Central Extensions of Supersymmetry in Four and Three Dimensions

Sergio Ferrara\textsuperscript{a,b} \footnote{e-mail: ferraras@vxcern.cern.ch} and Massimo Porrati\textsuperscript{c,d} \footnote{e-mail: massimo.porrati@nyu.edu}

\textit{a} Theory Division, CERN, CH-1211 Geneva 23, Switzerland

\textit{b} Dept. of Physics UCLA, 405 Hilgard Av. Los Angeles, CA 90024, USA

\textit{c} Department of Physics, NYU, 4 Washington Pl., New York NY 10003, USA \footnote{Permanent address.}

\textit{d} Rockefeller University, New York NY 10021-6399, USA

ABSTRACT

We consider the maximal central extension of the supertranslation algebra in d=4 and 3, which includes tensor central charges associated to topological defects such as domain walls (membranes) and strings. We show that for all $N$-extended superalgebras these charges are related to nontrivial configurations on the scalar moduli space. For $N = 2$ theories obtained from compactification on Calabi-Yau threefolds, we give an explicit realization of the moduli-dependent charges in terms of wrapped branes.
1 Introduction

Central extensions of extended supersymmetry algebras in any dimensions have been considered in recent years in connection to the dynamics of BPS saturates p-branes. Generically, for an $N$-extended supersymmetry algebra in $d$ dimensions, the structure of the central extension of the supertranslation algebra has the form:

$$\{Q^A_\alpha, Q^B_\beta\} = \sum_P Z^{AB}_{\mu_1..\mu_p} (\gamma^{\mu_1..\mu_p}C)_{\alpha\beta}, \quad A, B = 1, .., N$$  \hspace{1cm} (1)

where, for $p = 1$, $Z^{AB}$ must contain a singlet under the automorphism group of the supercharges (the R-symmetry). This singlet is a linear combination of the translation generator $P_\mu$ and a singlet vector charge $Z_\mu$. This central charge can be absorbed in a redefinition of $P_\mu$, but only when spacetime is uncompactified [1]. Upon compactification, its spectrum differs, in general, from the Kaluza Klein spectrum of $P_\mu$. The total number of central charges, including $P_\mu$, correctly adds up to the dimension of the symmetric product of $N$ spinorial representations: $(N \dim s)(N \dim s + 1)/2$, where $\dim s$ is the dimension of an irreducible spinor representation in space-time dimension $d$. The existence of tensor charges in the presence of p-branes has been derived in ref. [2]; for $N = 8$, the algebra in eq. (1) has been found in [3].

The central extension of Poincaré supersymmetry limits the central extension to Lorentz scalars, i.e. $p = 0$. On the other hand, already some years ago it has been realized that p-branes, i.e. extended objects present in consistent extensions of supergravity theories, can give rise to nonzero tensor central charges [2].

Tensorial central charges can be divided in two categories: charges that are sources of dynamical gauge fields, and charges that have no associated dynamical gauge field, and correspond to topological defects of the theory.

We will study charges in both categories. Particularly: a) vector charges in 4 dimensions, associated to rank-2 antisymmetric tensors, which are dual to 4-d scalars; b) rank-2 tensor charges, associated to rank 3 antisymmetric tensors, which are non-dynamical in 4-d. The latter case has received quite some attention in recent times in the context of supersymmetric Yang-Mills theories [4], since the associated extended object is a BPS saturated membrane.

In this paper, we first consider the central extension of the supersymmetry algebra in dimensions 4 and we consider particular physical situations in which these new central charges are non-vanishing. For the particular case of type II Calabi-Yau compactifications, we show that the extended objects source of the tensorial charges lead to new phenomena in the hypermultiplet moduli space. These phenomena can be described in a general setting valid for all $N$-extended theories. We also discuss central extensions of the supertranslation algebra in 3 dimensions, and its relation to three-dimensional U-duality.
2 Central Extension of Supersymmetry Algebras in d=4

The Haag-Lopuszanski-Sohnius central extension of the super-Poincaré algebra for $N$-extended supersymmetry is [5]:

$$\{ Q^A_{\alpha}, Q_B^{\dot{\alpha}} \} = \sigma^\mu_{\alpha\dot{\alpha}} P^A_{\mu} \delta_B^A, $$

$$\{ Q^A_{\alpha}, Q^B_{\beta} \} = \epsilon_{\alpha\beta} Z^{[AB]}.$$  \hspace{1cm} (2)

By counting the spinor components in the left-hand side of these equations, it is evident that some terms have been omitted in the right-hand side. These additional terms are central charges of the supertranslation algebra, rather than of the super-Poincaré algebra. The complete algebra has additional terms, as in equation (1), namely

$$\{ Q^A_{\alpha}, Q_B^{\dot{\alpha}} \} = \sigma^\mu_{\alpha\dot{\alpha}} P^A_{\mu} \delta_B^A + \sigma^\mu_{\alpha\dot{\alpha}} Z^{A}_{\mu A}, $$

$$\{ Q^A_{\alpha}, Q^B_{\beta} \} = \epsilon_{\alpha\beta} Z^{[AB]} + \sigma^\mu_{\alpha\dot{\alpha}} Z^{(AB)}.$$  \hspace{1cm} (3)

These new terms correspond to string charges in the adjoint of $SU(N)$, and membrane charges in the twofold symmetric of $SU(N)$.

One can easily see that the total number of bosonic generators in the rhs of eqs. (3) adds up to $8N^2 + 2N$. This is the same dimension as the lhs, which is a symmetric $4N \times 4N$ matrix.

The tensor charges in eq. (3) can be easily related to geometrical quantities that depend on the asymptotic value of massless scalar fields (moduli). To see this, we need only recall that in any theory with local supersymmetry, the supersymmetry charge can be expressed in a Gauss-like form which involves only the gravitino field $(\psi^A_{\mu \alpha}, \bar{\psi}^{\dot{A}}_{\mu \dot{\alpha}})$:

$$Q^A_{\alpha} = \int_S dx^\mu \wedge dx^\nu \sigma_{\mu \alpha \dot{\alpha}} \bar{\psi}^{\dot{A}}_{\nu \dot{\alpha}}, \quad Q^A_{\dot{\alpha}} = \int_S dx^\mu \wedge dx^\nu \sigma_{\mu \alpha \dot{\alpha}} \psi^A_{\nu \alpha}.$$  \hspace{1cm} (4)

$S$ is a large surface inside a 3-dimensional space-like hypersurface. Quite generally, one may write the change of the gravitino under a supersymmetry transformation as:

$$\delta \bar{\psi}^A_{\mu \dot{\alpha}} = D_\mu \epsilon^A_{\dot{\alpha}} + V^A_{B \mu} \epsilon^B_{\dot{\alpha}} + \frac{1}{2} \sigma_{\mu \alpha \dot{\alpha}} M^{AB} \epsilon^A_{\dot{\alpha}} + ...$$  \hspace{1cm} (5)

An analogous law holds for $\psi^A_{\mu \alpha}$. Here, $D_\mu$ is the standard Lorentz-covariant derivative, the scalar $M^{AB}$ is the gravitino mass matrix, while $V^A_{B \mu}$ is a composite gauge vector, function of the scalar fields in the theory. Notice that it can be decomposed into a traceless part, in the adjoint of the R-symmetry $SU(N)$, plus a singlet. The commutators of two supersymmetry charges can be found most easily by varying eq. (3) with respect to a supersymmetry transformation, and using eq. (5). The result is

$$\{ Q^A_{\alpha}, Q_B^{\dot{\alpha}} \} = \int_S dx^\mu \wedge dx^\nu \sigma_{\mu \alpha \dot{\alpha}} V^A_{B \nu} + ... $$

$$\{ Q^A_{\alpha}, Q^B_{\beta} \} = \int_S dx^\mu \wedge dx^\nu \sigma_{\mu \alpha \dot{\alpha}} M^{AB} + ...$$  \hspace{1cm} (6)
Here the ellipsis stand for the standard terms in the supersymmetry algebra (\(P_\mu\) and the scalar central charges \(Z^{[AB]}\)). These equations give the explicit relation between tensor charges and scalar moduli.

Eqs. (6) shows that for generic \(N\), the central charge of the domain wall is given in terms of the gravitino mass matrix, \(M^{AB}\), which belongs to the twofold symmetric representation of the R-symmetry \(SU(N)\), while the central charge of the strings is generated by the composite gauge connection of the R-symmetry.

It is easy to show that in the presence of a string there can exist a nonzero vector charge \(Z_\mu\). Explicitly, in the presence of an infinite string (either fundamental or solitonic), one may choose as surface of integration \(S\) a cylinder coaxial with the string. For an infinite string in uncompactified space, the total vector charge diverges, but the charge density per unit length, \(dZ^{A}_\mu B/dl\), is finite. At any point along the string it reads:

\[
dZ^{A}_i B/dl = \hat{l}_i \oint dx^j V^{A}_{j B}, \quad i, j = 1, 2, 3.
\] (7)

Here \(\hat{l}\) is a unit vector tangent to the string and the integral is taken around any loop orthogonal to \(\hat{l}\) and encircling the string once.

Notice that eq. (7) gives not only the vector charge in the adjoint of \(SU(N)\), but also the singlet vector charge. As we said before, even though this charge is not algebraically independent from \(P_\mu\), and it is indistinguishable from it in uncompactified space, it becomes an independent charge upon compactification on non-simply connected manifolds.

Similarly, using eqs. (6), we can find that the charge \(Z^{(AB)}_{\mu\nu}\) is nonzero in the presence of a domain wall. Consider for simplicity a flat domain wall located at \(z = \text{const}\) in Cartesian coordinates. Then, the second of eqs. (6) gives us the following expression for the tensor-charge density (i.e. the charge per unit area of the membrane):

\[
Z^{(AB)}_{ij} = [M^{AB}(z = +\infty) - M^{AB}(z = -\infty)], \quad Z^{(AB)}_{ij} = 0 \text{ otherwise.}
\] (8)

In \(N = 1\) (rigid) supersymmetry, there exist examples of both BPS saturated strings (the \(N = 1\) Abelian Higgs model) and BPS saturated membranes. In local supersymmetry, the tensor charges associated to these objects can be expressed in a model-independent way by using the superconformal tensor calculus (see for instance [4]), which allows us to write the off-shell gravitino transformation law as:

\[
\delta_\epsilon \bar{\psi}_{\mu\dot{\alpha}} = D_\mu \epsilon_{\dot{\alpha}} + \frac{3}{4} i A_\mu \epsilon_{\dot{\alpha}} - \frac{3}{4} i \theta(\Phi, \epsilon) \bar{\psi}_{\mu\dot{\alpha}} - \sigma_{\mu\dot{\alpha}} \eta^\alpha(\Phi, \epsilon).
\] (9)

In this universal equation, \(\epsilon, \eta\) are the fermionic generators of the superconformal group; \(A_\mu\) and \(\theta\) are, respectively, the gauge field and the generator of the local R-symmetry. The parameters \(\eta\) and \(\theta\) are not independent, rather, in any specific model, their dependence on \(\epsilon\) and the fields...
of the theory, $\Phi$, is fixed by choosing a conformal gauge in which the Einstein action and the gravitino kinetic term are canonically normalized \cite{7}. The gauge field $A_\mu$ is non-dynamical; by solving its algebraic equations of motion it can be expressed as a function of the $\Phi$ and their covariant derivatives. Eqs. (4,9) give a universal formula for the charge density of strings and domain walls. For domain walls the formula reads (cfr. eq. (8)):

$$Z_{12} = \frac{1}{2}[\epsilon_{\alpha\beta}\frac{\delta \eta^\alpha}{\delta \epsilon_\beta}(z = +\infty) - \epsilon_{\alpha\beta}\frac{\delta \eta^\alpha}{\delta \epsilon_\beta}(z = -\infty)], \quad Z_{ij} = 0 \text{ otherwise.}$$

(10)

This formula applies in particular to the the case in which the domain wall is due to strong-coupling phenomena or fermion condensates \cite{4}. It holds also when the supergravity Lagrangian contains higher curvature terms, as the ones due to string or M-theory corrections.

In $N = 2$ supersymmetry, the membrane charges and the algebraically independent string charges form a triplet of $SU(2)$. The string charges in the triplet of $SU(2)$ are given by the Wilson loop $\oint dx^j[V^A_{j B} - (1/2)\delta^A_B V^C_{j C}]$. They do not depend at all on the vector multiplets, since only hypermultiplets contribute to the composite gauge connection of the $SU(2)$ R-symmetry \cite{8}. The string charge singlet of $SU(2)$ instead, is a function of vector multiplets only, since hypermultiplets do not appear in the $N = 2$ formula for $V^C_{\mu C}$ \cite{8}.

The membrane central charge depends on both hypermultiplet and vector-multiplet moduli, since the formula for the gravitino mass matrix reads \cite{8, 9, 10}

$$M^{AB} = L^A P^{AB}_\Lambda,$$

(11)

and $L^A$ is a covariantly holomorphic section of the $U(1)$ bundle associated to the vector moduli. This is in agreement with (in fact, it is required by) the centrally extended algebra in eqs. (3). Indeed, since $Z^{(AB)}_{\mu\nu}$ enters in the $\{Q^A_\alpha, Q^B_\beta\}$ commutators, it must carry the same $U(1)$ weight as the scalar charge. This is indeed the case, since \cite{10}

$$Z^{[AB]} = \epsilon^{AB} Z, \quad Z = L^\Lambda q_\Lambda - F_{\Lambda\mu} p^\Lambda.$$

(12)

The string charge $Z^A_{\mu B}$, instead, is neutral under $U(1)$, since it carries the same $U(1)$ as the translations $P_\mu$.

An explicit realization of extended objects with moduli-dependent charge densities can be obtained in type II superstrings compactified on Calabi-Yau threefold. The study of these compactifications also provides a check of the properties discovered above, namely that: a) the charge density of triplet strings only depends on hypermultiplets, b) the charge density of singlet strings only depends on vector multiplets, c) the charge density of domain walls depends on both vector and hypermultiplet moduli.

Let us introduce the following notation: $\mu_1$ is the string charge density, $\mu_2$ is the domain wall charge density; $J$ is the Kähler form of the CY space, $\Omega$ is the CY holomorphic 3-form, and
\(a_k\) are the homology \(k\)-cycles of the CY space. The 4-d string dilaton, \(\hat{\phi}\), is a hypermultiplet both in type IIA and type IIB theory. It is related to the 10-d dilaton, \(\phi\), by

\[
\hat{\phi} = \phi + K_J/2, \quad K_J \equiv - \log(\int_{a_6} J \wedge J \wedge J).
\]

We consider first type IIB superstrings. In this case, the Kähler form is a function of the hypermultiplet moduli only, while \(\Omega\) depends only on the vector moduli. The 4-D strings present in this theory are:

1. The fundamental 10-d string. In the 4-d Einstein frame its charge density is

\[
\mu_1 = \exp(2\hat{\phi}).
\]

2. The D-string. Its charge density is

\[
\mu_1 = \exp(\hat{\phi}) \exp(K_J/2).
\]

3. The D3 brane wrapped on a 2-cycle. Here:

\[
\mu_1 = \exp(\hat{\phi}) \exp(K_J/2) \int_{a_2} J.
\]

4. The D5 brane wrapped on a 4-cycle:

\[
\mu_1 = \exp(\hat{\phi}) \exp(K_J/2) \int_{a_4} J \wedge J.
\]

5. The D7 brane wrapped on the CY threefold:

\[
\mu_1 = \exp(\hat{\phi}) \exp(-K_J/2).
\]

6. The NS5 brane wrapped on a 4-cycle.

\[
\mu_1 = \exp(K_J) \int_{a_4} J \wedge J.
\]

All the charge densities listed above are functions of hypermultiplets only; thus all the strings of type IIa on CY are triplets.

To get a domain wall in 4-d, on the other hand, one must wrap a D5 or NS5 brane on a 3-cycle. The charge densities now read

\[
\mu_2 = \exp(\hat{\phi}) \exp(K_J) V_3, \text{ for NS5}, \quad \mu_2 = \exp(2\hat{\phi}) \exp(K_J/2) V_3, \text{ for D5}.
\]

The volume of the minimal-volume 3-cycle is

\[
V_3 = \exp\left(K_\Omega/2 - K_J/2\right) |\int_{a_3} \Omega|, \quad K_\Omega = - \log(i \int \Omega \wedge \Omega).
\]
Since \( V_3 \) depends on vector moduli, \( \mu_2 \) is a function of both vectors and hypermultiplets, as predicted by the general SUSY algebra in eq. (3).

It is interesting to compare the above formulae with the general formula eq. (7), in which the charges are expressed by integrating the \( SU(2) \) connection of the quaternionic manifold. As examples, let us consider the fundamental string, the D-string, and the NS 5-brane wrapped on a 4-cycle. For this purpose, we may use the general formulae [12] of quaternionic manifolds obtained by c-map from special manifolds [13]. By writing the \( SU(2) \) connection in terms of sigma matrices,

\[
V_B^A = V^I \sigma_{IB},
\]

(22)

the differentials from the NS-NS sector give a contribution to \( V^3 \), while the R-R differentials contribute to \( V^1, V^2 \). For the fundamental and D-strings in the universal hypermultiplet we get, respectively:

\[
V^3 \sim (S + \bar{S})^{-1} da, \quad V^1 \sim (S + \bar{S})^{-1/2} \exp(K_J/2) dC.
\]

(23)

Here \( a \) is the space-time axion and \( S \equiv ia + \exp(-2\hat{\phi}) \). Integrating these formulas we reproduce eqs. (14,15). Also, for the NS 5-brane we obtain

\[
V^3 \sim Q,
\]

(24)

where \( Q \) is the connection one-form of the \( U(1) \) Hodge bundle of the special geometry associated by c-map to the quaternionic manifold. In the case at hand, in special coordinates,

\[
Q \sim \exp(K_J) d_{ABC} \text{Re} Z^A \text{Re} Z^B d \text{Im} Z^C.
\]

(25)

Integrating \( Q \) we reproduce eq. (19).

The type IIA string provides us with another subtle check of our formulae. In type IIA superstrings, indeed, \( J \) is a function of the vector moduli, while \( \Omega \) depends on the hypermultiplet moduli only. The 4-d strings in this case come from:

1. The fundamental 10-d string, \( \mu_1 = \exp(2\hat{\phi}) \).

2. D4 branes wrapped on 3-cycles, with charge density:

\[
\mu_1 = \exp(\hat{\phi}) \exp(K_J/2) V_3 = \exp(\hat{\phi}) \exp(K_{\Omega}/2) | \int_{a_3} \Omega |.
\]

(26)

3. NS5 branes wrapped on 4-cycles:

\[
\mu_1 = \exp(K_J) \int_{a_4} J \wedge J.
\]

(27)
Notice that the string obtained from the D4 brane is a triplet of \( SU(2) \), since its charge density eq. (26) depends only on hypermultiplets, but the string obtained from the NS5 brane is a singlet, since its \( \mu_1 \) depends only on vector moduli.

Finally, the 4-d domain walls of type IIA are:

1. D2 branes. Their charge density is: \( \mu_2 = \exp(2\hat{\phi}) \exp(K_J/2) \)
2. D4 branes wrapped on 2-cycles: \( \mu_2 = \exp(2\hat{\phi}) \exp(K_J/2) \int_{a_2} J. \)
3. D6 branes wrapped on 4-cycles: \( \mu_2 = \exp(2\hat{\phi}) \exp(K_J/2) \int_{a_4} J \wedge J. \)
4. D8 branes wrapped on the CY threefold: \( \mu_2 = \exp(2\hat{\phi}) \exp(-K_J/2). \)
5. NS5 branes wrapped on 3-cycles: \( \mu_2 \) is given by the first equation in formula (20).

As expected, they depend on both hypermultiplets and vector multiplets.

3 Central Extension in \( d=3 \)

The analysis done above for four-dimensional extended supersymmetry can be repeated in three dimensions. In \( d=3 \), the R-symmetry of the \( N \) extended supertranslation algebra is \( O(N) \), rather than \( SU(N) \), and its maximal central extension reads

\[
\{ Q^A_\alpha, Q^B_\beta \} = (\gamma^\mu C)_{\alpha\beta} P_\mu \delta^{AB} + (\gamma^\mu C)_{\alpha\beta} Z^{(AB)} + C_{\alpha\beta} Z^{[AB]}, \quad (Z^{(AB)} \delta_{AB} = 0). \tag{28}
\]

We see again that by adding to the translations a vector charge, traceless and symmetric under \( O(N) \), together with a scalar charge, in the adjoint of \( O(N) \), we find \( N(2N+1) \) bosonic charges. This is the dimension of the lhs of eq. (28).

The adjoint of \( O(2N) \) decomposes as follows under \( U(N) \subset O(2N) \):

\[
\text{adj } O(2N) = \text{adj } U(N) + [2] + [2], \tag{29}
\]

where \( [2] \) denotes the twofold antisymmetric of \( U(N) \). This property shows that the scalar central charges in 3 dimensions come from the scalar central charges in four dimensions plus the extra scalar charges coming from the dimensional reduction of \( Z^{A}_\mu B \) and \( P_\mu \).

The simplest model with a nonzero central charge \( Z^{[AB]} \) is the \( N = 2 \) Abelian Higgs model. It is obtained by dimensional reduction of the 4-d Abelian Higgs model, which contains a BPS saturated string. Upon dimensional reduction, the associated string charge gives rise to a scalar central charge \( Z^{[AB]} = e^{[AB]} Z \). Physically, this is the charge of the 3-d vortex generated by the dimensionally reduced BPS string.
In 3 dimensions, the U-duality group of the $N = 16$ theory is $E_{8(8)}$. This strongly suggests that the 120 scalar charges –in the adjoint of $O(16)$– together with the charges of the 128 “elementary” massless fields –in the left spinor representation of $O(16)$– should complete the 248-dimensional fundamental representation of $E_{8(8)}$.

4 Conclusions

In this paper, we found an explicit universal formula relating the tensor central charges to the geometry of scalar moduli fields. These formulae are easily generalized to take into account the possible existence of non-vanishing fermion condensates. We presented this generalization explicitly for the (off-shell) $N = 1$ superalgebra. Condensates are expected in strongly interacting theories. In rigid supersymmetry, indeed, it has been shown explicitly that they give rise to membrane central charges [1].

We must also note that all formulae for string tensions of type IIB on CY spaces obtained in this paper receive string quantum corrections, because the quaternionic geometry is corrected both perturbatively [14] and non-perturbatively [11, 15, 16, 17]. However, eq. (7) is non-perturbative in nature, and allows to determine the charge of all 4-d strings in terms of the quantum-corrected quaternionic geometry. This is parallel to the case of zero branes, where the central-charge formula [10] allows to study non-perturbative properties of type IIB strings near a conifold point [16], or fixed scalars at the black hole horizon [18].

Finally, we must point out that when the volume of the CY space is finite, all moduli are dynamical. The 4-d strings and membranes modify the equations of motion of these scalars, driving them to some fixed points. An interesting question, that we would like to address in the future, is whether fixed points exist where the string or domain-wall tension is neither zero nor infinite. These points exist in the analogous case of 4-d zero branes, where moduli can flow to fixed point with finite mass [18]. Tree-level formulae as in eqs. (14-19) do not allow for nontrivial fixed points, but the quantum corrected ones may.

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