Experimental determining of horizontal movements of the water tower reservoirs by GB-RAR

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Abstract. Ground-based radar interferometry (GBRI) with ground-based real aperture radar (GB-RAR) is most often used for monitoring vertical deflections of bridge structures caused by vehicle passages. This paper presents an experimental determining of the horizontal dynamic movements of water tower reservoirs by GB-RAR. Determining the dynamic movements of water tower reservoirs is more complicated precisely because the movement of the reservoir is influenced not only by external influences, such as wind, but also by the movement of water mass in the reservoir. The resulting oscillation is then a composite oscillation of multiple frequencies. Next, in the case of routine determination of vertical deflections of bridge structures, it is reasonable to assume a predominant deflection of the structure in this one particular direction. But in the case of tower structures such as reservoirs, it is necessary to assume their movements (oscillations) in the entire horizontal plane. The movements can be circular, elliptical, straight, spiral, or even completely irregular. This means using at least two radars to simultaneously determine 2D movements (in both perpendicular directions of the horizontal plane). In the optimal case, the radars aim at the monitored object in approximately perpendicular directions to each other, and the resulting motion vectors in the horizontal plane are calculated from LOS measurements. The processing of measurements from both radars raises other problems, namely accurate time synchronization of radar measurements. In case of tower structures, time synchronization cannot be solved by coincidence of oscillation amplitude peaks, since the peaks from different radar views may not occur simultaneously. Therefore, alternative solution is offered in this contribution. Purpose of this contribution is to design and verify a procedure for accurate determination of horizontal movements of tower reservoirs with sufficiently accurate oscillation characteristics. The procedure was experimentally verified in practice on a real water reservoir in central Bohemia. The results of the experiment confirm the expected benefits of simultaneous measurements by two radars for determining horizontal dynamic movements of water tower reservoirs by GB-RAR.

1. Introduction

Over the last 10 years, ground-based radar interferometry (GBRI) with ground-based real aperture radar (GB-RAR) has become a frequently used technology for determining the dynamic deflections of various types of building structures. This method has the ability to measure real-time deflections / movements at normal operating conditions. Furthermore, it can dynamically capture and detect frequency and amplitude of vibration of the monitored object in the frequency range from 0.05 to 50 Hz (when measuring at shorter distances up to 250Hz). This method has ability to determine the deflection size with precision better than 0.1mm (up to several microns). Deflections of a monitored object can be simultaneously determined at multiple locations. So it is possible to obtain both general
and detailed information on the behaviour of the structure under its dynamic load. For example, on a bridge of length of 100 m it is possible simultaneously monitor up to about 100 points.

GBRI method is most often used for monitoring dynamical vertical deflections of bridge structures caused by vehicle passages. Nevertheless, it is also often used to monitor movements of buildings, ancient towers, chimneys or wind farms. A review with the systematic survey of some older case studies reported in the literature until 2013 is given in [1].

Basic principles and examples application of GB-RAR technology to determine deflection of bridges are given in recent literature, e.g. in [2] and [3]. An example of using GB-RAR technology to determine the deflections of metal rail bridge constructions caused by both temperature changes and vehicle passages (dynamical loads) is given in [4]. Examples of monitoring communications towers, ancient towers and high-rise buildings are given in [5], [6], [7], [8], and [9]. Monitoring of factory chimneys and wind power plant pylons are given in [10] and [11].

Combinations of multiple methods are also often used. Joint use of terrestrial laser scanning (TLS), configured in line scanner mode, and GB-RAR technology for determination of vibration frequencies and oscillation amplitudes of tall structures is given in [12]. Joint use of a GB-RAR and accelerometers for monitoring of ancient towers is given in [13], and [14]. Utilisation of satellite InSAR (Synthetic Aperture Radar Interferometry) method to study coal mining areas and areas with groundwater withdrawal for monitoring transmission power lines and their towers is given in [15] and [16]. A review of Global Navigation Satellite System-based positioning technology for structural health monitoring including e.g. case study of a chimney is given in [17].

This contribution is focused on measurement of dynamical horizontal movements of water tower reservoirs (tanks) by GB-RAR method. A photo of such monitored reservoir is shown in Figure 1 together with a typical measurement diagram. The measurement was done by two IBIS-FS interferometric radars of the Italian manufacturer Ingegneria Dei Sistemi. More details about this instrument are presented e.g. in [18]. From the above listed literature, a similar topic is mentioned only in [10].

![Figure 1](image-url)  

**Figure 1.** Line of sight movement ($d_{LOS}$) and horizontal movement ($d$) in the direction of the radar.
2. Basic principles of determining movements in horizontal plane

One of the basic shortcomings of the GB-RAR method is that the radar measures displacements (movements) in the line of sight (LOS) only. These LOS displacements are recalculated into displacements in some expected direction based on the assumption of the predominant deflection of the structure in this one particular direction. In the case of bridges, the expected direction is usually vertical, in case of tower structures, the expected direction is horizontal. Standard geometrical situation of measurement of a water tower reservoir by just one radar is shown in Figure 1. The horizontal displacement \( d \) in the direction of the radar sight, can be easily calculated from the LOS displacement \( d_{\text{LOS}} \) by means formula (1)

\[
d = d_{\text{LOS}} \cdot \frac{R}{H}
\]

where \( R \) is straight distance of the measured point from the radar and \( H \) stands for the horizontal distance of the radar from the measured point.

Nevertheless, in the case of tower structures such as reservoirs or chimneys, it is necessary to consider their movements (oscillations) as two-dimensional in horizontal plane and not only in the direction of the radar sight. The movements can be circular, elliptical, straight, spiral, or even very often completely irregular. This means using at least two radars to simultaneously determine 2D movements in horizontal plane and thus reveal the actual movement of the tower structure. In the optimal case, the radars aim at the monitored object in approximately perpendicular directions to each other, and the resulting displacement vectors in the horizontal plane are determined by a suitable calculation. Simultaneous measurements by two radars are not mentioned in the commonly available professional literature. Only the principle of calculation of real displacements when measuring with two radars is given in [19]. The topic of time synchronization of measurements, which is crucial for correct calculation of real displacements is not mentioned there.

In addition to the already mentioned limitation resulting from the radar's ability to measure only LOS displacements, certain other principles of use need to be followed in order to successfully implement this technique in practice. These policies are listed e.g. in [4] and [10].

Simultaneous measurement by two radars brings several technical problems that need to be solved. It is mainly determining spatial configuration of radars and the measured object (in our case the water tower reservoir), enabling calculation of real displacements. Furthermore, it is the time synchronization of measurements of both radars.

2.1. Method of calculation of horizontal displacements

If we assume that the monitored object moves in the horizontal plane, it is possible to determine the actual horizontal displacements and their individual vector components by simultaneous measurement with two radars. Figure 2 shows the geometric relationships between the horizontal displacements in the directions of the radars \( d_1, d_2 \) and the actual horizontal displacements \( P = [s_X, s_Y] \) of the point \( P \) when measured from two different radar positions. The components of the displacement vector of the monitored point on the reservoir can be calculated according to formulas (2).

\[
\begin{align*}
    s_X &= \frac{d_1}{\sin(\psi)} \\
    s_Y &= \frac{-d_1 \cos(\psi) + d_2}{\sin(\psi)}
\end{align*}
\]

Where:

- \( d_1, d_2 \) … horizontal displacements in the directions of the radars sight,
- \( \psi \) … horizontal angle between radars,
- \( s_X, s_Y \) … cartesian coordinates of the point \( P \) (actual horizontal displacement vector).
The coordinate system used for the components of the vector of the actual horizontal displacement, has the X axis in the direction of the radar R₁ and the Y axis perpendicular to it. The origin is in the initial position of the monitored point P.

Figure 2. Relationship between horizontal displacements $d_1$, $d_2$ and $s_x$, $s_y$ in the Cartesian coordinate system. Points $R_1$, $R_2$ indicate the positions of radars.

In this way, it is possible to determine the actual (real) horizontal displacements of the monitored point in the whole horizontal plane. However, the condition is that the displacements $d_1$ and $d_2$ are related to the same point P, i.e. measured (determined) at the same time.

2.2. Time synchronization of two radars

When measuring with two radars, there is a practical problem how to recognize displacements measured at the same time in two different time series of the acquired LOS (hence $d_1$ and $d_2$) displacements. Therefore, the time series have to be synchronized to find time correspondence of the acquired LOS displacements. If the measurement is performed with sampling rate e.g. 200Hz, then the synchronization must be performed with the appropriate accuracy, i.e. ± 0.0025s.

One possible solution of the synchronization is based on identification of maximum deflections values (displacements) in the both time series. It is assumed that the positions of these maximums correspond to the same moments of their acquisitions. Thus, the synchronization could be performed simply as a time shift obtained after the oscillation amplitude peaks of the both time series have been identified. However, in the case of horizontal displacements of tower structures, the values of LOS displacements obtained in both time series will almost certainly not reach their maximums at the same moments. This is due to the position of radars directions of which are approximately mutually perpendicular. This case would occur only when the tower oscillates in a straight line back and forth. Such a case is almost unrealistic in practice. Therefore, solving time synchronization by means of coincidence of maximums is not possible in the case of tower structures.

Due to the above disadvantage, another method of synchronization was designed. This method utilizes system times of operating laptops that control measurement processes of the radars and on which the measured data are stored. Synchronizing the two radars therefore means synchronizing the system time of their operating laptops. The radar operating software obtains the exact time from the laptop's operating system and saves the measurement start time in a file with the measured values from the radar. By comparing the time data stored in the measurement files, it is therefore possible to identify the measured values that were taken at the same time.
3. Experimental case study determining horizontal movements of the water tower reservoir

Determining the dynamic horizontal movements of water tower reservoirs is more complicated precisely because the movement of the reservoir is influenced not only by external influences, such as wind, but also by the movement of water mass inside the reservoir. The resulting oscillation will be then a composite oscillation of multiple frequencies. For this reason, a water tower reservoir was chosen for the case study of determining the horizontal movements of tower structures. The chosen tower reservoir is located in the village of Zeleneč, northeast of Prague, Czechia. The tower reservoir is 30 m high and is shown in Figure 1. The support tube is made of steel and the ring welds on it can be used as natural reflectors of the radar signal. It is thus possible to simultaneously measure the horizontal movements of almost the entire tube at different heights, as shown in the diagram in Figure 1.

3.1. Performing measurements

The measurement was performed on August 12, 2020 in the period from 3:02 pm to 3:47 pm. Climatic conditions during the measurement: temperature 30.1°C - 30.9°C, humidity 33 - 37%, wind speed 0 - 1 m/s in a variable direction, mostly northern. Two Radars IBIS Italian manufacturer *Ingegneria Dei Sistemi* (IDS) were used for the measurements. Radar R1: IDS Radar IBIS – FS Plus, radar R2: IDS Radar IBIS – RU 172. In both cases, IBIS-ANT3-H17V15 antennas were used. See Table 1 for more details on radar settings. The local situation was mapped by a detailed terrestrial laser scan, and then a spatial (geometric) 3D model shown in Figure 3 was created. The model contains not only the tower reservoir itself, but also the positions of both radars.

![Figure 3](image_url)

*Figure 3.* left: 3D model of the local situation including the tower reservoir, positions of both radars and highlighting of Rbins selected for evaluation, right: identification of individual ring welds (natural reflectors) together with selected Rbins, welds selected for evaluation are indicated by an arrow.
Table 1. Settings of used radars.

|                     | R₁: IDS Radar IBIS – FS Plus | R₂: IDS Radar IBIS – RU 172 |
|---------------------|------------------------------|-----------------------------|
| **Sampling frequency** | 200 Hz                       | 199.2 Hz *                 |
| **Signal range (max. distance)** | 35 m                     | 35 m                      |
| **Rbin (range resolution area)** | 0.75 m                   | 0.75 m                |

*The set value was 200 Hz, the actual value (corrected by the radar control software) was 199.2 Hz.

3.2. Processing and evaluation

Using the spatial 3D model shown in Figure 3, individual Rbins and ring welds on the tube were identified. The ring welds here serve as natural reflectors of the radar signal. Subsequently, significant maxims were selected on the range profiles (SNR profiles) see Figure 4. These selected maxims determine the selection of Rbins for evaluation. These are highlighted in color in the 3D model in Figure 3. For these selected Rbins, the horizontal displacements $d_1$ and $d_2$ in the directions of the radars were computed according to equation (1). These displacements are shown in Figure 5. For clarity, only a short period of 75s is shown.

\[ \text{displacement} = \text{maxim} \]

**Figure 4.** Selected significant maxims on the SNR profiles of the radars R₁ (left) and R₂ (right).
Figure 5. Horizontal displacements $d_1$ and $d_2$ in the directions of the radars $R_1$ and $R_2$, for the selected Rbins and during time period of 75s.

It is clear from the graphs that as the height of the observed Rbin decreases, the magnitude of the displacements of these Rbins decreases too. At height of 22.5 m, the tower oscillates in the range of ± 1.5 mm, while at height of 11 m only in the range of ± 0.5 mm. This observation confirms expected known fact that at higher relative height the tower oscillates more. Furthermore, it is clear that oscillations are a composition of multiple frequencies because some cycles contain multiple local maximum deviations. Therefore, a frequency analysis was performed using a discrete Fourier transform (DFT) - see e.g. [20]. The result of the analysis is the periodogram in Figure 6. Three main frequencies of the tower reservoir oscillations were detected: 0.28 Hz, 0.52 Hz and 0.64Hz.

Figure 6. The resulting periodogram - detected three main frequencies of the tower reservoir: 0.28 Hz, 0.52 Hz, 0.64Hz.
The determination of the actual displacements was performed according to formulas (2). This determined the components $s_X$ and $s_Y$ of the actual horizontal displacement vectors. The plan view of the radars positions configuration is shown in Figure 7. The angle $\psi$ made by the radars was 88°.

![Figure 7](image)

**Figure 7.** The plan view of the radars positions configuration during measurements.

The resulting actual horizontal displacements during time period of 75s at the highest Rbins on the reservoir tube ($R_1$ Rbin 35 and $R_2$ Rbin 37) are shown in Figure 8. It can be seen that this is an irregular movement probably caused by the folding of the three detected main oscillation frequencies.

![Figure 8](image)

**Figure 8.** The plan view of the resulting actual horizontal displacements during time period of 75s at the highest Rbins on the reservoir tube ($R_1$ Rbin 35 and $R_2$ Rbin 37).
4. Results and discussions

Using a band-pass filter (BPF) - see e.g. [21], the displacements were divided into 3 sections with the following frequency intervals: 0.2-0.4 Hz, 0.4-0.6 Hz and 0.6-0.7 Hz. The resulting partial horizontal displacements caused by oscillations at individual frequency intervals can be seen in Figures 9 and 10. For greater clarity, Figure 11 shows the total displacements and Figures 12-14 show the partial displacements after their individual cycles.

Figure 9. The plan view of the resulting partial horizontal displacements caused by oscillations at individual frequency intervals: (from left) 0.2-0.4 Hz, 0.4-0.6 Hz and 0.6-0.7 Hz. The displacements are displayed at the same scale to highlight their partial effect on the total resulting displacements.

Figure 10. The plan view of the resulting partial horizontal displacements caused by oscillations at individual frequency intervals: 0.4-0.6 Hz (left) and 0.6-0.7 Hz (right) shown for better clarity at various scales.

It is clear from the graphs that the partial movements have the character of smooth curves resembling changing ellipses. And it is the composition of these partial smooth movements that creates an irregular final movement. The oscillation at the lowest main frequency of 0.28 Hz contributes the most to the resulting movements. The oscillation at the highest frequency of 0.64 Hz contributes the least. This confirms the results obtained by the frequency analysis (DFT) shown in the periodogram in Figure 6. There was almost no wind during the measurement, so the resulting horizontal displacements were in the range of only 3 mm. However, this made it possible to better examine the effect of composing partial movements on the resulting movement.
Figure 11. The plan view of the total resulting actual horizontal displacements during time period of 9 cycles (32.13s) at the highest Rbins on the reservoir's tube (R₁, Rbin 35 and R₂, Rbin 37). For greater clarity, the display of these displacements is divided into individual cycles of 3.57s. It is a visualization of the irregular movement caused by the composition of three partial movements on three main frequencies which are shown below.
Figure 12. The plan view of the partial resulting actual horizontal displacements caused by oscillations at frequency interval 0.2-0.4 Hz, during time period of 9 cycles (32.13s), at the highest Rbins on the reservoir's tube (R₁ Rbin 35 and R₂ Rbin 37). For greater clarity, the display of these displacements is divided into individual cycles of 3.57s. It is a visualization of the partial movement with the character of a smooth curve, which contributes the most to the resulting irregular movement.
Figure 13. The plan view of the partial resulting actual horizontal displacements caused by oscillations at frequency interval 0.4-0.6 Hz, during time period of 9 cycles (17.37s), at the highest Rbins on the reservoir's tube (R1 Rbin 35 and R2 Rbin 37). For greater clarity, the display of these displacements is divided into individual cycles of 1.93s and next a larger scale is used. It is a visualization of the partial movement with the character of a smooth curve, which also contributes to the resulting irregular movement.
Figure 14. The plan view of the partial resulting actual horizontal displacements caused by oscillations at frequency interval $0.6-0.7$ Hz, during time period of 9 cycles (14.04s), at the highest Rbins on the reservoir’s tube ($R_1$ Rbin 35 and $R_2$ Rbin 37). For greater clarity, the display of these displacements is divided into individual cycles of 1.56s and next a larger scale is used. It is a visualization of the partial movement with the character of a smooth curve, which contributes the least to the resulting irregular movement.

5. Conclusions
GBRI measurements were used for experimental determination of horizontal movements of the water tower reservoirs (tanks). The measurement of the tower reservoir took place in the village of Zeleneč, northeast of Prague, Czechia. The key was using two GB-RAR radars and precise time synchronization of their simultaneous measurements. The reason for using at least two radars is that the monitored object moves in horizontal plane and not only in one horizontal direction. It was
confirmed that the accuracy of measurements in a field practice is sufficient to detect the main oscillating frequencies, which together contribute to the resulting irregular movement of the tower reservoir. The resulting movement, together with the partial movements at the individual main oscillation frequencies, can be very well documented. The results thus confirm the expected advantages of simultaneous measurement by two radars for determining the horizontal dynamic movements of tower reservoirs using GB-RAR.

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