Relativistic effects in the double S- and P-wave charmonium production in $e^+e^-$ annihilation

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On the basis of perturbative QCD and the relativistic quark model we calculate relativistic and bound state corrections in the production processes of a pair of S-wave and P-wave charmonium states. Relativistic factors in the production amplitude connected with the relative motion of heavy quarks and the transformation law of the bound state wave function to the reference frame of the moving S- and P-wave mesons are taken into account. For the gluon and quark propagators entering the production vertex function we use a truncated expansion in the ratio of the relative quark momenta to the center-of-mass energy $\sqrt{s}$ up to the second order. The exact relativistic treatment of the wave functions makes all such second order terms convergent, thus allowing the reliable calculation of their contributions to the production cross section. Relativistic corrections to the quark bound state wave functions in the rest frame are considered by means of the Breit-like potential. It turns out that the examined effects change essentially the nonrelativistic results of the cross section for the reaction

$e^+ + e^- \rightarrow J/\Psi(\eta_c) + \chi_{cJ}(h_c)$

at the center-of-mass energy $\sqrt{s} = 10.6$ GeV.

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I. INTRODUCTION

The large rate for the exclusive double charmonium production measured at the Belle and BaBar experiments [1, 2] reveals definite problems in the theoretical description of these processes [3, 4, 5]. Many theoretical efforts were made in order to improve the calculation of the production cross section $e^+ + e^- \rightarrow J/\Psi + \eta_c$. They included the analysis of other production mechanisms for the state $J/\Psi + \eta_c$ [6, 7] and the calculation of different corrections which could change essentially the initial nonrelativistic result [8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18]. Despite the evident successes achieved on the basis of NRQCD, the light cone method, quark potential models in order to resolve the discrepancy between the theory and experiment, the double charmonium production in $e^+e^-$ annihilation remains an interesting task. On the one hand, the reason is that there exist the production processes of the P- and D-wave charmonium states which should be investigated as the production of S-wave states. On the other hand, the variety of the used approaches and the model parameters in this problem raises the question about the comparison of the obtained results resulting in a better understanding of the quark-gluon dynamics. Two sources of the enhancement of the nonrelativistic cross section for the double charmonium production are revealed to the present: the radiative corrections of order $O(\alpha_s)$ and relative motion of c-quarks forming...
The production amplitude of a pair of S- and P-wave charmonium states in $e^+e^-$ annihilation. S denotes the S-state meson and P the P-wave meson. The wavy line shows the virtual photon and the dashed line corresponds to the gluon. $\Gamma$ is the production vertex function.

In this work we continue the investigation of the exclusive double charmonium production in $e^+e^-$ annihilation on the basis of a relativistic quark model [14, 19, 20, 21] in the case of S- and P-wave charmonium states. The relativistic quark model provides the solution in many tasks of heavy quark physics. In particular, it gives the possibility to study the question about a broadening of the meson wave functions due to the account of special corrections which can lead to the increase of the double charmonium production cross sections. Thus, the aim of this study consists in the calculation of the relativistic effects in the processes $e^+e^- \rightarrow J/\Psi(\eta_c) + \chi_{cJ}(h_c)$ on the basis of a relativistic approach to the quarkonium production suggested in Refs. [14, 19].

II. GENERAL FORMALISM

We consider the following reactions $e^+e^- \rightarrow J/\Psi(\eta_c) + \chi_{cJ}(h_c)$, where the final state consists of the pair of S-wave ($J/\Psi$ or $\eta_c$) and P-wave ($\chi_{c0}$, $\chi_{c1}$, $\chi_{c2}$ or $h_c$) charm mesons. The diagrams that give contributions to the amplitude of these processes in the leading order of the QCD coupling constant $\alpha_s$ are presented in Fig.1. Two other diagrams can be obtained by corresponding permutations. There are two stages of the production process. In the first stage, which is described by perturbative QCD, the virtual photon $\gamma^*$ produces four heavy c-quarks and $\bar{c}$-antiquarks with the following four-momenta:

$$p_{1,2} = \frac{1}{2} P \pm p, \quad (p \cdot P) = 0; \quad q_{1,2} = \frac{1}{2} Q \pm q, \quad (q \cdot Q) = 0,$$

where $P(Q)$ are the total four-momenta, $p = L_P(0, p)$, $q = L_P(0, q)$ are the relative four-momenta obtained from the rest frame four-momenta $(0, p)$ and $(0, q)$ by the Lorentz transformation to the system moving with the momenta $P, Q$. In the second nonperturbative stage, quark-antiquark pairs form the final mesons.

Let consider the production amplitude of the S-wave vector state ($J/\Psi$) and P-wave states $\chi_{cJ}$ ($J=0,1,2$), which can be presented in the form [14, 21, 22, 23]:

$$M(p_-, p_+, P, Q) = \frac{8\pi^2 \alpha \alpha_s Q_c}{3s} \bar{v}(p_+)\gamma^\beta u(p_-) \int \frac{dp}{(2\pi)^3} \int \frac{dq}{(2\pi)^3} \times$$

$$\times Sp \left\{ \Psi^S(p, P) \Gamma_1^{\beta\nu}(p, q, P, Q) \Psi^P(q, Q) \gamma_\nu + \Psi^P(p, Q) \Gamma_2^{\beta\nu}(p, q, P, Q) \Psi^S(p, P) \gamma_\nu \right\},$$

where $\alpha_s$ is the QCD coupling constant, $\alpha$ is the fine structure constant and $Q_c$ is the $c$ quark electric charge. The relativistic S- and P-wave functions of the bound quarks $\Psi^{S,P}$
accounting for the transformation from the rest frame to the moving one with four momenta $P, Q$, are

$$\Psi^S(p, P) = \frac{\Psi_0^S(p)}{\frac{\epsilon(p)}{m} + \frac{\epsilon(p)\epsilon(p + m)}{2m}} \left[ \frac{\hat{v}_1 - \hat{q}_1 + m}{2} + \hat{v}_1 \frac{p^2}{2m(\epsilon(p) + m)} - \frac{\hat{p}}{2m} \right],$$

$$\Psi^P(q, Q) = \frac{\Psi_0^P(q)}{\frac{\epsilon(q)}{m} + \frac{\epsilon(q)\epsilon(q + m)}{2m}} \left[ \frac{\hat{v}_2 - \hat{q}_2 + m}{2} + \hat{v}_2 \frac{q^2}{2m(\epsilon(q) + m)} + \frac{\hat{q}}{2m} \right],$$

where $v_1 = P/M_S, v_2 = Q/M_P; \varepsilon_S$ is the polarization vector of the vector charmonium $J/\Psi$; $\varepsilon_P(Q, S_z)$ is the polarization vector of the spin-triplet state $\chi_{cJ}, \epsilon(p) = \sqrt{p^2 + m^2}$ and $m$ is the $c$ quark mass. At the leading order in $\alpha_s$, the vertex functions $\Gamma^{\mu}_1(p, P; q, Q)$ can be written as

$$\Gamma^{\mu}_1(p, P; q, Q) = \frac{\gamma_{\mu}(\hat{l} - \hat{q}_1 + m)}{(l - q_1)^2 - m^2 + i\epsilon} \gamma_{\beta} D^{\mu\nu}(k_1) + \frac{\gamma_{\beta}(\hat{p}_1 - \hat{l} + m)}{(l - p_1)^2 - m^2 + i\epsilon} \gamma_{\mu} D^{\mu\nu}(k_1),$$

$$\Gamma^{\mu}_2(p, P; q, Q) = \frac{\gamma_{\beta}(\hat{q}_2 - \hat{l} + m)}{(l - q_2)^2 - m^2 + i\epsilon} \gamma_{\mu} D^{\mu\nu}(k_2) + \frac{\gamma_{\mu}(\hat{l} - \hat{p}_2 + m)}{(l - p_2)^2 - m^2 + i\epsilon} \gamma_{\beta} D^{\mu\nu}(k_2),$$

where the gluon momenta are $k_1 = p_1 + q_1, k_2 = p_2 + q_2$ and $l^2 = s = (P + Q)^2 = (p_- + p_+)^2, p_-, p_+$ are four momenta of the electron and positron. The dependence on the relative momenta of $c$-quarks is presented both in the gluon propagator $D_{\mu\nu}(k)$ and quark propagators as well as in the relativistic wave functions (3), (4). Taking into account that the ratio of the relative quark momenta $p$ and $q$ to the energy $\sqrt{s}$ is small, we expand the inverse denominators of quark and gluon propagators as follows:

$$\frac{1}{(l - q_{1,2})^2 - m^2} = \frac{2}{s} \left[ 1 - \frac{2M^2_{\perp} - M^2_{\parallel} - 4m^2}{2s} - \frac{2q^2}{s} \pm \frac{4(lq)}{s^2} + \frac{16(lq)^2}{s^4} + \cdots \right],$$

$$\frac{1}{(l - p_{1,2})^2 - m^2} = \frac{2}{s} \left[ 1 - \frac{2M^2_{\perp} - M^2_{\parallel} - 4m^2}{2s} - \frac{2p^2}{s} \pm \frac{4(lp)}{s^2} + \frac{16(lp)^2}{s^4} + \cdots \right],$$

$$\frac{1}{k^2_{\perp,1}} = \frac{4}{s} \left[ 1 - \frac{4(p^2 + q^2 + 2pq)}{s} \pm \frac{4(lp + lq)}{s} + \frac{16}{s^2} [(lp)^2 + (lq)^2 + 2(lp)(lq)] + \cdots \right].$$

In the expansions (7)-(9) we accounted for terms of second order in relative momenta $p$ and third order in relative momenta $q$. Substituting (7)-(9), (3)-(4) in (2) we preserve relativistic factors entering the denominators of the relativistic wave functions (3)-(4), but in the numerator of the amplitude (2) we take into account corrections of second order in $|p|/m$ and up to fourth order in $|q|/m$. This provides the convergence of the resulting momentum integrals. Then the angular integrals are calculated using the following relations:

$$\int p_\mu p_\nu \Psi_0^S(p) \frac{dp}{(2\pi)^3} = -\frac{\sqrt{4\pi}}{3} (g_{\mu\nu} - v_1 \mu v_1 \nu) \int_0^\infty p^4 R_S(p) dp,$$
\[
\int q_\mu \Psi_0^P(q) \frac{d\mathbf{q}}{(2\pi)^3} = -i \varepsilon_{\mu} \rho(Q, L_z) \frac{1}{\pi \sqrt{6}} \int_0^\infty q^3 R_\rho(q) dq,
\]
(11)
\[
\int q_\alpha q_\beta q_\gamma \Psi_0^P(q) \frac{d\mathbf{q}}{(2\pi)^3} = \frac{i}{5 \pi \sqrt{6}} \varepsilon(Q, L_z) P_{\alpha \beta} + \varepsilon_{\alpha}(Q, L_z) P_{\gamma \beta} + \varepsilon_{\beta}(Q, L_z) P_{\alpha \gamma} \right] \int_0^\infty q^5 R_\rho(q) dq,
\]
(12)
where \( P_{\alpha \beta} = (g_{\alpha \beta} - v_2 \alpha v_2 \beta) \), \( R_S(p) \), \( R_P(q) \) are the radial momentum wave functions of S- and P-wave charmonium states, \( \varepsilon_{\mu}(Q, L_z) \) is the polarization vector in orbital space. The integrals in (10) and (12) look formally divergent, but the original momentum integrals contain also definite relativistic factors which lead to their convergence. For a specific P-wave state, summing over \( S_z \) and \( L_z \) in the amplitude (2) can be further simplified as [24]

\[
\sum_{S_z, L_z} <1, L_z; 1, S_z| J, J_z > \varepsilon_{\alpha}(Q, L_z) \varepsilon_{\beta}(Q, S_z) = \begin{cases} 
\frac{1}{\sqrt{2}} (g_{\alpha \beta} - v_2 \alpha v_2 \beta), & J = 0, \\
\frac{1}{\sqrt{2}} \varepsilon_{\alpha \beta \rho} v_2^{\rho} \varepsilon_{\alpha}(Q, J_z), & J = 1, \\
\varepsilon_{\alpha \beta}(Q, J_z), & J = 2,
\end{cases}
\]
(13)
where \( <1, L_z; 1, S_z| J, J_z > \) are the Clebsch-Gordon coefficients. Calculating the trace in the amplitude (2) by means of expressions (3)-(6), (13) and the system FORM [25], we find that the tensor parts of the four amplitudes describing the production of S- and P-wave charmonium states in the used approximation have the following structure:

\[
S_{1,\beta}(J/\Psi + \chi_{c0}) = A_1 \varepsilon_S^\star \beta + A_2 v_1 \beta (v_2 \varepsilon_S^\star) + A_3 v_2 \beta (v_2 \varepsilon_S^\star),
\]
(14)
\[
S_{2,\beta}(J/\Psi + \chi_{c1}) = B_1 \varepsilon_{\alpha \lambda \gamma \sigma} v_1^{\alpha} v_2^{\lambda} \varepsilon_S^\star \gamma(Q, J_z) (v_2 \varepsilon_S^\star) + B_2 \varepsilon_{\alpha \lambda \gamma \sigma} v_2^{\lambda} \varepsilon_S^\star \gamma \varepsilon_S^\star \gamma(Q, J_z) + B_3 v_1 \beta \varepsilon_{\alpha \lambda \gamma \sigma} v_2^{\lambda} \varepsilon_S^\star \gamma \varepsilon_S^\star \gamma \varepsilon_S^\star \gamma(Q, J_z),
\]
(15)
\[
S_{3,\beta}(J/\Psi + \chi_{c2}) = \varepsilon_{\alpha \sigma}(Q, J_z) [C_1 \varepsilon_S^\star \alpha g_{\sigma \beta} + C_2 v_1^{\alpha} \varepsilon_S^\star \gamma g_{\sigma \beta} + C_3 v_2^{\alpha} \varepsilon_S^\star \gamma v_1 \beta + C_4 v_2^{\alpha} \varepsilon_S^\star \gamma v_2 \beta + C_5 v_1^{\alpha} \varepsilon_S^\star \gamma v_2^{\alpha} + C_6 v_2^{\alpha} \varepsilon_S^\star \gamma v_2^{\alpha} (v_2 \varepsilon_S^\star)],
\]
(16)
\[
S_{4,\beta}(\eta_c + h_c) = D_1 v_1 \beta (v_2 \varepsilon_S^\star (Q, L_z)) + D_2 v_2 \beta (v_2 \varepsilon_S^\star (Q, L_z)) + D_3 \varepsilon_S^\star (Q, L_z),
\]
(17)
where the coefficients \( A_i, B_i, C_i, D_i \) can be presented as sums of terms containing the factors \( u = M_P/(M_P + M_S) \), \( \kappa = m/(M_P + M_S) \) and \( C_{ij} = c'(p)c'(q) = (|m - \varepsilon(p)|/(m + \varepsilon(p)))^2[(m - \varepsilon(q))/(m + \varepsilon(q))]^2 \), preserving terms with \( i + j \leq 2 \), and \( r^2 = (M_P + M_S)^2/s \) up to terms of order \( O(r^4) \). Exact analytical expressions for these coefficients are sufficiently lengthy (compare with the results written in Appendix A of our previous paper [19]), so, we present in Appendix A of this work only their approximate numerical form using the observed meson masses and the c-quark mass \( m = 1.55 \text{ GeV} \).

Introducing the scattering angle \( \theta \) between the electron momentum \( \mathbf{p}_e \) and the momentum \( \mathbf{P} \) of the \( J/\Psi \) meson, we can calculate the differential cross section \( d\sigma/d\cos\theta \) and then the total cross section \( \sigma \) as a function of \( r^2 \). We find it useful to present the charmonium production cross sections in the following form \( (k = 0, 1, 2, 3 \) corresponds to \( \chi_{c0}, \chi_{c1}, \chi_{c2} \) and \( h_c)\):

\[
\sigma(J/\Psi(\eta_c) + \chi_{cJ}(h_c)) = \frac{\alpha_s^2 \alpha_s^2 Q_c^2 \pi r^2 \sqrt{1 - r^2} \sqrt{1 - r^2(2u - 1)}}{6912 \kappa^2 u^9(1 - u)^9} \left| \tilde{R}_S(0) \right|^2 \left| \tilde{R}_P(0) \right|^2 s(M_P + M_S)^8 \sum_{i=0}^{7} F_i^{(k)}(r^2) \omega_i,
\]
(18)
where the functions $F^{i(k)}$ (k=0,1,2,3) are written explicitly in Appendix B,

$$
\tilde{R}_S(0) = \frac{1}{2\pi^2} \int_0^\infty p^2 R_S(p) \frac{(\epsilon(p) + m)}{2\epsilon(p)} dp,
$$

$$
\tilde{R}'_P(0) = \frac{1}{3\sqrt{2/\pi}} \int_0^\infty q^3 R_P(q) \frac{(\epsilon(q) + m)}{2\epsilon(q)} dq.
$$

The parameters $\omega_i$ can be expressed in terms of momentum integrals $I_n$, $J_n$ as follows:

$$
I_n = \int_0^\infty p^2 R_S(p) \frac{(\epsilon(p) + m)}{2\epsilon(p)} \left( \frac{m - \epsilon(p)}{m + \epsilon(p)} \right)^n dp,
J_n = \int_0^\infty q^3 R_P(q) \frac{(\epsilon(q) + m)}{2\epsilon(q)} \left( \frac{m - \epsilon(q)}{m + \epsilon(q)} \right)^n dq,
$$

$$
\omega_0 = 1, \quad \omega_1 = \frac{I_1}{I_0}, \quad \omega_2 = \frac{I_2}{I_0}, \quad \omega_3 = \omega_1^2, \quad \omega_4 = \frac{J_1}{J_0}, \quad \omega_5 = \frac{J_2}{J_0}, \quad \omega_6 = \omega_4^2, \quad \omega_7 = \omega_1\omega_4.
$$

On the one side, in the potential quark model the relativistic corrections, connected with the relative motion of heavy c-quarks, enter the production amplitude (2) and the cross section (18) through the different relativistic factors. They are determined in the final expression (18) by the specific parameters $\omega_i$. The momentum integrals which determine the parameters $\omega_i$ are convergent and we calculate them numerically, using the wave functions obtained by the numerical solution of the Schrödinger equation. The exact form of the wave functions $\Psi_0^S(p)$ and $\Psi_0^P(q)$ is important for improving the accuracy of the calculation of the relativistic effects. It is sufficient to note that the double charmonium production cross section $\sigma(s)$ in the nonrelativistic approximation contains the factor $|R_S(0)|^2|R_P'(0)|^2$. Small changes of the numerical values of the bound state wave functions at the origin lead to substantial changes of the final results. In the approach based on nonrelativistic QCD this problem is closely related to the determination of the color-singlet matrix elements for the charmonium [26]. Thus, on the other side, there are relativistic corrections to the bound state wave functions $\Psi_0^S(p)$, $\Psi_0^P(q)$. In order to take them into account, we suppose that the dynamics of a cc-pair is determined by the QCD generalization of the standard Breit Hamiltonian [27, 28, 29]:

$$
H = H_0 + \Delta U_1 + \Delta U_2, \quad H_0 = 2\sqrt{p^2 + m^2} - 2m - \frac{C_F\alpha_s}{r} + Ar + B,
$$

$$
\Delta U_1(r) = -\frac{C_F\alpha_s^2}{4\pi r} \left[ 2\beta_0 \ln(\mu r) + a_1 + 2\gamma_E \beta_0 \right], \quad a_1 = \frac{31}{3} - \frac{10}{9} n_f, \quad \beta_0 = 11 - \frac{2}{3} n_f,
$$

$$
\Delta U_2(r) = -\frac{C_F\alpha_s}{2m^2 r} \left[ p^2 + \frac{r(r)p}{r^2} \right] + \frac{\pi C_F\alpha_s}{m^2} \delta(r) + \frac{3C_F\alpha_s}{2m^2 r^3} (SL) - \frac{C_F\alpha_s}{2m^2} \left[ \frac{S^2}{r^3} - \frac{3(Sr)^2}{r^5} - \frac{4\pi}{3}(2S^2 - 3)\delta(r) \right] - \frac{C_A C_F\alpha_s^2}{2m^2 r^2},
$$

where $n_f$ is the number of flavors, $C_A = 3$ and $C_F = 4/3$ are the color factors of the SU(3) color group. For the dependence of the QCD coupling constant $\alpha_s(\mu^2)$ on the renormalization point $\mu^2$ we use the leading order result

$$
\alpha_s(\mu^2) = \frac{4\pi}{\beta_0 \ln(\mu^2/\Lambda^2)}.
$$
The typical momentum transfer scale in a quarkonium is of order of the quark mass, so we set the renormalization scale $\mu = m$ and $\Lambda = 0.168$ GeV, which gives $\alpha_s = 0.314$ for the charmonium states. The parameters of the linear potential $A = 0.18$ GeV$^2$ and $B = -0.16$ GeV have usual values of quark models. Starting with the Hamiltonian (23) we construct the effective potential model based on the Schrödinger equation and find its numerical solutions in the case of S- and P-wave charmonium [30]. The details of the used model are presented in Appendix C. Then we calculate the matrix elements entering in the expressions for the parameters $\omega_i$ and obtain the value of the production cross sections at $\sqrt{s}=10.6$ GeV. Basic parameters which determine our numerical results are collected in Table I. The comparison of the obtained results with the previous calculations [3, 4, 31, 32] and experimental data [1, 2] is presented in Table II.

### III. NUMERICAL RESULTS AND DISCUSSION

In this paper we have investigated the role of relativistic effects in the production processes of S- and P-wave mesons ($c\bar{c}$) in the quark model. In the present study of the production amplitude (2) we kept relativistic corrections of two types. The first type is determined by several functions depending on the relative quark momenta $p$ and $q$ arising from the gluon propagator, the quark propagator and the relativistic meson wave functions. The second type of corrections originates from the perturbative treatment of the quark-antiquark interaction operator which leads to the different wave functions $\Psi_0^S(p)$ and $\Psi_0^P(q)$ for the S-wave and P-wave charmonium states, respectively. In addition, we systematically accounted for the bound state corrections working with the observed masses of S-wave mesons ($J/\Psi, \eta_c$) and P-wave mesons ($\chi_{cJ}, h_c$). The calculated masses of S-wave and P-wave charmonium states agree well with experimental values [33] (see Table III). Note that the basic parameters of the model are kept fixed from the previous calculations of the meson mass spectra and decay widths [21, 34, 35]. The strong coupling constant entering the production amplitude (2) is taken to be $\alpha_s=0.24$ in accordance with the relation (26) at $\mu = 2m$.

Numerical results and their comparison with several previous calculations and experimental data are presented in Table II. Theoretically, there were two studies of the production $J/\Psi(\eta_c) + \chi_{cJ}(h_c)$ in $e^+e^-$ annihiliation in NRQCD [3, 4]. They give 2.4 fb and 6.7 fb for the production of $J/\Psi + \chi_{c0}$. Such spread in results is explained by the different numerical values of the used parameters, i.e. the matrix elements, the mass of c-quark $m$ and the

| Meson ($c\bar{c}$) | $n^{2S+1}L_J$ | $J^{PC}$ | $\tilde{R}_S(0)$, GeV$^{3/2}$ | $\tilde{R}_P(0)$, GeV$^{5/2}$ | $\omega_1(S)$ or $\omega_4(P)$ | $\omega_2(S)$ or $\omega_5(P)$ |
|-------------------|----------------|--------|----------------|----------------|----------------|----------------|
| $J/\Psi$          | $1^3S_1$       | 1$^{--}$ | 0.81           | —              | -0.20          | 0.0078         |
| $\eta_c$          | $1^1S_0$       | 0$^{--}$ | 0.92           | —              | -0.20          | 0.0087         |
| $\chi_{c0}$       | $1^3P_0$       | 0$^{++}$ | —              | 0.19           | -0.15          | 0.0065         |
| $\chi_{c1}$       | $1^3P_1$       | 1$^{++}$ | —              | 0.18           | -0.14          | 0.0065         |
| $\chi_{c2}$       | $1^3P_2$       | 2$^{++}$ | —              | 0.18           | -0.15          | 0.0065         |
| $h_c$             | $1^1P_1$       | 1$^{+-}$ | —              | 0.18           | -0.14          | 0.0065         |
strong coupling constant $\alpha_s$. The third investigation of the process $J/\Psi + \chi_{c0}$ was done in the light-front formalism in $[31]$ where the result 14.4 fb was obtained. The essential growth of the cross section in $[31]$ at $\sqrt{s} = 10.6$ GeV is connected with the use of specific light cone wave functions describing the relative motion of heavy $c$-quarks. The fourth study of the reaction $e^+ + e^- \rightarrow J/\Psi + \chi_{c0}$ was devoted to the next-to-leading order QCD corrections $[32]$. Here it was shown that a sharp increase of the production cross section ($\sigma = 17.9$ fb) can be derived with the account of NLO in $\alpha_s$ contributions.

The exclusive double charmonium production cross section presented in the form (18) is convenient for a comparison with the results of NRQCD. Indeed, in the nonrelativistic limit, when $u = 1/2$, $\kappa = 1/4$, $\omega_i = 0 \ (i \geq 1)$, $r^2 = 16 m^2 / s$, the cross section (18) coincides with the calculation in $[3]$. In this limit the functions $F_0^{(k)}(r^2)$ transform into corresponding functions $F_k$ from $[3]$. When we take into account bound state corrections working with observed meson masses, we get $u = M_P/(M_P + M_S) \neq 1/2$, $\kappa = m/(M_P + M_S) \neq 1/4$. This leads to the modification of the general factor in (18) in comparison with the nonrelativistic theory and the form of the functions $F_0^{(k)}$ (see $[3]$). It follows from the numerical values of the parameters $\omega_i$, presented in Table I, that the relativistic corrections amount to $15 \div 20\%$ in the production amplitude. Moreover, the relativistic effects decrease the values of the parameters $R_S(0)$, $R_P(0)$, which transform into $\tilde{R}_S(0)$, $\tilde{R}_P(0)$. In all considered reactions $e^+ + e^- \rightarrow J/\Psi(\eta_c) + \chi_{cJ}(h_c)$ the relativistic effects increase the nonrelativistic cross section, but in the case of the production $\eta_c + h_c$ the sum of bound state plus relativistic corrections decreases the nonrelativistic cross section. It is necessary to point out once again that the essential effect on the value of the production cross sections $J/\Psi(\eta_c) + \chi_{cJ}(h_c)$ belongs to the parameters $\tilde{R}_S(0)$, $\tilde{R}_P(0)$, $\alpha_s, m$. Small changes in their values can lead to significant changes for the production cross sections. Comparing the values of the parameters $R_S(0)$ ($|R_{J/\Psi,\eta_c}(0)|^2 = 0.9; 1.2 GeV^{3/2}$), $R_P(0)$ ($|R_P(0)|^2 = 0.043 GeV^{5/2}$) obtained in this study on the basis of quark model and in $[32]$ we see that the values of the radial wave functions at the origin are very close, but our value for the derivative of the radial wave function at the origin is slightly smaller. The calculation of radiative corrections $O(\alpha_s)$ to the nonrelativistic cross section of the production $J/\Psi + \chi_{c0}$ was done recently in $[32]$. It evidently shows that one-loop corrections are considerable (factor $K = 2.8$ to nonrelativistic result). As a result, the total value of the cross section $J/\Psi + \chi_{c0}$ significantly increases. It should be noted that the difference between the theory and both BaBar and Belle experiments became threatening.

We presented a systematic treatment of relativistic effects in the S- and P-wave double charmonium production in $e^+ e^-$ annihilation. We explicitly separated two different types of relativistic contributions to the production amplitudes. The first type includes the relativistic $v/c$ corrections to the wave functions and their relativistic transformations. The second type includes the relativistic $p/\sqrt{s}$ corrections emerging from the expansion of the quark and gluon propagators. The latter corrections were taken into account up to the second order. It is important to note that the expansion parameter $p/\sqrt{s}$ is very small. In our analysis of the production amplitudes we correctly take into account relativistic contributions of order $O(v^2/c^2)$ for the S-wave meson and corrections of orders $O(v^2/c^2)$ and $O(v^4/c^4)$ for the P-wave mesons. We cannot keep corrections of order $O(v^4/c^4)$ for the S-wave part of the amplitude (2) because they become divergent if we use expansions (7)-(9). Therefore the basic theoretical uncertainty of our calculation is connected with the omitted terms of order $O(p^4/m^4)$. Taking into account that the average value of the heavy quark velocity squared in the charmonium is $<v^2> = 0.3$, we expect that they should not exceed 30% of the obtained relativistic contribution. These theoretical errors in the calculated production
TABLE II: Comparison of the obtained results with previous theoretical predictions and experimental data.

| State    | $\sigma_{\text{BaBar}}$ | $\sigma_{\text{Belle}}$ | $\sigma_{\text{NRQCD}}$ | $\sigma_{(fb)}$ | $\sigma_{(fb)}$ | $\sigma_{(fb)}$ | Our result $\sigma_{(fb)}$ |
|----------|-------------------------|-------------------------|--------------------------|----------------|----------------|----------------|---------------------------|
| $J/\Psi + \chi_{c0}$ | $10.3 \pm 2.5_{-1.8}^{+1.4}$ | $6.4 \pm 1.7 \pm 1.0$ | $2.40 \pm 1.02$ | $6.7$ | $14.4$ | $17.9_{6.35}$ | $4.79 \pm 0.80$ |
| $J/\Psi + \chi_{c1}$ | $0.38 \pm 0.12$ | $1.1$ | $1.07 \pm 0.23$ |
| $J/\Psi + \chi_{c2}$ | $0.69 \pm 0.13$ | $1.6$ | $1.10 \pm 0.13$ |
| $\eta_c + h_c$ | $0.308 \pm 0.017$ | | $0.24 \pm 0.02$ |

The cross section at $\sqrt{s} = 10.6$ GeV are shown directly in Table II. We have neglected the terms in the cross section (18) containing the product of $I_n$ and $J_n$ with summary index $\geq 2$ because their contribution has been found negligibly small. There are no another comparable uncertainties related to the choice of $m$ or any other parameters of the model, since their values were fixed from our previous consideration of meson and baryon properties [21, 34].

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APPENDIX A: THE COEFFICIENTS $A_i$, $B_i$, $C_i$, $D_i$ ENTERING IN THE PRODUCTION AMPLITUDES (14)-(17)

These coefficients are the sums of the terms containing the parameters $u = M_P/(M_P + M_S)$ and $\kappa = m/(M_P + M_S)$. We present $A_i$, $B_i$, $C_i$, $D_i$ in numerical form using the observed meson masses and the mass of c-quark $m = 1.55$ GeV.

\[ e^+ + e^- \rightarrow J/\Psi + \chi_{c0} \]

\[
A_1 = -7.05 + \frac{18.55}{r^2} + 0.013 r^2 + C_{20}(7.05 - \frac{18.55}{r^2} + 0.013 r^2) + \\
+C_{02}(5.42 - \frac{5.41}{r^2} - 0.013 r^2) + C_{10}(68.16 - \frac{63.92}{r^2} - 12.15 r^2 + 0.089 r^4) + \\
+C_{01}(23.29 - \frac{13.71}{r^2} - 8.11 r^2 + 0.066 r^4) + C_{11}(-76.83 + \frac{41.48}{r^2} + 62.06 r^2 - 12.66 r^4),
\]

\[
A_2 = -8.35 + 2.34 r^2 + C_{20}(8.35 - 2.34 r^2) + C_{02}(1.80 - 2.34 r^2) + \\
+C_{10}(30.38 - 26.93 r^2 + 4.00 r^4) + C_{01}(5.96 - 9.62 r^2 + 2.69 r^4) + C_{11}(-19.22 + 36.18 r^2 - 23.80 r^4),
\]

\[
A_3 = -0.99 + 0.99 r^2 + C_{20}(0.99 - 0.99 r^2) + C_{02}(1.98 - 0.99 r^2) + \\
+C_{10}(1.65 - 9.72 r^2 + 1.50 r^4) + C_{01}(0.97 - 2.21 r^2 + 0.94 r^4) + C_{11}(-1.62 + 4.02 r^2 - 5.24 r^4).
\]
\[ e^+ + e^- \rightarrow J/\Psi + \chi_c \]

\[ B_1 = -0.002 + 2.66r^2 + C_{20}(0.002 - 2.66r^2) - 2.66r^2C_{02} + \]
\[ +C_{10}(-6.17 - 14.23r^2 + 4.59r^4) + C_{01}(-4.62 - 7.14r^2 + 3.13r^4) + C_{11}(11.56 + 28.13r^2 - 27.18r^4), \]
\[ B_2 = -2.10 - \frac{1.004}{r^2} + C_{20}(2.10 - \frac{1.004}{r^2}) + C_{10}(14.99 - \frac{12.39}{r^2} - 3.34r^2 + 0.016r^4) + \]
\[ +C_{01}(7.29 - \frac{9.28}{r^2} - 0.83r^2 + 0.01r^4) + C_{11}(-24.44 + \frac{23.21}{r^2} + 10.13r^2 - 0.98r^4), \]
\[ B_3 = -1.63r^2 + 1.63r^2C_{20} + 1.63r^2C_{02} + C_{10}(10.64r^2 - 2.87r^4) + \]
\[ +C_{01}(5.43r^2 - 1.20r^4) + C_{11}(-16.41r^2 + 13.44r^4), \]
\[ B_4 = -0.002 + 0.82r^2 + C_{20}(0.002 - 0.82r^2) - 0.82r^2C_{02} + \]
\[ +C_{10}(-6.17 - 1.23r^2 + 1.77r^4) + C_{11}(11.56 + 1.35r^2 - 11.46r^4). \]

\[ e^+ + e^- \rightarrow J/\Psi + \chi_c \]

\[ C_1 = -1.84 + 1.84C_{20} + C_{10}(7.50 - 1.87r^2 + 0.01r^4) + \]
\[ +C_{01}(3.72 - 0.97r^2 + 0.02r^4) + C_{11}(-23.15 + 11.46r^2 - 0.95r^4), \]
\[ C_2 = 2.63r^2 - 2.63r^2C_{20} - 2.63r^2C_{02} + C_{10}(-13.28r^2 + 4.55r^4) + \]
\[ +C_{01}(-7.72r^2 + 2.96r^4) + C_{11}(44.33r^2 - 30.95r^4), \]
\[ C_3 = 1.59r^2 - 1.59r^2C_{20} - 1.59r^2C_{02} + C_{10}(-5.58r^2 + 2.85r^4) + \]
\[ +C_{01}(-3.91r^2 + 1.10r^4) + C_{11}(12.99r^2 - 14.29r^4), \]
\[ C_4 = -0.80r^2 - 0.80r^2C_{20} + 0.80r^2C_{02} + C_{10}(1.41r^2 - 1.75r^4) + \]
\[ +C_{01}(4.02r^2 - 1.51r^4) + C_{11}(-4.43r^2 + 12.19r^4), \]
\[ C_5 = -2.39r^2 - 2.39r^2C_{20} + 2.39r^2C_{02} + C_{10}(6.91r^2 - 4.33r^4) + \]
\[ +C_{01}(10.29r^2 - 2.98r^4) + C_{11}(-32.09r^2 + 32.60r^4), \]
\[ C_6 = -2.37r^4C_{10} + C_{11}r^2(-12.59r^2 + 1.65r^4), C_7 = 3.29r^4C_{10} + C_{11}r^2(-12.59r^2 + 1.65r^4). \]

\[ e^+ + e^- \rightarrow \eta_c + h_c \]

\[ D_1 = -1.58 + 1.29r^2 + C_{20}(1.58 - 1.29r^2) + C_{02}(1.58 - 1.29r^2) + \]
\[ +C_{10}(0.70 - 9.33r^2 + 2.97r^4) + C_{01}(1.35 - 5.29r^2 + 1.01r^4) + C_{11}(-0.60 + 19.42r^2 - 16.51r^4), \]
\[ D_2 = 0.14 - 1.14r^2 + C_{20}(-0.14 + 1.14r^2) + C_{02}(-3.73 + 1.14r^2) + \]
\[ +C_{10}(7.11 + 2.77r^2 + 2.37r^4) + C_{01}(-0.12 + 4.72r^2 - 1.54r^4) + C_{11}(-6.08 - 3.69r^2 - 12.92r^4), \]
\[ D_3 = -1.49 + \frac{3.47}{r^2} + C_{20}(1.49 - \frac{3.47}{r^2}) + C_{10}(12.21 - \frac{15.99}{r^2} - 2.35r^2 + 0.02r^4) + \]
\[ +C_{01}(6.76 - \frac{2.97}{r^2} - 0.80r^2 + 0.02r^4) + C_{11}(-32.42 + \frac{13.67}{r^2} + 13.22r^2 - 1.35r^4). \]
APPENDIX B: THE FUNCTIONS $F_i^{(k)}(r^2)$ (K=0,1,2,3) ENTERING IN THE PRODUCTION CROSS SECTION (18)

\[ e^+ + e^- \rightarrow J/\Psi + X_{c0} \]

\[ F_0^{(0)} = 12.94r^2 + 690.48r^4 - 391.86r^6 + 39.82r^8 + 6.36r^{10}, \]
\[ F_1^{(0)} = -43.18r^2 - 4554.04r^4 + 5941.64r^6 - 1942.48r^8 + 89.61r^{10}, \]
\[ F_2^{(0)} = -25.87r^2 - 1380.96r^4 + 783.71r^6 - 79.64r^8 - 12.72r^{10}, \]
\[ F_3^{(0)} = 36.04r^2 + 7634.68r^4 - 15478.8r^6 + 10011.2r^8 - 2095.47r^{10}, \]
\[ F_4^{(0)} = -25.29r^2 - 996.50r^4 + 1961.67r^6 - 957.35r^8 + 97.10r^{10}, \]
\[ F_5^{(0)} = -38.81r^2 - 447.77r^4 + 601.11r^6 - 101.74r^8 - 12.87r^{10}, \]
\[ F_6^{(0)} = 12.36r^2 + 349.41r^4 - 1216.3r^6 + 1370.5r^8 - 579.05r^{10}, \]
\[ F_7^{(0)} = 84.44r^2 + 6382.47r^4 - 15011.8r^6 + 13175.8r^8 - 4056.79r^{10}. \]

\[ e^+ + e^- \rightarrow J/\Psi + X_{c1} \]

\[ F_0^{(1)} = 165.06r^4 - 248.34r^6 + 75.88r^8 + 17.33r^{10}, \]
\[ F_1^{(1)} = -1655.33r^4 + 3347.61r^6 - 1829.98r^8 + 80.46r^{10}, \]
\[ F_2^{(1)} = -330.12r^4 + 496.69r^6 - 151.76r^8 - 34.67r^{10}, \]
\[ F_3^{(1)} = 4263.79r^4 - 10572.2r^6 + 8306.62r^8 - 1953.72r^{10}, \]
\[ F_4^{(1)} = -752.14r^4 + 1660.5r^6 - 979.66r^8 + 60.00r^{10}, \]
\[ F_5^{(1)} = -237.01r^4 + 390.21r^6 - 117.85r^8 - 35.64r^{10}, \]
\[ F_6^{(1)} = 920.55r^4 - 2520.4r^6 + 2230.78r^8 - 612.50r^{10}, \]
\[ F_7^{(1)} = 7012.32r^4 - 18599.5r^6 + 15704.7r^8 - 4122.93r^{10}. \]

\[ e^+ + e^- \rightarrow J/\Psi + X_{c2} \]

\[ F_0^{(2)} = 23.41r^2 + 99.63r^4 - 247.93r^6 + 110.30r^8 + 27.28r^{10}, \]
\[ F_1^{(2)} = -78.15r^2 - 931.82r^4 + 2499.9r^6 - 1804.84r^8 + 137.34r^{10}, \]
\[ F_2^{(2)} = -46.82r^2 - 199.27r^4 + 495.86r^6 - 220.60r^8 - 54.56r^{10}, \]
\[ F_3^{(2)} = 65.23r^2 + 1952.35r^4 - 5759.03r^6 + 5433.52r^8 - 1541.06r^{10}, \]
\[ F_4^{(2)} = -94.75r^2 - 1014.54r^4 + 2475.82r^6 - 1509.41r^8 + 50.08r^{10}, \]
\[ F_5^{(2)} = -70.23r^2 - 213.25r^4 + 608.04r^6 - 269.46r^8 - 55.66r^{10}, \]
\[ F_6^{(2)} = 95.89r^2 + 2191.11r^4 - 5632.06r^6 + 4350.3r^8 - 954.85r^{10}, \]
\[ F_7^{(2)} = 316.34r^2 + 7972.42r^4 - 22933.8r^6 + 20774.7r^8 - 5658.04r^{10}. \]
\[ e^+ + e^- \rightarrow \eta_c + h_c \]

\[ F_0^{(3)} = 11.69r^2 - 27.29r^4 + 35.00r^6 - 22.48r^8 + 6.02r^{10}, \quad (B25) \]

\[ F_1^{(3)} = -10.35r^2 + 41.40r^4 - 185.00r^6 + 250.24r^8 - 141.63r^{10}, \quad (B26) \]

\[ F_2^{(3)} = -23.375r^2 + 54.57r^4 - 70.00r^6 + 44.97r^8 - 12.04r^{10}, \quad (B27) \]

\[ F_3^{(3)} = 2.29r^2 + 88.53r^4 + 5.76r^6 - 348.17r^8 + 451.27r^{10}, \quad (B28) \]

\[ F_4^{(3)} = -19.99r^2 + 118.20r^4 - 215.81r^6 + 199.84r^8 - 86.92r^{10}, \quad (B29) \]

\[ F_5^{(3)} = -35.06r^2 + 101.73r^4 - 115.65r^6 + 61.33r^8 - 12.52r^{10}, \quad (B30) \]

\[ F_6^{(3)} = 8.55r^2 - 81.12r^4 + 268.40r^6 - 363.38r^8 + 234.48r^{10}, \quad (B31) \]

\[ F_7^{(3)} = 17.70r^2 - 263.94r^4 + 959.68r^6 - 1671.7r^8 + 1317.3r^{10}. \quad (B32) \]

**APPENDIX C: EFFECTIVE RELATIVISTIC HAMILTONIAN**

For the calculation of the relativistic corrections in the bound state wave functions \( \Psi_0^S \), \( \Psi_0^P \), we consider the Breit potential (23). It contains a number of terms which should be transformed in order to use the program of numerical solution of the Schrödinger equation \[30\]. The rationalization of the kinetic energy operator can be done in the following form \[36\]:

\[ T = 2\sqrt{\mathbf{p}^2 + m^2} = 2\frac{\mathbf{P}^2 + m^2}{\sqrt{\mathbf{P}^2 + m^2}} \approx \frac{\mathbf{P}^2}{\tilde{m}} + \frac{2m^2}{E}, \quad (C1) \]

where \( \tilde{m} \) is the effective mass of heavy quarks,

\[ \tilde{m} = \frac{E}{2} = \sqrt{\mathbf{p}_{eff}^2 + m^2}. \quad (C2) \]

\( \mathbf{p}_{eff}^2 \) should be considered as a new parameter which effectively accounts for relativistic corrections in (C1). Numerical values of \( \mathbf{p}_{eff}^2 \) for S- and P-wave charmonium states discussed in \[13, 18, 20\] are presented in Table III. In the case of S-wave states it is necessary to transform the \( \delta \)-like terms of the potential. For this aim, we use the known smeared \( \delta \)-function of the Gaussian form \[37\]:

\[ \tilde{\delta}(\mathbf{r}) = \frac{b^3}{\pi^{3/2}} e^{-b^2r^2} \quad (C3) \]

with the additional parameter \( b \) which defines the hyperfine splitting in the \((c\bar{c})\) system. Since the numerical results are practically not dependent on \( b \) in the range of commonly used values \( 1.5 \div 2.2 \), we take \( b = 1.5 \) GeV. The second term in the Breit potential (23), which also has to be transformed, takes the form:

\[ \Delta U = -\frac{2\alpha_s}{3m^2r} \left[ \frac{\mathbf{P}^2}{d^2dr^2} \right]. \quad (C4) \]
TABLE III: The parameters of the effective relativistic Hamiltonian.

| Meson ($c\bar{c}$) | $n^{2S+1}L_J$ | $p_{\text{eff}}^2$, $\text{GeV}^2$ | $\bar{n}$, $\text{GeV}$ | $E$, $\text{GeV}$ | $\beta$, $\text{GeV}$ | $b$, $\text{GeV}$ | $M^\text{th}$, $\text{GeV}$ | $M^\text{exp}$, $\text{GeV}$ |
|---------------------|----------------|-----------------|-----------------|----------------|----------------|----------------|----------------|----------------|
| $J/\Psi$            | $1^3S_1$       | 0.5             | 0.85            | 0.087          | 0.75           | 1.5            | 3.044          | 3.097          |
| $\eta_c$           | $1^1S_0$       | 0.5             | 0.85            | 0.087          | 0.75           | 1.5            | 2.989          | 2.980          |
| $\chi_{c0}$        | $1^3P_0$       | 0.6             | 0.87            | 0.479          | 0.55           | --             | 3.437          | 3.415          |
| $\chi_{c1}$        | $1^3P_1$       | 0.6             | 0.87            | 0.479          | 0.55           | --             | 3.479          | 3.511          |
| $\chi_{c2}$        | $1^3P_2$       | 0.6             | 0.87            | 0.479          | 0.55           | --             | 3.520          | 3.556          |
| $h_c$              | $1^1P_1$       | 0.6             | 0.87            | 0.479          | 0.55           | --             | 3.486          | 3.526          |

In order to replace it by the effective term containing the power-like potentials, we use the approximate charmonium wave functions which can be written for S- and P-wave states as

$$\Psi^S_0(r) = \frac{\beta^{3/2}}{\pi^{3/4}} e^{-\frac{1}{2} \beta^2 r^2}, \quad \Psi^P_0(r) = \sqrt{\frac{8}{3}} \frac{\beta^{3/2}}{\pi^{3/4}} \beta r e^{-\frac{1}{2} \beta^2 r^2} Y_{1m}(\theta, \phi).$$  \hspace{1cm} (C5)

The wave functions (C5) give a good approximation of the true quark bound state wave functions in the region of nonrelativistic momenta. Using (C5), we transform (C4) as follows:

$$\Delta \tilde{U} \rightarrow \Delta \tilde{U}^\text{eff} = -\frac{2\alpha_s}{3m} \left( (mE - mB + \beta^2) - \frac{8\alpha_s^2}{9mr^2} + \frac{2\alpha_s A}{3m} + \frac{2\alpha_s^4}{3m^2} r \right),$$  \hspace{1cm} (C6)

where $E$ is the bound state energy of quarks which can be obtained from the Schrödinger equation with the Hamiltonian $H_0$. In order to derive (C6) we changed the operator $\mathbf{p}^2$ by its nonrelativistic expression: $\mathbf{p}^2 \Psi_0 = m[E + \frac{4\alpha_s}{3r} - A r - B] \Psi_0$. As a result of such transformations the potential $\Delta U_2$ takes the following form in the case of S-states:

$$\Delta U_2(r) = \frac{20\pi \alpha_s}{9m^2} \frac{b^3}{\pi^{3/2}} e^{-\frac{b^2 r^2}{2m}} - \frac{2\alpha_s}{3m^2} (mE - mB + \beta^2) - \frac{26\alpha_s^2}{9mr^2} + \frac{2\alpha_s A}{3m} + \frac{2\alpha_s^4}{3m^2} r,$$  \hspace{1cm} (C7)

$\eta_c$-meson:

$$\Delta U_2(r) = \frac{4\pi \alpha_s}{3m^2} \frac{b^3}{\pi^{3/2}} e^{-\frac{b^2 r^2}{2m}} - \frac{2\alpha_s}{3m^2} (mE - mB + \beta^2) - \frac{26\alpha_s^2}{9mr^2} + \frac{2\alpha_s A}{3m} + \frac{2\alpha_s^4}{3m^2} r.$$  \hspace{1cm} (C8)

A similar transformation of the Breit Hamiltonian can be done for the P-wave states. In Table III we present the results of the calculation of the charmonium mass spectrum and a comparison with the existing experimental data. The obtained masses agree with the experimental ones within an accuracy $1 \div 2$ per cent. So, we can suppose that the obtained effective Hamiltonian allows to account relativistic corrections in the bound state wave functions with sufficiently good accuracy.

[1] K. Abe, et al., Phys. Rev. D 70, 071102 (2004).
[2] B. Aubert, et al., Phys. Rev. D 72, 031101 (2005).
[3] E. Braaten, J. Lee, Phys. Rev. D 67, 054007 (2003); Phys. Rev. D 72, 099901(E) (2005).
[4] K.-Y. Liu, Z.-G. He, K.-T. Chao, Phys. Lett. B 557, 45 (2003).
[5] K. Hagiwara, E. Kou, C.-F. Qiao, Phys. Lett. B 570, 39 (2003).
[6] G.T. Bodwin, J. Lee, E. Braaten, Phys. Rev. Lett. 90, 162001 (2003).
[7] S.J. Brodsky, A.S. Goldhaber, J. Lee, Phys. Rev. Lett. 91, 112001 (2003).
[8] K.-Y. Liu, Z.-G. He, K.-T. Chao, Phys. Rev. D 77, 014002 (2008).
[9] J.P. Ma, Z.G. Si, Phys. Rev. D 70, 074007 (2004).
[10] A.E. Bondar, V.L. Chernyak, Phys. Lett. B 612, 215 (2005).
[11] V.V. Braguta, A.K. Likhoded, A.V. Luchinsky, Phys. Rev. D 72, 074019 (2005).
[12] Y.-J. Zhang, Y.-J. Gao, K.-T. Chao, Phys. Rev. Lett. 96, 092001 (2006).
[13] G.T. Bodwin, D. Kang, J. Lee, Phys.Rev. D 74, 114028 (2006).
[14] D. Ebert, A.P. Martynenko, Phys. Rev. D 74, 054008 (2006).
[15] H.-M. Choi, Ch.-R. Ji, Phys. Rev. D 76, 094010, (2007).
[16] Z.-G. He, Y. Fan, K.-T. Chao, Phys. Rev. D 75, 074011 (2007).
[17] A.V. Berezhnoy, Phys. Atom. Nucl. 71, 1803 (2007).
[18] G.T. Bodwin, J. Lee, Ch. Yu, Phys. Rev. D77, 094018 (2008).
[19] D. Ebert, R.N. Faustov, V.O. Galkin, A.P. Martynenko, Phys. Lett. B672, 264 (2009).
[20] A.P. Martynenko, Phys. Rev. D 72, 074022 (2005).
[21] D. Ebert, R.N. Faustov, V.O. Galkin, A.P. Martynenko, Phys. Rev. D 70, 014018 (2004).
[22] R.N. Faustov, Ann. Phys. 78, 176 (1973).
[23] S.J. Brodsky, J.R. Primack, Ann. Phys. 52, 315 (1969).
[24] J.H. Kuhn, J. Kaplan, El J. O. Safiani, Nucl. Phys. B157, 125 (1979).
[25] J.A.M. Vermaseren, FORM, e-preprint [math-ph/0010025].
[26] G.T. Bodwin, E. Braaten, G.P. Lepage, Phys. Rev. D51, 1125 (1995).
[27] N. Brambilla, A. Pineda, J. Soto, A. Vairo, Rev. Mod. Phys. 77, 1423 (2005).
[28] B. Kniehl, A.A. Penin, V.A. Smirnov, M. Steinhauser, Nucl. Phys. B635, 357 (2002).
[29] K. Melnikov, A. Yelkhovsky, Phys. Rev. D59, 114009 (1999).
[30] P. Falkensteiner, H. Grosse, F.F. Schöberl, P. Hertel, Comp. Phys. Comm. 34, 287 (1985).
[31] V.V. Braguta, A.K. Likhoded, A.V. Luchinsky, Phys. Lett. B635, 299 (2006).
[32] Y.-J. Zhang, Y.-Q. Ma, K.-T. Chao, Phys. Rev. D78, 054006 (2008).
[33] Particle Data Group, J. Phys. G 33, 1 (2006).
[34] D. Ebert, R.N. Faustov, V.O. Galkin, Phys. Rev. D67, 014027 (2002).
[35] N. Brambilla et al. Heavy Quarkonium Physics, FERMILAB Report, Report No. FERMILAB-FN-0779, CERN Yellow Report, Report No. CERN-2005-005.
[36] W. Lucha, F.F. Schöberl, M. Moser, Preprint HEPHY-PUB 594/93.
[37] I.M. Narodetskii, Yu.A. Simonov, V.P. Yurov, Yad. Fiz. 55, 2818 (1992).