Brans-Dicke Scalar Field as a Chameleon

Sudipta Das and Narayan Banerjee
Relativity and Cosmology Research Centre,
Department of Physics, Jadavpur University,
Calcutta - 700 032, India.

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Abstract

In this paper it is shown that in Brans - Dicke theory, if one considers a non-minimal coupling between the matter and the scalar field, it can give rise to a late time accelerated expansion for the universe preceded by a decelerated expansion for very high values of the Brans - Dicke parameter $\omega$.

1 Introduction

During the present decade, the speculation that the universe at present is undergoing an accelerated phase of expansion has turned into a certainty. The high precision observational data regarding the luminosity - redshift relation of type Ia supernovae [1], the Cosmic Microwave Background Radiation (CMBR) probes [2] suggest this acceleration very strongly. This is confirmed by the very recent WMAP data [3] as well. This observation leads to a vigorous search for some form of matter, popularly called dark energy, which can drive this acceleration as normal matter cannot give rise to accelerated expansion due to its attractive gravitational properties. A large number of possible candidates for this dark energy has already appeared in the literature and their detailed behaviours are being studied extensively. For excellent reviews, see [4].

Although the expansion of the universe is accelerated at present, it must have had a decelerated expansion in the early phase of the evolution so as to accommodate for nucleosynthesis in the radiation dominated era. The early matter dominated era also must have seen a decelerated phase for the formation of galaxies in the universe. There are observational evidences too that beyond a certain value of the redshift $z$, the universe surely had a positive value for the deceleration parameter ($q = \frac{-\ddot{a}/a^2}{\dot{a}/a^3} > 0$) [5]. It has also been indicated that unless there is a signature flip from a positive to a negative value of $q$, the supernovae data are not a definite indicator of an accelerated expansion considering the error bars of the observation [6].

So, we are very much in need of some form of matter, the dark energy, which maintained a low profile in the early part of the history of the universe but evolved to dominate the dynamics.
of the universe later in such a way that the universe smoothly transits from a decelerated to an accelerated phase of expansion during the later part of matter dominated regime. Apart from the Cosmological Constant $\Lambda$, which can indeed generate a sufficient negative pressure and hence drive this acceleration, the most talked about amongst the dark energy models are perhaps the ‘quintessence models’ - a scalar field endowed with a potential such that the potential term evolves to dominate over the kinetic term in the later stages of evolution generating sufficient negative pressure which drives the acceleration. A large number of quintessence potentials have appeared in the literature (for an extensive review see [7]). However, most of the quintessence potentials do not have a sound background from field theory explaining their genesis. Hence it might appear more appealing to employ a scalar field which is already there in the realm of the theory. This is where the non-minimally coupled scalar field models step in as the driver of this alleged late time acceleration. Brans - Dicke (BD) theory [8] is arguably the most natural choice as the scalar tensor generalization of general relativity (GR). BD theory or its modifications have already proved to be useful in providing clues to the solutions for some of the outstanding problems in cosmology (see [9] and [10]) and could generate sufficient acceleration in the matter dominated era even without the help of quintessence field [11]. Attempts have also been made to obtain a non-decelerating expansion phase for the universe at present by considering some interaction between the dark matter and the geometrical scalar field in generalised Brans - Dicke theory [12]. However, the form of interaction chosen was ad-hoc and did not follow from any action principle.

A different approach is now being considered in general relativity, where the quintessence scalar field is allowed to interact non-minimally with matter sector rather than with geometry and this interaction is introduced through an interference term in the action. This type of scalar field is given the name ‘chameleon field’ [13]. Many interesting possibilities with this chameleon field has been recently studied [14]. It has also been shown recently [15] that this chameleon field can provide a very smooth transition from a decelerated to an accelerated phase of expansion of the universe. For similar work where the scalar field is strongly coupled to matter, see also [16]. However, the problem remains the same as that of the genesis of the scalar field.

In Brans-Dicke theory or its modifications, there is an interaction between the scalar field and geometry. The chameleon field is also “nonminimally coupled”, but to the normal matter sector rather than with geometry. It deserves mention at this stage that there are attempts also to build up models where the dark energy and the dark matter do not conserve themselves individually, but has an interaction amongst them [17]. One important motivation of considering these interactions is of course to seek for a solution of the coincidence problem - why the dark energy sector dominates over the dark matter sector now.

The motivation of the present work is to investigate the interacting models in a more general framework. A Brans - Dicke framework is considered, where there is already a nonminimal coupling between the scalar field and geometry. The action is modified to include a nonminimal coupling of the scalar field with the matter sector as well. This work is actually motivated by the recent work by Clifton and Barrow [18] where they studied the behaviour of an isotropic cosmological model in the early as well as in the late time limits in this framework. However, the nonminimal coupling of a scalar field with both of geometry and the matter sector has been in use for quite a long time, courtesy the dilaton gravity, the low energy limit of string theory. The present work uses the ansatz for a particular purpose, namely to check if the required signature flip in the deceleration parameter $q$ can be obtained from this model. The actual form of the
coupling of the scalar field with matter certainly has to be introduced by hand, but the model has the advantage of having the scalar field in the theory itself.

As already mentioned, although Brans-Dicke theory proved useful for the solution for many a cosmological problem, it has the serious drawback that the Brans-Dicke parameter $\omega$ has to have a small value of order unity. This squarely contradicts the local astronomical requirement of a pretty high value of $\omega$. It has been shown that the present model works even for very high values of $\omega$ ($\sim 10^4$) and thus can have good agreement with the observational limits [19, 20]. Hence this kind of general interaction has features, which might solve the cosmological problems as well as take care of the observations on the solar systems etc.

In the next section the model is described and it is shown that this type of non-minimally coupled interacting models can provide a smooth transition from decelerated to accelerated phase of expansion for a wide range of values of the BD parameter $\omega$. Section 3 gives two cases of exact solutions and the last section discusses the results.

## 2 Field Equations and Results:

The relevant action in BD theory is given by

$$A = \int \sqrt{-g} \, d^4x \left[ \frac{\phi R}{16\pi G} + \frac{\omega}{\phi} \phi^{\mu\nu} \phi_{\mu\nu} + L_m f(\phi) \right],$$

(1)

where $R$ is the Ricci scalar, $G$ is the Newtonian constant of gravitation, $\phi$ is the BD scalar field which is non-minimally coupled to gravity, $\omega$ is the dimensionless BD parameter. The last term in the action indicates the interaction between the matter Lagrangian $L_m$ and some arbitrary function $f(\phi)$ of the BD scalar field. If $f(\phi) = \text{constant} = 1$, one gets back the usual BD action. For a spatially flat FRW model of the universe, the line element is given by

$$ds^2 = dt^2 - a(t)^2 \left[ dr^2 + r^2 d\theta^2 + r^2 \sin^2 \theta d\phi^2 \right],$$

(2)

where $a(t)$ is the scale factor of the universe.

Variation of the action (1) with respect to the metric components yields the field equations as

$$3 \frac{\dot{a}^2}{a^2} = \frac{\rho_m f}{\phi} + \frac{\omega}{2} \frac{\dot{\phi}^2}{\phi^2} - 3 \frac{\dot{a}}{a} \frac{\dot{\phi}}{\phi},$$

(3)

$$2 \frac{\ddot{a}}{a} + \frac{\dot{a}^2}{a^2} = -\frac{\omega}{2} \frac{\dot{\phi}^2}{\phi^2} - \frac{\ddot{\phi}}{\phi} - 2 \frac{\dot{a}}{a} \frac{\dot{\phi}}{\phi}.$$  

(4)

Here $\rho_m$ is the energy density of dark matter and as the universe at present is dominated by matter, the fluid is taken in the form of pressureless dust, i.e., $p_m = 0$. Here, a dot indicates differentiation with respect to the cosmic time $t$.

Also, variation of action (1) with respect to the Brans-Dicke scalar field $\phi$ yields the wave equation as

$$(2\omega + 3) \left( \ddot{\phi} + 3 \frac{\dot{a}}{a} \dot{\phi} \right) = \rho_m f + \rho_m f' \phi,$$

(5)
where a prime indicates differentiation with respect to \( \phi \).

From the two field equations and the wave equation one can arrive at the matter conservation equation which comes out as

\[
\dot{\rho}_m + 3 \frac{\dot{a}}{a} \rho_m = -\frac{3}{2} \rho_m \frac{\dot{f}}{f},
\]

and readily integrates to yield

\[
\rho_m = \frac{\rho_0}{a^3 f^{3/2}},
\]

where \( \rho_0 \) is a constant of integration.

It is evident from equation (7) that the usual matter conservation equation gets modified here because the scalar field is now coupled to both geometry and matter.

Out of equations (3), (4), (5) and (7), only three are independent equations as the fourth one can be derived from the other three in view of the Bianchi identity. On the other hand, we have four unknowns - \( a, \rho_m, f(\phi) \) and \( \phi \) to solve for.

In order to close the system of equations, we make an ansatz

\[
\frac{\dot{\phi}}{\dot{\phi}} = -\frac{\alpha}{H},
\]

where \( \alpha \) is an arbitrary positive constant. There is no a priori physical motivation for this choice, this is purely phenomenological which leads to the desired behaviour of the deceleration parameter \( q \) of attaining a negative value at the present epoch from a positive value during a recent past. The system of equations is closed now. Some characteristics of the model can now be discussed even without solving the system. In the remaining part of the section, the possibility of having a transition of the mode of the expansion from a decelerated to an accelerated one is studied.

With the assumption (8), equation (4) easily yields an expression for the deceleration parameter \( q \) as

\[
q = \frac{H^2 + \left(\frac{\omega}{2} + 1\right) \frac{\alpha^2}{H^2} - 3\alpha}{2H^2 + \alpha}.
\]

In obtaining equation (9), the relation

\[
\dot{H} = -H^2(q + 1)
\]

has been used. \( H \) gradually decreases with time from a very large value at the beginning of the evolution. The equation (9) indicates that \( q = \frac{1}{2} \) at \( H \rightarrow \infty \), i.e, the model starts exactly the same way as a matter dominated spatially flat model does.

Now the deceleration parameter \( q \) is plotted against \( H \) (Figure 1(a) - 1(h)) for different values of \( \omega \) and \( \alpha \). They clearly show that the required signature flip in \( q \) can be obtained for any negative value of \( \omega \) and also for small positive values of \( \omega \) in some recent past \( (H > 1) \). The nature of the behaviour of \( q \) against \( H \) is hardly affected by a small change in the value of \( \omega \), which can only shift the epoch at which the acceleration sets in. This can again be adjusted by choosing the value of \( \alpha \) properly which is a free parameter. It must be mentioned that as \( \frac{1}{H} \) is a measure of the age of the universe and \( H \) is a monotonically decreasing function of time \( t \), 'future' is given by \( H < 1 \), 'past' by \( H > 1 \) if \( H \) is scaled by the present value \( H_0 \), i.e, \( H = 1 \).
Figure 1: The $q$ vs. $H$ plot for different values of $\omega$ and $\alpha$. 

- **Fig 1a**: $\omega = -7000000$, $\alpha = 0.001$
  
- **Fig 1b**: $\omega = -10000$, $\alpha = 0.02$
  
- **Fig 1c**: $\omega = -1000$, $\alpha = 0.075$
  
- **Fig 1d**: $2\omega + 3 = -8$, $\alpha = 0.5$
  
- **Fig 1e**: $2\omega + 3 = 0$, $\alpha = 0.75$
  
- **Fig 1f**: $\omega = 0.01$, $\alpha = 1$
  
- **Fig 1g**: $\omega = 1$, $\alpha = 1$
  
- **Fig 1h**: $\omega = 1.5$, $\alpha = 0.5$
at the present epoch.

Figures 1(e) - 1(h) have the additional feature that \( q \) has two signature flips. For example, in figure 1(e) (i.e., for \( \omega = -\frac{3}{2} \)), the flips take place around \( H \approx 1.5 \) (i.e., past) and \( H \approx 0.25 \) (future). So, in all these cases the universe reenters a decelerated phase of expansion again in near future and thus a ‘phantom menace’ is avoided, i.e, the universe does not show a singularity of infinite volume and infinite rate of expansion in a ‘finite future’. The cases 1(a) - 1(d), however, do not have this “double signature flip”, which can be seen from equation (9). If \( q \) is put equal to zero, the combinations of values of \( \omega \) and \( \alpha \) used in these figures do not yield two real positive roots for \( H \).

3 Two specific examples:

Equation (8) along with equation (10) can be written as

\[
\frac{d}{dH} \left( \ln \phi \right) = \frac{\alpha}{H^3(q+1)} .
\]

Replacing the expression for \( q \) from equation (9), the above equation takes the form

\[
\frac{d}{dH} \left( \ln \phi \right) = \frac{\alpha \left(2H^2 + \alpha\right)}{3H \left[H^4 - \frac{2\alpha}{3}H^2 + \frac{1}{3} \left(\frac{\omega}{2} + 1\right)\alpha^2\right]} .
\]

We try to solve equation (12) analytically for two special cases.

**Case I :** \( 2\omega + 3 = 0 \)

With this choice of \( \omega \), equation (5) immediately gives

\[
f = \frac{\phi_0}{\phi},
\]

\( \phi_0 \) being a constant of integration.

Equation (12) then leads to the solution for \( \phi \) as

\[
\phi = A \frac{H^4 \left(H^2 - \frac{\alpha}{2}\right)^2}{\left(H^2 - \frac{\alpha}{6}\right)^4} ,
\]

\( A \) being a constant of integration.

Using this expression for \( \phi \), one can obtain the solutions for \( a, \rho_m \) and \( H \) as

\[
a = a_0 \frac{\left(H^2 - \frac{\alpha}{6}\right)^\frac{7}{3}}{\left[\left(H^2 - \frac{\alpha}{2}\right)\left(3H^4 - 3\alpha H^2 + \frac{3\alpha^2}{4}\right)\right]^\frac{1}{3}} ,
\]

\[
\rho_m = \frac{3A^2 H^6 \left(H^2 - \frac{\alpha}{2}\right)^6}{\phi_0 \left(H^2 - \frac{\alpha}{6}\right)^8} .
\]
and \( \left( \frac{H - \sqrt{\frac{\alpha}{2}}}{H + \sqrt{\frac{\alpha}{2}}} \right) \left( \frac{H + \sqrt{\frac{\alpha}{2}}}{H - \sqrt{\frac{\alpha}{2}}} \right)^{\frac{1}{3}} = \exp \left( \sqrt{\frac{\alpha}{2}} (t_0 - t) \right) \). \tag{17} 

It deserves mention that \( \omega = -\frac{3}{2} \) is a special case because in conformally transformed version of the BD theory, \( 2\omega + 3 = 0 \) indicates that the kinetic part of the energy contribution from the scalar field sector is exactly zero \([20]\).

As shown in equation (13), this particular choice of \( \omega \) gives \( f(\phi) \sim \frac{1}{\phi} \). The converse is also true. If one starts by assuming \( f(\phi) \sim \frac{1}{\phi} \), \( \omega \) can take only one value, i.e., \( -\frac{3}{2} \) and one arrives at the same results.

As already mentioned this choice of \( \omega = -\frac{3}{2} \) provides the important feature of ‘future’ deceleration and thus does not suffer from the problem of ‘big rip’.

**Case II: \( 2\omega + 3 = -8 \)**

In this case, equation (12) yields the solution for \( \phi \) as

\[
\phi = \phi_0 \left( 1 - \frac{7\alpha}{6H^2} \right)^{\frac{4}{3}},
\tag{18}
\]

\( \phi_0 \) being a constant of integration.

Using this expression for \( \phi \), from equation (8) one can obtain the solutions for the Hubble parameter \( H \) and the scale factor \( a \) as

\[
H = \sqrt{\frac{7\alpha}{6}} \coth \left( \frac{3}{2} \sqrt{\frac{7\alpha}{6}} t \right),
\tag{19}
\]

\[
a = a_0 \left[ \sinh \left( \frac{3}{2} \sqrt{\frac{7\alpha}{6}} t \right) \right]^{\frac{2}{3}}.
\tag{20}
\]

The interesting feature of equation (20) is that for small \( t \), \( a \sim t^{\frac{2}{3}} \) which is same as that for a dust dominated era. On the other hand, for high values of \( t \), \( a \sim e^{\frac{3}{2} \sqrt{\frac{\alpha}{2}} t} \) and thus gives an accelerated expansion for the universe.

From the field equations, the solutions for \( f \) and \( \rho_m \) also comes out as

\[
f(\phi) = \frac{196\rho_0^2}{a_0^6 \alpha^2 \phi^2} \left[ \frac{1}{40 \left( \frac{\phi}{\phi_0} \right)^{-2} - 24 + 33 \left( \frac{\phi}{\phi_0} \right)^{\frac{2}{3}}} \right]^2 \tag{21}
\]

and

\[
\rho_m(t) = \frac{a_0^6 \alpha^3 \phi_0^3}{2744 \rho_0^2} \left( \text{sech}^{14} X \right) \left[ 40 \coth^2 X - 24 \coth^2 X + 33 \text{sech}^2 X \coth^2 X \right] \tag{22}
\]

where \( X = \frac{3}{2} \sqrt{\frac{\alpha}{6}} t \).

Although here the equation system has been completely solved for only small negative values of \( \omega \), this model works even for high values of \( \omega \) ( \( \sim 10^5 \) ) as shown in figure 1(a) - 1(c). This
is consistent with the limits imposed by solar system experiments which predict the value of $\omega$ to be of the order of tens of thousands ($|\omega|\geq 40000$) [19].

It deserves mention that in figure 1(a) or 1(b), where values of $\omega$ chosen are very high ($\omega = 10^6$ in fig 1(a) and $\omega = 10^4$ in fig 1(b)), the corresponding values of $\alpha$ required are very low ($\alpha = 10^{-3}$ in fig 1(a) and $\alpha = 0.02$ in fig 1(b)) in order to adjust the time of signature flip in observationally consistent region.

### 4 Discussion:

Thus we see that for a spatially flat FRW universe ($k = 0$), we can construct a presently accelerating model with the history of a deceleration in the past in Brans - Dicke theory by considering a coupling between the matter Lagrangian and the geometric scalar field. The salient feature of this model is that no dark energy sector is required here to drive this alleged acceleration. Also it deserves mention that the nature of the $q$ vs. $H$ plot is not crucially sensitive to the value of $\omega$ chosen; only the ‘time’ when the signature flip in $q$ occurs shifts a little but that too can be adjusted by properly choosing the value of $\alpha$, which is a parameter of the model.

The matter conservation equation obviously gets modified in this framework due to the coupling between matter and the scalar field, i.e., matter is no longer conserved by itself. The right hand side of equation (6) indicates that a transfer of energy between the matter and the scalar field takes place due to the coupling factor $f(\phi)$. One may have an idea about the direction and amount of this energy transfer if $f(\phi)$ is exactly known. In the two specific examples discussed in the present work, the energy in fact flows from the dark matter to the scalar field sector. In case I, with $2\omega + 3 = 0$, $f(\phi)$ is given by equation (13) which yields (with equation (8))

$$\frac{\dot{f}}{f} = -\frac{\dot{\phi}}{\phi} = +\frac{\alpha}{H}.$$

So the right hand side of equation (6) is negative and hence $\rho_m$ decreases more rapidly than what is expected for a self-conserved matter sector. As we are working in units where the present value of $H$ is equal to 1 and $\alpha$ is less than one ($\alpha = 0.75$ as used in figure 1(e)), the present transfer rate is obviously less than the Hubble rate of expansion. In case II, where $2\omega + 3 = -8$, one can use equations (21) and (6) to find $\dot{f}/f$, and if $\alpha < 1$, the transfer rate is of the same order of magnitude as the Hubble expansion rate. In this case also, the present $\dot{f}/f$ is positive and hence the energy flows from the dark matter sector to the scalar field sector. If $f(\phi) = $ constant, this interaction vanishes and the matter sector conserves itself as usual.

As the nonminimally coupled scalar field theories allow for a variation of the strength of gravitational interaction, it is worthwhile to comment on this aspect as well. As $\frac{1}{\phi}$ behaves as the effective Newtonian gravitational constant $G$, one has

$$\frac{\dot{G}}{G} = -\frac{\dot{\phi}}{\phi} = +\frac{\alpha}{H}.$$

As the present value of $H = 1$, and $\alpha \leq 1$ in all the examples discussed, $\frac{\dot{G}}{G}$ at present is less than the Hubble rate of expansion. As already mentioned, the signature flip in $q$ in the figures 1(a) to 1(h) can still be obtained with other choices of the pair of $\omega$ and $\alpha$, the value of $\frac{\dot{G}}{G}$ can
be further lowered. In the early stages, when $H$ had a very high value, $\dot{G}$ was in fact negligible. In far future when $H \to 0$, $\dot{G}$ may have high values, but at that epoch the hierarchy between gravitational and electroweak couplings will hardly matter.

As mentioned earlier, this particular model works for a wide range of values of $\omega$ and even for high values of $\omega$ ($\sim 10^4$). So this model is capable of solving two major problems at one go - the first one is to obtain the smooth transition from a decelerated to an accelerated phase of expansion in the recent past without any dark energy sector and the second one is to solve the nagging problem of discrepancy in the values of $\omega$ as suggested by local experiments and that required in the cosmological context. It had been shown before that Brans-Dicke scalar field interacting with dark matter can indeed generate an acceleration [12] where $\omega$ is not severely restricted to low values, but the parameter $\omega$ had to be taken as a function of the scalar field $\phi$. The Brans-Dicke scalar field interacting nonminimally with dark energy sector also has a possibility of having an arbitrary value of $\omega$ [22]. But again that required a dark energy sector as the driver of the acceleration.

It deserves mention at this stage that the belief that BD theory goes over to GR in the high $\omega$ limit suffered a jolt [23]. But in the weak field regime, relevant for the observations on the solar system, a high value of $\omega$ is still warranted [19, 20]. So the present work, and the work by Clifton and Barrow [18] indeed opens up the possibility of seeking solutions to cosmological problems in BD theory. It should be noted that in view of the coupling between $\phi$ and $L_m$ as $f(\phi)L_m$ in the action, it is required that the said weak field approximation of the field equations be re-visited. In the presence of a potential $V = V(\phi)$ where $\phi$ is the BD field, such investigations have already been there [24], where the results depend on derivatives of the potential. For the present work, the details of the calculations will be different as there is no potential $V(\phi)$ as such. Investigations in this direction to find the actual order of magnitude of $\omega$ which passes the astronomical ‘fitness test’ are in progress. However, it appears that although the expression for the post-Newtonian corrections for various modifications of BD theory will have different features, all will require a large value of $\omega$ for the local astronomical tests [24, 25].

In the context of the present accelerated expansion of the universe, non-linear contribution from the Ricci scalar $\hat{R}$ in the action has attracted a lot of interest [26]. This form of action, very widely dubbed as $f(R)$ gravity, has been shown to be formally equivalent to a Brans-Dicke action endowed with an additional potential $V$ which is a function of the BD scalar field for a particular value of the BD parameter $\omega$, namely $\omega = -3/2$ [27]. This is particularly true for the Palatini kind of variation for the $f(R)$ gravity action. The present work does not contain a $V = V(\phi)$, but the term $f(\phi)L_m$ in the action serves as an effective potential and may serve a similar purpose.

The particular interaction chosen in this work is contrived, but it at least serves as a toy model, where the very existence of the particular scalar field is not questioned, it is there in the theory. A form of $f(\phi)$ for which the dark matter sector redshifts close to $a^{-3}$, and the acceleration takes place for quite a high value of $\omega$, could indeed be a very interesting possibility. Furthermore, the model has a lot of features, and has the promise to reproduce other forms of modifications of gravity as special cases.
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