Neutrino-photon Scattering in a Magnetic Field
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Abstract
Previous calculations of photon-neutrino scattering in a constant background magnetic field are extended and corrected.

1. Introduction

For massless neutrinos, the leading contribution to low energy neutrino-photon scattering vanishes because the amplitude can be written in the form of a vector or axial-vector current coupling to the photons. The largest low energy interaction between photons and neutrinos is therefore $\gamma \nu \rightarrow \gamma \gamma \nu$ and the crossed channels. Rates for these processes have been calculated, for energies less than the electron mass, $m_e$, by using an effective action [1] and, for larger energies, by a straight-forward calculation of box diagrams [2].

In the presence of magnetic fields neutrino photon elastic scattering does exist because the magnetic field gives an additional interaction – it effectively replaces one of the photons in $\gamma \nu \rightarrow \gamma \gamma \nu$ with a zero momentum insertion. This too has been calculated; for low energies using the effective action for the $2 \rightarrow 3$ processes [3] and, at higher energies, by calculating triangle diagrams in the external field [4, 5].

Our purpose here is to criticize mildly and extend slightly these external field results. In particular the effective action paper [3] gave results for only one channel ($\gamma \gamma \rightarrow \nu \bar{\nu}$). We extend this to all channels with $B$ perpendicular and parallel for each channel. Results of the more general calculation of Ref. [4] are given in three figures. We believe the curves in these figures are somewhat in error; for example, they sometimes disagree at low energies with our extended effective action results. Both references [3] and [4] express their results in terms of the critical magnetic field $B_c = m_e^2/e = 4.41 \times 10^{13}$ gauss. In each case the authors have taken $e$ to be in unrationalized units, $e^2 = 1/137$. whereas it should have been taken to be in natural units $e^2 = 4\pi/137$. Thus all their results are too large by a factor of $4\pi$. 
2. Neutrino-photon cross sections in a uniform magnetic field

The calculation of Ref. [3] uses the effective interaction given in Ref. [1]. In this interaction, as in the Euler-Heisenberg Lagrangian upon which it is based, $\alpha$ equals $e^2/4\pi$ so the coupling of the magnetic field has been taken as this $e$. As discussed in Ref. [6] the critical field, when written as $m_e^2/e$, means that in natural (or rationalized) units 1 tesla = 195 eV$^2$. The result of Ref. [3] agrees in every respect with the result given below for $\gamma\gamma \rightarrow \nu\bar{\nu}$ with the external field perpendicular to the direction of the photons except in the replacement of $B_c^2$ by $m_e^4/4\pi\alpha$ where it should be replaced by $m_e^4/4\pi\alpha$. Since the curve $\gamma\gamma \rightarrow \nu\bar{\nu}$ in Fig. 2 of Ref. [4] agrees, at low energy, with the result of Ref. [3] these authors too must have made the wrong substitution for $B_c$.

2.1 Low energy interactions

The explicit calculations at low energy, using the effective action, are unremarkable and give the following results. For $\gamma\gamma \rightarrow \nu\bar{\nu}$ and $\nu\bar{\nu} \rightarrow \gamma\gamma$ the cross sections with center of mass energy $\omega$ and scattering angle $z$ are given by

$$\frac{d\sigma}{dz} = \sigma_0 \frac{A}{4\pi} \left( \frac{\omega}{m_e} \right)^6 \left( \frac{B}{B_c} \right)^2 \left[ 2312 \left( 1 - N_1^2 \right) + 816 N_1 N_2 z - N_2^2 (744 + 72z^2) \right] ,$$

(1)

where $A = 1$ for $\gamma\gamma \rightarrow \nu\bar{\nu}$ and 1/2 for $\nu\bar{\nu} \rightarrow \gamma\gamma$. For electron neutrinos, including both the $W$ and $Z$-boson contributions, $a = 1/2 + 2\sin^2 \theta_W$ and

$$\sigma_0 = \frac{G_F^2 a^2 m_e^2}{(180)^2 \pi^2} = 2.12 \times 10^{-54} \text{cm}^2 ,$$

(2)

where $G_F$ is the Fermi constant and $\theta_W$ is the weak mixing angle. $N_1$ and $N_2$ depend on whether the $B$ field is parallel or perpendicular to the direction of the initial particles and are given in Table I.

The cross section for $\gamma\nu \rightarrow \gamma\nu$ is given by

$$\frac{d\sigma}{dz} = \sigma_0 \frac{A}{4\pi} \left( \frac{\omega}{m_e} \right)^6 \left( \frac{B}{B_c} \right)^2 \left[ 1211 - 143z^2 - \left( N_1^2 + N_2^2 \right) (896 + 62z^2) + N_1 N_2 z (859 - 11z^2) \right]$$

(3)

where terms odd in $z$ have been dropped. Using the $N_1$ and $N_2$ values and completing the integral over $z$ gives

$$\sigma = \frac{\sigma_0}{4\pi} \left( \frac{\omega}{m_e} \right)^6 \left( \frac{B}{B_c} \right)^2 X ,$$

(4)
where the \( X \) values for the six cases are given in Table I. Note that the result for \( \gamma\gamma \rightarrow \nu\nu \) with perpendicular \( B \) agrees with Ref. [3] if the \( 4\pi \) is ignored. Also note that the differential cross sections for parallel \( B \) go as \( 1 - z^2 \) as they must.

The \( X \) values show that the cross sections for \( \gamma\nu \rightarrow \gamma\nu \) differ by a factor of about 4 depending on whether the magnetic field is parallel or perpendicular. The cross sections for \( \nu\nu \rightarrow \gamma\gamma \) differ by a factor of about 200. These differences are not present in the figures of Ref. [4].

2.2 Interactions at arbitrary center of mass energies

The authors of Refs. [4, 5] do a nice job of deriving the lowest order constant magnetic field contribution to photon-neutrino scattering, both from the correction to the electron propagator and from the phase factor. Their expression for the matrix element [5] seems entirely correct with the exception of a relative minus sign in the tensor which multiplies the coefficient function \( C_{11} \). The authors of Ref. [5] inform us that this is a typographical error [7]. We agree with all the coefficient functions \( C_1, \ldots, C_{11} \).

The results of our calculation of neutrino-photon scattering in a constant magnetic field following the methods of Refs. [4] and [5] are shown in Figures 1, 2, and 3 for the three channels. All calculations are for \( B = 0.1B_c \) as is the case in Ref.[4]. At low energies the cross sections agree almost exactly with Eq. (4). Note the relative factor of about 4 between the magnetic field directions for \( \gamma\nu \rightarrow \gamma\nu \) and the similar factor of about 200 for \( \nu\nu \rightarrow \gamma\gamma \). Also note the infrared divergence at \( \omega = m_e \) in the \( \gamma\gamma \leftrightarrow \nu\nu \) channels where the magnetic field interaction acts as a zero energy insertion on an external leg.

For large energy all of the cross sections grow as \( \omega^2 \) except for \( \gamma\gamma \rightarrow \nu\nu \) with \( B \) parallel to the photon direction, which decreases. After summation over photon polarizations, the cross section for this channel can be written as

\[
\sigma = A_1 F^{\mu\nu} F_{\mu\nu} + A_2 (k_1^\mu F_{\mu\alpha} F^{\alpha\nu} k_{1\nu} + k_2^\mu F_{\mu\alpha} F^{\alpha\nu} k_{2\nu}) \\
+ A_3 k_1^\mu F_{\mu\alpha} F^{\alpha\nu} k_{2\nu} + A_4 (k_1^\mu F_{\mu\nu} k_2^\nu)^2, \tag{5}
\]

where \( F^{\mu\nu} \) is the field strength of the magnetic field and \( k_1 \) and \( k_2 \) are the momenta of the photons. The \( A_i \) are combinations of the coefficient functions \( C_1, C_2, \ldots, C_{11} \). For a magnetic field in a direction \( \vec{n} \) this reduces to

\[
\sigma \sim 2A_1 - \omega^2 A_2 [2 - (\hat{k}_1 \cdot \vec{n})^2 - (\hat{k}_2 \cdot \vec{n})^2] + \omega^2 A_3 [1 + \hat{k}_1 \cdot \vec{n} \hat{k}_2 \cdot \vec{n}], \tag{6}
\]
Thus for $\gamma \gamma \rightarrow \nu \bar{\nu}$ with $\vec{n}$ parallel to $\hat{k}_1$ and $\hat{k}_2$ the coefficients of $A_2$ and $A_3$ vanish. $A_1$, which in this channel depends only on $C_1$, falls as $\ln^2(\omega^2/m_e^2)/\omega^2$ for $\omega >> m_e$, giving the decrease at large $\omega$ seen in Fig. (1). $A_2$ and $A_3$ do not vanish for $\vec{n}$ perpendicular to $\hat{k}_1$ and $\hat{k}_2$ nor do they vanish for either $B$ direction in the other channels.

3. Conclusions

We have extended the low energy effective Lagrangian calculation of Ref. [3] to include all channels and directions of the magnetic field $B$. In Figs. (1-3), we have given correct values for the cross sections at all energies, which, while shown for $B = 0.1B_c$, can be scaled by $B^2$. (Of course, following Ref. [4] and [5], we have essentially done perturbation theory in $B/B_c$ so our results are not valid for much larger $B$.) All calculations were performed for electron neutrinos but this too can be easily changed by using $a = 1/2 - 2\sin^2\theta_W$ for the other neutrino types.

It turns out that neutrino photon scattering in an external magnetic field is less important, relative to the $2 \rightarrow 3$ processes, than previously thought [4]. Since the effects of these processes on stars was already calculated [4] to be small we have not repeated the determination of the stellar energy loss rates or mean free paths.

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Tables

| Process          | $N_1$   | $N_2$ | $X$       |
|------------------|---------|-------|-----------|
| $(\gamma \gamma \rightarrow \nu \bar{\nu})_\perp$ | $y/\sqrt{2}$ | 0     | $9248/3$  |
| $(\gamma \gamma \rightarrow \nu \bar{\nu})_\parallel$ | $z$     | 1     | $6272/3$  |
| $(\nu \bar{\nu} \rightarrow \gamma \gamma)_\perp$ | 0       | $y/\sqrt{2}$ | $10296/5$ |
| $(\nu \bar{\nu} \rightarrow \gamma \gamma)_\parallel$ | 1       | $z$   | $48/5$    |
| $(\gamma \nu \rightarrow \gamma \nu)_\perp$     | 0       | $y/\sqrt{2}$ | $25816/15$ |
| $(\gamma \nu \rightarrow \gamma \nu)_\parallel$ | 1       | $z$   | $6592/15$ |

Table 1: The subscript $\perp$ or $\parallel$ on the processes indicate the direction of the magnetic field perpendicular or parallel to the momentum of the initial particles. $y$ is the sine of the scattering angle, $y^2 = 1 - z^2$.

Figures

Figure 1: The cross section $\sigma(\gamma \gamma \rightarrow \nu \bar{\nu})$ in the presence of a constant magnetic field $B$ is shown as a function of the photon energy $\omega$. Two magnet field orientations, parallel and perpendicular to the direction incoming photons in the center of mass are indicated. The solid line is the cross section for the $2 \rightarrow 3$ process $\gamma \gamma \rightarrow \nu \bar{\nu} \gamma$. 
Figure 2: The cross section $\sigma(\nu\bar{\nu} \rightarrow \gamma\gamma)$ in the presence of a constant magnetic field $B$ is shown as a function of the photon energy $\omega$. Two magnet field orientations, parallel and perpendicular to the direction incoming photons in the center of mass are indicated. The solid line is the cross section for the $2 \rightarrow 3$ process $\nu\bar{\nu} \rightarrow \gamma\gamma\gamma$.

Figure 3: The cross section $\sigma(\gamma\nu \rightarrow \gamma\nu)$ in the presence of a constant magnetic field $B$ is shown as a function of the photon energy $\omega$. Two magnet field orientations, parallel and perpendicular to the direction incoming photons in the center of mass are indicated. The solid line is the cross section for the $2 \rightarrow 3$ process $\gamma\nu \rightarrow \gamma\gamma\nu$. 