Parameter Analysis and Simulation Experiment of Ant Colony Optimization on Small-scale TSP Problem

Lin Yang¹, Yongjie Wang¹,∗ and Jun Zhang¹

¹ College of Electromagnetic Countermeasure, National University of Defense Technology, Hefei, China

*Corresponding author e-mail: w_yong_j@189.cn

Abstract. Ant colony optimization (ACO) is a simple and efficient bionic intelligent algorithm, which can be applied to various optimization problems. There are many controllable parameters in the ant colony optimization, so the parameter setting has a great impact on the efficiency of the algorithm. In order to further improve the efficiency of the algorithm, many researchers have proposed various improvements. However, the current improved method of ACO ignored the consideration of its original parameter settings. In order to analyze the effect of parameter setting on the efficiency of the ant colony optimization, this article uses the ant colony optimization to solve small-scale TSP problems as an example for simulation experiments. Explore the impact of three parameter pairs on the efficiency of algorithm optimization by setting variable pairs. The experimental results show that when the ant colony size is set to approximately 0.6 times the TSP problem size, a better algorithm efficiency can be obtained. Within the value range of [1, 10], setting the information heuristic factor to 1, and the expected heuristic factor to a range of [5, 10] can get a better efficiency. In the last experiments, the pheromone volatility coefficient is generally better to be set greater than 0.2, and in a small-scale TSP problem, totally randomness has little effect on the optimization accuracy of the algorithm, and the value of pheromone enhancement coefficient has little effect on the efficiency of the algorithm.

1. Introduction

ACO algorithm is a simulation evolution algorithm based on ant colony first proposed by Italian scholar Dorigo in 1991 [1]. The algorithm is inspired by the foraging behavior of the ant colony, and the concept of pheromone is used. The ants use the pheromone to exchange and transfer information during the foraging process, and can automatically choose the next hop according to the length of the path and pheromone concentration, and show a positive feedback phenomenon, this positive feedback mechanism can help ants find the optimal foraging path faster.

As an intelligent bionic evolutionary algorithm, ant colony optimization has been widely applied in many fields, such as Traveling Salesman Problem (TSP), quadratic programming problem, function optimization, network routing optimization, robot path planning, data mining, Work flow planning, graphic coloring and other fields, with a good performance [2].

2. Application of ACO in TSP problem

ACO algorithm was first proposed to solve TSP problems, which is an NP-hard problem. ACO belongs to the class of meta-heuristics, which is showing some advantages in solving NP-hard
combinatorial optimization problems [3]. If there are \( n \) nodes in the TSP problem, then there will be \((n-1)!\) feasible solutions, and the time complexity of the algorithm to solve this problem is \( O(n!) \).

ACO algorithm performs better when solving small-scale TSP problems, but the performance decreases sharply when solving large-scale TSP problems, so it ACO easy to fall into a local optimum and the optimization process terminated.

2.1. Basic process of ACO to solve TSP problem

Traditional ACO algorithm includes two basic procedures: state transition and pheromone updating. According to different pheromone update rules, ACO is usually divided into three models, namely the Ant-Cycle model, Ant-Quantity model and Ant-Density model, the three have differences in pheromone increment, update time and update form, but the general idea of these models is the same.

The model usually sets the number of ant colonies as \( m \) and the size of TSP nodes as \( n \). The classic pheromone update model for solving TSP problems is the ant week model. The algorithm flow chart is shown in Figure 1.

![Figure 1. ACO based on Ant-Cycle model for solving TSP problem](image)

### 2.1.1. State transition
At time \( t \), ants randomly select the next node. The probability that the ant \( k \) at the node \( i \) chooses node \( j \) as the transfer target could be calculated as follows

\[
\rho^{ij}_k(t) = \begin{cases} 
\frac{\tau^{ij}_k(t)\eta^{ij}_k(t)}{\sum_{r \in \text{allow}_k} \tau^{ij}_r(t)\eta^{ij}_r(t)}, & j \in \text{allow}_k \\
0, & j \notin \text{allow}_k 
\end{cases}
\]  

(1)
In this formula, $\tau_{ij}$ is the pheromone concentration on path $(i, j)$ at time $t$, and $\eta_{ij}$ is the heuristic information for the distance of the path $(i, j)$, usually defined as $\eta_{ij} = 1/d_{ij}$. $allow_k$ is the set of nodes that ant $k$ has not visited at time $t$, and usually uses a tabu list to restrict access.

2.1.2. Pheromone update. In order to prevent the accumulation of pheromones from leading to the enhancement of information eliciting, which drowns out the expected eliciting effect, each ant needs to accumulate and volatilize the pheromones along the path after completing a step or an iteration. The pheromone update formula of path $(i, j)$ at time $t + \Delta t$ is:

$$
\tau_{ij}(t + \Delta t) = (1 - \rho)\tau_{ij}(t) + \Delta \tau_{ij}(t)
$$

(2)

Where $\rho$ is the pheromone volatility factor, $(1 - \rho)\tau_{ij}(t)$ is the volatile remaining pheromone, and $\Delta \tau_{ij}(t)$ is the pheromone increment at time $t$.

2.2. Research on the parameters of ACO

The Ant colony optimization uses the principle of random heuristics to improve the optimal solution search ability through a combination of "random" and "heuristic". Heuristic factors include information heuristic factors and expectation heuristic factors, which are used to reflect two types of reference standards: empirical and real-time.

Except for the TSP node size $n$, which is an uncontrollable parameter, the controllable parameters of the traditional ACO are defined in Table 1.

| symbol | parameter | Description |
|--------|-----------|-------------|
| $m$    | Ant colony size | Ant colony size involved in finding the optimal solution |
| $\alpha$ | Information heuristic factor | The importance of pheromone |
| $\beta$ | Expect Heuristic Factor | The extent to which locally shorter paths are valued |
| $\rho$ | Pheromone volatility factor | Pheromone volatility on the path |
| $Q$    | Pheromone enhancement factor | Release of pheromone in a single update |

Generally, when the ant colony size $m$ is larger, the search ability of the algorithm is stronger, but it is easy to produce many repeated solutions. When the algorithm converges to the vicinity of the optimal value, too many repeated optimizations will consume considerable resources.

For the heuristic factor, the larger the value of the information heuristic factor $\alpha$, the greater the probability that the ant will choose a local path based on the pheromone concentration. The larger the value of the expected heuristic factor $\beta$, the more ants tend to choose a local shorter path, the algorithm tend to be greedy, and the randomness of the search will also decrease.

The range of pheromone volatility factor $\rho$ is in $(0, 1)$. When $\rho$ is too small, the pheromone on the path will not volatilize in time, resulting in excessive pheromone on the path, which will affect the algorithm's convergence efficiency. When $\rho$ is too large, the pheromone on the path cannot be retained, the ant colony will lose the experience information of previous iterations [4].

When the pheromone enhancement coefficient $Q$ become larger, more pheromone released by the ant in a single update, the faster the pheromone accumulation on the path, which will also cause the proportion of the state transition probability to be affected by the pheromone to increase and easily fall into the local optimum.
Although it is currently popular to use the online parameter adjustment method for parameter setting, and it can indeed improve the robustness of the algorithm [5], however, whether it is a static setting or a dynamic adjustment, it is necessary to understand the inherent law of the parameters' influence on the algorithm. How to set controllable parameters to achieve the optimal efficiency of ACO algorithm? Many researchers have studied this problem.

Reference [6] studied the optimal parameter settings for TSP problems of different scales, as shown in Table 2.

| Parameter    | Small-scale (198) | Medium-scale (1291) | Large-scale (2392) |
|--------------|-------------------|---------------------|--------------------|
| $\alpha$     | 0.5               | 2                   | 0.5                |
| $\beta$      | 0.5               | 4                   | 3                  |
| $\rho$       | 0.999             | 0.999               | 0.999              |

Many researchers have studied the optimal parameter settings of one or several different pheromone update models, as shown in Table 3.

| References   | Model             | $m$       | $\alpha$ | $\beta$ | $\rho$ | $Q$ |
|--------------|-------------------|-----------|----------|---------|--------|-----|
| References [7]| Ant-Cycle         | -         | 1        | 5       | 0.5    | -   |
| References   | Ant-Quantity      | -         | 1        | 5       | 0.999  | -   |
| References   | Ant-Density       | -         | 1        | 10      | 0.9    | -   |
| References [8]| Ant-Cycle         | -         | 0.5      | 5       | 0.3    | -   |
| References [9]| Ant-Cycle         | [0.6$n$,0.9$n$] | [1,2] | [4,6]  | [0.5,0.8] | 100 |
| References [10]| Ant-Cycle      | 35        | 6        | 7       | 0.1    | 50  |
| References [11]| Ant-Cycle      | 31        | 5        | 1       | 0.1    | 30  |
| References [12]| Ant-Cycle      | $n$ or 1.5$n$ | [0.5]  | [0.5]  | [0.1,0.99] | [10,1000] |

3. TSP simulation experiment

The controllable parameters of the ACO algorithm in the TSP problem are analyzed above, and it can be found that previous researchers have different opinions on the optimal value of the parameters. In order to discover the influence of controllable parameters on the algorithm, the following verification through several sets of simulation experiments.

3.1. Optimal m-n value experiment

3.1.1. Method and parameter setting instructions. This experiment explores the impact of ant colony size on the optimal precision under different city sizes.

Set the variable parameters to city size $n$ and ant colony size $m$. Because the results of TSP problems for city's size within 4 are certain, a TSP model with a city size $n$ of 4 to 40 is randomly generated. At the same time, for each city size model, the ant colony size $m$ is set to 1 to 2$n$ for optimization.

The controllable parameter settings are shown in Table 4.

| $m$       | $\alpha$ | $\beta$ | $\rho$ | $Q$ |
|-----------|----------|---------|--------|-----|
| [1,2$n$] | 1        | 2       | 0.5    | 1   |
Initialize the value of each cell of the pheromone matrix to 9.1, set the maximum number of iterations $NC_{\text{max}}=100$, for each certain $n$ and $m$, perform 5 TSP optimization experiments, and take the average value as the corresponding $(n, m)$ binary Results of optimization experiments.

3.1.2. Experimental process and result analysis. The experimental optimal solution requires that when the city size $n$ is determined, the optimal ant colony size $m$ must minimize the obtained optimal value.

When the city size is set to 25 and 40, the average optimal solution changes with the ant colony size, as shown in Figure 2.

**Figure 2.** Changes in optimizing ability with ant colony size at 25 and 40

It can be seen that as the size of the ant colony increases, the algorithm's optimization ability increases logarithmically. However, increasing the size of the ant colony will reduce the efficiency of the algorithm. Therefore, it is necessary to find a suitable $m-n$ ratio, which can improve the optimization ability and maximize algorithm efficiency.

When the error cannot be tolerated, the $m-n$ ratio when TSP problems of different scales get the optimal solution is shown in Figure 3.

**Figure 3.** $m-n$ ratio at the optimal solution of TSP problems of different scales

It can be seen from the figure 3 that the optimal $m-n$ ratio is not stable, so a certain error range needs to be introduced. The error range is set from zero to 0.02 times the optimal value. The standard deviation of the optimal $m-n$ set changes as the error range expands is shown in Figure 4.
Figure 4. Variation of standard deviation of the optimal $m-n$ set with as the error changes

It can be seen from Figure 4 that when the error is greater than +0.008, the standard deviation of the optimal $m-n$ set gradually stabilizes. At this time, the difference fo the ratio of $m-n$ when the optimal solution for TSP problems of different scales between +0.008 error range and zero error is shown in Figure 5.

Figure 5. $m-n$ ratio at the optimal solution of TSP of different scales under two errors range

When the error range is +0.008, the average ratio of $m-n$ is 0.5791, that is, in small-scale problems, the ant colony size is usually set to 0.5791 times (about 0.6 times) the size of the city, in order to achieved a better efficiency.

3.2. Optimal $\alpha$-$\beta$ value experiment

3.2.1. Method and parameter setting instructions. This experiment compares the effect of the values of $\alpha$ and $\beta$ on the accuracy of the algorithm under different TSP models. Setting the variables $\alpha$ and $\beta$ from 1 to 10, a total of 100 combinations, and the controllable parameter settings are shown in Table 5.

Table 5. Controllable parameter settings for $\alpha$-$\beta$ experiments

| $m$  | $\alpha$ | $\beta$ | $\rho$ | $Q$ |
|------|----------|---------|-------|-----|
| $n$  | [1,10]   | [1,10]  | 0.5   | 1   |
Similarly, the value of each element of the pheromone matrix is initialized to 9.1, and the maximum number of iterations is set to $NC_{\text{max}}=100$. Experiments are performed 5 times for each group, and the average value is taken as the result of obtaining the optimal solution for the group.

3.2.2. Experimental process and result analysis. Experiments were performed on the CHN31 model and the Berlin52 model, and the experimental results are shown in Figure 6.

It can be seen from the above figure that as $\alpha$ increases, the optimization precision of the algorithm will decrease; and as $\beta$ increases, the efficiency of the algorithm will increase. As the size of the TSP model increases, for all cases where $\beta > 1$, its impact on the optimization accuracy of the algorithm gradually stabilizes.

In fact, the optimal solution of the CHN31 model appears in (1, 5) and (1, 7), while the optimal solution of the Berlin52 model appears in (1, 2) and (1, 5).

In order to analyze whether there is an optimal relationship with the ratio in the value range of [1,10], all the data with the same ratio are used as a class to find the average value of the optimal solution. The experimental results obtained are shown in Figure 7.

As can be seen from the figure 7, within the range of [1, 10], as the ratio of $\alpha$-$\beta$ increases, the algorithm's optimization accuracy becomes lower and lower. Therefore, under the constraints of this experiment, the optimal value of $\alpha$ is 1 and the optimal value range of $\beta$ is [5, 10].
3.3. Optimal $\rho$-$Q$ value experiment

3.3.1. Method and parameter setting instructions. This experiment compares the effect of the values of $\rho$ and $Q$ on the accuracy of the algorithm in different TSP models. The variable $\rho$ is in $[0.1,0.9]$ and the increment is 0.1; the variable $Q$ is in $[1,100]$ and the increment is 1. The controllable parameter settings are shown in Table 6.

| $m$  | $\alpha$ | $\beta$ | $\rho$  | $Q$ |
|-----|--------|------|------|-----|
|     | 1      | 2    | [0, 1]| [1, 100] |

In order to observe the two special cases of none volatility and complete volatility, the cases of $\rho = 0$ and $\rho = 1$ were retained. The other settings are the same as the experiments above: the value of each element of the initialization pheromone matrix is 9.1; the maximum number of iterations $NC_{\text{max}} = 100$, 5 experiments are performed for each group of $\rho$ and $Q$, and the average value is taken as the result of obtaining the optimal solution for this group.

The experimental results on the CHN31 and Berlin52 model are shown in Figure 8.

![Fig. 8. Experimental results on the CHN31 and Berlin52 model](image)

From the above experimental results, it can be seen that when $\rho \geq 0.2$, the algorithm can achieve high efficiency and the optimization ability is basically stable. Moreover, within the value range of $[1,100]$, the pheromone enhancement coefficient $Q$ has little influence on the efficiency of the algorithm. However, when $\rho = 1$, the pheromone in the ant colony optimization algorithm cannot be used, which will result in loss of empirical information, so $\rho$ should be set between $[0.2, 1)$.

4. Conclusion

In this paper, through three sets of parameter experiments of $m$-$n$, $\alpha$-$\beta$ and $\rho$-$Q$, a large number of experimental results were analyzed, and the basic rules of parameter setting in small-scale TSP problems are found. The summary is shown in table 7.

| $m$  | $\alpha$ | $\beta$ | $\rho$  | $Q$ |
|-----|--------|------|------|-----|
| 0.6n| 1      | [5,10]| [0.2, 1)| [1, 100] |

Experiments prove that when solving small-scale TSP problems, setting the ant colony optimization algorithm parameters according to the above table can effectively improve the efficiency of the algorithm.
ACO has many controllable parameters. In addition to the parameters discussed in this article, there are also the initial pheromone concentration and the maximum number of iterations. These two parameters will also affect the optimization efficiency of the ant colony algorithm. This article only uses the method of setting parameter pairs to discover the parameter setting regulation and the connection between the two parameters, without analyzing the parameters as a whole, so it needs to be carefully considered in subsequent studies.

References
[1] A. Colorni, M. Dorigo, V. Maniezzo, Distributed Optimization by Ant Colonies, Elsevier Publishing, Paris, 1991, 134-142.
[2] X. Y. Xia, Y. R. Zhou. Advances in Theoretical Research of Ant Colony Optimization. CAAI Transactions on Intelligent Systems. 11(2016), 27-36.
[3] S. F. Liu, H. Leng, L. Han, Pheromone Model Selection in Ant Colony Optimization for the Travelling Salesman Problem, Chinese Journal of Electronics, 26(2017), 223-229.
[4] J. Q. He, X. J. Sun, W. Li, et al. A new pheromone update strategy for ant colony optimization, Journal of Intelligent & Fuzzy Systems, 32(2017), 3355-3364.
[5] A. M. Abdelbar, K. M. Salama, Parameter Self-Adaptation in an Ant Colony Algorithm for Continuous Optimization, IEEE Access, 7(2019), 18464-18479.
[6] Q. W. Hu, Y. C. Liu. Parameters Setting of Ant Colony Algorithm in TSP Problem. Computer Knowledge and Technology. 7(2011), 4944-4946.
[7] Z. W. Ye, Z. B. Zheng. Configuration of Parameters $\alpha,\beta,\rho$ in Ant Algorithm. Geomatics and Information Science of Wuhan University. (2004), 597-601.
[8] G. Liu, X. H. Guo, Z. H. Feng, et al. Research on the Optimal Parameter Selection of the Ant Colony System Algorithm and its Application in TSP. JOURNAL OF SOOCHOW UNIVERSITY (ENGINEERING SCIENCE EDITION). (2007), 56-59.
[9] H. M. Xu, Y. B. Chen, J. G. Liu, et al. The research on the parameters of the ant colony algorithm. Journal of Shandong University of Technology (Natural Science Edition). (2008), 7-11.
[10] F. Shi, H. Wang. 30 Case Studies of MATLAB Intelligent Algorithm. Beijing: Beijing University Press, 2011.
[11] H. Yang, L. T. Han, Y. H. Lei, et al. Parameters Setting of Ant Colony Algorithm for CTSP Problem. Computer & Digital Engineering. 44(2016), 791-794.
[12] R. R. Yang, Y. Wang. Research on the Basic Principles and Parameter Setting of Ant Colony Algorithm. China Southern Agricultural Machinery. 49(2018), 38-39.