Overshooting and $L^1$-Norms of a Class of Nyquist Filters

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Abstract—To tightly control the signal envelope, estimating the peak regrowth between FFT samples is an important sub-problem in multicarrier communications. While the problem is well-investigated for trigonometric polynomials (i.e. OFDM), the impact of an aperiodic transmit filter is important too and typically neglected in the peak regrowth analysis. In this paper, we provide new bounds on the overshooting between samples for general multicarrier signals improving on available bounds for small oversampling factors. In particular, we generalize a result of [1, Theorem 4.10]. Our results will be extended to bound overshooting of a class of Nyquist filters as well. Eventually, results are related to some respective $L^1$-properties of these filters with application to filter design.

Index Terms—PAPR, peak value, peak regrowth, Nyquist filter, $L_p$-norms, noise enhancement

I. INTRODUCTION

The high peak-to-average power ratio (PAPR) of OFDM (or more general: multicarrier) transmit signals is a crucial problem. It has re-attracted much attention recently due to 5G [2], [3]. Multicarrier transmit signals are efficiently generated using some (oversampled) FFT processing. However, to tightly control the signal envelope, estimating the overshooting between FFT samples is an important sub-problem which has been well-investigated in the literature [4], [5], particularly for trigonometric polynomials. On the other hand, the impact of the aperiodic transmit filter is important too and typically neglected in the peak regrowth analysis (representing not trigonometric polynomials). Another problem is that available bounds are quite loose for small oversampling factors which will be typical in upcoming 5G multicarrier systems [2] e.g. in FBMC transmission using very large FFTs.

In this paper we provide new bounds on the overshooting between samples for general multicarrier signals improving on available bounds for small oversampling factors. Our results will be extended to bound overshooting of a class of Nyquist filters as well. Eventually, results are related to some respective $L^1$-properties of these filters with application to filter design.

Notation: The collection of signals whose $p^{th}$ power is integrable is denoted by $L^p(\mathbb{R})$ with the common norm $\| \cdot \|_p$. For $p = \infty$ the norm is given by the supremum norm. For further purposes let us also introduce the space of bounded, continuous signals over $\mathbb{R}$, denoted by $C(\mathbb{R})$ and endowed with the supremum norm.

II. PRELIMINARIES

A. Band-limited signals

We start by investigating the standard band-limited setting: A signal is called band-limited with bandwidth $B$ if the Fourier transform is supported on $[-B, B]$. The set of band-limited signals with bandwidth $B$ in $L^p(\mathbb{R})$ form the Paley-Wiener space $\mathcal{PW}^B_p$. The spaces $L^p(\mathbb{R}), C(\mathbb{R}), C_T(\mathbb{R})$ contain generally signals of which the spectrum cannot be defined in the classical sense so that it becomes distributional. Note that the inclusions $\mathcal{PW}^B_1 \subset \mathcal{PW}^B_2 \subset \ldots \subset \mathcal{PW}^B_{\infty}$ hold for Paley-Wiener spaces.

A signal $f \in \mathcal{PW}^B_p, 1 \leq p < \infty$, can be recovered by its samples by applying the Shannon sampling series; denoting the Nyquist-rate (critical) samples $t_i = \frac{n}{B}, l \in \mathbb{Z}$, we have

$$ f(t) = \sum_{l=-\infty}^{\infty} f(t_l) \frac{\sin (B(t-t_l))}{B(t-t_l)}. $$

Unfortunately, the sampling series [1] fails to converge in general for $p = \infty$. In this case, for every signal $f \in \mathcal{PW}^B_{\infty}$ Schönhage’s sampling series

$$ f(t) = f'(0) \frac{\sin (B\theta)}{B} + f(0) \frac{\sin (B\theta)}{B\theta} + t \sum_{l=-\infty, l \neq 0}^{\infty} \frac{f(t_l) \sin (B(t-t_l))}{B(t-t_l)} $$

can be applied converging uniformly on compact subsets of $\mathbb{R}$. Let us define the following family of kernels:

**Definition 1.** A set $\mathcal{M}^B_L$, is called a reproducing kernel set if

$$ \mathcal{M}^B_L := \left\{ g \in L^1(\mathbb{R}), \hat{g}(\omega) = \begin{cases} \frac{1}{\hat{g}_d(\omega)}, & 0 \leq |\omega| \leq B, \\ 0, & B \leq |\omega| \leq L_c B, \end{cases} \right\}, $$

where $\hat{g}_d(\omega)$ is a real function with $0 \leq \hat{g}_d(\omega) \leq 1$, $\hat{g}_d(B) = 1$, $\hat{g}_d(L_c,B) = 0$.

The real number $L_c \geq 1$ is called the bandwidth expansion factor.

For some reasons that will become clear later on we assume $\hat{g}_d(\omega)$ to be a non-increasing function. The set $\mathcal{M}^B_L$ is in fact not empty; an example of a kernel is given by the trapezoidal
kernel

\[ S_{t_\epsilon}(t) = \frac{2 \sin \left( \frac{(L_\epsilon+1)Bt}{2} \right) \sin \left( \frac{(L_\epsilon-1)Bt}{2} \right)}{\pi (L_\epsilon - 1) B t^2} , \]  

of which the Fourier transform is depicted in Fig. [1].

The reproducing kernel set \( \mathcal{M}_{t_\epsilon}^B \) can be used together with oversampling (beyond Nyquist-rate sampling) for alternative representations of Paley-Wiener spaces. Define \( t_{1,L} := \frac{\pi L}{2B} \) where the real number \( L \geq 1 \) is the oversampling factor. By a simple application of the bounded convergence theorem and provided that \( L \geq L_\epsilon \) we have

\[ f(t) = \frac{\pi}{LB} \sum_{l=-\infty}^{+\infty} f(l_{1,L}) g(t - l_{1,L}) , \quad f \in \mathcal{PW}_B^\infty , \]

i.e. the sampling of \( f \) together with the kernel \( g \) reproduces \( f \). The following theorem is a generalization.

**Theorem 1.** For any \( f \in \mathcal{PW}_B^\infty \) we have:

\[ f(t) = \frac{\pi}{LB} \sum_{l=-\infty}^{+\infty} f(l_{1,L}) g(t - l_{1,L}) \]

where \( g \in \mathcal{M}_{t_\epsilon}^B, L \geq \frac{L_\epsilon + 1}{2} \).

**Proof:** Since \( g \in L^1(\mathbb{R}) \) this can be shown using the classical Poisson sum formula.

**Remark 1.** Notably, if the kernel satisfies the Nyquist intersymbol interference (ISI) criterion such that for some \((L_\epsilon + 1)B/2 \leq LB \leq L_\epsilon B\)

\[ \sum_{k=-\infty}^{+\infty} \hat{g}(\omega - 2LB) = C_N \quad \forall \omega , \]

then the kernel is called a Nyquist filter, where \( C_N > 0 \) is some constant independent of \( \omega \). \( LB \) is called the Nyquist (angular) frequency (half the sampling rate). Moreover, if such a filter is sampled higher than as dictated by [4] then it is called faster than Nyquist (FTN) signaling.

**B. Problem statement: Overshooting**

Overshooting is a classical problem in multicarrier communications and will be referred to here as the peak value problem (PVP). PVP investigates peak growth between the samples of a (possibly oversampled) band-limited signal. For a formal definition of the problem let \( \|f\|_{\Delta t,g} \) be \( l_p \)-sequence norm obtained from sampling \( f \) with rate \( 2LB \).

**Problem 1.** Peak Value Problem: Find a “good” (i.e. tight) upper bound on the constant

\[ C_1(L) := \sup_{\|f\|_{t_{1,L},\infty} \leq 1, f \in \mathcal{PW}_B^\infty} \|f\|_{\infty} \cdot \]

It is interesting to note that \( C_1(L) \) is independent of \( B \). We further note that \( C_1(L) \) is not defined for \( L = 1 \) (i.e. \( C_1(1) = +\infty \)) [5]. Eventually, existence of a function \( f \) such that \( C_1(L) = \|f\|_{\infty} \) is guaranteed [6]. We have from [5]:

**Theorem 2.** Let \( L > 1 \). We have:

\[ C_1(L) = f^*_L(t^*_L) \leq \frac{1}{\cos \left( \frac{\pi}{\pi L} \right)} , \]

Moreover, if \( L \in \mathbb{N} \), then

\[ t^*_L = \frac{1}{2L}, \quad f^*_L(t) = \frac{\cos \left[ \frac{\pi}{\pi L} \right]}{\cos \left[ \frac{\pi}{\pi L} \right]} \]

and

\[ C_1(L) = \frac{1}{\cos \left( \frac{\pi}{\pi L} \right)} \]

Note that PVB is solved when restricting \( \mathcal{PW}_B^\infty \) to

\[ T_N := \left\{ f \in \mathcal{PW}_N^\infty ; f(\theta) = \frac{a_0}{2} + \sum_{k=1}^{N} a_k \cos (k\theta) \right. \]

\[ + b_k \sin (k\theta) , \quad a_k, b_k \in \mathbb{R} \}, \]

containing all degree \( N \) trigonometric polynomials (with normalized angular bandwidth \( N \)). Here, we define

\[ C^{TN}_1(N_1) := \sup_{\|f\|_{\pi L,\infty} \leq 1, f \in T_N} \|f\|_{\infty} \],

i.e. the optimizer set in the peak value problem is restricted to \( T_N \). The extremal signals \( f_2 \) of [6] are denoted as \( T-(N, N_1) \)-extremal which can be exactly calculated [7].

Altogether, PVP is quite well-understood. However, two problems occur in the context of general multicarrier signals:
1) The bounds in [5], [6] are quite good for \( L \geq 2 \); if \( 1 \leq L \leq 2 \) the provided bounds are quite loose
2) The theorems require \( L \geq L_\epsilon \) which is typically not the case for Nyquist filters

The latter observations are a main motivation for the new approach derived in the following sections.

**C. A related problem**

From [6], we obtain the following inequality

\[ \sup_{t \in \mathbb{R}} |f(t)| \leq \|f\|_{t_{1,L},\infty} \cdot \sup_{t \in [0, \frac{\pi}{\pi L}]} \frac{\pi}{LB} \sum_{l=-\infty}^{+\infty} |g(t - l_{1,L})| < \infty , \]

i.e. every kernel \( g \in \mathcal{M}_{t_\epsilon}^B \) defines a bounded, linear operator \( T_{g,L} : \mathcal{PW}_B^\infty \to \mathcal{PW}_{LB}^\infty \). In communication context, the samples do not represent the samples of a band-limited signal with respect to the bandwidth defined by \( B \). It is therefore reasonable to extend the definition region of the operator \( T_{g,L} \) to the space \( C(\mathbb{R}) \), i.e.

\[ T^L_g : C(\mathbb{R}) \to \mathcal{PW}_{LB}^\infty , \quad f \mapsto \frac{\pi}{LB} \sum_{l=-\infty}^{+\infty} f(l_{1,L}) g(t - l_{1,L}) . \]

The norm of this operator is given by

\[ |T^L_g| = \sup_{\|f\|_{\leq 1} \in C(\mathbb{R})} \|T^L_g f\|_{\infty} . \]

The operator norm represents the enhancement of errors in the samples. This leads us to the problem:
Problem 2. Related problem: Find a “good” upper bound on the operator norm

\[ C_2(L, L_\epsilon) := \inf_{g \in \mathcal{M}_L^{\epsilon}} |T_g^L|, \]

If \( L = L_\epsilon \) then \( C_2(L, L_\epsilon) := C_2(L) \).

Again, note that \( C_2(L, L_\epsilon) \) is independent of \( B \). For the purposes of filter design it is also interesting which kernel actually attains this bound. These filters will be called extremal filters and their existence is established in the next theorem.

Theorem 3. For any \( L, L_\epsilon > 1 \) there is an extremal signal \( f^* \), a time instance \( t^* \), and a kernel \( g^* \in \mathcal{M}_L^{\epsilon} \) such that \( C_2(L, L_\epsilon) = (T_{g^*}^L f^*) (t^*) \).

The proof is omitted and can be found in [8].

The main connection of the related problem to PVB is clearly

\[ C_1(L) := C_2(L, L_\epsilon), \quad \text{provided } L \geq \frac{L_\epsilon + 1}{2}, \tag{7} \]

e.i. \( C_1(L) \) represents a lower bound on what can be achieved for \( C_2(L, L_\epsilon) \). Using this approach it was shown in [5] that:

Theorem 4. Suppose \( L > 1 \). Then:

\[ C_2(L, L_\epsilon) \leq \sqrt{\frac{L_\epsilon + 1}{L_\epsilon - 1}} \tag{8} \]

Furthermore, for very small \( L_\epsilon \):

\[ C_2(L, L_\epsilon) \approx \frac{2}{\pi} \log \left( \frac{2L_\epsilon}{L_\epsilon - 1} \right) + O(1) \]

Setting \( L = L_\epsilon \) (as done in [5]), by virtue of (7) we have

\[ C_1(L) \leq C_2(L, L_\epsilon) \leq \sqrt{\frac{L + 1}{L - 1}} \]

and the bound is better for \( L < 2 \) compared to the \( 1/\cos(\frac{\pi}{2L}) \) law but still quite loose. However, a careful analysis reveals that by Theorem 1 only \( L \geq \frac{L_\epsilon + 1}{2} \) is required so that for the same \( L \) the expansion factor \( L_\epsilon \) can be pushed to \( L_\epsilon \leq 2L - 1 \) and since the RHS of (8) is monotone in \( L \) we obtain:

\[ C_1(L) \leq \sqrt{\frac{2L}{2L - 2}} = \sqrt{\frac{L}{L - 1}} \tag{9} \]

Remark 2. In this setting, i.e. \( L = 2, L_\epsilon = 2L - 1 = 3 \) the trapezoidal kernel is the (optimal) extremal filter \( g^*_L \), for the operator norm \( |T_{g^*_L}^L| \) since:

\[ C_1(2) = \frac{1}{\cos \left( \frac{\pi}{2} \right)} = \sqrt{\frac{2}{2 - 1}} = \sqrt{2} \]

We are now improving on this result.

III. MAIN RESULTS

In order to get an upper bound we make the following reasoning: Assume without loss of generality \( B = \pi \), let \( K_n \in \mathcal{P} \mathcal{W}_n^2 \cap \mathcal{P} \mathcal{W}_n^1 \) for some natural \( n \geq 1 \) be given as

\[ K_n(t) := \frac{2n \sin^2 \left( \frac{\pi t}{2n} \right)}{\pi n^2} \geq 0, \]

hence some triangle kernel. The reason for the normalization of the bandwidth to \( \frac{\pi}{n} \) will become clear later on. At this point we could have used \( \pi \) (or any other value) as well. We need the following sub-sampling property.

Lemma 1. Let \( K_n \in \mathcal{P} \mathcal{W}_n^2 \cap \mathcal{P} \mathcal{W}_n^1 \) for some natural \( n \geq 1 \), and \( K_n(0) = 1, K_n \left( \frac{\pi}{n} \right) = 0. \) Then, we have for some positive real \( a \leq 2 \)

\[ \sum_{l=-\infty}^{+\infty} K_n(t - lan) = \frac{1}{an}. \]

Proof: Let \( \hat{K}_n \) be the spectrum of \( K_n \in L^1(\mathbb{R}) \). By the Poisson sum formula we have:

\[ \sum_{m=-\infty}^{+\infty} K_n(t - man) = \frac{1}{an} \sum_{k=-\infty}^{+\infty} \hat{K}_n(\omega_k) e^{i\omega_k t} \]

with \( \omega_k = \frac{2\pi k}{an} \). By assumption \( \hat{K}_n(\omega_k) = 0 \) for \( k \geq 1 \), which gives the final result. \( \square \)

The following theorem is an upper bound on \( C_2(L) \) generalizing Theorem 4.10.

Theorem 5. Let \( L_\epsilon = \frac{n+1}{n}, L = \frac{n+m}{m}, n \in \mathbb{N}, m \in \mathbb{N} \cup \{1\} \).

Then:

\[ C_2(L, L_\epsilon) \leq \frac{1}{2(n+m)} \sum_{l=0}^{2(n+m)-1} \sum_{k=-n}^{n} e^{j k \left( \frac{n}{n+m} - \frac{\pi}{2} \right)} \]
Moreover:

\[ C_2 (L) \geq C_1^{\text{T}N} (N_1) \]

Here, \( N, N_1 \) are taken from any uniform sampling of (frequency) support \([0, \frac{n+1}{n} \pi] \) of some \( g \in \mathcal{M}_{\frac{N}{m}}^\pi \) where all frequencies that fall in the interval where the response equals unity gives the highest in-band frequency \( N \) and all that fall out-of-band give \( N_2 - N \).

**Proof:** For the purpose of practical applicability let us assume a more general setting. We assume that the kernel is in \( g \in \mathcal{M}_{L_e}^\pi \), where \( L_e := \frac{n+1}{n} \), \( n \in \mathbb{N} \), and let the oversampling factor to be of the form

\[ L = \frac{n + m}{n} \]

for some \( n \in \mathbb{N}, m \in \mathbb{N} \cup \{ \frac{1}{2} \} \). Then, the trapezoidal kernel can be represented as

\[ g_{L_e} (t) = \sum_{k=-n}^{n} K_n (t) e^{j \frac{2 \pi k}{n}} \]

where \( K_n \) is a kernel given by

\[ K_n (t) = \frac{2 n \sin^2 (\frac{2 \pi t}{n})}{\pi^2 t^2} \geq 0, \]

the so-called triangle kernel, which can be seen as a special case of the trapezoidal kernel [9]. This is illustrated in Fig. 1b. Clearly, \( K_n \in \mathcal{P}\mathcal{W}_{\frac{\pi}{2}}^k \cap \mathcal{P}\mathcal{W}_{\frac{\pi}{2}}^1 \). Hence, we obtain

\[ C_2 (L) = \sup_{l \in \Omega} \frac{1}{L} \sum_{t=-\infty}^{\infty} |g_{L_e} (t - \frac{1}{L})| \]

\[ = \max_{l \in \Omega} \frac{1}{2 (n + m)} \sum_{l=0}^{2(n+m)-1} \sum_{k=-n}^{n} e^{j \frac{2 \pi k}{n} (\frac{nl}{n+m} - \frac{2 \pi}{n+m})} \]

where \( \Omega = \{ t : \frac{-2(n+1)}{2(n+1)} \leq t \leq \frac{n}{2(n+1)} \} \). Since the term \( |\sum_{k=-n}^{n} e^{j \frac{2 \pi k}{n} (\frac{nl}{n+m} - \frac{2 \pi}{n+m})}| \) is periodic with \( 2(n+m) \) (we sample it at \( \frac{\pi}{n+m} \)), \( K (t) \geq 0, \) and finally

\[ \frac{n}{n+k} \sum_{t=-\infty}^{\infty} K_n \left( t - \frac{nl \cdot 2(n+m)}{n+m} \right) \]

\[ = \frac{1}{2(n+m)} \]

due to Lemma 1.

The construction of the lower bound is omitted due to lack of space.

For \( L_e \neq L \) we obtain the following corollary:

**Corollary 1.** Let \( L = \frac{n+1}{n}, \) \( n \) even. Then:

\[ C_1 (L) \leq \max_{\frac{n}{n+m} \leq \frac{2 \pi}{(n+m)2 \pi}} \frac{1}{n+1} \sum_{l=0}^{n/2} \sum_{k=-n/2}^{n/2} e^{j \frac{2 \pi k}{n} (\frac{nl}{n+m} - \frac{2 \pi}{n+m})} \]

**Proof:** Setting \( m = 1/2 \) yields \( L = (2n+1)/2n \). Replace \( n' = 2n \) so that \( L_e \) can be pushed towards \( L_e := (n' + 2)/n' \) yields the result provided \( n' \) is even and since \( L \geq (L_e + 1)/2 \) for any \( n \).

Numerical computations of the operator norm for different trigonometric polynomials were carried out and shown in Fig. 2 along with the bounds in (5), (6), (8) and the new bound in (10) where, for the sake of simplicity in both figures, the curves are depicted over \( \mathbb{R} \). It is observed that there is strong improvement of the new over the existing bounds.

**IV. EXTENSIONS: OVERSHOOTING AND \( L^1 \)-NORMS OF NYQUIST FILTERS**

**A. Overshooting of Nyquist filters**

Trapezoidal filters have been used in [9] as Nyquist filters. Obviously, to bound the overshooting for Nyquist filters we cannot use the classical PVP results since \( L \leq L_e \) when the Nyquist criterion is enforced, i.e. the transmit signal is actually undersampled with respect to the transmit signal’s bandwidth. On the other hand, the new approach can be used, specifically (10) and (11). More specifically, if \( L_e = \frac{n+1}{n}, L = \frac{n+m}{n}, n \in \mathbb{N}, m \in \mathbb{N} \cup \{ \frac{1}{2} \}, \) overshooting between the samples (i.e. data sequence) is upperbounded by (without loss of generality still \( B = \pi \))

\[ \min \left\{ \sqrt{2n+1}, \max_{\frac{n}{n+m} \leq \frac{2 \pi}{(n+m)2 \pi}} \frac{1}{2(n+m)} \sum_{l=0}^{2(n+m)-1} \sum_{k=-n}^{n} e^{j \frac{2 \pi k}{n} (\frac{nl}{n+m} - \frac{2 \pi}{n+m})} \right\} \]

where we tacitly assumed that the data is bounded by unity. It is also possible to construct more general filters which are “layerwise” composed of trapezoidal filters (or approximations of them), see the Fig. 1b. Then we can bound the overshooting as follows:
\[ C_2 (L) = \sup_{-\pi \leq t \leq \pi} \frac{1}{L} \sum_{l=1}^{n-1} \left| \langle g \rangle_t (t - \frac{l}{L}) \right| \]

\[ \leq \sum_{k=0}^{n-1} (X(\omega_k) - X(\omega_{k+1})) \]

Here, recall that \( S_{L_k} \) is the trapezoidal kernel family, \( X(\omega_k) \) are the frequency sampling points in the decay region, such that \( X(\omega_k) - X(\omega_{k+1}) \geq 0 \) (setting \( X(\omega_{k+1}) = 0 \)), and \( L_k \) are the extension factors, respectively. Hence, we conclude that for such general filters the overshoot is just an average of the individual trapezoidal layers with extension factors \( L_k^k, k = 0, \ldots, n - 1 \).

**B. \( L^1 \)-Norms of Nyquist filters**

The Nyquist criterion is a fundamental property ensures that samples are not interfering with each other. In a practical system nevertheless the samples actually do so as a consequence of the impairments of the communication channel such as channel induced ISI, time/frequency offsets etc. Notably, it is deliberately induced in FTN signalling. The impact of these effects is measured by the opening in the eye diagram of the overlayed signal, and is directly related to the \( L^1 \)-norm (tails) of the used Nyquist or (FTN) filter.

Let us comment on the relation to the \( L^1 \)-norm of trapezoidal filters and related families which bounds the ISI. Note first that, for the triangle kernel, this norm is actually independent and unity for all \( B \) since:

\[ \| g \|_1 = \frac{B}{2\pi} \int_0^1 \frac{\sin^2 \left( \frac{B t}{2} \right)}{2\pi} dt = 1 \]

Here, we defined \( \sin(At) = \sin(At)/At, A > 0 \). It is easy to prove that no filter with \( g(0) = 1 \) can fall below this value. Hence, we can argue that for any filter \( \| g \|_1 > 1 \) (after proper normalization), and that the \( L^1 \)-norm measures the (inverse) distance to the sinc kernel for which clearly \( \| g \|_1 = \infty \) holds. Suppose we want to “shift” the triangle filter kernel to the optimal sinc kernel while not loosing the favorable properties of the triangle kernel. This can achieved by the trapezoidal kernel family which has close to optimal \( L^1 \)-norm behaviour as follows:

**Theorem 6.** We have:

\[ \inf_{g \in \mathcal{M}_B^{t_k}} \| g \|_1 \leq C_2 (L) \]

**Proof:** It is easy to see that we have for any \( g \in \mathcal{P} \mathcal{W}_\nu^1 \):

\[ \| g \|_1 = \int_\mathbb{R} |g(t)| dt = \prod_{t=-\infty}^{t_{1,L}} \int_0^{t_{1,L}} |g(t-t_{1,L})| dt \]

By the bounded convergence theorem:

\[ \sup_{t_{1,L}} \int_0^{t_{1,L}} |g(t-t_{1,L})| dt = \int_0^{t_{1,L}} \sum_{t=-\infty}^{t_{1,L}} |g(t-t_{1,L})| dt \]

Taking the limes inferior on both sides yields the result.

We have the following observations which are quite convenient for filter design in 5G:

- The \( L^1 \)-norm is almost independent of the actual filter bandwidth so that tail properties can be tightly controlled when scaling.
- Trapezoidal kernels seemingly yield a good comprise between properties of both the extreme sinc and triangle kernels.
- \( L^1 \)-norm are seemingly quite good but optimality is yet to be proven.

**V. Conclusions**

In this paper we provided new bounds on the overshooting between samples of bandlimited signals for small oversampling factors improving on former results. Moreover, we discussed some extension to overshooting of Nyquist filters and related ISI bounds.

**Acknowledgements**

This work was supported by German Research Foundation/Deutsche Forschungsgemeinschaft (DFG) under grant WU 598/3-1.

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