We consider a small partially reflecting vibrating mirror coupled dispersively to a single optical mode of a high finesse cavity. We show this arrangement can be used to implement quantum squeezing of the mechanically oscillating mirror.

PACS numbers: 07.10.Cm, 42.50.Pq, 06.30.Bp, 04.80.Nn

I. INTRODUCTION

Experimental advances in nanofabrication and in laser cooling and trapping have turned optomechanical systems into viable laboratories for the observation of quantum mechanics at macroscopic scales. Non-equilibrium cooling of small movable mirrors using laser-driven cavities has been demonstrated experimentally by a number of groups [1-7]. Likewise, theory has shown that in principle these methods should be able to lower the mirror to its quantum mechanical ground state [8, 9, 10, 11, 12, 13]. The preparation of that state of the mirror is an important first step in exploring characteristic features of quantum mechanics such as superposition [14] and entanglement in macroscopic systems [15].

Squeezed states have also attracted much attention, due to their favorable quantum noise properties [16]. Squeezed states of light are expected to find applications in precision measurements [17] and optical communications [18, 19]. In a parallel development, the squeezing of classical noise in mechanical oscillators has been demonstrated in optomechanical cavities [20], ion traps [21, 22], optical lattices [23] and other systems [24, 25]. Quantum squeezing of phonons has been achieved in ion traps [26] and in crystals [27, 28]. Proposals to realize squeezed states of nanomechanical oscillators in the quantum regime have been made involving two-mirror cavities [29], parametric mixing in solid state circuits [30, 31], parametric driving [32], microwave coupling to a charge qubit [33], and the parametric modulation of a mechanical spring [34]. Their application to gravitational interferometry has also been discussed [35]. Other nonclassical states such as Schrödinger ‘cats’ have been proposed using movable cavity mirrors [36].

This article shows how to realize a squeezed state of a mechanically moving mirror in a high finesse optical cavity. Previous proposals to achieve this goal have relied on the mathematical analogy between an optical resonator with a moving mirror and a Kerr medium, and the mechanism of squeezing has been parametric driving. Here we invoke compression as an alternative route to squeezing [38]. In that scheme squeezing of the mirror motion relies on coupling it dispersively with the cavity, a possibility that has recently been pointed out [39] and analyzed in detail [40]. To provide a complete discussion we consider not one but two modes of the cavity, the moving mirror being coupled dispersively to one of the modes and dissipatively to the other. This configuration was recently proposed as an efficient cooling and trapping configuration for semi-transparent mirrors [41]; here we show that this configuration also allows for displacing and squeezing the error ellipse of the oscillating mirror in phase space.

The remainder of the paper is organized as follows. Section II introduces the physical system and its model Hamiltonian, section III discusses the corresponding evolution operator and the resulting displacement and squeezing assuming that the moving mirror starts from its quantum mechanical ground state. Section IV discusses the effects of squeezing in the presence of noise and damping. Section V supplies a conclusion and an outlook.

II. THE HAMILTONIAN

We consider a high finesse cavity with two perfectly reflecting fixed end mirrors, and a partially reflective movable middle mirror as shown in Fig.1. The middle mirror is assumed to execute small harmonic oscillations $q$ about its equilibrium position. It couples dissipatively (linearly in $q$) to a cavity mode $a$ of frequency $\omega_D$ and dispersively (quadratically in $q$) to a second mode $b$ of frequency $\omega_S$. The Hamiltonian $H'$ modelling the system is derived in Ref. [40], and is given explicitly by

$$H' = \hbar \omega_D (a^\dagger a + \frac{1}{2}) + \hbar \omega_S (b^\dagger b + \frac{1}{2}) + \frac{\dot{q}^2}{2m} + \frac{1}{2} m \omega_m^2 q^2 + \hbar \xi_D a^\dagger aq + \hbar \xi_S b^\dagger bq^2,$$
FIG. 1: The three-mirror cavity arrangement. The two outer mirrors are fixed and the middle mirror vibrates harmonically about its equilibrium position $q_0$. $P_{in}$ is the laser power coupling into the cavity.

where $\omega_m$ is the oscillation frequency of the middle mirror,

$$|\xi_D| = \frac{\sin 2k_nq_0}{\sqrt{(1-T)^{-1}-\cos^2 2k_nq_0}} \xi,$$

with $q_0$ the equilibrium position of the moving mirror of transmissivity $T$, $\omega_n = n\pi c/L$, $\xi = \omega_n/L$ and $k_n = \omega_n/c$, and

$$|\xi_S| = \frac{\tau \xi^2}{2} \left(\frac{1-T}{T}\right)^{1/2},$$

where $\tau = 2L/c$. The frequencies of the modes $a$ and $b$ can be chosen such that $\xi_{D,S}$ are either positive or negative. In the case of $\xi_S$ this corresponds to the use of trapping and anti-trapping modes, respectively [40].

For $\xi_D < 0$ we have

$$\omega_D = \omega_n - \frac{\pi}{\tau} \left[ \sin^{-1} \left( \sqrt{1-T} \right) - \sin^{-1} \left( \sqrt{1-T} \cos 2k_nq_0 \right) \right],$$

and for $\xi_D > 0$

$$\omega_D = \omega_n + \frac{\pi}{\tau} \left[ \sin^{-1} \left( \sqrt{1-T} \right) + \sin^{-1} \left( \sqrt{1-T} \cos 2k_nq_0 \right) \right].$$

Similarly,

$$\omega_S = \omega_n$$

for $\xi_S < 0$ and

$$\omega_S = \omega_n + \frac{2}{\tau} \cos^{-1}(1-T)^{1/2}.$$ for $\xi_S > 0$.

The first two terms in the Hamiltonian $H'$ describe the energies of the optical modes, the next two the energy of the oscillating mirror, and the last two the dissipative and dispersive coupling energies. The bosonic modes obey the commutation relations $[a, a^\dagger] = 1$ and $[b, b^\dagger] = 1$, and the dynamical variables of the oscillating mirror follow the canonical commutation relation $[q, p] = i\hbar$.

The Hamiltonian (1) indicates that for low values of $\xi_S$ the spring potential energy dominates the anti-trapping due to radiation pressure, hence the middle mirror still behaves as a harmonic oscillator, but of lower frequency. However, for $\xi_s < 0$ increasing $|\xi_S|$ leads to a point

$$C_S = -\omega_m/2,$$
where the mirror behaves as a free particle. For even higher values of $|\xi_S|$ radiation pressure-induced anti-trapping dominates and the mirror behaves like an inverted harmonic oscillator \[45\]. We do not consider that regime in this paper. This is consistent with the assumption of small mirror displacements $q$ used to derive the Hamiltonian \[11\], as well as with requirements of stability.

In the following we consider a semiclassical version of the Hamiltonian $H'$ valid for situations where the optical modes can be treated classically. In that case

$$a \rightarrow \alpha, \quad b \rightarrow \beta,$$

and expressing the mirror displacement in terms of raising and lowering operators

$$q = \sqrt{\frac{\hbar}{2m\omega_m}} (c^\dagger + c),$$

we have

$$H' \rightarrow H = \hbar C_D (c + c^\dagger) + 2\hbar C_R K_0 + \hbar C_S (K_- + K_+).$$

where we have removed a constant energy $E_0 = \omega_D(|\alpha|^2 + \frac{1}{2}) + \omega_S(|\beta|^2 + \frac{1}{2})$, and

$$C_D = \frac{\xi_D|\alpha|^2}{\sqrt{2m\omega_m/\hbar}},$$

$$C_S = \frac{\hbar\xi_S|\beta|^2}{m\omega_m},$$

$$C_R = C_S + \omega_m.$$ (12)

In the semiclassical Hamiltonian $H$ we have also introduced the operators

$$K_0 = (c^\dagger c + cc^\dagger)/4, \quad K_- = c^2/2, \quad K_+ = c^\dagger^2/2,$$

which together with $c$ and $c^\dagger$ form the basis of the so-called two-photon Lie algebra \[43\], with

$$[K_0, K_\pm] = \pm K_\pm, \quad [K_-, K_+] = 2K_0,$$

$$[K_-, c] = [K_+, c^\dagger] = 0,$$

$$[K_-, c^\dagger] = c, \quad [K_0, c]^\dagger = c^\dagger/2.$$ (14)

As is well known, the operators \{c, c^\dagger\} and \{K_0, K_\pm\} form two sub-algebras, the associated operators forming the generators of coherent states and of squeezed states, respectively. The Hamiltonian \[11\] has previously been studied in some detail in the context of molecular translational-vibrational interactions \[46\] and laser-plasma scattering \[47\] and very recently in the context of atomic vapors inside resonators \[48\]. See also \[41, 50\] for additional discussions of this model.

### III. TIME EVOLUTION

Using the Lie-algebraic symmetries of $H$, the associated evolution operator can be disentangled as \[43\]

$$U = \exp[-iHt/\hbar] = e^{i\delta} D(\nu) R(\phi) S(\kappa),$$

where $\delta$ is an unimportant overall phase, and

$$D(\nu) = e^{\nu c^\dagger - \nu^* c},$$

is a displacement operator, with \[43\]

$$\nu = \frac{C_D}{\chi} \left[ \frac{\omega_m}{\chi} (\cos \chi t - 1) - i \sin \chi t \right]$$

and

$$\chi = \sqrt{C_R^2 - C_S^2} = [\omega_m(\omega_m + 2C_S)]^{1/2}. \quad (18)$$
FIG. 2: The modulus of the dimensionless displacement amplitude $|\nu|$ defined using Eq. (17) as a function of time. The parameters used are cavity length $L = 5\text{mm}$, laser wavelength $\lambda = 514\text{nm}$, and a middle mirror of mass $m = 1\mu\text{g}$, vibration frequency $\omega_m = 2\pi 2.5\text{kHz}$, damping constant $D_m = 0.02\mu\text{gHz}$, transmissivity $T = 10^{-4}$, equilibrium position $\lambda/10$ and base temperature $T_e = 100\text{mK}$. The end mirror transmissivity has been taken to be $10^{-5}$ and the power coupling into the mode $1\text{mW}$.

In the bound oscillator regime, i.e. for $C_S > -\omega_m/2$, we have $\chi^2 > 0$, and we can choose $\chi > 0$ without loss of generality. That parameter largely determines the time scale of the mirror dynamics; in the absence of squeezing ($C_S = 0$) it is just the harmonic oscillator period. The factor in parentheses in Eq. (18) quantifies the mismatch from the condition $C_S = -\omega_m/2$ [Eq. (8)] which demarcates the regimes of qualitatively different physical behaviors in the system. From Eqs. (17) and (18) $C_S$ can both increase or decrease the characteristic time scale of the displacement as well as its magnitude. The displacement in phase space is given by the absolute value of $\nu$. A plot of $|\nu|$ versus time for typical experimental parameters is shown in Fig. 2. Near the first minimum i.e. for

$$t \ll 1/\chi,$$

the displacement is linear in time to lowest order, i.e.

$$|\nu| \approx |C_D| \left( t - \frac{\omega_m}{3} \left( \frac{\omega_m}{8} + C_S \right) t^3 \right) + \mathcal{O} \left[ t^5 \right],$$

and the effects of squeezing come in at third order. Interestingly, by adjusting the squeezing such that $C_S = -\omega_m/8$, which is still in the $\chi^2 > 0$ regime, the third-order time dependence of the displacement can be removed. Qualitatively similar behavior can be seen near every minimum in Fig. 2. We note that in the absence of squeezing ($C_S = 0$),

$$|\nu| \approx \left| \frac{2C_D}{\omega_m} \sin \frac{\omega_m t}{2} \right|,$$

while for large squeezing ($\omega_m/\chi \ll 1$),

$$|\nu| \approx \left| \frac{2C_D}{\chi} \sin \chi t \right|$$

For typical parameters we have $|\frac{2C_D}{\omega_m}| \sim 10^9, |\frac{2C_D}{\chi}| \sim 10^{11}$. Therefore for both small and large squeezing a coherent mechanical state of the middle mirror of relatively large amplitude can be produced starting from the ground state.
Returning to the various components of the evolution operator $U(t)$ we observe that

$$R(\phi) = e^{i\phi K_0}$$  \hspace{1cm} (23)

is a rotation operator, and

$$S(\kappa) = e^{\kappa^* K_+ - \kappa K_+},$$  \hspace{1cm} (24)

is a squeezing operator, with

$$|\kappa| = \left| \sinh^{-1} \left( \frac{C_S}{\chi} \sin \chi t \right) \right|.$$  \hspace{1cm} (25)

As expected that operator does not depend on $C_D$, i.e. displacement does not affect squeezing.

It turns out that the rotation angle $\phi$ in Eq. (23) is exactly opposite the angle at which the squeeze operator tilts the error ellipse of the moving mirror in phase space [41], i.e.

$$\phi = -\left[ \frac{\text{phase}(\kappa) + \pi}{2} \right].$$  \hspace{1cm} (26)

The two rotations therefore cancel each other out and $\phi$ effectively plays no role in the dynamics. It is in fact intuitively clear that the effects of rotation should cancel out, i.e. the axes of the final error ellipse should be aligned along $p$ and $q$ in phase space. This is because Eq. (1) stipulates that position is the only quadrature of the oscillating middle mirror that can be squeezed or anti-squeezed, the latter situation corresponding to momentum squeezing.

Figure 3 shows $|\kappa|$ versus time for typical experimental parameters. As can be seen from that plot the squeezing first grows linearly in time. This can be confirmed by analytically expanding Eq. (25) for the case of $t \ll 1/\chi$

$$|\kappa| \simeq \left| C_S \left[ t - \frac{(\omega_m + C_S)^2}{6} t^3 \right] + \mathcal{O}[t^5] \right|. $$  \hspace{1cm} (27)

We note that the third-order time dependence can be removed for $C_S = -\omega_m$. Actually under this condition it can readily be seen from Eq. (25) that all higher orders vanish and squeezing is purely linear in time : $|\kappa| = \omega_m t$. However that case corresponds to $\chi^2 < 0$, a situation where the mirror does not behave as a bound harmonic oscillator.
IV. SQUEEZING OF THERMAL STATES

If the middle mirror is prepared in its quantum mechanical ground state, the squeezing operator \( \alpha \) produces a squeezed vacuum \([42]\). From Fig. 3 the maximum value of \( \kappa \) is approximately 4, which implies a maximum squeezing of \( R = e^{-3} \sim 0.018 \), or \( \log_{10}(0.018) \sim 18 \) dB of squeezing.

However, the placement of a macroscopic nano-oscillator in its ground state has not yet been achieved experimentally, so we also consider thermal states of the middle mirror. They are characterized by a thermal phonon number given by the Bose distribution

\[
n_T = \left[ \exp \left( \frac{\hbar \omega_m}{k_B T_e} \right) - 1 \right]^{-1},
\]

where \( T_e \) is the mirror equilibrium temperature and \( k_B \) is Boltzmann’s constant. Any realistic model should also include the damping of the mirror. We estimate these effects by including noise and damping in the Heisenberg equations of the mirror in a manner consistent with the fluctuation-dissipation theorem. This produces the corresponding quantum Langevin equations from Eq. (1) in a standard way. Setting \( \alpha = 0 \) for simplicity and concentrating therefore solely on the squeezing part of the Hamiltonian \( H \) the quantum Langevin equations turn out to be

\[
\dot{q} = p/m, \\
\dot{p} = -m\omega_m^2 q - \frac{D_m}{m} p + \epsilon(t),
\]

where \( D_m \) is the damping constant of the mirror and \( \epsilon(t) \) represents Brownian noise with average zero and fluctuations correlated as

\[
\langle \delta \epsilon(t) \delta \epsilon(t') \rangle = D_m \int_{-\infty}^{\infty} \frac{d\omega}{2\pi} e^{-i\omega(t-t')} \hbar \omega \left[ 1 + \coth \left( \frac{\hbar \omega}{2k_B T_e} \right) \right].
\]

For a high mechanical quality factor, the Brownian force becomes delta-correlated in the time domain \([44]\). In Fourier space the correlation can then be written as

\[
\langle \delta \epsilon(\omega) \delta \epsilon(\omega') \rangle = 2D_m \hbar \omega_m (2n_T + 1) \delta(\omega + \omega').
\]

By setting the time derivatives equal to zero the steady-state solutions to Eq. (29) can easily be found to be

\[
q_s = p_s = 0.
\]

Linearizing all operators in Eq. (29) as sums of a semiclassical steady-state value and a small quantum fluctuation (i.e. \( q = q_s + \delta q \)) we obtain linear dynamical equations for the fluctuations. Using Fourier transforms and Eq. (31) we can solve the fluctuation equations to obtain \( \delta q(\omega) \), etc. We can therefore also find the (equal-time) correlation function for the position

\[
\langle \delta q^2 \rangle = (2n_T + 1) \frac{\hbar \omega_m}{2m\chi^2},
\]

which is independent of time since the noise process we have considered is stationary \([Eq. (30)]\). This result for the position uncertainty has followed from a linear response analysis, however it agrees to first order with results from more sophisticated computations \([52]\). For example in the absence of squeezing \( (C_S = 0) \), and at high temperatures, \( (n_T \sim k_B T_e / \hbar \omega_m \gg 1) \),

\[
\langle \delta q^2 \rangle = k_B T_e / m\omega_m^2.
\]

On the other hand for \( C_S = 0 \) and low temperatures \( (n_T \ll 1) \),

\[
\langle \delta q^2 \rangle = \hbar / 2m\omega_m,
\]

which is just the square of the oscillator length of the ground state of the moving mirror. The results in Eqs. (34) and (35) agree with an earlier and more rigorous derivation \([52]\). Using Eq. (33) in the presence of squeezing \( (C_S \neq 0) \) and defining a position uncertainty \( R \) in terms of the ground state oscillator length we find

\[
R = \frac{\langle \delta q^2 \rangle^{1/2}}{\sqrt{\hbar / 2m\omega_m}} = \left[ (2n_T + 1) \frac{\omega_m}{\omega_m + 2C_S} \right]^{1/2} \sim \left( \frac{k_B T_e}{\hbar C_S} \right)^{1/2},
\]
where the last expression has been written in the limit of high temperature and high squeezing. $R$ needs to be lower than 1 for squeezing to be present, i.e. the fluctuations in the position need to be smaller than the ground state uncertainty in position. For milliKelvin temperatures and hundreds of milliwatts of laser power, $R = 0.15$ and about 8dB of mechanical squeezing can be obtained, which is still considerable.

V. CONCLUSION

In conclusion we have considered a partially reflective vibrating mirror coupled dispersively to an optical mode of a high finesse cavity. We have shown that quantum squeezing of the mechanical motion of the mirror can be achieved in this way. We have described the unitary dynamics of the oscillator in some detail and shown that the squeezing remains non-negligible in the presence of noise and damping. Clearly the squeezing field itself can be employed in a time-dependent fashion although we have not investigated such a scenario.

It was also shown that the oscillator can be displaced by a second field to which it is coupled dissipatively. The dynamics of the displacement can be influenced by the squeezing field, although the converse is not true.

It would be interesting to explore the effects of the fully quantum mechanical Hamiltonian [Eq. (1)] without making the semiclassical approximation of Eq. (9). This may lead to highly non-classical states of the mirror-field system as found in the case of purely dissipative coupling [37]. We are currently also working on generalizing the present proposal to the case of multiple mirrors in the same cavity.

VI. ACKNOWLEDGEMENTS

This work is supported in part by the US Office of Naval Research, by the National Science Foundation and by the US Army Research Office. We thank H. Uys and O. Dutta for useful discussions.

References

[1] Cohadon P F, Heidmann A, Pinard M 1999 Phys. Rev. lett. 83 3174-3177
[2] Metzger C H and Karrai K 2004 Nature 432 1002
[3] Gigan S, Böhm H R, Paternostro M, Blaser F, Langer G, Hertzberg J B, Schwab K C, Bäuerle D, Aspelmeyer M. and Zeilinger A 2006 Nature 444 67
[4] Kleckner D and Bouweemeester D 2006 Nature 444 75
[5] Arcizet O, Cohadon P -F, Briant T, Pinard M and Heidmann A, 2006 Nature 444 71
[6] Schliesser A, Del'Haye P, Nooshi N, Vahala K J and Kippenberg T J 2006 Phys. Rev. Lett. 97 243905
[7] Corbitt T, Chen Y, Innerhofer E, Muller-Elhardt H, Ottaway D, Rehbein H, Sigg D, Whitcomb S, Wipf C and Mavalvala N 2007 Phys. Rev. Lett. 98 150802
[8] Poggio M, Degen C L, Mamin H J and Rugar D 2007 Phys. Rev. Lett. 99 017201 (2007)
[9] Vitali D, Mancini S, Ribichini L and Tombesi P 2003 J. Opt. Soc. Am. B 20 1054
[10] Bhattacharya M and Meystre P 2007 Phys. Rev. Lett. 99 073601
[11] Bhattacharya M and Meystre P 2007 Phys. Rev. Lett. 99 153603
[12] Wilson-Rae I, Nooshi N, Zwerger W and Kippenberg T J 2007 Phys. Rev. Lett. 99 093901
[13] Marquardt D, Chen J P, Clerk A A and Girvin S M 2007 Phys. Rev. Lett. 99 093902
[14] Leggett A J 2002 J. Phys.: Condens. Matter 14 R415-R451
[15] Mancini S, Giovannetti V, Vitali D and Tombesi P 2002 Phys. Rev. Lett. 88 120401
[16] Dodonov V V and Man’ko V I (ed) 2003 Theory of Nonclassical States of Light (London: Taylor and Francis)
[17] Caves C 1981 Phys. Rev. D 23 1693
[18] Balakrishnan A V 1965 (ed) 1961 Advances in Communication Systems, Vol. 1(Academic: New York)
[19] Yuen H P 1976 Phys. Rev. A 13 2226
[20] Briant T, Cohadon P F, Pinard M and Heidmann A Eur. Phys. J. D 22 131-140
[21] Mechkof D M, Monroe C, King B E, Itano W M and Wineland D J 2007 Phys. Rev. Lett. 99 093902
[22] Hu X and Nori F 1997 Phys. Rev. Lett. 79 4605-4608
[23] Garrett G A, Rojo A G, Sood A K, Whitaker J F and Merlin R 1997 Science 275 1638-1640
[24] DiFilippo F, Natarajan V, Boyce K and Pritchard D E 1992 Phys. Rev. Lett. 68 28592862
[25] Natarajan V, DiFilippo F and Pritchard D E 1995 Phys. Rev. Lett. 74 28552858
[26] Raithel G, Phillips W D and Rolston S L 1997 Phys. Rev. Lett. 78 2928
[27] Rugger D and Grutter P 1991 Phys. Rev. Lett. 67 699-702
[28] Sidles J A and Rugger D 1993 Phys. Rev. Lett. 70 3506-3509
[29] Hollenhorst J N 1979 Phys. Rev. D 19 001669
[30] Vitali D, Mancini S, Ribichini L and Tombesi P 1993 Phys. Rev. A 65 063803
[31] Huo W Y and Long G L 2007 Generating squeezed states of nanomechanical resonator [arXiv:0704.0960v2]
[32] Xue F, Liu Y -X, Sun C P and Nori F 2007 Two mode squeezed states and entangled states of two mechanical resonator [arXiv:quant-ph/0701209v2]
[33] Tian L and Simmonds R W 2006 Coupled macroscopic quantum resonators: entanglement and squeezed state [arXiv:cond-mat/0606787v1]
[34] Rabl P, Shnirman A and Zoller P 2004 Phys. Rev. A 70 205304
[35] Ruskov R, Schwab K and Korotkov A N 2005 Phys. Rev. A 71 235407
[36] Blencowe M P and Wybourne M N 2000 Physica B 280 555-556
[37] Bose S, Jacobs K and Knight P L 1997 Phys. Rev. A 56 4175-4186
[38] Both mechanisms of squeezing have been implemented on the same system in Ref. [26], for instance.
[39] Thompson J D, Zwickl B M, Jayich A M, Marquardt F, Girvin S M and Harris J G E 2007 Strong dispersive coupling of a high finesse cavity to a micromechanical membrane arXiv:quant-ph0707.1724v2
[40] Bhattacharya M, Uys H and Meystre P 2007 Optomechanical cooling and trapping of partially reflective mirrors arXiv:quant-ph0708.4078v1
[41] Mandel L and Wolf E 1995 Optical coherence and quantum optics (Cambridge: Cambridge University Press)
[42] Scully M O and Zubairy M S 1997 Quantum Optics (Cambridge: Cambridge University Press)
[43] Wunsche A 2002 J. Opt. B: Quantum Semiclass. Opt. 4 1-14
[44] Vitali D, Mancini S and Tombesi P 2007 J. Phys. A: Math. Theor. 40 8055
[45] Lo C F 1991 Quantum Opt. 3 333-340
[46] Gilmore R and Yuan J -M 1987 J. Chem. Phys. 86 130-139
[47] Ben-Aryeh Y and Mann A 1985 Phys. Rev. Lett. 70 1020-1022
[48] Ian H Gong Z R Liu Y X Sun C P and Nori F 2007 Cavity optomechanical coupling assisted by an atomic gas arXiv:quant-ph0803.0776v1
[49] Dattoli G, Richetta M and Torre A 1988 Phys. Rev. A 37 2007-2011
[50] Fernandez F M 1989 Phys. Rev. A 40 41-44
[51] Dattoli G, Dipace A and Torre A Phys. Rev. A 33 4387-4389
[52] Grabert H and Weiss U 1984 Z. Phys. B - Condensed Matter 5587-94