Abstract
A new formalism of beam-optics and polarization has been recently presented, based on an exact matrix representation of the Maxwell equations. This is described in Part-I and Part-II. In this Part, we present the application of the above formalism to the specific example of the axially symmetric graded index fiber. This formalism leads to the wavelength-dependent modifications of the six aberrations present in the traditional prescriptions and further gives rise to the remaining three aberrations permitted by the axial symmetry. Besides, it also gives rise to a wavelength-dependent image rotation. The three extra aberrations and the image rotation are not found in any of the traditional approaches.

1 Introduction
In Part-I and Part-II we presented the exact matrix representation of the Maxwell equations in a medium with varying permittivity and permeability [1, 2]. From this we derived an exact optical Hamiltonian, which was shown to be in close algebraic analogy with the Dirac equation. This enabled us to apply the machinery of the Foldy-Wouthuysen transformation and we obtained an expansion for the beam-optical Hamiltonian which works to all orders. Formal expressions were obtained for the paraxial Hamiltonian and the leading order aberrating Hamiltonian, without assuming any form for the refractive index. Even at the paraxial level the wavelength-dependent effects manifest by the presence of a matrix term coupled to the logarithmic gradient of the refractive index. This matrix term is very similar to the spin
term in the Dirac equation and we call it as the polarizing term in our formal-
ism. The aberrating Hamiltonian contains numerous wavelength-dependent
terms in two guises: One of these is the explicit wavelength-dependent terms
coming from the commutators inbuilt in the formalism with $\lambda$ playing the
role played by $\hbar$ in quantum mechanics. The other set arises from the the
polarizing term.

Now, we apply the formalism to specific examples. One is the medium
with constant refractive index. This is perhaps the only problem which can
be solved exactly in a closed form expression. This is just to illustrate how
the aberration expansion in our formalism can be summed to give the familiar
exact result.

The next example is that of the axially symmetric graded index medium.
This example enables us to demonstrate the power of the formalism, repro-
ducing the familiar results from the traditional approaches and further giving
rise to new results, dependent on the wavelength.

2 Medium with Constant Refractive Index

Constant refractive index is the simplest possible system. In our formalism,
this is perhaps the only case where it is possible to do an exact diagonal-
ization. This is very similar to the exact diagonalization of the free Dirac
Hamiltonian. From the experience of the Dirac theory we know that there
are hardly any situations where one can do the exact diagonalization. One
necessarily has to resort to some approximate diagonalization procedure. The Foldy-Wouthuysen transformation scheme provides the most convenient
and accurate diagonalization to any desired degree of accuracy. So we have
adopted the Foldy-Wouthuysen scheme in our formalism.

For a medium with constant refractive index, $n(r) = n_c$, we have,

$$\hat{H}_c = -n_c\beta + i(M_y p_x - M_x p_y), \quad (1)$$

which is exactly diagonalized by the following transform,

$$T^\pm = \exp \left[ i (\pm i \beta) \hat{O}\theta \right] = \exp \left[ \mp i \beta (M_y p_x - M_x p_y) \theta \right]$$

$$= \cosh (||\hat{p}_\perp||\theta) \mp i \beta \frac{(M_y p_x - M_x p_y)}{||\hat{p}_\perp||} \sinh (||\hat{p}_\perp||\theta) \quad (2)$$
We choose,
\[ \tanh(2|\hat{p}_\perp|\theta) = \frac{|\hat{p}_\perp|}{n_c} \]  
(3)
then
\[ T^\pm = \frac{(n_c + P_z) \mp i\beta (M_y p_x - M_x p_y)}{\sqrt{2} P_z (n_c + P_z)} \]  
(4)
where \( P_z = +\sqrt{(n_c^2 - \hat{p}_\perp^2)} \). Then we obtain,
\[ \hat{H}_c^{\text{diagonal}} = T^+ \hat{H}_c T^- 
= T^+ \{ -n_c \beta + i (M_y p_x - M_x p_y) \} T^- 
= - \left\{ n_c^2 - \hat{p}_\perp^2 \right\}^{1/2} \beta \]  
(5)
We next, compare the exact result thus obtained with the approximate ones, obtained through the systematic series procedure we have developed.
\[ \hat{H}_c^{(4)} = -n_c \left\{ 1 - \frac{1}{2n_c^2} \hat{p}_\perp^2 - \frac{1}{8n_c^4} \hat{p}_\perp^4 - \cdots \right\} \beta \]
\[ \approx -n_c \left\{ 1 - \frac{1}{n_c^2} \hat{p}_\perp^2 \right\}^{1/2} \beta \]
\[ = - \left\{ n_c^2 - \hat{p}_\perp^2 \right\}^{1/2} \beta \]
\[ = \hat{H}_c^{\text{diagonal}}. \]  
(6)
Knowing the Hamiltonian, we can compute the transfer maps. The transfer operator between any pair of points \{\((z'', z') | z'' > z'\)\} on the \(z\)-axis, is formally given by
\[ |\psi(z'', z')\rangle = \hat{T}(z'', z') |\psi(z'', z')\rangle, \]  
(7)
with
\[ i\hbar \frac{\partial}{\partial z} \hat{T}(z'', z') = \hat{H} \hat{T}(z'', z'), \quad \hat{T}(z'', z') = \hat{I}, \]
\[\hat{T}(z'', z') = \varphi \left\{ \exp \left[ -\frac{i}{\lambda} \int_{z'}^{z''} dz \hat{H}(z) \right] \right\} \]

\[\hat{T} = \hat{I} - \frac{i}{\lambda} \int_{z'}^{z''} dz \hat{H}(z) \]

\[+ \left(-\frac{1}{\lambda}\right)^2 \int_{z'}^{z''} dz \int_{z'}^{z''} dz' \hat{H}(z) \hat{H}(z') \]

\[+ \ldots, \]  \hspace{1cm} (8)

where \(\hat{I}\) is the identity operator and \(\varphi\) denotes the path-ordered exponential. There is no closed form expression for \(\hat{T}(z'', z')\) for an arbitrary choice of the refractive index \(n(r)\). In such a situation the most convenient form of the expression for the \(z\)-evolution operator \(\hat{T}(z'', z')\), or the \(z\)-propagator, is

\[\hat{T}(z'', z') = \exp \left[ -\frac{i}{\lambda} \hat{T}(z'', z') \right], \]  \hspace{1cm} (9)

with

\[\hat{T}(z'', z') = \int_{z'}^{z''} dz \hat{H}(z) \]

\[+ \frac{1}{2} \left(-\frac{1}{\lambda}\right) \int_{z'}^{z''} dz \int_{z'}^{z''} dz' \left[ \hat{H}(z), \hat{H}(z') \right] \]

\[+ \ldots, \]  \hspace{1cm} (10)

as given by the Magnus formula [3]. We shall be needing these expressions in the next example where the refractive index is not a constant.

Using the procedure outlined above we compute the transfer operator,

\[\hat{U}_c(z_{\text{out}}, z_{\text{in}}) = \exp \left[ -\frac{i}{\lambda} \Delta z \hat{H}_c \right] \]

\[= \exp \left[ +\frac{i}{\lambda} n_c \Delta z \left\{ 1 - \frac{1}{2} \frac{\hat{p}_z^2}{n_c^2} - \frac{1}{8} \left( \frac{\hat{p}_z^2}{n_c^2} \right)^2 - \cdots \right\} \right]. \]  \hspace{1cm} (11)

where, \(\Delta z = (z_{\text{out}}, z_{\text{in}})\). Using (11), we compute the transfer maps

\[\begin{pmatrix} \langle r_\perp \rangle \\ \langle p_\perp \rangle \end{pmatrix}_{\text{out}} = \begin{pmatrix} 1 & 0 \\ \frac{1}{\sqrt{n_c^2 - \hat{p}_z^2}} \Delta z \\ 0 & 1 \end{pmatrix} \begin{pmatrix} \langle r_\perp \rangle \\ \langle p_\perp \rangle \end{pmatrix}_{\text{in}}. \]  \hspace{1cm} (12)

The beam-optical Hamiltonian is intrinsically aberrating. Even for simplest situation of a constant refractive index, we have aberrations to all orders.
3 Axially Symmetric Graded Index Medium

The refractive index of an axially symmetric graded-index material can be most generally described by the following polynomial (see, pp. 117 in [4])

\[ n(r) = n_0 + \alpha_2(z)r_\perp^2 + \alpha_4(z)r_\perp^4 + \cdots, \] (13)

where, we have assumed the axis of symmetry to coincide with the optic-axis, namely the \( z \)-axis without any loss of generality. We note,

\[ \hat{\mathcal{E}} = -\left\{ \alpha_2(z)r_\perp^2 + \alpha_4(z)r_\perp^4 + \cdots \right\} \beta - i\lambda\beta \Sigma \cdot \mathbf{u} \]
\[ \hat{\mathcal{O}} = i(M_y p_x - M_x p_y) \]
\[ = \beta(M_\perp \cdot \hat{p}_\perp) \] (14)

where

\[ \Sigma \cdot \mathbf{u} = -\frac{1}{n_0}\alpha_2(z)\Sigma_\perp \cdot \mathbf{r}_\perp - \frac{1}{2n_0} \left( \frac{d}{dz}\alpha_2(z) \right) \Sigma_z r_\perp^2 \] (15)

To simplify the formal expression for the beam-optical Hamiltonian \( \hat{\mathcal{H}}^{(4)} \) given in (24-25) in Part-II, we make use of the following:

\[ (M_\perp \cdot \hat{p}_\perp)^2 = \hat{p}_\perp^2, \quad \hat{\mathcal{O}}^2 = -\hat{p}_\perp^2, \quad \frac{\partial}{\partial z} \hat{\mathcal{O}} = 0, \]
\[ (M_\perp \cdot \hat{p}_\perp) r_\perp^2 (M_\perp \cdot \hat{p}_\perp) = \frac{1}{2} \left( r_\perp^2 \hat{p}_\perp^2 + \hat{p}_\perp^2 r_\perp^2 \right) + 2\lambda\beta\hat{L}_z + 2\lambda^2, \] (16)

where, \( \hat{L}_z \) is the angular momentum. Finally, the beam-optical Hamiltonian to order \( \left( \frac{1}{n_0^2}\hat{p}_\perp^2 \right)^2 \) is

\[ \hat{\mathcal{H}} = \hat{H}_{0,p} + \hat{H}_{0,(4)} + \hat{H}_{0,(2)}^{(\lambda)} + \hat{H}_{0,(4)}^{(\lambda)} + \hat{H}^{(\lambda,\sigma)} \]
\[ \hat{H}_{0,p} = -n_0 + \frac{1}{2n_0} \hat{p}_\perp^2 - \alpha_2(z)r_\perp^2 \]
\[ \hat{H}_{0,(4)} = \frac{1}{8n_0^3}\hat{p}_\perp^4 \]
\[ -\frac{\alpha_2(z)}{4n_0^2} \left( r_\perp^2 \hat{p}_\perp^2 + \hat{p}_\perp^2 r_\perp^2 \right) \]
\[ \hat{H}_{0, (2)}^{(\lambda)} = -\frac{\alpha_4(z)}{2n_0^2} r_\perp^4 - \frac{\lambda}{2n_0^2} \alpha_2(z) \tilde{L}_z + \frac{\lambda^2}{2n_0^3} \alpha_2^2(z) r_\perp^2 \]

\[ \hat{H}_{0, (4)}^{(\lambda)} = \frac{\lambda}{4n_0^3} \alpha_2^2(z) \left( r_\perp^2 \tilde{L}_z + \tilde{L}_z r_\perp^2 \right) + \frac{\lambda^2}{2n_0^3} \alpha_2(z) \alpha_4(z) r_\perp^4 \]

\[ \hat{H}^{(\lambda, \sigma)} = \frac{i\lambda^3}{2n_0^3} \left\{ \frac{d}{dz} \alpha_2(z) \right\} \beta \Sigma_z \]

\[ + \frac{i\lambda^2}{4n_0^3} \alpha_2(z) \left( \Sigma_x p_y - \Sigma_y p_x \right) \]

\[ + \frac{i\lambda^3}{2n_0^3} \left\{ \frac{d}{dz} \alpha_2(z) \right\} \Sigma_z \tilde{L}_z \]

\[ + \frac{i\lambda}{4n_0^3} \alpha_2(z) \beta \left[ \Sigma_\perp \cdot r_\perp, \tilde{p}_\perp^2 \right]_+ \]

\[ + \frac{i\lambda}{8n_0^3} \left\{ \frac{d}{dz} \alpha_2(z) \right\} \beta \Sigma_z \left[ r_\perp^2, \tilde{p}_\perp^2 \right]_+ \]

\[ + \cdots \quad (17) \]

where \([A, B] = (AB + BA)\) and ‘\(\cdots\)’ are the numerous other terms arising from the polarization term. We have retained only the leading order of such terms above for illustration. All these matrix terms, related to the polarization, will be addressed elsewhere.

The reasons for partitioning the beam-optical Hamiltonian \(\hat{H}\) in the above manner are as follows. The paraxial Hamiltonian, \(\hat{H}_{0, p}\), describes the ideal behaviour. \(\hat{H}_{0, (4)}^{(\lambda)}\) is responsible for the third-order aberrations. Both of these Hamiltonians are modified by the wavelength-dependent contributions given in \(\hat{H}_{0, (2)}^{(\lambda)}\) and \(\hat{H}_{0, (4)}^{(\lambda)}\), respectively. Lastly, we have \(\hat{H}^{(\lambda, \sigma)}\), which is associated with the polarization.

From these sub-Hamiltonians we make several observations: The term \(\frac{\lambda}{2n_0^2} \alpha_2(z) \tilde{L}_z\) which contributes to the paraxial Hamiltonian, gives rise to an image rotation by an angle \(\theta(z)\):

\[ \theta(z'', z') = \frac{\lambda}{2n_0^2} \int_{z'}^{z''} dz \alpha_2(z). \quad (18) \]

This image rotation (which need not be small) has no analogue in the square-root approach [4, 5] and the scalar approach [6, 7].
The Hamiltonian $\hat{H}_{0,(4)}$ is the one we have in the traditional prescriptions and is responsible for the six aberrations. $\hat{H}_{0,(4)}^{(\lambda)}$ modifies the above six aberrations by wavelength-dependent contributions and further gives rise to the remaining three aberrations permitted by the axial symmetry. Before proceeding further we enumerate all the nine aberrations permitted by the axial symmetry. The axial symmetry permits exactly nine third-order aberrations which are:

| Symbol | Polynomial | Name                  |
|--------|------------|-----------------------|
| $C$    | $\hat{p}_\perp^4$ | Spherical Aberration |
| $K$    | $[\hat{p}_\perp^2, (\hat{p}_\perp \cdot \hat{r}_\perp + \hat{r}_\perp \cdot \hat{p}_\perp)]_+$ | Coma |
| $k$    | $\hat{p}_\perp^2 \hat{L}_z$ | Anisotropic Coma     |
| $A$    | $(\hat{p}_\perp \cdot \hat{r}_\perp + \hat{r}_\perp \cdot \hat{p}_\perp)^2$ | Astigmatism          |
| $a$    | $(\hat{p}_\perp \cdot \hat{r}_\perp + \hat{r}_\perp \cdot \hat{p}_\perp) \hat{L}_z$ | Anisotropic Astigmatism |
| $F$    | $(\hat{p}_\perp^2 \hat{r}_\perp^2 + \hat{r}_\perp^2 \hat{p}_\perp^2)$ | Curvature of Field   |
| $D$    | $[\hat{r}_\perp^2, (\hat{p}_\perp \cdot \hat{r}_\perp + \hat{r}_\perp \cdot \hat{p}_\perp)]_+$ | Distortion           |
| $d$    | $\hat{r}_\perp^4 \hat{L}_z$ | Anisotropic Distortion |
| $E$    | $\hat{r}_\perp^4$ | Nameless? or POCUS     |

The name POCUS is used in [4] on page 137.

The axial symmetry allows only the terms (in the Hamiltonian) which are produced out of, $\hat{p}_\perp^2$, $\hat{r}_\perp^2$, $(\hat{p}_\perp \cdot \hat{r}_\perp + \hat{r}_\perp \cdot \hat{p}_\perp)$ and $\hat{L}_z$. Combinatorially, to fourth-order one would get ten terms including $\hat{L}_z^2$. We have listed nine of them in the table above. The tenth one namely,

$$\hat{L}_z^2 = \frac{1}{2} (\hat{p}_\perp^2 \hat{r}_\perp^2 + \hat{r}_\perp^2 \hat{p}_\perp^2) - \frac{1}{4} (\hat{p}_\perp \cdot \hat{r}_\perp + \hat{r}_\perp \cdot \hat{p}_\perp)^2 + \lambda^2$$

So, $\hat{L}_z^2$ is not listed separately. Hence, we have only nine third-order aberrations permitted by axial symmetry, as stated earlier.

The paraxial transfer maps are given by

$$\left( \begin{array}{c} \langle r_\perp \rangle \\ \langle p_\perp \rangle \end{array} \right)_{\text{out}} = \left( \begin{array}{cc} P & Q \\ R & S \end{array} \right) \left( \begin{array}{c} \langle r_\perp \rangle \\ \langle p_\perp \rangle \end{array} \right)_{\text{in}}, \quad (20)$$

where $P$, $Q$, $R$ and $S$ are the solutions of the paraxial Hamiltonian (17). The symplecticity condition tells us that $PS - QR = 1$. In this particular case from the structure of the paraxial equations we can further conclude that: $R = P'$ and $S = Q'$ where '$$ denotes the $z$-derivative.
The transfer operator is most accurately expressed in terms of the paraxial solutions, $P$, $Q$, $R$ and $S$, via the interaction picture [8].

$$
\hat{T}(z, z_0) = \exp \left[-\frac{i}{\lambda} \hat{T}_0(z, z_0)\right],
$$

$$
= \exp \left[-\frac{i}{\lambda} \left\{ C(z'', z') \hat{p}_\perp^4 + K(z'', z') \left[ \hat{p}_\perp \cdot (\hat{p}_\perp \cdot r_\perp + r_\perp \cdot \hat{p}_\perp) \right] + k(z'', z') \hat{p}_\perp^2 \hat{L}_z + A(z'', z') (\hat{p}_\perp \cdot r_\perp + r_\perp \cdot \hat{p}_\perp)^2 + a(z'', z') (\hat{p}_\perp \cdot r_\perp + r_\perp \cdot \hat{p}_\perp) \hat{L}_z + F(z'', z') (\hat{p}_\perp^2 r_\perp^2 + \hat{r}_\perp^2 \hat{p}_\perp^2) + D(z'', z') \left[ \hat{r}_\perp^2 \cdot (\hat{p}_\perp \cdot r_\perp + r_\perp \cdot \hat{p}_\perp) \right] + d(z'', z') \hat{r}_\perp^2 \hat{L}_z + E(z'', z') \hat{r}_\perp^4 \right\} \right].
$$

(21)

The nine aberration coefficients are given by,

$$
C(z'', z') = \int_{z''}^{z'''} dz \left\{ \frac{1}{8n_0^3} S^4 - \frac{\alpha_2(z)}{2n_0^2} Q^2 S^2 - \alpha_4(z) Q^4 + \frac{\lambda^2}{2n_0^2} \alpha_2(z) \alpha_4(z) Q^4 \right\}
$$

$$
K(z'', z') = \int_{z''}^{z'''} dz \left\{ \frac{1}{8n_0^3} R S^3 - \frac{\alpha_2(z)}{4n_0^2} Q S (P S + Q R) - \alpha_4(z) P Q^3 + \frac{\lambda^2}{2n_0^2} \alpha_2(z) \alpha_4(z) P Q^3 \right\}
$$

$$
k(z'', z') = \frac{\lambda}{2n_0^3} \int_{z''}^{z'''} dz \alpha_2(z) Q^2
$$

$$
A(z'', z') = \int_{z''}^{z'''} dz \left\{ \frac{1}{8n_0^3} R^2 S^2 - \frac{\alpha_2(z)}{2n_0^2} P Q R S - \alpha_4(z) P^2 Q^2 + \frac{\lambda^2}{2n_0^2} \alpha_2(z) \alpha_4(z) P^2 Q^2 \right\}
$$
\[
a(z'', z') = \frac{\bar{\lambda}}{2n_0^2} \int_{z'}^{z''} dz \alpha_2(z) PQ
\]

\[
F(z'', z') = \int_{z'}^{z''} dz \left\{ \frac{1}{8n_0^2} R^2 S^2 - \frac{\alpha_2(z)}{4n_0^2} (P^2 S^2 + Q^2 R^2) - \alpha_4(z) P^2 Q^2 \right. \\
\left. + \frac{\bar{\lambda}^2}{2n_0^2} \alpha_2(z) \alpha_4(z) P^2 Q^2 \right\}
\]

\[
D(z'', z') = \int_{z'}^{z''} dz \left\{ \frac{1}{8n_0^2} R^3 S - \frac{\alpha_2(z)}{4n_0^2} PR (PS + QR) - \alpha_4(z) P^3 Q \right. \\
\left. + \frac{\bar{\lambda}^2}{2n_0^2} \alpha_2(z) \alpha_4(z) P^3 Q \right\}
\]

\[
d(z'', z') = \frac{\bar{\lambda}}{2n_0^2} \int_{z'}^{z''} dz \alpha_2(z) P^2
\]

\[
E(z'', z') = \int_{z'}^{z''} dz \left\{ \frac{1}{8n_0^2} R^4 - \frac{\alpha_2(z)}{2n_0^2} P^2 R^2 - \alpha_4(z) P^4 \right. \\
\left. + \frac{\bar{\lambda}^2}{2n_0^2} \alpha_2(z) \alpha_4(z) P^4 \right\}.
\]

(22)

Thus we see that the current approach gives rise to all the nine permissible aberrations. The six aberrations, familiar from the traditional prescriptions get modified by the wavelength-dependent contributions. The extra three \((k, a \text{ and } d \text{ are all anisotropic!})\) are all pure wavelength-dependent aberrations and totally absent in the traditional square-root approach \[4, 5\] and the recently developed scalar approach \[6, 7\]. A detailed account on the classification of aberrations is available in \[8-12\].

4 Conclusions

In Part-I and Part-II, we developed an exact matrix representation of the Maxwell equations which became the basis for an exact formalism of Maxwell optics. An exact optical Hamiltonian, with an algebraic structure in direct correspondence with the Dirac equation of the electron was derived. Then following a Foldy-Wouthuysens transformation technique, a procedure was developed to obtain the beam optical Hamiltonians to any desired degree of accuracy. Formal expressions were obtained for the paraxial and leading
order aberrating Hamiltonians, without making any assumption on the form of the refractive index. In this Part we look at the applications of the above formalism.

First of the two examples is the medium with a constant refractive index. This is perhaps the only problem which can be solved exactly, in a closed form expression. This example is primarily for illustrating certain aspects of the machinery we have used.

The second, and the more interesting example is that of the axially symmetric graded index medium. For this example, in the traditional approaches one gets only six aberrations. In our formalism we get all the nine aberrations permitted by the axial symmetry. The six aberration coefficients of the traditional approaches get modified by the wavelength-dependent contributions.

It is very interesting to note that apart from the wavelength-dependent modifications of the aberrations, this approach also gives rise to the image rotation. This image rotation is proportional to the wavelength and we have derived an explicit relationship for the angle in \( \text{(18)} \). Such, an image rotation has no analogue/counterpart in any of the traditional prescriptions. It would be worthwhile to experimentally look for the predicted image rotation. The existence of the nine aberrations and image rotation are well-known in axially symmetric magnetic electron lenses, even when treated classically. The quantum treatment of the same system leads to the wavelength-dependent modifications \[ \text{(13)} \].

The optical Hamiltonian has two components: Beam-Optics and Polarization. We have addressed the former in some detail and shall do the later soon. The formalism initiated in this article provides a natural framework for the study of light polarization. This would provide a unified treatment for the beam-optics and the polarization. It also promises a possible generalization of the substitution result in \( \text{(14)} \). We shall present this approach soon \[ \text{(17)} \].

The close analogy between geometrical optics and charged-particle has been known for too long a time. Until recently it was possible to see this analogy only between the geometrical optics and classical prescriptions of charge-particle optics. A quantum theory of charged-particle optics was presented in recent years \[ \text{(18, 19, 20, 13)} \]. With the current development of the non-traditional prescriptions of Helmholtz optics \[ \text{(8, 9)} \] and the matrix formulation of Maxwell optics (in these three Parts), using the rich algebraic
machinery of quantum mechanics it is now possible to see a parallel of the analogy at each level. The non-traditional prescription of the Helmholtz optics is in close analogy with the quantum theory of charged-particles based on the Klein-Gordon equation. The matrix formulation of Maxwell optics presented here is in close analogy with the quantum theory of charged-particles based on the Dirac equation. We shall narrate and examine the parallel of these analogies soon [21].

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