Uplink NOMA in Large-Scale Systems: Coverage and Physical Layer Security

Gerardo Gomez, Francisco J. Martin-Vega, F. Javier Lopez-Martinez, Yuanwei Liu and Maged Elkashlan

Abstract

In this paper, the physical layer security of uplink non-orthogonal multiple access (NOMA) in large-scale networks is analyzed. An stochastic geometry approach is applied to derive new analytical expressions for the coverage probability and the effective secrecy throughput (EST) of the kth NOMA user, which may use a fixed or an adaptive transmission rate. We consider a protected zone around the legitimate terminals to establish an eavesdropper-exclusion area. We assume that the channel state information associated with eavesdroppers is not available at the base station. The impact of an imperfect successive interference cancellation is also taken into account in this work. Our analysis allows an easy selection of the wiretap code rates that maximizes the EST. In addition, our framework also allows an optimum selection of other system parameters like the transmit power or the eavesdropper-exclusion radius.

Index Terms

Effective secrecy throughput (EST), non-orthogonal multiple access (NOMA), physical layer security, stochastic geometry.

I. INTRODUCTION

Non-orthogonal multiple access (NOMA) has recently been introduced as a new feature intended to increase the spectrum efficiency in 5G networks [1][2]. This technique allows serving

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multiple users simultaneously using the same spectrum resources at the cost of increased intra-cell interferences [3]. NOMA may use the power domain jointly with interference cancellation techniques to separate signals, exploiting the path loss differences among users.

In uplink (UL) NOMA, a set of users transmits simultaneously their signals to their associated base station (BS). As a consequence, the received signal of a particular user suffers from intra-cluster interference, which is a function of the channel statistics of other users. In order to minimize such interference, the BS may apply successive interference cancellation (SIC) to decode signals. SIC technique requires that different message signals arrive to the receiver (BS) with a sufficient power difference so that SIC may be successfully applied. This is typically achieved in the downlink (DL) by means of different weigths at the transmitter. However, since the UL channel gains already provide sufficient distinctness between the received signals, such weights are not necessary. In fact, the conventional UL transmit power control intended to equalize the received signal powers of users is not recommended for UL NOMA transmissions since it may remove the channel distinctness [3].

SIC technique in UL NOMA works as follows. The BS first decodes the strongest signal by considering the signals from other users as noise. However, the user with the weakest signal enjoys zero intra-cluster interference since the BS has previously canceled interfering signals (considering ideal conditions). If we consider the possibility of a SIC failure, the error is propagated to all remaining messages. UL NOMA was firstly presented in [4], by considering the minimum mean squared error (MMSE)-based SIC decoding at the BS. A novel dynamic power allocation scheme for DL and UL NOMA is proposed in [5]. The outage performance and the achievable sum data rate for UL NOMA is theoretically analyzed in [6]. In [7], a framework to analyze multi-cell UL NOMA with stochastic geometry is presented. In [8], the optimum received UL power levels using a SIC detector is determined analytically for any number of transmitters.

The possibility of having a secure communication in NOMA-based scenarios is also a current hot topic. The presence of eavesdroppers is a classical problem in communication theory, ever since Wyner introduced the wiretap channel [9]. In the last years, the field of physical layer security over different scenarios has taken an important interest in the research community as a means to provide reliable secure communications, relaxing the complexity and complementing the performance of the required cryptographic technologies. For instance, [10] considers the secure transmission of information over an ergodic fading channel in the presence of an
eavesdropper. An extension of this work considering a multiple-input multiple-output (MIMO) wiretap channel is analyzed in [11]. In [12], an analysis is conducted on the probability of secrecy capacity for wireless communications over the Rician fading channels. The communication between two legitimate peers in the presence of an external eavesdropper in the context of free-space optical (FSO) communications is analyzed in [13]. In [14], a comprehensive survey on various multiple-antenna techniques in physical layer security is provided, with an emphasis on transmit beamforming designs for multiple-antenna nodes. An overview on the state-of-the-art works on physical layer security technologies that can provide secure communications in wireless systems is given in [15].

A. Motivation and Contributions

In the particular field of physical layer security with NOMA, a small number of contributions are available. An analysis of the optimal power allocation policy that maximizes the secrecy sum rate for a DL NOMA scenario is presented in [16]. The work in [17] analyzes the secrecy outage probability (SOP) in a single-cell DL NOMA scenario in which the eavesdroppers are not part of the cellular system. [18] extends previous work by proposing several mechanisms to enhance the SOP in a DL NOMA multi-antenna aided transmission. However, up to the authors knowledge, physical layer security in UL NOMA scenario has not been addressed yet.

In this work, we provide a characterization of the physical layer security of UL NOMA in large scale systems. In particular, we provide the following contributions:

1) We first provide a connection level analysis of the different system components using an stochastic geometry approach. In particular, we obtain new expressions for the coverage probability of legitimate users (LUs) and eavesdroppers. We consider a protected zone around the LUs to establish an eavesdropper exclusion area.

2) We analyze the effective secrecy throughput (EST) [19] for uplink NOMA as a performance metric that captures the two key features of wiretap channels (reliability and secrecy) for any number of legitimate users.

3) We analyze previous metrics under two different scenarios: fixed and adaptive transmission schemes from LUs. In the case of fixed transmission rate, the impact of assuming a perfect or imperfect SIC is studied. Our analysis allows determining the wiretap code rates that achieve the locally maximum EST for both scenarios.
B. Organization and Notation

The remainder of this paper is organized as follows. The system model under analysis is introduced in Section II. The analysis of the Signal-to-Interference plus Noise Ratio (SINR) distributions for both legitimate users and eavesdroppers is presented in Section III. In Section IV, analytical expressions for the EST under different scenarios are derived. Numerical results are shown and described in Section V. Finally, we draw conclusions in Section VI.

**Notation:** Throughout this paper, $E[\cdot]$ stands for the expectation operator and $\mathbb{P}$ for the probability measure. Random variables (RV) are represented with capital letters whereas lower case is reserved for deterministic values and parameters. If $X$ is a RV, $f_X(\cdot)$, $F_X(\cdot)$, $\bar{F}_X(\cdot)$ and $L_X(\cdot)$ represent its probability density function (pdf), cumulative distribution function (cdf), complementary cdf (ccdf) and Laplace transform of its pdf, respectively.

II. System Model

We focus on the UL communication scenario in which LUs are connected to a base station (BS) of radius $r_c$ and centered at the origin. We assume a single cell scenario, as considered in most previous studies related to NOMA [1, 5, 6, 8, 16–18, 20]. A number of eavesdroppers (EDs) are distributed along the whole plane, attempting to intercept the communication between LUs and BS. The spatial distribution of EDs is modeled using a homogeneous Poisson Point Process (PPP) uniformly distributed in $\mathbb{R}^2$, which is denoted by $\Phi_e$ and associated with a density $\lambda_e$. An eavesdropper-exclusion zone of radius $r_p$ (in which no eavesdroppers are allowed to roam) is introduced around the LUs for improving the secrecy performance, as it is also considered in [18] for the downlink. Fig. 1 shows the system model under analysis.

At each radio resource, the BS gives service to $N$ simultaneous LUs (using NOMA), whose positions are random inside the cell. We assume a random scheduling, i.e. the BS selects randomly the set of $N$ LUs to be scheduled in a given radio resource according to NOMA. The locations of the LUs that are scheduled in a single radio resource are assumed to be uniformly distributed in the cell. Hence, we consider that the resulting set of points (LUs) inside the disk $B(0, r_c)$ is a Binomial Point Process (BPP) $\Phi_B$ with $N$ points, as it is normally assumed in the literature [6, 20].

We assume that transmitters (LUs), receiver (BS) and eavesdroppers are equipped with a single antenna each. We also assume that LUs’ channels and EDs’ channels are subject to
independent quasi-static Rayleigh fading with equal block length. UL transmit power control is not recommended as justified in the introduction section, and hence, it is not used.

As stated in [3], the impact of the path-loss factor is generally more dominant than channel fading effects. Hence, for tractability reasons, we assume that ordering of the received signal powers can be approximately achieved by ordering the distances of the users to their serving BS. Let $R_k$ be the distance between the $k$th user and the BS, being $R_1 \leq R_k \leq R_N$. Power loss due to propagation is modeled using a standard path loss model with $\alpha > 2$, whereas a Rayleigh model is assumed for small-scale fading. Hence, the received signal power at a distance $R_k$ can be simply computed as $H_k R_k^{-\alpha}$, where $H_k$ is the fading coefficient that follows an exponential distribution with unitary mean.

We consider a scenario in which EDs are not a part of the cellular system (passive eavesdropping) and therefore, the channel state information (CSI) associated with EDs’ channel is not available at the base station. In addition, we address two different cases regarding the LUs transmission mode:

- **Fixed transmission rate**: LUs transmit their information towards their BS at a fixed rate. In this scenario, we find the optimum values for the wiretap code rates, taking into account the reliability outage probability that occurs when the selected fixed rate exceeds the instantaneous channel capacity.

- **Adaptive transmission rate**: the BS enforces an adaptive secure transmission from LUs
assuming a perfect channel estimation. In this scenario, we find the optimum value of the redundancy rate, $R_e$, that maximizes the secrecy performance.

III. ANALYSIS OF THE SINR DISTRIBUTIONS

First, we analyze the connection related statistics of this scenario using an stochastic geometry approach. We assume that the BS applies SIC to detect the UL transmission from the nearest user first, and afterwards, it continues decoding the information from other users up to user $N$. The received instantaneous SINR at the BS of the $k$th user can be written as:

$$\gamma_k = \frac{H_k R_k^{-\alpha}}{I + 1/\rho_b}$$

where $I = \sum_{j=k+1}^N H_j R_j^{-\alpha}$ represents the intra-cluster interference due to other NOMA users; $\rho_b$ represents the transmit signal-to-noise ratio (SNR) defined as $\rho_b = \frac{P_T}{\sigma^2_b}$, being $P_T$ transmit power at the user terminal and $\sigma^2_b$ the additive white Gaussian noise (AWGN) power received at the BS. Note that (1) represents the SINR associated with the decoding process of the message from user $k$ subject to the correct decoding process from previous NOMA users (from user 1 to $k-1$) so that their intra-cluster interference has been successfully canceled. Also note that the SINR expression for the last user is simplified to $\gamma_N = \rho_b H_N R_N^{-\alpha}$ since the intra-cluster interference has been completely canceled.

A. Distribution of the SINR of Legitimate Users

In this section we compute the coverage probability of the legitimate users, i.e. the complementary distribution function (ccdf) of their received SINR at the BS, which represents the probability for a user to have a SINR higher than a given threshold $t$.

**Lemma 1.** The ccdf of the SINR for the $k$th user, $p_k(t)$, is given by

$$F_{\gamma_k}(t) = \int_0^{r_c} e^{-tr_k^\alpha/\rho_b} \left( 2 \cdot 2F_1 \left[ 1, \frac{\alpha+2}{\alpha}, \frac{r_c^\alpha}{tr_k^\alpha} - r_k^{\alpha+2} \cdot 2F_1 \left[ 1, \frac{\alpha+2}{\alpha}, \frac{r_c^\alpha}{tr_k^\alpha} - \frac{1}{t} \right] \right] \right)^{N-k} \times \frac{2 \cdot \Gamma \left( k + \frac{1}{2} \right) \Gamma (N + 1)}{\Gamma (k) \Gamma (N + \frac{3}{2})} \cdot \beta \left( \frac{r_c^2}{r_k^2}; k + \frac{1}{2}, N - k + 1 \right) \, dr_k$$

where $2F_1(\cdot, \cdot, \cdot, \cdot)$ is the Gauss hypergeometric function defined in [21] (Ch. 15), $\Gamma(z) = \int_0^\infty t^{z-1}e^{-t}dt$ stands for the Euler Gamma function, $\beta(x; a, b)$ is the beta density function defined as $\beta(x; a, b) = (1/B(a, b))x^{a-1}(1 - x)^{b-1}$, being $B(a, b)$ the beta function, which is expressible.
in terms of Gamma functions as $B(a,b) = \Gamma(a)\Gamma(b)/\Gamma(a+b)$. Note that (2) just includes a finite integral, which can be also computed by the Gaussian-Chebyshev quadrature relationship [22].

**Proof.** See Appendix A.

**Corollary 1.** For the farthest user ($N$) the coverage probability is simplified to

$$
\bar{F}_{\gamma_N}(t) = \frac{2N}{\alpha r_c^{2N}} \left( \frac{t}{\rho_b} \right)^{-\frac{2N}{\alpha}} \left( \Gamma \left[ \frac{2N}{\alpha} \right] - \Gamma \left[ \frac{2N}{\alpha}, \frac{r_c^\alpha t}{\rho_b} \right] \right) 
$$

(3)

**Proof.** The farthest user ($N$) experiences no intra-cluster interference, so its coverage probability can be expressed as

$$
\bar{F}_{\gamma_N}(t) = \int_0^{r_c} e^{-\frac{tr_c^\alpha}{\rho_b} f_{R_N}(r_N)} dr_N 
$$

(4)

Solving the finite integral, the proof is complete.

**B. Distribution of the SNR of eavesdroppers**

We address the worst-case scenario of large-scale networks, in which eavesdroppers are assumed to have strong detection capabilities. Specifically, by applying multi-user detection techniques, the multi-user data stream received at the BS can be also distinguished by the eavesdroppers.

We consider the most detrimental eavesdropper, which is not necessarily the nearest one, but the one having the best channel to the LU that is transmitting towards the BS. Therefore, the instantaneous received SNR at the most detrimental eavesdropper (with respect with any LU) can be expressed as follows:

$$
\gamma_e = \max_{e \in \Phi_e} \left\{ \rho_e H_e R_e^{-\alpha} \right\} 
$$

(5)

where $\rho_e$ represents the transmit SNR defined as $\rho_e = \frac{P_e}{\sigma_e^2}$, being $P_T$ transmit power at the LU and $\sigma_e^2$ the AWGN power received at the eavesdropper.

**Lemma 2.** Assuming an eavesdropper-exclusion zone or radius $r_p$ around the LUs, the cdf of the SNR for the most detrimental eavesdropper can be computed as follows:

$$
F_{\gamma_e}(t) = \exp \left[ -\frac{2\pi \lambda e \Gamma \left[ \frac{2}{\alpha}, r_p^\alpha t/\rho_e \right]}{\alpha (t/\rho_e)^{2/\alpha}} \right] 
$$

(6)
Proof. Taking into account that EDs follow a PPP distribution, we can express the cdf of the SNR for the most detrimental eavesdropper as follows:

$$F_{\gamma_e}(t) = 1 - p_e(t) = E_{\Phi_e} \left\{ \prod_{e \in \Phi_e} F_{H_e}(tr_e^\alpha / \rho_e) \right\}$$

$$= \exp \left[ -\lambda_e \int_{R^2} (1 - F_{H_e}(tr_e^\alpha / \rho_e)) r_e dr_e \right]$$

$$= \exp \left[ -2\pi \lambda_e \int_{r_p}^\infty r_p e^{-tr_e^\alpha / \rho_e} dr_e \right] \quad (7)$$

where \((a)\) comes from the Probability Generating Functional (PGFL) [23]. Solving the last integral, the proof is complete. □

In the particular case of no eavesdropper-exclusion zone, \((6)\) is simplified to:

$$F_{\gamma_e}(t) \big|_{r_p=0} = \exp \left[ -\frac{2\pi \lambda_e \Gamma \left[ 2/\alpha \right]}{\alpha (t/\rho_e)^{2/\alpha} } \right] \quad (8)$$

IV. Secrecy Rate Metrics

Let \(R_s\) be the secrecy rate in a legitimate link, i.e. the rate of transmitted confidential information. This rate can be computed as:

$$R_s \triangleq R_b - R_e \geq 0 \quad (9)$$

where \(R_b\) represents the codeword rate from the LU to the BS, i.e. rate at which the codeword is transmitted, including the confidential message and redundancy; \(R_e\) quantifies the redundancy rate, i.e. rate associated with redundant information for providing physical layer security in the message transmission. Roughly, a larger \(R_e\) provides a higher secrecy level.

On the one hand, if we select a codeword rate such that \(R_b \leq C_b\) (being \(C_b\) the capacity of the legitimate channel), a reliability constraint is ensured. On the other hand, if the redundancy rate is above the capacity of the eavesdropper’s channel, i.e. \(R_e > C_e\), a secrecy constraint is achieved.

Depending on whether the CSI ofLU and ED links are available at the BS, such rates can be adapted to the channel or not. Most of previous works on physical layer security computes the secrecy capacity as \(C_s = C_b - C_e\) [24], although this definition implicitly requires that both \(C_b\) and \(C_e\) are available. In our scenario, this assumption is not realistic since EDs are not part of the cellular system. Subsequently, we do not use the typical information-theoretic formulation
related to the secrecy capacity but a recent formulation of a new metric, referred to as the effective secrecy throughput (EST) \[19\], which captures both the reliability constraint and the secrecy constraint as independent terms. The EST of a wiretap channel quantifies the average secrecy rate at which the messages are transmitted from the LUs to the BS without being leaked to the eavesdroppers, and can be defined as

\[
\Phi(R_b, R_e) = (R_b - R_e) \left[ 1 - O_r(R_b) \right] \left[ 1 - O_s(R_e) \right]
\]  

(10)

where the term \((R_b - R_e)\) represents the rate of transmitted confidential information, i.e. \(R_s\); and the term \([1 - O_r(R_b)] [1 - O_s(R_e)]\) quantifies the probability that the information is securely transmitted from the LUs to the BSs, being \([1 - O_r(R_b)]\) associated with the reliability constraint and \([1 - O_s(R_e)]\) associated with the secrecy constraint. We assume a normalized bandwidth \(W = 1\), and therefore, secrecy rate and capacity metrics are measured in bits/s.

A. Adaptive Transmission Rate

In this scenario, the BS enforces an adaptive transmission scheme from LUs in the UL.

**Theorem 1.** The EST for the NOMA \(k\)th user in case of adaptive transmission is given by

\[
\Phi_k(R_e) = \left( \frac{1}{\ln 2} \int_0^\infty \frac{F_{\gamma_k}(z)}{1 + z} \, dz - R_e \right) \left( 2^{R_e} - 1 \right) 
\]

(11)

where \(F_{\gamma_k}(\cdot)\) and \(F_{\gamma_e}(\cdot)\) were given in (2) and (6), respectively.

**Proof.** In case of adaptive transmission, \(R_b\) can be optimally chosen as \(R_b = C_b\), and hence, the reliability constraint can be always guaranteed, i.e. the reliability outage probability is zero: \(O_r(R_b) = 0\). Therefore, the EST for the NOMA \(k\)th user can be defined as

\[
\Phi_k(R_e) = (C_k - R_e) \left[ 1 - O_s(R_e) \right]
\]  

(12)

where the term \(C_k\) represents the ergodic capacity for the \(k\)th user. Note that, in the adaptive transmission scheme, \(R_e\) is adjusted within the constraint \(0 < R_e < C_k\). Assuming a normalized channel bandwidth \(W = 1\), the ergodic capacity for the \(k\)th user, \(C_k\), can be expressed as

\[
C_k = \int_0^\infty \log_2 (1 + \gamma) f_{\gamma_k}(\gamma) d\gamma.
\]

(13)

Using integration by parts with \(u = \log_2(1 + \gamma)\), \(dv = f_{\gamma_k}(\gamma)\) and \(v = -(1 - F_{\gamma_k}(\gamma))\), the ergodic capacity can be also expressed as

\[
C_k = \frac{1}{\ln 2} \int_0^\infty \frac{F_{\gamma_k}(z)}{1 + z} \, dz 
\]

(14)
The secrecy outage probability term in (12) can be computed as

\[ O_s(R_e) = P(R_e < C_e) = P(\gamma_e > 2^{R_e} - 1) = 1 - F_{\gamma_e}(2^{R_e} - 1). \]  
(15)

Substituting (15) and (14) into (12), the proof is complete.

**Remark 1** (Impact of eavesdroppers density, \( \lambda_e \)). In view of Theorem 1, it can be deduced that, for \( \lambda_e = 0 \), the term associated with the secrecy constraint, \( O_s(R_e) \), is null; hence, the EST is mainly determined by the capacity of the LU’s link. On the other hand, the EST tends to zero as \( \lambda_e \) grows since expression (11) always satisfies that \( \lim_{\lambda_e \to \infty} \Phi_k(R_e) = 0, \forall r_p \in [0, \infty) \); this is due to the fact that, although the average received SNR at the most detrimental eavesdropper is increased for higher \( \lambda_e \) values, the fading distribution introduces a non-null probability of having a higher instantaneous capacity for the eavesdropper than for the legitimate user.

**Remark 2** (Impact of eavesdropper-exclusion radius, \( r_p \)). In view of expression (11), it can be noted that the only term that depends on \( r_p \) is the cdf of the SNR of the worst eavesdropper, \( F_{\gamma_e}(2^{R_e} - 1) \); for \( r_p = 0 \), this term is simplified to (8), whereas for \( r_p \to \infty \), this term satisfies that \( F_{\gamma_e}(2^{R_e} - 1) |_{r_p \to \infty} = 1 \), that is, eavesdroppers do not have any impact on the EST performance.

**B. Fixed Transmission Rate**

In case the LUs use a fixed transmission rate, the reliability constraint cannot be always guaranteed, i.e. a reliability outage must be taken into account as

\[ O_r(R_b) = P(R_b > C_b) \]  
(16)

Therefore, an outage may occur whenever a message transmission is either unreliable or non-secure.

Regarding the reliability constraint term, \( O_r(R_b) \), we address in our analysis the impact of imperfect SIC and detection probability for NOMA. Note that the signals from the intra-cluster interfering users may or may not be decoded perfectly; therefore, SIC may or may not be performed in a perfect fashion. As a consequence, we distinguish two cases: perfect and imperfect SIC.
The reliability constraint term for user $k$ in the case of perfect SIC, named as $p^{(P)}_k$, is given by

$$p^{(P)}_k(R_b) = 1 - O_{r_k}(R_b) = 1 - P(R_b > C_k) = 1 - P(\gamma_k < 2^{R_b} - 1) = \bar{F}_{\gamma_k}(2^{R_b} - 1)$$ (17)

That is, the reliability constraint term represents the detection probability for user $k$, whose expression was obtained in [2].

Finally, the EST for user $k$ in case of perfect SIC can be expressed as

$$\Phi^{(P)}_k(R_b, R_e) = (R_b - R_e) \bar{F}_{\gamma_k} (2^{R_b} - 1) F_{\gamma_e} (2^{R_e} - 1)$$ (18)

However, in the case of imperfect SIC, the intra-cluster interference experienced by the $k$th user depends on whether the detection for the $k-1$ nearest users were successful or not, which complicates the model significantly. In this paper we assume the worst case of imperfect SIC, which considers that the decoding of the $k$th user is always unsuccessful whenever the decoding of his relative $k-1$ closest users is unsuccessful [7]. Therefore, the reliability constraint term for the worst-case detection probability of $k$th user is given by:

$$p^{(I)}_k(R_b) = \prod_{i=1}^{k} \bar{F}_{\gamma_i} (2^{R_b} - 1)$$ (19)

Finally, the EST for user $k$ in case of imperfect SIC can be expressed as

$$\Phi^{(I)}_k(R_b, R_e) = (R_b - R_e) \prod_{i=1}^{k} \bar{F}_{\gamma_i} (2^{R_b} - 1) F_{\gamma_e} (2^{R_e} - 1)$$ (20)

Note that the EST expression is the same for the first NOMA user independently of the SIC assumption, i.e. $\Phi^{(P)}_1 = \Phi^{(I)}_1$, since potential detection errors occur from the second user up to the $N$th user.

A summary of secrecy metric expressions for different scenarios in shown in Table I.

V. NUMERICAL RESULTS

In this section, analytical results are illustrated and validated with extensive Monte Carlo simulations in order to assess the physical layer security in UL NOMA. We conduct a thorough performance comparison between the adaptive and fixed rate transmission schemes in terms of EST. Main parameters are presented in Table I unless otherwise stated.
TABLE I
SUMMARY OF SECRECY METRIC EXPRESSIONS FOR DIFFERENT SCENARIOS

| Scenario                  | Reliability constraint, \([1 - O_r(R_b)]\) | Secrecy constraint, \([1 - O_s(R_e)]\) | Effective Secrecy Throughput (EST), \(\Phi_k\) |
|---------------------------|------------------------------------------|------------------------------------------|------------------------------------------|
| Adaptive rate             | 1                                        | \(F_{\gamma_k}(2R_e - 1)\)               | \(\Phi_k(R_e) = \left(\frac{1}{ln2} \int_0^\infty \frac{\bar{F}_{\gamma_k}(z)}{1+z} dz - R_e\right) F_{\gamma_k}(2R_e - 1)\) |
| Fixed rate with perfect SIC| \(\bar{F}_{\gamma_k}(2R_b - 1)\)          | \(F_{\gamma_e}(2R_e - 1)\)               | \(\Phi_k^{(P)}(R_b, R_e) = (R_b - R_e) \bar{F}_{\gamma_k}(2R_b - 1) F_{\gamma_e}(2R_e - 1)\) |
| Fixed rate with imperfect SIC | \(\prod_{i=1}^{k} F_{\gamma_i}(2R_b - 1)\) | \(F_{\gamma_e}(2R_e - 1)\)               | \(\Phi_k^{(I)}(R_b, R_e) = (R_b - R_e) \prod_{i=1}^{k} F_{\gamma_i}(2R_b - 1) F_{\gamma_e}(2R_e - 1)\) |

TABLE II
MAIN CONFIGURATION PARAMETERS

| Parameter      | Value |
|----------------|-------|
| \(r_c\) (m)   | 500   |
| \(\alpha\)    | 3.8   |
| \(\rho_b\) (dB)| 110   |
| \(\rho_c\) (dB)| 90    |
| \(\lambda_e\) (points/m²)| 1e-5  |

A. Fixed Transmission Rate

In the case of fixed transmission rate, the reliability constraint term (or equivalently, the detection probability) plays an important role in NOMA performance. Let us analyze first this contribution separately.

Fig. 2 shows the detection probability results for legitimate users with perfect SIC, \(p_k^{(P)}\), and imperfect SIC, \(p_k^{(I)}\). In this case, we have considered a high number of NOMA LUs \((N = 6)\) randomly positioned according to a BPP in order to evaluate the performance as \(k\) grows. In the case of perfect SIC, results show that detection probability is not a monotonically decreasing function with \(k\) (i.e. with the distance from the \(k\)th user to the BS); instead, farthest LUs are boosted since the intra-cluster interference term has been partially (or totally) canceled. Note that the best result is achieved for the farthest user, \(k = N = 6\), since perfect SIC assumes that intra-cluster interference is fully canceled. However, in the case of imperfect SIC, the intra-
cluster interference experienced by the $k$th user depends on whether the detection for $k - 1$ nearer users were successful or not, thus providing a monotonically decreasing function with $k$. Note also that higher values of $R_b$ leads to a lower detection probability.

Fig. 2. Detection probability for LUs with perfect and imperfect SIC for each user $k$ with $N = 6$.

Fig. 3 shows the detection probability results for legitimate users with imperfect SIC, $p_k^{(I)}$, and for eavesdroppers, $p_e$, versus the SINR threshold $t = 2^{R_b} - 1$. Results for $p_k^{(I)}$ are obtained from [19] considering $N = 4$ NOMA LUs. The detection probability of eavesdroppers, $p_e$, is also shown for different values of the exclusion area radius, $r_p$. Since we consider the most detrimental eavesdropper, i.e. the one receiving the best channel quality from the LU, results without exclusion area outperform the results of LUs for a density of eavesdroppers of $\lambda_e = 1$ (default value). This undesirable scenario can be compensated by increasing the exclusion area radius.

Fig. 4 shows the EST for fixed rate transmission scheme and perfect SIC versus $R_b$ and $R_e$, $\Phi_k^{(P)}(R_b, R_e)$. We observe that there is a unique pair of $R_b$ and $R_e$ that maximizes the EST. Also note that EST is null for $R_e \leq R_b$.

The value of $R_e$ that maximizes the EST, noted as $R_e^\dagger$, has been determined numerically and
Fig. 3. Detection probability for LUs with imperfect SIC, $p_k^{(I)}$, and for eavesdroppers, $p_e$, versus $t = 2^{R_b - 1}$, with $N = 4$.

shown in Fig. 5 as a function of $R_b$ and $\lambda_e$, with $N = 2$, $k = 1$ and $r_p = 50$ m. Note that the ratio between $R_b^\dagger_e$ and $R_b$ is not linear. We also observe that a higher density of eavesdroppers requires a higher redundancy rate to optimize the EST.

Fig. 6 shows a comparison between the EST for fixed rate transmission with perfect SIC, $\Phi_k^{(P)}$, and imperfect SIC, $\Phi_k^{(I)}$. EST results are shown for $N = 2$ NOMA users as a function of $R_b$, assuming a value of $R_e = 3$ bps and $r_p = 50$ m. We observe that the results for the first user ($k = 1$) are the same for perfect and imperfect SIC since imperfect SIC models the propagation of decoding errors from previous decoded users. We also observe that, in the case of perfect SIC, the maximum EST for the second user is not degraded significantly compared to the first user, as the larger distance to the BS is compensated by the fact that the second user does not experience (ideally) any intra-cluster interference. However, in the case of imperfect SIC, the second user is highly degraded compared to the first user due to SIC error propagation from the previous decoded user. Note also that the value of $R_b$ that maximizes the EST is different of each LU, so optimum code rate selection at the base station must be done per LU.
B. Adaptive Transmission Rate

In this section we provide performance results in case the BS uses the CSI of LUs to enforce an adaptive transmission scheme.

Fig. 7 shows EST results of the first user \((k = 1)\) as a function of the redundancy rate, \(R_e\), assuming \(N = 2\) NOMA users. In this case, \(R_b\) is adapted to the channel capacity, i.e. \(R_b = C_b\), whereas the value of \(R_e\) must be properly designed. In that sense, there is a value of \(R_e\) that maximizes the EST. We also observe that higher eavesdropper-exclusion radii enhance the EST. As mentioned before, in case of adaptive transmission, the reliability constraint does not affect the secrecy performance; therefore, no SIC errors are considered in this case. Note that if the difference between \(\rho_b\) and \(\rho_e\) is higher (due to the value of \(\sigma_e^2\) compared to \(\sigma_b^2\)), the EST is considerably increased.

The impact of the eavesdropper-exclusion radius on the EST is depicted in Fig. 8. We observe an increasing S-shape behavior as \(r_p\) grows, since the most detrimental eavesdropper reduces its detection capabilities for higher \(r_p\) values. Results match perfectly with Remark 2, which stated that for \(r_p \to \infty\), eavesdroppers do not have any impact on the performance.
EST results as a function of the density of eavesdroppers, $\lambda_e$, is shown in Fig. 9. We observe an exponential decreasing behavior with $\lambda_e$. As stated in Remark 1 when $\lambda_e$ tends to zero, the EST is mainly determined by the capacity of the LU’s link; on the contrary, when $\lambda_e$ tends to infinity, the EST is zero, although higher eavesdropper-exclusion radii lead to a slower EST degradation.

Fig. 10 shows the EST for adaptive transmission for the $k$th user as a function of the transmission power ($P_T$) of the LU measured in dBm/Hz. We observe an optimum value of $P_T$, which depends on the specific values of $\rho_e$ and $k$. We have considered an eavesdropper-exclusion radius of $r_p = 50$ m and an average noise power received at the BS of $\sigma_b^2 = -160$ dBm/Hz; note that the default value of $\rho_b = 110$ dB would give a value of $P_T = -50$ dBm/Hz, or equivalently, a $P_T = 23$ dBm for a bandwidth of 20 MHz, which is a typical power value for a microcell. Results show that very low $P_T$ values lead to a very poor performance since the average SINR of the LUs is very low (reliability constraint); on the other hand. When then transmit power is increased, there is a optimum value above which the EST starts decreasing, since the eavesdroppers are also increasing their detecting capabilities (secrecy constraint). Results also

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**Fig. 5.** Optimum value of $R_e$ that maximizes the EST as a function of $R_b$ and $\lambda_e$, considering fixed transmission with $N = 2$, $k = 1$ and $r_p = 50$ m.
show that higher values of $\rho_e = \frac{P_T}{\sigma_e^2}$ (considering a higher noise power at EDs than at the BS) degrades considerably the EST. We also observe that the performance of the first and second LUs differs significantly as $\rho_e$ is increased. We must recall that in the adaptive transmission, the last user is ideally free of intra-cluster interference, and hence, its performance is limited by noise. Therefore, the second user is much more affected by the value of $\rho_e$. In case of high noise power at eavesdroppers (low $\rho_e$) the second user is shown to outperform the first user despite being further from the BS.

Fig. 10 shows the EST of the first NOMA user ($k = 1$) versus the transmission power $P_t$ for adaptive transmission as a function of the eavesdropper density, $\lambda_e$. We observe that the optimum transmit power value is very affected by $\lambda_e$. In fact, lower eavesdropper densities lead to higher EST, although an adjustment of the transmit power is critical to achieve such maximum. For the limit case of no eavesdroppers ($\lambda_e = 0$) there is no EST degradation for high $P_T$ values, as the secrecy constraint is null.
VI. Conclusions

In this paper, we analyzed the performance of UL NOMA for a generic number of simultaneous users, both from a connection level perspective and from a physical layer security viewpoint. We considered a passive eavesdropping scenario in which the BS and LUs are not aware of their CSI, and different cases depending on whether the LUs use a fixed or an adaptive transmission scheme. Our analysis includes the impact of an imperfect SIC during NOMA detection and an eavesdropper-exclusion radius to enhance the secrecy metrics.

We obtained new analytical expressions for the coverage probability in the uplink for LUs and eavesdroppers. In addition, we provide simple analytical expressions for the EST, which captures explicitly the reliability constraint and secrecy constraint of wiretap channels. Our analysis allows determining the wiretap code rates that achieve the maximum EST. Performance results also help designing optimum values of the transmit power \((P_T)\) and the eavesdropper-exclusion radius \((r_p)\) in order to enhance the overall EST.
Fig. 8. EST of the $k$th user versus $r_p$ for adaptive transmission with $N = 2$ and $\rho_b = 110$ dB.

Fig. 9. EST of the $k$th user versus $\lambda_e$ for adaptive transmission with $N = 2$ and $R_e = 1$. 
Fig. 10. EST of the $k$th user versus the transmission power $P_t$ for adaptive transmission as a function of $\rho_e$, with $N = 2$, $R_e = 1$, $r_p = 50$ m, $\lambda_e = 10^{-5}$ points/m$^2$ and $\sigma_b^2 = -160$ dBm/Hz.

APPENDIX A

PROOF OF LEMMA 1

The ccdf of the SINR for the $k$th user, $p_k(t)$, can be expressed as

$$
\bar{F}_{\gamma_k}(t) = \mathbb{P} [\gamma_k > t] 
= \int_0^{r_c} \mathbb{P} [h_k > t| r_k] f_{R_k}(r_k) dr_k 
= \int_0^{r_c} \mathbb{P} [h_k > t(I + \rho_b^{-1}) r_k^\alpha | r_k] f_{R_k}(r_k) dr_k 
= \int_0^{r_c} \mathbb{E}_{\mathbb{I}} [\mathbb{P} [h_k > t(i + \rho_b^{-1}) r_k^\alpha | r_k, i]] f_{R_k}(r_k) dr_k 
= \int_0^{r_c} e^{-tr_k^\alpha/\rho_b} \mathbb{E}_{\mathbb{I}_{|r_k}} [e^{-tr_k^\alpha | r_k}] f_{R_k}(r_k) dr_k 
= \int_0^{r_c} e^{-tr_k^\alpha/\rho_b} \mathcal{L}_{\mathbb{I}_{|r_k}} (tr_k^\alpha) f_{R_k}(r_k) dr_k
$$

where (a) and (b) follow from the total probability theorem [25], while (c) follows from the fact that $H_k$ has an exponential distribution with mean 1.
Fig. 11. EST of the first NOMA user \((k = 1)\) versus the transmission power \(P_t\) for adaptive transmission as a function of \(\lambda_c\), with \(N = 2\), \(R_e = 1\), \(\rho_e = 90\) dB, \(r_p = 50\) m and \(\sigma_b^2 = -160\) dBm/Hz.

The term \(\mathcal{L}_{l|r_k}(s) = \mathbb{E}_{l|r_k} [e^{l|r_k}]\) represents the Laplace transform of the intra-cluster interference conditioned on \(r_k\), which can be expressed as

\[
\mathcal{L}_{l|r_k}(s) = \mathbb{E}_{r_j|r_k,h_j} \left[ \exp \left( -s \sum_{j=k+1}^{N} h_j r_j^{-\alpha} \right) \right]
\]

\[
= \mathbb{E}_{r_j|r_k,h_j} \prod_{j=k+1}^{N} \exp \left( -sh_j r_j^{-\alpha} \right)
\]

\[
(a) \prod_{j=k+1}^{N} \mathbb{E}_{r_j|r_k,h_j} \left[ \exp \left( -sh_j r_j^{-\alpha} \right) \right]
\]

\[
= \left( \mathbb{E}_{r_j|r_k} \left[ \frac{1}{1 + s r_j^{-\alpha}} \right] \right)^{N-k}
\]

\[
(b) \left( \int_{r_k}^{r_c} \frac{1}{1 + s r_j^{-\alpha}} \frac{2r_j}{r_c^2 - r_j^2} dr_j \right)^{N-k}
\]

\[
= \left( \frac{2 \left( r_c^\alpha + 2 \Omega (-r_c^\alpha/s) - r_k^\alpha + 2 \Omega (-r_k^\alpha/s) \right)}{s \left( r_c^2 - r_k^2 \right) (\alpha + 2)} \right)^{N-k}
\]

(22)
being \( \Omega (x) = 2F_1 \left[ 1, \frac{\alpha + 2}{\alpha}, 2 + \frac{2}{\alpha}, x \right] \). Step \((a)\) comes from the fact that the fading is independent of the BPP and, although \(j\)th users’ location are correlated with \(k\)th user when their distances are ordered, the computation of the interference can be obtained considering that the \(N - k\) NOMA interfering users are located within a disk whose inner radius is \(r_k\) and outer radius \(r_c\).

Step \((b)\) comes from the fact that the pdf of the distance from a randomly located point within that disk is given by \(f_{R_j|R_k}(r_j|r_k) = 2r_j/(r_c^2 - r_k^2)\).

In [26], the marginal pdf of the \(k\)th nearest point to the origin of a BPP is given. In particular, this work shows that, in a BPP consisting of \(N\) points randomly distributed in a 2-dimensional ball of radius \(r_c\) centered at the origin, the Euclidean distance \(R_k\) from the origin to its \(k\)th nearest point follows a generalized beta distribution

\[
f_{R_k}(r_k) = \frac{2}{r_c} \frac{\Gamma \left( k + \frac{1}{2} \right) \Gamma (N + 1)}{\Gamma (k) \Gamma (N + \frac{3}{2})} \beta \left( \frac{r_k^2}{r_c^2}; k + \frac{1}{2}, N - k + 1 \right)
\]  

(23)

Substituting (22) and (23) into (21) the proof is complete.

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