Formalism of gauge invariant curvatures and constructing the cubic vertices for massive spin-$3/2$ field in AdS$_4$ space

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Abstract

We study the interaction of massive spin-3/2 field with electromagnetic and gravitational fields in the four dimensional AdS space and construct the corresponding cubic vertices. Construction is based on generalization of Fradkin-Vasiliev formalism, developed for massless higher spin fields, to massive fermionic higher spin fields. The main ingredients of this formalism are the gauge invariant curvatures. We build such curvatures for the massive theory under consideration and show how the cubic vertices are written in their terms.

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Introduction

Till now most of the investigations of consistent cubic interaction vertices for higher spin fields were devoted to bosonic massless fields (e.g. \[1, 2, 3, 4, 5, 6, 7, 8, 9\]). The results on massive higher spin interactions are not so numerous, although there exist classification of massive and massless cubic vertices in flat space by Metsaev \[10, 11, 12\] as well as some concrete examples \[13, 14, 15, 16, 17, 18, 19, 20, 21\]. At the same time there exists just a small number of papers devoted to fermionic higher spin interactions \[22, 23, 24\]. At least one of the reason for this is that technically such investigations appears to be much more involved. In this paper, using massive spin \(\frac{3}{2}\) as a simple but physically interesting and non-trivial example of massive fermionic higher spin fields, we apply the so-called Fradkin-Vasiliev formalism \[25, 26\] (see also \[27, 28, 29, 30, 31, 32, 21\]) to the construction of electromagnetic and gravitational cubic vertices.

Let us briefly remind basic properties of this formalism. Higher spin particle is describes by a set of fields that we collectively denote as \(\Phi\) here and for each field one can construct curvature and denote as \(R\). Moreover with help of these curvatures free Lagrangian can be rewritten in explicitly gauge invariant form as:

\[
L_0 \sim \sum R \wedge R
\]

Using these ingredients one can construct two types of non-trivial cubic vertices:

- Abelian vertices that have the form:

  \[
  L \sim R \wedge R \wedge \Phi
  \]

- non-Abelian ones that look like:

  \[
  L \sim R \wedge \Phi \wedge \Phi
  \]

For the massless bosonic fields Vasiliev \[33\] has shown that any such non-Abelian vertex can be obtained as a result of deformation procedure that can be described as follows.

- Construct quadratic in fields deformations for the curvatures and linear corrections to the gauge transformations

  \[
  \Delta R \sim \Phi \wedge \Phi, \quad \delta_1 \Phi \sim \Phi \xi
  \]

  so that deformed curvatures \(\hat{R} = R + \Delta R\) transform covariantly

  \[
  \delta \hat{R} = \delta_1 R + \delta_0 \Delta R \sim R \xi
  \]

  At this step most of the arbitrary parameters are fixed, however, there still remains some ambiguity. The reason is that covariance of transformations for deformed curvatures guarantees that the equations of motion will be gauge invariant but it does not guarantee that they will be Lagrangian.

\footnote{We call vertex trivial if it can be constructed using gauge invariant curvatures only, i.e. \(L \sim R \wedge R \wedge R\)}
Non-Abelian cubic vertex arises then one put these deformed curvatures into the free Lagrangian and using explicit form of the gauge transformations requires it to be gauge invariant

\[ \mathcal{L} \sim \sum \hat{R} \hat{\mathcal{R}} \quad \Leftrightarrow \quad \delta \mathcal{L} \sim \sum \hat{R} \mathcal{R} \xi = 0 \]

Note at last that Vasiliev has shown [33] that any non-trivial cubic vertex for three fields with spins \( s_1, s_2, s_3 \) having up to \( s_1 + s_2 + s_3 - 2 \) derivatives can be constructed as some combination of Abelian and non-Abelian vertices. However, one point out that the Vasiliev construction [33] was initially formulated only for massless higher spin field theories. In this paper we generalize the Vasiliev approach to massive higher spin theories on the example of cubic electromagnetic and gravitational coupling for massive spin-\( \frac{3}{2} \) field in the AdS_4 space.

The paper is organized as follows. In Section 1 we give all necessary kinematics formulae. Here we introduce two gauge invariant objects (for the physical field \( \psi_\mu \) and Stueckelberg one \( \phi \)) and rewrite the free Lagrangian using these objects. Section 2 devoted to the electromagnetic interaction for such massive spin-\( \frac{3}{2} \) field (for previous results see e.g. [34, 35, 36]). In subsection 2.1, as a independent check for our calculations as well as for instructive comparison, we consider this task using straightforward constructive approach. Then in subsection 2.2 we consider the same task in the Fradkin-Vasiliev formalism. At last in Section 3 we construct gravitational cubic vertex in the Fradkin-Vasiliev formalism.

**Notations and conventions**

We work in four-dimensional Anti-de Sitter space with cosmological constant \( \Lambda \). The AdS covariant derivatives \( D_\mu \) is normalized so that

\[
[D_\mu, D_\nu] \xi^a = \lambda^2 (\epsilon^a_{[\mu} \xi_{\nu]} + \frac{1}{2} \Gamma_{\mu\nu} \xi^a), \quad \lambda^2 = -\frac{\Lambda}{3}
\]

In [3] \( \epsilon^a_{\mu} \) is background (non-dynamical) tetrad linking world and local indexes and has a standard definition \( \epsilon^a_{\mu} e^b_{\nu} g^{ab} = g_{\mu\nu} \), where \( g_{\mu\nu} \) and \( g^{ab} \) are respectively curved world AdS metrics and flat local one. Wherever it is convenient local indices are converted into the world ones by \( \epsilon^a_{\mu} \) and its inverse \( e^a_{\mu} \), in particular the matrix \( \gamma_\mu \) with world index is understood as \( \gamma_\mu = \epsilon^a_{\mu} \gamma^a \). Also we introduce the following notation for antisymmetric combinations of \( \epsilon^a_{\mu} \)

\[
\{ \mu \nu \}_{ab} = e^{[\mu}_{a} e^{\nu]_{b}}, \quad \{ \mu \nu \alpha \}_{abc} = e^{[\mu}_{a} e^{\nu}_{b} e^{\alpha}_{c}], \quad \{ \mu \nu \alpha \beta \}_{abcd} = e^{[\mu}_{a} e^{\nu}_{b} e^{\alpha}_{c} e^{\beta}_{d]}
\]

In expression [3] \( \Gamma^{ab} = \frac{1}{2} \gamma^{[a} \gamma^{b]} \) is antisymmetric combination of two gamma matrices and is a particular case of the more general definition which we will use

\[
\Gamma^{a_1 a_2 \ldots a_n} = \frac{1}{n!} \gamma^{[a_1 \ldots a_n]}
\]

where \( n = 2, 3, 4 \) for four-dimensional space. Let us present the main properties of such

---

\[ \text{We use Greek letters } \mu, \nu, \ldots \text{ for world indices and Latin letters } a, b, \ldots \text{ for local ones. Summation over any repeated indices is implied. For indices in (square) round brackets we use convention of complete (anti)-symmetrization without normalization factor. Spinor indices of (tensor)-spinor fields are omitted.} \]
\[ \Gamma_{ab_1...b_n} = -g^{[b_1} \Gamma_{b_2...b_n]} + \gamma^a \Gamma_{b_1...b_n} \]
\[ \Gamma_{b_1...b_n} = g^{[b_1} \Gamma_{b_2...b_n]} + (-1)^n \Gamma_{b_1...b_n} \gamma^a \]
\[ \gamma^a \Gamma_{b_1...b_n} = 2g^{[b_1} \Gamma_{b_2...b_n]} + (-1)^n \Gamma_{b_1...b_n} \gamma^a \]
\[ \gamma_a \Gamma_{b_1...b_n} = (d - n) \Gamma_{b_1...b_n} \]
\[ \Gamma_{ab_1...b_n} \gamma_a = (-1)^n (d - n) \Gamma_{b_1...b_n} \]

The identities (4) are used in the paper to verify the gauge invariance of the Lagrangians.

All fermionic fields are anticommuting and Majorana. We also use Majorana representation of \( \gamma \)-matrices with hermitian conjugation defined as follows:

\[ (\gamma^a)\dagger = \gamma^0 \gamma^a \gamma^0 \]
\[ (\Gamma_{a_1a_2...a_n})\dagger = \gamma^0 \Gamma_{a_2...a_1} \gamma^0 = -\gamma^0 \Gamma_{a_1a_2...a_n} \gamma^0 \]

In this case \( \gamma^0, \gamma^a \) and \( \gamma^0 \Gamma^a \) are symmetric in spinor indices while \( \gamma^0, \gamma^0 \Gamma^{abc}, \) and \( \gamma^0 \Gamma^{abcd} \) are antisymmetric.

## 1 Kinematic of massive spin-3/2 field

For gauge-invariant description of massive spin-3/2 field besides the master vector-spinor field \( \Psi_\mu \) one has also introduce Stueckelberg spinor field \( \phi \). The free Lagrangian is known and has the form:

\[ L_0 = -\frac{i}{2} \{ \mu \nu \sigma \} \bar{\psi}_\mu \Gamma^{abc} D_\nu \psi_\sigma + \frac{i}{2} e_\mu^a \bar{\phi} \gamma^a D_\mu \phi \]
\[ + 3im \epsilon_\mu \bar{\psi}_\mu \gamma^a \phi - \frac{3M}{2} \{ \mu \nu \} \bar{\psi}_\mu \Gamma^{ab} \Psi_\nu - M \bar{\phi} \phi \]

This Lagrangian is invariant under the following gauge transformations

\[ \delta_0 \psi_\mu = D_\mu \xi + \frac{iM}{2} \gamma_\mu \xi \]
\[ \delta_0 \phi = 3m \xi \]

where \( M^2 = m^2 + \lambda^2 = m^2 - \Lambda \) and \( m \) is a mass parameter. Note that such description works in de Sitter space as well provided \( m^2 > \Lambda \), \( m^2 = \Lambda \) being the boundary of unitarity region.

Using the explicit form of gauge transformations one can construct two gauge-invariant objects (curvatures):

\[ \Psi_{\mu \nu} = D_{[\mu} \psi_{\nu]} + \frac{m}{6} \Gamma_{\mu \nu} \phi + \frac{iM}{2} \gamma_{[\mu} \psi_{\nu]} \]
\[ \Phi_\mu = D_\mu \phi - 3m \psi_\mu + \frac{iM}{2} \gamma_\mu \phi \]
which satisfy the Bianchi identities

\[
D_{[\mu} \Psi_{\nu]a} = \frac{m}{6} \Gamma_{[\mu} \Phi_{\alpha]} - \frac{i M}{2} \gamma_{[\mu} \Psi_{\nu]a}.
\]

\[
D_{[\mu} \Phi_{\nu]} = -3 m \Psi_{\mu\nu} - \frac{i M}{2} \gamma_{[\mu} \Phi_{\nu]}.
\]  (7)

Note that curvatures (6) are closely related with the Lagrangean equations of motion. It can be seen from the total variation of Lagrangian (5) that can be written as follows:

\[
\delta L_0 = -\frac{i}{2} \{ \mu \nu \alpha \beta \} \bar{\Psi} \mu \nu \Gamma_{abc} \delta \psi_\alpha - i e^\mu_a \bar{\Phi} \mu \gamma^a \delta \phi.
\]  (8)

Moreover using these curvatures the free Lagrangian (5) can be rewritten in explicitly gauge-invariant form. The most general ansatz looks like:

\[
L_0 = c_1 \{ \mu \nu \alpha \beta \} \bar{\Psi} \mu \nu \Gamma_{abc} \Phi_\alpha + i c_2 \{ \mu \alpha \} \bar{\Psi} \mu \alpha \Gamma^{abc} \Phi_\alpha + c_3 \{ \mu \alpha \} \bar{\Phi} \mu \Gamma^{ab} \Phi_\nu
\]  (9)

The requirement to reproduce the original Lagrangian (5) partially fixes the parameters

\[
3 c_3 = -8 c_1, \quad 6 c_2 m = \frac{1}{2} - 16 c_1 M.
\]  (10)

The remaining freedom in parameters is related with the identity:

\[
\{ \mu \nu \alpha \beta \} D_{[\mu} \bar{\Psi}_{\nu]a} \Gamma^{abc} \Phi_\beta = 0
\]

Using the Bianchi identities for the curvatures (7) we obtain

\[
-3 m \{ \mu \nu \alpha \beta \} \bar{\Psi} \mu \nu \Gamma^{abc} \Phi_\alpha + 8 i M \{ \mu \alpha \} \bar{\Psi} \mu \alpha \Gamma^{abc} \Phi_\alpha + 8 m \{ \mu \alpha \} \bar{\Phi} \mu \Gamma^{ab} \Phi_\nu = 0
\]

As it will be seen in the next section sometimes this ambiguity may be important in order to reproduce the most general cubic vertex so we will not fix it here.

## 2 Electromagnetic interaction

### 2.1 Constructive approach

We prefer to work with Majorana fermions thus for the description of electromagnetic interactions we will use pair of them $\psi_i^\mu$, $\phi^i$, where $i = 1, 2$ is the $SO(2)$ index. Let us switch minimal electromagnetic interaction by standard rule

\[
D_\mu \psi_\nu^i \Rightarrow D_\mu \psi_\nu^i + e_0 \varepsilon^{ij} A_\mu \psi_j^i
\]

and similarly for $\phi^i$. The Lagrangian (5) with above replacement for covariant derivative is not invariant under the free gauge transformations, variation of the Lagrangian has the form:

\[
\delta_0 L_0 = -\frac{i e_0}{2} \varepsilon^{ij} \{ \mu \nu \alpha \beta \} \bar{\psi} \mu i \Gamma^{abc} F_{\nu a} \xi^j = -3 i e_0 \varepsilon^{ij} e^\mu_a \bar{\psi} \mu i \Gamma^{abc} F_{bc} \xi^j
\]  (11)
To compensate for this non-invariance let us introduce non-minimal interactions as follows:

\[ \mathcal{L}_1 = \varepsilon^{ij}\left\{ \{\mu^a_{\ ab}\} \bar{\psi}_\mu^i (b_1 F^{ab} + b_2 \Gamma^{abcd} F_{cd}) \psi_\nu^j + i e^\mu_a \bar{\psi}_\mu^i (b_3 F^{ab} \gamma^b + b_4 \Gamma^{abc} F_{bc}) \phi^j + b_5 \bar{\phi}^i (\Gamma F) \phi^j \right\} \]  

(12)

as well as corresponding corrections to gauge transformations:

\[ \delta_1 \psi_\mu^i = i \alpha_1 \varepsilon^{ij} (\Gamma F)_{\gamma^\mu} \xi^i, \quad \delta_1 \phi^j = \alpha_2 \varepsilon^{ij} (\Gamma F) \xi^j \]

\[ \delta_1 A_\mu = \beta_1 \varepsilon^{ij} (\bar{\psi}_\mu^i \xi^j) + i \beta_2 \varepsilon^{ij} (\bar{\phi}^i \gamma^\mu \xi^j) \]  

(13)

By direct calculations one can check that gauge invariance can be restored that gives the solution for arbitrary coefficients:

\[ b_1 = 6 \alpha_1, \quad b_2 = 3 \alpha_1, \quad b_3 = -2 \alpha_2, \quad b_4 = \alpha_2, \quad b_5 = \frac{M \alpha_2}{3 m}, \quad \beta_1 = -24 \alpha_1, \quad \beta_2 = -2 \alpha_1 \]

provided the following important relation holds:

\[ 4 M \alpha_1 + 2 m \alpha_2 = e_0 \]  

(14)

Lagrangian (12) together with correction, stipulated by minimal switch of interaction in free Lagrangian, is the final cubic electromagnetic interaction vertex for spin-3/2 field. As we see, this vertex contains two arbitrary parameters.

A few comments are in order.

- Calculating commutator of two gauge transformations we obtain

\[ [\delta_1, \delta_2] A_\mu = -8 \varepsilon^{ij} (6 \alpha_1^2 + \alpha_2^2) (\xi^i \gamma^\nu \xi^j) F_{\mu\nu} \]

and it means that for non-zero electric charge \( e_0 \) any such model must be component of some (spontaneously broken) supergravity.

- From the supergravity point of view the remaining freedom in the parameters \( \alpha_1 \) and \( \alpha_2 \) is clear: in general our vector field is a superposition of graviphoton (i.e. vector field from the supermultiplet \((\frac{3}{2}, 1)\)) and goldstino’s superpartner (i.e. vector field from the supermultiplet \((1, \frac{3}{2})\)).

- From the expression for the parameter \( b_5 \) one can see that we have an ambiguity between flat and massless limits. Indeed, in the flat case \( b_5 = \frac{\alpha_2}{3} \) and does not depend on the mass any more, while for the non-zero cosmological term this parameter is singular in the massless limit.

- The most simple case — \( \alpha_2 = 0 \), i.e. vector field is just graviphoton. In this case electric charge \( e_0 = \frac{2}{3} M \alpha_1 \) becomes zero at the boundary of the unitarity region.
2.2 Formulation in terms of curvatures

Now we reformulate the results of the subsection 2.1 on the base of Fradkin-Vasiliev formalism. Following general procedure we begin with deformation of curvatures so that they transform covariantly (1), (2). In the case under consideration \( \mathcal{R} = \{ \Psi_{\mu \nu}^i, \Phi_\mu^i, F_{\mu \nu} \} \). For the fermionic curvatures deformations correspond simply to the minimal substitution:

\[
\Delta \Psi_{\mu \nu}^i = e_0 \varepsilon^{ij} A_{[\mu} \psi_{\nu]}^i, \quad \Delta \Phi_\mu^i = e_0 \varepsilon^{ij} A_{\mu} \phi^j
\]

(15)

while transformations for the deformed curvatures look like:

\[
\delta \tilde{\Psi}_{\mu \nu}^i = e_0 \varepsilon^{ij} F_{\mu \nu} \xi^j, \quad \delta \tilde{\Phi}_\mu^i = 0
\]

(16)

where \( \tilde{\Psi}_{\mu \nu}^i = \Psi_{\mu \nu}^i + \Delta \Psi_{\mu \nu}^i, \tilde{\Phi}_\mu^i = \Phi_\mu^i + \Delta \Phi_\mu^i \) and \( \Psi_{\mu \nu}^i, \Phi_\mu^i \) are given by (6).

The most general deformation for the electromagnetic fields strength quadratic in fermionic fields can be written as follows:

\[
\Delta F_{\mu \nu} = \varepsilon^{ij} [a_1 \tilde{\psi}_{[\mu}^i \psi_{\nu]}^j + i a_2 \tilde{\psi}_{[\mu}^i \gamma_{\nu]} \phi^j + a_3 \phi^i \Gamma_{\mu \nu} \phi^j]
\]

(17)

Corresponding corrections to the gauge transformation have the form:

\[
\delta_1 A_\mu = \varepsilon^{ij} [2 a_1 \tilde{\psi}_\mu^i \xi^j - i a_2 \phi^j \gamma_\mu \xi^j]
\]

(18)

The deformed curvatures will transform covariantly

\[
\delta_1 \hat{F}_{\mu \nu} = \varepsilon^{ij} [2 a_1 \tilde{\psi}_{\mu \nu}^i \xi^j - i a_2 \phi^j \gamma_{\mu \nu} \xi^j]
\]

(19)

provided the following relation holds:

\[
3 m a_3 = -\frac{m a_1}{6} - M a_2
\]

(20)

Here \( \hat{F}_{\mu \nu} = F_{\mu \nu} + \Delta F_{\mu \nu} \).

Now let us consider the Lagrangian

\[
\mathcal{L}_1 = -\frac{1}{4} \hat{F}_{\mu \nu} \hat{F}_{\mu \nu} + c_1 \left\{ \psi_{\mu \nu} \right\}_{abcd} \hat{\Psi}_{\mu \nu} i \Gamma_{abcd} \hat{\Psi}_{\mu \nu} i
\]

\[
+ c_2 \left\{ \psi_{\mu \nu} \right\}_{ab} \hat{\Psi}_{\mu \nu} i \Gamma_{abc} \hat{\Phi}_a^i + c_3 \left\{ \psi_{\mu \nu} \right\}_{ab} \hat{\Phi}_a^i i \Gamma_{ab} \hat{\Phi}_b^i
\]

(21)

where all initial curvatures (including electromagnetic field strength) are replaced by the deformed ones, and require that this Lagrangian be invariant. Non-trivial variations arise only with respect to transformations with the spinor parameter \( \xi^i \). Using explicit form of the covariant transformations (16), (19) we obtain

\[
\delta \mathcal{L} = -\frac{1}{2} \varepsilon^{ij} \left\{ \right\}_{ab} \Psi_{\mu \nu}^i (a_1 F_{ab} - 48 e_0 c_1 \Gamma_{abcd} F_{cd}) \xi^j
\]

\[
+ i \varepsilon^{ij} c_a \Phi_\mu^i (a_2 \gamma^b F_{ab} + 6 e_0 c_2 \Gamma_{abc} F_{bc}) \xi^j
\]

(22)
To compensate these terms one need to introduce non-minimal corrections (note that they have exactly the same form as in the previous subsection):

\[
\delta_1 \psi^i = i\alpha_1 \varepsilon^{ji} (\Gamma F) \gamma^i, \quad \delta_1 \phi^i = \alpha_2 \varepsilon^{ji} (\Gamma F) \xi^j
\]  

which in turn produce the following variations:

\[
\delta_1 \mathcal{L}_0 = -\alpha_1 \varepsilon^{ij} \left\{ \frac{\mu}{ab} \bar{\psi}_\mu^i \psi^j (6F^{abc} + 3F^{de} \Gamma^{abcde}) \xi^j - i\alpha_2 \varepsilon^{ij} \epsilon^a \bar{\psi}_\mu^i (2\gamma^b F^{abc} + \Gamma^{abc} F^{de}) \xi^j \right\}
\]

Comparing with (22) we conclude

\[
a_1 = -12\alpha_1, \quad a_2 = 2\alpha_2, \quad \alpha_1 = 8\varepsilon_0 c_1, \quad \alpha_2 = 6\varepsilon_0 c_2
\]

Thus we see that the choice of the parameters \(\alpha_1\) and \(\alpha_2\) is related with the choice of parameters \(c_1\) and \(c_2\) in the free Lagrangian. Moreover, if we use the relation

\[
6mc_2 = \frac{1}{2} - 16Mc_1
\]

we again obtain

\[
4M\alpha_1 + 2m\alpha_2 = \varepsilon_0
\]

Let us extract the cubic vertex. Using relations (10), (25) we get

\[
\mathcal{L}_1 = 3\alpha_1 \varepsilon^{ij} \left\{ \frac{\mu}{ab} \bar{\psi}_\mu^i (2\alpha_1 F^{abc} + \Gamma^{abcd} F^{de}) \psi^j \right\} + i\alpha_2 \varepsilon^{ij} \epsilon^a \bar{\psi}_\mu^i (2\gamma^b F^{abc} - \Gamma^{abc} F^{de}) \psi^j + \frac{M\alpha_2}{3m} \varepsilon^{ij} \phi^i (\Gamma F) \phi^j + \frac{i\varepsilon_0}{2} \varepsilon^{ij} \left\{ \frac{\mu}{abc} \right\} A_\mu \bar{\psi}_\mu^i \Gamma^{abc} \psi_\mu^j + \frac{i\varepsilon_0}{2} \varepsilon^{ij} \epsilon^a A_\mu \bar{\psi}_\mu^i \gamma^a \phi^j
\]

Here the last line corresponds to the minimal interactions while the other terms are non-minimal corrections.

The Lagrangian \(\mathcal{L}_1\) up to the minimal interaction is the same one obtained in the subsection 2.1 for the electromagnetic cubic vertex.

### 3 Gravitational interaction

#### 3.1 Kinematics for gravity

Let us briefly review basic features of gravity in the AdS\(_4\) space at free level. In the frame formulation the gravitational field is described by dynamical frame \(h_{\mu}^a\) and Lorentz connection \(\omega_{\mu}^{ab}\) being antisymmetric in local indices. Free Lagrangian in AdS space have the form

\[
\mathcal{L}_0 = \frac{1}{2} \{ \mu \nu \}_{ab} \omega_{\mu}^{ac} \omega_{\nu}^{bc} - \frac{1}{2} \{ \mu \rho \}_{abc} \omega_{\mu}^{ab} D_\nu h_\rho^{c} + \lambda^2 \{ \mu \nu \}_{ab} h_{\mu}^{a} h_{\nu}^{b}
\]

and is invariant under the gauge transformations

\[
\delta_0 \omega_{\mu}^{ab} = D_\mu \tilde{\xi}^{ab} - \lambda^2 e_{[a}^{\ [\alpha} \xi^{b]} \]

\[
\delta_0 h_{\mu}^{a} = D_\mu \tilde{\xi}^{a} + \tilde{\eta}_{\mu}^{a}
\]  

(28)
The gauge invariant objects (linearized curvature and torsion) have the form

\[ R_{\mu\nu}^{ab} = D_{[\mu} \omega_{\nu]}^{ab} - \lambda^2 e_{[\mu}^{[a} h_{\nu]}^{b]} \]
\[ T_{\mu\nu}^{a} = D_{[\mu} h_{\nu]}^{a} - \omega_{[\mu,\nu]}^{a} \]  (29)

They satisfy the Bianchi identities

\[ D_{[\mu} R_{\nu\alpha]^{ab}} = \lambda^2 e_{[\mu}^{[a} T_{\nu\alpha]}^{b]} \]
\[ D_{[\mu} T_{\nu\alpha]}^{a} = - R_{[\mu\nu,\alpha]}^{a} \]  (30)

Note that on the mass shell for the auxiliary field \( \omega_{\mu}^{ab} \) by virtue of (30) we have

\[ T_{\mu\nu}^{a} = 0 \Rightarrow R_{[\mu\nu,\alpha]}^{a} = 0, \quad D_{[\mu} R_{\nu\alpha]}^{ab} = 0 \]  (31)

At last the free Lagrangian can be rewritten as follows

\[ L_0 = c_0 \begin{bmatrix} \mu\nu\alpha\beta \\ abcd \end{bmatrix} R_{\mu\nu}^{ab} R_{\alpha\beta}^{cd}, \quad c_0 = \frac{1}{32\lambda^2} \]  (32)

### 3.2 Gravitational coupling for massive spin-3/2

Following general scheme (11), (12) we begin with the deformation of the curvatures that in the case under consideration are \( \mathcal{R} = \{ \Psi_{\mu\nu}, \Phi_{\mu}, R_{\mu\nu}^{ab}, T_{\mu\nu}^{a}\} \). As in the case of electromagnetic interactions deformations for spin-3/2 curvatures correspond to the minimal substitution rules, i.e. to the replacement of covariant derivative \( D \rightarrow D + \omega \) and background tetrad \( e_\mu^a \rightarrow e_\mu^a + h_\mu^a \):

\[ \Delta \Psi_{\mu\nu} = a_1 (\omega_{[\mu}^{ab} \Gamma_{ab} \psi_{\nu]} + 2 M i h_{[\mu}^{a} \gamma_{a} \psi_{\nu]} - \frac{2m}{3} h_{[\mu}^{a} \Gamma_{\nu]}^a \phi) \]
\[ \Delta \Phi_{\mu} = a_1 (\omega_{\mu}^{ab} \Gamma_{ab} \phi + 2 M i h_{\mu}^{a} \gamma_{a} \phi) \]  (33)

 Corrections to the gauge transformations will look like:

\[ \delta_1 \Psi_{\mu} = -a_1 (\Gamma_{ab} \psi_{\mu} \hat{\eta}_{ab} + 2 i M \gamma^{a} \psi_{\mu} \hat{\xi}_{a} - \frac{2m}{3} \Gamma_{\mu}^a \phi \hat{\xi}_{a} - \omega_{\mu}^{ab} \Gamma_{ab} \phi) \]
\[ \delta_1 \Phi_{\mu} = -a_1 (\Gamma_{ab} \psi_{\mu} \hat{\eta}_{ab} + 2 i M \gamma^{a} \phi \hat{\xi}_{a}) \]  (34)

while transformations for the deformed curvatures will have the form:

\[ \delta \hat{\Psi}_{\mu\nu} = -a_1 (\Gamma_{ab} \Psi_{\mu\nu} \hat{\eta}_{ab} + 2 i M \gamma^{a} \Psi_{\mu\nu} \hat{\xi}_{a} + \frac{2m}{3} \Gamma_{[\mu} \Phi_{\nu]} \hat{\xi}_{a}) \]
\[ R_{\mu\nu}^{ab} \Gamma_{ab} \xi - 2 i M T_{\mu\nu}^{a} \gamma_{a} \xi) \]
\[ \delta \hat{\Phi}_{\mu} = -a_1 (\Gamma_{ab} \Phi_{\mu} \eta_{ab} + 2 i M \gamma^{a} \Phi_{\mu} \hat{\xi}_{a}) \]  (35)

Here \( \hat{\Psi}_{\mu\nu} = \Psi_{\mu\nu} + \Delta \Psi_{\mu\nu}, \hat{\Phi}_{\mu} = \Phi_{\mu} + \Delta \Phi_{\mu} \) and \( \Psi_{\mu\nu}^i, \Phi^i \) are given by (6).
Now let us consider the most general deformations for gravitational curvature and torsion quadratic in spin-3/2 fields:

\[
\begin{align*}
\Delta R_{\mu\nu}^{ab} &= b_1 \bar{\psi}_\mu \Gamma^{ab} \bar{\psi}_\nu + ib_2 e_{[\mu} \bar{\psi}_{\nu]} \gamma^b \phi + ib_3 \bar{\psi}_\mu \Gamma^{ab}_\nu \phi + b_4 e_{[\mu} \bar{\psi}_{\nu]} b^a \phi + b_5 \phi \Gamma_{\mu\nu}^{ab} \phi \\
\Delta T_{\mu\nu}^a &= ib_6 \bar{\psi}_\mu \gamma^a \bar{\psi}_\nu + b_7 e_{[\mu} \bar{\psi}_{\nu]} b^a \phi + b_8 \bar{\psi}_\mu \Gamma^{ab}_\nu \phi + ib_9 \phi \Gamma_{\mu\nu}^{ab} \phi
\end{align*}
\]

(36)

Note that there are three possible field redefinitions:

\[
\omega_\mu^{ab} \Rightarrow \omega_\mu^{ab} + i\kappa_1 \bar{\phi} \Gamma^{ab} \phi, \quad h_\mu^a \Rightarrow h_\mu^a + \kappa_2 \bar{\psi} \gamma^a \phi + \kappa_3 e^{a} \phi
\]

(37)

that shift parameters $b_3, b_6$, and $b_7$.

In order that deformed curvatures transform covariantly we have to introduce the following corrections to the gauge transformations:

\[
\begin{align*}
\delta_1 \omega_\mu^{ab} &= 2b_1 \bar{\psi}_\mu \Gamma^{ab} \xi - ib_2 e_{[\mu} \bar{\psi}_{\nu]} \gamma^a \xi - ib_3 \phi \Gamma^{ab}_\nu \xi \\
\delta_1 h_\mu^a &= 2ib_6 \bar{\psi}_\mu \gamma^a \xi + b_7 e^{a} \phi \xi + b_8 \phi \Gamma^{ab}_\mu \xi
\end{align*}
\]

(38)

Then the curvature and torsion will transform as follows:

\[
\begin{align*}
\delta \hat{R}_{\mu\nu}^{ab} &= 2b_1 \bar{\psi}_\mu \Gamma^{ab} \xi + ib_2 e_{[\mu} \bar{\psi}_{\nu]} \gamma^b \xi - ib_3 \bar{\psi} \Gamma^{ab}_\nu \xi \\
\delta \hat{T}_{\mu\nu}^a &= 2ib_6 \bar{\psi}_\mu \gamma^a \xi - b_7 e^{a} \bar{\psi} \xi + b_8 \bar{\psi} \Gamma^{ab}_\nu \xi
\end{align*}
\]

(39)

provided the following restrictions on the arbitrary parameters hold:

\[
\begin{align*}
3mb_2 &= b_0 (M^2 - m^2) - b_1 M \\
6mb_4 &= \frac{mb_1}{3} + 2b_7 (M^2 - m^2) \\
6mb_5 &= -\frac{mb_1}{3} - 2b_3 M
\end{align*}
\]

(40)

Here $\hat{R}_{\mu\nu}^{ab} = R_{\mu\nu}^{ab} + \Delta R_{\mu\nu}^{ab}$, $\hat{T}_{\mu\nu}^a = T_{\mu\nu}^a + \Delta T_{\mu\nu}^a$ and $R_{\mu\nu}^{ab}, T_{\mu\nu}^a$ are given by (29).

General solution to these relations has four free parameters, for example, $b_{1,3,6,7}$. But as we have already noted three of them are related with possible field redefinitions, so we have one non-trivial parameter $b_1$ only.

Let us consider the following Lagrangian:

\[
\mathcal{L} = c_1 \left\{ \frac{\mu\nu\alpha\beta}{abcd} \right\} \bar{\psi}_\mu \Gamma^{abcd} \hat{\psi}_\nu + ic_2 \left\{ \frac{\mu\nu\alpha}{abcd} \right\} \bar{\psi}_\mu \Gamma^{ab} \hat{\phi}_\alpha + c_3 \left\{ \frac{\mu\nu}{ab} \right\} \bar{\phi}_\mu \Gamma^{ab} \hat{\phi}_\nu + c_0 \left\{ \frac{\mu\nu\alpha\beta}{abcd} \right\} \hat{R}_{\mu\nu}^{ab} \hat{R}_{\alpha\beta}^{cd} + ic_4 \left\{ \frac{\mu\nu\alpha\beta}{abcd} \right\} \bar{\psi}_\mu \Gamma^{ab} \hat{\phi}_\alpha \phi_{\beta}^{d} + c_5 \left\{ \frac{\mu\nu\alpha}{abcd} \right\} \bar{\phi}_\mu \Gamma^{ab} \hat{\phi}_\nu \phi_{\alpha}^{e}
\]

(41)

where the first four terms are just the sum of free Lagrangians for massive spin-$\frac{3}{2}$ and massless spin-2 with the initial curvatures replaced by the deformed ones while the last two terms are possible Abelian vertices. Note that in dimensions $d > 4$ we would have to introduce one more Abelian vertex:

\[
\left\{ \frac{\mu\nu\alpha\beta\gamma}{abced} \right\} \bar{\psi}_\mu \Gamma^{abcd} \psi_{\alpha\beta} h_{\gamma}^{e}
\]
To compensate these variations one can use the following corrections

\[ \delta \mathcal{L} = \left( -24c_1a_1 + 4b_1c_0 \right) \left\{ \frac{1}{2} \bar{\Psi}_\alpha \Gamma^{abcd} \Psi_\alpha \xi^d \right\} + i(6c_2a_1 - 8c_0b_2) \left\{ \frac{1}{2} \bar{\Psi}_\alpha \Gamma^{abcd} \Phi_\alpha + i(6c_2a_1 - 8c_0b_2) \right\} \]

Taking into account (10), (40), (42), (43) we obtain simply:

\[ 0 = \left\{ \frac{1}{2} \bar{\Psi}_\alpha \Gamma^{abcd} \Phi_\alpha = 3 \left\{ \frac{1}{2} \bar{\Psi}_\alpha \Gamma^{abcd} \Phi_\alpha \right\} \]

The first term does not vanish on-shell so we have to put:

\[ b_1c_0 = 6c_1a_1 \]  

Calculating variations for \( \eta^{ab} \)-transformations we obtain:

\[ \delta \mathcal{L} = -8c_1a_1 \left\{ \frac{1}{2} \bar{\Psi}_\mu \Gamma^{abcd} \Psi_\alpha \eta_i^d \right\} - i(12c_2a_1 - 3c_4) \left\{ \frac{1}{2} \bar{\Psi}_\mu \Gamma^{abcd} \Phi_\alpha \eta_i^d \right\} + (8c_3a_1 - 2c_5) \left\{ \frac{1}{2} \bar{\Psi}_\mu \Gamma^{abcde} \Psi_\alpha \eta_i^{de} \right\} \]

The first term vanishes in \( d = 4 \) due to the identity

\[ 0 = \left\{ \frac{1}{2} \bar{\Psi}_\mu \Gamma^{abcd} \Psi_\alpha \eta_i^d \right\} - i(12c_2a_1 - 3c_4) \left\{ \frac{1}{2} \bar{\Psi}_\mu \Gamma^{abcd} \Phi_\alpha \eta_i^d \right\} + (8c_3a_1 - 2c_5) \left\{ \frac{1}{2} \bar{\Psi}_\mu \Gamma^{abcde} \Psi_\alpha \eta_i^{de} \right\} \]

while the last two terms give restrictions

\[ c_4 = 4c_2a_1, \quad c_5 = 4c_3a_1 \]

At last we consider variations under the \( \hat{\xi}^a \)-transformations:

\[ \delta \mathcal{L} = -i(16c_1a_1M + \frac{3}{2}mc_4) \left\{ \frac{1}{2} \bar{\Psi}_\mu \Gamma^{abcd} \Psi_\alpha \hat{\xi}^d \right\} + (32c_1a_1m + 3mc_5) \left\{ \frac{1}{2} \bar{\Psi}_\mu \Gamma^{abcd} \Phi_\alpha \hat{\xi}^c \right\} + (-4M_1a_1c_2 + M_4) \left\{ \frac{1}{2} \bar{\Psi}_\mu \Gamma^{abcd} \Phi_\alpha \hat{\xi}^d \right\} + (4a_1c_2M_1 - m_4 - c_5) \left\{ \frac{1}{2} \bar{\Psi}_\mu \Gamma^{abcd} \Phi_\alpha \hat{\xi}^c \right\} + i(16a_1c_2m - 8mc_3a_1 + 2mc_4 - 4M_1c_5) \left\{ \frac{1}{2} \bar{\Psi}_\mu \Gamma^{abcd} \Phi_\alpha \hat{\xi}^b \right\} \]

Taking into account (10), (40), (42), (43) we obtain simply:

\[ \delta \mathcal{L} = -\frac{i}{2} \left\{ \frac{1}{2} \bar{\Psi}_\mu \Gamma^{abcd} \Psi_\alpha \hat{\xi}^d \right\} + 2ia_1 \left\{ \frac{1}{2} \bar{\Psi}_\mu \Gamma^{abcd} \Phi_\alpha \hat{\xi}^c \right\} \]

To compensate these variations one can use the following corrections

\[ \delta_1 \psi_\mu \sim \Psi_\mu \hat{\xi}^\mu, \quad \delta_1 \phi \sim \Phi_\mu \hat{\xi}^\mu \]

Thus the requirement that the Lagrangian be gauge invariant fixes the coefficients \( c_{4,5} \) for the Abelian vertices and also relates coefficients \( b_1 \) and \( c_1 \) (which is just a manifestation of
universality of gravitational interaction). Note that by Metsaev classification [11] in general dimensions \(d > 4\) we would have two non-minimal vertices with two derivatives as well as the minimal one. But in \(d = 4\) dimensions these two derivative vertices are absent (on-shell and up to possible field redefinitions) as we will now show.

**Vertex \(\frac{3}{2} - \frac{3}{2} - 2\).** For this vertex we obtain:

\[
\mathcal{L} = (-48c_1a_1 + 8c_0b_1) \left\{ \frac{\mu\nu\alpha\beta}{abcd} \right\} D_\mu \omega_\nu^{ab} \bar{\psi}_\alpha \Gamma^{cd} \psi_\beta = 0
\]

This expression vanishes due to obtained expressions and restrictions for the arbitrary coefficients.

**Vertex \(\frac{1}{2} - \frac{1}{2} - 2\).** In this case we have

\[
\mathcal{L} = (-c_3a_1 + 8c_0b_5) \left\{ \frac{\mu\nu}{ab} \right\} D_\mu \omega_\nu^{cd} \bar{\phi}^{abcd} \phi + (8c_3a_1 - 2c_5) \left\{ \frac{\mu\nu}{ab} \right\} \omega_\mu^{ac} D_\nu \bar{\phi} \Gamma^{bc} \phi \\
+ (2c_3a_1 + 16c_0b_4) \left\{ \frac{\mu\nu}{ab} \right\} D_\mu \omega_\nu^{ab} \bar{\phi} \phi
\]

The second term drops out due to relations among the coefficients, the first term vanishes on-shell for gravitational field due to identity

\[
0 = \left\{ \frac{\mu\nu\alpha}{abc} \right\} R_{\mu\nu,\alpha} \phi^a \Gamma^{abcd} \phi = 2 \left\{ \frac{\mu\nu\alpha}{abc} \right\} (D_\mu \omega_\nu^{cd} \phi + \lambda^2 \epsilon_{\mu d} h_{\nu,\alpha}) \bar{\phi} \Gamma^{abcd} \phi = 6 \left\{ \frac{\mu\nu}{ab} \right\} D_\mu \omega_\nu^{cd} \bar{\phi} \Gamma^{abcd} \phi
\]

while the third term can be removed by field redefinition

\[
h_\mu^a \rightarrow h_\mu^a + \kappa_3 e_\mu^a \bar{\phi} \phi
\]

**Vertex \(\frac{3}{2} - \frac{3}{2} - 2\).** For the last possibility we get:

\[
\mathcal{L} = i(-24c_2a_1 + 6c_4) \left\{ \frac{\mu\nu\alpha}{abc} \right\} \omega_\alpha^d D_\mu \bar{\psi}_\nu \Gamma^{bcd} \phi + \\
i(16c_0b_2 - 12c_2a_1) \left\{ \frac{\mu\nu}{ab} \right\} D_\nu \omega_\alpha^{ab} \bar{\psi}_\mu \Gamma^{\alpha c} \phi - i(16c_0b_3 - 12c_2a_1) \left\{ \frac{\mu\nu}{abc} \right\} D_\nu \omega_\alpha^{dc} \bar{\psi}_\mu \Gamma^{abcd} \phi
\]

The first term drops out due to relations among the coefficients, the last term vanishes on-shell for gravitational field, and the second term can be removed by the field redefinition

\[
h_\mu^a \rightarrow h_\mu^a + \kappa_2 \bar{\psi}_\mu \gamma^a \phi
\]

As a result, the expression (11) is the final form for Lagrangian of coupled spin-2 and spin-3/2 fields in \(AdS_4\) space including the cubic interaction vertex. The parameters \(c_1, c_2, c_3\) are fixed in the free gravitational field Lagrangian (27), the parameter \(c_0\) is given by (32). The parameters \(c_4, c_5\) and the parameters of gauge transformations are expressed through the single parameter \(a_1\) which is the only free parameter of the theory.

**Conclusion**

In this paper we have constructed the cubic vertices for massive spin-3/2 field coupled to electromagnetic and gravitational fields in \(AdS_4\) space. The corresponding vertices are gauge invariant due to presence of Stueckelberg auxiliary fields and contain some number of free parameters. The results are given by the expressions (12) or (26) for electromagnetic interaction and (11) for gravitational interaction.
Construction of vertices is based on generalization of Fradkin-Vasiliev formalism [25, 26] where the main building blocks of Lagrangians are the gauge invariant curvatures. Although this formalism was known only for massless higher spin theories, we have shown, on the example of spin-3/2 field, that the gauge invariant curvatures can in principle be constructed for massive higher spin fields as well. Certainly the case of massive fermionic fields appears to be the most technically involved one. Nevertheless it seem worth to apply this formalism to massive fermionic fields with spins higher than 3/2.

As we pointed out, the constructed vertices contain some number of arbitrary parameters (two for electromagnetic coupling and one for gravitational coupling) which can not be fixed only form gauge invariance. In principle one can hope that the additional constraints for those parameters can appear from requirements of causality. The possibility of such constraints was already demonstrated in the papers [17, 20].

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