Rapid entry trajectory planning without segmentation

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Abstract. A new rapid entry trajectory planning algorithm for hypersonic glide vehicles is presented in this paper. Firstly, build a new technique to calculate the boundary of the control variable that satisfy the path and control constraints without using quasi-equilibrium glide condition (QEGC). Secondly the optimal control variable is expressed as function of current state. Then combine two of them by control coordination. Finally, introduce one or two constant parameters to satisfy the final state condition. The approach is tested using the CAV model. This algorithm is efficient and will provide feasible trajectory without reversal of bank angle. It can apply for entire entry process, segmentation and the QEGC is unnecessary.

1. Introduction

Due to trajectory generation directly based on the optimization method[1-3] often requires a huge amount of computation, techniques such as segmentation strategy, quasi-equilibrium glide condition and lateral azimuth error deadband are widely used in the onboard trajectory generation of hypersonic glide vehicles.

The entry trajectory of the space shuttle[4] is divided into five phases that fitting with different formulas, and try to approximate constraint boundary. Zimmerman[5] divided the trajectory into two phases. The first phase adopted constant heat flow rate constraint to generate analytic solution, while the second phase did not consider the constraint and generated the trajectory by shooting method. Shen[6] employed the quasi-equilibrium glide condition to convert the path constraint into the constraint of bank angle, and single-point or two-point bank angle reversal was used for lateral planning. Li ZhaoYing[7] divides the entry trajectory into the initial descent phase, the constant heat flow rate phase, the constant overload phase and the interim phase, and each phase adopts different analytical formulas and adjusts some parameters online.

Although trajectory segmentation simplifies the planning in single phase, it increases the total number of parameters to be adjusted and the complexity of guidance, which is not conducive to the further development of the method, and the specific profile form will also limit the flight capability of the spacecraft itself.

Moreover, the application range of quasi-equilibrium glide condition is limited[8], which cannot be applied to the whole entry process. And reversal of bank angle usually brings about control tracking problems and has certain limitations on large lateral range[9].

In order to establish an efficient approach that avoid these problems, a new technique is invented in this paper to calculate the boundary of the control variable that satisfy the path and control constraints, which imply a function that restrict the arbitrary control variables to the available control variables. And an optimal control model is established, but instead of trying to find an explicit solution, the
optimal control variable is regard as the function of the state variables. The composition of the two functions will lead to an initial value problem. The only thing left is to satisfy the final state condition while integrate the initial value problem. By introduce one or two parameters, and represent the entire problem as to solve the zero point of a function of those parameters, it can be done easily.

2. Entry dynamics
Ignoring the earth rotation, the hypersonic unpowered glide entry dynamics can be expressed as equation (1-6). The effect of the earth's rotation can be taken into account later.

\[
\begin{align*}
\dot{r} &= v \sin \theta \\
\dot{v} &= -D - g \sin \theta \\
\dot{\theta} &= \frac{1}{v} \left[ L \cos \nu + \left( \frac{v^2}{r} - g \right) \cos \theta \right] \\
\dot{\Psi} &= \frac{1}{v} \left[ L \sin \nu \frac{v^2}{r} + \frac{1}{r} \cos \theta \sin \Psi \tan \phi \right] \\
\dot{\lambda} &= \frac{v \cos \theta \sin \Psi}{r \cos \phi} \\
\dot{\phi} &= \frac{v \cos \theta \cos \Psi}{r} 
\end{align*}
\]

where \( \lambda \) and \( \phi \) are the longitude and latitude, \( \theta \) is flight path angle, \( \Psi \) is the heading angle, \( \nu \) is the bank angle. The lift and drag accelerations, denoted as \( L \) and \( D \), are given by equation (7)

\[
\begin{align*}
L &= C_L(\alpha, Ma(v, r)) \rho \frac{v^2}{r} \frac{S}{2m} \\
D &= C_D(\alpha, Ma(v, r)) \rho \frac{v^2}{r} \frac{S}{2m}
\end{align*}
\]

The density variation with altitude is given by equation (8)

\[
\rho = \rho_0 \exp \left[ \frac{(r - R_0)}{h_s} \right]
\]

3. Path and control constraints
Considering heat flow, dynamic pressure, and total overload as mainly constraints, they are given by equation (9-11)

\[
\begin{align*}
\dot{Q} &= k_p \rho^{0.5} v^{1.15} \leq \dot{Q}_{\text{max}} \\
q &= \frac{1}{2} \rho v^2 \leq q_{\text{max}} \\
n &= \sqrt{L^2 + D^2} / g_0 \leq \frac{C_L^2 + C_D^2 \rho v^2 S}{2mg_0} \leq n_{\text{max}}
\end{align*}
\]

From (8) and (9), altitude constraint at a given speed are equation (12)

\[
h \geq h_{\text{min}}(v) = \max \left\{ 2h_s \log \left( \frac{K_n \sqrt{P_0 v^\nu}}{Q_{\text{max}}} \right), h_s \log \left( \frac{\rho_0 v^2}{2q_{\text{max}}} \right) \right\}
\]

3.1. Control constraints
Using (3) divided by (1) and arrange to get equation (13)

\[
\sin(\theta) d\theta = \left( \frac{C_L \cos(\nu) \rho S}{2m} + \left( \frac{1}{r} - \frac{g}{r} \right) \cos(\theta) \right) dr
\]

Assuming that the vehicle drops at a constant value of \( C_L \cos(\nu) \) and is just tangent to the boundary curve of the \( h_{\text{min}}(v) \) in (11), as shown in figure 1.
This tangent condition can be approximated to (14)

$$h_{low} \geq h_{min}(v_{low})$$

where $h_{low}$ and $v_{low}$ are the height and velocity of the local nadir. Integrate by assuming $k_h = \frac{1}{r - g/v^2} \cos(\theta)$ is constant will provide equation (15)

$$\cos(\theta_i) - \cos(\theta_f) = \left( -\frac{C_L \cos(\nu) e}{2m} \right) \int h \rho v S_r + k_h h_i$$

At local nadir, $\theta_i = \theta_f = 0$, and $k_h$, $h_i$ can be estimated based on the current state equation (16)

$$k_h = k_{th} = \frac{1 - \frac{g}{v_0^2}}{r_0} \cos(\theta_0)$$

$$h_i = h_{min}(v_0)$$

Apply with will provide equation (17)

$$C_L \cos(\nu)_{\min} = \frac{2m(k_h h_i + \cos(\theta_i) - 1)}{h S_r \rho v (\exp(-h_i / h_i) - \exp(-h_i / h_i))}$$

The maximum value of $C_L$ which constrained by (10), can denoted as equation (18)

$$C_L = C_{L,max}(r, \nu)$$

Within (16) and, all path constraints are satisfied.

For testing accuracy of this constraint-handling technique, we force the $C_L \cos(\nu)$ to be the minimum that provided by (16) and (15) in simulation, it turns out that the h-v curve almost precisely track the $h_{min}(v)$ without crossing it, see figure 2 which just indicated that our method is accurate and effective.

**Figure 1.** h-v constraints.

**Figure 2.** h-v curve that track the boundary.

4. Optimal control model

By dividing the planning problem into vertical and lateral, the corresponding control variable are $C_L \cos(\nu)$ and $C_L \sin(\nu)$. 
4.1. Lateral planning
Define line-of-sight error $\Delta \psi = \psi_{los} - \psi$, where $\psi_{los}$ is the angle of rotation from the current line-of-sight direction counterclockwise to the direct north direction.

Notice the range differential $R \approx v \cos(\theta)$, take the derivative of $\Delta \psi$, and apply (4) equation (19)

$$\frac{d\Delta \psi}{dR} \approx \frac{\psi_{los} \sin(\nu) \rho S_y}{V \cos(\theta)} - \frac{\tan(\phi) \sin(\psi)}{r} \overset{def}{=} u_z$$

Use equation (20) performance index

$$\min J = \frac{1}{2} \int_0^{\ell_{max}} u_z^2 dR$$

such that equation (21)

$$\Delta \psi \bigg|_{R=R_{\text{rest}}} = 0$$

then according to optimal control theory[10] equation (22)

$$u_z = -\frac{\Delta \psi}{r_{\text{rest}}}$$

where $r_{\text{rest}}$ can be estimation by spherical trigonometry. According (21) and (18), we have equation (23)

$$C_v \sin(\nu) = -\frac{2m \left( u_z - \frac{\psi_{los} \sec(\theta)}{V} + \frac{\sin(\psi) \tan(\phi)}{r} \right)}{\rho \sec^2(\theta) S_y}$$

4.2. Vertical planning
Similar to lateral planning, considering $\theta$ is very small, vertical dynamic respect to range can denoted by equation (24)

$$\begin{align*}
\frac{dr}{dR} &= \frac{\dot{r}}{R} = \tan(\theta) \approx \theta \\
\frac{d\theta}{dR} &= \frac{\dot{\theta}}{R} = \frac{C_v \cos(\nu) \rho S_y}{2m} - \frac{R_f}{v^2} + \frac{1}{r} \overset{def}{=} u_y
\end{align*}$$

Use the equation (25) performance index

$$\min J = \frac{1}{2} \int_0^{\ell_{max}} u_y^2 dR$$

such that equation (26)

$$\begin{align*}
\left[ r \right]_{R=R_{\text{rest}}} &= r_f \\
\left[ \theta \right]_{R=R_{\text{rest}}} &= \theta_f
\end{align*}$$

Solve (24) with (25) to get equation (27)

$$u_y = -\frac{2 \left( 3r - 3r_f + R_{\text{rest}} (2\theta_f + \theta_f) \right)}{R_{\text{rest}}^2}$$

combine (23) and (26) will provide equation (28)

$$C_v \cos(\nu) = \frac{2m \left( u_z + \frac{g}{v^2} - \frac{1}{r} \right)}{\rho S_y}$$

Given any state, there are corresponding control variables lead the vehicle to the target. It can be represented as equation (29)

$$u_{\text{opt}} = \text{opt}(x)$$

5. Construct an initial value problem
5.1. Control coordinator
The control variables calculated from (27) and (22) may be out of boundary that calculated from (16) and (17). The control coordinator force them into it, while maintain their continuity. New control variables denote as $C_{Lv}$ and $C_{Ll}$, corresponding to $C_{L} \cos(\nu)$ and $C_{L} \sin(\nu)$.

The algorithm employed by the control coordinator can simply describe as follow, ordered by priority.

(1) keep safe, in other words $C_{Lv} \geq (C_{L} \cos(\nu))_{\text{min}}$
(2) try to keep lateral to be optimum, in other words minimize $|C_{Ll} - (C_{L} \sin(\nu))|$
(3) try to keep vertical to be optimum, in other words minimize $|C_{Lv} - (C_{L} \cos(\nu))|$

Given any state and any control variables, there are new corresponding control variables that guarantee safe. It can be represented as a function

$u = cc(x, u_{org})$

when there is enough lift force, the coordinator will be an identity function of $u_{org}$.

5.2. Initial value problem
Given any state, there are corresponding control variables $C_{Lv}$ and $C_{Ll}$ that can guarantee safe and lead the vehicle to the target. It can be represented as a function

$u = f(x) = cc(x, opt(x))$

Dynamics (1)–(6) can denoted as

$\dot{x} = F(x, u)$

Combine (30) and (31) will provide an initial value problem

$\dot{x} = F(x, f(x))$

5.3. Reach the final state condition
Since the path constraints is solved, only the final state condition needs to be considered when integrating the equation (32). Due to the terminal time is uncertain, but the terminal energy $E = v^2/2 - \mu/r$ is known at the given altitude and speed, it is more convenient to integrate the equation with energy as the independent variable[11]. At the end point of integration, since the energy is known and there usually is enough lift force at the end, the altitude can be accurately satisfied, so that the speed can be accurately satisfied.

The remaining major problem is the horizontal position error. Note that the constraint control coordinator always gives priority to the lateral optimization when there is enough lift available, which enables the horizontal trajectory (or its extension line) to pass through the desired position $(\lambda_f, \phi_f)$ accurately. Therefore, the horizontal position error is actually a voyage problem. By adjusting the resistance, the terminal point can be just landed at point $(\lambda_f, \phi_f)$.

To do this, simply introduce two factor $[k_1, k_2]^T$ and multiply the control variables before passing it into the coordination function. That is, update (30) with

$u = f(x) = cc(x, [k_1, k_2]^T \otimes opt(x))$

where $\otimes$ represent multiply by elements. Now, Define the integral end error function

$g(k_1, k_2) = R_{\text{rest}} \cos(\Delta \psi')$

where $R_{\text{rest}}$ and $\Delta \psi'$ represent range-to-go and line-of-sight error at integral end. If endpoint is in front of $(\lambda_f, \phi_f)$, $\Delta \psi' < 90^\circ$, otherwise $\Delta \psi' \geq 90^\circ$. In particular, if endpoint is exactly $(\lambda_f, \phi_f)$, then

$g(k_1, k_2) = 0$

So the entire trajectory planning problem will be solved by find roots of (35).

6. Testing and discussion
For test the performance of the algorithm, the general aircraft model[12] CAV-H were adopted. It has a reference area of 0.48 m$^2$, and a mass of about 907kg. And the maximum heat flow density
constraint $\dot{Q}_{\text{max}} = 2MW/m^2$, maximum dynamic pressure constraint $q_{\text{max}} = 100kPa$, maximum total overload constraint $n_{\text{max}} = 3g$, attack range $\alpha \in [10^\circ, 20^\circ]$, attack change rate $|\dot{\alpha}| \leq 2^\circ/s$, bank angle $|\rho| \leq 60^\circ$, bank angle rate $|\dot{\rho}| \leq 20^\circ/s$. The initial conditions of the simulation are set as, the initial height $h_0 = 60km$, speed $v_0 = 6500m/s$, the flight path angle $\theta_0 = 0$, the initial longitude and latitude are both 0, the heading angle $\psi_0 = 90^\circ$, the target longitude and latitude(800,200), the target altitude is 30km, the target speed is 1965m/s, and target flight path angle is 0.

The zero point of $g(k_1,k_2)$ is not unique, in this example we set $k_1 = 1$, and random choose initial value of $k_2$ from [1, 5], then find zero point of $g(1,k_2)$ through Newton's method. After 7 iterations (8 times of integrals), it converges to $k_2 = 2.057$. The terminal position and height error are both 0, and the velocity error is 0.3m/s, which is a relatively accurate feasible solution. The corresponding trajectory parameter curve is shown in figure 3~figure 10.
7. Conclusion
In this paper, a new trajectory planning method for hypersonic glide entry is proposed, and the relationship between path constraints and control variables is derived directly based on the dynamic equation. Compared with the previous method, it has the following advantages: (1) it is not limited by the QEGC, does not need segmentation, and has good applicability in the whole entry flight. (2) no need for bank angle, provide smooth trajectory and continuous control quantity. (3) reliable convergence, good calculation performance, suitable for onboard calculation.

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