Spectrum of Eleven-dimensional Supergravity on a PP-wave Background

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Abstract

We calculate the spectrum of the linearized supergravity around the maximally supersymmetric pp-wave background in eleven dimensions. The resulting spectrum agrees with that of zero-mode Hamiltonian of a supermembrane theory on the pp-wave background. We also discuss the connection with the Kaluza-Klein zero modes of $AdS_4 \times S^7$ background.

Keywords: supergravity, pp-wave background, spectrum
1 Introduction

Recently, superstring and supermembrane on pp-wave backgrounds are focused upon. It was shown in [1] that the type IIB superstring theory on the maximally supersymmetric pp-wave background [2] is exactly solvable, and the spectrum of type IIB string theory was compared with the result calculated from the viewpoint of the linearized supergravity around the pp-wave background [3].

In addition, a matrix model on maximally supersymmetric pp-wave background was proposed in [4]. By using the matrix regularization [5], this matrix model is closely related to the supermembrane theory on pp-waves, see [8] or [9].) In the paper [10] the spectrum of the zero-mode Hamiltonian in the supermembrane theory on the pp-wave background was obtained by following the method of [5] and solving the system quantum mechanically. Motivated by the work [10], we calculate the spectrum of the linearized supergravity around the maximally supersymmetric pp-wave solution in eleven dimensions [11]. In the flat case, it is known that the spectrum of the zero-mode Hamiltonian corresponds to that of the linearized supergravity around Minkowski spacetime, and hence such a correspondence should be shown in the case of pp-wave. In this scenario, we will see that the spectrum is given by $(1 + 28 + 35) \times 2 (= 128)$ for graviton and three-form gauge field (bosons), and $(8 + 56) \times 2 (= 128)$ for gravitino (fermions), and then that the spectrum obtained from the linearized supergravity agrees with that of the zero-mode Hamiltonian. Notably, the resulting spectrum is identical with the Kaluza-Klein zero-modes in the eleven-dimensional supergravity on seven-sphere $S^7$ [12] as noted in [10].

This paper is organized as follows: In section 2 we prepare classical field equations in eleven-dimensional supergravity on the maximally supersymmetric pp-wave background. The Hamiltonian of Klein-Gordon type field equations for the fluctuation fields is also discussed. In section 3, we derive linearized field equations for fluctuation fields around the pp-wave background and calculate the spectrum of graviton, three-form gauge field and gravitino. The resulting spectrum agrees with that of the zero-mode Hamiltonian in the supermembrane theory on pp-wave background obtained in [10]. We find that the spectrum corresponds to the Kaluza-Klein zero-modes in the supergravity on $S^7$ [12]. Section 4 is devoted to conclusion and discussion. In Appendix A, our notation and convention are summarized.

2 Setup

We are interested in the spectrum of the linearized supergravity around the maximally supersymmetric pp-wave background (often called Kowalski-Glikman (KG) background [11]). In order to carry out our
calculation, we shall introduce the classical equations of motion in eleven-dimensional supergravity. For the later convenience, we also discuss the energy eigenvalue of the Hamiltonian of the fluctuation fields.

Eleven-dimensional supergravity consists of metric $g_{MN}$ (or vielbein $e_M$), three-form gauge field $C_{MNP}$ and gravitino $\Psi_M$ as dynamical fields. Equations of motion for these fields are\(^1\)

\[ 0 = \frac{1}{2} g_{MN} \mathcal{R} - \mathcal{R}_{MN} - \frac{1}{96} g_{MN} F_{PQRS} F^{PQRS} + \frac{1}{12} F_{MPQR} F_N^{PQR}, \quad (2.1a) \]

\[ 0 = \tilde{\Gamma}^{MNP} D_N \Psi_P - \frac{1}{96} \tilde{\Gamma}^{MNPQRS} \Psi_N F_{PQRS}, \quad (2.1b) \]

\[ 0 = \nabla^Q \{ e F_{QMN} \} + \frac{18}{(144)^2} g_{MZ} g_{NK} g_{PL} \epsilon_{ZKLQRSU} W_{X} Y F_{QRS} F_{VWXY}, \quad (2.1c) \]

where we wrote down the equation of motion in terms of $g_{MN}$. We also neglected the terms derived from torsion, quadratic and higher order terms with respect to gravitino. Such higher order terms do not contribute to field equations of fluctuation modes in the first-order approximation.

Now let us discuss the Hamiltonian and its energy eigenvalue. We need to calculate and solve field equations for fluctuation modes around the KG background (for the KG background, see Appendix A.3) in the next section. Then we will encounter Klein-Gordon type equations of motion and have to evaluate its energy spectrum.

We shall consider a Klein-Gordon type equation of motion for a field $\phi(x)$:

\[ (\Box - \alpha \mu i \partial_-) \phi(x^+, x^-, x^I) = 0, \quad (2.2) \]

where $\alpha$ is an arbitrary constant and $x^+$ is an evolution parameter. The d’Alembertian $\Box$ on the KG background is given by

\[ \Box = -\frac{1}{\sqrt{-g}} \partial_M (\sqrt{-g} g^{MN} \partial_N) = 2 \partial_+ \partial_- + G_{++}(\partial_-)^2 - (\partial_K)^2. \quad (2.3) \]

The above Klein-Gordon type field equation will appear later as equations of motion of fluctuation modes. Fourier transformed expression of $\phi(x)$

\[ \phi(x^+, x^-, x^I) = \int \frac{dp_- d^3 p_I}{\sqrt{(2\pi)^{10}}} e^{i(p_- x^- + p_I x^I)} \tilde{\phi}(x^+, p_-, p_I) \]

leads to the following expression:

\[ 0 = 2p_- i \partial_+ + \tilde{G}_{++}(p_-)^2 + (p_I)^2 + \alpha \mu p_-, \quad (2.4) \]

where $\tilde{G}_{++}$ is defined as

\[ \tilde{G}_{++} \equiv \sum_{I=1}^3 \left( \frac{\mu}{3} \right)^2 (\partial_{p_I})^2 + \sum_{I'=4}^9 \left( \frac{\mu}{6} \right)^2 (\partial_{p_{I'}})^2. \quad (2.5) \]

\(^1\)The convention and field contents on supergravity action is denoted in Appendix A.
By rewriting the above equation and \( H = i \partial_+ \), we can obtain the explicit expression of Hamiltonian:

\[
H = \frac{1}{-2p_-} \{ (p_I)^2 - \tilde{G}_{++}^{-1}(p_-)^2 + \alpha \mu p_- \}. \tag{2.6}
\]

The energy spectrum of this Hamiltonian can be derived by using the standard technique of harmonic oscillators. Now we define “creation/annihilation” operators

\[
a^\tilde{I} \equiv \frac{1}{\sqrt{2m}} \{ p_{\tilde{I}} + \tilde{m} \partial_{p_{\tilde{I}}} \}, \quad \tilde{a}^\tilde{I} \equiv \frac{1}{\sqrt{2m}} \{ p_{\tilde{I}} - \tilde{m} \partial_{p_{\tilde{I}}} \}, \quad \tilde{m} \equiv -\frac{1}{3} \mu p_-, \tag{2.7a}
\]

\[
a^{I'} \equiv \frac{1}{\sqrt{2m'}} \{ p_{I'} + m' \partial_{p_{I'}} \}, \quad a^{I'} \equiv \frac{1}{\sqrt{2m'}} \{ p_{I'} - m' \partial_{p_{I'}} \}, \quad m' \equiv -\frac{1}{6} \mu p-, \tag{2.7b}
\]

whose commutation relations are represented by

\[
[a^{\tilde{I}}, \tilde{a}^\tilde{J}] = \delta^{\tilde{I}\tilde{J}}, \quad [a^{I'}, \tilde{a}^{I'}] = \delta^{I' I''}, \quad [a^{\tilde{I}}, \tilde{a}^{I'}] = [a^{I'}, \tilde{a}^{\tilde{I}}] = 0.
\]

Thus we express the Hamiltonian in terms of the above oscillators:

\[
H = \frac{1}{3} \mu \sum_{\tilde{I}} \tilde{a}^{\tilde{I}} \tilde{a}^{\tilde{I}} + \frac{1}{6} \mu \sum_{I'} a^{I'} a^{I'} + \frac{1}{2} \mu (2 - \alpha). \tag{2.8}
\]

Note that the last term implies the zero-mode energy \( E_0 \) of the system, which is represented by

\[
E_0 = \frac{1}{2} \mu \mathcal{E}_0(\phi), \quad \mathcal{E}_0(\phi) = 2 - \alpha. \tag{2.9}
\]

In the next section, we will use \( \mathcal{E}_0 \) to evaluate the energy of the zero-modes of fluctuation fields.

## 3 Spectrum on PP-wave Background

In this section we discuss the spectrum of bosonic and fermionic fields in eleven-dimensional supergravity on the pp-wave background.

Let us derive field equations for fluctuation fields. Fluctuation fields are expanded around classical fields as follows:

\[
g_{MN} \rightarrow g_{MN} + h_{MN}, \quad \Psi_M \rightarrow 0 + \psi_M, \quad C_{MNP} \rightarrow C_{MNP} + C_{MNP}, \tag{3.1}
\]

where \( g_{MN} \) is the Kowalski-Glikman (KG) metric on the pp-wave background and \( C_{MNP} \) satisfies

\[
4 \partial_+ C_{123} = F_{+123} = \mu. \quad \text{Substituting (3.1) into classical field equations (2.1), we obtain linearized field equations for fluctuation fields around the pp-wave background:}
\]

\[
0 = \frac{1}{2} g_{MN} \{ h^{PQ} R_{PQ} - \nabla^P \nabla^Q h_{PQ} + \nabla^P \nabla_P h_{QQ} \}
- \frac{1}{2} \{ \nabla^P \nabla_M h_{NP} + \nabla^P \nabla_N h_{MP} - \nabla_M \nabla_N h_P^P - \nabla^P \nabla_P h_{MN} \}
\]

3
\[-\frac{1}{24}g_{MN}\{2F^{PQRS}_M \partial_P C_{QRS} - F_{PQRS} F_{UQR}^P h_{PU}\} + \frac{1}{3}F_{M}^{PQR} \partial_{[N} C_{PQR]} + \frac{1}{3}F_{N}^{PQR} \partial_{[M} C_{PQR]} - \frac{1}{4}F_{MPQR} F_{NUQR}^P h_{PU}, \quad (3.2a)\]

\[0 = \tilde{\Gamma}^{MNP} D_N \psi_P - \frac{1}{4} \tilde{\Gamma}^{MN+123} \psi_N - \frac{1}{4} \mu \left\{ g^{M+} (\tilde{\Gamma}^{12} g^{3N} + \tilde{\Gamma}^{13} g^{2N}) - g^{M1} (\tilde{\Gamma}^{23} g^{+N} + \tilde{\Gamma}^{31} g^{+2N}) + g^{M2} (\tilde{\Gamma}^{+3} g^{1N} + \tilde{\Gamma}^{+1} g^{13N}) - g^{M3} (\tilde{\Gamma}^{+12} g^{2N} + \tilde{\Gamma}^{+2} g^{3N}) \right\} \psi_N, \quad (3.2b)\]

\[0 = 4g^{QR}\{ \partial_R \partial_{[Q} C_{MNP]} - \Gamma^{S}_{RQ} \partial_{[S} C_{MNP]} - \Gamma^{S}_{RM} \partial_{[Q} C_{SNP]} - \Gamma^{S}_{RN} \partial_{[Q} C_{MSP]} - \Gamma^{S}_{RP} \partial_{[Q} C_{MNS]} \} - \frac{1}{2} g^{QR}\left\{ F_{SMNP} (\nabla_R h^Q S + \nabla_Q h^R S - \nabla^S h_{RQ}) + F_{QSNP} (\nabla_R h^M S + \nabla_M h^R S - \nabla^S h_{RM}) + F_{QMSP} (\nabla_R h^N S + \nabla_N h^R S - \nabla^S h_{RN}) + F_{QMNS} (\nabla_R h^P S + \nabla_P h^R S - \nabla^S h_{RP}) \right\} + \frac{1}{144} g_{MZ} g_{NK} g_{PL} \epsilon^{ZKQRS_{UVWX}Y} F_{QRUS} \partial_V C_{WXY}. \quad (3.2c)\]

In the following subsection we will solve the above equations for the fluctuations by using some gauge-fixing conditions.

### 3.1 Physical Modes of Bosonic Fields

Here the bosonic spectrum is derived by using the light-cone gauge-fixing: \( h_{-M} = C_{-MN} = 0 \). All we have to do is to solve various physical and unphysical modes from fluctuation field equations.

First, we find a traceless condition

\[0 = h_{II} \quad (3.3)\]

from the \((-\cdot)\) component of (3.2a). Substituting (3.3) into the \((-I)\) component of (3.2a) leads to

\[h_{I+} = \frac{1}{\partial_-} \partial_J h_{IJ}, \quad (3.4)\]

and so we see that \( h_{I+} \) is non-dynamical. In the same way, we can read off the following condition from the \((+ - I)\) component of (3.2a): \( \partial_J C_{+IJ} = 0 \). The \((-IJ)\) component of (3.2c) leads to the expression for the field \( C_{+IJ} \):

\[C_{+IJ} = \frac{1}{\partial_-} \partial_K C_{IJK}, \quad (3.5)\]

and hence we see that \( C_{+IJ} \) is also non-dynamical. Under the above conditions, the trace of (3.2a) gives an expression of a non-dynamical field \( h_{++} \):

\[h_{++} = \frac{1}{(\partial_-)^2} \partial_I \partial_J h_{IJ} - \frac{1}{3 \partial_-} \mu C_{123}. \quad (3.6)\]
In contrast to the IIB supergravity case \[3\], \(h_{++}\) includes the term proportional to \(\mu\). The appearance of this term is characteristic of our case \[2\].

By the use of the light-cone gauge-fixing conditions and the above-mentioned conditions for the non-dynamical modes, we can reduce field equations for \(h_{MN}\) and \(C_{MNP}\) as follows:

\[
\begin{align*}
(\tilde{I}\tilde{J}) \text{ component of } (3.2a) : 0 &= \Box h_{\tilde{I}\tilde{J}} + \frac{2}{3}\mu \delta_{\tilde{I}\tilde{J}} \partial_- C, \\
(\tilde{I}\tilde{J}') \text{ component of } (3.2a) : 0 &= \Box h_{\tilde{I}\tilde{J}'} + \mu \partial_- C_{\tilde{I}\tilde{J}'}, \\
(I'J') \text{ component of } (3.2a) : 0 &= \Box h_{I'J'} - \frac{1}{3}\mu \delta_{I'J'} \partial_- C,\end{align*}
\]

\[
\begin{align*}
(\tilde{I}\tilde{J}K) \text{ component of } (3.2a) : 0 &= \Box C - 2\mu \partial_- h_{\tilde{I}\tilde{J}}, \\
(\tilde{I}\tilde{J}K') \text{ component of } (3.2a) : 0 &= \Box C_{\tilde{I}\tilde{J}K'}, \\
(\tilde{I}'J'K') \text{ component of } (3.2a) : 0 &= \Box C_{\tilde{I}'J'K'} - \frac{1}{6}\mu \varepsilon^{\tilde{I}'\tilde{J}'\tilde{K}'\tilde{W}'\tilde{X}'\tilde{Y}'} \partial_- C_{\tilde{W}'\tilde{X}'\tilde{Y}'}, \end{align*}
\]

where \(\varepsilon^{\tilde{I}'\tilde{J}'\tilde{K}'\tilde{W}'\tilde{X}'\tilde{Y}'}\) is the \(SO(6)\) Levi-Civita symbol whose normalization is \(\varepsilon^{456789}_{456789} = 1\). Note that we wrote the above equations in terms of the following two quantities defined by

\[
C_{\tilde{I}\tilde{J}} \equiv \frac{1}{2}\varepsilon^{\tilde{I}\tilde{K}\tilde{L}} C_{\tilde{K}\tilde{L}J}, \quad C \equiv 2C_{123},
\]

where we introduced the \(SO(3)\) Levi-Civita symbol \(\varepsilon_{\tilde{I}\tilde{J}\tilde{K}} (\varepsilon_{123} = \varepsilon^{123} = 1)\).

Now let us solve the above reduced equations of motion for fluctuation modes, and derive the zero-mode energy spectrum and degrees of freedom of bosonic fields.

First we consider the field \(C_{\tilde{I}\tilde{J}K'}\). From the above equation \(3.7d\), we find that this field does not couple to the other fields. So the zero-mode energy \(\mathcal{E}_0(C_{\tilde{I}\tilde{J}K'})\) and degrees of freedom \(\mathcal{D}(C_{\tilde{I}\tilde{J}K'})\) are given by

\[
\mathcal{E}_0(C_{\tilde{I}\tilde{J}K'}) = 2, \quad \mathcal{D}(C_{\tilde{I}\tilde{J}K'}) = 45. \tag{3.8}
\]

Next, we consider \(SO(3) \times SO(6)\) tensor fields \(h_{\tilde{I}\tilde{J}}\) and \(C_{\tilde{I}\tilde{J}}\) coupled to each other. In order to diagonalize these coupled fields, we define two complex fields \(H_{\tilde{I}\tilde{J}}\) and \(\overline{H}_{\tilde{I}\tilde{J}}\) as

\[
H_{\tilde{I}\tilde{J}} = h_{\tilde{I}\tilde{J}} + iC_{\tilde{I}\tilde{J}}, \quad \overline{H}_{\tilde{I}\tilde{J}} = h_{\tilde{I}\tilde{J}} - iC_{\tilde{I}\tilde{J}}. \tag{3.9}
\]

By using these fields, \(3.7d\) and \(3.7c\) can be rewritten as

\[
0 = (\Box - \mu i\partial_-)H_{\tilde{I}\tilde{J}}, \quad 0 = (\Box + \mu i\partial_-)\overline{H}_{\tilde{I}\tilde{J}}. \tag{3.10}
\]

The spectrum of type IIA string theory and linearized supergravity is studied in \[13\]. In this case \(h_{++}\) contains the additional term proportional to \(\mu\).
Thus the zero-mode energies and degrees of freedom of $H_{I^I}$ and $\overline{H}_{I^I}$ are given by

$$E_0(H_{I^I}) = 1, \quad E_0(\overline{H}_{I^I}) = 3, \quad D(H_{I^I}) = D(\overline{H}_{I^I}) = 18. \quad (3.11)$$

Then we will solve the field equations (3.7a), (3.7c) and (3.7d) concerning $h_{I^I}$, $h_{I^I'}$ and $C$. Since these fields are coupled to one another, we have to diagonalize these fields in order to solve the equations. Hence let us introduce the following fields:

$$h_{I^I}^\perp \equiv h_{I^I} - \frac{1}{3} \delta_{I^I} h_{K^K}, \quad h_{I^I'}^\perp \equiv h_{I^I'} - \frac{1}{6} \delta_{I^I'} h_{K^K'}, \quad (3.12a)$$

$$h \equiv h_{I^I} + iC, \quad \overline{h} \equiv h_{I^I} - iC. \quad (3.12b)$$

Note that $h_{I^I}^\perp$ and $h_{I^I'}^\perp$ are transverse modes and two complex scalar fields $h$ and $\overline{h}$ are trace modes. In this re-definition we find $\Box h_{I^I}^\perp = 0$, and so its energy and degrees of freedom are given by

$$E_0(h_{I^I}^\perp) = 2, \quad D(h_{I^I}^\perp) = 5. \quad (3.13)$$

Since we also find $\Box h_{I^I'}^\perp = 0$, we obtain the energy and degrees of freedom of zero-mode $h_{I^I'}^\perp$:

$$E_0(h_{I^I'}^\perp) = 2, \quad D(h_{I^I'}^\perp) = 20. \quad (3.14)$$

Similarly the field equations for $h$ and $\overline{h}$ are described by

$$(\Box - 2\mu i\partial_-)h = 0, \quad (\Box + 2\mu i\partial_-)\overline{h} = 0. \quad (3.15)$$

Thus the energies and degrees of freedom of them are

$$E_0(h) = 0, \quad E_0(\overline{h}) = 4, \quad D(h) = D(\overline{h}) = 1. \quad (3.16)$$

Finally we consider (3.15) by decomposing $C_{I^I',K'}$ into self-dual part and anti-self-dual part as follows: $C_{I^I',K'} = C_{I^I',K'}^{\oplus} + C_{I^I',K'}^{\ominus}$, where $C_{I^I',K'}^{\oplus}$ is a self-dual part and $C_{I^I',K'}^{\ominus}$ is an anti-self-dual part. These are defined by, respectively,

$$C_{I^I',K'}^{\oplus} = \frac{i}{3!} \varepsilon_{I^I'J^J'K'W^W'X^X'Y^Y'} C_{W^W',X^X',Y^Y'}^{\oplus} \quad C_{I^I',K'}^{\ominus} = -\frac{i}{3!} \varepsilon_{I^I'J^J'K'W^W'X^X'Y^Y'} C_{W^W',X^X',Y^Y'}^{\ominus}. \quad (3.17)$$

Due to this decomposition, the field equations of them are expressed as

$$(\Box + \mu i\partial_-)C_{I^I',K'}^{\oplus} = 0, \quad (\Box - \mu i\partial_-)C_{I^I',K'}^{\ominus} = 0, \quad (3.18)$$

and hence we find the energies and degrees of freedom of $C_{I^I',K'}^{\oplus}$ and $C_{I^I',K'}^{\ominus}$:

$$E_0(C_{I^I',K'}^{\oplus}) = 3, \quad E_0(C_{I^I',K'}^{\ominus}) = 1, \quad D(C_{I^I',K'}^{\oplus}) = D(C_{I^I',K'}^{\ominus}) = 10. \quad (3.19)$$
Now we have fully solved the field equations for bosonic fluctuations and have derived the spectrum of graviton and three-form gauge field. The resulting spectrum is splitting with a certain energy difference in contrast to the flat case. The resulting spectrum is completely identical with that of zero-mode Hamiltonian in the supermembrane theory on the KG background when we take account of the difference between $E_0$ and $E_0$. We conclude this subsection by summarizing the spectrum of bosonic fields in Table 1:

| energy $\mathcal{E}_0$ | bosonic fields $(D)$ | degrees of freedom |
|------------------------|----------------------|--------------------|
| 4                      | $\bar{h}(1)$         | 1                  |
| 3                      | $\bar{H}_{\bar{i},\bar{j}}(18)$, $\bar{C}_{\bar{i},\bar{j}'}K'(10)$ | 28                 |
| 2                      | $\bar{C}_{\bar{i},\bar{j}'K'}(45)$, $\bar{h}_{\bar{i},\bar{j}}(5)$, $\bar{h}_{\bar{i},\bar{j}}(20)$ | 70                 |
| 1                      | $\bar{H}_{\bar{i},\bar{j}}(18)$, $\bar{C}_{\bar{i},\bar{j}'}K'(10)$ | 28                 |
| 0                      | $h(1)$               | 1                  |

Table 1: Bosonic zero-modes in eleven-dimensional supergravity on pp-wave background.

### 3.2 Physical Modes of Fermionic Fields

In this subsection let us solve the field equations of the fluctuations of gravitino by imposing the light-cone gauge-fixing: $\psi_- = 0$. We will study physical modes of gravitino and obtain its spectrum.

First, we consider the $M = -$ component of (3.2b), which is rewritten as

$$\hat{\Gamma}^N D_- \psi_N = \frac{1}{9} \hat{\Gamma}_- \hat{\Gamma}^N J^N,$$

where $J^N$ is defined as

$$\hat{\Gamma}^{MNP} D_N \psi_P \equiv J^M, \quad (3.21a)$$

$$J^+ = -J_- = 0, \quad (3.21b)$$

$$J^- = -\frac{1}{4} \mu \hat{\Gamma}^{+-123} \psi_P - \frac{1}{2} \mu \hat{\epsilon}^\gamma_{\bar{i},\bar{j}K} \hat{\Gamma}_{\bar{i},\bar{j}} \psi_K, \quad (3.21c)$$

$$J^\tilde{i} = J_{\tilde{i}} = \frac{1}{4} \mu \hat{\Gamma}^{+123} (\delta_{\tilde{i},\bar{j}} - \hat{\Gamma}_{\tilde{i},\bar{j}}) \psi_{\bar{j}}, \quad (3.21d)$$

$$J^\prime = J_{\prime} = -\frac{1}{4} \mu \hat{\Gamma}^{+123} (\delta_{\prime,\bar{j}} - \hat{\Gamma}_{\prime,\bar{j}}) \psi_\bar{j}. \quad (3.21e)$$

Then we can simplify as $\hat{\Gamma}^N D_- \psi_N = \partial_- (\hat{\Gamma}^N \psi_N)$, and find that $\partial_- (\hat{\Gamma}^N \psi_N)$ vanishes. Thus we obtain

$$\hat{\Gamma}^N \psi_N = 0. \quad (3.22)$$
Next, we consider the $M = +$ component of (3.2b), which can be rewritten as

$$0 = g^{P+}\hat{\Gamma}^N D_N\psi_P - g^{PN}\hat{\Gamma}^+ D_N\psi_P + \frac{1}{2}(\hat{\Gamma}^+\hat{\Gamma}^N - \hat{\Gamma}^N\hat{\Gamma}^+)\hat{\Gamma}^P D_N\psi_P.$$  (3.23)

Then the first and third term is deleted by the light-cone gauge-fixing, and we can reduce (3.23) to

$$0 = \hat{\Gamma}^+(-\partial_+\psi_+ + \partial_I\psi_I).$$

Thus $\psi_+$ can be expressed as

$$\psi_+ = \frac{1}{\partial_-}\partial_I\psi_I,$$  (3.24)

and we see that $\psi_+$ is a non-dynamical field.

Here we shall reduce the $M = \tilde{I}$ component of (3.2b) to

$$0 = \hat{\Gamma}^+(\partial_+ + \frac{1}{2}G_{++}\partial_-)\psi^\oplus_I + \hat{\Gamma}^-\partial_-\psi^\ominus_I + \hat{\Gamma}^K\partial_K(\psi^\oplus_I + \psi^\ominus_I) - \frac{1}{4}\mu\hat{\Gamma}^{i\bar{j}3}(\delta_{i\bar{j}} - \hat{\Gamma}_{i\bar{j}}\hat{\Gamma}^j)\psi^\ominus_I,$$  (3.25)

where we decomposed gravitino as $\psi_I = \psi^\oplus_I + \psi^\ominus_I$. The $\psi^\oplus_I$ and $\psi^\ominus_I$ are defined as

$$\psi^\oplus_I = -\frac{1}{2}\hat{\Gamma}^-\hat{\Gamma}^+\psi_I, \quad \psi^\ominus_I = \frac{1}{2}\hat{\Gamma}^+\hat{\Gamma}^-\psi_I,$$  (3.26)

which satisfy the projection conditions: $\hat{\Gamma}^-\psi^\oplus_I = \hat{\Gamma}^+\psi^\ominus_I = 0$. When we act $\hat{\Gamma}^+$ on (3.25) from the left, $\psi^\ominus_I$ can be expressed in terms $\psi^\oplus_I$ as follows:

$$\psi^\ominus_I = \frac{1}{2\partial_-}\hat{\Gamma}^+\hat{\Gamma}^K\partial_K\psi^\oplus_I.$$  (3.27)

Thus $\psi^\ominus_I$ is not independent of $\psi^\oplus_I$. Similarly, when we act $\hat{\Gamma}^-$ on (3.25) from the left and utilize (3.27), we obtain the following equation:

$$0 = \Box\psi^\ominus_I - \frac{1}{2}\mu\hat{\Gamma}^{i\bar{j}3}(\delta_{i\bar{j}} - \hat{\Gamma}_{i\bar{j}}\hat{\Gamma}^j)\partial_-\psi^\ominus_I.$$  (3.28)

In order to solve this equation, we shall introduce the following fields:

$$\psi^\ominus_I = (\delta_{i\bar{j}} - \frac{1}{3}\hat{\Gamma}^i\tilde{\Gamma}^j_\bar{j})\psi^\ominus_I, \quad \psi^\parallel_I = \hat{\Gamma}^I\psi^\ominus_I = \hat{\Gamma}^I\tilde{\psi}^\ominus_I,$$  (3.29)

and decompose $\psi^\ominus_I$ in to the $\tilde{\Gamma}$-transverse mode and $\tilde{\Gamma}$-parallel mode. Acting $\hat{\Gamma}^I$ on (3.28) from the left and contracting the index $I$, we get

$$0 = \Box\psi^\parallel_I - \mu\hat{\Gamma}^{i\bar{j}3}\partial_-\psi^\parallel_I.$$  (3.30)

On the other hand, when we act $(\delta_{i\bar{j}} - \frac{1}{3}\hat{\Gamma}^i\tilde{\Gamma}^j_\bar{j})$ on (3.28), we find

$$0 = \Box\psi^\perp_K - \frac{1}{2}\mu\hat{\Gamma}^{i\bar{j}3}\partial_-\psi^\perp_K.$$  (3.31)
Moreover, in order to solve (3.30) and (3.31), we decompose \( \psi^{\perp}_{\perp} \) and \( \psi^{\parallel}_{\parallel} \) according to the chirality in terms of \( i\bar{\Gamma}^{123} \) as follows:

\[
\begin{align*}
\psi^{\perp}_{\perp} & \equiv \frac{1 + i\bar{\Gamma}^{123}}{2} \psi^{\perp}_{\perp}, & \psi^{\perp}_{\parallel} & \equiv \frac{1 - i\bar{\Gamma}^{123}}{2} \psi^{\perp}_{\parallel}, \\
\psi^{\parallel}_{\parallel} & \equiv \frac{1 + i\bar{\Gamma}^{123}}{2} \psi^{\parallel}_{\parallel}, & \psi^{\parallel}_{\perp} & \equiv \frac{1 - i\bar{\Gamma}^{123}}{2} \psi^{\parallel}_{\perp}.
\end{align*}
\]

These variables satisfy the following chirality conditions:

\[
\begin{align*}
i\bar{\Gamma}^{123} \psi^{\perp}_{\perp} & = +\psi^{\perp}_{\perp}, & i\bar{\Gamma}^{123} \psi^{\perp}_{\parallel} & = -\psi^{\perp}_{\parallel}, \\
i\bar{\Gamma}^{123} \psi^{\parallel}_{\parallel} & = +\psi^{\parallel}_{\parallel}, & i\bar{\Gamma}^{123} \psi^{\parallel}_{\perp} & = -\psi^{\parallel}_{\perp}.
\end{align*}
\]

Multiplying \( \frac{1}{2}(1 \pm i\bar{\Gamma}^{123}) \) to (3.30) on the left, we get

\[
0 = \left( \Box + \mu i \partial_\perp \right) \psi^{\parallel}_{\parallel}, \quad 0 = \left( \Box - \mu i \partial_\perp \right) \psi^{\parallel}_{\perp}.
\]

Similarly, when we multiply \( \frac{1}{2}(1 \pm i\bar{\Gamma}^{123}) \) to (3.30) on the left, we obtain

\[
0 = \left( \Box + \frac{1}{2} \mu i \partial_\perp \right) \psi^{\perp}_{\perp}, \quad 0 = \left( \Box - \frac{1}{2} \mu i \partial_\perp \right) \psi^{\perp}_{\parallel}.
\]

From these equations, we can read off the zero-mode energies and degrees of freedom of \( \psi^{\perp}_{\perp} \) and \( \psi^{\parallel}_{\perp} \):

\[
E_0(\psi^{\perp}_{\perp}) = \frac{5}{2}, \quad E_0(\psi^{\perp}_{\parallel}) = \frac{3}{2}, \quad D(\psi^{\perp}_{\perp}) = D(\psi^{\perp}_{\parallel}) = 8 \times (3 - 1) = 16.
\]

We will discuss these quantities of \( \psi^{\parallel}_{\parallel} \) and \( \psi^{\parallel}_{\perp} \) later.

Then let us investigate the \( M = I' \) component of (3.32):

\[
0 = \left\{ \bar{\Gamma}^+ \left( \partial_+ + \frac{1}{2} G_{++} \partial_- \right) + \bar{\Gamma}^- \partial_- + \bar{\Gamma}^K \partial_K \right\} \psi_{I'} + \frac{1}{4} \mu \bar{\Gamma}^{123} \left( \delta_{I' J'} - \bar{\Gamma}_{I'} \bar{\Gamma}_{J'} \right) \psi_{J'}.
\]

In the same way as the case of \( \psi_{I'} \), we decompose \( \psi_{I'} \) into the \( \bar{\Gamma} \)-parallel mode and \( \bar{\Gamma} \)-transverse mode, and obtain

\[
\begin{align*}
0 & = \left( \Box - \frac{5}{2} \mu i \partial_- \right) \psi^{\parallel}_{2R}, & 0 & = \left( \Box + \frac{5}{2} \mu i \partial_- \right) \psi^{\parallel}_{2L}, \\
0 & = \left( \Box + \frac{1}{2} \mu i \partial_- \right) \psi^{\perp}_{2R}, & 0 & = \left( \Box - \frac{1}{2} \mu i \partial_- \right) \psi^{\perp}_{2L},
\end{align*}
\]

where the \( \bar{\Gamma} \)-transverse mode and \( \bar{\Gamma} \)-parallel mode are defined as

\[
\begin{align*}
\psi^{\parallel}_{I'} & = -\frac{1}{2} \bar{\Gamma}^- \bar{\Gamma}^+ \psi_{I'}, & \psi^{\perp}_{I'} & = \frac{1 + i\bar{\Gamma}^{123}}{2} \psi^{\perp}_{I'}, \\
\psi^{\perp}_{I'R} & = \frac{1 + i\bar{\Gamma}^{123}}{2} \psi^{\perp}_{2R}, & \psi^{\parallel}_{I'L} & = \frac{1 - i\bar{\Gamma}^{123}}{2} \psi^{\parallel}_{2L}.
\end{align*}
\]
From (3.38b), we find that the zero-mode energies and degrees of freedom of are given by $\psi_{IR}^{\oplus\perp}$ and $\psi_{IL}^{\oplus\perp}$:

$$E_0(\psi_{IR}^{\oplus\perp}) = \frac{3}{2}, \quad E_0(\psi_{IL}^{\oplus\perp}) = \frac{5}{2}, \quad D(\psi_{IR}^{\oplus\perp}) = D(\psi_{IL}^{\oplus\perp}) = 8 \times (6 - 1) = 40.$$  \tag{3.40}

Now we have finished the study of the $\hat{\Gamma}$-transverse part. The remaining task is to analyze the $\hat{\Gamma}$-parallel mode. In order to investigate the $\hat{\Gamma}$-parallel mode, we perform a linear combination of (3.34) and (3.38a), and define new $\hat{\Gamma}$-parallel modes as

$$\psi_{IR}^{\parallel} \equiv 2\frac{5}{2} \psi_{1R}^{\parallel} - \psi_{2R}^{\parallel}, \quad \psi_{IL}^{\parallel} \equiv 2\frac{5}{2} \psi_{1L}^{\parallel} - \psi_{2L}^{\parallel}.$$  \tag{3.41}

Then, by the use of (3.24), we can easily see that the re-defined fermions satisfy the equations:

$$0 = \left(\Box - \frac{3}{2} \mu i \partial_+\right)\psi_{IR}^{\parallel}, \quad 0 = \left(\Box + \frac{3}{2} \mu i \partial_+\right)\psi_{IL}^{\parallel}.$$  \tag{3.42}

Thus the zero-mode energies and degrees of freedom of them are represented by

$$E_0(\psi_{IR}^{\parallel}) = \frac{1}{2}, \quad E_0(\psi_{IL}^{\parallel}) = \frac{7}{2}, \quad D(\psi_{IR}^{\parallel}) = D(\psi_{IL}^{\parallel}) = 8.$$  \tag{3.43}

Now we have fully solved the field equations for fermionic fluctuations, and have derived the spectrum of gravitino in the case of pp-wave. As a result, we have found that the spectrum is splitting with a certain energy difference in the same manner with the spectrum of bosons. Summarizing (3.36), (3.40) and (3.43), we obtain the spectrum of gravitino as in Table 2:

| energy $E_0$ | fermionic fields $(D)$ | degrees of freedom |
|-------------|------------------------|-------------------|
| $\frac{7}{2}$ | $\psi_{IR}^{\parallel}$ (8) | 8 |
| $\frac{5}{2}$ | $\psi_{IL}^{\parallel}$ (16) | 56 |
| $\frac{3}{2}$ | $\psi_{IR}^{\parallel}$ (16) $\psi_{IL}^{\parallel}$ (40) | 56 |
| $\frac{1}{2}$ | $\psi_{IR}^{\parallel}$ (8) | 8 |

Table 2: Fermionic zero-modes in eleven-dimensional supergravity on pp-wave background.

This spectrum completely agrees with that of the zero-mode Hamiltonian in the supermembrane theory on the KG background [10].

In conclusion, we have shown that the spectrum of the linearized supergravity around the KG background is completely identical with that of the zero-mode Hamiltonian in the supermembrane theory on this background [10]. It should be remarked that the spectrum obtained here is also identical with the Kaluza-Klein zero-modes in the supergravity on $S^7$. The $AdS_4 \times S^7$ background is related to
the KG background \cite{14} via the Penrose limit \cite{15}. Hence the spectrum on the KG background would have a connection with that on the $AdS_4 \times S^7$. In fact, the relationship of the spectra between the $AdS_4 \times S^7$ and KG background is discussed from the viewpoint of superalgebra \cite{16}.

4 Conclusion and Discussions

We have calculated the spectrum of the linearized supergravity around the maximally supersymmetric pp-wave background. As a result, we have found that the spectrum is given by $(1+28+35) \times 2 \ (=128)$ for graviton and three-form gauge field (bosons), and $(8+56) \times 2 \ (=128)$ for gravitino (fermions), and then that the spectrum obtained from the linearized supergravity agrees with that of the zero-mode Hamiltonian. Notably, the resulting spectrum is identical with the Kaluza-Klein zero-modes in the eleven-dimensional supergravity on seven-sphere $S^7$.

Our considerations in this paper have clarified the spectrum of the supergravity multiplet splitting with a certain energy difference. As an application of our results, it would be possible to calculate propagators and energy-momentum tensors of graviton, gauge field and gravitino according to our classification of matter contents by following the method of \cite{17}. In contrast to the flat case, the polarization tensor is non-trivially decomposed. Hence we need a concrete classification of physical fields in order to calculate the propagators and energy-momentum tensors. If the calculation of them has been completed, we can evaluate the scattering amplitudes in the linearized supergravity around the pp-wave background. It is one of the most important problems to calculate the scattering amplitude in the matrix model and to compare the result with the amplitudes obtained from the supergravity side. Thus, we believe that our work will make an important contribution to study in this direction. We will continue to study in this direction and calculate the propagators and energy-momentum tensors \cite{18}.

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Appendix

A Convention

In this appendix we summarize some conventions.

A.1 Convention of gamma matrices

We use the convention that $M, N, P, \cdots$ refer to eleven-dimensional world indices and $\dot{A}, \dot{B}, \dot{C}, \cdots$ refer to eleven-dimensional tangent space indices. The eleven-dimensional gamma matrices satisfy the following relations

$$\{\hat{\Gamma}^{\dot{A}}, \hat{\Gamma}^{\dot{B}}\} = 2\eta^{\dot{A}\dot{B}}, \quad (A.1a)$$

$$(\hat{\Gamma}^{\dot{A}})^\dagger = \hat{\Gamma}_\dot{A} = -\hat{\Gamma}^{\dot{0}}\hat{\Gamma}(\hat{\Gamma}^{\dot{0}})^{-1}, \quad (A.1b)$$

$$\hat{\Gamma}_{M_1M_2\cdots M_n} \equiv \hat{\Gamma}_{[M_1}\hat{\Gamma}_{M_2} \cdots \hat{\Gamma}_{M_n]} = \frac{1}{n!}\text{sgn}(\sigma)\hat{\Gamma}_{M_{\sigma(1)}}\hat{\Gamma}_{M_{\sigma(2)}} \cdots \hat{\Gamma}_{M_{\sigma(n)}}, \quad (A.1c)$$

where $\eta^\dot{A}\dot{B}$ is the tangent space metric. Irreducible spinors in eleven-dimensional tangent space are anti-commuting variables and satisfy the Majorana condition:

$$\bar{\Psi} = \Psi^T C, \quad (A.2)$$

where the charge conjugation matrix $C$ is antisymmetric and defined by

$$CT^{\dot{A}}C^{-1} = -(\hat{\Gamma}^{\dot{A}})^T, \quad (A.3a)$$

$$CT^{\dot{A}_1\cdots\dot{A}_{2n}}C^{-1} = -(\hat{\Gamma}^{\dot{A}_1\cdots\dot{A}_{2n}})^T, \quad (A.3b)$$

$$CT^{\dot{A}_1\cdots\dot{A}_{2n+1}}C^{-1} = (\hat{\Gamma}^{\dot{A}_1\cdots\dot{A}_{2n+1}})^T. \quad (A.3c)$$

A.2 Eleven-dimensional supergravity

Eleven-dimensional supergravity contains graviton, three-form gauge field and gravitino. The degrees of freedom is $128$ (bosons) + $128$ (fermions). These fields are described by

$$e_M^{\dot{A}} : \text{vielbein} , \quad E_A^{\dot{M}} : \text{inverse vielbein} , \quad (A.4a)$$

$$\Psi_M : \text{gravitino (Majorana spinor)} , \quad (A.4b)$$

$$C_{MNP} : \text{completely antisymmetric tensor} . \quad (A.4c)$$

The Lagrangian of eleven-dimensional supergravity (without torsion) is given by

$$\mathcal{L} = eR - \frac{1}{2} e\bar{\Psi}_M \hat{\Gamma}^{MNP} D_N \Psi_P - \frac{1}{48} e F_{MNPQ} F^{MNPQ}$$
where the covariant derivative for local Lorentz transformation is defined as

\[ D_N \Psi_P = \partial_N \Psi_P - \frac{1}{4} \omega_{NP} \hat{\Gamma}^{AB} \tilde{\Gamma}_{AB} \Psi_P. \]

We also introduced the following expressions:

\[ g_{MN} = e_M{}^A e_N{}^B \eta_{AB}, \quad \eta_{AB} = E_A{}^M E_B{}^N g_{MN}, \quad e = \det e_M{}^A, \quad \overline{\Psi} = i \Psi \hat{\Gamma}^0, \]

and \( \tilde{\Gamma}^{NPQR} \) and \( \tilde{\Gamma}^{MNPQRS} \) are written as

\[ \tilde{\Gamma}^{NPQR} = \tilde{\Gamma}^{NPQR} - 8 \delta_M{}^N \tilde{\Gamma}^{PQR}, \quad \tilde{\Gamma}^{MNPQRS} = \tilde{\Gamma}^{MNPQRS} + 12 g^{[P} \tilde{\Gamma}^{QR} g^{S]N}. \]

The eleven-dimensional Levi-Civita symbol, \( \varepsilon^{MNPQRSTUWXY} \) is normalized as

\[ \varepsilon^{012\cdots10} = 1. \]

The Lagrangian written above is invariant under the supersymmetry transformations (up to torsion):

\[ \delta e_M{}^A = -\frac{1}{2} \tilde{\Gamma}^A \Psi_M, \quad (A.6a) \]
\[ \delta \Psi_M = 2 D_M \varepsilon + \frac{1}{144} F_{NPQR} \tilde{\Gamma}^{NPQR} \Psi_M, \quad (A.6b) \]
\[ \delta C_{MNP} = -\frac{3}{2} \tilde{\Gamma}_{[MN} \Psi_P. \quad (A.6c) \]

### A.3 Kowalski-Glikman background

Here we will summarize several properties of the maximally supersymmetric pp-wave background. This solution was found by Kowalski-Glikman \(^{[11]}\) and often called the KG solution. This is the unique pp-wave type solution preserving maximal supersymmetries. The metric of this background is given by

\[ ds^2 = -2dx^+dx^- + G_{++}(dx^+)^2 + \sum_{I=1}^9 (dx^I)^2, \quad (A.7) \]
\[ G_{++} = -\left[ \left( \frac{\mu}{3} \right)^2 \sum_{I=1}^3 (x^I)^2 + \left( \frac{\mu}{6} \right)^2 \sum_{I'=4}^9 (x^{I'})^2 \right], \]

which is equipped with the constant flux

\[ \mu = F_{+123} \neq 0. \]
Here the light-cone coordinates are defined as
\[ x^\pm = \frac{1}{\sqrt{2}} (x^0 \pm x^1) . \]

Affine connection \( \Gamma^P_{MN} \), Riemann curvature tensor \( R^R_{PMN} \) and spin connection \( \omega^A_{\dot{M} \dot{B}} \) are defined by
\[
\begin{align*}
\Gamma^P_{MN} &= \frac{1}{2} g^{PR} (\partial_M g_{NR} + \partial_N g_{MR} - \partial_R g_{MN}) , \\
R^R_{PMN} &= \partial_M \Gamma^R_{PN} - \partial_N \Gamma^R_{PM} + \Gamma^R_{QM} \Gamma^Q_{PN} - \Gamma^R_{QN} \Gamma^Q_{PM} , \\
\omega^A_{\dot{M} \dot{B}} &= -\eta^{\dot{A} \dot{C}} \epsilon^N \{ \partial_M e_N^\dot{A} - \Gamma^P_{NM} e^\dot{P} \} .
\end{align*}
\]

In our consideration the contribution from torsion is not included, i.e., affine connection is symmetric under lower indices: \( \Gamma^P_{MN} = \Gamma^P_{NM} \). For the KG metric, the above quantities are written as
\[
\begin{align*}
\tilde{\Gamma}^{\tilde{I}}_{++} &= \left( \frac{\mu}{3} \right)^2 x^{\tilde{I}} = -\frac{1}{2} \theta^{\tilde{I}} G_{++} , & \tilde{\Gamma}^{\tilde{I}}_{++} &= \left( \frac{\mu}{6} \right)^2 x^{\tilde{I}} = -\frac{1}{2} \theta^{\tilde{I}} G_{++} , \\
\Gamma^-_{\tilde{I}+} &= \Gamma^-_{+\tilde{I}} = \left( \frac{\mu}{3} \right)^2 x^{\tilde{I}} = \Gamma^{\tilde{I}}_{++} , & \Gamma^-_{+\tilde{I}} &= \Gamma^-_{-\tilde{I}} = \left( \frac{\mu}{6} \right)^2 x^{\tilde{I}} = \Gamma^{\tilde{I}}_{++} , \\
R^{\tilde{I}}_{++I+} &= \delta_{I\tilde{I}} \left( \frac{\mu}{3} \right)^2 , & R^{\tilde{I}}_{++I+} &= \delta_{I\tilde{I}} \left( \frac{\mu}{6} \right)^2 , \\
\omega^+_{\tilde{I}+} &= \left( \frac{\mu}{3} \right)^2 x^{\tilde{I}} , & \omega^+_{\tilde{I}+} &= \left( \frac{\mu}{6} \right)^2 x^{\tilde{I}} .
\end{align*}
\]

It should be noted that the scalar curvature vanishes and the Ricci tensor is constant and proportional to \( \mu^2 \). These are given by
\[
\mathcal{R}_{++} = \mathcal{R}^- = \frac{1}{2} \mu^2 , \quad \mathcal{R} = 0 . \quad (A.9)
\]

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