Modeling and Prediction of Effects of Seafloor Sediment on Sound Velocity Based on Multi-parameter Analysis

Qijun Xiao¹, Zhonghui Luo²*, Bin Peng², Yongshun Luo², Min Wang², Yanghua Liu², Hanzhang Liu²

¹School of Electronic and Electrical Engineering, Zhaoqing University, Zhaoqing, China
²School of Mechanical and Electronic, Guangdong Polytechnic Normal University, Guangzhou, China

*Corresponding author: Zhonghui Luo, e-mail: lzh2382002@163.com

Abstract. An multi-parameter equation to predict the effects of seafloor sediment on sound velocity is established by means of principal component analysis. The empirical equations in previous reports are analyzed and studied, and calculation errors are also analyzed. The principal component regression model is established based on the data of seafloor sediment of the continental slope in the Southern South China Sea. Theoretical research is performed on the method to eliminate interrelated parameters among the physical parameters affecting the sound velocity and select a few independent physical parameters with significant impact on the sound velocity. A three-parameter acoustic speed prediction formula is established with the selected physical parameters, i.e. medium diameter $\phi_d$, porosity $\eta$ and plastic limitation $W_p$ of sediment of the continental slope in the Southern South China Sea. Study results show that the minimum relative error of predicted value is -3.77% to 4.48%, which is less than that of the traditional prediction equation.

1. Introduction

Study on the sound features of seafloor sediment is not only a subject of ocean acoustics. Along with the development of such disciplines as marine science, geological oceanography and oceanics and due to the needs of marine engineering and marine development, more and more attention is paid to the relationship between the acoustic parameters of seafloor sediment and other physical parameters [1]-[3]. Remarkable progress has been made in such studies both at home and abroad in the past decade, and various empirical formulas have been established for the relation between the acoustic speed and physical parameters of submarine sediment. The empirical formulas are obtained by measuring the physical parameters of submarine sediment and the acoustic speed, performing regression analysis of a large number of data, inducing the formulas of the sound velocity depending on single (or double) physical parameters and selecting the best one from a variety of formulas [4]-[10]. They are not systemic and complete in theory but diverse in form. This paper aims at in-depth analysis of the relation between the acoustic speed and physical parameters of submarine sediment by means of principal component analysis and from the perspective of multivariate statistical theory, and provides a principal component regression calculation model.
2. Sound Velocity Empirical Formula of Seafloor Sediment

2.1. Physical Parameters and Acoustic features of Sediment

The commonly used physical parameters of sediment include density, porosity $n$ ($\%$), void ratio, saturation ($\%$), particle size and medium diameter $M_d$. They are related to each other, however, the correlation varies from the sedimentation environment and conditions [11].

The parameters reflecting the mechanical properties of soil including plasticity limitation ($\%$), liquid limitation ($\%$), plastic index ($\%$), and compressive strength (kpa). Liquid limitation, plastic limitation and plastic index reflect plasticity and also water content. In addition, the formula is applied to evaluate the allowable bearing capacity of sediment [11].

The main parameters reflecting the acoustic characteristics include sound velocity $C_p$, acoustic impedance, acoustic attenuation coefficient, and acoustic reflection coefficient[11].

2.2. Sound Velocity Empirical Formulas of Sediment

2.2.1. Single-parameter Formulas.

Many researchers at home and abroad have analyzed the relationship between the sound velocity and physical parameters of seafloor sediment and induced single-parameter and double-parameter formulas. Main single-parameter formulas are as follows.

(1) Relationship between the water content and sound velocity

Lu Bo et al. studied the relationship (the correlation coefficient R is 0.85) between the water content and longitudinal wave sound velocity of seafloor sediment in the coastal area of China and the south of the South China Sea [11]:

$$C_p = 1809.7 - 11.17\omega + 0.08\omega^2$$

(2) Relationship between the porosity ($n$) and sound velocity

Hamilton, Anderson, Orsi et al. once widely studied the influence of physical and chemical properties of seafloor sediment of world oceans on the sound velocity. They think that no satisfactory results are obtained except for the porosity [12]-[14]. Zhou Zhiyu et al. pointed out that the porosity-based single-parameter empirical formula and the measured sound velocity are subject to great nonlinear difference [15]. Zhou Zhiyu et al. and Tang Yonglu added the sound velocity $C_0$ of underlying seawater as a correction term into the above empirical formula [16]. The main empirical formula showing the relationship between the porosity and longitudinal wave sound velocity is as follows:

Hamilton formula [12]:

$$C_p = 2455.9 - 21.716n + 0.126n^2$$

(2)

Anderson formula [13]:

$$C_p = 2506 - 27.58n + 0.186n^2$$

(3)

Orsi et al. formula [18]:

$$C_p = 2527.4 - 27.132n + 0.1782n^2$$

(4)

Zhou Zhiyu et al.[15]:

$$C_p = 962 + C_0 - 27.58n + 0.186n^2$$

(5)

Tang Yonglu [16]:

$$C_p = 942 + C_0 - 25.02n + 0.156n^2$$

(6)

Lu Bo et al. [17]:

$$C_p = 2369.07 - 25.53n + 0.185n^2$$

(7)

(3) Main formula of the relationship between the particle size ($M_d$) and sound velocity:

Hamilton formula [12]:

$$C_p = 1927.27 - 75.82M_d + 3.21M_d^2$$

(8)

Chen Minben et al. [18]:

$$C_p = 2860.449 - 721.958M_d + 147.54M_d^2 - 10.157M_d^3$$

(9)

(4) For the relationship between the density ($\rho$) and sound velocity, Orsi gives a similar formula:

Orsi et al.formula [14]:

$$C_p = 2855.7 - 1885.98\rho + 652.06\rho^2$$

(10)

(5) Relationship between the liquid limit/plastic limit and sound velocity

Atterberg thinks that the liquid limit ($W_L$) and plastic limitation ($W_P$) can make known the sediment compressibility and expansibility [19]. Lu Bo et al. think that the sound wave can only be transmitted in compressive and expandable sediment, and the sediment compressibility has decisive influence on sound velocity changes. They concluded the following empirical formula [19]:
Lu Bo et al.: \( C_p = 1770.8 - 12.78L + 0.11L^2 \) (11) 
where the correlation coefficient \( R \) is 0.81.

Lu Bo et al.: \( C_p = 1738.8 - 17.75pW + 0.215pW^2 \) (12) 
where the correlation coefficient \( R \) is 0.84.

2.2.2. Double-parameter Formulas

(1) Relationship between the porosity/unconfined compressive strength and sound velocity

For double-parameter formulas, Liang Yuanbo et al. compared the research results of predecessors. They thought it more accurate to predict the sound velocity with one natural physical parameter and one mechanical parameter, and thus concluded the empirical formula involving the porosity (\( \% \)) and unconfined compressive strength (\( qu \)) (compliant to the continental slope in China's South-East coastal areas). The sound velocity deviation in fitting of experimental data is smaller [20].

\[ C_p = 4195 - 90.5833n + 0.7695n^2 - 94.6968qu + 64.4603qu^2 \] (13)

(2) Relationship between the density/porosity and sound velocity

The research team of Lu Bo analyzed and studied the relationship of the sound velocity and the density (\( \rho \)) and porosity (\( n \)) of seafloor sediment in the south of the South China Sea, and concluded the following double-parameter formula [11].

\[ C_p = 2362.98 - 31.42n + 0.22n^2 + 14.56L - 0.39L^2 + 0.12LW \] (14)

(3) Relationship between the porosity/water content and sound velocity

Zou Dapeng et al. [21] put forward the double-parameter formula involving the porosity (\( n \)) and water content (\( \omega \)), and concluded the equation (15) by the least squares fitting of the data of the continental shelf in the “Luhuitou” of Hainan Island of the South China Sea. In the equation, the correlation coefficient (\( r \)) is 0.893, the standard deviation is 9.983, and the relative deviation is -1.14% to 0.1%.

\[ C_p = 1973.333 - 7.9462n + 0.473929\omega \] (15)

2.2.3. Multi-parameter Formulas

Anderson divides the seafloor sediment into four categories based on the depth: less than 1500m, 1500 - 3000m, 3000 - 4200m and more than 4200m, and provides a few regression equations for each kind of sediment. Parameters of the equations include the porosity, void ratio, average granularity and carbon content. The equations include single-parameter and double-parameter equations. His analysis results show that it is better to predict the sound velocity based on the porosity and multiple parameters are not conducive to sound velocity prediction [13]. Liang Yuanbo and Lu Bo also studied this problem and think that there is a correlation between the physical parameters of the seafloor sediment. The study shows that the correlation coefficient of the equation which is fitted by three parameters, has almost no effect of improvement [20].

3. Principle of Principal Component Analysis

Principal component analysis is one of the most widely used method of multivariate statistics, and can help to reduce the dimensions of a data matrix with a large number of dimensions and reduce a large number of variable indicators of test data into a few comprehensive indicators by means of multivariate statistics. The object of study is the matrix which consists of test process data. The data is written in the form of a matrix in which each column represents a variable and each row represents a sample. The principal component regression model can be expressed as:

\[ Y = b_1t_1 + b_2t_2 + \cdots + b_kt_k = T_kB \] (16)

Where \( B = [b_1, b_2, \cdots, b_k]^T \) is the coefficient of the principal component regression model, and \( B = (T_k^TT_k)^{-1}T_k^TY \) can be obtained by data fitting. As principal components are orthogonal, the interference caused by multicollinearity can be overcome in the above model. The model with mapping of original data mapped as input variables is as follows:

\[ Y = T_kB = XP_kB = X\theta \] (17)
where $\theta$ is the model parameter of the original variable, $\theta = P_k B = P_k (T_k^T T_k)^{-1} T_k^T Y$, $P_k = [p_1, p_2, \cdots, p_k]$. $Y$ is the formula of the principal component regression model.

4. Principal Component Regression Calculation Model for Sound Velocity of Seafloor Sediment

Table 1 shows the data of the acoustic and physical property of the continental shelf in the south of the South China Sea, which obtained by Lu Bo et al. from the continental shelf in the south of the South China Sea[17]. According to the literature, the physical parameters affecting the Acoustic velocity $C_p$ of sediment mainly include void ratio $e$, the porosity $n$ (%), water content $\omega$ (%), medium diameter $M_d$, density $\rho$ (g/cm$^3$), liquid limit ($W_L$), compressive strength $q_u$ (kpa), and plastic limitation $W_p$[11]. The physical parameters with significant influence on $C_p$ include the porosity $n$, water content $\omega$ and medium diameter $M_d$. According to the literature, the sound velocity decreases along with the increases of the porosity and water content. Based on the definition, the void ratio, the porosity and water content are interrelated. The following regression formula can be obtained by regression analysis of $n$, $\omega$ and $e$ in Table 1 in the unary linear regression method: $(m/s)$

$$n = 41.8726 + 0.258\omega$$

where the correlation coefficient $R$ is 0.9088.

$$\omega = -2.2659 + 43.912e$$

where the correlation coefficient $R$ is 0.9422.

### Tab.1  Physical parameters of continental slope sediments in South China Sea

| Station No. | $C_p$ (m/s$^{-1}$) | $n$ (%) | $\omega$ (%) | $e$ | $\rho$ (g/cm$^3$) | $M_d$ | $q_u$ (kpa) | $W_L$ (%) | $W_p$ (%) |
|------------|-----------------|--------|--------------|-----|-------------------|------|------------|-----------|----------|
| 13-5-7     | 1490            | 80.3   | 155.6        | 4.072 | 1.31              | 6.126 | 6.1        | 78.8      | 20.9     |
| 13-5-8     | 1506            | 71.9   | 95.0         | 2.563 | 1.45              | 5.809 | 16.2       | 67.7      | 25.4     |
| 13-5-9     | 1522            | 63.3   | 62.0         | 1.723 | 1.63              | 5.809 | 19.3       | 48.1      | 25.8     |
| 13-5-11    | 1508            | 76.2   | 118.9        | 3.203 | 1.38              | 5.588 | 12.2       | 70.5      | 61.7     |
| 13-5-12    | 1514            | 72.6   | 98.0         | 0.74  | 1.46              | 4.804 | 11.2       | 67.7      | 25.3     |
| 13-5-13    | 1591            | 72.0   | 95.2         | 2.565 | 1.50              | 6.038 | 18.3       | 64.9      | 50.5     |
| 13-5-15    | 1484            | 76.6   | 124.3        | 3.276 | 1.39              | 5.381 | 7.5        | 80.2      | 51.3     |
| 14-12-13   | 1471            | 75.4   | 124.8        | 3.069 | 1.37              | 5.735 | 0.1        | 77.6      | 32.6     |
| 14-12-18   | 1460            | 79.9   | 147.5        | 3.968 | 1.34              | 5.566 | 0.1        | 77.9      | 24.7     |
| 14-12-19   | 1476            | 81.9   | 168.7        | 4.517 | 1.31              | 5.877 | 0.1        | 90.4      | 36.3     |
| 14-12-59   | 1466            | 78.5   | 153.5        | 3.650 | 1.33              | 6.044 | 0.2        | 83.4      | 28.9     |
| 14-12-63   | 1483            | 80.6   | 163.0        | 4.159 | 1.31              | 5.969 | 1.4        | 96.4      | 25.8     |
| 14-12-65   | 1542            | 80.3   | 160.2        | 4.064 | 1.31              | 6.047 | 1.1        | 100.5     | 43.4     |
| 14-12-67   | 1426            | 81.7   | 175.5        | 4.446 | 1.30              | 6.079 | 0.4        | 98.4      | 56.2     |
| 14-12-69   | 1429            | 81.6   | 174.4        | 4.424 | 1.30              | 6.100 | 0.8        | 100.8     | 36.7     |
| 14-12-71   | 1441            | 81.3   | 164.2        | 4.344 | 1.30              | 6.099 | 0.4        | 99.6      | 39.5     |
| 14-12-72   | 1431            | 81.5   | 173.1        | 4.398 | 1.29              | 6.207 | 1.1        | 95.7      | 40.0     |
| 14-12-47   | 1505            | 79.9   | 157.9        | 3.983 | 1.33              | 6.548 | 0.9        | 85.2      | 28.7     |
| 14-12-49   | 1541            | 81.9   | 182.5        | 4.541 | 1.30              | 5.970 | 0.1        | 97.7      | 68.6     |

According to the theory of principal component analysis, a $12 \times 4$ data matrix is formed with $n$, $\omega$, $e$ and $\rho$ as column vectors. Then the correlation coefficient $R$ of the data matrix and the eigenvalues of $R$ are calculated. regression formula obtained in this paper:
When \( n \), \( M_d \) and \( W_p \) in Table 1 are substituted in Formula (20), the predicted sound velocity is:

\[
C_p' = 1774 - 5.094n + 12.499M_d + 0.998W_p
\]

(20)

The comparison and analysis of the calculation results of the empirical equations provided by Zhou Zhiyu, Tang Yonglu, Lu Bo et al. and Zou Dapeng et al. are as follows:

1. When the equation (5) summarized by Zhou Zhiyu is applied, the predicted sound velocity \( C_p' \) is:
   \[
   C_p' = [1486.7, 1477.1, 1512.6, 1573.7, 1486.9, 1678.7, 1485.8, 1574.5, 1546.5, 1481.4, 1486.1, 1559.5]
   \]
   The deviation of the predicted value \( C_p' \) and measured value is: -195.5 to 123.7, and the relative error is: -11.22% to 7.95%. According to the literature [20], here, \( C_0 = 1529.9 \text{ (m/s)} \).

2. When the equation (6) summarized by Tang Yonglu is applied, the predicted sound velocity \( C_p' \) is:
   \[
   C_p' = [1502.3, 1486.1, 1536, 1604.3, 1469.2, 1711.9, 1500.9, 1605.2, 1574.8, 1468.6, 1501.4, 1589.1]
   \]
   The deviation of the predicted value \( C_p' \) and measured value is: -167.19 to 156.89, and the relative error is: -9.59% to 10.09%.

3. When the equation (7) summarized by Lu Bo et al. is applied, the predicted sound velocity \( C_p' \) is:
   \[
   C_p' = [1490.6, 1488.4, 1505.3, 1550.2, 1520.5, 1636.4, 1490.2, 1550.9, 1529.4, 1511.5, 1490.3, 1539.3]
   \]
   The deviation of the predicted value \( C_p' \) and measured value is: -212.64 to 81.45, and the relative error is: -12.21% to 5.24%.

It can be seen from the above calculations that the error of the sound velocity prediction equation based on principal component analysis in this paper is the smallest.

5. Discussion

Natural physical and mechanical parameters are the most fundamental parameters of seafloor sediment and as a whole, have decisive influence on the acoustic parameters of sound propagation. However, sound propagation should be analyzed according to the structural characteristics. The dynamic elastic properties are demonstrated by sediment under the sound wave effects and have a certain relationship with the natural physical and mechanical properties of sediment.

The porosity \( n \) depends on the volume ratio of liquid and solid of sediment. If the ratio varies, the composition of solid and liquid will change, which will directly result in different speeds of sound propagation. The porosity also reflects the structural density of sediment to a certain extent. Therefore, the porosity is more suitable to predict the acoustic velocity.

6. Conclusions

This paper introduces principal component analysis into the research on the multi-parameter sound velocity prediction formula of seafloor sediment, and studies how to exclude interrelated parameters among the physical parameters affecting the sound velocity of sediment and select a few independent physical parameters with significant impact on the sound velocity. The ternary first-order sound velocity prediction equation is established with the selected physical parameters, i.e. porosity \( n \), medium diameter \( M_d(\phi) \) and plastic limitation \( W_p \) of sediment of the continental shelf and continental slope in the Southern China Sea. According to the established model, the minimum prediction error is 3. is only -3.77% to 4.48%.

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