EFFECTIVE POTENTIAL CALCULATION OF THE MSSM LIGHTEST CP-EVEN HIGGS BOSON MASS∗

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In memory of Prof. Xi-De Xie.

I summarize results of two-loop effective potential calculations of the lightest CP-even Higgs boson mass in the minimal supersymmetric standard model.

Computing the lightest CP-even Higgs boson mass is the most important loop calculation in the minimal supersymmetric standard model because of the paramount importance of a precise $m_{h^0}$ value to the Higgs boson experimental discovery. Tree-level supersymmetry relations require that the Higgs field quartic coupling be related to the electroweak gauge couplings; therefore they impose a strict upper bound $m_{h^0} \leq m_Z$, which is already in conflict with the current lower limit from LEP 2.

It is well-known that this tree-level limit can be drastically changed by radiative corrections. One-loop calculations show that incomplete cancellations of the top and stop loops give positive corrections of the form

$$\Delta m_{h^0}^2 = \frac{3h^2m_t^2}{4\pi^2} \ln \frac{m^2_t}{\tilde{m}_t^2},$$  \hspace{1cm} (1)

where $m_t$ and $\tilde{m}_t$ are top and stop masses respectively. This formula, however, suffers from an ambiguity in the definition of $m_t$. Numerically, using running or on-shell top-quark mass can amount to about 20% difference in $\Delta m_{h^0}^2$. The problem can only be resolved by an explicit two-loop calculation.

Two-loop calculations in the existing literature have used two different approaches: (a) a renormalization group resummation approach and (b) a two-loop diagrammatic approach. In the first approach, leading and next-to-leading logarithmic corrections are calculated by integrating one- and two-loop renormalization group equations. However, two-loop non-logarithmic finite corrections are not calculable in principle. The second approach was initiated by Hempfling and Hoang using an effective potential method; they restricted their calculation to specific choice of supersymmetry parameters: i.e. large $\tan \beta \rightarrow \infty$ and zero left-right stop mixing. Two-loop QCD corrections were later computed at more general cases in the effective potential

∗Contribution to PASCOS99: 7th International Symposium on Particles, Strings and Cosmology, Granlibakken, Tahoe City, California, 10-16 Dec 1999.
approach. $m_{h^0}$ to the same two-loop QCD order was also computed using an explicit diagrammatic method. These calculations incorporate both two-loop logarithmic and non-logarithmic finite corrections. In the following, I shall concentrate on the effective potential approach.

The general way of calculating corrections to CP-even Higgs boson mass is to compute Higgs self-energy and tadpole diagrams to the required loop order. In an effective potential approach, these diagrams can be derived from a generating functional, i.e. the effective potential, by taking explicit derivatives with respect to the Higgs fields. These quantities then enter the MS SM CP-even Higgs boson mass-squared matrix as follows

$$
\mathcal{M}_h^2 = \begin{bmatrix} m_Z^2 c_\beta + m_A^2 s_\beta + \Delta M_{11}^2 & -(m_Z^2 + m_A^2) s_\beta c_\beta + \Delta M_{12}^2 \\ -(m_Z^2 + m_A^2) s_\beta c_\beta + \Delta M_{21}^2 & m_Z^2 s_\beta + m_A^2 c_\beta + \Delta M_{22}^2 \end{bmatrix},
$$

where $\Delta M_{ij}^2$ represents radiative corrections to the $ij$-entry. We note that all these corrections are computed at the zero external momentum limit; sometimes it is necessary to calculate self-energy diagrams directly to account for corrections from non-zero external momenta.

The CP-even Higgs boson masses can be calculated by diagonalizing the above matrix in eq. (2). This computation is tedious but can be greatly simplified when one considers the case $m_A^0 \gg m_Z$, where $m_A^0$ is the mass of the pseudoscalar $A^0$. In this case, we find the corrections to $m_{h^0}^2$ is

$$
\Delta m_{h^0}^2 = \frac{3m_t^4}{2\pi^2 v^2} \left( \ln \frac{m_{h^0}^2}{m_t^2} + \frac{\hat{X}_t^4}{12} \right)
$$

where $V$ is the effective potential, $v$ the Higgs field VEV, and the last two terms account for non-zero external momentum corrections.

We have carried out this calculation procedure to the two-loop order including leading QCD and top Yukawa corrections. To illustrate our analysis, we present an approximation formula which is derived under the following assumptions: the soft masses for left and right stops, gluino, heavy Higgs bosons and Higgsinos have a common mass $M_S$, where $M_S$ can be identified as the supersymmetry scale. The two eigenvalues and mixing angle of stops are then accordingly $m_{\tilde{t}_1}^2 = m_{\tilde{t}_1}^2 + m_t X_t$, $m_{\tilde{t}_2}^2 = m_{\tilde{t}_2}^2 - m_t X_t$ and $s_t = c_t = \frac{1}{\sqrt{2}}$, where the average top-squark mass $m_{\tilde{t}}^2 = M_S^2 + m_t^2$, and $X_t = A_t + \mu / \tan \beta$ is the left-right stop mixing parameter.

We find the approximation formula for two-loop QCD+top Yukawa corrections is (in terms of on-shell mass parameters)

$$
\Delta m_{h^0}^2 = \frac{3m_t^4}{2\pi^2 v^2} \left( \ln \frac{m_{h^0}^2}{m_t^2} + \frac{\hat{X}_t^4}{12} \right)
$$
\[
\alpha s m_t^4 \left( -3 \ln^2 \frac{m_t^2}{m_t^2} - 6 \ln \frac{m_t^2}{m_t^2} + 6 \hat{X}_t - 3 \hat{X}_t^2 \ln \frac{m_t^2}{m_t^2} - \frac{3 \hat{X}_t^4}{4} \right) \\
+ \frac{3 \alpha m_t^4}{16 \pi^3 v^2} \left \{ s_\beta^2 \left( 3 \ln^2 \frac{M_S^2}{m_t^2} + 13 \ln \frac{M_S^2}{m_t^2} \right) - 1 \right \} \left( 1 + \frac{61}{12} s_\beta^2 \right) \hat{X}_t^2 + \frac{2 \pi}{\sqrt{3}} \hat{X}_t^4 \\
+ \frac{c_\beta^2}{2} \left[ (3 - 16 K - \sqrt{3} \pi)(4 \hat{X}_t \hat{Y}_t + \hat{Y}_t^2) + \left( 16 K + \frac{2 \pi}{\sqrt{3}} \right) \hat{X}_t^3 \hat{Y}_t \right] \\
+ \left( -\frac{4}{3} + 24 K + \sqrt{3} \pi \right) \hat{X}_t \hat{Y}_t^2 - \left( \frac{7}{12} + 8 K + \frac{\pi}{2 \sqrt{3}} \right) \hat{X}_t^4 \hat{Y}_t^2 \right \},
\]

where the constant \( K \approx -0.195 \). We note that two-loop QCD corrections depend only on \( \hat{X}_t = X_t/m_t \) while the top Yukawa corrections depend on \( \hat{Y}_t = (A_t - \mu \tan \beta)/m_t \) as well. This approximation formula is good to a level of 0.5 GeV for most of the parameter space.

Figure 1. Higgs boson mass \( m_{h^0} \) versus \( \hat{X}_t \). The dotted, dot-dashed and solid lines correspond to Higgs boson masses calculated to the orders of one-loop, two-loop QCD and two-loop QCD+top Yukawa respectively.

Fig. 1 shows the Higgs boson mass \( m_{h^0} \) vs. the stop mixing parameter \( \hat{X}_t \), for different choices of \( M_S, \mu \), and \( \tan \beta \). The two-loop QCD corrections agree well with other approaches. They generally decrease \( m_{h^0} \) from their
one-loop values by $10 - 20$ GeV depending on the parameter choice. Two-loop Yukawa corrections are sizeable for large stop mixings, in particular, for $\hat{X}_t \simeq \pm 2$ two-loop Yukawa corrections can increase $m_{h^0}$ by about 5 GeV.

Another interesting feature observed in the literature is that two-loop corrections shift the maximal mixing peaks. At the one-loop level, these peaks are at $\hat{X}_t = \pm \sqrt{6}$. It is easy to see from eq. (4) that the size of shifts is about 10%, i.e. the peaks move to $\hat{X}_t \simeq \pm 2$. This is confirmed by Fig. 1.

Finally, renormalization group resummation technique can be used to derive a particularly nice mass correction formula which has clearer physical interpretations. We find eq. (4) can be transformed into the following form by using solutions to the renormalization group equations

$$\Delta m_{h^0}^2 = \frac{3m_t^4(Q_t)}{2\pi^2 \alpha(Q^*_1)} \ln \frac{m_t^2(Q_{th})}{m_t^2(Q^*_1)} + \frac{3m_t^4(Q_{th})}{2\pi^2 \alpha(Q^*_2)} \left[ \frac{\hat{X}_t^2(Q_{th}) - \hat{X}_t^2(Q_{th})}{12} \right] + \Delta_{th}^{(2)},$$

(5)

where $Q^*_1 = e^{-1/3}m_t$, $Q^*_2 = e^{1/3}m_t$, $Q_t = \sqrt{m_t m_{\tilde{t}}}$, $Q_t^* = (m_t m_{\tilde{t}})^{1/3}$ and $Q_{th} = m_{\tilde{t}}$, $\alpha$ and $\alpha$ are the Standard Model $\overline{MS}$ parameters. These choices of scales for evaluating one-loop corrections automatically take into account two-loop leading and next-to-leading logarithmic effects. The leftover finite correction term $\Delta_{th}^{(2)}$ is understood as two-loop threshold corrections and numerically small; its detail form can be found in a forthcoming paper.

I thank J. R. Espinosa for collaborations. This work was supported in part by a DOE grant No. DE-FG02-95ER40896 and in part by the Wisconsin Alumni Research Foundation.

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