Vibration response of a piezoelectrically actuated microcantilever subjected to tip-sample interaction

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Abstract Piezoelectric microcantilevers (MCs) are types of MCs which can be used in Atomic Force Microscopy (AFM) as a micro-robot, sensor, and imaging actuator. In this paper, the vibrating motion of piezoelectric MCs in AFM application is analyzed. With respect to the geometrical discontinuities, due to the piezoelectric layer, as well as tip, a non-uniform beam model is chosen for analysis. At first, to determine the accuracy of the non-uniform beam model in simulating the vibrating motion of piezoelectric MC, the simulation results are compared with the experimental ones in the absence of the tip-sample force. Good agreement of these results indicates the ability of this model in modeling this type of MC. A numerical solution and a Multiple Time Scale (MTS) method are used to study the nonlinear response of the MC near the sample surface. Comparison of results, at the non-contact mode, shows good agreement between the two solving methods at normal equilibrium distances (d ≥ 2 nm). The effects of the angle of MC, the probe length, and the geometric dimensions of the piezoelectric layer on the nonlinearity of the system are studied and it then becomes clear that they can affect the frequency response curvature of the curve and the nonlinearity of the system.

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1. Introduction

Nowadays, the Atomic Force Microscope (AFM) has become a useful tool for direct measurements of intermolecular forces with atomic precision. This microscope can be used in various fields such as electronics, semiconductors, manufacturing, polymeric materials, bio-analysis, biomaterials, and in the study of metal surfaces [1]. AFM is a powerful tool for nano-level evaluation, biomaterials diagnosis, nano description of materials and equipments, and assembly at nano-scale [2–4]. AFM is composed of a MC with a probe of a very fine tip, by which the information from the sample and tip interaction can be obtained. Recently, a new generation of MCs has been developed for AFM imaging [5–7]. These types of MCs are equipped with a piezoelectric layer that can be used for actuation, sensing, and actuation-sensing, simultaneously. The advantages of piezoelectric MCs compared with bulky piezotube actuators, has made them an appropriate choice for high-speed imaging of AFM [6]. Other advantages of this type of MC are compacting AFM, when the MC is used as a sensor, and the possibility of using multiple probes in parallel functions [8]. The topographical image of the surface can be measured in AFM by quantifying and recording the cantilever deformation. This measurement is typically conducted by a high precision laser interferometer. Although this method of measurement enjoys high accuracy, it is massive and expensive. Therefore, compact and inexpensive methods are preferred. There are different methods of measurement which can be introduced as substitutes for the laser interferometer in AFM, including capacitive [9], conductive [10], piezoelectric [5–7] and piezoresistive [11–13]. The capacitive and conductive methods can be used only for conductive materials, and the piezoelectric sensor is an appropriate method for amplitude mode. In this method, the piezoelectric layers are used as actuator and sensor. Due to the reverse effects of piezoelectric materials caused by the AC current, the cantilever will be vibrated and, along with the deformation emanating from the direct effect of piezoelectric materials, a charge will be...
Itoh and Suga [5] presented a piezoelectric MC for determination of force gradients as self-sensing and self-actuating. Adams et al. [7] presented a simple electrical circuit to make possible the commercial use of piezoelectric MCs as a self-sensing mode in AFM tapping mode. Lee et al. [14,15] introduced a microcantilever with a PZT piezoelectric layer in order to be used in the AFM in dynamic mode, and employed it as three building blocks of AFM, namely, microcantilever, actuator and deformation sensor. They then succeeded in capturing high-contrast images from the selected sample in the amplitude mode. Mahmoodi et al. [16,17] theoretically investigated the flexural vibration of the piezoelectric MC in the absence of the sample force for non-AFM applications using the theory of nonlinear uniform beam and compared the flexural responses with experimental results. Fung and Huang [18] simulated the piezoelectric microcantilever using the finite element method. Typically, a piezoelectric MC is constituted from a sandwiched piezoelectric layer between two electrodes on its surface, whose layer does not necessarily cover the entire MC surface. In the common configuration of this kind of MC [5,19], the main body of the MC is designed wider due to the presence of the piezoelectric layer, while the tip region is manufactured narrower in order to improve the measurement of tip deformation. Therefore, a piezoelectric MC is constituted from three segments in its cross section: the first step is the wide region of MC, which includes the piezoelectric and the electrodes, the second step is the wide region of the MC without the piezoelectric and the third step is the tip region. These discontinuities change the modal characteristics of the beam compared with a uniform beam and have a dramatic impact on it. Hence, in order to raise computational precision, the non-uniform beam method must be used in the analysis. The accuracy of power estimation based on AFM measured information depends on the selected dynamic model for the microcantilever. With its effect on the controlling system, the dynamic system model directly affects image resolution. Most of the mathematical models which have been used for piezoelectric MCs to date [12,20] are lumped-mass spring models, while it has been proved that [21] nonlinear lumped mass models, which approximate continuous dynamic systems with nonlinear boundary conditions, may encounter substantial errors. MC vibrating analysis has been studied at the specific attachment of the piezoelectric (throughout the layer) with a simple uniform cantilever [18,22], while considering the simple continuous beam model in non-contact mode using the Lennard-Jones model.

In this paper, the vibration response of the piezoelectric AFM MC is analyzed in two self-sensing and self-actuating modes. With respect to the discontinuities of the MC, due to the presence of the piezoelectric layer and tip, modeling of the vibrating motion is performed. The nonlinear response of MC in the self-sensing and self-actuating modes in the presence of surface interaction force is studied. The effects of different parameters, such as surface roughness, inclined angle, probe length and geometric characteristics, on the system nonlinearity are investigated. The non-uniform beam model is selected for vibration analysis. At first, the results of this model will be compared with the results of the uniform beam model in self-actuating mode, so that the difference between these two models would be determined during the simulation of the piezoelectric MC vibrating motion. To do so, MC is modeled with the help of the continuous beam theory of Euler–Bernoulli, by considering the existing discontinuities in the MC to increase the accuracy. The inclined MC has a piezoelectric layer confined between two electrodes. It is vibrated through actuating voltage. The Lennard-Jones potential model is selected to
describe the interaction between tip and sample [22]. The governing equation of motion is changed into the nonlinear and ordinary differential equation using Galerkin approximation and is solved using the MTS method and numerical solution. The output current of the piezoelectric as a parameter for qualifying the cantilever deformation will be introduced, and the behavior of this parameter interacting with the sample will be investigated.

2. Dynamic modeling

In order to model the MC, a discontinuous beam, shown in Figure 1a, with a piezoelectric layer on its top surface is considered. There are two layers on top and beneath the piezoelectric layer that act as an electrode. The beam is inclined towards the sample surface, as shown in Figure 1b. It is clamped at one end; the other end is free and subjected to interaction force between tip and surface.

In addition, it is assumed that the motion of MC is governed by the Euler–Bernoulli theory, therefore shear deformation and rotary inertia terms are negligible. The Hamilton’s approach is adopted and used in order to drive the equation of motion. In Appendix, we give the details of the solution to drive this equation. With Eq. (A.8), the equation of motion in non-dimensional form is expressed as:

\[ \ddot{u} + P_1 \left[ K (x) u'' \right]' + P_2 \dot{u} + P_3 C_i P_d (t) = F_{EM} \delta (x - 1) + \int F_{EM} \delta (x - 1) \, dx. \]  

Eq. (1) has been resulted from the integration of MC mechanical and electrical equations of the piezoelectric layer, in which the \( C_i \) coefficient has entered the voltage effect of the stimulation applied to the piezoelectric layer in the MC equation of motion as an actuator.

Regarding the layout of the piezoelectric layer on MC, the main body is usually made wider and its tip section is made narrower to improve the deformation measurement. On the other hand, the piezoelectric layer is not necessarily extended to the end. Such discontinuities can affect the frequency response through the mode shapes. Therefore, in order to achieve a more accurate dynamic model for actuator piezoelectric MC, discontinuities of the MC must be considered in modeling. The length of MC is divided into four uniform beams consisting of a multilayer beam with a piezoelectric layer and three plain beams with different cross sectional areas in the tip.

The linear dynamic approximation is a method employed by many authors [22–25] in order to solve the nonlinear differential equation of AFM, and has been turned into a common method of solving this type of equation. This method has been also compared with the experimental results [23] and it has been demonstrated that it can be used in the analysis of AFM MC with appropriate accuracy. Hence, here, it is used to solve the differential equation:

\[ u (x, t) = \sum_{n=1}^{\infty} U_n (x) q_n (t), \]  

where \( q_n \) are the generalized time-dependent coordinates and \( U_n (x) \) are nth mode shapes. Since the total length of MC is divided into four uniform beams, mode shapes can be written as:

\[ U_n (x) = \begin{cases} 
A_n^{(1)} \sin \beta_n x + B_n^{(1)} \cos \beta_n x, & 0 < x < \frac{L_1}{L}, \\
C_n^{(1)} \sin \beta_n x + D_n^{(1)} \cos \beta_n x, & \frac{L_1}{L} < x < \frac{L_2}{L}, \\
A_n^{(2)} \sin \beta_n x + B_n^{(2)} \cos \beta_n x, & \frac{L_2}{L} < x < \frac{L_3}{L}, \\
C_n^{(2)} \sin \beta_n x + D_n^{(2)} \cos \beta_n x, & \frac{L_3}{L} < x < 1 
\end{cases} \]  

The continuity conditions include the deflection and slope of the deflection, the bending moment and the shear stress of MC at the stepped points. By substituting Eqs. (2) and (3) into Eq. (1), taking the inner product of the resulting equation with mode shapes, \( U_n (x) \), integrating over the length of MC, the time-dependent part of the equation of motion can be expressed as [16,17,23,24]:

\[ \ddot{q}_n + \alpha_n^2 \dot{q}_n + \mu_n q_n - g_s q_n^2 - g_s q_n + g_s P_d (t) = 0. \]
where:
\[ \omega_n^2 = \int_0^1 U_n \left[ P_1 (K(x) U_n')^2 - P_2 U_n \cos \delta (x - 1) \right. \]
\[ \left. - P_3 h \sin \alpha [U_n \delta (x - 1)]' \right] dx, \]  
Eq. (6a)
\[ g_1 = \int_0^1 U_n \left[ P_1 U_n^2 \cos \delta (x - 1) \right. \]
\[ \left. + P_2 h \sin \alpha [U_n \delta (x - 1)]' \right] dx, \]  
Eq. (6b)
\[ g_2 = \int_0^1 U_n \left[ P_3 U_n^2 \cos \delta (x - 1) \right. \]
\[ \left. + P_2 h \sin \alpha [U_n \delta (x - 1)]' \right] dx, \]  
Eq. (6c)
\[ g_3 = 2 \]  
Eq. (6d).

3. Primary resonance response

3.1. MTS method

The method of multiple time scales is selected to solve Eq. (5). According to MTS formulation, the steady state solution can be expanded as:
\[ q_n(t) = \varepsilon q_{n1}(T_0, T_1, T_2) + \varepsilon^2 q_{n2}(T_0, T_1, T_2) + \varepsilon^3 q_{n3}(T_0, T_1, T_2) + o(\varepsilon^4), \]  
Eq. (7)
where \( T_0 = e^{it} \) and \( \varepsilon \) is introduced as a small book keeping parameter to show infinitesimal quantity in the equation. The time derivative becomes:
\[ \frac{d}{dt} = D_0 + \varepsilon D_1 + \varepsilon^2 D_1 + o(\varepsilon^3), \]  
Eq. (8)
where \( D_n = \partial / \partial T_n \). To obtain the frequency response, excitation voltage and modal damping terms are scaled to become the same as the order of the perturbation problem. Therefore, we let:
\[ P_n = \varepsilon^p P_n, \quad \mu_n = \varepsilon^\mu \mu_n. \]  
Eq. (9)
Substituting Eqs. (7)–(9) into Eq. (5) and separating similar powers of \( \varepsilon \), yields:
\[ O(\varepsilon) : D_0^p q_{n1} + \mu_n^p q_{n1} = 0, \]  
Eq. (10)
\[ O(\varepsilon^2) : D_0^p q_{n2} + \mu_n^p q_{n2} + 2D_0^p D_1^p q_{n1} - g_1 q_{n1} = 0, \]  
Eq. (11)
\[ O(\varepsilon^3) : D_0^p q_{n3} + \mu_n^p q_{n3} + 2D_0^p D_1^p q_{n2} + 2D_0^p D_2^p q_{n1} + \mu_n D_0^p q_{n1} + D_1^p q_{n1} + g_1 P_0 + g_2 q_{n1} - 2g_3 q_{n2} + g_4 q_{n3} = 0. \]  
Eq. (12)
The solution of Eq. (10) can be assumed as:
\[ q_{n1} = A_n (T_1, T_2) e^{i\omega_n T_0} + cc, \]  
Eq. (13)
where \( A_n \) is a complex amplitude and \( cc \) represents the complex conjugate of the preceding terms. Substituting Eq. (13) into (11) yields:
\[ D_0^p q_{n2} + \mu_n^p q_{n2} + 2i\omega_n D_1 A_n e^{i\omega_n T_0} - g_1 (A_n^2 e^{i\omega_n T_0} + A_n A_n^*) + cc = 0. \]  
Eq. (14)
To get steady-state solutions, the secular terms of Eq. (14), which have coefficients of \( e^{i\omega_n T_0} \), must be eliminated; as a consequence, \( D_1 A_n = 0 \). \( A_n \) will be a function of \( T_2 \) only.

Considering this result, the solution of Eq. (14) can be obtained as:
\[ q_{n2} = -\frac{g_1 A_n^2}{3\omega_n^2} e^{2i\omega_n T_0} + 2A_n A_n^* + cc. \]  
Eq. (15)
The harmonic excitation voltage is taken to be \( P_n(T_0) = \frac{1}{2} P_0 e^{i\omega_n T_0} + cc \), where \( P_n \) and \( \Omega \) denote the magnitude and frequency of the input voltage. It is assumed that excitation frequency remains near the natural frequency of vibration by the following relation:
\[ \Omega = \omega_n + \varepsilon^2 \sigma, \]  
Eq. (16)
where the detuning parameter, \( \sigma \), indicates the deviation of the excitation frequency to the natural frequency. By substituting assumed excitation into Eq. (12) and eliminating the secular terms, the solution of Eq. (12) is given by:
\[ 2iD_2 A_n + i\mu_n A_n + \frac{g_1 P_0}{2\omega_n} e^{i\omega_n T_0} - 8A_n^2 \gamma \gamma = 0, \]  
Eq. (17)
in which:
\[ \gamma = \frac{10}{80} g_2^2 + 3g_3. \]  
Eq. (18)
\( \gamma \) is a coefficient that indicates nonlinearity in the system. When this coefficient is positive, the softening phenomenon appears in the frequency response, and, when \( \gamma \) is negative, large amplitude motions would occur at excitation frequencies greater than natural frequency. Since \( \gamma \) in Eq. (18) is positive, it can be concluded that nonlinear interaction force between tip and sample always causes the softening phenomenon in the frequency response.

It is better that \( A_n \) is written in the polar form:
\[ A_n = \frac{1}{2} a_n e^{i\beta_n}, \quad A_n^* = \frac{1}{2} a_n e^{-i\beta_n}. \]  
Eq. (19)
Substituting Eq. (19) into Eq. (16) yields the following modulation equation of the amplitude and frequency:
\[ a_n \dot{\tau}_n - a_n \sigma + \frac{1}{2\omega_n} P_0 \delta_1 \cos (\tau_n) - \gamma \dot{\beta}_n^3 = 0, \]  
Eq. (20)
\[ \dot{\beta}_n - \frac{1}{2\omega_n} P_0 \delta_1 \sin (\tau_n) + \frac{1}{2} \mu_n a_n = 0, \]  
Eq. (20)
in which \( \tau_n = \sigma T_n - \beta_n \). In order to investigate the steady-state amplitude response, the coefficients, \( \dot{\alpha}_n \) and \( \dot{\tau}_n \), must be set to zero. By eliminating \( \tau_n \), the following nonlinear frequency-response equation can be obtained:
\[ \left( \frac{1}{2} \mu_n a_n \right)^2 + (\dot{a}_n \sigma + \gamma \dot{\beta}_n^3)^2 = \left( \frac{1}{2\omega_n} P_0 \delta_1 \right)^2. \]  
Eq. (21)
The obtained frequency response equation yields two branches of the frequency response of MC near the natural frequency. The bending of the frequency response curve obviously results from this equation. Because of specific values of the detuning parameter, there may be more than one amplitude. When an AC voltage is directly applied to the piezoelectric layer to vibrate the cantilever, as a result of the direct effect of piezoelectric materials, charge will be generated in this layer. Therefore, this output charge can be used to measure the amount of cantilever deformation, when the piezoelectric cantilever is used as a self sensor.
3.2. Numerical solution

In the numerical solution, instead of the Taylor series expansion of $F_{n,}$ force equation, its main equation (Eq. (A.13)) is used. Therefore, Eq. (5) can be rewritten as follows:

$$\ddot{q}_n + \omega^2 q_n + \mu_n q_n + g_3 P_d (t) = f_n,$$

(22)

where:

$$\omega_n^2 = \int_0^1 U_n \left[ P_1 (K (x) U_n^2)\right] dx,$$

(23a)

$$f_n = \int_0^1 \left[ \frac{U_n}{\rho A \omega_n^2} \left( F_{n,y} - F_{n,y}^{\text{avg}} \right) \delta (x - L) + \left[ (F_{n,x} - F_{n,y}^{\text{avg}} h \delta (x - L) \right) \right] dx,$$

(23b)

To solve Eq. (22), the Runge–Kutta method is used by applying the ode45 command of Matlab.

4. Piezoelectric MC in self-sensing mode

The output charge appearing on the electrodes of the piezoelectric material is given by Itoh and Suga [14]:

$$Q = W_1 d_1 E_2 z_p \int_{l_1}^{l_2} u'' dx,$$

(24)

where $z_p$ is the distance from the plane of zero strain to the neutral plane of the piezoelectric layer. By substituting Eq. (2) into Eq. (24), and integrating over the length of MC, the output charge can be expressed as:

$$Q = W_1 d_1 E_2 z_p \sum_{n=1}^{\infty} \left( \psi_n (l_2) - \psi_n (l_1) \right) q_n (t),$$

(25)

and the piezoelectric charge current output ($I_p$) can be obtained:

$$I_p = W_1 d_1 E_2 z_p \sum_{n=1}^{\infty} \left( \psi_n (l_2) - \psi_n (l_1) \right) \dot{q}_n (t).$$

(26)

The bending of piezoelectric MCs in vibrating motion leads to the development of an electric charge output from the piezoelectric layer. Regarding Eq. (24), the output charge can be associated with the flexural deformation of the MC. In the sample surface topography, the amplitude of vibrating motion changes as a result of surface roughness and this, in turn, results in the change of the electric charge output from the piezoelectric layer. So, the change of the electric charge output can be used for measuring surface roughness by a simple electric circuit [7].

5. Simulation and discussion

5.1. Analytical simulation and validation

In order to study the obtained differential equation numerically, it is supposed that MC is made of silicon and a piezoelectric layer. The piezoelectric layer has been confined between two electrodes made of Ti/Au with the thickness of 0.25 μm. The required geometric information and mechanical properties are provided in Table 1. Coefficients of Lennard-Jones are selected as $\sigma = 0.34$ (nm) and $H = 10^{-18}$ (J), based on Ref. [22].

| $E$ (GPa) | $\rho$ (kg/m$^3$) | $h$ (μm) | $W$ (μm) | $L$ (μm) |
|-----------|-------------------|----------|----------|----------|
| Base layer | 185              | 2330     | 3.5      | 250      | 375      |
| Lower electrode | 78              | 19,300   | 0.25     | 130      | 330      |
| Piezoelectric layer | 104            | 6390     | 3.5      | 130      | 330      |
| Upper electrode | 78              | 19,300   | 0.25     | 130      | 330      |
| Tip       | 185              | 2330     | 3.5      | 55       | 125      |

The natural frequency of discontinuous MC can be calculated through the boundary conditions, continuity of deformation, slope, bending moment, shearing force, and taking the determinant of the coefficients equal to zero. This can be used for calculating the frequency response of the piezoelectric cantilever. To validate these calculations, a DMASP microcantilever was used [19]. Experimental results show that the first natural frequency of this MC is equal to 52.0 (kHz) [17]. The theoretical calculations which were made with regard to the discontinuous beam method for MC show that the first natural frequency is equal to 53.2 (kHz) with only 2.3% error in comparison with the experimental results. In order to compare the results with the continuous beam method, the method used in [26] is utilized to obtain the resonance frequency of the system. In this case, the frequency is calculated to be 31.535 (kHz), which indicates a significant error when compared with experimental results.

To determine the accuracy of a non-uniform beam model in modeling the vibrating motion of a discontinuous piezoelectric microcantilever, the results obtained from the simulation, in the absence of tip-sample force, are with the experimental results of [17]. Figure 2 depicts the experimental and simulated frequency responses of the microcantilever to the applied chirp signal. Since, in this article, only the behavior of MC near the resonance frequency will be studied, we have chosen only that part of the frequency response of Ref. [17], which is near the first resonance frequency (Figure 2b), and have used it for validation. Accordance between the obtained results and experimental results shows that the selected discontinuous model can appropriately model the vibrating motion of MC in discontinuity.

To compare the solution made in this paper with that of Wolf and Gottlieb [22], it is assumed that the MC has the specifications according to [22]. Figure 3 shows the frequency response of the MC in the presence of Lennard-Jones force at two equilibrium distances (1 and 2 nm). Comparison of the results shows a very good agreement between the two solving methods.

5.2. Comparison between the results from simulating uniform and non-uniform beam models

Figure 4 shows the comparison between the frequency responses of MC in uniform and non-uniform (discontinuous) beam models. As can be seen, not only are the results of these two models different in calculating the value of MC natural frequency, but they are also different in calculating the amplitude of vibrating motion. Figure 5 shows the frequency response of MC at the equilibrium distance of $d = 2$ nm, using the two uniform and non-uniform models. Comparing the results shows that system nonlinearity (curvature of frequency response curve) in the discontinuous beam model is more than in the uniform model.
5.3. Comparison between the results from MTS method and numerical solution

Taylor series expansion of the Lennard-Jones equation around the selected equilibrium distances was used in the analysis of the vibrating motion of MC, using the MTS method. The Taylor series expansion of a nonlinear function around a certain point has an appropriate accuracy within certain distances from the selected point. Therefore, the MTS method, within the range in which Taylor series expansion does not agree with the nonlinear function, does not have adequate accuracy. Using Figure 6, we can determine the interval at which Taylor series expansion conforms to the non-linear force function. In these figures, the Lennard-Jones curve (solid curve) is estimated using its Taylor series expansion (dashed curves) at equilibrium distances of 5, 2, and 1 nm. Such curves are used in the following analyses in selecting the magnitude of exciting voltage, and the amplitude of oscillatory motion.

Figure 7 shows the vibrating motion of MC at equilibrium distances (d) 5, 2, and 1 nm. Exciting voltage is selected for 25, 12 and 6 (µV), respectively, and exciting frequency is the MC’s natural frequency. The amplitude of the vibrating motion is obtained through both MTS and numerical solutions to solve the nonlinear equation of motion. As seen, at equilibrium distances 5, 2 nm, the obtained results through two methods have good agreement, and for the equilibrium distance of 1 nm, little differences arise between the two solutions. Therefore, with the decrease of equilibrium distance and, consequently, the increase of tip-sample force and the nonlinearity of this force, the difference between both the MTS method and the numerical solution will be greater.

As the vibrating motions studied in Figure 7 are of the non-contact type with the surface of the sample, the Van der Waals attractive force has greater effect on the vibrating motion of MC in this region. Such a force always attracts the MC tip toward the sample. Since the tip-sample force increases by approaching the
Figure 6: Approximation of tip-sample interaction force curve using third order Taylor series expansion at equilibrium distances: (a) $d = 5$ nm, (b) $d = 2$ nm, (c) $d = 1$ nm.

MC tip to the sample surface in the vibrating motion, the MC time response curve becomes asymmetric.

As Figure 7c shows, one can achieve the asymmetric time response of the MC at very short equilibrium distances to the surface of the sample only through the numerical solution of a nonlinear differential equation. Therefore, it can be concluded that the little difference between the time responses, obtained from both the MTS and the numerical solution in Figure 7c, is because the time response curve becomes asymmetric, which can be achieved only through the numerical solution.

5.4. Piezoelectric MC response to uneven surface

Figure 8 shows a single-line schematic representation of an uneven surface passing through where we want to study the vibrating piezoelectric MC response. For this purpose the set point is selected 20 nm and excitation voltages are selected 10, 12, 20 mV for the first three harmonic modes, respectively. Figure 9 illustrates the numerical time response of the piezoelectric MC to the uneven surface. As can be seen, along
with passing the probe on the uneven surface and the distance increase, the vibration amplitude will be also increased. Since the piezoelectric MC has high mass inertia, its response to the uneven surface of the first mode takes place slowly, but, at higher modes, at which the vibration is performed with higher frequencies, the response is faster. Therefore, it can be concluded that in the case of piezoelectric MC application in surface topography, to get the higher speed response, it is better to excite MC in the higher harmonic modes.

5.5. Frequency response of the piezoelectric MC

Since the installation of MC in an inclined state is practically possible and MC installation in a horizontal state is difficult, it is necessary to study the effect of an inclined angle on the frequency response of this kind of microcantilever. The investigations into the influence of an inclined angle on the vibration response of the MC in previous work are limited to Lin et al. studies [27,28], in which the influence of an inclined angle is only investigated on the frequency shift of common MCs (without piezoelectric layer). In this section, the influence of an inclined angle on the frequency response and softening phenomenon, with the presence of shear force and bending moment, due to force components between the sample and the tip, is investigated.

In order to study the effect of an inclined angle on the frequency response, the ZnO layer is considered, and actuation is performed at the set point of 2 nm with voltages of 13 µV. The results obtained in this condition show that the increase of inclined angle leads to a reduction in the nonlinear effects of the force (softening phenomenon) (Figure 10). Inclined MC will cause nonlinear moment to be formed by the force between the beam and sample surface. The increase of inclined angle, on the one hand, reduces the applied shear force to the end, and, on the other hand, it will increase the moment, along with a function in the direction opposite to that of the shear force. For this reason, with the increase of MC angle, the nonlinearity of the system, and, thus, the amount of softening in the frequency response, are reduced.

Not only do the equilibrium distance and the inclined angle affect the amount of created moment by contraction force, but also the probe height affects this moment. In fact, the larger the probe height is, the more the moment will be. To do so, a 60° inclined angle is chosen.

Figure 11 shows the probe height effect on the frequency response. As seen, increasing the probe height leads to a reduction in the softening phenomenon. This has a negligible effect on the maximum of amplitude. This shows that by increasing the probe height, the effect of interaction force on the frequency response is slightly decreased. In fact, decreasing the angle of cantilever and increasing the set point sample will fade the effects of this parameter. Therefore, it can be concluded that at low set point distances, to make the cantilever’s vibrating motion more sensitive to the nonlinearity of the interaction force, the height of the probe and the inclined angle should be selected as low as possible.
5.6. Reviewing the effects of specifications of piezoelectric layer on nonlinear vibration response

The nonlinear behavior of MC at the proximity of the sample surface is created due to nonlinear interaction force. This force brings its nonlinear behavior onto the MC response through the nonlinear coefficient, \( \gamma_f \) (Eq. (18)), hence the amount of softening in the frequency response curve is determined using this coefficient. Since the value of this coefficient has a dramatic effect on the nonlinear behavior of the MC, in this section, the investigation of the effect of the geometric dimensions and the material of the piezoelectric layer (MC actuator) on \( \gamma_f \) will be studied. Figure 12 shows the variations of \( \gamma_f \) versus the length of the Zno piezoelectric layer in different layer thicknesses. As seen, in the short length of the piezoelectric layer, the value of coefficient \( \gamma_f \) is increased with an increment in the layer length. Also, after closing to a maximum length, coefficient \( \gamma_f \) is decreased with the increment of the layer length. By increasing the layer thickness, this maximum value of \( \gamma_f \) occurs in the larger values of layer length. Figure 12 shows how the piezoelectric layer thickness affects \( \gamma_f \) in different values for the length of the layer. As can be observed, the influence of thickness on \( \gamma_f \) differs in different lengths of the layer. In some values of layer length, the increase of thickness brings about the increase of \( \gamma_f \), and in some others, the increase of the thickness is accompanied with the decrease of \( \gamma_f \).

Figure 13 illustrates the variations of \( \gamma_f \) versus the width of the Zno piezoelectric layer in different layer thicknesses. As shown in this figure, the amount of this coefficient decreases by increasing the width of the layer in different thicknesses. By comparing Figures 12 and 13, it can be concluded that the influence of piezoelectric layer length on \( \gamma_f \) is much more considerable compared with its width.

5.7. Piezoelectric MC response in self-sensing mode

When the MC piezoelectric is used in the self-sensing mode, the output charge of the piezoelectric layer is used as a criterion for measuring the cantilever deformation; hence, as the amplitude of the vibrating motion of the MC varies in crossing the surface roughness, the output charge also should be varied in relation to its roughness. Figure 14 shows how the output charge varies relative to surface roughness (Figure 8) in the first three harmonics. As indicated, at the start of the surface roughness, the charge value increases, and after the end of the surface roughness, the charge returns to its initial value. The comparison of the diagrams in the first three harmonics shows that the charge response to roughness is faster in higher modes and its value increases in the first three harmonics.

6. Conclusion

A flexural vibration motion equation of an inclined piezoelectrically actuated microcantilever, under the nonlinear force between the tip and the sample surface, was used to study...
different parameters on the frequency response was studied, and the following results were obtained:

1. A discontinuous beam model leads to more accurate results in simulating the piezoelectric MC vibration response.
2. The increase of MC inclined angle in proportion to the sample surface decreases the amount of frequency response curvature.
3. As the probe length is increased and the amount of force moment is intensified, the softening phenomenon is reduced and this reduction is little, with regard to the smallness of the probe length.
4. The results of the geometric dimension effect of a piezoelectric layer on the nonlinear coefficient showed that the amount of this coefficient decreases with increasing the width of the layer in different thicknesses. For small amounts of length of piezoelectric layer, the value of coefficient $\gamma_f$ increases with increasing the layer length. After reaching a maximum length, it decreases with increasing layer length. Along with increasing layer thickness, the maximum value of $\gamma_f$ occurs in the larger values of layer length.

Appendix

We derive the governing equation of motion of a piezoelectrically driven microcantilever, shown in Figure 1, by using Hamilton’s principle. The kinetic energy of MC can be written as:

$$T = \frac{1}{2} \int_0^L m \dot{v}^2 \, dx,$$

where:

$$m = \rho_1 h_1 W_1 (H_0 - H_1) + (\rho_2 h_2 W_2 + \rho_3 h_3 W_3 + \rho_4 h_4 W_4 + \rho_5 h_5 W_5) \left( H_1 - H_2 \right) + \rho_1 h_1 W_1 (H_3 - H_4).$$

and $H_i$ is the Heaviside function:

$$H_i = H(x - L_i).$$

The potential energy of MC can be obtained by stress–strain relations. The linear constitutive equations of the piezoelectric for the particular geometry are as follows [29]:

$$\sigma_{11}^p = E_3 S_{11}^p - e_{21} E_2,$n

$$D = \varepsilon_{22} E_2 + \varepsilon_{21} S_{11}^p,$$

in which $E_2$ and $D$ are electric field and electric displacement. The linear material constants of the piezoelectric are as follows:

$$E_3 = \frac{1}{s_{11}},$$

$$\varepsilon_{21} = \frac{d_{21}}{s_{11}},$$

The superscripts indicate constant stress ($T$), constant strain ($L$), and constant strain ($L$). The total potential energy of MC can now be formulated as follows:

$$U = \frac{1}{2} \int_0^L \int_A \sigma_{11}^p S_{11}^p \, dA \, dx + \frac{1}{2} \int_A \int_0^L (\sigma_{11}^p S_{11}^p + E_2 D_2) \, dA \, dx + \frac{1}{2} \int_0^L \int_A \sigma_{11}^p S_{11}^p \, dA \, dx.$$
in which superscripts indicate lower electrode \( (e_t) \), upper electrode \( (e_u) \) and upper layer \( (u) \). Using Eqs. (A.1)–(A.6), the Lagrangian of the MC can be written as:

\[
L = \frac{1}{2} \int_{0}^{L} (m \dot{v}^2 - K (x) v^2) dx - C_d \dot{v}^2 (t) - 2C_d P (t) v^\nu dx,
\]  
\[(A.7)\]

where:

\[
K (x) = E_i \frac{W_i h_i^3}{12} (H_0 - H_i) + E_i (H_i - H_{i-1}) + E_i \frac{W_i h_i^3}{12} (H_{i-1} - H_1),
\]
\[(A.8)\]

\[
C_d = -\frac{W_i}{h_i} \varepsilon_{22},
\]
\[(A.9)\]

\[
C_v (x) = \varepsilon_{22} W_3 \left( h_1 + h_2 + \frac{1}{2} h_3 - y_0 \right) (H_1 - H_i).
\]
\[(A.10)\]

\[
EI = \sum_{k=1}^{4} E_k W_k h_i \left\{ \frac{h_k^2}{12} + \left[ y_n - \left( \sum_{j=1}^{k} h_j - \frac{h_k}{2} \right) \right]^2 \right\}
+ \frac{1}{12} W_i h_i^3 \varepsilon_{22} \varepsilon_{12} + \sum_{k=1}^{4} E_k h_i W_i \left( \frac{1}{\sum_{j=1}^{k} h_j - \frac{h_k}{2}} \right).
\]
\[(A.11)\]

\[
y_0 = \sum_{i=1}^{4} E_i h_i W_i.
\]
\[(A.12)\]

Note that the numbering of the layers is done from the lowest to the highest. The interaction model we use for the tip-sample force assumes a Lennard-Jones type, as shown below:

\[
F = \frac{HR}{6a^2} \left[ \frac{1}{30} \left( \frac{\sigma}{\bar{\tau}} \right)^8 - \left( \frac{\sigma}{\bar{\tau}} \right)^2 \right].
\]
\[(A.13)\]

The virtual work, done by the concentrated tip-sample force \( F \), associated with its virtual displacement is:

\[
\delta W = F \delta v (L, t) - F \delta h \delta \varepsilon_{(L,t)}.
\]
\[(A.14)\]

Using Hamilton’s principle, the governing equation of motion can be obtained as:

\[
\rho A \ddot{v} + \left[ K (x) v^\nu \right] + C_v (x) P_t (t) = F(L, t) \delta (x - L) + [F(L, h \delta (x - L)]'.
\]
\[(A.15)\]

In addition, the static deflection \( \bar{v}(x) \) of MC is calculated by:

\[
[ K (x) \bar{v}^\nu (x) ] + [ C_v (x) P_t (t) ] = F_{L,L} \delta (x - L) + [ F_{L,L} h \delta (x - L) ]',
\]
\[(A.16)\]

where \( P_t \) is the voltage that controls the static equilibrium orientations of the tip \([22]\). The equilibrium distance between tip and sample is denoted by \( d = Y + \bar{v}(x) \cos \alpha \), and total deflection of MC is expressed as:

\[
v (x, t) = \bar{v} (x) + u (x, t),
\]
\[(A.17)\]

where \( u(x, t) \) is the dynamic microcantilever deflection. By applying Taylor series expansion, the tip-sample force can be rewritten as:

\[
F = f_0 + f_1 u (L) + f_2 u^2 (L) + f_3 u^3 (L),
\]
\[(A.18)\]

\[
f_0 = \frac{H R a^6}{180 d^6} - \frac{H R}{6a^2},
\]
\[(A.19a)\]

\[
f_1 = \frac{2 H R a^6}{45 d^6} - \frac{H R}{3a^2} \cos \alpha,
\]
\[(A.19b)\]

\[
f_2 = \frac{H R a^6}{5 d^6} - \frac{H R}{2d^4} \cos^2 \alpha,
\]
\[(A.19c)\]

\[
f_3 = \frac{2 H R a^6}{3 d^6} - \frac{2 H R}{d^4} \cos^3 \alpha.
\]
\[(A.19d)\]

For convenience in understanding, and for representing the equations in a more clear form, the following non-dimensional variables are introduced and used:

\[
x^* = \frac{x}{L}, \quad t^* = \omega_0 t, \quad u^* = \frac{u}{L}, \quad d^* = \frac{d}{L},
\]
\[(A.20)\]

where \( \omega_0 = \sqrt{\frac{k}{\mu_0 d^4}} \). By substituting Eqs. (A.16)–(A.18) into Eq. (A.15) and using non-dimensional variables, the equation of motion can be written as:

\[
\ddot{u} + P_1 [K (x) u^\nu ] + P_2 \dot{u} + P_3 C_v (x) P_t (t) = F_{L,L} \delta (x - 1) + [F_{L,L} \delta (x - 1) ]',
\]
\[(A.21)\]

where:

\[
P_1 = \frac{1}{m L^2 \omega_0^2}, \quad P_2 = \frac{C}{m \omega_0}, \quad P_3 = \frac{1}{m L^3 \omega_0^2},
\]
\[P_4 = \frac{f_1}{m L^2 \omega_0^2}, \quad P_5 = \frac{f_2}{m L^2 \omega_0^2}, \quad P_6 = \frac{f_3}{m L^3 \omega_0^2}.
\]
\[(A.22a)\]

And boundary conditions are:

\[
u (0) = 0, \quad u' (0) = 0, \quad u'' (1) = 0, \quad u'''' (1) = 0.
\]
\[(A.23)\]

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