LINE OVERLAP AND SELF-SHIELDING OF MOLECULAR HYDROGEN IN GALAXIES

NICKOLAY Y. GNEDIN1,2,3 AND BRUCE T. DRAINE4
1 Particle Astrophysics Center, Fermi National Accelerator Laboratory, Batavia, IL 60510, USA; gnedin@fnal.gov
2 Kavli Institute for Cosmological Physics, The University of Chicago, Chicago, IL 60637, USA; andrey@oddjob.uchicago.edu
3 Department of Astronomy & Astrophysics, The University of Chicago, Chicago, IL 60637, USA
4 Princeton University Observatory, Princeton, NJ 08544-1001, USA; draine@astro.princeton.edu

Received 2014 June 12; accepted 2014 September 3; published 2014 October 9

ABSTRACT

The effect of line overlap in the Lyman and Werner bands, often ignored in galactic studies of the atomic-to-molecular transition, greatly enhances molecular hydrogen self-shielding in low metallicity environments and dominates over dust shielding for metallicities below about 10% solar. We implement that effect in cosmological hydrodynamics simulations with an empirical model, calibrated against the observational data, and provide fitting formulae for the molecular hydrogen fraction as a function of gas density on various spatial scales and in environments with varied dust abundance and interstellar radiation field. We find that line overlap, while important for detailed radiative transfer in the Lyman and Werner bands, has only a minor effect on star formation on galactic scales, which, to a much larger degree, is regulated by stellar feedback.

Key words: cosmology: theory – galaxies: evolution – galaxies: formation – methods: numerical – stars: formation

Online-only material: color figures

1. INTRODUCTION

In the last several years, an important advance has been made in understanding star formation on galactic scales. Both local (Leroy et al. 2008; Bigiel et al. 2008; Bolatto et al. 2011; Bigiel et al. 2011; Leroy et al. 2012, 2013) and intermediate redshift (Genzel et al. 2010; Daddi et al. 2010; Tacconi et al. 2013) observational studies find that the star formation rate surface density on kiloparsec scales correlates well, and approximately linearly, with the surface density of molecular gas.

Hence, from a theoretical perspective, tracing the formation of molecular hydrogen in cosmological and galactic-scale simulations is a prerequisite for modeling star formation and the subsequent stellar feedback. Several groups have recently implemented models of atomic-to-molecular gas transition in cosmological simulations codes and explored their predictions (Pelupessy et al. 2006; Robertson & Kravtsov 2008; Gnedin et al. 2009; Pelupessy & Papadopoulos 2009; Gnedin & Kravtsov 2011; Christensen et al. 2012; Kuhlen et al. 2012, 2013; Thompson et al. 2014). However, all these models either completely ignored the self-shielding of molecular hydrogen or included it in the approximation where each of the Lyman and Werner band absorption lines is treated as isolated—in that limit, shielding by cosmic dust is important and the characteristic column density of the atomic-to-molecular transition scales inversely proportional to the dust abundance.

Such an approximation is appropriate for sufficiently low column densities of molecular gas. However, for \( N_{\text{H}_2} \gg 10^{21} \text{ cm}^{-2} \), the damping wings of individual Lyman and Werner band absorption lines begin to overlap (Stecher & Williams 1967; Black & Dalgarro 1977; Draine & Bertoldi 1996). The process of line overlap, when applied to a molecular interstellar medium (ISM) with supersonic turbulence, results in a significant enhancement in the role of self-shielding; in sufficiently low metallicity environments, self-shielding may, in fact, dominate over the dust shielding, as we demonstrate below.

In complex environments of realistic galaxies, there are many other physical processes that affect the chemical and dynamical state of the ISM; molecular hydrogen self-shielding may or may not be an important process in such environments, but a study is needed to explore its role. This paper aims at addressing a part of this question—namely, the role of line overlap in the transition from atomic to molecular hydrogen in galaxies.

2. SELF-SHIELDING OF MOLECULAR HYDROGEN

The molecular hydrogen photodissociation rate \( \zeta_{\text{pd}} \) can be written as

\[ \zeta_{\text{pd}} = S_{\text{H}_2} \times S_{\text{dust}} \times \zeta_{\text{pd}}^0, \]

(1)

where \( \zeta_{\text{pd}}^0 \) is the “free space” rate, \( S_{\text{dust}} \) is the reduction in the rate due to shielding by dust, and \( S_{\text{H}_2} \) is the reduction factor due to \( \text{H}_2 \) self-shielding. Self-shielding of molecular hydrogen has been studied extensively since the pioneering work of Stecher & Williams (1967). A commonly used formula that conveniently parameterizes the self-shielding effects for \( \text{H}_2 \) with (local) one-dimensional velocity dispersion \( \sigma_v \) was given by Draine & Bertoldi (1996),

\[ S_{\text{H}_2} = \frac{0.965}{(1 + x/b_s)^2} + \frac{0.035}{\sqrt{1 + x}} \exp \left( -\frac{\sqrt{1 + x}}{1180} \right), \]

(2)

where \( x \equiv N_{\text{H}_2}/5 \times 10^{14} \text{ cm}^{-2} \) and \( b_s \equiv b/k\text{m s}^{-1} \), with \( b \equiv \sqrt{2} \sigma_v \). Here, \( N_{\text{H}_2} \) is the column density of \( \text{H}_2 \) between the point of interest and the sources of 111–91 nm radiation that can dissociate \( \text{H}_2 \) with \( v = 0 \) and \( J = 0, 1, 2 \). The accuracy of this shielding function has been confirmed by recent work by Sternberg et al. (2014, see their Figure 5), who also explicitly accounted for line overlap in the Lyman and Werner bands.

Equation (2) is suitable for the idealized case of a uniform slab of gas with no internal motions. Real molecular clouds are, however, supersonically turbulent on scales above the sonic length, \( L_s \lesssim 1 \text{ pc} \). If we consider two fluid elements in the molecular cloud separated by a large distance \( L \gg L_s \), then the velocity difference \( \Delta v \sim b (L/l)^{\gamma} \) (with \( \gamma \approx 0.5 \) from the Larson Law; McKee & Ostriker 2007) between them would be large, much larger than the local Doppler width \( b \) of each Lyman and Werner band line. Hence, the Doppler cores of these
two fluid elements would not shield each other even if they contain substantial amounts of molecular hydrogen.

In other words, a typical fluid element of size $l_e$ in the molecular cloud can be strongly shielded by another fluid element only if it accidentally happens to fall at the same line-of-sight velocity. In turbulent gas with velocity dispersion $\Delta v$, the probability of another fluid element being within the line width $b$ from any given fluid element is about $b/\Delta v$. Hence, a typical fluid element at depth $L$ inside a molecular cloud should be shielded by $dN_{H_2}/d\nu \sim N_{H_2}/\Delta v \sim (n_{H_2})_L/L/\Delta v$, where $(n_{H_2})_L$ is the average molecular hydrogen density on a sonic scale and $\Delta v = b(L/L_c)^{1/2}$ corresponds to the velocity width at depth $L$. For the Larson law with a slope of $\gamma \approx 0.5$ that translates into a shielding column density which is the geometric mean of the total column density at depth $L$ and the column density on sonic scale, $N_{H_2} \sim \langle n_{H_2} \rangle_x (L/L_c)^{1/2}$, which is much less than the total column density through the cloud, since $l_e \ll L$.

However, there is a major flaw in this argument. Absorption lines are narrow only at low column densities, before the damping wings become important. At sufficiently large column densities, absorption lines in the Lyman and Werner bands become broad enough to begin to overlap (Black & Dalgarno 1977; Draine & Bertoldi 1996), effectively rendering relative velocity shifts between different fluid elements unimportant. In other words, at sufficiently large column densities, line radiative transfer in the Lyman and Werner bands behaves as continuum radiative transfer and the effective length over which the column density is accumulated becomes the size of the whole cloud.

In Equation (2), the line overlap is described by the second term. To account for the supersonic turbulence inside the molecular cloud, Equation (2) can be modified by introducing two variables, $x_1$ and $x_2$, in place of $x$ as

$$S_{H_2}(N_{H_2}) = \frac{0.965}{(1 + x_1/b)^2} + \frac{0.035}{\sqrt{1 + x_2}} \exp \left(-\frac{\sqrt{1 + x_2}}{1180}\right), \quad (3)$$

where $x_1 = (N_{H_2}N_e)^{1/2}/5 \times 10^{14}$ cm$^{-2}$, $x_2 = N_{H_2}/5 \times 10^{14}$ cm$^{-2}$, $N_e = \langle n_{H_2} \rangle_x L / (n_{H_2})_L$. The good news is that line overlap becomes completely subdominant to the self-shielding of molecular hydrogen.

3. MODELING MOLECULAR HYDROGEN ON GALACTIC SCALES

Equation (3) applies to a given location in the molecular cloud. Modern cosmological or galactic-scale simulations may not resolve molecular clouds at all, or may resolve them only down to parsec scales. Hence, it is unlikely that Equation (3) can be used directly. Instead, we can imagine the whole space being tessellated into regions (say, simulation cells, not necessarily all of the same size), some of which include pieces of molecular clouds. Each such region $j$ has a full distribution of column densities inside it, $\phi_j(N)$ (which samples both different locations inside the region and different directions at a given location). Hence, the average shielding factor is

$$\langle S_{H_2} \rangle_j = \int S_{H_2}(N) \phi_j(N) dN,$$

which, by the first mean value theorem for integration, can be represented as

$$\langle S_{H_2} \rangle_j = S_{H_2}(N_{eff}) \int \phi_j(N) dN = S_{H_2}(N_{eff})$$

(since $\phi_j$ is normalized to unity by definition). If the distribution $\phi_j$ was known, one could compute the effective column density $N_{eff}$ but, at present, there are no models that attempt to determine $\phi_j$. Hence, we need to come up with an ansatz for $N_{eff}$.

Following the Gnedin & Kravtsov (2011, hereafter GK11) approach, we adopt a simple “Sobolev-like” ansatz,

$$N_{eff} \approx n_{H_2}/L_{Sob},$$

where

$$L_{Sob} = \frac{\rho}{2|\nabla \rho|}$$

is calibrated from exact ray-tracing calculations (see GK11). With such an approximation, the complete set of equations is obtained.

In order to explore the effect of line overlap on galactic scales, we follow the methodology described in GK11. We refer the reader to that paper for full details; here, we only mention that we use the Adaptive Refinement Tree (ART) code (Kravtsov 1999; Kravtsov et al. 2002; Rudd et al. 2008) to follow a region of the universe containing a couple dozen galaxies of various masses up to $3 \times 10^{11}$ $M_\odot$ at $z \sim 3$ with a mass resolution of $1.3 \times 10^6$ $M_\odot$ and a peak spatial resolution of 260 comoving parsecs (65 pc in physical units at $z = 3$). In order to explore the environmental dependence of the atomic-to-molecular transition, we run the simulations in the “fixed ISM” mode, in which we impose a 91.2–111 nm radiation field in and a fixed dust-to-gas ratio throughout the computational domain.

We parameterize the 91.2–111 nm interstellar radiation field and the dust-to-gas ratio in our “fixed ISM” simulations in units of their values in the Milky Way so that $U_{MW} = 1$ and $D_{MW} = 1$ corresponds to the Milky Way ISM, $U_{MW} = 10$ and $D_{MW} = 0.5$ corresponds to the ISM conditions with 10 times higher radiation field and half the Milky Way dust-to-gas ratio, etc.

The complete description of the numerical model for the formation of molecular hydrogen is presented in the Appendix of GK11. The only changes to that model in this paper are (1) the cooling and heating of the gas are modeled with more physically realistic cooling and heating functions of Gnedin & Hollon (2012), and, most importantly, (2) the self-shielding factor for molecular hydrogen $S_{H_2}$ is determined from Equation (3) above rather than from Equation (A11) of GK11, which, using a power-law approximation to the self-shielding factor,

$$S_{H_2} \approx \begin{cases} 1, & \text{for } N_{H_2} < 10^{14} \text{ cm}^{-2}, \\ \left(\frac{N_{H_2}}{10^{14} \text{ cm}^{-2}}\right)^{-3/4}, & \text{for } N_{H_2} > 10^{14} \text{ cm}^{-2}, \end{cases} \quad (4)$$

does not explicitly account for line overlap (the exponential factor in Equations (2) and (3)).

The H$_2$ formation model dependents on two parameters: the clumping factor $C_p = (n_{H_2}^{cell}/n_{H_2}^{cell}_0)$ of gas inside a simulation cell (which is not resolved, and hence needs to be parameterized) and the sonic length $l_s$. The good news is that line overlap effectively eliminates any dependence on $l_s$, so, in practice, just one parameter, the clumping factor, matters.
4. CALIBRATION OF THE H$_2$ FORMATION MODEL

While one can come up with reasonable estimates for the model parameters, the ultimate parameter choice is dictated by comparison with observations, given that our model is essentially an empirical one. For calibrating the model, we use two types of observational data: measurements of gas fractions along lines of sight to individual stars for atomic (Goldsmith & Li 2005) and molecular gas in the Milky Way and Magellanic Clouds (Tumlinson et al. 2002; Gillmon et al. 2006; Wolfire et al. 2008; Bolatto et al. 2011), and the measurements of atomic and molecular gas surface densities in nearby spirals from Wong & Blitz (2002).

The best-fit H$_2$ formation model is shown in Figures 1 and 2 for two values of the clumping factor $C_\rho = 10$ and 30. Both values provide acceptable fits to the data; the $C_\rho = 10$ case fits the Wong & Blitz (2002) measurement perfectly while the $C_\rho = 30$ is slightly better for the translucent clouds. The model is insensitive to the value of the sonic length $l_s$ as long as $l_s < 10$ pc.

One can reasonably wonder whether the calibration data we use actually test the effect of line overlap. The Milky Way data and Wong & Blitz (2002) galaxies have too high metallicities for the line overlap to be important. Hence, only the SMC data have any discriminating potential for models with different treatments of line overlap. To verify that, in the right panel of Figure 1 we also show an SMC-like model that neglects line overlap (i.e., uses Equation (2) instead of Equation (3) for modeling H$_2$ self-shielding).

For our best-fit parameters, the model without line overlap is clearly disfavored by the SMC data. One can recalibrate the model and make the no-line-overlap model fit the SMC data better, but such a model would then fail the Milky Way calibration. Hence, the combination of the Milky Way and SMC constraints appears to favor the model with nonlocal line overlap treatment, thus justifying our approach.
One may note, however, that GK11 did manage to reach a reasonable match for both the Milky Way and the SMC calibration data without line overlap. That apparent discrepancy is due to the fact that GK11 had no access to Bolatto et al. (2011) data (small black points on the left panel of Figure 1), but only to the older data from Tumlinson et al. (2002). The latter data set is indeed in acceptable agreement with the blue points from the left panel of Figure 1. Hence, the measurements of Bolatto et al. (2011) are the only data set that prefers the model with nonlocal line overlap over that without it.

5. RESULTS

With the complete model in hand, we repeat the procedure of GK11: sampling the interstellar UV radiation field strength and dust-to-gas ratios over a grid of values to investigate the dependence of the atomic-to-molecular hydrogen transition on these two physical parameters.

Figure 3 shows the first main result of this paper: the dependence of the characteristic density of the atomic-to-molecular hydrogen transition $n_{1/2}$ (defined as the density at which the mean H$_2$ fraction reaches 50%) on the interstellar UV radiation field and the dust-to-gas ratio. It can be compared directly to Figure 2 of GK11.

The primary feature of Figure 3 is the saturation of $n_{1/2}$ as a function of the dust-to-gas ratio for $D_{MW} \lesssim 0.1$—this is a direct consequence of line overlap, which dominates over dust shielding for low dust abundances. At extremely low dust abundances ($D_{MW} \lesssim 10^{-3}$), the timescale for molecular hydrogen formation (which is proportional to the dust abundance) becomes long compared to other galactic timescales (rotation period, accretion timescale, etc.) and the situation becomes more complex (Krumholz 2012). In addition, the timescale for dust formation also becomes long (Draine 2009; Inoue 2011), and in such a dynamical environment there is no alternative to full time-dependent nonequilibrium treatment.

At the higher dust-to-gas ratios considered here, the atomic-to-molecular transition is well-behaved, and a useful approximation can be developed as an alternative to the detailed calculation of molecular chemistry.

Following the approach of GK11, we can fit $n_{1/2}$ as a function of $U_{MW}$ and $D_{MW}$. In addition, we also consider the effect of averaging our numerical results over a larger spatial scale $L$, which would be useful if these fits are to be used in lower-resolution simulations.

It is important to note, however, that the spatial averaging of a high-resolution simulation is different from using the full chemical model in a lower-resolution simulation. Namely, as GK11 noted, the full chemical model is reasonably resolution-independent and can be used in simulations with spatial resolutions better than about 130 pc, while coarser resolved simulations should not use the full chemical model at all, and that statement is not affected by our account of the line overlap. If, however, a simulation is designed to use our fitting formulae, then the simulation resolution needs to be explicitly accounted for, since spatial averaging is explicitly dependent on the averaging scale. Consider, for example, a simulation with a spatial resolution of, say, 260 pc. One can imagine that each (260 pc)$^3$ cell in such a simulation contains $4^3$ of the 65 pc cells we use here. Hence, in order to provide atomic and molecular abundances in each cell of the low-resolution simulation, we should sum up the atomic and molecular abundances over $4^3$ of our high-resolution cells. Since, in the low-resolution simulation, the physical quantities are only defined in (260 pc)$^3$ cells, we would like to express the total atomic and molecular abundances in our $4^3$ cells as functions of atomic and molecular abundances in the low-resolution simulation; such a parameterization will not, of course, be exact and will explicitly depend on the averaging scale.

For our highest resolution of 65 pc, we use the simulated values directly. For larger spatial scales $L$, we average simulation cells in groups of eight, progressively sampling the scales $L = 130$ pc, 260 pc, etc. The following fitting formulae describe our simulated results reasonably well (we maintain the functional dependence used by GK11 as it is justified on physical grounds):

\[
n_{1/2} = n_* \frac{\Lambda}{g}, \tag{5}
\]

where

\[
\Lambda = \ln \left( 1 + (0.05/g + U_{MW})^{1/2} \frac{g^{1/3}}{U_*} \right),
\]

\[
g = \left( D_{MW}^2 + D_*^2 \right)^{1/2},
\]

and

\[
n_* = 14 \text{ cm}^{-3} \frac{D_{1/2}^{1/2}}{S},
\]

\[
U_* = \frac{D_*}{S},
\]

\[
D_* = 0.17 \frac{2 + S^5}{1 + S^5},
\]

and $S \equiv L/100$ pc.

With the known value of $n_{1/2}$, the full simulation results for the average molecular fraction as a function of the total hydrogen...
The Astrophysical Journal, 795:37 (6pp), 2014 November 1

Gnedin & Draine

Figure 4. Molecular hydrogen fraction as a function of variable $x$ (Equation (7)). The black line shows the approximation average $f_{H_2}$ for each value of $x$. The pink band shows the full rms scatter of $f_{H_2}$ as a function of $x$. The blue line tracks the approximation of Equation (6).

(A color version of this figure is available in the online journal.)

density $n_{H_1}$ can be fitted with the following simple expression:

$$
\langle f_{H_2}(n_{H_1}) \rangle = \frac{1}{1 + \exp(-x(1 - 0.02x + 0.001x^2))},
$$

where

$$
x = w \ln \left( \frac{n_{H_1}}{n_{1/2}} \right),
$$

and

$$
w = 0.8 + \frac{\Lambda^{1/2}}{S^{1/3}}.
$$

The accuracy of this approximation is demonstrated in Figure 4, together with the rms scatter $f_{H_2}$ as a function of $x$. Approximation (6) and (7) replaces Equations (6)–(9) of GK11.

In numerical simulations, parameterizing the molecular hydrogen abundance in terms of the gas density is most convenient, as the density is always directly followed in a simulation. For semi-analytical models, a more appropriate quantity is the gas surface density. Often it is more convenient to use a ratio of molecular-to-atomic surface densities $R \equiv \Sigma_{H_2}/\Sigma_{H_1}$ instead of individual surface densities, both because the ratio is known to scale simply with galactic properties (Blitz & Rosolowsky 2006) and because the ratio is easier to fit.

All of our simulations can be fitted on sufficiently large scales ($L \gtrsim 500$ pc) with a simple power-law dependence of $R$ on the surface density of neutral gas,

$$
R \approx \left( \frac{\Sigma_{H_1+H_2}}{\Sigma_{R=1}} \right)^\alpha,
$$

where

$$
\alpha = 0.5 + \frac{1}{1 + \sqrt{U_{MW} D_{MW}^2 / 600}},
$$

and the surface density at which the molecular and atomic fractions become equal

$$
\Sigma_{R=1} = \frac{50 M_\odot}{pc^2} \left( \frac{0.01 + U_{MW}}{g} \right)^{1/2} \left( \frac{0.01 + U_{MW}}{1 + 0.69 (0.01 + U_{MW})^{1/2}} \right).
$$

Note that in the limit of high column density, the functional form for the atomic hydrogen surface density,

$$
\Sigma_{H_1} = \frac{\Sigma_{H_1+H_2}}{1 + R},
$$

is not constant if $\alpha$ is not equal to one. Hence, contrary to the assumption of GK11, the atomic surface density does not saturate at high densities, but may continue to decrease gradually. This behavior is, indeed, in better agreement with the observational data of Wong & Blitz (2002), which show a variation in the asymptotic behavior of the $H_1$ surface density.

Figure 5 shows the accuracy of this approximation. The rms scatter around the mean relation is rather small at around $\Sigma_{R=1}$ but increases towards high and, even more significantly, toward low surface densities. For $R \lesssim 0.1$, the approximation of Equation (8) effectively breaks down as the scatter becomes too large.

6. CONCLUSIONS

We discuss the effect of line overlap in the Lyman and Werner bands on the atomic-to-molecular hydrogen transition on galactic scales. While in Milky-Way-like environments dust shielding is important and cannot be neglected, line overlap significantly enhances self-shielding and makes it dominant over dust shielding in those environments with dust-to-gas ratios below about 10% of the Milky Way value (i.e., in galaxies like SMC and smaller).
So far, all of the cosmological simulations that have modeled the formation of molecular hydrogen ignored line overlap. In particular, the results of GK11 were found (Krumholz & Gnedin 2011) to be fully consistent with the fitting formulae of Krumholz et al. (2009). Since we find that line overlap has a large effect on the atomic-to-molecular transition at dust abundances below about 10% of the Milky Way value, models of this transition similar to GK11 and Krumholz et al. (2009) underestimate molecular abundance in extremely dust-poor galaxies.

How important this effect is to the detailed physics of galaxy formation is currently unclear. For example, in an early galaxy with fully atomic ISM, the formation of the very first molecular gas should still be controlled by dust shielding; however, having formed, line overlap makes the molecular gas more resistant to photo-destruction. Full exploration of the role of line overlap in galaxy formation is well beyond the scope of a single paper. Here, we only present a simple illustrative example of the “cosmo I” simulation from GK11 with a single modification to the stellar feedback model. In the original GK11 simulation, only the thermal energy feedback was implemented, and such implementation is known to be inefficient. In the simulations used here, we adopt a currently widely used “blastwave” or “delayed cooling” feedback model (Stinson et al. 2009, 2013; Governato et al. 2010; Agertz et al. 2011, 2013; Brook et al. 2012) with the delay time parameter set to 30 Myr. Such a model for feedback is known to produce galaxies with overall properties (stellar masses, rotation curves, etc.) in reasonable agreement with observations (Munshi et al. 2013).

A stellar mass-total mass relation for two illustrative simulations—one with nonlocal line overlap included and another one with line overlap neglected (using the power-law approximation of Equation (4) for the self-shielding factor)—is presented in Figure 6. Line overlap has only a minor effect on the stellar masses of simulated galaxies, and that result is not very surprising. After all, it is well established that the stellar masses and star formation rates of galaxies are controlled by feedback. Any increase in star formation due to line overlap is going to be offset by stronger feedback on a timescale of several tens of Myr, and the average, long-term efficiency of star formation will remain at the “self-regulated” value (see Hopkins et al. 2011; Agertz et al. 2013; Hopkins et al. 2013).

We are grateful to Andrey Kravtsov, Dan Welty, Amiel Sternberg, and an anonymous referee for valuable comments and suggestions that significantly improved the original manuscript.

Fermilab is operated by the Fermi Research Alliance, LLC, under contract No. DE-AC02-07CH11359 with the United States Department of Energy. N.Y.G.’s work was also supported in part by the NSF grant AST-1211190. B.T.D. was supported in part by NSF grant AST-1008570.

REFERENCES

Agertz, O., Kravtsov, A. V., Leitner, S. N., & Gnedin, N. Y. 2013, ApJ, 770, 25
Agertz, O., Teysier, R., & Moore, B. 2011, MNRAS, 410, 1391
Behroozi, P. S., Wechsler, R. H., & Conroy, C. 2013, ApJ, 770, 57
Bigiel, F., Leroy, A., Walter, F., et al. 2008, AJ, 136, 2846
Bigiel, F., Leroy, A. K., Walter, F., et al. 2011, ApJL, 730, L13
Black, J. H., & Dalgalno, A. 1977, ApJS, 34, 405
Blitz, L., & Rosolowsky, E. 2006, ApJ, 650, 933
Bolatto, A. D., Leroy, A. K., Jameson, K., et al. 2011, ApJ, 741, 12
Brook, C. B., Stinson, G. S., Gibson, B. K., et al. 2012, MNRAS, 426, 690
Christensen, C., Quinn, T., Governato, F., et al. 2012, MNRAS, 425, 3058
Daddi, E., Bournaud, F., Walter, F., et al. 2010, ApJ, 713, 686
Draine, B. T. 2009, in ASP Conf. Ser. 414, Cosmic Dust: Near and Far, ed. T. Henning, E. Grün, & J. Steinacker (San Francisco, CA: ASP), 453
Draine, B. T., & Bertoldi, F. 1996, ApJ, 468, 269
Genzel, R., Tacconi, L. J., Gracia-Carpio, J., et al. 2010, MNRAS, 407, 2091
Gillmon, K., Shull, J. M., Tumlinson, J., & Danforth, C. 2006, ApJ, 636, 891
Gnedin, N. Y., & Hollen, N. 2012, ApJS, 202, 13
Gnedin, N. Y., & Kravtsov, A. V. 2011, ApJ, 728, 88
Gnedin, N. Y., Tassis, K., & Kravtsov, A. V. 2009, ApJ, 697, 55
Goldsmith, P. F., & Li, D. 2005, ApJ, 622, 938
Governato, F., Brook, C., Mayer, L., et al. 2012, MNRAS, 426, 690
Inoue, A. K. 2011, EPS, 63, 1027
Krumholz, A. V. 1999, PhD thesis, New Mexico State Univ.
Krumholz, A. V., Klypin, A., & Hoffman, Y. 2002, ApJ, 571, 563
Krumholz, M. R. 2012, ApJ, 759, 9
Krumholz, M. R., & Gnedin, N. Y. 2011, ApJ, 729, 36
Krumholz, M. R., McKee, C. F., & Tumlinson, J. 2009, ApJ, 693, 216
Kuhlen, M., Krumholz, M. R., Madau, P., Smith, B. D., & Wise, J. 2012, ApJ, 749, 36
Kuhlen, M., Madau, P., & Krumholz, M. 2013, ApJ, 776, 34
Leroy, A. K., Bigiel, F., de Blok, W. J. G., et al. 2012, ApJ, 144, 3
Leroy, A. K., Bolatto, A., Stanimirovic, S., et al. 2007, ApJ, 658, 1027
Leroy, A. K., Walter, F., Brinks, E., et al. 2008, AJ, 136, 2782
Leroy, A. K., Walter, F., Sandstrom, K., et al. 2013, ApJ, 146, 19
Mckee, C. F., & Ostriker, E. C. 2007, ARA&A, 45, 565
Munshi, F., Governato, F., Brooks, A. M., et al. 2013, ApJ, 766, 56
Pelupessy, F. I., & Papadopoulos, P. P. 2009, ApJ, 707, 954
Pelupessy, F. I., Papadopoulos, P. P., & van der Werf, P. 2006, ApJ, 645, 1024
Robertson, B. E., & Kravtsov, A. V. 2008, ApJ, 680, 1083
Rudd, D. H., Zentner, A. R., & Kravtsov, A. V. 2008, ApJ, 672, 19
Stecher, T. P., & Williams, D. A. 1967, ApJ, 149, L29
Sternberg, A., Le Petit, F., Rochev, E., & Le Bourlot, J. 2014, ApJ, 790, 10
Stinson, G. S., Brook, C., Macciò, A. V., et al. 2013, MNRAS, 428, 129
Stinson, G. S., Dalcanton, J. J., Quinn, T., et al. 2009, MNRAS, 395, 1455
Tacconi, L. J., Neri, R., Genzel, R., et al. 2013, ApJ, 768, 74
Thompson, R., Nagamine, K., Jaacks, J., & Choi, J.-H. 2014, ApJ, 780, 145
Tumlinson, J., Shull, J. M., Rachford, B. L., et al. 2002, ApJ, 566, 857
Welty, D. E., Xue, R., & Wong, T. 2012, ApJ, 745, 173
Wolfire, M. G., Tielens, A. G. G. M., Hollenbach, D., & Kaufman, M. J. 2008, ApJ, 680, 384
Wong, T., & Blitz, L. 2002, ApJ, 569, 157
Equation (10) in the published version of this article should read:

$$\Sigma_{R=1} = \frac{50 \, M_\odot}{\text{pc}^2} \frac{(0.001 + 0.1U_{\text{MW}})^{1/2}}{g} \left( 1 + 1.69(0.001 + 0.1U_{\text{MW}})^{1/2} \right).$$  \hspace{1cm} (10)

An even more accurate and simpler fit is offered by the following expressions,

$$R \approx q \frac{1 + \eta q}{1 + \eta}, \quad q = \left( \frac{\Sigma_{\text{gas}}}{\Sigma_{R=1}} \right)^{\alpha},$$  \hspace{1cm} (8a)

where $\eta \approx 0$ on kpc scale and $\eta = 0.25$ on 500 pc scale,

$$\alpha = 1 + 0.7 \frac{s^{1/2}}{1 + s},$$  \hspace{1cm} (9a)

and

$$\Sigma_{R=1} = \frac{40 \, M_\odot}{\text{pc}^2} \frac{s}{g} \left( 1 + s \right)$$  \hspace{1cm} (10a)

with $s = (0.001 + 0.1U_{\text{MW}})^{0.7}$. 