BARYONS, IN$_c$

RICHARD F. LEBED

Department of Physics & Astronomy,
Arizona State University,
Tempe, AZ 85287-1504, USA
E-mail: richard.lebed@asu.edu

Excited baryons may be analyzed in the $1/N_c$ expansion as true resonances in scattering amplitudes. The key idea making this program possible is a generalization of methods originally applied to chiral soliton models in the 1980’s. One finds model-independent relations among amplitudes that impose mass and width degeneracies among resonances of various quantum numbers. Phenomenological evidence demonstrates that patterns of resonant decay predicted by $1/N_c$ agree with data. The analysis can be extended to subleading orders in $1/N_c$, where again agreement with data is evident.

1. Introduction

Although 100 of the 1000 pages in the Review of Particle Properties$^1$ consist of compiled measured properties of baryon resonances, these states have resisted a model-independent analysis for decades. No one really understands ab initio from QCD why baryon resonances exist at all, much less their peculiar observed multiplicities, spacings, and branching fractions. Even the unambiguous existence of numerous resonances remains open to debate, as evidenced by the infamous 1- to 4-star classification system.

Baryon resonances are exceptionally difficult to study precisely because they are resonances, i.e., unstable under strong decay. As a particular example, quark potential models are strictly speaking applicable only when $qar{q}$ pair production from the vacuum is suppressed, but this mechanism is the means by which baryon resonances are produced in meson-baryon scattering.

The most natural description of excited baryons in large $N_c$ employs the $N_c$ valence quark baryon picture: Since the ground-state baryon multiplets ($J^P = \frac{1}{2}^+$ and $\frac{3}{2}^+$ for $N_c = 3$) neatly fill a single multiplet completely symmetric under combined spin-flavor symmetry [the SU(6) $56$], one may suppose that the ground state of $O(N_c)$ quarks is also completely spin-
flavor symmetric.\textsuperscript{a} Then, in analogy to the nuclear shell model, excited states may be formed by promoting a small number $[O(N_c^0)]$ of quarks into orbitally or radially excited orbitals. For example, the generalization of the SU(6) × O(3) multiplet $(70, 1^{-})$ consists of $N_c - 1$ quarks in the ground state and one in a relative $\ell = 1$ state. One may then analyze observables such as masses and axial-vector couplings by constructing a Hamiltonian whose terms possess definite transformation properties under the spin-flavor symmetry and are accompanied by known powers of $N_c$. By means of the Wigner-Eckart theorem, one then relates observables for different states in each multiplet. This approach has been extensively studied\textsuperscript{3,4,5,6,7,8} (see Ref. 9 for a short review), but it falls short in two important respects:

First, a Hamiltonian formalism is not entirely appropriate to unstable particles, since it refers to matrix elements between asymptotic external states. Indeed, a resonance is properly represented by a complex-valued pole in a scattering amplitude, for which the real and imaginary parts indicate the mass and width, respectively. Moreover, the Hamiltonian approach makes no reference to resonances as excitations of ground-state baryons.

Second, even if one were to construct a Hamiltonian respecting the instability of the resonances, it is not clear that the simple quark-shell baryon multiplets would be eigenstates of this Hamiltonian. Just as in the nuclear shell model, the possibility of configuration mixing means that the true eigenstates might consist of mixtures with 1, 2, or more excited quarks.

In contrast to quark potential models, chiral soliton models naturally accommodate baryon resonances as excitations resulting from scattering of mesons off ground-state baryons. Such models are consistent with the large $N_c$ limit because the solitons are heavy, semiclassical objects compared to the mesons. As has been known for many years,\textsuperscript{10} a number of predictions following from the Skyrme and other chiral soliton models are independent of the details of the soliton structure, and may be interpreted as group-theoretical, model-independent large $N_c$ results. Indeed, the equivalence of group-theoretical results for ground-state baryons in the Skyrme and quark models in the large $N_c$ limit was demonstrated\textsuperscript{11} long ago.

In the remainder of this paper I discuss how the chiral soliton picture may be used to study baryon resonances as well as the full scattering amplitudes in which they appear. It is based upon a series of papers written in collaboration with Tom Cohen (and more recently his students).\textsuperscript{12,13,14,15,16}

\textsuperscript{a}This is reasonable because SU(6) spin-flavor symmetry for ground-state baryons becomes exact in the large $N_c$ limit.\textsuperscript{2}
2. Amplitude Relations

In the mid-1980’s a series of papers\textsuperscript{17,18,19,20,21} uncovered a number of linear relations between meson-baryon scattering amplitudes in chiral soliton models. It became apparent that these results are consistent with the large $N_c$ limit because of their fundamentally group-theoretical nature.

Standard $N_c$ counting\textsuperscript{22} shows that ground-state baryons have masses of $O(N_c^1)$, but meson-baryon scattering amplitudes are $O(N_c^0)$. Therefore, the characteristic resonant energy of excitation above the ground state and resonance widths are both generically expected to be $O(N_c^0)$. To say that two baryon resonances are degenerate to leading order in $1/N_c$ thus actually means that their masses are equal at both the $O(N_c^1)$ and $O(N_c^0)$ level.

An archetype of these linear relations was first derived in Ref. 19. For a ground-state (\(N\) or \(\Delta\)) baryon of spin = isospin \(R\) scattering with a meson (indicated by the superscript) of relative orbital angular momentum \(L\) (and primes for analogous final-state quantum numbers) through a combined channel of isospin \(I\) and spin \(J\), the full scattering amplitudes \(S\) may be expanded in terms of a smaller set of “reduced” scattering amplitudes \(s\):

\[
S_{\pi}^{LL'RR'1J} = (-1)^{R'-R} \sqrt{|R||R'|} \sum_K \left\{ K \begin{pmatrix} I & J \\ R' & L' \end{pmatrix}\right\} \left\{ K \begin{pmatrix} I & J \\ R & L \end{pmatrix}\right\} s_{KL'L}^{\pi},
\]

\[
S_{\eta}^{LRJ} = \sum_K \delta_{KL} \delta (LRJ) s_{II}^{\eta},
\]

where \([X] \equiv 2X + 1\), and \(\delta(j_1j_2j_3)\) indicates the angular momentum triangle rule.\textsuperscript{b} The fundamental feature inherited from chiral soliton models is the quantum number \(K\) (grand spin), with \(K \equiv I + J\), conserved by the underlying hedgehog configuration, which breaks \(I\) and \(J\) separately. The physical baryon state consists of a linear combination of \(K\) eigenstates that is an eigenstate of both \(I\) and \(J\) but no longer \(K\). \(K\) is thus a good (albeit hidden) quantum number that labels the reduced amplitudes \(s\). The dynamical content of relations such as Eqs. (1)–(2) lies in the \(s\) amplitudes, which are independent for each value of \(K\) allowed by \(\delta(IJK)\).

In fact, \(K\) conservation turns out to be equivalent to the $1/N_c$ limit. The proof\textsuperscript{12} begins with the observation that the leading-order (in $1/N_c$) \(t\)-channel exchanges have $I_t = J_t$,\textsuperscript{24} which in turn is proved using large $N_c$ consistency conditions\textsuperscript{25}—essentially, unitarity order-by-order in $N_c$ in meson-baryon scattering processes. However, (\(s\)-channel) \(K\) conservation

\textsuperscript{b}Both are consequences of a more general formula\textsuperscript{23} involving \(9j\) symbols that holds for mesons of arbitrary spin and isospin, which for brevity we decline to include.
was found—years earlier—to be equivalent to the \((t\text{-channel}) I_t = J_t\) rule.\(^{21}\) The proof of this last statement relies on the famous Biedenharn-Elliott sum rule,\(^{26}\) an SU(2) identity.

The significance of Eqs. (1)–(2) lies in the fact that more full observable scattering amplitudes \(S\) than reduced amplitudes \(s\) exist. Therefore, one finds a number of linear relations among the measured amplitudes holding at leading \([O(N_c^0)]\) order. In particular, a resonant pole appearing in some physical amplitude must appear in at least one reduced amplitude; but this same amplitude contributes to a number of other physical amplitudes, implying a degeneracy between the masses and widths of resonances in several channels.\(^{12}\) For example, we apply Eqs. (1)–(2) to negative-parity \(I = \frac{1}{2}, J = \frac{1}{2}\) and \(\frac{3}{2}\) states (called \(N_{1/2}, N_{3/2}\)) in Table 1. Noting that neither the orbital angular momenta \(L, L'\) nor the mesons \(\pi, \eta\) that comprise the asymptotic states can affect the compound state except by limiting available total quantum numbers \((I, J, K)\), one concludes that a resonance in the \(S_{11}^{NN}\) channel \((K = 1)\) implies a degenerate pole in \(D_{13}^{πNN}\), because the latter contains a \(K = 1\) amplitude. One thus obtains towers of degenerate

| State | Quark Model Mass | \(S_{11}^{NN}\) | \(D_{11}^{πΔ}\) | \(S_{11}^{ππ}\) |
|-------|------------------|-----------------|-----------------|-----------------|
| \(N_{1/2}\) | \(m_0, m_1\) | \(s_{100}^π\) | \(s_{122}^π\) | \(s_{1}^π\) |
| \(N_{3/2}\) | \(m_1, m_2\) | \(\frac{1}{2}(s_{122}^π + s_{221}^π)\) | \(\frac{1}{2}(s_{122}^π - s_{221}^π)\) | \(s_{100}^π\) |

It is now fruitful to consider the quark-shell large \(N_c\) analogue of the first excited negative-parity multiplet [the \((70, 1^-)\)]. Just as for \(N_c = 3\),

\[^{13}\text{Parity enters by restricting allowed values of } L, L'.\]
there are 2 $N_{1/2}$ and 2 $N_{3/2}$ states. If one computes the masses to $O(N_c^0)$ for the entire multiplet in which these states appear, one finds only three distinct eigenvalues,\textsuperscript{6,12,27} which are labeled $m_0$, $m_1$, and $m_2$ and listed in Table 1. Upon examining an analogous table containing all the states in this multiplet,\textsuperscript{12} one quickly concludes that exactly the required resonant poles are obtained if each $K$ amplitude, $K = 0, 1, 2$, contains precisely one pole, which located at the value $m_K$. The lowest quark-shell multiplet of negative-parity excited baryons is found to be compatible with, \textit{i.e.}, consist of a complete set of, multiplets classified by $K$.

One can prove\textsuperscript{13} this compatibility for all nonstrange multiplets of baryons in the SU(6)×O(3) shell picture.\textsuperscript{4} It is important to point out that compatibility does not imply SU(6) is an exact symmetry at large $N_c$ for resonances as it is for ground states;\textsuperscript{2} Instead, it says that SU(6)×O(3) multiplets are reducible multiplets at large $N_c$. In the example given above, $m_{0,1,2}$ each lie only $O(N_c^0)$ above the ground state, but are separated by amounts of $O(N_c^0)$.

We point out that large $N_c$ by itself does not mandate the existence of any resonances at all; rather, it merely tells us that if even one exists, it must be a member of a well-defined multiplet. Although the soliton and quark pictures both have well-defined large $N_c$ limits, compatibility is a remarkable feature that combines them in a particularly elegant fashion.

### 3. Phenomenology

Confronting these formal large $N_c$ results with experiment poses two significant challenges, both of which originate from neglecting $O(1/N_c)$ corrections. First, the lowest multiplet of nonstrange negative-parity states covers quite a small mass range (only 1535–1700 MeV), while $O(1/N_c)$ mass splittings can generically be as large as $O(100$ MeV). Any claims that two such states are degenerate while two others are not must be carefully scrutinized.

Second, the number of states in each multiplet increases with $N_c$, meaning that a number of large $N_c$ states are spurious in $N_c = 3$ phenomenology. For example, for $N_c \geq 7$ the analogue of the 70 contains 3 $\Delta_{3/2}$ states, but only 1 $[\Delta(1700)]$ for $N_c = 3$. As $N_c$ is tuned down from large values toward 3, the spurious states must decouple through the appearance of factors such as $(1 - 3/N_c)$, which in turn requires one to understand simultaneously leading and subleading terms in the $1/N_c$ expansion.

\textsuperscript{d}Studies to extend these results to flavor SU(3) are underway; while the group theory is more complicated, it remains tractable.
Nevertheless, it is possible to obtain testable predictions for the decay channels, even with just the leading-order results. For example, note from Table 1 that the \( K = 0(1) N_{1/2} \) resonance couples only to \( \eta(\pi) \). Indeed, despite lying barely above the \( \eta N \) threshold, the \( N(1535) \) resonance decays through this channel 30–55% of the time, while the \( N(1650) \), which has much more comparable phase space for \( \pi N \) and \( \eta N \), decays to \( \eta N \) only 3–10% of the time. This pattern clearly suggests that the \( \pi \)-phobic \( N(1535) \) should be identified with \( K = 0 \) and the \( \eta \)-phobic \( N(1650) \) with \( K = 1 \), the first fully field theory-based explanation for these phenomenological facts.

4. Configuration Mixing

As mentioned above, quark-shell baryon states with a fixed number of excited quarks are not expected to be eigenstates of the full QCD Hamiltonian. Rather, configuration mixing is expected to cloud the situation. Consider, for example, the statement that baryon resonances are expected to have generically broad \( \mathcal{O}(N_c^0) \) widths. One may ask whether some states might escape this restriction and turn out to be narrow in the large \( N_c \) limit. Indeed, some of the first work\(^5\) on excited baryons combined large \( N_c \) consistency conditions and a quark description of excited baryon states to predict that baryons in the 70-analogue have widths of \( \mathcal{O}(1/N_c) \), while states in an excited negative-parity spin-flavor symmetric multiplet (56\(^{′}\)) have \( \mathcal{O}(N_c^0) \) widths.

In fact there exist, even in the quark-shell picture, operators responsible for configuration mixing between these multiplets.\(^{14}\) The spin-orbit and spin-flavor tensor operators (respectively \( \ell s \) and \( \ell^{(2)} g G_c \) in the notation of Refs. 6, 7, 27), which appear at \( \mathcal{O}(N_c^0) \) and are responsible for splitting the eigenvalues \( m_0, m_1, \) and \( m_2 \), give nonvanishing transition matrix elements between the 70 and 56\(^{′}\). Since states in the latter multiplet are broad, configuration mixing forces at least some states in the former multiplet to be broad as well. One concludes that the possible existence of any excited baryon state narrow in the large \( N_c \) limit requires a fortuitous absence of significant configuration mixing.

5. Pentaquarks

The possible existence of a narrow isosinglet, strangeness +1 (and therefore exotic) baryon state \( \Theta^+(1540) \), claimed to be observed by numerous experimental groups (but not seen by several others), was much discussed at this meeting. Although the jury remains out on this important question,
one may nevertheless use the large \( N_c \) methods described above to determine what degenerate partners a state with these quantum numbers would possess.\(^{15} \) For example, if one imposes the theoretical prejudice \( J_0 = \frac{1}{2} \), then there must also be pentaquark states with \( I = 1, J = \frac{1}{2}, \frac{3}{2} \) and \( I = 2, J = \frac{3}{2}, \frac{5}{2} \), with masses and widths equal that of the \( \Theta^+ \), up to \( O(1/N_c) \) corrections.

The large \( N_c \) analogue of the “pentaquark” actually carries the quantum numbers of \( N_c+2 \) quarks, consisting of \( (N_c+1)/2 \) spin-singlet, isosinglet \( uu \) pairs and an \( \bar{s} \) quark. The operator picture, for example, shows the partner states we predict to belong to SU(3) multiplets \( 27 \ (I = 1) \) and \( 35 \ (I = 2) \).\(^{28} \) However, the existence of partners does not depend upon any particular picture for the resonance or any assumptions regarding configuration mixing. Since the generic width for such baryon resonances remains \( O(N_c^0) \), the surprisingly small reported width (\(< 10 \) MeV) does not appear to be explicable by large \( N_c \) considerations alone, but may be a convergence of small phase space and a small nonexotic-exotic-pion coupling.

6. \( 1/N_c \) Corrections

All the results exhibited thus far hold at the leading nontrivial order (\( N_c^0 \)) in the \( 1/N_c \) expansion. We saw in Sec. 3 that \( 1/N_c \) corrections are essential not only to explain the sizes of effects apparent in the data, but in the very enumeration of physical states. Clearly, if this analysis is to carry real phenomenological weight, one must demonstrate a clear path to characterize \( 1/N_c \) corrections to the scattering amplitudes. Fortunately, such a generalization is possible: As discussed in Sec. 2, the constraints on scattering amplitudes obtained from the large \( N_c \) limit are equivalent to the \( t \)-channel requirement \( I_t = J_t \). In fact, Refs. 24 showed not only that this result holds in the large \( N_c \) limit, but also that exchanges with \( |I_t - J_t| = n \) are suppressed by a relative factor \( 1/N_{c}^{n} \).

This result permits one to obtain relations for the scattering amplitudes including all effects up to and including \( O(1/N_c) \):

\[
S_{LL'RR'I,J,s} = \sum_J \begin{bmatrix} 1 \ R' \ I_s \\ R \ 1 \ J = J_t \end{bmatrix} \begin{bmatrix} L' \ R' \ J_s \\ R \ L \ J_t = J_t \end{bmatrix} s_{J,LL'}^{(1)/(+) \ (J)} + O(N_c^{0}), (4)
\]

\[
= -\frac{1}{N_{c}} \sum_J \begin{bmatrix} 1 \ R' \ I_s \\ R \ 1 \ I_t = J \end{bmatrix} \begin{bmatrix} L' \ R' \ J_s \\ R \ L \ J_t = J_t + 1 \end{bmatrix} s_{J,LL'}^{(-)} + O(N_c^{0}). (4)
\]

\[
= -\frac{1}{N_{c}} \sum_J \begin{bmatrix} 1 \ R' \ I_s \\ R \ 1 \ I_t = J \end{bmatrix} \begin{bmatrix} L' \ R' \ J_s \\ R \ L \ J_t = J_t - 1 \end{bmatrix} s_{J,LL'}^{(-)} + O(N_c^{0}). (4)
\]
One obtains this expression by first rewriting $s$-channel expressions such as Eqs. (1)–(2) in terms of $t$-channel amplitudes. The $6j$ symbols in this case contain $I_t$ and $J_t$ as arguments (which for the leading term are equal). One then introduces new $O(1/N_c)$-suppressed amplitudes $s^{t(\pm)}$ for which $J_t - I_t = \pm 1$. The square-bracketed $6j$ symbols in Eq. (4) differ from the usual ones only through normalization factors, and in particular obey the same triangle rules.

Relations between observable amplitudes that incorporate the larger set $s^t$, $s^{t(+)}$, and $s^{t(-)}$ are expected to hold a factor of 3 better than those merely including the leading $O(N_c^0)$ results. Indeed, this is dramatically evident in cases where sufficient data is available, $\pi N \to \pi \Delta$ scattering (Fig. 1). For example, (c) and (d) in the first 4 insets give the imaginary and real parts, respectively, of partial wave data for the channels $SD_{31}$ (\text{\textcircled{O}}) and $(1/\sqrt{5})DS_{13}$ (\text{\textsquare}), which are equal up to $O(1/N_c)$ corrections; in (c) and (d) of the second 4 insets, the \text{\textcircled{O}} points again are $SD_{31}$ data, while \text{\textdiamond} represent $-\sqrt{2}DS_{33}$, and by Eq. (4) these are equal up to $O(1/N_c^2)$ corrections.

7. Conclusions

The purpose of this talk has been to convince you that there now exist reliable calculational techniques to handle not only long-lived ground-state baryons, but also unstable baryon resonances and the scattering amplitudes in which they appear. This approach, originally noted in chiral soliton models but eventually shown to be a true consequence of large $N_c$ QCD, is found to have phenomenological consequences [such as the large $\eta$ branching fraction of the $N(1535)$] that compare favorably with real data.

The first few steps into studying and classifying $1/N_c$ corrections to the leading-order results, absolutely essential to obtaining comparison with the full data set, have begun. The measured scattering amplitudes appear to obey the constraints placed by these corrections, and more work along these lines will be forthcoming. For example, the means by which the spurious extra resonances of large $N_c$ decouple as one tunes the value of $N_c$ down to 3 is crucial and not yet understood.

All the results presented here, as mentioned in Sec. 2, have used only relations among states of fixed strangeness. Moving beyond this limitation means using flavor SU(3) group theory, which is rather more complicated than isospin SU(2) group theory. Nevertheless, this is merely a technical complication, and existing work shows that it can be overcome.\textsuperscript{7}
Figure 1. Real and imaginary parts of $\pi N \rightarrow \pi \Delta$ scattering amplitudes. The first 4 insets give two particular partial waves equal to leading order [hence indicating the size of $O(1/N_c)$ corrections]. The second 4 insets give two particular linear combinations of the same data good to $O(1/N_c^2)$.

At the time of this writing, all of the necessary tools appear to be in place to commence a full-scale analysis of baryon scattering and resonance parameters. One may envision a sort of resonance calculation factory, Baryons $IN_C$. With sufficient time and researchers, the whole baryon resonance spectrum can be disentangled using a solid, field-theoretical approach based upon a well-defined limit of QCD.

Acknowledgments
I would like to thank the organizers for their hospitality and inviting me to this most lively conference. The work described here was supported in
part by the National Science Foundation under Grant No. PHY-0140362.

References

1. Review of Particle Properties (K. Hagiwara et al.), Phys. Rev. D66, 010001 (2002).
2. R.F. Dashen and A.V. Manohar, Phys. Lett. B315, 425 (1993).
3. C.D. Carone, H. Georgi, L. Kaplan, and D. Morin, Phys. Rev. D50, 5793 (1994).
4. J.L. Goity, Phys. Lett. B414, 140 (1997).
5. D. Pirjol and T.-M. Yan, Phys. Rev. D57, 1449 (1998); D57, 5434 (1998).
6. C.E. Carlson, C.D. Carone, J.L. Goity, and R.F. Lebed, Phys. Lett. B438, 327 (1998); Phys. Rev. D59, 114008 (1999).
7. J.L. Goity, C. Schat, and N. Scoccola, Phys. Rev. Lett. 88, 102002 (2002); Phys. Rev. D66, 114014 (2002); Phys. Lett. B564, 83 (2003).
8. C.E. Carlson and C.D. Carone, Phys. Rev. D58, 053005 (1998); Phys. Lett. B441, 363 (1998); B484, 260 (2000).
9. R.F. Lebed, in NStar 2002: Proceedings of the Workshop on the Physics of Excited Nucleons, ed. by S.A. Dytman and E.S. Swanson, World Scientific, Singapore, 2003, p. 73 [hep-ph/0301279].
10. E. Witten, Nucl. Phys. B223, 433 (1983); G.S. Adkins, C.R. Nappi, and E. Witten, Nucl. Phys. B228, 552 (1983); G.S. Adkins and C.R. Nappi, Nucl. Phys. B249, 507 (1985).
11. A.V. Manohar, Nucl. Phys. B248, 19 (1984).
12. T.D. Cohen and R.F. Lebed, Phys. Rev. Lett. 91, 012001 (2003); Phys. Rev. D67, 012001 (2003).
13. T.D. Cohen and R.F. Lebed, Phys. Rev. D68, 056003 (2003).
14. T.D. Cohen, D.C. Dakin, A. Nellore, and R.F. Lebed, Phys. Rev. D69, 056001 (2004).
15. T.D. Cohen and R.F. Lebed, Phys. Lett. B578, 150 (2004).
16. T.D. Cohen, D.C. Dakin, A. Nellore, and R.F. Lebed, hep-ph/0403125.
17. A. Hayashi, G. Eckart, G. Holzwarth, H. Walliser, Phys. Lett. B147, 5 (1984).
18. M.P. Mattis and M. Karliner, Phys. Rev. D31, 2833 (1985).
19. M.P. Mattis and M.E. Peskin, Phys. Rev. D32, 58 (1985).
20. M.P. Mattis, Phys. Rev. Lett. 56, 1103 (1986); Phys. Rev. D39, 994 (1989); Phys. Rev. Lett. 63, 1455 (1989).
21. M.P. Mattis and M. Mukerjee, Phys. Rev. Lett. 61, 1344 (1988).
22. E. Witten, Nucl. Phys. B160, 57 (1979).
23. M.P. Mattis, Phys. Rev. Lett. 56, 1103 (1986).
24. D.B. Kaplan and M.J. Savage, Phys. Lett. B365, 244 (1996); D.B. Kaplan and A.V. Manohar, Phys. Rev. C56, 76 (1997).
25. R.F. Dashen, E. Jenkins, and A.V. Manohar, Phys. Rev. D49, 4713 (1994).
26. A.R. Edmonds, Angular Momentum in Quantum Mechanics (Princeton Univ. Press, Princeton, NJ, 1996) [Eq. (6.2.12)].
27. D. Pirjol and C. Schat, Phys. Rev. D67, 096009 (2003).
28. E. Jenkins and A.V. Manohar, hep-ph/0401190 and 0402024.