Effective Higgs Lagrangian and Constraints on Higgs Couplings

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Abstract

Probing the properties of the discovered Higgs boson may tell us whether or not it is the particle predicted by the Standard Model. We review a model-independent approach to parametrize deviations from the Standard Model in an effective theory framework. We start with a general dimension-6 effective Lagrangian including both CP-even and CP-odd operators. Requiring that the operators do not introduce power divergences in the oblique parameters, we reduce the number of independent couplings of the theory. We then use this framework to put updated constraints on the effective couplings of the Higgs to matter, using up-to-date Higgs rates data from ATLAS and CMS and electroweak precision data from LEP, SLC and Tevatron. We show that the current data is able to significantly constrain CP-even and some CP-odd operators of the effective Lagrangian.
Introduction

The recent experimental confirmation of the existence of a scalar particle [1] with mass $m_h$ of about 126 GeV and production cross-sections and decay rates compatible with the ones of the Standard Model (SM) Higgs boson, triggered many studies of its properties and put constraints on the SM and on some Beyond-the-Standard Model (BSM) theories. Many questions remain open concerning the fundamental nature of this boson and the dynamics of the electroweak symmetry breaking (EWSB), and BSM models are called to the rescue to attempt to answer to these questions. If we want to be able to know whether this discovered scalar particle is the SM or a SM-like Higgs boson as predicted by those BSM models, probing its properties with precision is mandatory. For studying its properties and their possible deviations from their SM predictions, two main strategies are available: either perform a study in the context of a specified model or use a model-independent approach via an effective theory framework, employed in this paper. The Wilson coefficients of the effective operators parametrize in a continuous way the possible deviations of the Higgs couplings from their SM values.

The purpose of this paper is twofold: we derive the usual phenomenological Higgs Lagrangian by using dimension-6 operators and conditions from oblique parameters, and we update the constraints on the parameters of this Lagrangian by proceeding to a global fit using a combination of up-to-date Higgs signal rates from the ATLAS and CMS experiments with electroweak (EW) precision measurements from LEP, SLC and Tevatron. Latest results on $t\bar{t}H$ Higgs production from ATLAS and CMS are included. As such, this paper is a continuation of many previous studies, amongst which some that mainly targeted only CP-conserving couplings [2, 3, 4] or others considering CP-conserving as well as CP-violating couplings [5, 6]. Albeit LHC data strongly suggests that the observed Higgs boson is in excellent agreement with the SM prediction and it indeed possesses the required quantum numbers for a scalar particle [7], we will nevertheless consider the possibility that this Higgs boson can be coupled with both CP-conserving and CP-violating couplings to bosons and fermions. We will observe that the CP-even and some of the CP-odd couplings are significantly constrained using the existing experimental data.

This paper is organised as follows. In the first section we review the effective Lagrangian introduced to study the departures of the Higgs couplings from their SM values. We derive it starting from a $SU(3)_C \times SU(2)_L \times U(1)_Y$-invariant dimension-6 Lagrangian for a weak doublet $H$. For the purpose of the global fit the effective Lagrangian contains a too large number of parameters, which is reduced in the next section by using extra relations amongst them. Those relations are of two kinds: the first ones come from the nature of the dimension-6 Lagrangian used as the starting point and the second ones are needed to remove power divergences to the oblique parameters. We find that some of their combinations are strongly constrained. A total of 7 independent effective
couplings in the CP-conserving sector and 6 independent ones in the CP-violating sector is obtained. LHC experiments provide relative Higgs signal strengths $\hat{\mu}$ in various production and decay channels. Comparing the theoretical signal strengths obtained in our effective framework with the measured ones, allow us to constrain the parameters of the effective Lagrangian. In the last sections we fit the CP-even and CP-odd couplings to the latest Higgs data from ATLAS and CMS, combined with EW precision measurements, and we discuss our results. Details of some computations are relegated to the corresponding appendices.

1 Effective Lagrangian

Several assumptions on the physics of the involved Higgs boson are made, amongst those the first and critical one being that new-physics (NP) should manifest at an energy scale much higher than the EW scale. The NP fields are integrated-out and give rise to higher dimension effective non-renormalizable operators in the expansion of the effective Lagrangian, inducing deviations of the leading-order (LO) Higgs couplings from their SM values. In this work we also require baryon and lepton numbers (BL) conservation and the absence of any source of flavour violation.

We assume that the Higgs boson $h$ is part of the Higgs field $H$ which transforms in the $(1, 2, \frac{1}{2})$ representation of the Standard Model $SU(3)_C \times SU(2)_L \times U(1)_Y$ gauge group and acquires an expectation value $v$. The effective Lagrangian is then expanded:

$$L_{\text{eff}} = L_{\text{SM}} + L_{D=5} + L_{D=6} + \ldots$$

(1.1)

where each part consists of gauge-invariant local operators of canonical dimension $D$ that are made uniquely of SM fields. The leading term in this expansion is the SM Lagrangian which contains operators up to dimension 4. At the level of dimension-5 operators there is only one respecting the SM gauge symmetry (Weinberg operator) which gives masses to neutrinos after EWSB and does not have any impact on the Higgs phenomenology; however it violates lepton number conservation so it will be removed from our study. The part of interest consists in the dimension-6 operators. Requiring BL conservation, it is known [8] that the dimension-6 basis contains 59 operators\footnote{Although their original list was known since the 1980's [9] (assuming BL conservation, 80 operators were found), it was pointed out by many analyses that some of them were redundant, and a complete minimal list of 59 operators was finally given by Grzadkowski et al. [8] in 2010.} for a single generation\footnote{This is the number of operators when considering only one generation; otherwise their number increase to 2499, see the review [10] for the details of counting.}. A choice of operator basis needs to be made because the operators can be redefined into other ones using equations of motion. We use here the basis employed by Contino \textit{et al.} [11].
In this framework the dimension-6 part of the effective Lagrangian can be written as:

\[ \mathcal{L}_{D=6} = \mathcal{L}_{CPC} + \mathcal{L}_{CPV} \] 

where the CP-conserving part is given by: \( \mathcal{L}_{CPC} = \Delta \mathcal{L}_{SILH} + \Delta \mathcal{L}_{F_1} + \Delta \mathcal{L}_{F_2} + \Delta \mathcal{L}_{4F} + \Delta \mathcal{L}_{Gauge} \) using the notations of [11] (see also their Equations 2.1, 2.2, 2.3, 2.4 and 2.6): \( \Delta \mathcal{L}_{SILH} \) is the Strongly-Interacting Light Higgs doublet Lagrangian (SILH) introduced by Giudice et al. [12], \( \Delta \mathcal{L}_{F_1} \) contains the 2-fermion vertex operators, and \( \Delta \mathcal{L}_{F_2} \) contains the 2-fermion dipole operators. \( \Delta \mathcal{L}_{4F} \) possesses twenty-two 4-fermion baryon-number-conserving operators and \( \Delta \mathcal{L}_{Gauge} \) contains gauge-boson self-interaction operators, which affect the gauge-boson propagators and self-interactions but they do not have any effect on Higgs physics [11]. The CP-violating part \( \mathcal{L}_{CPV} \) contains all the possible dimension-6 CP-odd operators (Equation C.96 of [11]).

Given the current sensitivity of the experiments, operators of dimension greater than 6 are not relevant. EW precision measurements from LEP strongly constrain the couplings of SM fermions to EW gauge bosons, which are modified in the presence of the 2-fermion operators \( \Delta \mathcal{L}_{F_1} \), so that there is not much room to affect LHC Higgs phenomenology with the current data. The same remark also applies for the 2-fermion dipole operators \( \Delta \mathcal{L}_{F_2} \) which contribute to electric and magnetic dipole moments (EDM and MDM), and also contribute to the 3-body Higgs decay: they are further suppressed and thus can be ignored. The \( \Delta \mathcal{L}_{F_1} \) and \( \Delta \mathcal{L}_{Gauge} \) parts of the Lagrangian will be ignored because they do not involve any Higgs boson; the gauge part modifying only the triple and quartic gauge boson couplings and the oblique parameters. The CP-violating part \( \mathcal{L}_{CPV} \) contains also gauge-boson self-interaction operators that are not taken into account here for the same reasons as \( \Delta \mathcal{L}_{Gauge} \). Finally the SILH Lagrangian contains a Higgs self-interaction operator of the form \( (H^\dagger H)^3 \) that is also removed in this work because current experimental precision is not sensitive enough to modifications of the Higgs self-couplings.

Considering all of these remarks the relevant dimension-6 Lagrangian can be rewritten as:

\[
\mathcal{L}_{CPC} = \frac{\bar{c}_H}{2v^2} \partial^\mu (H^\dagger H) \partial_\mu (H^\dagger H) + \frac{\bar{c}_T}{2v^2} \left( \left( H^\dagger \overleftrightarrow{D_\mu} H \right) \left( H^\dagger \overleftrightarrow{D_\mu} H \right) \right) \\
+ \frac{\bar{c}_W}{2m_W^2} \left( D^\gamma \omega \left( D^\mu H \right) \left( D^\nu H \right) \right) + \frac{\bar{c}_B}{2m_W^2} \left( D^\gamma B \right) \\
+ \frac{i \bar{c}_H W}{m_W^2} \left( D^\mu H \right) \left( D^\nu H \right) \left( D^\rho H \right) \\
+ \frac{i \bar{c}_B B}{m_W^2} \left( D^\mu H \right) \left( D^\nu H \right) \left( D^\rho H \right) + \bar{c}_\gamma g^2 \left( H^\dagger H \right) \left( B_\mu B_\mu \right) + \frac{\bar{c}_g g_2^2}{m_W^2} \left( H^\dagger H \right) G^a_{\mu \nu} G^{a \mu \nu} \tag{1.3}
\]
\[ \mathcal{L}_{CPV} = \frac{i\tilde{c}_{HW}}{m_W} (D^\mu H)^\dagger \sigma^i (D_i^\nu H) \tilde{W}^i_{\mu\nu} + \frac{i\tilde{c}_{HB}}{m_W} (D^\mu H)^\dagger (D_i^\nu H) \tilde{B}^i_{\mu\nu} \]
\[ + \frac{\tilde{c}_s g_s^2}{m_W} (H^\dagger H) B^i_{\mu\nu} \tilde{B}^i_{\mu\nu} + \frac{\tilde{c}_a g_s^2}{m_W} (H^\dagger H) G^a_{\mu\nu} \tilde{G}^{a\mu\nu} \]  

(1.4)

where we used the field-strength tensors \( F_{\mu\nu} \) and their duals \( \tilde{F}_{\mu\nu} \equiv \frac{1}{2} \epsilon_{\mu\nu\rho\sigma} F^{\rho\sigma} \), and defined \( H^c \equiv i \sigma_2 H^* \) and the anti-Hermitian derivative \( A^\dagger \tilde{c}_B B \equiv A^\dagger (D_\mu B) - (D_\mu A)^\dagger B \). \( g_S \) is the strong coupling constant of \( SU(3)_C \); \( g \) and \( g' \) are the \( SU(2)_L \) and \( U(1)_Y \) coupling constants, respectively.

It should be noted that we adopt the normalization convention of [11] instead of the one of [12] or the usual one: \( \frac{\tilde{c}_l}{M^2} \mathcal{O}_{D=6} \) where \( M \) is the new-physics scale, because it is more convenient for our computations. A naive power counting gives that the \( \tilde{c}_l \) coefficients are of order \( \mathcal{O} \left( \frac{m_W^2}{M^2} \right) \) or \( \mathcal{O} \left( \frac{v^2}{M^2} \right) \). Therefore those \( \tilde{c}_l \) coefficients are generally small as they describe little deviations from the SM since that experimentally, the data is already well described by the SM.

We suppose that the Higgs couples separately to each type of charged fermions (up and down-type quarks, charged leptons), so that: \( \tilde{c}_u = \tilde{c}_c = \tilde{c}_t \equiv \tilde{c}_a, \tilde{c}_d = \tilde{c}_u = \tilde{c}_b \equiv \tilde{c}_d \) and \( \tilde{c}_c = \tilde{c}_\mu = \tilde{c}_\tau \equiv \tilde{c}_t \). Furthermore these coefficients are in full generality complex-valued so that their real part (resp. their imaginary part) give rise to CP-conserving (resp. CP-violating) Higgs couplings to fermions.

To finish the build-up of the Higgs effective Lagrangian we expand \( \mathcal{L}_{eff} = \mathcal{L}_{SM} + \mathcal{L}_{D=6} \) after EWSB in the Higgs field \( h \) around its vacuum expectation value, in unitary gauge, writing: \( H \rightarrow \frac{v}{\sqrt{2}} (0; 1 + \frac{h}{v})^T \). The \( \tilde{c}_H \) term of the dimension-6 Lagrangian introduces a finite wave-function renormalization to the Higgs field, which needs to be rescaled to bring its kinetic term back into canonical normalization:

\[ h \rightarrow \frac{h}{\sqrt{1 + \tilde{c}_H}} \approx h \left( 1 - \frac{\tilde{c}_H}{2} \right) \]  

(1.5)

which effect is to give a universal resizing of all the partial Higgs decay widths.

Finally, the effective Lagrangian can be written as: \( \mathcal{L}_{eff} = \frac{1}{2} (\partial^\mu h)^2 - \frac{m_H^2}{2} h^2 + \mathcal{L}_0 + \mathcal{L}_1 + \cdots \) where the Lagrangian is expanded in powers of the physical Higgs field \( h \); we keep only linear Higgs interactions for the same reasons as explained before. The Higgs-independent part is given by:

\[ \mathcal{L}_0 = - \frac{1}{4} V_{\mu\nu} V^{\mu\nu} + i \sum (\overline{L}_L \not\! D L_L + \overline{L}_R \not\! D L_R) + \frac{m_L^2}{2} V_{\mu} V^\mu - \tilde{c}_T m_Z^2 Z_\mu Z^\mu \]
\[ + \tilde{c}_B Z^\mu \partial^\mu (\tan^2 \theta_W Z_{\mu\nu} - \tan \theta_W \gamma_{\mu\nu}) + \text{CP-Odd} \]
\[ + 2 \tilde{c}_\gamma \tan^2 \theta_W (s_w^2 Z_{\mu\nu} Z^{\mu\nu} + c_w^2 \gamma_{\mu\nu} \gamma^{\mu\nu} - 2 s_w c_w Z_{\mu\nu} \gamma_{\mu\nu}) + \text{CP-Odd} \]
\[ + 2 \tilde{c}_a g_s^2 G_{\mu\nu} G^{\mu\nu} + \text{CP-Odd} + \tilde{c}_{HB} \times 3\text{-boson} \]
\[ + \tilde{c}_W \left( \tan \theta_W Z^\mu \partial^\nu \gamma_{\mu\nu} + Z^\mu \partial^\nu Z_{\mu\nu} + W^\mu D^\nu W_{\mu\nu}^{\dagger} + \text{h.c.} \right) + \text{CP-Odd} \]  

(1.6)

where "CP-Odd" holds for the CP-odd-equivalent operators \( (V_{1\mu\nu} V_2^{\mu\nu} \rightarrow V_{1\mu\nu} \tilde{V}_2^{\mu\nu}) \), and "3-boson"
holds for operators containing a product of 3 gauge bosons or more. The linear part is:

\[
\mathcal{L}_1 = \frac{h}{v} \left[ 2c_W m_W^2 W^\mu W_\mu + c_Z m_Z^2 Z_\mu Z^\mu - \sum_{f=u,d,l} m_f \mathcal{F} (c_f + i \bar{c}_f \gamma_5) f \right]
\]

where \( s_w = \sin \theta_W \) and \( c_w = \cos \theta_W \) are the sine and cosine of the weak angle. No \( \gamma^\mu \partial^\nu \gamma_{\mu\nu} \) or \( \gamma^\mu \partial^\nu Z_{\mu\nu} \) terms are present in \( \mathcal{L}_1 \) because they break \( U(1)_{EM} \) symmetry, and also no CP-odd term \( \kappa D^\nu \bar{V}_{\mu\nu} \) because the Bianchi identity for the field-strength tensor \( V_{\mu\nu} \) cancels it [11].

The dictionary between the couplings \( \bar{c}_i \) and \( c_i \) of the Lagrangians 1.3, 1.4 and 1.7 reads:

- \( c_W = 1 - \frac{\bar{c}_H}{2} \) ; \( c_Z = 1 - \frac{\bar{c}_H}{2} - \bar{c}_T \)
- \( c_f = 1 - \frac{\bar{c}_H}{2} + \text{Re}(\bar{c}_f) \) ; \( \bar{c}_f = \text{Im}(\bar{c}_f) \) where \( f = u, d, l \)
- \( c_{WW} = 4\bar{c}_{HW} \) ; \( \bar{c}_{WW} = 4\bar{c}_{HW} \)
- \( c_{ZZ} = c_{WW} + 4 \left( \frac{s_w^2}{c_w^2} \bar{c}_{HB} - 4 \frac{s_w^4}{c_w^4} \bar{c}_\gamma \right) \) ; \( \bar{c}_{ZZ} = \bar{c}_{WW} + 4 \left( \frac{s_w^2}{c_w^2} \bar{c}_{HB} - 4 \frac{s_w^4}{c_w^4} \bar{c}_\gamma \right) \)
- \( c_{\gamma\gamma} = -16s_w^2 \bar{c}_\gamma \) ; \( \bar{c}_{\gamma\gamma} = -16s_w^2 \bar{c}_\gamma \)
- \( c_{Z\gamma} = 2 \frac{s_w}{c_w} \left( \bar{c}_{HW} - \bar{c}_{HB} + 8s_w \bar{c}_\gamma \right) \) ; \( \bar{c}_{Z\gamma} = 2 \frac{s_w}{c_w} \left( \bar{c}_{HW} - \bar{c}_{HB} + 8s_w \bar{c}_\gamma \right) \)
- \( c_{gg} = 16 \frac{g_5^2}{g^2} \bar{c}_g \) ; \( \bar{c}_{gg} = 16 \frac{g_5^2}{g^2} \bar{c}_g \)

and:

- \( \kappa_{Z\gamma} = -2 \frac{s_w}{c_w} \left( \bar{c}_{HW} + \bar{c}_W - \bar{c}_{HB} - \bar{c}_B \right) \),
- \( \kappa_{ZZ} = -2 \left( \bar{c}_{HW} + \bar{c}_W + \frac{s_w^2}{c_w^2} \bar{c}_{HB} + \frac{s_w^2}{c_w^2} \bar{c}_B \right) \),
- \( \kappa_{WW} = -2 \left( \bar{c}_{HW} + \bar{c}_W \right) \).

The SM Lagrangian has: \( c_W = c_Z = c_f = 1 \) and all the \( c_{ij} = 0 \), \( \bar{c}_i = 0 \) and \( \kappa_{ij} = 0 \). Furthermore, not all of the parameters of 1.7 are independent, because the SILH Lagrangian introduces some "accidental" custodial symmetry that translates into the following identities:

- \( c_{WW} = c_w^2 c_{ZZ} + 2c_w s_w c_{Z\gamma} + s_w^2 c_{\gamma\gamma} \),
- \( \bar{c}_{WW} = c_w^2 \bar{c}_{ZZ} + 2c_w s_w \bar{c}_{Z\gamma} + s_w^2 \bar{c}_{\gamma\gamma} \),
- \( \kappa_{WW} = c_w^2 \kappa_{ZZ} + c_w s_w \kappa_{Z\gamma} \).
2 Constraints on the Parameters – Electroweak Corrections

We want to obtain meaningful constraints on the effective Higgs couplings using publicly available Higgs rates data, mainly from the ATLAS and CMS collaborations. However, Higgs rates have up to now a limited power of discrimination between different tensor structures of the Higgs couplings to vector bosons, so we cannot use this method to put constraints on the couplings. Another way would be to further reduce their number. A way to proceed is to use existing EW constraints on electroweak precision observables and, in particular, requiring that all the power divergences in the oblique corrections vanish.

Many BSM models satisfy the following criteria about the nature of the new physics: new physics comes from heavy particles which intrinsic scale $M_{NP}$ is much larger than the EW scale $M_{EW}$ (for example, of the order of $M_{GUT}$ or $M_{Planck}$), no new EW-like gauge bosons exist at the EW scale (apart from $\gamma$, $Z^0$ and $W^\pm$) so that $SU(2)_L \times U(1)_Y$ is still the EW gauge group [13, 14, 3], and new physics couplings to light fermions are much smaller than those to gauge bosons. When those criteria are met, the dominant corrections due to new physics are the ones to the propagation of gauge bosons exchanged in 2-fermion scattering processes. These are the so-called electroweak oblique corrections conveniently parametrized by the Peskin-Takeuchi $S, T, U$ parameters [13].

Under the previous assumptions the gauge-boson two-point functions can be expanded around zero momentum in powers of $p^2$:

$$\Pi_{\mu\nu}(p^2) = g_{\mu\nu} \left( \Pi_{V_1V_2}^{(0)}(p^2) + \frac{p_\mu p_\nu}{m^2_W} \cdots \right)$$

where the $V_i = 1, 2, 3, B$ label the $SU(2)_L \times U(1)_Y$ gauge bosons $W_{1,2,3}$ and $B$, and $\Pi_{V_1V_2}^{(0)}(0) \equiv \frac{1}{k!} \left( \frac{\partial}{\partial p^2} \right)^k \Pi_{V_1V_2}(0)$. Now, denoting $\delta \Pi$ the shift of the corresponding two-point function from the SM value, the Peskin-Takeuchi parameters are defined as (in terms of the $W_{1,2,3}$ and $B$, or $W^\pm, Z^0$ and $\gamma$ bosons):

$$\alpha S = -4 s_w c_w \delta \Pi_{3B}^{(2)} = 4 s_w^2 c_w^2 \left( \delta \Pi_{ZZ}^{(2)} - \delta \Pi_{\gamma\gamma}^{(2)} - \frac{c_w^2 - s_w^2}{s_w c_w} \delta \Pi_{Z\gamma}^{(2)} \right),$$  

$$\alpha T = \frac{\delta \Pi_{11}^{(0)} - \delta \Pi_{33}^{(0)}}{m_W^2} = \frac{\delta \Pi_{WW}^{(0)}}{m_W^2} - \frac{c_w^2 \delta \Pi_{ZZ}^{(0)}}{m_W^2} \cdots$$

$$\alpha U = 4 s_w^2 \left( \delta \Pi_{11}^{(2)} - \delta \Pi_{33}^{(2)} \right) = 4 s_w^2 \left( \delta \Pi_{WW}^{(2)} - c_w^2 \delta \Pi_{ZZ}^{(2)} - s_w^2 \delta \Pi_{\gamma\gamma}^{(2)} - 2 c_w s_w \delta \Pi_{Z\gamma}^{(2)} \right).$$
2.1 Tree-level constraints

Using $\mathcal{L}_0$ (Eq. 1.6), the Lagrangian part that does not depend on the physical Higgs field $h$, we find:

$$\alpha S = 2s_w^2(\bar{c}_B + \bar{c}_W), \quad \alpha T = \bar{c}_T, \quad \alpha U = 0.$$  \hspace{2cm} (2.5)

At fixed $U = 0$, the up-to-date experimental limits [15] for the $S$ and $T$ parameters, determined from a fit for a reference Standard Model with $m_{t,\text{ref}} = 173$ GeV and $M_{H,\text{ref}} = 126$ GeV, are:

$$S = 0.05 \pm 0.09, \quad T = 0.08 \pm 0.07.$$  \hspace{2cm} (2.6)

corresponding to the following limits on the values of the combinations of the effective parameters:

$$|\bar{c}_B + \bar{c}_W| \leq (0.8 \pm 1.5) \times 10^{-3}, \quad |\bar{c}_T| \leq (6.3 \pm 5.5) \times 10^{-4}.$$  \hspace{2cm} (2.7)

2.2 One-loop constraints

Considering only linear Higgs couplings (not quadratic Higgs couplings or more) in this work, only the one-loop corrections of the form $V_1 -(V / H) - V_2$, namely: $Z -(Z / H) - Z$ and $Z -(\gamma / H) - Z$, $W -(W / H) - W$, $\gamma -(\gamma / H) - \gamma$ and $\gamma -(Z / H) - \gamma$, and the $Z / \gamma$ mixing $Z -(Z / H) - \gamma$ and $Z -(\gamma / H) - \gamma$ are evaluated. It is implicitly assumed that the momentum scale $\Lambda$ of the EW loop is larger than $M_{EW}$ but much smaller than the NP scale for the effective framework to be still valid, so that the effective $hV_1V_2$ vertices can be still considered as "true" vertices; otherwise the usage of NP fields would have been a necessity. We obtain the following power-divergent contributions to the $S$, $T$, $U$ parameters (see Appendices A.1 and A.2 for the details of the computations):

$$\alpha S = 2s_w^2c_w^2 \frac{\Lambda^2}{16\pi^2v^2} \left[ \frac{c_Z^2 - c_{\gamma Z}^2 - \bar{c}_{ZZ}^2 + \bar{c}_{\gamma Z}^2 + 2c_Z\kappa_{ZZ} + 3(c_Z\kappa_{ZZ} + c_{\gamma Z}\kappa_{Z\gamma})}{c_w^2 - s_w^2} (cz\kappa_{Z\gamma} + c_{ZZ}\gamma + c_{\gamma Z}\gamma - \bar{c}_{Z\gamma}\bar{c}_{Z\gamma} - \bar{c}_{Z\gamma}\bar{c}_{\gamma Z}) \right] + O \left( \ln \tilde{\Lambda}^2 \right),$$  \hspace{2cm} (2.8)

$$\alpha T = 3 \frac{\Lambda^4}{8 \pi^2 v^2} \left( \frac{\kappa_{Z\gamma}^2}{m_Z^2} - \frac{\kappa_{WW}^2}{m_W^2} \right) + \frac{\Lambda^2}{16\pi^2v^2} \left[ \frac{c_Z^2 - c_{\gamma Z}^2 + 3c_Z\kappa_{ZZ} - 3c_{\gamma Z}\kappa_{Z\gamma} - 3\kappa_{ZZ}^2 - \kappa_{WW}^2}{4} \right] + O \left( \ln \tilde{\Lambda}^2 \right),$$  \hspace{2cm} (2.9)
\[
\alpha U = 2s_w^2 \frac{\Lambda^2}{16\pi^2 v^2} \left[ c_{WW}^2 - \tilde{c}_{WW}^2 + 3c_{WW}\kappa_{WW} - 3c_{W}^2 (c_{Z\gamma KZ\gamma} + c_{Z\gamma KZ\gamma}) - 3c_w s_w (c_{\gamma\gamma KZ\gamma} + \kappa_{Z\gamma KZ\gamma} + c_{Z\gamma KZ\gamma}) + 2 \left( c_{WW} - c_w^2 (c_{Z\gamma KZ\gamma} - c_w s_w c_{Z\gamma\gamma}) + 13 c_{WW}^2 - \frac{c_{WW}^2}{3} \right) - \frac{4c_{WW}^2}{3} \right]
+ \mathcal{O}\left( \ln \Lambda^2 \right) . \quad (2.10)
\]

Given the current constraints on the oblique parameters [15], we require that loop-induced power-divergent corrections coming from the effective couplings cancel. We furthermore make a second hypothesis, namely that there are no fine-tuned cancellations between operators of different types, i.e. between the CP-even (generating the \( c_V \) and \( c_{VV} \) couplings) and CP-odd operators (generating the \( \tilde{c}_{VV} \) couplings) and the ones that generate the \( \kappa_i \) couplings.

Cancelling the quartic divergence in the \( T \) parameter requires\(^3\) that \( \kappa_{WW}^2 = c_w^2 (\kappa_{Z\gamma}^2 + \kappa_{Z\gamma}^2) \). However raising the constraint 1.12 to the square gives: \( \kappa_{WW}^2 = c_w^2 (c_{Z\gamma} + s_w c_{Z\gamma})^2 \), so that we obtain: \( \kappa_{Z\gamma}^2 + \kappa_{Z\gamma}^2 = (c_w c_{Z\gamma} + s_w c_{Z\gamma})^2 \). After expansion of the right-hand side and rewrite of this equation, we obtain: \( (s_w c_{Z\gamma} - c_w c_{Z\gamma})^2 = 0 \), so that \( \kappa_{Z\gamma} = \frac{s_w c_{Z\gamma}}{c_w c_{Z\gamma}} \). Reinjecting this relation in 1.12 gives: \( \kappa_{WW} = \kappa_{Z\gamma} \). Therefore each of the power-divergent part of the \( S, T, U \) parameters can be rewritten under the form:

\[
S, T, U = \frac{\Lambda^2}{16\pi^2 v^2} \mathcal{P}(c, \tilde{c}, \kappa_{Z\gamma}) + \mathcal{O}\left( \ln \Lambda^2 \right) \quad (2.11)
\]

where \( \mathcal{P} \) is a multinom of degree 2. The \( T \) parameter can be rewritten as:

\[
\alpha T = \frac{\Lambda^2}{16\pi^2 v^2} (c_Z^2 - c_W^2 + 3\kappa_{Z\gamma} (c_Z - c_W)) + \mathcal{O}\left( \ln \Lambda^2 \right) \quad (2.12)
\]

and removing the quadratic divergence from it can be done if \( c_Z = c_W \equiv c_V \) (first custodial relation).

The \( U \) parameter can be reexpressed as:

\[
\alpha U = 2s_w^2 \frac{\Lambda^2}{16\pi^2 v^2} \left[ c_{WW}^2 - \tilde{c}_{WW}^2 - (c_w c_{Z\gamma} + s_w c_{Z\gamma})^2 - (c_w c_{Z\gamma} + s_w c_{Z\gamma})^2 + (c_w c_{Z\gamma} + s_w c_{Z\gamma})^2 + 3\kappa_{Z\gamma} (c_{WW} - c_w^2 c_{Z\gamma} - s_w^2 c_{Z\gamma} - 2s_w c_w c_{Z\gamma}) \right] + \mathcal{O}\left( \ln \Lambda^2 \right) . \quad (2.13)
\]

The constraint 1.10 automatically cancels the last line of the previous equation, so that removing all the remaining power divergences in \( U \) without fine-tuning between the CP-even and CP-odd parts

\(^3\)Using: \( c_w^2 = \frac{m_W^2}{m_Z^2} \).
implies:

\[
e^2_{WW} = (c_w c_{Z\gamma} + s_w c_{Z\gamma})^2 + (c_w c_{Z\gamma} + s_w c_{Z\gamma})^2,  \tag{2.14}
\]

\[
e'^2_{WW} = (c_w c_{Z\gamma} + s_w c_{Z\gamma})^2 + (c_w c_{Z\gamma} + s_w c_{Z\gamma})^2. \tag{2.15}
\]

Squaring the constraints 1.10 and 1.11 and equating them with the previous equations, give us:

\[
c^2_{Z\gamma} \left(1 - 4c_w^2 s_w^2\right) - 2c_{Z\gamma} \frac{c_w^2 - s_w^2}{s_w c_w} (c_{Z\gamma} - c_{\gamma\gamma}) + (c_{Z\gamma} - c_{\gamma\gamma})^2 = 0 \text{ (and similar for } c_i) \text{, and since: } \left(1 - 4c_w^2 s_w^2\right) = \left(\frac{c_w^2 - s_w^2}{s_w c_w}\right)^2; \text{ we obtain:}
\]

\[
c_{Z\gamma} = \frac{s_w c_w}{c_w^2 - s_w^2} (c_{Z\gamma} - c_{\gamma\gamma}) \Rightarrow c_{Z\gamma} = c_{\gamma\gamma} + \frac{c^2_w - s_w^2}{s_w c_w} c_{Z\gamma}, \tag{2.16}
\]

\[
\tilde{c}_{Z\gamma} = \frac{s_w c_w}{c_w^2 - s_w^2} (\tilde{c}_{Z\gamma} - \tilde{c}_{\gamma\gamma}) \Rightarrow \tilde{c}_{Z\gamma} = \tilde{c}_{\gamma\gamma} + \frac{c^2_w - s_w^2}{s_w c_w} \tilde{c}_{Z\gamma}. \tag{2.17}
\]

Hence, Eqs. 2.14 and 2.15 are verified, and we removed all the quadratic divergences in \(U\). Reinjecting those relations into 1.10 and 1.11 give:

\[
c_{WW} = c_{\gamma\gamma} + \frac{c_w}{s_w} c_{Z\gamma} \text{ and } \tilde{c}_{WW} = \tilde{c}_{\gamma\gamma} + \frac{c_w}{s_w} \tilde{c}_{Z\gamma}. \tag{2.18}
\]

The \(S\) parameter can be reexpressed as:

\[
\alpha S = 2s_w c_w \frac{\Lambda^2}{16 \pi^2 v^2} \left[ c_{Z\gamma}^2 - c_{\gamma\gamma}^2 + \tilde{c}_{Z\gamma}^2 - \frac{c^2_w - s_w^2}{s_w c_w} (c_{Z\gamma} c_{Z\gamma} + c_{Z\gamma} c_{\gamma\gamma} - \tilde{c}_{Z\gamma} \tilde{c}_{Z\gamma} - \tilde{c}_{\gamma\gamma} \tilde{c}_{\gamma\gamma}) \right]
\]

\[
+ c_v \kappa_{Z\gamma} \left(1 + \frac{s_w^2}{c_w^2}\right) + 3 \kappa_{Z\gamma} \left(c_{Z\gamma} + \frac{c_{Z\gamma}}{2} \left(3 \frac{s_w^2}{c_w^2} - \frac{s_w}{c_w}\right)\right) + O \left(\ln \frac{\Lambda}{v}\right) \tag{2.19}
\]

All the power divergences in \(S\) disappear without fine-tuning between the CP-even and CP-odd parts if those relations are satisfied:

\[
c_{Z\gamma}^2 = c_{\gamma\gamma}^2 + \frac{c_w^2 - s_w^2}{s_w c_w} (c_{Z\gamma} c_{Z\gamma} + c_{Z\gamma} c_{\gamma\gamma}), \tag{2.20}
\]

\[
\tilde{c}_{Z\gamma}^2 = \tilde{c}_{\gamma\gamma}^2 + \frac{c_w^2 - s_w^2}{s_w c_w} (\tilde{c}_{Z\gamma} \tilde{c}_{Z\gamma} + \tilde{c}_{Z\gamma} \tilde{c}_{\gamma\gamma}). \tag{2.21}
\]

However these equations are precisely 2.16 and 2.17, multiplied by \(c_{Z\gamma} + c_{\gamma\gamma}\) or \(\tilde{c}_{Z\gamma} + \tilde{c}_{\gamma\gamma}\) respectively. Therefore the first line in \(\alpha S\) cancels and only a quadratic divergence proportional to \(\kappa_{Z\gamma}\) remains. The only way to remove it is to put \(\kappa_{Z\gamma} = 0\). Another alternative would be to keep only the \(c_i\) and
\( \tilde{c}_i \) terms by taking \(^4\kappa_{ZZ} = 0 \) right from the beginning, so that all of the \( \kappa_i = 0 \). Doing so we retrieve the same constraints as before.

Only pure logarithmic divergences therefore remain in the Peskin-Takeuchi parameters (see Appendix A.2) by using these relations, summarized here:

\[
\kappa_i = 0, \quad (2.22)
\]
\[
c_Z = c_W \equiv c_V, \quad (2.23)
\]
\[
c_{WW} = c_{\gamma\gamma} + \frac{c_w}{s_w} c_{Z\gamma} \quad \text{and similar for } \tilde{c}_{WW}, \quad (2.24)
\]
\[
c_{ZZ} = c_{\gamma\gamma} + \frac{c_w^2 - s_w^2}{s_w c_w} c_{Z\gamma} = c_{WW} - \frac{s_w}{c_w} c_{Z\gamma} \quad \text{and similar for } \tilde{c}_{ZZ}. \quad (2.25)
\]

Those relations can be seen as extended custodial relations that link the parameters of the effective Lagrangian 1.7, and they are equivalent to the following constraints on the SILH Lagrangian parameters:

\[
(2.22) \rightarrow \tilde{c}_{HB} + \tilde{c}_B = 0 = \tilde{c}_{HW} + \tilde{c}_W, \quad (2.26)
\]
\[
(2.23) \rightarrow \quad \tilde{c}_T = 0, \quad (2.27)
\]
\[
(2.24) \text{or (2.25)} \rightarrow \tilde{c}_{HW} + \tilde{c}_{HB} = 0 = \tilde{c}_{HW} + \tilde{c}_{HB}, \quad (2.28)
\]

leading to:

\[
\tilde{c}_{HW} = -\tilde{c}_W = -\tilde{c}_{HB} = \tilde{c}_B. \quad (2.29)
\]

Furthermore, injecting 2.28 into 2.26 and adding the two members of 2.26, we obtain: \( \tilde{c}_B + \tilde{c}_W = 0 \), meaning that, together with 2.27 and 2.5, the theory only generates oblique corrections starting at one-loop and not at tree-level.

The effective Higgs Lagrangian 1.7 therefore depends on 7 independent parameters in the CP-even sector:

\[
c_V, \quad c_u, \quad c_d, \quad c_l, \quad c_{gg}, \quad c_{\gamma\gamma}, \quad c_{Z\gamma} \quad (2.30)
\]

and 6 independent parameters in the CP-odd sector:

\[
\tilde{c}_u, \quad \tilde{c}_d, \quad \tilde{c}_l, \quad \tilde{c}_{gg}, \quad \tilde{c}_{\gamma\gamma}, \quad \tilde{c}_{Z\gamma} \quad (2.31)
\]

and the (tree-level) SM Higgs Lagrangian is retrieved when \( c_V = c_{f=u,d,l} = 1, \ c_{gg} = c_{\gamma\gamma} = c_{Z\gamma} = 0 \) and all the \( \tilde{c}_i = 0 \), the \( c_{ij} \) being generated only at loop-level.

\(^4\)This can be motivated by computing the \( W \)-parameter, introduced by Barbieri, Pomarol, Rattazzi and Strumia [14]. Its quadratic divergence is proportional to \( \kappa_{WW}^2 \equiv \kappa_{ZZ}^2 \). Requiring its vanishing implies \( \kappa_i = 0 \).
3 Connection of the theory with the experiment

LHC experiments usually provide the relative Higgs decay rates (signal strengths) in various channels, defined as: 
\[ \hat{\mu}_{XX}^{YY} = \frac{\sigma_{YY}^{h \rightarrow XX}}{\sigma_{YY}^{h \rightarrow XX,SM}}. \] 
The relative branching fraction reads: 
\[ \frac{\text{Br}(h \rightarrow XX)}{\text{Br}(h \rightarrow XX,SM)} = \frac{\Gamma_{XX}}{\Gamma_{XX,SM}} \frac{\Gamma_{tot,SM}}{\Gamma_{tot}}, \] 
where \( \Gamma_{tot} \) is the sum of all the partial widths. An effect of the presence of the effective operators is that both Higgs decay rates and production cross-sections in those channels are shifted from their SM values. Hence, we can constrain the parameters of the effective Lagrangian by comparing the theoretical rates from the SM with the measured ones. In the following we summarize how they depend on the parameters of the effective Lagrangian. We do not consider any contributions to the Higgs width other than Higgs decays into SM particles.

We use the following values for the SM constants (from PDG 2012 [16]):
\[ G_F = 1.16637 \times 10^{-5} \text{ GeV}^{-2}, \quad \alpha_{EW}(m_Z) = 7.8186 \times 10^{-3}, \quad \alpha_S(m_Z) = 5.01184, \]
\[ m_Z = 91.1876 \text{ GeV}, \quad m_h = 125.6 \text{ GeV}. \]

3.1 Relative decay widths

Generated at tree-level, the on-shell decay rate of the Higgs into (charged) a fermion \( f \) and its antifermion can be written as:
\[ \left( \frac{\Gamma}{\Gamma_{SM}} \right)_{h \rightarrow f\bar{f}} = |c_f|^2 + |\bar{c}_f|^2 \frac{m_h^2}{m_h^2 - 4m_f^2}, \]
\[ \simeq |c_f|^2 + |\bar{c}_f|^2 \quad \text{if } f \neq \text{top quark}. \tag{3.2} \]

Generated at one-loop level in the SM and receiving a tree-level contribution from the corresponding effective coupling, the decay rate of the Higgs into two vector bosons \( V_1 \) and \( V_2 \), where \( (V_1, V_2) = (g, g), (\gamma, \gamma) \) or \( (Z, \gamma) \), can be written as:
\[ \left( \frac{\Gamma}{\Gamma_{SM}} \right)_{h \rightarrow V_1V_2} \simeq \frac{|\hat{c}_{V_1V_2}|^2}{|c_{V_1V_2,SM}|^2}. \tag{3.3} \]
The hatted quantities are "bookkeeping" effective CP-even and CP-odd Higgs couplings to \( V_1 \) and \( V_2 \) that include two types of contributions that enter the decay amplitude at the same order in the effective theory: the tree-level contributions proportional to the effective couplings \( c_{ij} \) of the NLO Lagrangian, and the one-loop quantum corrections, proportional to the effective couplings \( c_i, \bar{c}_i \) of

\(^{5}\text{World’s average of } \alpha_S \text{ in 2012, see [17].}\)
the LO Lagrangian (that also include SM corrections). The hatted quantity noted "SM" is the value of the corresponding hatted effective coupling when using SM values for the $c_i$ couplings. For our level of precision, we can safely stop the expansion of the relative decay widths (and the relative cross-sections, see after) at NLO, because of the following reason: for $h \rightarrow \gamma\gamma$ and $h \rightarrow Z\gamma$, the NLO corrections arise in QED only and are of order 0.2%, and for $h \rightarrow gg$, even if QCD N$^{2+}$LO corrections are sizeable, they almost cancel in the relative signal strengths and leave only a $\approx 2\%$ correction [18].

We implemented the effective model with FeynRules 1.8 [19] and we used FeynArts 3.8 [20] and FormCalc 8.2 [21] to generate and numerically compute the relative decay widths. We obtain:

- $V_1 = V_2 = g$:
  \[
  \hat{c}_{gg} \simeq c_{gg} + 10^{-2}1.298c_t - 10^{-3}(0.765 - 1.077 i)c_b,
  \hat{\tilde{c}}_{gg} \simeq \tilde{c}_{gg} - 10^{-2}1.975\tilde{c}_t + 10^{-3}(0.875 - 1.084 i)\tilde{c}_b, \tag{3.4}
  \]
  \[|\hat{c}_{gg,SM}| \simeq 0.0123,\]
  which gives also the relative production cross-section via gluon-fusion $\frac{\sigma_{ggh}}{\sigma_{gg,SM}}$.

- $V_1 = V_2 = \gamma$:
  \[
  \hat{c}_{\gamma\gamma} \simeq c_{\gamma\gamma} + 10^{-2}(1.050c_V - 0.231c_t) + 10^{-5}(3.399 - 4.786 i)c_b + 10^{-5}(2.934 - 2.674 i)c_T,
  \hat{\tilde{c}}_{\gamma\gamma} \simeq \tilde{c}_{\gamma\gamma} - 10^{-3}3.509\tilde{c}_t - 10^{-5}(3.887 - 4.813 i)\tilde{c}_b - 10^{-5}(3.136 - 2.676 i)\tilde{c}_T,
  \]
  \[|\hat{c}_{\gamma\gamma,SM}| \simeq 0.0083. \tag{3.5}\]

- $V_1 = Z, V_2 = \gamma$:
  \[
  \hat{c}_{Z\gamma} \simeq c_{Z\gamma} + 10^{-2}(1.507c_V - 0.0784c_t) + 10^{-5}(2.063 - 1.210 i)c_b + 10^{-7}(3.570 - 1.535 i)c_T,
  \hat{\tilde{c}}_{Z\gamma} \simeq \tilde{c}_{Z\gamma} - 10^{-3}3.190\tilde{c}_t - 10^{-5}(2.414 - 1.213 i)\tilde{c}_b - 10^{-7}(4.008 - 1.536 i)\tilde{c}_T,
  \]
  \[|\hat{c}_{Z\gamma,SM}| \simeq 0.0143. \tag{3.6}\]

At tree-level, the Higgs boson decays also into $ZZ^*$ or $WW^*$, that subsequently decay into leptons. The corresponding relative decay widths were computed with MadGraph 5 [22]. Since MadGraph is a tree-level matrix element generator, effects of one-loop corrections in the computations were taken into account by using the hatted effective couplings. Using the extended-custodial relations $2.23$, $2.25$ and $2.24$ the expressions are written in terms of the CP-even variables $c_V$, $c_{\gamma\gamma}$, $c_{Z\gamma}$ and the corresponding CP-odd ones. After variation of the values of the $c_i$ and $\tilde{c}_i$ couplings and performing a fit on a multinom of the form given in 3.9, we obtain:
\[
\left( \frac{\Gamma}{\Gamma_{SM}} \right)_{ZZ^*\rightarrow 4l} \simeq c_V^2 + 0.022c_{\gamma\gamma}^2 + 0.035c_Z^2 + 0.253c_Vc_{\gamma\gamma} + 0.316c_Vc_Z + 0.056c_{\gamma\gamma}c_Z + 0.009c_{\gamma\gamma}^2 + 0.014c_Z^2 + 0.023c_{\gamma\gamma}c_Z.
\]

(3.7)

\[
\left( \frac{\Gamma}{\Gamma_{SM}} \right)_{WW^*\rightarrow 2l2\nu} \simeq c_V^2 + 0.051c_{\gamma\gamma}^2 + 0.166c_Z^2 + 0.380c_Vc_{\gamma\gamma} + 0.687c_Vc_Z + 0.184c_{\gamma\gamma}c_Z + 0.021c_{\gamma\gamma}^2 + 0.069c_Z^2 + 0.076c_{\gamma\gamma}c_Z.
\]

(3.8)

### 3.2 Relative production cross-sections

For gluon-fusion production mode, the expression of the relative production cross-section \( \frac{\sigma_{ggh}}{\sigma_{ggh,SM}} \) is taken to be the same as the one of the decay rate via gluon-fusion, Eq. 3.4.

In the case of the VBF or VH production modes, we simulated the production of Higgs via \( pp \) collisions at \( \sqrt{s} = 8 \) TeV with MadGraph; then the same procedure as for the relative widths 3.7 and 3.8 was used: after variation of the effective couplings the relative cross-sections was fitted with a multinom of the form:

\[
\left( \frac{\sigma}{\sigma_{SM}} \right)_{VBF} \simeq c_V^2 + \alpha_1c_{\gamma\gamma}^2 + \alpha_2c_Z^2 + \alpha_3c_Vc_{\gamma\gamma} + \alpha_4c_Vc_Z + \alpha_5c_{\gamma\gamma}c_Z + \beta_1c_{\gamma\gamma}^2 + \beta_2c_Z^2 + \beta_3c_{\gamma\gamma}c_Z.
\]

(3.9)

The coefficients \( \alpha_i \) and \( \beta_i \) depend on the sets of cuts on the kinematics of the final-state jets chosen to perform the analysis/selection. This fact can be understood as follows: the detected events contain both background and the signal of interest. When setting cuts on kinematical variables to maximize the signal over (signal plus) background ratio, a little part of the signal of interest is always removed. This defines the efficiency of the cuts, which translates into the modification of the "observed" relative cross-sections.

- **Vector boson fusion (VBF):** \( qq \rightarrow hqq \) with exchange of \( W \) or \( Z \) bosons \( \left( \frac{\sigma}{\sigma_{SM}} \right)_{VBF} \) of the form given by 3.9. For illustration purposes we show few simple cuts chosen by the ATLAS and CMS experiments in their analyses in Table 1, and the corresponding estimated values of the coefficients are given in Table 2. We notice that using different cuts modify the coefficients at the 10% level. In the global fits we used the "ATLAS" and the "CMS-2 (Optimal VBF)" cuts (in pink in the tables) for fitting ATLAS and CMS data respectively.

- **Vector boson associated production (VH):** \( q\bar{q} \rightarrow hV \), where \( V = W, Z \). We use the set of cuts
### Table 1: VBF cuts from the ATLAS and CMS experiments (in pink: chosen cuts).

| Cut      | \( p_T \) (GeV) | \( |\eta| \) | \( m_{jj} \) (GeV) | \( |\Delta \eta_{jj}| \) | \( \Delta R_{jj} \) | Ref. |
|----------|----------------|----------|-------------------|-----------------|----------------|------|
| ATLAS    | \( \geq 25 \)  | \( \leq 2.4 \) | \( \geq 500 \)    | \( \geq 2.8 \)   | \( \geq 0.4 \) | [23] |
|          | \( \geq 30 \)  | \( 2.4 \leq \cdots \leq 4.5 \) |                       |                 |                |      |
| CMS-1    | \( \geq 30 \)  | \( \leq 4.7 \) | \( \geq 500 \)    | \( \geq 3.5 \)   | \( \geq 0.5 \) | [24] |
| CMS-2\(^a\) | \( \geq 30 \) | \( \leq 4.7 \) | \( \geq 650 \)    | \( \geq 3.5 \)   | \( \geq 0.5 \) | [25] |
| CMS-3a\(^b\) | \( \geq 30 \) | \( \leq 4.7 \) | \( \geq 300 \)    | \( \geq 3.0 \)   | \( \geq 0.5 \) | [25] |
| CMS-3b\(^c\) | \( \geq 30 \) | \( \leq 4.7 \) | \( \geq 500 \)    | \( \geq 4.0 \)   | \( \geq 0.5 \) | [25] |

\(^a\) "Optimal VBF". \(^b\) "VBF loose", 8 TeV only. \(^c\) "VBF tight" for 8 TeV.

### Table 2: Table of coefficients \( \alpha_i \) and \( \beta_i \) for the VBF relative cross-section. For each coefficient, two values are given, the first one corresponds to a fit where cross-terms \( c_{WW}c_{ZZ/\gamma\gamma/Z\gamma} \) (and \( \tilde{c}_{WW}\tilde{c}_{ZZ/\gamma\gamma/Z\gamma} \)) were kept, whereas the parenthesized one corresponds to a fit where those cross-terms were removed, because they are approximately two order of magnitude less (in pink: chosen cuts).

| Cut      | \( \alpha_1 \) | \( \alpha_2 \) | \( \alpha_3 \) | \( \alpha_4 \) | \( \alpha_5 \) | \( \beta_1 \) | \( \beta_2 \) | \( \beta_3 \) |
|----------|----------------|----------------|----------------|----------------|----------------|--------------|--------------|--------------|
| ATLAS    | 1.483          | 3.065          | 0.327          | 0.569          | 3.329          | 1.367        | 2.765        | 3.004        |
|          | (1.490)        | (3.086)        | (0.328)        | (0.570)        | (3.352)        | (1.370)      | (2.777)      | (3.017)      |
| CMS-1    | 1.188          | 2.177          | 0.248          | 0.426          | 2.379          | 1.123        | 2.022        | 2.213        |
|          | (1.190)        | (2.186)        | (0.248)        | (0.427)        | (2.389)        | (1.125)      | (2.033)      | (2.223)      |
| CMS-2    | 1.281          | 2.419          | 0.270          | 0.465          | 2.635          | 1.210        | 2.243        | 2.451        |
|          | (1.284)        | (2.429)        | (0.270)        | (0.466)        | (2.645)        | (1.212)      | (2.255)      | (2.460)      |
| CMS-3a   | 1.272          | 2.459          | 0.274          | 0.472          | 2.684          | 1.191        | 2.258        | 2.469        |
|          | (1.276)        | (2.474)        | (0.274)        | (0.473)        | (2.700)        | (1.193)      | (2.268)      | (2.477)      |
| CMS-3b   | 1.010          | 1.629          | 0.193          | 0.327          | 1.798          | 0.969        | 1.533        | 1.696        |
|          | (1.011)        | (1.634)        | (0.193)        | (0.327)        | (1.804)        | (0.972)      | (1.546)      | (1.710)      |

featured in [26, 27]: \( p_{TH} \geq 200 \) GeV and \( p_{TW/Z} \geq 190 \) GeV ("boosted" Higgs).

\[
\left( \frac{\sigma}{\sigma_{SM}} \right)_{hW} \approx c_V^2 + 24.481 c_\gamma^2 + 79.810 c_{Z\gamma}^2 - 4.610 c_V c_\gamma - 8.324 c_V c_{Z\gamma} + 88.405 c_\gamma c_{Z\gamma} \\
+ 22.430 c_\gamma^2 + 73.122 c_{Z\gamma}^2 + 80.997 c_\gamma c_{Z\gamma},
\]

\[
\left( \frac{\sigma}{\sigma_{SM}} \right)_{hZ} \approx c_V^2 + 18.992 c_\gamma^2 + 57.969 c_{Z\gamma}^2 - 4.460 c_V c_\gamma - 6.708 c_V c_{Z\gamma} + 59.580 c_\gamma c_{Z\gamma} \\
+ 16.546 c_\gamma^2 + 50.645 c_{Z\gamma}^2 + 51.865 c_\gamma c_{Z\gamma}.
\]

### 4 Experimental data

In our global fit we include the latest LHC results from the ATLAS and CMS experiments, summarized in Table 3. For most of the decay channels where only the 95\% CL limits are quoted by the experiments, we reconstruct \( \hat{\mu} \) assuming Gaussian errors. There are also decay channels for which the
experiments give the full 2-dimensional (2D) likelihood functions defined in the \( \hat{\mu}_{ggH+ttH} - \hat{\mu}_{VBF+VH} \) plane: we use them because they encode the non-trivial correlations between the rates measured for the \( ggH/ttH \) or \( VBF/VH \) production modes. The quoted value of \( \hat{\mu} \) in the table, obtained after a basic recombination of \( \hat{\mu}_{ggH+ttH} \) and \( \hat{\mu}_{VBF+VH} \), is then provided for illustration purposes only. However, it happens sometimes that only the 95\% CL or 68\% CL contours of the 2D likelihoods are given for these channels. In this case, we reconstruct an approximate 2D likelihood function in the whole \( \hat{\mu}_{ggH+ttH} - \hat{\mu}_{VBF+VH} \) plane as described in Appendix B, such that correlations between those rates are taken into account.

**Table 3:** The LHC Higgs rates included in the fit. The "2D" production holds for \( ggH+ttH \) and \( VBF+VH \) production modes.

| ATLAS | CMS |
|-------|-----|
| **Production** | **Decay** | \( \hat{\mu} \) | **Ref.** | **Production** | **Decay** | \( \hat{\mu} \) | **Ref.** |
| 2D | \( \gamma\gamma \) | 1.55\(^{+0.33}_{-0.29}\) | [28, 29] | \( \gamma\gamma \) | 0.77\(^{+0.29}_{-0.26}\) | [38] |
| | \( ZZ \) | 1.41\(^{+0.42}_{-0.33}\) | [28, 30] | \( ZZ \) | 0.92\(^{+0.25}_{-0.22}\) | [39] |
| | \( WW \) | 0.98\(^{+0.33}_{-0.26}\) | [28, 31] | \( WW \) | 0.72\(^{+0.20}_{-0.18}\) | [40] |
| | \( \tau\tau \) | 1.36\(^{+0.43}_{-0.38}\) | [32] | \( \tau\tau \) | 0.97\(^{+0.27}_{-0.25}\) | [41] |
| VH | \( bb \) | 0.2\(^{+0.7}_{-0.6}\) | [33] | VH | \( bb \) | 1.0 \pm 0.5 | [42] |
| ttH | \( bb \) | 1.7 \pm 1.4 | [34] | VBF | \( bb \) | 0.7 \pm 1.4 | [43] |
| | \( \gamma\gamma \) | -1.39 \pm 3.18 | [35] | ttH | \( bb \) | 1.0\(^{+1.9}_{-2.0}\) | |
| inclusive | \( Z\gamma \) | 2.18 \pm 4.57 | [36] | inclusive | \( Z\gamma \) | -0.21 \pm 4.86 | [46] |
| | \( \mu\mu \) | 1.75 \pm 4.26 | [37] | | \( \mu\mu \) | 2.9\(^{+2.8}_{-2.7}\) | [25] |

The following combined Tevatron measurements [47] were also used: \( \hat{\mu}_{\gamma\gamma}^{\text{incl}} = 6.2^{+3.2}_{-3.2} \), \( \hat{\mu}_{WW}^{\text{incl}} = 0.9^{+0.9}_{-0.8} \), \( \hat{\mu}_{bb}^{VH} = 1.62^{+0.77}_{-0.77} \), \( \hat{\mu}_{\tau\tau}^{\text{incl}} = 2.1^{+2.2}_{-2.0} \), as well as electroweak precision measurements from LEP, SLC and Tevatron collected in Table 1 of Falkowski et al. [3]. Finally, we assume a cut-off scale \( \Lambda = 3 \) TeV to evaluate the logarithmically divergent corrections from the Higgs loops to the electroweak precision observables.

### 5 Global fit – Discussion

We use the previous definitions for building the theoretical expressions of the signal strengths \( \hat{\mu}^{th} \) for different channels, which depend on the effective couplings \( c_i \) and \( \tilde{c}_i \). For the decay channels...
where we know only each signal strength $\hat{\mu}^{exp} \pm \delta \mu$ separately, we assume the errors to be Gaussian and uncorrelated and we define a $\chi^2_1D(\hat{\mu}^{th}, \hat{\mu}^{exp} \pm \delta \mu) = \left(\frac{\hat{\mu}^{th} - \hat{\mu}^{exp}}{\delta \mu}\right)^2$. For other channels where we know the correlations between the rates for $ggH/\ttH$ or $VBF/VH$ production modes, we use the experimental 2D likelihood functions $\chi^2_{2D}$. For electroweak precision data the correlations are known and we use $\chi^2_{EWPT}$ [3, 48]. The fitting procedure then consists in minimizing the following $\chi^2$-function:

$$\chi^2(c_i, \tilde{c}_i) = \chi^2_{EWPT}(c_i, \tilde{c}_i) + \sum \chi^2_1D(\hat{\mu}^{th}, \hat{\mu}^{exp} \pm \delta \mu) + \sum \chi^2_{2D}(\hat{\mu}^{th}_{ggH+ttH}, \hat{\mu}^{th}_{VBF+VH}) + \chi^2(\lambda \pm \delta \lambda).$$

The $\chi^2_{EWPT}$ function can be approximated around its best-fit point:

$$\chi^2_{EWPT}(c_i) = 193.005 + C^T \begin{pmatrix} 1 & -0.940 & -0.934 \\ -0.940 & 1 & 0.999 \\ -0.934 & 0.999 & 1 \end{pmatrix} C. \quad (5.3)$$

We incorporate also the large uncertainty on the prediction of the SM $ggH$ production cross-section by introducing a nuisance parameter $\lambda$ with a Gaussian distribution around the central value: for the LHC at $\sqrt{s} = 8$ TeV we take [27] the scale error: $(+7.2\%, -7.8\%)$ and the PDF error: $(+7.5\%, -6.9\%)$ and add those two linearly.

### 5.1 Fit over the CP-even Parameters

The 7 CP-even parameters (Eq. 2.30) are fitted to the available Higgs and electroweak precision data, while fixing the CP-odd ones to zero. We find the following central values and 68% CL intervals for the parameters:

$$c_V = 1.04 \pm 0.03, \quad c_u = 1.31^{+0.10}_{-0.34}, \quad c_d = 0.92^{+0.22}_{-0.13}, \quad c_l = 1.09^{+0.13}_{-0.11},$$

$$c_{gg} = -0.0016^{+0.0021}_{-0.0022}, \quad c_{\gamma\gamma} = 0.0009^{+0.0008}_{-0.0010}, \quad c_{Z\gamma} = -0.0006^{+0.0183}_{-0.0240}. \quad (5.4)$$

with a $\Delta \chi^2 = \chi^2_{SM} - \chi^2_{\min} = 5.3$, which means that the SM gives a perfect fit to the Higgs and electroweak precision data. When quoting the confidence regions above we ignored degenerate minima.
of the likelihood function isolated from the SM point where a large 2-derivative Higgs coupling conspires with the SM loop contributions to produce a small shift of the Higgs observables. Remarkably, the current data already put meaningful limits on all 7 parameters. The strong constraint on $c_V$ is dominated by electroweak precision data, and ignoring them in the fit weakens the constraint, and one obtains $c_V = 1.05^{+0.08}_{-0.10}$. It can also be relaxed in the presence of additional tuned contributions to the S and T parameters that could arise from integrating out heavy new physics states.

The fit features an approximately flat region along the line: $c_{gg} + 0.012(c_u - 1) = 0$, which is the combination that sets the strength of the gluon fusion production mode. This is clearly visible in Fig. 1 where a 2D fit in the $c_u$–$c_{gg}$ plane is performed, whereas the other couplings are fixed to their SM values. This flat region is lifted by the $ttH$ production mode which depends on $c_u$ only. The recent ATLAS [34] and CMS [44] results in the $ttH$ channel add interesting constraints on $c_u$ independently of $c_{gg}$: they shift the preferred values towards the SM one, as shown in Fig. 1a and 1b: the 68% and 95% CL regions move towards the right and the best-fit point moves closer to the SM point. The fit shows also a strong preference for $c_d \neq 0$ even though the $h \to b\bar{b}$ decay has not been clearly observed. The reason is that $c_d$ determines $\Gamma_{bb}$ which dominates the total Higgs decay width and the latter is indirectly constrained by the Higgs rates measured in other decay channels.

![Figure 1](image1)

**Figure 1:** A fit in the $c_u$–$c_{gg}$ plane with the other couplings fixed at their Standard Model values, without (Left) and with (Right) ATLAS+CMS $ttH$ data. Dark green: 68% CL; light green: 95% CL. See details in the text. Since we fixed here as an illustration all the other couplings to their SM values, contrary to what was done in the global fit, we obtain slightly different best-fit points: a $(c_u < 1; c_{gg} > 0)$ point is preferred whereas in the global fit $(c_u > 1; c_{gg} < 0)$ is preferred.

The least stringent constraint is currently the one on $c_{Z\gamma}$ which reflects weak experimental limits on the $h \to Z\gamma$ decay rate. There are good prospects [49] of probing $c_{Z\gamma}$, and also $c_{\gamma\gamma}$, using
differential cross section measurements in the "Golden Channel" $h \rightarrow 4\ell$.

5.2 Fit over the CP-odd Parameters

We move towards an attempt to constrain some of the CP-odd parameters (Eq. 2.31) of the effective Lagrangian. We make the same assumptions about data errors as in the case of the CP-even fit and we continue to take into account the large uncertainty in the prediction of the SM $ggH$ production cross-section. The CP-even parameters are fixed to their SM value and we fit the CP-odd ones. The following central values and 68\% CL intervals for the parameters are obtained:

$$
\tilde{c}_u = \pm(0.87^{+0.33}_{-2.08}), \quad \tilde{c}_d = -0.0035^{+0.4608}_{-0.4581}, \quad \tilde{c}_l = \pm(0.37^{+0.25}_{-0.99}),
$$

$$
\tilde{c}_{gg} = 0.0004^{+0.0038}_{-0.0040}, \quad \tilde{c}_{\gamma\gamma} = \pm(0.0033^{+0.0017}_{-0.0028}), \quad \tilde{c}_{Z\gamma} = 0.0075^{+0.0200}_{-0.0345},
$$

with a $\Delta\chi^2 = 1.1$. We note that if Higgs-gauge couplings are meaningfully constrained by current data (yet the coupling to $\gamma\gamma$ is found to have a sign degeneracy), the up-type and leptonic couplings are not constrained by their sign (see also Figs. 2 and 3). This is due to the fact that the Higgs rate measurements from the LHC actually constrain the sum of the squares of the CP-even and CP-odd couplings (see Section 3.1).

![Figure 2](image)

**Figure 2:** A fit in the $c_u-\tilde{c}_u$ plane with the other couplings fixed at their Standard Model values, without (Left) and with (Right) ATLAS+CMS $ttH$ data. Dark green: 68\% CL; light green: 95\% CL. See details in the text. Since we fixed here as an illustration all the other couplings to their SM values, contrary to what was done in the global fit, we obtain smaller preferred regions with different best-fit points.

To break this degeneracy and improve the precision on the CP-odd couplings, other types of studies are needed. A first one is to study differential cross-section measurements, for example in
the "Golden Channel" [49], or via jet kinematics in the VBF [50] or in VH [51] production modes. Alternatively for the up-type coupling, methods involving mass distributions as well as top-quark polarization and spin correlations can be done in the $t\bar{t}H$, $tH$ and $\bar{t}H$ production channels [52]. A second one is to use EDMs as shown by J.Brod et al. [6]: assuming that the Higgs couples to the first generation of fermions with SM couplings, constraints on the $c_i$ and $\bar{c}_i$ couplings can be derived for the top and bottom quarks and tau lepton by using low-energy bounds on the EDMs of the electron and the neutron together with existing Higgs production data. It is shown that those limits can be dramatically enhanced if bounds on EDMs are improved from a factor of 100 to 300, from $|d_e/e| < 8.7 \times 10^{-29}$ cm to $< 10^{-30}$ cm for the electron and $|d_n/e| < 2.9 \times 10^{-26}$ cm to $< 10^{-28}$ cm for the neutron, while using 3000 fb$^{-1}$ of updated Higgs data from the 14 TeV high-luminosity LHC upgrade. With these expected improvements the following limits are obtained: $c_t = 1.00 \pm 0.03$ and $\bar{c}_t = 0.00 \pm 2 \times 10^{-4}$ (other couplings fixed to their SM values), and $c_b = 1.00 \pm 0.08$ and $\bar{c}_b = 0.00 \pm 0.02$. If the assumption that the Higgs couples to the first generation with SM couplings is removed, then constraints from the neutron EDM can be still used, and improving its bounds may still allow to constrain $\bar{c}_t$ for the top quark at the same level as before. We refer to their paper [6] for an extended discussion. Concerning Higgs leptonic couplings, electron EDM reduces the possibility for large values of the CP-odd $\tau$ lepton coupling $\bar{c}_\tau$, of order 0.01, while keeping a sign degeneracy on its CP-even coupling $c_\tau$. 

**Figure 3:** Fits in the fit in the $c_d-\bar{c}_d$ plane (Left), and in the $c_l-\bar{c}_l$ plane (Right). Current Higgs signal rates doesn’t significantly constrain the signs of the couplings and therefore other types of studies need to be done. The displayed best-fit points are found for a fit around the SM values.
Summary

In this work we employed a model-independent effective approach by using an effective Lagrangian for parametrizing small deviations of Higgs couplings to matter from their SM prediction. We did not use directly the phenomenological Lagrangian of the same form as 1.7 because otherwise all of the effective parameters would be free. Instead, we used the dimension-6 SILH Lagrangian of Giudice et al. [12] written in the basis employed by Contino et al. [11], where new "custodial"-like relations link some of the effective Wilson coefficients. Since they introduce power divergences in the oblique parameters at loop-level, we argue that, due to current EW constraints, those divergences must disappear. A hypothesis made is that there is no accidental mixing between operators of different types, namely the CP-even, CP-odd and the $\kappa_i$ ones. Doing so, this allows us to obtain extra constraints on the parameters of the theory that reduce the number of free parameters of the phenomenological Lagrangian: 7 parameters on the CP-even sector and 6 parameters on the CP-odd sector of the theory are obtained. They are then fitted to current Higgs data.

Using the current LHC Higgs rates with the latest ATLAS and CMS $ttH$ data, we are able to constrain CP-even parameters and some CP-odd ones. However one should note that, since until now the rate measurements only constrain $|c_f|^2$ or $|\tilde{c}_f|^2$, or $|\tilde{c}_V V_2|^2$ or $|\tilde{c}_V V_2|^2$, other CP-odd parameters are only constrained via their absolute values (i.e. no sign constraints) and more elaborate methods are needed to constrain the possible values of the (CP-even and) CP-odd parameters and break the sign degeneracies.

So far no indication for deviations from the SM are found in the fits since all of the fitted parameters are compatibles with their SM values withing 68% CL.

Note: During the final preparation of this paper a work by J. Ellis et al. [53] was published, in which the authors made an analysis of the CP-even part of the dimension-6 Lagrangian 1.3 and extracted bounds on the $\tilde{c}_i$ parameters using Higgs signal strenghths and kinematic and differential distributions.

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A One-loop corrections for the $S$, $T$, $U$ parameters

A.1 One-loop corrections to the gauge bosons propagators

Considering only linear Higgs couplings in this work, the evaluated one-loop corrections due to the effective Lagrangian present only one Higgs propagator.

![Diagram](image)

**Figure 4:** The one-loop correction $V_1-(V/H)-V_2$ with one Higgs.

We use a hard cut-off scheme ($\Lambda \to +\infty$) for computing the loop integral and we keep its divergence up to the $\ln \Lambda^2$ order. We define $\tilde{\Lambda} \equiv \frac{\Lambda}{M}$, $M$ being the EW scale to write a dimensionless quantity inside the logarithm. Now, after defining the auxiliary function:

$$f(\Lambda^2, p^2, m_H^2) = \frac{1}{16\pi^2 v^2} \left[ \Lambda^2 - \ln \tilde{\Lambda}^2 \left( m_H^2 + m_V^2 - \frac{p^2}{3} \right) \right]$$  \hspace{1cm} (A.1)

the analytical expressions of the corrections up to order $\ln \tilde{\Lambda}^2$ write:

- **$Z$–$Z$ correction:**

  $$Z-(Z/H)$$–$Z$ loop:

  $$\Pi_{\mu\nu} = \frac{-3\kappa_{ZZ}}{8} \frac{\Lambda^4}{16\pi^2 v^2} g_{\mu\nu} - \left[ c_{ZZ}^2 m_Z^2 + 3c_{ZZ} \kappa_{ZZ} m_Z^2 - \frac{3\kappa_{ZZ}^2}{4} (m_H^2 + m_Z^2) \right] g_{\mu\nu} f(\Lambda^2, p^2, m_Z^2)$$

  $$+ \left[ \frac{c_{ZZ}^2 - \kappa_{ZZ}^2}{2} + c_{ZZ} \kappa_{ZZ} + \frac{3c_{ZZ} \kappa_{ZZ}}{2} \right] \left( \frac{26 - 3\kappa_{ZZ}^2}{m_Z^2} \right) (p^2 g_{\mu\nu} - p_{\mu} p_{\nu}) f(\Lambda^2, p^2, m_Z^2)$$

  $$- \frac{\ln \tilde{\Lambda}^2}{16\pi^2 v^2} \left[ 4c_{ZZ}^2 m_Z^2 \left( m_Z^2 g_{\mu\nu} - \frac{p_{\mu} p_{\nu}}{3} \right) - \frac{3c_{ZZ} \kappa_{ZZ} m_H^2}{4} g_{\mu\nu} \left( m_Z^2 - \frac{p^2}{3} \right) + \frac{p^2}{3} (p^2 g_{\mu\nu} - p_{\mu} p_{\nu}) \left( c_{ZZ}^2 + \frac{8\kappa_{ZZ}^2}{3} \right) \right]$$

  \hspace{1cm} (A.2)

  $$Z-(\gamma/H)$$–$Z$ loop:

  $$\Pi_{\mu\nu} = \frac{-3\kappa_{Z\gamma}^2}{8} \frac{\Lambda^4}{16\pi^2 v^2} g_{\mu\nu} + \frac{3\kappa_{Z\gamma}^2}{4} m_H^2 g_{\mu\nu} f(\Lambda^2, p^2, 0) + \left[ \frac{c_{Z\gamma}^2 - \kappa_{Z\gamma}^2}{2} + \frac{3c_{Z\gamma} \kappa_{Z\gamma}}{2} + \frac{2\kappa_{Z\gamma}^2}{3} \right] (p^2 g_{\mu\nu} - p_{\mu} p_{\nu}) f(\Lambda^2, p^2, 0)$$

  $$- \frac{\ln \tilde{\Lambda}^2}{16\pi^2 v^2} \left[ \kappa_{Z\gamma}^2 m_H^2 \left( p^2 g_{\mu\nu} - p_{\mu} p_{\nu} \right) \left( m_H^2 + \frac{2p^2}{3} \right) \left( c_{Z\gamma}^2 \kappa_{Z\gamma} + \frac{\kappa_{Z\gamma}^2}{3} \right) \right]$$

  \hspace{1cm} (A.3)
• $W-W$ correction: There is only the $W-(W/H)-W$ loop:

$$
\Pi_{\mu\nu} = -3k_{WW}^2 \frac{\Lambda^4}{8} \frac{\lambda^4}{16\pi^2 v^2} g_{\mu\nu} - \left[ c_W m_W^2 + 3c_W k_{WW} m_W^2 - 3k_{WW}^2 \frac{m_H^2 + m_W^2}{4} \right] g_{\mu\nu} f(\Lambda^2, p^2, m_W^2) \\
+ \left[ \frac{c_{WW}^2 - c_{WW}^4}{2} + c_W k_{WW} \frac{3c_{WW} k_{WW}}{2} + \frac{k_{WW}^2}{12} \left( 26 - \frac{3p^2}{m^2_W} \right) \right] (p^2 g_{\mu\nu} - p_{\mu} p_{\nu}) f(\Lambda^2, p^2, m_W^2) \\
- \ln \frac{\Lambda^2}{16\pi^2 v^2} \left[ 4c_{WW}^2 m_W^2 \left( m_W^2 g_{\mu\nu} - \frac{p_{\mu} p_{\nu}}{3} \right) - \frac{3k_{WW}^2 m_H^2}{4} g_{\mu\nu} \left( m_W^2 - \frac{p^2}{3} \right) + \frac{p_{\mu}^2}{3} (p^2 g_{\mu\nu} - p_{\mu} p_{\nu}) \left( c_{WW}^2 + \frac{8k_{WW}^2}{3} \right) \right] \\
\ln \frac{\Lambda^2}{16\pi^2 v^2} \left[ -\frac{1}{3} (p^2 g_{\mu\nu} - p_{\mu} p_{\nu}) \left( m_W^2 c_W (6c_{WW} + 16k_{WW}) - (m_H^2 + m_W^2 + 3p^2) \left( \frac{3c_{WW}}{2} + \frac{k_{WW}^2}{3} \right) \right) \right]
$$

(A.4)

• $\gamma-\gamma$ correction:

$\gamma-(\gamma/H)-\gamma$ loop:

$$
\Pi_{\mu\nu} = \frac{c_{2\gamma}^2 - c_{2\gamma}^4}{2} (p^2 g_{\mu\nu} - p_{\mu} p_{\nu}) f(\Lambda^2, p^2, 0) - \ln \frac{\Lambda^2}{16\pi^2 v^2} \frac{c_{2\gamma}^2}{3} \frac{p^2}{3} (p^2 g_{\mu\nu} - p_{\mu} p_{\nu})
$$

(A.5)

$\gamma-(Z/H)-\gamma$ loop:

$$
\Pi_{\mu\nu} = \left[ \frac{c_{2\gamma}^2 - c_{2\gamma}^4}{2} - \kappa_{Z\gamma}^2 \frac{p^2}{4m_Z^2} \right] (p^2 g_{\mu\nu} - p_{\mu} p_{\nu}) f(\Lambda^2, p^2, m_Z^2) - \ln \frac{\Lambda^2}{16\pi^2 v^2} \frac{p^2}{3} (p^2 g_{\mu\nu} - p_{\mu} p_{\nu}) (c_{2\gamma}^2 + 3c_{Z\gamma}^2 k_{Z\gamma} + 3\kappa_{Z\gamma}^2)
$$

(A.6)

• $Z \rightarrow \gamma$ mixing:

$Z-(\gamma/H)-\gamma$ loop:

$$
\Pi_{\mu\nu} = \left[ \frac{c_{ZZ} c_{2\gamma} - c_{2\gamma}^4}{2} + \frac{c_{Z\gamma} k_{Z\gamma}}{4} - \frac{3}{4} (c_{Z\gamma} k_{Z\gamma} + \kappa_{Z\gamma} k_{Z\gamma}) - \kappa_{Z\gamma} k_{Z\gamma} \frac{p^2}{4m_Z^2} \right] (p^2 g_{\mu\nu} - p_{\mu} p_{\nu}) f(\Lambda^2, p^2, m_Z^2) \\
+ \ln \frac{\Lambda^2}{16\pi^2 v^2} (p^2 g_{\mu\nu} - p_{\mu} p_{\nu}) \left[ c_{2\gamma} m_Z^2 (c_{Z\gamma} + 2\kappa_{Z\gamma}) - \frac{c_{Z\gamma} k_{Z\gamma}}{4} (m_H^2 - m_Z^2 + 3p^2) \right] \\
- \frac{p^2}{6} (2c_{Z\gamma} c_{Z\gamma} + 3c_{Z\gamma} k_{Z\gamma} + 8\kappa_{Z\gamma} k_{Z\gamma})
$$

(A.7)

$Z-(\gamma/H)-\gamma$ loop:

$$
\Pi_{\mu\nu} = \left[ \frac{c_{Z\gamma} c_{2\gamma} - c_{2\gamma}^4}{2} + \frac{3c_{Z\gamma} k_{Z\gamma}}{4} \right] (p^2 g_{\mu\nu} - p_{\mu} p_{\nu}) f(\Lambda^2, p^2, 0) - \ln \frac{\Lambda^2}{16\pi^2 v^2} (p^2 g_{\mu\nu} - p_{\mu} p_{\nu}) c_{Z\gamma} \left[ \frac{c_{2\gamma} p^2}{3} + \frac{\kappa_{Z\gamma}}{4} (m_H^2 + p^2) \right]
$$

(A.8)

### A.2 Corrections to $S$, $T$, $U$

Using our previous results in the Peskin-Takeuchi parameters, using: $f(\Lambda^2, 0, 0) = \frac{1}{16\pi^2 v^2} \left( \Lambda^2 - m_H^2 \ln \Lambda^2 \right)$ and defining for a given vector boson $V$: $\epsilon(V) = (c_V^2 - 1) - 6c_V c_{VV} - 10c_V k_{VV} + 3c_{VV} k_{VV} + 10k_{VV}^2$, we are led to the following expressions:
\[\alpha U = 2 s_w^2 c_w^2 f(\Lambda^2, 0, 0) \left[ c_{\gamma\gamma} - c_{\gamma\gamma}^2 + c_{ZZ}^2 + c_{\gamma\gamma}^2 - \frac{c_w^2}{s_w c_w} (c_{\gamma\gamma} c_{\gamma\gamma} + c_{\gamma\gamma} c_{\gamma\gamma} - c_{\gamma\gamma} c_{\gamma\gamma} - c_{\gamma\gamma} c_{\gamma\gamma}) \right] \]

\[+ 2 \left( c_{\gamma\gamma} c_{\gamma\gamma} - c_{\gamma\gamma} c_{\gamma\gamma} + c_{\gamma\gamma} c_{\gamma\gamma} - \frac{c_w^2}{s_w c_w} (c_{\gamma\gamma} c_{\gamma\gamma} - c_{\gamma\gamma} c_{\gamma\gamma}) \right) \]

\[+ \frac{2 m_w^2}{3} \left( \epsilon(Z) + m_Z^2 \left( c_{\gamma\gamma}^2 - c_{\gamma\gamma}^2 - c_{\gamma\gamma}^2 \right) \right) + f(\Lambda^2, 0, 0) \left[ c_{\gamma\gamma} - c_{\gamma\gamma}^2 + 3 c_{\gamma\gamma} c_{\gamma\gamma} - 3 c_{\gamma\gamma} c_{\gamma\gamma} - 3 \frac{\kappa_{ZZ}^2 - \kappa_{WW}^2}{4} \right] \]

After removal of the power divergences in the \( S, T, U \) parameters by using relations 2.22, 2.23, 2.24 and 2.25, only the following logarithmic contributions remain present:

\[\alpha S = -s_w^2 c_w^2 \frac{\ln \Lambda^2}{8 \pi^2 v^2} m_Z^2 \left[ c_{\gamma\gamma} - c_{\gamma\gamma}^2 + c_{\gamma\gamma}^2 - \frac{c_w^2}{s_w c_w} (c_{\gamma\gamma} c_{\gamma\gamma} - c_{\gamma\gamma} c_{\gamma\gamma}) \right] \]

\[+ 2 \left( \frac{c_w^2}{3} - c_{\gamma\gamma} c_{\gamma\gamma} - c_{\gamma\gamma} c_{\gamma\gamma} \right)\]
\[ = -s_w^2 c_w \ln \tilde{\Lambda}^2 / 8\pi^2 v^2 m_Z^2 \left[ c_{\gamma \gamma}^2 - c_{Z \gamma}^2 - c_{Z \gamma}^2 + c_{Z \gamma}^2 + c_w^2 - s_w^2 \left( c_{\gamma \gamma} c_{Z \gamma} - \tilde{c}_{\gamma \gamma} \tilde{c}_{Z \gamma} \right) \right] + 2 \left( \frac{c_V^2 - 1}{3} - 2 c_V c_{\gamma \gamma} - \frac{c_w^2 - s_w^2}{s_w c_w} c_V c_{Z \gamma} \right), \]

(A.12)

\[ \alpha_T = 3 \ln \tilde{\Lambda}^2 / 16\pi^2 v^2 m_Z^2 s_w^2 (c_V^2 - 1), \]

(A.13)

\[ \alpha_U = 0. \]

(A.14)

**B About the 2D likelihoods**

The ATLAS and CMS experiments give Higgs signal rates \( \hat{\mu} \) in many forms: usually their central values with the 68% CL intervals are known, or 95% CL upper limits are given in which case we reconstruct them assuming Gaussian errors. For some decay channels (\( H \to \gamma \gamma, ZZ, WW \) and \( \tau\tau \)) they give instead 2D likelihood functions defined in the \( \hat{\mu}_{ggH+ttH} - \hat{\mu}_{VBF+VH} \) plane. Those functions encode the non-trivial correlations between the rates measured for the \( ggH/ttH \) or \( VBF/VH \) production modes.

For the \( H \to \gamma \gamma, ZZ \) and \(WW\) decay channels given by ATLAS [28, 29, 30, 31], numerical data for the 2D likelihoods are given for a limited range of signal strengths. In this case we use a basic polynomial interpolation of order 1 in the range of signal strengths of interest. However, for other channels, only the 95% CL or 68% CL contours of the 2D likelihoods are given, as in the case of \( H \to \tau\tau \) by ATLAS [32] and \( H \to \gamma \gamma, ZZ, WW \) and \( \tau\tau \) by CMS [38, 39, 40, 41].

When this happens we must build an approximate description for the 2D likelihood function: a chosen reconstruction method is described in the following. First, we fit a quadratic-cubic 2D likelihood polynomial inside the enclosed region of a given (closed) 95% CL contour in the \( \hat{\mu}_{ggH+ttH} - \hat{\mu}_{VBF+VH} \) plane, such that its section corresponding to 95% CL passes through almost all of the points of this contour. Then, this 95% CL contour is approximated by its minimal enclosing ellipse. The minimal enclosing ellipse construction is based on the original Welzl’s algorithm [54], but we use an improved version due to Gärtner et al. [55] of which an implementation was written in [56]. Finally, we compute a 2D quadratic polynomial such that its minimum coincides with the best-fit point for the given contour, and its section at 95% CL is precisely the computed minimal enclosing ellipse. The reconstructed 2D likelihood is then a piecewise function: the first part is the fitted quadratic-cubic 2D likelihood polynomial in the region \( \leq 95\% \) CL; the second part is made of all the points in the region between this contour and the minimal enclosing ellipse and are set to 95% CL. The third part consists of the fitted 2D quadratic polynomial in the region \( \geq 95\% \) CL.

It is expected from this construct that the correlations between the signal rates are better taken into account in the region \( \leq 95\% \) CL, which is circumscribed by the minimal enclosing ellipse (purple
line in the following graphs), contrary to with a simple quadratic approximation (orange lines in the graphs). Outside of this region the knowledge of the contours only is not sufficient to describe the signal strengths correlations, so we suppose that the 2D likelihood is Gaussian-like outside this region. A full qualitative study of the influence of the different contour shapes in the results of the fits is beyond the scope of this work.

Figure 5: CMS 2D likelihoods for $H \to \gamma\gamma$, $ZZ$, $WW$ and $\tau\tau$. The described construction is used.
**Figure 6:** ATLAS 2D likelihoods for $H \rightarrow \gamma\gamma$, $ZZ$, $WW$ (left), and $H \rightarrow \tau\tau$ (right). We use the numerical data provided by ATLAS for $\gamma\gamma$, $ZZ$ and $WW$, and the described construction for $\tau\tau$.

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