SU(3) gauge theory with 12 flavours in a twisted box

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Collaborators

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*Till November 2013
Strategy/challenges for the lattice search of IRFP

- Spectrum: Large finite-volume effects.
- Finite-size scaling \( a'la \) M. Fisher: universal curves.
- Running coupling: (slow) running within error.

\[
\frac{g^2(L)}{g(L)}
\]

\[
\frac{g^2(2L)}{g(L)}
\]

\[
\frac{g^2(3L/2)}{g(L)}
\]
The Twisted Polyakov Loop scheme

C.-J.D.L., K.Ogawa, H.Ohki, E.Shintani, 2012

K.Ogawa, lattice 2013

It is challenging to draw any conclusion from such a “noisy scheme”. 

without (L/a=12 → L/a = 24) 

systematics severely underestimated...

with (L/a=12 → L/a=24)
Outline

• Step scaling.
• Twisted box.
• Wilson flow and numerical results.
• Outlook.
The step-scaling method
The practice

- Massless unimproved staggered fermions with Wilson’s plaquette gauge action.
- Compute $g_{\text{lat}}^2$ at many $g_0^2$ for each volume, and then interpolate volume by volume.
- Very challenging to pin down percentage-level effects in $r_\sigma = \frac{\sigma(u)}{u}$.
Twisted box
removing the torons, no odd powers in g.

• **Gauge field:**

\[ U_\mu(x + \hat{v}L) = \Omega_\nu U_\mu(x) \Omega_\nu^\dagger, \quad \nu = 1, 2, \]

where the twist matrices \( \Omega_\nu \) satisfy

\[ \Omega_1 \Omega_2 = e^{2i\pi/3} \Omega_2 \Omega_1, \quad \Omega_\mu \Omega_\mu^\dagger = 1, \quad (\Omega_\mu)^3 = 1, \quad \text{Tr}(\Omega_\mu) = 0. \]

• **Fermion:** If \( \psi(x + \hat{v}L) = \Omega_\nu \psi(x) \)

\[ \Rightarrow \psi(x + \hat{v}L + \hat{\rho}L) = \Omega_\rho \Omega_\nu \psi(x) \neq \Omega_\nu \Omega_\rho \psi(x) \]

• **The fermion “smell” dof:** \( N_s = N_c \)

\[ \psi^{a}_\alpha(x + \hat{v}L) = e^{i\pi/3} \Omega^{ab}_\nu \psi^b_\beta(x)(\Omega_\nu)^\dagger_{\beta\alpha}. \]
The Gradient Flow scheme

- The quantity, $\langle E(t) \rangle = \frac{1}{4} \langle G_{\mu\nu}(t) G_{\mu\nu}(t) \rangle$, is finite when expressed in terms of renormalised coupling at positive flow time.

- In a colour-twisted box, can define,

$$\bar{g}_{GF}^2(L) = \mathcal{N}^{-1} t^2 \langle E(t) \rangle = \bar{g}_{MS}^2 + \mathcal{O}(\bar{g}_{MS}^4),$$

with tree-level improvement.

- Use the clover operator, as well as the plaquette, to extract $\langle E(t) \rangle$.

- Step scaling at fixed $c_\tau = \frac{\sqrt{8t}}{L}$.
Bare-coupling interpolation

NDP fit of the clover coupling

\[
g^2_{\text{latt}} \quad \text{vs} \quad g^2_0
\]

- \(L/a = 12, N_{\text{deg}} = 10\)
- \(L/a = 6, N_{\text{deg}} = 11\)
- \(c_r = 0.400\)

\[
g^2_{\text{latt}} \quad \text{vs} \quad g^2_0
\]

- \(L/a = 16, N_{\text{deg}} = 10\)
- \(L/a = 8, N_{\text{deg}} = 12\)
- \(c_r = 0.400\)

\[
g^2_{\text{latt}} \quad \text{vs} \quad g^2_0
\]

- \(L/a = 20, N_{\text{deg}} = 14\)
- \(L/a = 10, N_{\text{deg}} = 7\)
- \(c_r = 0.400\)

\[
g^2_{\text{latt}} \quad \text{vs} \quad g^2_0
\]

- \(L/a = 24, N_{\text{deg}} = 9\)
- \(L/a = 12, N_{\text{deg}} = 10\)
- \(c_r = 0.400\)
Bare-coupling interpolation

NDP fit of the plaquette coupling

\[ c_\tau = 0.250 \]

\[ \frac{g^2}{g^2_{\text{latt}}} \text{plaquett coupling} \]

\[ g_0^2 \]

\[ \text{L/a} = 12, N_{\text{deg}} = 10 \]
\[ \text{L/a} = 6, N_{\text{deg}} = 7 \]

\[ \text{L/a} = 16, N_{\text{deg}} = 12 \]
\[ \text{L/a} = 8, N_{\text{deg}} = 10 \]

\[ c_\tau = 0.250 \]

\[ \frac{g^2}{g^2_{\text{latt}}} \text{plaquett coupling} \]

\[ g_0^2 \]

\[ \text{L/a} = 20, N_{\text{deg}} = 14 \]
\[ \text{L/a} = 10, N_{\text{deg}} = 7 \]

\[ \text{L/a} = 24, N_{\text{deg}} = 10 \]
\[ \text{L/a} = 12, N_{\text{deg}} = 10 \]

\[ c_\tau = 0.250 \]
Bare-coupling interpolation

NDP fit of the plaquette coupling

- $L/a = 12, N_{\text{deg}} = 10$
- $L/a = 6, N_{\text{deg}} = 11$

$c_{\tau} = 0.400$

NDP fit of the plaquette coupling

- $L/a = 16, N_{\text{deg}} = 10$
- $L/a = 8, N_{\text{deg}} = 12$

$c_{\tau} = 0.400$

NDP fit of the plaquette coupling

- $L/a = 20, N_{\text{deg}} = 14$
- $L/a = 10, N_{\text{deg}} = 7$

$c_{\tau} = 0.400$

NDP fit of the plaquette coupling

- $L/a = 24, N_{\text{deg}} = 9$
- $L/a = 12, N_{\text{deg}} = 10$

$c_{\tau} = 0.400$
Continuum extrapolation

\[
\begin{align*}
&c_T = 0.250 \quad \text{Input } u = 0.4 \quad \text{clover coupling} \\
&c_T = 0.250 \quad \text{Input } u = 5.6 \quad \text{plaquette coupling}
\end{align*}
\]
Continuum extrapolation

$c_\tau = 0.400$  
Input $u = 0.4$  
clover coupling

3pt-linear extrapolation

$c_\tau = 0.400$  
Input $u = 0.4$  
plaquette coupling

3pt-linear extrapolation

$c_\tau = 0.400$  
Input $u = 5.6$  
clover coupling

3pt-linear extrapolation

$c_\tau = 0.400$  
Input $u = 5.6$  
plaquette coupling

3pt-linear extrapolation

Wednesday, June 25, 14
Results

(8 → 16, 10 → 20, 12 → 24) linear continuum extrapolation

$c_\tau = 0.250$

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$c_\tau = 0.300$

---

$c_\tau = 0.400$

---

$c_\tau = 0.450$

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$g^2(L)$ vs $g^2(2L)/g^2(L)$

- Clover
- Plaquette
Remarks and outlook

• Wilson Flow offers a very nice tool to perform the difficult task of the search for IRFP.

• In our work, we have to go to $c \sim 0.4$ to have the continuum extrapolation under control.

• From our work, it is still inclusive whether SU(3) gauge theory is QCD-like or conformal in the IR, although the running is very slow.

• Better data on the way...