ON SOFT SUSY BREAKING PARAMETERS IN STRING MODELS
WITH ANOMALOUS $U(1)$ SYMMETRY

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We study the magnitudes of soft SUSY breaking parameters in heterotic string models with anomalous $U(1)$ symmetry. In most cases, $D$-term contribution to soft scalar masses is expected to be comparable to or dominant over other contributions provided that supersymmetry breaking is mediated by the gravitational interaction and/or an anomalous $U(1)$ symmetry and the magnitude of vacuum energy is not more than of order $m_{3/2} M^2$.

1 Introduction

Superstring theories are powerful candidates for the unification theory of all forces including gravity. The supergravity theory (SUGRA) is effectively constructed from 4-dimensional (4D) string model using several methods. The structure of SUGRA is constrained by gauge symmetries including an anomalous $U(1)$ symmetry ($U(1)_A$) and stringy symmetries such as duality. 4D string models have several open questions.

1. There are thousands of effective theories corresponding to 4-D string models. They have, in general, large gauge groups including $U(1)_A$ and many matter multiplets compared with those of the minimal supersymmetric standard model (MSSM). We have not known how to select a realistic model from stringy theoretical point of view yet. The study on flat directions is important because effective theories have, in general, several flat directions in the SUSY limit. Large gauge symmetries can break into smaller ones and extra matter fields can get massive through such flat directions. Actually models with semi-realistic gauge groups and matter contents have been constructed based on heterotic $Z_3$ orbifold models.

2. What is the origin of supersymmetry (SUSY) breaking? Although interesting scenarios such as SUSY breaking mechanism due to gaugino condensation and Scherk-Schwarz mechanism have been proposed, realistic one has not been identified yet.

3. How is the vacuum expectation value (VEV) of dilaton field $S$ stabilized? It is difficult to realize the stabilization with a realistic VEV of $S$ using a Kähler potential at the tree level alone without any conspiracy among several terms which appear in the superpotential. A Kähler potential generally receives...
radiative corrections as well as non-perturbative ones. Such corrections may be sizable for the part related to $S$.

It is important to solve these enigmas in order not only to understand the structure of more fundamental theory at a high energy scale but also to know the complete SUSY particle spectrum at the weak scale, but it is not an easy task because of ignorance of the explicit forms of fully corrected total Kähler potential. At present, it would be meaningful to get any information on SUSY particle spectrum model-independently. On the second question, quite an interesting approach has been adopted. The formulae of soft SUSY breaking parameters have been derived in terms of a few number of free parameters under the assumption that the starting string model has the standard model gauge group and MSSM particle contents and SUSY is broken by $F$-term condensations of the dilaton field and/or moduli fields $M^i$. Their phenomenological implications have been studied intensively.

The soft SUSY breaking parameters can be a powerful probe for high energy physics beyond the MSSM because they are related to the structure of the underlying theory. For example, in SUSY grand unified theories, the pattern of gauge symmetry breakdown can be specified by checking certain sum rules among scalar masses. Specific relations among soft SUSY breaking parameters can play an important role to probe 4D string models.

In this paper, we study the magnitudes of soft SUSY breaking parameters in heterotic string models with $U(1)_A$ and derive model-independent predictions for them without specifying both SUSY breaking mechanism and the dilaton VEV fixing mechanism. We assume that SUSY is broken by $F$-term condensation of $S$, $M^i$ and/or matter fields with non-vanishing $U(1)_A$ charge since the scenario based on $U(1)_A$ as a mediator of SUSY breaking is also possible. Soft SUSY breaking terms are calculated after SUSY breaking and flat direction breakings of gauge symmetries in an effective SUGRA derived from 4D string model. In particular, we make a comparison of magnitudes between $D$-term contribution to scalar masses and $F$-term ones and a comparison of magnitudes among scalar masses, gaugino masses and $A$-parameters. The features of our analysis are as follows. The study is carried out in the framework of SUGRA model-independently, i.e., we do not specify SUSY breaking mechanism, extra matter contents, the structure of superpotential and the form of Kähler potential related to $S$. We treat all fields including $S$ and $M^i$ as dynamical fields.

The paper is organized as follows. In section 2, we explain our starting effective SUGRA derived from heterotic string models with $U(1)_A$. We study the magnitudes of soft SUSY breaking parameters model-independently in section 3. Section 4 is devoted to conclusions and some comments. In appendix, we explain a basic structure of SUGRA with assumptions of SUSY breaking.

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b The stability of $S$ and soft SUSY breaking parameters are discussed in the dilaton dominated SUSY breaking scenario.

c The model-dependent analyses are carried out. Main results in this paper have been reported.
2 Heterotic string model with anomalous $U(1)$

We explain our starting point and assumptions here. The starting theory is an effective SUGRA on 4D heterotic string model. The gauge group $G = G_{SM} \times U(1)_A$ originates from the breakdown of $E_8 \times E_8'$ gauge group in 10D heterotic string theory. $^d$ Here $G_{SM}$ is a standard model gauge group $SU(3)_C \times SU(2)_L \times U(1)_Y$ and $U(1)_A$ is an anomalous $U(1)$ symmetry. The anomaly is canceled by the Green-Schwarz mechanism. $^2$ Chiral multiplets are classified into two categories. One is a set of $G_{SM}$ singlet fields which the dilaton field $S$, the moduli fields $M^i$ and some of matter fields $\phi^m$ belong to. The other one is a set of $G_{SM}$ non-singlet fields $\phi^k$. We denote two types of matter multiplet as $\phi^\lambda = \{\phi^m, \phi^k\}$.

The dilaton field $S$ transforms as $S \to S - i \delta_{GS}^A M \theta(x)$ under $U(1)_A$ with a space-time dependent parameter $\theta(x)$. Here $M = M_{Pl}/\sqrt{\alpha'}$ with $M_{Pl}$ being the Planck mass and $\delta_{GS}^A$ is so-called Green-Schwarz coefficient of $U(1)_A$ which is given by

$$
\delta_{GS}^A = \frac{1}{96\pi^2} Tr Q^A = \frac{1}{96\pi^2} \sum_{\lambda} q^A_{\lambda},
$$

where $Q^A$ is a $U(1)_A$ charge operator, $q^A_{\lambda}$ is a $U(1)_A$ charge of $\phi^\lambda$ and the Kac-Moody level of $U(1)_A$ is rescaled as $k_A = 1$. We find $|\delta_{GS}^A/q_{\lambda m}^A| = O(10^{-1\sim-2})$ in explicit models.

The requirement of $U(1)_A$ gauge invariance yields the form of Kähler potential $K$ as,

$$
K = K(S + \bar{S} + \delta_{GS}^A V_A, M^i, \bar{M}^i, \bar{\phi}_\mu e^{iA}V_A, \phi^A)
$$

up to the dependence on $G_{SM}$ vector multiplets. We assume that derivatives of the Kähler potential $K$ with respect to fields including moduli fields or matter fields are at most of order unity in the units where $M$ is taken to be unity. However we do not specify the magnitude of derivatives of $K$ by $S$ alone. The VEVs of $S$ and $M^i$ are supposed to be fixed non-vanishing values by some non-perturbative effects. It is expected that the stabilization of $S$ is due to the physics at the gravitational scale $M$ or at the lower scale than $M$. Moreover we assume that the VEV of $K_S$ is much bigger than the weak scale, i.e., $O(m_{3/2}) \ll \langle K_S \rangle$. The non-trivial transformation property of $S$ under $U(1)_A$ implies that $U(1)_A$ is broken down at some high energy scale $M_f$. The breaking scale of $U(1)_A$ is defined by $M_f \equiv |\langle \phi^m \rangle|$.

Hereafter we consider only the case with overall modulus field $T$ for simplicity. $^e$ The Kähler potential is, in general, written by

$$
K = K^{(S)}(S + \bar{S} + \delta_{GS}^A V_A) + K^{(T)}(T + \bar{T}) + K^{(S,T)} + \sum_{\lambda,\mu} (s^\lambda_{\mu}(S + \bar{S} + \delta_{GS}^A V_A) + t^\lambda_{\mu}(T + \bar{T}) + u^\lambda_{\mu}(S,T)) \phi^\lambda \bar{\phi}_\mu + \cdots
$$

$^d$It is straightforward to apply our method to 4D string models with a more generic gauge group $G = G'_{SM} \times U(1)'^n \times U(1)_A \times H'$ where $G'_{SM}$ is a group which contains $G_{SM}$ as a subgroup, $U(1)_A$ non-anomalous $U(1)$ symmetries, and $H'$ a direct product of some non-abelian symmetries.

$^e$It is straightforward to apply our method to more complicated situations with multi-moduli fields.
where $K^{(S,T)}$ and $u^{u(S,T)}_\lambda$ are mixing terms between $S$ and $T$. The magnitudes of $\langle K^{(S,T)} \rangle$, $\langle s^\mu_\lambda \rangle$ and $\langle w^u(S,T) \rangle$ are assumed to be $O(\epsilon_1 M^2)$, $O(\epsilon_2)$ and $O(\epsilon_3)$ where $\epsilon_n$'s ($n = 1, 2, 3$) are model-dependent parameters whose orders are expected not to be more than one.\footnote{The existence of $s^\mu_\lambda \phi^\lambda \bar{\phi}_\mu$ term in $K$ and its contribution to soft scalar masses are discussed in 4D effective theory derived from the standard embedding from heterotic M-theory.\footnote{Based on the assumption that SUSY is broken by F-components of $S$ and/or a moduli field, properties of soft SUSY breaking scalar masses have been studied.}} We estimate the VEV of derivatives of $K$ in the form including $\epsilon_n$. For example, $\langle K^u_S \rangle \leq O(\epsilon_p/M)$ ($p = 2, 3$). Our consideration is applicable to models in which some of $\phi^\lambda$ are composite fields made of original matter multiplets in string models if the Kähler potential meets the above requirements. Using the Kähler potential\footnote{\footnote{}} $\hat{K}$, $\hat{D}^A$ is given by

$$
\hat{D}^A = -K_S \delta^A_G S M + \sum_{\lambda,\mu} K^\mu_\lambda \phi^\lambda (q^A \phi)^\lambda + \cdots.
$$

The breaking scale of $U(1)_A$ is estimated as $M_I = O((K_S)^3 \delta^A_G S M/q^A_m)^{1/2}$ from the requirement $\langle D^A \rangle \leq O(m_{3/2} M)$. We require that $M_I$ should be equal to or be less than $M$, and then we find that the VEV of $K_S$ has an upper bound such as $\langle K_S \rangle \leq O(q^A_m M/\delta^A_G S)$. The $U(1)_A$ gauge boson mass squared $(M^2_I)^A$ is given by

$$
(M^2_I)^A = 2g^2_A \left( \langle K^A_S \rangle (\delta^A_G S M)^2 + \sum_{m,n} q^A_m q^A_n \langle K^A_m \rangle \langle \phi^m \rangle \langle \bar{\phi}^n \rangle \right)
$$

where $g_A$ is a $U(1)_A$ gauge coupling constant. The magnitude of $(M^2_I)^A/g^2_A$ is estimated as $Max(O(M^2_I (\delta^A_G S M)^2), O(q^A_m M_I^2))$. We assume that the magnitude of $(M^2_I)^A/g^2_A$ is $O(q^A_m M_I/\delta^A_G S M)^2)$. It leads to the inequality $\langle K^A_S \rangle^3 \leq O((q^A_m M_I/\delta^A_G S M)^2)$.

We discuss the relations among $\langle K_S \rangle$, $\langle K^A_S \rangle$ and $\langle K^A_{SS} \rangle$ using SUSY breaking conditions, i.e., SUSY is broken by the mixture of $S$, $T$ and matter F-components such that $\langle (K^A_S)^{1/2} F_s \rangle$, $\langle F_T \rangle$, $\langle F_m \rangle = O(m_{3/2} M)$, and the stationary conditions of scalar potential. The following relation is derived

$$
\langle K^A_{SS} \rangle^{1/2} = O\left( \frac{\langle G_S \rangle}{M} \right) = O\left( \frac{\langle K_S \rangle}{M} + M \frac{\langle W_S \rangle}{\langle W \rangle} \right)
$$

by the use of the definition $\langle K_{SS} \rangle^2$ if the magnitude of $\langle K^A_{SS} \rangle^{1/2}$ is much bigger than those of $\langle K^A_S \rangle$ and $\langle K^2_S \rangle$. If $\langle K_S \rangle > M^2 (W_S)/\langle W \rangle$ and no cancellation happens among terms in $\langle K_S \rangle$ and $M^2 (W_S)/\langle W \rangle$, we find that $\langle K^A_{SS} \rangle^{1/2} = O(\langle K_S \rangle/M) \leq O(q^A_m M/\delta^A_G S M)$. Further we can get the following relation among $\langle K_S \rangle$, $\langle K^A_S \rangle$ and $\langle K^A_{SS} \rangle$ from the stationary conditions $\langle K_S \rangle$ and $\langle K^A_{SS} \rangle$,

$$
\frac{\langle K^A_{SS} \rangle}{\langle K_S \rangle} = Max\left( O\left( \frac{\langle K^A_S \rangle}{\langle K_S \rangle} \right), O\left( \frac{\langle K^A_{SS} \rangle^{1/2}}{M} \right) \right).
$$

3 Magnitudes of soft SUSY breaking parameters

We study the magnitudes of soft SUSY breaking parameters in SUGRA from heterotic string model with $U(1)_A$.\footnote{The existence of $s^\mu_\lambda \phi^\lambda \bar{\phi}_\mu$ term in $K$ and its contribution to soft scalar masses are discussed in 4D effective theory derived from the standard embedding from heterotic M-theory.\footnote{Based on the assumption that SUSY is broken by F-components of $S$ and/or a moduli field, properties of soft SUSY breaking scalar masses have been studied.}}
on $G_{SM}$ non-singlet fields is given by\[2\]

\[
(m^2)_k^I = (m_{3/2}^2 + \frac{V_F}{M^2})(K_k^I) + \langle F_I \rangle \langle F_J \rangle \langle (R_{II}^{jk}) + \langle X_{II}^{jk} \rangle),
\]
(8)

\[
\langle R_{II}^{jk} \rangle = \langle (K_k^I (K^{-1})^I_j K_{jk}^I - K_{II}^{jk}) \rangle,
\]
(9)

\[
\langle X_{II}^{jk} \rangle = q_k^I (2g_A^2/(M_C^2))((D^A)_I^J)(K_k^I).
\]
(10)

Here we neglect extra $F$-term contributions and so forth since they are model-dependent. The neglect of extra $F$-term contributions is justified if Yukawa couplings between heavy and light fields are small enough and the $R$-parity violation is also tiny enough. We have used Eq.[36] to derive the part related to $D$-term contribution. Note that the last term in r.h.s. of Eq.[36] is negligible when $(M^2_f^I/ q_A^2$ is much bigger than $m_{3/2}^2$. Using the above mass formula, the magnitudes of $\langle R_{II}^{jk} \rangle$ and $\langle X_{II}^{jk} \rangle$ are estimated and given in Table 1. Here we assume $q_k^I/q_m^A = O(1)$.

Now we obtain the following features on $(m^2)_k^I$.

(1) The order of magnitude of $\langle X_{II}^{jk} \rangle$ is equal to or bigger than that of $\langle R_{II}^{jk} \rangle$ except for an off-diagonal part $(I, J) = (S, T)$. Hence the magnitude of $D$-term contribution is comparable to or bigger than that of $F$-term contribution except for the universal part $(m_{3/2}^2 + V_F/M^2)(K_k^I)$.

(2) In case where the magnitude of $(F_m)$ is bigger than $O(m_{3/2}M_f)$ and $M > M_f$, we get the inequality $(m_{3/2}^2 k \equiv (F_I^I)(F_J^J)(X_{II}^{jk}) > O(m_{3/2}^2)$ since the magnitude of $\langle D^A \rangle$ is bigger than $(m_{3/2}^2)$.

(3) In order to get the inequality $O((m_{3/2}^2) k > O((m_{3/2}^2)$, the following conditions must be satisfied simultaneously, $(m_{3/2}^2 k \equiv (F_I^I)(F_J^J)(R_{II}^{jk})$)

\[
\langle F_T \rangle, \langle F_m \rangle \ll O(m_{3/2}M_f), \ \langle F_S \rangle = O\left(\frac{m_{3/2}M_f}{(K_S^S)^{1/2}}\right)
\]

\[
\frac{M^2(K_S^S)}{(K_S^S)/(K_S^S)} < O(1), \quad \frac{\epsilon_p}{(K_S^S)} < O(1), \quad (p = 2, 3)
\]
(11)

unless an accidental cancellation among terms in $\langle D^A \rangle$ happens. To fulfill the condition $\langle F_{T,m} \rangle \ll O(m_{3/2}M_f)$, a cancellation among various terms including $\langle K_I \rangle$ and $\langle M^2W_I/W \rangle$ is required. Note that the magnitudes of $\langle K_T \rangle$ and $\langle K_m \rangle$ are estimated as $O(M)$ and $O(M_f)$, respectively.

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**Table 1: The magnitudes of $\langle R_{II}^{jk} \rangle$ and $\langle X_{II}^{jk} \rangle$**

| $(I, J)$ | $(R_{II}^{jk})$ | $(X_{II}^{jk})$ |
|---------|----------------|----------------|
| $(S, S)$ | $O(\epsilon_p/M^2)$ | $\max(O(K_S^S)(K_S^S), O(\epsilon_p/M^2))$ |
| $(T, T)$ | $O(1/M^2)$ | $\max(O(\epsilon_1/(K_S^S)), O(1/M^2))$ |
| $(m, m)$ | $O(1/M^2)$ | $O(1/M^2)$ |
| $(S, T)$ | $O(\epsilon_p/M^2)$ | $\max(O(\epsilon_1/(K_S^S)), O(\epsilon_3/M^2))$ |
| $(S, m)$ | $O(\epsilon_p M_f/M^4)$ | $\max(O(\epsilon_p/(K_S^S)), O(\epsilon_p/(M M_f)))$ |
| $(T, m)$ | $O(M_f/M^4)$ | $\max(O(\epsilon_3/(K_S^S)), O(1/(M M_f)))$ |
The gauge kinetic function is given by

$$f_{\alpha\beta} = k_\alpha \frac{S}{M} \delta_{\alpha\beta} + \epsilon_\alpha \frac{T}{M} \delta_{\alpha\beta} + f^{(m)}_{\alpha\beta} (\phi^\lambda)$$  \hspace{1cm} (12)$$

where $k_\alpha$’s are Kac-Moody levels and $\epsilon_\alpha$ is a model-dependent parameter. The gauge coupling constants $g_\alpha$’s are related to the real part of gauge kinetic functions such that $g_\alpha^2 = (\text{Re} f_{\alpha\alpha})^{-1}$. The magnitudes of gaugino masses and $A$-parameters in MSSM particles are estimated using the formulae

$$M_\alpha = \langle F^I \rangle \langle h_{\alpha I} \rangle,$$

$$\langle h_{\alpha I} \rangle \equiv \langle \text{Re} f_{\alpha I} \rangle^{-1} \langle f_{\alpha I} \rangle / 2,$$

$$A_{kl\nu} = \langle F^{I'} \rangle \langle a_{kl\nu I} \rangle,$$

$$\langle a_{kl\nu I} \rangle \equiv \langle f_{kl\nu \cdot I} \rangle + \left( \frac{K_I}{M} \right) \langle f_{kl\nu} \rangle - \langle (K_{I})^{-1}_l \rangle \langle f_{kl\nu} \rangle.$$

The result is given in Table 2. Here we assume that $g_\alpha^2 = O(1)$.

In case that SUSY is broken by the mixture of $S, T$ and matter $F$-components such that $\langle (K^S_3)^{1/2} F_S \rangle, \langle F_T \rangle, \langle F_m \rangle = O(m_{3/2}^2 M)$, we get the following relations among soft SUSY breaking parameters

$$(m^2)_k \geq (m^2)_D = O \left( m_{3/2} \frac{M^2}{M^2} \right) \geq (A_{kl\nu})^2 = O(m_{3/2}^2),$$ \hspace{1cm} (17)

$$(M_\alpha)^2 = O(m_{3/2}^2) \cdot \text{Max} \left( O(\langle K^S_3 \rangle^{-1}), O(\epsilon_\alpha^2), O \left( \frac{M^2}{M^2} \right) \right).$$ \hspace{1cm} (18)

Finally we discuss three special cases of SUSY breaking scenario.

1. In the dilaton dominant SUSY breaking scenario

$$\langle (K^S_3)^{1/2} F_S \rangle = O(m_{3/2}^2 M) \gg \langle F_T \rangle, \langle F_m \rangle,$$ \hspace{1cm} (19)

the magnitudes of soft SUSY breaking parameters are estimated as

$$(m^2)_k = O(m_{3/2}^2) \cdot \text{Max} \left( O(1), O \left( \frac{M^2 \langle K^S_3 \rangle}{\langle K^S_3 \rangle \langle K^S \rangle} \right), O \left( \frac{\epsilon_p}{\langle K^S \rangle} \right) \right),$$

$$M_\alpha = O \left( \frac{m_{3/2}}{\langle K^S_3 \rangle^{1/2}} \right), \quad A_{kl\nu} = O(m_{3/2}) \cdot \text{Max} \left( O \left( \frac{\langle K^S \rangle}{M} \right), O(\epsilon_p) \right).$$

Hence we have a relation such that $O((m^2)_k) \geq O((A_{kl\nu})^2)$.  

| Table 2: The magnitudes of $\langle h_{\alpha I} \rangle$ and $\langle a_{kl\nu I} \rangle$ |
|------------------|------------------|
| $I$               | $\langle h_{\alpha I} \rangle$ | $\langle a_{kl\nu I} \rangle$ |
| $S$               | $O(1/M)$          | $\text{Max}(O((K^S_3)^{1/2}), O(\epsilon_p/M))$ |
| $T$               | $O(\epsilon_\alpha/M)$ | $O(1/M)$ |
| $m$               | $O(M^2)$          | $O(M^2)$ |
Gauginos can be heavier than scalar fields if \( \langle K_S^S \rangle \) is small enough and the inequality \( O(M^2 \langle K_S^S \rangle) < O(\langle K_S \rangle) \) satisfied. In this case, dangerous flavor changing neutral current (FCNC) effects from squark mass non-degeneracy are avoided because the radiative correction due to gauginos dominates in scalar masses at the weak scale. It is shown that gauginos are much lighter than scalar fields from the requirement of the condition of vanishing vacuum energy in the SUGRA version of model \( \mathbb{D} \) proposed by P. Binétruy and E. Dudas. We study it using the relations among the magnitudes of soft SUSY breaking parameters estimated as \( \langle \delta G_S^A \rangle \). We get the relation such that \( M^2 / \langle K_S \rangle \) is estimated as \( O(\langle W \rangle) \). This relation leads to \( M_1 = M \) from the third assumption and the relation such that \( M \langle K_S^S \rangle / \langle K_S \rangle = O(\langle W \rangle) \). We obtain the relation \( M \langle K_S^S \rangle / \langle K_S \rangle = O(\langle W \rangle) \) using Eq. (20). Hence it is shown that the magnitude of gaugino masses is comparable to or smaller than that of scalar masses. It is necessary to relax some of assumptions in order to have the SUSY spectrum at \( M_1 \) such that gauginos are much heavier than scalar fields. A possibility is to introduce a constant term in \( W_{\text{non}} \) is generally given by

\[
W_{\text{non}} = \sum_i a_i (\phi^\lambda, T) \exp \left( \frac{-b_i S}{\delta G_S^A M} \right)
\]

where \( a_i \)'s are some functions of \( \phi^\lambda \) and \( T \), and \( b_i \)'s are model-dependent parameters of \( O(\lambda^A) \). Using the second assumption, Eqs. (20) and (21), we get the relation \( \langle K_S^S \rangle / \langle W \rangle = O(\langle W \rangle) \). Hence it is shown that the magnitude of gaugino masses is comparable to or smaller than that of scalar masses.

2. In the moduli dominant SUSY breaking scenario

\[
\langle F_T \rangle = O(m_{3/2} M) \gg \langle (K_S^S)^{1/2} F_S \rangle, \langle F_m \rangle ;
\]

the magnitudes of soft SUSY breaking parameters are estimated as

\[
(m^2)_k = O(m^2_{3/2}) \cdot \max \left( O(1), O \left( \frac{\epsilon_1 M}{\langle K_S \rangle} \right) \right) ,
\]

\[
M_a = O(\epsilon_a m_{3/2}) , \quad A_{klm} = O(m_{3/2}).
\]

Hence we have a relation such that \( O((m^2)_k) \geq O((A_{kl})^2) \geq O((M_a)^2) \). The magnitude of \( \mu_{TT} \) is estimated as \( \mu_{TT} = O(m_{3/2}) \).

3. In the matter dominant SUSY breaking scenario

\[
\langle F_m \rangle = O(m_{3/2} M) \gg \langle (K_S^S)^{1/2} F_S \rangle, \langle F_T \rangle ,
\]

the magnitudes of soft SUSY breaking parameters are estimated as

\[
(m^2)_k = O \left( \frac{m^2_{3/2} M^2}{M_f^2} \right) , \quad M_a, A_{kl} = O \left( \frac{M_1}{M_f} \right).
\]

The relation \( (m^2)_k \gg O((M_a)^2) = O((A_{kl})^2) \) is derived when \( M \gg M_f \). The magnitude of \( \mu_{mn} \) is estimated as \( \mu_{mn} = O(m_{3/2} M/M_f) \).
4 Conclusions

We have studied the magnitudes of soft SUSY breaking parameters in heterotic string models with $G_{SM} \times U(1)_A$, which originates from the breakdown of $E_8 \times E_8$, and derive model-independent predictions for them without specifying both SUSY breaking mechanism and the dilaton VEV fixing mechanism. In particular, we have made a comparison of magnitudes between $D$-term contribution to scalar masses and $F$-term ones and a comparison of magnitudes among scalar masses, gaugino masses and $A$-parameters under the condition that $O(m_{3/2}) \ll \langle K_S \rangle \leq O(q^A M / \delta^2_{GS})$, $(M_k^2)^A / g^2_{\Lambda} = O(q^{A^2} M_f^2)$ and $\langle V \rangle \leq O(m_{3/2} M^2)$. The order of magnitude of $D$-term contribution of $U(1)_A$ to scalar masses is comparable to or bigger than that of $F$-term contribution $\langle F^I \rangle \langle F_J \rangle \langle R_{11} \rangle$ except for the universal part $(m_{3/2}^2 + (V_F) / M^2) \langle K_S \rangle$. If the magnitude of $F$-term condensation of matter fields $(F_m)$ is bigger than $O(m_{3/2} M_I)$, the magnitude of $D$-term contribution $(m_{D}^2)_{k}$ is bigger than $O(m_{3/2}^2)$. In general, it is difficult to realize the inequality $O((m_{D}^2)_{k}) < O((m_{F}^2)_{k})$ unless conditions such as Eq.11) are fulfilled. We have also discussed relations among soft SUSY breaking parameters in three special scenarios on SUSY breaking, i.e., dilaton dominant SUSY breaking scenario, moduli dominant SUSY breaking scenario and matter dominant SUSY breaking scenario. In the same way, we can estimate the magnitude of $D$-term contribution to scalar masses for non-anomalous diagonal charges. The magnitudes of $\langle X_{I}^{(k)} \rangle$ is given by $O(\epsilon_{F} / M^2)$, $O(1 / M^2)$, $O(1 / M_B^2)$, $O(\epsilon_{3} / M^2)$, $O(\epsilon_{F} / (M M_B))$ and $O(1 / (M M_B))$ for $(I, J) = (S, S)$, $(T, T)$, $(m, m)$, $(S, T)$, $(S, m)$ and $(T, m)$ where $M_B$ is a breaking scale of extra gauge symmetry.

The $D$-term contribution to scalar masses with different broken charges as well as the $F$-term contribution from the difference among modular weights can destroy universality among scalar masses. The non-degeneracy among squark masses of first and second families endangers the discussion of the suppression of FCNC process. On the other hand, the difference among broken charges is crucial for the scenario of fermion mass hierarchy generation. It seems to be difficult to make two discussions compatible. There are several way outs. The first one is to construct a model that the fermion mass hierarchy is generated due to non-anomalous $U(1)$ symmetries. In the model, $D$-term contributions of non-anomalous $U(1)$ symmetries vanish in the dilaton dominant SUSY breaking case and it is supposed that anomalies from contributions of the MSSM matter fields are canceled out by an addition of extra matter fields. The second one is to use “stringy” symmetries for fermion mass generation in the situation with degenerate soft scalar masses.

Finally we give a comment on moduli problem. If the masses of dilaton or moduli fields are of order of the weak scale, the standard nucleosynthesis should be modified because of a huge amount of entropy production. The dilaton field does not cause dangerous contributions in the case with $\langle (K_S^X)^{1/2} F_S \rangle = O(m_{3/2} M)$ if the magnitude of $\langle K_S^X \rangle$ is small enough, because the magnitudes of $(m_{D}^2)_{S}$ is given by $O(m_{3/2}^2 / \langle K_S^X \rangle^2)$.

\footnote{This possibility has been pointed out in the last reference in 13.}
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Appendix

We review the scalar potential part in SUGRA. It is specified by two functions, the total Kähler potential \( G(\phi, \bar{\phi}) \) and the gauge kinetic function \( f_{\alpha \beta}(\phi) \) with \( \alpha, \beta \) being indices of the adjoint representation of the gauge group. The former is a sum of the Kähler potential \( K(\phi, \bar{\phi}) \) and (the logarithm of) the superpotential \( W(\phi) \)

\[
G(\phi, \bar{\phi}) = K(\phi, \bar{\phi}) + M^2 \ln |W(\phi)/M^3|^2. \tag{23}
\]

We have denoted scalar fields in the chiral multiplets by \( \phi^I \) and their complex conjugate by \( \bar{\phi}^J \). The scalar potential is given by

\[
V = M^2 e^{G/M^2} (G_I (G^{-1})_J G^J - 3M^2) + \frac{1}{2} (Re f^{-1})_{\alpha \beta} \hat{D}^\alpha \hat{D}^\beta \tag{24}
\]

where

\[
\hat{D}^\alpha = G_I (T^\alpha \phi)^I = (\bar{\phi} T^\alpha) J G^J. \tag{25}
\]

Here \( G_I = \partial G/\partial \phi^I \), \( G^I = \partial G/\partial \bar{\phi}^I \) etc., and \( T^\alpha \) are gauge transformation generators. Also in the above, \( (Re f^{-1})_{\alpha \beta} \) and \( (G^{-1})_J^I \) are the inverse matrices of \( Re f_{\alpha \beta} \) and \( G^I_J \), respectively, and a summation over \( \alpha,... \) and \( I,... \) is understood. The last equality in Eq. (25) comes from the gauge invariance of the total Kähler potential. The \( F \)-auxiliary fields of the chiral multiplets and the \( D \)-auxiliary fields of the vector multiplets are given by

\[
F^I = Me^{G/2M^2} (G^{-1})_J G^J, \quad D^\alpha = (Re f^{-1})_{\alpha \beta} \hat{D}^\beta, \tag{26}
\]

respectively. Using \( F^I \) and \( D^\alpha \), the scalar potential is rewritten down by

\[
V = V_F + V_D, \quad V_F = F_I K^I_J F^J - 3M^4 e^{G/M^2}, \tag{27}
\]

\[
V_D = \frac{1}{2} Re f_{\alpha \beta} \hat{D}^\alpha \hat{D}^\beta. \tag{28}
\]

Let us next summarize our assumptions on SUSY breaking. The gravitino mass \( m_{3/2} \) is given by

\[
m_{3/2} = \langle Me^{G/2M^2} \rangle \tag{29}
\]

where \( \langle \cdots \rangle \) denotes the VEV. As a phase convention, it is taken to be real. We identify the gravitino mass with the weak scale in most cases. It is assumed that SUSY is spontaneously broken by some \( F \)-term condensations (\( \langle F \rangle \neq 0 \)) for singlet fields under the standard model gauge group and/or some \( D \)-term condensations.
\[ (\langle D \rangle \neq 0) \] for broken gauge symmetries. We require that the VEVs of \( F^I \) and \( D^\alpha \) should satisfy

\[
\langle (F^I K^J F^J)^{1/2} \rangle \leq O(m_{3/2}^2 M), \quad \langle D^\alpha \rangle \leq O(m_{3/2}^2 M)
\]

for each pair \((I, J)\). Note that we allow the non-zero vacuum energy \( \langle V \rangle \) of order \( m_{3/2}^2 M^2 \) at this level, which could be canceled by quantum corrections.

In order to discuss the magnitudes of several quantities, it is necessary to see consequences of the stationary condition \( \partial V / \partial \phi^I = 0 \). From Eq.\((24)\), we find

\[
\partial V / \partial \phi^I = G_I \left( \frac{V_F}{M^2} + 2 M^2 e^{G/M^2} \right) + M e^{G/2M^2} G_{IJ} F^J - F_I G^J G_{J'} F^J - \frac{1}{2} \left( \text{Ref}_{\alpha \beta} \right)_{IJ} D^\alpha D^\beta + D^\alpha (\bar{\phi} T^\alpha)_{J} G_{IJ}.
\]

Taking its VEV and using the stationary condition, we derive the formula

\[
m_{3/2} \langle G_{IJ} \rangle \langle F^J \rangle = - \langle G_I \rangle \left( \frac{V_F}{M^2} + m_{3/2}^2 \right) + \langle F_I \rangle \langle G^J \rangle \langle F^J \rangle + \frac{1}{2} \langle \text{Ref}_{\alpha \beta} \rangle_{IJ} \langle D^\alpha \rangle \langle D^\beta \rangle - \langle D^\alpha \rangle \langle \bar{\phi} T^\alpha \rangle \langle G^J \rangle.
\]

We can estimate the magnitude of SUSY mass parameter \( \mu_{IJ} \equiv m_{3/2} \langle \langle G_{IJ} \rangle \rangle / M^2 - \langle G_I \rangle \langle G_J \rangle / M^2 \) for broken gauge symmetries. Using Eq.\((32)\) and multiplying \( \langle T^\alpha \phi \rangle^I \) to Eq.\((33)\), a heavy-real direction is projected on. Using the identities derived from the gauge invariance of the total Kähler potential

\[
G_{IJ}(T^\alpha \phi)^J + G_J(T^\alpha)_{J} - K^J_I(\bar{\phi} T^\alpha)_{J} = 0,
\]

\[
K^J_I(T^\alpha \phi)^J = G^J_I(T^\alpha)_{J} - [G^J_J(\bar{\phi} T^\alpha)_{J}]^J = 0,
\]

we obtain

\[
\frac{\partial V}{\partial \bar{\phi}^I}(T^\alpha \phi)^J = \left( \frac{V_F}{M^2} + 2 M^2 e^{G/M^2} \right) \bar{D}^\alpha - F_I F^J(\bar{D}^\alpha)^J - \frac{1}{2} \langle \text{Ref}_{\alpha \beta} \rangle_{IJ} \langle D^\alpha \rangle^J \bar{D}^\beta + \langle \bar{\phi} T^\alpha \rangle_J \langle D^\alpha \rangle^J \bar{D}^\beta.
\]

Taking its VEV and using the stationary condition, we derive the formula

\[
\left( \frac{M^2}{2 g_{\alpha \beta}} \right) (\langle T^\alpha \phi \rangle)^I \langle \bar{D}^\alpha \rangle^J = \langle F_I \rangle \langle F^J \rangle \langle \bar{D}^\alpha \rangle^J,
\]

\[
+ \frac{1}{2} \langle \text{Ref}_{\alpha \beta} \rangle_{IJ} \langle (T^\alpha \phi) \rangle^I \langle D^\beta \rangle \langle D^\gamma \rangle.
\]

where \( (M^2)^{\alpha \beta} = 2 g_{\alpha \beta} (\bar{\phi} T^\beta_J) K^J_I (T^\alpha \phi)^I \) is the mass matrix of the gauge bosons and \( g_{\alpha} \) and \( g_{\beta} \) are the gauge coupling constants. Using Eq.\((34)\), we can estimate the magnitude of D-term condensations \( \langle D^\beta \rangle \). The formula of \( \langle D^\beta \rangle \) is given by

\[
\langle D^\beta \rangle = \langle F_I \rangle \langle F^J \rangle \frac{2 g_{\alpha} g_{\beta}}{(M^2)^{\alpha \beta}} \langle \bar{D}^\alpha \rangle^J
\]

in case where \( (M^2)^{\alpha \beta} / g_{\alpha} g_{\beta} \gg O(m_{3/2}^2) \).
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