Mass Formulae for Supernarrow Dibaryons and Exotic Baryons

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Abstract

The mass formulae for the supernarrow dibaryons and the exotic baryons with small masses are constructed. With this aim their self energies are calculated in one loop approximation using the dispersion relations with two subtractions. The values of the masses obtained in this approach are in a good agreement with the experimental data. The mass formula for the baryons is also used to calculate the mass of the $\Delta(1232)$ resonance.

1 Introduction

Supernarrow dibaryons (SNDs) are 6-quark states, a decay of which into two nucleons is forbidden by the Pauli exclusion principle \cite{1,2,3,4}. Such dibaryons satisfy the following condition:

\[ (-1)^{T+S}P = +1 \]  

where \( T \) is the isospin, \( S \) is the internal spin, and \( P \) is the dibaryon parity. These dibaryons with the masses \( M < 2m_N + m_\pi \) (\( m_N(m_\pi) \) is the nucleon (pion) mass) can mainly decay by emitting a photon. This is a new class of dibaryons with the decay widths < 1keV. The experimental discovery of such states would have important consequences for particle and nuclear physics and astrophysics \cite{5}.

The SNDs have been first experimentally observed in works \cite{6,7,8,9,10,11,12} with the masses \( M = 1904, 1926, \) and 1942 MeV. Unfortunately, existed theoretical models do not allow a correct calculation of dibaryon masses in the mass region under consideration.

On the other hand, in the reactions \( pd \to p+pX \) \cite{10,11,12} and \( pp \to \pi^+pX \) \cite{13}, connected with the SND production, peaks have been found in the missing mass spectra. These peaks have been interpreted in \cite{13} as a manifestation of new exotic baryons with anomaly small masses. The calculation of the masses of exotic baryons are usually realized in the framework of the quark cluster models \cite{14,15,16,17}. However, these models do not reproduce all baryon correctly enough. And although the question about the nature of these peaks is still open, it is important to construct a mass formula which allows a more correct analysis of them.

In the present paper we construct mass formulae for SNDs and exotic baryons. With this aim we calculate self energies of the SNDs and the exotic baryons in one loop approximation using dispersion relations with two subtractions.

In order to check an efficiency of this approach, the mass formula for the exotic baryons is also used to calculate the mass of the $\Delta(1232)$ resonance.
The application of the mass formulae constructed for a calculation of the masses of the SNDs and the exotic baryons results in a good agreement with the experimental data [12, 13].

The contents of the paper are the following. In Section 2 and 3 the works on the SNDs and the exotic baryons are reviewed, respectively. In Section 4 the mass formula for exotic baryons is constructed and the equation obtained is used for the calculation of the mass of the $\Delta(1232)$ resonance. The calculation of the masses of the exotic baryons is performed in Section 5. The construction of the mass formula for SNDs and the calculation their masses are given in Section 6. The main conclusion are given in Section 7.

2 Supernarrow dibaryons

In works [6-12], the reactions $pd \rightarrow p + pX_1$ and $pd \rightarrow p + dX_2$ were studied with the aim of searching for SNDs. The experiment was carried out at the Proton Linear Accelerator of INR with 305 MeV proton beam using the two-arm spectrometer TAMS. As was shown in [8-9], the nucleons and the deuteron from the SND decay into $\gamma NN$ and $\gamma d$ have to be emitted in a narrow angle cone with respect to the direction of the dibaryon motion. On the other hand, if a dibaryon mainly decays into two nucleons, then the expected angular cone of the emitted nucleons must be more than $50^\circ$. Therefore, a detection of the scattered proton in coincidence with the proton (or the deuteron) from the decay of the dibaryon at correlated angles allowed the authors to suppress the contribution of the background processes essentially and to increase the relative contribution of a possible SND production.

Several software cuts have been applied to the mass spectra in these works. In particular, the authors limited themselves by the consideration of an interval of the proton energy from the decay of the $pX_1$ states, which was determined by the kinematics of the SND decay into $\gamma NN$ channel. Such a cut is very important as it provides an additional possibility to suppress essentially the contribution from the background reactions and random coincidences.

In [10-12], CD$_2$ and $^{12}$C were used as targets. The scattered proton was detected in the left arm of the spectrometer TAMS at the angle $\theta_L = 70^\circ$. The second charged particle (either $p$ or $d$) was detected in the right arm by three telescopes located at $\theta_R = 34^\circ$, $36^\circ$, and $38^\circ$.

As a result, three narrow peaks in the missing mass $M_{pX_1}$ spectra have been observed at $M_{pX_1} = 1904 \pm 2, 1926 \pm 2$, and $1942 \pm 2$ MeV with widths equal to the experimental resolution ($\sim 5$ MeV) and with numbers of standard deviations of 6.0, 7.0, and 6.3, respectively. It should be noted that the dibaryon peaks at $M_{pX_1} = 1904$ and $1926$ MeV had been observed earlier by same authors in [6-8, 10] at somewhat different kinematical conditions. On the other hand, no noticeable signal of dibaryons has been observed in the missing mass $M_{dX_2}$ spectra of the reaction $pd \rightarrow p + dX_2$. The analysis of the angular distributions of the protons from the decay of the $pX_1$ states and the suppression observed of the SND decay into $\gamma d$ showed that the peaks found can be explained as a manifestation of the isovector SNDs, the decay of which into two nucleons is forbidden by the Pauli exclusion principle.

An additional information about the nature of the observed states was obtained by studying the missing mass $M_{X_1}$ spectra of the reaction $pd \rightarrow p + pX_1$. If the state found is a dibaryon mainly decaying into two nucleons then $X_1$ is a neutron and the mass $M_{X_1}$ is equal to the neutron mass $m_n$. If the value of $M_{X_1}$, obtained from the experiment, differs essentially from $m_n$ then $X_1 = \gamma + n$ and it is an additional indication that the dibaryon observed is the SND.
The simulation of the missing mass $M_{X_1}$ spectra of the reaction $pd \rightarrow p + pX_1$ has been performed \cite{10,11,12} assuming that the SND decays as $SND \rightarrow \gamma + ^{31}S_0 \rightarrow \gamma pn$ through two-nucleon singlet state $^{31}S_0$ \cite{2,9,12}. As a result, three narrow peaks at $M_{X_1} = 965, 987, \text{and} 1003 \text{ MeV}$ have been predicted. These peaks correspond to the decay of the isovector SNDs with the masses 1904, 1926, and 1942 MeV, respectively.

In the experimental missing mass $M_{X_1}$ spectrum besides the peak at the neutron mass caused by the process $pd \rightarrow p + pn$, a resonance-like behavior of the spectrum has been observed at $966 \pm 2, 986 \pm 2, \text{and} 1003 \pm 2 \text{ MeV}$ \cite{10,11,12}. These values of $M_{X_1}$ coincide with the ones obtained from the simulation and essentially differ from the value of the neutron mass (939.6 MeV). Hence, for all states under study, one has $X_1 = \gamma + n$ in support of the statement that the dibaryons found are SNDs.

In \cite{18} dibaryons with exotic quantum numbers were searched for in the process $pp \rightarrow pp\gamma\gamma$. The experiment was performed with a proton beam from the JINR Phasotron at an energy of about 216 MeV. The energy spectrum of the photons emitted at $90^\circ$ was measured. As a result, two peaks have been observed in this spectrum. This behavior of the photon energy spectrum was interpreted as a signature of the exotic dibaryon resonance with the mass of about 1956 MeV and a possible isospin $T = 2$.

On the other hand, an analysis \cite{19} of the Uppsala proton-proton bremsstrahlung data looking for the presence of a dibaryon in the mass range from 1900 to 1960 MeV gave only the upper limits of 10 and 3 nb for the dibaryon production cross section at proton beam energies of 200 and 310 MeV, respectively. This result agrees with the estimates of the cross section obtained at the conditions of this experiment in the framework of the model suggested in \cite{2,9} and does not contradict to the data of \cite{18}.

It should be noted that if this dibaryon has $T = 2$ then it could not be a SND, the decay of which into two nucleons is forbidden by the Pauli exclusion principle. The decay of such a dibaryon into two nucleons is forbidden by the value of the isospin $T = 2$.

The reactions $pd \rightarrow pdX_2$ and $pd \rightarrow ppX_1$ have been also investigated by Tamii et al. \cite{20} at the Research Center for Nuclear Physics at the proton energy 295 MeV in the mass region of 1896–1914 MeV. They did not observe any narrow structure in this mass region and obtained the upper limit of the production cross section of a NN-decoupled dibaryon was equal to $\sim 2 \mu b/sr$ if the dibaryon decay width $\Gamma_D \ll 1 \text{ MeV}$. This limit is smaller than the value of the cross section of $8 \pm 4 \mu b/sr$ declared in \cite{9}.

However, the latter value was overestimated that was caused by not taking into account angle fluctuations related to a beam position displacement on the CD$_2$ target during the run. As was shown in the next experimental runs, the real value of the cross sections of the production of the SND with the mass 1904 MeV had to be smaller by 2–3 times than that was estimated in \cite{9}.

On the other hand, the simulation showed that the energy distribution of the protons from the decay of the SND with the mass of 1904 MeV has to be rather narrow with the maximum at $\sim 74 \text{ MeV}$. This distribution occupies the energy region of 60–90 MeV. The experiments \cite{9,12} confirmed the result of this simulation. However, in \cite{20} the authors considered the region 74–130 MeV. Therefore, they could detect only a small part of the SND contribution. Moreover, they used a very large acceptance of the spectrometer which detected these protons, while the protons under consideration have to fly in a very narrow angle cone. As a result, the ratio of the effect to the background in this work is more than 10 times worse than in \cite{9,12}. Very big
errors and absence of a proper cut on the energy of the protons from the decay of the \( pX_1 \) state in \ref{20} did not allow the authors to observe any structure in the \( pX_1 \) mass spectrum.

It is worth noting that the reaction \( pd \to NX \) was investigated in other works too (see for example \ref{21}). However, in contrast to \ref{9,12}, the authors of these works did not study either the correlation between the parameters of the scattered proton and the second detected particle or the emission of the photon from the dibaryon decay. Therefore, in these works the relative contribution of the dibaryons under consideration was very small, which hampered their observation.

Now let us consider the attempts to calculate masses of SNDs and determine their quantum numbers.

In the framework of the MIT bag model, Mulders et al. \ref{14} calculated the masses of different dibaryons, in particular, \( NN \)-decoupled dibaryons. They predicted dibaryons \( D(T = 0; J^P = 0^-, 1^-, 2^-; M = 2.11 \text{ GeV}) \) and \( D(1; 1^-; 2.2 \text{ GeV}) \) corresponding to the forbidden states \( ^{13}P_J \) and \( ^{31}P_1 \) in the \( NN \) channel. However, the dibaryon masses obtained exceed the pion production threshold. Therefore, these dibaryons mainly decay into the \( \pi NN \) channel.

Using the chiral soliton model, Kopeliovich \ref{22} predicted that the masses of \( D(T = 1; J^P = 1^+) \) and \( D(0, 2^+) \) dibaryons exceeded the two nucleon mass by 60 and 90 MeV, respectively. These values are lower than the pion production threshold.

In the framework of the canonically quantized biskyrmion model Krupnovnickas et al. \ref{23} obtained an indication on the existence of one dibaryon with \( J=T=0 \) and two dibaryons with \( J=T=1 \) with masses smaller than \( 2m_N + m_\pi \).

Unfortunately, all results obtained for the dibaryon masses are very model dependent and cannot reproduce the experimental data.

### 3 Exotic baryons

As was shown above, in the missing mass \( M_X \) spectra three peaks at 966, 986, and 1003 MeV had been observed. On the other hand, the peak at \( M_X = 1003 \pm 2 \text{ MeV} \) corresponds to the peak found in \ref{13} which was attributed to an exotic baryon state \( N^* \). Such a baryon state was first suggested in \ref{24}.

In \ref{13} Tatischeff et al. investigated the reaction \( pp \to \pi^+X \). This experiment was carried out using the proton beam at the Saturn Synchrotron and SPES3 facility \ref{25}. The measurements were performed at energies of \( T_p = 1520, 1805, \) and \( 2100 \text{ MeV} \) and at six angles, for each energy, from \( 0^\circ \) up to \( 17^\circ \). Three peaks with widths about 5–8 MeV have been observed in the missing mass spectra of this reaction at \( M_X = 1004, 1044, \) and \( 1094 \text{ MeV} \) with a statistic significance between 17 and 2 standard deviations. Two of these masses are below the sum of the nucleon and pion masses.

If exotic baryons with anomalously small masses really exist, the peaks observed at 966, 986, and 1003 MeV in \ref{12} might be a manifestation of such states. This is not in contradiction to the interpretation of the peaks in the \( M_\mu X \) mass spectra of \ref{12} as SNDs, since in principle the SNDs could decay into \( NN^* \). In this case the SND decay width could be equal to a few MeV.

The existence of such exotic states, if proved to be true, will fundamentally change our understanding of the quark structure of hadrons.
Exotic baryon states with masses smaller than \( m_N + m_\pi \) can mainly decay with an emission of photons. If they decay into \( \gamma N \) then such states have to contribute to the Compton scattering on the nucleon. However, L’vov and Workman \[26\] showed that existing experimental data on this process "completely exclude" such exotic baryons as intermediate states in the Compton scattering on the nucleon. On the other hand, the early Compton scattering data were not accurate enough to rule out these baryon resonances. Moreover, a measurement of the process \( \gamma p \to \gamma p \) in the photon energy range \( 60 < E_\gamma < 160 \) MeV resulted in a peak at \( M \approx 1048 \) MeV with an experimental resolution of 5 MeV and with 3.5 standard deviations \[27\]. Unfortunately, the accuracy of this experiment is not enough to do an unambiguous conclusion about the \( N^* \) contribution to the Compton scattering on the nucleon.

In \[28\] it was assumed that these states could belong to the totally antisymmetric \( \mathbf{20} \)-plet of the spin-flavor \( SU(6)_{FS} \) symmetry. Such a \( N^* \) can transit into a nucleon only if two quarks from the \( N^* \) participate in the interaction \[29\]. Then the simplest decay of the exotic baryons with the small masses is \( N^* \to \gamma \gamma N \). The production of such baryons in reactions on the nucleon is possible only through states with the mixed spin-flavor symmetry. The lowest of them are \( D_{13}(1520) \) and \( S_{11}(1535) \) resonances.

The reaction \( ep \to e'\pi^+X^0 \) was studied in \[30\] to search for narrow baryon resonances. In this experiment the mass resolution of 2.0 MeV was achieved. A search for structures in the mass region of \( 0.97 < M_{X^0} < 1.06 \) GeV yielded no signal. This experiment was performed at the invariant mass of the \( \pi^+X^0 \) system equal to \( W \approx 1.44 \) GeV. However, if the excited baryons belong to the totally antisymmetric representation, this value of \( W \) is far enough from the position of the \( D_{13}(1520) \) and \( S_{11}(1535) \) resonances and the cross section of the \( N^* \) production could be small at this \( W \).

In \[31\] the \( ep \to e'\pi^+X \) and \( ed \to e'pX \) reactions were investigated at MAMI. The missing mass resolution was 0.6 to 1.6 MeV (FWHM) in the proton experiment and 0.9 to 1.3 MeV in the deuteron experiment. None of these measurements showed a signal for narrow resonances to a level of down to \( 10^{-4} \) with respect to the neutron peak in the missing mass spectra. Unfortunately, the value of \( W \) used in this experiment does not also allow the hypothesis of \[28\] to be checked.

It should be noted that the reaction \( ed \to e'pX \) is the analog of the process \( pd \to pD \to p\gamma d \) studied in \[12\]. It has been shown in this work that the SND decay channel \( D \to \gamma d \) is strongly suppressed. Therefore, the negative result of work \[31\] at the investigation of the reaction \( ed \to e'pX \) only supports the result of \[12\] and it is an additional argument that the SNDs observed in \[9,12\] are isovectors.

On the other hand, the \( N^* \)s were produced in \[12,13\], more probably, from the decay of 6-quark states, what is supported by the observation of the dibaryon resonances in \[12\]. Therefore, an exotic quark structure of the \( N^* \) could arise which suppressed, in particular, the decay \( N^* \to \gamma N \) and could be the reason that such states were not observed in reactions on separate nucleons up to now.

In order to clarify the question about an existence of such exotic baryons, different experiments were proposed, in particular, in \[32\].

In \[13\] it was shown that values of the masses of the baryon resonances, observed in this work, can be reproduced with good enough accuracy by the mass formula for two colored clusters of quarks at the end of a stretched bag which was derived in terms of color magnetic interactions \[14,15\]. There are two free parameters in this model which were fixed by requiring
the mass of the nucleon and that of the Roper resonance to be reproduced exactly. As a result of the calculations, the following values of the masses and possible isospin ($I$) and spin ($J$) of these baryons have been obtained:

$$M(I; J) = 1005(1/2; 1/2, 3/2), \quad 1039(1/2; 3/2).$$

N. Konno [17] pointed out that the masses of the exotic baryons from [13] can also be calculated with the help of the mass formula of the diquark cluster model [16]. Eight free parameters of this model were fixed using data of baryon masses and the $\pi d$ phase shift. This model predicted the following values of the masses and $I, J$:

$$M = 990(I = 1/2(J^P = 1/2^-); 3/2(1/2^-)),$$

$$M = 1050(1/2(1/2^-, 3/2^-); 3/2(1/2^-, 3/2^-)), \quad M = 1060(1/2(1/2^-, 3/2^-)).$$

However, these two models do not reproduce the values of masses: 966 and 986 MeV, obtained in [12].

Th. Walcher [33] noted that the masses taking all experiments together and including the neutron ground state and two additional masses at 1023 and 1069 MeV are equidistant within the errors with an average mass difference of $\Delta M = 21.2 \pm 2.6$ MeV. The author hypothesized the existence of a light Goldstone boson with the mass of 21 MeV consisting of light current quarks. It was assumed that the series of excited states is due to the nucleon in its ground state plus 1, 2, 3,... light Goldstone bosons as the quantum of excitation.

4 Mass formula for exotic baryons

In this section we construct a model which allows the calculation of the masses of all possible exotic baryon states below the $\pi$ meson production threshold. This model is based on the calculation of the contribution of a meson–baryon loop to the exotic baryon mass operator (fig. 1) with the help of dispersion relations with two subtractions.

An analysis of the mass shifts of experimentally well-known baryons due to meson-baryon loops was carried out in a set of works (see for references [34]). In these works, the self energy of a baryon was calculated, as a rule, in the framework of a perturbation theory. In this case, an underintegral expression diverges strongly and additional assumptions about a behavior of baryon–baryon-meson vertices are required.

We determine the mass operator as

$$G^{-1}(p) = \hat{p} - m - \Sigma(p), \quad \Sigma(p) = a\hat{p} + b \quad (2)$$
where $p$ is the 4-momentum of the $N^*$ under consideration, $m$ is the mass of the baryon in the intermediate state. Then the mass of the $N^*$ is equal to

$$M = m + Re \delta(M)$$

(3)

where

$$\delta(M) = \bar{u}(p)\Sigma(p)u(p) = \bar{u}(p)(aM + b)u(p) = aM + b,$$

(4)

$a$ and $b$ are scalar functions.

In order to find the mass $M$, we construct the dispersion relations over $M^2$ for $\delta(M)$ with two subtraction at $M^2 = m^2$. Then taking into account (3) we obtain the following nonlinear integral equation for $M$

$$M = m + Re \delta(m) + \left( M^2 - m^2 \right) \frac{dRe \delta(M)}{dM^2} \bigg|_{M=m} + \frac{(M^2 - m^2)^2}{\pi} \int_{(m+\mu)^2}^{\infty} \frac{Im \delta(x) dx}{(x-M^2)(x-m^2)^2}.\tag{5}$$

where $P$ means a principal part of the integral.

All particles in the intermediate state are on their mass shells. As the subtraction is performed at the mass shell of the baryon in the intermediate state, the subtraction constant $Re \delta(m)$ is equal to zero. We also assume that $dRe \delta(M)/dM^2|_{m} = 0$. This assumption corresponds to a supposition that the baryon with the mass $m$ is in the ground state. It should be noted that if one takes into account a few different baryons in the intermediate state, the subtraction constants could not be equal to zero because in this case the mass $m$ does not coincide with the mass shell for some of these baryons.

Two subtractions in the dispersion relations provide a very good convergence of the under-integral expression in (5). Therefore, we restrict ourselves to a consideration of one baryon and the pion only in the intermediate state. The calculations showed that the contribution of the $\sigma$ meson is negligible. Therefore, it is expected that the contribution of other heavy mesons in the mass region under consideration is negligible too. However, these contributions could be important in the mass region higher than the meson–baryon production threshold.

Equation (5) can be also used for the calculation of the masses of the well-known nucleon resonances. Therefore, to check an efficiency of the mass formula obtained, we will apply it to calculate the mass of $\Delta(1232)$ resonance.

The contribution of the pion–nucleon loop to $Im \delta_\Delta(M)$ can be written as

$$\bar{u}_\mu(p)Im \delta_\Delta(M)u_\mu(p) = \frac{1}{2} \left( \frac{g_\Delta^2(M)}{4\pi} \right) \frac{1}{\mu^2} \bar{u}_\mu(p)q_\mu(\hat{p}_1 + m)q_\lambda u_\lambda(p)$$\tag{6}

where $p_1$ and $m$ ($q$ and $\mu$) are the 4-momentum and the mass of the nucleon (pion) in the intermediate state, $p$ is the 4-momentum of the $\Delta(1232)$ resonance.

Multiplying this expression by $u_\nu(p)$ from the left and by $\bar{u}_\nu(p)$ from the right and using the condition $u_\nu(p)\bar{u}_\mu(p) = \Delta_{\nu\mu}(p)$ we have

$$\Delta_{\nu\mu}(p)Im \delta_\Delta(M) \Delta_{\mu\nu}(p) = \frac{1}{2} \left( \frac{g_\Delta^2(M)}{4\pi} \right) \frac{1}{\mu^2} \Delta_{\nu\mu}(p)q_\nu(\hat{p}_1 + m)q_\lambda \Delta_{\lambda\nu}(p)$$\tag{7}
where
\[ \Delta_{\nu\mu}(p) = \frac{1}{3M^2}(\hat{p} + M) \left[ 2p_{\nu}p_{\mu} - 3M^2g_{\nu\mu} + M^2\gamma_{\nu}\gamma_{\mu} + M(\gamma_{\nu}p_{\mu} - \gamma_{\mu}p_{\nu}) \right]. \]
Calculating traces in the left and right parts of (11) and taking into account that \( p^2 = M^2 \) we obtain
\[ Im\delta(M) = -\frac{1}{6} \left( \frac{g_\Delta^2(M)}{4\pi} \right) \frac{1}{\mu^2} |q|^2(m + E_1), \tag{8} \]
here \( |q| = \sqrt{E_1^2 - m^2} \).
In accordance with \[35, 36\] we use the following parametrization
\[ g_\Delta^2(M) = g_0^2 \frac{1 + R^2|q_\Delta|^2}{1 + R^2|q|^2}, \tag{9} \]
\[ |q_\Delta| = |q(M = M_\Delta)|, \quad M_\Delta = 1232 \text{ MeV}, \quad R = 5.5 \text{ GeV}^{-1}. \]
The coupling constant \( g_0^2 \) can be expressed through the decay width of the \( \Delta^+ (1232) \) resonance \( \Gamma_\Delta \) as
\[ g_0^2 \frac{1}{4\pi \mu^2} = \frac{9M_\Delta \Gamma_\Delta}{|q_\Delta|^3(E_\Delta + m)} \]
where \( E_\Delta = \sqrt{|q_\Delta|^2 + M_\Delta^2} \). This value of \( g_0^2/(4\pi)\mu^2 \) is equal to 1.178 \( \mu^2 \) for the decay channel \( \Delta^+ (1232) \rightarrow \pi^+ n \) and 1.055 \( \mu^2 \) for the \( \Delta^+ (1232) \rightarrow \pi^0 p \) channel.
Substituting (8) into (5) and taking into account the indicated parametrization we obtain for the mass of the \( \Delta \) resonance the value of 1247 MeV. It differs not strongly from the experimental value of 1232 MeV. This result confirms a good efficiency of this mass formula.

## 5 Calculation of the masses of the exotic baryons

We will consider here only baryons with the spin equal to 1/2. Then the function \( Im\delta \) for the baryon-pion loop can be written as
\[ \bar{u}(p)Im\delta(M)u(p) = \bar{u}(p)Im\Sigma(M)u(p) = N\bar{u}(p)(\mp\hat{p}_1 + m)u(p) \tag{10} \]
where \( N = 1/(g^2/4\pi)|p_1|/M \), \( p_1 \) is the 4-momentum of the baryon in the intermediate state. The sign minus (plus) at \( \hat{p}_1 \) corresponds to the same (opposite) parities of the final baryon and the baryon in the intermediate state. Multiplying this expression by \( u(p) \) from the left and by \( \bar{u}(p) \) from right and using the condition \( u(p)\bar{u}(p) = (\hat{p} + M)/2M \) we have
\[ (\hat{p} + M)Im\delta(M)(\hat{p} + M) = N(\hat{p} + M)(\mp\hat{p}_1 + m)(\hat{p} + M). \tag{11} \]
Calculating traces in the left and the right parts of this expression we obtain:
\[ Im\delta(M) = \frac{N}{2M^2}[\mp2(pp_1)M + (p^2 + M^2)m]. \tag{12} \]
As \( p^2 = M^2 \), \( 2(pp_1) = M^2 + m^2 - \mu^2 = 2ME \), we have
\[ Im\delta(M) = \frac{1}{2}g_\Omega^2 \frac{|p_1|}{4\pi M}(m \mp E), \tag{13} \]
Table 1: The masses and $J^P$ of the exotic baryon resonances.

| $N^*$ | $J^P$ | model $M$ (MeV) | experiment $M$ (MeV) | experimental works |
|-------|-------|------------------|----------------------|------------------|
| 1     | $\frac{1}{2}^-$ | 963.4            | 966 ± 2               | [12]             |
| 2     | $\frac{1}{2}^+$  | 987              | 986 ± 2               | [12]             |
| 3     | $\frac{1}{2}^-$  | 1010             | 1004                  | [13, 12]         |
| 4     | $\frac{1}{2}^+$  | 1033             | ?                     |                  |
| 5     | $\frac{1}{2}^-$  | 1056             | 1044                  | [13]             |
| 6     | $\frac{1}{2}^+$  | 1079             | ?                     |                  |

here $|p_1| = \sqrt{E^2 - m^2}$.

In our calculations we assumed that all baryons under consideration have the isotopic spin and the spin equal to 1/2. Then equation (5) has solutions in the mass region lower than the $\pi N$ production threshold only if the final baryon with the mass $M$ and the baryon in the intermediate state have opposite parities.

Taking the nucleon plus the pion ($\pi^+$ and $\pi^0$) in the intermediate state, we find for the first exotic baryon state the mass $M = 963.4$ MeV and $J^P = 1/2^-$. Then taking the first exotic baryon with $m = 963.4$ MeV, $J^P = 1/2^-$ and the $\pi$ meson in the intermediate state we obtain for the second exotic baryon state $M = 987.0$ MeV and $J^P = 1/2^+$. Continuing the same procedure, six possible exotic states of baryons have been found. The states obtained are listed in table 1 where the experimental data are also given.

As seen from this table, the results of the calculations are in a good agreement with the experimental data. The mass values of two still unobserved states at 1033 and 1079 MeV are close to the ones predicted in [33].

At the calculation we assumed that the coupling constant $g^2/4\pi$ in the vertices $NN^*\pi$ and $N_i^* N_f^* \pi$ is same for all $N^*$ and equal to the coupling constant of the $\pi NN$ interaction ($g_{\rho\rho\pi\pi}^2/4\pi = 14.6$). So, the increase of the difference between the model predictions for the masses and experimental data with a rise of the mass could be caused, in particular, by this assumption. Therefore, it is expected that the real value of the mass for the still unobserved state #6 should be smaller by $\sim 10$ MeV and so it has to be bellow the pion production threshold.

As follows from the table, the odd parity has been predicted for all baryon states found in [13]. It agrees with the comment of Th. Walcher [33] and is due to the kinematics of the experiment [13]. This experiment detected $\pi^+$ and $p$ from the reaction $pp \rightarrow \pi^+ + pX$ in coincidence in one spectrometer at small angles. Since the $N^*$ states were mainly observed in the forward direction with respect to the beam ($\theta \approx 5^\circ$), this means that the outgoing $\pi$ and $p$ carry no angular momentum and only the odd parity state of $N^*$ could be observed in this experiment. But the baryons with the odd parity cannot belong to the totally antisymmetric 20-plet [28].

It should be noted that the big value of the constant $g^2/4\pi$ could suppose the noticeable contribution of the $N^*$ to the elastic $\pi N$ scattering. However, the experimental data on this process do not support such a possibility [37]. Therefore, if the model is correct, there are two possibilities. The first, the constant $g^2/4\pi$ is the effective coupling constant of some complicated
interaction of a pion cloud with the baryon in the intermediate state and does not connect with the usual pion–nucleon coupling constant. The second, the exotic baryons with the anomaly small masses do not exist. In this case, equation (5) describes the effective masses corresponding to the positions of the narrow peaks in the phase space of the products of the SNDs decay. Such peaks are not real resonances and do not contribute to processes on separate nucleons and the value of the constant \( g^2/4\pi \) coincides with the coupling constant of the \( \pi NN \) interactions by accident.

6 Mass formula for the SNDs

Using the complete Green function of the dibaryons

\[
\Delta(p^2) = \frac{F(p)}{p^2 - m^2 - \delta_D(p^2)},
\]

we determine the SND mass as

\[
M^2 = m^2 + \text{Re} \delta_D(M^2)
\]

where \( \delta_D(M^2) \) is the self energy of the SND under study and \( m \) is mass of the dibaryon in the intermediate state.

The self energy of the lightest SND will be determined in one loop approximation through the interaction of the pion with the 6-quark state of the deuteron. The self energy of the next SND will be obtain through the interaction of the pion with the lightest SND and so on.

We will calculate the SND self energy with the help of the dispersion relations with two subtractions at \( M^2 = m^2 \). Then taking into account (14) we obtain the mass formula for the SNDs

\[
M^2 = m^2 + \text{Re} \delta_D(m^2) + (M^2 - m^2) \frac{d \text{Re} \delta_D(M^2)}{d M^2} \bigg|_{M^2=m^2} + \\
\frac{(M^2 - m^2)^2}{\pi} \int_0^\infty \frac{\text{Im} \delta_D(x) dx}{(x - M^2)(x - m^2)^2}.
\]  

As the subtraction is performed at the mass shell of the dibaryon in the intermediate state, the subtraction function \( \text{Re} \delta_D(m^2) \) is equal to zero. Assuming that this dibaryon is in the ground state, we have \( d \text{Re} \delta_D(M^2)/d M^2 \big|_{M^2=m^2} = 0 \).

There are two possibilities to calculate \( \text{Im} \delta_D(x) \). The first, when the SND under study and the dibaryon in the intermediate state have opposite parities. The second, when these parities are the same.

The vertices \( D'(1^+) \rightarrow \pi + D(1^\pm) \) and \( D'(1^\pm) \rightarrow \pi + D(1^\pm) \) can be respectively written as

\[
\Gamma(-) = \frac{g_1}{M} G_{\mu\nu} \Phi^{\mu\nu}, \quad \Gamma(+) = \frac{g_2}{M} G_{\mu\nu} \Phi^\sigma \Phi^{\lambda\sigma}
\]

where \( G_{\mu\nu} = v_{\mu} p_{\nu} - p_{\mu} v_{\nu}, \Phi_{\mu\nu} = w_{\mu} p_{1\nu} - p_{1\mu} w_{\nu}; v \) and \( w \) are the 4-vectors of the polarization of the final SND and the dibaryon in the intermediate states, respectively; \( p \) and \( p_1 \) are their 4-momenta.
As result of the calculations we have the following expression for the imaginary part of $\delta_D(x)$ when the final SND and dibaryon in the intermediate state have the opposite parities

$$Im \delta_D^{(-)}(x) = \frac{1}{3} \left( \frac{g_1^2}{4\pi} \right) \frac{q[(x + m^2 - \mu^2)^2 + 2m^2x]}{x^\frac{3}{2}}$$

(17)

where $q$ is the pion momentum equal to $q = [(x - (m + \mu)^2)(x - (m - \mu)^2)]^{1/2}/2x^{1/2}$.

For the dibaryons with the same parities we have

$$Im \delta_D^{(+)}(x) = \frac{8}{3} \left( \frac{g_2^2}{4\pi} \right) \frac{q[(x + m^2 - \mu^2)^2 - 4m^2x]}{x^\frac{3}{2}}$$

(18)

The coupling constant $g_1^2/4\pi$ in the vertex for transition of the 6-quark state of the deuteron ($D(0, 1^+)$) plus the pion to the SND $D(1, 1^-)$ has been fixed by requiring a reproduction of the mass $M = 1904$ MeV. It results in

$$\frac{g_1^2}{4\pi} = 26.585$$

(19)

As follows from estimations of the value of the cross section of the $D(1, 1^-, M = 1904)$ production in the process $pd \rightarrow p + pX$ [12, 20], the product $(g_1^2/4\pi \eta)$ is equal to $\sim 2 \times 10^{-3}$, where $\eta$ is the probability of the 6-quark state existence in the deuteron. Then taking into account the value [19] for the coupling constant $g_1^2/4\pi$, one obtains $\eta \sim 1 \times 10^{-4}$. This value of $\eta$ is essentially smaller than its estimation found in [38] ($\eta \leq 0.03$). As a result, the decay width of the SNDs with small masses might be considerably smaller than 1 eV.

We will assume that the all SNDs under consideration have the isospin $T = 1$ and $J^P = 1^\pm$. The coupling constants $g_1$ in the vertex $D(1, 1^\pm) \rightarrow D(1, 1^\pm) + \pi$ differs from $g_1$.

On the other hand, now two channels give the contribution to the imaginary part $Im \delta_D(x)$. For example, such a contribution to the self energy of a positive charged supernarrow dibaryon $D^+$ is given by the channels

$$D^0 + \pi^+ \quad \text{and} \quad D^+ + \pi^0.$$

The calculations in the frame of our model give the very close values of the masses of $D^0$ and $D^+$ dibaryons. Therefore, we will take them equal one to other.

In order to calculate the mass of the following SND, we take in the intermediate state the SND $D(1, 1^-)$ with $M = 1904$ MeV and the pion. If the constant $g_1^2/4\pi = 3/4(g_1^2/4\pi)$ then taken into account the contribution from two possible channels and using (17) we find $M = 1924$ MeV and $J^P = 1^+$. To calculate the mass of the third SND we consider in the intermediate state the SND $D(1, 1^+)$ with $M = 1924$ MeV and the pion ($\pi^+$ and $\pi^0$) and use the same value of $g_1^2/4\pi$ and (17). It results in $M = 1943$ and $J^P = 1^-$. Continuing such calculations with (17) and using the same value of $g_1$ we get other values of the SND masses. The results of the calculations of the masses and $J^P$ of the SNDs are listed in table 2.

As seen from this table, the values of the SND masses obtained are in a good agreement with the experimental data. The existence of the SND with the mass $M = 1982$ MeV has been predicted in [12, 39] as a consequence of the observation of the peak in the missing mass spectra of the reaction $pp \rightarrow \pi^+ pX$ [13] at $M_X = 1044$ MeV.

Our calculations predict also the possibility of the existence of the new SND $D(1, 1^+)$ with the mass $M = 2001$ MeV.
Table 2: The masses and $J^P$ of the SNDs.

| No | $J^P$ | $M$ (MeV) | experimental $M$ (MeV) | experimental works |
|----|-------|-----------|------------------------|-------------------|
| 1  | 1$^-$ | 1904      | 1904 ± 2               | 12                |
| 2  | 1$^+$ | 1924      | 1926 ± 2               | 12                |
| 3  | 1$^-$ | 1943      | 1942 ± 2               | 12                |
| 4  | 1$^+$ | 1962      | 1956 ± 6               | 18                |
| 5  | 1$^-$ | 1982      | 1982                   | predicted 12,39    |
| 6  | 1$^+$ | 2001      | ?                      |                   |

As the SNDs observed in $[9,12]$ decay into $NN^*$ with the small relative momentum between $N$ and $N^*$, the SND parity has to be determined by the parity of the $N^*$. As seen from tables 1 and 2, the SND parities found here agree with the results obtained for the $N^*$ states.

As for the calculations with $\text{Im} \delta(x)$, the results very close to the ones presented in table 2 can be obtained if $g_2^2/4\pi = 3g_1^2/4\pi$ and $\bar{g}_2^2/4\pi = 3\bar{g}_1^2/4\pi$.

Then the calculation of the mass of the SND due to the pion-deuteron loop has given $D(1,1^+)$ with $M = 1925$ MeV. To calculate the mass of the next SND, we take $D(1,1^-)$ with $M = 1904$ and the $\pi^+$ and $\pi^0$ mesons in the intermediate state. It results in $D(1,1^-)$ with the mass $M = 1939$ MeV. Then taking from table 2 the following SND $D(1,1^+)$ with $M = 1924$ MeV and the pions in the intermediate state, we obtain $D(1,1^+)$ with $M = 1958$ MeV. Continuing this procedure we find also $D(1,1^-)$ with $M = 1978$ MeV and $D(1,1^+)$ with $M = 1998$ MeV. These values of the masses obtained with the help of 18 are very close to the ones calculated using 17 for the imaginary part of $\delta(x)$ (see table 2). However, it is not possible to obtain SND $D(1,1^-)$ with $M = 1904$ by using eq. 18 only.

7 Conclusion

As a result of the study of the reaction $pd \rightarrow p + pX_1$, three narrow peaks at $M_{pX_1} = 1904$, 1926, and 1942 MeV have been observed $[9,12]$. The analysis of the angular distributions of the protons from the decay of the $pX_1$ states showed that the peaks found can be explained as a manifestation of the isovector SNDs, the decay of which into two nucleons is forbidden by the Pauli exclusion principle. The observation of the peaks in the missing mass $M_{X_1}$ spectra at 966, 986, and 1003 MeV is an additional indication that the dibaryons found are the SNDs.

On the other hand, these peaks in $M_{X_1}$ mass spectra and peaks observed in 13 in the reaction $pp \rightarrow \pi^+pX$ could be consider as the new exotic baryon states with small masses. However, additional experiments are necessary to understand the real nature of these peaks.

In the present paper, the mass formulae for the SNDs and the exotic baryons have been constructed using the dispersion relations with two subtraction for their self energies in the one loop approximation. These mass formulae were used to calculate the masses and determine parities of the SNDs and the exotic baryons. The obtained values of the masses are in a good agreement with the experimental data of 12,13,18. The application of the mass formula found to calculate mass of the $\Delta(1232)$ resonance has given the value close enough to the experimental one.
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References

[1] L.V. Fil’kov, Sov.Physics–Lebedev Inst. Reports No 11, 49 (1986); Sov. J. Nucl. Phys. 47, 437 (1988).
[2] D.M. Akhmedov and L.V. Fil’kov, Nucl. Phys. A544, 692 (1992).
[3] V.M. Alekseyev, S.N. Cherepnya, L.V. Fil’kov, and V.L. Kashevarov, Preprint of Lebedev Phys.Inst. No 52 (1996).
[4] V.M. Alekseyev, S.N. Cherepnya, L.V. Fil’kov, and V.L. Kashevarov, Kratkie Soobscheniya po fizike, FIAN, No. 1, 28 (1998); nucl-th/9812041.
[5] E.E. Kolomeitsev and D.N. Voskresensky, Phys. Rev. C 65, 015805 (2003).
[6] L.V. Fil’kov, E.S. Konobeevski et al., Preprint of INR No. 0923/96 (1996).
[7] E.S. Konobeevski et al., Izv. Ross. Akad. Nauk, Ser. Fiz. 62, 2171 (1998).
[8] L.V. Fil’kov, V.L. Kashevarov, E.S. Konobeevskiy et al., Phys. Atom. Nucl. 62, 2021 (1999).
[9] L.V. Fil’kov, V.L. Kashevarov, E.S. Konobeevski et al., Phys. Rev. C 61, 044004 (2000).
[10] L.V. Fil’kov, V.L. Kashevarov, E.S. Konobeevskiy et al., Proc. VII Conf. CIPANP2000, Quebec City, Canada, AIP Conf. Proceed. v.549, p.267 (2000); nucl-th/0009044.
[11] L.V. Fil’kov, V.L. Kashevarov, E.S. Konobeevskiy et al., Proceed. XV Intern. Seminar on High Energy Physics Problems ”Relativistic Nuclear Physics and Quantum Chromodynamics”, Dubna 2000, v.II p.153; nucl-th/0101021.
[12] L.V. Fil’kov, V.L. Kashevarov, E.S. Konobeevski et al. Eur. Phys. J. A 12, 369 (2001).
[13] B. Tatischeff et al., Phys. Rev. Lett. 79, 601 (1997); nucl-ex/0207003.
[14] P.J. Mulders, A.T. Aerts, and J.J. de Swart, Phys. Rev. D 19, 2635 (1979).
[15] C. Besliu, L. Popa, and V Popa, J. Phys. G 18, 807 (1992).
[16] Y. Uehara, N. Konno, H. Nakamura, and H. Noya, Nucl. Phys. A 606, 357 (1996).
[17] N. Konno, Nuovo Cimento A 111, 1393 (1998).
[18] A.S. Khrykin, et al. Phys. Rev. C 64, 034002 (2001).
[19] H. Calén, et al., Phys. Lett. B427, 248 (1998).
[20] A. Tamii et al., Phys. Rev. C 65, 047001 (2002).

[21] K.K. Set, in Proc. XII Int. Conf. on Few Body Problems in Physics, Vancouver, Canada, TRIUMF Report No TR-89-2, C35.

[22] V.B. Kopeliovich, Phys. At. Nucl. 56, 1084 (1993); Phys. At. Nucl. 58, 1237 (1995).

[23] T. Krupnovnickas, E. Norvaisas, and D.O. Riska, nucl-th/001063.

[24] Ya.I. Azimov, Phys. Lett. B 32, 499 (1970).

[25] M.P. Comets et al., Report No IPNO DRE 88–41.

[26] A.I. L'vov and R.L. Workman, Phys. Rev. Lett. 81, 1346 (1998).

[27] V. Olmos de León et al., Eur. Phys. J. A 10, 207 (2001).

[28] A.P. Kobushkin, nucl-th/9804069.

[29] R. Feynman, "Photon-Hadron Interactions", Ed. W.A. Benjamin, Inc., Massachusetts (1972).

[30] X. Jiang et al., Phys. Rev. C 67, 028201 (2003).

[31] M. Kohl et al., nucl-ex/0304013.

[32] R. Beck et al., Proc. of the Workshop on The Physics of Excited Nucleons, ”NSTAR 2001”, Mainz, Germany 2001, Word Scientific p. 247; nucl-th/0104070.

[33] Th. Walcher, hep-ph/0111279.

[34] D. Morel and S. Capstick, nucl-th/0204014; D. Morel, Ph.D. Thesis, nucl-th/0204028.

[35] I. Blomqvist and J. Larget, Nucl. Phys. A 280, 405 (1997).

[36] S.P. Baranov and A.A. Shikanyan, Sov.J.Nucl.Phys. 48, 1775 (1998).

[37] Ya.I. Azimov et al., nucl-th/0307088.

[38] L.A. Kondratyuk, M.I. Krivoruchenko, and M.G. Shchepkin, Sov.J.Nucl.Phys. 43, 899 (1986).

[39] B. Tatischeff et al., Phys. Rev. C 59, 1878 (1999).