Abstract. An analysis of the CKM matrix parameters within the \( R \)fit approach is presented using updated input values with special emphasis on the recent \( \sin 2\beta \) measurements from BABAR and Belle. The QCD Factorisation Approach describing \( B \to \pi\pi, K\pi \) decays has been implemented in the software package CKMfitter. Fits using branching ratios and CP asymmetries are discussed.

STATISTICAL FRAMEWORK AND INPUTS

In the Standard Model (SM) with three families, CP violation is generated by a single phase in the CKM matrix [1]. This picture can be probed quantitatively by means of a global fit to all quantities sensitive to CKM elements in the SM. The analysis presented here is performed within the \( R \)fit statistical approach [2], which is implemented in the software package CKMfitter [3].

The quantity \( \chi^2 = -2 \ln L(\gamma_{\text{mod}}) \) is minimized in the fit, where the likelihood function is defined by \( L(\gamma_{\text{mod}}) = L_{\text{exp}}(x_{\text{exp}} - x_{\text{theo}}(\gamma_{\text{mod}})) \cdot L_{\text{theo}}(y_{\text{QCD}}) \). The experimental part, \( L_{\text{exp}} \), depends on measurements, \( x_{\text{exp}} \), and theoretical predictions, \( x_{\text{theo}} \), which are functions of model parameters, \( \gamma_{\text{mod}} \). The theoretical part, \( L_{\text{theo}} \), describes the knowledge on the QCD parameters, \( y_{\text{QCD}} \in \{\gamma_{\text{mod}}\} \), where the theoretical uncertainties are considered as allowed ranges. The agreement between data and the SM is gauged by the global minimum \( \chi^2_{\text{min} ; \gamma_{\text{mod}}} \), determined by varying all model parameters \( \gamma_{\text{mod}} \). For \( \chi^2_{\text{min} ; \gamma_{\text{mod}}} \), a confidence level (CL), expressing the goodness-of-fit, is computed by means of a Monte Carlo simulation. If the hypothesis “the CKM picture of the SM is correct” is accepted, CLs in parameter subspaces \( a \), e.g. \( a = (\bar{r}, \bar{h}) \) [4], are evaluated. For fixed \( a \), one calculates \( \Delta \chi^2(a) = \chi^2_{\text{min} ; \mu}(a) - \chi^2_{\text{min} ; \gamma_{\text{mod}}} \), where \( \mu \) stands for all model parameters (including \( y_{\text{QCD}} \)) with the exception of \( a \). The corresponding CL is obtained from \( \text{CL}(a) = \text{Prob}(\Delta \chi^2(a), N_{\text{dof}}) \), where \( N_{\text{dof}} \) is the number of degrees of freedom, in general the dimension of the subspace \( a \). Since the CL depends on the choice of the ranges for the \( y_{\text{QCD}} \), the results obtained in the fit have to be interpreted with care.

The input values used in this analysis are listed in Tab. 1. For |\( V_{ub} \)|, inclusive measurements from LEP and exclusive measurements from CLEO have been used. The preliminary CLEO lepton endpoint analysis [5] using moments obtained from \( B \to X_c\gamma \) is not yet included. For |\( V_{cb} \)|, inclusive measurements from LEP, the measurements of \( B \to D^* \ell \nu \) at zero-recoil and the moments analysis from CLEO [5], using inclusive \( B \to X_c\ell \nu \) and \( B \to X_s \gamma \) decays, have been combined. The uncertainty on \( \Delta m_d \) has been significantly reduced due to the measurements from the \( B \)-factories [6]. However, the constraint on (\( \bar{r}, \bar{h} \)) is not improved since it is dominated by the theoretical uncertainty on \( f_{B_d} \sqrt{B_d} \).
For $\Delta m_3$, the most recent combined amplitude spectrum from [7] is included in the fit using a modified version of the standard amplitude method [2]. If the amplitude spectrum is translated into a likelihood ratio [8, 9], a stronger constraint is obtained. However, to our knowledge, it has not been demonstrated so far that the likelihood ratio can be inter-
FIGURE 2. Left: Constraints in the $(\bar{r}, \bar{h})$ plane using $R_\kappa$ (see text) and the bound on $R$. The CL is indicated by the shaded areas; dark-grey: $> 90\%$, gray: $90\% - 32\%$, light-grey: $32\% - 5\%$. Right: Hypothetical constraints for summer 2002.

interpreted as a probability density function. Hence, we use the more conservative method of Ref. [2] for the numerical analysis presented here. For $\sin^2\beta$, the world average is used. It should be noted that the most precise measurements from BABAR and Belle [6] differ presently by about two standard deviations.

FIT RESULTS

The global minimum of the CKM fit is found to be $\chi^2_{\min;\gamma_{\text{mod}}} = 2.3$, resulting in a goodness-of-fit of 71%. It quantifies the excellent agreement between experimental data and the CKM picture of the SM. Fig. [1] shows the $(\bar{r}, \bar{h})$ plane. Drawn are 5% CL contours from the single constraints using $\Delta m_d$, $\Delta m_s$, $|V_{ub}/V_{cb}|$, and $|\epsilon_K|$, respectively, and the 1$\sigma$- and 2$\sigma$-contours for the four-fold ambiguity on $\beta$ from $\sin^2\beta$. Shown in addition are the contours for the combined fit including $\sin^2\beta$. The statistical precision of the $\sin^2\beta$ measurement already competes with the indirect, theoretically limited constraints. Fig. [1] (right) illustrates the improved constraint when using the likelihood ratio for $\Delta m_s$. Selected numerical results are summarized in Tab. [1].

CHARMLESS TWO BODY DECays

Constraints on the angle $\gamma$ can be obtained from $B \to \pi\pi, K\pi$ decays. Based on color transparency arguments, theoretical calculations such as the QCD Factorisation Approach (FA) [10] and the QCD hard scattering approach [11] have been developed. Recently, the FA has been implemented in CKMfitter. At present, it is premature to in-
fer reliable constraints on the basis of these calculations due to open theoretical ques-
tions [10, 11, 12]. Data from the B-factories are not yet precise enough to probe the cal-
culations in detail. Hence, all fit results are marked by an appropriate “R&D” logo.

For this review, a global fit to $B \to \pi\pi, K\pi$ branching fractions and direct CP asym-
metries measured in self-tagging $B \to K\pi$ decays has been performed within the frame-
work of the FA, where most recent experimental results from BABAR [13], Belle [14]
and CLEO [15] have been used. The numerous theoretical parameters are let free to vary
within the ranges given in Ref. [10]. We find $c_\text{min; mod}^2 = 2.0$ and conclude that data are
consistently described within the FA. The best FA fits are found at $g \approx 80^\circ$ and are in
agreement with the constraints from the standard fit.

Using less theoretical assumptions, ratios of branching fractions can be formed to derive
constraints in the $(\bar{\rho}, \bar{\eta})$ plane. As an example, the CP-averaged ratio

$$R = \frac{\tau_{B^\pm} \mathcal{B}(B^0 \to K^{\pm}\pi^\mp)}{\tau_{B^+} \mathcal{B}(B^0 \to K^0\pi^\pm)},$$

provides the bound $R > \sin^2\gamma$ which is independent of the strong phases [16]. Unfor-
tunately, the present world average $R = 1.07^{+0.19}_{-0.15}$ leads to weak constraints only owing to
the tails of the experimental errors. The ratio

$$R_* = \frac{\mathcal{B}(B^\pm \to K^{0\pi^\pm})}{2 \cdot \mathcal{B}(B^\pm \to K^{\pm\pi^0})},$$

measured to be $R_* = 0.70^{+0.16}_{-0.13}$, can be used to derive bounds in the $(\bar{\rho}, \bar{\eta})$ plane [17].

An important input for the theoretical prediction of $R_*$ is the tree-to-penguin ratio (P/T)
which can be determined experimentally using the relation

$$\tilde{e}_{3/2} = R_{\text{th}} \cdot \tan\theta_C \cdot \frac{f_K}{f_\pi} \sqrt{\frac{2 \cdot \mathcal{B}(B^\pm \to \pi^\pm\pi^0)}{\mathcal{B}(B^0 \to K^{0\pi^\pm})}},$$

where $R_{\text{th}}$ stands for SU(3) breaking corrections estimated in the FA to be [10] $R_{\text{th}} = 0.98 \pm 0.05$. The bound on $R_*$ can be translated into a prediction if additional information
on the strong phases is inserted [14]. Adopting the values for theoretical ranges quoted
in Ref. [10], one obtains the constraints shown in Fig. 2. At present, the constraints re-
main rather weak due to the limited experimental precision. The slight deformation of
the shape pattern around $\gamma \approx 90^\circ$ is due to the bound on $R$. For summer 2002, an inte-
grated luminosity of 100 fb$^{-1}$ is expected to be collected by each experiment, BABAR
and Belle. The experimental precision will then start to provide interesting constraints,
as can be seen from Fig. 2 obtained assuming the present central experimental values
and appropriately rescaling their errors. However, the constraints would still rely on the
validity of some theoretical assumptions not yet fully explored.

Within the FA, the P/T ratio for $B \to \pi^+\pi^-$ is predicted. Compared to the present experi-
mental error on the time-dependent asymmetry $S_{\pi\pi} = 0.03^{+0.53}_{-0.56} \pm 0.11$ from BABAR,
the quoted theoretical uncertainty is much smaller [14]. In Fig. 3 (left), the constraints in
$(\bar{\rho}, \bar{\eta})$ from $S_{\pi\pi}$ are shown using P/T from FA where theoretical uncertainties have been
neglected. The right plot shows the constraints when also using $\sin2\beta$. 
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