Constraints on anomalous gauge couplings from present LEP1 and future LEP2, BNL data

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Abstract

We analyze, in a rather general model where anomalous triple gauge couplings are present, the visible effects in $R_b$ (measured at LEP1), in $W$ pair production (to be measured at LEP2) and in the muon anomalous magnetic moment (to be measured at BNL). From the combination of the three experiments a remarkable improvement on the pure LEP2 constraints is obtained.
At the end of the high precision measurements performed at LEP1, the Standard Model predictions concerning the Z boson-fermion interactions have been verified to a level of accuracy that is generally of a few permille \[ \text{(1)} \]. The fact that, within these limits, no signal of any possible kind of new physics has been detected does not necessarily imply, though, that the same conclusion should apply to different sectors of the Standard Model. This is particularly true for the purely gauge boson interactions whose form, in the Standard Model (SM), is severely constrained by the assumed requests of local gauge invariance and renormalizability.

The possibility that “anomalous” boson gauge couplings exist has been, in fact, the subject of a number of theoretical speculations \[ \text{(2)} \] and discussions \[ \text{(3)} \] that we shall not review in this short letter, particularly since they can be already found in excellent recent review papers \[ \text{(4)} \]. Here we shall follow the approach that was presented by Hagiwara, Ishihara, Szalapski and Zeppenfeld \[ \text{(5)} \]. It is based on the assumption that such anomalous couplings appear as residual effects of particles and interactions lying beyond the SM whose dynamics is not yet known and is generically called new physics (NP). It is assumed that this dynamics is characterized by a scale \( \Lambda \) which is much higher than the electroweak scale, \( \Lambda \gg M_W \). It is then reasonable to expect that at energies much lower than \( \Lambda \) these residual effects should preserve the standard \( SU(2) \times U(1) \) gauge invariance before spontaneous breaking occurs through the Higgs mechanism. Consequently these effects can be described by the effective lagrangian method \[ \text{(6)} \] and again because of the assumption that \( \Lambda \gg M_W \) only dimension six operators constructed with standard bosonic fields will be retained. We further restrict to CP-conserving interactions and this leaves seven possible operators listed in \[ \text{(7)} \]. Among them only three contribute to anomalous 3-gauge boson couplings. They are dubbed

\[
O_{WWW} = Tr[\hat{W}_\mu \hat{W}_\nu \hat{W}_\lambda \rho] , \quad (1)
\]
\[
O_W = (D_\mu \Phi)\dagger \hat{W}^{\mu\nu} (D_\nu \Phi) , \quad (2)
\]
\[
O_B = (D_\mu \Phi)\dagger \hat{B}^{\mu\nu} (D_\nu \Phi) , \quad (3)
\]

where

\[
D_\mu = \partial_\mu + i \frac{g'}{2} B_\mu + i \frac{g}{2} \sigma^a W^a_\mu \quad (5)
\]
\[
\hat{B}^{\mu\nu} = i \frac{g'}{2} B^{\mu\nu} \quad \hat{W}^{\mu\nu} = i \frac{g}{2} \sigma^a W^a_{\mu\nu} \quad (6)
\]

so that the effective lagrangian is written

\[
\mathcal{L} = \frac{1}{\Lambda^2} (f_{WWW} O_{WWW} + f_W O_W + f_B O_B) \quad (7)
\]

The general anomalous \( \gamma W W \) and \( Z W W \) couplings defined in \[ \text{(8)} \] can be expressed in terms of the three independent parameters appearing in the above
lagrangian. Only five of them are fed by the above three operators. In the notation of reference \[6\], they are:

\[
g_{1Z} = 1 + f_{W} \frac{M_{W}^2}{2\Lambda^2} \quad (8)
\]

\[
\kappa_{\gamma} = 1 + (f_{B} + f_{W}) \frac{M_{W}^2}{2\Lambda^2} \quad (9)
\]

\[
\kappa_{Z} = 1 + [f_{W} - s_{W}^2(f_{B} + f_{W})] \frac{M_{Z}^2}{2\Lambda^2} \quad (10)
\]

\[
\lambda_{\gamma} = \lambda_{Z} \equiv \lambda = \frac{3M_{W}^2 g^2}{2\Lambda^2} f_{WW}. \quad (11)
\]

One of the best places to search for the existence of anomalous $\gamma W W$ and $Z W W$ couplings is the process $e^{+}e^{-} \rightarrow W^{+}W^{-}$. This investigation is presently starting at LEP2. An analysis of what could be expected has been previously performed in refs. \[5, 8\]. The results of these analyses are represented by exclusion plots in the space of the three parameters. More precisely, one defines there the quantities:

\[
x = f_{B} \frac{M_{W}^2}{2\Lambda^2} \quad (12)
\]

\[
y = f_{W} \frac{M_{W}^2}{2\Lambda^2} \quad (13)
\]

(in ref.\[3\] they are denoted $\alpha_{B\Phi}$ and $\alpha_{W\Phi}$ respectively) and fits them together with $\lambda$ defined in eq.\(11\). In practice, one rather shows three bidimensional figures in the $(x, y)$, $(x, \lambda)$ and $(y, \lambda)$ planes representing the regions inside which $x$, $y$ and $\lambda$ would be constrained to lie, at 95% C.L., if no visible effects were seen in the final WW channel, that means that the experimental measurements agree with the SM prediction within the assumed uncertainty. The solid lines in Fig.1-3 show such 3-parameter constraints derived for $\sqrt{s} = 190$ GeV with an integrated luminosity of 500 pb$^{-1}$ and relying on WW events where at least one W decays leptonically (the inclusion of events where both W decays hadronically should slightly improve the constraints, roughly by a factor 1.4, but the precise analysis has not been done yet \[9\]).

The information on $x$, $y$ and $\lambda$ obtained by analyzing the LEP2 data is provided by a reaction where two real Ws are produces by a virtual V. This is not the only possible source of constraints. At least two different processes can lead to information on the same parameters, by an analysis of special one-loop contributions where the same VWWW vertex appears but virtual Ws (and V) are implied. We shall now list these two cases and consider them in some details.

1) Measurements of the ratio $R_{b} = \Gamma_{Z\rightarrow b\bar{b}}/\Gamma_{Z\rightarrow \text{hadrons}}$ on top of the $Z^{0}$ resonance.

It has been recently emphasized \[10, 11\] that the value of the ratio $R_{b}$ would be substantially affected by those anomalous couplings that modify
the Zb¯b vertex by virtual effects proportional to m_t^2. Such an m_t^2 enhancement is present when a "longitudinal" W_L component (also called "would-be-goldstone" component H) is involved in a loop together with a top quark and the Htb vertex. This already occurs in the SM and it can also occur through the operators O_W and O_B which generate "anomalous" ZWH and ZHH vertices. The conclusion of ref.[10], to which we defer for a complete discussion, is that the shift produced in R_b by the considered model of anomalous gauge couplings (AGC) would be:

\[ \Delta R_b^{AGC} = -\frac{4}{6} \left( 1 + \frac{g_s^2}{128\pi^2} \right) \left(\frac{m_t^2}{M_W^2}\right) \left(7y - \frac{s_W^2}{c_W^2}x\right) \ln \frac{\Lambda^2}{M_Z^2} \] (14)

where \( b = 1 - \frac{4}{3}s_W^2 \). For m_t^2 = 175 GeV and \( \Lambda = 1 \) TeV this gives numerically:

\[ \Delta R_b^{AGC} \simeq -0.04(y - 0.04x) \] (15)

showing that this effect is, in practice, providing constraints on \( y \) alone.

The numerical exploitation of eq.(15) is obviously very strongly dependent on the experimental value of R_b, as we fully attribute \( \Delta R_b \equiv R_b^{exp} - R_b^{SM} \) to \( \Delta R_b^{AGC} \). Both the central value and the uncertainty of the experimental measurement are important for the conclusion that we want to draw. As there have been recently some changes in the experimental results we will be careful about these points. If one uses the last (to our knowledge) officially communicated averaged LEP1/SLC value, that reads [1],[2]

\[ R_b = 0.2178 \pm 0.0011 \] (16)

and the SM prediction for m_t = 177 GeV

\[ R_b(SM) = 0.2158 \] (17)

one obtains

\[ \Delta R_b^{AGC} = 0.0020 \pm 0.0011 \] (18)

and from eq.(15), assuming \( \Lambda = 1 \) TeV, we derive a first bound on \( y \). Combined with the LEP2 constraint at 95% C.L. the resulting domains are shown in Fig.1a-3a (dotted lines).

In order to show the consequences of recent changes in \( R_b \) results on our domains we also make an illustration only using the most recent value published by ALEPH [12], which fully agrees with the SM prediction

\[ R_b = 0.2159 \pm 0.0009 \pm 0.0011 \] (19)

We obtain

\[ \Delta R_b^{AGC} = 0.0001 \pm 0.0020 \] (20)

and the domains shown in Fig.1b-3b (dotted lines). As one could expect, the results essentially differ by a shift of the domain concerning the parameter
However, because of the importance of the experimental uncertainty the order of magnitude of the constraints is rather similar to the one obtained in Fig.1a-3a.

One should also note that our results depend through eq.(15) on the assumed value of the new physics scale \( \Lambda \), taken as 1 TeV in the illustrations. This is a typical feature of 1-loop effects. The dependence in \( \Lambda \) is only logarithmic and for example increasing (which would seem more reasonable than decreasing) \( \Lambda \) by a factor of two would only affect the result by making the constraint roughly a relative twenty percent more stringent.

In conclusion of this first part one can say that the net effect of adding the LEP1 constraint from \( R_b \) to the LEP2 bounds corresponds to a strong reduction of the limits on \( y \), while the two other parameters are essentially unaffected.

II) (Future) measurement of the muon’s anomalous magnetic moment at BNL.

Naively, one would expect that anomalous \( \gamma W W \) couplings could affect to a sensible extent the measured value of the muon’s anomalous magnetic moment \( a_\mu = \frac{1}{2}(g - 2) \). In fact in the Standard Model, the contribution of the graph involving the \( \gamma W W \) vertex and neutrino exchange is rather large as compared to the expected future precision of the planned measurement at BNL [13].

To be more precise, a few details must be added at this point. The last experimental value of \( a_\mu \) is [14]:

\[
a_\mu^{\text{exp}} = 1 165 923(8.5) \times 10^{-9}
\]  

(21)

not in disagreement with the last available theoretical estimate [15]:

\[
a_\mu^{\text{th}} = 1 165 917(1.0) \times 10^{-9}.
\]  

(22)

Most of the theoretical uncertainty (88\% of the quadratic error) arises from the lowest order hadronic contribution to \( a_\mu \), which is inferred, through a dispersion relation, from data of \( e^+e^- \) annihilation into hadronic final states and of hadronic \( \tau \) decays. It is expected that the forthcoming BNL experiment will reduce the experimental error by more than one order of magnitude, i.e.:

\[
\delta a_\mu^{\text{exp}} = 0.4 \times 10^{-9}.
\]  

(23)

It should also be mentioned that an extra reduction of the theoretical error, coming from new measurements of the total cross section of \( e^+e^- \) annihilation in the very low energy region, would be, in principle, in the reach of the DAΦNE activities [16]. In fact, the \( \rho \) resonance region is responsible for more than 50\% of the quadratic error on the lowest order hadronic contribution to \( a_\mu \) [15]. A total integrated luminosity of less than 500 nb\(^{-1}\), which requires fourteen hours of running at a luminosity of \( 10^{30} \text{ cm}^{-2}\text{s}^{-1}\) (more than two orders of magnitude lower than the expected peak luminosity of DAΦNE), should
allows to measure $\sigma(e^+e^- \to \pi^+\pi^-)$ from threshold for hadro-production up to 1.08 GeV with enough precision to push down the error on the hadronic contribution to $a_\mu$ from this energy region to the value of $1.5 \times 10^{-10}$. This would lead, according to [13], to a 30% reduction of the error on the total hadronic contribution at one loop and, hence, to a 20% reduction of the current overall theoretical uncertainty. Thus, one can imagine that $a_\mu$ will be soon measured to a relative accuracy of less than a part per million. In this picture, the numerical value of the $\gamma W W$ vertex contribution is, in the Standard Model, more than 4 times larger that the expected precision on $a_\mu$. This means that not unfairly small anomalous contributions seem to be in a good shape to produce visible effects.

In fact, this feeling is verified in a number of rigorous calculations [17] that were performed some time ago, mostly in the unitary gauge. The various results produce a leading term, on which the agreement is general, and finite terms that depend essentially on the used regularization scheme and do not always coincide. For this reason, but also in order to present an alternative calculation of the gauge invariant quantity $a_\mu$, we have redone the calculation in the Feynmann ‘t Hooft $\xi = 1$ gauge within the dimensional regularization framework. The various technical details of the calculation are fully illustrated elsewhere [18]. Using the generally accepted prescription that sets the formal correspondence $\frac{2}{n-4} \to \ln \frac{\Lambda^2}{\mu^2}$, we arrived to the following result:

$$\Delta a^{AGC}_\mu = \frac{G_F}{8\sqrt{2}\pi^2} m_\mu^2 \left[ \Delta k_\gamma \left( \frac{1}{2} - \ln \frac{\Lambda^2}{M_W^2} \right) - \lambda_\gamma \right]$$

(24)

where $\Delta k_\gamma = k_\gamma - 1$ is equal, in our notation, to $x + y$.

A few comments on eq. (24) are, at this point, appropriate. First of all, the leading logarithmic term is in agreement, as expected, with all previous calculations. Concerning the extra finite terms, that multiplying $\lambda$ turns out to be in agreement with the corresponding calculation of the second of ref. [17] that was performed using dimensional regularization; this does not agree with the analogous terms computed using different regularization techniques, as exhaustively discussed in the second of ref. [17]. The finite term that multiplies $\Delta k_\gamma$, which is non zero in our calculation, would be zero in the first of ref. [14] (unitary gauge and cutoff). Again, this is not particularly surprising given the quite different regularization prescriptions. Since for $\Lambda = 1$ TeV, which is generally considered as the lowest acceptable value for the new physics scale, the relative size of this term is one order of magnitude smaller than that of the leading logarithmically divergent contribution, we have decided to ignore it as a first quite reasonable approximation, keeping in mind the fact that, for (unlikely) smaller $\Lambda$ values, this might be a slightly arbitrary attitude. For what concerns the finite contribution coming from $\lambda$, we decided to retain it in a first time and to try to quantify its possible (regularization scheme dependent) role. Indeed, we have verified, as expected, that the effect of the (ambiguous) finite contribution from $\lambda$ is completely negligible. Therefore,
at least in this special example, one can safely ignore it in eq.(24). With these prescriptions, we have finally taken eq.(24) without the finite terms and assumed no deviations from the Standard Model prediction and an overall (conservative) accuracy of $1 \cdot 10^{-9}$. This constraint is then added to the previous LEP1, LEP2 constraints. The results of our analysis are shown in Fig.1-3(a,b) (starred lines) at 95% C.L. From inspection of those figures the following feature of our analysis should be underlined:

a) for what concerns $y$, the addition of the $a_\mu$ input has a marginal effect of less than 10%;

b) The region allowed for $\lambda$ which is practically completely derivable from LEP2 is only reduced by a relative factor of about 20%;

c) in the case of $x$, a much more drastic reduction is induced by $a_\mu$, numerically equal to a relative factor of about 3;

d) the inclusion of $a_\mu$ makes the overall picture nicely uniform in the space of the three parameters $x, y$ and $\lambda$. In fact, from the combined analysis, the allowed intervals for $x, y$ and $\lambda$ are rather similar and, typically, of size not larger than about 0.2-0.3 (in modulus). This should be compared with the present bounds obtained from Tevatron analyses [19] that provide allowed ranges typically one order of magnitude larger.

In conclusion, we have seen that the information on possible anomalous gauge couplings that is already provided by the existing measurement of $R_b$ at LEP1, and that will be soon improved by the current measurements of WW production at LEP2, would be remarkably enriched by the addition of the foreseen improved determination of the muon anomalous magnetic moment at BNL. Should a deviation from the Standard Model values be present with a size, typically, of an “acceptable” twenty percent in the relevant couplings, it should not escape an accurate simultaneous overall fit. More generally, one should probably insist on the fact that the direct (tree level) tests provided by LEP2 do not have the same theoretical base as the indirect (1-loop) tests provided by LEP1 and by BNL. Any discrepancy between these two types of measurements would constitute a signal for another kind of virtual effects and stimulate further developments of high precision tests.

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References

[1] A. Böhm, “Results from the Measurements of Electroweak Processes at LEP1”, invited talk at the XXXIInd Rencontre de Moriond, Les Arcs/Savoie France, March 15-22, 1997

[2] A. Blondel, ICHEP96 (plenary talk), Warsaw, July 1996

[3] K. Gaemers and G. Gounaris, Z. Phys. C1 (1979) 259;
K. Hagiwara, K. Hikasa, R. Peccei and D. Zeppenfeld, Nucl. Phys. B282 (1987) 253;
P. Mery, M. Perrottet and F.M. Renard, Z. Phys. C36 (1988) 579

[4] A. De Rújula, M.G. Gavela, P. Hernandez and E. Massó, Nucl. Phys. B384 (1992) 3

[5] G. Gounaris, J.L. Kneur, D. Zeppenfeld et al. “Triple Gauge Boson couplings”, in “Physics at LEP2”, G. Altarelli and F. Zwirner eds., CERN Report 1996

[6] K. Hagiwara, S. Ishihara, R. Szalapski, D. Zeppenfeld, Phys. Rev. D48 (1993) 2182

[7] W. Buchmüller and D. Wyler, Nucl. Phys. B268 (1986) 621; C.J.C. Burgess and H.J. Schnitzer, Nucl. Phys. B228 (1983) 454; C.N. Leung, S.T. Love and S. Rao Z. Phys. C31 (1986) 433

[8] M. Bilenky, J.L. Kneur, F.M. Renard, D. Schildknecht, Nucl. Phys. B409 (1993) 22

[9] J.L. Kneur, private communication

[10] F.M. Renard and C. Verzegnassi, Phys. Lett. B345 (1995) 500

[11] O.J.P. Eboli, S.M. Lietti, M.C. Gonzalez-Garcia, S.F. Novaes, Phys. Lett. B339 (1994) 119

[12] The ALEPH collaboration, CERN-PPE/97-018 (1997).

[13] V.W. Hughes et al., “The anomalous magnetic moment of the muon” Bonn 1990, Proceedings, High energy spin physics, vol. 1, 367-382;
B.L. Roberts, “The new muon (g-2) experiment at Brookhaven”, Heidelberg 1991, Proceedings, The future of muon physics 101-108, Z. Phys. C56 (1992) Suppl. 101-108

[14] F.J.M. Farley, E. Picasso, “The muon g-2 experiments”, In T. Kinoshita, (ed.): Quantum electrodynamics 479-559

[15] R. Alemany, M. Davier and A. Höcker, LAL 97-02 (1997); see also S. Eidelman, F. Jegerlehner, Z. Phys. C67 (1995) 585
[16] P. Franzini, “The muon gyromagnetic ratio and $R_h$ at DAΦNE”, The Second DAΦNE Physics Handbook, INFN-LNF (1995) vol. 2, 471

[17] P. Méry, S.E. Moubarik, M. Perrottet, F.M. Renard, Z. Phys. C46 (1990) 229;
F. Boudjema, K. Hagiwara, C. Hamzaoui, K. Numata, Phys. Rev. D43 (1991) 2223;
C. Arzt, M.B. Einhorn and J. Wudka, Phys. Rev. D49 (1994) 1370
and references therein

[18] S. Spagnolo, “Non Standard Electroweak Contributions to the muon g-2 from Anomalous Gauge Boson Couplings”, INFN/TH-97/01 (1997)

[19] F. Abe et al, Phys. Rev. Lett. 75 (1995) 1017;
S.Abach et al Phys. Rev. Lett. 75 (1995) 1023, Phys. Rev. Lett. 77 (1996) 3303
Figure captions

Fig. 1 Constraints on the $x$ and $y$ anomalous couplings resulting from LEP2 forthcoming data (solid line), from the combined LEP2 and LEP1 $R_b$ data (dotted line) and from the overall information provided by LEP2, $R_b$ and future $a_\mu$ measurements at BNL (starred line). The LEP2 sensitivity limit refers to a center of mass energy $\sqrt{s} = 190$ GeV and to an integrated luminosity of $500 \text{ pb}^{-1}$. The supposed precision on the muon anomalous magnetic moment is $\delta a_\mu = 1 \cdot 10^{-9}$. The contours shown here represent the projection of a threedimensional region in the $x$, $y$ and $\lambda$ space allowed, at 95% C.L., by the experiments. The LEP1 $R_b$ data are taken
(a) from the Warsaw compilation, ref.[1],[2]
(b) from ALEPH, ref.[12].

Fig. 2 Projection in the $x - \lambda$ plane of the region allowed, at 95% C.L., to the three anomalous couplings present in the considered model by the combination of present LEP1 and future LEP2, BNL data. The meaning of the different curves is the same as in Fig.1a,b.

Fig. 3 Projection in the $y - \lambda$ plane of the allowed region, at 95% C.L., for the $x$, $y$ and $\lambda$ parameters from present LEP1 and future LEP2, BNL data. The meaning of the different curves is described in Fig.1a,b.
\[ y = f_W \frac{M_W^2}{2\Lambda^2} \]

\[ x = f_B \frac{M_W^2}{2\Lambda^2} \]

Fig.1a
\[ y = f_W \frac{M_W^2}{2\Lambda^2} \]

\[ x = f_B \frac{M_W^2}{2\Lambda^2} \]

Fig. 1b
\[ \lambda = f_W \frac{3g^2 M_W^2}{2\Lambda^2} \]

\[ x = f_B \frac{M_W^2}{2\Lambda^2} \]

Fig. 2a
\[ \lambda = f_W \frac{3g^2 M_W^2}{2\Lambda^2} \]

\[ x = f_B \frac{M_W^2}{2\Lambda^2} \]

Fig. 2b
\[ \lambda = f W \frac{3g^2 M_W^2}{2 \Lambda^2} \]

\[ y = f W \frac{M_W^2}{2 \Lambda^2} \]

Fig. 3a
\[ \lambda = f_W \frac{3g^2 M_W}{2 \Lambda^2} \]

\[ y = f_W \frac{M_W}{2 \Lambda^2} \]

Fig. 3b