Theory of Heavy Baryon Decay

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Abstract

We discuss various topics in the theory of heavy baryon decays. Among these are recent applications of the Relativistic Three Quark Model to semileptonic, nonleptonic, one-pion and one-photon transitions among heavy baryons, new higher order perturbative results on the correlator of two heavy baryon currents and on the semi-inclusive decay $\Lambda_b \rightarrow X_c + D^{(*)}_s$. 

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1 Introduction

This review contains four different topics on heavy baryon decays. First we discuss some recent theoretical determinations of the quasielastic $\Lambda_b \to \Lambda_c$ form factor in HQET where there are many different results in the literature. The quasielastic $\Lambda_b \to \Lambda_c$ form factor has also been calculated in the Relativistic Three Quark Model (RTQM) which is an all-encompassing tool for the description of exclusive heavy baryon decays. We briefly describe the RTQM model and discuss various applications of the RTQM. As concerns QCD sum rules we describe some recent three-loop results on the finite mass baryon current-current correlator at $O(\alpha_s)$ which is a new result important for QCD sum rule calculations. Finally we discuss the semi-inclusive decays $\Lambda_b \to X_c + D^{(*)} - s$ where $O(\alpha_s)$ and $O(1/m_b^2)$ have been recently calculated. This is an important mode for $\Lambda_b$-decays with an expected branching ratio of $\approx 10\%$.

2 The quasielastic $\Lambda_b \to \Lambda_c$ form factor

There exist many different results on the the quasielastic $\Lambda_b \to \Lambda_c$ form factor in the literature. The predicted slope values of the form factor range from $\rho^2 = 0.33$ to $\rho^2 = 2.35$. In the simplest approach one takes a heavy quark – light diquark model and describes the transition by a one-loop Feynman diagram \cite{1, 2, 3}. One takes $M_Q = m_Q$ and local point coupling factors $g_1$ and $g_2$ for the quark-diquark-baryon vertices whose strengths are fixed by the compositeness condition. The compositeness condition is nothing but the field theoretic equivalent of the familiar quantum mechanical concept of wave function normalization. In the heavy quark limit the result of such a calculation is given by the form factor $\Phi(\omega) = (\omega^2 - 1)^{-1/2} \ln(\omega + \sqrt{\omega^2 - 1})$ which is familiar from the $\omega$-dependent renormalization of the heavy quark current. An expansion in terms of powers of $(\omega - 1)$ shows that the form factor is correctly normalized at the zero recoil point $\omega = 1$, has a slope of $\rho^2 = 1/3$ and a convexity of $c = 2/15$. The form factor $\Phi(\omega)$ is rather flat
when e.g. compared to the heavy meson form factor where experimentally one finds slope values of $\approx 1$. $\Phi(\omega)$ lies within the inclusive HQET sum rule bounds derived by Chiang [4] but must nevertheless be discarded since it oversaturates the semileptonic inclusive rate $\Lambda_b \to X_c + l^- + \bar{\nu}_l$ as recently shown in [5].

Improvements on this simplest approach lead one to the Relativistic Three Quark Model (RTQM) [6]. In the improvements one incorporates binding effects by replacing $M_Q = m_Q$ by $M_Q = m_Q + \Lambda$. The vertex is softened through introduction of a nonlocal vertex and one introduces a true three-quark structure by replacing the $(ud)$-diquark by single $u$, $d$ quarks.

3 The Relativistic Three Quark Model

According to the changes mentioned above the RTQM treats the decay $\Lambda_b \to \Lambda_c + W_{\text{off-shell}}$ in terms of a two-loop Feynman diagram with nonlocal vertices including binding effects. The result is that the $\Lambda_b \to \Lambda_c$ form factor becomes steeper. Depending on the choice of spin vertex structure for the heavy baryons the slope increases from the aforementioned $\rho^2 = 1/3$ to $\rho^2 = 0.75 \div 1.35$ depending on the choice of spin vertex structure to be discussed later on.

The RTQM is an all-encompassing and versatile tool for the description of heavy baryon decays in terms of a Feynman diagram description. The number of parameters associated with the nonlocality of the vertices, the binding effects and the values of the constituent quark masses is reasonably small and their values lie within common expectations. Many of the parameters are already fixed from light baryon decays where the RTQM also applies. For the loop integrations one uses the $\alpha$-parametrization in its exponential form. This introduces $n$ $\alpha$-parameters $\alpha_1, \ldots, \alpha_n$ for $n$ propagators and consequently $n$ integrations. One introduces a Laplace transform to facilitate the vertex form factor integration which is left to the very end. The exponential $\alpha$-parametrization
allows one to do the tensor loop integrals directly through differentiation, i.e. without use of the Passarino-Veltman expansion. One transforms to spherical type variables which leaves one with one radial type integration and \((n - 1)\) angular type integrations. All \((n + 1)\) numerical integrations including the Laplace transform can be done with ease. In fact the spherical integrations can also be done analytically but the ease of the numerical integration does not warrant this effort. We shall now discuss several applications of the RTQM to heavy baryon decays.

The dependence of the Isgur-Wise function on the choice of the vertex spin structure for the heavy baryons was investigated in [7]. For the \(\Lambda_Q\)-type baryons both the effective couplings

\[
J_{1\Lambda_Q} = \bar{\psi}_Q \psi_{\bar{u}}^T C\gamma_5 \psi_d \\
J_{2\Lambda_Q} = \psi_Q \bar{\psi}_{\bar{u}}^T C\gamma_5 \bar{\psi} \psi_d
\]

(1)
correctly describe the coupling of the \(\Lambda_Q\) to a heavy on-shell quark and two light off-shell quarks in the limit of Heavy Quark Symmetry (HQS). When inserted into the relevant two-loop diagram both coupling structures reproduce the required leading order HQET form factor structure including the unit normalization at zero recoil. However, one finds that the slope of the Isgur-Wise function depends on the choice of vertex structure. For the three choices \(J_{1\Lambda_Q}\), \(\frac{1}{2}(J_{1\Lambda_Q} + J_{2\Lambda_Q})\) and \(J_{2\Lambda_Q}\) one finds slope values of \(\rho^2 = 1.35, 1.05\) and 0.75. The fact that \(\rho^2(J_{1\Lambda_Q}) > \rho^2(J_{2\Lambda_Q})\) agrees with the sum rule analysis of [8] although the difference in slope values in [8] is not as large. Note that taking the geometric mean of the two currents leads to a constituent type vertex structure where the projector \((\bar{v} + 1)/2\) projects onto the large components of the light quark fields.

Finite mass effects in heavy \(\Lambda_{Q_1} \rightarrow \Lambda_{Q_2}\) transitions were analyzed in [9] by replacing the heavy quark propagators in the Feynman diagrams by the full propagator. This was effected by the replacement

\[
\frac{i}{l \cdot v + \Lambda} \rightarrow \frac{i(P + l) + m_Q}{(P + l)^2 - m_Q^2}
\]

(2)
where \((P + l)\) is the momentum of the heavy quark and \(l\) is a loop momentum. For \(\Lambda_b \rightarrow \Lambda_c\) the rate is decreased by 9.3% in qualitative agreement with the findings of
The decrease was found to be even larger for $\Lambda_c \rightarrow \Lambda_s$ where the strange quark was treated as a heavy quark in the reference rate for the sake of comparison. It is clear that an expansion of the full propagator in terms of powers of $1/m_Q$ would allow one to systematically explore higher order $1/m_Q$-effects in these transitions as e.g. in the zero recoil normalization of the relevant zero recoil $\Lambda_{Q_1} \rightarrow \Lambda_{Q_2}$ form factor.

In [11] the RTQM was used to calculate exclusive nonleptonic decays of heavy baryons. There are so-called factorizing and nonfactorizing contributions to these decays. The nonfactorizing contributions had never been calculated before. In the Feynman diagram approach they involve a genuine three-loop calculation which was done in [11]. As a sample result one finds that in the nonleptonic decays $\Lambda_b \rightarrow \Lambda_c + \pi^-$ the nonfactorizing contributions amount to $-20\%$ and $-28\%$ in the parity violating and parity conserving amplitudes, respectively, with an ensuing reduction in rate of $\approx 40\%$. The nonfactorizing contributions are therefore not negligible. A multitude of exclusive nonleptonic decays have been calculated within the RTQM model involving $(\bar{s}c)(\bar{u}d)$, $(\bar{b}c)(\bar{u}d)$ and $(\bar{b}c)(\bar{c}s)$ transitions [11].

The RTQM model is also well suited for heavy flavour-conserving one-pion and one-photon transitions between heavy baryons. The one-pion transitions are described by two-loop diagrams where the pion couples to a single light quark line. The $1/f_\pi$ coupling of chiral perturbation theory effectively appears through the quark level Goldberger-Treiman relation $g_\pi = 2m_q/f_\pi$. Many one-pion transitions have been calculated including transitions from excited states [12, 13]. The results are remarkably close to the results of using the constituent quark model [15, 16, 17] for the light quarks even though the light quarks are fully off-shell in the RTQM model.

For one-photon transitions the transverse on-shell photon couples only to the light quarks in the leading order of the heavy quark expansion with a coupling strength given by the light quark charge. In addition one has to include contact graphs to assure gauge invariance of the one-photon transitions. These are generated according to the path
integral formalism of Mandelstam. Again the results of the RTQM model [13, 14] are remarkably close to the constituent quark model calculation [18, 19]. The relation of the RTQM description of one-photon transitions to the chiral approach remains to be explored, in particular to the recent calculation of [20] which contains also chiral loops.

What we have discussed so far are some basic applications of the RTQM in the heavy baryon sector. Further work is in progress on the decays of double heavy baryons, on magnetic moments of heavy baryons and on heavy flavour-conserving nonleptonic charm and bottom baryon decays.

4 Finite mass baryonic current-current correlator at \( \mathcal{O}(\alpha_s) \)

The calculation of the spectral density associated with the baryonic current-current correlator is important for QCD sum rule applications. We want to report on some advances we have made in the calculation of the \( \mathcal{O}(\alpha_s) \) radiative corrections to the spectral density with one finite mass quark mass and two zero quark masses [21]. The calculation involves the evaluation of two-scale three-loop Feynman diagrams which only became possible due to recent technical advances [22] in three-loop technology. Taking the appropriate limits we recover previous results derived for the zero mass case [23] and for the infinite mass case [24]. In the mesonic case the corresponding calculation has been done some time ago showing that radiative corrections to the spectral density can become quite important [25].

The basic object of study is the vacuum expectation value of the time-ordered product of two baryonic currents \( \langle T J(x) J(0) \rangle \). In spinor space its Fourier transform is expanded along the spinor matrix structures \( \gamma \) and \( m \) with coefficients \( \pi_q(q^2) \) and \( \pi_m(q^2) \). We concentrate on the invariant \( \pi_m(q^2) \) which has associated with it a spectral density \( \rho_m(s) \) for which we shall present two- and three-loop results. Using the simplest possible current
\[ J = \Psi(u^T C d) \] and writing

\[ \rho_m(s) = \frac{1}{128\pi^2} s^2 \left\{ \rho_0(s) \left( 1 + \frac{\alpha_s}{\pi} \ln \left( \frac{\mu^2}{m^2} \right) \right) + \frac{\alpha_s}{\pi} \rho_1(s) \right\} \] (3)

we obtain the Born term two-loop contribution \( \rho_0(q^2) = 1 + 9z - 9z^2 - z^3 + 6z(1+z) \ln z \)

where \( z = m^2/q^2 \). In the \( \overline{\text{MS}} \) scheme the radiative three-loop contribution is given by

\[ \rho_1(s) = 9 + \frac{665}{9} z - \frac{665}{9} z^2 - \frac{58}{3} z^3 - \left( \frac{58}{9} + 42z - 42z^2 - \frac{58}{9} z^3 \right) \ln(1-z) \]

\[ + \left( 2 + \frac{154}{9} z - \frac{22}{3} z^2 - \frac{58}{9} z^3 \right) \ln z + 4\left( \frac{1}{3} + 3z - 3z^2 - \frac{1}{3} z^3 \right) \ln(1-z) \ln z \]

\[ + 12z \left( 2 + 3z + \frac{1}{9} z^2 \right) \left( \frac{1}{2} \ln^2 z - \zeta(2) \right) + 4 \left( \frac{2}{3} + 12z + 3z^2 - \frac{1}{3} z^3 \right) \text{Li}_2(z) \]

\[ + 24z(1+z) \left( \text{Li}_3(z) - \zeta(3) - \frac{1}{3} \text{Li}_2(z) \ln z \right) \] (4)

Writing \( q^2 = (m + E)^2 \) we can perform a threshold expansion of the spectral density in terms of powers of \( E/m \). We write the leading order result in a factorized form in order to facilitate comparison with HQET. One has

\[ m \rho_m \overset{m \to \infty}{\to} \frac{1}{128\pi^4} E^5 \left\{ 1 + \frac{\alpha_s}{\pi} \left( \frac{1}{2} \ln \left( \frac{m^2}{\mu^2} \right) - \frac{2}{3} \right) \right\}^2 \times \]

\[ \times \left\{ 1 + \frac{\alpha_s}{\pi} \left( 4 \ln \left( \frac{\mu^2}{2E} \right) + \frac{2}{45} \left( 10\pi^2 + 273 \right) \right) \right\}. \] (5)

The first bracket is the square of the appropriate HQET matching coefficient \( C(m/\mu, \alpha_s) \) first derived in [8] and the second bracket is the appropriate result for the leading order HQET spectral density \( \rho^{\text{HQET}}(E, \mu) \) first derived in [24]. We have checked that the zero mass limit of the general spectral density reproduces the result of [23]. Work is in progress on the momentum spectral density \( \rho_q(s) \) and on correlators of baryonic currents with arbitrary spin structure and on sum rule applications of the spectral densities.

5 **The semi-inclusive decay \( \Lambda_b \to X_c + D_s^{(*)-} \)**

Following the analysis of the semi-inclusive B-meson decays \( \bar{B} \to X_c + D_s^{(*)-} \) in [20, 21] we looked at the corresponding semi-inclusive \( \Lambda_b \)-decays. The \( \Lambda_b \) decays are potentially
more interesting because of the possibility to observe $\Lambda_b$ polarization effects in this decay. At the leading order of $\alpha_s$ and the heavy quark mass expansion one expects branching ratios of $BR(\Lambda_b \to X_c + D^-_s) = 3.2\%$ and $BR(\Lambda_b \to X_c + D^{*-}_s) = (4.4(L) + 2.4(T))\%$ where we have separately listed the longitudinal ($L$) and transverse component ($T$) of the spin 1 $D^{*-}_s$. The two components can be separately measured by an angular analysis of the subsequent decays $D^{*-}_s \to D^-_s + \gamma$ and $D^{*-}_s \to D^-_s + \pi^0$. The same holds true for the measurement of polarization effects [28] which will not be discussed here.

In [28] we calculated the perturbative $O(\alpha_s)$ and the nonperturbative corrections to these decays using the factorization hypothesis. Numerically one finds:

$$\begin{align*}
\Lambda_b \to X_c + D^-_s : & \quad \hat{\Gamma}_S = (1 - 0.096 - 0.013) \\
\Lambda_b \to X_c + D^{*-}_s : & \quad \hat{\Gamma}_L = 0.65(1 - 0.110 - 0.034) \\
& \quad \hat{\Gamma}_T = 0.35(1 - 0.108 + 0.026) \\
& \quad \hat{\Gamma}_{L+T} = (1 - 0.096 - 0.009). \quad (6)
\end{align*}$$

In order to clearly exhibit the percentage changes the rates have been normalized to their respective Born term rates. The second and third figures in the round brackets of Eq.(6) refer to the perturbative $O(\alpha_s)$ corrections and the nonperturbative kinetic energy correction, respectively. The perturbative corrections are negative and quite uniform. They amount to $\approx 10\%$. The nonperturbative corrections range from 0.9\% to 3.4\% with differing signs. The longitudinal mode dominates the rate into $D^{*-}_s$’s. The $L/T$ rate ratio $\Gamma_L/\Gamma_T$ decreases by 6.8\% from 1.86 to 1.73 after applying the perturbative and nonperturbative corrections.

The corresponding semi-inclusive $b \to u$ decays $\Lambda_b \to X_u + D^{(*)}_s$ are suppressed due to the smallness of $V_{bu}$. They are nevertheless of interest for the analysis of so-called wrong sign $D^{(*)}_s$’s [26]. Numerically one finds [28]

$$\begin{align*}
\Lambda_b \to X_u + D^-_s : & \quad \hat{\Gamma}_S = (1 - 0.169 - 0.013) \\
\Lambda_b \to X_u + D^{*-}_s : & \quad \hat{\Gamma}_L = 0.73(1 - 0.178 - 0.029)
\end{align*}$$
\begin{align}
\hat{\Gamma}_T &= 0.27(1 - 0.115 + 0.030) \\
\hat{\Gamma}_{L+T} &= (1 - 0.161 - 0.001).
\end{align}

The dominance of the longitudinal mode in the decay $\Lambda_b \to X_u + D_s^{*-}$ is now more pronounced. Also the radiative corrections are no longer uniform leading to a substantial 13.3% change in the $L/T$ rate ratio due to the perturbative and nonperturbative corrections. It would be interesting to study these semi-inclusive $\Lambda_b$ decay modes including the $L/T$ composition of the $D_s^{*-}$’s at future colliders.

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