Dynamical friction in dwarf galaxies

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ABSTRACT

We present a simplified analytic approach to the problem of the spiraling of a massive body orbiting within the dark halo of a dwarf galaxy. This dark halo is treated as the core region of a King distribution of dark matter particles, in consistency with the observational result of dwarf galaxies having solid body rotation curves. Thus we derive a simple formula which provides a reliable and general first order solution to the problem, totally analogous to the one corresponding to the dynamical friction problem in an isothermal halo. This analytic approach allows a clear handling and a transparent understanding of the physics and the scaling of the problem. A comparison with the isothermal case shows that in the core regions of a King sphere, dynamical friction proceeds at a different rate, and is sensitive to the total core radius. Thus, in principle, observable consequences may result. In order to illustrate the possible effects, we apply this formula to the spiraling of globular cluster orbits in dwarf galaxies, and show how present day globular cluster systems could in principle be used to derive better limits on the structure of dark halos around dwarf galaxies, when the observational situation improves. As a second application, we study the way a massive black hole population forming a fraction of these dark halos would gradually concentrate towards the centre, with the consequent deformation of an originally solid body rotation curve. This effect allows us to set limits on the fraction/mass of any massive black hole minority component of the dark halos of dwarf galaxies. In essence, we take advantage of the way the global matter distribution fixes the local distribution function for the dark matter particles, which in turn determines the dynamical friction problem.

Key words: galaxies: compact – kinematics and dynamics – structure – dark matter

1 INTRODUCTION

Over the past few years it has become apparent that dwarf galactic systems distinguish themselves from the larger class of galaxies in more ways than their total luminosity. Rotation velocity studies in dwarf spirals and velocity dispersion studies in dwarf irregulars and dwarf spheroidals have revealed the presence of a dark, dynamically dominant component, much in excess of what a scaled down version of a large galaxy would show. The dark matter halo of a dwarf galaxy is more massive than an extrapolation of a large galaxy would show. The dark matter halo of a dwarf galaxy is more massive than an extrapolation of a single mass/luminosity relationship fitted to massive galaxies would suggest (eg, that of Kormendy 1986), but it also appears to have a distinct type of structure.

One of the best-studied dark matter distributions is that of the Sagittarius dSph galaxy (Ibata et al. 1997). Ibata et al (1997) analyzed their kinematics of this galaxy to derive a (radial) period of the orbit of Sgr about the Galaxy of less than ~ 1Gyr. This limit, together with lifetime constraints derived from the ages of the old stellar populations in Sgr, show it has survived for more than 10 orbits. Numerical models of the survival of a dSph galaxy orbiting inside a larger Galactic halo, however, imply complete tidal disruption in a few orbits (Velazquez and White 1995; Johnston et al 1995). The solution of this paradox suggested by Ibata etal is that the dark matter halo of Sgr, whose existence is derived directly from their kinematics, must have an approximately constant (Heaviside) density profile, out to a cutoff at the tidal radius. This model is consistent with all available constraints. The rotation curves of gas-rich dwarf spirals also typically show ‘solid-body’ linear behavior in their inner few kpc (eg, Casertano & van Albada 1990 or Burkert 1995). Other examples of galactic systems where observations suggest the presence of a constant density structure in the central regions are the rotation curves of de Blok et al. (1996) for LSB’s and Pryor & Kormendy (1990) for dSph density profiles. Additionally, there is the case of DDO 154, a nearby dwarf galaxy the rotation curve of which has been extensively studied in detail. This galaxy shows a core region, a maximum, and then an almost Keplerian decline, in substantial disagreement with models of dark halos obtained in cosmological simulations, as analyzed by Burkert & Silk (1997). These observational results are in disagreement with the singular profiles obtained in self-consistent cosmological simulations (e.g. Navarro et al. 1996) so that the detailed structure of dark matter halos is presently the object of some debate e.g. Burkert (1997). From a theoretical point of view, in our paper I (Hernandez & Gilmore 1997) we use a simple baryonic infall model to infer the initial dark halo profiles of late type galaxies. Using the recently observed rotation curves of these systems (de Blok et al. 1996) and other observational restrictions to calibrate the initial halo profiles,
we show that a King profile can accurately reproduce the observed rotation curves of these systems, as well as those of normal late type galaxies. In that paper we also show that centrally divergent density profiles are difficult to reconcile with the rotation curves of LSB galaxies. In as much as the problem is not settled, we explore the case of a constant density halo for dwarf galaxies, (as observations suggest) in terms of its possible implications for dynamical friction in those systems.

That is, in general, whereas more luminous spiral galaxies show a typically flat rotation curve out to several disk scale radii, indicating isothermal halos with density falling radially as \( r^{-2} \), dwarf galaxies appear to have approximately constant density halos out to the edge of the stellar distribution. Moreover, while the central regions of luminous galaxies are gravitationally dominated by stars, so that inferences about the inner structure of the dark matter are necessarily model-dependent, the central regions of dwarf galaxies are gravitationally dominated by dark matter. In this paper we consider the consequences of this relatively flat mass density profile for dynamical evolution in the dwarf galaxy, to consider the possibility that this dominance by dark matter can be exploited to constrain the nature of that dark matter, particularly by exploiting dynamical friction.

Dynamical friction has important consequences in shaping the orbital evolution of massive bodies within dark halos, such as globular clusters and any hypothetical massive black holes. This has been used to set some interesting constraints on the parameters of the massive objects which orbit in the dark halo systems of large galaxies, such as the allowed masses of black holes, required to be consistent with dynamical friction not having made their orbits decay over their lifetimes e.g. Hut & Rees (1992). Similarly, if the orbiting objects are better known, as in the case of globular clusters, the problem can be inverted, and used to set limits on the dark halo structure and the evolution of the globular cluster system (e.g. Aguilar, Hut and Ostriker 1988). Nothing which is not derivable from the complete form of the rotation curve can be derived about the distribution function of the dark matter itself. In the case of dwarf systems however, which remain dark matter dominated into their inner regions, we can use dynamical friction constraints to investigate the distribution function of the dark matter in the core region, determined by the total matter distribution, even though only a fraction of the rotation curve may be accessible to observations. It is therefore interesting to use dynamical friction constraints in the case of the dark matter dominated dwarf galaxies, to try to learn more about the dark halos in these systems.

Although the problem reflects the gravitational interactions of the massive body with the totality of halo particles, and strictly should be treated through n-body simulations, a few well grounded assumptions can simplify the problem to allow an analytical treatment. This has been done (e.g. Binney & Tremaine 1987) in the case of the isothermal halo, to provide a robust, general purpose analytical formula which can give a reliable first order solution, which although not as accurate as an n-body code, has the advantage of providing physical insight into the solution and the scaling of the problem. Dynamical friction is most sensitive to the low velocity particles, which are the ones which interact more strongly with the body undergoing this friction. The presence of a core in a dark halo means effectively changing the ratio between low and high velocity halo particles, towards more low velocity ones, as the high energy particles are mostly found on extended orbits. Hence, it seems reasonable to suspect that a distribution function corresponding to a system with a core will change the problem of orbital spiraling due to dynamical friction with respect to the case of an isothermal dark matter halo. Since dSph galaxies do have globular clusters in some cases (Sgr has four) one is interested to know if their continued existence provides any useful limits on the dark matter. In this paper we develop a formula totally analogous to the one of Binney & Tremaine (1987) for the gravitational friction on a massive body orbiting within an isothermal sphere, but appropriate to the dwarf galaxy dark matter problem.

In this paper the dynamical friction formulation for the core regions of dwarf galaxies is derived and applied in connection with two aspects of the dynamics of these systems. First, we try to provide information on the structure of these dark halos, from regions beyond those accessible through direct measurements of stellar velocities, using the global information contained in the local distribution function of the halo particles, through dynamical friction effects. The orbital spiraling of a globular cluster will depend on the global dark halo structure, as it is this which determines the local distribution function, which in turn determines the dynamical friction problem. We can find what range of halo structures is consistent with some observed globular cluster system age and orbital distribution. Second, we consider a more general case, where some part less than 100 percent of the dark halo is in the form of massive compact objects (black holes?), and derive some limits on the mass of any such black holes by requiring that their orbits should not have decayed over the lifetime of the system.

In dwarf galaxies, it can not be expected that the density profile inferred from the regions where the stars can be measured extends indefinitely in a shallow density profile. It seems more natural that we should be seeing only the core region of a distribution of dark matter, the rest of the halo being empty of stars, see for example Pryor & Kormendy (1990) and Lake (1990). King-model spheres represent a self-consistent solution to both a Boltzmann and a Poisson equation, and have been found to fit well the end products of N-body violent relaxation simulations, so we adopt here the plausible simplifying assumption that we may treat galactic dark halos as King spheres. We note that in our paper I we find King profiles to adequately reproduce the rotation curves of LSB and dwarf galaxies. In that paper we perform a detailed study of baryon dissipation within dark halos, calibrated using a variety of observational relations.

In Section 2) we present the derivation of the simple analytical formula describing orbital spiraling due to dynamical friction in the core regions of dark halos, which we apply in section 3) to the two problems mentioned above, in connection to dwarf galaxies. In section 4) we present the conclusions of this paper.

2 THEORETICAL APPROACH

Take a body \( A \) of mass \( M \) moving on a stable circular orbit around the centre of a spherically symmetrical mass distribution \( \rho(r) \), made up of an equilibrium distribution of self
gravitating particles of mass $m$, where $M > m$. Consider one gravitational encounter between the body $A$ and one of the background particles. This produces a small deviation in $A$’s orbit producing a negative change $-\Delta V$ in the original forward velocity, $V$, and a positive change $+\Delta V$ perpendicular to the original forward velocity, in the plane of the interaction. Consider now the totality of the background particles. In this case all the $+\Delta V$ deviations will average out to zero, and only a net $\Sigma(-\Delta V)$ will remain, in the direction of the original velocity. This deceleration is what constitutes dynamical friction.

If one now integrates over the effects of particles interacting with impact parameters from 0 to $b_{\text{max}}$, a maximum impact parameter relevant to the problem, and assumes that the background particles move isotropically, one obtains:

$$
\frac{dV}{dt} = -16\pi^2 \ln \left( \frac{M M}{V^2} \right) \int_0^V f(v) v^2 \, dv, \quad (1)
$$

for the deceleration parallel to $V$ experienced by $A$ as a result of the collective interactions with the background particles having a distribution function $f(v)$, isotropic in velocity space. In equation (1) $\Lambda = b_{\text{max}} V^2/(GM)$, and $V_T$ is a typical velocity of the system (see Binney & Tremaine 1987). It has been assumed that $\ln \Lambda$ is a constant, although this is not strictly true, but usually $\Lambda >> 1$, and $\ln \Lambda$ does not vary appreciably for most applications. Similarly, $\ln \Lambda$ is not sensitive to the choice of $V_T$, taken in this work as the circular velocity of the halo of background particles. Numerical experiments have confirmed the validity of (1), and of $\ln \Lambda = cte.$ e.g. Bontekoe & van Albada (1987) and Zaritsky & White (1988), see Binney & Tremaine (1987) for a more detailed discussion, and a derivation of equation (1).

As explained in the introduction, in order to use equation (1) in the case of dwarf galaxies, we shall assume their dark matter halos to be well represented by the core regions of a King distribution, approximated by a constant density region, i.e.,

$$\rho(r < r_0) = \rho_0 \quad (2)
$$

and

$$f(v) = \frac{n_0}{(2\pi\sigma^2)^{3/2}} \left( \exp(-v^2/2\sigma^2) - \exp(-v_e^2/2\sigma^2) \right), \quad (3)
$$

In equation (2) $\rho_0$ is the density within the core region, $r_0$ the core radius, defined as $r_0 = (9\sigma^2/4\pi G \rho_0)^{1/3}$, and $\sigma$ is the isotropic velocity dispersion of the halo particles. In equation (3), $n_0$ is a particle number density, $n_0 = \rho_0/m$, and $v_e = v_e(r)$ is the escape velocity of the halo. Introducing equation (3) into equation (1), one obtains:

$$
\frac{dV}{dt} = -16\pi^2 \ln \left( \frac{M}{V^2} \rho_0 M I(r) V^{-2} \right), \quad (4)
$$

where,

$$I(r) = \frac{1}{(2\pi\sigma^2)^{3/2}} \int_0^V \left( \exp(-v^2/2\sigma^2) - \exp(-v_e^2/2\sigma^2) \right) v^2 \, dv.
$$

Using the substitutions $X = v/(\sqrt{2}\sigma)$, $Y = v_e/(\sqrt{2}\sigma)$ and $X_A = (v_e/X)$, $I(r)$ becomes:

$$I(r) = \frac{1}{\pi^{3/2}} \int_0^{X_A} X^2 \left( e^{-X^2} - e^{-Y^2} \right) dX.
$$

$$\Rightarrow I(r) = \frac{1}{4\pi} \left( \text{erf}(X_A) - \frac{2X_A}{\sqrt{\pi}} e^{-X^2} - \frac{4X^3}{3\sqrt{\pi}} e^{-Y^2} \right) \quad (5)
$$

At this point we have to introduce an assumption about the orbit of body $A$, so that we can evaluate $X_A$. Since the dynamical friction drag removes energy from $A$ in proportion to $(dV)^2$, and angular momentum only in proportion to $dV$, as time progresses, $A$ will tend to settle into an orbit of maximum angular momentum. This process leads to the circularisation of the orbits of bodies under the influence of dynamical friction. In this work, we will assume that the orbits of spiraling bodies are always circular, (see for example Wahlde & Donner (1996), who study the more difficult problem of the influence of the disk on the dynamical friction problem, also under the assumption of circular orbits for the spiraling body) in accordance with the objective of deriving a simple formula for the process. In this case, $V = V_c$, where:

$$V_c^2(r) = \frac{GM_H(r)}{r} = \frac{4\pi}{3} G \rho_0 r^2
$$

$$\Rightarrow V_c(r) = V_0(r/r_0)
$$

Now $V_c(r_0) = V_0$, and $X_c = V_c/\sqrt{2}\sigma \equiv X$, and $M_H(r)$ refers to the dark halo mass internal to radius $r$, with $M_H(r_0) = M_H$, the total halo mass within the core radius.

Now we need $v_e$, the escape velocity. From equation (2),

$$\frac{1}{2} v_e^2(r) = \int_r^{r_0} \frac{GM_H(r)}{r^2} \, dr + \int_{r_0}^{\infty} \frac{GM_H}{r^2} \, dr.
$$

The second integral is an underestimate, as $\rho(r > r_0) \neq 0$ (except in a tidally truncated dSph, such as Sgr), but since we are interested only in the region $r < r_0$, which corresponds to $X < 1.23$, this underestimate introduces little error: notice the $X^3$ and the $\exp(-Y^2)$ in the relevant term in $I(r)$.

$$\frac{1}{2} v_e^2(r) = \frac{4\pi}{3} G \rho_0 \left( \frac{r_0 - r^2}{2} + \frac{r_0^2}{2} \right)
$$

$$v_e^2(r) = V_c^2 \left( \frac{3r_0^2}{r^2} - 1 \right) \quad (6)
$$

$$\Rightarrow Y^2 = X^2 \left( \frac{3r_0^2}{r^2} - 1 \right)
$$

Introducing $R = r_0/r$, $V_c(R) = (3^{1/2}\sigma R)$ and $Y^2 = (9/2 - X^2)$:

$$I(X) = \frac{1}{4\pi} \left( \text{erf}(X) - \frac{2X_A}{\sqrt{\pi}} \exp(-X^2) - \frac{4X^3}{3\sqrt{\pi}} \exp(X^2 - 9/2) \right) \quad (7)
$$

Now to calculate the spiraling of $A$, we use the assumption that it always orbits at $V_c$, therefore, $|L| = r M V_c$, and

$$\frac{d(L/M)}{dt} = V_c \frac{dr}{dt}$$
Also, the acceleration in equation (4) corresponds to a drag force \( M(dV_c/dt) \) parallel to \( V_c \), and therefore a torque

\[
\frac{d(L/M)}{dt} = r \frac{dV_c}{dt}
\]

\[
\Rightarrow \frac{dr}{dt} = \frac{r}{V_c} \frac{dV_c}{dt}
\]

(8)

Now define \( \tau \), the characteristic time-scale with which the orbit of \( A \) decays as:

\[
\tau = \frac{r}{2(dr/dt)_r}
\]

using equation (8),

\[
\tau = \frac{V_c}{2(dV_c/dt)_r}
\]

and from equation (4),

\[
\tau = \frac{V_c^3}{32\pi^2 \ln \Lambda G^2 M \rho_0 I(X)}
\]

Substituting \( V_c \) for \( \rho_0 \), \( V_0 \) for \( V_c \) and \( X \) for \( r \), we get:

\[
\tau = \left( \frac{V_0}{r_0} \right) \left( \frac{2}{3} \right)^{3/2} \frac{r_0^3}{24 \pi \ln \Lambda G} \left( \frac{X}{I(X)} \right)
\]

(9)

Now if \( F(X) = X^3/I(X) \), we see in Fig. 1 that \( F(0) = 16.89 \), remaining almost constant as \( X \) increases, reaching 19 for \( X = 0.5 \), and increasing slightly to reach 29 at \( X = 1 \), and increasing slightly faster beyond that point. This shows that orbital decay in the core region of a King sphere from \( R = 1 \) to around \( R = 0.8 \) proceeds faster than exponential, and after that it becomes essentially exponential, with time-scales:

\[
\tau = (20) \left( \frac{V_0}{r_0} \right) \left( \frac{2}{3} \right)^{3/2} \frac{r_0^3}{24 \pi \ln \Lambda G I(X)}
\]

which comes to:

\[
\tau_{DF} = \left( \frac{V_0}{r_0} \right) \frac{r_0^3}{3 \pi \ln \Lambda G} \text{Gyr},
\]

(10)

which is the final result of this section, where \([V_0] = \text{km/s}, [r_0] = \text{pc} \) and \([M] = 10^8 M_\odot \). The main differences between this result and that for the isothermal case in Binney & Tremaine is the density profile, and the fact that \( v/\sigma \) is a function of \( r \), rather than \( X \equiv 1 \). This yields an exponential time-scale independent of the radius at which the particle orbits and which is only a function of the central density of the halo, and the total core radius.

### 2.1 Comparison with the isothermal case

At this point, it is interesting to compare equation (10) with the corresponding expression describing the spiraling of a body of mass \( M \) moving on a circular orbit around the centre of an isothermal density distribution, characterized by a constant circular velocity \( V_c \),

\[
t_{DF} = \frac{2.64 r_0^3 V_c}{M \ln \Lambda} \text{Gyr},
\]

(11)

(Binney & Tremaine 1987), where \( t_{DF} \) is the total time it takes for the body to spiral from an initial radius \( r_0 \) to \( r = 0 \), and \( \Lambda \) and the units are the same as in equation (10). The main difference between the two cases, is that in the isothermal one the spiraling body reaches the centre in a finite amount of time, whereas in the core region case the orbital spiraling only asymptotically reaches the centre, with a half-orbit time, \( \tau_{DF} \).

To compare these two results, consider an observed body at a galactocentric distance of \( r_B \), orbiting at a velocity \( v_B \). We wish to compare the evolution of its orbit in an isothermal sphere, which is an adequate first approximation to the outer parts of a dark halo in at least a large galaxy, to the evolution of its orbit in a constant density core, which is the topical model for the inner regions of dwarf galaxies. For the constant density core,

\[
V_0 = v_B \left( \frac{r_0}{r_B} \right)
\]

and therefore, from equations (10) and (11),

\[
\tau_{DF} = \left( \frac{1}{8} \right) \left( \frac{r_0}{r_B} \right)^3
\]

In practice, the inner regions of large galaxies are baryon-dominated rather than dark-matter dominated, and even in dwarfs the baryon component is not always negligible, so that interactions with the baryonic component, which increase as the radius decreases, will substantially modify the orbit at late times. To avoid this unnecessary complication, we take four half-orbit times as representative of the orbital decay in the core region case. This fiducial number of half
lives is only used for this particular comparison with the isothermal case, and is not used again in any of what follows. After this time the body will have decayed into an orbit with a radius 1/16 of the initial radius. In this case,

$$\frac{4\tau_{DF}}{t_{DF}} = \left(\frac{1}{2}\right) \left(\frac{r_0}{r_B}\right)^3$$

We see that for the simplest assumption of $r_B = r_0$, dynamical friction decay timescales are twice as fast in the case of a constant density core than in the case of an isothermal distribution. That is, the effects can be relatively large, enough to be observable. For $(r_0/r_B) = 2^{1/3}$ the total orbital decay timescale is equal in both cases, and becomes progressively longer in the constant density case, as the true core radius becomes larger than the observed orbital radius. Whereas in the isothermal case the observed radial distance and orbital velocity of the body uniquely determine how long the dynamical friction process will take to drive the body to $r = 0$, in the core region case one requires the central density (a velocity radius pair within the region of interest) and the halo core radius. In the isothermal case equation (11) yields a fixed $t_{DF}$ for a body with an observed radial distance and rotation velocity, while equation (10) further requires an assumed core radius for the system, which could sometimes be inferred from consistency requirements (see the case of the Sgr dwarf below).

It is this dependence of the orbital decay process on the total core radius which could be used to derive structural halo parameters from orbital structures. In the isothermal case, only the starting radius determines the spiraling process: there is no sensitivity to the global parameters. Presented differently, the only parameter of the isothermal halo is $V_c$. It should be noted that the comparison between the two time scales is highly sensitive to $r_0/r_B$, and therefore, the isothermal formula is in general not a reliable estimate of the dynamical friction problem in cases where a constant density core is suspected. Additionally, in some applications the actual evolution of the orbit might be relevant, in which case the better fitting of equation (10) or equation (11) might reveal the presence of a core region in the dark halo in question.

Finally, it should be noted that neither of equation (10) nor (11) can be applied to the case of the orbital decay of dwarf spheroidal galaxies in the halo of our galaxy. Direct application of equation (10) would assume that only the halo matter is present, so that the baryonic component of our galaxy, which has an important dynamical contribution over any possibly interesting dark halo core region, would be ignored. This would exclude an important fraction of the rotation velocity from consideration, resulting in erroneously short times being predicted. Equation (11) assumes that all the matter responsible for the flat rotation curve contributes to the dynamical friction drag, which would again be an overestimate of the dynamical friction, as the disk material does not interact with the dwarf galaxy in the same way as the halo particles. Additionally, one would require a consistent distribution function for the halo particles, in the presence of dynamically important disk and bulge components. For the above reasons, equation (10) is only relevant to the internal dynamics of dwarf galaxies, or other clearly dark matter dominated systems, where a uniform radial density is suspected from observations. Such cases do exist (Ibata et al. 1997), and may even be the norm with LSB galaxies (paper I).

### 3 Applications

In this section we use equation (10) in two interesting problems, to present a theoretical framework which should allow us to derive important constraints on the structure of the dark halos of dwarf galaxies, and to set some constraints on the fraction of these halos which could be made up of massive compact objects.

#### 3.1 Core radii determination in dwarf galaxies

We can use equation (10) to obtain information on the size of the core region of dwarf galaxies by considering the effects of dynamical friction on the globular cluster systems of these galaxies. It is of course notoriously difficult to deduce the properties of a hypothetical destroyed parent population from few, or no, survivors. Nonetheless, our aim here is to illustrate the sensitivity of such analyses to assumptions made concerning the spatial density distribution of the dark matter, independent of current observational limitations.

For simplicity here, to illustrate the scale of the effect to zeroth order, we characterize luminous globular clusters as having uniform parameters, in particular masses close to $M = 10^5 M_\odot$ (see Harris 1991 for a review on the subject). Taking $M = 10^5 M_\odot$, representative of a typical globular cluster, $b_{max} = 3 \text{kpc}$, in the range of the solid body rotation regions of dwarf galaxies, and $V_T = 40 \text{ km/s}$, used only to calculate $\ln A$, which is only marginally sensitive to these values, we obtain:

$$\tau_{DF} = \frac{V_c}{27.8 M_\odot} \frac{r_0^3}{V_0^2} \text{Gyr}. \quad (12)$$

Suppose that a dwarf galaxy is observed, having a solid body rotation curve out to the last measured point, at $r = 2.5 \text{kpc}$, with $V_c(2.5 \text{kpc}) = 30 \text{ km/s}$ (typical values for these systems, for example Carignan & Beaulieu (1989) for the case of DDO 154 in which case the HI rotation curve was measured beyond the extent of the stellar content). The null hypothesis would be to assume that the core region of this galaxy measures $2.5 \text{kpc}$, extending only as far as the rotation curve could be measured, with $V_0 = 30 \text{ km/s}$, but this is clearly only a lower limit. If we take $r_0 = 2.5 \text{kpc}$ and $V_0 = 30 \text{ km/s}$, for globular clusters of $M = 10^5 M_\odot$, equation (12) gives:

$$\tau_{DF} = (30/2.5)(2.5^3/27.8) = 6.74 \text{ Gyr}.$$

This timescale is sufficiently short compared to the ages of dwarf galaxies in the Local Group as to be potentially interesting. In general, as the dwarf galaxies correspond to higher contrast initial fluctuations, they became bound structures, and perhaps initiated star formation, earlier than normal, larger galaxies. Additionally, direct stellar population studies in these systems have yielded population ages of about the age of the universe (e.g. Hodge 1989, Ibata et al. 1997). In view of the above, 12 Gyr seems like a suitable age for these systems. The application of equation (12) shows that if the core radius measured only 2.5 kpc, almost 2 orbit half-times have elapsed for the globular cluster system, which should therefore show significant dynamical friction
effects. Specifically, one expects a low abundance of globular clusters at large radial distances and to find them only concentrated very close to the centre of the system, where dynamical friction would drive them.

Were such a distribution observed, could one in fact reliably infer that dynamical friction might have been to blame? Suppose instead, that the actual dark halo core region is 3.5 kpc in size. In this case, as the density of the dark matter would not change, $(V_0/r_0)$ remains the same, and we obtain,

$$\tau_{DF} = 6.74(3.5/2.5)^3 = 18 \text{ Gyr}.$$  

This last value is larger than the age of the universe, and we would therefore expect to see no dynamical friction effects in the globular cluster system of this galaxy.

Clearly, the present spatial distribution of globular clusters in dwarf galaxies is a (one-way) test of the importance of dynamical friction, by investigating the incidence of spatially extended cluster systems in galaxies with dark-matter dominated, linear, rotation curves. This program is not applicable at the present time, as we lack information on the state and nature of globular cluster systems around dwarf galaxies. In fact, due to observational difficulties, together with the intrinsic rarity of globular cluster systems in dwarf galaxies, there are presently only a handful of detections of globular clusters around dwarf galaxies. It is therefore the use of accumulated deep HST and wide angle photometry and redshift surveys (e.g. SDSS, 2dF) that this problem might be explored.

As a specific example, we can take the case of the Sagittarius dwarf, the recently discovered dSph galaxy, whose dark matter distribution, as derived from stellar kinematics, is in good agreement with the core model discussed in this paper. The Sgr dwarf has four globular clusters, and has been studied fairly well kinematically (Ibata et. al 1997). From the observed velocity dispersion of stars in this galaxy, we adopt $V(1\text{kpc}) = 20 \text{km/s}$. Additionally, the tidal radius for this galaxy at its current position is $\leq 1 \text{kpc}$ (Ibata et. al. 1997). Applying equation (12) with these numbers, we obtain $\tau_{DF} = 0.72 \text{ Gyr}$, for globular clusters of $10^7 M_\odot$. Since the globular cluster system of this galaxy has not decayed completely, as such a short half life would suggest, the Sgr dwarf necessarily had originally a dark halo core radius of more than its present tidal radius. The dependence of $\tau_{DF}$ on the total core radius of the galaxy allows to reconcile theory with observations, as a larger core radius (not constrained by any direct observation) would result in more extended values for $\tau_{DF}$.

Quantifying the original size is necessarily inexact. If, for example, we assume that there have elapsed only 2 half lives for this globular cluster system, equation (12) yields an original core radius of $2.4 \text{kpc}$, which is larger than the observed $1 \text{kpc}$ tidal radius. This is consistent with the finding that the stellar population which has been associated with this galaxy presently spreads over $3 \text{kpc}$, showing signs of tidal disruption by the Galaxy. This is what would be expected if the original extent of this galaxy had been larger than its present tidal radius, as equation (12) suggests. Further, it being the only dSph with a globular cluster system also suggest the Sagittarius dwarf had a large original size. Thus, the existence of a globular cluster system, together with the constant density dark matter mass distribution derived from stellar kinematics, and the dynamical friction analysis of this paper, requires that Sgr is significantly tidally stripped.

Thus, through requiring that the dynamical friction decay time, $\tau_{DF}$ be consistent with the spatial extent of the globular cluster system over the age of the galaxy we have used equation (12) to set some constraints on the size of the original core region of this system. If we took the formula for the isothermal halo, we would obtain $\tau_{DF} = 8 \tau_{DF} = 5.7 \text{ Gyr}$, also significantly shorter than the age of this system. Since, in the isothermal case, there is no further dependence beyond the observed position and orbital velocity, there would be no way to reconcile theory with observations in this case. This example serves to illustrate the differences in the dynamical friction problem between the isothermal and constant density core cases, as well as to provide independent confirmation for the dynamical calculations of Ibata et al (1997), in the sense that the dark halo of the Sgr dwarf is probably characterized by a constant density profile.

### 3.2 The case of massive black holes in the dark halos of dwarf galaxies

In this sub-section we shall use equation (10) to set some limits on the fractional part of dwarf galactic halos which could be made up of massive black holes. In the review on the subject of baryonic dark matter by Carr (1994), it is shown that the only non-excluded baryonic dark matter candidates for the halos of galaxies (like our own) are brown dwarf stars and massive black holes in the range $10^3 - 10^7 M_\odot$. Gravitational microlensing searches (eg Alcock et al. 1996) are ideal probes at low masses, but are less sensitive to the very rare and very long time-scale events caused by massive black holes. We can provide a limit in the case that massive black holes make up only a part of the dark halo, by investigating the survival time of such a system against dynamical friction decay.

We calculate the time it would take for such massive black holes (making up a fraction $\gamma$ of the total dark halo) to spiral towards the centre of the dark halos of dwarf galaxies, as a result of dynamical friction with another dark matter component, made up of small particles. To first order, we can expect that after 1 orbit half-life all the black holes formerly part of the uniform density distribution within $r_0$ will have formed a new density distribution within $r_0/2$, leaving the remaining fraction of dark matter as it was originally. Clearly, this scenario for the evolution of the halo is only applicable to low black-hole mass fractions. If the black holes make up a large fraction of the halo, dynamical friction will only redistribute energy between the two components, making the particle distribution expand, with the black holes segregating only marginally towards the central regions, at which point dynamical friction would stop operating. In this way, this method is complementary to the micro-lensing approach, which is not sensitive to the possibility of only a small fraction of the halo being made up of massive black holes.

It is easy to show that the fractional increase in the rotation velocity at a radius $r_0/2^n$ after $n$ half-orbit times have elapsed, and a fraction $\gamma$ (for $\gamma << 1$) of the original constant density halo has concentrated interior to $r_0/2^n$ is:
\[ V_c^2 \left( r_0/2^n \right) = \left( (1 - \gamma) + \gamma 2^{3n} \right)^{1/2} V_c^2 \left( r_0/2^n \right). \]

This increment would only appear inwards of \( r_0/2^n \), the rotation curve beyond being reduced. It is this distortion of a solid body rotation curve, to one with a central enhancement, which would appear anomalous relative to observed rotation curves, and so provides the observational application here. In the limiting case, a galaxy which had collected all of its black holes in the center would show an inner Keplerian rotation curve, rather than the observed solid body rotation. Thus, this case can be excluded by inspection. Even for a black hole fraction as low as 0.05, after only two half-orbit time-scales, at \( r_0/4 \) the rotation curve would show an increment of a factor of 2, which could easily be detected. For larger black hole fractions, e.g. 0.125, by only one half-orbit time-scale the rotation velocity at \( r_0/2 \) would show an increase of a factor of 1.4, which could also be easily detectable. We take the maximum \( \gamma \) for which this approach should be valid as 1/8, at which fraction after 1 half orbit time the black hole component would concentrate interior to \( r_0/2 \), with an average density equal to that of the background halo density. Higher black hole fractions are not applicable, as this simple approach would predict dynamical friction would concentrate the black hole component to densities higher than the background halo densities, which is not physical.

We define the critical mass above which black holes can be ruled out, at a given fraction, as \( M_c(\gamma) \), from equation (12) to obtain:

\[ M_c(\gamma) = \frac{V_0^2 r_0^2 n}{417(1 - \gamma)} 10^5 M_\odot \]  

where \( n \) is the number of half-orbit time-scales which have elapsed since the formation of the system.

For a typical dwarf galaxy we can take \( V_0 = 30 \text{km/s} \) and \( r_0 = 2.5 \text{ kpc} \), if we evaluate equation (13) for \( n = 1 \) and \( \gamma = 1/8 \) we obtain \( M_c = 0.5 \times 10^5 M_\odot \). For black holes of greater mass, the distortion to the solid body rotation curve after one half life had passed would become noticeable. This shows that black holes forming less than 1/8 the mass of dwarf galactic halos, with individual masses of more than \( 0.5 \times 10^5 M_\odot \), can be excluded. Lowering the fraction on the halo which the black holes constitute, or requiring 2 half lives to have elapsed before the effects are noticeable only introduces a factor of about two, making the limit mass \( M_c = 1 \times 10^5 M_\odot \). Taking now \( V_0 = 15 \text{km/s} \), and \( r_0 = 1.0 \text{kpc} \), representative of a dSph galaxy, we obtain \( M_c = 4 \times 10^3 M_\odot \). Again, the uncertainties in the other parameters in equation (13) introduce a factor of 2 in this number. From this we see that black holes forming a fraction of less than 1/8 of the dark halos of dSph galaxies can be ruled out, for masses greater than \( 1 \times 10^4 M_\odot \). Ruling out such small fractions is not uninteresting, as it is here where other more direct methods become insensitive.

4 CONCLUSIONS

From the analysis introduced in section 2 and the applications of section 3, we can conclude the following:

1) The inward spiraling of a massive body orbiting within the core region of a dark halo due to dynamical friction proceeds in general at a different rate than in the isothermal case. Orbital decay in the constant density case is rapid at first, and slows down as time progresses, rather than starting slowly and accelerating with time, as in the isothermal case.

2) The dynamical friction decay time in a constant density dark halo core, presented in equation (10), can be applied to several problems involving dynamical friction within dwarf galaxies. The simple analytic nature and generality of this expression allows a clear understanding of the physics and of the scaling properties of dynamical friction in this case.

3) Determination and analysis of the spatial distribution of the globular cluster systems around dwarf galaxies can provide insight not only into the evolutionary history of these systems, but also into the structure of their dark halos. In the specific example of the Sagittarius dwarf spheroidal galaxy, the existence and spatial distribution of its globular cluster system provides direct evidence for substantial tidal stripping during the lifetime of the galaxy, as well as making an isothermal halo profile seem unlikely.

4) The observed smoothness of inner rotation curves in dwarf galaxies, together with the analysis here, tightly constrains any minority contribution to their dark halo from individual, high-mass, objects. Massive black holes with masses of more than \( 10^4 M_\odot \) forming a fraction of less than about 0.1 of the dark halos of dwarf galaxies can be excluded, as such a contribution to the mass distribution would evolve by dynamical friction to generate observable distortions to the rotation curves.

ACKNOWLEDGMENTS

The work of X. Hernandez was partly supported by a DGAPA-UNAM grant.

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APPENDIX A: SOLVING FOR THE DETAILED ORBITAL EVOLUTION

From equation (4) and equation (8),

\[
\frac{dr}{dt} = -16\pi^2 \frac{\ln AG^2 M \rho_0 r I(X)}{V_c^2} \tag{A1}
\]

substituting \( V_c \) for \( \rho_0 \), \( V_0 \) for \( V_c \) and \( X \) and \( r_0 \) for \( r \), we get:

\[
\frac{dX}{dt} = -A \frac{I(X)}{X^2},
\]

where:

\[
A = (2 \times 3^3)^{1/2} \frac{\pi G \ln M}{r_0^3} \frac{r_0}{V_0} = 277.3 \frac{M}{r_0^3} \frac{r_0}{V_0} \text{Gyr}^{-1}
\]

with \( [V_0] = km/s \), \( [r_0] = kpc \) and \( [M] = 10^5 M_\odot \).

Solving Eq(A1) numerically, one obtains:

\[
12\pi^{3/2}(a_1 \ln X + a_2 X^2 + a_3 X^3 + \ldots) = -At + C \tag{A2}
\]

Where \( C \) is given by the initial conditions, \( X \) at \( t=0 \), and \( a_1 = 0.25281 \), \( a_2 = 0.07811 \), \( a_3 = 0.01079 \) and so on. Notice that \( 12\pi^{3/2}a_1 = 16.89 \), which we already knew from \( F(X=0)=16.89 \).

In this way, equation (A2) can be used to trace the exact temporal evolution of the orbit of any massive body, within the core region of a King sphere.

APPENDIX B: THE EFFECT OF THE MATTER DISTRIBUTION BEYOND \( R_0 \)

To explore the dependence of the solution on having neglected the matter content beyond the core radius, in this subsection we calculate the escape velocity, \( v_e(r) \) considering an exponential cut-off starting at the core radius, i.e.,

\[
\rho(r) = \begin{cases} 
\rho_0 & r < r_0 \\
\rho_0 e^{-(R-1)} & r \geq r_0
\end{cases}
\]

(B1)

Which implies,

\[
\frac{1}{2} v_e^2(r) = \frac{4\pi}{3} G \rho_0 \left( \int_r^{r_0} r dr + r_0^3 \int_{r_0}^{\infty} \frac{dr}{r} \right) + 4\pi G \rho_0 r_0^2 \int_1^{\infty} \left( \frac{5}{R^2} - \frac{2}{R^2} e^{-(R-1)} - \frac{2}{R^2} e^{-(R-1)} - e^{-(R-1)} \right) dR
\]

which gives,

\[
\frac{1}{2} v_e^2(r) = \frac{4\pi}{3} G \rho_0 \left( \frac{r_0^3 - r^2}{2} + r_0^2 + 6r_0^2 \right)
\]

(B2)

\[
\Rightarrow v_e^2(r) = V_e^2(r) \left( \frac{15r_0^2}{r^2} - 1 \right),
\]

where \( V_e^2 \) is the equivalent of equation (6), and reflects a higher escape velocity. This leads to \( Y^2 = \frac{45}{2} - X^2 \), which makes the equivalent of equation (7) become:

\[
I'(X) = \frac{1}{4\pi} \left( erf(X) - \frac{2X}{\pi^{1/2}} e^{-X^2} - \frac{4X^3}{3\pi^{1/2}} e^{-(45/2 - X)} \right)
\]

(B3)

and

\[
\tau = \left( \frac{V_0}{r_0} \right) \left( \frac{2}{3} \right)^{3/2} \frac{r_0^3}{24\pi G \ln M G} F'(X),
\]

(B4)

where \( F'(X) = X^3/I'(X) \).

We find, \( 0 < (F(X) - F'(X)) < 1.65 \), for \( 0 < X < 1 \), with the difference between the two expressions decreasing rapidly as \( X \to 0 \), and increasing monotonically as \( X \) increases, as illustrated in Fig 1. Therefore, considering a gradual cut-off in the density distribution, even a quite abrupt exponential one, increases the escape velocity within the core region enough to reduce the third term in \( I(X) \) effectively to zero. This has the effect of lowering \( I(X) \) towards its value at \( X = 0 \), driving the solution closer to the exponential case, with the same half-life. This is what one might have expected, as the matter distribution exterior to the orbiting body does not enter into the dynamical friction problem, except in as much as it determines the global distribution function for the halo particles i.e. \( \sigma \). This dependence has already been taken into account in the expression for \( r_0 \) and the assumption of a global King distribution, and hence one should see no further dependence of the solution on the matter distribution exterior to the core region. The only approximation enters in considering the density within the core region of the King sphere as constant. In as much as the matter distribution beyond this region does not affect the dynamical friction problem, the distribution function, circular velocity and density profile of the halo model are self consistent.

Solving equation (B4) numerically, one gets an identical expansion to equation (A2), with coefficients: \( a_1 = 0.25 \), \( a_2 = 0.075 \), \( a_3 = 0.0091 \), etc. Notice that \( 12\pi^{3/2}a_1 = 16.70 \), which is consistent with \( F'(0) = 16.70 \) and not far from \( F'(0) = 16.89 \).

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