Possibility of reflectionless tunneling crossed transport at normal metal / superconductor double interfaces

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Abstract. – We investigate one dimensional models (the Blonder, Tinkham, Klapwijk model and a tight-binding model) of non local transport at normal metal / superconductor (NS) double interfaces. We find a negative elastic cotunneling crossed conductance, strongly enhanced by additional scatterers away from the interfaces, suggesting the possibility of reflectionless tunneling non local transport at double NS interfaces with contacts having a sufficiently small extension.

Introduction. – Single electron tunneling in a superconductor is prohibited if the applied bias voltage is smaller than the superconducting gap. However, an electron in the spin-up band can be reflected as a hole in the spin-down band, a phenomenon called Andreev reflection [1]. A charge $2e$ is transmitted in the superconductor at each Andreev reflection, so that the conductance of a highly transparent normal metal / superconductor (NS) contact is doubled compared to the one of the corresponding NN contact. The equilibrium properties of the superconductor (such as the value of the self-consistent superconducting gap) are modified by a normal electrode connected to the superconductor, a phenomenon called the inverse proximity effect. It is expected that most of the inverse proximity effect takes place on a length $a$ if the area of the contact $a^2$ is much smaller than the superconducting coherence length $\xi$ [2]. The influence of the inverse proximity effect on transport properties can then be neglected, and a single channel, ballistic, one-dimensional model with a step-function variation of the superconducting gap captures the essential physics of localized interfaces, as shown by Blonder, Tinkham, and Klapwijk (BTK) [2]. Moreover, BTK introduce a repulsive potential at the NS interface, characterized by the dimensionless parameter $Z$, being the strength of the repulsive potential normalized to the Fermi energy. Transparent interfaces correspond to $Z = 0$ and tunnel interfaces correspond to $Z \gg 1$.

Disorder in the normal metal modifies strongly subgap transport at a single normal metal / insulator / superconductor (NIS) interface [3,4]. The conductance can be enhanced by orders of magnitude by constructive interferences in which an electron can “try” the tunneling process a huge number of times [3]. This effect due to scattering by disorder is already present in simple double barrier one-dimensional models. Melsen and Beenakker [5] consider a NINIS
Fig. 1 – Schematic representation of the electrical circuit corresponding to the NISIN junction studied experimentally in Ref. [22]. The current $I_a$ through electrode “a” is determined in response to a voltage $V_b$ on electrode “b”, with $V_a = 0$.

double junction in one dimension, with, in the BTK language, barrier parameters $Z_1$ (for the NIN interface) and $Z_2$ (for the NIS interface). The conductance, averaged over the Fermi oscillations, shows a maximum for a value of $Z_1$ comparable to $Z_2$ as $Z_1$ increases while $Z_2 \gg 1$ is fixed [5]. This enhancement of the conductance shows that the double barrier model captures multiple scattering as in a disordered system.

We address here similar effects for non local transport in NISIN junctions [6–20] in which the normal electrode “a” is at potential $V_a$, the electrode “b” is at potential $V_b$, and the superconductor is at potential $V_S$ (see Fig. 1). The non local conductance $G_{a,b}(V_b)$ contains the information on how the current $I_a(V_b)$ in electrode “a” depends on the voltage $V_b$ on electrode “b”: $G_{a,b}(V_b) = \partial I_a(V_b)/\partial V_b$ [9, 13]. The superconductor is taken as the reference voltage ($V_S = 0$), and we focus on the case $V_a = 0$. Such devices have been realized in two recent experiments, performed in Karlsruhe by Beckmann et al. with ferromagnets [21], and in Delft by Russo et al. with a NISIN trilayer [22]. A sizeable crossed signal is measured in the latter [22], which is surprising in view of lowest order perturbation theory in the tunnel amplitudes predicting an exact cancellation between the electron-electron and electron-hole channel crossed conductances [13]. We take as a working hypothesis that non local transport with normal metals is described by higher order contributions in perturbation theory in the tunnel amplitudes. These were already evaluated in Ref. [17] within microscopic Green’s functions for localized interfaces. This approach was continued in Ref. [18] to account for extended interfaces with a large normal metal phase coherence length, giving rise to weak localization. Our task here is to investigate related issues in simple one dimensional models in the spirit of Ref. [5].

Blonder, Tinkham, Klapwijk (BTK) approach to a NISIN junction. – Let us first consider a one dimensional model of NISIN double interface within the BTK approach [16] (see Fig. 2). The gap of the superconductor is supposed to have a step-function variation: $\Delta(z) = \Delta_0(z - R/2)\delta(R/2 - z)$, and we suppose $\delta$-function scattering potentials at the interfaces: $V(z) = H\delta(z + R/2) + H\delta(z - R/2)$ [2]. The two-component wave-functions are given by

$$\psi_1(z) = \begin{pmatrix} 1 \\ 0 \end{pmatrix} e^{ik_F z} + a \begin{pmatrix} 0 \\ 1 \end{pmatrix} e^{ik_F z} + b \begin{pmatrix} 1 \\ 0 \end{pmatrix} e^{-ik_F z}$$  \hspace{1cm} (1)
\[ \psi_2(z) = c \begin{pmatrix} u_0 \\ v_0 \end{pmatrix} e^{ik_F(z+R/2)} e^{-(z+R/2)/\xi} + d \begin{pmatrix} v_0 \\ u_0 \end{pmatrix} e^{-ik_F(z+R/2)} e^{-(z+R/2)/\xi} \]
\[ + e' \begin{pmatrix} u_0 \\ v_0 \end{pmatrix} e^{-ik_F(z-R/2)} e^{(z-R/2)/\xi} + d' \begin{pmatrix} v_0 \\ u_0 \end{pmatrix} e^{ik_F(z-R/2)} e^{(z-R/2)/\xi} \]
\[ \psi_3(z) = a' \begin{pmatrix} 0 \\ 1 \end{pmatrix} e^{-ik_F(z-R/2)} + b' \begin{pmatrix} 1 \\ 0 \end{pmatrix} e^{ik_F(z-R/2)}, \]

where \( \psi_1(z) \), \( \psi_2(z) \) and \( \psi_3(z) \) correspond respectively to \( z < -R/2 \), \( -R/2 < z < R/2 \) and \( R/2 < z \), and \( u_0^2 = 1 - v_0^2 = (1 + i\sqrt{\Delta^2 - \omega^2}/\omega)/2 \) are the BCS coherence factors. We introduce the parameter \( Z = 2mH/h^2 k_F \). The unknown coefficients \( a, b, a', b', c, d, e', d' \) are determined from matching the wave-functions and their derivatives [2]. Assuming \( R \gg \xi \), we expand \( a' \) and \( b' \) to first order in \( \exp(-R/\xi) \), to find the transmission coefficients

\[ \int_0^{2\pi} \frac{d(k_F R)}{2\pi} |a'(k_F R)|^2 = \left( \frac{1}{2Z^4} - \frac{1}{2Z^6} + \frac{1}{2Z^8} + \ldots \right) \exp(-2R/\xi) + O(\exp(-4R/\xi)) \]
\[ \int_0^{2\pi} \frac{d(k_F R)}{2\pi} |b'(k_F R)|^2 = \left( \frac{1}{2Z^4} - \frac{1}{2Z^6} + \frac{5}{4Z^8} + \ldots \right) \exp(-2R/\xi) + O(\exp(-4R/\xi)) \]

at \( \omega = 0 \). We deduce the first non vanishing term in the large-\( R \), large-\( Z \) expansion of the non local transmission:

\[ T' = \int_0^{2\pi} \frac{d(k_F R)}{2\pi} \left( |a'(k_F R)|^2 - |b'(k_F R)|^2 \right) \]
\[ = -\frac{3}{4Z^8} \exp(-2R/\xi) + O(\exp(-4R/\xi)), \]

having a sign dominated by elastic cotunneling, in agreement with the Green’s function approach in which the first term in expansion of the non local transmission appears at order \( T^3 \exp(-2R/\xi) \), where the large-\( Z \) normal transmission coefficient is proportional to \( Z^{-2} \) [2].

In the case of highly transparent interfaces corresponding to \( Z = 0 \), we find no crossed Andreev reflection: \( a'(\omega) = 0 \), in agreement with the Green’s function approach in Ref. [17].
The elastic cotunneling transmission coefficient for $Z = 0$ is given by
\[ |b'(\omega)|^2 = \frac{4(\Delta^2 - \omega^2) \exp(-2R/\xi)}{3\omega^2[1 - \exp(-2R/\xi)] + \Delta^2[1 + \exp(-2R/\xi)/2]} . \] (8)

**NINISIN junction.** – To describe multiple scattering in the normal electrodes, we consider now two additional scatterers at positions $z_1 = -L_1/2$ in the left electrode and $z_2 = L_2/2$ in the right electrode, described by the potentials $V'(z) = H'\delta(z - z_1) + H'\delta(z - z_2)$, and leading to the barrier parameter $Z' = 2mH'/\hbar^2k_F$ (see Fig. 2, for the definitions of $Z$ and $Z'$). We average numerically the non local transmission coefficient over the Fermi oscillation phases $\varphi_1 = k_F(R - L_1)/2$, $\varphi = k FR$ and $\varphi_2 = k_F(R - L_2)$.

The variations of the crossed conductance at zero bias as a function of $Z'$ for a fixed $Z$ are shown on Fig. 3 as well as the corresponding crossed conductance for the NINISIN junction. The integration over the microscopic Fermi oscillation phases for the latter involves a double integral so that the accuracy is larger than for the NINISININ junction involving a triple integral. As it is visible from the curves (a) - (f) on Fig. 3 corresponding to an increasing precision in the evaluation of the integrals, the crossed conductance for $Z_1 = 0$ has not
converged to the limiting value obtained for the NINISIN junction, meaning that the change of sign in the crossed conductance at small $Z_1$ for the NINISIN junction is an artifact related to the lack of precision in the evaluation of the triple integral (the crossed conductance at $Z_1 = 0$ for the NINISIN junction is indeed negative). The variation of the crossed conductance on Fig. 3 shows a strong enhancement by the additional scatterers, suggestive of reflectionless tunneling, as for a NIS interface [5].

Green’s functions. – Now we consider the same one dimensional geometry within Green’s functions, and first evaluate the normal and superconducting Green’s functions with appropriate boundary conditions. In one dimension, the Nambu Green’s function of a superconductor at distance $R$ and energy $\omega$ is given by

\[ \hat{g}(R, \omega) = \begin{pmatrix} g_{1,1}(R, \omega) & g_{1,2}(R, \omega) \\ g_{2,1}(R, \omega) & g_{2,2}(R, \omega) \end{pmatrix}, \]

with

\[ g_{1,1}(R, \omega) = \frac{1}{2T} \left[ -\frac{\omega}{s} \cos(k_F R) + \sin(k_F R) \right] e^{-R/\xi(\omega)} \]
\[ g_{2,2}(R, \omega) = \frac{1}{2T} \left[ -\frac{\omega}{s} \cos(k_F R) - \sin(k_F R) \right] e^{-R/\xi(\omega)} \]
\[ g_{1,2}(R, \omega) = \frac{1}{2T} \left[ \Delta s \cos(k_F R) e^{-R/\xi(\omega)} \right] + \frac{1}{4D} \sin(2k_F R) \left[ \cos(k_F R) + \frac{\omega}{s} \sin(k_F R) \right] e^{-3R/\xi(\omega)} \]
\[ g_{2,1}(R, \omega) = \frac{1}{2T} \left[ 1 + \frac{1}{4D} \right] \left[ -\frac{\omega}{s} \cos(k_F R) + \sin(k_F R) \right] e^{-R/\xi(\omega)} \]
\[ + \frac{1}{2DT} \sin(2k_F R) \left[ \cos(k_F R) + \frac{\omega}{s} \sin(k_F R) \right] e^{-3R/\xi(\omega)} \]

with $s = \sqrt{\Delta^2 - \omega^2}$ and $\xi(\omega) = \hbar v_F/s$, where $T$ is the bulk hopping amplitude of the one dimensional tight-binding model, and $v_F$ the Fermi velocity.

The Green’s functions on the finite segment $[\alpha, \beta]$ can be deduced from Eq. (9) by introducing a self-energy that disconnects the chain [23]. With the notations on Fig 4 we find

\[ g_{\alpha,\beta}^{1,1} = \frac{1}{T} \left[ 1 + \frac{1}{4D} \left( 1 - e^{-4R/\xi(\omega)} \right) \right] \times \left[ -\frac{\omega}{s} \cos(k_F R) + \sin(k_F R) \right] e^{-R/\xi(\omega)} \]
\[ + \frac{1}{2DT} \sin(2k_F R) \left[ \cos(k_F R) + \frac{\omega}{s} \sin(k_F R) \right] e^{-3R/\xi(\omega)} \]
\[ g_{\alpha,\beta}^{1,2} = \frac{1}{T} \left[ 1 + \frac{1}{4D} \left( 1 - e^{-4R/\xi(\omega)} \right) \right] \times \frac{\Delta}{s} \cos(k_F R) e^{-R/\xi(\omega)} \]
\[ - \frac{1}{2DT} \sin(2k_F R) \frac{\Delta}{s} \sin(2k_F R) \sin(k_F R) e^{-3R/\xi(\omega)}, \]
Fig. 5 – (Color online.) Variation of the crossed conductance $G_{a,b}$ (in units of $e^2/h$) within the tight-binding model for the junction on Fig. 2d, as a function of $t'$ for $t/T = 0.0499$ (corresponding to $Z = 10$ in the BTK model). The curves (a), (b) and (c) correspond to an increasing precision in the evaluation of the integral.

with

$$D = \frac{1}{4} \left[ 1 + e^{-4R/\xi(\omega)} - 2 \cos (2k_F R)e^{-2R/\xi(\omega)} \right].$$

(15)

Similar expressions are obtained for $g_{\alpha,\beta}^2$ and $\hat{g}_{\alpha,\alpha}$.

Fig. 5 shows the Green’s functions result for the variation of the crossed conductance of the NINISININ junction as a function of $t'$ for a fixed $t$ (see Fig. 2d). The numerical convergence is much faster than for the corresponding BTK model calculation because of the reduced dimension of the matrix to be inverted. We find the same feature as for the BTK model: the crossed conductance is enhanced by additional scatterers, as in reflectionless tunneling.

Imposing the same normal conductance in the BTK and in the tight-binding models leads to the relation

$$Z = \frac{1 - (t/T)^2}{2t/T},$$

(16)

leading to a good (but not perfect) agreement for the crossed conductance when the tight-binding and BTK results are rescaled on each other.

Conclusions. – To conclude, we have investigated simple one-dimensional models consisting of NISIN double interfaces, with additional scatterers away from the two interfaces, in the
spirit of Ref. [5]. We find a strong enhancement of the crossed conductance by the additional scatterers, suggesting that non local transport at localized double NIS interfaces is enhanced by orders of magnitude, like in reflectionless tunneling at a single NIS interface. The geometry studied here is such that the Thouless energy associated to the dimension of the structure parallel to the interfaces is larger than the bias voltage. This reflectionless tunneling regime is not expected to correspond to the experiment in Ref. [22] because of the extended interfaces in this experiment, but may be probed in future experiments with disordered normal metals and interfaces of reduced extension. Finally, we also evaluated the crossed conductance as a function of energy, and found no sign change when the energy is increased for the BTK model: the crossed conductance is dominated by elastic cotunneling at all energies.

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