Variation theory-based mathematics teaching: The new method in improving higher order thinking skills

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Abstract. Variation theory-based mathematics teaching emphasizes variation as necessary condition for learners to be able to discern new aspects of an object of learning. The study aims to describe whether the variation theory-based mathematics teaching is more effective than mathematics teaching using scientific approach in terms of higher order thinking skills of vocational school. This study is quasi-experimental research whose population is all grade X, students of Cipta Bhakti Husada Vocational School. The data collecting instrument in this research was a test, consisting of the higher order thinking skills questions. The result of the study at significance level of 5% show that the variation theory-based mathematics teaching is more effective than mathematics teaching using scientific approach.

1. Introduction

The skills of 21st century is needed to support us to live prosperously and to achieve better education. The International Society for Technology in Education (ISTE) has identified six education technology standards that are needed in 21st century [1]. They are (1) creativity and innovation, (2) communication and collaboration, (3) research and informational fluency, (4) critical thinking, problem solving and decision making, (5) digital citizenship, and (6) technology operations and concepts. There are also 12 skills components that are grouped in three main group based on Partnership for 21st century skills [2]. They are learning and innovation skills, digital literacies skill, and life and career skills. Table 1 below shows the components.

| Learning and Innovation                      | Digital Literacies                      | Life and Career Skills                      |
|----------------------------------------------|----------------------------------------|---------------------------------------------|
| Core Subjects                                | Information Literacy                   | Flexibility and Adaptability                |
| Critical Thinking and Problem Solving        | Media Literacy                         | Initiative and Self-direction               |
| Communication and Collaboration              | Information and Communication Technology Literacy | Productivity and Accountability            |
| Creativity and Innovation                    |                                        | Leadership and Responsibility               |
2. Higher order thinking skills
Some of the skills in 21st century belong to higher order thinking skills (HOTS). HOTS focused on developing students’ abilities to be able to analyze effectively, evaluate by drawing inference from existing information, and creating (synthesizing) something new [3]. HOTS consist of cognitive and knowledge domain. As for the cognitive, it covers analysis, evaluate, and create [4]. Analysing and evaluating skills are related to critical thinking and problem solving, while creating skill is related to creativity and innovation. The processes are in the level of C4, C5, and C6 of Bloom’s taxonomy revised by Anderson & Krathwohl [5]. Some experts also agree that higher-order thinking is conceived as the top end of Bloom's cognitive taxonomy [6]. While in the knowledge domain, HOTS consist of conceptual knowledge, procedural knowledge, and metacognitive knowledge [5].

3. Variation theories

3.1. Variation Theory-Based Mathematics Teaching
HOTS are very important skills for students. There are some methods have been used to improve students’ HOTS in mathematics learning. One of the methods is variation theory-based mathematics teaching. Table 2 below shows the perspective of some experts in constructing the meaning of variation theory.

| Expert        | Perspective                                                                 |
|---------------|------------------------------------------------------------------------------|
| Akerlind      | Learning through variation theory is sees as an expansion in awareness, in which students become aware of critical aspects of a disciplinary concept, skill, or practice that they had not previously noticed [7] |
| Marton and Pang| Variation theory is simply a more coherent, explicit, and systematic framework for making use of variation and invariance [8] |
| Kullberg et al.| Variation theory-based mathematics teaching emphasizes variation as necessary condition for learners to be able to discern new aspects of an object of learning [9] |
| Askew         | Variation is the key to being able to discern [10]                           |

3.1.1. Patterns of variation

![Figure 1. The connection between variation patterns and HOTS.](image)
There are four patterns of variation according Marton and Pang [8]. The explanation below shows the details description about each variation pattern and connection between variation patterns and HOTS.

- **Contrast.** Comparing two concepts involves a particular pattern of variation called contrast. In order to experience something, a person must experience something else to compare it with. An example of a contrast can be seen when determining the definition of a shape called a triangle. Based on the contrast, the triangle must be compared with other shape such as circles or squares in order to get the meaning of the triangle shape itself [9]. Contrast brings students to observe examples and counter examples to determine critical aspects. This is closely related to deciding the results based on certain criteria and standards, and it belongs to evaluation skills (C5) in HOTS.

- **Separation.** In order to experience a certain aspect of something, and in order to separate this aspect from other aspects, it must vary while other aspects remain invariant. This is closely related to building systematic and coherent relationships between pieces of information, which is in HOTS, it is included in analysing skills (C4). The example of this pattern can be seen when identifying right triangles. Students have to separate a right triangle from a triangle in general to get the meaning of a right triangle.

- **Generalization.** A certain value, $X_i$ in one of the dimensions of variation $X$ cannot be discerned from other values in other dimensions of the variation unless $X_i$ remains invariant while the other dimensions vary [8]. Generalization is all about experience varying appearances. Using generalization, the teacher tried to visualize that the same principle is applied to different representations [11]. This is closely related to identifying the elements together into interrelated elements, which is included in analysing skills (C4). The example of this pattern of variation is when the child tries to understand “three”. The child must experience various appearances of 'three', such as three apples, three orangutans, three cars, three books, and many more [12]. It can also be seen in the concept of fractions, such as $\frac{1}{2}$ . The teacher can bring a variety of different appearances such as $\frac{1}{2}$ apple, $\frac{1}{2}$ hours, $\frac{1}{2}$ slice of pizza, $\frac{1}{2}$ liters of water, and so on.
• Fusion. If there are several critical aspects that the learner has to take into consideration at the same time, they must all be experiencing simultaneously. Marton dan Pang [8] stated that fusion is experiencing several critical aspects simultaneously. This is closely related to identifying the elements together into interrelated elements. It is belonged to analysing skills (C4). An example of fusion patterns in mathematics learning can be seen when the teacher explains the subject of fractions. At first, the teacher varies the numerator and keeps the denominator invariant. For instance, given \( \frac{1}{5}, \frac{2}{5}, \frac{3}{5}, \frac{4}{5} \), the students understand easily that the bigger the numerator, the bigger the fraction.

Then the teacher makes variations on the denominator and keeps the numerator invariant. Given \( \frac{1}{2}, \frac{1}{3}, \frac{1}{4}, \frac{1}{5} \), the student understands that the bigger the denominator, the smaller the fraction. It becomes confusing when the teacher asks students to compare between \( \frac{2}{5} \) and \( \frac{4}{7} \). Students who only consider at the numerator will state that the fraction \( \frac{4}{7} \) is the bigger fraction because the numerator is bigger. Meanwhile, students who only consider at the denominator would state that the fraction \( \frac{2}{5} \) is a bigger fraction because the denominator was smaller. Therefore, understanding the concept of comparing fractions requires simultaneous variation of both the numerator and denominator. This is the variation pattern that belongs to fusion. There are also two essential patterns of variation according Gu [13].

• Conceptual variation is the strategies that are used to discern essential features of a concept and to experience connotation of the concept. This is closely related to deciding the results based on certain criteria and standards. Therefore, it is included in the evaluation skills (C5). Here the example of conceptual variation about altitude of triangles.

![Figure 4. The example of conceptual variation.](image)

• Procedural variation

Procedural variation is creating variation problems or situation for students to explore in order to find solutions to problem or develop connections among different concepts step by step or from multiple approaches. Procedural variation can be seen in the procedure of drawing equilateral triangle by using protractor and compass for drawing circles. Procedural variation patterns lead students to determine varied procedures or situations so that students can explore for solutions. This is closely related to creativity skills (C6) in HOTS.
3.2. Scientific approach

Mathematics teaching with a scientific approach is mathematics teaching designed in such a way that students actively construct concepts through the stages of observing (to identify or find problems), formulate problems, propose or formulate hypotheses, collect data with various techniques, analyze data, draw conclusions and communicate concepts. According to Indonesian Ministry of Education and Culture, there are five steps of teaching with a scientific approach, including: observing, asking questions, collecting information, associating, and communicating [14].

Table 3. The connection between scientific approach and HOTS.

| Scientific approach      | The connection between scientific approach and HOTS                                                                                                                                                                                                 |
|--------------------------|-----------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| Observing                | The activity of observing will train sincerity, accuracy in the observed aspects. This is closely related to deciding the results based on certain criteria and standards. This is included in the evaluation skills (C5) in HOTS.                                       |
| Questioning              | The questioning activity invites students to ask questions about information that is not understood from what is observed. This is included in the skill of creating (C6) in HOTS.                                                                                               |
| Collecting Information   | The information collecting stage invites students to apply the ability to gather information in various ways that are learned. This is closely related to the ability to distinguish relevant and irrelevant information. Therefore, it is included in the analysing skills (C4) in HOTS                          |
| Associating              | Associating is processing information collected from adding breadth and depth to information processing that is looking for solutions. This is related to the skills to build systematic and coherent relationships between pieces of information. This is included in the analysing skills (C4) in HOTS                   |
| Communicating            | Communicating means delivering observations, conclusions based on the results of analysis orally, in writing, or other media. It relates to all aspects of HOTS: conveying what was analysed (C4), what was evaluated (C5), and what was created (C6).                                      |
3.3. Previous research

Previous research related to variation theory has been conducted by Ting Jing-jing et al. The result suggests that the use of Variation Theory Based Strategy (VTBS) in the algebra class is effective for improving students’ algebraic achievement but it does not increase the students’ motivation [15]. Another relevant study has been also conducted by Annmarie O’Neil and Helen Doerr in 2015. The result claimed that using variation theory in task design can support students in developing important capabilities for reasoning about logarithms in powerful ways [16].

4. Methods

This study was conducted in second semester of the 2018/2019 academic year. It was started on March 18, 2109 until April 30, 2019. The place was carried out in class X SMK Kesehatan Cipta Bhakti Husada which is located at Jalan Nitikan Baru UH VI No. 112 Yogyakarta. This research was conducted in 14 meetings including one meeting for the pre-test and one meeting for the post-test.

This study is quasi experimental. The population of this research is students grade 10 of Cipta Bhakti Husada Vocational School. The sampling method is cluster random sampling, in which obtaining two classes as the sample randomly. Class X KP 3 and class X KP 2 are the experimental class and the control class respectively. There are 17 students in X KP3 and 14 students in X KP 2. Both classes are given both pre-test and post-test. Both test are similar and were made based on indicator of HOTS. There are 12 essay questions of HOTS and table 4 shows the some of the questions.

| HOTS domain | Example of question |
|--------------|---------------------|
| Analyse      | If \( pq = p + q \), determine 
\[
\frac{1}{p} + \frac{1}{pq} + \frac{1}{pq^2} + \frac{1}{pq^3} + \ldots
\] |
| Evaluate     | Mulia Sehat Hospital plans to give free cataract surgery for 200 patients. In the first month, the hospital is going to give surgery on 7 cataract patients. Every month the hospital plans that the number of patients always increases by 4 from the previous month. Can Mulia Sehat Hospital be able to complete the cataract surgery process for all those patients for one year? Explain it. |
| Create       | Create two examples of an arithmetic sequence consisting of 6 terms with the condition that the fourth term is equal to five times the first term. |

This study will use mean score of both experimental and control class. Independent \( t \) test involved to decide the effectiveness of variation theory-based mathematics teaching. Before independent \( t \) test, normality and homogeneity test must be conducted first. Table 5 and table 6 below show the result of normality and homogeneity test including the interpretations.

| Class            | Data       | Sig.    | Interpretation                              |
|------------------|------------|---------|---------------------------------------------|
| Experimental Class | Pre-test   | 0.200   | Accept \( H_0 \) and data were distributed normally |
Post-tests 0.200 Accept $H_0$ and data were distributed normally
Control Class Pre-test 0.200 Accept $H_0$ and data were distributed normally
Post-test 0.153 Accept $H_0$ and data were distributed normally

| Data         | Sig. | Interpretation                                          |
|--------------|------|--------------------------------------------------------|
| Pre-test     | 0.209| Accept $H_0$ and the data comes from a population that has homogeneous variance |
| Post-test    | 0.087| Accept $H_0$ and the data comes from a population that has homogeneous variance |

Table 6. Homogeneity test result.

5. Result and discussion

The initial activities of the research are constructing lesson plan and making instrument for measuring HOTS. Before applying to the research, this instrument should be tested for validity and reliability. There are two lectures with the background of mathematics education study program had stated that this instrument is valid according to the rule of content validity. The reliability of instrument for measuring HOTS is indicated by the score of Cronbach’s Alpha. The Cronbach’s Alpha score is 0.661. It indicates that this instrument is reliable. The study is continued to normality and homogeneity test. Data from pre-test and post-test from both classes are distributed normally. Then, independent $t$ test can be conducted. Table 7 below shows the result of independent $t$ test.

| Class          | Mean | $T$  | $t_{(0.05;29)}$ |
|----------------|------|------|-----------------|
| Experimental   | 86.76| 4.78 | 1.699           |
| Control        | 77.07|      |                 |

The result of the study at significance level of 5% show that the variation theory-based mathematics teaching is more effective than mathematics teaching using scientific approach. The similarity between variation theory-based mathematics teaching and mathematics teaching with a scientific approach lies in the existence of observing activities, but the amount of observing activities in variation theory-based mathematics teaching is more than mathematics teaching using scientific approach due to the emphasis on powerful ways of seeing on a given variation pattern. The patterns of variation support students’ HOTS (analysing, evaluating, and creating skills). Examples of problems on students’ worksheet related to contrast variation pattern displayed in Table 8 and table 9.

Table 8. Example and non-example of arithmetic sequence.

| Example Arithmetic Sequence | Non-Example of Arithmetic Sequence |
|-----------------------------|-----------------------------------|
| Given                       | Given                             |
Table 9. Example and non-example of geometric sequence.

| Example Geometric Sequence | Non-Example of Geometric Sequence |
|----------------------------|-----------------------------------|
| Given 2,4,8,16,32          | Given 2,4,6,8,10                  |

Table 8 illustrates problems in distinguishing between examples and non-examples of arithmetic sequence. It encourages students to derive independently the definition of arithmetic sequence. Similar to previous problem, table 9 shows students answer in distinguishing between examples and non-examples of geometric sequence. This will empower students’ skills to decide based on certain criteria or standards, which according to Anderson and Krathwohl are considered as critical skills and it is belonged to evaluation skill [5].

In regards with arithmetic equations, table 10 displayed the various contexts such as sugar supply, saving money, and figural problem. These contexts embodied the generalization pattern which encourages students to make connections between pieces of information given. According to Anderson and Krathwohl, this generalization will support students analysing skills [5].

Table 10. Variation pattern of generalization (1).

| Problem 1                                                                 | Problem 2                                                                 | Problem 3                                                                 |
|--------------------------------------------------------------------------|--------------------------------------------------------------------------|--------------------------------------------------------------------------|
| In January 2018, the sugar factory supplied 50 kg of sugar to a grocery store. In the following month, the supply of sugar to the shop always increased by 2 kg from the previous month, every month during 2018. How much was the weight of the sugar supply in June 2018? | Andi's balance in December 2017 was IDR 250,000. Andi saved IDR 100,000 every month in 2018. How much was Andi's savings balance in May 2018? | How many shaded squares are there in the fifth term? |
Table 11. Variation pattern of generalization (2).

| Number | Statement                                                                 | Equation                      |
|--------|---------------------------------------------------------------------------|-------------------------------|
| 1      | The numbers between 5 and 25 that are divisible by 3                       | 6, 9, 12, 15, 18, 21, 24     |
| 2      | Natural numbers that are less than 20 and divisible by 2                   | ........................     |
| 3      | The leap years between 1991 and 2020                                      | ........................     |

Table 11 shows that the students are asked to construct arithmetic equation with different context. It affects students’ skills of making connection based on certain descriptions and criterion. According to Anderson and Krathwohl, this generalization will support students creating skills [5].

This effectivenes of variation theory-based mathematics teaching is in line with the research conducted by Jing et al. Their research concludes that variation theory-based strategy (VTBS) that applied in the algebra class is more effective for improving students’ algebraic achievement than the class with conventional teaching strategies (CTS).

6. Conclusion
According to independent $t$-test, $t$ score is 4.78. This score is greater than $t_{0.05; 29} = 1.699$. Rejecting $H_0$ means that the variation theory-based mathematics teaching is more effective than mathematics teaching using scientific approach in term of HOTS. Thus, the variation theory-based mathematics teaching can be used as the new method in improving HOTS. It also indicates that the existence of variation in mathematics learning is very crucial.

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