Propagation of fermions in curved space-time generates gravitational interaction due to coupling of its spin with space-time curvature connection. This gravitational interaction, which is an axial-four-vector multiplied by a four gravitational vector potential, appears as CPT violating term in the Lagrangian which generates an opposite sign and thus asymmetry between the left-handed and right-handed partners under CPT transformation. In the case of neutrinos this property can generate neutrino asymmetry in the Universe. If the background metric is of rotating black hole, Kerr geometry, this interaction for neutrino is non-zero. Therefore the dispersion energy relation for neutrino and its anti-neutrino are different which gives rise to the difference in their number densities and neutrino asymmetry in the Universe in addition to the known relic asymmetry.

KEY WORDS: neutrino asymmetry, rotating black hole, space-time curvature, CPT violation

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I. INTRODUCTION

The generation of neutrino asymmetry, i.e., the excess of neutrinos over anti-neutrinos, in early Universe is a well known fact. This essentially arises due to lepton number asymmetry, for e.g. via the Affleck-Dine mechanism [1], in the early Universe. A large neutrino asymmetry in early Universe can have interesting effects on various cosmological phenomena like big-bang nucleosynthesis and cosmic microwave background [2]. Massive neutrinos with large asymmetry can also offer to explain existence of cosmic radiation with energy greater than GZK cutoff [3]. Apart from such asymmetry arising in early Universe, one can always ask whether there is a possibility of neutrino asymmetry arising when the Universe has cooled down, let us say in the present era. In this paper we present one such scenario when neutrinos are propagating around Kerr black holes.

Since long, propagation of test particles with some inherent structure in curved space-time has been of keen interest at both classical and quantum realms. A spinning test particle, when propagates in the gravitational field, its spin couples with the connection of the background space-time and produces an interaction term [4–6]. A similar coupling effect gets transferred to the phase factor at the quantum mechanical level leading to an interesting geometrical phase shifts (see e.g. [7]). This interaction between the spin of particle and spin connection of the background field is analogous to that of electric current with the vector potential in the case of electromagnetic field. The way electro-
magnetic connection serves as a gauge field, in a similar manner, in case of fermions in curved space, gravitational interaction gives rise to some sort of gauge field [8].

The propagation of fermions in curved space-time is well studied in past by several authors (e.g. [8–15]). The interaction term does not seem to preserve CPT and is similar to the effective CPT as well as Lorentz violating terms as described in other contexts in previous works (e.g. [16–18]). Therefore the interaction due to curvature coupling of spinor will give rise to opposite sign for a left-handed and right-handed field which for the case of neutrinos can lead to an asymmetry.

In this paper, we show that such a neutrino asymmetry can arise even in the present epoch like in the black hole space-times. In fact, we would show, it is just the form of the background metric which is responsible for such an effect. If the background metric satisfies a particular form which we discuss below and if the temperature of the bath is large enough, then the favourable conditions for neutrino asymmetry exist. In this connection, obviously Dirac equation and corresponding Lagrangian in curved background comes into the picture. Under curved space-times Dirac spinors can break the Lorentz invariance in the local frame which provide a background where the ordinary rules of quantum field theory, e.g. CPT invariance, can break down. It is seen that coupling between fermionic spin and curvature of space-time gives rise to an extra interaction term in the Lagrangian apart from free part, even if no further interaction is there. This interaction term does not preserve CPT and Lorentz symmetry.

The basic requirement to generate neutrino asymmetry by this mechanism is that the background metric should deviate from spherical symmetry, like that of a Kerr black hole. If the black hole is chosen as non-rotating (e.g. Schwarzschild type), then the CPT violating interaction term disappears and neutrino asymmetry is ruled out. In next section, we give the mathematical formalism, which clearly shows the neutrino asymmetry is possible to generate in present era. In §3, we give an example, where this asymmetry can arise in black-hole space-time. At last, in §4, we make conclusions.

II. FORMALISM TO PRODUCE NEUTRINO ASYMMETRY

The general Dirac Lagrangian density, which shows the covariant coupling of fermion of spin-1/2 to gravity, can be given as

\[ \mathcal{L} = \sqrt{-g} \left( i \bar{\psi} \gamma^a D_a \psi - m \bar{\psi} \psi \right), \]

where the covariant derivative and spin connection are defined as

\[ D_a = \left( \partial_a - \frac{i}{4} \omega_{bca} \sigma^{bc} \right), \]

\[ \omega_{bca} = e_{b\lambda} \left( \partial_a e_{c\gamma} + \Gamma_{\gamma\mu}^{\lambda} e_{\gamma}^\mu e_{\mu}^a \right). \]

We would work in units of \( c = \hbar = k_B = 1 \). We have assumed a torsion-less space-time and the Lagrangian is invariant under the local Lorentz transformation of the vierbien, \( e^a_{\mu} \), and the spinor field, \( \psi(x) \), as \( e^a_{\mu}(x) \rightarrow \Lambda^a_b(x) e^b_{\mu}(x) \) and \( \psi(x) \rightarrow \exp(i e_{ab}(x) \sigma^{ab}) \psi(x) \), where \( \sigma^{bc} = \frac{i}{2} \left[ \gamma^a, \gamma^b \right] \) is the generator of tangent space Lorentz transformation, the Latin and Greek alphabets indicate the flat and curved space coordinate respectively. Also
\[ e_\mu^\mu e_\nu^\nu = g^{\mu\nu}, \quad e_\alpha^\mu e_\beta^\mu = \eta^{ab}, \quad \{\gamma^a, \gamma^b\} = 2\eta^{ab}, \] \hspace{1cm} (4)

where \( \eta^{ab} \) represents the inertial frame of Minkowski metric and \( g^{\mu\nu} \) is the curved space metric, here Kerr geometry.

Now from (1) and (2), it is clear that the product of three Dirac matrices appears in the Lagrangian and which is

\[ \gamma^a \gamma^b \gamma^c = \eta^{ab} \gamma^c + \eta^{bc} \gamma^a - \eta^{ac} \gamma^b - i\epsilon^{abcd} \gamma^5 \gamma_d. \] \hspace{1cm} (5)

Thus the spin connection terms are reduced into the combination of an anti-hermitian, \( \bar{\psi} A_\alpha \gamma^a \psi \), and a hermitian, \( \bar{\psi} B^d \gamma^5 \gamma_d \psi \), coupling terms. The anti-hermitian interaction term disappears when its conjugate part of Lagrangian is added to (1). The only interaction survives in \( L \) is the hermitian part and (1) reduces to

\[ L = \text{det}(e) \bar{\psi} \left[ (i\gamma^a \partial_a - m) + \gamma^a \gamma^5 B_a \right] \psi, \] \hspace{1cm} (6)

where

\[ B^d = \epsilon^{abcd} e_{b\lambda} \left( \partial_a e_c^\lambda + \Gamma^\lambda_{\alpha\beta} e^\alpha_c e_\mu^\beta \right) \] \hspace{1cm} (7)

and in terms of tetrads, Christoffel connection is reduced as

\[ \Gamma^\alpha_{\mu\nu} = \frac{1}{2} \eta^{ij} e_i^\alpha e_j^\beta \left[ (e_\beta^\lambda e_\mu^\alpha + e_\beta^\alpha e_\mu^\lambda) \eta_{jp} + (e_\beta^\lambda e_\nu^\alpha + e_\beta^\alpha e_\nu^\lambda) \eta_{jq} - (e_\mu^p e_\nu^q + e_\nu^p e_\mu^q) \eta_{pq} \right]. \] \hspace{1cm} (8)

Thus from (6), the free part of the Lagrangian is, \( L_f = \text{det}(e) \bar{\psi} (i\gamma^a \partial_a - m) \psi \), which is exactly same as the Dirac Lagrangian in flat space, and the interaction part due to the curvature of space-time is, \( L_I = \text{det}(e) \bar{\psi} \gamma^a \gamma^5 \psi B_a \). It is known that Lagrangian for any fermionic field is invariant only under local Lorentz transformation [13]. However, if the gravitational four vector field \( B_a \), is chosen as constant background in the local frame, then \( L_I \) violates CPT as well as particle Lorentz symmetry in the local frame. For example, if \( B_a \) is constant and space-like (what we will show later according to Kerr geometry), then the corresponding fermion will have different interactions if its direction of motion or spin orientation changes, and thus the breaking of Lorentz symmetry in the local frame is natural. Our present formalism is based on this conception. It should be noted that similar interaction terms are considered in CPT violating theories and string theory (e.g. [16], [19]). Here the terms come into the picture automatically, due to the interaction with background curvature, and therefore the physical origin is very clear. Following [13], [16], we call the interaction, \( L_I \), is observer Lorentz invariant but there the particle Lorentz symmetry is broken. Here, both the kinds of Lorentz symmetry are different obviously as neutrinos are considered moving under gravitational field and thus they are no longer free. Now \( L_I \) is CPT violating if it changes sign under CPT transformation. Actually under CPT transformation, \( \bar{\psi} \gamma^a \gamma^5 \psi \), which is an axial-vector (pseudo-vector), changes sign. If \( B_a \) does not change sign under CPT, then \( L_I \) is CPT violating (CPT odd) interaction as well otherwise the interaction is CPT even. It is the nature of background metric which determines whether the functional form of \( B_a(x, y, z, t) \) is odd (changes sign) under CPT or not. Overall we can say, \( L_I \) is CPT as well as particle Lorentz violating interaction (it can be noted that CPT violation necessarily implies the Lorentz violation in local field theory [20]). The four-vector \( B_a \) is treated as a Lorentz-violating and CPT-violating spurion. However, if \( B_a \) does not break the symmetry of particle Lorentz transformations in the local frame, the CPT also cannot be broken. As we would see, for the propagation of
fermions in Kerr black hole space-times the interaction term is CPT violating. Thus, the vector $B_a$ causes breakdown of Lorentz invariance and CPT violation.

We would here like to mention that our analysis is different from earlier studies of interactions violating Lorentz invariance but which were mainly CPT even [21]. These studies were based on interactions in flat space-time and thus excluded interactions of fermions with background gravitational field. The purpose of these studies was to have high energy high precision tests of special relativity. One can then obtain bound on terms in Lagrangian violating Lorentz invariance, through various experiments like cosmic ray observations, neutrino oscillations etc. We in this paper, focus on the general relativistic effects on propagation of fermions and we establish that the background gravitational field plays an interesting role in disguise of vector $B_a$ to cause CPT violation and hence neutrino—anti-neutrino asymmetry. Further, our analysis, unlike that of [21] is based on considering interaction terms which violate CPT. As applied to phenomenology our motivation would be to seek possible generation of neutrino—anti-neutrino asymmetry in the Universe by putting bounds on parameters of the background black hole space-times. It would be interesting to extend this analysis to study the phenomenological applications e.g. neutrino oscillation [22] as studied earlier [21].

The corresponding dispersion relation for left and right chirality fields arised due to Lorentz and CPT violating term can be written as

$$(p_a \pm B_a)^2 = m^2, \quad (9)$$

where the ‘+’ and ‘−’ signs correspond to left handed and right handed partners.

The effect of background gravitational field on the propagation of fermions is to modify the dispersion relation. The vector $B_a$ violates CPT, breaks Lorentz invariance and causes the above modification. The energies of left handed and right handed fermionic species propagating in a gravitational background can be obtained by expanding out (9) as

$$E_L = \pm \sqrt{\left| \vec{p} \right|^2 + 2 (B_0 p^0 + B_1 p^1 + B_2 p^2 + B_3 p^3) + B_a B^a - m^2}$$

$$E_R = \pm \sqrt{\left| \vec{p} \right|^2 - 2 (B_0 p^0 + B_1 p^1 + B_2 p^2 + B_3 p^3) + B_a B^a - m^2}. \quad (10)$$

Thus there would be an energy gap between left handed and right handed species, which would be proportional to the interaction term $B_a p^a$. In the case of $B_a \rightarrow 0$, this helicity energy gap would disappear.

Now for the present purpose, we specialize to the case of neutrinos. In this scenario we can identify left handed species as particles and right handed species as corresponding anti-particles. Thus, the energy of particles ($E_\nu$) and anti-particles ($E_\bar{\nu}$) becomes,

$$E_\nu = \sqrt{\left| \vec{p} \right|^2 + 2 (B_0 p^0 + B_1 p^1 + B_2 p^2 + B_3 p^3) + B_a B^a - m^2}$$

$$E_\bar{\nu} = \sqrt{\left| \vec{p} \right|^2 - 2 (B_0 p^0 + B_1 p^1 + B_2 p^2 + B_3 p^3) + B_a B^a - m^2}. \quad (11)$$

Neutrinos and anti-neutrinos propagating in gravitational fields would thus have different energies. This energy difference between particles and anti-particles is the direct result of the presence of $B_a$ which violates CPT. We can further evaluate the difference in number density of neutrinos and anti-neutrinos propagating in a gravitational background as
\[ \Delta n = \frac{g}{(2\pi)^3} \int_{R_i}^{R_f} dV \int d^3p \left[ \frac{1}{1 + \exp(E\nu/T)} - \frac{1}{1 + \exp(E\bar{\nu}/T)} \right], \]  

(12)

where \( R_i \) and \( R_f \) refer to two extreme points of the interval over which the asymmetry is measured and \( dV \) is the small volume element in that interval.

In the case, when \( B_0 \) is vanishing, the integrand is an odd function and hence \( \Delta n = 0 \). Any non-zero value of \( B_0 \) would yield a \( \Delta n \neq 0 \) and hence neutrino asymmetry. Thus to create any neutrino asymmetry, a non-zero \( B_0 \) is required, and it does not matter whether \( B_i \)'s \( (i = 1, 2, 3) \) are vanishing or not. This is the reason, why the metric should have a non-zero off-diagonal spatial components for neutrino asymmetry to occur.

### III. NEUTRINO ASYMMETRY AROUND BLACK HOLE

An example of origin of neutrino asymmetry in black hole space-time can be given for Kerr geometry. For simplicity of analysis we would write the Kerr metric in Cartesian form, i.e., our variables are \( t(= x_0), x(= x_1), y(= x_2), z(= x_3) \). We would here stress that the conclusions are independent of the choice of coordinate system to describe the background space-time, as we comment in §4. In the Cartesian form the Kerr metric with signature [+−−−] can be written as

\[ ds^2 = \eta_{ij} dx^i dx^j - \left[ \frac{2\alpha}{\rho} s_i v_j + \alpha^2 v_i v_j \right] dx^i dx^j \]  

(13)

where

\[ \alpha = \sqrt{2Mr/\rho}, \quad \rho^2 = r^2 + \frac{a^2z^2}{r^2}, \]  

(14)

\[ s_i = \left( 0, \frac{rx}{\sqrt{r^2 + a^2}}, \frac{ry}{\sqrt{r^2 + a^2}}, \frac{z\sqrt{r^2 + a^2}}{r} \right), \]  

(15)

\[ v_i = \left( 1, \frac{ay}{a^2 + r^2}, \frac{-ax}{a^2 + r^2}, 0 \right). \]  

(16)

Here \( a \) and \( M \) are respectively the specific angular momentum and mass of the Kerr black hole and \( r \) is positive definite satisfying the following equation,

\[ r^4 - r^2 \left( x^2 + y^2 + z^2 - a^2 \right) - a^2z^2 = 0. \]  

(17)

The corresponding non-vanishing component of tetrads (vierbiens) are [23]

\[ e^0_t = 1, \quad e^1_t = -\frac{\alpha}{\rho} s_1, \quad e^2_t = -\frac{\alpha}{\rho} s_2, \quad e^3_t = -\frac{\alpha}{\rho} s_3, \]

\[ e^1_x = 1 - \frac{\alpha}{\rho} s_1 v_1, \quad e^2_x = -\frac{\alpha}{\rho} s_2 v_1, \quad e^3_x = -\frac{\alpha}{\rho} s_3 v_1, \]

\[ e^1_y = -\frac{\alpha}{\rho} s_1 v_2, \quad e^2_y = 1 - \frac{\alpha}{\rho} s_2 v_2, \quad e^3_y = -\frac{\alpha}{\rho} s_3 v_2, \quad e^3_z = 1 - \frac{\alpha}{\rho} s_3 v_3. \]  

(18)

Using (7), (8), (13) and (18), the gravitational scalar potential can be evaluated as
\[ B^0 = e_{1\lambda} \left( \partial_3 e^\lambda_4 - \partial_2 e^\lambda_3 \right) + e_{2\lambda} \left( \partial_1 e^\lambda_4 - \partial_3 e^\lambda_1 \right) + e_{3\lambda} \left( \partial_2 e^\lambda_1 - \partial_1 e^\lambda_2 \right). \]  

(19)

Similarly, gravitational vector potentials \( B^1, B^2, B^3 \) can be calculated. From (19), it is clear that \( B_0 \) will become zero, if all the off-diagonal spatial components of the metric are zero (i.e. \( g_{ij} = 0 \), where, \( i \neq j \to 1, 2, 3 \)). In other words we can say, there should be a minimum space-space curvature coupling effect to give rise to a nonzero scalar potential, \( B^0 \).

One can easily check from (19) along with (18) that under CPT transformation, form of \( B_0 \) would not behave as odd function, more precisely, \( B_0 \) neither flips its sign \( [B_0(-x, -y, -z, -a, M) \neq -B_0(x, y, z, a, M)] \) nor be invariant \( [B_0(-x, -y, -z, -a, M) \neq B_0(x, y, z, a, M)] \). The same would hold for \( B_1, B_2, B_3 \). Therefore, \( B_a \) leads to CPT violation. As mentioned earlier, this nature of \( B_a \) under CPT totally depends on the choice of background metric, the space-time, where the neutrino is propagating. A case of space-time was studied earlier [15] where \( B_0 \) flips its sign (odd function) under CPT and thus overall \( L_I \) is CPT invariant. However, the present case where the space-time is chosen around a rotating black hole gives rise to an actual CPT and Lorentz violating situation.

The important factor for our application is that the interaction term (\( L_I \)) has different signs for left and right chiral fields. The coupling term for particles \( \psi \) and anti-particles \( \bar{\psi}^c \) may be expressed as

\[
\bar{\psi}^c \gamma^a \gamma^5 \psi = (\bar{\psi}^c)_R \gamma^a (\psi)_R - (\bar{\psi}^c)_L \gamma^a (\psi)_L.
\]

(20)

Now, if we consider the spinor field as neutrino and since according to the standard model, particles (neutrinos) have left chirality and anti-particles (anti-neutrinos) have only right chirality, the first term in (20) and the second term in (21) will not be present. Thus the interactions will have opposite sign for (left-handed) neutrino and (right-handed) anti-neutrino.

Now we will show, how does the above mentioned property of neutrino, along with the effect of curvature, generates its asymmetry. For simplicity, let us consider a special case of a black hole space-time with \( \vec{B} \cdot \vec{r} \ll B_0 \gamma^0 \) and the black hole curvature effect is such that \( B_a B^a \) term can be neglected, and thus only the first order curvature effect is important. Then in the ultra-relativistic regime, we get from (12),

\[
\Delta n = \frac{g}{(2\pi)^2} T^3 \int_{R_i}^{R_f} \int_0^{\infty} \int_0^{\pi} \left[ \frac{1}{1 + eu e^{B_0/T}} - \frac{1}{1 + e^{-B_0/T}} \right] u^2 \, d\theta \, du \, dV 
\]

(22)

where \( u = |\vec{p}|/T \). If \( B_0 \ll T \), then

\[
\Delta n \sim g T^3 \left( \frac{\overline{B_0}}{T} \right),
\]

(23)

\( \overline{B_0} \) indicates the integrated value of \( B_0 \) over the space.

It should be noted that the sign of above asymmetry would depend on the overall sign of \( \overline{B_0} \), which depends on details of mass and angular momentum of the black hole. A large asymmetry can be achieved in practical situations as in accretion disks and case of Hawking radiation bath. In the first case, the virial temperature of thermal bath for the neutrinos can be as high as \( 10^{12} \) K \( \sim 100 \) MeV. Therefore, to have a neutrino asymmetry around a Kerr black
hole, the space-time curvature effect has to be at least one order of magnitude weaker, say, $B_0 \leq 10$ MeV, than the energy of bath. Moreover, the phenomena of a Hawking bath looks very promising, where small primordial black holes are produced in copious amounts. We know, all the primordial black holes of mass less than $10^{15}$ gm have been evaporated already. Only the black holes of mass, $M > 10^{15}$ gm, still exist today. The temperature of Hawking bath can be given as

$$T = \frac{\hbar}{8\pi k_B M} \sim 10^{-7} K \left( \frac{M}{M_\odot} \right).$$

Thus the primordial black hole of masses of the order $10^{15}$ gm can generate Hawking temperature of the order $T \sim 10^{11}$ K $\sim 10$ MeV. Hence, to generate a neutrino asymmetry, the restriction on curvature effect should be, $B_0 \leq 1 MeV$.

If we consider, temperature of bath, $T \sim 10^{11}$ K $\sim 1.6 \times 10^{-5}$ erg, $B_0 \sim 1.6 \times 10^{-6}$ erg, then $\Delta n \sim 10^{-16}$. If there are typically $10^6$ number of black holes with same sign of $B_0$, $\Delta n \sim 10^{-10}$, which agrees with the observed neutrino asymmetry in the Universe.

**IV. CONCLUSION**

We have proposed a new mechanism to generate neutrino asymmetry in the present epoch of the Universe. Such a mechanism can provide neutrino asymmetry in addition to the relic neutrino asymmetry arising due to leptogenesis in early Universe. We have explicitly demonstrated this through an example where neutrinos are propagating around Kerr black holes. Here, for convenience, we have chosen the Kerr metric in Cartesian coordinates $(x, y, z, t)$. It is seen that, in presence of any off-diagonal spatial component of the metric ($g_{ij}, i \neq j \rightarrow 1, 2, 3$) the scalar potential part ($B_0$) of the space-time interaction is non-zero. According to the present mechanism, this scalar potential is actually responsible for neutrino asymmetry in the Universe. If all the $g_{ij}$s are zero, $B_0$ vanishes and hence $\Delta n = 0$. Although, this restriction on $g_{ij}$ as well as $B_0$, to have a non-zero neutrino asymmetry, is made here on the basis of a fixed coordinate system, in principle we can choose any other kind of coordinate system to describe the background geometry and to generate neutrino asymmetry. One can easily check that, in Boyer-Lindquist coordinate system [24], $B_0$ is zero. But in that case, at least one non-zero $B_i$ ($i \rightarrow 1, 2, 3 \equiv \rho, \theta, \phi$) is required i.e., for example, presence of a $g_{03}$ is enough, to give rise to neutrino asymmetry. Thus the restriction to generate neutrino asymmetry around black hole is that the black hole must be rotating and hence the system is symmetric axially.

The asymmetry can be produced in accretion disks or/and Hawking radiation baths, which provide high enough temperature for such an effect to occur. Assume that, there are $N_i$ number of $i$-type black holes in Universe, each producing a net curvature effect $B_{0i}$ in a typical temperature of the system $T_i$, then the neutrino asymmetry due to the presence of black hole of kind-$i$ can be given as

$$\Delta n_i = 10^{-10} \left( \frac{N_i}{10^6} \right) \left( \frac{B_{0i}}{10^{-6} \text{erg}} \right) \left( \frac{T_i}{10^{-5} \text{erg}} \right)^2.$$  

If all the black holes in Universe are of $i$-kind and there are $10^6$ such black holes, curvature effect and temperature of the systems are $10^{-6}$ erg and $10^{-5}$ erg respectively, then the neutrino asymmetry in Universe is $10^{-10}$. Any change of $N_i$, $B_{0i}$ and $T_i$ will affect $\Delta n$. In general the net neutrino asymmetry in the Universe can be written as
\[ \Delta n = \sum_i \Delta n_i, \]  

(26)

where \( B_{0i} \) can be positive as well as negative depending on the kind of black hole (parameters of background space-time).

It should be reminded that, this kind of neutrino asymmetry can be achieved in some other space-time geometry where \( B_0 \) is non-vanishing. The Kerr geometry is chosen as an example only in the present paper. However, as the number of black hole may be very high and the physics behind it is very well established, it is advantageous to consider black hole space-times to built up a real feeling about the physics behind this new mechanism. Also the advantage to deal with Cartesian coordinate system is that the structure of Dirac gamma matrices (\( \gamma^0, \gamma^i \)) are very well known there.

In our earth, the curvature effect is measured as \( 10^{-34} \text{ MeV} = 10^{-40} \text{ erg} \) \cite{15} and the temperature is about \( 10^{-14} \text{ erg} \sim 10^{-2} \text{ eV} \). Thus, according to (25), the neutrino asymmetry comes out as \( 10^{-68} \) which is too small effect to observe. However, in earth’s laboratory, neutrinos can be examined in a high temperature bath. As the asymmetry is proportional to the square of temperature, it can be enhanced by increasing temperature in the laboratory. Moreover, if there are large number of earth like systems exist in the Universe, overall \( \Delta n \) may also increase according to (26).

Our mechanism essentially works on the presence of a pseudo-vector term \( (\bar{\psi} \gamma^a \gamma^5 \psi) \) multiplied by a background curvature coupling \( (B_a) \). This is the CPT and Lorentz violating term, which picks up an opposite sign for a neutrino and an anti-neutrino. In vacuum space-time where \( M \) is absent, in case of non-rotating black hole etc., this term vanishes and there is no neutrino asymmetry. Thus the CPT violating nature of the gravitational interaction with spinor is an essential condition in success of the mechanism. Thus we propose, to generate neutrino asymmetry in presence of gravity, following criteria have to be satisfied as: (i) The space-time should be axially symmetric, (ii) the interaction Dirac Lagrangian must have a CPT violating term which may be an axial-four vector (or pseudo-four vector) multiplied by a curvature coupling four vector potential. (iii) the temperature scale of the system should be large with respect to the energy scale of the space-time curvature. If all these conditions are satisfied simultaneously, our mechanism will give rise to neutrino asymmetry in Universe. It would be interesting to explore the further theoretical and phenomenological consequences of the role of background gravitational curvature for neutrinos, which might offer new insights in the interplay of gravity and standard model interactions and specially of neutrino physics.

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