Monogamy of Information Causality

Li-Yi Hsu

Department of Physics, Chung Yuan Christian University, Chung-li 32023, Taiwan

Abstract

We consider information causality in the multi-receiver random access codes. Therein, no receiver can gain any information only from classical communication. We claim the following statement. Information causality still holds with the help of the multi-partite physical non-local resource. That is, the summation of all revivers’ information gain cannot be greater than the amount of classical communication. The distributive multi-party physical nonlocal resource can be exploited only for information splitting. It is proved that such trade-off leads to the monogamy of entanglement. Finally the connection between information causality and spin-glass Bethe lattice is discussed.
For almost a hundred years, quantum theory has been a successful theory in describing microscopic physics. Quantum theory is essentially grounded on the mathematical axioms. Recently attempts have been made to reformulate quantum physics using physical principles. In contrast to classical physics, quantum theory can be featured by no-cloning, secret privacy, no-broadcasting, however, which are common among generic nonlocal theories \[1, 2\]. However, so far the only nonlocal theory that is physically realized is quantum theory \[3, 4\]. There should be a profound physical principle. Such principle can be exploited as the basis of quantum theory. It can also single out quantum theory as a physical theory.

In the hunting for the hidden principle, two potential candidates, macroscopic locality \[7\] and information causality \[8\], have been proposed recently. To simply put, macroscopic locality states that the Bell-type experiment with a sufficient number of detectors can falsify quantum mechanics. In other words, the statistics of coarse-grained outcome correlation should admit a local hidden variable model. On the other hand, information causality states that one can gain no more than \(m\)-bit information of the other’s database, once \(m\)-bit classical communication is allowed. Notably, even in the bipartite scenario, macroscopic locality and information causality are inequivalent with multi-level qudits \[9\]. As for the fulfillment of macroscopic locality, the global covariance matrix should be tested semidefinite positive.

In information processing, no-signaling correlation as physical resource can be quite useful. For example, communication complexity can be trivial if a physical theory were to allow maximal nonlocal correlation \[5, 6\]. From the information theoretic viewpoint, a relevant question can be stated as follows. Why can quantum theory not be more nonlocal? Historically, Popescu and Rohrlich demonstrated that maximal nonlocality can be achieved without violating the no-signaling principle, due to the request of relativity \[10\]. In the bipartite communication protocol, according to the no-signaling principle, distant Alice and Bob each cannot gain the other’s information only with accessible nonlocal resource. With classical communication, nonlocality can help gain the other’s information. Intuitively, one can gain more information if the resource were more nonlocal. As a physical principle, information gain is upper-bounded by information causality \[8\]. In this way, a nonlocal theory can be calibrated into a physical one, i.e. quantum theory, using information causality. Any theory more nonlocal than the quantum theory must violate information causality and hence be unphysical.
In this Letter, we investigate information causality in the framework of distributive \((n, k)\) random access codes \((n, k)\)-RAC \([11]\). Wherein Alice can directly access a local \(k\)-bit database, which comprises the components of the vector \(\vec{a} := (a_0, a_1, \cdots, a_{k-1})\), with \(a_i\) being the random variable and \(a_i \in \{0, 1\} \ \forall i\). There are \(n\) distant parties, Bob\(_1\), Bob\(_2\),\( \cdots\), Bob\(_n\). \(\forall j\), Bob\(_j\) is given the random variable \(b_j \in \{0, 1, \cdots, k-1\}\), and his task is to optimally guess \(a_{b_j}\). In the following Alice is allowed to public announce a bit, \(c\), via classical communication. Denote Bob\(_j\)’s local information gain as \(I_j = \sum_{l=0}^{k-1} I(a_l : \beta_j | b_j = l)\), where \(I(a_l : \beta_j | b_j = l)\) is the Shannon mutual information between \(a_l\) and Bob\(_j\)’s guessing answer \(\beta_j\) under the condition that Bob\(_j\) has received \(b_j = l\). Ideally, \(I(a_l : \beta_j | b_j = l) = 1\) if \(a_l = \beta_j\) always holds. That is, Bob\(_j\) always guesses \(a_l\) correctly. The information causality quantity is defined as

\[
I = \sum_{j=1}^{n} I_j
\]  

The \(n = 1\) case has been fully studied in \([8, 12]\). Therein, information causality can be stated as follows. Under the limit of the one-bit broadcasting, the nonlocal resource can be physically realized, if

\[
I \leq 1,
\]

Notably, the above inequality can be realized either classically or quantumly. However, it is not the case once \(n \geq 2\). For example, Alice can send the bit \(c = a_0\) and Bob\(_j\)’s answer \(\beta_j\) is always equal to \(c\). Hence \(I(a_l : \beta_j = c | b_j = l) = \delta_{l,0}\) and \(I = n\). In this case, \(I\) is an increasing function of \(n\) and goes infinite as \(n \to \infty\). In the following it is required that \(I(a_l : c) = 0 \ \forall l\). The reasons are twofold. Firstly, any receiver gains no information from the communication bit. The non-zero information gain must intrinsically come from the accessible nonlocal resource. Secondly, in cryptography, if the accessibility of the database is restrictive to the legitimate users, no information leakage should be allowed via classical communication. Eventually, if there is no nonlocal resource distributed between distant parties, \(I = 0\).

To perform the \((n, k)\)-RAC, Alice and Bob\(_j\) \(\forall j\) each input some bits into the no-signaling boxes, which then output a bit. The output correlation can be either local or nonlocal without violating the no-signaling principle. In this Letter, we claim the monogamy of information causality as follows. \textit{With broadcasting one classical bit carrying no information, the nonlocal boxes can be physically realized if the condition \([2]\) still holds in the multi-receiver}
scenario. In other words, even exploiting multi-partite entangled states cannot gain more information than one bit. Rather, nonlocal resource can only split information into different receivers.

The monogamy or shareability of entanglement or nonlocal correlation has been studied about a decade. Coffman et al considered the trade-off relation of three-qubit system using the measure of bipartite entanglement known as tangle [14]. Later such trade-off relation of N-qubit system was studied by Osborne and Verstraete [15]. The limited share-ability of generic correlations was also studied [16, 17]. Koashi and Winter considered the trade-off between quantum and classical correlations [18]. Notably, these monogamy relations are linear. Toner and Verstraete originally considered the monogamy relation of three-qubit system from the aspect of bipartite nonlocality [19]. The summation of the quadratic Clauser-Horne-Shimony-Holt (CHSH) values [20] is limited. Recently Kurzyński et al investigated the monogamy relation of N-qubit system using multipartite nonlocality [21]. Therein the summations of the quadratic Seevenick-Bell [23] or Mermin-Bell [24] values are also limited. In this Letter, it will be shown that information monogamy is deeply connected with that of entanglement [19, 21]. Specifically the monogamy relations of three- and four-qubit entanglement can be directly derived using the monogamy of information causality in the (3, 2)-RAC. Consequently monogamy of entanglement can be operationally meaningful from the information theoretic viewpoint. In [21], quadratic monogamy of entanglement is studied with two measurement settings on each qubit. Such trade-off relation is essentially related to local realistic description of the correlation functions, which lead to

\[
\sum_{k_1 \ldots k_N = x, y} T_{k_1 \ldots k_N}^2 \leq 1, \text{ where } T_{k_1 \ldots k_N} = \frac{1}{n} Tr(\rho \sigma_{k_1} \otimes \ldots \otimes \sigma_{k_N}), \rho \text{ is the } N\text{-qubit density matrix, and, under local unitary, } \sigma_{k_i} \text{ can be set as the Pauli operator either } \sigma_x \text{ or } \sigma_y \text{ for } i\text{-th qubit} \text{[13]. In this Letter the multi-setting monogamy of entanglement can be also derived using Ineq. [21].}
\]

On the other hand, monogamy of information causality can guarantee the unconditional security in quantum cryptography. Actually, the maximal \(I\) depends on the amount of classical communication, rather than the number of nonlocal boxes. Suppose the one-bit sender Alice wants to share the secret bits with Bob\(_1\) via quantum channels. Whereas Bob\(_2\), Bob\(_3\), \ldots Bob\(_n\) are regarded as collaborative eavesdroppers. Once \(I_1 > 1/2 > \sum_{j=2}^{n} I_j\), using any attack equipped with unlimited physical multipartite nonlocal resource, these
eavesdroppers can gain no more information than the receiver. Quantumly, $I_1 > 1/2$ can be done if Alice and Bob$_1$ share more entanglement than that shared between Alice and all other Bob$_j$s [22]. With afterward error correction and privacy amplification [25], Bob$_1$ can access these bits in secure. Here we take BB84 protocol as a simple example with $n = 2$. Therein, Bob$_1$ and Bob$_2$ are the receiver and the eavesdropper, respectively. Once the sender Alice announces 1-bit information of preparation basis, which is irrelevant to the secret bit. It is straightforward that $I_2 = -Q \log_2 Q - (1 - Q) \log_2(1 - Q) = 1 - I_1$, where $Q$ is the quantum bit error rate. As a result, $I = 1$.

As a final remark, the bipartite nonlocal correlation exploited in the $(1,k)$-RAC has been fully studied [12]. Therein Ineq. (2) can be reduced as a convex optimization problem [27]. Hence information causality can be numerically validated by semidefinite programming [12, 28]. However, such reduction is unknown in general $(n,k)$-RAC, since multipartite correlation is involved. Eventually, as a physical hypothesis, monogamy of information causality must be verified or falsified only by physical experiments. Hereafter, the addition is the addition modulo two.

$(n,2)$-RAC — Before further proceeding, $(1,2)$-RAC is reviewed as follows. Firstly Alice inputs the bit $x = a_0 + a_1$ into the box which outputs $A$; Bob$_1$ inputs the bit $y_1 = b_1$ into the accessible box which outputs $B_1$. Alice sends the bit $c = a_0 + A$ and Bob$_1$’s guess answer is $\beta_1 = c + B_1$. As for its physical realization, two entangled qubits are exploited as a nonlocal box (NL-box). The inputs and outputs correspond to the measurement settings and the outcomes, respectively [8, 13]. In the $(n,2)$-RAC case, similar to Bob$_1$’s processing, Bob$_j$ inputs the bit $y_j = b_j$ into the accessible box which outputs $B_j$. Then Bob$_j$’s guessing answer is $\beta_j = c + B_j$. If the unphysical Popescu-Rohrlich boxes are exploited such that, in the bipartite scenario, $A + B_j = xy_j$. Bob$_i$ can access both $a_0$ and $a_1$ and hence $I_i(a_l : \beta_j|b_j = l) = 1 \forall l \in \{0,1\}$. Given $b_j$, let the $B_j = xy_j + A$ with probability $\frac{1}{2}(1 + \xi_{j,l})$. Here Bob$_i$’s box is regarded a gate that errs with the probability $\frac{1}{2}(1 - \xi_{j,l})$. That is,

$$\xi_{j,l} = P(A + B_j = xy_j) - P(A + B_j = xy_j + 1). \tag{3}$$

Before proceeding further Evans-Schulman lemma should be introduced as follows [29, 30].

Lemma : Let $X$ and $Y$ be random variables. Let the binary symmetric channel $C$ be

$$
\begin{pmatrix}
\frac{1+\xi}{2} & \frac{1-\xi}{2} \\
\frac{1-\xi}{2} & \frac{1+\xi}{2}
\end{pmatrix}.
$$

(4)
Let $Y$ and $Z$ be the random variable input and output of the symmetric channel $C$, respectively. Let the random variables $Q$ such that $Z$ is independent of $(Q, X)$. Then

$$\frac{I(X; Z|Q)}{I(X; Y|Q)} \leq \xi^2.$$  

Proof: See Ref. [29, 30] for the detailed rigorous proof. Therein, $Q$ is not necessarily binary.

In the following Bob’s noisy box is decomposed as the perfect box attached with the noisy channel $C$. Specifically, the output of perfect box, $xy_j + A$, is the input of the channel, which outputs $B_j$. According to (4), $B_j$ is equal to $xy_j + A$ with probability $\frac{1+\xi}{2}$ with $\xi = \xi_{j,l}$. Such the disturbing noise in the physical boxes is intrinsic and hence unavoidable in the physical implementation. Regarding Bob’s information gain, $Q$ is set as the given condition $b_j = l$, and $X$ is set as $a_l$. $Y$ is set as the addition of the communication bit $c$ and the output of the perfect box. In other words, $Y = c + (xy_j + A) = a_l$ and $I(a_l; Y|b_j = l) = 1$. Finally $Z = c + B_j$ and with the flipping probability equal to $\frac{1}{2}(1 - \xi_{j,l})$. Consequently we have

$$I(a_k; \beta_l|b_i = l) \leq \xi^2_{j,l},$$

and hence $I_j \leq \sum_{l=0}^{k-1} \xi^2_{j,l}$. For the quantum correspondence, the inputs $x$ and $y_j (= b_j)$ correspond to physical observables $\hat{A}_x$ and $\hat{B}_{j,y_j}$, respectively, and the outputs to the measurement outcomes. Once the output 0 and 1 are mapped into 1 and -1, according to (3), we have $\xi_{j,l} = \frac{1}{2} \sum_{x=0,1} (-1)^x \langle \hat{A}_x \hat{B}_{j,l} \rangle$. Using Cauchy–Schwarz inequality and denoting $\mathcal{CHSH}_{AB_j} = 2(\xi_{j,0} + \xi_{j,1})$ [20], we have

$$\sum_{j=1}^{n} \mathcal{CHSH}_{AB_j}^2 \leq 8. \quad (5)$$

Notably the $n = 2$ case is exactly equal to the monogamy of three-qubit entanglement [19].

$(n, 2)$-RAC can be alternatively processed as follows. Alice exploits two boxes, where $x_1 = a_0$ and $x_2 = a_1$ are the inputs and $A_1$ and $A_2$ are the outputs respectively. Then Alice sends the bit $c = A_1 + A_2 + a_0 + a_0a_1$. Similarly, Bob input $y_j = b_j$ to the box that outputs $B_j$. Finally $\beta_j = B_j + c$ if $y_j = 0$ and $\beta_j = B_j + c + 1$ if $y_j = 1$. (Another very similar $(1, k)$-RAC has been recently proposed [26].) Notably, if $A_1 + A_2 + B_j = x_1x_2y_j + \overline{x_1} \overline{x_2} \overline{y_j}$, once the output 0 and 1 are mapped into 1 and -1, respectively, the Seevinck-Bell
value $SB = \sum_{x_1,x_2,y_j=0}^1 (-1)^{x_1 x_2 y_j + x_1 x_2 y_j} \langle x_1 x_2 y_j \rangle$ can be saturated up to 8, whereas its Tsirelson bound is $4\sqrt{2}$. Quantumly, the corresponding physical observables of inputs $x_1$ and $x_2$ are $\hat{A}_{x_1}$ and $\hat{A}'_{x_2}$, respectively, and as a result, $\xi_{j,0} = \frac{1}{4} \sum_{x_1,x_2=0}^1 (-1)^{x_1 x_2} \langle \hat{A}_{x_1} \hat{A}'_{x_2} \hat{B}_{j,0} \rangle$, $\xi_{j,1} = \frac{1}{4} \sum_{x_1,x_2=0}^1 (-1)^{x_1 x_2} \langle \hat{A}_{x_1} \hat{A}'_{x_2} \hat{B}_{j,1} \rangle$ and hence $SB_{A_1 A_2 B_j} = 4(\xi_{j,0} + \xi_{j,1})$. Then we have

$$\sum_{j=1}^n SB_{A_1 A_2 B_j}^2 \leq 32 \quad (6)$$

For $n = 2$ case with exchanging Alice’ boxes with Bob$_1$ and Bob$_2$’s boxes, we have

$$SB_{A_1 A_2 B_1}^2 + SB_{A_1 A_2 B_2}^2 + SB_{B_1 B_2 A_1}^2 + SB_{B_1 B_2 A_2}^2 \leq 64. \quad (7)$$

An alternative form is Mermin three-qubit monogamy $M_{A_1 A_2 B_1}^2 + M_{A_1 A_2 B_2}^2 + M_{B_1 B_2 A_1}^2 + M_{B_1 B_2 A_2}^2 \leq 16 \quad [21]$, where three-qubit Mermin-Bell value $M_{A_1 A_2 B_j} = \langle \hat{A}_1 \hat{A}_0 \hat{B}_{j,0} \rangle + \langle \hat{A}_0 \hat{A}_1 \hat{B}_{j,1} \rangle - \langle \hat{A}_1 \hat{A}_0 \hat{B}_{j,1} \rangle + \langle \hat{A}_0 \hat{A}_1 \hat{B}_{j,1} \rangle [24]$. Using Cauchy-Schwarz inequality and permuting the subscripts $0\leftrightarrow1$, Mermin monogamy can be lead to Ineq. (7).

Before further proceedings of $(n,k)$-$RAC$, here we argue the validity of the monogamy of information causality. In the original proposal of information causality, the bipartite protocol of $(1,k)$-$RAC$ is studied. Therein, $b$ is given to Bob to guess $a_b$. Since the multipartite nonlocal resource is accessible, now Bob is required to guess $n$ elements $a_{b_1}, \ldots, a_{b_n}$, of $\vec{a}$. As for Bob, he divides the boxes in $n$ groups. Those boxes in $i$-th group are exploited for guessing $a_{b_i}$. Notably, if $n > k$, there must be some $p$ and $q$, such that $b_p = b_q$. Bob can exploit the boxes in the $p$-th and $q$-th groups jointly to guess out $a_{b_p}$. According to definition of $I$ and information causality, Bob’s information gain is no more than one bit. As for the multi-partite protocol of $(n,k)$-$RAC$, boxes of different groups are spatially separated. No other extra classical-bit broadcast from Bob$_i$ $\forall i$ is allowed. In this condition, the value $I$ cannot be increased. Hence the monogamy of information causality should be guaranteed.

$(n,k)$-$RAC$ — The protocol is similar that of $(1,k)$-$RAC$ as follows. Alice inputs the $(k - 1)$-bits $\alpha_1, \ldots, \alpha_{k-1}$, where $\alpha_i = a_0 + a_i$, into the box. Bob$_j$ input the random variable $b_j$ into the box that outputs $B_j$. After Alice announces the bit $c = a_0 + A$, Bob$_j$’s guess answer $\beta_j = c + B_j = a_0 + A + B_j$.

Let the $k$-bit vector $\vec{a} = (a_0 \ldots a_{k-1})$ with $a_0$ always being 0. Assuming that $b_j = j'$, let another $k$-bit vector $\vec{b}_j = (b_{j,0} \ldots b_{j,k-1})$, where $b_{j,0} = \delta_{j,j'}$. Notably the Hamming weight
of $\vec{b}_j$ is always 1. As a result, $a_{b_j} = \beta_j$ if $A + B_j = \vec{\alpha} \cdot \vec{b}_j = a_0 + a_j'$. Quantumly, the corresponding observables of $\vec{\alpha}$ and $\vec{b}_j$ are $\hat{M}_{\vec{\alpha}}$ and $\hat{N}_{\vec{b}_j}$, respectively. With straight calculation,

$$\xi_{j, \vec{b}_j} = \frac{1}{2^{k-1}} \sum_{\{\vec{\alpha}\}} (-1)^{\vec{\alpha} \cdot \vec{b}_j} \left\langle \hat{M}_{\vec{\alpha}} \hat{N}_{\vec{b}_j} \right\rangle$$

and hence

$$I_j = \sum_{\{\vec{b}_j\}} \xi_{j, \vec{b}_j}^2.$$  \hspace{1cm} (8)

Here we consider the $n = 1$ case. Denote $\mathcal{IC}_{AB_j} = \sum_{\{\vec{\alpha}, \vec{b}_j\}} (-1)^{\vec{\alpha} \cdot \vec{b}_j} \left\langle \hat{M}_{\vec{\alpha}} \hat{N}_{\vec{b}_j} \right\rangle$ as the Bell value. Again, using the Cauchy-Schwarz inequality and straight calculation, a nonlocal physical theory must obey the following inequality

$$\sum_{m=0}^{k-1} \binom{k-1}{m} |k - 2m| \leq |\mathcal{IC}_{AB_1}| \leq 2^{k-1} \sqrt{k}.$$ \hspace{1cm} (10)

Both local realism and superquantum correlations each violate the above inequality in the different ways. As for arbitrary $n$, Ineqs. (2) and (9) lead to the following trade-off relation

$$\sum_{j=1}^{n} \mathcal{IC}_{AB_j}^2 \leq 4^{k-1} k.$$  

Notably, an alternative processing of $(1, k)$-RAC has been proposed by Pawłowski et al. \cite{8}. For simplicity, set $k = 2^p$. Notably, the nonlocal boxes accessible for each receiver are locally exploited as noisy gates comprising the circuit $G$, which corresponds to the 2-ary $p$-depth complete tree. Here we can regard $G$ as Bethe lattice with open boundary \cite{31}. The spin value $s_i$ for each vertex $i$ is assigned as $1(-1)$ if the corresponding box outputs $1(0)$. As a result the spin configuration is the ground state of the spin glass Ising model with Hamiltonian

$$H = - \sum_{(i, j) \in E} J_{ij} s_i s_j,$$ \hspace{1cm} (11)

where $(i, j) \in E$ if vertex $i$ and $j$ are connected. At the zero temperature ($T = 0$), $s_i = s_i^0$ for the vertex $i$, which is the noiseless output of the corresponding box. Artificially the coupling strength $J_{ij}$ is set as $s_i^0 s_j^0 J$, where $J > 0$. Hence $J = \pm J$ with probability $\frac{1}{2}$, and the spin configuration at $T = 0$ composes the ground state. The random noise of the boxes corresponds to the thermal fluctuation. Consequently $\xi$ in (11) is equal to $| < s_i > |$, which
is decreased with increasing $T$, and $n$. The lattice $G$ is paramagnetic and all boxes are local at $T \rightarrow \infty$. Here we conjecture as follows. The lattice $G$ comprised by physical nonlocal boxes are always \textit{paramagnetic} and never be in spin glass or ferromagnetic phase [32]. In other words, $I = 1$ may imply the occurrence of phase transition in the Bethe lattice $G$.

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