Three-dimensional Forward-fit Modeling of the Hard X-Ray and Microwave Emissions of the 2015 June 22 M6.5 Flare

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Abstract

The well-established notion of a “common population” of the accelerated electrons simultaneously producing the hard X-ray (HXR) and microwave (MW) emission during the flare impulsive phase has been challenged by some studies reporting the discrepancies between the HXR-inferred and MW-inferred electron energy spectra. The traditional methods of spectral inversion have some problems that can be mainly attributed to the unrealistic and oversimplified treatment of the flare emission. To properly address this problem, we use a nonlinear force-free field (NLFFF) model extrapolated from an observed photospheric magnetogram as input to the three-dimensional, multiwavelength modeling platform GX Simulator and create a unified electron population model that can simultaneously reproduce the observed HXR and MW observations. We model the end of the impulsive phase of the 2015 June 22 M6.5 flare and constrain the modeled electron spatial and energy parameters using observations made by the highest-resolving instruments currently available in two wavelengths, the Reuven Ramaty High Energy Solar Spectroscopic Imager for HXR and the Expanded Owens Valley Solar Array for MW. Our results suggest that the HXR-emitting electron population model fits the standard flare model with a broken power-law spectrum ($E_{\text{break}} \sim 200$ keV) that simultaneously produces the HXR footpoint emission and the MW high-frequency emission. The model also includes an “HXR-invisible” population of nonthermal electrons that are trapped in a large volume of magnetic field above the HXR-emitting loops, which is observable by its gyrosynchrotron radiation emitting mainly in the MW low-frequency range.

Key words: methods: numerical – radiation mechanisms: non-thermal – Sun: flares – Sun: particle emission – Sun: radio radiation – Sun: X-rays, gamma rays

1. Introduction

It has been recognized that microwave (MW) and hard X-ray (HXR) emission observed during impulsive phases of solar flares show very similar temporal behaviors (White et al. 2011 and references therein). This signature in general suggests that these two emissions, although observed in very different spectral windows, are produced by a common population of particles under a process of energization during the impulsive phase of solar flares. The dominant emission mechanism in the two wavelengths is likely to be different: the HXR emission is produced by bremsstrahlung, and the MW emission is mostly produced by gyrosynchrotron (GS) radiation. The former is produced when high-speed electrons lose energy by collisions with more-stationary targets within the ambient plasma, producing photons with HXR energies. Thus, the HXR emission is mostly dependent on and tells us about the ambient plasma density at the flare site and the energy of the accelerated electrons that collide with them. The latter is produced when moving electrons gyrate due to the Lorentz force in the magnetized plasma. Thus, the MW emission is mostly dependent on the energy of mildly relativistic electrons and the strength of the magnetic fields around which these electrons gyrate, i.e., the magnetic field strength at the flare site.

Considering the two emission mechanisms, one may assume that the two observations should converge to the same energy spectrum for the accelerated electrons, and in some simple (single-loop) flares, this is the case (e.g., Fleishman et al. 2016b). However, there is a record of observations that seems to suggest otherwise; the indices of nonthermal electron energy spectra inferred from HXR and MW observations are different (Kundu et al. 1994; Silva et al. 2000). Generally, the studies that found a difference between the HXR-inferred and MW-inferred electron energy spectral index found that the latter is harder than the former by $\sim 2$ (Kundu et al. 1994; Silva et al. 2000). Some suggest that considering two different electron energy distributions residing in different energy ranges (i.e., there is a break in the spectrum) could solve this problem, since MW-emitting electrons are thought to have higher energy than those emitting in HXR (Takakura 1972; Kundu et al. 1994). There are some difficulties with inferring the electron energy spectrum from the observed HXR and MW spectrum, however. On the observational side, flares that enable us to invert the observed HXR photon spectrum to the electron energy spectrum extending above a few hundred keV have to be relatively large (e.g., high M- or X-class flares) to obtain enough photon counts above the background level at high energy, and large flares complicate the HXR spectral inversion because the thermal part of the HXR spectrum dominates the nonthermal part at lower energies, sometimes up to 30 keV or higher, thus leading to the inaccurate calculation of the number of nonthermal electrons. There have been some observations of giant flares showing the HXR photon spectrum extending to several hundred keV, but the results are mixed regarding the possible break in the electron energy spectrum from event to event. The 1980 June 4 event introduced in Dennis (1988) showed a hardening break of $\sim Z$ at $\sim 300$ keV in the HXR photon spectrum spanning from $\sim 20$ keV to $\sim 20$ MeV, while the 2002 July 23 event observed by the Reuven Ramaty High Energy Solar Spectroscopic Imager (RHESSI) up to 8.5 MeV did not show such a break (Lin et al. 2003; Smith et al. 2003; White et al. 2003). A “cold” 2002
March 10 flare showed a breakdown energy spectrum with the break energy $\sim 100$ keV in the photon spectrum (Fleishman et al. 2016a). For MW, the frequency resolution of most of the instruments that record the total solar radio intensity spectrum has been limited, and it has sometimes been difficult to determine an accurate turnover frequency, resulting in uncertainties in the spectral index of the optically thin part of the MW spectrum that is used to determine the nonthermal electron energy spectral index (Kundu et al. 1994; White et al. 2003; Kundu et al. 2009). The difference in the source locations of HXR and MW emissions (footpoint and/or above the loop top for the former, whole loop and/or footprint for the latter) further complicates the simultaneous analysis. On the modeling side, for MW, inverting the electron energy spectrum from the observed MW spectrum has in the past been oversimplified for quantitative analysis. Therefore, the shape of the energy spectrum of the common population of electrons producing HXR and MW could be different from event to event, and whether or not it has a break at some energy remains inconclusive.

In this study, we create one unified multi-loop electron population model\(^3\) that can simultaneously reproduce the observed HXR and MW images and spectra at one point in time during a flare. Unlike many previous studies, we use a realistic three-dimensional magnetic field data cube based on magnetic field measurements, positioned at the actual location of the active region at the time of the observation. We constrain the model by using observations from the highest-resolving instruments available in both HXR and MW wavelengths. In Section 2, we introduce these instruments and the HXR and MW observations obtained from them. In Section 3, we introduce the modeling platform and the workflow of our simulation for the flare. In Section 4, we present the results of the simulation with the quantitative parameters of the electrons. In Section 5, we discuss what the results suggest in terms of the spatial and energy distributions of high-energy electrons in the flare. We note that, since this simulation is a forward-fitting simulation, the model presented in this study is one of many possible solutions, although we endeavor to create the simplest possible model with the minimum number of electron populations needed to fit the data, as will be shown in Section 3.

2. Data

2.1. Instruments

In this study, we use four observational data sources to constrain the HXR and MW emission model: HXR images, the HXR spatially integrated spectrum, MW interferometric observations, and the MW spatially integrated spectrum. We also employ an observation-based three-dimensional magnetic field model. For the HXR images and spatially integrated spectrum, the data from RHESSI are used. RHESSI is capable of producing HXR images and spectra at photon energies from 3 to 400 keV with angular resolutions as fine as 2 arcsec, a spectral resolution of $\sim 1$ keV, and a temporal cadence of typically 4 s (the spacecraft rotation period, although shorter times are possible using demodulation techniques). For this study, collimators 2 and 4 were excluded due to their insensitivity to below $\sim 20$ keV and lack of segmentation, respectively, and the sensitivities of all the other collimators were lowered to as low as 76% of the launch value (the lowest being collimator 1). The MW data are taken from the newly expanded solar-dedicated radio array known as the Expanded Owens Valley Solar Array (EOVSA). Formerly known as the Owens Valley Solar Array (OVSA), EOVSA is currently being commissioned to have an unprecedented imaging spectroscopic capability in the frequency range of 2.5–18 GHz at more than 300 frequency channels, with a spatial resolution of $\sim 60$ arcsec/$f_{\text{GHz}}$ (finest $\sim 3.3$ arcsec at 18 GHz), 1 s time cadence, and four polarizations. At the time of the event used for this study, EOVSA was recording total radio flux intensity and cross-correlated amplitudes from nine baselines from seven antennas. We used the total intensity MW spectrum (the spatially integrated spectrum at 162 frequency channels within 2.5–18 GHz) and the relative visibilities (Gary & Hurford 1989) calculated from the cross-correlated amplitudes as a spatial constraint replacement for images, which were not available yet.

The cross-correlated amplitudes from an interferometric baseline can be converted into one-dimensional relative visibility spatial information to determine the characteristic source size in the direction of the baseline orientation, assuming a simple Gaussian source shape. We use the relationship (see Appendix A for derivation)

$$
\ln(V_{\text{Rel}}) = \ln\left(\frac{a_i}{\sqrt{\Delta a_{ij}}}\right) = -8.393 \times 10^{-11} B_i^2 d^2
$$

$$
= -9.325 \times 10^{-14} B_{\text{cm}}^2 d^2 f_{\text{GHz}}^2,
$$

where $a_i$ is the cross-correlation amplitude from the baseline consisting of antennas $i$ and $j$, $a_i$ and $a_{ij}$ are the autocorrelation total power amplitudes from antennas $i$ and $j$, respectively; $B_i$ is the projected baseline length measured in wavelengths; $d$ is the one-dimensional characteristic source size in arcseconds; $B_{\text{cm}}$ is the projected baseline length in cm; and $f_{\text{GHz}}$ is the observing frequency in GHz. Note that, with this definition, $V_{\text{Rel}}$ is independent of calibration because antenna-based gains cancel within $\frac{a_i}{\sqrt{\Delta a_{ij}}}$. This shows that if the plot of $\ln\left(\frac{a_i}{\sqrt{\Delta a_{ij}}}\right)$ versus $B_i^2$ shows a linear dependence with a negative slope, then the one-dimensional characteristic source size can be estimated by a simple relationship. The steeper the slope, the larger the source size in the direction of that baseline’s orientation. In reality, the source could be elongated more in one direction than another. Such size variance will be projected onto the baselines with different orientation angles; the relative visibility plots from different baselines with different orientations can reveal the characteristic source size in different directions. At the time of the event for this study, the prototype correlator was not producing valid auto-correlations. Therefore, we used the cross-correlated amplitudes from the shortest baseline as a proxy for auto-correlated total power amplitudes for each baseline (i.e., $\sqrt{a_{ij}a_{jj}} \sim a_{i2}$, where antennas 1 and 2 constituted the shortest baseline within the array). The length of this baseline at the time of the observation was $B_1 \sim 508$ at the highest frequency (18 GHz), which yields the minimum fringe spacing of $\sim 400$ arcsec. This is confirmed to be well above the size of the target active region, which means that this

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\(^3\) The model and the documentation explaining how to reproduce the results presented in this paper are available at http://www.ioffe.ru/LEA/SF_AR/models/3dmodel3.html.
baseline should not be resolving any flaring sources. Therefore, the cross-correlated amplitude from this baseline can be used as a good approximation for the total power from each antenna, although this approximation now has the disadvantage that $V_{\text{rel}}$ is no longer independent of calibration. We use the frequency dependence of the baseline length for individual antenna pairs to convert $V_{\text{rel}}(\lambda)$ to $V_{\text{rel}}(B_\lambda)$ and will compare the observed and simulated visibilities at different frequencies. This approach accounts for the possible variation of source size with frequency.

For the magnetic field model, we used the nonlinear force-free field (NLFFF) model extrapolated from the SHARP Cylindrical Equal-Area (CEA) photospheric vector magnetogram from the Helioseismic and Magnetic Imager (HMI; Schou et al. 2012) onboard the Solar Dynamics Observatory (SDO; Pesnell et al. 2012) (available every 12 minutes). The net force and torque in the observed photospheric field are first minimized by a preprocessing procedure in order to obtain the chromosphere-like data that meets the force-free condition (Wiegelmann et al. 2006); for advantages and disadvantages of this approach, see Fleishman et al. (2017a). The weighted optimization method (Wheatland et al. 2000; Wiegelmann 2004) is then applied to the preprocessed photospheric boundary to perform the NLFFF extrapolation within a box of $256 \times 180 \times 200$ uniform grid points, corresponding to $\sim 230 \times 160 \times 180$ Mm$^3$. The performance of the extrapolation was verified by visually comparing the model field lines with the 171 Å channel images from the Atmospheric Imaging Assembly (AIA; Lemen et al. 2012) onboard the SDO.

2.2. Observations

In this study, we analyze a well-observed M6.5 flare that occurred on 2015 June 22 (Jing et al. 2016; Liu et al. 2016; Wang et al. 2017). The flare started around 17:50 UT, reached the soft X-ray (SXR) peak (the 1.0–8.0 Å channel of the Geostationary Operational Environment Satellite (GOES) X-ray monitor) at around 18:23 UT, and subsided to background level at around 02:00 UT the next day. Figures 1(a)–(c) show the light curves from GOES, RHESSI, and EOVSA for this flare from 17:36 to 18:43 UT. There were two precursors at around 17:24 and 17:42 UT (Wang et al. 2017; only the latter is shown in Figure 1). The parent active region 12371 was located at N13W06 at the time of the flare.

As seen in Figure 1(b), RHESSI missed most of the impulsive phase of the flare due to its passage through the South Atlantic Anomaly (SAA). However, EOVSA coverage during this period shows multiple sharp peaks leading to the GOES SXR maximum. In this study, we must choose a time covered by both RHESSI and EOVSA for the model. Since HXR images are crucial in this modeling, as will be shown in a later section, we choose the earliest time at which RHESSI’s HXR 25–50 keV light curve showed peaky behavior after the spacecraft came out of the SAA. This was 18:05:32 UT, which is indicated by the red vertical line in Figure 1. The total power dynamic spectrum from EOVSA containing this peak, from 18:03 to 18:08 UT, is shown in Figure 1(d). Its smooth time and frequency variation suggest that the emission is dominated by a GS mechanism over the entire frequency spectrum. The dominance of GS emission is expected at these frequencies based on statistics of MW bursts (Nita et al. 2004a, 2004b), which show that coherent emission mechanisms are rare above 2.6 GHz.

Figure 2(a) shows RHESSI image contours (50%, 70%, and 90%) in 6–12, 20–35, and 50–75 keV obtained at 18:05:32 UT, integrated over 2 minutes, reconstructed with the CLEAN algorithm, and overlaid onto the HMI line-of-sight (LOS) magnetogram taken at 18:04:26 UT. The image shows clear double-footpoint emissions in 50–75 keV rooted in regions of opposite magnetic field polarity. The region joining these footpoints is filled by a 6–12 keV source, presumably coming from the loop filled with heated chromospheric plasma, although it cannot be determined if this loop is footed exactly at the 50–75 keV sources. The intermediate-energy 20–35 keV image shows an interesting morphology: two of the sources are nearly cophasal with the 50–75 keV footpoint sources, but one is located between them, giving the appearance of three sources tracing out one loop that connects them. The centroid of this middle 20–35 keV source and that of the 6–12 keV source (the center of its northeastern bulge) seem to be slightly shifted, and since there seems to be no region of strong magnetic field corresponding to the location of the middle 20–35 keV source (in contrast to the case of the 50–75 keV source), we interpret this source as the so-called above-the-loop-top (ALT) HXR source (e.g., Masuda et al. 1994). We estimated the difference between the centroid of this source and that of the 6–12 keV source based on the field-line geometry of the NLFFF extrapolation model and found that it is about 20,000 km. This is in agreement with past studies that measured the size of the current sheet formed between the coronal HXR source and thermal loop-top source in the flare of similar magnitude (M1.2, 17,500–33,000 km; Sui & Holman 2003). It is possible that this source is a thermal source, and we will briefly discuss the variation in the model corresponding to this interpretation in a later section (Section 4.3). Figure 2(b) shows the background-subtracted (the background time range was 17:11–17:18 UT) RHESSI HXR photon spectrum created by the Object Spectral Executive (OSPEX; Schwartz et al. 2002) software, integrated over 8 s centered at 18:05:32 UT. The spectral fit was done using OSPEX, with a single-temperature thermal bremsstrahlung radiation function (“vth”) and the thick-target nonthermal bremsstrahlung with an isotropic pitch-angle distribution (“thick2”). The fitted parameters and goodness-of-fit value are listed in Table 1.

Figure 3(c) shows the background-subtracted (the background’s time range was 17:47:43–17:48:33 UT) MW total intensity spectrum taken from EOVSA at 18:05:32 UT (red). The spectra before and after this time, 17:58:48 and 18:12:28 UT, indicated by short magenta and green vertical lines Figure 1(c), are also shown in magenta and green, respectively. In Figure 3(a), the green curve shows the ln($V_{\text{rel}}$) versus $B_\lambda^2$ plot from the longest baseline, which is able to resolve the smallest feature among all baselines, and the green curve in Figure 3(b) shows one of the shorter baselines with a different orientation angle. As introduced in the previous section, if this plot shows a linear dependence, then from the slope we can estimate the one-dimensional characteristic source size in the direction determined by the baseline orientation: $-53^\circ$ for the former and $71^\circ$ for the latter, measured clockwise from the heliocentric-Cartesian x-axis. Note that $\gamma$ values above zero (at the low $B_\lambda^2$ end) are not considered in the analysis for both baselines, since the numerator of $\frac{a}{\sqrt{B_\lambda^2}}$ should not be larger than the denominator in general. We believe that they are
coming from the effect of dissimilar calibration at those frequencies among four antennas involved. The longer baseline shows a linear dependence in $B_{l} \lesssim 7 \times 10^{7}$ (corresponding to $6 < f_{\text{GHz}} < 15$, as $B_{l}$ has a one-to-one correspondence with frequency as shown in Equation (1)), whose slope indicates a size of $\sim$16 arcsec (the least-squares fit to the range determined by eye). There seems to be a steeper slope in the lower $B_{l}$ range, although it is difficult to determine the exact source size because of the scatter in the data points (estimates vary between $\sim$20 and $\sim$35 arcsec, depending on the fit). The plot above $B_{l}^{2} \sim 7 \times 10^{7}$ looks rather flat or even increasing, which would indicate that there is a new source with competing intensity but different characteristic size appearing above this frequency. Since such a judgment is not possible unless we extend the plot beyond the high-frequency limit of the instrument, we will not use this part of the relative visibility plot for the observational constraint for our model. The shorter baseline also shows a relatively straight slope in $4 \times 10^{5} < B_{l}^{2} < 1.5 \times 10^{6}$ ($8 < f_{\text{GHz}} < 15$), which corresponds to the source size of $\sim$27 arcsec. In summary, these two plots

Figure 1. (a)–(c) Light curves from GOES, RHESSI, and EOVSA, respectively, for the 2015 June 22 X6.5 flare. RHESSI’s data gap is due to its passage through the SAA, and the drop at $\sim$18:27 UT is due to the spacecraft’s night time. The vertical red line indicates 18:05:32 UT, the time at which the simultaneous modeling of the HXR and MW observations was conducted. The total intensity spectra taken at this time, at 17:58:48 UT (short vertical magenta line) and 18:12:28 UT (short vertical green line), are shown in Figure 3(c). (d) Portion of the EOVSA dynamic spectrum (162 frequency channels in the 2.5–18 GHz range, 1 s time resolution) corresponding to the 18:03–18:08 UT time range indicated by the two vertical black lines in the light-curve plot above.
tell us that our target source has a simple, slightly elongated shape from $\sim$8 to $\sim$15 GHz and perhaps a slightly larger size in $\leq$6 GHz in the $\pm 53^\circ$ direction.

3. Simulation Platform and Workflow

As mentioned in the Introduction, in this study, we create a sophisticated three-dimensional model that places within a realistic three-dimensional magnetic field data cube a set of electron populations that reproduce observed HXR and MW emissions. The simulation platform we use is the GX Simulator (Nita et al. 2015, 2017). It is an IDL-based, graphical user interface platform that has a highly diverse functionality, of which we employ the following here.

First, we import an externally defined NLFFF magnetic field model into the simulator. Then, we investigate the magnetic field topology and create magnetic flux tubes using the observed HXR images for guidance. This flux tube construction process is an iterative process, and we try to find the magnetic field model that contains the field lines that best connect the observed HXR 50–75 keV footpoint sources by eye. After several trial-and-error iterations, we select the model cube extrapolated at 18:24 UT. The models at other times—for example, the one at 18:00 UT—had too much shear in the overall field-line geometry, and we could not obtain the desired source connectivity. Since 18:24 UT is later than our modeling time (18:05:32 UT), this cube must contain more post-reconnection loops, which should be more suitable for modeling MW emissions that presumably come from trapped electron populations. The flux tubes are developed using these central guiding field lines.

Next, we populate the flux tube with thermal and nonthermal populations of electrons, defined to have the required spatial and energy distribution functions. Here the spatial distributions of the electrons are assigned based on the observed HXR images in Figure 2(a), and the energy distribution functions are

![Figure 2](image-url)

**Figure 2.** (a) RHESSI CLEAN image contours (integrated over 2 minutes) at 18:05:32 UT, overplotted onto the HMI LOS magnetogram taken at 18:04:26 UT. (b) RHESSI HXR photon spectrum taken at 18:05:32 UT, accumulated over 8 s using the front end of collimators 3, 5, 7, and 8. The background was taken from 17:11 to 17:18 UT. The spectral fit was done combining the vth and thick2 functions in 12–67 keV (dashed lines). The goodness-of-fit value was 2.20, with the normalized residuals plotted at the bottom.

| Parameter                        | vth + thick2                  |
|----------------------------------|-------------------------------|
| **Thermal (vth)**                |                               |
| EM                               | $1.3 \times 10^{49}$ cm$^{-5}$|
| Temperature                      | $2.0 \times 10^7$ K           |
| **Nonthermal (thick-target)**    |                               |
| Total integrated electron flux   | $2.7 \times 10^{39}$ s$^{-1}$ |
| $E_{\text{cutoff}}$              | 22.1 keV                      |
| $\delta_1$                      | 3.3                           |
| $E_{\text{Ed}}$                  | 36 keV                        |
| $\delta_2$                      | 5.5                           |
| $E_{\text{max}}$                 | 32,000 keV                    |
| $\chi^2$                        | 2.20                          |

**Table 1** Summary of the OSPEX Fitted Parameters for the Photon Spectrum Taken at 18:05:32 UT, with vth + thick2

*Note.* The upper energy limit of 32,000 keV was chosen arbitrarily and fixed during the fit.
Figure 3. (a) and (b) EOVSA’s $\ln(V_{\text{Re}})$ vs. $B_A^2$ plot calculated from the cross-correlated amplitudes taken from the longest baseline (a) and one of the shorter baselines (b) that were available on 2015 June 22. The straight negative slope can be used to calculate the characteristic source size in the direction of the baseline orientation ($\theta$, clockwise from the heliocentric-Cartesian x-axis). Note that y values above zero are not considered in the analysis (see Section 2.2). The red lines are the least-squares fits to the ranges of $B_A^2$ determined by eye (each corresponding to $6 < \lambda_{H\alpha} < 15$ and $8 < \lambda_{H\alpha} < 15$, respectively, as $B_A$ has a one-to-one correspondence with frequency). Here $d$ is the one-dimensional characteristic source size calculated from those fitted slopes. Blue curves are those calculated from the model (described later). (c) EOVSA background-subtracted total intensity spectrum plot taken at 17:58:48 (magenta), 18:05:32 (red, modeled time), and 18:12:28 UT (green). These times are also marked by three vertical lines (magenta, red, and green) in Figure 1(c).

$\theta = -53^\circ$
$d \sim 16''$  

$\theta = 71^\circ$
$d \sim 27''$

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assigned guided by OSPEX’s spectral fit to the observed HXR spectrum in Figure 2(b). For spatial distribution, we use the default functions provided by the simulator (see Appendix B). In cases where some modification is necessary, the simulator allows users to define the electron spatial distribution functions separately for thermal and nonthermal electron populations. For particle energy distribution, the simulator provides a predefined list of well-known functions, such as thermal–plus–single power law, thermal–plus–double power law, Kappa, and others, as well as the choices of isotropic or anisotropic pitch-angle distribution (the full list is provided in Fleishman & Kuznetsov 2010; Nita et al. 2015, 2017). The simulator also has the ability to define chromospheric layers with parameters such as plasma density, temperature, and depth. For this study, the initial parameters for the energy distribution for each of the electron populations were taken from the RHESSI spectral fit that was introduced in Section 2.2, which is a thermal–plus–double power law consisting of single-temperature thermal bremsstrahlung and thick-target nonthermal bremsstrahlung radiation functions. The thermal and nonthermal parts were appropriately assigned based on our interpretation of the nature of the observed sources.

Then, we generate 2D HXR and MW images and spectra through calculations using internal codes. The simulator’s HXR code calculates the observable flux of HXR photons at 1 au by summing the combination of thermal and nonthermal bremsstrahlung radiation from each voxel. The thermal part of the X-ray code calculates the total bremsstrahlung power radiated from the plasma with a single temperature $T$, taking into account collisions with hydrogen and other atoms, free-free and free-bound transitions, and various line emissions (Schwartz et al. 2002). The nonthermal code uses the instantaneous bremsstrahlung expression for voxels located at coronal height and the thick-target bremsstrahlung expression for voxels located at the height of the transition region. The expression for the former is

$$I(\epsilon, r) = \frac{n_p(r) V}{4\pi R^2} \int_0^\infty Q(E, \epsilon) F(E, r) dE, \quad (2)$$

where $n_p$ is the plasma proton density, $R$ is the 1 au distance, $V$ is the volume of the voxel, $\epsilon$ is the photon energy, $F(E, r)$ is the electron flux density distribution over energy in the given voxel, and $Q(E, \epsilon)$ is the angle-averaged bremsstrahlung cross section introduced by Haug (1997). The expression for the latter adopts the OSPEX representation of the thick-target model, which is based on the theory described in Brown (1971) but without the consideration of anisotropic pitch-angle distribution. The electron number density in each interface voxel is converted to the electron flux $F(E) = \int_{E}^{\infty} Q(E, \epsilon) F(E, r) d\epsilon$, where $A$ is the surface area of the top of the highest chromospheric voxel and $v(E)$ is the velocity of the electron with energy $E$. Then, provided that the HXR emission is optically thin, the individual contributions are added up along the LOS to form a set of HXR images at various energies. The emission can then be integrated over the image to yield the total power HXR spectrum for comparison with Figure 2(b). Currently, the HXR code only calculates electron-ion bremsstrahlung and does not account for Compton scattering or photoelectric absorption of HXRs in the solar
atmosphere, with the latter known to produce a broad hump on the photon spectrum around 30–50 keV (Bai & Ramaty 1978).

The simulator’s radio emission code is based on the fast GS algorithm developed by Fleishman & Kuznetsov (2010), which accounts for GS and free–free radio emission and absorption in a thermal plasma (e.g., the Razin effect is included) within the modeled cube (vacuum outside). The fast GS code is a generalization of a numerical Petrovian–Klein (PK) approximation of the exact GS equations, which is more precise than that approximation and also valid for an anisotropic pitch-angle distribution. It can reproduce discrete harmonic structures at low frequencies if requested by the user or averages over them otherwise. The 2D image at a given frequency is calculated by solving the radiative transfer equation along the LOS, and it includes frequency-dependent mode coupling in addition to emissivity and absorption.

Lastly, we compare the simulated HXR and MW images and spectra with the observed images and spectra. The simulator has the ability to convolve the pixelated two-dimensional model image with a user-defined point-spread function/beam, which enables the user to directly compare the model images to the observed images that go through instrumental responses. For HXR, the model images are convolved with a Gaussian point-spread function with FWHM of 6.79 arcsec, according to the nominal FWHM resolution of the finest resolving collimator that was used to reconstruct the observed RHESSI image (collimator 3). For MW, the model visibility was obtained by convolving the visibility of the pixelated model image from the simulator with the sampling function of the EOVSIA array at the modeling time. We fine-tune the model in HXR first, then in MW. The HXR model images and spectra are first created and tested against the observed HXR emission spectra and images by visual inspection. The parameters that are allowed to vary during the fine-tuning process are the plasma temperature and spatial distribution parameters for the thermal population and the spatial and energy distribution parameters for the nonthermal population (see Table 2). During the fine-tuning process, the emission measure is constrained by comparing the emission measure calculated by the simulator (the square of the thermal particle density integrated over the model volume) with the emission measure calculated from the OSPEX vih function. After fine-tuning this HXR-constrained model, we use it to produce the model MW images and spectra, which are later tested against the observed MW relative visibility and spatially integrated spectrum. The same parameters are allowed to vary, but in this step, we have the freedom to choose an additional flux tube if necessary, as long as the relative visibility calculated from the model images agrees with the observed relative visibility. The modeled relative visibility is compared to the observed relative visibility by eye. The modeled spectrum is fine-tuned so that the numerical value calculated from the difference between the modeled spectrum and the observed spectrum (standard deviation) stays as small as possible without causing a significant mismatch in relative visibility (i.e., even if we obtained a model that shows a better numerical match in the MW spectrum than in previous models, if this resulted in a significant mismatch in relative visibility, this model was rejected and we rolled back to the previous model). This MW-constraining step slightly alters the match obtained in the HXR fine-tuning, so another small HXR fine-tuning is run again, and so on. We iterate these fine-tunings several times to converge to the unified model that can simultaneously reproduce the observed HXR and MW images and spectra. This workflow, which is based on the framework introduced by Gary et al. (2013), is illustrated in Figure 4.

4. Model Construction

4.1. Constraining in HXR

As the first step in this modeling workflow, we created three flux tubes to reproduce HXR emission in three different energy channels, guided by the RHESSI HXR images. We could not reproduce all observed sources with a one- or two-loop model because the field line rooted in the 50–75 keV footprint source locations did not trace out the 6–12 keV source or cross the 20–35 keV ALT source.

Figure 5 shows each flux tube and its respective thermal and nonthermal populations. Note that we will be representing the non-ALT 20–35 keV source by the footpoint emissions from the flux tube representing the 50–75 keV sources, since they are spatially close to the 50–75 keV footpoint source locations in the RHESSI image and creating another loop footed at the non-ALT 20–35 keV sources will overcomplicate the model. The first flux tube, in Figure 5(a), represents the 6–12 keV source, lying low in the corona with an apex height of $\sim$8200 km from the photosphere and filled with a thermal population slightly concentrated at the loop top. We note that it was necessary for us to choose such a low-lying structure based on the observed 6–12 keV image; any field lines higher than the selected field line resulted in the misorientation of the model source compared to the observed source. We call this loop the “thermal-only loop.” For the thermal population, we kept the temperature predicted by OSPEX (20 MK) but altered the vertical density spatial distribution by adding a simple Gaussian-like function to the default function (Equation (13)) to match the observed source shape. Then, we fine-tuned the density so that the thermal part of the HXR spectrum is well reproduced. As a double check of the model validity, we checked that the emission measure calculated from the simulator is consistent with the one predicted from OSPEX. As a result, we found that the thermal population has a density of $1.1 \times 10^{11}$ cm$^{-3}$ at the bottom and $1.6 \times 10^{11}$ cm$^{-3}$ at the top of the loop. We assign this flux tube zero nonthermal particles, assuming that this loop is dominated by thermal plasma from chromospheric evaporation—an expected outcome from earlier episodes of particle acceleration.

The second flux tube, Figures 5(b) and (e), represents the 50–75 keV double-footpoint sources. Both thermal and nonthermal populations are assigned with default spatial distribution functions (essentially uniform for the former and a simple Gaussian-like distribution for the latter) in the loop with a height of $\sim$21,000 km from the photosphere. Note that the HXR emission from this loop will be dominated by footpoints even though the nonthermal electrons are filling the loop, due to a dense chromospheric layer within the magnetic field cube (defined with plasma density $n_0 = 10^{13}$ cm$^{-3}$, $T = 3500$ K, and a depth of $\sim$2000 km, the default values assigned by the simulator). We call this loop the “lower loop” and assign its thermal populations a density of $n_0 = 5.0 \times 10^{6}$ cm$^{-3}$, which is the default value of the simulator, about an order of magnitude lower than the value assigned for the thermal-only loop. For the temperature, we assign 20 MK. These two values are, however, not strictly constrained, since the emission measure is solely
Table 2
Summary of the Modeled Parameters for All Four Loops in the Final Model

| Loop Designation | Radius (grid pts) | \(B_{\text{max}}\) (G) | \(B_{\text{min}}\) (G) | \(n_0\) (cm\(^{-3}\)) | \(T\) (MK) | \(p_0\) | \(p_1\) | \(q_0\) | \(q_2\) | \(n_p\) (cm\(^{-3}\)) | \(p_0\) | \(p_1\) | \(q_0\) | \(q_2\) | \(E_{\text{cutoff}}\) (keV) | \(\delta_1\) | \(E_{\text{break}}\) (keV) | \(\delta_2\) | \(E_{\text{max}}\) (MeV) |
|------------------|------------------|------------------|------------------|------------------|------------------|------------------|------------------|------------------|------------------|------------------|------------------|------------------|------------------|------------------|------------------|------------------|------------------|------------------|
| Thermal-only     | 5                | 856              | 1925             | \((1.1-1.6) \times 10^{11}\) | 20               | 0.6              | 3                | 1.3\(^a\) | \(-0.28\)          | 0                | ...              | ...              | ...              | ...              | ...              | ...              | ...              | ...              |
| Lower            | 10               | 401              | 1847             | \(5.0 \times 10^{9}\) | 20\(^b\)         | 2                | 2                | ...              | ...              | \(3.0 \times 10^{7}\) | 2                | 2                | 3.9              | 0.02             | 22.1             | 5.0              | 200\(^e\)       | 3.0              | 10              |
| Higher           | 10               | 250              | 1984             | \(5.0 \times 10^{9}\) | 20\(^b\)         | 2                | 2                | ...              | ...              | \(4.5 \times 10^{9}\) | 2                | 2                | 15               | 0                | 22.1             | 3.6              | 43               | 6.8              | 10              |
| Overarching      | 23               | 85               | 225              | \(5.0 \times 10^{9}\) | 20\(^b\)         | 2                | 2                | ...              | ...              | \(6.1 \times 10^{10}\) | 3.5              | 3.5              | 16               | 0                | 22.1\(^e\)       | 2.5              | ...              | ...              | 5               |

Notes. The ranges for the parameters that are not strictly constrained are introduced in footnotes b–e.

\(^a\) Modified from the default function by adding a simple Gaussian-like function used for the nonthermal population (see the Appendix, Equation (13)).

\(^b\) The lower and higher loops: \(T < 50\) MK and \(n_0 < 2 \times 10^{10}\) cm\(^{-3}\). The overarching loop: \(T < 30\) MK and \(n_0 < 2 \times 10^{10}\) cm\(^{-3}\). The thermal parameters for these loops are not strictly constrained, since the emission measure is solely constrained by the thermal-only loop. See Section 4.3 for details.

\(^c\) Range: \(180\) keV \(\lesssim E_{\text{break}} \lesssim 220\) keV. The choice of \(E_{\text{break}} = 200\) keV is arbitrary, since this is above the statistically significant energy range of the HXR observation (67 keV). See Section 4.3 for details.

\(^d\) Range: \(\sim 10^9 < n_0 < 2 \times 10^{10}\) cm\(^{-3}\). The lower limit is considered based on the upper limit of the thermal density, while the upper limit is equal to the upper limit of the thermal density. See Section 4.3 for details.

\(^e\) Range: \(4 \times 10^9 < n_0 < 6.7 \times 10^9\) cm\(^{-3}\), with \(E_{\text{cutoff}} \sim 600\) keV corresponding to \(n_0 \sim 4 \times 10^9\). The nonthermal electron energy spectrum for the overarching loop cannot be constrained under several hundred keV, since the emission from this loop is only visible in the MW low-frequency range. See Section 4.3 for details.
constrained by the thermal-only loop. We therefore consider the allowable range for these two parameters later. For the energy distribution, we kept the lower cutoff energy of 22.1 keV from OSPEX and fine-tuned the density to be $3.0 \times 10^7$ cm$^{-3}$ and the low-energy spectral index to be 5.0 in order for the thick-target emission from the footpoint to match the observed spectrum. Furthermore, we assigned a harder spectral index of 3.0 for energy above 200 keV to simultaneously reproduce the negative spectral slope in the high-frequency part of the observed MW spectrum by this population.

The third flux tube, Figures 5(c) and (f), represents the 20–35 keV ALT source, with thermal electrons uniformly
distributed (Figure 5(c)) and nonthermal electrons highly concentrated near the highest point of the loop (Figure 5(f)), which is \( \sim 27,300 \text{ km} \) above the photosphere. We call this loop the “higher loop.” We assign the thermal population of this loop the same parameters as the lower loop \((T = 20 \text{ MK} \text{ and } n_0 = 5.0 \times 10^9 \text{ cm}^{-3})\). However, the density of the nonthermal electrons concentrated at the loop top is set to be much higher than that of the lower loop: \(4.5 \times 10^8 \text{ cm}^{-3}\). This high density is required to make the ALT source intensity competitive with the footpoint emissions produced by the lower loop, which are innately bright due to the interaction of their nonthermal particles with the dense chromosphere (thick-target emission). Furthermore, we found that this dense source has to be concentrated energetically as well, since the ALT source does not appear at all in the 50–75 keV image. This requires the nonthermal energy distribution to have a double power law with a softening break of +3.2 at 43 keV.

**Figure 5.** Magnetic flux tubes and corresponding thermal and nonthermal electron populations placed within the NLFFF extrapolation cube taken at 18:24 UT, based on the RHESSI image from Figure 2(a). (a) Thermal population occupying the flux tube representing the 6–12 keV source, slightly concentrated at the top of the loop. (b) and (c) Thermal population occupying the flux tube representing the 50–75 keV double-footpoint source and 20–35 keV ALT source, respectively. (d) Central field lines of three flux tubes shown together within the model. (e) Nonthermal population occupying the flux tube representing the 50–75 keV double-footpoint sources. The footpoint will be enhanced in the model HXR image, since a dense chromosphere (not shown) will be included in the final calculation. (f) Nonthermal population occupying the flux tube representing the 20–35 keV ALT HXR source, highly concentrated at the top of the loop. The nonthermal population for the flux tube (a) is not shown because this loop is assigned with zero nonthermal electron density, assuming that it is dominated by thermal electrons. Note that colors are used only for visual purposes and are scaled individually for each plot.

**Figure 6.** Comparison between the observed and modeled image contours (50%, 70%, and 90%) in three HXR photon energy ranges. The observed image contours are the same as those in Figure 2(a). The model images are produced as pixelated images by the simulator, so they are further convolved with a Gaussian point-spread function with the size according to the nominal FWHM resolution of the finest RHESSI collimator used in image reconstruction (collimator 3; 6.79 arcsec).
visible,” i.e., does not emit significantly in HXR. This is the “overarching loop” that we investigated. This source cannot be created by a thermal population, since the observed MW spectrum’s optically thick flux density requires a high brightness temperature (>~100 MK), which is unrealistically high for a true thermal temperature. Therefore, this source must be created by a nonthermal population. As mentioned in Section 2.2, the emission over the entire frequency range is produced by a GS mechanism. For the GS emission from nonthermal particles to fill the low-frequency part of the spectrum, the magnetic field strength has to be relatively weak to lower the peak frequency. Considering this, together with the abovementioned condition that the area must be larger than the HXR-constrained loops, we created a flux tube that is located above the HXR-emitting loops and has an ~10–12 times larger volume than the two other nonthermal loops. Figure 9 shows our final four-loop model that includes the three HXR-constrained loops and the MW-constrained overarching loop. We initially assigned to the overarching loop the identical thermal particle population as the other two loops (T = 20 MK and $n_0 = 5.0 \times 10^6$ cm$^{-3}$). Fine-tuning this model loop requires the use of the relative visibility plots from Figures 3 (a) and (b), and after several iterations (including the second fine-tuning in HXR in both images and spectrum), we obtained a nonthermal population that has a density of $6.1 \times 10^6$ cm$^{-3}$ concentrated at an intermediate height in the loop, where the magnetic field is weak. The concentrated density spatial distribution was required by the characteristic source-size constraint derived from the observed relative visibilities. The relative location of this population within the flux tube, which is close to the three HXR-emitting loops (Figure 9, right), suggests that this population may have been part of the population occupying the HXR-emitting loops but was transported and accumulated in the region of weaker magnetic field. Its nonthermal energy distribution is found to have a spectral index of 2.5, if we assume a single power law, and a cutoff energy of 22.1 keV like the other two nonthermal loops. Figure 3 (blue curves) shows the ln($V_{rel}$) versus $B_2^2$ plots from the model. It is evident that the slopes of the model plots are in reasonable agreement with those of the observation in both baselines with different orientation angles, validating that our model is successfully reproducing the observed source size in two different directions. A slight size difference between the model and the observation in the shorter baseline (b) is calculated to be about 4 arcsec.

The HXR emission contribution from this loop is evidently lower than that from the other three HXR-constrained loops (Figure 7, yellow), and it is so compared to the lower loop because of the difference in the emitting region: chromosphere for the lower loop and high corona for the overarching loop. The former is thick-target emission, while the latter is thin-target emission. Compared to the higher loop, on the other hand, the emission is smaller in the overarching loop simply due to the lower nonthermal particle density; both populations are concentrated within the coronal part of the loop, but the higher loop has $10^8$ cm$^{-3}$ nonthermal electrons, while the overarching loop has $10^6$ cm$^{-3}$ nonthermal electrons. The modeled emission intensity from the overarching loop is lower than the observed emission intensity by an order of magnitude or more for most of the statistically meaningful energy range (less than ~67 keV), and this may be the reason this source is “invisible” in HXR, as the dynamic range of RHESSI is about significant to the HXR-constrained loops, we created a flux tube that is located above the HXR-emitting loops and has an ~10–12 times larger volume than the two other nonthermal loops. Figure 9 shows our final four-loop model that includes the three HXR-constrained loops and the MW-constrained overarching loop. We initially assigned to the overarching loop the identical thermal particle population as the other two loops (T = 20 MK and $n_0 = 5.0 \times 10^6$ cm$^{-3}$). Fine-tuning this model loop requires the use of the relative visibility plots from Figures 3 (a) and (b), and after several iterations (including the second fine-tuning in HXR in both images and spectrum), we obtained a nonthermal population that has a density of $6.1 \times 10^6$ cm$^{-3}$ concentrated at an intermediate height in the loop, where the magnetic field is weak. The concentrated density spatial distribution was required by the characteristic source-size constraint derived from the observed relative visibilities. The relative location of this population within the flux tube, which is close to the three HXR-emitting loops (Figure 9, right), suggests that this population may have been part of the population occupying the HXR-emitting loops but was transported and accumulated in the region of weaker magnetic field. Its nonthermal energy distribution is found to have a spectral index of 2.5, if we assume a single power law, and a cutoff energy of 22.1 keV like the other two nonthermal loops. Figure 3 (blue curves) shows the ln($V_{rel}$) versus $B_2^2$ plots from the model. It is evident that the slopes of the model plots are in reasonable agreement with those of the observation in both baselines with different orientation angles, validating that our model is successfully reproducing the observed source size in two different directions. A slight size difference between the model and the observation in the shorter baseline (b) is calculated to be about 4 arcsec.

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We now discuss the possible ranges of some of the modeled electron parameters that were not strictly constrained in the model construction process presented above. First, the density and temperature of the thermal population for the three nonthermal loops (the higher, lower, and overarching loops) are not strictly constrained, since the emission measure of the entire model is solely constrained by the thermal-only loop. We obtain the ranges for the density and temperature by fixing one of the two parameters at the values assigned during the model construction and varying the other (the target parameter) until noticeable changes start to appear in the spatially integrated HXR model spectrum. We separately test the overarching loop and the combination of the lower and higher loops, since their effective volumes are largely different. For the combination of the lower and higher loops, we find the upper limit of the temperature to be \( \sim 50 \text{ MK} \) (excess in \( \sim 10-20 \text{ keV} \)) and of the density to be \( \sim 2 \times 10^{10} \text{ cm}^{-3} \) (excess in \( \sim 20-40 \text{ keV} \)). For the overarching loop, we find the upper limit of the temperature to be \( \sim 30 \text{ MK} \) (excess in \( \sim 10-20 \text{ keV} \)) and of the density to be \( \sim 2 \times 10^{10} \text{ cm}^{-3} \) (excess below \( \sim 20 \text{ keV} \)). We find virtually no lower limit for the two parameters, since any emission measure values below the observational constraint are allowed for these loops.

For the nonthermal population, the density of the higher loop and the minimum energy of the overarching loop are not strictly constrained. For the nonthermal density of the higher loop, we must consider its lower limit corresponding to the upper limit for the thermal density to keep the same level of nonthermal bremsstrahlung from the thin-target coronal source. We find this lower limit to be \( \sim 10^8 \text{ cm}^{-3} \). The upper limit should be that of the thermal density, \( \sim 2 \times 10^{10} \text{ cm}^{-3} \). For the overarching loop, the electron spectrum was modeled only in the MW range, and since the effective energy of the MW-emitting electrons is above several hundred keV, the spectrum below this energy cannot be strictly constrained. We obtain the more strictly constrained energy range and the number density by testing several different values of cutoff energy and the corresponding normalized number density. As a result, we find that at least \( 4 \times 10^7 \text{ cm}^{-3} \) nonthermal electrons above \( E_{\text{cutoff}} \sim 600 \text{ keV} \) are needed to match the low-frequency MW observations.

Lastly, we note that our proposed \( E_{\text{break}} = 200 \text{ keV} \) for the lower-loop nonthermal population is rather arbitrary, since this is above the statistically meaningful energy range of the \textit{RHESSI} observation (67 keV). To investigate this further, we have eliminated the \( \delta_2 \) above 200 keV from this loop and evaluated whether the high-frequency part of the observed MW spectrum can be reproduced solely by the overarching loop. As a result, we found that the overarching loop can reproduce the total intensity spectrum but cannot solely reproduce the observed relative visibility; the source size suggested from the modeled relative visibility becomes too large. We also note that, as shown in Figure 3(c), the observed spectra taken at different times during the flare suggest the existence of two components: one above \( \sim 8 \text{ GHz} \) and another below \( \sim 8 \text{ GHz} \) that shows a different temporal variation. Therefore, we strongly believe that the high-frequency part of the observed MW emission has to be produced by a flatter electron spectral index in the lower loop (the higher loop cannot produce it due to the steep \( \delta_2 \) required by the HXR observational constraint).

Having convinced ourselves of the need for a spectral break in the lower loop, we investigate the allowable range of \( E_{\text{break}} \) by testing several \( E_{\text{break}} \) values against the relative visibility calculated from the newly fine-tuned MW model. We find that we can obtain an acceptable relative visibility spectrum with \( E_{\text{break}} \) up to \( \sim 220 \text{ keV} \), with a slightly increased nonthermal electron density for the overarching loop, \( 6.7 \times 10^6 \text{ cm}^{-3} \), to compensate for the deficit caused by such an increase in \( E_{\text{break}} \) in the lower loop. We find the lower limit to be \( \sim 180 \text{ keV} \), at which the model relative visibility is acceptable but the model HXR spectrum starts to show the upward break below 67 keV. For this \( E_{\text{break}} \) value, a slightly lower nonthermal electron density of \( 5.7 \times 10^6 \text{ cm}^{-3} \) is required for the overarching loop to counterbalance the increase caused by such a decrease in \( E_{\text{break}} \) in the lower loop.

Table 2 summarizes the spatial and energy parameters of each loop and their thermal and nonthermal electron populations, with possible ranges for the unconstrained parameters discussed above shown in red. As stated in the Introduction, since this simulation is a forward-fitting simulation, there could be other “solutions” of flux tubes that can equally reproduce the
observed HXR and MW emissions (even more than four loops). In this respect, we consider our model to be a “minimally optimized” flux tube model and believe that our methodology, which starts from the construction of the model based on the HXR observation, was the best approach in obtaining a simultaneous fit to the observational constraints that were available for this event. Also, we coincidentally found several remote brightenings in an AIA 1600 A channel image taken at the modeling time that correspond to the western footpoints of the overarching loop in our model, as seen in Figure 10 (yellow arrows). This correspondence could be interpreted as the signature of the precipitation of the nonthermal particles in the overarching loop into the chromosphere. The fact that the brightenings appear only at the western end of the loop may be due to the large difference in the magnetic field strength (thus the mirror ratio) at the two ends: they are ∼2000 and ∼1000 G at the eastern and western end, respectively, and only the magnetically weaker end is allowing the particles to precipitate through. The fact that the locations of these remote brightenings closely match with the furthest end of the model loop strongly supports that the size and extent of our overarching loop is correctly representing the actual flaring loop. For the possible variation in the HXR-constrained model, we briefly consider how our model may vary if we interpret the 20–35 keV ALT source as a thermal source, as mentioned in Section 2.2. We find that modeling the 20–35 keV ALT source with $n_0 \sim 10^{10} \text{ cm}^{-3}$ and $T \sim 60 \text{ MK}$ can equally produce the observed HXR images and spectrum without violating the observed emission measure. We cannot, however, currently test the validity of this model, since it will require setting two different temperature values within one loop.

5. Discussion and Conclusions

Although a number of solar flares have been analyzed in three dimensions using the GX Simulator (Fleishman et al. 2016a, 2016b, 2017b), all previous studies relied on potential field or linear force-free field extrapolation. We present here the first flare model that contains multiple flaring loops inside a single NLFFF data cube. Based on the results of our modeling, we draw the following conclusions about the spatial and energy distributions of the energetic electrons producing HXR and MW emissions at the end of the impulsive phase of the 2015 June 22 M6.5 flare.

First, based on the observed HXR emission sources, the magnetic field configuration that best represents the flaring loop geometry in our study was found to be the post-reconnection loop configuration. We compared the field-line connectivity of the NLFFF extrapolation cube created near the modeling time of 18:05:32 UT and found that field lines contained in the cube created at the earlier time did not have the desired connectivity for our observed HXR sources due to its sheared overall field-line geometry. It is evident that our modeled peak has not yet started at 18:00 UT (see Figure 1(c)), so it is reasonable to think that the shear reduced due to the reconnection event responsible for our modeled peak. The chosen cube at 18:24 UT should contain more post-reconnection loops, and although the field model with a finer time cadence considering the dynamics of the flaring loop may still improve the accuracy of the model, our results show that this post-reconnection cube can reproduce the relative locations of the observed HXR sources and the characteristic size of the observed MW source well.

Second, the low-frequency part of the MW spectrum is dominated by the emission from an “HXR-invisible” source containing a nonnegligible number of nonthermal electrons in a relatively large volume with relatively weak magnetic field. The nonthermal particle population in the overarching loop fills a volume ∼10–12 times larger than the other two nonthermal loops. The total number of nonthermal electrons contained in this loop is calculated to be ∼10$^{34}$, which is in the same order of magnitude as that calculated from the lower loop, ∼5 × 10$^{34}$, the main contributor of both HXR and MW high-frequency emission. We conclude that the primary reason that this MW low-frequency source is “HXR invisible,” although it contains up to 6.1 × 10$^6$ cm$^{-3}$ nonthermal electrons, is because the emitting electrons are trapped in the high corona, and in HXR, the emission from this source is overcome by the bright thick-target footpoint emission. This population has an interesting nonthermal energy spectrum with a spectral index of 2.5, the hardest of all three nonthermal populations. We interpret this as a result of particle accumulation and trapping at that location above the main loops caused by some transport process underway throughout the impulsive phase. Such an interpretation is also reasonable if one notices that the low-frequency part of the MW spectrum grew over several minutes only toward the end of the impulsive phase, as evident in the 4.43 GHz light curve (red) in Figure 1(c). Our results also showed that the parameters of this overarching loop are essentially insensitive to the possible variation in the model parameters of the lower loop, which contributes to the high-frequency part of the spectrum, because the lower loop is too small to provide the GX flux level at low frequencies (see Figure 8 (dark blue curve); also see Fleishman et al. 2017b, Equation (1)). In other words, the deficit in the low-frequency part of the MW spectrum will be present as long as we confine the main HXR loops compactly at lower heights based on the observed HXR images.

Third, the overall geometry and the locations of the electron populations in the three HXR-emitting loops in our model are consistent with the standard flare scenario. The thermal-only loop can be interpreted as a result of chromospheric evaporation in response to earlier electron acceleration episodes; dense (10$^{13}$ cm$^{-3}$) thermal plasma is concentrated toward the loop top. It is interesting that this loop was found to lie relatively low, and this may be a result of our choice of the NLFFF magnetic field model from a time (18:24 UT) that is later than 18:05:32 UT, which contains more post-reconnection loops. However, considering that our modeling time is already near the end of the impulsive phase, when most of the loops have reconnected and became low-lying post-reconnection loops, we consider that our modeled geometry is correctly representing the actual geometry of the flare loops. The lower-loop population is the major contributor in the “HXR-visible” nonthermal population and can be considered the traditional “common” population of nonthermal electrons if its energy spectrum has a broken power-law distribution. The low-energy end of this population produces HXR thick-target emission from the chromosphere with a soft spectrum, and the high-energy end of this population produces high-frequency MW emission from the corona with a hard spectrum. The break energy of this population is modeled to be in the range of
∼180–220 keV. We cannot test whether the spectral hardening of \(-2\) at these energies was already present coming out of the acceleration region, or some of the lower-energy population moved to the higher-energy population via second-stage acceleration that preferentially accelerates higher-energy electrons during the propagation and trapping, or both.

In summary, our results show that our three-dimensional forward-fit modeling of the flare HXR and MW emission can reveal the properties of the nonthermal particles in the flare in much greater quantitative detail than those obtained by conventional means, both within and outside of the standard flare model. We would like to emphasize the importance of our finding the existence of the “HXR-invisible” nonthermal particles that can only be investigated through the properties of the MW low-frequency emission. This suggestion is not new (e.g., Lee et al. 1994; Fleishman et al. 2016a, 2017b) but has been largely neglected in the standard flare model, because the focus of the flare-accelerated electrons has been mainly in the HXR range (and their counterpart in the MW high-frequency range) where the bulk of their energy is deposited. Even though these low-frequency MW-emitting electrons are still at the “tail” of the electron number distribution, their high energy and trapped condition may make them an energetically important player in the overall flare energetic scenario, even after the impulsive phase. For instance, these trapped high-energy electrons (see Fleishman et al. 2016a, 2017b) may escape directly or be further accelerated by CME shocks and become solar energetic particles. Our modeling also stimulates further investigation into the possible spectral break in the population emitting in both the HXR and MW range. Our model for this particular flare seems to support the existence of the break, but other flares may not show such a break. Running this type of modeling for many other flares could lead to finding the properties of the flare that may or may not result in a break. Such information should enable us to discriminate among the competing models of flare particle acceleration, trapping, and escape.

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Figure 9. Visual representation of thermal (left) and nonthermal (right) electron populations occupying four loops found in the final model. The base image is the HMI photospheric magnetogram taken at 18:24 UT, and the red lines are the central field lines of the four loops. The colors are not in actual density scale. The detailed electron parameters for each named loop are shown in Table 2.

Figure 10. Top view of our four-loop model overlaid on an AIA 1600 Å image taken at 18:05:28 UT. The blue lines are the central guiding field lines of the three HXR-constrained loops, the red line is the central guiding field line of the overarching loop, and the green lines are the enveloping field lines of the overarching loop. The red ellipse in the middle is the top view of the circular cross section of the overarching loop defining the extent of the green field lines. Only the field lines contained in the overarching loop are shown. The western end of this loop seems to match the locations of the remote brightenings indicated by the yellow arrows, which can be interpreted as the precipitation of the nonthermal electrons into the chromosphere on the magnetically weaker side of this loop.
Appendix A
Derivation of One-dimensional Relative Visibilities for a Simple Gaussian Source

Visibility on a particular baseline is defined as the Fourier transform of the sky brightness distribution. Let us assume a simple Gaussian source flux function,

\[ S(x) = ae^{-\frac{x-x_0^2}{\alpha^2}}, \]

where \( S(x) \) is the flux intensity as a function of spatial coordinate \( x \), \( a \) is a unit amplitude at \( x = x_0 \), and \( \alpha \) is the \( 1/e \) width of the unit amplitude (half-power beam width). Then, the visibility as a function of spatial frequency \( x \) (\( \lambda \)) is

\[ V(s) = \int_{-\infty}^{\infty} S(x) e^{-2\pi isx} dx = ae^{-s_0^2/\alpha^2} e^{\left(\frac{Ns}{\alpha^2} - Ns^2\right)} \alpha^2 \]

\[ \times \int_{-\infty}^{\infty} e^{-\frac{x^2}{2\alpha^2}} = a\sqrt{\pi} e^{-s_0^2/\alpha^2} e^{-2\pi isx}. \] (4)

Relative visibility is defined as the visibility divided by the total power \( (s \to 0) \),

\[ V_{\text{Rel}} = \frac{V(s)}{V(0)} = e^{-s_0^2/\alpha^2}, \] (5)

where \( V_{\text{Rel}} \) is a unit amplitude \( (x_0 = 0) \). Plugging in the definition of spatial frequency \( s \) and the conversion factor between \( \alpha \) and the FWHM of the source, \( d \),

\[ s = \frac{1}{\theta} \text{rad}^{-1} \sim B_{\lambda} \text{rad}^{-1} = \frac{B_{\text{cm}} f_{\text{GHz}}}{30} \text{rad}^{-1} \]

\[ = -8.351 \times 10^{-11} B_{\lambda}^2 \text{arcsec}^{-1} \]

\[ = 1.62 \times 10^{-7} B_{\text{cm}} f_{\text{GHz}} \text{arcsec}^{-1}, \]

\[ \alpha \sim 0.6d \]

Equation (5) becomes

\[ V_{\text{Rel}} = e^{-8.351 \times 10^{-11} B_{\text{cm}}^2 f_{\text{GHz}}^2} \] (6)

where \( \theta \) is the fringe spacing, \( B_{\lambda} \) is the projected baseline length in number of wavelengths, \( B_{\text{cm}} \) is the projected baseline length in cm, \( f_{\text{GHz}} \) is the observing frequency in GHz, and \( d \) is in arcseconds. Note that \( V_{\text{Rel}} \) here is a unit amplitude \( (x_0 = 0) \). In practice, \( V(s) \) is the cross-correlated amplitude from a particular baseline, and \( V(0) \) is the geometric mean of the total power from the two antennas on that baseline. That is,

\[ V_{\text{Rel}} = \frac{a_{ij}}{\sqrt{a_{i}a_{j}}}, \] (7)

where \( a_{ij} \) is the cross-correlated amplitude from the baseline consisting of antennas \( i \) and \( j \) and \( a\) are the autocorrelated total power amplitudes from antennas \( i \) and \( j \), respectively. Combining Equations (6) and (7), we have

\[ V_{\text{Rel}} = \frac{a_{ij}}{\sqrt{a_{i}a_{j}}} = e^{-8.351 \times 10^{-11} B_{\text{cm}}^2 f_{\text{GHz}}^2}. \] (8)

\[
\ln(V_{\text{Rel}}) = \ln\left(\frac{a_{ij}}{\sqrt{a_{i}a_{j}}}\right) = -8.393 \times 10^{-11} B_{\lambda}^2 d^2
\]

\[ = -9.325 \times 10^{-14} B_{\text{cm}}^2 d^2 f_{\text{GHz}}^2. \] (9)

Appendix B
List of Spatial Distribution Functions Used for this Study

For the spatial distribution of the electron population modeled in this study, we used the default functions provided by the GX Simulator, presented hereafter and available with more details in Nita et al. (2015, 2017).

Thermal population. The default distribution function for thermal population is a product of a normalization factor (density, \( n_0 \)), a unit-amplitude radial distribution across the flux tube \( (n_r) \), and a unit-amplitude vertical distribution \( (n_z) \):

\[ n_\text{th}(x, y, z) = n_0 n_r(x/a, y/b)n_z(z(s)/R), \] (10)

where

\[ n_r(x, y) = \exp\left[-\left(\frac{p_0 x^2}{a^2} - \left(\frac{p_1 y^2}{b^2}\right)\right)\right]. \] (11)

where \( x \) and \( y \) are the cross-sectional coordinates of the flux tube normalized by the ellipse semiaxes of the cross-sections \( a \) and \( b \) and \( z(s) \) is the vertical coordinate intersecting the longitudinal coordinate \( s \) of the flux tube. The vertical density distribution, \( n_z \), is a simple hydrostatic formula adapted from Section 3.1 of Aschwanden (2004),

\[ n(z) = \exp\left[-\frac{z(s)/R}{6.7576 \times 10^{-8} T_0}\right]. \] (12)

where \( R \) is the solar radius and \( T_0 \) is the temperature of the population. The parameters \( p_0 \) and \( p_1 \) determine the radial extent of the population across the flux tube (the smaller the parameter, the broader the extent), while \( T_0 \) determines the vertical distribution of the population.

Note that, for the thermal-only loop in this study, we used a customized \( n(z) \) to match the observed HXR 6–12 keV source shape. The equation is the sum of the default function (Equation (12)) and a simple Gaussian-like function used for the nonthermal population (Equation (15), below) multiplied by a factor of 1.5,

\[ n(z) = \exp\left[-\frac{z(s)/R}{6.7576 \times 10^{-8} T_0}\right] + 1.5 \exp\left\{-\left[\frac{q_0 \left(\frac{s - s_0}{l} + q_2\right)}{l}\right]^2\right\}. \] (13)

where \( q_0 \) and \( q_2 \) determine the longitudinal distribution (see below).

Nonthermal population. The default distribution function for the nonthermal population is a product of a normalization factor (density, \( n_n \)), a unit-amplitude radial distribution across the flux tube \( (n_r) \) (Equation (11)), and a unit-amplitude longitudinal distribution along the central field line \( (n_z) \):

\[ n_{nth}(x, y, z) = n_b n_r(x/a, y/b)n_z(s/l), \] (14)
where \( a, b, \) and \( s \) are defined as above and \( l \) is the length of the central field line of the flux tube. The longitudinal function is a simple Gaussian-like function of the form

\[
n_s(s) = \exp\left\{-\left[q_0 \left(\frac{s - s_0}{l} + q_2\right)\right]^2\right\},
\]

where \( s_0 \) is the reference longitudinal point along the central field line. The parameter \( q_0 \) determines the longitudinal distribution of the population along the flux tube, while \( q_2 \) determines the location of the peak of such a distribution.

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