Crack Level Estimation Approach for Planetary Gear Sets Based on Simulation Signal and GRA

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Abstract. The planetary gearbox is a critical mechanism in helicopter transmission systems. Tooth failures in planetary gear sets will cause great risk to helicopter operations. A crack level estimation methodology has been devised in this paper by integrating a physical model for simulation signal generation and a grey relational analysis (GRA) algorithm for damage level estimation. The proposed method was calibrated firstly with fault seeded test data and then validated with the data of other tests from a helicopter transmission test rig. The estimation results of test data coincide with the actual test records, showing the effectiveness and accuracy of the method in providing a novel way to hybrid model based methods and signal analysis methods for more accurate health monitoring and condition prediction.

Key Words. Planetary gear sets, Fatigue crack, Physical model, Feature selection, GRA, Damage level estimation

1. Introduction

The majority of mishaps in helicopters are caused by engine and drive train failures. To reduce these mechanically induced failures and excessive maintenance, it is vital to accurately identify and diagnose developing faults in the mechanical system. Planetary gear sets are common mechanical components and are widely used to transmit power and change speed and/or direction in rotary aircrafts. One of the most common causes of planetary gear sets failure is tooth fatigue crack of sun gear due to excessive stress conditions. It results in progressive damage to gear teeth and ultimately leads to the complete failure of the planetary gear sets. This fault is particularly challenging as it is located deep inside the main transmission, suggesting it would be difficult to detect earlier.

Because of the high importance and challenge, the subject of damage level estimation for planetary gear sets has been studied intensively and resulted in a number of advanced papers published in several key journals and at conferences. In general, methods reported in these papers could be viewed through two categories: data-driven approach and model-based approach. A simple model is presented from the assumption that for each combination of crack location and inspector there is a threshold crack size such that all cracks above this size will be detected and all below that size will be missed. The proposed model adjusts the threshold crack size according to the difficulty associated with the crack location and the competence of inspectors (Coppe et al., 2008). A method to estimate the size of a tooth transverse crack for a spur gear in operation is developed. Using gear vibrations measured

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from an actual gear accelerated test, this study examined existing gear condition indices to identify those which correlated well to crack size and established their utility for crack size estimation through index fusion using a neural network (Choi and Li, 2006). A model-based demodulation scheme is developed. It is used to exploit the information contained in wideband gear vibrations and compared it to a state of the art technique that uses a vibration average of a gear with two defects of different sizes (Ma and Li, 1996). A method is proposed to classify the different levels of gear cracks automatically and reliably. The proposed method is applied to identifying the gear crack levels and the results obtained demonstrate the effectiveness of the method (Lei and Zuo, 2009).

In general, the data-driven methods aforementioned provide little guarantee of their estimation accuracy. Moreover, they are usually vague about the relationship between the index and damage severity since they assume some kind of simple black-box model. On the other hand, the physical-based approach has high computational costs associated with the physical models and these methods are only suitable for off-line applications. In addition, the datasets used for the damage severity estimation of planetary gear sets interfere strongly with environmental noise and many frequency components of other moving parts and the damage feature information is totally different from the ordinary gear train. Thus the feature selection is another challenge to be faced in this field.

In this paper, a novel fatigue crack level estimation method is presented for 2K-H planetary gear sets based on simulation signal and grey relational analysis (GRA), which is composed of an analytical model for dynamic response and damage feature information analysis grey relational analysis algorithm for damage level estimation. The rest of the content will address the method development according to these three phases.

2. Physical model of planetary gears with defects

Modeling of the gear tooth failure can help to analyze this dynamic change in order to give suitable tools to diagnose such failures. Whilst the modeling of healthy gear systems nowadays is extensively carried out, the failure modeling is still subject to many research papers. Gear tooth failures are assessed by the determination of the tooth stiffness reduction. The finite elements method is the most frequently used technique to do this (Pimsarn and Kazerounian, 2002; Wang, 2003; Ambarisha and Parker, 2007; Lin and Parker, 1999), but it requires, in certain applications, mesh refinements and then much computations time. Analytical methods can be a good alternative to model tooth failures. Some literature focuses on the tooth stiffness reduction due to damage by considering qualitative proportional reduction (Chaari et al., 2006; Chaari and Baccar, 2008; Hbaieb et al., 2005). This research is based on the analytical method.

2.1. Physical model of healthy planetary gear set

The epicyclic stage of the transmission is more complex due to its multiple components and the orbital motion of the planets. Noting that the stiffness of the planet gear support is very rigid and the internal ring gear is fixed to the top of the transmission casing, the central displacement of planet-gears and ring gear is ignored to simplify the model of 2K-H planetary gear sets. Then a lumped parameter, pure torsional dynamical formulation is employed to develop the physical model of 2K-H planetary gear sets. As shown in figure 1, \(K_{sp}\) denotes mesh stiffness between sun-gear and planet gear; \(K_{rp}\) denotes mesh stiffness between planet gear and ring gear. \(C_{sp}\) denotes mesh damping between sun-gear and planet gear. \(C_{rp}\) denotes mesh damping between planet gear and ring gear. \(\theta_s\), \(\theta_p\), and \(\theta_c\) denote the rotation angle of sun-gear, planet gear and carrier respectively. \(T_s\) and \(T_r\) denote driving torque and loading torque respectively. \(s\), \(r\), \(p1\) and \(c\) are the subscripts denoting sun-gear, ring gear, the \(i\)th planet gear and carrier. By ignoring mesh errors and defining internal meshing side clearance and external meshing side clearance as \(2h_{sp}\) and \(2h_{rp}\), respectively, the adhesive engaging force \(D\) and the elastic engaging force \(P\) are represented as
Where $K(t)$ is time varying mesh stiffness, $C$ is mesh damping constant, $r_p$ is radius of basic circle, $f(x,b)$ is nonlinear clearance function. $f(x,b)$ is defined by

$$f(x,b) = \begin{cases} 
  x - b & (x > b) \\
  0 & (-b \leq x \leq b) \\
  x + b & (x < -b) 
\end{cases}$$

Where $x$ is variable parameter, $b$ is clearance.

Dynamical differential equations of 2K-H planetary gear sets could be deduced from Lagrange equations:

$$\begin{align*}
I_s \ddot{\theta}_s + \sum_{i=1}^{N_s} (D_{sp} + P_{sp}) r_{sp} &= T_d \\
I_p \ddot{\theta}_p - \sum_{i=1}^{N_p} (D_{sp} - D_{pi} + P_{sp} - P_{pi}) r_{pi} &= 0 \\
I_c \sum_{i=1}^{N_c} m_i r_c^2 \dot{\theta}_c - \sum_{i=1}^{N_c} (D_{sp} + D_{pi} + P_{sp} + P_{pi}) r_c \cos \alpha &= -T_L
\end{align*}$$

Where $I$ is the rotational inertia of the subcomponents, $m$ is mass, $\alpha$ is pressure angle, $N$ is number of planet gear in a 2K-H planetary gear set.

Equation (3) is positive semi-definite, nonlinear equations, which has $N+2$ degrees-of-freedom (dof’s), with angle displacements of rigid bodies in a coordinated system. For translating angle displacements of rigid bodies into relative linear displacement, the relative displacements between sun-gear and planet gear $x_{p_i}$, and the relative displacement between sun-gear and carrier $x_c$ are defined as

$$\begin{cases} 
  x_{p_i} = \theta_{p_i} r_{p_i} - \theta_{p_{pi}} r_{p_{pi}} - \theta_c r_c \cos \alpha \\
  x_c = \theta_c r_c - 2 \theta_c r_c \cos \alpha 
\end{cases}$$

After this, equivalent mass is supposed as $M$, $M = \frac{I}{r_c^2} + \sum_{i=1}^{N} \frac{m_{p_i}}{\cos^2 \alpha}$.

Substituting equation (1) and equation (3) into equation (4) and simplifying the equations obtained, then the dynamic model of planetary gear sets is represented as:
2.2. Physical model of healthy planetary gears with damage

In this section, a common tooth fault in planetary gear sets is modeled: sun gear tooth fatigue crack. Common causes include cyclic stressing of the gear tooth material beyond its endurance fatigue limit. Bending fatigue crack starts in the root section and progresses until the tooth or part of it breaks off. Fatigue crack can occur in areas such as the tooth top or the pitch line, other than the usual tooth root fillet section, as can be seen in figure 2.

![Figure 2. Gear-mesh stiffness variation of gear pair with tooth crack. (a) Tooth root crack, (b) Time vary mesh stiffness.](image)

In this paper, a fatigue crack on tooth root fillet section of sun gear is considered. To simplify the model of damage, at each section of the tooth, the shape of tooth root crack is approximated with straight line defined by the length \( l \) and the direction angle \( \theta \) of the crack, as shown in figure 2. We define the level of tooth crack as \( s \), which is determined by \( l \) and \( \theta \), \( s = s(l, \theta) \). For simplifying the dynamical model of planetary gear sets, define the size and the threshold value of crack as \( b_i \) and \( b_f \), \( \cos / 2 \), \( \sum_i b_i \) = \( b_f \). And then \( s = s(l, \theta) \). Referenced to Literature [16, 17], the gear mesh stiffness of the gear pair, which is composed of sun gear and planet gear, is calculated by taking into account the geometric changes due to the tooth crack.

Gear mesh stiffness evolution caused by sun gear tooth crack is defined as \( \Delta K(f_i, t) \), so the time varying mesh stiffness with sun gear tooth crack is given by

\[
K_i(t) = K_0(f_i, t) + \Delta K(f_i, t)
\]

Where \( K_0(f_i, t) \) is mesh stiffness of gear pair in healthy case, \( f_i \) is mesh frequency of sun gear.
The time vary mesh stiffness is illustrated in figure 2. The dynamical model of 2K-H planetary gear set with sun gear tooth crack is acquired by substituting equation (6) into equation (5).

2.3. Simulation of physical models
The parameters in the physical models above should be set up according to Table 1, for the faulty case, s is varied as [0%:5%:100%]. The simulation is executed in Matlab, and four-order Runge-Kutta method is selected as the solution for the models. The duration and step size in solving the equations is set to 1 second and 0.0001 second respectively.

Figure 3 shows typical dynamic responses in both the time domain and the frequency domain which are registered on the internal ring gear for a healthy planetary gear set (s=0%) and 3 different levels of crack severity(s=0%, 30%, 60%, 90%). The healthy case is characterized by the dominance of the gear mesh frequency, denoted by 1X and its harmonics of 2X, 3X etc. For damage case, amplitude modulation of the gear mesh signal is clearly observed, which occurs once a revolution. As a consequence, many new frequency components appear at the sides of the dominant frequency. It can be seen that the amplitude of new frequency components increases with the growth of crack level. In the mean time, it decreases as the frequency increases and almost disappearing by the 3X of the dominant frequency. These features are very close to experimental observations in the previous study literatures (Pimsarn and Kazerounian, 2002; Wang, 2003; Ambarisha and Parker, 2007; Lin and Parker, 1999).

3. Feature selection
From literature review, 27 features have found in different cases of gearbox condition monitoring. In this study they are all explored to obtain an optimal subset for the detection and estimation of damage severity in planetary gear sets and organized into four groups and assigned with serial numbers.

(1) Features derived from the time domain are used most frequently in gearbox diagnosis (Dempsey et al., 2007). They include root mean squared(RMS), crest factor(CF), energy ratio(ER), kurtosis, standard deviation, energy operator, absolute mean value, clearance factor and impulse factor, which are assigned with serial numbers 1 to 9 respectively for the ease of identification in the process of feature selection.

| Parameter Name (Unit)            | Value |
|----------------------------------|-------|
| Modulus (mm)                    | 2.5   |
| Tooth Number of Sun Gear        | 28    |
| Tooth Number of Planet Gear     | 32    |
| Tooth Number of Ring Gear       | 92    |
| Number of Planet Gear           | 4     |
| Tooth Width (mm)                | 12    |
| Pressure Angle (Deg)            | 20    |
| Driving Torque (N•m)            | 100   |
| Loading Torque (N•m)            | 220   |
| Material                        | 40Cr  |
There are many other traditional statistical feature parameters for damage detection, such as FM0, FM4, NA4, M6A, M8A, NB4, NA4*, NB4*, M6A* and M8A*, generally used in planetary gearbox condition monitoring (Samuel and Pines, 2005). The serial numbers of these features are 10 to 19.

Other kinds of feature parameters based on frequency spectrum of vibration signal are widely used to detect and diagnose faults in helicopter power trains, such as mean frequency(MF), frequency centre(FC), root mean square frequency(RMSF) and standard deviation frequency(STD) (Saxena and Wu, 2005). The serial numbers of these features are 20 to 23.

Other than the features presented above, there are some important features which have been validated in literature, which are named Intra-Revolution Energy Variance(IREV) (Wu and Saxena, 2004), spectrum kurtosis(SK) (Barszcz and Randall, 2009), local spectrum kurtosis (Cheng el al., 2010), NSR (Cheng el al., 2010). The serial numbers of these features are 24 to 27.

Each of the simulation signals generated by the dynamical models is processed to obtain these feature parameters.

In this research, target features were selected from the 27 features above. A statistic algorithm named two-sample Z-test is commonly used to measure the distance of two class case (Cheng el al., 2011). In this research, this algorithm is applied to select the features.

4. Damage level estimation based on GRA

4.1. Grey relational analysis for damage

A system which has none of information is defined as a black system, while a system which is full of information is called white. Thus, when the information of a system is either incomplete or undetermined, it is defined as grey system. The grey number in grey system represents a number with less complete information. The grey element represents an element with incomplete information. The grey relation is the relation with incomplete information. In grey theory, GRA is reckoned as a contrasting way, in wholeness, equipped with reference for contrasting. Differing from the traditional mathematical analysis, GRA provides a simple scheme to analyze the series relationships or the system behaviours. The detailed illustration of GRA is presented according to Deng, 2001.

For mechanical system, GRA is used to analyze stochastic variables, the damage in planetary gear sets could be identified based on the GRA of unknown mode and normal mode. Before damage relational analysis, normal damage mode matrix $X_{n}(k)$ should be created as:

\[ X_{n}(k) = \begin{bmatrix} \end{bmatrix} \]
Where $K$ is damage level number, for detecting damage, $k$=2; for identifying damage level, $k$>=3; $i$ is feature parameter number in each normal mode.

Define the feature vector of unknown signal as $X(i)=[X_1(i), X_2(i), ..., X_i(i)]^T$, $j\in\{1,2,...,K\}$, $j$ is unknown number of damage level. Grey relational coefficient (GRC) is expressed as:

$$
\eta_0(k) = \frac{\min_{i} \min_{j} A_0(k) + \beta \max_{i} \max_{j} A_0(k)}{A_0(k) + \beta \max_{i} \max_{j} A_0(k)}
$$

(8)

Where $A_0(k)=[X_0(k) - X_i(j)]$, $\eta_0(k)$ is grey relational coefficient of $X_0(k)$ and $X_i(j)$, $\beta$ is distinguishing coefficient, usually $\beta=0.5$.

Supposing $\gamma_i$ as the grey relational grade (GRG) of $X_i(j)$ and $X_0(k)$, then

$$
\gamma_{i0}(k) = \frac{1}{I} \sum_{i=1}^{I} \eta_0(k)
$$

(9)

Where $I$ is feature number.

4.2. Damage level estimation

Generally, the ideal aim of quantitative detection is to determine the severity of damage accurately, but as there are many uncertainty factors in sensing signals, it is difficult or even impossible to yield the deterministic value of damage severity. As a result of that, it would be more feasible to evaluate the damage level with a certain confidence.

After selection and weighting, the feature parameter set used for damage level estimation and is remarked as $F = \{F_1, F_2, ..., F_i, ..., F_I\}$, and the weight set of the corresponding feature parameter set is $\bar{W} = \{W_1, W_2, ..., W_i, ..., W_I\}$, I is the number of feature parameters in the set. The feature parameter set and the weight set are used to do GRA for damage level estimation.

In this research, simulation signals of different crack levels are acquired from the damage seeded models. Then calculating the feature parameters $\bar{F}$ of simulation signals and forming the normal damage mode matrix $F_0(k)$, $k$ is the crack levels number of simulation signals.

The test signal to be detected and estimated is labeled as $d$, and the feature vector of $s$ is $F_i(d)$. Using equation (8) and equation (9) to calculate the GRC of $F_0(k)$ and $F_i(s)$. And then the GRG is calculated by

$$
\gamma_{i0}(k) = \frac{1}{I} \sum_{i=1}^{I} \eta_0(k)W_i
$$

(10)

After that a GRG vector $\gamma_{i0}(\gamma_{i0}(1), \gamma_{i0}(2), ..., \gamma_{i0}(k), ..., \gamma_{i0}(K))$ is obtained and the level number $k$ of $\max[\gamma_{i0}(k)]$ is the damage level of $d$.

5. Damage level estimation based on GRA

5.1. Helicopter transmission test rig

The validation of this research is carried out in the lab. The test rig simulates the transmission system of general helicopters. The transmission provides speed and torque reduction through three stages. The first stage consists of a spur-bevel configuration. The horizontal input shaft holds an 18-tooth spiral-bevel pinion which drives a 36-tooth spur-bevel gear on a vertical intermediate shaft. The directional change in shaft rotation totals 90 degrees and occurs in this first stage. The second stage uses a
planetary gearbox, which composed of two 2K-H planetary gear sets. The first planetary gear sets has one 32-sungears, three 40-planetgears and a stationary 112-tooth ring gear, while the second one has one 28-sungears, three 34-planetgears and a stationary 96-tooth ring gear. Both the ring gears are splined to the top of the transmission casing. The rotating planet gears drive the carrier, which is attached to the output shaft. The output shaft of planetary gearbox is attached to the spur gear box. Considering the first two stages only, the total reduction ratio is 39.86:1.

5.2. Test data validation
Before the validation with test data, feature selection and weighting have been carried out with simulation signal from dynamical models. The feature selection results show that only a few very features are kept and used in this study.

A number of faults seeded experiments have been conducted in this research. Referring to figure 5, the test rig consists of two electrical motors, a pair of spur-bevel gears, a planetary gearbox, two pairs of spur gears, a power supply unit with the necessary speed control electronics and the data acquisition system. The characteristics of the planetary gear sets are given in Table 1. The vibration signal generated by the planetary gearbox was picked up by an accelerometer bolted on the top of the planetary gearbox casing and the electrical signal was transferred to the data acquisition system, which has a fore-charge-amplifier. The sampling frequency \( f_s \) is 10 kHz. The signal was low-pass filtered at 5 kHz through a 4th order Bessel type filter, in order to limit aliasing distortion and retain waveform integrity as much as possible. Data was stored for post processing to a PC. A number of 10240 data points have been acquired in all experiments corresponding to a time-history length of 1s.

To validate the damage level estimation approach in this research, 6 test data \{ \( S_1, S_2, S_{x1}, S_{x2}, S_{x3}, S_{x4} \) \} were selected to be analysed. \( S_1 \) and \( S_2 \) are the test signals of which severity is 0% and 100%, while the other signals’ severity is unknown. \( S_1 \) and \( S_2 \) are used to calibrate and normalize feature vectors. We extracted 20 samples from each signal, and calculated the means of features for each sample.

Figure 6 shows the results of damage level estimation. It can be seen that the peak points of normalized GRG curves are corresponding to the related crack levels. To confirm the result precision, we check the test record and get the crack levels of the 4 test signals above as \( s_1=5\% \), \( s_2=10\% \), \( s_3=22.5\% \), \( s_4=35\% \). These records agree well with the results of crack level estimation.

6. Conclusion
In this paper, a new methodology has been developed to estimate the crack level of 2K-H planetary gear sets. The proposed method is firstly calibrated with fault seeded test data and then validated with the data of other tests. The crack level estimation results of test data agree with the actual test records.
It has demonstrated the potential of the hybrid models in providing an effective technique to improving the performance of health monitoring and condition prediction.

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