More on closed string induced higher derivative interactions on D-branes

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Abstract

In our continued efforts of matching full string computations with the corresponding effective field theory computations, we evaluate string theory correlators in closed forms. In particular, we consider a correlator between three SYM vertex operators and one Ramond-Ramond C-field vertex operator:

\[ \langle V_C V_\phi V_A V_A \rangle \].

We show that the infinite number of massless poles of this amplitude can be reproduced by the Born-Infeld action, the Wess-Zumino terms, and their higher derivative corrections. More specifically we find, up to an on-shell ambiguity, two scalar field and two gauge field couplings to all orders in \( \alpha' \) such that the infinite number of massless poles of the field theory amplitude exactly match the infinite number of massless poles of S-matrix elements of \( \langle V_C V_\phi V_A V_A \rangle \). We comment on close intertwinedness of an open string and a closed string that must be behind the matching.
1 Introduction

D-brane physics \cite{1,2,3} has played a central role in theoretical high energy physics for more than one and a half decades by now. In the open string description, a $D_p$-brane with a $(p+1)$-dimensional world volume is realized as a hypersurface in flat spacetime with the appropriate boundary condition on the string coordinates: Dirichlet boundary conditions on the directions transverse to the $D_p$-brane and Neumann boundary conditions along the worldvolume of the $D_p$-brane \cite{1}. The bosonic action for multiple $D_p$-branes was given by Myers \cite{5}. Note that a supersymmetric generalization is still unknown; see however \cite{6}. The effective action for a single bosonic $D_p$-brane was found in \cite{7}. The supersymmetric action for a single $D_p$-brane was derived in \cite{8}. See \cite{9} for more details on Born-Infeld, Chern-Simons actions and their higher derivative corrections. Section 5 of \cite{10} has a review of Chern-Simons action.

The advent of $D_p$-brane physics has greatly promoted the significance of an open string bringing numerous new results, and is likely to continue bringing exciting new physics in the future. A conjecture put forward in \cite{11} may be an example: quantum effects of open strings moving on $D_p$-branes should produce the curvature of the host $D_p$-branes.

Having concrete tools for various multi-point string amplitudes is important for many purposes, including the first-principle derivation of AdS/CFT. Previous works on scattering that involve $D_p$-branes and some applications of $D_p$-branes include \cite{12}. Given that a close interplay between an open string and a closed string must be behind AdS/CFT, amplitudes involving a mixture of open string states and closed string states should be especially worth studying. In this work, we continue our previous endeavours of computing amplitudes of one Ramond-Ramond $C$-field vertex and three massless open string vertices \cite{9}. The amplitudes that we specifically consider are $<V_CV_\phi V_AV_A>$ and $<V_CV_\phi V_AV_AV_A>$ \cite{9} (We focus on $<V_CV_\phi V_AV_A>$ in the main text presenting the simpler case $<V_CV_\phi V_A>$ in Appendix B.)

Applying the same methodology as that of the previous works \cite{9,14} and \cite{15}, we find below the precise (up to an on-shell ambiguity) forms of the two scalar two gauge field

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1 One may wonder whether it would be possible by applying string T-duality to deduce the result of $<V_CV_\phi V_AV_A>$ from \cite{9} in which $<V_CV_AV_AV_A>$ was analysed. Applying T-duality in present types of computations can be subtle. It is definitely so in loop-level computations \cite{13}. Although the present computations are all at tree-level the presence of a closed string state can make things subtle, and indeed does so.
vertices such that the infinite number of massless poles of the field theory amplitude exactly match the infinite number of massless poles of S-matrix elements of $< V_C V_\phi V_A V_A >$ to all orders in $\alpha'$. To obtain the infinite number of massless poles for this case, one needs to determine the higher derivative couplings of two scalar two gauge field couplings to all orders in $\alpha'$. (This is analogous to the situation in [9] where $< V_C V_A V_A V_A >$ was analyzed. The precise form of the four gauge field vertices was determined for the field theory amplitudes to match with the corresponding string theory computations.) We obtain all the infinite number of massless poles in field theory and find consistent results with the string theory counterpart.

Additionally, remarks on a subtlety are in order. The subtlety may be tied with the profound relation between an open and closed string. The present work is another demonstration that the pure SYM vertices (such as two scalar two gauge field couplings of this work) produce the same massless poles as the corresponding correlator of one RR field vertex $V_C$ and three SYM vertices.

This phenomenon seems quite universal and must have deep origins going back to the intertwinedness of an open string and a closed string. The intertwinedness should originate from the composite nature of a closed string state in terms of open string states. The compositeness is (implicitly) exploited in a common practice in literature, identification of both sets of closed oscillators with those of an open string’s. We will briefly comment on this issue in section 2 and contemplate the issue further in the conclusion.

The organization of the paper is as follows. In section 2, we calculate a tree-level four-point string scattering of one RR vertex, one scalar field vertex operator and two gauge field vertex operators, $< V_C V_\phi V_A V_A >$. In section 3, we consider the low energy effective field theory and determine the interaction vertices that, with Myers’ terms, produce the same massless poles as those of the string amplitude. In the conclusion, we summarize the results and ponder on the profound relation between an open string and a closed string as a reason for the matching. We end with comments on the future directions. In Appendix A, a summary of our conventions is presented. Appendix B contains a parallel analysis for the case of $< V_C V_\phi V_A >$. 

2
2 String amplitude computations

By applying the conformal field theory technique, we carry out the string scattering amplitude of one closed string Ramond-Ramond field, one scalar field and two gauge fields on the world volume of BPS $D_p$-branes in type II super string theory within a flat background. Some efforts for the tree level scattering amplitudes have been done [10, 17, 18, 9, 14, 15]. To compute a S-matrix element, one has to know the picture of the vertex operators so that the sum of the super ghost charges must be -2 for disk level amplitudes.

The vertex operators that we will need are given by

$$
V^{(0)}_\phi(x) = \xi_i \left( \partial X^i(x) + \alpha' k^i \psi \phi (x) \right) e^{\alpha' i k \cdot X(x)}, \\
V^{(-1)}_\phi(y) = \xi_\psi(y) e^{-\phi(y)} e^{\alpha' i q \cdot X(y)}, \\
V^{(0)}_A(x) = \xi_a \left( \partial X^a(x) + \alpha' q^a \psi \phi (x) \right) e^{\alpha' i q \cdot X(x)}, \\
V^{(-1)}_A(y) = \xi_\psi^a(y) e^{-\phi(y)} e^{\alpha' i q \cdot X(y)}
$$

where $(k, q, p)$ are the momenta of the scalar field, gauge field and $C$-field respectively; they satisfy the on-shell condition $k^2 = q^2 = p^2 = 0$. Our notation is such that the spinorial indices are raised by the charge conjugation matrix, $C^{\alpha\beta}$

$$(P_- \mathcal{H}_{(n)} M_p)^{\alpha\beta} = C^{\alpha\delta} (P_- \mathcal{H}_{(n)})_{\delta\beta}$$

(2)

In particular, the trace is defined by

$$
\text{Tr} (P_- \mathcal{H}_{(n)} M_p \gamma^k) \equiv (P_- \mathcal{H}_{(n)} M_p)^{\alpha\beta} (\gamma^k C^{-1})_{\alpha\beta} \\
\text{Tr} (P_- \mathcal{H}_{(n)} M_p \Gamma^{jai}) \equiv (P_- \mathcal{H}_{(n)} M_p)^{\alpha\beta} (\Gamma^{jai} C^{-1})_{\alpha\beta}
$$

(3)

where $P_-$ is a projection operator, $P_- = \frac{1}{2} (1 - \gamma^{11})$, and

$$
\mathcal{H}_{(n)} = \frac{a_n}{n!} H_{\mu_1 \ldots \mu_n} \gamma^\mu_1 \ldots \gamma^\mu_n,
$$

with $n = 2, 4$ for type IIA and $n = 1, 3, 5$ for type IIB. $a_n = i$ for IIA and $a_n = 1$ for IIB theory. To employ standard holomorphic worldsheet correlators, we implement the usual doubling trick. For more details see Appendix A of [10].

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2 We keep $\alpha'$ explicitly in this work. One may set $\alpha' = 2$ to simplify the expressions.
2.1 Computation of $< V_C V_A V_A >$

The S-matrix element of one closed string C field, one scalar field and two gauge fields is given by the following correlation function

$$A^{\phi AA} \sim \int dx_1 dx_2 dx_3 dz \langle V_{\phi}^{-1} (x_1) V_A^{(0)} (x_2) V_A^{(0)} (x_3) V_{RR}^{(-\frac{1}{2}, -\frac{1}{2})} (z, \bar{z}) \rangle,$$

(4)

The open string vertex operators are inserted at the boundary of the disk worldsheet while the closed string vertex operator is inserted inside. The amplitude reduces to the following correlators for 123 ordering

$$A^{\phi AA} \sim \int dx_1 dx_2 dx_3 dx_4 dx_5 (P-H_{(n)} M_p)^{\alpha \beta} \xi_1 \xi_2 \xi_{34} x_5^{-1/4} (x_{14} x_{15})^{-1/2} \times (I_1 + I_2 + I_3 + I_4) \text{Tr} (\lambda_1 \lambda_2 \lambda_3),$$

(5)

where $x_{ij} = x_i - x_j$, and

$I_1 = < : e^{\alpha' \lambda_1} : \partial X^a (x_2) e^{\alpha' \lambda_2} : \partial X^b (x_3) e^{\alpha' \lambda_3} : e^{i \frac{4}{\sqrt{2}} p \cdot X (x_4)} : e^{i \frac{4}{\sqrt{2}} p \cdot D \cdot X (x_5)} : >$

$\times < : S_\alpha (x_4) : S_\beta (x_5) : \psi^i (x_1) : >$,

$I_2 = < : e^{\alpha' \lambda_1} : e^{\alpha' \lambda_2} : \partial X^b (x_3) e^{\alpha' \lambda_3} : e^{i \frac{4}{\sqrt{2}} p \cdot X (x_4)} : e^{i \frac{4}{\sqrt{2}} p \cdot D \cdot X (x_5)} : >$

$\times < : S_\alpha (x_4) : S_\beta (x_5) : \psi^i (x_1) : \alpha' \lambda_2 \cdot \psi^a (x_2) : >$,

$I_3 = < : e^{\alpha' \lambda_1} : \partial X^a (x_2) e^{\alpha' \lambda_2} : e^{\alpha' \lambda_3} : e^{i \frac{4}{\sqrt{2}} p \cdot X (x_4)} : e^{i \frac{4}{\sqrt{2}} p \cdot D \cdot X (x_5)} : >$

$\times < : S_\alpha (x_4) : S_\beta (x_5) : \psi^i (x_1) : \alpha' \lambda_3 \cdot \psi^b (x_3) : >$,

$I_4 = < : e^{\alpha' \lambda_1} : e^{\alpha' \lambda_2} : e^{\alpha' \lambda_3} : e^{i \frac{4}{\sqrt{2}} p \cdot X (x_4)} : e^{i \frac{4}{\sqrt{2}} p \cdot D \cdot X (x_5)} : >$

$\times < : S_\alpha (x_4) : S_\beta (x_5) : \psi^i (x_1) : \alpha' \lambda_2 \cdot \psi^a (x_2) : \alpha' \lambda_3 \cdot \psi^b (x_3) : >$.$$ (6)

These correlators can be computed in straightforward fashion using Wick’s theorem. To obtain correlation function between two spin operators and several fermion fields and currents see Appendix A of [10].

The correlation function between two spin fields, two currents and one worldsheet fermion - which we call $I_6^{\text{bdaci}}$ - is more complicated:

$$I_6^{\text{bdaci}} = \langle : S_\alpha (x_4) : S_\beta (x_5) : \psi^i (x_1) : \psi^a (x_2) : \psi^b (x_3) : \rangle$$

$$= \left\{ \Gamma^{\text{bdaci}} C^{-1} \right\}_{\alpha \beta} + \alpha' r_1 \frac{\text{Re} [x_{24} x_{35}]}{x_{23} x_{45}} + \alpha^2 r_2 \left( \frac{\text{Re} [x_{24} x_{35}]}{x_{23} x_{45}} \right)^2 \right\} 2^{-5/2} x_{45}^{-5/4} (x_{14} x_{15})^{-1/2},$$

(7)
where
\[
\begin{align*}
\mathcal{A}^{\phi AA} &\sim \int dx_1 dx_2 dx_3 dx_4 dx_5 (P_\mu H_{(n)}(M_\mu)_{ab} | x_{14}x_{15})^{-1/2} \biggl( I_1 (-\eta^{ab} x_{23}^2 + a^b x_{24} + a^b x_{25} - \alpha^2 k_{2c} k_{3d} I_0^{abi}) \biggr) \text{Tr} (\lambda_1 \lambda_2 \lambda_3),
\end{align*}
\]
where
\[
\begin{align*}
I &= |x_{12}|^{\alpha^2 k_1 k_2} |x_{13}|^{\alpha^2 k_1 k_3} |x_{14}x_{15}|^{\alpha^2 k_2 k_3} |x_{23}|^{\alpha^2 k_1 k_4} |x_{24}x_{25}|^{\alpha^2 k_2 k_4} |x_{34}x_{35}|^{\alpha^2 k_1 k_5} |x_{45}|^{\alpha^2 k_1 k_5},
\end{align*}
\]
\[
\begin{align*}
a_1^a &= i k_1^a \left( \frac{x_{14}}{x_{12}x_{24}} + \frac{x_{15}}{x_{12}x_{25}} \right) + i k_2^a \left( \frac{x_{43}}{x_{24}x_{23}} + \frac{x_{53}}{x_{25}x_{23}} \right),
\end{align*}
\]
\[
\begin{align*}
a_2^b &= i k_1^b \left( \frac{x_{14}}{x_{13}x_{34}} + \frac{x_{15}}{x_{13}x_{35}} \right) + i k_2^b \left( \frac{x_{24}}{x_{34}x_{23}} + \frac{x_{25}}{x_{35}x_{23}} \right),
\end{align*}
\]
\[
\begin{align*}
a_3^{bi} &= \alpha i k_{3d} 2^{-3/2} x_{45} \xi_{14}x_{35}^{-1} (x_{14}x_{15})^{-1/2} \left( \Gamma^{abi} C^{-1} \right)_{ab},
\end{align*}
\]
\[
\begin{align*}
a_4^{ai} &= \alpha i k_{2c} 2^{-3/2} x_{45} \xi_{14}x_{25}^{-1} (x_{14}x_{15})^{-1/2} \left( \Gamma^{aci} C^{-1} \right)_{ab},
\end{align*}
\]
\[
\begin{align*}
I_1 &= \langle S_\alpha(x_4) : S_\beta(x_5) : \psi(x_1) : \rangle = 2^{-1/2} x_{45}^{-3/4} (x_{14}x_{15})^{-1/2} (\gamma^i C^{-1})_{ab}.
\end{align*}
\]

Explicitly evaluating the integrals over the closed string location(to deal with the integrals and to provide more details see Appendix B of [10]), one can write the amplitude (9) as
\[
\begin{align*}
\mathcal{A}^{\phi AAC} &= \mathcal{A}_1 + \mathcal{A}_2 + \mathcal{A}_3 + \mathcal{A}_4 + \mathcal{A}_5
\end{align*}
\]
where
\[
\begin{align*}
\mathcal{A}_1 &\sim -2^{-1/2} \xi_{14} \xi_{2a} \xi_{3b} \left[ k_{3d} k_{2c} \text{Tr} (P_\mu H_{(n)}(M_\mu)_{ab} \Gamma^{abi}) \right] L_1,
\end{align*}
\]
\[
\begin{align*}
\mathcal{A}_2 &\sim 2^{-1/2} \text{Tr} (P_\mu H_{(n)}(M_\mu)_{ab} \Gamma^{abi}) \xi_{14} \xi_{2a} \left[ 2 k_{1,2} L_2 - 2 k_{1,3} L_5 \right]
\end{align*}
\]
\[
\begin{align*}
\mathcal{A}_3 &\sim -2^{-1/2} \text{Tr} (P_\mu H_{(n)}(M_\mu)_{ab} \Gamma^{aci}) \xi_{14} \xi_{2a} k_{2c} \left[ -2 k_{2,3} L_5 + 2 k_{1,3} L_3 \right]
\end{align*}
\]
\[
\begin{align*}
\mathcal{A}_4 &\sim -2^{-1/2} L_5 \left\{ \xi_{3b} \xi_{14} \xi_{2a} \text{Tr} (P_\mu H_{(n)}(M_\mu)_{ab} u + 2 k_{2,3} k_{3d} \xi_{14} \xi_{2a} \text{Tr} (P_\mu H_{(n)}(M_\mu)_{ab} \Gamma^{abi})
\right.
\end{align*}
\]
where the functions $L_1, L_2, L_3, L_5, L_6$ are

$$
L_1 = (2)^{-2(t+s+u)} \sum_{n} \frac{\Gamma(-u + \frac{1}{2})\Gamma(-s + \frac{1}{2})\Gamma(-t + \frac{1}{2})\Gamma(-t - s - u + 1)}{\Gamma(-u - t + 1)\Gamma(-t - s + 1)\Gamma(-s - u + 1)},
$$

$$
L_2 = (2)^{-2(t+s+u)} \sum_{n} \frac{\Gamma(-u + 1)\Gamma(-s + 1)\Gamma(-t)\Gamma(-t - s - u + \frac{1}{2})}{\Gamma(-u - t + 1)\Gamma(-t - s + 1)\Gamma(-s - u + 1)},
$$

$$
L_3 = (2)^{-2(t+s+u)} \sum_{n} \frac{\Gamma(-u + 1)\Gamma(-s)\Gamma(-t + 1)\Gamma(-t - s - u + \frac{1}{2})}{\Gamma(-u - t + 1)\Gamma(-t - s + 1)\Gamma(-s - u + 1)},
$$

$$
L_5 = (2)^{-2(t+s+u)} \sum_{n} \frac{\Gamma(-u)\Gamma(-s + 1)\Gamma(-t + 1)\Gamma(-t - s - u)}{\Gamma(-u - t + 1)\Gamma(-t - s + 1)\Gamma(-s - u + 1)},
$$

$$
L_6 = (2)^{-2(t+s+u)} \sum_{n} \frac{\Gamma(-u + \frac{1}{2})\Gamma(-s + \frac{1}{2})\Gamma(-t + \frac{1}{2})\Gamma(-t - s - u)}{\Gamma(-u - t + 1)\Gamma(-t - s + 1)\Gamma(-s - u + 1)},
$$

It is possible to further simplify the result above:

$$
\mathcal{A}^{\phi A A C} = \hat{A}_1 + \hat{A}_2 + \hat{A}_3 + \hat{A}_4,
$$

where

$$
\hat{A}_1 \sim 2^{1/2} \xi_{1i} \xi_{2a} \xi_{3b} \left[k_{3d} k_{2c} \text{Tr} \left( P_- H_{(n)} M_p \Gamma^{bdcl} \right) \right] (t + s + u) \hat{L}_1,
$$

$$
\hat{A}_2 \sim 2^{-1/2} \text{Tr} \left( P_- H_{(n)} M_p \Gamma^{bd} \right) \xi_{1i} \left[k_{3d} \xi_{3b} \xi_{2a} \hat{L}_2 + 2 k_{3c} \xi_{3b} \xi_{2a} k_{1d} \hat{L}_5 - [2 \leftrightarrow 3] \right],
$$

$$
\hat{A}_3 \sim -2^{-1/2} \hat{L}_5 \left\{ \xi_{3b} \xi_{1i} \xi_{2a} \left[ \text{Tr} \left( P_- H_{(n)} M_p \Gamma^{ba} \right) \right] \xi_{2a} \hat{L}_2 + \xi_{1i} \left[ \xi_{2a} \xi_{3b} \xi_{2a} \hat{L}_2 + 2 k_{3c} \xi_{3b} \xi_{2a} k_{1d} \hat{L}_5 - [2 \leftrightarrow 3] \right] \right\},
$$

$$
\hat{A}_4 \sim 2^{-3/2} \text{Tr} \left( P_- H_{(n)} M_p \right) \xi_{1i} \left\{ \xi_{3b} \xi_{2a} \left[ \text{Tr} \left( P_- H_{(n)} M_p \Gamma^{ba} \right) \right] \xi_{3b} \hat{L}_2 + \xi_{1i} \left[ \xi_{3b} \xi_{2a} \xi_{3b} \xi_{2a} \hat{L}_2 + 2 k_{3c} \xi_{3b} \xi_{2a} k_{1d} \hat{L}_5 - [2 \leftrightarrow 3] \right] \right\},
$$

The functions $\hat{L}_1, \hat{L}_2$ are ($\hat{L}_5 = L_5$)

$$
\hat{L}_1 = (2)^{-2(t+s+u)} \sum_{n} \frac{\Gamma(-u + \frac{1}{2})\Gamma(-s + \frac{1}{2})\Gamma(-t + \frac{1}{2})\Gamma(-t - s - u)}{\Gamma(-u - t + 1)\Gamma(-t - s + 1)\Gamma(-s - u + 1)},
$$

$$
\hat{L}_2 = (2)^{-2(t+s+u)} \sum_{n} \frac{\Gamma(-u + 1)\Gamma(-s + 1)\Gamma(-t)\Gamma(-t - s - u + \frac{1}{2})}{\Gamma(-u - t + 1)\Gamma(-t - s + 1)\Gamma(-s - u + 1)}
$$

The amplitude satisfies ward identities as it should: by replacing $\xi_{2a} \rightarrow k_{2a}$ and $\xi_{3b} \rightarrow k_{3b}$, it vanishes. As in previous works, the amplitude is non-vanishing for certain values of $p$ and $n$: $n = p - 2, n = p + 2$ and $p = n$. The amplitude has an infinite number of massless scalar and gauge field poles and in addition it has infinite contact interactions. We must expand it at low energy limit where the momentum expansions have been introduced in detail in [9].
Expansion of the functions $\hat{L}_1, \hat{L}_2$ around the $t, s, u \to 0$ is

$$
\hat{L}_1 = -\pi^{5/2} \left( \sum_{n=0}^{\infty} c_n (s + t + u)^n + \frac{\sum_{n,m=0}^{\infty} c_{n,m} [s^n t^m + s^m t^n]}{(t + s + u)} \right)
+ \sum_{p,n,m=0}^{\infty} f_{p,n,m} (s + t + u)^p [(s + t)^n (st)^m]
$$

$$
\hat{L}_2 = -\pi^{3/2} \sum_{n=-1}^{\infty} b_n t^{n+1} + \sum_{p,n,m=0}^{\infty} e_{p,n,m} t^p (s u)^n (s + u)^m.
$$

where some of the coefficients $b_n, e_{p,n,m}, c_n, c_{n,m}$ and $f_{p,n,m}$ are

$$
b_{-1} = 1, b_0 = 0, b_1 = \frac{1}{6} \pi^2, b_2 = 2\zeta(3), c_0 = 0, c_1 = -\frac{\pi^2}{6},
$$

$$
e_{2,0,0} = e_{0,1,0} = 2\zeta(3), e_{1,0,0} = \frac{1}{6} \pi^2, e_{1,0,2} = \frac{19}{60} \pi^4, e_{1,0,1} = e_{0,0,2} = 6\zeta(3),
$$

$$
e_{0,0,1} = \frac{1}{3} \pi^2, e_{3,0,0} = \frac{19}{360} \pi^4, e_{0,0,3} = e_{2,0,1} = \frac{19}{90} \pi^4, e_{1,1,0} = e_{0,1,1} = \frac{1}{30} \pi^4,
$$

$$
c_2 = -2\zeta(3), c_{1,1} = \frac{\pi^2}{6}, c_{0,0} = \frac{1}{2}, c_{3,1} = c_{1,3} = \frac{2}{15} \pi^4, c_{2,2} = \frac{1}{5} \pi^4,
$$

$$
c_{1,0} = c_{0,1} = 0, c_{3,0} = c_{0,3} = 0, c_{2,0} = c_{0,2} = \frac{\pi^2}{6}, c_{1,2} = c_{2,1} = -4\zeta(3),
$$

$$
f_{0,1,0} = \frac{\pi^2}{3}, f_{0,2,0} = -f_{1,1,0} = -6\zeta(3), f_{0,0,1} = -2\zeta(3), c_{4,0} = c_{0,4} = \frac{1}{15} \pi^4.
$$

Note that the coefficients $b_n$ are exactly the coefficients that appear in the momentum expansion of the S-matrix element of CAAC [9]. $c_n, c_{n,m}, f_{p,n,m}$ are different from those coefficients that appeared in [15].

3 Two scalar and two gauge field couplings

The closed form result of $< V_{RR} V_\phi V_A V_A >$ above can be momentum-expanded to be compared with the corresponding effective field theory computations. In the earlier work [9], it was shown that the string theory result of $< V_{RR} V_A V_A V_A >$ is reproduced by the following SYM vertices

$$
-T_p (2\pi \alpha')^4 S Tr \left( -\frac{1}{8} F_{bd} F^{df} F_{fh} F^{hb} + \frac{1}{32} (F_{ab} F^{ba})^2 \right).
$$

Extension of these interaction vertices to higher derivative couplings reproduced both the massless pole terms and the contact terms.
Below, we will show that the same is true for the couplings two scalar fields and two gauge fields. First, we show that the correct couplings between two gauge fields and two scalar fields are

\[ -\frac{T_p}{2(2\pi\alpha')^4} \text{STr} \left( D_a\phi^i D_b\phi_i F^{ac} F_{bc} - \frac{1}{4} (D_a\phi^i D_a\phi_i F^{bc} F_{bc}) \right). \]  

(19)

In order to find the infinite higher derivative corrections to two scalar fields and two gauge fields, one must find the amplitude of two scalars and two gauge fields. Computing this amplitude and working out details, we could find the higher derivative corrections of two scalars and two gauge fields to all orders of \( \alpha' \) as the following:

\[ (2\pi\alpha')^4 \frac{1}{2\pi^2} T_p (\alpha')^{n+m} \sum_{m,n=0}^{\infty} (\mathcal{L}_{1}^{nm} + \mathcal{L}_{2}^{nm} + \mathcal{L}_{3}^{nm}), \]  

(20)

\[ \mathcal{L}_{1}^{nm} = -\text{Tr} \left( a_{nm} D_{nm} [D_a\phi^i D^b\phi_i F^{ac} F_{bc}] + b_{nm} D'_{nm} [D_a\phi^i F^{ac} D^b\phi_i F_{bc}] + h.c. \right), \]

\[ \mathcal{L}_{2}^{nm} = -\text{Tr} \left( a_{nm} D_{nm} [D_a\phi^i D^b\phi_i F^{ac} F_{bc}] + b_{nm} D'_{nm} [D_a\phi^i F_{bc} D^b\phi_i F^{ac}] + h.c. \right), \]

\[ \mathcal{L}_{3}^{nm} = \frac{1}{2} \text{Tr} \left( a_{nm} D_{nm} [D_a\phi^i D_a\phi_i F^{bc} F_{bc}] + b_{nm} D'_{nm} [D_a\phi^i F_{bc} D^a\phi_i F^{bc}] + h.c. \right), \]

where the higher derivative operators \( D_{nm} \) and \( D'_{nm} \) are defined \[9\] as

\[ D_{nm}(EFGH) \equiv D_{b_1} \ldots D_{b_m} D_{a_1} \ldots D_{a_n} E F G D_{a_1} \ldots D_{a_n} G D_{b_1} \ldots D_{b_m} H, \]

\[ D'_{nm}(EFGH) \equiv D_{b_1} \ldots D_{b_m} D_{a_1} \ldots D_{a_n} E D_{a_1} \ldots D_{a_n} F G D_{b_1} \ldots D_{b_m} H. \]

As usual, the above couplings are valid up to total derivative terms and terms such as \( \partial_a \partial^a F F D\phi D\phi \) that vanish on-shell. They have no effect on the massless poles of S-matrix elements, because by canceling \( k^2 \) with the massless propagator, they produce contact terms. We now turn to verification of (20) and the terms coming from the DBI part.

### 3.1 Infinite number of massless scalar poles for \( p + 2 = n \) case

In this subsection, we check that the two gauge two scalar interaction vertices \[20\] produce an infinite number of massless scalar poles of the string theory S-matrix element that are in the \( (s + t + u) \)-channel. Specifically, the goal is to show that the massless poles of the string computation are reproduced by the following WZ coupling (that was found in \[19\]),

\[ \lambda_{\mu\nu} \int d^{p+1}\sigma \frac{1}{(p+1)!} (\varepsilon^\nu)^{a_0 \cdots a_p} \text{Tr} \left( \phi^i \right) H^{(p+2)}_{ia_0 \cdots a_p}(\sigma) \]  

(21)
and by the higher derivative two gauge field and two scalar couplings that have been found in \([20]\). To that end, let us consider the amplitude of the decay of one R-R field to one scalar and two gauge fields in the world volume theory of the BPS branes. In the (Feynman gauge) Feynman diagrammatic rules, it is given by

\[
\mathcal{A} = V^i_\alpha(C_{p+1}, \phi) G^{ij}_{\alpha\beta}(\phi) V^j_\beta(\phi, \phi_1, A_2, A_3),
\]

where

\[
G^{ij}_{\alpha\beta}(\phi) = -i\delta_{\alpha\beta}\delta^{ij} \frac{T_p(2\pi\alpha')^2k^2}{T_p(2\pi\alpha')^2(t+s+u)},
\]

\[
V^i_\alpha(C_{p+1}, \phi) = i(2\pi\alpha')\mu_p \frac{1}{(p+1)!}(\varepsilon^u)_{a_0...a_p} H^{(n+2)}(\phi) \text{Tr}(\lambda_\alpha).
\]

We have replaced \(k^2\) by \((t+s+u)\) in the first equation of \([23]\). In the second equation of \([23]\), \(\text{Tr}(\lambda_\alpha)\) is non-zero for the abelian matrix \(\lambda_\alpha\). Noting the fact that the off-shell scalar field must be abelian, one finds the higher derivative vertex \(V^j_\beta(\phi, \phi_1, A_2, A_3)\) from the higher derivative couplings in \([20]\):

\[
V^j_\beta(\phi, \phi_1, A_2, A_3) = \xi^i_\beta I_8 (\alpha')^{n+m}(a_{n,m} + b_{n,m}) \left( (k_3 \cdot k_1)^n (k_1 \cdot k_2)^m + (k_3 \cdot k_2)^m (k_1 \cdot k_2)^n \right)
\]

\[
+ (k_1 \cdot k_3)^n (k_1 \cdot k_2)^m + (k \cdot k_3)^n (k \cdot k_2)^m
\]

where \(k\) is the momentum of the off-shell scalar field, and

\[
I_8 = (2\pi\alpha')^4 T_p \text{Tr}(\lambda_1 \lambda_2 \lambda_3 \lambda_\beta) \left[ \frac{sf}{2} \xi_2 \xi_3 + tk_1 \xi_3 k_3 \xi_2 + sk_1 \xi_2 k_2 \xi_3 - uk_1 \xi_2 k_1 \xi_3 \right]
\]

For the reason explained in \([1]\), \(b_{n,m}\) is symmetric. Note that we must consider two permutations,

\[
\text{Tr}(\lambda_1 \lambda_3 \lambda_2 \lambda_3), \text{Tr}(\lambda_3 \lambda_1 \lambda_2 \lambda_3)
\]

to obtain the desired 123 ordering of the amplitude. Let us list some of the coefficients \(a_{n,m}\) and \(b_{n,m}\) for convenience:

\[
a_{0,0} = -\frac{\pi^2}{6}, b_{0,0} = -\frac{\pi^2}{12}, a_{1,0} = 2\zeta(3), a_{0,1} = 0, b_{0,1} = -\zeta(3), a_{1,1} = a_{0,2} = -7\pi^4/90, a_{2,0} = -4\pi^4/90, b_{1,1} = -\pi^4/180, b_{0,2} = -\pi^4/45, a_{0,4} = -31\pi^6/945, a_{4,0} = -16\pi^6/945, a_{1,2} = a_{2,1} = 8\zeta(5) + 4\pi^2\zeta(3)/3, a_{0,3} = 0, a_{3,0} = 8\zeta(5), b_{1,3} = -(12\pi^6 - 7560\zeta(3)^2)/1890, a_{3,1} = -(52\pi^6 - 7560\zeta(3)^2)/945, b_{0,3} = -4\zeta(5), b_{1,2} = -8\zeta(5) + 2\pi^2\zeta(3)/3, b_{0,4} = -16\pi^6/1890.
\]

(27)
(They were computed in [9] for the four gauge field amplitude. In retrospect, it must be due to field theory T-duality that they remain valid for the present two scalars and two gauge fields amplitude.) Now one can write

\[ k_3 \cdot k = k_2.k_1 - (k^2)/2 \]

and

\[ k_2 \cdot k = k_1.k_3 - (k^2)/2. \]

The terms \( k_2 \) in the vertex (24) will be cancelled with the \( k_2 \) in the denominator of the scalar field propagator producing contact terms. They will not be explicitly considered in this work. Setting them aside, one finds the following result of an infinite number of massless poles,

\[
16\pi\mu_p \frac{e^{a_0 \cdots a_p} \varepsilon_{i}^i H_{\mu_0 \cdots \mu_p}}{(p+1)! (s + t + u)} \text{Tr} \left( \lambda_1 \lambda_2 \lambda_3 \right) \sum_{n,m=0}^{\infty} \left( (a_{n,m} + b_{n,m}) [s^m t^n + s^n t^m] \right) \]

\[
2st \xi_2 \xi_3 + 4tk_1.\xi_3k_3.\xi_2 + 4sk_1.\xi_2k_2.\xi_3 - 4uk_1.\xi_2k_1.\xi_3 \]

(28)

As a check of our calculations let us compare the above amplitude with the infinite number of massless poles in the string theory result. We will take several values of \( n, m \) for illustrations. Common factors of the string and field theory amplitudes will be omitted. For \( n = m = 0 \), the amplitude (28) has the following numerical factor

\[-2(a_{0,0} + b_{0,0}) = -2(\frac{-\pi^2}{6} + \frac{-\pi^2}{12}) = \frac{\pi^2}{2}\]

There is a corresponding term in the string amplitude, and it has the numerical factor \( 2\frac{\pi^2}{2}c_{0,0} \) which is equal to the number above. At the order of \( \alpha' \), the amplitude (28) has the following numerical factor

\[-(a_{1,0} + a_{0,1} + b_{1,0} + b_{0,1})(s + t) = 0\]

The corresponding term in string amplitude is proportional to \( \frac{\pi^2}{2}(c_{1,0} + c_{0,1})(s + t) \) which indeed vanishes. At the order of \( (\alpha')^2 \), the amplitude (28) has the following factor

\[-2(a_{1,1} + b_{1,1})st - (a_{0,2} + a_{2,0} + b_{0,2} + b_{2,0})[s^2 + t^2] \]

\[= \frac{\pi^4}{6}(st) + \frac{\pi^4}{6}(s^2 + t^2)\]

The string result has the numerical factor \( \frac{\pi^2}{2}[c_{1,1}(2st) + (c_{2,0} + c_{0,2})(s^2 + t^2)] \), which is equal to the above factor using the coefficients in (17). At the order of \( \alpha'^3 \), this amplitude has the following factor

\[-(a_{3,0} + a_{0,3} + b_{0,3} + b_{3,0})[s^3 + t^3] - (a_{1,2} + a_{2,1} + b_{1,2} + b_{2,1})st(s + t) \]

\[= -4\pi^2\zeta(3)st(s + t)\]
which is equal to the corresponding term, i.e., \(\frac{\pi^2}{15}[(c_{0,3} + c_{3,0})[s^3 + t^3] + (c_{2,1} + c_{1,2})st(s + t)]\). At the order of \((\alpha')^4\), the amplitude (28) has the following factor

\[-(a_{4,0} + a_{0,4} + b_{0,4} + b_{4,0})(s^4 + t^4) - (a_{3,1} + a_{1,3} + b_{3,1} + b_{1,3})[st(s^2 + t^2)]\]

\[-2(a_{2,2} + b_{2,2})s^2t^2 = \frac{\pi^6}{15}(s^4 + t^4 + 2(s^3t + t^3s) + 3s^2t^2)\]

The string result has the numerical factor \(\frac{\pi^2}{2}[(c_{4,0} + c_{0,4})(s^4 + t^4) + (c_{1,3} + c_{3,1})(s^3t + t^3s) + 2c_{2,2}s^2t^2]\) which is equal to the above factor using the coefficients in (17). Note that the string amplitude has been rescaled by \(2^{1/2}\pi^{1/2}\mu_p\).

Similar comparisons can be carried out for all higher orders of \(\alpha'\): the field theory amplitude (28) exactly reproduces the infinite number of massless scalar poles of the string theory amplitude of \(<V_CV_AV_AV_A>\). This shows that the higher derivative couplings of two scalar two gauge fields are exact up to terms that vanish on-shell. They are also consistent with the momentum expansion of the amplitude of \(AA\phi\phi\).

### 3.2 Infinite number of massless scalar poles for \(p = n\) case

Substituting the expansion of \(\hat{L}_2\) mentioned in (16) into the amplitude, it is possible to obtain all massless scalar poles in the string theory side:

\[\hat{A}_2 \sim 2^{-1/2}\Tr(P_{-}\hat{H}_{(n)}M_p\Gamma^{bdi})\xi_{1i}\left\{2k_1\xi_2\xi_3b^1\xi_3dL_2 - [2 \leftrightarrow 3]\right\}\]

(29)

The trace can be calculated straightforwardly:

\[\Tr\left(P_{-}\hat{H}_{(n)}M_p\Gamma^{bdi}\right) = \pm\frac{32}{p!}\epsilon_{a_0...a_{p-2}b}H_{i}\]

Substituting it into the amplitude and keeping all the scalar poles, one gets

\[A^{C\phi AA} = \pm\frac{32}{2p!}\mu_p\pi^2\xi_{1i}\epsilon_{a_0...a_{p-2}b}H_{i}\]

\[\left\{\sum_{n=-1}^{\infty} \frac{1}{t}b_n(u + s)^{n+1}(2k_1\xi_2)\xi_3b^1\xi_3d\right\}\Tr(\lambda_1\lambda_2\lambda_3).\]

(30)

where the amplitude is rescaled by \(2^{1/2}\pi^{1/2}\mu_p\). The amplitude is antisymmetric under the interchange of the gauge fields; therefore, the whole amplitude vanishes for the abelian gauge group. The amplitude satisfies the Ward identity under \(\xi_{3b} \rightarrow k_{3b}\). Note that we

---

3The same checks have been done for finding an infinite number of massless poles of \(<V_CV_AV_AV_A>\) in [9].
kept only the massless poles; the other terms are contact terms infinite in number. Let us analyze all orders of the massless scalar poles.

Because the amplitudes in s and t-channels are similar, we analyze the amplitude in the t-channel in detail; by interchanging the labels of momentum and polarization vectors, \((2 \leftrightarrow 3)\), the other massless poles in the s channel can easily be obtained. The relevant vertices for this case in the field theory side is

\[
S^{(1)} = i\lambda \mu_p \int \text{STr} \left( FP \left[ C^{(p-1)}(\sigma, \phi) \right] \right) \tag{31}
\]

\[
= i\lambda^2 \mu_p \int d^{p+1} \sigma \frac{1}{p!} (\varepsilon^v)^{a_0 \cdots a_p} \left[ p \text{Tr} \left( F_{a_0 a_1} \partial k C_{a_2 \cdots a_p}^{(p-1)}(\sigma) \right) \right].
\]

Where the scalar field comes from Taylor expansion (see section 5 of [10]). With this vertex, the massless poles in the t-channel should be reproduced as

\[
A = V^i_{\alpha}(C_{p-1}, A_3, \phi) G_{\alpha \beta}^{ij}(\phi) V^j_{\beta}(\phi, A_2, \phi_1), \tag{32}
\]

The vertices and propagator above are

\[
V^i_{\alpha}(C_{p-1}, A_3, \phi) = \frac{i N \lambda^2 \mu_p}{(p)!} (\varepsilon^v)^{a_0 \cdots a_p} (H^{(p)})^i_{a_2 \cdots a_p} \xi_{3a_1} k_{3a_0} \text{Tr} (\lambda_3 \lambda_\alpha) \sum_{n=-1}^{\infty} b_n (\alpha' k_3, k)^{n+1}, \tag{33}
\]

with

\[
(V^{\phi \phi_1 A_2})^j_{\beta} = -2i\lambda^2 T_p \text{Tr} (\xi_2 \cdot k_1 [\xi^j_1, T_\beta]) \tag{34}
\]

\[
(G^{\phi})^{ij}_{\alpha \beta} = -\frac{i}{N \lambda^2 T_p} \delta^{ij} \delta_{\alpha \beta} k^2,
\]

where \(k\) is the momentum of the off-shell scalar field and it satisfies \(k^2 = (k_1 + k_2)^2 = -t\). We also have written \(\zeta^i = \zeta^i_\alpha T_\alpha\) where \(T_\alpha\) are the \(U(N)\) generators with normalization \(\text{Tr} (T_\alpha T_\beta) = N \delta_{\alpha \beta}\). The propagator is derived from the standard gauge kinetic term resulting in the expansion of the Born-Infeld action. The vertex \(V^j_{\beta}(\phi, A_2, \phi_1)\) has found from the standard non-abelian kinetic term of the scalar field; the vertex \(V^i_{\alpha}(C_{p-1}, A_3, \phi)\) is found from the higher derivative extension of the WZ coupling \(\text{Tr} (\partial \_ C_{p-1} \wedge F^k)\). The important point is that the vertex \(V^j_{\beta}(\phi, A_2, \phi_1)\) has no higher derivative correction as it comes from the kinetic term of the scalar field.

Substituting all vertices in the amplitude (32), one finds

\[
A = \mu_p (2\pi \alpha')^2 \frac{1}{(p)!} \text{Tr} (\lambda_1 \lambda_2 \lambda_3) (\varepsilon^{a_0 \cdots a_p - 2ba} H^{a_1 \cdots a_p} \sum_{n=-1}^{\infty} b_n (\alpha' n + 1) (s + u)^{n+1}
\]

\[
\times \left[ -2k_1 \xi_2 \xi_{11} \xi_{3b} k_{3a} \right]. \tag{35}
\]
It describes exactly the same infinite number of massless poles of the string theory amplitude in t-channel; there is precise agreement between the field theory calculation and string result. A similar comparison in s-channel also yields agreement.

### 3.3 Infinite number of massless gauge field poles for $p = n$ case

The expansion of the function $\hat{L}_5$ is given by

$$\hat{L}_5 = -\frac{1}{2} \sum_{n=1}^{\infty} b_n (t+s)^{n+1} + \sum_{p,n,m=0}^\infty e_{p,n,m} u^p(s)^n(s+t)^m. \quad (36)$$

One can read off the infinite massless gauge field poles for the amplitude of $\langle V_CV_{\phi}V_AV_A \rangle$:

$$\hat{A}_2 \sim 2^{-1/2}L_5 \left( \text{Tr} (P_+ H (n)_M p^{\Gamma^{i\alpha}}) \xi_{i1} \xi_{3b} k_{3d}(-2k_3, \xi_2) - \text{Tr} (P_+ H (n)_M p^{\Gamma^{i\alpha}}) \xi_{i1} \xi_{2a} k_{2c}(-2k_2, \xi_3) -2k_2 \xi_3 k_{3d} \xi_{12} \text{Tr} (P_+ H (n)_M p^{\Gamma^{i\alpha}}) -2k_3 \xi_2 k_{2c} \xi_{1b} \text{Tr} (P_+ H (n)_M p^{\Gamma^{i\alpha}}) + \text{Tr} (P_+ H (n)_M p^{\Gamma^{i\alpha}}) k_{3d} k_{2c} \xi_{11}(2\xi_2, \xi_3) \right) \quad (37)$$

Working out the trace, one finds the infinite gauge field poles

$$A^{C\phi AA} = \frac{32}{2p!} \mu_p n^2 \xi_{i1} \xi_{a0 \ldots a_{p-2} dc} H_{a_0 \ldots a_{p-2}}^{i} \text{Tr} \left( \lambda_1 \lambda_2 \lambda_3 \right) \sum_{n=1}^{\infty} b_n (t+s)^{n+1} \left\{ [2\xi_2 \xi_3 k_{3d} k_{2c} -2k_3 \xi_2 k_{3d} (k_{2c} + k_{3c}) + 2k_2 \xi_3 k_{2d} (k_{2c} + k_{3c})] \right\}. \quad (38)$$

The amplitude is antisymmetric under the interchange of the gauge fields; it vanishes for the abelian gauge group. We have kept the entire massless poles; the other terms are infinite contact terms. Let us focus on the massless gauge field poles. (A similar analysis can be done for the contact terms.) The relevant WZ coupling is

$$S^{(2)} = i\lambda \mu_p \int \text{STr} (FP \left[ C^{(p-1)}(\sigma, \phi) \right]) \quad (39)$$

$$= i\lambda^2 \mu_p \int d^{p+1} \sigma \frac{1}{p!} \left( \varepsilon^v \right)^{a_0 \ldots a_p} \text{Tr} \left( F_{a_0 a_1} \phi^k \right) \partial_k C^{(p-1)}(\sigma) \right).$$

An infinite number of massless poles in the u-channel should be reproduced in field theory according to the following Feynman rule; the u-channel amplitude can be written as

$$A = V_{\alpha}^a(C_{p-1}, \phi_1, A) C^{ab}_{\alpha\beta}(A) V_{\beta}^a(A, A_2, A_3). \quad (40)$$

The vertices and propagator can be read from the effective action are

$$V_{\alpha}^a(C_{p-1}, \phi_1, A) = \frac{i\lambda^2 \mu_p}{(p)!} \left( \varepsilon^v \right)^{a_0 \ldots a_{p-1}} \text{Tr} \left( H^{(p)} \right)^{a_1 \ldots a_{p-1}} \xi_{1k} k_{a_0} \text{Tr} \left( \lambda_1 \lambda_1 \right) \sum_{n=1}^{\infty} b_n(\alpha' k_{1,k})^{n+1}, \quad (41)$$

$$13$$
with
\[ V^b_\beta(A, A_2, A_3) = -iT_p(2\pi\alpha')^2\text{Tr} \left( \lambda_2\lambda_3\lambda_\beta \left[ \xi^b_2(k_2 - k)\xi_3 + \xi^b_3(k - k_3)\xi_2 + \xi_3\xi_2(k_3 - k_2)^b \right] \right), \]
\[ G^{ab}_{\alpha\beta}(A) = \frac{i\delta\alpha\beta\delta^{ab}}{(2\pi\alpha')^2T_p(k)^2}, \]

where \( k \) is the momentum of the off-shell gauge field and satisfies \( k^2 = (k_2 + k_3)^2 = -u \). The vertex \( V^a_\alpha(C_{p-1}, \phi_1, A) \) has found from the higher derivative extension of the WZ coupling \( \text{Tr} \left( \partial_k C_{p-1} \wedge F\phi \right) \). Substituting these vertices in the amplitude, one gets
\[ \mathcal{A} = \mu_p(2\pi\alpha')^2 \frac{1}{(p)!u} \text{Tr} \left( \lambda_1\lambda_2\lambda_3 \epsilon^{a_0...a_{p-2}b} (H^{(p)})^i_{a_0...a_{p-2}} \sum_{n=-1}^{\infty} b_n \left( \frac{\alpha'}{2} \right)^{n+1} (s + t)^{n+1} \right) \times \left[ 2k_3\xi_2\xi_1,\xi_3b^1kd - 2k_2\xi_3\xi_1,\xi_2b^1kd - 2\xi_3,\xi_2,\xi_1,\xi_2b^1kd \right] \] \hspace{1cm} (42)

It again displays precise agreement with the infinite \( u \)-channel poles in the string amplitude, \[38\].

4 Conclusion

Following \[9\] in which a similar analysis was done for \( \langle VCV_\phi V_\Lambda V_\Lambda \rangle \), we have computed \( \langle VCV_\phi V_\Lambda V_\Lambda \rangle \) in closed form. By performing the momentum expansion, the corresponding low energy SYM vertices have been determined in all orders in \( \alpha' \). We believe that the result of this paper will provide the basis for future research on, e.g., next-to-leading order dielectric effect and other related topics in string theory \[20\].

For the simple scalar poles, since there is no correction to \( \text{Tr} \left( \phi^i \right) H^{i(p+2)}_{a_0...a_p} \), the non-leading scalar poles should give information about the higher derivative corrections to the couplings of two scalars and two gauge fields where we found them up to all orders of \( \alpha' \) in \[20\].

Although we did not analyse the contact terms of the string amplitude, they would give information about the higher derivative corrections to \( \text{Tr} \left( \partial_k C_{p-3} \wedge F \wedge F\phi^k \right) \). There are several other cases in which one could carry out analogous analyses.

Another line of research is associated with the subtlety brought up in the introduction with regards to any amplitude that contains both open string vertex operators and closed string vertex operators. In this work, we have followed a step that seems to be the commonly implemented in literature, which we are getting to now. Closed string coordinates have two
sets of oscillators; let us denote them by $\alpha_n$ and $\tilde{\alpha}_n$ collectively. For an amplitude that involves both open string states and closed string states, the computations are typically done in a path integral setup where the Green function is determined using conformal field theory techniques.

This is for a good reason. Once one attempts the computations in the oscillator, what to do with the second set of closed string oscillators, $\tilde{\alpha}_n$ is not clear in the framework of the first quantized string. The only viable option seems to be to use one set of oscillators as was commented below (3.4) in [21]. In other words, the two sets of the closed string oscillators are identified with each other, and in turn identified with the open string oscillators as well. As far as we can see, a certain “analytic continuation” is involved in the prescription of identifying the second set of the closed string oscillators as the first set. We believe that there is a room for better understanding and systematic study of this step. In effect, the identification makes the closed string state a composite state of the open string fields. At the effective field theory level, this means that the supergravity background fields that are present in the DBI action should become composite, namely, they must be functions of the SYM fields. Those background fields can then be ”Taylor-expanded” as discussed in the Myers’ work [2].

\footnote{The reason stated in [21] for the identification was the presence of fractional branes. However, the same practice seems to be adopted by other groups in later related works where fractional branes are not present.}

\footnote{To determine the forms of the functions, that is, the proper background, it would be necessary to rely on the full open string setup and to go beyond the tree level: it is expected that quantum effects would play an important role as proposed in [11].}
Appendix A: Conventions

Our index conventions are such that lowercase Greek indices take values in the whole ten-dimensional spacetime, e.g.,

\[ \mu, \nu = 0, 1, \ldots, 9 \]  

(A.1)

Early Latin indices run along the world-volume,

\[ a, b, c = 0, 1, \ldots, p \]  

(A.2)

while middle Latin indices represent the transverse space

\[ i, j = p + 1, \ldots, 9. \]  

(A.3)

Doubling trick is implemented according to

\[ \tilde{X}^\mu(\bar{z}) \to D^\mu_{\nu}X^\nu(\bar{z}), \quad \tilde{\psi}^\mu(\bar{z}) \to D^\mu_{\nu}\psi^\nu(\bar{z}), \quad \tilde{\phi}(\bar{z}) \to \phi(\bar{z}), \quad \text{and} \quad \tilde{S}_\alpha(\bar{z}) \to M_\alpha^\beta S_\beta(\bar{z}), \]

where

\[ D = \begin{pmatrix} -1_{9-p} & 0 & 0 \\ 0 & 1_{p+1} \end{pmatrix}, \quad \text{and} \quad M_p = \begin{cases} \pm i \gamma^{a_1} \gamma^{a_2} \ldots \gamma^{a_{p+1}} \epsilon_{a_1 \ldots a_{p+1}} & \text{for } p \text{ even} \\ \pm \frac{1}{(p+1)!} \gamma^{a_1} \gamma^{a_2} \ldots \gamma^{a_{p+1}} \gamma_1 \epsilon_{a_1 \ldots a_{p+1}} & \text{for } p \text{ odd} \end{cases} \]

The basic holomorphic correlators for the world-sheet fields \( X^\mu, \psi^\mu, \phi \) are

\[
\langle X^\mu(z)X^\nu(w) \rangle = -\frac{\alpha'}{2} \eta^{\mu\nu} \log(z - w), \\
\langle \psi^\mu(z)\psi^\nu(w) \rangle = -\frac{\alpha'}{2} \eta^{\mu\nu}(z - w)^{-1}, \\
\langle \phi(z)\phi(w) \rangle = -\frac{\alpha'}{2} \log(z - w). \quad \text{(A.4)}
\]

For convenience, we introduce

\[ x_4 \equiv z = x + iy, \quad x_5 \equiv \bar{z} = x - iy \]  

(A.5)

SL(2,R) symmetry is fixed by choosing the positions of the open string vertex operators

\[ x_1 = 0, \quad x_2 = 1, \quad x_3 \to \infty, \quad dx_1 dx_2 dx_3 \to x_3^2. \]  

(A.6)

In section 2, one encounters the following integral

\[ \int d^2z |1 - z|^a |z|^b (z - \bar{z})^c (z + \bar{z})^d, \quad \text{(A.7)} \]
where $d = 0, 1, 2$. The region of integration is the upper half of the complex plane. For $d = 0, 1$ the integral was evaluated in [22] and for $d = 2$ in [9]. The Mandelstam variables are defined by

$$
\begin{align*}
  s &= -\frac{\alpha'}{2}(k_1 + k_3)^2, \\
  t &= -\frac{\alpha'}{2}(k_1 + k_2)^2, \\
  u &= -\frac{\alpha'}{2}(k_2 + k_3)^2.
\end{align*}
$$

$T_\alpha$ denotes the $U(N)$ generators with normalization

$$
\text{Tr}(T_\alpha T_\beta) = N\delta_{\alpha\beta}.
$$

### Appendix B: Analysis of $\langle VCV_\phi V_A \rangle$

In the main text, we have analyzed $\langle VCV_\phi V_A V_A \rangle$. Here we carry out a parallel analysis for the simpler case of $\langle VCV_\phi V_A \rangle$ and we find all infinite contact terms of this amplitude. The S-matrix element is given by the following correlation function

$$
A_{\phi A} \sim \int dx_1 dx_2 dz d\bar{z} \langle (V^{(0)}_\phi(x_1)) V_A^{(-1)}(x_2) V_C^{(-\frac{1}{2}, -\frac{1}{2})}(z, \bar{z}) \rangle, \quad (B.1)
$$

Substituting the vertex operators, (B.1) can be written as

$$
A_{\phi A} \sim \int dx_1 dx_2 dx_4 dx_5 (P - \mathcal{H}) (n) M_p)^{\alpha\beta} \xi_{i1} \xi_{i2} x_{45}^{-1/4} (x_{24} x_{25})^{-1/2} \times (I_1 + I_2) \quad (B.2)
$$

with

$$
I_1 = \langle <X_i(x_1) e^{\alpha' i k_1 X(x_1)} : e^{\alpha' i k_2 X(x_2)} : e^{\frac{\alpha' p}{x_2} X(x_2)} : e^{\frac{\alpha' p}{x_2} D X(x_5)} : > < S_\alpha(x_4) : S_\beta(x_5) : \psi^a(x_2) : >, \\
I_2 = \langle < e^{\alpha' i k_1 X(x_1)} : e^{\alpha' i k_2 X(x_2)} : e^{\frac{\alpha' p}{x_2} X(x_2)} : e^{\frac{\alpha' p}{x_2} D X(x_5)} : > < S_\alpha(x_4) : S_\beta(x_5) : \alpha' i k_{1b} \psi^b \psi^i(x_1) : \psi^a(x_2) : >. \quad (B.3)
$$

Using Wick's theorem, one can show that

$$
I_1^a = \langle < S_\alpha(x_4) : S_\beta(x_5) : \psi^a(x_2) : > = 2^{-1/2} x_{45}^{-3/4} (x_{24} x_{25})^{-1/2} (\gamma^a C^{-1})_{\alpha\beta}. \quad (B.4)
$$

The Wick-like rule [23, 24] has been generalized to find the correlation function of two spin operators and some number of currents [9, 10].
Using these results, one can obtain the correlation function between two spin operators, one current and one world-sheet fermion as follows

\[
I_{aib}^2 = < S_\alpha(x_4) : S_\beta(x_5) : \psi^b \gamma^i(x_1) : \psi^a(x_2) > \\
= \left\{ (\Gamma_{aib} C^{-1})_{\alpha\beta} + \alpha' Re [x_{14} x_{25}] (\eta_{\alpha\beta} \gamma^i C^{-1})_{\alpha\beta} \right\} \\
\times 2^{-3/2} x_{45}^{1/4} (x_{14} x_{15})^{-1} (x_{24} x_{25})^{-1/2}.
\] (B.5)

Replacing the above spin correlators in the amplitude and performing the correlators over \( X \), one finds:

\[
A^{\phi A} \sim \int dx_1 dx_2 dx_4 dx_5 (P_\text{-}H (n) M_p)^{\alpha\beta} \xi_1 \xi_2 a^{-1/4} (x_{24} x_{25})^{-1/2} \\
\times \left( I_1 (a_1^a) + i \alpha' k_{1b} I_{aib} \right),
\] (B.6)

where

\[
I = |x_{12}|^{\alpha'^2 k_{1} \cdot p} |x_{14} x_{15}| \frac{x_{12}^2}{x_{1} |x_{14} x_{15}|} \frac{x_{15}^2}{x_{14} |x_{14} x_{15}|} \\
a_1^i = ip^i \frac{x_{54}}{x_{14} x_{15}}.
\] (B.7)

The amplitude has \( SL(2, R) \) invariance; let us gauge fix it as

\[
(x_1, x_2, x_4, x_5) = (x, -x, i, -i), dx_1 dx_2 dx_4 dx_5 = -2i(1 + x^2) dx
\] (B.8)

With this gauge fixing, the amplitude takes

\[
A^{\phi A} = \int_{-\infty}^{\infty} dx (x^2 + 1)^{2t-1} (2x)^{-2t} (2 \xi_1 \xi_2 a 2^{-1/2}) \\
\times \left[ - p^i Tr (P_\text{-}H (n) M_p \gamma^i a) + k_{1b} Tr (P_\text{-}H (n) M_p \Gamma_{bai}) \right]
\] (B.9)

Note that the term \( \frac{\alpha' Re [x_{14} x_{25}]}{x_{12} x_{45}} \) does not have any contribution to the amplitude because the integrand is odd but we have to take integration over the whole space time.

Having found the integral, the final result for the amplitude is given by

\[
A^{\phi A} = (2 \xi_1 \xi_2 a 2^{-1/2} \pi^{1/2}) \frac{\Gamma(-t + \frac{1}{2})}{\Gamma(-t + 1)} \\
\times \left[ - p^i Tr (P_\text{-}H (n) M_p \gamma^i a) + k_{1b} Tr (P_\text{-}H (n) M_p \Gamma_{bai}) \right]
\] (B.10)

where \( t = -(k_1 + k_2)^2 \) and we used the momentum conservation as well. The expansion is low energy expansion which can be achieved by sending the Mandelstam variable \( t \) to
zero which means that we took the $\alpha' \to 0$ limit of the string amplitude. One can then understand that the amplitude is non zero for $n = p$. Thus it is clear from the gamma function that the scattering amplitude has just infinite contact interactions. The expansion of the overall factor for the above amplitude is as follows

$$\sqrt{\pi} \frac{\Gamma(-t + \frac{1}{2})}{\Gamma(-t + 1)} = \pi \sum_{n=-1}^{\infty} c_n(t)^{n+1}$$

(B.11)

where some coefficients may be written down as

$$c_{-1} = 1, \quad c_0 = 2ln(2), \quad c_1 = \frac{\pi^2}{6} + 2ln(2)^2$$

(B.12)

To begin we focus on the Chern-Simons action (see eq. (3) of [10]). The interactions will include a bulk RR field $(p-1)$-form potential, one scalar field and one gauge field in the world-volume of brane. Let us extract the first term in (B.10)

$$A_1^C\phi^A = (2^{-1/2}\pi\mu_p)(2\xi_{11}\xi_{2a}2^{-1/2}\pi) \sum_{n=-1}^{\infty} c_n(t)^{n+1}$$

$$\times \left( \frac{32}{2p!} \right)^2 (\varepsilon^v)_{a_0...a_p-1} H_{a_0...a_p-1}^{(p)}$$

(B.13)

where $(2^{-1/2}\pi\mu_p)$ is the normalization factor. The leading contact interactions can be produced by the following coupling

$$S^{(3)} = \frac{\lambda^2\mu_p}{(p)!} \int d^{p+1}\sigma (\varepsilon^v)_{a_0...a_p} Tr (F_{a_0a_1} \phi^i) \partial_i C_{a_2...a_p}^{(p-1)}(\sigma)$$

The scalar comes from the Taylor expansion and field strength comes from Chern-Simons action (for more details see introduction and section 5 of [10]). Note with an integration by parts, one can re express the above coupling as

$$S^{(3)} = \frac{\lambda^2\mu_p}{(p)!} \int d^{p+1}\sigma (\varepsilon^v)_{a_0...a_p} Tr (A_{a_0} \phi^i) \partial_i H_{a_1...a_p}^{(p)}(\sigma)$$

Having normalized the amplitude, we are able to produce all infinite contact interactions for the first term of this amplitude (B.10) by the following field theory vertices

$$S^{(3)} = \frac{\lambda^2\mu_p}{(p)!} \int d^{p+1}\sigma (\varepsilon^v)_{a_0...a_p} \sum_{n=0}^{\infty} c_n(\alpha')^{n+1} Tr (\partial a_1...\partial a_{n+1} A_{a_0} \partial_1...\partial_{a_n+1} \phi^i) \partial_j H_{a_1...a_p}^{(p)}(\sigma)$$

where $H^{(p)} = dC^{(p-1)}$. To confirm all infinite contact interactions of the second term of (B.10) we have to take into account the following contact interactions in the low energy
effective action. Indeed the leading low energy terms of the amplitude will be reproduced by

\[
S^{(4)} = \frac{\lambda^2 \mu_p}{(p)!} \int d^{p+1} \sigma (\varepsilon^\nu)^{a_0 \cdots a_p} \text{Tr} (F_{a_0 a_1} D_{a_2} \Phi^i) C_{i_a \cdots a_p}^{(p-1)} (\sigma) p(p - 1)
\]

Taking integration by parts and considering the antisymmetric property of \((\varepsilon^\nu)^{a_0 \cdots a_p}\) we can show that the all infinite contact terms of the second term of (B.10) are reproduced by the following contact interactions

\[
S^{(4)} = \frac{\lambda^2 \mu_p}{(p)!} \int d^{p+1} \sigma (\varepsilon^\nu)^{a_0 \cdots a_p} (p) H^{(p)}_{i_{a_2} \cdots a_p} (\sigma)
\times \sum_{n=-1}^{\infty} c_n (\alpha')^{n+1} \text{Tr} (\partial_{a_1} \cdots \partial_{a_{n+1}} A_{a_0} \partial_{a_1} \cdots \partial_{a_{n+1}} D_{a_1} \Phi^i)
\]

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