Research Article

Perturbation Analysis of Population Growth Rates in Taiwan

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The total fertility rate in Taiwan has fallen below 1.3 since 2003. The objective of the study is to use perturbation analysis with census data from 1992 to 2017 to identify which demographic parameters are most important to target for population management. The research shows that the fertility of older ages plays an important role for declining population in Taiwan. From a practical viewpoint, population management policy having a substantial impact on the survival of prereproductive females and the fertility of order females is likely to produce the most dramatic change in population trends. Therefore, the perturbation analysis is useful in understanding the relative importance of vital rates to increase management effectiveness.

1. Introduction

A quantitative understanding of complex changes in population structure, defining some of the important time scales for population dynamics, is crucial to sustainable population management in a country. Taiwan’s population was characterized by both quite high fertility and mortality during the Japanese colonial period [1]. Since the implementation of family planning program in 1964, Taiwan’s fertility rate has rapidly declined [2, 3]. Taiwan currently has one of the world’s lowest fertility rates and the total fertility rate (TFR) has reached the below-replacement level in 1984. Since 2003, the TFR in Taiwan has fallen below 1.3, which is often described as lowest-low fertility [4], leading to projections of rapid population aging and negative socioeconomic effects.

Many studies pointed out that lowest-low fertility in East Asia is mainly attributable to delayed marriage and childbearing, and increasing proportion remained single [5–8]. However, to judge the relative importance of demographic parameters for population management and to assess how variation in vital rates contributes to variation in population growth rate (PGR) have been little explored until now.

Matrix population models (MPMs) are the primary models used to understand demographics and guide management of populations [9–11]. To model the dynamics of a population requires a link between vital rates and population processes. That link is provided by age-structured population models since age differences are most important. Furthermore, Fujiwara and Diaz-Lopez [12] recommend using an age-structured MPM for investigating the status of a population when life table data are available. Life tables are now available for nearly all countries and are routinely produced by government statistical offices, insurance companies, and academic demographers. Therefore, the data in life table consisting of the number of births, number of deaths, and the exposed population all by age are employed in this study (see, e.g., [13, 14]).

The life table can be converted to the population projection matrix (PPM) which extends the dichotomy of the life table to consider individuals with different characteristics. The asymptotic population growth rate (PGR) λ, the rate at which population size would change if vital rates remained constant over time, is given by the dominant eigenvalue of a PPM. That is, the multiple vital rates are integrated into a single metric, the PGR. Projection is frequently followed by analytical and numerical perturbation analysis to understand the relative influence of vital rates. The results of perturbation analysis are often expressed as sensitivities and elasticities. Sensitivities quantify the absolute change in λ resulting from an infinitesimal absolute change in transition elements, whereas elasticities quantify the proportional change in λ resulting from an infinitesimal
proportional change in transition elements. Therefore, elasticity can be used to compare the effects of changes in vital rates that are measured on different scales (e.g., survival, which is close to one, and fertility, which is much less than one in this study). Because the elasticities sum to unity, they can be interpreted as the relative contributions of the vital rates to PGR [15, 16]. The effect that each vital rate has on population growth can be used as an index for evaluating the importance of certain vital rates for management and research. In general, management strategies are recommended that improve those vital rates that are most sensitive or elastic with respect to the PGR.

Under constant fertility and mortality rates, the asymptotic population dynamics are independent of initial conditions. This property is known as ergodicity [17, 18], implying that population patterns might reveal something about processes rather than initial conditions. Therefore, in a limited sense, the population management may be explored. The approach employed would be characterized as prospective analysis that focuses on the functional mathematical dependence of \( \lambda \) on parameters.

The objective of the study is to use MPMs with census data from 1992 to 2017 to identify which demographic parameters are most important to target for population management. The status is investigated by comparing differences in important pieces of information with year, such as PGRs and the generation time, calculated using parameters most important to target for population management. The status is investigated by comparing differences in important pieces of information with year, such as PGRs and the generation time, calculated using age-specific tabulations of midyear population size, deaths, and births. The following three items are used to construct the life tables and age-specific fertility schedules: midyear population size between age \( x \) and \( x + n \) \((\_xK_x)\), number of deaths between age \( x \) and \( x + n \) \((\_xD_x)\), and number of births at age \( x \) \((\_b_x)\) for \( x \in \{0, 1, 4, 9, \ldots, 94, 99\} \). Age one is shown because mortality at age zero is very different from mortality at age one to five. We can directly obtain TFR and period central death rate \( nM_x \), as TFR = \( n\sum_{i}(\_b_x/\_xK_x) \), and \( nM_x = \_xD_x/\_xK_x \), respectively.

The life table is constructed using the Keyfitz-Flieger graduation [22] and contains nine columns: age class \((x)\), person-years lived by those dying between age \( x \) and \( x + n \) \((\_a_x)\), period central death rate \((\_xM_x)\), interval mortality probability \((\_d_x = \_xD_x/\_xK_x)\), probability of surviving from birth to age \( x \) \((\_L_x)\), difference in number of survivors for ages \( n \) years apart \((\_D_x = I_x - I_{x+n})\), person-years lived between age \( x \) and \( x + n \) \((\_L_x = \int_0^{\infty} L(t) dt)\), person-years of remaining life at age \( x \) \((T_x = e_{cx}L_x)\), and life expectancy at age \( x \) \((e_x = T_x/L_x)\). Some researchers [18, 20, 23, 24] have provided sufficient discussion on making a life table.

2.2. Projections. Converting age-based life-table data to an age-classified PPM can show how age structure changes during population growth. The PPM \( A \) based on five-year age structure has survival probability \( P_x \), on the subdiagonal and fertility \( F_i \) in the first row and zeros elsewhere and can be expressed as

\[
A = \begin{bmatrix}
0 & 0 & 0 & F_1 & F_2 & F_3 & F_4 & F_5 & F_6 & F_7 & F_8 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & P_3 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & P_4 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & P_5 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & P_6 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & P_7 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & P_8 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & P_9 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & P_{10} & 0 & 0 \\
\end{bmatrix}
\]

The asymptotic PGR \( \lambda \) is given by the dominant eigenvalue of \( A \) \((\lambda > 1)\): increasing population; \((\lambda < 1)\): declining population. The long-term population dynamics are described by \( \lambda \) and the stable age distribution \( w \), which is the right eigenvector corresponding to \( \lambda \). The following Euler-Lotka equation is used to obtain an exact solution for \( \lambda \) [19]:

\[
1 = \sum_{i=0}^{n} \lambda_i \left( \prod_{j=1}^{\infty} P_j \right) F_i,
\]

where \( a \) is the age at first reproduction and \( \beta \) is the age at last reproduction.

Vector of reproductive values is the ratio of the number of offspring produced by individuals of some specific age or older and the number of individuals of that age in a stable age distribution [25]. The dominant left eigenvector \( v^* \) of the PPM is the reproductive value vector [9], which is scaled so that \( v^* w = 1 \) where \( v^* \) is the complex conjugate transpose of \( v \).

The net reproduction number \( R_0 \) is the number of girl children expected to be born to a girl child just born. Generation time \( T \) is the amount of time that it takes a typical female to produce \( R_0 \) offspring, or equivalently, the

2. Methods

2.1. Life Tables. In constructing a life table, three assumptions need to be made [20]: (1) no migration or the migrants as a whole have the same mortality regime specified by the life table, (2) annual age-specific death rates do not change over time, and (3) annual number of births remains constant over time. While the life table oversimplifies demographic mechanisms, a rich variety of useful results is based on it [21].

The life table is used as a unifying technique in demography [21]. In this study, the analysis commences by constructing the female life tables and age-specific fertility schedules using age-specific tabulations of midyear population size, deaths, and births. The following three items are used to construct the life tables and age-specific fertility schedules: midyear population size between age \( x \) and \( x + n \) \((\_xK_x)\), number of deaths between age \( x \) and \( x + n \) \((\_xD_x)\), and number of births at age \( x \) \((\_b_x)\) for \( x \in \{0, 1, 4, 9, \ldots, 94, 99\} \). Age one is shown because mortality at age zero is very different from mortality at age one to five. We can directly obtain TFR and period central death rate \( nM_x \), as TFR = \( n\sum_{i}(\_b_x/\_xK_x) \), and \( nM_x = \_xD_x/\_xK_x \), respectively.

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The net reproduction number \( R_0 \) is the number of girl children expected to be born to a girl child just born. Generation time \( T \) is the amount of time that it takes a typical female to produce \( R_0 \) offspring, or equivalently, the
time it takes a population growing at instantaneous rate $r$ to increase by a factor of $R_0$. Thus, the net reproduction number can be defined as $R_0 = \frac{\lambda}{e^{rT}}$ over an $n$-year period [9]. The intrinsic rate of increase measures the rate of increase as a function of absolute time, whereas $R_0$ measures increase as a function of generation time. Therefore, if we want to calculate intrinsic rate of increase $r$, we must scale $R_0$ to account for generation time.

2.3. Perturbation Analysis. The rate of increase of a population is a function of the age-specific vital rates. Demographic perturbation analysis examines the PGR $\lambda$ of the model to changes in the vital rates $a_{ij}$ (PPM elements). The results of perturbation analysis are often expressed as sensitivities and elasticities. The sensitivity $s_{ij}$ of $\lambda$ to transition elements $a_{ij}$ is $s_{ij} = (\partial \lambda / \partial a_{ij}) = \nu_i \nu_j$. The elasticity $e_{ij}$ of $\lambda$ to transition elements $a_{ij}$ is $e_{ij} = (a_{ij} / \lambda) (\partial \lambda / \partial a_{ij})$. Caswell [26] derived a formula for the second derivative of $\lambda$ to investigate the change in sensitivities as follows:

$$\frac{\partial^2 \lambda}{\partial a_{ij} \partial a_{kl}} = s_{ij}^{(1)} \sum_{m \neq 1} \frac{s_{kj}^{(m)}}{\lambda - \lambda^{(m)}} + s_{ij}^{(1)} \sum_{m \neq 1} \frac{s_{il}^{(m)}}{\lambda - \lambda^{(m)}}.$$  

While sensitivities may be used to compare the absolute effects on $\lambda$ of the same absolute changes in vital rates, elasticities may be used to predict changes in $\lambda$ as a result of changes in vital rates. Therefore, elasticity analysis is often used to give directions for population management [9, 10].

Among the modern software package or computer language, for example, MATLAB [27], R [28], SAS [29], SPSS [30], and Stata [31], the high-level programming language R stands out because it allows both statistical and mathematical modeling. In addition, it is free and has excellent graphics capabilities [32]. For all the analysis in this paper, the R package demogR developed by Jones [33] is used to construct and analyze the age-structured population of Taiwan.

3. Results and Discussion

3.1. Vital Rates and Projections. In 1992, the goal of the Guideline for Population Policy was modified from reducing population growth to maintaining a reasonable growth of population. Therefore, six sets of census data from 1992, 1997, 2002, 2007, 2012, and 2017 are selected for this study. The statistical data of midyear population size, number of deaths, and number of births are available from the website of Ministry of the Interior [34] of Taiwan. Table 1 shows all the census counts and death tabulations for female population of Taiwan from the discussed six years.

Age-specific mortality rates for the six populations, as shown in Figure 1, display the pattern of high and declining early mortality. The most rapidly changing portion is the ages under four and especially from birth to the first birthday. The age-specific mortality rates indicate a steady decline trend with year for later ages.

Figure 1 shows that the fertility rate falls more rapidly at age class 25–29 than at others. While the fertility rates of younger ages decrease with year, the fertility rates of older ages increase with year, indicating a phenomenon of aging of fertility due to the postponement of marriage. The peak in fertility occurred at an older age class 30–34 in 2017 with a lower TFR, compared to the peaks in 1992 at age class 26–30 with higher TFR. The highest fertility values shifted toward older ages with year.

Plotting survivorship against age shows high survivorship during young and intermediate ages, as shown in Figure 2. The age extends only to age 99 because midyear population size and number of deaths beyond that age before 1997 are not available. The decline in survivorship is more moderate before the age of 69. The survivorship is consistent with the trend of age-specific mortality rates. The age-specific life expectancies indicate a steady lengthening trend, as shown in Figure 2. The promotion of National Health Insurance in 1995 has effectively reduced the chances of death among all ages and is reflected in a significant increase in life expectancies. Age-based life-table data are converted to age-classified PPMs, as shown in Table 2.

The Euler-Lotka equation and age-structured model show the similar values for PGRs. This suggests there is no bias introduced by discretizing age. PGR ranges from 0.912 to 0.975. The fluctuation in the intrinsic rate of increase is more substantial than that in TFR, as shown in Figure 3. The intrinsic rate of increase tells us how quickly the declining population over time. Despite the fluctuations, the rebounded fertility in 2012 echoed the effect of the Dragon years on fertility behavior in Taiwan.

From Figure 3, we can see that the generation time is extended over time. The average woman in Taiwan after 1992 replaced herself with less one daughter and took more than 30 years to do so. Therefore, it is also the expected number of offspring produced by an individual during her lifetime.

Since the population is declining, the stable age distribution is skewed toward older ages, as shown in Figure 4. Declining populations have larger proportions of old people. For increasing population, relative frequencies in the stable age distribution always decrease with later ages. That is, newborns are the most common age, and the oldest age is most rare.

The reproductive value vector is usually scaled so that the reproductive value of newborns always equals one, to measure reproductive value relative to a newborn individual. Curves of reproductive value for females in Taiwan show no peak near the age of first reproduction, as shown in Figure 5, reflecting the increased reproduction ages and survival of order ages. Before age class 20–24, the line with highest reproductive value is that with the highest $\lambda$; after age class 20–24, the line with highest reproductive value is that with the lowest $\lambda$. That is, the influence on projected $\lambda$ compared with newborn is larger and larger for older age class.

3.2. Perturbation Analysis. Sensitivities to variation in elements of the PPM are given by Table 3. While the fertility sensitivity increases with age and grows faster with year, the survival sensitivity declines or remains the same with age and shrinks slightly with year, as shown in Figure 6. The PGR
Table 1: Midyear population size, number of deaths, and number of births of Taiwan (1992–2017).

| Age Class | 1992 | 1997 | 2002 | 2007 | 2012 | 2017 |
|-----------|------|------|------|------|------|------|
| Kx        | Dx   | bx   | Kx   | Dx   | bx   | Kx   |
| 0         | 147356 717 | 146330 988 | 115418 650 | 91808 443 | 97887 385 | 91590 363 |
| 1         | 154191 145 | 134812 96 | 99093 49 | 88758 34 | 102500 29 |
| 2–4       | 464325 200 | 414713 145 | 320733 65 | 278539 66 | 308456 35 |
| 5–9       | 847953 238 | 774933 168 | 772768 124 | 669329 101 | 519036 77 |
| 10–14     | 978288 262 | 837993 205 | 775539 118 | 776093 88 | 669180 79 |
| 15–19     | 903112 447 | 794787 453 | 719832 192 | 775452 174 | 68830 148 |
| 20–24     | 909546 509 | 896260 415 | 716260 392 | 763477 275 | 56821 127 |
| 25–29     | 951476 616 | 903078 536 | 874920 443 | 996651 476 | 85480 376 |
| 30–34     | 898636 771 | 903078 536 | 775539 118 | 776093 88 | 669180 79 |
| 35–39     | 949454 933 | 931417 992 | 950255 887 | 916807 747 | 87997 748 |
| 40–44     | 664160 1084 | 846357 1275 | 927101 1247 | 948103 1182 | 923424 1054 |
| 45–49     | 436912 1136 | 658309 1508 | 839730 1659 | 921845 1745 | 947141 1711 |
| 50–54     | 436912 1136 | 658309 1508 | 839730 1659 | 921845 1745 | 947141 1711 |
| 55–59     | 951476 616 | 903078 536 | 874920 443 | 996651 476 | 85480 376 |
| 60–64     | 951476 616 | 903078 536 | 874920 443 | 996651 476 | 85480 376 |
| 65–69     | 247233 4358 | 296121 4686 | 35726 4539 | 36923 4525 | 37646 4521 |
| 70–74     | 175615 5559 | 217330 5611 | 269127 6334 | 31116 6140 | 36430 6128 |
| 75–79     | 114103 6484 | 142027 6658 | 184658 7202 | 233440 7915 | 275107 8444 |
| 80–84     | 64410 6102 | 80268 6550 | 106527 7371 | 141477 8604 | 187533 10259 |
| 85–89     | 25460 3717 | 36643 4888 | 48351 5797 | 68872 7239 | 97358 9468 |
| 90–94     | 8188 1574 | 10482 2099 | 16075 2953 | 22995 4055 | 34755 5920 |
| 95–99     | 1768 422 | 2451 592 | 3123 777 | 5150 1280 | 7485 1982 |

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is more sensitive to changes in fertility than to changes in survival for all ages. Sensitivities vary by as much as three orders of magnitude in 2007, with the lowest PGR. The highest sensitivities are to change in fertility at age class 45–49. \( \lambda \) is most sensitive to fertility element \( F_8 \), which has larger effect on \( \lambda \) than the most important survival element \( P_1 \). The ratio \( F_8/P_1 \) ranges from 1.3 to 2.7. Note that the value of the sensitivity to first period survival declines with year. The intuition behind this is that, in low-fertility populations, the age structure is more heavily weighted toward the older ages. Demetrius [35] showed that the sensitivity to fertility is a declining function of the age if the PGR is greater than the greatest survival rate. Caswell [9] also showed that the sensitivity to fertility in an increasing
population modeled by a PPM cannot increase with age. However, in this study, the PGR is smaller than the greatest survival rate and the sensitivity to fertility increases from one age to the next, as shown in Figure 6.

Table 4 shows elasticities to variation in elements of the PPM. The most important coefficients to $\lambda$ are survivals, which together account for 84.60% to 86.48% of $\lambda$ and the fertilities for only 13.52% to 15.40%. While the importance of survivals increases with year, the importance of fertilities decreases with year. While the changes in fertility at age class 20–29 have the larger impact on $\lambda$ before 2007, the changes in fertility at age class 25–34 have the larger impact on $\lambda$ after 2007. Figure 7 identifies that the elasticity to survival probability consistently declines with age and that the highest elasticities are those of prereproductive survival. That is, the smallest changes in survival at prereproductive age will produce the bigger change in PGR. The elasticity to fertility firstly increases and then declines. The elasticities to survival from younger ages are much greater than the elasticities to fertility from younger ages. However, the elasticity to fertility from older ages is slightly greater than the elasticity to survival from older ages. Caswell [9] showed that the elasticity to survival cannot increase with age and that the elasticity to survival from any age is greater than or equal to the elasticity to fertility from any older age. For a decreasing population, Figure 7 shows that the elasticity to survival is greater than the elasticity to fertility only before middle in the reproductive period. However, the elasticity to survival into the oldest age is equal to the elasticity to fertility from the oldest age. This is to be expected, since these two demographic processes belong to the same life cycles and the elasticities to all processes within a cycle are the same [9, 36]. The peak of the elasticity to fertility shifts to order ages with year, indicating that the
Figure 4: Stable age distributions for the six populations.

Figure 5: Age-specific reproductive values for the six populations.
### Table 3: Sensitivity matrices with the first row and subdiagonal elements (1992–2017).

| Year | 0  | 2–4 | 5–9 | 10–14 | 15–19 | 20–24 | 25–29 | 30–34 | 35–39 | 40–44 | 45–49 |
|------|----|-----|-----|-------|-------|-------|-------|-------|-------|-------|-------|
| 1992 | 0.0000 | 0.0000 | 0.0000 | 0.1670 | 0.1717 | 0.1763 | 0.1810 | 0.1857 | 0.1903 | 0.1946 | 0.1982 |
| 1997 | 0.0000 | 0.0000 | 0.0000 | 0.1624 | 0.1663 | 0.1702 | 0.1741 | 0.1780 | 0.1818 | 0.1853 | 0.1883 |
| 2002 | 0.0000 | 0.0000 | 0.0000 | 0.1806 | 0.1928 | 0.2057 | 0.2193 | 0.2337 | 0.2488 | 0.2642 | 0.2803 |
| 2007 | 0.0000 | 0.0000 | 0.0000 | 0.1364 | 0.1202 | 0.0806 | 0.0343 | 0.0078 | 0.0009 | 0.0000 | 0.3491 |
| 2012 | 0.0000 | 0.0000 | 0.0000 | 0.1693 | 0.1812 | 0.1938 | 0.2071 | 0.2213 | 0.2361 | 0.2517 | 0.2675 |
| 2017 | 0.1244 | 0.1244 | 0.1244 | 0.1236 | 0.1179 | 0.0981 | 0.0594 | 0.0208 | 0.0032 | 0.0001 | 0.3053 |

**Figure 6: Sensitivities to age-specific survival in black and fertility in grey.**
importance of fertility at order ages to PGR is getting more
evident. Note that as fertility shifts to order ages, the relative elasticity of early survival is reduced.

\[ \frac{d^2 \lambda}{dF_i^2} \]

Table 4: Elasticity matrices with the first row and subdiagonal elements (1992–2017).

| Year | 0  | 2–4 | 5–9 | 10–14 | 15–19 | 20–24 | 25–29 | 30–34 | 35–39 | 40–44 | 45–49 | Total |
|------|----|-----|-----|-------|-------|-------|-------|-------|-------|-------|-------|-------|
| 1992 | 0.0000 | 0.0000 | 0.0000 | 0.0035 | 0.0230 | 0.0525 | 0.0496 | 0.0204 | 0.0044 | 0.0006 | 0.0000 | 0.1540 |
| 1997 | 0.1540 | 0.1540 | 0.1540 | 0.1505 | 0.1275 | 0.0975 | 0.0254 | 0.0050 | 0.0056 | 0.0006 | 0.0000 | 0.0000 |
| 2002 | 0.0000 | 0.0000 | 0.0000 | 0.0031 | 0.0197 | 0.0478 | 0.0505 | 0.0241 | 0.0055 | 0.0006 | 0.0006 | 0.0000 |
| 2007 | 0.0000 | 0.0000 | 0.0000 | 0.0029 | 0.0174 | 0.0422 | 0.0495 | 0.0282 | 0.0074 | 0.0010 | 0.0000 | 0.0000 |
| 2012 | 0.1429 | 0.1429 | 0.1429 | 0.1415 | 0.1282 | 0.0860 | 0.0365 | 0.0084 | 0.0010 | 0.0000 | 0.0000 | 0.0000 |
| 2017 | 0.0000 | 0.0000 | 0.0000 | 0.0009 | 0.0070 | 0.0264 | 0.0475 | 0.0400 | 0.0146 | 0.0020 | 0.0000 | 0.0000 |

Figure 7: Elasticities to age-specific survival in black and fertility in grey.

The second derivatives \( \frac{d^2 \lambda}{dF_i^2} \) denoted with small circles in Figure 8 decrease with age, implying nonlinear rates of change in elasticity values, and become negative.
soon after the beginning of the life cycle. Note that \( \frac{\partial^2 \lambda}{\partial F_i \partial F_j} > 0 \) as expected [37]. The rates of increase \( \lambda \) are a convex function of \( F_j \) for the age class 10–14 and a concave function of \( F_j \) for later ages. The second derivative of \( \lambda \) with respect to changes in the fertility \( \frac{\partial^2 \lambda}{\partial F_i \partial F_j} \) decreases with both \( i \) and \( j \); this means that declining fertility at any age shifts the age distribution to older ages, because it decreases \( \lambda \). Despite the fluctuation in declining rates of the second derivative with respect to the fertility, the rebounded rate in 2012 echoed the effect of the Dragon years on fertility behavior in Taiwan.

The value of \( \frac{\partial^2 \lambda}{\partial P_i \partial P_j} \) denoted with small circles in Figure 9 is negative for all age, indicating that PGR is a concave function of \( P_i \), especially at young ages. However, these values are getting closer to zero with year. That is, changes in survival at these ages have some effect on the sensitivity of \( \lambda \) to further changes in survival. However, the effect decreases with year. The value of \( \frac{\partial^2 \lambda}{\partial P_i \partial P_j} \) at old ages is

![Figure 8: PGR second derivatives with respect to pairs of fertilities.](image-url)
very small; that is, changes in survival at later ages have little or no effect on the sensitivity of \( \lambda \) to further changes in survival. The second derivatives of \( \lambda \) with respect to changes in the survival \( \frac{\partial^2 \lambda}{\partial P_i \partial P_j} \) may be positive and decrease with age for early ages. However, the declining rate is getting more and more moderate. At early ages, the survival second derivatives show alternation in sign. That is, as survival at one age increases, further increases in survival sensitivity at that age become less important, and increases in survival sensitivity at other ages become more important.

Overall, the way that survival and fertility combine to influence \( \lambda \) at later ages is highest, as shown in Figure 10, but declines as PGR declines. There is a great deal of diversity in these six populations in the magnitude of the eigenvalue second derivatives to survival and fertility. For example, the curves at age class 20–24 show that an increase in fertility at age class 20–24 reduces the sensitivity to survival at age class 20–24 and all subsequent ages and increases the sensitivity to survival before age class 20–24.

Figure 9: PGR second derivatives with respect to pairs of survival probabilities.
4. Conclusions

This study provides guidance about the relative importance of demographic parameters that are most likely to influence population dynamics in Taiwan. The research results are different from some previously proposed constraints on sensitivity and elasticity for increasing population. For increasing population, elasticity analysis inevitably concludes that management strategies improving survival are best applied to younger ages and indicates that changes to the fertility of older ages are mathematically constrained to have less impact on population growth than equivalent changes to the fertility of younger ages. However, the fertility of older ages plays an important role for declining population in Taiwan. PGR is most sensitive to fertility at order ages, which is consistent with the result of the elasticity analysis; the importance of fertility at order ages to PGR is getting more evident. The skewness toward older ages is also reflected in
the stable age distribution. The reproductive value vector reveals that reproductive females theoretically contribute proportionally less to future population growth because these females reproduce less than one in their lives. Survival sensitivity declines or remains the same with age and shrinks slightly with year. However, the largest elasticities are those for the probability of prereproductive survival. From a practical viewpoint, population management policy having a substantial impact on the survival of prereproductive females and the fertility of order females is likely to produce the most dramatic change in population trends.

PGR and generation time are two of the most important pieces of information in determining the status of threatened species [38]. The perturbation analysis is useful in understanding the relative importance of vital rates to increase management effectiveness. The survivorships during young and intermediate ages in Taiwan are high and show a steady increasing trend with year for later ages. The long-term PGR in Taiwan is most sensitive to fertility at older ages, suggesting that management should aim to increase such fertility.

Data Availability

The statistical data of midyear population size, number of deaths, and number of births are available from the website of Ministry of the Interior of Taiwan (https://www.ris.gov.tw/app/en/3911).

Conflicts of Interest

The authors declare that they have no conflicts of interest.

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