CT System Parameter Calibration and Imaging Issues
Jie Peng
North China Electric Power University Baoding 07100, China
pengjie@ncepu.cn

Abstract
The article takes the A model of the National University Student Modeling Contest in 2017 as an example to study CT system parameter calibration and imaging issues. From the questions we can see, the topic is to calibrate the parameters of the CT system according to the sample (template) of known structure and obtain the related CT scan information (position, geometry, absorption rate) of other unknown objects.

Keywords
The basic principle of CT imaging; Radon transform; Attenuation factor; Rotation center.

1. Introduction
When the CT system is installed, there are often errors, which affect the imaging quality. Therefore, it is necessary to calibrate the installed CT system, that is, to calibrate the parameters of the CT system by means of a sample of known structure (called a template), and according to the unknown structure The sample was imaged.

2. Organization of the Text
2.1 Model establishment and solution
Calibrate the CT system based on the relevant information of the calibration template of the known information. According to the questions on the first question, we use the radar transform to calculate. Distance light that is a unit pixel passes through the image, then the radon transform calculates the integral through the image length as:

\[ R_y(x') = \int_{-\infty}^{\infty} f(x' \cos \theta - y' \sin \theta, x' \sin \theta + y' \cos \theta) dy \]  \hfill (1)

The line integral of \( f(x,y) \) in the vertical direction is \( f(X,Y) \) projected onto the X axis; the integration in the horizontal direction is \( f(X,Y) \) projected onto the Y axis. The projection can be calculated at any angle \( \theta \). The figure below illustrates the geometry of the Radon transform along the angle \( \theta \).

![Fig.1 Parallel beam projection geometry](image)

The parameters in Fig.1 are explained as follows:
1. XOY is a Cartesian coordinate system with the origin at point O.
2. XOY is a rotating coordinate system with the origin at point O.
The angle between the Xr axis and the X axis is Φ, γ, θ are polar coordinates, and the angle between the radial direction r and the x axis is ∅.

Let the image to be built be a(x,y), and its two-dimensional Fourier transform is A(ω₁, ω₂), then:

\[
a(x, y) = \frac{1}{4\pi^2} \int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} A(\omega_1, \omega_2) e^{j(x\omega_1 + y\omega_2)} d\omega_1 d\omega_2
\]  

(2)

Convert each physical quantity in (4) to the quantity expressed in polar coordinates

\[
(\omega_1, \omega_2) \rightarrow P(f, \phi)
\]

(3)

\[x\omega_1 + y\omega_2 = 2x\pi\cos\phi + 2y\pi\sin\phi = 2\pi\cos(\theta - \phi)
\]

(4)

To convert the expressions of dω₁ and dω₂ to the expression represented by df and dφ, it is necessary to introduce the Jacobian determinant and then convert it, that is:

\[
a(x, y) = \frac{1}{4\pi^2} \int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} A(\omega_1, \omega_2) e^{j(x\omega_1 + y\omega_2)} d\omega_1 d\omega_2
\]

(5)

Note the physical meaning of Eq.2 (3.9), look at the second integral:

\[|f| P(f, \phi) e^{j2\pi f \cos(\theta - \phi)} df
\]

(6)

This can be written as the inverse Fourier transform of the spatial domain variable Xr:

\[
\int_{-\infty}^{+\infty} |f| P(f, \phi) e^{j2\pi f \cos(\theta - \phi)} df = h(x_r) * P(x_r, \phi) = g(x_r, \phi)
\]

(7)

H (Xr) and P (Xr, Φ ) are Fu Liye transforms of |f| and P (F, Φ). The physical meaning of Eq.7 is exactly the value of the modified projection g (Xr,Φ) obtained by filtering the projection P (Xr, Φ) through a filter whose transfer function is |f| in the frequency domain when Xr = ycos (θ,Φ) is satisfied. The Xr = ycos (0,Φ) is exactly the ray equation given to the fixed point (y,0).

2.2 Model Using

The data in Annex 2 are plotted using MATLAB to obtain the absorption intensity image as shown in Fig. 2 and compared with the template schematic diagram provided. In the absorption intensity image, the coordinates of the X axis represent the rotation angle, and the coordinates of the Y axis represent the vertical distance of the sensor. The longer the ray passes through the object, the greater the energy absorbed by the object. At the same time, it also represents the greater absorption intensity. The data is processed by matlab, and the point A of the maximum data is obtained. The coordinates of the point A are 151, 223. Similarly, when projected from the short axis of the ellipse, the data is highly symmetrical, and its data coordinate is B (61,235), and the width of the object is L₁ = 73-46 = 27, L₂ = 277-169 = 108, L₈ = 307 for two scans.
Observed data in two directions have been obtained. The direction of the line connecting the two circle centers to the right is the positive direction of the X axis, the direction of the long axis of the ellipse is the positive direction of the Y axis, and the center of the ellipse is the origin. The angle of each rotation is 1 degrees. The distance between detector elements is \( L = 8/28 = 0.2857 \text{mm} \). The position of the rotation center of the CT system in the square pallet.

- **X direction**: \( \Delta X = (512/2 - 223) \times \Delta L = -9.1806 \text{mm} \)
- **Y direction**: \( \Delta Y = (512/2 - 235) \times \Delta L = 5.8422 \text{mm} \)

3. **Image Radon transform**

The data is brought into the `iradon` function in matlab to obtain an image (Fig. 3) of the absorbed intensity of the object in this plane.

![Fig. 3 Two or more references](image)

Use matlab to find the four leftmost, rightmost, top, and bottom boundary points of the large ellipse, and find the centroid position \((x_1, y_1)\) of the ellipse, and then calculate the distance from the centroid in the red region of the graph. The size thus finds the end position \((x_2, y_2)\) of the long axis of the large ellipse, and then uses the geometric relationship to find the relative position of the center of the remaining square tray.

Substitute data to obtain:
- **Shape center position** \((205, 210.5)\)
- **Long axis end point** \((292, 91)\)
- **Error in X direction** \( \Delta X_1 = 2.283 \times \Delta L = 0.6352 \text{mm} \)
- **Error in Y direction** \( \Delta Y_1 = 8.7778 \times \Delta L = 2.4420 \text{mm} \)
- **The angle error** \( \Delta \theta_1 = 75.42^\circ \) (the large ellipse long axis along the angle from the small elliptical hole to the x-axis of the pallet coordinate system)

The geometry of the unknown medium is a large elliptical object with two small elliptical circular holes on one side as shown in the Fig4.
4. Conclusion

The model mainly analyzes the CT imaging system, analyzes the distribution of uniform medium by using Radon transform and inverse transform, calibrates the square plate, and uses the calibrated parameters to calculate the decay rate of the unknown medium. Evaluation, and finally improved the model on the actual situation, improving the accuracy and stability of the CT system. When applied in production, it can increase economic efficiency.

The model is a two-dimensional image evaluation and analysis, and the actual production application requires image reconstruction of voxels. It has a wide range of applications in medicine. The reconstructed CT image can truly reproduce the organization of the body to ensure that the correct diagnosis can be made. The quality of the CT image is not only related to the fixed factors such as the performance of the CT machine, but also some variable factors directly affect the CT image. Quality. This model can be studied to further determine the distribution of body tissues.

References

[1] Hui Miao. Cone beam 3D XCT reconstruction algorithm [D]. North Central University, 2007. (In Chinese).

[2] Zhang Xuesong, Zhao Baishan. Cup artifact correction for CT images based on Radon transform[J/OL]. CT theory and application, 2016, 25 (05): 539-546. (In Chinese)

[3] Yanshulin, Niu Yantao. Explanation of medical imaging technical terms [J]. Chinese Journal of Medical Imaging, 2010, 18 (06): 524. (In Chinese)

[4] Jiang Qiyuan, Xie Xing Jin, Ye Jun. mathematical model [M]. Beijing: Higher Education Press, 2003. 8. (In Chinese)

[5] Huang Ke Chun. CT mathematical model of system parameter calibration [J]. Journal of Jiaying University, 2018, 36 (08): 16-21. (In Chinese)