Effects of fermion-boson interaction in neutral atomic systems

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We investigate the collective excitations of $^3$He-$^4$He mixture films at zero temperature within random phase approximation and linear response theory. In low concentration regime of $^4$He, a level repulsion between zero sound and third sound mode is derived, which opens the possibility to observe quantum mechanical coherence between $^4$He particle-hole pair and the condensate of $^3$He. We also investigate the $^3$He-$^4$He vertex corrections in ladder approximation and show that the third branch, the combined mode of fermionic particle-hole pair and the third sound quanta, provides a unique correction to Landau $f$ function. Some implications to the fermion-boson mixture of alkali atoms in a potential trap are discussed.

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Two dimensional (2D) Fermi liquid is one of the most important subjects in condensed matter physics; they are in deep connection with high-temperature superconductivity in cuprates and alkali-doped fullerene, and with quantum Hall effect. Free surface of superfluid $^4$He bulk liquid or films provides ideal fields for 2D fermi systems free from impurities and inhomogeneities in chemical potential due to randomness of walls or substrates. In particular, phase separated $^3$He-$^4$He mixture films has been a subject of theoretical and experimental interest for the last two decades [1]. At very low temperature, it is well-known that $^3$He atoms are bound to the surface of superfluid $^4$He [2], and behave as a well-defined 2D Fermi liquid [3,4]. Therefore, it is regarded as a good candidate of 2D fermion-boson systems. The nature of $^3$He and $^4$He films depends strongly on system parameters, such as concentrations, temperature, and van der Waals potential from substrates. This richness of the parameters has provided various suggestions such as Cooper pairing [5] and dimerization [6,7,8] of $^3$He, as well as suppression of the superfluidity [9] and Casimir effect [10,11] of superfluid $^4$He films.

In this 2D fermion-boson system, $^3$He particles interact with one another through thickness variation of $^4$He films, i.e., third sound driven by van der Waals potential [12], as well as direct interactions. A fascinating feature of this system is that characteristic energies can be easily tuned by varying the thickness of $^3$He and $^4$He films continuously. Therefore, it is expected that interference between excitations is enhanced, and nonadiabatic effect becomes relevant when suitable parameters are chosen. In this paper, we report a theoretical study of response to the spectrum of collective excitations when thickness of the mixture films is varied at zero temperature. We will show that a level repulsion between zero sound and third sound mode takes place. Moreover, the third branch is derived, which we interpret as combined mode of fermionic particle-hole pair and third sound phonon, by calculating the vertex function in some detail. We also calculate the contribution of this collective excitation to Landau $f$ function.

We assume that the system forms a phase-separated double layer, i.e., normal $^3$He liquid covering $^4$He which consist of a superfluid layer and a non-superfluid “inert layer” [12]. In this system, particles are sensitive to the modulation of substrate van der Waals potential due to the thickness variation of $^3$He and $^4$He films, especially at low concentrations. We take account of this effect into interactions between particles and include other effects in hydrodynamic masses. The effective Hamiltonian which consists of third sound phonons and $^3$He quasiparticles interacting with one another and phonons was derived by one of the authors [12], and has the following form

$$ H_{\text{eff}} = \sum_{\mathbf{k},\sigma} \frac{\hbar^2 k^2}{2m_3} c_{\mathbf{k},\sigma}^\dagger c_{\mathbf{k},\sigma} + v_3 \sum_{\mathbf{q}} \rho_{\mathbf{q}}^{\uparrow} \rho_{-\mathbf{q}}^{\downarrow} + \sum_{\mathbf{q}} \hbar \omega_{\mathbf{q}} b_{\mathbf{q}}^\dagger b_{\mathbf{q}} + \sum_{\mathbf{q},\sigma} g_{\mathbf{q}} (b_{\mathbf{q}} + b_{-\mathbf{q}}^\dagger) \rho_{-\mathbf{q},\sigma}, $$

where correlation between superfluid and inert layers and surface tension of the films are neglected for simplicity. These approximation may be allowed, except the region where the structure normal to substrate becomes remarkable [13]. Here, $\rho_{\mathbf{q},\sigma} = \sum_{\mathbf{k}} c_{\mathbf{k},\sigma}^\dagger c_{\mathbf{k}+\mathbf{q},\sigma}$ is Fourier transformation of density operator and $\sigma = \uparrow, \downarrow$ and $N_4$ are spin indices and number of $^4$He atoms respectively. The spectrum of third sound phonon and coupling energies are respectively given by

$$ (\hbar \omega_{\mathbf{q}})^2 = (\hbar c_B g)^2 + \left( \frac{\hbar^2 q^2}{2m_4} \right)^2 $$

$$ v_3 = \frac{3u_3}{2n_3^2 (d + h_3 + h_4)^4 S} $$

$$ g_{\mathbf{q}} = \frac{1}{\sqrt{N_4} u_3 (d + h_3 + h_4)^4} \left( \frac{\hbar^2 q^2}{2m_4 \hbar \omega_{\mathbf{q}}} \right)^{1/2}, $$

where the subscripts 3 and 4 denote $^3$He and $^4$He; $n_i$, $u_i$ are average bulk densities and average thickness of the films, and $m_1$, $u_i$ are hydrodynamic masses and characteristic van der Waals energies. $S$ and $d$ are surface area and thickness of inert layer. In the equations above, $c_B$ is the velocity of third sound phonon which strongly depends on concentrations through $h_3$ and $h_4$, and can be
obtained by \[ \chi_3(q, \omega) = \frac{S}{(2\pi)^2} \int d^2k \frac{n(\epsilon_k - q/2) - n(\epsilon_k + q/2)}{\epsilon_k - q/2 - \epsilon_{k+q/2} + \hbar \omega + i\epsilon} \]

where \( \epsilon_k = \hbar^2 k^2/2m_3 - \epsilon_F \) with \( \epsilon_F \) being Fermi energy and \( n(\epsilon_k), v_F, N(0) = m_3S/\pi \hbar^2 \) are Fermi-Dirac distribution, Fermi velocity and density of states at Fermi surface of \(^3\text{He}\), respectively. In the equations above, terms of order \( q^4 \) are neglected since we are interested in low energy excitations.

Within the RPA treatment, the susceptibility matrix of mixture films can be obtained as a solution of \( \tilde{\chi} = \tilde{\chi}^{(0)} + \tilde{\chi}^{(0)} \tilde{V} \tilde{\chi}^{(0)} \), which is

\[
\tilde{\chi}_{\text{RPA}} = \frac{1}{1 - v_3 \chi_3 - 2g_q^2 \chi_3 \chi_4} \times \left( \begin{array}{cc} \chi_3 & g_q \chi_3 \chi_4 \\ g_q \chi_3 \chi_4 & \chi_4 (1 - v_3 \chi_3) \end{array} \right),
\]

where the factor 2 in the last term of the denominator comes as a result of summing over the spin indices. If \( g_q = 0 \), it can be easily seen that this susceptibility matrix becomes diagonal whose elements are usual RPA susceptibility, \( \chi_3/(1 - v_3 \chi_3) \), and \( \chi_4 \). Therefore, two poles of this matrix correspond to the spectrum of two eigenmodes; one is zero sound branch of \(^3\text{He}\) due to the nonlinearity of van der Waals potential, and the other is third sound branch of \(^3\text{He}\). For the third sound phonon, this RPA treatment is equivalent to solving Dyson’s equation in Migdal approximation, where effective self-energy of third sound is \( 2g_q^2 \chi_3/(1 - v_3 \chi_3) \). At zero temperature and long wave length limit, dispersion relation and damping rate are calculated numerically and the results are shown in Fig.1. Here, we employs the parameters...
$v_i$ and $d$, modeling the double layer system on the flat surface of 4He films, and alter the $^3$He concentration from a small fraction of the monolayer to one layer (a monolayer corresponds to the density 10.6 $\text{mmol/m}^2$ for $^3$He and 12.9 $\text{mmol/m}^2$ for $^4$He). One can see that these two branches exhibit a level repulsion characteristic of reactively coupled oscillators. Hence, the two branches are well-hybridized and show the clear level splitting around which nonadiabatic effects are relevant and new qualitative phenomena arise from vertex corrections. Here, we treat the vertex function in ladder approximation, i.e., as a solution of Bethe-Salpeter equation

$$\Gamma(q, i\omega_n) = g_q - k_B T \sum_{k\mu \nu} g_{k - p} \times \chi_4(p - k, ip - i\omega_n) G(k, ik_n) \times G(k - q, i\omega_n) \Gamma(q, i\omega_n), \quad (11)$$

where $G(k, ik_n)$ is a bare single-particle propagator of $^3$He quasiparticles with $ik_n$, $i\omega_n$ being fermionic and bosonic Matsubara frequency. We assume that the vertex function depends only on the energy-momentum transfer. Here, external fermion frequency $ip$ is set to zero after analytic continuation. This approximation may be justified when low energy particle-hole excitations give dominant contribution. As for the external wave vector $p$, we introduce $q_c$ as a cutoff of wave number transfer $|p - k|$, which is of order of inverse of coherence length of 4He films. At zero temperature and long wave length limit, this equation is solved analytically and the vertex function has the following form

$$\Gamma(q, \omega) = \frac{g_q}{1 + (\frac{\omega}{\Omega_c})^2 (C(q, \omega) + \frac{\omega}{\Omega_c})}, \quad (12)$$

$$C(q, \omega) = \frac{1}{2} \left( \frac{1}{1 - \left( \frac{\omega - Q_c}{q_c} \right)^2} + \frac{1}{1 - \left( \frac{\omega + Q_c}{q_c} \right)^2} \right). \quad (13)$$

Here, we neglect polarization contribution of the second term of right hand side in Eq.(11), which has the same dynamical ($\omega \rightarrow 0$, $q = 0$) and static limit ($\omega = 0$, $q \rightarrow 0$), because it gives only quantitative difference in the denominator of vertex function. Obviously, Migdal approximation breaks down qualitatively in the region $\omega \approx \nu_p q$ and the vertex function diverges to $\pm \infty$, as $\omega \rightarrow \omega_C$ from above and below respectively. Here, $\omega_C$ is obtained as

$$\omega_C \simeq \frac{1 + \frac{\lambda}{2}}{\sqrt{1 + \lambda}} \left( \frac{\nu_p q}{q_c} - \frac{\lambda(1 + \lambda^2/2)}{8(1 + \lambda/2)^2} \right). \quad (14)$$

This means that dynamical phonon-mediated $^3$He-$^4$He interaction becomes strongly enhanced and even changes its sign. Its behavior resembles Feshbach resonance in alkali atom gasses, but this frequency dependent "potential" cannot be used in a Hamiltonian formalism like a scattering length of alkali atoms since the strong frequency dependence of interaction imposes one to take the strong retardation effect into account. We expect this break down of perturbation theory not to lead to the violation of Fermi liquid theory. Indeed, the pole provides only a subdominant correction to imaginary part of single-particle self-energy that behaves as $\text{Im} \Sigma(p, \omega) \sim \lambda^3 \epsilon F(\hbar\omega/\epsilon F)^3$ for $\omega \rightarrow 0^+$ (subscript $p$ denotes the pole contribution). Moreover, nonzero quasi-particle residue $Z$ at the Fermi surface can be derived.

$$Z(\omega) = \frac{1 + \frac{\lambda}{2}}{\sqrt{1 + \lambda}} \left( \frac{\nu_p q}{q_c} - \frac{\lambda(1 + \lambda^2/2)}{8(1 + \lambda/2)^2} \right). \quad (14)$$
with use of Kramers-Kronig relation. Thus the quasiparticle is well-defined, even with the singularity in vertex function.

We shall investigate the effect of this phonon exchange vertex function to the spectrum of excitations. The singularity of $\Gamma(q,\omega)$ in long wave length limit implies the existence of low-lying excitation which obeys Bose-Einstein statistics. We interpret the pole as an excitation energy of an additional collective mode.Physically, this third branch corresponds to a combined mode of $^3$He particle-hole pair and third sound phonon. It may be considered as the pole of $D_C(q,\omega)$, which is Fourier transformation of $-\sum_\sigma(T_1(\phi_\sigma q(t)\rho_{-\sigma,q}(0)) + \rho_{\sigma,q}(t)\phi_{-\sigma}(0))$. Here, $T_i$ is usual Wick’s time ordering operator, $\phi_q(t)$ is field operator of third sound phonon, and $\langle \cdots \rangle$ denotes the average for the ground state of mixture. It should be noted that this “susceptibility” contains multiphonon processes, thus relation with area density fluctuations is no longer linear. The third spectrum obtained by numerical calculation lies above particle-hole continuum and between upper and lower branch. Furthermore, one can see from dynamical structure function $S^C(q,\omega) = -1/\pi \Im\Gamma_C$ that the combined mode branch has considerable spectral weight except the region where zero sound and third sound are well-hybridized. The behavior of this spectral weight causes the resonant nature of $S^C(q,\omega)$ at $\omega \approx c_{3q}$. Indeed, the combined mode is completely suppressed as $\omega_C \rightarrow c_{3q}$. Physically, this behavior causes the mode-mode repulsion [15].

We also investigate the correction of the combined mode to Landau Fermi liquid parameter. At zero temperature, it is obtained as $f_{\sigma',\sigma}(\theta) = \delta(\sigma_k,\delta n_\sigma)\delta n_{\sigma'}(\delta k')$ with $k \cdot k' = k^2\cos\theta$. Since the combined mode does not contribute to $f_{\uparrow\downarrow}(\theta)$, we can obtain the Landau $f$ function as follows:

$$f_{\uparrow\downarrow}(\theta) = f_{\uparrow\downarrow}(\theta) - P \int \frac{d\lambda}{\pi} \frac{\lambda \Im\Gamma(k-k',s)}{\beta k - k'}. \tag{15}$$

Here, $P$ denotes the principal value of the integral. The pole contribution, i.e. the collective mode contribution, comes from delta function part of $\Im\Gamma$, and is obtained as

$$N(0)(f^P_{\uparrow\downarrow}(\theta) - f^P_{\uparrow\downarrow}(\theta)) \approx \frac{\lambda^3}{16} \left( 1 + \frac{\lambda (v_F k_F)^2}{2 c_{Bq}\lambda} \sin^2 \left( \frac{\theta}{2} \right) \right). \tag{16}$$

In addition, contribution from the particle-hole continuum of vertex function can be obtained from the real part of $\Gamma$ with use of Kramers-Kronig relation, and has the following form

$$N(0)(f^p_{\uparrow\downarrow}(\theta) - f^p_{\uparrow\downarrow}(\theta)) \approx \frac{-\lambda}{1 + \frac{\lambda}{2} C(2k_F\sin(\frac{\theta}{2}),0)}. \tag{17}$$

These contributions are not singular. Additionally, the pole gives rise an additional factor $\lambda^2$ to correction for Landau $f$ function. Therefore, third branch contribution may become important at strong coupling region.

![FIG. 3: Dynamical structure function of the combined mode, $S^C(q,\omega)$, is plotted for three parameter regimes as a function of $\omega/q$. One can see that there exists a sharp spectral weight between third sound and zero sound, which does not occur within RPA treatment. At $\omega_C/c_{3q} = 1.4$, spectral weight of third branch is found to be about 30% of all. At $\omega_C/c_{3q} = 1.4$, spectral weight of third branch is found to be about 30% of all. At the region $\omega_C \approx c_{3q}$, the combined mode is strongly suppressed by mode-mode repulsion effect.](image)
and finite level splitting in the spectrum. Furthermore, damping of third sound, resonant decay into particle-hole pair excitation, is also shown to be significant when the third sound velocity is smaller than $v_F$. The other is that Migdal’s theorem breaks down when nonadiabatic effect of $^3$He-$^4$He interaction is considered. A third branch comes in: this may be interpreted as a combined mode of particle-hole pair and third sound quanta. The concentration dependence of dynamical structure factor and the correction to Landau Fermi liquid parameter of this collective excitation are also discussed.

In the discussions above, we have focused on low-energy collective behavior and we have not considered two-body scattering problem. It is expected that interaction between $^3$He and $^4$He induces the effective $^3$He-$^3$He attraction and leads to spin-singlet and neutral superfluid formation. We consider in future work the possibility that nonadiabatic phonon exchange may either drive the system into a superfluid transition or polaron formation.

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