Enhancement of surge-induced synchronized switch harvesting on inductor strategy

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Abstract

We propose and demonstrate a novel method to enhance vibration harvesting based on surge-induced synchronized switch harvesting on inductor (S\textsuperscript{3}HI). S\textsuperscript{3}HI allows harvesting of a large amount of energy even from low-amplitude vibrations by inducing a surge voltage during the voltage inversion of a synchronized switch harvesting on inductor (SSHI). The surge voltage and the voltage amplification from the conventional voltage inversion improve energy harvesting. S\textsuperscript{3}HI modifies SSHI by both rewiring the circuit without adding components and using a novel switching pattern for voltage inversion, thus maintaining the simplicity of SSHI. We propose a novel switching strategy and circuit topology and analyze six methods that constitute the S\textsuperscript{3}HI family, which includes traditional S\textsuperscript{3}HI and high-frequency S\textsuperscript{3}HI. We demonstrate that the six methods suitably harvest energy even from low-amplitude vibrations. Nevertheless, the harvestable energy per vibration cycle depends on the switching pattern and storage-capacitor voltage. The use of the proposed switching strategy, which allows energy harvesting before energy-dissipative voltage inversion, substantially increases the harvestable energy per vibration cycle. In the typical case considered in this study, the said increase is on the order of 11\%--31\% and 15\%--450\% compared to the traditional and existing high-frequency S\textsuperscript{3}HI methods, respectively, depending on the storage-capacitor voltage. Additionally, the proposed circuit can be used as a traditional circuit. It could be considered a promising alternative to S\textsuperscript{3}HI methods owing to its potential auto-reboot capability, which is not found in traditional S\textsuperscript{3}HI circuit.

Keywords: energy harvesting, piezoelectric, low-amplitude vibration, surge voltage

(Some figures may appear in colour only in the online journal)

1. Introduction

Technologies to generate electric power from an ambient source have attracted increasing interest for over a decade, with the aim of enabling applications such as sensor networks, environment monitoring in remote locations, and health monitoring of structures in inaccessible areas. Such a technology may replace batteries and power lines to achieve standalone systems that require low or no maintenance. Ambient energy
sources include sunlight, temperature gradient, wind, tides, and vibration. Vibration-energy harvesting using piezoelectric materials is being actively studied, given its high efficiency. The standard method for energy harvesting involves connecting a full-bridge rectifier (FBR) and a storage capacitor to a piezoelectric element attached to a vibrating structure, but the harvesting performance is low. Ottman et al [1, 2] integrated a DC/DC converter into this simple circuit and controlled the converter for maximum power harvesting. Lefevre et al [3] proposed synchronous electric charge extraction (SECE) for harvesting the energy from a piezoelectric element into a storage capacitor by using an inductor and a switch. Based on a semi-passive vibration control method called synchronized switch damping on inductor [4], Guyomar et al [5] proposed a nonlinear harvesting method called synchronized switch harvesting on inductor (SSI). In SSI, the polarity of the voltage across the piezoelectric element is inverted by switching an inductive shunt circuit in synchrony with the structure vibration. Hence, SSI increases the absolute value of the voltage across the piezoelectric element for energy harvesting. This method attracted research attention owing to its simplicity and high performance, driving various developments on vibration-energy harvesting using piezoelectric elements.

Lefevre et al [6] studied a series SSI method with the piezoelectric element series-connected to an inductor. Lallart et al [7] proposed a hybrid SSI method to accommodate a wider range of load impedances. Numerous strategies were subsequently developed, including synchronized switching and discharging to a storage capacitor through an inductor [8], double synchronized switch harvesting [9], enhanced synchronized switch harvesting (ESSE) [10], and energy injection [11]. Guyomar and Lallart [12] reviewed and evaluated some of these approaches. To increase the frequency range of the exciting force, nonlinear mechanical structures have been proposed, as reviewed by Tran et al [13]. Recently, researches to improve and extend SECE and SSI have been actively conducted. Lallart et al [14] proposed synchronous inversion and charge extraction (SICE). Du and Seshia [15] proposed a technique called synchronized switch harvesting on capacitors, which can reduce the volume of circuits drastically. Badr et al [16] proposed a parallel-type SSI method using a negative voltage converter. As mentioned above, several developments have successfully increased the available power for harvesting from vibration generators under various conditions.

Various applications of vibration-energy harvesters present intermittent bursty consumption of large amounts of energy. For example, health-monitoring devices transmit data intermittently and consume more energy during communication. The amplitude of vibration may vary depending on the environmental conditions. Even when the vibration amplitude is reduced, the monitoring devices need to communicate at the same time intervals. For such applications, a higher energy-harvesting rate (i.e. harvested energy per period of vibration cycle) and higher upper limit of storable energy from low-amplitude vibrations are desired. This is equivalent to maintaining a high energy-harvesting rate over a wide range of storage-capacitor voltages covering higher voltage, even when the vibration amplitude is small. However, in the classical SSSI, the storage capacitor is charged only when the voltage across the piezoelectric device exceeds that of the storage capacitor plus the forward voltage drop (FVD) of the diodes in the bridge rectifier, thus limiting energy storage, especially under small-amplitude vibrations.

Makihara et al [17] proposed a switching strategy for SSSI considering vibration supression, and Yoshimizu et al [18] proposed its adaptive version to increase the upper limit of storable energy by waiting for vibration amplitude recovery from the depressed status by energy harvesting—they achieved successful results. However, in principle, the amount of gain is limited to the recovery of loss.

Remarkably, Kwon et al [19] proposed charging the storage capacitor even under a small vibration amplitude, a high-voltage charge of the storage capacitor, and a non-negligible FVD. They called this method surge-inducing synchronized switch harvesting (S¹HI). In S¹HI, a surge voltage is generated by turning the switch off while current still flows through the inductor at the end of SSSI voltage inversion. By exploiting this surge voltage, more energy can be stored compared with the conventional SSSI. Moreover, in the high-frequency S¹HI method, a higher harvesting rate is achievable over time if the switch is turned off several times during voltage inversion [20]. The S¹HI methods are suitable for systems that intermittently demand high consumption.

The objective of this study is to further enhance the energy harvesting rate and upper limit of storable energy of the S¹HI method, even under small-amplitude vibrations with a non-negligible FVD. In this study, we considered a conventional and a high-frequency S¹HI as two methods in the S¹HI family. In addition, we aimed to develop and demonstrate new methods from the S¹HI family to further increase the harvesting performance. Consequently, we propose a switching strategy to harvest energy at the beginning of voltage inversion, unlike conventional and high-frequency S¹HI. As voltage inversion dissipates energy in the piezoelectric element, the proposed method may enhance the efficiency and harvesting rate. In addition, we propose and evaluate a circuit topology for energy harvesting. Then, we thoroughly characterize the methods in the S¹HI family. Specifically, we theoretically and experimentally analyze six combinations of two circuits (the proposed circuit and that used in [19] and [20]) and three switching patterns (the proposed pattern and those in [19] and [20]) considering various scenarios.

In this study, the amplitude of mechanical vibrations remains constant, irrespective of the energy-harvesting action performed. Therefore, this study is limited to the case involving a weak coupling between the mechanical and electrical systems. Cases involving strong coupling are to be taken up in a future study. Further, the interaction between the exiting forces and mechanical vibrations (such as resonance) is out of the scope of this study.

The remainder of this paper is organized as follows. In section 2, we describe the conventional SSSI method. Section 3 presents the operation of each method in the S¹HI
family, including the method combining the proposed switching strategy and circuit topology. In addition, we approximate algebraic equations to estimate the performances of the methods, only omitting lengthy derivations. In section 4, we report numerical simulations and their results, which are compared with approximate solutions to verify their consistency. In section 5, we report experiments and their results, which are compared with the simulation results. In section 6, the performance and characteristics of each method in the S\textsuperscript{2}HI family for various scenarios are investigated by using the previously verified numerical simulations. Finally, we draw conclusions in section 7.

2. Conventional S\textsuperscript{2}HI

We describe the typical S\textsuperscript{2}HI behavior and approximate the harvesting performance by obtaining the harvesting rate. Figure 1 shows the system outline for the S\textsuperscript{2}HI method. The circuit consists of a piezoelectric element attached to the vibrating structure, an inductor $L$, a switch, and a storage capacitor $C_h$ that is connected via an FBR. The vibrating structure is modeled by a mass and a spring. The piezoelectric element is modeled by a capacitance $C_p$ and a voltage generator with voltage $V_p$. $C_p$ denotes the capacitance of the piezoelectric element when the displacement of the mass shown in figure 1 is restricted; and $V_a$ denotes the open-circuit voltage of the piezoelectric element. The mass displacement is given by $u = u_0 \cos(\omega_s t)$, where $u_0$ is the constant vibration amplitude and $\omega_s$ is the angular frequency of the structural vibration. Then, the open-circuit voltage of the generator is given by

$$V_a = V_{oc} \cos(\omega_s t),$$

(1)

where $V_{oc} = b_u u_0$ is the amplitude of the open-circuit voltage of the piezoelectric element and $b_u$ represents a piezoelectric constant.

Let $Q_p$ denote the charge in the piezoelectric element. The voltage across the piezoelectric element is given by

$$V_p = V_a + Q_p/C_p.$$  

(2)

The characteristics of each diode of the bridge rectifier are modeled as follows:

$$i = (V_{ap} - V_d)/R_d, \text{ if } V_{ap} > V_d,$$

$$i = 0, \text{ if } V_{ap} \leq V_d,$$

(3)

where $V_{ap}$ is the applied voltage, $i$ is the forward current, $V_d$ is the FVD, and $R_d$ is the on-state resistance of the diode. In the S\textsuperscript{2}HI method, the switch is turned on whenever $V_d$ reaches its maximum or minimum, and the on-state remains for half the period of the electrical vibration in the main circuit. If switching is repeated, the $V_d$ amplitude increases, and some energy is stored in capacitor $C_h$ when the absolute value of $V_p$ exceeds $2V_d + V_h$, where $V_h$ is the voltage of $C_h$.

We assume that each component in the circuit, including the piezoelectric element, has an equivalent series resistance [21]. We denote the equivalent series resistances of the piezoelectric element, inductor, and on-state switch as $R_p$, $R_L$, and $R_s$, respectively. Figure 2 shows typical steady behaviors of $V_p$ and $\dot{Q}_p$. At $t = 0$, we assume that $V_p = V_{p0}$ and

$$|V_p| \leq 2V_d + V_h.$$  

(4)

Hence, $Q_p$ and $\dot{Q}_p$ at this instant are given by

$$Q_{p0} = C_p (V_{p0} - V_{oc}),$$

$$\dot{Q}_{p0} = 0.$$  

(5)

(6)

Let us further assume that $V_d$ reaches maximum $V_{oc}$, and the switch is turned on at this moment. The subsequent behavior of $Q_p$ is described as

$$L \ddot{Q}_p + (R_p + R_L + R_s) \dot{Q}_p + Q_p/C_p + V_a = 0.$$  

(7)
Moreover, we assume that the system is designed such that \( \omega_s \ll \omega_e \), and thus we can regard \( V_s \) as constant during the subsequent period of \( \pi/\omega_e \), where

\[
\zeta = (R_p + R_L + R_s) / (2L),
\]

\[
\omega_e = \sqrt{(LC_p)^{-1} - \zeta^2}^{1/2}.
\]

Considering the initial values, the solution of equation (7) can be obtained as follows:

\[
Q_p = C_p V_{p0} \exp \left( -\zeta t \right) \left( \zeta \omega_e^{-1} \sin (\omega_e t) + \cos (\omega_e t) \right) - C_p V_{oc}.
\]

Then, the values of \( V_p \), \( Q_p \), and \( \dot{Q}_p \) at \( t = \pi/\omega_e \) are derived as

\[
V_{p1} = -\gamma V_{p0},
\]

\[
Q_{p1} = C_p ( -\gamma V_{p0} - V_{oc} ),
\]

and \( \dot{Q}_{p1} = 0 \), where \( \gamma = \exp \left( -\pi \zeta/\omega_e \right) \) is an inversion factor. Therefore, as shown in figure 2, the value of \( V_p \) jumps from \( V_{p0} \) to \( V_{p1} \), and its polarity inverts almost immediately.

In the subsequent half period of structural vibration, \( \pi/\omega_e \), the switch is kept open, and thus \( Q_p \) remains constant, whereas \( V_p \) varies toward \( V_{p1} - 2V_{oc} \) as \( V_s \) varies from \( V_{oc} \) to \( -V_{oc} \). Once \( V_p \) attains the value of \( -V_h - 2V_d \), electric current \( \dot{Q}_h \) flows through the bridge rectifier and storage capacitor, as shown in figure 2, and \( C_h \) is charged. Voltage \( V_h \) can be considered constant because capacitance \( C_h \) is large and the effect of \( R_d \) is negligible. Hence, \( V_p \) remains at \( -V_h - 2V_d \), as shown in figure 2. Moreover, the electric charge of \( C_p ( -V_{p1} + 2V_{oc} - 2V_d - V_h ) \) is added to \( C_h \) by \( t = \pi/\omega_e \). Equation (4) holds at this instant, and the switch is turned on again. The subsequent behavior is the same as that described above except for the inverted polarity of some variables.

From the steady-state assumption and considering the behavior in figure 2, we can note that \( V_{p0} = V_h + 2V_d \). After some manipulations of equations based on this relation, we observe that the following energy term:

\[
\Delta E = 2C_p V_h \{ 2V_{oc} - (1 - \gamma)(2V_d + V_h) \}
\]

(13)

is additionally stored in \( C_h \) per structural vibration cycle. Equation (13) shows that no energy is additionally stored when \( V_h \) increases to \( 2V_{oc}(1 - \gamma)^{-1} - 2V_d \) unless the stored energy is consumed or transferred to another storage component.

3. S^3HI

3.1. Overview

When the current in an inductor varies quickly, the inductor generates a high voltage called a surge voltage. In SSSI, the switch is turned off when the current in the inductor is zero. Therefore, a surge voltage is not generated. Even when a surge voltage appears, the circuit in figure 1 cannot use it to increase the stored energy in \( C_h \). Therefore, to exploit the surge voltage, the SSSI system should be modified in two aspects.

First, the circuit is modified such that the surge voltage enhances energy storage. Figure 3 depicts two such circuits. No components have been added to the original SSSI circuit. A rectifier can be connected to the main circuit in three ways: (a) across the piezo element (circuit P in figure 1), (b) across the inductor (circuit I in figure 3(a)), or (c) across the switch (circuit S in figure 3(b)). The SSSI approach uses circuit P, whereas the conventional and high-frequency S^3HI approaches [19, 20] use circuit I. The last circuit, S, has not been investigated using the S^3HI approach. In addition, when the switch in circuit S remains open, the circuit appears similar to that used in the standard FBR method. Therefore, given a large vibration amplitude, circuit S might harvest some energy without the need for switching control, thereby resulting in an auto-reboot of a system undergoing shutdown owing to complete battery discharge. Therefore, it is important to investigate how circuit S performs in accordance with the S^3HI strategy, albeit the realization of the auto-reboot feature is a topic for future research. Because the only difference between the three circuits is the rectifier connection, their behaviors can be described by a single equation if no current flows through the rectifier.

Second, the switching pattern for voltage inversion is modified to generate surge voltage. Figure 4 shows four switching patterns. Pattern S keeps the switch turned on for period \( \pi/\omega_e \) synchronized with half the period of the electric vibration, as described in section 2, and thus it does not generate surge voltage. Pattern E keeps the switch turned on for period \( \pi_T \), which is shorter than \( \pi/\omega_e \). This pattern generates a surge voltage at the end of the voltage inversion. Pattern H turns
the switch on and off at a high frequency during the period \( \tau_T \). We propose pattern B, which maintains the switch in the ON state for period \( \tau_N \), turns it OFF for a short period \( \tau_F \), and finally maintains it in the ON state for period \( \tau_{N2} \). This switching pattern generates a surge voltage close to the beginning of the voltage inversion. Voltage inversion decreases the energy stored in the piezoelectric element to \( \gamma^2 \) times (i.e. less than \( 2/3 \) even when \( \gamma \) is as large as 0.8). Pattern B aims to transfer the energy before it dissipates.

We analyzed the combinations of both circuits (I and S) in figure 3 with every switching pattern, E, H, and B, in figure 4. Note that the conventional SSHI combines circuit P and switching pattern S. The combinations of circuit I with switching pattern E (S\(^3\)HI) [19] and circuit I with pattern H (high-frequency S\(^3\)HI) [20] have already been studied. Note that we refer to all the combinations exploiting surge voltage as S\(^3\)HI and indicate the circuit and switching pattern to identify the combination (e.g. IE indicates S\(^3\)HI combining circuit I with switching pattern E). The circuit S and switching pattern B constitute our proposed configuration in this study.

In the remainder of this section, we analyze the typical operation of the different methods in the S\(^3\)HI family. Approximate performance estimates are also derived, except for the cumbersome combinations IH and SH. Figure 5 shows the typical behaviors of voltage \( V_p \) and currents \( \dot{Q}_p \) and \( \dot{Q}_h \) in long and short time scales. \( V_{pm} \) in figure 5(a) denotes the value of \( V_p \) upon completion of voltage inversion, and its value according to the combination is indicated in the figure legend.

### 3.2. IE

IE uses circuit I and switching pattern E. Similar to SSHI, IE turns the switch on when \( V_p \) reaches its maximum or minimum and turns it off after period \( \tau_T \). In IE, \( \tau_T = \beta \pi / \omega_c \) for \( 0.5 < \beta < 1 \) to generate surge voltage.

We use the assumptions from section 2 unless otherwise stated. If no current flows through the bridge rectifier of circuit I, the electric behavior of circuit I is described by equation (7).

Therefore, as described in section 2, the values of \( Q_p \), \( \dot{Q}_p \), and \( V_p \) at \( t = \tau_T \) can be derived from equation (7) as follows:

\[
Q_{p1} = C_p V_{p0} \exp \left( -\zeta \beta \pi / \omega_c \right) \left\{ (\zeta / \omega_c) \sin (\beta \pi) + \cos (\beta \pi) \right\} - C_p V_{oc}, \tag{14}
\]

\[
\dot{Q}_{p1} = -C_p V_{p0} \exp \left( -\zeta \beta \pi / \omega_c \right) \left\{ (\zeta / \omega_c) + \omega_c \right\} \sin (\beta \pi), \tag{15}
\]

\[
V_{p1} = -\gamma_\beta V_{p0}, \tag{16}
\]

where \( \gamma_\beta = \exp \left( -\zeta \beta \pi / \omega_c \right) \left\{ (\zeta / \omega_c) \sin (\beta \pi) + \cos (\beta \pi) \right\}. \)

The switch is turned off at \( \tau_T \). Then, it is clear from figure 3(a) that subsequently \( Q_h = 0 \) and \( V_p = V_{p1} \) hold. However, the current in the inductor cannot vary instantly. Therefore, the induced surge voltage overcomes voltage barrier \( 2V_d + V_h \).

![Figure 3](image_url)

**Figure 3.** Circuit topologies to exploit surge voltage—(a) circuit I (bridge rectifier connected across inductor) and (b) circuit S (bridge rectifier connected across switch).

![Figure 4](image_url)

**Figure 4.** Switching patterns for conventional SSHI and surge-induced synchronized switch harvesting on inductor (S\(^3\)HI).
Figure 5. Typical behaviors of $V_p$, $\dot{Q}_p$, and $\dot{Q}_h$ according to $S^3$HI combination—(a) general behavior over a long period along with specific behaviors over a short period for combinations of (b) IE, (c) IB, (d) IH, (e) SE, (f) SB, and (g) SH.

The subsequent behavior of $Q_h$ is described as

$$L \ddot{Q}_h + R_T \dot{Q}_h + V_{B1} = 0,$$

(17)

where $R_T = R_a + R_l$, $V_{B1} = V_h + 2V_d$, and $R_h$ is the total series resistance of two diodes and the storage capacitor. We assume that $C_h$ is sufficiently large, and $V_h$ can be considered constant. After some manipulations of equation (17), we observe that $\dot{Q}_h$ ceases to flow at $t = \tau_T + \tau_{z1}$, and the increment of charge in $C_h$ by this moment is given by

$$\Delta Q_h = \left(\frac{L}{R_T}\right) \{Q_{p1} - (V_{B1}/R_T)\ln (1 + Q_{p1}R_T/V_{B1})\},$$

(18)

where $\tau_{z1} = (L/R_T)\ln (1 + Q_{p1}R_T/V_{B1})$. In the next half period of structural vibration ($\pi/\omega_e$) the switch is kept open, and thus $Q_h$ remains constant. In contrast, $V_p$ varies from $V_{p1}$ to $V_{p1} - 2V_{oc}$ as $V_a$ varies from $V_{oc}$ to $-V_{oc}$. As the system operates in steady state, $V_{p1} - 2V_{oc} = -V_{p0}$, as shown in figure 5(a). Therefore, from equation (16), we obtain

$$V_{p0} = 2V_{oc}/(1 - \gamma/\beta).$$

(19)

In the next half period of structural vibration, $C_h$ is additionally charged by the amount described in equation (18). Therefore, the energy stored in $C_h$ per cycle of structural vibration is given by

$$\Delta E = 2 \left(\frac{V_{p0}L}{R_{T1}}\right) \{Q_{p1} - (V_{B1}/R_{T1})\ln (1 + Q_{p1}R_{T1}/V_{B1})\},$$

(20)

where $\Delta E$ is obtained by substituting equations (15) and (19) into equation (20). Note that the above analysis is limited to the typical case in which $V_{p0}$ given by equation (19) satisfies equation (4).

3.3. IB

IB uses circuit I and switching pattern B for the energy in the piezoelectric element to be harvested at the beginning of voltage inversion. We use the assumptions from section 3.2 unless otherwise stated. If the switch is turned on at $t = 0$ and turned off at $t = \tau_N = \beta\pi/\omega_e$, the charge given by equation (18) is additionally stored in storage capacitor $C_h$ during subsequent period $\tau_1$, and the values of $Q_p$, $Q_h$, and $V_p$ at $t = \tau_N + \tau_L$ are respectively given by equation (14), $Q_p = 0$, and
equation (16) assuming that \( \tau_p \leq \tau_f \). For IB, we assume that 0 < \( \beta \) < 0.5, which typically results in \( V_{p1} > 0 \), as shown in figure 5(c). IB then turns the switch on again at \( t = \gamma_N + \tau_f \) to complete voltage inversion. Then, as described in section 2, voltage inversion is completed at \( t = \gamma_N + \tau_f + \pi/\omega_c \).

IB typically turns the switch off at this moment. As in section 3.2, \( V_p \) at this moment can be derived from equation (11) as follows:

\[
V_{p2} = -\gamma V_{p1} + \gamma \beta V_p 0. \tag{21}
\]

As in section 3.2, it is clear that \( V_{p2} - 2V_{oc} = -V_{p0} \), and thus \( V_{p0} \) is given by

\[
V_{p0} = 2V_{oc}/(1 + \gamma \beta). \tag{22}
\]

The amount of energy stored per cycle of structural vibration is given by equation (20) and can be obtained from equations (15) and (22).

### 3.4. SE

SE uses circuit S and switching pattern E. We use the assumptions from section 3.2 unless otherwise stated. If no current flows through the bridge rectifier, the behavior of circuit S can be described by equation (7). Therefore, the values of \( \dot{Q}_p \), \( \ddot{Q}_p \), and \( V_p \) at \( t = \tau_f = \beta \pi/\omega_c \) (i.e., \( \dot{Q}_p \), \( \ddot{Q}_p \), and \( V_p \)) are respectively given by equations (14)–(16). SE turns the switch off at this moment. As current \( \dot{Q}_p \) cannot vary instantly, it starts to flow through the piezoelectric element, inductor, diode, storage capacitor, and next diode, charging the storage capacitor as shown in figure 5(e). Hence, \( \dot{Q}_h = -\dot{Q}_p \), and the subsequent behavior of \( \dot{Q}_h \) is described as

\[
L \ddot{Q}_h + R \tau_f \dot{Q}_h + Q_h/C_p^{-1} + V_{B2} = 0, \tag{23}
\]

where \( V_{B2} = V_{oc} - V_h - 2V_d \) and \( R \tau_f = R_p + R_L + R_h \). After some manipulations of equation (23), we can see that \( \dot{Q}_h = 0 \) at \( t = \tau_f + \tau_{2} \), as shown in figure 5(e), completing voltage inversion. In addition, \( \dot{Q}_h \) at this moment is given by

\[
\dot{Q}_{p11} = e^{-\zeta_2 \tau_2} \left\{ (\dot{Q}_{p1} + \zeta_2 \dot{Q}_{pp}) \omega_2^{-1} \sin(\omega_2 \tau_2) + \dot{Q}_{pp} \cos(\omega_2 \tau_2) \right\} - C_p V_{B2}, \tag{24}
\]

with

\[
\tau_2 = \omega_2^{-1} \tan^{-1} \left\{ \dot{Q}_{p1} / \left[ \zeta_2 \omega_2^{-1} (\dot{Q}_{p1} + \zeta_2 \dot{Q}_{pp}) + \omega_2 \dot{Q}_{pp} \right] \right\}, \tag{25}
\]

where \( \dot{Q}_{pp} = \dot{Q}_{p1} + C_p V_{B2} \), \( \zeta_2 = R \tau_f/(2L) \), and \( \omega_2 = \sqrt{(LC_p)^{-1} - \zeta_2^2} \). Moreover, \( V_p \) at this moment is given by

\[
V_{p11} = Q_{p11}/C_p + V_{oc}. \tag{26}
\]

The additionally stored charge in the storage capacitor within this period \( \tau_2 \) is equal to the corresponding decrease in \( Q_p \):

\[
\Delta Q_h = Q_{p1} - Q_{p11}. \tag{27}
\]

As shown in figure 5(a), after voltage inversion, \( V_p \) varies from \( V_{p1} \) to \( V_{p11} - 2V_{oc} \), and \( V_h \) varies from \( V_{oc} \) to \( -V_{oc} \). If \( V_{p11} - 2V_{oc} < -2V_d - V_h \), current \( \dot{Q}_p \) starts to flow when \( V_v \) reaches \( 2V_{oc} - 2V_d - V_h \) while varying toward \( V_{p11} - 2V_{oc} \). However, we omit this case because it is rare, as analyzed below. Hence, we assume that \( V_{p11} - 2V_{oc} > -2V_d - V_h \) and focus on the typical case. As the system is in steady state,

\[
V_{p11} - 2V_{oc} = -V_{p0}. \tag{28}
\]

Unlike IE and IB, we need to solve algebraic equation (28) and the above equations to obtain the value of \( V_{p0} \) for SE. Once we determine \( V_{p0} \), \( \Delta Q_h \) can be obtained by substituting equations (14) and (24) into equation (27). In the next half period of structural vibration, \( C_h \) is additionally charged by the amount described in equation (27), and the energy stored per period of structural vibration is given by

\[
\Delta E = 2V_h \Delta Q_h. \tag{29}
\]

### 3.5. SB

SB drives the switch in the proposed circuit S according to the proposed switching pattern B shown in figure 4. We use the assumptions from section 3.4 unless otherwise stated. Again, if the switch is turned on at \( t = 0 \) and turned off at \( t = \gamma_N = \beta \pi/\omega_c \), the charge given by equation (27) is additionally stored in capacitor \( C_h \) during the next period \( \tau_2 \), and \( \dot{Q}_p \), \( \ddot{Q}_p \), and \( V_p \) at \( t = \gamma_N + \tau_f \) are respectively given by equation (24), \( \dot{Q}_p = 0 \), and equation (26), provided that \( \tau_2 < \tau_f \).

For SB, we assume that 0 < \( \beta \) < 0.5, and therefore \( V_{p11} \) is positive as shown in figure 5(f). SB turns the switch on again at this moment and turns it off at \( t = \gamma_N + \tau_f + \pi/\omega_c \). Subsequently, \( \dot{Q}_p \) becomes zero, and this completes the voltage inversion, as shown in figure 5(f). The value of \( V_p \) at this moment is given based on equation (11) by

\[
V_{p21} = -\gamma V_{p11}. \tag{30}
\]

We assume that \( V_{p21} - 2V_{oc} \geq -V_h - 2V_d \) to focus on the typical case. Then, as described in section 3.4, it is clear that

\[
V_{p21} - 2V_{oc} = -V_{p0}. \tag{31}
\]

In the steady state. Therefore, \( V_{p0} \) can be obtained by solving equation (31). The amount of energy stored per period of structural vibration is obtained from equation (29) along with equations (14), (24), and (27), and \( V_{p0} \) can be obtained accordingly.

### 3.6. IH and SH

IH turns the switch of circuit I on at \( t = 0 \), off at \( t = \gamma_N \), on at \( t = \gamma_N + \tau_f \), off at \( t = 2\gamma_N + \tau_f \), and so on. Typically, voltage inversion is expected to complete when the switch is turned off at \( t = (N_p + 1) \gamma_N + N_F \tau_f \), where \( N_p \) is the number of periods \( \gamma_f \).

SH controls the switch of circuit S like IH. The typical behaviors of \( V_p \), \( \dot{Q}_p \), and \( \dot{Q}_h \) are illustrated in figures 5(d) and
(g) for IH and SH, respectively. The energy stored per cycle of structural vibration can be derived by using the equations derived in the previous sections. However, they are not shown here for brevity.

4. Numerical simulations

In section 3, algebraic equations to estimate the performance of each method are derived for typical cases. To investigate and evaluate thoroughly the methods in the $S^3$HI family including the omitted cases, we conducted numerical simulations. We derived the governing equations from Kirchhoff’s law and performed numerical integration using the Runge–Kutta method.

4.1. Governing equations

As the considered system includes diodes, it is convenient to describe the governing equations for the six states listed in table 1. Let $i_t$ denote the current flowing through the bridge rectifier as shown in figures 1 and 3. To write the equations compactly, we introduce variable $S$, whose value is shown in table 1 according to the switch state and polarity of $i_t$. Table 1 lists the applicable governing equations per state:

\[ \dot{Q}_p + S \dot{Q}_h = 0, \]  
\[ \dot{Q}_h = 0, \]  
\[ L (\dot{Q}_p + S \dot{Q}_h) + (R_p + R_s + R_L) \dot{Q}_p + (R_L + R_s) S \dot{Q}_h + V_a + C_p^{-1} Q_p = 0, \]  
\[ \dot{Q}_p = 0, \]  
\[ L \dot{Q}_h + (R_L + R_h) \dot{Q}_h + 2 V_d + C_h^{-1} Q_h = 0, \]  
\[ (R_p + R_s) \dot{Q}_p - S (2 V_d + R_h \dot{Q}_h + C_h^{-1} Q_h) + V_a + C_p^{-1} Q_p = 0, \]  
\[ R_s (\dot{Q}_h + S \dot{Q}_p) + R_h \dot{Q}_h + 2 V_d + C_h^{-1} Q_h = 0, \]  
\[ L \dot{Q}_p + (R_p + R_L + R_s) \dot{Q}_p + S R_s \dot{Q}_h + V_a + C_p^{-1} Q_p = 0. \]

\[ L (\dot{Q}_p + S \dot{Q}_h) + (R_p + R_s + R_L) \dot{Q}_p + R_L S \dot{Q}_h + V_a + C_p^{-1} Q_p = 0, \]  
\[ L \dot{Q}_p + (R_p + R_L) \dot{Q}_p - S (2 V_d + R_h \dot{Q}_h + C_h^{-1} Q_h) + V_a + C_p^{-1} Q_p = 0, \]  
\[ L \dot{Q}_p + (R_p + R_L + R_s) \dot{Q}_p + S R_s \dot{Q}_h + V_a + C_p^{-1} Q_p = 0. \]

\[ (R_p + R_s) \dot{Q}_p - S (2 V_d + R_h \dot{Q}_h + C_h^{-1} Q_h) + V_a + C_p^{-1} Q_p = 0, \]

4.2. Simulation program

The simulation program was scripted in Fortran wherein the applicable governing equations were selected in accordance with table 1 based on the switch and current $i_t$ states. The equations were numerically integrated to evaluate $Q_p$, $\dot{Q}_p$, $\dot{Q}_h$, and $\dot{Q}_h$. The states of the switch and $i_t$ were checked at each successive time instant, and if any state demonstrated a change, the corresponding governing equation to be integrated was changed to its applicable form. When the governing equations were changed, the current flowing through the inductor and the charge of the capacitors were maintained at a constant value. Updated values of other variables were

Table 1. Switch states, values of $S$, and applicable governing equations per case considered in numerical simulations.

| Switch state | Polarity of $i_t$ | $S$ | Circuit P | Circuit I | Circuit S |
|--------------|-------------------|-----|-----------|-----------|-----------|
| Off          | $i_t > 0$         | $S = 1$ | (32), (33) | (36), (37) | (32), (40) |
|              | $i_t < 0$         | $S = -1$ |           |           |           |
| On           | $i_t > 0$         | $S = 1$ | (32), (34) | (35), (39) | (32), (41) |
|              | $i_t < 0$         | $S = -1$ |           |           |           |
|              | $i_t = 0$         | $S = 0$ | (34), (35) | (34), (39) | (34), (42) |

Table 2. Parameters of energy harvesting circuit.

| Parameter | Value | Unit |
|-----------|-------|------|
| $C_p$     | 1.41  | µF   |
| $C_h$     | 4.7   | µF   |
| $L$       | 10.5  | mH   |
| $R_p$     | 4.5   | Ω    |
| $R_s$     | 0.5   | Ω    |
| $R_L$     | 5.5   | Ω    |
| $R_d$     | 10    | Ω    |
| $V_d$     | 0.65  | V    |
| $C_1$     | 100   | nF   |
| $C_2$     | 200   | nF   |
| $R_1$     | 420   | Ω    |
| $R_2$     | 340   | Ω    |
Table 3. Switching timing to obtain simulation and approximate results.

| Method  | SSHI | IE and SE | IB and SB |
|---------|------|-----------|-----------|
| Parameter | $\tau$ | $T$ | $\omega_e/\pi$ | $\tau$ | $T$ | $\omega_e/\pi$ |
| Value    | 1    | 0.781    | 0.206    | 0.126    | 1.310 |

Table 4. Near-optimal switching times obtained from experiments at $V_h/V_{oc} = 10$.

| Method  | $\tau$ | $T$ | $\omega_e/\pi$ | $\Delta E/C_pV_{oc}^2$ | Additional parameters |
|---------|------|-----|----------------|------------------------|----------------------|
| IE      | —    | —   | 0.781          | 5.12                   | —                   |
| SE      | —    | —   | 0.794          | 5.23                   | —                   |
| IH      | 0.264| 0.0096| 1.045        | 5.45                   | $N_F = 3.82$, duty = 0.964 |
| SH      | 0.264| 0.0096| 1.045        | 5.58                   | $N_F = 3.82$, duty = 0.964 |
| IB      | 0.206| 0.126| 1.310        | 6.60                   | $\tau_N\omega_e/\pi = 0.979$, duty = 0.904 |
| SB      | 0.182| 0.107| 1.241        | 6.62                   | $\tau_N\omega_e/\pi = 0.955$, duty = 0.916 |

Note: Duty is defined in the footnote of table 5.

calculated using the governing equations prior to proceeding to the next time step. All simulations were performed under double precision on a PC (Panasonic CF-LX3; Osaka, Japan). The magnitude of the time-step increment equaled $7.64 \times 10^{-9}$ s.

4.3. Results from simulations and approximate equations

To evaluate the approximate equations and numerical simulations, we obtained the corresponding energy harvesting performances of SSHI, IE, SE, IB, and SB using the same input data. The circuit parameters are listed in table 2. The parameter values were determined by measuring those of actual components from the corresponding experimental setup. In addition, the switching timing listed in table 3 was used to obtain the performances. The timing values were obtained from the values in table 4, which lists near-optimal experimental values. Because we assumed that $C_h \ll C_p$ when an approximate result is obtained, the value of $C_h/C_p$ was set to 10 000 during simulations performed in this study. We also analyzed excitation frequencies of 100 and 25 Hz.

Figure 6 depicts the results obtained from numerical simulations and approximations. For the sake of clarity, the results obtained for IE and SB are not shown. The values of the normalized energy harvested per structural-vibration cycle (i.e. $\Delta E/(C_pV_{oc}^2)$) are plotted against those of $V_h/V_{oc}$. As can be observed, the assumptions considered to derive the approximate equation are satisfied in all cases. When considering IE and SB as well, the maximum difference between the values of $\Delta E/(C_pV_{oc}^2)$ obtained using the approximate equation and numerical simulation equals 0.67 at a 100 Hz excitation frequency, whereas it remains below 0.1 at a 25 Hz excitation frequency. This is consistent with the assumption $\omega_e \gg \omega_s$ considered to derive the approximate equation. Therefore, the approximate equations demonstrate good agreement with the numerical-simulation results.

4.4. Numerical-simulation results

To verify the experimental results reported in section 5, we obtained the corresponding results from numerical simulations. The circuit parameters listed in table 2 were used, except for $V_d$ and $R_d$ for SSHI, which were set as described in section 5.2. In addition, the experimental switching timing listed in table 4 was used, and $C_h/C_p$ was set to 10 000. First, $C_h$ was charged up to voltage $V_h$ to set an initial condition, and integration continued over 50 cycles of mechanical vibration. After confirming that the system reached the steady state, the stored charge per cycle was obtained by integrating $\dot{Q}_h$. 
5. Experiments

To determine if each method in the S*HI family operates as expected, we conducted verification experiments.

5.1. Experimental setup

Figure 7 depicts the mechanical setup for vibration-energy harvesting. As can be seen in figure 7(a), a conical metal fitting adheres to each end of a stack piezoelectric element (PICMA P-885.51; PI Ceramic, Lederhose, Germany). Additionally, the semiconductor strain gauges (KSP-1-350-E4; Kyowa, Tokyo, Japan) adhere to the side surfaces of the element, thereby evaluating the axial strain therein. Subsequently, the element was clamped between the dints on the support structure and cantilevered plate. Thus, the element was compressively preloaded, as depicted in figures 7(b) and (c). The support structure comprised a rod with adjustable length. An 8 kg mass was mounted on the plate, which was sinusoidally excited at 100 Hz frequency by an electromagnetic shaker (WF1974; Akashi Seisakusho, Tokyo, Japan), whereas the resonant frequency was 144 Hz.

Figure 8 shows a diagram of the electrical setup for the experiments. Channel 1 of the function generator (WF1974; NF corporation, Kanagawa, Japan) provided a 100 Hz sinusoidal signal, which was amplified and fed to the shaker to induce mechanical vibration. The switch shown in figures 1 and 3 was implemented using two field effect transistors (FETs; 2SK4017; Toshiba, Tokyo, Japan). Channel 2 of the function generator provided a 200 Hz square wave, whose amplitude, duty, and phase delay with respect to channel 1 were controllable. To drive the FET switch, the output from channel 2 was directly used when applying switching pattern S or E (figure 4). For switching pattern B, the output from channel 2 was modified by applying the output from a clock generator (TLA494IN; Texas Instruments, Dallas, TX, USA), which provided a square signal. The frequency and duty of the output were adjustable by modifying the resistance and capacitance of the clock generator (not shown in figure 8). Therefore, all the circuits, P, I and S (figures 1 and 3), and all the switching patterns, S, E, H, and B (figure 4), could be implemented by adjusting the function generator, variable resistances of the RC delay circuits, capacitor, and variable resistance connected to the clock generator, and three snap switches shown in figure 8. The inductor in the circuit was a model ELC18B103L (Panasonic, Osaka, Japan), and the diodes composing the rectifier were of model IN1448TR (Vishay Intertechnology, Malvern, PA, USA). The various voltages were measured typically via unit-gain buffers using an Analog-Digital converter board (LPC-320724; Interface, Hiroshima, Japan) connected to a desktop PC. The unit-gain buffers used OPA445AP (Texas Instruments, Dallas, TX, USA) as the operational amplifier. A signal conditioner (CDV-230 C, Kyowa, Tokyo, Japan) was used for strain measurements. Figure 9 shows a picture of the circuit, where the jumper lines for voltage measurement at various points were removed. The phase difference between the outputs from channel 2 of the function generator and clock generator was not controllable.

5.2. Experimental parameters

The parameters of the components for the experiments were measured using an LCR meter at a frequency of 1.3 kHz, which is close to $\omega_c/(2\pi) = 1.31$ kHz. The relation between the applied voltage and current of the diode was measured as shown in figure 10. In this experiment of S*HI, the current through the diodes was approximated to 5 mA from preliminary experiments and simulations. Therefore, the values of $V_d$ and $R_d$ were estimated from the tangent around 5 mA, as shown in figure 10. The identified parameters are listed in table 2, which provides the important parameters of this system, including $\gamma = 0.826$, $\omega_c = 8.20 \times 10^3$ s$^{-1}$, and $V_d/V_{oc} = 3.25$ at $V_{oc} = 0.2$ V. On the other hand, the current through the diodes was estimated to be below 10 $\mu$A when applying SSHI. Therefore, $V_d$ and $R_d$ for SSHI were 0.45 V and 630 $\Omega$, respectively, as obtained from figure 10. These values were used only when we compared the experimental and simulation results.

5.3. Experimental procedure

Because S*HI facilitates energy harvesting from low-amplitude vibrations, the output amplitude from channel 1 was adjusted such that the amplitude of open-circuit voltage of the piezoelectric element $V_{oc}$ equaled 0.2 V. This value of $V_{oc}$ is equivalent to a strain of 1.74 $\mu$ε for an 18 mm long piezoelectric element. The value of $V_{oc}$ was manually adjusted by monitoring the temporal evolution of $V_p$ and/or strain values obtained from the strain gauges attached to the piezoelectric element. For each method in the S*HI family, load resistor $R_{load}$, which makes the value of $V_b$ to be approximately 2 V, was first connected to the storage capacitor as shown in figure 8. Then, the switching timing was manually adjusted to maximize $V_b$. The obtained and measured switching timings are listed in table 4. Typical values of $C_1$, $C_2$, $R_1$ and $R_2$ in figure 8 obtained via tuning are listed in table 2. As these values were manually obtained during experiments, they can be considered near-optimal.
Subsequently, by maintaining this condition for each method, the load resistance was varied with different resistors. By measuring $V_h$, the energy stored in a period of structural vibration was obtained as $\Delta E = 2V_h^2 \pi / (R_{\text{load}} \omega_s)$. To characterize the operation of each method, we measured various voltages, including $V_p$, voltage across the inductor, and voltage across the switch, through unity-gain buffers.

5.4. Comparison between experimental and simulation results

Figure 11 shows the experimental and simulation results of energy harvested per vibration cycle obtained for each evaluated method. The normalized performance, $\Delta E / (C_p V_{oc}^2)$, is shown according to the normalized voltage of the storage capacitor, $V_h/V_{oc}$. The experimental results are mostly consistent with the simulation results. The performance depends on the switching pattern but is almost independent of the circuit topology (i.e. circuit I or S). These results may not correspond to the maximum performance of each method because the switching timing was roughly optimized to $V_h/V_{oc} \approx 10$.

When IE or SE is applied, figure 11 shows that the experimental and simulation results suitably agree. Figure 12 shows a close-up view of the experimental and simulation evolution of the input voltage to the bridge rectifier of circuit I (i.e. voltage across the inductor) for IE. The curves virtually coincide. When the switch is turned off after keeping it on for period $\tau_N$, a high voltage is induced, and the storage capacitor is charged at the end of voltage inversion, overcoming the voltage barrier of $V_h + 2V_d = 3.34$ V.
Figure 10. Voltage–current relation for diode 1N4148TR.

The experimental and simulation results differ by the high-frequency vibration in the experimental curve. This vibration may be generated by the stray capacitance and the inductor. The vibration frequency of 112 kHz and inductance of 10.5 mH result in a stray capacitance of 0.192 nF. The vibration amplitude suggests an energy dissipation of approximately $7.01 \times 10^{-10}$ J, being around 0.15% of the mechanical vibration converted into electrical energy. Therefore, the dissipation is negligible. Figure 13 shows the evolution of $V_p$ and $V_h$ in a longer time scale. The value of $V_h$ jumps at every voltage inversion, showing that some energy is added as intended in the proposed design.

Figure 11 reveals that the discrepancy between the experimental and simulation results of the performance estimation for IH and SH is relatively large when $10 < V_h/V_{oc} < 20$, possibly due to the estimation error of $\tau_N$. As $\tau_N$ is shorter than 4 $\mu$s, it is difficult to obtain an accurate measure from data sampled at 1.54 MHz. If we consider a shorter $\tau_N$, the discrepancy decreases.

Figure 14 shows the evolution of the experimentally obtained input voltage to the bridge rectifier during voltage inversion for IH. The switch is turned off four times within period $\tau_T$, generating a high voltage and charging the storage capacitor four times during one voltage inversion process.

Figure 11 shows that IB and SB outperform the other S$^3$HI methods. Figure 15 shows the experimental and theoretical evolution of the input voltage in the bridge rectifier of circuit S (i.e., voltage across the switch) for SB. As is the case for IE, both curves coincide except for the high-frequency vibration that appears in the experimental results whenever the current ceases to flow. Such high-frequency vibration may also be related to the stray capacitance, and its effect on harvesting is negligible.

Figure 16 shows the evolution of the experimentally obtained $V_p$ and $V_h$ in long and short time scales. Figures 15 and 16 show that a high surge voltage is induced, and the storage capacitor is charged near the beginning of voltage inversion, overcoming the barrier of $V_h + 2V_d = 3.77$ V as designed. The voltage pulse at the end of period $\tau_{N2}$ indicates that $\tau_{N2}$ is slightly shorter or longer than $\pi/\omega_c$.

6. Evaluation of S$^3$HI methods

6.1. Effect of precise switching timing optimization

The preceding sections of this paper discussed the investigation and comparison of the performance of the different S$^3$HI methods using near-optimal switching times obtained via experiments. This section discusses the corresponding
investigations and comparisons performed after precise optimization of the switching time. We determined two sets of optimal switching timings through simulations to maximize $\Delta E/C_p V_{oc}^2$ at $V_h/V_{oc} = 10$ and $V_h/V_{oc} = 30$. We fixed $V_{oc}$ to 0.2 V like in the experiments. For IB and SB, the optimal value of $\tau N_2 \omega_e / \pi$ is approximately 0.99–0.98 in many cases. However, even when the other two parameters are optimized while maintaining $\tau N_2 \omega_e / \pi$ at 1.0, $\Delta E/C_p V_{oc}^2$ decreases only by 0.1% compared with the optimal value. Therefore, we fixed $\tau N_2 \omega_e / \pi$ to 1.0 and considered the other two parameters as design parameters. In IH and SH, $N_F$ was considered to be an integer, and the optimal values of the other two independent parameters were searched for $N_F = 2, 3, \text{ and } 4$. Table 5 lists the obtained optimal timings, maximum values of $\Delta E/C_p V_{oc}^2$ at $V_h/V_{oc} = 10$, and increments of $\Delta E/C_p V_{oc}^2$ due to the precise optimization.

By using the optimal switching timing, we performed various numerical simulations. Figure 17 shows the performances of the SSHI and S$^3$HI methods at various values of $V_h/V_{oc}$. For IH and SH, $\Delta E/C_p V_{oc}^2$ is almost independent of $N_F$, as listed in table 5. Therefore, figure 17 only shows the case for
Figure 16. Evolution of $V_p$ and $V_h$ for SB.

Table 5. Optimal switching timing at $V_h/V_{oc} = 10$, corresponding $\Delta E/C_p V_{oc}^2$, and effect of precise optimization.

| Method | $N_F$ | $\tau\omega_c/\pi$ | Duty | $\Delta E/C_p V_{oc}^2$ | Increment |
|--------|-------|----------------------|------|-------------------------|-----------|
| IE     | —     | 0.807                | —    | 5.20                    | 0.08      |
| SE     | —     | 0.779                | —    | 5.27                    | 0.04      |
| IB     | —     | 1.245                | 0.952| 6.73                    | 0.13      |
| SB     | —     | 1.229                | 0.939| 6.90                    | 0.28      |
| IH     | 2     | 1.020                | 0.970| 5.76                    | 0.31      |
|        | 3     | 1.020                | 0.970| 5.79                    | 0.34      |
|        | 4     | 1.010                | 0.970| 5.80                    | 0.35      |
| SH     | 2     | 0.985                | 0.968| 5.87                    | 0.29      |
|        | 3     | 0.985                | 0.970| 5.89                    | 0.31      |
|        | 4     | 0.980                | 0.970| 5.90                    | 0.32      |

Note: For IB and SB, duty = $(\tau_N + \tau_E)/\tau_T$; for IH and SH, duty = $\tau_N/(\tau_N + \tau_I)$. The increment in $\Delta E/C_p V_{oc}^2$ (last column) is due to precise optimization.

$N_F = 3$. As shown in table 5 and by comparing figures 17 and 11, the performance improvement due to the precise optimization of the switching timing is negligible. Hence, the methods in the S$^3$HI family are relatively robust to variations in switching timing, with IE and SE being the most robust, IB and SB being moderately robust, and IH and SH being the least robust methods.

6.2. Comparison of the methods under low-amplitude vibration or large FVD

Figure 17 shows that SSHI can harvest energy in a limited range of $V_h/V_{oc}$, whereas any method in the S$^3$HI family achieves harvesting in a wider range beyond the values shown in the figure and can harvest more energy. Even for cases in which SSHI can harvest, its performance (i.e. $\Delta E/C_p V_{oc}^2$) is much lower than that of the methods in the S$^3$HI family. In addition, the curve showing the relation between $\Delta E/C_p V_{oc}^2$ and $V_h/V_{oc}$ depends on the value of $V_h/V_{oc}$ at which switching timing is optimized. This dependency is strong in IH and SH and weak in IE and SE. Regarding both peak performance and average performance, SB and IB provide the highest values. Hence, the proposed switching pattern B substantially enhances the performance of S$^3$HI. Figure 17 reveals that when $4 < V_h/V_{oc}$, the switching pattern B demonstrates a harvesting performance that has increased by 11%–31% and 15%–450% compared to patterns E and H, respectively, depending on the $V_h/V_{oc}$ value.

As observed, SE, SH, and SB demonstrate slightly better performance compared to the IE, IH, and IB, respectively, at $V_h/V_{oc}$ values corresponding to optimum switching times, as indicated by the value of $\Delta E/C_p V_{oc}^2$ listed in table 5. However,
the observed differences in table 5 and figure 17 are marginal. This demonstrates that circuit S is a good alternative to circuit I.

6.3. Performance of methods in S³HI family under large-amplitude vibrations or small FVD

We have exclusively focused on small-amplitude vibrations and large FVD (i.e. $V_d/V_{oc} = 3.25$) in the simulations and experiments reported thus far in this paper, because this is the main operating condition considered in this study. However, we have also investigated the performance and characteristics of the different methods in the S³HI family under conditions of large-amplitude vibration and/or small FVD (i.e. $V_d/V_{oc} = 0.1$). More specifically, we set $V_{oc} = 6.5\text{ V}$, and the other parameters were set as listed in tables 2 and 5. Hence, the switching timing was not optimized for $V_d/V_{oc} = 0.1$.

Figure 18 shows the harvesting performance of SSHI, SE, SH, and SB for $V_d/V_{oc} = 0.1$. Similar to figure 17, figure 18 reveals that SSHI can harvest energy only when $V_h/V_{oc}$ is lower than a certain value, whereas all the methods in the S³HI family achieve harvesting over a wide range of $V_h/V_{oc}$ values beyond that shown in the figure. Therefore, S³HI can store significantly more energy than SSHI when $V_d/a = 0.1$. Unlike the case wherein $V_d/V_{oc} = 3.25$, SSHI provides the highest performance when $V_h/V_{oc} < 7$. Otherwise, SB provides the highest performance. If the switching timing is optimized for $V_d/V_{oc} = 0.1$, the performance of each method in the S³HI family increases. However, the performance increase at $V_h/V_{oc} = 10$ and $V_h/V_{oc} = 30$ due to this optimization is below 0.12 for SE and 0.75 for SB. In practice, this increase is considered negligible. Hence, the timing parameters determined at $V_{oc} = 0.2\text{ V}$ for SE and SB can be used in practice even when $V_{oc}$ assumes a large value of 6.5\text{ V}. This demonstrates the robustness of the proposed strategy to variations in the vibration amplitude.

As discussed above, we have demonstrated that the performance of the proposed circuit S is equivalent to that of circuit I. Nevertheless, the use of circuit S might provide additional advantages when vibration amplitude is large or $V_d/V_{oc}$ is small. Figure 18 demonstrates the performance of the SS method, which is an energy-harvesting method based on the combination of circuit S and switching-pattern S. Therefore, when circuit S is used, SS and SB become interchangeable via a simple change in the switching pattern from S to B and vice versa. Figure 18 reveals that the performances of SS and SSHI are identical and that they outperform other methods when $V_h/V_{oc} < 7$. When $V_h/V_{oc} > 7$, SB outperforms other methods. This fact indicates that one can always attain the best performance by using circuit S and adaptively selecting the preferred switching pattern between B and S depending on the $V_h/V_{oc}$ value.

6.4. Comparison of performance of SB from S³HI family with that of passive FBR and certain other methods

Figure 19 compares the performances of passive FBR, SSHI, and SB (a typical member of the S³HI family). The FBR circuit was implemented by neglecting the switch and inductor of the circuit P shown in figure 1. Thus, the performance of FBR was obtained using the simulation program described in section 4.2, keeping the switch open. The parameter values listed in table 1, which were used in all the numerical simulations in this study, were used in this simulation as well. The value of $V_d$ was 0.65\text{ V}. The switching timings for SB were $\tau T/\pi = 1.25$ and duty = 0.97, which are optimal at $V_b/V_{oc} = 30$ and $V_d/V_{oc} = 3.25$. The value of FVD was $V_d = 0.65\text{ V}$, and the vibration frequency was 100 Hz. Figure 19(a) shows the case of small-amplitude vibration, which is the target condition of this study. The figure shows that the FBR harvests no energy in this case, irrespective of the storage-capacitor voltage. However, SB achieves a high harvesting rate even when $V_h/V_{oc}$ is large and the vibration amplitude is small. This figure depicts well that SB performs effectively from the perspective of our objective. Figure 19(b) shows the case of large-amplitude vibration; in this case, FBR can harvest some energy only when $V_h/V_{oc}$ is relatively small. SB realizes a high harvesting rate when $V_h/V_{oc}$ is large, in this case as well.

Table 6 summarizes figure 19. Power enhancement is defined as $\max \Delta P_{max}/\max \Delta P_{ FBR}$ in this study; $\max \Delta P_{max}$ and $\max \Delta P_{ FBR}$ are defined in the footnote of table 6. When the amplitude of open-circuit voltage $V_{oc}$ is as small as $V_d/V_{oc} = 3.25$, power enhancement is infinity because $\max \Delta P_{max} = 0$. $V_h$ denotes the maximum value of $V_h$ at which $0 < \Delta E$ holds, and $E_{max}$ denotes the upper limit of storable energy. Because no energy is additionally harvested when $V_h < V_h$, the upper limit is given by $E_{max} = C_hV_h^2/2$. 

Figure 18. Performance of SSHI, SE, SH, SB, and SS under large-amplitude vibration and/or small FVD ($V_d/V_{oc} = 0.1$).
Table 6. Comparison of performances of SBR, SSHI, and S$^3$HI-SB.

| Publication | $V_d/V_{oc}$ | $\gamma$ | Method  | $\max \Delta E/C_p V_{oc}^2$ | Power enhancement | $V_h/\max V_{oc}$ | $E_{\max}/C_p V_{oc}^2$ |
|-------------|--------------|----------|---------|-------------------------------|-------------------|------------------|-----------------|
| This work   | 3.25         | 0.826    | S$^3$HI-SB | 11.4                          | $\infty$         | $>1000$          | $>5.0 \times 10^5$ |
|             | 0.1          | 0.826    | S$^3$HI-SB | 11.5                          | 18.0              | $>1000$          | $>5.0 \times 10^5$ |
| [3]         | 0            | —        | SECE    | 4.0                           | 4.0               | $\infty$         | $\infty$        |
| [6]         | 0            | 0.822    | Series SSHI | 10                            | 10                | 1.0              | 0.50            |
| [7]         | 0            | 0.74     | Hybrid SSHI $^1$ | 7.7                          | 7.7               | 30               | 450             |
| [10]        | 0            | 0.8      | ESSE $^2$ | 8.0                           | 8.0               | $\infty$         | $\infty$        |
| [14]        | 0            | 0.7      | SICE $^3$ | 6.4                           | 6.4               | $\infty$         | $\infty$        |

Note: Power enhancement: $\max \Delta E/\max \Delta E_{\text{FBR}}$, $\max$: maximum value with respect to $V_h/V_{oc}$, $\Delta E$: energy harvested per vibration cycle, $\Delta E_{\text{FBR}}$: value of $\Delta E$ of FBR. 1) Transformer ratio $m = 30$, 2) efficiency of buck-boost converter $\gamma_c = 0.9$, 3) energy transfer efficiency $\eta = 0.85$.

Table 6 shows the performance of certain previously proposed methods as well. When the value of $V_h$ was not included in a study, it was calculated from the corresponding values of load resistance and harvested power in each study. Unfortunately, values of parameters such as $V_d/V_{oc}$ and $\gamma$ are different in each case. However, table 6 shows that $E_{\max}$ is very large when S3HI-SB, SECE, ESSE, or SICE is applied. In principle, $\max \Delta E/C_p V_{oc}^2$ decreases when $V_d/V_{oc}$ increases. However, table 6 shows that when S3HI-SB is applied, value of $\max \Delta E/C_p V_{oc}^2$ is larger than those of prior methods even though $V_d/V_{oc} = 0$ in them. The objective of this study was not only to obtain a large value for $\Delta E/C_p V_{oc}^2$, but also to increase the upper limit of the stored energy. Therefore, table 6 demonstrates that our objective is achieved well by, e.g. SB of the S$^3$HI family.

7. Conclusions

In this paper, various vibration-energy harvesting methods based on piezoelectric transducers, establishing the S$^3$HI family, were characterized and analyzed. These methods exploit the surge voltage generated by turning off the switch for short periods during the voltage inversion of the conventional SSHI. The underlying circuits for the methods in the S$^3$HI family are as simple as the circuit for the SSHI method. By incorporating a novel circuit S and switching pattern B, we derived six methods in the S$^3$HI family.

Simulation and experimental results as well as theoretical derivations demonstrate that the methods in the S$^3$HI family generate surge voltage as designed and achieve efficient vibration-energy harvesting even when the open-circuit voltage of the piezoelectric element is much smaller than the FVD of the diodes in the bridge rectifier. This feature contrasts with the SSHI operation.

Of the six methods from the S$^3$HI family evaluated in this study, SB and IB demonstrate equivalent performance. Therefore, the proposed switching pattern B outperforms the existing patterns E and H when implemented in either the proposed or conventional energy-harvesting circuits. In the case of low-amplitude vibrations, the proposed pattern increases harvesting performance by 11%–31% and 15%–450% compared to the existing patterns E and H depending on the $V_h/a$ value. The proposed circuit S harvests the same energy per
vibration period as a traditional circuit. However, it affords a potential auto-reboot capability. In addition, using circuit S, one can realize the best energy-harvesting performance by selecting an appropriate switching pattern. Therefore, the proposed circuit is a promising alternative to the $S^{3}H$ configuration, although the realization of these concepts is a topic for future research. Lastly, this paper presented the characteristics of each method in the $S^{3}H$ family. These included robustness against switching-time errors as well as that against variations in the vibration amplitude and storage-capacitor voltage.

Data availability statement

All data that support the findings of this study are included within the article.

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Conflicts of interest

The authors have no conflicts of interest to declare.

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