Non-Gaussianity from false vacuum inflation: Old curvaton scenario

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Abstract

We calculate the three-point correlation function of the comoving curvature perturbation generated during an inflationary epoch driven by false vacuum energy. We get a novel false vacuum shape bispectrum, which peaks in the equilateral limit. Using this result, we propose a scenario which we call “old curvaton”. The shape of the resulting bispectrum lies between the local and the false vacuum shapes. In addition we have a large running of the spectral index.
1 Introduction

Inflation \[1,2\] nowadays is accepted as the standard paradigm to solve many cosmological problems, and to provide the appropriate initial conditions for the hot big bang cosmology. One of the strongest supports of this early era of inflation is the observed primordial curvature perturbation \( R_c \), whose power spectrum \( P_R \) is nearly scale invariant with its index being \( n_R \approx 0.96 \), with almost perfect Gaussian statistics \[3\]. Conversely, these observations have been acting as a powerful discriminator among the models of inflation based on particle physics \[4\] and placing strong constraints on the realization of inflation, e.g. in the context of supergravity \[5\]. In the next few years, we will obtain more precise data on the primordial density perturbations by ongoing and forthcoming experiments like the Wilkinson Microwave Anisotropy Probe (WMAP) and the Planck satellites, and thus will be able to constrain the models of inflation even stronger than now.

In the light of these upcoming observations, there have been increasing interests in the non-Gaussian signature of the primordial perturbations. Currently, the non-Gaussianity of the cosmic microwave background (CMB) fluctuations is constrained to be \( |f_{\text{NL}}| \lesssim 100 \) \[3\], with \( f_{\text{NL}} \) being the non-linear parameter \[6\], which translates into less than 0.1% of non-Gaussian contribution. The sensitivity is expected to be improved at the level of \( f_{\text{NL}} \sim \mathcal{O}(1) \) on the CMB scales \[7\], and indeed primordial non-Gaussianity will be another powerful probe of inflation: if substantial non-Gaussianity were to be detected, we should look for new models beyond the simplest single field slow-roll inflation models, where \( f_{\text{NL}} \ll \mathcal{O}(1) \) \[8\]. A short list of the models where large non-Gaussianity may arise includes Dirac-Born-Infeld (DBI) inflation \[9\], single field inflation with features \[11\], multi-field inflation \[13\], curvaton \[14\], and so on.

In this note, we study a new possibility of generating large amount of highly non-Gaussian curvature perturbation: we consider a period of inflation supported by a non-zero vacuum energy. This is in fact the original model of inflation \[1\] and has received a renewed interest in the context of vast string landscape \[15\]. But \( R_c \) generated during this stage by vacuum fluctuations has been unknown until very recently \[16\]. The difficulty is that while the inflaton field \( \phi \) is well anchored in a false vacuum, the comoving curvature perturbation, which is given by

\[
R_c \sim H_\dot{\phi} \delta \phi, \tag{1}
\]

is ill defined. The origin of this difficulty is that during false vacuum inflation the inflaton \( \phi \) is classically not moving, and there is no preferred time direction to define comoving hypersurfaces. But this phase should be broken, and indeed in Ref. \[16\] \( P_R \) was calculated by incorporating a regularizing mass which scales as \( a^{-2} \). The results are such that \( P_R \) is dependent on the scale of the regularizing mass, with steeply blue index \( n_R = 4 \) \[16\] and nearly \( \chi^2 \) distribution \[17\]. Thus, we need an additional mechanism to generate \( R_c \) to match observations.

In addition to false vacuum inflation, we incorporate the curvaton mechanism \[19\] to gener-

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ate the Gaussian contribution of \( R_c \). In this combined scenario of false vacuum inflation and the curvaton mechanism, or shortly “old curvaton” scenario, as we will see we can build a concrete model with observationally consistent \( P_R \), while providing substantial degree of non-Gaussian signature. Especially, the bispectrum lies between the so-called equilateral and local types, giving a concrete example of mixed non-Gaussianity. The structure of this note is as follows. In Section 2 we explicitly calculate the three-point correlation function, or the bispectrum of \( R_c \) using the results of Ref. [16]. Then in Section 3 we incorporate the curvaton mechanism and present the resulting power spectrum and the bispectrum in the old curvaton scenario. We conclude in Section 4. Some technical detail is given in Appendix.

2 Bispectrum from false vacuum inflation

We consider a theory where the inflaton sector is described by the Einstein gravity with scalar Lagrangian,
\[
\mathcal{L} = \frac{m_{\text{Pl}}^2}{2} R - \frac{1}{2} \phi^\mu \phi_{,\mu} - V(\phi),
\]
where \( m_{\text{Pl}} = (8\pi G)^{-1/2} \approx 2.4 \times 10^{18}\text{GeV} \). The potential is assumed to have the form
\[
V(\phi) = V_0 + \frac{1}{2} \left( -m_\phi^2 + \frac{\mu^2}{a^2} \right) \phi^2.
\]
We are interested in the epoch while \( m_{\text{eff}}^2 = -m_\phi^2 + \mu^2/a^2 > 0 \) so that \( \phi \) is well anchored at \( \phi = 0 \). During this stage, as shown in Appendix A, we can find the comoving curvature perturbation \( R_c \) as
\[
R_c(k) = \frac{1}{2a^2(\rho + p)_{\text{ren}}(\mu\eta)^2} \int \frac{d^3q}{(2\pi)^3} \phi_q \phi_{k-q} \left[ \mu^2 + q \cdot (q - k) \right].
\]
The convolution in the momentum space will lead to a loop integral if we calculate the two-point or three-point correlation function. It’s worth to note that this kind of loop integral is not the real loop integral in interactional quantum field theory. It is the so-called c-loop [20]. As can be read from \( \mathcal{L} \), the curvature perturbation is quadratic in \( \delta \phi \), so we need not follow the conventional approach [8]. The bispectrum arise from c-loop integration between different modes of curvature perturbation. The three-point correlation function of \( R_c \) is explicitly written as
\[
\langle R_c(k_1)R_c(k_2)R_c(k_3) \rangle = \frac{1}{[2a^2(\rho + p)_{\text{ren}}(\mu\eta)^2]^2} \int \frac{d^3q_1}{(2\pi)^3} \frac{d^3q_2}{(2\pi)^3} \frac{d^3q_3}{(2\pi)^3} \langle \phi_{q_1} \phi_{k_1-q_1} \phi_{q_2} \phi_{k_2-q_2} \phi_{q_3} \phi_{k_3-q_3} \rangle \times \left[ \mu^2 + q_1 \cdot (q_1 - k_1) \right] \left[ \mu^2 + q_2 \cdot (q_2 - k_2) \right] \left[ \mu^2 + q_3 \cdot (q_3 - k_3) \right].
\]
We can after some straightforward calculations find the expectation value by explicitly plugging \( \mathcal{L} \) into \( \mathcal{R} \), but we can anticipate the result by considering the possible contractions between \( \phi \)’s. It is obvious that there are 8 possible physically meaningful contractions: \( R_c(k_1) \) to \( R_c(k_2) \) or \( R_c(k_3) \), so we have 2 choices. Further, since each \( R_c(k_i) \) has 2 field contents, there are 2 choices for each: one of the fields in (say) \( R_c(k_1) \) can be correlated to one in (say) \( R_c(k_2) \) and
the other in $\mathcal{R}_c(k_1)$ can be correlated to one in $\mathcal{R}_c(k_3)$. This is to avoid self-contraction of $\mathcal{R}_c(k_3)$. Thus total $2 \times 2 \times 2 = 8$ different ways of contraction. These are explicitly written as

\[
\langle (\phi_{k_1}^* \phi_{k_1-q_1})(\phi_{k_2}^* \phi_{k_2-q_2})(\phi_{k_3}^* \phi_{k_3-q_3}) \rangle + \langle (\phi_{k_1} \phi_{k_1-q_1})(\phi_{k_2} \phi_{k_2-q_2})(\phi_{k_3} \phi_{k_3-q_3}) \rangle \\
+ \langle (\phi_{k_1} \phi_{k_1-q_1})(\phi_{k_2}^* \phi_{k_2-q_2})(\phi_{k_3}^* \phi_{k_3-q_3}) \rangle + \langle (\phi_{k_1}^* \phi_{k_1-q_1})(\phi_{k_2} \phi_{k_2-q_2})(\phi_{k_3} \phi_{k_3-q_3}) \rangle \\
+ \langle (\phi_{k_1} \phi_{k_1-q_1})(\phi_{k_2} \phi_{k_2-q_2})(\phi_{k_3}^* \phi_{k_3-q_3}) \rangle + \langle (\phi_{k_1} \phi_{k_1-q_1})(\phi_{k_2}^* \phi_{k_2-q_2})(\phi_{k_3} \phi_{k_3-q_3}) \rangle \\
+ \langle (\phi_{k_1} \phi_{k_1-q_1})(\phi_{k_2} \phi_{k_2-q_2})(\phi_{k_3} \phi_{k_3-q_3}) \rangle + \langle (\phi_{k_1}^* \phi_{k_1-q_1})(\phi_{k_2} \phi_{k_2-q_2})(\phi_{k_3} \phi_{k_3-q_3}) \rangle ,
\]

and these terms exactly correspond to the terms with $\delta^{(3)}(k_1 + k_2 + k_3)$ which we can obtain by direct computations. Indeed, after some trivial calculations, we find that (5) can be written as

\[
\langle \mathcal{R}_c(k_1)\mathcal{R}_c(k_2)\mathcal{R}_c(k_3) \rangle \\
= \frac{4\delta^{(3)}(k_1 + k_2 + k_3)}{[2a^2(p + p)_{\text{ren}}(\mu \eta)^2]^3} \\
\times \int d^3 q \left\{ |\varphi_q|^2 |\varphi_{|k_1-q|}|^2 |\varphi_{|k_2+q|}|^2 \left[ \mu^2 + q \cdot (q - k_1) \right] \left[ \mu^2 + q \cdot (k_2 + q) \right] \left[ \mu^2 + (q - k_1) \cdot (k_2 + q) \right] \\
+ |\varphi_q|^2 |\varphi_{|k_1-q|}|^2 |\varphi_{|k_3+q|}|^2 \left[ \mu^2 + q \cdot (q - k_1) \right] \left[ \mu^2 + q \cdot (k_3 + q) \right] \left[ \mu^2 + (q - k_1) \cdot (k_3 + q) \right] \right\} .
\]

Where $\varphi_k$ is the mode function of perturbation $\phi_k$ (see (A.11) for example),

\[
\phi_k = a_k \varphi_k + a_k^\dagger \varphi_k^*. \tag{8}
\]

Since the two terms of the integrand of (7) are exactly the same with the permutation of $k_2$ and $k_3$, we only consider the first term.

As can be read, (7) is a very complex function of three momenta. However, since we are interested in large scales, we can expand each term of (7) and only keep the lowest non-zero contribution. Then, explicitly using the asymptotic form of the mode function $\varphi_k$ when $|\eta| \gg 1$,

\[
|\varphi_k|^2 \xrightarrow{|\eta| \gg 1} \frac{(H \eta)^2}{2 \sqrt{k^2 + \mu^2}} ,
\]

and integrating over angles, up to linear order of $k$ we can find

\[
\int d\Omega |\varphi_q|^2 |\varphi_{|k_1-q|}|^2 |\varphi_{|k_2+q|}|^2 \left[ \mu^2 + q \cdot (q - k_1) \right] \left[ \mu^2 + q \cdot (k_2 + q) \right] \left[ \mu^2 + (q - k_1) \cdot (k_2 + q) \right] \\
= \frac{\pi}{2} (H \eta)^6 (q^2 + \mu^2)^{3/2} ,
\]

and $k$ dependence appears only beyond quadratic order. Now we have to perform the integration with respect to the magnitude $q = |q|$. If one naively integrates from 0 to infinity, the integral
badly diverges. However, actually we should appropriately regularize the integral as follows: we are interested in the large scale curvature perturbation produced during the false vacuum inflation stage. Thus, we can trace the momentum up to the scale at which false vacuum inflation ends. That is, we should consider up to the scale $k_*$ which crosses the horizon at the moment the potential becomes concave down. This can be found from the potential (3): it becomes concave down when the effective mass is zero, i.e. $m_\phi^2 = \mu^2/a_*^2$, which in turn gives the scale factor at this moment as $a_* = \mu/m_\phi$. The momentum scale which crosses the horizon at this moment is thus $k_* = a_* H = (H/m_\phi)\mu$. The modes with larger wavenumbers $k > k_*$ exit the horizon after the false vacuum inflation phase. So for the purpose of bispectrum produced during false vacuum inflation, one should cutoff the integration at $k = k_*$. After imposing the cutoff at $k_*$, the integration becomes

$$
\int d^3q |\tilde{\varphi}_q|^2 |\tilde{\varphi}_{q_1-q}|^2 |\tilde{\varphi}_{q_2+q}|^2 \left[ \mu^2 + q \cdot (q - k_1) \right] \left[ \mu^2 + q \cdot (k_2 + q) \right] \left[ \mu^2 + (q - k_1) \cdot (k_2 + q) \right] = \frac{\pi}{96} (H\eta)^6 \mu^6 f \left( \frac{k_*}{\mu} \right),
$$

where

$$
f \left( \frac{k_*}{\mu} \right) \equiv \frac{k_*}{\mu} \sqrt{1 + \frac{k_*^2}{\mu^2}} \left[ 3 + 14 \frac{k_*^2}{\mu^2} + 8 \frac{k_*^4}{\mu^4} \right] - 3 \log \left[ 2 \left( \frac{k_*}{\mu} + \sqrt{1 + \frac{k_*^2}{\mu^2}} \right) \right].
$$

For example, when $m_\phi = H$, we have $f(k_*/\mu) \approx 30.6318$. Thus, with the other term of (7) for which we can simply replace $k_2$ by $k_3$, the full three-point correlation function of $\mathcal{R}_c$ is found to be, up to an $\mathcal{O}(1)$ constant,

$$
\langle \mathcal{R}_c(k_1)\mathcal{R}_c(k_2)\mathcal{R}_c(k_3) \rangle = \delta^{(3)}(k_1 + k_2 + k_3) \frac{(2\pi)^7}{3A^3} \frac{H^6}{m_\phi^6} f \left( \frac{k_*}{\mu} \right),
$$

where we have used (31). Comparing this with the definition of the bispectrum

$$
\langle \mathcal{R}_c(k_1)\mathcal{R}_c(k_2)\mathcal{R}_c(k_3) \rangle \equiv (2\pi)^3 \delta^{(3)}(k_1 + k_2 + k_3) B_{\mathcal{R}}(k_1, k_2, k_3),
$$

we can find the bispectrum of the curvature perturbation $\mathcal{R}_c$ produced during false vacuum inflation as

$$
B_{\mathcal{R}}(k_1, k_2, k_3) = \frac{(2\pi)^4}{3A^3} \frac{H^6}{m_\phi^6} f \left( \frac{k_*}{\mu} \right).
$$

Note that $B_{\mathcal{R}}(k_1, k_2, k_3)$ is independent of $k_i$ at linear order of $k_i/\mu$. This is very different from non-Gaussianity produced by any other known models.

In Fig. 11 we show the dimensionless shape function $(k_1k_2k_3)^2 B_{\mathcal{R}}(k_1, k_2, k_3)$ as a function of $k_2/k_1$ and $k_3/k_1$. It is clear that the bispectrum exhibits its maximum value at the equilateral limit $k_1 = k_2 = k_3$. This means, unlike the so-called local type non-Gaussianity where non-linearity arises due to the classical super-horizon evolution of the curvature perturbation, the curvature perturbation itself is intrinsically highly non-Gaussian. Indeed, as we can see from (4), $\mathcal{R}_c \propto \phi^2$ with $\phi$ being nearly Gaussian, thus $\mathcal{R}_c$ follows $\chi^2$ statistics. A more close study will follow separately [17].
Figure 1: The shape of the bispectrum produced during false vacuum inflation. We have the maximum value of the bispectrum when all the momenta are of comparable magnitude.

3 Old curvaton scenario

In the previous section, we calculated the bispectrum of $R_c$ produced during false vacuum inflation. However, false vacuum inflation cannot reproduce the observed nearly scale invariant curvature perturbation with nearly Gaussian distribution. To generate the Gaussian contribution, we introduce a curvaton field $\sigma$. As we will see shortly, in this “old curvaton” scenario we can obtain the desired nearly scale invariant power spectrum of the Gaussian curvature perturbation, along with novel shape of the three-point correlation function as well as large running of the spectral index.

We employ the standard curvaton scenario for producing the nearly scale invariant power spectrum. The curvaton $\sigma$ has very small mass, no direct coupling with the inflaton field, and sub-dominant energy density during inflation. After inflation, the curvaton field eventually dominates the energy density either during curvaton oscillation stage or a secondary inflation stage. Then the isocurvature perturbation stored in the curvaton field is converted to the curvature perturbation.

In the following calculation, we will denote as $R_\phi$ and $R_\sigma$ the curvature perturbations on the comoving slices of the inflaton (including possible radiation component) and the curvaton, respectively, and use $R_c$ to denote the curvature perturbation on the comoving slice of the total energy-momentum tensor. As shown in Ref. [21], $R_\phi$ and $R_\sigma$ are separately conserved quantities as long as there is no direct coupling between the inflaton $\phi$ and the curvaton $\sigma$, which is assumed in our scenario as well as in the standard curvaton scenario.

The comoving curvature perturbation defined from the total energy-momentum tensor can be written as

$$R_c = (1 - r)R_\phi + rR_\sigma,$$

(16)
where the constant \( r \) is given by
\[
    r = \frac{\dot{\rho}_\sigma}{\dot{\rho}_\phi + \dot{\rho}_\sigma}|_{\text{dec}},
\]
which is evaluated when curvaton decays: after then, \( \mathcal{R}_e \) becomes constant. From \([16]\), the power spectrum can be written as
\[
    \mathcal{P}_\mathcal{R} = (1 - r)^2 \mathcal{P}_{\mathcal{R}_\phi} + r^2 \mathcal{P}_{\mathcal{R}_\sigma},
\]
where we have assumed no cross correlation between \( \phi \) and \( \sigma \). Actually, such correlation can be induced by the gravitational coupling. But this cross correlation can be absorbed into the redefinition of \( \mathcal{R}_\phi \) and \( \mathcal{R}_\sigma \) without modifying any other calculation present in this paper: we will present more discussion on this redefinition at the end of this section. The spectral index is
\[
    n_\mathcal{R} - 1 \equiv \frac{d \log \mathcal{P}_\mathcal{R}}{d \log k} = 3 + (n_\sigma - 4)r^2 \frac{\mathcal{P}_{\mathcal{R}_\sigma}}{\mathcal{P}_\mathcal{R}},
\]
where \( n_\sigma \equiv d \log \mathcal{P}_{\mathcal{R}_\sigma}/d \log k \). The running of the spectral index is written as
\[
    \alpha_\mathcal{R} \equiv \frac{dn_\mathcal{R}}{d \log k} = (n_\mathcal{R} - 1) - (n_\mathcal{R} - 1)^2 + 6 \left[ \alpha_\sigma - (n_\sigma - 1) + (n_\sigma - 1)^2 - 6 \right] \frac{r^2 \mathcal{P}_{\mathcal{R}_\sigma}}{\mathcal{P}_\mathcal{R}},
\]
where \( \alpha_\sigma \equiv dn_\sigma/d \log k \). Note that the running of the index is of the same order of magnitude as the spectral index. We can obtain a large running in the old curvaton scenario.

The total three-point correlation function in the old curvaton scenario can be calculated as
\[
    \langle \mathcal{R}_e(k_1)\mathcal{R}_e(k_2)\mathcal{R}_e(k_3) \rangle = (1 - r)^3 \langle \mathcal{R}_\phi(k_1)\mathcal{R}_\phi(k_2)\mathcal{R}_\phi(k_3) \rangle + r^3 \langle \mathcal{R}_\sigma(k_1)\mathcal{R}_\sigma(k_2)\mathcal{R}_\sigma(k_3) \rangle,
\]
with \( \langle \mathcal{R}_\phi(k_1)\mathcal{R}_\phi(k_2)\mathcal{R}_\phi(k_3) \rangle \) given by \([13]\). \( \langle \mathcal{R}_\sigma(k_1)\mathcal{R}_\sigma(k_2)\mathcal{R}_\sigma(k_3) \rangle \) has a typical local shape, which is standard in the curvaton scenario written as \([14]\).

\[
    r^3 \langle \mathcal{R}_\sigma(k_1)\mathcal{R}_\sigma(k_2)\mathcal{R}_\sigma(k_3) \rangle = (2\pi)^7 \delta^3(k_1 + k_2 + k_3) \left( -\frac{3}{10} \mathcal{P}_\mathcal{R} \right) \left( \frac{5}{4r} - \frac{5}{3} - \frac{5r}{6} \right) \sum_i k_i^3. \]

Depending on which term is dominant in \([21]\), the total three-point correlation function in the old curvaton scenario can exhibit different shape: if the inflaton contribution is larger, the shape coincides with strong equilateral type shown in Fig. 1, while the curvaton is dominating we have the well-known local shape as shown in Fig. 2. More generally, in the old curvaton scenario in general we have a mixed shape, lying between these two extremes. An example of mixed shape is illustrated in Fig. 3.

We can also match our mixed shape non-Gaussianity with the local shape at \( k_1 = k_2 = k_3 \) to obtain a non-Gaussian estimator \( f_{\text{NL}}^{\text{mix}} \) as
\[
    f_{\text{NL}}^{\text{mix}} = \frac{5}{4r} - \frac{5}{3} - \frac{5r}{6} - (1 - r)^3 \frac{10}{27A^3} \mathcal{P}_\mathcal{R}^2 \mathcal{P}_{\mathcal{R}_\phi} m_\phi^2 H^6 \left( \frac{k_\sigma}{\mu} \right)^2. \]

Using the expression of \( \mathcal{P}_{\mathcal{R}_\phi} \) \([16]\), \([23]\) can be further simplified to
\[
    f_{\text{NL}}^{\text{mix}} = \frac{5}{4r} - \frac{5}{3} - \frac{5r}{6} - (1 - r)^3 \frac{10A}{27B^2} \mathcal{P}_{\mathcal{R}_\phi}^2 \mathcal{P}_\mathcal{R} \left( \frac{k_\sigma}{\mu} \right)^2 \left( \frac{\mathcal{P}_{\mathcal{R}_\phi}}{\mathcal{P}_\mathcal{R}} \right)^2, \]

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Figure 2: The local shape of bispectrum produced by the nonlinearity of curvaton. The bispectrum is maximum when either $k_2$ or $k_3$ is comparable to $k_1$, while the other one negligible.

Figure 3: An example of the mixed shape between the local and the false vacuum shapes. The momentum dependence of the bispectrum is of the form $B_R(k_1, k_2, k_3) \sim 20 + \sum_i k_i / \prod_i k_i$, where the former and the latter are the false vacuum and the curvaton contributions, respectively.
where $B = \mathcal{O}(0.1)$, and $m_\phi$ is a free parameter in our scenario. We can get a positive and large $f_{NL}$ by tuning $m^2_\phi \gg H^2$. Note that $f_{NL}^{\text{mix}}$ is strongly scale dependent: the scale dependence is encoded in the fraction $(\mathcal{P}_{\mathcal{R}_\phi}/\mathcal{P}_\mathcal{R})^2$.

Finally, we would like to make some remarks on the aforementioned redefinition of the curvature perturbations. In the calculation, we have ignored the gravitational coupling between $\phi$ and $\sigma$, and assume these two fields are uncorrelated. Actually, they should interact gravitationally so that part of $\delta \sigma$ is induced from $\phi$ via $\mathcal{R}_\phi$ at second order. However, the curvature perturbation from this part of $\delta \sigma$ is proportional to $\mathcal{R}_\phi$, thus can be absorbed into the redefined $\mathcal{R}_\phi$. To be explicit, one can decompose $\mathcal{R}_\sigma$ into two parts,

$$\mathcal{R}_\sigma = \mathcal{R}_\sigma^{(\text{vac})} + \mathcal{R}_\sigma^{(\text{induced})},$$

(25)

where $\mathcal{R}_\sigma^{(\text{vac})}$ originates from the vacuum fluctuation of the curvaton, thus has nearly Gaussian distribution, while $\mathcal{R}_\sigma^{(\text{induced})}$ is induced by the gravitation potential during the false vacuum stage of inflation, thus follows $\chi^2$ distribution. Then the total curvature perturbation can be rewritten as

$$\mathcal{R}_c = (1 - r)\mathcal{R}_\phi + r \mathcal{R}_\sigma$$

$$= (1 - r) \left[ \mathcal{R}_\phi + \frac{r}{1 - r} \mathcal{R}_\sigma^{(\text{induced})} \right] + r \mathcal{R}_\sigma^{(\text{vac})}$$

$$\equiv (1 - r) \tilde{\mathcal{R}}_\phi + r \tilde{\mathcal{R}}_\sigma,$$

(26)

where we have redefined $\tilde{\mathcal{R}}_\phi$ and $\tilde{\mathcal{R}}_\sigma$. After this redefinition, all the calculations presented in this section can be safely applied.

4 Conclusions and discussions

In this paper, we calculated the bispectrum of the curvature perturbation $\mathcal{R}_c$ produced during false vacuum inflation. Because $\mathcal{R}_c$ is intrinsically highly non-Gaussian, the corresponding bispectrum exhibits a novel shape. Using this result, we proposed the “old curvaton” scenario, where the Gaussian contribution of the total curvature perturbation consistent with the current observations is provided by the curvaton. The resulting shape of the bispectrum is a mixed type, i.e. the combination of the novel “false vacuum” and the well known local shapes. In addition to the novel bispectrum, in the old curvaton scenario we also find relatively large running of the spectral index.

In the old curvaton scenario, both the inflaton and the curvaton play key roles in producing observable effects. Moreover, there are possibly thermal components to confine the inflaton at the top of the potential. These components set up interesting connections between our work and some other recent progress on inflation and non-Gaussianity:

- **Quasi-single field inflation**: In the old curvaton scenario, the curvaton field slowly rolls down its potential. One could think of this scenario in an alternative way: to interchange the name of the curvaton and the inflaton, so that inflation is driven by the curvaton field $\sigma$. Then our model describes a slow roll inflation direction, plus another direction along which the field is trapped at the minimum of the potential. In this viewpoint, our model looks
similar to quasi-single field inflation [12]. However, in quasi-single field inflation, the transition from the isocurvature perturbation to the curvature perturbation comes from direct coupling between the two directions. While in the old curvaton scenario, the transition comes from the non-conservation of curvature perturbation after the end of inflation, which is standard in the curvaton scenario. The origin of non-Gaussianity is also different. In quasi-single field inflation, non-Gaussianity originates from the interaction between the scalar fields. While in the old curvaton scenario, the non-Gaussianity originates from the non-linear mapping from the scalar field fluctuation to the curvature perturbation.

- **Thermal non-Gaussianity**: In the old curvaton scenario, the effective potential (3) can be obtained in several ways. A simple way is that the inflaton is trapped at the origin for some $e$-folds by thermal radiations [22]. It is possible that they also have non-Gaussianities [23], which could be transferred to the curvature perturbation by gravitational or other couplings. Here we note this possibility, and at the same time emphasize that this effect is under control in our model. Thus we can turn off these contributions, and our previous calculations give the leading non-Gaussianity. Nevertheless, it would be interesting to see whether there are parameter regions where these thermal effects can affect the running and the non-Gaussianity, as well as the power spectrum and the spectral index [24].

- **Multi-stream inflation**: At the end of false vacuum inflation, the potential (3) becomes unstable. Then, the field could roll towards either $\phi > 0$ or $\phi < 0$ at different causal patches. As discussed in multi-stream inflation [25], these bifurcation does not necessarily lead to disasters, and can rather probably give rise to some interesting observational effects, such as features, non-Gaussianities, and the CMB asymmetries at the bifurcation scale. However, these effects do not change the calculation and conclusion in our present paper, because they are associated with perturbations on smaller scales.

Finally, we would like to mention that the old curvaton scenario and the related works we discussed above all belong to the attempts assuming that the inflationary dynamics is more complicated than the simplest single field slow roll inflation. Recent progress in string theory, especially the string landscape [26] and related cosmology [27], provides a theoretical playground, and also higher a priori expectation for the complicated inflationary dynamics. Hopefully, the old curvaton, together with a great number of other models and mechanisms, can serve as a building block for a realistic description of inflation in the string landscape.

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A \( \mathcal{R}_c \) in terms of \( \phi \) during false vacuum inflation

In this appendix\footnote{Some parts of this section come from Ref. \cite{17}.}, we provide a little more detailed argument on how we obtained (4). We know from Ref. \cite{16} that the gauge invariant energy density perturbation on the comoving hypersurfaces \( \rho \Delta \) is related to the gauge invariant intrinsic spatial curvature perturbation \( \Phi \) in the longitudinal gauge by a Poisson-like equation,

\[
\rho \Delta = -2m_{\text{Pl}}^2 (H\eta)^2 \nabla^2 \Phi . \tag{27}
\]

To extract the functional behaviour of \( \Phi \), we can notice that from the energy density correlation function,

\[
D(x, x') = 4m_{\text{Pl}}^4 (H\eta)^4 \langle \nabla_x^4 \Phi(x) \nabla_{x'}^4 \Phi(x') \rangle \sim \frac{e^{-2\mu r}}{r^3} , \tag{28}
\]

so that \( \Phi \) has an exponential factor \( e^{-\mu r} \). Since we are interested in the super-horizon separations, the Laplacian operator primarily picks the terms with the least power of \( r \) in the denominator. Thus we can approximate

\[
\nabla^2 \Phi \approx \mu^2 \Phi . \tag{29}
\]

Further, \( \Phi \) can be written in terms of the comoving curvature perturbation \( \mathcal{R}_c \) as

\[
\Phi = -\frac{A}{32\pi^2 m_{\text{Pl}}^2} (\mu \eta)^2 \mathcal{R}_c = -\frac{\langle \rho + p \rangle_{\text{ren}}}{2m_{\text{Pl}}^2 H^2} \mathcal{R}_c , \tag{30}
\]

where \( A \) is a constant of \( O(1) \), and \( \langle \rho + p \rangle_{\text{ren}} \) is the “renormalized” expectation value of \( \rho + p \), given by

\[
\langle \rho + p \rangle_{\text{ren}} = A \frac{H^4}{16\pi^2} \frac{m_{\text{Pl}}^2}{H^2} (\mu \eta)^2 . \tag{31}
\]

Therefore, combining (29) and (30), from (27) we can write \( \mathcal{R}_c \) in terms of the density perturbation \( \rho \Delta \) as

\[
\mathcal{R}_c \approx \frac{\rho \Delta}{\langle \rho + p \rangle_{\text{ren}} (\mu \eta)^{-2}} . \tag{32}
\]

We can move to the Fourier space more conveniently by noticing that for \( D(x, x') \) the most significant contributions come from the terms which do not contain any time derivative. This means,

\[
\nabla^2 (\rho \Delta) = \nabla^2 \left[ \frac{\dot{\phi}^2}{2} + \frac{(\nabla \phi)^2}{2a^2} + V(\phi) \right] + 3H\dot{\phi} \nabla^2 \phi \approx \nabla^2 \left[ \frac{(\nabla \phi)^2}{2a^2} + V(\phi) \right] . \tag{33}
\]

Further, as we are interested in the false vacuum inflation stage where \( m_{\text{eff}}^2 \) is positive, \( 1/a^2 \) term in (33) completely dominates the potential. Thus, after all, we have

\[
\rho \Delta \approx \frac{1}{2a^2} \left[ (\nabla \phi)^2 + \mu^2 \phi^2 \right] . \tag{34}
\]
Decomposing $\phi$ in terms of the Fourier mode

$$
\phi(x) = \int \frac{d^3k}{(2\pi)^3} e^{i k \cdot x} \phi_k(\eta),
$$

we can easily find the Fourier component of $\rho\Delta$ as

$$
(\rho\Delta)_k = \frac{1}{2a^2} \int \frac{d^3q}{(2\pi)^3} \phi_q \phi_{|k-q|} [\mu^2 + q \cdot (q - k)] .
$$

Substituting (36) into (32), we can obtain (4).

Before we finish, we note that since the potential is quadratic, we can promote $\phi$ as a quantum harmonic oscillator and expand the Fourier mode $\phi_k$ in terms of the annihilation and creation operators, $a_k$ and $a_k^\dagger$ respectively, as

$$
\phi(x) = \int \frac{d^3k}{(2\pi)^3} e^{i k \cdot x} \phi_k(\eta) = \int \frac{d^3k}{(2\pi)^3} e^{i k \cdot x} \left[ a_k \phi_k(\eta) + a_k^\dagger \phi_k^*(\eta) \right],
$$

where $a_k$ and $a_k^\dagger$ satisfy the canonical commutation relation

$$
[a_k, a_q^\dagger] = (2\pi)^3 \delta^{(3)}(k - q) .
$$

The solution of the mode function $\phi_k(\eta)$ satisfies the asymptotic behaviour (9) in the limit $|\eta| \gg 1$.

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