Nonparametric Method for Aircraft Flight Control

V Glasov, E Zybin and V Kosyanchuk
State Research Institute of Aviation Systems, Moscow, Russia
E-mail: vvglasov@gosniias.ru

Abstract. This paper is a part of a series of articles on nonparametric methods in dynamic systems theory. Here the authors propose a new method for inverse dynamic problem solution based only on some past measurements of controlled system input-output signals. The described nonparametric control method needs no any priori information on controlled system model parameters and does not require identification, training or statistical calculations. An example of nonparametric aircraft flight control is presented.

1. Introduction
Parametric (model-based) and nonparametric (model-free) methods are the two main parts of a complete dynamic systems control theory. The broad classification of all control methods according to the accuracy of controlled dynamic system mathematical models is presented in figure 1 [1].

![Control Methods Diagram](image)

**Figure 1.** The classification of control methods.

Parametric control methods are the most powerful ones, but applicable only when accurate models are available or models are inaccurate and contains some uncertainties. Such mathematical models are
priori obtained from the first principles and their accurate parameters have to be calibrated on-line or off-line using measured data or estimated during identification [2–4]. Modern dynamic systems become more and more complicated. So modeling, whether by the first principles or by identification from data, is an approximation of the true system, and some error is inevitable and unmodeled dynamics always exist in the modeling and controlling processes [5]. For this reason, designing a controller using an inaccurate model with arbitrarily small errors could lead to either arbitrarily bad performance or an unstable closed-loop system [1]. Moreover, there are systems and situations when the identification problem is unsolvable in principle. Unfortunately, all the modern aircrafts belong to such systems, because their mathematical open-loop models parameters are unidentifiable in normal closed-loop operation conditions without prior information [6].

In cases when a model contains some uncertainties or inaccuracies and the required level of accuracy is higher than the classical control methods allow to achieve, the methods of adaptive or robust control are used [1, 7]. These methods are also used to control systems which models contain a large number of nonlinearities or are described by too high order complex equations. Nevertheless, adaptive control model may show unexpected behavior in the presence of certain unmodeled dynamics, and application of model-based robust control design techniques lack of adequate, practical uncertainty descriptions [1].

For controlled systems with difficult to establish or unavailable mathematical models the relatively new nonparametric control methods are used. The theory of nonparametric control is a continuation of the traditional (parametric) theory and received its development with an increase in the level of computing power of modern information technology. Nonparametric control methods can be used for systems with a well-known mathematical model and is not incompatible with parametric theory. Nevertheless, the main feature of nonparametric control theory is that it allows to solve a wide range of control tasks for any systems, including the conditions of complete lack of model parameters information. The stability and convergence of nonparametric algorithms do not depend upon the accuracy of the model, the twinborn problem of unmodeled dynamics and robustness in traditional parametric theory do not exist under nonparametric framework [1].

Although the studies on nonparametric control are still at in the inception phase, they have attracted a great deal of attentions of many researchers. At this moment a large number of nonparametric control methods are presented, which have different names depending on the group of researchers offering the method, problem statement and data usage: data-driven control (DDC), data-based control (DBC), modeless control (MLC), model-free adaptive control (MFAC), iterative feedback tuning (IFT), virtual reference feedback tuning (VRFT), iterative learning control (ILC), nonparametric model with nonparametric regulator, nonparametric dual control, nonparametric control with PID-regulator, etc. [1, 8–12]. Despite the large variety of names and approaches for solving nonparametric control problems, all these methods are united by one common condition. They find the controls based not on the known model parameters, but only on the past few measurements of its input $u$ and output $x$ signals (figure 2).

![Data points](image.png)

**Figure 2.** The general idea of nonparametric methods.

In this paper the authors propose a new method for nonparametric control problem solution, significantly expanding the applicability of nonparametric dynamic systems theory [13–16].
2. Nonparametric aircraft flight control problem formulation and solution

Let’s an aircraft dynamics is described by a discrete state space model in the form the following parametric vector
\[ x_{i+1} = Ax_i + Bu_i, \]
or matrix
\[ [x_{i-h} \ldots x_i \ x_{i+1}] = A[x_{i-h-1} \ldots x_{i-1} \ x_i] + B[u_{i-h-1} \ldots u_{i-1} \ u_i], \]
expressions, where \( A, B \) – eigen-dynamics and control efficiency matrices, \( x \) – state vector, \( u \) – control vector, \( h \) – number of observations, \( i \) – discrete time.

The scheme of the aircraft dynamic model is presented in figure 3.

![Figure 3. The scheme of a discrete aircraft dynamic model.](image)

The problem of nonparametric control of the aircraft, described by the equation (1), is to find the controls \( u_i \), knowing only the desired states \( x_i \), without any priori information on the aircraft model parameters \( A, B \).

To solve this problem, let’s write the equation (1) in a block-matrix form
\[ [X_i \ \ x_{i+1}] = A[X_{i-h} \ldots x_{i-1} \ x_i] + B[U_{i-h} \ldots u_{i-1} \ u_i], \]
where \( X_i = [x_{i-h} \ldots x_i] \), \( X_{i-h} = [x_{i-h-1} \ldots x_{i-1}] \), \( U_{i-h} = [u_{i-h-1} \ldots u_{i-1}] \), and convert it to the following linear matrix equation
\[ [I - A][X_i \ \ x_{i+1}] = B[U_{i-h} \ldots u_{i-1} \ u_i], \]
representing the aircraft eigen-dynamics parameters identification problem.

Equation (3) has the form of a right-hand side matrix equation
\[ ZC = D, \]
which is solvable if and only if
\[ DC^R = 0, \]
where \( C^R \) – the right-hand full rank zero divisor of the matrix \( C \), such that \( C C^R = 0 \), which formally describes all the linear dependent columns of matrix \( C \) [6, 13–16].

According to (4), we always can find such \( h \), that the necessary and sufficient solvability condition of (3) has the form of equality
\[ B[U_{i-h} \ldots u_{i-1}][X_i \ \ x_{i+1}]^R = 0, \]
which can be always achieved also by fulfilling the sufficient condition
\[ [U_{i-h} \ldots u_{i-1}][X_i \ \ x_{i+1}]^R = [U_{i-h} \ldots u_{i-1}][r_i \ 1]^R = 0, \]
where
\[ [r_i \ 1] = [X_i \ \ x_{i+1}]^R \]
is the single-column normalized right-hand zero divisor, such that

$$\begin{bmatrix} X_i & x_{i+1} & 1 \end{bmatrix}^T r = 0. \quad (8)$$

So, from (6) we can find the nonparametric aircraft control algorithm of the form

$$u_i = -U_{i,j} r_i, \quad (9)$$

allowing obtaining the required controls $u_i$ based only on some $h$ past measurements and the desired state $x_{i+1}$ without any information on the model parameters $A$ and $B$.

3. Nonparametric aircraft flight control example

To verify the proposed nonparametric control method let’s simulate a midrange passenger aircraft dynamics by discrete state-space model (1), where $x = [\Delta V, \Delta \alpha, \Delta \beta, \Delta \gamma, \Delta \omega_x, \Delta \omega_y, \Delta \omega_z]^T$; $V$ – flying speed (km/h); $\alpha, \beta, \gamma$ – angles of attack, pitch, and roll (deg); $\omega_x, \omega_y, \omega_z$ – angular rates of roll, yaw, and pitch (deg/s); $u = [u_{e,j}, u_{e,r}, u_{a,l}, u_{a,r}, u_{a,l}, u_{a,r}, u_{i,l}, u_{i,r}]^T$; $u_{e,j}, u_{e,r}, u_{a,l}, u_{a,r}, u_{a,l}, u_{a,r}, u_{i,l}, u_{i,r}$ – deflection angles of right and left elevators, stabilizer, rudder, right and left ailerons, right and left interceptors (deg).

$$A = \begin{bmatrix}
0.99770 & -1.02099 & -0.98098 & 0 & 0 & 0 & 0 & 0 \\
-0.00057 & 0.90700 & 0.00008 & 0.10000 & 0 & 0 & 0 & 0 \\
0.00057 & 0.09300 & 0.99992 & 0 & 0 & 0 & 0 & 0 \\
-0.00002 & -0.07897 & -0.00001 & 0.97279 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 1.00278 & 0.01139 & 0.09876 & 0.01370 \\
0 & 0 & 0 & 0 & -0.34455 & 0.65043 & -0.08993 & 0 \\
0 & 0 & 0 & 0 & -0.19870 & 0.02042 & 0.89593 & 0 \\
0 & 0 & 0 & 0 & 0 & 0.10000 & -0.01087 & 1
\end{bmatrix};$$

$$B = \begin{bmatrix}
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0.00052 & 0.00052 \\
0.00178 & 0.00178 & -0.00084 & 0 & -0.00010 & -0.00010 & -0.00070 & -0.00070 \\
-0.00178 & -0.00178 & 0.00084 & 0 & 0.00010 & 0.00010 & 0.00070 & 0.00070 \\
-0.03378 & -0.03378 & -0.13433 & 0 & -0.00501 & -0.00501 & 0.01271 & 0.01271 \\
0 & 0 & 0 & 0 & -0.00054 & 0 & 0 & -0.00022 & 0.00022 \\
0.02052 & -0.02052 & 0 & -0.01973 & 0.05419 & -0.05419 & -0.16357 & 0.16357 \\
-0.00092 & 0.00092 & 0 & -0.31463 & -0.00579 & 0.00579 & 0.02973 & -0.02973 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0
\end{bmatrix}. $$

Figure 4 shows the scheme of the dynamic aircraft model with a nonparametric control device.

Figure 5 depicts the parallel simulation results of parametric (blue line) and nonparametric (red line) control algorithms with simulation time of 5 seconds and sample rate $\Delta t = 0.1$ seconds. Note that the nonparametric controls appear only at the 14th simulation step. This is due to the fact that in the current situation for the existence of the right-hand zero divisor (8) it is necessary to accumulate 13 columns of past states and controls to detect their linear dependences. So, the sample width $h = 13$, and the nonparametric algorithm tune time $t_{\text{tune}} = \Delta t(h + 1) = 1.4 \text{ sec}$. The simulation results show that nonparametric controls completely coincide with reference ones after tune time.
Figure 4. The scheme of aircraft discrete dynamic model with a nonparametric control.

Figure 5. Aircraft’s controls and states.

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