Abstract

We apply the physically more appealing MIT Bag boundary conditions to study the Casimir effect on the lattice. Employing the formalism of Ref. [1] to calculate the Casimir energy for free lattice fermions, we show that the results for the naive, Wilson and overlap fermions match the continuum expressions precisely in the zero lattice spacing limit, as expected from universality. In contrast to Ref. [1] where the result for the naive fermions rapidly oscillates with the lattice size for both, the periodic (P) and antiperiodic (AP) boundary conditions, no oscillations are observed with the lattice size. Furthermore, the apparent violation of the universality for naive fermion in Ref. [1] is shown to be cured by applying suitable series extrapolation techniques, thus demonstrating that the Casimir energy for the naive fermions with periodic/antiperiodic boundary conditions agrees with the results for other free lattice fermions, and can be used to obtain the results for the Dirac fermion in the zero limit of the lattice spacing.

Keywords: Casimir effect, lattice fermions, MIT Bag model
1. Introduction

The Casimir effect arises as a consequence of restricting the allowed zero-point fluctuations in the vacuum to discrete values due to the experimental set-up. This vacuum distortion manifests as an attractive force due to a finite lowering of the vacuum energy. These forces have been measured between two parallel plates, and the phenomenon was confirmed to a high degree of precision in several experiments recently. One might conclude quantum phenomena like these to be esoteric, with a limited practical consequence. But, as characteristic distances get smaller, their effects have become increasingly significant in nanotechnology, for example, the silicon integrated circuits based on micro-and nano-electromechanical systems. The Casimir effect also established the fact that energy density in certain regions of space is negative relative to the ordinary vacuum energy. This has exciting consequences, as such effects contribute to the stability of hadrons and model colour-confinement, might make it possible to stabilize a traversable wormhole, and provide significant insights into the cosmological constant problem. The Casimir effect has also been generalized to the scalar and fermion fields.

The strong coupling between quark and gluon fields gives rise to various non-perturbative phenomena in Quantum Chromodynamics. Given the role Casimir effect plays in the MIT Bag model, it would be interesting to investigate its role in the confinement of quarks in hadrons and spontaneous symmetry breaking using the formalism defined in Ref. Clearly using a spacetime lattice for such investigations appears to be a natural choice. An important motivation for studying the Casimir effect for various lattice fermions is that their form also appears in condensed matter systems, like Dirac semi-metals, topological insulators and ultracold atom systems, which are currently active areas of research. Chern Insulators are also shown to exhibit a repulsive Casimir effect. The negative mass Wilson fermions and overlap fermions correspond to the bulk and surface fermions of the topological insulator, respectively. Studying the Casimir effect for these lattice fermions is equivalent to studying it for corresponding topological insulators, which can then be experimentally observed for very small lattice sizes.

The Casimir effect for free lattice fermions was first studied in Ref. using periodic and antiperiodic boundary conditions. Periodic and antiperiodic boundary conditions are often used as a standard theoretical
setup in lattice simulations or condensed matter physics. While the Casimir energy for the Wilson and overlap fermions exactly matched the continuum expressions expected for the fermionic Casimir effect in the zero limit of the lattice spacing, it was observed that the result for the naive fermion rapidly oscillated and approached two different expressions in the continuum limit for the odd and even lattice sizes respectively. This lead Ref. [1] to claim that the naive fermion cannot be used to compute Casimir energy for the Dirac fermion, which is an apparent violation of the universality.

Motivated by the physically appealing fact that the MIT Bag boundary conditions [7] prevent the fermionic current from crossing the plates, thereby ensuring the confinement of fermions within the bag, we adopt these boundary conditions in this work to study the fermionic Casimir energy. If the fermion field $\psi$ is subjected to the MIT Bag boundary conditions, and confined between the two parallel plates placed at $x^1 = 0$ and $x^1 = d$, $\psi$ satisfies the following equation

$$(1 + i\gamma^1)\psi|_{x^1=0,d} = 0.$$  \hspace{2cm} (1)

By substituting the standard positive and negative frequency solutions of the Dirac equation in the equation for the MIT Bag boundary conditions one obtains the following transcendental relations for the corresponding momentum $k_1$ [16, 17]:

$$md \sin(k_1d)/k_1d + \cos(k_1d) = 0.$$  \hspace{2cm} (2)

For the massless fermion case, this reduces to

$$k_1d = \left(n + \frac{1}{2}\right) \pi \quad n \in \mathbb{Z}.$$  \hspace{2cm} (3)

These boundary conditions will be used to calculate Casimir energy for the lattice fermions in section (2).

The results we obtained from the MIT Bag boundary conditions were encouraging and precisely matched the continuum results for all lattice fermions, including the naive fermions, in the zero limit of the lattice spacing. As expected, doubling was observed for the results for the naive fermion. In order to investigate whether the choice of boundary conditions is crucial for our results above, we focused our attention on the periodic and antiperiodic boundary conditions for naive fermions again. Instead of examining the even and the odd lattice size series separately we treat them together as one series and applied known series extrapolation techniques. Contrary to the claim of Ref. [1] of an apparent violation of the universality for the naive fermion,
we succeeded in demonstrating numerically that the Casimir effect for Dirac fermions can also be computed from the naive fermion in the zero limit of the lattice spacing since looked upon this way, the series converges to the same result as given by other lattice fermions in the continuum limit. We mention the known continuum results for both massless and massive fermions for the MIT Bag boundary conditions for reference and comparison.

1.1. Massless fermions in (3+1)-dimensional spacetime

In continuum theory, one computes the zero-point energy using dimensional regularization by summing over the odd integer modes obtained in (3). We obtain the characteristic Casimir energy for massless fermions in (1+1)- and (3+1)-dimensional spacetime as \[ E_{\text{Cas}}^{3+1,\text{cont}},f(d) = \frac{7\pi^2}{2880d^3} \] and \[ E_{\text{Cas}}^{1+1,\text{cont}},f(d) = \frac{\pi}{24d} \] (\( \hbar = c = \kappa_B = 1 \)).

1.2. Massive fermions in (3+1)-dimensional spacetime

One obtains approximate solutions for the Casimir energy of massive fermions \( (mf) \) in continuum theory for two limiting cases, namely \( md \ll 1 \) and \( md \gg 1 \). The Casimir force per unit area in the limit \( md \gg 1 \), which we shall verify for the lattice fermions is \[ E_{\text{Cas}}^{3+1,\text{cont}},mf(d) = -\frac{3}{32}\sqrt{\frac{m}{\pi^3d^5}}e^{-2md} \]

where the Casimir energy decays exponentially with \( md \).

2. Casimir effect for lattice fermion fields in slab-bag

In this letter, out of the \( D \) latticized spatial dimensions, we consider only one \( (x^1) \) compactified by a boundary condition at \( x^1 = 0, d \). The corresponding spatial momentum component \( p_1 \) is discretized, while the other momenta components remain continuous. Initially, modelling time is kept continuous and temporal components of momentum are unaffected by latticization. The non-zero lattice spacing is ‘\( a \)’, with an ultraviolet cut-off scale ‘\( 1/a \)’ in momentum space. The energy - momentum dispersion relation for lattice fermions is obtained from the Dirac operator defined in a relativistic fermion action. The energy is defined as:

\[ aE(ap) = a\sqrt{\mathcal{D}^\dagger\mathcal{D}} \]
The Dirac operator, $\mathcal{D}$ includes the spatial momenta and mass but not the temporal momenta. The boundary condition obtained in (3) constrains the allowed modes and ensures that there is no particle current through the walls, thereby confining the fermion fields inside the slab-bag. The MIT Bag (B) boundary conditions are realized on the lattice as follows:

$$ap_1 \rightarrow ap_1^B(n) = \left( n + \frac{1}{2} \right) \frac{\pi a}{d} = \left( n + \frac{1}{2} \right) \frac{\pi}{N}$$  \hspace{1cm} (7)

Here, $N = d/a$ is the lattice size in the compactified spatial direction. The discretized momentum component under periodic (P) and antiperiodic (AP) boundary conditions, on the other hand, is:

$$ap_1 \rightarrow ap_1^P(n) = \frac{2n\pi}{N}; \hspace{0.5cm} ap_1 \rightarrow ap_1^{AP}(n) = \frac{(2n+1)\pi}{N}$$  \hspace{1cm} (8)

Restriction of the momentum to the Brillouin zone bounds the integer $n$ for periodic, antiperiodic boundary conditions is, $0 \leq n^{P,AP} < N$, and MIT Bag boundary conditions is $0 \leq n^B < 2N$. The Casimir energy on lattice is also calculated by subtracting the zero-point energy at finite lattice size $aE_0(N \rightarrow \infty)$ from the one at infinite lattice size $aE_0(N)$ [1]:

$$aE_{Cas}^{3+1} = aE_0(N) - aE_0(N \rightarrow \infty)$$

$$= c_{deg} \int_{BZ} \frac{d^2ap_\perp}{(2\pi)^2} \left[ -\sum_n aE(ap_\perp, ap_1(n)) + N \int_{BZ} \frac{dap_1}{2\pi}aE(ap) \right]$$  \hspace{1cm} (9)

The Casimir energy for the naive fermion is $2^D$ times the result for the Wilson fermion due to the doubling multiplicity for the naive fermion. Here, $c_{deg}$ is the degeneracy factor which accounts for the spin of fermion and the naive fermion doubling. All comparisons are made taking care of this factor, so that the equivalence between analytic results of naive and Wilson can be established in (15) and (18) below.

2.1. Naive Fermions

The dispersion relation for naive fermions in $(D+1)$-dimensions obtained from (6) is [21]:

$$a^2E_{nf}(ap) = \sum_{k=1}^{D} \sin^2(ap_k) + (am_f)^2$$  \hspace{1cm} (10)
As in Ref. [1], the (1 + 1)-dimensions is treated analytically. By substituting (10) in the definition (9), the expression for Casimir energy of naive fermion with MIT Bag boundary conditions in (D + 1)-dimensional spacetime is:

\[ aE^{D+1,B,nf}_{Cas} \equiv aE^{D+1,B,nf}_0(N) - aE^{D+1,B,nf}_0(N \to \infty) \]  

\[ = c_{deg} \int_{BZ} \frac{d^{D-1}p_\perp}{(2\pi)^2} \left[ -\sum_n \sqrt{\sin^2 (n + 1/2)\pi/N} + \sum_{k=2}^D \sin^2 (ap_k) + (am_f)^2 + \frac{\sin^2 (ap_k) + (am_f)^2}{N} \right] \]  

In the case for \( D = 1 \), the series expansions are substituted, and the exact expression is calculated using the Abel-Plana formulae for finite range [22, 23], to be:

\[ aE^{1+1,B,nf}_{Cas} = \frac{N}{\pi} - \frac{1}{2} \csc \left( \frac{\pi}{2N} \right) \]  

\[ = -4 \left\{ -\frac{d}{4a\pi} + \frac{1}{8} \left\{ \frac{2}{a\pi} + \frac{1}{6} \frac{a\pi}{2d} + \frac{7}{360} \left( \frac{a\pi}{2d} \right)^3 + \ldots \right\} \right\} \]  

\[ \Rightarrow E^{1+1,B,nf}_{Cas} = -\frac{\pi}{24d} - \frac{7\pi^3 a^2}{5760d^3} + O(a^4) \]  

Therefore, the Casimir energy obtained per unit area in the continuum limit \( a \to 0 \), is:

\[ \lim_{a \to 0} E^{1+1,B,nf}_{Cas} = -\frac{\pi}{24d} \]  

As expected, taking the naive fermion doubling correction in one spatial dimension into account, the above result in continuum limit is equivalent to the expression obtained for Dirac fermion [4]. These expressions for Casimir energy of naive fermions, without the doubling factor are plotted in Fig. 1. Studies for the naive fermion for \( D > 1 \) are done numerically. Results for (2+1)- and (3+1)- dimensions have been verified to be exactly equal to the continuum expressions. Only results for (3+1)-dimensions have been shown in Fig. 1.

2.2. Wilson Fermion

We shall now discuss the Casimir energy for Wilson lattice fermion in (1 + 1)-dimensional spacetime. The Wilson term proportional to \( r \) in the
Figure 1: Casimir energy for massless and positively massive Wilson and naive fermion with MIT Bag boundary conditions. [(a),(b)] and [(c),(d)] represent the MIT Bag Casimir energy for naive and Wilson fermion in (1+1)- and (3+1)-dimension respectively. Subfigure (d) also confirms the exponential decay of Casimir energy for massive fermions with energy for naive and Wilson fermion in (1+1)- and (3+1)-dimension respectively. Subfigure (d) also confirms the exponential decay of Casimir energy for massive fermions with

Dirac operator breaks the chiral symmetry and acts as a momentum dependent mass term introduced to eliminate fermion doubling. The expression for Casimir energy of Wilson fermion in \((D + 1)\) is similar to the one for naive fermion, where the Wilson Dispersion relation, obtained using (6), is substituted in (9). Using the Abel-Plana formulae for finite range like in the naive fermion case, setting \(r = 1\), the following expressions are obtained:

\[
aE_{\text{Cas}}^{1+1,\text{B},\text{W}} = \frac{4N}{\pi} - \csc\left(\frac{\pi}{4N}\right) = -4\left[-\frac{d}{a\pi} + \frac{1}{4}\left\{\frac{4d}{a\pi} + \frac{1}{6}\frac{a\pi}{4d} + \frac{7}{360}\left(\frac{a\pi}{4d}\right)^3 + \ldots\right\}\right]
\]

\[
\Rightarrow \quad E_{\text{Cas}}^{1+1,\text{B},\text{W}} = -\frac{\pi}{24d} - \frac{7\pi^3a^2}{23040d^3} + \mathcal{O}(a^4)
\]

Therefore, In the limit \(a \to 0\), the Casimir energy obtained per unit area is:

\[
\lim_{a \to 0} E_{\text{Cas}}^{1+1,\text{B},\text{W}} = -\frac{\pi}{24d}
\]
The exact expressions obtained for the massless naive and Wilson fermion using MIT Bag boundary conditions in (1 + 1)-dimension are \( \text{(13)} \) and \( \text{(16)} \) respectively. The leading terms of the order \( 1/N \) in the zero lattice spacing limit of these analytic results and the numerical results for naive lattice fermion in (3+1)-dimensions are plotted in Fig. 1. Note that the exponential suppression of Casimir energy for massive fermionic fields, which is derived for the fermion mass limit \( m_d \gg 1 \) in \( \text{(5)} \), is also verified in numerical calculations on a lattice as shown in Fig. 1(d) for comparison.

In \((D+1)\)-dimensional spacetime, the smaller windows in each plot represent the coefficient of Casimir energy, \( N^D \cdot a \cdot E_{\text{Cas}} \), which is constant in continuum and thus is used to observe the precision of agreement between the lattice and continuum results. The result obtained for the Wilson fermion matches with the continuum result \( \text{(4)} \) exactly.

### 2.3. Overlap Fermion

The analytic calculation of the Casimir effect for overlap fermions in (1+1)-dimensions turned out to be intractable. Thus, the results for the overlap fermion are numerically calculated for all dimensions, using the MIT Bag boundary conditions by substituting the corresponding Dirac operator into the definition \( \text{(9)} \). The Dirac operator for overlap fermion is defined as:

\[
aD_{\text{OV}} \equiv 2M_0 \times \frac{(1 + am_f) + (1 - am_f)V}{2}
\]

where \( am_f \) is the fermion mass and \( V \) is defined as:

\[
V \equiv \gamma_5 \text{sign}(\gamma_5 aD_W) = \frac{D_W}{\sqrt{D_W^\dagger D_W}}
\]
with $D_W$ as the previously defined Wilson Dirac operator with $r = 1$ and a negative mass parameter $-M_0$. We used $M_0 = 0.5, 1.0$ and $1.5$. The results obtained for (1 + 1)- and (3 + 1)-dimensions are plotted in Fig. 2. It is easy to notice in (1 + 1)- dimensions that the overlap fermion $M_0 = 1.0$ case and massless ($am_f = 0$) Wilson fermion case have equivalent dispersion relations. The expressions obtained numerically for overlap fermions in higher dimensions, also match precisely with the naive and Wilson fermions in the zero lattice spacing limit $a \rightarrow 0$ and, subsequently, also with the continuum result.

3. Naive fermion and Series Extrapolation in large $N$-Limit

While it is reassuring that one obtains the same results for the naive, Wilson and overlap fermions for the physically appealing MIT bag boundary conditions, as shown in the section above, it still is a bit disturbing that such validation of universality is not seen in the results of Ref. [1] which employed periodic and antiperiodic boundary conditions. We therefore now turn to examine the naive fermion case with those boundary conditions in an attempt to shed more light on this problem. The exact expressions for Casimir energy $aE_{\text{Cas}}$ of massless naive fermion on lattice in (1 + 1)-dimensional spacetime with periodic and antiperiodic boundary conditions, were first obtained in Ref [1, 14]. Their results are:

$$
\lim_{a \rightarrow 0} E_{\text{Cas}}^{1+1,P,nf} = \frac{\pi}{6d} \quad (\text{odd } N) ; \quad \lim_{a \rightarrow 0} E_{\text{Cas}}^{1+1,P,nf} = \frac{2\pi}{3d} \quad (\text{even } N) \quad (21)
$$

for periodic boundary. Similarly, for antiperiodic boundary conditions, the result is:

$$
\lim_{a \rightarrow 0} E_{\text{Cas}}^{1+1,AP,nf} = \frac{\pi}{6d} \quad (\text{odd } N) ; \quad \lim_{a \rightarrow 0} E_{\text{Cas}}^{1+1,AP,nf} = -\frac{\pi}{6d} \quad (\text{even } N) \quad (22)
$$

The continuum Casimir energy for massless Dirac fermion using periodic and antiperiodic conditions in (8) is found to be:

$$
E_{\text{Cas}}^{1+1,\text{cont},P} = \frac{\pi}{3d} ; \quad E_{\text{Cas}}^{1+1,\text{cont},AP} = -\frac{\pi}{6d} \quad (23)
$$

Thus, the Casimir energy depends on the type of boundary conditions. Comparing (1) and (23) one observes a peculiar apparent violation of universality as first shown in Ref [1]. The results depend on the whether the lattice size
is odd or even, and their respective continuum limits do not agree with (23).

Since $d = N \times a$, where $N$ now can be even or odd, $aE \propto 1/N$ with differing constants for the continuum result as well as odd/even lattice results, as seen in (21, 23). These curves are shown with appropriate labels in the panels of Fig. 3 for both periodic and antiperiodic boundary conditions.

In the limit of large lattice size $N$, one can compute the difference between the Casimir energy for odd and even lattice sizes from Ref. [1], as

$$aE_{\text{Cas}}^{1+1,nf} \text{(odd } N) - aE_{\text{Cas}}^{1+1,nf} \text{(even } N) = \mp 4 \tan \left( \frac{\pi}{4N} \right)$$

for periodic and antiperiodic boundary conditions respectively, which is an oscillating yet rapidly converging function. The oscillations as $N$ increases from odd to even are depicted by the black dotted function in Fig. 3 in both the panels. The continuum expressions are represented by the continuous green line in Fig. 3 and are seen to average out the oscillations well. This observation provided us a hint to look for suitable methods to understand the behaviour of Casimir energy for naive fermion as part of a single series. In (9), the Casimir energy is expressed as a difference between the integral and summation of the same function in one spatial dimension on the lattice. It is the sum-part which contributes to the rapidly oscillating behaviour above. The Euler-Maclaurin formula is a series acceleration technique often applied

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3The routine attribute of numerical summation in Wolfram Mathematica, "NSum" uses the Euler-Maclaurin formulae using "Method \rightarrow EulerMaclaurin" to extrapolate the series after a specified number of terms in the summation using the "NSumTerms" attribute.
in such cases, where the last term of the series tends to 0 as \( n \to \infty \), and expresses the Euler-Maclaurin sum of a function as an infinite series in terms of integral of the same function and higher-order derivative differences.

Employing the Euler-Maclaurin method as implemented in Wolfram Mathematica, we obtained the results as exhibited in Fig. 4 (a),(b). The oscillatory Casimir energy data points for \( N < 15 \) are obtained from definition \([9]\) for periodic and antiperiodic boundary conditions. The extrapolated results are represented by hollow markers in Fig. 4 for \( N > 15 \), and signal their precise agreement with the continuum results in \([23]\). The Euler-Maclaurin method of numerical sum extrapolates the data for points greater than the prescribed number to terms to be considered using the NSumTerms attribute of Wolfram Mathematica. In Fig. 4 this attribute is set to 15 for the agreement between the extrapolated points and continuum results to be clearly visible. Changing the attribute to higher values, say 20, does not alter the results qualitatively. We find that the Casimir energy for naive fermion converge to a single function which is exactly equal to the continuum result. Moreover,
these expressions also match the Wilson and overlap fermion results for both the periodic and antiperiodic boundary conditions, as expected from universality. In contrast to Ref. [1], we therefore argue that the continuum limit of the Casimir energy expression (11) must be treated as a combined oscillatory expression of the odd and even lattice sizes ($N$).

A similar extrapolation works for the $(2 + 1)$-dimensions as shown in Fig 4(c),(d). However, some lattice points are seen to be off the mark as the numerical computation involves calculating the integral of a sum approximated by an expression of higher-order derivatives of the function $|\sin(2\pi n/N)|$, over a continuous variable. This function is not differentiable at the lattice points and thus leads to this discrepancy. Nonetheless, it is sufficiently clear that the oscillating series indeed converges to the same continuum expression as $N$ increases.

In [14], it was shown that the negative mass Wilson fermions behave differently according to phases of the fermion mass and represent the bulk fermions, whereas the overlap fermions represent the surface fermions in a topological insulator. They also exhibit oscillation of Casimir energy on odd and even lattice sizes in certain phases. In all such phases, it is observed that these oscillatory results approach different expressions in the continuum limit. We have verified that applying the same principle of treating both odd and even $N$ as part of the same series, and applying the Euler-Maclaurin series extrapolation method for the Casimir energy of the negative mass Wilson fermions also leads to the correct result in the continuum limit. Therefore, it is shown that the apparent universality violations in all known cases so far are cured when the rapid oscillations between odd and even $N$ are treated as a single oscillatory function.

4. Conclusion

In this paper, we first studied the continuum limit of the Casimir effect for common lattice Dirac fermions in the light of the MIT Bag model. In section 2, the MIT Bag boundary conditions were realized on a lattice for the first time and used to calculate the Casimir effect for lattice fermions as per the formalism developed in Ref. [1]. Following the studies done in Ref. [1, 14], analytic expressions for the Casimir energy of naive (with doubling correction) and Wilson fermions are obtained, along with the numerical results for overlap fermions with MIT Bag boundary conditions. These expressions matched the continuum results for Dirac fermions exactly as expected from
universality. Results in higher dimensions are also similar to the \((1 + 1)\)-dimensions. Moreover, unlike the Casimir energy oscillations in the naive fermion case, seen for both periodic and antiperiodic boundary conditions, MIT Bag boundary conditions do not exhibit any such oscillations, neither do the Dirac fermions in the continuum, nor the other lattice fermions like Wilson and overlap fermions. Interestingly, negative mass Wilson fermions also exhibit such oscillatory behaviour \([14]\) for a certain range of mass parameter \(am_f\), suggesting it to be possibly related to specific structure of the dispersion relations.

In Ref. \([1, 14]\), the analytic expressions of Casimir energy for odd and even lattice sizes for naive fermion were obtained and interpreted separately. This led to different results for odd and even lattice sizes, which in turn did not agree with the continuum results, suggesting an apparent universality violation. We noticed that the difference between the Casimir energy for odd and even lattice sizes given in \([24]\), converges rapidly. This led us to propose that the results obtained from two analytic expressions formed a single oscillatory series. In the continuum limit of this single oscillatory series, we obtained the same result as for other lattice fermions agreeing with the continuum results, thus restoring universality. Thus, Casimir energy for naive fermions can also be used to obtain results in the continuum limit, in contrast to the claim in \([1, 14]\). Employing the Euler-Maclaurin series extrapolation method, better suited for strongly oscillating series like the one obtained, we showed the convergence of the oscillating analytic expression between odd and even lattice sizes to the same expression as continuum. This makes even the naive fermions with periodic/antiperiodic boundary conditions suitable for such calculations.

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