Dynamic continuous-wave spectroscopy of coherent population trapping at phase-jump modulation

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A method of dynamic continuous-wave spectroscopy of coherent population trapping (CPT) resonances using phase modulation of the jump type is developed. The time evolution of the spectroscopic signal is investigated. A method for the formation of an error signal for frequency stabilization is proposed. The experimental results are in good qualitative agreement with theoretical predictions based on a mathematical model of a three-level Λ system in a bi-chromatic field. This method can be used in atomic frequency standards (including chip-scale atomic clocks).

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I. INTRODUCTION

Atomic clocks is one of the rapidly developing directions of investigation in quantum metrology. They have numerous fundamental and industrial applications: they can be used in verification of basic theoretical models, ultra-precision measurements, navigation, telecommunications, geodesy, etc. [1–4]. At present, major efforts in the field of compact atomic clocks of the microwave range are focused on using coherent population trapping (CPT) resonances [5–9] for frequency stabilization of local oscillator. An advantage of such devices is a fully optical excitation scheme of rf resonance without a microwave cavity. This allows one to considerably decrease the physical package size (up to the chip scale) [12–14] and the energy consumption. At the same time, CPT clocks have very good metrological characteristics. For instance, a stability of several units of $10^{-13} \tau^{-1/2}$ was demonstrated in [10, 11].

The evolution of atomic clocks leads to the development of new spectroscopic methods. At present, there is growing interest not only in standard stabilization schemes in which relatively slow harmonic modulation is used to form the error signal, but also in stabilization modes using the dynamic response of the quantum system. For instance, a frequency standard based on the transition process in a spectroscopic signal with frequency-step modulation was proposed and implemented in [15, 16].

In this paper, an alternative dynamic approach to the continuous-wave spectroscopy of CPT resonances based on phase-jump modulation is developed. On the interval with a constant phase, atoms are pumped into a dark state. Then a phase jump takes place, which leads to the transition process in the absorption signal. In this case, the time dynamics of such a spectroscopic signal depends on the frequency of a local oscillator, which makes it possible to generate an error signal for the feedback loop. The method being proposed holds much promise for use in CPT atomic clocks (including chip-scale ones).

II. THEORETICAL MODEL

Let us consider, as a theoretical model of the atomic medium, a closed three-level Λ system (see Fig. 1) inter-
acting with a bichromatic field:

\[ E(t) = E_1 e^{-i[\omega_1 t + \phi_1(t)]} + E_2 e^{-i[\omega_2 t + \phi_2(t)]} + \text{c.c.,} \quad (1) \]

under conditions of coherent population trapping, that is, when the difference of frequencies \( \omega_1 - \omega_2 \) is scanned at the frequency \( \omega_{\text{hfs}} \) of the transition between the lower states of the \( \Lambda \) system. We will describe the time dynamics of the \( \Lambda \) system using the formalism of an atomic density matrix, which has the following form in the basis of states \( \{|j\rangle\} \):

\[ \dot{\rho}(t) = \sum_{m,n} |m\rangle \rho_{mn}(t) \langle n| \]. \quad (2)

In the rotating wave approximation the density matrix elements satisfy the following equations:

\[
\begin{align*}
\dot{\rho}_{31} & = (-\gamma_{\text{opt}} + i\delta_1) \rho_{31} + i\Omega_1 e^{-i\phi_1(t)}(\rho_{11} - \rho_{33}) + i\Omega_2 e^{-i\phi_2(t)} \rho_{21} , \\
\dot{\rho}_{32} & = (-\gamma_{\text{opt}} + i\delta_2) \rho_{32} + i\Omega_2 e^{-i\phi_2(t)}(\rho_{22} - \rho_{33}) + i\Omega_1 e^{-i\phi_1(t)} \rho_{12} , \\
\dot{\rho}_{21} & = (-\Gamma + i\delta_R) \rho_{21} - i\Omega_1 e^{-i\phi_1(t)} \rho_{23} + i\Omega_2 e^{i\phi_2(t)} \rho_{31} , \\
\dot{\rho}_{11} & = \frac{\Gamma}{2} \text{Tr}(\dot{\rho}) - \Gamma \rho_{11} + \gamma_1 \rho_{33} - i\Omega_1 e^{-i\phi_1(t)} \rho_{13} + i\Omega_1 e^{i\phi_1(t)} \rho_{31} , \\
\dot{\rho}_{12} & = \frac{\Gamma}{2} \text{Tr}(\dot{\rho}) - \Gamma \rho_{22} + \gamma_2 \rho_{33} - i\Omega_2 e^{-i\phi_2(t)} \rho_{23} + i\Omega_2 e^{i\phi_2(t)} \rho_{32} , \\
\dot{\rho}_{13} & = -\gamma_{\text{sp}} + \Gamma \rho_{33} + i\Omega_1 e^{-i\phi_1(t)} \rho_{13} - i\Omega_1 e^{i\phi_1(t)} \rho_{31} + i\Omega_2 e^{-i\phi_2(t)} \rho_{23} - i\Omega_2 e^{i\phi_2(t)} \rho_{32} , \\
\rho_{23} & = \rho_{23}^*, \quad \rho_{13} = \rho_{31}^*, \quad \rho_{23} = \rho_{32}^* , \quad (3)
\end{align*}
\]

under the condition of normalization (conservation of the total population):

\[ \text{Tr}(\rho) = \rho_{11} + \rho_{22} + \rho_{33} = 1. \quad (4) \]

Let us introduce the following notation: \( \Omega_1 = d_{31} E_1 / \hbar \) and \( \Omega_2 = d_{32} E_2 / \hbar \), are Rabi frequencies for the transitions \( \{1\} \leftrightarrow \{3\} \) and \( \{2\} \leftrightarrow \{3\} \), respectively (\( d_{31} \) and \( d_{32} \) are matrix elements of the operator of electric dipole interaction); \( \delta_1 = \omega_1 - \omega_{31} \) and \( \delta_2 = \omega_2 - \omega_{32} \) are single-photon detunings of laser fields; \( \delta_R = \delta_1 - \delta_2 = \omega_1 - \omega_2 - \omega_{\text{hfs}} \) is two-photon (Raman) detuning; \( \gamma_{\text{opt}} \) is the damping rate of optical coherences (due to spontaneous decay processes, collisions with buffer gas atoms, etc.), \( \gamma_1 \) and \( \gamma_2 \) are the rates of incoherent population transfer from a state \( \{3\} \) to states \( \{1\} \) and \( \{2\} \), respectively; \( \gamma_{\text{sp}} = \gamma_1 + \gamma_2 \) is the spontaneous decay rate of the excited state \( \{3\} \); and the constant \( \Gamma \) describes the relaxation of atoms (for instance, due to transit effects) to an equilibrium isotropic distribution over the lower energy levels of the \( \Lambda \) system.

From the components of the density matrix \( \dot{\rho}(t) \), we form the following vector-column, \( \tilde{\rho}(t) \):

\[ \tilde{\rho} = (\rho_{11}, \rho_{12}, \rho_{13}, \rho_{21}, \rho_{22}, \rho_{23}, \rho_{31}, \rho_{32}, \rho_{33})^T . \quad (5) \]

Then the system of equations (3) can be rewritten as

\[ \dot{\tilde{\rho}}(t) = \hat{L} \tilde{\rho}(t) ; \quad \rho_{11} + \rho_{22} + \rho_{33} = 1, \quad (6) \]

where the matrix \( \hat{L} \) is determined by the coefficients of the system of equations (3):

\[
\hat{L} = \begin{pmatrix}
-\Gamma/2 & 0 & -i\Omega_1 e^{-i\varphi_1} & 0 & \Gamma/2 & 0 & i\Omega_1 e^{i\varphi_1} & 0 & \gamma_1 + \Gamma/2 \\
0 & -\Gamma - i\delta_R & -i\Omega_2 e^{-i\varphi_2} & 0 & 0 & 0 & 0 & i\Omega_1 e^{i\varphi_1} & 0 \\
-i\Omega_1 e^{i\varphi_1} & -i\Omega_2 e^{i\varphi_2} & -\gamma_{\text{opt}} - i\delta_1 & 0 & 0 & 0 & 0 & 0 & i\Omega_2 e^{i\varphi_2} \\
\Gamma/2 & 0 & 0 & 0 & -\Gamma/2 & -i\Omega_1 e^{-i\varphi_1} & i\Omega_2 e^{i\varphi_2} & 0 & 0 \\
0 & 0 & 0 & i\Omega_1 e^{i\varphi_1} & -i\Omega_2 e^{i\varphi_2} & -\gamma_{\text{opt}} - i\delta_2 & 0 & 0 & i\Omega_2 e^{-i\varphi_2} \\
i\Omega_1 e^{-i\varphi_1} & 0 & 0 & 0 & i\Omega_2 e^{-i\varphi_2} & 0 & -\gamma_{\text{opt}} + i\delta_1 & 0 & -i\Omega_1 e^{i\varphi_1} \\
0 & i\Omega_1 e^{-i\varphi_1} & 0 & 0 & i\Omega_2 e^{-i\varphi_2} & 0 & 0 & -\gamma_{\text{opt}} + i\delta_2 & -i\Omega_2 e^{i\varphi_2} \\
0 & 0 & i\Omega_1 e^{-i\varphi_1} & 0 & 0 & 0 & 0 & -i\Omega_1 e^{i\varphi_1} & -i\Omega_2 e^{i\varphi_2}
\end{pmatrix} . \quad (7)
\]

As a spectroscopic signal, we will study the absorbed power which, in the case of an optically thin medium, is proportional to

\[ A(t) = 2 \text{Im}\{\Omega_1 e^{i\phi_1(t)} \rho_{31} + \Omega_2 e^{i\phi_2(t)} \rho_{32}\} = \partial_1 \rho_{33} + (\gamma_{\text{sp}} + \Gamma) \rho_{33} . \quad (8) \]

In this case the inverted signal \(-A(t)\) corresponds to the dynamic part of the transmission signal at the output of the atomic medium.
FIG. 2: Coherent population trapping resonance. Model parameters: $\Omega_1 = \Omega_2 = 0.1 \gamma_{sp}$, $\gamma_1 = \gamma_2 = \gamma_{sp}/2$, $\gamma_{opt} = 50 \gamma_{sp}$, $\Gamma = 10^{-1} \gamma_{sp}$, $\delta_1 = -\delta_2$.

### III. STEADY-STATE CPT RESONANCE

Under steady state conditions ($\partial \bar{\rho}/\partial t = 0$), the dependence of the absorption signal $A$ on two-photon detuning $\delta (\text{as shown in Ref. [17]})$ is described (at $|\delta| \ll \gamma_{opt}$) by the function

$$A_{st}(\delta) = C_0 + C_1 \frac{\gamma_2^2}{(\delta - \delta_0)^2 + \gamma_2^2} + C_2 \frac{(\delta - \delta_0) \gamma_2^2}{(\delta - \delta_0)^2 + \gamma_2^2}.$$  

(9)

where the quantities $C_0$, $C_1$, $C_2$, $\gamma_2$, and $\delta_0$ depend on the model parameters ($\Omega_1$, $\Omega_2$, $\delta_1$, $\delta_2$, $\gamma_{sp}$, $\gamma_{opt}$, and $\Gamma$).

In the case of small single-photon detunings ($|\delta_1|, |\delta_2| \ll \gamma_{opt}$) the antisymmetric contribution to the CPT resonance shape [the last term in the expression (9)] becomes negligibly small, that is, $C_2 \approx 0$.

Under an additional assumption that the decay rate of the dark state is much less than the damping rate of optical coherences and the excited state population ($\Gamma \ll \gamma_{sp}$, $\gamma_{opt}$), for the half-width of the CPT resonance $\gamma_{CPT}$ (see Fig. 2) we obtain the following analytical expression:

$$\gamma_{CPT} \approx \left(\frac{\Gamma + \frac{\Omega_1^2 + \Omega_2^2}{\gamma_{opt}}}{\sqrt{1 + \frac{12\Omega_1^2\Omega_2^2}{\gamma_{sp}\gamma_{opt}} \left(\frac{\Gamma + \frac{2\gamma_2\Omega_1^2 + 2\gamma_1\Omega_2^2}{\gamma_{sp}\gamma_{opt}}}{\gamma_{sp}}\right)}}\right).  \tag{10}$$

The atomic clocks typically have a low saturation regime ($\Omega_{1,2}^2/\gamma_{sp}\gamma_{opt} \ll 1$), in which case the following relation holds:

$$\frac{12\Omega_1^2\Omega_2^2}{\gamma_{sp}\gamma_{opt}} \ll \Gamma + \frac{2\gamma_2\Omega_1^2 + 2\gamma_1\Omega_2^2}{\gamma_{sp}\gamma_{opt}}. \tag{11}$$

Then we find from (10) with (11) that the half-width of the dark resonance in the low-saturation regime can be described by the following simple formula:

$$\gamma_{CPT} \approx \left(\frac{\Gamma + \frac{\Omega_1^2 + \Omega_2^2}{\gamma_{opt}}}{\sqrt{1 + \frac{12\Omega_1^2\Omega_2^2}{\gamma_{sp}\gamma_{opt}} \left(\frac{\Gamma + \frac{2\gamma_2\Omega_1^2 + 2\gamma_1\Omega_2^2}{\gamma_{sp}\gamma_{opt}}}{\gamma_{sp}}\right)}}\right). \tag{12}$$

Note, that the theory of CPT resonances taking into account Zeeman structure of hyperfine energy levels was developed in Ref. [18].

### IV. PHASE-JUMP MODULATION

In our approach, the dynamic (that is, time-dependent) response of the quantum system is due to the modulation of the relative phase of the bichromatic field:

$$\varphi_r(t) = \varphi_1(t) - \varphi_2(t) \tag{13}$$

according to the jump law:

$$\varphi_r(t) = \begin{cases} \varphi_0, & \text{if } t < t_0; \\ \varphi_0 + \Delta \varphi, & \text{if } t \geq t_0, \end{cases} \tag{14}$$

where $\varphi_0$ is the initial phase difference of the two-frequency field, and $\Delta \varphi$ is the value of phase jump. A scheme of modulation of the relative phase $\varphi_r$ is shown in Fig. 3. First the atoms are pumped to a steady state. Then the relative phase changes, in a jump, by some value, which causes a transition process in the absorption signal. Fig. 3 presents the calculated absorption signal $A$ versus time for two opposite values of the phase jump: $A(t, +\Delta \varphi)$ and $A(t, -\Delta \varphi)$. In this case there are three locations of Raman detuning relative to dark resonance: at the top, near the top, and at half-width. The curves show that at an exact two-photon resonance ($\delta R = 0$) the dynamics of the transition process does not depend on the sign of the jump (see Fig. 3b). However, at detuning from resonance ($\delta R \neq 0$) the time evolution of the spectroscopic signal becomes different for phase jumps of opposite signs (see Fig. 3b, c). These features of the transition process in the absorption signal make it possible...
FIG. 4: The dynamic part of the transmission signal $-A(t)$ versus time for three values of two-photon detuning: (a) $\delta_R = 0$, (b) $\delta_R = 0.25\gamma_{CPT}$, and (c) $\delta_R = \gamma_{CPT}$. $\Delta \varphi = +\pi/2$ (red solid curve), $\Delta \varphi = -\pi/2$ (blue dashed curve). Model parameters: $\Omega_1 = \Omega_2 = 0.1\gamma_{sp}$, $\gamma_1 = \gamma_2 = \gamma_{sp}/2$, $\gamma_{opt} = 5\gamma_{sp}$, $\Gamma = 10^{-4}\gamma_{sp}$, $\gamma_{CPT} \approx 5 \times 10^{-4}\gamma_{sp}$, $\delta_1 = -\delta_2$.

FIG. 5: Error signal. Model parameters: $\Delta \varphi = \pi/2$, $\Omega_1 = \Omega_2 = 0.1\gamma_{sp}$, $\gamma_1 = \gamma_2 = \gamma_{sp}/2$, $\gamma_{opt} = 50\gamma_{sp}$, $\Gamma = 10^{-4}\gamma_{sp}$, $\delta_1 = -\delta_2$, $t_0 = 0$, $\tau_d = 12000\gamma_{sp}^{-1}$.

FIG. 6: Slope of the error signal linear part section $[17]$ versus detection time of spectroscopic signal $\tau_d$. Model parameters: $\Delta \varphi = \pi/2$, $\Omega_1 = \Omega_2 = 0.1\gamma_{sp}$, $\gamma_1 = \gamma_2 = \gamma_{sp}/2$, $\gamma_{opt} = 50\gamma_{sp}$, $\Gamma = 10^{-4}\gamma_{sp}$, $\delta_1 = -\delta_2$, $t_0 = 0$.

The key characteristic of the error signal that affects the stability of the frequency standard is the slope of the linear part in the center of the spectral line: $S_{err}(\delta_R)$. Under conditions of single-photon resonance ($\delta_{1,2} \ll \gamma_{opt}$), we have the following symmetry relation:

$$A(t, \delta_R, \Delta \varphi) = A(t, -\delta_R, -\Delta \varphi).$$

By virtue of (16), the dependence of the error signal (15) on two-photon detuning has an antisymmetric form, that is, $S_{err}(-\delta_R) = -S_{err}(\delta_R)$.

The key characteristic of the error signal that affects the stability of the frequency standard is the slope of the linear part in the center of the spectral line:

$$K_{err} = \left. \frac{\partial S_{err}}{\partial \delta_R} \right|_{\delta_R=0}. \quad (17)$$

It follows from the numerical analysis that

$$S_{err} \propto \sin (\Delta \varphi). \quad (18)$$

Hence, the maximal slope $K_{err}^{max}$ of the error signal corresponds to the variation of the relative phase $\Delta \varphi = \pi/2$.

Fig. [15] shows the error signal slope $K_{err}$ versus the spectroscopic signal accumulation time $\tau_d$ [see (15)]. One can see that the slope increases with increasing signal accumulation time to some value, and then it practically does...
not change. Comparing the curves in Figs. 4 and 6 we can conclude that the error signal slope reaches a maximum value when \( \tau_d \) corresponds to the time of reaching a steady state (and exceeds it).

Note that our approach is radically different from the dynamic frequency-step method [15, 16]. Indeed, if we present phase-jump modulation in its frequency equivalent (\( \nu = d\varphi/dt \)), we will have a time representation of alternating \( \delta \)-functions with different signs at the times of phase jumps.

V. PERIODIC PHASE-JUMP MODULATION (PHASE MEANDER)

In the experiment, at frequency stabilization we, as a rule, use the signal accumulated in a sufficiently large number of successive spectroscopic measurements for each frequency value. That is, a periodic process of the quantum system excitation takes place. The above spectroscopic scheme can be considered as a particular case of periodic phase-jump modulation (phase meander) if the modulation period is much greater than the time of pumping of the atoms to a steady state (\( T \gg \gamma_{\text{CPT}}^{-1} \)).

Fig. 7 presents a periodic sequence of phase jumps. To construct a periodic solution to equation (6) we used a method of [19], which is based on a concept of “dynamic steady state”. According to the approach described in [19], the periodic solution for the vector \( \vec{\rho}(t) \) corresponds to the eigenvector of the evolution operator \( \hat{W}(t+T,t) \) with an eigenvalue equal to unity:

\[
\hat{W}(t+T,t)\vec{\rho}(t) = \vec{\rho}(t), \quad \sum_{j=1}^{3} \rho_{jj}(t) = 1. \tag{19}
\]

A solution for the vector \( \vec{\rho}(t) \) at time \( t = t_0 \) in the case of the periodic sequence shown in Fig. 7 can be found from the following equation:

\[
\left( e^{(T/2)\hat{L}^{(\varphi_0+\Delta\varphi)}} e^{(T/2)\hat{L}^{\varphi_0}} - \hat{I} \right) \vec{\rho}(t_0) = 0,
\]

\[
\sum_{j=1}^{3} \rho_{jj}(t_0) = 1, \tag{20}
\]

where \( \hat{I} \) is a unit matrix. Next, we calculate the value of the vector \( \vec{\rho}(t) \) at time \( t = t_0 + T/2 \):

\[
\vec{\rho}(t_0 + T/2) = e^{(T/2)\hat{L}^{(\varphi_0+\Delta\varphi)}} \vec{\rho}(t_0). \tag{21}
\]

For convenience, we choose the beginning of the period at time \( t_0 = 0 \). Then at an arbitrary time inside the period \( 0 \leq t \leq T \) we have the following expression for \( \vec{\rho}(t) \):

\[
\vec{\rho}(t) = \begin{cases} 
 e^{\hat{L}^{\varphi_0}} \vec{\rho}(0), & 0 \leq t < T/2, \\
 e^{(t-T/2)\hat{L}^{\varphi_0}} \vec{\rho}(T/2), & T/2 \leq t < T. 
\end{cases} \tag{22}
\]

Without loss of generality, in the further calculations we can set \( \varphi_0 = 0 \) in equations (21)–(22).

In the case of periodic modulation of the spectroscopic signal, we should investigate the error signal averaged over the period \( T \):

\[
S^{(T)}_{\text{err}}(\delta R) = \frac{1}{T} \left[ \int_{t_0}^{t_0+\tau_d} A(t, \varphi_0 + \Delta\varphi) dt - \int_{t_0+T/2+\tau_d}^{t_0+T/2} A(t, \varphi_0) dt \right]. \tag{23}
\]

It follows from an analysis for jumpwise change in the relative phase \( \varphi_r \) performed in Section 4 that the largest error signal slope \( K^{(T)}_{\text{err}} \) will be achieved at the maximum possible detection time of the spectroscopic signal (see Fig. 8), which corresponds to \( \tau_d = T/2 \). Fig. 8 shows the error signal slope (23) versus the modulation period. One can see that there exists an optimal modulation period in which there is a maximum slope. In the low saturation mode and near the single-photon resonance we have \( T_{\text{opt}} \approx 3.7/\gamma_{\text{CPT}} \).
A comparison of an error signal formed by phase jumps and an error signal based on harmonic modulation at optimal parameters (see [24]) is given in Fig. 9, in which one can see that the maximum possible slope for jump modulation is more than 30% greater than the maximum slope for harmonic modulation.

VI. EXPERIMENT

Fig. 10 shows a scheme of the experimental setup. We use a single-mode vertical-cavity surface-emitting laser (VCSEL), generating at a wavelength of 795 nm, which corresponds to the D1-absorption line of $^{87}$Rb. The pump current is modulated at a frequency of 3.417 GHz by a MW generator so that the laser radiation spectrum becomes polychromatic. The CPT resonance is excited by the first-order components of the spectrum. The $\lambda/2$ plate and the polarizer are used to adjust the radiation power. Linearly polarized laser radiation passing through the $\lambda/4$ plate acquires circular polarization needed for the formation of the resonance. Radiation passing through the atomic cell is registered by the photodetector. Signal from the photodetector is processed using a data acquisition board (DAQ) (sample rate 2 MS/s) and LabView software (National Instruments). A feedback loop is used to stabilize the laser frequency, controlling its temperature. The power of laser radiation entering the atomic cell is 47 $\mu$W, and the beam diameter is about 3 mm.

The atomic cell, heater, and solenoid are placed in a three-layer magnetic shield. The heater and servo loop system maintain the cell at a temperature of 64 °C with an accuracy of 0.01 °C. A magnetic field of 0.1 G produced by the solenoid is used to exclude the influence of magnetically dependent resonances on the metrological one. A cylindrical cell has a diameter of 8 mm and a length of 15 mm and is filled with $^{87}$Rb vapor and a mixture of Ar-N2 buffer gases with a total pressure of 29 Torr.

The modulation of the relative phase difference between the components of the optical spectrum forming the CPT resonance is carried out by step modulation of the microwave signal phase provided by the built-in option of the generator (Agilent E8257C). The control signal is a meander with a period of 9 ms supplied by the function generator. The same signal is used as a trigger for DAQ. The chosen phase modulation period satisfies the condition of the maximum slope of the linear part of the error signal $T_{\text{opt}} \approx 3.7/\gamma_{\text{CPT}}$ for the CPT resonance width of 840 Hz.

Fig. 11 shows the transmission of the atomic cell versus time at different frequency detunings from the CPT resonance peak. The experimental curves demonstrate good qualitative agreement with the theoretical ones. As noted in Section IV, an error signal for frequency stabilization of the microwave generator is formed by subtracting the integrals of the signals corresponding to phase jumps with opposite signs.

It is shown that the slope coefficient of the linear part of an error signal obtained in this way depends on the detection time $\tau_d$ (Fig. 12). As the detection time increases, the slope increases and reaches a constant level. The maximum slope is achieved by integrating the entire spectroscopic signal corresponding to a constant value of the phase difference, that is, before the next switching. The shape of the experimental curve also corresponds to the theory. A comparison of Figs. 12 and 13 confirms a conclusion made in Section IV that the maximum slope is achieved when the detection time is equal to or greater than the time in which a steady state is reached.
at a frequency of 3.417 GHz, 110 Hz (taking into account that RF modulation occurs
crowave field at a frequency of 170 Hz with a deviation of
error signal is formed by harmonic modulation of the mi-
modulation of the microwave signal. In the first case, an
by frequency (black curve) and phase-jump (red curve)
in Fig. 10). The inset shows the laser spectrum entering the cel l,
Synchronous detection. In the second case, an error sig-
components of the spectrum
= 1
π/2 (blue curves), +
π/2 (red curves). The inset shows the laser spectrum entering the cell, which is provided by a Fabry-Perot interferometer (not shown in Fig. 10).

Fig. 11 presents a comparison of error signals obtained by frequency (black curve) and phase-jump (red curve) modulation of the microwave signal. In the first case, an error signal is formed by harmonic modulation of the microwave field at a frequency of 170 Hz with a deviation of 110 Hz (taking into account that RF modulation occurs at a frequency of 3.417 GHz, \(f_m = 1.5\Gamma, M = 1.3\): the condition of maximum signal slope \([20]\) ) and subsequent synchronous detection. In the second case, an error sig-

FIG. 11: Transmission versus time for three values of two-photon detuning: 0, 100, and 420 Hz. The curves are ob-
tained by averaging 100 periods of phase modulation. The zero on the horizontal axis corresponds to the moment of phase switching; relative phase difference of the first-order components of the spectrum \(\pm \pi/2\) (blue curves), \(\pm \pi/2\) (red curves). The inset shows the laser spectrum entering the cell, which is provided by a Fabry-Perot interferometer (not shown in Fig. 10).

The slope of the linear part of the signals. The inset shows the linear part of the signals.

FIG. 12: Slope of the linear part of error signal versus detection time \(\tau_d\). Values for normalized error signals are indicated.

FIG. 13: Error signals obtained by frequency harmonic mod-
ulation (black curve) and phase jump method (red curve). The inset shows the linear part of the signals.

Fig. 13 presents a comparison of error signals obtained by frequency (black curve) and phase-jump (red curve) modulation of the microwave signal. In the first case, an error signal is formed by harmonic modulation of the microwave field at a frequency of 170 Hz with a deviation of 110 Hz (taking into account that RF modulation occurs at a frequency of 3.417 GHz, \(f_m = 1.5\Gamma, M = 1.3\): the condition of maximum signal slope \([20]\) ) and subsequent synchronous detection. In the second case, an error sig-

The inset shows the linear part of the signals.

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FIG. 12: Slope of the linear part of error signal versus detection time \(\tau_d\). Values for normalized error signals are indicated.
VII. CONCLUSION

We have proposed and studied a previously unexplored method of error signal formation in continuous-wave spectroscopy. This method is based on the excitation of CPT resonances in a bichromatic field with jump modulation of the relative phase and subsequent detection of the time dynamics of the spectroscopic signal. A feature of the approach is that phase-jump modulation corresponds to the $\delta$-function in terms of frequency modulation. The parameters at which the slope of the error signal has a maximum value have been determined. The theoretical predictions are in good agreement with the experimental results.

In addition, the preliminary estimations show that the proposed phase-jump technique has a reduced sensitivity to the artificial shift due to asymmetry of the CPT resonance. It can lead to the improvement of metrological characteristics (long-term stability and accuracy) in comparison with common method using harmonic frequency modulation to form an error signal. Therefore, the developed method requires further careful theoretical and experimental studies, and can find wide use in atomic CPT clocks (including chip-scale ones).

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