Computing Wilson lines with dielectric branes

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ABSTRACT

Wilson lines in $\mathcal{N} = 4$ SYM can be computed in terms of branes carrying electric flux, i.e. F-strings dissolved in their worldvolumes. It is then natural to think that those configurations are the effective description of strings expanding due to dielectric effect to D-branes. In this note we explicitly show this for a class of such configurations, namely those dual to Wilson lines either in the symmetric or in the antisymmetric tensor product of fundamentals.
1 Introduction

The $AdS/CFT$ correspondence \cite{1} relates gravity theories in $AdS$ spaces with certain field theories. In particular, it identifies type IIB string theory on $AdS_5 \times S^5$ with $\mathcal{N} = 4$ SYM.

In the field theory side of the correspondence, a very interesting operator is the Wilson loop. Recently, very much progress has been done in understanding these kind of operators, both from the string theory side and the CFT side (e.g. \cite{2}, \cite{3}, ...).

In \cite{4} and \cite{5} a specific proposal to compute Wilson loops in terms of the dual string theory was put forward. To be precise, the strong coupling version of the Wilson loop is computed by means of the area of the worldsheet of a string which, at the boundary of $AdS$, terminates on the loop. However, with these techniques, one cannot obtain the subleading corrections which arise when one considers coincident Wilson loops, multiply wound Wilson loops or Wilson loops in a higher dimensional representation \cite{6}. In a beautiful paper, Drukker and Fiol \cite{7} showed that it is possible to compute a class of these loops (for example those with many coincident Wilson loops) using D branes carrying a large fundamental string charge dissolved on their worldvolume pinching off at the boundary of the $AdS$ on the Wilson loop. The idea behind this proposal is that the D-brane can be seen as the effective description of a multi-string configuration when the number of strings is large, in very much of the spirit of \cite{8}, \cite{9} and \cite{10}. Thus, the D3 brane represents a way of taking into account all the interactions among the strings.

Similar configurations but with D5 branes carrying electric DBI flux were recently considered in \cite{11} and \cite{12}. In the first reference, a D5 brane in the euclidian Schwarzchild black hole in $AdS$ carrying electric flux was proposed as dual to the thermal Polyakov loop. One can see that there are several possible configurations for such a brane. The extremal version of one of them is proposed in the second reference as the dual of a Wilson loop on an antisymmetric tensor product of the fundamental representation. In \cite{15} more work along this line was done, showing how the group-theoretic structure of the associated Wilson line arises. The final picture for this

\footnote{Let us also point that this type of branes has been considered in the literature for other purposes (\cite{13}).}
Wilson line coming from those papers is that the infinite straight Wilson line in a symmetric tensor product of fundamentals can be described by means of a single D3 brane carrying F1 strings dissolved, while the infinite straight Wilson line in an antisymmetric tensor product of fundamentals can be computed by means of a D5 brane with fundamental strings on it. Clearly, both branes should pinch off on the Wilson line at the boundary in order to capture the desired field theoretic configuration. In some sense, the brane duals arising from this picture are in great parallelism to the giant graviton phenomenon of \cite{16}, to which also a microscopic description along the lines presented in this note was given in a series of papers \cite{17}.

In this note we will concentrate on the straight Wilson line, which preserve one half of the total supersymmetries. We will explicitly show how this branes should be seen as the effective description of a collective of strings pinching off on the loop at the boundary (and thus following the original recipe to compute the Wilson loop in the AdS/CFT context). Once the interactions between them are taken into account, due to dielectric effect (\cite{10}), they undergo an expansion into higher dimensional objects which, in the adequate limit, are effectively described with the so far mentioned branes. In a sense, this properly justifies why the these branes capture the desired Wilson loops, since they are nothing but the effective description of a collection of strings following the prescription in \cite{4} and \cite{5}.

In the case of the symmetric product representation Wilson line, since fundamental strings naturally expand to D3 branes; one should expect to find a microscopical description in terms of dielectric fundamental strings expanding to a D3 brane, in such a way that once a suitable limit is taken, the macroscopic D3 of \cite{7} is recovered as the effective description. Making use of the S-duality invariance of the background, we will actually compute the expansion of coincident D1 branes into a D3. Thus we will use the action for dielectric D1 branes of \cite{10}. Then, due to the S-duality invariance of the background, the functional form of the action for such a configuration will be the same as the one we are interested in. In any case, we should be aware that strictly speaking we are computing a ’t Hooft line. We will explicitly show this, along the lines of \cite{7}. However, there is a way of computing directly the Wilson loop, namely by constructing an action for dielectric fundamental strings. This action can be constructed (\cite{18}) by means of a chain of dualities starting with the action for dielectric gravitons in M-theory.
presented in [19]. However, as it should be expected, once we particularize it for our background it reduces exactly to Myers action, thus giving the same functional form for both the Wilson and ’t Hooft line.

The case of the D5 is more subtle, since naturally F1 strings do not polarize into D5 branes, but into NS5 branes. The natural candidate to find a microscopical description for the D5 is the D1 brane. We will see explicitly how we can indeed achieve a microscopical description of the D5 brane configurations in [11], [12], [15] in terms of dielectric D1 branes. In turn, we will have to add the F1 charge as an electric DBI vector on the worldvolume of such a dielectric configuration. In addition, in order to capture the desired macroscopic configurations, we should restrict to those configurations in which the net D1 brane charge is zero. Furthermore, inspired by the latter construction in which we have at the same time both D1 and F1 charge pinching off on the line at the boundary, an interesting possibility arises, namely that of computing Wilson-’t Hooft lines. We will see that if we do not remove the extra D1 charge what we get is a configuration in which what pinches off on the boundary is not F1 charge but a dyonic string, carrying also D1 brane charge. This should be capturing information about the associated Wilson-’t Hooft line. Indeed, a similar computation can be done in the symmetric product representation Wilson line, in terms of a D3 brane which carries both F1 and D1 charge, which should also contain information about the Wilson-’t Hooft line in a symmetric product representation.

It is worth to mention that since we will be interested in the blowing up of strings to D-branes, we will focus in the bulk computation. The boundary terms to add in order to have the suitable boundary conditions will be exactly as those in the literature, and we refer the reader to the corresponding papers.

2 Infinite straight Wilson line in the symmetric tensor product representation

In this section we will use as coordinates for $AdS_5$

$$ds^2 = \frac{L^2}{y^2}(dy^2 + d\vec{x}^2).$$  

We will take the infinite Wilson line in the symmetric tensor product of
fundamental representation to be extended along $x^1$. Clearly, there is an $SO(3)$ rotational invariance around this line.

As it is well-known, for this Wilson line $\langle W \rangle = 1$ (see [2]), and, following [2], we would like to see this rather than with a single fundamental string ending on the line at the boundary, with a D3 brane carrying electric flux on it representing $M$ strings and pinching off at the boundary on the line. In order to do this, it is convenient to switch to the following coordinates

$$ds^2 = \frac{L^2}{y^2} (dy^2 + dt^2 + dr^2 + r^2 d\Omega_2^2).$$

(2)

In addition, there is a 4-form RR potential given by

$$C^{(4)} = \frac{L^4 r^2}{y^4} dt \wedge dr \wedge d\omega_2,$$

(3)

where $d\omega_2$ stands for the volume form on the $S^2$.

Because of the symmetries of the problem, it is clear that we should consider as dual of the infinite Wilson line a D3 brane whose worldvolume coordinates are $\{t, r, \Omega_2\}$, and take $y = y(r)$. As one can see in [2], this D3 brane pinches off at the boundary over the Wilson line along $t$.

In addition, we have to add electric DBI flux on the brane representing the strings. Therefore we will take a non-zero electric DBI field $F_{tr} = F$. For such a D brane the action is easily seen to be

$$S = \int dt dr \ 2T_1 \frac{L^4 r^2}{y^4} \left( \sqrt{1 + y'^2} - \frac{y^4}{L^4} F^2 - 1 \right),$$

(4)

where, following [11], we have taken $F \rightarrow iF$ in order to properly represent $F$ strings.

For future convenience, we re-write everything in terms of the 1-brane tension $T_1$. In addition, we are taking units so that $2\pi l_s^2 = 1$.

The prime represents derivative with respect to $r$.

Since in the action the DBI potential does not explicitly appear in the action, we can change the DBI field strength for its conserved momentum $P$ to obtain a “routhian”.

$$\mathcal{R}_{Macro} = \int dt dr \ 2T_1 \frac{L^4 r^2}{y^4} \left( \sqrt{1 + y'^2} \sqrt{1 + \frac{y^4 P^2}{4r^4 T_1^2 L^4}} - 1 \right),$$

(5)
Here the conserved momentum represents the number of dissolved strings, so, in units of $T_1$, it will be an integer $M$.

Although it is not the main aim of this work, let us mention that, once the appropriate boundary terms are added, in order to ensure the right boundary conditions, the resulting action leads to an equation of motion which has as a solution $y \sim r$. This clearly displays the fact that the brane pinches off on the line, while once we evaluate the action of the brane in the solution we get that $S = 0$, which leads to $\langle W \rangle = 1$ ([2], [7]).

The configuration above was engineered in such a way that it captures the behavior of a large number of fundamental strings. In view of the dielectric effect, it then is natural to search for a dielectric configuration of strings expanding to the D3 brane configuration. Since our background is S-duality invariant, we will describe microscopically our configurations using the action for coincident D1 strings [10]. We will take the strings along the coordinates $\{t, r\}$, and assume that they expand to the transverse $S^2$. In addition, we will consider a worldvolume scalar field $y = y(r)$. In cartesian coordinates, the background reads

$$ds^2 = \frac{L^2}{y^2}(dy^2 + dt^2 + dr^2 + r^2dx^2) \ ,$$  

(6)

and

$$C(4) = \frac{L^4r^2}{y^4}x^k\epsilon_{ijk} dt \wedge dr \wedge dx^i \wedge dx^j \ ,$$  

(7)

where $\epsilon_{ijk}$ is the completely antisymmetric tensor, and $i, j, k = i, 2, 3$.

As non-commutative ansatz we will take

$$X^i = \frac{1}{\sqrt{C_2}}J^i \ ,$$  

(8)

where the $J^i$ are the generators of and $M$ dimensional representation of $SU(2)$ whose Casimir is $C_2$. Clearly, this way it is satisfied that $\bar{X}^2 = 1$ as a matrix identity.

Upon particularizing Myers action to our particular set up, after taking the trace, we arrive to the following action

$$\mathcal{R}_{\text{micro}} = \int dt dr \ 2T_1 \frac{L^4r^2}{y^4}\left(\sqrt{1 + y'^2} \sqrt{\frac{M^2}{C_2} + \frac{y'^2M^2}{4r^4L^4}} - 1 \right) \ ,$$  

(9)
In order to arrive to this expression we have made use of the properties of the symmetrized trace, which we have computed up to order $O(C_2^{-2})$, since we are interested in comparing with the effective macroscopical computation. Clearly, once we take the Myers limit of large $M$, since $\frac{M}{\sqrt{C_2}} \sim 1$, both the expression for the macroscopic and microscopic configurations match perfectly.

The microscopical computation was done in terms of dielectric D1 strings. However, we arrive to a perfect matching with the macroscopical computation, which carries F string charge due to the S-duality invariance of the configuration (namely, because the dilaton and the 2-forms are zero, and the D3 is self S-dual). Strictly speaking we are computing a 't Hooft line. Following [7], let us do the proper dual macroscopical description dual to the 't Hooft line, which will be now in terms of a D3 with magnetic DBI field on it simulating $M$ units of D1 string charge to the brane. In the CS action we would have

$$S_{CS} = T_3 \int F \int dt dr \left(C^{(2)} + C^{(0)} B_{NS}\right)_{xr} .$$

Therefore it is clear that we should take $F = \frac{M}{2} d\omega_2$. Then, after a straightforward computation, it is easy to see that the action for such a brane is simply

$$\mathcal{R}_{t \text{ Hooft}} = \int dt dr \frac{L_4^4 r^2}{y^4} \left(\sqrt{1 + y^2} \sqrt{1 + \frac{y^4 M^2}{4r^4 L_4^4}} - 1\right) .$$

This expression is directly the same as the Legendre transformed of the D3 with dissolved F1, and in turn the same expression as the action for the dielectric D1 strings, as expected due to the referred S-duality invariance of the considered background.

It is possible to directly describe the Wilson loop by means of an action for dielectric fundamental strings (which naturally expand also to a D3 brane). This action can be constructed ([18]) by means of a chain of dualities starting with the action for coincident gravitons in M-theory in [19], and, once particularized for the case at hand, it coincides with Myers action, thus giving the same result. This is in agreement with what has been described here following [7], and is to be expected due to the S-duality invariance of this D3-brane configuration.
3 Polyakov lines and infinite straight Wilson lines in the antisymmetric product representation

The Polyakov loop, defined as the $SU(N)$ holonomy around the Euclidean time, is a very useful quantity in trying to determine the order of the phase transition between strong and weak coupling of the $\mathcal{N} = 4$ SYM theory on $S^3 \times S^1$ (see e.g. [20]). In [11] it was proposed that the multiply wound Polyakov loop can be calculated along the lines of [7], i.e. using D-branes with large dissolved string charge. Following the same logic as before, since these configurations are also designed to pinch off on the Wilson loop while carrying string charge, it should be possible to give a microscopical description of them in terms of the dielectric effect.

Since we are analyzing thermal loops, we will work with the euclidian Schwarzschild black hole in $AdS$, given by the metric

$$ds^2 = R^2 f dt^2 + \frac{R^2}{f} dr^2 + R^2 r^2 (d\alpha^2 + \sin^2 \alpha d\Omega_2^2) + R^2 d\Omega_5^2,$$

(12)

where

$$f = 1 - \frac{r_+^2 (1 + r_+^2)}{r^2} + r^2.$$

(13)

The extremal limit corresponds to $r_+ \to 0$, which reduces to $AdS_5 \times S^5$.

In [11] it was realized that the suitable brane to describe such a configuration was not a 3-brane but a 5-brane. In [12] and [15], it was further proposed that this D5-brane, which pinches off on the boundary and wraps an $S^4$ inside the $S^5$ of the background, indeed represents the infinite straight Wilson line in an antisymmetric tensor product of fundamentals. Evidence for this in terms of the group-theoretic structure of the associated loop was given in [15], which also confirms the analogy between Wilson line operators and giant gravitons in the sense that both correspond either to symmetric or antisymmetric product representations of half BPS operators which in the gravity side can be described as branes carrying some dissolved charge. Exactly as in the symmetric tensor product, we will see that these branes are to be seen as the effective description of a multi-string configuration pinching off on the loop at the boundary, following the classical recipe for computing the Wilson loop. In the appendix we also consider the D3-brane
configuration of [11], which was shown not to be the right candidate for this configuration. However, it should also be regarded as the effective description of a multi-F1 configuration.

### 3.1 D5 branes

Let us consider the D5 brane configurations of [11] and [12] (indeed, they have been already considered in the literature previously [13]). In suitable coordinates the background reads

\[
\begin{align*}
\text{d}s^2 &= R^2 f \text{d}t^2 + \frac{R^2}{f} \text{d}r^2 + R^2 r^2 \text{d}\omega_3^2 + R^2 \text{d}\gamma^2 + R^2 \sin^2 \gamma \text{d}\vec{x}^2, \\
(14)
\end{align*}
\]

where \(\vec{x}\) are cartesian coordinates for the \(S^4\), and satisfy \(\vec{x}^2 = 1\).

The relevant piece of the RR potential is

\[
C^{(4)}_{ijkl} = R^4 D(\gamma) x^m \epsilon_{ijklm};
\]

where

\[
D(\gamma) = \sin^2 \gamma \cos \gamma + \frac{3}{2} \cos \gamma \sin \gamma - \frac{3(\gamma - \pi)}{2}.
\]

(16)

Since the natural candidates to find a microscopical description of the D5 are now D1 strings, let us consider D1-branes with worldvolume coordinates \(\{t, r\}\) and take \(\gamma = \gamma(r)\). In addition, let us add a DBI field on the strings \(F_{tr} = F\), which stands for the presence of the fundamental strings.

We will take the ansatz that our D1 strings expand to the transverse \(S^4\), which will be a fuzzy 4-sphere of radius 1. We will closely follow the construction of the fuzzy 4-sphere of [21].

#### 3.1.1 The fuzzy \(S^4\)

Without going into very much detail, let us simply point out that in order to have a fuzzy \(S^4\) we have to find 5 matrices such that \(\vec{X}^2 = 1\) as a matrix identity. We can achieve such matrices by taking the \(n\)-fold symmetric tensor

\[
\text{ Indeed, in [13], a microscopical description is given once we expand the Myers DBI action in a power series. As we will see, the full action has specialties (some instanton number in particular) which are fundamental when describing this Wilson loop.}

9
product of the $\Gamma$ matrices of the Clifford algebra underlying the $SO(5)$ group, i.e.

$$G_i^{(n)} = (\Gamma_i \otimes 1 \otimes 1 + 1 \otimes \Gamma_i \otimes 1 \otimes 1 + \cdots 1 \otimes 1 \otimes \Gamma_i)_{\text{Sym}}.$$ (17)

These matrices satisfy the property that $\Sigma_i (G_i^{(n)})^2 = n(n + 4) \times 1$.

The dimension of the representation of the fuzzy $S^4$ can be seen to be

$$Tr(1) = N = \frac{(n+1)(n+2)(n+3)}{6}.$$ (18)

If we call $c = n(n + 4)$, it is then clear that we can construct our unit fuzzy $S^4$ as

$$X^i = \frac{1}{\sqrt{c}} G_i^{(n)}.$$ (19)

A useful property is then

$$\epsilon_{ijklm} X^i X^k X^l X^m = \frac{8n + 16}{n} \times 1.$$ (20)

In addition, we define

$$G_{ij} = \frac{1}{2} [G_i, G_j],$$ (21)

which will be useful when computing the action of our configuration. These $G_{ij}$ satisfy the properties stated in the appendix A of [21].

For further details on the construction of the fuzzy $S^4$, we refer the reader to [21].

Once we make use of the fuzzy $S^4$ construction, after a straightforward computation, we arrive to the following action for our system, which in the large $n$ limit in which we are interested in order to compare with the macroscopic description reads

$$S = \int dt dr \left[ \frac{n^3}{6} + \frac{2n}{3} R^4 \sin^4 \gamma \right] \sqrt{1 + f \gamma'^2 - \frac{F^2}{R^4}} - \int dt dr \frac{2n}{3} R^4 D(\gamma) F.$$ (22)
Again, the prime denotes derivative with respect to $r$, and following \[11\] we have taken $F \rightarrow iF$.

As we will see, our construction is very similar to that in \[22\] in what microscopically we are capturing an extra charge of the brane, which macroscopically will show up as dissolved instantons in the worldvolume. In that reference, a microscopical description of the M5 giant graviton in $AdS_4 \times S^7$ was found in terms of dielectric gravitons expanding to a fuzzy $S^5$, which was constructed as a fibration of an $S^1$ over a fuzzy $CP^2$. The giant graviton carries momentum along a cycle in the $S^7$. However, in the microscopical description although the action used already assumes momentum along one direction, the computation captures momentum also along the isometry of the fibering. This can be seen by comparing with the action for a wrapped macroscopical M5, which carries momentum along both the 11th direction (as instantons) and along an extra direction. Since the configuration of giant graviton carries momentum in just one direction, by comparing with the macroscopical description one learns how to unplug this extra charge. In our situation a similar situation occurs. In the case at hand removing this extra charge (also instantons from the macroscopical point of view) amounts, as we will see, to keep just

$$S = n\left\{ \int dt dr \frac{2T}{3} R^6 \left( \sin^4 \gamma \sqrt{1 + f \gamma'^2 - \frac{F^2}{R^4} - \frac{D(\gamma)F}{R^2}} \right) \right\}. \quad (23)$$

Let us note what we recover is not the action for a single 5 brane (see next subsection), but the action for $n$ of them. This means that we are actually describing the coincident D5 brane configurations in \[15\].

### 3.1.2 Macroscopical D5 with instantons

In order to see the macroscopical counterpart of the above microscopical configuration, instead of directly consider those branes on \[11\], we will also add a magnetic DBI field on the $S^4$ satisfying that $F_{mag} = \ast F_{mag}$, where the star is taken with respect to the unit $S^4$ metric. The existence of such an instanton on $S^4$ can be seen for example in \[22\].

It is straightforward to see that for a D5 with worlvolume coordinates $\{t, r, \Omega_4\}$ and $\gamma = \gamma(r)$ with both electric ($F_{tr}$) and magnetic (let's call it $B$, satisfying as mentioned $B = \ast B$), the action reads
\[ S = \int dt dr \left( \frac{2T_1}{3} R^2 (B \wedge B + R^4 \sin^4 \gamma) \sqrt{1 + f \gamma^2 - \frac{F^2}{R^4}} - \int dt dr \frac{2T_1}{3} R^4 D(\gamma) F \right) . \] (24)

Note that (24) is exactly of the form of (22). From here we see that the first term in the latter corresponds with the instanton charge in the macroscopic version. In order to describe the configurations in (21) we have to eliminate those instantons, which leads us to the following macroscopic configuration

\[ S = \int dt dr \left( \frac{2T_1}{3} R^6 \left( \sin^4 \gamma \sqrt{1 + f \gamma^2 - \frac{F^2}{R^4}} - \frac{D(\gamma) F}{R^2} \right) \right), \] (25)

which matches perfectly with that of (23) once we take into account that the latter represents indeed several coincident D5 branes.

Let us now analyze the meaning of the magnetic DBI field. In the CS piece of the action for our D5 we would have a coupling to \( C^{(2)} + C^{(0)} B_{NS} \) like

\[ S_{CS} = \frac{T_5}{2} \int_{S^4} B \wedge B \int dt dr (C^{(2)} + C^{(0)} B_{NS})_{tr} . \] (26)

Therefore we see that the effect of adding these instantons on our brane is to give it an extra D1 charge along the \( t, r \) directions. From this point of view, since on the D1 branes we also have an electric DBI field which has the meaning of F1 strings along the \( t, r \) directions, these kind of configurations would describe dyonic strings. Since the macroscopic configurations we are interested in just carry F-string charge and no D1 charge, we have to unplug it by setting the instanton number to zero, which in turn means to eliminate the extra term in (22).

### 4 Dyonic branes

As we have just seen, in order to capture the Wilson line in the antisymmetric product representation we have to unplug an extra D1 charge. These D1 are extended along the same direction in which we have the F1, namely the \( t, r \). Therefore, for the complete configuration, the charge carried by the brane when pinching off on the loop at the boundary is not just F1 charge (Wilson
loop) but also D1 charge. This suggests that (25) and its microscopical counterpart indeed capture a Wilson-‘t Hooft line.

Also, we could perform an analog computation for the symmetric tensor product case. Focusing for simplicity in the macroscopical description, we can consider the D3 brane with both electric and magnetic DBI field, thus with F1 and D1 charge dissolved on the brane. Since we are at $g_s = 1$, and the D3 brane is self S-dual, as expected the value of the “routhian” for such a brane is

$$\mathcal{R} = \int dt \, dr \, 2T_1 \frac{L^4 r^2}{y^4} \left( \sqrt{1 + y^2} \sqrt{1 + \frac{y^4}{4r^4L^4}(M^2 + \frac{P^2}{T_1^2})} - 1 \right),$$  \tag{27}

which is invariant under the exchange of the two charges ($M$ (magnetic) $\leftrightarrow \frac{P}{T_1}$ (electric)).

5 Conclusions

In [7] D3 branes with electric flux representing F strings where used to compute certain Wilson loops. The strategy was essentially to capture the all genus Wilson loop by means of a D3 brane since this D3 brane is assumed to be the effective description of the multi-string configuration. Therefore it incorporates in a smooth manner all the interactions between the strings, which amount to the subleading corrections to the Wilson loop.

Restricting ourselves to the Wilson (or Polyakov) lines, which in the dual picture are represented by branes which pinch off at the boundary on a line, as described in [24], [12] and [15], the line in the symmetric tensor product representation can be computed by means of a D3 brane carrying F1 strings, while the line in the antisymmetric product can be computed with D5 with electric flux representing F1 strings.

For the case of the Wilson line in the symmetric product representation, since F1 strings naturally polarize into D3 branes, it is to be expected that there should exist a microscopical description in terms of fundamental strings expanding due to dielectric effect to the D3, which in a suitable limit overlaps with the effective description in terms of the D3 brane with dissolved F1. We have explicitly shown this. Indeed, we have worked with the S-dual picture, which captures the ‘t Hooft line. However, due to the
manifest S-duality invariance, the functional form will be the same for both
the 't Hooft and the Wilson line. Actually, it can be seen that an action for
dielectric F1 can be constructed by means of a chain of dualities starting
with the action of \[19\], which however overlaps with Myers action in this
background. This is precisely a consequence of the anticipated fact (see \[7\])
that both the Wilson and 't Hooft lines have the same functional form.

The case of the Wilson line in the antisymmetric product representa-
tion is somehow more involved, since naturally F1 strings do not expand
to D5 branes. In turn, D1 strings expand naturally to D5 branes. Thus,
we achieved a microscopical description of such lines in terms of dielectric
D1 strings carrying the necessary electric flux, which expand to a fuzzy $S^4$.
Once we make sure that the total D1 charge is zero, we have a perfect agree-
ment, again in a suitable limit, with the effective description in terms of a
D5 with electric flux.

It is worth to note that, along the lines in \[21\], microscopically we obtain
the action for $n$ D5 rather that the action for a single D5. Thus, we would
be describing the multi-D5 configurations of \[15\].

Inspired by the D5-brane case, in which we have the possibility of switch-
ing an instantonic DBI field which represents D1 charge in addition to the
desired F1 charge, we can speculate on the possibility of capturing Wilson-'t
Hooft lines in terms of D3 (D5) branes for the symmetric (antisymmetric)
tensor product representation. Since the charge pinching off at the bound-
dary on the loop is both F1 and D1, we expect that these branes capture
some information about the Wilson-'t Hooft line. It would be interesting to
further study this configurations.

In addition, this picture of the Wilson line opens a new perspective,
since in principle it is possible to compute the loop at any charge using
the work in \[25\] to compute at finite $N$ the symmetrized trace. This might
be more interesting for the circular Wilson loop, which is described by a
random matrix model. The circular Wilson loop calculated in \[2\], breaks
another set of supersymmetries as the one considered here. Since this loop can
be represented by means of a D3 with electric flux pinching off on the loop
at the boundary, in close parallelism with the Wilson line in the symmetric
product representation, we belive that a similar microscopical description
should be possible for that loop, which in turn could be related to the
matrix model to which the circular loop is dual.
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A (No) D3-branes

Let us analyze the configurations of D3 branes studied in [11]. Motivated by [7], and given (12), in [11] it is suggested that the thermal multiply wound Polyakov loop is represented by a D3 brane with worldvolume coordinates \{t, \alpha, \Omega_2\}, with \( r = r(\alpha) \) and non-vanishing \( F_{t\alpha} \). However, as shown in [11], this configuration does not behave as expected for such a loop.

In addition to the metric background of (12), there is a 4-form potential. The relevant piece for the case at hand is

\[
C^{(4)} = -i R^4 r^4 \sin^2 \alpha \, dt \wedge d\alpha \wedge d\omega^2 ,
\]

(28)

It is straightforward to check that the action for the system is

\[
S = \int d\tau d\alpha \, 4\pi T_3 R^4 r^2 \sin^2 \alpha \sqrt{r^2 f + r'^2} - \frac{F^2}{R^4} - \int dld\alpha \, 4\pi T_3 R^4 r^4 \sin^2 \alpha .
\]

(29)

In (29) we are setting \( F_{t\alpha} = iF \), following [11]. In addition, the prime stands for derivative with respect to \( \alpha \).

Since in (29) the DBI potential does not appear, we can Legendre transform to get a “routhian” in terms of a conserved momentum \( \frac{\partial L}{\partial F} = P \). It is easy to check that such a routhian is

\[
R_{Macro} = \int d\tau d\alpha \, T_1 R^4 \sqrt{r^2 f + r'^2} \left( \frac{P^2}{T_1^2 R^4} + 4r^4 \sin^4 \alpha \right) - \int d\tau d\alpha \, 2T_1 R^4 r^4 \sin^2 \alpha .
\]

(30)

In (30) we have made use of the fact that in our conventions \( 2\pi l_s^2 = 1 \) to write the routhian in terms of \( T_1 \) for later convenience.

In order to find a microscopical description of this configuration as in the subsection it is convenient to go to cartesian coordinates
\[ ds^2 = R^2 f dt^2 + \frac{R^2}{f} dr^2 + R^2 r^2 (d\alpha^2 + \sin^2 \alpha d\vec{x}^2) + R^2 d\Omega_5^2 , \quad (31) \]

being \( f \) the same as above.

In addition, the relevant piece of the RR potential is

\[ C^{(4)} = -i R^4 r^4 \sin^2 \alpha x^k \epsilon_{ijk} dt \wedge d\alpha \wedge dx^i \wedge dx^j , \quad (32) \]

Note that we are assuming that \( \vec{x}^2 = 1 \).

Given the macroscopic configuration, it is natural to assume that it can be regarded as the effective theory of dielectric strings stretching along the \( \{ t, \alpha \} \) directions, and expanding to the transverse 2-sphere whose cartesian coordinates are the \( \vec{x} \). In addition, we will assume that \( r = r(\alpha) \).

Upon particularizing Myers action for our background we arrive to the following action

\[ \mathcal{R}_{\text{micro}} = \int dt d\alpha T_1 R^4 \left( \sqrt{r^2 f + r'^2} \sqrt{\frac{M^2}{R^4} + \frac{M^2}{C^2} 4 r^4 \sin^4 \alpha - \frac{M}{\sqrt{C^2 g_s}} 2 r^4 \sin^2 \alpha} \right) . \quad (33) \]

If we now take the Myers limit, in which \((30)\) and \((33)\) are expected to agree, we see that once we are in the large \( M \) limit, when \( \frac{M}{\sqrt{C^2 g_s}} \to 1 \), this is indeed the case, and we have a perfect matching between both descriptions.

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