Error correction of spectral reflectance based on constrained linear spectral mixture model

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Abstract. With the rapid development of remote sensing technology, spatial information extraction from remote sensing images is playing an increasingly important role. However, spectral signals of various ground objects are inevitably recorded in pixel. The classification accuracy is reduced and the development of quantitative remote sensing is restricted. In this paper, four groups of mixed pixel samples are designed, using the ASD FieldSpec3 spectroradiometer and the constrained linear spectral model to analyse the error distribution and the error correction methods are proposed. The results show that the constrained linear spectral model can be used in experiments where the illumination is uniform and the measured objects are smooth, and model has strong applicability. After correction, the average error of each group was reduced by 0.00847, and this method can effectively reduce the error caused by wavelength variation in the actual spectral measurement experiment.

1. Introduction
In remote sensing images, mixed pixels increase the difficulty of remote sensing interpretation, resulting in the reduction of classification accuracy, which is an important factor restricting the development of quantitative remote sensing \textsuperscript{[1]}. Many mixed pixel decomposition models have been proposed by researchers at home and abroad, the constrained linear spectral model is widely used because it is closer to the theory of remote sensing transmission and which is convenient and practical \textsuperscript{[2-3]}. A large number of studies show that under the condition of uniform illumination and smooth light, the light transmission is mainly single scattering, and multiple scattering has little effect on reflectivity. In this case, the decomposition results of constrained linear spectral mixing model are superior to those of other methods \textsuperscript{[4]}. Thus, the study and analysis the errors sources of constrained linear spectral mixture model and using the corresponding modified model to reduce the error are important in improving the model applicability. This paper designed four groups samples which use the black, white for endmembers, the distribution characteristics of errors are analysed by using the constrained linear spectral model. At the same time, the corresponding correction model is proposed, it can provide a scientific basis for the optimization model and model application in the future.
2. Materials and methods

2.1. ASD FieldSpec3 Spectroradiometer
The FieldSpec3 spectrometer used in this experiment is an optical measuring equipment produced by Analytical Spectral Devices in the United States. It can be used in remote sensing, crop monitoring, forest survey, mineral surveys [5-6]. The main components of the instrument include the front optical system probe, the illuminating optical fiber, the illumination reflector light and spectral scanning system. The field of view angle in front of the spectrometer is 25°, the observed wavelength range is 350-2500 nm, the spectral resampling interval is 1.4 nm in the range of 350-1000 nm and 2 nm in the range of 1000-2500 nm. The spectral resolution is 3 nm in the range of 350-1000 nm and 10 nm in the range of 1000-2500 nm.

2.2. Sample design and spectral measurement
Four groups of samples were designed with black and white as endmembers. Each group consisted of 19 samples. In order to minimize the measurement error and ensure that the spectrometer probe was aligned with the sample. The diameter of the sample is 22 mm, which is the same as that of the probe, the inner diameter of the black endless ring is 22 mm and the outer diameter is 28 mm, which is the same as the outside diameter of the probe. After printing out the designed sample on the common A4 paper, the probe is aligned in the laboratory to measure the end-component spectrum and the mixed spectrum respectively, and each sample collects 3 spectral curves. After smoothing, the average value is taken as the original spectral reflectance data of each sample. In order to avoid the effects of Rayleigh scattering, the range data of 0.44-0.78μm is used as the research data of this paper.

2.3. Constrained linear spectral mixture model (CLSMM)
Linear spectral mixture model (LSMM) is a common model in the field of mixed pixel research [7], the model assumes that the spectral reflectance of pixel spectral reflectance is pixel inside each endmember to a percentage of the pixel area as a linear combination of weight coefficient, as shown in Eq.1. On the basis of LSMM, adding constraints a constraint, linear spectral mixture model [8] (CLSMM). Formulas as follows:

\[ R_\lambda = \sum_{i=1}^{n} a_i f_{\lambda i} + \varepsilon_\lambda \]  
(1)

\[ \sum_{i=1}^{n} a_i = 1 \quad (0 \leq a_i \leq 1) \]  
(2)

\[ \varepsilon_\lambda = |R_\lambda - R'_\lambda| \]  
(3)

In the above formulas, \( R_\lambda \) represents the theoretical spectral reflectivity of the mixed pixel in the \( \lambda \) band, \( R'_\lambda \) represents the measured spectral reflectivity of the pixel in the \( \lambda \) band, and \( \varepsilon_\lambda \) is the error corresponding to the \( \lambda \) band of the pixel, \( n \) is the number of endmembers contained in the pixel, \( a_i \) is the percentage of the area of the \( i \)-th component in the pixel, \( f_{\lambda i} \) is the spectral reflectivity of the \( i \)-th endmembers in the \( \lambda \) band.

2.4. Selection and Evaluation of modified Model
The correlation between error and wavelength is analyzed, error as dependent variable, wavelength as independent variable, then linear function, exponential function, logarithmic function, quadratic function, cubic function and power function were used to fit the curve [9,10]. The model was selected by using the regression model's goodness of fit \( R^2 \), \( F \) and \( P \) value, the model was evaluated by the variation of the error sum of the sample after fitting.
3. Error correction and effect evaluation

3.1. Distribution characteristics of errors

It can be seen from figure 1 that the spectral characteristics of theoretical spectral curves calculated by the constrained linear spectral model are basically the same as those of the measured spectral curves. The results show that the model has strong applicability under certain experimental conditions, but there are errors in different bands of each sample. The results of calculation show that the error distribution of each sample shows a tendency of first decreasing and then increasing. The error decreases gradually with wavelength in range of 0.44-0.6μm, and reaches the minimum at 0.6μm. In the range of 0.6-0.78μm, the error increases gradually with the wavelength. Because the error value of the same group of samples is obviously different, the error is further divided according to the sample. After calculating the average, with wavelength as independent variables, the original error as the dependent variable, were fitted with a linear function of the six models, according to the $R^2$, $F$ and $P$ value to select the optimal model.

![Figure 1](image_url)

Figure 1. The distribution characteristic of the error in sample 1-4.

3.2. Selection of error correction model and evaluation of its effect

It can be seen from Table 1 that there is a certain correlation between wavelength and error in each sample. The best fitting model of four groups of samples is a quadratic function, and all the models have passed the test. The fitting degree $R^2$ was greater than 0.71, $P$ value was less than 0.005. The confidence of the model is more than 95%. In the band range of 0.44-0.78μm, it is the best to correct the error by using the quadratic function model. The error and average error of each sample is reduced by 0.00847 after correction. As shown in figure 2, the correction effect of the error is better by using the quadratic function model, and the error correction model is used to correct the actual measurement. The correction of the spectral data shows that the sample error and average error of each group are
reduced by 0.00847, and the error caused by the wavelength variation in the actual spectral measurement is reduced.

### Table 1. Wavelength and error Correction Models of sample 1-4.

| Model                  | Test parameters of the model | Changes of total error |
|------------------------|------------------------------|------------------------|
| Sample 1               | y=0.423x²-0.497x+0.174       | 0.399*                 |
|                        |                             | 0.741                  |
|                        |                             | 44.318                 |
|                        |                             | 0.000                  |
|                        |                             | 0.01720                |
| Sample 2               | y=0.490x²-0.572x+0.199       | 0.441**                |
|                        |                             | 0.730                  |
|                        |                             | 41.866                 |
|                        |                             | 0.000                  |
|                        |                             | 0.01646                |
| Sample 3               | y=0.703x²-0.815x+0.292       | 0.479**                |
|                        |                             | 0.719                  |
|                        |                             | 39.747                 |
|                        |                             | 0.000                  |
|                        |                             | 0.00022                |
| Sample 4               | y=0.738x²-0.881x+0.325       | 0.239*                 |
|                        |                             | 0.716                  |
|                        |                             | 39.029                 |
|                        |                             | 0.000                  |
|                        |                             | 0.00013                |

**Note:** The above table "*" indicates significant correlation at level 0.05 (bilateral), "**" indicates significant correlation at level 0.01 (bilateral).

![Figure 2. Results of error correction in sample 1-4.](image)

4. **Conclusion**

The spectral information of mixed pixel can be well simulated and retrieved by using constrained linear spectral mixing model. The theoretical spectral reflectivity curve of each sample is basically consistent with the spectral reflectivity curve measured in practice. The results show that the model can be used in the experiments where the illumination is uniform and the measured object is smooth. In the range of 0.44-0.78μm, there is a significant correlation between the spectral error and the wavelength, the error distribution showed the tendency of decreasing first and then increasing. The
correction effect of the error was better by using the quadratic function model and established error correction model was used to correct the measured spectral data. The sample error and average error of each group are reduced by 0.00847, and the error caused by wavelength variation in actual spectral measurement is reduced, which provides a scientific basis for model optimization and model application in the future.

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