An analysis of the models informativeness in parametric identification problems

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Abstract. Solving problems of parametric identification with uncertainty in the initial data due to their inaccuracy and incompleteness complicates, and, in some cases, even rules out the possibility of using classic mathematical and statistical methods as the latter are meant to deal with sufficiently large amounts of observations. These conditions involve the use of specific methods to directly determine the exact type of functional link and to verify its qualitative characteristics. A set of such characteristics also forms an idea of the informativeness of a model. This paper considers the characterization of this concept in relation to parametric identification problems and presents a method of their solution where the procedure of the analysis of informativeness of the obtained models is formalized. Approbation of the developed technique was carried out on the example of parametric identification of the time series model. This time series cannot be linearized. A small number of observations were also known. All these conditions precluded the use of statistical methods to construct a time series model.

1. Introduction

When applied to mathematical models, the concept of "informativeness" has no explicit interpretation. It is used in accordance with the general logic [1, 2], according to which this term denotes saturation of information, with the subject area of research considered [3–6]. For example, in the theory of measurements, informativeness is understood as a measure of measurement accuracy [7], in the theory of systems, statistical physics, thermodynamics and quantum mechanics – as an information measure [8, 9]. The common thing while using this term is that the informativeness assessment is carried out by means of quantitative criteria, the values of which serve as the basis for the conclusion about qualitative features of interest to the researcher. And, first of all, such signs include accuracy and stability.

Model accuracy is a numerical criterion value that characterizes the degree of proximity of calculated and experimental data. At the same time, the smaller the value of this numerical criterion, the higher the accuracy of the model estimation, so the obvious desire is to achieve its minimum.

In the case of parametric identification problems, the concept of model stability means a slight variation of its resulting characteristics with small parameter changes [10, 11]. In other words, if small...
changes in a parameter (or several parameters) resulting in similar but still new models retain their adequacy and acceptable accuracy, then the original model can be considered as having a stability property. The development of computer engineering at the present stage has led to the use of the computational procedures for the analysis of the stability of models, during which the behavior of the obtained solution is investigated while the parameters vary near their fixed values accepted as optimal.

The analysis of the informativeness of the identified dependencies is of particular importance in the case of uncertainty in the initial data due to their inaccuracy and incompleteness. Inaccuracy results from the fact that the initial data are obtained from measurements that do not, a priori, produce absolutely accurate values of the observed values, and incompleteness is characteristic of research objects, for which it is not possible to carry out a large number of experiments due to, for example, their high cost or fundamental non-feasibility. In such situations, any factor can have a significant impact on model identification results.

In view of the above, the purpose of this paper is to develop a method of parametric identification with uncertainty in the initial data, where the synthesis of the two problems solution will be carried out: the problem of determining the model parameters and the problem of analysing the informativeness of the obtained solution.

2. Methods and models
To solve the problem of parametric identification is to determine in some sense the best parameters of functional dependence:

\[ y = f(\bar{\alpha}, \bar{x}). \]  

(1)

At the same time, the existence of a set of experimental data \( \Upsilon \) and an identification algorithm \( \mathcal{M} \) are mandatory.

**Definition 1.** Under the analysis of model informativeness we will understand the quantitative estimation of the impact of the basic elements of the identified dependence (1), the set \( \Upsilon \) and the algorithm \( \mathcal{M} \) on its qualitative characteristics.

By the term "qualitative characteristics" we will mean the criteria of interest to the researcher, which will be denoted through \( K \). We will consider accuracy and stability as such criteria in this paper.

The basic elements of dependence (1) will include variables ( \( y \) and \( \bar{x} \)), parameters \( \bar{\alpha} \), and constraints ( \( G = \{g_i(\bar{x}, y) \leq 0, l = 1, L\} \) ). These basic elements are essentially defined by the model specification (1). The entered notations allow us to write as follows: \( K = K(f, \Upsilon, \mathcal{M}) \).

**Definition 2.** Let's consider the model (1) informative if with preset \( f \), \( \Upsilon \) and \( \mathcal{M} \) \( \exists \pi: K \in K^\pi \) where \( K^\pi \) is a set of acceptable values of qualitative characteristics.

Let’s explain how the listed elements of the model affect its informativeness. The set of \( f, \Upsilon \) \( \mathcal{M} \) defines a set of permissible values of the model (1) variables. If no point in this set provides the performance of at least one qualitative characteristic at an acceptable level, the model cannot be considered informative. Possible reasons for this may be too strict constraints of the model, its inaccurate specification, where some important factors, incorrectly specified numerical parameters or coefficients, etc. were not taken into account.

If, in the case of the set of optimal solutions \( \Lambda^\prime = \{\bar{\alpha}: K = K^\pi\} \), the system of constraints is such that a large part of the conditions is not satisfied as equations, and the discrepancy between the left and right parts of such constraints is significant, this is indicative of either the unreasonableness of the use of these constraints or the need to revise their specification to improve the relevance. Thus, for example, in the case of Figure 1, corresponding to a hypothetical linear model, the constraints given by the semiplanes bounded by the lines \( k, l, m \) were not affected by the formation of a set of permissible values \( S \). This gives rise to a clarification of either the method of formalizing the above-
mentioned constraints or some of their coefficients, which may not have been sufficiently determined on the basis of the available experimental information.

Figure 1. Geometric interpretation of constraint effects.

If the set of optimal solutions \( \Lambda^* \) is large, there are many potentially acceptable ways to set the final form of the model. This means that the set \( \Lambda^* \) has a high degree of uncertainty, so that the information value of the model (1) cannot be considered high. The degree of uncertainty of the set \( \Lambda^* \) can be estimated, for example, by the diameter of this set. This type of uncertainty can be reduced by changing the kind of optimization criterion in the identification algorithm \( M \) (for example, in case of linear problems, a slight variation of target functions coefficients can result in the set of optimal solutions given by the polyhedron being brought to a single point).

The informativeness analysis should result in quantitative estimations, on the basis of which conclusions can be drawn on the need to refine individual experimental data, carry out additional experiments, check or change model elements, etc.

In conditions of uncertainty in the initial data, it is relevant to use non-statistical methods to solve parametric identification problems [12–14]. This is due to the fact that statistical methods are initially aimed at the study of numerous similar observations, for which a number of properties must be fulfilled, such as, for example, the same distribution of observed quantities and their independence. Obviously, in the case of limited experimental data, verification of such properties may not, in principle, be feasible.

One approach to solving parametric identification problems in conditions of uncertainty in the initial data is developing in the context of the ideas expressed in the paper of L. V. Kantorovich [15] and is based on the search of exact bilateral boundaries for the desired parameters of mathematical models, which ensure fulfillment of the specified requirements for mathematical description [16–18]. According to this approach, the parametric identification algorithm \( M \) can be represented by three stages.

In the first stage, the maximum deviation of calculated and experimental values is determined - the maximum permissible approximation error \( \xi^* \), which ensures the fairness of the inequality system

\[
\left| \hat{y}_t - y_t \right| \leq \xi^* , \quad t = 1, m .
\]

It is proposed to calculate the maximum permissible approximation error \( \xi^* \) using the model:

\[
\xi \rightarrow \min_{\pi \in \Lambda^*},
\]
where \( \Lambda^0 \) is a set of initial approximations of the vector \( \bar{a} \).

In the second stage - for each parameter of the identified dependence the maximum permissible estimates are calculated, which are the boundaries of uncertainty intervals.

**Definition 3.** We will term the line \( \left[ a_j, \bar{a}_j \right] = \left[ a_j, (\bar{a}_j, \bar{a}_j^t) \right] : \forall a^0_j \notin \left[ a_j, \bar{a}_j \right] \exists \bar{a} = \{a_1, \ldots, a_{j-1}, a^0_j, a_{j+1}, \ldots, a_p\} : \left| \hat{y}_\tau - y_\tau \right| \leq \bar{\varepsilon}^*, t = 1, m \) as the uncertainty interval by the parameter \( a_j, j = 1, p \).

The maximum permissible parameter estimates can be found on the basis of the model:

\[
\begin{align*}
\hat{a}_j & \rightarrow \min_{\pi \in \Lambda^0} \left( \max_{\pi \in \Lambda^0} \right), \ j = 1, p \\
\left| \hat{y}_\tau - y_\tau \right| & \leq \bar{\varepsilon}^*, t = 1, m, \\
g_j(\bar{x}, y) & \leq 0, l = 1, L.
\end{align*}
\]

In the third stage, in the set of uncertainties

\[
\Lambda = [a_1, \bar{a}_1] \times \ldots \times [a_p, \bar{a}_p],
\]

representing an external approximation of all the range of the parameter \( \bar{a} = \{a_1, \ldots, a_p\} \) values, that guarantee fulfillment of the conditions (2), an optimal vector of the \( \bar{a}^* \) parameters is searched.

This is achieved either by solving the optimization problem or by a computational experiment. In the case of the latter, on a set of uncertainties, a discrete grid \( \{\bar{a}^k\} \) should be introduced and a computational experiment should be done to identify a subset \( I \subset \{\bar{a}^k\} \), the elements of which ensure the fairness of the model (4) constraints. (Let's term \( I \) as an information set.) An optimal set of parameters \( \bar{a}^* \) must then be established from the information set \( I \) based on the rule formulated in accordance with the researcher’s views.

### 3. Analysis of informativeness

An analysis of informativeness will be carried out from the points of view of the above mentioned qualitative characteristics - accuracy and stability. To do this, we will formulate (C1-C4) informativeness conditions taking into account the basic elements of the model.

**C1.** Variables and parameters of model (1) are informative if the achieved accuracy of experimental data description according to the researcher is acceptable.

In the context of the above parametric identification algorithm, \( \bar{\varepsilon}^* \) characterizes the greatest divergence of calculated and experimental data, and is therefore a characteristic of the absolute accuracy of the model (1). Thus, if the researcher is satisfied with the value of the maximum permissible approximation error, the C1 condition can be considered fulfilled.

In addition to the absolute accuracy index of the model, we can use the relative accuracy measure - the average approximation error:

\[
\bar{A} = \frac{1}{m} \sum_{i=1}^{m} \left| \frac{\hat{y}_\tau - y_\tau}{y_\tau} \right|,
\]

where \( m \) is the number of experimental points.
Limits of the acceptable values are known for this indicator [19]. Therefore, in the case where the values of criterion (5) fall within the range of 8-10%, the relative accuracy should be considered satisfactory and the information value condition C1 satisfied. Note that the range of variation of the values of the criterion (5) for the elements of the set $\Lambda^*$ (or information set $I$) \[ \left[ \min_{\Lambda^*} A - \max_{\Lambda^*} A \right] \] is also in a sense a characteristic of the uncertainty measure of the obtained solution of the parametric identification problem.

C2. The model (1) constraint \( l \) is informative if it is involved in the formation of a set of optimal solutions to the $\Lambda^*$ parametric identification problem (the information set \( I \)).

The verification of this condition should be carried out in the third stage of solving the parametric identification problem. In particular, when conducting a computational experiment to define the set \( I \), it is necessary to record information on exactly how constraints are fulfilled at nodal points: as equalities, or as strict inequalities. If it is found that at all nodal points of the grid on the set of uncertainties $\Lambda$ some constraints are fulfilled as strict inequalities of one sign, this may lead to their revision or correction. In addition, for the optimal solution $\bar{a}^*$, the most informative constraints can be identified, which should include those that are fulfilled as equalities. Since these are such constraints that do not improve the values of the objective functions in the problems (3) and (4), this may lead to their adjustment or refinement.

C3. A set of potentially acceptable parameters for identifying a model is informative if its degree of uncertainty satisfies the researcher. This condition is formalized as:

\[
\text{diam } \Lambda^* \leq d^*,
\]

where $d^*$ is a fixed threshold value.

C4. The set of experimental data $\mathbf{Y}$ is informative if the model \( y = f(\bar{a}^*, \bar{x}) \) identified by the results of the application of the algorithm $\mathbf{M}$ has the property of stability in terms of parameters and the initial data.

It is proposed to check this informativeness condition on the basis of the analysis of the set of values of the initial data, with which the set problem of parametric identification is solved taking into account the "strengthening" of requirements to the achieved level of $\xi^*$ description accuracy. The formalized representation of this set is as follows:

\[
\mathbf{X}^* = \{ \bar{\mathbf{y}}_t, t = 1, m \mid |\bar{\mathbf{y}}_t - \mathbf{y}_t| \leq \xi^* \mathbf{\lambda}, 0 \leq \lambda < 1 \}.
\]

The definition of the precise area $\mathbf{X}'$ boundaries for large dimension tasks can be reduced to cumbersome calculations. For this reason, it may be more reasonable to identify the boundaries of the approximation set $\mathbf{X}^*$ having a simpler structure [20, 21]. It is proposed to identify the set $\mathbf{X}^*$ based on the following model:

\[
\zeta \rightarrow \min_{\zeta, |\delta_i|, |\gamma_t|, |\varphi_j|} \left\{ \begin{array}{l}
|\bar{\mathbf{y}}_t - \mathbf{y}_t| \leq \xi^* \mathbf{\lambda}, t = 1, m \\
|1 - \delta_t| \leq \zeta, t = 1, m, \\
|1 - \gamma_t| \leq \zeta, i = 1, n \\
|1 - \varphi_j| \leq \zeta, j = 1, p
\end{array} \right\}
\]

(6)
where \( t \geq 0,  y_i \geq 0, \, \partial_j \geq 0, \, \zeta \geq 0 \),

\[
y'_i = \delta_i y_i, \quad \hat{y}'_i = f(\vec{a}', \vec{x}'_i), \quad \vec{x}'_i = \{x'_i | x'_i = y_i, x_i, i = 1, n\}, \quad \vec{a}' = \{a'_j | a'_j = \partial_j, \, j = 1, p\}, \quad \lambda \text{ is a fixed numerical parameter } (0 \leq \lambda < 1),
\]

which characterizes the desired improvement of the maximum permissible approximation error. If small variations \( \xi^* \) (i.e. values \( \lambda \) close to 1) correspond to small variations of the initial data and there is a set of parameter values slightly different from \( \vec{a}'^* \), the set of experimental data \( \Upsilon \) of the parametric identification task will be considered informative and the condition C4 fulfilled.

The conclusion on the model informativeness as a whole should be formulated on the basis of the set of results obtained as part of the verification of the C1-C4 conditions.

4. Approbation

We implement the above method of informativeness analysis in terms of an example of parametric identification of time series (Table 1).

| \( t \) | Initial data, \( y_t \) | Calculated values, \( \hat{y}_t \) | Residuals, \( \hat{y}_t - y_t \) | Relative errors, \( \frac{\hat{y}_t - y_t}{y_t} \) | \% |
|-----|------------------|------------------|------------------|------------------|-----|
| 1   | 1.6              | 1.84             | 0.24             | 14.8             |
| 2   | 3.6              | 3.15             | -0.45            | 12.5             |
| 3   | 3.7              | 4.15             | 0.45             | 12.1             |
| 4   | 5.4              | 4.99             | -0.41            | 7.7              |
| 5   | 6.1              | 5.72             | -0.38            | 6.2              |
| 6   | 6.2              | 6.38             | 0.18             | 2.9              |
| 7   | 6.6              | 6.99             | 0.39             | 5.9              |
| 8   | 8.0              | 7.55             | -0.45            | 5.6              |
| 9   | 8.3              | 8.08             | -0.22            | 2.7              |
| 10  | 8.7              | 8.58             | -0.12            | 1.4              |

Based on the time series visualization results (Figure 2), the use of the following model specification was assumed:

\[
y_i = a \cdot t^b + c.
\]

Geometric interpretation of time series presented in the Figure 2: "1" is designation of initial data, "2" is curve of calculated values according to the model (9).
The model (7) cannot be linearized. Moreover, the amount of the available data (10 per three parameters to be evaluated) is insufficient to obtain significant characteristics in the case of using mathematical and statistical methods. As an additional limitation for the model (7) parameter values, a formalized representation of the requirement to the time series level was used at $t = 12$: $9.2 \leq y_{12} \leq 9.5$. (Such conditions may be based on the objective data or expert opinions, and their a priori consideration may increase confidence in the resulting models.)

The model (3) for the functional dependence (7) will be:

$$\min_{a, b, c, \xi} \xi,$$

$$\left| a \cdot t^b + c - y_i \right| \leq \xi, \quad t = 1, 10,$$

$$9.2 \leq y_{12} \leq 9.5. \tag{8}$$

Based on the results of the numerical implementation of the model (8), the maximum permissible approximation error was $\xi^* = 0.449$. The corresponding values of the parameters $a$, $b$ and $c$ allowed us to obtain the following time series model:

$$\hat{y}_i = 3.308 \cdot t^{0.482} - 1.472. \tag{9}$$

The model (4) for calculating maximum permissible estimates of the parameter $a$ has the following form:

$$\min_{a, b, c} \left( \max_{a, b, c} \right),$$

$$\left| a \cdot t^b + c - y_i \right| \leq \xi^*, \quad t = 1, 10,$$

$$9.2 \leq y_{12} \leq 9.5.$$

The calculations showed that $a = \bar{a} = 3.308$. The similar calculations performed for the parameters $b$ and $c$ also showed that the lower and upper estimates for each are the same: $b = \bar{b} = 0.482$, $c = \bar{c} = -1.472$.

Thus, the function (7) parametric identification resulted in the model (9). Its visual analysis (Figure 2) shows that the resulting relationship describes the initial data quite well, as also indicated by the value of the average approximation error $\bar{A} = 7.18\%$.

![Figure 2. Geometric interpretation of time series model.](Figure2.jpg)
The obtained values $\xi^*$ and $\bar{A}$ allow us to conclude that the C1 informativeness condition is met.

The model (9) is such that the calculated value $\hat{y}_2 = 9.5$. This means that the constraint $\hat{y}_2 \leq 9.5$ has the greatest impact on the achieved level $\xi^*$. This constraint is more informative than the condition $9.2 \leq \hat{y}_2$, and it is this constraint that can be recommended for correction in the first place in order to improve the accuracy of the initial data description. On the basis of the above, the C2 condition should be considered fulfilled.

Since the uncertainty intervals for each of the model parameters (7) are pulled to a single point, the set $\Lambda^*$ consists of a single point $\{3.308, 0.482, -1.472\}$. Thus, $\text{diam} \Lambda^* = 0$, which means that the C3 condition fulfilled.

To check the informativeness of the initial ones, we specify the model form (6) for our task, assuming the parameter $\lambda$ to be 0.95:

$$
\zeta \rightarrow \min_{\zeta, (\delta_i, (\delta_j))} \left[ \hat{y}_t - y_t \right] \leq 0.95 \xi^*, \ t = 1,10,
$$

$$
9.2 \leq \hat{y}_t \leq 9.5, \ t = 1,10,
$$

$$
|1 - \delta_i| \leq \zeta, \ t = 1,10,
$$

$$
|1 - \delta_j| \leq \zeta, \ j = 1,3
$$

$$
\zeta_i \geq 0, \ \delta_j \geq 0, \ \zeta \geq 0,
$$

where $y_t = \delta_i y_i, \ \hat{y}_t = a' \cdot t^b + c', \ a' = \delta a^* = 3.308 \delta, \ b' = \delta b^* = 0.482 \delta, \ c' = \delta c^* = -1.472 \delta$.

The results of the task (10) solution showed (Table 2) that the improvement of the maximum permissible approximation error $\xi^*$ by 5% can be achieved due to the initial time series levels variation not more than 0.552%. The changes in the parameters of the new time series model with respect to the time series parameters (9) values will vary in the range of 0.014-0.552%.

| Model constraint group (10) (variables designation) | Variable number in the group | Variable value | Change of the corresponding parameter designation | value gain, % |
|------------------------------------------------------|-------------------------------|----------------|-----------------------------------------------|---------------|
| By the initial data ($\delta_i$)                     | 1                             | 1.006          | $y_1$                                | -0.552         |
|                                                     | 2                             | 0.994          | $y_2$                                | 0.552          |
|                                                     | 3                             | 1.006          | $y_3$                                | -0.552         |
|                                                     | 4                             | 0.994          | $y_4$                                | 0.552          |
|                                                     | 5                             | 0.994          | $y_5$                                | 0.552          |
|                                                     | 6                             | 1.006          | $y_6$                                | -0.552         |
|                                                     | 7                             | 1.006          | $y_7$                                | -0.552         |
|                                                     | 8                             | 0.994          | $y_8$                                | 0.552          |
|                                                     | 9                             | 0.999          | $y_9$                                | 0.124          |
|                                                     | 10                            | 0.994          | $y_{10}$                             | 0.552          |
| By time series parameters (9) ($\delta_j$)           | 1                             | 1.000          | $a$                                   | 0.014          |
|                                                     | 2                             | 0.997          | $b$                                   | 0.316          |
|                                                     | 3                             | 0.994          | $c$                                   | 0.552          |
Thus, the adjusted time series model will be:

$$\hat{y}_t = 3.308 \cdot t^{0.481} - 1.463.$$ (11)

For the model (11), the maximum permissible approximation error is 0.427, the average approximation error $\bar{A} = 6.78\%$, the predicted time series level $\hat{y}_{12} = 9.465$.

Small changes in the initial time series levels and its parameters after assuming a 5% decrease in the maximum permissible approximation error make it possible to conclude that the C4 informativeness condition is fulfilled.

Based on the set of calculations performed, it is possible to conclude on the acceptable qualitative characteristics of the obtained time series model (11) and its good informativeness.

5. Conclusion

A parametric identification method based on the calculation of the maximum permissible estimates of the desired dependence parameters is presented. The method has been devised in the context of the development of the idea of L. V. Kantorovich on the advantages of determining the areas of location of the sought-for values, providing the required quality of the initial data description. The characteristic of the method is determined by the possibility of its application in conditions of inaccuracy of the initial data and their small amount, which does not allow using classical approaches of statistical analysis. These uncertainties make it important to analyse the informativeness of the model. To this end, informativeness conditions are set out and their verification procedures are formalized in this paper.

The presented method is implemented by time series modeling based on 10 observations. Its application made it possible to establish parameter values that provide an acceptable quality of the initial data description taking into account the additional condition relative to the perspective analysis data, and to conclude on the informativeness of the obtained model. An important feature of the method is that it can be used if the identified model specification does not allow linearization (this was exactly what was demonstrated in the example discussed).

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