COSMOLOGICAL REDSHIFT DISTORTION OF CORRELATION FUNCTIONS AS A PROBE OF THE DENSITY PARAMETER AND THE COSMOLOGICAL CONSTANT

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Received 1996 April 11; accepted 1996 July 26

ABSTRACT

We propose cosmological redshift-space distortion of correlation functions of galaxies and quasars as a probe of both the density parameter \( \Omega_0 \) and the cosmological constant \( \lambda_0 \). In particular, we show that redshift-space distortion of quasar correlation functions at \( z \sim 2 \) can in principle set a constraint on the value of \( \lambda_0 \). This is in contrast to the popular analysis of galaxy correlation functions in redshift space which basically determines \( \Omega_0 b^2 / \lambda_0 \), where \( b \) is the bias parameter, but is insensitive to \( \lambda_0 \). For specific applications, we present redshift-space distortion of correlation functions both in cold dark matter models and in power-law correlation function models, and discuss the extent to which one can discriminate between the different \( \lambda_0 \) models.

Subject headings: cosmology: theory — large-scale structure of universe — methods: statistical

1. INTRODUCTION

Redshift-space distortion of two-point correlation functions of a galaxy two-point correlation functions is known as a powerful tool in estimating the cosmological density parameter \( \Omega_0 \); on nonlinear scales Davis & Peebles (1983) computed the relative peculiar velocity dispersions of pairs of galaxies around \( 1 \ h^{-1} \) Mpc from the anisotropies in the correlation functions of the CfA 1 galaxy redshift survey, and then concluded that \( \Omega_0 = 0.20e^{0.04} \) (also see Mo, Jing, & Börner 1993; Suto 1993; Ratcliffe et al. 1996). In linear theory Kaiser (1987) showed that the peculiar velocity field systematically distorts the correlation function observed in redshift space; the averaged redshift-space correlation function \( \xi^{(0)}(x) \) of galaxies is related to its real-space counterpart \( \xi^{(0)}(x) \) as

\[
\xi^{(0)}(x) = \left( 1 + \frac{2}{3} \beta + \frac{1}{5} \beta^2 \right) \xi^{(0)}(x),
\]

where \( b \) is the real-space counterpart \( b \) of the linear growth rate with respect to the scale factor \( a \) (eq. [15] below) at \( z = 0 \). As is clear from the empirical fitting formula in the second line by Lahav et al. (1991), however, this formula depends on \( \Omega_0 \) but is practically insensitive to the cosmological constant \( \lambda_0 \).

At higher redshift \( z \gtrsim 1 \), another anisotropy due to the geometrical effect of the spatial curvature becomes important. We derive a formula for the cosmological redshift-space distortion in linear theory. This formula turns out to be a straightforward generalization of that derived by Hamilton (1992) for \( z = 0 \). It provides a promising method to estimate both \( \Omega_0 \) and \( \lambda_0 \) from future redshift surveys of galaxies and quasars including the Sloan Digital Sky Survey (SDSS) and the Anglo-Australian 2-Degree-Field Survey (2dF). A very similar idea was put forward independently by Ballinger, Peacock, & Heavens (1996), in which they considered the anisotropy of the power spectrum, while we developed a formulation in terms of two-point correlation functions. We outline the derivation in § 2, and present some examples of results in cold dark matter (CDM) models and in power-law correlation function models.

2. COSMOLOGICAL REDSHIFT DISTORTION

Throughout the present analysis, we assume a standard Robertson-Walker metric of the form

\[
ds^2 = -dt^2 + a(t)^2 \left( d\chi^2 + S(\chi)^2 (d\theta^2 + \sin^2 \theta d\phi^2) \right),
\]

where \( S(\chi) \) is determined by the sign of the spatial curvature \( K \) as

\[
S(\chi) = \begin{cases} \sin (\sqrt{K} \chi) / \sqrt{K} & (K > 0), \\ \chi & (K = 0), \\ \sinh (\sqrt{-K} \chi) / \sqrt{-K} & (K < 0). \end{cases}
\]

In our notation, \( K \) is written in terms of the scale factor at present, \( a_0 \), and the Hubble constant, \( H_0 \), as

\[
K = a_0^2 H_0^2 (\Omega_0 + \lambda_0 - 1).
\]
The radial distance \( \chi(z) \) is computed by

\[
\chi(z) = \int^{z} dt \frac{1}{a(t)} = \frac{1}{a_0} \int^{z} dz H(z),
\]

where we introduce the Hubble parameter at redshift \( z \):

\[
H(z) = H_0 \sqrt{\Omega_0 (1 + z)^3 + (1 - \Omega_0 - \lambda_0)(1 + z)^2 + \lambda_0}.
\]

Consider a pair of galaxies or quasars located at redshifts \( z_1 \) and \( z_2 \). If both the redshift difference \( \delta z = z_1 - z_2 \) and the angular separation of the pair \( \delta \theta \) are much less than unity, the comoving separations of the pair parallel and perpendicular to the line-of-sight direction, \( x_1 \) and \( x_2 \), are given by

\[
x_1(z) = \frac{dx_1(z)}{dz} \delta z = \frac{c_1(z)}{a_0 H_0} \delta z, \quad x_2(z) = S(\chi(z)) \delta \theta = \frac{c_2(z)}{a_0 H_0} \delta \theta,
\]

where \( c_1(z) = H_0/H(z) \), \( c_2(z) = a_0 H_0 S(\chi(z)) \), and \( z = (z_1 + z_2)/2 \). Thus their ratio becomes

\[
\frac{x_1(z)}{x_2(z)} = \frac{c_1(z)}{c_2(z)} \frac{\delta z}{\delta \theta} = \frac{\eta(z)}{z} \frac{\delta z}{\delta \theta}.
\]

Since \( x_1/x_2 \) should approach \( \delta z/(z \delta \theta) \) for \( z \ll 1 \), \( \eta(z) \) can be regarded as representing the correction factor for the cosmological redshift distortion (Fig. 1, upper panel). This was first pointed out in explicit form by Alcock & Paczynski (1979).

Although \( \eta(z) \) is a potentially sensitive probe of \( \Omega_0 \) and especially \( \lambda_0 \), this is not directly observable unless one has an independent estimate of the ratio \( x_1/x_2 \). Therefore, Alcock & Paczynski (1979) proposed to apply the result to intrinsically spherical structure resulting from gravitational clustering. More recently, Ryden (1995) proposed to apply the test to statistically spherical voids around \( z \approx 0.5 \). It is not likely, however, that the intrinsic shape of the objects formed via gravitational clustering is spherical even in a statistical sense. Thus their proposal would be seriously contaminated by the aspherical shape distribution of the objects considered. In what follows, we propose to use the clustering of quasars and galaxies in \( z \), which should be safely assumed to be statistically isotropic, in estimating \( \eta(z) \). Also, we take account of the distortion due to the peculiar velocity field using linear theory.

If an object located at redshift \( z_1 \) has a peculiar recession velocity \( v_{\parallel,1} \) with respect to us, we actually observe its redshift as

\[
1 + z_{\text{obs},1} = (1 + z_1)(1 + v_{\parallel,1}/c_{\parallel,1}),
\]

where \( v_{\parallel,1} \) is our (observer’s) peculiar velocity, and we assume that all peculiar velocities are nonrelativistic. Let us consider a sample of galaxies or quasars located in the range \( z - dz \) to \( z + dz \) with \( dz \ll z \) distributed over an angular radius comparable to the proper distance corresponding to \( dz \). Then we set the locally Euclidean coordinates in real space (comoving), \( x \), and in redshift space, \( s \), which share the same origin at redshift \( z \). Since we assume that these coordinates are far from us, we can use the distant-observer approximation. If we take the third axis along the line of sight, the above two coordinates are related to each other as

\[
s_1 = \frac{x_1}{c_1(z)}, \quad s_2 = \frac{x_2}{c_2(z)},
\]

\[
s_1 = \frac{z_{\text{obs},1} - z}{H_0} = \frac{1}{c_1(z)} \left[ x_1 + \frac{1 + z}{H(z)} (v_{\parallel} - v_{\parallel,0}) \right].
\]

Note that in the last expression, we assume that \( x_1 \) is sufficiently small and replace \( 1 + z + H x_1 \) by \( 1 + z \). The number densities of objects in these coordinates are related by the Jacobian of the above transformation. In linear order of density contrast and peculiar velocity, we obtain

\[
\delta^{(i)}(s(x)) = \delta^{(i)}(x) - \frac{1 + z}{H(z)} \delta_{\parallel} v_{\parallel}.
\]

According to linear theory, the peculiar velocity is related to the mass density fluctuation \( \delta_m \) (e.g., Peebles 1993) as

\[
v_{\parallel}(x) = -\frac{H(z)}{1 + z} f(z) \Delta^{-1} \delta_m(x),
\]

where \( \Delta^{-1} \) is the inverse Laplacian;

\[
f(z) = \frac{d \ln D(z)}{d \ln a} \simeq \Omega(z)^{0.6} + \frac{\lambda(z)}{70} \left[ 1 + \frac{\Omega(z)}{2} \right];
\]
and the linear growth rate \(D(z)\) [normalized as \(D(z) = 1/(1 + z)\) for \(z \to \infty\)] is

\[
D(z) = \frac{5 \Omega_0 H_0^2}{2} \frac{1}{H(z)} \int_0^z \frac{1 + z'}{H(z')}^3 \, dz'.
\]

(16)

In the above, \(\Omega(z)\) and \(\lambda(z)\) are the density parameter and the dimensionless cosmological constant at \(z\):

\[
\Omega(z) = \left[ \frac{H_0}{H(z)} \right]^2 (1 + z)^3 \Omega_0, \quad \lambda(z) = \left[ \frac{H_0}{H(z)} \right]^2 \lambda_0.
\]

(17)

Allowing for the possibility that the density contrast of mass, \(\delta_m\), differs from that of objects, \(\delta^{(o)}\), we assume linear biasing \(\delta^{(o)} = b \delta_m\), where \(b(z)\) is a bias factor which now depends on \(z\). Then equation (13) simply generalizes Kaiser’s (1987) result to \(z \neq 0\):

\[
\delta^{(o)}(s(x)) = \int \frac{d^3k}{(2\pi)^3} \left[ 1 + \beta(z) \frac{k^2}{k^2_m} \right] e^{ik \cdot \delta^{(o)}(k)},
\]

(18)

where \(\beta(z) = f(z)/b(z)\), and \(\delta^{(o)}\) is the Fourier transform of the density contrast. From equation (18), we derive the following relation for the two-point correlation function at \(z \neq 0\), which also generalizes the formula of Hamilton (1992) at \(z = 0\):

\[
\xi^{(o)}(s, s_z) = \left\{ 1 + \frac{2}{3} \beta(z) + \frac{1}{5} \left( \beta(z) \right)^2 \right\} \xi_0(x) P_0(\mu) - \left\{ \frac{4}{3} \beta(z) + \frac{4}{7} \left( \beta(z) \right)^2 \right\} \xi_0(x) P_1(\mu) + \frac{8}{35} \left( \beta(z) \right)^2 \xi_0(x) P_2(\mu),
\]

(19)

where \(x = (c^2 s^2 + c^2 s_z^2)^{1/2}\), \(\mu = c s / x\) (\(s = s_z = s^2 + s_z^2\)), the \(P_n\) are the Legendre polynomials, and

\[
\xi_0(x) = \frac{1}{2\pi^3} \int_0^\infty dk k^2 j_m(kx) P(k; z) = \frac{(-1)^l}{x^{2l+1}} \left( \int_0^x dx \right)^l x^{2l} \left( \frac{d}{dx} \right)^l x^l \xi^{(o)}(x; z).
\]

(20)

Again in linear theory and for linear biasing, the power spectrum \(P(k; z)\) of objects at \(z\) is related to that of mass \(P^{(m)}(k)\) of matter at \(z = 0\) as

\[
P(k; z) = [b(z)]^2 \left[ \frac{D(z)}{D(0)} \right]^2 P^{(m)}(k),
\]

(21)

3. SIMPLE MODEL PREDICTIONS

The lower panel in Figure 1 plots \(f(z)\). Here and in what follows, we use the fitting formula for \(f(z)\) (eq. [15]), whose mean error is 2%. As expected from equation (2), \(f(z = 0)\) is insensitive to the value of \(\lambda_0\) and basically determined by \(\Omega_0\) only. At higher
redshifts, however, \( f(z) \) becomes sensitive to \( \lambda_0 \) as pointed out earlier by Lahav et al. (1991), especially if \( \Omega_0 \ll 1 \). Therefore, the behavior of \( f(z) \) at low and high \( z \) is a potentially good discriminator of \( \Omega_0 \) and \( \lambda_0 \), respectively. What we propose here is that a careful analysis of the redshift distortion in correlation functions of galaxies at low redshifts and quasars at high redshifts can probe \( \Omega_0 \) and \( \lambda_0 \) through \( f(z) \) as well as \( \eta(z) \).

For specific examples, we compute \( \xi_0(s, s) \) in linear theory applying equations (19) and (20) in CDM models with \( H_0 = 70 \) km s\(^{-1}\) Mpc\(^{-1}\) (Tanvir et al. 1995). The resulting contours are plotted in Figure 2 (Plate L1). The four sets of values of \( \Omega_0 \) and \( \lambda_0 \) are the same as those adopted in Figure 1, and we adopt the COBE normalization (Sugiyama 1995; White & Scott 1996).

In order to quantify the cosmological redshift distortion in Figure 2, let us introduce the anisotropy parameter \( \xi_0(s) = \xi_0(s, 0) \) and \( \xi_0(s) = \xi_0(0, s) \). The four left-hand panels in Figure 3 show the anisotropy parameter against \( z \) in CDM models. Since the evolution of bias is largely unknown, we assume \( b = 1 \) and 2 just for definiteness. The effect of the evolution of bias (e.g., Fry 1996) will be considered elsewhere (Suto & Matsubara 1996). This clearly exhibits the extent to which one can discriminate the different \( \lambda_0 \) models on the basis of the anisotropies in \( \xi_0(s) \) at high redshifts.

For comparison, let us consider a simple power-law model \( \xi_0(s) = A(z)x^{-\gamma}(\gamma > 0) \). Then equation (19) reduces to

\[
\frac{\xi_0(s, s)}{\xi_0(s)} = 1 + \frac{2(1 - \gamma \mu^2)}{3 - \gamma} \beta(z) + \frac{\gamma(\gamma + 2)\mu^2 - 6\gamma\mu^2 + 3}{(3 - \gamma)(5 - \gamma)} \beta(z)^2.
\]

In this case the anisotropy parameter is given by

\[
\frac{\xi_0(s)}{\xi_0(s)} = \eta(z)^{-\gamma} \frac{(3 - \gamma)(5 - \gamma) + 2(1 - \gamma)(5 - \gamma)\beta(z) + (3 - \gamma)(1 - \gamma)\beta(z)^2}{(3 - \gamma)(5 - \gamma) + 2(5 - \gamma)\beta(z) + 3\beta(z)^2},
\]

and is independent of the scale \( s \).

The four right-hand panels in Figure 3 show that the behavior of the anisotropy parameter is sensitive to the power-law index \( \gamma \). This is partly because \( \Omega_0 = 1 \) CDM models which we consider are already nonlinear on \( 10^{-2} \) h\(^{-1}\) Mpc (Fig. 2), and then the linear theory prediction is not reliable enough. On the other hand, \( \Omega = 0.1 \) CDM models are well described in linear theory on the scales of our interest \( \sim 1 \) h\(^{-1}\) Mpc. Since most observations do suggest that \( \Omega_0 \) in our universe is around 0.1–0.3 (e.g., Peebles 1993; Suto 1993; Ratcliffe et al. 1996), this is encouraging for our present result on the basis of linear theory. Incidentally, Figure 3 also implies that \( \Omega = 0.1 \) CDM models in the linear regime are well approximated by the power-law model with \( \gamma \sim 1 \). This is reasonable, since the CDM spectral index of \( P(k) \) is around \( 2 \), which corresponds to \( \gamma \sim 1 \).

4. CONCLUSIONS

Redshift-space distortion of galaxy correlation functions has attracted much attention as a tool to determine \( \Omega_0^{\lambda \times} / b \) (Kaiser 1987; Hamilton 1992). We obtained an expression to describe the cosmological redshift-space distortion at high \( z \) in linear theory, taking proper account of the spatial curvature \( K \). Then we showed that in principle this can discriminate the value of \( \lambda_0 \) via the \( z \) dependence of the \( \beta(z) \) and \( \eta(z) \) parameters.

Recent analysis, for instance of the Durham/UKST galaxy redshift survey on the basis of equation (1), yields \( \Omega_0^{\lambda \times} / b = 0.55 \pm 0.12 \) (Ratcliffe et al. 1996) with \( \sim 2500 \) galaxies. Their Figure 4a clearly exhibits that the observational data at \( z = 0 \) are already statistically reliable for the direct quantitative comparison with the upper panels in our Figure 2, although we do not attempt it at this point. SDSS, for instance, is expected to have a million galaxy redshifts up to \( z \approx 0.2 \) that would be able to determine \( \Omega_0^{\lambda \times} / b \) to better than \( 10\% \) percent accuracy. Again, upon completion of SDSS, \( \sim 10^7 \) quasar samples become available, and the anisotropy

![Image of Figure 3](image-url)
in quasar correlation functions at $z \sim 2$ will put a constraint on $\lambda_0$, given $\Omega_0$ determined from the galaxy correlation functions. The quasar luminosity function of Boyle, Shanks, & Peterson (1988) predicts that the number of quasars per $\pi$ steradians brighter than 19th $B$ magnitude is about 4500 either in $z = 0.9–1.1$ or in $z = 1.9–2.1$ (for the $\Omega_0 = 1$ and $\lambda_0 = 0$ model). Therefore, we expect that the resulting statistics are even better than those obtained by Ratcliffe et al. (1996) if unknown evolution effects and other potential systematics interfere.

Although the linear theory which we used throughout the paper becomes less restrictive at higher redshifts, it is observationally easier to detect clustering features in the nonlinear regime. Thus the analysis of nonlinear redshift-space distortion is another important area for research (Suto & Sugino, 1991; Matsubara & Suto, 1994; Suto & Matsubara, 1994; Matsubara, 1994). In addition, it is important to examine the possible systematic errors due to the finite volume size and the shape of the survey region. This can be best investigated by the direct analysis of the numerical simulations. This is partly considered by Ballinger et al. (1996), although in $k$-space. We plan to return to this in a later paper (Magira, Matsubara, & Suto, 1996).

We thank Naoshi Sugiyama for discussions. This research was supported in part by grants-in-aid by the Ministry of Education, Science, and Culture of Japan (07740183, 07CE2002). After we submitted the present paper, we became aware of the similar independent work by Ballinger et al. (1996). We are grateful to John Peacock for calling our attention to that paper and for useful comments.

REFERENCES

Alcock, C., & Paczyński, B. 1979, Nature, 281, 358
Ballinger, W. E., Peacock, J. A., & Heavens, A. F. 1996, MNRAS, in press
Boyle, B. J., Shanks, T., & Peterson, B. A. 1988, MNRAS, 235, 935
Davis, M., & Peebles, P. J. E. 1983, ApJ, 267, 465
Fry, J. 1996, ApJ, 461, L65
Hamilton, A. J. S. 1992, ApJ, 385, L5
Kaiser, N. 1987, MNRAS, 227, 1
Magira, H., Matsubara, T., & Suto, Y. 1996, in preparation
Matsubara, T. 1994, ApJ, 424, 30
Mo, H. J., Jing, Y. P., & Börner, G. 1993, MNRAS, 264, 825
Lahav, O., Lilje, P. B., Primack, J. R., & Rees, M. J. 1991, MNRAS, 251, 128
Peebles, P. J. E. 1993, Principles of Physical Cosmology (Princeton: Princeton Univ. Press)
Ratcliffe, A., et al. 1996, preprint, astro-ph/9602062
Ryden, B. 1995, ApJ, 452, 25
Sugiyama, N. 1995, ApJ, 100, 281
Suto, Y. 1993, Prog. Theor. Phys., 90, 1173
Suto, Y., & Matsubara, T. 1994, ApJ, 420, 504
———. 1996, preprint, astro-ph/9607102
Suto, Y., & Sugino, T. 1991, ApJ, 370, L15
Tanvir, N. R., Shanks, T., Ferguson, H. C., & Robinson, D. R. T. 1995, Nature, 377, 27
White, M., & Scott, D. 1996, ApJ, 459, 415
FIG. 2.—Contours of $\xi^{(\|)}(s, s)$ in CDM models at $z = 0$ (upper panels) and $z = 3$ (lower panels). $H_0 = 70$ km s$^{-1}$ Mpc$^{-1}$ is assumed, and the amplitude of the fluctuation spectrum is normalized according to the COBE 2 yr data. Solid and dashed lines indicate the positive and negative $\xi^{(\|)}$, respectively. Contour spacings are $\Delta \log_{10} |\xi| = 0.25$. Thick lines in all $\Omega_0 = 1$ models represent $\xi^{(\|)} = 10, 1, 0.1, -0.1,$ and $-1$. Thick lines in $\Omega_0 = 0.1$ models indicate that $\xi^{(\|)} = 0.01$ and 0.001 for $\lambda_0 = 0$ at $z = 0$, $\xi^{(\|)} = 1, 0.1,$ and 0.01 for $\lambda_0 = 0.9$ at $z = 0$, $\xi^{(\|)} = 0.01$ for $\lambda_0 = 0$ at $z = 3$, and $\xi^{(\|)} = 0.1$ for $\lambda_0 = 0.9$ at $z = 3$.

Matsubara & Suto (see 470, L4)