CP violation of $\tau \to K\pi(\eta, \eta')\nu$ decays

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Abstract

We study direct CP violation of $\tau \to K\pi(\eta, \eta')\nu$ decays. By studying the forward and backward asymmetry, the interference of $L = 0$ and $L = 1$ amplitudes of the hadronic system can be extracted. By including the scalar and vector mesons into the chiral Lagrangian, we compute the form factors which correspond to $L = 0$ and $L = 1$ angular momentum state of the hadronic system. We include real and imaginary parts of the one loop corrections to the self-energies of the scalar and vector mesons. The direct CP violation of the forward and backward asymmetry is computed using a two Higgs doublet model in which a new CP violating phase is introduced. We show how the CP violation of the forward and the backward asymmetry may depend on the new CP violating phase and the strong phase of the form factors.

PACS numbers: 13.35.Dx, 11.30.Er, 12.39.Fe, 12.60.Fr
I. INTRODUCTION

Two B factories, both Belle and Babar have accumulated the very large samples of \( \tau \) decays. The \( \tau \) lepton physics beyond the standard model, such as \( \tau \) lepton number violation and time reversal violation through the measurement of electric dipole moment have been studied. The CP violation of the hadronic \( \tau \) decay also has been investigated both theoretically [1] and experimentally [2]. Various angular distributions including the quantities using the \( \tau \) spin polarization have been also proposed [1, 3]. Recently, Belle and Babar reported the precise measurements of the branching fractions of \( \tau \to K_s \pi \nu \) [4] and \( \tau^- \to K^- \pi^0 \nu \) [5]. The improved measurement of the branching fraction for \( \tau \to K \eta \nu \) has been also obtained [6]. About the \( \tau \to K \pi \nu \) decays, the hadronic invariant mass spectrum has been measured.

Motivated by these measurements, we study the direct CP violation of \( \tau^\pm \to K^\pm P \bar{\nu}(\nu) \) with \( P = \pi^0, \eta, \eta' \) and \( \tau^\pm \to K_s \pi^\pm \bar{\nu}(\nu) \). Non-vanishing direct CP violation in the processes may arise with some new source of the CP violation in addition to Kobayashi Maskawa phase [7] and the strong phase shifts of the final states of hadrons.

In \( \tau \to KP \nu \) decays, the hadronic system \( KP \) may have the angular momentum \( L = 0(\text{s wave}) \) and \( L = 1(\text{p wave}) \). The interference term of them can be extracted from the forward and backward asymmetry [8]. In the present paper, we define the asymmetry as the difference of the numbers of events for K meson scattered into the forward and backward directions with respect to the incoming \( \tau \) momentum in the hadronic CM frame. By comparing the forward and backward asymmetries for CP conjugate processes, the direct CP violation can be defined. The s wave and p wave of the hadronic amplitudes are related to the scalar and vector form factors in the time like region which have their own strong phases. To evaluate them, we use a chiral Lagrangian including scalar and vector meson resonances such as \( \kappa(800) \) and \( K^*(890) \). We compute the both real and imaginary parts of the one loop corrections to the self-energy of the resonances and obtain the strong phase shifts. We include the pseudoscalar meson loop correction and scalar and vector meson loop correction. The latter may give an important contribution to the form factors at higher invariant mass regions above 1 GeV up to \( m_\tau \sim 1.7(\text{GeV}) \).

As a new physics effect, we study a two Higgs doublet model with non-minimal Yukawa couplings to the charged leptons. The two Higgs doublets contribute to the charged lepton mass through the Yukawa couplings. In the non-minimal model, the interaction of the
charged Higgs boson to the $\tau$ lepton family can be CP violating. The interaction generates the amplitude $\tau_R \to \nu_\tau L H^- \to \nu_\tau L (\bar{u}_LS_R)_{s-wave}$. The interference with the charged current interaction due to $W$ boson exchanged diagram may lead to the direct CP violation which can be measured in the forward and backward asymmetry.

The paper is organized as follows. In section II, we show the hadronic chiral Lagrangian including scalar and vector resonances. In section III, we derive the form factors. In section IV, by fixing the finite renormalization constants, we numerically evaluate the form factors and the hadroic invariant mass spectrum. In section V, we introduce the two Higgs doublet model and present the direct CP violation. Section VI is devoted to conclusion and discussion.

II. CHIRAL LAGRANGIAN INCLUDING SCALAR AND VECTOR MESONS

In this section, we show the chiral Lagrangian with vector and scalar resonances. The following aspects are the main feature of the chiral Lagrangian.

- U(1)$_A$ breaking effect is taken into account so that we can apply the Lagrangian to $\tau$ decays into the final states including $K\eta$ and $K\eta'$.

- SU(3) breaking of the vector mesons are taken into account.

About the inclusion of the scalar resonances, we followed the approach of Ref.[9]. About the vector meson sector, our Lagrangian is equivalent to the one in Ref.[10] except SU(3) breaking effect for vector mesons. The chiral Lagrangian is given by,

$$\mathcal{L} = \frac{f_2^2}{4} \text{Tr} DU DU^\dagger + B \text{Tr} M(U + U^\dagger) - ig_2 \text{Tr} (\xi M \xi - \xi^\dagger M \xi^\dagger) \eta_0 - \frac{M_0^2}{2} \eta_0^2$$

$$+ \text{Tr} D_\mu S D^\mu S - M_s^2 \text{Tr} S^2$$

$$+ \frac{g_1}{4} \text{Tr}(D_\mu UD^\mu U^\dagger)(\xi S \xi^\dagger) + g_2 \text{Tr} ((\xi M \xi + \xi^\dagger M \xi^\dagger) S)$$

$$- \frac{1}{2} \text{Tr} F_{\mu\nu} F^{\mu\nu} + M_v^2 \text{Tr} (V_\mu - \frac{\alpha_\mu}{g})^2 + g_1 V \text{Tr} S (V_\mu - \frac{\alpha_\mu}{g})^2,$$

(1)

where $S$ and $V$ are the scalar nonets and vector nonets respectively. (See appendix A.) $U$ is the chiral field and is given as $U = \exp(2i\pi f) = \xi^2$. $\pi$ is SU(3) octet pseudo Nambu Goldstone boson and $\eta_0$ corresponds to U(1)$_A$ pseudoscalar of which mass is denoted by $M_0$. The U(1)$_A$ symmetry is broken by the mass term explicitly. The covariant derivatives for
the chiral field and the scalar field are given as,

\[ D_\mu U = (\partial_\mu + iA_{L\mu})U, \]  

\[ D_\mu S = \partial_\mu S + i[\alpha_\mu, S], \]  

\[ \alpha_\mu = \alpha_\mu^0 + \frac{\xi^\dagger A_{L\mu}}{2}\xi, \]  

\[ \alpha_\mu^0 = \frac{\xi^\dagger \partial_\mu \xi + \xi \partial_\mu^\dagger \xi}{2i}, \]  

where \( A_L \) denotes the external vector field corresponds to SU(3)_L. \( M \) in Eq.(1) is the chiral breaking term for the light quarks and is given by,

\[ M = \text{diag.}(m_u, m_d, m_s) = m_s \cdot \text{diag.}(\Delta, \Delta_d, 1). \]  

\( \Delta \) denotes \( \frac{m_u}{m_s} \) and \( \Delta_d = \frac{m_d}{m_s} \). In this work, we work in the isospin limit, \( m_u = m_d \). Below we explain how we determine the parameters in the Lagrangian of Eq.(1).

- \( B, g_1, g_2 \)

In the isospin limit, the vacuum expectation values of the scalar fields are given as,

\[ S_{01} = S_{02} = \frac{g_2 m_u}{M^2}, \quad S_{03} = \frac{g_2 m_s}{M^2}, \]  

which leads to the SU(3) breaking of the wave function renormalization constants and the decay constants of the pseudo Nambu Goldstone bosons,

\[ Z_{ij} = 1 + g_1 \frac{S_{0i} + S_{0j}}{2f^2}, \]  

\[ F_K = f \sqrt{Z_{13}}, \quad F_\pi = f \sqrt{Z_{11}}. \]  

The decay constant for \( \eta_8 \) is written as,

\[ F_8 = f \sqrt{\frac{Z_{11}}{3} + \frac{2Z_{33}}{3}} = F_\pi \sqrt{\frac{4}{3} R^2 - \frac{1}{3}}, \]
where $R$ is $\frac{F_K}{F_\pi}$. The generalized Gell-Mann Oakes Renner relation becomes,

$$m_K^2 F_K^0 = (m_u + m_s) (2B + g_2 (S_{01} + S_{03})),$$

(7a)

$$m_\pi^2 F_\pi^2 = 2m_u (2B + 2g_2 S_{01}),$$

(7b)

which can be used to express $g_2$, $g_1$ and $B$ with Eq.(4) and Eq.(5) in terms of the physical quantities as,

$$g_2 m_s = \frac{M_\sigma F_K m_K}{\sqrt{1 - \Delta}} \sqrt{\frac{1}{1 + \Delta} - R^{-2} \frac{m_\pi^2}{m_K^2} \frac{1}{2\Delta}},$$

(8a)

$$g_1 = 2 \frac{M_\sigma}{m_K} F_K (1 - R^{-2}) \sqrt{\frac{1}{1 + \Delta} - \frac{1}{2\Delta R^2} \frac{m_\pi^2}{m_K^2}} \sqrt{1 - \Delta},$$

(8b)

$$B m_s = \frac{1}{4\Delta} \frac{1 + \Delta}{1 - \Delta} m_\pi^2 F_\pi^2 - \frac{1}{1 + \Delta} \frac{\Delta}{1 - \Delta} m_K^2 F_K^2.$$

(8c)

- $\eta$ and $\eta'$ mesons and octet and singlet mixing angle $\theta_{08}$

$g_{2p}$ in Eq. (1) leads to the $\eta_0$ and $\eta_8$ mixing. The mass matrix for $\eta_0$ and $\eta_8$ sector is diagonalized as,

$$L_{08} = -\frac{1}{2} (\eta_8, \eta_0) \begin{pmatrix} M_{88}^2 & M_{80}^2 \\ M_{80}^2 & M_{00}^2 \end{pmatrix} \begin{pmatrix} \eta_8 \\ \eta_0 \end{pmatrix} = -\frac{1}{2} (\eta, \eta') \begin{pmatrix} M_\eta^2 & 0 \\ 0 & M_{\eta'}^2 \end{pmatrix} \begin{pmatrix} \eta \\ \eta' \end{pmatrix},$$

(9a)

$$\begin{pmatrix} \eta \\ \eta' \end{pmatrix} = \begin{pmatrix} \cos \theta_{08} & -\sin \theta_{08} \\ \sin \theta_{08} & \cos \theta_{08} \end{pmatrix} \begin{pmatrix} \eta_8 \\ \eta_0 \end{pmatrix}.$$ 

(9b)

where $M_{08}^2 = \frac{4}{\sqrt{3}} \frac{m_s - m_u}{F_\pi} g_{2p}$ and we take the convention $M_{08}^2 < 0$. Because the octet mass $M_{88}$ is given by

$$M_{88}^2 = \frac{1}{3 F_8^2} \left( \frac{8}{1 + \Delta} m_K^2 F_K^2 - \frac{2}{\Delta} m_\pi^2 F_\pi^2 + m_\pi^2 F_\pi^2 \right),$$

(10)

$M_{88}^2$ can be determined by $F_K$, $F_\pi$, $M_K$, $M_\pi$, and $\Delta$. With $M_{88}^2$ given by Eq. (10), the parameters $M_{00}^2$ and $M_{08}^2$ are also determined by using the masses of $\eta$ and $\eta'$ as,

$$M_{00}^2 = M_\eta^2 + M_{\eta'}^2 - M_{88}^2,$$

(11a)
\[ M_{08}^2 = -\sqrt{M_{08}^2 M_{88}^2 - M_{08}^2 M_{\eta'}^2}. \]  

(11b)

Therefore, one may predict the \( \eta_0 \) and \( \eta_8 \) mixing angle with the relation,

\[ \theta_{08} = -\frac{1}{2} \arctan \frac{2|M_{08}^2|}{M_{08}^2 - M_{88}^2}. \]  

(12)

The prediction of the mixing angle is rather close to the one experimentally extracted from \( J/\psi \to \gamma \eta(\eta') \) decays. (See for example [11].)

\[ \theta_{08\text{th}} = -22.38(\Delta = \frac{1}{24}), \quad -21.49(\Delta = \frac{1}{25}), \]  

(13a)

\[ \theta_{08\text{exp}} = -\arctan \left( \frac{\Gamma[J/\psi \to \gamma \eta]}{\Gamma[J/\psi \to \gamma \eta']} \right) \left( \frac{M_{08}^2 - M_{\eta'}^2}{M_{08}^2 - M_{\eta}^2} \right)^{\frac{3}{2}} \approx -(22.36^{+1.12}_{-1.21}). \]  

(13b)

**Vector meson mass spectrum**

The vector meson masses are given by the following formulae.

\[ M_{V_{ij}}^2 = M_V^2 + g_{1V} S_{0i} + S_{0j}. \]  

(14)

With this formulae, the U(3) nonets vector mesons masses are given by,

\[ M_{\rho}^2 = M_\omega^2 = M_V^2 + g_{1V} S_{01}, \]  

(15a)

\[ M_{\phi}^2 = M_V^2 + g_{1V} S_{03}, \]  

(15b)

\[ M_{K^*}^2 = M_V^2 + g_{1V} S_{01} + S_{03}. \]  

(15c)

One can fix the parameter \( g_{1V} \) as,

\[ g_{1V} = 2 \frac{M_{\phi}^2 - M_{K^*}^2}{\Delta S}, \]  

(16)

where \( \Delta S \) is the difference of the vacuum expectation values in Eq.(4),

\[ \Delta S = S_{03} - S_{01} = \frac{g_2 m_s(1 - \Delta)}{M_\sigma^2}. \]  

(17)

One can also derive the following relation by using Eq.(15),

\[ M_\rho = \sqrt{2M_{K^*}^2 - M_\phi^2}. \]  

(18)

The relation leads to the prediction \( M_\rho = 743 \text{ MeV} \) which is about \(-4\%\) smaller than the measured value. In table II we summarize the numerical values for the parameters in the chiral Lagrangian of Eq. (1).
M_κ = M_σ (MeV) & 800 & 840 & 760 \\
g_2m_s (MeV^3) & 2.65 \times 10^7 & 2.79 \times 10^7 & 2.52 \times 10^7 \\
g_1(MeV) & 215 & 225 & 204 \\
Bm_s (MeV^4) & 9.24 \times 10^8 & 9.24 \times 10^8 & 9.24 \times 10^8 \\
\Delta = \frac{m_u}{m_s} & \frac{1}{25} & \frac{1}{25} & \frac{1}{25} \\
\Delta S = S_{03} - S_{01}(MeV) & 39.8 & 37.9 & 41.9 \\
g_{1V}(MeV) & 12200 & 12800 & 11600 \\
g & 5.90 & 5.90 & 5.90 \\

TABLE I: The numerical values for the parameters in the chiral Lagrangian. We use M_κ = 494(MeV), M_π = 135(MeV), F_K = 113 (MeV) , F_π = 92.2(MeV) and Γ_{K^*} = 50.8(MeV) as input. g is determined with the width of K^*.

III. FORM FACTORS

The hadronic form factors relevant for the processes \( \tau^+ \to \bar{\nu}K^+P(P = \pi^0, \eta, \eta') \) are,

\[
\langle K^+(p_K)P(p_P)|\bar{u}\gamma_\mu s|0\rangle = F^{K^+P}(Q^2)q^\mu + \left(F_s^{K^+P}(Q^2) - \frac{\Delta_{KP}}{Q^2}F^{K^+P}(Q^2)\right)Q^\mu ,
\]

with \( Q^\mu = (p_K + p_P)^\mu \) and \( \Delta_{KP} = m_K^2 - m_P^2 \). The form factor denoted by \( F \) is the vector form factor and \( F_s \) is the scalar form factor. The form factors have been computed by using the variety of the methods, Ref. [12, 13, 14, 15]. In this work, we have used the hadronic chiral Lagrangian including the vector and the scalar resonances in Eq.(1). We compute the loop corrections to the self-energy of the vector and the scalar resonances. The real part of the self-energy is divergent and we need to subtract the divergence. Corresponding to the subtractions, we have added the polynomials. Some of the coefficients of the polynomials are determined by the pole positions and the residues of the propagator for the resonances.

To compute the form factors, let us write the V-A charged current in terms of hadrons. By differentiating Eq.(1) with the external vector fields \( A_L \), we obtain the current as,

\[
\bar{q}_jL\gamma_\mu q_iL = -i\frac{f^2}{2}(U\partial_\mu U^\dagger)_{ij} + \frac{M^2}{g} \left(\xi(V_\mu - \frac{\alpha^0_\mu}{g})\xi^\dagger\right)_{ij} + i\frac{g_1}{4} \{U\partial_\mu U^\dagger, \xi S\xi^\dagger\}_{ij} + \frac{g_{1V}}{2g} \{S, \xi(V_\mu - \frac{\alpha^0_\mu}{g})\xi^\dagger\}_{ij} - i(\xi[S, \partial_\mu S]\xi^\dagger)_{ij} - (\xi[S, [S, \alpha^0_\mu]]\xi^\dagger)_{ij}.
\]
We first show the results of the form factors for $K\pi$ final state.

\[
F_{K^+\pi^0}(Q^2) = \frac{1}{\sqrt{2}} \left\{ -\frac{R + R^{-1}}{2} + \frac{(\Delta S)^2}{2F_KF_\pi} + \frac{M_{K^*}^2}{2g^2 F_KF_\pi} \left( 1 - \frac{M_{K^*}^2}{A_R} \right) \right. \\
+ \left. \frac{\Pi_{VS}^T}{2g^2 F_KF_\pi}(1 - \frac{2M_{K^*}^2}{A_R}) \right\},
\]

(21a)

\[
F_s^{K^+\pi^0}(Q^2) = \frac{\Delta_{K\pi}}{Q^2} F_{K^+\pi^0}(Q^2) \\
+ \frac{1}{2\sqrt{2}} \left\{ R^{-1} - R + \frac{M_{K^*}^2}{g^2 F_KF_\pi} A_R \frac{\Delta_{K\pi}(B_RD_R - C_R^2)}{(A_R + Q^2 B_R)D_R - Q^2 C_R^2} \\
+ \frac{1}{2\sqrt{2}} \frac{\Delta_{K\pi}(Q^2) M_{K^*}^2 + M_{K^*}^2 \Delta_{K\pi} \Delta S}{gF_KF_\pi} \left( A_R + Q^2 B_R \right) D_R - Q^2 C_R^2 \\
+ \frac{1}{2\sqrt{2}} \frac{\Delta_{K\pi}(Q^2) (\Pi_{VS}^L - \Pi_{VS}^T)(1 - \frac{2M_{K^*}^2}{A_R}) + 2\Delta_{K\pi}}{A_R \Pi_{VS}^L} \right\}. \quad (21b)
\]

The form factors include the contribution of the Feynman diagrams shown in Fig. 1. In Fig. 1 the propagators for $K^*$ and $\kappa$ mesons are represented by the thick solid lines which include the one loop corrections to the self-energy. Let us consider the propagators for $K^*$ and $\kappa$. They are obtained by inverting the inverse propagators for $K^*$ and $\kappa$.

\[
\begin{pmatrix}
(g^{\mu\nu}A_R(Q^2) + Q^\mu Q^\nu B_R(Q^2)) & Q^\mu C_R(Q^2) \\
Q^\mu C_R(Q^2) & D_R(Q^2)
\end{pmatrix}
\begin{pmatrix}
K_\mu \\
\kappa
\end{pmatrix} = \begin{pmatrix}
J_\mu \\
J
\end{pmatrix},
\]

(22)
where \( J_\mu \) and \( J \) are source terms for \( K^* \) and \( \kappa \) respectively. \( A_R g^{\mu\nu} + B_R Q^\mu Q^\nu \) is the inverse propagator for \( K^* \) and \( D_R \) denotes the inverse propagator for the \( \kappa \). \( C_R \) denotes the mixing between the \( K^* \) and \( \kappa \). Inverting Eq. (22), one can obtain the propagator.

\[
\begin{pmatrix}
K^\mu \\
\kappa
\end{pmatrix} = \begin{pmatrix}
g^{\mu\nu} A_R^{-1} + Q^\mu Q^\nu \frac{D_R}{Q^\mu C_R} & -Q^\mu C_R \\
\frac{Q^\mu C_R}{(A_R+Q^2 B_R) D_R - Q^2 C_R^2} & -A_R^{-1}
\end{pmatrix} \begin{pmatrix}
J_\nu \\
J
\end{pmatrix}.
\] (23)

To obtain the contributions to the form factors from Feynman diagrams Fig.(1-a) \sim Fig.(1-e), we set \( i = s, j = u \) in Eq. (20),

\[
\bar{u}_L \gamma_\mu s_L = -\frac{1}{\sqrt{2}} F_K \partial_\mu K^- + \frac{M^2_{K^*}}{2g} \bar{K}^* - ig \frac{\Delta S}{M^2_{K^*}} \partial_\mu \kappa^- \\
- i \frac{1}{2\sqrt{2}} \left( K^- \partial_\mu \eta^0 \left( \frac{F_\pi}{F_K} \frac{F_{K^*} + g^2(\Delta S)^2}{2g^2 F_K F_\pi} \right) - \partial_\mu K^- \eta^0 \left( \frac{F_K}{F_\pi} \frac{M^2_{K^*} + g^2(\Delta S)^2}{2g^2 F_K F_\pi} \right) \right) \\
- i \frac{\sqrt{3}}{2\sqrt{2}} \left( K^- \partial_\mu \eta_8 \left( \frac{F_8}{F_K} \frac{M^2_{K^*} + g^2(\Delta S)^2}{2g^2 F_K F_\pi} \right) - \partial_\mu K^- \eta_8 \left( \frac{F_K}{F_\pi} \frac{M^2_{K^*} + g^2(\Delta S)^2}{2g^2 F_K F_\pi} \right) \right) \\
- i \frac{1}{2} \left( \bar{K}^0 \partial_\mu \kappa^- \left( \frac{F_\pi}{F_K} \frac{M^2_{K^*} + g^2(\Delta S)^2}{2g^2 F_K F_\pi} \right) - \partial_\mu \bar{K} \kappa^- \left( \frac{F_K}{F_\pi} \frac{M^2_{K^*} + g^2(\Delta S)^2}{2g^2 F_K F_\pi} \right) \right) \\
+ \frac{g_{IV}}{4g} \left( \frac{1}{\sqrt{2}} \kappa^- (\rho_\mu + \omega_\mu) \right) + \bar{\kappa}^0 \rho_\mu + f^0 K^* \\
+ \frac{1}{\sqrt{2}} K^* \left( \bar{a}^0 + \sigma \right) \right)
\] (24)

For the diagram in Fig.(1-a), the direct coupling of the charged current to \( K\pi \) can be easily extracted from Eq. (24). For the other diagrams, the amplitudes are obtained by multiplying the propagators in Eq. (23) with the production amplitudes of \( K^* \) and \( \kappa \) and the amplitudes corresponding to their decays.

The matrix elements corresponding to Fig.(1-a) \sim Fig.(1-e) are given as,

\[
\begin{align*}
\langle K^{+0} | \bar{u} \gamma_\mu s | 0 \rangle |_{(1-a)} &= \frac{1}{\sqrt{2}} \left\{ Q_\mu \frac{R^{-1} - R}{2} + q_\mu \left( -\frac{R^{-1} + R}{2} + \frac{M^2_{K^*} + g^2(\Delta S)^2}{2g^2 F_K F_\pi} \right) \right\}, \\
\langle K^{+0} | \bar{u} \gamma_\mu s | 0 \rangle |_{(1-b)} &= \frac{M^4_{K^*}}{2\sqrt{2} g^2 F_K F_\pi} \left( -q_\mu A_R^{-1} + Q_\mu \frac{\Delta S_{K\pi}}{A_R (A_R + Q^2 B_R) D_R - Q^2 C_R^2} \right), \\
\langle K^{+0} | \bar{u} \gamma_\mu s | 0 \rangle |_{(1-c)} &= -\frac{\Delta S g_{J\kappa K\pi}(Q^2)}{2\sqrt{2} F_K F_\pi} \frac{1}{D_R - \frac{Q^2 C_R^2}{A_R + Q^2 B_R}} Q_\mu, \\
\langle K^{+0} | \bar{u} \gamma_\mu s | 0 \rangle |_{(1-d)} &= \frac{M^2_{K^*}}{2\sqrt{2} g^2 F_K F_\pi} \frac{\Delta S_{\Delta K\kappa \pi C_R}}{(A_R + Q^2 B_R) D_R - Q^2 C_R^2} Q_\mu, \\
\langle K^{+0} | \bar{u} \gamma_\mu s | 0 \rangle |_{(1-e)} &= \frac{M^2_{K^*}}{2\sqrt{2} g^2 F_K F_\pi} \frac{g_{J\kappa K\pi}(Q^2) C_R}{(A_R + Q^2 B_R) D_R - Q^2 C_R^2} Q_\mu.
\end{align*}
\] (25)
FIG. 2: The Feynman diagrams of scalar and vector mesons loop which contribute to the form factor for $\langle KP | \bar{u} \gamma^\mu s | 0 \rangle$. They can be written in terms of the self-energy correction function $\Pi_{VS}$.

To derive Eq. (25), we have used the production amplitudes for $K^*$ and $\kappa$ due to the vector current $\bar{u} \gamma^\mu s$,

$$\langle K^*^\rho | \bar{u} \gamma^\rho s | 0 \rangle = g_{\rho\mu} \sqrt{2} M_{K^*}^2 / g,$$

$$\langle \kappa | \bar{u} \gamma^\mu s | 0 \rangle = Q_{\mu} \sqrt{2} \Delta S.$$  \hspace{1cm} (26)

We also have used the strong interaction vertices which are given as,

$$\langle K^+ \pi^0 | L_{VPP} | K^*^\sigma^+ \rangle = \frac{M_{K^*}^2}{4 g F_K F_\pi} q_{\sigma},$$

$$\langle K^+ \pi^0 | L_{SPP} | \kappa^+ \rangle = \frac{1}{4 g F_K F_\pi} g_{\kappa K \pi}(Q^2),$$  \hspace{1cm} (27)

with $q = p_K - p_{\pi}$. $g_{\kappa K \pi}(Q^2)$ is the strong coupling for $\kappa \rightarrow K \pi$ defined by,

$$g_{\kappa K \pi}(Q^2) = g_1 \frac{m_{K}^2 + m_{\pi}^2 - Q^2}{2} - g_2 (3m_u + m_s) + \Delta_{K \pi}(\Delta S).$$  \hspace{1cm} (28)

In addition to the pseudoscalar loops, we have taken into account the vector and scalar meson loops denoted by $\Pi_{VS}$. Each contribution is given by,

$$\langle K^+ \pi^0 | \bar{u} \gamma^\mu s | 0 \rangle |_{(2-a)} = \frac{q_{\rho} \Pi_{V S}^{\mu \rho}}{2 \sqrt{2} F_K F_\pi g^2},$$

$$\langle K^+ \pi^0 | \bar{u} \gamma^\mu s | 0 \rangle |_{(2-b)} = - \frac{M_{K^*}^2}{A} q_{\rho} \Pi_{V S}^{\mu \rho} (\delta_{\mu} - \frac{Q_{\mu} Q_{\nu}}{M_{K^*}^2}) \frac{1}{2 \sqrt{2} F_K F_\pi g^2},$$

$$\langle K^+ \pi^0 | \bar{u} \gamma^\mu s | 0 \rangle |_{(2-c)} = - \frac{M_{K^*}^2}{A} q^\mu (g_{\rho \nu} - \frac{Q_{\rho} Q_{\nu}}{M_{K^*}^2}) \frac{1}{2 \sqrt{2} F_K F_\pi g^2} \Pi_{V S}^{\mu \nu},$$  \hspace{1cm} (29)

where $\Pi_{VS}$ is identical to the self-energy function in Fig. (3-d),

$$\Pi_{V S}^{\mu \nu} = (g^{\mu \nu} - \frac{Q^{\mu} Q^{\nu}}{Q^2}) \Pi_{V S}^T + \frac{Q^{\mu} Q^{\nu}}{Q^2} \Pi_{V S}^L.$$  \hspace{1cm} (30)

By denoting the self-energy corrections as $\delta A_R, \delta B_R, C_R$ and $\Pi_{VS}$ corresponding to the
FIG. 3: (3-a): The pseudoscalar meson loop corrections to the self-energy for $K^*$ ($\delta A_R, \delta B_R$).
(3-b): The self-energy for $\kappa$ ($\delta D_R$). (3-c): The mixing term ($C_R$). (3-d): $\Pi^V$. Feynman diagrams in Fig. 3, one obtains

\[ A_R(s) = M_{K^*}^2 - s - iM_K\Gamma_{K^*}(s) + \text{Re}(\delta A_R(s)) + \Pi_T^{VS} + (a_0 + k_0) + (a_1 + k_1)s, \]
\[ sB_R(s) = s(1 + b_0 + l_1) + i\left(\frac{\nu_{K\pi}^3}{48\pi^2 s^2} + \frac{\nu_{K\eta}^3 \Delta_{K\eta}^2}{16\pi^2 s^2}\right) 3g_{K^*\kappa}^2 \]
\[ + i\left(\frac{\nu_{K\eta}^3}{48\pi^2 s^2} + \frac{\nu_{K\eta'}^3 \Delta_{K\eta'}^2}{16\pi^2 s^2}\right) 3g_{K^*\kappa}^2 \left(\frac{F_{\pi}}{F_S}\right)^2 \cos^2 \theta_{08} \]
\[ + i\left(\frac{\nu_{K\eta}^3}{48\pi^2 s^2} + \frac{\nu_{K\eta'}^3 \Delta_{K\eta'}^2}{16\pi^2 s^2}\right) 3g_{K^*\kappa}^2 \left(\frac{F_{\pi}}{F_S}\right)^2 \sin^2 \theta_{08} + s\text{Re}(\delta B_R(s)) + \Pi_{SV}^T - \Pi_{SV}^T, \]
\[ C_R(s) = \text{Re}(C_R(s)) + c_0 + i\frac{\nu_{K\eta}(s)\Delta_{K\eta}F_{\pi}}{16\pi^2 s^2 4F_M F_{\pi}} 3g_{K^*\kappa}(s)g_{K^*\kappa}, \]
\[ -i\frac{\nu_{K\eta}(s)\Delta_{K\eta}F_{\pi}}{16\pi^2 s^2 4F_M F_{\pi}} 3g_{K^*\kappa}(s)g_{K^*\kappa} \cos^2 \theta_{08} \]
\[ -i\frac{\nu_{K\eta'}(s)\Delta_{K\eta'}F_{\pi}}{16\pi^2 s^2 4F_M F_{\pi}} 3g_{K^*\kappa}(s)g_{K^*\kappa} \sin^2 \theta_{08}, \]
\[ D_R(s) = s - M^2_k + iM_k\Gamma_k(s) + \text{Re}(\delta D_R(s)) + d_0 + d_1 s + d_2 s^2, \]  
(31)

where $s = Q^2$ and $g_{K^*\kappa} = \frac{M_{K^*}^2}{4F_M F_{\pi}}$. The momentum dependent widths $\Gamma_{K^*}(s)$ and $\Gamma_k(s)$ are given as,

\[ \Gamma_{K^*}(s) = 3\frac{1}{4\pi M_{K^*}} \left(\frac{\nu_{K\pi}^3}{s^2} + \cos^2 \theta_{08} \frac{\nu_{K\eta}^3}{s} \left(\frac{F_{\pi}}{F_S}\right)^2 + \sin^2 \theta_{08} \frac{\nu_{K\eta'}^3}{s^2} \left(\frac{F_{\pi}}{F_S}\right)^2\right) g_{K^*\kappa}^2, \]
\[ \Gamma_k(s) = 3\frac{\nu_{K\eta} g_{K\eta}(s)}{16\pi s M_k} \left(\frac{1}{4F_M F_{\pi}}\right)^2 \]
\[ + \left(\cos^2 \theta_{08} \frac{\nu_{K\eta} g_{K\eta}(s)}{16\pi s M_k} + \sin^2 \theta_{08} \frac{\nu_{K\eta'} g_{K\eta'}(s)}{16\pi s M_k} \right) \left(\frac{1}{4\sqrt{3} F_M F_S}\right)^2, \]  
(32)
where \( \nu_{KP} = \sqrt{s^2 - 2s(m_K^2 + m_p^2) + \Delta_{KP}^2} (P = \pi, \eta, \eta') \) and is related to the momentum of kaon \( p_K \) in the hadronic rest frame as \( p_K = \frac{\nu_{KP}}{2\sqrt{s}} \). The real part of the self-energy corrections are divergent. We have subtracted the divergences at zero momentum as,

\[
\delta A_R(s) = \delta A(s) - \delta A(0) - sA'(0),
\]

\[
\delta B_R(s) = \delta B(s) - \delta B(0),
\]

\[
C_R(s) = C(s) - C(0),
\]

\[
\delta D_R(s) = \delta D(s) - \delta D(0) - sD'(0) - \frac{s^2}{2}D''(0).
\]  \( \text{(33)} \)

Then we have added the polynomials with respect to \( s \) which coefficients are finite renormalization constants. We have added the polynomial \( a_0 + a_1 s \) which corresponds to the twice subtractions for \( \delta A_R \) in Eq. \( \text{(33)} \). For \( \delta B_R \) and \( C_R \), we have added a finite constant to each denoted by \( b_0 \) and \( c_0 \) respectively. About the self-energy correction of the scalar meson \( \delta D_R(s) \), we need to subtract divergences up to \( s^2 \). Therefore, we need to add the polynomial \( d_0 + d_1 s + d_2 s^2 \) which is quadratic with respect to \( s \).

The self-energy corrections in Eq. \( \text{(33)} \), \( \delta A_R \sim C_R \), are given by,

\[
\delta A_R(s) = \frac{3g_{K\pi}^2}{16\pi^2} \left\{ \left( -\Sigma_{K\pi} \left( \frac{1}{2} + \log \frac{s}{m_K m_\pi} \right) + \frac{s}{3} \left( -1 + \log \frac{s}{m_K m_\pi} \right) \right) - 2sR_{K\pi} \right\}
\]

\[
- \frac{s^6}{3\Delta_{K\pi}^3} \left( m_K^6 + m_\pi^6 - 3m_K^2 m_\pi^2 \Sigma_{K\pi} \right) \log \frac{m_K}{m_\pi} + \frac{s}{18\Delta_{K\pi}^2} \left( 5m_K^4 + 5m_\eta^4 - 22m_K^2 m_\eta^2 \right)
\]

\[
+ \frac{\Sigma_{K\pi}^2 + \Delta_{K\pi}^2}{4\Delta_{K\pi}} \log \frac{m_K^2}{m_\pi^2} \right) \left( \left( \frac{F_\pi}{F_8} \right)^2 \cos^2 \theta_{06} \left( \frac{s}{18\Delta_{K\eta}^2} \right) \left( 5m_K^4 + 5m_\eta^4 - 22m_K^2 m_\eta^2 \right)
\]

\[
- \Sigma_{K\eta} \left( \frac{1}{2} + \log \frac{s}{m_K m_\eta} \right) + \frac{\Sigma_{K\eta}^2 + \Delta_{K\eta}^2}{4\Delta_{K\eta}} \log \frac{m_K^2}{m_\eta^2} + \frac{s}{3} \left( -1 + \log \frac{s}{m_K m_\eta} \right)
\]

\[
- 2sR_{K\eta} - \frac{s^6}{3\Delta_{K\eta}^3} \left( m_K^6 + m_\eta^6 - 3m_K^2 m_\eta^2 \Sigma_{K\eta} \right) \log \frac{m_K}{m_\eta} + \sin^2 \theta_{06} \left( \frac{F_\pi}{F_8} \right)^2 \left( 5m_K^4 + 5m_\eta^4 - 22m_K^2 m_\eta^2 \right)
\]

\[
- \Sigma_{K\eta'} \left( \frac{1}{2} + \log \frac{s}{m_K m_\eta} \right) + \frac{\Sigma_{K\eta'}^2 + \Delta_{K\eta'}^2}{4\Delta_{K\eta'}} \log \frac{m_K^2}{m_\eta'} + \frac{s}{3} \left( -1 + \log \frac{s}{m_K m_\eta} \right)
\]

\[
- \frac{s^6}{3\Delta_{K\eta'}^3} \left( m_K^6 + m_\eta^6 - 3m_K^2 m_\eta^2 \Sigma_{K\eta'} \right) \log \frac{m_K}{m_\eta'} + \frac{s}{18\Delta_{K\eta'}^2} \left( 5m_K^4 + 5m_\eta^4 - 22m_K^2 m_\eta^2 \right)
\]  \( \text{(34)} \)
\[ \delta B_R(s) = \frac{3g_{K^*K\pi}^2}{16\pi^2} \left\{ \frac{1}{3} \log \frac{s}{m_Km_\pi} - R'_{K^\pi} + \frac{\Sigma^3_{K^\pi}}{6\Delta^3_{K^\pi}} \log \frac{m^2_K}{m^2_\pi} - \frac{4m^4_K + m^2_Km^2_\pi + m^4_\pi}{9\Delta^2_{K^\pi}} \right\} + \cos^2 \theta_{08} \left( \frac{F_\pi}{F_8} \right)^2 \left( \frac{1}{3} \log \frac{s}{m_Km_\eta} - R'_{K\eta} + \frac{\Sigma^3_{K\eta}}{6\Delta^3_{K\eta}} \log \frac{m^2_K}{m^2_\eta} - \frac{4m^4_K + m^2_Km^2_\eta + m^4_\eta}{9\Delta^2_{K\eta}} \right) + \sin^2 \theta_{08} \left( \frac{F_\pi}{F_8} \right)^2 \left( \frac{1}{3} \log \frac{s}{m_Km_{\eta'}} - R'_{K\eta'} + \frac{\Sigma^3_{K\eta'}}{6\Delta^3_{K\eta'}} \log \frac{m^2_K}{m^2_{\eta'}} - \frac{4m^4_K + m^2_Km^2_{\eta'} + m^4_{\eta'}}{9\Delta^2_{K\eta'}} \right) \right\}, \]

(35)

\[ C_R(s) = \frac{3g_{K^*K\pi}}{64\pi^2F_\pi^2} \left\{ -g_{K^\pi}(s)(2R^1_{K^\pi} - R^0_{K^\pi}) + g_{K^\pi}(0) \left( \frac{m^2_Km^2_\pi \log \frac{m^2_K}{m^2_\pi}}{\Delta^2_{K^\pi}} - \frac{2\Sigma_{K^\pi}}{2\Delta_{K^\pi}} \right) \right\} - \cos^2 \theta_{08} \left( \frac{3g_{K^*K\pi}F_\pi}{64\pi^2F_\pi F_8^2} \right) \left\{ -g_{K\eta}(s)(2R^1_{K\eta} - R^0_{K\eta}) + g_{K\eta}(0) \left( \frac{m^2_Km^2_\eta \log \frac{m^2_K}{m^2_\eta}}{\Delta^2_{K\eta}} - \frac{\Sigma_{K\eta}}{2\Delta_{K\eta}} \right) \right\} - \sin^2 \theta_{08} \left( \frac{3g_{K^*K\pi}F_\pi}{64\pi^2F_\pi F_8^2} \right) \left\{ -g_{K\eta'}(s)(2R^1_{K\eta'} - R^0_{K\eta'}) + g_{K\eta'}(0) \left( \frac{m^2_Km^2_{\eta'} \log \frac{m^2_K}{m^2_{\eta'}}}{\Delta^2_{K\eta'}} - \frac{\Sigma_{K\eta'}}{2\Delta_{K\eta'}} \right) \right\}, \]

(36)

where \( R_{PQ} \) and \( R'_{PQ} \) are defined as,

\[ R_{PQ} = \int_0^1 \! dx \left( x^2 - x(1 + \frac{\Delta_{PQ}}{s}) + \frac{M_p^2}{s} \right) \log \left( x^2 - x(1 + \frac{\Delta_{PQ}}{s}) + \frac{M_p^2}{s} - i\epsilon \right), \]

\[ R'_{PQ} = 4R^2_{PQ} - 4R^1_{PQ} + R^0_{PQ}, \]

(37)

with

\[ R^{(n)}_{PQ} = \int_0^1 \! dx \ x^n \log \left( x^2 - x(1 + \frac{\Delta_{PQ}}{s}) + \frac{M_p^2}{s} - i\epsilon \right). \]

(38)

We give the explicit forms for \( R^{(n)}_{PQ} \) \((n = 0 \sim 2)\) in appendix B. The inverse propagator for
the scalar meson is,

\[
\delta D_R (s) = \frac{3}{(4 F_K F_\pi)^2} \left\{ g_{K \pi}^2 (s) \bar{J}_{K \pi} (s) - g_{K \pi}^2 (0) J'_{K \pi} (0) \right\} + \frac{1}{2} g_{K \pi}^2 (0)^2 J''_{K \pi} (0) \right\} + \frac{\cos^2 \theta_{08}}{(4 \sqrt{3} F_F F_{38})^2} \left\{ g_{K \eta}^2 (s) \bar{J}_{K \eta} (s) - g_{K \eta}^2 (0) J'_{K \eta} (0) \right\} 
- s^2 \left( 2 g_{K \eta}^2 (0) g_{K \eta'} (0) J'_{K \eta'} (0) + \frac{1}{2} g_{K \eta'}^2 (0) J''_{K \eta'} (0) \right) \right\}, \tag{39}
\]

where

\[
\bar{J}_{PQ} (s) = \frac{\theta (s - (M_P + M_Q)^2)}{32 \pi^2} \left\{ 2 + \frac{\Delta_{PQ}}{s} - \frac{\Sigma_{PQ}}{2 \Delta_{PQ}} \log \frac{M_P^2}{M_P^2 - \nu_{PQ}^2} + \frac{\Sigma_{PQ}}{2 \Delta_{PQ}} \log \frac{M_P^2}{M_P^2 - \nu_{PQ}^2} - \frac{\Sigma_{PQ}}{2 \Delta_{PQ}} \log \frac{M_P^2}{M_P^2 - \nu_{PQ}^2} \right\} + \frac{\nu_{PQ}^2}{s} \left\{ \frac{\Delta_{PQ}}{s} - \frac{\Sigma_{PQ}}{2 \Delta_{PQ}} \log \frac{M_P^2}{M_P^2 - \nu_{PQ}^2} \right\} \right\}, \tag{40}
\]

with \( \Sigma_{PQ} = m_P^2 + m_Q^2 \) and \( \nu_{PQ}^2 = s^2 - 2 s \Sigma_{PQ} + \Delta_{PQ}^2 = (s - (M_P + M_Q)^2)(s - (M_P - M_Q)^2) \). We also note,

\[
J'_{PQ} (0) = \frac{1}{32 \pi^2} \left\{ \frac{\Sigma_{PQ}}{\Delta_{PQ}^2} + 2 \frac{M_P^2 M_Q^2}{\Delta_{PQ}^2} \log \frac{M_P^2}{M_P^2} \right\}, \]

\[
J''_{PQ} (0) = \frac{1}{32 \pi^2} \left\{ \frac{2}{3 \Delta_{PQ}} (3 \Sigma_{PQ}^2 - 2 \Delta_{PQ}^2) + 4 \frac{M_P^2 M_Q^2}{\Delta_{PQ}^2} \Sigma_{PQ} \log \frac{M_P^2}{M_P^2} \right\}. \tag{41}
\]

The self-energy corrections due to the vector and the scalar meson loop are also divergent,

\[
\Pi^{\mu \nu} = -\frac{g_{1V}^2}{8} \sum_{VS} C_{VS} \mu^{d-4} \int \frac{d^d k}{(2 \pi)^d i} \frac{g^{\mu \nu} - k^{\mu} k^{\nu}}{m_V^2 - m_Q^2}, \tag{42}
\]

where \( C_{VS} \) are factors determined by scalar and vector mesons which contribute to the loop and are given as,

\[
(C_{K^{* \sigma}}, C_{\omega K}, C_{\rho K}, C_{K^{* \alpha}}, C_{\phi K}, C_{K^{* f_0}}) = \left( \frac{1}{2}, \frac{1}{2}, \frac{3}{2}, \frac{3}{2}, 1, 1 \right). \tag{43}
\]
We subtracted the divergences of $K_1$ and $K_2$ as,

$$K_{1VS} = K_1(s) - K_1(0) - K'_1(0)s,$$

$$K_{2VS} = K_2(s) - K_2(0).$$

Using $K_{1VS}$ and $K_{2VS}$, we write the self-energies $\Pi^T_{VS}$ and $\Pi^T_{VS}$ in Eq. (30) as,

$$\Pi^T_{VS} = -\frac{g_{1V}^2}{8} \sum_{VS} C_{VS} K_{1VS},$$

$$\frac{\Pi^T_{VS} - \Pi^T_{VS}}{s} = \frac{g_{1V}^2}{8} \sum_{VS} C_{VS} K_{2VS}. \quad (45)$$

The explicit forms for $K_{iVS}$ ($i = 1, 2$) are given as,

$$K_{1VS} = \frac{1}{16\pi^2} \left\{ -\frac{1}{2} \frac{R^{(0)}_V}{2M^2_V} - \frac{s + \Delta_{VS} R^{(1)}_V}{2M^2_V} + \frac{s}{2M^2_V} R^{(2)}_V - 1 + \frac{s}{12M^2_V} + \frac{\Sigma_{VS}}{8M^2_V} - \frac{\Sigma_{VS}}{2s} \right\},$$

$$+ \frac{s}{12M^2_V} \left( -\frac{3}{2} + \frac{2M^4_V + M^2_V M^2_S + M^4_S}{\Delta^2_{VS}} + \log \frac{M_V M_S}{s} \left( 1 - \frac{\Sigma_{VS}}{4M^2_V} + \frac{s}{12M^2_V} \right) \right),$$

$$+ \log \frac{M_V}{M_S} \left( \frac{\Sigma_{VS}}{\Delta_{VS}} - \frac{M^4_V + M^4_S}{4\Delta_{VS} M^2_V} + \frac{2M^2_V M^2_S}{s\Delta_{VS}} + \frac{s}{12M^2_V} \frac{M^6_V + M^6_S - 3M^2_V M^2_S \Sigma_{VS}}{\Delta^3_{VS}} \right) \},$$

$$K_{2VS} = \frac{1}{16\pi^2 M^2_V \Delta^3_{VS}} \left\{ \frac{M^6_V}{3} - \frac{11}{18} M^6_V - \frac{M^6_S \log M^2_S}{s} + \frac{M^6_S}{9} \right\} \right\},$$

$$- \frac{M^2_S M^2_V}{2} + M^4_V M^2_S - M^2_V M^2_S \Delta_{VS} \log \frac{M^2_S}{s} \right\} - \frac{1}{16\pi^2 M^2_V} R^{(2)}_V. \quad (46)$$

The absorptive parts of $K_{iVS}$ ($i = 1, 2$) are written as,

$$\text{Im}(K_{1VS}) = \frac{\nu_{VS}}{32\pi s} \left( 1 + \frac{s}{6M^2_V} + \frac{\Delta_{VS}}{2M^2_V} + \frac{\Sigma_{VS}}{6M^2_V} + \frac{\Delta^2_{VS}}{6M^2_V} \right),$$

$$\text{Im}(K_{2VS}) = -\frac{\nu_{VS}}{48\pi s M^2_V} (1 - \frac{\Sigma_{VS}}{2s} + \frac{3\Delta_{VS}}{2s} + \frac{\Delta^2_{VS}}{s^2} ). \quad (47)$$

In $A_R$ and $B_R$ of Eq. (32), we have added the polynomial $l_s$ to $\Pi^T_{VS} - \Pi^T_{VS}$ and $k_0 + k_1 s$ to $\Pi^T_{VS}$. The polynomials corresponds to the subtraction of the divergent parts of Eq. (44).

The above derivation of the form factors for $K\pi$ final state is easily extended to $K\eta$ and $K\eta'$ case.

$$F^{K+\eta}(Q^2) = \cos \theta_{0S} \frac{\sqrt{3}}{\sqrt{2}} \left\{ -\frac{R_s + R_s^{-1}}{2} + \frac{(\Delta S)^2}{2F_K F_s} + \frac{\Pi^T_{VS}}{2g^2 F_K F_s} \left( 1 - \frac{M^2_K}{A_R} \right) + \frac{M^2_{K*}}{2F_K F_s} \left( 1 - \frac{M^2_{K*}}{A_R} \right) \} \right\}, \quad (48a)$$
\[ F^{K+\eta}(Q^2) = \frac{\Delta_{K\eta}}{Q^2} F^{K+\eta}(Q^2) \]

\[ + \cos \theta_{08} \frac{\sqrt{3}}{2\sqrt{2}} \left\{ R_8^{-1} - R_8 + \frac{M_{K^*}}{g^2 F_K F_8} \frac{M_{K^*}}{A_R} \frac{\Delta_{K\eta}(B_R D_R - C_R^2)}{(A_R + Q^2 B_R)D - Q^2 C_R^2} \right\} \]

\[ - \frac{1}{A_R + Q^2 B_R} \frac{Q^2 C_R}{3F_K F_8} \]

\[ - \frac{g_{K\eta}(Q^2)^2}{3g F_K F_8} \Delta S \]

\[ \frac{1}{A_R + Q^2 B_R} \frac{C_R}{D - Q^2 C_R^2} \]

\[ + \frac{1}{F_K F_8 g^2} \left( \frac{\Delta_{K\eta}}{s} (\Pi'_{V'S} - \Pi'_{V'S})(1 - \frac{2M_{K^*}^2}{A_R}) + \frac{2\Delta_{K\eta}}{A_R} \Pi'_{V'S} \right) \}, \quad (48b) \]

\[ F^{K+\eta'}(Q^2) = \sin \theta_{08} \frac{\sqrt{3}}{2\sqrt{2}} \left\{ \frac{-R_8 + R_8^{-1}}{2} + \frac{(\Delta S)^2}{2F_K F_8} + \frac{M_{K^*}^2}{2g^2 F_K F_8} \left(1 - \frac{M_{K^*}^2}{A_R}\right) + \right\} \]

\[ \frac{\Pi'_{V'S}}{2g^2 F_K F_8} \left(1 - \frac{2M_{K^*}^2}{A_R}\right) \}, \quad (49a) \]

\[ F^{K+\eta'}(Q^2) = \frac{\Delta_{K\eta'}}{Q^2} F^{K+\eta'}(Q^2) \]

\[ + \sin \theta_{08} \frac{\sqrt{3}}{2\sqrt{2}} \left\{ \frac{-R_8 + R_8^{-1}}{2} + \frac{(\Delta S)^2}{2F_K F_8} + \frac{M_{K^*}^2}{2g^2 F_K F_8} \left(1 - \frac{M_{K^*}^2}{A_R}\right) + \right\} \]

\[ \frac{-\frac{Q^2 C_R}{A_R + Q^2 B_R}}{3F_K F_8} \Delta S \]

\[ \frac{3g F_K F_8}{C_R} \frac{(A_R + Q^2 B_R)D - Q^2 C_R^2}{D - Q^2 C_R^2} \]

\[ + \frac{1}{F_K F_8 g^2} \left( \frac{\Delta_{K\eta'}}{s} (\Pi'_{V'S} - \Pi'_{V'S})(1 - \frac{2M_{K^*}^2}{A_R}) + \frac{2\Delta_{K\eta'}}{A_R} \Pi'_{V'S} \right) \}, \quad (49b) \]

with \( R_8 = \frac{F_K}{F_8} \) and \( \Delta_{K\eta'} = m_{K^*}^2 - m_{\eta'}^2 \). To derive Eq. (48a) and Eq. (49a), we used the strong interaction vertices for \( K^* \to K\eta \) and \( K^* \to K\eta' \),

\[ \langle K^+\eta | \mathcal{L}_{VPP} | K^+_\sigma \rangle = \cos \theta_{08} \frac{M_{K^*}}{4g F_K F_8} \sqrt{3}q_\sigma, \]

\[ \langle K^+\eta' | \mathcal{L}_{VPP} | K^+_\sigma \rangle = \sin \theta_{08} \frac{M_{K^*}}{4g F_K F_8} \sqrt{3}q_\sigma, \]

(50)

where \( q = p_k - p_P \) (\( P = \eta, \eta' \)). We also use \( \kappa \to K\eta^{'} \) vertices which are given by,

\[ \langle K^+\eta | \mathcal{L}_{SPP} | K^+_\sigma \rangle = \frac{-\cos \theta_{08}}{4\sqrt{3}F_K F_8} g_{K\eta}(Q^2), \]

\[ \langle K^+\eta' | \mathcal{L}_{SPP} | K^+_\sigma \rangle = \frac{-\sin \theta_{08}}{4\sqrt{3}F_K F_8} g_{K\eta'}(Q^2), \]

\[ g_{K\eta^{(')}(Q^2)} = g_1 \frac{m_{K^*}^2 + m_{\eta^{(')}}^2 - Q^2}{2} - g_2(5m_s - m_u) - 3\Delta_{K\eta^{(')}}(\Delta S). \]
TABLE II: The numerical values for masses, widths and $V_{us}$ used in our numerical analysis. The units of mass and width are MeV.

| $m_K$ | $m_{\eta}$ | $m_{\eta'}$ | $m_\phi$ | $M_\rho$ | $M_\tau$ | $M_\omega$ | $V_{us}$ | $\Gamma_{K^*}$ |
|-------|------------|--------------|-----------|----------|----------|------------|---------|---------------|
| 493.7 | 957.8      | 135.0        | 775.5     | 891.7    | 1777     | 782.7      | 0.2257  | 50.8          |

IV. NUMERICAL ANALYSIS OF THE FORM FACTORS

In this section, we give the numerical results of the form factors. We summarize the numerical values for the masses of hadrons and widths which are used for the numerical analysis in table II. We first determine the finite renormalization constants. There are ten constants. We first renormalize $\text{Re}.A_R$ and $\text{Re}.D_R$ so that each inverse propagator has zero at the on-shell mass $s = M_{K^*}^2$ and $s = M_{\kappa}^2$ respectively. And then we require the residues of the propagators $\text{Re}.\frac{1}{A_R}$ and $\text{Re}.\frac{1}{D_R}$ are unity on their pole masses. Because of the four conditions, we can constrain the parameters $a_0 + k_0, a_1 + k_1, d_0, d_1$ and $d_2$. We show the four conditions below,

$$
\begin{align*}
 a_0 + k_0 + (a_1 + k_1)M_{K^*}^2 &= -\text{Re}.\delta A_R(M_{K^*}^2) - \text{Re}.\Pi^{T}_{VS}(M_{K^*}^2), \\
 a_1 + k_1 &= -\frac{d\text{Re}[(\delta A_R + \Pi^{T}_{VS})]}{ds}|_{s=M_{K^*}^2}, \\
 d_0 + d_1M_{K}^2 + d_2M_{\kappa}^4 &= -\text{Re}.\delta D_R|_{s=M_{\kappa}^2}, \\
 d_1 + 2M_{K}^2d_2 &= -\frac{d\text{Re}.\delta D_R}{ds}|_{s=M_{\kappa}^2}. 
\end{align*}
$$

Moreover we set four parameters $c_0, b_0, l_1, d_2$ to be zeros. Below we show the numerical values of constants for $M_{\kappa} = 800$ MeV case.

$$
\begin{align*}
a_0 + k_0 &= -5.68 \times 10^5 (\text{MeV}^2), \\
a_1 + k_1 &= 0.345, \\
d_1 &= -0.140, \\
d_0 &= -1.87 \times 10^3 (\text{MeV}^2). 
\end{align*}
$$
\[ a_1 + k_1 \quad 0.33961 \quad 0.34471 \quad 0.35100 \\
\[ a_0 + k_0 \quad -63519.7 \quad -56822.9 \quad -52534.2 \\
\[ d_1 \quad -0.139154 \quad -0.140467 \quad -0.134301 \\
\[ d_0 \quad 6485.53 \quad -1871.23 \quad -16362.1 \\
\[ k_0 \quad -4.16 \times 10^5 \quad -4.03 \times 10^5 \quad -3.92 \times 10^5 \\
\[ k_1 \quad 0.6558 \quad 0.6561 \quad 0.6564 \\
\[ \text{Br}(K \pi^0) \quad 0.00416079 \quad 0.00416079 \quad 0.00415823 \\
\[ \text{Br}(K \eta) \quad 0.000162063 \quad 0.000162015 \quad 0.000162325 \\
\[ \text{Br}(K \eta') \quad 4.14285 \times 10^{-6} \quad 3.87288 \times 10^{-6} \quad 3.94976 \times 10^{-6} \\
\]

**TABLE III:** The numerical values for the finite renormalization constants. They are chosen so that the branching fractions of \( \tau \to K \pi \nu \) and \( \tau \to K \eta \nu \) can be reproduced. We also show the values of the constants in Table III for \( M_\kappa = 760 \text{ MeV} \) and 840 MeV. Because the other parameters were set to be zero, there are only two undetermined parameters.

We choose \( k_0 \) and \( k_1 \) as the parameters to be adjusted. They are fixed so that the branching fractions for \( \tau \to K \pi \nu \) and \( \tau \to K \eta \nu \) can be reproduced. For \((k_0, k_1) = (-4.03 \times 10^5, 0.656)\), we obtain

\[
\begin{align*}
\text{Br}(\tau^+ \to K^+ \pi^0 \nu) &= 0.416 \times 10^{-2}, \\
\text{Br}(\tau^\pm \to K^\mp \eta \nu) &= 1.62 \times 10^{-4},
\end{align*}
\]

which are close to the experimental results \[5\] and \[6\],

\[
\begin{align*}
\text{Br}(\tau^- \to K^- \pi^0 \nu) &= 0.416 \pm 0.003 \pm 0.018 \times 10^{-2}, \\
\text{Br}(\tau^- \to K^- \eta \nu) &= (1.62 \pm 0.05 \pm 0.09) \times 10^{-4}.
\end{align*}
\]

The branching fraction for \( \tau \to K \eta' \nu \) becomes,

\[
\text{Br}(\tau \to K \eta' \nu) = 3.87 \times 10^{-6}.
\]

We have plotted the hadronic invariant mass spectrum for \( K \pi, K \eta \) and \( K \eta' \) cases in Fig. 1.
The formulae can be found in [1],

\[
\frac{d\text{Br}(\tau^\pm \rightarrow K^\pm P\bar{\nu}(\nu))}{d\sqrt{s}} = \frac{p_K G_F^2 |V_{us}|^2 (m_\tau^2 - s)^2}{\Gamma_\tau \ 2 \pi^3 \ m_\tau^2} \left( \frac{2m_\tau^2}{3s} + \frac{4}{3} p_K^2 |F^{KP}(s)|^2 + \frac{m_\tau^2}{2}|F^{KP}(s)|^2 \right),
\]

where \( p_K \) is the momentum of kaon in the hadronic CM frame. In the hadronic invariant mass spectrum for \( K^\pm \pi^0 \) at low invariant mass region, \( K^* \) resonance can be seen. Just below \( K^* \), we can see the effect of \( \kappa(800) \). At the high invariant mass region, the new thresholds due to the vector and scalar channels are open and these effects can be seen in \( K\pi \), \( K\eta \) and \( K\eta' \) cases in Fig. [4].

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{mass_spectrum.png}
\caption{The hadronic invariant mass spectrum \( \frac{d\text{Br}}{d\sqrt{s}} \) for \( K\pi^0 \) (solid line), \( K\eta \) (dashed line) and \( K\eta' \) (short dashed line) cases. We choose \( M_\kappa = 800 \) MeV and the other parameters are fixed as in the corresponding columns of Table [III].}
\end{figure}

V. FORWARD AND BACKWARD ASYMMETRY AND CP VIOLATION

In this section, we compute the forward and backward asymmetry [8] and the direct CP violation for \( \tau \rightarrow KP\nu \) decay. We start with the double differential rate of the unpolarized
\[ \frac{d\text{Br}}{d\sqrt{s}d\cos\theta} = \frac{1}{\Gamma_\tau} \frac{G_F^2 |V_{us}|^2 (m_\tau^2 - s)^2}{2\pi^3 m_\tau^2} \left\{ \left( \frac{m_\tau^2}{s} \cos^2 \theta + \sin^2 \theta \right) p_K^2 |F_{K^P}(s)|^2 + \frac{G_\tau^2}{4} |F_s|_{K^P}^2 - \frac{m_\tau^2}{\sqrt{s}} p_K \cos \theta \text{Re}(F_{K^P}(s) F_{sK^P}(s)^* ) \right\}, \quad (58) \]

where \( \theta \) is the scattering angle of kaon with respect to the incoming \( \tau \) in the hadronic CM frame. The forward and backward asymmetry extracts the interference term of the vector form factor and the scalar form factor.

\[ A_{FB}(s) = \int_{0}^{1} d\cos \theta \frac{d\text{Br}}{d\sqrt{s}d\cos\theta} - \int_{-1}^{0} d\cos \theta \frac{d\text{Br}}{d\sqrt{s}d\cos\theta} = -\frac{p_K}{\sqrt{s}} \frac{|F_{K^P}|}{|F_{sK^P}|} \cos \delta_{st}^{KP}, \quad (59) \]

with \( \delta_{st}^{KP} = \arg \left( \frac{F_{K^P}}{F_{sK^P}} \right) \). As we can see from Eq. (59), the forward and the backward asymmetry is determined by the ratio of the scalar and the vector form factors. It is also proportional to cosine of the strong phase shift \( \delta_{st} \). In Fig. 5 we show the vector and the scalar form factors and their ratio for \( K\pi \) case. We also show the strong phase shifts in Fig. 6. The forward and backward asymmetries for \( K\pi, K\eta \) and \( K\eta' \) cases are shown in Fig. 7. As can be seen from Fig. 7, the forward and backward asymmetry for \( K\pi \) case is large near the threshold region where the scalar contribution is also large. (See Fig. 5) We can expect about 50% asymmetry for \( \tau \rightarrow K\pi\nu \) decay.

![Graphs](image-url)

**FIG. 5:** Left: The vector form factor \( |F_{K^\pi}| \) (thick solid line) and the scalar form factor \( 10 \times |F_{sK^\pi}| \) (thin solid line). Right: The ratio \( \frac{|F_{K^\pi}|}{|F_{sK^\pi}|} \).

By including new physics source of CP violation, we can predict the direct CP violation of the forward and backward asymmetry. To be definite, we start with non-minimal two...
FIG. 6: The phase of the vector form factor: \( \delta_{V}^{K\pi} = \arg F_{s}^{K\pi} \) (short dashed line) and the phase of the scalar form factor \( \delta_{S}^{K\pi} = \arg F_{s}^{K\pi} \) (dashed line). The strong phase shift \( \delta_{st}^{K\pi} = \delta_{V}^{K\pi} - \delta_{S}^{K\pi} \) is shown with thick solid line. \( \delta_{st}^{K\eta} \) and \( \delta_{st}^{K\eta'} \) are shown with long short dashed line and solid line respectively.

FIG. 7: The predictions of the forward and backward asymmetries of \( \tau \to K\pi\nu \) (thick solid line) \( \tau \to K\eta\nu \) (dashed line) and \( \tau \to K\eta'\nu \) (short dashed line) in the standard model.

Higgs doublet model in which the two Higgs doublets have couplings to the charge leptons,

\[
-\mathcal{L} = y_{1ij}e_{Ri}^{\dagger}H_{1}^{\dagger}L_{Lj} + y_{2ij}e_{Ri}^{\dagger}H_{2}^{\dagger}L_{Lj} + \frac{y_{u}^{d}}{2}d_{Ri}^{\dagger}H_{1}^{\dagger}Q_{Lj} + \frac{y_{u}^{u}}{2}u_{Ri}^{\dagger}H_{2}^{\dagger}Q_{Lj} + \frac{M_{N}}{2}\nu_{Ri}^{\dagger}\nu_{Rj} + \text{h.c.} \tag{60}
\]

where we assume that \( H_{2} \) is coupled with neutrinos and both \( H_{1} \) and \( H_{2} \) are coupled with the charged leptons. The neutrino mass is given by the seesaw mechanism. However, the right-handed Majorana neutrino with the mass \( M_{N} \) much heavier than the electro-weak...
scale does not affect on the interaction terms with the mass dimension equal to four at all. Therefore, we keep the terms which are not suppressed by a factor $\frac{1}{M_N}$ in discussion below.

In two Higgs doublet model, without loss of generality, one can parametrize the Higgs fields as,

$$
H_2 = e^{i\frac{\theta_{CP}}{2}} \left( -\cos \beta H^+ \frac{v_2 + h_2 - i \cos \beta A}{\sqrt{2}} \right), \quad \tilde{H}_1 = i r_2 H_1^* = e^{-i\frac{\theta_{CP}}{2}} \left( -\sin \beta H^+ \frac{-v_1 + h_1 + i \sin \beta A}{\sqrt{2}} \right),
$$

(61)

where $\theta_{CP}$ is the relative phase of the vacuum expectation values of Higgs and its value can be determined from Higgs potential. The charged current interactions of the model are,

$$
\mathcal{L} = H^+ \overline{L}_i R_j \left( \frac{Y_{2i}^* e^{i\frac{\theta_{CP}}{2}}}{\cos \beta} - \delta_{ij} \frac{g \tan \beta m_j}{\sqrt{2} M_W} \right)
- \frac{g}{\sqrt{2}} \overline{U}_i W^- \overline{L}_j \gamma^\mu u_L - \frac{g}{\sqrt{2}} \overline{U}_i W^+ \gamma_\mu l_L + M_W^2 W^+ W^- + h.c.,
$$

(62)

where the charged lepton masses $m_l = \text{diagonal}(m_e, m_\mu, m_\tau)$ are obtained by the diagonalization.

$$
V_R \frac{1}{\sqrt{2}} (-y_1 v_1 e^{i\frac{\theta_{CP}}{2}} + y_2 v_2 e^{-i\frac{\theta_{CP}}{2}}) V_L^\dagger = m_l.
$$

(63)

Eq. (63) can be used to express $y_1$ in terms of the charged lepton mass and the other Yukawa coupling $y_2$. By introducing,

$$
Y_2 = V_R y_2 V_L^\dagger,
$$

(64)

one can obtain Eq. (62). The four fermi interactions induced by the charged Higgs exchange are,

$$
\mathcal{L}_H = \frac{G_F \tan^2 \beta}{\sqrt{2}} \frac{1}{M_W^2} \left\{ \overline{\nu}_i \left( \delta_{ij} - r_{2ij} \frac{1}{\sin \beta} \right) m_{lj}(1 + \gamma_5) l_j \right\}
\times \left\{ \overline{d} V_{KM}^\dagger (m_d(1 - \gamma_5) + m_u \cot^2 \beta(1 + \gamma_5)) u \right\} + h.c.,
$$

(65)

where $r_{2ij}$ denotes the non-minimal couplings of charged Higgs boson between the charged lepton $l_j$ to the neutrino $\nu_i$,

$$
r_{2ij} = \frac{Y_{2ji}^* e^{i\frac{\theta_{CP}}{2}}}{Y_{ji}^SM}.
$$

(66)
where $Y^{SM}$ denotes the standard model Yukawa couplings for charged leptons, $Y^{SM}_{ij} = m_{ij}$. The amplitude of the two Higgs doublet model is,

$$
\text{Amp.} (\tau^- \to \nu_i K^- P) = -\frac{G_F}{\sqrt{2}} V^*_{us} \bar{u}_i \gamma^\mu (1 - \gamma_5) u_\tau \left\{ \delta_{i\tau} (q_\mu - \frac{\Delta K P Q_{\mu}}{Q^2}) F^{KP}_s + \left( \delta_{i\tau} (1 - \frac{Q^2}{M^2_H} \tan^2 \beta) + \frac{Q^2 \tan^2 \beta}{M^2_H \sin \beta} r_{2\tau} \right) F^{KP}_s Q_{\mu} \right\}, \quad (67)
$$

where the matrix element of the scalar current is given by,

$$
\langle K^- \pi^0 | \bar{s} u | 0 \rangle = \frac{Q^2}{m_s - m_u} F^{KP}_s. \quad (68)
$$

Then $\tau \to K P \nu_i$ branching fraction is,

$$
\frac{d\text{Br}(\tau^- \to K^- P \nu_i)}{d\sqrt{s} d\cos \theta} = \frac{1}{\Gamma} \frac{G^2_F (m^2_\tau - s)^2}{2^5 \pi^3 m^2_\tau} \left\{ \left( \frac{m^2_\tau}{s} \cos^2 \theta + \sin^2 \theta \right) p_K^2 |F^{KP}_s|^2 \delta_{i\tau} + \frac{m^2_\tau}{4} \left| F^{KP}_{s\tau\tau} \right|^2 \right\} - \frac{\delta_{i\tau} m^2_\tau}{\sqrt{s}} p_K \cos \theta \text{Re.} (F^{KP}_s F^{KP*}_{s\tau\tau}) \quad (69)
$$

where we define

$$
F^{KP}_{s\tau\tau} = \left\{ \delta_{i\tau} \left( 1 - \frac{Q^2}{M^2_H} \tan^2 \beta \right) + \frac{Q^2}{M^2_H} \tan^2 \beta r_{2\tau} \right\} F^{KP}_s \quad (70)
$$

for $i = e, \mu$ and $\tau$. We neglect the small corrections proportional to up quark mass. For CP conjugate processes $\tau^+ \to K^+ \pi^0 \bar{\nu}_i$ is obtained by replacing $r_2$ in the amplitude $\tau^- \to K^- \pi^0 \nu_i$ with its complex conjugate $r_2^*$. Therefore the direct CP violation of the forward and backward asymmetries is given as,

$$
A_{FB} - \bar{A}_{FB} = \frac{2p_K (|F^{KP}_s|^2 |F^{KP*}_{s\tau\tau}| \sin \delta_{st}}{(2m^2_{\tau} + 4\frac{p_K^2}{m^2_\tau} + \frac{1}{2} \sum_i \frac{|F^{KP}_{s\tau\tau}|^2}{M^2_H \sin \beta}) \left| r_{2\tau} \right| \sin \theta_{2\tau}}, \quad (71)
$$

where we parametrize CP violating phase of the flavor diagonal coupling as,

$$
r_{2\tau} = |r_{2\tau}| e^{i \theta_{2\tau}}. \quad (72)
$$

We set the flavor off-diagonal couplings in $r_{2\tau}$ to be zeros. We note that in the isospin limit $m_u = m_d$, the CP asymmetry of Eq. (71) for $K^+ \pi^0$ case is identical to the direct CP violation of $\tau^+ \to K^0 \pi^- \nu_i$ and $\tau^- \to K^- \pi^0 \bar{\nu}_i$. Contrary to the CP violation of the total branching ratios which is sensitive to the CP violation of the mixing of $K^0$ and $\bar{K}^0$, the CP violation of the forward and backward asymmetries does not depend on the mixing induced CP violation.
The CP violation of the forward and backward asymmetry is shown for $K^{\mp}\pi^0$ case in Fig. 8. By taking $|r_{2\tau\tau}| = 1$, one can see that CP asymmetries can be as large as a few %. At low invariant mass region $\sqrt{s} < 900$(MeV), the direct CP violation is negative while at high invariant mass region $\sqrt{s} > 900$ (MeV), the CP violation is positive. The sign is correlated to $\sin\delta_{st}$ as can be seen in Eq. (71). From Fig. 6, we can see $\sin\delta_{st}$ also changes its sign around $\sqrt{s} = 900$(MeV). We also change the charged Higgs boson mass. For $M_H > 500$ (GeV), CP violation is suppressed to less than 1 %. In Fig. 9, we also show the CP asymmetries for $K\eta$ and $K\eta'$ cases. We note the sign of the CP violation is opposite to the sign at high invariant mass region of $K\pi$ case.

FIG. 8: CP violation for the forward and backward asymmetries of $\tau \to K\pi\nu$. The charged Higgs boson mass is changed as $M_H$(GeV) = 200 (thick solid line), 250 (dashed line), 500 (short dashed line) and 700 (solid line). The other parameters are $\tan\beta = 50$, $|\tau_{2\tau\tau}| = 1$ and $\theta_{2\tau\tau} = \frac{\pi}{2}$.

VI. CONCLUSION AND DISCUSSION

We have studied CP violation of $\tau \to K P\nu$ ($P = \pi^0, \eta, \eta'$) decays and $\tau \to K_s\pi\nu$. CP violation for the forward and backward asymmetries is computed using the two Higgs doublet
The effect on the direct CP violation of the non-minimal coupling is studied. For the non-minimal Yukawa coupling as large as that of the standard model Yukawa coupling of τ lepton, we have predicted a few % CP asymmetries within the parameter region with $M_H = 200 \sim 300$(GeV) and with $\tan \beta \sim 50$. 

model which has the non-minimal Yukawa couplings to the charged leptons. The effect of the CP violation is numerically studied. The direct CP violation depends on the strong phase shifts $\sin(\delta_V - \delta_s)$. To evaluate the phase shifts, we use the chiral lagrangian including the vector and the scalar resonances and take into account of the one loop corrections. We compute both real part and imaginary part of their self-energies. The divergences of the real part is subtracted properly. We have determined the finite renormalization constants so that the branching fractions for $K\pi$ and $K\eta$ modes are reproduced. We also take into account of $U(1)_A$ breaking so that $\eta_0$ and $\eta_8$ mixing can be incorporated. With those improved treatment, we can predict the hadronic invariant mass spectrum and CP violation even at high invariant mass region for $K\pi$ case as well as the same observables of $K\eta$ and $K\eta'$ cases.

FIG. 9: CP violation for the forward and backward asymmetries of $\tau \to K\pi\nu$ (thick solid line), $\tau \to K\eta\nu$ (dashed line) and $\tau \to K\eta'\nu$ (short dashed line). We choose the parameters as $M_H = 300$, $\tan \beta = 50$, $|r_{2\tau\tau}| = 1$, $\theta_{2\tau\tau} = \frac{\pi}{2}$.
Acknowledgments

We would like to thank K. Hayasaka, K. Inami, T. Onogi and T. Oshima for discussion and encouragement.

This work was supported by the Korea Research Foundation Grant funded by the Korean Government (MOEHRD, Basic Research Promotion Fund, KRF-2007-C00145) and the BK21 program of Ministry of Education (K.Y.L.).

The work of T. M. was supported by KAKENHI, Grant in Aid for Scientific Research on Priority Areas ”New Development for Flavor Physics”, No.19034008 and No.20039008, MEXT, Japan. We also thank YITP for support from international molecule type visitor program by the Yukawa International Programs for Quark-Hadron Sciences. The preliminary results of the paper were presented in BNM2006, YITP workshop; Towards the precise predictions of CP violation (YITP-W-07-21), BNM2008, New Development of flavor physics 2008, YongPyong 2008 and HQL2008.

APPENDIX A: SCALAR AND VECTOR MESON NONETS

Here we show the scalar and vector meson nonets.

\[
S = \begin{pmatrix}
\frac{a_0 + \sigma}{2} & \frac{a^+}{\sqrt{2}} & \frac{\kappa^+}{\sqrt{2}} \\
\frac{a^-}{\sqrt{2}} & -\frac{a_0 + \sigma}{2} & \frac{\kappa^0}{\sqrt{2}} \\
\frac{\kappa^-}{\sqrt{2}} & -\frac{i \rho}{\sqrt{2}} & f^0
\end{pmatrix}, \quad V = \begin{pmatrix}
\frac{\rho + \omega}{2} & \frac{\rho^+}{\sqrt{2}} & \frac{K^{*+}}{\sqrt{2}} \\
\frac{\rho^-}{\sqrt{2}} & -\frac{\rho + \omega}{2} & \frac{K^{*0}}{\sqrt{2}} \\
\frac{K^{*-}}{\sqrt{2}} & -\frac{i \phi}{\sqrt{2}} & \frac{\phi}{2}
\end{pmatrix}.
\]  

\( \text{(A1)} \)

APPENDIX B: FUNCTIONS \( R_{PQ}^{(n)} \)

The function \( R_{PQ}^{(n)} \) is defined as,

\[
R_{PQ}^{(n)} = \int_0^1 dx x^n \log(x^2 - (1 + \frac{\Delta_{PQ}}{s})x + \frac{M_P^2}{s} - i\epsilon). \]

\( \text{(B1)} \)

There are two regions for \( s \) of interests depending on below the threshold, i.e., (1) \( (M_P - M_Q)^2 \leq s \leq (M_P + M_Q)^2 \) or the above threshold (2) \( (M_P + M_Q)^2 \geq s \).

For the case (2),

\[
R_{PQ}^{(n)} = \int_0^1 x^n \log((x - a)(x - b) - i\epsilon)dx, \]

\( \text{(B2)} \)
with $a$ and $b$ are given as,

\[
a = \frac{1 + \frac{\Delta PQ}{s}}{2} - \frac{\nu PQ}{2s},
\]

\[
b = \frac{1 + \frac{\Delta PQ}{s}}{2} + \frac{\nu PQ}{2s},
\]  

with $0 \leq a, b \leq 1$. We show the real part of $R^{(n)}_{PQ}$,

\[
\text{Re}(R^{(0)}_{PQ}) = -2 + \log(1 - a)(1 - b) - a \log \frac{1 - a}{a} - b \log \frac{1 - b}{b},
\]

\[
\text{Re}(R^{(1)}_{PQ}) = \frac{a^2}{2} \log \frac{a}{1 - a} + \frac{1}{2} \log(1 - a) - \frac{1}{4} - \frac{a^2}{2} + \frac{b^2}{2} \log \frac{b}{1 - b} + \frac{1}{2} \log(1 - b) - \frac{1}{4} - \frac{b}{2},
\]

\[
\text{Re}(R^{(2)}_{PQ}) = -\frac{6a^2 + 3a + 2}{18} + \frac{1}{3} \log(1 - a) + \frac{a^3}{3} \log \frac{a}{1 - a} - \frac{6b^2 + 3b + 2}{18} + \frac{1}{3} \log(1 - b) + \frac{b^3}{3} \log \frac{b}{1 - b}.
\]  

(B4)

For the case (1),

\[
R^{(n)}_{PQ} = \int_0^1 dx x^n \log((x - \beta)^2 + \alpha^2)
\]

\[
= \int_{-\beta}^{1-\beta} dy (\beta + y)^n \log(y^2 + \alpha^2),
\]  

where,

\[
\alpha = \sqrt{((M_P + M_Q)^2 - s)((s - (M_P - M_Q)^2)} = \sqrt{-\frac{\nu PQ}{2s}},
\]

\[
\beta = \frac{s + \Delta PQ}{2s}.
\]  

(B6)

We define the indefinite integrals,

\[
r^{(n)}_{PQ}(y) = \int y^n dyy \log(y^2 + \alpha^2).
\]  

(B7)

Using the integrals, one can write,

\[
R^{(0)}_{PQ} = r^{(0)}_{PQ}(1 - \beta) - r^{(0)}_{PQ}(-\beta),
\]

\[
R^{(1)}_{PQ} = r^{(1)}_{PQ}(1 - \beta) - r^{(1)}_{PQ}(-\beta) + \beta R^{(0)}_{PQ},
\]

\[
R^{(2)}_{PQ} = r^{(2)}_{PQ}(1 - \beta) - r^{(2)}_{PQ}(-\beta) + 2\beta R^{(1)}_{PQ} - \beta^2 R^{(0)}_{PQ}.
\]  

(B8)

The indefinite integrals are given by,

\[
r^{(2)}_{PQ} = \frac{1}{3}(y^3 \log(y^2 + \alpha^2) - \frac{2y^3}{3} + 2\alpha^2 y - 2\alpha^3 \arctan \frac{y}{\alpha}),
\]

\[
r^{(1)}_{PQ} = \frac{1}{2}((y^2 + \alpha^2) \log(y^2 + \alpha^2) - y^2),
\]

\[
r^{(0)}_{PQ} = y \log(y^2 + \alpha^2) - 2y + 2\alpha \arctan \frac{y}{\alpha}.
\]  

(B9)
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