Phase mixing of propagating Alfvén waves in a stratified atmosphere: solar spicules

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Abstract Alfvénic waves are thought to play an important role in coronal heating and solar wind acceleration. Recent observations by Hinode/SOT showed that the spicules mostly exhibit upward propagating high frequency waves. Here we investigate the dissipation of such waves due to phase mixing in stratified environment of solar spicules. Since they are highly dynamic structures with speeds at about significant fractions of the Alfvén phase speed, we take into account the effects of steady flows. Our numerical simulations show that in the presence of stratification due to gravity, damping takes place in space than in time. The exponential damping low, $\exp(-At^3)$, is valid under spicule conditions, however the calculated damping time is much longer than the reported spicule lifetimes from observations.

Keywords Sun: spicules · Alfvén waves: phase mixing · Stratification

1 Introduction

The coronal heating (Edlén 1943) mechanism is one of the major unsolved problems in solar physics. Since the energy flux carried by acoustic waves is too small, the possibility of heating by MHD waves has been investigated intensively as the magnetic structure of the solar corona can play an important role here (Hood et al. 1997). The propagation of Alfvén waves is one of the candidate mechanisms that can carry energy to large distances from the surface, and heat the solar corona (Wentzel 1974; Kudoh and Shibata 1999). However, a heating theory based on the Alfvén waves faces a couple of difficulties: Firstly, the waves have to transport enough energy flux, and secondly, they have to dissipate efficiently in order to deposit the right amount of energy at the right place (De Moortel et al. 1999). The Alfvén waves may reach the corona even in the absence of highly stratified atmosphere but with lesser propagation speed. The damping length of Alfvén modes is defined by various dissipative processes such as phase mixing (Heyvaerts and Priest 1983; Browning 1991), resonance absorption (Ionson 1978), and nonlinear mode conversion (Hollweg 1982). Phase mixing is a mechanism for dissipating Alfvén waves, which was first proposed by Heyvaerts and Priest (1983). When Alfvén waves propagating in an inhomogeneous medium, on each magnetic field line, a wave propagates with its own local Alfvén speed. After a certain distance or after enough time, these neighboring perturbations will be out of phase. This ultimately results in a strong enhancement of the dissipation of Alfvén waves energy via both viscosity and resistivity. Hood et al. (2002) and Heyvaerts and Priest (1983) analytically showed that in both the strong phase mixing limit and the weak damping approximation, the amplitude of Alfvén waves decays with time as $\exp(-t^3)$. Karami and Ebrahimi (2009) calculated
numerically the damping times of oscillations in the presence of viscosity and resistivity in coronal loops. They concluded that the above exponential damping law in time is valid for the Lundquist numbers higher than $10^7$. De Moortel et al. (1999, 2000) studied the effect of stratification due to gravity on phase mixing, and found that the wavelengths lengthen when Alfvén waves propagate through a stratified plasma. They concluded that a vertical stratification of density makes phase mixing by ohmic heating less important in coronal heating problem. They also found that in a stratified atmosphere, the heat will be deposited higher up than in an unstratified atmosphere, and that the viscous heating will be the dominant component in the heating processes at lower heights. However, in coronal conditions the effect of stratification on efficiency of phase mixing would still be large as the height at which most heat would be deposited through ohmic dissipation is increased considerably by stratification. Moreover, depending on the value of pressure scale height, phase mixing can either be more or less efficient than in the uniform case.

Spicules have long been investigated as a coronal heating agent (Hollweg 1982; Athay and Holzer 1982). They are grass-like spiky features seen in chromospheric spectral lines at the solar limb (Zaqarashvili and Erdélyi 2009). These spiky dynamic jets are propelled upwards at speeds of about 20–25 km s$^{-1}$ from photosphere into the magnetized low atmosphere of the sun. Their diameter varies from spicule to spicule having the values from 400 km to 1500 km. The mean length of classical spicules varies from 5000 km to 9000 km, and the typical lifetime of them is 5–15 min. The typical electron density at heights where the spicules are observed is $3.5 \times 10^{10} - 2 \times 10^{11}$ cm$^{-3}$, and their temperatures are estimated $\sim 5000$–$8000$ K (Beckers 1968). Kukhianidze et al. (2006), Zaqarashvili et al. (2007) by analyzing the height series of Hα spectra in solar limb spicules observed their transverse oscillations. The period of them estimated 20–55 and 75–110 s. They concluded that these oscillations can be caused by propagating kink waves in spicules. De Pontieu et al. (2007) based on Hinode observations concluded that the most expected periods of transverse oscillations lay between 100–500 s, which interpreted as signatures of Alfvén waves. Okamoto and De Pontieu (2011) used Hinode/SOT observations of spicules, and concluded that upward propagating, downward propagating and standing waves occurred at the rates of about 60 %, 20 % and 20 %, respectively. Furthermore, they found that upward propagating waves dominate at lower latitudes, and the medians of amplitude, period, and velocity amplitude are 55 km, 45 s, and 7.4 km s$^{-1}$, respectively. More recently Ebadi et al. (2012) made time-slice images of spicules, which were observed by Hinode/SOT. They concluded that the energy flux stored in spicule axis oscillations is of the order of coronal energy loss in quiet sun. These results motivated us to study the phase mixing of upward propagating Alfvén waves in a stratified atmosphere. To do so, the Sect. 2 gives the basic equations and the theoretical model. In Sect. 3 the numerical results are presented and discussed, and a brief summary is followed in Sect. 4.

2 Theoretical modeling

In the present work we keep the effects of stratification due to gravity in 2D x–z plane. The phase mixing and dissipation of propagating Alfvén waves in a region with non-uniform Alfvén velocity is studied. MHD equations governing the plasma dynamics are as follows:

$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \mathbf{v}) = 0, \quad (1)$$

$$\rho \frac{\partial \mathbf{v}}{\partial t} + \rho (\mathbf{v} \cdot \nabla) \mathbf{v} = - \nabla p + \rho \mathbf{g} + \frac{1}{\mu} (\nabla \times \mathbf{B}) \times \mathbf{B} + \rho \nu \nabla^2 \mathbf{v}, \quad \text{(2)}$$

$$\frac{\partial \mathbf{B}}{\partial t} = \nabla \times (\mathbf{v} \times \mathbf{B}) + \eta \nabla^2 \mathbf{B}, \quad \text{(3)}$$

$$p = \frac{\rho RT}{\mu}, \quad \text{(4)}$$

$$\nabla \cdot \mathbf{v} = 0, \quad \text{(5)}$$

where $\nu$ and $\eta$ are constant viscosity and resistivity coefficients, and other quantities have the usual meaning. In particular, typical values for $\eta$ in the solar chromosphere and corona are $8 \times 10^9 T^{-3/2}$ and $10^9 T^{-3/2}$ m$^2$ s$^{-1}$, respectively. The value of $\rho \nu$ for a fully ionized $H$ plasma is $2.2 \times 10^{-17} T^{5/2}$ kg m$^{-1}$ s$^{-1}$ (Priest 1982). We assume that the spicules are highly dynamic with speeds that are significant fractions of the Alfvén speed. The perturbations are assumed independent of $y$, with a polarization in $\hat{y}$ direction, i.e.:

$$\mathbf{v} = v_0 \hat{k} + v_y(x, z, t) \hat{j}, \quad \text{(6)}$$

$$\mathbf{B} = B_0 \hat{k} + b_y(x, z, t) \hat{j}. \quad \text{(7)}$$

Therefore, the pressure gradient is balanced by the gravity force, which is assumed to be $\mathbf{g} = g \hat{k}$ via this equation:

$$- \nabla p_0 + \rho \mathbf{g} = 0, \quad \text{(7)}$$

and the pressure in an equilibrium state is:

$$p_0 = p_0(x) e^{-z/H}, \quad \text{(8)}$$

The density profile is in the form of:

$$\rho_0 = \rho_0(x) e^{-z/H}, \quad \text{(9)}$$
with

\[ H = \frac{RT}{\mu g}, \]  

(10)

where \( H \) is the pressure scale height. The linearized dimensionless MHD equations with these assumptions are:

\[ \frac{\partial v_y}{\partial t} + v_0 \frac{\partial v_y}{\partial z} = V_A^2(x, z) \frac{\partial b_y}{\partial z} + \nu \nabla^2 v_y, \]  

(11)

and

\[ \frac{\partial b_y}{\partial t} + v_0 \frac{\partial b_y}{\partial z} = \frac{\partial v_y}{\partial z} + \eta \nabla^2 b_y, \]  

(12)

where the velocities, the magnetic field, time and space coordinates are normalized to \( V_{A0} = B_0/\sqrt{\mu \rho_0} \) (with \( \rho_0 \) as the plasma density at \( z = 0 \)), \( B_0 \), \( t_A \) (the period of Alfvén waves), \( a \) (spicule radios), respectively. Also the resistivity and viscosity coefficients are normalized to \( a^2/\tau \). The second terms in the left hand side of Eqs. (11), and (12) present the effect of steady flows. \( V_A(x, z) \) is the Alfvén velocity, which for a phase mixed and stratified atmosphere due to gravity is assumed to be (De Moortel et al. 1999; Karami and Ebrahimi 2009):

\[ V_A(x, z) = V_{A0} e^{z/2H} [2 + \tanh[a(x - 1)]], \]  

(13)

where parameter \( a \) controls the size of inhomogeneity across the magnetic field. The set of Eqs. (11), and (12) should be solved under these initial and boundary conditions:

\[ v_y(x, z, t = 0) = V_{A0} \exp \left[ -\frac{1}{2} \left( \frac{x - 1}{d} \right)^2 \right] \sin(kz) e^{z/4H}, \]  

(14)

\[ b_y(x, z, t = 0) = A \sin(\pi x) \sin(\pi z), \]

where \( d \) is the width of the initial packet and \( A = 10^{-7} \). Figure 1 is the plot of initial wave packet given by Eq. (14) for \( d = 0.3a \) (\( a \) is the spicule radius). The parameter \( k \) is chosen in such a way to have upward propagating Alfvén wave.

\[ v_y(x = 0, z, t) = v_y(x = 2, z, t) = 0, \]

\[ b_y(x = 0, z, t) = b_y(x = 2, z, t) = 0, \]

\[ \left. \frac{\partial v_y}{\partial t} \right|_{t = 0} = 0. \]  

3 Numerical results and discussion

To solve the coupled Eqs. (11), and (12) numerically, the finite difference and the Fourth-order Runge-Kutta methods are used to take the space and time derivatives, respectively. We set the number of mesh grid points as \( 256 \times 256 \). In addition, the time step is chosen as \( 0.002 \) (time is normalized to the Alfvén time, \( t_A \)), and the system length in the \( x \) and \( z \) dimensions (simulation box sizes) are set to be \( 2000 \) km and \( 6000 \) km. The parameters in spicule environment are as follows (Murawski and Zaqarashvili 2010; Ebadi et al. 2012): \( a \) (spicule radios) = 500 km, \( d = 0.3a = 150 \) km (the width of Gaussian packet), \( L = 6000 \) km (Spicule length), \( v_0 = 25 \) km/s, \( B_0 = 10G \), \( n_e = 10^{11} \) cm\(^{-3} \), \( T = 8000 \) K, \( g = 272 \) m s\(^{-2} \), \( R = 8300 \) m\(^2\) s\(^{-1} \) k\(^{-1} \) (universal gas constant), \( V_{A0} = 40 \) km/s, \( k = 4\pi/3 \) (dimensionless wavenumber normalized to \( a \)), \( \nu = 10^3 \) m\(^2\) s\(^{-1} \), \( \eta = 10^3 \) m\(^2\) s\(^{-1} \), \( \mu = 0.6 \), \( H = 500 \) km, \( t_A = 37.5 \) s (the period of Alfvén waves), and \( \alpha = 2 \) (Okamoto and De Pontieu 2011).

Figure 2 shows the perturbed velocity variations with respect to time in \( x = 1000 \) km, \( z = 1000 \) km; \( x = 1000 \) km, \( z = 3000 \) km; and \( x = 1000 \) km, \( z = 5000 \) km respectively. We presented the perturbed magnetic field variations obtained from our numerical analysis in Fig. 3 for \( x = 1000 \) km, \( z = 1000 \) km; \( x = 1000 \) km, \( z = 3000 \) km; and \( x = 1000 \) km, \( z = 5000 \) km respectively. In these plots the perturbed velocity and magnetic field is normalized to \( V_{A0} \) and \( B_0 \) respectively. In each set of plots it is appeared that both the perturbed velocity and magnetic field are damped in the developed stage of phase mixing.

Further, at the first height (1000 km), total amplitude of both velocity and magnetic field oscillations have values near to the initial ones. As height increases, the perturbed velocity amplitude does increase in contrast to the behavior of perturbed magnetic field. Nonetheless, exponentially damping behavior is obvious in both cases. This means that with an increase in height, amplitude of velocity oscillations is expanded due to significant decrease in density, which acts as inertia against oscillations. Similar results are observed by time-distance analysis of Solar spicule oscillations (Ebadi et al. 2012). It is worth to note that the density stratification influence on the magnetic field is negligible, which is in agreement with Solar Optical Telescope observations of Solar spicules (Verth et al. 2011).
Fig. 2 The perturbed velocity variations with respect to time in $x = 1000 \text{ km}$, $z = 1000 \text{ km}$; $x = 1000 \text{ km}$, $z = 3000 \text{ km}$; and $x = 1000 \text{ km}$, $z = 5000 \text{ km}$ respectively from top to bottom are showed.

Fig. 3 The same as in Fig. 2 but for the perturbed magnetic field.

Figures 4, 5 show the 3D plots of the perturbed velocity and magnetic field with respect to $x$, $z$ for $t = 30t_A$, $t = 60t_A$, and $t = 80t_A$. They show that in the presence of stratification due to gravity, the damping takes place in space than in time as an important point of these graphs. It should be emphasized that the damping time scale of the velocity field pattern is longer than the corresponding magnetic field pattern due to the initial conditions. In other words, in spite of standing waves, propagating waves are stable and dissape after some periods due to phase mixing (Heyvaerts and Priest 1983).

For calculating the damping time it is suitable to calculate the total energy (kinetic energy plus magnetic energy) per unit of length in $y$ direction as:

$$E_{tot}(t) = \frac{16\pi}{B_0^2aL} \int_0^2 dx \int_0^6 dz [\rho(x,z)v_y^2(x,z) + b_y^2(x,z)].$$

(17)
Fig. 4 (Color online) The perturbed velocity in $x$–$z$ space is presented. The panels from top to bottom correspond to $t = 30t_A$, $t = 60t_A$, and $t = 80t_A$, respectively.

In Fig. 6 we plot the normalized total energy for three initial wave packet widths, i.e. $d = 0.1a$, $d = 0.3a$, and $d = 0.8a$.

Since the treatments of different profiles of total energy are similar and the damping times are very close to each other, hence, we continue our calculations with $d = 0.3a$, which is more logical compared with tube radius. In Fig. 7 the kinetic energy, magnetic energy, and total energy normalized to the initial total energy are presented respectively from top to bottom. The damping time calculated from total energy profile is 1050 s. Actually, spicules have short lifetimes, and are transient phenomena. We claim that in such circumstances, phase mixing can occur in space rather than in time. This is in agreement with previous works (De Moortel et al. 1999). We can definitely consider it as an uncertain rule of thumb.

The total energy profile is best fitted to the exponential damping function in time, $\exp(-At^B)$, with $A = 10^{-9}$.

$B = 2.85$. Since the spicules are structures with low resistivity and viscosity coefficients, so this is in agreement with Heyvaerts and Priest (1983) work.
Fig. 7 Time variations of normalized kinetic energy, magnetic energy, and total energy for \( d = 0.3a \) are presented from top to bottom respectively.

So far the presented results in this paper are taken with considering a steady flow (\( v_0 = 25 \text{ km/s} \)). Actually, it is of interest to investigate the differences in our numerical analysis outputs if the flow effect is omitted. To do that, we repeated the numerical analysis with zero-steady flow. Interestingly, the relevant results did not show significant difference in respect to the case of non-zero steady flow. A short theoretical scaling may help to clarify this result; comparing two dimensionless terms in Eq. (11): the convective term, which is associated with the steady flow, \( v_0 \frac{\partial v_y}{\partial z} \), and the Lorentz term, \( V_A^2(x, z) \frac{\partial b_y}{\partial z} \), yields:

\[
\left( V_A^2(x, z) \frac{\partial b_y}{\partial z} \right) / \left( v_0 \frac{\partial v_y}{\partial z} \right) \sim \left( V_A^2 e^{z/(2H)} \frac{\tilde{b}_y}{L} \right) / \tilde{v}_y \sim e^{z/H} \frac{\tilde{b}_y}{\tilde{v}_y},
\]

Here \( \tilde{b}_y, \tilde{v}_y \) are the average values of \( b_y, v_y \) normalized to \( B_0 \) and \( V_A \). Also, \( v_0 \) is normalized to \( V_A \) assuming that both have the same order of magnitudes. Moreover, \( V_A \approx V_A e^{z/(2H)} \) has been taken into account. Now with the large values of \( z \), and assuming that \( \tilde{b}_y \) and \( \tilde{v}_y \) have a same order of magnitudes, \( e^{z/H} \frac{\tilde{b}_y}{\tilde{v}_y} \gg 1 \). Meaning that the effect of the Lorentz term which is associated with the stratification effect is more important than the dominated convective term bringing into account the steady flow effects. Thus as long as \( z/H \gg 1 \), one might ignore the steady flow effects here. Similar scaling can be done in Eq. (12) to show that in the presence of stratification the steady flow effects are ignorable.

4 Conclusion

In our simple model, we assume that spicules are small scale structures (relative to coronal loops and other mega structures) with a uniform magnetic field, and a uniform temperature in all heights, and the transition region between chromosphere and corona has been neglected. Density change along spicule axis is considerable, and stratification due to gravity is significant. As a result, the medium is dense in its lower heights, but it becomes rare and rare as height increases. Also, spicules have short lifetimes, and are transient phenomena. We claim that in such circumstances, phase mixing can occur in space rather than in time. This is in agreement with previous studies. In ordinary heights of a spicule, there is enough space to amplitude of a wave to damp, and to transfer its energy to the medium. Furthermore, we observed that the amplitude of perturbed velocity field, magnetic field and total energy decreases with height exponentially. Our numerical analysis show that the main phase of evolution dynamics occurs only in the first one third of the spicule height. Furthermore, we repeated the numerical analysis with zero-steady flow. Interestingly, the relevant results did not show significant difference in respect to the case of non-zero steady flow.

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