Perturbed N=(2,2) supersymmetric sigma models on Lie groups

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Abstract

We perturbed N=(2,2) supersymmetric WZW and sigma models on Lie groups by adding a term to their actions. Then by using non-coordinate basis we obtain conditions, from the algebraic point of view, under which the N=(2,2) supersymmetry is preserved. By applying this method, we have obtained conditions on the existence of N=(2,2) supersymmetry on the Drinfeld action (master action for the Poisson-Lie T-dual sigma models).

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1 Introduction

String theories with N=2 worldsheet supersymmetry give rise to spacetime physics which is itself supersymmetric. Spacetime supersymmetry is our best hope for addressing the hierarchy problem and hence has strong theoretical motivation. Furthermore, supersymmetric sigma models are of interest, e.g., as gauge-fixed actions, for their intimate connection to complex geometry of target manifold [1], [2] and for their role as effective low-energy actions for supergravity scalars. From the geometrical point of view the N=(2,2) extended supersymmetry in sigma model is equivalent to the existence of bi-Hermitian structure on the target manifold [1] (see also [3] and references therein). Furthermore, it is shown that the algebraic structures related to these bi-Hermitian relations for the N=(2,2) supersymmetric WZW models are the Manin triples [4], [5]. Meanwhile the algebraic structure associated to the bi-Hermitian geometry for N=(2,2) supersymmetric sigma models on Lie groups are found recently in [6]. On the other hand, T-duality is the most important symmetries of string theory [7]. In this way, Poisson-Lie T-duality, a generalization of T-duality, does not require existence of isometry in the original target manifold (as in usual T-duality [7], [8], [9]). There are some attempts to find the effect of T-duality on the N=(2,2) supersymmetry. In [10] and [11], it is shown that the N=(2,2) supersymmetry is preserved under Abelian T-duality; both in the cases where complex structures are independent and dependent on the coordinates to which the T-duality is performed. In the dependency case it is shown that the extended worldsheet supersymmetry is non-local under T-duality transformation [11]. In [12] there are some attempts for studying the effect of Poisson-Lie T-duality on N=(2,2) supersymmetry. Here, we try to have one step forward in this direction by studying conditions on the existence of N=(2,2) supersymmetry on the Drinfeld action (master action for the Poisson-Lie T-dual sigma models). The paper is organized as follows.

In section two, to introduce the notations and selfcontaining the paper we review the N=(2,2) supersymmetric WZW and sigma models on Lie groups from geometrical and algebraic point of view. Then, in section three we first perturb N=(2,2) supersymmetric WZW and sigma models on Lie groups by adding a general term to their action; then by using of non-coordinate bases we obtain from the algebraic point of view, conditions under which the N=(2,2) supersymmetry is preserved. In section four we obtain conditions on the existence of N=(2,2) supersymmetry on Drinfeld action (master action for the Poisson-Lie T-dual sigma models) [9].

2 Review of $N = (2, 2)$ supersymmetric WZW and sigma models

In this section we review the results of N=(2,2) supersymmetric sigma models on the manifolds [1] in general and on the Lie groups as special case (see for example [3], [5]). We start from the general N=1 supersymmetric sigma models action on the manifold M as follows:

$$S = \int d^{2}\sigma d^{2}\theta D_{+}\Phi^{\mu} D_{-}\Phi^{\nu}(G_{\mu\nu}(\Phi) + B_{\mu\nu}(\Phi)), \quad (1)$$

where $\Phi^{\mu}$ are N=1 superfields with bosonic parts as coordinates of the manifold M, meanwhile the bosonic parts of $G_{\mu\nu}$ and $B_{\mu\nu}$ are respectively metric and antisymmetric tensor on M. The action (1) is manifestly invariant under supersymmetry transformations

$$\delta^{1}(\epsilon)\Phi^{\mu} = i(\epsilon^{+}Q_{+} + \epsilon^{-}Q_{-})\Phi^{\mu}, \quad (2)$$

furthermore this action has additional non manifest supersymmetry of the form

$$\delta^{2}(\epsilon)\Phi^{\mu} = \epsilon^{+} D_{+}\Phi^{\mu} J_{+\nu}^{\mu}(\Phi) + \epsilon^{-} D_{-}\Phi^{\mu} J_{-\nu}^{\mu}(\Phi), \quad (3)$$

where in the above relations $Q_{\pm}$ and $D_{\pm}$ are supersymmetry generators and superderivatives respectively furthermore $\epsilon^{\pm}$ are parameters of supersymmetry transformations and $J_{\pm\sigma}^{\mu} \in TM \otimes T^{*} M$. Invariance of the action (1) under the transformations (3) imposes the following conditions on $J_{\pm\sigma}^{\mu}$:

$$J_{\pm\lambda}^{\mu} J_{\pm\nu}^{\lambda} = -\delta_{\mu}^{\nu}, \quad (4)$$

$$J_{\pm\mu}^{\nu} G_{\mu\nu} = -G_{\mu\rho} J_{\pm\nu}^{\mu}, \quad (5)$$

2
\[ \nabla_{\rho}^{(\pm)} J_{\pm \nu}^{\mu} = J_{\pm \nu, \rho}^{\mu} + \Gamma_{\rho \sigma}^{\pm \mu} J_{\pm \nu}^{\sigma} - \Gamma_{\rho \nu}^{\pm \sigma} J_{\pm \sigma}^{\mu} = 0, \]  

(6)

where

\[ \Gamma_{\rho \nu}^{\pm \mu} = \Gamma_{\rho \nu}^{\mu} \pm G^{\mu \sigma} H_{\sigma \rho \nu}, \]  

(7)

such that \( \Gamma_{\rho \nu}^{\mu} \) is the usual Christoffel symbols and \( H \) being the torsion three form

\[ H_{\mu \rho \sigma} = \frac{1}{2} (B_{\mu \rho, \sigma} + B_{\rho \sigma, \mu} + B_{\sigma \mu, \rho}), \]  

(8)

In order to have a closed on-shell supersymmetry algebra with generators (2) and (3) we must have zero Nijenhuis tensor \([14] \) for \( J_{\pm \nu}^{\mu} \) \([1], [2] \);

\[ N_{\mu \nu}^{\rho} (J_{\pm}) = J_{\pm \nu}^{\mu} \partial_{[\gamma} J_{\pm \mu \nu]}^{\rho} - J_{\pm \nu}^{\gamma} \partial_{[\gamma} J_{\pm \mu \nu]}^{\rho} = 0. \]  

(9)

In this way, having an \( N=(2,2) \) supersymmetric sigma models on the manifold \( M \) is geometrically equivalent to having two bi-Hermitian complex structure \( J_{\pm} \) such that their covariant derivations with respect to connections \( \Gamma_{\rho \nu}^{\pm \mu} \) are equal to zero (6). The vanishing of the Nijenhuis tensor (9) (the integrability condition) and condition (6) implies that the complex structures \( J_{\pm} \) should preserve the torsion \([1-3] \);

\[ H_{\mu \nu \lambda} = J_{\pm \lambda}^{\mu} J_{\pm \mu}^{\nu} H_{\sigma \rho \lambda} + J_{\pm \lambda}^{\nu} J_{\pm \nu}^{\mu} H_{\sigma \rho \lambda} + J_{\pm \mu}^{\nu} J_{\pm \nu}^{\rho} H_{\sigma \rho \lambda}. \]  

(10)

In the case that \( M \) is a Lie group \( G \), then using non-coordinate bases, we have

\[ G_{\mu \nu} = L^{A}_{\mu} L^{B}_{\nu} G_{AB} = R^{A}_{\mu} R^{B}_{\nu} G_{AB}, \]  

(11)

\[ f_{AB}^{C} = L^{C}_{\nu} (L^{A}_{\mu} \partial_{\mu} L^{B}_{\nu} - L^{B}_{\mu} \partial_{\mu} L^{A}_{\nu}) = R^{C}_{\nu} (R^{A}_{\mu} \partial_{\mu} R^{B}_{\nu} - R^{B}_{\mu} \partial_{\mu} R^{A}_{\nu}), \]  

(12)

\[ H_{\mu \nu \lambda} = L^{A}_{\mu} L^{B}_{\nu} L^{C}_{\lambda} H_{ABC} = R^{A}_{\mu} R^{B}_{\nu} R^{C}_{\lambda} H_{ABC}, \]  

(13)

\[ J_{\mu \nu}^{A} = L^{A}_{\mu} J_{\nu}^{B} L^{B}_{\nu}, \quad J_{\mu \nu}^{\nu} = R^{A}_{\mu} J_{\nu}^{B} R^{B}_{\nu}, \]  

(14)

where \( G_{AB} \) is symmetric ad-invariant non-degenerate bilinear form and \( H_{ABC} \) is antisymmetric tensor on Lie algebra \( \mathfrak{g} \) \([1] \).

Furthermore \( L^{A}_{\mu} (R^{A}_{\mu}) \) and \( L^{A}_{\nu} (R^{A}_{\nu}) \) are left (right) invariant vielbein and their inverses on the Lie group \( G \) respectively, and \( f_{AB}^{C} \) is the structure constant of Lie algebra and \( J \) is a Lie algebraic map: \( J: \mathfrak{g} \rightarrow \mathfrak{g} \).

In this case by use of (11-14) the relations (4-6), (9) and (10) can be rewritten in the following algebraic form \([6] \):

\[ J^{2} = -I, \]  

(15)

\[ \chi_{A} + J^{t} \chi_{A} J^{t} + J^{B} A \chi_{B} J^{t} - J^{B} A J^{t} \chi_{B} = 0, \]  

(16)

\[ J^{t} G J = G, \]  

(17)

\[ H_{A} = J^{t} (H_{B} J_{A}^{B}) + J^{t} H_{A} J + (H_{B} J_{A}^{B}) J, \]  

(18)

\[ J^{t} (H_{A} + \chi_{A} G) = (J^{t} (H_{A} + \chi_{A} G))^{t}, \]  

(19)

where \( (\chi_{A})^{C}_{B} = -f_{AB}^{C} \) is the adjoint representation and \( (H_{A})^{B}_{C} = H_{ABC} : G_{AB} \) is the ad-invariant metric on Lie algebra \( \mathfrak{g} \), such that:

\[ (\chi_{A} G)^{t} = -\chi_{A} G. \]  

(20)

These relations shows that \( N=(2,2) \) supersymmetric sigma models on Lie groups \( G \) have a geometric bi-Hermitian structures on the Lie group \( G \) \([1] \) or equivalently an algebraic bi-Hermitian structures \((J, G, H)\) on Lie algebra \( \mathfrak{g} \) \([6] \).

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1 Note that \( G_{\mu \nu} (G_{AB}) \) raise and lower the target space (Lie algebra) indices.
For N=2 supersymmetric WZW models on the Lie group G we have (up to constant)
\[ H_{\mu\nu\lambda} = L_\mu^A L_\nu^B L_\lambda^C f_{ABC} = R_\mu^A R_\nu^B R_\lambda^C f_{ABC}, \tag{21} \]
i.e. \( H_{ABC} = f_{ABC} \). In this case relation (16) shows that we have a Lie bialgebra structures on \( g \); and relation (18) reduce to (16) and (19) is automatically satisfied i.e. Lie bialgebra structure is a special case of algebraic bi-Hermitian structure \((J,G,H)\) with \( H_{ABC} = f_{ABC} \).

3 Perturbed N=(2,2) supersymmetric WZW and sigma models on Lie groups.

In this section we find conditions such that the perturbed N=(2,2) supersymmetric WZW and sigma models on Lie groups preserve N=(2,2) supersymmetry. We assume that the action (1) (as sigma models on Lie group or WZW model) has N=(2,2) supersymmetry, and is perturbed with the following general term:
\[ S' = \int d^2\sigma d^2\theta D_+ \Phi D_- \Phi' (G''_{\mu\nu}(\Phi) + B''_{\mu\nu}(\Phi)), \tag{22} \]
where the bosonic part of \( G''_{\mu\nu} \) and \( B''_{\mu\nu} \) are symmetric and antisymmetric tensors on the Lie group G. In this case we have the following action:
\[ S'' = S + S' = \int d^2\sigma d^2\theta D_+ \Phi D_- \Phi' (G''_{\mu\nu}(\Phi) + B''_{\mu\nu}(\Phi)), \tag{23} \]
such that \( G''_{\mu\nu} = G'_{\mu\nu} + G''_{\mu\nu} \) and \( B''_{\mu\nu} = B'_{\mu\nu} + B''_{\mu\nu} \). Now we find conditions under which the action \( S' \) have N=(2,2) supersymmetry; i.e. the relations (4)-(6) and (10) holds for \( G''_{\mu\nu} \) and \( H''_{\mu\nu} \).

Similar to the previous section we use (11)-(14) and the following relations:
\[ G'_{\mu\nu} = L_\mu^A L_\nu^B G''_{AB}, \quad G''_{\mu\nu} = L_\mu^A L_\nu^B G_{AB}, \tag{24} \]
\[ H'_{\mu\nu\lambda} = L_\mu^A L_\nu^B L_\lambda^C H''_{ABC}, \quad H''_{\mu\nu\lambda} = L_\mu^A L_\nu^B L_\lambda^C H_{ABC}, \tag{25} \]
\[ H''_{\mu\nu\lambda} = L_\mu^A L_\nu^B L_\lambda^C H''_{ABC} = R_\mu^A R_\nu^B R_\lambda^C H''_{ABC}. \tag{26} \]
with the assumptions that \( G_{AB}, G''_{AB} \) and \( G''_{AB} \) are constant (independent from coordinates of Lie group G and its Lie algebra \( g \)) and invertible. Now by use of the above relations one can discussed about perturbed algebraic bi-Hermitian structure instead of perturbed \( N = (2,2) \) supersymmetry. We perform these in the following seven cases: \[ \text{Case a) The algebraic bi-Hermitian structure \((J,G,H)\) is perturbed with \((J',G',H')\). Now we must impose the following conditions on \((J',G',H')\) such that the structure\((J'',G'',H'')\) become bi-Hermitian (i.e.relations (15)-(20) holds for \((J'',G'',H'')\)). From relation (15) we obtain:} \]
\[ \{J', J + J'/2\} = 0, \tag{27} \]
where in the case that \( J' \) is a complex structure we have
\[ \{J, J'\} = 1. \tag{28} \]

From relation (16) we obtain
\[ J' \chi_A J'' + J'' \chi_A J' + J'' \chi_A J'' + J'' A \chi_B J'' + J'' A \chi_B J' + J'' A \chi_B J'' - J'' A \chi_B J' = 0, \tag{29} \]

\[ ^2 \text{Note that relation (9) is equivalent to relation (10)} \]
\[ ^3 \text{Note that the five cases b)-g) are especial case of a), but we discusses them for further details} \]

\[ \]
in the case that \( J' \) is a complex structure we have

\[ N(J, J') = J' \chi_A J^n + J^n \chi_A J' + + J^B_A \chi_B J^n + J^n B_A \chi_B J' - J^B_A J^n \chi_B - J^n B_A J' \chi_B - \chi_A = 0, \quad (30) \]

where \( N(J, J') \) is the Nijenhuis concomitant of \( J \) and \( J' \) \cite{13, 14}. Furthermore, one can obtain the following relation from (17):

\[ J^i G J' + J^i G' J + J^i G J' + J^n G J + J^n G' J = G', \quad (31) \]

where in the case that \((J', G')\) is bi-Hermitian complex structure we have

\[ J^i G J' + J^i G' J + J^i G J' + J^n G J + J^n G' J + J^n G' J = 0. \quad (32) \]

Finally from (18) and (19) in the case that \((J', G', H')\) is algebraic bi-Hermitian structure, we obtain the following relations respectively:

\[ J^i H_B J'^B A + J^i H'_B J^B A + J^i H'_B J'^B A + J^n H_B J^B A + J^n H'_B J'^B A + J^n H'_B J^B A \quad (33) \]

\[ + J^i H_A J' + J^i H'_A J' + J^i H'_A J' + J^n H_A J + J^n H'_A J + J^n H'_A J + J^i H_B J^B A J' + H_B J^B A J + H^B A J + H^B A J' + H^B A J = 0, \]

\[ J^i (H'_A + \chi_A G') + J^n (H_A + \chi_A G) = -(H'_A + \chi_A G')J - (H_A + \chi_A G)J'. \quad (34) \]

Meanwhile from (20) we see that \( G' \) must be ad-invariant metric i.e.

\[ \chi_A G' = -(\chi_A G')^t. \quad (35) \]

**Case b)** The algebraic bi-Hermitian structure \((J, G, H)\) is perturbed with \((0, G', H')\). We see that this case is special case of the above case. So from above relations we see that if \((J, G', H')\) is an algebraic bi-Hermitian structure, then \((J, G'', H'')\) is an algebraic bi-Hermitian structure.

**Case c)** The algebraic bi-Hermitian structure \((J, G, H)\) is perturbed with \((J', 0, H')\). This case is a special case of case a) when \( G' = 0 \). So relations (27),(29) are also satisfied for this case and instead of (31) we must have

\[ J^i G J' + J^n G J + J^n G J' = 0. \quad (36) \]

Furthermore, relation (33) is also must be imposed on \((J', G, H')\) and instead of (34) we must have

\[ J^i H'_A + J^n (H_A + \chi_A G) + J^n H'_A = -H'_A J - (H_A + \chi_A G)J' - H'_A J', \quad (37) \]

such that by assuming \((J', G, H')\) is a bi-Hermitian structure, instead of (36) and (37) we have

\[ J^i G J' + (J^i G J')^t = -G, \]

\[ (H'_A J + H_A J')^t = H'_A J + H_A J'. \]

**Case d)** The algebraic bi-Hermitian structure \((J, G, H)\) is perturbed with \((J', G', 0)\). This case is a special case of case a) when \( H' = 0 \). By assuming \( J'^2 = -1 \) and that \((J', G, H')\) is a bi-Hermitian structure, the relations (27)-(32) are also satisfied for this case and instead of (33) and (34) we have:

\[ J^i H_B J'^B A + J^n H_B J^B A + J^i H_A J' + J^n H_A J + H_B J^B A J' - H_B J'^B A J + H_A = 0, \quad (38) \]

\[ J^n \chi_A G - (J^n \chi_A G)^t = -(J^i \chi_A G' - (J^i \chi_A G')^t), \quad (39) \]

and relation (35) is also must be satisfied for this case.
i.e. for having \((J'' , G , H)\) as a bi-Hermitian structure we must have \((J' , G , H)\) as bi-Hermitian structure such that relations \((28) , (30) , (38)\), and \((40)\) must be satisfied.

**Case f)** The algebraic bi-Hermitian structure \((J , G , H)\) is perturbed with \((0 , G' , 0)\). In this case from (17) we have

\[
J^t G' J = G',
\]

and from (19) we obtain:

\[
(\chi_A G' J)^t = \chi_A G' J,
\]

furthermore from (20) we have

\[
\chi_A G' = -(\chi_A G')^t,
\]

i.e. for obtaining \((J , G' , H)\) as an algebraic bi-Hermitian structure we must have \(G'\) as an ad-invariant metric such that \((J , G' , H)\) is also bi-Hermitian complex structure and the matrix \(\chi_A G' J\) must be symmetric matrix.

**Case g)** The algebraic bi-Hermitian structure \((J , G , H)\) is perturbed with \((0 , 0 , H')\). From (19) we obtain

\[
(H' A J)^t = H' A J.
\]

Note that in this case by assuming that \((J , G , H')\) is bi-Hermitian structure then from relation (18) we can not obtain a new result. In this way, for having \((J , G , H'')\) as an algebraic bi-Hermitian structure we must have \(H' A J\) as a symmetric matrix.

4 **Conditions on the existence of N=(2,2) supersymmetry on Drinfeld action**

In this section, we consider Drinfeld super action as an example of perturbed N=(2,2) supersymmetric WZW model. The form of Drinfeld super action as an action on Drinfeld Lie group \(D\) is as follow [15]:

\[
S = I_0(L) + S',
\]

with

\[
S' = -\frac{1}{2(2\pi)^2} \int [L^{-1} D_+ L | R | L^{-1} D_- L] d^2 \sigma d^2 \theta,
\]

where \(I_0(L)\) is the \(N=1\) supersymmetric WZW action and \(L\) is an extension of Lie group element such that its bosonic part is \(l \in D\) [15]; furthermore the operator \(R\) in terms of bases \(R^\pm\) has the following form [15]:

\[
R = |R^+_a| \eta^{ab} \langle R^+_b | + |R^-_a| \eta^{ab} \langle R^-_b |,
\]

such that :

\[
\langle R^+_a | R^+_b \rangle , \langle R^-_a | R^-_b \rangle = 0,
\]

\[
|R^+_a| \eta^{ab} \langle R^+_b | - |R^-_a| \eta^{ab} \langle R^-_b | = I,
\]

\[
R^\pm_a = T_a \pm (E^\pm_0)_{ab} T^b , \eta_{ab} = (E^+_0)_{ab} + (E^-_0)_{ab},
\]

where \(\{T_a\}\) and \(\{T^a\}\) are the bases of the Lie algebra \(g\) and \(\bar{g}\) such that \(D = g \oplus \bar{g}\) is a Lie algebra of Drinfeld double \(\bar{D}\). The matrix \(E^+_0\) is a arbitrary constant matrix and \(E^-_0\) is its transpose. Note that the action (45) is the master action for the Poisson-Lie T-dual sigma models [9]. Indeed for the decompositions \(L = \hat{g} h (L = \hat{g} h)\) one can obtain the sigma model (and its T-dual) as follows:

\[
S = \int \frac{1}{|E^+_0|} \Pi(g)|_{ij}^{-1} (\partial_+ g g^{-1})^i (\partial_- g g^{-1})^j d\xi^+ d\xi^- d^2 \theta
\]
and
\[ \tilde{S} = \int \left[ E_0^+ \pm \tilde{\Pi}(\tilde{g}) \right]_{ij}^{-1} (\partial_+ \tilde{g}^{-1})^i (\partial_- \tilde{g}^{-1})^j \, d\xi^+ d\xi^- d^2\theta \]  \tag{51}

with
\[ \Pi(g) = b(g)a^{-1}(g) \]  \tag{52}
such that:
\[ g^{-1}T_ag = a(g)_a bT_b, \quad g^{-1}T^a = b(g)^{ab}T_b + (a^{-1}(g))_b a^b \]  \tag{53}
and in the same way for \( \tilde{\Pi}(\tilde{g}), \tilde{g}^{-1}T_a \tilde{g} \) and \( \tilde{g}^{-1}T^a \tilde{g} \).

Note that in this case the WZW action \( I_0(L) \) has \( N = (2, 2) \) supersymmetry, because we have Lie bialgebra structure \((g, \tilde{g})\) [5]. Now using above relation and using \( L^{-1}D_+L = L_a^{\mu}A_\mu X^\mu T_a \) where \( X^\mu \) are superfield with bosonic sections as coordinates of Lie group \( D \); and by use of the following decomposition for \( D \):
\[ L_\mu^A T_A = L_\mu^a T_a + L_{\mu, n+a} \tilde{T}^a \]  \tag{54}
and using of isotropy condition on inner product i.e.
\[ < T_a, T_b > = < \tilde{T}^a, \tilde{T}^b > = 0, \quad < T_a, \tilde{T}^b > = \delta_a^b \]  \tag{55}
the perturbed terms can be rewritten as follows:
\[ S' = -\frac{1}{2\Pi} \int L_\mu^A E'^{AB}_{\mu} L_\nu^B D_+ X^\mu D_- X^\nu d^2\sigma d^2\theta, \]  \tag{56}
such that the background matrix \( E'^{AB}_{\mu} \) has the following form:
\[ E'^{AB}_{\mu} = \left( \begin{array}{cc} E_0^+ \eta^{-1}E_0^- + E_0^+ \eta^{-1}E_0^- & -(E_0^+ - E_0^-)\eta^{-1} \\
\eta^{-1}(E_0^- - E_0^+) & 2\eta^{-1} \end{array} \right) \]  \tag{57}
Note that this matrix is symmetric, i.e. assuming \( E'^{AB}_{\mu} = G'^{AB}_{\mu} + B'^{AB}_{\mu} \); then for this example \( B'^{AB}_{\mu} = 0 \) and consequently \( H'^{ABC}_{\mu} = 0 \). Now by applying the formalism of the previous section for the above example, we see that this is an example of the cases d) or f); i.e. we must impose condition (41)-(43) for having \( N = (2, 2) \) supersymmetry on the action \( S \) (45).

The next step for investigating the invariance of \( N = (2, 2) \) supersymmetry structure under Poisson-Lie T-duality is of obtaining the sigma model action (50) and its dual (51) by using the decompositions \( L = \tilde{g}\hbar \) and \( L = \tilde{g}h \) in the action (45)(with the above restrictions on \( G' \)); then one can investigate the \( N = (2, 2) \) structure on these actions. Note that for these sigma models the background matrix \( E_{\mu AB}(g)(\tilde{E}(\tilde{g})) \) is dependent on the Lie group coordinates and one can not use the above algebraic formulation; and one must control the conditions (4)-(6) and (9) directly for these models.

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