DISTRIBUTED FAULT-TOLERANT CONSENSUS TRACKING FOR NETWORKED NON-IDENTICAL MOTORS

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Abstract. This paper investigates a distributed fault-tolerant consensus tracking algorithm for a group non-identical motors with unmeasured angular speed and unknown failures. First, the failures are modeled by nonlinear functions, and sliding mode observer is designed to estimate the angular speed and nonlinear failures. Then, in order to achieve the desired results, a novel distributed fault-tolerant algorithm is constructed based on the estimated angular speed and reconstructed failures. Theoretical analysis illustrates the stability and globally exponentially asymptotically convergence of the proposed observer and controller. The numerical simulations verify the high estimation accuracy, effectiveness and robustness of the proposed methods. The semi-physical experiments based on RT-LAB real-time simulator further test the system and controller with accurate performance in real-time.

1. Introduction. With the development of technologies, the size and complexity of multi-agent systems increase rapidly, which involves a number of agent failures, actuator failures and other system failures which might cause undesired reactions. Therefore, fault-tolerant is becoming one of the main problems that arise in the development of system reliability and stability [6, 15].

In high-risk industrial systems, such as high-speed printing, aerospace and military, servomotor driven systems are the cornerstones. These motors should be capable of continued functional operation, even when insulation failures, open-circuit of windings or other failures occur. For such a purpose, many researches dedicated to studying complex systems that are subjected to failures. Reference [12] constructed a bank of unknown input observers for fault detection and isolation in the network of interconnected second-order systems. Different from fault diagnosis and isolation, many literatures focus on the tolerance of systems in spite of failures. Reference [18] proposed a new scheme for estimating the actuator and sensor fault for Lipschitz...
nonlinear systems with unstructured uncertainties using the sliding mode observer technique. By adjusting some weights of the cooperative protocol, the target point can still be reached in the presence of agent faults \cite{17}. When it comes to the networked motors, object failures that change the system dynamics and actuator failures that influence the control input should be considered \cite{11,12}. The coordination behavior of multi-agent system has been widely studied in recent years \cite{8,9,10,16}, and the coordination of linear multi-agent system with agent actuator failures was analyzed in detail \cite{7}. An adaptive fault tolerant fuzzy tracking control problem was studied for networked high-order multi-agent with time-varying actuator failures \cite{13}. Another adaptive control protocol with time varying adaptive gain was developed to guarantee the consensus under all possible actuator failures \cite{19}. Fault estimation module to compensate the effect of faults has been investigated for a class of nonlinear systems \cite{4}. However, complex systems might encounter multiple faults simultaneously, the forementioned researches only discussed one certain fault. And the forementioned driven system was verified by non-real-time simulation tools, such as MATLAB/Simulink, which might prone many troubles from off-line simulation to real prototype implementation. These two concerns motivate this work.

In this paper, we focus on a kind of consensus tracking problem for networked non-identified motors with generalized failures including object failures and actuator failures. A novel distributed fault-tolerant protocol is proposed in the presence of generalized failures. Additionally, based on RT-LAB platform where the virtual motor driven is connected to a physical controller, Hardware-In-the-Loop simulation is further designed to test the valid of the system and controller \cite{3}.

The main contributions of the present work are as follows. (1) Let the estimated angular speed replace the actual angular, which decrease sensor numbers as well as sensor faults possibilities. (2) Strictly communication constraints is considered. (3) Fault-tolerant controller based on sliding mode variable structure observer is proposed, one can use the on-line identified angular speed and failures for control loop compensation and robustness improvement. (4) Hardware-In-the-Loop simulation based on RT-LAB is done for a demonstration of accuracy and validity of the proposed control strategy.

The remainder of this paper is organized as follows. Section II presents some basic graph theory notations and introduces the problem to be solved in this paper. Section III is the main part of this paper focusing on the distributed fault-tolerant consensus algorithm and theoretical analysis. Section IV provides some simulation and experiment results. Section V concludes this paper.

2. System model and problem statement. Consider \( n + 1 \) members in the multi-motor team with distinct dynamics, each servo motor stands for an agent with actual motion ability, then multiple servo systems can be regarded as a multi-motor system.

For this multi-motor system that consists of \( n \) followers labeled as 1-n, and a leader labeled as 0(L), the graph theory is introduced to describe the information exchange among team members. Let \( G_{n+1} = (v_{n+1}, \varepsilon_{n+1}) \) be an directed graph which consists of nonempty set of nodes \( v_{n+1} = \{0, 1, \cdots, n\} \) and set of edges \( \varepsilon_{n+1} \subseteq v_{n+1} \times v_{n+1} \). Its adjacency matrix is defined as \( A_{n+1} = [a_{ij}] \in R^{(n+1)\times(n+1)} \), \( a_{ij} > 0 \) if \((j, i) \in \varepsilon_{n+1} \); otherwise \( a_{ij} = 0 \), here we assume \( a_{ii} = 0, \forall i = 0, 1, \cdots, n \); \( a_{0j} = 0, \forall j = 0, 1, \cdots, n \). Let \( G_n = (v_n, \varepsilon_n) \) represent the communication topology.
among followers, and $A_n = [a_{ij}] \in \mathbb{R}^{n \times n}$ is the corresponding adjacency matrix. Define Laplacian matrix $L_n = D_n - A_n$, where $D_n = [d_{ij}] \in \mathbb{R}^{n \times n}$ is the in-degree matrix given as $d_{ij} = 0, i \neq j$, and $d_{ii} = \sum_{j=1}^{n} a_{ij}, i = 1, 2, \cdots, n$.

In this paper, the fixed undirected communication graph for followers is connected and only one $a_{i0} (i = 1, 2, \cdots, n)$ is positive, that is, $A_n$ and $L_n$ are symmetric, $0 = \lambda_1(L_n) \leq \lambda_2(L_n) \leq \cdots \leq \lambda_n(L_n)$, and the algebraic connectivity $\lambda_2(L_n)$ which quantifies the convergence rate of consensus algorithm is positive. Given a matrix $M = L_n + \text{diag}(a_{10}, a_{20}, \cdots, a_{n0})$, then $M$ is symmetric positive definite.

Suppose motor $i$ takes the following dynamic

$$
\begin{align*}
\dot{\theta}_i &= \omega_i \\
\dot{\omega}_i &= -K_{J_i R_i} \omega_i + K_{K_i} u_i \\
\end{align*}
$$

where $i = 0, 1, \cdots, n$, $-K_{J_i} K_{ci}/(J_i R_i) = k_i + \Delta k_i$, $K_{ei}/(J_i R_i) = b_i + \Delta b_i$, $k_i$ and $b_i$ represent nominal value, $\Delta k_i$ and $\Delta b_i$ represent the parameter variation, $\theta_i$, $\omega_i$ and $u_i$ denote, respectively, the rotor position, the electrical angular speed and control input. $R_i$, $J_i$, $K_{ii}$ and $K_{ei}$ denote, respectively, the resistance, rotor moment of inertia, electromagnetic torque coefficient and voltage feedback coefficient.

The distributed consensus tracking control for multi-motor system is divided into two parts.

(I) Tracking control for the virtual leader (motor 0)

$$
\begin{align*}
\dot{\theta}_0 &= \omega_0 \\
\dot{\omega}_0 &= k_0 \omega_0 + b_0 u_0 \\
\end{align*}
$$

The first control goal is to design a tracking control algorithm $u_0$ for virtual leader to receive

$$
\lim_{t \to \infty} \| \theta_0 - \theta^d \| = 0, \lim_{t \to \infty} \| \omega_0 - \dot{\theta}^d \| = 0
$$

where $u_0 = l_p (\theta^d - \theta_0) + l_i \int (\theta^d - \theta_0) dt + l_d (d(\theta^d - \theta_0)/dt)$, $\theta^d$ is reference signal, $l_p$, $l_i$ and $l_d$ are positive parameters being chosen properly.

(II) Distributed coordination control for the followers (motor $i$)

From (1), the dynamic of follower motor $i$ with failures is given by

$$
\begin{align*}
\dot{\theta}_i &= \omega_i \\
\dot{\omega}_i &= k_i \omega_i + b_i u_i + F_{ai} \\
y_i &= \theta_i
\end{align*}
$$

where $i = 1, 2, \cdots, n$, $y_i$ is measurable output, $F_{ai}$ is generalized failure, which is modeled by nonlinear function, if the controlled object is faulty, $F_{ai} = \Delta k_i \omega_i + \Delta b_i u_i$; when actuators become faulty, $F_{ei} = b_i \Delta u_i$, $\Delta u_i$ represents the actuator failure. If object failures and actuator failures occur simultaneously, $F_{ai} = \Delta k_i \omega_i + \Delta b_i u_i + b_i \Delta u_i$.

Equation (3) can be written in the form of

$$
\begin{align*}
\dot{\theta} &= \omega \\
\dot{\omega} &= k \omega + b u + F_a \\
y &= \theta
\end{align*}
$$

where $\theta = [\theta_1, \theta_2, \cdots, \theta_n]^T \in \mathbb{R}^{n \times 1}$, $b = \text{diag}(b_1, b_2, \cdots, b_n)$, $k = \text{diag}(k_1, k_2, \cdots, k_n)$, $\omega = [\omega_1, \omega_2, \cdots, \omega_n]^T \in \mathbb{R}^{n \times 1}$, $u = [u_1, u_2, \cdots, u_n]^T \in \mathbb{R}^{n \times 1}$, $F_a = [F_{a1}, F_{a2}, \cdots, F_{an}]^T \in \mathbb{R}^{n \times 1}$.
The other control goal is to design distributed fault-tolerant consensus algorithm \( u_i \) for system \( i \) to achieve
\[
\lim_{t \to \infty} \| \theta_i - \theta_0 \| = 0, \quad \lim_{t \to \infty} \| \dot{\omega}_i - \omega_0 \| = 0
\]
in the presence of unknown angular speed and generalized failures.

3. **Main results.** The basic distributed consensus tracking algorithm can be given by
\[
u_{1i} = \frac{1}{b_i} \left( - \sum_{j=0}^{n} a_{ij} \beta_i (\theta_i - \theta_j) - c_i \text{sgn} \left( \sum_{j=0}^{n} a_{ij} (\theta_i - \theta_j) + \sum_{j=0}^{n} a_{ij} (\omega_i - \omega_j) \right) \right)
\]
where \( i = 1, 2, \cdots, n \), \( \beta_i \) and \( c_i \) are positive control gains, and \( \text{sgn}(\cdot) \) denotes a sign function.

However, algorithm 6 cannot deal with dynamic system 3 with unknown generalized failure \( F_a \) and unmeasurable angular speed \( \omega \), which motivate us to reconstruct the generalized faults and estimate the angular speed for control input compensation. The system fault-tolerant control diagram is shown in Fig. 1 where \( \theta_\Sigma \) represents the linear combination of \( \theta_1, \theta_{i-1}, \theta_{i+1}, \dot{\omega}_{i-1}, \dot{\omega}_{i+1} \), and \( \theta_0, \omega_0 \), if follower \( i \) can receive the leaders information, otherwise it is a linear combination of \( \theta_1, \theta_{i-1}, \theta_{i+1}, \dot{\omega}_{i-1}, \dot{\omega}_{i+1} \).

![Figure 1](image_url)

**Figure 1.** The system fault-tolerant control diagram for follower motor \( i \)

3.1. **Observer design.** To identify the unmeasurable angular speed \( \omega \) and the unknown nonlinear failure \( F_a \), a sliding mode observer is designed in the form of
\[
\begin{align*}
\dot{\theta} &= \dot{\omega} + \text{vsgn} (e_1) \\
\dot{\omega} &= k \dot{\omega} + \dot{u} + \text{usgn} (e_2)
\end{align*}
\]
where \( \hat{\theta} = [\hat{\theta}_1, \hat{\theta}_2, \cdots, \hat{\theta}_n]^T \in R^{n\times 1}, \hat{\omega} = [\hat{\omega}_1, \hat{\omega}_2, \cdots, \hat{\omega}_n]^T \in R^{n\times 1}, e_1 = \theta - \hat{\theta} \in R^{n\times 1}, e_2 = \omega - \hat{\omega} \in R^{n\times 1}, v = \text{diag}(v_1, v_2, \cdots, v_n) \) and \( w = \text{diag}(w_1, w_2, \cdots, w_n) \). \( v_i \) and \( w_i \) \( (i = 1, 2, \cdots, n) \) are positive sliding gains to be designed.

**Theorem 3.1.** With the proposed observer 7, we can find positive sliding gain \( v_i \) and \( w_i \) \( (i = 1, 2, \cdots, n) \) sufficiently large such that the angular speed and generalized failures can be identified in finite time, i.e., \( \hat{\omega} = \omega, \hat{F}_a = \text{usgn} (\text{vsgn} (e_1)) \).

**Proof of Theorem 3.1.** From Equations 4 and 7, the estimation dynamic of \( e_1 \) and \( e_2 \) can be written as
\[
\begin{align*}
\dot{e}_1 &= e_2 - \text{vsgn} (e_1) \\
\dot{e}_2 &= ke_2 + F_a - \text{usgn} (e_2)
\end{align*}
\]
Step 1. Consider a sliding mode surface \( s_1 = e_1 \), by defining a Lyapunov function \( V_1 = \frac{1}{2} s_1^T s_1 \), it has

\[
\dot{V}_1 = s_1^T \dot{s}_1 = e_1^T \dot{e}_1 = e_1^T (e_2 - e_1 \text{sgn}(e_1)) \
\leq e_1^T e_2 - v_{i, \text{min}} \| e_1 \| \leq \| e_1 \| (\| e_2 \| - v_{i, \text{min}})
\]

(9)

where \( v_{i, \text{min}} = \min \{ v_1, \ldots, v_i, \ldots, v_n \} \).

Choosing \( v_{i, \text{min}} \geq \| e_2 \|_{\text{max}} + \eta_1 \), where \( \eta_1 \) is a positive constant, one has

\[
\dot{V}_1 \leq -\eta_1 \| e_1 \|
\]

(10)

which implies that the sliding mode can be reached in finite time. Based on the equivalent principle [14], it has \( s_1 = \dot{s}_1 = 0 \) during the sliding motion, that is

\[
\theta = \dot{\theta} \\
\dot{e}_2 = \text{sgn}(e_1)
\]

(11)

Step 2. Consider another sliding mode surface \( s_2 = e_2 \), by defining a Lyapunov function \( V_2 = V_1 + s_2^T s_2 / 2 \), it has

\[
\dot{V}_2 = V_1 + s_2^T \dot{s}_2 \leq e_2^T \dot{e}_2 = e_2^T k e_2 + e_2^T F_a - e_2^T \text{sgn}(e_2) \
\leq \| e_2 \| (\| F_a \| - w_{i, \text{min}})
\]

(12)

where \( w_{i, \text{min}} = \min \{ w_1, \ldots, w_i, \ldots, w_n \} \).

Choosing \( w_{i, \text{min}} \geq \| F_a \|_{\text{max}} + \eta_2 \), where \( \eta_2 \) is a positive constant, one has

\[
\dot{V}_2 \leq -\eta_2 \| e_2 \|
\]

(13)

which implies that the sliding mode can be reached in finite time. Based on the equivalent principle [14], it has \( s_2 = \dot{s}_2 = 0 \) during the sliding motion, that is

\[
\omega = \dot{\omega} \\
\dot{F}_a = \text{sgn}(\text{sgn}(e_1))
\]

(14)

Obviously, from [14], \( \dot{F}_a \) is bounded. In general, the switching function \( \text{sgn}(s) \) is replaced by a sigmoid function \( H(s) = \frac{1}{1 + e^{-r s}} - 1 \) which can efficiently reduce the chattering of the system, where \( s \) represents the sliding mode surface, and \( r \) is a positive parameter.

3.2. Distributed fault-tolerant consensus algorithm. With the identified angular speed and the reconstructed failures, the distributed consensus algorithm [6] can be reconstructed in the form of

\[
u_i = u_{i,1} + u_{i,2}
\]

(15)

where

\[
u_{i,1} = \frac{1}{k_i} \left( - \frac{1}{n} \sum_{j=0}^{n} a_{ij} \beta_i (\theta_i - \theta_j) - c_i \text{sgn} \left( \sum_{j=0}^{n} a_{ij} (\theta_i - \theta_j) + \sum_{j=0}^{n} a_{ij} (\dot{\omega}_i - \dot{\omega}_j) \right) \right),
\]

and

\[
u_{i,2} = - \frac{1}{n} \dot{F}_{ai}.
\]

Substituting the fault-tolerant consensus algorithm [15] into system dynamic (3) gives

\[
\dot{\theta}_i = \omega_i \\
\dot{\omega}_i = k_i \omega_i - \sum_{j=0}^{n} a_{ij} \beta_i (\theta_i - \theta_j) - c_i \text{sgn} \left( \sum_{j=0}^{n} a_{ij} (\theta_i - \theta_j) + \sum_{j=0}^{n} a_{ij} (\dot{\omega}_i - \dot{\omega}_j) \right)
\]

(16)

\[+ F_{ai} - \dot{F}_{ai} \]
Let $\hat{\theta}_i = \theta_i - \theta_0$, $\hat{\omega}_i = \omega_i - \omega_0$, $\hat{F}_{ai} = F_{ai} - \hat{F}_{ai}$, the error dynamic can be written as
\begin{align}
\dot{\hat{\theta}} &= \hat{\omega} \\
\dot{\hat{\omega}} &= K\hat{\omega} - \beta M \hat{\theta} - C \text{sgn} \left( M \hat{\theta} + M \hat{\omega} \right) + \hat{F}
\end{align}
(17)
where $\hat{\theta} = [\hat{\theta}_1, \hat{\theta}_2, \cdots, \hat{\theta}_n]^T$, $\hat{\omega} = [\hat{\omega}_1, \hat{\omega}_2, \cdots, \hat{\omega}_n]^T$, $\beta = \text{diag} (\beta_1, \beta_2, \cdots, \beta_n)$, $C = \text{diag} (c_1, c_2, \cdots, c_n)$, $M = L_n + \text{diag} (a_{01}, a_{02}, \cdots, a_{0n})$, $\hat{F} = \hat{F}_a - \hat{\omega}_0 1_n + k \omega_0 1_n$, $\hat{F}_a = \text{diag} \left( \hat{F}_{a1}, \hat{F}_{a2}, \cdots, \hat{F}_{an} \right)$.

**Theorem 3.2.** Suppose the fixed undirected communication graph for followers is connected and only one element of $a_{0i} (i = 1, 2, \cdots, n)$ is positive, using [15] for $[k]$, if $c_i$ is sufficiently large, and $\beta_i > \max \{ 0, (4k_i + 1)/4\lambda_i(M) \}$ holds, then, $\lim_{t \to \infty} \| \hat{\theta}_i - \theta_0 \| = 0$, $\lim_{t \to \infty} \| \hat{\omega}_i - \omega_0 \| = 0$, $\forall i = 1, 2, \cdots, n$.

**Proof of Theorem 3.2.** Let $P = \begin{bmatrix} (-KM + \beta M^2)/2 & M/2 \\ M/2 & M/2 \end{bmatrix}$, $Q = \begin{bmatrix} \beta M^2 & 0_{n \times n} \\ 0_{n \times n} & -KM \end{bmatrix}$, it shows that $P$ and $Q$ are symmetric, we can get that $P$ and $Q$ are symmetric positive definite if $\beta_i > \max \{ 0, (4k_i + 1)/4\lambda_i(M) \}$ holds. Define the error vector $X_e = [\hat{\theta}^T, \hat{\omega}^T]^T$, the Lyapunov function is given by
\begin{equation}
V = X_e^T P X_e = \frac{1}{2} \hat{\theta}^T (-KM + \beta M^2) \hat{\theta} + \hat{\theta}^T M \hat{\omega} + \frac{1}{2} \hat{\omega}^T M \hat{\omega}
\end{equation}
(18)
which satisfies $\lambda_{\min} (P) \| X_e \|^2 \leq V \leq \lambda_{\max} (P) \| X_e \|^2$, one has
\begin{align}
\dot{V} &= \hat{\theta}^T (-KM + \beta M^2) \hat{\omega} + \hat{\theta}^T M \hat{\omega} + \hat{\omega}^T M \hat{\omega} \\
&= \hat{\theta}^T (-KM + \beta M^2) \hat{\omega} + \hat{\theta}^T M \left( K \hat{\omega} - \beta M \hat{\theta} - C \text{sgn} \left( M \hat{\theta} + M \hat{\omega} \right) + \hat{F} \right) \\
&+ \hat{\omega}^T M \left( K \hat{\omega} - \beta M \hat{\theta} - C \text{sgn} \left( M \hat{\theta} + M \hat{\omega} \right) + \hat{F} \right) \\
&= -\hat{\theta}^T \beta M \hat{\theta} + \hat{\omega}^T M \hat{\omega} + \left( \hat{\theta}^T M + \hat{\omega}^T M \right) \left( -C \text{sgn} \left( M \hat{\theta} + M \hat{\omega} \right) + \hat{F} \right) \\
&\leq -X_e^T Q X_e + \left\| M \hat{\theta} + M \hat{\omega} \right\| \left( \left\| \hat{F} \right\| - c_{\min} \right)
\end{align}
(19)
where $c_{\min} = \min \{ c_1, \cdots, c_1, \cdots, c_n \}$.

Suppose $F_a$ is bounded in practical engineering, then with Equation (14), we know $\hat{F}$ is bounded, choosing $c_{\min} > \left\| \hat{F} \right\|$, it gives
\begin{equation}
\dot{V} \leq -X_e^T Q X_e - \lambda_{\min} (Q) \| X_e \|^2
\end{equation}
(20)
Therefore, based on the above theoretical analysis and Theorem 4.10 [5], it has $X_e \to 0$, as $t \to \infty$, that is, $\theta_i \to \theta_0$ and $\omega_i \to \omega_0$ as $t \to \infty$. Note that $V \leq \lambda_{\max} (P) \| X_e \|^2$, then from (20), it gives $\dot{V} \leq - \left( \lambda_{\min} (Q)/\lambda_{\max} (P) \right) V$, one has $V(t) \leq V(0) e^{-\left( \lambda_{\min} (Q)/\lambda_{\max} (P) \right) t}$, Thus, we can conclude that the equilibrium point $X_e = 0$ is globally exponentially asymptotically stable. \hfill \Box

### 4. Simulations and experiments.

In this section, we present simulations and semi-physical experiments to validate the main results in this paper. Consider a group of five members with the communication graph given in Fig. 2 and parameters given in Table 1. Note that the undirected graph for follower 1 to 4 is connected and the virtual leader 0 is a neighbor of follower 2. Let $a_{ij} = 1$ if $j$ is a neighbor of $i$, otherwise $a_{ij} = 0$. 


Figure 2. Communication topology for a group of four followers with a virtual leader

Table 1 Parameters of Five Driven Motors

| Motor  | Motor 0(L) | Motor 1 | Motor 2 | Motor 3 | Motor 4 |
|--------|------------|---------|---------|---------|---------|
| R(Ω)   | 0.2        | 0.5     | 0.4     | 0.6     | 0.7     |
| $K_l$  | 0.005      | 0.01    | 0.008   | 0.015   | 0.02    |
| $J(k_g \cdot m^2)$ | 0.02 | 0.03 | 0.025 | 0.05 | 0.04 |
| $K_e$  | 0.1        | 0.2     | 0.2     | 0.18    | 0.25    |
| Initial $\theta$ (rad) | 0.5 | -0.8 | 1.3 | -2.0 | -2.4 |

4.1. Simulations. To demonstrate the proposed observer performance, suppose follower motor 1, 2 and 3 occur, respectively, the incipient failure, the intermittent abrupt failure and high-frequency stochastic failure. Let follower motor 4 occur the above three faults simultaneously. To verify the consensus tracking performance, we select two typical reference signals: $\theta^{d1} = 2t$(rad) and $\theta^{d2} = 3\sin(\pi t/2)$ (rad), then the corresponding desired angular speed can be calculated by Equation (1): $\omega^{d1} = 2$(rad/s) and $\omega^{d2} = 3\pi \sin(\pi t/2)/2$(rad/s). Let $F_{a1} = f_1 f_2$, where $f_1$ is a chirp signal, which starts with 0.03Hz, ends with 0.5Hz, and the target time is 20s. $f_2 = 0.6\sin(\pi t/20)$. Given $F_{a3} = 4(\sin(10\pi t) + \sin \pi t + \sin(6\pi t))$ as a high-frequency stochastic failure, which starts at 3s.

Let $F_{a2} =$ \begin{align*}
0 N \cdot m & \quad 0 \leq t \leq 3 \\
10 N \cdot m & \quad 3 < t \leq 5 \\
0 N \cdot m & \quad 5 < t \leq 9 \\
8 N \cdot m & \quad 9 < t \leq 11 \\
-2 N \cdot m & \quad 11 < t \leq 12 \\
1 N \cdot m & \quad 12 < t
\end{align*}

$F_{a4} = F_{a1} + F_{a2} + F_{a3}$.

4.1.1. Failure reconstruction and angular speed estimation. Let $\theta^{d1} = 2t$(rad) and $\theta^{d2} = 3\sin(\pi t/2)$ (rad), Fig. 3 shows the unknown nonlinear failures reconstruction. In spite of different given reference signals, observer (7) can precisely estimate the four given different type of nonlinear failures in real time.

Fig. 4 and Fig. 5 show the unknown angular speed estimation. From Fig. 4 we can see that the unmeasurable angular speed can be successfully identified in finite time with reference $\theta^{d1}$, the comparisons between the estimated value and the actual angular speed illustrate the accuracy of the designed observer. Fig. 5 shows the unknown angular speed estimation with reference $\theta^{d2}$. Fig. 4 and Fig. 5 illustrate that the angular speed can both be estimated precisely according to different reference signals.

4.1.2. Consensus tracking based on estimated failures and angular speed. Based on the estimated failures and angular speed, the consensus tracking trajectories with different reference signals are given below, respectively. Given the reference signal $\theta^{d1} = 2t$(rad), Fig. 6(a) and Fig. 6(b) show, respectively, the consensus tracking
Figure 3. The unknown nonlinear failures estimation in both cases using sliding mode observer (7).

Figure 4. Angular speed estimation using observer (7) with $\theta_d^1 = 2t{\text{rad}}$
for desired rotor position and angular speed. It is clear that the virtual leader can track the reference signal, and the networked motors can track the virtual leader with the identified generalized failures in Fig. 3 and estimated angular speed in Fig. 4. Given the reference signal $\theta^d = 3 \sin (\pi t/2)$ (rad), Fig. 7(a) and Fig. 7(b) show, respectively, that the consensus tracking for desired rotor position and angular speed are reached with the identified generalized failures in Fig. 3 and estimated angular speed in Fig. 5.

Figure 5. Angular speed estimation using observer (7) with $\theta^d = 3 \sin (\pi t/2)$ (rad)

Figure 6. Consensus tracking for rotor position and angular speed with protocol (15) when $\theta^d = 2t$ (rad)
4.2. Experiments. The aim of building the semi-physical simulation system of networked motors is to make the whole simulation process to be similar to the real engineering environment, and evaluation accuracy of the control performance. RT-LAB is a modular real-time simulation platform which can achieve hardware-in-the-loop simulation (HILS). The HILS of networked motors system can be realized by downloading the compiled code from the networked motors model for running in RT-LAB, and by downloading the C-code generated from the designed controller model for running in DSP. RT-LAB OP5600 is selected to simulate the networked motors, and TMS320F2812 is selected as the controller. PWM carrier frequency is set to 5 KHz, the sampling period is set to 0.5 ms.

The parameters, failures and other initial conditions are chosen the same as above in order to compare the experiment results with simulation results.

4.2.1. Failure reconstruction and angular speed estimation. Let \( \theta_d^1 = 2t \) (rad) and \( \theta_d^2 = 3 \sin \left( \frac{\pi t}{2} \right) \) (rad), the same as Fig. 3, Fig. 8 shows the nonlinear generalized failures which are identified successfully with different reference signals. Fig. 9 and Fig. 10 shows the experimental results of angular speed estimation according to different reference signals. Fig. 9 corresponds to \( \theta_d^1 \), and Fig. 10 corresponds to \( \theta_d^2 \). The same as Fig. 4 and Fig. 5 the unknown angular speed is estimated in real time with high accuracy.

4.2.2. Consensus tracking based on estimated failures and angular speed. Given the reference signal \( \theta_d^1 = 2t \) (rad), Fig. 11 show the consensus tracking of desired rotor position and angular speed, which are both successfully reached with the identified generalized failures in Fig. 8 and estimated angular speed in Fig. 9.

Given the reference signal \( \theta_d^2 = 3 \sin \left( \frac{\pi t}{2} \right) \) (rad), Fig. 12 show that consensus tracking of desired rotor position and angular speed are both reached with the identified generalized failures in Fig. 8 and estimated angular speed in Fig. 10.

5. Conclusion. This paper deals with a class of non-identical networked motors with unmeasurable angular speed and multiple unknown failures. Based on the measurable rotor position and control input, the sliding mode variable structure observer is designed for fault reconstruction and angular speed estimation. Furthermore, a distributed fault-tolerant tracking consensus protocol is developed for
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Figure 8. The unknown nonlinear failures estimation in both cases using sliding mode observer (7) ($F_{a1}, \hat{F}_{a1}$: 2/unit; $F_{a2}, \hat{F}_{a2}$: 10/unit; $F_{a3}, \hat{F}_{a3}$: 14.2/unit; $F_{a4}, \hat{F}_{a4}$: 19/unit)

Figure 9. Angular speed estimation using observer (7) with $\theta^{d1} = 2t(\text{rad})$ ($\omega$: 10rad/s/unit)

Figure 10. Angular speed estimation using observer (7) with $\theta^{d2} = 3\sin(\pi t)(\text{rad})$ ($\omega$: 7.85rad/s/unit)

System functioning well in spite of failures. Theoretical analysis illustrates the correctness and robustness of the proposed method. Simulations and experiments show that the proposed observer and fault-tolerant controller are capable of eliminating the adverse effects of unknown nonlinear generalized failures.
Figure 11. Consensus tracking for rotor position and angular speed with protocol (15) when $\theta^d_1 = 2t$ (rad) ($\theta$: 90 rad/unit, $\omega$: 10 rad/s/unit)

Figure 12. Consensus tracking for rotor position and angular speed with protocol (15) when $\theta^d_2 = 3\sin(\pi t)$ (rad) ($\theta$: 3.75 rad/unit, $\omega$: 7.85 rad/s/unit)
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