Stimulated emission from superradiant atoms in waveguide QED

Rui Asaoka¹, Julio Gea-Banacloche², Yuuki Tokunaga¹, and Kazuki Koshino³
¹Computer and Data Science Laboratories, NTT Corporation, Musashino 180-8585, Japan
²Department of Physics, University of Arkansas, Fayetteville, Arkansas 72701, USA
³College of Liberal Arts and Sciences, Tokyo Medical and Dental University, Ichikawa 272-0827, Japan
rui.asaoka.st@hco.ntt.co.jp

Abstract: We investigate the stimulated emission from superradiant atoms coupled to a waveguide induced by a coherent-state pulse. We provide the dynamical properties of this system and also evaluate the system as a amplifier. © 2022 The Author(s)

1. Introduction

Collective effects via the interaction between quantum emitters and reservoirs have attracted a great deal of attention as a consequence of quantum mechanics and are now an indispensable part of modern technology. One of the most fundamental paradigms of them is stimulated emission, which is a phenomenon essentially reflecting the indistinguishability of photons and multiphoton interference. Stimulated emission is the chief mechanism underlying lasers and masers and has also been studied from the perspective of photon cloning [1], and the control of flying photons [2], which have a potential to develop the quantum network. This type of phenomenon has been difficult to observe in the past, but the recent development of waveguide quantum electrodynamics (wQED) is opening the possibility because of its high coherence between the radiation field from atoms and propagating photons [3].

In this work, we investigate the stimulated emission from atoms coupled to a waveguide induced by a coherent-state pulse. Stimulated emission by a coherent state has been well studied within single-mode analysis assuming cavity-QED systems, especially in the context of lasers. On the other hand, along with the development of the wQED system, previous studies have provided useful insights into the interaction between a single emitter and a single photon pulse [4]. Our work here is another fundamental case that can be observed in wQED systems: stimulated emission by a quasi-classical pulse, namely a coherent-state pulse. The dynamics and the output state are still unclear in such situation, where the output state is affected by strong emitter-reservoir interaction in addition to indistinguishability of photons. Furthermore, we extend our interest to a more general case, stimulated emission from multiple atoms.

Our study shows that the atoms emit photons coherently into the waveguide for intense $\pi$-pulses, regardless of pulse shape. Moreover, the stimulated emission amplifies the fluctuations of an incident pulse anisotropically in phase space, and in particular, the increment of the fluctuation takes place almost entirely in the phase direction. This property lets the signal-to-noise ratio improve after the amplification for sufficiently short pulses. These results provide useful insights into the energy extraction from a cluster of emitters and may prove helpful in passive control of flying bosonic states in quantum processing.

2. Results and Discussion

Here we consider an infinitely long waveguide system that couples with $N$ two-level atoms at the same position ($r = 0$), as illustrated in Fig. 1(a). Defining propagating channels to the right (left) along the waveguide by the creation operators $a_1$ ($b_1$) and the annihilation operators $a$ ($b$), the Hamiltonian of the waveguide system coupled with $N$ atoms is given by

$$
H = \omega_a J_z + \int_{-\infty}^{\infty} dk \left[ k a_k^\dagger a_k + i \frac{\gamma}{4\pi} (J_+ a_k - a_k^\dagger J_-) \right] + \int_{-\infty}^{\infty} dk \left[ k b_k^\dagger b_k + i \frac{\gamma}{4\pi} (J_+ b_k - b_k^\dagger J_-) \right],
$$

(1)

where $J_\pm$ and $J_z$ are the spin operators of the total atomic system defined as $J_+ = \sum_j \sigma_1^j$, $J_- = \sum_j \sigma_j^1$, and $J_z = \sum_j \sigma_1^j \sigma_1^j - N/2$ ($\sigma_j^1$ is the annihilation (creation) operator of each atom). When all atoms are initially
excited, Heisenberg equations of the total atomic spin are given by
\[
\frac{d\hat{J}_p}{dt} = -\left(\imath\omega_p + \gamma\right)\hat{J}_p + \gamma\hat{J}_p \hat{J}_m - \sqrt{2}\gamma \hat{J}_m [\hat{a}^\dagger \hat{a} + \hat{b}^\dagger \hat{b}] + \frac{\gamma^2}{2} [\hat{a}^\dagger \hat{a}^\dagger + \hat{b}^\dagger \hat{b}^\dagger] [\hat{J}_m + \hat{J}_m^\dagger] + \frac{\gamma^2}{2} [\hat{a}^\dagger \hat{a} \hat{J}_m + \hat{b}^\dagger \hat{b} \hat{J}_m^\dagger],
\]
where \(\hat{a}(t)\) and \(\hat{b}(t)\) are the annihilation operators of propagating photons at \(t = 0\).

When an intense rectangular \(\pi\)-pulse is incident, from Eqs. (2) and (3), we analytically derive the average photon number of coherent photons included in the output pulses into the right and left:
\[
\int_0^\infty dt |\langle \hat{a}_{\text{out}}(t) \rangle|^2 = |\hat{\epsilon}|^2 t_p + N + \frac{\pi\gamma N^2}{16\Omega},
\]
\[
\int_0^\infty dt |\langle \hat{b}_{\text{out}}(t) \rangle|^2 = \frac{\pi\gamma N^2}{16\Omega},
\]
where \(\hat{\epsilon}\) and \(t_p\) are the amplitude and the pulse duration of the incident pulse. \(\Omega\) is the Rabi frequency defined as \(\Omega \equiv \sqrt{2\gamma|\hat{\epsilon}|}\). The third term on the right hand in Eq. (4) and the component emitted to the left are negligible for an intense pulse; therefore, this result indicates that the atoms emit photons coherently in the same direction as the incident pulse. Thus, all energy of the atomic cluster can be extracted, encouraged by the stimulated emission.

We also evaluate our system as an amplifier, based on the rigorous simulation of the Heisenberg equations; the coherent-state pulse is amplified in phase-preserving manner [7], where the increment of the fluctuation is forced almost in the phase direction in phase space (see Figs. 1(b) and (c)), while the fluctuation is amplified isotropically in general phase-preserving linear amplifiers [7]. This property lets the signal-to-noise ratio improve after the amplification for sufficiently short pulses. These properties suggests the possibility to control the average photon number of bosonic states in phase preserving manner with minimal changes to the photon-number fluctuation.

In the end of the abstract, we note the amplified pulse is in a near-coherent state after the stimulated emission, which is not the photon-added coherent state (PACS) [5]. The PACS is defined by \(|\alpha, m\rangle = \hat{a}^\dagger m |\alpha\rangle\) \((\hat{a}^\dagger \hat{a} = N)\); it is generated via the stimulated emission in an OPA [6]. The PACS has a sub-Poisson distribution resulting in squeezed fluctuation in the signal direction, while our results does not. Thus, the stimulated emission in our system is a quite different process from the photon addition through an OPA, because the photon addition by an OPA involves a somewhat complicated process with significantly weak crystal-light interaction and the projection into the state \(a^\dagger |\alpha\rangle\) by the detection of the idler photon. On the other hand, our study shows dynamical properties of pure stimulated emission.

**References**

1. A. Lamas-Linares, C. Simon, J. C. Howell, and D. Bouwmeester, Science 296, 712 (2002).
2. M. Mirhosseini, E. Kim, X. Zhang, A. Sipahigil, P. B. Dieterle, A. J. Keller, A. Asenjo-Garcia, D. E. Chang, and O. Painter, Nature (London) 569, 692 (2019).
3. D. Roy, C. M. Wilson, and O. Firstenberg, Rev. Mod. Phys. 89, 021001 (2017).
4. E. Rephaeli and S. Fan, Phys. Rev. Lett. 108, 143602 (2012).
5. G. S. Agarwal and K. Tara, Phys. Rev. A 43, 492 (1991).
6. A. Zavatta, S. Viciani, and M. Bellini, Science 306, 660 (2004).
7. C. M. Caves, Phys. Rev. D 26, 1817 (1982).