The entanglement of the four-photon cluster state

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Abstract. Measurement based quantum computation draws its power from the remarkable entanglement properties of the cluster states. This contribution analyses the entanglement and its persistency of an experimentally realized four-photon cluster state. We describe how spontaneous parametric down conversion together with linear optics quantum logic can be used for the experimental study.

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1. Introduction

Whenever a computer performs a computation, it modifies a given input information according to definite rules, that will finally yield a desired output information. From a physical point of view, the computation is associated with the time evolution of the initial (input) state of a physical system, to a final state which is to be read out. With the advent of quantum computers, it seems plausible to associate this time evolution with a unitary process transforming the quantum state of the computer.

Surprisingly, the (unitary) evolution of quantum information can very well go together with absolutely non-unitary, irreversible and destructive measurement processes—provided the information is embedded in entangled states [1]. This culminated in the discovery by Raussendorf and Briegel, suggesting universal computation by an algorithm-specific sequence of single-qubit measurements in combination with classical feed-forward transformations on the qubits of a, what they named, cluster state. In all of these so-called measurement based quantum computation schemes multi-qubit entanglement of the cluster state, or more generally, of graph states plays a central role [2].

Starting from the basic definition of cluster states, we show in this contribution how a simple photonic cluster state can be experimentally observed using the process of spontaneous parametric down conversion (SPDC) and linear optics quantum logic (LOQL). In particular, we focus on the state’s entanglement properties and its peculiarities with respect to measurement and loss of qubits. Finally, we demonstrate that the entangling operation of a controlled not gate (CNOT) can be emulated by single measurements on two qubits of the cluster state.

2. Cluster states

A cluster state is an entangled state of many qubits that can be represented by a schematic presentation of its generation prescription, i.e. a cubic lattice in which the vertices are formed by two-level quantum systems and the grid lines correspond to next neighbour Ising type interactions between them (see figure 1(a)). Cluster states $|\mathcal{C}\rangle$ are fully described by a set of eigenvalue equations,

$$\hat{O}^i |\mathcal{C}\rangle = (-1)^{k_i} |\mathcal{C}\rangle,$$

where the operator $\hat{O}^i = \sigma_x^i \otimes \prod_{j \in \text{nn}(i)} \sigma_z^j$ means the application of $\sigma_x$ on every qubit $i$ of the lattice and $\sigma_z$ on all its next neighbours (nn) $j$ [3]. Therein $\sigma_x$ and $\sigma_z$ are the usual Pauli spin operators and $\{k\} = \{k_i \in \{0, 1\}\}$ is an additional parameter that characterizes the particular cluster state.

For a simple example of a linear four-qubit cluster state (see figure 1(b)) with $\{k\} = \{0, 0, 0, 0\}$ we get the following equations,

$$\left(\sigma_x \otimes \sigma_z \otimes \mathbb{I} \otimes \mathbb{I}\right) |\mathcal{C}_4\rangle = |\mathcal{C}_4\rangle,$$

$$\left(\sigma_z \otimes \sigma_x \otimes \mathbb{I} \otimes \mathbb{I}\right) |\mathcal{C}_4\rangle = |\mathcal{C}_4\rangle,$$

$$\left(\mathbb{I} \otimes \sigma_z \otimes \sigma_x \otimes \sigma_z\right) |\mathcal{C}_4\rangle = |\mathcal{C}_4\rangle,$$

$$\left(\mathbb{I} \otimes \mathbb{I} \otimes \sigma_z \otimes \sigma_x\right) |\mathcal{C}_4\rangle = |\mathcal{C}_4\rangle.$$
Figure 1. (a) Graphical representation of a cluster state. The vertices represent qubits and the edges next neighbour Ising type interactions. (b) Representation of the linear four-qubit cluster. (c) An experimental way to create a cluster state. The qubits are initially prepared in the state $|+\rangle$ and a CPHASE operation is applied between them. (d) An alternative way for the experimental generation of a four-qubit cluster state, starting from two entangled pairs of qubits.

which have the unique solution

$$|C_4\rangle = \frac{1}{2}(|+\rangle_1|0\rangle_2|+\rangle_3|0\rangle_4 + |+\rangle_1|0\rangle_2|-\rangle_3|1\rangle_4 + |-\rangle_1|1\rangle_2|-\rangle_3|0\rangle_4 + |-\rangle_1|1\rangle_2|+\rangle_3|1\rangle_4),$$

(6)

where $|0\rangle$, $|1\rangle$ or $|\pm\rangle = \frac{1}{\sqrt{2}}(|0\rangle \pm |1\rangle)$ represent the eigenvectors of $\sigma_z$ or $\sigma_x$ for positive and negative eigenvalue, respectively.

The generic way of initializing such a cluster state is to prepare four qubits in the state $|+\rangle$ and sequentially apply a controlled phase gate (CPHASE) in the computational basis between them (see figure 1(c)),

$$\text{CPHASE} = \frac{1}{2} (I \otimes I + \sigma_z \otimes I + I \otimes \sigma_z - \sigma_z \otimes \sigma_z).$$

(7)

This two-qubit gate induces a $\pi$-phase shift, once both qubits are in the logical state 1. From equation (6) one can see that for the generation of the linear four-qubit cluster the CPHASE has to be applied three times. In a photonic qubit system the application of two-qubit gates is in general a difficult task, as the required photon–photon interaction is not of reasonable coupling strength. However, as we intend to observe a four-photon cluster state, we will deal in the following with a state $|C_4\rangle$, that is local-unitary equivalent to the linear four-qubit cluster:

$$|C_4\rangle = \frac{1}{2}(|0\rangle_1|0\rangle_2|0\rangle_3|0\rangle_4 + |0\rangle_1|0\rangle_2|1\rangle_3|1\rangle_4 + |1\rangle_1|1\rangle_2|0\rangle_3|0\rangle_4 - |1\rangle_1|1\rangle_2|1\rangle_3|1\rangle_4).$$

(8)

$|C_4\rangle$ and $|C_4\rangle$ are transformed into each other by a Hadamard gate, $H = \frac{1}{\sqrt{2}}(\sigma_x + \sigma_z)$, acting on qubits 1 and 4. The advantage of $|C_4\rangle$ is, that it can be generated from the product of two Bell pairs $|\phi^+\rangle_{ij} = \frac{1}{\sqrt{2}}(|00\rangle_{ij} + |11\rangle_{ij})$ and a single application of CPHASE on qubits 2 and 3. For the experimental observation we can thus profit from the entanglement inherent in the photon pairs produced by SPDC (see figure 1(d)).
3. Experimental implementation

3.1. The cphase gate

As introduced by Knill, Laflamme and Milburn (KLM), all-optical two-qubit quantum logic can be achieved using linear optics in combination with conditional detection [4]. Our approach for the implementation of the cphase is also based on these principles [5], however, simpler in the experimental realization than the original KLM proposal. We use polarization encoding, i.e. |0⟩ and |1⟩ are represented by |H⟩ and |V⟩, where H and V denote horizontal and vertical linear polarization states, respectively. In order to achieve the desired gate operation, which corresponds to a π-phase shift for the input |VV⟩, we apply a second-order interference on a polarization dependent beam-splitter (PDBS₀) (see figure 2).

Two input modes b and c are overlapped at the PDBS₀. The transmission of 1/3 for vertical polarization results in a total amplitude of −1/3 for the |VV⟩ output terms, as can be seen by adding the amplitudes for a coincident detection of one photon in each of the output modes b and c:

\[ (t^b_V \cdot t^c_V) + (i r^b_V \cdot i r^c_V) = \sqrt{\frac{1}{3}} \sqrt{\frac{1}{3}} - \sqrt{\frac{2}{3}} \sqrt{\frac{2}{3}} = -1/3, \]

where \( t^x_i \) (\( r^x_i \)) is the amplitude for transmission (reflection) of state |i⟩ in mode x. Perfect transmission of horizontal polarization prevents interference for the contributions |HH⟩, |HV⟩ and |VH⟩. As the absolute values of the amplitudes need to be equal for any input, we attenuate the contributions that include horizontal polarization. This is achieved by placing a PDBS_{b/c} with the transmission 1/3 for horizontal polarization and perfect transmission for vertical polarization.
in each output mode. Altogether we find a probability of $\frac{1}{9}$ to obtain a coincidence in the outputs and thus a gate operation with perfect fidelity.

To experimentally test the gate operation we used photon pairs emitted from SPDC. A 2 mm thick $\beta$-barium borate (BBO) crystal was pumped by UV pulses with a central wavelength of 390 nm and an average power of 700 mW from a frequency-doubled mode-locked Ti:sapphire laser (pulse length 130 fs). The emission is filtered with polarizers to prepare input product states with high quality. We couple the photon pairs into single mode fibres for selection of the spatial modes. This guarantees identical beam modes which eases the alignment of spatial mode matching at PDBSO. The spectral mode selection is improved via 2 nm bandwidth filters behind the gate. Information about the indistinguishability of the photons at the PDBSO is obtained from a Hong–Ou–Mandel (HOM) [6] dip-measurement. We call the ratio of the experimentally observed and the theoretically expected dip-visibility $V$ overlap quality $Q = \frac{V_{\text{exp}}}{V_{\text{th}}}$. For $|VV\rangle$-input we achieved $Q = 91.0 \pm 0.9\%$. Note that for non-perfect interference, the probability for a $VV$ coincidence is enhanced and leads to an additional admixture of vertically polarized photon pairs (further referred to as $|VV\rangle\langle VV|$-noise). In order to get a more quantitative estimate for the effects of the noise on the observation of the cluster state, we have performed a quantum process tomography of our gate [7, 8]. The tomography data together with a fitted theoretical model including experimental parameters provides us with a matrix that characterizes the realized $\text{cphase}$ process (for details see Kiesel et al in [5]). On the basis of the matrix, simulations of the gate performance in further experiments can be carried out (see next section).

3.2. The four-photon cluster state

In order to observe the state $|C_4\rangle$, the SPDC set-up described in the previous section was modified such that two pairs

$$|\phi^+\rangle = \frac{1}{\sqrt{2}} (|HH\rangle + |VV\rangle)$$

are emitted in four spatial modes $a$, $b$ and $c$, $d$, by a double pass configuration (see figure 2) [9]. One photon of each pair (mode $b$ and $c$) is fed as input into the phase gate. Consequently, under the condition of detecting one photon in each mode the gate operation is applied and the cluster state is observed.

For its characterization, polarization analysis (PA) is performed in all four output modes. The photons are detected by silicon-avalanche photo diodes (Si-APD) and the signals are fed into a multi-channel coincidence unit which allows to simultaneously register any possible coincidence detection between the inputs. Each of the 16 characteristic four-fold coincidence events has a certain detection efficiency (given by the product of the efficiencies of the four corresponding detectors). The coincidence count rates are corrected for these efficiencies such that the total count rate is left unchanged. The errors on the independently determined efficiencies enter into the errors of the count rates via Gaussian error propagation.

Figure 3 displays the counts obtained for the experimental four-photon state (red bars). In addition, the counts of a simulation, incorporating the $\text{cphase}$ model matrix and contributions of higher order SPDC emissions (black bars), are shown. For comparison we added as well the outcomes (grey bars) of the ideal cluster state (equation (8)) for which we multiplied the theoretical probabilities by the experimental average count rate. In the computational basis, one clearly observes the four-term structure with peaks at $HHHH$, $HHVV$, $VWHH$ and $VVVV$. The
Figure 3. Displayed are the correlation measurements (red bars) for $|C_4\rangle$ when qubits are measured (a) all in the $\sigma_z$-basis (b) $a$, $b$ in $\sigma_x$- and $c$, $d$ in $\sigma_z$-basis (c) $a$, $b$ in $\sigma_z$- and $c$, $d$ in $\sigma_x$-basis (measurement time: 120 min per basis). In addition the results are shown of a simulation (black bars), that includes a theoretical model fit of the experimental phase gate and noise contributions of higher-order emissions from the SPDC. For better comparison the outcomes for an ideal state are shown as grey bars.

$VVVV$-contribution is enhanced, mainly due to the $|VV\rangle\langle VV|$-noise of the phase gate (figure 3(a)). A slight asymmetry in the transmission amplitudes for vertical polarization in the two modes of the phase gate causes a raising of the $HHVV$ term. The measurement of the other bases proves the contributions to be in a coherent superposition. Exemplarily we show the four-photon coincidence counts when the photons in mode $a$, $b$ (figure 3(b)), or the photons in mode $c$, $d$ (figure 3(c)), respectively, are measured along $\pm 45^\circ$. The clear four term structure is present here as well, however, the imperfect interference results in additional terms of detections with $xxVV$ (b), or $VVxx$ (c), respectively ($x = +45^\circ / -45^\circ$).

In order to quantify the imperfect interference for the simulation, we performed a dip-measurement similar to the one described in section 3.1, but conditioned on the detection of four photons, one in each mode $a$, $b$, $c$, $d$. From this, we obtain the quality factor $Q_4 = 0.845$ for the four-photon case. Using $Q_4$, the measured data in the computational basis is matched very well by the simulation. Though there are minor deviations in the remaining two bases, they lie mostly within the statistical measurement error. Thus the model does not account for every detail of the experimental set-up but conclusively identifies lack of interference as the main source of errors in the observation of the cluster state. This will be confirmed as well in the next section when discussing the state’s fidelity.

4. Analysis

A general limitation in experiments on multi-photon entanglement is the fact that one relies on coincidence detection and therefore suffers from low count rates caused by limited detection efficiencies. This is particularly relevant for the previously described set-up, due to the probabilistic functionality of the implemented phase gate. As the effort required for full quantum state tomography as a function of raising the number of qubits scales exponentially, we restrict ourselves in the following section to efficient, non-tomographic tools for the analysis of the observed state.
4.1. Four-photon entanglement and quality of the state

Concerning the entanglement properties, graph states offer the possibility for the construction of efficient (i.e. low number of measurement settings) entanglement witnesses, testing their multi-partite entanglement [10]. For the state \(|C_4\rangle\) that results in

\[
W_{C_4} := 3 \cdot \mathbb{1} \otimes 4 - \frac{1}{2} (\sigma_z^{(a)} \sigma_z^{(b)} + \mathbb{1}) (\sigma_z^{(b)} \sigma_z^{(c)} \sigma_z^{(d)} + \mathbb{1}) - \frac{1}{2} (\sigma_z^{(a)} \sigma_z^{(b)} \sigma_z^{(c)} + \mathbb{1}) (\sigma_z^{(c)} \sigma_z^{(d)} + \mathbb{1}),
\]

(11)

with the theoretically optimal value of \(\text{Tr}[W_{C_4} \rho_{\text{th}}] = -1\) [11]. From the structure of (11) we see, that the correlations in the two basis settings of figures 3(b) and (c) suffice to evaluate the entanglement witness. Experimentally, we find \(\text{Tr}[W_{C_4} \rho_{\text{exp}}] = -0.299 \pm 0.050\) clearly proving genuine four-photon entanglement of the observed state \(\rho_{\text{exp}}\).

In order to evaluate the quality of \(\rho_{\text{exp}}\), the fidelity \(F_{C_4} = \langle C_4 | \rho_{\text{exp}} | C_4 \rangle\) is the tool of choice. In general, full knowledge of the experimental state, and therefore a complete state tomography would be necessary to calculate the fidelity between two states. However, one can profit from the fact that the cluster state, as a graph state, is completely describable by its stabilizers. These are the operators forming the group that is generated by the operators of the characteristic eigenvalue equations. The fidelity for the cluster state (as for any graph state) equals the average absolute expectation value of these stabilizer operators. This can be easily seen by decomposing \(\rho_{\text{exp}}\) in terms of the Pauli operators,

\[
\rho_{\text{exp}} = \frac{1}{16} \sum_{ijkl} c_{ijkl} (\sigma_i \otimes \sigma_j \otimes \sigma_k \otimes \sigma_l),
\]

(12)

with \(i, j, k, l \in \{0, x, y, z\}\) and \(\sigma_0 = \mathbb{1}\). It follows that \(F_{C_4} = \frac{1}{16} \sum_{ijkl} c_{ijkl} \langle C_4 | (\sigma_i \otimes \sigma_j \otimes \sigma_k \otimes \sigma_l) | C_4 \rangle\).

Thus, a measurement of the respective correlations \(c_{ijkl}\) for which \(\langle C_4 | (\sigma_i \otimes \sigma_j \otimes \sigma_k \otimes \sigma_l) | C_4 \rangle \neq 0\) is sufficient to evaluate the state fidelity (figure 4(a)). In our case, these are 16 correlations, instead of 81 for full tomography, resulting in a value of \(F_{C_4} = 0.741 \pm 0.013\). It is difficult to draw conclusions about the consequences of a reduced fidelity with respect to measurement based computation in general. The performance of the cluster state to that effect might depend on the kind of noise, responsible for the reduction of the state’s quality, and on the algorithm that should be implemented [12]. However, we can determine the relevant experimental parameters that influence the fidelity in the presented set-up. If we assume perfect coherence in the phase gate, i.e. \(Q_4 = 1\), and just consider the actual and not ideal transmission and reflection amplitudes we obtain from the simulation \(F_{C_4}^{\text{sim}} = 0.993\). The case of imperfect coherence, \(Q_4 = 0.845\), leads to a simulated fidelity of \(F_{C_4}^{\text{sim}} = 0.779\). Taking into account additionally higher-order emissions of the SPDC reduces this value further to \(F_{C_4}^{\text{sim}} = 0.747\), which is in good agreement with the measured fidelity. Thus, as already conjectured in the previous section, it is mainly the imperfect interference in the phase gate that leads to a loss of quality for the experimental state.

4.2. Measurement and loss of qubits

As mentioned before, the entanglement contained in the cluster state is the essential resource in the measurement based quantum computation scheme. As in this process single qubits are
projected out of the cluster state, in the following, we are going to study the entanglement properties of our experimentally observed state with respect to measurement and loss of one and two qubits, respectively.

A projective measurement of one qubit in the $\sigma_x$-basis means, with regard to a computation, that this qubit is removed or disconnected from the cluster. The resulting state is again a cluster state of a lower qubit number. Depending on the result of the projection measurement (i.e. ‘$|+\rangle$’ or ‘$|−\rangle$’, exemplarily in mode $d$), $|C_4\rangle$ reduces to

$$|C_3\rangle_{\pm abc} = \left( |HH\rangle_{abc} \pm |VV\rangle_{abc} \right) / \sqrt{2} = \left( |\phi^+\rangle_{ab} |H\rangle_c \pm |\phi^-\rangle_{ab} |V\rangle_c \right) / \sqrt{2},$$

(13)

with $|\phi^-\rangle = \frac{1}{\sqrt{2}} (|HH\rangle - |VV\rangle)$.

To test whether the respective states are genuinely three-partite entangled, we apply entanglement witnesses, which again follow from the stabilizer formalism [11, 13]:

$$W_{C_3,abc} = \frac{3}{2} \mathbb{I} \otimes^3 - \sigma_x^{(a)} \sigma_x^{(b)} \sigma_x^{(c)} - \frac{1}{2} \left( \sigma_z^{(a)} \sigma_z^{(b)} \mathbb{I} \otimes^c + \sigma_z^{(a)} \mathbb{I} \otimes^b \sigma_z^{(c)} + \mathbb{I} \otimes^a \sigma_z^{(b)} \sigma_x^{(c)} \right).$$

(14)

In the experiment we obtain expectation values of the entanglement witnesses of $\langle W_{C_3,abc} \rangle = -0.362 \pm 0.090$ and $\langle W_{C_{3,abc}} \rangle = -0.392 \pm 0.082$ clearly proving the three-partite entanglement. Similar results for the witnesses are achieved when projecting other photons. Three qubit cluster states belong to the GHZ-class, what can be shown for the experimentally observed states by measuring their stabilizers.

The loss of a photon in mode $d$ results in the incoherent sum of the terms of equation (13), denoted by $\rho_{a,b,c}$. As a mixture of two bi-separable states, it is not tri-partite entangled anymore. However the contained bi-partite entanglement can be shown by a witness again, $W_{\rho_{abc}} = \frac{3}{2} \mathbb{I} \otimes^3 - \sigma_z^{(a)} \sigma_z^{(b)} \mathbb{I} \otimes^c - \sigma_x^{(a)} \sigma_x^{(b)} \sigma_x^{(c)}$ with $\langle W_{\rho_{abc}} \rangle = -0.648 \pm 0.057$. 

Figure 4. Absolute value of the measured expectation values for (a) the 16 stabilizing operators of $|C_4\rangle$ and (b) the eight stabilizing operators of $|C_3\rangle_{\pm abc}$ respectively.
Figure 5. Real part of the measured two-qubit density matrices of (a) qubits $a, b$ after the measurement \( \left( \frac{1}{2} + \sigma_z \right)_c \otimes \left( \frac{1}{2} + \sigma_z \right)_d \) (b) qubits $a, b$ after the measurement \( \left( \frac{1}{2} - \sigma_z \right)_c \otimes \left( \frac{1}{2} - \sigma_z \right)_d \) (c) qubits $a, b$ after the loss of photons in modes $c, d$ and (d) qubits $a, d$ after the loss of photons in modes $b, c$.

The residual state after measurement or loss of two qubits depends on the chosen pair of photons. This can be better understood if we take a closer look at the symmetry of $|C_4\rangle$ with respect to permutation of particles before we continue the examination of the entanglement persistency. Therefore we introduce the particle permutation operator

\[
S_{ji} = \frac{1}{2} \left( \mathds{1}^{(i)} \otimes \mathds{1}^{(j)} + \sigma^{(i)}_x \otimes \sigma^{(j)}_x + \sigma^{(i)}_y \otimes \sigma^{(j)}_y + \sigma^{(i)}_z \otimes \sigma^{(j)}_z \right),
\]

swapping qubits in mode $i$ and $j$. For four qubits we find $24$ possible permutations. They form a group and can be obtained by concatenation of swap operators between different pairs of qubits. Thus, we need to consider only the three generators of the group and choose the ones acting on neighbouring qubits, i.e. $S_{badc}$, $S_{acbd}$ and $S_{abdc}$. To quantify the symmetry of the state $|C_4\rangle$ under a certain permutation $S$ we determine the corresponding expectation value $s = \text{Tr}[S\rho]$. Experimentally we find

\[
s_{badc} = 0.98 \pm 0.07 \ (1.0), \quad s_{acbd} = 0.62 \pm 0.09 \ (0.5), \quad s_{abdc} = 0.98 \pm 0.08 \ (1.0),
\]

where theoretically expected values are shown in parenthesis. The cluster state exhibits a reduced symmetry under permutation of qubits $b$ and $c$. The slightly higher symmetry in the experimentally observed state can be attributed to the symmetric but incoherent $|VV\rangle\langleVV|$-noise. In contrast, we expect the state $|C_4\rangle$ to be fully symmetric under exchange of qubits $a, b$ and $c, d$, which is indeed fulfilled very well in the experiment. Due to this experimentally verified symmetry we can assume the same entanglement in the residual state after projective measurements or loss in any of the qubit pairs $\{a, c\}$, $\{a, d\}$, $\{b, c\}$ and $\{b, d\}$. A different kind of behaviour is expected, though, for the qubit pairs $\{a, b\}$ and $\{c, d\}$.

Let us start with the consideration of pairs from the sets $\{a, b\}$ or $\{c, d\}$. It is possible to obtain a maximally entangled bipartite residual state if both qubits are measured in the $\sigma_z$-basis. Depending on the measurement result, $|HH\rangle_{c,d}$ or $|VV\rangle_{c,d}$, we obtain the state $|\phi^+\rangle_{a,b}$ or $|\phi^-\rangle_{a,b}$, respectively. The result is shown in figures 5(a) and (b). There is a strong difference in the quality of the two observed states, which can be quantified by the fidelity and the logarithmic negativity $N$ [14]: $F_{\phi^+} = 0.963 \pm 0.054$, $N_{\phi^+} = 0.959 \pm 0.064$; $F_{\phi^-} = 0.772 \pm 0.027$, $N_{\phi^-} = 0.629 \pm 0.058$. The reason is the $|VV\rangle\langleVV|$-noise of the phase gate devastating the state in case of a projection on $|VV\rangle_{c,d}$ while being irrelevant for a projection on $|HH\rangle_{c,d}$. However, as expected, both states are entangled.

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A measurement of qubits $b, c$ in the basis $\sigma_x \otimes \sigma_x$ yields a maximally entangled state in modes $a, d$. These states correspond to the outcome of a CNOT operation, acting in the basis $\sigma_z \otimes \sigma_x$ on qubits $b, c$.

| Input                      | CNOT$_{zx}$ | Output                      | Experimentally obtained |
|---------------------------|-------------|-----------------------------|-------------------------|
| $|H + V\rangle | |+\rangle$         | $|H\rangle |+\rangle + |V\rangle |−\rangle$ | $0.767 \pm 0.031$ | $0.64 \pm 0.06$ |
| $|H + V\rangle | |−\rangle$         | $|H\rangle |−\rangle + |V\rangle |+\rangle$ | $0.758 \pm 0.029$ | $0.63 \pm 0.06$ |
| $|H − V\rangle | |+\rangle$         | $|H\rangle |+\rangle − |V\rangle |−\rangle$ | $0.736 \pm 0.031$ | $0.60 \pm 0.06$ |
| $|H − V\rangle | |−\rangle$         | $|H\rangle |−\rangle − |V\rangle |+\rangle$ | $0.809 \pm 0.027$ | $0.73 \pm 0.04$ |

In case qubits $c$ and $d$ are lost one obtains an incoherent mixture of the two states $|\phi^+_{a,b}\rangle$ and $|\phi^-_{a,b}\rangle$. In this sum the coherence terms cancel each other resulting in the separable state $\rho_{a,b} = \frac{1}{2}(|HH\rangle \langle HH| + |VV\rangle \langle VV|)$. Figure 5(c) shows what we obtain experimentally. The imaginary part is negligible and hardly any coherence is left. The smallest eigenvalue of the partially transposed matrix (PTM) is $\lambda_{\text{min}} = -0.024 \pm 0.021$ indicating that also the entanglement has cancelled almost completely.

In contrast, for the loss of photons in modes $b$ and $c$ we expect white noise in modes $a$ and $d$. The experimental result for this case is shown in figure 5(d). As before, there are hardly any coherence terms visible (the imaginary part is negligible as well, except for one entry of 0.067). Here, we can conclusively prove that the state is indeed completely disentangled with the smallest eigenvalue of the PTM, $\lambda_{\text{min}} = 0.159 \pm 0.007$.

### 4.3. Measurement based quantum computation

Of particular interest with respect to computation is the measurement of qubits $b$ and $c$. If these photons are measured, the residual state of qubits $a$ and $d$ equals the output of a CNOT operating in the basis $\sigma_z \otimes \sigma_x$ (CNOT$_{zx}$) with $b$ as control and $c$ as target bit. It is well known that a CNOT gate can be used to entangle two qubits, once the control bit is in a superposition of the computational basis states. That means that a measurement in the basis $\sigma_x \otimes \sigma_x$ in modes $b, c$, should lead to four orthogonal maximally entangled bipartite states in modes $a, d$, depending on the four possible projection results. Table 1 illustrates the gate entangling operation and shows the obtained fidelities as well as the logarithmic negativity for the experimental states. As can be seen, all four states are indeed entangled and show fidelities of up to 80% to the theoretically expected ones.

This clearly proves the implementation of a measurement based CNOT operation by the use of the experimentally obtained cluster state.

### 5. Conclusion

The one-way quantum computation scheme, which relies solely on single-qubit measurements and classical feed-forward, offers a feasible way for the processing of quantum information, once the required cluster state is initialized. This is possible by exploiting the entanglement contained in the emission of SPDC and the application of a simple linear optics CPHASE gate. The state obtained with our set-up exhibits the characteristic entanglement properties of the ideal state with a high fidelity. It is genuinely four-partite entangled, and the entanglement is
not completely destroyed if a photon is lost. Furthermore, characteristic for cluster states, the projection of one or two qubits can result in maximally entangled states. In this respect it was of particular interest that a two qubit CNOT operation can be realized by projective measurements on two qubits of the cluster, resulting in Bell-states of the other two with high fidelity.

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