Dynamical Quark Effects on Light Quark Masses

A. Ali Khan\(^1\), S. Aoki\(^2\), G. Boyd\(^1\), R. Burkhalter\(^{1,2}\), S. Ejiri\(^1\), M. Fukugita\(^3\), S. Hashimoto\(^4\), N. Ishizuka\(^{1,2}\), Y. Iwasaki\(^{1,2}\), K. Kanaya\(^{1,2}\), T. Kaneko\(^1\), Y. Kuramashi\(^4\), T. Manke\(^1\), K. Nagai\(^1\), M. Okawa\(^3\), H.P. Shanahan\(^1\), A. Ukawa\(^{1,2}\) and T. Yoshi\(^{1,2}\)

(CP-PACS Collaboration)

\(^1\)Center for Computational Physics, University of Tsukuba, Tsukuba, Ibaraki 305-8577, Japan
\(^2\)Institute of Physics, University of Tsukuba, Tsukuba, Ibaraki 305-8571, Japan
\(^3\)Institute for Cosmic Ray Research, University of Tokyo, Tanashi, Tokyo 188-8502, Japan
\(^4\)High Energy Accelerator Research Organization (KEK), Tsukuba, Ibaraki 305-0801, Japan

(April 2000, revised July 2000)

We present results for light quark masses from a systematic lattice QCD study with two degenerate flavors of dynamical quarks. Simulations are made with a renormalization-group improved gauge action and a mean-field improved clover quark action for sea quark masses corresponding to \(m_{PS}/m_V \approx 0.8-0.6\) and the lattice spacing \(a = 0.22-0.11\) fm. In the continuum limit we find \(m_{PS}^{MS}(2\text{GeV}) = 3.44^{+0.14}_{-0.22}\) MeV using the \(\pi\) and \(\rho\) meson masses as physical input, and \(m_{PS}^{MS}(2\text{GeV}) = 88^{+4}_{-6}\) MeV or \(90^{+5}_{-7}\) MeV with the \(K\) or \(\phi\) meson mass as additional input. The quoted errors represent statistical and systematic combined, the latter including those from continuum and chiral extrapolations, and from renormalization factors. Compared to quenched results, two flavors of dynamical quarks reduce quark masses by about 25%.

PACS numbers: 12.38.Gc, 12.15.Ff, 14.65.Bt

Masses of light quarks belong to the most fundamental parameters of the Standard Model [1], and yet their precise values have been difficult to determine due to quark confinement. Lattice QCD provides a fundamental approach to overcome this problem [2] since it enables first principle calculations of hadron masses as functions of quark masses. This approach has progressed considerably recently [3–5] through high statistics calculations that have allowed the continuum limit to be taken for quark masses, and the development of non-perturbative renormalization techniques for a reliable conversion of lattice bare quark masses to those in the continuum.

These studies, however, have been carried out within the quenched approximation which ignores effects of sea quarks. A limitation, shown in Ref. [4], is that the quark part we choose the “clover” improvement of the plaquette and Wilson quark actions. We therefore aim at a continuum extrapolation needed for the standard plaquette and Wilson quark actions. We therefore aim at a continuum extrapolation from simulations made at \(a^{-1} \approx 1–2\text{ GeV}\).

We employ improved actions both for the gluon part and the quark part. The gluon action consists of \(1 \times 1\) and \(1 \times 2\) Wilson loops whose coefficients are determined by an approximate renormalization-group analysis [8]. For the quark part we choose the “clover” improvement of the Wilson action [9], adopting, for the clover coefficient, a mean-field value \(c_{SW} = P^{-3/4}\). We substitute the one-loop result \(P = 1 - 0.1402g^2\) [8] for the plaquette \(P\), which agrees within 8% with the values measured in our runs. The one-loop result of \(c_{SW}[10]\) is found to be close to our choice.

The improved action described here was tested in our preparatory full QCD study [11]. We found that scaling violation in hadron masses is small with this action already at \(a^{-1} \approx 1\text{ GeV}\), as compared to \(a^{-1} \geq 2\text{ GeV}\) needed for the standard plaquette and Wilson quark actions. We therefore aim at a continuum extrapolation from simulations made at \(a^{-1} \approx 1–2\text{ GeV}\).

We make runs at three values of the coupling \(\beta \leq 6/g^2 = 1.8, 1.95, 2.1\) to cover this range, employing lattices of a similar physical spatial size \(La \approx 2.5\) fm, as listed in Table I. For each \(\beta\), gauge configurations are generated by the hybrid Monte Carlo algorithm at four values of the sea quark hopping parameter \(\kappa\) corresponding to the pseudoscalar (PS) to vector (V) meson mass ratio of \(M_{PS}/M_V \approx 0.8, 0.75, 0.7, 0.6\). For each sea quark mass, we calculate hadron masses at five values of the valence quark hopping parameter \(\kappa_{val}\) corresponding to \(M_{PS}/M_V \approx 0.8, 0.75, 0.7, 0.6\) and 0.5, taking unequal as well as equal quark mass cases. Masses are extracted from hadron propagators with the standard correlated \(\chi^2\) fit. Errors are estimated with the jackknife procedure with a bin size of 50 trajectories, derived from an autocorrelation study.
For the Wilson-type quark action including the clover action, different definitions of quark masses lead to results that differ at finite lattice spacing due to explicit breaking of chiral symmetry. We employ three definitions in the present work, checking consistency among them for reliability of results: (i) using the axial vector Ward identity, we define

\[ m_q^{\text{AWI}} = \lim_{t \to \infty} \frac{\langle \nabla_4 A_4^{\text{imp}}(t) P(0) \rangle}{2 \langle P(t) P(0) \rangle} , \quad (1) \]

with \( P \) the pseudoscalar density and \( A_4^{\text{imp}} = A_4 + c_A \nabla_4 P \) the axial vector current improved to \( O(a) \); (ii) another possibility, suggested by the vector Ward identity and naturally appearing in perturbative analyses, reads

\[ m_q^{\text{VWI}} = \frac{1}{(1/\kappa - 1/\kappa_c)/2} \]

where \( \kappa_c \) represents the critical hopping parameter at which the PS meson mass vanishes \( M_{PS}(\kappa_{val} = \kappa_{sea} = \kappa_c) = 0 \); (iii) a third possibility, suggested in [12] and denoted by \( m_q^{\text{MS}1,\text{PQ}} \), replaces \( \kappa_c \) in (ii) by the “partially quenched” critical value \( \kappa_c^{\text{PQ}} \) when the PS meson mass vanishes as a function of the valence hopping parameter \( \kappa_{val} \) when \( \kappa_{sea} \) for sea quark is fixed to the physical point of \( u \) and \( d \) quark, \( M_{PS}(\kappa_{val} = \kappa_c^{\text{PQ}}; \kappa_{sea} = \kappa_{ud}) = 0 \).

We express the PS meson mass \( M_{PS} \) in terms of quark masses by a general quadratic ansatz of the form,

\[ M_{PS} a^2 = b_s m_{\text{sea}} a + b_v m_{\text{val}} a + c_s (m_{\text{sea}} a)^2 + c_v (m_{\text{val}} a)^2 + c_{sv} m_{\text{sea}} a m_{\text{val}} a . \quad (2) \]

Here \( m_{\text{sea}} = \frac{m_{\text{val},1} + m_{\text{val},2}}{2} \) with \( m_{\text{val},i} (i = 1, 2) \) the bare mass of the valence quark and antiquark of the PS meson, and \( m_{\text{sea}} \) the mass of sea quark. We mention that details vary depending on the definitions, e.g., terms depending only on \( m_{\text{sea}} \) are absent for the case of AWI mass since with (1) \( M_{PS} \) is zero for vanishing \( m_{\text{val}} \), and a cross term \( m_{\text{val},1} a m_{\text{val},2} a \) is found to be necessary in the case of VWI mass.

The vector meson mass \( M_V \) is written in a similar manner, adopting, however, PS meson masses as independent variables. We fit data with the formula

\[ M_V a = A + B_s m_{\text{sea}} a + B_v m_{\text{val}} a + C_s m_{\text{sea}} a^2 + C_v m_{\text{val}} a^2 + C_{sv} m_{\text{sea}} a m_{\text{val}} a . \quad (3) \]

Here \( m_{\text{val}} = (\mu_1 + \mu_2)/2 \) represents the average of PS meson mass squared \( \mu_i = M_{PS}^2 (\kappa_{val,i}; \kappa_{val,i}; \kappa_{sea}) \) made of a degenerate quark-antiquark pair of the valence hopping parameter \( \kappa_{val,i} \), and \( m_{\text{sea}} = M_{PS}^2 (\kappa_{sea}; \kappa_{sea}; \kappa_{sea}) \).

Fitting our hadron mass data with (2) and (3) we find reasonable results with \( \chi^2/N_{\text{DF}} \) in the range 0.6–2.3 (except for (2) for VWI quark mass at \( \beta = 1.8 \) for which \( \chi^2/N_{\text{DF}} = 4.0 \)). We then determine the bare lattice value of the average \( u \) and \( d \) quark mass \( m_{\text{ud}} \) by fixing the valence and sea quark masses to be degenerate in (2) and (3), and requiring the experimental value for the ratio \( M_{\pi}/M_{\rho} = 0.1757 \). For the \( s \) quark mass \( m_s a \) we use either \( M_{K}/M_{\rho} = 0.6477 \) or \( M_{\phi}/M_{\rho} = 1.3267 \) while keeping the sea quark mass \( m_{\text{sea}} \) at the value \( m_{\text{ud}} \) determined above. The lattice scale \( a^{-1} \) is set using \( M_{\rho} = 0.7684 \text{ GeV} \) as input.

We convert bare quark masses calculated above to renormalized quark masses in the \( \overline{MS} \) scheme at \( \mu = 1/a \) through \( m_{\text{ud}}^{\text{VWI}} = Z_m(g^2) m_{\text{ud}}^{\text{VWI}} \) and \( m_{\text{ud}}^{\text{AWI}} = (Z_A(g^2)/Z_P(g^2)) m_{\text{ud}}^{\text{AWI}} \). In these relations, the \( O(a) \) improvement terms with the coefficients \( b_m, b_A \) and \( b_P \) are also included. For renormalization factors and improvement coefficients, including that for \( c_A \), one-loop perturbative values for massless quark [13] are used. For the coupling constant we adopt a mean-field improved scheme in the \( \overline{MS} \) scheme appropriate for the RG-improved gluon action (see Table I for numerical values): \( g_0^{-2} \langle 1/a \rangle = (3.648 W_1 x_1 - 2.648 W_1 x_2) \beta/6 - 0.1066 + 0.03149 N_f \) where measured values extrapolated to zero sea quark mass are substituted for the Wilson loops. The results for quark masses are run from \( \mu = 1/a \) to \( \mu = 2 \text{ GeV} \) using the 3-loop beta function for \( N_f = 2 \) [14]. Numerical values of quark masses at each \( \beta \) are given in Table II.

In Fig. 1 we show our two-flavor full QCD results for \( m_{\text{ud}}^{\overline{MS}} (2 \text{ GeV}) \) with filled symbols. The values for the three definitions, while differing sizably at finite lattice spacings [6], tend to converge toward the continuum limit. A similar trend has been seen in the quenched data for the standard action [4], reproduced in Fig. 1 with thin open symbols.

For our choice of the improved action, the scaling violation starts at \( O(g_0^{-2} (1/a)) \) for the quark masses at the scale \( \mu = 2 \text{ GeV} \). We therefore make a continuum extrapolation linear in \( a \). The results are given in Table II. The masses for the three definitions are consistent with each other at two-sigma level of statistics. Hence we carry out a combined linear fit, as shown in Fig. 1 by dashed lines, obtaining \( m_{\text{ud}} = 3.44(9) \) with \( \chi^2/N_{\text{DF}} = 2.9 \) where the error is only statistical. The systematic error of the continuum extrapolation is estimated from the spread of values obtained by separate fits of data for the three definitions. The fractional error thus calculated is given in Table III.

This table lists our estimate for two more systematic errors that we need to incorporate. One is an uncertainty due to chiral extrapolations. We estimate this error from the change of the combined linear fit in the continuum limit when the quadratic term \( \mu^2 \) in the vector mass formula (3) is replaced by \( \mu^{3/2} \) or cubic terms \( m^3 \) are included in the PS mass formula (2). Another is the error due to the use of one-loop perturbative values for the renormalization factors. As non-perturbative values are not yet available, we estimate the effect of higher order contributions by recalculating masses while either shifting the matching scale from \( \mu = 1/a \) to \( \mu = \pi/a \)
or using an alternative definition of coupling given by

\[ g^2 = 1/a = W_{1x1}\beta/6 + 0.2402 + 0.03149N_f \]

using only the plaquette.

Combining the statistical error and the systematic errors listed in Table III by quadrature to obtain the total error, we find for our final value,

\[ m_{ud}^{\overline{MS}}(2 \text{ GeV}) = 3.44^{+0.14}_{-0.22} \text{ MeV} \quad (N_f = 2). \quad (4) \]

Our full QCD result for the average u-d quark mass is considerably lower than our previous quenched result for the standard action given in [4] as

\[ m_{ud}^{\overline{MS}}(2 \text{ GeV}) = 4.57(18) \text{ MeV} \quad (N_f = 0, \text{ standard}), \quad (5) \]

where the error is only statistical. In order to confirm that the decrease is a dynamical sea quark effect, we carry out a quenched simulation using the same improved gluon and quark actions as for the full QCD runs. This simulation is made at 10 values of \( \beta \) chosen so that the string tension matches that of two-flavor full QCD for each simulated value of sea quark mass and for the chiral limit at \( \beta = 1.95 \) and 2.1.

Analyses leading from hadron masses to quark masses parallel those for full QCD. In particular we employ polynomial chiral expansions of the form (2)–(3), except that terms referring to sea quark masses are dropped. As a cross-check we also make an analysis parallel to the one in [4], employing quenched chiral perturbation theory formulae, and obtain consistent results.

We plot results of this quenched analysis with thick lines. The standard action with quenched chiral perturbation theory to one loop. The quenched values for the standard and improved actions are mutually consistent for each choice of the input. This confirms the existence of a systematic uncertainty of 20–30% in the value of \( m_{ud}^{\overline{MS}} \) in quenched QCD [4].

One of our important results for the strange quark mass is that this uncertainty disappears within an error of 10% by the inclusion of two flavors of sea quarks. The consistency reflects a closer agreement of the \( K - K^* \) and \( K - \phi \) mass splittings with experiment in our two-flavor QCD results compared to the quenched case [7].

Another important result is that dynamical quark effects reduce the value of \( m_{ud}^{\overline{MS}} \) significantly, from the range 110–140 MeV in quenched QCD to 90 MeV for two-flavor full QCD. It will be interesting to see whether the inclusion of dynamical effects of the strange quark itself would decrease the value of \( m_{ud}^{\overline{MS}} \) even further. This is an important issue to settle as our two-flavor results are already close to the lower bounds estimated from the positivity of spectral functions [16].

Clearly, establishing the values of light quark masses incorporating three flavors of dynamical quarks will be one of the main tasks of future lattice QCD calculations.

This work is supported in part by Grants-in-Aid of the Ministry of Education (Nos. 09304029, 10640246, 10640248, 10740107, 11640250, 11640294, 11740162). AAK and TM are supported by the JSPS Research for the Future Program (No. JSPS-RFTF 97P01102). GB, SE, TK, KN and HPS are JSPS Research Fellows.

\[ m_s^{\overline{MS}}(2 \text{ GeV}) = 110^{+3}_{-4} \text{ MeV} \quad (N_f = 0, \text{ improved}) \quad (9) \]

\[ = 116(3) \text{ MeV} \quad (N_f = 0, \text{ standard}) \quad (10) \]

\[ m_s^{\overline{MS}}(2 \text{ GeV}) = 132^{+4}_{-5} \text{ MeV} \quad (N_f = 0, \text{ improved}) \quad (11) \]

\[ = 146(4) \text{ MeV} \quad (N_f = 0, \text{ standard}), \quad (12) \]

where the values for the standard action [4] are also quoted for comparison.

[1] S. Weinberg, Trans. New York Acad. Sci. 38, 185 (1977).
[2] Early studies were reviewed in, A. Ukawa, Nucl. Phys. B (Proc. Suppl.) 30, 3 (1993). For a recent review, see R. Gupta and T. Bhattacharya, Nucl. Phys. B (Proc. Suppl.) 63, 95 (1998); Phys. Rev. D 55, 7203 (1997).
[3] B. J. Gough et al., Phys. Rev. Lett. 79, 1622 (1997).
[4] S. Aoki et al. (CP-PACS Collaboration), Phys. Rev. Lett. 84, 238 (2000); Nucl. Phys. B (Proc. Suppl.) 73, 189 (1999).
FIG. 1. Continuum extrapolation of the average up and down quark mass $m_{ud}$ for full QCD (filled symbols) and quenched QCD (thick open symbols) obtained with the improved action, and quenched results with the standard action (thin open symbols). Lines show combined fits linear in $a$.

FIG. 2. Continuum extrapolation of the strange quark mass $m_s$ using $M_K$ as input. Symbols have the same meaning as in Fig. 1.

FIG. 3. Continuum extrapolation of the strange quark mass $m_s$ using $M_\phi$ as input. Symbols have the same meaning as in Fig. 1.
TABLE I. Run parameters of two-flavor full QCD simulations. The scale $a$ is set by $M_\rho = 768.4$ MeV.

| $\beta$ | $g_{MS}^2(\mu = 1/a)$ | $L^3 \times T$ | $c_{SW}$ | $a$(fm) | $La$(fm) | $m_{PS}/m_V$ for sea quarks : #traj. |
|---------|------------------------|-----------------|---------|---------|----------|-----------------------------------|
| 1.80    | 3.168                  | $12^3 \times 24$ | 1.60    | 0.215(2) | 2.58(3)  | 0.807(1):6250 0.753(1):5000 0.694(2):7000 0.547(4):5250 |
| 1.95    | 2.816                  | $16^3 \times 32$ | 1.53    | 0.155(2) | 2.48(3)  | 0.804(1):7000 0.752(1):7000 0.690(1):7000 0.582(3):5000 |
| 2.10    | 2.565                  | $24^3 \times 48$ | 1.47    | 0.108(1) | 2.58(3)  | 0.806(1):4000 0.755(2):4000 0.691(3):4000 0.576(3):4000 |

TABLE II. Renormalized quark masses (in MeV) in the $\overline{MS}$-scheme at $\mu = 2$ GeV for each $\beta$, and in the continuum obtained by linear fits in $a$ (for these fits $\chi^2/N_{DF}$ is also given). All errors are statistical.

| $\beta$ | $m_{ud}^{VWI}$ | $m_{ud}^{AWI}$ | $m_{ud}^{VWI}$ | $m_{ud}^{AWI}$ | $m_{u}^{VWI}$ ($K$) | $m_{u}^{VWI}$ ($\phi$) | $m_{s}^{VWI}$ ($K$) | $m_{s}^{VWI}$ ($\phi$) | $m_{s}^{AWI}$ ($\phi$) | $\chi^2/N_{DF}$ |
|---------|----------------|---------------|----------------|---------------|---------------------|----------------------|---------------------|---------------------|---------------------|----------------|
| 1.8     | 2.277(27)      | 4.183(42)     | 3.322(37)      | 102.92(92)    | 104.54(93)          | 88.0(1.0)            | 129.1(2.2)          | 130.7(2.2)          | 113.9(2.4)          | 10.8            |
| 1.95    | 2.489(38)      | 4.064(43)     | 3.321(38)      | 100.65(98)    | 102.08(99)          | 87.2(1.0)            | 123.1(1.7)          | 124.5(1.7)          | 109.8(1.7)          | 2.4             |
| 2.1     | 2.966(55)      | 3.816(47)     | 3.344(46)      | 95.6(1.1)     | 96.4(1.1)           | 87.0(1.2)            | 108.0(2.2)          | 108.8(2.2)          | 100.0(2.2)          | 0.07            |
| $a \to 0$ | 3.47(10)    | 3.50(10)      | 3.36(9)        | 89.4(2.3)     | 89.5(2.3)           | 85.8(2.4)            | 90.1(4.9)           | 90.3(4.9)           | 88.1(4.9)           |                 |
| $\chi^2/N_{DF}$ | 10.8 | 2.4 | 0.07 | 2.1 | 2.7 | 0.03 | 6.0 | 6.5 | 2.4 |

TABLE III. Contributions to total error in continuum limit.

| $m_{ud}$ | $m_{s}$ ($K$-input) | $m_{s}$ ($\phi$-input) |
|----------|---------------------|------------------------|
| statistical | +2.6% | +2.4% | +4.8% |
| chiral | +1.2% | +1.6% | +1.5% |
| $Z$-factor | +2.3% | +2.2% | +1.7% |
| cont. extrap. | +1.7% | +1.4% | +0.9% |
| $\chi^2/N_{DF}$ | -2.6% | -2.4% | -4.8% |
| -2.3% | -2.2% | -7.6% |
| -5.0% | -5.6% | -6.9% |
| -2.3% | -2.8% | -1.6% |