Sensitivity of the squark flavor mixing to the CP violation of $K$, $B^0$ and $B_s$ mesons

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Abstract

We study the sensitivity of the squark flavor mixing to the CP violating phenomena of $K$, $B^0$ and $B_s$ mesons in the framework of the split-family scenario, where the first and second family squarks are very heavy, $\mathcal{O}(10)$ TeV, on the other hand, the third family squark masses are at $\mathcal{O}(1)$ TeV. In order to constrain the gluino-sbottom-quark mixing parameters, we input the experimental data of the CP violations of $K$, $B^0$, and $B_s$ mesons, that is $\epsilon_K$, $\phi_d$, and $\phi_s$. The experimental upper bound of the chromo-EDM of the strange quark is also input. In addition, we take account of the observed values $\Delta M_{B^0}$, $\Delta M_{B_s}$, the CKM mixing $|V_{ub}|$, and the branching ratio of $b \to s\gamma$. The allowed region of the mixing parameters are obtained as $|\delta_{13}^{dL(dR)}| = 0 \sim 0.01$ and $|\delta_{23}^{dL(dR)}| = 0 \sim 0.04$. By using these values, the deviations from the SM are estimated in the CP violations of the $B^0$ and $B_s$ decays. The deviation from the SM one is tiny in the CP asymmetries of $B^0 \to \phi K_S$ and $B^0 \to \eta' K^0$ due to the chromo-EDM of the strange quark. On the other hand, the CP asymmetries $B_s \to \phi \phi$ and $B_s \to \phi \eta'$ could be largely deviated from the SM predictions. We also predict the time dependent CP asymmetry of $B^0 \to K^0 \bar{K}^0$ and the semi-leptonic CP asymmetries of $B^0 \to \mu^- X$ and $B_s \to \mu^- X$. We expect those precise measurements at Belle II, which will provide us interesting tests for the squark flavor mixing.

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1 Introduction

The flavor physics is on the new stage in the light of LHCb data. The LHCb collaboration has reported new data of the CP violation of the $B_s$ meson and the branching ratios of rare $B_s$ decays [1]-[12]. For many years the CP violation in the $K$ and $B^0$ mesons has been successfully understood within the framework of the standard model (SM), so called Kobayashi-Maskawa (KM) model, where the source of the CP violation is the KM phase in the quark sector with three families. However, the new physics has been expected to be indirectly discovered in the precise data of $B^0$ and $B_s$ meson decays at the LHCb experiment and the further coming experiment, Belle II.

The supersymmetry (SUSY) is one of the most attractive candidates for the new physics. The SUSY signals have not been observed yet although the Higgs-like events have been confirmed [13]. Since the lower bounds of the superparticle masses increase gradually, the squark and the gluino masses are supposed to be at the TeV scale [14]. While, there are new sources of the CP violation if the SM is extended to the SUSY models. The soft squark mass matrices contain the CP-violating phases, which contribute to the flavor changing neutral current (FCNC) with the CP violation. Therefore, we expect the effect of the SUSY contribution in the CP-violating phenomena. However, the clear deviation from the SM prediction has not been observed yet in the LHCb experiment [1]-[12].

The LHCb collaboration presented the time dependent CP asymmetry in the non-leptonic $B_s \rightarrow J/\psi \phi$ decay [11,12,4], which gives a constraint of the SUSY contribution on the $b \rightarrow s$ transition. They have also reported the first measurement of the CP violating phase in the $B_s \rightarrow \phi \phi$ decay [2]. This decay process is occurred at the one-loop level in the SM, where the CP violating phase is very small. On the other hand, the gluino-squark mediated flavor changing process provides new CP violating phases. Thus, the CP asymmetry of $B_s \rightarrow \phi \phi$ is expected to be deviated considerably from the SM one. In this work, we discuss the sensitivity of the SUSY contribution to the CP asymmetry of $B_s \rightarrow \phi \phi$ and $B_s \rightarrow \phi \eta'$ by taking account of constraints from other experimental data of the CP violation. For these decay modes, the most important process of the SUSY contribution is the gluino-squark mediated flavor changing process [15]-[26]. This FCNC effect is constrained by the CP violations in $B^0 \rightarrow J/\psi K_S$ and $B_s \rightarrow J/\psi \phi$ decays. The CP violation of $K$ meson, $\epsilon_K$, also provides a severe constraint to the gluino-squark mediated FCNC. In the SM, $\epsilon_K$ is proportional to $\sin(2\beta)$ which is derived from the time dependent CP asymmetry in $B^0 \rightarrow J/\psi K_s$ decay [27]. The relation between $\epsilon_K$ and $\sin(2\beta)$ is examined by taking account of the gluino-squark mediated FCNC [28].

The time dependent CP asymmetry of $B^0 \rightarrow \phi K_S$, $B^0 \rightarrow \eta' K^0$, and $B^0 \rightarrow K^0 \bar{K}^0$ decays are also attractive ones to search for the gluino-squark mediated FCNC because the penguin amplitude dominates this process as well as $B_s \rightarrow \phi \phi$. Furthermore, we discuss the FCNC with the CP violation in the semileptonic CP asymmetries of $B^0$ and $B_s$ mesons.

In addition, it is remarked that the upper-bound of the chromo-EDM(cEDM) of the strange quark gives a severe constraint for the gluino-squark mediated $b \rightarrow s$ transition [29]-[32].

The lower bounds of the squark masses increase gradually. The gluino mass is expected to be larger than 1.3 TeV, and the squarks of the first and second families are also heavier.
than 1.4 TeV \cite{[14]}. Therefore, we take the split-family scenario, in which the first and second family squarks are very heavy, $\mathcal{O}(10)$ TeV, while the third family squark masses are at $\mathcal{O}(1)$ TeV. Then, the $s \to d$ transition mediated by the first and second family squarks is naturally suppressed by their heavy masses, and competing process is mediated by the second order contribution of the third family squark. In order to estimate the gluino-squark mediated FCNC for the $K$, $B^0$ and $B_s$ meson decays comprehensively, we work in the basis of the squark mass eigenstate. Then, the $6 \times 6$ mixing matrix among down-squarks and down-quarks is studied by input of the experimental constraints.

In section 2, we present the formulation of the gluino-squark mediated transition in our split-family scenario. In section 3, we discuss the gluino-squark mediated FCNC contribution to $\epsilon_K$. In section 4, we discuss the sensitivity of the gluino-squark mediated FCNC to the CP violation of the non-leptonic and the semi-leptonic decays of $B^0$ and $B_s$ mesons. Section 5 is devoted to the summary.

2 CP violation through squark flavor mixing

2.1 Squark flavor mixing

Let us discuss the gluino-squark mediated flavor changing process as the dominate SUSY contribution. We give the $6 \times 6$ squark mass matrix to be $M_{\tilde{q}}$ ($\tilde{q} = \tilde{u}, \tilde{d}$) in the super-CKM basis. In order to go to the diagonal basis of the squark mass matrix, we rotate $M_{\tilde{q}}$ as

$$\tilde{m}^2_{\tilde{q} \text{dia}} = \Gamma^{(q)}_G M^2_{\tilde{q}} \Gamma^{(q)\dagger}_G ,$$

where $\Gamma^{(q)}_G$ is the $6 \times 6$ unitary matrix, and we decompose it into the $3 \times 6$ matrices as $\Gamma^{(q)}_G = (\Gamma^{(q)}_{GL}, \Gamma^{(q)}_{GR})^T$ in the following expressions. Then, the gluino-squark-quark interaction is given as

$$\mathcal{L}_{\text{int}}(\tilde{g}q\tilde{q}) = -i\sqrt{2}g_s \sum_{\{q\}} \tilde{q}^* (T^a) \tilde{G}^a \left[ (\Gamma^{(q)}_{GL})_{ij} L + (\Gamma^{(q)}_{GR})_{ij} R \right] q_j + \text{h.c.} ,$$

where $\tilde{G}^a$ denotes the gluino field, and $L$ and $R$ are projection operators. This interaction leads to the gluino-squark mediated flavor changing process with $\Delta F = 2$ and $\Delta F = 1$ through the box and penguin diagrams.

In our framework, the squarks of the first and second families are heavier than multi-TeV, on the other hand, the masses of the third family squarks, stop and sbottom, are around 1 TeV. Therefore, the first and second squark contribution is suppressed in the gluino-squark mediated flavor changing process by their heavy masses. The stop and sbottom interactions dominate the gluino-squark mediated flavor changing process. Then, the sbottom interaction dominates $\Delta B = 2$ and $\Delta B = 1$ processes. We take a suitable parametrizations of $\Gamma^{(d)}_{GL}$ and
\( \Gamma_{GR}^{(d)} \) as follows \[28\]:

\[
\Gamma_{GR}^{(d)} = \begin{pmatrix}
1 & 0 & \delta_{13}^{dL} c_\theta & 0 & 0 & -\delta_{13}^{dL} s_\theta e^{i\phi} \\
0 & 1 & \delta_{23}^{dL} c_\theta & 0 & 0 & -\delta_{23}^{dL} s_\theta e^{i\phi} \\
-\delta_{13}^{dL*} & -\delta_{23}^{dL*} c_\theta & 0 & 0 & -s_\theta e^{i\phi}
\end{pmatrix},
\]

\[
\Gamma_{GR}^{(d)} = \begin{pmatrix}
0 & 0 & \delta_{13}^{dR} s_\theta e^{-i\phi} & 1 & 0 & \delta_{13}^{dR} c_\theta \\
0 & 0 & \delta_{23}^{dR} s_\theta e^{-i\phi} & 0 & 1 & \delta_{23}^{dR} c_\theta \\
0 & 0 & s_\theta e^{-i\phi} & -\delta_{13}^{dR*} & -\delta_{23}^{dR*} c_\theta & 0
\end{pmatrix},
\]

(3)

where \( c_\theta = \cos \theta \) and \( s_\theta = \sin \theta \), with the mixing angle \( \theta \) in the \( \tilde{b}_{L,R} \) sector and \( \delta_{j3}^{dL}, \delta_{j3}^{dR} \) are the couplings responsible for the flavor transitions. By using these rotation matrices, we estimate the gluino-sbottom mediated flavor changing amplitudes in the \( K, B^0, \) and \( B_s \) meson decays.

For the numerical analysis, we fix sbottom masses. The third family squarks can have substantial mixing between the left-handed squark and the right-handed one due to large Yukawa couplings. In our numerical calculation, we take the typical mass eigenvalues \( m_{\tilde{b}_1}, m_{\tilde{b}_2}, \) and the gluino mass \( m_{\tilde{g}} \) as follows:

\[
m_{\tilde{b}_1} = 1 \text{ TeV}, \quad m_{\tilde{b}_2} = 1.1 \text{ TeV}, \quad m_{\tilde{g}} = 2 \text{ TeV},
\]

(4)

where we take account of the present experimental bounds \[14\]. Then, we can roughly estimate the mixing angle \( \theta \) between the left-handed sbottom and the right-handed one by using RGE’s under the assumption of the universal mass of the GUT scale although it depends on the SUSY parameters in details \[33\]. Therefore, we scatter the left and right mixing \( \theta \) in the range of \( 10^\circ - 35^\circ \) in our numerical calculations. The mixing parameters \( \delta_{13}^{dL} \) and \( \delta_{23}^{dL} \) are complex, and will be constrained by the experimental data. For simplicity, we take

\[
|\delta_{13}^{dR}| = |\delta_{13}^{dL}|, \quad |\delta_{23}^{dR}| = |\delta_{23}^{dL}|,
\]

(5)

on the other hand, the phases of \( \delta_{23}^{dR} \) and \( \delta_{13}^{dR} \), and the phase \( \phi \) are free parameters. Therefore, we have three mixing angles and five phases in the mixing matrices of Eq.(3), which are free parameters in our calculations.

### 2.2 CP violation in \( \Delta B = 2 \) and \( \Delta B = 1 \) processes

Let us discuss the SUSY contribution in the \( \Delta B = 2 \) process. The contribution of new physics to the dispersive part \( M_{12}^q \) is parameterized as

\[
M_{12}^q = M_{12}^{q,SM} + M_{12}^{q,SUSY} = M_{12}^{q,SM}(1 + h_q e^{2i\sigma_q}), \quad (q = d, s)
\]

(6)

where \( M_{12}^{q,SM} \) and \( M_{12}^{q,SUSY} \) are the SM and the SUSY contributions. The parameters \( h_q \) and \( \sigma_q \) are given in terms of mixing parameters of Eq.(3). The \( M_{12}^{q,SUSY} \) are given explicitly in Appendix A. By inputting experimental data of \( \epsilon_K, \Delta M_{B^0}, \Delta M_{B_s}, \) and \( \sin(2\beta), \sin(2\beta_s) \), we constrain the magnitude \( h_q \) and the phase \( \sigma_q \).
The indirect CP violation leads to the non-zero asymmetry \( a_{sl}^q \) in the semileptonic decays \( B_q \rightarrow \mu^- X(q=d,s) \) with “wrong-sign” such as:

\[
    a_{sl}^q \equiv \frac{\Gamma(\bar{B}_q \rightarrow \mu^+ X) - \Gamma(\bar{B}_q \rightarrow \mu^- X)}{\Gamma(\bar{B}_q \rightarrow \mu^+ X) + \Gamma(\bar{B}_q \rightarrow \mu^- X)} \approx \text{Im} \left( \frac{\Gamma_{12}^q}{M_{12}^q} \right) = \frac{\Gamma_{12}^q}{M_{12}^q} \sin \phi_{sl}^q, \tag{7}
\]

where \( \Gamma_{12}^q \) is the absorptive part in the effective Hamiltonian of the \( B_q-\bar{B}_q \) system, where \( B_d \) is denoted as the \( B^0 \) meson in this paper. The SM contribution to the absorptive part \( \Gamma_{12}^q \) is dominated by tree-level decay \( b \rightarrow c\bar{c}s \) etc. Therefore, we assume \( \Gamma_{12}^q = \Gamma_{12}^{SM} \) in our calculation. In the SM, the CP phases are read \[34\],

\[
    \phi_{sl}^{SM} = (3.84 \pm 1.05) \times 10^{-3}, \quad \phi_{sl}^{dSM} = -(7.50 \pm 2.44) \times 10^{-2}, \tag{8}
\]

which correspond to

\[
    a_{sl}^{SM} = (1.9 \pm 0.3) \times 10^{-5}, \quad a_{sl}^{dSM} = -(4.1 \pm 0.6) \times 10^{-4}. \tag{9}
\]

The recent experimental data of these asymmetries are given as \[8, 35\]

\[
    a_{sl}^s = (-0.24 \pm 0.54 \pm 0.33) \times 10^{-2}, \quad a_{sl}^d = (-0.3 \pm 2.1) \times 10^{-3}. \tag{10}
\]

There are many interesting non-leptonic CP violating decays to search for new physics. The effective Hamiltonian for the \( \Delta B = 1 \) process is given as follows:

\[
    H_{\text{eff}} = \frac{4 G_F}{\sqrt{2}} \left[ \sum_{q' = u,c} \sum_{i=1,2} C_i O_i^{(q')} - V_{tb} V_{tq}^* \sum_{i=3-6,7,8G} \left( C_i O_i + \tilde{C}_i \tilde{O}_i \right) \right], \tag{11}
\]

where \( q = s,d \). The local operators are given as

\[
    O_1^{(q')} = (\bar{q}_a \gamma_\mu P_L q_\beta')(\bar{q}_\beta' \gamma_\mu P_L b_\alpha), \quad O_2^{(q')} = (\bar{q}_a \gamma_\mu P_L q_\beta')(\bar{q}_\beta' \gamma_\mu P_L b_\beta),
\]

\[
    O_3 = (\bar{q}_a \gamma_\mu P_L b_\alpha) \sum_Q (\bar{Q}_\beta' \gamma_\mu P_L Q_\alpha), \quad O_4 = (\bar{q}_a \gamma_\mu P_L b_\beta) \sum_Q (\bar{Q}_\beta \gamma_\mu P_L Q_\alpha),
\]

\[
    O_5 = (\bar{q}_a \gamma_\mu P_L b_\beta) \sum_Q (\bar{Q}_\beta \gamma_\mu P_R Q_\alpha), \quad O_6 = (\bar{q}_a \gamma_\mu P_L b_\beta) \sum_Q (\bar{Q}_\beta \gamma_\mu P_R Q_\alpha),
\]

\[
    O_{7,8} = \frac{e}{16\pi^2} m_b \bar{q}_a \sigma^{\mu\nu} P_R b_\alpha F_{\mu\nu}, \quad O_{8G} = \frac{g_s}{16\pi^2} m_b \bar{q}_a \gamma_5 \sigma^{\mu\nu} P_R T^a_{\alpha\beta} b_\beta G_\mu^a, \tag{12}
\]

where \( P_R = (1 + \gamma_5)/2 \), \( P_L = (1 - \gamma_5)/2 \), and \( \alpha, \beta \) are color indices, and \( Q \) is taken to be \( u,d,s,c \) quarks. Here, \( C_i \)’s and \( \tilde{C}_i \)’s are the Wilson coefficients at the relevant mass scale, and \( \tilde{O}_i \)’s are the operators by replacing \( L(R) \) with \( R(L) \) in \( O_i \). In this paper, \( C_i \) includes both SM contribution and squark-gluino one, such as \( C_i = C_i^{SM} + C_i^g \), where \( C_i^{SM} \)’s are given in Ref. \[36\]. The Wilson coefficients of the gluino-squark contribution \( C_7^g \) and \( C_8^g \) are presented in Appendix B, where it is remarked that the magnitudes of \( C_7^g(m_b) \) and \( C_8^g(m_b) \) are reduced by the cancellation between the contributions of two sbottom \( \tilde{b}_1 \) and \( \tilde{b}_2 \).
The Wilson coefficients of $C^q_{7\gamma}(m_b)$ and $C^q_{8G}(m_b)$ at the $m_b$ scale are given at the leading order of QCD as follows \[36\] :

\[
C^q_{7\gamma}(m_b) = \zeta C^q_{7\gamma}(m_\bar{g}) + \frac{8}{3} (\eta - \zeta) C^q_{8G}(m_\bar{g}), \\
C^q_{8G}(m_b) = \eta C^q_{8G}(m_\bar{g}), \tag{13}
\]

where

\[
\zeta = \left(\frac{\alpha_s(m_\bar{g})}{\alpha_s(m_t)}\right)^{\frac{16}{25}} \left(\frac{\alpha_s(m_t)}{\alpha_s(m_b)}\right)^{\frac{16}{25}}, \quad \eta = \left(\frac{\alpha_s(m_\bar{g})}{\alpha_s(m_t)}\right)^{\frac{14}{25}} \left(\frac{\alpha_s(m_t)}{\alpha_s(m_b)}\right)^{\frac{14}{25}}. \tag{14}
\]

Let us discuss the time dependent CP asymmetries of $B^0$ and $B_s$ decaying into the final state $f$, which are defined as \[37\]

\[S_f = \frac{2\text{Im}\lambda_f}{1 + |\lambda_f|^2}, \tag{15}\]

where

\[\lambda_f = \frac{q}{p} \tilde{\rho}, \quad \frac{q}{p} \simeq \sqrt{\frac{M_{12}^q}{M_{12}^q}}, \quad \tilde{\rho} \equiv \frac{A(B^0_q \to f)}{A(B^0_q \to f)).} \tag{16}\]

Here $M_{12}^q(q = s, d)$ include the SUSY contribution in addition to the SM one.

In the $B^0 \to J/\psi K_S$ and $B_s \to J/\psi \phi$ decays, we write $\lambda_{J/\psi K_S}$ and $\lambda_{J/\psi \phi}$ in terms of phase factors, respectively:

\[\lambda_{J/\psi K_S} \equiv -e^{-i\phi_d}, \quad \lambda_{J/\psi \phi} \equiv e^{-i\phi_s}. \tag{17}\]

In the SM, the angle $\phi_d$ is given as $\phi_d = 2\beta$, in which $\beta$ is one angle of the unitarity triangle with respect to $B^0$. On the other hand, $\phi_s$ is given as $\phi_s = -2\beta_s$, in which $\beta_s$ is one angle of the unitarity triangle for $B_s$. Once $\phi_d$ is input, the SM predicts $\phi_s$ as \[38\]

\[\phi_s = -0.0363 \pm 0.0017. \tag{18}\]

The recent experimental data of these phases are \[41, 39\]

\[
\sin \phi_d = 0.679 \pm 0.020, \quad \phi_s = 0.07 \pm 0.09 \pm 0.01, \tag{19}\]

in which the contribution of the gluino-squark-quark interaction is expected to be found because of

\[\phi_d = 2\beta + \arg(1 + h_d e^{2i\phi_d}), \quad \phi_s = -2\beta_s + \arg(1 + h_s e^{2i\phi_s}), \tag{20}\]

where $\beta(\beta_s)$ is given in terms of the CKM matrix elements. These experimental values also constrain the mixing parameters in Eq. \[31\].

Let us consider the contribution from the gluino-sbottom-quark interaction in the non-leptonic decays of the $B^0$ meson. Since the $B^0 \to J/\psi K_S$ process occurs at the tree level in the SM, the CP asymmetry in this process mainly originates from $M_{12}^d$. The CP asymmetries of the penguin dominated decays $B^0 \to \phi K_S$ and $B^0 \to \eta' K^0$ also come from $M_{12}^d$ in the SM. Then, the CP asymmetries of $B^0 \to J/\psi K_S$, $B^0 \to \phi K_S$, and $B^0 \to \eta' K^0$ decays are
expected to be the same magnitude within 10%. On the other hand, if the gluino-sbottom-quark interaction contributes to the decay at the one-loop level, its magnitude could be comparable to the SM penguin one in $B^0 \to \phi K_S$ and $B^0 \to \eta' K^0$ decays, but the effect of the gluino-sbottom-quark interaction is tiny in the $B^0 \to J/\psi K_S$ decay because this process is at the tree level in the SM. Therefore, there is a possibility to find the SUSY contribution by observing the different CP asymmetries among those processes \[40, 41\].

The time dependent CP asymmetry $S_{J/\psi K_S}$ has been precisely measured. We take the data of these time dependent CP asymmetries in HFAG \[39\], which are

$$S_{J/\psi K_S} = 0.679 \pm 0.020, \quad S_{\phi K_S} = 0.74^{+0.11}_{-0.13}, \quad S_{\eta' K^0} = 0.59 \pm 0.07.$$  \tag{21}$$

These values may be regarded to be same within the experimental error-bar. Thus, the experimental values are consistent with the prediction of the SM. In other words, these data severely may constrain the flavor mixing parameter $\delta_{23}^{4L(dR)}$.

Recently, LHCb reported the first flavor-tagged measurement of the time-dependent CP-violating asymmetry in the $B_s$ decay \[2\]. In this decay process, the CP-violating weak phase arises due to the CP violation in the interference between $B_s - B \bar{s}$ mixing and the $b \to s s \bar{s}$ gluonic penguin decay amplitude. The CP-violating phase $\phi_s$ is measured to be in the interval

$$\phi_s = [-2.46, -0.76] \text{ rad},$$  \tag{22}$$
at 68\%C.L. \[2\]. We expect that the precise data will be presented in the near future.

### 2.3 The $b \to s$ transition

The CP asymmetries $S_f$ for $B^0 \to \phi K_S$ and $B^0 \to \eta' K^0$ are given in terms of $\lambda_f$ in Eq. \[10\]:

$$\lambda_{\phi K_S, \eta' K^0} = -e^{-i\phi_d} \sum_{i=3-6,7,8G} \left( C_i^{SM}\langle O_{i}\rangle + C_{i}^{\tilde{g}}\langle O_{i}\rangle + \tilde{C}_{i}^{\tilde{g}}\langle \tilde{O}_{i}\rangle \right) \sum_{i=3-6,7,8G} \left( C_i^{SM*}\langle O_{i}\rangle + C_{i}^{\tilde{g}*}\langle O_{i}\rangle + \tilde{C}_{i}^{\tilde{g}*}\langle \tilde{O}_{i}\rangle \right), \tag{23}$$

where $\langle O_{i}\rangle$ is the abbreviation of $\langle f|O_{i}|B^0\rangle$. It is noticed $\langle \phi K_S|O_{i}|B^0\rangle = \langle \phi K_S|\tilde{O}_{i}|B^0\rangle$ and $\langle \eta' K^0|O_{i}|B^0\rangle = -\langle \eta' K^0|\tilde{O}_{i}|B^0\rangle$, because these final states have different parities \[41\].

Since the dominant term comes from the gluon penguin $C_{8G}^{\tilde{g}}$, the decay amplitudes of $f = \phi K_S$ and $f = \eta' K^0$ are given as follows:

$$\tilde{A}(B^0 \to \phi K_S) \propto C_{SG}(m_b) + \bar{C}_{SG}(m_b),$$

$$\tilde{A}(B^0 \to \eta' K^0) \propto C_{SG}(m_b) - \bar{C}_{SG}(m_b). \tag{24}$$

Since $\bar{C}_{SG}(m_b)$ is suppressed compared to $C_{SG}(m_b)$ in the SM, the magnitudes of the time dependent CP asymmetries $S_f (f = J/\psi \phi, \phi K_S, \eta' K^0)$ are almost same in the SM prediction. However, the squark flavor mixing gives the unsuppressed $\tilde{C}_{SG}(m_b)$, then, the CP asymmetries in those decays are expected to be deviated among them. Therefore, those experimental data give us the tight constraint for $C_{SG}(m_b)$ and $\bar{C}_{SG}(m_b)$. 


We have also $\lambda_f$ for $B_s \rightarrow \phi \phi$ and $B_s \rightarrow \phi \eta'$ as follow:

$$
\lambda_{\phi \phi, \phi \eta'} = e^{-i\phi_f} \frac{\sum_{i=3-6,7,8G} C_{SM}^i \langle O_i \rangle + C_{SM}^i \langle \bar{O}_i \rangle + \tilde{C}_{SM}^i \langle \bar{O}_i \rangle}{\sum_{i=3-6,7,8G} C_{SM}^i \langle O_i \rangle + C_{SM}^i \langle \bar{O}_i \rangle + \tilde{C}_{SM}^i \langle \bar{O}_i \rangle},
$$

(25)

with $\langle \phi \phi | O_i | B_s \rangle = -\langle \phi \phi | \bar{O}_i | B_s \rangle$ and $\langle \phi \eta' | O_i | B_s \rangle = \langle \phi \eta' | \bar{O}_i | B_s \rangle$. The decay amplitudes of $f = \phi \phi$ and $f = \phi \eta'$ are given as follows:

$$
\bar{A}(B_s \rightarrow \phi \phi) \propto C_{SG}(m_b) - \tilde{C}_{SG}(m_b),
$$

$$
\bar{A}(B_s \rightarrow \phi \eta') \propto C_{SG}(m_b) + \tilde{C}_{SG}(m_b).
$$

(26)

Since $C_{SG}(O_{SG})$ and $\tilde{C}_{SG}(\bar{O}_{SG})$ dominate these amplitudes, our numerical results are insensitive to the hadronic matrix elements. In order to obtain precise results, we also take account of the small contributions from other Wilson coefficients $C_i$ ($i = 3, 4, 5, 6$) and $\tilde{C}_i$ ($i = 3, 4, 5, 6$) in our calculations. We estimate each hadronic matrix element by using the factorization relations in Ref. [42]:

$$
\langle O_3 \rangle = \langle O_4 \rangle = \left(1 + \frac{1}{N_c} \right) \langle O_5 \rangle, \quad \langle O_6 \rangle = \frac{1}{N_c} \langle O_5 \rangle,
$$

$$
\langle O_{SG} \rangle = \frac{\alpha_s(m_b)}{8\pi} \left( -\frac{2m_b}{\sqrt{\langle q^2 \rangle}} \right) \left( \langle O_4 \rangle + \langle O_6 \rangle - \frac{1}{N_c} (\langle O_3 \rangle + \langle O_5 \rangle) \right),
$$

(27)

where $\langle q^2 \rangle = 6.3$ GeV$^2$ and $N_c = 3$ is the number of colors. One may worry about the reliability of these naive factorization relations. However, this approximation has been justified numerically in the relevant $b \rightarrow s$ transition as seen in the calculation of PQCD [13].

### 2.4 The $b \rightarrow d$ transition

The time dependent CP asymmetry $S_{K^0\bar{K}^0}$ in the $B^0 \rightarrow K^0\bar{K}^0$ decay is also the interesting one to search for the new physics since there is no tree process of the SM in the $B^0 \rightarrow K^0\bar{K}^0$ decay [44, 45]. The amplitude $\bar{A}(B^0 \rightarrow K^0\bar{K}^0)$ is given in Ref. [44], in which the QCD factorization is taken for the hadronic matrix elements [46], as

$$
\bar{A}(B^0 \rightarrow K^0\bar{K}^0) \approx \frac{4G_F}{\sqrt{2}} \sum_{q=u,c} V_{qb}V_{qd}^* \left[ a_4^q(m_b) + r_\chi a_6^q(m_b) \right] X.
$$

(28)

Here $X$ is the factorized matrix element (See Ref. [44], as

$$
X = -if_K F_0(m_K^2) (m_B^2 - m_K^2),
$$

(29)

\footnote{Improved analyses with $SU(3)$ flavor symmetry were presented in Refs. [47, 48, 49].}
where \( f_K \) and \( F_0(m_K^2) \) denote the decay coupling constant of the \( K \) meson and the form factor, respectively, and \( r_\chi = 2m_K^2/((m_b - m_s)(m_s + m_d)) \) denotes the chiral enhancement factor. The coefficients \( a_i^q \)'s are given as \[44, 46\]

\[
\begin{align*}
a_1^q(m_b) &= (C_4 - \bar{C}_4) + \frac{(C_3 - \bar{C}_3)}{N_c} + \frac{\alpha_s(m_b) C_F}{4\pi N_c} \left[ (C_3 - \bar{C}_3) [F_K + G_K(s_d) + G_K(s_b)] ight. \\
&
+ C_2G_K(s_q) + \left[ (C_4 - \bar{C}_4) + (C_6 - \bar{C}_6) \right] \sum_{f=u} G_K(s_f) + (C_{SG} - \bar{C}_{SG})G_{K,g} \right], \\
a_2^q(m_b) &= (C_6 - \bar{C}_6) + \frac{(C_5 - \bar{C}_5)}{N_c} + \frac{\alpha_s(m_b) C_F}{4\pi N_c} \left[ (C_3 - \bar{C}_3) [G_K'(s_d) + G_K'(s_b)] ight. \\
&
+ C_2G_K'(s_q) + \left[ (C_4 - \bar{C}_4) + (C_6 - \bar{C}_6) \right] \sum_{f=u} G_K'(s_f) + (C_{SG} - \bar{C}_{SG})G_{K,g}' \right],
\end{align*}
\tag{30}
\]

where \( q \) takes \( u \) and \( c \) quarks, \( C_F = (N_c^2 - 1)/(2N_c) \), and the loop functions \( F_K \), \( G_K \), \( G_{K,g} \), \( G_K' \), and \( G_{K,g}' \) are given in Refs. \[44, 46\]. The internal quark mass in the penguin diagrams enters as \( s_f = m_f^2/m_b^2 \). \(^2\) The minus sign in front of \( \tilde{C}_i \) \( (i = 3-6,G) \) comes from the parity of the final state. The CP asymmetry \( S_{K^0\bar{K}^0} \) is given in terms of \( \lambda_{K^0\bar{K}^0} \):

\[
\lambda_{K^0\bar{K}^0} = -e^{-i\phi_d} \frac{\bar{A}(B^0 \to K^0\bar{K}^0)}{A(B^0 \to K^0\bar{K}^0)}.
\tag{31}
\]

### 2.5 Chromo EDM of strange quark

In addition to the CP violating processes with \( \Delta B = 2, 1 \), we should discuss the T violation of flavor conserving process, that is the electric dipole moment. The T violation is expected to be observed in the electric dipole moment of the neutron and the electron. The experimental upper bound of the electric dipole moment of the neutron provides us the upper-bound of the chromo-EDM (cEDM) of the strange quark \[29]-[32].

The cEDM of the strange quark \( d_s^C \) is given in terms of the gluino-sbottom-quark interactions as seen in Appendix C. The upper bound of the cEDM of the strange quark is given by the experimental upper bound of the neutron EDM as \[32]\n
\[
e|d_s^C| < 0.5 \times 10^{-25} \text{ ecm}.
\tag{32}
\]

This bound severely constrains phases of the mixing parameters \( \delta_{23}^{dL(dR)} \) of Eq. (3).

### 3 Tension between \( \epsilon_K \) and \( \sin 2\beta \)

We start our numerical discussion by looking at the \( \epsilon_K \) parameter, which is given in the following theoretical formula

\[
\epsilon_K = e^{i\phi_e} \sin \phi_e \left( \frac{\text{Im}(M_{K^0})}{\Delta M_K} + \xi \right), \quad \xi = \frac{\text{Im}A_0}{\text{Re}A_0}, \quad \phi_e = \tan^{-1} \left( \frac{2\Delta M_K}{\Delta \Gamma_K} \right),
\tag{33}
\]

\(^2\)The \( C_i^q \) in Eq. (30) should be replaced with \( [(V_{tb}V_{ts}^\ast)/(V_{tb}V_{td}^\ast)]C_i^q \) in Appendix B.
with $A_0$ being the isospin zero amplitude in $K \to \pi\pi$ decays. Here, $M_{12}^K$ is the dispersive part of the $K^0 - \bar{K}^0$ mixing, $\Delta M_K$ is the mass difference in the neutral K meson. An effect of suppression factor $\kappa_\epsilon$ which indicates effects of $\xi \neq 0$ and $\phi_\epsilon < \pi/4$, was given by Buras and Guadagnoli [27] as:

$$\kappa_\epsilon = 0.92 \pm 0.02 .$$

(34)

In the SM, the dispersive part $M_{12}^K$ is given as follows,

$$M_{12}^K = (K | \mathcal{M}_{\Delta F=2} | \bar{K})$$

$$= -\frac{4}{3} \left( \frac{G_F}{4\pi} \right)^2 m_K^2 \hat{B}_F M_K \left( \eta_{cc} \lambda_c^2 E(x_c) + \eta_{tt} \lambda_t^2 E(x_t) + 2\eta_{ct} \lambda_c \lambda_t E(x_c, x_t) \right) ,$$

(35)

where $\lambda_c = V_{cs} V_{cd}^*$, $\lambda_t = V_{ts} V_{td}^*$, and $E(x)$'s are the one-loop functions [50]. Then, we obtain $|\epsilon_{K}^{SM}|$ in terms of the Wolfenstein parameters $\lambda$, $\rho$ and $\eta$ as follows:

$$|\epsilon_{K}^{SM}| = \kappa_\epsilon C_\epsilon \hat{B}_K |V_{cb}|^2 \frac{\lambda^2}{2} \left( |V_{cb}|^2 (1 - \bar{\rho}) \eta_{tt} E(x_t) - \eta_{ct} E(x_c) + \eta_{ct} E(x_c, x_t) \right)$$

$$= \kappa_\epsilon C_\epsilon \hat{B}_K |V_{cb}|^2 \lambda^2 \left( \frac{1}{2} |V_{cb}|^2 R_t^2 \sin(2\beta) \eta_{tt} E(x_t) + R_t \sin \beta (-\eta_{ct} E(x_c) + \eta_{ct} E(x_c, x_t)) \right) ,$$

(36)

where

$$C_\epsilon = \frac{G_F^2 F_K^2 m_K M_W^2}{6 \sqrt{2} \pi^2 \Delta M_K} ,$$

(37)

and

$$\bar{\rho} = \rho \left( 1 - \frac{1}{2} \lambda^2 \right) , \quad \bar{\eta} = \eta \left( 1 - \frac{1}{2} \lambda^2 \right) .$$

(38)

In Eq.(38), we use the relation:

$$R_t \sin \beta = \bar{\eta}, \quad R_t \cos \beta = 1 - \bar{\rho} ,$$

(39)

where $R_t$ is

$$R_t = \frac{1}{\lambda} \frac{|V_{td}|}{|V_{ts}|} = \frac{1}{\lambda} \frac{F_{B_s} \sqrt{B_s}}{F_B \sqrt{B}} \sqrt{\frac{M_{B_s}}{M_{B_0}} \frac{\Delta M_{B_0}^{\exp}}{\Delta M_{B_s}^{\exp}}} .$$

(40)

As seen in Eq.(36), $|\epsilon_{K}^{SM}|$ is given in terms of $\sin(2\beta)$ because there is only one CP violating phase in the SM.

If we take into account the gluino-sbottom-quark interaction, $\epsilon_K$ is modified as

$$\epsilon_K = \epsilon_{K}^{SM} + \epsilon_{K}^{\tilde{g}} .$$

(41)

Here, $\epsilon_{K}^{\tilde{g}}$ is given by the imaginary part of the gluino-sbottom box diagram, which is presented in Appendix A. The magnitude of $\epsilon_{K}^{\tilde{g}}$ is proportional to the product $|\delta_{13}^{dL(d\bar{R})} \times \delta_{23}^{dL(d\bar{R})}|$ because
the first and second families are decoupled in the gluino-sbottom box diagrams. We should also modify \( R_t \) as follows:

\[
R_t = \frac{1}{\lambda} \frac{F_{B_s}}{F_B} \sqrt{\frac{M_{B_s}}{M_B}} \sqrt{\frac{\Delta M_{B_s}^{\text{exp}}}{\Delta M_{B_s}^\text{ud}}} \sqrt{\frac{C_s}{C_d}},
\]

where

\[
C_q = 1 + h_q e^{2i\sigma_q}, \quad (q = d, s).
\]

Now, we remark the non-perturbative parameter \( \hat{B}_K \) in eq.(36). Recently, the error of this parameter shrank dramatically in the lattice calculations. The most updated value is presented as \[51, 52\]

\[
\hat{B}_K = 0.73 \pm 0.03.
\]

By inputting this value, we can calculate \( |\epsilon_{K}^{\text{SM}}| \) for the fixed \( \sin(2\beta) \). In other words, we can test numerically the overlap region of among \( |\epsilon_{K}|, \Delta M_{B^0}/\Delta M_{B_s} \) and \( \sin(2\beta) \) in the unitarity triangle of the SM.

![Figure 1: The predicted region on sin 2β-|εK|/B̂K plane in SM. Solid and dotted lines denote the experimental best fit and bounds with 90% C.L.](image)

We obtain the relation between \( \sin(2\beta) \) and \( |\epsilon_{K}^{\text{SM}}/\hat{B}_K| \), which is shown with the experimental allowed region with 90% C.L. in Figure 1. It is noticed that the consistency between the SM prediction and the experimental data in \( \sin(2\beta) \) and \( |\epsilon_{K}^{\text{SM}}/\hat{B}_K| \) is marginal. This fact was pointed out by Buras and Guadagnoli [27], and called as the tension between \( |\epsilon_{K}| \) and \( \sin(2\beta) \). This situation may indicate the new physics. We will show that this tension is understood by taking account of the SUSY box diagram through the gluino-sbottom-quark interaction, which also predicts the deviation from the SM in the CP violations of \( B^0 \to \phi K_S, B^0 \to \eta' K^0, B_s \to \phi \phi, B_s \to \phi \eta', B^0 \to K^0 \bar{K}^0, B^0 \to \mu^{-} X, \) and \( B_s \to \mu^{-} X \) decays.

## 4 Numerical Results

Let us present the numerical results. In order to constrain the gluino-sbottom-quark mixing parameters, we input the experimental data of the CP violations, \( \epsilon_K, \phi_d, \) and \( \phi_s \). The experimental upper bound of cEDM of the strange quark is also put. In addition to these experimental data of the CP violations and T violation, we take account of the observed values \( \Delta M_{B^0}, \Delta M_{B_s}, \) the CKM mixing \( |V_{ub}| \), and the branching ratio of \( b \to s \gamma \). The input
parameters in our calculation are summarized in Table 1.

At first, we present the allowed region on the plane of $|\epsilon_K^{SM}|/\hat{B}_K$ and $\sin(2\beta)$ in Figure 2 where SM components in Eqs. (20) and (41) are only shown. The present experimental data of $\sin\phi_d$ in Eq. (19) allows the range of $\sin(2\beta) = 0.57 - 0.88$, where $\beta$ is one angle of the unitarity triangle. Once we take account of the contribution of the gluino-sbottom-quark interaction, the allowed regions of $|\epsilon_K|$ and $\sin\phi_d$ converge within the experimental error-bars.

When $\sin 2\beta$ and $\Delta M_{B_0}^{SM}/\Delta M_{B_s}^{SM}$ are fixed, the $|V_{ub}|$ is predicted. In Figure 3 we show the relation between $\sin(2\beta)$ and $|V_{ub}|$, where the outside the experimental error-bar of $|V_{ub}|$ are cut. In the present measurement of $|V_{ub}|$, there is 2.6 $\sigma$ discrepancy in the exclusive and inclusive decays as follows [51]:

$$|V_{ub}| = (3.28 \pm 0.30) \times 10^{-3} \text{ (exclusive)}, \quad |V_{ub}| = (4.40 \pm 0.31) \times 10^{-3} \text{ (inclusive)}, \quad (45)$$

although the average value is $|V_{ub}| = (3.82 \pm 0.56) \times 10^{-3}$. The precise observation of $|V_{ub}|$ leads to the determination of $\sin(2\beta)$.

The allowed region of the mixing parameters $|\delta_{13}^{dL(dR)}|$ and $|\delta_{23}^{dL(dR)}|$ are shown in Figure 4 where we input the experimental data of the CP violations, $\epsilon_K$, $\phi_d$, and $\phi_s$. The experimental upper bound of cEDM of the strange quark is also input. We also take account of the observed values $\Delta M_{B_0}$, $\Delta M_{B_s}$, the CKM mixing $|V_{ub}|$, and the branching ratio of $b \to s\gamma$.

As seen in Figure 4 we obtain the allowed region of

$$|\delta_{13}^{dL(dR)}| = 0 \sim 0.01, \quad |\delta_{23}^{dL(dR)}| = 0 \sim 0.04. \quad (46)$$

By using these values, we discuss the sensitivity of the SUSY contribution to the CP violation of the $B^0$ and $B_s$ decays.
\[ \alpha_s(M_Z) = 0.1184 \quad [35] \\
m_c(m_c) = 1.275 \text{ GeV} \quad [35] \\
m_t(m_c) = 1.275 \text{ GeV (}\bar{M}\text{)} \quad [35] \\
M_{B_s} = 5.36677(24) \text{ GeV} \quad [35] \\
\Delta M_s = (116.942 \pm 0.1564) \times 10^{-13} \text{ GeV} \quad [7] \\
\Delta M_d = (3.337 \pm 0.033) \times 10^{-13} \text{ GeV} \quad [35] \\
f_{B_s} = (233 \pm 10) \text{ MeV} \quad [51] \\
f_{B_s}/f_{B^0} = 1.200 \pm 0.02 \quad [51] \\
\xi_s = 1.21(6) \quad [27] \\
\lambda = 0.2255(7) \quad [35] \\
|V_{cb}| = (4.12 \pm 0.11) \times 10^{-2} \quad [51] \\
\eta_{cc} = 1.43(23) \quad [27] \\
\eta_{ct} = 0.47(4) \quad [27] \\
\eta_{tt} = 0.5765(65) \quad [27] \\
f_K = (156.1 \pm 1.1) \text{ MeV} \quad [35] \\
\kappa_s = 0.92(2) \quad [27] \\
\]

Table 1: Input parameters in our calculation.

Let us discuss the time dependent CP asymmetries \( S_{\phi K_s} \) and \( S_{\eta' K^0} \). The SM leads to \( S_{J/\psi K_S}^{(\text{SM})} \simeq S_{\phi K_S}^{(\text{SM})} = S_{\eta' K^0}^{(\text{SM})} \), while the present data of these time dependent CP asymmetries are given in Eq. (21). We predict the deviation from the SM in Figure 5 for the two cases, where the constraint of the cEDM of the strange quark is imposed or is not imposed. It is clearly seen that the cEDM of the strange quark reduces the deviation from the SM. Thus, it is very difficult to observe the gluino-sbottom-quark contribution in these non-leptonic decays.

In Figure 6, we show the prediction of the time dependent CP asymmetries \( S_{\phi\phi} \) and \( S_{\phi\eta'} \), where the constraint of the cEDM is imposed. We use the experimental result of \( S_{J/\psi\phi} \) for the phase \( \phi_s \), which is given in Eq. (19), in our calculations. We denote the small pink region as the SM value \( S_{J/\psi\phi}^{(\text{SM})} = -0.0363 \pm 0.0017 \quad [38] \) in the figure. It is found that
$S_{J/\psi\phi}$ is almost proportional to $S_{\phi\phi}$. If the $\Delta B = 1$ SUSY contribution is sizable, these asymmetries should be different each other as seen in Eq. (25). That is, the gluino interaction induced $\Delta B = 1$ contribution is very small. On the other hand, the gluino induced $\Delta B = 2$ contribution (SUSY box diagrams) could be detectable as seen in Eqs. (20) and (23). This situation is understandable because the magnitude of $C_{8G}(m_b)$ is reduced by the cancellation between the contributions of two sbottom $\tilde{b}_1$ and $\tilde{b}_2$ as seen in Appendix B. In conclusion, we predict 

$$-0.1 \lesssim S_{\phi\phi} \lesssim 0.2$$

and

$$-0.1 \lesssim S_{\phi\eta'} \lesssim 0.2$$

respectively. Since the phase $\phi_s$ has still large experimental error bar, our prediction will be improved if the precise experimental data of $S_{J/\psi\phi}$ will be given in the near future at LHCb. In order to see this situation clearly, we show the $S_{J/\psi\phi}$ dependence for the predicted $S_{\phi\phi}$ in Figure 7.

LHCb reported the first flavor-tagged measurement of the time-dependent CP-violating asymmetry in $B_s \to \phi\phi$ decay \[2\]. The CP-violating phase is measured to be in the interval $\phi_s = [-2.46, -0.76]$ rad as seen in Eq.(22). The precision of the CP violating phase measurement is dominated by the statistical uncertainty and is expected to improve with larger LHCb data in the near future.

![Figure 5](image.png)

Figure 5: The predicted time dependent CP asymmetries on $S_{\phi K_s} - S_{\eta' K_0}$ plane without/with the constraint of cEDM of the strange quark. The SM prediction $S_{J/\psi K_s} \simeq S_{\phi K_s} = S_{\eta' K_0}$ is plotted by the pink slant lines. The experimental data with error-bar is plotted by the red solid lines at 90% C.L.

In Figure 8, we show the prediction of the time dependent CP asymmetry $S_{K^0\bar{K}^0}$ depending on $|a_{13}^{dL(dR)}|$. The predicted region is $-0.4 \leq S_{K^0\bar{K}^0} \leq 0.3$, on the other hand, one predicts $S_{K^0\bar{K}^0}(\text{SM}) \simeq 0.06$ in the SM \[44\]. The present experimental data are given as $S_{K^0\bar{K}^0}(\text{exp}) = -0.8 \pm 0.5$ \[35\]. Since the SM predicted value is tiny, we have a chance to observe the SUSY contribution by the precise experimental data in the near future.

At last, we present the prediction of the indirect CP violation $a_{sl}^d$ and $a_{sl}^s$ in Figure 9. The predicted region is $a_{sl}^d = -0.0017 \sim 0.002$ and $a_{sl}^s = -0.001 \sim 0.001$, on the other hand, the SM gives $a_{sl}^{SM} = -(4.1 \pm 0.6) \times 10^{-4}$ and $a_{sl}^{SM} = (1.9 \pm 0.3) \times 10^{-5}$ as shown in Eq. (27). The experimental data still have large error-bars \[8, 35\]. The precise measurement of the semi-leptonic asymmetry $a_{sl}^d$ at Belle II will provide us an interesting test of the SUSY contribution.
5 Summary

We have discussed the sensitivity of the gluino-sbottom-quark interaction to the CP violating phenomena of the $K$, $B^0$ and $B_s$ mesons. We take the split-family scenario, which is consistent with the LHC data. In this scenario, the first and second family squarks are very heavy, $\mathcal{O}(10)$ TeV, on the other hand, the third family squark masses are at $\mathcal{O}(1)$ TeV. Then, the $s \to d$ transition is mediated by the second order contribution of the third family sbottom. We have used $m_{\tilde{g}} = 2$ TeV, $m_{\tilde{b}_1} = 1$ TeV, and $m_{\tilde{b}_2} = 1.1$ TeV. In order to constrain the gluino-sbottom-quark mixing parameters, we input the experimental data of the CP violations, $\epsilon_K$, $\phi_d$, and $\phi_s$. The experimental upper bound of the cEDM of the strange quark is also input. In addition, we take account of the observed values $\Delta M_{B^0}$, $\Delta M_{B_s}$, the CKM mixing $|V_{ub}|$, and the branching ratio of $b \to s\gamma$. 

Figure 6: The predicted time dependent CP asymmetries on $S_{\phi'\phi} - S_{\phi\phi}$ plane. The small pink region denotes the SM prediction.

Figure 7: The predicted $S_{\phi\phi}$ versus $S_{J/\psi\phi}$ plane, where $S_{J/\psi\phi}$ is plotted within the experimental error at 90% C.L. The small pink region denotes the SM prediction.

Figure 8: The predicted time dependent CP asymmetry $S_{K^0\bar{K}^0}$ versus $|\delta_{13}^{dL(\bar{d}R)}|$. The red solid and red dotted lines denote the best fit value and the experimental bound with 90% C.L., respectively.

Figure 9: Predicted semi-leptonic CP asymmetries $a_{sl}^d$ and $a_{sl}^s$. The red solid and red dotted lines denote the best fit value and the experimental bounds with 90% C.L., respectively.
By using the non-perturbative parameter $\hat{B}_K = 0.73 \pm 0.03$, which is the most updated value in the lattice calculations, it is clearly presented that the consistency between the SM prediction and the experimental data is marginal on the $\sin(2\beta) - |\epsilon_K^{\text{SM}}|$ plane. This tension has been solved by taking account of the gluino-sbottom-quark interaction.

The allowed region of the mixing parameters are obtained as $|\delta_{13}^{dL(dR)}| = 0 \sim 0.01$ and $|\delta_{23}^{dL(dR)}| = 0 \sim 0.04$. By using these values, the deviations from the SM prediction are estimated in the CP violation of $B^0$ and $B_s$ decays. The CP asymmetries of the $B^0 \to \phi K_S$ and $B^0 \to \eta' K^0$ decays are found to be tiny due to the cEDM constraint of the strange quark.

On the other hand, the CP asymmetries of the $B_s \to \phi\phi$ and $B_s \to \phi\eta'$ decays could be largely deviated from the SM predictions such as $-0.1 \lesssim S_{\phi\phi} \lesssim 0.2$ and $-0.1 \lesssim S_{\phi\eta'} \lesssim 0.2$. It is remarked that the time dependent CP asymmetry $S_{\phi\phi}$ is almost proportional to $S_{\phi\eta'}$. That is, the gluino-sbottom interaction induced $\Delta B = 1$ transition is very small, but the $\Delta B = 2$ transition could be detectable. Since the phase $\phi_s$ has still large experimental error-bar, our prediction will be improved if the precise experimental data of $S_{J/\psi\phi}$ will be given in the near future at the LHCb experiment.

We also predict the time dependent CP asymmetry $S_{K^0\bar{K}^0}$ as $-0.4 \leq S_{K^0\bar{K}^0} \leq 0.3$ while one predicts $S_{K^0\bar{K}^0}(\text{SM}) \approx 0.06$ in the SM. More precise data will test the SUSY contribution in the near future. The semi-leptonic CP asymmetries $a^d_{sl}$ and $a^s_{sl}$ are predicted as $a^d_{sl} = -0.0017 \sim 0.002$ and $a^s_{sl} = -0.001 \sim 0.001$, while the SM predicts $a^d_{sl}^{\text{SM}} = -(4.1 \pm 0.6) \times 10^{-4}$ and $a^s_{sl}^{\text{SM}} = (1.9 \pm 0.3) \times 10^{-5}$. We expect the precise measurement of the $a^d_{sl}$ at Belle II, which will provide us interesting tests of the squark flavor mixing.

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Appendix

A Squark contribution in $\Delta F = 2$ process

The $\Delta F = 2$ effective Lagrangian from the gluino-sbottom-quark interaction is given as

$$\mathcal{L}^{\Delta F=2}_{\text{eff}} = -\frac{1}{2} \left[ C_{VLL} O_{VLL} + C_{VRR} O_{VRR} \right] - \frac{1}{2} \sum_{i=1}^{2} \left[ C_{SLL}^{(i)} O_{SLL}^{(i)} + C_{SRR}^{(i)} O_{SR}^{(i)} + C_{SLR}^{(i)} O_{SRL}^{(i)} \right],$$

(47)

then, the $P^0 - \bar{P}^0$ mixing, $M_{12}$, is written as

$$M_{12} = -\frac{1}{2m_P} \langle P^0 | \mathcal{L}^{\Delta F=2}_{\text{eff}} | \bar{P}^0 \rangle.$$ 

(48)
The hadronic matrix elements are given in terms of the non-perturbative parameters $B_i$ as:

$$
\langle P^0 | O_{VLL} | \bar{P}^0 \rangle = \frac{2}{3} m_P^2 f_P^2 B_1, \quad \langle P^0 | O_{VRR} | \bar{P}^0 \rangle = \langle P^0 | O_{VLL} | \bar{P}^0 \rangle,
$$

$$
\langle P^0 | O_{SLL}^{(1)} | \bar{P}^0 \rangle = -\frac{5}{12} m_P^2 f_P^2 R_P B_2, \quad \langle P^0 | O_{SRR}^{(1)} | \bar{P}^0 \rangle = \langle P^0 | O_{SLL}^{(1)} | \bar{P}^0 \rangle,
$$

$$
\langle P^0 | O_{SLL}^{(2)} | \bar{P}^0 \rangle = \frac{1}{12} m_P^2 f_P^2 R_P B_3, \quad \langle P^0 | O_{SRR}^{(2)} | \bar{P}^0 \rangle = \langle P^0 | O_{SLL}^{(2)} | \bar{P}^0 \rangle,
$$

$$
\langle P^0 | O_{SLL}^{(1)} | \bar{P}^0 \rangle = \frac{1}{2} m_P^2 f_P^2 R_P B_4, \quad \langle P^0 | O_{SLL}^{(2)} | \bar{P}^0 \rangle = \frac{1}{6} m_P^2 f_P^2 R_P B_5,
$$

where

$$
R_P = \left( \frac{m_P}{m_Q + m_q} \right)^2,
$$

with $(P, Q, q) = (B_d, b, d), (B_s, b, s), (K, s, d)$.

The Wilson coefficients for the gluino contribution in Eq. (47) are written as

\[
C_{VLL}(m_{\tilde{g}}) = \frac{\alpha_s^2}{m_{\tilde{g}}^2} \sum_{i,J=1}^{6} (\lambda^{(d)}_{GLL})_{ij}^l (\lambda^{(d)}_{GLL})_{ij}^l \left[ \frac{11}{18} g_{2[1]}(x_i^\tilde{g}, x_j^\tilde{g}) + \frac{2}{9} g_{1[1]}(x_i^\tilde{g}, x_j^\tilde{g}) \right],
\]

\[
C_{VRR}(m_{\tilde{g}}) = C_{VLL}(m_{\tilde{g}})(L \leftrightarrow R),
\]

\[
C_{SRR}^{(1)}(m_{\tilde{g}}) = C_{SRR}^{(1)}(m_{\tilde{g}})(L \leftrightarrow R),
\]

\[
C_{SLL}^{(1)}(m_{\tilde{g}}) = C_{SRR}^{(1)}(m_{\tilde{g}})(L \leftrightarrow R),
\]

\[
C_{SRR}^{(2)}(m_{\tilde{g}}) = C_{SRR}^{(2)}(m_{\tilde{g}})(L \leftrightarrow R),
\]

\[
C_{SLL}^{(2)}(m_{\tilde{g}}) = C_{SRR}^{(2)}(m_{\tilde{g}})(L \leftrightarrow R),
\]

\[
C_{SLL}^{(1)}(m_{\tilde{g}}) = \frac{\alpha_s^2}{m_{\tilde{g}}^2} \sum_{i,J=1}^{6} \left\{ (\lambda^{(d)}_{GLR})_{ij}^l (\lambda^{(d)}_{GLR})_{ij}^l \left[ -\frac{1}{3} g_{1[1]}(x_i^\tilde{g}, x_j^\tilde{g}) + \frac{14}{3} g_{2[1]}(x_i^\tilde{g}, x_j^\tilde{g}) - \frac{2}{3} g_{2[1]}(x_i^\tilde{g}, x_j^\tilde{g}) \right] \right\},
\]

\[
C_{SRR}^{(2)}(m_{\tilde{g}}) = \frac{\alpha_s^2}{m_{\tilde{g}}^2} \sum_{i,J=1}^{6} \left\{ (\lambda^{(d)}_{GLR})_{ij}^l (\lambda^{(d)}_{GRR})_{ij}^l \left[ -\frac{5}{3} g_{2[1]}(x_i^\tilde{g}, x_j^\tilde{g}) + \frac{2}{9} g_{1[1]}(x_i^\tilde{g}, x_j^\tilde{g}) + \frac{10}{9} g_{2[1]}(x_i^\tilde{g}, x_j^\tilde{g}) \right] \right\},
\]

where

\[
(\lambda^{(d)}_{GLL})_{ij}^l = (\Gamma^{(d)})_{ij}^{\dagger} (\Gamma^{(d)})_{ij}^l K (\Gamma^{(d)})_{ij}^l, \quad (\lambda^{(d)}_{GLR})_{ij}^l = (\Gamma^{(d)})_{ij}^{\dagger} (\Gamma^{(d)})_{ij}^l K (\Gamma^{(d)})_{ij}^l,
\]

\[
(\lambda^{(d)}_{GLR})_{ij}^l = (\Gamma^{(d)})_{ij}^{\dagger} (\Gamma^{(d)})_{ij}^l K (\Gamma^{(d)})_{ij}^l, \quad (\lambda^{(d)}_{GRR})_{ij}^l = (\Gamma^{(d)})_{ij}^{\dagger} (\Gamma^{(d)})_{ij}^l K (\Gamma^{(d)})_{ij}^l.
\]
Here we take \((i, j) = (1, 3), (2, 3), (1, 2)\) which correspond to \(B^0, B_s,\) and \(K^0\) mesons, respectively. The loop functions are given as follows:

- If \(x_I^g \neq x_J^g\) \((x_{I,j}^g = m_{d_{I,j}}^2/m_{g}^2)\),

\[
g_{1/(1)}(x_I^g, x_J^g) = \frac{1}{x_I^g - x_J^g} \left( \frac{x_I^g \log x_I^g}{(x_I^g - 1)^2} - \frac{x_J^g \log x_J^g}{(x_J^g - 1)^2} + 1 \right),
\]

\[
g_{2/(1)}(x_I^g, x_J^g) = \frac{1}{x_I^g - x_J^g} \left( \frac{(x_I^g)^2 \log x_I^g}{(x_I^g - 1)^2} - \frac{1}{x_I^g} \frac{(x_J^g)^2 \log x_J^g}{(x_J^g - 1)^2} + 1 \right). \tag{53}
\]

- If \(x_I^g = x_J^g\),

\[
g_{1/(1)}(x_I^g, x_J^g) = -\frac{(x_I^g + 1) \log x_I^g}{(x_I^g - 1)^3} + \frac{2}{(x_I^g - 1)^2},
\]

\[
g_{2/(1)}(x_I^g, x_J^g) = -\frac{2x_I^g \log x_I^g}{(x_I^g - 1)^3} + \frac{x_I^g + 1}{(x_I^g - 1)^2}. \tag{54}
\]

In this paper, we take \((I, J) = (3, 3), (3, 6), (6, 3), (6, 6)\), because we assume the split-family. The effective Wilson coefficients are given at the leading order of QCD as follows:

\[
C_{VLL}(m_b(\Lambda = 2 \text{ GeV})) = \eta_{VLL}^{B(K)} C_{VLL}(m_{g}), \quad C_{VRR}(m_b(\Lambda = 2 \text{ GeV})) = \eta_{VRR}^{B(K)} C_{VLL}(m_{g}),
\]

\[
\begin{pmatrix}
C_{SLL}^{(1)}(m_b(\Lambda = 2 \text{ GeV})) \\
C_{SLL}^{(2)}(m_b(\Lambda = 2 \text{ GeV}))
\end{pmatrix} =
\begin{pmatrix}
C_{SLL}^{(1)}(m_g) \\
C_{SLL}^{(2)}(m_g)
\end{pmatrix} X_{LL}^{-1} \eta_{LL}^{B(K)} X_{LL},
\]

\[
\begin{pmatrix}
C_{SRR}^{(1)}(m_b(\Lambda = 2 \text{ GeV})) \\
C_{SRR}^{(2)}(m_b(\Lambda = 2 \text{ GeV}))
\end{pmatrix} =
\begin{pmatrix}
C_{SRR}^{(1)}(m_g) \\
C_{SRR}^{(2)}(m_g)
\end{pmatrix} X_{RR}^{-1} \eta_{RR}^{B(K)} X_{RR},
\]

\[
\begin{pmatrix}
C_{SLR}^{(1)}(m_b(\Lambda = 2 \text{ GeV})) \\
C_{SLR}^{(2)}(m_b(\Lambda = 2 \text{ GeV}))
\end{pmatrix} =
\begin{pmatrix}
C_{SLR}^{(1)}(m_g) \\
C_{SLR}^{(2)}(m_g)
\end{pmatrix} X_{LR}^{-1} \eta_{LR}^{B(K)} X_{LR}, \tag{55}
\]

where

\[
\eta_{VLL}^{B} = \eta_{VRR}^{B} = \left( \frac{\alpha_s(m_g)}{\alpha_s(m_t)} \right)^{\frac{6}{2\pi}} \left( \frac{\alpha_s(m_t)}{\alpha_s(m_b)} \right)^{\frac{6}{2\pi}},
\]

\[
\eta_{LL}^{B} = \eta_{RR}^{B} = S_{LL} \begin{pmatrix}
\eta_{B g}^{d_{1 LL}} & 0 \\
0 & \eta_{B g}^{d_{2 LL}}
\end{pmatrix} S_{LL}^{-1}, \quad \eta_{LR}^{B} = S_{LR} \begin{pmatrix}
\eta_{B g}^{d_{1 LR}} & 0 \\
0 & \eta_{B g}^{d_{2 LR}}
\end{pmatrix} S_{LR}^{-1},
\]

\[
\eta_{B g} = \left( \frac{\alpha_s(m_g)}{\alpha_s(m_t)} \right)^{\frac{1}{12\pi}} \left( \frac{\alpha_s(m_t)}{\alpha_s(m_b)} \right)^{\frac{4}{9\pi}},
\]

\[
\eta_{VLL}^{K} = \eta_{VRR}^{K} = \left( \frac{\alpha_s(m_g)}{\alpha_s(m_t)} \right)^{\frac{6}{2\pi}} \left( \frac{\alpha_s(m_t)}{\alpha_s(m_b)} \right)^{\frac{6}{2\pi}} \left( \frac{\alpha_s(m_b)}{\alpha_s(\Lambda = 2 \text{ GeV})} \right)^{\frac{6}{2\pi}},
\]

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\[ \eta_{LL}^K = \eta_{RR}^K = S_{LL} \left( \begin{array}{cc} d_{LL}^1 & 0 \\ 0 & \eta_{\tilde{g}g}^L \end{array} \right) S_{LL}^{-1}, \quad \eta_{LR}^K = S_{LR} \left( \begin{array}{cc} d_{LR}^1 & 0 \\ 0 & \eta_{\tilde{g}g}^L \end{array} \right) S_{LR}^{-1}, \]

\[ \eta_{\tilde{g}g} = \left( \frac{\alpha_s(m_{\tilde{g}})}{\alpha_s(m_t)} \right)^{\frac{1}{11}} \left( \frac{\alpha_s(m_{\tilde{g}})}{\alpha_s(m_b)} \right)^{\frac{\alpha_s(m_b)}{\alpha_s(L = 2 \text{ GeV})}} \]

\[ d_{LL}^1 = \frac{2}{3} \left( 1 - \sqrt{241} \right), \quad d_{LL}^2 = \frac{2}{3} \left( 1 + \sqrt{241} \right), \quad d_{LR}^1 = -16, \quad d_{LR}^2 = 2, \]

\[ S_{LL} = \left( \frac{16+\sqrt{241}}{60}, \frac{16-\sqrt{241}}{60} \right), \quad S_{LR} = \left( \begin{array}{cc} -2 & 1 \\ 3 & 0 \end{array} \right), \]

\[ X_{LL} = X_{RR} = \left( \begin{array}{cc} 1 & 0 \\ 4 & 8 \end{array} \right), \quad X_{LR} = \left( \begin{array}{cc} 0 & -2 \\ 1 & 0 \end{array} \right). \]  

(56)

For the parameters \( B_i^{(d)}(i = 2 - 5) \) of \( B \) mesons, we use values in [51] as follows:

\[ B_2^{(d)}(m_b) = 0.79(2)(4), \quad B_3^{(d)}(m_b) = 0.92(2)(4), \]
\[ B_4^{(d)}(m_b) = 1.15(3)(+5), \quad B_5^{(d)}(m_b) = 1.72(4)(+20), \]
\[ B_2^{(s)}(m_b) = 0.80(1)(4), \quad B_3^{(s)}(m_b) = 0.93(3)(8), \]
\[ B_4^{(s)}(m_b) = 1.16(2)(+5), \quad B_5^{(s)}(m_b) = 1.75(3)(+21). \]  

(57)

On the other hand, we use the most updated values for \( \hat{B}_1^{(d)} \) and \( \hat{B}_1^{(s)} \) as [51] [52]

\[ \hat{B}_1^{(d)} = 1.33 \pm 0.06, \quad \hat{B}_1^{(d)} / \hat{B}_1^{(d)} = 1.05 \pm 0.07. \]  

(58)

For the parameters \( B_i^K(i = 2 - 5) \), we use following values [53],

\[ B_2^{(K)}(2 \text{ GeV}) = 0.66 \pm 0.04, \quad B_3^{(K)}(2 \text{ GeV}) = 1.05 \pm 0.12, \]
\[ B_4^{(K)}(2 \text{ GeV}) = 1.03 \pm 0.06, \quad B_5^{(K)}(2 \text{ GeV}) = 0.73 \pm 0.10, \]  

(59)

and we take recent value of Eq. [44] for deriving \( B_1^{(K)}(2 \text{ GeV}) \).

### B Squark contribution in \( \Delta F = 1 \) process

The Wilson coefficients for the gluino contribution in Eq.(11) are written as [53]

\[ C_{77}^g(m_{\tilde{g}}) = \frac{8 \sqrt{2} \alpha_s \pi}{32 G_F V_{tb} V_{td}^*} \]
\[ \times \left[ \left( \frac{\Gamma_{GL}^{(d)}}{m_{d_3}^2} \right)^* \left\{ \left( \Gamma_{GL}^{(d)} \right)_{33} \left( -\frac{1}{3} F_2(x_t^2) \right) + \frac{m_2}{m_b} \left( \Gamma_{GR}^{(d)} \right)_{33} \left( -\frac{1}{3} F_2(x_t^2) \right) \right\} \right] + \left[ \left( \frac{\Gamma_{GL}^{(d)}}{m_{d_6}^2} \right)^* \left\{ \left( \Gamma_{GL}^{(d)} \right)_{36} \left( -\frac{1}{3} F_2(x_t^2) \right) + \frac{m_2}{m_b} \left( \Gamma_{GR}^{(d)} \right)_{36} \left( -\frac{1}{3} F_2(x_t^2) \right) \right\} \right], \]  

(60)
The cEDM of the strange quark from gluino contribution is given by [53]

\[
\begin{align*}
C_{\text{cEDM}}(m_{\tilde{g}}) &= \frac{8 \sqrt{2} \alpha_s \pi}{3} \frac{\left( \Gamma^{(d)}_{\text{GL}} \right)^*_{k_3}}{m_{\tilde{d}_3}^2} \left\{ \left( \Gamma^{(d)}_{\text{GL}} \right)_{33} \left( -\frac{9}{8} F_1(x^3_{\tilde{g}}) - \frac{1}{8} F_2(x^3_{\tilde{g}}) \right) + \frac{m_{\tilde{g}}}{m_b} \left( \Gamma^{(d)}_{\text{GR}} \right)_{33} \left( -\frac{9}{8} F_3(x^3_{\tilde{g}}) - \frac{1}{8} F_4(x^3_{\tilde{g}}) \right) \right\} \\
&+ \frac{m_{\tilde{g}}}{m_{\tilde{d}_6}} \left\{ \left( \Gamma^{(d)}_{\text{GL}} \right)_{36} \left( -\frac{9}{8} F_3(x^6_{\tilde{g}}) - \frac{1}{8} F_4(x^6_{\tilde{g}}) \right) \right\},
\end{align*}
\]

where \( k = 2, 1 \) correspond to \( b \to q \) (\( q = s, d \)) transitions, respectively. The loop functions \( F_i(x^I_{\tilde{g}}) \) are given as

\[
\begin{align*}
F_1(x^I_{\tilde{g}}) &= \frac{x^I_{\tilde{g}} \log x^I_{\tilde{g}}}{2(x^I_{\tilde{g}} - 1)^4} + \frac{(x^I_{\tilde{g}})^2 - 5x^I_{\tilde{g}} - 2}{12(x^I_{\tilde{g}} - 1)^3}, \\
F_2(x^I_{\tilde{g}}) &= -\frac{(x^I_{\tilde{g}})^2 \log x^I_{\tilde{g}}}{2(x^I_{\tilde{g}} - 1)^4} + \frac{2(x^I_{\tilde{g}})^2 + 5x^I_{\tilde{g}} - 1}{12(x^I_{\tilde{g}} - 1)^3}, \\
F_3(x^I_{\tilde{g}}) &= -\frac{\log x^I_{\tilde{g}}}{(x^I_{\tilde{g}} - 1)^3} + \frac{x^I_{\tilde{g}} - 3}{2(x^I_{\tilde{g}} - 1)^2}, \\
F_4(x^I_{\tilde{g}}) &= -\frac{x^I_{\tilde{g}} \log x^I_{\tilde{g}}}{(x^I_{\tilde{g}} - 1)^3} + \frac{x^I_{\tilde{g}} + 1}{2(x^I_{\tilde{g}} - 1)^2} = \frac{1}{2} g_{2|1}(x^I_{\tilde{g}}, x^I_{\tilde{g}}),
\end{align*}
\]

with \( x^I_{\tilde{g}} = m_{\tilde{g}}^2/m_{\tilde{d}_I}^2 \) (\( I = 3, 6 \)).

\[\text{C cEDM}\]

The cEDM of the strange quark from gluino contribution is given by [53]

\[
d^C_s = -2 \sqrt{4 \pi \alpha_s(m_{\tilde{g}})} \text{Im}[A^g_{22}],
\]

where

\[
A^g_{22} = -\frac{\alpha_s(m_{\tilde{g}})}{4\pi} \left\{ \frac{1}{3} \frac{1}{2m_{\tilde{d}_3}^2} \left\{ m_s(\lambda^{(d)}_{\text{GLL}})^2_{32} + m_s(\lambda^{(d)}_{\text{GRR}})^2_{32} \right\} \left( 9F_1(x^3_{\tilde{g}}) + F_2(x^3_{\tilde{g}}) \right) + m_{\tilde{g}}(\lambda^{(d)}_{\text{GLR}})^2_{33} \left( 9F_3(x^3_{\tilde{g}}) + F_4(x^3_{\tilde{g}}) \right) \right\} \\
+ \frac{1}{2m_{\tilde{d}_6}^2} \left\{ m_s(\lambda^{(d)}_{\text{GLL}})^2_{62} + m_s(\lambda^{(d)}_{\text{GRR}})^2_{62} \right\} \left( 9F_1(x^6_{\tilde{g}}) + F_2(x^6_{\tilde{g}}) \right) + m_{\tilde{g}}(\lambda^{(d)}_{\text{GLR}})^2_{66} \left( 9F_3(x^6_{\tilde{g}}) + F_4(x^6_{\tilde{g}}) \right)\right\}.
\]
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