Light Propagation in a Background Field for Time-Space Noncommutativity and Axionic Noncommutative QED

N. Chatillon\textsuperscript{1} and A. Pinzul\textsuperscript{2}

Department of Physics, Syracuse University, Syracuse, NY 13244-1130, USA.

Abstract

We study the low-energy effects of space-time non-commutativity on light propagation in a background electromagnetic field. Contrary to some of the previous claims, we find no polarization rotation for vanishing time-space commutator $[\hat{x}^i, \hat{x}^0] = 0$, although dispersion relation is modified, allowing for propagation faster than the vacuum speed of light. For non-zero $[\hat{x}^i, \hat{x}^0]$, as allowed with a proper quantization, a naive rotation effect is found to be actually absent when physical fields are defined through Seiberg-Witten map. We also consider non-commutative QED weakly coupled to small mass particles such as axions. Non-commutativity is found to dominate the inverse oscillation length, compared to axion mass and QED effects, for mixing particle masses smaller than $10^{-12}$ eV. Conventional constraints on axion coupling based on photon-axion transition rates are unmodified, however induced ellipticity is proportional to the non-commutativity squared length scale. This last effect is found to be too small to account for the ellipticity reported by the PVLAS experiment, yet unexplained by conventional QED or axion physics.

1 Introduction

The idea that space-time may be described by non-commutative (NC) coordinates at short distance scales has attracted an increasing interest. The expectation that, in a quantum gravity framework, coordinates should obey an uncertainty relation \[1\] of the form $\Delta \hat{x}^\mu \Delta \hat{x}^\nu \geq \frac{1}{2} |\theta^{\mu\nu}|$, leads to the commutation relations

\[ [\hat{x}^\mu, \hat{x}^\nu] = i\theta^{\mu\nu} \quad (1.1) \]

with $\theta^{\mu\nu}$ an antisymmetric set of constants. A similar result is obtained in string theory in a non-vanishing background of the NS-NS 2-form $B_{\mu\nu}$ \[2–4\]. This algebra of abstract coordinates $\hat{x}^\mu$ can be realized on functions of the commutative space-time coordinates, $x^\mu$, by replacing all products with a star product $\ast$, such that

\[ [x^\mu, x^\nu]_\ast = x^\mu \ast x^\nu - x^\nu \ast x^\mu = i\theta^{\mu\nu} = [\hat{x}^\mu, \hat{x}^\nu]. \quad (1.2) \]

\textsuperscript{1}nchatill@physics.syr.edu
\textsuperscript{2}apinzul@physics.syr.edu
The specific choice of the Moyal product
\[ f \star g = f \exp \left( \frac{i}{2} \theta_{\mu
u} \partial_{\mu} \partial_{\nu} \right) g = fg + \sum_{n=1}^{\infty} \frac{1}{n!} \left( \frac{i}{2} \right)^n \theta_{\mu_1 \nu_1} \ldots \theta_{\mu_n \nu_n} (\partial_{\mu_1} \ldots \partial_{\mu_n} f)(\partial_{\nu_1} \ldots \partial_{\nu_n} g). \] (1.3)
corresponds to a symmetric ordering prescription when mapping a function of the commutative \( x^\mu \) to an abstract function of the NC \( \hat{x}^\mu \), e.g. \( x^\mu x^\nu \to \frac{1}{2}(\hat{x}^\mu \hat{x}^\nu + \hat{x}^\nu \hat{x}^\mu) \). This product has the important properties of breaking Lorentz invariance due to the presence of the constant tensor \( \theta^{\mu\nu} \), and introducing non-locality due to the infinite number of derivatives involved. A twisted form of Lorentz invariance is however formally preserved [5].

Following this approach, non-commutative QED had been built [6], as well as a non-commutative Standard Model [7]. Its tree-level structure is identical to the commutative SM, plus \( O(\theta) \) corrections of dimension 6 and higher. The strongest bounds on the NC energy scale \( \theta^{-1/2} \) come from the low energy constraints on Lorentz violation. Assuming that \( \theta^{\mu\nu} \) is constant in the CMB frame, its orientation varies in an Earth based laboratory frame, essentially due to Earth rotation. The spacelike components \( \theta^{ij} \) couple to atomic and nuclear angular momenta similarly to a magnetic field, while the bounds on the timelike components \( \theta^{0i} \) are usually much weaker, if considered at all. Clock comparison experiments searching for a sidereal time variation of the atomic Zeeman splitting yield a lower bound on \( \theta^{-1/2} \), that we will refer to as the spacelike component scale in what follows. Tree-level analysis of the effect of dimension 6 operators in NC QED implies \( \theta^{-1/2} > O(10 \, \text{TeV}) \), or \( O(10 \, \text{GeV}) \) for the \( \theta^{0i} \) components [8]. An estimation of the tree-level NC QCD sector gives the much more stringent bound \( \theta^{-1/2} > 5 \times 10^{14} \, \text{GeV} \) [9], from generated operators of the form \( \theta^{\mu\nu} \bar{N} \gamma_{\mu\nu} N \) involving nucleons.

Lorentz violating loop-induced operators of dimension 4 and lower have also been considered. In a Lorentz-violating theory, these very constrained operators are expected to be naturally large, and grow with the UV momentum cut-off \( \Lambda \), although non-intuitive behavior also appears from the UV/IR connection. In the NC QED sector, the constraint \( \theta^{-1/2} > 10^{-10} \Lambda \) is obtained from the operator \( m_e \theta^{\mu\nu} \bar{\psi} \gamma_{\mu\nu} \psi \) [10], and the stronger one \( \theta^{-1/2} > 10^{13} \Lambda \) in the case of a supersymmetric regularization where \( \Lambda \) is the superpartner mass scale [11]. A similar analysis in the NC QCD sector produces the even stronger \( \theta^{-1/2} > 10^{14-15} \Lambda \) from clock comparison experiments [12]. Thus, even for a UV cut-off as small as \( \text{TeV} \), the worst bounds push \( \theta^{-1/2} \) to the Planck scale. Also, the large dimensionless ratio of the NC scale to the cut-off may be considered as unnatural.

Taking this into account, finding new signals of non-commutativity is certainly extremely challenging. In the best case, assuming that NC does not apply at all to the QCD sector, and that loop-induced operators are cancelled by a fine-tuning of the counterterms, one may retain \( \theta^{-1/2} > 10 \, \text{TeV} \).

It appears that some of the most constraining tests of space-time non-commutativity involve dynamics in a background field\(^3\). It is interesting in particular to study the propagation of electromagnetic waves in a non-zero background. This was done in [14] for the purely magnetic case and [15] including an electric background too. Both assumed

\(^3\)See also the recent work [13] on wave solutions of the NC Einstein-Maxwell equations in absence of a background field.
\( \theta^{0i} = 0 \), with the conclusion that the dispersion relation is modified according to

\[
\omega^2 = k^2(1 - 2g \vec{\theta}_T \cdot \vec{B}_T)
\]

(1.4)

to first order in \( \theta \). We have defined the 3-vector \( \theta^i = \frac{1}{2} \epsilon^{ijk} \theta^{jk} \); here the \( T \) indices refer to the projection transverse to the wave momentum. A first important consequence is that depending on the sign of \( \vec{\theta}_T \cdot \vec{B}_T \) the propagation may become superluminal, i.e. the group velocity \( \frac{d\omega}{dk} > 1 \). A similar phenomenon has been found for NC solitons which may have superluminal velocities on arbitrarily large distances [16, 17]. It is discussed in [17] that this does not necessarily imply a violation of causality because in that case the faster-than-light propagation is associated to a preferred frame where \( \theta^{0i} = 0 \).

It has been argued [18] that unitarity is violated in NC theories unless there exist a frame where \( \theta^{0i} = 0 \), or frame independently,

\[
\theta_{\mu\nu} \theta^{\mu\nu} = \theta^{ij} \theta^{ij} - 2\theta^{0i} \theta^{0i} > 0
\]

and

\[
\epsilon_{\mu\nu\rho\sigma} \theta^{\mu\nu} \theta^{\rho\sigma} = 0,
\]

(1.5)

which has lead many authors to ignore the \( \theta^{0i} \) components. We stress that even in the case when there is such a frame, it may be different from the laboratory frame, although taking \( \theta^{0i} = 0 \) e.g. in the CMB frame would lead to only a very small contribution to \( \theta^{0i} \) in the Earth frame, with \( \beta \sim 10^{-3} \). Also, unitarity no longer requires (1.5) if a proper quantization is applied [19]. As a result, the theories with \( \theta^{0i} \neq 0 \) have become an area of active research (see [20–24] for a non-exhaustive list of related works dealing with different aspects of time-space noncommutativity). Therefore it is natural that the properties of wave propagation with non-zero \( \theta^{0i} \) should be considered too. This was studied in [25, 26] for a magnetic background.

The recent claim, by the PVLAS collaboration, of having observed a non-zero rotation and ellipticity of initially polarized light propagating in a strong magnetic field [27] provides another motivation for reconsidering NC wave propagation with non-vanishing \( \theta^{0i} \). Such effects typically result from self-interaction terms of the electromagnetic field. They are expected in ordinary QED from the loop-level photon splitting and vacuum polarization graphs in a background field [28]. The experiment was actually designed to observe this last effect, but obtained instead larger results unexplainable in the Standard Model, in particular an ellipticity signal four orders of magnitude larger than the expected QED one. There was a claim [29], that the pure NC QED in the case of the space-space non-commutativity (\( \theta^{0i} = 0 \)) might be (partially) responsible for this effect. In this paper, we revisit this proposal and arrive at a different conclusion, namely that there is no rotation effect due to NC. In section 2, we present the analysis of the general case including \( \theta^{0i} \neq 0 \) and find that using the NC gauge field variable, a qualitatively new effect seems to appear, with a polarization rotation. However this effect is an artifact of using the transverse gauge inconsistently for NC gauge field variables, which would involve an infinite number of time derivatives. We show that the effect is still vanishing when using physical gauge fields obtained via the Seiberg-Witten map.

Another possible source of polarization rotation and ellipticity generation is the mixing of photons with a light scalar or pseudo-scalar such as an axion [30]. The PVLAS results could be explained by an axion mass \( m \sim 10^{-3} \text{ eV} \) and a photon-axion coupling scale.
\( M^{-1} \sim (10^6 \text{ GeV})^{-1} \), however this last value would induce a globular cluster star cooling rate \([31]\), and a solar axion flux in the CAST detector \([32]\) by Primakoff photon-axion conversion, by four orders of magnitude too large. Astrophysical arguments \([33–35]\) have been proposed recently to evade the previous bounds, along with simple alternatives to axion models based on the mixing with extra vector fields \([36,37]\). In section 3, we study wave propagation in a background field in axionic NC QED. We show that the axion-photon inverse oscillation length, which is the relevant parameter for induced ellipticity, can be dominated by non-commutativity for small enough axion masses, and still be larger than the ordinary QED contribution. The axion coupling \( M \) needed is however too small to reproduce the ellipticity observed in the specific PVLAS setup. Finally, we discuss the constraints on the various scales relevant for the problem.

2 Wave propagation in a background field for general non-commutativity

2.1 Noncommutative vs. physical gauge field variables

We consider non-commutative QED, characterized by the constants 
\( i\theta^{\mu\nu} = [\hat{x}^\mu, \hat{x}^\nu] \). Alternatively, one may use the magnetic part 
\( \theta^i = \frac{1}{2} \epsilon^{ijk} \theta^{jk} \) and the electric part 
\( \xi^i = \theta^i \). The action is obtained by substitution of Moyal products \( \star \) to ordinary products:

\[
S_{NC \text{ QED}} = \int d^4x \left( -\frac{1}{4} F^{\mu\nu} \star F_{\mu\nu} \right)
\]

\[
F^{\mu\nu} = \partial^{[\mu} A^{\nu]} + ig[A^\mu, A^\nu] \approx \partial^{[\mu} A^{\nu]} - g\theta^{\alpha\beta} \partial_\alpha A^\mu \partial_\beta A^\nu + O(\theta^2) \quad (2.1)
\]

with \( g \) the \( U(1) \) gauge coupling. It can be seen that even abelian gauge theories possess photon self-couplings, suppressed by the NC energy scale. A single \( \star \) may be removed from every term in the action, up to a total derivative. This action does not enjoy the usual \( U(1) \) gauge invariance, as can be seen in particular from the fact that it does not depend only on the "commutative" field-strength \( \partial_{[\mu} A_{\nu]} \). The full NC gauge invariance is expressed as

\[
A_\mu \rightarrow U^\dagger \star (A_\mu + \frac{1}{ig} \partial_\mu) \star U
\]

\[
(U \star U^\dagger)(x) = (U^\dagger \star U)(x) = 1. \quad (2.2)
\]

This can be traced back to the fact that the (Moyal) commutator of two infinitesimal gauge transformations involves now the commutator of functions:

\[
[\epsilon^a_1(x) T_a, \epsilon^b_2(x) T_b]_\star = \frac{1}{2} \{ \epsilon^a_1(x), \epsilon^b_2(x) \}_\star [T_a, T_b] + \frac{1}{2} [\epsilon^a_1(x), \epsilon^b_2(x)]_\star \{T_a, T_b\} \quad (2.3)
\]

and is thus no longer vanishing even in the \( U(1)_{NC} \) case where \( [T_a, T_b] = 0 \). The standard trick to recover the ordinary gauge symmetry is to perform a nonlocal redefinition of the gauge (and matter) fields according to the Seiberg-Witten map \([4]\):

\[
A_\mu = A_\mu - \frac{1}{2} g\theta^{\alpha\beta} A_\alpha (\partial_\beta A_\mu + F_{\beta\mu}) + O(\theta^2)
\]

\[
F_{\mu\nu} = \partial_{[\mu} A_{\nu]} \quad, \quad (2.4)
\]
which is unique up to some homogeneous terms that for our purpose can be consistently set to zero [38]. The field $A_\mu$ is the commutative $U(1)$ gauge field and the gauge invariant action is

$$S_{NC \text{ QED}} = \int d^4x \left( -\frac{1}{4} F^{\mu\nu} F_{\mu\nu} + \frac{1}{8} g \theta^{\alpha\beta} F_{\alpha\beta} F^{\mu\nu} F_{\mu\nu} - \frac{1}{2} g \theta^{\alpha\beta} F_{\mu\alpha} F_{\nu\beta} F^{\mu\nu} \right) + O(\theta^2). \quad (2.5)$$

It is interesting to compare this approach to NC QED with the one in which calculations are done working directly with the non-commutative gauge field $A$ (this is the approach used in [29]). Though it enjoys an obvious advantage of the possibility to perform calculation exact in $\theta$, in the case of the time-space non-commutativity, $\theta^{0i} \neq 0$, this approach meets serious difficulties related to the gauge fixing (when $\theta^{0i} = 0$, this problem is absent). The ‘natural’ gauge used in [29] - the temporal transverse gauge $A_0 = \partial_i A^i = 0$ - is now inconsistent with the equations of motion, for $\vec{\xi} \neq 0$. The equations of motion for the field perturbation read, prior to gauge fixing, and to all order in $\theta^{\mu\nu}$ for constant background field strength:

$$\Box \eta^{\mu\nu} - \partial^{\mu} \partial^{\nu} + g \theta^{\sigma\rho}(\partial_{\mu} A^{\omega}_B)(2 \eta^{\mu\nu} \partial_{\tau} - \partial^{\mu} \delta^{\nu}_{\tau} - \partial^{\nu} \delta^{\mu}_{\tau}) \partial_{\sigma}$$

$$+ g^2 (\theta^{\rho\sigma} \partial_{\mu} A^i_B)(\theta^{\sigma\rho} A^i_B) \eta^{\mu\nu} \partial_{\omega} \partial_{\sigma} - g^2 (\theta^{\rho\sigma} \partial_{\mu} A^j_B)(\theta^{\sigma\rho} \partial_{\nu} A^i_B) \partial_{\omega} \partial_{\sigma} \big) A_\mu + O(A^2) = 0(2.6)$$

where $A^i_B$ is the background vector potential. From this the transverse temporal gauge would impose

$$\vec{B} \times \vec{\xi} \cdot \partial_0^2 \vec{A} = 0,$$

which would kill one of the two required degrees of freedom. So the naive ‘natural’ gauge choice is inconsistent.\(^4\) Therefore we choose to work instead using (2.5) with commutative gauge field variables obtained from the Seiberg-Witten map, at the cost of being limited to work at finite order in $\theta^{\mu\nu}$.

### 2.2 Explicit derivation of the dispersion relation for $\theta^{0i} \neq 0$

We consider now the quadratic expansion of the action (2.5) around a constant non-zero background, $F_{\mu\nu} = F_{\mu\nu}^0 + f_{\mu\nu}$ with $f \ll F$. It turns out to be convenient to work with explicitly gauge invariant variables $E^i$ and $B^k$, which are defined as in the commutative case

$$F^{i0} = E^i, \quad F_{ij} = -\epsilon_{ijk} B^k. \quad (2.8)$$

As above, we will use two 3-d vectors, $\xi^i$ and $\theta^k$, characterizing non-commutativity:

$$\theta^{0i} = \xi^i, \quad \theta^{ij} = \epsilon^{ijk} \theta^k. \quad (2.9)$$

So we explicitly take into account the possibility of time-space non-commutativity\(^5\). In the case when $\xi^i = 0$, our analysis reduces to the one presented in [15].

\(^4\)This is important: using this gauge, one would have arrived to the conclusion that when $\theta^{0i} \neq 0$ there is rotation of the polarization. Which is not the case as we will see in the next section.

\(^5\)After completion of this work, reference [25] which considered the dispersion relations in the $\xi^i \neq 0$ case for a constant magnetic background, purpose of the section 2.2, was brought to our attention. The work [26] showed later that the same result could be found using covariant coordinates.
In terms of the definitions (2.8) and (2.9) the Lagrangian from Eq.(2.5) becomes
\[
L = \frac{1}{2} (1 + g\tilde{\zeta} \cdot \vec{E} - g\tilde{\theta} \cdot \vec{B})(\vec{E}^2 - \vec{B}^2) + (g\tilde{\zeta} \cdot \vec{B} + g\tilde{\theta} \cdot \vec{E})(\vec{E} \cdot \vec{B}) \tag{2.10}
\]

Varying (2.5) with respect to \(A_i\) and \(A_0\) gives, as usual, equation of motion and a Gauss law constraint respectively
\[
\frac{\delta S}{\delta A_i} = -\partial_0 \left( \frac{\partial L}{\partial E^i} \right) + \epsilon^{ijk} \partial_k \left( \frac{\partial L}{\partial B^j} \right) \quad \text{and} \quad \frac{\delta S}{\delta A_0} = \partial_i \left( \frac{\partial L}{\partial E^i} \right) \tag{2.11}
\]

To write (2.11) explicitly, it is convenient to introduce ‘displacement field’ \(\vec{D}\) and ‘magnetic field’ \(\vec{H}\)
\[
\vec{D} = (1 + g\tilde{\zeta} \cdot \vec{E} - g\tilde{\theta} \cdot \vec{B})\vec{E} + (g\tilde{\zeta} \cdot \vec{B} + g\tilde{\theta} \cdot \vec{E})\vec{B} + \frac{1}{2}(\vec{E}^2 - \vec{B}^2)g\tilde{\zeta} + (\vec{E} \cdot \vec{B})g\tilde{\theta} \tag{2.12}
\]
\[
\vec{H} = (1 + g\tilde{\zeta} \cdot \vec{E} - g\tilde{\theta} \cdot \vec{B})\vec{B} - (g\tilde{\zeta} \cdot \vec{B} + g\tilde{\theta} \cdot \vec{E})\vec{E} + \frac{1}{2}(\vec{E}^2 - \vec{B}^2)g\tilde{\theta} - (\vec{E} \cdot \vec{B})g\tilde{\zeta}.
\]

Then Eq.(2.11) will have the form of the sourceless Maxwell equations
\[
\partial_0 \vec{D} - \vec{\partial} \times \vec{H} = 0 \quad \text{and} \quad \vec{\partial} \cdot \vec{D} = 0
\]
\[
\vec{\partial} \cdot \vec{D} = 0 \quad \text{and} \quad \vec{\partial} \cdot \vec{B} = 0 \tag{2.13}
\]

We are interested in the plane-wave solution of (2.13), \(\vec{E} = \vec{E}(\omega t - \vec{k} \cdot \vec{r})\), in the presence of the constant magnetic background \(\vec{B}\). Then we have from (2.13)
\[
\vec{B} = -\frac{\vec{k}}{\omega} \times \vec{E} + \vec{\bar{B}} \quad \text{and} \quad \vec{D} = -\frac{\vec{k}}{\omega} \times \vec{H} + \vec{\bar{D}} \tag{2.14}
\]

The non-vanishing constant background field \(\vec{D}\) is needed to satisfy (2.13) for a vanishing background, \(\vec{E} = \vec{0}\).

We will not attempt to solve (2.12)-(2.14) exactly in fields. We rather will consider the linearized system which is enough for our purpose - finding the dispersion relation. Writing \(\vec{B} = \vec{\bar{B}} + \vec{\delta B}\) and \(\vec{H} = \vec{\bar{H}} + \vec{\delta H}\), we have the following linearized system
\[
\vec{D}_i = \varepsilon^{ij} \varepsilon_j + \rho^{ij} \delta B^j, \quad \vec{H}_i = \mu^{ij} \varepsilon_j + \eta^{ij} \delta B^j \tag{2.15}
\]

where
\[
\varepsilon^{ij} = \delta^{ij}(1 - g\tilde{\theta} \cdot \vec{B}) + g\tilde{\theta}^i \delta B^j + g\tilde{\theta}^j \delta B^i, \quad \rho^{ij} = \delta^{ij}(g\tilde{\zeta} \cdot \vec{B}) + g\tilde{\zeta}^i \delta B^j + g\tilde{\zeta}^j \delta B^i, \tag{2.16}
\]
\[
\mu^{ij} = \delta^{ij}(1 - g\tilde{\theta} \cdot \vec{B}) - g\tilde{\theta}^i \delta B^j - g\tilde{\theta}^j \delta B^i, \quad \eta^{ij} = -\delta^{ij}(g\tilde{\zeta} \cdot \vec{B}) + g\tilde{\zeta}^i \delta B^j - g\tilde{\zeta}^j \delta B^i.
\]

Let us choose the third direction along \(\vec{k}\). Then we have \(\vec{k} \cdot \vec{E} = 0\). Now using the (homogeneous part of) Eq. (2.14) in Eq. (2.15) after some algebra one arrives at the following dispersion relation
\[
\frac{\vec{k}^2}{\omega^2} = \left(1 + 2g\tilde{\theta} T \cdot \vec{B}_T - 2(g\tilde{\xi} \times \vec{B})_3\right) + O(\theta^2) \tag{2.17}
\]

\(^6\)It is true that longitudinal component of \(\vec{E}, \varepsilon_3\), is not zero. But because it is of the order of \(\theta\), one cannot catch it in the linearized approach. But this does not affect the calculation of the dispersion relation up to order \(O(\theta^2)\).
where the subscript $T$ means the transverse component of the corresponding vector. The important property of Eq. (2.17) is that RHS is actually independent of the polarization of the plane wave. This means that the only effect of the noncommutativity is the change of ‘speed of light’, the same for both polarization. So there is no rotation of the polarization of light in this approximation ($O(\theta^2)$) in contrast with the conclusion of [29].

2.3 Calculation from the Lorentz-violating extended Standard Model

These results may be cross-checked and generalized using the framework of the Lorentz-violating extension of QED [39], described by the renormalizable Lagrangian

$$\mathcal{L}_{LV \text{ QED}} = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} - \frac{1}{4} (k_F)_{\mu\nu\rho\sigma} F^{\mu\nu} F^{\rho\sigma} + \frac{1}{2} (k_{AF})^\mu_{\mu\nu\rho\sigma} A^\nu F^{\rho\sigma}$$

(2.18)

where $k_F$ and $k_{AF}$ are Lorentz-breaking constant tensors, and one can assume the same symmetries as the Riemann tensor for $k_F$. In the case of non-commutative space-time with constant background field $F_{\mu\nu}$, one can treat $A_\mu$ in the action above as the perturbation around this background. The CPT-odd term $(k_{AF})^\mu = 0$ vanishes and

$$(k_F)_{\mu\nu\rho\sigma} = \frac{1}{8} (T_{[\mu\nu][\rho\sigma]} + \mu\nu \leftrightarrow \rho\sigma)$$

$$T_{\mu\nu\rho\sigma} \equiv -\frac{1}{2} g \theta^{\alpha\beta} F_{\alpha\beta} \eta_{\mu\rho\sigma} - g \theta_{\mu\nu} F_{\rho\sigma} + 4 g \theta_{\alpha\nu} F^\alpha_{\rho\sigma} \eta_{\mu\sigma} + 2 g \theta_{\nu\sigma} F_{\mu\rho} + O(\theta^2)$$

(2.19)

where we defined $M_{[\mu\nu]} \equiv M_{\mu\nu} - M_{\nu\mu}$. The photon dispersion relation is modified, for the two polarizations, as follows:

$$\omega_{\pm} = (1 + \rho \pm \sigma) |\vec{k}|$$

(2.20)

with

$$\rho = \frac{1}{2} \bar{k}^\alpha \bar{k}_\alpha$$

$$\sigma^2 = \frac{1}{2} \bar{k}_{\alpha\beta} \bar{k}^{\alpha\beta} - \rho^2$$

$$\bar{k}_{\mu\rho} \equiv (k_F)_{\mu\nu\rho\sigma} \frac{k^\nu}{|k|} \frac{k^\sigma}{|k|}$$

(2.21)

The polarization rotation arises from the speed difference $2\sigma$ between the polarization orthogonal and parallel to the background field, and is in general non-zero. However in the NC case, we find $\sigma = 0$ to first order in $\theta$, leading to no rotation.

We obtain the overall change in the speed of light from

$$\frac{d\omega}{dk} = 1 + \rho = g \theta_{\alpha\nu} F^\alpha_{\nu} \frac{k^\mu}{|k|} \frac{k^\nu}{|k|} + O(\theta^2)$$

(2.22)

where to leading order in $\theta$, the photon momentum $k^\mu$ on the right-hand side can be taken to obey the unmodified dispersion relation $k^2 = 0$. Again, this can take both signs and illustrates superluminal propagation in NC field theories.
It is easy to see that (2.22) is nothing but our result (2.17), including also a constant background electric field. Thus calculation in the framework of the Lorentz-violating extension of QED confirms our conclusion and contradicts the one of [29] where a non-zero rotation was claimed in spite of a vanishing $\sigma$.

3 Non-commutative axionic electrodynamics

3.1 Equations of motion

Let us consider now the coupling of a pseudoscalar $\phi$ to electromagnetism in the NC framework. Although we will refer to it as an axion in what follows, we will not assume that it is a Peccei-Quinn axion introduced to solve the strong CP problem [40–43], and we will keep $M$ and $m$ below as independent parameters. Similarly, we will assume that the matter couplings to the pseudo-scalar are vanishing or small enough to evade existing constraints that would be stronger than those from the pure axion-gauge sector. The corresponding action in NC field variables is

$$ S_{\text{axion}} = \int d^4x \left( \frac{1}{2} (\partial \phi)^2 - \frac{m^2}{2} \phi^2 + \frac{\phi}{8M} \epsilon_{\mu \nu \rho \sigma} (F^\mu \nu \star F^\rho \sigma) \right) $$

where one star product has been removed in each term, which leaves the action invariant up to a boundary term. Notice that for constant $\phi$, the last term is still a topological invariant. Applying the Seiberg-Witten map $A[A]$ and neglecting $O(\theta \times \frac{1}{M})$ terms, the action reduces to the commutative one:

$$ S_{\text{axion}} = \int d^4x \left( \frac{1}{2} (\partial \phi)^2 - \frac{m^2}{2} \phi^2 + \frac{\phi}{8M} \epsilon_{\mu \nu \rho \sigma} F^\mu \nu F^\rho \sigma \right) + O(\frac{\theta}{M}) $$

(3.1)

We expand around a constant background $F_{\mu \nu} = F_{\mu \nu} + f_{\mu \nu}$, and we assume $\tilde{F}_{\mu \nu} F^{\mu \nu} = 0$. The total action is

$$ S = \int d^4x \left( -\frac{1}{4} \eta_{\mu \nu} \eta_{\rho \sigma} + (k_F)_{\mu \nu \rho \sigma} f^{\mu \nu} f^{\rho \sigma} + \frac{1}{2} (\partial \phi)^2 - \frac{m^2}{2} \phi^2 + \frac{1}{2M} \tilde{F}_{\mu \nu} f^{\mu \nu} \phi \right) $$

$$ + O(\theta^2, \frac{\theta}{M}, f^3) $$

(3.3)

with $k_F$ given in (2.19), and $\tilde{F}_{\mu \nu} \equiv \frac{1}{2} \epsilon_{\mu \nu \rho \sigma} F^{\rho \sigma}$ is the dual background electromagnetic field strength.

We specialize now to the PVLAS-like configurations, with purely magnetic background orthogonal to the wave propagation, and take for simplicity pure space-space non-commutativity $\theta^0 = 0, \theta^i = \frac{1}{2} \epsilon^{ijk} \theta^{jk}$. The equations of motion are, to first order in
\[ E_L = O(\theta) \]
\[ (1 - \frac{\vec{k}^2}{\omega^2} + 2g\vec{\theta} \cdot \vec{B}) E_{T\perp} = 0 \]
\[ (1 - \frac{\vec{k}^2}{\omega^2} + 2g\vec{\theta} \cdot \vec{B}) E_{T\parallel} - \frac{\vec{B}}{M} \phi = 0 \]
\[ (\omega^2 - \vec{k}^2 - m^2) \phi - \frac{\vec{B}}{M} E_{T\parallel} = 0. \]  

(3.4)

Here \( E_L \) is the longitudinal (to \( \vec{k} \)) component of the electric field, \( E_{T\perp} \) is the transverse component orthogonal to the background magnetic field, and \( E_{T\parallel} \) is the transverse component projected on the background magnetic field; these three fields form an orthogonal basis. The longitudinal component, non-vanishing here, is a function of the transverse ones and does not represent a third degree of freedom for the electromagnetic field.

The dispersion relation for the transverse component orthogonal to \( \vec{B} \), \( E_{T\perp} \), is the same as in the case of the pure NC QED

\[ \omega^2_{\perp} = \left[ 1 - 2g(\vec{\theta} \cdot \vec{B}) \right] \vec{k}^2. \]  

(3.5)

Searching for plane wave solutions for the \( \phi-E_{T\parallel} \) coupled system, we obtain the energy eigenvalues

\[ (\omega^\pm)^2 = \frac{1}{2} \left( \omega^2_{\perp} + \omega^2_{\phi} + \frac{\vec{B}^2}{M^2} \right) \pm \frac{1}{2} \sqrt{\left( \omega^2_{\perp} + \omega^2_{\phi} + \frac{\vec{B}^2}{M^2} \right)^2 - 4\omega^2_{\perp} \omega^2_{\phi}} \]  

(3.6)

where we introduced the notation

\[ \omega^2_{\phi} \equiv \vec{k}^2 + m^2 \]  

(3.7)

which is the axion squared energy in the absence of mixing, and \( \tilde{M} \equiv M(1 + g\vec{\theta} \cdot \vec{B}) \approx M \) to lowest order. The effect of non-commutativity is thus the modification \((3.5)\) of the photon dispersion relation. In the \( \theta \to 0 \) limit for finite \( M \), the usual commutative axion-photon mixing occurs \([30]\).

In the \( M \to \infty \) limit for finite \( \theta \), the axion and parallel photon decouple, leading to the known

\[ \omega^2_{\text{axion}} = \omega^2_{\phi} = \vec{k}^2 + m^2 \]
\[ \omega^2_{\text{photon}} = \omega^2_{\perp} = \left[ 1 - 2g(\vec{\theta} \cdot \vec{B}) \right] \vec{k}^2 \]  

(3.8)

where speed of light is equally shifted for both polarizations.

### 3.2 Physical scales and constraints

Let us consider now the scales involved in the problem. The beam pulsation corresponds to infrared radiation \( \omega \sim 1.2 \text{ eV} \). We assume that axions are relativistic, \( m^2 \ll \vec{k}^2 \sim \).
$1.2 \text{ eV}^2$. These conditions exclude the detection of resonant axion production stimulated by an hypothetical galactic halo dark matter axion background [44], which would require $\frac{\omega}{m} - 1 \leq \frac{1}{2} (\frac{\omega}{m})_{DM} \sim 10^{-6}$. The background magnetic field is of order $|\mathbf{B}| \sim 5.5 \times 10^{-5} \Rightarrow g|\mathbf{B}| \sim (18 \text{ eV})^2$. Weak axion coupling such that $\mathbf{B}^2/M^2 \ll \omega^2$ is assumed, implying $gM \gg 15 \text{ eV}$, obviously satisfied when considering other experimental constraints on $M$.

Let us remind the allowed ranges of the axion parameters $M, m$. The strongest model-independent constraints on $M$, which make no assumption on the axion matter couplings and keep $M, m$ as independent parameters, come from the Primakoff photon to axion conversion in stars. Avoiding excessive star cooling by this process in the globular clusters, which would modify the star evolution, imposes [31]

$$M \geq 1.7 \times 10^{10} \text{ GeV.}$$

A bound of the same order is obtained from the non-observation of an axion flux from the Sun by the CAST helioscope [32], which reconverts them into photons in a strong magnetic field:

$$M \geq 0.86 \times 10^{10} \text{ GeV.}$$

More specifically now, the QCD axion required to solve the strong CP problem has matter couplings almost completely characterized by the Peccei-Quinn symmetry breaking scale $f$, related to its photon coupling $M^{-1}$ by $f = g_\gamma \frac{g_\phi^2}{4\pi} M$ with $g_\gamma$ a model-dependent coefficient of order unity. Its mass $m$ is given by

$$m = (0.62 \times 10^{-3} \text{ eV}) \frac{10^{10} \text{ GeV}}{f} \sim 2.7 g_\gamma^{-1} \times 10^{-3} \text{ eV} \frac{10^{10} \text{ GeV}}{M}. $$

Furthermore, its Yukawa coupling to nucleons $g_{\phi N} \sim \frac{1}{f} \text{ GeV}$ is constrained to be $g_{\phi N} \leq 3 \times 10^{-7}$ to avoid excessive supernovae cooling [45], which pushes higher the bound (3.9) by roughly one order of magnitude. Correspondingly, this translates into

$$m \leq 10^{-2} \text{ eV} $$

for the QCD axion mass. A lower bound

$$m \geq 10^{-6} \text{ eV} $$

can be set by requiring that the QCD axions, produced by the vacuum misalignment mechanism [46] or by axionic string radiation [47], do not lead to overclosure of the universe.

As we are going to consider rather small NC mass scales, it is no longer relevant to restrain ourselves to the QCD axion case. We will therefore keep only the model-independent bound (3.9) on $M$. Similarly, the usual 'axion window' for $m$ mentioned above should be rederived in the new framework and will not be taken into account here.
3.3 Oscillation length, ellipticity and polarization rotation

Since $|ω^+ − ω| ≪ ω$ and $|ω^− − ω| ≪ ω$, one can approximate (3.6) with

$$ω^+ = ω^− + \frac{1}{2} \left( \frac{m^2}{2ω} + g\tilde{θ} \cdot \vec{B}ω \right) \pm \frac{1}{2} \sqrt{\left( \frac{m^2}{2ω} + g\tilde{θ} \cdot \vec{B}ω \right)^2 + \vec{B}^2 \frac{m^2}{M^2}}$$

$$= ω^− + \frac{1}{2} \left( \frac{m^2}{2ω} + g\tilde{θ} \cdot \vec{B}ω \right) \left( 1 \pm \frac{1}{\cos(2α)} \right) \quad (3.14)$$

where

$$α = \frac{1}{2} \arctan \left( \frac{||\vec{B}|/M}{\frac{m^2}{2ω} + g\tilde{θ} \cdot \vec{B}ω} \right) \in [0, \frac{π}{4}] \quad (3.15)$$

is the photon-axion mixing angle. The weak mixing condition $α ≪ 1$, or

$$\frac{m^2}{2ω} + g\tilde{θ} \cdot \vec{B}ω \gg |\vec{B}|/M \quad (3.16)$$

translates, in the usual case of vanishing or subdominant non-commutativity compared to the axion mass, into a constraint on the axion parameters,

$$m^2M \gg |\vec{B}|ω. \quad (3.17)$$

However, we are interested in the case of dominant non-commutativity in the left-hand side of (3.16), which is related to the oscillation length $L_{osc}$ by

$$2πL_{osc}^{-1} = \frac{m^2}{2ω} + g\tilde{θ} \cdot \vec{B}ω \equiv 2πL_{θ}^{-1} + 2πL_{QED}^{-1}. \quad (3.18)$$

Assuming $θ$ as large as allowed by the tree-level NC QED bound $θ \leq (10 \, TeV)^{-2}$, as discussed in the introduction, the $θ$ term dominates the inverse oscillation length for $m ≲ ω\sqrt{2g\tilde{θ} \cdot \vec{B}} \sim 10^{-12} \, eV$, a nearly massless axion, and the weak mixing condition implies then an almost Planck-size axion coupling $gM \geq 10^{17} \, GeV$. Although this is extremely small, note that the QED contribution to the inverse oscillation length, that should have been included in principle in (3.18), is still subdominant,

$$2πL_{QED}^{-1} \sim \frac{2α^2_{QED}|\vec{B}|^2ω}{15m_e^2} < 2πL_{θ}^{-1} \sim g|\tilde{θ} \cdot \vec{B}|ω \quad (3.19)$$

(with $m_e$ the electron mass) for a background magnetic field $|\vec{B}|$ of order 0.1 $T$ or lower. The Cotton-Mouton contribution $L_{CM}^{-1}$ to the inverse oscillation length, due to the residual gas in the cavity, has to be even smaller in any experiment designed to observe the QED contributions, which can be achieved for small enough gas pressure.

Under all these conditions, one should observe a rotation $ψ$ of the polarization plane after $N$ traversals of a cavity of length $L$, for an initial angle at 45° of the magnetic field,

$$ψ \, [rad] \approx \frac{1}{8} \frac{NL^2|\vec{B}|^2}{M^2}, \quad (3.20)$$
which is independent of the oscillation length and an ellipticity (ratio of minor to major axis)

$$\mathcal{E} \approx \frac{\pi}{12} \frac{NL^2|\vec{B}|^2}{M^2} \frac{L}{L_{osc}} \approx \frac{1}{24} \frac{NL^3|\vec{B}|^2}{M^2} g\vec{\theta} \cdot \vec{B} \omega. \quad (3.21)$$

The expansions above are allowed by the fact that we are largely in the coherence regime $L \ll L_{osc}$ necessary for the experiment. The rotation angle $\psi$ is independent of the oscillation length and is thus the same as in the commutative axion case. It originates from a depletion of the photon component parallel to the background field as it transforms into axions, while the orthogonal component remains the same. It is equivalently a measure of the photon-axion transition rate depending essentially on $M$, which is exactly what the CAST experiment, the star cooling rates, or the less constraining $\gamma \rightarrow \phi \rightarrow \gamma$ 'shine light through walls' [48] experiments measure. The polarization rotation $\psi$ is thus not the most relevant number in PVLAS-type oscillation experiments: only the induced ellipticity $\mathcal{E}$ is sensitive to the oscillation length, and thus can provide more complete information on the detailed physics involved such as the axion mass $m$ in the commutative case, or the NC scale $\theta^{-1/2}$ here. Incidentally, this shows that known lower bounds on $M$ should be unchanged by non-commutativity.

Unfortunately here, the weak mixing condition pushes $M$ to almost Planckian values, and as we required also a practically massless axion, the effect of mixing should be roughly the same as that of photon-graviton mixing, unobservably small. Practically, $N \sim 10^5$ and $L = 1 \, m$, so that we expect for weak mixing

$$\psi_{weak} \sim 10^{-32} \left(\frac{M_P}{M}\right)^2 \left(\frac{|\vec{B}|}{5.5 \, T}\right)^2 \ll (1 - 4) \times 10^{-7} \, \text{rad}$$

$$\mathcal{E}_{weak} \sim 10^{-49} \left(\frac{M_P}{M}\right)^2 \left(\frac{|\vec{B}|}{5.5 \, T}\right)^3 \ll 10^{-8} - 2 \times 10^{-7} \quad (3.22)$$

where the last numbers given are the ones measured by PVLAS. Also, the ellipticity does not have the observed $\vec{B}^2$ dependence in the background field. Finally, we note that the experiment magnet is actually on a slowly rotating turntable such that the magnetic field rotates in the horizontal plane, the beam propagating vertically. The measured ellipticity would fluctuate in phase with the magnetic field as the $\vec{\theta} \cdot \vec{B}$ term varies.

We may also consider the case of maximal mixing

$$\frac{m^2}{2\omega} + g\vec{\theta} \cdot \vec{B} \omega \ll \frac{|\vec{B}|}{M}. \quad (3.23)$$

In this case we do not expect the NC scale to play any role. Effectively, no ellipticity is generated, and the only effect is a polarization rotation, again unsensitive to the oscillation length and identical to the commutative axion case.

### 4 Conclusions

We have studied noncommutative electromagnetic wave propagation in a constant background field, and shown that contrary to previous claims, no polarization rotation occurs...
for general $\theta^{\mu\nu}$. We illustrated the practical importance of using the commutative field variables derived from the Seiberg-Witten map. The result has been cross-checked using the general Lorentz-violating extension of QED. We have then studied how axion-photon oscillations in a strong background field may probe noncommutativity of space-time. Processes which are only sensitive to the axion-photon coupling $M$ are unaffected, while for small enough axion mass, the inverse oscillation length may be dominated by the NC scale. PVLAS-type experiments measuring induced ellipticity on linearly polarized light propagating in a strong magnetic field can directly probe this scale in principle. We found however that the almost Planckian axion-photon coupling required, even for a very fine-tuned NC scale $\theta^{-1/2} \sim 10$ TeV, makes the effect unobservably small.

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