The Measurement of the Muon’s Anomalous Magnetic Moment Isn’t

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Recent results announced as measurements of the muon’s anomalous magnetic moment are in fact measurements of the muon’s anomalous spin precession frequency. This precession frequency receives contributions from both the muon’s anomalous magnetic and electric dipole moments. We note that all existing data cannot resolve this ambiguity, and the current deviation from standard model predictions may equally well be interpreted as evidence for new physics in the muon’s anomalous magnetic moment, new physics in the muon’s electric dipole moment, or both.

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Recently the Muon (g − 2) Collaboration announced a new measurement of the muon’s anomalous magnetic moment $\mu$. More precisely, however, what has been measured is the muon’s anomalous spin precession frequency. This receives contributions from both the muon’s anomalous magnetic and electric dipole moments, and we point out that the reported data and all existing constraints cannot distinguish between the two.

The recent result is the latest tour de force from the Muon (g − 2) Experiment [2]. This experiment measures the anomalous spin precession frequency of muons circulating in a perpendicular and uniform magnetic field. For fermions with gyromagnetic ratio $g = 2$, the cyclotron and spin precession frequencies are identical. Measurements of the anomalous spin precession frequency have therefore been reported as measurements of the anomalous magnetic dipole moment (MDM) $a_\mu = (g_\mu - 2)/2$.

The spin precession frequency is also sensitive to the muon’s electric dipole moment (EDM), however [3, 4]. For a muon traveling with velocity $\beta$ perpendicular to both a magnetic field $B$ and an electric field $E$, the anomalous spin precession vector is

$$\omega_a = -a_\mu \frac{e}{m_\mu} B - d_\mu \frac{2c}{\hbar} \beta \times B - \frac{e}{m_\mu c} \left( \frac{1}{\gamma^2 - 1} - a_\mu \right) \beta \times E - d_\mu \frac{2}{\hbar} E,$$

where $m_\mu$ and $d_\mu$ are the muon’s mass and EDM. In recent experiments, the $\beta \times E$ term of Eq. 1 is removed by running at the ‘magic’ $\gamma \approx 29.3$, and the last term is negligible. For highly relativistic muons with $|\beta| \approx 1$, then, the anomalous spin precession frequency is

$$|\omega_a| \approx |B| \left[ \left( \frac{e}{m_\mu} \right)^2 a_\mu^2 + \left( \frac{2c}{\hbar} \right)^2 d_\mu^2 \right]^{1/2},$$

and it constrains only a combination of $d_\mu$ and $a_\mu$.

In Fig. 1 we show regions of the $(d_\mu, a_\mu)$ plane that are consistent with the new $|\omega_a|$ measurement. Also shown are the latest standard model (SM) predictions [3, 4, 5].

Assuming a negligible $d_\mu$, the measurement shows tentative evidence for new physics in $a_\mu$ with uncertain significance, given the spread in theoretical predictions. However, the $|\omega_a|$ result could just as well be taken as evidence for new physics in $d_\mu$. The best direct bound on $d_\mu$ [3] is also shown. Clearly it does not resolve this ambiguity; if anything, it favors the EDM interpretation. In fact, even taking the lowest SM prediction for $a_\mu$, a striking and unambiguous conclusion is that, barring a fine-tuned cancelation, the Muon (g − 2) Experiment has now set the most stringent upper bound on the muon’s electric dipole moment with $|d_\mu| < 3.2 \times 10^{-19}$ e cm.

The MDM/EDM ambiguity may be resolved by appealing to theoretical prejudice that $d_\mu$ is small. In supersymmetry, for example, the maximal value of $a_\mu$ is $a_\mu_{\text{max}} \sim 10^{-7}$, assuming only flavor conservation [5]. By a phase rotation of the relevant operator, this implies a maximal EDM of roughly $a_\mu^\text{max} \sim (e\hbar/2m_\mu c)d_\mu^\text{max} \sim$...
$10^{-20}$ e cm. This conclusion is far from universal, however. For example, large muon EDMs are possible in models where EDMs scale approximately as $d_f \propto m_f^3$ \footnote{See, e.g., K. S. Babu, S. M. Barr and I. Dorsner, Phys. Rev. D 64, 053009 (2001) [hep-ph/0012303].}. Given our current profound ignorance of the origins of electroweak symmetry breaking, flavor, and CP violation, no definitive statement can be made.

The effects of $d_\mu$ and $a_\mu$ are, of course, distinguishable \footnote{J. Miller, private communication.}: $a_\mu$ causes precession around the magnetic field’s axis, but $d_\mu$ leads to oscillation of the muon’s spin above and below the plane of motion. A search for up-down asymmetry in the current data is in progress \footnote{J. Bailey et al. [CERN-Mainz-Daresbury Collaboration], Nucl. Phys. B 150, 1 (1979).}. A dedicated EDM experiment \footnote{Y. K. Semertzidis et al., hep-ph/0012087; J. L. Feng, K. T. Matchev and Y. Shadmi, in Proc. of the APS/DPF/DPB Summer Study on the Future of Particle Physics (Snowmass 2001) ed. R. Davidson and C. Quigg, hep-ph/0110157.} would provide a conclusive resolution by either measuring a non-vanishing $d_\mu$ or constraining the contribution of $d_\mu$ to $|\omega_a|$ to be insignificant.

For now, however, the reported data is not a model-independent measurement of the muon’s anomalous magnetic moment. If measurements of precession frequency are interpreted as measurements of $a_\mu$, the assumption of a negligible muon EDM is best made explicit. Alternatively, the experimental status may be summarized without theoretical assumptions as in Fig. 1.

\footnote{Incumbent of a Technion Management Career Development Chair.}

\begin{thebibliography}{99}
\bibitem{1} G. W. Bennett [Muon g-2 Collaboration], Phys. Rev. Lett. 89, 101804 (2002) [hep-ex/0208001].
\bibitem{2} For previous results, see R. M. Carey et al., Phys. Rev. Lett. 82, 1632 (1999); H. N. Brown et al. [Muon (g-2) Collaboration], Phys. Rev. D 62, 091101 (2000) [hep-ex/0009029]; H. N. Brown et al. [Muon g-2 Collaboration], Phys. Rev. Lett. 86, 2227 (2001) [hep-ex/0102017].
\bibitem{3} J. Bailey et al. [CERN-Mainz-Daresbury Collaboration], Nucl. Phys. B 150, 1 (1979).
\bibitem{4} J. L. Feng, K. T. Matchev and Y. Shadmi, Nucl. Phys. B 613, 366 (2001) [hep-ph/0107182].
\bibitem{5} F. Jegerlehner, talk given at the Workshop Centre de Physique Théorique Marseille, France, 14-16 March 2002.
\bibitem{6} K. Hagiwara, A. D. Martin, D. Nomura and T. Teubner, talk given by T. Teubner at ICHEP’02, Amsterdam, Netherlands, 24-31 July 2002.
\bibitem{7} M. Davier, S. Eidelman, A. Hocker and Z. Zhang, hep-ph/0208177.
\bibitem{8} J. L. Feng and K. T. Matchev, Phys. Rev. Lett. 86, 3480 (2001) [hep-ph/0102146].
\bibitem{9} See, e.g., K. S. Babu, S. M. Barr and I. Dorsner, Phys. Rev. D 64, 053009 (2001) [hep-ph/0012303].
\bibitem{10} J. Miller, private communication.
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