Simple Cellular Model of Long-Range Multiplicity and $p_t$ Correlations in High-Energy Nuclear Collisions

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Abstract

A simple cellular model for the description of the long-range multiplicity and $p_t$ correlations in high-energy nuclear collisions originating from the string fusion model is proposed. Three versions of the model: without fusion, with local and with global string fusion are formulated.

A Gauss approximation which enables explicit analytical calculations of the correlation functions in some asymptotic cases in the framework of the model is developed. The assumptions of the model and the validity of a Gauss approximation are checked up in the simplest (no fusion) case when the explicit solution of the model can be found.

The role of the size of cells is anylised. The modification of the results in the case of non-Poissonian distributions is also discussed.
1 Introduction.

The colour strings approach is widely applied for the description of the soft part of the hadronic and nuclear interactions at high energies.

In the framework of this approach the string fusion model was suggested. Later it was developed and applied for the description of the long-range multiplicity and \( p_t \) correlations in relativistic nuclear collisions.

The aim of the present paper is to formulate some simple cellular analog of the model, which enables explicit analytical calculations of the correlation functions in some asymptotic cases and drastically simplifies calculations in the case of real nucleus collisions.

We check up the assumptions of the cellular model and the validity of a suggested Gauss approximation in the simplest (no fusion) case when the explicit solution of the model can be found.

The paper organized as follows. Next section is devoted to the formulation of the a cellular analog of the string fusion model. The version of the model with a local string fusion is considered.

The section 3 deals with the no string fusion limit of the model. The correspondence with the previous results is demonstrated.

In the section 4 a Gauss approximation for the correlation function calculations is formulated. The results of the calculations in this approximation is compared with exact solution, which can be found in the no fusion case.

In the section 5 the role of the size of cells is anylised. The cluster size dependence is considered and the version of the model with global string fusion is formulated.

The modification of the results in the case of non-Poissonian distributions is discussed in the section 6.

2 Cellular approach to the string fusion phenomenon.

Let us consider the collision of nuclei in two stage scenario when at first stage the colour strings are formed, and at the second stage these strings (or some other (higher colour) strings formed due to fusion of primary strings) are decaying, emitting observed particles.

We’ll consider three possibilities: without string fusion, with local and with global string fusion. The case with a local fusion corresponds to the model, where colour fields are summing up only locally and the global fusion case corresponds to the model, where colour fields are summing up globally - all over the cluster area - into one average colour field, the last case corresponds to the summing of the sources colour charges. (In section 5 we are refering to these cases as A) and B) correspondingly.)

In the transverse plane depending on the impact parameter \( b \) we have some interaction area \( S(b) \). Let us split this area on the cells of order of the transverse string size. Then we’ll have \( M = S(b)/\sigma_0 \) cells, where \( \sigma_0 = \pi r_0^2 \) is the transverse square of the string and \( r_0 \approx 0.2 fm \) is the string radius.

**Local string fusion.** At first let us consider the case with a local fusion. In this case the assumption of the model is that if the number of strings belonging to the \( i \)-th
cell is $\eta_i$, then they form higher colour string, which emits in average $\mu_0 \sqrt{\eta_i}$ particles with mean $p_t^2$ equal to $p^2 \sqrt{\eta_i}$, compared with $\mu_0$ particles with $\langle p_t^2 \rangle = p^2$ emitting by a single string. (Note that situation when some $\eta_i = 0$ is also admitted.)

Let us denote by $n_i$ and $\overline{n}_i$ - the number and the average number of particles emitted by the higher string from $i$-th cell in a given rapidity interval, then

$$\overline{n}_i = \mu_0 \sqrt{\eta_i}$$  \hspace{1cm} (1)

From event to event the number of strings $\eta_i$ in $i$-th cell will fluctuate around some average value - $\overline{\eta}_i$. Clear that in the case of real nuclear collisions these average values $\overline{\eta}_i$ will be different for different cells. They will depend on the position ($s$) of the $i$-th cell in the interaction area ($s$ is two dimensional vector in transverse plane). To get the physical answer we have to sum the contributions from different cells, which corresponds to integration over $s$ in transverse plane.

The average local density of primary strings $\overline{\eta}_i$ in the point $s$ of transverse plane is uniquely determined by the distributions of nuclear densities and the value of the impact parameter - $b$. They can be calculated, for example, in Glauber approximation. We’ll do this later in a separate paper. In present paper we consider that all $\overline{\eta}_i$ are already fixed from these considerations at given value of the impact parameter - $b$.

If we introduce:

$$N = \sum_{i=1}^{M} \eta_i, \quad \overline{N} = \sum_{i=1}^{M} \overline{n}_i$$  \hspace{1cm} (2)

then clear that $N$ is the number of strings in the given event and $\overline{N}$ is the mean number of strings for this type of events (at the fixed impact parameter $b$).

To go to long-range rapidity correlations let us consider two rapidity windows $F$ (forward) and $B$ (backward). Each event corresponds to a certain configuration $\{\eta_1, ..., \eta_M\}$ of strings and certain numbers of charged particles $\{n_1, ..., n_M\}$ emitted by these strings in the forward rapidity window. Then the total number of particles produced in the forward rapidity window will be equal to $n_F$:

$$n_F = \sum_{i=1}^{M} n_i$$  \hspace{1cm} (3)

The probability to detect $n_F$ particles in the forward rapidity window for a given configuration $\{\eta_1, ..., \eta_M\}$ of strings is equal to

$$P_{\{\eta_1, ..., \eta_M\}}(n_F) = \sum_{\{n_1, ..., n_M\}} \delta_{n_F, \sum_i n_i} \prod_{i=1}^{M} p_{\eta_i}(n_i)$$  \hspace{1cm} (4)

where $p_{\eta_i}(n_i)$ is the probability of the emission of $n_i$ particles by the string $\eta_i$ in the forward rapidity window. By our assumption (1)

$$\overline{n}_i \equiv \sum_{n_i=0}^{\infty} n_i p_{\eta_i}(n_i) = \mu_0 \sqrt{\eta_i}$$  \hspace{1cm} (5)

If we denote else by $W(\eta_1, ..., \eta_M)$ the probability of realization of the string configuration $\{\eta_1, ..., \eta_M\}$ in the given event, then the average value of some quantity $O$
under condition of the production of \( n_F \) particles in the forward window will be equal to

\[
\langle O \rangle_{n_F} = \frac{\sum \langle O \rangle_{\eta_1, ..., \eta_M} n_F W(\eta_1, ..., \eta_M) P(\eta_1, ..., \eta_M)}{\sum \langle O \rangle_{\eta_1, ..., \eta_M} W(\eta_1, ..., \eta_M) P(\eta_1, ..., \eta_M)}(n_F)
\]  

(6)

One has to omit in this \( M \)-fold sums one term, when all \( \eta_i = 0 \), which corresponds to the absence of inelastic interaction between the nucleons of the colliding nuclei.

If the \( O \) in the number of particles produced in the backward rapidity window \( n_B \) in the given event, then (for \( \langle n_B \rangle_{n_F} \) correlations) we have to use

\[
\langle n_B \rangle_{\eta_1, ..., \eta_M, n_F} = \mu_0 \sum_{i=1}^M \sqrt{\eta_i}
\]  

(7)

If the \( O \) in the mean squared transverse momentum of particles produced in the backward rapidity window \( p_{tB}^2 \) in the given event, then (for \( \langle p_{tB}^2 \rangle_{n_F} \) correlations) we have to use:

\[
\langle p_{tB}^2 \rangle_{\eta_1, ..., \eta_M, n_F} = \sum_{i=1}^M \frac{\sqrt{\eta_i}}{\sum_{i=1}^M \sqrt{\eta_i}} p^2 \sqrt{\eta_i} = p^2 \frac{\sum_{i=1}^M \eta_i}{\sum_{i=1}^M \sqrt{\eta_i}}
\]  

(8)

Later we’ll assume that numbers of primary strings in each cell \( \eta_i \) fluctuate independently around some average quantities \( \overline{\eta}_i \) uniquely determined by the distributions of nuclear densities and the value of the impact parameter - \( b \) (see above). then

\[
W(\eta_1, ..., \eta_M) = \prod_{i=1}^M w(\eta_i), \quad \sum_{i=1}^M \eta_i w(\eta_i) = \overline{\eta}_i
\]  

(9)

For clareness we’ll sometimes address to a simple ”homogeneous” case, when all \( \overline{\eta}_i \) (but not the \( \eta_i \), which fluctuate!) is equal each other in the interaction area \( \overline{\eta}_i = \eta \). The parameter \( \eta \) coincides in this case with the parameter \( \eta \) used in the papers [6, 8, 9] and has the meaning of the mean number of strings per area of one string \( \eta = (\text{mean string density}) \times \sigma_0 \). In general case the parameters \( \overline{\eta}_i \) have the same meaning, but with mean string density depending on the point \( s \) in the transverse interaction plane \( \langle \overline{\eta}_i \rangle = (\text{mean string density in the point } s) \times \sigma_0 \).

If we assume else the Poissonian form of \( p_{\eta_i} (n_i) \) \( (\rho_{\alpha} (x) \text{ is the Poisson distribution with } \alpha = a) \):

\[
p_{\eta_i} (n_i) = \rho_{\mu_0 \sqrt{\eta_i}} (n_i) = e^{-\mu_0 \sqrt{\eta_i}} \left( \frac{\mu_0 \sqrt{\eta_i}}{n_i} \right)^{n_i} \frac{1}{n_i!}
\]  

(10)

then we find for \( P(\eta_1, ..., \eta_M) (n_F) \):

\[
P(\eta_1, ..., \eta_M) (n_F) = \rho_{\mu_0} \sum_{i=1}^M \sqrt{\eta_i} (n_F)
\]  

(11)

3 No string fusion. Correspondence with the previous results.

No string fusion. In the no fusion case we have the same formulae, but instead of (5), (7) and (8), we have to use

\[
\pi_i = \sum_{n_i=0}^\infty n_i p_{\eta_i} (n_i) = \mu_0 \eta_i
\]  

(12)
\begin{align}
\langle n_B \rangle_{\{\eta_1, \ldots, \eta_M\}}^{n_F} &= \mu_0 \sum_{i=1}^{M} \eta_i \\
\langle p_{1B}^2 \rangle_{\{\eta_1, \ldots, \eta_M\}}^{n_F} &= \sum_{i=1}^{M} \frac{\eta_i}{\sum_{i=1}^{M} \eta_i} p^2 = p^2
\end{align}

which immediately leads to the absence of \( \langle p_{1B}^2 \rangle_{n_F} \) correlations.

In this case we have to use also
\[ p_{\eta_i}(n_i) = \rho_{\mu_0 \eta_i}(n_i) \equiv e^{-\mu_0 \eta_i} \left( \frac{\mu_0 \eta_i}{n_i!} \right) \]
and
\[ P_{\{\eta_1, \ldots, \eta_M\}}(n_F) = \rho_{\mu_0} \sum_{i} \eta_i(n_F) \]
instead of (10) and (11). Then for \( \langle n_B \rangle_{n_F} \) correlations we find
\begin{align}
\langle n_B \rangle_{n_F} &= \frac{\mu_0 \sum_{i} \eta_i(n_M) \prod_{i} \sum_{\eta_i} w(\eta_i) \rho_{\mu_0} \sum_{i} \eta_i(n_F)}{\sum_{\{\eta_1, \ldots, \eta_M\}} \prod_{i} w(\eta_i) \rho_{\mu_0} \sum_{i} \eta_i(n_F)}
\end{align}

Introducing under the sums
\[ 1 = \sum_{N} \delta_{N, \sum_i \eta_i} \]
and putting everywhere \( \sum_i \eta_i = N \) we find
\begin{align}
\langle n_B \rangle_{n_F} &= \frac{\mu_0 \sum_{N} \sum_{\eta_i} w(\eta_i) \rho_{\mu_0} N(\eta_i) \rho_{\mu_0} N(n_F)}{\sum_{N} \sum_{\eta_i} w(\eta_i) \rho_{\mu_0} N(n_F)}
\end{align}

where
\[ W(N) = \sum_{\{\eta_1, \ldots, \eta_M\}} \delta_{N, \sum_i \eta_i} \prod_{i} w(\eta_i) \]
If we also admit the Poissonian form for \( w(\eta_i) \) and that all \( \eta_i = \eta \) (the homogeneous case):
\[ w(\eta_i) = \rho_\eta(\eta_i) \equiv e^{-\eta} \left( \frac{\eta^n}{\eta_i!} \right) \]
then we find
\[ W(N) = \rho_{M \eta}(N) \]
and (19) has the form
\begin{align}
\langle n_B \rangle_{n_F} &= \frac{\mu_0 \sum_{N} \sum_{\eta_i} \rho_{M \eta}(\eta_i) \rho_{\mu_0 \eta} N(n_F)}{\sum_{N} \rho_{M \eta}(\eta_i) \rho_{\mu_0 \eta} N(n_F)}
\end{align}

As \( N \equiv \sum_{N} \sum_{\eta_i} \rho_{M \eta}(\eta_i) \rho_{\mu_0 \eta} N(n_F) \) we see, that (23) coincides with the formula from the paper [10], where the notation \( \pi = \mu_0 \) for the mean multiplicity from one string (emitter). The \( N \) and \( \bar{N} \) is the number and the mean number of strings (emitters) was used.
\begin{align}
\langle n_B \rangle_{n_F} &= \frac{\bar{\pi} \sum_{N} \sum_{\eta_i} \rho_{\bar{N}}(\eta_i) \rho_{\pi N} N(n_F)}{\sum_{N} \rho_{\bar{N}}(\eta_i) \rho_{\pi N} N(n_F)}
\end{align}
Let us now evaluate (24) at $N \gg 1$ and $\mu_0N \gg 1$. At these assumptions we can replace $\sum N$ by $\int dN$ and the Poissonian distributions by Gaussian distributions ($g_{a,\sigma}(x)$ is the Gauss distribution with $x = a$ and $x^2 - \bar{x}^2 = \sigma^2$).

$$\rho_{\mu_0N}(n_F) \to g_{(n_F)_N,\sigma_F}(n_F) \equiv \frac{1}{\sqrt{2\pi\sigma_F}} e^{-\frac{(n_F - \langle n_F \rangle_N)^2}{2\sigma_F^2}}$$  \hspace{1cm} (25)

with $\sigma_F^2 = \langle n_F \rangle_N = \mu_0N$. Similarly

$$\rho_N(N) \to g_{N,\sigma_N}(N) \equiv \frac{1}{\sqrt{2\pi\sigma_N}} e^{-\frac{(N - \bar{N})^2}{2\sigma_N^2}}$$  \hspace{1cm} (26)

with $\sigma_N^2 = \bar{N}$.

So we find

$$\langle n_B \rangle_{n_F} = \frac{\mu_0}{\int dN} \int dN \frac{e^{-\varphi(N,n_F)}}{\sqrt{N} e^{-\varphi(N,n_F)}}$$  \hspace{1cm} (27)

with

$$\varphi(N,n_F) = \frac{(N - N)^2}{2N} + \frac{(n_F - \mu_0N)^2}{2\mu_0N}$$  \hspace{1cm} (28)

and

$$\frac{d\varphi}{dN} = \frac{N - 1 + \mu_0}{2} - \frac{n_F^2}{2\mu_0N^2}$$  \hspace{1cm} (29)

Let us denote by $N_*$ the point where $\frac{d\varphi}{dN} = 0$. Then we can evaluate (27) as follows

$$\langle n_B \rangle_{n_F} = \mu_0N_*(n_F)$$  \hspace{1cm} (30)

In relative variables we can rewrite condition $\frac{d\varphi}{dN} = 0$ as follows

$$z^3 - z^2 = \frac{\mu_0}{2}(f^2 - z^2)$$  \hspace{1cm} (31)

which defines $z$ as function of $f$, where $z = N_*/\bar{N}$ and $f = n_F/\langle n_F \rangle = n_F/(\mu_0N)$ and then

$$\langle n_B \rangle_{n_F} = \mu_0N_*(n_F) = \mu_0\bar{N}z(f) = \langle n_F \rangle z(f)$$  \hspace{1cm} (32)

Let us define correlation coefficient as

$$b \equiv \frac{d\langle n_B \rangle_{n_F}}{dn_F}_{|_{n_F = \langle n_F \rangle}} = \frac{dz}{df} \bigg|_{f = 1}$$  \hspace{1cm} (33)

From (31) we have

$$\frac{dz}{df} = \frac{\mu_0f}{3z^2 + z(\mu_0 - 2)}$$  \hspace{1cm} (34)

and then

$$b = \frac{\mu_0}{\mu_0 + 1}$$  \hspace{1cm} (35)

Because as clear from (31) at $f = 1$ one has $z = 1$. 

4 Gauss approximation.
In reality for one string $\mu_0 = \frac{d\mu}{dy} \Delta y$, where $\frac{d\mu}{dy} \simeq 1.0 \div 1.2$ and $\Delta y$ is the width of the rapidity window. If one chooses backward and forward windows of a different width $\Delta y_B \neq \Delta y_F$, then $\mu_{0B} \neq \mu_{0F}$, and instead of (33) we have

$$\frac{d\langle n_B \rangle_{n_F}}{dn_F} \Bigg|_{n_F = \langle n_F \rangle} \equiv b = \frac{\mu_{0B}}{\mu_{0F} + 1}$$

or for "relative" quantities

$$\frac{d\langle n_B \rangle_{n_F}/\langle n_B \rangle}{dn_F/\langle n_F \rangle} \Bigg|_{n_F = \langle n_F \rangle} \equiv \tilde{b} = \frac{\mu_{0F}}{\mu_{0F} + 1}$$

We see that in the last case correlation coefficient depends only on value of $\mu_{0F}$ in the forward window (see physical explanation of this fact in the end of the section).

In Figs.1-2 we present the results of the exact (dashed lines) and approximate (in the Gauss approximation)(solid lines) calculations of the function $\langle n_B \rangle_{n_F}$ using formulas (24) and (32) correspondingly at the different values of the mean number of strings $\bar{N} = 4$ especially in the region $n_F = \langle n_F \rangle$, where the most of experimental points lay and where correlation coefficient is defined.

One has also to pay attention on the approximate linearity of the correlation functions obtained here in the case of two Poisson distributions by use of the formula (24), which coincides with the corresponding formula from [10], the problem dealt with in that paper.

In Figs.3 we present the results of the exact (dashed lines) and approximate (in the Gauss approximation)(solid lines) calculations of the correlation coefficient $b$ defined as function of $\mu_0$ using formulas (24) and (33) correspondingly at the different values of the mean number of strings $\bar{N} = 4$ in the no fusion case. We see again that Gauss approximation works very well starting from $\bar{N} = 4$ for any $\mu_0$.

As one can see from Figs.1-3 we have $\bar{N}$-independence for correlation functions and correlation coefficient $\tilde{b}$ starting very early (from $\bar{N} = 4$). It’s interesting to mention that as one can see from Figs.4,5 at that the resulting distributions $P(n_F)$ are changing drastically with $\bar{N}$ from $\bar{N} = 4$ to $\bar{N} = 128$.

We see also that we have practically ideal Gauss distribution for $P(n_F)$ at $\bar{N} = 128$. This is in agreement with the experimental data. Unlike the case of pp-interactions (where NBD for $P(n_F)$ takes place) in the case of nuclear collisions with a large number of emitting centers the ideal Gauss distribution for $P(n_F)$ has been observed experimentally for central PbPb collisions (i.e. at fixed value of impact parameter $b$ and hence fixed $S(b)$) (see, for example, Fig.6 in [11]). Note that for nuclear collisions in Glauber approximation we have also Gauss distribution for $P(n_F)$ at fixed value of impact parameter $b$ (see, for example, [12]).

**Physical interpretation.** Let us to discuss in the conclusion of the section why the correlation coefficient for "relative" quantities $\tilde{b}$ (37) depends only on the multiplicity in the forward rapidity window.

The correlations between $n_B$ and $n_F$ in the model under consideration arise only through fluctuations in the number of strings $N$. At large values of $\mu_{0F} \gg 1$ it’s more probable that fluctuation in the number of forward produced particles $n_F$ was caused...
by the fluctuation in the number of strings $N$ than by the fluctuations in $n_i$ and vice versa at small values of $\mu_0F \ll 1$ it’s more probable that fluctuation in the number of forward produced particles $n_F$ was caused not by the fluctuation in the number of strings $N$ but by the fluctuations in $n_i$.

Formally we can see it from analysis of the maximum $N_*$ of the function $\varphi(N, n_F)$ (28), in which the first term originates from the fluctuations of $N$ and the second term originates from the fluctuations of $n_i$. If we have some fluctuation (i.e. $n_F \neq \mu_0N$), then at $\mu_0F \gg 1 \ N_* \rightarrow n_F/\mu_0$ and at $\mu_0F \ll 1 \ N_* \rightarrow N$.

So if every string emits in average large number of particles ($\mu_0F \gg 1$) in the given forward interval $\Delta y_F$ then based on information of $n_F$ we can do the justified conclusion on the number of strings $N$ in the given event and hence expect corresponding change in the $n_B$. And if every string emits in average small number of particles ($\mu_0F \ll 1$) in the given interval $\Delta y_F$ then based on information of $n_F$ we can’t do any conclusions on the number of string $N$ in the given event.

Note that more detail analysis shows that this conclusion is not based on the specific (Poissonian) form of the distributions. At small $\pi_i \ll 1$ one always will have $\sigma_i \gg \pi_i$ due to discrete nature of $n_i$.

5 Cell size, cluster size, global string fusion.

Let us go back to the string fusion and consider the case, when $r_c$ - correlation radius (cluster size) is not equal to the string radius - $r_0 (\gamma \equiv r_c/r_0 > 1)$, then the square of the cluster in a transverse plane will be equal to $\Delta S = \gamma^2 \sigma_0$ and $\gamma^2$ is the square of the cluster in string square units. If $j$-th cluster was formed in the given event by $m_j$ strings, then it will have $\eta_{cj} = m_j\sigma_0/\Delta S = m_j/\gamma^2$, where $j = 1, ..., M_c$ and $M_c$ is the number of the clusters: $M_c = S(b)/\Delta S = \gamma^2 = M/\gamma^2$. (We keep notation $M$ for the quantity $S(b)/\sigma_0 = M$.) The $S(b)$ is the transverse square of interaction area of nuclei at given impact parameter $b$. Note that in given approach we consider the clusters of the fixed (by hands) area ($\Delta S$) with fluctuating from event to event number of strings $m_j$ forming it (i.e. the $\eta_{cj}$ fluctuates around $\eta$ in the homogeneous case).

Now the elementary emitters will be not strings, but clusters. The mean number of particles emitted by such cluster will be equal to $\mu_c\sqrt{\eta_{cj}}$ (in the case with string fusion), where $\mu_c = \mu_0\Delta S/\sigma_0 = \mu_0\gamma^2 = \frac{4\gamma}{\sqrt{\gamma}}\Delta y\gamma^2$. So to study the dependence on the cluster size $\Delta S$ in the given model we have to

1) increase the luminosity of elementary emitters $\mu_c\sqrt{\eta_{cj}} \rightarrow \mu_c\sqrt{\eta_{cj}} = \mu_0\gamma^2\sqrt{m_j}/\gamma^2 = \mu_0\gamma\sqrt{m_j}$

2) simultaneously reduce the number of clusters $M \rightarrow M_c = S(b)/\Delta S = S(b)/(\sigma_0\gamma^2) = M/\gamma^2$.

Note that there is no sense to introduce the clusters $\Delta S > \sigma_0$ (i.e. correlated fluctuations of the $\eta_j$ within area $\Delta S$) in the case without string fusion. Similarly it’s no physical reasons to consider clusters at small values of $\eta$ ($\eta < 1$) even in the case with string fusion.

On the contrary at large $\eta$ ($\eta >> 1$) there are physical reasons in the case with string fusion to consider two possibilities:

A) $\Delta S = \sigma_0, \ \gamma^2 = \Delta S/\sigma_0 = 1, \ M_c = M = S(b)/\sigma_0$

B) $\Delta S = S(b), \ \gamma^2 = \Delta S/\sigma_0 = M, \ M_c = 1$ (one cluster).
The first one corresponds to the model, where colour string fields are summing up only locally (formulated in section 2) and the second one corresponds to the model, where colour string fields are summing up all over the interaction area \(S(b)\) into one colour field.

In the case A) at the first stage we have \(M = S(b)/\sigma_0\) with \(\eta_i, i = 1, \ldots, M\) fluctuated around \(\eta\). Then we have to generate particles from each area \(\sigma_0\) with average multiplicities equal to \(\mu_0\sqrt{\eta_i}\) (see section 2).

**Global string fusion.** In the case B) we must MODIFY our formulae, as at first stage we also have \(M = S(b)/\sigma_0\) (like in the case A)) with \(\eta_i, i = 1, \ldots, M\) fluctuated around \(\eta\). Then (unlike the case A)) we have to find average \(\eta_c = \frac{1}{M} \sum_i \eta_i = \frac{N}{M}\) for given event, and then to generate particles from one cluster with average multiplicity equal to \(\mu_0\sqrt{\eta_c}\). So in the fusion case with one cluster (case B)) we can write simple formulæ as in the no fusion case. Namely we have

\[
\langle O \rangle_{n_F} = \frac{\sum_{\{\eta_1, \ldots, \eta_M\}} \langle O \rangle_{\{\eta_1, \ldots, \eta_M\}} W(\eta_1, \ldots, \eta_M) \mu_c \sqrt{\eta}^c \sum_i \eta_i}{\sum_{\{\eta_1, \ldots, \eta_M\}} W(\eta_1, \ldots, \eta_M) \mu_c \sqrt{\eta}^c \sum_i \eta_i} (n_F)
\]

In the case B) we must ALSO MODIFY our expressions for \(\langle O \rangle_{\{\eta_1, \ldots, \eta_M\}}\) - the rates of the backward production from configuration \(\{\eta_1, \ldots, \eta_M\}\). We have to use instead of (3) for \(\langle n_B \rangle_{n_F}\) correlations:

\[
\langle n_B \rangle_{\{\eta_1, \ldots, \eta_M\}} = \mu_c \sqrt{\eta}^c = \mu_0 M \sqrt{\frac{1}{M} \sum_i \eta_i}
\]

with \(M = S(b)/\sigma_0\) and instead of (8) for \(\langle p_{B}^2 \rangle_{n_F}\) correlations:

\[
\langle p_{B}^2 \rangle_{\{\eta_1, \ldots, \eta_M\}} = p^2 \sqrt{\eta}^c = p^2 \sqrt{\frac{1}{M} \sum_i \eta_i}
\]

Again we see that the difference with the case A) consists in replace \(\frac{1}{M} \sum_i \sqrt{\eta_i} \rightarrow \sqrt{\frac{1}{M} \sum_i \eta_i}\). As a consequence calculations in the case B) are much more simple as we can reduce all sums \(\sum_{\{\eta_1, \ldots, \eta_M\}}\) to one sum \(\sum_N\) using identity (18) as in the no fusion case.

So in the fusion case with one cluster (case B)) we can write simple formulæ as in the no fusion case. Namely we have

\[
\langle n_B \rangle_{n_F} = \frac{\mu_0 \sqrt{M} \sum_N \sqrt{\sum_N W(N) p_{\mu_0 \sqrt{M} \sqrt{\eta}}} (n_F)}{\sum_N W(N) p_{\mu_0 \sqrt{M} \sqrt{\eta}}} (n_F)
\]

and

\[
\langle p_{B}^2 \rangle_{n_F} = \frac{p^2 \sqrt{\sum_N W(N) p_{\mu_0 \sqrt{M} \sqrt{\eta}}} (n_F)}{\sum_N W(N) p_{\mu_0 \sqrt{M} \sqrt{\eta}}} (n_F)
\]

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with \( M = S(b)/\sigma_0 \) and \( W(N) \) is given by the formula (20). We see that in this case B) (one cluster at large \( \eta \)) n-n and pt-n correlations are connected

\[
\langle p_{TB}^2 \rangle_{n_F} = \frac{p^2}{\mu_0 M} \langle n_B \rangle_{n_F}
\] (43)

Note also that in the case A) for Poissonian distributions \( p_{\eta_i}(n_i) \) we have for \( P_{\eta_1,...,\eta_M}(n_F) \) the formula (11) which is very similar to (38) but with \( \frac{1}{M} \sum \sqrt{\eta_i} \) instead of \( \sqrt{\frac{1}{M} \sum \eta_i} \). More over we can fulfil in the case A) the summation (4) over \( \{n_1,...,n_M\} \) and get the similar formula for \( P_{\eta_1,...,\eta_M}(n_F) \) also in the cases of binomial and negative binomial distributions (see the next section). In these formulas the rate of \( n_F \) production depends only on some average quantities for configuration \( \{\eta_1,...,\eta_M\} \) (on \( \frac{1}{M} \sum \sqrt{\eta_i} \) in the case A) and on \( \sqrt{\frac{1}{M} \sum \eta_i} \) in the case B)) So we expect that we’ll have very similar results for n-n and pt-n correlations in the cases A) and B).

6 Non-Poissonian (binomial, negative binomial) distributions.

Important note: Even if we have no Poisson distributions (13) for \( p_{\eta_i}(n_i) \) we still can get Gauss type formula (22) for \( P_{\eta_1,...,\eta_M}(n_F) \) at \( M \gg 1 \) using (4) and the central limit theorem of the probability theory

\[
P_{\eta_1,...,\eta_M}(n_F) \rightarrow g_{\langle n_F \rangle,\sigma_F}(n_F)
\] (44)

with \( \langle n_F \rangle = \sum_i \pi_i \) and \( \sigma_F^2 = \sum_i \sigma_i^2 \), but now \( \sigma_F^2 \) can be not equal to \( \langle n_F \rangle \).

Let us consider, as an example, the binomial distributions instead of Poisson distributions.

\[
p_{\eta_i}(n_i) = \beta_{k_i,\lambda}(n_i) = C_{k_i}^{n_i} \lambda^{n_i} (1-\lambda)^{k_i-n_i}
\] (45)

Unlike Poisson distribution there are two parameters in binomial distribution. We choose the same parameter \( \lambda \) for all \( p_{\eta_i}(n_i) \) distributions. In this case we can sum these distributions like Poisson distributions due to formula

\[
\beta_{\sum_i k_i,\lambda}(n_F) = \sum_{\{n_1,...,n_M\}} \delta_{n_F,\sum_i n_i} \prod_{i=1}^M \beta_{k_i,\lambda}(n_i)
\] (46)

For binomial distributions we have

\[
\pi_i = k_i \lambda \quad \sigma_i^2 = k_i \lambda (1-\lambda) = \pi_i (1-\lambda)
\] (47)

In the no fusion case we also have \( \pi_i = \mu_0 \eta_i \) and hence \( k_i = \pi_i / \lambda = \frac{\mu_0}{\lambda} \eta_i \). So we have to choose \( \lambda \) so that to ensure the \( \frac{\mu_0}{\lambda} \) will be integer, in other respects the parameter \( \lambda \) is arbitrary. (Clear that \( n_{i_{\text{max}}} = k_i \) and the case of small \( \lambda \), when \( k_i = n_{i_{\text{max}}} \gg \pi_i \), corresponds to the Poisson case.) Then we find

\[
P_{\eta_1,...,\eta_M}(n_F) = \beta_{\langle n_F \rangle,\lambda}(n_F)
\] (48)
with $\langle n_F \rangle = \sum_i n_i = \mu_0 \sum_i \eta_i$ and $\sigma_{n_F}^2 = \sum_i \sigma_i^2 = (1 - \lambda) \mu_0 \sum_i \eta_i$. We see that for binomial distribution $\sigma_{n_F}^2 = (1 - \lambda) \langle n_F \rangle$ with $0 < \lambda < 1$. Note that case $\lambda \to 1$ corresponds to the situation, when all $n_i = n_i^{\text{max}} = k_i$ and there are no dispersion all $\sigma_i^2 = 0$.

Similarly for $w(\eta)$ starting instead of (21) from

$$w(\eta_i) = \beta_{r_i, \lambda_\eta}(\eta_i)$$

with

$$\overline{\eta}_i = r_i \lambda_\eta = \eta$$

$$\sigma_{\eta_i}^2 = r_i \lambda_\eta (1 - \lambda_\eta) = \overline{\eta}_i (1 - \lambda_\eta)$$

where now $\eta/\lambda_\eta$ must be integer. Then we find instead of (22)

$$W(N) = \beta_{\overline{\eta}_N, \lambda_\eta}(N)$$

with $N = \sum_i \overline{\eta}_i = M \eta$ and $\sigma_N^2 = \sum_i \sigma_{\eta_i}^2 = (1 - \lambda_\eta) M \eta$ and again $\sigma_N^2 = (1 - \lambda_\eta) N$ with $0 < \lambda_\eta < 1$. Clear that the case $\lambda_\eta \to 0$ corresponds to the Poissonian case and the case $\lambda_\eta \to 1$ corresponds to the situation, when $N = \overline{N}$ and there are no dispersion $\sigma_N^2 = 0$ (fixed number of strings).

Then instead of (24) we find

$$\langle n_B \rangle_{n_F} = \frac{\mu_0 \sum N \cdot \overline{\eta}_N \beta_{\overline{\eta}_N, \lambda_\eta}(N) \beta_{\overline{\eta}_N, \lambda_\eta}(n_F)}{\mu_0 \sum N \cdot \overline{\eta}_N \beta_{\overline{\eta}_N, \lambda_\eta}(N) \beta_{\overline{\eta}_N, \lambda_\eta}(n_F)}$$

In Gauss approximation we have the same formulas (23) and (26) but now with $\sigma_N^2 = \langle n_F \rangle_N (1 - \lambda) = \mu_0 N (1 - \lambda)$ and $\sigma_N^2 = \overline{N} (1 - \lambda_\eta)$. We have also the same formula (27) but now with

$$\varphi(N, n_F) = \frac{(N - \overline{N})^2}{2N (1 - \lambda_\eta)} + \frac{(n_F - \mu_0 N)^2}{2\mu_0 N (1 - \lambda)}$$

Clear that the case $\lambda \to 1$ corresponds to $\delta(n_F - \mu_0 N)$ and $N = n_F/\mu_0$. Each string emits exactly $\mu_0$ particles and the number of strings $N$ in the given event can be uniquely reconstructed on the value of $n_F$ which leads to 100% correlations (see discussion in the end of the section 4). The case $\lambda_\eta \to 1$ corresponds to $\delta(N - \overline{N})$ and $N = \overline{N}$, which corresponds to the fixed number of strings. There are no dependence of $N$ on the value of $n_F$ in this case which leads to no correlations (note that we consider the no fusion case).

We’ll have all the same formulae (34, 32, 33) for $\langle n_B \rangle_{n_F}$ and $b$, but with modified equation for $z(f)$:

$$z^3 - z^2 = \frac{\mu_0 \kappa}{2} (f^2 - z^2)$$

where

$$\kappa = \frac{1 - \lambda_\eta}{1 - \lambda}$$

We see again that at $\lambda \to 1$ we have $\kappa \to \infty$ and $z = f$; at $\lambda_\eta \to 1$ we have $\kappa \to 0$ and $z = 1$ (doesn’t depend on $f$). Instead of (34) and (35) we have

$$\frac{dz}{df} = \frac{\mu_0 \kappa f}{3z^2 + z(\mu_0 \kappa - 2)}$$
and then

\[ b = \frac{\mu_0 \kappa}{\mu_0 \kappa + 1} \]  

(57)

We see that really at \( \lambda \to 1 \) we have \( \kappa \to \infty \) and the correlation coefficient \( b = 1 \); at \( \lambda_i \to 1 \) we have \( \kappa \to 0 \) and the correlation coefficient \( b = 0 \).

Clear that we can do all the same for negative binomial distributions. We can also use any combinations of these distributions - one type for \( p_{\eta_i}(n_i) \) and another type for \( w(\eta_i) \).

7 Conclusion. Acknowledgments.

We see that suggested simple cellular model for the description of the \( p_t \) and multiplicity correlations in high-energy nuclear collisions and the Gauss approximation which enables explicit analytical calculations of the correlation functions in some asymptotic cases give the adequate results in the no fusion case.

In the next paper we plan to present the results of the correlation functions calculations with taking into account the string fusion phenomenon, based both on numerical summations on the configurations \( \{\eta_1, ..., \eta_M\} \) using formulae of the section 2 and on analytical calculations using the Gauss approximation discussed in section 4.

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Figure 1: $\langle n_B \rangle_{n_F}$ correlation functions in the no fusion case, $N$-independence. The check of validity of the Gauss approximation. The $\langle N \rangle = N$ is the mean number of strings, the $\mu_0 = \mu_{0F}$ is the mean number of particles emitted by one string in the forward rapidity window, $\mu_0 = 1$. (The $p(n_i)$ and $w(\eta_i)$ are both Poissonian.)

Figure 2: The same as in Fig.1, but for $\mu_0 = 2$. 
Figure 3: \((n_B)_{n_F}\) correlation coefficient \(T\) \((37)\) in the no fusion case as function of \(\mu_0\), \(\overline{N}\)-independence. The check of validity of the Gauss approximation. The \(\langle N \rangle = \overline{N}\) is the mean number of strings, the \(\mu_0 = \mu_0F\) is the mean number of particles emitted by one string in the forward rapidity window. (The \(p(n_i)\) and \(w(\eta_i)\) are both Poissonian.)

Figure 4: The same as in Fig.1, but plots for \(P(n_F)\) added (not normalized, arbitrary units).
mu_0=2.0, both Poisson, no fusion

Gauss <N>=\infty

exact <N>=4

exact <N>=8

exact <N>=128

Figure 5: The same as in Fig.2, but plots for $P(n_F)$ added (not normalized, arbitrary units).