Abstract

It is shown that Quantum Mechanics is ambiguous when predicting relative frequencies for an entangled system if the measurements of both subsystems are performed in spatially separated events. This ambiguity gives way to unphysical consequences: the projection rule could be applied in one or the other temporal (?) order of measurements (being non local in any case), but symmetry of the roles of both subsystems would be broken.

An alternative theory is presented in which this ambiguity does not exist. Observable relative frequencies differ from those of orthodox Quantum Mechanics, and a gendaken experiment is proposed to falsify one or the other theory. In the alternative theory, each subsystem has an individual state in its own Hilbert space, and the total system state is direct product (rank one) of both, so there is no entanglement. Correlation between subsystems appears through a hidden label that prescribes the output of arbitrary hypothetical measurements.

Measurement is treated as a usual reversible interaction, and this postulate allows to determine relative frequencies when the value of a magnitude is known without in any way perturbing the system, by measurement of the correlated companion.

It is predicted the existence of an accompanying system, the de Broglie wave, introduced in order to preserve the action reaction principle in indirect measurements, when there is no interaction of detector and particle. Some action on the detector, different from the one cause by a particle, should be observable.

1 Introduction

In the author’s alternative interpretation of Quantum Mechanics [6], both spinless point particles in phase space and 1/2 spin particles in the space of spin states were represented in an enlarged Hilbert space, where physical states can have simultaneously definite values of two or more non commuting magnitudes, position and momentum in phase space, two or more spin directions in spin space. Observable predictions of Quantum Mechanics (the marginal amplitudes of probability) were reproduced in individual systems, but the existence of an additional physical ingredient, the de Broglie wave [7], was proposed in order to preserve the action reaction principle, which seems to be violated in Quantum Mechanics for indirect measurements. The state of the particle with respect to a given magnitude was described through a label $\lambda$, attached to one of the non null components of the vector state as linear combination of eigenstates of the magnitude (position, momentum, spin in a given direction). For particles correlated in spin variables, entanglement in the product of Hilbert spaces of
both particles was replaced by a $\lambda$ correlation prescribing a total null spin, and the composite state was direct product of individual ones.

In the former interpretation there is no scientific (observable) contradiction with orthodox Quantum Mechanics, relative frequencies for measurements of individual systems match the known values. The local $\lambda$ description of correlation for spin variables is also an interpretation, Aspect’s experiment [1] with polarization of photons and Bell’s *gendaken* experiment [3] with spin are satisfactorily reproduced.

Measurement over an eigenstate (change of state of the apparatus but not of the measured system) suggest the existence of hidden variables in order to preserve the action reaction principle. As a new observable system (but not contradictory with the orthodox theory), it was predicted the existence of a real, physical wave, interacting with the detector when virtual paths of a process are discarded because the particle is not detected, and the projection rule is applied to take into account this fact. Detector and particle do not interact, but there is an observable change of state in the particle. The suppressed (or phase shift of) wave component would explain this change of state in case some measurable effect could be observed on the detector.

The theory presented in this article applies the former ideas, states with simultaneously definite values of non commuting magnitudes, hidden label prescribing the output of measurements, $\lambda$ correlation, to a generic quantum system, state in a Hilbert space, set of self adjoint operators. While there is no observable distinction with orthodox Quantum Mechanics for individual systems (up to the undetected de Broglie wave), a different table of relative frequencies is obtained for generic correlated systems and variables, although this difference disappears in particular cases, e.g., for spin and polarisation variables. In fact, there is ambiguity in the standard theory for generic cases in the prediction of the distribution of probabilities for measurement events that are spatially separated; only if probabilities for both possible temporal orders of measurement coincide the ambiguity disappears.

In the proposed Quantum Mechanics with label, the distribution becomes different from the orthodox one even when there is well established time order in the measurement events and the projection rule should be applied, because in the alternative theory there is no projection of state in the system not being measured, and the type of space time separation between measurements is irrelevant. For two correlated systems $S_a$ and $S_b$, measurement of system $S_b$ is just a source of information about outputs of an hypothetical measurement on system $S_a$. Measurement is considered a usual Hamiltonian, reversible interaction; the distribution in the $S_a$ state of probabilities for the (correlated) variable (the one measured at $S_b$) is preserved under the real $S_a$ measurement, that is, knowledge of it after measurement (real interaction, projection of state) determines the distribution previous to the interaction. This postulate of reversibility, a kind of “virtual” (non interacting) measurement, allows to obtain predictions of relative frequencies; they match the standard ones in absence of projection rule, and determine the statistical correlation with the other system’s measurement outputs.
Next section presents the theory in the general case for individual systems. The previous examples of phase space for a point like spinless particle and spin state space are revisited. They are certainly the most relevant; if entanglement is replaced by direct product of individual states, as described in section 3, every quantum system can be decomposed into its elementary subsystems (elementary particles). Section 3 introduces the description of correlated systems through the \( \lambda \) mechanism, and when Born’s rule is applied, both in real and “virtual” measurements, we get relative frequencies to be compared with the orthodox theory. A gendaken two sit experiment, with an additional system correlated to the right–left slit variable of the particle, shows that, while in the standard theory the diffraction pattern disappears for early measurement of the other system, when it is known before reaching the final screen which slit the particle is coming from, in this formulation diffraction pattern is preserved, even when the correlated system’s very early measurement event allows to know in advance which slit the particle will go through at the first screen.

2 Description of a generic quantum system

Let \( S \) be a quantum system, \( F = \{A, B, C, \ldots\} \) physical magnitudes (and their self adjoint representation in the Hilbert space \( H \) of the system), \( |S> \) a particular quantum state of \( S \), unit vector (or ray) in \( H \). We will denote by \( a_i, b_j, \ldots \), eigenvalues of \( A, B \), etc., and \( |a_i>, |b_j> \) eigenvectors of the shown eigenvalue. If \( |S> = \sum_i z_i^A |a_i> \), all \( a_i \) with non vanishing \( z_i^A \) are possible outputs of a measurement of magnitude \( A \) on \( |S> \). For a given family \( F \) of not necessarily commuting magnitudes, the set of \( n \)-tuples of consistently (in \( |S> \)) joint values for these magnitudes is defined by

\[
\mathcal{M}_{|S>} = \{(a_i, b_j, c_k, \ldots)| < a_i|b_j> \neq 0, < a_i|c_k> \neq 0, \ldots\}
\]

i.e., \( a_i \) and \( b_j \) do not belong to the same element of \( \mathcal{M}_{|S>} \) if pure state \( |a_i> \) has not \( b_j \) as possible output of a \( B \)-measurement (and vice versa). The same requirement applies to all pairs of values for different magnitudes. These values \( \{a_i, b_j, c_k, \ldots\} \) are formally defined, and can not be jointly and consistently measured on \( |S> \) for non commuting magnitudes. We will denote by \( \pi_A : \mathcal{M}_{|S>} \rightarrow \mathcal{E}_A = \{a_i\} \) the projection over the set of \( A \) eigenvalues, \( \pi_B \) over \( \mathcal{E}_B \), etc.

In orthodox Quantum Mechanics, it is on \( \mathcal{E}_A, \mathcal{E}_B, \ldots \), and not on \( \mathcal{M}_{|S>} \), where a distribution of amplitudes of probability is defined, and this is just the quantum state \( |S> \), with corresponding amplitudes \( z_i^A, z_i^B, \) etc. If \( M_A(|S>) \) represents the output of an \( A \) measurement performed on \( |S> \), \( M_A(|S>) \in \mathcal{E}_A \), the distribution of probabilities for these outputs is \( P(a_i) = |< a_i|S>|^2 \). After measurement, the new state of the system is \( |a_i> \) when \( M_A(|S>) = a_i \).

In the usual interpretation, a state \( |S> \neq |a_i> \) has no definite value of magnitude \( A \) (but it can have forbidden values \( z_i^A = 0 \)), and it is at measurement when a precise value is achieved. We could put a label on the element of \( \mathcal{E}_A \) output of measurement. In alternative interpretations, the label exists (but hidden)
before measurement is performed, and determines its output. This hypothesis
is introduced in order to interpret measurement as a usual interaction, and the
projection rule as both a prescription of the new state (after interaction with
the apparatus) and a probabilistic rule over a family of non identical physical
systems.

Obviously, a hidden label without any other modification in the mathemati-
cal machinery, is just an interpretative matter. But, if the label is present
(an element of reality \[4\]) before measurement, the pure state \(|S>|\) must be a
sample of non identical physical systems, with the label in different positions.
Moreover, while we put the label generated at measurement once at a time
for each successive measurement of non commuting observables, if it is always
present there must be all the time a label at each \(E_A, E_B, \ldots\) ready to deter-
mine the output of an arbitrary measurement. Taking into account the previous
consistency requirement, the label will be an element \(\lambda \in M_{|S>}\).

Let us consider the pure quantum state as a sample, family of physical
systems, \(|S>=\{|S; \lambda >; \lambda \in M_{|S>}\}, all of them represented by the same state
vector \(|S>\) in \(\mathcal{H}\), but each one with different label. The label completely (for
all considered magnitudes) determines the output of arbitrary measurements of
the system. While \(M_A(|S>) \in \mathcal{E}_A\) is probabilistic, \(M_A(|S; \lambda >) = \pi_A(\lambda)\) is
fixed for an hypothetically known label. Obviously, the probabilistic character
of the theory does not disappear, it is just displaced from each \(E_A, E_B, \ldots\), onto
\(M_{|S>}: in this notation, \(P(\pi_A(\lambda)) = |<\pi_A(\lambda)|S>|^2\) replaces \(P(a_i) =
|<a_i|S>|^2\) when applied to the statistical “universe” \(|S>=\{|S; \lambda >\}. The
former relation means: “there is probability \(P(\pi_A(\lambda) = a_i)\) that a randomly
chosen element \(|S; \lambda > of \(|S>\) has \(\pi_A(\lambda) = a_i”\).

We can ask ourselves if there is a distribution of probabilities in \(M_{|S>}, P(\lambda),\)
such that

1. Marginal probabilities match the quantum theory values

\[
\sum_{\lambda \in \pi_A(\lambda) = a_i} P(\lambda) = P(\pi_A(\lambda)) = P(a_i) = |z^A_i|^2
\]

The previous sum is restricted to labels fulfilling the given condition. A
similar condition must hold for all considered magnitudes \(B, C, \ldots\)

2. The condition of consistency, contained in the definition of \(M_{|S>}, which
is a subset of \(\mathcal{E}_A \times \mathcal{E}_B \times \cdots\)

\(P(\lambda) = 0 \quad \text{if} \quad <\pi_A(\lambda)|\pi_B(\lambda) = 0 \quad \text{or} \quad \ldots\)

i.e., \(P(\lambda)\) vanishes if some pair of projected values are incompatible.

3. Positivity, \(P(\lambda) \geq 0.\) From 1 and 3 we get \(P(\lambda) = 0\) when \(P(\pi_A(\lambda)) = 0\)
for some \(A.\)
It is well known that the third requirement cannot be accomplished in generic cases, as Bell’s type inequalities theorems prove [3]. The first and second requirements are linear equations, generically with more unknowns than equations, and have usually many solutions. Symmetry considerations can establish a preferred solution. For two magnitudes it is easy to find a general solution, with notation \( W \) instead of \( P \) because of its quasi probability (weight, or Wigner [5]) character,

\[
W(a_i, b_j) = \frac{1}{2} \left( z_i^A z_j^B < a_i | b_j > + z_j^B z_i^A < b_j | a_i > \right)
\]

from where

\[
\sum_j W(a_i, b_j) = \frac{1}{2} \left( z_i^A < a_i | \sum_j z_j^B b_j > + z_i^A (\sum_j z_j^B < b_j | a_i >) \right) = \frac{1}{2} \left( z_i^A < a_i | S > + z_i^A < S | a_i > \right) = z_i^A z_i^A = P(a_i)
\]

For example, Wigner’s quasi probability distribution in phase space is obtained from

\[
1 = \frac{1}{2} \left( \int dx_1 \Psi^\ast(x_1) < x_1 | \int dp \xi(p) | p > + \int dp \xi^\ast(p) < p | \int dx_1 \Psi(x_1) | x_1 > \right) = \frac{1}{2} \left( \int dx_1 \int dp \int dx_2 \Psi^\ast(x_1) \Psi(x_2) e^{i/\hbar p(x_1-x_2)} + \text{conjugate} \right)
\]

and, after change of variables \( x_1 = x + s/2, x_2 = x - s/2 \), we find the well known Wigner’s kernel \( W(x, p) \),

\[
1 = \int dx \int dp \int ds \Psi^\ast(x + s/2) \Psi(x - s/2) e^{i/\hbar s} = \int dx \int dp W(x, p)
\]

There is no physical interpretation for these weights; moreover, they are secondary ingredients, built from the amplitudes. Although there is neither a clear physical interpretation of amplitudes of probability, the path integral formalism [2] suggest its association with some physical wave, the de Broglie wave [6] of which phases are added in a diffusion, propagation process with the typical interference phenomenon. We can ask ourselves if there is a distribution \( Z(\lambda) \), of amplitudes of probability in \( \mathcal{M}_{|S>} \), representation of the wave, and such that

1. Marginal amplitudes match the quantum theory

\[
\sum_{\lambda \in \pi_\lambda(a_i)=a_i} Z(\lambda) = z_i^A
\]

for each magnitude \( A, B, \ldots \)
2. Consistency in $\mathcal{M}_{|S>}$

$$Z(\lambda) = 0 \quad \text{if} \quad <\pi_A(\lambda)|\pi_B(\lambda)> = 0 \ldots$$

i.e., whenever some $<a_i|b_j> = 0$.

3. No positivity requirement, amplitudes are not even real numbers!

The third point allows (does not prevent) the existence of consistent solutions, contrarily to the probabilities distribution problem. Again, they are linear equations with many solutions in general. However, only one solution would have physical meaning if these amplitudes (i.e., relative frequencies through Born’s rule) were observable. For two magnitudes $A$ and $B$, with amplitude $Z_{ij} = Z(a_i, b_j)$ and associated joint probability (always positive!) $|Z_{ij}|^2$, the relative frequencies can not be observed if $A$ and $B$ do not commute. Measurement of $A$ necessarily modifies $\pi_B(\lambda)$, and vice versa. We can only observe $P(a_i) = |z_i^A|^2$, and then $P(b_j|a_i) = |<b_j|a_i>|^2$ on the new state $|a_i>$ after projection. Similarly, $P(b_j) = |z_j^B|^2$ for $B$ measurement, and $P(a_i|b_j)$ (which equals $P(b_j|a_i)$) on the new state $|b_j>$ after projection. $P(a_i)P(b_j|a_i) \neq P(b_j)P(a_i|b_j)$ unless $|z_i^A|^2 = |z_j^B|^2$.

Let us consider a simple measurement of magnitude $A$, performed on a known eigenstate $|a_i>$ of $A$. Before measurement, the distribution of amplitudes of probability for $B$ is $<b_j|a_i>$. The label in a particular system $|a_i, \lambda>$ of the given state is an unknown $b_j = \pi_B(\lambda)$ together with the known $a_i, \lambda = (a_i, b_j)$. In this case we know the distribution of amplitudes $Z_{ij} = Z(\lambda = (a_i, b_j)) = <b_j|a_i>$ (or $Z_{ij} = \delta_{ij} <b_j|a_i>$) and probabilities $|<b_j|a_i>|^2$, according to Born’s rule, before measurement. This is the distribution to be found if $B$ were measured in a sample of systems (identical in the standard interpretation, with different $\lambda$ here). As we measure $A$, the output is obviously $a_i$, and there is no change of pure quantum state. The new distribution matches the initial one.

If measurement is a usual Hamiltonian interaction with Hamiltonian $H(A, Y)$, depending on the measured magnitude $A$ and some variable $Y$ of the apparatus, action over the measurement system (new state of the pointer) is accompanied by a reaction over $|a_i, \lambda>$, i.e., an output system $|a_i, \lambda'>$ of the same quantum state, both $\pi_B(\lambda') = b_j$ and $\pi_B(\lambda') = b_j'$ unknown, with $b_j' \neq b_j$.

The relevant point is that the distribution $Z(\lambda')$ equals $Z(\lambda)$, according to quantum rules.

Notice that $\pi_A(\lambda) = a_i = \pi_A(\lambda')$ is preserved in an arbitrary measurement of magnitude $A$, from an input $|S, \lambda>$ with $\pi_A(\lambda) = a_i$ unknown before measurement ($|S \neq |a_i>$ in a generic case), onto the output $|a_i, \lambda'>$: the ideal Hamiltonian of interaction commutes with $A$. By reversibility of Hamiltonian interactions, when $A$ is measured the statistical distribution of the $b_j$ is not modified, although the label changes from $\pi_B(\lambda) = b_j$ to $\pi_B(\lambda') = b_j'$ at

---

1In indirect measurements there is no physical interaction of particle, the label, with the detector. Therefore, the label does not change. It is the de Broglie wave that physically interacts, and either it is suppressed by the obstacle or there is a phase shift. This phenomenon, necessary to maintain the action reaction principle, has not been observed.
each particular measurement event. That is, as we get an output $|a_i, \lambda'>$ with known distribution $<b_j|a_i>$ for the $b_j$, the same distribution is the initial one in state $|S, \lambda>$, a state of which we know (a posteriori) that it fulfilled $\pi_A(\lambda) = a_i = \pi_A(\lambda')$.

We postulate that the conditional distribution $Z_{ij} = Z(a_i, b_j)$ for a state $|S, \lambda>$ with $\pi_A(\lambda) = a_i$ matches the distribution $<b_j|a_i>$ of the eigenstate $|a_i>$. For the joint distribution of amplitudes of probability

$$|Z_{ij}|^2 = |<b_j|a_i>|^2$$

Similarly, $\pi_B(\lambda) = b_j$ conditional distribution for $Z_{ij}$ matches $<b_j|a_i>$,

$$|Z_{ij}|^2 = |<b_j|a_i>|^2$$

from where we find

$$\sum_{j'}^N |Z_{ij'}|^2 = \sum_{j'}^N |Z_{ij'}|^2 = \frac{1}{N}$$

in a normalised total state $\sum_{j'}^N |Z_{ij'}|^2 = 1$.

An initial measurement of $A$ has marginal distribution of amplitudes and probabilities

$$z_i^A = \sum_{j'} Z_{ij'} \quad P(a_i) = |z_i^A|^2 = \sum_{j'} |Z_{ij'}|^2$$

or, equivalently,

$$P(a_i) = |\sum_j z_j^B <a_i|b_j>|^2$$

(1)

On the other hand, an initial measurement of $B$ gives

$$P(b_j) = |z_j^B|^2 = \sum_{j'} |Z_{ij'}|^2$$

and, if later on we measure $A$ on output states $|b_j>$, we find

$$P(a_i|b_j) = |<a_i|b_j>|^2 \quad P(b_j) P(a_i|b_j) = |z_j^B|^2 |<a_i|b_j>|^2$$

with marginal probability

$$P'(a_i) = \sum_j P(b_j) P(a_i|b_j) = \sum_j |z_j^B|^2 |<a_i|b_j>|^2$$

(2)

This is a well known result: there is interference of all $b_j$ components in $z_i^A$ and $P(a_i)$, while there is a sum of classical independent probabilities in $P'(a_i)$. But,
contrarily to the usual interpretation, interference at $P(a_i)$ does not contradict
an assignment of (hidden) $b_j$ values to the initial state. The label contains
a particular $b_j$ value, but the associated de Broglie wave, represented by the
orthodox quantum state, has all $b_j$ components.

The previous postulate allows to find an essentially unique solution for $Z_s$;
the whole information is contained in the orthodox quantum state. Its physical
foundation is time reversibility of dynamical evolution, including measurement.
At all times, not just under measurement, the label is attached to some $n$–tuple
of values (with some kind of stochastic time evolution) of the physical magni-
tudes. We can consider at any time a “virtual” measurement of an arbitrary
magnitude $A$. The theory will be consistent with orthodox Quantum Mechanics
if the conditional amplitude and probability distribution for any other magni-
tude $B$ in the (real) state before the virtual measurement matches the one of
the (virtual) quantum state for the projection rule applied according to the
hypothetical output.

It is important to distinguish between virtual and real measurements. The
first one is used to formally derive a distribution of probabilities without physical
effects on the system, while in the second case there is real interaction, and
therefore physical effects. What is forbidden at individual systems, simultaneous
or consistent measurement of non commuting magnitudes, becomes observable
in correlated states, where a real measurement in one system plays the role of
virtual one in the companion.

A distribution of amplitudes in the space of spin states, with arbitrary num-
ber of directions of spin (obviously, all of them non commuting), was given in
\[6\]. In the continuum limit, we consider a measurable function $f : S^2 \to \{+, -\}$,skew $f(-n) = -f(n)$, representing the hidden state of spin; $f(n)$ is the $\pi_n \in
\{+,-\}$ projection of the label $\lambda \equiv f$, the result of an hypothetical measure-
ment in $n$ direction, and $|f>$ represents the elementary labelled state. The
amplitude of probability for $|f>$ is

$$
\Psi(f) = \int d\Omega f(n) N
$$

where $N = (n \cdot i)(n \cdot j)(n \cdot k)K$ is a quaternion without real component. An
orthodox quantum state $|n_0>$, with positive spin in direction $n_0$, is represented
by

$$
||n_0 >= \int_{f(n_0)=1} Df \Psi(f)||f>
$$

i.e., the family of all states $|n_0, f>$ fulfilling the condition $f(n_0) = 1$, and its
associated distribution of amplitudes $\Psi(f)$.

Similarly, a solution in phase space for a point like spinless particle, alterna-
tive to the one given in \[6\], is

$$
||S >= \int dx \int dp \Psi(x|\xi(p)e^{ix(p-x_0)}|x'>|p'>
$$
$(x_0, p_0)$ is a reference value at which $\Psi(x_0) \neq 0 \neq \xi(p_0)$, and we have taken advantage of the gauge freedom, even after normalisation, to modify the phase of an orthonormal basis of eigenstates, $|x \rangle \rightarrow \exp(ipx_0/\hbar)|x \rangle$ and $|p \rangle \rightarrow \exp(-ipx_0/\hbar)|p \rangle$. Trace over the second component

$$|S \rangle = \Psi(x_0) \int dx \Psi(x) e^{i\pi pr^2/\hbar} |x \rangle$$

reproduces the quantum state ray.

### 3 Correlated systems

Measurement is an interaction, a usual Hamiltonian interaction. Measurement is an interaction in which the initial and a family of final states for one of the interacting systems, the measurement apparatus, can be distinguished. Then, any interaction is a potential measurement, if we were able to distinguish among states of one of the interacting systems. Being able to distinguish among states is a possible definition of macroscopic state, and there is nothing fundamental on it. In fact, it evolves with the available technology, and we can now distinguish states that were indistinguishable in the past.

There are three ways in which we can get (necessarily partial) information about the state of a system, the position of the label. First, by direct interaction with the particle, direct measurement. Second, by indirect interaction with other components of the accompanying wave, indirect measurement, when some virtual paths of the particle are discarded but there is no direct interaction of the particle with the obstacle or detector. In both cases the state of the (compound particle plus wave) system is unavoidably modified by interaction with particle or wave. In indirect measurement, the label does not jump because it is attached to the unperturbed particle state. But the new labelled state has less (or shifted) wave components.

Third, we can get information on the label in a system $S_a$ through measurement on another system $S_b$ that has interacted in the past with $S_a$, and some correlation between variables of both systems has been established. In this case, we get information “without in any way disturbing the system” [4] $S_a$, i.e., its individual quantum state does not change because of measurement on $S_b$, no matter if that measurement event happens in the past, future or with spatial separation with respect to a measurement on the system $S_a$. Of course, if there is an instrumental arrangement in such a way that outputs of measurement on $S_b$ generate some planned physical reaction reaching $S_a$ before it is measured, the system could change of state through a real interaction. But projection of state is applied to $S_b$ because of interaction with its measurement apparatus, and it will have in general localised effects, but not over $S_a$.

Let us consider two quantum systems $S_a$ and $S_b$, and two correlated magnitudes $B_a$ with $B_b$ because of a previous interaction between them. Without loss of generality, we consider that the state of the compound system is an eigenvector of $B_a + B_b$ with null eigenvalue, $(B_a + B_b)|S_T \rangle = 0$. 
In a basis of eigenvectors,

$$|S_T> = \sum_j z_j^B |b_j^a> \times |b_j^b> = -b_j^a >$$

in the product Hilbert space. $|S_T>$ is not of rank one, and a change of state (e.g., through measurement, but also through other interactions) in system $S_b$ will unavoidably modify the state and table of relative frequencies for subsequent measurements over system $S_a$.

With the introduction of the label, we can replace the previous entanglement by a $\lambda$ correlation. Systems $S_a$ and $S_b$ have each a quantum state $|S_a> = \sum_j z_j^B |b_j^a>$ and $|S_b> = \sum_j z_j^B |b_j^b>$ describing amplitudes and relative frequencies, but correlated elements of $|S_a>$ and $|S_b>$ have also a particular label ($|S_a, \lambda_a>$, $|S_b, \lambda_b>$), such that $\pi_B(\lambda_a) = -\pi_B(\lambda_b)$, a condition that appears at the initial interaction event. Then, measurements of magnitude $B$ on a jointly generated pair will present perfect correlation.

In the space of spin states, the total null spin for particles $S_a$ and $S_b$ is prescribed through correlation $|S_a, \lambda_a = f >$ and $|S_b, \lambda_b = -f >$. A statistical sample of measurements on both particles in arbitrarily chosen directions reproduces the quantum correlation without projection of state.

Let us consider another magnitude $A$ of $S_a$. If magnitude $A$ is measured in $S_a$, and no measurement in $S_b$ is performed, we can ignore the correlation (i.e., trace over the second component), and the distribution of probability is

$$P(a_i) = |\sum_j z_j^B <a_i|b_j^a>|^2$$

in both the standard theory and the theory with label.

On the contrary, if magnitude $B$ is measured in $S_b$, and in a future event magnitude $A$ is measured in $S_a$, according to the standard projection rule (which applies to correlated systems because of the entanglement representation) it is equivalent to performing both measurements in $S_a$ in the exposed order. The marginal probability for $a_i$ is

$$P'(a_i) = \sum_j P(b_j)P(a_i|b_j) = \sum_j |z_j^B <a_i|b_j>|^2$$

In the theory with label, measurement of $B$ in $S_b$ is understood as a “virtual” measurement, i.e., just information, about the value $\pi_B(\lambda_a)$. Without projection rule, the distribution of probability for $a_i$ remains $P(a_i)$. We can calculate the correlation with measurement outputs in $S_b$,

$$P(b_j|a_i) = |<b_j|a_i>|^2 \quad \text{or} \quad P(b_j|a_i) = \frac{|Z_{ij}|^2}{\sum_{j'}|Z_{ij'}|^2}$$

when using joint probability distributions.

A *gendaken* experiment where both theories have different predictions is a two slit experiment with particle $S_a$, and a second system $S_b$ correlated with
the particle slit variable \( L \) or \( R \). Both systems are generated at the origin and move in opposite directions; position (or some other variable) of \( S_b \) is supposed perfectly correlated with \( S_a \) going through \( L \) or \( R \) slit (in an ideal case). An early measurement event of \( S_b \) can lie in the past of the arrival event of \( S_a \) to the final screen, or even in the past of its arrival to the slits screen. Knowledge of the \( L-R \) variable before \( S_a \) reaches the final screen gives way to projection of its state in the usual treatment, therefore, disappearance of the diffraction pattern. On the other side, if \( S_b \) variable is not measured (or if it is measured in the future of the arrival event of \( S_a \) to the screen), the diffraction pattern is preserved. There is ambiguity about the predicted behaviour (diffraction pattern or not) of \( S_a \) when there is spatial separation between both measurement events. Let us fix the \( S_a \) measurement event; initial and final boundary points, of the interval in which there is spatial separation with \( S_a \) measurement event, do not match. There is no physical argument to select one or the other behaviour, i.e., to extrapolate one or the other boundary behaviours onto the spatial separation interval.

In the theory with label, the knowledge of the slit variable is obtained without in any way perturbing the system, and the diffraction pattern is always preserved, no matter the temporal relation with the other particle measurement. That is, even for very early \( S_b \) measurements, when we know in advance which slit is going to go \( S_a \) through, its individual state is not perturbed. There is no ambiguity in this prediction, the other measurement event is irrelevant for the behaviour of \( S_a \). There is also a contradiction with the standard prediction for early measurements of \( S_b \).

More specifically, with \( \Psi(r_i, L) \) and \( \Psi(r_i, R) \) denoting the amplitudes at position \( r_i \) of the screen for waves from \( L \) and \( R \) slit respectively, the total distribution on the screen is \( \left| \Psi(r_i, L) + \Psi(r_i, R) \right|^2 \) (diffraction pattern); the conditional distribution, for particles arriving to \( r_i \), between labels \( L \) and \( R \) (information obtained through measurements of \( S_b \)) is

\[
P(L|r_i) = \frac{|\Psi(r_i, L)|^2}{|\Psi(r_i, L)|^2 + |\Psi(r_i, R)|^2} \quad P(R|r_i) = \frac{|\Psi(r_i, R)|^2}{|\Psi(r_i, L)|^2 + |\Psi(r_i, R)|^2}
\]

4 Summary and outlook

Two observable predictions have been obtained from the Quantum Mechanics with label: a distribution of probabilities without projection of state for correlated systems, and existence of an additional physical system, the de Broglie wave, accompanying the particle even when isolated (so, an æther).

The first one solves an ambiguity of the standard Quantum Mechanics that appears, in generic cases, at spatially separated measurements of entangled systems, and contradicts the standard theory when the orthodox projection rule
should be applied for well established time order of measurements. Some additional interpretative issues are: non local interaction does not happen in this theory; measurement is a Hamiltonian, reversible interaction; quantum systems have (hidden) values of non commuting magnitudes simultaneously. However, the alternative theory of Quantum Mechanics with label becomes, at the end of the day, a single modification of the projection rule for practical purposes: it is not applied to the state of a system when measurement is performed in a correlated system.

The de Broglie wave has perhaps higher relevance. Introduced to preserve the action reaction principle in indirect measurements, its hypothetical existence opens new insight into a bunch of issues. Tunnel effect, and in general energy fluctuation, would be an energy interchange between particle and accompanying wave, with strict conservation of total energy. Wave particle duality would be just wave and particle. Spontaneous emission for isolated (meta)stationary excited systems could be understood as induced emission of the system in a “thermal” bath, the surrounding æther.

The path integral formalism would have a more direct interpretation, a diffusion process of the wave. Inspired in the path integral formalism, where paths could be grouped by final position and velocity (or momentum) of the particle, the theory with label can be applied as it is to Relativistic Quantum Mechanics and Quantum Field Theory: particles and fields have precise, but hidden, values of non commuting physical magnitudes. The joint use of position and velocity variables in Relativistic Quantum Mechanics could allow to formulate it in a curved, instead of Minkowski’s flat, background.

Vacuum energy effects are amply accepted as observable in Quantum Field Theory (Casimir energy) and Cosmology (dark energy), at both extreme scales of length. The de Broglie wave, and perhaps also dark matter, can be additional observable effects of the vacuum æther at intermediate scales.

5 Acknowledgements

Financial support from research project MAT2011-22719 is acknowledged. I also kindly acknowledge helpful comments from members of the Department of Theoretical Physics, University of Zaragoza, where I presented some results of this research in April 9th 2015.

References

[1] A. Aspect, P. Grangier and G. Roger : Phys. Rev. Lett. 49 1804 (1982)
[2] R. P. Feynman and A. R. Hibbs : QM and Path Integrals McGraw–Hill, New York (1965)
[3] J. S. Bell : Physics 1, 195 (1964)
[4] A. Einstein, B. Podolsky and N. Rosen : Phys. Rev.47, 777 (1935)
[5] E. P. Wigner : *Phys. Rev.* **40** 749 (1932)

[6] C. López : sent to *Foundations of Physics*, Jan. 2015.

[7] L. de Broglie : *Annales de la Fondation Louis de Broglie* **12**(4) 1 (1987)