A time-varying distributed unit hydrograph and its application in flood simulation of small mountainous watersheds

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Abstract. Flood forecasting is an important non-engineering measure to reduce flood damage. Small mountainous watersheds are characterized by rapid flow concentration and steeply rising and falling floods. Their flow concentration process exhibits strong time dependence, which is not well-captured by conventional unit hydrographs. In this paper, four mountainous watersheds were studied, and an algorithm for generating a time-varying distributed unit hydrograph (TVDUH) was proposed. The algorithm was integrated with the China Flash Flood Hydrological Model (CNFF-HM) to simulate small watersheds in the study area. The results showed relatively large Nash–Sutcliffe efficiency (NSE) of about 0.8 for the watersheds during the calibration and validation periods. Then the TVDUH was compared with the time-invariant one. The results showed that the TVDUH could better reflect the flow concentration characteristics of small mountainous watersheds, as the method took into account the effect of rainfall intensity on flow concentration during the computation process. And the TVDUH has great application potential in flood forecasting in mountainous watersheds to reduce the loss of flood disasters.

1. Introduction

The simulation of watershed flow concentration has become a major focus of hydrologists around the world. At present, flow concentration simulation usually deploys isochronal or unit hydrograph methods [1]. In 1996, Maidment [2] proposed a distributed unit hydrograph that reflects the terrain and geomorphology of the watershed and the spatial distribution of flow concentration paths. This unit hydrograph uses an algorithm based Digital Elevation Model (DEM) to calculate the times that rain droplets at various points of the watershed arrive at the watershed outlet (i.e., time of concentration) and finds the time of concentration–area relationship for the watershed. The unit hydrographs obtained by this method are closely related to the properties of the underlying surface of a basin. The fact that there is only one unit hydrograph for flow concentration in a specific basin implies the following hypothesis: watershed flow concentration only depends on constant parameters of the underlying surface (such as slope and roughness) and not on time-varying quantities (such as discharge, storage capacity, and rainfall intensity). Thus, flow concentration possesses the characteristics of a linear, time-invariant system. However, in practical applications, the flow concentration system of a natural watershed is neither linear nor time-invariant, particularly in small- and medium-sized watersheds. It is found that rainfall intensity is the most important factor leading to nonlinearity.
Ye and Fang [3] integrated the time-varying rainfall intensity directly into the computation function for the unit hydrograph, proposed a three-parameter, variable unit hydrograph, and further illustrated this model with a medium-sized watershed in Hubei Province. Li et al. [4] established a time-varying instantaneous unit hydrograph model by taking into full consideration the changes in rainfall intensity and the spatial distribution of rainfall, and they confirmed the parameters of the model using a genetic algorithm. The research on the nonlinearity of the above unit hydrographs is entirely based on the Nash unit hydrograph model. Little research has been conducted on the nonlinearity of distributed unit hydrographs. In this paper, a non-linear algorithm, the time-varying distributed unit hydrograph (TVDUH), is formulated for the construction of distributed unit hydrographs.

2. Study Area and Data

In China, flood disasters in the mountains usually occur in places with an average annual rainfall of above 400 mm, that is, humid and semi-humid regions. Based on the distribution of mountain flood disasters in China, four small basins, Luanchuan, Lichuan, Yuexi, and Siqian, were chosen as the subjects of this study for their high quality and completeness of data. The geographical locations of these four small basins are shown in Figure 1.

![Figure 1. Location of study areas](image)

DEM, land use, vegetation coverage, and soil type data were collected for the hydrological simulation. DEM data with a resolution of 25 m was obtained from the STRM Dataset (http://srtm.csi.cgiar.org). Land use and soil type data were provided by the Cold and Arid Regions Sciences Data Center in Lanzhou, China (http://westdc.westgis.ac.cn/), and climate data were retrieved from the China Meteorological Data Service Center (http://data.cma.cn/). The areas of Luanchuan, Lichuan, Yuexi, and Siqian basins are all less than 500 km² with an average slope of above 0.2. The lands in these four basins are primarily forested lands or arable lands. The basins also have diverse types of soil, with predominantly loam and silt in Luanchuan, sandy clay and clay loam in Lichuan, and clay loam in Siqian. The basins all have annual rainfall levels of above 800 mm and annual evaporation levels below the annual rainfall. Additional details can be found in Table 1.

| Watershed characteristics | Luanchuan | Lichuan | Yuexi | Siqian |
|---------------------------|-----------|---------|-------|--------|
| **Watershed structure**   |           |         |       |        |
| Area (km²)                | 343       | 500     | 152   | 139    |
| Altitude (m)              | 1110      | 1071    | 403   | 663    |
| Average slope             | 0.40      | 0.24    | 0.26  | 0.43   |
| Drainage density (km/km²) | 0.33      | 0.35    | 0.24  | 0.27   |
| **Land use and vegetation coverage** | | | | |
| Forested land (%)         | 91.71     | 37      | 58.6  | 89.31  |
| Arable land (%)           | 6.81      | 30      | 30.7  | 8.86   |
| Land for housing and building construction (%) | 1.44 | 0.3 | 1.54 | 0.44 |
| Others (%)                | 0.04      | 30      | 9.16  | 1.39   |
Soil type

- Loamy clay (%) 11.7
- Sandy clay loam (%) 9.88 10.17
- Sandy Clay (%) 29.04 58.79 0.73
- Gravel (%) 61
- Loam (%) 44.66 12.08 3.05
- Clay loam (%) 1.04 29.13 3.32 88.57
- Sandy loam (%) 7.53 0.35
- Silty loam (%) 25.26 3.52

Climate

- Average annual rainfall (mm) 864 1600 1478 2183
- Average annual evaporation (mm) 751 720 871 965
- Average annual temperature (°C) 13.7 16.2 14.4 17.6

The hydrometeorological data of the study area were obtained from the Hydrological Information Center of the Bureau of Hydrology, Ministry of Water Resources, China (http://10.1.17.50/hsnew/default.aspx#/Main). The hydrometeorological data of the basins are given in Table 2.

### Table 2. Hydrometeorological data of small watersheds

| Watershed | Rainfall stations | Hydrological stations | Flood events | Data Period |
|-----------|-------------------|-----------------------|--------------|-------------|
| Yuexi     | 4                 | 1                     | 42           | 1990–2012   |
| Luanchuan | 7                 | 1                     | 25           | 1996–2012   |
| Lichuan   | 5                 | 1                     | 28           | 1979–2011   |
| Siqian    | 4                 | 1                     | 36           | 1974–1992   |

3. Methods

3.1. Velocity calculation for time-varying unit hydrograph

Based on the achievements in the past, a new equation for the flow velocity in a grid is found:

$$V = kS^{0.5}(i/I)^m$$

where $k$ is the velocity coefficient (m/s), which is an empirical parameter attributed to the friction and resistance imposed by land use on the flow velocity. Its values are given in Table 3; $i$ is the effective rainfall intensity (mm/h); $I$ is the critical effective rainfall intensity (mm/h); and $m$ is a positive number less than 1.

### Table 3. Velocity coefficients for different land use types

| Land use              | Velocity coefficient $k$ (m/s) | Land use                                      | Velocity coefficient $k$ (m/s) |
|-----------------------|--------------------------------|----------------------------------------------|--------------------------------|
| Paddy field           | 6.5                            | Man-made grassland                           | 0.3                            |
| Dry land              | 1.4                            | Land for transportation                      | 6.5                            |
| Garden                | 0.45                           | Land for water bodies and water conservation facilities | 6.5                            |
| Forested land         | 0.45                           | Housing and building construction (zone)     | 6.5                            |
| Shrub land            | 0.2                            | Structure                                    | 6.5                            |
| Other forested land   | 0.35                           | Human excavation activity                    | 6                              |
| High-coverage grassland | 0.3                      | Saline-alkali soil, sandy land, bare land   | 6                              |
| Medium-coverage grassland | 0.55                 | Rocks, gravel                                | 6.5                            |
| Low-coverage grassland | 0.65                         | Marshland                                    | 0.65                           |
This equation has the following characteristics: 1) both sides of the equation have the same dimension and contain all three major independent factors influencing the velocity of overland flow: land use type (reflected in the value of $k$), slope of the landform ($S$), and rainfall intensity ($i, I$); and 2) the dimensionless term $(i/I)^m$ is added as a comprehensive measure of the effect of rainfall intensity on flow velocity. It shows the increase in flow velocity with rainfall intensity. When $i < I$, the increase is rapid. When $i > I$, the increase slows down. At $i = I$, the equation becomes exactly $V = k \sqrt{S}$. Following previous research on distributed unit hydrographs of the watersheds in the study area, here $m$ and $I$ adopt a value of 0.4 and 40 mm/h, respectively.

3.2. Extraction of time-varying unit hydrograph

The effective rainfall in each DEM grid always flows to the adjacent grid along the direction of maximum slope. The flow concentration path of the effective rainfall from the grid to the outlet could be determined based on this principle. Using the grid size and the flow velocity in the grid, the lag time of the runoff in each grid can be calculated by Equation (2),

$$\Delta \tau = \frac{L}{V} \quad \text{or} \quad \Delta \tau = \frac{\sqrt{2} L}{V}$$

where $\Delta \tau$ is the lag time of the runoff in a grid, and $L$ is the length of the side of the grid. Along the flow concentration path of the effective rainfall, the sum of lag times of the grids is the time the effective rainfall in the grids reaches the watershed outlet, that is, time of concentration.

The probability density distribution for the time of concentration in a small watershed was obtained statistically from the time of concentration in each grid of the watershed. This probability density distribution was the instantaneous unit hydrograph, which was converted to a unit hydrograph with arbitrary duration using Equation (3).

$$q(\Delta t, t) = \frac{10 F}{3.6 \Delta t} \left[ S(t) - S(t - \Delta t) \right]$$

Here, $\Delta t$ is the duration covered by the unit hydrograph; $q(\Delta t, i)$ is the unit hydrograph; $F$ is the area of the watershed; and $S(t)$ is the S-curve from the instantaneous unit hydrograph.

3.3. Integration of time-varying unit hydrograph with hydrological model

Unique TVDUHs were obtained for each watershed under different rainfall intensities. To validate the time-varying unit hydrographs obtained, the described method was integrated into a distributed hydrological model, CNFF-HM. CNFF-HM is a distributed hydrological model developed by the China Institute of Water Resources and Hydropower Research in 2014 for flood simulation and forecasting in small mountainous watersheds. It features a user-friendly modular structure and incorporates a number of runoff generation and flow concentration algorithms, making it a powerful tool for runoff generation and flow route simulation under different climate types. The model uses sub-watersheds as basic computational units. A detailed description of the CNFF-HM model can be found in references [5, 6]. The algorithms used here for each module of CNFF-HM are shown in Figure 2. The time-varying unit hydrograph was integrated into the CNFF-HM model in the flow concentration stage.
4. Results and Discussion

4.1. Simulation results using time-varying unit hydrograph

The NSE, the relative errors in peak discharge and runoff depth, and the error in time to peak were graphed for all the flood events in the four small basins of the study area simulated by the CNFF-HM model based on the TVDUH. The results are shown in Figure 3. As seen in the graph, the small basins all featured high mean values of NSE during both the calibration and validation periods. For most of the flood events in the basins, NSE values of about 0.8 were found. The relative errors in peak discharge were fairly concentrated at around −20~20% for the calibration and validation periods. For the majority of the basins, the relative errors in runoff depth were concentrated at around −10~10% during the calibration period. Slightly larger relative errors in runoff depth were seen in the validation period, clustered at around −20~20%.

The error in time to peak is an important index in the evaluation of hydrological forecasting accuracy. Small mountainous watersheds frequently plagued by mountain floods usually have an area of less than 500 km² and a time of concentration below 6 h [6]. For these watersheds, a small error in the time to peak is highly desirable. The forecasting is considered accurate with an error in time to peak of less than 2 h. The errors in time to peak for most basins in the study area fell within this range during both the calibration and validation periods.

In conclusion, the CNFF-HM model based on the TVDUH produced largely accurate simulations of the flood process in the basins. This flood forecasting model (scheme) is reliable and can be applied to most basins in the study area.

4.2. Comparison of simulation results by time-varying and time-invariant distributed unit hydrographs

Runoff generation and flow concentration were modeled based on the time-invariant distributed unit hydrograph (DUH), and the DUH algorithm is described in detail by Kong et al. [7]. In the modeling process, the algorithms and parameters used for the runoff generation module were the same as in Section 4.1. In the flow concentration module, the DUH was used to simulate the concentration of overland flow. The concentration of other flows was simulated as in Section 4.1.
Forecasts made with the CNFF-HM model based on the TVDUH were compared with those made with the CNFF-HM model based on DUH (Table 4). The forecasts made with the TVDUH were of significantly higher quality than those made with DUH. For most of the basins, the forecasts made by the former method were of Grade II or better as indicated by the pass rates. The latter, however, had lower pass rates, and only 50% of the forecasts were of Grade II or better. This shows the clear advantage of the TVDUH-based CNFF-HM model over its time-invariant counterpart for the simulation of flood in small mountainous watersheds.

Table 4. Comparison of pass rates of the model before and after modification

| Watershed | DUH Calibration period Pass rate Grade | Validation period Pass rate Grade | TVDUH Calibration period Pass rate Grade | Validation period Pass rate Grade |
|-----------|----------------------------------------|----------------------------------|----------------------------------------|----------------------------------|
| Yuexi     | 61% III                                | 60% III                          | 76% II                                 | 72% II                           |
| Luanchuan | 62% III                                | 66% III                          | 79% II                                 | 92% I                            |
| Lichuan   | 73% II                                 | 70% II                           | 78% II                                 | 70% II                           |
| Siqian    | 63% III                                | 67% III                          | 75% II                                 | 75% II                           |

This clear dominance of the TVDUH in flood modeling could be attributed to the following two reasons. First, the TVDUH takes into account the influence of rainfall intensity in addition to the slope and vegetation coverage of the watershed in its computation of flow velocity. The non-linear characteristic of flow concentration in the watershed is thus better captured. The DUH, however, only considers the slope and vegetation of the watershed in the computation of flow velocity and not the rainfall intensity. Second, when the TVDUH is used in CNFF-HM for flow concentration simulation, the unit hydrograph of corresponding rainfall intensity is automatically selected according to the rainfall process of the incoming flood event. In contrast, the DUH utilizes only one unit hydrograph for each small watershed. Regardless of the precipitation conditions, the same unit hydrograph is used for flow concentration simulation by the model. In this sense, the TVDUH is a more sophisticated algorithm compared to the time-invariant one, and it adequately depicts the abrupt rise and fall in floods in small mountainous watersheds.

5. Conclusions
In order to reduce the loss of flood disasters and improve the accuracy of flood forecasting, the principle and methodology of the TVDUH are described in detail in this paper. A distributed hydrological model
CNFF-HM) is constructed using four small mountainous watersheds as the study areas. The simulation results of CNFF-HM integrated with the TVDUH are analyzed. The following conclusions are drawn:

(1) The CNFF-HM model based on the TVDUH offers superb flood simulation results for the four watersheds. Relatively large Nash–Sutcliffe efficiency (NSE) of about 0.8 for the watersheds during the calibration and validation periods are obtained. And the relative errors in runoff depth and peak discharge and the error in time to peak are small.

(2) Further comparison of the simulation results based on the TVDUH and time-invariant distributed unit hydrograph shows that the unit hydrograph extracted by the first method better reflects the time-dependent characteristics of flow concentration in small mountainous watersheds due to its incorporation of rainfall intensity.

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