Field-direction Dependence of Majorana-mediated Spin Transport

Hirokazu Taguchi¹, Akihisa Koga¹, Yuta Murakami², Joji Nasu³, Hiroki Tsuchiura⁴

¹Department of Physics, Tokyo Institute of Technology, Meguro, Tokyo 152-8551, Japan
²Center for Emergent Matter Science, RIKEN, Wako 351-0198, Japan
³Department of Physics, Tohoku University, Sendai 980-8578, Japan
⁴Department of Applied Physics, Tohoku University, Sendai 980-8579, Japan

E-mail: koga@phys.titech.ac.jp

(Received June 3, 2022)

We study the field-direction dependence of the Majorana-mediated spin transport in the Kitaev clusters with zigzag and armchair edges, applying a static magnetic field to one of the edges and a magnetic pulsed field to the other edges. By means of the exact diagonalization method, we calculate the time-evolution of the spin moments in both edge regions to clarify how the directions of two fields and shape of the edges affect the Majorana-mediated spin transport.

KEYWORDS: Majorana fermions, Kitaev model, spin transport

1. Introduction

Transport phenomena of spin excitations have attracted considerable attention in condensed matter physics. In insulating magnets, spin excitations are mainly carried by magnons. It was recently suggested that, in the nonmagnetic ground state of the one-dimensional Heisenberg system, the spin transport is realized by spinons, which are elementary excitations fractionalized from spins. This has been observed in the spin Seebeck experiments for the cuprate Sr₂CuO₃ [1]. The Kitaev quantum spin model [2] is another candidate of nonmagnetic systems. In the system, the elementary excitations are described by two kinds of quasiparticles: itinerant and localized Majorana fermions [3], and their roles have extensively been discussed [4–10]. It has also been clarified that the itinerant Majorana fermions carry the spin excitations although no magnetic moments appear in the genuine Kitaev model [11–13]. Such a spin transport can be examined in the time-evolution of the Kitaev cluster with edges; the introduction of the magnetic pulse in one of the edges does not induce the spin oscillations in the bulk region, but induces them in the other edge with the static magnetic field. In the previous studies, this unusual phenomenon has been clarified in the cluster, where both magnetic pulse and static fields are applied along the S_z direction. Then, a question arises: how do similar spin oscillations appear even when the magnetic pulse are introduced along the S_x direction? Do itinerant Majorana fermions carry the direction of the magnetic field pulse as well as the spin excitations? This should be important to observe spin oscillations in the realistic systems.

Motivated by this, we consider the Kitaev clusters, where the magnetic fields in two edge regions are applied in the distinct directions. We study the field-direction dependent spin propagation by means of the exact diagonalization method. The difference in the shape of edges is also addressed.
Fig. 1. Kitaev clusters with (a) zigzag edges and (b) armchair edges. Green, red, and blue lines indicate $x$, $y$, and $z$ bonds, respectively. The static magnetic field $h_R$ is applied in the right (R) region, and no magnetic field is applied in the middle (M) region. A time-dependent pulsed magnetic field is introduced in the left (L) region. (c) Plaquette $P$ with sites marked $P_1$, $P_2$, $P_3$, $P_4$, $P_5$, $P_6$ shown for the operator $W_P$.

2. Model and Method

We consider the Kitaev clusters with zigzag and armchair edges, which are schematically shown in Figs. 1(a) and (b). The Hamiltonian is given as

$$H(t) = -J \sum_{\langle i,j \rangle_x} S_i^x S_j^x - J \sum_{\langle i,j \rangle_y} S_i^y S_j^y - J \sum_{\langle i,j \rangle_z} S_i^z S_j^z - h_R \sum_{i \in R} S_i^z - h_L(t) \sum_{i \in L} \mathbf{n} \cdot \mathbf{S}_i,$$

where $\langle i, j \rangle_\mu$ indicates the nearest-neighbor pair on the $\mu = x, y, z$-bonds. The $x$-, $y$-, and $z$-bonds are shown as green, red, and blue lines in Fig. 1. $S_i^\mu$ is the $\mu$ component of an $S = 1/2$ spin operator at site $i$. $J$ is the exchange coupling between the nearest neighbor sites. $h_R(= 0.01J)$ represents the static magnetic field in the R region. For simplicity, its direction is fixed as the $S_z$ direction. In the L region, the time-dependent magnetic field is applied in the direction $\mathbf{n}$ and its magnitude is given by the Gaussian form as

$$h_L(t) = \frac{A}{\sqrt{2\pi}\sigma} \exp \left[ \frac{t^2}{2\sigma^2} \right],$$

where $A$ and $\sigma$ are strength and width of the pulse. Here, we set $\sigma = 2/J$ and $A = 1$.

Now, we consider the famous local operator as

$$W_P = 2^6 S_{P_1}^x S_{P_2}^y S_{P_3}^z S_{P_4}^x S_{P_5}^y S_{P_6}^z,$$

where $P_i$ ($i = 1, 2, \ldots, 6$) is the site in the plaquette $P$ [see Fig. 1(c)]. Each local operator commutes with the Kitaev Hamiltonian eq. (1) with $h_R = h_L(t) = 0$. This means the existence of the local conserved quantities, and each eigenstate of the model is classified by the subspace with $\{w_P\}$, where $w_P$ is the eigenvalue of $W_P$. Furthermore, it is known that, in the ground state, $w_P = 1$ for each plaquette $P$ [2]. This leads to the important features for the Kitaev model. For example, one can prove that $\langle S_{P_1}^y \rangle = \langle S_{P_1}^z \rangle = \langle S_{P_2}^z \rangle = \langle S_{P_2}^x \rangle = \cdots = 0$ for the state with $w_P = 1$ on an isolated plaquette.
Furthermore, the Zeeman terms does not commute with the local operator moment at site 9 in the Kitaev cluster with zigzag edges even when absence of the moment in terms of the local conserved quantities, eg., the y-component of the spin moment at site 9 in the Kitaev cluster with zigzag edges even when $h_R = h_L(t) = 0$ [see Fig. 1(a)]. Furthermore, the Zeeman terms does not commute with the local operator $W_P$, the spin moments, in general, appear in both L and R regions under magnetic fields. These facts are important to observe the spin oscillations in the Kitaev system. In the following, we focus on the spin moments in the L and R regions. Here, we calculate the change in the moment as

$$\Delta S(t) = \langle S(t) \rangle - \langle S(-\infty) \rangle,$$

where $S(t)$ is the spin moment at time $t$.

In this study, we discuss how the shape of the edges and/or direction of the magnetic field affects the spin transport. To this end, we treat the 24-site Kitaev clusters with armchair and zigzag edges, as shown in Figs. 1(a) and (b). Then, using the Lanczos method [14–19], we examine the time evolution of the system after the magnetic pulse is applied in the $S^x$ and $S^z$ directions.

3. Results

First, we treat the zigzag-edge Kitaev cluster to discuss how the direction of the magnetic pulse induces the spin oscillations in the R region where the direction of the static magnetic field is fixed in the $S^z$ direction. Figure 2 shows the time-evolution of the change in the spin moments. When the magnetic pulse with $n = z$ is introduced in the L region, it is naively expected that the spin oscillations in the $S^z$ direction appear in the L region. In fact, the magnetic moments are induced around $t = 0$, as shown in Fig. 2(a). The subpeak-like structure around $t = 12/J$ should originate from complex factors such as the oscillation in the flux next of edges, the amplitude and width of the Gaussian pulse, finite size effect in the cluster, etc. This is beyond the main scope of our study, and will be discussed in the future. In the middle region bounded by the L and R regions, there are local conserved quantities $W_p$, and thereby magnetic moments never appear (not shown). In the R region, the spin oscillations are induced only in the $S^z$ direction around $t = 5/J$. This is consistent with the fact that the Hamiltonian is invariant under the symmetry operation $[1, U_i^z, 1]$, where $U_i^z = \exp[i\pi S_i^z]$ is the $\pi$-rotation along the $S^z$ direction for the $i$th spin. It has been clarified that this unusual spin propagation is mediated by the itinerant Majorana fermions, which is one of the fractionalized particles in the Kitaev model [11]. On the other hand, different behavior appears when $n = x$. In the L region, the oscillations in magnetic moments are induced along the applied field, as shown in Fig. 2(c). Namely, the curves of $z$ component in (a) and of $x$ component in (c) are the same due to the symmetry of the system. On the other hand, in the R region, the magnetic moments are induced along the $S^z$ direction. In this case, the direction of the spin oscillations is not related to the magnetic pulse introduced in the L region but the static field in the R region. This can be explained by the following symmetry argument. The Kitaev system can be regarded as the linked zigzag chains parallel to the edges. The Hamiltonian with $n = x$ is invariant under the symmetry operation $[1, U_i^x, \prod_{ee} U_i^z]$, where $o$ (e) represents the set of spins in odd (even) numbered zigzag chains from the left edge. Therefore, we can conclude the absence of the x and y components in the induced moment in the R region. This may suggest that the itinerant Majorana fermions propagate the wave packet triggered by the magnetic pulse, while cannot propagate its direction.

We also consider the Kitaev cluster with armchair edges. The spin oscillations in the Kitaev cluster with armchair edges are shown in Fig. 3. When the Gaussian magnetic pulse with $n = z$ is introduced in the L region at $t = 0$, the magnetic moments in the region L are immediately induced in the $S^z$ direction, as shown in Fig. 3(a). In the middle region, magnetic moments are never induced.
(not shown). In the R region under the static magnetic field, the finite spin oscillations appear in the $S^z$ direction, as shown in Fig. 3(b). When the magnetic pulse field is applied along the $S^x$ direction, the magnetic moments are induced along the $S^x$ direction in the L region, as shown in Fig. 3(c). In the R region, the induced magnetic moments have the components in the $S^z$ direction, as shown in Fig. 3(d). This fact is essentially the same as the result for the zigzag-edge Kitaev cluster discussed above. We have sometimes observed oscillation behavior in $S^x$ and $S^y$ directions (not shown). We have also found that this phenomenon strongly depend on the initial state in the Lanczos method, while the oscillation in the $S^z$ direction never changes. This can be explained by the following. There exists a local $Z_2$ operator at the edge: $X = \sigma^y_{11} \sigma^z_{12} \sigma^z_{6} \sigma^x_{5}$ in the Kitaev system with the armchair edges [see Fig. 1(b)]. This operator commutes with the Hamiltonian with the finite field $h_R$, which is similar to the operator $W_P$ in the middle region. Therefore, one can prove that $\langle S^i_1 \rangle = \langle S^i_2 \rangle = 0$ ($i = 6, 12$) for a certain eigenstate with $x = \pm 1$. However, in contrast to the fact that $w_P = 1$ in the ground state, the eigenvalue of $X$ is not uniquely determined, meaning the existence of the ground state degeneracy originated from the edges. This leads to the initial state dependent oscillations in the $S^x$ and $S^y$ directions, while the oscillation in the $S^z$ direction parallel to the applied field does not depend on the edge degeneracy. This can be clarified in the following. When the doubly degenerate normalized states $|\pm\rangle$ are defined such as $H|\pm\rangle = E_g|\pm\rangle$, $X|\pm\rangle = \pm|\pm\rangle$, and $\langle +| - \rangle = 0$, the ground state is, in
Fig. 3. Real-time evolution of the Kitaev cluster with armchair edges. (a) and (b) represent the change in the spin moments in the L and R regions when the direction of the injected magnetic pulse is along the $S^z$ direction $n = z$. (c) and (d) represent the change in the spin moments in the L and R regions when $n = x$.

general, given as

$$|\psi\rangle = |\alpha\rangle + |\beta\rangle,$$

where $|\alpha|^2 + |\beta|^2 = 1$. Now, we furthermore introduce the operator $Y$, which satisfies $Y^2 = 1$, $[X, Y] = 0$, $[Y, H] = 0$, and $Y|\pm\rangle = e^{\pm i\theta} |\mp\rangle$ with a real constant $\theta$. One of the examples is $Y = \sigma^z_{12}\sigma^z_{11}\sigma^y_{10}\sigma^z_{09}\sigma^y_8\sigma^z_{14}\sigma^z_{13}\sigma^y_{19}$. In the case, we prove $\langle\psi|\sigma^z_{12}|\psi\rangle = \langle +|\sigma^z_{12}||+\rangle$ since $\langle +|Y|\sigma^z_{12}||+\rangle = 0$ and $[\sigma^z_{12}, Y] = 0$. Therefore, we can say that the $z$ component of the moment at the site 12 does not depend on the initial state in the Lanczos method. On the other hand, the other components depends on $\alpha$ and $\beta$ at $t = 0$. By these reasons, we conclude that the itinerant Majorana fermions propagate the wave packet triggered by the magnetic pulse, while cannot propagate its direction.

4. Summary

We have investigated the Majorana-mediated spin transport in the Kitaev clusters with zigzag and armchair edges, applying the static magnetic field to one of the edges and magnetic pulsed field to the other edges. By means of the exact diagonalization methods, the time-evolution of the systems has been examined. In the Kitaev model with zigzag and armchair edges, the spin oscillations in the R region appear along the $S^z$ direction even when the magnetic pulse with $n = x$ is applied to the L region. Therefore, we conclude that the itinerant Majorana fermions propagate the spin excitation
injected by the magnetic pulse, while cannot propagate its direction. The present results clarifying the spin propagation independent on field directions suggest that the information of a spin component is mainly stored by the localized Majorana fermions rather than itinerant ones.

Acknowledgements

Parts of the numerical calculations are performed in the supercomputing systems in ISSP, the University of Tokyo. This work was supported by Grant-in-Aid for Scientific Research from JSPS, KAKENHI Grant Nos. JP17K05536, JP19H05821, JP21H01025, JP22K03525 (A.K.), JP20K14412, JP21H05017 (Y.M.), JP19K03742, JP20H00122, JP22H01175 (J.N.), JP21H01025 (H.T.), and JST CREST Grant Nos. JPMJCR1901 (Y.M.), JPMJCR17J5 (H.T.), and JST PRESTO Grant No. JP-MJPR19L5 (J.N.).

References

[1] D. Hirobe, M. Sato, T. Kawamata, Y. Shiomi, K. i. Uchida, R. Iguchi, Y. Koike, S. Maekawa, and E. Saitoh: Nat. Phys. 13 (2017) 30.
[2] A. Kitaev: Ann. Phys. 321 (2006) 2.
[3] Y. Motome and J. Nasu: J. Phys. Soc. Jpn 89 (2020) 012002.
[4] J. Nasu, M. Udagawa, and Y. Motome: Phys. Rev. B 92 (2015) 115122.
[5] J. Feldmeier, W. Natori, M. Knap, and J. Knolle: Phys. Rev. B 102 (2020) 134423.
[6] E. J. König, M. T. Randeria, and B. Jäck: Phys. Rev. Lett. 125 (2020) 267206.
[7] R. G. Pereira and R. Egger: Phys. Rev. Lett. 125 (2020) 227202.
[8] M. Udagawa, S. Takayoshi, and T. Oka: Phys. Rev. Lett. 126 (2021) 127201.
[9] A. Koga, Y. Murakami, and J. Nasu: Phys. Rev. B 103 (2021) 214421.
[10] S.-H. Jang, Y. Kato, and Y. Motome: Phys. Rev. B 104 (2021) 085142.
[11] T. Minakawa, Y. Murakami, A. Koga, and J. Nasu: Phys. Rev. Lett. 125 (2020) 047204.
[12] H. Taguchi, Y. Murakami, A. Koga, and J. Nasu: Phys. Rev. B 104 (2021) 125139.
[13] H. Taguchi, Y. Murakami, and A. Koga: Phys. Rev. B 105 (2022) 125137.
[14] T. J. Park and J. C. Light: J. Chem. Phys. 85 (1986) 5870.
[15] Y. Saad: SIAM J. Numer. Anal. 29 (1992) 209.
[16] V. Druskin and L. Knizhnerman: Numerical Linear Algebra with Applications 2 (1995) 205.
[17] M. Hochbruck and C. Lubich: SIAM J. Numer. Anal. 34 (1997) 1911.
[18] M. Hochbruck, C. Lubich, and H. Selhofer: SIAM J. Sci. Comp. 19 (1998) 1552.
[19] M. Hochbruck and C. Lubich: BIT Numerical Mathematics 39 (1999) 620.