Jets and elliptic flow correlations at low and high transverse momenta in ultrarelativistic $A+A$ collisions

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The LHC data on elliptic flow correlations at low and high $p_T$ from Pb+Pb collisions at $\sqrt{s_{NN}}=5.02$ TeV are analyzed and interpreted in the framework of the HYDJET++ model. This model allows us to describe simultaneously the region of both low and high transverse momenta and, therefore, to reproduce the experimentally observed nontrivial centrality dependence of elliptic flow correlations. The origin of the correlations between low and high-$p_T$ flow components in peripheral lead-lead collisions is traced to correlations of particles in jets.

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I. INTRODUCTION

A number of exquisite and intriguing phenomena, which have never been systematically studied at the accelerators of previous generation, has been observed after the start of Relativistic Heavy Ion Collider (RHIC) and Large Hadron Collider (LHC) heavy-ion program. Azimuthal anisotropy of multi-particle production is among them as a powerful probe of collective properties of a new state of matter, quark-gluon plasma (QGP), see, e.g., [1]. It is commonly described by the Fourier decomposition of the invariant cross section in a form [2, 3]:

$$E \frac{d^3N}{dp^3} = \frac{d^2N}{2\pi p_T dp_T d\eta} \times \left\{ 1 + 2 \sum_{n=1}^{\infty} v_n(p_T, \eta) \cos \left[ n(\phi - \Psi^{PP}_n) \right] \right\} \tag{1}$$

where $p_T$ is the transverse momentum, $\eta$ is the pseudorapidity, $\phi$ is the azimuthal angle with respect to the participant plane $\Psi^{PP}_n$ of $n$-th order, and $v_n$ are the Fourier coefficients:

$$v_n = \langle \cos \left[ n(\phi - \Psi^{PP}_n) \right] \rangle. \tag{2}$$

The averaging in the last equation is performed over all particles in a single event and over all events.

The second harmonic, $v_2$, typically referred to as elliptic flow, is the most thoroughly investigated one, for review see [4] and references therein. The reason is obvious: it directly relates the anisotropic shape of the overlap region of the colliding nuclei to the corresponding anisotropy of the outgoing momentum distribution. At relatively low transverse momenta, $p_T < 3 \div 4$ GeV/$c$, the azimuthal anisotropy results from a pressure-driven anisotropic expansion of the created matter, with more particles emitted in the direction of the largest pressure gradients [5]. At higher transverse momenta, this anisotropy is understood to arise from the path-length dependent energy loss of partonic jets as they traverse the matter, with more jet particles emitted in the direction of shortest path-length [6]. The correlations between soft and hard contributions to anisotropic flow has attracted a lot of attention, see, e.g., [7, 8] and references therein.

Recently, the interesting correlations between $v_2$ values at high and low transverse momenta for different centralities has been reported by the CMS Collaboration [9]. These correlations can also be seen in the ATLAS data [10]. In the present paper we analyze and interpret this intriguing experimental observation within the HYDJET++ model, which allows us to perform such analysis due to its remarkable feature, namely the presence of soft and hard physics simultaneously.

The paper is organized as follows. Basic features of the model are sketched in Sec. II. Here the origin of elliptic flow in the model, and interplay between the soft and hard physics are discussed. Section III presents the results of model calculations of hadronic elliptic flow in Pb+Pb collisions at $\sqrt{s_{NN}} = 5.02$ TeV. The calculations are in fair agreement with the experimental data, and the role of jets is clarified. Finally, conclusions are drawn in Sec. IV.

II. HYDJET++ MODEL

The model HYDJET++ [12] is the popular and known event generator, which describes successfully the large
number of physical observables measured in heavy-ion collisions during the RHIC and LHC operation. Among them are centrality and pseudorapidity dependence of inclusive charged particle multiplicity, transverse momentum spectra and $\pi^+\pi^-$ correlation radii in central Pb+Pb collisions \[12\], momentum and centrality dependence of elliptic and higher order harmonic coefficients \[14\] \[18\], flow fluctuations \[19\], angular dihadron correlations \[20\], forward-backward multiplicity correlations \[21\], jet quenching effects \[22\] \[23\] and heavy meson production \[24\] \[26\]. Details of the model can be found in the hydjet++ manual \[12\].

The event generator includes two independent components: the soft, hydro-type state and the hard state resulting from in-medium multi-parton fragmentation. The soft component is the thermal hadronic state generated on the chemical and thermal freeze-out hypersurfaces prescribed by the parametrization of relativistic hydrodynamics with preset freeze-out conditions. It represents the adapted version of the event generator FASTMC \[27\] \[28\]. Particle multiplicities are calculated using the effective thermal volume approach and Poisson multiplicity distribution around its mean value, which is supposed to be proportional to the number of participating nucleons for a given impact parameter in a $A+A$ collision.

To simulate the elliptic flow effect, the hydro-inspired parametrization for the momentum and spatial anisotropy of soft hadron emission source is implemented \[12\] \[29\]. Note that there are two parameters which govern the strength and the direction of the elliptic flow in original hydjet++ version \[12\]. The first one is the spatial anisotropy $\epsilon(b)$. It is responsible for the elliptic modulation of the final freeze-out hyper-surface at a given impact parameter $b$. The second one is the momentum anisotropy $\delta(b)$, dealing with the modulation of the flow velocity profile. One can treat these two parameters as independent ones and, therefore, adjust it separately for each centrality by comparing to data. Although providing better agreement with the data, this procedure leads to significant increase of the parameters to be tuned. Therefore, for the sake of simplicity we opted for another scenario, which is implemented in basic version of the model. According to it, both parameters are correlated through the dependence of the elliptic flow coefficient $v_2$ on both $\epsilon(b)$ and $\delta(b)$, obtained in the hydrodynamic approach \[29\] :

$$v_2(\epsilon, \delta) \propto \frac{2(\delta - \epsilon)}{(1 - \delta^2)(1 - \epsilon^2)} .$$  \hspace{1cm} (3)

Because $v_2(b)$ is proportional to the initial ellipticity $\epsilon_0(b) = b/2R_A$, where $R_A$ is the radius of colliding nucleus, the relation between $\epsilon(b)$ and $\delta(b)$ reads \[12\] :

$$\delta = \frac{\sqrt{1 + 4B(\epsilon + B)} - 1}{2B} , \quad B = C(1 - \epsilon^2)\epsilon , \quad \epsilon = k\epsilon_0 .$$  \hspace{1cm} (4)

Two new parameters $C$ and $k$, entering the last expression, are independent on centrality and, therefore, should be obtained from the fit to the experimental data.

In hard sector the model propagates the hard partons through the expanding quark-gluon plasma and takes into account both collisional loss and gluon radiation due to parton rescattering. It is based on the PYQUEN partonic energy loss model \[30\]. The number of jets is generated according to the binomial distribution. Their mean number in an $A+A$ event is calculated as a product of the number of binary nucleon-nucleon (NN) sub-collisions at a given impact parameter and the integral cross section of the hard process in NN collisions with the minimum transverse momentum transfer $p_T^{min}$. The latter is the input parameter of the model. In the hydjet++ framework, partons produced in (semi)hard processes with the momentum transfer lower than $p_T^{min}$ are considered as being “thermalized”, and their hadronization products are automatically included in the soft component of the event.

Recall that there are many competing event generators successfully describing the soft \[31\] \[38\] and hard \[39\] \[41\] momentum components of particle production in heavy-ion collisions separately. The hydjet++ is among the few ones \[42\] \[43\] which allows us to study the soft and hard physics simultaneously.

III. RESULTS

To measure azimuthal correlations and to extract the Fourier coefficients the CMS Collaboration employs the cumulant and the scalar product (SP) methods. The 2- and 4-particle correlations are defined as:

$$\langle\langle 2 \rangle\rangle = \langle\langle e^{in(\varphi_1 - \varphi_2)} \rangle\rangle , \quad \langle\langle 4 \rangle\rangle = \langle\langle e^{in(\varphi_1 + \varphi_2 - \varphi_3 - \varphi_4)} \rangle\rangle .$$  \hspace{1cm} (5)

Here the double averaging is performed over all particle combinations and over all events. The multi-particle cumulant method is applied to measure $v_2$ from 4-particle correlations. The 2-nd order and 4-th order cumulants, $c_n\{2\}$ and $c_n\{4\}$, respectively, are given as \[44\] :

$$c_n\{2\} = \langle\langle 2 \rangle\rangle , \quad c_n\{4\} = \langle\langle 4 \rangle\rangle - 2 \cdot \langle\langle 2 \rangle\rangle^2 .$$  \hspace{1cm} (6)

The cumulants are expressed in terms of the corresponding $Q_n$ vectors \[45\]. For differential flow calculations the restricted 2- and 4-particle correlations, $\langle\langle 2' \rangle\rangle$ and $\langle\langle 4' \rangle\rangle$, are defined, where transverse momentum of one of the particles is limited to being within a certain $p_T$ bin. The differential cumulant $d_n\{4\}$ reads

$$d_n\{4\} = \langle\langle 4' \rangle\rangle - 2 \cdot \langle\langle 2' \rangle\rangle \cdot \langle\langle 2 \rangle\rangle .$$  \hspace{1cm} (7)

Finally, the differential $v_n\{4\}(p_T)$ coefficients are derived as

$$v_n\{4\}(p_T) = -d_n\{4\} \cdot (-c_n\{4\})^{-3/4} .$$  \hspace{1cm} (8)

This technique is used by the CMS Collaboration \[10\]. The same technique is applied in the present paper to calculate the corresponding differential $v_2\{4\}(p_T)$ coefficients in the hydjet++ model.
To compare model calculations with the data we generated Pb+Pb collisions at \( \sqrt{s_{NN}} = 5.02 \) TeV in seven centrality bins: \( \sigma/\sigma_{\text{pp}} = 5–10, 10–15, 15–20, 20–30, 30–40, 40–50 \) and 50–60\%. Statistics of generated events varies from 2 million for semi-central to 7 million for very periperal collisions, respectively. Figure 1 shows the elliptic flow restored by 4-cumulant method from the CMS data for Pb+Pb collisions at \( \sqrt{s_{NN}} = 5.02 \) TeV for the centrality 30–40\%. Lines are drawn to guide the eye.

FIG. 1: (Color online). The comparison of CMS data for \( v_2(4)(p_T) \) (triangles) and HYDJET++ calculations for \( v_2(4)(p_T) \) (squares) and \( v_2^{\text{RP}}(p_T) \) (circles) in Pb+Pb collisions at \( \sqrt{s_{NN}} = 5.02 \) TeV for the centrality 30–40\%. Lines are drawn to guide the eye.

FIG. 2: (Color online). The elliptic flow soft (triangles) and hard (squares) components and resulting total flow \( v_2^{\text{RP}}(p_T) \) (circles) in HYDJET++ for Pb+Pb collisions at \( \sqrt{s_{NN}} = 5.02 \) TeV with the centrality 30–40\%.

back in the intermediate momentum region. Originally, the model includes the adequate description of soft and hard physics. In the intermediate \( p_T \) region the result is obtained by a simple superposition of two independent contributions. The crosslinking is regulated by one parameter, \( p_T^{\text{min}} \), which is the minimum transverse momentum transfer in a hard parton subprocess. For the transverse momentum spectra this procedure is painless since the crosslinking takes place for a continuous function, and its fracture is smoothed out by the overlapping of particles from both contributions. As a result, we describe effectively the transverse momentum spectra even in the intermediate region also \[13\] without any additional mechanism. For elliptic flow coefficients the crosslinking takes place for a “discontinuous” function, see the crosslinking region of \( p_T \approx 10 \) GeV/c, where \( v_2^{\text{RP}}(p_T)(\text{soft}) \) and \( v_2^{\text{RP}}(p_T)(\text{hard}) \) have the quite different values. Thus, the simple smoothing is not enough to describe this region by a simple superposition. It means that some improvements of the model are required to describe successfully the whole \( p_T \)-region. For instance, it can be a minijet production or some other mechanism. Fortunately, this “problematic” region of the model is not used in the present flow correlation analysis.

Now we focus on the high transverse momentum region, to study which the large number of events should be generated. In this region the HYDJET++ flow, restored by cumulant method, is close to the measured one, but it is visibly larger than the original elliptic flow generated in the model. This observation requires an explanation. The azimuthal anisotropy arises in the model discr. jet as a result of jet quenching, see \[22, 24\]. Due to the path-length dependent energy loss of partonic jets as they traverse the matter, the jet particles become to be correlated with
the reaction plane. However, the particle correlations relative to the jet axis remain also. Unlike the region of small transverse momentum, there are at least two singled out directions, the reaction plane and the jet axis, respectively, relative to which particles are correlated. In this case we can decompose the azimuthal distribution in a form

\[
\frac{dN}{d\phi} = N_0 \left( 1 + 2v_2^{RP} \cos 2(\phi - \Psi_2^{RP}) \right) + 2v_2^{jet} \cos 2(\phi - \Psi_2^{jet}), \tag{9}
\]

where the direction of jet axis \(\Psi_2^{jet}\) is randomly oriented with respect to the reaction plane \(\Psi_2^{RP}\), which is set to zero in the model calculations. \(N_0\) is the normalization factor. The 2- and 4-particle correlations are estimated as

\[
\langle \langle 2 \rangle \rangle \approx (v_2^{RP})^2 + (v_2^{jet})^2, \quad \langle \langle 4 \rangle \rangle \approx [(v_2^{RP})^2 + (v_2^{jet})^2]^2 \tag{10}
\]

and they are sensitive to both types of correlations discussed above. Herewith \(\langle \langle 2 \rangle \rangle^{1/2}\) and \(\langle \langle 4 \rangle \rangle^{1/4}\) are always larger than the “true” elliptic coefficient \(v_2^{RP}(p_T)\) which we are interested in.

The correlations between the low-\(p_T\) and high-\(p_T\) elliptic flow which are the main issue of our study are displayed in Fig. 3(a). The picked up intervals are \(14 < p_T < 20\) GeV/c for high and \(1.0 < p_T < 1.25\) GeV/c for low transverse momenta, respectively. One can see that the model calculations, \(v_2\{4\}(p_T)\) (HYDJET++), reproduce the experimentally observed centrality dependence of the flow correlations fairly well except the last centrality bin, while the generated original elliptic flow \(v_2^{RP}(p_T)\) at high \(p_T\) goes always lower than the restored flow in accordance with the explanation above. The deviations become significant for the centralities larger than 30–40%. In this centrality interval the anisotropy caused by the jet quenching becomes to die out, but particles remain correlated with the axes of jets. Note that in the low-\(p_T\) region the integrated values of \(v_2\) in the model remain lower than the data. This is due to simplification, discussed in Sec. II, aiming to reduce the number of free parameters in the model. To reach better quantitative agreement with the data one has to treat both anisotropy parameters, \(c(z)\) and \(\delta(b)\), as independent ones, see [19].

This circumstance, however, does not affect the main results associated with identifying the role of jets on the behavior of cumulants. Note also, that in the high-\(p_T\) sector the agreement between the model results and the data is good.

Figure 3(b) demonstrates the comparison of model calculations without jet quenching effects with the data. In this case \(v_2^{RP} = 0\) and the correlations relative to the jet axis contributes to the 4-th order cumulant only, but the magnitude of jet correlations is not strong enough to reproduce data. Thus, in the collisions with centrality up to 30–40% the azimuthal anisotropy due to jet quenching reveals itself, whereas in more peripheral collisions the jet correlations contribute significantly to the 4-th order cumulant. The reason is as follows. Since in peripheral heavy-ion collisions there are simply quantitatively fewer nucleon-nucleon collisions, then, fewer jets are produced. In the limiting case there is one back-to-back pair, and the method sees this axis and anisotropy. In more central collisions there are many jet pairs. They are all distributed randomly in azimuth, therefore, the anisotropy caused by jets tends to zero.

It is worth noting that jets were the main source of violation of the number-of-constituent quark (NCQ) scaling in HYDJET++ calculations of differential elliptic [14, 15] and triangular [16] flow. The linear fit [11], performed by CMS Collaboration to data on elliptic flow correlations at low and high transverse momenta, also indicates some sort of scaling behavior. In contrast to situation with the NCQ scaling, here jets work toward the scaling fulfillment.

IV. CONCLUSIONS

The phenomenological analysis of elliptic flow correlations at low and high \(p_T\) in Pb+Pb collisions at center-of-mass energy 5.02 TeV per nucleon pair has been per-

FIG. 3: (Color online). The correlation between \(v_2\) values at low and high \(p_T\) in Pb+Pb collisions at \(\sqrt{s_{NN}} = 5.02\) TeV as function of centrality. The points represent the centrality bins 5–10, 10–15, 15–20, 20–30, 30–40, 40–50, and 50–60%. The model results are presented for \(v_2^{RP}\) (circles) and \(v_2\{4\}\) (squares). Data (triangles) are taken from [11]. Model calculations are performed (a) with and (b) without the jet quenching.
formed within the two-component HYDJET++ model. These correlations are stipulated by the fact that the magnitudes of anisotropy at low and high \( p_T \) are mainly determined by the value of initial ellipticity of the nuclei overlapping. At relatively low transverse momenta, \( p_T < 3 \div 4 \text{ GeV}/c \), the model generated elliptic flow \( v_2^{RP}(p_T) \) and its value restored by 4-cumulant method, \( v_2^{4}(p_T) \) (HYDJET++), are very close to the differential elliptic flow \( v_2(p_T) \) measured by the CMS Collaboration. At high transverse momenta \( p_T > 10 \text{ GeV}/c \) the cumulants are sensitive to both the anisotropy due to jet quenching, \( v_2^{RP} \), and the particle correlations with the jet axis, \( v_2^j \). In the collisions with centrality up to 30–40\% the 4-cumulant method “measures” mainly the azimuthal anisotropy due to jet quenching, whereas in more peripheral collisions it is affected by the jet correlations primarily. The model calculations restored by this method, \( v_2^{4}(p_T) \) (HYDJET++), reproduce the experimentally observed centrality dependence of elliptic flow correlations rather well without any additional tunes of model parameters.

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