Delocalization in two dimensional disordered Bose systems and depinning transition in the vortex state in superconductors.

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(Dated: March 22, 2022)

We investigate two-dimensional Bose system with the long range interactions in the presence of disorder. Formation of the bound states at strong impurity sites gives rise to an additional depletion of the superfluid density \( n_s \). We demonstrate the existence of the intermediate superfluid state where the condensate and localized bosons present simultaneously. We find that interactions suppress localization and that with the increase of the boson density the system experiences a sharp \textit{delocalization crossover} into a state where all bosons are delocalized. We map our results onto the three dimensional system of vortices in type II superconductors in the presence of columnar defects; the intermediate superfluid state maps to an intermediate vortex liquid where vortex liquid neighbors pinned vortices. We predict the \textit{depinning transition} within the vortex liquid and \textit{depinning induced} vortex lattice/Bose glass melting.

Effects of disorder on strongly correlated systems are central to contemporary condensed matter physics. Disordered Bose systems are of special interest. Superfluid \(^4\)He and \(^3\)He remain one of the favorite and important experimental tools for studies macroscopic quantum effects. Moreover, quantum mechanical mapping that relates quantum Bose system to thermodynamics of superconducting vortices in the space of the one dimension up, motivates research of Bose fluids in random environment by the hope for further progress in vortex physics.

There has been a significant recent advance in understanding disordered Bose systems started by who discussed a continuum model of the dilute interacting Bose gas in a random potential. The proposed model described the quasiparticle dissipation and depletion of superfluidity at zero temperature. Later we developed a systematic diagrammatic perturbation theory for the dilute Bose gas with weak disorder that enabled us to extend the description of superfluid thermodynamics on finite temperatures. Disorder corrections to the thermodynamic potential and the disorder-induced shift of the condensation temperature resulting from disorder scattering of quasiparticles were found. In particular, we obtained that the superfluid density decreases monotonically with the temperature.

Yet there are many questions that remain open. First, more close examination of models shows that the described depletion of superfluid density reflects only \textit{scattering} of zero energy quasiparticles by random potential, while possible contribution from the quasiparticles bound states was overlooked. Further, the fact that some quasiparticles leave condensate and get localized suggests the existence of the novel \textit{intermediate} state where superfluid and localized components present simultaneously. This poses the next question: if this intermediate state exists, what part of the phase diagram does it cover? In other words the question is: would even arbitrarily weak disorder localize the part of the condensate, or else this localization effect \textit{vanishes} if disorder is too weak or if the Boson interactions are sufficiently strong?

In this Letter we examine the behavior of 2D interacting Bose-system subject to strong disorder having in mind application of our results (via quantum mechanical mapping) to the system of 3D superconducting vortices in type II superconductors pinned by columnar defects. It is well established that at low temperatures where the pristine vortex structure would form Abrikosov lattice, columnar defects cause formation of strongly pinned Bose glass. At the melting line \( B_m(T) \), Bose glass melts into a vortex liquid. In the related quantum mechanical 2D system vortex liquid corresponds to a superfluid Bosons. The depletion of the superfluid density by disorder corresponds to enhancement of the average tilt modulus of the related vortex system. Moreover, one can expect that this stiffening is accompanied by the change in transport properties: the part of the vortices remain pinned and does not participate in transport. This may be interpreted as the intermediate vortex phase where both vortex liquid and pinned vortices coexist.

We focus on the most interesting case where the applied magnetic filed is not too close to \( H_{c1} \) such that distance between the vortices is less than their interaction range, which coincides with the magnetic field penetration length \( \lambda \). We show that upon increasing the temperature (or, equivalently, magnetic field ), the system undergoes a sharp crossover between the phase containing vortices pinned by the columnar defects and the phase where all vortices are delocalized and find the expression for the crossover line:

\[
\rho = \frac{e \kappa^4}{\lambda^2} \ln(\lambda/L) \exp \left[ -\frac{\sqrt{2} \lambda^2}{\kappa \tau_T} \right],
\]

where \( \rho \) is the density of vortices, \( \kappa = \lambda/\xi \gg 1 \) is the ra-
tio of the magnetic penetration and coherence lengths, $r_c$ is the characteristic size of the columnar defect, $\epsilon$ is the anisotropy parameter, the length $L$ is determined by the temperature $T$ as $L_T = (\Phi_0/4\pi)^2/T$, and $\xi$ is a numerical coefficient of the order one. The localization length $L$ that enters the logarithmic factor is defined below in Eq. (10). The corresponding phase diagram is shown on Fig. 1 where the delocalization line is drawn along with the vortex lattice melting line. In what follows we formulate our model in terms of vortex configuration, establish the existence of the delocalization crossover, and derive equation (1) for the crossover line.

Model. A free energy of the vortex configuration can be written as a functional of the vortex displacements $r_i(z)$ as

$$F = \int_0^L dz \left[ \sum_i \frac{\epsilon_1}{2} \left( \frac{\partial r_i}{\partial z} \right)^2 + U(r_i) + \frac{1}{2} \sum_{i,j} v(r_{ij}) \right]$$

(2)

where the interaction between the vortices is

$$v(r) = \epsilon_0 K_0(r/\lambda),$$

(3)

with $\epsilon_0 = (\Phi_0/4\pi \lambda)^2$ and $K_0$ being the Bessel function. The vortex tension $\epsilon_1$ is related to the anisotropy parameter $\epsilon$ and $\epsilon_0$ as $\epsilon_1 = \epsilon^2 \epsilon_0$ and the disorder potential $U(r)$ can be written as a sum of the potential wells representing the columnar defects

$$U(r) = \sum_i u(r - r_i),$$

(4)

where $r_i$ is the coordinate of the $i$-th defect.

Statistical mechanics of vortices can be mapped onto the quantum mechanical problem of interacting two-dimensional bosons described by the Hamiltonian

$$\hat{H} = \int d^2r \begin{bmatrix} \hat{\psi}^\dagger(r) [\hat{\psi}^\dagger(r) \hat{\psi}(r)] + \frac{1}{2} \int d^2r_1 d^2r_2 \hat{n}(r_1) v(r_1 - r_2) \hat{n}(r_2) \end{bmatrix},$$

(5)

where $\hat{\psi}^\dagger, \hat{\psi}$ are boson creation and annihilation operators, $\mu$ is the chemical potential and $\hat{n}(r) = \hat{\psi}^\dagger(r) \hat{\psi}(r)$ is the density operator of the Bose gas. Results obtained in the Bose gas representation can be translated onto vortex language by the substitute $\hbar \rightarrow T, m \rightarrow \epsilon_1$. The energy of the Bose gas corresponds to the free energy per unit length of the vortex lattice. Keeping in mind the application to vortices we will make several assumptions about the form of a single potential well: We will consider temperatures $T \approx T_c$, in this regime pinning potential is given by

$$u(r) = \frac{\epsilon_0}{2} \frac{r_c^2}{r^2 + 2\xi^2},$$

(6)

where $\xi$ is the coherence length and $r_c$ is the scale that characterizes the size of the columnar defect. The strength of the pinning potential can be characterized by the dimensionless coefficient

$$\beta = r_c^2 \epsilon_1 \epsilon_0 / 2T^2.$$  

(7)

We will assume that pinning is weak such that $\beta \ll 1$. In this limit the solution of the Schrödinger equation

$$[-\nabla_r/2\epsilon_1 + u(r)] \hat{\psi}(r) = E_1 \hat{\psi}(r),$$

(8)

results in the pinning energy

$$E_1 \sim T^2 / \epsilon_1 \xi^2 e^{-1/\sqrt{T}}.$$  

(9)

The localization length of the quantum sate is related to the bound state energy as $L^2 \sim \hbar^2 / |E_1| m$, translating this relation on vortex language we define the localization length of the vortex as on the columnar defect as

$$L = \xi e^{1/2\sqrt{T}}.$$  

(10)

Further, we assume that the interaction between the bosons (vortices) is strong enough such that the bound state of the well can be only single occupied. This condition is satisfied when the energy of the double occupied state $E_2$ is much larger than the the absolute value of the energy of the single occupied state $E_1$. The interaction energy of the double occupied state is

$$E_2 = \frac{1}{2} \int d^2r_1 d^2r_2 \phi_1^\dagger(r_1) \phi_1(r_1) v(r_1 - r_2) \phi_2^\dagger(r_2) \phi_1(r_2),$$

(11)

where $\phi_1$ is the wave function of the bound state. In the case when the localization length $L$ is smaller than the interaction range $\lambda$ the energy $E_2$ can be estimated as

$$E_2 \sim \epsilon_0 \ln(\lambda/L),$$

(12)

where we have used the asymptotic of the Bessel function $K_0(x) \approx \ln(1/x)$. The condition of no double occupation can be now written as

$$\epsilon_0 \epsilon_1 \ln(\lambda/L) \gg (T/L)^2.$$  

(13)

In case of low concentration of defects we can consider the contributions of different potential wells separately, thus we end up with the problem of the interacting Bose gas and single potential well (or a single columnar defect placed in the vortex liquid). Leaving only one potential well we can easily diagonalize the non-interacting Hamiltonian and formulate the problem in terms of eigenfunctions of the noninteracting Hamiltonian

$$\hat{\psi} = \sum_k b_k \phi_k,$$

(14)

where $\phi_k$ are the eigenfunctions of the non-interacting Hamiltonian

$$[\hat{p}^2 / 2m + u(r)] \phi_k(r) = E_k \phi_k(r),$$

(15)
and $E_k$ is the energy of the $k$–th state. The bound state corresponds to the index $k = 1$. Although this state has the lowest energy, the Bose-condensation to this state cannot occur due to our assumption of no-double occupation. Thus, the condensation takes place to the extended state with the lowest energy (which we label with $k = 0$). The normalization conditions for eigenfunctions $\phi_k$ are as follows:

$$
\int d^2r \phi_k^* \phi_k = \begin{cases} V, & E_k > 0 \\ 1, & k = 1 \end{cases} \quad (16)
$$

with $V$ being the volume. In the absence of the current the basis can be chosen real, thus we will assume further on that $\phi^* = \phi$. The \psi operators that enter the Hamiltonian \[(14)\] can be presented as a sum of three terms

$$
\hat{\psi} = \phi_0 \hat{b}_0 + \phi_1 \hat{b}_1 + \sum_{E_k > 0} \phi_k \hat{b}_k, \quad (17)
$$

representing the condensate, the bound state, and all other states having energy larger than the condensate energy. The effective temperature of the Bose gas is related to the inverse physical length of the superconducting sample in the $z$ direction. We will assume that the size of the sample is microscopically large, this correspond to zero temperature limit for bosons. We will consider the most interesting case where the applied magnetic field is not too close to $H_{c1}$, such that the average distance between the vortices is smaller than the screening length $\lambda$. In this regime the mean field Bogolubov approximation, that we will use, is justified for the case of sufficiently high density of vortices. \[11\] Thus, the third term in Eq.\[(17)\] representing the particles that do not belong to the condensate is small and can be omitted in the leading order.

Now we turn to the analysis of the population of the bound state in the presence of the condensate. Inserting the representation Eq.\[(17)\] into the Hamiltonian \[(15)\] and keeping only two first terms in the representation \[(17)\] we get the effective Hamiltonian

$$
\hat{H}_{eff} = b_1^\dag (E_1 + \alpha - \mu) b_1 + \gamma (b_1^\dag b_1 + b_1^\dag b_1), \quad (18)
$$

where the terms $b_1^\dag b_1$ were not written due to no double occupancy condition. The coefficients $\alpha$ and $\gamma$ in the Hamiltonian \[(18)\] are

$$
\alpha = b_0^3 \int d^2 r_1 d^2 r_2 \phi_1^2 (r_1) v(r_1 - r_2) \phi_0^2 (r_2) + \phi_1 (r_1) \phi_0 (r_1) v(r_1 - r_2) \phi_1 (r_2) \phi_0 (r_2), \quad (19)
$$

$$
\gamma = b_0^3 \int d^2 r_1 d^2 r_2 \phi_0^3 (r_1) v(r_1 - r_2) \phi_1 (r_2), \quad (20)
$$

where the average value $b_0$ of the operator $\hat{b}_0$ is related to the condensate density $\rho_0$ as $b_0^2 = \rho_0$. The model \[(18)\] can be easily solved: Presenting the wave functions in the form

$$
\psi = a_0 |0> + a_1 |1>, \quad (21)
$$

with $a_0$ and $a_1$ being the amplitudes of the zero and single occupied states we find two eigenstates corresponding to the energy levels

$$
E_\pm = \frac{E \pm \sqrt{E^2 + 4\gamma^2}}{2} \quad (22)
$$

where $E = E_1 + \alpha - \mu$. We see that $E_+ > 0$ and $E_- < 0$ for any sign of the energy $E$, thus the state with energy $E_-$ is always occupied while $E_+$ is always empty. The occupation of the center is determined by the state with the lowest energy ($E_-$) and is given by

$$
n = a_1^2 = \frac{2}{4 + (E/\gamma)^2 + (E/\gamma)\sqrt{(E/\gamma)^2 + 4}}. \quad (23)
$$

One can easily see that $n \rightarrow 1$ when $E/\gamma \ll -1$ and $n \rightarrow 0$ when $E/\gamma \gg 1$. Thus the quantity $E/\gamma$ controls the occupation such that at $E = 0$ localization-delocalization crossover takes place. Making use of Eq.\[(19)\] and relationship between the chemical potential, condensate density and interaction $\mu = b_0^2 \nu_0$ where $\nu_0 = \int d^2 r v(r)$, for the parameter $E$ we find $E = E_1 + \delta E$ with

$$
\delta E = b_0^2 \int d^2 r_1 d^2 r_2 \phi_0 (r_1) v(r_1 - r_2) \phi_1 (r_2) \approx \rho L^2 c_0 \ln(\lambda/L) \quad (24)
$$

Now, using the relationship between the bound state energy and localization length, from the condition $E = 0$, we get the vortex density at the localization-delocalization crossover

$$
\rho = \frac{c T^2}{\epsilon_0 c_1 L^4 \ln(\lambda/L)}, \quad (25)
$$

where $c \sim 1$ is a numerical constant. Plugging in the localization length from Eq.\[(10)\] we arrive at the expression \[(11)\] for the delocalization crossover line which can be rewritten in terms of the depinning field for vortices as

$$
B_{dep} \simeq \Phi_0 \frac{2 \pi c T^2}{\epsilon_0 c_1 \xi^4 \ln(\lambda/L)} \exp \left( -\frac{T}{T_0} \right), \quad (26)
$$

where $T_0$ is the effective temperature dependent depinning energy $T_0 = r_r \sqrt{\epsilon_1 \epsilon_0 / 8}$. One can easily check that the assumed condition of no double occupancy \[(13)\] always satisfies near the crossover as long as $\rho L^2 < 1$.

As we see from Eq.\[(26)\], the occupation of the center changes continuously as long as "hybridization parameter" $\gamma$ is larger than zero. Therefore the phase boundary between the localized and delocalized phases should be viewed as a crossover rather than the true transition line. Note, however, that in case of finite concentration of defects the delocalization transition may result in significant change of the condensate density, that may induce the true superfluid-bose glass transition studied in \[(10)\].

While the ultimate type of the crossover remains an open question, experimentally it can appear very sharp
and undistinguishable from the true first order transition. Indeed, the sharpness of the crossover is controlled by the ratio of \( \gamma \) and \( \delta E \). Formula (20) determining \( \gamma \), should be corrected to include the screening of the potential by the surrounding Bose gas. In the dense limit the screening length is given by \( l_{sc} = \lambda (1/2 \pi \rho \epsilon^2 l_T^2)^{1/3} \). The parameter \( \gamma \) is estimated then as

\[
\gamma \approx \frac{\rho L \lambda^2 \epsilon_0}{l_T \epsilon \sqrt{2\pi}} \tag{27}
\]

and the ratio \( \gamma/\delta E \) becomes:

\[
\frac{\gamma}{\delta E} \sim \frac{\lambda^2}{(\lambda L \ln(\lambda/L))} \tag{28}
\]

and is always small due to the no-double occupancy condition [12]. We thus conclude that the crossover between the localized and non-localized phases is sharp.

We are now in a position to construct a phase diagram for the disordered Bose system. We use the corresponding 'superconductor vortex language': the schematic vortex phase diagram for the system with columnar defects is presented in Fig 1. Shown are the melting line of the pristine sample \( B_M^{(0)} \), and melting line \( B_M \) in the sample with the columnar defects shifted, as it should be, upwards as compared to \( B_M^{(0)} \). Note that because of the exponential dependence of temperature, \( B_{dep} \) drops much faster than \( B_M \) as temperature approaches \( T_c \). Thus the depinning line that lies in the vortex liquid domain at high sufficiently fields hits and then crosses the melting line \( B_M \), traversing then to \( T_c \) and crossing the pristine melting line at very low fields.

In the liquid phase the depinning line marks the crossover separating the intermediate state with the columnar defects occupied by vortices (i.e. vortex liquid coexists with the pinned vortices), and the phase where all vortices are delocalized from the columnar defects. In the strip confined between the \( B_M^{(0)} \) and \( B_M \) lines, \( B_{dep} \) marks melting of the vortex lattice. Indeed, at \( B_M^{(0)} < B < B_M \) lattice is stabilized by vortices pinned by columnar defects. As soon as depinning at \( B_{dep} \) occurs, columnar defects become inessential, and lattice looses its stability since in the absence of defects vortex liquid is a stable thermodynamic state above \( B_M^{(0)} \). We therefore discover a novel type of the vortex lattice melting, the *depinning induced melting*, that occurs in the interval \( B_M^{(0)} < B < B_M \).

This new kind of vortex lattice melting may well explain the origin of the low-field kink in the melting line that clearly indicates a switch between the different melting mechanisms. This kink has been observed in almost all experiments on the Bose glass melting [8], and was addressed specifically in the recent study of the melting of "porous" vortex matter [12]. Although these results obtained for pronounced vortex lines may be applied only with caution and many reservations to BSCCO, it is interesting to note that at temperatures around 80K the characteristic energy \( T_0 \) for BSCCO parameters can be estimated as \( T_0 \approx 5K \), close enough to that observed in [12].

In conclusion we have shown that in a disordered Bose system with the long range interactions an intermediate phase where both superfluid and localized bosons can exist. We demonstrated that a sharp delocalization crossover occurs with the increase of bosons density. We apply our results to description of a vortex system in type II superconductors in the presence of columnar defects and show that this delocalization crossover describes the depinning line separating two liquid phases, the intermediate phase containing pinned vortices and 'fully molten' liquid. We predict a new kind of the first order vortex lattice melting, depinning induced melting, at low fields.

It is a pleasure to thank S. Banerjee, A. Koshelev, D. Maslov, and E. Zeldov for illuminating discussions. This work was supported by the U.S. Department of Energy Office of Science through contract No. W-31-109-ENG-38.

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