Splitting and broadening techniques for SFQ-pulse driver based on SQIF

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Abstract. We present the detailed analysis of the Single Flux Quantum (SFQ) pulse splitting structure for the output SFQ-pulse driver suggested earlier. The splitter tree is an important part of the driver based on series Superconducting Quantum Interference Filter (SQIF). Conflicting requirements to the circuit parameters and possible approaches to meet the challenges are presented and discussed. Importance of the pulse broadening technique for the driver operation is also discussed.

1. Introduction

Recently we proposed a new approach for the design of high performance SFQ-pulse amplifier (driver) based on Superconducting Quantum Interference Filter (SQIF) [1]. The output driver capable of operating at frequency up to several tens of GHz can be used as an interface between RSFQ digital electronics and semiconductor circuits. The considered driver consists of three major parts: splitter tree structure, series SQIF and output line (Fig. 1). The splitter tree plays critical role in the proposed driver design. In this paper, we present results of detailed analysis of the splitter structure and discuss its principal characteristics.

2. Simulation Results

2.1. JTL-model of Splitter Tree

The splitter structure (Fig. 2a) consisting of \(N\) columns provides splitting of input SFQ pulses into the set of \(k^N\) synchronous SFQ pulses each to be applied to the series SQIF, where \(k\) is a splitting coefficient (\(k = 2\) for the structure shown in Fig. 2a). A model (equivalent circuit) of the splitter structure is Josephson Transmission Line (JTL) with varying cell parameters (Fig. 2b): inductance \(L_n = L/q^n\) and critical current \(I_{c_n} = I_C \cdot q^n\), where \(L\) and \(I_C\) are inductance and critical current of a splitter cell accordingly, \(q\) is a transforming coefficient, and, \(n\) - the splitter column number. Each JTL cell corresponds to columns of the splitter structure. In case of homogeneous splitter structure with equal inductances and critical currents: \(L_1 = L_2 = \ldots L_n\), \(I_{c_1} = I_{c_2} = \ldots I_{c_n}\), the splitting and transforming coefficients coincide, i.e. \(q = k\). The decrease (increase) of the transforming coefficient \(q\) in comparison with \(k\) takes place in monotonic decrease (increase) of \(I_C\), accompanied by proportional increase (decrease) of \(L\), with the splitter column number \(n\). For example, to come to homogeneous JTL-model of the splitter by reducing \(q\) down to 1, we have to change the column parameters by factor
In case of \( k = 2 \) and \( N = 7 \), the parameters of output column will differ from the ones of the first element by factor \( M = 2^7 = 128 \). It seems rather impossible to fabricate. At the same time, the reducing \( q \) down to 3/2 looks more realistic since it will make \( M \approx 10 \).

### 2.2. Basic Distinctions

First, let us consider the case of usual JTL with \( q = 1 \). Fig. 3a shows normalized time \( \tau \) of the SFQ-pulse propagation (accompanied by splitting in a splitter structure) via the line versus the applied bias current at different inductances \( L \). The propagation time increases sharply with the bias current decrease down to a threshold value. Below this threshold the energy contribution providing by the bias current source is less than the energy to move the SFQ, and therefore the SFQ is trapped. The threshold value increases monotonically with inductance \( L \), crossing level \( 0.8I_c \) at \( l = 20 \) (see the curve “\( q = 1 \)” in Fig. 3d).

At \( q = 2 \), which corresponds in particular to homogeneous slitter structure of order \( k = 2 \), the threshold values of bias current are drastically higher, and at \( q = 3 \) the values become extremely high, with the especially strong rise for low inductances (see Fig. 3b and Fig. 3d). On the one hand, this results from the fact that critical current rapidly rises with the cell number \( n \), while increase in circular current \( I_q \sim \Phi_0/L \) through a cell junction is less because of progressing shunting by the next cells. To compensate it, an increase of bias currents is required. On the other hand, the dramatic rise of the threshold at low inductances follows from increasing of radius of the cell interaction with decrease of \( L \). If inductance \( L \) is reasonably small, all cells of the structure act about simultaneously. In this case input SFQ pulse ought to “switch” all cells, i.e. to cause the phases of all junctions to exceed \( \pi/2 \). To make it possible the considerable increase in the bias currents is needed to shift the starting phase values towards \( \pi/2 \). Such a rise of the threshold values of \( I_b \) limits the acceptable range for \( I_b \), which should be much wider than fabrication process spread in the critical currents. To reduce the problem one can decrease \( q \), for example down to 3/2, introducing appropriate monotonic transformation of \( L \) and \( I_c \) with \( n \).

Fig. 2. Splitter structure with splitting coefficient \( k = 2 \) (a) and JTL-model of the splitter structure (b).

Fig. 1. Principal parts of the driver proposed: splitter tree structure, series SQIF, and output line.
2.3. Pulse Repetition Rate

In the case of low inductances when the cell interaction radius is large, period $T$ of the SFQ pulse repetition rate must exceed the time $\tau$ of the pulse propagation in the splitter structure to avoid strong mutual interaction between successive pulses and keep correct regime of the splitter operation (time interval between output pulses must be the same as the one at input). With decrease of the interaction radius resulting from increase of $L$, this restriction weakens and period $T$ can be less than $W$. For example, at $l=7$, $q=3/2$ and $N=7$ period $T$ may be as short as $W/2$. At the same time if we take into account dependence of $W$ on $L$ and $I_B$ shown in Fig. 3c for $q=3/2$, we come to approximately the same limitation on the period $T$ for the whole range $l=2…7$ at $I_B=0.7…0.8$. This limitation corresponds to the pulse repetition rate $\omega_o/\omega_C=0.2…0.3$. The most pronounced impact on the repetition rate restriction comes from coefficient $q$ (compare Fig. 3c and Fig. 3b). Moreover, the increase of $q$ leads to significant narrowing of an acceptable range for bias current $I_B$.

2.4. Impact of Small Capacitance

It is well known that RSFQ logic circuitry is constructed with overdamped Josephson junctions with non-hysteretic IV-curve corresponding to McCumber parameter $\beta_c<1$ (here $\beta_c=(2e/\eta)I_cR_C^2/C$). We have considered an impact of a finite Josephson junction capacitance on the bias current threshold value. The capacitance gives some threshold decrease which vanishes with increase of inductance $l$. It has been found that at $q=2$ the maximum decrease is about 10 percent (at $l=1$). The threshold reduction increases with increase of $\beta_c$ up to 0.5. The splitter works well even at slightly hysteretic IV-curve ($\beta_c=1…2$), while the threshold reduction stops.

One should emphasize that the described threshold decrease reduces propagation time $\tau$ only near the threshold and does not change $\tau$ at higher values of bias current $I_B$. It means that the small capacitance does not influence on the pulse repetition rate limitation discussed above.

3. Broadening technique

The suggested approach to the driver design implies the resistive initial state of SQIF with the cells each operates as conventional dc SQUID. This means that the rate of the input signal change should be less than the frequency of Josephson oscillation $\omega_j$ of the junctions of SQIF: $d\Phi/m/dt<\omega_j$ ($\Phi_m$ is input
magnetic flux for a cell of SQIF). To fulfill these conditions, the pulse-width broadening in the output column of the splitter structure could be used (to decrease $d\Phi_m/dt$), as well as the SQIF characteristic frequency $F_c$ (and the frequency of Josephson oscillation $\omega_J$, accordingly) should be as high as possible. The broadening of the SFQ voltage pulse, which has constant area $\int V dt = \Phi_0$, means the respective decrease in the voltage pulse amplitude as well as the pulse spectrum depletion. Despite of the voltage pulse amplitude reduction, the amplitude of current pulse through the loop inductance does not decrease; moreover it increases with the bias current increase in the splitter cell (see Fig. 4a,b). This means that the energy transferring by the pulse grows up as a square of the current amplitude $W = \int \frac{1}{2} LI^2 dt$ and concentrates in the low spectrum harmonic components.

In such a way, the broadening of the output current pulse gives both the required reduction of $d\Phi_m/dt$ and the additional gain of the driver. It is seen from Fig. 4a, for the pulse repetition rate $\omega/\omega_c \approx 0.022$, the initial pulse spectrum extends beyond frequency $\omega/\omega_c \approx 0.6$, while the broadened pulse spectrum ranges only up to frequency about $\omega/\omega_c \approx 0.06$. It simplifies substantially the problem of the output signal filtering in order to cut off Josephson oscillations and keeping the signal shape fixed. It is clear that the maximum broadening is limited by the pulse repetition rate.

The proposed pulse broadening technique is achieved by additional shunting of Josephson junctions in output cells which connected with the SQIF. Numerical simulation shows that each output cell must be separated from the last cell of the splitter tree by at least two buffer JTL cells.

One should also draw attention to inevitable fabrication process spread in Josephson junction critical currents $\delta I_c$ (typically about 5%) that will result in the time shifts $\delta t$ between output pulses of the splitter tree. According to our estimations, $\delta t$ is of the order of the regular SFQ pulse width or even more, but much less than the broadened pulse width. In such a way the broadening technique allows to compensate the negative influence of the pulse time-shifting and to provide the efficient adding up of the pulses by series SQIF as well.

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References
[1] V. K. Kornev, I. I. Soloviev, and O. A. Mukhanov, *IEEE Trans. Applied Superconductivity*, 2005, 15, p. 388-391.