Simulation results for eight factors and two fake factors are summarized in Figure 1. While power is still near 1.0 for identification of up to six main effects (Figure 1a), our ability to identify active second-order effects has improved. Keeping power above about 0.8, we can identify three two-factor interactions in the presence of six active main effects and two quadratic effects, as shown in Figure 1b. Similarly, three quadratic effects can be reliably identified in the presence of five active main effects and two active two-factor interactions, as shown in Figure 1b. In the interest of brevity, we do not discuss explicitly the minimum-run-size case for eight factors, but the drop in power that results is evident from an examination of the “Fake Factors” panels in each of Figures 1b and 1c.

Figure 1: Conditional effects plots of the power for detecting various effects for DSD for eight factors with plots corresponding to four added runs (i.e., two fake factors)

Results for 10 factors and two fake factors are shown in Figure 2. The main effects situation
(Figure 2a) is virtually unchanged—the power for identifying up to six factors is still essentially 1.0 in the presence of three active two-factor interactions and three active quadratic effects. We note that seven through nine main effects can still be identified with very high power, however the simulation times required for those cases precluded their inclusion. As shown in Figures 2b and 2c, respectively, power is high for identifying three two-factor interactions in the presence of six active main effects and three active quadratic effects; it is also high for identifying three quadratic effects in the presence of six active main effects and three active two-factor interactions.

Figure 2: Conditional effects plots of the power for detecting various effects for DSD for ten factors with plots corresponding to four added runs (i.e., two fake factors)
Table 1 contains the timing study data for full $3^3$ factorial described in Section 5 of the paper.

| Run Number | A  | B  | C  | Time (seconds) |
|------------|----|----|----|----------------|
| 1          | -1 | -1 | -1 | 2.367          |
| 2          | -1 | -1 | 0  | 3.633          |
| 3          | -1 | -1 | 1  | 3.983          |
| 4          | -1 | 0  | -1 | 3.633          |
| 5          | -1 | 0  | 0  | 4.517          |
| 6          | -1 | 0  | 1  | 4.783          |
| 7          | -1 | 1  | -1 | 4.600          |
| 8          | -1 | 1  | 0  | 4.567          |
| 9          | -1 | 1  | 1  | 4.767          |
| 10         | 0  | -1 | -1 | 4.167          |
| 11         | 0  | -1 | 0  | 7.650          |
| 12         | 0  | -1 | 1  | 13.600         |
| 13         | 0  | 0  | -1 | 8.100          |
| 14         | 0  | 0  | 0  | 13.783         |
| 15         | 0  | 0  | 1  | 14.483         |
| 16         | 0  | 1  | -1 | 14.633         |
| 17         | 0  | 1  | 0  | 15.067         |
| 18         | 0  | 1  | 1  | 15.783         |
| 19         | 1  | -1 | -1 | 7.717          |
| 20         | 1  | -1 | 0  | 17.333         |
| 21         | 1  | -1 | 1  | 33.367         |
| 22         | 1  | 0  | -1 | 16.417         |
| 23         | 1  | 0  | 0  | 37.833         |
| 24         | 1  | 0  | 1  | 43.100         |
| 25         | 1  | 1  | -1 | 42.800         |
| 26         | 1  | 1  | 0  | 45.383         |
| 27         | 1  | 1  | 1  | 48.167         |

Table 1: Timing study data for full $3^3$ factorial. Times are execution times in seconds for 1,000 analyses.
Figure 3 shows the results of each of the phases of the model selection procedure as implemented in the early adopter version of JMP 13 for the timing study of Section 5.

### Fit Definitive Screening for Time

#### Stage 1 - Main Effect Estimates

| Term | Estimate | Std Error | t Ratio | Prob>|t| |
|------|----------|-----------|---------|-------|
| A    | 14.813   | 0.5401    | 27.427  | <.0001* |
| B    | 6.4691   | 0.5401    | 11.978  | 0.0003* |
| C    | 3.219    | 0.5401    | 5.9601  | 0.0040* |
| E    | -2.423   | 0.5401    | -4.486  | 0.0109* |

| Statistic | Value |
|-----------|-------|
| RMSE      | 2.0208 |
| DF        | 4     |

#### Stage 2 - Even Order Effect Estimates

| Term   | Estimate | Std Error | t Ratio | Prob>|t| |
|--------|----------|-----------|---------|-------|
| Intercept | 13.105   | 0.8475    | 15.462  | 0.0001* |
| A*B     | 6.1682   | 0.4081    | 15.114  | 0.0001* |
| A*C     | 3.9251   | 0.4081    | 9.6178  | 0.0007* |
| B*C     | -2.791   | 0.4398    | -6.346  | 0.0032* |
| A^A     | 5.2954   | 0.9458    | 5.5986  | 0.0050* |

| Statistic | Value |
|-----------|-------|
| RMSE      | 1.3773 |
| DF        | 4     |

#### Combined Model Parameter Estimates

| Term   | Estimate | Std Error | t Ratio | Prob>|t| |
|--------|----------|-----------|---------|-------|
| Intercept | 13.105   | 1.0641    | 12.315  | <.0001* |
| A      | 14.813   | 0.4622    | 32.051  | <.0001* |
| B      | 6.4691   | 0.4622    | 13.997  | <.0001* |
| C      | 3.219    | 0.4622    | 6.965   | 0.0001* |
| E      | -2.423   | 0.4622    | -5.242  | 0.0008* |
| A*B    | 6.1682   | 0.5124    | 12.038  | <.0001* |
| A*C    | 3.9251   | 0.5124    | 7.6605  | <.0001* |
| B*C    | -2.791   | 0.5521    | -5.054  | 0.0010* |
| A^A    | 5.2954   | 1.1875    | 4.4593  | 0.0021* |

| Statistic | Value |
|-----------|-------|
| RMSE      | 1.7293 |
| DF        | 8     |

Figure 3: Model selection procedure for timing study