Turbulence visualization using reflective flakes

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Abstract. The physical mechanism of flow visualizations using reflective flakes is investigated. First, we derive theoretically the governing equations of flake motion based on the assumption that flakes are infinitely thin elliptic disks. Secondly, we verify numerically and experimentally that these equations describe the flake behavior excellently. An important indication of these equations is that the temporal evolution of flake orientations, which determine the intensity in a visualized image, is identical to that of the infinitesimal material surface elements. Since the orientation of material surface elements is governed by velocity gradients, and since the velocity gradient field of turbulent flows is accompanied by coherent vortical structures at the Kolmogorov length, it is expected that such coherent structures in turbulence may be visualized by reflective flakes. It is numerically demonstrated that a flake visualization with appropriate light thickness, indeed, captures the clusters of the coherent structures in isotropic turbulence.

1. Introduction

Reflective flakes (tiny aluminum plates; TiO₂-coated mica, see figure 1; commercial products such as Kalliroscope; and so on) are frequently used in laboratory experiments to visualize structures not only in steady flows but also in turbulence. For example, it is impressive that reflective flakes clearly visualize the stationary or wavy Taylor vortices in the gap between two concentric rotating cylinders (see the album by Van Dyke 1982 as well as the recent experiment by Abcha et al. 2008); whereas Schwarz (1990) claimed that coherent, sheet-like, vortical structures at the Kolmogorov length η in isotropic turbulence were visualized by reflective flakes. In the former case of the Taylor-Couette flow, the coincidence between the flow structure, i.e. Taylor vortex, and the visualized pattern might be evident, but there is no clear general answer to the question about the correspondence between flow structures and visualized patterns. This general question is interesting especially in the case that visualized flow is turbulence because it is widely recognized that turbulence consists of coherent structures. So the present paper aims at examining the possibility to visualize coherent structures in turbulence by reflective flakes.

For this purpose, it is essential to clarify the mechanism of flake visualizations. This simple but rather difficult problem has been investigated by many authors (Savas, 1985; Gauthier et al., 1998; Abcha et al., 2008; Hecht et al., 2010; Wilkinson et al., 2009; Bezuglyy et al., 2010; Wilkinson et al., 2011), and important developments were made recently for flake visualizations of two-dimensional flows (Wilkinson et al., 2009), and for those of three-dimensional flows (Goto et al., 2011). As described briefly in the next section, the main conclusions of Goto et al. (2011)
are that (i) the pattern observed in a visualized image by reflective flakes stems from the non-uniform distribution of flake orientations, and (ii) the temporal evolution of the flake orientation is identical to that for infinitesimal material surface elements.

These conclusions enable us to numerically simulate flake visualizations of turbulence in the laboratory by tracking flake particles in numerical turbulence. So, in the present paper, we conduct such a numerical simulation to conclude that the flake visualization of turbulence indeed reflects the existence of the coherent vortical structures in turbulence.

2. Theory and its verification

2.1. Derivation of governing equations for flake motions

To investigate the physical mechanism of flow visualization by reflective flakes, we derive the governing equations for flakes in a fluid. Our approach is similar to the previous studies; that is, we start with the solution of the Stokes flow around an ellipsoid derived by Jeffery (1922), but here we assume that flakes are infinitely thin elliptic disks. This bold but appropriate assumption [since the typical size of the flakes used in our experiments is about (10μm)² × 0.1μm; see the microscope image shown in figure 1] makes the arguments drastically simple. Then, a straightforward calculation (see Goto et al. 2011 for details) leads to the conclusion that the angular velocity $\Omega$ of such a thin flake, without the moment of inertia, is written as

$$\Omega = -(N_f \times \nabla)(N_f \cdot u),$$

(1)

where $N_f$ is the normal vector of the flake, and $u$ is the fluid velocity at the position $X_f$ of the flake. Then, we obtain the governing equation of flake orientation:

$$\frac{dN_f}{dt} = -(N_f \times \nabla)(N_f \cdot u) \times N_f.$$ 

(2)

Here, it is important to note that the above equation (2) is rewritten as

$$\frac{dN_f}{dt} = -\nabla(N_f \cdot u) + \text{(a term proportional to } N_f).$$ 

(3)

Let us compare (3) with the governing equation for the infinitesimal material surface elements (the infinitely small surface frozen in the fluid),

$$\frac{ds}{dt} = -\nabla(s \cdot u);$$

(4)
see Batchelor (1967) for the derivation. In (4), \( s \) denotes the normal vector of a material surface element whose magnitude is proportional to its area. It is, then, seen that the temporal evolution of the orientation \( N_f \) of a flake is identical to that of an infinitesimal material surface element.

As for the translational motion of flakes, under the assumption that they do not have inertia, we obtain

\[
\frac{dX_f}{dt} = u,
\]

which implies that flakes are passively advected by fluid motion.

If equations (2) and (5) describe flake motions (and this is indeed the case as shown in the next subsection), bright parts in a flake visualization are the regions where a number of flakes are oriented in the direction for the incident rays to be reflected to the observer rather than the regions where they accumulate. This is because (5) implies that the distribution of flakes are always spatially uniform in an incompressible fluid, if the initial distribution is so. Furthermore, the equivalence of the temporal evolutions of flakes and surface elements is intuitive, and helps us to predict the temporal evolution of flake orientations easily.

2.2. Verification of the theory by laboratory and numerical experiments

Here, we briefly summarize the verification of the governing equations (2) and (5) for flake motions. See Goto et al. (2011) for more detailed arguments.

In order to compare direct numerical simulations (DNS), based on (2) and (5), of flake visualizations with real images recorded in experiments, we have conducted DNS and laboratory experiments of the fluid motion in a precessing sphere (figure 2). Here, the term precession denotes the rotational motion of the spin axis around another axis (the precession axis). In our experiments, the precession is driven by rotating (in a constant magnitude \( \Omega_s \) of angular velocity) a spherical vessel on a turntable which also rotates in a constant angular velocity \( \Omega_p \).

Note that control parameters of the motion of fluid confined in the sphere are the Reynolds number,

\[
Re = a^2 \Omega_s / \nu, \tag{6}
\]

and the Poincaré number,

\[
\Gamma = \Omega_p / \Omega_s. \tag{7}
\]

Here, \( \Omega_s \) and \( \Omega_p \) are the magnitudes of the spin and precession angular velocities respectively, \( a \) is the sphere radius, and \( \nu \) is the kinematic viscosity of the fluid. Since it is easy to set these two parameters accurately in the laboratory, this system has excellent reproducibility. Furthermore, it is possible to conduct DNS under the precisely same condition because the boundary condition is simple.

We have made comparisons between the laboratory visualization of a steady flow in the precessing sphere and the corresponding numerically simulated visualization. More precisely, in the laboratory, the mica flakes shown in figure 1 are seeded into the flow, and their reflections of the incident laser sheet (which goes through the centre of the sphere in the perpendicular direction to the spin axis) are recorded by a digital camera from the perspective parallel to the spin axis. On the other hand, in DNS, we employ the spectral method to simulate the steady flow in the sphere (see Kida & Nakayama 2008 for the numerical scheme); and simulate flake motions by numerically integrating (2) and (5). Then, the intensity at each location in the visualized image is simulated by estimating the probability for flake orientations to be in the direction to reflect the incident rays to the observer.

An example of the comparison is shown in figure 3; (a) is the experimental result, whereas (b) is simulated flake visualization. The control parameters are set the same \( (Re = 8 \times 10^4 \) and \( \Gamma = 2 \times 10^{-3} \) both in these laboratory and numerical experiments. These two images shown in figure 3 are surprisingly similar to each other, and this example clearly indicates that (2) and (5) well describe the motion of flakes in fluids.
Figure 2. Precessing spherical vessel, the spin axis of which rotates around the precession axis. The control parameters of the confined fluid motion are the Reynolds number (6) and the Poincaré number (7). Incident rays for the flake visualization (figure 3) are the thin green laser sheet perpendicular to the spin axis.

Figure 3. Flake visualizations of a steady flow in a precessing sphere by (a) experiment and (b) DNS. In the experiment, we use the mica flakes shown in figure 1 and a laser sheet shown in figure 2. In DNS, we track flakes by integrating (2) and (5) in the velocity field simulated by the spectral method, and estimate the light intensity by the probability of the orientation of $\mathbf{N}_f$ in the bisectional direction between the incident rays and the perspective.

3. Application to turbulence visualization

Based on the argument developed in the preceding section, we may investigate turbulence visualization using reflective flakes. We employ homogeneous isotropic turbulence in a periodic cube as an example. The turbulence is simulated by the Fourier spectral method, and the flake motions are numerically tracked according to (2) and (5). Figure 4 shows the visualization of this turbulence numerically simulated by the similar method as in figure 3(b). This image looks
Figure 4. Flake visualization of isotropic turbulence (Taylor-length based Reynolds number is 380). DNS. The size of shown region is about $3000\eta \times 1500\eta \approx 80\lambda \times 40\lambda$. The light thickness is set to be $80\eta$.

Figure 5. Iso-surface of enstrophy on the same cross section as in figure 4. We can observe qualitative coincidence of the bright regions in figure 4 and the clusters of coherent vortical structures.

similar to turbulence visualizations in experiments.

Recall that the intensity in the image indicates the probability of the orientation of flake normals in the bisectional direction between the incident rays and the perspective. Therefore, we can in principle identify flake orientations based on the experimentally recorded light intensity distribution, although it requires multiple incidental rays or perspectives in different directions as proposed by Thoroddsen & Bauer (1999) and Bezuglyy et al. (2010).

Recall also that governing equation (2), equivalently (3), for the normal vectors of flakes implies that their orientations (after sufficiently long duration) are identical to those of the infinitesimal material surface elements. The alignment between the material elements and the velocity gradient tensor in turbulence is a long-standing problem since the study by Batchelor (1952), and it has been shown (Girimaji & Pope, 1990) that there is no one-to-one correspondence between the surface element orientation and the eigenvectors of the strain tensor or the vorticity vector. It is, therefore, impossible to identify the velocity gradient tensor only from flake visualizations; even if we can identify flake orientations accurately.

However, it is known that coherent vortical structures in tubular shape (figure 5) always exist in turbulence, and their characteristics are likely to be universal. More precisely, irrespective of the Reynolds number, their radius is about 5 times the Kolmogorov length $\eta$ (Goto & Kida, 2003), and the typical size of their clusters is in the order of the Taylor length $\lambda$. It is also important that the life of individual vortex tube is much longer than the timescale, i.e. the Kolmogorov time, of its swirling motion. Therefore, flake orientations may well be governed by these coherent vortex tubes; like they are governed by Taylor vortices in the Taylor-Couette flow. For example, the qualitative coincidence are observed in the intensity of the flake visualization (figure 4) and the location of the cluster of coherent tubular vortices (figure 5), when we set the thickness of the incident light sheet to be $80\eta$. Note that this is much thicker than the radius of individual vortex tubes. Hence, we may identify the characteristic length scales such as $\lambda$ of turbulence from the pattern of flake visualizations, if we appropriately set the thickness of incident rays.

4. Conclusion

The translational and rotational motions of reflective flakes in fluids are governed by (5) and (2), respectively. Hence, the intensity of an image visualized using reflective flakes is determined by flake orientations. These governing equations also imply that the temporal evolution of flakes is governed by local velocity gradients, and the evolution is identical to that of infinitesimal material surface elements. Therefore, flake orientations may reflect the smallest-scale eddies in
fluid motions. In other words, flake visualizations are likely to reflect coherent structures at the Kolmogorov length in turbulence. Although it is difficult, in principle, to visualize individual coherent vortices by a single pair of light source and perspective, the preliminary numerical result (figures 4 and 5) indicates the possibility of flake visualization of the cluster of coherent vortices in isotropic turbulence.

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