Prediction of thermal characteristics of turbulent spot using Large Eddy Simulation

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Abstract. This research presents the prediction of thermal behavior of a turbulent spot on a flat plate having a constant heat flux of 1893 W/m² using Large Eddy Simulation (LES) with 7.1 million grid points. In this study, the water is used as a working fluid. The mainstream flow on the heating plate having the width of 0.2 m x the length of 0.4 m, corresponding to the local Reynolds number between 42,200 and 98,400. A laminar boundary layer on the test plate was transformed into Bypass transition by injecting the water to initiate the turbulent spot in upward direction and perpendicular to the mainstream flow with the velocity of 26.31 m/s and the period of 0.01 seconds through a 1 mm diameter hole. The results are presented as the contours of Nusselt number and temperature of the spot in the top and elevation views, respectively. They show that the heated near wall water is accumulated by the spot and cause the increase of the temperature inside the spot body in the higher layer flow. The near wall water is replaced by the water from the upper layer and makes the decrease of the surface temperature underneath the spot. Consequently, this leads to the increase of the Nusselt number within the spot bound above the laminar state. The yielded convective coefficient, spot celerities, and half spreading angle from the LES agree well with experimental results reported by other researchers. Thus, this obtained information is a strong evidence to confirm that the LES can provide an accurate prediction of the characteristics of the artificially initiated turbulent spot.

1. Introduction

A turbulent patch or turbulent spot under the boundary layer transition over flat surface can be directly initiated by injecting the water jet in upward direction, perpendicular to the mainstream flow. The spot has been found to have a potential to enhance the heat transfer because it can boost the heat transfer up to 25% above the laminar state with a small input power [1]. The heat transfer is still increasing when the spot propagates further downstream. This turbulent patch or turbulent spot was discovered in 1951 [2]. The turbulent spot plays a key role on boundary layer transition over flat plate and the turbulent flow is performed when a single spot merges with the neighbours. The turbulent spot is a combination of the horseshoe or hairpin vortices [3]. Its structure is arrowhead like shape and this shape alters under the streamwise pressure gradient [4, 5]. The leading and trailing edges of the turbulent spot propagates downstream with the celerity of 74% and 53% of the freestream velocity, respectively [6]. Because the turbulent spot is under the unsteady flow and depends on the instability of the flow, Direct Numerical Simulation – DNS has been used for the prediction of its behavior [7, 8]. However, since the average characteristics of the turbulent spot at a specific moment was reported by Rakpakdee and Chaiworapuek...
Thus, the Large Eddy Simulation-LES is found to have a big potential to simulate the presence of the turbulent spot. It also consumes the computing resource less than the DNS. The Large Eddy Simulation is a mathematical model, used in computational fluid dynamics - CFD. It was first proposed by Smagorinsky [10] in 1963 to simulate atmospheric air current and first utilized by Deardoff [11] in 1970. It is currently applied in many engineering applications such as combustion [12], acoustics [13] and boundary-layer flows [14] etc.

The LES employs the Navier-Stokes equation to resolve a very wide range of time and length scales, directly affect the fluid flows. The detailed flow behaviour such as the velocity fluctuation can be calculated by the LES instead of the Reynolds Averaged Navier Stokes – RANS while the computational cost can be reduced by ignoring the small length scales comparing with the DNS. In LES, these small length scales are neglected by low-pass filtering. They can be viewed as time and spatial averaging from the numerical simulation. Thus, the average behaviour of the detailed turbulent flow can be captured by the LES. This consequently leads to the simulation capability to predict the average behaviour of a turbulent patch, surrounding by the laminar flow. Hence, the current study focuses on the numerical simulation of an artificially turbulent spot over a constant heat flux surface in 3D flow domain. The simulation will be set up as the experiment of Rakpakdee and Chaiworapuek [9] and the results are mutually compared and discussed. The obtained information will unveil the performance of LES in order to predict the characteristics of turbulent spot under the boundary layer transition.

2. Numerical methods and validation

The computational domain in the present study models the transitional flow from the experiment of Rakpakdee and Chaiworapuek [9]. It is rectangular domain, having 0.514 m length in x direction, 0.08 m height in y direction, and 0.2 m width in z direction. The x, y, and z axes refer to the streamwise, heightwise, and spanwise directions, respectively as shown in figure 1. From the setup of Rakpakdee and Chaiworapuek [9], this numerical domain is modeled during x = 0.186 – 0.7 m from the plate leading edge to reduce the computing resource. This streamwise distance corresponds to the local Reynolds number, Rex between 26,200 and 98,400. A 1 mm diameter hole locates on the bottom wall at x = 0.3 m or the Rex of 42,200. It is used to inject the water in perpendicular direction of the mainstream with the speed and duration of 26.31 m/s and 0.01 s, respectively. Beyond the location of the injection hole, the heat flux of 1,893 W/m² is applied on the bottom wall to establish a constant heat flux condition of the flat plate. The outlet is set as pressure outlet of 0 Pa. The left and right boundaries are symmetry while the upper wall and the inlet are the velocity inlet, having the magnitude of 0.179 m/s in the direction of the mainstream flow.

Figure 1. A water flow domain.
The cubic grid, having the size of \( l_x = 0.001 \) m, \( l_y = 0.001 \) m, and \( l_z = 0.001 \) m was meshed through the flow domain. Thus, these caused the 434x81x201 elements or 7.1 million grid points as depicted as figure 2. This study resolved 80% of the turbulent kinetic energy, causing \( l_0 / l_0 = 0.42 \). The \( l_0 \) is the integral length scale that the turbulent kinetic energy is maximum. It can be given as:

\[
l_0 = k^{0.5} / (C_\mu \omega)
\]

where \( k = 0.000071 \), resolved from RANS
\( C_\mu = 0.09 \)
\( \omega = 13.022408 \), resolved from RANS

![Figure 2. The grid in the flow domain.](image)

In the current study, the local Reynolds number on the flat plate can be determined as:

\[
Re_x = \rho u_0 \alpha x / \mu
\]

where \( Re_x \) = local Reynolds number
\( \rho \) = water density (kg/m³)
\( U_0 \) = freestream velocity (m/s)
\( \alpha \) = streamwise distance from the leading edge (m)
\( \mu \) = dynamic viscosity of water (kg/m·s)

As the boundary condition of the flow domain, the local Nusselt number can be predicted following Kays et al. [15] as:

\[
Nu_x = (0.453 Re_x^{1/2} Pr^{1/3}) / \left[ 1 - (\xi/\alpha)^{3/4} \right]^{1/3}
\]

where \( Nu_x \) = local Nusselt number
\( Pr \) = Prandtl number
\( \xi \) = unheated starting length (m)

In this simulation, the convective coefficient and the local Nusselt number are determined as:

\[
h_x = q / (T_s - T_0)
\]

and

\[
Nu_x = h_x \alpha / k
\]

where \( h_x \) = local heat transfer coefficient (W/m²·°C)
\[ q = \text{heat flux (W/m}^2\text{)} \]
\[ T_s = \text{surface temperature (°C)} \]
\[ T_a = \text{freestream temperature (°C)} \]
\[ k = \text{thermal conductivity (W/m\text{•°C})} \]

The local Nusselt number from this simulation is validated by comparing with those determined from equation (3) as depicted in figure 3. The figure shows that they agree well with each other.

![Figure 3](image)

Figure 3. The comparison between the local Nusselt number from the simulation and the correlation of Kays et al. [15].

For laminar boundary layers over a flat plate, the Blasius solution gives:

\[ \delta(x) = \frac{5x}{Re_{x}^{1/2}} \]  
(6)

where \( \delta = \) the overall thickness of the boundary layer (m)

The thickness and velocity can be normalized as:

\[ \eta = \frac{y}{\delta(x)} \]  
(7)

and

\[ \frac{df}{d\eta} = \frac{u}{u_a} \]  
(8)

where \( \eta = \) normalized thickness
\( y = \) the distance from the wall to a point (m)
\( df/d\eta = \) normalized velocity

The velocity profiles from the simulation are validated with Blasius profile at \( Re_x = 42200, 56200, 70300, 84300 \) and \( 98400 \) as illustrated in figures 4. The comparison shows that the obtained profiles are consistent with the Blasius ones as shown in figure 5. At the location of spot generator, the boundary layer displacement thickness is 0.002 m. It is employed to normalize the length of the flow domain through the simulation.
Figure 4. The location of the flow domain at $Re_x = 42200, 56200, 70300, 84300$ and $98400$.

Moreover, the average convective coefficient calculated inside the bound of the turbulent spot from the LES is validated with the experimental results of Rakpakdee and Chaiworapuek [9] as shown in figure 6. The figure shows that the average heat transfer coefficient from both simulation and experiment is decreasing with time but the numerical results are approximately 10% higher. This is because the heat power, reported by Rakpakdee and Chaiworapuek [9] is the total power of the heater. This power can loss to the other directions even it was well insulated. The actual power, transferring into the water is only partial amount. Thus, the difference, occurring through the comparison can be caused by this reason.
Figure 6. The comparison between the local heat transfer coefficient from LES and experiment of Rakpakdee and Chaiworapuek [9]

3. Numerical results
Figure 7(a) – (b) shows the contour of the instantaneous convective coefficient at the region of the turbulent spot at $t = 3.04$ s. The turbulent spot appears as streaky structure and the maximum magnitude of 650 W/m$^2$·°C locates at the core of frontal streaks as depicted in figure 7(a). These are in agreement with the contour of the spot footprint, ensemble averaged from 30 turbulent spots by Rakpakdee and Chaiworapuek [9] as illustrated in figure 7(b). The size of both spots is not different at this time. However, the maximum convective coefficient from the ensemble averaging method is less than the instantaneous turbulent spot.
Figure 7. Convective coefficient contours from (a) LES and (b) experiment of Rakpakdee and Chaiworapuk [9] in the region of turbulent spot.

In this simulation, the time is counted after injecting the water through a 1 mm diameter hole and the dimensionless time was defined by

\[ \tau = t / (\delta^*/u_0) \]  

where \( \tau \) = the dimensionless time
\( t \) = time (s)
\( \delta^* \) = boundary layer displacement thickness at the water injection (m)

Figure 8(a) - (f) presents the contour of local Nusselt number on the heating surface at \( \tau = 42, 81, 123, 162, 204 \) and 243, respectively. This can refer to the turbulent spot footprint. The x axis is the local Reynolds number from 42,200 to 98,400 while y axis shows the normalized spanwise distance, \( Z = z/\delta^* \) between -50 and 50. The magnitude of the local Nusselt number can be obtained from the colour bar. At \( \tau = 42 - 123 \), the relatively high Nusselt number appears inside the spot bound, consistent with the results reported by Chaiworapuek and Kittichaikarn [1]. This occurs from the process of the spot generation. Beyond this time, the maximum \( N_u \) inside the spot decreases because the effect from the spot initiation fade out and it increases again after \( \tau = 204 \) by the spot maturity. Furthermore, the spot
structure spreads out in spanwise direction with the half spreading angle of 6.45° and stretches in streamwise direction when the spot convects further downstream.

![Figure 8](image.png)

**Figure 8.** The contour of the Nusselt number at \( \tau = \) (a) 42, (b) 81, (c) 123, (d) 162, (e) 204, and (f) 243

Figure 9(a) – (d) shows the contour of the instantaneous temperature inside the turbulent spot at \( \tau = 123, 162, 204, \) and 243. In this figure, x axis is \( Re_x \), in the same range as figure 8. Meanwhile, y axis presents the normalized heightwise distance, \( Y = y / \delta^* \) between 0 and 30. It is noted that these figures are yielded from the subtraction process between temperature contours with and without the presence of the spot. Then, these distributions can present the turbulent spot structure in heightwise direction and the colour bar presents the temperature difference between 0° and 1°. The results show that the instantaneous turbulent spot accumulates the heated near wall water inside itself when it convects downstream. The spot shape, appearing in the figure is not the single structure because the turbulent spot is a combination of the hairpin vortices. These vortices alternately emerge and are consistent to the substructures, described by Sankaran et al. [6]. If the spot structure is reconstructed using the ensemble averaging method, the shape will appear as those reported by Van Atta and Helland [16]. Besides, the
results show that the spot also grows in y direction, meaning that this turbulent patch enlarges in streamwise, spanwise, and heightwise directions simultaneously.

![Figure 9](image_url)

**Figure 9.** The contour of the temperature difference (side view) at $\tau = 123, 162, 204, 243$

The average Nusselt number, $\text{Nu}_{\text{avg}}$, is then calculated within the turbulent spot bound and plotted with the Nusselt number of the heating surface, averaged in the same area as shown in figure 10. The figure presents that the $\text{Nu}_{\text{avg}}$ increases with time and the turbulent spot boosts the heat transfer above the laminar state. At $\tau = 250$, the heat transfer capability of the spot is higher than the level of laminar approximately 21.86% and this difference is increasing with the time. Hence, this increase will eventually lead to the heat transfer level of the turbulent flow.

![Figure 10](image_url)

**Figure 10.** The average Nusselt number within the turbulent spot bound and the Nusselt number of the heating surface, averaged in the same area.
Figure 11 presents the position in streamwise direction of the leading and trailing edges of turbulent spot against the dimensionless time. With this distances, the velocity of the leading and trailing edges of the turbulent spot footprint can be calculated as 0.74 and 0.49 of the freestream velocity, respectively during $\tau = 110 – 250$. These velocities are also in agreement with those obtained by Sankaran et al. [6]. The difference of the velocity of leading and trailing edges causes the enlargement of the turbulent spot in streamwise direction. Thus, the obtained information is a strong evidence to confirm that the LES can provide an accurate prediction in order to determine the characteristics of the artificially initiated turbulent spot.

**Figure 11.** The position of the leading and trailing edges of turbulent spot.

4. **Conclusion**

This study presents the numerical investigation of the turbulent spot characteristics over a constant heat flux surface using Large Eddy Simulation during the $Re_x$ between 42,200 and 98,400. In this simulation, the spot was initiated by injecting the water in upward direction and perpendicular to the mainstream flow with the velocity of 26.31 m/s and the period of 0.01 s through a 1 mm diameter hole. The results are presented as Nusselt number and temperature distributions of the turbulent spot. It shows that the heated near wall water is accumulated by the spot and causes the increase of the temperature inside the spot body. The near wall water is replaced by the water from the upper layer flow and this mechanism makes the decrease of the surface temperature underneath the spot. Consequently, it gains the Nusselt number within the spot bound above the laminar state. The yielded convective coefficient, spot celerities, and half spreading angle from the LES agree well with those from the experiment of other researchers. Thus, this obtained information is a strong evidence to confirm that the LES can provide an accurate prediction for the characteristics of the artificially initiated turbulent spot.

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