One-dimensional completed scattering and quantum nonlocality of entangled states

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Abstract

Entanglement is usually associated with compound systems. We first show that a one-dimensional (1D) completed scattering of a particle on a static potential barrier represents an entanglement of two alternative one-particle sub-processes, transmission and reflection, macroscopically distinct at the final stage of scattering. The wave function for the whole ensemble of scattering particles can be uniquely presented as the sum of two isometrically evolved wave packets to describe the (to-be-)transmitted and (to-be-)reflected subensembles of particles at all stages of scattering. A noninvasive Larmor-clock timing procedure adapted to either subensemble shows that namely the dwell time gives the time spent, on the average, by a particle in the barrier region, and it denies the Hartman effect. As regards the group time, it cannot be measured and hence it cannot be accepted as a measure of the tunneling time. We argue that nonlocality of entangled states appears in quantum mechanics due to inconsistency of its superposition principle with the corpuscular properties of a particle. For example, this principle associates a 1D completed scattering with a single (one-way) process, while a particle, as an indivisible object, cannot take part in transmission and reflection, simultaneously.

Key words: transmission, reflection, Larmor, nonlocality, entanglement

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1 Introduction

For a long time scattering a particle on one-dimensional (1D) static potential barriers have been considered in quantum mechanics as a representative of well-understood phenomena. However, solving the so-called tunneling time problem (TTP) for a 1D completed scattering (see reviews [1,2,3,4,5,6,7] and references therein) showed that this is not the case.

At present there is a variety of approaches to introduce characteristic times for the process. They are the group (Wigner) tunneling times (more known as
the "phase" tunneling times) [1,8,9,10,11], different variants of the dwell time
[10,12,13,14,15,16,17,18,19,20], the Larmor time [15,21,22,23,24,25,26], and
the concept of the time of arrival which is based on introducing either a suitable
time operator (see, e.g., [27,28,29,30]) or the positive operator valued measure
[5,31] (see also [32,33,34,35]). A particular class of approaches to study the
temporal aspects of a 1D scattering includes the Bohmian [32,36,37,38,39],
Feynman and Wigner ones (see [40,41,42,43,44] as well as [2,5] and references
therein). One has also point out the papers [45,46,47] to study the character-
istic times of "the forerunner preceding the main tunneling signal of the wave
created by a source with a sharp onset”.

The source of a long-lived controversy in solving the TTP, which still persists,
is usually associated with the absence of a Hermitian time operator. However,
our analysis shows that this problem is closely connected to the mystery of
quantum nonlocality of entangled states [48,49]. As is known, the main pecu-
liarity of such states is the availability of nonzero correlations between two
events separated with space-like intervals.

The main intrigue is that, though this prediction of quantum theory con-
tradicts special relativity, now it has been reliably stated (theoretically and
experimentally [50]) that nonlocality is indeed an inherent property of existing
quantum mechanics (a deep analysis of this question is done in [51,52]).

It is now widely accepted that nonlocal correlations of entangled states do
not violate special relativity, for they are not associated with a superluminal
transmission of signals (see, e.g., [53,54]). However, with regards to this 'no-
signalling' interpretation, Bell pointed out that "... we have lost the idea that
correlations can be explained, or at least this idea awaits reformulation. More
importantly, the no signaling notion rests on concepts which are desperately
vague, or vaguely applicable..." (quoted from [51]).

We agree entirely with this doubt: if nonzero correlations between two events
are not a consequence of a causal relationship between them, then the very
notion of 'correlations' becomes physically meaningless. It is just the main
challenge of quantum mechanics that its principles imply introducing such
strange correlations. So that, it is worthwhile to reveal an imperfectness in
the foundation of quantum theory, which creates such a paradoxical situation.

In this paper, the origin of quantum nonlocality is analyzed in the case of a
1D completed scattering. Studying this particular problem suggests the way of
how to reconcile quantum mechanics with special relativity. We show (Section
2) that existing quantum mechanics does not allow any consistent model of this
process. Its superposition principle, applied to entangled states, contradicts
corpuscular properties of particles. A new, consistent model of a 1D completed
scattering, free of nonlocality, is presented in Sections 3 and 4.
Towards a local model of a 1D completed scattering.

2.1 On the inconsistency of the existing model of a 1D completed scattering.

It is evident that a proper theoretical description of any physical phenomenon must obey the following three requirements which are connected with each other: (i) it must explain the phenomenon; (ii) it must be consistent; (iii) it can be verified experimentally. However, the existing quantum-mechanical model of a 1D completed scattering does not obey these requirements.

Firstly, existing quantum mechanics endows a 1D completed scattering with quantum nonlocality whose reality is questionable.

Some manifestations of nonlocality, arose in the existing approaches, have been pointed out and analyzed by Leavens and co-workers (see [23,33,35,36]). For example, the Bohmian model of a 1D completed scattering predicts that the fate of the incident particle (to be transmitted or to be reflected by the barrier) depends on the coordinate of its starting point (see [36]). In this case, that of the critical spatial point to separate the starting regions of to-be-transmitted and to-be-reflected particles depends on the shape of the potential barrier, though it is located at a considerable distance from the particle’s source.

Further, the time-of-arrival concept [31] predicts a nonzero probability of arriving a particle at the spatial regions where the probability density is \textit{a priori} zero (see [33,35]). The Larmor time concept predicts the precession of the average spin of reflected particles, under the magnetic field localized beyond the barrier, on the side of transmission where reflected particles are absent \textit{a priori} (see [23]).

However, perhaps the most known manifestation of quantum nonlocality, predicted by the existing model of a 1D completed scattering, is the so-called Hartman effect (and its versions) which is associated with the anomalously short (or even negative) times of tunneling a particle through the barrier region (see, e.g., [9,55,56,57,58,59,60,61,62,63,64]).

The existing explanations of this effect (see, e.g., [63,61]) are made, in fact, in the spirit of the ‘no-signalling’ theories. They suggest that anomalously short dwell and group times do not mean a \textit{superluminal} transmission of a particle through the barrier region. In fact, this means that the notions of the dwell and group times, as characteristics of the particle’s dynamics in the barrier region, loses theirs initial physical sense.

So, in the existing form, conventional quantum mechanics endows a 1D completed scattering with quantum nonlocality. Our next step is to show that this...
prediction results from inconsistency of the quantum-mechanical principles.

**Secondly,** *within the existing framework of quantum mechanics, any procedure of timing the motion of a scattering particle (both without and with distinguishing transmission and reflection) is a priory inconsistent.*

On the one hand, the main feature of a particle, as an indivisible object, implies that it cannot be simultaneously transmitted and reflected by the potential barrier. So that a 1D scattering should be considered as a combined process to consist from two alternative sub-processes, transmission and reflection, macroscopically distinct at the final stage of scattering. And, thus, there should be two experimenters for studying the subensembles of transmitted or reflected particles.

In this problem, introducing characteristic times and other observables, common for these two subensembles, has no physical sense. Such quantities simply cannot be measured, since they describe neither transmitted nor reflected particles. Their introduction necessitates quantum nonlocality, and they cannot be properly interpreted (about the interpretation problem for the dwell time, see in [10,63]). For example, the average value of the particle’s position (or momentum), calculated for the whole ensemble of scattering particles, does not give the expectation (i.e., most probable) value of this quantity.

On the other hand, the superposition principle, as it stands, demands of treating a 1D completed scattering as a single one-particle process, even at its final stage. By this principle, the set of one-particle’s observables should be introduced namely for the whole ensemble of scattering particles, i.e., without distinguishing transmission and reflection.

One has to stress that the existing model of a 1D completed scattering denies, not only on the conceptual level, introducing individual characteristic times and observables for transmission and reflection. This model does not provide any description of these sub-processes at all stages of scattering. All the existing approaches, which notwithstanding introduce the transmission (or reflection) time, deal in fact with the subensembles, in which the number of particles is not conserved.

**Thirdly,** *existing quantum mechanics does not allow a consistent procedure of measuring the time spent by a particle in the barrier region.*

This equally concerns experiments on photonic tunneling which are at present more reliable than those for electronic tunneling. As is known (see, e.g., [63]), such experiments imply two steps. At the fist step, a light pulse is sent through a barrier-free region. The arrival time of the peak of this pulse at a detector is needed as a reference time. At the second step, an investigated potential barrier is inserted in the path of the pulse. The arrival time of the transmitted
The main difficulty of measuring this asymptotic characteristic time is usually associated with reshaping the incident light pulse (or, wave packet) in the barrier region. At the same time there is once more problem which has remained obscure. It relates to the fact that the above procedure is based on the implicit assumption that the transmitted and free-evolved peaks start from the same spatial point.

However, as it follows from our model of a 1D competed scattering, this is not the case even for the resonant tunneling. So that this procedure gives the time delay neither for transmitted nor for reflected parts of the incident wave packet. We are sure that the same is valid for photonic tunneling. Moreover, as will be seen from our analysis, there is a reason by which the group time is a physical quantity of secondary importance.

2.2 How to reconcile a quantum model of a 1D scattering with special relativity?

So, as it follows from the above analysis, a principal shortcoming of the existing quantum model of a 1D completed scattering is that it endows a particle with the properties to contradict its corpuscular nature.

In this paper we present a new model of this process, which is based on two main ideas: (i) the state of a particle taking part in a 1D completed scattering is an entangled (combined) state; (ii) quantum mechanics must distinguish, on the conceptual level, entangled and unentangled (elementary) states.

We first show that in the problem under consideration, for a given potential and initial state of a particle, the wave function to describe the whole ensemble of particles can be uniquely presented as a sum of two isometrically evolved wave packets which describe alternative sub-processes, transmission and reflection, at all stages of scattering.

Note, at present all quantum-mechanical rules are equally applied to macroscopically distinct states and their superpositions. However, the main lesson of solving the TTP is just that this rule is erroneous. A single system (however, macroscopic or microscopic) cannot take part simultaneously in two or more macroscopically distinct sub-processes. This means that the averaging rule (Born’s formula) is not applicable to entangled states.

The phenomenon of quantum nonlocality results from ignoring this restric-
tion. In other words, it appears when one attempts to associate, contrary to the nature of entangled states, the interference pattern formed by a superposition of macroscopically distinct sub-processes with a single causally evolved process.

3 Wave functions for transmission and reflection

3.1 Setting the problem

Let us consider a particle incident from the left on the static potential barrier 

\[ V(x) \] 

confined to the finite spatial interval \([a, b]\) \((a > 0)\); \(d = b - a\) is the barrier width. Let its in-state, \(\psi_{in}(x)\), at \(t = 0\) be a normalized function to belong to the set \(S_\infty\) consisting from infinitely differentiable functions vanishing exponentially in the limit \(|x| \to \infty\). The Fourier-transform of such functions are known to belong to the set \(S_\infty\), too. In this case the position, \(\hat{x}\), and momentum, \(\hat{p}\), operators both are well-defined. Without loss of generality we will suppose that

\[
<\psi_{in}|\hat{x}|\psi_{in}> = 0, \quad <\psi_{in}|\hat{p}|\psi_{in}> = \hbar k_0 > 0, \quad <\psi_{in}|\hat{x}^2|\psi_{in}> = l_0^2; \quad (1)
\]

here \(l_0\) is the wave-packet’s half-width at \(t = 0\) \((l_0 << a)\).

We consider a completed scattering. This means that the average velocity, \(\hbar k_0/m\), is large enough, so that the transmitted and reflected wave packets do not overlap each other at late times. As for the rest, the relation of the average energy of a particle to the barrier’s height may be any by value.

We begin our analysis with the derivation of expressions for the incident, transmitted and reflected wave packets to describe, in the problem at hand, the whole ensemble of particles. For this purpose we will use the variant (see [66]) of the well-known transfer matrix method [67]. Let the wave function \(\psi_{full}(x, k)\) to describe the stationary state of a particle in the out-of-barrier regions be written in the form

\[
\psi_{full}(x; k) = e^{ikx} + b_{out}(k)e^{ik(2a-x)}, \quad \text{for} \quad x \leq a; \quad (2)
\]

\[
\psi_{full}(x; k) = a_{out}(k)e^{ik(x-d)}, \quad \text{for} \quad x > b. \quad (3)
\]

The coefficients entering this solution are connected by the transfer matrix \(Y\):
\[
\begin{pmatrix}
1 \\
b_{out}e^{2ika}
\end{pmatrix} = Y 
\begin{pmatrix}
a_{out}e^{-ikd} \\
0
\end{pmatrix},
\quad Y = \begin{pmatrix}
q & p \\
p^* & q^*
\end{pmatrix};
\tag{4}
\]

\[
q = \frac{1}{\sqrt{T(k)}} \exp [i(kd - J(k))], \quad p = \sqrt{\frac{R(k)}{T(k)}} \exp \left[ i \left( \frac{\pi}{2} + F(k) - ks \right) \right]
\tag{5}
\]

where \(T, J\) and \(F\) are the real tunneling parameters: \(T(k)\) (the transmission coefficient) and \(J(k)\) (phase) are even and odd functions of \(k\), respectively; \(F(-k) = \pi - F(k)\); \(R(k) = 1 - T(k)\); \(s = a + b\). We will suppose that the tunneling parameters have already been calculated.

In the case of many-barrier structures, for this purpose one may use the recurrence relations obtained in [66] just for these real parameters. For the rectangular barrier of height \(V_0\),

\[
T = \left[ 1 + \vartheta^2 \sin^2(\kappa d) \right]^{-1}, \quad J = \arctan \left( \vartheta (-) \tanh(\kappa d) \right),
\]

\[
F = 0, \quad \kappa = \sqrt{2m(V_0 - E)/\hbar}, \tag{6}
\]

if \(E < V_0\); and

\[
T = \left[ 1 + \vartheta^2 \sin^2(\kappa d) \right]^{-1}, \quad J = \arctan \left( \vartheta (+) \tan(\kappa d) \right),
\]

\[
F = \begin{cases} 
0, & \text{if } \vartheta (-) \sin(\kappa d) \geq 0 \\
\pi, & \text{otherwise,}
\end{cases} \quad \kappa = \sqrt{2m(E - V_0)/\hbar}, \tag{7}
\]

if \(E \geq V_0\); in both cases \(\vartheta(\pm) = \frac{1}{2} \left( \frac{k}{\kappa} \pm \frac{\kappa}{k} \right)\) (see [66]).

Now, taking into account Exps. (4) and (5), we can write down in-asymptote \(\psi_{in}(x,t)\) and out-asymptote \(\psi_{out}(x,t)\) for the time-dependent scattering problem (see [68]):

\[
\psi_{in}(x,t) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} f_{in}(k,t)e^{ikx} dk, \quad f_{in} = A_{in}(k) \exp[-iE(k)t/\hbar];
\tag{8}
\]

\[
\psi_{out}(x,t) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} f_{out}(k,t)e^{ikx} dk, \quad f_{out} = f_{out}^{tr} + f_{out}^{ref}; \tag{9}
\]
\[ f^{tr}_{\text{out}} = \sqrt{T(k)}A_{\text{in}}(k) \exp[i(J(k) - kd - E(k)t/\hbar)] \]

\[ f^{\text{ref}}_{\text{out}} = \sqrt{R(k)}A_{\text{in}}(-k) \exp[-i(J(k) - F(k) - \frac{\pi}{2} + 2ka + E(k)t/\hbar)] \]

where Exps. (8), (10) and (11) describe, respectively, the incident, transmitted and reflected wave packets. Here \( A_{\text{in}}(k) \) is the Fourier-transform of \( \psi_{\text{in}}(x) \). For example, for the Gaussian wave packet to obey condition (1), \( A_{\text{in}}(k) = c \cdot \exp(-l_0^2(k - k_0)^2); c \) is a normalization constant.

Let us now show that by the final states (9)-(11) one can uniquely reconstruct the prehistory of the subensembles of transmitted and reflected particles, impinging the barrier from the left, at all stages of scattering. Let \( \psi_{\text{tr}}(x,t) \) and \( \psi_{\text{ref}}(x,t) \) be searched-for wave packets to describe transmission (TWF) and reflection (RWF), respectively. By our approach their sum should give the (full) wave function \( \psi_{\text{full}}(x,t) \) to describe the whole 1D completed scattering:

\[ \psi_{\text{full}}(x,t) = \psi_{\text{tr}}(x,t) + \psi_{\text{ref}}(x,t). \] (12)

### 3.2 Incoming waves for transmission and reflection

We begin our analysis with searching for the stationary wave functions for reflection, \( \psi_{\text{ref}}(x;k) \), and transmission, \( \psi_{\text{tr}}(x;k) \), in the region \( x \leq a \). Let us write down these two solutions to the stationary Schrödinger equation in the form,

\[ \psi_{\text{ref}}(x;k) = A^{\text{ref}}_{\text{in}} e^{ikx} + B^{\text{ref}}_{\text{out}} e^{-ikx}, \quad \psi_{\text{tr}}(x;k) = A^{\text{tr}}_{\text{in}} e^{ikx} + B^{\text{tr}}_{\text{out}} e^{-ikx}, \]

where coefficients obey the following conditions:

\[ A^{\text{tr}}_{\text{in}} + A^{\text{ref}}_{\text{in}} = 1; \quad B^{\text{tr}}_{\text{out}} = 0; \quad B^{\text{ref}}_{\text{out}} = b_{\text{out}} e^{2ika}. \] (14)

Besides, we suppose that reflected particles do not cross the barrier region, and, hence, the probability flux for \( \psi_{\text{ref}}(x;k) \) should be equal to zero:

\[ |A^{\text{ref}}_{\text{in}}|^2 = |b_{\text{out}}|^2 \equiv R(k). \] (15)

By the same reason, the probability flux for \( \psi_{\text{full}}(x;k) \) and \( \psi_{\text{tr}}(x;k) \) should be the same,
\[ |A_{tr}^2| = |a_{out}|^2 = T(k) \] (16)

Taking into account that \( \psi_{tr} = \psi_{full} - \psi_{ref} \), we can exclude \( \psi_{tr} \) from Eq. (16). As a result, we obtain

\[ \Re \left( A_{in}^{ref} \right) - |b_{out}|^2 = 0. \] (17)

Thus, from Eqs. (15) and (17) it follows that \( A_{in}^{tr} = \sqrt{T}(\sqrt{T} \pm i\sqrt{R}) \); \( A_{in}^{ref} = \sqrt{R}(\sqrt{R} \pm i\sqrt{T}) \equiv \sqrt{R}\exp(i\lambda) \); \( \lambda = \pm \arctan(\sqrt{T}/R) \).

So, a coherent superposition of the incoming waves to describe transmission and reflection, for a given \( E \), yields the incoming wave of unite amplitude, that describes the whole ensemble of incident particles. In this case, not only \( A_{in}^{tr} + A_{in}^{ref} = 1 \), but also \( |A_{in}^{tr}|^2 + |A_{in}^{ref}|^2 = 1 \). Besides, the phase difference for the incoming waves to describe reflection and transmission equals \( \pi/2 \) irrespective of the value of \( E \).

Our next step is to show that only one root of \( \lambda \) gives a searched-for \( \psi_{ref}(x; k) \). For this purpose the above solution should be extended into the region \( x > a \). To do this, we will further restrict ourselves by symmetric potential barriers, though the above analysis is valid in the general case.

### 3.3 Wave functions for transmission and reflection in the region of a symmetric potential barrier

Let \( V(x) \) be such that \( V(x - x_c) = V(x_c - x) \); \( x_c = (a + b)/2 \). As is known, for the region of a symmetric potential barrier, one can always find odd, \( u(x - x_c) \), and even, \( v(x - x_c) \), solutions to the Schrödinger equation. We will suppose here that these functions are known. For example, for the rectangular potential barrier (see Exps. (6) and (7)),

\[
\begin{align*}
    u(x) &= \sinh(\kappa x), & v(x) &= \cosh(\kappa x), & \text{if } E \leq V_0; \\
    u(x) &= \sin(\kappa x), & v(x) &= \cos(\kappa x), & \text{if } E \geq V_0.
\end{align*}
\]

Note, \( \frac{dv}{dx} - \frac{du}{dx} \) is a constant, which equals \( \kappa \) in the case of the rectangular barrier. Without loss of generality we will keep this notation for any symmetric potential barrier.

Before finding \( \psi_{ref}(x; k) \) and \( \psi_{tr}(x; k) \) in the barrier region, we have firstly to derive expressions for the tunneling parameters of symmetric barriers. Let in the barrier region \( \psi_{full}(x; k) = a_{full} \cdot u(x - x_c, k) + b_{full} \cdot v(x - x_c, k) \). "Sewing"
this expression together with Exps. (2) and (3) at the points \( x = a \) and \( x = b \), respectively, we obtain

\[
a_{full} = \frac{1}{\kappa} (P + P^* b_{out}) e^{ika} = -\frac{1}{\kappa} P^* a_{out} e^{ika},
\]
\[
b_{full} = \frac{1}{\kappa} (Q + Q^* b_{out}) e^{ika} = \frac{1}{\kappa} Q^* a_{out} e^{ika};
\]

\[
Q = \left. \left( \frac{du(x - x_c)}{dx} +iku(x - x_c) \right) \right|_{x=b};
\]
\[
P = \left. \left( \frac{dv(x - x_c)}{dx} +ikv(x - x_c) \right) \right|_{x=b}.
\]

As a result,

\[
a_{out} = \frac{1}{2} \left( \frac{Q}{Q^*} - \frac{P}{P^*} \right); \quad b_{out} = -\frac{1}{2} \left( \frac{Q}{Q^*} + \frac{P}{P^*} \right). \tag{18}
\]

As it follows from (4), \( a_{out} = \sqrt{T} \exp(iJ) \), \( b_{out} = \sqrt{R} \exp\left(i\left(J - F - \frac{\pi}{2}\right)\right)\).

Hence \( T = |a_{out}|^2 \), \( R = |b_{out}|^2 \), \( J = \arg(a_{out}) \). Besides, for symmetric potential barriers \( F = 0 \) when \( \Re(QP^*) > 0 \); otherwise, \( F = \pi \).

Then, one can show that "sewing" the general solution \( \psi_{\text{ref}}(x; k) \) in the barrier region together with Exp. (13) at \( x = a \), for both the roots of \( \lambda \), gives odd and even functions in this region. For the problem considered, only the former has a physical meaning. The corresponding roots for \( A_{\text{ref}}^{\text{in}} \) and \( A_{\text{in}}^{\text{tr}} \) read as

\[
A_{\text{in}}^{\text{ref}} = b_{out} (b_{out}^* - a_{out}^*); \quad A_{\text{in}}^{\text{tr}} = a_{out}^* (a_{out} + b_{out}) \tag{19}
\]

One can easily show that in this case

\[
\frac{Q^*}{Q} = -\frac{A_{\text{in}}^{\text{ref}}}{b_{out}} = \frac{A_{\text{in}}^{\text{tr}}}{a_{out}}; \tag{20}
\]

for \( a \leq x \leq b \)

\[
\psi_{\text{ref}} = \frac{1}{\kappa} \left( P A_{\text{in}}^{\text{ref}} + P^* b_{out} \right) e^{ika} u(x - x_c, k). \tag{21}
\]

The extension of this solution onto the region \( x \geq b \) gives
\[ \psi_{\text{ref}} = -b_{\text{out}} e^{ik(x-d)} - A_{\text{in}}^{\text{ref}} e^{-ik(x-s)}. \]  

(22)

So, Exps. (13), (21) and (22) give the solution to the Schrödinger equation, which we expect to describe reflection. Then, the corresponding solution for transmission is \( \psi_{\text{tr}}(x; k) = \psi_{\text{full}}(x; k) - \psi_{\text{ref}}(x; k). \)

Note that \( \psi_{\text{full}}(x; k) \) does not contain an incoming wave impinging the barrier from the right, while the found TWF and RWF include such waves. That is, the superposition of these probability waves leads, due to interference, to their macroscopical reconstruction: in the superposition, both outgoing waves are connected only with the left source of particles. One can show that, in this case, the reflected and transmitted waves are connected causally with the incoming waves \( A_{\text{in}}^{\text{ref}} e^{ikx} \) and \( A_{\text{in}}^{\text{tr}} e^{ikx} \), respectively.

Indeed, let us firstly consider reflection. As is seen from Exp. (21), \( \psi_{\text{ref}}(x; k) \) is equal to zero at the point \( x = x_c \), for all values of \( k \). As a result, the probability flux, for any time-dependent wave function formed from \( \psi_{\text{ref}}(x; k) \), is equal to zero at this point, for any value of time. This means that, in the case of reflection, particles impinging the symmetric barrier from the left do not enter the region \( x \geq x_c \). In other words, the wave packet, \( \tilde{\psi}_{\text{ref}}(x; k) \), to describe such particles can be written in the form

\[ \tilde{\psi}_{\text{ref}}(x; k) \equiv \psi_{\text{ref}}(x; k) \text{ for } x \leq x_c; \quad \tilde{\psi}_{\text{ref}}(x; k) \equiv 0 \text{ for } x \geq x_c. \]  

(23)

Note, for a given potential, \( \tilde{\psi}_{\text{ref}}(x; k) \) does not obey the Schrödinger equation at the point \( x = x_c \). Nevertheless the probability density for this function is everywhere continuous and the probability flux is everywhere equal to zero. This means that the wave packet, \( \tilde{\psi}_{\text{ref}}(x, t) \), formed from the functions \( \tilde{\psi}_{\text{ref}}(x; k) \) with different \( k \), despite discontinuity its first derivation at the point \( x_c \), is everywhere continuous and evolves with a fixed norm. As is said above, namely this packet describes the subensemble of particles which impinge the barrier from the left and are reflected by it.

The above suggests that the subensemble of incident particles to be transmitted by the barrier is described by the incident wave \( A_{\text{in}}^{\text{tr}} e^{ikx} \) of the solution \( \psi_{\text{tr}}(x; k) \). Namely this incident wave is causally connected with the transmitted one \( a_{\text{out}}(k) e^{ik(x-d)} \) of the solution \( \psi_{\text{full}}(x; k) \).

One can easily show that the function \( \tilde{\psi}_{\text{tr}}(x; k) \), where \( \tilde{\psi}_{\text{tr}}(x; k) = \psi_{\text{full}}(x; k) - \psi_{\text{ref}}(x; k) \), is everywhere continuous and the corresponding probability flux is everywhere constant. In this case,

\[ \tilde{\psi}_{\text{tr}}(x; k) \equiv \psi_{\text{tr}}(x; k) \text{ for } x \leq x_c; \quad \tilde{\psi}_{\text{tr}}(x; k) \equiv \psi_{\text{full}}(x; k) \text{ for } x \geq x_c. \]  

(24)
As in the case of reflection, the wave packet, $\tilde{\psi}_{tr}(x, t)$, formed from the functions $\tilde{\psi}_{tr}(x; k)$ (despite discontinuity its first derivation at the point $x_c$) is everywhere continuous and evolves with a fixed norm. Hence namely this packet describes the subensemble of particles which impinge the barrier from the left and are transmitted by it.

One can easily show that

$$\tilde{\psi}_{tr}(x; k) = a_{tr}^l u(x - x_c, k) + b_{tr} v(x - x_c, k) \quad \text{for} \quad a \leq x \leq x_c; \quad (25)$$

$$\tilde{\psi}_{tr}(x; k) = a_{tr}^r u(x - x_c, k) + b_{tr} v(x - x_c, k) \quad \text{for} \quad x_c \leq x \leq b; \quad (26)$$

$$\tilde{\psi}_{tr}(x; k) = a_{out} e^{i k (x - d)} \quad \text{for} \quad x \geq b; \quad (27)$$

where

$$a_{tr}^l = \frac{1}{\kappa} P A_{tr}^i e^{ika}, \quad b_{tr} = b_{full} = \frac{1}{\kappa} Q^{*} a_{out} e^{ika}, \quad a_{tr}^r = a_{full} = -\frac{1}{\kappa} P^{*} a_{out} e^{ika}$$

Note, for any value of $t$

$$T = \langle \tilde{\psi}_{tr}(x, t) | \tilde{\psi}_{tr}(x, t) \rangle = const; \quad R = \langle \tilde{\psi}_{ref}(x, t) | \tilde{\psi}_{ref}(x, t) \rangle = const;$$

$T$ and $R$ are the average transmission and reflection coefficients, respectively. Besides,

$$\langle \psi_{full}(x, t) | \psi_{full}(x, t) \rangle = T + R = 1. \quad (28)$$

From this it follows, in particular, that the scalar product of the wave packets for transmission and reflection, $\langle \tilde{\psi}_{tr}(x, t) | \tilde{\psi}_{ref}(x, t) \rangle$, is a purely imaginary quantity to approach zero when $t \to \infty$.

We have to stress that these wave packets are not solutions to the Schrödinger equation for a given potential, just as transmission and reflection described by them are not independent quantum processes. These wave packets may be considered only as parts of an entangled state to describe a 1D completed scattering, like the sub-processes may be considered only as two different alternatives to constitute the same one-particle scattering process.
Of importance is that namely these two wave packets describe the (to-be-)
transmitted and (to-be-)reflected subensembles of particles at all stages of
scattering. In this case

\[ \psi_{\text{full}}(x, t) = \psi_{\text{tr}}(x, t) + \psi_{\text{ref}}(x, t) = \tilde{\psi}_{\text{ref}}(x, t) + \tilde{\psi}_{\text{tr}}(x, t). \]  

(29)

(Below we will deal only with \( \tilde{\psi}_{\text{ref}} \) and \( \tilde{\psi}_{\text{tr}} \). For this reason these notations
will be used without tilde.)

Now we can proceed to the study of temporal aspects of a 1D completed
scattering. The found wave packets for transmission and reflection permit us
to introduce characteristic times for either sub-process. As will be seen from
the following, the motion of either subensemble of particles in the barrier
region can be investigated with help of the Larmor-clock timing procedure
adapted to the sub-processes.

4 Characteristic times for transmission and reflection

So, our main purpose now is to find, for each sub-process, the time spent, on
the average, by a particle in the barrier region. In doing so, we have to take
into account that a chosen timing procedure must not influence an original
value of the characteristic time.

Under such conditions, perhaps, the only way to measure the tunneling time
for a completed scattering is to exploit internal degrees of freedom of quantum
particles. As is known, namely this idea underlies the Larmor-time concept
based on the Larmor precession of the particle’s spin under the infinitesimal
magnetic field.

However, as will be seen from the following, the Larmor-time concept is di-
rectly connected to the dwell time to describe the stationary scattering prob-
lem. By this reason, we define firstly the dwell times for transmission and
reflection for a particle in the stationary state.

4.1 Dwell times for transmission and reflection

Note, in the case of transmission the density of the probability flux, \( I_{\text{tr}} \),
for \( \psi_{\text{tr}}(x; k) \) is everywhere constant and equal to \( T \cdot \hbar k/m \). The velocity,
\( v_{\text{tr}}(x, k) \), of an infinitesimal element of the flux, at the point \( x \), equals \( v_{\text{tr}}(x) = I_{\text{tr}}/|\psi_{\text{tr}}(x; k)|^2 \). Outside the barrier region the velocity is everywhere con-
stant: \( v_{\text{tr}} = \hbar k/m \). In the barrier region it depends on \( x \). In the case of an
opaque rectangular potential barrier, $v_{tr}(x)$ decreases exponentially when the infinitesimal element approaches the midpoint $x_c$. One can easily show that $|\psi_{tr}(a; k)| = |\psi_{tr}(b; k)| = \sqrt{T}$, but $|\psi_{tr}(x_c; k)| \sim \sqrt{T} \exp(\kappa d/2)$.

Thus, any selected infinitesimal element of the flux passes the barrier region for the time $\tau_{tr}^{dwell}$, where

$$\tau_{tr}^{dwell}(k) = \frac{1}{I_{tr}} \int_a^b |\psi_{tr}(x; k)|^2 dx. \quad (30)$$

By analogy with [15] we will call this time scale the dwell time for transmission.

For the rectangular barrier this time reads (for $E < V_0$ and $E \geq V_0$, respectively) as

$$\tau_{tr}^{dwell} = \frac{m}{2\hbar k \kappa^3} \left[ (\kappa^2 - k^2) \kappa d + \kappa_0^2 \sinh(\kappa d) \right], \quad (31)$$

$$\tau_{tr}^{dwell} = \frac{m}{2\hbar k \kappa^3} \left[ (\kappa^2 + k^2) \kappa d - \beta \kappa_0^2 \sin(\kappa d) \right]. \quad (32)$$

In the case of reflection the situation is less simple. The above arguments are not applicable here, for the probability flux for $\psi_{ref}(x, k)$ is zero. However, as is seen, the dwell time for transmission coincides, in fact, with Buttiker’s dwell time introduced however on the basis of the wave function for transmission. Therefore, making use of the arguments by Buttiker, let us define the dwell time for reflection, $\tau_{ref}^{dwell}$, as

$$\tau_{ref}^{dwell}(k) = \frac{1}{I_{ref}} \int_a^{x_c} |\psi_{ref}(x, k)|^2 dx; \quad (33)$$

where $I_{ref} = R \cdot \hbar k / m$ is the incident probability flux for reflection.

Again, for the rectangular barrier

$$\tau_{ref}^{dwell} = \frac{mk}{\hbar k} \cdot \frac{\sinh(\kappa d) - \kappa d}{\kappa^2 + \kappa_0^2 \sinh^2(\kappa d/2)} \quad \text{for } E < V_0; \quad (34)$$

$$\tau_{ref}^{dwell} = \frac{mk}{\hbar k} \cdot \frac{\kappa d - \sin(\kappa d)}{\kappa^2 + \beta \kappa_0^2 \sin^2(\kappa d/2)} \quad \text{for } E \geq V_0. \quad (35)$$
As is seen, for rectangular barriers the dwell times for transmission and reflection do not coincide with each other, unlike the asymptotic group times.

We have to stress once more that Exps. (30) and (33), unlike Smith’s, Buttiker’s and Bohmian dwell times, are defined in terms of the TWF and RWF. As will be seen from the following, the dwell times introduced can be justified in the framework of the Larmor-time concept.

4.2 Larmor times for transmission and reflection

As was said above, both the group (see [65]) and dwell time concepts do not give the way of measuring the time spent by a particle in the barrier region. This task can be solved in the framework of the Larmor time concept. As is known, the idea to use the Larmor precession as clocks was proposed by Baz’ [21] and developed later by Rybachenko [22] and Büttiker [15] (see also [23,25]).

However, we have to stress that the existing concept of the Larmor time was introduced on the basis of incoming and outgoing waves (see [15,23,25]). In this connection, our next step is to redefine the Larmor times for the barrier region, making use the expressions of the corresponding wave functions just for this region.

4.2.1 Preliminaries

Let us consider the quantum ensemble of electrons moving along the $x$-axis and interacting with the symmetrical time-independent potential barrier $V(x)$ and small magnetic field (parallel to the $z$-axis) confined to the finite spatial interval $[a, b]$. Let this ensemble be a mixture of two parts. One of them consists from electrons with spin parallel to the magnetic field. Another is formed from particles with antiparallel spin.

Let at $t = 0$ the in state of this mixture be described by the spinor

$$
\Psi_{in}(x) = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 \\ 1 \end{pmatrix} \psi_{in}(x),
$$

(36)

where $\psi_{in}(x)$ is a normalized function to satisfy conditions (1). So that we will consider the case, when the spin coherent in state (36) is the eigenvector of $\sigma_x$ with the eigenvalue 1 (the average spin of the ensemble of incident particles is oriented along the $x$-direction); hereinafter, $\sigma_x$, $\sigma_y$ and $\sigma_z$ are the Pauli spin matrices.
For electrons with spin up (down), the potential barrier effectively decreases (increases), in height, by the value $\hbar \omega_L/2$; here $\omega_L$ is the frequency of the Larmor precession; $\omega_L = 2 \mu B/\hbar$, $\mu$ denotes the magnetic moment. The corresponding Hamiltonian has the following form,

$$\hat{H} = \frac{\hat{p}^2}{2m} + V(x) - \frac{\hbar \omega_L}{2} \sigma_z, \quad \text{if } x \in [a, b]; \quad \hat{H} = \frac{\hat{p}^2}{2m}, \quad \text{otherwise.} \quad (37)$$

For $t > 0$, due to the influence of the magnetic field, the states of particles with spin up and down become different. The probability to pass the barrier is different for them. Let for any value of $t$ the spinor to describe the state of particles read as

$$\Psi_{full}(x, t) = \frac{1}{\sqrt{2}} \begin{pmatrix} \psi_{full}^{(\uparrow)}(x, t) \\ \psi_{full}^{(\downarrow)}(x, t) \end{pmatrix}. \quad (38)$$

In accordance with (12) (or (29)), either spinor component can be uniquely presented as a coherent superposition of two probability fields to describe transmission and reflection:

$$\psi_{full}^{(\uparrow\downarrow)}(x, t) = \psi_{tr}^{(\uparrow\downarrow)}(x, t) + \psi_{ref}^{(\uparrow\downarrow)}(x, t); \quad (39)$$

note that $\psi_{ref}^{(\uparrow\downarrow)}(x, t) \equiv 0$ for $x \geq x_c$. As a consequence, the same decomposition takes place for spinor (38): $\Psi_{full}(x, t) = \Psi_{tr}(x, t) + \Psi_{ref}(x, t)$.

We will suppose that all the wave functions for transmission and reflection are known. It is important to stress here (see (28) that

$$<\psi_{full}^{(\uparrow)}(x, t)|\psi_{full}^{(\downarrow)}(x, t)> = T^{(\uparrow\downarrow)} + R^{(\uparrow\downarrow)} = 1;$$
$$T^{(\uparrow\downarrow)} = <\psi_{tr}^{(\uparrow)}(x, t)|\psi_{tr}^{(\downarrow)}(x, t)> = \text{const};$$
$$R^{(\uparrow\downarrow)} = <\psi_{ref}^{(\uparrow)}(x, t)|\psi_{ref}^{(\downarrow)}(x, t)> = \text{const}; \quad (40)$$

$T^{(\uparrow\downarrow)}$ and $R^{(\uparrow\downarrow)}$ are the (real) transmission and reflection coefficients, respectively, for particles with spin up ($\uparrow$) and down ($\downarrow$). Let further $T = (T^{(\uparrow)} + T^{(\downarrow)})/2$ and $R = (R^{(\uparrow)} + R^{(\downarrow)})/2$ be quantities to describe all particles.

### 4.2.2 Time evolution of the spin polarization of particles

To study the time evolution of the average particle’s spin, we have to find the expectation values of the spin projections $\hat{S}_x$, $\hat{S}_y$ and $\hat{S}_z$. Note, for any $t$
\[ < \hat{S}_x >_{\text{full}} = \frac{\hbar}{2} \sin(\theta_{\text{full}}) \cos(\phi_{\text{full}}) = \hbar \cdot \Re(< \psi_{\text{full}}^{(t)}|\psi_{\text{full}}^{(t)}>); \]

\[ < \hat{S}_y >_{\text{full}} = \frac{\hbar}{2} \sin(\theta_{\text{full}}) \sin(\phi_{\text{full}}) = \hbar \cdot \Im(< \psi_{\text{full}}^{(t)}|\psi_{\text{full}}^{(t)}>); \]

\[ < \hat{S}_z >_{\text{full}} = \frac{\hbar}{2} \cos(\theta_{\text{full}}) = \frac{\hbar}{2} \left[ < \psi_{\text{full}}^{(t)}|\psi_{\text{full}}^{(t)}> - < \psi_{\text{full}}^{(t)}|\psi_{\text{full}}^{(t)}> \right]. \]

Similar expressions are valid for transmission and reflection:

\[ < \hat{S}_x >_{\text{tr}} = \frac{\hbar}{T} \Re(< \psi_{\text{tr}}^{(t)}|\psi_{\text{tr}}^{(t)}>), \quad < \hat{S}_y >_{\text{tr}} = \frac{\hbar}{T} \Im(< \psi_{\text{tr}}^{(t)}|\psi_{\text{tr}}^{(t)}>), \]
\[ < \hat{S}_z >_{\text{tr}} = \frac{\hbar}{2T} \left( < \psi_{\text{tr}}^{(t)}|\psi_{\text{tr}}^{(t)}> - < \psi_{\text{tr}}^{(t)}|\psi_{\text{tr}}^{(t)}> \right), \]

\[ < \hat{S}_x >_{\text{ref}} = \frac{\hbar}{R} \Re(< \psi_{\text{ref}}^{(t)}|\psi_{\text{ref}}^{(t)}>), \quad < \hat{S}_y >_{\text{ref}} = \frac{\hbar}{R} \Im(< \psi_{\text{ref}}^{(t)}|\psi_{\text{ref}}^{(t)}>), \]
\[ < \hat{S}_z >_{\text{ref}} = \frac{\hbar}{2R} \left( < \psi_{\text{ref}}^{(t)}|\psi_{\text{ref}}^{(t)}> - < \psi_{\text{ref}}^{(t)}|\psi_{\text{ref}}^{(t)}> \right). \]

Note, \( \theta_{\text{full}} = \pi/2, \phi_{\text{full}} = 0 \) at \( t = 0 \). However, this is not the case for transmission and reflection. Namely, for \( t = 0 \) we have

\[ \phi_{\text{tr,ref}}^{(0)} = \arctan \left( \frac{\Im(< \psi_{\text{tr,ref}}^{(t)}|\psi_{\text{tr,ref}}^{(t)}(x,0)>)}{\Re(< \psi_{\text{tr,ref}}^{(t)}|\psi_{\text{tr,ref}}^{(t)}(x,0)>)} \right); \]

\[ \theta_{\text{tr,ref}}^{(0)} = \arccos \left( < \psi_{\text{tr,ref}}^{(t)}(x,0)|\psi_{\text{tr,ref}}^{(t)}(x,0) > - < \psi_{\text{tr,ref}}^{(t)}(x,0)|\psi_{\text{tr,ref}}^{(t)}(x,0) > \right); \]

Since the norms of \( \psi_{\text{tr}}^{(t)}(x,t) \) and \( \psi_{\text{ref}}^{(t)}(x,t) \) are constant, \( \theta_{\text{tr}}(t) = \theta_{\text{tr}}^{(0)} \) and \( \theta_{\text{ref}}(t) = \theta_{\text{ref}}^{(0)} \) for any value of \( t \). For the z-components of spin we have

\[ < \hat{S}_z >_{\text{tr}}(t) = \hbar \frac{T^{(t)} - T^{(t)}}{T^{(t)} + T^{(t)}}, \quad < \hat{S}_z >_{\text{ref}}(t) = \hbar \frac{R^{(t)} - R^{(t)}}{R^{(t)} + R^{(t)}}. \]
So, since the operator $\hat{S}_z$ commutes with Hamiltonian (37), this projection of the particle’s spin should be constant, on the average, both for transmission and reflection. From the most beginning the subensembles of transmitted and reflected particles possess a nonzero average $z$-component of spin (though it equals zero for the whole ensemble of particles, for the case considered) to be conserved in the course of scattering. By our approach the angles $\theta_{tr}^{(0)}$ and $\theta_{ref}^{(0)}$ cannot be used as a measure of the time spent by a particle in the barrier region.

4.2.3 Larmor precession caused by the infinitesimal magnetic field confined to the barrier region

As in [15,25], we will suppose further that the applied magnetic field is infinitesimal. In order to introduce characteristic times let us find the derivations $d\phi_{tr}/dt$ and $d\phi_{ref}/dt$. For this purpose we will use the Ehrenfest equations for the average spin of particles:

$$\frac{d < \hat{S}_x >_{tr}}{dt} = -\frac{\hbar \omega_L}{T} \int_a^b \Im[(\psi_{tr}^{(1)}(x,t))^{*}\psi_{tr}^{(1)}(x,t)] dx$$

$$\frac{d < \hat{S}_y >_{tr}}{dt} = \frac{\hbar \omega_L}{T} \int_a^b \Re[(\psi_{tr}^{(1)}(x,t))^{*}\psi_{tr}^{(1)}(x,t)] dx$$

$$\frac{d < \hat{S}_x >_{ref}}{dt} = -\frac{\hbar \omega_L}{R} \int_a^{x_c} \Im[(\psi_{ref}^{(1)}(x,t))^{*}\psi_{ref}^{(1)}(x,t)] dx$$

$$\frac{d < \hat{S}_y >_{ref}}{dt} = \frac{\hbar \omega_L}{R} \int_a^{x_c} \Re[(\psi_{ref}^{(1)}(x,t))^{*}\psi_{ref}^{(1)}(x,t)] dx.$$  

Note, $\phi_{tr,ref} = \arctan(\frac{< \hat{S}_y >_{tr,ref}}{< \hat{S}_x >_{tr,ref}})$. Hence, considering that the magnetic field is infinitesimal and $|< \hat{S}_y >_{tr,ref}| \ll |< \hat{S}_x >_{tr,ref}|$, we have

$$\frac{d\phi_{tr}}{dt} = \frac{1}{< \hat{S}_x >_{tr}} \frac{d < \hat{S}_y >_{tr}}{dt}; \quad \frac{d\phi_{ref}}{dt} = \frac{1}{< \hat{S}_x >_{ref}} \frac{d < \hat{S}_y >_{ref}}{dt}.$$  

Then, considering the above expressions for the spin projections and their derivatives on $t$, we obtain

$$\frac{d\phi_{tr}}{dt} = \omega_L \int_a^b \Re[(\psi_{tr}^{(1)}(x,t))^{*}\psi_{tr}^{(1)}(x,t)] dx; \quad \int_a^{\infty} \Re[(\psi_{tr}^{(1)}(x,t))^{*}\psi_{tr}^{(1)}(x,t)] dx.$$
\[
\frac{d\phi_{\text{ref}}}{dt} = \omega_L \frac{\int_a^c \Re[(\psi_{\text{ref}}^{(t)}(x,t))^*\psi_{\text{ref}}^{(t)}(x,t)]dx}{\int_{-\infty}^c \Re[(\psi_{\text{ref}}^{(t)}(x,t))^*\psi_{\text{ref}}^{(t)}(x,t)]dx}.
\]

Or, taking into account that in the first order approximation on \(\omega_L\), when 
\[\psi_{\text{tr}}^{(t)}(x,t) = \psi_{\text{tr}}^{(t)}(x,t)\] and \(\psi_{\text{ref}}^{(t)}(x,t) = \psi_{\text{ref}}^{(t)}(x,t)\), we have

\[
\frac{d\phi_{\text{tr}}}{dt} \approx \frac{\omega_L}{T} \int_a^b |\psi_{\text{tr}}(x,t)|^2 dx; \quad \frac{d\phi_{\text{ref}}}{dt} \approx \frac{\omega_L}{R} \int_a^c |\psi_{\text{ref}}(x,t)|^2 dx;
\]

note, in this limit, \(T \to T\) and \(R \to R\).

As is supposed in our setting the problem, both at the initial and final instants of time, a particle does not interact with the potential barrier and magnetic field. In this case, without loss of exactness, the angles of rotation (\(\Delta \phi_{\text{tr}}\) and \(\Delta \phi_{\text{ref}}\)) of spin under the magnetic field, in the course of a completed scattering, can be written in the form,

\[
\Delta \phi_{\text{tr}} = \frac{\omega_L}{T} \int_{-\infty}^\infty dt \int_a^b dx |\psi_{\text{tr}}(x,t)|^2, \quad \Delta \phi_{\text{ref}} = \frac{\omega_L}{R} \int_{-\infty}^\infty dt \int_a^c dx |\psi_{\text{ref}}(x,t)|^2 \tag{43}
\]

On the other hand, both the quantities can be written in the form: \(\Delta \phi_{\text{tr}} = \omega_L \tau_{\text{tr}}^L\) and \(\Delta \phi_{\text{ref}} = \omega_L \tau_{\text{ref}}^L\), where \(\tau_{\text{tr}}^L\) and \(\tau_{\text{ref}}^L\) are the Larmor times for transmission and reflection. Comparing these expressions with (43), we eventually obtain

\[
\tau_{\text{tr}}^L = \frac{1}{T} \int_{-\infty}^\infty dt \int_a^b dx |\psi_{\text{tr}}(x,t)|^2, \quad \tau_{\text{ref}}^L = \frac{1}{R} \int_{-\infty}^\infty dt \int_a^c dx |\psi_{\text{ref}}(x,t)|^2. \tag{44}
\]

These are just the searched-for definitions of the Larmor times for transmission and reflection.

Let us write down the wave packets for transmission and reflection in the form,

\[
\psi_{\text{tr},\text{ref}}(x,t) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^\infty A_{in}(k)\psi_{\text{tr},\text{ref}}(x,k)e^{-iE(k)t/h}dk;
\]

expressions for \(\psi_{\text{tr}}(x,k)\) and \(\psi_{\text{ref}}(x,k)\) see in Section 3. Then Exps. (44) can be rewritten in terms of dwell times (30) and (33):
\[
\tau_{tr}^L = \frac{1}{T} \int_0^\infty \varpi(k)T(k)\tau_{dwell}^r dk, \quad \tau_{ref}^L = \frac{1}{R} \int_0^\infty \varpi(k)R(k)\tau_{dwell}^r dk.
\] (45)

where \(\varpi(k) = |A_{in}(k)|^2 - |A_{in}(-k)|^2\). Thus, the Larmor times for transmission and reflection are, in fact, the average values of dwell times (30) and (33), respectively.

In the end of this section it is useful to address the rectangular barrier. For the stationary case, in addition to Larmor times (31), (32), (34) and (35), we present explicit expressions for the initial angles \(\theta_{tr}^{(0)}\) and \(\phi_{tr}^{(0)}\). To the first order in \(\omega_L\), we have

\[
\theta_{tr}^{(0)} = \frac{\pi}{2} - \omega_L T_z, \quad \phi_{tr}^{(0)} = \omega_L T_0, \quad \theta_{ref}^{(0)} = \frac{\pi}{2} + \omega_L T_z \quad \text{and} \quad \phi_{tr}^{(0)} = -\omega_L T_0,
\]

where

\[
T_z = \frac{m\kappa^2_0}{\hbar^2} \cdot \frac{(\kappa^2 - k^2) \sinh(\kappa d) + \kappa_0^2 \kappa d \cosh(\kappa d)}{4k^2 \kappa^2 + \kappa_0^4 \sinh^2(\kappa d)} \sinh(\kappa d)
\]

\[
T_0 = \frac{2mk}{\hbar \kappa} \cdot \frac{\kappa_0^2 \kappa d \cos(\kappa d) - \beta(\kappa^2 + k^2) \sin(\kappa d)}{4k^2 \kappa^2 + \kappa_0^4 \sin^2(\kappa d)}
\] (46)

for \(E < V_0\) and \(E \geq V_0\), respectively.

Note that \(T_z\) is just the characteristic time introduced in [15] (see Exp. (2.20a)). However, we have to stress once more that this quantity does not describe the duration of the scattering process (see the end of Section 4.2.2). As regards \(T_0\), this quantity is directly associated with timing a particle in the barrier region. It describes the initial position of the "clock-pointers" which they have before entering a particle into this region.

### 4.3 Tunneling a particle through an opaque rectangular barrier

Note, the problem of scattering a particle, with a well defined energy, on an opaque rectangular potential barrier is the most suitable case for verifying our approach. Let us denote final measured azimuthal angles, for transmission and reflection, as \(\phi_{tr}^{(\infty)}\) and \(\phi_{ref}^{(\infty)}\), respectively. By our approach \(\phi_{tr,ref}^{(\infty)} = \phi_{tr,ref}^{(0)} + \Delta\phi_{tr,ref}\). That is, the final times are expected to be registered by the Larmor
clock, for transmission and reflection, should be equal to $\tau^L_{tr} + \tau_0$ and $\tau^L_{ref} - \tau_0$, respectively.

Note, for a particle scattering on an opaque rectangular barrier (when $\kappa d \gg 1$) we have $|\tau_0| \ll \tau^L_{ref} \ll \tau^L_{tr}$ (see Exps. (31), (34) and (46)). As is known, Smith’s dwell time $\tau^\text{Smith}_{dwell}$ (which coincides with the ”phase” time) and Buttiker’s dwell time saturate in this case with increasing the barrier’s width (see Exps. (3.2) and (2.20b) in [15]). Just this property of the tunneling times is interpreted as the Hartman effect.

At the same time, our approach denies the existence of this effect: transmission time (31) increases as exponent when $d \to \infty$. Of course, reflection time (34) is naturally to saturate in this case.

As regards the Bohmian approach, it formally denies this effect, too. It predicts that the average time, $\tau^\text{Bohm}_{dwell}$, spent by a transmitted particle in the opaque rectangular barrier is

$$\tau^\text{Bohm}_{dwell} \equiv \frac{1}{T} \tau^\text{Smith}_{dwell} = \frac{m}{2\hbar k^3 \kappa^3} \left[ (\kappa^2 - k^2) k^2 \kappa d + \kappa^4_0 \sinh(2\kappa d)/2 \right].$$

Thus, for $\kappa d \gg 1$ we have $\tau^\text{Bohm}_{dwell}/\tau^\text{tr}_{dwell} \sim \cosh(\kappa d)$, i.e.,

$$\tau^\text{Bohm}_{dwell} \gg \tau^\text{tr}_{dwell} \gg \tau^\text{Smith}_{dwell} \sim \tau^\text{Butt}_{dwell}.$$

As is seen, in comparison with our definition, $\tau^\text{Bohm}_{dwell}$ overestimates the duration of dwelling transmitted particles in the barrier region. Of course, at this point we can remind that the existing Bohmian model of the scattering process is inconsistent, since it contains nonlocality.

However, it is useful also to point out that $\tau^\text{Bohm}_{dwell}$ to describe transmission was obtained in terms of $\psi_{\text{full}}$. One can show that the input of to-be-reflected particles into

$$\int_a^b |\psi_{\text{full}}(x,k)|^2 dx$$

is dominant inside the region of an opaque potential barrier. Therefore treating this time scale as the characteristic time for transmission has no basis.

So, we state that the ”causal” trajectories of transmitted and reflected particles introduced in the Bohmian mechanics are, in fact, ill-defined. However, we have to stress that our approach does not at all deny the Bohmian mechanics. It rather says that ”causal” trajectories for scattered particles should be redefined. Indeed, an incident particle should have two possibility (both to be transmitted and to reflected by the barrier) irrespective of the location of its starting point. This means that just two causal trajectories should evolve from each staring point. Both sets of causal trajectories must be defined on the basis of $\psi_{tr}(x,t)$ and $\psi_{ref}(x,t)$. As to the rest, all mathematical tools developed
in the Bohmian mechanics (see, e.g., [38,39]) remain in force.

In the end of this section it is very important to stress that the group transmission and reflection times (see [65]), which are coincident for symmetric potential barriers, lead to the Hartman effect, as the previous approaches. Thus, our model reveals a deep difference between the dwell and group times. Only one of them has a physical sense, and the Larmor-clock timing procedure resolves this dilemma in favor of the former. As regards the group time, it cannot be measured for scattering particles. And, hence, it says nothing about the effective velocity of passing a particle (signal, information) through the barrier region.

5 Conclusion

It is shown that a 1D completed scattering can be considered as an entanglement of two alternative sub-processes, transmission and reflection, macroscopically distinct at the final stage of scattering. For this quantum process, the (entangled) state of the whole ensemble of particles can be uniquely presented as a sum of two solutions to the Schrödinger equation to possess all needed information about the time evolution of either sub-process, at all stages of scattering.

We develop the Larmor timing procedure to allow measuring the average time spent by particles, of either subensemble, in the barrier region. This procedure shows that namely the dwell time gives the time spent, on the average, by a particle in the barrier region. As regards the group time, for scattering particles it cannot be measured with the Larmor clock, and, hence, it has no physical sense in this case.

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