Analysis of Reflective Student Analogy Reasoning in Solving Geometry Problems

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Abstract. In math subjects, students must involve an analogy reasoning in solving geometry problems. Where students were required to use the ability of reasoning analogy which one of them in solving the building of curved side space (BRSL) with a previously studied concept was a flat building. The ability of reasoning analogy was thought to be influenced by the characteristics of students, that was cognitive style. The purpose of this study was to analyze the reasoning of analogies that reflective students had in solving geometry problems. The study was conducted on 27 students with varying cognitive styles. The research began with a Matching Familiar Figure Test (MFFT) to classify reflective cognitive styles, followed by the Mathematical Ability Test (TKM). Then one reflective student was selected at high-skilled, one moderate-skilled student, and one low-skilled student. Furthermore, a BRSL analogy reasoning test (TPABRSL) was conducted and concluded with an interview. Research shows that High Reflective Students (SRT) can pass through all stages of analogy reasoning, namely encoding, inferring, mapping, and applying. Meanwhile, Moderate Reflective Students (SRS) and Low Reflective Students (SRR) were unable to pass all stages of analogy reasoning appropriately due to errors in the use of existing concepts and procedures.

1. Introduction
Geometry is one of the abstract knowledge in mathematics that learns about lines, volumes, and spaces relate to each other. The introduction of Geometry introduces early, from the early stages to the college. This is in line with Ozerem which explains that geometry materials in schools are important to provide and to assist students in analyzing and interpreting the world around them [1]. Geometry is classified into two that is flat build and build space. Build space geometry consists of building a flat side space and building a curved side space.

Students at the middle level are weak when absorbing the information or solve geometry problems. This evidence from the national exam results report on geometry materials, indications of what happens due to students' knowledge and experience are still limited in using their reasoning. Adholpus explains the weakness of students in geometry materials, continuing to work on the same question [2]. So the importance of using individual reasoning capabilities in developing problem-solving. This problem can be caused by the opposite result between the knowledge that has been learned and the new information that has been obtained by students, resulting in a systemic error [3].

The National of Council Teacher Mathematics reveals reasoning in mathematics is an important aspect that can influence a person's mindset so that existing materials in mathematics will produce systematic and sustainable thinking [4]. The variation of reasoning which can use that is mathematical...
reasoning. Mofidi, et al. states mathematical reasoning can map into three, namely inductive reasoning, deductive reasoning, and analogy reasoning [5]. Analogy reasoning according to Lailiyah, et al. is narrowly a decision-making process that is linked based on identification from the source domain to the target domain, whereas it is widely interpreted as a form of reason based on commonality [6]. The Programme For International Assessment (PISA) in 2015 can further prove how weak are students' reasoning skills with the reports that show an average mathematics grade in Indonesia only reaches a score of 386 [7]. Even the average score of mathematics in Indonesia in 2018 has decreased compared to 2015, namely Indonesia is ranked 7th from the bottom (73) with an average score of 379 [8]. So in math learning, it takes a more dominant reasoning analogy. It is also supported by Lee & Sriraman which states the use of analogy reasoning can be an efficient way of solving problems [9]. Cognitive processes in analogy reasoning have four stages of solving problems, namely Encoding, Inferring, Mapping, and Applying. The systematic lack of analogy leads to students' mistakes in solving math problems. So it is common for students to hate math and their achievements are bad in that field just because the grades are low. This is relevant to what Maswar stated that students' resentment of mathematics will increase when their grades are low on the lesson and allow its achievement to get worse as well [10].

Einsenmann, et al. finding less careful solutions can also cause problems that students are facing and cannot assist the students in developing, improving, and growing mathematical minds [11]. Susanto explains the reasoning process can be influenced by different characteristics in students. One characteristic is the cognitive style [12]. One of the kinds in cognitive styles can be distinguished based on conceptual tempo, namely reflective conclusive style and impulsive cognitive style displayed [13]. Cai & Ding explains that the reasoning analogy ability is important to be analyzed and developed through problem-solving activities to help the effectiveness of problem workmanship, one of them is in geometry materials [14]. Because the results of problem-solving are not only spontaneously create from memorization by students and it should be supported with the right concept [15]. Meanwhile, Purwanti also explained in his research that the reasoning ability of student analogies in geometry materials still shows a low category, resulting from students' inability to solve math problems [16]. Based on differences in analogy reasoning skills, cognitive styles, and mathematical abilities in students, the question in this study is "How does the reasoning of a student analogy be reflective in solving geometry problems?".

2. Methodology
This research was a qualitative descriptive approach. Creswel mentioned this research was conducted to explore previously unknown circumstances to produce new circumstances [17]. Subject determination techniques use systematic sampling techniques. This research material focused on curved side space building material (BRSL). The research was begun by providing matching familiar figure tests (MFFT) to determine the reflective cognitive style in students. Then followed by the Mathematical Ability Test (TKM) which validated by three experts namely 2 competent lecturers in their field and 1 experienced math teacher. Validation results were shown at a score of 4 which means it is Valid and worth. Then the test was given to 27 students of Grade X (ten) Islamic High School Sabilurrosyad, Malang. And then, 3 reflective students based on mathematical abilities (1 high-ability student, 1 medium-ability student, and 1 low-ability student) were selected to be given the Curved Side Room Build Analogy Reasoning Test (TPABRSL) which were validated by a previously mentioned expert with validation results being on a score of 4 which means Valid and worth. The test conducted in writing then followed by an interview to find a deepen reasoning analysis of the student analogy. According to Tohir et al. commonly, the data presentation in qualitative research is narrative text [18] [19]. But in this research, the presentation of data was done by collecting data, reducing data, presenting data then drawing conclusions. The stage of reasoning analogy that had been seen was Encoding, Inferring, Mapping, and Applying which has been presented [20].
3. Result and Discussion

Based on the result of that data analysis test and interview it showed that student with a high reflection skill (SRT) in performing the following stages of analogy reasoning

| Analogical Reasoning Stages | Indicator | Test | Interview |
|-----------------------------|-----------|------|-----------|
| **Encoding**                | Identification of traits or structures on known issues and target issues | SRT can understand the problem and can write down the concept of the source correctly | SRT can properly mention the concept of the question with correctly and confident |
| **Inferring**               | Relationship search which found on known issues | SRT can see similarities in form between source issues and target issues | SRT can mention the similarity of the question form and can also look for a relationship between the source problem and the target problem. |
| **Mapping**                 | Searching the same relationship between known issues and target issues | SRT can use the same concept to solve target problems with the same pattern as the source problem | SRT can explain that when working on a target issue he remembers the same workmanship process as the source problem. |
| **Applying**                | Selection of the right concepts to answer between known problems with target issues | SRT can select and apply the right concept to resolve questions on source issues and target issues. |

Based on Table 1 above, it shows that the SRT had mastered the material that had been studied before so that students can identify the question precisely and know the concept of BRSL correctly. Vakil et al. explained that a person who can identify the characteristics of the source problem
According to the one remembered, then the encoding stage can be skipped by that person [20]. This evidenced by the interview and concludes that SRT understands the basis of BRSL to be used to solve the problem. Thus, SRT performs the encoding stage. The SRT can do similar forms between source problems and target problems. Clement explained that a person who can find a link between the source problem and the target problem, then the inferring stage can be bypassed by the person [21]. This was done with the seriousness of students in following the learning of the previous material namely flat build, so that still embedded concept despite having passed with a long time. Thus, SRT performs the inferring stage. The SRT can search for the same relationship between the source problem and the target issue. Mozzer & Justi explained that a person who can see the relationship between the source problem and the target problem, then the mapping stage can be passed by the person [22]. This evidenced by the mastery of the concept between the source problem and the target problem and can solve it in detail with the illustration of the image presented in the answer result. Thus, SRT performs mapping stages. The SRT can choose the right concept to answer the target problem by using the concept of source problems. Kapon & DiSessa explained that a person can choose and apply the answer, then the applying stage can be skipped by that person [23]. This evidenced by the explanation of the relationship in the interview then SRT applied it correctly. Thus, SRT performs the applying stage.

Based on the results of analyst test data and interviews it is known that reflective students with the moderate ability (SRS) in conducting the following stages of analogy reasoning can be described in table 2 below.

| Analogue Reasoning Stages | Indicator | Test | Interview |
|---------------------------|-----------|------|-----------|
| Encoding                  | Identification of traits or structures on known problems and target issues. | SRS can understand the question, mention and write keywords on the question. | SRS can correctly mention the concept of the source problem. |
| Inferring                 | A search the relationship on a known issue. | SRS can infer the relationship between known numbers by performing calculations according to the concepts that have been studied before. | SRS explains the question link in the question. |
| Mapping                   | Search for the same relationship between known issues and target issues. | SRS writes the similarities incomplete. | SRS is only able to analyze solution completion with the final concept. |
| Applying                  | Choose the right concept to answer between known problems and target issues. | SRS cannot apply the right concept, only able to write down with known numbers on the question. | |

Based on Table 2 above, it shows that the SRS can understand the question, mention and write keywords on the question. Vakil et al. explained that a person who can identify the characteristics of the source problem according to the one remembered, then the encoding stage can be skipped by that person [20]. This evidenced from the interview SRS can correctly mention the concept contained in the source problem. Thus, SRS performs the encoding stage. The SRS can infer known problems by performing calculations according to the concept related Mozzer & Justi explained that a person who can find a relationship between the source problem and the target problem, then the inferring stage can be skipped by the person [22]. This evidenced by the efforts that SRS made to explain appropriately. Thus, SRS performs the inferring stage. The SRS may write similarities but incompletely. This evidenced by the lack of mastery of concepts so that SRS was only able to analyze without using clear mapping. Kapon & DiSessa
explained that a person who cannot see the relationship between the source problem and the target problem, then the mapping stage cannot be passed by that person [23]. Thus, SRS does not perform mapping stages. The SRS cannot implement the right concept to answer the target problem. This evidenced by the analysis of solution completion with the final concept on the right theorem. Clement explained that a person who cannot choose and apply the alleged answer, then the applying stage cannot be passed by that person [21]. Thus, SRS does not perform the applying stage.

Based on the results of analyst test data and interviews it is known that reflective students with the moderate ability (SRR) in conducting the following stages of analogy reasoning can be described in table 3 below.

**Table 3. SRR Analogy Reasoning Stage Analysis**

| Analogical Reasoning Stages | Indicator | Test | Interview |
|-----------------------------|-----------|------|-----------|
| **Encoding**                | Identification of traits or structures on known issues and target issues | SRR can understand the problem and can understand the difference in images on the problem. | SRR can correctly mention what is known about the source |
| **Inferring**               | Relationship search which found on known issues | SRR cannot infer the concept of a relationship but is wrong in calculating the results | SRR cannot mention the similarity of the question form. |
| **Mapping**                | Search for the same relationship between known issues and target issues. | SRR cannot analyze the image as a source problem, so to resolve the problem the target cannot search for the relationship correctly | SRR cannot map what is in question. |
| **Applying**               | Selection of the right concepts to answer between known problems with target issues | SRR cannot choose the concept properly, due to minimal knowledge | |

Based on Table 3 above, it shows that the SRR can understand and mention source problems even if it is incomplete. Vakil et al. explained that a person who can identify the characteristics of the source problem according to the one remembered, then the encoding stage can be skipped by that person [20]. Thus, SRR performs the encoding stage. The SRR cannot infer the concept of relationships and is wrong in calculating the results. This evidenced from interviews stating that SRR was not serious in following the learning in the previous material namely flat build, so it was not an embedded concept in a long time. Clement explained that a person who cannot find a link between the source problem and the target problem, then the inferring stage cannot be skipped by that person [21]. Thus, SRR does not perform inferring stages. The SRR cannot search for the same relationship between the source problem and the target issue. This evidenced by SRR not being able to analyze the image on the source issue, so to resolve the target problem. Mozzer & Justi explained that a person who cannot see the relationship between the source problem and the target problem, then the mapping stage cannot be passed by that person [22]. Thus, SRR did not perform mapping stages. The SRR cannot choose the concept properly, due to minimal knowledge. This evidenced by SRR's difficulty in giving arguments because understanding less-than-maximal concepts, as well as the selection of words at the time of the interview, made it difficult to study. Kapon & DiSessa explained that a person who cannot choose and apply the answer, then the applying stage cannot be passed by that person [23]. Thus, SRR does not perform the applying stage.

From the above discussion, we can conclude achievements that have been made at each stage of analogy reasoning in table 4 as follows.
Tabel 4. Achievement of Analogy Reasoning Stage

| Tahap Penalaran Analogi | SRT | SRS | SRR |
|------------------------|-----|-----|-----|
| Encoding               | √   | √   | √   |
| Inferring              | √   | √   | ×   |
| Mapping                | √   | ×   | ×   |
| Applying               | √   | ×   | ×   |

Highly qualified reflective students (SRT) have met all the criteria of analogy reasoning stages namely encoding, inferring, mapping, and applying stages. Then, reflective students with moderate ability (SRS) were only able to meet the two criteria of analogy reasoning stage, namely encoding stage, and inferring. Meanwhile, reflective students with a low ability (SRR) were only able to meet one criterion, namely encoding stage. This difference occurs due to inappropriate procedure and incorrect use of concepts.

4. Conclusion

Based on the results of analysis and discussion, the conclusions of this study were: (1) at the encoding stage, high, medium, and low reflective students can understand source problems and target problems based on the learning experience that had been skipped. (2) at the inferring stage, reflective students with high ability and moderate ability were performing using procedural concepts. While reflective students with low ability can only conclude without using basic concepts that had been studied so that there was an error in doing calculations. (3) at the mapping stage, high ability reflective students can find and explain the relationship of concepts and can draw conclusions from several ideas. While students reflective of moderate and low abilities were simply write down incomplete relationship similarities and perform incorrect calculations due to lack of understanding of concepts that had been studied before. (4) at the applying stage, high ability reflective students can apply the right relationships to solve the problem. This is in contrast to reflective students with moderate and low abilities.

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