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Spin-orbit coupling and operation of multi-valley spin qubits

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Spin qubits composed of either one or three electrons are realized in a quantum dot formed at a Si/SiO2 interface in isotopically enriched silicon. Using pulsed electron spin resonance, we perform coherent control of both types of qubits, addressing them via an electric field dependent g-factor. We perform randomized benchmarking and find that both qubits can be operated with high fidelity. Surprisingly, we find that the g-factors of the one-electron and three-electron qubits have an approximately linear but opposite dependence as a function of the applied dc electric field. We develop a theory to explain this g-factor behavior based on the spin-valley coupling that results from the sharp interface. The outer “shell” electron in the three-electron qubit exists in the higher of the two available conduction-band valley states, in contrast with the one-electron case, where the electron is in the lower valley. We formulate a modified effective mass theory and propose that inter-valley spin-flip tunneling dominates over intra-valley spin-flips in this system, leading to a direct correlation between the spin-orbit coupling parameters and the g-factors in the two valleys. In addition to offering all-electrical tuning for single-qubit gates, the g-factor physics revealed here for one-electron and three-electron qubits offers potential opportunities for new qubit control approaches.

Silicon is known to have small spin-orbit coupling (SOC), a beneficial fact for silicon quantum computing, since charge noise is largely decoupled from information stored in the spin1. Furthermore, silicon can be isotopically enriched and chemically purified to 28Si, thereby removing nuclear spin background fluctuations and so silicon is often referred to as a semiconductor vacuum2. These two facts have motivated intense research on silicon qubits, leading to recent realizations of single-qubit3–7 and two-qubit8 logic gates. Despite the small SOC, the tunability of the g-factor via gate-controlled electric fields allows one to electrostatically turn on and off the spin rotations that constitute single-qubit gates9–11, thereby providing an important tool for quantum computation.

The low-energy subspace in silicon quantum dot (QD) systems is governed by two spin-degenerate valley states. When these valley states are quasi-degenerate, qubit operation becomes complex8, and coupling qubits is even more challenging10. However, the valley states can be separated using a vertical electric field and the sharp potential of an interface, and their energy separation can be electrically controlled over several hundreds of μeV7–11. While one-electron spin qubits are naturally operated in the lowest valley state6,7, it is intriguing to consider the performance of qubits operated in the higher valley state, which has a long spin lifetime when orbital relaxation is suppressed11. When spatial confinement in the QD is strong, the orbital excited states are lifted high in energy and qubit operation in the upper valley state is possible by populating three electrons in the quantum dot, assuming a single-particle description (see, e.g. Refs.10,12). In this mode two electrons form a singlet in the lower valley state and the third electron is operated in the upper valley (see Fig.1a and 1b). It has been suggested that such multi-electron qubits could enhance the gate fidelity, due to partial screening of electrical noise13.

Here we study the valley structure of silicon and spin-
orbit coupling by high-fidelity operation of one- and three-electron spin qubits, operated in the lower and upper valley, respectively. Using electron-spin-resonance (ESR) we map out the qubit frequency as a function of the applied perpendicular electric field. We experimentally demonstrate and theoretically explain how inter-valley spin-orbit coupling at the Si/SiO₂ interface results in an opposite dependence of the g-factor for the two valleys. Via the direct g-factor coupling to the electric field the three-electron (upper valley) qubit is about twice as sensitive to external field fluctuations compared to the one-electron qubit, leading to a different decoherence mechanism than discussed in Ref.12 and resulting in a lower three electron-qubit fidelity. Randomized benchmarking supports this observation, while showing that both qubit systems are capable of fidelities above 99%, approaching the surface code thresholds for fault-tolerant quantum computing14.

The QD structure is fabricated on an epitaxially grown, isotopically purified ²⁸Si epilayer with a residual concentration of ²⁹Si at 800 ppm using multi-level gate stack silicon metal-oxide-semiconductor (SIMOS) technology15, see Fig. 1. The charge stability diagram of the quantum dot is shown in Fig. 1c. From a Ramsey sequence on the three-electron qubit we find a dephasing time T₂ = 70 μs, which is slightly less than we have previously measured for the one-electron qubit, which had T₂ = 120 μs7.

We have demonstrated electric field control over the resonance frequency ν_ESR of the one-electron qubit7, showing tunability over several MHz that appears linear in electric field, corresponding to more than 3000 times the 2.4 kHz ESR line-width. We find that spin-valley mixing of the QD eigenstates due to interface (local) roughness16 would predict a modification of the electron g-factor that is two orders of magnitude smaller than is found experimentally, together with a non-linear dependence close to the anticrossing point of the spin-valley states that we do not observe.

Here we propose and analyze a model where the g-factor modification proceeds via inter-valley spin-flip tunneling, mediated by the strong z-confinement at the interface. The Si/SiO₂ (001) interface of silicon MOS quantum dots can be described with a Hamiltonian that consists of a bulk term H₀ and an interface term H_if. The reduction of the bulk Si crystal symmetry at the interface, in the presence of strong perpendicular electron confinement induced by an applied electric field F_z, lifts the six-fold valley degeneracy, leaving two low-lying Δ-valleys at ±k₀₀. These are then mixed via enhanced inter-valley tunneling due to the strong z-confinement at the interface16,17. The consequent effective two-valley Hamiltonian acts on the four-component vector \( \Phi(r) = |r⟩, |r⟩/2⟩, |r⟩, |r⟩/2⟩T \equiv \Phi(r) \), where the bulk part (spin and valley degenerate) is given by

\[
H₀ = \sum_{j=x,y,z} \frac{\hbar^2 k_j^2}{2m_j} + U_{x,y} + U_z \times \hat{I}_z
\]

with the quasi-momentum operators \( \hat{k}_j \equiv -i\partial_j \); and \( U_{x,y} = \frac{m_j}{2} \omega_0^2 (x^2 + y^2) \) and \( U_z = |e|F_z \) are the in-plane and perpendicular confinement electron potentials, respectively. Here \( m_j, m_l \) are the Si effective Δ-valley electron masses, \( |e| \) is the electron charge, and \( \hbar \) is the reduced Planck constant. Taking into account the large band offset of Si/SiO₂, the interface term is

\[
H_{if} = -\frac{\hbar^2}{2Rm_l} \delta(z - z₀) - i \frac{\hbar^2}{2m_l} \delta(z - z₀) \hat{k}_z + \delta(z - z₀) \hat{V}_{if}(k),
\]

where \( R \) is a parameter with dimension of length, characterizing an abrupt interface18,19, and \( |R| \ll |l_z| \ll |l_D| \); here \( l_z = (\hbar^2/2m_l|e|F_z)^{1/3} \) and \( l_D = (\hbar/m_l\omega_0)^{1/2} \) are the perpendicular and in-plane confinement lengths (assuming much stronger \( \hat{z} \)-confinement). For \( R \approx 0 \) the interface Hamiltonian (Eq. 2) corresponds to the standard infinite boundary condition (BC) \( \Phi(z) \big|_{z=0} = 0 \), while for finite \( R \) it generates spin and valley mixing at the interface, also preserving the hermiticity of the Hamiltonian in the half-space18,20, \( z \geq z₀ \). Following the symmetry reasoning of Refs.21,22 the spin-valley mixing interface matrix \( \hat{V}_{if}(k) \) can be expressed via the \( C_{2v} \) invariants \( H_{R}(k) = \sigma_x k_y - \sigma_y k_x, H_{D}(k) = \sigma_x k_x - \sigma_y k_y \), resulting in

\[
\hat{V}_{if}(k) = \begin{bmatrix}
A(k) & \tilde{V}\hat{I}_2 + B(k) \\
\tilde{V}^*\hat{I}_2 + B^†(k) & A(k)
\end{bmatrix}.
\]

In Eq. (3) the \( 2 \times 2 \) block-diagonal element \( A(k) = \delta D H_D(k) + s_R H_R(k) \) corresponds to intra-valley spin-flipping transitions, while the off-diagonal elements are related to inter-valley tunneling (in momentum space) with no spin-flipping amplitude \( V = |V|e^{iφ} \) with phase22,23, or with a spin-flipping process, \( B(k) = -\chi_D H_D(k) + \chi_R H_R(k) \).

Since experimentally the valley splitting (≈ \( |V| \)) is generally large with respect to the spin-flipping terms, we diagonalize with respect to the leading \( V \)-matrix element and via a unitary transformation we find

\[
\hat{V}_{if}^U(k) = \begin{bmatrix}
|V| + \frac{1}{2}B_{d} & h.c. \frac{1}{2}B_{off} \\
-|V| + A - \frac{1}{2}B_d & B_{d}
\end{bmatrix}.
\]
This matrix is approximately diagonal in the valley basis $|v_1\rangle$, $|v_2\rangle$, with a calculated valley splitting energy $E_{VS} = 2|V|R^2|\phi'(0)|^2 = 2|V|R^2l_0^{-3} \propto F_z$. We neglect the off-diagonal contribution $B_{\text{eff}} \equiv B - B_{l}e^{2i\phi_D}$ in Eq.(4), since it is suppressed as $\sim 1/E_{VS}$ and $E_{VS}$ is typically several hundred of $\mu eV$ in MOS quantum dots.\(^7\)\(^1\)\(^1\) Thus, in the valley subspaces $|v_1\rangle$, $|v_2\rangle$, one can consider two independent boundary conditions as in Eq.(2), with spin-flipping interface matrices $\hat{V}_{v_1,v_2} = A + \frac{1}{2}\hat{B}_d \equiv A + \frac{1}{2}(B e^{-i\phi_D} + B e^{i\phi_D})$, in which the inter-valley spin-flip tunneling element changes sign between $v_1$ and $v_2$.

The effective 2D spin-orbit Hamiltonians (proportional to the Rashba and Dresselhaus forms, $H_R(k)$, $H_D(k)$) are calculated by recasting the BC, Eq.(2), to a standard one via a suitable unitary transform, $\tilde{\Phi} |z=\pm 0 \rangle = \hat{G}_{bc} \Phi |z=\pm 0 \rangle$, and obtaining a smooth perturbing Hamiltonian: $\delta \mathcal{H} \simeq R^2 \frac{2m}{\hbar^2} \tilde{V}_{v}(k) \partial_z U_z$.

The corresponding 2D SOC parameters change due to sign flipping:

$$\begin{align*}
\alpha_{R,v_1,v_2} & = 2[s_R \mp |x_R| \cos(\phi_R - \phi_V)] R^2 |\phi'(0)|^2 \\
\beta_{D,v_1,v_2} & = 2[s_D \mp |x_D| \cos(\phi_D - \phi_V)] R^2 |\phi'(0)|^2.
\end{align*}$$

(5)

The scaling of the spin-orbit terms with the electric field $F_z$ is linear, as is the valley splitting, $E_{VS}$. Here, we have introduced the phases $\phi_R$ and $\phi_D$ for the Rashba and Dresselhaus terms and $|\phi'(0)|$ is the derivative of the $z$-component of an eigenstate of the bulk Hamiltonian $\mathcal{H}_0$. These results are similar to the strong field limit results of Ref.\(^2\)\(^2\).

Explicit calculation of the $g$-factor change, based on the interface Hamiltonian Eq.(2) and the fact that an in-plane magnetic field mixes the perpendicular and in-plane motion, shows that the in-plane interface $g$-factor renormalization $\delta g_{z}^{\text{fl}}$ is proportional to $\alpha_R$, $\beta_D$; non-parabolicity effects\(^2\)\(^3\) are estimated to be much smaller, to be presented elsewhere. We find for the magnetic field parallel to the [110] direction:

$$\delta g_{z}^{\text{fl}}_{v_1,v_2} = -\frac{\langle \alpha_{R,v_1,v_2} - \beta_{D,v_1,v_2} \rangle | e |}{\hbar \mu B} \langle z \rangle$$

(6)

where $\mu B$ is the Bohr magneton and $\langle z \rangle \simeq 1.5587 l_0$ is an average of the $z$-motion in the lowest subband, see Eq.(1).

The $g$-factor scales as $F_z^{2/3}$, which is close to a linear scaling over the range ($\sim 10\%$) of the experimentally applied electric fields, see Fig. 3b.

We therefore expect from Eq.(6) that the renormalization $\delta g_{z}$ will be of opposing sign for the two valleys, following the sign change of the SOC parameters in Eq.(5). In particular, the change will be exactly opposite for zero intra-valley spin-flip coupling, $s_R, s_D = 0$:

$$\delta g_{v_1} = -\delta g_{v_2}.$$  

(7)

Relatively smaller corrections due to non-zero intra-valley spin flipping, $s_R, s_D \neq 0$ will generally violate Eq.(7), leaving the $g$-factor changes opposite in sign, but with different absolute value, $|\delta g_{v_1}| \neq |\delta g_{v_2}|$.

To observe this experimentally, we control the quantum dot electric field via the plunger gate $\hat{P}G$ and the confinement gate $CG$, see Fig. 1. In Fig. 3a we show the magnetic field dependence and in Fig. 3b we show electrical control over the qubit resonance frequency $\nu_{ESR}$. The opposite field dependence of the $g$-factor for the two valleys is in qualitative agreement with the prediction of Eq.(7). Since the resonance frequency of the one-electron qubit increases with the electric field, while the resonance frequency of the three-electron qubit decreases (see Fig. 3b), we infer from Eq.(6) that the Rashba and Dresselhaus contributions are in this experiment subject to the constraints: $\delta \chi_{\text{inter-val}} \equiv |x_R| \cos(\phi_R - \phi_V) - |x_D| \cos(\phi_D - \phi_V) > 0$, $\delta \chi_{\text{intra-val}} \equiv s_R - s_D < 0$. The change in sign of $\delta g_{z}$ is evidence that the inter-valley spin-flip contributions dominate the intra-valley spin-flip processes and from the $\delta g_{z}$-dependence we estimate the ratio $\delta \chi_{\text{inter-val}}/|\delta \chi_{\text{intra-val}}| \approx 2.6$. This observation is consistent with tight-binding calculations on SiGe quantum wells\(^2\)\(^2\), which predict that the inter-valley spin-flip processes and from the $\delta g_{z}$-dependence can be expected due to the greater band-edge offset in Si/SiGe, disorder\(^2\)\(^5\), and built-in electric fields.

In order to explore the qubit performance, we have performed (interleaved) randomized benchmarking (RB)\(^2\)\(^6\)\(^2\) on the one-electron qubit\(^7\) and three-electron qubit, and all results are shown in Fig. 4. In order to eliminate the fitting parameter $B_{\text{RB}}$, which is a constant offset parameter present in standard RB fits, we plot the sequence fidelity combination $F = F_3 + F_1 - 1$, which approaches zero for infinite sequence length when the assumptions of RB hold\(^2\)\(^8\). When the noise is gate independent, an exponential decay is expected. However, when low-frequency noise is present, non-exponential decays arise\(^2\)\(^8\). This non-exponential decay is due to slow drifts in the resonance frequency, such that the time ensemble is averaged over sequences with small detuning (resulting in a high
FIG. 4: Clifford based randomized benchmarking (a) Sequence fidelity as a function of the sequence length and (b) schematic representation of randomized benchmarking, where \( H \) is an interleaved test gate. In (a), the black filled circles correspond to standard randomized benchmarking and the green open circles to the average Clifford gate length. Both data sets are fitted with a two-fidelity model (see text) and the results are shown in (c), where the standard error is smaller than the corresponding gate error.

The faster decay of the sequence fidelity of the three-electron qubit vs. one-electron qubit is consistent with a larger sensitivity to electrical noise, as revealed by the larger frequency shift with gate voltage, \( |\delta \nu_{\text{ee}}/\delta V| \approx 2.2 |\delta \nu_{\text{ee}}/\delta V| \), shown in Fig.3b. The frequency detuning caused by electrical noise results in rotations around the \( z \)-axis of the qubit Bloch sphere and perpendicular to the Rabi driving axis. In the small-angle approximation, this would result in an error rate that is around 5 times larger for the three-electron qubit, comparable with the difference in fidelities between the one-electron and three-electron qubits. It is therefore likely that both qubits are ultimately limited by high-frequency electrical noise, possibly due to charge noise from the aluminium-SiO\(_2\) interface.

The realizations of single- and two-qubit gates using isotopically purified silicon quantum dots\(^{18}\) are now revealing the early promises of silicon as a platform for quantum computation and the possibility of qubit operation with either one-electron or three-electrons allows more flexibility in scaling these systems. The ultra-narrow spin-resonance linewidth of these qubits has pushed silicon into a new regime, where the weak spin-orbit coupling in silicon becomes not only visible, but also forms a new tool to control the spin states, as shown here. Further qubit optimization may be achieved by reducing the spin-orbit interaction, for example by changing the magnetic field amplitude or orientation, while the remarkably large electric field control in SiMOS quantum dots provides additional motivation to explore spin-orbit coupling in silicon for qubit control and spin manipulation.

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