Cosmic age problem revisited in the holographic dark energy model

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Because of an old quasar APM 08279 + 5255 at z = 3.91, some dark energy models face the challenge of the cosmic age problem. It has been shown by Wei and Zhang [Phys. Rev. D 76, 063003 (2007)] that the holographic dark energy model is also troubled with such a cosmic age problem. In order to accommodate this old quasar and solve the age problem, we propose in this Letter to consider the interacting holographic dark energy in a non-flat universe. We show that the cosmic age problem can be eliminated when the interaction and spatial curvature are both involved in the holographic dark energy model.

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I. INTRODUCTION

The fact that our universe is undergoing accelerated expansion has been confirmed by lots of astronomical observations such as type Ia supernovae (SNIa) [1], large scale structure (LSS) [2] and cosmic microwave background (CMB) anisotropy [3]. It is the most accepted idea that this cosmic acceleration is caused by some kind of negative-pressure matter known as dark energy whose energy density has been dominant in the universe. The combined analysis of cosmological observations indicates that the universe today consists of about 70% dark energy, 30% dust matter (cold dark matter plus baryons), and negligible radiation. The famous cosmological constant $\lambda$ introduced first by Einstein is the simplest candidate for dark energy. However, the cosmological constant scenario has to face the so-called “fine-tuning problem” and “cosmic coincidence problem” [4]. Many dark energy models have been proposed, while the nature of dark energy is still obscure. Besides quintessence [5], a wide variety of scalar-field dark energy models have been studied including k-essence [6], hessence [7], phantom [8], tachyon [9], quintom [10], ghost condensate [11], etc. In addition, there are other proposals on dark energy such as interacting dark energy models [12], brane world models [13], Chaplygin gas models [14], Yang-Mills condensate models [15], and so on.

The dark energy problem is essentially an issue of quantum gravity, owing to the concern of the vacuum expectation value of some quantum fields in a gravity universe. However, by far, we have no a complete theory of quantum gravity yet. So, it seems that we have to consider the effects of gravity in some effective quantum field theory in which some fundamental principles of quantum gravity could be taken into account. It is commonly believed that the holographic principle [16] is just a fundamental principle of quantum gravity. Based on the effective quantum field theory, Cohen et al. [17] pointed out that the quantum zero-point energy of a system with size $L$ should not exceed the mass of a black hole with the same size, i.e., $L^3 \Lambda^4 \leq L M_{Pl}^2$, where $\Lambda$ is the ultraviolet (UV) cutoff of the effective quantum field theory, which is closely related to the quantum zero-point energy density, and $M_{Pl} \equiv 1/\sqrt{8\pi G}$ is the reduced Planck mass. This observation relates the UV cutoff of a system to its infrared (IR) cutoff. When we take the whole universe into account, the vacuum energy related to this holographic principle can be viewed as dark energy (its energy density is denoted as $\rho_{\Lambda}$ hereafter). The largest IR cutoff $L$ is chosen by saturating the inequality, so that we get the holographic dark energy density

$$\rho_{\Lambda} = 3c^2 M_{Pl}^2 L^{-2}$$

where $c$ is a numerical constant characterizing all of the uncertainties of the theory, and its value can only be determined by observations. If we take $L$ as the size of the current universe, say, the Hubble radius $H^{-1}$, then the dark energy density will be close to the observational result. However, Hsu [18] pointed out that this yields a wrong equation of state for dark energy. Subsequently, Li [19] suggested to choose the future event horizon of the universe as the IR cutoff of this theory. This choice not only gives a reasonable value for dark energy density, but also leads to an accelerated universe. Moreover, the cosmic coincidence problem can also be explained successfully in this model, provided that the inflation lasts for more than 60 $e$-folds. Most recently, a calculation of the Casimir energy of the photon field in a de Sitter space is performed [20], and it is a surprising result that the Casimir energy is indeed proportional to the size of the horizon (the usual Casimir energy in a cavity is inversely proportional to the size of the cavity), in agreement with the holographic dark energy model.

Up to now, the holographic dark energy model has been tested by various observational data including SNIa [21], SNIa+BAO+CMB [22, 23], X-ray gas mass fraction of galaxy clusters [24], differential ages of passively evolving galaxies [25], Sandage-Leob test [26], and so on [27]. These analyses show that the holographic dark energy model is consistent with the observational data. However, Wei and Zhang [28] used some old high redshift objects (OHROs) to test the holographic dark energy model and found that the original holographic dark energy model can be ruled out unless a lower Hubble constant (e.g., $h = 0.56$) is taken. So, according to Ref. [28], there is a cosmic age crisis in the holographic dark energy model.

In fact, many dark energy models are in the face of such a cosmic age problem. In history, the cosmic age problem has been focused in cosmology for several times. At present, the cosmic age crisis coming from some OHROs appears again in cosmological models, even though dark energy is involved in the models. In cosmology there is a very basic principle that the universe cannot be younger than
its constituents. So, if the age of some astronomical object (at some redshift) is measured accurately, then it can be used to test cosmological models according to this simple age principle. Now, there are some OHROs discovered, for example, the 3.5 Gyr old galaxy LBDS 53W091 at redshift $z = 1.55$ [29] and the 4.0 Gyr old galaxy LBDS 53W069 at redshift $z = 1.43$ [30]. In particular, the old quasar APM 08279 + 5255 at redshift $z = 3.91$ is an important one, which has been used as a “cosmic clock” to constrain cosmological models. Its age is estimated to be $2.0 - 3.0$ Gyr [31]. These three OHROs at $z = 1.43, 1.55$ and 3.91 have been used to test many dark energy models, including the ΛCDM model [32], the general EoS dark energy model [33], the scalar-tensor quintessence model [34], the $f(R) = \sqrt{R^2 - R_0^2}$ model [35], the DGP braneworld model [36], the power-law parameterized quintessence model [37], the Yang-Mills condensate model [38], the holographic dark energy model [28], the agegraphic dark energy model [39], and so on. These investigations show that the two OHROs at $z = 1.43$ and 1.55 can be easily accommodated in most dark energy models, whereas the OHRO at $z = 3.91$ cannot, even in the ΛCDM model [32] and the holographic dark energy model [28].

In this Letter, we revisit the cosmic age problem in the holographic dark energy model. We consider an interacting holographic dark energy model in a non-flat universe. We will show that the age crisis in the original holographic dark energy model can be avoided when the interaction and the spatial curvature are involved in the holographic dark energy model.

II. THE HOLOGRAPHIC DARK ENERGY MODEL WITH SPATIAL CURVATURE AND INTERACTION

In this section we describe the interacting holographic dark energy in a non-flat universe. In a spatially non-flat Friedmann-Robertson-Walker (FRW) universe, the Friedmann equation reads

$$3M_p^2H^2 = \rho_\Lambda + \rho_m - \frac{3M_p^2k}{a^2},$$

(2)

where $\rho_\Lambda = 3c^2M_p^2 L^{-2}$ is the holographic dark energy density, and $\rho_m$ is the energy density of matter. We define

$$\Omega_k = -\frac{k}{H^2a^2} = \Omega_0\left(\frac{H_0}{aH}\right)^2, \quad \Omega_\Lambda = \frac{\rho_\Lambda}{\rho_c}, \quad \Omega_m = \frac{\rho_m}{\rho_c},$$

(3)

where $\rho_c = 3M_p^2H^2$ is the critical density of the universe, thus we have

$$\Omega_m + \Omega_\Lambda + \Omega_k = 1.$$  

(4)

Now, let us consider some interaction between holographic dark energy and matter:

$$\dot{\rho}_m + 3H\rho_m = Q,$$

(5)

$$\dot{\rho}_\Lambda + 3H(\rho_\Lambda + p_\Lambda) = -Q,$$

(6)

where $Q$ denotes the phenomenological interaction term. Owing to the lack of the knowledge of micro-origin of the interaction, we simply follow other work on the interacting holographic dark energy and parameterize the interaction term generally as $Q = 3H(\alpha\rho_\Lambda + \beta\rho_m)$, where $\alpha$ and $\beta$ are the dimensionless coupling constants. For reducing the complication and the number of parameters, one often considers the following three cases: (i) $\beta = 0$, and thus $Q = 3\alpha H\rho_\Lambda$, (ii) $\alpha = \beta$, and thus $Q = 3\alpha H(\rho_\Lambda + \rho_m)$, and (iii) $\alpha = 0$, and thus $Q = 3\beta H\rho_m$. Note that in these three cases, according to our convention, $\alpha > 0$ (or $\beta > 0$) means that dark energy decays to matter. Moreover, it should be pointed out that $\alpha < 0$ (or $\beta < 0$) will lead to unphysical consequences in physics, since $\rho_m$ will become negative and $\Omega_\Lambda$ will be greater than 1 in the far future. So, in the present Letter, we only consider the physically reasonable situations, namely, $\alpha > 0$ or $\beta > 0$ in the above three cases. In the rest of this section, we will formulate the model generally (by using both $\alpha$ and $\beta$), but in the next section we will only consider the above three simpler cases due to the aforementioned reason.

From the definition of holographic dark energy (1), we have

$$\Omega_\Lambda = \frac{c^2}{H^2L^2},$$

(7)

or equivalently,

$$L = \frac{c}{H\sqrt{\Omega_\Lambda}}.$$  

(8)

Thus, we easily get

$$L = -\frac{c}{H\sqrt{\Omega_\Lambda}}\left(\frac{\dot{H}}{H^2} + \frac{\Omega_\Lambda}{2\Omega_\Lambda}\right).$$

(9)

Following Ref. [40], in a non-flat universe the IR cutoff length scale $L$ takes the form

$$L = ar(t),$$

(10)

and $r(t)$ satisfies

$$\int_0^{\alpha(t)} \frac{dr}{\sqrt{1 - kr^2}} = \int_t^{+\infty} \frac{dt}{a(t)}.$$  

(11)

Consequently, we have

$$r(t) = \frac{1}{\sqrt{k}} H^{-1}\sin\left(\sqrt{k} \int_t^{+\infty} \frac{dt}{a}\right) = \frac{1}{\sqrt{k}} H^{-1}\sin\left(\sqrt{k} \int_0^{\alpha(t)} \frac{da}{Ha^2}\right).$$

(12)

Equation (10) leads to another equation about $r(t)$, namely,

$$r(t) = \frac{L}{a} = \frac{c}{\sqrt{\Omega_\Lambda} Ha}.$$  

(13)

Combining Eqs. (12) and (13) yields

$$\sqrt{k} \int_t^{+\infty} \frac{dt}{a} = \arcsin\frac{c}{\sqrt{\Omega_\Lambda} aH}.$$  

(14)
Taking the derivative of Eq. (14) with respect to $t$, one can get

$$\sqrt{\frac{\Omega_\Lambda H^2}{c^2} - \frac{k}{a^2}} = \frac{\Omega_\Lambda}{2\Omega_\Lambda} + \frac{H}{\dot{H}}. \quad (15)$$

Let us combine Eqs. (5) and (6), and then we have $(\dot{\rho}_\Lambda + \dot{\rho}_m) + 3H(\rho_\Lambda + \rho_m + p_\Lambda) = 0$, which is equivalent to the equation $(\dot{\rho}_\Lambda - \dot{\rho}_m) + 3H(\rho_\Lambda - \rho_m + p_\Lambda) = 0$. From this equation, we can obtain the form of $p_\Lambda$:

$$p_\Lambda = -\frac{1}{3H}\left(\frac{2H}{H} \rho_\Lambda + \frac{2\dot{a}}{a} \rho_k\right) - \rho_c + \rho_k. \quad (16)$$

Combining this equation with Eq. (15), we eventually obtain the following equations governing the dynamical evolution of the interacting holographic dark energy in a non-flat universe,

$$\frac{1}{H/\bar{H}_0} \frac{d}{dz} \left(\frac{H}{\bar{H}_0}\right) = -\Omega_\Lambda \left(\frac{\Omega_\Lambda - 3 + \frac{\Omega_\Lambda(1+z)^2}{(H/\bar{H}_0)^2} + 3\alpha\Omega_\Lambda + 3\beta(1 - \Omega_\Lambda - \frac{\Omega_\Lambda(1+z)^2}{(H/\bar{H}_0)^2})}{2\Omega_\Lambda} + \frac{\Omega_\Lambda(1+z)^2}{(H/\bar{H}_0)^2}\right), \quad (19)$$

$$\frac{d\Omega_\Lambda}{dz} = -\frac{\Omega_\Lambda(1 - \Omega_\Lambda)}{1 + z} \left(2\sqrt{\frac{\Omega_\Lambda(1+z)^2}{(H/\bar{H}_0)^2}} + 1 - \frac{3\alpha\Omega_\Lambda + \frac{(1+z)^2}{(H/\bar{H}_0)^2} + 3\beta(1 - \Omega_\Lambda - \frac{\Omega_\Lambda(1+z)^2}{(H/\bar{H}_0)^2})}{1 - \Omega_\Lambda}\right). \quad (20)$$

These two equations can be solved numerically, and the solutions, $\Omega_\Lambda(z)$ and $H(z)$, determine the expansion history of the universe in the holographic dark energy model.

The holographic dark energy model with spatial curvature and interaction described in this section has been strictly constrained in Ref. [23] by using the current observational data including the SNIa Constitution data, the shift parameter of the CMB given by the five-year WMAP observations, and the BAO measurement from the SDSS. The main fitting results were summarized as Table I and Figs. 1-5 of Ref. [23]. In the following discussions, we restrict the values of parameters to the observational constraint results derived by Ref. [23]. Note that our definition of $\Omega_\Lambda$, $\alpha$ and $\beta$ are different from that of Ref. [23] by a minus sign.

III. TESTING THE MODEL WITH THE OHRO

The age of the universe at redshift $z$ is given by

$$\tau(z) = \int_0^z \frac{dz'}{(1 + z')H(z')} \quad (21)$$

For convenience, we introduce the dimensionless cosmic age

$$T_{cos}(z) \equiv H_0\tau(z) = \int_0^z \frac{dz'}{(1 + z')E(z')} \quad (22)$$

where $E(z) \equiv H(z)/H_0$, and for the holographic dark energy model it is given by the solutions of Eqs. (19) and (20). At any redshift, the age of the universe should be larger than, or at least equal to, the age of the OHRO, namely $T_{cos}(z) \geq T_{obj}(z) \equiv H_0 t_{obj}(z)$, where $t_{obj}(z)$ is the age of the OHRO at redshift $z$. Following Ref. [28], we define a dimensionless quantity, the ratio of the cosmic age and the OHRO age,

$$\tau(z) \equiv \frac{T_{cos}(z)}{T_{obj}(z)} = H_0^{-1} t_{obj}(z) \int_0^z \frac{dz'}{(1 + z')E(z')} \quad (23)$$

So, the condition $T_{cos}(z) \geq T_{obj}(z)$ is translated into $\tau(z) \geq 1$. From Eq. (23), it is easy to see that given the age of OHRO $t_{obj}(z)$, the lower $H_0$, the higher $\tau(z)$; given the Hubble constant $H_0$, the smaller $t_{obj}(z)$, the larger $\tau(z)$.

In the work of Wei and Zhang [28], the original holographic dark energy model (neither spatial curvature nor interaction is involved) has been examined by using the three OHROs, the old galaxy LBDS 53W091 at redshift $z = 1.55$, the old galaxy LBDS 53W069 at redshift $z = 1.43$, and the old quasar APM 08279 + 5255 at redshift $z = 3.91$. It is found in Ref. [28] that the former two OHROs, the old galaxy LBDS 53W091 at redshift $z = 1.55$ and the old galaxy LBDS 53W069 at redshift $z = 1.43$, can be easily accommodated, but the last one, the old quasar APM 08279 + 5255 at redshift $z = 3.91$, cannot be accommodated in the model. In the present Letter, we extend the holographic dark energy model to involving the spatial curvature and the interaction, as described in the previous section, and we shall examine whether the OHRO, the old quasar APM 08279 + 5255 at redshift $z = 3.91$, is consistent with such a sophisticated holographic dark energy model.

For the age of the OHRO at $z = 3.91$, following Ref. [28], we use the lower bound estimated, $t_{obj}(3.91) = 2.0$ Gyr. For the holographic dark energy model, since the main goal of this Letter is to probe the effects of spatial curvature and interaction in fighting against the cosmic age crisis, we keep the
values of $c$ and $\Omega_{\text{cdm}}$ fixed in the whole Letter. We take $c = 0.8$ and $\Omega_{\text{cdm}} = 0.28$ that are consistent with the observational constraint results of Ref. [23]. For decreasing the complication, let us close some parameters in turn. We shall consider the following three cases: (a) the model of holographic dark energy with spatial curvature but without interaction (namely, $\Omega_{\text{k0}} \neq 0$ but $Q = 0$), denoted as KHDE; (b) the model of holographic dark energy with interaction but without spatial curvature (namely, $Q \neq 0$ but $\Omega_{\text{k0}} = 0$), denoted as IHDE; (c) the model of holographic dark energy with both interaction and spatial curvature (namely, $Q \neq 0$ and $\Omega_{\text{k0}} \neq 0$), denoted as KIHDE. Next, let us discuss the use of the Hubble constant $H_0 = 100h$ km s$^{-1}$ Mpc$^{-1}$. Based on the HST key project, Freedman et al. [41] give the result $h = 0.72 \pm 0.08$. However, recently, many authors argue for a lower Hubble constant, say, $h = 0.68 \pm 0.07$ ($2\sigma$) [42]. Moreover, the final result of the 15-year HST program given by Sandage et al. [43] is $h = 0.623 \pm 0.063$ which has attracted more and more attention. Furthermore, when the holographic dark energy model is fitted via observational data (SNIa+BAO+CMB), a lower value of $h$ ($h \sim 0.65$) is obtained [23] (the latest fit value is $h = 0.686$ [44]). It should also be mentioned that the result of the 7-year WMAP observations (WMAP+BAO+$H_0$) is $h = 0.704_{-0.014}^{+0.013}$ [45], which is derived based on a $\Lambda$CDM model. In this Letter, we follow Ref. [23] and take $h = 0.64$ that is the lower bound of Freedman et al. [41]. We will also extend our discussion by taking some higher values of $h$ into account (say, we will also consider $h = 0.72$, the central value of Freedman et al. [41], which is high enough for our discussion, since it is even higher than the upper bound of WMAP 7-year result). Note that $T_{\text{obj}}(3.91) = 0.131$ is obtained according to $T_{\text{obj}}(3.91) = 2.0$ Gyr and $h = 0.64$.

First, we test the KHDE model. The current observational constraint result of the KHDE model is [23]: $-0.02 \lesssim \Omega_{\text{k0}} \lesssim 0.02$ ($1\sigma$). When we take $\Omega_{\text{k0}} = 0.02$, we find $\tau(3.91) = 0.866$, less than 1; when we take $\Omega_{\text{k0}} = -0.02$, we obtain $\tau(3.91) = 0.872$, still less than 1. So, we find that the spatial curvature is hard to help solve the cosmic age crisis for the holographic dark energy model. From the above example, we find that the value of $\tau$ in a closed space is greater than that in an open space. Thus, let us increase the value of $|\Omega_{\text{k0}}|$ in a closed space geometry in order to see whether the problem can be solved in some extremal cases. Our efforts can be found in Table I. In this table, we see that even the value of $\Omega_{\text{k0}}$ is taken to be $-0.1$, the value of $\tau$ derived is merely 0.883, far from solving the cosmic age problem. In addition, we also plot the $T_{\text{cav}}(z)$ curves for the KHDE model in Fig. 1. It can be explicitly seen from this figure that the cosmic age problem is still acute in the KHDE model. Therefore, the conclusion is that the cosmic age crisis cannot be avoided by only considering the spacial curvature in the holographic dark energy model.

For the IHDE model, we consider the aforementioned three cases: (I) $\beta = 0$, named IHDE1; (II) $\alpha = \beta$, named IHDE2; (III) $\alpha = 0$, named IHDE3. To see how the interaction influences the cosmic age in the holographic dark energy model, we calculate the age for these three cases in Table II. From this table, we see that with the increase of the interaction parameter $\alpha$ or $\beta$, the cosmic age $T_{\text{cav}}$ also increases. It is clear that the value of $\tau(3.91)$ can be greater than 1 when the value of $\alpha$ (or $\beta$) is large enough. For example, for the case of IHDE2 (Case II), when $\alpha$ is taken to be 0.03, the value of $\tau(3.91)$ obtained is 1.005. The cosmic age $T_{\text{cav}}$ versus redshift $z$ in the case of IHDE2 is also displayed in Fig. 2. This figure shows explicitly that the age problem can be overcome when the interaction is involved in the holographic dark energy model. However, it should be pointed out that the parameter values making $\tau(3.91) > 1$ actually exceed the $2\sigma$ regions given by Ref. [23]. Therefore, if we confine our discussions in the parameter space constrained by current observational data, the problem is not so easy as it looks. Nevertheless, it is found in Ref. [23] that when simultaneously considering the interaction and spatial curvature in the holographic dark energy model, the parameter space is amplified, especially, the ranges of $\alpha$ (or $\beta$) and $\Omega_{\text{k0}}$ are enlarged by 10 times comparing to the IHDE and KHDE models. Based on this fact, it can be expected that the age problem could be solved when the interaction and spatial curvature are both taken into account.

Now, let us consider the KIHDE model. For simplicity, in our discussion we fix $\Omega_{\text{k0}} = -0.06$. The three phenomenological interaction cases are the same as in the IHDE model. Since the parameter space of the KIHDE model is greatly amplified, the interaction parameter can be chosen to be some large
TABLE II: The values of $T_{\text{cor}}(3.91)$ and $\tau(3.91)$ in the IHDE models with $c = 0.8, \Omega_{\text{mat}} = 0.28$ and $h = 0.64$.

| Case I ($\beta = 0$) | $\alpha$ | $\tau_{\text{cor}}(3.91)$ | $\tau(3.91)$ |
|---------------------|---------|--------------------------|-------------|
| $\beta$             | 0.02    | 0.1172 0.1246 0.1335 0.1475 | 0.894 0.951 1.019 1.126 |

| Case II ($\alpha = \beta$) | $\alpha$ | $\tau_{\text{cor}}(3.91)$ | $\tau(3.91)$ |
|---------------------------|---------|--------------------------|-------------|
| $\beta$                   | 0.01    | 0.1194 0.1253 0.1316 0.1456 | 0.912 0.957 1.005 1.111 |

| Case III ($\alpha = 0$) | $\beta$ | $\tau_{\text{cor}}(3.91)$ | $\tau(3.91)$ |
|------------------------|---------|--------------------------|-------------|
| $\beta$                | 0.01    | 0.1177 0.1259 0.1346 0.1440 | 0.899 0.961 1.028 1.099 |

values. For example, for the KIHDE2 case, one can choose $\alpha = 0.05$ that is allowed by current observations, then the result $\tau(3.91) = 1.137$ is obtained. Some typical examples for all the three cases are shown in Table III, where the values of the interaction parameters are taken within the $2\sigma$ ranges of the observational constrains given by Ref. [23]. It is explicitly shown that the cosmic age problem can be successfully solved in the KIHDE model. For clarity, we plot the curves of $T_{\text{cor}}(z)$ in Fig. 3. This figure shows a direct comparison of HDE, KHDE, IHDE, and KIHDE (the Case II of interaction is taken as an example in this figure). It should be noted that $\alpha = 0.05$ is not allowed in the IHDE model but is allowed in the KIHDE model, from the viewpoint of observation.

From Fig. 3, we also see that the age problem can be evaded in the KIHDE even a much larger value of $h$ is taken, for instance, when $h = 0.72$, we get $T_{\text{cor}}(3.91) = 0.147, T_{\text{cor}}(3.91) = 0.149$, and thus $\tau > 1$ in this case. Moreover, when a larger age of the quasar is taken, say, $T_{\text{cor}}(3.91) = 2.1$ Gyr, the age problem can also be overcome in the KIHDE model; in Table IV one can find the values of $T_{\text{cor}}(3.91)$ corresponding to $T_{\text{cor}}(3.91) = 2.1$ Gyr for $h = 0.64$ and 0.72. Therefore, the cosmic age crisis can be avoided in the holographic dark energy model when the interaction and spatial curvature are both taken into account. Nevertheless, we have to admit that the price for solving the age problem has been paid by the holographic dark energy model, i.e., there are too many free parameters have to be considered in the model. This would inevitably weaken, to some extent, the plausibility of the model.

TABLE III: The values of $T_{\text{cor}}(3.91)$ and $\tau(3.91)$ in the KIHDE models with $c = 0.8, \Omega_{\text{mat}} = 0.28, \Omega_{\text{de}} = -0.06$ and $h = 0.64$.

| Case I ($\beta = 0$) | $\alpha$ | $T_{\text{cor}}(3.91)$ | $\tau(3.91)$ |
|---------------------|---------|---------------------|-------------|
| $\beta$             | 0.1     | 0.1375 0.1489 0.1593 | 1.049 1.137 1.521 |

| Case II ($\alpha = \beta$) | $\alpha$ | $T_{\text{cor}}(3.91)$ | $\tau(3.91)$ |
|-----------------------------|---------|---------------------|-------------|
| $\beta$                     | 0.05    | 0.1364 0.1523       | 1.041 1.239 |

| Case III ($\alpha = 0$) | $\beta$ | $T_{\text{cor}}(3.91)$ | $\tau(3.91)$ |
|-------------------------|---------|---------------------|-------------|
| $\beta$                 | 0.05    | 0.1364 0.1523       | 1.041 1.239 |

Of course, to be honest, it should also be confessed that the age problem would still exist if one considers some extremal cases such as a much larger possible age of the quasar $t_{\text{obj}}$ with a larger $h$. Consider the upper limit of the quasar age, $T_{\text{cor}}(3.91) = 3.0$ Gyr. For this extreme case, when $h = 0.64$, we have $T_{\text{cor}}(3.91) = 0.196$; when $h = 0.72$, we have $T_{\text{cor}}(3.91) = 0.220$; see also Table IV. So, we have to admit that for the limit case of $T_{\text{cor}}(3.91) = 3.0$ Gyr and $h = 0.72$ the age problem cannot be solved yet even in the KIHDE model.
TABLE IV: The values of $T_{ob}/(3.91)$ corresponding to different $h$ and $t_{ob}/(3.91)$.

| $h$   | $t_{ob}/(3.91)$/Gyr | $T_{ob}/(3.91)$ |
|-------|---------------------|-----------------|
| 2.0   | 0.131               |                 |
| 0.64  | 2.1                | 0.137           |
| 3.0   |                    | 0.196           |
| 0.72  | 2.0                | 0.147           |
| 3.0   |                    | 0.220           |

IV. CONCLUSION

In this Letter, we have revisited the cosmic age problem in the holographic dark energy model. The cosmic age problem brought by the old quasar APM 08279 + 5255 has caused trouble to many cosmological models, and the holographic dark energy model is not an exception either [28]. In order to accommodate the old quasar APM 08279 + 5255 in the holographic dark energy model, we propose to consider the interaction between dark energy and matter in the model. We have shown that the quasar indeed can be accommodated in the holographic dark energy model when an appropriate interaction strength is chosen. Taking the current observational constraints [23] into account, we have demonstrated that both interaction and spatial curvature should be simultaneously involved in the holographic dark energy model. It has been shown that if such a sophisticated case is considered the quasar APM 08279 + 5255 can be accommodated and the cosmic age problem can thus be avoided in the holographic dark energy model. The price of solving the age problem in this way is also apparent, i.e., the model involves too many free parameters, which may weaken the plausibility of the model, to some extent.

It is well known that the consideration of interaction in the holographic dark energy can be used to avoid the future big-rip singularity caused by $c < 1$ [23, 46]. In this Letter we have provided another advantage for the consideration of interaction in the holographic dark energy, i.e., the interaction between dark energy and matter can also be used to avoid the age problem caused by the old quasar. So, our result can be viewed as a further support to the interacting holographic dark energy model.

Of course, we have to confess that the age problem would still exist if some extreme cases are taken into account, say, a much larger possible age of the quasar $t_{ob}$ with a larger $h$. It is remarkable that the age of the old quasar APM 08279 + 5255 has not been measured accurately yet, and the age problem caused by this quasar has troubled many dark energy models (including the $\Lambda$CDM model). It is expected that the future accurate measurement on the age of this old quasar would eliminate the cosmic age crisis in dark energy models.

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