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Gender differences in the volatility of work hours and labor demand

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Abstract

This paper examines the role of heterogeneity in a real business cycle model, which traditionally has not fully captured the relative volatility of hours to output. Men and women have different cyclical volatilities in hours worked, which is robust to different filtering methods. This empirical regularity is used to motivate a standard RBC model augmented to allow for two different agents following Jaimovich et al. (2013). These two agents have identical utility functions, but face different elasticities of labor demand due to their different complementarities with capital. These estimated elasticities find that women are more complementary to capital. The calibrated model generates the cyclical volatility of work hours by gender and for the total hours worked that matches the U.S. data better than the traditional representative agent model. I then explore other extensions to this model including investigating the stability of the estimated labor demand elasticities and allowing for various Frisch elasticities of labor supply. This paper demonstrates that allowing for even broad levels of heterogeneity in a simple framework can increase the model's tractability with the data. Since gender is important to explain U.S. business cycle dynamics, we need to carefully consider heterogeneity when analyzing counter-cyclical economic policy, as it may not have symmetric effects across assorted groups.

1. Introduction

The 20th Century saw a large influx of women into the labor force for both supply and demand reasons. There is a deep literature identifying the causes for this change in trend including lack of male employees during WWII (Acemoglu et al., 2004), decrease labor intensity of home production due to increased technology and decreased price of technology (Greenwood et al., 2005), decrease child care costs and decrease of the gender wage gap (Attanasio et al., 2008), and increase of schooling and wages (Eckstein and Lifshitz, 2011). There have been previous models focusing on gender difference in a theoretical framework for the shorter run focusing on labor supply issues, such as differing levels of altruism towards work (Chiappori, 1992), women facing child care and home production tradeoffs (Mulligan, 1998), and two working member households facing an additional budget constraint (Cho and Rogerson, 1988). This paper focuses on the labor demand differences between male and female workers to examine the effectiveness of a two-agent real business cycle (RBC) model in capturing the U.S. business cycle statistics.

There are a variety of reasons why firms may view male and female workers as different components in the production function with differing labor demand elasticities. Men and women vary in occupations (Blau et al., 2013; Polachek, 1981) and...
Fig. 1. Aggregate Hours Worked. The natural logs of the data series for total aggregate hours and aggregate hours by gender, as described in Section 4.1 are graphed above. The shaded regions indicate NBER U.S. recession dates.

levels of education (Charles and Luoh, 2003; He et al., 2011; Pekkarinen, 2012). Additionally, recent research has shown that the elasticity of labor demand has changed in the U.S. as jobs and employer requirements have evolved (Olivetti, 2006) due to shifts from manufacturing to services (Goldin, 1990), increase in computerization (Galor and Weil, 1996), change in society’s attitudes (Fernández et al., 2004), and a rise in female educational attainment (He et al., 2011). Black and Spitz-Oener (2010) found that women have seen a relative increase in nonroutine analytic and interactive tasks due to technological change. This is similar to Galor and Weil (1996) who make the argument for female-biased technological change, as work requirements have shifted from physical to intellectual requirements (Rendall, 2017). Even at midcentury, Acemoglu et al. (2004) found that women were more substitutable for high school educated men than for lower skilled men.

Understanding how and why men and women differ across the business cycle is important for effective policy decisions. Recent research has found recessions to be gendered (Engemann and Wall, 2009; Hoynes et al., 2012) with some finding that the slowing of female labor force participation can be linked to the jobless recoveries of the 1990s and 2000s (Fukui et al., 2020; Albanesi, 2019). This discussion is continuing to happen considering the unequal gendered effects of COVID-19 (Alon et al., 2020).

In order to provide evidence for the model specification, I first show that aggregate hours varies by gender. I use average hours for non-agricultural industries separated by gender from the Current Population Survey (CPS) transforming each series into a quarterly series of aggregate hours carefully taking into account possible outliers and seasonality issues following Cociuba et al. (2009). I find that the volatility of the cyclical component is greater for men than women using a sample from 1976Q3 to 2015Q2; this result is robust to different filtering methods. This is consistent with recent research that has found larger recessionary effects on men than women, which is not only a feature of the most recent recession (Engemann and Wall, 2009; Hoynes et al., 2012).

Motivated by this empirical result, I set up a model with two different agents that compose a representative household, as specified by Jaimovich et al. (2013). Since these agents have identical utility functions and additive separability, consumption is the same among household members. These agents are only different in the demand for their labor in the baseline model. Motivated by the capital and skill complementarity literature (Acemoglu, 1998; Krusell et al., 2000), which finds that differences in elasticities of substitution can explain a large portion of the wage differences between skilled and unskilled labor, I allow the elasticities of substitution to vary between men and women in the firms’ production function. This specification is consistent with Olivetti (2006) who finds that the elasticity of labor demand has shifted in favor of women as employers require more office work than manual labor. Additionally, allowing for different elasticities of men and women workers has been used in an overlapping generations model by Galor and Weil (1996) to explain fertility and growth within a country.

These labor demand elasticities are the key parameters in the model. Using annual micro-level income data from the March supplement of the CPS and national data from the Bureau of Economic Analysis from 1964 to 2014, I am able to estimate the elasticities directly from the first order conditions of the representative firm's problem. I find that the elasticity of substitution between female workers and capital is greater than the elasticity between female workers and male workers, which is consistent with this explanation that women are more complementary with capital than men. Additionally, I explore the stability and sensitivity of

1 For a literature review of gender in the labor market see Altonji and Blank (1999).
these estimated elasticities. Using standard values for all other parameters, I compare the performance of the model with HP filtered U.S. data statistics, which is standard in the literature. I find that the relative volatility of hours for males and females generated by the model (1.34 and 1.19) closely matches the U.S. data (1.32 and 0.96). As a consequence of matching the volatilities of the gender groups, the model (1.24) generated relative volatility of aggregate hours that matches the data (1.12), as well. This result is much improved over the classical RBC model, which finds the volatility of hours to be around half the volatility of output (King et al., 1988). Understanding that men and women can differ on the labor supply side as well, I then explore the concurrence of my baseline results to the estimated elasticities and to various Frisch elasticities taken from the literature and find that my results are largely robust to differing labor supply elasticities. Therefore, accounting for difference in the labor demand for the two groups, specifically defining female labor as being more complementary to capital, resolves the classical RBC's inability to generate sufficient hours worked volatility.

The paper is organized as follows: Section 2 explores the gender differences of work hours using three different filtering methods. Section 3 describes the real business cycle model augmented to allow for two types of workers, men and women. Section 4 discusses the estimation and sensitivity of the elasticities of substitution of labor demand. Section 5 outlines the results of the baseline model and extensions of the model. Finally, Section 6 concludes.

2. Empirical evidence

It has been well documented that Real Business Cycle models have not been able to generate the relative volatility of work hours with relation to output that is seen in the data (Kydland and Prescott, 1982; Prescott, 1986). While others have looked into adding different types of frictions to the model to generate more volatility in the labor market, primarily on the labor supply side, this paper will look into the possible benefits of heterogeneity in explaining the labor market dynamics. There is a longstanding debate about the merits (or lack thereof) of aggregation (Theil, 1954; Grunfeld and Griliches, 1960). Some research suggests that we may lose information when we choose to aggregate across heterogeneous groups (Pesaran and Barker, 1990). Therefore, this section will investigate the volatility of aggregate hours disaggregated by gender.

In order to evaluate the business cycle volatility, the data on aggregate hours worked by gender needs to be decomposed into a permanent or trend component and a transitory or cyclical component. Different filters might lead to divergent decompositions of the same data series. Therefore, following the conclusions of Canova (1998) this section will evaluate business cycle volatility by using multiple detrending methods in order to extract a transitory or business cycle series of aggregate hours worked. The three types of filters used are the Hodrick–Prescott (HP) filter (Hodrick and Prescott, 1997), the Baxter–King Band Pass (BP) Filter (Baxter and King, 1999), and the correlated univariate unobserved components (UC) model (Morley et al., 2003) using specifications standard in the literature.

As the focus of this paper is to explain the movement across the business cycle, it is important to use a data frequency that is appropriate to capture these dynamics. The Bureau of Labor Statistics (BLS) has seasonally unadjusted monthly data for average

Fig. 2. Variability in Hours. This figure plots the HP cyclical component for men and women with the NBER recessions displayed by the shaded area. These series can be interpreted as the percent deviations from the trend.
Table 1

Variability in hours.

| Group | HP  | BP  | UC  |
|-------|-----|-----|-----|
| All   | 1.61| 1.49| 0.90|
| Males | 1.91| 1.77| 2.20|
| Females | 1.38| 1.23| 1.39|
| observations | 131 | 131 | 131 |

All calculations are the standard deviations of the cyclical component. The first column (HP) shows the results for the Hodrick Prescott filter, the second column (BP) shows the results for the Baxter–King Band Pass filter, and the third column (UC) shows the results for the univariate unobserved components model. In order to have the same number of observations, the sample goes from 1979Q3 to 2012Q2.

hours at work disaggregated by men and women from June 1976. Following Cociuba et al. (2009) the average, monthly data is transformed into a seasonally adjusted, aggregate quarterly series. This method takes into account monthly outliers, which could be due to a holiday falling on a reference week leading to an under-reporting hours worked. The natural logs of the data series for total aggregate hours and aggregate hours by gender are graphed on Fig. 1, where the shaded regions are NBER U.S. recession dates. As can be seen in the figure, the aggregate hours worked does seem to fall sharply during the recessionary times for all groups.

Table 1 shows the standard deviation of the cyclical components of each series using the methods. The data used to estimate these models is from 1976Q3 to 2015Q2; however, since the Baxter–King Band Pass Filter required 12 observations at the beginning and end of sample for estimation, the sample presented is from 1979Q3 to 2012Q2. It is important to note that the aggregate cyclical variability in hours is relatively similar across the HP and BP filtering methods, 1.61 and 1.49, respectively. However, the aggregate volatility for the univariate UC model is much less than the other two filters at 0.90. This may be due to the fact that this is the only filter that allows for the correlation between the innovations of the two components (trend and cycle).

Consistent with Morley et al. (2003), I find that the covariance between the innovations of the components to be negative; although, the covariance for males is not statistically significant.

Comparing the relative volatilities between the gender groups by filtering methods in Table 1, we can see that the univariate UC method estimated the largest volatility for both males and females, the BP filter estimated the smallest variability for both groups. However, despite the filtering method used, business cycle volatility for males is consistently greater than females.

The HP filter has known problems such as overstating the cyclical components in some cases, the poor end of sample properties, and an arbitrary smoothing parameter (Hamilton, 2018; Cogley and Nason, 1995; Gordon, 1993). However, since the relative percent deviations from the trend based on gender results are relatively consistent and as the HP filter is standard in the RBC literature, this paper will motivate the model using the results from the HP filter. Fig. 2 plots the HP cyclical component for men and women with NBER recessions displayed by the shaded area. These series can be interpreted as the percent deviations from the trend. As seen in the figure, male work hours falls by more than women work hours during recessions, but rises by more during expansions. These results are visually consistent with the estimated volatilities, which find that the volatility of the cyclical component is greater for men than women over the sample. This result is consistent with recent research that have found larger recessionary effects on men than women in the recent recessions in the US (Engemann and Wall, 2009; Hoynes et al., 2012) and in Europe (Périmier, 2018; Verdugo, 2020).

3. Model

This section describes an augmented RBC model introduced by Jaimovich et al. (2013) that allows for two types of workers that form a representative household. I recast this model to define the two types of workers as males and females, and assume that households are comprised of both types of workers. While these household members are assumed to be identical on the supply side, the representative firm’s elasticities of substitution on the demand side of the problem are allowed to vary, as male and female workers enter the production function as separate labor inputs.

3.1. Households

The economy is populated with many identical, infinitely-lived households. The households are composed of family members, which sum to unity. Heterogeneity is introduced in the model by allowing for two types of family members, males and females,

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2 The series used are LNU02033120, LNU02033510 (women), and LNU02033447 (men).
3 Model results are available from the author by request.
4 It is important to note that these results may be influenced by other differences that are manifested in gender, such as occupational differences (Blau et al., 2013; Polachek, 1981). Albanesi and Şahin (2018) find that males tend to have a higher share of workers in production occupations, while females tend to have a higher share of workers in sales and office occupations. However, how to appropriately define work, either by occupations, sectors, or tasks, has been an interesting topic of discussion (Acemoglu and Autor, 2011; Guvenen et al., 2020).
where the share of males is denoted by $s_m$ and the share of females is $1-s_m$. Family members derive utility from consumption, $C_t$, and disutility from hours spent working, $N_i$, where $i \in \{m,f\}$. The representative household maximizes

$$E_t \sum_{i=1}^{\infty} \beta^{t-i} \left[ s_m U_m(C_{m,t}, N_{m,t}) + (1-s_m)U_f(C_{f,t}, N_{f,t}) \right].$$

subject to

$$s_mC_{m,t} + (1-s_m)C_{f,t} + K_{t+1} = (1-\delta)K_t + r_tK_t + s_mW_{m,t}N_{m,t} + (1-s_m)W_{f,t}N_{f,t},$$

for a date, $t$, where $U_t$ is the utility function, $K_t$ is capital, $r_t$ is the rental rate of capital, and $W_{it}$ is the wage rate (which can vary across members). I assume that households take all prices as given. The discount rate, $\beta$ and the depreciation rate of capital, $\delta$, is between 0 and 1.

The utility function is defined as:

$$U_t = \log C_t - \psi_t N^{1+\theta_t} - \delta_t,$$

where $\psi_t > 0$ is used to calibrate the steady-state hours worked for males and females and $\theta_t \geq 0$ are the Frisch elasticities of labor supply. The time endowment of the household is normalized to unity, such that $0 \leq N_m, N_f \leq 1$. As seen in Eq. (3.3), the utility function for males and females has the same functional form. Therefore, differences between males and females are not arising from differences in their utility structure. However, this assumption is relaxed in Section 5.3, when I allow for different Frisch elasticities.

Since the preferences represented in the utility function are additively separable, and because the household is maximizing utility by choosing consumption, the solution will have consumption equated across all household members:

$$C_{it} = C_{ft} = C_t.$$ (3.4)

Households choose $C_t$, $K_{t+1}$, $N_{m,t}$, and $N_{f,t}$ to maximize Eq. (3.1) subject to Eq. (3.2). The first order conditions (FOCs) are:

$$\frac{1}{C_t} = \beta E_t \frac{1}{C_{t+1}} (r_{t+1} + 1 - \delta),$$

$$W_{it} = \psi_t C_t N^{\theta_t}_{it},$$

where Eq. (3.5) combines the first order conditions with respect to consumption and investment and Eq. (3.6) is the FOC with respect to hours worked for males and females.

3.2. Firms

As members of the household are identical on the supply side, they must differ on the demand side in order to deviate from the typical representative agent model. Specifically, males and females enter into the production function as different factors of production. I assume a nested CES production function with multiple labor inputs following the capital skills literature (Krusell et al., 2000). Goods in the economy are produced by perfectly competitive firms according to the following three-factor production function:

$$Y_t = [\mu(A_tH_{m,t})^\sigma + (1-\mu)(\lambda K_t^\sigma + (1-\lambda)(A_tH_{f,t})^\sigma)^\rho]^\psi.$$ (3.7)

The three factors of production are labor hours of males ($H_{m,t}$), labor hours of females ($H_{f,t}$), and capital ($K_t$), and $\sigma$ and $\rho$ are the substitution parameters, which are less than unity in order for the isoquants to have the appropriate convexity (Solow, 1956).

Unlike the standard Cobb–Douglas production function, this CES production function does not require that all factors inputs are necessary for production. The technology, $A_t$, evolves according to the following process:

$$A_t = \exp(g_t + z_t),$$ (3.8)

where $g$ is the trend growth rate and $z_t$ is the stationary shock. This shock follows an AR(1) process ($z_t = \phi z_{t-1} + \varepsilon_t$), where $E(\varepsilon_t) = 0$ and $\phi$ are the autoregressive coefficient is between 0 and 1. These perfectly competitive firms choose the level of the factors in order to maximize profits, where the first order conditions are:

$$r_t = Y_t^{1-\sigma}(1-\mu)\Omega_t \lambda K_t^{\sigma-1},$$

$$W_{f,t} = Y_t^{1-\sigma}(1-\mu)\Omega_t (1-\lambda)A_t^{\rho-1} H_{f,t}^{\psi},$$

$$W_{m,t} = Y_t^{1-\sigma}\mu A_t^{\rho} H_{m,t}^{\psi-1},$$

where $\Omega_t \equiv [\lambda K_t^{\sigma} + (1-\lambda)(A_tH_{f,t})^\rho]^\psi$.

As shown above, marginal revenues are set equal to factor prices.

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5 Nested CES production functions have been found to lead to incongruous long-run predictions (Klump and de La Grandville, 2000; Klump et al., 2007). However, this paper’s focus is on explaining the business cycle volatilities of hours worked.
As shown in (3.7), the substitution parameters are allowed to vary; however, the elasticity between female workers and male workers is restricted to be the same as the elasticity between capital and male workers. This is similar to the specifications in the capital-skills complementarity literature where labor is disaggregated by education level, skill level, or experience level (Griliches, 1969; Krusell et al., 2000; Jaimovich et al., 2013), rather than gender. The elasticity of substitution between capital and female workers is given by \(1/(1 - \rho)\) and the elasticity of substitution between female workers (or capital) and male workers is \(1/(1 - \sigma)\). In order for the capital-skills complementary interpretation to hold, \(\sigma > \rho\) must be true.

The conjecture that female labor is more complementary to capital than is male labor is consistent with recent research which has shown that elasticity of labor demand has changed in the U.S. as jobs and requirements have changed for employers. Employers are demanding more office work, which favors the fine motor skills where females have a comparative advantage, over manual labor where males have a comparative advantage (Olivetti, 2006). This change in labor demand has been linked to a shift from the manufacturing sector and towards the services sector (Goldin, 1990), a change in society’s attitudes towards females in the workplace (Fernández et al., 2004), and a rise in women’s educational attainment (He et al., 2011). Additionally, allowing for men and women to be separate labor inputs in the production function and allowing for different elasticities of substitution has been implemented in an overlapping generations framework to explain the rapid decline in fertility and increase in output growth (Galer and Weil, 1996). Furthermore, Rendall (2017) argues that the recent labor demand changes that favor women are in part due to the job requirements shifting away from physical and toward intellectual attributes in conjunction with women overtaking men in college educational attainment. These changes lead to what she coins as “brain biased technical change” that can explain the change in women’s labor force participation, wages, and educational attainment. Black and Spitz-Oener (2010) document a relative increase for women in nonroutine and interactive tasks in West Germany and Acemoglu et al. (2004) found that at midcentury women were more substitutable for high school men than for lower skilled men. While Verdugo (2020) found relatively greater decline in demand for male workers in Europe, they were unable to find a similar imbalance in the US, albeit using a narrower sample from 1995–2013. While there is substantial evidence that female labor is more complementary to capital, the alternative nesting is also estimated in Section 4.3.

In order to understand the mechanism at work to generate a greater volatility of hours, assume that female labor is a perfect complement to capital or that \(1/(1 - \rho)\) tends towards unity and male labor is not a perfect complement to capital or that \(1/(1 - \rho) > 1/(1 - \sigma)\), so that the following holds: \(\sigma > \rho\). If there is a productivity shock, then firms will want to reduce the amount of goods they are producing. Assuming that capital is inelastic in the short run, that leads firms to adjust the quantity of labor demanded. Since female workers and capital are perfect complements in production (in this example), this will lead firms to adjust the quantity of labor demanded of male workers. The result is that male workers will be more volatile over the business cycle than female workers.

### 3.3. Equilibrium

The competitive equilibrium is a set of quantities, \(\{C_t, N_{mt}, N_{ft}, K_t, Y_t, H_{mt}, H_{ft}\}\) and prices, \(\{r_t, W_{mt}, W_{ft}\}\), such that the representative household chooses \(C_t, N_{mt},\) and \(N_{ft}\), given \(r_t, W_{mt},\) and \(W_{ft}\); the representative firm chooses \(K_t, Y_t, H_{mt},\) and \(H_{ft}\) given \(r_t, W_{mt},\) and \(W_{ft}\); and all markets clear. In equilibrium, the following conditions hold:

\[
K_0 > 0, \quad (3.12)
\]

\[
H_{mt} = s_m N_{mt}, \quad (3.13)
\]

\[
H_{ft} = (1 - s_m) N_{ft}, \quad (3.14)
\]

\[
C_t + K_{t+1} = Y_t + (1 - \delta)K_t, \quad (3.15)
\]

for all \(t\), such that the rental market, the labor markets, and the goods market clear. Aggregate hours worked in the economy is defined as the sum of female and male labor hours worked or \(H_t = H_{ft} + H_{mt}\).

### 4. Estimation

I use standard calibrated values from the literature for the remaining parameters whenever possible. However, the main difference between the two types of labor in this model rests on the differences of the elasticities of substitution in the production function. Therefore, following Krusell et al. (2000) and Jaimovich et al. (2013), I estimate these values using the CPS March Supplement Survey (from IPUMS) and other macroeconomic data series from FRED (Federal Reserve Bank of St. Louis) and the Bureau of Economic Analysis. This section describes the data used, the estimation specification, sensitivity analysis, and other calibrated values used for the model.

#### 4.1. Data

While Section 2 uses a quarterly data series to explore the business cycle relationships of the data, due to data limitations this section uses an annual series. Specifically, I use the CPS March Supplement Survey from IPUMS which includes annual individual

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6 While the Bureau of Labor Statistics provides aggregate monthly, average wages disaggregated by men and women starting in 1976, these series only includes full-time workers. Average wages for part time workers is not available until 2000. Therefore, it is necessary to use the micro-level data to explore the answer to this question.
observations on demographic characteristics, hours worked, weeks worked, and wage and salary income.\textsuperscript{7} I keep only observations from non-self-employed, civilians between the ages of 15 and 70 from 1964 to 2014.\textsuperscript{8} While this section implicitly assumes that the elasticities are not changing over time,\textsuperscript{9} this assumption is investigated in Section 4.3.

There are a few modifications I make to the data in order to exploit increased precision due to survey updates and to correct for reporting discrepancies. First, in 1976 the survey started recording the number of weeks worked the previous year (WKSWORK1) as two digit numeric values. Prior to this, the number of weeks worked the previous year (WKSWORK2) is recorded as an interval with eight bin options. In order to use the longest series possible and take advantage of the more precise estimates from 1976 onward, I use the median weeks worked in the interval prior to 1976 and use the numeric values from 1976 onward.

In a similar vein to the previous change, in 1976 a new question was included in the survey to ask what is the \textit{usual} hours a week a person works, if they worked the previous year (UHRSWORK). Prior to this, the survey recorded just the hours worked last week (HRSWORK) during a given reference week. However, there could be some bias if the person happened to not work last week or worked less or more hours last week than usual. Therefore, for observations prior to 1976, I use the hours worked last week, and from 1976 onward I use the usual hours worked.

Finally, following Jaimovich et al. (2013), I correct for the consistency of reporting for hours worked using the workers’ full-time or part-time status. Specifically, if a person claims to be part-time and has hours worked between 1 and 34 h there is no change to the work hours; however, if they report 0 or more than 34 h, they are given the group average of hours worked. Similarly, if a person claims to be full-time and has reported working 35 h or more, then there is no change; however, if they report less than 35 h, they are given the group average of hours worked. Similar to Krusell et al. (2000) and Jaimovich et al. (2013), I create groups based on age (eleven five-year age bins), education (five bins), gender, and race (2 bins). Therefore, there is a total of 220 possible groups, \( g \), where the weighted group average, \( h_g \), of individual \( i \)'s hours worked is given by:

\[
h_g = \frac{\sum_{l=1}^{g} h_l \mu_l}{\sum_{l=1}^{g} \mu_l},
\]

where \( h_l \) is the hours worked reported by the individual, \( l \), that matches the full-time or part-time status reported and \( \mu_l \) is the CPS person-level weight. Jaimovich et al. (2013) find that this correction method avoids an under-reporting of hours when comparing the corrected hours to the usual hours a week a person works prior to 1976. This correction is similar in spirit to the correction in Section 2, due to the under-reporting of hours worked for a given reference week.

4.2. Elasticity estimation specification

This section discusses the methodology for estimating the substitution parameters from the production function, \( \sigma \) and \( \rho \), which are the main differences between the two types of workers, males and females. In order to estimate the substitution parameters from the production function, I use the first order conditions from the firm’s problem, Eqs. (3.9) through (3.11), following a similar methodology by Jaimovich et al. (2013). Starting with the estimation of \( \sigma \), I take the logged first difference of Eq. (3.11), which becomes:

\[
\Delta \log W_{mf} = a_0 + (1 - \sigma) \Delta \log Y_t + (\sigma - 1) \Delta \log H_{mf} + \sigma \epsilon_t.
\]

In this equation, \( \log \) refers to the natural log, \( \Delta \) refers to the first difference, \( a_0 \) is the constant term, and \( \epsilon_t \) includes all the shock innovations. Since the annual CPS data does not include a wage variable for the whole series directly, I estimate the following variant:

\[
\Delta \log Inc_{mf} - \Delta \log Y_t = a_1 + \sigma (\Delta \log H_{mf} - \Delta \log Y_t) + \sigma \epsilon_t,
\]

where \( Inc_{mf} \) is the labor income of men, which is equivalent to weekly hours multiplied by the number of weeks worked last year. With this specification, \( Y_t \) is available from FRED and \( Inc_{mf} \) and \( H_{mf} \) are available directly from the IPUMS CPS data.

I follow a similar procedure to estimate \( \rho \). First I take the logged first difference of the first order conditions with respect to \( K_t \) and \( H_{f1} \), Eqs. (3.10) and (3.9), and then take the difference of those two equations to get:

\[
\Delta \log W_{f1} - \Delta \log r_{f1} = a_2 + (\rho - 1) (\Delta \log H_{f1} - \Delta \log K_t) + \rho \epsilon_t.
\]

Again in order to use data directly from the survey, I estimate a variation of the above equation:

\[
\Delta \log Q_{f1} - \Delta \log Q_{Kt} = a_3 + \rho (\Delta \log H_{f1} - \Delta \log K_t) + \rho \epsilon_t.
\]

In this equation, \( Q_{f1} \) is the national income share of women and \( Q_{Kt} \) the national income share of capital, which can be estimated from data from the Bureau of Economic Analysis. Specifically, the national income share of labor (males and females combined) is estimated from the NIPA data from the BEA. The national income share of capital is defined as one minus the national income share of labor. Finally, the average of the percent of total income by gender from IPUMS is used to estimate male and female income shares.

\textsuperscript{7} There was no correction for the top coding of wages. Although, it was adjusted for inflation using the CPI99 variable from IPUMS.

\textsuperscript{8} The wage and salary income is for the previous year; therefore, the actual sample includes observations from 1963 to 2013.

\textsuperscript{9} While this is a standard assumption in the literature, due to the difficult nature of estimating elasticities generally (Duffy et al., 2004), there is some research to believe that these elasticities may not be constant over time due to the changing work demand brought on by computerization of the workforce in the 1970s (Olivetti, 2006). However, since this data set is starting in 1965, the changes would have occurred during this sample period.
### Table 2
Substitution parameter estimates.

| Parameter | Coefficient | Std errors |
|-----------|-------------|------------|
| $\sigma$  | 0.758$^{***}$ | 0.151      |
| $\rho$    | 0.697$^{***}$ | 0.071      |

Estimates are from Eqs. (4.3) and (4.5) using OLS. Robust standard errors are reported. The $^*$ denotes $p < 0.1$, $^{**}$ denotes $p < 0.05$, and $^{***}$ denotes $p < 0.01$.

### Table 3
Substitution parameter estimates — exogeneity.

| IV - Lagged birth rates | $\sigma$ | $\rho$ |
|-------------------------|---------|-------|
| 15                      | 1.574 (0.2158) | 6.974$^{**}$ (0.011) |
| 20                      | 0.000 (0.997) | 7.025$^{**}$ (0.011) |
| 22                      | 0.105 (0.748) | 6.221$^{**}$ (0.016) |
| 25                      | 0.989 (0.325) | 5.529$^{**}$ (0.02)  |
| 30                      | 2.167 (0.148) | 7.220$^{***}$ (0.010) |
| 35                      | 0.391 (0.535) | 2.324 (0.134) |
| 40                      | 2.527 (0.119) | 2.561 (0.116) |
| 50                      | 7.116$^{**}$ (0.010) | 0.460 (0.501) |

Estimates are from Eqs. (4.3) and (4.5) using 2SLS with the instrument being lagged birth rates. The first column indicates the number of years the birth rates are lagged for the IV. The next two columns indicate the robust regression F-statistic for the test of endogeneity, where the null hypothesis is that the variables are exogenous. The $p$-value is reported in parenthesis below the F-statistic. The $^*$ denotes $p < 0.1$, $^{**}$ denotes $p < 0.05$, and $^{***}$ denotes $p < 0.01$.

Using the data described earlier, Eqs. (4.3) and (4.5) can be estimated to yield parameter values for $\sigma$ and $\rho$. However, the equations above are assuming exogeneity of the right hand side variables. Table 2 shows the results of the estimates elasticities, $\sigma$ and $\rho$, and their robust standard errors. I estimate $\sigma$ to be 0.758 and $\rho$ to be 0.697 with both estimated parameters are found to be statistically significant at the one percent level. These estimates are within the range of elasticities in found in other specifications.\(^\text{10}\)

It is important to note that the estimation method used in this paper does not put any restrictions of the values that $\sigma$ and $\rho$ can take. However, the estimated result that $\sigma > \rho$ is consistent with the capital-skill complementarity literature (Krusell et al., 2000).

While the estimation method is assuming exogeneity of the right hand side variables, it is important to consider that there might be some endogeneity in the setup. Since the error term also includes all past technology shocks, there is a concern that the error term may be correlated with the explanatory variables or that there may be endogeneity with the specification. Following previous literature,\(^\text{11}\) I investigate using lagged birthrates as a possible instrumental variable. Using data from the Census’s Statistical Abstract of the United States, I construct a continuous series of birthrates in the U.S. from 1909. Using two-stage least squares, I estimate Eqs. (4.3) and (4.5) using one instrument at a time and conduct a test for exogeneity along with considering whether the instruments are weak. Table 3 shows the test for exogeneity by Durbin (1954) and Wooldridge (1995), where the null hypothesis is that the variables are exogenous. While the null hypothesis was not rejected for most lagged instruments for $\sigma$, it was rejected for most lagged instruments of $\rho$.

Based on these results, it appears that I would want to use an IV for my specification of $\rho$; however, using weak IVs could be cause more harm than good due to a lack of consistency (Bound et al., 1995; Chao and Swanson, 2005). Following Staiger and Stock (1997), I should be concerned about weak instruments, if my first-stage F-statistic is less than 10. Table 4 reports the first-stage F-statistic, and none of the values reach the cutoff as defined by Staiger and Stock (1997). Therefore, using the weak instruments may lead to less stable and inconsistent estimates. I look into the stability of my results by conducting sensitivity tests to $\sigma$ and $\rho$ in Section 5.2.

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\(^{10}\) While Krusell et al. (2000) estimated demand elasticities between skilled and unskilled labor, $\sigma$, and equipment and unskilled labor, $\rho$, to be 0.401 and −0.495, respectively. Alternatively, Jaimovich et al. (2013) estimated demand elasticities between experienced and inexperienced labor, $\sigma$, and capital and inexperienced labor, $\rho$, to be 0.662 and 0.201, respectively.

\(^{11}\) For some example of papers using lagged birthrates as an instrument, see Shimer (2001), Foote (2007), Beaudry and Green (2003), and Jaimovich et al. (2013).
Table 4
Substitution parameter estimates — IV first-stage.

| IV - Lagged birth rates | $\sigma$  | $\rho$ |
|-------------------------|-----------|--------|
| 15                      | 4.957**   | 0.057  |
| 20                      | 2.408     | 0.443  |
| 22                      | 1.834     | 0.298  |
| 25                      | 2.225     | 0.001  |
| 30                      | 2.146     | 0.022  |
| 35                      | 1.875     | 0.258  |
| 40                      | 2.945*    | 0.442  |
| 50                      | 1.141     | 0.002  |

Estimates are from Eqs. (4.3) and (4.5) using 2SLS with the instrument being lagged birth rates. The first column indicates the number of years the birth rates are lagged in the IV. The next two columns indicate the robust regression F-statistic for the first-stage regression. The * denotes $p < 0.1$, ** denotes $p < 0.05$, and *** denotes $p < 0.01$.

4.3. Sensitivity analysis

This section details the robustness of these estimated substitution parameters including checking the stability of these estimates over time and estimating the estimates under the reverse assumption (that the elasticity of male workers and female workers is restricted to be the same as the elasticity between capital and female workers.).

The first assumption of the model that I am investigating is the assumption that the elasticities are constant over time. While this is a standard assumption in the literature, there is some research to believe that these elasticities may not be constant over time due to the changing work demand brought on by computerization of the workforce in the 1970s (Olivetti, 2006). Therefore, I re-estimate Eqs. (4.3) and (4.5) using a rolling regression window, where I look at a sample window of 31 observations, that moves across full sample from 1964 to 2014. This technique has been used to investigate the stability of many macroeconomic relationships, including Okun’s Law (Edward S. Knotek, 2008), inflation forecasts (Meese and Rogoff, 1988), and other univariate and bivariate forecasting relationships (Stock and Watson, 1996). The first regression estimates of $\sigma$ and $\rho$ are from 1964 to 1994 and the last estimates use a sample from 1984 to 2014, which leads to 21 estimates over the full sample.

Fig. 3 graphs the point estimates of $\sigma$, the solid line, and the 95 percent confidence intervals, the dotted line. The year associated with each point estimate is the start of the sample. As can be seen in the figure, the estimate of $\sigma$ stays relatively constant throughout the time period with a minimum estimate of 0.687 in 1984 and a maximum estimate of 0.924 of in 1976. This estimate is also statistically significant across the rolling regression horizon. Fig. 4 graphs the point estimates of $\rho$, where the minimum estimated
value is 0.614 in 1964 and a maximum value of 0.769 in 1984. Therefore, from the rolling regression analysis, both estimated elasticities remain relatively stable and statistically significant at the one percent significant level throughout the sample period.12

While it is important that the elasticities remain relatively constant throughout the full sample, this analysis also lends itself to test the hypothesis that the difference in labor demand is due to differing complementarity with capital, which requires that $\sigma > \rho$. It is important to note that the original methodology and the rolling regression methodology does not place any restrictions on the 

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12 It is interesting that the last rolling regression, sample from 1984 to 2014, provided the extreme estimates for both $\sigma$ and $\rho$ with a lower limit for $\sigma$ an upper limit for $\rho$. This may be due to the most recent recession, but it is an interesting topic for future research.
where \( \frac{1}{\Delta_l\log Q} \) is the labor income of women, which is equivalent to weekly hours multiplied by the number of weeks worked last year. With this specification, \( Y_t \) is available from FRED and \( Inc_{ft} \) is available from the CPS.

In order to estimate \( \gamma \), I can estimate the following variation from the first order conditions:

\[
\Delta \log Q_{ft} - \Delta \log Y_t = a_3 + \gamma(\Delta \log H_{ft} - \Delta \log Y_t) + \sigma \epsilon_t \tag{4.7}
\]

In this equation, \( Q_{mf} \) is the national income share of men and \( Q_{fk} \) the national income share of capital, which can be estimated from data from the Bureau of Economic Analysis.

Table 5 shows the estimated substitution parameters under the assumption that men are more capital complementary. Both estimated parameters are significantly significant at one percent significance level with point estimates of 1.022 for \( \psi \) and 0.800 for \( \gamma \). While the estimated coefficients displays the relative magnitudes necessary for capital experience complementarity, \( \psi > \gamma \), the substitution parameters must be less than one to generate feasible elasticities of substitution. I can reject the null hypothesis that \( \psi \) is equal to one at the 5 percent significance level versus the alternative that \( \psi \) is greater than one. Therefore, the assumption that men are more complementary than capital does not lead to feasible estimated elasticities, since an estimated substitution parameter greater than one leads to isoquants that have the wrong convexity (Solow, 1956).

4.4. Calibration

The remaining parameters use standard values or methods in the literature, and are summarized in Table 6. \( \beta \) and \( \delta \) are set at 0.99 and 0.025, respectively, which is standard for quarterly time periods. The Frisch labor supply elasticities, \( \theta_j \) and \( \theta_m \), are

\[\begin{array}{ccc}
\text{Parameter} & \text{Value} & \text{Source} \\
\beta & 0.99 & \text{Jaimovich et al. (2013)} \\
\delta & 0.025 & \text{Jaimovich et al. (2013)} \\
\theta_j & 0.00001 & \text{Jaimovich et al. (2013)} \\
\phi & 0.94 & \text{Jaimovich et al. (2013)} \\
\omega & 0.353 & \text{estimated from BEA} \\
\omega_f & 0.228 & \text{estimated from CPS & BEA} \\
\sigma & 0.570 & \text{estimated from CPS} \\
\rho & 0.758 & \text{estimated March CPS & BEA} \\
\sigma & 0.697 & \text{estimated March CPS & BEA} \\
\end{array}\]

Above is a summary of the remaining parameters used to calibrate the baseline model, which are standard values or methods in the literature for quarterly time periods.

Relative size of \( \sigma \) and \( \rho \). Fig. 5 plots both \( \sigma \) and \( \rho \) on the same graph. As can be seen in the graph, \( \sigma > \rho \) for every estimate, except the last one with a sample from 1984 to 2014. Therefore, for 20 out of the 21 estimates, the explanation that the differences in labor demand is due to differing complementarity is not violated.

While there is recent literature to support the idea that women may be more complementary to capital than males (Galor and Weil, 1996; Olivetti, 2006; He et al., 2011), Polachek (1981) noted that women tend to have intermittent labor supply as opposed to men, who have continuous labor supply. Therefore, when women leave the labor market, they have an atrophy of skills, which causes them to return to the labor market at a lower wage and skill level. If this hypothesis is correct, then we would expect men to be more complementary with capital, since they will have more continuous on-the-job training and acquisition of skills. I estimate the elasticities assuming the opposite restriction, specifically, that the elasticity of substitution between males and females is the same as the elasticity of substitution between capital and females.

Under this identification scheme, the elasticity of substitution between capital and male workers is given by \( 1/(1 - \gamma) \) and the elasticity of substitution between male workers (or capital) and female workers is \( 1/(1 - \psi) \). In order for the capital experience complementary interpretation hold, the following must be true \( \psi > \gamma \). Therefore, following the similar methodology in Section 4.2, to estimate \( \psi \), I can estimate the following equation, which is a modification of the firm's first order conditions:

\[
\Delta \log Inc_{ft} - \Delta \log Y_t = a_1 + \psi(\Delta \log H_{ft} - \Delta \log Y_t) + \sigma \epsilon_t \tag{4.6}
\]

where \( Inc_{ft} \) is the labor income of women, which is equivalent to weekly hours multiplied by the number of weeks worked last year. With this specification, \( Y_t \) is available from FRED and \( Inc_{ft} \) and \( H_{ft} \) are available from the CPS.

In order to estimate \( \gamma \), I can estimate the following variation from the first order conditions:

\[
\Delta \log Q_{ft} - \Delta \log Q_{kt} = a_3 + \gamma(\Delta \log H_{ft} - \Delta \log K_t) + \rho \epsilon_t \tag{4.7}
\]

In this equation, \( Q_{mf} \) is the national income share of men and \( Q_{fk} \) the national income share of capital, which can be estimated from data from the Bureau of Economic Analysis.

\[\begin{array}{ccc}
\text{Parameter} & \text{Coefficient} & \text{Std errors} \\
\psi & 1.022^{***} & 0.011 \\
\gamma & 0.800^{***} & 0.065 \\
\end{array}\]

Estimates are from Eqs. (4.6) and (4.7) using OLS. Robust standard errors are reported. The * denotes \( p < 0.1 \), ** denotes \( p < 0.05 \), and *** denotes \( p < 0.01 \).
set to 0.00001 following Jaimovich et al. (2013), since there are many complications with the precision of the microeconomic estimates (Rogerson, 1988). Following Krusell et al. (2000) and Jaimovich et al. (2013), I calibrate $\mu$ and $\theta$ to match national income shares using labor income from the NIPA from the BEA and the March supplement of the CPS from 1964 to 2014. Using these data series, I estimate the income share of capital to be 0.353 and the income share of female labor to be 0.228. Finally, the persistence of the technology is set to 0.94 with a standard deviation of 0.0064, following Jaimovich et al. (2013).

5. Results

This section evaluates the model with heterogeneity in household members, where household members vary due to differences in their complementarity with capital. The model’s performance is evaluated on the ability to match the business cycle volatility of hours worked in the U.S., where the business cycle is defined as the cyclical component of the HP filter. Additional expansions to the model, analyzing the sensitivity of the model’s results when changing the substitution parameters on the demand side and Frisch elasticities of labor supply, are also explored.

5.1. Baseline results

The results are reported in Table 7, where the first column, Data, represents U.S. business cycle statistics. The standard deviation of hours worked are calculated from the BLS data referenced in Section 2 and the standard deviation of real GDP, $Y$, is from the Federal Reserve Bank of St. Louis’s FRED database. As mentioned earlier, males have greater volatility in hours worked as compared with women. Additionally, the aggregate hours worked volatility, $H$, is greater than the volatility of real GDP, $Y$, which is the feature that traditional RBC models fail to predict. As a comparison, the third column, KPR, reports the relative volatility of hours worked and output from King et al. (1988). This statistic comes from a simplified version of the Kydland and Prescott (1982) model in which time-to-build in investment, non-separable utility in leisure, and technology shocks that include both a permanent and a transitory component are eliminated. Similar to the findings of other RBC models, the KPR model fails to capture the relative volatility of hours in the data with the standard deviation of hours being less than half of the standard deviation of output.

The second column of Table 7 reports the result of the model in Section 3. Assuming the standard calibration values outlined in Section 4.4, the model generates volatility of aggregate hours that are slightly above what is estimated in the data, 1.24 compared to 1.12. This slight over-prediction appears to come from the model generating a larger volatility of female hours than seen in the data, with the standard deviation of female hours relative to the standard deviation of output being 1.19 in the model and 0.96 in the data. However, the model is able to accurately predict the volatility of male hours with the standard deviation of male hours relative to the standard deviation of output being 1.34 in the model and 1.32 in the data. Despite this slight over prediction of the volatility of female hours, overall, the model is able to generate volatility of total hours worked that closely resemble the data, and is a large improvement over the standard RBC predictions.

| Parameter Data Model KPR       |
|-------------------------------|------------------|------------------|
| Column                        | (1)              | (2)              | (3)              |
| \(\text{std}(H_f)/\text{std}(H_m)\) | 0.73             | 0.83             |
| \(\text{std}(H_f)/\text{std}(Y)\)  | 0.96             | 1.19             |
| \(\text{std}(H_m)/\text{std}(Y)\)  | 1.32             | 1.34             |
| \(\text{std}(H)/\text{std}(Y)\)    | 1.12             | 1.24             | 0.48             |
| \(\text{std}(W_f)/\text{std}(W_m)\) | 0.80             | 1.00             |
| \(\text{std}(W_f)/\text{std}(Y)\)    | 0.66             | 0.16             |
| \(\text{std}(W_m)/\text{std}(Y)\)    | 0.82             | 0.16             |
| \(\text{std}(I)/\text{std}(Y)\)     | 3.46             | 3.57             | 2.31             |
| \(\text{std}(C)/\text{std}(Y)\)     | 0.76             | 0.16             | 0.64             |

This table focuses only on the cyclical component of the HP-filtered data. The Data column is calculated from CPS and BEA data. The KPR column reports values from King et al. (1988) (Table 4). Wages are from IPUMS data from 1976 to 2014. The data used to estimate investment is private nonresidential fixed investment and consumption is PCE from 1976Q3 to 2015Q2 available on FRED.

14 Sensitivity tests to various microeconomic estimates of the Frisch elasticities can be found in Section 5.3.
15 This is similar to the estimate from Jaimovich et al. (2013) of 0.37 using data from 1964 to 2010.
16 This would lead to an estimate of the income share of men to be 0.419, which means that the total income share of labor is 0.647, which is standard in the literature.
5.2. Parameter sensitivity extension

As noted in Section 4.2, the specification of $\sigma$ and $\rho$ may suffer from endogeneity, particularly $\rho$, based on the test of exogeneity (Durbin, 1954; Wooldridge, 1995). Therefore, I will investigate the model’s ability to match the data if $\rho$ is allowed to be smaller than the estimated value, which is consistent with other papers using nested CES production functions. In particular, I will set $\rho$ to be 0.201 (which is the substitution parameter used in Jaimovich et al. (2013)) and –0.495 (which is the substitution parameter used in Krusell et al. (2000)). It is important to note that neither of these substitution parameters were estimated for a gender breakdown, but breakdowns by age and skill-level, respectively. In addition to the values mentioned above, $\rho$ was also set to equal 0.449, which is half way between my estimate (0.697) and the estimate of Jaimovich et al. (2013).

The results for the model can be found in Table 8, where the Data and Model columns are the same as from Table 7. As can be seen in the table, allowing the $\rho$ to decrease leads to a lower relative volatility of female hours and a greater relative volatility of males hours. Additionally, Models (2) and (3) both find relative aggregate hour volatility greater than 1; however, Model (3) underestimates the females hours volatility and overestimates male hours of volatility. The rest of the statistics are fairly similar across all specifications. Therefore, it appears that an estimated $\rho$ between 0.449 (Model 2) and 0.697 (baseline specification, Model 1) generates relative volatility of hours that matches the U.S. statistics.

5.3. Labor supply extension

Another interesting dimension to test the baseline model, is to relax the assumption of the Frisch elasticities to be equal to 0, but allow them to deviate from zero and vary between agents. Allowing for this elasticity to deviate from between men and women can capture gendered differences related to home production (Greenwood et al., 2005), childcare (Attanasio et al., 2008), and change in societies attitudes (Fernández et al., 2004). There has been a wide debate comparing macro and micro labor demand elasticities (Chetty et al., 2011; Chetty, 2012; Keane and Rogerson, 2012), and it is well documented that macroeconomic RBC models have large elasticities compared to what is found in the micro literature. Breaking down aggregate hours labor supply elasticities into intensive and extensive margins, Chetty et al. (2011) find that macro models have a Frisch elasticity of aggregate hours of 2.84, compared with micro studies of 0.82. Additionally, there is no consensus about the estimated labor supply elasticities of men and women, with men having estimated elasticities between –0.07 and 0.45 (Pencavel, 1986), and women having –0.48 to 0.48 (Blundell and MaCurdy, 1999). Blundell et al. (2016) find female labor supply elasticities vary by education, marital status, and children with lower labor supply elasticities found for women with higher levels of education and women without children.

This paper will look at various Frisch labor supply elasticities for men and women. Specifically, I will investigate four specific combinations of elasticities by gender based on the micro literature. The labor supply elasticities chosen are all smaller than what is generally found in macro papers Chetty et al. (2011). Additionally, they all follow a similar trend of male elasticity being closer to zero and female elasticity higher. Following Jacobsen (2007), I will calibrate the model for the male elasticity of labor supply to be –0.09 and the female elasticity of 0.77, Blau and Kahn (2007) finds that female labor supply elasticities have been decreasing over time, while male elasticity have stayed relatively stable over the same time period. Therefore, I will use the mean of their model’s estimates for the 1979 to 1981 period and the 1999 to 2001 period, which have elasticities of 0.825 and 0.386 for women and 0.040 and 0.072 for men, respectively. Finally, I will calibrate a model assuming that the Frisch elasticities of men and women are the same using the point estimates of 0.54 as documented by Chetty et al. (2011).

The results for the various Frisch elasticity specifications can be found in Table 9, where the U.S. moment estimates are followed by the baseline model results in Column (1), which has the same calibration as in Table 7. Column (2) shows the results using the estimates from Jacobsen (2007), which has the lowest elasticity for men and the second highest elasticity for women. This model underestimated the relative volatility of hours of women and overestimated the relative volatility of men. This pattern continued for most of the specifications with Column (4) generating the closest relative volatilities of hours worked, but still underestimated

### Table 8
Results — changing $\rho$.

| Parameter | Data Model | $\rho = 0.449$ | $\rho = 0.201$ | $\rho = 0.495$ |
|-----------|------------|----------------|----------------|----------------|
| $\sigma$ | 0.73       | 0.83           | 0.44           | 0.26           | 0.09           |
| $\sigma$ | 0.96       | 1.19           | 0.66           | 0.40           | 0.15           |
| $\sigma$ | 1.32       | 1.34           | 1.48           | 1.56           | 1.67           |
| $\sigma$ | 1.12       | 1.24           | 1.10           | 1.02           | 0.91           |
| $\sigma$ | 0.80       | 1.00           | 1.00           | 1.00           | 1.00           |
| $\sigma$ | 0.66       | 0.16           | 0.20           | 0.21           | 0.23           |
| $\sigma$ | 0.82       | 0.16           | 0.20           | 0.21           | 0.23           |
| $\sigma$ | 3.46       | 3.57           | 3.44           | 3.39           | 3.33           |
| $\sigma$ | 0.76       | 0.16           | 0.20           | 0.21           | 0.22           |

This table focuses only on the cyclical component of the HP-filtered data. The Data column is calculated from CPS and BEA data. Wages are from IPUMS data from 1976 to 2014. The data used to estimate investment is private nonresidential fixed investment and consumption is PCE from 1976Q3 to 2015Q2 available on FRED. $\rho$ is allowed to be 0.201 following Jaimovich et al. (2013), –0.495 following Krusell et al. (2000), and 0.449, which is half way between my estimate (0.697) and the estimate of Jaimovich et al. (2013).
the volatility of hours worked for men. It is interesting to note that while the estimates of labor supply elasticities by gender lack consistency across methods and time periods, overall, I find most specifications can still match the standard deviation of male hours relative to the standard deviation output.

Similar to previous work by Jaimovich et al. (2013), I find that allowing the Frisch elasticities to deviate from zero generates more wage dynamics in the model. However, all the models generate relative volatility of wages that is greater for women than men, when the data finds the opposite result. This inconsistency could also be due to the lack of consistency in the data with women’s wages increasing over the sample (Olivetti, 2006; Blau and Kahn, 2007) and the presence of the gendered wage gap (Attanasio et al., 2008). Additionally, these data statistics were calculated using annual level data, which is not ideal for business cycle dynamics. The BLS does produce a quarterly wage series disaggregated by gender from 1976; however, it only includes full-time workers. Finally the investment and consumption volatilities remain comparatively stable across the different specifications.

6. Conclusion

This paper examines the role of heterogeneity in a real business cycle model. Using quarterly data, this paper first shows that men and women have different cyclical volatilities in work hours. Specifically, using a Hodrick–Prescott filter (Hodrick and Prescott, 1997), Baxter–King Band Pass Filter (Baxter and King, 1999), and univariate unobserved components model, I find that the volatility of the cyclical component is greater for men than women over the sample of 1976Q3 to 2015Q2. This result is consistent with recent research that have found larger recessionary effects on men than women in the recent recessions (Hoynes et al., 2012).

Motivated by this empirical result, I utilize a model with a representative household that consists of two types of members from Jaimovich et al. (2013), which I characterize as males and females. In the baseline model, these two members are identical in their utility functions. The difference between the members comes from different elasticities of labor demand, which is due to their different complementarity with capital. The elasticity between female workers and male workers is restricted to be the same as the elasticity between capital and male workers. This is similar to the specifications in the capital-skills complementarity literature where instead of labor disaggregated by gender, it is instead disaggregated by education, skill, or experience (Griliches, 1969; Krusell et al., 2000; Jaimovich et al., 2013). This specification of women being more capital complementary is consistent with recent research which has shown that the elasticity of labor demand has changed in the U.S. as jobs and employer requirements have evolved (Olivetti, 2006) due to shifts from manufacturing to services (Goldin, 1990), increase in computerization (Galor and Weil, 1996), change in society’s attitudes (Fernández et al., 2004), and a rise in female educational attainment (He et al., 2011). Once these elasticities are estimated from the first order conditions of the firm’s problem, the model generates cyclical volatilities of hours worked that match the U.S. data better than the traditional representative agent model. Additional robustness checks were conducted on the substitution parameters including exploring an alternative specification of complementarity, examining instruments, and assessing the stability over time. After identifying a baseline model which is able to capture, not only, the relative volatilities of hours worked by gender, but also the relative volatility of the aggregate hours worked with respect to output. I then explore the sensitivity of the substitution parameter and the Frisch elasticities of substitution. While the discrepancies between micro and macro labor supply elasticities have been noted to be very large (Chetty et al., 2011; Chetty, 2012; Keane and Rogerson, 2012),

A series for the wages of part-time workers disaggregated by gender is available from 2000.

| Parameter | Data | (1) | (2) | (3) | (4) | (5) |
|-----------|------|-----|-----|-----|-----|-----|
| $\theta_f$ | -- | 0.00001 | 0.77 | 0.825 | 0.368 | 0.54 |
| $\theta_w$ | -- | 0.00001 | --0.09 | 0.040 | 0.072 | 0.54 |
| $\text{std}(H_f)/\text{std}(H_w)$ | 0.73 | 0.83 | 0.14 | 0.24 | 0.46 | 0.91 |
| $\text{std}(H_f)/\text{std}(Y)$ | 0.96 | 1.19 | 0.27 | 0.36 | 0.62 | 0.71 |
| $\text{std}(H_f)/\text{std}(f)$ | 1.32 | 1.34 | 1.92 | 1.48 | 1.33 | 0.78 |
| $\text{std}(H_f)/\text{std}(Y)$ | 1.12 | 1.24 | 1.16 | 0.96 | 1.00 | 0.74 |
| $\text{std}(W_f)/\text{std}(W_w)$ | 0.80 | 1.00 | 3.18 | 1.89 | 1.43 | 0.94 |
| $\text{std}(W_f)/\text{std}(Y)$ | 0.66 | 0.16 | 0.33 | 0.49 | 0.42 | 0.63 |
| $\text{std}(W_f)/\text{std}(Y)$ | 0.82 | 0.16 | 0.11 | 0.26 | 0.29 | 0.66 |
| $\text{std}(I)/\text{std}(Y)$ | 3.46 | 3.57 | 3.73 | 3.37 | 3.37 | 3.17 |
| $\text{std}(C)/\text{std}(Y)$ | 0.76 | 0.16 | 0.14 | 0.20 | 0.21 | 0.26 |

This table focuses only on the cyclical component of the HP-filtered data. The Data column is calculated from CPS and BEA data. Wages are from IPUMS data from 1976 to 2014. The data used to estimate investment is private nonresidential fixed investment and consumption is PCE from 1976Q3 to 2015Q2 available on FRED. Column (1) shows the baseline model with Frisch elasticities of labor supply equal to zero. Column (2) shows the model results when using elasticities from Jacobsen (2007). Column (3) uses elasticities from Blau and Kahn (2007) in 1980 and column (4) uses the elasticities from 2000. Column (5) uses the elasticities from Chetty et al. (2011).
I calibrate the model to various estimates from the micro literature, and find that allowing the Frisch elasticity of substitution to vary across agents allows the model to still capture the relative volatility of aggregate hours to output and leads to more wage dynamics.

Overall, I find that a simple RBC model augmented to allow for two types of workers, males and females, in a nested CES production function can not only match the relative volatility of hours by gender, but also for total hours. This paper emphasizes that males and females adjust differently across the business cycle in terms of the intensive margin of the labor market through the firm’s labor demand process. These results accentuate the fact that the labor market consists of different agents, and allowing for even broad levels of heterogeneity can increase the model’s tractability with the data. Understanding how and why men and women differ across the business cycle is important for effective policy. More work needs to be done to consider appropriate counter-cyclical economic policy decisions, especially considering the unequal effects of COVID-19 across gender (Alon et al., 2020) and race (Fairlie et al., 2020).

Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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