Whistler Waves in the Foot of Quasi-Perpendicular Supercritical Shocks

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Abstract  Whistler waves are thought to play an essential role in the dynamics of collisionless shocks. We use the magnetospheric multiscale spacecraft to study whistler waves around the lower hybrid frequency, upstream of 11 quasi-perpendicular supercritical shocks. We apply the 4-spacecraft timing method to unambiguously determine the wave vector $\mathbf{k}$ of whistler waves. We find that the waves are oblique to the background magnetic field with a wave-normal angle between 20° and 42°, and a wavelength of around 100 km, which is close to the ion inertial length. We also find that $\mathbf{k}$ is predominantly in the same plane as the magnetic field and the normal to the shock. By combining this precise knowledge of $\mathbf{k}$ with high-resolution measurements of the 3D ion velocity distribution, we show that a reflected ion beam is in resonance with the waves, opening up the possibility for wave-particle interaction between the reflected ions and the observed whistlers. The linear stability analysis of a system mimicking the observed distribution suggests that such a system can produce the observed waves.

Plain Language Summary  The interaction between waves and particles is proposed to be one of the main mechanisms for energy dissipation at collisionless plasma shock waves. Of particular interest are a type of wave called whistlers, they fall in a frequency range that allows interactions with both electrons and ions, making them important for energy transfer between the two species. Their mechanism of generation is still not fully understood. We use data from the 4 magnetospheric multiscale spacecraft to unambiguously characterize whistler waves upstream of quasi-perpendicular supercritical shocks. We find that the waves are oblique to the background magnetic field with a wavelength around the width of the shock and a frequency around the lower hybrid frequency. We also find that the shock-reflected ions are at a velocity that allows them to exchange energy with the waves, making them a likely source. We confirm this conclusion by using a computer code to model the system and study its stability.

1. Introduction

Collisionless shocks, despite their ubiquity in astrophysical plasmas (Bykov & Treumann, 2011; Treumann, 2009), are not yet fully understood, in particular which mechanisms provide plasma thermalization in the absence of collisions. One of the main mechanisms proposed is wave-particle interactions. Whistler waves are known to play an integral part in the dynamics and evolution of collisionless shocks. Their spatial and temporal scale allows them to mediate energy between ions and electrons, paving the way for the thermalization of cold solar wind plasma as it passes the shock.

Whistler waves were first observed close to the earth's bow shock by the OGO 5 spacecraft (Heppner et al., 1967). Later, they were observed at planetary bow shocks of Venus, Mercury, and Saturn (Russell (2007) and references therein) as well as at interplanetary shocks (Wilson et al., 2012). The waves can be separated into different categories according to their frequency. The low-frequency whistlers ($f \sim 10^{-2}$ Hz; Fairfield, 1969) are connected to the shock and generated locally in the ion foreshock by the ion-ion two stream instability. The high-frequency whistlers ($f \sim 10^{2}$ Hz) are observed near the foot and the ramp of the shock (Hull et al., 2012; Tokar et al., 1984) and are considered to be generated by the whistler anisotropy instability. At intermediate frequencies are whistlers around the lower hybrid frequency $f \sim f_{\text{LH}} \sim 10^0 \sim 10^1$ Hz, first observed by Fairfield (1974). For supercritical quasi-perpendicular shocks, three possible mechanisms have been proposed to explain the generation of...
those whistlers: internal shock generation (Dimmock et al., 2013; Fairfield, 1974; Krasnoselskikh et al., 1991; Sundkvist et al., 2012), generation by shock macro-dynamics and nonstationarity (Balikhin et al., 1997; Galeev et al., 1989; Krasnoselskikh, 1985), and generation by ion or electron microinstabilities in the foot/ramp region (Krauss-Varban et al., 1995; Orlowski et al., 1995). One particular instability of interest is the kinetic cross-field streaming instability (KCFSI) between the reflected ion beam and the incoming solar wind electrons (Wu et al., 1983) (also known as the modified two stream instability). Several observational (Dimmock et al., 2013; Hoppe et al., 1981; Wilson et al., 2012) and simulation (Hellinger et al., 1996; Matsukiyo & Scholer, 2006; Muschietti & Lembège, 2017; Umeda et al., 2012a, 2012b) studies suggested that this instability is responsible for the generation of the intermediate-frequency whistlers.

To investigate the source of the observed whistlers, it is necessary to characterize the wave properties, that is, their frequency, phase velocity ($V_{\text{phase}}$) and polarization in the plasma frame, as well as the corresponding particle distributions. Using single-spacecraft measurements, one can determine the wave normal direction using the minimum variance analysis (MVA) of the magnetic field, but with a π ambiguity in direction (Sonnerup & Scheible, 1998). The earliest multi-spacecraft measurement was done by Russell (1988). They used 2 spacecraft to characterize two types of waves, whistler waves attached to the shock and upstream turbulence waves, upstream of both bow shocks and interplanetary (IP) shocks. Balikhin et al. (1997) and Dimmock et al. (2013) used 2 spacecraft to study whistlers upstream of supercritical quasi-perpendicular shocks. By timing the difference in the measured signal between the 2 spacecraft and using the MVA, they were able to get $V_{\text{phase}}$ and hence determine the wave characteristics in the solar wind frame. Overall, the wave normals and spectra of the waves have been characterized well, but getting $V_{\text{phase}}$ in the plasma frame was challenging; its determination was restricted to rare occurrences where spacecraft separation was adequate. Furthermore, particle distributions were typically unresolved. Because of those technical challenges, testing for the various generation mechanisms with observation was not always feasible, and the question on which of the proposed mechanisms explains the observed whistlers upstream of quasi-perpendicular supercritical shocks in the heliosphere remains open.

The magnetospheric multiscale (MMS) spacecraft (Burch et al., 2016) is a constellation of 4 spacecraft in a tetrahedral formation equipped with high-resolution field and particle instruments that allow the exploration of the microphysics of collisionless shocks in an unprecedented way. Hull et al. (2020) used MMS data to conduct a case study about the generation mechanism and energetics of whistler waves upstream of quasi-perpendicular and supercritical shocks with the Alfvénic Mach number $M_A$ much larger than the nonlinear whistler critical Mach number

$$M_{\text{run}} = \frac{|\cos \theta_{bn}|}{\sqrt{2m_i/m_e}},$$

in the cold plasma limit, where $\theta_{bn}$ is the angle between the upstream magnetic field and the normal to the shock, and $m_i$ and $m_e$ are the ion and electron mass, respectively. In this $M_A$ limit, no upstream whistlers are to be found and nonstationary behavior of the shock is expected (Krasnoselskikh et al., 2002). They found that the KCFSI of the reflected ions with the solar wind electrons is the likely source of the observed whistlers. Here, we extend this study to a different regime of shocks with $M_A \leq M_{\text{run}}$.

2. Observation

We analyze whistler waves upstream of 11 supercritical and quasi-perpendicular shocks with $M_A$ ranging between 3.5 and 9.8, the fast mode Mach number, $M_f$, between 1.7 and 5.4, and $\theta_{bn}$ between 55° and 82° (Table 1). We calculate the ratio of the downstream flow velocity in the normal direction $V_{\text{dn}}$ to the downstream sound speed $C_{sd}$ (not shown) and find that $V_{\text{dn}}/C_{sd} < 1$, which means that all shocks analyzed are supercritical. For the magnetic field measurement, we use the fluxgate magnetometer (FGM) (Russell et al., 2016) with a sampling frequency up to 128 Hz. The fast plasma investigation (FPI) (Pollock et al., 2016) measures the 3D distribution functions of electrons and ions with a cadence of 30 and 150 ms, respectively, and with an energy range from 10 eV to 30 keV.

To illustrate our upstream whistler waves’ characterization, we use the shock observed by MMS on 24 November 2017 as shown in Figure 1. This shock has $\theta_{bn} = 82^\circ$ and $M_A = 4.2$. All vectorial quantities are in the ($n, t_1, t_2$) coordinate system with $n$ being the normal to the shock determined using the coplanarity theorem with the jumps in the velocity and the magnetic field, mixed mode 3 method given by Equation 10.17 in Schwartz (1998).
\( \hat{\mathbf{A}_t} = \hat{n} \times \hat{B} \) and \( \mathbf{f}_2 = \mathbf{f}_1 \times \hat{n} \). Furthermore, to make sure that the first quadrant in the \( \hat{n} - \mathbf{f}_1 \) plane contains the upstream magnetic field, we flip the sign of \( \mathbf{f}_2 \) whenever the upstream magnetic field has a negative \( \hat{n} \) component. Between 23:20:16 and 23:20:22 UT, we see a slight increase in the magnetic field and density (panels a and b) and a slight decrease in the ion velocity (panel c). Concurrently, a reflected ion component (a component with positive normal velocity) is observed (panel d) and is delimited by the vertical dashed lines in panels (b–e). All of these are signatures of a localized foot region. This is followed by the shock ramp identified by an abrupt increase in the magnetic field and ion and electron densities and an abrupt decrease in the ion velocity between 23:20:22 and 23:20:25 UT. For this event, \( M_1 \) is comparable to \( M_{\text{lim}} \) given the uncertainties (Table 1).

### 2.1. Wave Characterization

Between 23:19:54 and 23:20:22 UT, upstream whistler waves are clearly visible in panel a. A wavelet power spectrum of the magnetic fluctuations is shown in Figure 1e. The waves have a frequency slightly below \( f_{\text{w}} \) with the highest intensity near the edge of the shock foot and as we go upstream the intensity of the waves decreases. Using singular value decomposition (SVD; Santolik et al., 2003; Taubeschuss et al., 2014), we calculate the degree of polarization (panel g), planarity (panel h), and ellipticity (panel i) of the waves. We see that these waves are highly polarized with predominant planar polarization as it is evident from panels g–h. This indicates that the ellipticity calculation is reliable. From panel (i), it is evident that the waves have circular right-hand polarization. Noting that the polarization in the plasma frame (discussed later) is also right handed, we conclude that these are whistler waves.

Using 4-spacecraft measurements, we can unambiguously determine the wave vector \( \mathbf{k} \). We apply a running wavelet transform to the \( \mathbf{B} \) measurement at each of the spacecraft for the interval shaded in Figure 2a, giving a power spectrum evolution with time. Then, in wavelet space, at each frequency and time step, phase shifts, \( \Delta \Phi (\omega, t) \), between the signals are calculated. Knowing the distance between the spacecraft (\( \Delta \mathbf{R} \)) and taking advantage of its tetrahedral formation, we calculate \( \mathbf{k} \) using

\[
\Delta \Phi (\omega, t) = \mathbf{k} (\omega, t) \cdot \Delta \mathbf{R}.
\]

To avoid spatial aliasing, the separation between the spacecraft \( \sim 15 \) km (Figure 2f) should be smaller than half of the smallest wavelength that we are trying to resolve. Therefore, we select events having a consistent small phase shift between the spacecraft, which is expected for waves with a sufficiently long wavelength. We then demonstrate a posteriori that the observed waves have the wavelength larger than twice the spacecraft separation and thus the measurement is not affected by aliasing.

Figure 2b shows the wave vector obtained by a weighted averaging of \( \mathbf{k}(\omega, t) \) over the frequencies at each time step and using the wavelet power as the weight. A weighted standard deviation provides an error in the measurement. Averaging \( \mathbf{k} \) over the shaded time interval and normalizing we get

\[
\langle \hat{k} \rangle = \langle \mathbf{k} \rangle / \langle \mathbf{k} \rangle = (0.76, 0.65, -0.05) \pm (0.15, 0.05, 0.08).
\]

This vector is mostly in the coplanarity, \( \hat{n} \sim \hat{f}_1 \) plane. Figure 2g shows a schematic of the spacecraft position, \( \langle \hat{k} \rangle \) and upstream \( \mathbf{B} \) projected on the ecliptic plane.

Figures 2c–2e show the angle between \( \langle \mathbf{k} \rangle \) and \( \mathbf{B} \), the wavelength and the phase speed in the spacecraft frame. Averaging these quantities over the same time interval, we get \( \theta_{\mathbf{k}} = 42^\circ \pm 6^\circ \) showing that the whistlers are oblique. We find an average \( \lambda \) of \( 70 \pm 6 \) km, which is comparable to the ion inertial length \( l_i = 85 \) km. The average phase speed in the spacecraft frame is \( V_{\phi,sc} = 340 \pm 70 \text{ km s}^{-1} \). To go to the plasma...
frame, we use \( \mathbf{V}_{\text{p,pf}} = \mathbf{V}_{\text{p,sc}} - (\mathbf{V} \cdot \langle \hat{k} \rangle) \langle \hat{k} \rangle \), where \( \mathbf{V} \) is the relative velocity between frames, here taken to be the measured ion velocity averaged over the same interval \( \mathbf{V}_i = (-293, 250, 28) \pm (30, 10, 30) \) km s\(^{-1}\). \( \langle \hat{k} \rangle \) is almost perpendicular to \( \mathbf{V}_i \), making the dot product in the above equation small. The resultant \( \mathbf{V}_{\text{p,pf}} \) has a magnitude of 400 km s\(^{-1}\) and has a positive normal component, showing that these waves propagate upstream. Finally, using \( f_{\text{pf}} = f_{\text{sc}} - \mathbf{V} \cdot \langle \hat{k} \rangle / 2\pi, \) we find the average frequency in the plasma frame \( f_{\text{pf}} \) to be \( 5.9 \pm 1.1 \) Hz, where \( f_{\text{sc}} \) is the spacecraft-frame frequency.

Applying the same analysis to all 11 events (Table 1), we find that the upstream waves always propagate upstream at an oblique \( \theta_{\text{ka}} \) that varies between 20° and 42° with a wavelength ranging from 0.7 to 1.7 \( d_i \), and the plasma-frame frequency ranging from 0.3 to 1.2 \( f_{\text{Ly}} \). Two of the events had left-hand polarized waves in the spacecraft frame, which flip to right-hand polarization in the plasma frame consistent with the other events. The wave

Figure 1. Overview of a shock crossing by MMS on 2017-11-24, 23:20 UT and wave polarization analysis. (a) Magnetic field enlarged around the foot and the ramp, (b) magnetic field, (c) electron and ion densities, (d) ion velocity, (e) 1D velocity distribution function reduced in the shock normal direction, (f) power spectrum of the magnetic field, (g) degree of polarization, (h) planarity, and (i) ellipticity. Overlaid on top of panels (f–i) is the lower hybrid frequency. The black solid line in panel (e) shows the time of the 2D distribution of Figures 3a and 4a corresponding to a peak in the current. The dashed lines in panels (b–e) mark the edges of the region with reflected ion components in the ion velocity distribution function (VDF).
Vectors are in the \( \hat{n} - \hat{t} \) plane within \( \pm 20^\circ \). All of the wave vectors are in the first quadrant (positive \( k_n \) and \( k_t \)), that is, pointing upstream of the shock. Knowing that the group velocity for whistler waves always lies in between \( \mathbf{B} \) and \( \mathbf{k} \), we conclude that it is also pointing in the upstream direction.

Finally, looking at Table 1, we see that the standard deviation of the measured values of \( \lambda \) is small compared to the mean. Using Faraday’s law, we can use the estimated \( \mathbf{k} \) to reconstruct \( \mathbf{B} \) and then compare it with that measured wave \( \mathbf{B} \). Doing so shows excellent agreement between the two. We also check for consistency between our measured wavelength and that predicted from the cold plasma whistler dispersion relation (e.g., Wilson III et al., 2017), which also shows excellent agreement. All of this provides conclusive evidence that our multi-spacecraft wave-vector estimates are not affected by spatial aliasing.

### 2.2. Ion and Electron Velocity Distributions

The foot region of quasi-perpendicular supercritical shocks contains 3 primary plasma components: the incoming solar wind, reflected ions, and the electrons. Such a system can be unstable to a KCFSI (Wu et al., 1983). In what
follows, we show that the most likely source of the observed whistlers is the instability generated by the relative drift between the reflected ion beam and the solar wind plasma.

For the system under investigation, the scale length of the foot is of the order of the ion gyroradius. At these scales, the ions are unmagnetized. In that case, when the reflected ion beam satisfies the resonance condition:

$$V_{\text{phase}} = V_{\text{beam}} \cdot \hat{k},$$

the plasma and the waves will be able to exchange energy. Furthermore, for the waves to grow, the slope of the ion velocity distribution function (VDF) in the wave vector direction should be positive, $\frac{\partial f}{\partial \theta} > 0$. To verify whether the resonance condition (Equation 3) is satisfied by the observed VDFs, we show in Figure 3 the 2D ion VDF reduced in the $\hat{k} - \hat{\mathbf{r}}_2$ plane and 1D ion VDF reduced in the $\mathbf{k}$ direction for 3 events with different $\theta_{\text{BN}}$. The black line shows the wave phase speed and the pink-shaded area shows its 2$\sigma$ interval. The times indicate the center of the 150 ms acquisition interval of the ions VDFs.

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The 2D Ion VDFs reduced in the $\hat{k} - \hat{\mathbf{r}}_2$ plane (top row) and 1D velocity distribution functions (VDFs) reduced in the $\mathbf{k}$ direction (bottom row) for three different events with $\theta_{\text{BN}} = 82^\circ$ (a, d), $68^\circ$ (b, e), and $55^\circ$ (c, f). The black line shows the wave phase speed and the pink-shaded area shows its 2$\sigma$ interval. The times indicate the center of the 150 ms acquisition interval of the ions VDFs.

We also investigate the electron distribution for the presence of backstreaming electron beams (Meziane et al., 2019; Wu, 1984), which can also contribute to wave generation (Inan & Tkalecovic, 1982). In Figure 4, we plot the 2D electron VDF reduced in the $\hat{\mathbf{b}} - \hat{\mathbf{r}}_2$ plane in panels (a–c), where $\hat{\mathbf{b}}$ is the field aligned direction and the 1D electron VDF reduced in the $\hat{\mathbf{b}}$ direction in panels (d–f) for the same 3 events shown in Figure 3. The $y$ axis of panels (d–f) has been set to a logarithmic scale to better visualize the existence or absence of an electron beam. From $\theta_{\text{BN}}$ and the ratio of the upstream magnetic field to the maximum magnetic field at the overshoot, one can estimate a lower limit on the reflected beam parallel velocity $V_{rb}$ (Wu, 1984). We overlay $V_{rb}$ on top of panels (a–f) in blue. It is worth noting that $V_{rb} > 0$ corresponds to the propagation upstream of the shock for the first
two shocks, while upstream propagation corresponds to \( V_b < 0 \) for the last shock in Figure 4 (this can be seen in Table 1 as well). Furthermore, to check for the resonance condition, we also overlay the phase speed parallel to the magnetic field \( \vec{A} \cdot \vec{k} \) of the observed waves along with the \( 2 \sigma \) interval. Note that there is an order of magnitude difference between the phase speed with its \( 2 \sigma \) interval and the thermal speed of electrons in the observed distributions; this makes the pink-shaded area in panels (d–f) to look as a single thick black line.

For the shock with \( \theta_{\text{Bn}} = 82^\circ \), we see an enhancement in the parallel to B electrons in the predicted velocity range around \( V_{\text{rb}} \sim 5 \times 10^3 \text{ km s}^{-1} \) (Figure 4d). For the other two shocks with smaller \( \theta_{\text{Bn}} \), the expected beam velocities fall in the range of velocities, which are not well resolved by FPI (the minimum energy that FPI can measure is \( \sim 10 \text{ eV} \), so speeds in the range \( \sim [-2, 2] \times 10^3 \text{ km s}^{-1} \) cannot be properly resolved, making the ring shape at the center of the distributions an instrumental effect.

### 3. Linear Model Analysis

To verify that the observed VDFs are unstable and can generate waves with the observed properties, we use a simplified model where the three plasma components, that is, the electrons, the incoming, and reflected ions, are represented by Maxwellian distributions and study the wave growth using the Bo kinetic dispersion solver (Xie, 2019). Older dispersion solvers, like WHAMP (Rönmark, 1982), do not take into consideration cross-field drifts. On the other hand, Bo solves for the roots of the magnetized kinetic dispersion relation allowing for cross-field drifts, opening up the possibility to explore the stability of more complex plasma configurations than what was allowed by older solvers.

**Figure 4.** 2D electron velocity distribution functions (VDFs) reduced in the \( \vec{k} \cdot \vec{t} \) plane (top row) and 1D VDFs reduced in the \( k \) direction (bottom row) for three different events with \( \theta_{\text{Bn}} = 82^\circ \) (a, d), \( 68^\circ \) (b, e), and \( 55^\circ \) (c, f). The black line shows the wave phase speed and the pink-shaded area shows its \( 2 \sigma \) interval. The blue lines show the expected velocity of the reflected beam according to Wu (1984). The times indicate the center of the 30 ms acquisition interval of the electron VDFs. It is worth noting that the minimum resolution of FPI is \( \sim 10 \text{ eV} \), so speeds in the range \( \sim [-2, 2] \times 10^3 \text{ km s}^{-1} \) cannot be properly resolved, making the ring shape at the center of the distributions an instrumental effect.
We use measured values of B, density, and temperature for the event shown in Figure 1 as input parameters for the solver. Since FPI was designed for the measurement of primarily magnetosheath and magnetosphere plasmas, it has limitations when it comes to measuring solar wind distributions. Therefore, some of the electrons and ion moments measured by FPI are not reliable. In particular, FPI does not resolve the bulk of the electron distribution, which will cause it to overestimate the electron velocity. Furthermore, the logarithmic spacing between the FPI energy bins is as big as the solar wind ion temperature; hence, it overestimates the ion temperature. Below, we describe the way we measure each of the needed parameters. For ions, we separate the VDF into an incoming (solar wind) and reflected beam component and calculate their moments separately. We obtain the reflected and incoming ion density from the corresponding zeroth moment. The electron density is given by the sum of the two components.

The velocities are transformed into a field-aligned coordinate system and to the rest frame of the electrons. For the temperature, we use FPI measurement for electrons. For ions, the solar wind beam is too narrow for FPI to measure first-order moments of the VDF of each of the ion species. While for the electrons, FPI overestimates the velocity since it does not resolve low-energy electrons. Therefore, we calculate the velocity from the current obtained by the curlometer method (Robert et al., 1998), \( \mathbf{V}_i = \mathbf{J}/(N_e) \). The velocities are transformed into a field-aligned coordinate system and to the rest frame of the electrons. For the temperature, we use FPI measurement for electrons. For ions, the solar wind beam is too narrow for FPI to properly characterize the temperature, so we use a time-shifted measurement from OMNI data. We treat the reflected ion beam temperature as a free parameter varied to give the best agreement with the observed wave characteristics. Figure 5a shows the modeled ion VDFs for the case of the shock event as shown in Figure 1, which can be compared to the observed VDF in Figure 3a. The input parameters were taken from the point corresponding to a peak in the current in the foot and closest to the ion reflection point. This is marked by the black line in panel (e) of Figure 1. A summary of the input parameters is presented in Table 2.

The results of the solver obtained for a reflected beam temperature of 7.5 eV (Figure 5b) show that the system is unstable to whistler generation with the maximum growth rate \( \gamma_{\text{max}} \sim 0.1 \omega_i = 3 \) (rad s\(^{-1}\)). However, for the waves to grow to large amplitudes, they need to stay in the unstable region for a long enough time. We therefore calculate the spatial growth rate \( \gamma_s = \gamma/\mathbf{V}_{\perp i} \), where \( \mathbf{V}_{\perp i} \) is the normal component of the group velocity in the shock reference frame. We then compare \( \gamma_s \) to the width of the foot \( L_i = 0.68V_i/\omega_i = 0.68 \times 340/0.9 = 260 \) km (Woods, 1971) where \( V_i \) is the upstream ion velocity in the shock frame. The product \( \gamma_s L_i \) is the number of e-foldings of the wave, while it propagates through the foot, which will then determine if

Table 2

| Input Parameters to the Dispersion Solver | \( N \) (cm\(^{-3}\)) | \( T \) (eV) | \( \mathbf{V}^* \) (km s\(^{-1}\)) |
|----------------------------------------|----------------|---------|-----------------|
| Incoming ions                          | 10.8           | 2.5     | \(-6 - 210 - 70\) |
| Reflected ions                         | 2.1            | 7.5     | 423 0 24        |
| Incoming electrons                     | 12.9           | 15.2    | 0 0 0           |

*Note. Background magnetic field used is \( B = 9.3 \) [0 0 1] nT. All velocities are in the electron reference frame.
the waves have sufficient time to grow to large amplitudes or not. For \( \theta_{\text{br}} = 40^\circ \), \( \lambda = 64 \text{ km} \), and \( \beta f_{\text{LH}} L = 0.65 \), which are on the numerical dispersion surface and in closest agreement with the observed wave parameters, the model gives \( V_{\text{wn}} \sim 100 \text{ km s}^{-1} \) with the spatial growth rate \( \gamma_s = 0.03 \text{ (rad km}^{-1}) \) and \( \gamma_{L_{\text{c}}} \approx 7.8 \), while the maximum \( \gamma_{L_{\text{c}}} \approx 17 \). Thus, this linear model predicts that the waves can reach large amplitudes while propagating upstream within the foot region. This is consistent with the observation in Figures 1b–1e where the amplitude of the waves is small at the reflection point (rightmost dashed line in panels b–e), increases throughout the foot, and reaches a maximum near the upstream edge of the foot (leftmost dashed line in panels b–e). The results of this model suggest the large drift between the reflected ion beams and the incoming solar wind electrons as a possible driver of the observed whistlers.

4. Discussion

When using a dispersion solver, it is important to mention some of its caveats. First, although the reflected beam is not a simple Maxwellian, approximating it by one can still qualitatively reproduce the physical behavior in terms of the wave properties and linear growth. Despite its complicated shape (see Figure 3), the distribution is a monotonic function, and a Maxwellian captures qualitatively the gradients necessary for wave growth and damping. The dispersion solver only provides approximate values for the growth rate, wave vector, and frequency as the phase-space gradients driving the waves are somewhat different from the observed gradients. Furthermore, we note that we have considered the linear growth in homogeneous plasma, and our model does not include the effects of inhomogeneity and nonlinearity. These can be important if we intend to study the growth and propagation of large-amplitude waves in realistic shocks, and these effects can be addressed only in full kinetic simulations, which are outside of the scope of this paper.

We note that Hull et al. (2020) studied whistler generation at a shock with \( \theta_{\text{br}} = 82^\circ \) and \( M_A = 10 \). They found that the reflected ion beam is in resonance with the waves, generating a whistler at frequency \( \sim \omega_{\text{fi}} \) through the KCFSI. In their case, \( M_A \) exceeds the nonlinear whistler critical Mach number, while our study is limited to shocks with \( M_A \lesssim M_{\text{wn}} \). Thus, the generation mechanism can be the same for shocks with Mach numbers both lower and larger than the nonlinear whistler critical Mach number.

Apart from the instability discussed above, there are other mechanisms that can potentially generate oblique whistlers. For shock macrodynamics (Balikhin et al., 1997; Galeev et al., 1989; Krasnoselskikh, 1985), we study supercritical shocks with \( M_A \lesssim M_{\text{wn}} \) so no large-scale dynamics are expected; hence, we can rule out this mechanism as a source for the observed whistlers.

Furthermore, other microinstabilities of ions can be active in the foot/ramp region generating waves in the same frequency range. The incoming solar wind ions also have a drift with respect to the solar wind electrons. This drift can be the source of free energy to an instability generating whistler waves traveling toward the downstream at a predominantly perpendicular angle to the background magnetic field (Wu et al., 1984). Such wave properties are not consistent with the observed waves, which are oblique and travel upstream of the shock.

Other ion instabilities that are potentially active in the foot/ramp region are the lower hybrid drift instability or the ion-ion cross-field drift instability (Scudder et al., 1986; Wu et al., 1984). The lower hybrid drift instability predicts that the waves are oblique to the background magnetic field and are perpendicular to the coplanarity plane, while the ion-ion drift instability predicts waves that are perpendicular to the background magnetic field. Those predictions do not match the observed properties of the waves as is seen in Table 1; hence, those instabilities could not be behind the observed whistlers.

As for instabilities involving electrons, they could be either due to temperature anisotropy or loss-cone distributions or due to an electron beam (Hull et al., 2012; Tokar et al., 1984). The former generates whistlers that are predominantly parallel to the magnetic field, which does not match the oblique whistlers observed. While to check for the generation by an electron beam, we examine the electron VDF shown in Figure 4. As mentioned in Section 2.2, the expected velocity of an electron beam (Wu, 1984), shown as the blue vertical line, is well outside the resonance interval of the waves. It is likely that only the thermal electrons are the ones in resonance. This will provide damping to the waves instead of growth, a damping that the ions have to overcome for the waves to grow (Muschietti and Lembège, 2017).
Finally, the whistlers can be generated by the shock itself and dispersively run upstream (Krasnoselskikh et al., 2002). This model has two regimes separated by the linear whistler Mach number

\[ M_w = \frac{|\cos \theta_B|}{2 \sqrt{m_e/m_i}}, \]

(Krasnoselskikh et al., 2002). When \( M_A < M_w \) the wavelength of the waves is predicted to be around the ion skin depth. On the other hand, when \( M_A > M_w \), the wavelength of the waves is predicted to be around the electron skin depth. Looking at Table 1, and taking into consideration the uncertainties, all events studied have \( M_A \lesssim M_w \), so for them, the predicted wavelength is similar to the observed one. Another key parameter that characterizes the waves would be the wave vector direction. As it stands now, the dispersive shock model is a 1D model that assumes that the wave vector is parallel to the shock normal, which is clearly in contrast to what we observe (see Table 1). Unfortunately, there is no 2D or 3D model of this mechanism available as of today, so we have no prediction for the wave normal direction that can be tested against observation in order to support or reject this mechanism. It is worth mentioning that Wilson III et al. (2017) studied the structure of 145 low Mach number (<3) low \( \beta \) (<1) shocks. At such low Mach numbers, those shocks are considered subcritical with minimal ion reflection; hence, upstream whistler waves are considered to be dispersively generated by the shock itself. They found that such whistler waves travel at angles that are oblique to both the shock normal and to the background magnetic field consistent with our observation. Yet they travel at a large angle to the coplanarity plane with 79% of their observed waves travel at an angle of at least 20°. If one takes this observation as a general characteristic of whistler waves generated dispersively by the shock, that would rule out this model as a possible explanation of the whistler waves that we observe, which are closely aligned with the coplanarity plane (within ±20°). But further theoretical, simulation, and experimental studies are required to make any solid conclusions.

5. Conclusions

We use the MMS spacecraft to study upstream whistler waves around the lower hybrid frequency at quasi-perpendicular supercritical shocks. We select 11 shock events with narrow band upstream whistler waves with an Alfvénic Mach number ranging between 3.5 and 9.8 (all \( \lesssim M_{\text{cwn}} \)) and \( \theta_B \) between 55° and 82° for which the whistlers can be characterized with high precision using the multi-spacecraft methods.

Our main findings are:

1. The wavelengths of these waves range from 0.7 to 1.7 ion inertial length and the wave-normal angle range from 20° to 42° with a \( \mathbf{k} \) directed upstream of the shock and close to the shock coplanarity plane. The frequency of the waves in the solar wind frame ranges between 0.3 \( f_{\text{LH}} \) and 1.2 \( f_{\text{LH}} \).
2. The highest wave amplitude is found in the foot, where we found the shock-reflected ion component in the distribution function. After reducing the observed 3D ion VDF in the direction of \( \mathbf{k} \), we find that the reflected ion component of the VDF is in Landau resonance with the observed waves, which indicates that the reflected ion beam is interacting with the observed whistlers and could be behind the generation of those waves.
3. Using a linear kinetic dispersion solver, we find that a VDF composed of a reflected ion beam on top of incoming solar wind, with parameters taken from the observation, is unstable to the generation of whistler waves with properties close to what we observe. This supports the kinetic cross-field streaming instability between the reflected ions beam and the incoming solar wind electrons as a likely generation mechanism.

We have found that the waves are in resonance with the shock reflected ions, and the linear stability analysis shows that the waves can potentially grow to large amplitudes driven by the drift between the reflected ions and the solar wind electrons. But this analysis obviously does not include the effects of nonlinearity and inhomogeneity, which can affect the wave growth. Moreover, there is another possible generation mechanism (dispersive shock model), which predicts the waves with the observed wavelength. In order to determine which mechanism is generating the observed whistlers, one has to compare the results of a full kinetic simulation to the observations. In this manuscript, we have presented the wave properties with high accuracy as well as the relevant observations of the ion VDFs, which can be used for such a comparison.
Data Availability Statement
MMS data are available at https://lasp.colorado.edu/mms/sdc/public/data/ following the directories: mms#/fgm/brst/12 for FGM data, mms#/fpi/brst/12/dist for FPI ion distributions, mms#/fpi/brst/12/des-moms for FPI electron moments. OMNI data used are available at https://omniweb.gsfc.nasa.gov/. The Bo solver can be found at http://dx.doi.org/10.17632/cvbrztyfz5.1. Data analysis was performed using the IRFU-Matlab analysis package. No new data have been produced as part of this project.

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