Confinement-deconfinement and universal string effects from random percolation

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The ’t Hooft criterion leading to confinement out of a percolating cluster of central vortices suggests defining a novel three-dimensional gauge theory directly on a random percolation process. Wilson loop is viewed as a counter of topological linking with the random clusters. Beyond the percolation threshold large Wilson loops decay with an area law and show the universal shape effects due to flux tube fluctuations. Wilson loop correlators define a non-trivial glueball spectrum. The crumbling of the percolating cluster when one periodic direction narrows accounts for the finite temperature deconfinement, which belongs to 2D percolation universality class.

1. INTRODUCTION

Understanding confinement in gauge theories is a major challenge of particle physics. Center vortices are believed to play an important role in explaining this phenomenon. Long time ago ’t Hooft proposed a criterion for confinement [1] based on three main ingredients: assuming that i) there is a percolating cluster of central vortices, ii) the Wilson loops measure the linking with the vortex lines and iii) the vortices at different places are weakly correlated, the sought after area decay law follows.

The flux of a vortex is conserved modulo $N$, where $N$ is the number of elements of the center of the gauge group. This implies that a) the vortices are closed lines and that b) the coordination number of the intersection points is a multiple of $N$. While the former property is essential for defining topological linking, the latter does not take part in the argumentation.

The unsolved difficulty in order to demonstrate confinement is to replace the numerical evidence of the weakness of the correlation among central vortices with a convincing proof.

This suggests to reverse the argument using random percolation to define weakly correlated loops and regarding them as the central vortices of a suitable gauge theory.

We define the following purely geometric setting: generate a sample of possible states $\{C_1, C_2, \ldots\}$ simply by populating each of the links of a 3D lattice independently with occupation probability $p$. The physical observables of this system, that we call still Wilson operators $W_\gamma$, are associated to arbitrary loops $\gamma$ of the dual lattice with the following rule

1. $W_\gamma(C_i) = 1$ if no cluster of the configuration $C_i$ is topologically linked to $\gamma$;
2. $W_\gamma(C_i) = 0$ otherwise.

The vacuum expectation value of this operator is defined by

$$\langle W_\gamma \rangle = \lim_{n \to \infty} \frac{1}{n} \sum_{i=1}^{n} W_\gamma(C_i) .$$

Note that only the closed paths of occupied links can contribute to $W_\gamma$, while the dangling ends do not play any role, thus in this theory the center vortices have to be identified with the loops of the random clusters.

If $p$ is greater than the percolation threshold $p_c$ there is a percolating cluster in the infinite lattice and large Wilson loops obey the area law by construction.

The numerical implementation of this system is straightforward by comparison with usual sim-
ulations of ordinary gauge systems: no Markov process is needed to perform importance sampling and there are no thermalization problems and no critical slowing down.

On the theoretical side, both the partition function $Z$ and the gauge group are trivial: $Z \equiv 1$, $G = \mathbb{Z}_1$. Some obvious questions arise: $i)$ does the model have a well-defined continuum limit? $ii)$ do the Wilson loops exhibit the Lüscher term and the other universal shape effects like in ordinary gauge theories? $iii)$ does the theory have a non-trivial glueball spectrum? $iv)$ is it possible to define a finite temperature deconfinement transition in the same percolation picture? The answer to all these questions is affirmative.

2. THE STRING TENSION

We estimated the string tension $\sigma$ by fitting the mean values of the Wilson loops associated to squares of side $R$ to the function

$$\langle W(R) \rangle = a R^{2+} \exp(-b R - \sigma R^2) \ . \quad (2)$$

The fits for not too small $R$ are very good (see Fig.1).

If the model under study has a well-behaved continuum limit, the scaling form of $\sigma$ near $p_c$ should be

$$\sigma(p) = \sigma_o (p - p_c)^{2\nu} \ , \quad (3)$$

where $p_c = 0.2488126(5)$ on the cubic lattice [2] and $\nu$ is the correlation length critical exponent of 3D percolation. We used the value $\nu = 0.8765(16)(2)$ [3]. In the range $0.258 \leq p \leq 0.270$ a one-parameter fit yields $\sigma_o = 8.90(3)$ with $\chi^2/d.o.f \sim 0.4$. The fit is reported in Fig.2.

Disregarding the “string” factor $R^{2+}$ in $\langle W \rangle$ yields much worse fits. This suggests that the percolation process could also account for the universal shape corrections ascribed to effective string fluctuations.

A suitable quantity which is sensible to these effects is

$$f = \exp(-n^2 \sigma) \frac{\langle W(L - n, L + n) \rangle}{\langle W(L, L) \rangle} \ , \quad (4)$$

which asymptotically (large $L$ and $L - n$) should be, in the effective string picture, a known function $f(t)$ of the ratio $t = \frac{n}{L}$ without any adjustable parameters [4]. Fig.3 shows a nice agreement to this conjecture.

![Figure 1](image1.png)

Figure 1. Expectation values of square Wilson loops $W(R)$ in a $64^3$ lattice at $p = 0.26 > p_c$ as a function of $R$. The solid line is a fit to Eq.2.

![Figure 2](image2.png)

Figure 2. The string tension as a function of $p$. The line is a one-parameter fit to Eq.3.
Figure 3. The quantity defined in Eq. (4) for three different values of \( p \). The line accounts for the universal shape effects due to effective string vibrations. No adjustable parameter is involved.

Table 1

| \( p \)  | \( m_a \)    | \( m/\sqrt{\sigma} \) |
|---------|-------------|----------------------|
| 0.26    | 0.2189(2)   | 3.75(3)              |
| 0.27    | 0.3870(5)   | 3.81(2)              |

3. GLUEBALLS

Though the occupied bond connected correlator is exactly zero by construction, the correlator among the occupied bonds belonging to a loop is non-zero, because of the constraint of being part of a closed path. The exponential decay rate of this correlator gives an estimate of the mass of the lowest scalar glueball state. We performed two numerical experiments at \( p = 0.26 \) and \( p = 0.27 \). The results reported in Tab. 1 show that the deviation from the expected scaling behavior is rather small.

4. DECONFINEMENT AT FINITE T

Though the usual argument for deconfinement at finite \( T \) based on spontaneous breaking of center symmetry here is inapplicable, because there is no way to break \( \mathbb{Z}_1 \), it is possible to show that the system under study goes through a continuous, finite temperature, deconfining transition.

The argument runs as follows. Consider a lattice endowed with a finite temperature geometry \( L \times L \times 1/T \) with fixed \( p > p_c \) and start to vary \( T \). At low temperature (\( L \sim 1/T \)) there is a percolating cluster, so the system is confining. As the temperature increases, the system becomes more and more dominated by the 2D geometry. The crucial point is now to observe that the percolation threshold is a decreasing function of the space dimension. This implies that there is a critical value \( T(p) \gg 1/L \) where the percolating cluster crumbles away and the system no longer confines. Clearly this transition belongs to the universality class of 2D percolation.

There is now an efficient Monte Carlo algorithm for studying percolation on any lattice [5]. It allows to calculate wrapping probabilities over the entire range of \( p \) in a single run. We applied this algorithm to evaluate the wrapping probability around the large \( L \) directions. We chose \( L = 40, 50, 64, 128 \) and \( 1/T = 6 \) or \( 1/T = 8 \).

Finite size scaling allows us to extrapolate the results to \( L \to \infty \) limit, where wrapping probability is equal to percolation threshold. The results are reported in Tab. 2.

Table 2

| \( T \) | \( p(T) \)     | \( T_c/\sqrt{\sigma} \) |
|-------|--------------|----------------------|
| 6     | 0.272355(5)  | 1.497(4)             |
| 8     | 0.265615(5)  | 1.510(4)             |

REFERENCES

1. 't Hooft, G., Nucl. Phys. B \textbf{B138} (1978) 1.
2. Lorenz, C.D. and Ziff, R.M. Phys. Rev. E \textbf{57} (1998) 230.
3. Ballesteros, H.G., Fernandez, L.A., Martin-Mayor, L.A.V., Munoz-Sudupe, A., Parisi, G. and Ruiz-Lorenzo, J.J., J. Phys. A \textbf{32} (1999) 1.
4. Caselle, M., Fiore, R., Gliozzi, F., Hasenbusch, M. and Provero, P., Nucl. Phys. B \textbf{486} (1997) 245.
5. Newman, M.E.J. and Ziff, R.M., Phys. Rev. Lett. \textbf{85} (2000) 4104.