Study of Fusion Dynamics Using Skyrme Energy Density Formalism with Different Surface Corrections

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Within the framework of Skyrme energy density formalism, we investigate the role of surface corrections on the fusion of colliding nuclei. The coefficient of surface correction is varied between 1/36 and 4/36, and its impact is studied on about 180 reactions. The detailed investigations indicate a linear relationship between the fusion barrier heights and strength of the surface corrections. Our analysis of the fusion barriers advocate the strength of surface correction of 1/36.

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The usefulness of the Skyrme energy density formalism (SEDF) in understanding the fusion dynamics has been widely accepted.1−3 Several ion-ion potential models and parametrizations have been suggested based on the same formalism.1−3 Interestingly, along with SEDF, many other models/formalisms based on either microscopic/macroscopic or phenomenological picture are found to reproduce the fusion barrier heights within ±10%. In a recent study,4 we employed as many as 16 versions of different proximity potentials and found that one can reproduce the barrier heights across the periodic table within ±8%. Even in the last few years, many sets of the Skyrme forces that reproduce the ground state properties of large number of nuclei have also been proposed.1−3 Within the same SEDF, the use of different Skyrme forces can yield difference of ±10% for fusion barriers. The same Skyrme forces have also been used at intermediate energies to investigate many rare phenomena.5 The strength and form of nuclear potential is also very important in the study of cluster decay.6

Generally, SEDF consists of Hamiltonian which depends on the nucleonic density, kinetic energy density as well as on the spin density.1 Among all of them, the form and strength of the kinetic energy density has always been controversial in the literature.7 Mostly reported works used approximations based on the Thomas–Fermi approximation.7−9

In the literature, additional correction in terms of gradient term has been suggested over and above the Thomas–Fermi approximation.8−10 Though all studies advocate the inclusion of this term, its strength has yet not been resolved and many different strengths are available in the literature.6,10,11 We plan to explore the role of this surface correction term in heavy-ion collisions via fusion process and try to understand whether one can narrow down the choice of this parameter or not. We shall also attempt to present a simple parametrization of this term for fusion barriers. This aim is achieved by employing SEDF within the proximity concept discussed in the following.

In the Skyrme energy density formalism,1 the nuclear part of the interaction potential \( V_N(r) \) is calculated as a difference of the energy expectation value at a distance \( r \) and at infinity (i.e. at \( r = \infty \)):

\[
V_N(r) = E(r) - E(\infty),
\]

with

\[
E = \int H(r)dr.
\]

In our formalism, the energy density functional \( H(r) \) reads

\[
H(\rho, \tau, J) = \hbar^2 \left( \frac{1}{2m} \frac{\partial^2}{\partial \rho^2} + \frac{1}{2} \left( \frac{1}{2} x_0 \right) \rho^2 \right) - \left( \frac{1}{2} \left( \frac{1}{2} x_0 \right) \left( \rho_n^2 + \rho_p^2 \right) \right) + \frac{1}{4} (t_1 + t_2) \rho \tau + \frac{1}{8} (t_2 - t_1) (\rho_n \tau_n + \rho_p \tau_p) + \frac{1}{16} (t_2 - 3t_1) \rho \nabla^2 \rho + \frac{1}{32} (3t_1 + t_2) (\rho_n \nabla^2 \rho_n + \rho_p \nabla^2 \rho_p) + \frac{1}{4} t_3 \rho_n \rho_p \rho - \frac{1}{2} W_0 (\rho \nabla \cdot J + \rho_n \nabla \cdot J_n + \rho_p \nabla \cdot J_p),
\]

where \( \rho \) is the nucleon density taken to be two-parameter Fermi density and \( J \) is the spin density generalized by Puri et al. for spin-unsaturated nuclei.1 The remaining term is the kinetic energy density \( \tau \). The most widely used Skyrme force type SIII12 is used in the present analysis.

The evaluation of kinetic energy density term was performed within the Thomas–Fermi (TF) approximation which is a well-known alternative to the Hartree–Fock (HF) method. As shown by Myers and Swiatecki17 and Hilf and Süssman,8 the kinetic energy density \( \tau \) can be separated into volume term \( \tau_0 \) and surface term \( \tau_s \) plus reminder. In the first-order approximation, one can limit to \( \tau_0 = \frac{3}{8} \left( \frac{\hbar^2}{m} \right)^2 \rho^2 \) term
only. However, due to the serious drawback in explaining the nuclear surface as well as the densities, a demand of further correction to this term was realized. Bethe and Brueckner\cite{6} proposed that this difficulty can be overcome by adding some gradient term and correction proportional to $(\nabla \rho)^2$; derived long ago by von-Weizsäcker.\cite{10} Alternative to this is the gradient extension method that also yields similar results. The coefficient in the above term was first thought to be close to $1/4 = (\pi/9)/36$.\cite{6} Later it was found that this strength does not give proper results for kinetic energy density. Following this, several authors suggested different values for surface correction\cite{6-11} that ranges from 1/36 to 9/36. The kinetic energy density can therefore be written as

$$\tau = \tau_0 + \tau_\lambda = \tau_0 + \lambda (\nabla \rho)^2 \rho,$$

where $\lambda$ is an adjustable parameter. As discussed by Gräf,\cite{6} its value is supposed to be between $1/36$ and $9/36$. In last few years, a large number of theoretical models however used the value of $\lambda = 1/36$\cite{2,11,13} One will wonder how $\lambda$ can alter the results of fusion barriers and further whether we can understand its effect in terms of some parameterized form or not. It is worth mentioning that different strengths of $\lambda$ can be thought to be close to Yukawa term used in addition to the Skyrme forces at intermediate energies.\cite{5} Therefore, this term is very important for reproducing the surface properties of nuclei as well as for multifragmentation. We shall calculate the ion-ion potential using the above formalism for different values of $\lambda$ and then extract the fusion barrier heights and positions.

In the present study, as many as 180 reactions involving even-even masses between 24 and 246 are taken for analysis. As noted, symmetric as well as asymmetric colliding nuclei have been taken into account. We calculated the ion-ion potential for all 180 reactions using different surface strengths. The heights and positions of the barriers were calculated using the conditions:

$$\frac{dV_F(r)}{dr} \bigg|_{r=R_B} = 0, \quad \text{and} \quad \frac{d^2V_F(r)}{dr^2} \bigg|_{r=R_B} \leq 0. \quad (5)$$

For the present analysis, we took $\lambda = 1/36$, $2/36$, $3/36$, and $4/36$. The barrier height is denoted by $V_B$ and its corresponding position is marked as $R_B$. We shall also attempt to parameterize the fusion barriers, thus, obtained and will present unique correlation between different values of $\lambda$.

In Fig. 1, we display the percentage difference of the barrier heights and positions calculated using different values of $\lambda$ (between $1/36$ and $4/36$) over experimental values defined as

$$\Delta V_B(\%) = \frac{V_B^{\text{theor}} - V_B^{\text{expt}}}{V_B^{\text{expt}}} \times 100, \quad (6)$$

as a function of $Z_1Z_2$. The experimental data are taken from Refs.\cite{1,14-24}. We see that different values of $\lambda$ can explain barrier heights within $\pm10\%$ of the experimental values. However, due to large uncertainty in the measurement of barrier positions the deviation in some cases can be quite large $\pm30\%$. These data have been of controversy in the literature.\cite{1} Due to different experimental setups, measurements do not yield few barrier positions as per known trend. A more careful look reveals that fusion barrier heights calculated with $\lambda = 1/36$ yield better results on average as compared to other values of surface correction used. From this figure, which is spanned over the entire periodic table, it is evident that $\lambda = 1/36$ may be preferred as the coefficient of the gradient term in the kinetic energy density correction. This finding is in agreement with other large number of studies based on the structural aspect of heavy-ion reactions.\cite{2,11,13} However slight deviations in some cases may be due to the experimental uncertainties reported in many papers\cite{24} or it may also be due to the influence of additional higher order correction terms that we have not taken into account in the present calculations.

We shall now attempt to parametrize the fusion barrier heights and positions using different $\lambda$ values in terms of charges and masses of the colliding pair. In Fig. 2, we display the barrier positions $R_B$ as a function of $(A_1^{1/3} + A_2^{1/3})$ and $V_B$ as a function of $Z_1Z_2e^2(1 - 1/R_{\text{expt}})$, where $R_{\text{expt}}$ is the analytical barrier position obtained after parametrization have been performed over $R_B$ results. When we increase the value of $\lambda$ from 0 to $1/36$, $2/36$, $3/36$, and $4/36$, a monotonous increase in the nuclear part of the potential results in the reduction of the height of the
barrier and therefore, the barrier positions are pushed outward. The addition of the strength $\lambda$ increases the attractive part of the interaction potential. In other words, one can counterbalance the repulsive Coulomb potential at larger distances, therefore, pushing the barrier outwards. As a result, net decrease in the barrier height occurs.

$$V_1 = \frac{1}{2} \epsilon \left( \frac{1}{R_1^{1/3}} + \frac{1}{R_2^{1/3}} \right),$$

$$V_2 = \frac{1}{2} \epsilon \left( \frac{1}{R_1^{1/3}} + \frac{1}{R_2^{1/3}} \right).$$

The solid lines represent the straight line linear fit over the points.

In Fig. 2, we display the results with $\lambda = \frac{1}{36}$, $\frac{1}{36}$, and $\frac{1}{36}$. The left part is for the barrier positions whereas right part is for the barrier heights. The barrier heights and positions are parametrized in terms of the following relations:

$$R_B^{\text{anal}} = a' + b'X_1; \quad V_B^{\text{anal}} = c'X_2,$$

where $X_1 = (A_1^{1/3} + A_2^{1/3})$, and $X_2 = \frac{Z_1Z_2e^2}{R_B^{\text{anal}}} \left(1 - \frac{1}{R_B^{\text{anal}}} \right)$. Here $a'$, $b'$, and $c'$ are the constants displayed in the figure. We see a linear increase in the barrier heights and positions with the masses of the colliding nuclei. This is in agreement with many previous studies. With the increase in the strength of $\lambda$, a monotonous decrease in the barrier height and increase in the barrier positions can be seen. In all the cases, a linear fit can explain the effect of $\lambda$ on the barrier positions very well.

In Fig. 3, we display the percentage difference between the analytical and exact values as a function of $Z_1Z_2$. Very interestingly, we note that we can reproduce the actual barriers (heights as well as positions) within $\pm 2.5\%$ for all values of $\lambda$. This introduces great simplification in calculating the barrier positions using different strengths $\lambda$. It would be of further interest to understand how different values of $\lambda$ affects the fusion barriers in different reacting nuclei.

$$V_B^{\text{anal}} = \alpha \left(1 - \frac{\lambda}{36}\right),$$

where $\alpha$, $\delta$, and $\lambda$ are parameters describing the analytical fusion barriers in different reacting nuclei.

![Graph showing the percentage difference between the analytical and theoretical values as a function of $Z_1Z_2$ for different values of surface correction $\lambda$.](image-url)

**Fig. 3.** The percentage difference between the analytical and theoretical values as a function of $Z_1Z_2$ for different values of surface correction $\lambda$.

![Graph showing the analytical barrier heights $V_B^{\text{anal}}$ as a function of $\lambda/36$ for the reactions of $^{24}\text{Mg} + ^{24}\text{Mg}$, $^{28}\text{Si} + ^{28}\text{Si}$, $^{48}\text{Ca} + ^{48}\text{Ca}$, $^{64}\text{Ni} + ^{64}\text{Ni}$, $^{40}\text{Ca} + ^{50}\text{Ti}$, and $^{48}\text{Ti} + ^{58}\text{Ni}$.](image-url)

**Fig. 4.** The analytical barrier heights $V_B^{\text{anal}}$ as a function of $\lambda/36$ for the reactions of $^{24}\text{Mg} + ^{24}\text{Mg}$, $^{28}\text{Si} + ^{28}\text{Si}$, $^{48}\text{Ca} + ^{48}\text{Ca}$, $^{64}\text{Ni} + ^{64}\text{Ni}$, $^{40}\text{Ca} + ^{50}\text{Ti}$, and $^{48}\text{Ti} + ^{58}\text{Ni}$. Interestingly, we note that the barrier heights reduce systematically as one move from $\lambda = 0.0$ to $\lambda = \frac{1}{36}$.
where $\alpha$ and $\delta$ are the constants whose values depend upon the colliding pair (see Fig. 4). Therefore, decrease in the barrier height with $\lambda$ is on expected lines, a linear reduction is of particular interest.

![Diagram](image)

**Fig. 5.** (Color online) The fusion cross sections $\sigma_{\text{fus}}$ (mb) as a function of the center-of-mass energy $E_{\text{c.m.}}$ for the reactions of $^{24}\text{Mg} + ^{24}\text{Mg}$, $^{28}\text{Si} + ^{28}\text{Si}$, $^{48}\text{Ca} + ^{48}\text{Ca}$, $^{64}\text{Ni} + ^{64}\text{Ni}$, $^{40}\text{Ca} + ^{50}\text{Ti}$, and $^{48}\text{Ti} + ^{58}\text{Ni}$ using different values of surface correction $\lambda$.

Finally, in Fig. 5, we display the fusion cross sections $\sigma_{\text{fus}}$ calculated using the formula of Wong et al. as a function of the c.m., $E_{\text{c.m.}}$, for the reactions of $^{24}\text{Mg} + ^{24}\text{Mg}$, $^{28}\text{Si} + ^{28}\text{Si}$, $^{48}\text{Ca} + ^{48}\text{Ca}$, $^{64}\text{Ni} + ^{64}\text{Ni}$, $^{40}\text{Ca} + ^{50}\text{Ti}$, and $^{48}\text{Ti} + ^{58}\text{Ni}$. We see that no particular value of $\lambda$ explains the fusion cross sections. In few cases, however, $\lambda = \frac{1}{\kappa}$ yields better results whereas in other cases, higher values of $\lambda$ has better edge.

In summary, we have investigated the effect of surface corrections on the fusion process. Our finding over 180 reactions reveals that $\lambda = \frac{1}{\kappa}$ can be a better choice for the surface correction and it is in agreement with earlier attempts. We also obtain the parameterized form of the fusion barrier heights and positions for different strengths of $\lambda$. We further find that the fusion barrier heights depend linearly on the strength of the coefficient.

**References**

[1] Puri R K and Dhiman N K 2005 *Eur. Phys. J. A* 23 429
[2] Arora R et al 2000 *Eur. Phys. J. A* 8 103
[3] Puri R K et al 1998 *Eur. Phys. J. A* 3 277
[4] Puri R K et al 1992 *Phys. Rev. C* 45 1837
[5] Puri R K et al 1991 *Phys. Rev. C* 43 315
[6] Puri R K, Chattopadhyay P, and Gupta R K 1991 *Nucl. Phys. A* 589 315
[7] Puri R K and Gupta R K 1992 *Phys. Rev. C* 45 1837
[8] Dobrowolski A, Pomorski K, and Bartel J 2003 *Nucl. Phys. A* 729 713
[9] Zagrebaev V I 2004 *Nucl. Phys. A* 734 164
[10] Wang N, Li J and Zhao E 2008 *Phys. Rev. C* 78 054607
[11] Shen Q, Han Y and Guo H 2009 *Phys. Rev. C* 80 024604
[12] Feng Z, Jin G, and Zhang F 2008 *Nucl. Phys. A* 802 91
[13] Dutt I and Puri R K 2010 *Phys. Rev. C* 81 014615
[14] Dutt I and Puri R K 2010 *Phys. Rev. C* 81 064609
[15] Vermesi Y K and Puri R K 2009 *J. Phys. G: Nucl. Part. Phys.* 36 105103
[16] Kumar S et al 2010 *Phys. Rev. C* 81 014601
[17] Puri Y K et al 2010 *J. Phys. G: Nucl. Part. Phys.* 37 015105
[18] Kumar S et al 2010 *Phys. Rev. C* 81 014611
[19] Puri Y K et al 2009 *Europhys. Lett.* 85 62001
[20] Dhawan J et al 2008 *Phys. Rev. C* 78 064602
[21] Dhawan J et al 2007 *Phys. Rev. C* 75 057601
[22] Sood A D et al 2009 *Phys. Rev. C* 79 064618
[23] Puri Y K et al 2009 *Europhys. Lett.* 85 62001
[24] Dhawan J et al 2007 *Phys. Rev. C* 75 057601
[25] Singh J et al 2000 *Phys. Rev. C* 62 046617
[26] Kumar S et al 1998 *Phys. Rev. C* 58 3494
[27] Puri R K et al 1996 *Phys. Rev. C* 54 R28
[28] Puri R K et al 1994 *Nucl. Phys. A* 575 733
[29] Gupta R K et al 1993 *Phys. Rev. C* 47 561
[30] Puri R K et al 1992 *J. Phys. G: Nucl. Part. Phys.* 18 1533
[31] Malik S S et al 1989 *Pramana J. Phys.* 32 419
[32] Puri R K et al 1989 *Europhys. Lett.* 7 767
[33] Puri R K et al 1992 *J. Phys. G: Nucl. Part. Phys.* 18 1533
[34] Gräf H 1980 *Nucl. Phys. A* 343 91
[35] Hilf E and Süsemann G 1966 *Phys. Lett.* 21 654
[36] Bethe H A 1968 *Phys. Rev.* 167 879
[37] von Weizsäcker C F 1935 *Z. Phys.* 96 431
[38] Chattopadhyay P and Gupta R K 1984 *Phys. Rev. C* 30 1191
[39] Beiner M, Flocard H, Van Giai N and Quentin P 1975 *Nucl. Phys. A* 238 29
[40] Brack M, Guet C and Håkansson H 1985 *Phys. Rev. C* 32 123
[41] Skalski J 2007 *Phys. Rev. C* 76 044603
[42] Mitsuoka S et al 2007 *Phys. Rev. Lett.* 99 182701
[43] Jachcinski C M et al 1981 *Phys. Rev. C* 24 2070
[44] Gary S and Volant C 1982 *Phys. Rev. C* 25 1877
[45] DiCenzo S B, Petersen J F and Betts R R 1981 *Phys. Rev. C* 23 2561
[46] Aguilera E F, Kolata J D, DeYoung P A and Vega J J 1986 *Phys. Rev. C* 33 1961
[47] Stefanini A M et al 2009 *Phys. Lett. B* 679 R5
[48] Trotta M et al 2001 *Phys. Rev. C* 65 011601(R)
[49] Ichikawa T, Hagino K and Iwamoto A 2009 *Phys. Rev. Lett.* 103 202701
[50] Sonzogni A A et al 1998 *Phys. Rev. C* 57 722
[51] Vinoth Kumar A M et al 1996 *Phys. Rev. C* 53 803
[52] Trotta M et al 2001 *Phys. Rev. C* 65 011601(R)
[53] Aljuwair H A et al 1984 *Phys. Rev. C* 30 1223
[54] Vaz L C, Alexander J M, and Satchler G R 1981 *Phys. Rep.* 69 373
[55] Kovar D G et al 1979 *Phys. Rev. C* 20 1305
[56] Christensen P R and Winget A 1976 *Phys. Lett. B* 65 19
[57] Nicolis N G 2004 *Eur. Phys. J. A* 21 265
[58] Wong C Y 1972 *Phys. Lett. B* 42 186
[59] Wong C Y 1973 *Phys. Rev. Lett.* 31 766