Numerical simulation of the Saffman-Taylor instability in the Hele-Show cell

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Abstract. The process of displacing one viscous liquid with a less viscous one in porous medium is accompanied by folds formation on the border of liquids division. These folds called 'viscous fingers' grow with not the same velocity. This phenomenon doesn't allow extracting all oil while displacing it with water. The process of formation and growth of viscous fingers has a critical sustainability which depends on a number of parameters. One of these parameters is characteristics of imposed oscillations. Practical field experiments demonstrate the impact that imposed oscillations have on the watercut of extracted oil. Two-dimensional analogue of displacing oil with water is displacing viscous liquid from Hele-Show cell. The impact of characteristics of initial oscillation on the ‘viscous fingers’ formation and development process is studied in this paper. Dependencies of relative growth velocities of ‘viscous fingers’ on amplitude and frequency of influence are presented below.

1. Introduction
Oil extraction is characterized by low oil recovery ratio (less than 30%). One of the reasons of that is that viscous oil is extracted from porous medium by displacing it with water (a liquid which is less viscous than oil). Physically this process may be called Saffman-Taylor instability [1]. With a displacement like that a border of two liquids division is deformed and forms a few folds. These folds grow with not the same velocity, forming so-called ‘viscous fingers’. ‘Viscous fingers’ of water after reaching extracting wells make a breakthrough, rising a watercut of extracted oil drastically. Sections that do not participate in filtration are formed in the stratum. Obviously, Saffman-Taylor instability is defined by properties of moving liquid and porous medium. To solve this problem of oil extracting industry, different gels are used instead of water (liquids that are more viscous than oil or solutions of surface-active substances which provide decrease in surface tension). The main idea of this paper is that sustainability of the process can be ruled with an influence from the outside. Selecting an appropriate influence mode allows to achieve sustainability of the process of displacing a high-viscosity liquid with the one with lower viscosity. To solve this problem, knowledge of instability development and external factors influence on the process of formation and growth of ‘viscous fingers’ is necessary. Computer technology development allows to broaden the area of solving problems with moving borders. The paper [2] presents the idea of displaying time and space so that borders can develop exponentially fast in a new time scale with a constant square. With a number of
different flows in Hele-Show cell as an example, the possibility of an accurate modeling of dynamics of slow flows is demonstrated. A group of accurate solutions providing linear stability of division border between two liquids with different viscosities in Hele-Show cell is defined in the paper [3]. A short review of condition in the area of Saffman-Taylor instability shows an active research of this phenomenon in different forms by different scientists groups. The question of ruling instability by external influence is barely examined at the moment.

The influence of elastic vibrations on productive layers and studies of intensification of oil extraction due to using wave influence on layers started in the middle of XX century. The paper [4] presents quite full review of techniques of using wave effects for oil extraction in both trade and laboratory conditions. A research conducted previously by the authors of this paper has shown that the influence of elastic vibrations allows not only to increase oil output, but also to decrease its watercut [5]. This circumstance is an indirect confirmation of the impact that external influence has on the process of displacing oil with water.

2. Formulation of the problem

One of the techniques of researching the process of displacing oil with water in laboratory conditions is examining the displacement of viscous liquid with the one that has a lower viscosity in Hele-Show cell. Hele-Show cell consists of two parallel plates with the distance between them much smaller than width of the plate. In this paper we are examining the process of displacing viscous oil with water by conducting numerical modeling. The problem of this paper is studying the impact that oscillations of liquid division border have on the process of formation and development of ‘viscous fingers’.

As an explored liquid we used motor oil which was being displaced by water in Hele-Show cell in room conditions. The preparation stage of research included measuring viscosity of a researched motor oil of grade 10W40 with room temperature on vibrations viscosimeter. Calculated 3D models of Hele-Show cell were created in software SolidWorks. The initial oscillation of liquid division border was described by the equation of entering border, which water was going through to the cell, as

\[ x = A \cos 2\pi \frac{t}{T} \]

where \( A \) and \( T \) are amplitude and period of oscillations. Scheme of the cell is depicted in Fig. 1. Cells were created with the same sizes of width \( h=176 \text{ mm} \), length \( l=400 \text{ mm} \), thickness \( d=0.81 \text{ mm} \) and different \( A \) and \( T \).

![Figure 1. Calculation area](image)

Numerical modelling was conducted in software FlowVision. A mathematical model was used – laminar flow, described by Navier-Stokes equations and equation of continuity. The displacing liquid is pure water with temperature 20 °C and the displaced liquid is motor oil with viscosity \( \mu_0 = 133 \text{ mPa}\cdot\text{s} \) with the same temperature. Boundary conditions: at the inlet velocity of water is \( U_0 = 3 \text{ mm/s} \), at the outlet there is free exit, condition of adhesion was set on the rest of borders. The results of
numerical calculations are pictures of distribution of liquid concentration in the plane of flow, according to which the process of formation and growth of ‘viscous fingers’ can be easily visualized.

3. Results of numerical studies
To process the data it was decided to examine the dependence of growth velocity of the longest ‘viscous finger’ on amplitude and oscillation period, that was imposed on liquid division border. The longest finger out of obtained pictures of casting at every specified moment was chosen and its length was measured. Considering that liquid started being supplied at the moment \( t=0 \), then growth velocity of the longest finger can be calculated as \( U=\frac{l_f}{t} \) where \( l_f \) is length of the longest ‘viscous fingers’ at the moment \( t \). Average velocity \( U_{avg}=\frac{l_{max}}{t} \), where \( l_{max} \) is maximum length of the ‘viscous fingers’ and \( t \) is the time that finger took to reach maximum length. As a result of the modeling process different pictures of casting were obtained depending on parameters of supplied division of oscillation on the border. As an example, Fig. 2 presents pictures of liquid concentration distribution at the same time for the unperturbed (upper image) and the disturbed (bottom image) initial interface of liquids. Oil is shown in blue color and water is red. Obtained calculated data indicate that initial oscillations are the source of formation and growth of ‘viscous fingers’. In the majority of cases as viscous fingers grow, the new ones are formed at their tops. With further growth the closest ‘viscous fingers’ get blended into the one. Its growth velocity increases. In Fig. 3 shows an example of the difference in the development of the instability for different periods of the initial perturbation of the liquid interface.

Figure 2. The development of instability without an initial perturbation and under perturbation
Figures 4 and 5 present dependencies of changes of growth velocity of 'viscous fingers' on frequency of initial oscillation and on amplitude. It is visible that the bigger amplitude of oscillation, the higher growth velocity of 'viscous finger' (Fig. 5). A particularly visible dependency of changes of growth velocity of 'viscous fingers' on action period is not found (Fig. 4). It probably has to do with the fact that they blend together as a result of growth and a smaller number of 'viscous fingers' participates in liquid motion.
4. Conclusion
The dependencies of ‘viscous fingers’ growth were studied based on conducted series of numerical experiment with displacing oil with water in Hele-Show cell. It was established that characteristics of initial oscillation have an impact mainly on the process of ‘viscous fingers’ formation and almost do not influence their further development. It was found that at initial stages of experiment growth velocity of ‘viscous fingers’ increases acutely but after reaching relative temperature $t=0.1$, change of velocity is minor. In researched ranges of varied parameters relative growth velocity of ‘viscous fingers’ changes from 2 to 4.

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References
[1] Saffman P and Taylor G 1958 Proc. Roy. Soc. London. Ser. A. 245 (1242) 312-29.
[2] Li S, Lowengrub J and Leo P 2007 Journal of Computational Physics. 225 (1) 554-67
[3] Alvarez-Lacalle E, Ortin J and Casademunt J 2004 Physical Review Letters 92 (5) 545011-545014
[4] Beresnev I and Johnson P 1994 Geophysics 59 1000-1017
[5] Marfin E, Kravtsov Y, Abdrashitov A, Gataullin R and Galimzyanova A 2015 Pet. Sci. Tech. 33(15) 1526-1532