GAMMA RAYS FROM SMALL SCALE
STRUCTURES ON LONG COSMIC STRINGS

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ABSTRACT

The formation of cusps on long cosmic strings is discussed and the probability of
cusp formation is estimated. The energy distribution of the gamma-ray background
due to cusp annihilation on long strings is calculated and compared to observations.
Under optimistic assumptions about the cusp formation rate, we find that strings
with a mass per unit length $\mu$ less than $G\mu = 10^{-14}$ will have an observable effect.
However, it is shown that the gamma-ray bursters can not be attributed to long
ordinary strings (or loops).

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1. Introduction

Grand Unified Theories (GUT) of particle physics predict that the Universe underwent a series of phase transitions during its early stages of evolution. In many such theories, the vacuum manifold after symmetry breaking has nontrivial topology and hence gives rise to the formation of topological defects after the phase transition. Specifically, in many models the vacuum manifold has nontrivial first homotopy group, in which case the defects are linear - cosmic strings [1].

There are various types of cosmic strings. Depending on whether the symmetry which is broken is a global or local symmetry we get global or local cosmic strings, and local cosmic strings may be non-superconducting (ordinary) or superconducting [2]. In this paper we focus on models with ordinary local cosmic strings. The key parameter in such models is the mass per unit length $\mu$. All the purely gravitational consequences of cosmic strings depend only on this parameter.

For a value of $\mu$ given by

$$G\mu \simeq 10^{-6}, \quad (1.1)$$

cosmic strings give rise to a successful model for cosmological structure formation [3]. The above value of $\mu$, determined on the basis of purely cosmological considerations, coincides with the scale of symmetry breaking in the simplest GUT models compatible with the running of the coupling constants of the Standard Model of particle physics [4]. However, in more complicated GUT models, in particular in models with several stages of symmetry breaking, cosmic strings may arise with a value of $\mu$ smaller than the one given in Eq. (1.1).

One of the motivations for this paper is to search for bounds on (and observational signatures for) models with a value of $\mu$ smaller than the one given in Eq. (1.1). Another motivation for the research is to check the consistency of the cosmic string model with the high energy gamma ray observations (both the high energy background and the frequency of gamma ray bursters), and to see whether cosmic strings can be responsible for any of the observations.

The reason why we might expect useful constraints is the following: cosmic strings typically will lead to cusps [5]. A cusp is a point on the string which reaches the
speed of light and where the derivative of $x^\mu(\sigma, t)$ with respect to $\sigma$ vanishes. Here, $x^\mu(\sigma, t)$ is the string world sheet, and $\sigma$ is the affine parameter of the string. At a cusp, the two segments of the string overlap (see below), there is no topological barrier to decay, and hence we expect most of the energy of the cusp to radiate in a short, high intensity burst [6] which will include a substantial amount of high energy electromagnetic radiation. Cusp annihilations will thus give rise to gamma ray bursts, and will contribute to the gamma ray background.

Previous work has focused on the effects of cosmic string loops. In [7, 8], the expected contribution of string loops to the neutrino background was calculated, and in [9], this work was extended to a calculation of the gamma ray background. In [10], the probability of cusp annihilations on loops giving rise to observable bursters was estimated and found to be much smaller than the observed rate of bursters.

In this work, we estimate the effects of cusps on long strings (strings which have a radius of curvature comparable or larger than the horizon). We analyze the way in which small scale structure on long strings (in particular kinks) can give rise to cusps, and we calculate the observational consequences for both gamma ray background and rate of gamma ray bursters.

Kinks are points on the string where the tangent vector to the string changes discontinuously by a non zero angle. They are usually formed when strings intercommute and cross each other. Kinks propagate along the string and decay due to gravitational radiation [11].

One immediate incentive for this work was the recent discovery by the BATSE instrument on the NASA GRO observatory of a more or less uniform distribution of gamma ray bursts in the range of energy $100KeV - 1MeV$ [12]. Attributing the bursts to galactic phenomena like neutron stars colliding with nomadic small asteroids is not possible since the distribution of these objects is not uniform over the sky. However, cosmic strings have a uniform distribution and it is hence tempting to speculate whether the gamma ray bursts (GRB) can be attributed to cosmic string cusp annihilation.

In this paper, we study the effects of cusp annihilation on long strings on the high energy gamma ray background radiation and on the gamma ray bursts. We
focus exclusively on ordinary (i.e. non-superconducting) cosmic strings. Bursts from superconducting cosmic strings have been discussed in [13]. The superconducting cosmic string scenario, however, is now very tightly constrained by the absence of spectral distortions in the cosmic microwave background [14].

We propose a new mechanism for cusp formation on long strings: small scale structures on the long strings like wiggles or kinks will propagate with the speed of light and collide with each other. The distribution function for the shape of the small scale structures is not known. If we assume a random distribution for their shapes, then the probability of cusp formation is close to 50%.

Using the methods developed in [7,9], we calculate the gamma ray background and the gamma ray burst frequency due to cusp annihilation on long strings. As in the case of string loops [10] we conclude that our mechanism cannot produce the observed GRB, the reason being that there are not enough strings sufficiently close to the observer. However, the mechanism has the potential to enhance the background gamma ray radiation appreciably.

In Section 2, we review cusps and kinks and discuss how they are formed. In the next section, we summarize the analysis of [7,9] of gamma ray emission by cusp annihilation. In Section 4, we calculate the gamma ray background produced by the distribution of cosmic strings, assuming that the scaling solution [15] describes the network of strings. Section 5 answers the questions concerning gamma ray bursts from long strings. The final section contains our conclusions.

2. Cusp Formation on Long Strings

The fields making up the cosmic string evolve according to the equations of motion derived from the field theory action. For a global $U(1)$ string e.g., the action is

$$S = \int d^4x \sqrt{-g} \left[ \frac{1}{2} \partial_\mu \phi \partial^\mu \phi - f(|\phi|^2 - \sigma^2)^2 \right] ,$$

(2.1)

where $\phi$ is a complex scalar field and $f$ is the self coupling constant. Assuming that the field configuration corresponds to a string with world sheet $\chi^\mu(s, \tau)$ in four
dimensional space-time, where $s, \tau$ and $h_{ij}$ are its world sheet coordinates and metric, the above field theory action reduces [16] to the Nambu action for the world sheet:

$$ S = \mu \int d^2s \sqrt{h} \partial_i \chi^\mu(s, \tau) \partial_j \chi^\mu(s, \tau) h^{ij} $$  \hspace{1cm} (2.2)

where $i$ and $j$ are world sheet indices. This action to a good accuracy describes the motion of the string world sheet provided $wR \ll 1$, where $w$ and $R$ are the width and curvature radius of the string, respectively. $\mu$ is the mass per length of the string. The width $w$ of the string is given in terms of $\mu$ by $w \sim \mu^{-1/2}$ [17].

In the flat space-time limit, the equation of motion which follows from (2.2) reduces to

$$ \ddot{\chi}(s, \tau) = \chi''(s, \tau) $$  \hspace{1cm} (2.3)

where $\cdot$ and $'$ are the derivatives with respect to $\tau$ and $s$, respectively. $\tau$ can be chosen to be time, and the gauge conditions are

$$ \dot{\chi} \cdot \chi' = 0 $$
$$ \dot{\chi}^2 + \chi'^2 = 0. $$  \hspace{1cm} (2.4)

The general solution of (2.3) can be written as the superposition of a right and a left moving wave:

$$ \chi^\mu(s, \tau) = \frac{1}{2} [\chi^\mu_+(s - \tau) + \chi^\mu_-(s + \tau)] $$  \hspace{1cm} (2.5)

with the gauge conditions yielding

$$ |\chi'_-|^2 = |\chi'_+|^2 = 1. $$  \hspace{1cm} (2.6)

Here and in the following $\chi$ will denote the spatial three vector corresponding to the original four vector.
A cusp is a point on the string with $|\dot{\chi}| = 1$ and $\chi' = 0$, i.e. a point where the velocity of the string reaches the speed of light and where the string doubles back on itself.

As first shown in [5], cusps on cosmic string loops generically arise once per oscillation time. The vectors $\pm \chi_\pm$ are arbitrary functions of $(\sigma \pm \tau)$, with the condition (2.6) for their first derivatives. These vectors describe closed curves on the unit sphere which satisfy the periodicity condition

$$\int \chi'_\pm = 0. \quad (2.7)$$

As a result, the paths on the unit sphere will pass through both hemispheres. Two such curves generically self intersect which means that a cusp will be formed.

We will now generalize the above analysis to the case of small scale perturbations on long strings. The small scale structure is a remnant of self intersections of strings at early times. We expect small scale structure to survive on all scales larger than a minimum scale determined by the decay of string excitations via gravitational radiation.

The string solution can still be decomposed into left and right moving waves. However, the periodicity condition (2.7) is no longer valid. Consider a long string with a right moving and a left moving perturbation approaching each other. In this case, far from the regions of support of the perturbations, $\chi'_\pm$ are at the north and south poles of the unit sphere, respectively. In the regions of support of the perturbations, $\chi'_\pm$ execute closed paths on the unit sphere.

The exact properties of the closed paths on the unit sphere traced out by $\chi'_\pm$ are not known except through numerical simulations, We will make the assumption that even on small scales the perturbations look locally like a random walk. This corresponds to a fractal dimension of 2, a result not supported by some of the numerical cosmic string evolution simulations [18], which show a significantly smaller fractal dimension on small scales. However, it is still unclear if all of the small scale structure on cosmic strings is treated adequately in the present numerical simulations, and if
nothing else the results of this paper might encourage more detailed numerical studies of small scale structure on strings.

With our assumption, the distribution of the paths $\chi'_\pm$ can be represented by a function $f(\chi_\pm)$ which is that of white noise i.e. $f(\chi'_\pm) = f(\theta_\pm, \phi_\pm) = \text{const.}$ or by normalization considerations

$$f(\theta_\pm, \phi_\pm) = \frac{1}{4\pi} \quad (2.8)$$

The cusp condition is

$$\chi'\big|_{s=cusp} = -\chi'_+|_{s=cusp}$$

or $\chi'_- \cdot \chi'_+ + 1 = 0 \quad (2.9)$

Therefore, the probability that the two independent random paths cross each other will be

$$P_{cusp} = \int d\Omega \chi'_- \int d\Omega \chi'_+ f(\chi'_-) f(\chi'_+) \delta(\chi'_- \cdot \chi'_+ + 1) \quad (2.10)$$

It is important to note that the parameter $s$ does not have to be the same when the two random walks cross each other, since $\chi_\pm$ are traveling waves. Thus, Eq.(9) is the probability of cusp formation on long strings.

The integral (9) can be easily done with the result

$$P_{cusp} = \frac{1}{2} \quad (2.11)$$

Therefore, the probability of cusp formation on infinite strings when two arbitrary wiggles collide is about 50%, provided that our rather stringent assumptions on the small scale structure are satisfied.

In Figure 1 an example of two travelling waves which form a cusp is shown. Figure 2 shows the corresponding paths of the two waves of Fig. 1 on the unit sphere.
3. Radiation from Cusps

Cusps are artifacts of the Nambu approximation for strings. When the curvature radius of the string becomes comparable to the width (which is the case at a cusp), the internal structure of the string becomes important. For a cusp of a cosmic string with finite width $w$, there is a region of comoving length $\ell_c$ where the two segments of the string overlap (see Figure 3). No topological criterion prevents this segment of the string from annihilating into a collection of quanta of the constituent fields of the string. The total energy of this region of the cusp (and hence [6] the maximal energy which can be released in the cusp annihilation process) is

$$E_{\text{cusp}} \simeq \mu \ell_c$$  (3.1)

For loops, $\ell_c$ has been calculated in [19]. A field theoretic derivation of Eq. (3.1) for the energy radiated by cusp annihilation is given in [20].

For long strings, the length $\ell_c$ can be computed in analogy to the case of loops. In the calculations of [19], the radius $R$ of the loop plays the same role as the length $l$ of the small scale structures radius. Hence,

$$\ell_c \sim w^{1/3} l^{2/3}.$$  (3.2)

Cusps annihilate into scalar particles and gauge bosons corresponding to the fields which are excited in the string. These particles subsequently decay into jets of lower mass particles, like quarks, gluons and leptons. The empirical QCD multiplicity function [21] can be applied to find the energy spectrum of created particles. For example, consider the decay of a single cusp into particles with initial energy $Q_f$. Then, the number distribution of photons with energy $E = x Q_f (0 \leq x < 1)$ produced by the neutral pions is [7]

$$\frac{dN}{dE} = \frac{15}{16} \mu \ell_c \left( \frac{16}{3} - 2x^{1/2} - 4x^{-1/2} + \frac{2}{3} x^{-3/2} \right) \big|_{x = E/Q_f}$$  (3.3)

Eq.(3.3) will be used later to determine the density of photons from the distribution of cusps.
4. Gamma Ray and UHE Background

As discussed before, cusps on long strings can form when two travelling waves on the string collide. These travelling waves (called “wiggles” from now on) can be produced by interactions of the string network at earlier times. The number density of wiggles obeys the scaling solution \([15]\). The distribution of wiggles is hence on dimensional grounds given by

\[
K(l, t) = \frac{t}{l^2}
\]  

(4.1)

where \(l\) is the size of the kink and \(t\) is the horizon length. \(K(l, t)dl\) is the number of small scale wiggles of length in the interval \([l, l + dl]\) on a long string of length \(t\).

The flux \(F(E)\) of the gamma ray background (number density of photons per unit area and time of energy \(E\) per \(E\) interval) can be obtained by integrating in time over the contributions of all the sources (here cusps annihilations) in the past in the range of energy which after redshifting corresponds to \([E, E + dE]\):

\[
F(E) = \int_{t_{\text{rec}}}^{t_0} dt \, f(t, E z(t)) z(t)^{-3}
\]

(4.2)

where \(z(t)\) is the redshift factor at time \(t\), \(E\) the gamma ray energy and \(f(t, zE)\) is the number of photons of energy \(E z\) per unit physical volume at time \(t\) per unit time emitted at \(t\). \(t_{\text{rec}}\) is the time of recombination, when the universe becomes transparent to photons.

For long strings, the above function \(f\) can be obtained by integrating over the contributions from all small scale structures with size \(l\) smaller than \(t\) which give rise to cusp formation with probability \(P_c\).

\[
f(t, zE) = \frac{z}{Q_f} \frac{dN}{dx} \bigg|_{x = \frac{E}{\sigma}} \int_{l_{\text{min}}}^{t} dl \, K(l, t) \frac{1}{l} n_{ls}(t) P_c,
\]

(4.3)

where \(\frac{dN}{dE}\) is the number of photons with energy \(E\) produced by each cusp annihilation, \(\frac{1}{t}\) is the frequency that two wiggles meet each other, \(n_{ls}\) is the number of long strings.
per 3- volume at time $t$ i.e.

$$n_{ls} = \frac{\nu}{t^3}, \quad (4.4)$$

where $\nu$ is a constant ($\nu \simeq 100$), $P_c$ is the cusp formation probability and the lower limit for the integration is

$$l_{\text{min}} \simeq \gamma G\mu t, \quad (4.5)$$

since kinks with $l < l_{\text{min}}$ will be smoothed by gravitational radiation. In Eq. (4.5) $\gamma \simeq 100$.

Using (3.2) and the formula for $\frac{dN}{dE}$ in the limit of $E \ll Q_f$ we obtain

$$f(t, zE) = \frac{45}{32} \frac{z}{Q_f^2} \frac{(Q_f)^{3/2} \mu w^{1/3}}{t^{3+1/3}} \frac{1}{(\gamma G\mu)^{4/3}}. \quad (4.6)$$

Plugging (4.6) into (4.2) and using the redshift formula $z(t) = \left(\frac{t_0}{t}\right)^{2/3}$, where $t_0$ is the present time, the gamma ray energy density flux becomes

$$E^3 F(E) = \frac{45}{16} [G^{-5/6}] \gamma^{-4/3} \nu \ln(z_{\text{rec}}) \left(\frac{Q_f}{10^{15}\text{GeV}}\right)^{-1/2} \left(\frac{E}{1\text{GeV}}\right)^{3/2} (G\mu)^{-1/2} (10^{-9}\text{GeV}). \quad (4.7)$$

For $G = .7 \times 10^{-38}\text{GeV}^{-2}$, $t_0 = 4.73 \times 10^{17}\text{sec}$ ($7.188 \times 10^{41}\text{GeV}^{-1}$) and $z_{\text{rec}} = 1380$ we will have

$$\log_{10} E^3 F(E) = -0.63 - \frac{1}{2} \log_{10}(G\mu) + \frac{3}{2} \log_{10}(\frac{E}{1\text{GeV}}) - \frac{1}{2} \log_{10}(\frac{Q_f}{10^{15}\text{GeV}}) \quad (4.8)$$

where the units for $E^3 F(E)$ are chosen to be $(eV)^2 m^{-2} sec^{-1}$.

In Figure 4 we have drawn the curves for Eq. (4.8) for various values of $G\mu$. It is obvious that in order that the radiation from string does not exceed the observed background radiation we should have the lower limit

$$G\mu > 10^{-14}. \quad (4.9)$$
5. Gamma Ray Bursts From Long Cosmic Strings

Here, we answer the question whether we can attribute the recently observed gamma ray bursters to cusp annihilations on long ordinary cosmic strings.

It has been shown [9,10] that in order for the cusp annihilations from cosmic strings loops to be detected as gamma ray bursts, they should occur very close to the detector on cosmological distance scales. The basic reason is that the cascade of the particle decay due to cusps annihilation results in a wide angular distribution jet [9]. Therefore, gamma rays from the cusp annihilation will be distributed in a wide angle and will be too diluted to be detected unless the distance to the burst is small. The maximum distance of the burst corresponds to a minimal emission time $t_{\text{min}} = t_0(1 - \epsilon)$ with $\epsilon \sim 10^{-1}$. The time difference between $t_0$ and $t_{\text{min}}$ will be denoted by $\Delta t$.

For long strings, we can apply the same line of reasoning as for loops [10] and find the number of detected bursters per unit time to be

$$n_{\text{burst}} \simeq (\Delta t)^2 N_k(l, t) \omega_c d_c^2(t_{\text{min}}) P_c. \quad (5.1)$$

Here, $N_k(l, t)$ is the number density of the wiggles with size $l$ at time $t$

$$N_k(l, t) \sim K(l, t)t^{-3}. \quad (5.2)$$

$\omega_c$ is the frequency of cusp formation (here $\omega_c \simeq \frac{1}{l}$) and $d_c(t)^2$ is the area of the past light cone at time $t$ in comoving coordinates. Hence, the number of bursts per unit time will be

$$n_{\text{bursts}} \sim (\Delta t)^2 K(t_0, t_0)t_0^{-3} \frac{1}{t_0} d_c^2(t_{\text{min}}) P_c. \quad (5.3)$$

Evaluating $d_c$ for radiation dominated universe $d_c(t) = 2t_0^{2/3}(t_0^{1/3} - t^{1/3})$ the above inequality becomes
\[ n_{\text{bursts}} \sim e^{A t_0^{-1} P_c}. \]  

(5.4)

This is comparable to the value for loops. Therefore, cusp annihilations on long ordinary cosmic strings are not responsible for GRBs.

6. Conclusions

We have studied a mechanism for cusp production on long strings which is based on small scale wiggles on these strings travelling in opposite directions meeting each other and forming a cusp in a similar way to how cusps form on cosmic string loops. Assuming that the small scale structure on the long strings has the form of a random walk, we estimate the cusp formation probability to be close to 50%. The small scale structure on long strings is due to long string intersections and loop production in the past. These processes generically produce traveling waves along the string.

Using an upper limit for the energy produced by cusp annihilation into mesons, we have calculated the contribution of long ordinary cosmic string cusp annihilation to the gamma ray background. We find that the gamma ray background increases as \( \mu \) decreases and that we can therefore set a lower bound on \( \mu \). Given our optimistic assumptions about both energy release from cusps and on the small scale structure, we find this lower bound to be: \( G \mu > 10^{-14} \). In particular, we find that cusp annihilation on long strings may dominates over cusp annihilation on loops. This is a consequence of the fact that most of the energy density in the string distribution is in long strings.

We have also estimated the number of gamma ray bursters which can be attributed to cusp annihilations on long strings. As is the case of loops [10], this number is much too small to account for the recent observations.

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FIGURES

FIGURE 1. Two incoming waves on a long string in z direction that can form a cusp when they collide.

FIGURE 2. The unit sphere with paths made by the tips of the vectors $\pm \chi'$. The two curves intersect and therefore cusps will be formed.

FIGURE 3. The structure of cusps on cosmic strings. $\ell_c$ is the cusp length and $w$ is the thickness of the cosmic string.

FIGURE 4. The predicted photon background fluxes by long strings as a function of $E$ for various values of $G\mu$, with $Q_f = 10^{15}$GeV. The UHE data points and the dashed line are the limits for the gamma-ray background.

FIGURE 5. The predicted photon background fluxes by long strings as a function of $E$ for various values of $Q_f$, with $G\mu = 10^{-6}$

FIGURE 6. The predicted photon background fluxes by long strings with $Q_f = \mu^{1/2}$, for various values of $G\mu$. 

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