The process $\pi + A$ leading to a pair of mini-jets at high relative transverse momentum ($k_t \gtrsim 2 \text{ GeV}$) while leaving the nucleus in its ground state was selected as a definitive test of the existence of color transparency by Frankfurt, Miller and Strikman in 1993. The preliminary results of Fermilab experiment E791, led by Ashery and Weiss–Babai, show a strong A-dependence, consistent with the notion of color transparency.

1 Introduction

The process we discuss is $\pi + A \rightarrow q\bar{q} + A$ (ground state)? The selection of the final state as a $q\bar{q}$ pair moving at high relative momentum ($k_t \gtrsim 2 \text{ GeV}$) plus the nuclear ground state causes the $q\bar{q}$ component of the pion to be the dominant component. At 500 GeV, the pion breaks up into a $q\bar{q}$ pair well before hitting the target. Since $k_t$ is large, the transverse separation $b$ between the $q$ and $\bar{q}$ is small. The gluon emission from a color singlet $q\bar{q}$ pair (or any color–singlet, quark–gluon system) of small spatial extent is suppressed.

The forward scattering amplitude $f \sim ib^2 \sim i\sigma_0 \langle b^2 \rangle$, where $\sigma_0$ is a normal size cross section ($\sim 20 - 30 \text{ mb}$) and $\langle b^2 \rangle$ is the pionic average of the squared transverse of the $q\bar{q}$ pair. The origin of $f$ is the exchange of two dipole gluons with the nuclear target. Two gluons are needed to maintain the color neutrality of the projectile and target. For large values of $k_t$, $f \sim -i\sigma_0/\langle b^2 \rangle$.
q\bar{q} production process (which simultaneously leaves the nucleon in its ground state) occurs rarely. Thus the dominant nuclear amplitude occurs via only one interaction of $f$. There are no other initial or final state interactions because of the small size of the $q\bar{q}$ pair. This is color transparency. The single interaction of $f$ can occur on any nucleon and, because the final state is the nuclear ground state, is a coherent process. The nuclear amplitude $M_A \propto Af$. This is a huge nuclear enhancement.

In the following, I’ll discuss our 1993 calculation, the experiment and their relationship.

3 Nucleon Amplitude

The amplitude, $M(N)$, for the process $\pi N \rightarrow q\bar{q}N$ to occur on a nucleon $N$ is given, for large enough values of $k_t$ by

$$M(N) = \int d^2 b \psi_\pi(x, b) \frac{b^2}{<b^2>} e^{-ik_t \cdot b},$$

(1)

where $\psi_\pi(x, b)$ is the pion wave function and $x$ is the fraction of the pion longitudinal momentum carried by the final state quark. The anti-quark carries a fraction $1 - x$. The $b^2$ operator can be replaced by $-\nabla_{k_t}^2$ acting on the Fourier transform, $\tilde{\psi}_\pi(x, k_t)$, of the pion wave function. For wave functions with a power fall–off in $k_t$, the term $b^2$ then acts as $\sim -1/k_t^2$. Note that $<b^2> \approx 0.24 \text{fm}^2$, while $1/k_t^2 \sim 0.01 \text{fm}^2$, so the interaction is weak indeed. For the asymptotic pion wave function we find

$$M(N) \sim \frac{i}{k_t^2} \frac{x(1-x)}{k_t^2} \alpha_s(k_t^2).$$

(2)

In this case $M(N) \propto \tilde{\psi}_\pi(x, k_t) = \alpha_s(k_t^2) \frac{x(1-x)}{k_t^2}$. This especially simple relation occurs only if the final $q\bar{q}$ pair do not interact with each other by the exchange of a high momentum gluon.

4 Nuclear Amplitude

The picture we have is that the pion becomes a $q\bar{q}$ pair of essentially zero transverse extent well before hitting the nuclear target. This point like configuration PLC can move through the entire nucleus without expanding. The $q\bar{q}$ can interact with one nucleon and can pass undisturbed through any other nucleon. For zero momentum transfer, $q_t$, to the nucleus, the amplitude $M(A)$ takes the form
\[ \mathcal{M}(A) = \mathcal{M}(N) \left( 1 + \frac{\epsilon}{\langle b^2 \rangle} A^{1/3} \right) \equiv \mathcal{M}(N) \gamma, \quad (3) \]

where the real number \( \epsilon > 0 \) and \( \gamma > 1 \). Observe the factor \( A \) which is the dominant effect here. The \( \epsilon \) correction term arises from a soft rescattering which can occur as the PLC moves through the nuclear length \((R_A \propto A^{1/3})\). The action of \( f \) produces the \( 1/\langle b^2 \rangle \) factor.

The differential cross section takes the form

\[ \frac{d\sigma(A)}{dq_t^2} = A^2 \gamma^2 \frac{d\sigma}{dq_t^2}(N)e^{-q_t^2 R_A^2/3}. \quad (4) \]

One measures the integral

\[ \sigma(A) = \int dq_\perp^2 \frac{d\sigma(A)}{dq_t^2} = \frac{3}{R_A^3} A^2 \gamma^2 \sigma(N). \quad (5) \]

A typical procedure is to parametrize \( \sigma(A) \) as

\[ \sigma(A) = \sigma_1 A^\alpha \quad (6) \]

in which \( \sigma_1 \) is a constant independent of \( A \). For the \( R_A \) corresponding to the two targets Pt (\( A = 195 \)) and C (\( A = 12 \)) of E791, one finds \( \alpha \approx 1.45 \), if \( \gamma \) is taken to be unity. Including the value of \( \gamma \) using the results of 15 leads to

\[ \alpha \approx 1.58. \quad (7) \]

That \( \gamma > 1 \) was a somewhat surprising feature of our 1993 calculation because the usual second order scattering reduces cross sections. The key features of the usual first order term are: \( f = i\sigma_0 \), and those of the second order term are \( if^2 = -i\sigma_0^2 \). Note the opposite signs. For us here \( f = i\sigma_0 b^2/\langle b^2 \rangle > 0 \), which for very large values of \( k_t^2 \) becomes \( f = -i\sigma_0/(\langle b^2 \rangle > k_t^2) \). The second order term depends on \( i f^2 = i[-i\sigma_0/(\langle b^2 \rangle > k_t^2)]^2 \), which now has the same sign as the first–order term.

5 Requirements for Color Transparency

These were discussed in Ref. 17. The two jets should have total transverse momentum to be very small. The relative transverse momentum should be \( \lesssim 2 \) GeV/c. One must identify the final nucleus as the target ground state. This is done by isolating the \( q_t^2 \)-dependence of the elastic diffractive peak. We need substantial A-dependence, \( \sigma \equiv \sigma_1 A^{1.6} \), for large enough values of
The background processes involving nuclear excitation vary as $A$, so an unwanted counting of such would actually weaken the signal we seek. The amplitude varies as $M(A) \sim \alpha_s/k_t^4$ and $\sigma(A) \sim \alpha_s^2/k_t^8$; this should be checked experimentally. For the amplitude discussed here, $\sigma(A) \sim (x(1-x))^2$. See however 6.

6 The experiment – ELAB E791

All of the information I have is from Ashery’s webpage. This discusses the excellent resolution of the transverse momentum, shows the identification of the di-jet using the Jade algorithm, and displays the identification of the diffractive peak by the $q_t^2$ dependence for very low $q_t^2$. This dependence is consistent with that obtained from the previously measured radii $R_c = 2.44$ fm, and $R_P = 5.27$ fm.

Their preliminary result is

$$\alpha \approx 1.55 \pm 0.05,$$

which is remarkably close to the theoretical value shown in Eq.(9). The $k_t^2$ dependence of the scattering amplitude has not been checked, but we are told that this is both feasible and in progress.

The measured cross section does have an $x$-dependence which is reasonably well described by $[x(1-x)]^2$. This lends support to the notion that the asymptotic behavior is manifest in nature. However, $M(A)$ is only approximately proportional to the quark distribution amplitude.

7 Electromagnetic Background

Because very low values of $q_t^2$ are involved, one could ask if the process occurs by one photon exchange (Primakoff effect) instead of two gluon exchange. Even if the Primakoff process were dominant, the unusual A-dependence would be a consequence of color transparency. However, we can estimate the relative importance of the two effects. We have

$$M_P(N) = \frac{e^2 \tilde{\psi}_\pi(x, k_t) Z}{q_t^2},$$

which should be compared with the amplitude of Eq. (2) written as

$$M(N) = \tilde{\psi}_\pi(x, k_t) A \frac{\sigma_0}{b^2 > k_t^2}$$

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using $q_t^2 \approx 0.02 \text{ GeV}^2$, $Z = 78$, $e^2 = 1/137$, $k_t = 2 \text{ GeV}$, and $\langle b^2 \rangle \approx 10$ gives
\[
\frac{\mathcal{M}_p(N)}{\mathcal{M}(N)} = -0.06i .
\] (11)

The Primakoff term is small and, because of its real nature, does not interfere with the larger strong amplitude.

8 Meaning of $\alpha = 1.55$

The use of $\alpha = 1.55$ in Eq. (6) leads to
\[
\frac{\sigma(Pt)}{\sigma(C)} = 75 .
\] (12)

The usual dependence of a diffractive process is $\approx A^{2/3}$ or $\alpha = 2/3$. This would give
\[
\frac{\sigma_{\text{USUAL}}(Pt)}{\sigma_{\text{USUAL}}(C)} = 7.
\] (13)

Thus color transparency causes a factor of 10 enhancement! This seems to be the huge effect of color transparency that many of us have been hoping to find. A word of caution must be sounded. The $k_t^2$ dependence of the cross section must be verified at least roughly to be consistent with the $(\alpha_s/k_t^4)^2$ behavior discussed above.

9 Implications

Suppose color transparency has been correctly observed in $\pi + A \to q\bar{q} + A$ (ground state). So what?

There are many implications. The spectacular enhancement of the cross section would be a new novel effect. The point like configurations PLC would be proved to exist. This is one more verification of the concept and implications of the idea of color. Previous experiments showing hints of color transparency (for a review see Ref. 3) probably do show color transparency. Efforts to observe color transparency at Jefferson Lab should be increased.

The idea that a nucleon is a composite object is emphasized by these findings. If there are PLC, there must also exist blob–like configurations of Huskyons. These different configurations have wildly different interactions with a nucleus, so that the nucleon in the nucleus can be very different from a free nucleon. This leads to an entirely new view of the nucleus, one in which the nucleus is made out of oscillating, pulsating, vibrating, color singlet, composite objects.
Acknowledgments

This work is partially supported by the USDOE, and is based on a collaboration with L. Frankfurt and M. Strikman.

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