Comparative Numerical Study of SBA (Somé Blaise-Abbo) Method and Homotopy Perturbation Method (HPM) on Biomathematical Models Type Lotka-Volterra

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Abstract

In this work the Homotopy Perturbation Method (HPM) is used to find an exact or approximate solutions of Lotka-Volterra models. Then we compare the HPM solution with the solution given by SBA (Somé Blaise Abbo) method.

Keywords: Lotka-Volterra models, Homotopy Perturbation Method (HPM), SBA (Somé Blaise Abbo) method

1. Introduction

We are interested in the resolution of nonlinear systems of general form:

\[ \begin{align*}
\frac{du}{dt} &= f(u,v) + g(u,v) \\
\frac{dv}{dt} &= \varphi(u,v) + \psi(u,v)
\end{align*} \]

in a suitable Hilbert space, where \( f(u,v) \) et \( \varphi(u,v) \) are the linear functions and \( \psi(u,v) \) are nonlinear function.

These systems model a large number of phenomena in biomathematics. They are also Lotka-Volterra equations type of the form:

\[ \begin{align*}
\frac{du}{dt} &= au + buv \\
\frac{dv}{dt} &= -cv + buv
\end{align*} \]

In this article, we use Homotopy Perturbation Method (BAGAYOGO, M, 2019; M.S.H. Chowdhury & et al., 2010) to solve Lotka-Volterra models. Then we compare the HPM solutions with the solution given by SBA (Somé Blaise Abbo) method (MINOUNGOU,Y., 2019; PARE, Y., 2010). SBA (MINOUNGOU,Y., 2019; PARE, Y., 2010; YARO, R., 2016) method is a numerical method for obtaining exact or approximate solutions of linear or nonlinear equations.

The paper is organized as follows: in the following section the homotopy perturbation method is explained. In Section 3 we solve two problems. Numerical results are reported in Section 4. Finally, the paper is concluded in Section 5.

2. HPM for System of ODEs

To illustrate the basic idea of the HPM for system of PDEs, we consider the following non-homogeneous, non-linear system of ODEs (M.S.H. Chowdhury & et al., 2010; M.S.H. Chowdhury & et al., 2015).

\[ \begin{align*}
\frac{du_1}{dt} + g_1(t, u_1, u_2, \cdots, u_m) &= f_1(t) \\
\frac{du_2}{dt} + g_2(t, u_1, u_2, \cdots, u_m) &= f_2(t)
\end{align*} \]
\[ \frac{du_m}{dt} + g_m(t, u_1, u_2, \ldots, u_m) = f_m(t) \quad (3) \]

subject to the initial conditions
\[ u_1(0) = c_1, u_2(0) = c_2, \ldots, u_m(0) = c_m \quad (4) \]

where \( u_m = u_m(t) \) and \( f_m = f_m(t) \)

First write system (1)-(3) in the operator form:
\[ L(u_1) + N_1(u_1, u_2, \ldots, u_m) = f_1 = 0 \quad (5) \]
\[ L(u_2) + N_2(u_1, u_2, \ldots, u_m) = f_2 = 0 \quad (6) \]
\[ \vdots \]
\[ L(u_m) + N_m(u_1, u_2, \ldots, u_m) = f_m = 0 \quad (7) \]

subject to the initial conditions (4), where \( L(\cdot) = \frac{d(\cdot)}{dt} \) is linear operator and \( N_1, N_2, \ldots, N_m \) are nonlinear operators.

According to HPM (BAGAYOGO, M., 2019; BAGAYOGO, M., 2018; BAGAYOGO, M. & et al., 2019), we construct a homotopy for (5)-(7) which satisfies the following relations:
\[ L(u_1) - L(v_1) + pL(v_1) + p[N_1(u_1, u_2, \ldots, u_m) - f_1] = 0 \]
\[ L(u_2) - L(v_2) + pL(v_2) + p[N_2(u_1, u_2, \ldots, u_m) - f_2] = 0 \]
\[ \vdots \]
\[ L(u_m) - L(v_m) + pL(v_m) + p[N_m(u_1, u_2, \ldots, u_m) - f_m] = 0 \quad (8) \]

where \( p \in [0, 1] \) is an embedding parameter and \( v_1, v_2, \ldots, v_m \) are initial approximations which satisfying the given conditions. Consider the initial approximations as follows:
\[ u_{1,0}(t) = v_1(t) = u_1(0) = c_1 \]
\[ u_{2,0}(t) = v_2(t) = u_2(0) = c_2 \]
\[ \vdots \]
\[ u_{m,0}(t) = v_m(t) = u_m(0) = c_m \]

and
\[ u_1(t) = u_{1,0}(t) + pu_{1,1}(t) + p^2u_{1,2}(t) + \cdots \]
\[ u_2(t) = u_{2,0}(t) + pu_{2,1}(t) + p^2u_{2,2}(t) + \cdots \]
\[ \vdots \]
\[ u_m(t) = u_{m,0}(t) + pu_{m,1}(t) + p^2u_{m,2}(t) + \cdots \quad (9) \]

where \( u_{i,j} (i = 1, 2, \ldots, m; \ j = 1, 2, \ldots, \cdots) \) are functions yet to be determined. Substituting (9) into (8) arranging the coefficients of the same powers of \( p \), obtain
\[ L(u_{1,1}) + L(v_1) + N_1(u_{1,0}, u_{2,0}, \ldots, u_{m,0}) - f_1 = 0, \ u_{1,1}(0) = 0 \]
\[ L(u_{2,1}) + L(v_2) + N_2(u_{1,0}, u_{2,0}, \ldots, u_{m,0}) - f_2 = 0, \ u_{2,1}(0) = 0 \]
\[ \vdots \]
\[ L(u_{m,1}) + L(v_m) + N_m(u_{1,0}, u_{2,0}, \ldots, u_{m,0}) - f_m = 0, \ u_{m,1}(0) = 0 \]
\[ L(u_{1,2}) + N_1(u_{1,1}, u_{2,1}, \ldots, u_{m,1}) - f_1 = 0, \ u_{1,2}(0) = 0 \]
\[ L(u_{2,2}) + N_2(u_{1,1}, u_{2,1}, \cdots, u_{m,1}) - f_2 = 0, \quad u_{2,2}(0) = 0 \]
\[
\vdots
\]
\[ L(u_{m,2}) + N_m(u_{1,1}, u_{2,1}, \cdots, u_{m,1}) - f_m = 0, \quad u_{m,2}(0) = 0 \]

etc

Now solve the above systems of equations for the unknowns \( u_{ij} (i = 1, 2, \cdots, m; j = 1, 2, \cdots, \cdots) \). Therefore, according to HPM the \( n \)-term approximations for the solutions of (5)-(7) can be expressed as

\[
\phi_{1,n}(t) = u_1(t) = \lim_{p \to 1} u_1(t) = \sum_{k=0}^{n-1} u_{1,k}(t)
\]
\[
\phi_{2,n}(t) = u_2(t) = \lim_{p \to 1} u_2(t) = \sum_{k=0}^{n-1} u_{2,k}(t)
\]
\[
\vdots
\]
\[
\phi_{m,n}(t) = u_m(t) = \lim_{p \to 1} u_m(t) = \sum_{k=0}^{n-1} u_{m,k}(t)
\]

3. Application

3.1 Example 1

Consider a Lotka-Volterra model which describe predators-prey interactions (ABBO, B., 2007):

\[
\begin{align*}
\frac{du}{dt} &= \alpha u - c_2 u^2 + c_1 uv
\frac{dv}{dt} &= \alpha v - c_1 v^2 - c_2 uv
\end{align*}
\]

(10)

with the initial conditions \( u(0) = c_1 \) et \( v(0) = c_2 \).

The exact solution of (10) obtained by SBA method is (ABBO, B., 2007):

\[
(u, v) = (c_1 e^{\alpha t}, c_2 e^{\alpha t})
\]

In order to apply the homotopy perturbation method, we construct the following homotopy equations:

\[
(1 - p) \left[ \frac{du}{dt} - \alpha u \right] + p \left[ \frac{du}{dt} - \alpha u + c_2 u^2 + c_1 uv \right] = 0
\]
\[
(1 - p) \left[ \frac{dv}{dt} - \alpha v \right] + p \left[ \frac{dv}{dt} - \alpha v + c_1 v^2 + c_2 uv \right] = 0
\]

(11)

We obtain the following homotopy system:

\[
\begin{align*}
\frac{du}{dt} - \alpha u + pc_2 u^2 u - pc_1 uv &= 0
\frac{dv}{dt} - \alpha v - c_1 v^2 + c_2 uv &= 0
\end{align*}
\]

Assume the solution of (10) to be in the form:

\[
\begin{align*}
u(t) &= u_0 + pu_1 + p^2 u_2 + p^3 u_3 + \cdots \\
v(t) &= v_0 + pv_1 + p^2 v_2 + p^3 v_3 + \cdots
\end{align*}
\]

(12)
Substituting (12) into (11) and equating the coefficients of like \( p \), we get the following set of differential equations:

\[
\begin{align*}
\text{For} \quad p^0 & : \\
\frac{du_0}{dt} - \alpha u_0 &= 0 \\
\frac{dv_0}{dt} - \alpha v_0 &= 0 \quad \text{where} \quad u_0(0) = c_1, \quad v_0(0) = c_2 \\
\frac{d u_0}{dt} &= 0 \\
\frac{d v_0}{dt} &= 0 \\
\end{align*}
\]

\[
\begin{align*}
\text{For} \quad p^1 & : \\
\frac{du_1}{dt} - \alpha u_1 - c_1 u_0 v_0 + c_2 u_0^2 &= 0 \\
\frac{dv_1}{dt} - \alpha v_1 - c_1 v_0^2 + c_2 u_0 v_0 &= 0 \\
\frac{d u_1}{dt} &= 0 \\
\frac{d v_1}{dt} &= 0 \quad \text{where} \quad u_1(0) = 0, \quad v_1(0) = 0 \\
\text{For} \quad p^2 & : \\
\frac{du_2}{dt} - \alpha u_2 - c_1 u_0 v_1 - c_1 u_1 v_0 + 2 c_2 u_0 u_1 &= 0 \\
\frac{dv_2}{dt} - \alpha v_2 - 2 c_1 v_0 v_1 + c_2 u_0 v_1 + c_2 u_1 v_0 &= 0 \\
\frac{d u_2}{dt} &= 0 \\
\frac{d v_2}{dt} &= 0 \quad \text{where} \quad u_2(0) = 0, \quad v_2(0) = 0 \\
\text{For} \quad p^n & : \\
\frac{d u_n}{dt} - \alpha u_n - c_1 u_0 v_{n-1} - c_1 u_{n-1} v_0 + 2 c_2 u_0 u_{n-1} &= 0 \\
\frac{d v_n}{dt} - \alpha v_n - 2 c_1 v_0 v_{n-1} + c_2 u_0 v_{n-1} + c_2 u_{n-1} v_0 &= 0 \\
\frac{d u_n}{dt} &= 0 \\
\frac{d v_n}{dt} &= 0 \quad \text{where} \quad u_n(0) = 0, \quad v_n(0) = 0 \\
\end{align*}
\]

Solving the above equations, we obtain:

\[
\begin{align*}
\frac{du}{dt} &= c_1 e^{\alpha t} \\
\frac{dv}{dt} &= c_2 e^{\alpha t} \\
\end{align*}
\]

For \( n \geq 1 \), we have \( u_n(t) = v_n(t) = 0 \)

Hence the solution of (10) by HPM is given by

\[
\frac{du}{dt} = \lim_{p \to 1} \left[ u_0(t) + p u_1(t) + p^2 u_2(t) + \cdots \right] \]

\[
\frac{dv}{dt} = \lim_{p \to 1} \left[ v_0(t) + p v_1(t) + p^2 v_2(t) + \cdots \right]
\]

Therefore

\[
\frac{du}{dt} = c_1 e^{\alpha t} \quad \text{et} \quad \frac{dv}{dt} = c_2 e^{\alpha t}
\]

3.2 Example 2

Consider a Lotka-Volterra model, describing competitors (YARO, R., 2016)

\[
\begin{align*}
\frac{du}{dt} &= \frac{1}{7} (3u + 10v) + 2uv(u + v - (\alpha + \beta)e^t) \\
\frac{dv}{dt} &= \frac{1}{7} (4u - 3v) - 2uv(2u - 5v - (\alpha + \beta)e^-t) \\
\end{align*}
\]

with the initial conditions \( u(0) = \alpha = 6 \cdot 10^6 \) and \( v(0) = \beta = 10^6 \) where \( \alpha = 6\beta \)
The exact solution of (16) obtained by SBA method is (YARO, R., (2016) :

\[ (u, v) = \left( \alpha ch(t) + \frac{1}{7} (3\alpha + 10\beta) \ sh(t), \beta ch(t) + \frac{1}{7} (4\alpha - 3\beta) \ sh(t) \right) \]

According to HPM, we construct the following homotopy equations :

\[
(1 - p) \left[ \frac{du}{dt} - \frac{3}{7}u + p \left[ \frac{du}{dt} - \frac{3}{7}u - \frac{1}{7} (3u + 10v) - 2uv(u + v - (\alpha + \beta)e^t) \right] \right] = 0
\]

\[
(1 - p) \left[ \frac{dv}{dt} - \frac{4}{7}v + p \left[ \frac{dv}{dt} - \frac{4}{7}v - \frac{1}{7} (4u - 3v) + 2uv(2u - 5v - (\alpha + \beta)e^{-t}) \right] \right] = 0
\]

We obtain the homotopy system:

\[
\begin{align*}
\frac{du}{dt} - \frac{3}{7}u - p \frac{10}{7} v - 2puv(u + v - (\alpha + \beta)e^t) &= 0 \\
\frac{dv}{dt} - \frac{4}{7}v - p \frac{4}{7}u + 2puv(2u - 5v - (\alpha + \beta)e^{-t}) &= 0
\end{align*}
\]

(17)

Suppose the solution of (16) have the form:

\[
\begin{align*}
u(t) &= u_0 + pu_1 + p^2u_2 + p^3u_3 + \cdots \\
v(t) &= v_0 + pv_1 + p^2v_2 + p^3v_3 + \cdots
\end{align*}
\]

(18)

Then substituting (18) into (17) and rearranging based on powers of p-terms, we have:

\[
\begin{align*}
p^0 : \quad & \frac{du_0}{dt} - \frac{3}{7}u_0 = 0 \\
& \frac{dv_0}{dt} - \frac{4}{7}v_0 = 0 \quad u_0(0) = \alpha, \ v_0(0) = \beta
\end{align*}
\]

(19)

\[
\begin{align*}
p^1 : \quad & \frac{du_1}{dt} - \frac{3}{7}u_1 + 2u_0v_0[u_0 + v_0 + (\alpha + \beta)e^t] - \frac{10}{7} v_0 = 0 \\
& \frac{dv_1}{dt} - \frac{4}{7}v_1 + 2u_0v_0[2u_0 - 5v_0 - (\alpha + \beta)e^{-t}] + \frac{4}{7}u_0 = 0 \\
& u_1(0) = v_1(0) = 0
\end{align*}
\]

(20)

\[
\begin{align*}
p^2 : \quad & \frac{du_2}{dt} - \frac{3}{7}u_2 + 2u_0v_1(\alpha + \beta)e^t + 2u_1v_0(\alpha + \beta)e^t - 4u_0v_0(u_1 + v_1) \\
& \quad - 2u_0v_1(u_0 + v_0) - \frac{10}{7} v_1 = 0 \\
& \frac{dv_2}{dt} - \frac{4}{7}v_2 - 2u_0v_1(\alpha + \beta)e^{-t} + 2u_1v_0(\alpha + \beta)e^{-t} - 4u_0v_0(2u_1 - 5v_1) \\
& \quad + 2u_0v_1(2u_0 - 5v_0) + \frac{4}{7}u_1 = 0 \\
& u_2(0) = v_2(0) = 0
\end{align*}
\]

(21)

The solutions of above equations are as follows:

\[
\begin{align*}
u_0(t) &= \alpha e^{\frac{2t}{7}} \\
v_0(t) &= \beta e^{\frac{2t}{7}} \\
u_1(t) &= \frac{14 e^{2t} \alpha \beta^2}{11} + \frac{7 e^{\frac{3t}{7}} \alpha \beta^2}{4} - \frac{21 e^{\frac{4t}{7}} \alpha \beta^2}{44} - \frac{14 e^{2t} \alpha^2 \beta}{11} + 2 e^{\frac{3t}{7}} \alpha^2 \beta \\
& \quad - \frac{8 e^{\frac{3t}{7}} \alpha^2 \beta}{11} + 10 e^{\frac{4t}{7}} \beta - 10 e^{\frac{3t}{7}} \beta
\end{align*}
\]
\[\begin{align*}
v_1(t) &= 10e^{\frac{\alpha}{2}}\beta^2 - 13e^{\frac{\alpha}{2}}\beta^2 - 7\alpha^2 - \frac{14}{3}e^{\frac{\alpha}{2}}\beta^2 + 49e^{\frac{\alpha}{2}}\beta^2 - \frac{7}{2}\alpha^2 \\
&\quad - 4e^{\frac{\alpha}{2}}\alpha + 4e^{\frac{\beta}{2}}
\end{align*}\]

\[\begin{align*}
u_2(t) &= \frac{28e^{\frac{\alpha}{2}}\beta^4}{33} - \frac{931e^{\frac{\alpha}{2}}\beta^2}{396} + \frac{49e^{\frac{\alpha}{2}}\beta^4}{30} + \frac{147e^{\frac{\alpha}{2}}\beta^4}{220} - \frac{21e^{\frac{\alpha}{2}}\beta^4}{2} \\
&\quad + \frac{7e^{\frac{\alpha}{2}}\beta^4}{45} + \frac{56e^{\frac{\alpha}{2}}\beta^2}{33} - \frac{931e^{\frac{\alpha}{2}}\beta^2}{396} - \frac{2240e^{\frac{\alpha}{2}}\beta^2}{187} + \frac{51e^{\frac{\alpha}{2}}\beta^2}{2} \\
&\quad + \frac{2373e^{\frac{1}{2}}\beta^3}{11} + \frac{302e^{\frac{1}{2}}\beta^3}{33} + \frac{196e^{\frac{1}{2}}\beta^3}{3} - \frac{98e^{\frac{1}{2}}\beta^3}{2} \\
&\quad + \frac{23354e^{\frac{1}{2}}\beta^3}{765} - \frac{140e^{\frac{1}{2}}\beta^3}{11} + \frac{14e^{\frac{1}{2}}\beta^3}{2} - \frac{35e^{\frac{1}{2}}\beta^3}{2} - 20e^{\frac{1}{2}}\beta^3 \\
&\quad + \frac{27e^{\frac{1}{2}}\beta^3}{22} + \frac{28e^{\frac{1}{2}}\beta^3}{33} - \frac{2240e^{\frac{1}{2}}\beta^3}{187} + \frac{49e^{\frac{1}{2}}\beta^3}{12} + \frac{196e^{\frac{1}{2}}\beta^3}{39} \\
&\quad - \frac{217e^{\frac{1}{2}}\beta^3}{2} + \frac{98e^{\frac{1}{2}}\beta^3}{3} - \frac{49e^{\frac{1}{2}}\beta^3}{2} - \frac{98e^{\frac{1}{2}}\beta^3}{2} - \frac{49e^{\frac{1}{2}}\beta^3}{2} \\
&\quad + \frac{132881e^{\frac{1}{2}}\beta^3}{4420} - \frac{140e^{\frac{1}{2}}\beta^3}{11} + \frac{14e^{\frac{1}{2}}\beta^3}{2} + \frac{380e^{\frac{1}{2}}\beta^3}{7} \\
&\quad - \frac{140e^{\frac{1}{2}}\beta^3}{3} - \frac{65e^{\frac{1}{2}}\beta^3}{7} - \frac{924e^{\frac{1}{2}}\beta^3}{4} + \frac{12e^{\frac{1}{2}}\beta^3}{2} - \frac{49e^{\frac{1}{2}}\beta^3}{2} \\
&\quad - \frac{49e^{\frac{1}{2}}\beta^3}{9} - \frac{343e^{\frac{1}{2}}\beta^3}{3} + \frac{245e^{\frac{1}{2}}\beta^3}{9} - \frac{49e^{\frac{1}{2}}\beta^3}{2} + \frac{1813e^{\frac{1}{2}}\beta^3}{180} \\
&\quad + \frac{28e^{\frac{1}{2}}\beta^3}{5} + \frac{56e^{\frac{1}{2}}\beta^3}{9} - \frac{16e^{\frac{1}{2}}\beta^3}{9} + \frac{98e^{\frac{1}{2}}\beta^3}{3} + \frac{51e^{\frac{1}{2}}\beta^3}{3} \\
&\quad - \frac{181e^{\frac{1}{2}}\beta^3}{60} + \frac{35e^{\frac{1}{2}}\beta^3}{4} + \frac{28e^{\frac{1}{2}}\beta^3}{5} + \frac{56e^{\frac{1}{2}}\beta^3}{9} - \frac{28e^{\frac{1}{2}}\beta^3}{3} + \frac{56e^{\frac{1}{2}}\beta^3}{3} \\
&\quad - \frac{56e^{\frac{1}{2}}\alpha^3}{45} - \frac{40e^{\frac{1}{2}}\alpha}{7} + 40e^{\frac{1}{2}}\alpha - 40e^{\frac{1}{2}}\alpha \\
&\quad + 490e^{\frac{1}{2}}\beta^4 - \frac{49e^{\frac{1}{2}}\beta^4}{6} + \frac{161e^{\frac{1}{2}}\beta^4}{2} + \frac{49e^{\frac{1}{2}}\beta^4}{8} - \frac{133e^{\frac{1}{2}}\beta^4}{36} \\
&\quad + \frac{147e^{\frac{1}{2}}\beta^4}{88} + \frac{490e^{\frac{1}{2}}\beta^4}{187} + \frac{784e^{\frac{1}{2}}\beta^4}{103} - \frac{1566e^{\frac{1}{2}}\beta^4}{11} \\
&\quad + \frac{49e^{\frac{1}{2}}\beta^4}{11} + \frac{49e^{\frac{1}{2}}\beta^4}{8} + \frac{322e^{\frac{1}{2}}\beta^4}{3} + \frac{127031e^{\frac{1}{2}}\beta^4}{1224} \\
&\quad + \frac{49e^{\frac{1}{2}}\beta^4}{8} + \frac{2373e^{\frac{1}{2}}\beta^4}{175} + \frac{175e^{\frac{1}{2}}\beta^4}{2} - \frac{100e^{\frac{1}{2}}\beta^4}{6} + \frac{145e^{\frac{1}{2}}\beta^4}{6} \\
&\quad + \frac{140e^{\frac{1}{2}}\beta^4}{3} + \frac{35e^{\frac{1}{2}}\beta^4}{3} + \frac{384e^{\frac{1}{2}}\beta^4}{187} - \frac{3136e^{\frac{1}{2}}\beta^4}{39} + \frac{5306e^{\frac{1}{2}}\beta^4}{33} \\
&\quad + \frac{1225e^{\frac{1}{2}}\beta^4}{33} - \frac{322e^{\frac{1}{2}}\beta^4}{3} + \frac{49e^{\frac{1}{2}}\beta^4}{3} + \frac{105161e^{\frac{1}{2}}\beta^4}{2652} \\
&\quad + \frac{49e^{\frac{1}{2}}\beta^4}{4} - \frac{217e^{\frac{1}{2}}\beta^4}{66} + \frac{28e^{\frac{1}{2}}\beta^4}{55} - \frac{81e^{\frac{1}{2}}\beta^4}{3} + \frac{280e^{\frac{1}{2}}\beta^4}{3} \\
&\quad + \frac{11e^{\frac{1}{2}}\beta^4}{15} - \frac{21e^{\frac{1}{2}}\beta^4}{11} + \frac{35e^{\frac{1}{2}}\beta^4}{3} + \frac{98e^{\frac{1}{2}}\beta^4}{9} + \frac{343e^{\frac{1}{2}}\beta^4}{9} \\
&\quad + \frac{49e^{\frac{1}{2}}\beta^4}{3} + \frac{240e^{\frac{1}{2}}\beta^4}{72} + \frac{49e^{\frac{1}{2}}\beta^4}{8} - \frac{343e^{\frac{1}{2}}\beta^4}{12} \\
&\quad + \frac{28e^{\frac{1}{2}}\beta^4}{55} - \frac{80e^{\frac{1}{2}}\beta^4}{5} + 92e^{\frac{1}{2}}\beta^4 - \frac{62e^{\frac{1}{2}}\beta^4}{5} - \frac{32e^{\frac{1}{2}}\beta^4}{11} \\
&\quad - \frac{56e^{\frac{1}{2}}\alpha^2\beta}{5} + 14\alpha^2 - \frac{40e^{\frac{1}{2}}\beta}{7} + 40e^{\frac{1}{2}}\beta - 40e^{\frac{1}{2}}\beta + \frac{56e^{\frac{1}{2}}\alpha^3}{3}
\end{align*}\]
The approximate solution of (16) by HPM is given by:
\[
 u(t) \approx \lim_{p \to 1} \left[ u_0(t) + pu_1(t) + p^2 u_2(t) \right] = u_0(t) + u_1(t) + u_2(t)
\]
and
\[
 v(t) \approx \lim_{p \to 1} \left[ v_0(t) + v_1(t) + p^2 v_2(t) \right] = v_0(t) + v_1(t) + v_2(t)
\]

4. Numerical Results and Discussion

For the example 1, we get its exact solution with both methods (HPM and SBA method). For example 2, in order to verify the efficiency of the proposed method in comparison with the exact solution, we calculate the values of these solutions for different values of \( t \).

The differences between HPM solution and exact solution are shown in tables (1), (2) and the figures (1), (2), and (3). We can see a good agreement between the results of HPM and the SBA method, which confirms the validity of HPM.

Table 1. Example 2: approximate solution of \( u \) by SBA and HPM

| \( t \)  | \( u_{sba} \)          | \( u_{hpm} \)          | \( |u_{sba} - u_{hpm}| \) |
|--------|------------------------|------------------------|--------------------------|
| 0      | 0.0060×10^9            | 0.0060×10^9            | 0                        |
| 0.5    | 0.0349×10^9            | 0.0349×10^9            | 0                        |
| 1.0    | 0.0727×10^9            | 0.0727×10^9            | 0                        |
| 1.5    | 0.1291×10^9            | 0.1291×10^9            | 0                        |
| 2.0    | 0.2184×10^9            | 0.2183×10^9            | 0.0010×10^8              |
| 2.5    | 0.3635×10^9            | 0.3629×10^9            | 0.0060×10^8              |
| 3.0    | 0.6014×10^9            | 0.5981×10^9            | 0.0330×10^8              |
| 3.5    | 0.9927×10^9            | 0.9791×10^9            | 0.1360×10^8              |
| 4.0    | 1.6375×10^9            | 1.5902×10^9            | 0.4730×10^8              |
| 4.5    | 2.7002×10^9            | 2.5573×10^9            | 1.4290×10^8              |
| 5.0    | 4.4522×10^9            | 4.0639×10^9            | 3.8830×10^8              |

Table 2. Example 2: approximate solution of \( v \) by SBA and HPM

| \( t \)  | \( v_{sba} \)          | \( v_{hpm} \)          | \( |v_{sba} - v_{hpm}| \) |
|--------|------------------------|------------------------|--------------------------|
| 0      | 0.0360×10^9            | 0.0360×10^9            | 0                        |
| 0.5    | 0.0343×10^9            | 0.0343×10^9            | 0                        |
| 1.0    | 0.0414×10^9            | 0.0414×10^9            | 0                        |
| 1.5    | 0.0591×10^9            | 0.0591×10^9            | 0                        |
| 2.0    | 0.0919×10^9            | 0.0919×10^9            | 0                        |
| 2.5    | 0.1482×10^9            | 0.1482×10^9            | 0                        |
| 3.0    | 0.2422×10^9            | 0.2423×10^9            | 0.0100×10^7              |
| 3.5    | 0.3981×10^9            | 0.3980×10^9            | 0.0100×10^7              |
| 4.0    | 0.6556×10^9            | 0.6539×10^9            | 0.1700×10^7              |
| 4.5    | 1.0805×10^9            | 1.0712×10^9            | 0.9300×10^7              |
| 5.0    | 1.7811×10^9            | 1.7450×10^9            | 3.6100×10^7              |
Figure 1. Example2: Comparison of HPM and SBA method solution

Figure 2. Example2: Comparison of HPM and SBA method solution

Figure 3. Example2: Comparison of HPM and SBA method solution
5. Conclusion

In this paper, the homotopy perturbation method was used for finding exact or approximate solutions of Lotka-Volterra models. Through the examples studied, we have shown that we obtain practically the same solutions with HPM and the SBA method.

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