Testing new strategies in finite density

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A new approach for non zero chemical potential simulations is tested in the Gross-Neveu model for infinite flavor number, where the critical line is reconstructed in a large $\mu/T$ interval. A comparison with results from standard imaginary chemical potential approach as well as first results for $N_f = 4$ QCD are presented.

Recently we proposed a new method for simulate systems at non zero chemical potential \cite{1}. The starting point is to note that the temporal part of the Dirac operator (the one that contains the $\mu$ dependence) can be written as the sum of two contributions: the first, proportional to $\cosh(\mu)$, multiplies an antihermitian operator, the second, proportional to $\sinh(\mu)$, multiplies an hermitian operator. Since the spacial part of the fermionic operator is antihermitian too, we see that for non zero $\mu$ the Dirac matrix looses its global antihermiticity and this is the origin of the complex action problem.

Clearly the imaginary chemical potential solves the problem: $\sinh(\mu)$ gets an extra factor $i$ and converts the corresponding operator to an antiermitian one. Introducing an imaginary chemical potential we look for a critical line in the $(\beta, i\mu)$ plane to analytically continue in the $(\beta, \mu)$ one: in doing that we are forced to consider larger values of the physical temperature $T$ than the ones of physical relevance.

In our proposal we promote $\cosh(\mu)$ and $\sinh(\mu)$ to independent variables $x$ and $y$ and explore the phase structure as a function of such variables to recover the physical critical point along the line that corresponds to $x = \cosh(\mu)$, $y = \sinh(\mu)$. To do that we perform standard (i.e. with real and positive action) simulations at pure imaginary $y$ and extend the results to real $y$. This can be achieved in two different ways \cite{1}.

The first is to fix the inverse gauge coupling and (i) find the critical line in the $(x, iy)$ plane, (ii) fit this line with an even polynomial of $y$, (iii) analytically continue the critical line to the $(x, y)$ plane, (iv) from the intersection of the analytically continued critical line with the $x^2 - y^2 = 1$ line (which corresponds to real and positive values of $\mu$) it is possible to read the critical value of $\mu$.

Working at fixed lattice temporal extent $(N_t)$ this is equivalent to do simulations at fixed $T$ i.e. the same physical temperature of the physical critical point.

The second approach is to choose a value for $\mu$ and fix the $x$ variable to $\cosh(\mu)$. By standard simulations we find the critical line in the $(\beta, iy)$ plane and proceed in close analogy with previous case. Only point (iv) changes: to determine the critical value of $\beta$ at given $\mu$ we look for the intersection of the analytically continued critical line with $y = \sinh(\mu)$. In this case, for a given $N_t$, this is equivalent to search for the transition point doing simulations at fixed $\mu/T$.

Comparing the standard imaginary chemical potential approach \cite{2} and our approach(es) we can note that in both case we have to rely on an analytical continuation of the critical line (with all the uncontrolled systematics that this procedure can in principle introduce).

The simulation cost of our method(s) is bigger since we need to explore a two dimensional space.
1.05
0.15
0.25
0.35
0.4
0
0.05
0.1
0.15
0.2
0.25
0.3
0.35
0.4
0.5
0.6
0.7
0.8
0.9
1
1.1
1.15
1.2
1.25
1.3
1.35
1.4
1.45
1.5
1.55
1.6
1.65
1.7
1.75
1.8
1.9
2

Figure 1. G-N model: analytical continuation of the critical line from imaginary to real $y$ and determination of the critical $\mu$.

The above actions agree with the standard one for $x = \cosh(\mu)$, $y = \sinh(\mu)$. In the $N_f \to \infty$ limit the generalized model can be analytically solved by the gap equation

$$\Sigma = -g^2 \langle \bar{\psi} \psi \rangle \quad (1)$$

where $\Sigma$ is the expectation value of the scalar field.

We used Fig. 1 in the infinite spacial volume limit to find the critical line in the $(x, iy)$ and $(\beta, iy)$ planes and then apply our procedures to extract the critical line as a function of $y$. These results can be directly compared with the exact ones we can extract from the gap equation in order to check, in a non trivial case, the potentialities of our approach. The final goal is to reconstruct the critical line in the $(\beta, \mu)$ plane or, using adimensional variables, in the $(\mu/\Sigma_0, T/\Sigma_0)$ one where $\Sigma_0$ is the infinite volume chiral condensate at $\mu = 0$.

For $N_t = 4$ Fig. 1 is an example of how the method works. The points are the exact gap equation solution for the critical line (both for real and imaginary $y$) and a two parameter quadratic fit is satisfactory (upper continuous
Figure 3. G-N model: comparison of our results from $(x, iy)$ and $(\beta, iy)$ analysis and exact result in the $\mu/\Sigma_0 - T/\Sigma_0$ plane.

Figure 4. As in fig. 1: determination of the critical $\mu$ for $N_f = 4$ QCD (the symbols are the critical points at imaginary $y$).

The analytical continuation to real $y$ gives the decreasing continuous line and its intersection with the $x^2 - y^2 = 1$ line allows to determine the critical point. Fig. 2 summarizes the results in the $(\beta, \mu)$ plane compared with the exact ones as well as with the imaginary chemical potential results based on a quadratic approximation of the critical line.

We get similar results in the $(\beta, y)$ plane: the results are reported in fig. 3 and compared with the $(x, y)$ approach for the $N_t = 8$ case. Results from $(x, iy)$ plane are closer to the exact solution respect to $(\beta, iy)$ ones and, when comparing with the standard imaginary chemical potential approach (considering in both cases an analytical continuation based on a quadratic two parameter fit), we see a clear improvement when the critical chemical potential is large (see fig. 2). Indeed we pick up the correct result up to $\mu/T \sim 8$. From considering different $N_t$ values we realize that the quality of our results seems to decrease increasing $N_t$ and/or increasing the critical value of the chemical potential. We considered up to $N_t = 12$ (far beyond any optimistic prevision for large scale simulations in finite density QCD in the next years) and, up to this value, the quantitative agreement is good.

We have the first results for $N_f = 4$ full QCD in a $8^3 \times 4$ lattice at $m_q = 0.06$. The critical point is located from peaks in the susceptibility of plaquette, Polyakov loop or chiral condensate (they all agree). Fig. 4 shows the determination of the critical $\mu$ based on continuation to real $y$ of a quadratic fit to the critical line in the $(x, iy)$ plane at $\beta = 5.05$. A consistent result is obtained from an independent analysis carried out from a different data set based on numerical simulations at fixed $\mu$ to determine the $(\beta, iy)$ critical line.

REFERENCES

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