Alternative to $R_\lambda$-scaling of Small-Scale Turbulence Statistics

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Traditionally, trends of universal turbulence statistics are presented versus $R_\lambda$, which is the Reynolds number based on Taylor’s scale $\lambda$ and the root-mean-squared (rms) velocity $u_{rms}$. $\lambda$ and $u_{rms}$, and hence $R_\lambda$, do not have the attribute of universality. The ratio of rms fluid-particle acceleration to rms viscous acceleration, $R_a$, is an alternative to $R_\lambda$ that has the advantage of being determined by the small scales of turbulence. This ratio has the following attributes: $R_a$ is a Reynolds number, it is composed of statistics of the small scales of turbulence, can be evaluated with single-wire hot-wire anemometry, and like $R_\lambda$, can be partially evaluated by means of flow similarity. For isotropic turbulence the relationship between $R_a$ and $R_\lambda$ is given. Graphs of velocity derivative flatness measured in anisotropic turbulence driven by counter rotating blades have different appearances depending on whether $R_a$ or $R_\lambda$ is the abscissa.

I. INTRODUCTION

Reynolds [1] sought, from the Navier-Stokes equation, “the dependence of the character of motion on a relation between the dimensional properties and the external circumstances of motion.” Assuming that the motion depends on a single velocity scale $U$ and length scale $c$, Reynolds found that the accelerations are of two distinct types and thereby deduced that the relevant solution of the Navier-Stokes equation “would show the birth of eddies to depend on some definite value of $cU/\mu$.” [1] where $\rho$ is the mass density of the fluid and $\mu$ is the coefficient of viscosity. Reynolds performed exhaustive experiments that demonstrated his deduction, as well as experiments on the stabilization of fluctuating flow. [1]. He discovered the sudden onset of flow instability [1]. The Navier-Stokes equations [1]; that is, the flow geometry becomes negligible in the limit that the Reynolds number is infinite. Discovering the statistics of the small scales, when appropriately scaled, should become equal as Reynolds number increases [3]. The observed trends as real turbulent flows is discussed by Nelkin [3] and Sreenivasan and Antonia [3].

The Reynolds number based on the root-mean-square (rms) of the longitudinal-velocity component $u_{rms}$ is given by

$$R_\lambda \equiv \langle u_1^2 \rangle^{1/2} / \langle \nu^2 \rangle$$

and

$$R_{\lambda} \equiv \langle \nu^2 \rangle$$

where $\langle \cdot \rangle$ denotes an average. $R_\lambda$ is a Reynolds number in Batchelor’s concept, and $R_{\lambda}$ is a parameterization of the relative magnitudes of $\nu^2 u$. The latter parameterization does not technically lead to a Reynolds number, but it will be shown that the two parameterizations become equivalent at large Reynolds numbers. We therefore introduce the two ratios:

$$R_a \equiv \langle a \cdot a \rangle^{1/2} / \langle \nu^2 \rangle$$

and

$$R_{\lambda} \equiv \langle \nu^2 \rangle$$

where $\langle \cdot \rangle$ denotes an average. $R_a$ is a Reynolds number in Batchelor’s concept, and $R_{\lambda}$ is a parameterization of the relative magnitudes of $\nu^2 u$.

To paraphrase Nelkin’s [3] description of Reynolds number scaling: if two turbulent flows have the same geometry and the same Reynolds number, then their statistics, when appropriately scaled, should be equal. A statistic of the small scales of turbulence is an average of quantities that contain only products of differences, such as two-point velocity difference or derivatives of velocity. Universality of the small scales of turbulence is the hypothesis that statistics of the small scales, when appropriately scaled, should become equal as Reynolds number increases [3]; that is, the flow geometry becomes negligible in the limit that the Reynolds number is infinite. Discovering the appropriate scaling that results in universality is the topic of a vast amount of research [3] and will not be pursued here. The relevance of universality to real turbulent flows is discussed by Nelkin [3] and Sreenivasan and Antonia [3].

The Reynolds number based on the root-mean-square (rms) of the longitudinal-velocity component $u_{rms}$ is given by

$$R_\lambda \equiv \langle u_1^2 \rangle^{1/2} / \langle \nu^2 \rangle$$

and

$$R_{\lambda} \equiv \langle \nu^2 \rangle$$

Here, $u_1$ and $x_1$ are the components of velocity and spatial coordinate in the direction of the 1-axis. For decades, $R_\lambda$ has been used as the abscissa for presenting statistics that are believed to be universal aspects of small-scale turbulence (such as velocity derivative statistics normalized by powers of $\langle (\partial u_1/\partial x_1)^2 \rangle$). The observed trends as $R_\lambda$ increases are an often-sought quantification of scaling universality. $R_\lambda$ has the advantage of being easily measured because it requires only measurement of $u_1$ (which yields $\partial u_1/\partial x_1$ by means of Taylor’s hypothesis); that measurement can be obtained with a single hot-wire anemometere. Alternatively, flow similarity can be used to estimate the energy
dissipation rate $\varepsilon$, and by substituting the local-isotropy relationship that $\varepsilon = 15\nu \langle (\partial u_1/\partial x_1)^2 \rangle$, $R_\lambda$ can be obtained from $R_\lambda = \bar{u}_{rms}^2 / (\varepsilon \nu / 15)^{1/2}$. Because $R_\lambda$ depends on $u_{rms}$, it depends on large-scale geometry of the flow. Nelkin discussed the nonuniversal attributes of $R_\lambda$. As a result of the nonuniversality of $R_\lambda$, statistics of the small scales, e.g., normalized derivative moments, when graphed with $R_\lambda$ on the abscissa, can have different curves corresponding to dissimilar flows.

II. ALTERNATIVE

In addition to graphing such statistics with $R_\lambda$ on the abscissa, it would seem advantageous to use a quantity on the abscissa that is solely a property of the small scales of turbulence. That advantage has long been recognized. Here, we consider the alternative $R_\alpha$ defined in (3), and determine how it can be measured with an instrument no more complex than a single-wire hot-wire anemometer. Because the intended application is to statistical

$$R_\alpha = \sqrt{1 + R_{vp}^2} \text{ and } R_{vp} = \sqrt{R_{\alpha}^2 - 1}. \quad (2)$$

Because $\langle \nabla p \cdot \nabla p \rangle \gg \langle \nu^2 (\nabla^2 u) \cdot (\nabla^2 u) \rangle$ at high Reynolds numbers, (3) becomes $R_\alpha \approx R_{vp}$.

A. Menu for the variance of viscous acceleration

The correlation of viscous acceleration is

$$V_{ij} (r) = \langle \nu^2 \nabla^2 u_r \nabla^2 u_i' \rangle = -\frac{\nu^2}{2} \nabla^2 \nabla^2 D_{ij} (r), \quad (3)$$

where prime denotes evaluation at a point $x'$; $x$ and $x'$ are independent variables; $r = x - x'$; $\nabla^2 \nabla^2$ is the Laplacian operator in $r$-space; the right-most expression in (3) is obtained on the basis of local homogeneity. Let $\varepsilon$ denote the energy dissipation rate, and $D_{ij} (r)$ and $D_{ijk} (r)$ denote the second- and third-order velocity structure functions. The Navier-Stokes equation and local isotropy give (4) $\partial_t D_{ij} (r) + \partial_{x_k} D_{ijk} (r) + (4/3) \varepsilon = 2\nu \nabla^2 D_{ij} (r)$; applying the operator $-(\nu/4) \nabla^2$ to that equation and comparison with (3) gives,

$$V_{ij} (r) = -\left( \frac{\nu}{4} \right) \nabla^2 \left[ \partial_t D_{ij} (r) + \partial_{x_k} D_{ijk} (r) \right]. \quad (4)$$

Derivative operators are abbreviated; e.g., $\partial_t = \partial/\partial t$, $\partial^2 = \partial^2/\partial x^2$, etc. Summation is implied by repeated indexes. Performing the contraction of (3) and (4) such that the terms become functions of $r$ and $\nabla^2 \nabla^2 \rightarrow \partial^2 + (2/r) \partial_r$, we have, on the basis of local isotropy,

$$V_{ii} (r) = -\nu \left[ \frac{\partial^2}{r^2} + \frac{1}{r} \frac{\partial^2}{\partial r^2} \right] D_{ij} (r) = \frac{-\nu^2}{2} \left( r \partial_t^3 + 11 \partial_r^4 + \frac{24}{r} \partial_r^3 \right) D_{11} (r)$$

$$= -\partial_t \left[ \frac{\partial^2}{2} + \frac{1}{r} \partial_r \right] D_{ii} (r) + \nu \left[ -\frac{1}{r^2} D_{111} (r) + \left( -\partial_t^2 \frac{7}{r^2} \partial_r^2 - \frac{3}{r^2} \partial_r + \frac{6}{r^3} \right) D_{133} (r) \right]. \quad (5)$$

Power-series expansion of the structure functions followed by differentiation and taking the limit $r \rightarrow 0$ gives various formulas for the viscous acceleration variance:

$$V_{ii} (0) = \langle \nu^2 (\nabla^2 u) \cdot (\nabla^2 u) \rangle = 12\nu^2 \langle (\partial^2_{x_1} u_\beta)^2 \rangle = 35\nu^2 \langle (\partial^2_{x_1} u_1)^2 \rangle$$

$$= -\frac{1}{2} \partial_t \varepsilon - \frac{105}{4} \nu \langle (\partial_{x_1} u_\beta)^2 \partial_{x_1} u_1 \rangle = -\frac{1}{2} \partial_t \varepsilon - \frac{35}{2} \nu \langle (\partial_{x_1} u_1)^3 \rangle, \quad (6)$$

where $\partial^2_{x_1} \equiv \partial^2/\partial x^2_1$. The term $-(1/2) \partial_t \varepsilon$, which vanishes for local stationarity, is included above, but was neglected by Ref. (3). The enstrophy generation rate can be written as: $(-35/2) \langle (\partial_{x_1} u_1)^3 \rangle = \langle \omega_j \omega_j s_{ij} \rangle = (-4/3) s_{ij} s_{jk} s_{ki}$; comparing this with the right-most expression in (3) gives expressions that can be evaluated by DNS or multi-wire anemometers (1) (2):
Asymptotic expressions for \( \langle \nabla \cdot (\nabla \mathbf{u}) \rangle \) and Wilczak [15] pointed out that the utility of determining \( H \) of Hill and Wilczak [15] argued that the ratio and negative parts of the integrand, i.e., between \( r \) and \( D \) of the integral where \( \mathbf{h} \) is known such that (6) gives 
\[
\langle \nu^2 (\nabla^2 \mathbf{u}) \cdot (\nabla^2 \mathbf{u}) \rangle = -\frac{1}{2} \partial_t \varepsilon + \nu \langle \omega_i \omega_j s_{ij} \rangle = -\frac{1}{2} \partial_t \varepsilon - \frac{4}{3} \nu \langle s_{ij} s_{jk} s_{ki} \rangle,
\]
where \( \omega_i \) is vorticity vector and \( s_{ij} \) is rate-of-strain tensor. The velocity-derivative skewness is 
\[
S \equiv \langle (\partial_{x_i} u_1)^3 \rangle / \langle (\partial_{x_i} u_1)^2 \rangle \equiv \langle (\partial_{x_i} u_1)^3 \rangle / \langle \varepsilon \rangle / \langle \nu \rangle^{3/2},
\]
such that (3) gives 
\[
\langle \nu^2 (\nabla^2 \mathbf{u}) \cdot (\nabla^2 \mathbf{u}) \rangle = -\frac{1}{2} \partial_t \varepsilon + 0.3 \varepsilon^{3/2} \nu^{-1/2} |S|.
\]
The variation of \( |S| \) with \( R_\lambda \) is known. Herring and Kerr [13] [14] show \( |S| \) increasing from 0.674 at \( R_\lambda = 0.46 \) to become constant at 0.5 for \( R_\lambda > 20 \) (see Ref. [15]), whereas at \( R_\lambda > 400 \) Antonia et al. [16] show \( |S| \) increasing from 0.5 as \( |S| \approx 0.8 R_\lambda^{0.11} \). If \( \varepsilon \) and \( u_{rms} \) are known from measurements or from flow similarity, then \( R_\lambda = u_{rms} / \langle \varepsilon \nu \rangle^{1/2} \) and hence \( |S| \) is known such that (3) can determine \( \langle \nu^2 (\nabla^2 \mathbf{u}) \cdot (\nabla^2 \mathbf{u}) \rangle \). The term - (1/2) \( \partial_t \varepsilon \) can be neglected if it is not evaluated. The DNS data of Herring and Kerr [13] [14] suggest that \( |S| \approx R_\lambda/5 \) for \( R_\lambda < 1 \), then \( \langle \nu^2 (\nabla^2 \mathbf{u}) \cdot (\nabla^2 \mathbf{u}) \rangle \approx 0.06 \varepsilon^{3/2} \nu^{-1/2} R_\lambda \) for \( R_\lambda < 1 \).

### B. Menu for the variance of pressure-gradient acceleration

Poisson’s equation, local homogeneity and local isotropy, but no other approximations, result in [16]
\[
\langle \mathbf{\nabla} \cdot \mathbf{\nabla} \rangle = 4 \int_0^\infty r^{-3} [D_{1111} (r) + D_{aaaa} (r) - 6 D_{11\beta \beta} (r)] dr,
\]
where \( D_{1111} (r) \), \( D_{aaaa} (r) \), and \( D_{11\beta \beta} (r) \) are components of the fourth-order velocity structure-function tensor, which is defined by \( D_{ijkl} (r) \equiv \langle (u_i - u'_i) (u_j - u'_j) (u_k - u'_k) (u_l - u'_l) \rangle \); the 1-axis is parallel to the separation vector \( r \); \( \alpha \) and \( \beta \) denote the Cartesian axes perpendicular to the 1-axis. Thus, \( \alpha \) and \( \beta \) are 2 or 3; equally valid options under local isotropy are \( \alpha = \beta \) or \( \alpha \neq \beta \).

There is enough cancellation between the positive and negative parts of the integrand, i.e., between \( r^{-3} [D_{1111} (r) + D_{aaaa} (r)] \) and \( -r^{-3} (D_{11\beta \beta} (r)) \), to make evaluation of the integral \( r^{-3} [D_{1111} (r) + D_{aaaa} (r) - 6 D_{11\beta \beta} (r)] dr \) difficult by means of experimental or DNS data [15] [16] [17]. Hill and Wilczak [18] argued that the ratio \( H_\chi \equiv \int_0^\infty r^{-3} [D_{1111} (r) + D_{aaaa} (r) - 6 D_{11\beta \beta} (r)] dr / \int_0^\infty r^{-3} D_{1111} (r) dr \) is a universal constant at high Reynolds numbers. Universality of \( H_\chi \) is equivalent to the assertion that \( \langle \mathbf{\nabla} \cdot \mathbf{\nabla} \rangle \) scales with \( \int_0^\infty r^{-3} D_{1111} (r) \) at high Reynolds numbers. Hill and Wilczak [19] pointed out that the utility of determining \( H_\chi \) is that the pressure-gradient variance can then be measured with a single-wire hot-wire anemometer by means of 
\[
\langle \mathbf{\nabla} \cdot \mathbf{\nabla} \rangle = 4 H_\chi \int_0^\infty r^{-3} D_{1111} (r) dr.
\]
Asymptotic expressions for \( \langle \mathbf{\nabla} \cdot \mathbf{\nabla} \rangle \) derived from (11) are [20] 
\[
\langle \mathbf{\nabla} \cdot \mathbf{\nabla} \rangle \approx 3.1 H_\chi \varepsilon^{3/2} \nu^{-1/2} F^{0.79} \approx 3.9 H_\chi \varepsilon^{3/2} \nu^{-1/2} R_\lambda^{0.25} \text{ for } R_\lambda \gtrsim 400,
\]
where \( F \equiv \langle (\partial_{x_i} u_1)^4 \rangle / \langle (\partial_{x_i} u_1)^2 \rangle^2 \), and the right-most expression is obtained from the data for \( F \) of Antonia et al. [21], and, for low Reynolds numbers \( F \approx 0.11 \varepsilon^{3/2} \nu^{-1/2} R_\lambda \) for \( R_\lambda \lesssim 20 \).
Using DNS data, the preferable evaluation of $H_\chi$ is via $H_\chi = \langle \nabla p \cdot \nabla p \rangle / \left[ 4 \int_0^\infty r^{-3} D_{1111}(r) \, dr \right]$ so as to avoid the statistical uncertainty caused by the cancellations within the integrand of (10). Vedula and Yeung [19] evaluated $H_\chi$ using DNS data with $R_\lambda < 230$ and obtained the variation from $H_\chi \approx 0.55$ at $R_\lambda = 20$ to a constant value of $H_\chi \approx 0.65$ in the range $80 < R_\lambda < 230$. For $H_\chi = 0.65$, [12] agrees quantitatively with the DNS data in Table 1 of Gotoh and Fukayama [20] for $R_\lambda \geq 387$, and [12] is a good approximation for $R_\lambda \gtrsim 200$. [18] Since the asymptotic formula (12) is thereby verified with $H_\chi = 0.65$, the range of validity of $H_\chi = 0.65$ seems to extend to the highest $R_\lambda$ for which Antonia et al. [7] obtained $F$, namely $R_\lambda \approx 10^{14}$, but it might extend to $R_\lambda = \infty$. On that basis, we have the increase from the low-Reynolds-number asymptote [2] $H_\chi = 0.36$ as $R_\lambda \to 0$, to $H_\chi \approx 0.55$ at $R_\lambda = 20$, to $H_\chi \approx 0.65$ in the range $R_\lambda > 80$.

Thus, the menu for calculating $(\nabla p \cdot \nabla p)$ is (11), or (11) with the empirically known values of $H_\chi$ as a function of $R_\lambda$, or (12)-(13) with the DNS data of Vedula and Yeung [19] and Gotoh and Fukayama [20] used in the range $20 \gtrsim R_\lambda \gtrsim 400$.

III. ISOTROPIC TURBULENCE

For isotropic turbulence there is a one-to-one relationship between $R_\lambda$ and $R_\lambda$; it is shown in Fig. 1 with $R_{\nabla p}$ included. Fig. 1 was obtained from (3) by use of (12)-(13) and (9) and the discussions following those equations. That is: for $R_\lambda \lesssim 1$, $R_{\nabla p} \approx (12R_\lambda/35|S|)^{1/2} \approx (12/7)^{1/2} \approx 1.3$; for $1 \lesssim R_\lambda \lesssim 20$, $R_{\nabla p} \approx (12R_\lambda/35|S|)^{1/2}$ with data of Kerr [14] for $|S|$; for $20 \lesssim R_\lambda \lesssim 400$, $R_{\nabla p} \approx 2.6 \left( (\nabla p \cdot \nabla p)/\lambda^3/\nu^{-1/2} \right)^{1/2}$ with data tabulated by Vedula and Yeung [19] and Gotoh and Fukayama [20]; for $R_\lambda \gtrsim 400$, $R_{\nabla p} \approx \left[ (3.1H_\chi^3/\nu^{-1/2} F_{0.79}^2) / (0.35^{3/2} \nu^{-1/2} |S|) \right]^{1/2} \approx 3.3R_\lambda^{0.79}$; where the above mentioned $R_\lambda$ variation of $F$ and $|S|$ of Ref. [3] was used; finally, $R_\lambda = (1 + R_{\nabla p}^2)^{-1/2}$. In Fig. 1, $R_{\nabla p} \approx 3.3R_\lambda^{0.79}$ is extended to $R_\lambda = 100$ to show, by comparison with the data of Vedula and Yeung and Gotoh and Fukayama, that it is a good approximation for $R_\lambda \gtrsim 200$. From $R_\lambda = 1$ to $10^4$ in Fig. 1, $R_{\nabla p}$ changes by one decade and $R_\lambda$ less so. Fig. 1 is based on nearly isotropic data only for $R_\lambda \lesssim 400$; for $R_\lambda \gtrsim 400$ the data used for $F$ in [14] and $S$ in [3] is that of Antonia et al. [7] which consists, in part, of atmospheric surface layer data at $R_\lambda \gtrsim 2000$. The assumption is that for $R_\lambda \gtrsim 400$ the turbulence is locally isotropic and that the relationships of $F$ and $S$ to $R_\lambda$ measured by Antonia et al. [7] do not differ significantly from what they would be for isotropic turbulence. Some support for the assumption is that the asymptote $R_{\nabla p} \approx 3.3R_\lambda^{0.79}$ in Fig. 1 agrees with the nearly isotropic data to $R_\lambda \approx 200$. However, further confirmation must await DNS and experiments on nearly isotropic turbulence at higher Reynolds numbers than have been attained to date.

A. Relationship to recent data

The DNS data of Vedula and Yeung [19], Gotoh and Rogallo [22], and Gotoh and Fukayama [20] are in the range of $R_\lambda$ where $|S| \approx 0.5$. Since their DNS are steady state, (3) gives $\langle v^2 (\nabla^2 u) \cdot (\nabla^2 u) \rangle \approx 0.15\epsilon^3/2\nu^{-1/2}$. In their Fig. 1, Vedula and Yeung [19] show a ratio that they call $\zeta$, which equals $R_{\nabla p}^2$, as well as a quantity $a_0^{(I)} \equiv \langle a \cdot a \rangle / (3\epsilon^3/2\nu^{-1/2}) \approx 0.05R_{\nabla p}^2$, where $\langle v^2 (\nabla^2 u) \cdot (\nabla^2 u) \rangle \approx 0.15\epsilon^3/2\nu^{-1/2}$ was used; they graphed both $\zeta$ and $a_0^{(I)}$ versus $R_\lambda$. Similarly, Gotoh and Rogallo [22] and Gotoh and Fukayama [20] show $F_{\nabla p} = a_0^{(I)} \approx 0.15R_a^2$ versus $R_\lambda$ in their Figs. 1 and 2. Therefore, the above-mentioned graphs show $R_{\nabla p}^2$ and $R_a^2$ as $R_\lambda$ varies. Reversing the role of ordinates and abscissas in their graphs and in Fig. 1, the graphs show $R_\lambda$ for nearly isotropic turbulence as the universal Reynolds number $R_a$ varies.

IV. ANISOTROPIC TURBULENCE

For anisotropic turbulence, the value of $\langle u_1^2 \rangle$ depends by definition of anisotropy, on the direction of $r$. Thus, $R_a$ does also. The appearance of a statistic graphed versus $R_\lambda$ can change depending on which velocity component is used to calculate $R_a$. For many flow geometries, both $R_{\nabla p}$ and $R_a$ increase as $R_\lambda$ increases. It is appropriate to change perspective: The nonuniversal abscissa $R_\lambda$ typically, but not necessarily, increases as the universal abscissas $R_{\nabla p}$ and $R_a$ increase.

The turbulent flow in a cylinder driven by counter rotating blades has several modes of large-scale structure [23, 24] and is anisotropic at large scales, but local isotropy is approached [25] at high Reynolds numbers. Belin
et al. [26] measured $F$ in the flow between counter-rotating blades; their $F$ graphed with $R_\lambda$ on the abscissa has a maximum and minimum, shown here in Fig. 2a. That behaviour of $F$ has been called controversial [24] because it has not been previously observed, and was not observed by Pearson and Antonia [28] who obtained $F$ in the range of $R_\lambda$ where Belin et al. observe the maximum and minimum. The Belin et al. [20] data for $F$ is thus a good test of whether or not $R_a$ produces a different organization of a small-scale statistic than $R_\lambda$. Only the single lowest Reynolds-number datum of Belin et al. is below $R_\lambda = 200$. Figure 1 shows that (12) is accurate for $R_\lambda = 200$. From the menus we therefore select (1) and (12) and use $H_\lambda = 0.65$; then $R_\nabla p = (2.0 F^{0.79}/0.3 |S|)^{1/2}$. Thus their data for $F$ and $S$ are used to calculate $R_\nabla p$, then $R_a$ is obtained from (1), although $R_a \approx R_\nabla p$ in this case. In Fig. 2a the data of Belin et al. for $F$ versus $R_\lambda$ is shown using distinct symbols in several ranges of $R_\lambda$. Figure 2b shows their $F$ versus $R_a$ using the same symbols for the same data points. The maximum and minimum in Fig. 2a is absent in Fig.2b, and the symbols are significantly rearranged in Fig. 2b relative to Fig. 2a. Figure 2b is not just $(F^{0.79})^{1/2}$ vs. $F$ because that would be a straight line. The data is scattered in Fig. 2b because of the scatter in $S$. Their $S$-values at nearly the same value of $R_\lambda$ vary by as much as 28%. The important point is shown by comparing Fig. 2a with 2b: The use of $R_a$ on the abscissa rather than $R_\lambda$ can give a significantly different distribution of the data of a small-scale statistic.

V. SUMMARY AND COMMENT

As defined in (1), $R_a$ is a Reynolds number; it is the ratio of rms fluid-particle acceleration to rms viscous acceleration; it is composed of statistics of the small scales of turbulence; it can be used as a universal abscissa for judging the universality of turbulence statistics. Although $R_\nabla p$ is not strictly a Reynolds number, it can also be used as a universal abscissa. At high Reynolds numbers, $R_\nabla p \approx R_a$. $R_\nabla p$ can be evaluated with single-wire hot-wire anemometry, and by flow similarity; use of (4) then determines $R_a$. If one chooses to measure $S$ for the purpose of obtaining the viscous acceleration variance from (4), then accurate values of $S$ require many samples and high spatial resolution. This does not require an enormous data set because the data can be stored as a histogram having one bin for each of the digitizer’s distinct outputs with a bin for digitizer overflow and one for underflow.

Models of the small-scale statistics of turbulence should be expressed in terms of universal attributes instead of in terms of $R_\lambda$. For example, in Table II of Belin et al. [20], the model by Pullin and Saffman [29] is in good agreement with data when judged in terms of power laws between derivative moments, but it is in relatively poor agreement with data when judged in terms of power laws between normalized derivative moments and $R_\lambda$. $R_\lambda$ can be specific to the flow geometry.

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Fig. 1 Relation of $R_a$ (upper symbols) and $R_\nabla p$ (lower symbols) to $R_\lambda$ for isotropic turbulence. Data of Refs. [14], [19], [20] produce the pluses, asterisks, and triangles, respectively. Solid lines are the low- and high-Reynolds number asymptotes.

Fig. 2 a) The velocity-derivative flatness of Belin et al. [20] versus $R_\lambda$ with different symbols for several ranges of $R_\lambda$; b) the same data and symbols for each point, but versus $R_a$.

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