Learning Probabilistic Reward Machines from Non-Markovian Stochastic Reward Processes

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Abstract

The success of reinforcement learning in typical settings is, in part, predicated on underlying Markovian assumptions on the reward signal by which an agent learns optimal policies. In recent years, the use of reward machines has relaxed this assumption by enabling a structured representation of non-Markovian rewards. In particular, such representations can be used to augment the state space of the underlying decision process, thereby facilitating non-Markovian reinforcement learning. However, these reward machines cannot capture the semantics of stochastic reward signals. In this paper, we make progress on this front by introducing probabilistic reward machines (PRMs) as a representation of non-Markovian stochastic rewards. We present an algorithm to learn PRMs from the underlying decision process as well as to learn the PRM representation of a given decision-making policy.

1. Introduction

Traditionally, reinforcement learning techniques have relied on strong Markovian assumptions of the underlying decision process from which an optimal policy is to be derived. In particular, the reward signal by which an agent learns through positive or negative reinforcement of action outcomes is generally defined over the current state and action of an agent. The history of potentially meaningful states observed by the agent are not considered in such settings. However, many problem domains require taking into account this history for the decision problem at hand. Examples include learning in settings with sparse rewards (Neider et al., 2021), rewards defined as regular expressions and formal logics (Camacho et al., 2019), and decision-making under partial observability (Toro Icarte et al., 2019). In such settings, the non-Markovian reward signal may not be known, but can be learned from traces of behavior. To that end, the problem of learning a structured representation from the reward signal of a decision process has received significant attention in recent years (Neider et al., 2021; Xu et al., 2020a; Gaon & Brafman, 2020; Xu et al., 2020b; Toro Icarte et al., 2019). These structures are commonly represented in the form of weighted automata called reward machines (Icarte et al., 2018a; 2020). Reward machines serve to efficiently augment the state space of a non-Markovian reward decision process in order to reason about it as a Markov Decision Process (MDP) (Gaon & Brafman, 2020; Xu et al., 2020c). They also serve as a useful memory mechanism for reasoning over partially observable environments (Toro Icarte et al., 2019), are useful for defining reward shaping functions to mitigate sparse reward signals (Camacho et al., 2019; Velasquez & Melcer, 2020; Velasquez et al., 2021), and can be used to facilitate explanations of reinforcement learning systems (Xu et al., 2020c).

While much progress has been made on learning and leveraging reward machines for non-Markovian reward decision processes, the more general problem of rewards exhibiting both non-Markovian and stochastic dynamics has not been addressed. In this paper, we make progress on this front. First, we propose probabilistic reward machines (PRMs) as an extension of traditional reward machines that can encode non-Markovian stochastic rewards of a decision process. Second, we demonstrate how these PRMs can be learned using the \( \mathcal{L}^* \) algorithm (Angluin, 1987) by leveraging recent results on the use of \( \mathcal{L}^* \) for learning deterministic Markov Decision Processes (MDPs) (Tappler et al., 2019) and reward machines (Xu et al., 2020c). Finally, we adopt passive automaton learning techniques to derive a PRM for a given policy. By doing so, we can facilitate explainable AI approaches through a PRM representation of policy behavior.

2. Related Work

The classical problem of grammatical inference seeks to learn grammars, or some representation thereof, from a finite number of samples (Horning, 1969; De la Higuera, 2010). Approaches for learning such grammars are generally categorized as either active or passive. Passive grammatical inference seeks to mine the underlying specification
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from a static dataset of observed traces. Approaches in this area generally rely on state-merging procedures and can be used to learn probabilistic automata (Carrasco & Oncina, 1994), MDPs in the context of model checking (Mao et al., 2012), and timed automata (Mao et al., 2016), among others. State-merging approaches have also been combined with sampling to learn regular decision processes (Abadi & Brafman, 2020).

Active methods differ in that the System Under Learning (SUL) can be actively queried to guide the learning process. The $L^*$ algorithm for learning regular languages is the quintessential exemplar of this (Angluin, 1987). The $L^*$ algorithm assumes the existence of a minimally adequate teacher capable of answering two types of queries known as membership and equivalence queries. The former entails the learner asking whether some trace is in the language of the system to be learned, whereas the latter takes as input a hypothesis automaton from the learner and answers whether said automaton captures the language to be learned. When the hypothesis automaton and the language to be learned differ, the teacher can provide a counter-example trace in the symmetric difference of these two languages. The learner can then leverage this counter-example to ask more informative membership queries until the desired language is learned. This approach has been broadly adopted to learn interface automata (Aarts & Vaandrager, 2010), deterministic Mealy machines (Niese, 2003), automaton representations of recurrent neural networks (Weiss et al., 2018; 2019), and deterministic MDPs (Tappler et al., 2019), among others.

More recently, grammar inference has become a popular problem in the context of reinforcement learning, particularly for learning reward signals. While most literature in this area assumes that a reward machine is given (Icarte et al., 2018a; Hahn et al., 2019), the problem of actually learning the machine itself from observations has only been considered in the past few years. The work in (Xu et al., 2020a) (Neider et al., 2021) explores a satisfiability solving approach where, during the execution of the reinforcement learning algorithm (in this case Q-learning), traces of behavior are stored and the problem of deriving a minimal reward machine consistent with said traces can be extracted from a satisfiable assignment to a Boolean formula. Approaches that merge Q-learning and $L^*$ for the learning of reward machines are explored in (Gaon & Brafman, 2020) and (Xu et al., 2020c). In (Gaon & Brafman, 2020), the automaton learned using $L^*$ is leveraged to augment the state space of the underlying decision process in order to enable the adoption of Markovian reinforcement learning for solving the task at hand. However, consider the membership and equivalence queries used by $L^*$ in this setting. While the equivalence query can be obtained through the standard process of exploration (counter-examples to the currently learned automaton are obtained naturally through the policy learning process), it is the membership queries that pose a challenge here. Indeed, the problem of answering whether a trace (over some alphabet $\Sigma = 2^\mathcal{AP}$ of atomic propositions $\mathcal{AP}$) is the language of the reward signal requires the search for a trajectory of states in $\mathcal{S}$ whose corresponding trace affirmatively answers the query. A simple heuristic is employed in (Gaon & Brafman, 2020) to make answering membership queries tractable. In particular, the query is answered negatively if some threshold of attempts to answer it is exceeded. The work in (Xu et al., 2020c) handles this membership query problem in a different manner by posing it as a reinforcement learning problem. Indeed, their proposed approach defines two Q functions from which policies are derived to solve the original reinforcement learning problem as well as the problem of generating a trajectory of states whose induced trace answers the underlying membership query. The latter is accomplished by rewarding the agent as it observes elements of the membership query in the correct order.

In (Toro Icarte et al., 2019), the problem of learning a reward machine is viewed through the lens of discrete optimization. In particular, the problem is posed as that of finding a reward machine that maximizes the log-likelihood of predicting future observations. Tabu search is then performed, starting with a random reward machine and evaluating other reward machines within some neighborhood of it in the direction of the objective function.

Our work differs from the foregoing in a number of ways. To the best of our knowledge, we present the first approach to learn PRMs as a representation of an underlying non-Markovian stochastic reward signal in reinforcement learning. Furthermore, our approach can generate approximate PRMs and, as a special case, approximate non-probabilistic reward machines. This mitigates the problem of requiring vast amounts of data for learning an exact reward machine.

3. Probabilistic Reward Machines

We first present an example to motivate the study of Probabilistic Reward Machines (PRMs). Consider the office grid-world environment found in (Icarte et al., 2018b), a diagram of which is shown in Figure 1a. The original task, which takes the form of a deterministic reward machine, requires that the agent acquire coffee at location $c$, and then deliver the coffee to office $o$, at which point the agent receives a reward of 1. However, if the agent steps on a decoration, which is marked with the symbol $\dagger$, the agent will fail the task and observe a reward of 0.

Now, we present a task in which the reward signal is probabilistic in nature. In this task, there is a 10% chance that the coffee machine has malfunctioned, thus producing weak coffee which will be rejected with a reward of 0 when delivered
A policy is a function \( \pi : (X \times A) \times X \rightarrow \text{Dist}(A) \) specifying a distribution over the set of actions following a run. A policy is pure if, for any run \( w \), there exists and action \( a \) such that \( \pi(w, a) = 1 \). A policy is stationary if, for any run \( w = x_0a_1\cdots a_nx_n \) and action \( a \), it holds that \( \pi(w, a) = \pi(x_n, a) \). If a policy is both pure and stationary, it is called positional. Imposing a fixed policy on an NMDP and selecting a state induces a unique probability measure \( \mathbb{P}_\pi : \text{Path}(M) \rightarrow [0, 1] \) defined as \( \mathbb{P}_\pi(w) = \prod_{k=1}^{n} \pi(x_k-1, a_k) \cdot p(x_k-1, a_k, x_k) \), for \( w = x_0a_1x_2\cdots x_n \). A run \( w \in \text{Path}(M) \) is consistent with a policy \( \pi \) if it has non-zero probability: \( \mathbb{P}_\pi(w) > 0 \).

Given an NMDP \( M = (X, x_1, A, p, AP, L, R) \), we seek to construct a PRM, defined below, that encodes the underlying reward \( R \).

**Definition 2** (Probabilistic Reward Machine). A Probabilistic Reward Machine (PRM) \( H \) is a tuple \((AP, \Gamma, Y, y_1, \tau, \rho)\) where \( AP \) is a set of atomic propositions and \( 2^\text{AP} \) is the input alphabet, \( \Gamma \subseteq \mathbb{R} \) is a finite set of rewards, \( Y \) is a finite set of states, \( y_1 \) is a distinguished initial state, \( \tau : Y \times 2^\text{AP} \rightarrow \text{Dist}(Y) \) is a probabilistic transition function, and \( \rho : Y \times 2^\text{AP} \stackrel{\rightarrow}{\rightarrow} \Gamma \) is a function mapping each pair of state and input symbol to a reward from \( \Gamma \).

Given this definition, we return to the probabilistic office gridworld task described earlier. A graphical representation of the PRM for this task is shown in Figure 1b.

**Figure 1.** Office gridworld environment with a probabilistic task to the office.

We begin our treatment of the proposed PRMs with some preliminary definitions pertaining to the underlying decision process from which samples are obtained to estimate the PRM representation of the objective.

**Definition 1** (Non-Markovian Decision Process). A Non-Markovian Decision Process (NMDP) is a tuple \((X, x_I, A, p, AP, L, R)\) where \( X \) is a finite set of states, \( x_I \in X \) is a distinguished initial state, \( A \) is a finite set of actions, \( p : X \times A \rightarrow \text{Dist}(X) \) is a probabilistic transition function, \( AP \) is a finite set of atomic propositions, \( L : X \times A \times X \rightarrow 2^\text{AP} \) is a function labelling each transition with a subset of \( AP \), and \( R : X \times (A \times X)^+ \rightarrow \text{Dist}(\mathbb{R}) \) is a non-Markovian stochastic reward function, where \( \text{Dist}(\mathbb{R}) \) denotes the set of possible probability distributions over \( \mathbb{R} \).

A run of an NMDP is a sequence \( x_0a_1x_1\cdots a_nx_n \in X \times (A \times X)^* \) such that \( \prod_{k=1}^{n} p(x_k-1, a_k, x_k) > 0 \). Let \( \text{Path}(M) \) denote the set of all runs in \( M \). For a run \( x_0a_1x_1\cdots a_nx_n \), its corresponding label sequence is \( \ell_1\ell_2\cdots\ell_n \), where \( \ell_k = L(x_k-1, a_k, x_k) \), and the corresponding reward sequence is \( r_1r_2\cdots r_n \), where \( r_k = R(x_0\cdots a_kx_k) \). A trajectory is a run for which \( x_0 = x_I \).

A policy is a function \( \pi : (X \times A) \times X \rightarrow \text{Dist}(A) \).
and identify the induced conditional distribution as
\[ P_H(\gamma \mid \ell_1 \cdots \ell_k) = y^T \mathbf{H}(\gamma \mid \ell_1 \cdots \ell_k) \mathbf{1}. \]

The PRM \( H \) is said to encode the reward function \( R \) of an NMDP \( M \) if for any trajectory \( x_0a_1x_1 \cdots a_nx_n \) in \( M \) with corresponding label sequence \( \ell_1 \cdots \ell_n \) and reward sequence \( r_1 \cdots r_n \), the following equality holds for all \( 1 < k \leq n \):
\[ P_H(r_k \mid \ell_1 \cdots \ell_{k-1}) = \Pr[R(x_0 \cdots a_kx_k) = r_k]. \]

4. An Excursus on \( L^* \)

In the seminal work of (Angluin, 1987), an algorithm called \( L^* \) is proposed by which a deterministic finite automaton (DFA) for a given regular language \( L \) can be learned through the use of a minimally adequate teacher that can answer membership and equivalence queries. When the learner executes a membership query, it presents a word \( w \in (2^A)^+ \) to the teacher and the teacher outputs whether \( w \) belongs to the language to be learned. Alternatively, if the learner executes an equivalence query, it presents the currently learned automaton representation of the language to the teacher and the teacher must answer whether this automaton encodes the language to be learned. If not, the teacher generates a counter-example in the form of a word in which the two languages differ.

The \( L^* \) algorithm keeps track of state words \( S \subseteq (2^A)^+ \), which are closed under prefix operations (e.g. if \( ab \in S \), then \( a \in S \)) and test words \( E \subseteq (2^A)^+ \), which are closed under suffix operations (e.g. if \( ab \in E \), then \( b \in E \)). In the beginning, we have \( S = E = \{\epsilon\} \), where \( \epsilon \) is the empty string. As the algorithm proceeds, there are two critical correctness and completeness properties that must be tracked which revolve around the notion of \( E \)-equivalence, all defined below.

**Definition 3** (E-Equivalence). Given two words \( w, w' \in (2^A)^+ \) and test set \( E \subseteq (2^A)^+ \), the words \( w \) and \( w' \) are said to be \( E \)-equivalent, denoted by \( w \equiv_E w' \), if \( w \cdot e \in \mathcal{L} \iff w' \cdot e \in \mathcal{L} \) holds for every test word \( e \in E \), where \( \mathcal{L} \) is the language to be learned.

**Definition 4** (Consistency). Given \( (S, E) \), the consistency property holds if and only if there are no two state words in \( S \) that are \( E \)-equivalent. That is \( \forall s \in S, \exists s' \in S, s \equiv_E s' \).

**Definition 5** (Closedness). Given \( (S, E) \), the closedness property holds if and only if for all \( s \in S \) and \( \ell \in 2^A \), there exists some \( s' \in S \) such that \( s \cdot \ell \equiv_E s' \).

Given a closed and consistent \( (S, E) \), its corresponding automaton, in this case a DFA, can be derived by taking each \( s \in S \) to be a node, with the empty string \( \epsilon \) as the starting node. The transition function is defined using the closedness property. That is, whenever we have \( s \cdot \ell \equiv_E s' \) per the closedness property, then we also have the transition from \( s \) to \( s' \) upon observing the label \( \ell \) in the DFA. Furthermore, it follows from the consistency property that \( s' \) is unique. The accepting nodes in the automaton are those in the language of the teacher.

The \( L^* \) algorithm ensures that \( (S, E) \) remains closed and consistent until the target language is learned. This is done by formulating the DFA of the currently learned language from \( (S, E) \). If a counter-example is produced, it is used to modify \( (S, E) \) by adding the counter-example and all its prefixes to \( S \). Membership queries are then used to determine the values of the new entries.

Consider again the reinforcement learning setting where we wish to learn the PRM representation of the objective as given by the reward signal. Note that the case where there is a language to be learned as denoted by accepting states in the automaton representation is a special case of PRMs. Indeed, this would correspond to only some subset of states in the PRM having non-zero rewards and self-loops upon observance. Some challenges arise in this reinforcement learning setting. Indeed, note that the seemingly innocuous membership queries must now be answered indirectly through the NMDP within which the agent interacts. This introduces two challenges to the adoption of the \( L^* \) algorithm within reinforcement learning. First, there is no obvious way to perform membership queries. Indeed, consider asking the question of whether \( w \) belongs to the target language. The construction of a meaningful \( w \) is not obvious on the part of the learner and answering this query is difficult on the part of the teacher. This is because we can only reason about these words indirectly through the NMDP, whereas traditional approaches reason directly over the target language. For example, if asking the question of whether \( \text{(red, blue, red)} \) is in the target language, the teacher would have to generate a trajectory of states and actions following the transition dynamics of the NMDP such that its corresponding sequence of labels is \( \text{(red, blue, red)} \) and respond based on the reward observed by said trajectory.

The second challenge lies in performing the equivalence queries since the teacher cannot answer the question of whether the currently learned PRM encodes the reward of the underlying NMDP. However, it is worth noting that counter-examples are obtained naturally through the reinforcement learning process. These counter-examples are words \( w \in (2^A)^+ \) that either have not been seen before or whose corresponding reward sequence differs from other encounters of the same word. The latter case arises due to the stochastic nature of the reward signal. Thus, the answers to equivalence queries are obtained implicitly through the reinforcement learning process. We conjecture that the capacity to answer such queries explicitly may improve the performance of the PRM algorithm and leave this as an open question.
avenue for future work.

5. Learning PRMs from NMRDPs

In this section, we describe an algorithm for learning NMMDPs with stochastic reward signals. Our presentation builds upon the active MDP learning approach of (Tappler et al., 2019) and the work of (Xu et al., 2020c) on combining active learning with reinforcement learning for deterministic non-Markovian rewards.

5.1. Observation Table and Hypothesis Construction

For an underlying PRM \((AP, \Gamma, Y, y_I, \tau, \rho)\), an observation table (Angluin, 1987) is a tuple \((S, E, T)\), where \(S, E \subset (2^AP)^+\) are finite sets of words called samples and experiments, and \(T : S \times (\varepsilon \cup 2^AP) \times E \rightarrow (\Gamma \rightarrow \mathbb{N})\) is a function mapping input sequences to reward frequencies. Denote by \(\text{row}(s)\) the set of all \(T(s \cdot e)\) for \(e \in E\).

We say that the test sequences \(s, s' \in (2^AP)^+\) are statistically different (Tappler et al., 2019), and we write \(\text{Diff}(f, s, s') \in \{0, 1\}\) with respect to a function \(f : (2^AP)^+ \rightarrow (\Gamma \rightarrow \mathbb{N})\) if:

1. \(0 < N = \sum_{\gamma \in \Gamma} f(s)(\gamma)\),
2. \(0 < N' = \sum_{\gamma \in \Gamma} f(s')(\gamma)\), and
3. there exists \(\gamma \in \Gamma\) such that

\[
\left| \frac{f(s)(\gamma)}{N} - \frac{f(s')(\gamma)}{N'} \right| > \sqrt{\frac{1}{2} \ln \frac{2}{\alpha}} \left( \sqrt{\frac{1}{N}} + \sqrt{\frac{1}{N'}} \right),
\]

where \(\alpha\) defines the confidence level for the preceding Hoeffding bound. We choose \(\alpha = \frac{1}{10^6}\) with \(M = \sum_{s \in S} \text{Sample}(s)\) being the total number of samples taken thus far.

Two cells \(s \cdot e\) and \(s' \cdot e'\) of an observation table \((S, E, T)\) are compatible, denoted \(\text{Compatible}(s \cdot e, s' \cdot e')\), if they are not different (i.e. \(\text{Diff}(T, s \cdot e, s' \cdot e') = 0\)). Similarly, we say that two rows \(s, s'\) are compatible, denoted \(\text{CompatibleRow}(s, s')\) if \(\text{Compatible}(s \cdot e, s' \cdot e)\) holds for all \(e \in E\). We leverage these definitions and adapt sampling-based notions of closedness and consistency from (Tappler et al., 2019) to our PRM learning setting.

Definition 6 (Sampling Closedness). An observation table is closed if, for all \(s \in S \times (\varepsilon \cup 2^AP)\), there exists \(s' \in S\) such that \(\text{CompatibleRow}(s, s')\) holds.

Definition 7 (Sampling Consistency). An observation table is consistent if, for all compatible pairs \(\{(s, s') \in S \times S : \text{Compatible}(s, s')\}\) and all \(\ell, \gamma \in 2^AP \times \Gamma\), either \(T(s \cdot \ell)(\gamma) = 0\) or \(T(s' \cdot \ell)(\gamma) = 0\), or \(\text{CompatibleRow}(s \cdot \ell, s' \cdot \ell)\).

Given a closed and consistent observation table \((S, E, T)\), we can construct a PRM \(H = (AP, \Gamma, Y, y_I, \tau, \rho)\) as follows, where \(\bot\) denotes a failure state and transitions to said state are taken with probability 1 when there is not sufficient sampled data for the given transition. Furthermore, note that the samples in \(S\) can be partitioned into compatibility classes based on the \(\text{Compatible}(s, s')\) function. Within these compatibility classes, we can assign a representative \(\text{rep}(s) = \arg\max_{s' \in S} \{\text{rank}(s') : \text{CompatibleRow}(s, s')\}\) for any given \(s \in S\), where \(\text{rank}(s) = \sum_{\ell \in 2^\gamma} \sum_{\gamma \in \Gamma} T(s \cdot \ell)(\gamma)\). That is, the representative sample for a given compatibility class is the one for which we have the most information as given by its rank. We denote the set of all such representatives by \(U = \{\text{rep}(s) : s \in S\}\).

- \(Y = \{((\gamma, \text{row}(s)) : s \in U, \gamma \in \Gamma) \cup \{\bot\}\},\)
- \(y_I = (0, \text{row}(\varepsilon))\),
- \(\tau(y, \ell, y') = \begin{cases} 0 & \text{if } y = \bot \text{ and } y' \neq \bot, \\ 1 & \text{if } y = y' = \bot \text{ or if } y = (\gamma, \text{row}(s)) \text{ and } y' = \bot \text{ and } \text{Sample}(s) < N_{\text{check}}^1, \\ \frac{T(s)(\gamma')}{\sum_{s' \in \text{Check}(s)} T(s')}\end{cases}\text{if } (\gamma, \text{row}(s)) \text{ and } y' = (\gamma', \text{row}(s\ell)),\)
- \(\rho((\gamma, \text{row}(s)), \ell) = \gamma,\)

5.2. Membership and Equivalence Queries

In order to construct the hypothesis machine, we need a statistically significant amount of queries on the probabilistic reward machine to be able to approximate the reward distributions over given query. Unfortunately, in the RL setting these queries to the PRM must be posed via the MDP, and hence the learner may not directly choose the membership queries. Following the work of (Xu et al., 2020b), we “prime” the RL algorithm to generate a large number of membership queries required by the learning algorithm for PRMs, by incentivizing the RL agent for outputs matching the query by using RL. We thus create a pseudo-"reward machine" from the membership query in the following way.

Definition 8 (Membership Reward Machine). Given a query string \(\zeta = \ell_1 \cdots \ell_n\), the corresponding membership query machine is \(H_\zeta = (AP, \Gamma, Y, y_I, \tau, \rho)\) where

- \(Y = \{(y_0, y_1, \ldots, y_n)\},\)
- \(y_I = y_0.\)

\(^1\)In other words, if there is not sufficient data in the observation table to estimate a distribution over \(\Gamma\) for \(s\).
Algorithm 1. The algorithm uses membership reward machines as each transition in the reward machine has exactly one value associated with it as enforced by the underlying deterministic transition dynamics. However, note that in the case of traditional non-probabilistic reward machines as each transition in the reward machine has exactly one value associated with it as enforced by the underlying deterministic transition dynamics. However, note that in our setting, the observance of the same sequence of labels yielding different rewards is handled since this reflects the stochastic nature of the underlying reward signal as captured by the PRM being learned.

Note that, during the reinforcement learning process, encountering some sequence $s \in (2^{AP})^+$ which has not been encountered before or for which different reward values are observed when compared to previous observations of $s$ constitutes a counterexample to the currently learned PRM (Algorithm 3, Line 11). This counterexample $s$ is then used in further refinement of the hypothesis PRM in Algorithm 1 via additional membership queries. Experimental results can be found in the Appendix.

Figure 2. Reconstructed reward machine for an optimal policy in the probabilistic office world example

Any convergent RL algorithm working on the product of the MDP and membership reward machine, will converge to a policy generating the desired membership query with optimal probability. Similarly, any convergent RL algorithm working on the product of the MDP and hypothesis reward machine, will converge to the optimal policy with respect to the rewards consistent with the hypothesis reward machine. This product is defined as follows.

**Definition 9.** Let $M = (X, x_1, A, p, \text{AP}, L, R)$ be an NMDP and let $H = (\text{AP}, \Gamma, Y, y_1, \tau, \rho)$ be an PRM. The product $M \times H$ is an MDP $M_H = (X', x_1', A, p', \text{AP}, L', R')$ with a Markovian reward function $R' : X \times A \times X \rightarrow \mathbb{R}$ such that:

- $X' = X \times Y$,
- $x_1' = (x_1, y_1)$,
- $p'(y(x, y), a, (x', y')) = p(x, a, x')\tau(y, L(x, a, x'), y')$,
- $L'(y(x, y), a, (x', y')) = L(x, a, x')$, and
- $R'(y(x, y), a, (x', y')) = \rho(y, L(x, a, x'))$.

### 5.3. Putting it all Together

Our active reinforcement learning algorithm is shown in Algorithm 1. The algorithm uses membership reward machines to prime an RL agent to answer membership queries. The RL algorithm continues to optimize on the product of the MDP and the membership reward machine, and continues to observe the reward signals from the current PRM. In doing so, the policy that maximizes these rewards is guided towards answering the membership query by finding a trajectory of states in the decision process whose induced trace of labels answers the membership query.

Recall that membership queries are given as a sequence over $2^{AP}$. The work in (Xu et al., 2020c) includes reward values in such sequences. Indeed, consider how membership queries are answered in (Xu et al., 2020c). Given a membership query $((\ell_1, 0), (\ell_2, 1), (\ell_3, 1))$, where $\ell_i \in 2^{AP}$ and the second entry in each pair denotes a reward, if a trace $((\ell_1, 0), (\ell_2, 0))$ is observed, the membership query returns a 0, thereby negatively answering the query. This makes sense in the case of traditional non-probabilistic reward machines as each transition in the reward machine has exactly one value associated with it as enforced by the underlying deterministic transition dynamics. However, note that in our setting, the observance of the same sequence of labels yielding different rewards is handled since this reflects the stochastic nature of the underlying reward signal as captured by the PRM being learned.

Note that, during the reinforcement learning process, encountering some sequence $s \in (2^{AP})^+$ which has not been encountered before or for which different reward values are observed when compared to previous observations of $s$ constitutes a counterexample to the currently learned PRM (Algorithm 3, Line 11). This counterexample $s$ is then used in further refinement of the hypothesis PRM in Algorithm 1 via additional membership queries. Experimental results can be found in the Appendix.

Figure 2. Reconstructed reward machine for an optimal policy in the probabilistic office world example
We demonstrate the utility of the proposed approach by invoking Algorithm 5 on the probabilistic office gridworld. PRM is not complete. For example, because the policy avoids stepping on a decoration, no transitions are provided.

Algorithm 2 MembershipQuery
1: Input: $\zeta, O = (S, E, T)$
2: counter $\leftarrow 0$
3: Construct query machine $H_\zeta$
4: while $Sample(\zeta) < N_{\text{check}}$ and counter $< N_{\text{query}}$ do
5: $\lambda, Q_m$ $\leftarrow$ Teacher.Query($Q_m, H_\zeta$, membership)
6: for prefix $\ell_1 \ell_2 \cdots \ell_k r_k$ of $\lambda$ do
7: $T(\ell_1 \cdots \ell_k)(r_k) \leftarrow T(\ell_1 \cdots \ell_k)(r_k) + 1$
8: $Sample(\ell_1 \cdots \ell_k) \leftarrow Sample(\ell_1 \cdots \ell_k) + 1$
9: end for
10: counter $\leftarrow$ counter + 1
11: end while

Algorithm 3 EquivalenceQuery
1: Input: $O$
2: Construct hypothesis $H$ from table $O$
3: counter $\leftarrow 0$
4: repeat
5: counter $\leftarrow$ counter + 1
6: $\lambda, Q_H$ $\leftarrow$ Teacher.Query($Q_H, H$, equivalence)
7: for prefix $\ell_1 \ell_2 \cdots \ell_k r_k$ of $\lambda$ do
8: $T(\ell_1 \cdots \ell_k)(r_k) \leftarrow T(\ell_1 \cdots \ell_k)(r_k) + 1$
9: $Sample(\ell_1 \cdots \ell_k) \leftarrow Sample(\ell_1 \cdots \ell_k) + 1$
10: end for
11: until $\lambda$ is a counterexample or counter $\geq N_{\text{stop}}$

6. Learning the Reward Signal of a Policy

It may be the case that a policy has already been synthesized, and we wish to understand the reward signal induced by the policy. This could be useful, for example, if we have already derived a policy using an off-the-shelf reinforcement learning approach, and would like to explore the reward signal of the corresponding Markov chain. We may also want to design a policy to elicit certain aspects of the environment’s reward signal which we would like to better understand. To this end, we propose the passive approach shown in Algorithm 5. This algorithm runs $M$ episodes using the provided policy $\pi$, and builds an observation table from the resulting traces. This table can then be used to construct a PRM.

We demonstrate the utility of the proposed approach by invoking Algorithm 5 on the probabilistic office gridworld example of Figure 1. We use the optimal policy with the shortest number of steps, marked in Figure 1a with a blue line, and run $M = 10^4$ episodes. The reconstructed PRM is shown in Figure 2. It is clear from this PRM that delivery of coffee does not always result in a reward. An operator examining this PRM may find this result unexpected, thus prompting an investigation to better understand the reason for the failure. We note that, as intended, the reconstructed PRM is not complete. For example, because the policy avoids stepping on a decoration, no transitions are provided for the $\ast$ labels. Here, we are only interested in understanding the reward signal for the ideal scenario in which the coffee is delivered to the office, while avoiding extraneous states and transitions.

7. Conclusion

In this paper, we have proposed Probabilistic Reward Machines (PRMs) as an extension of reward machines in order to capture reward signals exhibiting both non-Markovian and stochastic dynamics. An algorithm was proposed to learn PRMs from the underlying decision process in reinforcement learning problems. The proposed approach has the added benefit of learning an approximate representation of the PRM when insufficient data is available for an exact representation.

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Algorithm 5  Passive Algorithm for Learning the Reward Signal of a Policy

1: Input: Policy $\pi$. Number of episodes $M$.
2: Initialize observation table $O = (S, E, T)$
3: for $1 \leq i \leq M$ do
4:  Execute policy $\pi$ to get trace $\zeta = \ell_1 r_1 \cdots \ell_k r_k$
5:  Add all suffixes of trace $\zeta$ to $E$
6: for $1 \leq j \leq k$ do
7:  $T(\ell_1 \cdots \ell_j)(r_j) = T(\ell_1 \cdots \ell_j)(r_j) + 1$
8:  end for
9: end for
10: while $\exists s \in S$ and $\ell \in 2^{AP}$ such that $\forall s' \in S :$ 
11:  $\neg$CompatibleRow$(s\ell, s')$ do
12:  $S \leftarrow S \cup \{s\ell\}$
13: end while
14: return $O = (S, E, T)$

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