Non-linear Bragg trap interferometer

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We propose a scheme for trapped atom interferometry using an interacting Bose-Einstein condensate. The condensate is controlled and spatially split in two confined external momentum modes through a series of Bragg pulses. The proposed scheme allows the generation of large entanglement in a trapped-interferometer configuration via one-axis twisting dynamic induced by interatomic interaction, and it avoids the suppression of interactions during the interferometer sequence by a careful manipulation of the state before and after phase encoding. The interferometer can be used for the measurement of gravity with a sensitivity beyond the standard quantum limit.

Introduction. Matter-wave atom interferometers are ideal tools for inertial measurements [1, 2]: they enable tests of fundamental theories [3–8], as well as practical applications such as gravimeters [9–13], gradiometers [14–16] and gyroscopes [17–22]. In the wider context of Grand Unification theory [23], dual-species matter-wave atom interferometers have been proposed to test in a unique way the weak equivalence principle [24–28] where gravity can be tested within a quantum framework working with state-of-the-art classical technologies [29–31].

The use of entangled probe states [32–34] has been proposed as a viable method to increase the sensitivity of atom interferometers beyond the shot noise limit imposed by uncorrelated-atoms probes [35–43]. However, so far, sub-shot noise sensitivities have been mainly studied in proof-of-principle experiments [35] that might not be compatible with the strict experimental conditions imposed by the specific application [44]. For instance, gravimeters require the creation of entangled atoms in controllable and separable momentum modes. To generate such states, recent proposals exploited the use of high-finesse optical cavities [45–47] or particle-particle interaction in Bose-Einstein condensates (BECs) where entanglement into internal levels is converted to external degrees of freedom via Raman addressing [36, 48].

In this manuscript, we propose a trapped atom interferometer for the measurement of inertial forces and gravity with a sensitivity beyond the standard quantum limit. The interferometer uses a trapped interacting BEC with beam-splitters implemented by Bragg pulses [49–51], see Fig. 1. Particle entanglement is generated in trapped momentum modes via elastic atom-atom interaction, which is kept during the interferometer sequence. We show that sub-shot-noise sensitivities can be reached, in our scheme [also referred to as non-linear atom interferometer (NLAI)], thanks to a careful rotation of the state before and after the interferometer sequence that accounts for the growth of phase fluctuations generated by interatomic collisions. This avoids the exploitation of a Feshbach resonance to suppress the scattering length between BEC atoms [37] during the interferometer operations, which may introduce substantial systematic effects [52, 53]. It thus paves the way toward practical applications of ultra-sensitive trapped-atom interferometry.

Interferometer scheme. The atom interferometer discussed in this manuscript is shown schematically in Fig. 1. It starts with a BEC of N atoms initially at rest in the bottom of a harmonic dipole potential. The trap is kept on during the full interferometer process, until the final readout. At t = 0, a Bragg pulse coherently splits the BEC in two momentum state, ±k0, where the effective wave vectors correspond to the two-photon transition k0 = 2kL. Each particle in the BEC is in a quantum superposition of momenta ±k0, such that the state after the Bragg pulse is described by the coherent spin state |ψin⟩ = 2−S/2 S n=0 (2S n)1/2 |S, S − n⟩, with S = N/2 and the state |S, S − n⟩ indicating S ± n atoms with momentum ±nhk0, respectively. Notice that we neglect the possible extra momentum mode generated at each laser pulses [51, 54]. To this aim, different configuration can be considered such as double-Bragg pulses [50] or the combination of optical lattice and single-Bragg pulses [49, 51]. Here we assume an infinitely narrow momentum distribution of the input state and justify the use of BEC instead of thermal ensemble [55].

The system is described by the field operator ˆΨ(r, t) = Φ+(r, t) ˆa+ + Φ−(r, t) ˆa+, where Φ+(r, t) carries the spatial evolution of the two wave-function and ˆa± (a† ±) is the bosonic annihilation (creation) operator of the ± mode. It is convenient to introduce the SU(2) pseudo-spin operators of the Lie’s algebra [56], ˆSx = (a† 0a0 + a† 0a0)/2, ˆSy = (a† 0a0 − a† 0a0)/2i and ˆSz = (a† 0a0 − a† 0a0)/2, satisfying the commutation relation [ ˆSx, ˆSy ] = iεijk ˆSj with εijk the Levi-Civita symbol. Bragg pulses, considered instantaneous at particular time tpp, are characterised by an effective Rabi frequency ΩR and phase φL and are described by the linear Hamiltonian ˆH0(t) = hΩRδ(tpp)[cos(φL) ˆSx + sin(φL) ˆSz] + hδ ˆθ, where δ(tpp) is the Dirac delta-function at time tpp and ˆθ is the precession of the state due to the phase accumulation. Particle-particle interaction is described by ˆHint(t) = hχ(t) ˆS2, where the time dependence in the coefficient χ(t) is associated to the dynamics of the wave function, see supplementary information [57].

During the state preparation, no phase is accumulated.
where \( m_T \) is the mass of the atom, \( \omega_{x,y,z} \) the trap frequencies, \( \hbar \) is the Plank constant and \( \gamma = \omega_{x,y}/\omega_z \) is the trap aspect ratio. In practice a fine tuning of \( \tau_m \) can be obtained by tuning the trap aspect ratio \( \gamma = \omega_{x,y}/\omega_z \) and frequency \( \omega_z \). Furthermore, \( \tau_m^S \) linearly increases with the number \( m \) of back-and-forth oscillations of the two spatial modes in the trap, see Fig. 1. In Fig. 2 we compare \( \tau_m \) with the approximated \( \tau_m^S \) for the case \( m = 1/2 \), as a function of the trap frequency, panel (a), and trap aspect ratio (b) [59]. The entangling evolution (1) can generate a substantial amount of spin squeezing in the state \( |\psi_s(mT)\rangle \), which can be quantified by the Wineland parameter \( \xi^s = N(\Delta \hat{S}_z)^2/(\langle \hat{S}_z \rangle^2 + \langle \hat{S}_y \rangle^2) \) [60]. In particular, the horizontal dot-dashed lines in Fig. 2 denotes \( \tau_{opt} \approx 1.2 N^{2/3} \) leading to the minimum value \( \min[\xi(\tau_{opt})] = N^{-1/3} \) [32]. It is important to compare our scheme with that of Ref. [36], where entanglement has been generated between a two-component BEC (different internal state of \(^{87}\)Rb addressed by Raman transitions, see also [48]) thanks to a state-dependent potential. Here, entanglement is generated between two different momentum states of a single component BEC (same internal state) where the overlap of the two modes does not inhibit the generation of squeezing.

The interferometer operations consists of two Bragg pulses at time \( mT \) and \( mT + T/2 \), described by \( R_s(\pi/2) = e^{-i\pi/2S_z} \) and \( R_t(\pi/2) \), respectively. In between, the angular trap frequency is changed from \( \omega_z \) to \( \tilde{\omega}_z \), with \( T = 2\pi/\tilde{\omega}_z \) indicating the period of the new trap, see Fig. 1. In this configuration the phase accumulated after half a period is \( \theta = 2k_0g(1/\tilde{\omega}_z^2 - \omega_z^2) \).
used to limit the impact of the sequence. In the following we denote by \( \hat{\alpha} \) implemented experimentally. The case \( \hat{\alpha} = 0 \) always optimized to maximize the sensitivity gain while \( \hat{\alpha} \neq 0 \) denotes the accumulated extra non-linear coefficient during the interferometry sequence.

The nonlinear parameters \( \tau_m \) (for state preparation) and \( \hat{\tau} \) (for the interferometer sequence) can be tuned independently from each other. Notice that the current trap for an extra time \( \tilde{\tau} \). Notice that the current state of the full interferometer sequence is described by the transformation

\[
|\psi_f\rangle = e^{-i\hat{S}_z} e^{-i(S_2^x + \theta S_3^z)} e^{-i\alpha \hat{S}_i} |\psi_e(mT)\rangle
\]

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Figure 3 shows the sensitivity gain optimized with respect to $\alpha$ and $\beta$ for different trap aspect ratio (panel a) and different trap frequency (panel b) in the case where the input spin squeezed state is generated by $m = 1/2$ (red) or $m = 1$ (blue) back-and-forth oscillations. In both cases, increasing the number of back-and-forth oscillations benefitted to the maximum sensitivity gain where the non-linear terms are controlled though the different traps. Indeed, even so $\tau_{1/2}$ is small, after $m$ back-and-forth oscillations $\tau_m = 2m\tau_{1/2}$ while $\tilde{\tau} = \tau_{1/2}$. The oscillations of the maximum sensitivity gain, highlighted in panel b, are a direct consequence of the trade-off discussed above. In the case of high trap frequencies and “over-squeezed” input spin squeezed state, $\tau > \tau_{opt}$, the non-linear term, $\tilde{\tau}$, can “un-squeeze” the state explaining the sudden increase of the optimized sensitivity gain at high trap frequency observed in panel b.

Discussion and Conclusions. Above, we have assumed a perfectly harmonic trap configuration. Indeed non-harmonic traps (magnetic or optic) do not prohibit the two modes to overlap classically but could induce a non-identical shape deformation of each mode limiting therefore the efficiency of the different Bragg pulses. Nevertheless, one can expect that the different back-and-forth oscillation would impact the shape of each mode in a similar manner on average limiting the detrimental effect of a shape deformation. In addition, in the case of dipole trap configuration large harmonic traps can be accessible through the “painted potential” technique at the cost of laser power [63, 64].

We have also considered a constant number of atoms while, in practice, the shot-to-shot fluctuation in atom number between two consecutive runs cannot be avoided. Such fluctuations can be dramatics in regards of the full optimization of the interferometer sequence where the pre- and post-rotation of the state is directly linked to the number of atoms. Though this paper we have shown in the regime of weak-interactions ($N = 10^3$ atoms in a $2\pi\{20, 20, 100\}$ Hz trap), that the pre-rotation of the state can be avoided ($\alpha = 0$) to reach the best sensitivity gain, $G_{0,\beta_{opt}} = 3.5$. The strategy proposed is then consistent with the current technology development and feasibility in current lab experiments [36] where only the final beamsplitter has to be optimized to exhibit sub-shot noise sensitivity measurements.

The trapped atom interferometer proposed in this paper reaches sub-shot noise sensitivities without requiring the suppression of the particle-particle scattering length.
during phase encoding. The presence of non-linearity during the interferometer operations can be mitigated via optimal rotations of the state on the Bloch sphere. Our result are supported by analytical calculations in the regime of small interaction and numerically in the regime of strong interactions. Larger trap aspect ratios and/or weaker trap frequency accessible today in microgravity environment [65, 66] would benefit to the propose interferometer where arbitrary input spin-squeezed state could be made available but not impacted by a non-linear interferometer sequence.

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Time evolution of the Hamiltonian

As introduced in the main text, we consider the two mode field operator \( \hat{\Psi}(\mathbf{r}, t) = \Phi_+(\mathbf{r}, t) \hat{a}_+ + \Phi_-(\mathbf{r}, t) \hat{a}_- \), where \( \Phi_{\pm}(\mathbf{r}, t) \) carry the spatial evolution of the two wave-functions and \( \hat{a}_{\pm} (\hat{a}_{\pm}^\dagger) \) is the bosonic annihilation (creation) operator of the \( \pm \) mode and obey the usual bosonic commutation rules: \( [a_i, a_j^\dagger] = \delta_{ij} \) and \( [a_i, a_j] = [a_i^\dagger, a_j^\dagger] = 0 \).

Linear Hamiltonian

The linear Hamiltonian explicitly read

\[
\hat{H}_0(t) = \int_{-\infty}^{\infty} dr \hat{\Psi}^\dagger(\mathbf{r}, t)[H_K + H_P + H_B] \hat{\Psi}(\mathbf{r}, t) \tag{6}
\]

where \( H_K \) is the kinetic Hamiltonian, \( H_P \) the total external potential Hamiltonian, \( H_B = M \omega_{x}^2 x^2 / 2 + M \omega_{y}^2 y^2 / 2 + M \omega_{z}^2 (z - z_s)^2 / 2 \), \( M \) being the mass of the atom, \( \omega_i \) the angular trap frequency in the direction \( i = \{x, y, z\} \) and \( z_s = -g/\omega_{z}^2 \) denotes the gravitational sag with \( g \) the gravity constant. Though the paper the effect of non-harmonic potential are neglected due the possibility to generate wide harmonic potential with painted potential at the expense of laser power [63, 64]. \( H_B \) denotes the Bragg potential Hamiltonian characterized though its effective Rabi frequency, \( \Omega_B \) and phase \( \phi_B \). General treatment of Bragg pulse can be found in [49, 51]. Introducing the SU(2) pseudo-spin operators of the Lie’s algebra [56]

\[
\begin{align*}
\hat{S}_x &= \left( \hat{a}_+^\dagger \hat{a}_- + \hat{a}_-^\dagger \hat{a}_+ \right) / 2, \\
\hat{S}_y &= \left( \hat{a}_+^\dagger \hat{a}_- - \hat{a}_-^\dagger \hat{a}_+ \right) / 2i, \\
\hat{S}_z &= \left( \hat{a}_+^\dagger \hat{a}_+ - \hat{a}_-^\dagger \hat{a}_- \right) / 2,
\end{align*}
\tag{7}
\]

satisfying the commutation relation \( [S_i, S_j] = i\epsilon_{ijk}S_k \) with \( \epsilon_{ijk} \) the Levi-Civita symbol, it is convenient to decompose \( \hat{H}_0(t) \) as

\[
\hat{H}_0(t) = \hbar \Omega_B \delta(t_p)[\cos(\phi_B)\hat{S}_x + \sin(\phi_B)\hat{S}_y] + \hbar \delta(t)\hat{S}_z, \tag{8}
\]

where the Bragg-pulse (first two terms) have been considered instantaneous at particular time \( t_p \) with \( \delta \) the Dirac delta-function. Depending on the choice of the laser phase, rotation of the state can be performed around \( \hat{S}_x (\phi_B = 0) \), \( \hat{S}_y (\phi_B = \pi / 2) \) or a combination of both by an angle defined by the strength of the laser pulse. A \( \pi / 2 \)-pulses at time \( t_p \) for instance is defined by \( \int dt \Omega_B \delta(t_p - t') = \pi / 2 \delta(t_p) \). Here the contribution of \( H_K + H_P \) to \( \hat{S}_x \) and \( \hat{S}_y \) have been neglected when the two modes overlap. The phase accumulated between two consecutive pulses at time \( t_1 \) and \( t_2 \) read [61, 62]

\[
\theta = \int_{t_1}^{t_2} dt \delta(t) = 2k_0(z(t_2) - z(t_1)), \tag{9}
\]

where \( z(t_1) \) and \( z(t_2) \) denote the overlap position of the two momentum state. In the case where the trap frequency is constant we find \( z(t_2) = z(t_1) \) and no phase due to gravity is accumulated.
Non-linear Hamiltonian

The non-linear Hamiltonian describes particle-particle elastic collision explicitly and read

\[ \hat{H}_{\text{int}}(t) = \frac{2\pi\hbar^2a}{M} \int_{-\infty}^{\infty} dr \left( \hat{\Psi}^\dagger(r, t) \right)^2 \left( \hat{\Psi}(r, t) \right)^2, \]  

(10)

Restricting to the case of two momentum modes, the main contribution to the non-linear Hamiltonian considered in this study read

\[ \hat{\Psi}^\dagger \hat{\Psi} \hat{\Psi} = \Phi_+^4 \hat{a}_+^\dagger \hat{a}_+^\dagger \hat{a}_+ \hat{a}_+ + \Phi_-^4 \hat{a}_-^\dagger \hat{a}_-^\dagger \hat{a}_- \hat{a}_- + 4\Phi_+^2 \Phi_-^2 \hat{a}_-^\dagger \hat{a}_+^\dagger \hat{a}_- \hat{a}_+ \]  

(11)

where we recognized the SPM modulation term (sum of the two first term) and the CPM modulation term (third term). Using the relation,

\[ N + 1 = N = \hat{a}_+^\dagger \hat{a}_+ + \hat{a}_-^\dagger \hat{a}_- \]  

(12)

with \( \hat{N} = \hat{a}_+^\dagger \hat{a}_+ + \hat{a}_-^\dagger \hat{a}_- \) we find:

\[ \hat{a}_+^\dagger \hat{a}_+^\dagger \hat{a}_+ \hat{a}_+ + \hat{a}_-^\dagger \hat{a}_-^\dagger \hat{a}_- \hat{a}_- = \frac{\hat{N}^2}{2} + \frac{1}{2} (\hat{a}_+^\dagger \hat{a}_+ - \hat{a}_-^\dagger \hat{a}_-)^2 - \hat{N} \propto 2S_z^2, \]  

(13a)

\[ 4 \hat{a}_+^\dagger \hat{a}_+ \hat{a}_+ + \hat{a}_-^\dagger \hat{a}_- = \frac{\hat{N}^2}{2} - (\hat{a}_+^\dagger \hat{a}_+ - \hat{a}_-^\dagger \hat{a}_-)^2 \propto -4S_z^2. \]  

(13b)

Using the relation \( \int_{-\infty}^{\infty} dr |\Phi_+(r, t)|^4 = \int_{-\infty}^{\infty} dr |\Phi_-(r, t)|^4 \) and Eq. (10) we can rewrite \( \hat{H}_{\text{int}}(t) \) as

\[ \hat{H}_{\text{int}}(t) = \hbar \chi_S(t) \hat{H}_S - 2 \hbar \chi_C(t) \hat{H}_C \equiv \hbar \chi(t) S_z^2, \]  

(14)

where the first term,

\[ \hat{H}_S = \frac{1}{2} (\hat{a}_+^\dagger \hat{a}_+) + \frac{1}{2} (\hat{a}_-^\dagger \hat{a}_-) \]  

(15)

is a self-phase modulation (SPM) that contains non-linearity that are local in each mode with

\[ \chi_S(t) = \frac{4\pi\hbar a}{M} \int_{-\infty}^{\infty} dr |\Phi_{\pm}(r, t)|^4. \]  

(16)

and the second term,

\[ \hat{H}_C = 2 \hat{a}_+^\dagger \hat{a}_+ \hat{a}_+ + \hat{a}_-^\dagger \hat{a}_-, \]  

(17)

is a cross-phase modulation (CPM) which describes correlation between the two mode with

\[ \chi_C(t) = \frac{4\pi\hbar a}{M} \int_{-\infty}^{\infty} dr |\Phi_+(r, t)|^2 |\Phi_-(r, t)|^2. \]  

(18)

Finally we find \( \chi(t) = \chi_S(t) - 2 \chi_C(t) \).

Non-linear coefficient

In the case of small interaction, one can describe the spatial mode though a variational calculation and a Gaussian trial function approach with re-scaled oscillator length. Within this approach the characteristic sizes of the Gaussian \( \sigma_i = b R_i \) where \( R_i \) is the Thomas-Fermi radii in the direction \( i \in \{x, y, z\} \), \( R_i^2 = 2\mu/m\omega_i^2 \) with \( \mu = \hbar\omega_0/2 \left( 15Na\sqrt{m\omega_0/h} \right)^{2/5} \) the chemical potential, \( \omega_0 = (\omega_x\omega_y\omega_z)^{1/3} \) the geometric trap frequency and \( b = (2/15\pi)^{1/10}/\sqrt{2} \approx 1/\sqrt{7} [67] \). The 3D Gaussian density distribution normalized to 1,

\[ |\Phi_{\pm}^G(r, t)|^2 = \frac{1}{(2\pi)^{3/2}\sigma_x\sigma_y\sigma_z} \exp \left[ -\frac{x^2}{2\sigma_x^2} - \frac{y^2}{2\sigma_y^2} - \left( z - z_0 \right)^2 + \frac{(z + z_0)^2}{2\sigma_z^2} \right], \]  

(19)
with $\pm z_0$ the position of the $\pm$ momentum mode. After a direct integration we find

$$\chi_{\text{max}} = \frac{g}{\hbar} \int_{-\infty}^{\infty} dr \left| \Phi_{\pm}^G(r, t) \right|^4 = \frac{1}{(4\pi \varrho^2)^{3/2}} \frac{g}{\hbar R_x R_y R_z}. \quad (20)$$

In the case of high atom number the density distribution of the spatial mode can be described by the 3D Thomas-Fermi distribution [67],

$$\left| \Phi_{\pm}^{TF}(r) \right|^2 = \frac{\mu}{N g} \left( 1 - \frac{x^2}{R_x^2} - \frac{y^2}{R_y^2} - \frac{(z \mp z_0)^2}{R_z^2} \right), \quad (21)$$

in the region where the right hand side is positive and zero otherwise. A direct integration leads to

$$\chi_{\text{max}} = \frac{g}{\hbar} \int_{-\infty}^{\infty} dr \left| \Phi_{\pm}^{TF}(r, t) \right|^4 = \frac{15}{14\pi} \frac{g}{\hbar R_x R_y R_z}. \quad (22)$$

One can notice that both approaches lead to the same scaling and differ only by 8%.

In the case of a variational calculation approach (Eq.19), the total non-linear coefficient, $\chi(t) = \chi_S(t) - 2\chi_C(t)$ at time $t$ read as,

$$\chi(t) = \chi_{\text{max}} \left( 1 - 2 \exp \left[ -\frac{z_0^2}{\sigma_z^2} \right] \right). \quad (23)$$

The accumulated non-linear coefficient responsible to the one-axis twisting dynamic at time $t'$ is given by

$$\tau_m = \int_0^{mT} \chi(t) \, dt. \quad (24)$$

For a total time of $t_{\text{tot}} = m\pi/\omega_z$, with $m$ the number of back and force oscillation of the two spatial mode in the trap, the maximum accumulated non-linear coefficient read

$$\tau_m^S = \frac{2 m \pi}{7} \left( 15 a \gamma^2 \sqrt{\frac{m}{\hbar}} \right)^{2/5} \left( \frac{\omega_z}{N^3} \right)^{1/5}. \quad (25)$$

with $\gamma$ the trap ratio: $\gamma = \omega_{x,y}/\omega_z$.

**Interferometer sequence**

For further clarity, we detail and list here the operations of the non-linear interferometer:

1. The coherent spin state is generated at $t = 0$ by a first $\pi/2$ Bragg pulse where the phase of the laser is fixed to $\phi_L = 0$ with an amplitude of $\int dt \Omega_R \delta(t) = \pi/2$. This step can be described by $\pi/2$ rotation around $\hat{S}_y$. 

![FIG. 5. Evolution of the non-linear coefficient with time (see Fig. 1).](image)
2. The back and forth oscillation of the two spatial mode between $t = 0$ and $t = mT$ generated entanglement through the “one-axis-twisting” dynamics.

3. The orientation of the spin-squeezed state is made possible by changing the laser phase to $\phi_L = 0$ with an amplitude of $\int dt \Omega_R \delta(t) = \alpha$. This step can be described by $\alpha$ rotation around $\hat{S}_x$.

4. The interferometer sequence consists then on two consecutive $\pi/2$-pulse at time $t = mT$ and $t = mT + \tilde{T}/2$ with $\phi_L = \pi/2$ and ($\pi/2$ rotation around $\hat{S}_x$).

5. A final pulse at $t = mT + \tilde{T}/2$ optimized the rotation of the output state as described step 3 with $\int dt \Omega_R \delta(t) = \beta$.

Calculation of the gain of the interferometer.

Input state preparation and linear interferometry sequence.

We consider the initial coherent spin state (CSS)

$$|\psi_0\rangle = 2^{-S} \sum_{n=0}^{2S} \left( \frac{2S}{n} \right)^{1/2} |S, S - n\rangle,$$

(26)

where $S = N/2$ and we define $|\psi_e\rangle = \hat{U}|\psi_0\rangle$ with $\hat{U} = e^{-i\tau \hat{S}_z}$. The mean value of a generic operator $\hat{Q}$ at the input port of the interferometer read as

$$\langle \hat{Q} \rangle_\alpha = \langle \psi_e | \hat{Q}_\alpha |\psi_e\rangle$$

(27)

where $\hat{Q}_\alpha = R^\dagger_{z/2}(\alpha) \hat{Q} R_{x/2}(\alpha)$ and $R_{x}(\alpha) = e^{-i\alpha \hat{S}_x}$. We recall the relation,

$$R^\dagger_{z/2}(\alpha) S_{y/z} R_{x}(\alpha) = \cos(\alpha) S_{y/z} \mp \sin(\alpha) S_{z/y}.$$ 

(28)

In order to evaluate the gain at the input of the interferometer, i.e., the spin-squeezing parameter introduced the main text, we have to evaluate the pseudo-spin operators $\{\hat{S}_x, \hat{S}_y, \hat{S}_z\}$ in addition to $\{\hat{S}_x^2, \hat{S}_y^2, \hat{S}_z^2\}$ where the operators can be expressed with respect to the raising and lowering operators $S_{\pm}$: $S_x = (S_+ + S_-)/2$, $S_y = (S_+ - S_-)/2i$, $S_z = (S_+ S_- - S_- S_+)/2$, $\hat{S}_z^2 = (\pm \hat{S}_z^2 + \pm \hat{S}_z^2 + 2(S_+ S_- - 2S_z))/4$, $\hat{S}_z^2 = S^2 - \hat{S}_x^2 - S_y^2$. 

Using the relations $\hat{U}^\dagger S_+ \hat{U} = S_+ e^{i\tau (2S_x + 1)}$ and $\hat{S}_- = \begin{bmatrix} \hat{S}_+ \end{bmatrix}^\dagger$, it is convenient to calculate the following list:

\[
\langle \psi_e | \hat{S}_+^n | \psi_e \rangle \equiv \langle \psi_0 | \hat{S}_+^n e^{i2\tau (\hat{S}_x + \frac{1}{2})} | \psi_0 \rangle = S \cos (\tau) \cos (2S_x - 1)
\]  
\[
\langle \psi_e | \hat{S}_z^2 | \psi_e \rangle \equiv \langle \psi_0 | \hat{S}_+^2 e^{i4\tau (\hat{S}_x + 1)} | \psi_0 \rangle = S \left( S - \frac{1}{2} \right) \cos (2\tau) \cos (2S_x - 3)
\]  
\[
\langle \psi_e | \hat{S}_z^3 | \psi_e \rangle \equiv \langle \psi_0 | \hat{S}_+^3 e^{i6\tau (\hat{S}_x + \frac{1}{2})} | \psi_0 \rangle = S \left( S - \frac{1}{2} \right) (S - 1) \cos (3\tau) \cos (2S_x - 3)
\]  
\[
\langle \psi_e | \hat{S}_+ \hat{S}_- | \psi_e \rangle \equiv \langle \psi_0 | \hat{S}_+ \hat{S}_- e^{i2\tau (\hat{S}_x + \frac{1}{2})} | \psi_0 \rangle = iS \left( S - \frac{1}{2} \right) (S - 1) \cos (2\tau) \cos (2S_x - 3)
\]  
\[
\langle \psi_e | \hat{S}_-^2 \hat{S}_+ | \psi_e \rangle \equiv \langle \psi_0 | \hat{S}_-^2 \hat{S}_+ e^{i4\tau (\hat{S}_x + 1)} | \psi_0 \rangle = iS \left( S - \frac{1}{2} \right) (S - 1) \cos (2\tau) \cos (2S_x - 3)
\]  
\[
\langle \psi_e | \hat{S}_+^2 \hat{S}_-^2 | \psi_e \rangle \equiv \langle \psi_0 | \hat{S}_+^2 \hat{S}_-^2 e^{i2\tau (\hat{S}_x + 1)} | \psi_0 \rangle = \frac{S}{4} \cos (\tau) \cos (2S_x - 1) + \frac{S}{2} \left( S - \frac{1}{2} \right) \cos (\tau) \cos (2S_x - 1)
\]  
\[
\langle \psi_e | \hat{S}_+ \hat{S}_- | \psi_e \rangle \equiv \langle \psi_0 | \hat{S}_+ \hat{S}_- e^{i2\tau (\hat{S}_x + \frac{1}{2})} | \psi_0 \rangle = \frac{S}{2} \left( S - \frac{1}{2} \right)
\]  
\[
\langle \psi_e | \hat{S}_-^2 | \psi_e \rangle \equiv \langle \psi_0 | \hat{S}_-^2 | \psi_0 \rangle = 0
\]  
\[
\langle \psi_e | \hat{S}_z^2 | \psi_e \rangle \equiv \langle \psi_0 | \hat{S}_z^2 | \psi_0 \rangle = \frac{S}{2}
\]  

Using Eq. (28), the different operators are then given by [58]:

\[
\langle S_x \rangle_\alpha = S \cos (\tau) \cos (2S_x - 1),
\]  
\[
\langle S_y/z \rangle_\alpha = 0,
\]  
\[
\langle S_x^2 \rangle_\alpha = \frac{S}{2} \left[ 2S - (S - \frac{1}{2}) A \right]
\]  
\[
\langle S_y/z^2 \rangle_\alpha = \frac{S}{2} \left( 1 + \frac{2S - 1}{4} \right) \left[ A \pm \sqrt{A^2 + B^2} \cos (2\alpha + 2\delta) \right]
\]

with $A = 1 - \cos (2\tau) \cos (2S_x - 2)$, $B = 4 \sin (\tau) \cos (2S_x - 2)$ and $\delta = \arctan (B/A)/2$. The spin-squeezing parameter therefore read as:

\[
\xi^2 = \frac{4 + (2S - 1)(A - \sqrt{A^2 + B^2})}{4 \cos (\tau) \cos (2S_x - 2)},
\]

and lead in the case of a linear atom interferometer sequence to the gain $G_L = 1/\xi$.

**Non-linear interferometry sequence**

The linear error propagation

The sensitivity of the interferometer, $\Delta \theta$, is evaluated via the linear error propagation formula:

\[
(\Delta \theta)^2 = \frac{(\Delta S_x^2)_{\text{out}}}{(d(S_x^2)/d\theta)^2},
\]

where the state at the end of the interferometer sequence is given by:

\[
|\Psi_{\text{out}}\rangle = e^{i\beta / 2 \hat{S}_z} e^{-i \delta \hat{S}_z} e^{-i \eta \hat{S}_z^2} e^{-i \delta \hat{S}_z} e^{-i \tau / 2 \hat{S}_z} e^{-i \alpha \hat{S}_z} |\Psi_e\rangle \equiv e^{-i \beta \hat{S}_z} e^{-i \tau \hat{S}_z^2} e^{-i \delta \hat{S}_z} e^{-i \alpha \hat{S}_z} |\Psi_e\rangle.
\]
In this case, \( \langle \hat{S}_z \rangle_{\text{out}} = \langle \Psi_e | e^{i \alpha \hat{S}_z} e^{i \theta \hat{S}_z} e^{i \tau \hat{S}_z} e^{i \beta \hat{S}_z} \hat{S}_z e^{-i \beta \hat{S}_z} e^{-i \theta \hat{S}_z} e^{-i \alpha \hat{S}_z} | \Psi_e \rangle \) can be simplified to
\[
\langle \hat{S}_z \rangle_{\text{out}} = \langle \Psi_e | e^{i \alpha \hat{S}_z} e^{i \tau \hat{S}_z} \left( \cos(\theta) \hat{S}_z - \sin(\theta) \hat{S}_z \right) e^{-i \tau \hat{S}_z} e^{-i \alpha \hat{S}_z} | \Psi_e \rangle \cos(\beta) + \langle \Psi_e | e^{i \alpha \hat{S}_z} \hat{S}_y e^{-i \alpha \hat{S}_z} | \Psi_e \rangle \sin(\beta),
\]
where we have used the relations, \( e^{i \beta \hat{S}_z} \hat{S}_z e^{-i \beta \hat{S}_z} = \cos(\beta) \hat{S}_z - \sin(\beta) \hat{S}_y \) and \( e^{i \theta \hat{S}_z} \hat{S}_e^{-i \theta \hat{S}_z} = \cos(\theta) \hat{S}_z - \sin(\theta) \hat{S}_x \). The denominator of Eq. 32 read then as
\[
\frac{d}{d\theta} \langle \hat{S}_z \rangle_{\text{out}} = -\langle \Psi_e | e^{i \alpha \hat{S}_z} e^{i \tau \hat{S}_z} \left( \sin(\theta) \hat{S}_z + \cos(\theta) \hat{S}_x \right) e^{-i \tau \hat{S}_z} e^{-i \alpha \hat{S}_z} | \Psi_e \rangle \cos(\beta).
\]
In the particular case \( \theta = 0 \), we find
\[
\frac{d}{d\theta} \langle \hat{S}_z \rangle_{\text{out}} \bigg|_{\theta=0} = -\langle \Psi_e | e^{i \alpha \hat{S}_z} e^{i \tau \hat{S}_z} \hat{S}_y e^{-i \alpha \hat{S}_z} | \Psi_e \rangle \cos(\beta) \equiv -\cos(\beta) \langle \hat{S}_z \rangle_{\text{out}},
\]
where we have used the relation: \( e^{i \beta \hat{S}_z} \hat{S}_x e^{-i \beta \hat{S}_z} = \hat{S}_z \). Fm Eqs. 32 and 36 we define the sensitivity gain as \( \Delta \theta = 1/\sqrt{N G_{\alpha,\beta}} \), with
\[
(G_{\alpha,\beta})^2 = \frac{\cos^2(\beta) \langle \hat{S}_z \rangle_{\text{out}}}{N(\Delta \hat{S}_z)^2_{\text{out}}},
\]

**Calculation of the different part**

In this section we want to calculate the gain at the output port of the interferometer sequence in the case where \( \tau \neq 0 \). We want to evaluate the quantity \( \langle Q \rangle_{\alpha,\beta} = \langle \Psi_{\text{out}} | Q | \Psi_{\text{out}} \rangle_{\alpha,\beta} \). In the case where \( \theta \) and \( \tau \) are considered small we have
\[
\langle \hat{Q} \rangle_{\alpha,\beta} = \langle \Psi_e | \hat{Q}_{\alpha,\beta} | \psi_e \rangle i \theta \langle \psi_e | R_1^\dagger(\alpha) \hat{S}_y R_2(\alpha), \hat{Q}_{\alpha,\beta} | \psi_e \rangle + i \tau \langle \psi_e | R_1^\dagger(\alpha) \hat{S}_y R_2(\alpha), \hat{Q}_{\alpha,\beta} | \psi_e \rangle,
\]
with \( \hat{Q} \) being \( \{ \hat{S}_x, \hat{S}_y \} \) and \( \hat{Q}_{\alpha,\beta} = R_1^\dagger(\alpha + \beta) \hat{Q} R_2(\alpha + \beta) \). Using Eq. (28) we have to calculate
\[
\langle \hat{S}_z \rangle_{\alpha,\beta} = \langle \psi_e | \hat{S}_z | \psi_e \rangle + i \theta \langle \psi_e | \left[ \cos(\alpha) \hat{S}_y - \sin(\alpha) \hat{S}_z, \hat{S}_z \right] | \psi_e \rangle + i \tau \langle \psi_e | \left[ \cos^2(\alpha) \hat{S}_y^2 + \sin^2(\alpha) \hat{S}_z^2 - \frac{\sin(2\alpha)}{2} \right] (\hat{S}_y \hat{S}_z + \hat{S}_z \hat{S}_y), \hat{S}_z \rangle | \psi_e \rangle,
\]
and
\[
\langle \hat{S}_z^2 \rangle_{\alpha,\beta} = \langle \psi_e | \cos^2(\alpha + \beta) \hat{S}_y^2 + \sin^2(\alpha + \beta) \hat{S}_z^2 + \frac{\sin(2\alpha + 2\beta)}{2} (\hat{S}_y \hat{S}_z + \hat{S}_z \hat{S}_y) | \psi_e \rangle + \langle \psi_e | \left[ \cos(\alpha) \hat{S}_y - \sin(\alpha) \hat{S}_z, \cos^2(\alpha + \beta) \hat{S}_y^2 + \sin^2(\alpha + \beta) \hat{S}_z^2 + \frac{\sin(2\alpha + 2\beta)}{2} (\hat{S}_y \hat{S}_z + \hat{S}_z \hat{S}_y) \right] | \psi_e \rangle + \langle \psi_e | \left[ \cos^2(\alpha) \hat{S}_y^2 + \sin^2(\alpha) \hat{S}_z^2 - \frac{\sin(2\alpha)}{2} (\hat{S}_y \hat{S}_z + \hat{S}_z \hat{S}_y), \cos^2(\alpha + \beta) \hat{S}_y^2 + \sin^2(\alpha + \beta) \hat{S}_z^2 + \frac{\sin(2\alpha + 2\beta)}{2} (\hat{S}_y \hat{S}_z + \hat{S}_z \hat{S}_y) \right] | \psi_e \rangle.
\]
The commutators appearing in the above equation are calculated using
\[
\begin{align*}
[\hat{S}_y, \hat{S}_z] &= -i \hat{S}_z \\
[\hat{S}_z, \hat{S}_x] &= i \hat{S}_y \\
[\hat{S}_y^2, \hat{S}_x] &= \left( \hat{S}_z + \frac{1}{2} \right) \hat{S}_- - \hat{S}_+ \left( \hat{S}_z + \frac{1}{2} \right) \\
[\hat{S}_z^2, \hat{S}_x] &= \hat{S}_+ \left( \hat{S}_z + \frac{1}{2} \right) - \left( \hat{S}_z + \frac{1}{2} \right) \hat{S}_- \\
\hat{S}_y \hat{S}_z + \hat{S}_z \hat{S}_y &= i \left( \hat{S}_z + \frac{1}{2} \right) \hat{S}_- - i \hat{S}_+ \left( \hat{S}_z + \frac{1}{2} \right) \\
\hat{S}_y \hat{S}_z + \hat{S}_z \hat{S}_y &= \frac{1}{2i} \left( \hat{S}_+^2 + \hat{S}_-^2 + 4 \hat{S}_x^2 + 2 \hat{S}_z - 2 \hat{S}_+ \hat{S}_- \right)
\end{align*}
\]
and

\[
\begin{align*}
\{\hat{S}_y, \hat{S}_y^2\} &= i\hat{S}_+ \left(\hat{S}_z + \frac{1}{2}\right) + i \left(\hat{S}_z + \frac{1}{2}\right) \hat{S}_- \quad (42a) \\
\{\hat{S}_x, \hat{S}_y^2\} &= \frac{\hat{S}_z^2 - \hat{S}_z^2}{2} \quad (42b) \\
\{\hat{S}_+, \hat{S}_y, \hat{S}_z + \hat{S}_x\hat{S}_y\} &= \frac{\hat{S}_z^2 - \hat{S}_z^2}{2} \quad (42c) \\
\{\hat{S}_+, \hat{S}_y, \hat{S}_z + \hat{S}_x\hat{S}_y\} &= -i\hat{S}_+ \left(\hat{S}_z + \frac{1}{2}\right) - i \left(\hat{S}_z + \frac{1}{2}\right) \hat{S}_- \quad (42d) \\
\{\hat{S}_+, \hat{S}_z, \hat{S}_x\} &= \hat{S}_+^2 (\hat{S}_z + 1) - (\hat{S}_z + 1) \hat{S}_z^2 \quad (42e) \\
\{\hat{S}_y, \hat{S}_z + \hat{S}_x\hat{S}_y, \hat{S}_x^2\} &= \frac{1}{2i} \left(4\hat{S}_+^2 \hat{S}_z^2 + 4\hat{S}_+ \hat{S}_z + \hat{S}_+ + 4\hat{S}_z^2 \hat{S}_- + 4\hat{S}_z \hat{S}_- + \hat{S}_-\right) \quad (42f) \\
\{\hat{S}_y, \hat{S}_z + \hat{S}_x, \hat{S}_x, \hat{S}_y\} &= \frac{1}{2i} \left(\hat{S}_x^2 \hat{S}_- + \hat{S}_x \hat{S}_-^2 - 2(\hat{S}_+ \hat{S}_z + \hat{S}_z \hat{S}_-) - \hat{S}_x \hat{S}_-^2 - \hat{S}_- - \hat{S}_+ - \hat{S}_-\right) \quad (42g)
\end{align*}
\]

**Evaluation of the sensitivity gain in the perturbative regime**

For safe of simplicity it is convenient to introduce the notation \(\langle \hat{X}\rangle_{\alpha,\beta} = \langle \hat{X}^{(0)}\rangle_{\alpha,\beta} + \hat{\tau} \langle \hat{X}^{(1)}\rangle_{\alpha,\beta}\). Equation 37 becomes

\[
\begin{align*}
(G_{\alpha,\beta})^2 &= \frac{\cos^2(\beta)}{N} \frac{\left(\langle \hat{S}_x^{(0)}\rangle_{\alpha,\beta} + \hat{\tau} \langle \hat{S}_x^{(1)}\rangle_{\alpha,\beta}\right)^2}{\langle \hat{S}_x^{(2)}\rangle_{\alpha,\beta}, \langle \hat{S}_x^{(2)}\rangle_{\alpha,\beta}} \\
&= \cos^2(\beta) \frac{\left(\langle \hat{S}_x^{(0)}\rangle_{\alpha,\beta} + \hat{\tau} \langle \hat{S}_x^{(1)}\rangle_{\alpha,\beta}\right)^2}{N \langle \hat{S}_x^{(2)}\rangle_{\alpha,\beta}} \\
&= \cos^2(\beta) \left(1 + 2\hat{\tau} \langle \hat{S}_x^{(1)}\rangle_{\alpha,\beta} - \hat{\tau} \langle \hat{S}_x^{(2)}\rangle_{\alpha,\beta}, \langle \hat{S}_x^{(2)}\rangle_{\alpha,\beta}\right),
\end{align*}
\]

where \(\langle \hat{S}_x^{(0)}\rangle_{\alpha,\beta}, \langle \hat{S}_x^{(2)}\rangle_{\alpha,\beta} = 1/\xi^2\) is the spin-squeezing parameter introduced in [68]. After calculation we find,

\[
\begin{align*}
\langle \hat{S}_x^{(0)}\rangle_{\alpha,\beta} &= S \cos(\tau)^{2S-1}, \quad (44a) \\
\langle \hat{S}_x^{(1)}\rangle_{\alpha,\beta} &= S \left(S - \frac{1}{2}\right) \left[2 \cos(\tau)^{2S-2} \sin(\tau) \cos(2\alpha) + (1 - \cos(2\tau)^{2S-2}) \frac{\sin(2\alpha)}{2}\right], \quad (44b) \\
\langle \hat{S}_x^{(2)}\rangle_{\alpha,\beta} &= \frac{S}{2} \cos^2(\alpha + \beta) + S(2S - 1) \cos(\tau)^{2S-2} \sin(\tau) \frac{\sin(2\alpha + 2\beta)}{2} \quad (44c) \\
&+ S \left[1 + 2S - (2S - 1) \cos(2\tau)^{2S-2}\right] \frac{\sin(\alpha + \beta)^2}{4}, \\
\langle \hat{S}_x^{(1)}\rangle_{\alpha,\beta} &= -S \left(S - \frac{1}{2}\right) \left[2(2S - 1) \cos(2\tau)^{2S-3} \sin(2\tau) \cos(2\alpha + \beta) - (S - 1) \cos(3\tau)^{2S-3} \cos(\alpha) \sin(\alpha + \beta) \quad (44d) \\
&+ \cos(\tau)^{2S-3} \left[-2 \cos(\alpha + \beta)(\cos(\tau)^2 - 2(S - 1) \sin(\tau)^2) \sin(\alpha) + (S + \cos(2\tau) \cos(\alpha) \sin(\alpha + \beta))\right]\right].
\end{align*}
\]

To first order in \(\tau\) we have:

\[
\begin{align*}
\langle \hat{S}_x^{(0)}\rangle_{\alpha,\beta}^2 &= 1 - (2S - 1) \sin(2\alpha + 2\beta) \tau, \quad (45a) \\
\langle \hat{S}_x^{(1)}\rangle_{\alpha,\beta} &= (2S - 1) \cos(2\alpha) \tau, \quad (45b) \\
\langle \hat{S}_x^{(2)}\rangle_{\alpha,\beta} &= -(2S - 1) \left[\sin(2\beta) + \cos(\beta) \{2(2S - 3) \cos(2\alpha + \beta) + (2S - 1) \cos(2\alpha + 3\beta)\}\tau\right]. \quad (45c)
\end{align*}
\]

The sensitivity gain therefore read to first order in \(\tau\) and \(\hat{\tau}\) as

\[
(G_{\alpha,\beta})^2 = [1 + (2S - 1) \{\sin(2\beta) \hat{\tau} - \sin(2\alpha + 2\beta) \tau\}] \cos^2(\beta). \quad (46)
\]
In the case where $\alpha = 0$ we have:
\[
(G_{0,\beta})^2 = [1 + (2S - 1)\sin(2\beta)(\bar{\tau} - \tau)] \cos^2(\beta).
\]  
(47)

In the case where $\alpha = \pi/2$ we have:
\[
(G_{\pi/2,\beta})^2 = [1 + (2S - 1)\sin(2\beta)(\tau + \bar{\tau})] \cos^2(\beta).
\]  
(48)

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