A Multi-Target Track-Before-Detect Particle Filter Using Superpositional Data in Non-Gaussian Noise

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Abstract—In this paper, we propose a general and tractable approach to multi-target track-before-detect based on the particle filter. It is even applicable to general superpositional sensor signals, and/or in the presence of non-Gaussian observation noise. Superpositional sensor signals depend on the sum of general nonlinear target contributions, and arise in diverse domains, such as radio-frequency (RF) tomography, wireless communications, and array signal processing. Moreover, the proposed method realizes MTT for an unknown, time-varying number of targets, in an online manner, without knowing their initial states. We conducted a simulation involving superpositional sensor signals in the context of RF tomography. The proposed method was shown to outperform the state-of-the-art approximate cardinalized probability hypothesis density filter for superpositional sensor signals ($\Sigma$-CPHD) in terms of the optimal subpattern assignment (OSPA) metric by a factor of approximately two to five.

Index Terms—Multi-target tracking (MTT), track-before-detect, superpositional data, particle filter, birth/death process.

I. INTRODUCTION

Multi-target tracking (MTT) aims to estimate time-varying states of multiple targets jointly from available observations, where these states typically include kinematic states (e.g., the position/velocity/acceleration). MTT constitutes one of the most active areas in statistical signal processing, and myriads of methods have been proposed. These methods encompass multiple hypothesis tracking (MHT) [1], joint probabilistic data association (JPDA) [2], probabilistic multitarget tracking (PMHT) [3], random finite sets (RFS) [4], [5], [6], multi-target particle filters [3], [9], and sequential Markov chain Monte Carlo (MCMC) [10].

The conventional approach to MTT is based on a two-step procedure, consisting of detection and tracking steps. In the detection step, sensor signals are preprocessed for detection by, e.g., thresholding, so that each detection corresponds to a single target or a clutter. These detections are fed into the subsequent tracking step. In such a procedure, tracking performance heavily depends on detection performance. Unfortunately, the latter degrades severely under adverse conditions, e.g., at a low signal-to-noise ratio (SNR), and thus, so does the former. Note that, in this approach, information contained in observed time series is not being fully exploited for detection, because it is performed based solely on observations at the current time step without reference to past time steps.

Under adverse conditions, it is significantly advantageous to operate directly on raw sensor signals for joint detection and tracking, which is known in the literature track-before-detect. Compared to the two-step procedure, this approach can exploit information from not only the current but also past time steps in performing detection, leading to better robustness under adverse conditions.

Salmond et al. [11] proposed a track-before-detect method for at most one target in the framework of recursive Bayesian estimation. Since then several authors have considered multi-target extensions of track-before-detect. Kreucher et al. and Vo et al. focused on a restricted class of sensor signals, where targets contribute in a binary [12] or a disjoint [13] manner. Mahler [14], [15] derived a RFS-based filter for multi-target track-before-detect using superpositional sensor signals, which depend on the sum of general nonlinear target contributions. This method is called a superpositional cardinalized probability hypothesis density ($\Sigma$-CPHD) filter [15]. Moreover, Nannuru et al. [16] developed a tractable approximate implementation of the $\Sigma$-CPHD filter based on the particle filter, but it is limited to additive Gaussian observation noise. Boers et al. [17] and Lepoutre et al. [18] considered superpositional sensor signals in the presence of unknown target amplitudes, but again this is limited to additive Gaussian observation noise.

In this paper, we propose a general and tractable approach to multi-target track-before-detect based on the particle filter. It is even applicable to general superpositional sensor signals and/or in the presence of non-Gaussian observation noise. Moreover, it realizes MTT for an unknown, time-varying number of targets, in an online manner, without knowing their initial states. The proposed method can be viewed as a multi-target extension of Salmond et al.’s single-target track-before-detect [11], and includes it as a particular example up to specific implementation of the particle filter.

II. PROPOSED PARTICLE FILTER FOR MULTI-TARGET TRACK-BEFORE-DETECT

This section describes the proposed particle filter for multi-target track-before-detect. It is based on a state-space model with time index $t$, unknown states $\mathbf{x}_t (t = 0, 1, 2, \ldots)$, a given initial distribution $p(\mathbf{x}_0)$, a given transition distribution $p(\mathbf{x}_t | \mathbf{x}_{t-1})$, given observations $\mathbf{z}_t (t = 1, 2, \ldots)$, and a given observation distribution $p(\mathbf{z}_t | \mathbf{x}_t)$.

A. Modeling Multi-Target States

Our states and their modeling follow Septier et al. [10]. In MTT, a target may enter/leave the region observed by sensors or start/cease to emit a signal at any time, referred to as target birth/death. Consequently, the number of active targets is unknown and time-varying in general. To deal with such a general setting, we prepare $n_{\text{max}}$ target models, which may be active or inactive at each time $t$, with $n_{\text{max}}$ being the
maximum possible number (given) of simultaneously active targets. Each target model \( j \in \{1, \ldots, n_{\text{max}}\} \) has a discrete state \( a_{jt} \in \{0,1\} \) indicating whether it is active \( (a_{jt} = 1) \) or not \( (a_{jt} = 0) \) at each time \( t \). It also has continuous states \( x_{jt} \in \mathbb{R}^{n_x} \), which usually include kinematic states and possibly target signal amplitude \([19], [20]\) or any other physical quantities. Our states \( x_t \) consist of \( x_t := (x_{1t}, \ldots, x_{n_{\text{max}},t}) \in \mathbb{R}^{n_{\text{max}}n_x} \) and \( a_t := (a_{1t}, \ldots, a_{n_{\text{max}}t}) \in \{0,1\}^{n_{\text{max}}} \).

We assume that the transition distribution factorizes as

\[
p(x_t, a_t \mid x_{t-1}, a_{t-1}) = \prod_{j=1}^{n_{\text{max}}} p(x_{jt}, a_{jt} \mid x_{j,t-1}, a_{j,t-1})
\]

(1)

where \( \pi_b \) and \( \pi_d \) are birth and death probabilities (given), respectively. The factor \( p(x_{jt} \mid x_{j,t-1}, a_{jt}, a_{j,t-1}) \) is modeled as

\[
p(x_{jt} \mid x_{j,t-1}, a_{jt}, a_{j,t-1}) = \begin{cases} p_b(x_{jt}), & \text{if } (a_{jt}, a_{j,t-1}) = (1,1) \\ p_b(x_{jt}), & \text{if } (a_{jt}, a_{j,t-1}) = (1,0) \\ p_d(x_{jt}), & \text{if } a_{jt} = 0. \end{cases}
\]

(4)

Here, \( p_b \) and \( p_d \) are given densities corresponding to target survival and birth, respectively, where \( p_d \) may be non-Gaussian and involve nonlinearity. As we will see later, \( p_d \) is actually not used in our particle filter at all, and therefore does not need to be specified.

### B. Modeling Observations

In our track-before-detect setting, observations \( z_t \in \mathbb{C}^{n_z} \) consist in raw sensor signals, where \( n_z \) denotes the number of sensor signals. Let us first consider the case of additive noise for simplicity, where \( z_t \) is modeled as

\[
z_t = \sum_{j=1}^{n_{\text{max}}} a_{jt} h(x_{jt}) + v_t = \sum_{j:a_{jt}=1} h(x_{jt}) + v_t.
\]

Here, \( h(x_{jt}) \) is the target signal from target \( j \) with \( h: \mathbb{R}^{n_x} \rightarrow \mathbb{C}^{n_z} \) being a given, possibly nonlinear function, and \( \sum_{j=1}^{n_{\text{max}}} a_{jt} h(x_{jt}) \) the sum of the target signals from all active targets. Additive noise \( v_t \) is assumed to be independent of time step to time step and have a given, possibly non-Gaussian distribution \( p_v \). In this case, the observation distribution is given by

\[
p(z_t \mid x_t, a_t) = p_v(z_t - \sum_{j=1}^{n_{\text{max}}} a_{jt} h(x_{jt})).
\]

(5)

This model corresponds to Mahler [14], and specifically to Nannuru et al. [16] in the Gaussian case.

However, here we consider a more general observation distribution, which is such that it depends on \( x_t = (x_t, a_t) \) only through \( \sum_{j=1}^{n_{\text{max}}} a_{jt} h(x_{jt}) \). That is, we consider an observation distribution of form

\[
p(z_t \mid x_t, a_t) = p_o \left( z_t - \sum_{j=1}^{n_{\text{max}}} a_{jt} h(x_{jt}) \right),
\]

(6)

where \( p_o \) is a given distribution possibly involving nonlinearity and non-Gaussianity.

### C. Recursive Bayesian Estimation

In the Bayesian framework, we aim to obtain a posterior distribution \( p(x_t, a_t \mid z_{1:t}) \) of the states given all observations up to the current time \( t \), where \( 1 : t \) is a shorthand notation for \( 1, \ldots, t \). This can be done recursively by alternating prediction and update steps, which can be carried out for our hybrid discrete-continuous states \( (x_t, a_t) \) as well in a similar manner to [21], [22], [23], [17].

**Algorithm 1 Proposed auxiliary particle filter for multi-target track-before-detect**

**Input:** \( \{x^k_{t-1}, a^k_{t-1}, w^k_{t-1}\}_{k=1}^{n_p} \)

**Output:** \( \{x^k_t, a^k_t, w^k_t\}_{k=1}^{n_p} \)

1. for \( k = 1 : n_p \) do
   2. for \( j = 1 : n_{\text{max}} \) do
      3. Draw \( \tilde{a}^k_{jt} \sim P(a_{jt} \mid a_{j,t-1}^k) \)
      4. if \( (\tilde{a}^k_{jt}, a_{j,t-1}^k) = (1,1) \) then
         5. Draw \( \tilde{x}^k_{jt} \sim p_a(x_{jt} \mid x_{j,t-1}^k) \)
      6. end if
      7. if \( (\tilde{a}^k_{jt}, a_{j,t-1}^k) = (1,0) \) then
         8. Draw \( \tilde{z}^k_{jt} \sim p_o(z_{jt} \mid \tilde{x}^k_{jt}) \)
      9. end if
   10. end for
   11. \( \tilde{s}^k_t \leftarrow \sum_j \tilde{a}^k_{jt} = 1 h(\tilde{x}^k_{jt}) \)
   12. \( \tilde{w}^k_t \leftarrow p_o(z_t \mid \tilde{s}^k_t w^k_{t-1} \)
   13. end for
14. Resample from \( \{\tilde{w}^k_{t-1}\}_{k=1}^{n_p} \) to get \( \{k^l\}_{l=1}^{n_p} \), where \( k^l \) denotes the index of the parent of the \( l \)th resampled particle
15. for \( l = 1 : n_p \) do
   16. for \( j = 1 : n_{\text{max}} \) do
      17. Draw \( a^l_{jt} \sim P(a_{jt} \mid a^k_{jt}) \)
      18. if \( (a^l_{jt}, a^k_{jt}) = (1,1) \) then
         19. Draw \( x^l_{jt} \sim p_a(x_{jt} \mid x^k_{jt}) \)
      20. end if
      21. if \( (a^l_{jt}, a^k_{jt}) = (1,0) \) then
         22. Draw \( x^l_{jt} \sim p_o(x_{jt} \mid x^l_{jt}) \)
      23. end if
   24. end for
   25. \( s^l_t \leftarrow \sum_j a^l_{jt} = 1 h(x^l_{jt}) \)
   26. \( w^l_t \leftarrow p_o(z_t \mid s^l_t) \)
   27. end for
28. Normalize \( \{w^l_t\}_{l=1}^{n_p} \) so that \( \sum_{l=1}^{n_p} w^l_t = 1 \)

Suppose the posterior distribution \( p(x_{t-1}, a_{t-1} \mid z_{1:t-1}) \) at time \( t - 1 \) is available. The prediction step uses the transi-
tion distribution to obtain a prediction distribution \( p(x_t, a_t \mid z_{1:t-1}) \) by the Chapman-Kolmogorov equation:

\[
p(x_t, a_t \mid z_{1:t-1}) = \sum_{a_{t-1}} p(x_t, a_t \mid x_{t-1}, a_{t-1}) p(x_{t-1}, a_{t-1} \mid z_{1:t-1}) dX_{t-1}.
\]

Here, \( \sum_{a_{t-1}} \) denotes the sum over \( a_{t-1} \in \{0, 1\}^{n_{max}} \), and we define \( p(x_0, a_0 \mid z_{1:0}) := p(x_0, a_0) \) with similar notations defined analogously. The update step combines this prediction distribution with the observation distribution to obtain the posterior distribution \( p(x_t, a_t \mid z_{1:t}) \) at time \( t \) by the Bayes theorem:

\[
p(x_t, a_t \mid z_{1:t}) = \frac{p(z_t \mid x_t, a_t) p(x_t, a_t \mid z_{1:t-1})}{p(z_t \mid z_{1:t-1})}.
\]

Here, the normalizing constant in the denominator writes \( p(z_t \mid z_{1:t-1}) = \sum_{a_t} \int p(z_t \mid x_t, a_t) p(x_t, a_t \mid z_{1:t-1}) dx_t \).

**D. Particle Filter Implementation**

The Bayesian recursion in (7) and (8) can be implemented by using the particle filter (also known as sequential Monte Carlo) [24], [25], [26], [27]. It is a versatile framework applicable to the general nonlinear, non-Gaussian state-space model, where the posterior distribution \( p(x_t, a_t \mid z_{1:t}) \) is approximated by using \( n_p \) point masses (or “particles”) as

\[
p(x_t, a_t \mid z_{1:t}) \approx \sum_{k=1}^{n_p} w_k \delta_{x_k}(x_t) \delta_{a_k}(a_t).
\]

Here, \( \{x_k^t, a_k^t\}_{k=1}^{n_p} \) denotes particle locations, \( \{w_k\}_{k=1}^{n_p} \) probability masses located at the particle locations satisfying \( \sum_{k=1}^{n_p} w_k = 1 \), \( \delta_{x_k}(x_t) \) the Dirac delta function located at \( x_k^t \), and \( \delta_{a_k}(a_t) \) the Kronecker delta

\[
\delta_{a_k}(a_t) = \begin{cases} 1, & \text{if } a_t = a_k^t \\ 0, & \text{otherwise.}
\end{cases}
\]

The particle filter recursively computes particles \( \{x_k^t, a_k^t, w_k^t\}_{k=1}^{n_p} \) at each time \( t \), given particles \( \{x_{k-1}^t, a_{k-1}^t, w_{k-1}^t\}_{k=1}^{n_p} \) at the previous time \( t-1 \) and observations \( z_t \).

There are several implementations of the particle filter, and here we use an auxiliary particle filter [28] (see also [27]). This implementation takes account of observations at time \( t \) when generating particle locations at time \( t \), and can be more effective than the simple sequential importance resampling (SIR) implementation. Algorithm 1 presents a pseudocode of one iteration of the proposed auxiliary particle filter for multi-target track-before-detect. Before applying Algorithm 1 we initialize the particles by \( (x_0^0, a_0^0) \sim p(x_0, a_0) \), \( w_0^0 = 1/n_p \) \((k = 1, \ldots, n_p)\). It is assumed that \( h \) can be evaluated at any point, and so does \( p_o \) up to a normalizing constant. It is also assumed that it is possible to sample realizations from \( p_o, p_0 \), and \( p(x_0, a_0) \). We perform ancestral sampling based on the factorization (2) to sample realizations from the transition distribution \( p(x_t, a_t \mid x_{t-1}, a_{t-1}) \).

**E. Point Estimation**

Once a particle representation of the posterior probability is obtained, it can be used to compute various point estimates of the states. In this paper, we focus on minimum mean square error (MMSE) type estimates. The MMSE estimate of \( x_{jt} \) conditional to \( a_{jt} = 1 \) can be computed by

\[
\hat{x}_{jt}^{MMSE} := E[x_{jt} \mid z_{1:t}, a_{jt} = 1] = \frac{\sum_{k=1}^{n_p} w_k a_k^{jt} x_k^t}{\sum_{k=1}^{n_p} w_k a_k^{jt}}.
\]

Moreover, the MMSE estimate of \( a_{jt} \) can be computed as

\[
\hat{a}_{jt}^{MMSE} := u \left( \sum_{k=1}^{n_p} w_k a_k^{jt} - \frac{1}{2} \right),
\]

where \( u \) denotes the step function

\[
u(x) := \begin{cases} 1, & x \geq 0 \\ 0, & x < 0.
\end{cases}
\]

**III. SIMULATION: MTT FOR RADIO-FREQUENCY TOMOGRAPHY**

As an example, we considered a challenging task of MTT for an unknown, time-varying number of targets with unknown initial positions in the context of radio-frequency (RF) tomography as in [16]. RF tomography [29], [30] aims to localize/track targets (e.g., persons) in a surveillance region by using a network of RF antennas. As in Fig. 1 we employed \( n_a = 24 \) antennas (nodes) on the perimeter of a square surveillance region of dimensions \( 20 \text{m} \times 20 \text{m} \). In RF tomography, signals communicated between RF antennas are used instead of those emanating from targets. These signals contain target location information in the form of attenuation in received signal strength (RSS), which can be exploited for localization/tracking. Hence, we used as sensor signals \( z_t \), RSS.
where $p$ with covariance matrix $\Sigma_w$ was modeled by a superposition of each target $j$, where $x_{jt}$ and $y_{jt}$ are Cartesian coordinates of target $j$ and $\dot{x}_{jt}$ and $\dot{y}_{jt}$ its velocities.

Transition of the states $x_{jt}$ for a surviving target was described by a linear Gaussian model $x_{jt} = F x_{jt-1} + G w_{j,t-1}$ [16]. Here,

$$F := \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \otimes \begin{pmatrix} 1 & T \\ 0 & 1 \end{pmatrix}, \quad G := \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \otimes \begin{pmatrix} \frac{T^2}{2} & T \\ T & \frac{T^2}{2} \end{pmatrix},$$

where $F$ is the Kronecker product, $T = 0.25$ s the sampling period, and $w_{j,t-1}$ zero-mean white Gaussian noise with covariance matrix $\Sigma_w = 0.35 I$. The distribution $p_b$ for a newborn target was modeled by $p_b(x_0, a_0) = U(x, y) N(x \mid 0, 1) N(y \mid 0, 1)$, where $U$ denotes the uniform distribution over the surveillance region. The transition probability matrix $\Pi$ for $a_t$ was given by $\pi_0 = 0.2$ and $\pi_d = 0.1$. The initial state distribution was modeled by $p(x_0, a_0) = \prod_{j=1}^{n_{\text{max}}} p(x_{j,0}) P(a_{j,0})$, where $p(x_{j,0})$ was defined in the same way as $p_b$ and $P(a_{j,0} = 0) = 1$. The observation distribution was given by the superpositional model in [15] with zero-mean white Gaussian noise with covariance matrix $\Sigma_v = \sigma_v^2 I$. The nonlinear function

$$h = (h_1, \ldots, h_{n_z})$$

was given by $h_j(\xi) = \phi \exp(-d_i(\xi)/\sigma_h)$, where $d_i(\xi)$ is an elliptical distance [31] between the $i$th link and a target with states $\xi$ and $\phi = 5$ and $\sigma_h = 0.2$ are empirically determined hyperparameters.

Sensor signals were generated as follows. Temporal behavior of $a_t$ was deterministically scheduled so that the number of active targets started from one, then increased gradually up to four, and finally decreased gradually down to one. On the other hand, $x_t$ was randomly generated based on the above model, and so was $\pi_t$. We adjusted $\sigma^2$ to give a desired signal-to-noise ratio (SNR), where $\text{SNR}(\text{dB}) := 10 \log_{10}(\sum_{j=1}^{n_{\text{max}}} a_j h(x_{jt})^2) - 10 \log_{10}(\|v_t\|_2)$ with $(\cdot)$ being temporal averaging. The number of time steps were 200, corresponding to 50s. Figure [2] shows an example of observed signals.

The proposed method was compared with Nannuru et al.’s approximate $\Sigma$-CPHD filter [16], which we hereafter call the conventional method. The number of particles was fixed to $n_p = 2000$ in the proposed method, and time-varying with 500 particles per target plus 500 particles for proposing newborn targets in the conventional method. In both methods, an auxiliary particle filter with residual resampling [32] was employed. The maximum possible number of targets was set to $n_{\text{max}} = 4$ in the proposed method, and to 10 in the conventional method. In the conventional method, the probability of a target being born was set to 0.03, and the probability of each target surviving to 0.985.

Figures [3] and [4] show estimated x- and y-coordinates versus time for the proposed and the conventional methods, respectively. These estimates were obtained by MMSE estimation and $k$-means clustering in the proposed and the conventional methods, respectively. Particles are also shown by gray dots with associated weights expressed by darkness. Figure [5] shows the estimation error in terms of optimal subpattern assignment (OSPA) metric [33] as a function of the SNR. The error bar shows (the mean) ± (one standard deviation) for 100 trials.

**IV. CONCLUSION**

In this paper, we proposed a particle filter for multi-target track-before-detect using superpositional sensor signals. A simulation example of MTT for RF tomography clearly showed effectiveness of the proposed method. Future work includes state augmentation with unknown signal amplitudes [19, 20] and estimation of static parameters (e.g., $\pi_b$ and $\pi_d$).
