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Heat-transfer enhancement in AC electro-osmotic micro-flows

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Abstract.

Heat transfer in micro-flows is essential to emerging technologies as advanced micro-electronics cooling systems and chemical processes in lab-on-a-chip applications. The present study explores the potential of AC electro-osmotic (ACEO) flow forcing, a promising technique for the actuation and manipulation of micro-flows, for heat-transfer enhancement. Subjects of investigation include the 3D flow structure due to ACEO forcing via an array of electrodes in a micro-channel by way of 3D velocity measurements. Presence and properties of vortical structures of the 3D flow are quantified in laboratory experiments. Typical outcomes of the experimental study result from a number of 3D particle trajectories obtained by using 3D micro-Particle-Tracking Velocimetry (3D µ-PTV). The steady nature of the flow enables combination of results from a series of measurements into one dense data set. This facilitates accurate evaluation of quantities relevant for heat transfer by data-processing methods. The primary circulation is given above one half of an electrode in terms of the spanwise component of vorticity. The outline of the vortex boundary is determined via the eigenvalues of the strain-rate tensor. To estimate convective heat transfer, wall shear rate above one half of an electrode is quantitatively analyzed as function of voltage amplitude and frequency. These results yield first insights into the characteristics of 3D ACEO flows and ways to exploit and manipulate them for heat-transfer enhancement.

1. Introduction

Heat transfer in micro-flows is essential to emerging technologies as advanced micro-electronics cooling systems and chemical processes in lab-on-a-chip applications, for e.g. biotechnology and molecular analysis [1]. The heat transfer is, due to the small length- and velocity scales of micro-flows, essentially laminar [2]. In addition, smaller micro devices lead to higher power densities and higher heat dissipation rates, where a heat removal capacity in the order of hundreds of W/cm². Single-phase liquid flow is a promising cooling approach, due to its reliability and high efficiency [3]. However, generating fluid motion at micro scales of order 100µm poses a considerable challenge, since conventional pressure-driven flow technology is no longer an option due to excessive pressures. This motivates the present study on alternative flow-forcing methods.

Electro-hydrodynamics (EHD) forcing is a promising technique for the actuation and manipulation of micro-flows, and is therefore an interesting option for micro-scale heat-transfer enhancement [4]. EHD forcing, instead of using the pressure gradient or moving parts, drives the flow by way of electro-kinetic mechanisms [5]. Here we adopt AC electro-osmosis (ACEO) as particular EHD forcing method, which sets the fluid into motion via a slip velocity induced at
electrodes on the channel wall by a low amplitude AC electric field \cite{6}. Various applications of ACEO flow in microfluidic devices have been developed so far, including micro-pumping \cite{7, 9}, micromixing \cite{10} and manipulation of polarizable particles \cite{11, 12}.

AC electro-osmosis is in essence flow forcing by electro-kinetic effects induced via a low-voltage AC electric field. So far, experimental observations and numerical simulations exhibit great discrepancies, and results vary with the geometry of the device and properties of the fluids \cite{16}. Moreover, to date there is no reliable mathematical model that describes and predicts the ACEO velocity profile in a quantitatively adequate way. In-depth experimental investigations of the 2D and 3D velocity distributions are an essential step in further exploring ACEO as a flow-forcing technique for heat-transfer enhancement.

In this paper, the 3D flow structure of ACEO flows is studied in laboratory experiments by way of 3D velocity measurements using 3D micro-Particle-Tracking Velocimetry (3D \(\mu\)-PTV) \cite{20, 21}. Investigated are presence and properties of vortical structures, which are expected to promote convective heat transfer. By virtue of the Reynolds-Colburn analogy, the heat-transfer rates are quantitatively estimated in terms of wall shear rates and mean Nusselt number at the bottom wall. The experimental results will serve as valuable reference for follow-up studies on ACEO flow and its effect on heat-transfer characteristics in microchannels.

2. Problem definition

The flow domain consists of a straight rectangular channel, shown schematically in Fig.1a. Periodic symmetric electrode arrays are aligned on the bottom of the channel at a 10-degree angle with respect to the axial direction of the main channel. This electrode pattern is under investigation for application in microscopic particle separation \cite{12}. The electro-hydrodynamics in this device is studied in the present work. To this end we, in a first approximation, focus on the bulk flow and assume effects of the channel side walls to be negligible. The \(x\)-\(z\) plane of the coordinate system is chosen perpendicular to the surface of the electrodes, and the \(y\)-axis runs parallel to the electrode edge. Each spatial period encompasses the horizontal extent \(L = W + G\), with \(W\) and \(G\) the electrode and gap widths, respectively. The symmetry of the electrodes and the electric field yield two symmetrical counter-rotating vortices above each electrode (Fig.1b).

![Figure 1. Schematic diagrams of: (a) the channel, where each electrode pair consists of an electrode width \(W\) and a gap width \(G\), respectively. (b) electro-osmosis flow in symmetric electrode array.](image)

The Reynolds number is defined as \(Re = \frac{\bar{u}_x L}{\nu}\), with \(\nu\) the kinematic viscosity, and \(\bar{u}_x\) the mean slip velocity, \(\bar{u}_x = \frac{1}{L^2} \int_0^L u_x \big|_{z=0} dx\). The spanwise component of the vorticity is described by \(\omega_y = \partial u_x / \partial z - \partial u_z / \partial x\). The circulation, \(\text{i.e.}\) strength, of each vortex is given via the area integral, \(\Gamma = \int_A \omega_y dA\), where the area \(A\) is determined via eigenvalues of the strain-rate tensor (\(\lambda_2\)-method) \cite{17, 18}.

According to the Reynolds-Colburn analogy \cite{8}, the Nusselt number indicative of heat-transfer rates at the wall can be calculated from \(Nu = \frac{C_f Re Pr^{1/3}}{2}\), where \(Pr = \nu / \alpha\) is the Prandtl number,
and $\alpha$ is the thermal diffusivity of the fluid. The skin friction factor, $C_f$, is given via $C_f = \frac{2 \mu}{\nu_m} \dot{\gamma}$, where $\dot{\gamma}$ is the viscous shear rate at the bottom wall, $\dot{\gamma} = \frac{\partial u}{\partial z} |_{z=0}$.

3. Experimental methods

3.1. Laboratory set-up

The microfluidic device shown schematically in Fig.1 was fabricated by using standard photolithography techniques. Indium tin oxide (ITO) layers with 120 nm thickness are employed as electrode. Refer to [12] for details of the fabrication. The dimensions of each electrode are $W = 60 \mu m$ and $G = 10 \mu m$, and the whole channel is 60 $\mu m$ high and 200 $\mu m$ wide.

The device was mounted on a chip holder and was connected via a silicon tube with an inner diameter of 0.79 mm (Masterflex L/S, NL) to the syringe. Potassium hydroxide (KOH) solutions with a concentration of 0.1 mM (Sigma-Aldrich Co., USA) were used as working fluid. Fluorescent polymer micro-particles of diameter 2 $\mu m$ (Fluoro-Max, Duke Scientific Corp., CA, 1% solids), with a density of 1.21 $g/cm^3$, were used as tracer particles to measure fluid velocity. In the experiment, the channel was filled with the solution and subsequently closed. A function generator (Sefram 4422, NL) provides an AC signal to the electrode arrays. Its amplitude and frequency were measured by a digital oscilloscope (Hewlett-Packard, USA). The particle movement was observed using a fluorescence microscope with the Nd:YAG laser generation (Quantel ICE450) producing a pulsed monochromatic laser beam with a wavelength of 532 nm. Images of the fluorescent tracer particles were recorded by the digital camera (12-bit PCO SensiCam, QE).

3.2. Measurement technique

![Figure 2. Schematic diagram of 3D $\mu$-PTV setup.](image)

The measurement is based on the astigmatism or wavefront-deformation micro-particle tracking velocimetry (3D $\mu$-PTV) [20, 21]. To this end, a cylindrical lens with focal length of $f_c = 150 mm$ was added in front of a CCD camera. This lens causes the particles in the images to deform into ellipses, where the ellipticity is a measure for the particle position normal to the focal plane. Thus the 3D particle positions can be determined using a single camera. The schematic of the 3D $\mu$-PTV setup is shown in Fig.2. The parameters of the experimental set-up are given in Table 1. The algorithm by [20, 21] was implemented in MATLAB program so as to analyze the particle trajectories. Associated with the particle velocity, a Lagrangian average 2D velocity field of ACEO flow is obtained. Measurement errors like sticking particles and spurious vectors are eliminated by the algorithm, involving two steps: (i) the velocity interpolation onto a regular grid by Gaussian weight average; (ii) the interpolated velocity field smoothed by a combination of discrete cosine transforms and the penalized least-squares approach [22].

The calibration of the particle images in the $x$- and $y$-direction was performed by a series of images of the particles at different relative depth $z$ along the optical axis (Fig.3a). The resulting
Table 1. Parameters of 3D $\mu$-PTV system

| Parameter | Value | unit | Description |
|-----------|-------|------|-------------|
| $f_c$     | 150   | mm   | Focal length of cylindrical lens |
| $\lambda$ | 612   | nm   | Wavelength of the light emitted by the particles |
| $d_p$     | 2     | $\mu$m | Diameter of the particles |
| $f_0$     | 7.9   | mm   | Focal length of the objective lens |
| $M$       | 20    | [-]  | Magnification of the optical system |
| $n_{air}$ | 1     | [-]  | Refractive index of the air |
| $NA$      | 0.4   | [-]  | Numerical aperture of the objective lens |

relation between $x$-wise and $y$-wise particle size and relative depth $z$ is shown in Fig.3b, where the curve corresponds to a fifth-order polynomial fit. The RMS error of the particle depth is less than 0.5$\mu m$ for a measurable depth of 42$\mu m$.

Figure 3. (a): Series of 2$\mu m$ fluorescent particle scanning images, the black number indicating the relative position of particle in $\mu m$, and (b): the size of the deformed particle images in pixel and the corresponding calibration fitting with a fifth order polynomial.

4. Results and Discussion

4.1. Particle trajectories

The tracer particles were uniformly distributed in the micro-channel before the start of experiment. When an AC voltage is employed, most particles move in the vortex at the range of the voltage from $2V_{PP}$ to $4V_{PP}$ for all frequencies and only few particles tend to stick to the edges of the electrodes. Fig.4a depicts the 3D trajectories of several particles at a voltage of $V_0 = 2V_{PP}$ and a frequency of 1500Hz for the time interval from $t = 0s$ to 25s. The figure demonstrates that the particles follow the fluid flow well.

In general, tracer particles in applied electric fields are subjected to electro-osmotic flow and dielectrophoretic force (DEP). Both effects are straightly related to the frequency and gradient of the applied electric field. Assuming the complex Clausius-Mossotti factor equal to 1 (the complex Clausius-Mossotti factor is defined as $K(\omega) = (\tilde{\varepsilon}_p - \tilde{\varepsilon}_f) / (\tilde{\varepsilon}_p + 2\tilde{\varepsilon}_f)$, where $\omega$ is the frequency, $\tilde{\varepsilon}_p$ and $\tilde{\varepsilon}_f$ are the complex permittivities of the particle and the fluid), the contribution ratio of DEP force to ACEO flow on the movement of our polystyrene tracer particle at low frequency can be given and simplified [26] as

$$\frac{u_{DEP}}{u_{ACEO}} = \frac{8\sqrt{\varepsilon_\varepsilon}(1 + k_e)^2 d_p^2}{3\pi^2 \frac{r^2}{r_2}},$$

where $u_{DEP}$ is the particle velocity due to DEP, $u_{ACEO}$ the fluid velocity induced by electro-osmosis, $k_e$ the width ratio between the electrodes, $r$ the distance to the centre of the gap and
Figure 4. (a): 3D particle trajectories at applied voltage of 2V_{pp} and frequency of 1500Hz, the yellow area indicating the electrodes, (b): Quasi-2D velocity vectors of particles in the segment including two half electrodes, black solid line indicating the electrodes.

4.2. Flow field and slip velocity

The particle velocity was calculated for each position in Fig.4a, where the x-component of particle velocity is indicated by the color bar. Contrary to $u_x$ and $u_z$, the value of $u_y$ remains small everywhere in the bulk flow, meaning that the particle can to good approximation be considered a quasi-2D flow, perpendicular to the electrode edge. The quasi-2D velocity vectors of the particles are given in Fig.4b, where the black solid lines at the bottom indicate two neighboring electrodes and colors correspond with the magnitude of $u_x$. This exposes two symmetric counter-rotating vortices above the electrode surface. The magnitude of $u_x$ increases significantly and reaches a maximum close to the electrode edges when the particle approaches the edges of the electrodes. It falls off rapidly with distance from the edge along the electrode surface and vanishes at the centre of the electrodes, where the stagnation line of the slip velocity is located. The maximum of $u_z$ is above the gaps between the electrodes. The velocity magnitude, $U = \sqrt{u_x^2 + u_z^2}$, in the bulk domain was calculated and attains a maximum value of about 170\(\mu\)m/s close to the electrode edge. The corresponding $Re$ number is calculated, yielding $Re = U_{max}L/2\nu \sim 10^{-2}$.

Figure 5. Slip velocity as a function of frequency at 2V_{pp} and at 4V_{pp}, respectively.
By interpolating these particle velocity vectors into an equidistant grid with the spacing of \( \Delta x = 1 \mu m \) and \( \Delta z = 1 \mu m \) respectively, a Lagrangian average 2D velocity field of ACEO flow in the \( x-z \) plane is obtained. Fig. 5 shows the average axial velocity profile of ACEO flow near the bottom surface versus frequency at voltages of \( 2V_{PP} \) and \( 4V_{PP} \). These results are qualitatively consistent with literature [13], although significant quantitative differences between numerical and experimental observations exist [16]. The latter is also observed in literature and in e.g. [14] accounted for by a fitting parameter. The velocity magnitude at the bottom varies nonlinearly with frequency by first increasing and subsequently decreasing when the frequency exceeds the frequency of about \( 600 \sim 1000Hz \). It must be noted that for frequency fixed, the velocity increases approximately by a factor 2 upon raising the applied voltage from \( 2V_{PP} \) to \( 4V_{PP} \), which is significantly different from the linear theoretical prediction \( u \sim V_{0}^{2} \) [15, 25]. This may be caused by the nonlinear electrokinetic phenomena at large voltages, where resulting electric field and induced surface ions are large enough to violate the assumptions of the classical theory, such as nonlinear capacitance and surface conductivity due to crowding effects on the distribution of ions in EDL [16].

4.3. Vortex structure

The vortices may set up a strong convection in the flow, thereby enhancing the heat transfer. Fig. 6 shows the spanwise component of vorticity above one half of an electrode at a voltage of \( 2V_{PP} \) and a frequency of \( 1500Hz \). Here the vortex core, outlined by the red curve, is determined from the eigenvalues of the strain-rate tensor. The vortex centre \( (x_c = \frac{1}{r} \int_{A} \omega_{y} x dA, z_c = \frac{1}{r} \int_{A} \omega_{y} z dA) \) is roughly at a distance of \( 2.8 \mu m \) from the electrode edge and \( 5.8 \mu m \) from the bottom. The circulation of the vortex is about \( 1449 \mu m^2/s \). The vorticity distribution reveals that vorticity is generated near the electrode edge and dissipated towards the top wall and electrode centre. These results demonstrate that ACEO yields strong circulations above the electrode surface, which could be exploited for heat-transfer enhancement.

Figure 6. Vorticity and circulation at voltage of \( 2V_{PP} \) and frequency of \( 1500Hz \), the red solid line indicates the boundary of the ACEO vortex, \( \lambda_2 = -10 \).

4.4. Estimation of the heat-transfer rate

Heat-transfer enhancement by ACEO flow is estimated by way of the Reynolds-Colburn analogy. To this end, the wall shear rate \( \dot{\gamma} \) is calculated above the bottom surfaces as a function of frequency at \( 2V_{PP} \) and \( 4V_{PP} \) (Fig.7). The shear rate exhibits considerable and essentially nonlinear variation along the electrode. For instance, at a voltage of \( 2V_{PP} \) and a frequency of \( 1500Hz \), its magnitude reaches a distinct peak just off the electrode edge at \( x = 0.5 \mu m \), \( \dot{\gamma} \approx 17s^{-1} \), then decreases significantly and finally vanishes at \( 20 \mu m \). This variation of wall shear rate is due to the fact that the ACEO induced slip velocity is higher than the surrounding fluid nearby the electrode edge and becomes smaller with distance at downstream as it falls significant along the surface. For \( 4V_{PP} \) the magnitude increases significantly and the region of
high shear rates becomes more spread out. Comparing the wall shear rate at the frequencies from 600 Hz to 3000 Hz, the profile of $\dot{\gamma}$ is basically the same, except that the maximum value decrease from 25 s$^{-1}$ to 11 s$^{-1}$ at 2VPP and from 53 s$^{-1}$ to 31 s$^{-1}$ at 4VPP, respectively.

![Figure 7](image1.png)  
**Figure 7.** Wall shear rate $\dot{\gamma}$ as a function of frequency at 2VPP and at 4VPP, respectively.

The mean wall shear rate, $\frac{1}{L} \int_0^L |\dot{\gamma}| \, dx$, as a function of frequency at 2VPP and 4VPP is shown in Fig.8. This reveals a variation by about a factor of approximately 3.4 and 2.5 at 2VPP and at 4VPP. Increasing the voltage from 2VPP up to 4VPP, the mean wall shear rate increases approximately by a factor of 2.7 – 3.6 at the fixed frequency.

![Figure 8](image2.png)  
**Figure 8.** Mean wall shear rate versus the frequency and amplitude of applied voltage.

![Figure 9](image3.png)  
**Figure 9.** Mean Nu number versus the frequency and amplitude of applied voltage.

When ACEO flow is induced by the electric field, the electric current in the fluid generates internal Joule heating in the fluid directly above the electrode surfaces. The measured conductivity of the 0.1 mM KOH solution is 15 $\mu$S/cm and the RMS electric field strength is about 0.14 MV/m (corresponding to an AC voltage of 4Vpp applied over a gap of 10$\mu$m). The heat generation per unit volume can then be estimated by $W_J = \sigma E_{rms}^2$ [6], and is about $3 \times 10^7$ W/m$^3$. This allows us to estimate a temperature rise in the following manner. If the fluid is stagnant and the temperature field quickly attains a steady-state condition at AC voltage, the heat equation can be simplified to $k \nabla^2 T + \sigma E_{rms}^2 = 0$, where $k$ is the thermal conductivity of the solution (for water $k = 0.58 W/(m \cdot K)$) [6, 25]. An order of magnitude estimate of the temperature rise then follows from $\Delta T \approx \frac{W_{rms}^2}{k}$, where $V_{rms}$ is the RMS voltage. Its value is estimated to be $\Delta T \approx 5 \times 10^{-3} K$, meaning that the Joule heating can be neglected.

The mean Nu number is estimated by the Reynolds-Colburn analogy, using the mean velocity as the characteristic velocity. Fig.9 shows the mean Nu number evaluated as a function of frequency.
frequency at voltages of $2V_{PP}$ and $4V_{PP}$. The dependence of $Nu$ on the frequency is the same as of mean wall shear rate. The maximum of $Nu$ is at the characteristic frequency, and becomes smaller when the frequency is low compared to the characteristic frequency. For all frequencies in the experiment, it reveals a Nusselt number $Nu > 10$, where $Nu \approx 10$ corresponds to a thermally developing laminar flow in the rectangular microchannels at a high $Re$ number ($Re \sim 10^5$) [23, 24]. It thus demonstrates that the ACEO flow can significantly enhance the heat transfer in the microchannels.

5. Conclusion
In this paper, we experimentally investigated the potential of heat-transfer enhancement due to convective transport via AC electro-osmosis (ACEO) flow. To this end the properties of the ACEO flow have been analyzed using 3D micro-Particle-Tracking Velocimetry (3D μ-PTV) setup. Through the velocity field, the strength and position of ACEO-induced vortices have been quantified. This revealed that pairs of counter-rotating vortices form above each electrode setup. Through the velocity field, the strength and position of ACEO-induced vortices have been analyzed using 3D micro-Particle-Tracking Velocimetry (3D μ-PTV).

The heat-transfer rate has been estimated from the wall shear rate. This shear rate has been quantified. This revealed that pairs of counter-rotating vortices form above each electrode. The heat-transfer rate has been estimated from the wall shear rate. This shear rate varies considerably with frequency and amplitude of applied voltage. The mean wall shear rate is found to increase approximately by a factor of 2.7 – 3.6 if the voltage increases from $2V_{PP}$ to $4V_{PP}$. The Reynolds-Colburn analogy yields a mean Nusselt number, $Nu > 10$, signifying that the ACEO flow induces higher heat transfer rates at the channel bottom compared to the laminar flow in the microchannel. This work thus demonstrates that the heat transfer in micro-flows can in principle be enhanced via ACEO flow.

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