RADIATION BACKREACTION IN SPINNING BINARIES

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The evolution under radiation backreaction of a binary system consisting of a black hole and a companion is studied in the limiting case when the spin of the companion is negligible compared with the spin $S$ of the black hole. To first order in the spin, the motion of the reduced-mass particle excluding radiation effects, is characterized by three constants: the energy $E$, the magnitude $L$ of the angular momentum and the projection $L_S$ of the angular momentum along the spin $S$. This motion is quasiperiodic with a period determined by $r_{\text{min}}$ and $r_{\text{max}}$. We introduce a new parametrization, making the integration over a period of a generic orbit especially simple. We give the averaged losses in terms of the 'constants of motion' during one period for generic orbits, to linear order in spin.

1 Introduction

We describe here the gravitational radiation backreaction on two kinds of binary systems: a black hole accompanied either by another black hole with comparable mass (compact binary, CB) or by a neutron star viewed as a test particle in the Lense-Thirring (LT) picture.

Our recently developed method can be applied whenever the Newtonian evolution of the binary system is perturbed in such a way that an uncoupled radial equation exists. Then for any bounded orbit the turning points $r_{\text{max}}$ are at $\dot{r} = 0$. The half period is defined as the time elapsed between consecutive turning points. We introduce the true anomaly parametrization $r = r(\chi)$ for the evaluation of integrands of the type: $F/r^{2+n}$, where $n$ is a positive integer, defined as:

$$\frac{dr}{d(\cos \chi)} = -(\gamma_0 + S \gamma_1)r^2, \quad r(0) = r_{\text{min}} \quad \text{and} \quad r(\pi) = r_{\text{max}}$$

where $\gamma_0, \gamma_1$ are constants. The integrals over one period are conveniently evaluated by computing the residues enclosed in the circle $\zeta = e^{i\chi}$. This parametrization has the nice feature that there is only one pole, at $\zeta = 0$. We replace the usual polar angles by new, monotonously changing angle variables.

We employ a post-Newtonian ($PN$) and additional expansions in both cases: In the LT limit an additional expansion over the small parameter $\eta = m_2/m_1$ is necessary. For a CB, the spin $S$ is of $PN^{1/2}$ order smaller than the orbital angular momentum $L$.

Our main result is that we obtain the leading spin terms in the averaged losses of the constants of motion on generic orbits.
2 The orbit in the absence of radiation

The equations of motion in the presence of the spin can be derived from the second order Lagrangian:

\[ L = \frac{\mu v^2}{2} + \frac{g m \mu}{r} + \frac{2(1 + \eta) g \mu}{c^2 r^3} (\mathbf{r} \times \mathbf{S}) + \frac{\eta \mu}{2 c^2 m} (\mathbf{a} \times \mathbf{S}) \] (2)

where \( r = |\mathbf{r}| \) is the relative distance, \( v \) the relative velocity, \( \mu = m_1 m_2 / (m_1 + m_2) \) the reduced mass and \( m = m_1 + m_2 \) the total mass of the system. The parameter \( \eta \) vanishes in the LT case.

Up to linear terms in the spin there are three constants of motion: the energy \( E \), the magnitude \( L \) and the spin projection \( L \cdot \mathbf{S} \) of the orbital angular momentum:

\[
E = \frac{\mu v^2}{2} - \frac{g m \mu}{r} + \eta \frac{gL \cdot \mathbf{S}}{c^2 r^3} = \frac{\mu}{2} [r^2 + \eta g (\dot{\theta}^2 + \sin^2 \theta \dot{\phi}^2)] - \frac{g m \mu}{r} + \eta \frac{gL \cdot \mathbf{S}}{c^2 r^3}
\]

\[
L^2 = \mu r^2 (\dot{\theta}^2 + \sin^2 \theta \dot{\phi}^2) - 4 \eta \frac{g L \cdot \mathbf{S}}{c^2 r} + \frac{2 \eta}{c^2 m} E L \cdot \mathbf{S} 
\]

\[
L \cdot \mathbf{S} = L \cos \kappa .
\] (3)

From these a pure radial equation follows:

\[
\dot{r}^2 = 2 \frac{E}{\mu} - \frac{2 g m \mu}{r} + \frac{L^2}{\mu r^3} - 2 \eta \frac{E L \cdot \mathbf{S}}{c^2 m \mu^2 r^2} - 2 (2 + \eta) \frac{g L \cdot \mathbf{S}}{c^2 \mu^3} .
\] (4)

The true anomaly parametrization, by [1], is:

\[
r = \frac{L^2}{\mu (g m \mu + A_0 \cos \chi)} - \frac{2 \eta L \cdot \mathbf{S}}{c^2 m \mu^2 A_0} \frac{g m \mu^2 A_0 + (g^2 m^2 \mu^3 + E L^2) \cos \chi}{(g m \mu + A_0 \cos \chi)^2} \\
+ \frac{2 (2 + \eta) g L \cdot \mathbf{S}}{c^2 L^2 A_0} \frac{A_0 (2 g^2 m^2 \mu^3 + E L^2) + g m \mu (2 g^2 m^2 \mu^3 + 3 E L^2) \cos \chi}{(g m \mu + A_0 \cos \chi)^2}
\] (5)

where \( A_0 = (g^2 m^2 \mu^2 + 2 E L^2 / \mu)^{1/2} \). By introducing three Euler angle variables, \( \Psi \) (the argument of the latitude), \( \iota_N \) (the inclination of the orbit) and \( \Phi \) (the longitude of the node), the equations of motion became simpler. From among these angles only the zeroth order part (in the spin) of the angle \( \Psi = \Psi_0 + \chi \) enters the radiative losses.

3 Averaged radiation losses

The instantaneous radiation losses of the constants of motion are evaluated employing the radiative multipole tensors of Kidder, Will and Wiseman, which originate in the Blanchet-Damour-Iyer formalism. The instantaneous losses for the energy \( E \) and total angular momentum \( J \) were given by Kidder. From these we derive both in the LT case and in the CB case the instantaneous losses of the constants \( E, L \) and \( L_S \).
Now in the averaging process we replace the time integration by the parameter integration, then we use the residue theorem to find the averaged radiative losses in the constants of motion. For instance, the power is

\[
\langle \frac{dE}{dt} \rangle = -\frac{g^2 m (-2\mu E)^{3/2}}{15c^5 L^7} \left(148E^2L^4 + 732g^2m^2\mu^3EL^2 + 425g^4m^4\mu^6 \right) + S\left(-2\mu E\right)^{3/2}g^2 \cos \kappa \times \frac{10c^7 L^{10}}{15c^5 L^7}
\]

\[
\left[520E^3L^6 + 10740g^2m^2\mu^3E^2L^4 + 24990g^4m^4\mu^6EL^2 + 12579g^6m^6\mu^9 \right] + \eta \left[256E^3L^6 + 6660g^2m^2\mu^3E^2L^4 + 16660g^4m^4\mu^6EL^2 + 8673g^6m^6\mu^9 \right] \right].
\]

The LT limit, \( \eta \to 0 \), includes the results by Peters and Mathews \cite{7,8} for the special case of a nonspinning black hole; by Shibata \cite{9} for an equatorial orbit about a spinning black hole; and by Ryan for a circular and generic orbit \cite{10,11} about a spinning black hole, respectively. With the proper definitions of orbital parameters we find perfect agreement.

Complete details of our computations including some subtleties and the radiative losses in \( E, L \) and \( L_S \) both in terms of the constants of motion and orbital parameters are described elsewhere \cite{12,13}.

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