Impact of partial slip and lateral walls on peristaltic transport of a couple stress fluid in a rectangular duct

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Abstract
The impact of lateral walls and partial slip with different waveforms on peristaltic pumping of couple stress fluid in a rectangular duct with different waveforms has been discussed in the current article. By means of a wave frame of reference the flow is explored travelling away from a fixed frame with velocity c. Peristaltic waves generated on horizontal surface walls of rectangular duct are considered using lubrication technique. Mathematical modelling of couple fluid for three-dimensional flow are first discussed in detail. Lubrication approaches are used to simplify the proposed problem. Exact solutions of pressure gradient, pressure rise, velocity and stream function have been calculated. Numerical and graphical descriptions are displayed to look at the behaviour of diverse emerging parameters.

Keywords
Couple stress fluid, exact solution, partial slip condition, peristaltic flow, rectangular duct

Introduction
Much attention has been received for the research in the domain of Non-Newtonian flows among the researchers due to the fact that this has mutual grounds in plenty of areas such physics, technological and industrial related applications. Fluids of this nature show a relationship of stress with the change of rate of the strain which is nonlinear in its kind. Non-Newtonian liquids may include
semiliquid mixtures, macromolecular fluids, medical solutions related to pharmacy, paints, bio and beauty products, etc. Unquestionably engineers, researchers, biologists, mathematicians and analysts are most of the times challenged by the Non-Newtonian liquids’ mechanics. These flows are not only significant because of their technological importance but it is also the interesting mathematics and features involved in these non-Newtonian fluids. While focusing on the rheology, these fluids show a very complex behaviour, and it becomes almost impossible to seek a generic fundamental relation applicable for all the non-Newtonian fluids. 1966 was the year when Stokes managed to develop one theory amongst non-Newtonian theories which was termed as Couple stress theory. This fluid theory is placed with other polar theories which happens to consider the couple stresses in the presence of classical Cauchy stress. It surely became the most generic and simplest explanation of the classical model of such related fluids that can allow polar effects. Investigations relevant to the fluid behavioural aspects of the couple stress are very beneficial due to the fact that such investigations tend to have this potential for much improved explanations of the rheology of complex fluids. Considering a significant relevance in the wide range of applications, many scholars have investigated the flows in multiple geometries.

Because of the complications linked with the structures of the organs and their materials, studies relevant to the making up of mathematical models of biological systems have been the most typical areas of work. For a human body and its organs, peristalsis is one of the most significant mechanisms for the transportation of biofluids. This mechanism does the transport of the fluids, by the help of stretching walls’ contraction and expansion. Biomechanics and the investigators related to this area, have found this domain tempting to work on in the last couple of decades as multiple applications are relevant in the areas of bio medics and engineering sciences. The mechanism is deployed in multiple applications in a living body like when one swallows the food via oesophagus, flowing of blood and the chyme’s movement in the intestinal tracts. Further, the mechanism can also be used for blood pumping in machines related to heart or lungs. Since the very initial work investigated by Latham, other researchers have studied the concerns and issues related to the peristaltic passages by considering numerous geometries, symmetric and non-symmetric flows, uniform and non-uniform tubes.

On the contrary, Reddy et al. have presented an idea that for a better approximation of uterus’ sagittal cross section can be done through rectangular cross sectioned tubes. They also showed significance of lateral walls in rectangle shaped ducts for the peristaltic flows. Mandviwalla and Archer worked on the extension of the same idea and deliberated on the effect’s peristaltic pumps along the slip bounded conditions in the rectangular shaped ducts. Down the years, many researchers have worked on the extension of the ideas presented in Subba Reddy et al. and Mandviwalla and Archer by considering Newtonian fluids and non-Newtonian fluid models. To the best of our knowledge, the study for peristaltic passages especially designed for the rectangle shaped ducts with slip bounded condition is still being researched. Due to this, we have deliberated the idea related
to partial slip in the same shaped ducts for couple stressed fluids. Pressure gradients and velocities have been worked upon and their exact solutions have been determined. Furthermore, using numerical methods, calculations have been done to find the expression for the pressure rise. To experience the physical behaviours of these pertinent constraints, graphical representations have been made.

Mathematical composition

We are considering peristaltic flow of an incompressible couple stress fluid in a rectangular duct possess channel height $2a$ and width $2d$. Cartesian coordinates system are taking into consideration so that $X-$ axis, $Y-$ axis and $Z-$ axis is taking in axial direction, lateral direction and vertical direction of rectangular duct respectively (Figure 1). Geometry of walls surfaces are representing as

$$Z = H(X, t) = \pm a \pm b \cos \left[ \frac{2\pi}{\lambda} (X - ct) \right], \quad (1)$$

here $(a, b), \lambda, c$, and $t$ are magnitudes of waves, wavelength, velocity propagation and time respectively. The walls parallel to $XZ-$ plane stay continuous and are no longer be dependent upon any peristalsis wave movement. We are considering lateral velocity equal to zero due to the fact there’s no alter in lateral path of duct cross section. The velocity for a rectangular duct is $(U, 0, W)$.

The present flow is governed by the following equations

$$\frac{\partial U}{\partial X} + \frac{\partial W}{\partial Z} = 0, \quad (2)$$

$$\rho \left( \frac{\partial U}{\partial t} + U \frac{\partial U}{\partial X} + W \frac{\partial U}{\partial Z} \right) = - \frac{\partial P}{\partial X} + \mu \left( \frac{\partial^2 U}{\partial X^2} + \frac{\partial^2 U}{\partial Y^2} + \frac{\partial^2 U}{\partial Z^2} \right)$$

$$- \eta \left( \frac{\partial^4 U}{\partial X^4} + \frac{\partial^4 U}{\partial Y^4} + \frac{\partial^4 U}{\partial Z^4} + 2 \frac{\partial^4 U}{\partial X^2 \partial Y^2} + 2 \frac{\partial^4 U}{\partial X^2 \partial Z^2} + 2 \frac{\partial^4 U}{\partial Y^2 \partial Z^2} \right), \quad (3)$$

Figure 1. Geometry of the problem.
\[ 0 = -\frac{\partial P}{\partial Y}, \tag{4} \]

\[ \rho\left(\frac{\partial W}{\partial t} + U\frac{\partial W}{\partial X} + W\frac{\partial W}{\partial Z}\right) = -\frac{\partial P}{\partial Z} + \mu\left(\frac{\partial^2 W}{\partial X^2} + \frac{\partial^2 W}{\partial Y^2} + \frac{\partial^2 W}{\partial Z^2}\right) \]

\[ -\eta\left(\frac{\partial^4 W}{\partial X^4} + \frac{\partial^4 W}{\partial Y^4} + \frac{\partial^4 W}{\partial Z^4} + 2\frac{\partial^4 W}{\partial X^2\partial Y^2} + 2\frac{\partial^4 W}{\partial X^2\partial Z^2} + 2\frac{\partial^4 W}{\partial Y^2\partial Z^2}\right), \tag{5} \]

where \( \rho \) is density and \( P \) is pressure.

The two frames of references (wave and fixed frame) are linked as

\[ x = X - ct, y = Y, \quad z = Z, \quad u = U - c, \quad w = W, \quad p(x, z) = P(X, Z, t). \tag{6} \]

Defining for the following quantities

\[ \tilde{x} = \frac{x}{\lambda}, \quad \tilde{z} = \frac{z}{a}, \quad \tilde{y} = \frac{y}{d}, \quad \tilde{u} = \frac{u}{c}, \quad \tilde{w} = \frac{w}{c\delta}, \quad \tilde{t} = \frac{ct}{\lambda}, \quad h = \frac{H}{a}, \quad \tilde{\rho} = \frac{\rho a^2}{\mu c\lambda}, \]

\[ \text{Re} = \frac{\rho ac\delta}{\mu}, \quad \delta = \frac{a}{\lambda}, \quad \bar{L}_{\text{slip}} = \frac{L_{\text{slip}}}{d}, \quad \beta = \frac{a}{d}, \quad \gamma = \sqrt{\frac{\mu}{\eta}} a. \tag{7} \]

by means of equations (6) and (7), the equations (2)–(5) can be modified as (after dropping bars)

\[ \frac{\partial u}{\partial x} + \frac{\partial w}{\partial z} = 0, \tag{8} \]

\[ \text{Re}\left(u\frac{\partial u}{\partial x} + w\frac{\partial u}{\partial z}\right) = -\frac{\partial p}{\partial x} + \delta^2\frac{\partial^2 u}{\partial x^2} + \beta^2\frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2} \]

\[ -\frac{1}{\gamma^2}\left(\delta^4\frac{\partial^4 u}{\partial x^4} + \beta^4\frac{\partial^4 u}{\partial y^4} + \beta^4\frac{\partial^4 u}{\partial z^4} + 2\delta^2\beta^2\frac{\partial^4 u}{\partial x^2\partial y^2} + 2\frac{\partial^4 u}{\partial x^2\partial z^2} + 2\frac{\partial^4 u}{\partial y^2\partial z^2}\right), \tag{9} \]

\[ 0 = -\frac{\partial p}{\partial y}, \tag{10} \]

\[ \text{Re}\delta^2\left(u\frac{\partial w}{\partial x} + w\frac{\partial w}{\partial z}\right) = -\frac{\partial p}{\partial z} + \delta^2\left(\delta^2\frac{\partial^2 w}{\partial x^2} + \beta^2\frac{\partial^2 w}{\partial y^2} + \frac{\partial^2 w}{\partial z^2}\right) \]

\[ -\frac{1}{\gamma^2}\left(\delta^6\frac{\partial^4 w}{\partial x^4} + \delta^4\beta^4\frac{\partial^4 w}{\partial y^4} + \delta^4\frac{\partial^4 w}{\partial z^4} + 2\delta^4\beta^2\frac{\partial^4 w}{\partial x^2\partial y^2} + 2\frac{\partial^4 w}{\partial x^2\partial z^2} + 2\frac{\partial^4 w}{\partial y^2\partial z^2}\right). \tag{11} \]

Applying lubrication approach (long wavelength \( \delta \leq 1 \) and low Reynolds number), overlooking order of \( \delta \) terms and higher, equations (8)–(12) reduces to the following form
\[
\frac{dp}{dx} = \beta \frac{\partial^2 u}{\partial y^2} + \frac{\partial^3 u}{\partial z^3} - \frac{1}{\gamma^2} \left( \beta^4 \frac{\partial^4 u}{\partial y^4} + \frac{\partial^4 u}{\partial z^4} + 2\beta^2 \frac{\partial^4 u}{\partial y^2 \partial z^2} \right), \tag{12}
\]

\[
\frac{\partial u}{\partial x} + \frac{\partial w}{\partial z} = 0 \tag{13}
\]

The consequent boundary conditions are

\[
\begin{align*}
\frac{\partial u}{\partial y} & = -1 \text{ at } y = 1, \quad \frac{\partial^2 u}{\partial y^2} = 0 \text{ at } y = 1, \quad \tag{14} \\
\frac{\partial u}{\partial y} & = -1 \text{ at } y = -1, \quad \frac{\partial^2 u}{\partial y^2} = 0 \text{ at } y = -1, \quad \tag{15} \\
\frac{\partial^2 u}{\partial y^2} & = 0 \text{ at } y = 1, \quad \frac{\partial^2 u}{\partial y^2} = 0 \text{ at } y = -1, \quad \tag{16}
\end{align*}
\]

here \(0 \leq \phi \leq 1\), for straight duct \(\phi = 0\) and \(\phi = 1\) for total occlusion.

### Solution of proposed problem

Using the similar procedure as down in,\(^{36-37}\) the exact solution of equation (12) satisfying the boundary conditions (14)–(16) can be defined as

\[
\begin{align*}
u & = -1 + \frac{1}{2} \frac{dp}{dx} \left( \frac{2}{\gamma^2} - h^2(x) + z^2 \right) - \frac{dp}{dx} \left( \frac{sech[yh] \cos h[yz]}{\gamma^2} \right) \\
& + 2h^2(x) \frac{dp}{dx} \left( \sum_{n=1}^{\infty} (-1)^n \frac{\cos \left( \frac{\zeta_n}{h(x)} \right) \cos h \left( \frac{\zeta_n}{\gamma h(x)} y \right)}{\zeta_n^3 \left( \cos h \left( \frac{\zeta_n}{\gamma h(x)} y \right) + \frac{\zeta_n L_{slip}}{\gamma h(x)} \sin h \left( \frac{\zeta_n}{\gamma h(x)} y \right) \right)} \right), \tag{17}
\end{align*}
\]

here \(\zeta_n = \frac{(2n-1)\pi}{2}\).

The flow rate is given by

\[
\hat{q} = \int_0^1 \int_0^h u \, dz \, dy = -h(x)
\]

\[
\begin{align*}
&+ \frac{dp}{dx} \left( 2\beta^2 h^5(x) \sum_{n=1}^{\infty} \sin \left( \frac{\xi_n}{\beta h(x)} \right) \frac{\sin h \left( \frac{\xi_n}{\beta h(x)} \right)}{\xi_n^2 \left( \gamma^2 h(x) \cos h \left( \frac{\xi_n}{\beta h(x)} \right) + \frac{\xi_n L_{slip}}{\gamma h(x)} \sin h \left( \frac{\xi_n}{\gamma h(x)} \right) \right)} \right) \\
&- \frac{h^3(x)}{3} - \frac{\tan h(\gamma h)}{\gamma^2} + \frac{h(x)}{\gamma^2}, \tag{18}
\end{align*}
\]

The flux is followed by
\[
\dot{Q} = \int_0^1 \int_0 (u + 1)dzdy = \dot{q} + h(x). \tag{19}
\]

Over one period \((T = \frac{1}{C})\) average volume flow rate of peristaltic wave can be determined as

\[
Q = \frac{1}{T} \int_0^T \dot{Q}dt = \dot{q} + 1. \tag{20}
\]

From equations (18) and (20), pressure gradient is determined as

\[
\frac{dp}{dx} = \frac{Q - 1 + h(x)}{(\frac{h(x)}{\gamma} - \frac{h^3(x)}{3}) + 2\beta \gamma^2 h^5(x) \sum_{n=1}^{\infty} (-1)^n \frac{\sin (\xi_n)\sin h(\frac{\xi_n}{\gamma h(x)})}{\xi_n^2 (\gamma^2 h(x)\cos h(\frac{\xi_n}{\gamma h(x)}) + \xi_n L_{\text{slip}} \sin h(\frac{\xi_n}{\gamma h(x)}) - \frac{\tan h(\gamma h(x))}{\gamma^3})}. \tag{21}
\]

Integrating equation (21) over one wavelength concurs

\[
\Delta p = \int_0^1 \frac{dp}{dx} dx. \tag{22}
\]

**Special cases**

In the absence of slip parameter \(L_{\text{slip}}\) it is noted that \(\beta \to 0\) (holding \(a\) fixed and \(d \to \infty\)), rectangular duct can be reduced to two-dimensional channel. Moreover, for square duct \(\beta = 1\).

**Graphical illustration**

This section is devoted to examining the variational impact of important physical parameters on various flow characteristics, like, pressure rise per wavelength \(\Delta p\), pressure gradient \(\frac{dp}{dx}\), axial velocity \(u(y)\) and streamlines. Various graphs are plotted in Figures 2 to 13 to see the variation trends of dimensionless parameters, for example, material parameter \(\gamma\), wave amplitude, wall slip parameter and the aspect ratio of the channel \(\beta\). Furthermore, a comparative analysis among different wave shapes, namely, multi-sinusoidal, trapezoidal, square and triangular shaped waves is also provided. Figures 2 and 3 are reserved to see the variational trend of fluid material parameter \(\gamma\) and wave amplitude \(\phi\) on the pressure-rise per wavelength \(\Delta p\). It is reflected from Figure 2 that higher values of couple stress fluid parameter
establish an upsurge on $\Delta p$ in the peristaltic pumping zone but an opposite conduct is noticed in the co-pumping area. In the free pumping zone, the impact of $\gamma$ on $\Delta p$ is trivial. From Figure 3 it is noticed that $\Delta p$ enhances for large values of wave amplitude $\varphi$ in peristaltic pumping and free pumping region, whereas this behaviour is reverse in the augmented pumping part. It is also observed that in the pumping and co-pumping channel parts, the magnitude volumetric flow rate rises when large values of $\varphi$ are considered. Since the energy carried by the peristaltic wave retains a direct relation with wave amplitude that induces an augmentation in the volumetric flow rate in the elastic duct. From Figures 4 to 6, we consider the pressure gradient against the dimensionless independent variable $x$ by varying different parameters. Figure 4 illustrates that high values of couple stress parameter $\gamma$ develops an adverse pressure gradient ($\frac{\Delta p}{dx}$), and this rise is consistent in the downstream. Since higher values of $\gamma$ indicate the weakening Couple-stress fluid.
viscosity, therefore, it is concluded from here that Couple-stress fluid hinders the pressure gradient as compare to the viscous fluid. However, to balance the flow rate, an adequate favourable pressure gradient is needed in the deep duct portion. Large amplitude values generate denser and intense waves which endorse peristaltic activity (Figure 5). This increase is more influential in the wider duct zone than in the core part. In Figure 6, a comparative glamp of effect of amplitude ratio ($\varphi$) on $\frac{dp}{dx}$ for various wave shapes. Here it is clear that $\frac{dp}{dx}$ is maximum sinusoidal/multi-sinusoidal and minimum for square wave the deep duct portion. Figure 7(i–ii) demonstrates the two- and three-dimensional view of axial velocity for various values of volumetric flow rate ($Q$), respectively. It is seen that the higher values of $Q$ cause an intensification in axial velocity through the duct. This increase is more

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**Figure 4.** Pressure gradient is plotted against $x$ for various values of $\gamma$.

**Figure 5.** Pressure gradient is plotted against $x$ for various values of $\varphi$.
substantial in the central part of the elastic duct as compared to boundaries. Figure 8(i–ii) reveals that slip parameter ($L_{slip}$) supports fluid flow near the centre and elastic boundary of the duct. Here it can be seen that the magnitude of fluid velocity enhances when large values of $L_{slip}$ are considered. This conduct of velocity is obvious in pressure driven flows. Figure 9(i–ii) describes that the small values of

Figure 6. Pressure gradient is plotted against $x$ for different wave forms.

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couple stress fluid parameter impede fluid velocity throughout the duct, but this hindrance is more noticeable in the contracted portion of the duct. Since the smaller values of material parameter corresponds to large Couple-stress viscosity, therefore, Couple-stress fluid reduce the fluid movement. The negative values of velocity profile indicate the reverse flow.

Trapping is described as the development of internally revolving fluid mass confined by streamlines of propulsive waves. This enclosed bolus is drove ahead along with the progressive wave of contraction and expansion. Figure 10 depicts that the trapped bolus enlarges in size when large values of aspect ratio ($\beta$) are taken into consideration. From Figures 11 and 12, it is concluded that the trapped bolus expands in size when higher values of $\gamma$ and smaller values of $L_{slip}$ are selected.

**Figure 7.** Velocity profile is plotted against $z$ in case of two-dimensional and $(y, z)$ in case of three-dimensional for various values of $Q$: (i) for two-dimensional and (ii) for three-dimensional.
Figure 13 reflects the streamlines adopt the similar wave pattern as the type of wave configuration considered.

Concluding remarks

The impact of lateral walls and partial slip with different wave forms on peristaltic pumping of couple stress fluid in a rectangular duct with different wave forms has been discussed in the current article. Peristaltic waves generating on horizontal...
Figure 9. Velocity profile is plotted against $z$ in case of two-dimensional and $(y, z)$ in case of three-dimensional for various values of $\gamma$: (i) for two-dimensional and (ii) for three-dimensional.

Figure 10. Impact of streamlines for $\beta$. 

$\beta = 1.5$ $\beta = 3$
surface walls of rectangular duct are considered under lubrication technique. Mathematical modelling of couple fluid for three-dimensional flow is first discussed in detail. Exact solutions of pressure gradient, pressure rise, velocity and stream function have been calculated. Graphical descriptions are displayed to look at the behaviour of diverse emerging parameters. The main finding are as follows:

- The pressure rise raises in the peristaltic pumping ($\Delta p > 0, Q > 0$) region with the increasing values of couple stress parameter $\gamma$ and amplitude ratio $\phi$, whereas the behaviour is quite opposite when $\Delta p < 0, Q < 0$.
- The pressure gradient increases with the increasing values of couple stress parameter $\gamma$ and amplitude ratio $\phi$. Moreover, it is also noted that when $x \in [0.22, 0.29]$ pressure gradient is same for all values of amplitude ratio $\phi$.
- The magnitude of velocity profile increases with an enlargement in values of volume flow rate $Q$, slip parameter $L_{slip}$ and couple stress parameter $\gamma$.

**Figure 11.** Impact of streamlines for $\gamma$.

**Figure 12.** Impact of streamlines for $L_{slip}$. 

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The number of trapping bolus reduces with the increasing values of aspect ratio $\beta$ and bolus size increases with the increasing values of couple stress parameter $\gamma$.

- The trapped bolus size diminishes with an enlargement of slip parameter $L_{slip}$.

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