PROBING THE LARGE-SCALE VELOCITY FIELD
WITH CLUSTERS OF GALAXIES

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ABSTRACT

What is the role of clusters of galaxies in probing the large-scale velocity field of the universe? We investigate the distribution of peculiar velocities of clusters of galaxies in the popular low-density ($\Omega = 0.3$) flat Cold-Dark-Matter (CDM) cosmological model, which best fits many large-scale structure observations. An $\Omega = 1$ CDM model is also studied for comparison. We find that clusters of galaxies are efficient tracers of the large-scale velocity field. The clusters exhibit a Maxwellian distribution of peculiar velocities, as expected from Gaussian initial density fluctuations. The cluster 3-D velocity distribution for the $\Omega = 0.3$ model peaks at $v \sim 400$ km s$^{-1}$, and extends to high velocities of $v \sim 1200$ km s$^{-1}$. The rms peculiar velocity of the clusters is 440 km s$^{-1}$. Approximately 10% of all model clusters move with high peculiar velocities of $v \geq 700$ km s$^{-1}$. The observed velocity distribution of clusters of galaxies is compared with the predictions from cosmological models. The observed data exhibit a larger velocity tail than seen in the model simulations; however, due to the large observational uncertainties, the data are consistent at a $\sim 3\sigma$ level with the model predictions, and with a Gaussian initial density field. The large peculiar velocities reported for some clusters of galaxies ($v \geq 3000$ km s$^{-1}$) are likely to be overestimated, if the current model is viable.

Subject headings: cosmology: theory – galaxies: clustering
1. INTRODUCTION

Clusters of galaxies are an efficient tracer of the large-scale structure of the universe. The strong correlation function of cluster of galaxies (Bahcall & Soneira 1983; Klypin & Kopylov 1983; Postman et al. 1992; Peacock & West 1992) and the superclustering properties of clusters (Bahcall & Soneira 1984; Postman et al. 1992) provided some of the first evidence for the existence of organized structure on large scales.

Are clusters of galaxies also useful tracers of the large-scale peculiar velocity field in the universe? The peculiar velocity field results most likely from the gravitational acceleration that develops from initial density fluctuations in the early universe. The velocity field can place strong constraints on cosmological models, including the mass-density of the universe (Bertschinger & Dekel 1989; Dekel 1994). Our galaxy moves with $\sim 600 \text{ km s}^{-1}$ relative to the cosmic frame defined by the Cosmic Microwave Background (CMB), as implied by the dipole observations of the CMB (Smoot et al. 1977). Peculiar motions of other galaxies have been detected by measuring their velocity deviations from the Hubble flow by using relative distance indicators that predict the Hubble velocities of the galaxies (Rubin et al. 1976; Dressler et al. 1987; Burstein et al. 1987; Faber et al. 1989). The observational determination of cluster motions can be more accurate than the determination of peculiar velocities of individual galaxies since the distance to each cluster is determined from a large number of member galaxies. Moreover, rich clusters of galaxies, unlike galaxies themselves, trace the large-scale linear density field of the universe, thus directly reflecting the initial conditions.

The possible existence of large peculiar velocities of clusters in some superclusters, with $v_r \sim 10^3 \text{ km s}^{-1}$, has been suggested (Bahcall et al. 1986). Using relative distance
indicators, clusters with similarly high peculiar velocities, \( v_r \sim 10^3 \) km s\(^{-1}\), have been observed (Faber et al. 1989; Mould et al. 1991, 1993). A large bulk-flow of relatively nearby rich clusters of galaxies, with \( v_r \sim 700 \) km s\(^{-1}\), has recently been reported (Lauer & Postman 1994).

In this Letter, we investigate the expected motions of clusters of galaxies in some popular cosmological models. We first study the velocity distribution of clusters of galaxies in a low-density CDM model, which best fits most of the large-scale structure observations, including the galaxy and the cluster correlation functions, the galaxy power-spectrum, the mass-function of clusters of galaxies, the small-scale pairwise velocities of galaxies, as well as galaxy formation and thermodynamic properties (Graham 1988; Maddox et al. 1990; Bahcall & Cen 1992; Cen et al. 1993; Kofman et al. 1993). We find that clusters of galaxies move with considerable speeds; they provide an important tracer of the large-scale velocities. We compare the model expectations with observations of group and cluster velocities. We find that the data exhibit a tail of larger velocities than predicted by the model; however, within the large observational uncertainties, the available data are consistent with the model predictions. An \( \Omega = 1 \) CDM model yields similar results but with somewhat larger velocities.

2. PECULIAR VELOCITIES OF CLUSTERS OF GALAXIES

A large-scale Particle-Mesh code with a box size of 800h\(^{-1}\) Mpc is used to simulate the evolution of the matter. A large simulation box is needed in order to insure that:

1) contributions to velocities from waves larger than the box size are small; and 2) uncer-
tainties due to fluctuations in the small number of large waves in the box are minimized.

We find that these conditions are satisfied with the $800h^{-1}\text{Mpc}$ box size (see below).

The simulation box contains $500^3$ cells and $250^3 = 10^{7.2}$ matter particles. The spatial resolution is $1.6h^{-1}\text{Mpc}$. [A higher resolution ($0.8h^{-1}\text{Mpc}$) smaller box ($400h^{-1}\text{Mpc}$) is also studied for comparison.] Details of the simulations are discussed by Cen (1992). A low-density CDM model with a mass density $\Omega = 0.3$, a cosmological constant $\Omega_\Lambda = 1 - \Omega = 0.7$, a Hubble constant $h = H_0/100 = 0.67$, and a normalization of the mass fluctuations on $8h^{-1}\text{Mpc}$ scale $\sigma_8 = 2/3$ (as determined by the CMB anisotropy measurements of COBE; Smoot et al. 1992) is used. For comparison we also investigate the standard $\Omega = 1$ CDM model, with $h = 0.5$ and $\sigma_8 = 1.05$ (COBE normalization).

Clusters are selected in the simulation using an adaptive linkage algorithm (Bahcall & Cen 1992). The threshold for the cluster mass, within $r = 1.5h^{-1}\text{Mpc}$ of the cluster center, is selected to correspond to a number density of clusters comparable to the observed density of rich ($R \geq 1$) clusters, $n_{cl} \sim 6 \times 10^{-6} h^3 \text{Mpc}^{-3}$ (Bahcall & Cen 1993). We also use, for comparison, a lower threshold that corresponds to the observed number density of small groups of galaxies, $n_{gr} \sim 10^{-4} h^3 \text{Mpc}^{-3}$ (Ramella, Geller & Huchra 1989; Bahcall & Cen 1993).

A total of $\sim 3000$ rich clusters of galaxies and $\sim 5 \times 10^4$ groups are obtained in the $800h^{-1}\text{Mpc}$ simulation box. The peculiar velocity of each of these clusters (or groups) relative to the co-moving cosmic frame is determined from the simulation; these velocities are used in the analysis described below.
The peculiar velocity distribution of clusters of galaxies, $P(v)$, is presented in Figure 1a. It represents the probability distribution, or the normalized number density, of clusters with peculiar velocities in the range $v \pm dv$, per unit $dv$, as a function of $v$. The velocity $v$ refers to the three-dimensional peculiar velocity of the cluster relative to the cosmic background frame. The solid line represents the velocity distribution of rich clusters of galaxies (with $n_{cl} = 6 \times 10^{-6} \text{ h}^3 \text{ Mpc}^{-3}$); the dashed line corresponds to the distribution of the lower-threshold groups of galaxies (with $n_{gr} = 10^{-4} \text{ h}^3 \text{ Mpc}^{-3}$). The similarity of the two distributions implies that the cluster velocity distribution is insensitive to the threshold (i.e. richness) of the selected clusters.

Figure 1 shows that clusters of galaxies move with significant speeds. The cluster velocities peak at $v \sim 400 \text{ km s}^{-1}$ (comparable to the velocity of our own Galaxy); their rms peculiar velocity is $440 \text{ km s}^{-1}$. The velocity distribution exhibits a moderate fall-off at high velocities ($v \sim 500 - 1000 \text{ km s}^{-1}$), and a faster drop at small velocities ($v \leq 300 \text{ km s}^{-1}$). Approximately 35% of all clusters exhibit peculiar velocities in the range $v \approx 500 \pm 100 \text{ km s}^{-1}$. About 10% of all clusters have peculiar velocities in the range $v \approx 1000 \pm 300 \text{ km s}^{-1}$; the velocity distribution tail reaches $\sim 1200 \text{ km s}^{-1}$.

The integrated velocity distribution of clusters of galaxies, $P(> v)$ (representing the probability distribution of clusters with peculiar velocities larger than $v$), is presented in Figure 1b. The integrated function emphasizes the high-velocity tail of the distribution. It reveals that a significant fraction of all model clusters ($\sim 10\%$) exhibit high peculiar velocities of $v \geq 700 \text{ km s}^{-1}$; the tail of the cluster velocity distribution, to $\sim 1200 \text{ km s}^{-1}$, is clearly seen.
The shape of the cluster velocity distribution is well matched by a Maxwellian distribution \( P(v) \sim v^2 \exp(-v^2/2\sigma^2) \) (dotted line in Figures 1a and 1b). The velocity dispersion, \( \sigma \), is determined from the model simulation such that the fit and the model have the same rms velocity. A Maxwellian distribution is expected from a Gaussian initial density fluctuation field. The above results suggest that clusters of galaxies provide an important tracer of the large-scale velocity field and can help to test the type of the initial density field (Gaussian or non-Gaussian).

How does the velocity distribution of clusters of galaxies depend on the specifics of the model? We studied an \( \Omega = 1 \) CDM model, with \( h = 0.5 \) and \( \sigma_8 = 1.05 \) (as required for a COBE normalization). This model, however, unlike the low-density CDM model, is inconsistent with several large-scale structure observations such as the number-density and correlation function of clusters of galaxies and the small-scale pairwise velocities of galaxies (Maddox et al. 1990; Bahcall & Cen 1992; Cen & Ostriker 1992; Ostriker 1993). We find that the velocity distribution of clusters in the \( \Omega = 1 \) CDM model is similar to that of the \( \Omega = 0.3 \) model but shifted to higher velocities: the peak velocity is \( v \sim 600 \) km s\(^{-1}\), the velocity tail extends to \( \sim 2000 \) km s\(^{-1}\), and the rms peculiar velocity is 762 km s\(^{-1}\) (3D). Both model cluster velocity distributions are well approximated by a Maxwellian.

The simulation results are consistent with expectations from linear theory (Graham et al. 1994). Using linear theory we find that contributions from waves larger than \( 800h^{-1} \) Mpc are small; the rms velocities integrated to \( \lambda = 800h^{-1} \) Mpc and to infinity are 416 and 426 km s\(^{-1}\), respectively, for the \( \Omega = 0.3 \) CDM model. The
sensitivity of the results to the resolution of the simulation is tested by comparing the simulations with a 0.8h$^{-1}$ Mpc nominal resolution and with a 1.6h$^{-1}$ Mpc nominal resolution (both in a 400h$^{-1}$ Mpc box). The resulting velocity distributions of groups and clusters are consistent with each other (to within $\leq 60$ km s$^{-1}$) for both resolutions.

3. COMPARISON WITH OBSERVATIONS

How does the predicted cluster velocity distribution compare with observations?

Observational determination of peculiar velocities of galaxies, groups, and clusters is difficult, since the true distances of the objects and hence their Hubble velocities are uncertain. However, data are available for the peculiar velocities of some samples of groups and clusters of galaxies. We use the group and cluster peculiar velocities observed by the Tully-Fisher (TF) method for distance indicators (Aaronson et al. 1986; Mould et al. 1991, 1993; Mathewson et al. 1992), and by the $D_n - \sigma$ method (Faber et al. 1989). Groups with observational velocity uncertainties $\geq 900$ km s$^{-1}$ are excluded from the analysis. A total of 48 group and cluster peculiar velocities are available from the TF method, and 91 from $D_n - \sigma$. The total sample of peculiar velocities includes 123 non-overlapping groups and clusters. These data are used to determine the observed velocity distribution of groups of galaxies. As seen in the simulations, the velocity distribution is insensitive to the group threshold: small groups and rich clusters yield similar results.

The observed differential and integrated group velocity distributions are presented in Figure 2a-b. Here we use the one-dimensional velocities, as observed ($v_{1D}$); they are
compared with the 1-D velocities in the models (as opposed to the 3-D velocities used in the previous discussion, since the 3-D velocities are not directly observed). We present in different symbols the data obtained from the TF, $D_n - \sigma$, and total (combined) samples. The results from the different sub-samples are approximately consistent with each other; the total sample is thus used for comparison with model expectations.

The number of rich clusters in the observed sample is small ($\sim 18 \ R \geq 0 \ clusters$). Within the large statistical uncertainties of such a small sample, the observed rich cluster velocity distribution is consistent with that of the groups, and thus consistent with the model comparisons discussed below.

The observed velocity distribution is superimposed on the 1-D velocity distribution expected for groups (dotted line; Fig. 2). The shape of the functions differs from those of Figures 1 since the 1-D velocity distribution is plotted instead of the 3-D. The 3-D distribution is proportional to $v^2$ at small velocities; instead, the 1-D velocities exhibit a Gaussian distribution, as expected for a 3-D Maxwellian. In comparing model expectations with observations, the model velocity distribution (dotted line) is convolved with the observational uncertainties; each model cluster is given an uncertainty drawn at random from the actual distribution of observed uncertainties of the total sample. The convolved model distribution of groups is shown by the dashed line in Fig. 2; the convolved rich cluster distribution is shown by the solid line, for comparison. The convolution flattens the model distributions, as expected, and produces a high-velocity tail. The rms peculiar velocities of model groups versus observations (total sample and T-F only) are summarized in Table 1. The models are convolved with the observational uncertainties of each sample (total and TF) separately.
A comparison between the data and the convolved model suggests that the observed and model velocity distributions are consistent with each other at $\sim 3\sigma$ level. A K-S test of the velocity distribution indicates that the model is consistent with the data at a significance level of $\sim 3\%$ (or $\sim 70\%$ if only the T-F velocities are used). The $\Omega = 1$ model is consistent with the data at a 5% level (or $\sim 46\%$ for the T-F data).

The observations exhibit a long tail of high velocity groups and clusters, to $v_{1D} \sim 3000$ km s$^{-1}$. This high velocity tail is not consistent with the convolved models. Since the current observational uncertainties are large, and the effect of model convolution is strong (in fact it yields most of the high-velocity tail), more accurate velocity data are needed in order to further constrain the cosmological models. One effect of large observational errors is to produce an artificial high-velocity tail. It seems likely that more accurate cluster velocities will yield smaller values than currently suggested by the observed high-velocity tail of Figure 2. If, however, the high velocities ($v_{1D} \geq 2000$ km s$^{-1}$) are confirmed, with $P \geq 0.01$, it will suggest that the above models are unlikely.

4. SUMMARY AND CONCLUSIONS

We find that rich clusters of galaxies exhibit a robust, Maxwellian distribution of peculiar velocities in the cosmological models studied. The distribution peaks at $v \sim 400$ km s$^{-1}$ for the $\Omega = 0.3$ model, and extends to high velocities of $v \sim 1200$ km s$^{-1}$. Approximately 10% of all clusters move with high peculiar velocities of $v \geq 700$ km s$^{-1}$. The velocity distribution of model clusters is insensitive to the cluster selection threshold (i.e., richness). The observed distribution of peculiar velocities of clusters of galaxies exhibits a larger velocity tail than seen in the model; however, due to the large
observational uncertainties, the data are consistent at a $\sim 3\sigma$ level with the COBE-normalized low-density CDM model predictions when convolved with the observational uncertainties. This model fits well other large-scale structure observations. The observed velocity distribution is consistent with Gaussian initial density fluctuations.

There are large uncertainties in the existing measurements of peculiar velocities; these uncertainties do not currently allow significant constraints of the cosmological models. More accurate observations for groups and for clusters of galaxies are likely to yield a lower velocity tail than suggested by the existing observations. These observations of peculiar velocities can help to constrain cosmological models and to test whether the initial density field was Gaussian.

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FIGURE CAPTIONS

FIGURE 1. The 3-D peculiar velocity distribution of clusters (solid line) and groups of galaxies (dashed line) in the low-density CDM model. The Maxwellian distribution for the clusters is represented by the dotted line. (a) Differential velocity distributions; (b) Integrated distributions.

FIGURE 2. Comparison of observations and model predictions. The observed velocity distributions (in 1-D velocities) of groups of galaxies as determined from Tully-Fisher (TF) and $D_n - \sigma$ distance-indicators (stars and open circles, respectively), and for the combined sample (dark circles with $\sqrt{N}$ uncertainties indicated) are presented. The model 1-D group velocity distribution (dotted line), and its convolved distribution (dashed line; convolved with the observational velocity uncertainties) are shown. The convolved distribution of simulated rich clusters (solid line) is also shown, for comparison. The observations should be compared with the convolved model group simulations (dashed line). (a) Differential velocity distributions; (b) Integrated distributions.