G-warm inflation

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Abstract. A warm inflationary universe in the context of Galileon model or G-model is studied. Under a general formalism we study the inflationary dynamics and the cosmological perturbations considering a coupling of the form $G(\phi, X) = g(\phi) X$. As a concrete example, we consider an exponential potential together with the cases in which the dissipation and Galilean coefficients are constants. Also, we study the weak regime given by the condition $R < 1 + 3gH\dot{\phi}$, and the strong regime in which $1 < R + 3gH\dot{\phi}$. Additionally, we obtain constraints on the parameters during the evolution of G-warm inflation, assuming the condition for warm inflation in which the temperature $T > H$, the conditions or the weak and strong regimes, together with the consistency relation $r = r(n_s)$ from Planck data.

Keywords: alternatives to inflation, cosmology of theories beyond the SM, inflation

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1 Introduction

It is well known that in the evolution of the early universe, it presented a rapid but finite period of the expansion called inflationary period or simply inflation [1–5]. In this context, inflation gives an elegant proposal for solving some problems of the standard model of the universe; the flatness, horizon, monopoles, among other. Nevertheless, an important feature of the inflationary epoch is that inflation gives account of the Large-Scale Structure (LSS) of the universe [6–9], as well as gives a causal explanation of the source of the anisotropies observed in the Cosmic Microwave Background (CMB) radiation [10–18].

Among the different models that give account of the dynamical evolution of the inflation, we can mention the model of warm inflation. In the stage of warm inflation the early universe is filled with a self-interacting scalar field (inflaton) and the radiation. In this sense, the model of warm inflation has the special feature that it omits the reheating stage at the end of inflation, since this model introduces a decay of a scalar field into radiation [19, 20]. Thus, the scenario of warm inflation differs from the standard inflation or the cold inflaton [21]. In this form, during the stage of warm inflation the decay takes place from the dissipative effects, and are caused from a friction term introduced on the background equations. Another important characteristic of warm inflation are the thermal fluctuations produced from the inflationary epoch. These thermal fluctuations play an important role in the initial thermal fluctuations essential for the seeds of the LSS formation [22–24]. As warm inflation incorporates in its evolution a radiation field with temperature $T$, and the quantum and thermal fluctuations of the scalar field are directly proportional to the Hubble parameter $H$ and the temperature, and considering that during warm inflation the thermal fluctuations of the scalar field predominates over the quantum fluctuations, then the condition $T > H$ could be satisfied, see refs. [19, 20, 22, 25, 26]. Also, we mention that warm inflation ends when the universe stops inflating and smoothly goes into the radiation era of the standard Big-Bang model. For a review of models of warm inflation model, see refs. [25, 26], and for a representative list of recent references, see refs. [27–38].

On the other hand, one general class of inflationary models in which the expansion of the universe is driven by a minimally coupled scalar field are the Galilean models of inflation or G-inflation. The model of the G-inflation includes the canonical and non-canonical
scalar field models of inflation, and in the literature are known as kinetic gravity braiding models [39, 40]. However, we can mention that the most general class of inflationary model corresponds to a generalized G-inflation or $G^2$-inflation that was studied in ref. [41], here the authors shown the equivalence with the Horndeski theory [42]. The importance of these models is that the field equations still include derivatives only up to second order [43]. The action of the G-model incorporate an additional term of the form $G(\phi, X) \Box \phi$ to the standard action of the non canonical scalar field [39, 40]. Moreover, the model of G-inflation is an intriguing type of inflationary universe, since it has a blue spectrum of the primordial tensor modes which violates the null energy condition stability [40, 44]. However, considering the slow-roll approximation together with some effective potentials, these models cannot generate a blue tensor spectrum [45]. For example, in the model of Higgs G-inflation that corresponds to a modification of the standard Higgs inflation, in which $G(\phi, X) \propto \phi X$, this situation does not occur [46]. Similarly, in ref. [47] was studied the case of the graceful exist from Higgs G-inflation and also the blue spectrum is prohibited, see also ref. [47] for the generalized Higgs G-inflation. In particular for the case in which the potential is exactly flat was studied in G-inflation, this model was called ultra slow roll G-inflation [48], and here the blue tensor perturbation and the null energy condition are prohibited. The same way, an effective potential of the form power law can be found in ref. [49], and depending of the parameters-space the consistency relation could be violated. Recently, the reheating and the particle production at the end of inflation in G-inflation was studied in ref. [50], see also [51], and the difference from string gas cosmology in ref. [52]. Also, in the context of the dark energy these G-models was studied e.g. in refs. [53–56].

The goal of this paper is to study the model of the warm inflation in the context of the Galilean model or G-model, and how a dissipative coefficient $\Gamma$ coming from an interaction between a standard scalar field and a radiation field influences the standard G-cold model. Under a general formalism, we will find the dynamics and the cosmological perturbations; scalar perturbation and tensor perturbation in which the function $G(\phi, X)$ is given by $G(\phi, X) = g(\phi) X$. In order to consider the model of G-warm inflation at the background level, we will analyze the modified motion of equations assuming the slow roll approximation. Also, from the Langevin equation in this framework, we will obtain an analytical expression of the power spectrum and its spectral index. As a concrete example and in order to obtain analytical quantities, we consider an exponential potential, together with a dissipative coefficient and Galileon parameter constants. Here, we will consider a G-warm inflation for two regimes, namely the weak and strong regimes, where the standard weak and the strong dissipative stages together with the Galileon effect are analyzed. In both stages, we will obtain constraints on the different parameters of our G-warm model, considering the condition for warm inflation in which $T > H$, the condition for the weak regime in which $R < 1 + 3gH\dot{\phi}$ (or the strong regime where $1 < R + 3gH\dot{\phi}$), together with the $r - n_s$ plane from Planck data [16].

The outline of the paper is as follows: the next section shows the dynamics of the model during the scenario of G-warm inflation assuming the slow roll approximation. In the section III, we find the cosmological perturbations specifically, we obtain explicit expressions for the scalar power spectrum, spectral index and tensor to scalar ratio. In section IV we analyze a concrete example for our model, in which we consider an exponential potential together with a dissipation coefficient constant and a Galileon parameter constant. Here, we study the weak and strong regimes, respectively. Finally, section V resumes our results and exhibits our conclusions. We chose units so that $c = \hbar = 8\pi = 1$. 
2 G-warm inflation: background equations

We begin with the 4-dimensional action of the form

\[
S = \int \sqrt{-g_4} d^4x \left( \frac{M_p^2}{2} R + K(\phi, X) - G(\phi, X) \Box \phi \right) + S_\gamma + S_{\text{int}},
\]

(2.1)

where \( g_4 \) corresponds to the determinant of the space-time metric \( g_{\mu\nu} \), \( M_p \) is the Planck mass, \( R \) is the Ricci scalar and \( X = -g^{\mu\nu} \partial_\mu \phi \partial_\nu \phi / 2 \). Here, the \( K \) and \( G \) are arbitrary functions of \( X \) and \( \phi \), where \( \phi \) corresponds to the scalar field. Additionally, we consider the radiation action \( S_\gamma \) and the interaction action \( S_{\text{int}} \). The action \( S_{\text{int}} \) represents the interaction of the scalar field with the other fields \([57-61]\).

By considering a spatially flat Friedmann Robertson Walker (FRW) metric, together with a scalar field homogeneous i.e., \( \phi = \phi(t) \), the standard Friedmann equation can be written as

\[
3H^2 = \kappa \rho = \kappa [\rho_{\phi s} + \rho_\gamma],
\]

(2.2)

where \( H = \frac{\dot{a}}{a} \) corresponds to the Hubble parameter, \( a \) is the scalar factor and \( \kappa = 1 / M_p^2 \). Also, here we assume a two-component system, namely, radiation field, described by an energy density \( \rho_\gamma \), and a scalar field \( \phi \) with energy density \( \rho_{\phi s} \). As said before, the universe is filled with a self-interacting radiation field and a scalar field. Also the total energy density \( \rho \) is given by \( \rho = \rho_{\phi s} + \rho_\gamma \), and the dots denote differentiation with respect to the time.

In this context, from the action given by eq. (2.1), the energy density and the pressure associated to the scalar field are \([39, 40]\]

\[
\rho_{\phi s} = 2K_X X - K + 3G_X H^2 \phi^3 - 2G_\phi X,
\]

(2.3)

and

\[
p_{\phi s} = K - 2(G_\phi + G_X \phi) X,
\]

(2.4)

respectively. In the following, we will consider the subscript \( K_X \) corresponds to \( K_X = \partial K / \partial X \), \( G_\phi \) to \( G_\phi = \partial G / \partial \phi \), \( K_{XX} = \partial^2 K / \partial X^2 \) etc.

From refs. \([19, 20]\) the dynamical equations for \( \dot{\rho}_{\phi s} \) and \( \dot{\rho}_\gamma \) during warm inflation can be written as

\[
\dot{\rho}_{\phi s} + 3H (\rho_{\phi s} + p_{\phi s}) = -\Gamma \dot{\phi}^2,
\]

(2.5)

and

\[
\dot{\rho}_\gamma + 4H \rho_\gamma = \Gamma \dot{\phi}^2.
\]

(2.6)

Note that the the continuity equation for the total energy density becomes \( \dot{\rho} + 3H (\rho + p) = 0 \).

Also, we note that combining eqs. (2.3) and (2.4), the dynamical equation for scalar field given by eq. (2.5) can be rewritten as

\[
K_X \Box \phi + 2K_{XX} X \ddot{\phi} + 2K_X X - K - 2(G_\phi - G_X \phi) \Box \phi + 6G_X (\dot{H} X + \dot{X} H + 3H^2 X) + 6HG_{XX} X \dddot{\phi} - 2G_{\phi \phi} X - 4G_{X \phi} X \dddot{\phi} = -\Gamma \dot{\phi}, \quad \Box \phi = \dddot{\phi} + 3H \dot{\phi}.
\]

(2.7)

In these equations, the coefficient \( \Gamma \) denotes the dissipation coefficient and from the second law of thermodynamics the coefficient \( \Gamma > 0 \) \([19, 20]\). This coefficient is responsible for the decay of the scalar field into radiation during the process inflationary of the universe. In general, this coefficient can be defined as a function of the temperature of the thermal bath \( T \) and the scalar field \( \phi \) i.e., \( \Gamma = \Gamma(T, \phi) \). In particular, \( \Gamma \) can be expressed only as
function the temperature, or function the scalar field or merely a constant [19, 20, 62–74].
Also, we mention that for the cases in which \(K = X - V(\phi)\) and \(G = 0\), we obtained the standard model of warm inflation. Here, \(V(\phi)\) corresponds to the effective potential.

In the following we will consider a special case of G-model in which the functions \(K(\phi, X)\) and \(G(\phi, X)\) are given by

\[
K(\phi, X) = X - V(\phi), \quad \text{and} \quad G(\phi, X) = g(\phi)X.
\]  

(2.8)

On the other hand, during the inflationary expansion of the universe, the energy density \(\rho_\phi\) predominates over the density \(\rho_\gamma\) [19, 20, 22]. At the same time, we consider from refs. [39, 40] that the effective potential dominates over the quantities \(X, |G_X H \dot{\phi}|\) and \(|G_\phi X|\), i.e., \(\rho_\phi \sim V(\phi)\). In this approximation the Friedmann equation given by eq. (2.2) can be rewritten as

\[
3H^2 \approx \kappa \rho_\phi \approx \kappa V(\phi).
\]  

(2.9)

Defining the slow-roll parameters in G-inflation as [40]

\[
\epsilon_1 = -\frac{\ddot{H}}{H^2}, \quad \epsilon_2 = -\frac{\dddot{\phi}}{H \dot{\phi}}, \quad \epsilon_3 = \frac{g_{\phi \phi}}{gH}, \quad \text{and} \quad \epsilon_4 = \frac{g_{\phi X}^2}{V_\phi},
\]  

(2.10)

and replacing the functions \(K\) and \(G\) given by eq. (2.8) into eq. (2.7) and considering the slow roll parameters of eq. (2.10) we get

\[
3H \dot{\phi}(1 + R - \epsilon_2/3 - gH \dot{\phi}[3 - \epsilon_1 - 2\epsilon_2 - 2\epsilon_2 \epsilon_3/3]) = -(1 - 2\epsilon_4) V_\phi,
\]  

(2.11)

where \(R\) denotes the ratio between the dissipative parameter \(\Gamma\) and the Hubble parameter, and is specified as

\[
R = \frac{\Gamma}{3H}.
\]  

(2.12)

Assuming that the slow-roll parameters \(\epsilon_1, |\epsilon_2|, |\epsilon_3|, |\epsilon_4| \ll 1\), then the slow-roll equation of motion for the field \(\phi\) given by eq. (2.11) results

\[
3H \dot{\phi}(1 + R + 3gH \dot{\phi}) \simeq -V_\phi.
\]  

(2.13)

Here, we observe different limiting cases. In the context of the dynamical equations, the limits \(R + 3gH \dot{\phi} < 1\) and \(1 + 3gH \dot{\phi} < R\), are the standard weak and strong dissipative regimes in warm inflation. In the limit \(1 + R < |gH \dot{\phi}|\) the Galileon effect predominates, and then the dynamics of warm inflation is modified. However, there are two interesting limits in which the standard weak and strong dissipative regimes are combined with the Galileon effect, and these are the limits are \(R < 1 + 3gH \dot{\phi}\) and \(1 < R + 3gH \dot{\phi}\), respectively. Here, we call the weak regime to the condition \(R < 1 + 3gH \dot{\phi}\) and strong regime to \(1 < R + 3gH \dot{\phi}\).

Also, we consider that during warm inflation the radiation production is quasi-stable in which \(\dot{\rho}_\gamma < 4H \rho_\gamma\) and \(\dot{\rho}_\gamma < \Gamma \dot{\phi}^2\), see refs. [19, 20, 22]. In this form, from eq. (2.6) the energy density of the radiation field can be written as

\[
\rho_\gamma = C_\gamma T^4 \simeq \frac{\Gamma X}{2H},
\]  

(2.14)

where \(C_\gamma = \pi^2 g_*/30\) is a constant, and \(g_*\) denotes the number of relativistic degrees of freedom [19, 20].
From eq. (2.13) the rate between the velocity of the scalar field $\dot{\phi}$ and the Hubble parameter results
\[
\frac{\dot{\phi}}{H} = -\frac{(1 + R)}{2\kappa g V} \left[ 1 - \left( 1 - \frac{4 g V_\phi}{(1 + R)^2} \right)^{1/2} \right].
\] (2.15)

Here, we choose the negative sign of the square root, in order to obtain the limit appropriate of eq. (2.15). Thus, taking the limit $g \to 0$, the eq. (2.15) coincides with that corresponding to the standard warm inflation, where $\frac{\dot{\phi}}{H}\mid_{g \to 0} = -\frac{V_\phi}{\kappa V(1+R)}$, see refs. [19, 20].

Also, from eq. (2.10) the slow-roll parameter $\epsilon_1$ results
\[
\epsilon_1 = -\frac{\dot{H}}{H^2} \approx \frac{(1 + R)}{4\kappa g} \left( \frac{V_\phi}{V^2} \right) \left[ 1 - \left( 1 - \frac{4 g V_\phi}{(1 + R)^2} \right)^{1/2} \right].
\] (2.16)

Again, we note that in the limit $g \to 0$, the eq. (2.16) agrees with the standard slow-roll parameter $\epsilon_1$ of warm inflation.

On the other hand, considering eqs. (2.12) and (2.16), the relation between the energy density of the radiation field $\rho_\gamma$, and the energy density of the scalar field $\rho_{\phi S}$, is given by
\[
\rho_\gamma \approx 4 \kappa R \epsilon_1^2 \frac{V^3}{V^2} \approx 4 \kappa \epsilon_1^2 \frac{\rho_{\phi S}}{\rho_{\phi S}^3},
\] (2.17)
where we have used that $\rho_{\phi S} \approx V$, then $\rho_{\phi S} \phi \approx V_\phi$. Since the inflationary scenario takes place provided that $\epsilon_1 < 1$ (or equivalently $\dot{a} > 0$) then $\frac{\rho_{\phi S}}{\rho_{\phi S}^3} > \frac{\rho_\gamma}{4\kappa R}$.

An important quantity corresponds to the number of e-folds $N$ at the end of inflation that is defined as
\[
N = \int_{\phi_*}^{\phi_f} H dt \approx -2\kappa \int_{\phi_*}^{\phi_f} \frac{g V}{(1 + R)} \left[ 1 - \left( 1 - \frac{4 g V_\phi}{(1 + R)^2} \right)^{1/2} \right]^{-1} d\phi.
\] (2.18)
In the following, the subscripts $*$ and $f$ are used to note the epoch when the cosmological scales exit the horizon and the end of inflation, respectively.

3 Cosmological perturbations

In this section we will analyze the cosmological perturbations in our G-warm inflationary model. It is well known that the source of the density fluctuations correspond to thermal fluctuations during the scenario of warm inflation. Therefore, in the evolution of the warm inflation the fluctuations of the scalar field $\delta \phi$ are dominantly thermal rather than quantum, see refs. [19, 20, 25, 26]. In this scenario, the curvature and entropy perturbations are present, since the mixture of the scalar and radiation fields evolve at the perturbative levels. However, the entropy perturbations on the large scales decay during warm inflation and are much smaller that its curvature (adiabatic modes) counterpart, see refs. [19, 20, 25, 26, 75–77]. In the following, we will only study the curvature perturbations that can contribute on the large scales. In this context, following refs. [19, 20] the power spectrum of the scalar perturbation is given by $P_R \simeq (H / \dot{\phi})^2 \delta \phi^2$, where $\delta \phi$ corresponds to the thermal fluctuations of the scalar field (see also ref. [78] the expression of $P_R$ for the case of a noncanonical scalar field in warm inflation).
In order to obtain an analytical expression for the fluctuations $\delta\phi$, we need to calculate the freeze-out wave number $k_F$, since the relation between $\delta\phi$ and $k_F$ is given by $\delta\phi^2 \propto k_F T$ [19, 20]. In this context, we consider the Langevin equation that includes a thermal stochastic noise $\xi(k, t)$. The stochastic noise satisfies $<\xi(k, t)> = 0$, and in the temperature limit $T \to \infty$, the stochastic noise is Markovian, such that $<\xi(k, t)\xi(k', t)> = 2\Gamma T (2\pi)^3 \delta^3(k - k') \delta(t - t')$ [19, 20, 79, 80]. Here, the correlation functions can be measured by probability averages, and $k$ corresponds to the wave number vector.

In this form, assuming the slow roll scenario and with the incorporation of the stochastic noise together with the spatial Laplacian term, the eq. (2.13) can be rewritten as

$$\frac{d\delta\phi(k, t)}{dt} \approx \frac{1}{(3H + \Gamma + 18gH^2\phi)} \left[ -(k^2 + V_{\phi\phi})\delta\phi(k, t) + \xi(k, t) \right]. \tag{3.1}$$

Here, we have considered up to first order in $\delta\phi(k, t)$ and the Fourier transformation. The approximate solution of eq. (3.1) is given by

$$\delta\phi(k, t) \approx \Theta(k, t) \int_{t_0}^{t} \frac{\xi(k, t')}{(3H + \Gamma + 18gH^2\phi)} \Theta^{-1}(k, t') \, dt' + \Theta(k, t)\delta\phi(kek^{-H(t-t_0)}, t_0), \tag{3.2}$$

where the quantity $\Theta(k, t)$ is defined as

$$\Theta(k, t) = \exp \left[ -\int_{t_0}^{t} \left( \frac{k^2 + V_{\phi\phi}}{(3H + \Gamma + 18gH^2\phi)} \right) \, dt' \right]. \tag{3.3}$$

From eq. (3.2) we mention that the first term on the right hand side realizes the thermalization of fluctuations $\delta\phi$ and considers the efficiency during the thermal process. The other term corresponds to the memory-term that archives the state of the mode in the beginning $t_0$ of process. Following refs. [19, 20, 25, 26] during the time interval $\sim H^{-1}$, and from eq. (3.3), the freeze-out wave number is defined by the condition $\frac{k_F^2}{(3H + \Gamma + 18gH^2\phi)H} = 1$. Here, we have considered that the mass term given by $V_{\phi\phi}$ is negligible in relation to the momentum $k^2$. In this form, the fluctuations of the scalar field $\delta\phi$ can be written as

$$\delta\phi^2 = \frac{k_F T}{2\pi^2} = \sqrt{3H^2 + HT + 18gH^3\dot{\phi}} \, T. \tag{3.4}$$

Note that in the limit $g \to 0$, the fluctuations given by eq. (3.4) reduces to the fluctuations obtained in refs. [19, 20].

In this way, from eq. (3.4) we find that the power spectrum of the scalar perturbation is given by

$$P_R \simeq \frac{1}{2\pi^2} \left( \frac{H}{\phi} \right)^2 \left[ \frac{\Gamma X}{2C_{\gamma}H} \right]^{1/4} \sqrt{3H^2 (1 + 6gH\dot{\phi}) + HT}. \tag{3.5}$$

Here, we have considered eq. (2.14).

On the other hand, the spectral index $n_s$ is defined as $n_s = 1 + \frac{d\ln P_R}{d\ln k}$, and considering eq. (3.5) the index $n_s$ becomes

$$n_s \simeq 1 - \frac{\epsilon_1}{2} \left[ \frac{7}{2} + \frac{1 + 12gH\dot{\phi}}{(1 + R + 6gH\dot{\phi})} \right] + 3\epsilon_2 \left[ \frac{1}{4} - \frac{gH\dot{\phi}}{(1 + R + 6gH\dot{\phi})} \right] + \epsilon_3 \left[ \frac{3gH\dot{\phi}}{(1 + R + 6gH\dot{\phi})} \right]$$

$$+ \frac{\epsilon_5}{2} \left[ \frac{1}{2} + \frac{R}{(1 + R + 6gH\dot{\phi})} \right], \tag{3.6}$$

where $\epsilon_1 = \frac{H}{2\pi^2}$.
where the quantity $\epsilon_5$ is defined as

$$\epsilon_5 = \left( \frac{\dot{\phi}}{H} \right) \left( \frac{\Gamma}{H} \right),$$

and the ratio $\frac{\dot{\phi}}{H}$ is given by eq. (2.15).

On the other hand, the tensor-perturbation during inflation would generate gravitational wave [81]. In particular during G-warm inflation the amplitude of the tensor perturbation is equivalent to the standard inflationary models and the corresponding spectrum becomes

$$\mathcal{P}_G = 8\kappa \left( \frac{H}{2\pi} \right)^2 \approx \frac{2\kappa^2 V}{3\pi^2}. \quad (3.7)$$

Here we have used eq. (2.9).

A crucial observational quantity is the tensor-scalar ratio $r$, which is specified as $r = \mathcal{P}_G/\mathcal{P}_R$. Considering eqs. (3.5) and (3.7) we find that the tensor to scalar ratio is given by

$$r = 4\kappa X \left( \frac{2C_{\gamma H}}{\Gamma X} \right)^{1/4} [3H^2(1 + 6gH\dot{\phi}) + H\Gamma]^{-1/2}. \quad (3.8)$$

In the following, we will analyze the G-warm inflation for an exponential potential, together with a dissipative coefficient $\Gamma = \Gamma_0$ constant [19, 20], and the Galileon parameter $g = g_0 = \text{constant}$ [82, 83]. Also, we will restrict ourselves to the weak and strong dissipative regimes together with the Galileon effect i.e., $R < 1 + 3gH\dot{\phi}$ and $1 < R + 3gH\dot{\phi}$.

4 The weak and strong dissipative regimes together with the Galileon effect: exponential potential

In this section we apply the formalism of above to G-warm inflation model, considering the standard weak and strong dissipative regimes in which $R = \frac{\Gamma}{3H} < 1$, and $R > 1$, together with the Galileon effect. In this sense, we call the weak regime to the condition $R < 1 + 3gH\dot{\phi}$ and the strong regime to $1 < R + 3gH\dot{\phi}$. In order to obtain analytical solutions in both regimes, we consider the simplest case in which the parameter $g = g_0 = \text{constant}$ (with units of mass $^{-3}$), and the dissipative coefficient $\Gamma = \Gamma_0 = \text{constant}$ (with units of mass).

For the specific potential $V(\phi)$, we assume an exponential potential defined as

$$V(\phi) = V_0 e^{-\alpha \phi},$$

where $\alpha$ (with units of mass $^{-1}$) and $V_0$ are free parameters. In the following we shall take $\alpha > 0$ and $g_0 > 0$. We should mention that this potential does not have a minimum, then the scalar field does not oscillate around this minimum [84–89], a fundamental requirement for standard mechanism of reheating in the evolution of cold inflation [90]. However, during warm inflation there is not separate reheating scenario as we have mentioned previously, then the problem of non-oscillating from the exponential potential is not present here.

4.1 The weak regime: $R < 1 + 3gH\dot{\phi}$

Assuming that the model of G-warm inflation evolves in the regime in which $R < 1 + 3gH\dot{\phi}$ i.e., in the weak regime, where the weak dissipative ($R < 1$) and the Galileon effect dominate the dynamics of G-warm inflation.
By considering eq. (2.15), we find that \( \dot{\phi} \) becomes
\[
\dot{\phi} = \frac{1}{2\sqrt{3\kappa g_0} V^{1/2}} \left[ (1 + 4g_0\alpha V)^{1/2} - 1 \right],
\]
and assuming the exponential potential, then the solution of eq. (4.1) can be written as
\[
V_0^{-1/2} e^{\alpha\phi/2}(1 + \sqrt{1 + 4\alpha g_0 V_0 e^{-\alpha\phi}}) - 2\sqrt{\alpha g_0} \arcsin(2\sqrt{\alpha g_0 V_0 e^{-\alpha\phi}}) = \frac{\alpha^2}{\sqrt{3}\kappa} t + C_1,
\]
where \( C_1 \) is an integration constant.

The inflationary scenario ends when the universe heats up at a time when the slow-roll parameter \( \epsilon_{1W} \simeq 1 \) (or equivalently \( \ddot{a} \simeq 0 \)), where the parameter \( \epsilon_{1W} \) from eq. (2.10) is given by
\[
\epsilon_{1W} \simeq \frac{\alpha}{4\kappa g_0 V} \left[ (1 + 4\alpha g_0 V)^{1/2} - 1 \right].
\]
In this way, the potential at the end of inflation considering the condition \( \epsilon_{1W}(\phi = \phi_f) \simeq 1 \), results
\[
V_f = V_0 e^{-\alpha\phi_f} \simeq \frac{\alpha}{2\kappa g_0} \left[ \frac{\alpha^2}{2\kappa} - 1 \right].
\]
Here, we note that as \( V_f > 0 \), then we obtain a lower bound for the parameter \( \alpha \) given by \( \alpha > \sqrt{2\kappa} \).

During this regime, the number of e-folds at the end of inflation becomes
\[
N = \frac{\kappa}{\alpha^2} \left[ \sqrt{1 + 4\alpha g_0 V_*} - \sqrt{1 + 4\alpha g_0 V_f} + \ln \left( \frac{\sqrt{1 + 4\alpha g_0 V_*} - 1}{\sqrt{1 + 4\alpha g_0 V_f} - 1} \right) \right],
\]
where \( V_f \) is given by eq. (4.2). Here we have used that \( d\phi = dV/V_\phi = -\alpha^{-1}d(\ln V) \) for the exponential potential.
3.6 plots we have used the value \( \alpha \). The dashed, dotted and solid lines correspond to the pairs \((\alpha, 1)\) for three different values of the parameter \( g \) during this scenario. In both panels, the dashed, dotted and solid lines correspond to the pairs \((\alpha = 1.42 M_p^{-1}, \Gamma_0 = 5.1 \times 10^{-6} M_p), \) \((\alpha = 1.43 M_p^{-1}, \Gamma_0 = 1.1 \times 10^{-10} M_p), \) and \((\alpha = 1.42 M_p^{-1}, \Gamma_0 = 1.2 \times 10^{-12} M_p), \) respectively. In these plots we have used the value \( C_\gamma = 70 \).

**Figure 2.** The dependence of the ratio \( T/H \) versus the scalar spectral index \( n_s \) (left panel) and the dependence of the ratio \((1 + 3 g_0 H \dot{\phi})/R \) versus the scalar spectral index \( n_s \) (right panel) during the weak regime, for three different values of the parameter \( g_0 \) (in units of \( M_p^{-3} \)).

On the other hand, from eq. (3.5) the power spectrum of the scalar perturbation \( P_R \) during this scenario results

\[
P_R \simeq \frac{2g_0^2}{\pi^2} \frac{\kappa^{5/2} V^{5/2}}{[(1 + 4 g_0 \alpha V)^{1/2} - 1]^2} \left[ \frac{\sqrt{3 \Gamma_0 X}}{2C_\gamma \sqrt{\kappa V}} \right]^{1/4} (1 + 2g_0 \sqrt{3 \kappa V} \dot{\phi})^{1/2},
\]

(4.4)

where \( \dot{\phi} \) in terms of the potential \( V \) (or scalar field \( \phi \), since \( V = V_0 e^{-\alpha \phi} \)) is given by eq. (4.1).

The spectral index \( n_s \) from eq. (3.6) is given by

\[
n_s \simeq 1 - \frac{3 \epsilon_{1W}}{4} \left[ \frac{3 + 22g_0 \sqrt{3 \kappa V} \dot{\phi}/3}{1 + 2g_0 \sqrt{3 \kappa V} \dot{\phi}} \right] + \frac{3 \epsilon_{2W}}{4} \left[ \frac{1 + 2g_0 \sqrt{3 \kappa V} \dot{\phi}/3}{1 + 2g_0 \sqrt{3 \kappa V} \dot{\phi}} \right],
\]

(4.5)
where the slow parameter $\epsilon_{2W}$ during the weak scenario is defined as

$$\epsilon_{2W} \simeq \left[ \frac{\alpha}{\kappa V(1 + 2\sqrt{3}\kappa V\phi)} \right] \left[ \frac{\alpha V - \sqrt{3}\kappa V\phi'}{2} - 6\kappa g_0 V\phi'^2 \right].$$

From eq. (3.8) we obtain that the tensor to scalar ratio during the weak regime can be written as

$$r \simeq \left( \frac{2C_{\gamma} \sqrt{\kappa V}}{\sqrt{3}\Gamma_0} \right)^{1/4} \left( \frac{16\kappa X^{3/2}}{V(1 + 2g_0\sqrt{3}\kappa V\phi)} \right)^{1/2}. \quad (4.6)$$

In figure 1 we show the dependence of the rate $T/H$ and the spectral index $n_s$ on the exponential potential $V(\phi) = V_0 e^{-\alpha \phi}$ (in units of $M_p^4$) during the regime in which $R < 1 + 3g_0H\dot{\phi}$ (weak regime). In both plots we have used three different values of the parameter $g_0$ (in units of $M_p^{-3}$). The left panel shows the essential condition for warm inflation in which the temperature of the thermal bath $T > H$ in terms of the exponential potential $V(\phi)$ (or the scalar field). The right panel shows the dependence of the spectral index $n_s$ in terms of the exponential potential $V(\phi)$. In order to write down the ratio $T/H$ and the index $n_s$ as a function of the potential $V(\phi)$ during this regime, we consider eqs. (2.9), (2.14), and (4.1), and then we obtain the rate between the temperature of the thermal bath $T$ and the Hubble parameter $H$, as a function of the exponential potential $V(\phi)$. Analogously, we consider eqs. (4.1) and (4.5), and together with the slow roll parameters $\epsilon_{1W}$ and $\epsilon_{2W}$, we find the curve of the spectral index $n_s$ as a function of the effective potential $V(\phi)$. In both panels we have used the value of $C_{\gamma} = 70$. From the left panel, we note that the essential condition for warm inflation $T > H$ occurs for the values of the potential $V < 9 \times 10^{-11}M_p^4$ for the curve in which $g_0 = 10^{19}M_p^{-3}$. Analogously, the condition $T > H$ occurs for $V < 4 \times 10^{-11}M_p^4$ in the case $g_0 = 10^{17}M_p^{-3}$, and $V < 4 \times 10^{-12}M_p^4$ for the curve in which $g_0 = 9 \times 10^{14}M_p^{-3}$. In the right panel, we observe that the spectral index $n_s = n_s(\phi)$ is always $n_s \lesssim 1$. Also, we note that the index $n_s$ takes the observational value $n_s = 0.967$ for the value of the potential $V \simeq 5 \times 10^{-16}M_p^4$ for the curve in which $g_0 = 10^{19}M_p^{-3}$, $V \simeq 5 \times 10^{-14}M_p^4$ in the case in which $g_0 = 10^{17}M_p^{-3}$, and $V \simeq 6 \times 10^{-12}M_p^4$ in the case in which $g_0 = 9 \times 10^{14}M_p^{-3}$.

On the other hand, in order to obtain the parameter-pair $(\alpha, \Gamma_0)$ for a given value of the parameter $g_0$, we consider the observational data in which $n_s \simeq 0.967$ and $P_R \simeq 2.2 \times 10^{-9}$ from Planck experiment data, and also we assume that the number of e-folds $N = 60$. In this form, we numerically consider eqs. (4.3), (4.4) and (4.5) and we find that the parameter $\alpha \simeq 1.42M_p^{-1}$ and $\Gamma_0 \simeq 5.1 \times 10^{-6}M_p$ for the value of the parameter $g_0 = 10^{10}M_p^{-3}$ for which $n_s \simeq 0.967$, $P_R \simeq 2.2 \times 10^{-9}$ and $N = 60$. Analogously, for the value $g_0 = 10^{17}M_p^{-3}$ corresponds to $\alpha \simeq 1.43M_p^{-1}$ and $\Gamma_0 \simeq 1.1 \times 10^{-10}M_p$ and for the case in which $g_0 = 9 \times 10^{14}M_p^{-3}$ we have $\alpha \simeq 1.42M_p^{-1}$ and $\Gamma_0 \simeq 1.2 \times 10^{-12}M_p$.

In the following we will analyze the plots of the essential condition for warm inflation, and the condition of weak (or strong) regime in terms of the spectral index $n_s$. In this form, we study the conditions in our model with the help of the observational data of the spectral index $n_s$, since it has a very well defined range from the marginalized constraints Planck data (68% and 95% confidence levels) [16].

In figure 2 we show the dependence of the rates $T/H$ and $(1 + 3g_0H\dot{\phi})/R$ on the spectral index $n_s$ during the weak regime i.e., $R < 1 + 3g_0H\dot{\phi}$. In both plots we have used three different values of the parameter $g_0$ (in units of $M_p^{-3}$). The left panel shows the essential condition for warm inflation in which the temperature of the thermal bath $T > H$. Here,
we confirm that the condition $T > H$ takes place. The right panel shows the dependence of the dimensionless quantity $(1 + 3g_0H\dot\phi)/R$ during inflation and we check that the model evolves in the weak regime in warm inflation. In order to write down the rates $T/H$ and $(1 + 3g_0H\dot\phi)/R$ in terms of the scalar spectral index $n_s$ during this regime, we take into account eqs. (2.9), (2.14), (4.1) and (4.5), and we numerically obtain the parametric plot for the rate between the temperature of the thermal bath $T$ and the Hubble parameter $H$, as a function of the scalar spectral index $n_s$. Similarly, we use eqs. (2.9), (4.1) and (4.5), and we numerically find the parametric plot of the curve $(1 + 3g_0H\dot\phi)/R$ as a function of the spectral index $n_s$. In both panels we have considered the value of $C_\gamma = 70$.

In figure 3 we show the consistency relation $r = r(n_s)$ during the weak regime, for three different values of the parameter $g_0$. In particular, for the value of tensor to scalar ratio $r = r(n_s = 0.967)$, we find that for the value $g_0 = 10^{19}M_p^{-3}$ (together with the pair $\alpha \simeq 1.42M_p^{-1}$ and $\Gamma_0 \simeq 5.1 \times 10^{-6}M_p$) the ratio $r = r(n_s = 0.967) \simeq 2.32 \times 10^{-8}$, for the value $g_0 = 10^{17}M_p^{-3}$ corresponds to $r = r(n_s = 0.967) \simeq 1.95 \times 10^{-6}$ and for the case in which $g_0 = 9 \times 10^{14}M_p^{-3}$ corresponds to $r = r(n_s = 0.967) \simeq 3.85 \times 10^{-5}$. Thus, for the different values of $g_0$ (and the pairs $(\alpha, \Gamma_0)$), we find that the tensor to scalar ratio are well corroborated from Planck data where $r < 0.1$. Also, we observe that the values of the consistency relation $r = r(n_s) \sim 0$. In this form, we note that the tensor to scalar ratio gives $r \sim 0$, and we conclude that the consistency relation $r = r(n_s)$ does not add a new constraint on the parameter $g_0$ during this stage.

In this form, from left panel of the figure 2, we find a lower bound for $g_0 > 9 \times 10^{14}M_p^{-3}$, considering the essential condition for the scenario of warm inflation takes place i.e., $T/H > 1$. Now from the right panel, we obtain an upper bound for the parameter $g_0 < 10^{19}M_p^{-3}$ from the condition that the model evolves in the weak regime in which $R < (1 + 3g_0H\dot\phi)$. In this way, we achieve that the interval of the parameter $g_0$ during the weak regime is given by $9 \times 10^{14} < g_0 M_p^{-3} < 10^{19}$. The same form for the dissipative parameter $\Gamma_0$ we find that the range is given by $1.2 \times 10^{-2} < \Gamma_0 M_p^{-1} < 5.1 \times 10^{-6}$ and for the parameter $\alpha$ a value very close to $\alpha \sim \sqrt{2\kappa}$ (see condition of eq. (4.2)).

4.2 The strong regime: $1 < R + 3gH\dot\phi$

Considering that the model of G-warm inflation develops in the strong regime in which the strong dissipative regime together with the Galileon effect $(1 < R + 3gH\dot\phi)$ predominate the evolution of warm inflation, we find that the $\dot\phi$ can be written as

$$\dot\phi = \frac{\Gamma_0}{6\kappa g_0 V} \left[ \left( 1 + \frac{12\kappa g_0 \alpha V^2}{\Gamma_0^2} \right)^{1/2} - 1 \right],$$

and the solution of the scalar field in terms of the cosmological time can be written as

$$e^{\alpha\phi}[1 + (1 + b_1 e^{-2\alpha\phi})^{1/2}] + \sqrt{b_1} \arcsinh(\sqrt{b_1} e^{-\alpha\phi}) = \frac{2\alpha^2 V_0}{\Gamma_0} [t + C_1],$$

where the dimensionless constant $b_1$ is defined as $b_1 = \frac{12\kappa g_0 \kappa V_0^2}{\Gamma_0^2}$, and $C_1$ is an integration constant. Here we have considered that $\dot\phi = \frac{\dot V}{V} = -\frac{\dot V}{\alpha V}$.

As before, assuming that the inflationary scenario ends when the universe heats up at a time when the slow-roll parameter $\epsilon_{1 s} \simeq 1$, where now the parameter $\epsilon_{1 s}$ from eq. (2.10)
is defined as
\[ \epsilon_{1s} \simeq \frac{\alpha \Gamma_0}{4\sqrt{3}g_0\kappa^{3/2}V^{3/2}} \left[ 1 + \frac{12g_0\alpha \kappa V^2}{\Gamma_0^2} \right] - 1, \]
during this regime. Thus, under the condition \( \epsilon_{1s} \simeq 1 \), we find that the real solution for the potential at the end of inflation \( V_f \) can be written as
\[
V_f = V_0 e^{-\alpha \phi_f} \simeq \frac{2A^2B}{3} + \frac{A^4B^2}{3D} + \frac{D}{3},
\]
where the constants \( A, B \) and \( D \) are defined as
\[
A = \frac{\alpha \Gamma_0}{4\sqrt{3}\kappa^{3/2}g_0}, \quad B = \frac{b_1}{V_0^2} = \frac{12\alpha g_0 \kappa}{\Gamma_0^2}, \quad \text{and} \quad D = [54A^2 + A^6B^3 + 6A^2\sqrt{3}(27 + A^4B^3)]^{1/3},
\]
respectively.

Also, during this scenario the number of e-folds \( N \) is given by
\[
N = \frac{1}{AB^{3/4}} [\Xi(V_f) - \Xi(V_*)],
\]
where the function \( \Xi(V) \) is defined as
\[
\Xi(V) = \frac{1 + \sqrt{1 + BV^2} - B^{1/4}V^{1/2}}{B^{1/4}V^{1/2}} \frac{2F_1[1/4, 1/2, 3/2, 1 + BV^2]}{B^{1/4}V^{1/2}},
\]
and \( 2F_1 \) is the hypergeometric function \([91]\). Here, \( V_f \) is given by eq. (4.8) and also we have used that \( d\phi = dV/V_\phi = -\alpha^{-1}d(\ln V) \).

On the other hand, considering that the fluctuations of the scalar field can be written as
\[
\delta\phi^2 = \left( \frac{1}{2\pi^2} \right) \frac{\sqrt{\Gamma_0}X}{2C_\gamma\sqrt{\kappa V}} \right)^{1/4} \sqrt{H_s(\Gamma_0 + 6\kappa g_0 V \dot{\phi})} \quad \text{during this stage, then the power spectrum of the scalar perturbation} \ P_R, \quad \text{from eq. (3.5) becomes}
\]
\[
P_R \simeq \frac{2 \times 3^{7/8}}{\pi^2} \left( \frac{g_0}{\Gamma_0[(1 + BV^2)^{1/2} - 1]} \right)^2 \left[ \frac{\Gamma_0 X(\kappa V)^{25/2}}{2C_\gamma} \right]^{1/4} \left[ \Gamma_0 + 6\kappa g_0 V \dot{\phi}^2 \right]^{1/2},
\]
where \( \dot{\phi} = \dot{\phi}(V) \) is given by eq. (4.7).

From eq. (3.6) the scalar spectral index \( n_s \) during this scenario results
\[
n_s \simeq 1 - \frac{\epsilon_{1s} \alpha}{4} \left[ \frac{7R + 22g_0\sqrt{3}\kappa V \dot{\phi}}{R + 2g_0\sqrt{3}\kappa V \dot{\phi}} \right] + \frac{3\epsilon_{2s} \alpha}{4} \left[ \frac{R - 2g_0\sqrt{3}\kappa V \dot{\phi}/3}{R + 2g_0\sqrt{3}\kappa V \dot{\phi}} \right],
\]
where the slow-roll parameter \( \epsilon_2 \), in the strong regime \( \epsilon_{2s} \) (for \( \Gamma + 9gH^2 \dot{\phi} > 3H \)) becomes
\[
\epsilon_{2s} \simeq \frac{\sqrt{3}V_s \alpha}{\sqrt{\kappa}} \left[ \frac{\alpha - 3g_0\dot{\phi}^2}{\Gamma_0 + 6\kappa g_0 V \dot{\phi}^2} \right].
\]

For the tensor-to-scalar ratio \( r = P_T/P_R \), we have
\[
r \simeq \left[ \frac{2^9 \times 3 \sqrt{3} C_\gamma \kappa^{7/4} X^4}{\Gamma_0 \sqrt{V}} \right]^{1/4} \left[ \Gamma_0 + 6\kappa g_0 V \dot{\phi} \right]^{-1/2}.
\]

the solid, dotted and dashed lines correspond to the pairs \((\alpha \simeq 0.34 M_p^{-1}, \Gamma_0 \simeq 6.1 \times 10^{-7} M_p)\), \((\alpha \simeq 0.21 M_p^{-1}, \Gamma_0 \simeq 5.1 \times 10^{-8} M_p)\), and \((\alpha \simeq 0.11 M_p^{-1}, \Gamma_0 \simeq 6.9 \times 10^{-9} M_p)\), respectively. In these plots we have used the value \(C_\gamma = 70\).

Here we have used eq. (3.8).

In figure 4 we show the evolution of the rates \(T/H\) (left panel) and \(R + 3g_0 H \dot{\phi}\) (right panel) on the spectral index \(n_s\) during the strong regime in which \(1 < R + 3g_0 H \dot{\phi}\). As before, we take into account three values of the parameter \(g_0\) (in units of \(M_p^{-3}\)). Considering eqs. (2.14), (4.7) and (4.11) we numerically obtain the parametric plot for the rate \(T/H = \frac{T}{H}(n_s)\) and the quantity \([R + 3g_0 H \dot{\phi}] = [R + 3g_0 H \dot{\phi}](n_s)\) during warm inflation. In addition, as before from eqs. (4.9) (4.10) and (4.11) we numerically find that the parameter \(\alpha \simeq 0.34 M_p^{-1}\) and \(\Gamma_0 \simeq 6.1 \times 10^{-7} M_p\) for the value of the parameter \(g_0 = 10^{14} M_p^{-3}\) for which the spectral index \(n_s = 0.967\), \(P_R = 2.2 \times 10^{-9}\) and the number of e-folds \(N = 60\). Similarly, for the value of the parameter \(g_0 = 10^{11} M_p^{-3}\) gives the values \(\alpha \simeq 0.21 M_p^{-1}\) and \(\Gamma_0 \simeq 5.1 \times 10^{-8} M_p\) and for the case in which \(g_0 = 5 \times 10^8 M_p^{-3}\) corresponds to \(\alpha \simeq 0.11 M_p^{-1}\) and \(\Gamma_0 \simeq 6.9 \times 10^{-9} M_p\), respectively. Thus, in the plots of figure 4 the solid, dotted and dashed lines correspond to the pairs \((\alpha \simeq 0.34 M_p^{-1}, \Gamma_0 \simeq 6.1 \times 10^{-7} M_p)\), \((\alpha \simeq 0.21 M_p^{-1}, \Gamma_0 \simeq 5.1 \times 10^{-8} M_p)\), and \((\alpha \simeq 0.11 M_p^{-1}, \Gamma_0 \simeq 6.9 \times 10^{-9} M_p)\), for the values of \(g_0 = 10^{14} M_p^{-3}\), \(g_0 = 10^{11} M_p^{-3}\) and \(g_0 = 5 \times 10^8 M_p^{-3}\), respectively. As before in these plots we have used the value \(C_\gamma = 70\).

On the other hand, we consider eqs. (4.11) and (4.12) and we numerically obtain the parametric relationship of the consistency relation \(r = r(n_s)\) during the strong regime. For the value of the parameter \(g_0 = 10^{14} M_p^{-3}\) (together with the pair \((\alpha \simeq 0.34 M_p^{-1}, \Gamma_0 \simeq 6.1 \times 10^{-7} M_p)\)) we numerically find that the tensor to scalar ratio is \(r = r(n_s = 0.967) \simeq 3.73 \times 10^{-5}\). For the case in which the parameter \(g_0 = 10^{11} M_p^{-3}\) we obtain \(r = r(n_s = 0.967) \simeq 2.34 \times 10^{-3}\) and for the value \(5 \times 10^8 M_p^{-3}\) corresponds to \(r = r(n_s = 0.967) \simeq 0.42\) and this value is disproved from observational data, since \(r < 0.1\) (Planck satellite). Thus, we note that for the values \(g_0 = 10^{14} M_p^{-3}\) and \(g_0 = 10^{11} M_p^{-3}\), the tensor to scalar ratio during the strong regime becomes \(r \sim 0\), and these values of \(r\) are well corroborated with the Planck data (figure not shown).

In this way, from the left panel of figure 3 we observe that for value of the parameter \(g_0 > 5 \times 10^8 M_p^{-3}\) (a lower bound) is well supported by the essential condition of warm inflation \(T > H\), but is disapproved by Planck data. However, from the right panel we also
obtain a lower bound for the parameter \( g_0 > 10^{11}M_p^{-3} \) from the condition \( 1 < R + 3g_0H\dot{\phi} \)
i.e., the strong regime. Thus, we only can get a lower limit for the parameter \( g_0 \) given by \( 10^{11} < g_0M_p^{-3} \). Similarly, we find that for the parameters \( \alpha \) and \( \Gamma_0 \) the limits are given by \( \alpha > 0.21M_p^{-1} \) and \( \Gamma_0 > 5.1 \times 10^{-8}M_p \), respectively. Additionally, we numerically observe that for values of the parameter \( g_0 > 10^{11}M_p^{-3} \), the tensor to scalar ratio \( r \rightarrow 0 \).

5 Conclusions

In this paper we have investigated the model of G-inflation in the framework of the warm inflation. Under a general formalism we have found the dynamics and the cosmological perturbations in our G-warm inflationary model in the context of the slow roll approximation. In this general analysis we have obtained from the Langevin equation the fluctuations of the scalar field \( \delta\phi \) in order to obtain an analytical expression for the power spectrum. As a concrete example we have considered an exponential potential and we have applied our results considering the standard weak and strong dissipative regimes together with the Galileon effect. Also, in order to obtain analytical solutions in both regimes, we have consider the simplest case in which the parameter \( g = g_0 = \text{constant} \) and the dissipative coefficient \( \Gamma = \Gamma_0 = \text{constant} \). From these parameters we have obtained analytical quantities in the slow roll approximation for the corresponding scalar field, number of e-folds, power spectrum, spectral index and tensor to scalar ratio. Thus, from these expressions and in both scenarios we have found the constraints on the parameters in our model. In this sense, in both regimes, we have obtained constraints for the parameters considering the condition of warm inflation in which the temperature of the thermal bath \( T > H \); from the conditions of the weak and strong regime where \( R < 1 + 3gH\dot{\phi} \) and \( 1 < R + 3gH\dot{\phi} \), and also from the Planck data in which we have considered the constraint on the consistency relation \( r = r(n_s) \).

In our analysis for the both regimes, we have numerically found the pair \( (\alpha, \Gamma_0) \) for a given value of the parameter \( g_0 \), from the observational Planck-data in which the spectral index \( n_s \approx 0.967 \), \( P_R \approx 2.2 \times 10^{-9} \), and also we have considered the number of e-folds \( N = 60 \).

For the weak regime in which \( R < 1 + 3gH\dot{\phi} \) we have found the constraints on the parameters \( g_0, \alpha \) and \( \Gamma_0 \), from the essential condition of warm inflation \( T > H \) which establishes a lower bound, and the condition of the weak regime i.e., \( R < 1 + 3gH\dot{\phi} \) which gives an upper bound. Also, we have observed that the tensor to scalar ratio results \( r \sim 0 \) and the consistency relation \( r = r(n_s) \) does not add a new constraint on the parameters during this regime. For the case of the strong regime \( (1 < R + 3gH\dot{\phi}) \) we have found the constraints on the parameters, only from the condition \( 1 < R + 3gH\dot{\phi} \) (or strong regime) which gives a lower limit on the parameters. In this sense, the essential condition of warm inflation and the consistency relation do not give information on the constraints during this regime.

We mention that the G-warm inflation models are less restricted than analogous pure warm inflation and that pure generalized G-inflation due to the introduction of a new parameter; \( g_0 \) during warm inflation and \( \Gamma_0 \) in the case of standard G—inflation. The inclusion of these parameters gives us a freedom that allows us to change the pure warm or pure generalized G inflationary scenarios by simply modifying the corresponding values of the parameters. We conclude with some comments concerning the way to distinguish our model and pure warm inflation or pure generalized G-inflation. In our model, beginning with the background dynamics we noted that from eq. (2.15) that the velocity of the scalar field (or equivalently the kinetic term) is much smaller than the standard warm inflation. Also, in particular for the exponential potential, we noted that the velocity \( \dot{\phi} \) in the case of the pure
generalized G-inflation becomes independent of the potential, since $\dot{\phi}^2 \propto \alpha/g$, in contradistinction to G-warm and for values of $g \mathcal{M}_p^3 \gg 1$, the velocity is much smaller than our model. This suggests that the number of e-folds $N$ at the end inflation during the model of pure generalized G-inflation is much bigger than our model and than during standard warm inflation. From the point of view of the perturbative dynamics we have found that in our model the consistency relation $r(n_s) \sim 0$, and is similar to the result obtained for the case of pure warm inflation, see ref. [80]. However, one may expect that in the case of the pure generalized G-inflation the consistency relation could be much bigger than in our model [47].

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