Inclusive One Jet Production With Multiple Interactions in the Regge Limit of pQCD

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Abstract. DIS on a two nucleon system in the regge limit is considered. In this framework a review is given of a pQCD approach for the computation of the corrections to the inclusive one jet production cross section at finite number of colors and discuss the general results.

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INTRODUCTION

After experiments at HERA put in evidence the possibility of having kinematical regions characterized by high density hadronic matter, more investigations have been undertaken with nuclei at RHIC. Signals of high density effects will appear also in proton proton collision at LHC in the deep forward region. Typically tagging a not too hard jet in such a situation one expects multiple interaction between the emitted jet and one proton, which are typically classified as higher twist effects but which may be not negligible, power suppression being compensated by the resummation of some large energy logs. Such a situation can be somewhat more easily understood in the framework of photon-deuton scattering considering, due to the particular kinematics, small $x$ resummation techniques developed in a series of works after the pioneering BFKL [1] approach. The study of multiple interaction both in total or jet inclusive cross section is also of great theoretical interest. In particular the computation requires to include some ingredients which are also needed to restore the unitarity, badly spoiled by too crude approximations (as e.g. the violation of the Froissart bound). In this analysis one also meets the need to discuss the AGK [2] rules in the context of pQCD which are deeply interconnected to the inclusive one jet analysis [3, 4].

I will first review an approach developed to study the total cross section case and after that describe how to procedure to compute the cross section when one gluon jet is fixed at some rapidity. The results obtained should be compared with other approaches [5, 6, 7, 8, 9].

TOTAL CROSS SECTION

The total cross section for deep inelastic scattering on a nucleus consisting of two weakly bound nucleons is constructed from the imaginary part of the corresponding amplitude. Following the discussion in [2], there are three contributions illustrated in the left part in Fig.1. They are usually referred to as 'diffractive cut' (Fig.1a), 'single cut' (Fig.1b),
and 'double cut' (Fig. 1c). The total cross section is obtained from the sum of these terms plus some others obtained by permutations among the nucleons.

In order to make explicit calculations it is convenient to introduce the light cone momenta $q'$ and $p$ constructed from the external kinematical variables and use the Sudakov decomposition for any momenta $k_i = \alpha_i q' + \beta_i p + k_{i\perp}$ so that gluonic subamplitudes depend on the Sudakov parameters $\beta_i, k_{i\perp}$. In our case subamplitudes depends on 4 gluons and therefore on $\beta_1, \beta_2$ and $\beta_3$. The integration in these variables always involve a path associated to the discontinuity related to produced particles while the other two path are real. The gauge invariance leads to a good behavior on the $\beta_i$ planes at infinity and deforming the contours one can show that in all situations of Fig.1 after integration one obtains the same function of transverse momenta $N_4(k_{1\perp}, k_{2\perp}, k_{3\perp}, k_{4\perp})$. This fact leads to the fulfillment of the AGK rules so that the in the high energy limit the correction due to double interaction to the cross section are negative and constructed from Fig.1 using different weights: $-1 = +1 - 4 + 2$. Another useful fact is that the single discontinuity amplitudes can be constructed from the triple discontinuity ones \[4\]. Therefore just providing the correct phases one can define the total cross section from one triple discontinuity calculation.

The triple discontinuity which is proportional to the total cross section is defined by a set of coupled Bethe-Salpeter evolution equations \[10, 11\] for two, three and four reggeized gluon subamplitudes together with a set of initial conditions. The solution of these equations can be decomposed in a reducible reggeized part and a term satisfying Ward identities, characterized by an effective 2-to-4 conformal covariant transition vertex and a 4-gluon Green’s function, which may be exactly constructed only in the large $N_c$ approximation, when integrability appears. In the nuclei for large $N_c$ the Green functions factorize in two BFKL pomeron Green’s functions and in the Möbius representation the effective vertex provides a triple pomeron interaction \[11, 12\] which is mostly studied in the from given in the non linear BK equation \[13\].

**ONE JET INCLUSIVE CROSS SECTION**

Let us now consider the one jet inclusive cross section. When a jet is fixed in its rapidity and transverse momentum, among the produced gluons in the generalized LLA approximation, one faces a new situation which is depicted in the right part of Fig. 1. One can show that, since the situation is less inclusive compared to the previous case, after the integration in the $\beta_i$ Sudakov parameters the subamplitudes are no more fully symmetric but still exhibit a symmetry on both sides of the cut. This means that the three
subamplitudes corresponding to Fig.1a-c on the right side are different. The inclusive cross section is therefore constructed summing the different contribution each with a suitable phase factor and no factorization is possible as before. The good news is that it is still possible to use the triple discontinuities to reconstruct the three subamplitudes.

The task of computing the three different triple discontinuities is accomplished generalizing the approach for the total cross section case. Again one can write three coupled Bethe-Salpeter equations \([4]\) with suitable initial conditions at the rapidity point where the jet is emitted. Such conditions can be written in terms of the subamplitudes encountered in the total cross section analysis and of gluon production operators.

The subamplitudes \(iZ_n\) which solve the coupled system of equations, \(i\) denoting the position of the cut and \(n\) the number of reggeized gluons, are written as a sum of a reggeized part (constructed from a linear combination of solutions with a number of reggeized gluons \(< n\)) and a irreducible part which satisfies Ward identities, have correct symmetry properties and define the effective vertices followed by Green’s function evolution:

\[
iZ_n = iZ^R_n + iZ^I_n
\]  

The analysis of the equations becomes non trivial already at the three gluon level. Indeed complete reggeization breaks down and considering, due to azimuthal symmetry in the jet momentum, an even function of the latter, one can show that new terms beyond the reggeized ones do appear, nevertheless containing effective 2-to-3 transition vertices which nicely satisfy Ward identities. These vertices, denoted \(i\mathcal{G}_3\) in \([4]\), are not present in total cross sections, in agreement with signature conservation.

The four gluon subamplitudes can be analyzed following the decomposition above. The reggeizing terms are of different kinds: there are terms similar to the total cross section case, the jet being emitted at a rung of the BFKL pomeron ladder, wherein the presence of the jet is not disturbing reggeization. There is also a new term where the jet emission is taking place in the effective vertices \(i\mathcal{G}_3\) followed by a rapidity evolution governed by three gluon pomeron states. Such Pomeron evolution is dual to the odderon evolution \([14]\).

Among the irreducible terms one finds that two of them are the ones naively expected, the jet being emitted again along a BFKL pomeron ladder or in a rung of the interacting 4 reggeized gluons system corresponding to the Green’s function evolution. The remaining two terms are less trivial. One can be interpreted as characterized by jet emission from an effective production vertex, denoted \(i\mathcal{V}_4\) in \([4]\). This vertex again has nice symmetry properties and satisfies Ward Identities, which are crucial to pass eventually to a Möbius representation \([15]\). Finally the last term is characterized by the jet emission from the effective vertex \(i\mathcal{G}_3\), followed by an evolution of the reggeized gluons which at some point split by means of a new disconnected effective vertex in a 4 gluon state. The latter \(3 \rightarrow 4\) effective vertex satisfies clearly Ward identities and has the same symmetry properties. It has been denoted by \(i\mathcal{W}_4\) and its expression may be found in \([4]\). We give here pictorially the structure of the solution:
\[
\langle Z_4 \rangle = \sum \left( \langle X \rangle + \langle X \rangle + \langle X \rangle + \langle X \rangle \right) + \langle \langle X \rangle \rangle + \langle \langle X \rangle \rangle + \ldots \ .
\] (2)

We remind that the reggeized terms have a clear physical meaning, they play the role of higher order correlators inside the target.

At this stage the different single cut subamplitudes present a pattern of contribution which is similar to what has appeared in other computations, apart from a new term which was the latter described. In order to obtain a physical one jet inclusive cross section the different subamplitudes should be added, each multiplied with a suitable phase factor, as described in details in [4]. After that to compare explicitly the result with the other approaches and see if the latter term with the jet emitted at the \(2 \rightarrow 3\) effective vertex is physically present in the one jet inclusive cross section, or is only relevant for the different multiplicities in the final state, one should pass to the Möbius representation. After that, as it was the case for the comparison to the dipole picture [16, 17], one expects huge simplifications in the long operatorial expressions. This work will be carried on soon.

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