EQUATION OF STATE OF AN ANYON GAS IN A STRONG MAGNETIC FIELD

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Abstract:

The statistical mechanics of an anyon gas in a magnetic field is addressed. An harmonic regulator is used to define a proper thermodynamic limit. When the magnetic field is sufficiently strong, only exact $N$-anyon groundstates, where anyons occupy the lowest Landau level, contribute to the equation of state. Particular attention is paid to the interval of definition of the statistical parameter $\alpha \in [-1, 0]$ where a gap exists. Interestingly enough, one finds that at the critical filling $\nu = -1/\alpha$ where the pressure diverges, the external magnetic field is entirely screened by the flux tubes carried by the anyons.

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Introduction: It is now widely accepted that anyons [1] should play a role in the Quantum Hall effect [2]. In the case of the Fractional Quantum Hall effect, Laughlin wavefunctions for the ground state of \( N \) electrons in a strong magnetic field with filling \( \nu = 1/m \) provide an interesting compromise between Fermi degeneracy and Coulomb correlations. A physical interpretation is that at the critical fractional filling electrons carry exactly \( m \) quanta of flux \( \phi_o \) \( (m \text{ odd}) \), \( m-1 \) quanta screening the external applied field. One is left with usual fermions (i.e. anyons carrying one quantum of flux) in an effective magnetic field with filling 1, or free bosons with no magnetic field. Anyons with intermediate statistics \( 1/m \) enter the game when localized excitations above the groundstate are recognized as carrying fractional charge and statistics. The presence of a gap in the spectrum is crucial for explaining the absence of dissipation on the Hall plateaux.

In the case of the Integer Quantum Hall effect, on the other hand, one considers a gas of non interacting electrons filling exactly \( n \) Landau levels. The groundstate is not degenerate, one has automatically a cyclotron gap and the Coulomb interaction can be neglected.

In this letter we calculate the equation of state of an anyon gas in a strong magnetic field. We argue that considering boson based anyons the \( N \)-anyon groundstate problem is entirely solvable in terms of known linear states [3,4], which end up being product of one-body Landau groundstate. Particular care is given to the interval of definition of the statistical parameter \( \alpha \in [-1,0] \) in order the gap above the groundstate being under control. This allows for the analytical derivation of the equation of state of an anyon gas in a strong magnetic field at low temperature. We find that the pressure diverges when the filling factor \( \nu \) takes its maximal value \( \nu = -1/\alpha \), suggesting that everything happens as if
at most $-1/\alpha$ anyons can occupy a given one-body Landau groundstate. At the critical value of the filling factor, the anyon gas completely screens the external applied magnetic field, leaving a free Bose gas. Moreover, the system is incompressible (both nondegenerate and with a gap). When $\alpha = -1/n$ where $n$ is an integer, at the critical filling $\nu = n$ the one-body Landau groundstate has been filled exactly $n$ times, suggesting a possible reinterpretation of the integer quantum Hall effect in terms of critical anyons of statistics $-1/n$.

The model: Let us consider in the symmetric gauge the Hamiltonian of $N$ anyons (charge $e$, flux $\phi$) in a constant magnetic field $B$ ($\vec{k}$ is the unit vector perpendicular to the plane, $\vec{r}_{ij} = \vec{r}_i - \vec{r}_j$)

$$H_N = \sum_{i=1}^{N} \frac{1}{2m} \left( \vec{p}_i - \alpha \sum_{j \neq i} \frac{\vec{k} \times \vec{r}_{ij}}{r_{ij}^2} - eB\frac{\vec{k} \times \vec{r}_i}{2} \right)^2$$

The statistical parameter, $\alpha = e\phi/2\pi$, measures the algebraic fraction of quantum of flux $\phi_o = 2\pi/|e|$ carried by each anyon. One deals with boson based anyons, meaning that the wavefunctions $\psi$ are symmetric. The anyons are coupled to the external magnetic field by their electric charge $e$. Coulomb interactions between anyons are ignored. This will be justified a posteriori when the anyon gas will be taken at its critical filling where the groundstate is non degenerate and has a gap. The Hamiltonian being invariant under $(x_i, y_i, \alpha, \epsilon) \rightarrow (x_i, -y_i, -\alpha, -\epsilon)$, where $\epsilon = eB/|eB|$, the spectrum and thus the partition function are invariant under $(\alpha, \epsilon) \rightarrow (-\alpha, -\epsilon)$. They depend only on $|\alpha|, \epsilon\alpha$ and $\omega_c = |eB|/2m$ (half the cyclotron frequency). One chooses $\epsilon = +1$; in the opposite case one would simply change $\alpha \rightarrow -\alpha$. The shift $\alpha \rightarrow \alpha + 2$ is equivalent to the regular gauge transformation $\psi \rightarrow \exp(-2i\sum_{i<j} \theta_{ij})\psi$, which does not affect the symmetry of the

\[3\] after completion of this work, we noticed that a similar conclusion has been reached in [5], by a qualitative scaling argument using one-component plasma analogy.
wavefunctions. The spectrum is thus periodic in $\alpha$ with period 2.

What is the $N$-anyon groundstate in a magnetic field? Let us reexamine this question more closely by paying particular attention to the domain of definition of $\alpha$. It will turn out that $\alpha$ has to be taken in the interval $[-1,0]$, meaning that the magnetic field is antiparallel to the flux tubes carried by each anyon\textsuperscript{4}.

The creation and annihilation operators (in complex coordinate $z_i = x_i + iy_i$

\begin{align}
a_i^+ &= \frac{1}{\sqrt{m\omega_c}} \left( \partial_i - \frac{\alpha}{2} \sum_{j \neq i} \frac{1}{z_{ij}} \right) - \frac{\sqrt{m\omega_c}}{2} \bar{z}_i \\
a_i &= \frac{1}{\sqrt{m\omega_c}} \left( -\bar{\partial}_i - \frac{\alpha}{2} \sum_{j \neq i} \frac{1}{\bar{z}_{ij}} \right) - \frac{\sqrt{m\omega_c}}{2} z_i
\end{align}

are well defined since they act on anyonic eigenstates which vanish at coinciding points $z_{ij} = 0$. It follows that they obey the usual commutation rules $[a_i, a_j^+] = \delta_{ij}$. The Hamiltonian reads

$$H_N = N\omega_c + 2\omega_c \sum_{i=1}^{N} a_i^+ a_i$$

Since the operator $a_i^+ a_i$ is positive, the lowest energy is $N\omega_c$ and the $N$-anyon groundstates are annihilated by the $a_i$’s. One finds the groundstate basis

$$\psi = z^l \prod_{i<j} r_{ij}^{-\alpha} \prod_{i<j} m_{ij}^{z_{ij}} \exp(-\frac{m\omega_c}{2} \sum_{i} \bar{z}_i z_i)$$

where $z = \sum_{i} z_i/N$ is the center of mass coordinate. The total angular momentum is $L = l + \sum_{i<j} m_{ij}$. Clearly, the $m_{ij}$’s can be choosen as independent orbital quantum numbers. Moreover, since a bosonic representation has been choosen, the wavefunctions must be symmetrized, leading to additional constraints on the $m_{ij}$’s.

\textsuperscript{4}Because of the periodicity $\alpha \to \alpha + 2$, opposite direction has to be understood to a given even number quanta of flux.
If $\alpha$ is not an integer, the eigenstates (5) are entirely contained in the class I $[4]$ of
exact eigenstates of the $N$-anyon problem. If $\alpha$ is an integer $\alpha_o$, the $m_{ij} > \alpha_o$ states are
contained in class I, the $m_{ij} = \alpha_o$ states are contained in class II, and the remaining states
do not belong either to class I or to class II. The latter states should be obtained at $\alpha = \alpha_o$
from the non linear states which are not analytically known. More precisely, those states
in (5) which are not in class I if $\alpha = \alpha_o$ (i.e. one or more of the $m_{ij}$’s equal to $\alpha_o$) are not
obtained from the states in (5) when $\alpha \rightarrow \alpha_o$ by superior value. On the contrary, when
$\alpha \rightarrow \alpha_o$ by inferior value, there is a one to one mapping with the $\alpha = \alpha_o$ states. This
failure to map exactly all states when $\alpha \rightarrow \alpha_o$ by superior value is pertinent only when
$\alpha_o$ is an even integer (Bose case). When $\alpha_o$ is odd (Fermi case), one finds that the states
which are not mapped, either fall in class I, or simply vanish, after proper symmetrization.
Thus they can simply be ignored. This reflects the effect of the exclusion principle since
the remaining states vanish as $m_{ij} - \alpha_o \geq 1$ when two particles coincide.

It follows that if one wants to control the gap above the $N$-anyon groundstate (5), one
should accordingly constrain the interval of definition of $\alpha$. First of all, since we consider
boson based anyons, $\alpha_o$ should be even, say $\alpha_o = 0$ (remember that the system is periodic
in $\alpha$ with period 2). Second, one has to consider anyon eigenstates obtained by negative
value $\alpha \leq 0$. Finally, the range of variation of $\alpha$ has to end at $\alpha = -1$, where the statistics
is fermionic. In this interval, the groundstates (5) interpolate continuously between the
bosonic and fermionic groundstates when $\alpha$ decreases from 0 to $-1$. Of course, one should
also consider the interval $[-2, -1]$. However, one knows for sure that when $\alpha \rightarrow -2$
by superior value some unknown non linear states enter the game, as the gap between
these peculiar states and the groundstate decreases when $\alpha \rightarrow -2$ and finally vanishes at
$\alpha = -2$. Clearly, in this region it is not possible to consider a consistent thermodynamic
of the system in the groundstate. Semi-classical [6] and numerical analysis [7] for the few-
anyon problem confort this analysis. In particular, if the semiclassical analysis indicates that the gap above the groundstate is indeed of order $2\omega_c$ in the interval $\alpha \in [-1, 0]$, it clearly shows that excited states merge in the groundstate when $\alpha \to -2$.

To conclude this discussion, in a regime of strong magnetic field and low temperature, where the thermal energy $1/\beta$ is assumed to be smaller than the cyclotron gap, the thermal probability $\exp(-2\beta\omega_c)$ to have an excited state is negligible in the interval $\alpha \in [-1, 0]$. The system is projected into the Hilbert space of the groundstate. In fact, if one leaves aside the anyonic prefactor $\prod_{i<j} r_{ij}^{-\alpha}$, the $N$-anyon groundstate basis (5) can be rewritten as the direct product $\{ \otimes_{i=1}^N \varphi_{n_i,\ell_i}(z_i) \}_{n_i=0, \ell_i \geq 0}$ of the one-body Landau groundstates of energy $\omega_c$ and angular momentum $\ell_i$

$$\varphi_{n,\ell}(z) = z^\ell L_n^\ell(m\omega_c z \bar{z}) \exp(-\frac{1}{2}m\omega_c z \bar{z})$$
$$\epsilon_{n,\ell} = \omega_c(2n + 1) \quad (6)$$

In this sense, the groundstate of $N$ anyons in a magnetic field ($L = \sum \ell_i$) is constructed in terms of one-body eigenstate in the lowest Landau level. As far as exact anyonic eigenstates are concerned, the $m_{ij}$’s basis has naturally prevailed. However, when symmetrizing the Fock space to derive the equation of state, the $\ell_i$’s basis will be definitively well adapted.

- The equation of state : Since one knows exactly the $N$-anyon groundstate spectrum, one can compute the $N$-anyon partition function $Z_N$ in the regime of strong magnetic field and low temperature. From the $Z_N$’s one in principle deduces the cluster coefficients $b_N$. However, this algorithm happens to be quite tedious when $N$ becomes large. Instead, one can choose to derive directly the equation of state in a second quantized formalism as a power series expansion in $\alpha$. Both methods will be used below.

In order to study the statistical mechanics of an anyon gas, one should regularize the system at long distance to define a proper thermodynamic limit [8]. This is obviously
still needed in the presence of the magnetic field. One confines the anyons by a harmonic attraction, adding \( \sum_{i=1}^{N} \frac{1}{2} m \omega^2 r_i^2 \) to the Hamiltonian \( \mathcal{H} \). The thermodynamic limit is obtained when \( \omega \to 0 \). In the presence of the harmonic regulator, the groundstate problem is simply solved by replacing in \( \omega_c \to \omega_t = \sqrt{\omega^2 + \omega_c^2} \). The effect of the regulator is to partially lift the degeneracy of the groundstate spectrum

\[
N \omega_c \to N \omega_t + \{ l + \sum_{i \neq j} (m_{ij} - \alpha) \} (\omega_t - \omega_c) \tag{7}
\]

by \( (L - \alpha N(N - 1)/2)(\omega_t - \omega_c) \), where \( L - \alpha N(N - 1)/2 \) is interpreted as a total orbital angular momentum of the \( N \)-anyon groundstate in the singular gauge. Again, if one leaves aside the anyonic prefactor, the eigenstates can be rewritten in terms of one-body harmonic Landau eigenstates

\[
\varphi_{n,\ell}^\omega(z) = z^\ell L_n^\ell (m \omega_t z \bar{z}) \exp(-\frac{1}{2} m \omega_t z \bar{z})
\]

\[
\epsilon_{n,\ell} = \omega_t (2n + 1) + (\omega_t - \omega_c) \ell \tag{8}
\]

As already stressed above, \( N \)-anyon states have to be symmetrized in the case of boson based anyons. However, as far as symmetry is concerned, \( N \)-anyon states and \( N \)-boson states are identical. A \( N \)-anyon state is entirely characterized by the number \( n_\ell \) of one-body Landau state of angular momentum \( \ell = 0, 1, ... \infty \), with the constraint \( \sum_\ell n_\ell = N \), and its energy is nothing else but the sum of one-body harmonic Landau levels \( \sum_\ell n_\ell \epsilon_{0,\ell} \) shifted by the constant \(-N(N - 1)\alpha(\omega_t - \omega_c)/2\). Since one is interested by the equation of state, symmetrization is done at the level of partition functions, in a way quite similar to the bosonic oscillator equation of state. The grand partition function for a gas of bosonic oscillators in the lowest Landau level is

\[
Z^b = \prod_{\ell=0}^{\infty} \frac{1}{1 - z \exp(-\beta \epsilon_{0,\ell})} \tag{9}
\]
By definition, $Z^b = \sum_{N=0}^{\infty} z^N Z^b_N$, where $z$ is the fugacity and $Z^b_N$ is the $N$-boson partition function. For a linear spectrum one can use the identity $(1 - e^{-\beta \omega t}) Z^b \rightarrow Z^b$ when $z \rightarrow z e^{-\beta(\omega t - \omega_c)}$. One deduces that $Z^b_N - e^{-\beta \omega t} Z^b_{N-1} = e^{-N \beta(\omega t - \omega_c)} Z^b_N$ and finally

$$Z^b_N = \frac{e^{-N \beta \omega t}}{(1 - e^{-\beta(\omega t - \omega_c)})(1 - e^{-2\beta(\omega t - \omega_c)}) \cdots (1 - e^{-N \beta(\omega t - \omega_c)})} \tag{10}$$

Since the anyonic interaction shifts the $N$-body groundstate spectrum by $-N(N-1)\alpha(\omega t - \omega_c)/2$, the $N$-anyon partition function reads

$$Z_N = e^{\beta N(N-1)\alpha(\omega t - \omega_c)} Z^b_N \tag{11}$$

The thermodynamic limit, $\omega \rightarrow 0$, is understood as $1/((\beta \omega)^2 \rightarrow V/\lambda^2$, where the cluster coefficients $b_N$ ($b_2 = Z_2 - \frac{1}{2} Z^2_1$, $b_3 = Z_3 - Z_2 Z_1 + \frac{1}{3} Z^3_1$, ...) are multiplied by $N$ accordingly [8,9,10]. One infers

$$b_N = \frac{V}{\lambda^2} 2 \beta \omega_c \frac{(N \alpha + 1)(N \alpha + 2) \cdots (N \alpha + N - 1)}{N!} e^{-N \beta \omega_c} \tag{12}$$

where $V/\lambda^2$ is the volume in unit of the thermal wavelength $\lambda \equiv \sqrt{2 \pi \beta/m}$.

One can see more directly how the volume factor, in the thermodynamic limit, materializes in the cluster coefficients using a second quantization language. One has to perform a perturbative expansion in $\alpha$ of the thermodynamical potential. In this context, short distance singularities of the anyon interaction $\frac{\alpha^2}{r^2_{ij}}$ should be treated before the perturbative analysis can proceed. These singularities manifest themselves in the non-analyticity in $|\alpha|$ of the $N$-anyon spectrum, and the fact that $N$-anyon states have to vanish when two anyons approach each other. A perturbative analysis in $\alpha$ is possible [9] if the $N$-anyon wavefunction is rewritten as

$$\psi(\vec{r}_1, \cdots, \vec{r}_N) = \prod_{i<j} r_{ij}^{-|\alpha|/2} \tilde{\psi}(\vec{r}_1, \cdots, \vec{r}_N) \tag{13}$$
In (13), the exclusion of the diagonal of the configuration space, a non perturbative effect in \(|\alpha|\), has been encoded, by hand, in the \(N\)-anyon wavefunction (\(\tilde{\psi}\) is assumed to be non singular). The Hamiltonian \(\tilde{H}_N\) acting on \(\tilde{\psi}\) is known to generate the correct perturbative expansion in \(\alpha\) [9,10]. One notes that the redefinition (13) applied to the groundstate (5) precisely factors out the anyonic prefactor \(\prod_{i<j} r_{ij}^{-\alpha}\) in \(\tilde{\psi}\). Thus, the \(\tilde{\psi}\) groundstate basis does not depend on \(\alpha\), and is identical to the unperturbed basis. In the presence of the harmonic regulator \(\tilde{H}_N^\omega\) reads

\[
\tilde{H}_N^\omega = \sum_{i=1}^N \left[ -\frac{2}{m} \partial_i \bar{\partial}_i + \frac{m}{2} \omega_t^2 z_i \bar{z}_i - \omega_c (z_i \partial_i - \bar{z}_i \bar{\partial}_i) \right] \\
+ \sum_{i<j} \left[ -\frac{|\alpha| - \alpha \partial_i - \bar{\partial}_i}{m} \frac{z_i - z_j}{z_i - \bar{z}_i} - \frac{|\alpha| + \alpha \partial_i - \partial_j}{m} \frac{\bar{z}_i - \bar{z}_j}{z_i - \bar{z}_j} + \alpha \omega_c \right] 
\]

(14)

When acting on the groundstate basis it becomes a sum of one-body Hamiltonian \(\sum_i -\frac{2}{m} \partial_i \bar{\partial}_i + \frac{m}{2} \omega_t^2 z_i \bar{z}_i - \omega_c (z_i \partial_i - \bar{z}_i \bar{\partial}_i)\) with total energy \(\sum_i \epsilon_{0,\ell_i}\) shifted by \(-\sum_{i<j} \alpha (\omega_t - \omega_c)\).

Second quantizing \(\tilde{H}_N^\omega\) [10], the 2-anyon vertex is simply the constant shift \(-\alpha (\omega_t - \omega_c)/2\). One uses one particle Green’s function in the lowest Landau level

\[
G_\beta(\vec{r}_2, \vec{r}_1) = \sum_{\ell=0}^\infty \varphi_{0,\ell}(z_2) \exp(-\beta \epsilon_{0,\ell}) \bar{\varphi}_{0,\ell}(z_1) \\
= \frac{m \omega_t}{\pi e^{\beta \omega_t}} \exp\left( -\frac{m \omega_t^2}{2 e^{\beta \omega_t}} (e^{\beta \omega_t} r_{21}^2 + (e^{\beta \omega_t} - e^{\beta \omega_c}) (r_{11}^2 + r_{22}^2) + 2ie^{\beta \omega_c} \vec{k}.(\vec{r}_2 \times \vec{r}_1)) \right) 
\]

(15)

and computes the diagrammatic expansion of the thermodynamical potential \(\Omega \equiv -\ln \sum_{N} Z_N z^N\) as a power series in \(\alpha\). At a given order \(\alpha^n\), the leading connected diagrams (which are the diagrams connected with \(n+1\) loops) are indeed behaving as \(1/(\beta \omega)^2\) when \(\omega \to 0\). Also, at this order, non vanishing diagrams start contributing in the cluster coefficient
\[ b_{n+1} \]. Finally, the thermodynamical potential is found to be

\[ \Omega \equiv - \sum_{N=1}^{\infty} b_N z^N = - \frac{V}{\lambda^2} 2 \beta \omega_c \ln y(z e^{-\beta \omega_c}) \] (16)

where \( y(z') \) is solution of \( y - z' y^{\alpha+1} = 1 \) with \( y(z') \to 1 \) when \( z' \to 0 \) [11]. The thermodynamical potential for bosons (fermions) is correctly reproduced when \( \alpha = 0 \) since \( y = 1/(1 - z') \) (respectively \( \alpha = -1 \) since \( y = 1 + z' \)). The filling factor \( \nu \equiv \rho/\rho_L \) (where \( \rho_L = 2 \beta \omega_c / \lambda^2 \) is the Landau degeneracy per unit volume) as a function of \( z \) is given by \( y(z e^{-\beta \omega_c}) = 1 + \nu/(1 + \alpha \nu) \). It is monotonically increasing with \( z \) from 0 to \(-1/\alpha\). One deduces the equation of state

\[ P\beta = \rho_L \ln \left( 1 + \frac{\nu}{1 + \alpha \nu} \right) \] (17)

When expanding the pressure as a power series in the density \( \rho \), one verifies that the expression of the second virial coefficient \( a_2 = \frac{V}{\lambda^2} \frac{1}{2 \beta \omega_c} (1 + 2 \alpha) \) is reproduced [10,12] in the limit where the Boltzman weight \( \exp(-2 \beta \omega_c) \) is neglected\footnote{One finds \( a_3 = \frac{V}{\lambda^2} \frac{1}{3 \beta^2 \omega_c^2} (1 + 3 \alpha + 3 \alpha^2), a_4 = \cdots \).}. Moreover, the first order expansion in \( \alpha \) of (17) coincides with the perturbative result in a strong magnetic field given in [10]. The magnetization per unit volume is

\[ \mathcal{M} = - \mu_0 \rho + 2 \frac{\mu_0}{\lambda^2} \ln \left( 1 + \frac{\nu}{1 + \alpha \nu} \right) \] (18)

where \( \mu_0 \equiv |e|/2m \) is the Bohr magneton. Except near the singularity \( \nu = -1/\alpha \), the ratio of the logarithmic correction with the De Haas-Van Alphen magnetisation [13] \( \mathcal{M} = - \mu_0 \rho \) is of order \((\beta \omega_c)^{-1}\), and thus negligible.

Both pressure and magnetization diverge at \( \nu = -1/\alpha \). In the case \( \alpha = 0 \), any value of \( \nu \) is allowed due to Bose condensation. On the other hand, in the case of Fermi statistics \( \alpha = -1 \), Pauli exclusion implies that the lowest Landau level is completely filled when
\( \nu = 1 \). At a particular \( \alpha \), the critical value \( \nu = -1/\alpha \) can be interpreted as at most 
\(-1/\alpha\) anyons of statistics \( \alpha \) can occupy a given lowest Landau level. Since transitions to 
excited levels are by construction forbidden, the pressure necessarily diverges when the 
lowest Landau level is fully occupied such that any additional particle is excluded. In this 
situation the gas is incompressible. Indeed the isothermal compressibility coefficient \( \chi_T = 
-1/V \left( \frac{\partial V}{\partial P} \right)_{T,B} \) vanishes at the critical filling (except when \( \alpha = 0 \) where \( \chi_T \rightarrow \lambda^2/2\omega_c \)). The 
groundstate is clearly nondegenerate as can be shown by extracting from \( \Omega \) the canonical 
partition function of the critical system \( Z_{<N>_{cr}} = \exp(-\beta <N>_{cr} \omega_c) \) where \( <N>_{cr} = V\rho_L(-1/\alpha) \). Last but not least, one can get some information on the quantum numbers of 
the critical nondegenerate groundstate. In the fermionic case \( \alpha = -1 \), the non degenerate 
groundstate is known to be a Vandermonde determinant, built from one body Landau 
eigentstates \( \ell_i = 0 \) implying a minimal total angular momentum \( <N>_{cr} (<N>_{cr} -1)/2 \) 
in the singular gauge. By analogy, in the case \( \alpha \in [-1,0] \) one infers that the state (5) 
with the \( \ell_i \)’s all equal to 0 is the critical nondegenerate groundstate with total angular 
momentum \(-\alpha <N>_{cr} (<N>_{cr} -1)/2 \).

-Discussion : At the critical filling the magnetic field is entirely screened by the flux 
tubes carried by the anyons: each anyon carrying \( \alpha \phi_o \) individual flux, one gets at the 
critical filling \( \nu = -1/\alpha \) that the flux of the magnetic field is precisely \(-N\alpha\phi_o \).

As already emphasized in the introduction, a similar magnetic screening is at the origin 
of the mean-field Chern-Simons-Landau-Ginzburg theory of the fractional Quantum 
Hall effect [2]. If \( m \) in an odd integer, one can gauge transforms the Hall electrons in boson 
based anyons carrying \( m \) quanta of flux. The mean field solution is meaningful when the 
external magnetic field is completely screened by the flux tubes carried by the anyons. It 
precisely describes the \( \nu = 1/m \) fractional Hall liquid. When \( m = 1 \), one has the \( \nu = 1 \) 
integer quantum Hall effect where the lowest Landau level is entirely filled. In the present
situation, each anyon carries $m = -\alpha$ quanta of flux, where $\alpha \in [-1, 0]$. At the critical filling $\nu = -1/\alpha$, again a magnetic screening occurs; it corresponds to the maximum filling of the lowest Landau level.

It would certainly be interesting to find out if something special happens in the particular case $\alpha = -1/n$ with $n$ an integer ($n > 1$). At the critical filling $\nu = n$, the many-body Landau groundstate has been filled exactly $n$ times. So one has a non-degenerate groundstate, with a cyclotron gap, and an integer filling $n$. This suggests a possible reinterpretation of the $n$ integer quantum Hall effect in terms of a critical anyon gas of statistics $-1/n$ in a strong magnetic field with Coulomb interactions ignored. In the usual picture of electrons filling $n$ Landau levels, the external magnetic field is not screened. Here, on the contrary, the magnetic field is screened by the critical anyon gas, quite similarly to the Laughlin wavefunctions in the fractional quantum Hall effect.
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