Enhancing Predicate Pairing with Abstraction for Relational Verification

Emanuele De Angelis\(^1\), Fabio Fioravanti\(^1\), Alberto Pettorossi\(^2,3\), and Maurizio Proietti\(^3\)

\(^1\) DEC, University ‘G. d’Annunzio’, Chieti-Pescara, Italy
\{emanuele.deangelis,fabio.fioravanti\}@unich.it
\(^2\) DICII, University of Rome ‘Tor Vergata’, Italy
pettorossi@info.uniroma2.it
\(^3\) IASI-CNR, Rome, Italy
maurizio.proietti@iasi.cnr.it

Abstract. Relational verification is a technique that aims at proving properties that relate two different program fragments, or two different program runs. It has been shown that constrained Horn clauses (CHCs) can effectively be used for relational verification by applying a CHC transformation, called predicate pairing, which allows the CHC solver to infer relations among arguments of different predicates. In this paper we study how the effects of the predicate pairing transformation can be enhanced by using various abstract domains based on linear arithmetic (i.e., the domain of convex polyhedra and some of its subdomains) during the transformation. After presenting an algorithm for predicate pairing with abstraction, we report on the experiments we have performed on over a hundred relational verification problems by using various abstract domains. The experiments have been performed by using the VeriMAP transformation and verification system, together with the Parma Polyhedra Library (PPL) and the Z3 solver for CHCs.

1 Introduction

Relational program properties are properties that relate two different programs or two executions of the same program. Relational properties that have been studied in the literature include program equivalence, non-interference for software security, and relative correctness \([4, 5, 30]\).

Recent work has advocated the use of Constrained Horn Clauses (CHCs) for the verification of relational program properties \([16, 21, 30]\). These methods translate a verification problem into a set of Horn clauses with constraints in a suitable domain (usually, Linear Arithmetic), and then verify the satisfiability of those clauses by using SMT solvers for Horn clauses, called here CHC solvers, such as Z3 \([17]\) or Eldarica \([27]\).

\(^*\) This work has been partially funded by INdAM-GNCS (Italy). E. De Angelis, F. Fioravanti, and A. Pettorossi are research associates at IASI-CNR, Rome, Italy.
The main difficulty encountered by CHC solvers when verifying relational properties is that these solvers find models of single predicates expressed in terms of Linear Arithmetic constraints, whereas the proof of relational properties often requires the discovery of relations among arguments of two (or more) distinct predicates. To mitigate this difficulty, Predicate Pairing transforms a set of clauses defining two predicates, say $p$ and $q$, into a new set of clauses defining a new predicate, say $r$, equivalent to the conjunction of $p$ and $q$ \cite{16}. Thus, when the CHC solver finds a model for the predicate $r$, it discovers relations among the arguments of $p$ and $q$.

In the approach presented in this paper we use Predicate Pairing together with Abstraction. Abstraction is a technique that is often used in program analysis and transformation, and consists in mapping the concrete semantics of a program into an abstract domain, where some program properties can more easily be verified \cite{8}. In the context of the verification of relational properties, Predicate Pairing combined with a basic form of abstraction has been introduced in a previous paper \cite{16}. In that paper, in fact, Predicate Pairing is performed by introducing new definitions whose bodies are made out of two atoms whose predicates are the ones to be paired, together with the equalities between the arguments of these predicates, and these equality constraints can be viewed as an abstraction into the domain of equalities.

Abstraction is also used by CHC Specialization, which is another transformation technique that has been proposed to increase the effectiveness of CHC solvers \cite{11, 28}. CHC Specialization propagates constraints through the clauses of the program, and since propagation often causes strengthening of the constraints, it is possible that by first specializing programs, the task of CHC solving is much facilitated. However, the impact of the specialization process very much depends on the choices of the particular abstract domain and associated widening operator, which are used when specialized predicates are introduced or manipulated.

In this paper we address the problem of evaluating various combinations of Predicate Pairing, Abstraction, and CHC Specialization for the specific objective of verifying relational properties of programs. In order to do so, we introduce a general algorithm for Predicate Pairing that is parametric with respect to the abstract constraint domain that is used. This domain is taken to be a subdomain of Linear Arithmetic, such as Convex Polyhedra, Boxes, Bounded Differences, and Octagons \cite{3, 9, 35}. Our parametric Abstraction-based Predicate Pairing algorithm generalizes the one that makes use of equalities between variables that has been used in a previous paper of ours \cite{16}. We also consider a CHC Specialization algorithm that is parametric with respect to the abstract constraint domain. Then we consider various compositions of Abstraction-based Predicate Pairing and CHC Specialization, by varying the abstract constraint domain used.

These compositions of transformations are applied to sets of CHCs encoding relational properties of imperative programs. The lesson one may learn from the results of our experiments is that the transformations achieving the best results make use of constraint domains that are expressive enough to describe relations
between variables (such as, Bounded Differences and Octagons), without requiring the higher precision of Convex Polyhedra. Moreover, Abstraction-based Predicate Pairing essentially incorporates the effect of CHC Specialization, and thus extra specializations (before or after Abstraction-based Predicate Pairing) are not cost-effective.

The paper is organized as follows. In Section 2 we present an introductory example showing the usefulness of abstraction. Then, in Section 3 we present the various abstract constraint domains, that is, Convex Polyhedra, Boxes, Bounded Differences, and Octagons, and the operations defined on them. In Section 4 we present the Abstraction-based transformation techniques for CHCs and we prove that they preserve satisfiability (and unsatisfiability). The implementation of the verification method, based on the Parma Polyhedra Library for constraint manipulation and on the Z3 solver for CHC satisfiability, and its experimental evaluation are reported in Section 5. Finally, in Section 6 we discuss the related work on program transformation and verification.

2 An Introductory Example

In this section we present the proof of equivalence of two imperative programs acting on integer variables. This equivalence can be proved by using our method based on the Predicate Pairing strategy using a suitable abstract constraint domain. By using Predicate Pairing without abstraction the proof cannot be performed.

Let us consider the programs \( P_1 \) and \( P_2 \) shown in Figure 2, where program \( P_2 \) is obtained from program \( P_1 \) by applying a compiler optimization technique, called software pipelining. This technique has the effect of allowing more parallelism during execution.

Software pipelining takes as input a program with a loop and produces an improved program with a new loop whose instructions are taken from different iterations of the original loop. Combined with other optimizations, software pipelining can produce loops whose instructions have no read/write dependencies and thus can be executed in parallel. For example, in program \( P_2 \) derived by pipelining from program \( P_1 \), the dependency on \( x \) in the instructions of the loop in \( P_2 \) can be removed by: (i) introducing a fresh variable \( u \) initialized to the value of \( x \), and (ii) replacing \( x \) by \( u \) on the right-hand side of the assignments in the loop. After this replacement the resulting instructions ‘\( y = y+u; \)’ and ‘\( a = a+1; x = u+a; \)’ can be safely executed in parallel.

The equivalence of programs \( P_1 \) and \( P_2 \) with respect to the output value of \( x \), can be expressed by the following clause \( F \):

\[
F: \quad \text{false} \leftarrow X_1 \neq X_2, \ s_{11}(A, B, X, Y, X_1, Y_1), \ s_{21}(A, B, X, Y, X_2, Y_2)
\]

where: (i) predicates \( s_{11} \) and \( s_{21} \) represent the input/output relation of programs \( P_1 \) and \( P_2 \), respectively; (ii) \( A, B, X, Y \) represent the values of the integer variables \( a, b, x, y \) at the beginning of program execution; (iii) \( X_1, Y_1 \) and \( X_2, Y_2 \) represent the values of the integer variables \( x \) and \( y \) at the end of the
execution of $P_1$ and $P_2$, respectively. In order to be processed by the constraint-based techniques presented in this paper, which do not consider disequality ($\neq$), clause $F$ is split into the following two clauses:

$F_1$: $false \leftarrow X_1 \leq X_2 - 1$, $s11(A, B, X, Y, X1, Y1)$, $s21(A, B, X, Y, X2, Y2)$

$F_2$: $false \leftarrow X_2 \leq X_1 - 1$, $s11(A, B, X, Y, X1, Y1)$, $s21(A, B, X, Y, X2, Y2)$

The clauses defining $s11$ and $s21$ (and the predicates $s12$, $s22$, and $s23$ on which they depend) are reported below. (The unfamiliar reader may refer, for instance, to [15], where the technique for generating clauses from imperative programs is presented.) Note, for example, that predicates $s12$ and $s23$ correspond to the loops in programs $P_1$ and $P_2$, respectively. Note also that strict inequalities occurring in programs (for instance, $a < b$ in program $P_1$) are represented by using non-strict inequalities in clauses ($A \leq B - 1$ in the first clause for $s12$).

$s11(A, B, X, Y, X2, Y2) \leftarrow s12(A, B, X, Y, A2, B2, X2, Y2)$,

$s12(A, B, X, Y, A2, B2, X2, Y2) \leftarrow A \leq B - 1$, $X1 = A + X$, $Y1 = Y + X1$, $A1 = A + 1$, $s12(A1, B, X1, Y1, A2, B2, X2, Y2)$.

$s12(A, B, X, Y, A, B, X, Y) \leftarrow A \geq B$.

$s21(A, B, X, Y, X2, Y2) \leftarrow s22(A, B, X, Y, A2, B2, X2, Y2)$,

$s22(A, B, X, Y, A2, B2, X2, Y2) \leftarrow A \leq B - 1$, $X1 = X + A$, $s23(A, B, X1, Y, A2, B2, X2, Y2)$.

$s22(A, B, X, Y, A, B, X, Y) \leftarrow A \geq B$.

$s23(A, B, X, Y, A2, B, X, Y2) \leftarrow A \geq B - 1$, $Y2 = Y + X$, $A2 = A + 1$.

$s23(A, B, X, Y, A2, B2, X2, Y2) \leftarrow A \leq B - 2$, $Y1 = Y + X$, $A1 = A + 1$,

$X1 = X + A1$, $s23(A1, B, X1, Y1, A2, B2, X2, Y2)$.

Let $P$ be the set of clauses defining $s11$ and $s21$ and the predicates on which they depend. By proving the satisfiability of $\{F_1, F_2\} \cup P$, we prove that programs $P_1$ and $P_2$ produce identical values for $x$ as output, when provided with the same input values. For reasons of simplicity we only consider the subset $\{F_1\} \cup P$. The satisfiability of the larger set can be proved by applying the same technique.

Unfortunately, a state-of-the-art CHC solver such as Z3 [17] fails to prove the satisfiability of $\{F_1\} \cup P$. This inability is due to the fact that Z3 computes models of single predicates expressed in terms of linear constraints, and it can be shown that non-linear constraints among the arguments of each predicate $s11$.
and $s_{21}$ need be discovered to prove that the two atoms in the body of clause $F_1$ imply $X_1 > X_2 - 1$.

The Predicate Pairing strategy we have introduced in a previous paper [16] may help in overcoming this difficulty. By Predicate Pairing we may introduce new predicates defined in terms of two (or more) atoms, and infer relations among arguments of distinct predicates. Indeed, models of single predicates derived by Predicate Pairing, correspond to models of pairs (that is, conjunctions) of predicates before Predicate Pairing. Unfortunately, it is the case that also the set of clauses derived from $\{F_1\} \cup P$ by ‘simple’ Predicate Pairing, that is, without adding any constraint to the arguments of the paired predicates, cannot be proved satisfiable by Z3.

However, if we enhance our Predicate Pairing strategy by allowing in the new definitions the addition of constraints taken from the abstract domain of Convex Polyhedra, then Z3 is indeed able to prove the satisfiability of the derived set of clauses, and hence of the equisatisfiable set $\{F_1\} \cup P$.

To get this result, we have implemented in the VeriMAP transformation system [12] a new version of the Predicate Pairing strategy enhanced with abstraction techniques based on Convex Polyhedra (as well as other abstract constraint domains such as Boxes, Bounded Differences, and Octagons). In our example, Predicate Pairing enhanced with abstraction on Convex Polyhedra introduces the following definitions (variables are automatically renamed by VeriMAP):

$\text{pp}_1(A, B, C, D, E, F, G, H, A, B, C, D, I, J, K, L) \leftarrow G \leq K - 1,$
$s_{12}(A, B, C, D, E, F, G, H), s_{22}(A, B, C, D, I, J, K, L).$

$\text{pp}_2(A, B, C, D, E, F, G, H, A, B, K, D, M, N, O, P) \leftarrow G \leq O - 1, A \leq B - 1,$
$K = A + C, s_{12}(A, B, C, D, E, F, G, H), s_{23}(A, B, K, D, M, N, O, P).$

and derives the following final set of clauses:

false $\leftarrow X_1 \leq X_2 - 1,$ $\text{pp}_1(A, B, C, D, E, F, X_1, H, A, B, C, D, I, L, X_2, N),$

$\text{pp}_1(A, B, C, D, E, F, G, H, A, B, C, D, I, J, K, L) \leftarrow G \leq K - 1, A \leq B - 1,$
$M = A + C, \text{pp}_2(A, B, C, D, E, F, G, H, A, B, M, D, I, J, K, L).$

$\text{pp}_2(A, B, C, D, E, F, G, H, A, B, K, D, M, N, O, P) \leftarrow G \leq O - 1, A \leq B - 2,$
$K = A + C, R = A + 1, T = A + C, S = D + T, X = A + 1, W = K + X,$
$Y = D + K, \text{pp}_2(R, B, T, S, E, F, G, H, X, B, W, Y, M, N, O, P).$

Now, the satisfiability of this set of clauses is trivial, and is easily checked by Z3, because it contains no constrained facts (that is, clauses with only constraints in their body), and hence a model is obtained by taking both $\text{pp}_1$ and $\text{pp}_2$ to be false.

The constraints occurring in the new predicate definitions are crucial for deriving a set of clauses without constrained facts. Indeed, it can easily be shown that if we introduce a predicate defined by a conjunction of predicates with distinct variables as arguments (that is, we perform ‘simple’ Predicate Pairing), then by unfolding the definition of that predicate we generate constrained facts derived from those of $s_{12}$ and $s_{22}$.

The equivalence of the two programs $P_1$ and $P_2$ with respect to the output value of $y$ can be proved in a similar way.
3 Constrained Horn Clauses over Numerical Domains

Let us recall some basic notions about: (i) abstract domains often used in static program analysis [8], and (ii) constrained Horn clauses (CHCs).

We consider the abstract domain of Convex (Closed) Polyhedra [3][10], CP for short, whose atomic constraints are of the form \( a_1 x_1 + \ldots + a_n x_n \leq a \), where \( a_i \)'s and \( x_i \)'s are real coefficients and variables, respectively. A constraint \( c \) is either true, or false, or an atomic constraint, or a conjunction of constraints.

Given a formula \( F \), by \( \forall(F) \) and \( \exists(F) \) we denote its universal and existential closure, respectively. By \( \text{vars}(F) \) we denote the set of variables occurring in \( F \). A constraint \( c \) is said to be satisfiable if \( \text{CP} \models \exists(c) \). Given two constraints \( c \) and \( d \), we say that \( c \) entails \( d \), and we write \( c \sqsubseteq d \), if \( \text{CP} \models \forall(c \rightarrow d) \). We say that \( c \) and \( d \) are equivalent if \( c \sqsubseteq d \) and \( d \sqsubseteq c \).

We also consider some other abstract constraint domains, namely Octagons, Bounded Differences, and Boxes, which are subdomains of Convex Polyhedra in the sense that they are defined by putting restrictions on the form of the polyhedra associated with the atomic constraints. These abstract domains have all true and false as constraints and are closed under conjunction.

The atomic constraints of the Octagons domain are inequalities of the form
\[ a_1 x_1 + a_2 x_2 \leq a, \quad a \in \mathbb{R} \quad \text{and} \quad a_i \in \{-1, 0, 1\} \quad \text{for} \ i = 1, 2. \]

The atomic constraints of the Bounded Differences domain are inequalities of the form
\[ a_1 x_1 + a_2 x_2 \leq a, \quad a \in \mathbb{R} \quad \text{and} \quad a_i \in \{-1, 0, 1\} \quad \text{for} \ i = 1, 2, \quad \text{and} \ a_1 \text{ is different from } a_2. \]

The atomic constraints of the Boxes domain are inequalities of the form \( x \leq a, \quad a \in \mathbb{R} \).

Each abstract constraint domain \( D \subseteq \text{CP} \) is endowed with some operators that we now define. Let \( c \) and \( d \) be two constraints in \( D \), or \( D \)-constraints.

The least upper bound operator is a function \( \sqcup : D \times D \rightarrow D \) such that
(i) \( c \sqsubseteq c \sqcup d \), (ii) \( d \sqsubseteq c \sqcup d \) and (iii) for all \( D \)-constraints \( c \), if \( c \sqsubseteq e \) and \( d \sqsubseteq e \), then \( c \sqcup d \sqsubseteq e \).

A widening operator is a function \( \nabla : D \times D \rightarrow D \) such that
(i) \( c \sqsubseteq c \nabla d \), (ii) \( d \sqsubseteq c \nabla d \), and (iii) for all chains \( y_0 \sqsubseteq y_1 \sqsubseteq \ldots \), the chain \( x_0 \sqsubseteq x_1 \sqsubseteq \ldots \), where \( x_0 = y_0 \quad \text{and} \quad \text{for} \ i > 0, x_{i+1} = x_i \nabla y_{i+1} \), is finite.

The abstraction operator for a subdomain \( D \) of \( \text{CP} \), is a function \( \alpha : \text{CP} \rightarrow D \) such that
(i) \( c \sqsubseteq \alpha(c) \), and (ii) for all \( D \)-constraints \( c \), if \( c \sqsubseteq e \), then \( \alpha(c) \sqsubseteq e \).

The projection of a \( D \)-constraint \( c \) onto a set \( X \) of variables, denoted \( c \downarrow X \), is a \( D \)-constraint \( c' \), with variables in \( X \), which is equivalent to \( \exists Y. c \), where \( Y = \text{vars}(c) - X \). Clearly, \( c \sqsubseteq c' \).

An atom is a formula of the form \( p(X_1, \ldots, X_m) \), where \( p \) is a predicate symbol different from ‘\( ' \quad \text{and} \quad X_1, \ldots, X_m \) are distinct variables.

A constrained Horn clause (or simply, a clause, or a CHC) is an implication of the form \( A \leftarrow c, G \) (comma denotes conjunction), where the conclusion (or head) \( A \) is either an atom or false, the premise (or body) is the conjunction of a constraint \( c \) and a (possibly empty) conjunction \( G \) of atoms. The empty conjunction is identified with true.
We assume that variables occur distinct in the atoms of a clause, although in the examples, for ease of reading, we feel free to write clauses with repeated occurrences of the same variable.

A set $S$ of CHCs is said to be satisfiable if $S \cup CP$ has a model, or equivalently, $S \cup CP \neq false$ (as done above, we identify convex polyhedra with linear arithmetic formulas).

4 Predicate Pairing with Abstraction

In this section we present an algorithm for transforming CHCs, called Abstraction-based Predicate Pairing (or APP strategy, for short), which combines Predicate Pairing [16] with abstraction operators acting on a given constraint domain (see Figure 2). The APP transformation strategy preserves satisfiability of clauses and has the objective of increasing the effectiveness of the satisfiability check that is performed by the subsequent application of a CHC solver.

The APP transformation strategy tuples together two or more predicates into a single new predicate which is equivalent to their conjunction. As shown in the example of Section 2, the interaction between the Predicate Pairing technique and the constraints among the variables of the predicates paired together (or tupled together, if more than two), may ease the discovery of the relations existing among the arguments of the individual predicates.

The APP strategy is parametric with respect to: (i) the abstract constraint domain which is considered, and (ii) a Partition operator that determines, given a clause, which are the atoms to be tupled together by splitting any conjunction $G$ of atoms in the body of the clause into $n$ ($\geq 1$) subconjunctions $G_1, \ldots, G_n$. By choosing the abstract constraint domain and the Partition operator in a suitable way, we can derive other transformations, and in particular, the Predicate Pairing strategy introduced in a previous paper [16], Linearization [13], and CHC Specialization [11, 28]. For example, the Predicate Pairing strategy is derived by taking the constraint domain to be the set of equalities between variables, and the Partition operator to be the one which, given a clause, returns a suitable set of pairs of atoms in its body [16].

The APP strategy is realized by performing a sequence of applications of the well-known unfold/fold rules [19]. In order to be self-contained, now we present the version of the Unfolding rule used in this paper. The other rules will be presented when describing the APP strategy. In the definition of the Unfolding rule and in the definitions of the other rules as well, we assume, without loss of generality, that: (i) every atom in a clause has distinct variables as arguments, and (ii) two atoms have pairwise disjoint sets of variables, and hence the relations among variables (including equalities) are explicitly written as constraints in the body of the clause.

Unfolding Rule. Let $P$ be a set of clauses and $C$ be a clause of the form $H \leftarrow c, L, A, R$, where $A$ is an atom and $L$ and $R$ are (possibly empty) conjunctions of atoms. Let us consider the set $\{A \leftarrow c_i, B_i \mid i = 1, \ldots, m\}$ made out
of all the clauses in $P$ whose head is $A$ (after renaming). By unfolding $C$ w.r.t. $A$ using $P$, we derive the set of clauses \{$(H \leftarrow c_i, L, B_i, R) \mid i = 1, \ldots, m$\}.

The APP strategy constructs a tree $Defs$ of definitions, that is, clauses whose head predicates do not occur in the input set $P$ of clauses. A definition $D$ is said to be a child of a definition $C$, and equivalently, $C$ is said to be the parent of $D$, if $D$ is introduced to fold a clause derived by unfolding from clause $C$.

The ancestor relation on $Defs$ is the reflexive, transitive closure of the parent relation.

**Input**: A set $P \cup \{C\}$ of clauses where $C$ is a clause whose head predicate does not occur in $P$.

**Output**: A set $TransfCls$ of clauses.

**Initialization**: $InCls := \{C\}$; $Defs$ is the tree made out of the root clause $C$ only; $TransfCls := P$;

\[\text{while there is a clause } C \text{ in } InCls \text{ of the form } H \leftarrow c, B \text{ do}\]

- **Unfolding**: From clause $C$ derive a set $U(C)$ of clauses by unfolding $C$ with respect to each atom occurring in its body using $P$;

- **Clause Deletion**: Remove from $U(C)$ all clauses with an unsatisfiable constraint;

- **Definition & Folding**: for every clause $E \in U(C)$ of the form $H \leftarrow d, G$ do

  \[\text{Partition the conjunction } G \text{ into } n (\geq 1) \text{ subconjunctions } G_1, \ldots, G_n;\]

  \[\text{for } i = 1, \ldots, n \text{ do}\]

  \[d_i := \alpha(d) \downarrow V_i, \text{ where } V_i \text{ is the set of variables in } G_i;\]

  \[\text{if in }Defs \text{ there is no clause } newp_i(V_i) \leftarrow e_i, G_i \text{ such that } d_i \sqsubseteq e_i \text{ then}\]

  \[\text{if in }Defs \text{ there is an ancestor clause of } C \text{ of the form } newq(V_i) \leftarrow f_i, G_i \text{ then } D_i := (newp_i(V_i) \leftarrow a_i, G_i), \text{ where } a_i = f_i \triangledown (f_i \sqcup d_i)\]

  \[\text{else } D_i := (newp_i(V_i) \leftarrow d_i, G_i);\]

  \[InCls := InCls \cup \{D_i\}; \quad \text{add } D_i \text{ as a child of } C \text{ in }Defs;\]

end-for;

$TransfCls := TransfCls \cup \{H \leftarrow d, newp_1(V_1), \ldots, newp_n(V_n)\}$;

$U(C) := U(C) \setminus \{E\}$;

end-for;

$InCls := InCls \setminus \{C\}$;

end-while

\[\text{Fig. 2. The Predicate Pairing with Abstraction algorithm (APP strategy)}\]

Note that, by construction, every constraint, either $a_i$ or $d_i$, occurring in a new definition $D_i$ introduced during the Definition & Folding phase, belongs to the abstract constraint domain.

The APP strategy terminates if there exists an integer $k$ such that, for any application of the Partition operator producing $G_1, \ldots, G_n$, the size of each $G_i$ is bounded by $k$. For instance, the Predicate Pairing strategy presented in a previous paper \[16\] guarantees that $k=2$ is such an integer bound. Indeed, if this bound exists, by the properties of the widening operator, the set of definitions that can be introduced by APP (and hence the number of executions of the while loop of the strategy) is finite.
Moreover, from well-known correctness results for the unfold/fold rules [19], it follows that the APP strategy preserves satisfiability.

**Theorem 1 (Soundness of Predicate Pairing with Abstraction).** Let the set $P \cup \{C\}$ of clauses be the input of the APP strategy. If the strategy terminates and returns a set TransfCls of clauses, then $P \cup \{C\}$ is satisfiable iff TransfCls is satisfiable.

A CHC Specialization strategy with Abstraction, which we call ASp strategy, can be derived by instantiating the APP strategy. The ASp strategy is obtained by using the Partition operator that, given a conjunction of atoms $A_1, \ldots, A_n$ in the body of a clause, returns $n$ subconjunctions, each consisting of a single atom $A_i$, for $i = 1, \ldots, n$ (that is, no predicate pairing is made). It has been shown that CHC Specialization can be very useful for increasing the effectiveness of satisfiability checkers [11, 15, 28].

In the next section we will present the results of the experiments we have performed by combining the APP strategy and the ASp strategy in various ways. In particular, these results illustrate the role of abstract constraint domains for Predicate Pairing, and they also show the usefulness of the APP strategy for making CHC solvers more effective when performing satisfiability checking.

## 5 Experimental Evaluation

In this section we present the results of the experimental evaluation we have performed for assessing the effectiveness and the efficiency of the APP strategy.

We have implemented that strategy using the VeriMAP transformation system [12] and the Parma Polyhedra Library [3]. Then, in order to check the satisfiability of the clauses generated by the APP strategy we have used the Z3 solver [17]. The verification process is depicted in Figure 3.

![Fig. 3. The verification process based on: (i) the APP strategy, (ii) the Parma Polyhedra Library, and (iii) the CHC solver Z3.](image)

*Implementation of the APP strategy.* We have ported the VeriMAP tool from SICStus Prolog 3.12.5 to SWI-Prolog 7.4.2 and we have extended its transformation engine so to use the abstract domains and the associated operations provided by the Parma Polyhedra Library (PPL).
In particular, we have considered the following abstract domains: (i) Universe (that is, the whole $n$-dimensional space), (ii) Boxes, (iii) Bounded Differences (also called Bounded Difference Shapes, or BDS, for short), (iv) Octagons (also called Octagonal Shapes, or OS, for short), and (v) Convex Polyhedra (also called Convex Polyhedra, or CP, for short), together with the operations of projection, least upper bound, inclusion check (that is, entailment), emptiness check (that is, satisfiability), and widening. We have used Convex Polyhedra with the widening operator of Halbwachs [25] (CP-H), and with the widening operator of Bagnara et al. [2] (CP-B). Predicate Pairing with the Universe domain is the ‘simple’ Predicate Pairing we mentioned in Section 2, that introduces new definitions having no constraints in their body.

Since VeriMAP natively represents constraints using the Constraint Logic Programming (CLP) syntax, we had to use the facilities provided by PPL for constructing objects of the abstract domains starting from constraints represented in CLP, and vice versa, for constructing constraints represented in CLP starting from PPL objects. We are currently working for overcoming the inefficiency due to these translations of representation.

**Benchmark suite.** We have considered a benchmark suite consisting of 136 sets of CHCs, for a total number of 1655 clauses, representing verification problems of various relational properties, such as loop optimizations, equivalence, monotonicity, injectivity, functional dependency, and non-interference [4,5,13,16,21].

**Experiments.** We have performed the following six experiments, where the parameter $x$ stands for an abstract constraint domain, which is either Universe, or Boxes, or BDS, or OS, or CP-H, or CP-B:

0. Z3
1. ASP($x$); Z3
2. ASP($x$); APP($x$); Z3
3. ASP($x$); APP($x$); ASP($x$); Z3
4. APP($x$); Z3
5. APP(Universe); ASP($x$); Z3

Experiment 0 consists in running Z3 directly on the 1655 clauses that encode the 136 problems. Z3 solved 28 problems, either positively or negatively, by providing the answer ‘satisfiable’ or ‘unsatisfiable’, respectively, in an average time of 2.36 seconds per problem (see Frame 0 in Table 1).

The other experiments with domain $x$ have been performed as we now explain for Experiments 1 and 2 with domain $x = OS$ (see Frames 1 and 2, respectively, in Table 1). By using VeriMAP we have run ASP(OS) on the initial 1655 clauses. Then, on the resulting 3540 clauses (with an average time of 0.73 seconds per problem), either (Experiment 1) we have run Z3 that solved 28 problems (with an average time of 4.10 seconds per problem) or (Experiment 2) by using again VeriMAP, we have run APP(OS) that produced 20361 clauses (with an average time of 4.60 seconds per problem). Finally, acting on these 20361 clauses, Z3 solved 121 problems out of 136 (with an average time of 3.90 seconds per solved problem). In Column SizeRatio we reported the increase in the number of clauses due to the transformations performed by VeriMAP. In particular, ASP(OS) en-
larged the size of about $2.14 (= \frac{3540}{1655})$ times, and APP(OS) of about 5.83 times.

During an experiment it may be the case that VeriMAP does not complete the execution of the ASp(x) strategy, or the APP(x) strategy, within the given timeout of 300 seconds. In that case the clauses of the problem for which the timeout occurred, are not passed to the subsequent step of the experiment (and they are not considered in the computation of the average times and the size ratios).

We have used the Z3 4.5.0 solver with the Duality fixed-point engine on an Intel Xeon CPU E5-2640 2.00GHz processor with 64GB of memory under the GNU/Linux 64 bit operating system CentOS 7.

Results. The results of the experiments are summarized in Table 1. In order to have a more detailed understanding of the results, for the Experiments 2, 3, and 5, we have adopted an incremental presentation in the sense that, for instance, Frame 2 presents the results for ‘APP(x) ; Z3’, and the results concerning the first step ‘ASp(x)’ are to be read in the first five columns of Frame 1 where the experiment ‘ASp(x) ; Z3’ is presented.

First we observe that the use of the ASp and APP strategies significantly increases the number of problems which Z3 can solve (see Frames 0 and 2 of Table 1). Indeed, for Experiment 2 with domain OS the number of problems solved by Z3 is 121, while Z3 alone solved 28 problems only.

Our results show that the use of abstract domain Boxes is not very effective. Indeed, in Frame 2 we see that 73 problems only are solved (with respect to 121 for OS). Similarly, in Frame 3, only 77 problems are solved (with respect to 121 for BDS). The poor performance of Boxes with respect to those of OS and BDS can be explained by the fact that Boxes (that is, intervals of individual variables) are not expressive enough to represent relations among variables of programs. Hence, they are of little help for proving the given relational property. Note, however, that if precision is further increased, from BDS and OS to Convex Polyhedra, the performance does not increase. Indeed, the number of solved problems decreases because the computations required for CP-H and CP-B are more expensive and the timeout limit is reached more often.

Looking at frames 1, 2, and 3 (Columns for Z3) we see that the main contribution to the increase of efficacy in proving the desired properties is due to the APP strategy (when used with a suitable domain, such as BDS, OS, CP-H, and CP-B), rather then the ASp strategy. Indeed, the comparison of Frames 2 and 3 (Column SolvedProbs) shows that the use of ASp after APP does not make any significant change in the efficacy. Note also that applying ASp before APP for BDS and OS does not really pay off. Indeed, the comparison of Frame 2 with Frame 4 shows that more problems can be solved with lower verification time by directly applying the APP strategy starting from the clauses encoding the verification problems.

The comparison between Frames 4 and 5 (Columns for Z3) tells us that the effect of the APP(x) strategy, for a given abstract domain x, can also be obtained
| Domain x | InProbs | OutProbs | OutCls | AvgTime1 | SizeRatio | SolvedProbs | AvgTime2 |
|----------|---------|----------|--------|----------|-----------|-------------|----------|
| **1.**   | ASp(x) ; Z3 |          |        |          |           |              |          |
| Boxes    | 136     | 136      | 3111   | 0.67     | 1.88      | 28          | 3.15     |
| BDS      | 136     | 136      | 2629   | 0.66     | 1.59      | 28          | 3.79     |
| OS       | 136     | 136      | 3540   | 0.73     | 2.14      | 28          | 4.10     |
| CP-H     | 136     | 136      | 3021   | 0.66     | 1.83      | 34          | 3.95     |
| CP-B     | 136     | 136      | 3633   | 0.69     | 2.20      | 36          | 10.14    |

| **2.** Assuming ASp(x) already performed: | APP(x) ; Z3 |          |        |          |           |              |          |
| Boxes    | 136     | 134      | 27753  | 1.85     | 9.37      | 73          | 2.20     |
| BDS      | 136     | 136      | 12793  | 2.60     | 4.87      | 119         | 3.69     |
| OS       | 136     | 134      | 20361  | 4.60     | 5.83      | 121         | 3.90     |
| CP-H     | 136     | 135      | 16193  | 3.09     | 5.39      | 113         | 0.93     |
| CP-B     | 136     | 127      | 12554  | 2.82     | 3.80      | 114         | 3.65     |

| **3.** Assuming ASp(x) ; APP(x) already performed: | ASp(x) ; Z3 |          |        |          |           |              |          |
| Boxes    | 134     | 134      | 45970  | 2.57     | 1.66      | 77          | 3.54     |
| BDS      | 136     | 136      | 26683  | 3.30     | 2.09      | 121         | 3.86     |
| OS       | 134     | 134      | 36871  | 4.89     | 1.81      | 119         | 3.06     |
| CP-H     | 135     | 135      | 31521  | 3.91     | 1.95      | 115         | 2.05     |
| CP-B     | 127     | 127      | 25495  | 4.59     | 2.03      | 112         | 1.27     |

| **4.** | APP(x) ; Z3 |          |        |          |           |              |          |
| Boxes    | 136     | 136      | 40997  | 0.71     | 2.48      | 73          | 2.00     |
| BDS      | 136     | 136      | 20926  | 2.27     | 12.26     | 78          | 2.01     |
| OS       | 136     | 136      | 8630   | 1.38     | 5.21      | 121         | 2.45     |
| CP-H     | 136     | 135      | 13762  | 2.97     | 8.37      | 120         | 1.77     |
| CP-B     | 136     | 131      | 11718  | 2.22     | 7.35      | 113         | 2.19     |

| **5.** Assuming APP(Universe) already performed: | ASp(x) ; Z3 |          |        |          |           |              |          |
| Boxes    | 136     | 136      | 19932  | 2.23     | 4.87      | 74          | 3.07     |
| BDS      | 136     | 136      | 8387   | 1.46     | 2.05      | 120         | 1.63     |
| OS       | 136     | 135      | 14065  | 2.93     | 3.46      | 118         | 1.39     |
| CP-H     | 136     | 135      | 14111  | 2.57     | 3.47      | 112         | 1.44     |
| CP-B     | 136     | 129      | 9831   | 2.41     | 2.53      | 113         | 2.05     |

Table 1. Frame 0 reports the results obtained by using Z3 on the input clauses. Frames 1–5 report the results obtained by transforming the clauses using VeriMAP and then checking their satisfiability using Z3. Column ‘Domain x’ reports the chosen abstract domain. Column ‘InProbs’ (‘OutProbs’) reports the number of problems given as input to (produced as output by) VeriMAP. Columns ‘OutCls’ and ‘AvgTime1’ report the number of output clauses and the average time taken by VeriMAP. Column ‘SizeRatio’ reports the value OutCls/InCls, where InCls is the number of clauses of the problems for which VeriMAP terminates within the timeout. Columns ‘SolvedProbs’ and ‘AvgTime2’ report the number of solved problems and the average time taken by Z3. The times are the CPU seconds spent in user mode. A timeout of 300 seconds has been set for each run of VeriMAP and Z3.
in two steps: (i) performing ‘simple’ Predicate Pairing, that is, APP(Universe), and (ii) applying ASp(x) which realizes the abstraction on the domain x.

The large values in Column OutCls show that in our implementation more effort should be put in keeping the number of clauses generated by the APP and ASp strategies small, and indeed ongoing work is devoted to the design of new transformation techniques that have that effect.

6 Related Work and Conclusions

We have proposed various ways of combining transformation and abstraction techniques for constrained Horn clauses with the goal of verifying relational properties of imperative programs. To this aim we have presented Predicate Pairing and Specialization algorithms that are parameterized with respect to a given abstract constraint domain and its operators. Then we have presented an extensive experimental evaluation of CHC satisfiability problems encoding relational verification problems. Our experiments show that suitable combinations of transformations and abstraction dramatically increase the effectiveness of the Z3 solver on the given benchmark. The most effective techniques combine Predicate Pairing and Abstraction based on Bounded Differences or Octagons [2, 35], that is, constraint domains that are quite simple, but expressive enough to capture the relations between predicate arguments.

Relational verification has been extensively studied, and still receives much attention as a relevant problem in the field of software engineering [4, 6, 7, 16, 20, 21, 22, 30, 32, 39, 40, 41]. In particular, during the software development process it may be helpful to prove that the semantics of a new program version has some specified relation with the semantics of an old version.

Among the various methods to prove relational properties, those by Mordvinov and Fedyunkovich [36] and by Felsing et al. [21] are the most closely related to ours. The method proposed in the former paper [36] introduces the notion of CHC product (somewhat related to our Predicate Pairing strategy), that is, a CHC transformation that synchronizes computations to improve the effectiveness of the CHCs satisfiability check. The latter method proposed by Felsing et al. [21] presents proof rules for relations between imperative programs that are translated into constrained Horn clauses. The satisfiability of these clauses which entails the relation of interest, is then checked by state-of-the-art CHC solvers.

In the literature there are also partially automated approaches that reduce the problem of verifying relational properties (in particular, program equivalence) to a standard verification problem by using some composition operator between imperative programs [4, 30, 39, 41].

The reuse of existing verification techniques for proving relational program properties can also be realized by first computing summaries of the two programs, that is, relations between the input and output values of the programs, and then proving relational properties of these summaries. This approach is followed by: (i) Ganty et al. [22], whose method also supports a restricted class
of recursive programs, (ii) Hawblitzel et al. [26], who use automated theorem provers for proving relative termination, and (iii) Lopes and Monteiro [32], who use (possibly nonlinear) integer polynomials and recurrences for summarizing loops of programs acting on integers.

A method that, like ours, is parametric with respect to the semantics of the programming languages in which the programs to be verified are written, is proposed by S. Ciobăcă et al. [7], who present a language-independent deductive system for proving mutual equivalence of programs.

The Predicate Pairing technique we present in this paper is a descendant of well-known techniques for logic program transformation, such as Tupling [38] and Conjunctive Partial Deduction [18], which derive new predicates defined in terms of conjunctions of atoms. The goal of these techniques is the derivation of efficient logic programs by: (i) avoiding multiple traversals of data structures and repeated evaluations of predicate calls, and (ii) producing specialized program versions that take into account partial information on the input values. An integration of Conjunctive Partial Deduction and abstract interpretation, called Abstract Conjunctive Partial Deduction, has also been presented in the literature [31]. Recent work has shown that the extension of these transformation techniques to constrained Horn clauses can play a significant role in improving the effectiveness of CHC solvers for proving properties of imperative programs, and in particular for verifying relational properties [14, 16].

The CHC Specialization strategy we consider in this paper is a variant of specialization techniques for (constraint) logic programs which have been proposed to support program verification [1, 11, 13, 15, 24, 28, 29, 34, 37]. However, these techniques are focused on the verification of partial or total correctness of single programs, and not on the relational verification.

References

1. E. Albert, M. Gómez-Zamalloa, L. Hubert, and G. Puebla. Verification of Java bytecode using analysis and transformation of logic programs. In Proc. PADL ’07, LNCS 4354, pages 124–139. Springer, 2007.
2. R. Bagnara, P. M. Hill, E. Ricci, and E. Zaffanella. Precise widening operators for convex polyhedra. *Science of Computer Programming*, 58(1):28 – 56, 2005.
3. R. Bagnara, P. M. Hill, and E. Zaffanella. The Parma Polyhedra Library: Toward a complete set of numerical abstractions for the analysis and verification of hardware and software systems. *Science of Computer Programming*, 72(1–2):3–21, 2008.
4. G. Barthe, J. M. Crespo, and C. Kunz. Relational verification using product programs. In Proc. *FM ’11*, LNCS 6664, pages 200–214. Springer, 2011.
5. N. Benton. Simple relational correctness proofs for static analyses and program transformations. In Proc. *POPL ’04*, pages 14–25. ACM, 2004.
6. S. Chaki, A. Gurfinkel, and O. Strichman. Regression verification for multi-threaded programs. Proc. *VMCAI ’12*, LNCS 7148, pages 119–135. Springer, 2012.
7. S. Ciobăcă, D. Lucanu, V. Rusu, and G. Rosu. A language-independent proof system for mutual program equivalence. In Proc. *ICFEM ’14*, LNCS 8829, pages 75–90. Springer, 2014.
8. P. Cousot and R. Cousot. Abstract interpretation: A unified lattice model for static analysis of programs by construction of approximation of fixpoints. In Proc. POPL ’77, pages 238–252. ACM, 1977.
9. P. Cousot, R. Ganty, and J.-F. Raskin. Fixpoint-guided abstraction refinements. In Proc. SAS ’07, LNCS 4634, pages 333–348. Springer, 2007.
10. P. Cousot and N. Halbwachs. Automatic discovery of linear restraints among variables of a program. In Proc. POPL ’78, pages 84–96. ACM, 1978.
11. E. De Angelis, F. Fioravanti, A. Pettorossi, and M. Proietti. Program verification via iterated specialization. Sci. Comput. Program., 95, Part 2:149–175, 2014.
12. E. De Angelis, F. Fioravanti, A. Pettorossi, and M. Proietti. VeriMAP: A tool for verifying programs through transformations. Proc. TACAS ’14, LNCS 8413, pages 568–574. Springer, 2014. Available at: http://www.map.uniroma2.it/VeriMAP.
13. E. De Angelis, F. Fioravanti, A. Pettorossi, and M. Proietti. Proving correctness of imperative programs by linearizing constrained Horn clauses. Theory and Practice of Logic Programming, 15(4-5):635–650, 2015.
14. E. De Angelis, F. Fioravanti, A. Pettorossi, and M. Proietti. A rule-based verification strategy for array manipulating programs. Fundamenta Informaticae, 140(3-4):329–355, 2015.
15. E. De Angelis, F. Fioravanti, A. Pettorossi, and M. Proietti. Semantics-based generation of verification conditions by program specialization. In: Selected and Extended papers from the International Symposium on Principles and Practice of Declarative Programming 2015. Sci. Comput. Program., 147 (78–108), 2017.
16. E. De Angelis, F. Fioravanti, A. Pettorossi, and M. Proietti. Relational verification through Horn clause transformation. In Proc. SAS ’16, LNCS 9837, pages 147–169. Springer, 2016.
17. L. M. de Moura and N. Bjørner. Z3: An efficient SMT solver. In Proc. TACAS ’08, LNCS 4963, pages 337–340. Springer, 2008.
18. D. De Schreye, R. Glück, J. Jørgensen, M. Leuschel, B. Martens, and M. H. Sørensen. Conjunctive partial deduction: Foundations, control, algorithms, and experiments. Journal of Logic Programming, 41(2–3):231–277, 1999.
19. S. Etalle and M. Gabbrielli. Transformations of CLP modules. Theoretical Computer Science, 166:101–146, 1996.
20. G. Fedyukovich, A. Gurfinkel, and N. Sharygina. Property directed equivalence via abstract simulation. CAV ’16, LNCS 9780, pages 433–453. Springer, 2016.
21. D. Felsing, S. Grebing, V. Klebanov, P. Rümmer, and M. Ulbrich. Automating regression verification. Proc. ASE ’14, pages 349–360. ACM, 2014.
22. P. Ganty, R. Iosif, and F. Konečný. Underapproximation of procedure summaries for integer programs. TACAS ’13, LNCS 7795, pages 245–259. Springer, 2013.
23. B. Godlin and O. Strichman. Inference rules for proving the equivalence of recursive procedures. Acta Informatica, 45(6):403–439, 2008.
24. M. Gómez-Zamalloa, E. Albert, and G. Puebla. Decompilation of Java bytecode to Prolog by partial evaluation. Inf. Softw. Technol., 51(10):1409–1427, 2009.
25. N. Halbwachs, Y. E. Proy, and P. Roumanoff. Verification of real-time systems using linear relation analysis. Formal Methods in Sys. Design, 11:157–185, 1997.
26. C. Hawblitzel, M. Kawaguchi, S. K. Lahiri, and H. Rebélo. Towards modularly comparing programs using automated theorem provers. In Proc. CADE ’13, LNCS 7896, pages 282–299. Springer, 2013.
27. H. Hojjat, F. Konecný, F. Garnier, R. Iosif, V. Kuncak, and P. Rümmer. A verification toolkit for numerical transition systems. In Proc. FM ’12, LNCS 7436, pages 247–251. Springer, 2012.
28. B. Kafle and J. P. Gallagher. Constraint specialisation in Horn clause verification. *Science of Computer Programming*, 137:125–140, 2017.
29. B. Kafle and J. P. Gallagher. Horn clause verification with convex polyhedral abstraction and tree automata-based refinement. *Computer Languages, Systems & Structures*, 47:2–18, 2017.
30. S. K. Lahiri, K. L. McMillan, R. Sharma, and C. Hawblitzel. Differential assertion checking. In Proc. *ESEC/FSE ’13*, pages 345–355. ACM, 2013.
31. M. Leuschel. A framework for the integration of partial evaluation and abstract interpretation of logic programs. *ACM TOPLAS*, 26(3):413–463, 2004.
32. N. P. Lopes and J. Monteiro. Automatic equivalence checking of programs with uninterpreted functions and integer arithmetic. *Software Tools for Technology Transfer*, 18(4):359–374, 2016.
33. K. L. McMillan and A. Rybalchenko. Solving constrained Horn clauses using interpolation. MSR Technical Report 2013-6, Microsoft Report, 2013.
34. M. Méndez-Lojo, J. A. Navas, and M. V. Hermenegildo. A flexible, (C)LP-based approach to the analysis of object-oriented programs. In Proc. *LOPSTR ’07*, LNCS 4915, pages 154–168. Springer, 2008.
35. A. Miné. The octagon abstract domain. *Higher-Order and Symbolic Computation*, 19(1):31–100, 2006.
36. D. Mordvinov and G. Fedyukovich. Synchronizing constrained Horn clauses. In Proc. *LPAR ’17*, EPiC Series in Computing 46, pages 338–355. EasyChair, 2017.
37. J. C. Peralta, J. P. Gallagher, and H. Saglam. Analysis of imperative programs through analysis of constraint logic programs. In Proc. *SAS ’98*, LNCS 1503, pages 246–261. Springer, 1998.
38. A. Pettorossi and M. Proietti. Transformation of logic programs: Foundations and techniques. *Journal of Logic Programming*, 19,20:261–320, 1994.
39. O. Strichman and M. Veitsman. Regression verification for unbalanced recursive functions. In Proc. *FM ’16*, LNCS 9995, pages 645–658. Springer, 2016.
40. S. Verdoolaege, G. Jaussens, and M. Bruynooghe. Equivalence checking of static affine programs using widening to handle recurrences. *ACM TOPLAS*, 34(3):11, 2012.
41. A. Zaks and A. Pnueli. CoVaC: Compiler validation by program analysis of the cross-product. In Proc. *FM ’08*, LNCS 5014, pages 35–51. Springer, 2008.