Triangle Relation of Dark Matter, EDM and CP Violation in $B^0$ Mixing in a Supersymmetric $Q_6$ Model

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Abstract

We consider a recently proposed supersymmetric model based on the discrete $Q_6$ family group. Because of the family symmetry and spontaneous CP violation the electric dipole moment (EDM), the CP violation in the mixing of the neutral mesons and the dark matter mass $m_{DM}$ are closely related. This triangle relation is controlled by the size of the $\mu$ parameters. Loop effects can give rise to large contributions to the soft mass insertions, and we find that the model allows a large CP violation in the $B^0$ system. Its size is comparable with the recent experimental observations at D0 and CDF, and it could be observed at LHCb in the first years. If the parameter space is constrained by the neutron EDM, and flavor changing neutral currents (FCNC) and CP violations in $K^0$ as well as $B^0$ mixing, the triangle relation yields the following bound on the dark matter candidate: $0.12 \text{ TeV} < m_{DM} < 0.33 \text{ TeV}$, which is directly observable at LHC. We also compute $a_{s(d)}^{s} - a_{s(d)}^{d}$, which is observable at LHCb, where $a_{s(d)}^{s(d)}$ is the semi-leptonic CP asymmetry for the $B_{s(d)}$ system.

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I. INTRODUCTION

Family symmetry is a useful tool \cite{1-7} to suppress FCNCs in supersymmetric (SUSY) extensions of the standard model (SM) \textsuperscript{1}. If it is combined with spontaneous violation of CP in SUSY models, CP violation in these models can be suppressed, too \cite{4,5,7}. However, this theoretical idea may be conflict with the recent measurement of the CP violating dimuon asymmetry $A^b_{sl}$ by the D0 collaboration \cite{9}. Its measured value $A^b_{sl} = -(9.57\pm 2.51 \pm 1.46) \cdot 10^{-3}$ is a factor of 42 larger than the SM prediction $A^b_{sl} = -(2.3^{+0.5}_{-0.6}) \cdot 10^{-4}$ \cite{10}, which has stimulated a number of papers \cite{11,12} dealing with a large CP violation in $B^0$ mixing \textsuperscript{2}. Moreover, the CKM fitter group \cite{14,15} also obtained from a global fit to flavor observables a large value for the dimuon asymmetry; $A^b_{sl} = -(4.2^{+1.9}_{-1.8}) \cdot 10^{-3}$ \cite{15} \textsuperscript{3}. If the size of CP violation in a symmetry-based mechanism to suppress CP violation turns out be of the same order of the SM value above, we may be running into a dilemma between suppressed and large CP violation. In any case, the mechanism has to take care of small CP violation in $K^0$ mixing and at the same time allow large CP violation in $B^0$ mixing. See \cite{15} for a large list of references in which diverse theoretical possibilities for large CP violation in $B^0$ mixing have been proposed.

Recently, two of us \cite{12} considered a supersymmetric extension of the SM based on the discrete $Q_6$ family symmetry \cite{4,7} \textsuperscript{4}. Due to the family symmetry this model contains three pairs of $SU(2)_L$ doublet Higgs supermultiplets. We found that the one-loop effects of the extra Higgs multiplets to the soft mass insertions can generically give rise to large contributions to the soft mass insertions and that the model allows values for $A^b_{sl}$, that touch the 1 $\sigma$-range of the fit result from \cite{15}. In this paper we will continue with our investigation of this model. In this model the size of the $\mu$ parameters play an important role: It enters directly into the above mentioned one-loop corrections to the soft mass insertions and into the EDMs \cite{7}. If the neutralino LSP should be a dark matter candidate, then its mass also depends on the $\mu$ parameters. We are thus particularly interested in the triangle relationship between CP violation in $B^0$ mixing, the EDM and the mass of dark matter candidate.

II. THE MODEL

We start by considering the superpotential

$$ W = Y_{ij}^{uI} Q_i U_j^c H_u^I + Y_{ij}^{dI} Q_i D_j^c H_d^I + \mu^{IJ} H_u^I H_d^J, \quad (1) $$

\textsuperscript{1} For a recent review on family symmetry, see [8] for instance.
\textsuperscript{2} For earlier works see e.g. [13].
\textsuperscript{3} This value is for the New Physics scenario I of [15]. The UTfit group [16] and Lunghi and Soni [17] also reported large CP violating effects in $B^0$ mixing.
\textsuperscript{4} $Q_6$ was considered in past in [18].
With this extended Higgs sector one can break the flavor symmetry spontaneously. Moreover, the scalar potential of the original theory has turned out to have the \( Q \) term is \((H^u H_d^d + H^d H_d^u)\), and no \( H^u H^d \) and no mixing between the \( Q \) doublet and singlet Higgs multiplets are allowed. Therefore, there is an accidental global \( SU(2) \), implying the existence of Nambu-Goldstone modes. In \([19]\) the Higgs sector is extended to include a certain set of SM singlet Higgs multiplets to avoid this problem. With this extended Higgs sector one can break the flavor symmetry \( Q \) and CP invariance spontaneously. Moreover, the scalar potential of the original theory has turned out to have an accidental \( Z_2 \) invariance

\[
h^u_d = \frac{1}{\sqrt{2}} (h_1^{u,d} + h_2^{u,d}) \rightarrow h^u_d, \quad h^u_{-d} = \frac{1}{\sqrt{2}} (h_1^{u,d} - h_2^{u,d}) \rightarrow -h^u_{-d},
\]

where \( h \)'s are scalar components of \( H \)'s. After the singlet sector has been integrated out, we obtain an effective \( \mu \) term

\[
W_{\text{eff}} = \mu^{++} (H_1^u H_1^d + H_3^u H_3^d) + \mu^{++} (H_3^u H_3^d + H_1^u H_1^d)
\]

with \( H_\pm^{u,d} = (H_1^{u,d} \pm H_2^{u,d})/\sqrt{2} \), and the soft-supersymmetry-breaking Lagrangian

\[
\mathcal{L}_{\text{soft}} = m_{H_+}^2 \left( |h_+^u|^2 + |h_+^d|^2 \right) + m_{H_3}^2 |h_{3}^u|^2 + m_{H_3}^2 |h_{3}^d|^2 \left( |h_+^u|^2 + |h_{-d}^u|^2 \right) + m_{H_3}^2 |h_{3}^d|^2
\]

\[
+ \left[ B^{++} (h_+^u h_3^d + h_{-d}^u h_3^d) + B^{+-} h_+^u h_{3}^d + B^{3+} h_3^u h_3^d + h.c. \right].
\]

\( ^5 \) More details of the model can be found in \([2, 4]\).
(The $A$ terms are suppressed.) The parameters $\mu$’s and $B$’s are complex, they originate from the complex VEVs of the SM singlet Higgs fields of the original theory [19]. But because of the CP invariance of the original theory the Yukawa matrices and soft scalar masses are real. So, the effective superpotential (14) and the effective soft-supersymmetry-breaking Lagrangian (15) break $Q_6$ and CP softly. However, thanks to (3), the VEVs of the form

$$< h_{-u,d}^0 > = 0 \, , \, < h_{+u,d}^0 > = \frac{v_{u,d}^0}{\sqrt{2}} \exp i \theta_{u,d}^+ \, , \, < h_{3u,d}^0 > = \frac{v_3^{u,d}}{\sqrt{2}} \exp i \theta_3^{u,d}$$

(6)
can be realized, where the $SU(2)$ components of the Higgs fields are defined as

$$h_I^u = (h_{I+}^u, h_{I0}^u) \, , \, h_I^d = (h_{I0}^d, h_{I-}^d) \, .$$

(7)

To proceed with our discussion we make a phase rotation of the Higgs superfields so that their VEVs become real: $\tilde{H}_{u,d}^u = H_{u,d}^u e^{-i \theta_{u,d}^+}, \tilde{H}_3^{u,d} = H_3^{u,d} e^{-i \theta_3^{u,d}}$. Then we define

$$\left( \Phi_L^{u,d} \right)^T = \left( \begin{array}{c} \cos \gamma^{u,d} \sin \gamma^{u,d} 0 \\ -\sin \gamma^{u,d} \cos \gamma^{u,d} 0 \\ 0 0 1 \end{array} \right) \cdot \left( \begin{array}{c} \tilde{H}_3^{u,d} \\ \tilde{H}_+^{u,d} \\ H_-^{u,d} \end{array} \right),$$

(8)

where

$$\cos \gamma^{u,d} = v_3^{u,d}/v_{u,d} \, , \, \sin \gamma^{u,d} = v_+^{u,d}/v_{u,d} \, , \, v_{u,d} = \sqrt{(v_3^u)^2 + (v_+^u)^2} \, .$$

(9)

We further define the components of the $SU(2)$ doublet Higgs multiplets as

$$\Phi_I^u = \left( \begin{array}{c} \Phi_{I+}^u \\ \Phi_{I0}^u \end{array} \right) \, , \, \Phi_I^d = \left( \begin{array}{c} \Phi_{I0}^d \\ \Phi_{I-}^d \end{array} \right) \, \, , \, I = L, H, -.$$

(10)

The light and heavy MSSM-like Higgs scalars are then given by

$$H = (v + h - iX)/\sqrt{2} = (\phi_{L0}^d)^* \cos \beta + (\phi_{L0}^u)^* \sin \beta \, ,$$

$$H = (v + h + iX)/\sqrt{2} = -(\phi_{L0}^d)^* \sin \beta + (\phi_{L0}^u)^* \cos \beta \, ,$$

$$G^+ = - (\phi_{L-}^d)^* \cos \beta + (\phi_{L-}^u)^* \sin \beta \, , \, H^+ = (\phi_{L+}^d)^* \sin \beta + (\phi_{L+}^u)^* \cos \beta \, ,$$

(11)

where $X$ and $G^+$ are the Nambu-Goldstone fields, $\phi$’s are scalar components of $\Phi$’s of (14), and $v = \sqrt{v_u^2 + v_d^2} \ (\approx 246 \text{ GeV})$ and $\tan \beta = v_u/v_d$.

III. THE YUKAWA SECTOR IN THE QUARK MASS EIGENSTATES

The Yukawa sector in the quark mass eigenstates is needed to compute EDMs mediated by the Yukawa couplings. The quark mass matrices $m^u$ and $m^d$ can be read off from the
superpotential (1) along with (2) and (6). Then using the phase matrices defined below

\[ R_L = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & 1 & 0 \\ -1 & 1 & 0 \\ 0 & 0 & \sqrt{2} \end{pmatrix}, \quad R_R = \frac{1}{\sqrt{2}} \begin{pmatrix} -1 & -1 & 0 \\ -1 & 1 & 0 \\ 0 & 0 & \sqrt{2} \end{pmatrix}, \quad (12) \]

\[ P_L^q = \text{diag.}(1, \ exp i2\Delta\theta^q, \ exp i\Delta\theta^q), \]
\[ P_R^q = (-1) \exp i\theta_3^u \text{ diag.}(\exp i2\Delta\theta^u, 1, \ exp i\Delta\theta^u), \]
\[ P_R^d = \exp i\theta_3^d \text{ diag.}(\exp i2\Delta\theta^d, 1, \ exp i\Delta\theta^d), \quad (13) \]

\[ \Delta\theta^q = \theta_3^q - \theta_+^q, \quad (q = u, d) \]

we can bring \( m^q \) into a real form \( \hat{m}^q = P_L^q R_L^T m^q R_R^q P_R^q \). The mass matrix \( \hat{m}^u \) can then be diagonalized as \( O_L^{uT} \hat{m}^u O_R^u = \text{diag.}(m_u, m_c, m_t) \), and similarly for \( m^d \), where \( O_{L,R} \) are orthogonal matrices. So, the mass eigenstates \( u'_{L,R} = (u'_L, c'_L, t'_L) \) etc. can be obtained from \( q_L = U_L^q q_L', q_R = U_R^q q_R' \), where \( U_{L,R}^q = R_{L,R}(q)O_{L,R}(q) \). Therefore, the CKM matrix \( V_{\text{CKM}} \) is given by

\[ V_{\text{CKM}} = O_L^{uT} P_L^q P_L^d O_L^d = O_L^{uT} P_R^q O_L^d, \quad (14) \]

where

\[ P_q = \text{diag.}(1, \exp(i2\theta_q), \exp(i\theta_q)), \quad \theta_q = \theta_+^q - \theta_+^d - \theta_3^u + \theta_3^d. \quad (15) \]

There are nine independent theory parameters, which describe the CKM parameters and the quark masses: \( Y_{a_0 v_3^a}^u, Y_{c_0 v_3^c}^u, Y_{b_0 v_3^b}^u, Y_{a_0 v_3^a}^d, Y_{c_0 v_3^c}^d, Y_{b_0 v_3^b}^d \) and \( \theta_q \). The set of the theory parameters is thus over-constrained. Therefore, there is not much freedom in the parameter space, and so it is sufficient to consider a single point in the space of the theory parameters of this sector:

\[
\begin{align*}
Y_{a_0 v_3^a}^u &= 1.409 \ m_t, & Y_{c_0 v_3^c}^u &= 2.135 \times 10^{-4} \ m_t, & Y_{b_0 v_3^b}^u &= 0.0847 \ m_t, & Y_{a_0 v_3^a}^d &= 0.0879 \ m_t, \\
Y_{a_0 v_3^a}^d &= 1.258 \ m_b, & Y_{c_0 v_3^c}^d &= -6.037 \times 10^{-3} \ m_b, & Y_{b_0 v_3^b}^d &= 0.0495 \ m_b, & Y_{a_0 v_3^a}^{d'} &= 0.6447 \ m_b, \\
\theta_q &= -0.7125. \quad (16)
\end{align*}
\]

With these parameter values we obtain \( V_{\text{CKM}} \)

\[ m_u/m_t = 0.609 \times 10^{-5}, \quad m_c/m_t = 3.73 \times 10^{-3}, \quad m_d/m_b = 0.958 \times 10^{-3}, \quad (17) \]

\[ m_s/m_b = 1.69 \times 10^{-2}, \quad |V_{\text{CKM}}| = \begin{pmatrix}
0.9740 & 0.2266 & 0.00361 \\
0.2264 & 0.9731 & 0.0414 \\
0.00858 & 0.0407 & 0.9991
\end{pmatrix}, \quad (18) \]

\[ |V_{\text{us}}| = 0.211, \quad \sin 2\beta(\phi_1) = 0.695, \quad \bar{\rho} = 0.152, \quad \bar{\eta} = 0.343. \quad (19) \]

The mass ratio (18) is defined at \( M_Z \) and consistent with the recent updates of \( [21] \), and the CKM parameters above agree with those of Particle Data Group \( [22] \) and CKM fitter groups \( [14, 16] \). (See \( [23] \) for the predictions of the model in the lepton sector.)
In the basis of the fermion mass eigenstates the Higgs couplings have the following form:

\[
\mathcal{L}_Y = - \sum_{i=L,H,-} Y_{ij}^{u0I} (\phi_i^{u0})^* \bar{u}_{iL}' u_{jR}' + \sum_{i=L,H,-} Y_{ij}^{d-1} (\phi_i^{d-})^* \bar{d}_{iL}' d_{jR}'
- \sum_{i=L,H,-} Y_{ij}^{d0I} (\phi_i^{d0})^* \bar{d}_{iL}' d_{jR}' + \sum_{i=L,H,-} Y_{ij}^{u+I} (\phi_i^{u+})^* \bar{u}_{iL}' u_{jR}' + h.c.,
\]

(20)

where the Higgs fields are defined in (10), the Yukawa matrices \( Y^{u1} \) etc. are given in (2), and

\[
Y^{d0L} = O_{L}^{uT} R_{L}^{T} Y^{dL} R_{R} O_{R}^{d} = \sqrt{2}\text{diag.}(m_d, m_s, m_b)/v \cos \beta,
\]

\[
Y^{d0H} = O_{L}^{uT} R_{L}^{T} Y^{dH} R_{R} O_{R}^{d} , \quad Y^{d0} = \frac{1}{\sqrt{2}}O_{L}^{uT} R_{L}^{T} \left( Y^{d1} - Y^{d2} \right) R_{R} O_{R}^{d} ,
\]

\[
Y^{d-L} = O_{L}^{uT} P q R_{L}^{T} Y^{dL} R_{R} O_{R}^{d} , \quad Y^{d-H} = O_{L}^{uT} P q R_{L}^{T} Y^{dH} R_{R} O_{R}^{d} ,
\]

\[
Y^{d-} = \frac{1}{\sqrt{2}}O_{L}^{uT} P q R_{L}^{T} \left( Y^{d1} - Y^{d2} \right) R_{R} O_{R}^{d},
\]

\[
Y^{dL} = \left[ \frac{1}{\sqrt{2}} \sin \gamma^d (Y^{d1} + Y^{d2}) + \cos \gamma^d Y^{d3} \right],
\]

\[
Y^{dH} = \left[ \frac{1}{\sqrt{2}} \cos \gamma^d (Y^{d1} + Y^{d2}) - \sin \gamma^d Y^{d3} \right],
\]

(22)

and similarly for \( Y^{u*} \)'s, where the matrices other than the Yukawa matrices are defined in (2), (4) and (5). One finds that \( Y^{d0L} \) and \( Y^{d0H} \) are real and that the only phase appearing in \( Y^{d-L} \) and \( Y^{d-H} \) is \( \theta_q \) given in (5), which is the same phase entering into \( V_{CKM} \).

**IV. SOFT MASS INSERTIONS**

The \( A \) terms and soft scalar mass terms obey the \( Q_6 \) family symmetry in the effective theory. Therefore, the soft mass matrices have the form

\[
\tilde{m}_{QLL}^2 = m_a^2 \text{diag.} (a_L^a, a_L^a, b_L^a), \quad (a = q, l)
\]

\[
\tilde{m}_{RR}^2 = m_a^2 \text{diag.} (a_R^a, a_R^a, b_R^a), \quad (a = u, d, e)
\]

\[
\left( \tilde{m}_{LLR}^2 \right)_{ij} = A_{ij}^a (m^a)_{ij}, \quad (a = u, d, e)
\]

(23)

where \( m_a \) denote the average of the squark and slepton masses, respectively, \( (a_L^a, b_L^a) \) are dimensionless free real parameters, \( A_{ij}^a \) are real free parameters of dimension one, and \( m^a \) are the respective fermion mass matrices. According to \( \{24, 25\} \) we define the tree-level supersymmetry-breaking soft mass insertions as

\[
\delta_{LL}^{a0} = U_{L}^{aJ} \tilde{m}_{QLL}^{2} U_{L}^{aJ}/m_a^2,
\]

\[
\delta_{LR}^{a0} = U_{L}^{aJ} \left( \tilde{m}_{LLR}^2 - \mu^J < h_{J}^{0} > Y^{uJ} \right) U_{R}^{aJ}/m_a^2,
\]

\[
\delta_{LR}^{d0} = U_{L}^{dJ} \left( \tilde{m}_{LLR}^2 + \mu^J < h_{J}^{0} > Y^{dJ} \right) U_{R}^{dJ}/m_a^2,
\]

(24)

(25)

(26)
in the super CKM basis, where $U$’s are unitary matrices that diagonalize the quark mass matrices, and $h$’s are the neutral Higgs fields defined in (7). (We restrict ourselves to the quark sector.) The $\mu$ term and $A$ term contributions to $\delta_{LR}^{ii(0d)}$ are the first and second terms in (25) and (26), respectively. Note that because of the CP invariance, the A term contributions are real so that only the $\mu$ term contributes to EDMs.

For the input parameters given in (16) we obtain, for the down quark sector for instance,

\[
\begin{align*}
(\delta_{12}^{d})_{LL} &= (\delta_{21}^{d})^{*}_{LL} \simeq -2.6 \times 10^{-4} \Delta a_{L}^{q} , \quad (\delta_{13}^{d})_{LL} = (\delta_{31}^{d})^{*}_{LL} \simeq -8.7 \times 10^{-3} \Delta a_{L}^{q} , \\
(\delta_{23}^{d})_{LL} &= (\delta_{32}^{d})^{*}_{LL} \simeq -3.0 \times 10^{-2} \Delta a_{L}^{d} , \\
(\delta_{12}^{d})_{RR} &= (\delta_{21}^{d})^{*}_{RR} \simeq 5.0 \times 10^{-2} \Delta a_{R}^{d} , \quad (\delta_{13}^{d})_{RR} = (\delta_{31}^{d})^{*}_{RR} \simeq -0.10 \Delta a_{R}^{d} , \\
(\delta_{23}^{d})_{RR} &= (\delta_{32}^{d})^{*}_{RR} \simeq 0.39 \Delta a_{R}^{d} ,
\end{align*}
\]  

(27)

where $\Delta a_{L}^{q} = a_{L}^{q} - b_{L}^{q}$, $\Delta a_{R}^{d} = a_{R}^{d} - b_{R}^{d}$. The A term contributions to the left-right insertions are

\[
\begin{align*}
(\delta_{12}^{d})_{LR}(A) &\simeq 1.9(\tilde{A}_{1}^{d} - \tilde{A}_{2}^{d}) \times 10^{-5} , \quad (\delta_{21}^{d})_{LR}(A) \simeq (-2.2\tilde{A}_{1}^{d} + 1.7\tilde{A}_{2}^{d}) \times 10^{-5} , \\
(\delta_{13}^{d})_{LR}(A) &\simeq (1.0\tilde{A}_{1}^{d} + 4.0\tilde{A}_{2}^{d}) \times 10^{-5} , \quad (\delta_{31}^{d})_{LR}(A) \simeq 5.8\tilde{A}_{2}^{d} \times 10^{-4} , \\
(\delta_{23}^{d})_{LR}(A) &\simeq 1.4\tilde{A}_{2}^{d} \times 10^{-4} , \quad (\delta_{32}^{d})_{LR}(A) \simeq -2.3\tilde{A}_{2}^{d} \times 10^{-2} , \\
(\delta_{12}^{d})_{LR}(A) &\simeq 1.9(\tilde{A}_{1}^{d} - \tilde{A}_{2}^{d}) \times 10^{-5} , \quad (\delta_{21}^{d})_{LR}(A) \simeq (-2.2\tilde{A}_{1}^{d} + 1.7\tilde{A}_{2}^{d}) \times 10^{-5} , \\
(\delta_{13}^{d})_{LR}(A) &\simeq (1.0\tilde{A}_{1}^{d} + 4.0\tilde{A}_{2}^{d}) \times 10^{-5} , \quad (\delta_{31}^{d})_{LR}(A) \simeq 5.8\tilde{A}_{2}^{d} \times 10^{-4} , \\
(\delta_{23}^{d})_{LR}(A) &\simeq 1.4\tilde{A}_{2}^{d} \times 10^{-4} , \quad (\delta_{32}^{d})_{LR}(A) \simeq -2.3\tilde{A}_{2}^{d} \times 10^{-2} ,
\end{align*}
\]  

(28)

where $\tilde{A}_{i}^{d} = [A_{i}^{d}(\tilde{A}_{i}^{d})/m_{d}][0.5 \text{ TeV}/m_{d}]$, and the real parameters $A_{i}^{d}$ and $A_{i}^{d}$ represent four independent elements of $A_{ij}^{d}$ given in (23). The $\mu$ term contributions can be obtained from the second terms of (25) and (26).

The mass insertions above are the tree-level ones. In [12] it has been shown that the one-loop corrections to them, especially to $(\delta_{ij}^{d0})_{LL}$, can be large in the presence of more than one pair of the Higgs $SU(2)_{L}$ doublet. Moreover, it has been found that in the present model the one-loop corrections are needed to obtain a large CP violation in $B^{0}$ mixing that are comparable with the observations at Tevatron. These one-loop corrections depend strongly on the parameters in the Higgs sector, and we use the formula given in [12] to do the numerical analysis in the last section.

V. DARK MATTER, EDM AND $B^{0}$ MIXING

A. LSP and Dark matter

We assume that the LSP is a neutralino and is a dark matter candidate in this model. Because of $Z_{2}$ defined in (3) the higgsinos can also be grouped into the $Z_{2}$ even and odd
sectors. The higgsinos in the $Z_2$ odd sector have no mixing with the gauginos. If therefore the LSP belongs to the $Z_2$ odd sector, the LSP is a pure higgsino state with the mass $\mu^{++}$. For this LSP to be a dark matter candidate, $\mu^{++}$ has to be larger than $O(1)$ TeV and at the same time smaller than the other $\mu$’s and gaugino masses. This parameter region can not satisfy the EDM constraint without an extreme fine tuning because we need relatively small $\mu$’s to satisfy the EDM constraint in the present model. So, we may assume that the LSP belongs to the $Z_2$ even sector. The mass matrix of the neutralinos in $Z_2$ even sector is

$$M^F_{\text{Neven}} = \begin{pmatrix}
M_1 & 0 & s_W s_\beta M_Z & -s_W c_\beta M_Z & 0 & 0 \\
0 & M_2 & -c_W s_\beta M_Z & c_W c_\beta M_Z & 0 & 0 \\
s_W s_\beta M_Z & -c_W s_\beta M_Z & 0 & -\mu_L & 0 & -\mu_{LH} \\
-s_W c_\beta M_Z & c_W c_\beta M_Z & -\mu_L & 0 & -\mu_{HL} & 0 \\
0 & 0 & 0 & -\mu_{HL} & 0 & -\mu_H \\
0 & 0 & -\mu_{LH} & 0 & -\mu_H & 0
\end{pmatrix},$$ (29)

where $c_\beta = \cos \beta$, $c_W = \cos \theta_W$, and similarly for $s_\beta$ and $s_W$ ($\theta_W$ is the Weinberg angle). Because of the EDM constraint we expect the mass of the LSP is relatively light $O(\text{few100})$ GeV. Therefore, the LSP has to be a mixture of the higgsinos and the gauginos to obtain a desirable relic density $\Omega h^2 \simeq 0.11$. So, we require that the gaugino fraction of the LSP is in a range between 65% and 95% (see for instance [26]), and assume that if this is satisfied, the neutralino LSP can be a dark matter candidate in the present model.

B. EDM

Our concern here is the neutron EDM, $d_n$, because the electron EDM in this model is extremely suppressed. There are two sources for $d_n$: the Yukawa sector because of the multi Higgs structure and the SUSY breaking sector. Here we simply assume that $d_n$ can be obtained from $d_n = \frac{1}{3}(4d_d - d_u)$, where $d_{u(d)}$ is the EDM of the u(d) quark. The experimental upper bound is given by

$$d_n/e \lesssim 6.3 \cdot 10^{-26} \text{ cm}.$$ (30)

1. Yukawa contribution

We start in the Yukawa sector. The one-loop diagrams can be divided into: the photon is attached to a quark or a charged Higgs, and the internal Higgs is neutral or charged. The contribution to $d_n/e$ with the neutral Higgs boson exchange (satisfying the constraint (31))

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6 Since $Z_2$ is not an exact symmetry of the theory, the even and odd states will mix with each other in higher orders in perturbation theory.

7 See for instance [27].
is less than $O(10^{-31})$ cm as was previously found in [7]. We have computed the contribution with the charged Higgs boson exchange and found that it is slightly smaller than the upper bound (30). The result indeed depends on the mass of the heavy Higgs bosons. However, as we will argue in the last section, the heavy Higgs masses can not be freely increased in this model. Therefore, this model predicts $d_n/e$ which is close to the upper bound (30).

2. SUSY breaking contribution

The second source is the SUSY breaking sector. To obtain $d_n$ we use the approximate result of [29] which takes into account only the gluino contribution

$$
\frac{d_d}{e} = -\frac{2\alpha_s}{9\pi} \text{Im}(\delta_{11}^{a0})_{LR}, \quad \frac{d_u}{e} = \frac{4\alpha_s}{9\pi} \text{Im}(\delta_{11}^{a0})_{LR},
$$

(31)

where we have assumed that $m_\tilde{g} = m_\tilde{\tilde{u}} = m_\tilde{\tilde{d}} = m_\tilde{\tilde{q}}$, and $\xi \simeq 0.12$ is the QCD correction [28, 30]. Since the $A$'s are real, only the $\mu$ terms contribute to $\text{Im}(\delta_{11}^{a0})_{LR}$, and therefore,

$$
\text{Im}(\delta_{11}^{a0})_{LR} = \frac{1}{\tan \beta} \text{Im}(\mu_L)m_u + \frac{v \cos \beta}{\sqrt{2}} \text{Im}(\mu_{HL}) \ Y_{11}^{u0H} / m_\tilde{u}^2,
$$

(32)

$$
\text{Im}(\delta_{11}^{a0})_{LR} = \tan \beta \text{Im}(\mu_L)m_d + \frac{v \sin \beta}{\sqrt{2}} \text{Im}(\mu_{HL}) \ Y_{11}^{d0H} / m_\tilde{d}^2,
$$

(33)

where we have used (25) and (26), and $Y_{11}^{u0H}$ and $Y_{11}^{d0H}$ are defined in (21). In the last section the equations (32) and (33) will be used to relate the dark matter mass $m_{DM}$, the neutron EDM and the CP violation in $B^0$ mixing.

C. $B^0$ mixing

The tree-level contributions to the $B^0$ mixing coming from the heavy neutral Higgs boson exchange in this model are small if

$$
\cos \beta M_H \gtrsim 1.2 \text{ TeV},
$$

(34)

is satisfied [6, 7], where $M_H^2$ is the $(\varphi^d_H - \varphi^d_H)$ element of the inverse of the mass squared matrix of the neutral Higgs bosons in $Z_2$ even sector ($\varphi^d_H$ is the scalar component of $\Phi^d_H$ given in (10)). In the following discussion we assume this, so that the only relevant contribution comes from the SUSY breaking sector. Therefore, the total matrix element $M_{12}^q$ in the neutral meson mixing can be written as $M_{12}^q = M_{12}^{SM,q} + M_{12}^{SUSY,q}$, where $M_{12}^{SM,q}$ and $M_{12}^{SUSY,q}$ are the SM contribution and the SUSY contribution, respectively. We take into account only the dominant contribution (gluino exchange) for $M_{12}^{SUSY,q}$ given in [29]. (See e.g. [31] for a more refined calculation)

We follow [10] to parameterize new physics effects as

$$
M_{12}^{SM,q} + M_{12}^{SUSY,q} = M_{12}^{SM,q} \cdot \Delta_q,
$$

(35)
and consider $\Delta M_q$ and the flavor specific CP-asymmetry $a_{sl}^q$ in terms of the complex number $\Delta_q = |\Delta_q|e^{i\phi_q^\Delta}$, where $q = d, s$, and

$$
\Delta M_q = 2|M_{12}^{SM,q}| \cdot |\Delta_q|,
$$

$$
a_{sl}^q = \left| \frac{\Gamma_q^{12}}{|M_{12}^{SM,q}|} \right| \sin \phi_q \, \phi_q = \phi_q^{SM} + \phi_q^\Delta.
$$

(36)

The SM values are given e.g. in [10], in which the results of [32–36] are used:

$$
2M_{12}^{SM,d} = 0.56(1 \pm 0.45) \exp(i0.77) \text{ ps}^{-1} ,
$$

$$
2M_{12}^{SM,s} = 20.1(1 \pm 0.40) \exp(-i0.035) \text{ ps}^{-1} ,
$$

$$
\phi_d^{SM} = (-0.091^{+0.026}_{-0.038}) \text{ rad} , \quad \phi_s^{SM} = (4.2 \pm 1.4) \cdot 10^{-3} \text{ rad} ,
$$

where the errors are dominated by the uncertainty in the decay constants and bag parameters $^8$. We use the central values of (37) for our calculations, while requiring the (conservative) constraints

$$
0.6 < \frac{\Delta M_{db}}{\Delta M_{d,s}} < 1.4 , \quad 2 \frac{|M_{12}^{SU3,Y}|}{\Delta M_{K}} < 2 , \quad \frac{\text{Im}M_{12}^{SU3,Y}|}{\sqrt{2} \Delta M_{K} |\lambda_u|^2} < \epsilon_K = 2.2 \cdot 10^{-3} ,
$$

(38)

where $\lambda_u = (V_{CKM})_{us}^*(V_{CKM})_{ud}$.

The same sign dimuon asymmetry $A_{sl}^b$ measured at D0 [9] is a linear combination of the semileptonic CP-asymmetries in the $B_d$ and in the $B_s$ system:

$$
A_{sl}^b = (0.494 \pm 0.043) \cdot a_{sl}^s + (0.506 \pm 0.043) \cdot a_{sl}^d.
$$

(39)

The SM value for $A_{sl}^b$ is given by $A_{sl}^b = -2.3^{+0.5}_{-0.6} \cdot 10^{-4}$ [10], while the fit result yields [15]

$$
A_{sl}^b = -(4.2^{+1.9}_{-1.8}) \cdot 10^{-3} .
$$

(40)

VI. RESULT AND CONCLUSION

Most of the free parameters belong to the Higgs sector and the SUSY breaking sector. The parameter space is so large that it will be beyond the scope of the present paper to analyze the complete parameter space. Instead, we first look for a benchmark point in the parameter space that satisfies all the requirements (30), (34), (38) and (40). Then we consider neighbor points and look for a border beyond which the constraints are no longer simultaneously satisfied. The border is extended by a certain amount and the parameter space to be considered is defined as such that is surrounded by the extended border.

$^8$ Note that the values for $M_{12}^{SM,q}$ quoted above are those in the standard parameterization of the CKM matrix [22] and that the CKM matrix obtained from (14) is not in the standard parameterization. Therefore, we have to express the supersymmetric contribution $M_{12}^{SU3,Y}$ in the standard parameterization of the CKM matrix before actual calculations.
Note that a larger $\tan \beta$ means a smaller $\cos \beta$ which requires a finer fine tuning in the Higgs sector in order to satisfy (34). $\tan \beta = 10$ for instance would require $M_H \gtrsim 12$ TeV. In the following analysis we consider a benchmark value $\cos \beta = 0.3$ ($\tan \beta \simeq 3.18$), which implies $M_H \gtrsim 4$ TeV. Further, $\Delta q_{L,R}^{q,d}$ in (27) are $O(1)$ free parameters. We assume that $|\Delta q_{L,R}^{q,d}| \lesssim 15$.

We start with the dark matter mass $m_{DM}$ (the mass of the neutralino LSP). It is the smallest eigenvalue of (29) and depends on the gaugino masses and $\mu$ parameters. The $\mu$ parameters directly enter into EDM (see (32) and (33)), while the tree-level mass insertions $(\delta_{ij}^0)_{RR,LL}$ given in (27) do not depend on the $\mu$ parameters. However, their one-loop corrections do depend on them [12]. So, the dark matter mass $m_{DM}$ in the present model is constrained by EDM and by the mixing of the neutral meson systems. We find that $m_{DM}$ is indeed bounded above and below:

\[ 0.12 \ [TeV] \lesssim m_{DM} \lesssim 0.33 \ [TeV] , \tag{41} \]

where we have required (38) and (40) with $\cos \beta M_H \simeq 1.2$ TeV and used $m_\tilde{g} = m_\tilde{a} = m_\tilde{d} = m_\tilde{q} = 0.5$ TeV. The upper bound becomes larger if the size of the $\mu$ parameters increases. However, the size of the second term in the rhs of (26), in particular for $(\delta_{32}^0)_{LR}$, increases, too. The upper bound given in (41) corresponds to $|((\delta_{32}^0)_{LR})| \sim O(10^{-2})$ which is about the upper limit to satisfy the constraint from $b \to s\gamma$ [29]. Similarly, if we increase $\cos \beta M_H$, the one-loop effect becomes larger because of a larger SUSY breaking in the extra Higgs sector, and consequently (38) will be violated. To reduce the one-loop effect, we have to increase the size of the $\mu$ parameters to reduce the SUSY breaking. But this was not allowed because of the $b \to s\gamma$ constraint. Therefore, (41) should be regarded as the area of $m_{DM}$ of the present model. The phenomenological feature of the dark matter of the present model is basically the same as the one of the MSSM. Therefore, it could be observed in various future experiments [37].

Next we consider the extra phases $\phi_s$ and $\phi_d$ defined in (36), which are shown in Fig. 1. Also shown are the fit results of the CKMfitter group (purple) [15] and the UTfit group (blue) [16]. As we see from the figure, the theoretical values are comparable with the fit values and about one order of magnitude larger than the SM value (black dot). The same sign dimuon asymmetry $A_{sl}^d$ against $d_n/e$ is shown in Fig. 2. A large imaginary part of the $\mu$ parameters, on one hand, produces a large CP violation in $B^0$ mixing. On the other hand, the large imaginary part implies a large EDM. Fig. 2 shows that the SUSY contribution to $d_n$ in this model can be made very small, while allowing a large $A_{sl}^d$ which in magnitude is comparable with the fit result (41). As we see from Fig. 2 the error in $A_{sl}^d$ is very crucial to test the prediction of the model. We hope that the error will be reduced by the future experiments.

\[^9\] $|((\delta_{32}^0)_{LR})|$ is two orders of magnitude smaller than $|((\delta_{32}^0)_{LR})|$.  

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FIG. 1: The prediction in the $\phi_s - \phi_d$ plane. The fit result of the CKMfitter group (purple) [15] and that of the UTfit group (blue) [16] are also shown. The black dot is the SM value.

FIG. 2: The same sign dimuon asymmetry $A^b_{\text{sl}}$ against $d_n/e$. The fit result for $A^b_{\text{sl}}$ is $-4.2^{+1.9}_{-1.8} \times 10^{-3}$ (purple) [16], and the D0 result [9] is $A^b_{\text{sl}} = -(9.57 \pm 2.51 \pm 1.46) \times 10^{-3}$. The SM value is shown in black.

FIG. 3: The prediction of $a^s_{\text{sl}} - a^d_{\text{sl}}$, where the horizontal axis stands for $\phi_s$. The fit result for $a^s_{\text{sl}} - a^d_{\text{sl}}$ is $-3.9^{+2.4}_{-3.1} \times 10^{-3}$ (purple), while the SM value is $(0.793^{+0.066}_{-0.214}) \times 10^{-3}$ (black).

In Fig. 3 we plot the prediction of $a^s_{\text{sl}} - a^d_{\text{sl}}$ against $\phi_s$. This combination of the asymmetries can be measured at LHCb, and the experimental sensitivity with one fb$^{-1}$, which will be achieved in 2011 [38], is sufficient to test it.

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