The gravitational lens effect of galaxy clusters can produce large arcs from source galaxies in their background. Typical source redshifts of $\sim 1$ require clusters at $z \sim 0.3$ for arcs to form efficiently. Given the cluster abundance at the present epoch, the fewer clusters exist at $z \sim 0.3$ the higher $\Omega_0$ is, because the formation epoch of galaxy clusters strongly depends on $\Omega_0$. In addition, at fixed $\Omega_0$, clusters are less concentrated, and hence less efficient lenses, when the cosmological constant is positive, $\Omega_\Lambda > 0$. Numerical cluster simulations show that the expected number of arcs on the sky is indeed a sensitive function of $\Omega_0$ and $\Omega_\Lambda$. The numerical results are compatible with the statistics of observed arcs only in a universe with low matter density, $\Omega_0 \sim 0.3$, and zero cosmological constant. Other models fail by one or two orders of magnitude, rendering arc statistics a sensitive probe for cosmological parameters.

1 Introduction

Given the average mass of a rich galaxy cluster, $M_c$, and the spatial number density of such clusters in our neighbourhood, $n_c$, we can find the fraction of cosmic material contained in rich clusters today,

$$F'_c = \frac{M_c n_c}{\rho_c \Omega_0} \approx \frac{9 \times 10^{-3}}{\Omega_0},$$

where $\rho_c$ is the critical cosmic matter density, and $\Omega_0$ is the usual density parameter. The ansatz by Press & Schechter (1974) asserts that the fraction of cosmic material accumulated by clusters at redshift $z$ is

$$F_c(z) = \frac{1}{2} \text{erfc} \left( \frac{\delta_c}{\sqrt{2} \sigma_c D(z)} \right),$$

where $\sigma_c$ is the rms density fluctuation on the linear cluster scale today, and $D(z)$ is the growth factor for cosmic structures, normalized to $D(0) = 1$. $\delta_c$ is the linear density contrast of a spherical top-hat perturbation at collapse time, and erfc($x$) is the complementary error function. Demanding $F_c(0) = F'_c$, $\sigma_c$ is fixed to the local cluster abundance. The evolution of $F_c(z)$ with redshift then depends on cosmology because the growth factor $D(z)$ does. This leads
to the well-known results that (i) clusters form late in cosmic history, and (ii) cluster formation is significantly delayed in high-density compared to low-density universes (e.g. N. Bahcall, these proceedings).

The ability of a galaxy cluster to act as a strong gravitational lens (i.e., to produce arcs) depends on the geometry of the lens system. Let $D_{\text{eff}} = D_d D_{ds} D_s^{-1}$ be the effective lens distance, with $D_d, D_{ds}, D_s$ the angular-diameter distances between observer and lens, lens and source, and observer and source, respectively. $D_{\text{eff}}$ is a measure for the lensing efficiency of a given mass distribution. $D_{\text{eff}}$ peaks at $z \sim 0.2 - 0.3$ for sources at a typical redshift $z_s \sim 1$, quite independent of cosmology.

It follows that an efficient formation of arcs requires that there be sufficiently many clusters in place and compact enough at redshifts $z \sim 0.2 - 0.3$. This establishes the link between arc statistics and cosmology. Quite obviously, the number of efficient cluster lenses per unit redshift is estimated by

$$\frac{dN_{\text{lens}}}{dz} \propto F_c(z) \times (1 + z)^3 \times D_{\text{eff}}^2 \times \left| \frac{dV(z)}{dz} \right|,$$

where the square on $D_{\text{eff}}$ approximates the dependence of the lensing cross section on $D_{\text{eff}}$, and $dV(z)$ is the proper cosmic volume of a spherical shell of radius $z$ and width $dz$. Figure 1 illustrates $dN_{\text{lens}}/dz$ as a function of redshift for $\Omega_0 = 1$ and $\Omega_0 = 0.3$, both for $\Omega_\Lambda = 0$. Evidently, there is a huge difference of about two orders of magnitude, clusters in the low-density universe being much more efficient in producing arcs than in the Einstein-de Sitter universe. This straightforward argument leads one to expect that the number of observed arcs could be a sensitive discriminator between cosmological models.

Fig. 1 — Estimate of the number of lenses per unit redshift, $dN_{\text{lens}}/dz$, as defined in the text, for high- and low-density universes. Note the logarithmic scale of the ordinate. The curves illustrate that the number of efficient lenses is expected to be lower by about two orders of magnitude in high compared to the low-density model.
2 Simulations

We use numerical simulations to quantify the expected effect more precisely. Clusters are taken from four different cosmological simulations, the parameters of which are summarized in Tab. 1. The models indexed by 1 were kindly supplied by the GIF collaboration (cf. S.D.M. White or J.M. Colberg, these proceedings), those indexed by 2 were simulated with a different numerical algorithm. In total, we use nine simulated clusters for (S,Λ,O)CDM, and five for τCDM.

| Model     | Ω₀  | Ωₐ | h    | σₘ₀ | Γ  |
|-----------|-----|-----|------|-----|----|
| SCDM1     | 1.0 | 0.0 | 0.5  | 0.60| 0.50|
| τCDM1     | 1.0 | 0.0 | 0.5  | 0.60| 0.21|
| ΛCDM1     | 0.3 | 0.7 | 0.7  | 0.90| 0.21|
| OCDM1     | 0.3 | 0.0 | 0.7  | 0.85| 0.21|
| SCDM2     | 1.0 | 0.0 | 0.5  | 0.60| 0.50|
| ΛCDM2     | 0.3 | 0.7 | 0.7  | 1.12| 0.21|
| OCDM2     | 0.3 | 0.0 | 0.7  | 1.12| 0.21|

Tab. 1 — Summary of the parameters used for the cluster simulations. h is the Hubble constant in units of 100 km s⁻¹ Mpc⁻¹, Γ is the shape parameter of the power spectrum, and the other parameters have their conventional meaning.

Each cluster is studied at ten time steps between redshifts 1 and 0, projecting it along each of the three independent spatial directions, ending up with roughly $10^3$ lensing mass distributions. For each of these, the arc cross section is computed numerically, mapping elliptical sources at redshift $z_s = 1$ that are placed on an adaptive grid tracing the caustic curves of the lenses. In total, we classify all images of about $1.3 \times 10^6$ sources. This procedure yields arc cross sections as a function of cluster redshift, $\sigma(z)$, for the four cosmological models used. The arc optical depth, i.e., the probability for a source to be imaged as an arc with specified properties, is then given by a volume-weighted integral of $\sigma(z)$ over redshift, multiplied by the cluster number density $n_c$ and divided by the area of the source sphere.

3 Results

The optical depth normalized by the cluster number density, $n_c^{-1}\tau$, is shown in Fig. 2 for the models (S,Λ,O)CDM. The result for the τCDM model is almost identical to that for the SCDM model, and is therefore omitted. In summary, the optical depth for large arcs, i.e. such with a length-to-width ratio $\geq 10$, is highest for the OCDM model and lowest for the SCDM model, with differences of about an order of magnitude between each of the models.

Combining the optical depth with the number densities of observed clusters and of appropriately bright sources, we find that the number of arcs on the
whole sky expected from our simulations is

\[ N_{\text{arcs}} \sim \begin{cases} 2400 & \text{OCDM} \\ 280 & \text{ΛCDM} \\ 36 & \text{SCDM} \end{cases} \quad (4) \]

The observed number of arcs, estimated from the EMSS arc survey and extrapolated to the whole sky, falls within 1500 – 2300. We therefore conclude that the only of our cosmological models for which the expected number of arcs comes near the observed number is the open CDM model. The others fail by one or two orders of magnitude. This result can be understood by (i) the delayed cluster formation in high-Ω_0 universes, combined with lensing efficiency, and (ii) the higher concentration of clusters in low-density models without Ω_Λ compared to such with Ω_Λ > 0, combined with the sensitivity of lensing to compactness. For details, see Bartelmann et al. (1997).

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