Optimization of multiple quality characteristics for injection moulded polyamide helical gear via integration of Taguchi method and Grey relational analysis

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Abstract. Considering the great importance of injection moulding in plastic gear manufacturing, it is momentous to effectively control all the influential factors in the plastic injection moulding industry to improve the quality characteristics of the final gear part. As plastic materials exhibit extremely convoluted properties, the complexity of the injection moulding process makes it very challenging to attain the desired gear part properties. Since the intricate injection moulding process produces a wide range of parts with complex shapes within very narrow limits of tolerances, requires a great effort in order to keep the quality characteristics of moulded gears under control. In fact, the optimum properties of the plastic material cannot be achieved even with the most innovative part and mould design, and become meaningless without optimized processing parameters during the gear manufacturing. Therefore, the aim of this study is to propose the integration of Taguchi method/Grey relational analysis optimization approach in designing the gear part, setting up processing parameters, and selecting a suitable material for a helical gear via numerical simulation. The findings implied that an experimental design based on the integration of numerical simulation and Taguchi/Grey relational analysis is capable to enhance the multi-quality characteristic of the helical gear.

1. Introduction
To date, plastics have entrenched within the spectrum of modern materials. The versatility, variability, self-lubrication, cost efficient, light-weight, anti-corrosion and low maintenance of plastic allow them to be tailored to meet very specific technical requirement best suited for gear application in particular. Plastic gears are continuing to displace metal gears in a variety of applications [1-3]. The evolution of plastic gears in power and motion transmission application manifests itself in consumer electronic items, including copiers, printers, scanners, and washing machines. The resourcefulness of plastic materials has encouraged manufacturers of automotive components to use plastic gears, particularly in windshield wipers, power car seats and windows, for more light weight and cost-effective drive-train
Plastic gears can be produced by milling or hobbing, or alternatively, by injection moulding. With the continuous development of technology, plastic injection moulding has become one of more economical methods of mass production needed to meet the rapidly rising market demand for plastic gearing in various applications. Injection moulding has the highest efficiency, largest yield, and highest dimensional accuracy among all the processing methods. It is easy to actualize the automatization in injection moulding process for mass-producing plastic part with complex geometry. There are many types of plastic gear including helical gear which is widely used in application that demand silent and vibration free. In contrast with other type of gears, manufacturing the helical gear by injection moulding required an intrinsic task in order to obtain the accurate dimensional stability and good quality of the final part produced. The complexity of injection moulding process creates a very intense effort to keep the quality characteristic of helical gear under control. Generally, injection moulding process is a cyclic process which consist of four significant phases: filling, cooling, packing and ejection. During injection moulding process, the material selection, part and mould designs, and the processing parameters interact to determine the quality of plastic gear. Inappropriate material selection, bad part and mould design, or improper setting of processing parameters could negatively affect the final mechanical properties or the aesthetic appearance of moulded helical gear. Therefore, it is of utmost importance to effectively control all the influential factors in the plastic injection moulding industry to improve the quality characteristics of the final gear part. However, a fast and reliable optimization approach is imperatively needed for substituting the trial and error method which more rely on moulder or machine operators experience as well as depending on manual books to overcome the quality problem arise during the injection moulding process [7-9].

Comprehending to this fact, the aim of this study is to propose the integration of Taguchi method and Grey relational analysis (GRA) optimization approach in designing the gear part and setting up processing parameters, as well as selecting a suitable material for a helical gear via numerical simulation. The optimal set of multiple factors including different gate location and fibre types, as well as variation in processing parameters including melt temperature, injection time, injection pressure, filling pressure, packing time, cooling time and cooling liquid temperature was also determined in this study. The significant factors that influence the multi-quality characteristics of the moulded helical gear will be another aspect to be considered in this study.

2. Methodology
A helical gear with module 1.7, pressure angle of 20° teeth number of 20, face width of 10mm and helix angle of 120° was designed in Solidworks and then was used as a model, using a 3D mesh type with the aspect ratio maximum of 8.49, match percentage of 80.8% and reciprocal percentage of 85.4%. Figure 1 shows the geometry and specification of the helical gear. In this study, the Moldflow Plastic Insight (MPI) was used to conduct the numerical simulation for the 3D mesh helical gear model. The robust parameter design of Taguchi method and GRA were integrated in conducting the simulation and the overall procedures is illustrated in Figure 2 and will be furthered explain in following section.

![Figure 1. Geometry and specification of the helical gear](image)
2.1. Determination of quality characteristics

MPI is able to provide analyses on immesurable quality characteristics in real practice of injection moulding such as shrinkage, deflection or also known as warpage, weld line, air traps, shear stress, sink index and etc. In this study, shrinkage, deflection and sink mark are given the momentous priority to signify the quality characteristics of the helical gear. Due to the complexity of helical gear design, dimensional stability of the moulded gear will be a challenging task in producing a good quality of plastic helical gear. The dimensional stability of the part could be determined directly from the shrinkage and deflection analysis. On the other hand, sink index is an indication of the potential shrinkage due to a hot core. Higher sink marks, index value shows higher potential shrinkage. Therefore, shrinkage, deflection and sink index were determined as quality characteristics that present the objective of conducting the simulation runs.

2.2. Selection of influential factors

Several control factors were selected in this study including different gate types, material selection as well as processing parameters. The processing parameters that will be considered in this work including melt temperature, injection time, injection pressure, filling pressure, packing time, cooling time and cooling liquid temperature. The selected control factors and their levels are shown in Table 1.

| Column | Factor                  | Level 1         | Level 2               | Level 3         |
|--------|-------------------------|-----------------|-----------------------|-----------------|
| A      | Gate type               | Side            | Diaphragm             | Multi Pin       |
| B      | Fiber type              | Unfilled        | 15% Glass             | 15% Carbon      |
| C      | Melt Temperature (°C)   | 230             | 260                   | 290             |
| D      | Injection Time (s)      | 0.4             | 1.4                   | 2.4             |
| E      | Injection Pressure (MPa)| 95              | 115                   | 135             |
| F      | Filling Pressure (%)    | 60              | 80                    | 100             |
| G      | Filling Time (s)        | 0.4             | 1.4                   | 2.4             |
| H      | Cooling Time (s)        | 10              | 15                    | 20              |
| I      | Cooling liquid temperature (°C) | 20 | 25 | 30 |
2.3. Selection of orthogonal array (OA)
There are nine factors in total with three levels each. Each three-level factor has two DOF (DOF = number of levels - 1). The total DOF required was 18. The total of DOF of selected OA should be greater than or least equal to the total DOF of studied factors in the Taguchi method. Therefore, a L27 OA was selected in conducting the simulation.

2.4. Analysis on results by GRA
GRA is used to be integrated with Taguchi method in this study to analyze multi-quality characteristics for the helical gear including shrinkage, sink mark and deflection. The procedures of implementing GRA in analysing the results are as follows:

Step 1: Generation of raw data
Generation of raw data is a pre-process that convert the original data into comparable data. The theory of GRA is which Grey is a color that stand nowhere between white and black. Thus, the quality characteristic which is shrinkage, sink mark and deflection data are stand between 0 and 1. The normalization can be performed by using formula:

The larger the better
$$x_i^+(k) = \frac{x_i^{(o)}(k) - \min x_i^{(o)}(k)}{\max x_i^{(o)}(k) - \min x_i^{(o)}(k)} \quad (1)$$

The smaller the better
$$x_i^-(k) = \frac{\max x_i^{(o)}(k) - x_i^{(o)}(k)}{\max x_i^{(o)}(k) - \min x_i^{(o)}(k)} \quad (2)$$

The Nominal the better
$$x_i^0(k) = 1 - \frac{|x_i^{(o)}(k) - x_i(k)|}{\max x_i^{(o)}(k) - x_i(k)} \quad (3)$$

Where $x_i^+(k)$ is the value after Grey relational generation (data pre-processing), $\max x_i^{(o)}(k)$ is the largest value of the original sequence $x_i^{(o)}(k)$, $\min x_i^{(o)}$ is the smallest value of the original sequence $x_i^+(k)$ and $x_i^+(k)$ is the desired value.

Step 2: Computation of Grey relational coefficient of response variable
Following the data pre-processing, a Grey relational coefficient is calculated to express the relationship between the ideal and actual normalized experimental result. The Grey relational coefficient can be expressed as follows:

$$\xi_i(k) = \frac{\Delta min + \xi \Delta max}{\Delta _{0i}(k) + \xi \Delta max} \quad (4)$$

$$0 < \xi [x_i^+(k), x_i^+(k)] \leq 1$$

$\Delta _{0i}(k)$ is the deviation sequence of the reference sequence, $x_i^+(k)$ and comparability sequence, $x_i(k)$, $\xi$ is the distinguishing coefficient $\xi \in [0,1]$. 
Step 3: Computation of Grey relational grade

After obtaining the Grey relational coefficient, its average is calculated to obtain the Grey relational grade. The Grey relational grade is defined as follows:

$$
\gamma_i = \frac{1}{n} \sum_{k=1}^{n} \xi_i(k)
$$

Where, $\xi_i$ is the Grey relational coefficient of response variable and $n$ is the total number of response variable.

3. Results and discussion

3.1. Development of GRA

In this study, shrinkage, deflection and sink mark are considered as quality characteristics for the simulated helical gear. These quality characteristics are continuous and non-negative, and can be recognized as the smallest-the-better type. The results of these multiple quality characteristics are normalized to a unit value, ranging from 0 to 1 by using equation (1). The result from normalization were converted to deviation by subtracting the maximum normalization value with the normalized result and Grey relational coefficient (GRC) were then determined by using equation (4). Once the GRC was obtained for each trial, then the data was used in order to obtain Grey Relational Grade (GRG) by averaging the coefficient for three quality result for each trial. The GRG was determined from equation (5). All the results of normalization data, GRC and GRG for each trial is tabulated in Table 2.

| Trial | Shrinkage | Deflection | Sink Mark | Shrinkage | GRC | Deflection | Sink Mark | GRG |
|-------|-----------|------------|-----------|-----------|-----|------------|-----------|-----|
| 1     | 0.420     | 0.805      | 0.584     | 0.463     | 0.720 | 0.546      | 0.575     |
| 2     | 0.420     | 0.845      | 0.966     | 0.463     | 0.763 | 0.936      | 0.720     |
| 3     | 0.420     | 0.847      | 1.000     | 0.463     | 0.766 | 1.000      | 0.742     |
| 4     | 0.670     | 1.000      | 0.258     | 0.602     | 1.000 | 0.403      | 0.667     |
| 5     | 0.703     | 0.340      | 0.628     | 1.000     | 0.431 | 0.686      |           |
| 6     | 0.703     | 0.401      | 0.628     | 0.999     | 0.455 | 0.693      |           |
| 7     | 0.582     | 0.306      | 0.545     | 0.891     | 0.419 | 0.618      |           |
| 8     | 0.571     | 0.436      | 0.538     | 0.894     | 0.470 | 0.633      |           |
| 9     | 0.565     | 0.465      | 0.535     | 0.900     | 0.483 | 0.639      |           |
| 10    | 0.207     | 0.510      | 0.387     | 0.339     | 0.505 | 0.410      |           |
| 11    | 0.201     | 0.442      | 0.385     | 0.333     | 0.473 | 0.397      |           |
| 12    | 0.216     | 0.505      | 0.390     | 0.334     | 0.502 | 0.408      |           |
| 13    | 0.489     | 0.082      | 0.494     | 0.487     | 0.353 | 0.444      |           |
| 14    | 0.491     | 0.454      | 0.495     | 0.478     | 0.333 | 0.435      |           |
| 15    | 0.485     | 0.049      | 0.493     | 0.493     | 0.344 | 0.443      |           |
| 16    | 0.998     | 0.616      | 0.842     | 0.996     | 0.566 | 0.760      | 0.773     |
| 17    | 1.000     | 0.807      | 0.993     | 0.542     | 0.722 | 0.751      |           |
| 18    | 1.000     | 0.828      | 1.000     | 0.544     | 0.744 | 0.762      |           |
| 19    | 0.000     | 0.813      | 0.333     | 0.771     | 0.728 | 0.610      |           |
| 20    | 0.009     | 0.842      | 0.335     | 0.779     | 0.760 | 0.624      |           |
| 21    | 0.006     | 0.160      | 0.335     | 0.730     | 0.373 | 0.479      |           |
| 22    | 0.953     | 0.915      | 0.918     | 0.968     | 0.594 | 0.825      |           |
| 23    | 0.955     | 0.663      | 0.918     | 0.968     | 0.597 | 0.827      |           |
| 24    | 0.951     | 0.605      | 0.912     | 0.958     | 0.559 | 0.809      |           |
| 25    | 0.776     | 0.637      | 0.691     | 0.884     | 0.579 | 0.717      |           |
### 3.2. Main effects analysis and optimization of MQCI

For better understanding in identifying the optimal controlled factors, main effect analysis is exploited to examine the optimum performance of GRG obtained in Table 2. Generally, main effect analysis is the effect of response experiment (dependent variable) on the averaging of controlled factors (independent variable) according to their levels. The mean response at each level of controlled factors is computed by averaging the performance values of each factor at different levels. The main effects analysis is plotted in Figure 3.

![Main effect analysis plot](image)

**Figure 3.** Main effect analysis plot

Referring to Figure 3, it is clearly shown that the multiple quality characteristics which in this case indicated by GRG of the simulated helical gear are greatly influenced by the variation in gate types, material selection as well as adjustments of the processing parameters. Considering that the GRG represents the level of correlation between the reference and the comparability sequences, a larger GRG indicates that the comparability sequence exhibits a stronger correlation with the reference sequence. In other words, a larger GRG will result in better multiple quality characteristics which is in this case are shrinkage, deflection and sink mark.

In this study, three different gate types including side gate, diaphragm gate and multiple pin gate are being selected to examine the impact of different gate type on shrinkage, deflection and sink index of the helical gear. From Figure 3, it seems that with multiple pin gate system, the GRG value is the highest which represent the lowest shrinkage, deflection and sink index of the helical gear. Meanwhile, polyamide 12 (PA12) without any addition of filler assigned as B1, PA12 with 15% glass fiber assigned as B2 and PA12 with 15% carbon fiber assigned as B3 were selected as material for the simulated helical gear. Polyamide (PA) are known to have characteristics of strength, stiffness, and toughness which are present under unmodified, toughened, and reinforced conditions. These plastic materials also exhibit outstanding wear resistance, low coefficients of friction, excellent electrical properties, and chemical resistance. This has earned PAs a reputation as engineering plastics. PA gears operate at temperatures of up to 175 °C for glass-reinforced grades and up to 150 °C for unfilled ones. Some PAs are hygroscopic (absorb moisture), which has a negative effect on their strengths. These plastic materials, whose dimensions change as they absorb moisture and lubricants, are unsuitable for precision gears. In addition, PAs have poorer dimensional stabilities than other engineering plastics. The review from [10] elucidated the complexity of the PA materials' mechanical properties, which depend on temperature and humidity. Terashima et al. [11] showed that PA materials lose their tensile strength within the range of 5–10% when exposed to a temperature increase of 10 °C. As seen in

|   |   |   |   |   |   |   |
|---|---|---|---|---|---|---|
| 26 | 0.756 | 0.932 | 0.613 | 0.672 | 0.881 | 0.564 | 0.705 |
| 27 | 0.776 | 0.928 | 0.516 | 0.691 | 0.874 | 0.508 | 0.690 |
Figure 3, PA12 with 15% carbon fiber have the maximum GRG value which represent the better multi-quality characteristics of the helical gear. The reinforcement of PA12 with 15% carbon fiber could minimize the problem of shrinkage, deflection and sink index in the helical gear.

The variation in processing parameters setting also greatly influences the multi-quality characteristics of the helical gear. Figure 3 shows that lower melt temperature imparts good quality characteristics in terms of shrinkage, deflection and sink index in the helical gear. This is because higher melt temperature reduces the molecular orientation due to the molecular relaxation after the cessation of the flow and imparts the quality of the final part produced.

The best combination of factors and levels could easily be obtained from the main effects analysis by selecting the level of each factor with the highest GRG value. As a result, the optimal factors which statistically result in the minimum shrinkage, deflection and sink index for the simulated helical gear, are predicted to be A3B3C1D2E2F3G3H2I3. As seen in Figure 3, the optimal factors represent a gear type of multi pin gate, material of PA12 with 15% carbon fiber, 230 °C melting temperature, 1.4s injection time, 115 MPa injection pressure, 2.4 s filling time, 100% of filling pressure, 15s of cooling time and 30°C of cooling liquid.

3.3. Analysis of variance (ANOVA)
The Taguchi method not only can generate the response plots to illustrate the quality changes caused by varying each factor, but it can also perform the ANOVA to enable engineers to quantitatively estimate the relative contribution of each factor to the overall measured response. As there are three quality characteristics including shrinkage, deflection and sink index involved in this study, the ANOVA is performed for multiple quality characteristics towards the GRG. ANOVA is analyzed by computing the quantities degrees of freedom, (f), sum of squares (S), variance (V), F-ratio and percentage contribution of the GRG (%) and listed in Table 3.

| Column | Factor               | DOF | Sum Square | Variance | F-Ratio | Percentage,% |
|--------|----------------------|-----|------------|----------|---------|--------------|
| A      | Gate Location        | 2   | 0.132      | 0.066    | 41.236  | 26.700       |
| B      | Material Selection   | 2   | 0.100      | 0.050    | 31.418  | 20.343       |
| C      | Melt Temperature     | 2   | 0.209      | 0.105    | 65.530  | 42.430       |
| D      | Injection Time       | 2   | 0.021      | 0.011    | 6.614   | 4.283        |
| E      | Injection Pressure   | 2   | 0.001      | 0.001    | 0.374   | 0.242        |
| F      | Filling Pressure     | 2   | 0.002      | 0.001    | 0.691   | 0.447        |
| G      | Filling Time         | 2   | 0.013      | 0.006    | 3.927   | 2.543        |
| H      | Cooling Time         | 2   | 0.001      | 0.001    | 0.337   | 0.219        |
| I      | Cooling Liquid       | 2   | 0.001      | 0.001    | 0.314   | 0.204        |
| ERROR  |                      | 8   | 0.013      | 0.002    | -       | 2.590        |
| TOTAL  |                      | 26  | 0.493      | -        | -       | 100          |

In determining the relative contribution of each factor in ANOVA, the value of F-ratio of the factors which are greater than the F-table of specific confidence level is statistically considered as significant [12]. In this study, for level of significance (95% confidence), the obtained result F.05(2,8) = 4.459. Thus, in this case, any factor with the F-ratio higher than 4.459 is considered as significance. Referring to Table 3, out of nine factors, only four factors including melt temperature, gate location, material selection and injection time are considered as significant as their F-ratios are greater than the threshold values of F.05 (95% confidence level).

4. Conclusion
The integration of Taguchi method/GRA and numerical simulation offers a fast, reliable and systematic solution in determining the optimal and significant factors in producing a good quality of helical gear part. Injection moulding has been a challenging process for many manufacturers and researchers to produce a gear in particular to meet requirements at the lowest cost. Numerical
simulation aids the designers and engineers as it is capable to simulate the scenarios without carrying out in the real practices whereas the integration of Taguchi/GRA optimization approach offers the advantages in reducing the simulation experiments and analysing the results to solve multi-quality characteristic problem in injection moulding process with systematic way.

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