Supersymmetric contributions to the decay of an extra $Z$ boson

Tony Gherghetta*, Thomas A. Kaeding†, and Gordon L. Kane‡

Department of Physics, University of Michigan, Ann Arbor, Michigan 48109-1120
(December 1996)

Abstract

We analyse in detail the supersymmetric contributions to the decay of an extra $Z$ boson in effective rank 5 models, including the important effect of D-terms on sfermion masses. The inclusion of supersymmetric decay channels will reduce the $Z'$ branching ratio to standard model particles resulting in lower $Z'$ mass limits than those often quoted. In particular, the supersymmetric parameter space motivated by the recent Fermilab $ee\gamma\gamma$ event and other suggestive evidence results in a branching fraction $B(Z' \rightarrow e^+e^-) \simeq 2 - 4\%$. The expected cross sections and branching ratios could give a few events in the present data and we speculate on the connection to the three $e^+e^-$ events observed at Fermilab with large dielectron invariant mass.

*tgher@umich.edu
†kaeding@feynman.physics.lsa.umich.edu
‡gkane@umich.edu
I. INTRODUCTION

Low energy supersymmetry provides the most successful solution to the naturalness problems which plague the standard model (SM). This has led to an extensive study of the experimental consequences of low energy supersymmetry in the literature and the next generation of colliders will explore a substantial fraction of the supersymmetric parameter space. If superstring theory is assumed to be the underlying fundamental theory at the Planck scale, then not only is it responsible for low energy supersymmetry, but the possibility exists that the SM gauge group may be larger at the TeV scale. This is because a consistent string theory requires large gauge groups ($E_8 \times E_8$ or $SO(32)$). Depending on the particular compactification scenario, the resulting low energy $\mathbb{N}=1$ supersymmetric theory often has a gauge group with rank larger than four.

The simplest possible extension of the SM gauge group suggested by a gauge group of larger rank involves the introduction of one extra U(1) factor. This produces an extra neutral gauge boson, $Z'$, in the particle spectrum. The low energy phenomenology of $Z'$ bosons has been extensively discussed in the literature (see [1] and references therein). Recently, particularly strong motivation for having the mass of the $Z'$ below a TeV has been emphasized by Cvetic and Langacker [2]. However, most analyses of $Z'$ physics do not discuss supersymmetric contributions in detail. In this work we will examine the supersymmetric decay channels of the $Z'$ boson, including decays to neutralinos, charginos and supersymmetric Higgs bosons which are normally neglected. Our analysis will also include D-term corrections to the scalar masses which can have appreciable effects and are also not normally included [1].

The supersymmetric particle spectrum will be calculated by choosing typical values for soft masses and parameters which are consistent with recent suggestive evidence [1] for supersymmetry, such as the $ee\gamma\gamma$ event at the Fermilab Tevatron [3]. We prefer to focus on this region of parameters rather than survey the entire parameter space. This will lead to representative values of the $Z'$ branching fractions in the presence of low energy supersymmetry. If a $Z'$ exists then our analysis will provide a more realistic context in which to experimentally search for these particles.

The outline of this paper is as follows: We begin in Sec. II by systematically writing down all the supersymmetric Lagrangian terms relevant for $Z'$ boson decay in effective rank 5 models. Using these terms we will derive the decay widths including the phase space factors. A parametrisation of D-term contributions to the supersymmetric scalar masses in effective rank 5 models is also briefly discussed and we comment on possible radiative U(1) symmetry breaking scenarios. In Sec. III we present a generic set of branching fractions for various $Z'$ models. Using these values we show that the $Z'$ mass bounds from direct limits at Fermilab are lower than those normally quoted. Final comments and the conclusion will be presented in Sec. IV. An Appendix also follows which summarises the mass contributions

---

1. While we were writing this paper some analysis of D-term contributions to scalar masses in $E_6$ models was reported by E. Ma [3]. The possibility of large mass shifts from the effect of $Z'$ D-terms on sfermion masses has also been emphasized by J. Lykken [4].
II. Z' DECAY MODES WITH LOW ENERGY SUPERSYMMETRY

The low energy theory resulting from the breakdown of extended gauge groups can include additional gauge bosons in the spectrum. Since the grand unified gauge group $E_6$ has rank 6 and the SM gauge group has rank 4 there can be at most 2 additional gauge bosons. For simplicity we will consider an effective rank 5 low energy theory with only one additional gauge boson associated with an extra $U(1)$ and parametrised by

$$Z'(\theta) = Z_\psi \cos \theta - Z_\chi \sin \theta,$$

where $\theta$ is a mixing angle and

(i) $Z_\psi$ occurs when $E_6 \rightarrow SO(10) \times U(1)_\psi$,

(ii) $Z_\chi$ occurs when $SO(10) \rightarrow SU(5) \times U(1)_\chi$.

The orthogonal combination to (1) is assumed to occur at the intermediate or Planck scale.

When $E_6$ breaks directly to a rank 5 group (SM $\times$ $U(1)_\eta$) as in superstring inspired models via Wilson line breaking \cite{7}, the extra $Z$ boson is denoted $Z_\eta \equiv \sqrt{5/8} Z_\psi - \sqrt{3/8} Z_\chi$. We will also consider the model with $\theta = \eta - \pi/2$ which will be referred to as $Z_I$. This model occurs when $E_6$ breaks to $SU(6) \times SU(2)_I$ and is orthogonal to the $\eta$ model.

There are two types of mixing that can occur between the SM $Z$ boson and an extra $Z'$ boson. The first is associated with the fact that in general $Z$ and $Z'$ are not mass eigenstates. The mass squared matrix in the interaction basis has the form

$$M^2 = \begin{bmatrix} M_{Z_1}^2 & \Delta M^2 \\ \Delta M^2 & M_{Z_2}^2 \end{bmatrix} \equiv \begin{bmatrix} 2g_W^2 \sum_i T_{3i}^2 \langle \phi_i \rangle^2 & 2g_W g' \sum_i T_{3i} Q'_i \langle \phi_i \rangle^2 \\ 2g_W g' \sum_i T_{3i} Q'_i \langle \phi_i \rangle^2 & 2g'^2 \sum_i Q'^2_i \langle \phi_i \rangle^2 \end{bmatrix}$$

where $g_1, g_2, g'$ are the $U(1)_Y$, $SU(2)$, $U(1)'$ gauge couplings, $g_W = \sqrt{g_1^2 + g_2^2}$, and $\langle \phi_i \rangle \equiv \frac{v_i}{\sqrt{2}}$ is the vacuum expectation value (vev) of a Higgs field $\phi_i$ with weak isospin $T_{3i}$ and $U(1)'$ charge $Q'_i$. This matrix can be diagonalised by an orthogonal matrix which is parametrised by a mixing angle $\phi$ satisfying

$$\tan 2\phi = \frac{2 \Delta M^2}{M_{Z_1}^2 - M_{Z_2}^2},$$

where $M_{Z_1}$ and $M_{Z_2}$ are the mass eigenstates and

$$Z_1 = \cos \phi Z + \sin \phi Z'$$

$$Z_2 = -\sin \phi Z + \cos \phi Z'.$$

Recent analysis \cite{8} of the LEP results indicate that this mixing must be small, $\phi \lesssim 0.01$, because the measured $Z$ mass is quite close to the value predicted in the SM. Since the mixing angle in the effective rank 5 model is

$$\phi \simeq 2 \sin \theta_W (Q'_1 \cos^2 \beta - Q'_2 \sin^2 \beta) \frac{M_{Z_1}^2}{M_{Z_2}^2} \sim \frac{M_W^2}{M_{Z_2}^2},$$

for sfermions and Higgs bosons.
TABLE I. U(1) charges of the fields in the 27 of E₆.

| Field | Q - | 2√10 Qₓ | 2√6 Qψ | 2√15 Qη |
|-------|-----|---------|---------|---------|
| Q     | 1/6 | -1      | 1       | 2       |
| uᶜ    | -2/3| -1      | 1       | 2       |
| dᶜ    | 1/3 | 3       | 1       | -1      |
| L     | -1/2| 3       | 1       | -1      |
| eᶜ    | 1   | -1      | 1       | 2       |
| νᶜ    | 0   | -5      | 1       | 5       |
| H     | -1/2| -2      | -2      | -1      |
| Hᶜ    | 1/2 | 2       | -2      | -4      |
| D     | -1/3| 2       | -2      | -4      |
| Dᶜ    | 1/3 | -2      | -2      | -1      |
| Sᶜ    | 0   | 0       | 4       | 5       |

where tan β = v₂/v₁, the Z’ boson needs to be well above the electroweak scale. Except for the decay mode Z’ → W⁺W⁻ this mixing will be unimportant and for the most part we will ignore mixing effects and assume that Z₁ ≃ Z and Z₂ ≃ Z’.

The other type of mixing is associated with the kinetic energy terms when there are two U(1) factors in the Lagrangian [9–11]. This kinetic mixing will result in a shift of the U(1)’ charges and depends on the matter content of the theory. It will also not be important for the results that we will obtain.

We will assume that the matter superfields reside in the fundamental 27 of E₆, which consists of the left handed fields

\[ 27 = (Q, uᶜ, eᶜ, L, dᶜ, νᶜ, H, Dᶜ, Hᶜ, D, Sᶜ)_{L}, \]  

where Q includes the left-handed quarks u and d and L includes the left-handed leptons ν and e. The exotic matter superfield H is an electroweak doublet containing the exotic leptons N and E, while Hᶜ contains Eᶜ and Nᶜ. The exotic superfields D and Dᶜ are a pair of vector-like colour triplets, while Sᶜ is a standard model singlet. The SU(5) representations are

\[ 10(Q, uᶜ, eᶜ), \]
\[ 5(L, dᶜ), \]
\[ 1(νᶜ), \]
\[ 5(H, Dᶜ), \]
\[ 5(Hᶜ, D), \]
\[ 1(Sᶜ). \]  

The first three representations of these make up the 16 of SO(10), while the remainder form the 10 and 1. All the U(1) charge assignments of the fields contained in (7) are given in Table I. In general the U(1)’ charge of a field Φ will be denoted Q’(Φ) = Qₓ(Φ) cos θ − Qψ(Φ) sin θ.
A. Fermion/sfermion sector

The Lagrangian for \( Z' \) coupling to the fermions of the 27 is given by

\[
\mathcal{L} = g' \bar{f} L \gamma^\mu (v_f - a_f \gamma_5) f Z'_\mu 
\]  

(9)

where \( f = \left( \begin{array}{c} f_L \\ \bar{f}_L \end{array} \right) \) is a Dirac fermion and the vector, axial-vector couplings are

\[
v_f = \frac{1}{2} (Q'(f_L) + Q'(f_R)) \equiv \frac{1}{2} \left( (Q_\psi(f_L) + Q_\psi(f_R)) \cos \theta - (Q_\chi(f_L) + Q_\chi(f_R)) \sin \theta \right) 
\]

(10)

\[
a_f = \frac{1}{2} (Q'(f_L) - Q'(f_R)) \equiv \frac{1}{2} \left( (Q_\psi(f_L) - Q_\psi(f_R)) \cos \theta - (Q_\chi(f_L) - Q_\chi(f_R)) \sin \theta \right). 
\]

(11)

Note that in the above equations \( Q(f_R) = -Q(f_L) \) and we will assume that the \( U(1)' \) gauge coupling constant \( g'^2 = (5/3)g_4^2 \). The \( Z' \) decay rate into fermions is calculated to be

\[
\Gamma(Z' \rightarrow \tilde{f} f) = C_f \frac{g'^2}{12\pi} M_{Z'} \left[ v_f^2 \left( 1 + 2 \frac{m_f^2}{M_{Z'}^2} \right) + a_f^2 \left( 1 - 4 \frac{m_f^2}{M_{Z'}^2} \right) \right] \left( 1 - 4 \frac{m_f^2}{M_{Z'}^2} \right)^{1/2} 
\]

(12)

where \( C_f = 1(3) \) for leptons (quarks) \([1]\).

The Lagrangian describing the interaction of the \( Z' \) with sfermions is given by

\[
\mathcal{L} = ig'(v_f \pm a_f) \bar{f}_{L,R} \gamma^\mu \tilde{f}_{L,R} Z'_\mu \n\]

(13)

where the \(+(-)\) is for left (right) handed sfermions respectively. The decay rate for \( Z' \) decay to sfermions is calculated to be

\[
\Gamma(Z' \rightarrow \tilde{f}_{L,R} \tilde{f}_{L,R}) = C_f \frac{g'^2}{48\pi} M_{Z'} (v_f \pm a_f)^2 \left( 1 - 4 \frac{m_{\tilde{f}_{L,R}}^2}{M_{Z'}^2} \right)^{3/2} 
\]

(14)

where the colour factor \( C_f \) is defined as in the fermion case and \( m_{\tilde{f}_{L,R}} \) is the \( \tilde{f}_{L,R} \) sfermion mass. In the case of non-negligible sfermion mixing (such as the top squark) we must work with the Lagrangian in the mass eigenstate basis

\[
\mathcal{L} = ig' (v_f \pm a_f \cos 2\theta_f) \tilde{f}_{1,2} \gamma^\mu \tilde{f}_{1,2} Z'_\mu - ig' a_f \sin 2\theta_f (\tilde{f}_{1,2}^* \gamma^\mu \tilde{f}_{1,2} + \tilde{f}_{1,2}^* \gamma^\mu \tilde{f}_{1,2}) Z'_\mu 
\]

(15)

where \( \tilde{f}_{1,2} \) denotes the mass eigenstates (see Appendix). The decay width is similar to \( (14) \) with the appropriate couplings from \( (13) \), except that the phase space factor in the case of different mass decay products is \( [1 - 2(m_1^2 + m_2^2)/M_{Z'}^2 + (m_1^2 - m_2^2)^2/M_{Z'}^4]^{3/2} \).

1. D-term contributions to scalar masses

In general, breaking the \( U(1)' \) gauge symmetry with Higgs field vevs contributes to sfermion masses via \( U(1)' \) D-terms in the scalar potential \([12, 13, 14]\). Depending on the
number of Higgs fields which are used to spontaneously break the gauge symmetry the contribution to the scalar mass squared term has the form

$$\Delta \tilde{m}_a^2 = g'^2 Q'_a \sum_i Q'_i \langle \phi_i \rangle^2,$$  \hspace{1cm} (16)

where the sum is over all Higgs fields which obtain vevs, (recall that $\langle \phi_i \rangle = v_i/\sqrt{2}$). If in the pure rank 5 or $\eta$ model the $U(1)'$ symmetry is broken using only one SM $U(1)'$ singlet, then the D-term contribution to the scalar mass squared may be written

$$\Delta \tilde{m}_a^2 = -\frac{g'^2}{120} (v_1^2 + 4v_2^2 - 5v_3^2)(2\sqrt{15} Q' \eta(a)).$$  \hspace{1cm} (17)

However in the effective rank 5 models which are parametrised by the angle $\theta$, there are also orthogonal D-terms which come from breaking the extra $U(1)''$ at an intermediate or Planck scale. Since these orthogonal D-terms terms can raise the sparticle mass spectrum to energy scales much greater than a TeV, we will assume that their contribution is negligible. This can be achieved by using a mirrorlike pair of $U(1)''$ charged SM singlets with charges of opposite sign to ensure $D''$-flatness. This amounts to assuming degeneracy of the mirrorlike Higgs soft masses at some high energy scale. Thus in the effective rank 5 models the D-term contribution to the scalar masses (assuming again that $U(1)'$ is broken by one SM singlet) becomes

$$\Delta \tilde{m}_a^2 = \frac{g'^2}{2} (Q'_1 v_1^2 + Q'_2 v_2^2 + Q'_3 v_3^2) Q'(a).$$  \hspace{1cm} (18)

In the special case of $E_6$ breaking directly to a rank 5 gauge group we obtain the $\eta$ model and recover the result (17). The D-term contributions to the sfermion masses are listed in the Appendix.

Note that the exotic superfields in the $27$ will also contribute fermions and sfermions. However we will assume that these particles are heavier than the $Z'$ mass and do not contribute to the $Z'$ decay width.

B. The Higgs sector

The scalar Higgs doublets required for the spontaneous breakdown of the electroweak symmetry can be associated with the scalar doublet components of the superfields $H$ and $H^c$. In addition, the scalar component of the superfield $S^c$ is an electroweak singlet Higgs boson which is used to break the $U(1)'$ gauge symmetry and give mass to the $Z'$. We will denote these Higgs bosons by

$$\Phi_1 = \begin{pmatrix} \phi_1^0 \\ \phi_1^+ \end{pmatrix}, \quad \Phi_2 = \begin{pmatrix} \phi_2^0 \\ \phi_2^+ \end{pmatrix}, \quad \Phi_3 = \phi_3^0$$  \hspace{1cm} (19)

where $\phi_{1,2,3}$ are the scalar components of $H$, $H^c$, and $S^c$ respectively. In the ground state the vacuum expectation values (vev) of the Higgs fields will be denoted by $\langle \phi_i^0 \rangle = v_i/\sqrt{2}$ where $i = 1, 2, 3$. Note that only one generation of scalar doublets receive vevs. There are also scalar Higgs fields associated with the other two generations. These fields are referred to as “unHiggs” because discrete symmetries can be used to make sure that these other two generations of scalars do not acquire vevs [10,11].
1. Radiative $U(1)'$ symmetry breaking

While we can treat the vevs $v_i$ as phenomenological parameters it is also interesting to examine the consequences for effective rank 5 models if we assume that a radiative mechanism is responsible for breaking the $U(1)'$ gauge symmetry. This is similar to the analysis done by Cvetic and Langacker [2] where it was shown that it is possible to achieve an $M_{Z'}/M_Z$ hierarchy for general string models, without excessive fine tuning provided $M_{Z'} \lesssim 1$ TeV (see also [17]). We will consider the case of grand unification with effective rank 5 models parametrised by (1). If the radiative breaking of $U(1)'$ is due to one SM singlet and the superpotential contains a term $W = \lambda \Phi_1 \Phi_2 \Phi_3$, then at low energies we will assume that the scalar potential for the neutral Higgs bosons has the form

$$V = m_1^2 |\phi_1|^2 - m_2^2 |\phi_2|^2 - m_3^2 |\phi_3|^2 + (\lambda A \phi_1^0 \phi_2^0 \phi_3^0 + h.c) + \lambda^2 (|\phi_1|^2 |\phi_2|^2 + |\phi_1|^2 |\phi_3|^2 + |\phi_2|^2 |\phi_3|^2) + \frac{1}{8} g_W^2 (|\phi_1|^2 - |\phi_2|^2)^2$$

$$+ \frac{1}{2} g' (Q_1 |\phi_1|^2 + Q_2 |\phi_2|^2 + Q_3 |\phi_3|^2)^2$$

(20)

where $m_1, m_2, m_3$ are the soft supersymmetric Higgs masses and $A_3$ is a soft trilinear parameter. Given that $v_2 = v_1 \tan \beta$, the minimum of the potential (20) occurs at

$$v_1^2 \approx \frac{8}{g_W^2 (1 - \tan^2 \beta)^2} \left[ -m_1^2 + m_2^2 \tan^2 \beta - \frac{(Q_1 + Q_2 \tan^2 \beta)}{Q_3} m_3^2 \right]$$

(21)

$$v_3^2 \approx \frac{2m_3^2}{g'^2 Q_3^2} - \frac{(Q_1 + Q_2 \tan^2 \beta)}{Q_3} v_1^2,$$

(22)

where we have neglected the corrections from the potential terms involving $\lambda$. The matrix elements of the $Z - Z'$ mass mixing matrix become

$$M_Z^2 \approx 2\frac{(1 + \tan^2 \beta)}{(1 - \tan^2 \beta)^2} \left[ -m_1^2 + m_2^2 \tan^2 \beta - \frac{(Q_1 + Q_2 \tan^2 \beta)}{Q_3} m_3^2 \right]$$

(23)

$$\Delta M^2 \approx -4\frac{g'}{g_W} \frac{(Q_1 - Q_2 \tan^2 \beta)}{(1 - \tan^2 \beta)^2} \left[ -m_1^2 + m_2^2 \tan^2 \beta - \frac{(Q_1 + Q_2 \tan^2 \beta)}{Q_3} m_3^2 \right]$$

(24)

$$M_{Z'}^2 \approx 2m_3^2 + 8\frac{g'^2}{g_W} \frac{1}{(1 - \tan^2 \beta)^2} \left[ Q_1^2 (1 - \frac{Q_3}{Q_1}) + Q_2^2 (1 - \frac{Q_3}{Q_2}) \tan^2 \beta \right]$$

$$\times \left[ -m_1^2 + m_2^2 \tan^2 \beta - \frac{(Q_1 + Q_2 \tan^2 \beta)}{Q_3} m_3^2 \right].$$

(25)

If $\tan \beta = 0$ and $m_1^2 < 0$ then the above expressions agree with those in [2]. In order to achieve a reasonable hierarchy between $M_Z$ and $M_{Z'}$ for negligible $Z - Z'$ mixing $\phi$ we must have $v_1^2 \ll m_3^2$. If minima exist for $m_1^2 < 0$ in (20), then this can only occur when the charge combination $Q_1 + Q_2 \tan^2 \beta$ has the same relative sign as $Q_3$. If $\tan \beta \lesssim 1$ this will happen when $-\pi/2 \leq \theta \lesssim -\pi/3$. For example, an exact numerical determination of the minimum at $\theta = -1.161$ for the parameters $\tan \beta = 1.5, m_1 = m_2 = 100$ GeV, $m_3 = 500$ GeV, $A_3 = 1$ TeV and $\lambda = 0.02$ yields an acceptable $Z - Z'$ hierarchy with $M_{Z'} \approx 700$ GeV and $\phi = -0.0057$. 

7
At values of $\theta \gtrsim -\pi/3$ an extreme fine tuning of the soft breaking parameters is needed for a radiative mechanism to work. However if $m_1^2 > 0$ in the potential (20) then we do not necessarily need the charge combination $Q'_1 + Q'_2 \tan^2 \beta$ to have the same relative sign as $Q'_3$. In this case no fine tuning of the soft parameters is needed as $v_3^2$ can be made small by cancellation of the positive terms against $-m_1^2$ for all values of $\theta$. Particular scenarios for achieving these various symmetry breaking potentials at low energies requires a complete renormalisation group analysis.

Note also that $v_3 \to \infty$ when the charge $Q'_3 \to 0$, while $M_{Z'} \simeq g'Q_3v_3 \simeq \sqrt{2}m_3$. The condition $Q'_3 = 0$ actually occurs when $\theta = \pi/2$ and signifies that there is no minimum to the assumed form of the potential (20), since the stabilising quartic term is proportional to $Q'_3^2$ (of course, at the value $\theta = \pi/2$, $\phi_3^0$ can no longer break the U(1)$'$ gauge symmetry). Clearly there is a limit to how big one can tolerate $v_3$ in the above scenario and this will have consequences for the D-term contributions to the squark and slepton masses (see next section).

2. Higgs decay modes

After symmetry breaking the physical and unphysical charged Higgs bosons are given by

$$H^\pm = \sin \beta \phi_1^\pm + \cos \beta \phi_2^\pm$$
$$G^\pm = \cos \beta \phi_1^\pm - \sin \beta \phi_2^\pm$$

where $\tan \beta = v_2/v_1$ and $G^\pm$ is the charged Nambu-Goldstone boson which is absorbed by the $W^\pm$ gauge boson. In the unitary gauge ($G^\pm = 0$) we will then obtain $\phi_1^\pm = \sin \beta H^\pm$ and $\phi_2^\pm = \cos \beta H^\pm$. The physical charged Higgs boson masses are obtained by diagonalising the mass mixing matrix given in the Appendix [1], with the result

$$m_{H^\pm}^2 = \frac{2\lambda A v_3}{\sin 2\beta} + \left(1 - \frac{\lambda^2}{g_2^2}\right) m_W^2$$

where $A$ is the soft supersymmetry breaking parameter in the Higgs potential. The Lagrangian for the $Z'$ coupling to the charged Higgs states $H^\pm$ is

$$\mathcal{L} = ig'(Q'_1 \sin^2 \beta - Q'_2 \cos^2 \beta) H^\pm \partial^\mu H^- Z'_\mu$$
$$+ g'(Q'_1 + Q'_2) \sin \beta \cos \beta M_W(W^\mu H^- + W^\mu H^+)Z'_\mu$$

where $Q'_1 \equiv Q'(H)$ and $Q'_2 \equiv Q'(H^c)$. This gives rise to the decay widths

$$\Gamma(Z' \to H^+H^-) = \frac{g'^2}{48\pi}(Q'_1 \sin^2 \beta - Q'_2 \cos^2 \beta) M_{Z'}^2 \left(1 - \frac{4m_{H^\pm}^2}{M_{Z'}^2}\right)^{3/2}$$

$$\Gamma(Z' \to W^\pm H^\mp) = \frac{g'^2}{192\pi}(Q'_1 + Q'_2)^2 \sin \beta \cos \beta M_{Z'} \left[1 + \frac{2(5M_W^2 - m_{H^\pm}^2)}{M_{Z'}^2} + \frac{(M_W^2 - m_{H^\pm}^2)^2}{M_{Z'}^4}\right]$$

$$\times \sqrt{1 - \frac{2(M_W^2 + m_{H^\pm}^2)}{M_{Z'}^2} + \frac{(M_W^2 - m_{H^\pm}^2)^2}{M_{Z'}^4}}$$
where $m_{H^\pm}$ is the mass of the $H^\pm$ Higgs boson. This agrees with the result [18] in the limit $m_{H^\pm} \ll M_{Z'}$.

While the $Z'$ boson has no direct couplings to the W-boson, the small mixing angle discussed earlier in Eq.(6) induces a $Z'W^+W^-$ coupling via the small Z admixture. The decay amplitude into $W^+W^-$ pairs is calculated to be [19]

$$
\Gamma(Z_2 \to W^+W^-) = \frac{g^2}{192\pi} \cos^2 \theta_W \sin^2 \phi M_{Z_2} M_W^4 \left( 1 - 4 \frac{M_W^2}{M_{Z_2}^2} \right)^{3/2} \left( 1 + 20 \frac{M_W^2}{M_{Z_2}^2} + 12 \frac{M_W^4}{M_{Z_2}^4} \right).
$$

(32)

Recall that in the limit $v_3 \gg v_{1,2}$, the mixing angle (6) is $\phi \propto M_{Z_2}^2/M_W^2$ and so the width (32) in the equivalence theorem limit $\sqrt{s} \gg M_W$ becomes

$$
\Gamma(Z' \to W^+W^-) \simeq \Gamma(Z' \to G^+G^-) = \frac{g'^2}{48\pi} (Q'_1 \cos^2 \beta - Q'_2 \sin^2 \beta)^2 M_{Z'}.
$$

(33)

Note that in (33) the $Z-Z'$ mixing angle $\phi$ cancels the usual enhancement factor, $M_{Z_2}^2/M_W^4$, that one normally expects in the equivalence theorem limit. Similarly, in the limit $v_3 \gg v_{1,2}$ the equivalence theorem dictates that [15]

$$
\Gamma(Z' \to W^\pm H^\mp) \simeq \Gamma(Z' \to G^+G^-) = \frac{g'^2}{48\pi} (Q'_1 + Q'_2)^2 \sin^2 \beta \cos^2 \beta M_{Z'}.
$$

(34)

In the neutral Higgs boson sector, the Higgs fields are written as

$$
\phi^0_i = \frac{1}{\sqrt{2}} (v_i + \phi^0_{Ri} + i\phi^0_{Ai})
$$

(35)

where $\phi_{Ri}, \phi_{Ai}$ are the real and imaginary parts of $\phi^0_i$. Of the six neutral Higgs degrees of freedom, two ($G^0, G'^0$) will act as the Nambu-Goldstone bosons and will be absorbed by the $Z$ and $Z'$. The remaining four degrees of freedom will become the physical neutral Higgs bosons which consists of three scalars, $H^0_{i=1,2,3}$ and one pseudoscalar $P^0$. In the unitary gauge ($G, G' = 0$) we will obtain [20,21,1]

$$
\phi^0_{I1} = \frac{v_2 v_3}{N} P^0
$$

(36)

$$
\phi^0_{I2} = \frac{v_1 v_3}{N} P^0
$$

(37)

$$
\phi^0_{I3} = \frac{v_1 v_2}{N} P^0
$$

(38)

$$
\phi^0_{Ri} = \sum_{j=1}^3 U_{ij} H^0_j
$$

(39)

where $N = \sqrt{v_1^2 v_2^2 + v_1^2 v_3^2 + v_2^2 v_3^2}$ and $U$ is the inverse of the matrix that diagonalises the scalar Higgs ($H^0_i$) mass term. The Lagrangian relevant for $Z'$ boson decay to neutral Higgs bosons is
\[ \mathcal{L} = 2g' M_Z \sum_{i=1}^{3} (Q'_i \cos \beta U_{1i} - Q'_2 \sin \beta U_{2i}) Z'_\mu Z^\mu H_i^0 \quad (40) \]

\[ - g' \frac{v}{N} \sum_{i=1}^{3} (v_3 Q'_1 \sin \beta U_{1i} + v_3 Q'_2 \cos \beta U_{2i} + v Q'_3 \sin \beta \cos \beta U_{3i}) Z'_\mu H_i^0 \delta^\mu P_0, \quad (41) \]

where \( v = \sqrt{v_1^2 + v_2^2} \). The \( Z' \to Z H_i^0 \) decay width may be obtained in a similar manner to (30) and is given by

\[ \Gamma(Z' \to Z H_i^0) = \frac{g'^2}{48\pi} (Q'_1 \cos \beta U_{1i} - Q'_2 \sin \beta U_{2i})^2 M_{Z'} \left[ 1 + 2 \frac{(5M_Z^2 - m_{H_i^0}^2)}{M_{Z'}^2} + \frac{(M_Z^2 - m_{H_i^0}^2)^2}{M_{Z'}^4} \right] \]

\[ \times \left( 1 - 2 \frac{(M_Z^2 + m_{H_i^0}^2)}{M_{Z'}^2} + \frac{(M_Z^2 - m_{H_i^0}^2)^2}{M_{Z'}^4} \right) \quad (42) \]

where \( m_{H_i^0} \) is the mass of the scalar Higgs boson \( H_i^0 \). In general the \( 3 \times 3 \) scalar Higgs mass mixing matrix is complicated (see Appendix) and must be numerically evaluated. However, in the limit that \( v_3 \gg v_{1,2} \) the state \( \phi_0^0 \) decouples and the only mixing occurs between \( \phi_1^0 \) and \( \phi_2^0 \). If this mixing is parametrised as

\[ \left( \begin{array}{c} \phi_1^0 \phi_2^0 \\ \phi_1^0 \phi_2^0 \end{array} \right) = \left( \begin{array}{cc} \cos \alpha & -\sin \alpha \\ \sin \alpha & \cos \alpha \end{array} \right) \left( \begin{array}{c} H_1^0 \\ H_2^0 \end{array} \right), \quad (43) \]

then our results agree with those in the literature [13,14].

Similarly, the \( Z' \) decay into scalar-pseudoscalar Higgs bosons is

\[ \Gamma(Z' \to P_0 H_i^0) = \frac{g'^2}{48\pi} \frac{v^2}{N^2} (v_3 Q'_1 \sin \beta U_{1i} + v_3 Q'_2 \cos \beta U_{2i} + v Q'_3 \sin \beta \cos \beta U_{3i})^2 M_{Z'} \]

\[ \times \left[ 1 - 2 \frac{(m_{H_i^0}^2 + m_{P_0}^2)}{M_{Z'}^2} + \frac{(m_{H_i^0}^2 - m_{P_0}^2)^2}{M_{Z'}^4} \right]^{3/2} \quad (44) \]

where \( m_{P_0} \) is the mass of the pseudoscalar Higgs boson \( (P_0) \). The mass of \( P_0 \) is much easier to obtain in the exact limit. Using the mass matrix in the Appendix one finds that

\[ m_{P_0}^2 = \frac{2\lambda Av_3}{\sin 2\beta} \left( 1 + \frac{v^2}{4v_3^2} \sin^2 2\beta \right). \quad (45) \]

If one again considers the limit \( v_3 \gg v_{1,2} \) and the mixing (43), then the decay widths for \( Z' \to P^0 H_{i,2}^0 \) agree with the expressions obtained in [13,14].

**C. The neutralino sector**

In the \( E_6 \) models that we are considering there will be two extra neutralinos in addition to the four found in the minimal supersymmetric standard model (MSSM). An extra neutral gaugino comes from the extra \( Z' \) boson and an extra neutral Higgsino comes from the
Higgs boson which is used to break the U(1)' gauge symmetry. In the interaction basis
\(-iB, -i\tilde{W}_3, -i\tilde{B}', \Phi, \tilde{\Phi}_2, \tilde{\Phi}_3\), the 6 × 6 neutralino mass matrix is given by

\[
\begin{bmatrix}
M_1 & 0 & 0 & -\frac{i}{2}g_1v_1 & \frac{i}{2}g_1v_2 & 0 \\
0 & M_2 & 0 & \frac{i}{2}g_2v_1 & -\frac{i}{2}g_2v_2 & 0 \\
0 & 0 & M' & Q'_1g'v_1 & Q'_2g'v_2 & Q'_3g'v_3 \\
-\frac{i}{2}g_1v_1 & \frac{i}{2}g_2v_1 & Q'_1g'v_1 & 0 & \frac{1}{\sqrt{2}}\lambda v_3 & \frac{1}{\sqrt{2}}\lambda v_2 \\
\frac{i}{2}g_1v_2 & -\frac{i}{2}g_2v_2 & Q'_2g'v_2 & \frac{1}{\sqrt{2}}\lambda v_3 & 0 & \frac{1}{\sqrt{2}}\lambda v_1 \\
0 & 0 & Q'_3g'v_3 & \frac{1}{\sqrt{2}}\lambda v_2 & 0 & 0
\end{bmatrix}
\] (46)

where \(M_1, M_2,\) and \(M'\) are the soft gaugino masses for \(\tilde{B}, \tilde{W}_3\) and \(\tilde{B}'\), respectively. The
supersymmetric Higgsino mass terms come from the trilinear superpotential interaction
\(\lambda \Phi_1 \Phi_2 \Phi_3\) which gives rise to a \(\mu\) term with \(\mu = \lambda v_3/\sqrt{2}\). The neutralino mass eigenstates
\((\tilde{N}_1, \tilde{N}_2, \cdots, \tilde{N}_6)\) are obtained by diagonalising the mass matrix (16). A general analytic formula
for the neutralino masses does not exist. However, in certain limits such as \(v_3 \gg v_1, v_2\)
approximate expressions can be obtained [22]. Of course, one can simply diagonalise the
mass matrix (16) numerically, and this will be done for the results in the next section.

In terms of the mass eigenstates \((\tilde{N}_1, \tilde{N}_2, \cdots, \tilde{N}_6)\) we may parametrise the coupling be-
tween the neutralinos and the \(Z'\) boson as

\[
\mathcal{L} = \sum_{i,j} g_{ij} \tilde{N}_i \gamma^\mu \gamma_5 \tilde{N}_j Z'_\mu
\] (47)

where the coupling constants \(g_{ij}\) are obtained from the diagonalisation of the neutralino
mass matrix (16).

The expression for the \(Z'\) decay width into neutralinos can be obtained using the La-
grangian (17). The decay widths are

\[
\Gamma(Z' \to \tilde{N}_i \tilde{N}_j) = \frac{g_{ij}^2}{12\pi} M_{Z'} \left[ 1 - \frac{(m_i^2 + m_j^2)}{2M_{Z'}^2} - \frac{(m_i^2 - m_j^2)^2}{2M_{Z'}^4} - 3 \frac{m_i m_j}{M_{Z'}^2} \right] \times \sqrt{\left(1 - \frac{(m_i + m_j)^2}{M_{Z'}^2}\right) \left(1 - \frac{(m_i - m_j)^2}{M_{Z'}^2}\right)}
\] (48)

where the \(m_i\) refer to the neutralino masses. This agrees with the expression in [23]. The
coefficients \(g_{ij}\) will be determined numerically when we calculate the neutralino branching
fractions in the next section.

D. Chargino sector

Since the \(Z'\) boson and the singlet Higgs \(S^0\) supermultiplets are electromagnetically
neutral they do not contribute any extra particles to the chargino spectrum. Consequently
the chargino mass matrix remains the same as in the MSSM, namely

\[\text{\footnotesize \cite{CVETIC96}}\]

\[\text{\footnotesize \cite{CVETIC96}}\]
except that $\mu = \lambda v_3/\sqrt{2}$ as defined above. The $Z'$ can of course couple to the charged Higgsinos which leads to the chargino Lagrangian

$$\mathcal{L} = \frac{1}{2} g' \sum_{i,j=1}^2 \tilde{C}_i \gamma^\mu v_{ij} + a_{ij} \gamma_5 \tilde{C}_j Z'_\mu \tag{50}$$

where $\tilde{C}_i$ are the chargino mass eigenstates and

$$v_{11} = Q'_1 \sin^2 \phi_- - Q'_2 \sin^2 \phi_+,$$  
$$a_{11} = Q'_1 \sin^2 \phi_- + Q'_2 \sin^2 \phi_+,$$  
$$v_{12} = v_{21} = Q'_1 \sin \phi_- \cos \phi_- - \delta Q'_2 \sin \phi_+ \cos \phi_+,$$  
$$a_{12} = a_{21} = Q'_1 \sin \phi_- \cos \phi_- + \delta Q'_2 \sin \phi_+ \cos \phi_+,$$  
$$v_{22} = Q'_1 \cos^2 \phi_- - Q'_2 \cos^2 \phi_+,$$  
$$a_{22} = Q'_1 \cos^2 \phi_- + Q'_2 \cos^2 \phi_+.$$

In \((51)-(56)\), $\phi_{\pm}$ are the angles of the unitary transformation matrices used to diagonalize the chargino mass matrix \([22]\) and $\delta = n_1 n_2$, where $n_1 = \text{sgn}(m_{\tilde{C}_1^\pm})$ and $n_2 = \text{sgn}(m_{\tilde{C}_2^\pm})$. The $Z'$ decay rate into chargino pairs is calculated from the Lagrangian \((50)\) to be

$$\Gamma(Z' \to \tilde{C}_i \tilde{C}_j) = \frac{g'^2}{48\pi} M_{Z'} \left[ (v_{ij}^2 + a_{ij}^2) \left( 1 - \frac{(m_i^2 + m_j^2)}{2M_{Z'}^2} - \frac{(m_i^2 - m_j^2)^2}{2M_{Z'}^4} \right) + 3(v_{ij}^2 - a_{ij}^2) \frac{m_i m_j}{M_{Z'}^2} \right]$$

$$\times \left( 1 - \frac{(m_i + m_j)^2}{M_{Z'}^2} \right) \left( 1 - \frac{(m_i - m_j)^2}{M_{Z'}^2} \right). \tag{57}$$

In the limit that $M_{Z'} \gg m_{\tilde{C}_i}$ the decay width \((57)\) reduces to a form similar to the result \((12)\) that was obtained earlier in the fermion sector.

### III. $Z'$ PRODUCTION CROSS SECTION AND BRANCHING FRACTIONS

At the $\bar{p}p$ Tevatron collider the $Z'$ production mechanism is due to the Drell-Yan process. If the $u$ and $d$ quark couplings to the $Z'$ are obtained from \((9)\), then the expression for the Drell-Yan production differential cross section is \([24]\)

$$\frac{d\sigma}{dy}(AB \to Z'X) = \kappa \frac{\pi g'^2}{3 M_{Z'}^2} \sum_q (v_q^2 + a_q^2) x_a x_b q(x_a) \bar{q}(x_b) \tag{58}$$

where $y$ is the rapidity of the $Z'$ in the $AB$ c.m. frame, $\kappa \approx 1 + \frac{8\pi}{9} \alpha_s(M_{Z'})$ is a QCD correction factor, $x_a(x_b)$ is the momentum fraction of $q(\bar{q})$ in $A(B)$ and $q(x)$ is the quark distribution function. Similarly, the $Z'$ production cross section at the planned 14 TeV $pp$ LHC collider is also obtained from \((58)\). In Fig. 1 the cross section for $Z'$ production in the
FIG. 1. (a) The $Z'$ production cross section for various effective rank 5 models at the Fermilab Tevatron with $\sqrt{s} = 1.8$ TeV. (b) Similarly for the LHC with $\sqrt{s} = 14$ TeV.
TABLE II. Branching fractions of all possible $Z'$ decay channels in a supersymmetric framework for various $Z'$ models.

| $Z'$ decay channel | $Z_I$  | $Z_\psi$ | $Z_\eta$ |
|--------------------|--------|----------|----------|
| $e^+e^-$           | 0.0391 | 0.0280   | 0.0171   |
| $\mu^+\mu^-$      | 0.0391 | 0.0280   | 0.0171   |
| $\tau^+\tau^-$    | 0.0391 | 0.0280   | 0.0171   |
| $\tilde{\nu}_e\nu_e$ | 0.0782 | 0.0280   | 0.0890   |
| $\tilde{\nu}_\mu\nu_\mu$ | 0.0782 | 0.0280   | 0.0890   |
| $\tilde{\nu}_\tau\nu_\tau$ | 0.0782 | 0.0280   | 0.0890   |
| $\bar{\nu}_e\nu_e$ | 0.0000 | 0.0839   | 0.0820   |
| $\bar{\nu}_\mu\nu_\mu$ | 0.0000 | 0.0839   | 0.0820   |
| $\bar{\nu}_\tau\nu_\tau$ | 0.0000 | 0.0839   | 0.0820   |
| $\bar{\nu}_u\nu_u$ | 0.1174 | 0.0839   | 0.0513   |
| $\bar{\nu}_c\nu_c$ | 0.1174 | 0.0839   | 0.0513   |
| $\bar{\nu}_t\nu_t$ | 0.0000 | 0.0553   | 0.0540   |
| $\bar{\nu}_d\nu_d$ | 0.0000 | 0.0553   | 0.0540   |
| $\bar{\nu}_s\nu_s$ | 0.1174 | 0.0839   | 0.0513   |
| $\bar{\nu}_b\nu_b$ | 0.1174 | 0.0839   | 0.0513   |
| $\sum\tilde{u}_i\tilde{u}_j$ | 0.0000 | 0.0000   | 0.0000   |
| $\sum\tilde{d}_i\tilde{d}_j$ | 0.1746 | 0.0000   | 0.0000   |
| $H^+H^-$          | 0.0048 | 0.0213   | 0.0003   |
| $\sum P_0H_0^0$   | 0.0110 | 0.0038   | 0.0010   |
| $W^\pm H^\mp$     | 0.0018 | 0.0102   | 0.0039   |
| $\sum ZH_0^0$     | 0.0075 | 0.0500   | 0.0206   |
| $W^+W^-$          | 0.0028 | 0.0062   | 0.0155   |
| $\sum\tilde{C}_i\tilde{C}_j$ | 0.0182 | 0.0759   | 0.0329   |
| $\sum\tilde{N}_i\tilde{N}_j$ | 0.0753 | 0.2090   | 0.1077   |

$\psi$, $\chi$, and $\eta$ models at both the Tevatron and the LHC are depicted. The slight variation in the cross section for each $Z'$ model in Fig. 1 is due to the $\theta$ dependence of the quark-gauge boson couplings ($\theta^{(10)}$ and $\theta^{(11)}$).

Direct limits on $pp \to Z' \to e^+e^-$ at the Tevatron can place a lower bound on the $Z'$ mass depending on the value of the branching ratio $B(Z' \to e^+e^-)$ [\ref{25}]. In order to estimate the $e^+e^-$ branching ratio in a supersymmetric framework, all possible decay channels (such as decays to sparticles) need to be included. While there are many unknown parameters in the supersymmetric standard model it is possible to constrain parameters by requiring consistency with other indirect studies of supersymmetric parameter space. For example, in the recent supersymmetric interpretation of the $ee\gamma\gamma$ event, the supersymmetric parameters $\mu$, $\tan\beta$, $M_1$ and $M_2$ are constrained [\ref{26}]. The parameter ranges of these variables will in turn constrain the neutralino and chargino mass spectrum.

Similarly, by assuming typical ranges of supersymmetric parameters which are required, for example, in the constrained minimal supersymmetric standard model [\ref{27}] the branching fractions of all possible decay modes can be estimated. In Table II we list the branching fraction of all possible $Z'$ decay channels in various $Z(\theta)$ models for the following choice of
supersymmetric parameters:

\[ \tan \beta = 1.5, \quad \mu = -50 \text{ GeV} \]

\[ M_1 = 80 \text{ GeV}, \quad M_2 = 100 \text{ GeV}, \quad M' = 300 \text{ GeV} \]

\[ \tilde{m} = 500 \text{ GeV}, \quad A = 500 \text{ GeV} \]

where we have assumed common scalar mass \( \tilde{m} \) and \( A \) terms. In addition we also fix the \( Z' \) mass to be \( M_{Z'} \simeq 700 \text{ GeV} \), which sets the scale for all the kinematically allowed decays. Thus, in Table II when the branching fraction \( B(Z' \to \tilde{f}_i^* \tilde{f}_j) = 0 \) it is because the squark and slepton masses are too heavy to be kinematically allowed. Similarly only the sufficiently light Higgs bosons, neutralinos and charginos are included. All exotic particles such as the vectorlike quarks are also assumed to be very heavy.

In Fig. 2 we show the squark and slepton masses including the D-term contributions. The general effect of the D-term contributions is to make the squarks and sleptons heavy since the scale of the D-terms is set by \( M_{Z'} \) (or \( v_3 \)). However, in Fig. 2 there are special values of the mixing parameter \( \theta \) where the D-term contributions cancel and the squark and slepton masses become light. These are the only points that could be consistent with squarks and sleptons that would have appeared already at Fermilab [28]. Such sensitivities are very encouraging from the point of view of extracting information about new physics from limited data. Beyond these values the squared masses become negative and this signals the onset of charge and colour breaking minima. It may be possible that radiative corrections can stabilise the vacuum but the analysis of these corrections is beyond the scope of this paper.

When the \( Z' \) mixing parameter \( \theta \) approaches \( \pm \pi/2 \) the squark and slepton masses become unacceptably large. This is because the D-term contribution to the scalar masses from \( v_3 \) (Eq. (18)) is \( \Delta \tilde{m}^2_a \propto M_{Z'}^2/Q_3' \) and when \( Q_3' \to 0 \) we have \( \Delta \tilde{m}^2_a \to \pm \infty \). Thus, in effective rank 5 models with a U(1)\(_{\prime}\) symmetry breaking potential of the form (20) one can exclude \( \chi \)-like models (or regions near \( \theta \simeq \pm \pi/2 \)) because the squark and slepton masses become large and negative leading to charge and colour breaking minima (where we have neglected radiative corrections).

As far as determining a lower \( Z' \) mass bound from collider experiments, the main consequence of accurately including all supersymmetric decay channels is that the branching fraction into leptons and quarks is reduced. This lowers the \( Z' \) mass bound from that normally quoted in the literature. The \( e^+e^- \) branching fraction from Table II is in the range 2 – 4%, where typically in non-supersymmetric models one obtains 3 – 6% [24]. If we use the direct limit from a recent combined analysis of CDF and D0 data \( (\sigma \times B(Z' \to e^+e^-) \leq 0.28 \text{ pb}) [25]\), then for the branching fractions listed in Table II we obtain \( M_{Z'} \gtrsim 360, 370, 360 \text{ GeV} \) for \( Z_I, Z_\psi \) and \( Z_\eta \) respectively.

The importance of various decay channels can be determined from how the branching fractions in Table II vary as a function of the \( Z' \) mixing parameter \( \theta \). This dependence is shown in Fig. 3 for \( \tilde{m} = 500 \) and 1000 GeV. We have excluded the regions corresponding to negative squark and slepton masses which are near \( \theta = \pm \pi/2 \). As the soft mass parameter \( \tilde{m} \) becomes larger the charge and colour breaking regions shrink. Near the excluded regions (which correspond to \( Z_\eta \) and \( Z_I \) for \( \tilde{m} = 500 \text{ GeV} \)) the \( Z' \) branching fractions to squark and sleptons are maximised because their masses become light enough to be kinematically allowed. As \( \theta \to 0 \) the D-term corrections make the squarks and sleptons kinematically
FIG. 2. The squark and slepton masses squared as a function of the $Z'$ mixing parameter $\theta$. Mixing is only important for the top squarks and the solid line in the up squark plot represents $\tilde{t}_{1,2}$. 
FIG. 3. The $Z'$ decay branching fractions as a function of the mixing parameter $\theta$ for a representative set of supersymmetric parameters as defined in the text. The excluded regions correspond to values of the squark and slepton masses that lead to charge and colour breaking minima. The lower figure has $\tilde{m} = 1$ TeV.
inaccessible. A significant neutralino and chargino branching fraction occurs in models which are $\psi$-like (or $\theta \simeq 0$). Specifically, decays to the lightest neutralino can be as large as 10% and this would contribute greatly to the $Z'$ invisible width. As $\theta \rightarrow \pm \pi/2$ the branching fraction to neutralino and chargino pairs gets smaller while the neutrino branching fraction becomes larger. Decays to Higgs bosons become non-negligible for $\theta \lesssim 0$. One should also note that the branching fraction to leptons and quarks remains fairly constant. In particular the branching fraction to light quarks (u,c,d,s,b) is at most 0.45. Thus for light quarks $\sigma \times B(Z' \rightarrow \bar{q}q) \simeq 0.25$pb where we have assumed $m_{Z'} \simeq 700$ GeV. This is too small to be observable in the inclusive jet cross section. Note that the branching fraction to up quarks vanishes for $Z_I$-type models. This is a consequence of the fact that the up-quark $Z'$ coupling becomes zero.

It is also amusing to note that both CDF [30] and D0 [31] report events with dielectron invariant mass $\gtrsim 500$ GeV and CDF also has a second event with invariant mass $\simeq 350$ GeV. If one assumes that these high energy events are due to the decay of a $Z'$ boson, then given our range of $B(Z' \rightarrow e^+e^-)$, this would correspond to a $Z'$ mass $m_{Z'} \simeq 600 - 700$ GeV. One would also expect these events to be backward, i.e. $\cos \theta^* < 0$ where $\theta^*$ is the angle of the outgoing $e^-$ with respect to the quark in the $q\bar{q}$ centre of mass frame [29,32]. In Fig. 4 we plot the forward-backward asymmetry, $A_{FB}$ as a function of the mixing angle $\theta$. A very large asymmetry is expected for $Z_I$-type models. It is tantalising that the two events reported by CDF with invariant masses of 350 GeV and 504 GeV have $\cos \theta^*$ values of -0.14 and -0.27 respectively [30]. If we require that the probability of observing two backward events be at least $1/e$ then one would need $A_{FB} \lesssim -0.2$.

**IV. CONCLUSION**

We have examined in detail the supersymmetric contributions to the decay of an extra $Z$ boson in effective rank 5 models parametrised by the angle $\theta$, including the important effect of D-terms on sfermion masses. The supersymmetric particle spectrum was chosen so as to be consistent with other analyses such as the recent $ee\gamma\gamma$ event and the constrained minimal supersymmetric standard model. The main effect of including these contributions is that it reduces the $Z'$ branching fraction into lepton pairs. This, in turn reduces the lower bound on the $Z'$ mass obtained from direct limits at the Tevatron.

While the $Z'$ decay to neutralino and chargino pairs is always non-negligible, this is not necessarily the case for squarks and sleptons. When the squarks and sleptons are dominated by positive D-term contributions their masses can become very large and consequently they are no longer a kinematically accessible $Z'$ decay mode. However for special ranges in parameter space the D-term contributions become negative and give rise to light squark and slepton masses. At these values the branching fraction of squarks and sleptons is non-negligible. As $\theta$ approaches $\pm \pi/2$ various squark and slepton masses squared become negative leading to charge and colour breaking minima. Thus depending on the size of soft squark and slepton mass parameters, effective rank 5 models with regions of $\theta$ near $\pm \pi/2$ produce unacceptably large D-term contributions. If one also assumes a $U(1)'$ symmetry breaking potential (20) then it is possible to achieve a radiative breaking mechanism without any fine-tuning of the soft parameters.
FIG. 4. The forward-backward asymmetry for $e^+e^-$ pairs at the $\bar{p}p$ Tevatron ($\sqrt{s} = 1.8$ TeV) as a function of the $Z'$ mixing parameter $\theta$ with $M_{Z'} = 700$ GeV.
Finally, the forward-backward asymmetry of $e^+e^−$ pairs becomes quite large at the $Z_I$ model and the two reported CDF events are consistent with models of this type. The measurement of $A_{FB}$ will provide an interesting test for $Z'$ models at future colliders.

ACKNOWLEDGEMENTS

We would like to thank S. Ambrosanio, H. Frisch, G. Kribs, G. Mahlon and S. Martin for useful conversations. TG and GK were supported by the U.S. Department of Energy at the University of Michigan. TK thanks the Department of Physics at the University of Michigan for its hospitality during the completion of this work.

APPENDIX:

This appendix summarises the D-term contributions to the sfermions and the neutral scalar Higgs boson masses.

1. Sfermion masses

The sfermion mass matrix is parametrised as

$$M_{	ilde{f}}^2 = \begin{pmatrix}
M_{LL}^2 & M_{LR}^2 \\
M_{LR}^2 & M_{RR}^2
\end{pmatrix}.$$

(A1)

Defining the U(1)' D-term contribution to be $\tilde{m}_{D}^2 = \frac{1}{2}g'^2(Q_1v_1^2 + Q'_2v_2^2 + Q'_3v_3^2)$, the mass-mixing matrix elements for the up squarks $\tilde{u}_{L,R}$ are given by

$$M_{\tilde{u}LL}^2 = \tilde{M}_{\tilde{u}L}^2 + m_u^2 + \left(\frac{1}{2} - \frac{2}{3}x_W\right)M_Z^2 \cos 2\beta + Q'_{\tilde{u}L} \tilde{m}_{D'}^2$$

(A2)

$$M_{\tilde{u}RR}^2 = \tilde{M}_{\tilde{u}R}^2 + m_u^2 + \frac{2}{3}x_WM_Z^2 \cos 2\beta + Q'_{\tilde{u}R} \tilde{m}_{D'}^2$$

(A3)

$$M_{\tilde{u}LR}^2 = m_u(A_u - \mu \cot \beta)$$

(A4)

and for the down squarks $\tilde{d}_{L,R}$

$$M_{\tilde{d}LL}^2 = \tilde{M}_{\tilde{d}L}^2 + m_d^2 + \left(-\frac{1}{2} + \frac{1}{3}x_W\right)M_Z^2 \cos 2\beta + Q'_{\tilde{d}L} \tilde{m}_{D'}^2$$

(A5)

$$M_{\tilde{d}RR}^2 = \tilde{M}_{\tilde{d}R}^2 + m_d^2 - \frac{1}{3}x_WM_Z^2 \cos 2\beta + Q'_{\tilde{d}R} \tilde{m}_{D'}^2$$

(A6)

$$M_{\tilde{d}LR}^2 = m_d(A_d - \mu \tan \beta)$$

(A7)

where $x_W = \sin^2 \theta_W$. Similarly for the $\tilde{e}_{L,R}$ sleptons we obtain
where the mixing term negligible.

where we are assuming a superpotential term \( W = \lambda \Phi_1 \Phi_2 \Phi_3 \) with \( \Phi_i \) defined as in (19) and we have written the D-terms for the more general effective rank 5 model. The neutral Higgs bosons \( H_i^0 \) masses directly receive D-term contributions from the spontaneous symmetry breakdown of the extra \( U(1)' \). The mass mixing matrix is given by

\[
\mathcal{M}_{H^0}^2 = \frac{1}{2} \begin{bmatrix}
B_1 v_1^2 + \lambda A v_2 v_3/v_1 & B_2 v_1 v_2 - \lambda A v_3 & B_3 v_1 v_3 - \lambda A v_2 \\
B_2 v_1 v_2 - \lambda A v_3 & B_4 v_2^2 + \lambda A v_1 v_3/v_2 & B_5 v_2 v_3 - \lambda A v_1 \\
B_3 v_1 v_3 - \lambda A v_2 & B_5 v_2 v_3 - \lambda A v_1 & B_6 v_3^2 + \lambda A v_1 v_2/v_3
\end{bmatrix}
\]  

where

\[
B_1 = \frac{1}{2}(g_1^2 + g_2^2) + 2Q'_{12}^2 g^2 \\
B_2 = 2\lambda^2 - \frac{1}{2}(g_1^2 + g_2^2) + 2Q'_{12} g^2
\]  

This basis is really only important for the top squarks where the mixing term \( M_{LR}^2 \) is non-negligible.

2. Higgs masses

The Higgs boson masses are obtained from the Higgs potential [1][21]

\[
V = \mu_1^2 \Phi_1^\dagger \Phi_1 + \mu_2^2 \Phi_2^\dagger \Phi_2 + \mu_3^2 \Phi_3^\dagger \Phi_3 - \frac{i}{\sqrt{2}} A (\Phi_1^\dagger \tau_2 \Phi_2 \Phi_3 + h.c.)
\]

\[
+ \lambda^2 (\Phi_1^\dagger \Phi_2^\dagger \Phi_2 + \Phi_1^\dagger \Phi_3^\dagger \Phi_3 + \Phi_2^\dagger \Phi_2^\dagger \Phi_3^\dagger \Phi_3) + (\frac{g_2^2}{2} - \lambda^2) |\Phi_1^\dagger \Phi_2|^2
\]

\[
+ \frac{1}{8}(g_1^2 + g_2^2)(|\Phi_1^\dagger \Phi_1 - \Phi_2^\dagger \Phi_2|^2 + \frac{1}{2} g^2 (Q_1' \Phi_1^\dagger \Phi_1 + Q_2' \Phi_2^\dagger \Phi_2 + Q_3' \Phi_3^\dagger \Phi_3)^2
\]  

and for the \( \tilde{\nu}_{L,R} \) sleptons

\[
M_{LL}^2 = \tilde{M}_{\tilde{\nu}_L}^2 + m_{\tilde{\nu}_e}^2 + \frac{1}{2} M_Z^2 \cos 2\beta + Q'_{\tilde{\nu}_L} \bar{m}_{D'}^2 \\
M_{RR}^2 = \tilde{M}_{\tilde{\nu}_R}^2 + Q'_{\tilde{\nu}_R} \bar{m}_{D'}^2 \\
M_{LR}^2 = m_e (A_e - \mu \tan \beta),
\]

The sfermion mass eigenstates are given by

\[
\begin{pmatrix}
\tilde{f}_1 \\
\tilde{f}_1
\end{pmatrix} = \begin{pmatrix}
\cos \theta_{\tilde{f}} & \sin \theta_{\tilde{f}} \\
-\sin \theta_{\tilde{f}} & \cos \theta_{\tilde{f}}
\end{pmatrix}
\begin{pmatrix}
\tilde{f}_L \\
\tilde{f}_R
\end{pmatrix}.
\]  

(A8)

(A9)

(A10)

(A11)

(A12)

(A13)

(A14)

(A15)

(A16)

(A17)

(A18)

(A19)

(A20)
\[ B_3 = 2\lambda^2 + 2Q'_1Q'_3g'^2 \]  
(A21)

\[ B_4 = \frac{1}{2}(g'^2_1 + g'^2_2) + 2Q'^2_2g'^2 \]  
(A22)

\[ B_5 = 2\lambda^2 + 2Q'_2Q'_3g'^2 \]  
(A23)

\[ B_6 = 2Q'^2_3g'^2 \]  
(A24)

For completeness we also list the pseudoscalar and charged Higgs boson mass matrices \[ \mathcal{M}_{^P0}^2 = \frac{\lambda A v_3}{2} \begin{bmatrix} v_1/v_2 & 1 & v_1/v_3 \\ 1 & v_2/v_1 & v_2/v_3 \\ v_1/v_3 & v_2/v_3 & v_1 v_2/v_3^2 \end{bmatrix} \]  
(A25)

\[ \mathcal{M}_{^H\pm}^2 = \frac{1}{2} \begin{bmatrix} (g^2_2/2 - \lambda^2)v_1^2 + \lambda A v_1 v_3/v_2 & (g^2_2/2 - \lambda^2)v_1 v_2 + \lambda A v_3 \\ (g^2_2/2 - \lambda^2)v_1 v_2 + \lambda A v_3 & (g^2_2/2 - \lambda^2)v_2^2 + \lambda A v_2 v_3/v_1 \end{bmatrix} \]  
(A26)

where we have used the definitions in Sec.II.
REFERENCES

[1] J.L. Hewett and T.G. Rizzo, Phys. Rep. **183** 193, (1989).
[2] M. Cvetic and P. Langacker, Phys. Rev. **D54** 3570, (1996); hep-ph/9602424.
[3] E. Ma, hep-ph/9612470.
[4] J.D. Lykken, hep-ph/9610218.
[5] The supersymmetric parameters are mostly unchanged if the leptons are e’s, µ’s, τ’s or a mixture, so long as the event has isolated hard photons.
[6] D. Toback (for the CDF collaboration), “The Diphoton Missing $E_t$ Distribution at CDF”, FERMILAB-CONF-96-240-E.
[7] M.B. Green, J.H. Schwarz. and E. Witten, *Superstring Theory*, Volume 2, Cambridge University Press (1987).
[8] P. Langacker and M. Luo, Phys. Rev. **D45** 278, (1992).
[9] B. Holdom, Phys. Lett. **B166** 196, (1986).
[10] F. del Aguila, G.D. Coughlan, and M. Quiros, Nucl. Phys. **B307** 413, (1988).
[11] K. Babu, C. Kolda, and J. March-Russell, Phys. Rev. **D54** 4635, (1996).
[12] M. Drees, Phys. Lett. **B181** 279, (1986).
[13] Y. Kawamura and M. Tanaka, Prog. Theor. Phys. **91**, 949 (1994).
[14] H. C. Cheng and L. J. Hall, Phys. Rev. **D51** 5289, (1995).
[15] C. Kolda and S. P. Martin, Phys. Rev. **D53** 3871, (1996).
[16] J. Ellis, D.V. Nanopoulos, S.T. Petcov, and F. Zwirner, Nucl. Phys. **B283** 93, (1987).
[17] D. Suematsu and Y. Yamagishi, hep-ph/9411239, Int. J. Mod. Phys. **A10**,4521,(1995).
[18] N.G. Deshpande and J. Trampetic, Phys. Lett. **B206** 665, (1988).
[19] F. del Aguila, M. Quiros and F. Zwirner, Nucl. Phys. **B284** 530, (1987).
[20] M.M. Boyce, M.A. Doncheski, and H. König, hep-ph/9607376.
[21] V. Barger, and K. Whisnant, Int. J. Mod. Phys. **A3** 1907, (1988).
[22] S. Nandi, Phys. Lett. **B197** 144, (1987).
[23] H. Haber and G. Kane, Phys. Rep. **117** 75, (1985).
[24] V.D. Barger and R.J.N. Phillips, *Collider Physics*, Addison-Wesley (1987).
[25] F. Abe et al, Phys. Rev. **D51** 949, (1995); M. Pillai, hep-ex/9608013.
[26] S. Ambrosanio, G. Kane, G. Kribs, S. Martin and S. Mrenna, Phys.Rev.Lett. **76** 3498, (1996); hep-ph/9607414.
[27] G.L. Kane, C. Kolda, L. Roszkowski, J. D. Wells, Phys. Rev. **D49**, 6173 (1994).
[28] See for example [26]; G.L. Kane and S. Mrenna, hep-ph/9605351, Phys. Rev. Lett. **77**, 3502, (1996); M. Barnett and L. Hall, Phys. Rev. Lett. **77**, 3505, (1996).
[29] V. Barger, N.G. Deshpande, J.L. Rosner and K. Whisnant, Phys. Rev. **D35**, 2893 (1987).
[30] F. Abe et al, Phys. Rev. Lett. **77** 2616, (1996).
[31] C.E. Gerber (for the D0 collaboration), “Search for Heavy Neutral Gauge Bosons at D0”, FERMILAB-CONF-96-389-EF.
[32] J.L. Rosner, Phys. Rev. **D54**, 1078 (1996).