Emergence of time in power-counting renormalizable Riemannian theory of gravity

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Abstract

We suggest a new scenario of gravitation in which gravity at the fundamental level is described by a Riemannian (i.e. locally Euclidean) theory without the notion of time. The Lorentzian metric structure and the notion of time emerge as effective properties at long distances. On the other hand, at short distances, higher derivative terms compatible with the Riemannian diffeomorphism become important and the system is described by a power-counting renormalizable Riemannian theory.
1 Introduction

Reconciliation of gravity and quantum theory is one of the most outstanding problems in modern physics. Although general relativity (GR) has been successful to reproduce and predict gravitational phenomena over a vast range of scales, attempts to quantize GR lead to a number of problems. In particular, if we naively applied the perturbative quantum field theory approach to GR then we would end up with infinite number of counter terms and would lose predictability at short distances such as the Planck length. For this reason, various short-distance modifications to GR have been proposed in the literature.

Inclusion of higher derivative terms is one of such modifications. Actually, if forth-order derivative terms in the action dominate the system at short distances then one can render the theory of gravity power-counting renormalizable. In the literature of forth-order gravity theories, some evidences have been reported for renormalizability [1], for asymptotic freedom of dimensionless couplings [2, 3] and for asymptotic safety of Newton’s constant and the cosmological constant [4]. These theories, however, are known to have a serious problem: because of higher time derivatives present in the action, it is not obvious how to maintain unitarity. There is an attempt to remedy this problem by using the $1/p^2$-type propagator and including infinite number of interaction terms [5], but beta functions for this infinite set of couplings is not known at present and it is expected that not all couplings are asymptotically safe. The challenge is then how to reconcile asymptotically safe couplings with unitarity. It is thus fair to say that the issue of non-unitarity in higher derivative theories has not been settled [6].

The purpose of this paper is to point out a possible alternative way to get around the issue of non-unitarity while maintaining renormalizability. This new possibility is based on two ideas: (i) in Riemannian (i.e. locally Euclidean) theories with positive definite metrics there is no dynamics and thus higher derivative terms do not necessarily lead to a problem; (ii) the Lorentzian metric structure that we usually consider as fundamental may be an effective property that emerges only at long distances and only in some regions [7]. We thus propose a scenario in which gravity at short-distances is described by a power-counting renormalizable Riemannian theory of gravity with a positive definite metric and the Lorentzian metric structure emerges at long distances.

The rest of this paper is organized as follows. In Sec. 2 we describe a power-counting renormalizable Riemannian theory of gravity that potentially leads to emergence of time at long distances in some regions. This theory contains a Riemannian
metric with positive definite signature and a clock field, i.e. a scalar field playing the role of time by means of spontaneous symmetry breaking of Riemannian diffeomorphism. In Sec. 3 we consider the infrared (IR) limit of the theory and obtain the IR action. It is then shown in Sec. 4 that the IR action can be cast into the form of an effective action for a Lorentzian theory so that the Lorentzian metric structure can emerge. In Sec. 5 we consider a cosmological background in the effective Lorentzian theory and analyze its stability against tensor and scalar perturbations. Sec. 6 is devoted to a summary of the results and discussions.

2 Power-counting renormalizable theory

Let us consider a 4-dimensional Riemannian manifold $\mathcal{M}$ with a positive definite metric $g_{\mu\nu}^E$. The theory we shall consider on this manifold does not have the concept of time. In order to make the notion of time to emerge spontaneously, following [7], let us consider a real scalar field $\phi$ with shift symmetry. We shall see that the value of $\phi$ plays the role of time. For this reason, we call $\phi$ a clock field. The shift symmetry is necessary for the system after the emergence of time to have the time translation symmetry. We also demand that the theory respects the symmetry under the $Z_2$ transformation $\phi \rightarrow -\phi$ in order to ensure that the system after the emergence of time has the time reversal symmetry. We also demand that the theory is invariant under the 4-dimensional parity, i.e. $x^\mu \rightarrow -x^\mu$.

We demand that the short-distance behavior of the system is dominated by fourth-order derivative terms so that the scaling dimensions of $g_{\mu\nu}^E$ and $\phi$ become zero at short distances. This allows us to construct a power-counting renormalizable theory\footnote{In the previous work [7], to minimize the number of physical degrees of freedom and to simplify the system, it was imposed as a convenient assumption that equations of motion be second-order. On the other hand, in the present paper we seek a theory which has a potential to be renormalizable, asymptotically safe and thus UV complete. For this reason, we consider a power-counting renormalizable theory.} describing $g_{\mu\nu}^E$ and $\phi$. With the shift- and $Z_2$-symmetries and the 4-dimensional parity invariance mentioned above, the system at short distances is described by the action of the form

$$I_4 = \int dx^4 \sqrt{g_E} \left[ c_1 R_E^2 + c_2 R_{\mu\nu}^E R_{\mu\nu}^E + c_3 R_{\mu\nu\rho\sigma}^E R_{\mu\nu\rho\sigma}^E + c_4 X_E R_E + c_5 R_{\mu\nu}^E \partial_\mu \phi \partial_\nu \phi + c_6 X_E^2 + c_7 (\nabla_E^2 \phi)^2 + c_8 (\nabla_\mu^E \nabla_\nu^E \phi)^2 \right],$$

(2.1)

where $\nabla_\mu^E$, $R_{\mu\nu\rho\sigma}^E$, $R_{\mu\nu}^E$ and $R_E$ are the covariant derivative, Riemann curvature, Ricci
curvature and Ricci scalar of the Riemannian metric $g^E_{\mu \nu}$:

$$X_E \equiv g^E_{\mu \nu} \partial_\mu \phi \partial_\nu \phi, \quad (\nabla^E E_{\mu \nu} \phi)^2 \equiv (\nabla^E E^\mu_\mu \phi)(\nabla^E E^\nu_\nu \phi),$$

(2.2)

$g^E_{\mu \nu}$ and $g_E$ are the inverse and the determinant of $g^E_{\mu \nu}$, and $c_i \ (i = 1, \cdots, 8)$ are constants. Here, we have neglected total derivatives. At long distances, terms with less number of derivatives become important. There are two independent terms with two derivatives

$$I_2 = \int dx^4 \sqrt{g_E} [c_9 R_E + c_{10} X],$$

(2.3)

and a term without derivatives

$$I_0 = c_{11} \int dx^4 \sqrt{g_E},$$

(2.4)

where $c_i \ (i = 9, 10, 11)$ are constants.

Without loss of generality, one can set $c_5 = 0$ by integration by parts and redefinition of $c_{7,8}$. One can also set $c_6 = 1$ by rescaling of $\phi$, provided that $c_6$ before rescaling is positive. Hence, the total action is rewritten as

$$I = I_4 + I_2 + I_0$$

$$= \int dx^4 \sqrt{g_E} \left[ 2Z \Lambda_E - Z R_E + \frac{1}{2 \lambda} C^2_E - \frac{\omega}{3 \lambda} R^2_E + \frac{\theta}{\lambda} E_E \right.$$

$$\left. + X^2_E - 2X_E X + \alpha(\nabla^2 \phi)^2 + \beta(\nabla^E E^\mu_\mu \phi)^2 + \gamma X_E R_E \right],$$

(2.5)

where $C^2_E \equiv R^E_{\mu \nu \rho \sigma} R^E_{\mu \nu \rho \sigma} - 2R^E_{\mu \nu} R^E_{\mu \nu} + R^2_E / 3$ is the square of the Weyl tensor, $E_E \equiv R^E_{\mu \nu \rho \sigma} R^E_{\mu \nu \rho \sigma} - 4R^E_{\mu \nu} R^E_{\mu \nu} + R^2_E$ is the integrand of the Euler (or Gauss-Bonnet) term, and $(Z, \Lambda_E, \lambda, \omega, \theta, X_E, \alpha, \beta, \gamma)$ are constants.\(^2\)

The action (2.5) is power-counting renormalizable. It should be noted that, unlike Hořava-Lifshitz gravity [10], the anisotropic scaling is not invoked here. This is the reason why we need only up to fourth (not sixth) order derivatives in the action. Also, the theory enjoys the four-dimensional Riemannian diffeomorphism invariance and this significantly reduces the number of possible terms in the action. Another difference from Hořava-Lifshitz gravity is the absence of the so-called scalar graviton, again due to the four-dimensional Riemannian diffeomorphism invariance. Instead,

\(^2\)For the coefficients of purely geometrical terms in the action, we adopted the notation often used in the literature of the asymptotic safety scenario [6]. In particular, we keep the Euler term since it ceases to be topological when some regularization scheme such as the dimensional regularization is employed. On the other hand, the constant $X_E$ was introduced so that the polynomial of $X_E$ in the action is of the form $(X_E - X_s)^2 + const.$ as in ghost condensate [3] [4].
in the present theory the emergence of time and dynamics requires inclusion of the clock field $\phi$.

It is worthwhile to stress here again that the signature of the metric is positive definite and that there is no notion of time and dynamics at the fundamental level. Hence, higher derivative terms do not necessarily lead to a problem as far as the action is bounded from below.

3 IR action

Let us suppose a situation in which scalar invariants made of the curvature of $g_{\mu\nu}^E$ and derivatives of the curvature are low in the unit of $Z$ while $X_E$ may become relatively large. In this situation we can ignore the higher curvature terms $C_2^E, R_2^E$ and $E_E$. The system is then described by the action (2.5) without these higher curvature terms. If $\alpha = -\beta = -2\gamma$ then this is a special case of the shift- and $Z_2$-symmetric, Riemannian version of the covariant Galileon action considered in [7] and the corresponding equations of motion are second order. For general values of $\alpha$ and $\beta$, however, the equations of motion include terms containing higher derivatives. These higher derivative terms in the equations of motion are proportional to either $\alpha + 2\gamma$ or $\beta - 2\gamma$.

We are allowing for a relatively large $X_E$. On the other hand, let us suppose that scalar invariants made of second or higher covariant derivatives of $\phi$ are small in the unit of $Z$. In this case, while we should keep terms that stem from $\gamma X_E R_E$, we can safely neglect higher derivative terms in the equations of motion, which are proportional to either $\alpha + 2\gamma$ or $\beta - 2\gamma$. Hence, the long-distance behavior of the system can be described by the following IR action

$$ I_{\text{IR}} = \int dx^4 \sqrt{g^E} \left[ 2Z\Lambda_E - ZR_E ight. $$

$$ + X_E^2 - 2X_\ast X_E - 2\gamma (\nabla^2_E \phi)^2 + 2\gamma (\nabla^E_{\mu} \nabla^E_{\nu} \phi)^2 + \left. \gamma X_E R_E \right]. \quad (3.1) $$

As already stated above, this is a special case of the shift- and $Z_2$-symmetric, Riemannian version of the covariant Galileon action considered in [7] and the corresponding equations of motion are second order \footnote{The original Galileon theory was proposed in ref. [11]. The covariantized version of Galileon was then found in ref. [12] and is equivalent to Horndeski theory [13].}. (See (4.8) below for the precise correspondence between the IR limit (3.1) of the power-counting renormalizable action and the Riemannian Galileon action considered in [7].)
4 Effective Lorentzian action

The IR action (3.1) (as well as the full action (2.5)) is defined in terms of a Riemannian metric with positive definite signature and thus does not have the notion of time at the fundamental level. However, in a region where derivative of the clock field $\partial_\mu \phi$ does not vanish, one can consider the induced metric on each constant $\phi$ hypersurface and a sequence of the induced metrics parameterized by $\phi$. A priori, we do not know whether the differential equation describing such a sequence is elliptic or hyperbolic. In this and the next sections we show that the differential equation describing the sequence of induced metrics on constant $\phi$ surfaces can become hyperbolic when $\partial_\mu \phi$ is large enough. We call this phenomenon *emergence of time and dynamics.*

Let us suppose that $X_E \neq 0$ in a region $\mathcal{M}_0$ of the Riemannian manifold $\mathcal{M}$ with the positive definite metric $g^{E\mu\nu}$, where $X_E = g^{E\mu\nu} \partial_\mu \phi \partial_\nu \phi$. In this case there is a positive number $X_c$ such that $X_E > X_c$ in $\mathcal{M}_0$. Under this assumption, one can define a Lorentzian metric $g_{\mu\nu}$ with the signature $(-,+,+,+)$ given by

$$g_{\mu\nu} = g^{E\mu\nu} - \frac{\partial_\mu \phi \partial_\nu \phi}{X_c}, \quad g'^{\mu\nu} = g^{E\mu\nu} - \frac{g^{E\rho\sigma} g'_{E\rho\sigma} \partial_\rho \phi \partial_\sigma \phi}{X_E - X_c}. \quad (4.1)$$

As a result, we have the relation

$$\frac{1}{X} = \frac{1}{X_c} - \frac{1}{X_E}, \quad (4.2)$$

and the inequality

$$X > X_c, \quad (4.3)$$

where $X \equiv -g^{\mu\nu} \partial_\mu \phi \partial_\nu \phi$. Since we are interested in the differential equation describing the induced metric on constant $\phi$ surfaces, changing the basic variable from $g^{E\mu\nu}$ to $g_{\mu\nu}$ (defined by (4.1)) is not physically essential since these two metrics share the same induced metric on each constant $\phi$ hypersurface. What really matters is whether the differential equation is elliptic or hyperbolic. Nonetheless, it is still convenient for computational purposes (e.g. for the analysis in the next section) and also assuring to have an IR description that is manifestly covariant with respect to the four-dimensional Lorentzian diffeomorphism. It is in this sense that the introduction of the Lorentzian metric $g_{\mu\nu}$ is useful.

Like any gauge symmetries, (either Riemannian or Lorentzian) diffeomorphism invariance is nothing but redundancy of description and thus can be removed by gauge-fixing and can be restored by introduction of extra degrees of freedom. This simple fact suggests that there may exist a Lorentzian description for the sequence of the induced metrics on constant $\phi$ surfaces. Technically speaking, the construction
of such a Lorentzian description is achieved by adopting the so called unitary gauge, i.e. by choosing one of four coordinates $t$ as

$$t = \frac{\phi}{M^2}, \quad (4.4)$$

where $M$ is an arbitrary mass scale, and then undoing it. When adopting the unitary gauge, we lose a part of the original Riemannian diffeomorphism invariance. In the language of the Riemannian version of the Arnowitt-Deser-Misner (ADM) formalism, it is the lapse function that is removed (or, to be more precise, is written in terms of other quantities) by adopting the unitary gauge (4.4). On the other hand, when undoing the unitary gauge, we introduce the lapse function in the language of the Lorentzian ADM formalism. In this way we can restore the Lorentzian diffeomorphism invariance. Hence we obtain, so to speak, a duality between a Riemannian theory describing the Riemannian metric $g^E_{\mu\nu}$ and a Lorentzian theory describing the Lorentzian metric $g^\mu\nu$. The detailed derivation of the “duality” can be found in [7] and the result is that the Riemannian action of the form

$$I_{IR} = \int dx^4 \sqrt{g^E} \left\{ G_4(X_E) R^E + K(X_E) - 2G'_4(X_E) \left[ (\nabla^2_E\phi)^2 - (\nabla^E\nabla^E\phi)^2 \right] \right\} \quad (4.5)$$

is equivalent to the Lorentzian action

$$I_{IR} = \int dx^4 \sqrt{-g} \left\{ f(X) R + P(X) + 2f'(X) \left[ (\nabla^2\phi)^2 - (\nabla_{\mu}\nabla_{\nu}\phi)^2 \right] \right\}, \quad (4.6)$$

where $R$ and $g$ are the Ricci scalar and the determinant of the Lorentzian metric $g_{\mu\nu}$, and the functions $f(X)$ and $P(X)$ in the Lorentzian action are specified as

$$f(X) = \frac{G_4(X_E)}{\sqrt{X}}, \quad P(X) = \frac{K(X_E)}{\sqrt{X^E}}. \quad (4.7)$$

By setting

$$G_4(X_E) = \gamma X_E - Z, \quad K(X_E) = X_E - 2X_c X_E + 2Z\Lambda_E, \quad (4.8)$$

we obtain

$$f(X) = \left( \frac{\gamma X_c X}{X - X_c} - Z \right) \sqrt{\frac{X - X_c}{X_c}},$$

$$P(X) = \left[ \left( \frac{X_c X}{X - X_c} \right)^2 - 2X_c \left( \frac{X_c X}{X - X_c} \right) + 2Z\Lambda_E \right] \sqrt{\frac{X - X_c}{X_c}}. \quad (4.9)$$

The effective Lorentzian action (4.6) with (4.9) describes the long-distance behavior of the system in the region $M_0$.  


5 Stability (hyperbolicity)

In the previous section we have shown that the long-distance behavior of the theory \((2.5)\) is described by the effective Lorentzian action \((4.6)\) with \((4.9)\). This itself does not imply the emergence of time and dynamics as defined at the beginning of the previous section. Actually, what we need to show is that the differential equation describing the system is hyperbolic rather than elliptic, in some region of the Riemannian manifold. With the effective Lorentzian description at hand, this is equivalent to the existence of a well-defined background around which fluctuations are stable in the usual Lorentzian sense.

For this reason, we now analyze the stability of a flat Friedmann-Lemaître-Robertson-Walker (FLRW) background using the effective Lorentzian action \((4.6)\) with \((4.9)\).

5.1 Cosmological background

We consider a flat FLRW background spacetime for which the effective Lorentzian metric and the clock field are

\[
g_{\mu\nu}dx^\mu dx^\nu = -dt^2 + a(t)^2 \delta_{ij}dx^i dx^j, \quad \phi = \phi_0(t), \tag{5.1}
\]

where \(a\) is the scale factor.

The equations of motion for \(\phi\) and the metric are, respectively,

\[
\dot{J}_\phi + 3HJ_\phi = 0, \tag{5.2}
\]

and

\[
6(3\gamma X_E + Z)rH^2 = 3X_E^2 - 2X_cX_E - 2Z\Lambda_E, \tag{5.3}
\]

where \(H = \dot{a}/a\),

\[
J_\phi \dot{\phi}_0 = \frac{2X_E}{\sqrt{r}}\left(X_E - X_c - 3\gamma rH^2\right), \tag{5.4}
\]

\[
r \equiv \frac{X_E}{X} = \frac{X_E - X_c}{X_c} = \frac{X_c}{X - X_c}, \tag{5.5}
\]

and it is understood that \(X_E\) is a function of \(X\) as

\[
X_E = \frac{X_cX}{X - X_c}. \tag{5.6}
\]

The equation of motion \((5.2)\) implies that the shift charge density \(J_\phi\) decays as \(J_\phi \propto 1/a^3\) and approaches zero. By setting \(J_\phi = 0\), hence, one can obtain equations
defining attractors of the system as

\[(X_E - X_*)(3\gamma X_E - \gamma X_* + 2Z) = \gamma \left( X_*^2 - 2Z\Lambda_E \right), \]

\[3H^2 = \frac{X_c X_E - X_*}{\gamma X_E - X_c}. \]  

(5.7)

These equations allow two branches of solutions: the first equation is an algebraic equation for \(X_E\) and generically allows two solutions. The second equation then determines the value of the Hubble expansion rate.

If the r.h.s. of the second of (5.7) is positive then the universe at late time exhibits accelerated expansion. Moreover, deviation of \(J_\phi\) from zero generically introduces an \(O(J_\phi)\) correction to \(H^2\) and decays as \(1/a^3\). Intriguingly, this behavior is exactly like what we expect for dark matter. Therefore the late time behavior of the system is similar to that in the standard ΛCDM cosmology, at least at the background level.

5.2 Tensor perturbations

We now consider tensor perturbations around the FLRW background so that the metric and the clock field are given by

\[g_{\mu\nu} dx^\mu dx^\nu = -dt^2 + a(t)^2 \left[ e^h \right]_{ij} dx^i dx^j, \quad \phi = \phi_0(t), \]  

(5.8)

where \(h_{ij}\) is transverse and traceless (i.e. \(\partial_i h^i_k = 0 = \delta^{ij} h_{ij}\)).

In Fourier space, the quadratic action for each polarization of the tensor mode is given by

\[I^{(2)}_{T, k} = \frac{1}{8} \int dt a^3 \left[ M_{\text{eff}}^2 \dot{h}_k^2 - 2f \frac{k^2}{a^2} \dot{h}_k^2 \right], \]  

(5.9)

where

\[M_{\text{eff}}^2 = 2\sqrt{r}(\gamma X_E + Z), \quad f = \frac{1}{\sqrt{r}}(\gamma X_E - Z), \]  

(5.10)

where \(r\) is defined by (5.5) and it is again understood that \(X_E\) is a function of \(X\) as (5.6). Hence, the stability of the tensor sector requires that

\[\gamma X_E > |Z|. \]  

(5.11)

This in particular requires that \(\gamma > 0\). Note that the stability condition (5.11) applies to the system both on and away from the attractors (5.7).
5.3 Scalar perturbations

For scalar perturbations around the FLRW background, the metric and the clock field in the unitary gauge are given by

\[ g_{\mu \nu} dx^{\mu \nu} = -(1 + \alpha)^2 dt^2 + 2 \partial_i \beta dt dx^i + a(t)^2 e^{2\phi} \delta_{ij} dx^i dx^j, \quad \phi = \phi_0(t). \quad (5.12) \]

It is straightforward to calculate the quadratic action for perturbations, following the treatment in ref. [14]. Since the time derivatives of \( \alpha \) and \( \beta \) do not appear in the action, the equations of motion for \( \alpha \) and \( \beta \) are constraint equations. After solving those constraint equations with respect to \( \alpha \) and \( \beta \), one obtains the quadratic action for \( \zeta \) in Fourier space as

\[ I^{(2)}_{S,k} = \frac{1}{2} \int dt a^3 \left[ A \dot{\zeta}_k^2 - B \frac{k^2}{a^2} \zeta_k^2 \right], \quad (5.13) \]

where \( A \) and \( B \) are given by

\[ A = \frac{M_{\text{eff}}^2}{H^2 G^2} \left( 6 + M_{\text{eff}}^2 \mathcal{F} \right), \quad B = \frac{1}{a} \frac{d}{dt} \left( \frac{a M_{\text{eff}}^4}{H G^2} \right) + 4f, \quad (5.14) \]

\( \mathcal{F} \) and \( G \) are given by

\[ \mathcal{F} = \left[ 3 X_E^2 - X_E X_E - 3(6 \gamma X_E + Z) r H^2 \right] r^{3/2}, \]

\[ G = (3 \gamma X_E + Z)^{3/2}, \quad (5.15) \]

\( r \) is defined by (5.5), and it is again understood that \( X_E \) is a function of \( X \) as (5.6). Hence, the stability condition for the scalar perturbations is

\[ A > 0, \quad B > 0. \quad (5.16) \]

Note that this stability condition applies to the system both on and away from the attractors (5.7).

Let us now study the stability condition (5.16) on the attractors (5.7) describing the late time accelerated expansion of the universe. On the attractors, \( X_E \) and \( H \) are constant. This means that \( f, M_{\text{eff}}^2, G \) and \( \mathcal{F} \) are also constant and that

\[ B = \frac{M_{\text{eff}}^4}{G^2} + 4f. \quad (5.17) \]

Hence, under the stability condition (5.11) for the tensor sector, it follows that \( B > 0 \). Let us now investigate the other stability condition \( A > 0 \) on the attractors. For
simplicity let us suppose that $X_\star = O(Z)$, that $\gamma = O(1)$, that the expansion rate of the universe is low in the unit of $Z$ i.e. $H^2 \ll |Z|$ and that $r = O(1)$. In this case, the second of the attractor equations (5.7) implies that $|X_E - X_\star| = O(H^2) \ll |Z|$. Hence, it is shown that

$$F = \left[ 2X_\star^2 - \frac{1}{\gamma} (\gamma X_\star + Z) (X_E - X_\star) - 3(X_E - X_\star)^2 \right] r^{3/2}$$

$$= 2X_\star^2 r^{3/2} \left[ 1 + O(H^2/Z) \right] > 0. \quad (5.18)$$

Thus, under the stability condition (5.11) for the tensor sector, it follows that $\mathcal{A} > 0$.

### 5.4 Summary of stability condition

We have seen that the flat FLRW background behavior of the system is similar to that of the standard ΛCDM cosmology. We have then shown that tensor perturbations are stable (in the usual Lorentzian sense), provided that the parameter $\gamma$ (defined in (2.5) or (3.1)) is positive and that the background value of $X_\star$ is large enough. To be more precise, the stability condition for tensor perturbations is given by (5.11). The stability condition for the scalar sector shown in (5.16) is more involved but we have shown that it is always satisfied at the low-energy, late-time attractor, provided that the tensor sector is stable.

The stability (in the usual Lorentzian sense) in particular implies that the differential equation describing fluctuations of the system is hyperbolic rather than elliptic. This is achieved by a large enough background value of derivative of the clock filed, in the context of a purely Riemannian theory. As defined at the beginning of the previous section, we call this phenomenon *emergence of time and dynamics.*

### 6 Discussion

When we talk about the history or dynamics of the universe, we are actually taking about a sequence of configurations parameterized by time. We often ask fundamental questions such as those concerning the beginning of the geometrical description of the universe, and in this case we are forced to think about the initial singularity. We might speculate that the notion of space does not exist before the initial singularity and that the space may be emergent. If the space may be emergent, then how about the time? Can the notion of time be emergent?

In any diffeomorphism invariant theories of gravity, the Hamiltonian of the system is a sum of constraints associated with general coordinate transformations and thus
vanishes up to boundary terms. For this reason, there is no time evolution of quantum states in diffeomorphism invariant theories of quantum gravity. Therefore, dynamics should be encoded as correlations among various fields. In this case one of those fields should play the role of time. It is perhaps in this sense that the concepts of time and dynamics may be emergent.

In the present paper, based on the mechanism developed in ref. [7], we have proposed a new scenario of gravitation in which gravity at short-distances is described by a power-counting renormalizable Riemannian (i.e. locally Euclidean) theory without the fundamental notion of time. The Lorentzian metric structure and the notion of time emerge as effective properties at long distances.

At the fundamental level the theory includes a Riemannian (i.e. locally Euclidean) metric $g_{\mu\nu}^E$ and a clock field $\phi$ playing the role of time. The symmetries defining the theory are the 4-dimensional Riemannian diffeomorphism invariance, the 4-dimensional parity invariance, and the shift- and $Z_2$-symmetries of the clock field. We have written down the most general action that contains up to forth-order derivatives. The action is shown in (2.5) and contains 9 parameters, which are subject to running under the renormalization group (RG) flow. Since the ultraviolet (UV) behavior of the system is dominated by forth-order derivative terms, the scaling dimensions of $g_{\mu\nu}^E$ and $\phi$ are zero in the UV and, as a result, the theory described by the action (2.5) is power-counting renormalizable.

We have then considered the infrared (IR) limit of the system and obtained the IR action shown in (3.1), which turned out to be a special case of the shift- and $Z_2$-symmetric, Riemannian version of the covariant Galileon action. In ref. [7] this IR theory was shown to be equivalent to a Lorentzian theory in a region where the first derivative of the clock field is non-vanishing. The theory thus has an effective Lorentzian description valid in the IR in some regions. Therefore, the notion of time can emerge as an effective property at long distances. On the other hand, at short distances, forth-order derivative terms compatible with the Riemannian diffeomorphism become important and thus the system is described by the power-counting renormalizable Riemannian theory.

There are many issues to be addressed regarding theoretical consistency and phenomenological viability of the theory.

Quantum nature of the forth-order derivative theory of gravity without the clock field has been extensively studied in the literature. In particular, it has been reported that the theory is renormalizable [1], that the dimensionless couplings are asymptotically free [2] and that Newton’s constant and the cosmological constant appear to be asymptotically safe [4]. However, as already mentioned in Sec. 1, the Lorentzian
forth-order derivative theory has the serious issue of non-unitarity because of higher
time derivatives in the action; the challenge in the Lorentzian theory is then how to
reconcile asymptotically safe couplings with unitarity.

On the contrary, in the scenario proposed in the present paper the theory is
Riemannian (i.e. locally Euclidean) at the fundamental level and the metric has
the positive definite signature. The Lorentz metric signature emerges as an effective
property at long distances in some regions. Thus, at short distances the notion of
time and dynamics do not exist. For this reason, the existence of higher-derivative
terms is not necessarily a problem. However, emergence of the notion of time at long
distances requires the introduction of a clock field and, as a result, the action (2.5)
contains extra terms. It is thus necessary to revisit the issues of renormalizability
and asymptotic safety of the forth-order derivative theory of gravity, with the clock
field now included.

Having a new scenario of gravitation, it is important to investigate its cosmological
implications. In the case of Hořava-Lifshitz gravity [10], almost scale-invariant
cosmological perturbations can be generated even without inflation [15, 16]. The
mechanism relies on the fact that the scaling dimension of fields becomes zero in the
UV, and this property is shared by the power-counting renormalizable Riemannian
theory in the present paper, despite the fact that these two theories are quite differ-
ent. Hence, one might hope to find a similar mechanism for generation of cosmological
perturbations.

It is also intriguing to see if one can find a regime of parameters in which the
clock field behaves like ghost condensate [8, 9] and can drive ghost inflation [17].
(See footnote [2]) If this is possible then the power-counting renormalizable theory
proposed in the present paper might be considered as a possible UV completion.

In a Riemannian theory, in principle everything is determined by a boundary
condition. This may change our view on the cosmological constant problem. In some
sense, the cosmological constant problem is a tension between the initial condition
and the late time behavior of the universe. In Lorentzian theories, quantum gravity
may tell us something about the initial condition of the universe but it is hard to
imagine how it can address the late time behavior of the universe. On the contrary,
if the notion of time is an emergent phenomenon in a Riemannian theory then there
might be a possibility that what we call the past and what we call the future may
be ultimately related to each other by the boundary condition that determines the
whole system including many islands with the Lorentzian metric structure as well
as many other vast regions without notion of time. However, before addressing the
cosmological constant problem, we need to develop the quantum theory.
It is also interesting to combine the emergent time scenario with ideas of dimensional reduction such as Kaluza-Klein compactification and brane-world. We might end up with a landscape with various signatures and dimensions.

While we have shown that the Lorentzian metric structure can emerge as an effective property of the gravity sector at long distances, it is phenomenologically important to ensure that not only the Lorentzian signature but also the Lorentz symmetry can emerge in the matter sector at long distances [18]. For the emergence of Lorentzian signature in the matter sector, we thus need to couple the matter sector to derivatives of the clock field as in [7]. It is also necessary to develop mechanisms or symmetries to suppress Lorentz violating operators in the matter sector at low energies after emergence of time. It has been known that the RG flow allows a Lorentz invariant IR fixed point so that the Lorentz symmetry emerges as a low energy effective property of Lorentz violating theories [19]. Although the RG running towards the Lorentz invariant IR fixed point is typically logarithmic, it may be possible to enhance the RG running to a power-law type by strong dynamics [20]. Another possibility would be to forbid lower dimensional Lorentz violating operators by invoking supersymmetry [21].

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