Effect of in-plane magnetic field on magnetic phase transitions in $\nu = 2$ bilayer quantum Hall systems

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By using the effective bosonic spin theory, which is recently proposed by Demler and Das Sarma [Phys. Rev. Lett. 82, 3895 (1999)], we analyze the effect of an external in-plane magnetic field on the magnetic phase transitions of the bilayer quantum Hall system at filling factor $\nu = 2$. It is found that the quantum phase diagram is modified by the in-plane magnetic field. Therefore, quantum phase transitions can be induced simply by tilting the magnetic field. The general behavior of the critical tilted angle for different layer separations and interlayer tunneling amplitudes is shown. We find that the critical tilted angles being calculated agree very well with the reported values. Moreover, a universal critical exponent for the transition from the canted antiferromagnetic phase to the ferromagnetic phase is found to be equal to 1/2 within the present effective theory.

73.40.Hm, 73.20.Dx, 75.30.Kz

Based on a microscopic Hartree-Fock theory and a long wavelength field theory, recent theoretical works predict that, in the $\nu=2$ bilayer quantum Hall (QH) system under quite general experimental conditions, there can be three qualitatively different quantum magnetic phases: the fully spin polarized ferromagnetic phase (F), the paramagnetic symmetric or spin singlet (S) phase, and the intermediate canted antiferromagnetic (C) phase. There have been some encouraging experimental evidence for the existence of the novel C phase through inelastic light scattering spectroscopy. However, it is difficult to make a precise experimental verification of the proposed quantum phase transitions, because a given sample (with fixed values of the system parameters such as well widths, separations, etc.) is always at a fixed point in the quantum phase diagram calculated in Refs. [8]. Hence some extra parameters (for example, the in-plane magnetic field $H$ or the bias voltage $eV$) are necessary in order to allow experimentally a continuous tuning of the $\nu = 2$ QH system through the phase boundaries.

In this paper, we present a theoretical investigation of the influence of an external in-plane magnetic field on the quantum phase diagram of the bilayer system. Although the topology of the phase diagram given by the Hartree-Fock theory should be correct, the Hartree-Fock theory overestimates the stability of the C phase. In order to go beyond the Hartree-Fock theory, an effective hard-core boson theory was proposed by Demler and Das Sarma. Later, it is pointed out that the hard-core bosons introduced in Ref. [8] form an exact Schwinger boson representation of an effective spin-1/2 system, and thus theoretical techniques developed for spin models can be readily applied to the present problem. Hence we extend this effective model to the case that a parallel magnetic field is applied, and study its effect on the magnetic phase transitions of the bilayer QH system at $\nu = 2$. It is found that the system may make multiple transitions from the S phase to the C phase, and finally to the F phase as the parallel magnetic field increases. The general behavior of the critical tilted angle for different layer separations and interlayer tunneling amplitudes is shown, and we find that the critical tilted angles for the transition from the C phase to the F phase coincide quantitatively with the experimental values. Moreover, it is found that the critical exponent of this transition has a universal value, which is equal to 1/2 within the present model.

In this work we assume that the electron-electron interaction and Zeeman energies are much smaller than the Landau level splitting due to the strong perpendicular magnetic field $B_{\perp}$. In this paper, we assume that the orbitals of the double-layer system have the form $H = H_0 + H_1$ with

$$H_0 = -\frac{\Delta_{\text{SAS}}}{2} \sum_{k,\sigma} \left( e^{-iQk} c_{1,k,\sigma}^\dagger c_{2,k,\sigma} + h.c. \right) - \frac{\Delta}{2} \sum_{a,k,\sigma} \sigma c_{a,k,\sigma}^\dagger c_{a,k,\sigma}$$

and

$$H_1 = \frac{1}{2} \sum_{a_1,a_2} \sum_{k,p,q} \sum_{\pi_1,\pi_2} V^{a_1,a_2}(q,k-p) \times c_{a_1,k+q,\pi_1}^\dagger c_{a_2,p,\pi_2} c_{a_2,p+q,\pi_2} c_{a_1,k,\pi_1}$$

where $c_{a,k,\sigma}^\dagger$ creates an electron in the $a$-th layer ($a = 1, 2$) with spatial wavefunction $u_k(x,y) = e^{-iky} e^{-i(x-kL)^2/2\ell^2}/\sqrt{\pi\ell L}$ and spin $\sigma/2$ ($\sigma = \pm 1$) in the direction of the total magnetic field $B = B_0 \hat{y} + B_{\perp} \hat{z}$. ($L$ is the length of system along the $y$ direction and...
where $\Delta_{\text{SAS}}$ is the tunneling-induced symmetric-antisymmetric energy separation, and $\Delta_z$ is the Zeeman energy in the absence of $B_\parallel$. We notice that $B_\parallel$ induces an Aharonov-Bohm phase factor $\exp(\pm iQk_l^2)$ depending on the sense of interlayer tunneling with $Q = B_\parallel d / B_\perp l^2$.

The matrix elements of the intralayer Coulomb interaction are

$$V_A(p_1, p_2) = V^{11}(p_1, p_2) = V^{22}(p_1, p_2) = \sum_q v_A(q) \delta_{p_1, p_2} e^{-q^2 l^2 / 2} e^{i\varepsilon_p q l^2}, \quad (5)$$

and the matrix elements of the interlayer Coulomb interaction are

$$V_E(p_1, p_2) = V^{12}(p_1, p_2) = V^{21}(p_1, p_2) = \sum_q v_E(q) \delta_{p_1, p_2} e^{-q^2 l^2 / 2} e^{i\varepsilon_q q l^2}. \quad (6)$$

Here $v_A(q) = (2\pi e^2 / eq) F_A(q, b)$ and $v_E(q) = v_A(q) F_E(q, b) e^{-q d / 2}$ are the Fourier transforms of the intralayer and the interlayer Coulomb interaction potentials, respectively. $\varepsilon$ is the dielectric constant of the system, and $d$ is the interlayer separation. In order to make a detailed comparison between the experimental data and the theoretical predictions, we have also included finite-well-thickness correction by introducing the form factor $F_A(q, b)$ ($F_E(q, b)$) in the intralayer (interlayer) Coulomb matrix elements, where $F_A(q, b) = 2/bq - 2(1 - e^{-qb}) / b^2 q^2$, $F_E(q, b) = 4 \sin^2 (q b / 2) / b^2 q^2$, and $b$ is the width of a quantum well.

Due to the Aharonov-Bohm phase factor in the tunneling process, if the electrons of the bilayer system are in the symmetric state for $B_\parallel = 0$, the inclusion of an in-plane field twists the original interlayer phase coherence and results in an increase in the interlayer Coulomb energy.\[3\] This effect can be made explicit, if one absorbs the Aharonov-Bohm phase factor in $H_0$ and makes the matrix elements of $H_0$ real by redefining $\tilde{c}_{1, k, \sigma} = \exp(iQk_l^2 / 2) c_{1, k, \sigma}$ and $\tilde{c}_{2, k, \sigma} = \exp(-iQk_l^2 / 2) c_{2, k, \sigma}$, which can be considered as a kind of pseudospin rotation. In terms of the new operators $\tilde{c}_{a, k, \sigma}$, the matrix elements of the intralayer and the interlayer Coulomb interactions become

$$\tilde{V}^{11}(p_1, p_2) = \tilde{V}^{22}(p_1, p_2) = V_A(p_1, p_2),$$

$$\tilde{V}^{12}(p_1, p_2) = \left( \tilde{V}^{21}(p_1, p_2) \right)^* e^{iQp_1 q l^2}. \quad (7)$$

Notice that the matrix elements of the interlayer Coulomb interaction become functions of $B_\parallel$. Consequently, in term of the $\tilde{c}_{a, k, \sigma}$ operators, the microscopic Hamiltonian at $B_\parallel \neq 0$ can be considered as that at $B_\parallel = 0$ with the modified matrix elements $\Delta_{\text{SAS}}, \Delta_z$, and $\tilde{V}_{\text{a}, a_2}$. From these discussions, one can readily extend the effective hard-core boson theory to the $B_\parallel \neq 0$ case. We first give a brief review of the model for the $B_\parallel = 0$ case\[3\] and then provide the explicit relations between the matrix elements of the microscopic Hamiltonian and the parameters in the effective hard-core boson theory, such that the effect of an external in-plane magnetic field can be easily incorporated.

The Hamiltonian of a simple bilayer lattice model to describe the physics of the bilayer $\nu = 2$ QH system at $B_\parallel = 0$ may be written as

$$\mathcal{H} = -\frac{\Delta_{\text{SAS}}}{2} \sum_i (c_{1, i, \sigma}^\dagger c_{2, i, \sigma} + c_{2, i, \sigma}^\dagger c_{1, i, \sigma})$$

$$-\Delta_z \sum_i (S_{1, i}^x + S_{2, i}^x)$$

$$+ \frac{e_\epsilon}{2} \sum_i \left( (n_{1, i} - 1)^2 + (n_{2, i} - 1)^2 \right)$$

$$- J \sum_{(i,j)} (S_{1, i} S_{1, j} + S_{2, i} S_{2, j}) \quad (8)$$

where $i$ is the in-plane site (intra-Landau-level) index, and $\sigma$ is the spin index. $S_{1, i}^a = \sum_{\alpha, \beta} c_{1, i, \sigma, \alpha}^\dagger \left( \sigma_{a, \alpha}^\beta / 2 \right) c_{1, i, \sigma, \beta}$ and $n_{1, i} = \sum_{\sigma} c_{1, i, \sigma}^\dagger c_{1, i, \sigma}$ are spin and charge operators for layer 1, with analogous definitions for layer 2. The effective Heisenberg coupling $J$ and the local charging energy $e_\epsilon$ of this model can be estimated as follows. In order to have the same magnon spectrum in the F phase, one must impose that\[4\] $Ja^2 / 2 = A l^2$ with the lattice constant $a = \sqrt{2\pi l}$ and $A = (1 / 4) \sum_k v_A(k) |k|^2 l^2 \exp(-k^2 l^2 / 2)$. Thus

$$J = \frac{1}{4\pi} \sum_k v_A(k) |k|^2 l^2 e^{-k^2 l^2 / 2}. \quad (9)$$

On the other hand, $e_\epsilon$ can be estimated from $H_1$ under the Hartree-Fock approximation for the system in the F phase, which is given by

$$e_\epsilon = \sum_k V_A(k, 0) - \sum_k V_E(k, 0). \quad (10)$$

Under the simplifications in which the total charge fluctuations are left out and only the lowest two energy states for a given Landau orbital are kept, the effective bilayer lattice model can be further reduced to a hard-core boson theory\[4\] and it leads to a phase diagram which is more precise than that given by the Hartree-Fock theory and is actually exact within the reduced Hamiltonian.\[3\] The phase boundary separating the F and the C phases and that separating the C and the S phases can be written as

$$\Delta_z = -E_\epsilon - J(1 - \sin 2\theta), \quad (11)$$

$$\Delta_z = -E_\epsilon - J(1 + \sin 2\theta), \quad (12)$$
where the parameters $E_v$ and $\theta$ are:

$$E_v = \frac{\epsilon_c}{2} - \sqrt{\Delta_{\text{SAS}}^2 + (\epsilon_c/2)^2},$$  (13)

$$\theta = \tan^{-1}\left[\frac{\epsilon_c^2}{\Delta_{\text{SAS}} + \sqrt{\Delta_{\text{SAS}}^2 + (\epsilon_c/2)^2}}\right].$$  (14)

Now we turn to the case of the presence of an in-plane magnetic field. As mentioned before, when $B_{||} \neq 0$, one needs to replace $\Delta_{\text{SAS}}$ and $\Delta_{s}$ by $\Delta_{\text{SAS}}$ and $\Delta_{s}$, respectively. Moreover, the matrix elements of the interlayer Coulomb interaction become $V_E(p_1, p_2) = V_E(p_1, p_2) \exp(\pm ip_1 Q l^2)$. Therefore, the local charging energy becomes

$$\tilde{\epsilon}_c = \frac{e^2}{\ell} \int_0^\infty dx e^{-x^2/2} F_A(x/l, b) - \frac{e^2}{\ell} \int_0^\infty dx e^{-x^2/2} e^{-x d/l} F_E(x/l, b) J_0(Q x l)$$  (15)

($J_0(x)$ is the Bessel function), which is an increasing function of $B_{||}$.

Substituting these modified parameters into Eqs. (12)–(14), the phase boundaries at nonzero $B_{||}$ are obtained. The quantum phase diagram for several in-plane magnetic fields is shown in Fig. 1. (In the followings, the length and the energy units are chosen to be the magnetic length $l$ and the intralayer Coulomb energy $\epsilon_c^2/\ell$.) It is clear that the region of the F (S) phase is expanded (shrunken) as $B_{||}$ increases. The reason that the F phase is enhanced and the S phase is suppressed comes from the fact that the system at $B_{||} = 0$ has a weaker $\Delta_{\text{SAS}}$ and a stronger $\Delta_{s}$ compared to the system at $B_{||} = 0$. From this quantum phase diagram, one finds that a parallel magnetic field can be a useful control parameter to tune the system through the phase boundaries. For example, by applying $B_{||}$, the system can undergo a phase transition from the C phase to the F phase (or from the S phase to the C phase). Indeed, such transitions are observed in recent tilted-field experiments. The samples in Refs. [3] and [4] are initially in the C phase near the F-C phase boundary in the absence of $B_{||}$. The bilayer system transits to the F phase when the tilted angle $\Theta = \tan^{-1}(B_{\perp}/B_{||})$ reaches a certain critical value $\Theta_C$. Thus the present theory can provide a qualitative explanation of the observed transitions in these experiments.

Moreover, we are able to compare the calculated $\Theta_C$ with these experimental results quantitatively. Since it is relatively easy to vary $\Delta_{\text{SAS}}$ and $d$ in fabrication, we focus our attention on the dependence of $\Theta_C$ on them. Our theoretical prediction of the dependence of $\Theta_C$ on $\Delta_{\text{SAS}}$ (while the other parameters are fixed) is shown in Fig. 2. We find that $\Theta_C$ is a monotonic increasing function of $\Delta_{\text{SAS}}$. It is expected because the system with a larger $\Delta_{\text{SAS}}$ locates at the phase diagram with a farther distance to the F-C phase boundary, and a larger bending of the F-C phase boundary caused by a bigger $B_{||}$ is necessary to make a transition to the F phase. Adopting the sample parameters reported in Refs.[3] and [4], we obtain the theoretical values of $\Theta_C = 33.5^\circ$ and $48^\circ$, respectively (see Fig. 2), which agree very well with the measured values: $\Theta_C \approx 37^\circ$ in Ref.[3] and $\Theta_C \approx 50^\circ$ in Ref.[4]. The close agreement gives us confidence on the accuracy of the phase diagram calculated by using the effective bosonic spin theory. We also find that, near the transition point $\Delta_{\text{SAS}} = \Delta_{c(S)}$, both curves in Fig. 2 (and other curves for several sets of system parameters) fit very well with the function $\Theta_c \approx (\Delta_{\text{SAS}} - \Delta_{c(S)})^\alpha$, where $\alpha = 1/2$. Indeed, by using series expansion for small $\Theta_c$ and $\Delta_{\text{SAS}} - \Delta_{c(S)}$, this functional form can be derived from the formula of the F-C phase boundary. This result means that $\alpha$ is a universal critical exponent at the C-F phase transition, and its value ($\alpha = 1/2$) may reflect the mean-field character of the present approach.

The dependence of the critical tilted angle $\Theta_C$ on layer separation $d$ is shown in Fig. 3 for several different sets of sample parameters. It can be seen that the critical tilted angle drops slowly at large $d$. In fact, for some cases it seems that $\Theta_C$ may not drop to zero even when the layer separation approaches infinity, which means that the C phase is stabilized in the single-layer limit. This is an artifact due to fixing the interlayer tunneling amplitude $\Delta_{\text{SAS}}$ for a given curve. If the dependence of $\Delta_{\text{SAS}}(d)$ can be known and incorporated in the calculation, the critical tilted angles should eventually drop to zero since the C phase is expected to disappear when the two layers are completely decoupled. Therefore, the increase of $d$ in Fig. 3 should not be interpreted simply as a process of moving two layers apart. Rigorously speaking, a single curve in Fig. 3 is merely meant for samples with different values of $d$ but the same value of $\Delta_{\text{SAS}}$.

In conclusion, we have shown that the effective bosonic spin theory with no adjustable parameters can describe the experimental observations for the quantum phase transitions with quantitative accuracy. Moreover, we find that the critical exponent of the C-F phase transition has a universal value $\alpha = 1/2$ within the present effective model. It would be quite interesting to see whether future experiments support our prediction or not. Finally, the results presented here can be useful guidelines for experimentalists to design their samples for observing the magnetic phase transitions by tilting magnetic fields.

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