I. EMERGING SERVERLESS PROGRAMMING STYLE

Contemporary mainframes are huge data centers comprising tens or hundreds of thousands of interconnected physical servers and disk arrays. Usually, they are built according to the cloud architecture where almost everything is virtualized, starting with IaaS, PaaS and SaaS. The next virtualizations are the cloud architecture where almost everything is virtualized, servers and disk arrays. Usually, they are built according to tens or hundreds of interconnected physical servers and disk arrays. The next virtualizations are FaaS (Function as a Service) and BaaS (Backend as a Service) that constitute a new computing paradigm called Serverless. It is based on an execution model in which an application consists of interrelated individual functions and backend services that can be managed, scaled and executed by cloud provider by automatic and dynamic allocations of compute resources.

Using FaaS and BaaS, developers can effectively implement micro-services that comprise the applications. Cloud providers are responsible for deploying the code, and for an application runtime environment. The book by Brendan Burns 2018 [2] may serve as a classic (now a bit obsolete) introduction to this subject.

There are many serverless platforms provided by main players like Amazon Web Services, Microsoft Azure, Google Cloud Platform, IBM Cloud, Alibaba, as well as many others. They are trying to persuade developers do create applications mainly out of already existing and proprietary (more or less universal) functions and backend services by composing and orchestrating them into workflows. This gives rise to a new high-level programming paradigm, see, for example, Schleier-Smith et al. 2021 [3] where the following view was presented: “This next phase of cloud computing will change the way programmers work as dramatically as the first phase changed how operators work. [...] new general-purpose serverless abstractions will emerge...”

The physical servers are based on the von Neumann architecture. The programming languages (for developing functions and backend services) are still of von Neumann style. However, the overall high-level architecture of cloud systems, and the emerging (serverless) programming style (see for example Castro et al. 2019 [4]) are not von-Neumann. This resembles the idea of function-level programming postulated by John Backus 1978 [5].

The classic functional programming languages (based on lambda calculus, lazy evaluation and term rewriting) like Clojure, Erlang, Scala, Haskell and many others, are of limited use here. The essence of serverless computing is dynamic orchestration of functions and backend services into workflows where the functions are black boxes with clearly defined input and output interfaces, and functionalities.

We are going to show that abstract calculus of functionals (higher-order functions) and relations (proposed in [6], and then extended in [7]) may help to understand and formalize the emerging serverless programming paradigm. Functionals and relations correspond to generic mechanisms to compose and reconfigure serverless functions into complex and sophisticated workflows. They are abstractions that allow to cope with such complexity.

The necessity of such abstraction is the main idea of Open Application Model (https://oam.dev/) for defining cloud native apps. It is focused on application rather than container (Docker) or orchestrator (K8s), and brings modular, extensible, and portable design for modeling application deployment with higher level yet consistent API.

There are interesting speculations about the near future of serverless programming like The Serverless SuperComputer proposed by AWS Lambda creator Tim Wagner in 2019, and The Serverless End Game by Garcia et al. 2021 [8] predicting virtualization and transparency of computing resources eventually enabling unlimited flexible scaling.
II. MICROSERVICES

As a simple example of an application based on microservices, let us sketch briefly a shopping application composed of the following microservices:

- A frontend application interface.
- A search service that looks up products in a database based on user-generated search queries.
- A product-detail service that displays additional information about products that users click on.
- A shopping cart service to keep track of items that users place in their cart.
- A checkout service that handles the payment process.
- A service for maintaining inventory with a database.
- A service for shipping orders.

These microservices and corresponding databases interact via communication protocols. This is just an example; applications can be decomposed into microservices in a variety of ways.

The general principle of cloud-native approach is that microservice performs one service only, runs independently of the other parts of application, operates in its own environment, and stores its own data (in a separate back-end service). Microservices do not have to be small in size as the prefix micro may suggest. The one service it performs may be quite large and complex.

Microservices constitute an architectural pattern in which complex and sophisticated applications are composed from a collection of fine-grained, independent microservices implemented and deployed independently of each other. They communicate over a network using lightweight protocols. They can be scaled if needed, e.g. by replication.

Once a collection of such microservices is composed and orchestrated into a dynamic workflow, then it can be deployed on a cloud infrastructure, and be called a Cloud-Native Application (CNApp for short). Let us explore the consequences of the above architectural pattern.

The first conclusion is that a CNApp is a network application. That is, it consists of several parts (here, they are microservices) that communicate to each other using dedicated specific protocols implemented on a networking protocol stack; so far it is still the Internet TCP/UDP/IP.

A general definition of protocol in distributed systems is as follows.

A protocol defines the format and the order of messages exchanged between two or more communicating entities. The order, types, and the contents of messages depend on the current state of sender. Sending (reception) of a message may cause a change of the state of sender (recipient).

Each of the specific protocols of CNApp is based on the client-server model of communication. It means that the server component is running on a host with a fixed network address, and is listening on a fixed port waiting for clients. A client component initializes a communication session to the server; so that the client must know the address and the port number. Usually, the server can communicate with many clients at the same time in separate threads. TCP sockets and UDP sockets are the Internet APIs for such communications.

The notions of server and client (in the protocols) are here different from the notions of physical server (a hardware) and client as a host (computer) requesting a service from that server.

Note that we abstract from HTTP and REST-based protocols that (although commonly used for inter microservice communications) usually hard-code the format of messages of the real protocols. HTTP and the REST-based are here mere transport protocols for the factual protocols. For serverless computing they are also inefficient because of additional and large transport overhead. See also [9] The RESTless cloud by Pemberton et al. 2021 [https://sigops.org/s/conferences/hotos/2021/papers/hotos21-s03-pemberton.pdf] for an interesting discussion on serverless communication APIs.

A single microservice may implement and participate in multiple different protocols acting as a client and/or as a server.

Internal functionality of a microservice is event-driven and composed of serverless functions. Roughly, the incoming messages (as events) trigger serverless functions (dynamically composed into dataflows) to run, and then to produce outgoing messages. This will be explained in details in the next section.

Scaling by replication and reduction (closing replicas of microservice instances) of a microservice enforces its stateless. The reason is as follows. If a microservice is statefull, then closing a replica will result in the loss of its state if the state was not stored elsewhere before the closing.

Actually, the stateless is not a serious restriction, because a stateful microservice (with permanent stored data) may be decomposed into stateless part (where the basic functionality is performed), and back-end services where the permanent data are stored. It is a common practice in CNApp development. Here, the backend services should be (logically) the same for all replicas of the microservice in question. The communication with the backend services is done by specific dedicated protocols. The stateless microservice and its replicas implement clients whereas the backend storing services implement servers.

Hence, a replicate-able microservice can be roughly specified as collection of servers and/or clients of the protocols it participates in, and of its own internal functionality. Permanent data (state) of microservice are stored in dedicated backend storing services (BaaS). The internal functionality is composed of stateless elementary functions (FaaS, like the AWS Lambdas), and orchestrated into a dynamic dataflow.

Usually, communication protocols (at application layer) are defined in an abstract way independently of their implementation.

For our propose, protocol is defined as two tightly coupled software applications (implementing an abstract protocol): server $S$ and client $P$. Formally, let it be denoted by pair $(P,S)$ with appropriate superscripts and/or subscripts if needed.

The abstract input of a microservice can be defined by the
servers (of the protocols) it implements:

\[ IN := (S_1, S_2, \ldots S_k) \]

The abstract output of a microservice is defined by the clients (of the protocols) it implements:

\[ OUT := (P'_1, P'_2, \ldots P'_n) \]

Usually, at least one of the clients is for the communication with dedicated backend storing service.

To omit confusions, the server part and a client part of a protocol will be renamed. Components of abstract input will be called abstract sockets, whereas components of abstract output will be called abstract plugs.

An abstract plug may be connected to an abstract socket if they are of the same type, i.e., they are two complementary sides of the same communication protocol. There may be multiple abstract plugs to the same abstract socket. Let the connection (abstract plug \(\rightarrow\) abstract socket) be directed meaning that client has initialized a communication session (of protocol) with server.

Note that these abstract input and abstract output do not correspond necessarily to data flow. That is, data (messages) may be sent (according to a protocol) out from abstract input (an abstract socket) to an abstract output (an abstract plug).

Let us formalize the concept described above. 

**Microservice** is defined as

\[ \mathcal{M} := (IN, F, OUT) \]

where \( IN \) is the abstract input of the microservice, \( OUT \) is the abstract output, and \( F \) is the internal functionality of the microservice to be formally defined in Section III.
A. Proxy and manager of a microservice

Replication of a microservice is postulated as one of the main factors of the cloud-native approach.

CNApp is supposed to be formed by a collection of interconnected (via communication protocols) microservices. This is related to the concept of Service Mesh, i.e. a dedicated infrastructure layer for facilitating service-to-service communications. Examples of popular service meshes are: Linkerd, CNCF project, and Istio [https://istio.io/latest/docs/concepts/what-is-istio/] that is becoming a widely accepted open standard now. There are also interesting academic frameworks, for example, Kosińska and Zieliński 2020 [10] for autonomic management of CNApps.

A service mesh consists of network proxies (called also sidecars), each of them is paired with a microservice. There is also a collection of managements rules that coordinate the behavior of proxies and provide APIs to monitor and control the mesh.

Sidecar of microservice is the key concept of Service Mesh. It is attached (and loosely coupled) to a microservice and extends its outside functionalities.

All the incoming and outgoing network traffic to/from an individual microservice flows through its sidecar. So that, the sidecar manages the traffic flow, can gather telemetry data, and can enforce policies of cloud provider. Sidecar can also perform several functionalities like load balancing (by microservice replication), security control, content cache, monitoring and management. For more on microservice sidecar, see [https://dzone.com/articles/sidecar-design-pattern-in-your-microservices-ecosy-1].

Let us apply the concept of sidecar to any individual microservice, say \( M \), in CNApp, and call it manager & proxy (or sidecar) of \( M \), and denote it as \( M_{mp} \).

Sidecar \( M_{mp} \) can replicate \( M \) (or close some replicas of \( M \)) if needed, and manages (distributes) the input communication to \( M \) and all its replicas. Hence, it acts as a proxy, and as a load balancer. This is illustrated by Fig. 3 and Fig. 4.

Sidecar of a microservice can also act as a proxy for the abstract output of the microservice and all its replicas.

Let us present a simple example of (an abstract) CNApp. Its microservices along with their \( mkp \)-s can be composed into a CNApp in the way depicted in Fig. 5.

The following layers may be distinguished for a typical CNApp, see also Fig. 5.

- Frontend, i.e. GUIs for users of CNApp.
- API Gateways on the top.
- Middle layers for microservices performing the main functionality of CNApp.
- Backend on the bottom for data storing microservices.

The next Figures (i.e. Fig. 6, Fig. 7, and Fig. 8) illustrate consecutive abstractions leading to the concept we are going to introduce below.
That is, $P$ belongs to $\text{OUT}$ of $\mathcal{M}_1$, and $S$ belongs to $\text{IN}$ of $\mathcal{M}_2$.

Note that the abstract graph of CNApp is supposed to comprise all possible microservices and all edges (protocols) that can be used every time a CNApp is executed.

Multi graph means that there may be multiple edges (protocols) between two vertices.

We can assume that abstract graphs are acyclic. If there is a cycle, then a new protocol (breaking off the cycle) can replace a protocol in the cycle. In the new protocol, client and server are swapped, whereas the messages remain the same, and the essential functionality is preserved. This may require a bit more changes in the code, however, it is possible. For a loop in the graph, a client-server communication within a single application (microservice) can be replaced by OS interprocess communication, or by shared memory.

Initial vertices of the graph correspond to API Gateways, whereas the terminal vertices correspond to backend data storing microservices.

While running CNApp, some of edges of its abstract graph may be inactive for some time-intervals, i.e. the corresponding protocols sessions are closed or not invoked yet, or can not be invoked for some reasons. This means dynamic shrinking or expanding of the active part of abstract graph at the runtime. This can be done by the sidecars of microservices according to the polices determined by cloud provider.

Since sidecar $m&p$ acts as a proxy for a microservice, it can monitor protocol session invocations as well as session closings. This can be implemented (in simple cases) by checking ACK and FIN flags in TCP segments if protocol sessions are implemented on permanent TCP connections.

In general case, sidecar should have access to event monitor implemented inside its microservice and replicas to know their current communication sessions.

The abstract graph of a CNApp is defined as static one. Temporal reductions of the graph due to inactive vertices and/or edges should not be viewed as the only transformations of the graph. There could be also dynamic transformations of the graph at the runtime due to contents (data) of the incoming messages from the frontend (from external users) or/and from the backend where the current state of CNApp can be stored.

Amazon EventBridge may be seen as a universal platform for transforming functionality of running CNApp. It connects microservices using events. Roughly, an event is a signal that a system’s state has changed. The events may also correspond to incoming and outgoing messages of implicit communication protocols hardcoded in microservices. The platform is at software level, and does not scale if the number of microservices and connections is getting large.

In 2019, Netflix had over 700 microservices, whereas Uber had over 500 microservices. Hence, the number of microservices approaches thousands in CNApps created by big tech companies.

If the number of vertices and the number of edges of the abstract graph and its transformations are relatively small (say dozens), it can be done manually by implementing dedicated management mechanisms. In general case, when complexity increases dramatically, the management requires generic mechanisms, see [why-you-need-a-microservice-catalog/](https://www.opslevel.com/2020/04/21/why-you-need-a-microservice-catalog/). Since the abstract graphs are abstract compositions of abstract functions, these mechanisms (as higher level abstractions) should be related to abstract functionals, i.e. operations on abstract functions.

It seems that a calculus of functionals may serve as a high level abstraction for implementing generic management rules for Service Mesh like Istio and others. More pro arguments are in Section IV.

### III. SERVERLESS COMPUTING PARADIGM

**Serverless** means that application code (serverless function) is executed on-demand in response to triggers (based on events) that application developers provide in advance. Usually, serverless functions implement discrete units of application functionality. For an up to date introduction to the subject, see [microservices-vs-serverless-architecture/](https://www.sumologic.com/blog/microservices-vs-serverless-architecture/)

The serverless computing model includes also Backend as a Service (BaaS) component. It means that the entire backend
(database, storage, etc.) of a system is handled independently and offered as a service.

A. Combining serverless and microservices

Serverless and microservices are different sorts of technologies. Microservices are a way to design an application, while serverless is a way to run an application (or a part of an application).

Recently, there is an emerging trend to combine serverless and microservices approaches. Cloud providers (like Google, Amazon, Microsoft) found ways to bridge the gap between them. That is, a microservice can be developed as a collection of interrelated and orchestrated event-driven serverless functions (as a workflow) and stored on the third-party vendor’s infrastructure. It can be done, for example, with Logic Apps (Microsoft), Step Functions (Amazon), or AirFlow workflow engine (Google). They allow assigning triggers to microservices, and combine several functions into a service. All these workflow engines (as well as others) are at software level, and provide tools to orchestrate (compose) serverless functions into a microservice or a complete application. It seems to be a right solution if the number of these functions is relatively small, say, less than one hundred. If, additionally, the workflow requires its essential reconfiguration at the runtime, then a higher level of abstraction (above the software) is necessary to cope with complexity.

Both serverless functions and microservices (as a Service Mesh) require similar approaches to monitoring and management.

These approaches, like Amazon EventBridge, seem to be over-complicated. Let us present a different approach that can be scaled.

The crucial question to be answered is: What are events in a microservice, and what are they for?

Generally, events serve for dynamic (depending on these events) configurations of dataflows composed of serverless functions (including functions implementing protocols) that comprise functionality of microservice.

B. An abstract view of serverless architecture of microservice

Formal foundations of serverless computing is a hot research subject in academic community, see for example Jangda et al. 2019 [11], Obetz et al. 2020 [12], and Gerasimov 2019 [13]. They are based on classic (based on von Neumann programming style) formal techniques like operational semantics.

The key concept for the proposed abstract architecture of microservice is relation. An event may be defined as an evaluation of a relation to be either true or false.

Once proper relations are defined, the constructor if-then-else can be used to compose functions into dataflows.

Incoming messages to a microservice can be considered as coarse events to be processed in order to evaluate relations that (as conditions) trigger functions.

We need a formal framework to define functions and relations. Let us start with types, and let $T$ denote a type with superscripts and/or subscripts if needed. We might think of them as data types in programming languages; at this level of abstraction it is not essential. Let $a : T$ denote that object $a$ is of type $T$.

Let

$$f : (T_1; T_2; \ldots; T_k) \rightarrow (T'_1; T'_2; \ldots; T'_n)$$

denote function $f$ where $T_i$ (for $i = 1, 2, \ldots, k$) is the type of its $i$-th input, and $T'_j$ (for $j = 1, 2, \ldots, n$) is the type of its $j$-th output.

Note that $(T_1; T_2; \ldots; T_k) \rightarrow (T'_1; T'_2; \ldots; T'_n)$ denotes the type of function $f$.

Function may have multiple inputs, as well as multiple outputs.

Function application is denoted

$$f(e_1, e_2, \ldots, e_k)$$

where $e_i : T_i$ for $i = 1, 2, \ldots, k$. Result of this application is denoted $(d_1, d_2, \ldots, d_n)$ where $d_j : T'_j$ for $j = 1, 2, \ldots, n$; so that $f(e_1, e_2, \ldots, e_k) = (d_1, d_2, \ldots, d_n)$.

Relation, denoted $\phi$ with subscripts and/or superscripts if needed, may be viewed as a boolean-valued function. i.e.

$$\phi : (T_1; T_2; \ldots; T_k) \rightarrow \text{boolean}$$

and evaluated as a function. Type $\text{boolean}$ consists of two logical values: true and false. In type theory, a special sort Propositions is needed for evaluation of relations. For simplicity, the type $\text{boolean}$ may serve as a substitute here.

Events are defined as evaluations of these relations.

Functions that process either incoming messages, or produce outgoing messages are defined as follows.

- Function $f_{\text{in}}^S$, that processes messages incoming to abstract socket $S$ of protocol $(S, P)$. Let the type of such messages be denoted by $T_{\text{in}}^S$. Then, such a function is of the form

$$f_{\text{in}}^S : T_{\text{in}}^S \rightarrow (T_1; T_2; \ldots; T_n)$$

- Analogously, for function $f_{\text{in}}^P$ that processes messages incoming to abstract plug $P$ of protocol $(S, P)$. Let the type of such messages be denoted by $T_{\text{in}}^P$. Then,

$$f_{\text{in}}^P : T_{\text{in}}^P \rightarrow (T_1; T_2; \ldots; T_m)$$

- Function $g_{\text{out}}^S$ that generates messages outgoing from abstract socket $S$ of protocol $(S, P)$. Let the type of such messages be denoted by $T_{\text{out}}^S$. Then,

$$g_{\text{out}}^S : (T_1; T_2; \ldots; T_i) \rightarrow T_{\text{out}}^S$$

- Analogously, function $g_{\text{out}}^P$ that produces messages outgoing from abstract plug $P$ of protocol $(S, P)$. Let the type of such messages be denoted by $T_{\text{out}}^P$. Then,

$$g_{\text{out}}^P : (T_1; T_2; \ldots; T_k) \rightarrow T_{\text{out}}^P$$

Note that one of these protocols may realize a one way communication with external backend service (a clock) to deliver (in consecutive time slots) the current date and time to the microservice.
Since it is supposed that the state of a microservice is stored in associated backend services, the protocols (for communications with these backend services) may deliver events informing the microservice on the state change.

Putting all these pieces together, functionality $F$ of microservice may be defined on the basis of:

- functions, that process incoming messages, of the form $f_{in}$ with appropriate superscripts corresponding to protocols;
- repository of relations ($\phi_1, \phi_2, \ldots, \phi_m$);
- repository of functions ($h_1, h_2, \ldots, h_l$);
- functions, that generate outgoing messages, of the form $g_{out}$ with appropriate superscripts corresponding to protocols.

Functionality $F$ of microservice $M$ may be presented as dataflow graph where initial vertices are functions of the form $f_{in}$ that process incoming messages. Terminal vertices are functions of the form $g_{out}$ that produce outgoing messages.

The interior vertices of the graph are either functions from repository ($h_1, h_2, \ldots, h_l$) or constructors of the form if-then-else to be defined below.

Directed edges of the graph are just compositions, i.e. links (for data flows) directed from an output type of one function to an input type of another function. It means (a partial) composition of these two functions. The type of input and the type of output must be the same.

In Fig. 9 an idea of the constructor if-then-else is presented. Relation $\phi : (T_1; T_2 \ldots; T_n) \rightarrow boolean$ determines condition (event) if the values (data) $x_i : T_i$ for $i = 1, 2, \ldots n$ are delivered to the input of $\phi$. If the relation is evaluated as true, then the link from input of type $T$ to output of type $T'$ is established. Otherwise, the link to output of type $T'$ (the same type as $T$) is established.

A simple example of event-dependent dataflow graph is presented in Fig. 10. It is important to emphasize that this dataflow is, in fact, a function of type $(D; E) \rightarrow (B||C)$.

The dataflow graph for a microservice may be complex if the numbers of its functions and its relations are large. Then, construction, updating functions in the graph, and dynamic reconfiguration may require abstractions related to functionals as it is illustrated in Fig. 11 where the concept of type (as a board) is presented along with plugging in the function (as input) into the board.

In Fig. 11 functional $G$ (the shaded rectangle) consists of: INPUT (as the collection of six boards on the top), OUTPUT as the board on the bottom, and directed links between them. Once the functions (on the top): $f4_{in}, f3_{in}, h1, if[\phi]then \rightarrow else, g4_{out}, g3_{out}$, are plugged (in the very similar way as in Fig. 11) into the INPUT boards of $G$ (it is shown by dashed directed links), then the result (at the OUTPUT board of $G$) is exactly the function $f : (D; E) \rightarrow (B||C)$ from Fig. 10. It means that $G$, with the functions plugged in, is the same as the dataflow in Fig. 10. That is, $f = G(f4_{in}, f3_{in}, h1, if[\phi]then \rightarrow else, g4_{out}, g3_{out})$. To see so, follow the directed links from socket $D$ and socket $E$ of IN of OUTPUT (via the six INPUT boards on the top, and the plugged functions from the top) to either the plug $B$ or plug $C$ of OUT of OUTPUT. Hence, the functional $G$ represents the complete connectivity of the dataflow graph of Fig. 10 abstracted from the concrete functions placed in the vertices of the graph. These functions can be replaced by different functions if only their types are appropriate. Then, the resulting dataflow graph is syntactically correct, and can be executed if only data (incoming messages) are delivered to its inputs.

Hence, $G$ with the functions plugged in (as in Fig. 12) is the same as the dataflow in Fig. 10. It means that $G$ expresses the essence of the dataflow graph connectivity, i.e. any vertex (concrete function) of the graph may be changed by any function of the same type as the type of the vertex.
Static dataflow graph alone might not represent the entire functionality of a microservice if a recursion in involved. As an example, see Fig. 13 illustrating the idea of construction of higher order primitive recursion schema where dataflow graph is expanded dynamically depending on input data. On the right of Fig. 13 functional \( F \) (actually, a complex dataflow graph) is iterated, i.e. composed with itself \( n \)-times. The number of iterations is determined by input parameter \( n \) in the primitive recursion schema (picture in the middle). Also \( F \) is an input parameter. These parameters expand the dataflow graph of the schema. The graphs are composed from primitive functionals such that: \( \text{proj} \) for projection, \( \text{Copy} \) for duplication of the input parameter, \( \text{compose} \) for composing two functions given as input parameters, \( \text{Suc} \) for the successor function on natural number, \( \text{Pred} \) as the predecessor function, and \( \text{Iter} \) for function iteration. More details are in the next section.

There is a similar approach CIEL (a bit old) Murray et al. 2011 [14] where there are also similar data-dependent control flow constructors, and iterations as dynamic unfolding of acyclic dataflow graphs. In our approach we introduce much more, i.e. functionals as generic mechanisms for transformations of the graphs at the runtime.

Dynamic creation and/or closing abstract plugs of \( \text{OUT} \) of microservice transforms its dataflow graph at the runtime by adding or removing appropriate functions. New connections (initialized by outside clients, i.e. by abstract plugs of another microservices) to abstract sockets of the microservice expand the dataflow with appropriate functions. Closing connections of some of the outside clients results in removing corresponding functions.

Hence, in general case, functionality of microservice consists in dynamic transformations of the dataflow graphs.

The graph transformations may be caused also by the input from its backend services where the state of microservice can be stored. The input may dynamically change the basic functionality of the microservice at the runtime. Dynamic transformations of dataflow graphs can be viewed as functionals. Calculus of functionals and relations may be seen as a general framework for constructing dynamic dataflow graphs.

C. A bit more on functionals

At the basic level there are some primitive types, primitive functions and type constructors, i.e. the type of natural numbers, the successor function and the predecessor function, constant functions, projections, constructors for product and function types. The key primitive functionals correspond to application, composition, copy (replication), and iteration. It is crucial that these functionals can be constructed by (dynamic, in the case of iteration) establishing links between plugs (here corresponding to output types) and sockets (here corresponding to input types).

The type of functions from natural numbers into natural numbers (denoted by \( N^* \rightarrow N^p \)) may be realized as a simple board consisting of a socket and a plug, see Fig. 14 where also types of higher order are presented. Note that for the...
type \((A \to B) \to C\), the input \(A \to B\) becomes the socket. For the type \((A \to B) \to (C \to D)\), the output \(C \to D\) becomes the plug.

- \(N^p \to N^p\)
- \((A \to B) \to C\)
- \((A \to B) \to (C \to D)\)
- \(((A \to B); (B \to C)) \to (A \to C)\)

Fig. 13. An example of dataflow graph corresponding to the higher order primitive recursion schema constructed as a complex functional; the complete presentation is in \([7]\) and \([6]\).

For the type \((A \to B) \to (C \to D)\), the output \(C \to D\) becomes the plug.

Fig. 14. Function type is a board of sockets and plugs. Higher order types are nested boards of sockets and plugs.

Fig. 15. Higher order application of functional \(F\) of type \((A \to B) \to C\) to a function \(g: A \to B\). The result \(F(g)\) is an object of type \(C\).

Application of a functional \(F: (A \to B) \to C\) to a function \(g: A \to B\) is realized as follows. \(A \to B\) is the socket of the functional \(F\). The application is done (see Fig. \[15\]) by establishing appropriate directed connections (links). That is, the link between the socket \(A\) of the socket of \(F\) and the socket \(A\) of \(g\), and the link between the plug \(B\) of \(g\) and the plug of the socket of \(F\).

Composition functional (denoted by \(\text{compose}_{A,B,C}\)) for simple composition of two functions (the first function \(f\) of type \(A \to B\), and the second one \(g\) of type \(B \to C\)) is realized as two boards with appropriate links shown in Fig. \[16\]. It is easy to check (by following the links) that applying \(\text{compose}_{A,B,C}\) to two functions (see Fig. \[16\]) results in their composition.

Note that a higher order application (i.e. application of a functional to a function), and a functional for composition are constructed just by providing some links between sockets and plugs.

A functional of special interest is \(\text{Copy}\) as replication. So that \(\text{Copy}(a)\) returns two objects: the original \(a\), and its copy (replica) \(a'\). Although the meaning of \(\text{Copy}\) seems to be simple, its realization is not obvious in general case. In the clouds, there are no problems with replication of functions (as...
software) and data as bytes in memory.

**Iteration** is \( n \)-times composition of a function \( f : A \rightarrow A \) with itself. Note that \( n \) as a natural number is a parameter. The iteration is denoted by \( \text{Iter}_A \) and it is a functional of type \( (N : (A \rightarrow A)) \rightarrow (A \rightarrow A) \). So that \( \text{Iter}_A(n; f) \) is the function being \( n \)-time composition of \( f \). The realization of \( \text{Iter}_A \) requires \text{Copy} for making copies of \( f \), and \( (n - 1) \) copies of the composition functional \text{compose}, see Fig. 17, where the construction is done for \( n = 4 \).

**IV. SUMMARY AND CONCLUSIONS**

Let us present some related work on workflows and FaaS.

HyperFlow, Balis 2016 [15], provides an interesting (although explicitly not based on FaaS) model of computation for workflows. The abstraction there is still at the software level. HyperFlow was applied to serverless execution of scientific workflows Malawski et al. 2020 [16], and Pawlik et al. 2019 [17].

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**Triggerflow**, see Lopez et al. 2020 [18], is an interesting framework for composing event-based services. Although a formal model of workflow is presented as a Finite State Machine, its architecture and realization are done on the software level.

Bobbi et al. 2021 [19] presented an up to date overview of the existing literature on FaaS orchestrations, and their executions environments.

Ristov at al. 2021 [20] proposed an abstract function choreography language (AFCL) for serverless workflow specification. The authors claim that AFCL is at a high-level of abstraction. Its basic constructs correspond to the classic programming control-flow and data-flow expressions, and are: if-then-else, switch, sequence, for, while, parallel, and parallelFor. They are not enough for all possible dynamic (at the runtime) reconfigurations of data-flows, and to cope with complexity if the number of functions is large.

All Cloud providers are now offering cloud orchestration and function composition services, for example, IBM Composer, Amazon Step Functions, Azure Durable Functions, and Google Cloud Composer. All of them are essentially at software level, and do not provide higher-order abstractions to cope with complexity and dynamic reconstructions. The proposed abstract architecture of **serverless Cloud-Native Apps** consists of two layers of abstraction.

**Top level** concerns the so called service mesh. The abstract view of CNApp (consisting of interacting microservices) is presented in Section II-B as abstract multi graph of CNApp for orchestration of its microservices.

**Bottom level** is about the functionality of microservice. Dynamic event-driven dataflow graph (from the previous Section) is proposed as an abstract view of such functionality.

If the architecture is of some interest as a theoretical approach, then the following challenges emerge.

- Repository of standard generic functions
- Repository of standard communication protocols, and corresponding functions implementing the protocols.
- Repository of standard generic relations.
- Repository of standardized microservices build on the basis of the above repositories.
- Platforms and tools for developing these repositories.
- Tools and generic mechanisms for constructing the graphs and transforming them.
- Managing complexity of internal structure of microservice if the number of its functions is large.
- Managing complexity of CNApp if the number of its microservices in large.

Calculus of functionals and relations provides abstractions for reducing complexity of:

- Construction of the dataflow graphs of microservices, and the abstract graphs of CNApps.
- Dynamic reconfiguration and transformations of the graphs at the runtime.

It seems that for the both levels of abstraction, a calculus of functionals and relations (like the one proposed in [7] and [6]) may serve as a theoretical foundations for abstract architecture of CNApps.

**REFERENCES**

[1] “The Coq Proof Assistant,” 2022. [Online]. Available: [https://coq.inria.fr](https://coq.inria.fr)
[2] B. Burns, Designing Distributed Systems: Patterns and Paradigms for Scalable, Reliable Services, 1st ed. O’Reilly Media, Inc., 2018.
[3] J. Schleier-Smith, V. Sreekanti, A. Khandelwal, J. Carreira, N. J. Yadwadkar, R. A. Popa, J. E. Gonzalez, I. Stoica, and D. A. Patterson, “What serverless computing is and should become: The next phase of cloud computing,” Communications of the ACM, vol. 64, no. 5, pp. 76–84, 2021.
[4] P. Castro, V. Ishakian, V. Muthusamy, and A. Slominski, “The rise of serverless computing,” Communications of the ACM, vol. 62, no. 12, pp. 44–54, 2019.
[5] J. Backus, “Can programming be liberated from the Von Neumann Style?: A functional style and its algebra of programs,” Commun. ACM, vol. 21, no. 8, pp. 613–641, Aug. 1978.
[6] S. Ambroszkiewicz, “Primitive recursion on higher types,” in 2015 Computer Science and Information Technologies (CSIT). Proceedings of IEEE, 2015, pp. 23–32. [Online]. Available: https://doi.org/10.1109/csittechnol.2015.7358244

[7] ——, “Functionals and hardware,” 2015. [Online]. Available: https://arxiv.org/abs/1501.03043

[8] P. Garcia Lopez, A. Slominski, M. Behrendt, and B. Metzler, “Serverless predictions: 2021-2030,” arXiv e-prints, pp. arXiv–2104, 2021.

[9] N. Pemberton, J. Schleier-Smith, and J. E. Gonzalez, “The restless cloud,” in Proceedings of the Workshop on Hot Topics in Operating Systems, 2021, pp. 49–57.

[10] J. Kosińska and K. Zieliński, “Autonomic management framework for cloud-native applications,” Journal of Grid Computing, vol. 18, no. 4, pp. 779–796, 2020.

[11] A. Jangda, D. Firckney, Y. Brun, and A. Guha, “Formal foundations of serverless computing,” Proceedings of the ACM on Programming Languages, vol. 3, no. OOPSLA, pp. 1–26, 2019.

[12] M. Obetz, A. Das, T. Castiglia, S. Patterson, and A. Milanova, “Formalizing event-driven behavior of serverless applications,” in European Conference on Service-Oriented and Cloud Computing. Springer, 2020, pp. 19–29.

[14] D. G. Murray, M. Schwarzkopf, C. Snowton, S. Smith, A. Madhavapeddy, and S. Hand, “CIEL: a universal execution engine for distributed data-flow computing,” in Proc. 8th ACM/USENIX Symposium on Networked Systems Design and Implementation, 2011, pp. 113–126.

[15] B. Balis, “Hyperflow: A model of computation, programming approach and enactment engine for complex distributed workflows,” Future Generation Computer Systems, vol. 55, pp. 147–162, 2016.

[16] M. Malawski, A. Gajek, A. Zima, B. Balis, and K. Figiela, “Serverless execution of scientific workflows: Experiments with hyperflow, aws lambda and google cloud functions,” Future Generation Computer Systems, vol. 110, pp. 502–514, 2020. [Online]. Available: https://www.sciencedirect.com/science/article/pii/S0167739X1730047X

[17] M. Pawlik, P. Banach, and M. Malawski, “Adaptation of workflow application scheduling algorithm to serverless infrastructure,” in European Conference on Parallel Processing. Springer, 2019, pp. 345–356.

[18] P. G. López, A. Arjona, J. Sampa, A. Slominski, and L. Villard, “Triggerflow: trigger-based orchestration of serverless workflows,” in Proceedings of the 14th ACM International Conference on Distributed and Event-based Systems, 2020, pp. 3–14.

[19] A. Bocci, S. Forti, G.-L. Ferrari, and A. Brogi, “Secure FaaS orchestration in the fog: how far are we?” Computing, pp. 1–32, 2021.

[20] S. Ristov, S. Pedratscher, and T. Fahringer, “AFCL: An abstract function choreography language for serverless workflow specification,” Future Generation Computer Systems, vol. 114, pp. 368–382, 2021.