Probing short-lived fluctuations in hadrons and nuclei

Stéphane Munier

Centre de physique théorique, École Polytechnique, CNRS, Palaiseau, France

Abstract. We develop a picture of dipole-nucleus (namely dilute-dense) and dipole-dipole (dilute-dilute) scattering in the high-energy regime based on the analysis of the fluctuations in the quantum evolution. We emphasize the difference in the nature of the fluctuations probed in these two processes respectively, which, interestingly enough, leads to observable differences in the scattering amplitude profiles.

Keywords: Quantum chromodynamics, high-energy scattering, hadronic cross sections, parton evolution, color dipole model, fluctuations

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This paper introduces and summarizes the results recently published in Ref. [1], from a less technical viewpoint (see Ref. [2] for a complementary presentation), and with illustrations from numerical simulations (see Figs. 2 and 3 below). Our goal is to understand the qualitative properties of the short-lived and short-distance (with respect to $1/\Lambda_{\text{QCD}}$) quantum fluctuations, namely the ones that are probed most efficiently in deep-inelastic scattering experiments in the small-$x_B$ regime, or in observables in proton-proton and proton-nucleus scattering whose cross sections may be related to dipole amplitudes (see e.g. Ref. [3]). (A recent general review of high-energy QCD can be found in Ref. [4]).

We shall first describe qualitatively the scattering of two color dipoles and of a dipole off a nucleus, before turning to an analysis of the quantum fluctuations. We shall eventually review the most striking quantitative prediction derived from our discussion.

Our picture relies on the well-known color dipole model [5], which describes, in the framework of perturbative quantum chromodynamics, how the quantum state of a hadron builds up from a cascade of dipole splittings.

PICTURE OF THE INTERACTION OF A SMALL DIPOLE WITH QCD MATTER

Scattering of a dipole off a dilute target and off a dense target

We start with the scattering of two color dipoles (concretely, e.g. two quark-antiquark pairs) of respective transverse sizes $r_0$ and $R_0$ with the ordering $|r_0| < |R_0|$. At low energy, the forward elastic scattering amplitude of the dipoles consists in the exchange of a pair of gluons. Since the dipoles are colorless, this exchange can take place only if their sizes are comparable (on a logarithmic scale), and if the scattering occurs at coinciding impact parameters. Once these conditions are fulfilled, the cross section is parametrically proportional to $\alpha_s^2$.

Let us go to larger center-of-mass energies $\sqrt{s}$ by boosting the small dipole to the rapidity $y = \ln(sR_0^2)$. The most probable Fock state at the time of the interaction is then a dense state of gluons, which may be represented by a set of dipoles [5] (see the sketch in Fig. 1a). The amplitude is now enhanced by the number of these dipoles which have a size of the order of $R_0$. We may define a “one-event amplitude” $T_d^{\text{one event}}$, which is related to the probability that gluons are exchanged between one given realization of the dipole evolution and the target dipole. If $n(r_0, r|y)$ denotes the number of dipoles in a given realization of the evolution up to rapidity $y$ of size $r$, starting with a dipole of size $r_0$, then $T_d^{\text{one event}}(r_0, R_0|y) \simeq \alpha_s^2 \times n(r_0, R_0|y)$, where it is understood that the impact parameters of the dipoles which scatter need to coincide (up to a distance of order of the smallest size). Of course, the dipole number fluctuates from event to event, and so does $T_d^{\text{one event}}$. The physical amplitude measured in an experiment is proportional to the average of the latter over events, namely over realizations of the dipole evolution. We conclude that the scattering of two dipoles of respective sizes $r_0$ and $R_0$ probes the density of gluons of transverse size $R_0$, at a given impact parameter, in typical quantum fluctuations of a source dipole of size $r_0$ appearing in the quantum evolution over the rapidity $y$.

We turn to the case in which instead of the dipole of size $R_0$, the target consists in a large nucleus. At low
FIGURE 1. Diagrams contributing to the forward elastic scattering amplitude of a dipole off another dipole (a), and off a nucleus (b). In the latter case, the nucleons, represented by small disks, are assumed independent and present in large numbers. This picture is valid in the restframe of the target dipole and nucleus, in such a way that all quantum evolution takes place in the projectile dipole (initially of transverse size $r_0$). In the large number-of-color limit, the Fock state of the projectile is a set of color dipoles at the time of the interaction [5]. In the dipole-dipole case, the scattering consists in the exchange of a pair of gluons between one of the dipoles resulting from the evolution, and the target dipole. In the dipole-nucleus case, the dipoles whose size is larger than the inverse saturation momentum of the nucleus exchange an arbitrary number of pairs of gluons with independent nucleons in the nucleus, eventually leading to a resummation à la Glauber-Mueller, and physically, to absorption.

energy, the dipole now scatters through multiple gluon exchanges since the nucleus is dense. The Glauber-Mueller summation of the latter implemented in the McLerran-Venugopalan model [6] gives the scattering amplitude in the form $T_A(r_0) = 1 - e^{-r_0^2 Q_A^2/4}$. A new momentum scale $Q_A$ is generated: It is the saturation momentum of the nucleus, and grows with the nucleus mass number $A$ like $Q_A^2 \sim A^{1/3}$. This amplitude may actually be approximated by a Heaviside $\Theta$ function: $T_A(r_0) \approx \Theta \left[ \ln(\frac{r_0^2 Q_A^2}{4}) \right]$. This has the advantage of simplifying the interpretation of the scattering of the evolved dipole with the nucleus.

If one increases the center-of-mass energy by boosting the dipole, then at the time of the interaction, in each event, the nucleus “sees” a set of dipoles. The interaction takes place if and only if at least one dipole in this set has a size larger than the inverse saturation momentum. Hence $T_A^{\text{one event}}(r_0, Q_A | y) \approx 1$ if at least one of the dipoles in the evolution is larger than $\sim \frac{1}{Q_A}$, or 0 else. The cross section that is measured in an experiment by counting events (which technically is an averaging of $T_A$ over the events) is thus the probability that at least one dipole in the Fock state of the source dipole evolved to the time of the interaction has a size larger than $\sim \frac{1}{Q_A}$ (see Fig. 1b). Hence, while dipole-dipole scattering probes the bulk of the quantum state, the scattering of a dipole of size $r_0$ with a nucleus is sensitive to the probability distribution of the size of the largest fluctuation generated in the quantum evolution of the dipole, namely, in a mathematical language, to the statistics of extremes.

### Rapidity evolution of a dipole

We have just argued that scattering amplitudes can be thought of as probes of the quantum fluctuations of the dipole. The latter are best pictured with the help of the color dipole model. We shall now describe the way how these fluctuations build up (see Ref. [7] for an extensive discussion), in order to arrive at a model that leads to quantitative predictions for the scattering amplitudes.

In these matters, useful intuition can be gained from numerical implementations of the dipole model in the form of a Monte Carlo event generator [8]. Therefore, we shall illustrate our arguments with the help of simulations of the toy model constructed in Ref. [9].

The quantum evolution of the dipole proceeds through successive independent splittings $1 \rightarrow 2$, with rate per unit rapidity given by the probability distribution $\frac{d^P}{dy} = \bar{a} \frac{r_0^2}{r_1^2 (r_0 - r_1)^2} \frac{d^2 r_1}{2\pi}$, where $r_0$ is the size vector of the initial dipole,

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1 This statement relies on the fact that the exponential in the McLerran-Venugopalan formula is “steep”, in some sense. The full argument specifies what “steep” means, which is not completely straightforward, see the discussion in Ref. [1].
$r_1$ and $r_0 - r_1$ are the size vectors of the offspring, and $\bar{\alpha} \equiv \alpha_s N_c / \pi$. The stochastic process of dipole evolution is a branching diffusion. Indeed, if one looks at the distribution of the log of the dipole sizes at a given impact parameter, it has a Gaussian-like shape growing exponentially with the rapidity, see Fig. 2a. The stochasticity has a strong effect wherever the number of dipoles is of order unity, namely either in the beginning of the evolution (for low rapidities), or at the edges of the distribution. Elsewhere, in the bulk of the distribution, the law of large numbers applies, and therefore, the rapidity evolution of the dipole density is nearly deterministic.

According to these observations, in Ref. [1], we conjectured that there are essentially two types of fluctuations: The “front fluctuations” that come from the early stages of the evolution, and the “tip fluctuations” occurring throughout the evolution in the regions in which the dipole density is low. Both kinds of fluctuations shift the distribution of dipoles towards larger sizes, by say $\Delta$ and $\delta$ respectively, in logarithmic scale. We conjectured that the latter random variables are distributed exponentially, $p(\Delta) \sim e^{-y_{\Delta}}$ and $p(\delta) \sim e^{-y_{\delta}}$, where $y_{\delta} \simeq 0.63$ is a number determined by the eigenvalues of the BFKL kernel.

The way how the scattering probes the quantum evolution is represented in Figs. 2 and 3. Since the number of dipoles grows exponentially with the rapidity, $T$ would at some point violate unitarity if the formula for $T_d$ applied without restrictions. But we know that saturation effects (for example rescatterings, gluon recombination...) may occur in the course of the evolution so that the effective number of dipoles does never grow larger than $1/\alpha_s^2$ (see Fig. 2b). These saturation effects are crucial in the dipole case, since the scattering is sensitive to the shape of the bulk of the distribution of the quantum fluctuations. They make the averaging over events nontrivial.\(^2\)

In the nucleus case (Fig. 3), it is the size of the largest dipole which determines the shape of the amplitude. Therefore, both front and tip fluctuations must be taken into account. On the other hand, the shape of the amplitude is less sensitive to saturation effects in the dipole evolution, at variance with the dipole case, at least for rapidities parametrically less than $\frac{1}{\alpha} \ln^3 \frac{1}{\alpha_s^2}$.

\(^2\) In the dipole case, only the front fluctuations have a significant effect on the amplitude. This holds true for low enough rapidities, namely $\bar{\alpha} y \ll \ln^3(1/\alpha_s^2)$; see e.g. [10] for a discussion of how this scale comes about.
FIGURE 3. Two realizations (a) and (b) of the same dipole evolution without saturation in the simplified model of Ref. [9]. As in Fig. 2, we plot the dipole number $n(r_0, r|y)$ in bins of the logarithm of the dipole size. The shaded zone represents the target nucleus of saturation momentum $Q_A$, which absorbs all dipoles of size larger than $2/Q_A$. Hence this time, the scattering probability in one realization is 1 if at least one dipole is larger than $2/Q_A$, and 0 else. The solid lines represent the dipole density integrated over the impact parameter, to which the nucleus would actually be a priori sensitive, while the dashed lines represent the density at zero impact parameter. We see that the largest dipole is always at zero impact parameter, which illustrates that it is enough to understand the properties of the dipole distribution at fixed impact parameter in order to be able to compute the scattering amplitude. The (a)-realization turns out to have a larger density than the (b)-realization, since the scattering would occur for smaller values of $\bar{\alpha} y$ in the former case than in the latter case. Note that the scale on the $y$-axis is logarithmic in both cases.

PHENOMENOLOGICAL PREDICTIONS

From our model for the fluctuations, an elementary calculation leads to the shape of the scattering amplitudes in the dipole-dipole and dipole-nucleus cases:

$$\langle T(r_0|y) \rangle \sim \left( \frac{\gamma_0 Q_2(y)}{r_0^2 Q_2(y)} \right)^{\gamma_0} \begin{cases} 
\ln \frac{1}{r_0^2 Q_2(y)} & \text{if the target is a nucleus,} \\
\ln^2 \frac{1}{r_0^2 Q_2(y)} & \text{if the target is a dipole.}
\end{cases}$$

The calculation simply averages the amplitude $T_{\text{one dipole}}$ over events, assuming that the stochasticity is fully captured by the distribution of the random variables $\Delta$ and $\delta$, see again Ref. [1] for all details and more results.

The fact that the power of the logarithmic prefactor be different in the dipole-dipole and dipole-nucleus cases is our main result.

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