A Very Naive Model of Hadron with Negative Quintessence

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Abstract
A simple model of hadron which exhibits the quark confinement and the asymptotic freedom is described. The hadron is modelled as a sphere of radius equal to its Compton wavelength in which quarks occur surrounded by space with the uniformly distributed negative vacuum energy density characterized by $\lambda$. The acceleration stemming from this vacuum energy causes the quark confinement. However, in the center of hadron quarks behave as practically free particles. From the requirement that the Compton wavelength of a hadron should remain constant while the scale factor varies with time we estimate the strength of the vacuum energy density surrounding quarks. It follows that the inward pressure of this vacuum energy is large enough to keep quarks inside the hadron. \footnote{E-mail:fyziemar@savba.sk}

1 Introduction
The recent astronomical observations \cite{1} \cite{2} \cite{3} give increasing support for the accelerating and flat universe which consists of a mixture of a small part of the baryonic matter about one third non-relativistic dark matter (DM) and two thirds of a smooth component, called dark energy (DE). In this short communication, we put forward the hypothesis that quarks in hadrons are surrounded by negative DE whose inward pressure causes its confinement. This dark energy we modelled by negative cosmological constant $\Lambda$.

In the literature, DE is theoretically modelled by many ways, e.g. as (i) a very small cosmological constant (e.g.\cite{10}) (ii) quintessence (e.g.\cite{11}) (iii) Chaplygin gas (e.g.\cite{12}) (iv) tachyon field (e.g.\cite{13} \cite{15} \cite{16}) (v) interacting quintessence (e.g.\cite{14}), quaternionic field (e.g.\cite{17}), etc. It is unknown which of the said models will finally emerges as the successful one.

Physicists found by the late 1960s that hadrons were made of quarks which remain firmly locked together. Yet the deep-inelastic scattering experiments probed the mechanism of quarks confinement shown that the quarks behave practically as free particles
This fact called as asymptotic freedom refers to the vanishing of the strong nuclear force between quarks as the distance between them approaches zero [4] [5] [6].

2 The model

We model a hadron as a sphere of radius $R_m$ in which quarks occur surrounded by space of the uniformly distributed vacuum energy density characterized by $\lambda$. From the Friedmann equations, it follows that the total energy of a unit mass situated at a distance $R$ from the origin is given by the equation [7]

$$ \frac{1}{2} \left( \frac{dR}{dt} \right)^2 - \frac{GM}{R} - \frac{c^2 R^2 \lambda}{6} = -\frac{kc^2}{2}. $$

(1)

The first term is the kinetic energy, the second term is the gravitational potential energy due to the mass contained in a sphere of radius $R$, the third term may be interpreted as the potential energy due to that portion of the vacuum encompassed by the same sphere. The concomitant equation of motion is obtained by differentiating Eq. (1) with respect to $R$ [7]

$$ \ddot{R} = -\frac{GM}{R^2} + \frac{R\lambda}{3}. $$

(2)

According to Eq. (2), the acceleration between quarks in the field of the negative vacuum energy (negative quintessence) which is uniform distributed inside a hadron is

$$ \ddot{R} = -\frac{Gm_q}{R^2} - \frac{R\lambda}{3}, $$

(3)

where $m_q$ is the mass of quark. The average mass density of all quarks in a hadron is

$$ \rho = \sum_{i=1}^{3} \frac{m_q^i}{R_h^3} \quad \sum_{i=1}^{3} m_q^i = M_h, $$

where $m_q^i, i = 1, 2, 3$ are masses of quarks and $M_h$ is the mass of the whole hadron.

Here, the problem arises what value should be inserted for $\lambda$. To estimate this value we used the following argument. The scale factor $R(t)$ is a universal fundamental quantity depending on $t$ which for the given stress-energy tensor and cosmological constant is determined by the Einstein equations. All distances in the universe should change in accord with the time dependence of $R(t)$. It is generally supposed that the Compton wavelength of particles (except of photons of the relict radiation) does not varies with the scale factor (remains constant). Next, we point out that this fact can be also explained if the mass distribution in a hadron is equal to the negative vacuum energy density. We start with the standard Einstein field equations (c=1)

$$ R_{ij} - g_{ij}(1/2)R = 8\pi G(T_{ij}^{(m)} + T_{ij}^{(v)}), $$
where $T^{(m)}_{ij}$ is the energy-momentum tensor and

$$T^{(v)}_{ij} = \Lambda = \left( \frac{\lambda}{8\pi G} \right) g_{ij}$$

the cosmological term. In a homogeneous and isotropic medium characterized by the Friedmann-Robertson-Walker line element the Einstein equations with matter density $\rho$ and non-zero cosmical term $\Lambda$ acquire the following forms ($k=1$)

$$3 \frac{\dot{R}^2}{R(t)^2} = 8\pi G (\rho + \Lambda), \quad \lambda = 8\pi G \Lambda.$$  \hspace{1cm} (4)

The necessary condition for the constancy of hadron radius is $\dot{R} = 0$ which implies according to Eq.(4) $\rho = -\Lambda$.

Keeping in mind that $\lambda = 8\pi G \Lambda$ and $\Lambda \approx -\rho$, respectively, we have finally

$$\ddot{R} = T_1 + T_2 = -\frac{GmQ}{R^2} - \frac{8\pi GR\rho}{3}.$$  \hspace{1cm} (5)

We see that the first and second term $T_1$ and $T_2$ in Eq.4 represents the acceleration of quarks in their gravitation field and the acceleration due to negative cosmological constant, respectively. If the mass distribution within the sphere with the Compton wavelength of a hadron (e.g. neutron) $R_h \approx 10^{-15} m$ is homogeneous then the mass density $\rho_n \approx 10^{-45}$. Inserting $\rho_n$ into Eq.(5) and neglecting the gravitational interaction between quarks we obtain

$$\ddot{R} = -\frac{8\pi G \rho}{3} R.$$  \hspace{1cm} (6)

This is the familiar equation of motion of a harmonic oscillator

$$\ddot{R} = -\kappa R$$

with $\kappa \approx 5.10^{34}$. Taking for mass of a quark the value $\approx 10^{-27} kg$ then the force acting on it at $R_h$ reaches the value $\approx 10^7 N$ which is approximately $10^5$ larger than the force acting on a unit mass at the Earth surface. This means that quarks in a hadron are bound very strong.

### 3 Conclusions

(i) Quarks occurring in a medium consisting of negative vacuum energy might be confined due to the inward pressure of this vacuum energy.

(ii) From the requirement of constancy of the Compton wavelength of a hadron the density of the vacuum energy within the sphere surrounding quarks may be estimated.

(iii) The inward force acting on quarks due to negative vacuum is strong enough to confine them within hadron.

(iv) Quarks occurring around the center of the sphere of the negative vacuum energy
behave as practically free particles.

(v) Hadrons appear as islands of quarks surrounded by negative vacuum energy in a sea of the positive vacuum energy given by the common cosmological constant.

(vi) The energy balance of positive quark energy and negative vacuum energy surrounding them equals to zero.

(vii) The positive energy of quarks manifests itself by the gravitation field in the neighboring space.

Of course, there are nowadays sophisticated theories for explaining the asymptotical freedom of quarks. In this communication we only sketched the basic idea of a model of hadron which point out that the local concentration of negative vacuum energy might play an important role not only in astrophysics [9] [18] but also in the elementary particle physics.

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