Uplink Massive MIMO for Channels with Spatial Correlation

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Abstract—A massive MIMO system entails a large number of base station antennas $M$ serving a much smaller number of users. This leads to large gains in spectral and energy efficiency compared with other technologies. As the number of antennas $M$ grows, the performance of such systems gets limited by pilot contamination interference. Large Scale Fading Precoding/Postcoding (LSFP) was proposed for mitigation of pilot contamination, and it was shown that in channels without spatial correlation (uncorrelated base station antennas) LSFP leads to large spectral-efficiency gains. It was recently proven that if a correlation (CSC), pilot contamination is naturally mitigated, and it was shown that in channels without spatial correlation, SINRs tend to some finite limit [1].

In this work, we analyze the performance of Uplink (UL) transmission of massive MIMO systems with finitely many antennas $M$ for channels with spatial correlation. We extend the idea of LSFP to correlated channel models and derive SINR expressions that depend only on slow fading channel components for such systems with and without LSFP. These simple expressions lead us to simple algorithms for transmit power optimization. As a result, we obtain a multi-fold increase in data transmission rates.

II. SYSTEM MODEL

We assume that the network is comprised of $L$ cells and there are $K$ randomly located single antenna users in each cell. The $M \times 1$ channel vector between the $k$th user in the $l$th cell to the BS in the $j$th cell is denoted by 

$$\mathbf{h}_{jkl} = \mathbf{R}_{jkl}^{1/2} \mathbf{w}_{jkl},$$

where $\mathbf{R}_{jkl} = \mathbb{E}[\mathbf{h}_{jkl} \mathbf{h}_{jkl}^H]$ is the $M \times M$ covariance matrix (the slow fading component) and $\mathbf{w}_{jkl} \sim \mathcal{CN}(\mathbf{0}, \mathbf{I}_M)$ denotes the fast fading component. The covariance matrix $\mathbf{R}_{jkl}$ can be decomposed as 

$$\mathbf{R}_{jkl} = \beta_{jkl} \tilde{\mathbf{R}}_{jkl},$$

where $\beta_{jkl}$ is the path loss, and $\tilde{\mathbf{R}}_{jkl}$ depends on the propagation environment between the user and BS. We model $\tilde{\mathbf{R}}_{jkl}$ according to the one ring model shown in Fig. 1, where a user located at azimuth angle $\theta_{jkl}$ and distance $s$ is surrounded by a ring of scatterers of radius $r$ such that the angular spread $\Delta = \arctan \frac{r}{s}$. The correlation between antennas $1 \leq m, p \leq M$ is given by [6]

$$[\mathbf{R}_{jkl}]_{m,p} = \beta_{jkl} \frac{1}{2\Delta_{jkl}} \int_{-\Delta_{jkl}}^{\Delta_{jkl}} e^{i(k^T(\alpha + \theta_{jkl}))(\mathbf{u}_m - \mathbf{u}_p)} d\alpha,$$

where $\Delta_{jkl} = \arctan \frac{r}{s}$. This is followed by a conclusion in Section VII.

The rest of the paper is organized as follows. We first describe our system model in Section II, followed by MMSE channel estimation in Section III. In Section IV we formulate LSFP of CSC case and present our results on SINR expressions. In Section V we look at performance improvements attained via transmit power optimization. Finally, we present our results for a realistic cellular configuration in Section VI and compare uncorrelated channels and channels with spatial correlation with and without LSFP and power optimization.
where \( \mathbf{k}(\alpha) = \frac{2\pi}{\lambda} (\cos(\alpha) \sin(\alpha))^T \) is the wave vector for a planar wave impinging with AoA \( \alpha \), \( \lambda \) is the carrier wavelength, and \( \mathbf{u}_m, \mathbf{u}_p \in \mathbb{R}^2 \) are the vectors indicating the position of BS antennas \( m, p \) in the two-dimensional coordinate system.

Fig. 1. A user at AoA \( \theta \) with a scattering ring of radius \( r \) generating a two-sided AS \( \Delta \) with respect to the BS at origin.

We assume that the BS is equipped with a uniform linear array, resulting in the covariance matrix \( \mathbf{R}_{jkl} \)

\[
[\mathbf{R}_{jkl}]_{m,p} = \beta_{jkl} \frac{1}{2\Delta_{jkl}} \int_{-\Delta_{jkl}}^{\Delta_{jkl}} e^{j2\pi D \sin(\alpha + \theta_{jkl})(m-p)} d\alpha
\]

where \( D \) denotes the smallest distance between the BS antennas, normalized by the carrier wavelength \( \lambda \). \( \beta_{jkl} \) is modeled according to the 3GPP-LTE standard for urban macro with frequency \( f_c = 850 MHz \).

\[
10 \log_{10}(\beta_{jkl}) = -127.8 - 35 \log_{10}(d_{jkl}) + X_{jkl}
\]

where \( d_{jkl} \) is measured in kms and \( X_{jkl} \sim \mathcal{CN}(0, \sigma^2_{\text{mad}}) \) represents the shadowing.

III. UPLINK CHANNEL ESTIMATION

In order to ensure reliable communication between the users and the BS, the users send pilot signals which is used by the BS to estimate the channels \( \mathbf{h}_{jkl} \). We assume that the users in all the cells use the same training codebook \( \Phi = [\Phi_1, \Phi_2, \ldots, \Phi_K] \in \mathbb{C}^{K \times K} \) comprised of \( K \) orthonormal training vectors. The received signal at the \( l \)-th BS is

\[
t_l = \sum_{n=1}^{L} \mathbf{H}_{ln} \mathbf{P}_n^\frac{1}{2} \Phi + \mathbf{z}_l
\]

where \( \mathbf{H}_{ln} = [\mathbf{h}_{l1n}, \mathbf{h}_{l2n}, \ldots, \mathbf{h}_{lKn}] \), \( \mathbf{P}_n = \text{diag}(p_{1n}, p_{2n}, \ldots, p_{Kn}) \) is the diagonal channel matrix of the user powers in the \( n \)-th cell, and \( \mathbf{z}_l \sim \mathcal{CN}(0, \mathbf{I}_M) \) is AWGN.

Multiplying \( t_l \) by \( \mathbf{\Phi}^H \), and taking the \( k \)-th column we get

\[
t_{kl} = t_l \mathbf{\Phi}^H = \sum_{n=1}^{L} \mathbf{h}_{lkn} \sqrt{p_{kn}} + \mathbf{\hat{z}}_l, \quad \mathbf{\hat{z}}_l \sim \mathcal{CN}(0, \mathbf{I}_M).
\]

The MMSE estimate \( \mathbf{\hat{h}}_{lkm} \) of \( \mathbf{h}_{lkm} \) is

\[
\mathbf{\hat{h}}_{lkm} = \mathbf{E}[\mathbf{h}_{lkm} \mathbf{\Phi}^H] \mathbf{E}[t_{kl} \mathbf{\Phi}^H]^{-1} t_{kl} = \sqrt{p_{km}} \mathbf{R}_{lkm} K^{-1}_{kl} t_{kl},
\]

where \( K_{kl} = \mathbf{I}_M + \sum_{n=1}^{L} \mathbf{R}_{lkn} P_{kn} \). Thus, we have \( \mathbf{h}_{lkm} = \mathbf{h}_{lkm} + \mathbf{e}_{lkm} \) independent of \( \mathbf{h}_{lkm} \) and \( \mathbf{h}_{lkm} \sim \mathcal{CN}(0, \mathbf{R}_{lkm}) \), \( \mathbf{e}_{lkm} \sim \mathcal{CN}(0, \mathbf{R}_{lkm} - \mathbf{R}_{lkm} K^{-1}_{kl} \mathbf{R}_{lkm} P_{kn}) \). Note also that \( \mathbf{h}_{lkm} \) and \( \mathbf{h}_{lkl} \) are correlated, with

\[
\mathbf{E}[\mathbf{h}_{lkm} \mathbf{\hat{h}}_{lkl}^H] = \mathbf{R}_{lkm} K^{-1}_{kl} \mathbf{R}_{klm} P_{km}.
\]

IV. LARGE SCALE FADING POSTCODING

LSFP is a way of organizing cooperation between base stations so that this cooperation is based only on slow fading components. So the traffic needed for this cooperation is independent of \( M \) and OFDM tone index. The fact that slow fading components change about 40 times slower than fast fading components also reduces the needed communication traffic.

As it will be clear from the description presented below, we formulate all our results for LSFP with generic \( L \times L \) LSFP matrices \( \mathbf{A}_k = [\mathbf{a}_k, \mathbf{a}_{k2}, \ldots, \mathbf{a}_{KL}] \), \( k = 1, \ldots, K \). By \( a_{klp} \) we denote the element on the intersection of the \( l \)-th column and \( (p-1)L + j \)-th row of \( \mathbf{A}_k \). LSFP matrices with \( a_{klp} = 1 \) when \( j = p = l \) and \( a_{klp} = 0 \) for all other indices mean that there is no cooperation between base stations, that is, we do not use LSFP.

After transmitting pilots, all users transmit uplink data and the \( l \)-th BS receives the vector

\[
\mathbf{y}_l = \sum_{n=1}^{L} \sum_{m=1}^{K} \mathbf{h}_{lmn} \sqrt{q_{mn}} s_{mn} + \mathbf{z}_l
\]

where \( q_{mn} \) is the power of the \( n \)-th user in the \( n \)-th cell and \( s_{mn} \) is the corresponding data symbol. Next the \( l \)-th BS applies an \( M \)-dimensional receiver to \( \mathbf{y}_l \). In this work we assume that either Matched Filtering (MF) or Zero-Forcing (ZF) receivers are used. (The important case of MMSE receiver will be consider in a future work.) As a result, the \( l \)-th BS gets the estimate \( \mathbf{s}_{klp} \) of signals \( s_{kp} \). In particular, in the case of MF receiver,

\[
\mathbf{s}_{klp} = \mathbf{h}_{lkp}^H \mathbf{y}_l,
\]

and in the case of ZF receiver

\[
\mathbf{s}_{klp} = \mathbf{v}_{lklp}^H \mathbf{y}_l,
\]

were \( \mathbf{v}_{lklp} \) denotes the \((l-1)L + p \)-th column of

\[
\mathbf{V}_l = \mathbf{H}_l (\mathbf{H}_l^H \mathbf{H}_l)^{-1}, \quad \mathbf{H}_l = [\mathbf{h}_{l11}, \ldots, \mathbf{h}_{l1L}, \mathbf{h}_{l21}, \ldots, \hat{\mathbf{h}}_{lKL}]^T.
\]

Next the \( l \)-th BS sends the quantities \( \mathbf{s}_{klp} \) and \( \mathbf{E}[\mathbf{h}_{lkp} \mathbf{\hat{h}}_{lkn}] \), \( l,p,n = 1, \ldots, L \), and \( k = 1, \ldots, K \), to a central controller (SC). SC forms the \( L^2 \times 1 \) vector \( \mathbf{s}_k = [\mathbf{s}_{k11}, \ldots, \mathbf{s}_{kKL}, \mathbf{s}_{k21}, \ldots, \mathbf{s}_{kKL}]^T \), and computes estimates

\[
\mathbf{s}_k = [\mathbf{s}_{k11} \ldots \mathbf{s}_{kKL}]^T = \mathbf{A}_k^H \mathbf{s}_k,
\]

data of symbols sent by the \( k \)-th user in all cells.

Let SINR\(_{kl} \) be the SINR of the \( k \)-th user in the \( l \)-th cell. Our goal is to derive estimates for SINR\(_{kl} \) with different receivers as functions of only the slow fading components.
Such estimates are important for several reasons. They give an insight into the system performance, explicitly showing main sources of interference and further allowing to find bottlenecks that prevents us from further performance improvement. Next, they allow simple simulations of systems with large $M$, since we do not have to simulate $M$-dimensional receivers, but simply generate slow fading components and substitute them into the estimates of SINR$_{kl}$. Finally, and perhaps most importantly, such estimates allow us to use power optimization algorithms that depend only on slow fading components. Typically, such algorithms are simple and they allow updating power with much less frequency than algorithms based on fast fading components.

Let $\hat{A}_k = [\hat{a}_{kl1}, \ldots, \hat{a}_{klM}]$ be the matrix with entries $\hat{a}_{kljp} = a_{kljp} \sqrt{p_{kj}}$. We formulate our first result without further detail due to page limit.

**Theorem 1.** If $M$-dimensional MF receiver is used then

$$\text{SINR}_{kl} = \frac{\left[ \sum_{j=1}^{L} \sum_{p=1}^{L} \hat{a}_{kljp}^* \text{tr} \left( R_{jkp} K^{-1}_{kj} R_{jkp}^* \right) \right]^2}{p_{kj} q_{kn} (I_1 + I_2 + I_3)},$$

where

$$I_1 = \sum_{n=1, n \neq l}^{L} \left| \sum_{j=1}^{L} \sum_{p=1}^{L} \hat{a}_{kljp}^* \text{tr} \left( R_{jkp} K^{-1}_{kj} R_{jkp}^* \right) \right|^2 q_{kn},$$

$$I_2 = \sum_{m=1}^{K} \sum_{j=1}^{L} \sum_{p=1}^{L} \sum_{p'=1}^{L} \hat{a}_{kljp}^* \hat{a}_{kljp'} \text{tr} \left( R_{jkp} R_{jkp'} K^{-1}_{kj} R_{jkp}^* \right),$$

$$I_3 = \sum_{j=1}^{L} \sum_{p=1}^{L} \hat{a}_{kljp}^* \hat{a}_{kljp'} \text{tr} \left( R_{jkp} K^{-1}_{kj} R_{jkp}^* \right).$$

Let us consider now the case of $M$-dimensional ZF receiver. After substitution of $y_i$ from (10) into (12) and some computations, we obtain

$$\hat{s}_{kl} = \hat{a}_{kljp}^* \hat{y}_{kljp} = \sum_{j=1}^{L} \sum_{p=1}^{L} \hat{a}_{kljp}^* \hat{y}_{kljp}$$

$$= \sum_{j=1}^{L} a_{kljp}^* \sqrt{q_{kjn}} s_{kl} + \sum_{n=1, n \neq l}^{L} \sum_{j=1}^{L} a_{kljn}^* \sqrt{q_{kn}} s_{kn}$$

$$+ \sum_{n=1}^{K} \sum_{j=1}^{L} \sum_{p=1}^{L} a_{kljp}^* v_{jkp}^H e_{jmn} \sqrt{q_{mn}} s_{mn}$$

$$+ \sum_{j=1}^{L} \sum_{p=1}^{L} a_{kljp}^* v_{jkp}^H z_j.$$ (14)

Computing expectations in the above expression and using Jensen’s inequality, we obtain

$$R_{kl} = E_{v_i, \ell \in \{1, \ldots, L\}} \log_2 \left[ 1 + \frac{\left| \sum_{j=1}^{L} a_{kljp}^* v_{jkp} \right|^2}{I_1 + I_2 + I_3} \right],$$

where

$$I_1 = \sum_{n=1, n \neq l}^{L} \frac{\sum_{j=1}^{L} a_{kljn}^* \sum_{j=1}^{L} a_{kljn}}{q_{kn}},$$

$$I_2 = \sum_{n=1}^{K} \sum_{j=1}^{L} \sum_{p=1}^{L} a_{kljp}^* a_{kljp'}^* v_{jkp}^H \sum_{n=1}^{K} \sum_{m=1}^{L} (R_{jmn} K^{-1}_{mn} R_{jmn}) q_{mn} + I_M v_{jkp'}^H v_{jkp},$$

$$I_3 = \sum_{j=1}^{L} \sum_{p=1}^{L} a_{kljp}^* a_{kljp'}^*.$$ (16)

Using an approximation via random matrix theory, we obtain the following result.

**Theorem 2.** If ZF receiver is used then

$$\text{SINR}_{kl} \approx \text{SINR}_{kl}^{app} = \frac{\sum_{j=1}^{L} a_{kljn}^* \sum_{j=1}^{L} a_{kljn}}{I_1 + I_2},$$

where

$$I_1 = \sum_{j=1}^{L} \sum_{p=1}^{L} \frac{a_{kljn}^* \sum_{j=1}^{L} a_{kljn}}{q_{kn}},$$

$$I_2 = \sum_{j=1}^{L} \sum_{p=1}^{L} a_{kljp}^* a_{kljp'}^* e_p^T \Gamma_j e_p,$$ (18)

where $e_p$ is the $p$th column of the identity matrix $I_L$, and $\Gamma_j$ is provided at the very end of the Appendix, which is a function of the covariance matrices $R_{jk}$. A proof of this theorem is quite technical. We present a sketch of it in Appendix.

Theorems 1 and 2 give simple expressions for SINRs, which further allow us to find optimal LSPF matrices $A_k$. Let us define $c_{kn} = [c_{kn11}, \ldots, c_{kn1L}, c_{kn21}, \ldots, c_{kn2L}]$ with $c_{knjp} = \text{tr} (R_{jkp} K^{-1}_{kj} R_{jkp})$ and block diagonal matrices

$$D_k = \text{diag}(D_{1k}, D_{2k}, \ldots, D_{Lk})$$

with

$$[D_{jk}]_{p,p'} = \text{tr} (R_{jkp} K^{-1}_{kj} R_{jkp}^* + \sum_{m=1}^{K} \sum_{n=1}^{L} R_{jmn} R_{jkp} K^{-1}_{kj} R_{jkp} q_{mn}).$$

Let us further define $\eta_i$ to be the $i$th column of the matrix $1_L \otimes I_L$, and block diagonal matrices $E_k = \text{diag}(E_{1k}, E_{2k}, \ldots, E_{Lk})$ with

$$[E_{jk}]_{p,p'} = e_p^T \Gamma_j e_p.$$
Theorem 3. For MF receiver the optimal LSFP matrices are defined by vectors leading to giving

\[ \hat{a}_{kl} = \left( \sum_{n=1, n \neq l}^{L} c_{kn}c_{kn}^H p_{kn}q_{kn} + D_k \right)^{-1} c_{kl}, \]

leading to

\[ \text{SINR}^{(MF)}_{kl} = c_{kl}^H \left( \sum_{n=1, n \neq l}^{L} c_{kn}c_{kn}^H p_{kn}q_{kn} + D_k \right)^{-1} c_{kl} \cdot p_{kl} q_{kl}. \]  

For ZF receiver we have

\[ \hat{a}_{kl} = \left( \sum_{n=1, n \neq l}^{L} \eta_n \eta_n^H p_{kn}q_{kn} + E_k \right)^{-1} \eta_l, \]

giving

\[ \text{SINR}^{(ZF)}_{kl} = \eta_l^H \left( \sum_{n=1, n \neq l}^{L} \eta_n \eta_n^H p_{kn}q_{kn} + E_k \right)^{-1} \eta_l \cdot p_{kl} q_{kl}. \]  

V. TRANSMIT POWER OPTIMIZATION

SINR expressions presented in Theorem 3 allow us to find optimal transmit powers.

Due to space limit, we formulate results only for MF-receiver. Results for ZF-receiver are similar. We consider the following optimization problem

\[ \max_{q} \min_{k,l} \text{SINR}_{kl} = \max_{q} \min_{k,l} c_{kl}^H \left( \sum_{n=1, n \neq l}^{L} c_{kn}c_{kn}^H p_{kn}q_{kn} + D_k \right)^{-1} c_{kl} \cdot p_{kl} q_{kl}, \]

subject to \( 0 \leq q \leq Q_{\text{max}} \).

where \( q \) is the \( KL \times 1 \) vector of the user powers, and \( \mathbf{1} \) is a \( KL \times 1 \) vector of all ones. This optimization problem can be equivalently formulated as

\[ \max_{q} \gamma \]

subject to

\[ 0 \leq q \leq Q_{\text{max}} \mathbf{1}, \]

\[ c_{kl}^H \left( \sum_{n=1, n \neq l}^{L} c_{kn}c_{kn}^H p_{kn}q_{kn} + D_k \right)^{-1} c_{kl} \cdot p_{kl} q_{kl} \geq \gamma, \forall k, l. \]

This optimization problem can be solved with the following iterative bisection algorithm:

1) Set \( \gamma_{\text{max}} = \max_{k,l} ||c_{kl}||^2 P_{\text{max}} Q_{\text{max}} \) and \( \gamma_{\text{min}} = 0 \).
2) Set \( \gamma = (\gamma_{\text{max}} + \gamma_{\text{min}})/2 \).
3) Check the feasibility of constraints (22) and (23).
4) If \( \gamma \) is feasible, assign \( \gamma_{\text{min}} = \gamma \) and go to Step 6, else go to Step 5.

5) Set \( \gamma_{\text{max}} = \gamma \).
6) If \( \gamma_{\text{max}} - \gamma_{\text{min}} < \epsilon \) (\( \epsilon \) is a small number), stop and output \( \gamma_{\text{min}} \).

For checking feasibility at Step 3 of the above algorithm, we can use the following distributed power optimization algorithm. This algorithm can be also used on its own to achieve a desired SINR target for all users, and in fact it leads to better 5% outage rates (see Section VI). The distributed algorithm is as follows:

1) Set \( q = q^{(0)} \) and compute SINR^{(0)}_{kl} according to (19).
2) At iteration \( n \) compute \( q^{(n+1)}_{kl} = \min_{Q_{\text{max}}} \{Q_{\text{max}}, q^{(n)}_{kl}/\gamma/\text{SINR}^{(n-1)}_{kl} \} \).
3) If \( ||q^{(n)} - q^{(n-1)}||_2 < \epsilon ||q^{(n)}||_2, \forall k,l \) stop, else go to Step 2.

Theorem 4. The distributed algorithm always converges and, if \( \gamma \) is feasible, it converges to powers \( q_{kl} \) that minimize total power \( \sum_k \sum_l q_{kl} \).

VI. NUMERICAL RESULTS

We consider a cellular layout consisting of \( L = 7 \) cells, with \( M = 100 \) and \( K = 5 \) users. (We are currently working on results for large networks with \( L \geq 19 \) and hope to present them in the final version of this conference paper.) Each cell has a cell radius \( R_c = 1 \) km, with users generated randomly within the cellular coverage area. The user position determines the distance, angle of arrival and angular spread to all the base stations. The scattering radius is fixed at \( r = 20 \) m, and the covariance matrices are generated using (3). The path loss coefficients \( \beta \)'s are generated according to (5), and the variance of the log normal shadowing coefficient is taken to be \( \sigma_{\text{shad}} = 8 \) dB. The maximum transmit power of a user is taken to be \( Q_{\text{max}} = 200 \) mW. The noise variance is given as

\[ \text{Noise Var. (dBm)} = -174 + 10 \log_{10} B + NF + 2, \]  

where the bandwidth \( B = 20 \) MHz and \( NF = 4 \) is the noise figure at the BS. Based on these parameters, the SNR at the cell edge (neglecting the shadowing) is approximately \(-6 \) dB (taking into account a 2 dB antenna gain). The SINR expressions for LSFP with matched filtering MF and ZF receiver are computed according to (19) and (20) respectively. For SINR expressions without LSFP, we use (19) and (20) with \( a_{kl} \) such that \( a_{klijp} = 1 \) when \( j = p = l \). The pilot powers and transmission powers are equal to \( Q_{\text{max}} \) for all users.

Figure 2 shows a comparison between the CDF of the user rates with and without LSFP between correlated and uncorrelated channels. By uncorrelated channels, we mean the covariance matrices are given as \( R_{jjkl} = \beta_{jkl} I_M, \forall j, k, l \). The “dashed” curves correspond to the user rates without LSFP and the “solid” curves denote the user rates with LSFP. It can be seen from Fig. 2 that LSFP gives a significant improvement in the user rates compared to the scenario without LSFP. Also, user rates in the case of correlated channels are better than the uncorrelated case.

Figure 3 shows the fraction of users achieving a desired SINR target for transmission schemes with MF receiver. The
“dashed” and “solid” curves denote the results for without and with LSFP respectively. For obtaining the “black” and “magenta” curves, we solve the power optimization problem defined by (22) and (23) by fixing a target SINR, for uncorrelated and correlated channels respectively.

One can see that using proper power allocation schemes for MF in addition to LSFP enables increases outage SINR for both correlated and uncorrelated channels. For example, the 5% outage SINR for correlated channels without LSFP is -15 dB, whereas using LSFP, it can be increased to 2 dB, and further to 4 dB using proper power allocation. This translates to a 40 times increase in the data rates.

Figure 4 shows results for ZF receiver. Power optimization over LSFP does not yield significant gains in the outage SINR compared to LSFP without power optimization when channels are correlated. However, the 5% outage SINR for LSFP with ZF gives a 21 dB improvement over the MF scenario, corresponding to a 4.5 times increase in the data rates over LSFP with MF and a 185 times increase over no LSFP with MF.

VII. CONCLUSION

We have used LSFP to analyze the performance of uplink massive MIMO systems with finite number of antennas for channels with spatial correlation. We have derived SINR expressions that depend only on slow fading channel components with and without LSFP. In addition, we have used these expressions to provide simple algorithms for transmit power optimization, resulting in a multi-fold increase in data transmission rates.

VIII. APPENDIX

In this section, we provide an approximation to the quantity $I_2 = \psi_{j,kp}^H \Lambda_j \psi_{j,kp}$, $\Lambda_j = \sum_{m=1}^L \sum_{n=1}^K (R_{j,m,n} - R_{j,m,n} K_{m,j} \bar{R}_{j,m,n} p_{m,n}) q_{m,n} + I_M$, defined in (16), in the regime $M, K \to \infty$, $M/K = \text{const.}$

We remind that $\psi_{j,kp}$ denotes the $((k-1)L+p)^{th}$ column of $V_j$, where

$$V_j = \tilde{H}_j (\tilde{H}_j^H \tilde{H}_j)^{-1} = \lim_{z \to 0} (\tilde{H}_j^H \tilde{H}_j - M z I_M)^{-1} \tilde{H}_j.$$ 

Hence, our goal is equivalent to evaluation of

$$\lim_{z \to 0} \frac{1}{M} \tilde{h}_{j,kp}^H (B_M - z I_M)^{-1} \Lambda_j (B_M - z I_M)^{-1} \tilde{h}_{j,kp},$$

where $\bar{h}_{j,m,n} = \frac{1}{\sqrt{M}} h_{j,m,n} \sim \mathcal{CN}(0, \frac{1}{M} \bar{R}_{m,n} = R_{j,m,n} K_{m,j}^{-1} R_{j,m,n} p_{m,n})$ and $B_M = \sum_{m=1}^K \sum_{n=1}^L \bar{h}_{j,m,n} \bar{h}_{j,m,n}^H$.

We will omit $j$ to shorten notations. Below, we first obtain an approximation to the above quantity using tools from random matrix theory [7] for finite $z$ and then take the limit $z \to 0$. Note that the cross covariance between vectors $\tilde{h}_{k,p}$ and $\tilde{h}_{m,n}$ is given by

$$\mathbb{E}[\tilde{h}_{k,p}^H \tilde{h}_{m,n}] = \begin{cases} \frac{1}{M} R_{p,m} K_{k,m}^j R_{j,k,n} \sqrt{p_{k,n} p_{k,p}}, & k = m, \\ 0, & k \neq m. \end{cases}$$
We define $m_M(z, -\alpha \Lambda, \bar{R}_{qn}) = \frac{1}{M} \text{tr}(\bar{R}_{qn}(B_M - \alpha \Lambda - zI_M)^{-1})$, where $\alpha$ is a positive scalar. For a finite $L$, and $M, K \to \infty$, $M/K = \text{const}$, we define a deterministic equivalent for $m_M(z, -\alpha \Lambda, \bar{R}_{qn})$, denoted by $f_{qn, -\alpha \Lambda}$, by

$$m_M(z, -\alpha \Lambda, \bar{R}_{qn}) = f_{qn, -\alpha \Lambda} = 0 \quad \text{a.s.} \tag{25}$$

We further prove that

$$f_{qn, -\alpha \Lambda} = \frac{1}{M} \text{tr}(\bar{R}_{qn}(\sum_{k=1}^{K} \sum_{n=1}^{L} \sum_{p=1}^{L} e^{\alpha \Lambda}_{\bar{n}, \Lambda} \bar{R}_{kp} - \alpha \Lambda - zI_M)^{-1}),$$

with $e^{\alpha \Lambda}_{\bar{n}, \Lambda} = [e^{\alpha \Lambda}_{1, -\alpha \Lambda} \ldots e^{\alpha \Lambda}_{n, -\alpha \Lambda}]^T$, and $f_{qn, -\alpha \Lambda} = [f_{1, -\alpha \Lambda} \ldots f_{L, -\alpha \Lambda}]^T$ and $L \times L$ matrix $[F_{k, -\alpha \Lambda}]_{nq} = f_{qn, -\alpha \Lambda}$.

Let (26) be a fixed point equation and since $k = 1, \ldots, K$, $n, p = 1, \ldots, L$, we have $KL^2$ such equations. Hence we can find $f_{qn, -\alpha \Lambda}$'s as solutions of these equations.

We define $T_{\alpha \Lambda} = \sum_{k=1}^{K} \sum_{n=1}^{L} \sum_{p=1}^{L} [\bar{R}_{kn} - c^{\alpha \Lambda}_{n, \Lambda} \bar{R}_{kp}] - \alpha \Lambda - zI_M$, $T = T_0$, $f_{qn} = f_{qn, 0}$, $c_{pq}^{\alpha \Lambda} = e_{n, \Lambda}^{\alpha \Lambda}$. Differentiating (25) and taking the value at $\alpha = 0$, we get

$$d \left( m_M(z, -\alpha \Lambda, \bar{R}_{qn}) - f_{qn, -\alpha \Lambda} \right) = 0 \quad \text{a.s.} \tag{27}$$

where $\frac{d}{d\alpha} |_{\alpha=0} f^{\alpha \Lambda}_{qn} \Lambda = 0 = f^{\alpha \Lambda}_{qn} \Lambda$. We define vector $f^{\alpha \Lambda}_{n} = [f_{1n}^{\alpha \Lambda} \ldots f_{Ln}^{\alpha \Lambda}]^T$ and matrices $[F^{\alpha \Lambda}_{k}]_{nq} = f_{qn}^{\alpha \Lambda}$, $[F^{\alpha \Lambda}]_{nq} = f_{qn}^{\alpha \Lambda}$.

We find $f_{qn}^{\alpha \Lambda}$ by taking the derivative of both sides of (26) with respect to $\alpha$ and taking the limit $\alpha \to 0$. This gives

$$f_{qn}^{\alpha \Lambda} = \frac{1}{M} \text{tr}(\bar{R}_{qn} T_{\alpha \Lambda}^{-1} \Lambda T_{\alpha \Lambda}^{-1} + \sum_{m=1}^{L} \sum_{p=1}^{L} [u_{\Lambda}^{m} J_{\Lambda}^{m} - u_{\Lambda}^{m} I_{\Lambda}^{m} F_{\Lambda}^{\alpha \Lambda} w_{p}^{m}]), \tag{28}$$

where $w_{p}^{m} = [v_{p1}^{m} \ldots v_{pL}^{m}]^T$, $w_{p}^{m} = (I_L + F_{\Lambda}^{m})^{-1} f_{p}^{m}$, and $u_{\Lambda}^{m} = (I_L + F_{\Lambda}^{m})^{-1} u_{\Lambda}^{m}$ are such that $v_{p1}^{m} = \frac{1}{M} \text{tr}(\bar{R}_{qn} T_{\alpha \Lambda}^{-1} R_{kp}^{\alpha \Lambda})$. Note that (28) defines $KL^2$ linear equations. Hence, using $f_{qn}^{\alpha \Lambda}$, we can find $\tilde{f}_{qn}^{\alpha \Lambda}$.

One can see that the coefficients $f_{qn}^{\alpha \Lambda}$, $f_{qn}^{\alpha \Lambda}$ are functions of $z$. We define quantities $\tilde{f}_{qn} = \lim_{z \to 0} -z \tilde{f}_{qn}$ and $\tilde{f}_{qn}^{\alpha \Lambda} = \lim_{z \to 0} z^2 \tilde{f}_{qn}$ and matrices $[\tilde{F}_{\Lambda}]_{nq} = \tilde{f}_{qn}$, $[\tilde{F}_{\Lambda}]_{nq} = \tilde{f}_{qn^{\alpha \Lambda}}$.

To obtain $\tilde{f}_{qn^{\alpha \Lambda}}$, we multiply both sides of (26) by $-z$ and take the limit $\alpha, z \to 0$. Similarly, to obtain the quantity $\tilde{f}_{qn^{\alpha \Lambda}}$, we multiply both sides of (28) by $z^2$ and take the limit $z \to 0$.

Defining $B_{k,M} = B_M - \sum_{n=1}^{L} \hat{h}_{kn} H_{kn}^H$, and making use of (27), we get that

$$\tilde{h}_{kn} (B_{k,M} - zI_M)^{-1} \Lambda (B_{k,M} - zI_M)^{-1} \hat{h}_{kp} = \text{tr} \left[ \hat{h}_{kp} H_{kn} (B_{k,M} - zI_M)^{-1} \Lambda (B_{k,M} - zI_M)^{-1} \right] \tag{29}$$

Let $H_k = (\hat{h}_{k1}, \ldots, \hat{h}_{kL})$, $D_k = (B_{k,M} - zI_M)^{-1}$, $E_k = (I_L + H_k^H D_k H_k)^{-1}$, and $G_k = D_k H_k E_k H_k^H D_k$. Now, using (29), after rigorous computations, we obtain

$$\frac{1}{M} \tilde{h}_{kn} (B_{k,M} - zI_M)^{-1} \Lambda (B_{k,M} - zI_M)^{-1} \hat{h}_{kp} = \frac{1}{M} \hat{h}_{kn} (G_k L_k^2 \Lambda G_k^H \hat{h}_{kp} - \hat{h}_{kn} G_k D_k \hat{h}_{kp} - \hat{h}_{kn} D_k \Lambda G_k \hat{h}_{kp} + \hat{h}_{kn} G_k \Lambda G_k \hat{h}_{kp}) = \frac{1}{M} \hat{h}_{kn} G_k \Lambda G_k \hat{h}_{kp} = \hat{h}_{kn} G_k \Lambda G_k \hat{h}_{kp}$$

As $z \to 0$, we can prove that the above quantity reduces to

$$\lim_{z \to 0} \frac{1}{M} \tilde{h}_{kn} (B_{k,M} - zI_M)^{-1} \Lambda (B_{k,M} - zI_M)^{-1} \hat{h}_{kp} = e_n^T (\hat{F}_{\Lambda})^{-1} F_{\Lambda} e_p = e_n^T \Gamma_k e_p, \tag{30}$$

where $e_n$ is the $n^{\text{th}}$ column of $I_L$, and $\Gamma_k = (F_{\Lambda})^{-1} F_{\Lambda}$. Note that $\Gamma_k$ is $\Gamma_{jk}$ in notations of (18) (we dropped the index $j$ in the beginning of these derivations). Note also that $\Gamma_{jk}$ depends only on slow fading components.

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