Fluctuation electromagnetic interaction between a small rotating particle and a surface

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Abstract – We study the fluctuation electromagnetic interaction between a small neutral rotating particle and a polarizablesurface. The attraction force, friction torque and heating are produced by the particle polarization and the fluctuating near field of the surface. Closed analytical expressions are found for these quantities assuming that the particle and the surface are characterized by frequency-dependent polarizability, dielectric permittivity and different temperatures. It is shown that the stopping time of the rolling particle in the near field of the surface is much lower than the stopping time under uniform motion or rolling in vacuum space. This is of paramount importance in the development of NEMS where the corresponding interactions must be taken into account.

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Introduction. – The near-field fluctuation electromagnetic interaction between a particle in uniform motion and the surface is the subject of continuing interest demonstrating numerous new features relative to the simplest case of a resting particle. In particular, the origin of noncontact friction between the bodies in relative motion has been debated for a long time [1] (see also recent discussions [2–5]). Quite recently Manjavacas and Garcia de Abajo [6] considered the problem of a friction torque acting on a particle rotating in vacuum. The friction torque is produced by fluctuations of the vacuum electromagnetic field and the particle polarization. A related question are the friction and heating of a particle moving with constant velocity in the vacuum background [7,8].

The rotation of the particle leads to the frequency shifts in fluctuating fields involving the rotation frequency Ω. The attraction, friction and heating between rotating bodies will therefore be different in comparison with the static case. The aim of this paper is to examine the general problem of the fluctuation electromagnetic interaction between a spherical rotating particle and a surface, using the standard formalism of fluctuation electrodynamics [9].

Theory. – The system under consideration is shown in fig. 1. It consists of an isotropic particle at temperature $T_1$ placed in vacuum space at a distance $z_0$ from the surface at temperature $T_2$. The particle is assumed to be a point-like fluctuating nonrelativistic dipole rotating around its $z$-axis with the angular frequency $Ω$. In this case the following conditions are fulfilled ($ω_0$ is the characteristic absorption frequency of the particle, $c, k_B, c$, and $ℏ$ are the speed of light in vacuum, the Boltzmann constant and Planck’s constant, respectively): $R \ll \min \left\{ \frac{2\pi c}{ω_0}, \frac{2\pi c}{Ω}, \frac{2\pi ℏc}{k_B T_1}, \frac{2\pi ℏc}{k_B T_2} \right\}$.

Apart from the force of attraction to the surface $F_z$ and the rate of heat exchange $Q$, the interaction produces a frictional torque $M_z$ on the particle. All these quantities are calculated within the unified approach based on fluctuation electrodynamics that we have first proposed in [10] (for a review see [11]; for general relativistic results see in [2]). According to our method, we consider the contribution of all independent sources of electromagnetic fluctuations, namely $d^{sp}$ and $E^{sp}$ in this case, where the former and latter quantities are the fluctuating dipole moment of the particle and the fluctuating electromagnetic field of the surface.

Frictional torque. The torque is given by

$$M_z = \langle [d\vec{E}]_z \rangle = \langle [d^{sp} E^{sp}]_z \rangle + \langle [d^{in} E^{sp}]_z \rangle = M_z^{(1)} + M_z^{(2)},$$

where $\langle \rangle$ denotes the total quantum-statistical averaging, the superscripts “$sp$” and “$in$” denote spontaneous and induced quantities taken in the resting coordinate.
Using the Fourier transforms
\[
d^{sp}(t) = \int_{-\infty}^{+\infty} \frac{d\omega}{2\pi} d^{sp}(\omega) \exp(-i\omega t),
\]
\[
E^{in}(t) = \int \frac{d\omega}{2\pi} \frac{d^2k}{(2\pi)^2} E^{in}(\mathbf{k}, t) \exp(i(k_x x + k_y y - \omega t)),
\]
we take \( E^{sp}(t) \) at the particle location point \( r_0 = (0, 0, z_0) \). Inserting these transforms into (2) yields
\[
M^{(1)}_z = \int \frac{d\omega'}{2\pi} \frac{d^2k}{(2\pi)^2} \exp(-i(\omega + \omega') t)
\cdot \langle d^{sp}(\omega')^*E^{in}(\mathbf{k}, z_0) - d^{sp}(\omega')E^{in}(\mathbf{k}, z_0) \rangle.
\] (3)
The Fourier components \( E^{in}(\mathbf{k}, z) \) are expressed through the Fourier components of the induced potential of the surface by the equation \( E^{in} = -\nabla \varphi^{in} \):
\[
\varphi^{in}(\mathbf{k}, z) = \frac{2\pi}{k} \Delta(\omega) \exp(-k(z + z_0))
\cdot [ik_x d^{sp}_{zz}(\omega) + ik_y d^{sp}_{yy}(\omega) + kd^{sp}_{ww}(\omega)],
\]
\[
\Delta(\omega) = (\varepsilon(\omega) - 1) / (\varepsilon(\omega) + 1),
\]
\[
E^{in}(\mathbf{k}, z) = -ik_x \varphi^{in}(\mathbf{k}, z),
\]
\[
E^{in}_{yy}(\mathbf{k}, z) = -ik_y \varphi^{in}(\mathbf{k}, z),
\]
\[
E^{in}_{zz}(\mathbf{k}, z) = k \varphi^{in}(\mathbf{k}, z),
\] (4)
where \( \varepsilon(\omega) \) is the dielectric permittivity of the surface and \( k = |\mathbf{k}| = (k_x^2 + k_y^2 + k_z^2)^{1/2} \). The fluctuation-dissipation theorem for the dipole operator \( d^{i\alpha^\prime} \) is given in the rotating coordinate system \( \Sigma' \) of the particle,
\[
\langle d^{i\alpha^\prime \prime}(\omega')^*d^{i\alpha^\prime}(\omega) \rangle = 2\pi \delta_{i,k} \delta(\omega + \omega') \hbar \alpha^\prime(\omega) \coth \frac{\hbar \omega}{2kBT_1},
\] (6)
where \( T_1 \) and \( \alpha(\omega) \) are the particle temperature and polarizability in the coordinate system \( \Sigma' \) (\( \alpha^\prime(\omega) \) is the corresponding imaginary component). In order to calculate the correlator in eq. (3) using (6), we must express the Fourier transforms of \( d^{sp} \) in \( \Sigma \) through the Fourier transforms of \( d^{sp} \) in \( \Sigma' \). Any vector quantity \( \mathbf{A} \) in \( \Sigma \) is related to \( \mathbf{A}' \) in \( \Sigma' \) by the equation \( \mathbf{A} = \mathbf{B} \mathbf{A}' \), where \( \mathbf{B} \) is the rotation matrix
\[
\mathbf{B} = \begin{pmatrix}
\cos \Omega & -\sin \Omega & 0 \\
\sin \Omega & \cos \Omega & 0 \\
0 & 0 & 1
\end{pmatrix}.
\] (7)
Using the equation \( d^{sp} = \mathbf{B} d^{sp} \), we then take the Fourier transforms of \( d^{sp}(t) \) and substitute them into (4). Finally, using (3)–(6) yields (where \( \omega^\pm = \omega \pm \Omega \))
\[
M^{(1)}_z = -\frac{\hbar}{4\pi z_0^2} \int_0^{\infty} d\omega \Delta'(\omega) \left[ \alpha'(\omega^-) \coth \frac{\hbar \omega^-}{2kBT_1} - \alpha'(\omega^+) \coth \frac{\hbar \omega^+}{2kBT_1} \right].
\] (8)

Now we pass to the calculation of \( M^{(2)}_z \) in eq. (1) that is given by
\[
M^{(2)}_z = \langle d^{i\alpha^\prime \prime \prime \prime}(\omega')^*E^{sp}(\mathbf{r}_0, \omega') \rangle.
\] (9)
Substituting the Fourier expansions of \( d^{in}(t) \) and \( E^{sp}(t) \) in (9) yields
\[
M^{(2)}_z = \int \frac{d\omega'}{2\pi} \frac{d\omega}{2\pi} \exp(-i(\omega + \omega') t)
\times \langle d^{i\alpha^\prime \prime \prime \prime}(\omega')^*E^{in}_{yy}(\mathbf{r}_0, \omega') \rangle.
\] (10)
The induced components of \( d^{in}(t) \) in \( \Sigma \) and \( d^{in}(t) \) in \( \Sigma' \) are related by \( d^{in} = \mathbf{B}d^{in} \). Taking the Fourier transform of this equation we must take into account that
\[
d^{in}(\omega) = \alpha(\omega)E^{sp}(\mathbf{r}_0, \omega), \quad i = x, y, z.
\] (11)
To express \( E^{sp}(\mathbf{r}_0, \omega) \) through \( E^{sp}(\mathbf{r}_0, \omega) \), we must take the Fourier transform of \( E^{sp}(\mathbf{r}_0, \mathbf{t}) = \mathbf{B}^{-1}E^{sp}(\mathbf{r}_0, \mathbf{t}) \). Substituting the calculated Fourier components \( d^{i\alpha^\prime \prime \prime \prime}(\omega) \) and \( E^{sp}(\mathbf{r}_0, \omega) \) in (10) we obtain
\[
M^{(2)}_z = \frac{i}{2} \int \frac{d\omega'}{2\pi} \frac{d\omega}{2\pi} \exp(-i(\omega + \omega') t)(\alpha(\omega^+) - \alpha(\omega^-))
\cdot \left[ \langle E^{sp}_{yy}(\omega)E^{sp}(\omega') \rangle + \langle E^{sp}_{yy}(\omega)E^{sp}(\omega') \rangle \right].
\] (12)
Correlators of the electric-field fluctuations in eq. (12) are expressed through an imaginary part of the retarded
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Green function. In the configuration under consideration these are given by [11] (no summation over i)

\[
\langle E_i^{sp}(\omega) E_i^{sp}(\omega') \rangle = 2\pi \delta(\omega + \omega') (E_i^{sp})_i^2, \tag{13}
\]

\[
i = x, y, z,
\]

\[
(E_i^{sp})_i^2 = \int \frac{d^2 k}{(2\pi)^2} (E_i^{sp})_{i,k}^2, \tag{14}
\]

\[
(E_x^{sp})_{x,k}^2 = \hbar \cot \frac{\omega}{2k_BT} \Delta''(\omega) \frac{2\pi}{k} k^2 \exp(-2kz),
\]

\[
(E_y^{sp})_{y,k}^2 = \hbar \cot \frac{\omega}{2k_BT} \Delta''(\omega) \frac{2\pi}{k} k^2 \exp(-2kz),
\]

\[
(E_z^{sp})_{z,k}^2 = \hbar \cot \frac{\omega}{2k_BT} \Delta''(\omega) \frac{2\pi}{k} k^2 \exp(-2kz),
\]

where \(T_2\) is the surface temperature. The above equations define a spectral density of the fluctuating electromagnetic field at the particle location point \(r_0 = (0, 0, z_0)\). Substituting them in (12) and making use of simple transformations yields

\[
M^{(2)}_z = -\frac{\hbar}{4\pi z_0^3} \int_0^\infty d\omega \Delta''(\omega) \left[ \alpha''(\omega^+) \cot \frac{\omega}{2k_BT} - \alpha''(\omega^-) \cot \frac{\omega}{2k_BT} \right]. \tag{15}
\]

Finally, the torque resulting from the sum of (8) and (15) reduces to

\[
M_z = -\frac{\hbar}{4\pi z_0^3} \int_0^\infty d\omega \Delta''(\omega) \left\{ \alpha''(\omega^-) \left[ \cot \frac{\omega}{2k_BT} - \coth \frac{\omega}{2k_BT} \right] - \alpha''(\omega^+) \left[ \coth \frac{\omega}{2k_BT} - \coth \frac{\omega}{2k_BT} \right] \right\}. \tag{16}
\]

Transforming the integral limits in eq. (16) we obtain a more compact expression,

\[
M_z = \frac{\hbar}{4\pi z_0^3} \int_{-\infty}^{\infty} d\omega \Delta''(\omega) \alpha''(\omega^+) \left[ \coth \frac{\omega}{2k_BT_1} - \coth \frac{\omega}{2k_BT_2} \right]. \tag{17}
\]

In the limit \(\Omega \ll \omega_0\) and \(T_1 = T_2 = T\), the dominating term in eq. (17) which is linear in \(\Omega\) takes the form

\[
M_z = -\hbar \Omega \int_{-\infty}^{\infty} d\omega \Delta''(\omega) \alpha''(\omega) \left( -\frac{\partial}{\partial \omega} \right) \coth \frac{\omega}{2k_BT}. \tag{18}
\]

Attraction force and heating rate of the particle. The particle-surface attraction force and the rate of heat exchange can be calculated quite analogously to the calculation of \(M_z\). The starting expressions are given by

\[
F_z = \langle \partial_z (d^{sp}E^n) \rangle + \langle \partial_z (d^{sp}E^{sp}) \rangle, \tag{19}
\]

\[
\dot{Q} = \langle \dot{d}^{sp}E^{sp} \rangle + \langle \dot{d}^{sp}E^{sp} \rangle. \tag{20}
\]

The resulting equations have the form

\[
F_z = -\frac{3\hbar}{16\pi z_0^3} \int_{-\infty}^{\infty} d\omega \left\{ \Delta''(\omega) \alpha''(\omega) \coth \frac{\omega}{2k_BT} + \Delta''(\omega) \alpha''(\omega^+) \coth \frac{\omega}{2k_BT} + \Delta''(\omega) \alpha''(\omega^-) \coth \frac{\omega}{2k_BT} \right\} \tag{21}
\]

\[
\dot{Q} = \frac{\hbar}{8\pi z_0^3} \int_{-\infty}^{\infty} d\omega \left\{ \Delta''(\omega) \alpha''(\omega) \left[ \coth \frac{\omega}{2k_BT_1} - \coth \frac{\omega}{2k_BT_2} \right] + \Delta''(\omega) \alpha''(\omega^+) \left[ \coth \frac{\omega}{2k_BT_1} - \coth \frac{\omega}{2k_BT_2} \right] \right\}. \tag{22}
\]

Evaluation of stopping times of nanoparticles. First, it is interesting to compare the stopping time of a small rolling particle with that under uniform motion with constant velocity \(V\) parallel to the surface. Assuming the temperatures of the particle and the surface to be \(T\), the stopping force is given by [11,12]

\[
F_z = -\frac{3\hbar V}{8\pi z_0^3} \int_{-\infty}^{\infty} d\omega \Delta''(\omega) \alpha''(\omega) \left( -\frac{\partial}{\partial \omega} \right) \coth \frac{\omega}{2k_BT}. \tag{23}
\]

Using (18), (23) and Newton’s second law, the corresponding stopping times (assuming that the velocity diminishes by \(e\) times) are given by \(\tau_\Omega = \frac{32\pi^2}{3\pi^2} \rho R^3 z_0^3\) and \(\tau_V = \frac{32\pi^2}{3\pi^2} \rho R^3 z_0^3\), where \(R\) is the particle radius, \(\rho\) is the material density (the particle and the surface), and \(J\) is the frequency integral in (18) and (23). We also took into account the inertia moment of the spherical particle which is equal to \(8\pi\rho R^5/15\). Using the above expressions we obtain \(\tau_\Omega/\tau_V = \frac{3}{4} \left( \frac{z_0}{\nu} \right)^2\). Since typically \(R \ll z_0\), this implies that \(\tau_\Omega/\tau_V \ll 1\) and the decay time of rolling is much shorter than the decay time of uniform motion.

Second, it is interesting to compare \(\tau_\Omega\) near the surface and in the vacuum background (at equal temperature \(T\) of the particle and the background radiation). In the latter case the friction torque is given by formula (18) from [6] which can be written similar to (18) and (23),

\[
M_z^{(vac)} = -\frac{2\hbar}{3\pi e^3} \int_{-\infty}^{\infty} d\omega \omega^3 \alpha''(\omega) \left( -\frac{\partial}{\partial \omega} \right) \coth \frac{\omega}{2k_BT}. \tag{24}
\]

Using eq. (24), the stopping time is estimated to be

\[
\tau_\Omega^{(vac)} = \frac{4\pi^2}{5kJ_{vac}} \rho R^3 c^3,\]

where \(J_{vac}\) is the frequency integral.
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Comparing the structure of $J_{\text{vac}}$ and $J$, we can conclude that $J_{\text{vac}}/J \approx \omega W/\Delta''(\omega W)$ (with $\omega W = k_B T/h$ being the Wien frequency). Therefore, ignoring a numerical factor we obtain $\tau_{\Omega}/\tau_{\Omega}^{\text{(vac)}} \approx (\omega W z_0/c)^3/\Delta''(\omega W) \ll 1$ at room temperature and $z_0$ in nanometer range.

A more accurate numerical estimation of the stopping times is shown in fig. 2 in the particular case of a SiO$_2$ particle with $R=1\text{nm}$ near a quartz surface at $T=300\text{K}$ (or in the vacuum background with the same temperature). The dielectric function was approximated in a form similar to [13]

$$
\varepsilon(\omega) = \varepsilon_{\infty} + \sum_{j=1}^{2} \frac{\sigma_j}{\omega_0^2 - \omega^2 - i\omega \gamma_j},
$$

$$
\varepsilon_0 = 2.0014, \quad \sigma_1 = 0.00193(\varepsilon\text{V}), \quad \omega_{0,1} = 0.057\varepsilon\text{V},
$$

$$
\gamma_1 = 0.0021\text{eV}, \quad \sigma_2 = 0.0102(\varepsilon\text{V})^2, \quad \omega_{0,2} = 0.133\varepsilon\text{V},
$$

$$
\gamma_2 = 0.0055\varepsilon\text{V}.
$$

Very similar estimations are obtained in the case of material properties corresponding to SiC. As we can see from fig. 2, the stopping time for rolling in the near field of the surface is much shorter than in free vacuum. For metallic particles, the dominating contribution in eq. (24) results from the magnetic polarizability of the particle [6], that can be higher by 1–2 orders of magnitude than the contribution from electric polarizability. But generally, the estimation $\tau_{\Omega}/\tau_{\Omega}^{\text{(vac)}} \ll 1$ proves to be valid in this case, as well.

Conclusions. — Using a general background of the fluctuation electromagnetic theory, we have obtained closed nonrelativistic expressions for the friction torque, attraction force and heating rate of a small spherical particle rotating in the near field of the surface. Material properties of the particle and the surface are characterized by frequency-dependent polarizability and dielectric permittivity. The temperatures of the particle and the surface are assumed to be arbitrary. It is worth noting that the anisotropy of the particle polarizability has no appreciable physical importance in this case and may only change numerical factors in the quantities under study. Apparently, it is not difficult to generalize the obtained formulas in this case.

Assuming isothermal conditions, we have compared the stopping times of spherical SiO$_2$ particles corresponding to the rolling and uniform motion near a quartz surface and those for rolling near the surface and in the vacuum background. In both of the cases (at room and lower temperatures) the stopping times for rolling in the near field of the surface turn out to be smaller by 5 to 9 orders of magnitude than in vacuum and in uniform motion near the surface.

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![Fig. 2: Stopping times for SiO$_2$ particles ($R=1\text{nm}$) near a quartz surface at 300K. The solid and dashed curves correspond to rolling and uniform motion, the dash-dotted curve corresponds to rolling in free vacuum at 300K.](image)