Generalised Emergent Dark Energy Model: Confronting $\Lambda$ and PEDE

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ABSTRACT

We introduce a generalised form of an emergent dark energy model with one degree of freedom for the dark energy sector that has the flexibility to include both $\Lambda$CDM as well as the PEDE model (Phenomenologically Emergent Dark Energy) proposed by Li & Shafieloo (2019) as two of its special limits. This allows us to compare statistically these models in a straightforward way and following conventional Bayesian approach. The free parameter for the dark energy sector, namely $\Delta$, has the value of 0 for the case of the $\Lambda$ and 1 for the case of PEDE and its posterior fitting the generalised parametric form to different data can directly show the consistency of the models to the data. Fitting the introduced parametric form to Planck CMB data and most recent $H_0$ results from local observations of Cepheids and Supernovae (Riess et al. 2019), we show that the $\Delta = 0$ associated with the $\Lambda$CDM model would fall out of $4\sigma$ confidence limits of the derived posterior of the $\Delta$ parameter. In contrast, PEDE model can satisfy the combination of the observations. This is another support for the case of PEDE model with respect to the standard $\Lambda$CDM model if we trust the reliability of both Planck CMB data and local $H_0$ observations.

Keywords: Cosmology: observational - Dark Energy - Methods: statistical

1. INTRODUCTION

The standard $\Lambda$CDM model of cosmology has been greatly successful in describing cosmological observations including type Ia Supernovae (SNe Ia) (Riess et al. 1998, 2007; Scolnic et al. 2018), Baryon Acoustic Oscillation (BAO) (Beutler et al. 2011; Ross et al. 2015; Alam et al. 2017; Zhao et al. 2018) and Cosmic Microwave Background (CMB) (Ichiki & Nagata 2009; Komatsu et al. 2011; Ade et al. 2016; Aghanim et al. 2018). Despite of its success, comparing the Hubble constant values estimated from local observations of Cepheid in the Large Magellanic (LMC) (Riess et al. 2019) and the predicted values from Planck cosmic microwave background (CMB) observations assuming $\Lambda$CDM model (Aghanim et al. 2018) represent a serious discrepancy. Considering some most recent observations, the tension can reach to about 5$\sigma$ in significance. This implies that either there are considerable, but not accounted systematic-errors in our observations, or, we might need to consider modifications to the standard $\Lambda$CDM model.

Many ideas have been put forward to resolve the tensions, such as interaction dark energy models (Kumar & Nunes 2017; Di Valentino et al. 2017b; Zheng et al. 2017; Yang et al. 2018b,a; Kumar et al. 2019; Pan et al. 2019a), metastable dark energy models (Shafieloo et al. 2017; Li et al. 2019), Quintom dark energy model (Panpanich et al. 2019) and so on (Di Valentino et al. 2015, 2016, 2017a; Solà et al. 2017; Di Valentino et al. 2018a,b; Khosravi et al. 2019; Ó Colgáin et al. 2019; D’Eramo et al. 2018; Guo et al. 2019; Yang et al. 2019a; Poulin et al. 2019; Yang et al. 2019b; Di Valentino et al. 2019; Visinelli et al. 2019; Schöneberg et al. 2019; Kreisch et al. 2019) In the work of Li & Shafieloo (2019), the authors introduced a zero freedoms dark energy model -Phenomenologically Emergent Dark Energy (PEDE)– where dark energy has no effective presence in the past and emerges at the later times. This PEDE model was revisited with CMB, BAO and SNe Ia in Pan et al. (2019b) and Arendse et al. (2019). The results for PEDE model show that if there is no substantial systematics in either of SNe Ia, BAO or Planck CMB data and assuming reliability of the current local $H_0$ measurements, there is a very high probability that with slightly more precise measurement of the Hubble constant, PEDE model could rule out the cosmological constant with decisive statistical significance.
However, it has not been possible to compare these two models directly and statistically in a straightforward manner since they are distinct models with the same degrees of freedom without any flexibility for the dark energy sector. In other words unlike many extensions of the concordance model, \( \Lambda \) with \( w = -1 \) for the equation of state of dark energy is not a specific point in the parameter space of the PEDE model and vice versa.

In this work, we propose a generalised parameterization form for PEDE model that can include both cosmological constant and PEDE model. Based on PEDE model, we introduce two new parameters to describe the properties of dark energy evolution: one free parameter namely \( \Delta \) to describe the evolution slope of dark energy density and parameter \( z_t \) that describes the transition redshift where dark energy density equals to matter density. The transition redshift \( z_t \) locates where dark energy density equals to matter density and is not a free parameter. This Generalised Emergent Dark Energy (GEDE) model has the flexibility to include both \( \Lambda \)CDM model as well as the PEDE model as two of its special limits with \( \Delta = 0 \) and \( \Delta = 1 \), respectively.

We confront this model with CMB from Planck 2018 measurements (Aghanim et al. 2018) and most recent \( H_0 \) results from local observations of Cepheid in the Large Magellanic (LMC) (Riess et al. 2019). We show that the constraints on the parameter \( \Delta \) using the combination of local \( H_0 \) measurement and Planck 2018 CMB results, can rule out the standard \( \Lambda \)CDM model at more than 4\( \sigma \) level where the suggested data combination suggests an emergent behavior for dark energy.

This paper is organised as follows: in section 2 we briefly introduce the Friedmann equations for GEDE model and the observational data to be used. Section 3 contains our main results and some discussion. We conclude in section 4.

2. GENERALISED EMERGENT DARK ENERGY (GEDE) MODEL

Assuming a spatially flat universe and the Friedmann-Lemaître-Robertson-Walker (FLRW) metric, the Hubble parameter could be written as

\[
H^2(a) = H_0^2 \left[ \Omega_{\text{DE}}(a) + \Omega_{\text{m,0}} a^{-3} + \Omega_{\text{R,0}} a^{-4} \right] \tag{1}
\]

where \( a = 1/(1 + z) \) is the scale factor, \( \Omega_{\text{m,0}} \) and \( \Omega_{\text{R,0}} \) is the current matter density and radiation density, respectively. Here \( \Omega_{\text{DE}}(a) \) is defined as

\[
\Omega_{\text{DE}}(a) = \frac{\rho_{\text{DE}}(a)}{\rho_{\text{crit,0}}} = \frac{\rho_{\text{DE}}(a)}{\rho_{\text{crit}}(a)} x \frac{\rho_{\text{crit}}(a)}{\rho_{\text{crit,0}}} \tag{2}
\]

\[
= \Omega_{\text{DE}}(a) \times \frac{H^2(a)}{H_0^2} \tag{3}
\]

and \( \rho_{\text{crit},0} = \frac{3H_0^2}{8\pi G} \), \( \rho_{\text{crit}}(a) = \frac{3H^2(a)}{8\pi G} \). In \( \Lambda \)CDM model, \( \Omega_{\text{DE}}(a) = (1 - \Omega_{\text{m,0}} - \Omega_{\text{R,0}}) \) is constant.

In GEDE model, the evolution for dark energy density has the following form:

\[
\Omega_{\text{DE}}(z) = \frac{1 - \tanh \left( \Delta \times \log_{10}(\frac{1+z}{1+z_t}) \right)}{1 + \tanh \left( \Delta \times \log_{10}(1 + z_t) \right)} \tag{4}
\]

here \( \Omega_{\text{DE},0} = (1 - \Omega_{\text{m,0}} - \Omega_{\text{R,0}}) \) and transition redshift \( z_t \) can be derived by the condition of \( \Omega_{\text{DE}}(z_t) = \Omega_{\text{m,0}}(1 + z_t)^3 \) (hence it is not a free parameter). In this model, when setting \( \Delta = 0 \), this model recovers \( \Lambda \)CDM model and when setting \( \Delta = 1 \), it becomes the PEDE model which was introduced in Li & Shafieloo (2019), except that in Li & Shafieloo (2019) the authors set \( z_t = 0 \) for simplicity while parameter \( z_t \) in this work is treated as a transition redshift parameter related to matter density \( \Omega_{\text{m}} \) and \( \Delta \). In Figure 1, we show \( z_t \) as a function of matter density \( \Omega_{\text{m},0} \) for some certain values of \( \Delta \) for demonstration.

![Figure 1. Transition redshift \( z_t \) as a function of matter density \( \Omega_{\text{m},0} \) for different values of parameter \( \Delta \).](image)

We can derive the equation of state of GEDE model following:

\[
w(z) = \frac{1}{3} \frac{d\ln \Omega_{\text{DE}}(1 + z)}{dz} - 1 \tag{5}
\]

where we get,

\[
w(z) = -\frac{\Delta}{3\ln 10} \left( 1 + \tanh \left( \Delta \times \log_{10}(\frac{1+z}{1+z_t}) \right) \right) - 1. \tag{6}
\]

While the derived equation of state of dark energy seems to have a complicated form, it has in fact a simple physical behaviour related to dark energy density. We should note that for the two special cases of \( \Delta = 0 \) and
\( \Delta = 1 \) we would get the \( \Lambda \)CDM model and PEDE-CDM model, respectively.

Assuming a spatially flat universe and \( \Omega_{m,0} = 0.3 \), we show some properties for GEDE model as a function of redshift for some certain values of \( \Delta \) in Figure 2. Upper left plot shows the evolution of equation of state \( w(z) \) while upper right plot shows the evolution of dark energy density \( \Omega_{\text{DE}}(z) \) from redshift \( 10^{-3} \) to \( 10^2 \). Lower plots show the evolution of Expansion rate \( \dot{H}(z) = H(z)/H_0 \) (left) and deceleration parameter \( q(z) \) (right). Different colors correspond to parameter \( \Delta \) fixed at different values for demonstration. In terms of linear scale of redshift from 0 to 2.5, we show the evolution for \( w(z) \), \( h(z) \) and \( q(z) \) in each inner plot, respectively.

In our analysis, we consider CMB measurement from Planck TT, TE, EE+lowE data released in 2018 (Aghanim et al. 2018). In addition to CMB measurements, we add local measurement \( H_0 = 74.03 \pm 1.42 \) from Riess et al. (2019) in our analysis (denote as R19 hereafter). The constraint results are obtained with Markov Chain Monte Carlo (MCMC) estimation using CosmoMC (Lewis & Bridle 2002).

For quantitative comparison between GEDE model, PEDE model and \( \Lambda \)CDM model, we employ the deviance information criterion (DIC) (Spiegelhalter et al. 2002; Liddle 2007), defined as

\[
\text{DIC} = D(\hat{\theta}) + 2p_D = D(\theta) + p_D,
\]

where
\[
p_D = D(\hat{\theta}) - D(\hat{\theta}) \quad \text{and} \quad D(\hat{\theta}) = -2 \ln \mathcal{L} + C,
\]
here \( C \) is a ‘standardizing’ constant depending only on the data which will vanish from any derived quantity and \( D \) is the deviance of the likelihood.

We will show that by adding local measurement \( H_0 = 74.03 \pm 1.42 \) from Riess et al. (2019) to CMB measurements, PEDE model behave better than \( \Lambda \)CDM model and in the context of the GEDE parametric model \( \Lambda \)CDM model stays outside of the 4\( \sigma \) confidence limit.

3. RESULTS AND DISCUSSION

We summarize the best fit and the 1\( \sigma \) constrain results using CMB and CMB+R19 for \( \Lambda \)CDM model, PEDE model and GEDE model in Table 1. The three cosmological parameter denote with * (in the model), \( H_0 \), \( \Omega_{m,0} \) and \( r_{\text{drag}} \), are derived parameters. Parameter \( \Delta \) is fixed to 0 for \( \Lambda \)CDM model and 1 for PEDE model and set as free parameter for GEDE model. In the last two rows we also show \( \Delta \chi^2 \) and the \( \Delta \text{DIC} \) values with respect to \( \Lambda \)CDM model from same data combinations. In Figure 3 we present 1\( \sigma \) and 2\( \sigma \) contours from CMB and CMB+R19 for \( \Lambda \)CDM model (upper left plot), PEDE model (upper right plot) and GEDE model (bottom plot).

From Table 1 and Figure 3, it is obvious that with PEDE model and GEDE model, the constraints on \( H_0 \) from CMB and CMB+R19 is in agreement with each other and also agree with the local \( H_0 \) results from Riess et al. (2019). While \( \Lambda \)CDM behaves better than PEDE model with \( \Delta \chi^2_\text{bf} = 3.638 \) and \( \Delta \text{DIC} = 5.29 \) when using CMB observations alone. When adding local \( H_0 \) measurements, PEDE model is much more favored by the combined observations (CMB+R19), with \( \Delta \chi^2_\text{bf} = -15.332 \) and \( \Delta \text{DIC} = -6.02 \). With CMB measurement alone the constraints on \( \Delta \) parameter of the GEDE model do not distinguish between \( \Lambda \)CDM model and PEDE-CDM model. However, for the case of the combined CMB+R19 data, we can derive \( \Delta = 1.13 \pm 0.28 \) and it is very clear that \( \Lambda \)CDM model (\( \Delta = 0 \)) is now outside 4\( \sigma \) confidence level region.

We also show the 1\( \sigma \) and 2\( \sigma \) constrain contours on all the parameters for GEDE model in Figure 4.

4. CONCLUSION

In this work, PEDE model that was introduced in Li & Shafieloo (2019) is generalised to GEDE model which has one degree of freedom for the dark energy sector and has the flexibility to include both PEDE model and \( \Lambda \)CDM model as two of its special limits. We confront this model with CMB measurements from Planck 2018 (Aghanim et al. 2018) and \( H_0 \) result from local observations of Cepheid in the LMC (Riess et al. 2019), as two most important cosmological observations at high and low redshifts, and compare the results with the case of \( \Lambda \)CDM model using DIC analysis and find that, 1) our results are consistent with the previous analysis by Li & Shafieloo (2019) and Pan et al. (2019b) that PEDE model works better than \( \Lambda \)CDM when we use CMB measurements in combination with local \( H_0 \) measurements (CMB+R19) and this model can alleviate \( H_0 \) tension that is present in \( \Lambda \)CDM model. 2) using CMB+R19 data and within the context of the GEDE parametrisation, \( \Lambda \)CDM model (\( \Delta = 0 \)) is ruled out at 4\( \sigma \) confidence level.

We should note that future CMB measurements such as Advanced CMB Stage 4 (Abazajian et al. 2016) can surpass Planck CMB measurements in their ability to put tight constraints on cosmological parameters, including Hubble constant \( H_0 \) assuming any particular cosmological model. Furthermore, local \( H_0 \) measurement will be also improved with highly improved distance calibration from Gaia (Prusti et al. 2016) and improved techniques such as using the tip of the red giant branch to build the distance ladder (Freedman 2017) as well as using strong lens systems to measure the expansion rate (Suyu et al. 2017; Birrer et al. 2019; Liao et al.
Figure 2. Upper left: Equation of state of dark energy \( w(z) \) evolved with redshift. Upper right: the evolution of dark energy density \( \Omega_{DE}(z) \). lower left: Expansion rate \( h(z) = \frac{H(z)}{H_0} \) as a function of redshift \( z \). Lower right: deceleration parameter \( q(z) \) as a function of redshift \( z \). Inner plots show the evolution for each cosmological quantity in linear scale from \( z = 0 \) to \( z = 2.5 \). \( \Omega_m = 0.3 \) and flat universe is assumed and plots are mainly for demonstration.

Table 1. We report the 1σ constraint results on the free and derived parameters (with *) of ΛCDM, PEDE model and GEDE model using CMB and CMB+R19. In the last two rows of the table we also display the \( \Delta \chi^2 \) and the \( \Delta \text{DIC} \) values with respect to ΛCDM model from same data combinations.

| Parameters | CMB | CMB+R19 | CMB | CMB+R19 | CMB | CMB+R19 |
|------------|-----|---------|-----|---------|-----|---------|
| \( \Omega_b h^2 \) | 0.02236 ± 0.00015 | 0.02255 ± 0.00015 | 0.02233 ± 0.00015 | 0.02239 ± 0.00014 | 0.02236 ± 0.00015 | 0.02236 ± 0.00015 |
| \( \Omega_c h^2 \) | 0.1202 ± 0.0014 | 0.1179 ± 0.0013 | 0.1204 ± 0.0014 | 0.1197 ± 0.0012 | 0.1202 ± 0.0014 | 0.1201 ± 0.0014 |
| \( 100\theta_{MC} \) | 1.04091 ± 0.00031 | 1.04121 ± 0.00030 | 1.04088 ± 0.00031 | 1.04096 ± 0.00030 | 1.04090 ± 0.00031 | 1.04092 ± 0.00032 |
| \( \tau \) | 0.0543 ± 0.0079 | 0.0580±0.0074 | 0.0545 ± 0.0078 | 0.0558 ± 0.0079 | 0.0542 ± 0.0079 | 0.0552 ± 0.0079 |
| \( \ln(10^{10}A_s) \) | 3.045 ± 0.016 | 3.047 ± 0.017 | 3.046 ± 0.016 | 3.046 ± 0.016 | 3.044 ± 0.016 | 3.046 ± 0.016 |
| \( n_s \) | 0.9648 ± 0.0044 | 0.9704 ± 0.0043 | 0.9645 ± 0.0044 | 0.9662 ± 0.0041 | 0.9647 ± 0.0043 | 0.9653 ± 0.0044 |
| \( H_0^* \) | 67.28 ± 0.62 | 68.35 ± 0.58 | 72.24 ± 0.75 | 72.65 ± 0.67 | 66.76±0.76 | 73.2 ± 1.4 |
| \( \Omega_m,0^* \) | 0.3165 ± 0.0086 | 0.3021 ± 0.0076 | 0.2748 ± 0.0081 | 0.2705 ± 0.0071 | 0.323±0.029 | 0.2672±0.0097 |
| \( r_{drag}H_0^* \) | 9890 ± 100.0 | 10080 ± 100 | 10620 ± 130 | 10690 ± 110 | 9820±120 | 10770 ± 210 |
| \( \Delta \) | 0 | 0 | 1 | 1 | < 1.55 (3σ) | 1.13 ± 0.28 |
| \( \Delta \chi^2 \) | 0 | 0 | 3.638 | -15.332 | -1.42 | -15.716 |
| \( \Delta \text{DIC} \) | 0 | 0 | 5.29 | -6.02 | 0.6 | -2.88 |
All of these improvements will lead to higher precision of $H_0$ measurements and would finally shed light on the nature of the current tensions.

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Figure 3. Constrain results for ΛCDM model (upper left plot), PEDE model (upper right plot) and GEDE model (bottom plot) from CMB and CMB+R19. The cyan shadows show the 1σ $H_0$ results from Riess et al. (2019). In the 1D likelihood of GEDE for Δ, we show Δ = 0 for ΛCDM model in black vertical line and Δ = 1 for PEDE model in dark orange vertical line.
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