Charge Neutrality of the Color-Flavor Locked Phase from the Low Energy Effective Theory

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Abstract

We investigate the issue of charge neutrality of the CFL$^0$ phase of dense quark matter using the low energy effective theory of high density QCD. We show that the local electric and color charge neutrality of the ground state in a homogeneous color superconducting medium follows from its dynamics. We also consider the situation of a spatially inhomogeneous medium, such as may be found in a neutron star core. We find that spatial inhomogeneity results in the generation of electric fields, and positrons and/or electrons may be present in the ground state. We estimate the concentration of charged leptons in the ground state to be $n_e \sim 10^2 \text{cm}^{-3}$ and consider their influence on the opacity of the medium with respect to the modified photons.

I. INTRODUCTION

It is now understood that at asymptotically large baryon number densities, the ground state of 3-flavor, 3-color massless QCD is the Color-Flavor Locked (CFL) phase [1,2]. (See [3] for comprehensive reviews.) In this phase, all 9 species of quarks with momenta close to the Fermi surface undergo BCS-like pairing. The resulting quark-quark condensate spontaneously breaks the original $SU(3)_C \times SU(3)_L \times SU(3)_R$ symmetry down to the diagonal subgroup, $SU(3)_{C+L+R}$, causing the gauge bosons of the original $SU(3)_C$ group to become massive, and also breaks the $U(1)_B$ symmetry associated with conserved baryon number.$^1$ One linear combination of gluon and photon remains massless; the associated unbroken gauge symmetry is referred to as $U(1)_{\tilde{Q}}$, denoting the “modified” electromagnetism.

A nonzero mass for the strange quark encourages the system to reduce the strange quark number density relative to the density of up- and down-type quarks. It has been argued

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$^1$A local gauge symmetry cannot really be spontaneously broken [4]. We have to fix a gauge to define expectation values of gauge variant quantities.
that the system responds to this stress by forming a kaon condensate in the ground state [5–7]. Kaon condensation allows the strange quark number density to be decreased without the costly breaking of pairs in the CFL background.

In particular, it was found that in the presence of small enough chemical potentials for the electric and lepton charges $K^0$ condensate was present in the ground state (the CFL$K^0$ phase) [6,7]. The CFL and CFL$K^0$ phases are distinct, as hypercharge is spontaneously broken in the latter, but not in the former. Some recent reviews are listed in [8].

Recently, some attention has been devoted to the issue of realization of charge neutrality in the high baryon density matter. First, it was argued that for a macroscopic system the difference between being in a color singlet state and being in a state with equal number densities of red, blue and green color charges is negligible [9]. Alford and Rajagopal constructed an expression for the free energy that incorporated general features of the CFL phase [10], while Steiner, Reddy and Prakash used NJL model supplemented by diquark and the t’Hooft six-fermion interactions [11]. Neither approach included gauge bosons as dynamical fields. Instead, to ensure charge neutrality of the CFL phase, chemical potentials $\mu_3$ and $\mu_8$ were introduced for the charges corresponding to $T_3$ and $T_8$ generators of the $SU(3)_c$ group, along with $\mu_Q$, a chemical potential for the electric charge. In both papers it was found that $\mu_3$, $\mu_8$ and $\mu_Q$ had to be adjusted to ensure local charge neutrality of the bulk quark matter.

Motivated by the ideas of [10], in this paper we employ the low energy effective theory of QCD at high baryon number density developed in [12,13,6] to address the issue of charge neutrality in the high baryon density matter. In particular, in this approach gauge fields are treated as dynamical degrees of freedom. One expects that in a color superconducting state, color charge neutrality will follow from the low energy dynamics of the system, and that is what we find for a homogeneous case.

In section 2 we show that this is indeed the case for a homogeneous medium; in section 3 we present the analysis for the case of an inhomogeneous medium, such as one would expect in a compact star, and we find a surprising result that nonzero electric fields exist in the bulk in this case.

II. CHARGE NEUTRALITY IN HOMOGENEOUS MEDIUM

QCD at asymptotically high baryon density has several effective field theories valid for different ranges of excitation energies. For energies well below the quark number chemical potential $\mu$ heavy anti-quarks (mass $\sim 2\mu$) may be integrated out in a systematic manner leading to the High Density Effective Theory (HDET) developed in [15,16,6].

For energies smaller than the superconducting gap $\Delta$ quasi-particle and quasi-hole states in the vicinity of the Fermi surface may, in turn, be integrated out leaving the Nambu-Goldstone bosons due to breaking of various flavor symmetries in the ground state as the relevant degrees of freedom [12,13,6,17]. For instance, in the CFL phase we have an octet due to breaking of the chiral symmetry and a singlet due to $U(1)_B$ breaking. See [18] for a review of the subject. The low energy effective theory of dense QCD is similar in spirit to the effective theory of an ordinary (QED) superconductor (Ginzburg-Landau theory). See, for example, a pedagogical introduction by S. Weinberg [14].

The leading terms of the low energy effective Lagrangian of QCD at high baryon number density are
\[
\mathcal{L} = -\frac{f_\pi^2}{2} \left[ \text{Tr} \left( (X^\dagger D_0^L X)^2 - \frac{1}{3} (X^\dagger D_i^L X)^2 \right) + \text{Tr} \left( (Y^\dagger D_0^R Y)^2 - \frac{1}{3} (Y^\dagger D_i^R Y)^2 \right) \right] \\
- \frac{1}{4} F_{\mu\nu} F^{\mu\nu} - \frac{1}{2} \text{Tr}(G_{\mu\nu} G^{\mu\nu}),
\]

where \( X \) and \( Y \) are the \( 3 \times 3 \) unitary matrix valued composite fields describing oscillations of the left- and right-handed quark-quark condensates about the CFL ground state. Under the original symmetry group \( SU(3)_c \times SU(3)_L \times SU(3)_R \), \( X \) transforms as \((3,3,1)\) and \( Y \) as \((3,1,3)\). The pion decay constant \( f_\pi \approx 0.209\mu \) was computed in [13]. The gauge covariant derivatives are

\[
D_\nu^L X = \partial_\nu X + ig_\pi X(A_\nu^c)^T + ieA_\nu^{em}QX + i\delta_\nu^0 \frac{MM^\dagger}{2\mu} X,
\]

\[
D_\nu^R Y = \partial_\nu Y + ig_\pi Y(A_\nu^c)^T + ieA_\nu^{em}QY + i\delta_\nu^0 \frac{MM^\dagger}{2\mu} Y,
\]

where \( A_\nu^c \) and \( A_\nu^{em} \) are the gluon and photon gauge fields, respectively; \( A_\nu^c = A_\nu^{em} t_a \), where \( t_a \) is an \( SU(3)_c \) generator in the fundamental representation. We treat quark mass matrix \( M \) as an external (spurion) field that transforms as \((1,3,3)\). It was argued in [6] that \( M^2/2\mu \) terms should appear in the covariant derivatives (2), and, thus, that in the low energy effective theory \( m_s^2/2\mu \) plays the role of chemical potential for strangeness. These are the terms responsible for the formation of kaon condensate in the ground state. In this paper we neglect up and down quark masses and set

\[
M = \text{diag}(0,0,m_s).
\]

The quark electric charge matrix is \( Q = \text{diag}(2/3, -1/3, -1/3) \).

A generic term in the effective Lagrangian is made of various \( SU(3)_c \times SU(3)_L \times SU(3)_R \times U(1)_B \) invariant combinations of powers of \( f_\pi^2 \Delta^2 Y^\dagger(\frac{\Delta}{2})^k Y \), \( f_\pi^2 \Delta^2 X^\dagger(\frac{\Delta}{2})^n X \), \( G_{\mu\nu} \) and \( F_{\mu\nu} \). The terms involving both \( X \) and \( Y \) and the ones containing the field strengths are further suppressed by powers of \( g_\pi \) and/or \( e \) [13].

It is more convenient to rewrite the Lagrangian in the following form [12]

\[
\mathcal{L} = \mathcal{L}_{XYA} + \mathcal{L}_X,
\]

where

\[
\mathcal{L}_{XYA} = -\frac{f_\pi^2}{4} \text{Tr} \left( X^\dagger \dot{X} + Y^\dagger \dot{Y} + iX^\dagger \left( \frac{MM^\dagger}{2\mu} + eA_0^{em} Q \right) X + iY^\dagger \left( \frac{MM^\dagger}{2\mu} + eA_0^{em} Q \right) Y + 2ig_\pi (A_0^c)^T \right)^2 \\
+ \frac{f_\pi^2}{4} \frac{1}{3} \text{Tr} \left( X^\dagger \nabla_i X + Y^\dagger \nabla_i Y + i\delta_0 \frac{MM^\dagger}{2\mu} (X^\dagger QX + Y^\dagger QY) + 2ig_\pi (A_0^c)^T \right)^2 \\
- \frac{1}{4} F_{\mu\nu} F^{\mu\nu} - \frac{1}{2} \text{Tr}(G_{\mu\nu} G^{\mu\nu}),
\]

and

\[
\mathcal{L}_X = -\frac{f_\pi^2}{4} \text{Tr} \left( X^\dagger \dot{X} - Y^\dagger \dot{Y} + iX^\dagger \frac{MM^\dagger}{2\mu} X - iY^\dagger \frac{MM^\dagger}{2\mu} Y + ieA_0 \left( X^\dagger QX - Y^\dagger QY \right) \right)^2.
\]
Here

\[ W_L = \tilde{e}A_0^Q Q + \frac{MM^\dagger}{2\mu}, \]  

and

\[ W_R = \tilde{e}A_0^Q Q + \frac{M^\dagger M}{2\mu}, \]  

\( A_\mu^Q \) is the \( U_Q(1) \) gauge field with the coupling constant

\[ \tilde{e} = \frac{\sqrt{3eg_s}}{\sqrt{3g_s^2 + 4e^2}}. \]  

We have introduced the color singlet field \( \Sigma = X Y^\dagger = \exp[2i\pi^a t^a / f]\), where \( \pi^a \)'s are the pseudo-scalar octet of Nambu-Goldstone bosons which arise from the breaking of chiral symmetry. \( \Sigma \) transforms as \((1,3,\overline{3})\). We neglect the \( U_B(1) \) Nambu-Goldstone boson terms in the Lagrangian as they are irrelevant for the gauge charge neutrality.

We see that \( \mathcal{L}_\Sigma \) only depends on light fields, while \( \mathcal{L}_{XYA} \) has both light and heavy degrees of freedom in it. The convenience of the representation (4) is due to the fact that at low energy the heavy gauge fields may be integrated out. Then \( \mathcal{L}_{XYA} \) piece vanishes leaving us with \( \mathcal{L}_\Sigma \) only. Now one may proceed to solve equations of motion to determine values of various (classical) fields in the ground state. It was found in \([6,7]\) that in the ground state \( \Sigma \) field takes on a non trivial value

\[ \Sigma_{K^0} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & i \\ 0 & i & 0 \end{pmatrix} \]  

corresponding to CFLK\(^0\) phase. We note that in the approximation \( m_u = m_d = 0 \) the \( K^0 \) condensate in the ground state is maximal and formed for arbitrarily small value of \( m_s \) \([7]\).

From \( \mathcal{L}_{XYA} \) we see that \( X^\dagger \frac{M M^\dagger}{2\mu} X + Y^\dagger \frac{M^\dagger M}{2\mu} Y \) acts as a source term for the temporal components of gauge fields. This follows from \( M^2/2\mu \) terms being part of the covariant derivatives (2). We may also see this from the matching calculation of the coefficient of the operator

\[ g_s A_a^c \text{Tr} \left( (t_a)^T \mathcal{M} \right) , \]  

where
This is the HDET diagram that produces operator (11) at one loop level. Fermion lines with triangle insertions denote the anomalous quasi particle propagators. Hollow circle stands for insertion of $M$ defined in (12). Normal one loop diagram contribution vanishes.

\[ M \equiv \frac{X^\dagger M M^\dagger}{2\mu} X + \frac{Y^\dagger M^\dagger M Y}{2\mu}, \]  

in the low energy effective Lagrangian (4). The calculation yields a non zero coefficient of $f_\pi^2/4$ that comes from the anomalous contribution to the one-loop tadpole diagram with one insertion of $M$ as shown in Fig. 1. The diagram is superficially proportional to $\Delta^2$, but the loop integration is infrared sensitive and picks up a compensating $1/\Delta^2$.

We note that spatial components of gauge fields do not have sources and vanish in the ground state.

To proceed it is convenient to define a unitary matrix $\xi$, such that $\xi^2 = \Sigma$, and an $SU(3)$ matrix $U$. Then $X$ and $Y$ may be presented as

\[ X = \xi U, \quad Y = \xi^\dagger U \]  

We fix $SU(3)_{\text{C}}$ gauge by setting

\[ U = 1, \]  

which corresponds to the unitary gauge [12]. Then we have

\[ \xi_{K^0} = \frac{1}{\sqrt{2}} \begin{pmatrix} \sqrt{2} & 0 & 0 \\ 0 & 1 & i \\ 0 & i & 1 \end{pmatrix} \]  

and $X = \xi_{K^0}$, $Y = \xi_{K^0}^\dagger$.

In the homogeneous medium all fields are constants and the Lagrangian reduces to

\[ L = \frac{f_\pi^2}{4} \text{Tr} \left( \xi_{K^0}^\dagger \frac{M M^\dagger}{2\mu} \xi_{K^0}^\dagger + \xi_{K^0}^\dagger \frac{M^\dagger M}{2\mu} \xi_{K^0} + 2g_s (A^c_{0})^T + 2e A_{0}^e Q \right)^2 \]  

Now let us proceed to state the main result of the paper. The charge densities are defined as
\[ J_0 = -\frac{\partial}{\partial A_0} \mathcal{L}, \]  

(17)

while the equations of motion for the temporal components of the gauge fields in the homogeneous case read

\[ \frac{\delta}{\delta A_0} \mathcal{L} \equiv \frac{\partial}{\partial A_0} \mathcal{L} = 0. \]  

(18)

Then color and electromagnetic charge densities vanish in the ground state since the time components of the (classical) gauge fields must satisfy their equations of motion. Note that vanishing of charge densities in the ground state is a gauge independent result and does not depend on the particular (gauge dependent) values of the gauge fields in the ground state.

From the Lagrangian (16) we see that the charge density due to the nonzero strange quark mass (the first two terms in (16)) induces non zero \( A_0 \) fields in the ground state. So, when substituted into the expression for the charge density, the Debye screening term (the last two terms in (16)) and the \( m_s \)-induced charge density term cancel each other and the corresponding charge density \( J_0 \) vanishes. This is just what we expect to see in a superconductor. Qualitatively, this is similar to the picture from [10]. There the linear in the chemical potentials charge density terms (analogous to the Debye screening terms in (16)) cancel the \( m_s \)-induced quark charge density pieces in the expressions for the charge densities thus ensuring charge neutrality.

Now let us solve equations of motion for the \( A_0 \)'s. The formulas for the gauge fields will be needed in the next section. Let us note first that lagrangian (5) (and therefore (16)) does not depend on the \( A_0^Q \). This is a manifestation of the fact that the medium is \( U_Q(1) \) neutral. The equation of motion for \( A_0^Q \) is \( \nabla^2 A_0^Q = 0 \) and the solution is

\[ A_0^Q = C, \]  

(19)

where \( C \) is a constant. Then solving the equations of motion \( \frac{\delta}{\delta A_0^Q} \mathcal{L} = 0 \), with \( \mathcal{L} \) from (16), yields for \( A_0 \)'s \( (A_0 \equiv \phi) \) in the original basis

\[
\begin{align*}
\phi_3^c &= \frac{\bar{e}^2 m_s^2}{4\mu g_s e^2} - \frac{C\bar{e}}{g_s}, \\
\phi_8^c &= \frac{\bar{e}^2 m_s^2}{4\sqrt{3}\mu g_s e^2} - \frac{C\bar{e}}{\sqrt{3}g_s}, \\
\phi_{em} &= \frac{\bar{e}^2 m_s^2}{3\mu g_s^2 e} + \frac{C\bar{e}}{e}.
\end{align*}
\]  

(20)

All other gauge fields vanish. In the pure quark matter \( A_0^Q \) may take an arbitrary value. However, when one takes into account electrons, the requirement of CFLK\(^0 \) electric charge neutrality gives \(^2\)

\(^2\)I am grateful to M.Alford and K.Rajagopal for pointing out this fact to me.
\[ A_0^Q \equiv C = -\frac{\tilde{m}_s^2}{3\mu g_s^2} \]  

so that \( \phi^{em} = 0 \). The remaining fields take on the following values

\[
\phi^c_3 = \frac{m_s^2}{4\mu g_s}, \\
\phi^c_8 = \frac{m_s^2}{4\sqrt{3}\mu g_s}.
\]

Finally, let us make an important observation that due to the constraint imposed by the gauge invariance on the form of operators that are allowed to appear in the low energy effective theory Lagrangian, the expressions in (20) are not corrected by the neglected higher order terms in the effective Lagrangian (1).

III. SPATIAL INHOMOGENEITY AS A MECHANISM FOR GENERATING ELECTRIC FIELDS IN HIGH DENSITY QCD

Our formalism also allows us to consider the case of inhomogeneous medium such as may be found in a neutron star core. We assume \( \mu \) to be an unspecified, spherically symmetric function \( \mu(r) \) with some typical size of inhomogeneity \( L \). We use \( L = 1 \) km for the numerical estimates, which is the typical length scale of a neutron star core. The characteristic length scale of the ground state is the screening Debye length \( \lambda_D \sim (g_s\mu)^{-1} \sim 1 \text{ fm} \). Since \( \lambda_D/L \sim 10^{-18} \), the system is only very weakly inhomogeneous. We will analyze the system in an expansion in \( \epsilon = \lambda_D/L \). For now we will neglect the electrons and consider only the quark matter.

Let us state the main result right away. To leading order in \( \epsilon \), the equations of motion for the gauge fields are \( \frac{\partial}{\partial A_0} \mathcal{L} = 0 \), with \( \mathcal{L} \) given by (16), but with \( \mu \) replaced by \( \mu(r) \). The solutions are given by (20) with \( \mu \) replaced by \( \mu(r) \). Then

\[
\phi \sim m_s^2/\mu(r) + \text{const} + \mathcal{O}(\epsilon^2).
\]

We immediately notice that, for example,

\[ E_r \equiv -\partial_r \phi^{em} \sim \frac{m_s^2}{\mu L} \neq 0. \]

So, we make a surprising observation that the spatial inhomogeneity gives rise to non zero electric fields in the bulk of quark matter.

It is useful to consider formation of such weakly inhomogeneous state, that is a situation where \( \mu = \mu(t,r) \) evolves as schematically shown in Fig. 2. Starting from a homogeneous density profile at \( t = 0 \), the system grows denser in the central region as time elapses. We set the typical time evolution scale \( T \) to be much bigger than any relevant QCD time scale, the typical size of the inhomogeneity \( L \) is much greater than \( \lambda_D \) as discussed above.

Let us consider, for example, the equations of motion for \( A^m_\mu \). We have
FIG. 2. Starting from a homogeneous profile at \( t_1 = 0 \) density increases in the central region while remaining constant far away from the center. By the time \( t_4 \gg T \) density profile approaches its stationary asymptotic form \( \mu(r) \) and \( j_r \) defined in (30) vanishes, but the charge density, (28), remains non zero.

\[
\partial_\mu F^{\mu\nu} = -\frac{\partial}{\partial A^{em}_\nu} \mathcal{L},
\]

where \( \mathcal{L} \) is given by Eq. (4). We seek solutions in the form

\[
\phi^{em} = \phi_0 \epsilon^0 + \phi_2 \epsilon^2 + \ldots,
\]

\[
\vec{A}^{em} = \vec{A}_0 \epsilon^0 + \vec{A}_2 \epsilon^2 + \ldots
\]

(26)

To leading order in \( \epsilon \) the equations of motion for the gauge fields are \( \frac{\partial}{\partial A^{em}_\nu} \mathcal{L} = 0 \), with \( \mathcal{L} \) given by (16), but with \( \mu \) replaced by \( \mu(t, r) \). The temporal components of the fields are given by (20), and we find

\[
\phi_0 = \frac{m^2 s \tilde{e}^2}{3 e g_s^2 \mu(t, r)} + \frac{C \tilde{e}}{e},
\]

\[
\vec{A}_0 = 0
\]

(27)

where constant \( C \) is defined in (19). At order \( \epsilon^2 \) we find the following expressions for the electromagnetic current components

\[
j^0_{em} = -\nabla^2 \phi_0
\]

(28)

and

\[
j_{em} = \partial_t \nabla \phi_0.
\]

(29)

Note that \( \partial_\mu j^\mu_{em} = 0 \), so that the electric charge conservation is satisfied. Thus, there exists a current with nonzero radial component

\[
j_r(t, r) = \partial_t \partial_r \frac{m^2 s \tilde{e}^2}{3 e g_s^2 \mu(t, r)} \sim \frac{m^2_s}{\mu T L} \geq 0.
\]

(30)
Now we see that formation of an inhomogeneous density state is accompanied by the polarization of the system; the positive charges move outward and the negative ones - inward. Let us emphasize that during this process the system remains charge neutral. As the density profile reaches its stationary form, \( j_r(t, r) \) vanishes but the charge density, (28), remains non zero.

Let us consider a stationary inhomogeneous case \( \mu = \mu(r) \) such as the asymptotic density profile \( \mu(r, t_4) \) in Fig. 2. So, we consider a spherically symmetric inhomogeneous region of size \( R > L \) embedded into infinite homogeneous CFL\(^0\) medium.

As we have mentioned earlier, a spatial inhomogeneity of the density generates electric fields in the quark matter. Then it is possible that the light charged leptons, \( i.e. \) positrons and electrons, may be attracted into the system to neutralize the \( U_{em}(1) \) electric field created by the quark matter. To address this question we add

\[
\mathcal{L}_e = \bar{\psi}(i\gamma^\nu \partial_\nu - m_e)\psi - e\bar{\psi}\gamma^\nu \psi A^em_\nu
\]

(31)

to the Lagrangian (4), where \( \psi \) is the usual electron-positron Dirac field and \( m_e \) is the electron/positron mass. Now, using equations of motion we integrate out all the remaining heavy gauge fields. We are left with the low energy effective Lagrangian

\[
\mathcal{L} = \bar{\psi}(i\gamma^\nu \partial_\nu - m_e)\psi - \frac{1}{2}A^Q_0\nabla^2 A^Q_0 \\
- n_e \left( A^Q_0 \bar{c} + \frac{m^2_e \bar{c}^2}{3g^2 s_\mu(r)} \right) - \frac{2e^4 n^2_e}{3f^2 g^4 s^4},
\]

(32)

where \( n_e = \psi^\dagger \psi \) is the fermion density. We notice that fermions appear in the Lagrangian (32) as coupled to a gauge field with nonzero temporal component equal to

\[
A(r) = A^Q_0(r) + \frac{m^2_e \bar{c}^2}{3g^2 s_\mu(r)}.
\]

(33)

One may think of the fermions as being in the presence of an effective chemical potential \(-\bar{c}A\). Then we may assume that in the ground state fermions form a Fermi sphere with Fermi momentum \( k_F \) being a slow function of \( r \) (Thomas-Fermi approximation). In the uniform case the electric charge neutrality of CFL\(^0\) requires that no electrons should be present in the ground state meaning that

\[
A = A^Q_0 + \frac{m^2_e \bar{c}^2}{3g^2 s_\mu} = 0,
\]

(34)

which is consistent with (21).\(^3\)

We find the Fermi momentum from the equality of energy of a particle on the top of the Fermi sphere inside the system at a given point and a particle at rest far away from center at some \( r = R \), where the density is constant (uniform density region far away from the center in Fig. 2). The equation is

\[
3\text{Strictly speaking, the condition is } |\bar{c}A| < m_e. \text{ We neglect } m_e \text{ compared to } m^2_s/\mu \sim 50\text{MeV}.\)
\[
\sqrt{k_F(r)^2 + m_e^2 + \tilde{e}A(r)} = m_e + \tilde{e}A(R) = m_e. \tag{35}
\]

Then in Thomas-Fermi approximation, ignoring \(O(n_e^2)\) term in (32), we have

\[
n_e(r) = \frac{1}{3\pi^2} k_F(r)^3 = \frac{1}{3\pi^2} ((\tilde{e}A(r) - m_e)^2 - m_e^2)^3. \tag{36}
\]

Solving the equation of motion for \(\tilde{A}_0\)

\[
\tilde{\nabla}^2 \tilde{A}_0 = -\tilde{e}n_e(r) = -\frac{\tilde{e}}{3\pi^2}((\tilde{A}_0(r)\tilde{e} + \frac{m_s^2\tilde{e}^2}{3g_s^2\mu(r)} - m_e^2 (\tilde{e}^2) - m_e^2)^{3/2} \tag{37}
\]

by the expansion in \(\epsilon\) yields

\[
\tilde{A}_0 = -\frac{m_s^2\tilde{e}}{3g_s^2\mu(r)} + \mathcal{O}(\frac{m_s^{4/3}}{L^{4/3}\mu^{2/3}m_e}) \]

\[
n_e = \frac{\tilde{\nabla}^2 m_s^2}{3g_s^2\mu(r)} + \mathcal{O}(\frac{m_s^{4/3}}{L^{10/3}\mu^{2/3}m_e}). \tag{38}
\]

Since \(\tilde{\nabla}^2(1/\mu(r)) \sim 1/L^2\mu(r)\), we have \(n_e \sim m_s^2/L^2\mu \sim 10^2\text{cm}^{-3}\) and \(k_{Fe^+} \sim 10^{-7}\text{MeV}\) for \(\mu = 400\text{MeV}\) and \(m_s = 150\text{MeV}\). An \(a\ posteriori\) estimate gives \(\sim m_e m_s^2/L^2\mu\) for the size of the neglected \(n_e^2\) term, which is subleading in \(\lambda_D/L\) compared to the size of the \(n_e\tilde{e}A(r)\) term in (32) which we retained.

From (38) we can see that for the case of our toy model density profile the total electric charge enclosed in the inhomogeneous region vanishes by Gauss’ law and we don’t have to worry about the energy contribution from the long range electric fields.

We estimate mean free path \(l\) of \(\tilde{Q}\) photons propagating in such a medium. For the case of low energy incoherent scattering of photons off of \(e^+\) we get \(l = 1/n_e\sigma \sim m_s^2/\alpha_em_e \sim 10^{19}\text{cm}\) which is much larger than the radius of a star. So, the medium is still transparent. Let us stress, however, that taking into account the spatial inhomogeneity results in a qualitative change in the ground state. Namely, light \(\tilde{Q}\) charged particles (positrons and/or electrons) are present in the ground state, while the homogeneous \(\text{CFL}^0\) phase is a perfect \(\tilde{Q}\) insulator.

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**Postscript**

After this work had been completed and the paper was being prepared for submission I became aware of Ref. [19], where a similar conclusion about the charge neutrality of the color superconducting quark matter was reached.
REFERENCES

[1] M.G. Alford, K. Rajagopal, and F. Wilczek, Nucl. Phys. B537, (1999) 443
[2] T. Schaefer, Nucl. Phys. B575, (2000) 269; N. Evans, J. Hormuzdiar, S.D.H. Hsu and M. Schwetz, Nucl. Phys. B581, (2000) 391
[3] K. Rajagopal and F. Wilczek, hep-ph/0011333; M.G. Alford, Ann. Rev. Nucl. Part. Sci. 51, (2001) 131
[4] S. Elitzur, Phys. Rev. D 12, (1975) 3978
[5] T. Schaefer, Phys. Rev. Lett. 85, (2000) 5531
[6] P.F. Bedaque and T. Schaefer, Nucl. Phys. A697, (2002) 802
[7] D. Kaplan and S. Reddy, Phys. Rev. D 65, (2002) 054042
[8] T. Schaefer and E.V. Shuryak, Lect. Notes Phys. 578, (2001) 203; T. Schaefer, nucl-th/0201031; T. Schaefer, hep-ph/0304281
[9] Paolo Amore, Michael C. Birse, Judith A. McGovern, Niels R. Walet, Phys. Rev. D 65, (2002) 074005
[10] M. Alford and K. Rajagopal, JHEP 0206, (2002) 031
[11] Andrew W. Steiner, Sanjay Reddy, and Madappa Prakash, Phys. Rev. D 66, (2002) 094007
[12] R. Casalbuoni and R. Gatto, Phys. Lett. B464, (1999) 111
[13] D.T. Son and M. A. Stephanov, Phys. Rev. D 61, (2000) 074012 erratum, ibid. Phys. Rev. D 62, (2000) 059902
[14] S. Weinberg, Prog. Theor. Phys. Suppl. 86, (1986) 43
[15] D.K. Hong, Phys. Lett. B473, (2000) 118
[16] S.R. Beane, P. F. Bedaque, and Martin J. Savage, Phys. Lett. B483, (2000) 131
[17] R. Casalbuoni, R. Gatto, and G. Nardulli, Phys. Lett. B498, (2001) 179 erratum, ibid. Phys. Lett. B517, (2001) 483
[18] G. Nardulli, Riv. Nuovo Cim. 25N3, (2002) 1
[19] A. Gerhold, A. Rebhan, hep-ph/0305108