Abstract— A hallmark of visual analytics is its ability to support users in translating broad, open-ended analytic questions (e.g., “is our company succeeding?”) into verifiable hypotheses that can be tested from the available data (e.g., “our total revenue increased this past quarter”). However, the process of converting open-ended analytic questions into testable hypotheses is complex and often ill-defined. Beyond high-level descriptions, the visual analytics literature lacks a formalization that can be operationalized for analysis and automation. In this paper, we propose a novel grammar to express hypothesis-based analytic questions for visual analysis. Drawing from prior work in science and education, our grammar defines a formal way to express sets of verifiable hypotheses as a “hypothesis space”. Our proposed use of hypothesis spaces contributes a new lens to unify concepts of user goals, the capabilities of a dataset, visual analysis, and testable hypotheses. As a result, we can reformulate abstract classes of visual analysis goals, such as analytic and data-related tasks, in a way that is suitable for analysis and automation. We demonstrate use cases of our grammar in real-world analytic applications including VAST challenges, Kaggle competitions, and pre-existing task taxonomies. Finally, we provide design opportunities in which our grammar can be operationalized to articulate analysis tasks, evaluate visualization systems, and support hypothesis-based reasoning in visual analytic tools.

1 INTRODUCTION

Hypotheses play a central role in the visual analytics process. From hypothesis generation to refinement and testing, interactive visual analysis has been lauded to support a user in each step of this process. For example, in Illuminating the Path, visual analytics is described as a new science of interaction that supports the “hypothesis-guided discourse” of analytical reasoning [16]. Similarly, in Mastering the Information Age - Solving Problems with Visual Analytics, the authors describe the success of a visual analytics tool as the evaluation of its ability to support the “entire process of hypothesis forming, refinement, validation, and presentation” [33].

While the importance of supporting hypotheses in visual analytics is well touted, the question that remains unanswered is “what is a hypothesis?” In the science community, a hypothesis is a statement posing an explanation for some phenomena, and is considered valid under the conditions that it is both testable and falsifiable [45]. Because of this formalism, valid hypotheses are well-defined in science education. They must describe at least two variables whose relationship must be qualifiable, and an appropriate test should verify whether the hypothesis can be confirmed or rejected [58].

In visual analytics, the precise definition and criteria of a hypothesis is less clear. As observed by Sacha et al. [49], a hypothesis in visual analytics is a “[formulation of] an assumption about the problem domain that is subject to analysis.” In accordance with this observation, Deng et al. [18] report a hypothesis generated by environmental scientists using their visual analytics system: “The coal-fired power plants could be a severe air pollution source that had a considerable impact on remote regions.” Similarly, Choi et al. [12] report a hypothesis that could be formed with their tool in the business domain: “I expect New Business Density in Ireland to be lower than France, Australia, Netherlands Sweden, and Spain.” In these works, hypotheses used for visual analytics are more similar to open-ended analysis questions [41]. This stands in contrast to the formalism used in the science community.

Although the definitions of hypothesis in the visual analytics and science communities appear disparate, their relationship can be articulated with science education literature. For example, in a book by Sunal and Haas, the authors propose a “Hypothesis Quality Scale” for teaching and evaluating hypotheses (Table 1). With this scale, hypotheses in visual analytics can be thought of as underdeveloped scientific hypotheses. For instance, the Hypothesis Quality Scale would rank the two example hypotheses above (about air pollution and business) at level 2, because of their incomplete reference to qualifiable variables. Given a scale such as that proposed by Sunal and Haas, it becomes possible to reason about and improve these imprecise hypotheses. For example, the Hypothesis Quality Scale shows that providing a specific relationship that qualifies why the coal power plants would be a source of pollution and how such pollution could be measured would improve the air pollution hypothesis from underdeveloped (level 2) to more precise (level 4).

In addition to disambiguating the definitions of hypothesis, the ability to express high-level visual analytic hypotheses using the same language as scientific hypotheses can bring practical benefits. It allows us to formalize the process of translating high-level visual analytic hypotheses into well-formed scientific hypotheses and offers the opportunity to design tools to support this process. It allows us to compare different analytic hypotheses with each other to understand, for example, whether one is more complex or ambiguous than another.

To that end, this paper proposes the first mechanism for describing and transforming visual analytics hypotheses into scientific hypotheses. Drawing from prior work in science and education literature [24 37 53], we propose a grammar for expressing hypothesis-based analytic questions. We start with a grammar that expresses a scientific hypothesis—a specific, narrow, and testable question—and relax it to express sets of scientific hypotheses. We call these sets of hypotheses a “hypothesis space”. This allows us to decompose an open-ended visual analytics question such as “what is the state of the Rose Crested Blue Pipit’s nesting?” of the 2017 VAST Challenge [70] into a number of scientific hypotheses such as: Average number of nests has decreased between time_1 and time_2; Average number of nests has increased from location_1 to location_2, etc..

Our grammar is designed to capture the concept of different “hypothesis spaces” for visual analytics. We observe that, in the infinite space of all possible hypotheses in the universe, a visual analytics hypothesis is constrained by three considerations: (1) the hypotheses that a given dataset is able to answer, (2) the hypotheses that can be evaluated by a given visual analytics system, and (3) the questions that the analyst seeks to answer. Each of these constraints forms its own hypothesis space: a data hypothesis space, a visualization hypothesis space, and an analyst hypothesis space. We posit that a successful application of visual analytics relies on generating and evaluating hypotheses that lie within the intersection of these three hypothesis spaces. Our grammar enables more formal study of visual analytics by allowing us to examine the relationships between these three spaces and consider their implications to the field.

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To validate the efficacy of our proposed grammar, we apply it in three case studies. First, we reformulate the 2017 VAST Challenge questions using our grammar. Second, we examine the parallel between our work and the pre-existing research efforts into formalizing tasks in visual analytics. To highlight the interconnected nature of hypotheses and tasks, we describe Amar et al.’s analytic task taxonomy \(^1\) with our grammar and discuss the benefits and limits of these two approaches. Last, we extend the application of our grammar beyond visual analytics and consider its applicability in the broader scope of data science. We examine select Kaggle challenges that traditionally assume machine learning solutions through the lens of hypothesis testing.

To summarize, in this paper we make the following contributions:
- We introduce a Grammar of Hypotheses for visual analytics.
- We use the grammar to describe “hypothesis spaces” that can transform open-ended visual analytics questions into testable hypotheses.
- We present case studies to demonstrate applications of our grammar, including an examination of VAST Challenges, Kaggle competitions, and the analytic task taxonomy by Amar et al. \(^1\).
- We propose design opportunities afforded by our grammar including articulating analysis tasks, evaluating visualization systems, and supporting hypothesis-based reasoning with visual analytic tools.

## 2 What is a Hypothesis?

The concept of a “hypothesis” has varying implications depending on the context and community in which it is used. In this section, we provide a background on how hypotheses are defined and used in science and education. We outline the criteria for a “valid” scientific hypothesis and contrast it with the visual analytics community’s use of the same term. Based on these criteria and the needs of the visual analytics community we propose our grammar for hypotheses.

### 2.1 Scientific Hypotheses

A scientific hypothesis is the primary building block of the scientific method. In the most rigorous cases of its use, a scientific hypothesis is accepted by scholars as a proposition (or a set of propositions) proposed as a tentative explanation for an observed situation \(^15\)\(^23\).

Formally, first and foremost, a scientific hypothesis must be both testable and falsifiable to be considered valid \(^40\). Beyond this central tenet, researchers in both the science and education communities have proposed a number of criteria and requirements for evaluating the quality of a scientific hypothesis \(^37\)\(^40\)\(^64\). One accepted set of criteria was introduced in 1975 by Quinn and George \(^47\) which states that a “good” hypothesis must have the following properties:
- it makes sense;
- it is empirical, a (partial) scientific relation;
- it is adequate, a scientific relation between at least two variables;
- it is precise—a qualified and/or quantified relation;
- it states a test, an explicit statement of a test.

In 2002, Sunal and Haas \(^58\) proposed a Hypothesis Quality Scale, seen in Table 1 as a way to assess and provide rankings for a student’s hypothesis. This, and other similar criteria such as the one proposed by Kroze et al. \(^37\), not only provide a formalism to assess the quality of a hypothesis, but also suggest relations between the different levels of hypotheses and the mechanism for refinement.

### 2.2 Hypotheses in Other Domains

It is relevant to note that research into the formalism of a “hypothesis” is rich and nuanced. In an article by Jeong and Kwon \(^29\), the authors document some common uses and criteria for hypotheses across philosophy, science, and science education. Of particular relevance to this work is the distinction between scientific hypotheses and statistical hypotheses \(^60\). Scholars have argued that the two are distinct due to the use of confidence thresholds (e.g., \(p\)-values) and the reliance on null and alternate hypotheses in formulating statistical hypotheses – both of which would be inappropriate for scientific hypotheses \(^4\). For clarity, in this paper we use the definition and criteria for scientific hypotheses when developing our proposed grammar to avoid confusion.

| Level | Criterion |
|-------|-----------|
| 0     | No explanation provided. A nonsense statement, a question, an observation, or a single inference about a single event, person, or object. |
| 1     | An inappropriate explanation. For example, “...because it’s magic” or “...because the man pushed a button.” |
| 2     | Partial appropriate explanation. For example, incomplete reference to variables, a negative explanation, or analogy. |
| 3     | Appropriate explanation relating at least two variables in general or nonspecific terms. |
| 4     | Precise explanation with qualification of the variables. A specific relationship between the variables is provided. This is a hypothesis. |
| 5     | In addition to level 4, a test is given. |
| 6     | An explicit statement of a test of a hypothesis is given. An inference is made by the teacher that the student who states a test also hypothesizes adequately and precisely. |

### 2.3 Visual Analytics Hypotheses

Through the lens of scientific hypotheses, we can examine the use of “hypotheses” in the visual analytics literature. The relation between hypotheses and visual analysis has been formalized in theoretical models and documented in publications. In the Sensemaking process by Pirolli and Card \(^45\), hypotheses are a product of schematization and the step before the presentation of analysis findings. In the knowledge generation model by Sacha et al. \(^49\), a hypothesis is generated from knowledge and insight, and is the driving motivation behind a user’s (inter)actions with a visual analytics system.

In the article The Science of Interaction, Pike et al. \(^44\) similarly describe hypothesis generation in visual analytics as an “abductive process.” The authors wrote, “[in] constructing hypotheses or ‘abductions’, the analyst is engaged in exploration of the data space and the formation of mental models to explain observations.” This sentence is telling and reflective of the way that the visual analytics community considers a hypothesis. In this sentence, “hypothesis generation” is defined as the formation of a mental model, suggesting that a hypothesis is in fact a mental model for explaining an observation.

When compared to a scientific hypothesis, this definition of a visual analytics hypothesis is more similar to that of the psychology and the intelligence analysis communities. For example, Wason described his psychology study on the effects of confirmation bias in the article On the Failure to Eliminate Hypotheses in a Conceptual Task \(^68\). In this article, a (psychology) hypothesis is described as a “necessary-and-sufficient concept” that a participant was required to mentally construct and evaluate when completing the four-card problem (now known as the Wason selection task). In the book “Psychology of Intelligence Analysis” Heuer wrote extensively about the importance of generating and evaluating hypotheses in decision-making \(^25\). His examples of (intelligence analysis) hypotheses include “[Iraq] will sponsor some minor terrorist actions,” which does not resemble a valid scientific hypothesis (Table 1), but is more similar to the visual analytics’ definition of a hypothesis as a mental model.

Similar uses of the term hypothesis appear throughout visual analytics literature. In the 2017 VAST Challenge, contestants were asked to “provide a hypothesis of what the pattern [of vehicle activities] represents” in the Lekagul Nature Preserve dataset \(^70\). In a paper by Wang et al. on visual analysis of hypothesis-based evaluation of machine learning models, the authors describe hypothesis-testing as the testing of a high-level concept, such as “would having additional information about a geographical context improve the accuracy of building recognition?”

In contrast to a scientific hypothesis, which is required to be empirical, testable, and precise, the visual analytics community’s use of hypothesis is less rigorous and formal. However, we postulate that the lack of formalism might be necessary in order to reflect the broad nature of task types that visual analytics is designed to solve – open-ended challenges, “wicked problems,” and messy data \(^20\), which cannot be
described with a single, well-formed, but narrow scientific hypothesis. Literature in the science and education domains (such as those by Sunal and Hass [55]) provide the perspective for considering scientific hypotheses and visual analytics hypotheses along a spectrum (e.g., see Table 1). However, these literature fail to provide the concrete mechanisms for transforming one type of hypothesis into another. In this paper, we propose such a formalism using a grammar and present the methods that can unify the two definitions of hypotheses.

3 A Grammar of Hypotheses

We propose a grammar to express hypotheses in visual analysis. It is informed by our literature review of science and science education, established criteria for valid scientific hypotheses, and prior use of hypotheses in visual analytics. The key difference that enables this approach is that our hypotheses are designed to be verified with respect to a dataset that has already been collected and stored, rather than statements about the world at large, which requires data collection (e.g., from experimentation).

We first clarify our definition of data, which is based on the relational model [14]. We then introduce our grammar by starting from a trivial hypothesis and incrementally enriching its expressiveness. Finally, we describe how to ground a grammar to a dataset. Our grammar is based on Parsing Expression Grammars (PEG) [22], and follows its syntax.

3.1 What is the Data?

A hypothesis refers to characteristics of a dataset, thus we first describe what we mean by “data”. Many of these concepts and terminology are borrowed from relational databases [14].

A relation is defined by a schema—a sequence of attribute-type pairs—and records that conform to the schema. For instance, Relation 1 below is a marketing table with schema (id:int, month:int, market:string, cost:float, price:float) and six records. A relation is a relation that contains a finite number of rows—for instance, a CSV file or a table in a database. We assume every table has a unique id attribute. A function is a relation whose schema are its input and output attributes, that does not explicitly store its rows. For instance, Gaussian(x,y) → (mean,std) is a relation with four attributes, x,y are vectors of numbers, and the rest are the fit parameters. Invoking a function “looks up” the values of its output attributes in its corresponding “row”.

| id | month | market | cost | price |
|----|-------|--------|------|-------|
| r1 | 1     | US     | 1    | 7     |
| r2 | 2     | US     | 2    | 8     |
| r3 | 3     | US     | 3    | 9     |
| r4 | 4     | US     | 4    | 10    |
| r5 | 5     | EU     | 5    | 11    |
| r6 | 6     | EU     | 6    | 12    |

Relation 1

For simplicity, we will assume there is a single table that the user is analyzing, and that any data transformations (joining tables, extraction, cleaning, etc.) have already been performed. Supporting more tables or data transformations may add more rules to the grammar, but does not change the technical discussion; Section 7 discusses this in more depth.

3.2 Hypothesis Grammar

We introduce a single grammar, step by step, that parses boolean hypothesis statements. Starting from a trivial but valid scientific hypothesis that describes the relation between two values [47], we incrementally increase its expressiveness at each stage. Any time a new rule is introduced to our grammar throughout the text, we show it in blue.

Trivial Hypothesis: This grammar matches statements that equate two numbers (e.g., 2 = 2). The grammar is described by production rules (shown in our grammar bolded) that expand a non-terminal on the left side of the : to the sequence of symbols on the right side. number is a non-terminal that matches numeric strings.

\[ \text{hyp} :: \text{var} \mathbin{=} \text{"=} \text{const} \]

Attributes and Constants: We can use dot notation to reference the \( p \text{th} \) row’s attribute value: \( r2.\text{cost} = r2.\text{cost} \) checks that the second row’s cost is 2. attr expands to all possible string values (str), and string constants like ‘="' are wrapped in double quotes to distinguish them from PEG grammar syntax.

\[ \text{hyp} :: \text{var} \mathbin{=} \text{"=} \text{const} \]

Constants: We introduce constants (const) of different data types, and use e.g., number to refer to all strings that parse to a number. For instance, \( r2.\text{market} = \text{"US"} \) checks that market is ‘US’.

\[ \text{hyp} :: \text{var} \mathbin{=} \text{"=} \text{const} \]

Operators: We now extend statements to any comparison operator. For instance, \( r2.\text{cost} < 10 \) states that r2’s cost is less than 10.

\[ \text{hyp} :: \text{var} \mathbin{op} \text{const} \]

Non-scalar Variables: We often wish to make universal quantification statements. Consider the statement that all costs are less than 10: \( \text{cost} < 10 \). The \( < \) compares scalar values, so we “flatten” its operands to compare each scalar on the left and right sides. The statement is true if all evaluations are true. The grammar remains the same.

Predicates: We often only care about a subset of the dataset. We use predicates to restrict the rows that an attribute refers to. Predicates thus consist of a variable, an operator, a constant, and optionally another predicate (in the grammar, if \( x \) is optional we denote it (x)). For instance, \( \text{cost}[(\text{market} = \text{"US"}) \& \text{price} < 10] \) states that all US costs are below 10, and \( \text{cost}[(\text{market} = \text{"US"} \& \text{month} = 2) < 10] \) states that US costs in February are below 10. Note \( r2.\text{cost} \) is simply shorthand for the general cost[\text{id}=2]. We use the latter notation in this paper.

\[ \text{hyp} :: \text{var} \mathbin{op} \text{const} \]

Hypothesis-level Predicates: Both sides of the hypothesis operator may reference attributes. For instance, \( \text{cost}[(\text{market} = \text{"US"}) \& \text{price} < \text{cost}[(\text{market} = \text{"US"})] \) checks that US costs were less than US prices. Since the left and right operands are tables that match their respective predicates, we evaluate the inequality for every record in the dataset where either predicate is true. This means that expressions such as \( \text{cost}[(\text{market} = \text{"US"}) \& \text{price} < \text{cost}[(\text{market} = \text{"EU"})] \) are invalid, because the left and right operands do not refer to the same records. To avoid these errors, the predicate can be applied to the entire hypothesis, rather than both sides of the hypothesis operator.

\[ \text{hyp} :: \text{var} \mathbin{op} \text{var} \]

Functions: Operands may be expressions over an attribute as well. For instance, \( \text{avg}(\text{cost}) < 10 \) checks that the average cost is lower than 10; \( \text{avg}(\text{cost}) < \text{avg}(\text{price}) \) checks that the average cost is lower than the average price; and \( \text{avg}(\text{cost}) < \text{price} \) checks that the average cost is lower than every price. In these cases, hypotheses are expressed over expressions (expr), which are expressed as functions (func). We use func to refer to any pre-registered function name e.g., \( \text{avg}, \text{sum} \).

\[ \text{hyp} :: \text{expr} \mathbin{op} \text{expr} \]

Binary and Nullary Functions: Functions may take zero or two arguments. For example, \( \text{avg}(\text{cost}), \text{avg}(\text{price}) < 10 \) checks...
that average cost and price differ by less than 10, using prefix notation, and count() counts the number of rows that match a given predicate(s). In addition, \( EMD(cost, price) < 1 \) checks that the earth mover’s distance between cost and price is below 1.

\[
\text{hyp} :\ -\ expr\ op\ expr\ ("[\ "pred\"]")?
\text{expr} :\ -\ func\ "(\ "expr\ (,\ expr)?\ "\)"| var
\text{var} :\ -\ attr\ ([\ "pred\"])?\ | const
\text{const} :\ -\ \text{number}\ |\ \text{str}\ |\ \text{date}\ |\ ..
\text{op} :\ =\ |\ <\ |\ >\ |\ ...
\text{attr} :\ -\ \text{month}\ |\ \text{price}\ |\ ...
\]

Finally, we express the conjunction of multiple hypotheses by augmenting the hyp rule to support a list of hypotheses. For example, \((\text{avg}(\text{cost}),\ \text{avg}(\text{price})) < 10 \& \text{price} > 6\) checks that average cost and price differ by more than 6 and less than 10.

\[
\text{hyp} :\ -\ expr\ op\ expr\ ("[\ "pred\"]")?\ (\&\ hyp)?
...
\]

This brings us to the final version of our grammar that we refer to as the Base Grammar in the rest of the text and summarized in Table 2.

### 3.2.1 A Hypothesis is a Grammar

So far, each hypothesis we have looked at has been one statement that can be parsed by the underlying grammar. However, we observe that a hypothesis is a degenerate grammar that can parse a single statement. We will now illustrate this idea, as this perspective will be helpful when we discuss hypothesis spaces next.

Consider the statement \( KL(cost, price) < 10 \), a distributional hypothesis where \( KL \) refers to the KL divergence. It is equivalent to the following grammar that restricts non-terminals to concrete values.

\[
\text{hyp} :\ -\ \text{expr}\ \text{op}\ \text{expr}\ ("[\ "pred\"]")?
\text{func} :\ -\ \text{KL}
\text{expr1} :\ -\ \text{var}
\text{expr2} :\ -\ \text{var}
\text{attr1} :\ -\ \text{cost}
\text{attr2} :\ -\ \text{price}
\text{op} :\ -\ <\ 
\]

Notice that there is a copy of \( \text{expr} \), and its production rules, for each function argument. This ensures that each argument is restricted independently. We denote the \( \ell \)-th copy of a non-terminal \( T \) as \( T_i \).

As a short-hand, we can inline all production rules that expand to a constant, and concatenate adjacent string constants together:

\[
\text{hyp} :\ -\ \text{KL(cost,price)}<^{\text{\text{\&}}} 10
\]

### 3.3 Sets of Hypotheses as Hypothesis Spaces

A hypothesis space is defined as the set of all hypotheses expressible by a subset of the base grammar. While one hypothesis is equivalent to a grammar that parses a single statement, a hypothesis space is a grammar that parses many statements. Therefore, a hypothesis space defines a set of verifiable hypotheses, as opposed to just one hypothesis.

For instance, suppose our hypothesis is that cost and price are similar, and any similarity measure is acceptable to us. We might express this with the grammar below, which accepts any function name.

\[
\text{hyp} :\ -\ \text{expr}\ <^{\text{\&}} 10
\text{expr} :\ -\ \text{func}\ "(\ \text{cost},\text{price})"\ 
\text{func} :\ -\ \text{str}
\]

We could further limit the distance functions to KL-divergence, correlation, and earth movers distance. This reduces the number of hypothesis statements that can be parsed by the grammar.

\[
\text{hyp} :\ -\ \text{expr}\ <^{\text{\&}} 10
\text{expr} :\ -\ \text{func}\ "(\text{cost},\text{price})"\ 
\text{func} :\ -\ \text{KL} |\ \text{CORR} |\ \text{EMD}
\]

Finally, suppose we suspect an unknown (or arbitrary) attribute is similar to cost. We would replace price with the non-terminal \( \text{attr1} \), which expands to many possible attribute names.

\[
\text{hyp} :\ -\ \text{expr}\ <^{\text{\&}} 10
\text{expr} :\ -\ \text{func}\ "(\text{cost},\ \text{attr1})"\ 
\text{func} :\ -\ \text{KL} |\ \text{CORR} |\ \text{EMD}
\text{attr1} :\ -\ \text{"month"}|\ \text{"price"}|\ ...
\]

In English, the above hypothesis space contains one hypothesis for each attribute in the dataset that should be compared to cost, and for each of three similarity functions. For example, \( H_1 = \text{The correlation between cost and price is less than 10” (i.e., corr(cost, price) < 10), H_2 = \text{The KL distance between cost and month is less than 10” (i.e., KL(cost, month) < 10), ...} \). Because the statements produced by this grammar are enumerable, it is simple to automatically check that the grammar is a strict subset of the base grammar.

### 3.4 Grounding

Grounding a grammar amounts to binding all non-terminals to values in the underlying dataset. Specifically, \( \text{func} \) and \( \text{op} \) are bound to the pre-registered function names and operators, and \( \text{attr} \) is bound to the set of attributes in the dataset. All statements that do not type check are also rejected (e.g., a hypothesis containing “US” + 1, or \( KL(\text{market, price}) \)).

Grounding also means that we evaluate all predicates, inline production rules that evaluate to constants, and simplify all expressions. We can further restrict non-terminal operands that are compared with an attribute. Below we provide an example of grounding the base grammar to a dataset.

### Table 2: Breaking down each term in our proposed hypothesis grammar, as described in Section 3. We refer to this as our Base Grammar throughout the text.

| Term | Grammar Definition | Semantic Definition |
|------|--------------------|---------------------|
| hyp  | expr op expr (“["pred"]”)? ("&" hyp)? | A statement, which evaluates to either true or false, describing the relationship between at least one expression and one variable. |
| expr | func "(" (expr (, expr)? )? "")" | A nullary, unary, binary expression, or variable. |
| var  | attr (["pred"])? | An attribute reference or a constant. |
| pred | var op const (& pred)? | A condition expression that evaluates to true or false for a given var. |
| func | str | The name of a function with explicit input and output types from a predefined library or table. |
| op   | = | [ ] | > | ... | A boolean operation. |
| attr  | str | An attribute reference. Grounded to attributes in dataset. |
| const | str | number | .. | array | Constants are specified as part of the hypothesis, and may be any scalar, or array that isn’t derived from the database. |

### Example 1. Let’s ground the base grammar to Table 3 with the schema (\( \text{title:}\text{string}\), \( \text{year:}\text{int}\), \( \text{genre:}\text{string}\), \( \text{rating:}\text{float}\)). We pre-register the functions AVG (average), KS (Kolmogorov Smirnov test), \( \text{LM} \) (linear model), \( \text{SRES} \) (standard residuals), and \( \text{MAX} \) (maximum); and the operators \( =\), \( >\), and \( <\). As such, the grounded grammar is:

\[
\text{hyp} :\ -\ \text{expr}\ \text{op}\ \text{expr}\ (["\ "pred\"])?\ (&\ hyp)?
\text{expr} :\ -\ \text{func}\ "(" (expr (, expr)? )? ")"\ |\ var
\text{var} :\ -\ \text{attr}\ (["\ "pred\"])?\ |\ const
\text{pred} :\ -\ \text{var}\ \text{op}\ \text{const}\ (&\ pred)?
\text{func} :\ -\ \text{AVG}\ |\ \text{KS}\ |\ \text{LM}\ |\ \text{SRES}\ |\ \text{MAX}
\text{op} :\ -\ =\ |\ <\ |\ >
\]

### Table 3: A subset of the \textit{movies} dataset to illustrate grounding a grammar in Section 3.4.

| Title          | Year | Genre   | Rating |
|----------------|------|---------|--------|
| The Godfather  | 1972 | Crime   | 9.1    |
| Back to the Future | 1985 | Comedy  | 8.6    |
| Pulp Fiction   | 1994 | Comedy  | 8.8    |
| Fight Club      | 1999 | Action  | 8.7    |
| The Matrix      | 1999 | Science Fiction | 7.6    |
This grammar expresses a space of hypotheses that the dataset (and the pre-registered functions and operators) can be used to answer. Because the grammar is grounded to a specific dataset, we say that the collection of hypotheses parsable by this grammar forms a “Data Hypothesis Space.” In Section 4.4 we discuss the other types of hypothesis spaces and their implications. Below we provide example hypotheses of this data hypothesis space.

- Rating[Title='The Godfather'] = 9.1
  The Godfather's rating is 9.1.
- AVG(Rating)[Genre='Comedy'] > AVG(Rating)[Genre='Action']
  Comedies have a higher rating on average than Action films.
- KS_normal(Rating) > 0.05 [Genre='Comedy']
  The ratings for Comedy films follow a normal distribution with $\alpha = 0.05$ using the Kolmogorov-Smirnov test.
- SRES(fit_LM(Year, Rating), Year, Rating) $\in [-2,2]$
  There are no outliers to linear correlation between Year and Rating since the standardized residual (SRES) of all data points to the regression line (fit_LM) is in ±2.
- count() > 1 [Year=1999 & Genre='Action']
  There is more than one action movie in 1999.

In addition to these individual hypotheses, we can also express smaller hypothesis spaces. The following states that there is a Comedy with a higher rating than any other move genre. $const1$ will be bound to all ids in the data after grounding.

```
  hyp :- Rating[genre='Comedy' & id=const1] >
        MAX(Rating[genre='Comedy'])
```

The following states that newer films generally have higher ratings than older films, but without knowing the threshold between old and new. Notice that both operands expand to the same copy of const, so they must match the same value.

```
  hyp :- AVG(Rating[year=const1]) >
        AVG(Rating[year<const1])
```

$const1$ is a datetime

### 4 Hypothesis Spaces

A hypothesis space is defined by a grammar, possibly grounded to a dataset. This provides a common language to describe three important hypothesis spaces that can be defined for visual analytics: those bound by the dataset, the analyst goals, and the capabilities of the visualization. We further remark on interactions between these spaces.

#### 4.1 Full Hypothesis Space ($H_V$)

The full (infinite) hypothesis space is denoted as $H$ and represents all possible hypotheses that can be generated. Formally, it is defined as all statements that can be parsed by the base grammar (see Table 2).

#### 4.2 Data Hypothesis Space ($H_D$)

A data hypothesis space is denoted as $H_D$. The space is defined by grounding the base grammar to a dataset $D$. The mechanism for grounding and an example of generating hypotheses from $H_D$ are presented in Section 3.3. Informally, we say that $H_D$ describes all hypotheses that can be evaluated or verified using the dataset $D$, therefore, $H_D$ is a strict subset of $H$.

#### 4.3 Analyst Hypothesis Space ($H_A$)

An analyst hypothesis space is denoted as $H_A$. This hypothesis space represents all the analysis questions (expressed as hypotheses) that an analyst wants to answer independently of the data that is available to them, or the consideration of what visualization to use. In Section 5.1 we use the 2017 VAST Challenge to give an example of how an open-ended analysis question can be expressed formally using our grammar, resulting in $H_A$.

#### 4.4 Visualization Hypothesis Space ($H_V$)

The visualization hypothesis space, denoted $H_V$, represents all hypotheses that a visualization $V$ is able to evaluate. Here we consider $V$ independently of any dataset. Finding the limitations of human perception and cognition with different visual encodings is still an active research area in the visualization community. There is, at this time, no definitive mapping between all visualization designs and their task effectiveness. However, to illustrate the general concept of $H_V$, we provide examples of visualization hypothesis spaces, and point out the limitations of our formalism.

**Hypothesis Space of a Barchart with 3 Bars:** Consider $V_b$, a static barchart graphic that shows three bars (Figure 2(a)). When $V_b$ is rendered as an image, the image has specific property values for the canvas (width, height, border size, spacing, etc.) and contains three bars that have positional attributes, such as their position along the x-axis in pixels (xpos) and their height in pixels (pixel-height). For now, let’s only focus on these two attributes.

$V_b$ can be used to evaluate a few different hypotheses. For example, whether bar 1 is taller than bar 2 (pixelheight[id=1] > pixelheight[id=2]) or if the three bars show an increasing order in height (fit(pixelheight, xpos) > const). Given the limited hypotheses expressible by $V_b$, the hypothesis space should be relatively small. The following grammar refers to the rendered mark attributes, since $V_b$ is not grounded to any specific dataset:

```
  hyp :- (hyp1 | hyp2) (& hyp3)
  hyp1 :- pixelheight[id=1] > pixelheight[id=2]
  hyp2 :- fit(pixelheight, xpos) > const
  hyp3 :- pixelheight & xpos

  const1 :- number
  const2 :- number

  This grammar (and the hypothesis space) is specific to this instance of $V_b$, and will change if its design specification or dataset changes. For example, if $V_b$ now renders 100 bars, evaluating if the barchart exhibits a normal distribution may be a valid hypothesis. Conversely, it may no longer be valid to compare the heights of bars with bars5 because perceptually this becomes much harder to do, especially if the width of the visualization is large [59].

  **Grounding $V_b$:** Suppose $V_b$ now renders dataset [K] where month is mapped to the x-axis and price to the y-axis; now, the values of xpos and pixel-height are functions of the month and price values, respectively. Grounding $V_b$’s grammar allows us to also reference the data attributes month and price. In this way, comparative hypotheses may be based on the underlying data attributes, or based on the visual representation - as illustrated above. The hypothesis space is now:

```
  hyp :- (hyp1 & hyp2) (& hyp3) "(" & "pred")"...

  This grammar (and the hypothesis space) is specific to this instance of $V_b$, and will change if its design specification or dataset changes. For example, if $V_b$ now renders 100 bars, evaluating if the barchart exhibits a normal distribution may be a valid hypothesis. Conversely, it may no longer be valid to compare the heights of bars with bars5 because perceptually this becomes much harder to do, especially if the width of the visualization is large [59].

**Adding Interaction:** Unsurprisingly, adding interaction also impacts $H_b$ by modifying the grammar. Using Figure 2(b), a user can also pick the market from a dropdown and choose to compare price or cost with month. The grammar expresses this using non-terminals for the market and attributes[1]. Choosing “US” in the

```
that the data & vis can answer

(a) There are analysis hypotheses that the data & vis can answer
(b) Analyst hypotheses fully answerable by data and vis
(c) Inappropriate vis
(d) Ineffective vis
(e) Insufficient Data
(f) Overkill vis
(g) Underpowered Vis

Fig. 2: Relationships between hypothesis spaces correspond to successful or unsuccessful applications of visual analysis.

dropdown would be equivalent to binding const1 to “US” and restricts the hypothesis space to those in the US market.

hyp :: (hyp1 | hyp2) (& hyp)? "[market =" const1 "]"
const1 :: “US” | “EU”
attr :: “pixelheight” | “xpos” | “month” | “price” | “cost”

Visualization Hypothesis Space and Graphical Perception: The difference between a data and a visualization hypothesis space illuminates the distinction between hypotheses that can be evaluated computationally from those “computed” by the user (via a visualization). Although there does not exist an exhaustive list of the functions that a user perceptually and cognitively performs when reading a visualization, research has begun to propose some equivalences (e.g., [17, 25, 42, 48, 51, 75], to name a few). In addition, recent work such as Saket et al. [50], Kim et al. [55], and Mortiz et al. [59] have started to empirically evaluate and model the effectiveness of visualizations for analytics tasks. We will further discuss the implications of quantifying and evaluating visualization hypothesis spaces in Section 5.

4.5 Definitions of Hypothesis Spaces

We use set notation to describe several definitions of hypothesis spaces.

**Definition 1 (A Hypothesis).** A hypothesis statement $h$ is said to be in a hypothesis space $H_1$ (i.e., $h \in H_1$) if $h$ can be parsed and type-checked by the grammar associated with the hypothesis space.

**Definition 2 (Size of a Hypothesis Space).** The size of a hypothesis space $H_1$ is denoted as $|H_1|$. This represents the number of statements that can be parsed by the grammar associated with the hypothesis space. In general, it is exponential in the domains of the non-terminals in the grammar.

**Definition 3 (Containment).** Given two hypothesis spaces $H_1$ and $H_2$, if $H_1 \subset H_2$ then $|H_1| < |H_2|$. Further, any solution that can be used to answer hypotheses in $H_1$ can be used to answer $H_2$.

**Definition 4 (Intersection).** Given two hypothesis spaces $H_1$ and $H_2$, their intersection $H_1 \cap H_2$ represents hypotheses that can be parsed by both the grammar of $H_1$ and the grammar for $H_2$. $H_1 \cap H_2$ denotes that there is no intersection between the two spaces (i.e., $|H_1 \cap H_2| = 0$).

**Definition 5 (Grounding).** Grounding a hypothesis space $H_1$ to a dataset $D$ is defined as finding the intersection between $H_1$ and $H_D$, where $H_D$ is the hypothesis space of $D$. For example, grounding a visualization hypothesis space $H_V$ to $D$ results in a new hypothesis space $H_{DV}$ such that $H_{DV} = H_V \cap H_D$.

4.6 Unifying the Hypothesis Spaces

Figure 2 provides an illustration of the relationship between the hypothesis spaces, $H$, $H_D$, $H_V$, and $H_A$. With this perspective, we consider the roles of data, visualization, and users in visual analytics.

**Successful Visual Analytics Solutions.** A visual analytics solution is considered to be successful when the intersection between $H_D$, $H_V$, and $H_A$ is not null (i.e., $|H_D \cap H_V \cap H_A| > 0$, see Figure 2a). An ideal solution is when a user’s analysis questions can be fully answered by

the data and the visualization tool such that $H_A \subset H_D$, $H_A \subset H_V$, and $|H_D \cap H_V| > 0$ (see Figure 2b).

**Inappropriate Visualization.** The use of an inappropriate visualization occurs when $H_V \nsubseteq H_D$ (or $|H_V \cap H_D|$ is very small) where a visualization cannot support the hypotheses about the data (Figure 2c).

**Ineffective Visualization.** The use of an ineffective visualization occurs when $H_V \nsubseteq H_A$ (or $|H_V \cap H_A|$ is very small) where a visualization cannot support a user’s analysis questions (Figure 2d).

**Insufficient Data.** In the case where $H_A \nsubseteq H_D$, not all of a user’s analysis questions can be answered with the given data. (Figure 2e)

“Overkill” and Under-Powered Visualizations. If $H_V \nsubseteq H_A$ but $|H_A|$ is significantly larger than $|H_V|$ (i.e., $|H_A| \gg |H_V|$), the visualization is too limited for the user’s analysis needs (Figure 2f). Conversely, if $H_A \nsubseteq H_V$ but $|H_V|$ is significantly larger than $|H_A|$ (i.e., $|H_V| \gg |H_A|$), the visualization is likely too complex for the user’s needs (Figure 2g).

5 Demonstration of the Grammar

This section applies the Grammar of Hypotheses in three use cases. We first use the 2017 VAST Challenge to illustrate the process of expressing an open-ended analytic question as hypotheses. We then revisit the relationship between tasks in visual analytic literature and our grammar of hypotheses; we translate tasks by Amar et al. [1] into grammars. Finally, we translate data science questions from a Kaggle competition. For legibility, we will eschew quotation marks in the grammar when the meaning is clear.

5.1 2017 VAST Challenge (MC1)

The 2017 VAST Challenge consists of three mini challenges. In this example, we consider Mini-Challenge 1 (MC1). The goal of MC1 is to assess whether there are unusual vehicle activities that might be causing the decreased nesting of the Rose-Crested Blue Pipits in a fictitious national park. For brevity, we shorten the challenge description and data attribute values in our text. The full description of the challenge can be found online.

**Problem Statement:** The number of nesting pairs of the Rose-Crested Blue Pipit is decreasing. Their habitat, the Boonsong Lekagul Nature Preserve, has had odd behaviors of vehicles that are inconsistent with expected park visitors. Analyze the behaviors of vehicles through the park over time to investigate causes in the Pipit’s decreasing nesting.

**Dataset:** MC1 includes two datasets: a map of camp sites, ranger stations, gates, etc. throughout the preserve, and a sensor log of the vehicular movements that has the schema (Timestamp:datetime, Car-id:str, Car-type:str, Gate-name:str). The metadata and example data are shown below:

| Timestamp | Car-id | Car-type   | Gate-name   |
|-----------|--------|------------|-------------|
| 5/1/2015 7:50 | 523   | 2-axle-car | general-gate |
| 5/1/2015 7:52 | 669   | ranger-truck | ranger-base |
| 5/1/2015 7:53 | 647   | 2-axle-ranger-truck | entrance4 |
| 5/1/2015 7:58 | 751   | 3-axle-truck | ranger-stop0 |
| 5/1/2015 7:59 | 523   | 2-axle-truck | camping1 |

5.1.1 Expressing MC1 as Hypotheses

To express MC1, we first decompose it into analysis questions and then convert them into hypotheses. We synthesized many of these questions based on official Challenge submissions found online.

For simplicity, our grammar assumes that data transformations (joining tables, extraction, cleaning, etc.) have already been applied (see https://www.vacommunity.org/VAST+Challenge+2017).
Section [51]. Some of the following examples require deriving new data (e.g., computing duration from two consecutive timestamps). For those, we will separately describe the transformations. Finally, we omit quotation marks around terminals.

Q1: Are park visitors entering and exiting the park?

H1: Some park visitor passed through the entrance gates an odd number of times.

hyp :- (mod(count(), 2) = 1) [pred]

pred :- [Gate-name="entrance" & Car-id=const1]

This set of hypotheses compares the count of all gates whose names start with “entrance” against the total count across all ranger gates.

Q2: Are any gates used more frequently than ranger gates?

H2: All camping gates are used more frequently than ranger gates.

hyp :- count([Gate-name="camping"] > count([Gate-name="ranger"])

This set of hypotheses compares the count of all gates whose names start with “camping” against the total count across all ranger gates.

Q3: Is there illegal activity in the park?

H3: Non-ranger cars are going through ranger stops, gates, and bases.

hyp :- (expr1 > 0) ![Car-type="ranger!

expr1 :- count([Gate-name="ranger"]

Q4: Is the park more popular during Pipit mating season?

H4: There are more visitors in the Spring than some other season.

hyp :- count([Season="Spring"] > count([Season=const1])

const1 :- "Winter" | "Fall" | "Summer"

We first pre-compute Season:string based on the month in Timestamp.

Q5: Are cars speeding when rangers are off duty?

H5: Average car speed is highest between midnight and 7am.

hyp :- avg(Speed)[Hour∈[0,7]] > avg(Speed)[Hour∈[0,7]]

We first pre-compute Speed:float as the distance between two consecutive gate check-ins of the same car, divided by the time difference.

Q6: Are there any outliers in vehicle nightly activity?

H6: The number of gate checkins at night follow the same distribution as the distribution of the day.

hyp :- KL(DistrEvening, DistrDay) < 10

This hypothesis needs to pre-compute a distribution of the number of gate check-ins per hour in the evenings (DistrEvening) and day time (DistrDay). It then uses KL divergence to confirm that the difference between the two distributions is less than a threshold.

Q7: Is the length of visitor stay affecting nesting?

H7: Length of visitor stay is inversely correlated with nesting activity.

hyp :- Corr(StayLengths, NestingAmount) < -0.75

This hypothesis needs to pre-compute the average visitor stay lengths per week or month, and the average amount of nesting over the same time interval. Although the average visitor stay is computable, there is Insufficient Data ($H_7 \not\subset H_6$) to compute nesting activity, and thus to verify the hypothesis.

5.1.2 Analyst Hypothesis Space for MC1

Ignoring H7, the following analyst hypothesis space $H_A$ expresses H1-6 as well as other hypotheses:

\[
\begin{align*}
\text{hyp} & \text{ :- expr op expr "[" pred "]"} \\
\text{expr} & \text{ :- func "[" expr "[" expr "]" "]"} \\
\text{var} & \text{ :- attr "[" pred "]"} \\
\text{pred} & \text{ :- attr op const} \\
\text{func} & \text{ :- ! | % | avg | count | Corr | KL}
\end{align*}
\]

Note that the original data’s hypothesis space $H_D$ has little overlap with the analyst’s hypothesis space $H_A$, as is evident by the numerous attr values not present in the original data. By expressing MC1 as a grammar, it becomes clear what data transformations are necessary so that $H_A \subset H_D$. Similarly, H7 makes clear that more data needs to be collected before it is answerable.

By “pre-registering” her analyst hypothesis space, the analyst can know ahead of time 1) the data transformations or feature engineering they need to perform, and 2) what visualizations will be needed to visually verify her hypotheses. As a result, the analyst hypothesis space can act as a proxy for an initial “design requirements gathering” before designing a visualization system.

5.2 Analytic Tasks as Hypotheses?

In visualization and visual analytics, tasks have traditionally been used to characterize the operations an analyst would undertake to answer an open-ended question such as the VAST Challenge. Multiple task taxonomies have been proposed, both low-level [16,29,46] and high-level [58,43,54] that have served as a foundation to the design and evaluation of visualization systems.

We translated the tasks in Amar et al.’s taxonomy [1] to our hypothesis grammar. We found that the majority (Retrieve Value, Compute Derived Value, Find Extremum, Sort, Determine Range) are operations or queries, rather than hypotheses. Three tasks (Characterize distribution, Correlate, and Cluster) can be interpreted as either operations or hypotheses, and one task (Find Anomalies) is ambiguous and requires further refinement.

5.2.1 Non-Hypothesis Tasks

Retrieve Value: Given a set of specific cases, find attributes of those cases. Example: What is the mileage per gallon of the Audi TT?

\[
\text{expr} \text{ :- MPG[Name='Audi TT']}\]

This direct translation matches an expression (expr) rather than a hypothesis. Technically, we could express the task by exhaustively enumerating all possible values of MPG as: MPG[Name='Audi TT']=const1, but this lacks any meaningful interpretation of the task.

Compute Derived Value: Given a set of data cases, compute an aggregate numeric representation of those data cases. Example: What is the average calorie content of Post cereals?

\[
\text{expr} \text{ :- AVG(Calories)[Brand='Post']}\]

The **Compute Derived Value** task is similar to **Retrieve Value** but with the addition of an aggregation function in the hyp production rule. It is also inefficient to express this task as a hypothesis space.

Filter: Given concrete conditions on attribute values, find data cases satisfying those conditions. Example: What comedies have won awards?

\[
\text{expr} \text{ :- Film-name[Genre='Comedy' & Awards-won>0]}\]

The **Filter** task is similar to several questions (Q1-3) in Section 5.1. It can be represented as a hypothesis but is best expressed as an operation.

Find Extremum: Find data cases possessing an extreme value of an attribute over its range within the data set. Example: What director has won the most awards?

\[
\text{expr} \text{ :- Director[AwardsWon=MAX(AwardsWon)]}\]

We express the **Find Extremum** task similarly to the above tasks.

Sort: Given a set of data cases, rank them according to some ordinal metric. Example: Order the cars by weight.

\[
\text{hyp} \text{ :- Weight[id=car1] = Weight[id=car2] &}
\text{Weight[id=car2] = Weight[id=car3] & ...}
\]

The **Sort** task is another example of an operation that is difficult to express as a hypothesis. One possible strategy is to enumerate all
orderings in a pre-processing step and evaluate each one is true (a hypothesis for one possible ordering is shown above). However, we note that this is inefficient not appropriate to formulate as hypotheses.

**Determine Range:** Given a set of data cases and an attribute of interest, find the span of values. Example: What is the range of film lengths?

\[
\text{expr} : - (\text{MAX(FilmLength)}, \text{MIN(FilmLength)})
\]

**Determine Range** is another task that is better suited as an expression, which takes the difference between the MAX and MIN values.

### 5.2.2 Potentially Hypothesis-Oriented Tasks

**Characterize Distribution:** Given a set of data cases and a quantitative attribute of interest, characterize the distribution of that attribute’s values over the set. Example: What is the age distribution of shoppers?

\[
\text{hyp} : \text{func1(Age)} < 0.1
\]

\[
\text{func1} : \text{fit.Gaussian} \mid \text{fit.Powerlaw} \mid \text{fit.Linear} \mid \ldots
\]

This grammar assumes functions that fit data to different distributions (e.g., Gaussian, Powerlaw) and return the loss (e.g., mean squared error). A strict reading of the task implies an operation to fit a distribution to Age. We translate this to a hypothesis by also checking that the resulting loss is below a threshold.

**Correlate:** Given a set of data cases and two attributes, determine useful relationships between the values of those attributes. Example: Is there a correlation between carbohydrates and fat?

\[
\text{hyp} : \text{ABS(CORR(carbohydrates, fat))} > 0.7
\]

Again, a strict reading of this task implies an operation to compute a correlation coefficient. We translate this by setting a threshold of 0.7 for the absolute value (ABS) of the correlation.

**Cluster:** Given a set of data cases, find clusters of similar attribute values. Example: Are there groups of cereals with similar calories?

\[
\text{hyp} : \text{func1(calories)} < 0.1
\]

\[
\text{func1} : \text{fit.Kmeans} \mid \text{fit.Hierarchical} \mid \ldots
\]

The **Cluster** task is an operation that takes an array of calories values as input, and returns an array of labels—one for each row. It would typically be performed as a pre-processing transformation, but can be formulated as a hypothesis with a threshold on the goodness of fit.

### 5.2.3 Ambiguous Tasks

**Find Anomalies:** Identify any anomalies within a given set of data cases with respect to a given relationship or expectation, e.g. statistical outliers. Example: Are there any outliers in horsepowers?

The challenge is that what constitutes an outlier is ambiguous and depends on the user and the context, and there are multiple outlier-detection procedures in any given domain. However, if we restrict our interpretation to simple statistical outliers, we can express the task as a testable hypothesis. For example, the grammar below examines whether a car’s horsepower exceeds two standard deviations of the average; const3’s domain is every Name in the dataset.

\[
\text{hyp} : \text{ABS(Horsepower) [ const1, const2 ]} \{ \text{pred} \}
\]

\[
\text{pred} : \text{Name = const3}
\]

\[
\text{const1} : \text{avg(Horsepower)} - 2 \times \text{stdev(Horsepower)}
\]

\[
\text{const2} : \text{avg(Horsepower)} + 2 \times \text{stdev(Horsepower)}
\]

\[
\text{const3} : \text{str}
\]

### 5.2.4 Reflections

Our translation reflects some groupings within Amar et al.’s tasks [1]. Tasks such as **Retrieve Value**, **Compute Derived Value**, **Filter**, **Find Extremum**, **Sort**, and **Determine Range** represent queries or operations that do not relate well to hypothesis and hypothesis testing. Although these tasks may be expressed with our grammar, the result is inefficient and does not offer insight into the nature of the tasks. Conversely, tasks such as **Characterize Distribution**, **Correlate**, and **Cluster** are presented as operations, but can be translated into hypotheses by specifying a threshold for comparison. As noted in the Sunal and Haas’ Hypothesis Quality Scale (Table 1), a second variable for comparison is needed to constitute a hypothesis. Thus, the preceding tasks are considered “partial appropriate explanations”, or level 2 hypotheses. Lastly, we find the task **Find Anomalies** has the structure of a hypothesis, but is too open-ended and ill-defined for evaluation. If the task can be refined into sub-tasks or questions, it may be possible to express these as hypotheses (similar to the MC1 example).

### 5.3 Kaggle Competitions

Finally, we extend the application of our grammar to the greater data science community. We examine Kaggle Competitions, which are publicly available machine learning challenges. Unlike the 2017 VAST challenge which has an open-ended question, Kaggle competitions are often very well specified in terms of metrics and desired outcomes.

For this demonstration, we examine the 2020 Makridakis Forecasting Accuracy Competition (“M5”) sponsored by the University of Nicosia [1]. The **Problem Statement** asks data scientists to forecast the unit sales of Walmart retail goods for the next 28 days. We provide examples of the data and the list of hypotheses related to the data in the supplemental material. These hypotheses can be used to guide the development of a predictive model in forecasting sales. However, central to the requirement of the Kaggle challenge is the task of **Forecasting** (or prediction). We observe that such a task is an operation rather than a hypothesis, with similar limitations as those in Amar et al.’s taxonomy.

### 6 Design Implications

We propose design opportunities in which our grammar can be used to articulate analysis tasks, evaluate visualization systems, and support hypothesis-based reasoning in visual analytic tools.

**Articulating Visualization Tasks.** The visualization community has organized a multitude of analytic goals, questions, and procedures using the concept of tasks as a common framework. Many low-level tasks are similar to expressions or queries that make up a user’s analytic goals, rather than hypotheses (e.g., the operations tasks in the Amar et al. taxonomy). In contrast, high-level tasks are more focused on data understanding and question answering, thus expressible as hypothesis spaces when the tasks are decomposed into well-articulated questions.

Our hope is that a Grammar of Hypotheses can help reframe the abstract notion of tasks. Although a number of ‘low-level’ and ‘high-level’ task taxonomies and typologies have been contributed throughout our community [15][69][77], they typically lack a formal language to operationalize them that is amenable to data and analysis. We have shown how a grammar can express open-ended analytic questions, ground itself to a given dataset, and be used to represent and verify the set of plausible hypotheses.

Reframing tasks as grammars brings additional benefits as well. Considering a researcher that designs a new visualization system to aid a visual analytic task. By articulating the open-ended task as a grammar, we can automatically sample concrete hypotheses for a user to verify using the visualization system. Evaluations can now articulate metrics such as coverage, and identify regions of the hypothesis space where users encounter difficulty when using a new visualization design.

In addition, we can compare open-ended tasks based on their cardinalities and containment relationships. For instance, if Task A’s grammar is a subset of Task B’s grammar, then we can state that Task B is more open-ended, and potentially more ambiguous. Similarly, if hypotheses in a task’s space involve many expressions and nested function evaluations, then we can state that the hypotheses are complex, requiring multiple computation steps to evaluate. We could also generate tasks of varying ambiguity by starting with a single hypothesis (hypothesis space of size one) and replacing strings with non-terminals.

**Exploration as Space Refinement.** We observe that data exploration can potentially be modeled as a search within a hypothesis space. Starting with an initial (potentially unbounded) hypothesis space, the user’s goal is to verify one or a handful of “target” hypotheses that the user might not be aware of a priori. Throughout exploration, as the user makes concrete decisions with their interactions in a visualization (which data and metrics to focus on), she shrinks the hypothesis space until it is small enough to choose and evaluate a “target” hypothesis.

https://www.kaggle.com/c/m5-forecasting-accuracy
Under this model, a good visualization system is one that helps the user shrink the hypothesis space as quickly as possible in order to hone in on the “target” hypotheses. This observation is consistent with Shneiderman’s Information Seeking Mantra of “Overview first, zoom and filter, then details-on-demand [55].” The overview phase helps the user quickly identify a candidate subset of data, zoom and filter help the user quickly verify that the corresponding hypothesis subspace likely contains a “target” hypothesis, and details-on-demand verifies individual hypotheses.

Hypothesis-Driven Visualization Design. A concrete grammar can help aid the design of task-oriented visualization systems. For a single hypothesis, a design tool could automatically recommend a chart to help supply the evidence. However, a large hypothesis space may require an interactive visualization. To this end, Section 4.3 draws a connection between visualization interactions such as sliders and drop-downs and non-terminals in a grammar—when a user manipulates a widget or performs an interaction, it binds its corresponding non-terminal.

Recent work, such as Precision Interfaces [10] which automatically generates interactive visualizations from example SQL queries, or a similar framework may be used to translate task grammars into interactive visualizations. We leave opportunities for hypothesis-driven design of visualizations and visual analytics systems for future work.

7 LIMITATIONS

Boundedness of the grammar: We do not claim our grammar is capable of expressing every possible hypothesis that could be formulated with the English language. Hypotheses are a complex subset of natural language and have been an ongoing object of study. Previous grammars for hypotheses have been contributed in literature [13,63–65], but are typically limited to a specific domain application (e.g., electrical circuits [87]). Similarly, our hypothesis grammar is restricted to performing analysis and posing questions and statements about data.

Assumptions for the data: Our grammar makes several assumptions about data. For one, we assume the data is relational and can be accessed from a database (or a table). We also assume the data has been transformed, cleaned, and is not missing any values required by the grammar. This is obviously not always the case with “real-world” analysis. The existence of existing data transformation languages such as SQL, as well as non-relational query languages, suggest that extending the grammar to support data pre-processing is possible. However, there are trade-offs between grammar simplicity and expressiveness that must be carefully balanced.

Qualifying human judgement: Our hypothesis grammar does not take into account uncertain criteria for human judgement. For example, the hypothesis “action movies are better than comedies” is a verifiable hypothesis but a subjective one. Our grammar does not take into these hypotheses into consideration but instead focuses on quantifiable statements that can be posed about data, driven by the notion of scientific hypotheses.

Visualization’s ability to verify hypotheses: We postulate that visualizations can confirm or reject some subset of hypotheses given H₀ as illustrated by Wickham et al. with their LineUp technique [72]. However, currently we cannot enumerate all hypotheses that a visualization can be used to answer. There are many open-ended questions that our grammar therefore introduces. If a single visualization is capable of verifying a particular hypothesis, what is the translation to visualization tools in which many visualizations can be shown or suggested to users? Are there some visualizations that are more indicative of supporting a hypothesis than others? Prior work has sought to understand the effectiveness of a visualization in supporting tasks [59]. In future work we aim to investigate if the same can be done for hypothesis spaces.

Extensions to other concepts in visual analysis. In this work, we demonstrate applications of our grammar with real-world analytic challenges. We also map a commonly used analytic task taxonomy for visual analysis to our use of hypothesis grammars. However, there are other important concepts of data analysis and visualization that we did not have space to address. For example, visualization recommendation [58], how visualizations support hypothesis refinement [8], how uncertainty affects data understanding [56], data augmentation [8], perceptual models and experiments [19], and so on. We hope that our formalism for hypotheses in visualization can provide the springboard for research in all of these areas.

8 RELATED WORK

Our work on a grammar of hypotheses is inspired by and related to active research on related topics in the visualization community.

8.1 Historical Use of Hypothesis in Visual Analytics

Before visual analytics became a field in its own right around 2006, the human factors community was calling for the production of collaborative human-computer systems to support “information analysis tasks” [71]. In subsequent visual analytics research, the ability for a system to support analysts through “information analysis tasks” has been translated to broadly supporting analysts through “hypothesis formation and testing” [21,32–43,74].

In 2004, Chappell et al. developed the KANI system to specifically support analysts in constructing and testing models of “alternative hypotheses” (not in the statistical sense) [9]. The use of “alternative hypotheses” was instead meant to describe an alternate depiction or observation of the data. In 2006, Wright et al. developed Sandbox, an analysis tool intended to support the formation of hypotheses and analysis of competing hypotheses (ACH) [25]. Similar to KANI, Sandbox describes the process of developing and verifying hypotheses as the process of sense making and completing analytic tasks [73]. Jigsaw, a famous visual analytic tool, employs a Sandbox view with the express purpose of supporting hypothesis formation – that is, the development of an analyst’s “mental model for the data” [57].

8.2 Supporting Hypotheses with Visual Analytics

We often classify the purpose of a visualization tool as enabling exploratory or confirmatory analysis [61,62]. With respect to visual analysis, confirmatory data analysis (CDA) is described as the process of using a visualization to affirm or reject a priori hypotheses. In contrast, exploratory data analysis (EDA) is the process of using a visualization to gain new insights about data until ill-formed hypotheses can be refined for testing. Keim et al. declare CDA and EDA as two of the three major goals for visual analytics [55] and, thus, a multitude of visualization designs and tools have been proposed to support users through iterative hypothesis formation and validation. Recent visualization systems leverage graphical inference for hypothesis testing, combining EDA and CDA practices [28,67]. Wickham et al. argued for a tighter coupling between EDA and CDA and developed two novel graphical inference methods: Rorschach and LineUp [72]. Rorschach is designed to visualize null-hypothesis data (for comparison with potentially non-null data), and LineUp is a method for graphical hypothesis testing. Similarly, Hsu et al. developed a system for code verification based on “hypothesis-driven comparative and quantitative visualizations” [27]. Our proposed use of hypothesis spaces can similarly assist in the coupling of EDA and CDA.

8.3 Hypothesis Refinement with Visualization

Several visualization systems focus explicitly on supporting users in hypothesis refinement. Choi et al. performed a wizard-of-oz user study that investigated the effect of asking users to express their expectations of data (i.e., their pre-conceptions or hypotheses) prior to using an exploratory visualization system [11]. Interestingly, they found that users will often express hypotheses in ill-defined terms (i.e., refer to a phenomenon that multiple data attributes could represent, or a large H₀ in our grammar), or with respect to data not included in the dataset they were analyzing (i.e., H₀ ⊊ Hₚ). Operationalizing this work, Koonchanok et al. developed PredicMe, a tool that asks users to draw their expectations of a dataset before performing any analysis [36].
Their findings suggest that pre-registering hypotheses can result in more deliberate analyses.

In contrast to the work of Koonchanok et al., which does not ask the user to formalize their hypotheses of data beyond drawing, several systems focus explicitly on guiding users from conceptual mental models of data to formal statistical hypotheses. For example, Jun et al.’s Tea system is designed to take a user’s conceptual hypothesis of data, formalize it, and run the appropriate statistical tests. Notably, Tea expects users to already have a reasonably well-defined conceptual hypothesis that they can express in code. In a different work, Jun describes the process of moving from a conceptual hypothesis of data to a formal statistical hypothesis as a dual search task where users are constrained by their mental model of data and possible mathematical models of data. Our proposed grammar can serve a similar purpose, focusing on how analysts can compose verifiable hypotheses (not necessarily statistically valid) from open-ended analytic questions.

9 CONCLUSION
This paper presented a novel Grammar of Hypotheses to express hypothesis-based questions for visual analysis. The grammar draws from the science and education literature, and provides a formal language to write hypotheses over datasets. We show how an open-ended analysis question can be expressed as a Hypothesis Space that represents a set of potential hypotheses relevant to the question, and illustrate how a space is equivalent to all statements parsable by a grammar. This grammar helps unify the analyst’s questions, the available data, and the capabilities of a visualization system as hypothesis spaces.

We used this grammar to re-express the VAST 2017 Mini-Challenge using formal notation and found how it helps us immediately identify necessary data transformations, as well as questions that are unanswered by the available data. By translating an existing taxonomy as well as a Kaggle competition into hypotheses, we find that the majority of low-level tasks are operations or queries, rather than hypotheses. Overall, we believe a formal grammar can help articulate analysis tasks in ways that promote better clarity, can help evaluate visualization systems, and help support hypothesis-based reasoning in visual analytics.

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APPENDIX

9.1 Kaggle

The goal of the M5 challenge is to evaluate and compare the accuracy of different time series forecasting methods. For the sake of brevity, we assume all necessary data transformations have been completed.

9.1.1 Walmart Product Forecasting (M5)

The M5 Forecasting competition’s full description can be found at https://www.kaggle.com/c/m5-forecasting-accuracy.

Problem Statement: Forecasting the future value of commodities is always important for investors to make informed decisions. Data scientists are tasked with forecasting unit sales of Walmart retail goods for the next 28 days using advanced forecasting algorithms.

Dataset: Walmart provides three files detailing historical sales over five years for their products. We assume all data files have been joined and cleaned into a single table with the following metadata:

- **Item**: unique product ID.
- **Dept**: department ID for the product.
- **Categ**: category ID for the product.
- **Store**: store ID where the product was sold.
- **State**: state where the product was sold.
- **Date**: date when the product was sold.
- **Event**: (optional) event name when the product was sold.
- **Event-type**: (optional) event type when the product was sold.
- **SNAP**: a boolean indicating if the state allows SNAP purchases.
- **Price**: price of the product.
- **Sales**: total number of units sold of the product.

9.1.2 M5 Analyst Hypothesis Space ($H_{AM5}$)

We pose questions for exploratory data analysis that can be used to guide the development of a predictive model in forecasting sales.

Q1: How frequent are events in each state and month?

H1: The number of SNAP events are equal across each state.

\[ \text{hyp} \leftarrow \text{count} \left( \text{Event=SNAP & State="California"} \right) = \text{count} \left( \text{Event=SNAP & State="New York"} \right) = \text{count} \left( \text{Event=SNAP & State="Utah"} \right) = \ldots \]

Q2: How do sales compare across states?

H2: The average sales are higher in California than Maryland.

\[ \text{hyp} \leftarrow \text{expr1} > \text{expr2} \]

\[ \text{expr1} \leftarrow \text{AVG(Sales)} \left[ \text{State="California"} \right] \]

\[ \text{expr2} \leftarrow \text{AVG(Sales)} \left[ \text{State="Maryland"} \right] \]

Q3: Is one season more popular for shopping?

H3: Consumers purchase more total goods in Winter than Spring.

\[ \text{hyp} \leftarrow \text{expr1} > \text{expr2} \]

\[ \text{expr1} \leftarrow \text{SUM(Sales)} \left[ \text{Season="Winter"} \right] \]

\[ \text{expr2} \leftarrow \text{SUM(Sales)} \left[ \text{Season="Spring"} \right] \]

Q4: How many states support SNAP?

H4: At least 20 states support SNAP.

\[ \text{hyp} \leftarrow \text{count} \left( \text{SNAP=true} \right) > 20 \]

Q5: Are sales dependent on the type of event?

H5: Event sales are lowest on New Years Day and highest on Black Friday.

\[ \text{hyp} \leftarrow \text{expr1} = \text{expr2} & \text{expr3} = \text{expr4} \]

\[ \text{expr1} \leftarrow \text{Sales} \left[ \text{Event="New Years Day"} \right] \]

\[ \text{expr2} \leftarrow \text{MAX(Sales)} \]

\[ \text{expr3} \leftarrow \text{Sales} \left[ \text{Event="Black Friday"} \right] \]

\[ \text{expr4} \leftarrow \text{MIN(Sales)} \]

Q6: Does the department affect product sales?

H6: The average sales do not differ by more than $1000 between the Clothing and Bathroom departments.

\[ \text{hyp} \leftarrow \text{ABS} \left( \text{expr1} - \text{expr2} \right) < 1000 \]

\[ \text{expr1} \leftarrow \text{AVG(Sales)} \left[ \text{Dpt="Clothing"} \right] \]

\[ \text{expr2} \leftarrow \text{AVG(Sales)} \left[ \text{Dpt="Bathroom"} \right] \]

Q7: Is there any similarity between a product’s price and sales?

H7: There is a trend between price and sales.

\[ \text{hyp} \leftarrow \text{fit.LM(Price, Sales)} > \text{const} \]

\[ \text{const} \leftarrow \text{number} \]

Q8: Are there any differences between a product’s price and sales?

H8: The price and sales have the same distribution.

\[ \text{hyp} \leftarrow \text{KL(Price, Sales)} < 10 \]

Therefore, our final bounded grammar (and corresponding $H_{AM5}$) is:

\[ \text{hyp} \leftarrow \text{expr op expr} \left( \left[ \text{pred} \right] \right)? \left( \& \text{hyp}\right)? \]

\[ \text{expr} \leftarrow \text{func} \left( \right) | \text{func} \left( \text{expr} \right) \]

\[ \text{func} \left( \text{expr, expr} \right) | \text{var} \]

\[ \text{var} \leftarrow \text{attr} \left( \left[ \text{pred} \right] \right)? | \text{const} \]

\[ \text{pred} \leftarrow \text{var} \text{op} \text{const} \left( \& \text{pred}\right)? \]

\[ \text{func} \leftarrow \text{AVG} | \text{MIN} | \text{MAX} | \text{SUM} | \text{KL} | \text{fit.LM} | \text{count} \]

\[ \text{op} \leftarrow \text{=} | \text{<} | \text{>} \]

\[ \text{attr} \leftarrow \text{Dept} | \text{Sales} | \text{State} | \text{Event} | \text{Season} | \text{Cost} | \text{Price} \]

\[ \text{const} \leftarrow \text{number} | \text{string} | \text{datetime} | \text{bool} \]