HIGH ECLIPTIC LATITUDE SURVEY FOR SMALL MAIN-BELT ASTEROIDS

TSUYOSHI TERAI1, JUN TAKAHASHI2, AND YOICHI ITOH2

1 National Astronomical Observatory of Japan, 2-21-1 Osawa, Mitaka, Tokyo 181-8588, Japan; tsuyoshi.terai@nao.ac.jp
2 Center for Astronomy, University of Hyogo, 407-2 Nishigaichi, Sayo-cho, Sayo-gun, Hyogo 679-5313, Japan

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ABSTRACT

Main-belt asteroids have been continuously colliding with one another since they were formed. Their size distribution is primarily determined by the size dependence of asteroid strength against catastrophic impacts. The strength scaling law as a function of body size could depend on collision velocity, but the relationship remains unknown, especially under hypervelocity collisions comparable to 10 km s\(^{-1}\). We present a wide-field imaging survey at an ecliptic latitude of about 25\(^{\circ}\) for investigating the size distribution of small main-belt asteroids that have highly inclined orbits. The analysis technique allowing for efficient asteroid detections and high-accuracy photometric measurements provides sufficient sample data to estimate the size distribution of sub-kilometer asteroids with inclinations larger than 14\(^{\circ}\). The best-fit power-law slopes of the cumulative size distribution are 1.25 \pm 0.03 in the diameter range of 0.6–1.0 km and 1.84 \pm 0.27 in 1.0–3.0 km. We provide a simple size distribution model that takes into consideration the oscillations of the power-law slope due to the transition from the gravity-scaled regime to the strength-scaled regime. We find that the high-inclination population has a shallow slope of the primary components of the size distribution compared to the low-inclination populations. The asteroid population exposed to hypervelocity impacts undergoes collisional processes where large bodies have a higher disruptive strength and longer lifespan relative to tiny bodies than the ecliptic asteroids.

Key word: minor planets, asteroids: general

Online-only material: supplemental data

1. INTRODUCTION

Main-belt asteroids (MBAs) have continuously undergone self-collisional processes. The impact events are characterized by the target/impactor masses and collision velocity. When the kinetic energy of an impactor is larger than the critical specific energy \(Q^*\), the energy per unit target mass required to shatter the target and disperse half of its mass, the target is catastrophically disrupted (Davis et al. 2002). Otherwise, the target (largest fragment) retains a mass larger than half of the original, resulting in cratering or gravitational reaccumulation of collisional fragments after shattering. \(Q^*_D\) is an indicator of impact strength, which depends on the body size (e.g., Housen & Holsapple 1990; Durda et al. 1998; Benz & Asphaug 1999). \(Q^*_D\) decreases with increasing diameter for asteroids less than \(~0.1–1\) km, called the “strength-scaled regime.” In contrast, it increases with increasing diameter for the larger asteroids, called the “gravity-scaled regime.” The degree of change in \(Q^*_D\) with body size is the primary determinant of a power-law size distribution of the small body population in a collisional cascade (O’Brien & Greenberg 2003).

The size distribution of MBAs down to sub-kilometer size has been estimated by previous extensive surveys with ground-based telescopes such as the Palomar–Leiden Survey (van Houten et al. 1970), Spacewatch (Jedicke & Metcalfe 1998), Sloan Digital Sky Survey (SDSS; Ivezić et al. 2001), Subaru Main Belt Asteroid Survey (SMBAS; Yoshida et al. 2003; Yoshida & Nakamura 2007), and Sub-Kilometer Asteroid Diameter Survey (Gladman et al. 2009). In addition, infrared satellites including \(IRAS\) (Tedesco et al. 2002), \(AKARI\) (Usui et al. 2011), and \(WISE\) (Masiero et al. 2011) measured accurate diameters of numerous asteroids. Figure 1 shows the cumulative size distribution (CSD) of MBAs compiled from the Asteroid Orbital Elements Database\(^3\) (ASTORB; Bowell et al. 1994) and the survey results reported by the SDSS and SMBAS. Using the observed size distributions, many studies devoted effort to modeling the collisional evolution of MBAs via numerical simulations (Durda et al. 1998; Bottke et al. 2005a, 2005b; O’Brien & Greenberg 2005; de Elía & Brunini 2007). It should be noted that the \(Q^*_D\) law is supposed to be a function only of asteroid size in each model.

Petit et al. (2001) presented a dynamical evolution model for primordial asteroids in the early main belt that were dynamically excited due to gravitational perturbations from Jupiter and embedded planetary embryos. In this phase, collisions between asteroids occurred at higher velocities than at present (\(\sim4\) km s\(^{-1}\); Vedder 1998) because of the higher eccentricities and inclinations (Bottke et al. 2005b). Bottke et al. (2005a) pointed out that the \(Q^*_D\) law could be affected by varying collision velocities. They suggested a steeper \(Q^*_D\) curve in the gravity-scaled regime for 10 km s\(^{-1}\) collisions than that for slower collisions.

The asteroid collisional evolution among impacts with much higher velocities (i.e., \(\geq4\) km s\(^{-1}\); hereafter called “hypervelocity”) than the mean collisions in the main belt remains unknown. Because of technical difficulties, only a few laboratory experiments for hypervelocity collisions have been conducted (Kadono et al. 2010; Takasawa et al. 2011). The hydrocode simulations by Benz & Asphaug (1999) indicated that in the gravity-scaled regime, the \(Q^*_D\) for a basalt target has similar slopes between collisions of 3 km s\(^{-1}\) and 5 km s\(^{-1}\), while the \(Q^*_D\) for an icy target in 3 km s\(^{-1}\) collisions increases with size more steeply than that in 0.5 km s\(^{-1}\) collisions. However, another study with impact simulations showed that the slope of \(Q^*_D\)

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\(^3\) ftp://ftp.lowell.edu/pub/elgb/astorb.html
for a basalt target in 5 km s\(^{-1}\) collisions is shallower than that in 3 km s\(^{-1}\) collisions (Jutzi et al. 2010). The collision-velocity dependency of the \(Q_D\) law has not yet been confirmed.

Investigation of an asteroid population with highly inclined orbits is an effective means to understand the material properties against hypervelocity collisions. In the main belt, asteroids with orbits inclined at higher than 15° (hereafter called high-inclination MBAs) have mean collision velocities exceeding \(\sim 7\) km s\(^{-1}\) (Farinella & Davis 1992; Gil-Hutton 2006). Those high-inclination MBAs remain in the collisional evolution dominated by hypervelocity impacts. The size distribution of high-inclination MBAs enables us to examine the \(Q_D\) law under collisional processes at high velocity.

Terai & Itoh (2011) performed a survey focused on high-inclination small-size MBAs in 9.0 deg\(^2\) fields using data with a detection limit of \(r = 24.0\) mag obtained by the 8.2 m Subaru Telescope. They detected 178 MBA candidates 0.7–7 km in diameter and found that the size distribution of high-inclination MBAs is shallower than that of low-inclination MBAs over a wide diameter range of 0.7 km to 50 km. However, the faint-end slopes, less than 2 km in diameter based on their own survey data, potentially include large bias due to the non-uniform data taken at sky regions with various ecliptic latitudes and solar phase angles in uneven atmospheric conditions. Actually, the power-law index of the size distribution for low-inclination MBAs is inconsistent with that presented by previous studies (Ivezić et al. 2001; Yoshida & Nakamura 2007).

In this paper, we present the results of an additional survey to determine the size distribution of high-inclination MBAs down to sub-kilometer diameters. We note that the size distribution of MBAs is poorly represented by a single power law. As seen in Figure 1, the distribution has significant slope transitions around sub-kilometer diameters. We note that the size distribution of MBAs is shallower than that of low-inclination MBAs over a wide diameter range of 0.7 km to 50 km. However, the faint-end slopes, less than 2 km in diameter based on their own survey data, potentially include large bias due to the non-uniform data taken at sky regions with various ecliptic latitudes and solar phase angles in uneven atmospheric conditions. Actually, the power-law index of the size distribution for low-inclination MBAs is inconsistent with that presented by previous studies (Ivezić et al. 2001; Yoshida & Nakamura 2007).

We carried out uniform wide-field imaging at high ecliptic latitudes using the Subaru Telescope. This survey allows us to obtain a large amount of homogeneous data that gives three times more small high-inclination MBAs than the previous study. We evaluate the difference in size distribution between low- and high-inclination MBAs considering the wavy structure and taxonomic distribution. The results provide useful clues for understanding the collisional evolution of primordial asteroids in the early solar system, as well as that of planetesimals in some debris disks that have been found outside the solar system.

2. ASTEROID SURVEY

2.1. Observations

Our survey was performed on 2008 August 24 and 25 (UT) using the Suprime-Cam mounted on the 8.2 m Subaru Telescope. The Suprime-Cam is a mosaic camera with ten 2 k \(\times\) 4 k CCD chips and covers a 34′ \(\times\) 27′ field of view with a pixel scale of 0′.20 (Miyazaki et al. 2002). The data were taken at a sky area centered on R.A. (J2000) = 21^h\(40^m\) and decl. (J2000) = +14°00′, within 6° from opposition in ecliptic longitude. The region with ecliptic latitude of around 25° is suitable for detecting asteroids with inclinations of 15° or higher in the main belt. We imaged 104 fields that contained no bright background objects. Most of the fields overlap the sky coverage of the SDSS Data Release 9 (DR9). Each field was visited twice with 240 s exposures at an interval of 20 minutes using the r-band filter. The seeing size ranged from 0′.7 to 1′.0 in almost all of the data. The total surveyed area is 26.5 deg\(^2\).

2.2. Data Analysis

The images were processed using IRAF, produced by the National Optical Astronomy Observatories, and SDFRED2 (Ouchi et al. 2004). The standard procedure of data reduction includes overscan subtraction, flat-fielding, correction of geometric distortion, subtraction of sky background, and position matching between two images that were taken at a same field. Moving objects are searched by the image processing technique presented in Terai et al. (2007). The two-visit imaging with a 20 minute interval allows us to identify MBAs that have sky motion faster than \(\sim 30\) hr\(^{-1}\) at near opposition (see Figure 2).

We measured the positions and brightness of detected moving objects using the SExtractor (Bertin & Arnouts 1996) and the IRAF/APPHOT package, respectively. In the images acquired with 240 s exposures, most asteroids are trailed as seen in Figure 2. For precise photometry, we produced synthetic apertures appropriate to each object through the following procedures. (1) The circular aperture for absolute photometry of point sources in the image is determined. Its radius is set to \(\sim 2.5\) times the FWHM of the point sources. (2) The motion velocity of the moving object is estimated from the difference between its central coordinates in the two-visit images. (3) The circular aperture around the object’s center is evenly extended in both directions along the axis of the motion by the distance that the object moved during the exposure time. The “moving-circular” apertures formed in these ways are shown in Figure 2. We estimated the total flux within the given apertures using the POLYPHOT task.

We also conducted photometry on the field stars listed in the SDSS DR9 catalog with \(r = 19.0–20.0\) mag in the AB system using the same technique for the flux calibration. The

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Figure 1. Cumulative size distribution of main-belt asteroids estimated from the previous observations. The solid line represents the population of observed asteroids listed in the ASTORB database (Bowell et al. 1994). The circles and diamonds represent extrapolations based on the results of the Sloan Digital Sky Survey (Ivezić et al. 2001) and Subaru Main Belt Asteroid Survey (Yoshida & Nakamura 2007), respectively.

\[ \text{Cumulative number vs. Diameter (km)} \]

\[ 10^1 \quad 10^2 \quad 10^3 \]

\[ 0.1 \quad 1 \quad 10 \]

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\[ \frac{\text{Cumulative size distribution of main-belt asteroids estimated from}}{\text{the previous observations. The solid line represents the population of observed}} \]

\[ \text{ASTORB database (Bowell et al. 1994). The circles and diamonds represent}} \]

\[ \text{extrapolations based on the results of the Sloan Digital Sky Survey}} \]

\[ \text{(Ivezić et al. 2001) and Subaru Main Belt Asteroid Survey (Yoshida}} \]

\[ \text{& Nakamura 2007), respectively.} \]
The measured total flux and magnitude zero point of each image was determined from the original images taken with 240 s exposures at an interval of 20 minutes under a seeing size of 0′′.8. The field of view covers 25′′× 25′′ with north up and east to the left. The background objects have been masked (black regions). The moving objects have a sky motion of 38′′ hr−1 and brightness of $r = 23.07 ± 0.07$ mag. The circles surrounding the objects show the apertures for photometry.

Magnitude zero point of each image was determined from the measured total flux and r-band magnitude provided in the catalog. However, the filter transmission of the Suprime-Cam differs from that of SDSS. The difference in r-band magnitude of the field stars, $r_{\text{sup}}-r_{\text{sdss}}$, was corrected with the SDSS color $(r-i)_{\text{sdss}}$ using the transformation equation presented by Fumiaki Nakata:

$$r_{\text{sup}}-r_{\text{sdss}} = -0.00282 - 0.0498(r-i)_{\text{sdss}} - 0.0149(r-i)_{\text{sdss}}^2.$$  

The $(r-i)_{\text{sdss}}$ colors of the field stars were derived from the SDSS DR9 catalog. The resulting zero point of the Suprime-Cam filter system was applied to calculate the apparent magnitudes of detected moving objects in the frame.

To obtain the statistically homogeneous sample, we evaluated the limiting magnitude of the data. The detection efficiency was estimated using artificial asteroid trails implanted into the raw data with every 0.2 mag. The fractions of detection were represented by

$$\eta(r) = \frac{A}{2} \left[ 1 - \tanh \left( \frac{r - m_{\text{50}}}{w} \right) \right],$$  

where $A, m_{\text{50}},$ and $w$ are the maximum efficiency, half-maximum magnitude, and transition width, respectively (Gladman et al. 1998). The mean of $A$ is 0.84 due to the sky coverage of background objects. We defined $r = 24.4$ mag as the limiting magnitude in this survey. We excluded the data with the net detection efficiency $\eta'(r) = \eta(r)/A$ less than 50% at $r = 24.4$ mag. Figure 3 shows the $\eta'(r)$ curves with the minimum (filled circles) and maximum (open circles) values of the detected objects at 24.4 mag. The combined $\eta'(r)$ of all of the detected objects are also plotted (open triangles). The selected data cover 13.6 deg$^2$ in actual or 11.4 deg$^2$ in effect. Figure 4 shows histograms of the ecliptic longitude from the opposition and ecliptic latitude covered by the selected data.

3. RESULTS

Our exploration found 441 moving objects in the selected data with a 50% detection limit of $r > 24.4$ mag. Figure 5 shows the distribution of their sky motions in the geocentric ecliptic coordinate system. The major group with negative longitudinal motions of $\sim 30′′$–$45′′$ hr$^{-1}$ consists of MBAs, while the clump around 22′′ hr$^{-1}$ corresponds to Jovian Trojans.

At the region with geocentric ecliptic latitude $\beta$ and longitude with respect to opposition $\lambda'$, the lower inclination limit of detectable asteroids, $I_{\text{lim}}$, is

$$\sin I_{\text{lim}} = \frac{\Delta}{R} \sin \phi,$$  

where $\Delta$ is the transition width, $R$ is the distance from the Sun to Jupiter, and $\phi$ is the angular offset of $\beta$. The lower inclination limit of detectable asteroids is $0.0498$ deg$^{-1}$.

Figure 2. Detected moving objects in the combined image from two-visit images. The original images were taken with 240 s exposures at an interval of 20 minutes under a seeing size of 0′′.8. The field of view covers 25′′× 25′′ with north up and east to the left. The background objects have been masked (black regions). The moving objects have a sky motion of 38′′ hr$^{-1}$ and brightness of $r = 23.07 ± 0.07$ mag. The circles surrounding the objects show the apertures for photometry.

Figure 3. Net detection efficiency $\eta'(r)$ for moving objects as a function of apparent r magnitude. The detection efficiency reduces with motion velocity due to the decrease in flux density. In this work, the motion velocity was set to $40′′$ hr$^{-1}$, similar to or faster than those of most of the detected MBA candidates (see Figures 5 and 6). Only the data with $\eta'(r) \geq 0.5$ at $r = 24.4$ mag was used for our analysis of the asteroid population. The filled circles and open circles represent objects with minimum and maximum $\eta'(r)$ at $r = 24.4$ mag (dashed line in the selected data, respectively. The open triangles show combined $\eta'(r)$ of all of the selected data. This curve indicates that objects even close to $r = 24.4$ mag can be detected with efficiencies of about 0.7 or higher in most of the images. The faintest objects in our sample cause little increase in statistical uncertainty of measurement of the size distributions. The solid curves are best-fit functions given by Equation (2).

Figure 4. Distributions of the ecliptic longitude with respect to opposition ($\lambda - \lambda_{\text{opp}}$, left) and ecliptic latitude ($\beta$, right) covered by the selected data.

\[ \sin I_{\text{lim}} = \frac{\Delta}{R} \sin \phi, \]  

where $\Delta$ is the transition width, $R$ is the distance from the Sun to Jupiter, and $\phi$ is the angular offset of $\beta$. The lower inclination limit of detectable asteroids is $0.0498$ deg$^{-1}$. 
where $R$ is the heliocentric distance, $\Delta$ is the geocentric distance, and $\phi$ is the elongation angle between the Sun and the asteroid given by $\cos \phi = \cos \lambda \cos \beta$. In the survey area of $\beta \sim 25^\circ$ near the opposition, $I_{\text{lim}}$ is around $15^\circ$. Asteroids with inclination $I_{\text{lim}}$ show no motion along the ecliptic latitude at the opposition field. In contrast, latitudinally moving asteroids have inclinations higher than $I_{\text{lim}}$. The dotted lines in Figure 5 represent the motions of asteroids in circular orbits when they are observed at opposition ($\lambda'=0^\circ$) and $\beta=25^\circ$.

### 3.1. Estimation of Orbital Parameters

Two-visit positioning of moving objects in a night is insufficient for orbit determination. Instead, we estimated the semimajor axis and inclination of each asteroid from the sky motion assuming that the orbit is circular, namely, the eccentricity is zero. The adequacy of this assumption is evaluated below. Jedicke (1996) presents the expressions of ecliptic motion derived from the orbital elements and sky coordinates (see also the Appendix in Ivezic et al., 2001).

Let us consider an asteroid in a circular orbit with a semimajor axis $a$ and inclination $I$ located at a heliocentric ecliptic longitude with respect to opposition $l'$ and latitude $b$. We use the coordinate system defined in Figure 2 of Jedicke (1996). The position vector from the Sun, $\mathbf{R}$, and angular momentum vector, $\mathbf{h}$, are given by

$$\mathbf{R} = a \left( \cos l' \cos b, \sin l' \cos b, \sin b \right),$$  \hspace{1cm} (4)

$$\mathbf{h} = \sqrt{\mu a} \left( \sin \Omega \sin I, -\cos \Omega \sin I, \cos I \right),$$  \hspace{1cm} (5)

respectively, where $\mu$ is the product of the gravitational constant and mass of the Sun, and $\Omega$ is the longitude of the ascending node derived from $l'$, $b$, and $I$. The relative velocity with respect to the Earth is given by

$$\mathbf{v} = -\frac{\mathbf{R} \times \mathbf{h}}{a^2} - (0, \sqrt{\mu}, 0).$$  \hspace{1cm} (6)

The observed ecliptic motion is converted from $\mathbf{v}$ using the unit vectors representing the geocentric directions of increasing ecliptic longitude, latitude, and distance. These vectors are given by

$$\hat{\lambda}' = (-\sin \lambda', \cos \lambda', 0),$$  \hspace{1cm} (7a)

$$\hat{\beta} = (-\cos \lambda' \sin \beta, -\sin \lambda' \sin \beta, \cos \beta).$$  \hspace{1cm} (7b)

The rates of apparent motion are

$$\dot{\lambda}' = \frac{\mathbf{v}}{\Delta} \cdot \hat{\lambda}' ,$$  \hspace{1cm} (8a)

$$\dot{\beta} = \frac{\mathbf{v}}{\Delta} \cdot \hat{\beta}.$$  \hspace{1cm} (8b)

The orbital elements of a detected moving object are derived from the best-fit set of $a$ and $I$ for Equation (8b). Several objects with inclinations larger than $40^\circ$ were excluded because of the significant uncertainty of estimated $a$ and $I$ (see Figure 5). Figure 6 shows the distribution of detected asteroids in the semimajor axis versus inclination space. The dashed curve represents the inclination limit of detectable asteroids given by Equation (3). MBAs and Jovian Trojans can be identified clearly. We consider objects with $a=2.0$–3.3 AU to be MBA candidates.

The estimation accuracy of orbital elements of the MBA candidates was evaluated by Monte Carlo simulation of a virtual
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asteroid survey. We randomly generated 10,000 hypothetical asteroids in the area of $-5^\circ < \lambda' < +5^\circ$ and $+20^\circ < \beta < +30^\circ$ (see Figure 4). The orbital elements were given in the orbital parameter space in the range of $a = 2.3$–3.3 AU, $e = 0.0$–0.5, and $I = 15^\circ$–40$^\circ$ in inclination. These were governed by the probability distributions based on the orbital distribution of known MBAs with $I > 15^\circ$ listed in the ASTORB database. The simulation showed that the rms errors are 0.14 AU in the semimajor axis and 4.5 in inclination, which are comparable to the values presented by Nakamura & Yoshida (2002) for an ecliptic survey. The estimated heliocentric distances have an uncertainty of $\sim$0.37 AU. We note that the systematic errors in heliocentric distance and inclination are less than 0.05 AU and 0.1, respectively, which are small enough to be negligible compared to the random errors.

3.2. Estimation of Asteroid Size

The apparent $r$-band magnitude of the detected moving objects is converted into the absolute magnitude, $H_r$, by

$$H_r = r - 5 \log(R \cdot \Delta) - P(\alpha),$$

where $\alpha$ is the phase angle, the angle between the Sun and the Earth from the asteroid, and $P(\alpha)$ is the phase function. We used the $P(\alpha)$ expressions presented by Bowell et al. (1989) assuming the slope parameter $G = 0.15$. The phase angles of MBA candidates range from 6$^\circ$ to 14$^\circ$.

Asteroid diameter $D$ in kilometers is estimated from

$$\log D = 0.2 r_\odot - \log \left( \frac{\sqrt{\beta}}{2(\text{AU km}^{-1})} \right) - 0.2 H_r,$$

where $r_\odot$ and $p$ represent the $r$-band magnitude of the Sun in the AB system and the geometric albedo. In the SDSS photometric system, $r_\odot = -26.91$ mag and $(r-i)_\odot = 0.13$ mag (Fukugita et al. 2011), which are converted into the Suprime-Cam $r_\odot$ by Equation (1).

The geometric albedo was assigned the mean value of albedo-known asteroids. MBAs mainly consist of two major groups: redder/brighter asteroids dominated by S-type asteroids and bluer/darker asteroids dominated by C-type asteroids. These are hereafter called S-like asteroids and C-like asteroids, respectively. We used the mean albedos obtained from the AKARI All-Sky Survey observations: 0.22 for the S-like asteroids and 0.07 for the C-like asteroids (Usui et al. 2011). The mean albedo of all asteroids depends on the number ratio between S- and C-type asteroids, which varies with heliocentric distance.

Assuming that the heliocentric distribution of each group is constant with asteroid size, we estimated fractions of the S-/C-type asteroids from the SDSS Moving Object Catalog$^5$ (SDSS MOC; Ivezić et al. 2002). Ivezić et al. (2001) divided asteroids into the two color groups using a color index given by

$$a^* = 0.89(g - r) + 0.45(r - i) - 0.57.$$  

We classified the SDSS MOC asteroids with $a^* > 0$ as S-like asteroids, and those with $a^* < 0$ as C-like asteroids. The number ratios of the C-like to S-like derived from the orbit-known asteroids with $I > 15^\circ$ and $D > 5$ km (larger than the limiting size of complete detection for the both types) are 0.5 in the inner belt in the range of $R = 2.0$–2.5 AU, 1.7 in the middle belt in the range of $R = 2.5$–3.0 AU, and 10.0 in the outer belt in the range of $R = 3.0$–3.3 AU. The weighted mean albedos in the inner, middle, and outer belts are 0.17, 0.13, and 0.09, respectively.

As seen in Equation (9), $H_r$ is derived from measurements of the apparent $r$-band magnitude, $R$, and $\Delta$. The apparent magnitude includes 1$\sigma$ uncertainties of $\sim$0.15 mag at the faint end, namely $r \sim 24$ mag, where $\sigma$ is the standard deviation. The estimated $R$ and $\Delta$ have uncertainties of $\sim$0.37 AU (see Section 3.1). These errors cause the uncertainty in $H_r$ of $\sim$0.7 mag. It corresponds to a $\sim$30% error in $D$.

4. DISCUSSION

4.1. Sample Selection

Figure 7 shows the distribution in the semimajor axis versus absolute $r$ magnitude in the AB system for the main-belt asteroid candidates. The error bars represent photometric uncertainty. The dashed curve shows the 50% completed detection limit corresponding to an apparent magnitude of $r = 24.4$ mag.

4.2. Size Distribution

Figure 9 shows the CSD as a function of asteroid diameter obtained from the final sample. The cumulative number was
regions, 0.6–1.0 km and 1.0–3.0 km, for characterization of the MBAs by Yoshida & Nakamura (2007). Hence, we fixed two diameter limits, 0.56 km and 1.0 km, for characterization of low-inclination sub-kilometer MBAs with $H_r \leq 19.4$ mag ($H_r$ is absolute r magnitude in the AB system).

The CSD slopes were estimated by the maximum likelihood method (e.g., Irwin et al. 1995; Loredo 2004). The differential surface density of asteroids with diameter $D$ km is represented by

$$\Sigma(D) = b\Sigma(>1\text{ km})D^{-b-1}. \tag{12}$$

The likelihood function for $n$ objects is given by

$$L = \exp \left[ -\Omega \int \eta(r(D)) \Sigma(D|b) \, dD \right] \prod_{i=1}^{n} \sigma(D_i) \Sigma(D_i|b), \tag{13}$$

where $\Omega$ is the survey area and $\sigma(D_i)$ is the uncertainty in the diameter of object $i$. The function parameter of the detection efficiency $\eta$ is derived from that combined with the selected data (see Section 2.2). $r(D)$ denotes the apparent r magnitude converted from $D$ with $R = 2.9$ AU (the mean of the sample) and $p = 0.10$ (the weighted mean albedos in the whole main belt with $I > 15^\circ$).

The likelihood analysis gives slopes of $b = 1.25 \pm 0.03$ (1σ) in 0.6 km < $D$ < 1.0 km and $b = 1.84 \pm 0.27$ in 1.0 km < $D$ < 3.0 km. The best-fit power-law CSDs are shown in Figure 9. We evaluated the fitting of the power laws using the Anderson–Darling statistic (Anderson & Darling 1952), given by

$$AD = \int_{0}^{1} \frac{[S(D) - P(D)]^2}{P(D)[1 - P(D)]} \, dP(D), \tag{14}$$

where $P(D)$ is the cumulative detection probability for an object larger than $D$ in diameter, and $S(D)$ is the cumulative distribution function of the detected objects (Bernstein et al. 2004). The goodness of fit is decided by the probability $Pr(AD)$ of a random realization with an AD value higher than the real data. Low $Pr(AD)$ (less than 0.05) implies a poor fit of the distribution. Our calculation found $Pr(AD) = 0.52$, indicating that the power-law fitting well represents the observed CSD.

Terai & Itoh (2011) found that $b = 1.79 \pm 0.05$ for MBAs with $I < 15^\circ$ and $b = 1.62 \pm 0.07$ for MBAs with $I > 15^\circ$ in 0.7 km < $D$ < 2.0 km. The slope for low-$I$ MBAs is much steeper than that of Yoshida & Nakamura (2007), $b = 1.29 \pm 0.02$ in 0.6 km < $D$ < 1.0 km. This discrepancy seems to be due to significant observational bias caused by the use of mixed weighting methods.
data taken in different ecliptic latitudes as well as solar phase angles. In contrast, assuming that $b$ of high-I MBAs is ~0.1 smaller than that of low-I MBAs as shown by Terai & Itoh (2011), the result of this study, $b = 1.25$ for high-I MBAs, is consistent with the CSD slope for low-I sub-kilometer MBAs given by Yoshida & Nakamura (2007). It shows a significant improvement in measurement accuracy of the size distribution by an increase of the sample number, appropriate survey fields located around the opposition, homogeneity of the survey region and data quality, and precise photometric calibration using the background SDSS stars.

### 4.3. Power-law Slope

Finally, we examined the difference in CSD slopes between low- and high-I MBAs. Previous asteroid surveys showed that the size distribution of MBAs exhibits a wavy pattern. This structure is generated by the transition of impact strength between the strength- and gravity-scaled regimes (Davis et al. 1994; O’Brien & Greenberg 2003) as well as possibly by a small size cutoff due to the Poynting–Robertson drag and solar radiation pressure (Campo Bagatin et al. 1994). In addition, it has been indicated that the CSD slopes are different between S- and C-like asteroids (Ivezić et al. 2001; Yoshida & Nakamura 2007). In order to compare CSD slopes, the size range and number ratio of S- and C-like asteroids should be confirmed.

As a representative CSD slope of low-I MBAs, we cited the results of Yoshida & Nakamura (2007). The colorimetric asteroid survey in the field within $\pm 3^\circ$ from the ecliptic plane detected a thousand small MBAs, most of which have inclinations less than 10°. It presented CSD slopes of $b = 1.29 \pm 0.02, 1.33 \pm 0.02$ for S-like asteroids and $b = 0.6 \pm 0.02$ for C-like asteroids. Yoshida & Nakamura (2007) also showed the heliocentric distribution of both classes. In the outer region beyond ~2.6 AU where most of the sample asteroids in our survey are distributed, the fraction of S-like asteroids is ~0.2 and C-like asteroids is ~0.8.

We conducted Monte Carlo simulations to estimate the CSD for low-inclination MBAs with given fractions of the two groups. Tens of thousands of hypothetical S- and C-like asteroids are generated according to the abundance ratio of 1:4. Each asteroid is given a diameter ranging from 0.6 km to 40 km following the differential size distribution $dN/dD \propto D^{-b-1}$ with power-law indexes of $-2.29 \pm 0.02$ for S-like asteroids and $-2.33 \pm 0.02$ for C-like asteroids. The results showed that the compound CSD obeys a power-law distribution with $b = 1.32 \pm 0.02$ in 0.6 km $< D < 1.0$ km. It is significantly steeper than that obtained from our survey, $b = 1.25 \pm 0.03$, with a difference of $\Delta b = 0.07 \pm 0.04$. We confirmed that the high-inclination MBAs have a shallow CSD compared to the low-inclination MBAs, at least in the sub-kilometer size range.

On the other hand, it is difficult to compare the CSDs between Yoshida & Nakamura (2007) and this study in the larger size range of $D > 1.0$ km because of large uncertainties due to the small number of samples and the loss of some unmeasurable objects that are bright enough to reach saturation. Instead, we analyzed the SDSS MOC, including astrometric and photometric data for 471,569 moving objects. As in the case of our survey, most of them are unknown asteroids in orbit and albedos. We estimate orbital elements and the absolute magnitude of each SDSS MOC object using the same methods and assumptions as this study (see Sections 3.1 and 3.2).

The SDSS MOC objects are classified according to the following definitions. (1) MBAs are objects with $a = 2.1–3.3$ AU, (2) S-like asteroids are objects with $a^* > 0$, and C-like asteroids are objects with $a^* < 0$, where $a^*$ is the color index defined by Equation (11). (3) Low-inclination asteroids are objects with $I < 15^\circ$, and high-inclination asteroids are objects with $I > 15^\circ$. The diameter of each MBA is estimated assuming $p = 0.22$ for objects with $a^* > 0$ as S-like asteroids and $p = 0.07$ for objects with $a^* < 0$ as C-like asteroids. The SDSS MOC seems to keep the complete detection up to $r = 21.2$ mag corresponding to $D \leq 2.0$ km (Parker et al. 2008). In 2.0 km $< D < 5.0$ km, the CSDs of SDSS MOC MBAs have $b = 2.65 \pm 0.03$ for low-inclination S-like, $b = 2.17 \pm 0.04$ for high-inclination S-like, $b = 2.24 \pm 0.02$ for low-inclination C-like, and $b = 2.01 \pm 0.02$ for high-inclination C-like. We confirmed that high-inclination MBAs have a shallower CSD in both classes.

Then, model CSDs were generated from a mixture of S-like and C-like MBAs with a number ratio of 1:4 in each inclination population. The estimation of CSD slopes in 2.0 km $< D < 5.0$ km gives $b = 2.31 \pm 0.02$ for low-inclination MBAs and $b = 2.04 \pm 0.02$ for high-inclination MBAs. The difference of the slopes, $\Delta b = b(I < 15^\circ) - b(I > 15^\circ)$, is $0.27 \pm 0.03$, much larger than the value indicated in our survey ($\Delta b = 0.07 \pm 0.04$). The discrepancy in $\Delta b$ between the two size ranges can be explained by the difference in the wavy patterns in CSDs. We discuss the interpretations of this result in the following section.

### 4.4. Impact Strength Law

O’Brien & Greenberg (2003) presented an analytical model for steady-state size distributions resulting from a collisional cascade with the following two essential facts. First, the primary component of the CSD slope represented by a single power-law index $b_p$ is given by a simple expression of

$$b_p = \frac{5}{2 + s/3}, \quad (15)$$

where $s$ is the power-law index of the $Q_D^s$ law, namely, $Q_D^s \propto D^s$. Second, the transition of the $Q_D^s$ law at a diameter of $D_t = 0.1–1.0$ km induces the wavy structure on the MBA size distribution. In the gravity-scaled regime ($D > D_t$), the index $s_p$ is positive, i.e., $b_p < 2.5$. Conversely, in the strength-scaled regime ($D < D_t$), the index $s_p$ is negative, i.e., $b_p > 2.5$. The inflection in the $Q_D^s$ law results in wave-like oscillations about the CSD power law with an index $b_p$ in the gravity-scaled regime. The CSD shape in $D > D_t$ is determined by the primary slope $b_p$ as well as the phase and amplitude of the wave pattern. O’Brien & Greenberg (2005) found that the $Q_D^s$ law with $s \approx 1.40$ and $D_t \approx 0.2$ km reproduces the observed MBA size distribution.

The diameter range of CSD measurements in this study is 0.7–5.0 km covering from the first bump down to $D_t$. Terai & Itoh (2011) confirmed no significant difference in the peak/valley positions of the CSDs’ wavy pattern between low- and high-inclination MBAs. We suggest a simple model: the CSD shape varies only with $b_p$ and the wave amplitude between the two populations. The difference between a CSD slope in the gravity- and strength-scaled regimes ($D > D_t$ and $D < D_t$), is assumed to shift in proportion to wave amplitude. When the wave amplitude increases $k$ times, $b$ becomes $k(b(b_p) + b_p)$.

This model allows us to briefly express the relationships between $b_p$ and the estimated CSD slopes, $b_{1L}, b_{2L}, b_{1H},$ and $b_{2H},$ where 1 and 2 are the diameter ranges of 0.6–1.0 km and 2.0–5.0 km, respectively, and $L$ and $H$ are the low- and high-inclination populations, respectively.
The MBAs $Q_D^b$ law with $s_b = 1.40$ (O’Brien & Greenberg 2005) derives $b_p = 2.03$ using Equation (15). It corresponds to the primary CSD slope of low-inclination MBAs. The relational expressions of the CSD indexes in this model are given by

$$b_{i,H} - b_{p,H} = k(b_{i,L} - b_{p,L}), \quad i = 1, 2, \quad (16)$$

where $b_{p,L}$ and $b_{p,H}$ are the primary CSD slopes for low- and high-inclination MBAs, respectively ($b_{p,L} = 2.03$). Our analysis showed the local CSD slopes of $b_{i,L} = 1.32 \pm 0.02$, $b_{i,L} = 2.31 \pm 0.02$, $b_{1,H} = 1.25 \pm 0.03$, and $b_{2,H} = 2.04 \pm 0.02$. Those give solutions in Equation (16) of $b_{p,H} = 1.82 \pm 0.07$ and $k = 0.80 \pm 0.04$. This $b_{p,H}$ value is converted into $s_b = 2.2 \pm 0.3$ when Equation (15) is applied. However, the collisional evolution of high-inclination MBAs is dominated by collisions with low-inclination asteroids, though Equation (16) is based on collisional equilibrium in a self-contained system (O’Brien & Greenberg 2003). Numerical simulations for the collisional evolution are required to derive $s_b$ from $b_{p,H}$. The result in $b_{p,H} < b_{p,L}$ indicates a steep $Q_D^b$ law in the gravity-scaled regime of high-inclination MBAs. It leads to the conclusion that hypervelocity collisions on large bodies are relatively less disruptive. This may suggest that the inelasticity parameter determining the fraction of impact energy partitioned into fragment kinetic energy, generally denoted by $f_{KE}$ (Campo Bagatin et al. 2001; O’Brien & Greenberg 2005), decreases with collision velocity around several km s$^{-1}$.

Our results imply that the collisional evolution and the resulting size distribution of an asteroid population that suffered hypervelocity collisions are not the same as those of elliptic MBAs even if the compositions and internal structures are similar to each other. In the inner region of the planetesimal disk after the formation of giant planets and gas dissipation, small bodies are dynamically excited and collide with each other at high velocities. Bottke et al. (2005b) showed that in the primordial main-belt zone during the dynamical excitation phase caused by planetary embryos and Jupiter, collisions between remnant asteroids occur at 6–8 km s$^{-1}$ and collisions between remnant asteroids and depleted asteroids reach more than 10 km s$^{-1}$. The velocity-dependent $Q_D^b$ law should be introduced to investigate the ancient size distribution and its asteroid evolution at the final stage of planet formation processes. It allows us to more precisely estimate the impact rate and size distribution of meteorites colliding with the Earth and moon in the early solar system.

Besides high-inclination MBAs, near-Earth asteroids collide with each other at greater than 10 km s$^{-1}$ (Bottke et al. 1994). Jovian Trojans with high inclinations ($i \gtrsim 20^\circ$) also have mean collision velocities of ~6 km s$^{-1}$ or higher (Marzari et al. 1996). In addition, high-velocity collisional processes were confirmed in several planetesimal disks such as HD172555 (Lisse et al. 2009) and Epsilon Eridani systems (Thébault et al. 2002).

The relationship between the $Q_D^b$ law and collision velocity must be determined by a combination of further studies and observations, numerical simulations, and laboratory experiments. It provides insight into the collisional evolution of small-body populations located in regions close to the host star or distant from the ecliptic plane, as well as in the systems around a massive star or containing giant planets.

5. CONCLUSIONS

Our survey detected 441 asteroids in 13.6 deg$^2$ at high ecliptic latitudes with a 50% limiting magnitude of $r = 24.4$ mag. We obtained an unbiased sample consisting of 221 MBA candidates with inclinations of $i \gtrsim 14^\circ$ and absolute magnitudes of $H_i \lesssim 19.4$ mag. Although orbits and diameters of each asteroid cannot be determined and instead are estimated with the assumption of a circular orbit and a given albedo depending on radial regions, the sample yields a CSD of sufficient quality to measure its slope in the sub-kilometer size.

The CSD for high-inclination MBAs shows a roll-over at $D \sim 1.0$ km, which has also been indicated by Yoshida & Nakamura (2007) for low-inclination MBAs. The maximum likelihood analysis provided the best-fit power laws with $b = 1.25 \pm 0.03$ in 0.6 km $< D < 1.0$ km and $b = 1.84 \pm 0.27$ in 1.0 km $< D < 3.0$ km. Most of the MBA candidates are located beyond 2.6 AU where Yoshida & Nakamura (2007) showed the abundance ratio of S- and C-like asteroids is 1:4 for low-inclination sub-kilometer MBAs. The compound CSD with a number ratio of 1:4 has $b = 1.32 \pm 0.02$ in 0.6 km $< D < 1.0$ km, indicating that high-inclination MBAs have a shallower CSD in the sub-kilometer size.

We furthermore examined the CSDs in 2.0 km $< D < 5.0$ km using the SDSS MOC database. The slopes are $b = 2.31 \pm 0.02$ for low-inclination MBAs and $b = 2.04 \pm 0.02$ for high-inclination asteroids. Although high-inclination MBAs have shallower CSDs in both of the size ranges, the slope difference is larger in the larger size. This inconsistency is explained as being due to the difference of the wavy pattern on CSDs between low- and high-inclination populations. Assuming a simple model that both populations have a “bump” at the same position on the CSDs at a few kilometers diameter, high-inclination MBAs have a primary CSD slope of $b_p = 1.82 \pm 0.07$ over the diameter range of 0.6 km to 5.0 km. It is definitely shallower than that of low-inclination MBAs, indicating that hypervelocity collisions raise the relative strength of large bodies against catastrophic impacts.

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