The application of power quality signal de-noising based on the improved wavelet threshold function

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Abstract. The contradiction between de-noising and preserving the local features of the original signal simultaneously cannot be solved with the traditional wavelet threshold function (TF), and thus an improved combination of soft and hard threshold functions (TFs) of power quality signal de-noising is proposed in this paper. The improved wavelet TF is utilized to overcome the problem of “Pseudo-Gibbs phenomenon”, the deviation of wavelet coefficients, caused by the traditional wavelet TF, which fills a gap for the deficiency of wavelet soft and hard threshold de-noising, and with stronger adaptability. Finally, the de-noising tests of three typical signals of power quality are simulated based on MATLAB R2016a. Compared with the traditional wavelet TFs, the results obtained based on the proposed improved wavelet TF can guarantee better effectiveness and feasibility.

1. Introduction

With the extensive use of a large number of nonlinear and impact loads, the power quality of the power supply system has been seriously polluted and the electromagnetic environment of electrical equipment has been deteriorated. In addition, the highly automatic equipment and intelligent equipment with microelectronic control technology as the core are extremely sensitive and lack of anti-interference ability, which requires higher power quality. As a result, the problem of power quality has increasingly attracted the attention of the power apartments and power users. Therefore, how to detect various types of power quality events accurately, quickly and effectively is particularly important. However, in practical applications, the power quality signal obtained by the monitoring and data acquisition equipment may introduce random noise due to changes in internal and external conditions of the equipment operation. In addition, when measuring and channel transmitting data, the power quality signal may also be polluted by noise [1]. The existence of noise will worsen the effectiveness of disturbance detection and recognition methods, and sometimes even make them invalid [2]. Therefore, recently, the research on power quality signal denoising has attracted great attention.

The difficulty in denoising the power quality signal is to preserve the local characteristic information of the original signal while effectively removing the noise. In recent years, power quality signal de-noising has been researched extensively and deeply at home and abroad. A large number of denoising methods have emerged, such as linear filtering technology denoising, Gaussian filtering technology denoising, decision criterion of likelihood ratio denoising, EMD decomposition technology
denoising, wavelet threshold technology denoising, mathematical morphology method and singular value decomposition denoising, etc. [3-5], these methods have their own advantages and disadvantages, but they cannot overcome the power quality signal denoising difficulties. Linear filtering technology only has obvious smooth denoising effect for Gaussian noise, but it cannot achieve better denoising effect for common abrupt signal features, such as signal and high-frequency component different from Gaussian distribution, or even blur abrupt information so as to cause effective signal distortion. Gaussian filtering techniques require multiple iterations. For the decision criterion of likelihood ratio denoising, it is necessary to make a decision of the mutation point in advance to find the location of the mutation point. However, in the case of high noise intensity, it will be so difficult to determine the location of the mutation point, thereby affecting the denoising effect. EMD decomposition technology is mainly used to process nonlinear or non-stationary signals, and is not suitable for fixed parameter extraction of stationary signals. For the mathematical morphological method, because the actual pulse width is unknown, its morphological filter may mistake the pulse disturbance as noise filtering. The smoothing effect of the singular value decomposition denoising method is poor, and the noise content in the reconstructed signal is too large.

Given the deficiency of traditional wavelet of soft and hard TFs, related studies proposed a compromise TF, but the signal is still blurred at high frequencies after denoising. The improved TF proposed in [6], in specific applications, requires multiple tests to determine the value of the adjustment parameters for achieving better denoising effect. The improved TF proposed in reference [7] needs to optimize the parameters $\alpha$ and $\beta$ of the TF through particle swarm optimization algorithm, which requires a large amount of calculation. Given the above problems, an improved weighted TF combining soft and hard TFs is proposed in this paper, which fills a gap for the deficiency of soft TF at high frequency and gives full play to the advantage of hard TF at high frequency. The simulation results show the effectiveness and feasibility of the proposed improved TF.

2. Wavelet threshold denoising principle

The main basis of wavelet threshold denoising is that the energy distribution of the effective signal and noise after wavelet transform is different. After the time-frequency transform of the effective signal, the energy distribution is concentrated, regular and the wavelet coefficient is large. After the time-frequency transform, the noise is randomly distributed, irregular and the wavelet coefficient is small. In this way, an appropriate value can be selected as the threshold for dividing the effective signal and the wavelet coefficients after the wavelet transform. If the wavelet coefficient exceeds the threshold, denoising is performed and then used for signal reconstruction. If the coefficient is within the threshold, it is directly set to zero for denoising. In this way, the purpose of wavelet threshold denoising can be guaranteed.

Since the noise signal mixed in the power quality signal is a Gaussian white noise signal conforming to the Gaussian distribution, a one-dimensional noise signal model which shown in equation (1) can be established

$$g(t) = o_s(t) + e(t)$$  \hspace{1cm} (1)

Where $o_s(t)$ is the original electrical signal without noise, and $e(t)$ is a Gaussian white noise which obeys the normal distribution of $N(0,\sigma^2)$.

The basic flow chart of wavelet denoising is shown in Fig.1, and the detailed steps of wavelet threshold denoising can be found in reference [7], which will not be described here.

3. The selection of wavelet TFs and wavelet threshold

3.1. Selection of Wavelet TF

The soft and hard TFs are mainly included in the traditional wavelet, which can be expressed as equations (2) and (3), respectively.
In equations (2) and (3), $\omega_{j,k}$ is the wavelet coefficient, $\hat{\omega}_{j,k}$ is the estimated wavelet coefficient after denoising, and $T$ is the selected threshold.

The curves of denoising characteristic of the soft and hard TFs are illustrated in Fig.2 and Fig.3, respectively.

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**Figure 2.** Denoising characteristic curve of soft TF  
**Figure 3.** Denoising characteristic curve of hard TF

From the expressions and denoising characteristic curves of the two functions, it can be obtained that the hard TF is discontinuous, which will produce a breakpoint. Therefore, $\hat{\omega}_{j,k}$ has a breakpoint at the threshold $T$, and the signal reconstructed by $\hat{\omega}_{j,k}$ is also unstable. The signal oscillation will cause great hidden danger to the system and great harm to the power equipment in the power system. The soft TF is continuous, which can overcome the shortcoming of signal oscillation. The continuity of $\hat{\omega}_{j,k}$ calculated by soft TF is good. However, the soft TF can reduce the wavelet coefficient with a large absolute value, which makes the high frequency signal lost. And when $\hat{\omega}_{j,k} \geq T$, there is always a fixed deviation between $\hat{\omega}_{j,k}$ and $\omega_{j,k}$, which leads to the fuzzy edge of the signal, and the approximation degree between the reconstructed signal and the effective signal is lower.

### 3.2. Selection of wavelet threshold

The denoising effect is mainly hinges on the selection of the threshold, if the threshold is too high, the signal will be distorted, and if it is too low, the denoising is not complete. The traditional methods of threshold selection include sqrtwolog criterion, rigrsure criterion, heursure criterion and minimaxi criterion. Among the above four criterions, the minimaxi criterion and rigrsure criterion are relatively conservative in denoising, which are more suitable for the situation that the signal is concentrated and distributed in the low frequency. At this time, the two criterions are particularly suitable for the extraction of weak signals. The characteristic of using sqrtwolog criterion and heursure criterion to denoising is that the denoising is more thorough, but in the process of denoising, useful signals will be mistaken as noise signals. Combined with the above four threshold selection methods, Donoho and Johnstone proposed a more applicable threshold selection method, which is called unified threshold method, as expressed is equation (4):

$$T = \sigma \sqrt{2 \cdot \text{lg} n}$$

Because the threshold obtained by the unified threshold method is directly proportional to the variance of noise, it is especially suitable for the case of uncertain noise characteristics and distribution. During power quality signal de-noising, the noise signal is random, only according to the different
noise characteristics of the real-time transformation of the corresponding threshold value can get the ideal effect. Therefore, the unified threshold method is utilized in this paper.

3.3. Performance indicators for evaluating signal denoising effects

SNR (signal noise ratio) is generally selected to evaluate the denoising effect, and RMSE (root mean square error) can also be used to distinguish the denoising effect. Therefore, the indexes of SNR and RMSE are adopted to evaluate the denoising effect in this paper.

1) SNR

\[
SNR = 10 \log_{10} \left[ \frac{\sum g^2(n)}{\sum [g(n) - \hat{g}(n)]^2} \right] 
\]

where \(g(n)\) is the original signal and \(\hat{g}(n)\) is the reconstructed signal.

According to the definition of SNR, the higher the SNR is, the better the denoising effect is.

2) RMSE

\[
RMSE = \sqrt{\frac{1}{n} \sum [g(n) - \hat{g}(n)]^2}
\]

From the definition of RMSE, it can be known that after denoising a signal containing Gaussian white noise, the smaller RMSE is, the higher reliability is.

4. Improved wavelet TF

To solve the problems of wavelet coefficient deviation and “Pseudo-Gibbs phenomenon” caused by the de-noising of soft and hard TFs, an improved wavelet TF is proposed in this paper:

\[
\hat{\omega}_{j,k} = \begin{cases} 
(1 - \xi) \cdot \omega_{j,k} + \xi \cdot \text{sgn}(\omega_{j,k})(|\omega_{j,k}| - T), & |\omega_{j,k}| \geq T \\
0, & |\omega_{j,k}| < T 
\end{cases}
\]

Where \(\xi\) represents the weighting factor, and \(T\) represents the selected threshold.

When \(\xi=0.5\), the weighted TF is called compromise TF. The compromise TF can reduce the probability of signal loss, which caused by the soft TF at the high frequency signal and to some extent inhibit the signal mutation when the hard TF is de-noising, and the characteristic curve of denoising is illustrated in Fig.4.

Fig.4 shows that the compromise TF is still discontinuous, and the signal after \(\hat{\omega}_{j,k}\) recombination may generate oscillation, and when \(|\omega_{j,k}| \geq T\), a fixed deviation between \(\hat{\omega}_{j,k}\) and \(\omega_{j,k}\) is always exist, which will result the approximation degree between reconstructed signal and original signal is relatively low.

Therefore, in this paper, the \(\xi\) in equation (7) is

\[
\xi = \frac{T}{|\omega_{j,k}| \cdot 2 \sqrt{(|\omega_{j,k}| - T)^2 / (|\omega_{j,k}| + T)}}
\]

And the threshold \(T\) in equation (7) and (8) is selected by the unified threshold method, which is obtained by equation (4). The curve of denoising characteristic of the improved TF is presented in Fig.5.

Equation (7) and (8) show that when \(|\omega_{j,k}| = T\), \(\hat{\omega}_{j,k} = 0\), when \(|\omega_{j,k}|\) approaches \(T\), \(\xi\) approaches 1 and the improved TF denoising effect is infinitely close to the soft TF denoising. The effect, therefore, can achieve a smooth transition at the threshold, no signal oscillation and abrupt change. As the \(|\omega_{j,k}|\) increases, the improved TF becomes a combination of the soft and hard TFs, and with the increase of \(|\omega_{j,k}|\), \(\xi\) is decreasing, the improved TF is getting closer to the hard TF. When \(|\omega_{j,k}|\) tends to infinity, \(\xi\) approaches 0, and the improved TF denoising effect is infinitely close to the hard TF. The noise effect, therefore, can reduce the constant error generated in the high frequency portion during denoising.
It can be seen from the above analysis that in the process of denoising, with the change of the original signal frequency, the coefficients after wavelet transform will also change. The improved TF is always between the soft and hard TFs, so that the coefficient of wavelet estimation after denoising is maximized close to the coefficients of the effective signal.

5. Simulation example

5.1 Explanation of the example

The power quality signals in the power system are different from other signals, and have some special signal forms. Therefore, to test whether the denoising function is effective in power quality signal denoising, it is necessary to analyze some unique signals in power quality signal denoising. In the modern power system, voltage sag, voltage swell and voltage interruption signals have been recognized as the most serious power quality signals that affect the safety and normal operation of all kinds of electrical equipment, which is a new challenge to power quality in modern information society. Therefore, the de-noising simulations are mainly conducted for the above-mentioned three typical power quality signals, as shown in Fig.6-Fig.8, respectively.

5.2. Simulation result analysis

The original power quality signal is added to 10dB of Gaussian white noise, and then the noise signal is decomposed into three layers by db3 wavelet, and a set of coefficients are obtained. Then the improved TF is used to quantify the threshold, and a set of estimated coefficients are obtained. Finally, the estimated coefficients are utilized to reconstruct the signal through inverse wavelet transform.

For the noisy signal, the threshold is quantized by soft and hard TFs, compromise TF as well as improved TF, and signal reconstruction are performed. The de-noised signals and the comparison of the signal after de-noising with four kinds of TFs are shown in Fig.6-Fig.8.
It can be concluded from Fig.6-8, the traditional soft TF and hard TF denoising method can't accurately and completely reconstruct the original signal after denoising three typical power quality signals containing Gaussian white noise, especially in the large amplitude, the signal ambiguity is more obvious. The compromise TF can reconstruct the original signal well, but there is slight distortion at the peak and trough of the signal. The improved TF is utilized to de-noise the power quality signal containing white Gaussian noise, the reconstructed signal is smoother than the other three, and has a high similarity with the original signal, which can effectively de-noise and retain the information contained in the original power quality signal.

5.3 Effectiveness and reliability analysis

To further measure the performance of denoising by four different TFs, the SNR and RMSE after denoising by four different TFs are listed in the Table 1.

According to the data analysis in Table 1, it can be seen that by comparing the SNR and RMSE, it can be obtained that the compromise TF is combined with the soft and hard TFs, after denoising, the SNR is large and the RMSE is small. In addition, the de-noising effect is better and the reliability is higher. Further compare the SNR and RMSE after de-noising by the compromise TF and the improved TF, it can be concluded that the improved TF has obvious improvement in denoising performance and reliability.
Table 1. Comparison of SNR and RMSE among the four TFs denoising methods

| Signal type           | Noise | Soft TF denoising | Hard TF denoising | Compromised TF denoising | Improved TF denoising |
|-----------------------|-------|-------------------|-------------------|--------------------------|-----------------------|
|                       |       | SNR/dB | RMSE     | SNR/dB | RMSE     | SNR/dB | RMSE     | SNR/dB | RMSE     |
| Voltage sag           | 10dB  | 14.609  | 0.418    | 17.792 | 0.304    | 18.644 | 0.271    | 19.001 | 0.264    |
|                       | 15dB  | 15.825  | 0.367    | 18.930 | 0.268    | 20.072 | 0.231    | 20.474 | 0.224    |
| Voltage swell         | 10dB  | 16.649  | 0.441    | 19.587 | 0.326    | 20.892 | 0.277    | 21.166 | 0.271    |
|                       | 15dB  | 17.523  | 0.396    | 20.894 | 0.279    | 21.692 | 0.251    | 22.184 | 0.240    |
| Voltage interruption  | 10dB  | 14.134  | 0.434    | 16.788 | 0.335    | 18.459 | 0.271    | 18.679 | 0.266    |
|                       | 15dB  | 16.251  | 0.341    | 19.437 | 0.247    | 20.358 | 0.219    | 20.808 | 0.211    |

6. Conclusion
An improved wavelet TF is proposed to cope with the de-noising problem mainly for the three typical power quality signals, voltage sag, voltage swell, voltage interruption. The results are analyzed and compared based on SNR and RMSE, which prove the effectiveness and feasibility of the proposed improved TF for power quality signal denoising with Gaussian white noise. In addition, the other power quality signals can also be further verified with the proposed improved TF.

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