A General Constrained Shortest Path Approach for Virtual Path Embedding

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Technical Report

Abstract—Network virtualization has become a fundamental technology to deliver services for emerging data-intensive applications in fields such as bioinformatics and retail analytics hosted at multi-data center scales. To create and maintain a successful virtual network service, the problem of generating a constrained path manifests both in the management plane with a physical path creation (chains of virtual network functions or virtual link embedding) and in the data plane with on-demand path adaptation (traffic steering with Service Level Objective (SLO) guarantees). In this paper, we define the virtual path embedding problem to subsume the virtual link embedding and the constrained traffic steering problems, and propose a new scheme to solve it optimally. Specifically, we introduce a novel algorithm viz., ‘Neighborhood Method’ (NM) which provides an optimal loop-free physical path applying the k-shortest path algorithm [6], often with k = 1. Even when node and link embedding are considered jointly, heuristics do not guarantee that an optimal loop-free physical path will be found, even if a feasible embedding solution exists [7]. This in turn leads to lower physical network utilization, suboptimal path allocation and consequent energy wastage caused by over provisioning.

In essence, to create and maintain a successful virtual network service, the problem of generating a constrained path is fundamental both in the management plane with a physical path creation (NFV chain instantiation or virtual link embedding) and in the data plane with on-demand path adaptation (traffic steering with SLO guarantees). In this paper, we generalize the NFV chain instantiation, the virtual link embedding and the SLO-constrained traffic steering problems, and subsume them as a unique virtual path embedding problem. The Virtual Path Embedding problem is the problem of embedding a virtual path on a physical or logical constrained loop-free path minimizing the network over provisioning. Our notion of over provisioning (that hinders infrastructure providers’ revenue maximization) can be understood in a case where three switches are used for maintaining one virtual link (point-to-point path) when there is a solution alternative that needs only two switches. We define a path to be optimal if it satisfies all SLO constraints for the minimum number of hops. Other metrics such as bandwidth, delay and jitter directly reflect a service quality, and have to be declared in a SLO.

Solutions to particular instances of the virtual path embedding problem already exist in both management and data planes. Let us consider e.g., the multi-constrained path resource reservation problem, i.e., the link embedding phase of the VNE problem. After the source and destination physical nodes have been identified, most of the link embedding solutions (see [4], [5] for a survey) seek the reservation of a loop-free physical path applying the k-shortest path algorithm [6], often with k = 1. Even when node and link embedding are considered jointly, heuristics do not guarantee that an optimal loop-free physical path will be found, even if a feasible embedding solution exists [7]. This in turn leads to lower physical network utilization, suboptimal path allocation and consequent energy wastage caused by over provisioning.

Existing data plane traffic steering solutions also have limitations [8]. Some heuristics solve this NP-complete problem [9] considering only additive/multiplicative or path constraints [10], e.g., delay and jitter, while others consider concave, min/max or link constraints, such as bandwidth [11]. Even when such solutions consider both constraint types, they are often suboptimal, e.g., they may find longer paths [10], [12], or they are not guaranteed to find a feasible solution even if it exists [13].

I. INTRODUCTION

The advent of network virtualization has enabled new business models allowing service providers to share or lease their physical network infrastructure. Even the research community has leveraged such technology offerings within wide-area virtual network federated testbeds such as the Global Environment for Network Innovations (GENI) [1] and user communities such as High Energy Physics [2]. The virtual network service offerings should meet application Service Level Objectives (SLO) demands on shared (constrained) physical networks. The Service Level Objectives are the technical constraints of a Service Level Agreement (SLA) contract. Although the term SLO may include any technical agreement constraint, in this paper we restrict our focus to the constraints within a virtual network service as well as the policies that drive the path of a Virtual Network Functions (NFV) Service chain [3]. Examples of such SLO demands include guaranteed bandwidth, high reliability, or low latency, while examples of virtual network services include high-performance computing applications such as data-intensive clusters for bioinformatics, or retail analytics hosted at multiple data centers.

In order to host such virtual network services, infrastructure and cloud providers are required to run a fundamental management protocol commonly referred to as Virtual Network Embedding (VNE) [4], [5]. The VNE is the NP-hard graph matching problem of mapping a constrained virtual network on top of a shared physical network. Once a virtual network has been embedded, traffic steering techniques need to be enforced to avoid SLO violations during both the virtual network creation phase (within the management plane), as well as when a network service is already functional (within the data plane). Moreover, to connect the (virtual) instances of a chain of virtual network functions, many middleboxes need to be connected with different traffic policies (path constraints).

In essence, to create and maintain a successful virtual network service, the problem of generating a constrained path is fundamental both in the management plane with a physical path creation (NFV chain instantiation or virtual link embedding) and in the data plane with on-demand path adaptation (traffic steering with SLO guarantees). In this paper, we generalize the NFV chain instantiation, the virtual link embedding and the SLO-constrained traffic steering problems, and subsume them as a unique virtual path embedding problem. The Virtual Path Embedding problem is the problem of embedding a virtual path on a physical or logical constrained loop-free path minimizing the network over provisioning.
Our Contributions. In this paper, we present a general multi-constrained optimal (virtual) path finder method for the virtual path embedding problem, i.e., for both physical path creation (NFV chains and virtual link embedding) applications, and for on-demand path adaptation (traffic steering) techniques i.e., after a virtual network has been created. Our algorithm viz., “Neighborhoods Method” (NM), provides on-demand virtual path embedding guarantees while reducing over provisioning, and runs in polynomial time when accepting multiple link and a single path constraints.

When instead we seek paths with multiple link and multiple path constraints, our NM algorithm runs in exponential time and hence it is intractable for large-scale physical networks [14]. To show our approach benefits, we analyze the complexity of NM, and compare it with two related solutions: the Exhaustive Breadth-First Search (EBFS) [14] and the extended version of Dijkstra (EDijkstra) [15] algorithms. We also show how a path with similar constraints can be found only in exponential time if we use the EBFS algorithm instead.

We evaluate our NM by first showing how beneficial the approach is in terms of physical network utilization when applied to existing management plane (VNE) solutions. Then we show NM is beneficial for data plane SLO-adhering solutions such as on-demand traffic steering. For every tested VNE solution, we found that the number of embedded VN requests (and thereby the providers’ revenue) increase when using NM as link embedding. In particular, with NM we were able to allocate at best twice as many virtual links. To evaluate our proposal with respect to path adaptation data plane solutions, we compare instead NM against Extended-Dijkstra (EDijkstra) a well-known technique that combines links pruning phase with Dijkstra algorithm [15]. Our simulation results show that our approach leads to gains of up to 20% in physical network utilization, and up to 150% in energy efficiency.

The rest of the paper is organized as follows: In Section II we discuss a few virtual path embedding solutions pertaining to VNE and the multi-constrained path problem. In Section III we present details of our NM approach. Section IV describes our evaluation methodology, performance metrics and results which show NM’s effectiveness. Section V concludes the paper.

II. RELATED WORK

Our NM approach has general applicability to any instance of the virtual path embedding problem. In this section we relate our contributions to a limited set of path embedding strategies which focus either on the VNE management protocol, or on current traffic steering approaches for dynamic path adaptation in a data plane. The literature of both sub-areas is vast, and we only focus here on a few relevant papers which help us highlight our contributions. A complete survey of recent VNE solutions and NFV is discussed in [4], [5], [16], [17], while a survey on multi-constrained path solutions can be found in [8].

Virtual link embedding solutions. The VNE problem requires a constrained virtual network to be mapped on top of a physical network hosted by a single infrastructure provider, or by a federation of them. To solve this NP-hard [13] problem, researchers have proposed centralized [19], [20], [21], [22] and distributed [7], [23], [24], heuristics that either separate the node embedding from the link embedding phase [7], [19], [23], [24], or simultaneously apply the two phases [24]. Most of the (centralized or distributed) VNE solutions which separate the node from the link embedding use a k-shortest path algorithm, often with $k = 1$. A link embedding based on a k-shortest path may be suboptimal. The widely cited approach used by Yu et al. [19] for example, begins a k-shortest path embedding with $k = 1$, and then increases $k$ until a path which satisfies all constraints is found. Such a schema leads to an exhaustive search with recalculation of previously found paths at each iteration, while our NM finds the optimal paths if they exist with just a single pass. In Lischka et al. [22], the authors propose a VNE based on subgraph isomorphism detection of the original physical network graph. The link suboptimality in this case arises from the physical paths having a length lower than a predefined heuristically chosen value. The heuristic is necessary to reduce the search space that is otherwise exponential. Our NM approach applied to the link embedding is agnostic to the type of heuristic used to solve the NP-hard node embedding problem. To show that this is the case, in our evaluation section we compare a representative set of existing distributed VNE solutions; replacing the $k$-shortest path algorithm with our NM leads to a higher virtual network request acceptance rate, and therefore a higher infrastructure provider revenue, with slightly higher time to solution.

Multi-constrained Path Solutions. The problem of providing a path with multiple (SLO) constraints is NP-complete [12], and its complexity has inspired many heuristics. Most of these heuristics group multiple metrics into a single function to reduce the problem to a single constrained routing problem [9], and then solve the routing optimization problem using e.g., Lagrangian relaxation [13]. Depending on the constraints, these approaches may not reach an optimal route (while ours does), i.e., the duality gap introduced by the Lagrangian function may not be zero. Other suboptimal solutions use a $k$-constrained approach. In [10] for example, Yuli et al. use $k$ path constraints and attempt to find a solution in $O(|V|^2)$, where $V$ and $E$ are the number of vertices and edges of the physical network graph, respectively. NM’s running time is comparable with the above solutions in multiple link and a single path constraints case, but NM is exact and optimal.

The non-heuristic solution proposed by Jaffe et al. [12] offer a distributed path finder solution for a two-path constraint problem with $O(|V|^2 log(|V|))$, where $b$ is the largest weight of all links in the substrate network which makes the algorithm pseudo-polynomial. Wang et al. [15] leverage the Dijkstra shortest path algorithm to propose a bandwidth-delay constraint routing approach called “pre-routing schema”. During such pre-routing, their algorithm excludes links with unfeasible bandwidth constraints. Their algorithm provides a solution in $O(|V| log |V| + |E|)$. Despite EDijkstra runs in polynomial time it has to omit path length optimization due to delay constraint satisfaction, whereas our NM algorithm allows such optimization. In [14], the authors propose a “Constrained Bellman-Ford” which leverages EBFS. The algorithm finds (in exponential time) a path that satisfies constraints
provisioning. To solve this general problem with logical constrained loop-free path minimizing the network over the problem of embedding a virtual path on a physical or virtual path embedding problem.

The authors in [26] propose an exact algorithm for the NP-hard multiple path constraints problem, and apply several search space reduction techniques for the k-shortest path constraints and apply computational complexity [12], and scales poorly to multi data links and multiple path constraints, it has exponential computational complexity. Hop-count optimization techniques have been floated before, e.g., to QoS flow scheduling where bandwidth constraint are also taking into account [11]. NM instead finds the optimal hop-count (virtual) paths to solve the virtual path embedding problem.

### III. Neighborhood Method

**Problem definition.** The Virtual Path Embedding problem is the problem of embedding a virtual path on a physical or logical constrained loop-free path minimizing the network over provisioning. To solve this general problem with l link constraints and p path constraints, we devised the Neighborhood Method. We define a path to be optimal if it is the shortest (in terms of number of hops) that satisfies our SLO constraints hence minimizing the network over-provisioning.

**Why the name “Neighborhoods Method”?** Most path seeking algorithms require at least two inputs: (i) knowledge of neighbors, and (ii) awareness of all adjacent link costs, often dictated by SLO constraints or policies. Such constraints are then used by the path seeking algorithm to compute the lowest-cost paths. The Dijkstra algorithm for example finds the shortest path traversing the source neighbors and the neighbors of their neighbors, recursively. This recursive notion leads to our definition of “neighborhoods” in NM, i.e., a set of nodes that can be reached from the source node with the same number of hops, where each “neighborhood” contains unique elements.

**A. The NM General Case (l ⊕ p case)**

The general case of our NM algorithm accepts multiple links and multiple path constraints, it has exponential computational complexity [12], and scales poorly to multi data center virtual network service applications. Note however, that existing heuristics such as single mixed metric [9] or Lagrangian relaxation [13] can reduce the l ⊕ p to a l ⊕ 1 case.

Figure 1 and Algorithm 1 show the workflow of the general NM algorithm. NM is executed in three steps: (i) the forward pass or neighborhoods building, (ii) a backward pass, and a final (iii) back track path validation step. During the forward pass, NM builds the neighborhoods to estimate the path length. The backward pass is used to find end-to-end paths with a given length and the final constraints validation step is used to decide whether or not the path search should be extended to longer path candidates involving more neighbors.

#### Algorithm 1: General NM algorithm (l ⊕ p case)

**Input:** X := src Y := dest, C := constraints list (links and paths)  
**Output:** Shortest Route between X and Y satisfying C

```
begin
/* Build neighborhoods <NH> from X to Y */
/* Backward Pass to find whole set of routes <Route> with length equal <NH> size */
/* Validate QoS constraints; if not satisfied, add one neighborhood and repeat Backward Pass */
end
```

**Example 1.** To illustrate how our NM algorithm works in the l ⊕ p case, consider an example network consisting of 4 nodes X, Y, A and B (Figure 2a). Each link has two metrics - a link metric (first value) e.g., bandwidth bw and a path metric (second value) e.g., propagation delay. Our aim is to find a route from X to Y which satisfies constraints, e.g. bw ≥ 5 and delay ≤ 5. In the first step, we build neighborhoods starting from source node X until destination node Y is reached as shown in Figure 2b. As soon as we reach Y, we begin the second step where we perform backward pass to find the full set of shortest routes as shown in Figure 2c. If a route with satisfying cost among the shortest ones is found, NM will stop searching and returns this route. In our case, there is no route which satisfies all constraints among the shortest ones. If an appropriate route is not found, the NM builds an additional neighborhood as shown in Figure 2d. Following this, NM performs backward pass and finds all the shortest + 1 length routes as shown in Figure 2e. In the last step, NM looks for appropriate solution among found the shortest + 1 length routes, and if it is not there NM terminates due to the maximum path length violation. In our case, X → B → A → Y satisfies all constraints, and NM returns this solution.
Forward Pass for $l \oplus p$. In the first step, NM successively builds $<NH>$ from the source node $X$ to reach the destination node $Y$. Algorithm 2 describes in detail the forward pass of NM. To build a $NH$, we add therein neighbors (adjacent nodes) of each node $n$ in $cNH$ (line 6). For example, the first $NH$ includes nearest neighbors of the source node $X$ (which is in the zero $NH$), and the second $NH$ contains nearest neighbors of nodes in the first $NH$, and so on. The first step ends as soon as the destination node $Y$ appears in the $cNH$ (line 4), or a number of the $<NH>$ is more or equal to the number of nodes (line 9).

**Algorithm 2: Build Neighborhoods ($l \oplus p$ case)**

```plaintext
/* Returns: neighborhoods list $<NH>$ from $X$ to $Y$ if reachable */
Input: $X$ := src, $Y$ := dest
Output: $<NH>$ from $X$ to $Y$

begin
  $cNH$ := $X$
  $<NH>$ := $<NH>$ ∪ $cNH$
  while $Y \not\in cNH$ do
    $NH$ := $\emptyset$
    foreach Node $n$ ∈ $cNH$ do
      $NH$ := $NH$ ∪ neighbors of $n$
    end
    if $<NH$.size ≤ number of nodes then
      $<NH>$ := $<NH>$ ∪ $NH$
    end
    else
      $Y$ is unreachable
    end
  end
end
```

Backward Pass for $l \oplus p$. Algorithm 3 details the second step of NM - backward pass. This step works similar to the Breadth-First Search method with the only difference being that we do not process all neighbors of each node $n$ but only those which are within previous $NH$ (line 10). The first step is to find the intersection between neighbors of the destination node $Y$ with its previous $NH$ (line 5). This intersection is not an empty set, it contains at least one node $nh$. For all obtained nodes we again build the intersection of their neighbors with the penultimate $NH$ (line 16). The second step ends as soon as we hit the zero $NH$ (line 6), and as a result we obtain the collection of all paths with a length of $<NH$.size between the source node $X$ and the destination node $Y$. A distinctive feature of our NM is the construction of the intersection of two neighborhoods, one of which will always be the nearest one. Thus, our NM approach greatly reduces the number of comparisons i.e., the computational complexity of the analysis.

**Constraints Validation for $l \oplus p$.** Finally, we check relevant candidates i.e., paths that satisfy required cost among found ones. If appropriate paths are not found, we build one more $(N + 1)$ neighborhood (Figure ??) and repeat backward pass (line 3 of Algorithm 1) and subsequently obtain all shortest + 1 length paths. Again, we check that the retrieved routes satisfy all constraints, and if none of them satisfy, we build a neighborhood $N + 2$. Iteration continues until the solution length will be equal to the number of nodes.

**Algorithm 3: Perform Backward Pass ($l \oplus p$ case)**

```plaintext
Input: $<NH>$ - list of the sets of nodes, $l$ - link constraints
Output: All possible paths $<path>$ between $X$ and $Y$ of a given length equal to the size of $<NH>$

begin
  path := $Y$
  $<path>$ := $<path>$ ∪ path
  $k$ := 1
  $NH$ := $<NH>$ [size $k$]
  while $NH \neq <NH>$ [0] do
    $<tempPath>$ := \emptyset
    foreach path $<path>$ ∈ neighbors of n ∩ $NH$ do
      $<tempPath>$ := $<tempPath>$ ∪ path
    end
    $NH$ := $NH$ [size $k$]
  end
end
```

Fig. 3: Running example of NM with $l$ links and $p$ path constraint specified ($l \oplus 1$ case): (a) In the pre-routing phase NM prunes $B \rightarrow Y$ due a $bw$ violation; (b) In the forward phase NM finds the best length to $Y$ discarding previous results if better subpaths are found; (c) back trace phase identifies the valid path $X, B, A, Y$ by recursive predecessor visits.

**B. NM for $l$ links and a single path constraints ($l \oplus 1$ case)**

If we only have $l$ links and a single path constraint NM runs in polynomial time, as unnecessary iterations are avoided.

**Example 2.** Consider Figure 3a On the link $(X, A)$, the first value 5 refers to a link constraint in this case bandwidth, and the second 5 refers to a path metric, in this case end-to-end delay. NM finds a path from $X$ to $Y$ satisfying the two constraints $bw \geq 5$ and $delay \leq 5$ as follows: In the pre-routing phase (Figure 2a) In the pre-routing phase NM prunes $B \rightarrow Y$ due a $bw$ violation. In the forward phase (Figure 3b) NM finds the best length to $Y$ discarding previous results if better subpaths are found. During the back trace phase we then recursively construct the actual optimal path using the node identifiers found in the forward phase (Figure 3c).

**NM pseudocode for $l \oplus 1$.** Algorithm 4 describes the NM algorithm for the $l \oplus 1$ case with additive path metric. Note that multiplicitive constraints can be converted to additive by composing them with a logarithmic function. Each node $u$ contains information about its predecessor $\pi(u)$, the distance $D(u)$ and the location of its last neighborhood $L_{NH}(u)$. Note that we do not delete empty neighborhoods to be able detect a negative weight cycle (line 33). During the pre-routing phase, $\pi(u)$ and $L_{NH}(u)$ are set to NIL, whereas $D(u)$ is set to 0 (line 3-5). We then set all edge weights that do not satisfy $l$ constraints to $\infty$ to avoid using these links during the forward phase (line 19).
During forward phase we successively build the neighborhoods \(<NH>\), where for each current neighborhood \(cNH\) of \(u\) we exclude all neighbors \(v\) that do not satisfy the path constraint or that have previous distance lower than \(dist\) (line 31). In addition, we remove neighbor \(v\) from the previous neighborhood location \(L_{NH}(v)\) (line 24) and save a new neighborhood location of \(v\) (line 25). The forward phase ends as the destination node \(Y\) appears in the current neighborhood \(cNH\) (line 4). If the number of neighborhoods \(<NH>\) is equal to the nodes number (line 33) we terminate the algorithm concluding that a negative weight cycle is detected, or that a path does not exist (line 36). During the back track phase we recursively find the shortest possible path between \(X\) and \(Y\) which satisfies \(l \oplus 1\) (line 41).

**Algorithm 4: NM in \(l \oplus 1\) case**

Input: \(X := src\), \(Y := dest\), \(l\) link constraints, \(p\) single path constraint.
Output: The shortest possible path between \(X\) and \(Y\) which satisfies \(l \oplus 1\).

```plaintext
begin
    /* pre-routing phase: */
    foreach Vertex \(u \in [V]\) do
        \(\pi(u) \leftarrow NIL\)
        \(D(u) \leftarrow 0\)
        \(L_{NH}(u) \leftarrow NIL\)
    end

    foreach Edge \(u \rightarrow v \in [E]\) do
        if \(u \rightarrow v\) does not satisfy \(l\) then
            \(w(u \rightarrow v) \leftarrow \infty\)
        end
    end

    /* forwarding phase: */
    \(<NH> \leftarrow X\)
    \(<NH> \leftarrow <NH> \cup cNH\)
    while \(Y \notin cNH\) do
        \(NH \leftarrow \emptyset\)
        foreach Vertex \(u \in cNH\) do
            foreach Neighbor \(v \in adjacent(u)\) do
                \(dist \leftarrow D(u) + w(u \rightarrow v)\)
                if \(dist < D(v)\) and \(dist < p\) then
                    \(\pi(v) \leftarrow u\)
                    \(D(v) \leftarrow dist\)
                    \(NH \leftarrow NH \cup \{v\}\)
                    if \(L_{NH}(v) = NIL\) then
                        \(<NH> \leftarrow L_{NH}(v)\)
                        \(<NH> \leftarrow <NH> \cup cNH\)
                        \(L_{NH}(v) \leftarrow <NH> . size + 1\)
                    end
                end
            end
        end
        if \(NH \neq \emptyset\) and \(<NH> . size < |V|\) then
            \(<NH> := <NH> \cup NH\)
            \(cNH \leftarrow NH\)
        else if \(<NH> . size == |V|\) then
            Return Negative Weight Cycle is detected.
        else
            Return \(Y\) is unreachable.
        end
    end

    /* back track phase: */
    while \(\pi(path(1)) \neq NIL\) do
        \(\pi(path(1)) \leftarrow path\)
        \(path \leftarrow \pi(path(1)) \cup path\)
    end
end
```

C. NM Optimal Solution and Constraints Satisfaction

**Theorem 1.** (Theorem of the Optimal Solution)
NM always finds the optimal path if it exists.

**Proof:** To prove this theorem we need to prove: firstly, that the forward pass of the general NM checks constraints satisfaction for each possible path length between the source and the destination nodes; secondly, that the backward pass of NM can return any path of a given length. The first thesis can be proved by contradiction: assume that there is the optimal path with \((i)\) the minimum or \((ii)\) the maximum length, and for this length the constraints have not been checked. Firstly, the forward pass step starts building neighborhoods from the first neighborhood, meaning that we check for constraints satisfaction at the path length=1. If there is a path with length lower than 1, we have the special case in which the source node is also the destination assumed to be satisfied, which in turn contradicts with assumption \((i)\). Further, the forward pass step continues to build neighborhoods until their number is equal to the number of nodes, or the destination node with satisfied constraints is in the last neighborhood. In the first case, the forward pass ends by checking constraints satisfaction at the maximum possible length of a loop-free solution, and there will never be a case in which a complex path (with loops) can be provided due to a contradiction with the optimal solution definition. In the second case, the forward pass ends by finding the minimum length at which all constraints are satisfied, hence NM will provide this path which is by definition the optimal solution. In both cases we contradict assumption \((ii)\). The second thesis can be also proved by contradiction: assume that NM has not found a path with the \(N\) hops length between the source and the destination nodes. This is possible only if at least one of the path’s nodes has not appeared in the neighborhoods \(<NH>\). In this case, it suggests that this node is not accessible from the source node within \(N\) hops, i.e., if it is unreachable, or path does not have the \(N\) hops length, which contradicts our assumption. In summary, we proved that the forward pass of NM checks constraints satisfaction at each possible path length between the source and the destination nodes. At the same time, the backward pass can return any path of a given length which provided by the forward pass. Consequently, we can conclude that if an optimal solution exists, then NM will find it.

**Corollary 1.** (The \(l \oplus 1\) Constraints Satisfaction)
NM always provides a solution that satisfies \(l \oplus 1\) if such a solution exists.

**Proof:** Based on the Theorem\[1\] NM provides the optimal solution if it exists. Moreover, NM provides a solution which satisfies \(l\) constraints if it exists by pruning all links which do not satisfy them during pre-processing phase (Algorithm\[2\] line 9) Hence, we only need to show in the \(l \oplus 1\) case that NM always provides a solution that satisfies the path constraint if it exists. We prove this by contradiction. Assume NM provides a path which does not satisfy the requested path constraint. In this case, the destination node cannot appear in the last neighborhood owing to the path constraint violation (Algorithm\[4\] line 19). Hence, NM cannot provide
this solution. That is a contradiction, which confirms our original thesis.

IV. PERFORMANCE EVALUATION

In this section, we establish the practicality of NM by evaluating its performance in two complementary scenarios: (i) within the management plane, during a virtual network creation when a path is sought, and (ii) within the data plane in traffic steering solution when maintaining a given SLO in a virtual link. In particular, we compare the performance of NM when applied to the VNE management plane problem, as a link embedding phase. We also analyze the impact of NM during the lifetime of a virtual network and a single NFV chain of middleboxes (virtual nodes) by assessing the physical network utilization and the energy consumption, when adapting the data path to external physical network state changes.

Simulation Settings. For our simulations, we used a machine with an Intel(R) Xeon(R) processor with CPU 2.1 GHz and 1GB RAM, and running the Ubuntu OS GNU/Linux x86_64. We use the BRITE [28] topology generator to create our physical and virtual networks. Our results are consistent across physical networks which follow both Waxman and Barabasi-Albert models [29] [30]. All our results show 95% confidence interval, and our randomness lays in both the (virtual) link constraints and in the physical network topology.

To assess the impact of NM on the virtual network embedding, we include here the results obtained with a physical network of 100 nodes following Waxman connectivity model, where each physical node has 200 CPU units and 200 bandwidth units, and each virtual node and link have up to 20 units of CPU and bandwidth, respectively. We attempt to embed a pool of 15 VN requests with 14 virtual nodes and random virtual topologies, so that we could vary the number of virtual links from 1 (the VN has linear topology) to 13 (VN is a fully connected topology).  

To evaluate the impact of NM on data plane traffic steering applications, we simulate instead requests for remapping constraints virtual links. This scenario suits also traffic steering applications in a dynamic NFV chain reallocation when a new policy is declared. In particular, on a physical network topology of 10,000 nodes, where each physical link has bandwidth uniformly distributed between 1 and 9 Gbps, we attempt to find an optimal path for 1000 random pairs <src,dst>, where for each pair we allocate as many virtual links as possible with the same demands. We denote as low, medium and high bandwidth constraints, 1, 4 and 7 Gbps, respectively, which represent approximately 10%, 45% and 80% of the maximum physical link capacity, while with high, medium and low delay constraints we indicate 400%, 250%, and of 80% of the maximum physical link delay, respectively. An additional simulation involves using a fixed average physical node degree equal to 4 (which is common for the Internet [30]), different bandwidth (BW) constraints considered in the case of SLO demands while varying the delay constraint. We request a delay constraint from 400% to 50% of the maximum link delay.

Management plane evaluation metrics. To demonstrate the advantages of using NM as a path embedding solution, we compared three representative VNE distributed algorithms and replaced their proposed link embedding solutions with our NM. We have chosen Hub-and-Spoke (H&S) [24] as to our knowledge it is the first distributed VNE solution. In physical nodes elect the host of the hub virtual node, and then the host for the spokes virtual nodes. Then a virtual link (VL) embedding phase finds the shortest path among the hosts. If at least a VL constraint is violated or a virtual node cannot be hosted, the entire VN request is rejected. We also compare against PolyViNE (Poly) [23], the first policy-based distributed VNE solution. Physical nodes belonging to different infrastructure providers partition the VN and attempt to embed the largest possible VN partitions, and then create virtual links among the winner physical hosts. Finally, we compare against a Consensus-based Auction mechanism (CAD) [7], the first policy-based distributed VNE approximation algorithm with convergence and optimality guarantees. The link embedding of these algorithms runs a k-shortest path with k = 3 for Waxman and k = 1 for Barabasi-Albert topologies shown to be within optimal range of k.

In this simulation scenario we have tested the potential revenue loss by using a suboptimal distributed VNE algorithm by specifying the fraction of VN request accepted over the VN requested (allocation ratio), how many virtual links (VL) were accepted over the VL requested (link allocation ratio) and to what extent physical links were utilized (link utilization). Note that in all other link embedding solutions, all shortest paths are pre-computed, while NM computes them dynamically.

Data plane evaluation metrics. To evaluate the performance of NM for data plane traffic steering solutions we compare NM with the Extended Dijkstra algorithm or k-shortest path algorithm with k = 1, where both algorithms find a path on-demand. We compare NM across four metrics: total gained throughput of all reallocated VLs, energy efficiency, average path length per VL, and average (convergence) time to a stable path. We approximate the energy efficiency with the ratio between unused nodes and the total gained throughput:

\[ \text{Energy Efficiency} = \frac{N - \sum_{\text{used}}}{N} \cdot \frac{bw_{\text{total}}}{bw} \]  

where N is the total number of physical nodes, \(N_{\text{used}}\) is the number of physical used nodes to maintain all VLs, and \(bw_{\text{total}}\) is the number of VLs multiplied by their bw.

Results Summary. During the virtual network formation, we found that using NM to dynamically search a loop-free physical path to embed a constrained virtual link may significantly increase the virtual network allocation ratio (Figure 4a), resulting in higher physical link utilization (Figures 4c and 4d). During the virtual network lifetime instead, we show how embedding a path with NM is beneficial in terms of optimal path length, network utilization and energy efficiency. Such advantages come at different intensity levels, depending on the severity of the requested constraints, and bring along with them some interesting tradeoffs.

1 Using a dataset of 8 years of 64,000 real VN embedding requests to the Emulab [31] virtual network testbed, it has been shown in [7] that VNs have size of 14 nodes on average.
Path embedding during virtual networks creation. Except when the node embedding phase leaves no room for allocation ratio improvements (see e.g., the Hub&Spoke heuristic), NM brings higher link embedding acceptance rates (see Figure 4a). This is because, when a lower-cost alternative path is available, NM finds it (see Figure 4b). We conclude that seeking on-demand low-cost paths brings higher VN allocation ratios as well as higher overall network utilization, hence making the Cloud and infrastructure provider services more attractive to customers (see Figures 4c and 4d).

The price we pay to dynamically find the optimal loop-free hosting physical path is higher response time: on-demand path computation cannot be as fast as pre-computing a suboptimal path.

NM shows benefits in physical network utilization and energy efficiency. The next set of results analyze the performance of our NM during path maintenance in comparison with EDijkstra under physical network topologies with a different average node degree, as well as with different SLO constraints.

In Figure 5a and 5b we show how the total gained throughput of all reallocated VLs changes when multiple physical links become available (the average physical node degree increases). This dependence is an affine function with a negative coefficient: the maximum possible total gained throughput also increases linearly with the available physical links. Moreover, we found how the total gained throughput is higher for low SLO constraints. This is not surprising as the path constraints requirement becomes harder to satisfy.
Average path length tradeoff and over provisioning. We further investigate the reasons why we observed such gains in both throughput and energy efficiency w.r.t. EDijkstra (Figures 5c, 5d, 6c, and 6d). In particular, observing Figures 5c and 6c we note that there are almost 2 hops difference in the average path length between NM and EDijkstra for the high average node degree for Waxman topology and slightly lower for Barabasi-Albert topology, whereas for the medium SLO constraints this difference is reduced to circa one hop for Waxman topology and it is even lower for Barabasi-Albert topology. Finally, when the SLO constraints are high, there is no significant physical path length difference. This is confirmed by Figures 5d and 6d, where we show that NM has an higher average time per embedded path than EDijkstra for the low and medium SLO constraints, which is in line with NM’s quadratic time complexity of $O(n^2)$.  

To understand why the average path length changes with the constraint severity, note how the longer is an end-to-end physical path, the lower is the probability that the entire path satisfies the SLO constraints. On the other hand, the higher the number of hops, the higher is the number of candidates paths, and so the higher is the probability of finding one which satisfies these constraints. This explains the trade-offs in average path length behavior observed in Figures 5c and 6c.

**Correlation between SLO constraints.** To better understand the correlation between link (bandwidth) and path (delay) constraints, additional results are as follows. Figures 7c and 8c show a logarithmic function - with delay increasing the total gained throughput grows logarithmically. Again, NM gains
up to 1 Tbps throughput with respect to what EDijkstra produces when we set the bandwidth to low and delay to high constraints (6 Tbps versus 5 Tbps). This translates into an extra 20% of network utilization for Waxman topology and lesser difference for Barabasi-Albert topology.

Similar trade-offs can be observed in energy efficiency w.r.t. previous scenario (Figures 77 and 89) where NM shows 10% and 5% higher maximum energy efficiency than EDijkstra for Waxman and Barabasi-Albert topologies, respectively. Despite the performance in the medium bandwidth case, NM always shows better or equal energy efficiency. Note that in all simulations where the bandwidth is set to low values and delay constraint is set to high and medium values, we observe a trade off between total throughput (higher for NM) and energy efficiency (higher for EDijkstra). However, NM is never worse for both total gained throughput and energy efficiency. Further, Figures 77 and 89 again confirm optimality of NM solution. However, this time we may see logarithmic dependency from delay constraint is explained by unlikeliness of having longer path for low delay (high SLO constraint). Figures 77 and 89 also agree with NM’s quadratic time complexity of $\mathcal{T} + 1$ case.

V. CONCLUSION

In this paper we defined the Virtual Path Embedding problem, i.e., the problem of embedding a virtual path on a physical or logical constrained loop-free path minimizing the network over provisioning. To solve this problem, we proposed a novel on-demand path embedding scheme viz., “Neighborhoods Method” (NM). NM is suitable for both management plane mechanisms such as virtual network embedding and network functions virtualization, as well as for data plane mechanisms such as traffic steering satisfying SLOs. In particular, we show how NM provides on-demand SLO virtual path guarantees while reducing expensive physical network over provisioning, by considering multiple link and a single path constraints in polynomial time.

Via Monte-Carlo simulations for a set of diverse topology scenarios, we also show that our NM approach can lead to gains of up to 20% in network utilization during the virtual network creation phase within both the management and the data planes, and up to 150% in energy efficiency during the virtual network lifetime within the data plane, with respect to existing embedding solutions based on the multi-constrained $k$-shortest path or Extended Dijkstra.

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