Magnetic reconnection in plasma under inertial confinement fusion conditions driven by heat flux effects in Ohm’s law

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In the interaction of high-power laser beams with solid density plasma there are a number of mechanisms that generate strong magnetic fields. Such fields subsequently inhibit or redirect electron flows, but can themselves be advected by heat fluxes, resulting in complex interplay between thermal transport and magnetic fields. We show that for heating by multiple laser spots reconnection of magnetic field lines can occur, mediated by these heat fluxes, using a fully implicit 2D Vlasov-Fokker-Planck code. Under such conditions, the reconnection rate is dictated by heat flows rather than Alfvénic flows. We find that this mechanism is only relevant in a high \( \beta \) plasma. However, the Hall parameter \( \omega_c \tau_{ci} \) can be large so that thermal transport is strongly modified by these magnetic fields, which can impact longer time scale temperature homogeneity and ion dynamics in the system.

Understanding the \( \mathcal{O}(10^3) \) T magnetic fields that can develop in high-power-laser interactions with solid-density plasma \([1,2] \) is important because such fields significantly modify both the magnitude and direction of electron heat fluxes \([3] \). The dynamics of such fields evidently has consequences for inertial fusion energy applications \([4,5] \), as the coupling of the laser beams with the walls or pellet and the development of temperature inhomogeneities are critical to the uniformity of the implosion. There is a significant interplay between heat fluxes and magnetic fields: in semi-collisional plasmas heat flux can be the dominant mechanism for transporting magnetic field lines in addition to currents or bulk ion flow \([6] \). This effect, arising due to an electric field analogous to the Nernst-Ettingshausen effect in metals \([7] \), has been shown to be significant in laser heated plasma \([8,9] \). The Nernst effect in plasma \([10] \) arises as a consequence of the velocity dependent collision frequency of electrons in plasma. Since the faster, “hot”, population of electrons are essentially collisionless, the magnetic field is “frozen” to them, whereas the collisional, “cold”, portion of the distribution function is able to diffuse across field lines. Hence, magnetic fields can be advected with close to zero net current by “hot” electrons.

In heating plasma with a finite laser spot, an azimuthal magnetic field about the heated region arises through the Biermann battery effect \([12] \). For multiple spots in close proximity, as in inertial fusion, these magnetic fields will be in a configuration with oppositely directed field lines. Under such conditions, magnetic reconnection of field lines may be expected to arise. Magnetic reconnection has been intensely studied in space plasmas, but more recently, laser inertial fusion relevant scenarios have been investigated \([13–15] \).

In Sweet-Parker theory, a resistive region between plasma inflows with resistivity \( \eta \) allows magnetic field lines to diffuse and change topology, leading to jets outflowing at Alfvénic speeds, \( v_A \) \([16,17] \). However, observed reconnection rates are rarely well described by this model. In case of a small diffusion region \( L \) for which the Sweet-Parker width, \( \delta_{SP} = L/\sqrt{3} \) where \( S = v_A L \mu_0/\eta \) is the Lundquist number, is smaller than the ion inertial length, \( c/\omega_{pi} \), Hall physics is relevant and reconnection is no longer resistivity dominated. Rather, augmented by the inclusion of Hall physics, reconnection rates have been shown to be significantly faster suggesting that the dynamics at such small scale lengths contribute strongly to reconnection. Recently, researchers have been interested in the intermediate regime between collisionless and collisional reconnection \([21,22] \), where the Nernst effect is important. Given the analogous form of the Nernst term and Hall current term in Ohm’s law, it is natural to assume that the Nernst effect may enable reconnection in a similar manner to Hall reconnection, but with the electron currents replaced by heat fluxes.

In this Letter, we show that under conditions similar to those found in hohlraums, where heat flux effects in Ohm’s law are important \([10] \), reconnection of field lines can occur. The heat fluxes that are generated by the laser hot-spots drive reconnection through advection at the “Nernst” velocity \( v_T \). The Nernst effect allows magnetic field advection without an associated electron current, which is different than the standard Hall effect within the reconnection layer; this breaks the Alfvénic constraint (at least within the parameters considered) and allows characteristic reconnection rates of \( E_z/(B_0 v_T) \) rather than \( E_z/(B_0 v_A) \). We show that this can occur for conditions described by a dimensionless number describing the ratio of Nernst to electron flow velocities. We find that this mechanism is only relevant in a high \( \beta \) plasma, i.e. where the ratio of thermal pressure to magnetic pressure is large. However, the Hall parameter \( \omega_c \tau_{ci} \) can simulta-
neously be large so that thermal transport is strongly modified by magnetic fields, which can impact longer time scale temperature homogeneity and ion dynamics.

The Vlasov-Fokker-Planck (VFP) equation, is solved together with Ampere’s and Faraday’s Laws to describe the plasma. The code we use, IMPACTA [23, 20], uses a Cartesian tensor expansion [27], with the distribution function expanded as \( f(t, \mathbf{r}, \mathbf{v}) = f_0 + f_1 \cdot \mathbf{v} + f_2 \cdot \mathbf{v}^2 + \ldots \). This expansion can be truncated in a collisional plasma, as collisions tend to smooth out angular variations in the distribution function, resulting in a close to isotropic distribution in the centre of mass frame, represented by \( f_0 \). Higher orders are successively smaller perturbations, \( f_2 \ll f_1 \ll f_0 \). Using the Lorentz gas approximations, electron-ion collisions appear in the equation describing the evolution of \( f_1 \) in the ion center-of-momentum frame as an effective collision frequency \( \propto 1/v^3 \):

\[
\frac{\partial f_1}{\partial t} + v \cdot \nabla f_0 - \frac{eE}{m_e} \frac{\partial f_0}{\partial v} - \frac{eB}{m_e} \times f_1 + \frac{2}{5} v \nabla \cdot f_2 - \frac{2}{5v^2} \frac{\partial}{\partial v} \left( \frac{v^3}{m_e} \mathbf{E} \cdot f_1 \right) = -\frac{Y t}{v^3} f_1 \tag{1}
\]

where \( Y = 4\pi (e^2/4\pi e_0 m_e)^2 \ln \Lambda_{ei} \). In IMPACTA, terms up to and including \( f_2 \) are retained.

In reconnection studies, Ohm’s law is of crucial significance. We can formulate a generalized form of Ohm’s law for this velocity dependent collision operator by multiplying by \( v^3 \) and taking the current moment \((4\pi/3) \int_0^\infty \ldots v^3 dv \). To express Eqn. 2 in a more familiar form, if we assume that the distribution function is a Maxwellian speed distribution multiplied by a function of angle only, it can be shown that Eqn. 2 reduces to

\[
E = \eta j + \frac{\mathbf{j} \times \mathbf{B}}{en_e} - \nabla \left( \frac{n_e m_e \langle v^3 \rangle}{6en_e} \right) - \nabla \cdot \left( \frac{n_e m_e \langle \mathbf{v} v \rangle}{2en_e \langle v^3 \rangle} \right), \tag{2}
\]

where the effective resistivity is

\[
\eta = \frac{2\pi Z e^2 \ln \Lambda_{ei}}{(4\pi e_0)^2 m_e \langle v^3 \rangle),
\]

the magnetic convection velocity by heat flow [10] is

\[
\mathbf{v}_T = \frac{\langle \mathbf{v} v \rangle}{2\langle v^3 \rangle} + \frac{j}{en_e},
\]

the inertial term \((\partial f_1/\partial t)\) is neglected, valid for a sufficiently collisional system, a term contracting with \( E \), \( \langle \mathbf{v} v \rangle/(2\langle v^3 \rangle) \), is assumed to be small, and velocity moments are defined by

\[
\langle v^n \rangle = \frac{4\pi}{n} \int_0^\infty f_0 \, v^{n+2} \, dv, \\
\langle v^2 v \rangle = \frac{2\pi}{3n_e} \int_0^\infty f_1 \, v^{n+3} \, dv, \\
\langle v v v \rangle = \frac{8\pi}{15n_e} \int_0^\infty f_2 \, v^{n+4} \, dv.
\]

The last two terms in Eqn. 2 combined play the role of the pressure tensor term normally used in Ohm’s law.
This means that resistive effects will be small, and therefore anisotropic pressure-like \((f_x)\) effects must be included in Ohm’s law to support the electric field at the X-point. In ref. [22], Daughton et al included heat flux effects in their reconnection study, but for their system \(H_N \ll 1\) so the thermal contribution was small. Here we examine a situation where \(H_N \gg 1\) where heat flux effects dominate.

As in ref. [25,26], we use a normalization scheme with time normalized to \(\tau_n = 4r_{ci}/3\sqrt{\pi}\) and velocity normalized to \(v_{th0} = \sqrt{2kBTe0/m_e}\). As a result, lengths are normalized to the electron mean-free-path \(\lambda_{mfp} = v_{th}\tau_n\). The computation is performed in a domain defined over the range \(-100\lambda_{mfp} < y < 100\lambda_{mfp}\) and \(-1500\lambda_{mfp} < x < 1500\lambda_{mfp}\). The cells near the boundary in \(\hat{x}\) are exponentially increasing in step size such that they can be considered “far away”. The domain of interest in \(\hat{x}\), where the cell size is constant, is \(-400\lambda_{mfp} < x < 400\lambda_{mfp}\). The numerical resolution in the runs shown in the paper is \(\Delta x = 13.3333\lambda_{mfp}\), \(\Delta y = 3.125\lambda_{mfp}\), \(\Delta t = 0.0625v_{th}\).

The connection between the normalized quantities and real parameters is made through the ratios \(v_{th}/c\) and \(\omega_{pe}\tau_n\). Here, \(v_{th}/c = 0.08\) and \(\omega_{pe}\tau_n = 125\) are chosen in order to put the system into inertial confinement relevant conditions, corresponding to a temperature \(T_0 = 1.6\, \text{keV}\) and electron number density \(n_e = 2.5 \times 10^{22}\, \text{cm}^{-3}\). A magnetic field of \(B_0 = 1\) corresponds to a field strength of 400 T (4 MG).

The magnetic field is generated through the \(\nabla n_e \times \nabla T_e\) mechanism. We introduce an out of plane plasma density gradient of the form \(\nabla n(x,y)\) / \(\nabla z = L_n e^{-((z/20r_0)^2)}\left(e^{-((x+y_{max}/r_0)^2)} + e^{-((x-y_{max}/r_0)^2)}\right)\), where \(L_n = 50\) and \(r_0 = 50\), by adding a \(z\) component of electric field. This gradient is switched off at \(t = 800\tau_n\) to prevent excessive magnetic field generation. The temperature profile is accomplished by heating the plasma near the \(y\)-boundaries of the system using an inverse bremsstrahlung heating operator [29] with a profile \(H(x,y) = H_0 e^{-((y+y_{max}/r_0)^2)} e^{-((y-y_{max}/r_0)^2)} + e^{-((x-y_{max}/r_0)^2)}\), where \(H_0 = 0.5\), corresponding to a laser of intensity \(2.5 \times 10^{14}\, \text{Wcm}^{-2}\). The heated regions result in strong heat fluxes in the \(\hat{y}\) direction that advect the magnetic field lines inwardly towards the reconnection region. Ions are stationary in the simulation to isolate these effects, which may be justified physically in the case of the walls of a hohlraum as they are heavy ions (for example gold) [30]. Simulations run with ion motion show similar behavior. A thorough study of the Nernst and bulk flow advection of magnetic fields is in Ref. [10].

Figure [1] shows output from the simulation at a time \(19000\, \tau_n\), (a) shows the magnetization of the plasma, \(B\) and (b) illustrates the temperature profile of the system. The Nernst velocity \((c)\) is approximately \(10^2\) larger in magnitude than the maximum current (not shown). The flow direction of the Nernst velocity (calculated directly from the distribution function) indicates that thermal energy is being brought inwards in the \(y\)-direction towards the reconnection region and is subsequently redirected outwards in the \(x\)-direction, carrying the magnetic field with it. Distinct “jets” of heat flux are formed out of the reconnection region.
Figure 1(d) shows the magnetic field profile after the majority of the flux has reconnected, at a time $27000 \tau_n$ into the simulation, which corresponds to approximately 0.6 ns. The reconnected field lines are then advected by the Nernst jets towards the $\hat{x}$ boundaries. Figure 1(e) shows the temperature profile at the same time as the magnetic field in (d). The outward flow in $\hat{x}$ from the reconnection process causes the change in the temperature profile from (b) to (e).

Figure 2: Illustration of the contribution of the different components of Ohm’s Law in Equation (2) taken from the simulation at a time $t = 11000 \tau_n$.
(a) $E_z$ calculated from the code, (b) $\tilde{n}j_z$, (c) $|\mathbf{J}|$ (e) $|\nabla \times \mathbf{B}|$, (d) $|\mathbf{v}_T \times \mathbf{B}|$, (e): $\left[ \frac{\nabla \cdot \langle \mathbf{v} \mathbf{v}^\ast \rangle}{2\langle \mathbf{v}^\ast \rangle} \right]_z$, (f) Sum of all contributions (b-e).

The quantity $E_z$ is the rate at which magnetic flux crosses the neutral point. In the case of oppositely directed magnetic fields, $B_z$, the reconnecting magnetic field, $B_B$, is generated through Faraday’s Law by the out of plane electric field, $\partial E_z/\partial x$ in a 2-D Cartesian geometry. We can analyze the various contributions from the generalized Ohm’s Law, eq. (2) by directly calculating the velocity moments. Figure 2 shows the out-of-plane electric field, $E_z$, and four of the terms that contribute to it. Anisotropic pressure tensor-like terms almost entirely support $E_z$ at the X-point where the flows diverge, with a small contribution from the resistive term. The $\mathbf{v}_T \times \mathbf{B}$ term provides an analogue of the Hall current, with the actual Hall current $\mathbf{j} \times \mathbf{B}$ being negligible. The sum of just these moments of the numerical distribution function agrees well with the electric field taken from the code (which in these calculations includes electron inertia). Using the terms in eq. (3) instead, similar results are obtained, with the small difference being due to the non-Maxwellian distribution that develops in the reconnection region.

By convention, as in [23], the reconnection rate coefficient is reported as $E_z/Bv_A$ where $v_A$ is the Alfvén velocity and typical rates associated with fast reconnection are $E_z/Bv_A = 0.1 \sim 0.2$. In our simulation, the ions are fixed, and consequently, Alfvénic flows are nonexistent. The characteristic flow velocity for the flux is clearly $\mathbf{v}_T$. There is a marked increase in the strength of the magnetic field near the reconnection region. Fox et al. [23] account for this effect in the calculation of the local magnetic field, and we perform the same correction. We find that in our simulation, $E_z/Bv_T \approx 0.1$, as shown in fig. 3(a).

Figure 3(b) illustrates the evolution of $\beta$, the ratio of thermal pressure to magnetic pressure. The sharp peak arises due to rapid heating of the plasma, and then the subsequent decrease comes from the compression of the magnetic field flux before the reconnection process can begin. Once the anisotropic pressure-like term, fig. 2(e) supports the out-of-plane electric field, $E_z$, across the reconnection layer, the field compression is maintained and eventually reduced, while the plasma is heated due to the decrease in transport inhibition because of the reconnection process. This corresponds to the steady increase in $\beta$ as observed after $t = 13000 \tau_n$. Figure 3(c) illustrates the magnetization of the plasma over time. The initial rise in $\omega_c \tau_{ei}$ is due to the compression phase of the magnetic field. After this period, a plateau arises because while the magnetic field decompresses due to reconnection, the plasma heats in the reconnection region, effectively increasing $\tau_{ei}$. The steady decrease in the late-time behavior is attributed to the magnetic field decompression as the reconnected field lines relax from the Nernst outflows.

When simulations are performed with different values of $\omega_e/\nu_{ei}$ and $v_{th}/c$, i.e. different plasma density and temperature, they evolve similarly for fixed $\delta_a$, with a reconnection rate stabilizing close to $E_z/Bv_T \approx 0.1$, as expected from $H_N$, which only depends on $\delta_a$ for fixed $\omega_c \tau_{ei}$. Hence, the ratio of the skin depth to the collision mean-free-path is the important consideration for this mechanism. One significant difference with Hall recon-
nection that we wish to highlight is that quasi-neutrality can be maintained throughout the system and therefore there is no necessity for ion motion outside of the reconnection region to maintain dynamic equilibrium. Redirected heat flows by magnetic reconnection can result in a redistribution of thermal energy and reconnection of field lines can remove thermal transport barriers. Since strong heat flows and magnetic fields are expected in the interior of hohlraums, understanding this mechanism can be expected to be important for inertial fusion energy, in particular because reconnection may mitigate the thermal transport inhibition by magnetic fields that could affect the uniformity of the drive. However, the magnetic reconnection could also lead to the production of energetic electrons.

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[1] J. A. Stamper, K. Papadopoulos, R. N. Sudan, S. O. Dean, E. A. McLean, and J. M. Dawson, Phys. Rev. Lett. 26, 1012 (1971)
[2] M. Borghesi, A. J. Mackinnon, R. Gaillard, O. Willi, A. Pukhov, and J. Meyer-ter Vehn, Phys. Rev. Lett. 80, 5137 (1998)
[3] R. J. Kingham and A. R. Bell, Phys. Rev. Lett. 88, 045004 (2002)
[4] M. Tatarakis, I. Watts, F. N. Beg, E. L. Clark, A. E. Danor, A. Gopal, M. G. Haines, P. A. Norreys, U. Wagner, M. S. Wei, M. Zepf, and K. Krushelnick, Nature 415, 280 (2002)
[5] U. Wagner, M. Tatarakis, A. Gopal, F. N. Beg, E. L. Clark, A. E. Danor, R. G. Evans, M. G. Haines, S. P. D. Mangles, P. A. Norreys, M.-S. Wei, M. Zepf, and K. Krushelnick, Phys. Rev. E 70, 026401 (2004)
[6] S. I. Braginskii, Reviews of Plasma Physics 1, 205 (1965).
[7] J. R. Rygg, F. H. Seguin, C. K. Li, J. A. Frenje, J. M. E. Manuel, R. D. Petrasso, R. Betti, J. A. Delettrez, O. V. Gotchev, J. P. Knauer, D. D. Meyerhofer, F. J. Marshall, C. Stoeckl, and W. Theobald, Science 319, 1223 (2008)
[8] S. Glenzer, W. Alley, K. Estabrook, J. De Groot, M. Haines, J. Hammer, J. Jaudau, B. MacGowan, J. Moody, W. Rozmus, L. Suter, T. Weiland, and E. Williams, Physics of Plasmas 6, 2117 (1999) 40th Annual Meeting of the Division of Plasma Physics of the American-Physical-Society, NEW ORLEANS, LOUISIANA, NOV 16-20, 1998.
[9] C. K. Li, F. H. Seguin, J. A. Frenje, M. Rosenberg, R. D. Petrasso, P. A. Amendt, J. A. Koch, O. L. Landen, H. S. Park, H. F. Robey, R. P. J. Town, A. Casner, F. Philippe, R. Betti, J. P. Knauer, D. D. Meyerhofer, C. A. Back, J. D. Kilkenny, and A. Nikroo, Science 327, 1231 (2010).
[10] M. Haines, Plasma Physics and Controlled Fusion 28, 1705 (1986).
[11] A. v. Ettingshausen and W. Nernst, Annalen der Physik 265, 343 (1886)
[12] A. Nishiguchi, T. Yabe, M. Haines, M. Psimopoulos, and H. Takekaki, Phys. Rev. Lett. 53, 262 (1984)
[13] C. P. Ridgers, R. J. Kingham, and A. G. R. Thomas, Phys. Rev. Lett. 100 (2008), 10.1103/PhysRevLett.100.075003.
[14] L. Willingale, A. G. R. Thomas, P. M. Nilson, M. C. Kaluza, S. Bandypadhyay, A. E. Danor, R. G. Evans, P. Fernandes, M. G. Haines, C. Kamperidis, R. J. Kingham, S. Minard, M. Notley, C. P. Ridgers, W. Rozmus, M. Sherlock, M. Tatarakis, M. S. Wei, Z. Najmudin, and K. Krushelnick, Phys. Rev. Lett. 105 (2010), 10.1103/PhysRevLett.105.095001.
[15] A. Schluter and L. Biermann, Zeitschrift fur Naturforschung Section A--Journal of Physical Sciences 5, 237 (1950).
[16] P. M. Nilson, L. Willingale, M. C. Kaluza, C. Kamperidis, S. Minard, M. S. Wei, P. Fernandes, M. Notley, S. Bandypadhyay, M. Sherlock, R. J. Kingham, M. Tatarakis, Z. Najmudin, W. Rozmus, R. G. Evans, M. G. Haines, A. E. Danor, and K. Krushelnick, Phys. Rev. Lett. 97 (2006), 10.1103/PhysRevLett.97.255001.
[17] C. K. Li, F. H. Seguin, J. A. Frenje, J. R. Rygg, R. D. Petrasso, R. P. J. Town, O. L. Landen, J. P. Knauer, and V. A. Smalyuk, Phys. Rev. Lett. 99 (2007), 10.1103/PhysRevLett.99.055001.
[18] L. Willingale, P. M. Nilson, M. C. Kaluza, A. E. Danor, R. G. Evans, P. Fernandes, M. G. Haines, C. Kamperidis, R. J. Kingham, C. P. Ridgers, M. Sherlock, A. G. R. Thomas, M. S. Wei, Z. Najmudin, K. Krushelnick, S. Bandypadhyay, M. Notley, S. Minard, M. Tatarakis, and W. Rozmus, Phys. Plasmas 17, 043104 (2010).
[19] R. Green and P. Sweet, Astrophysical Journal 147, 1153 (1967).
[20] E. Parker, Astrophysical Journal 174, 499 (1972).
[21] P. A. Cassak, M. A. Shay, and J. F. Drake, Phys. Rev. Lett. 95, 235002 (2005).
[22] W. Daughton, V. Roytershteyn, B. J. Albright, H. Karimabadi, L. Yin, and K. J. Bowers, Physics of Plasmas 16 (2009), 10.1063/1.3191718.
[23] J. Kingham and A. Bell, Journal of Computational Physics 194, 1 (2004).
[24] W. Fox, A. Bhattacharjee, and K. Germaschewski, Phys. Rev. Lett. 106 (2011), 10.1103/PhysRevLett.106.215003.
[25] W. Fox, A. Bhattacharjee, and K. Germaschewski, Physics of Plasmas 19, 056309 (2012).
[26] R. Kingham and A. Bell, Journal of Computational Physics 194, 1 (2004).
[27] E. Epperlein and M. Haines, Physics of Fluids 29, 1029 (1986).
[28] A. Langdon, Phys. Rev. Lett. 44, 575 (1980).
[29] J. Lindl, P. Amendt, R. Berger, S. Glendinning, S. Glenzer, S. Haan, R. Kauffman, O. Landen, and L. Suter, Physics of Plasmas 11, 339 (2004).