Constituent Quark Scaling of Strangeness Enhancement in Heavy-Ion Collisions

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1. Introduction

Relativistic heavy-ion collisions aim at creating matter at extreme conditions of energy density and temperature, which is governed by the partonic degrees of freedom called quark-gluon plasma (QGP) [1]. The main focus of these studies is the observation of quark-hadron phase transition and exploration of quantum chromodynamics (QCD) phase diagram [2]. In the early phases of ultrarelativistic heavy-ion collisions, when a hot and dense region is formed at the core of the reaction zone, different quark flavors are produced. Then, the produced matter undergoes transverse expansion and multiple scattering among the produced particles. The formation of the hadrons from the partonic phase is accomplished through further expansion and cooling of the system. In proton-proton (p + p) collisions, the formation of QGP is not expected, whereas a possible formation of QGP is expected in nucleus-nucleus (A + A) collisions [3, 4]. Hence, a comparative study of produced particles in A + A collisions, with that of p + p collisions, could give better understanding of the properties of the medium formed in A + A collisions.

In the midrapidity region, strangeness enhancement has been proposed as a potential signature of QGP [5, 6]. Strange baryons are produced in strong interaction processes, and decay through weak interaction. It has been observed that multi-strange baryons, that is, Ω(sss), Ξ(ssd), and Λ(uds), are formed and decoupled from the system earlier in time [7]. Due to their different reaction rates in the medium, particles with different strangeness decouple at different times. Relativistic quantum molecular dynamics (RQMD) results suggest that the multi-strange baryons freeze out at energy densities more than 1 GeV/fm$^3$ [7] which correspond to the critical energy density predicted by lattice QCD calculations [8, 9]. The study of multi-strange baryons is of paramount importance in high-energy heavy-ion collisions because of their dominant strangeness content (s-quark). As the colliding species in nuclear collisions do not contain any strange valence quarks, particles with nonzero strange quarks can only be produced out of the collision process. The production of strange particles is enhanced in a QGP phase compared to a hadronic system. This is because the
production rate of $gg \rightarrow s\bar{s}$ (gluon fusion) is high in a QGP medium \cite{10,11}, which is absent in hadronic phase. In addition, multi-strange baryons are less suffered by hadronic rescatterings in the later stage of the evolution of the fireball because of their small hadronic interaction cross-sections. That is why multi-strange hadrons are good probes to carry the early-stage information \cite{7,12–16}. Hydrodynamic model estimations on hadron $p_T$ spectra suggest that the thermal freeze-out temperature of multi-strange baryons is close to their chemical freeze-out temperature ($T_{ch}$ $\approx$ 160 MeV), which is around the critical temperature, $T_c$, for deconfinement transition. This indicates that multi-strange baryons are almost not affected by hadronic rescatterings at the later stage of the heavy-ion collisions \cite{7,14,15,17}. Hence, multi-strange baryons could carry the information of the possible formation of a QGP phase. It could be envisaged that the multi-strange baryons are formed out of partonic interactions rather than nucleonic interactions. We will be justifying this in the following sections.

In this paper, both nuclei and nucleons are considered as superposition of constituent or "dressed" quarks (partons or valons). Baryons are composed of three quarks, and mesons are composed of two such quarks. The concept of constituent quarks is very well known \cite{18–20} and established in the realm of the discovery of constituent quark scaling of identified particles elliptic flow at RHIC \cite{21}. The constituent quark approach is successful in explaining many features of hadron-hadron, hadron-nucleus, and nucleus-nucleus collisions \cite{22}. These include global properties like the charged particle and the transverse energy density per participant pair \cite{23,24}. QCD calculations support the presence of three objects of size 0.1–0.3 fm inside a nucleon \cite{25}. Furthermore, it has been seen that nucleus-nucleus collisions and $p + p$ collisions have similar initial states if the results are scaled by the number of constituent quark participants \cite{26–28}. These observations also indicate that the particle production is essentially controlled by the number of constituent quarks pairs participating in the collision. In a constituent quark picture, nucleon-nucleon (NN) collision looks like a collision of two light nuclei with essentially one $q\bar{q}$ pair interacting in the collision, leaving other quarks as spectators. These quark spectators form hadrons in the nucleon fragmentation region with a part of the entire nucleon energy being used for the particle production ($\sqrt{s_{NN}}$ $\approx$ 160 MeV/3). But in $A + A$ collisions, due to the large size of the nucleus compared to the nucleon, there is a higher probability of $q\bar{q}$ interaction between the projectile and target nucleons. It has been observed at RHIC energies that the production rate of strange and multi-strange baryons in $Au + Au$ collisions at $\sqrt{s_{NN}}$ = 200 GeV, when scaled by the number of nucleon participants, is different (gets enhanced) when compared to similar measurements in $p + p$ collisions at the same energy \cite{29}. The observed enhancement increases with strangeness content of the baryons and also as a function of collision centrality, while going from peripheral to central collisions. Similar observations have been made at SPS energy for $Pb + Pb$ collisions at $\sqrt{s_{NN}}$ $= 17.3$ GeV, when compared with the corresponding measurements for $p + Be$ collisions at the same energy \cite{30–32}. In this paper, we have compared the centrality dependence of the number of nucleon participants normalized multi-strange baryon enhancement at SPS and RHIC energies with the expectations from HIJING, AMPT, and UrQMD models. The linear rise of the enhancement seen in the data gets converted into a spectacular quark-participant scaling behavior when normalized to the number of quark participants. Furthermore, we explore the strangeness scaling, where quark-participant normalized enhancement for different multi-strange baryons, when divided by the strangeness content, shows a strangeness-independent scaling behavior.

2. Calculation of the Number of Participants

The calculations of the mean number of nucleon/quark participants are done in the following way. In the nuclear overlap model, the mean number of participants, that is, $N_{\text{N-part}}$, in the collisions of a nucleus $A$ and a nucleus $B$ with the impact parameter $b$ is given by the following \cite{33,34}:

$$N_{\text{N-part}} = \int d^2sT_A(\vec{s}) \left\{ 1 - \left[ 1 - \frac{\sigma_{NN}T_B(\vec{b} - \vec{s})}{B} \right]^B \right\} + \int d^2sT_B(\vec{s}) \left\{ 1 - \left[ 1 - \frac{\sigma_{NN}T_A(\vec{s} - \vec{b})}{A} \right]^A \right\},$$  \hspace{1cm} (1)

where $T(b) = \int_{-\infty}^{\infty} dzn_B(\sqrt{\vec{s}^2 + z^2})$ is the thickness function, defined as the probability of having a nucleon-nucleon (NN) collision within the transverse element of area $A$. $1 - \sigma_{NN}(b) \times \frac{A}{A}$ is the probability for a nucleon to pass through the nucleus without any collision. $A$ and $B$ are the mass numbers of two nuclei participating in the collision process. We use the following Woods-Saxon nuclear density profile \cite{23,34}:

$$n_A(r) = \frac{n_0}{1 + \exp[(r - R)/d]},$$  \hspace{1cm} (2)

with parameters, where the normal nuclear density $n_0 = 0.17 \text{ fm}^{-3}$, the nuclear radius $R = (1.12A^{1/3} - 0.86^{-1/3})$ fm, and the skin depth $d = 0.54$ fm. The inelastic nucleon-nucleon cross-sections, that is, $\sigma_{NN}$, are 42 mb at $\sqrt{s_{NN}} = 200$ GeV and 30 mb at $\sqrt{s_{NN}} = 17.3$ GeV. Considering the proton as a point particle and the nucleus as an extended object in a $p + A$ collision, the number of participating nucleons is given by

$$N_{\text{N-part}} = \frac{1}{A} \left[ 1 - \frac{\sigma_{NN}T_B(b)}{A} \right]^A + T_A(b) \sigma_{NN}.$$  \hspace{1cm} (3)

In order to calculate the number of quark participants, $N_{q\text{-part}}$ in nucleus-nucleus collisions, the density for quarks inside the nucleus is changed to three times that of the nucleon density ($n_0^q = 3n_0 = 0.51$ fm$^{-3}$). Instead of nucleon-nucleon cross-section, quark-quark cross-section is used, which is 4.67 mb and 3.3 mb at $\sqrt{s_{NN}} = 200$ GeV and
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17.3 GeV, respectively [23, 35]. In $p+p(\bar{p})$ collisions, the quark participants are calculated by considering the proton and antiproton as hard sphere of radius 0.8 fm [36]. For asymmetric collisions, like $p+A$ collisions, the proton is considered as a hard sphere of radius 0.8 fm and the nucleus as an extended object with a Woods-Saxon density profile.

3. Results and Discussion

At lower center of mass energies, it has been found that the particle production scales with the number of participating nucleons, contrary to the case of high energies where hard processes dominate. Hard processes have much smaller cross-sections than those of the soft processes. However, the number of binary collisions increases with an increase in collision centrality faster than the number of participants. As a result, the particle production per participant nucleon increases with centrality. By using constituent quark approach, we are going to show how the particle production at higher energies depends on the participating quarks. For this, multi-strange particles are chosen because of the proposed signature of strangeness enhancement in QGP medium. Few of the mechanisms for strangeness enhancement include the following.

(i) The chemical and flavor equilibration time in gluon-rich plasma have been predicted to be shorter than a thermally equilibrated hadronic matter of $T \sim 160$ MeV [10, 11]. Gluon fusion ($gg \rightarrow s\bar{s}$) is a dominant production mechanism of strangeness in an equilibrated gluon-rich plasma [10, 11]. This might allow for strangeness equilibration within the lifetime of QGP and hence resulting in strong enhancement of strangeness compared to a hadronic system. This enhancement in strangeness can also be explained in the context of statistical mechanics. In $p+p$ system, the strangeness enhancement is not expected as the available volume is much smaller compared to nucleus-nucleus system. The $p+p$ system can be treated as a canonical system, and the net strangeness number should be conserved on an event-by-event basis [37]. So the $s\bar{s}$ pairs are created at the same point and are annihilated as well, resulting in the suppression of strange hadrons. But, in $Au+Au$ system, strange and antistrange hadrons are created independently and statistically distributed over the entire nuclear fireball, which could be treated in a grand canonical (GC) ensemble approach [38]. As the system thermalizes, the phase space suppression disappears, as the volume available is more. The volume here is linearly proportional to the number of participating nucleons, that is, $N_{N\text{-part}}$. So a relative strangeness enhancement is observed in $Au+Au$ collisions with respect to $p+p$ collisions [29].

(ii) Early-stage multiple scattering in heavy-ion collisions may lead to an increase of the color-electric field strength. In string-hadronic model, particles are produced through fragmenting color fields (strings) in Schwinger mechanism [39]. In high-energy heavy-ion collisions, the string density could be very high so that color flux tubes overlap leading to a superposition of color-electric fields. This results in enhanced production of multi-strange particles. This mechanism has been verified in the framework of UrQMD model with $\Omega$ showing an enhancement factor up to 100 [40].

To study the constituent quarks dependence of strangeness enhancement, we need to estimate the number of participating quarks, which has been done in the framework of nuclear overlap model. It is essential to check how good our estimation of the number of participating nucleons in the collision is. In order to do that, the mean number of participating nucleons, calculated in overlap model, is compared with the number estimated by the STAR experiment [41]. A very good agreement of nuclear overlap model calculations with that of STAR estimations has been observed. We have then estimated the number of quark participants within the prescription described in the previous section. The ratio of quark participants and nucleon participants has been found to increase monotonically with collision centrality [23]. This ratio shows a sharp increase for $N_{N\text{-part}} \leq 100$, which follows a type of linear rise while going from peripheral to central collisions.

The yield enhancement factor, $E(i)$, for particle species $i$ is given by

$$E(i) = \frac{\text{Yield}^{AA}(i) / \langle N_{AA}^{N_{N\text{-part}}} \rangle}{\text{Yield}^{NN}(i) / \langle N_{NN}^{N_{N\text{-part}}} \rangle},$$

where $\text{Yield}^{AA}(i)$ and $\text{Yield}^{NN}(i)$ are the yields of strange particles and $N_{N\text{-part}}^{AA}$ and $N_{N\text{-part}}^{NN}$ are the numbers of nucleon participants in nucleus-nucleus and nucleon-nucleon collisions, respectively. The number of nucleon participants, that is, $N_{N\text{-part}}$, is used to characterize the collision centrality. However, in the constituent quark framework, the nucleon participants no longer bear the meaning of sources of particle production. It is assumed that in this picture the constituent quarks are participating in the reaction and are the sources of interest for particle production. To understand the collision dynamics, the collision data are compared with models like HIJING-1.35, AMPT-v1.25s3, and UrQMD-3.3p1. HIJING is a heavy-ion jet interaction generator which includes multiple minijet production, nuclear shadowing of parton distribution functions, and mechanism of jet interaction with dense matter. This model is based on perturbative QCD [42, 43]. Motivated by perturbative QCD, AMPT (a multiphase transport) model includes both initial-state partonic and final-state hadronic interactions with quark-gluon to hadronic matter transition and their space-time evolution. Here, the partons are allowed to undergo scattering before they hadronize [44, 45]. UrQMD (ultrarelativistic quantum molecular dynamics) model is based on microscopic transport theory, where hadronic interactions play an important role describing the evolution of the system [46]. It should be mentioned here that the errors in the model calculations used in the subsequent sections are very small and within the marker size.

In Figure 1, $N_{N\text{-part}}$-normalized enhancements of (a) $\Lambda$ and (b) $\bar{\Lambda}$ baryons for $Au+Au$ collisions at $\sqrt{s_{NN}} = 200$ GeV...
are shown as a function of $N_{N\text{-part}}$. These collision data are from the STAR experiment [29] and are compared with the corresponding estimates of HIJING, AMPT, and UrQMD models. In this paper, the $p+p$ yields are taken from experimental data for the enhancement factor calculations [47]. It is observed that none of these models correctly describes the collision data. All of the models, that is, HIJING, AMPT, and UrQMD, show a centrality-independent behavior. It is observed that the $N_{N\text{-part}}$-normalized enhancement, obtained from AMPT model, is higher than the data obtained from HIJING and UrQMD models. This is because of the initial-state partonic interactions in AMPT model.

In Figure 2, $N_{N\text{-part}}$-normalized enhancements of (a) $\Xi^-$ and (b) $\Xi^0$ baryons at $\sqrt{s_{NN}} = 200$ GeV for $Au+Au$ collisions are shown as a function of $N_{N\text{-part}}$. The collision data points, obtained from STAR experiment [29], are compared with those of HIJING, AMPT, and UrQMD models. The enhancement observed from the model estimates is independent of centrality and is much less than the data (enhancement factor being close to one). Although AMPT model has partonic degrees of freedom, the enhancement of $\Xi^-$ and $\Xi^0$ being close to one, like HIJING and UrQMD models, indicates that AMPT model does not describe the enhancement of baryons of higher strangeness content.

The collision data obtained from STAR experiment [29] on $N_{N\text{-part}}$-normalized enhancements of $\Omega + \bar{\Omega}$ baryons at $\sqrt{s_{NN}} = 200$ GeV for $Au+Au$ collisions are shown in Figure 3 as a function of $N_{N\text{-part}}$ and are compared with the corresponding estimates of HIJING, AMPT, and UrQMD models. The enhancements observed from HIJING and AMPT models are independent of centrality, and the enhancement factor is close to one, whereas the corresponding estimates from UrQMD show a rise with centrality like experimental data, with that only difference being the enhancement factor is higher in case of data compared to UrQMD. A close observation of UrQMD estimation of $N_{N\text{-part}}$-normalized enhancement of strange baryons at $\sqrt{s_{NN}} = 200$ GeV for $Au+Au$ collisions (shown in Figures 1–3) shows a strangeness-dependent increase in the enhancement. This means that the $N_{N\text{-part}}$-normalized enhancement increases monotonically while going from $\Lambda$ to $\Omega$ baryons. This monotonic behavior of strangeness enhancement seems interesting in view of UrQMD being a microscopic transport model with hadronic scattering playing a vital role in describing the space-time evolution of the system. Multi-strange baryons are produced early in time, and also their freeze-out time is very small [7]. With hydro + UrQMD calculations and taking hadronic rescattering into account, the mean freeze-out times for $\Omega$, $\Xi$, and $\Lambda$ at top RHIC energy are 17.3, 32.2, and 27.4 fm/c, respectively [17]. The hadronic rescattering cross-sections for $\Omega$ and $\Xi$ particles and also their mean freeze-out times are much less compared to $\Lambda$ particles [48]. This shows that $\Omega$ comes out of the fireball almost without interacting with the hadronic medium, and, hence, the enhancement is much less affected compared to $\Xi$ and $\Lambda$ particles. Additionally, UrQMD calculations with a color flux tube breakup mechanism having strong color fields predict a dramatic enhancement in the multi-strange hadron production compared to $p+p$ interactions; for example, $\Lambda$‘s are enhanced by a factor of 7, whereas $\Omega$‘s are enhanced by a factor of 60 [40].

The enhancement ratio of baryons and antibaryons is usually affected by the net-baryon content or baryon stopping at midrapidity. This difference decreases with increase in collision energy. For this reason, the enhancements of baryons and antibaryons are different at SPS energies, which is not that much at RHIC energies. In Figures 4 and 5, $N_{N\text{-part}}$-normalized enhancements of $\Lambda$ and $\bar{\Lambda}$ baryons for $Pb+Pb$ collisions at $\sqrt{s_{NN}} = 17.3$ GeV are plotted as a function of $N_{N\text{-part}}$, respectively. The collision data at $\sqrt{s_{NN}} = 17.3$ GeV are from SPS NA57 experiment [30–32] and are compared

![Figure 1: $N_{N\text{-part}}$-normalized enhancements of (a) $\Lambda$ and (b) $\bar{\Lambda}$ as a function of collision centrality for $Au+Au$ collisions at $\sqrt{s_{NN}} = 200$ GeV are compared with HIJING, AMPT, and UrQMD models at midrapidity.](image)
with the corresponding estimates of HIJING, AMPT, and UrQMD models, as shown in Figures 4 and 5. The HIJING and AMPT data show a centrality-independent behavior, whereas UrQMD shows a weak centrality dependence. But the collision data of \( \Lambda \) show a remarkable centrality dependence, as shown in Figure 4. At the same time, the collision data and the HIJING, AMPT, and UrQMD data of \( \Xi^- \) are almost independent of centrality, which are shown in Figure 5. The difference in behavior of \( \Lambda \) and \( \Xi^- \) at \( \sqrt{s_{NN}} = 17.3 \) GeV could be due to different production mechanisms. This difference in the production mechanism of \( \Xi^- \) has been assigned to the following reasons. Firstly, the production threshold for antihyperons is larger than that of hyperons. Secondly, while going from higher to lower collision energy, baryon density of the system increases. This does not favor the production of particles without having a common valence quark with nucleons [30–32]. This difference in yields goes away as one moves from SPS energy to top RHIC energy.

In Figures 6 and 7, \( N_{N\text{part}} \)–normalized enhancement of \( \Xi^- \) and \( \Xi^+ \) baryons at \( \sqrt{s_{NN}} = 17.3 \) GeV for Pb + Pb collisions have been shown as a function of \( N_{N\text{part}} \). These data points are from NA57 experiment and are compared with...
Figure 5: $N_{N\text{-part}}$-normalized enhancements of $\Lambda$ as a function of collision centrality for Pb + Pb collisions at $\sqrt{s_{NN}} = 17.3$ GeV are compared with HIJING, AMPT, and UrQMD models at midrapidity.

Figure 6: $N_{N\text{-part}}$-normalized enhancements of $\Xi^-$ as a function of collision centrality for Pb + Pb collisions at $\sqrt{s_{NN}} = 17.3$ GeV are compared with HIJING, AMPT, and UrQMD models at midrapidity.

Figure 7: $N_{N\text{-part}}$-normalized enhancements of $\Xi^+$ as a function of collision centrality for Pb + Pb collisions at $\sqrt{s_{NN}} = 17.3$ GeV are compared with HIJING, AMPT, and UrQMD models at midrapidity.

Figure 8: $N_{N\text{-part}}$-normalized enhancements of $\Omega^+$ as a function of collision centrality for Pb + Pb collisions at $\sqrt{s_{NN}} = 17.3$ GeV compared with HIJING, AMPT, and UrQMD models at midrapidity.

Figure 9: $N_{q\text{-part}}$-normalized enhancements for multi-strange baryons for Au + Au collisions at $\sqrt{s_{NN}} = 200$ GeV have been shown as a function of collision centrality. These data points are from the STAR experiment at RHIC [29]. Here, we observe that, when the enhancement is normalized to the number of constituent quarks, it turns out to be a

the corresponding estimates of HIJING, AMPT, and UrQMD models [30–32]. At SPS energies, both $\Xi^-$ and $\Xi^+$ show strangeness enhancement which rises with centrality, unlike the models under discussion. However, all of the models with partonic and hadronic degrees of freedom show no enhancements of $\Xi^-$ and $\Xi^+$ baryons and are independent of collision centrality.

In Figure 8, $N_{N\text{-part}}$-normalized enhancements of $\Omega^+$ have been shown as a function of centrality for Pb + Pb collisions at $\sqrt{s_{NN}} = 17.3$ GeV for Pb + Pb collisions have been shown as a function of $N_{N\text{-part}}$. These data points are from the corresponding estimates of HIJING, AMPT, and UrQMD models [30–32]. A similar observation has been made for $\Omega^+ + \Omega^-$ baryons like the $\Xi$ baryons. To understand different enhancement profiles of multi-strange baryons at SPS energies, it is important to study their production dynamics. However, it could be observed that the shape of the enhancements for (anti-) baryons is similar, which goes in line with the predictions of a grand canonical (GC) ensemble approach [49, 50].
centrality-independent scaling behavior. This enhancement, however, depends on the particle mass and shows an increase with higher mass number. The former observation is very interesting in view of partonic degrees of freedom playing a crucial role in particle production especially for the multi-strange particles, the enhancement of which has been conjectured to be a signal of formation of a partonic phase. In other words, at top RHIC energy, the collision could be described at partonic level interactions. The number of quark participant scaling (NQ-scaling) works fine at intermediate of the elliptic flow of multi-strange particles, showing partonic collectivity at RHIC [51]. This supports present observation of centrality-independent scaling behavior of constituent quarks normalized multi-strange baryon enhancement. However, the top SPS energy shows a weak quark participant scaling towards higher strangeness content of the particles, which is shown in Figure 10. Hence, it could be inferred here that an onset of deconfinement phase transition might have taken place already at top SPS energies leading to a mixed phase of partons and hadrons. This goes in line with other experimental findings related to the onset of deconfinement or saturation of different observables starting from SPS energies [52, 53].

In Figure 10, $N_{q-part}$-normalized enhancements for multi-strange baryons at $\sqrt{s_{NN}} = 17.3$ GeV for Pb + Pb collisions have been shown as a function of collision centrality. These data points are from NA57 experiment [30–32]. Here, we observe that, for lower strangeness content particles, the quark participant normalized multi-strange enhancement turns out to be a centrality-independent scaling behavior. However, this is not true specifically for the $\Omega$ baryons, for which we observe a linear rise. This makes a difference between SPS and RHIC energies so far the constituent quark scaling of multi-strange baryons goes.

Statistical model estimation of strangeness enhancement in a broad range of energies spanning from $\sqrt{s_{NN}} = 8.73–130$ GeV suggests that (i) the absolute value of the enhancement decreases with increasing collision energy and increases with strangeness content and that (ii) the enhancement pattern as a function of centrality seems to be preserved at all energies [49, 50]. The fact that the chemical freeze-out temperature is almost constant at higher energies and there is little effect of baryon chemical potential on the strength and pattern of strangeness enhancement, going from 130 to 200 GeV collision energy, will not change these observations. At lower collision energies, the initial conditions do not favor a deconfinement transition. Hence, as discussed in [49, 50], strangeness enhancement and the pattern of enhancement are not a unique signal of deconfinement, which is a consequence of canonical suppression of strangeness in $p + p$ collisions. However, a quark participant scaling of strangeness enhancement could be considered as a signal of deconfinement transition and formation of a QGP phase, which is seen for the top RHIC energy. A weak quark participant scaling of strangeness enhancement seen at the top SPS energy in turn indicates a coexistence of partonic and hadronic phases.

The absolute values of the enhancement factor (after normalizing to the number of constituent quarks) as a function of strangeness content of different multi-strange baryons at top RHIC and SPS energies are shown in Figures 11 and 12, respectively. None of the models used here predicts any enhancement for $\Omega$’s except for UrQMD. The quark participant normalized enhancement factor at SPS energy is 5-6 times higher than that of the corresponding value at
top RHIC energy (as seen from Figures 11 and 12). This is because of the canonical suppression [37] and the number of quark participants at SPS and RHIC. The denominator in (4) governs the enhancement factor. The denominator represents the yield of $p + p(A)$ collisions at a particular energy. The $p + p(A)$ data are treated as a canonical ensemble. The canonical suppression factor decreases as we move from SPS energy to RHIC energy, which implies that the yields of multi-strange particles in $p + p(A)$ collisions increase from SPS to RHIC. This causes the enhancement of SPS data compared to RHIC data. In addition to this, the number of quark participants increases from SPS to RHIC because of the increase in the inelastic cross-section. This also helps in the enhancement of SPS data over RHIC data. The UrQMD model shows some enhancement because of the increase of the color-electric field strength due to multiple scattering in the early stage [40]. It is evident from these plots that the absolute enhancement number decreases with collision energy. This could be understood in a grand canonical ensemble approach, which predicts a significant decrease in (anti-) baryon enhancements with collision energy [29, 49, 50]. We do observe a monotonic rise in the number of constituent quarks normalized strangeness enhancement as a function of strangeness content for both energies (i.e., $E(Ω) > E(Ξ) > E(Λ)$). This could be understood as follows. The strangeness enhancement in $A + A$ collisions is always measured relative to $p + p$ or $p + A$ collisions after appropriately dividing by the number of participants. In $p + p$ collisions, even at higher energies, the available thermal or energy phase space is very less compared to $A + A$ collisions. The absolute yields of strange particles in $p + p$ collisions go down with increase in strangeness content because of canonical suppression. Compared to $A + A$ collisions, the gluon density in $p + p$ collisions is very less, which contributes to strangeness production ($gg \rightarrow s\bar{s}$) more abundantly. In $A + A$ collisions, there is enough phase space available for the formation of multi-strange particles. When we look at strangeness enhancement, particles with higher strangeness content are enhanced more, compared to the enhancement of particles with lower strangeness content [49, 50]. This also highlights the effect of the medium produced in $A + A$ collisions, where quark coalescence and gluon fusion play important roles in multi-strange particle production. However, when the enhancement factor is further normalized with respect to the strangeness content and is plotted as a function of strangeness content, a monotonic rise is observed for the top SPS energy. On the other hand, it remains almost flat for the top RHIC energy. This means that at topmost RHIC energy both constituent quark and strangeness scaling of multi-strange baryons have been observed. It can be observed from Figures 9 and 10 that the $N_{q\text{-part}}$ scaling is more prominent at RHIC compared to SPS. In view of the fact that onset of deconfinement starts at SPS energies and there are several observations of a mixed phase of partons and hadrons at SPS energies [52, 54, 55], weak $N_{q\text{-part}}$ scaling and strangeness scaling at these energies are expected. However, RHIC shows a strangeness scaling along with $N_{q\text{-part}}$ scaling. This gives an evidence of a purely partonic phase. These are shown in Figures 13 and 14. We expect that these scaling laws will hold good at LHC energies.

### 4. Summary and Conclusions

In this work, we have calculated the number of nucleon and quark participants in proton-proton, proton-nucleus, and nucleus-nucleus collisions in the framework of a nuclear overlap model. The data for the enhancement of multi-strange baryons (in $A + A$ collisions compared to $p + p$ or $p + Be$) at the top SPS and RHIC energies have been compared with HIJING, AMPT, and UrQMD models. We observe the $N_{q\text{-part}}$-normalized yields of multi-strange particles show a
monotonic increase with centrality. This turns out to be a centrality-independent scaling behavior when normalized to the number of constituent quarks participating in the collision. This geometrical scaling indicates the partonic degrees of freedom playing an important role in the production of multi-strange particles. Furthermore, we see that the constituent quark normalized yield when further divided by the strangeness content, shows a strangeness-independent scaling behavior at the top RHIC energy, whereas this scaling is not seen at the top SPS energy. This goes in line with other observations at RHIC towards the formation of a partonic phase and a mixed phase of partons and hadrons at SPS energies. These features are reproduced neither by explicit hadronic kinetic models, like UrQMD and HIJING, nor by AMPT model which treats the partonic phase on the basis of PQCD with massless partons and a noninteracting equation-of-state. A quark participant scaling of strangeness enhancement, in view of the above, may be a signal of a possible deconfined state of quark-gluon plasma.

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