Black holes and holography

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Abstract. The idea of holography in gravity arose from the fact that the entropy of black holes is given by their surface area. The holography encountered in gauge/gravity duality has no such relation however; the boundary surface can be placed at an arbitrary location in AdS space and its area does not give the entropy of the bulk. The essential issues are also different between the two cases: in black holes we get Hawking radiation from the ‘holographic surface’ which leads to the information issue, while in gauge/gravity duality there is no such radiation. To resolve the information paradox we need to show that there are real degrees of freedom at the horizon of the hole; this is achieved by the fuzzball construction. In gauge/gravity duality we have instead a field theory defined on an abstract dual space; there are no gravitational degrees of freedom at the holographic boundary. It is important to understand the relations and differences between these two notions of holography to get a full understanding of the lessons from the information paradox.

1. Introduction

The idea of holography arose in gravitational physics from the expression for the entropy of black holes [1]

\[ S_{bek} = \frac{A}{4G} \]  

Since the entropy is given by its surface area measured in units of \( l_p^2 \), it appeared plausible that there is ‘one bit of data per planck area of the horizon’. Because the degrees of freedom are given by the bounding area rather than the ‘volume’ of the hole, we use the term ‘holographic’ to characterize the gravitational physics of the hole.

In recent years the term ‘holography’ has been applied to the idea of gauge/gravity duality in AdS spacetimes [2]. This idea is depicted in fig.1. The dashed line in fig.1(a) is an arbitrary boundary surface \( S \). A field theory on \( S \) describes the gravitational physics of the entire region below \( S \). Again, it appears that physics is ‘holographic’.

Since the idea of gauge/gravity duality arose from studies of black holes in string theory, it is often assumed that these two uses of the term ‘holographic’ are the same. Stretching this further, one may think that the idea of gauge/gravity duality would somehow explain the mysteries associated with the entropy (1) and the related problem of the information paradox [3]. But as we now note, there are several differences between the above two notions of holography:

(i) For the black hole, the location of the boundary surface (the horizon) is fixed at \( r = 2M \). In gauge/gravity duality, the boundary surface can be placed anywhere in the AdS region.
Figure 1. (a) Branes create a geometry that is AdS in the ‘near’ region; the dual CFT lives on a boundary placed anywhere in the AdS region, and describes gravity in all the region below it. (b) The singularity at \( r = 0 \) can be avoided by moving to global AdS, where a 3-point function is computed by a simple path integral with no singularities. (c) If we have enough energy in global AdS, we make a black hole, and then we face the difficulties of Hawking’s argument again. (The vertical direction is time, and the surface of the inner cylinder is the black hole horizon.)

(ii) For the black hole, the area of the boundary surface gives the entropy (1) of the interior. But in gauge/gravity duality here is no such relation. The entropy of the interior spacetime depends on how much energy \( E \) we put in it. In particular if we take \( E = 0 \), we get empty AdS with entropy \( S = 0 \).

As we probe these differences deeper, we will uncover important aspects of gravitational degrees of freedom and of the structure of spacetime itself.

2. Lessons from the information paradox

A fundamental problem in black hole physics is the information paradox [3]. In a geometry with horizon, there is no time-independent foliation of spacetime. As shown in fig.2, spacelike slices are \( t = \text{constant} \) outside the horizon, but \( r = \text{constant} \) inside. ‘Later’ slices are obtained by ‘stretching’ the \( r = \text{constant} \) part of the slice; this stretching creates particle pairs, with one member being inside the horizon and one outside. The important aspect of this particle creation process is that the two members of the pair are in an entangled state, which we may write schematically as

\[
|\psi\rangle_{\text{pair}} = \frac{1}{\sqrt{2}} (|0\rangle_{\text{in}} |0\rangle_{\text{out}} + |1\rangle_{\text{in}} |1\rangle_{\text{out}})
\]

where \(|0\rangle, |1\rangle\) represent occupation numbers 0, 1 respectively for a given particle mode. Thus each step of the pair creation process generates an entanglement entropy \( S_{\text{ent}} = \ln 2 \), and after \( N \) steps of particle creation the entanglement is

\[
S_{\text{ent}} = N \ln 2
\]

This pair creation continues until the hole reaches planck size, at which point the emitted radiation is heavily entangled with the quanta in the hole. If the hole evaporates completely, the radiation cannot be attributed any quantum state – the radiation is entangled, but there is nothing that it is entangled with. This possibility is termed ‘information loss’ or ‘loss of unitarity’. If we are left with a remnant, then this remnant needs to have at least \( 2^N \) internal states to permit the entanglement (3). Since \( N \) is unbounded, our theory must permit arbitrarily
Figure 2. Eddington-Finkelstein coordinates for the Schwarzschild hole. Spacelike slices are $t = \text{const}$ outside the horizon and $r = \text{const}$ inside. Curvature length scale for a solar mass black hole is $\sim 3$ km all over the region of evolution covered by the slices $S_i$.

high degeneracy in a bounded volume with bounded energy budget; something that is hard to achieve with normal quantum theories.

Many relativists had reconciled themselves to admitting remnants of some sort, perhaps with baby Universes opening up inside the remnant where states could be hidden [4]. But such remnants have not been found in string theory. How do we avoid being forced to ‘information loss’, which would imply a breakdown of string theory?

Some string theorists have been seriously worried about this problem. But many others assumed that Hawking’s argument was somehow flawed. Among the latter, the most common belief was the following. Hawking computed the pair creation at leading order, but there can always be small quantum gravity corrections to the wavefunction (2)

$$|\psi\rangle_{\text{pair}} = \frac{1}{\sqrt{2}} (|0\rangle_{\text{in}}|0\rangle_{\text{out}} + |1\rangle_{\text{in}}|1\rangle_{\text{out}}) + \epsilon\frac{1}{\sqrt{2}} (|0\rangle_{\text{in}}|0\rangle_{\text{out}} - |1\rangle_{\text{in}}|1\rangle_{\text{out}})$$

(4)

where we have added a small amount of an orthogonal state for the pair. The correction $\epsilon$ for each pair must be small since the horizon geometry is smooth, but the number of emitted quanta is large ($\sim (M/m_p)^2$), and the net effect of the small corrections may cumulate in such a way that the overall state of the radiation would be un-entangled with the hole.

But in [5] it was shown that this hope is false; the change in entanglement is bounded as

$$\frac{\delta S_{\text{ent}}}{S_{\text{ent}}} < 2\epsilon$$

(5)

This inequality is the essential reason why the Hawking argument has proved so robust over the years – no small corrections can save the situation.

To summarize, it can be rigorously argued that we must choose between one of the following: (i) we have information loss or remnants (ii) we find a way to get order unity corrections to low energy physics at the horizon.
3. Gauge/gravity duality and the information paradox

With the above knowledge of the information problem, let us return to our analysis of holography. A common error is to argue the following: “Many computations support the idea that gravity is dual to a gauge theory. Since the latter is unitary, there cannot be information loss in black holes. Thus we have solved the information problem”.

As we will now see, this argument is completely incorrect, and arises from ignoring the power of the information paradox, encoded in (5). We can approach gauge/gravity duality from two sides:

(i) We know string theory at low energy gives gravity, which contains black holes. Low energy gravity amplitudes can be reproduced by the gauge theory. But if we put together enough energy to make an AdS-Schwarzschild black hole, then the Hawking argument tells us that we will get information loss. Thus unless we find some way to bypass the Hawking argument, gauge/gravity duality would fail at the same place where all other quantum gravity approaches fail: at the threshold of black hole formation.

(ii) We can define the gravity theory as the dual of the gauge theory. In this case we cannot lose unitarity. But now we cannot assume that the dual gravity theory has black holes. Low energy amplitudes in the gauge theory agree with gravity, but if we take a large energy excitation in the gauge theory then the natural timescale for dispersion of this energy is the crossing time of the black hole, not the much longer Hawking evaporation time. But if the energy disperses in order crossing time, we have no black hole in the theory.

As a concrete manifestation of the above points, we can look at the simplest manifestation of a gauge/gravity type correspondence: the 1-d matrix model which is dual to 1+1 dimensional gravity [6]. The low energy gravity theory is 1+1 dimensional dilaton gravity, in which we can make a black hole by infalling matter [7]. We find information loss or remnants, depending on how we complete the theory at the planck scale. The low energy gravity theory can be reproduced by a dual ‘matrix model’ which lives in 0+1 dimensions. This matrix model is unitary, but if we try to make a black hole we fail: the energy of a collapsing shell bounces off the origin and returns in a time of order crossing time. The differences from the low energy theory are caused by higher order quantum corrections, which are small at low energies, but grow large enough at the black hole threshold to prevent black hole formation altogether [8].

In short, gauge/gravity duality has no direct bearing on the information problem. As we noted in the introduction, the ‘holography’ in gauge/gravity duality is not the same as the ‘holography’ that arises for black holes from the entropy formula (1). The holography of gauge/gravity duality can be verified for low energy correlators, but above the threshold of black hole formation the duality cannot answer any quantum gravity questions until we understand the nature of gravitational states in this domain. The traditional approach of writing down the AdS-Schwarzschild metric for the black hole allows us to define thermal averages in the dual gauge theory, but cannot tell us anything about how the information paradox is to be resolved. For the latter question, we need to understand black hole microstates themselves.

4. Resolving the information paradox – fuzzballs

In string theory, we have to make black hole states from the objects present in the theory – strings and branes. It turns out that the size of brane bound states grows with the coupling and with the number of branes in the bound state – in such a way that the size of the state is always order horizon size [9]. Thus we do not get a traditional horizon with vacuum in its vicinity, as was assumed in the Hawking computation. The emission of low energy modes can be therefore modified by order unity, as required to solve the information paradox.

Further progress along these lines is obtained by taking specific states of the hole and constructing their gravity description. In each case we see that no horizon forms. It is interesting
to see these constructions in the context of the no-hair conjectures that suggested that the
traditional black hole geometry was unique. In string theory we have extra compact dimensions,
and a set of sources (branes). These objects are all crucial to the structure of microstates. A
compact circle ‘pinches off’ before reaching the horizon, creating a smooth end to the spacetime
gamey. This pinch-off generates a set of Kaluza-Klein monopoles and antimonopoles just
outside the place where the horizon would have formed, and fluxes corresponding to brane
charges wrap the topological cycles produced by this monopole structure. The simplest black
hole is the extremal 2-charge hole made from D1 and D5 branes. For this hole we find the
following:

(i) The number of extremal bound states of the D1D5 brane system can be counted by
abstract topological methods, and give a microscopic entropy

\[ S_{\text{micro}} = 4\pi \sqrt{n_1 n_5} \]

[10].

(ii) If we assume a spherically symmetric ansatz and a trivial factorization of the compact
directions, then the low energy supergravity action gives an extremal black hole with horizon,
with a Bekenstein-Wald entropy

\[ S_{\text{bek}} = 4\pi \sqrt{n_1 n_5} \]

[11].

(iii) The actual microstates of the D1D5 system can be constructed. It is found that they are
not spherically symmetric and the compact directions are locally nontrivially fibered, though
the net monopole charge of these fibrations vanishes. The solutions have no horizon and no
singularity [12].

(iv) The phase space of these horizonless gravitational solutions can be quantized, and yields
the entropy

\[ S = 4\pi \sqrt{n_1 n_5} \]

[13].

(v) Though there is no horizon for any microstate, the region where the typical microstates
exhibits their nontrivial structure has a boundary whose area \( A \) satisfies

\[ A/G \sim \sqrt{n_1 n_5} \sim S_{\text{bek}} \]

[14].

Work on more complicated extremal holes [15] has yielded a similar picture [16], though
all microstates have not been constructed yet. Some families of nonextremal microstates have
been constructed as well [17], and again they have no horizon or singularity. But they do have
ergoregions, and the rate of ergoregion emission [18] agrees exactly with the Hawking radiation
rate expected for these microstates [19]. But this time the radiation process does not lead to
information loss; the radiation is similar to that from a normal warm body.

5. The fate of a collapsing shell

Let us pause for a moment to see what the above discussion says about holography. We again
find a difference between the case of the black hole and the case of gauge/gravity duality:

(a) For the black hole, we have found that microstates do not have traditional horizons
where we would have found a featureless vacuum around the area of the horizon. Instead,
the information of the microstate is encoded in the detailed structure of the microstate
at the location where the horizon would have been. In short, there are real degrees of freedom
at the surface which we use to get the notion of holography.

(ii) In the case of gauge/gravity duality, there are no degrees of freedom apparent at the
location of the boundary used for holography. This fact is related to the observation that the
holographic boundary in this case can be moved to an arbitrary location in the AdS region.

We now turn to addressing a common question with black holes: what happens to a shell
that is collapsing to make a black hole?

Consider a shell of mass \( M \) that is collapsing through its horizon radius \( R \sim GM \). In ordinary
3+1 dimensional gravity the wavefunction of the shell moves in the expected way to smaller \( r \),
creating the traditional black hole geometry. But in string theory the \( e^{S_{\text{bek}}} \) states of the hole
give alternate wavefunctions with the same quantum numbers as the shell. There is a small
amplitude for the wavefunction of the shell to tunnel into one of these microstate wavefunctions. We may estimate this amplitude as $A \sim e^{-S_{\text{gravity}}}$ where $S_{\text{gravity}} = \frac{1}{16\pi G} \int R \sqrt{-g} d^4 x$ and we use $\sim GM$ for all length scales. This gives

$$S_{\text{gravity}} \sim \frac{1}{G} \int R \sqrt{-g} d^4 x \sim \frac{1}{G (GM)^4} (GM)^4 \sim GM^2 \sim \left(\frac{M}{m_p}\right)^2$$

Thus $A \sim e^{-(M/m_p)^2}$ is indeed tiny. But we must now multiply by the number of states that we can tunnel to, and this is given by $N \sim e^{S_{\text{bek}}}$ where

$$S_{\text{bek}} = \frac{A}{4G} \sim \frac{(GM)^2}{G} \sim \left(\frac{M}{m_p}\right)^2$$

Thus the smallness of the tunneling amplitude is offset by the remarkably large degeneracy of states that the black hole has [20]. The wavefunction of the shell tunnels into these ‘fuzzball’ states in a time much shorter than the evaporation time of the hole [21], and then these fuzzballs states radiate energy much like any other normal body. In short, the semiclassical approximation that leads to the standard black hole geometry gets invalidated by the large measure of phase space over which the wavefunction of the shell can spread.

6. The failure of the ‘good slicing’ argument

We can rephrase the above discussion in the following way. The information problem arose due to the ‘good slicing’ of the black hole, in which we keep the slice smooth while stretching it more and more (fig.2). This stretching creates the entangled pairs that lead to Hawking’s paradox. But with the above estimate of tunneling rates, we have a different situation. On an early time slice we can indeed arrange the wavefunction so that it describes a semiclassical spacetime containing a collapsing shell. But the wavefunction of a later slice has to be obtained by evolution (using the Hamiltonian constraint) from the wavefunction on the initial slice. If we had a simple theory of quantum gravity, like canonically quantized general relativity, then evolution of the earlier slice would indeed give the ‘stretched’ slice. But in string theory the situation is different. We have an enormous space of alternative solution to the gravity theory, where for example the compact directions can ‘pinch off’ to make monopole pairs. We can take the wavefunction on the initial slice to be peaked around the smooth gently curved manifold, but the evolution will force this initial wavefunction to spread over the space of fuzzball solutions. Thus instead of getting the ‘stretched’ slice, we get a linear combination of fuzzballs, which then radiate from their surface just like any other warm body. We depict this spreading of the wavefunction in fig.3.

We can also use this picture to address some related questions. Marolf has discussed the information problem in the following language. Suppose we assume that the gravity theory is holographically dual to a boundary field theory. Then the boundary theory contains all the information in the bulk, and we can connect the bulk state at early times to the bulk state at late times by just evolving unitarily in the boundary theory. Thus it seems impossible to have information loss in any theory with a gravity dual.

While this argument helps to frame the information problem sharply, note that we cannot use it to argue away the paradox itself. The reasons are the same as those we discussed in section 3. If we start with some simple theory of gravity, then we do not know that it will have a holographic dual at the energies where black holes will form. If we start with the field theory and define bulk gravity as its dual, then we do not know that we will get black holes.

But Marolf’s argument can be used to sharpen the puzzles arising from the paradox, as was done recently in a paper by Heemskerk, Marolf and Polchinski [23]. These authors asked if the dual field theory captured the state of a ‘Schrodinger’s cat’ that was behind the horizon.
Figure 3. (a) The stretching of ‘good slices’ in the traditional black hole geometry leads to pair creation by the Hawking process and the consequent information problem. (b) If there are $Exp[S]$ fuzzball solutions, the wavefunction giving semiclassical geometry on the initial slice spreads over this vast phase space of solutions after some evolution, and we no longer get the traditional pair creation with growing entanglement.

At early times when there was no black hole, the dual field theory presumably did capture all details of the bulk, and therefore captured the state of the cat. Evolution in the boundary is a known unitary evolution, so if the cat then evolved to be at a point inside the horizon, then its state would also be captured by the boundary theory. What appears puzzling though is that the radiation emitted from the hole should also capture the information of the cat, and one can draw a Cauchy slice that captures both this radiation and the cat behind the horizon. Since information cannot be duplicated, one expects a kind of ‘complementarity’ [24] which somehow does not create a contradiction between the presence of these two copies. This argument, of course, is just a restatement of the usual information problem of black holes; it appears sharper in the context of the dual field theory since this field theory is manifestly unitary and does not duplicate information.

With the help of fig.3 we can see how the puzzle is resolved with fuzzballs. Suppose the initial slice contains the collapsing shell and the cat, in their usual semiclassical wavefunctions. But the slice that captures a significant part of the radiation involves a lot of stretching, and this evolution spreads the wavefunction away from semiclassical slices to a linear combination of fuzzballs. The data of the cat is encoded in these fuzzballs, and is then carried away in the radiation from the fuzzballs, as in the process of ergoregion emission described above in section 4.

7. The infall problem and complementarity
It is interesting that we can get a notion of complementarity from the fuzzball picture. We have seen that spacetime ‘ends’ before the horizon is reached in black hole microstates. What happens when a heavy object ($E \gg kT$) falls onto the fuzzball? Thus question was addressed in [25], where it was argued that we excite collective modes of the fuzzball whose structure is relatively insensitive to the exact microstate that the hole was in. Further, applying ideas of Israel [26], Maldacena [27] and Van Raamsdonk [28], one can argue [25] that these collective modes will have the same spectrum as the spectrum of ‘empty space with a horizon’. Thus low energy ($E \sim kT$) modes see the detailed structure of the microstate, allowing information to be
carried out in Hawking radiation, while the approximate behavior of heavy infalling quanta can be approximated over the crossing timescale by the traditional geometry with vacuum around the horizon.

8. Summary

We have seen that the term ‘holography’ has been used in two different contexts in the study of gravity. In gauge/gravity duality, holography maps a gravity theory to an abstract theory at an arbitrarily placed boundary. There are no gravitational degrees of freedom at this boundary itself, its area does not give an entropy, and there is no radiation from this boundary leading to the Hawking problem. By contrast, the origins of holography lie in black hole thermodynamics, where the holographic boundary is the horizon. The horizon location is fixed, its area gives the entropy, and radiation from this horizon leads to the information paradox. If we confuse these two notions of holography, then we arrive at an erroneous conclusion that the information paradox is somehow magically evaded if we assume an imaginary surface at the horizon that carries the degrees of freedom of the hole. In actuality, the information paradox is evaded only because there are real degrees of freedom (hair) at the location of the horizon, and the construction of this hair is accomplished in string theory by the fuzzball construction.

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