Snowmass White Paper: Quantum Aspects of Black Holes and the Emergence of Spacetime

Raphael Bousso, 1 Xi Dong, 2 Netta Engelhardt, 3 Thomas Faulkner, 4 Thomas Hartman, 5 Stephen H. Shenker, 6 and Douglas Stanford 6

1 Center for Theoretical Physics and Department of Physics, University of California, Berkeley, CA 94720, U.S.A.
2 Department of Physics, University of California, Santa Barbara, CA 93106, U.S.A.
3 Center for Theoretical Physics, Massachusetts Institute of Technology, Cambridge, MA 02139, USA
4 Department of Physics, University of Illinois Urbana-Champaign, Urbana, IL 61801, USA
5 Department of Physics, Cornell University, Ithaca, NY, USA
6 Stanford Institute for Theoretical Physics and Department of Physics, Stanford University, Stanford, CA 94305, U.S.A.

E-mail: bousso@berkeley.edu, xidong@ucsb.edu, engeln@mit.edu, tomf@illinois.edu, hartman@cornell.edu, sshenker@stanford.edu, salguod@stanford.edu

Abstract:

Black holes provide a window into the microscopic structure of spacetime in quantum gravity. Recently the quantum information contained in Hawking radiation has been calculated, verifying a key aspect of the consistency of black hole evaporation with quantum mechanical unitarity.

This calculation relied crucially on recent progress in understanding the emergence of bulk spacetime from a boundary holographic description. Spacetime wormholes have played an important role in understanding the underpinnings of this result, and the precision study of such wormholes, in this and other contexts, has been enabled by the development of low-dimensional models of holography.

In this white paper we review these developments and describe some of the deep open questions in this subject. These include the nature of the black hole interior, potential applications to quantum cosmology, the gravitational explanation of the fine structure of black holes, and the development of further connections to quantum information and laboratory quantum simulation.
1 Introduction

Nearly a half-century ago, Hawking showed that black holes emit radiation [1], and ever since then the study of these objects has been a central part of our quest for a quantum theory of gravity. Hawking’s calculation also showed that the radiation left behind after black hole evaporation would be in a mixed (thermal) quantum state [2], even if the initial state of the matter forming the black hole was pure. The fundamental irreversibility implied by this result sets up a sharp conflict between the unitary time evolution of quantum mechanics and general relativity at the event horizon. This “black hole information paradox” arises in a seemingly controlled regime characterized by a weakly curved geometry and low energy quanta, making it even more puzzling.

The AdS/CFT correspondence (also known as gauge/gravity duality) [3–5] has provided us with a precise definition of nonperturbative quantum gravity, at least for
a certain class of spacetimes, and therefore serves as a testing ground for the information paradox as well as other deep problems in quantum gravity. This holographic correspondence relates a non-gravitational quantum system, often a conformal field theory (CFT), on the boundary of an asymptotically anti-de Sitter (AdS) spacetime to a theory of quantum gravity (string or M theory) in the bulk of the spacetime.

Many (but by no means all) of the entries in the dictionary relating bulk gravity and boundary quantum mechanics are known. One basic entry links a high energy thermal state of the boundary system to a black hole in the bulk \cite{6, 7}. The entropy of the boundary system is the entropy of the black hole which, according to a result of Bekenstein and Hawking \cite{1, 8}, is determined by the area of its horizon.

Another entry that has played a central role in recent developments is the gravity dual of the entanglement entropy of a subregion of the boundary field theory. Ryu and Takayanagi \cite{9–11} argued that this is again proportional to the area of a surface, called the RT surface, ending on the boundary subregion, as illustrated in Figure 1. Both of these relations take the form
\begin{equation}
S = \frac{\text{Area}}{4G\hbar} + \cdots
\end{equation}
for the entropy $S$, thus relating gravity, quantum mechanics, and information. More generally, surface area constrains the amount of quantum information in a spacetime region, indicating that the holographic encoding of information in quantum gravity is not limited to the AdS/CFT correspondence.

The region enclosed by an RT surface — or rather its generalization to include quantum effects, called a quantum extremal surface (QES) \cite{12} — is holographically encoded in the corresponding boundary subregion \cite{13–18}. This encoding can be very complex, and is only partially understood. The problem of ‘decoding the hologram’ is a research program known as bulk reconstruction that has illuminated deep links between quantum gravity and quantum information science. A striking example is the realization that information about the bulk is stored redundantly in the boundary theory via a quantum error-correcting code \cite{19}.

Rapid progress on the information paradox has been made in the last three years, building on the Ryu-Takayanagi formula and many parallel developments in the theory of black hole information over the last decade. The initial breakthrough was made by locating a new, unexpected, quantum extremal surface in an evaporating black hole \cite{20, 21}. This enabled the first gravitational derivation of a key signature of unitarity — the ‘Page curve’ \cite{22} — for the entropy of Hawking radiation. The ideas underlying these calculations also extend the method of bulk reconstruction to black holes in more general spacetimes, taking a significant step beyond the AdS/CFT correspondence towards a theory of emergent spacetime in more realistic models of quantum gravity.
Figure 1. Subregion-subregion duality relates a boundary subregion \( A \) with its entanglement wedge \( a \) (of which only a time slice is shown here). The entanglement wedge is bounded by a (quantum) extremal surface \( \chi_A \).

One important insight is that many of these results can also be understood from new nonperturbative saddle points in the semiclassical approximation to the gravitational path integral [23, 24]. Such semiclassical methods have a long history in this subject, going back to the evaluation of the entropy of a black hole from a Euclidean saddle point by Gibbons and Hawking [25]. In the Page curve calculation it turns out to be necessary to include the contribution of spacetime wormhole geometries in the semiclassical analysis.

An important catalyst for progress has been the construction of simple low dimensional models of quantum gravity where such wormhole effects can be studied in a controlled way. These include the the Sachdev-Ye-Kitaev (SYK) model [26–31] and its low energy limit, Jackiw-Teitelboim (JT) gravity [32, 33] in two dimensions.

The above threads share two common traits: a dramatic acceleration of progress over the past decade, and increasingly deep and central connections to quantum information science and quantum many-body physics. This progress is gratifying but many mysteries remain. What is the bulk dual of a typical state in the boundary system, and how is this related to the firewall paradox [34] that helped initiate these developments? What is the nature of the black hole singularity and what role does it play in this circle of ideas? How do these ideas extend beyond AdS spacetimes, especially to cosmologies resembling our world? What is the bulk explanation for the individual microstates of a black hole? Is it possible to construct model systems in the lab that would actually allow us to gain experimental insight into some of these issues?
2 Recent progress

2.1 Emergence of spacetime

Several related dualities\(^1\) have connected quantum gravity systems to non-gravitational degrees of freedom. In each case, the gravitational spacetime emerges from the collective behavior of the nongravitational degrees of freedom.

This idea is sharpest in the context of AdS/CFT, but even there, the basic mechanism has only been clarified recently – insights from a quantum information perspective have been central to these developments. In particular a dictionary has been developed relating bulk gravitational quantities to quantum information-theoretic quantities.

These ideas grew out of the Ryu-Takayanagi [9] proposal connecting the areas of minimal (RT) surfaces to entanglement entropies of boundary subregions. This proposal has been extensively developed [10, 11, 46–55]. In particular, quantum corrections have been understood and a semiclassical gravitational path integral derivation has been formulated [12, 56–61]. A key notion is the entanglement wedge of a boundary subregion [13–17]. This is a spacetime region in the bulk that is bounded by a (quantum) extremal surface [12] (classically, an RT surface) associated with the boundary subregion, as shown in Figure 1.

A large body of work has led to the central concept of subregion-subregion duality [13–18]: the quantum information present in a subregion of the boundary field theory is exactly the information needed to describe the bulk quantum state in the entanglement wedge. In particular, bulk operators in the entanglement wedge of a boundary subregion can be reconstructed as some boundary operators on that subregion [18].

A striking aspect of subregion-subregion duality is that the information about the bulk is stored redundantly in the boundary. It functions as a quantum error-correcting code [19]. A simple example of this is the “ABC” puzzle illustrated in Figure 2: a bulk operator at point \(p\) can be reconstructed using three different boundary operators\(^2\) supported on the distinct, different regions \(A \cup B, B \cup C,\) and \(A \cup C\). This redundant quantum encoding is the basic mechanism at work in quantum error-correcting codes.\(^3\)

Another approach to bulk reconstruction that highlights the emergence of bulk locality and causality involves the concept of modular flow [90–92]. This is a generalized

---

\(^1\)The chronology includes matrix models and 2d gravity [35–38] in the double-scaled limit [39–41], M(atrix) theory [42], and AdS/CFT [3–5]. See [43, 44] and [45] for connections.

\(^2\)This follows from the AdS-Rindler version of the Hamilton-Kabat-Lifschytz-Lowe (HKLL) [62] procedure of reconstructing bulk operators in terms of nonlocal boundary operators by solving the bulk equations of motion as operator equations.

\(^3\)Similar error-correcting properties have been realized in concrete toy models of holography built from tensor networks [63–67]. Also see [68–89] for further work on quantum error correction and information recovery in holography.
Figure 2. The “ABC” puzzle for reconstructing a bulk operator at point $p$ in three different ways is resolved by the insight that holography works as a quantum error-correcting code. The first panel shows the individual entanglement wedges of the three regions $A, B, C$. None of these contains the point $p$ so it cannot be reconstructed from information in a single region. The second, third and fourth panels illustrate the entanglement wedges of the regions $A \cup B$, $B \cup C$, and $A \cup C$ respectively. Each one of these regions contains $p$ so they provide the data for distinct, redundant, reconstructions of $p$.

notion of time evolution that treats the logarithm of a general density matrix as a Hamiltonian, also known as the modular Hamiltonian. This generalizes the Rindler time evolution near the black hole horizon generated by the ordinary CFT Hamiltonian [93–95]. A result that is basic to this program is that the bulk and boundary modular Hamiltonians can be identified up to an area term [60].

Operator reconstruction with modular flow allows one to reach operators everywhere inside the entanglement wedge [96], thus in many situations reaching behind causal horizons.4

A deeper connection between modular flow and bulk emergence follows from studying relations between bulk causality and analyticity in modular time [54, 98].5 For example, by studying analyticity of correlation functions in modular time one can establish a connection between the emergence of local bulk physics and the saturation of modular chaos bounds constraining the growth of these correlators. Such bounds are connected to the chaos bound [101], which constrains the growth of finite temperature out-of-time-ordered correlators. It is well known that gravity saturates the chaos bound, and that this is connected to the existence of a smooth black hole horizon. The modular chaos generalization attempts to move these results away from black hole horizons to general locations in spacetime.

Constraints from analyticity in modular time are also known to directly constrain dynamical aspects of the emergent gravitational physics, most notably via an interesting

---

4The Petz map [97] is a specific reconstruction map used in quantum error-correcting codes and also involves a kind of modular flow, achieving a similar outcome [19, 24, 73, 80] and thus connecting these approaches to bulk reconstruction.

5See [99, 100] for related approaches to bulk reconstruction.
connection to quantum energy conditions [102, 103]. These conditions generalize the well known null energy condition that plays an important role in constraining the causal structure of classical general relativity and underlies the celebrated Penrose singularity theorem [104].

Causal aspects of semi-classical gravity, which includes leading order quantum corrections, are often usefully constrained by the Quantum Focusing Conjecture [102, 105–108]. For example, the Quantum Focusing Conjecture can be used to prove a basic causal constraint on entanglement wedge reconstruction - that bulk regions should nest when the corresponding boundary regions nest [16, 109]. As shown in [54], entanglement wedge nesting is indeed connected to analyticity in modular time, a further constraint on the correlation functions discussed above that arises for theories saturating the modular chaos bound. This line of reasoning has been particularly fruitful, leading to proofs of energy conditions in QFT [103, 110] without gravity.6

These developments have highlighted a deep connection between AdS/CFT and a formal mathematical framework called Algebraic Quantum Field Theory, whose constructs are very natural from the quantum information theoretic point of view. This abstract approach has provided powerful new tools and new insights into these issues.7

2.2 The information paradox

In defiance of a longstanding expectation, new calculations [20, 21] have produced striking evidence that low energy, semiclassical gravity can detect unitarity in black hole evaporation (see [119] for a review). These recent developments are summarized in Figure 3. A sharp diagnostic of the information paradox is the von Neumann entropy of Hawking radiation,

\[ S_R = -\text{tr} \rho_R \ln \rho_R , \]  
(2.1)

where \( \rho_R \) is the density matrix of the radiation. Hawking’s calculation indicates that \( \rho_R \) is mixed and that \( S_R \) grows monotonically as the black hole evaporates, while the Bekenstein-Hawking entropy of the black hole,

\[ S_{BH} = \text{Area}/4G , \]  
(2.2)

decreases to zero. This leads to a contradiction: Unitarity requires \( S_R \to 0 \) at late times, because if the initial state of the black hole is pure then the final state of the radiation must be pure also. Furthermore, general properties of entangled quantum systems require \( S_R \leq S_{BH} \). Since Hawking’s calculation takes place in a region of low

---

6Further applications of modular flow to these issues can be found in [53, 55, 111–114].
7See for example [88, 91, 92, 115–118].
Figure 3. Progress on the information paradox. (a) Black hole evaporation. An island (green) bounded by a quantum extremal surface (blue) appears at late stages in the evaporation. (b) The von Neumann entropy of Hawking radiation: Hawking’s calculation (dashed red) leads to a paradox when the radiation entropy exceeds the black hole entropy (dashed gray). If black hole evaporation is unitary, then the true entropy should follow the Page curve (blue). (c) Two copies of the black hole can be used to probe the quantum information in the radiation. At late times, spacetime wormholes join the black hole interiors. This leads to the creation of islands, which in turn produce the unitary Page curve.

curvature, this contradiction constitutes an apparent violation of effective field theory in a regime where strong quantum gravity corrections should be suppressed.

The behavior of the radiation entropy expected from unitary evaporation is known as the Page curve [22] (Figure 3b). It is this universal curve that was determined quan-
titatively from recent semiclassical calculations using the quantum extremal surface (QES) formula for entropy in quantum gravity [12]:

$$S_R = \frac{{\text{Area}[\chi]}}{4G} + S_{\text{bulk}} \equiv S_{\text{gen}}[\chi]$$

(2.3)

Here $\chi$ is a QES: a surface where $S_{\text{gen}}$ is stationary under small perturbations. (If there are multiple stationary surfaces, $\chi$ is taken to be the one with minimal entropy.) The second term $S_{\text{bulk}}$ is the von Neumann entropy of quantum fields in a region bounded by $\chi$. In many situations the quantum correction $S_{\text{bulk}}$ has a small effect compared to the area term (which determines the classical RT surface). But at sufficiently late stages of the evaporation, when according to the Page curve the entropy $S_R$ should begin to decrease, $S_{\text{bulk}}$ has a dramatic effect. A new QES in the black hole interior becomes dominant, and its effect is to produce exactly the decreasing part of the Page curve.

The QES formula therefore agrees with unitary black hole evaporation, but in fact, it provides much more than just a formula for the entropy. An extension of the bulk reconstruction ideas discussed in Section 2.1 leads to the conclusion that the region behind the QES — an ‘island’ in the black hole interior — is actually encoded in the Hawking radiation [21, 120].

The island can be traced back to spacetime wormholes [128–130] in the gravitational path integral [23, 24]. This can be described in the context of the von Neumann entropy, but it is simpler to use the quantum purity, $\text{tr} \rho^2_R$. This is a convenient diagnostic for unitarity because in a pure state, $\text{tr} \rho^2_R = 1$, while in a mixed state, $\text{tr} \rho^2_R < 1$. Because it involves two copies of the density matrix, the purity is calculated by a gravitational path integral involving two copies of the black hole known as replicas (Figure 3c). At early times, the purity agrees with Hawking’s calculation of monotonically increasing entropy. However, at late times there is a dynamical transition to another saddle point in the gravitational path integral in which the two black holes are joined through their interiors by a spacetime wormhole. In the wormhole phase the purity is larger, and indeed it returns to unity as the black hole evaporates. This is consistent with a

---

The region must also be homologous to the region $R$ containing the radiation, and $S_{\text{bulk}}$ is by definition the von Neumann entropy calculated without gravitational instantons. From an information-theoretic viewpoint it is not the von Neumann entropy of the exact density matrix of the quantum fields, but of a density matrix defined on the code subspace appropriate to bulk reconstruction.

This is a concrete realization of earlier intuitions like $A = R_B$ and $\text{ER} = \text{EPR}$ [121–125] relating the radiation to the interior. It provides a geometric realization of the Hayden-Preskill protocol [126], whereby the information in the black hole interior can be decoded from the radiation at late times. Concrete reconstruction procedures have been discussed using the Petz map [24] and modular flow [127].
unitary final state. In the analogous replica calculation of the von Neumann entropy, the mouth of the wormhole becomes the island inside the black hole, and an evaluation of the wormhole action justifies the QES formula.\(^\text{10}\)

The appearance of the island in the entanglement wedge indicates that at late stages of the evaporation the black hole interior is encoded in the Hawking radiation.\(^\text{11}\) However, the relation between the island and the radiation constitutes a departure from the standard holographic dictionary: data in the island is not spatially connected to the radiation, so reconstruction is necessarily more subtle. A natural question, then, is what qualitatively sets apart the island — and more generally the deep black hole interior — from the rest of the bulk in terms of reconstruction. This question is at the root of decoding the black hole interior from the radiation.

Recent developments have pointed to the central role of quantum computational complexity in understanding this issue. General considerations [85, 126, 136, 137] imply that decoding the Hawking radiation is exponentially complex. More precisely, the state of the Hawking radiation cannot be reliably distinguished from a random state by any quantum circuit whose size is polynomial in the black hole entropy.

Ideas about tensor networks [63, 138] and the geometrization of quantum complexity [139–145] have led to a conjecture geometrizing reconstruction complexity in terms of QESs. The so-called Python’s Lunch conjecture states that reconstruction of bulk data in the entanglement wedge is exponentially complex if that data lies behind a subdominant (non-minimal) QES [146].\(^\text{12}\) This perspective unifies the developing geometric picture of QESs and both dominant and subdominant saddles of the gravitational path integral with earlier developments on the process and feasibility of decoding the Hawking radiation.

These calculations have opened new avenues of research in black hole information that are reshaping our approach to the information paradox. At the same time, they have raised many new questions about the interplay of quantum mechanics and gravity, some of which are discussed in Section 3.

\(^\text{10}\)This idea was initially tested in a doubly-holographic model [23] in which the evaporating black hole was holographically dual to a higher-dimensional purely classical bulk. The QES formula without islands was derived earlier from the gravitational path integral in [50, 56, 58, 59, 61].

\(^\text{11}\)For an alternative perspective, and a discussion of subtleties that arise in separating the radiation from the black hole, see [131–135].

\(^\text{12}\)The converse is also expected to be true [147, 148], which has implications for a holographic understanding of Hawking’s calculation.
2.3 Wormholes, low-dimensional gravity, and the SYK model

Some of the phenomena discussed above depend crucially on subtle nonperturbative effects in the gravitational path integral, for example the replica wormhole saddles that compute the Page curve. Such effects are difficult to study in a controlled way in general. A strategy employed extensively in recent years has been to use simple two-dimensional gravity models to study these effects, as well as other aspects of the quantum physics of black holes, with precision.

An important step here was the development of the Sachdev-Ye-Kitaev (SYK) model [26–30], an ensemble of simple but strongly-interacting quantum mechanical systems. Concretely, the SYK model describes the quantum mechanics of a collection of $N$ Majorana fermions coupled with generic four-fermion couplings which are drawn from a probabilistic ensemble. Among other things these systems display the maximally chaotic, fast scrambling behavior characteristic of black holes [101, 126, 149–153].

At low energies these systems are described by a two-dimensional gravity theory (Jackiw-Teitelboim or “JT” gravity [32, 33, 154–157]) that provides a universal description of near-extremal black holes. In the SYK model, this theory arises by a concrete change of variables starting from the quantum mechanics of Majorana fermions. This precise mapping has made it possible to sharpen the connections between gravity and quantum mechanics, to resolve some puzzles, and to generate new ones. In gravity variables, several of the areas of progress have involved the physics of wormholes.

One example of this is the role of replica wormholes in addressing the black hole information paradox. Another concerns the statistics of black hole energy levels, packaged together into a convenient function called the spectral form factor [158, 159]

$$Z(\beta + it)Z(\beta - it) = \sum_{n,m} e^{-\beta(E_n + E_m)} e^{it(E_n - E_m)}.$$ (2.4)

From the perspective of black hole physics, the LHS is computed by a pair of black hole geometries, and the answer apparently decreases forever as a function of $t$. But in a true quantum system, the RHS cannot decay forever; instead, it will rattle around erratically as the phases oscillate. This conflict is known as Maldacena’s black hole information problem [7]. Numerical studies of the SYK model and the concrete mapping to gravity showed that the lack of decay of the RHS arises due to a spacetime wormhole that can connect the two black holes together [162]. The functional form of this late time behavior is a signature of the random matrix statistics of the energy levels, which are

---

13The spectral form factor is related to the two-point correlation function with the operator matrix elements removed. For work on wormholes and correlation functions see [160, 161].
a universal property of quantum chaotic systems [158] and so should be a feature of black holes more broadly.\textsuperscript{14}

More precisely, this wormhole accurately computes the answer after averaging over the ensemble of SYK theories, smearing the erratic oscillations into a smooth function. This suggests a connection between simple gravity theories and ensembles of quantum systems, as exemplified by the exact duality between dilaton gravity theories like JT and quantum theories where the Hamiltonian is drawn from a random matrix ensemble.\textsuperscript{15} So wormholes solved one puzzle but created another: how can gravity describe a single quantum system, rather than an ensemble?\textsuperscript{16} We will discuss this issue further in Section 3.3.

Another application of wormholes involves the physics of entanglement. Black holes manifest certain patterns of entanglement via geometrical connections of spatial wormholes.\textsuperscript{17} Entanglement alone does not allow signalling, which is dual to the statement that the corresponding wormholes are not traversable. But a small interaction between the two black holes can lead to traversability [167]. This phenomenon was studied in the SYK model and in JT gravity [168], and eventually used to construct theoretical examples of traversable wormholes in our own four-dimensional world [169–172]. It has also created new connections to quantum information theory and quantum simulation. From a quantum information perspective, passing through the wormhole is a particular implementation of quantum teleportation, carried out by an elegant protocol invented by gravity itself. This protocol has been used to inspire and explain experiments carried out using noisy quantum simulators [173–175].

Progress has not been limited to the physics of wormholes, however. For example, precise computations of the density of states in JT gravity [176–181] were used in [182, 183] to resolve an old problem [184] related to the energy spectrum of near-extremal black holes.

\textsuperscript{14}More precisely, the wormhole describes the ramp part of the spectral form factor. The ultimate late time behavior, the plateau, has a more complicated origin.

\textsuperscript{15}This duality is formally analogous to the older random matrix description of low-dimensional string theories, reviewed in [163–165]. But here the perspective is different – the random matrix is the full boundary system Hamiltonian, not a single field in a field theory.

\textsuperscript{16}In fact the connection between wormholes and ensembles is an old one, going back to work on baby universes and wormholes in the 1980s [128, 129]. For a modern reformulation of these ideas in the AdS/CFT context see [166].

\textsuperscript{17}The basic example of this is the Einstein-Rosen bridge in the eternal Schwarzschild black hole. This is the bulk embodiment of entanglement in of the boundary thermofield double state [7].
3 Future directions

3.1 Behind the horizon

Classically the black hole horizon serves as a sharp division of spacetime into regions that are accessible and inaccessible to a distant observer. Deep questions exist about the nature of the region behind the horizon – the black hole interior.

3.1.1 The firewall paradox

In 2007, Hayden and Preskill [126] used modern tools from quantum information theory to show that information that falls into an old black hole should rapidly become recoverable from the radiation. In 2013, Almheiri, Marolf, Polchinski, and Sully (AMPS) [34] used this result to strengthen a previous argument of Mathur [185], deriving a paradox that motivated them to conjecture that the geometry of an old black hole should be dramatically altered behind the horizon: a “firewall” would form. Whether this actually happens is still an open question. However the island results discussed above suggest that gravity may evade the original entanglement-based argument by encoding the black hole interior in the radiation [121–125].

Other arguments [188, 189] for firewalls attempt to establish their existence in random states in Hilbert space, and these arguments are not obviously affected by such an identification. So an apparently sharp question remains completely open: what is the interior of a black hole in a random quantum state? Is the interior even uniquely determined by the state?

3.1.2 The black hole singularity

Putting firewalls aside, it is still certainly the case that classical geometry breaks down behind the horizon near the black hole singularity [104, 190]. Ever since the discovery of the Schwarzschild solution the nature of this singularity has been a mystery. The developments described in this white paper have not, to date, cast new light on this problem and it remains a central task of a theory of quantum gravity to understand it.

Classically the singularity represents an “end of time” behind the horizon, raising deep questions about the nature of ordinary quantum time evolution there. The intriguing black hole final state proposal [191–194] posits that the quantum state of the black hole is postselected to match a fixed final state at the singularity. This allows

\footnote{These ideas were partially anticipated in [186].}

\footnote{There has been extensive work on trying to resolve this question. Examples of approaches include [121–125, 187].}

\footnote{Time evolution outside the horizon proceeds without end and joins to the manifestly unitary quantum time evolution of the boundary theory.}
for a unitary black hole scattering matrix, but represents a deviation from ordinary quantum mechanics in the interior. This highlights a key question: what is the proper quantum description of bulk dynamics inside the horizon?

3.2 Cosmology

Up till now this white paper has focused on black holes and AdS spacetimes, but the geometry of our universe is quite different. Its cosmology at both early and late times seems to be an exponentially expanding one, consistent with de Sitter, not Anti-de Sitter, space.

Finding a nonperturbative description of quantum gravity in such cosmologies is a problem of central importance – some current approaches to this problem will be discussed in a separate white paper on cosmology and string theory. Here we will content ourselves with pointing out a few areas where the ideas discussed in this paper may be of some relevance to this problem.

A basic reason to expect a connection to black holes is the existence of horizons in de Sitter space. An observer sees a horizon whose area defines a de Sitter entropy and which semiclassical calculations show to radiate thermally [195] at the de Sitter temperature [196]. These thermal fluctuations are related to the primordial fluctuations that are directly observed via the cosmic microwave background (CMB).

One crucial difference between de Sitter space and black holes is that de Sitter observers are in the interior of their horizons. A second difference is the absence of an analog of the black hole singularity that observers encounter in the future. (There is a past singularity though, the big bang.) Despite these differences we can still ask whether QES notions continue to be useful here. Do islands play a role? How about wormholes?

There are cosmologies with future singularities (“big crunches”) that observers eventually encounter. These cosmologies typically have spherical regions where the matter entropy is large compared to the classical area [197, 198], suggesting that matter entanglement could perhaps compete with classical geometry [199], as in the island effect in black hole evaporation. And there are models where islands, replica wormholes and other related configurations, bra-ket wormholes, do appear [200–211]. It is an open question whether they occur in the more realistic, expanding cosmologies.

There are more basic questions. What degrees of freedom does the de Sitter entropy count and what does this concept mean in a geometry with exponentially expanding space? Bulk reconstruction, as it is presently understood, relies on a boundary region or auxiliary degrees of freedom that exist outside of the gravitating spacetime and which, through their entanglement structure, encode the properties of the emergent geometry. In a closed universe, or in an expanding cosmology with no spacelike boundary, there
is no clear separation between gravitating and non-gravitating regions, so this crutch
must be modified or abandoned. There are a number of proposals for formulating a
holographic description, but as of now they are incomplete. Further exploration of
holography in this setting is an important task for the future.

3.3 Fine structure of black holes

The boundary quantum description of a finite entropy black hole has a discrete spec-
trum of energy levels – each one of these levels describes a microstate of the black hole.
A complete description of the bulk must include a description of these states, but the
way this is realized in general is a mystery.\textsuperscript{21} It seems likely that understanding this
description will require substantial new insights, and will shed important new light on
the nature of quantum gravity.

This discreteness causes quantum noise \cite{176, 220–223} in addition to the signal
computed using gravitational structures, like wormholes. In certain quantities, like the
Page curve, we expect that this noise will be very small. But in others, like the ramp in
the spectral form factor or individual matrix elements of the radiation density matrix
$\rho_R$, or of the black hole S matrix itself, we expect the noise to be comparable to the
signal. The simplest way to isolate the gravitational contribution is by averaging over
an ensemble of boundary quantum systems. And in fact 2D JT gravity is precisely
dual to such an ensemble, in this case a random matrix ensemble \cite{224, 225}.

Such ensembles of theories can appear to violate basic rules of quantum mechan-
ics \cite{226, 227}; for example, observables in completely disjoint universes can nonetheless
be correlated by the ensemble average. In gravitational theories this correlation is cap-
tured by wormholes. This sharp tension between decoupled boundary theories and bulk
geometries that connect them is referred to as the factorization problem \cite{130, 228}. Its
resolution is likely to be related to an understanding of black hole microstructure.

These ideas raise several important questions. First, we do not expect the usual
examples of holographic duality to be precisely described by an ensemble – the boundary
theories are too special.\textsuperscript{22, 23} What ingredients need to be added to the bulk gravitational

\textsuperscript{21}Such a bulk description has been found for the microstates of certain extremal supersymmetric
black holes, as well as a few nonextremal examples \cite{212–216}. The active “fuzzball” program \cite{185, 217–
219} seeks to build on this success. The bulk realization of a typical state of a large non-extremal
black hole is currently unknown though, and there are reasons to suspect that the general case will be
qualitatively different from the extremal one.

\textsuperscript{22}An argument for the absence of a bulk ensemble has been presented in \cite{229}.

\textsuperscript{23}There are other ways to average over the noise that apply to systems with a fixed boundary
Hamiltonian. Averaging over time intervals on the ramp of the spectral form factor is an example.
Such averaged quantities are the ones that are expected to display universal random matrix behavior
in quantum chaotic systems.
description to describe the noise? A number of proposals have been discussed [166, 230–237], but the full story remains to be told.

Second, how much does the semiclassical gravitational description itself know about this noise? To what extent do wormholes in e.g., four-dimensional gravity provide a useful statistical description of it [238–240]? Are there further examples of precise dualities between ensembles of boundary systems and bulk gravitational ones [241–247]? What lessons can be drawn from thinking about these questions from the perspective of the Hilbert space of baby universes [128, 129, 166, 248–250]?

3.4 New connections to quantum information

3.4.1 The holographic code

Holography packages the quantum state of the bulk theory into the boundary theory according to a type of quantum error correcting code. Such codes must have remarkable properties, with the flexibility and precision to describe all of bulk physics. By trying to explain how basic features of the bulk theory are represented, we can try to reverse-engineer these codes and extract lessons both for quantum gravity and the theory of quantum error correction.

Some progress has been made on this problem recently. For example, non-flat entanglement spectra can be explained using codes with central commutative algebras [71, 76, 77, 84]. Internal symmetries have been incorporated [81, 251], and we are beginning to understand how the entanglement wedge and its area can emerge from properties of the code [71, 252]. State specific entanglement wedges have been understood [74, 87].

However, many puzzles remain, ranging from structural questions like the origin of local physics on sub-AdS scales [65] and the compatibility of bulk and boundary dynamics, to more detailed questions like the apparent ability of holography to defeat location-based cryptography with polynomial entanglement resources [253, 254].

3.4.2 String theory and quantum information

While ideas from quantum information have been given natural dual descriptions in semi-classical gravity, understanding the role of string theory in this duality is an important open question. As a well-established UV completion of gravity one might have expected it to play a larger role. For small string lengths the RT area formula gets corrections from higher derivative couplings in the effective action which have been extensively studied. However these perturbative corrections are not always sufficient to capture all stringy effects even at small string lengths. Indeed, just as stringy effects can be enhanced near black hole horizons due to boosted kinematics, they can also be enhanced when reconstructing operators inside the entanglement wedge [255].
In holography, string theory with nonzero string length offers an interpolation between emergent spacetime (for small string length) and more weakly-coupled boundary degrees of freedom (at large string length). How does the relationship between quantum information and spacetime play out along this axis? For example, can we study entanglement entropy and bulk reconstruction in string theory? Entanglement requires a notion of splitting a system – how do we locally split the string theory Hilbert space? To what extent can we understand the RT formula as arising from stringy edge modes? This connects to old ideas about explaining black hole entropy as strings ending on the horizon. A related idea is the possible transition between the large number of highly excited string states and black hole states, as a parameter is varied. Understanding this transition might shed light on the nature of black hole microstates and the various factorization puzzles that arise from using semiclassical gravity to describe the dual of entangled states of decoupled systems.

3.5 Quantum gravity in the lab

Quantum gravity has traditionally been a largely theoretical field. However, increasing control over laboratory quantum systems and near-term (NISQ) quantum devices may make it possible to simulate quantum systems with interesting holographic duals: “quantum gravity in the lab.” Progress in this direction has started already with. Following this example, we can expect that ideas from quantum gravity might help to inspire or explain experiments. An equally exciting prospect is that experiments on simulated quantum systems might challenge or inform our understanding of theoretically intractable (strongly coupled) limits of quantum gravity systems, with possible relevance to black hole physics, high energy scattering, or the early universe.

4 Outlook

Over the last decade progress in the areas discussed here has been impressively rapid. Cross-fertilization from different fields like quantum information theory and many-body physics has contributed new vitality to the subject, and has helped cause new avenues of research to multiply. As is often the case in rapidly developing interdisciplinary subjects, progress has come from surprising, unexpected directions. The continuing ferment in this field makes us optimistic that the coming decade will be similarly productive and surprising.

---

24Examples of work on this problem include [256–262].
Acknowledgements

RB is supported in part by the Berkeley Center for Theoretical Physics; by the Department of Energy, Office of Science, Office of High Energy Physics under QuantISED Award DE-SC0019380 and under contract DE-AC02-05CH11231; and by the National Science Foundation under Award Number 1820912. XD is supported in part by the Air Force Office of Scientific Research under award number FA9550-19-1-0360 and by funds from the University of California. NE is supported in part by NSF grant no. PHY-2011905, by the U.S. Department of Energy Early Career Award DE-SC0021886, by the U.S. Department of Energy grant DE-SC0020360 (Contract 578218), by the John Templeton Foundation and the Gordon and Betty Moore Foundation via the Black Hole Initiative, and by funds from the MIT department of physics. TF is supported by the Air Force Office of Scientific Research under award number FA9550-19-1-0360 and by the Department of Energy under award number DE-SC0019183. TH is supported by the Simons Foundation and NSF grant PHY-2014071. SS is supported in part by NSF grant PHY-1720397. DS is supported in part by DOE grant DE-SC0021085 and by the Sloan Foundation.

References

[1] S. W. Hawking, Particle Creation by Black Holes, Commun. Math. Phys. 43 (1975) 199.
[2] S. W. Hawking, Breakdown of Predictability in Gravitational Collapse, Phys. Rev. D 14 (1976) 2460.
[3] J. M. Maldacena, The Large N limit of superconformal field theories and supergravity, Int. J. Theor. Phys. 38 (1999) 1113 [hep-th/9711200].
[4] E. Witten, Anti-de Sitter space and holography, Adv. Theor. Math. Phys. 2 (1998) 253 [hep-th/9802150].
[5] S. Gubser, I. R. Klebanov and A. M. Polyakov, Gauge theory correlators from noncritical string theory, Phys. Lett. B 428 (1998) 105 [hep-th/9802109].
[6] E. Witten, Anti-de Sitter space, thermal phase transition, and confinement in gauge theories, Adv. Theor. Math. Phys. 2 (1998) 505 [hep-th/9803131].
[7] J. M. Maldacena, Eternal black holes in anti-de Sitter, JHEP 04 (2003) 021 [hep-th/0106112].
[8] J. D. Bekenstein, Black holes and the second law, Lett. Nuovo Cim. 4 (1972) 737.
[9] S. Ryu and T. Takayanagi, Holographic derivation of entanglement entropy from AdS/CFT, Phys. Rev. Lett. 96 (2006) 181602 [hep-th/0603001].
[10] V. E. Hubeny, M. Rangamani and T. Takayanagi, *A Covariant holographic entanglement entropy proposal*, JHEP **07** (2007) 062 [0705.0016].

[11] T. Nishioka, S. Ryu and T. Takayanagi, *Holographic Entanglement Entropy: An Overview*, J. Phys. A **42** (2009) 504008 [0905.0932].

[12] N. Engelhardt and A. C. Wall, *Quantum Extremal Surfaces: Holographic Entanglement Entropy beyond the Classical Regime*, JHEP **01** (2015) 073 [1408.3203].

[13] R. Bousso, S. Leichenauer and V. Rosenhaus, *Light-sheets and AdS/CFT*, Phys. Rev. D **86** (2012) 046009 [1203.6619].

[14] B. Czech, J. L. Karczmarek, F. Nogueira and M. Van Raamsdonk, *The Gravity Dual of a Density Matrix*, Class. Quant. Grav. **29** (2012) 155009 [1204.1330].

[15] R. Bousso, B. Freivogel, S. Leichenauer, V. Rosenhaus and C. Zukowski, *Null Geodesics, Local CFT Operators and AdS/CFT for Subregions*, Phys. Rev. D **88** (2013) 064057 [1209.4641].

[16] A. C. Wall, *Maximin Surfaces, and the Strong Subadditivity of the Covariant Holographic Entanglement Entropy*, Class. Quant. Grav. **31** (2014) 225007 [1211.3494].

[17] M. Headrick, V. E. Hubeny, A. Lawrence and M. Rangamani, *Causality & holographic entanglement entropy*, JHEP **12** (2014) 162 [1408.6300].

[18] X. Dong, D. Harlow and A. C. Wall, *Reconstruction of Bulk Operators within the Entanglement Wedge in Gauge-Gravity Duality*, Phys. Rev. Lett. **117** (2016) 021601 [1601.05416].

[19] A. Almheiri, X. Dong and D. Harlow, *Bulk Locality and Quantum Error Correction in AdS/CFT*, JHEP **04** (2015) 163 [1411.7041].

[20] A. Almheiri, N. Engelhardt, D. Marolf and H. Maxfield, *The entropy of bulk quantum fields and the entanglement wedge of an evaporating black hole*, 1905.08762.

[21] G. Penington, *Entanglement Wedge Reconstruction and the Information Paradox*, 1905.08255.

[22] D. N. Page, *Information in black hole radiation*, Phys. Rev. Lett. **71** (1993) 3743 [hep-th/9306083].

[23] A. Almheiri, T. Hartman, J. Maldacena, E. Shaghoulian and A. Tajdini, *Replica Wormholes and the Entropy of Hawking Radiation*, JHEP **05** (2020) 013 [1911.12333].

[24] G. Penington, S. H. Shenker, D. Stanford and Z. Yang, *Replica wormholes and the black hole interior*, 1911.11977.
[25] G. W. Gibbons and S. W. Hawking, *Action Integrals and Partition Functions in Quantum Gravity*, Phys. Rev. D 15 (1977) 2752.

[26] S. Sachdev and J.-w. Ye, *Gapless spin fluid ground state in a random, quantum Heisenberg magnet*, Phys. Rev. Lett. 70 (1993) 3339 [cond-mat/9212030].

[27] A. Kitaev, “A simple model of quantum holography”, talks at kitp april 2015: talk1 and talk2.”

[28] A. Kitaev and S. J. Suh, *The soft mode in the Sachdev-Ye-Kitaev model and its gravity dual*, JHEP 05 (2018) 183 [1711.08467].

[29] O. Parcollet and A. Georges, *Non-fermi-liquid regime of a doped mott insulator*, Phys. Rev. B 59 (1999) 5341.

[30] J. Maldacena and D. Stanford, *Remarks on the Sachdev-Ye-Kitaev model*, Phys. Rev. D 94 (2016) 106002 [1604.07818].

[31] G. Sárosi, *AdS$_2$ holography and the SYK model*, PoS Modave2017 (2018) 001 [1711.08482].

[32] R. Jackiw, *Lower Dimensional Gravity*, Nucl. Phys. B252 (1985) 343.

[33] C. Teitelboim, *Gravitation and Hamiltonian Structure in Two Space-Time Dimensions*, Phys. Lett. B126 (1983) 41.

[34] A. Almheiri, D. Marolf, J. Polchinski and J. Sully, *Black Holes: Complementarity or Firewalls?*, JHEP 02 (2013) 062 [1207.3123].

[35] F. David, *Planar Diagrams, Two-Dimensional Lattice Gravity and Surface Models*, Nucl. Phys. B257 (1985) 45.

[36] J. Ambjorn, B. Durhuus and J. Frohlich, *Diseases of Triangulated Random Surface Models, and Possible Cures*, Nucl. Phys. B257 (1985) 433.

[37] V. A. Kazakov, *Bilocal Regularization of Models of Random Surfaces*, Phys. Lett. 150B (1985) 282.

[38] V. A. Kazakov, A. A. Migdal and I. K. Kostov, *Critical Properties of Randomly Triangulated Planar Random Surfaces*, Phys. Lett. 157B (1985) 295.

[39] M. R. Douglas and S. H. Shenker, *Strings in Less Than One-Dimension*, Nucl. Phys. B335 (1990) 635.

[40] E. Brezin and V. A. Kazakov, *Exactly Solvable Field Theories of Closed Strings*, Phys. Lett. B236 (1990) 144.

[41] D. J. Gross and A. A. Migdal, *Nonperturbative Two-Dimensional Quantum Gravity*, Phys. Rev. Lett. 64 (1990) 127.

[42] T. Banks, W. Fischler, S. H. Shenker and L. Susskind, *M theory as a matrix model: A Conjecture*, Phys. Rev. D55 (1997) 5112 [hep-th/9610043].
N. Itzhaki, J. M. Maldacena, J. Sonnenschein and S. Yankielowicz, Supergravity and the large N limit of theories with sixteen supercharges, Phys. Rev. D 58 (1998) 046004 [hep-th/9802042].

J. Polchinski, M theory and the light cone, Prog. Theor. Phys. Suppl. 134 (1999) 158 [hep-th/9903165].

J. McGreevy and H. L. Verlinde, Strings from tachyons: The c=1 matrix reloaded, JHEP 12 (2003) 054 [hep-th/0304224].

M. Headrick, Entanglement Renyi entropies in holographic theories, Phys. Rev. D 82 (2010) 126010 [1006.0047].

X. Dong, Holographic Entanglement Entropy for General Higher Derivative Gravity, JHEP 01 (2014) 044 [1310.5713].

J. Camps, Generalized entropy and higher derivative Gravity, JHEP 03 (2014) 070 [1310.6659].

X. Dong, The Gravity Dual of Renyi Entropy, Nature Commun. 7 (2016) 12472 [1601.06788].

X. Dong, A. Lewkowycz and M. Rangamani, Deriving covariant holographic entanglement, JHEP 11 (2016) 028 [1607.07506].

M. Freedman and M. Headrick, Bit threads and holographic entanglement, Commun. Math. Phys. 352 (2017) 407 [1604.00354].

T. Takayanagi and K. Unemoto, Entanglement of purification through holographic duality, Nature Phys. 14 (2018) 573 [1708.09393].

Y. Chen, X. Dong, A. Lewkowycz and X.-L. Qi, Modular Flow as a Disentangler, JHEP 12 (2018) 083 [1806.09622].

T. Faulkner, M. Li and H. Wang, A modular toolkit for bulk reconstruction, JHEP 04 (2019) 119 [1806.10560].

S. Dutta and T. Faulkner, A canonical purification for the entanglement wedge cross-section, 1905.00577.

A. Lewkowycz and J. Maldacena, Generalized gravitational entropy, JHEP 08 (2013) 090 [1304.4926].

T. Faulkner, The Entanglement Renyi Entropies of Disjoint Intervals in AdS/CFT, 1303.7221.

T. Barrella, X. Dong, S. A. Hartnoll and V. L. Martin, Holographic entanglement beyond classical gravity, JHEP 09 (2013) 109 [1306.4682].

T. Faulkner, A. Lewkowycz and J. Maldacena, Quantum corrections to holographic entanglement entropy, JHEP 11 (2013) 074 [1307.2892].
[60] D. L. Jafferis, A. Lewkowycz, J. Maldacena and S. J. Suh, *Relative entropy equals bulk relative entropy*, JHEP 06 (2016) 004 [1512.06431].

[61] X. Dong and A. Lewkowycz, *Entropy, Extremality, Euclidean Variations, and the Equations of Motion*, JHEP 01 (2018) 081 [1705.08453].

[62] A. Hamilton, D. N. Kabat, G. Lifschytz and D. A. Lowe, *Holographic representation of local bulk operators*, Phys. Rev. D74 (2006) 066009 [hep-th/0606141].

[63] B. Swingle, *Entanglement Renormalization and Holography*, Phys. Rev. D 86 (2012) 065007 [0905.1317].

[64] F. Pastawski, B. Yoshida, D. Harlow and J. Preskill, *Holographic quantum error-correcting codes: Toy models for the bulk/boundary correspondence*, JHEP 06 (2015) 149 [1503.06237].

[65] P. Hayden, S. Nezami, X.-L. Qi, N. Thomas, M. Walter and Z. Yang, *Holographic duality from random tensor networks*, JHEP 11 (2016) 009 [1601.01694].

[66] W. Donnelly, B. Michel, D. Marolf and J. Wien, *Living on the Edge: A Toy Model for Holographic Reconstruction of Algebras with Centers*, JHEP 04 (2017) 093 [1611.05841].

[67] X.-L. Qi and Z. Yang, *Space-time random tensor networks and holographic duality*, 1801.05289.

[68] E. Mintun, J. Polchinski and V. Rosenhaus, *Bulk-Boundary Duality, Gauge Invariance, and Quantum Error Corrections*, Phys. Rev. Lett. 115 (2015) 151601 [1501.06577].

[69] B. Freivogel, R. Jefferson and L. Kabir, *Precursors, Gauge Invariance, and Quantum Error Correction in AdS/CFT*, JHEP 04 (2016) 119 [1602.04811].

[70] A. Almheiri, X. Dong and B. Swingle, *Linearity of Holographic Entanglement Entropy*, JHEP 02 (2017) 074 [1606.04537].

[71] D. Harlow, *The Ryu–Takayanagi Formula from Quantum Error Correction*, Commun. Math. Phys. 354 (2017) 865 [1607.03901].

[72] F. Pastawski and J. Preskill, *Code properties from holographic geometries*, Phys. Rev. X 7 (2017) 021022 [1612.00017].

[73] J. Cotler, P. Hayden, G. Penington, G. Salton, B. Swingle and M. Walter, *Entanglement Wedge Reconstruction via Universal Recovery Channels*, Phys. Rev. X 9 (2019) 031011 [1704.05839].

[74] P. Hayden and G. Penington, *Learning the Alpha-bits of Black Holes*, JHEP 12 (2019) 007 [1807.06041].
A. Almheiri, *Holographic Quantum Error Correction and the Projected Black Hole Interior*, 1810.02055.

C. Akers and P. Rath, *Holographic Renyi Entropy from Quantum Error Correction*, *JHEP* **05** (2019) 052 [1811.05171].

X. Dong, D. Harlow and D. Marolf, *Flat entanglement spectra in fixed-area states of quantum gravity*, *JHEP* **10** (2019) 240 [1811.05382].

B. Yoshida, *Soft mode and interior operator in the Hayden-Preskill thought experiment*, *Phys. Rev. D* **100** (2019) 086001 [1812.07353].

N. Bao, G. Penington, J. Sorce and A. C. Wall, *Beyond Toy Models: Distilling Tensor Networks in Full AdS/CFT*, *JHEP* **11** (2019) 069 [1811.05382].

X. Dong, D. Harlow and D. Marolf, *One-loop universality of holographic codes*, 1910.06329.

I. Kim, E. Tang and J. Preskill, *The ghost in the radiation: Robust encodings of the black hole interior*, *JHEP* **06** (2020) 031 [2003.05451].

T. Faulkner, S. Hollands, B. Swingle and Y. Wang, *Approximate recovery and relative entropy I. general von Neumann subalgebras*, 2006.08002.

C. Akers and G. Penington, *Leading order corrections to the quantum extremal surface prescription*, *JHEP* **04** (2021) 062 [2008.03319].

C.-F. Chen, G. Penington and G. Salton, *Entanglement Wedge Reconstruction using the Petz Map*, *JHEP* **01** (2020) 168 [1902.02844].

R. Haag, *Local quantum physics: Fields, particles, algebras*. Springer, 1992.

H. Casini, *Relative entropy and the Bekenstein bound*, *Class. Quant. Grav.* **25** (2008) 205021 [0804.2182].

E. Witten, *APS Medal for Exceptional Achievement in Research: Invited article on entanglement properties of quantum field theory*, *Rev. Mod. Phys.* **90** (2018) 045003 [1803.04993].
[93] J. J. Bisognano and E. H. Wichmann, *On the Duality Condition for Quantum Fields*, *J. Math. Phys.* **17** (1976) 303.

[94] P. D. Hislop and R. Longo, *Modular Structure of the Local Algebras Associated With the Free Massless Scalar Field Theory*, *Commun. Math. Phys.* **84** (1982) 71.

[95] H. Casini, M. Huerta and R. C. Myers, *Towards a derivation of holographic entanglement entropy*, *JHEP* **05** (2011) 036 [1102.0440].

[96] T. Faulkner and A. Lewkowycz, *Bulk locality from modular flow*, *JHEP* **07** (2017) 151 [1704.05464].

[97] D. Petz, *Quantum information theory and quantum statistics*. Springer Science & Business Media, 2007.

[98] J. De Boer and L. Lamprou, *Holographic Order from Modular Chaos*, *JHEP* **06** (2020) 024 [1912.02810].

[99] H. W. Lin, J. Maldacena and Y. Zhao, *Symmetries Near the Horizon*, *JHEP* **08** (2019) 049 [1904.12820].

[100] S. Leutheusser and H. Liu, *Causal connectability between quantum systems and the black hole interior in holographic duality*, 2110.05497.

[101] J. Maldacena, S. H. Shenker and D. Stanford, *A bound on chaos*, *JHEP* **08** (2016) 106 [1503.01409].

[102] R. Bousso, Z. Fisher, J. Koeller, S. Leichenauer and A. C. Wall, *Proof of the Quantum Null Energy Condition*, *Phys. Rev.* **D93** (2016) 024017 [1509.02542].

[103] S. Balakrishnan, T. Faulkner, Z. U. Khandker and H. Wang, *A General Proof of the Quantum Null Energy Condition*, *JHEP* **09** (2019) 020 [1706.09432].

[104] R. Penrose, *Gravitational collapse and space-time singularities*, *Phys. Rev. Lett.* **14** (1965) 57.

[105] R. Bousso, Z. Fisher, S. Leichenauer and A. C. Wall, *Quantum focusing conjecture*, *Phys. Rev.* **D93** (2016) 064044 [1506.02669].

[106] J. Koeller and S. Leichenauer, *Holographic Proof of the Quantum Null Energy Condition*, *Phys. Rev.* **D94** (2016) 024026 [1512.06109].

[107] S. Leichenauer, A. Levine and A. Shahbazi-Moghaddam, *Energy density from second shape variations of the von Neumann entropy*, *Phys. Rev. D* **98** (2018) 086013 [1802.02584].

[108] S. Balakrishnan, V. Chandrasekaran, T. Faulkner, A. Levine and A. Shahbazi-Moghaddam, *Entropy Variations and Light Ray Operators from Replica Defects*, 1906.08274.
[109] C. Akers, J. Koeller, S. Leichenauer and A. Levine, Geometric Constraints from Subregion Duality Beyond the Classical Regime, 1610.08968.

[110] F. Ceyhan and T. Faulkner, Recovering the QNEC from the ANEC, 1812.04683.

[111] N. Engelhardt and A. C. Wall, Coarse Graining Holographic Black Holes, JHEP 05 (2019) 160 [1806.01281].

[112] R. Bousso, V. Chandrasekaran and A. Shahbazi-Moghaddam, From black hole entropy to energy-minimizing states in QFT, Phys. Rev. D101 (2020) 046001 [1906.05299].

[113] R. Bousso, V. Chandrasekaran, P. Rath and A. Shahbazi-Moghaddam, Gravity dual of Connes cocycle flow, Phys. Rev. D 102 (2020) 066008 [2007.00230].

[114] A. Levine, A. Shahbazi-Moghaddam and R. M. Soni, Seeing the entanglement wedge, JHEP 06 (2021) 134 [2009.11305].

[115] R. Longo and F. Xu, Comment on the Bekenstein bound, J. Geom. Phys. 130 (2018) 113 [1802.07184].

[116] H. Casini, M. Huerta, J. M. Magán and D. Pontello, Entanglement entropy and superselection sectors. Part I. Global symmetries, JHEP 02 (2020) 014 [1905.10487].

[117] K. Furuya, N. Lashkari and S. Ouseph, Real-space RG, error correction and Petz map, 2012.14001.

[118] E. Witten, Gravity and the Crossed Product, 2112.12828.

[119] A. Almheiri, T. Hartman, J. Maldacena, E. Shaghoulian and A. Tajdini, The entropy of Hawking radiation, Rev. Mod. Phys. 93 (2021) 035002 [2006.06872].

[120] A. Almheiri, R. Mahajan, J. Maldacena and Y. Zhao, The Page curve of Hawking radiation from semiclassical geometry, JHEP 03 (2020) 149 [1908.10996].

[121] R. Bousso, Complementarity Is Not Enough, Phys. Rev. D 87 (2013) 124023 [1207.5192].

[122] Y. Nomura, J. Varela and S. J. Weinberg, Complementarity Endures: No Firewall for an Infalling Observer, JHEP 03 (2013) 059 [1207.6626].

[123] E. Verlinde and H. Verlinde, Black Hole Entanglement and Quantum Error Correction, JHEP 10 (2013) 107 [1211.6913].

[124] K. Papadodimas and S. Raju, An Infalling Observer in AdS/CFT, JHEP 10 (2013) 212 [1211.6767].

[125] J. Maldacena and L. Susskind, Cool horizons for entangled black holes, Fortsch. Phys. 61 (2013) 781 [1306.0533].

[126] P. Hayden and J. Preskill, Black holes as mirrors: Quantum information in random subsystems, JHEP 09 (2007) 120 [0708.4025].
[127] Y. Chen, *Pulling Out the Island with Modular Flow*, JHEP 03 (2020) 033 [1912.02210].

[128] S. R. Coleman, *Black Holes as Red Herrings: Topological Fluctuations and the Loss of Quantum Coherence*, Nucl. Phys. B307 (1988) 867.

[129] S. B. Giddings and A. Strominger, *Loss of Incoherence and Determination of Coupling Constants in Quantum Gravity*, Nucl. Phys. B 307 (1988) 854.

[130] J. M. Maldacena and L. Maoz, *Wormholes in AdS*, JHEP 02 (2004) 053 [hep-th/0401024].

[131] A. Laddha, S. G. Prabhu, S. Raju and P. Shrivastava, *The Holographic Nature of Null Infinity*, SciPost Phys. 10 (2021) 041 [2002.02448].

[132] H. Geng, A. Karch, C. Perez-Pardavila, S. Raju, L. Randall, M. Riojas et al., *Information Transfer with a Gravitating Bath*, SciPost Phys. 10 (2021) 103 [2012.04671].

[133] H. Geng, A. Karch, C. Perez-Pardavila, S. Raju, L. Randall, M. Riojas et al., *Inconsistency of islands in theories with long-range gravity*, JHEP 01 (2022) 182 [2107.03390].

[134] S. Raju, *Failure of the split property in gravity and the information paradox*, 2110.05470.

[135] H. Geng, A. Karch, C. Perez-Pardavila, S. Raju, L. Randall, M. Riojas et al., *Entanglement Phase Structure of a Holographic BCFT in a Black Hole Background*, 2112.09132.

[136] D. Harlow and P. Hayden, *Quantum Computation vs. Firewalls*, JHEP 06 (2013) 085 [1301.4504].

[137] S. Aaronson, *The Complexity of Quantum States and Transformations: From Quantum Money to Black Holes*, 7, 2016, 1607.05256.

[138] T. Hartman and J. Maldacena, *Time Evolution of Entanglement Entropy from Black Hole Interiors*, JHEP 05 (2013) 014 [1303.1080].

[139] L. Susskind, *Computational Complexity and Black Hole Horizons*, Fortsch. Phys. 64 (2016) 24 [1403.5695].

[140] L. Susskind and Y. Zhao, *Switchbacks and the Bridge to Nowhere*, 1408.2823.

[141] D. Stanford and L. Susskind, *Complexity and Shock Wave Geometries*, Phys. Rev. D 90 (2014) 126007 [1406.2678].

[142] D. A. Roberts, D. Stanford and L. Susskind, *Localized shocks*, JHEP 03 (2015) 051 [1409.8180].
[143] A. R. Brown, D. A. Roberts, L. Susskind, B. Swingle and Y. Zhao, Complexity, action, and black holes, Phys. Rev. D 93 (2016) 086006 [1512.04993].
[144] A. Bouland, B. Fefferman and U. Vazirani, Computational pseudorandomness, the wormhole growth paradox, and constraints on the AdS/CFT duality, 1910.14646.
[145] L. Susskind and Y. Zhao, Complexity and Momentum, JHEP 21 (2020) 239 [2006.03019].
[146] A. R. Brown, H. Gharibyan, G. Penington and L. Susskind, The Python’s Lunch: geometric obstructions to decoding Hawking radiation, JHEP 08 (2020) 121 [1912.00228].
[147] N. Engelhardt, G. Penington and A. Shahbazi-Moghaddam, A world without pythons would be so simple, Class. Quant. Grav. 38 (2021) 234001 [2102.07774].
[148] N. Engelhardt, G. Penington and A. Shahbazi-Moghaddam, Finding Pythons in Unexpected Places, 2105.09316.
[149] Y. Sekino and L. Susskind, Fast Scramblers, JHEP 10 (2008) 065 [0808.2096].
[150] S. H. Shenker and D. Stanford, Black holes and the butterfly effect, JHEP 03 (2014) 067 [1306.0622].
[151] S. H. Shenker and D. Stanford, Multiple Shocks, JHEP 12 (2014) 046 [1312.3296].
[152] A. Kitaev, “Hidden Correlations in the Hawking Radiation and Thermal Noise, talk at Fundamental Physics Prize Symposium, November 2014.”
[153] S. H. Shenker and D. Stanford, Stringy effects in scrambling, JHEP 05 (2015) 132 [1412.6087].
[154] A. Almheiri and J. Polchinski, Models of AdS$_2$ backreaction and holography, JHEP 11 (2015) 014 [1402.6334].
[155] K. Jensen, Chaos in AdS$_2$ Holography, Phys. Rev. Lett. 117 (2016) 111601 [1605.06098].
[156] J. Maldacena, D. Stanford and Z. Yang, Conformal symmetry and its breaking in two dimensional Nearly Anti-de-Sitter space, PTEP 2016 (2016) 12C104 [1606.01857].
[157] J. Engelsøy, T. G. Mertens and H. Verlinde, An investigation of AdS$_2$ backreaction and holography, JHEP 07 (2016) 139 [1606.03438].
[158] F. Haake, Quantum Signatures of Chaos; 3rd ed., Springer series in synergetics. Springer, Dordrecht, 2010, 10.1007/978-3-642-05428-0.
[159] K. Papadodimas and S. Raju, Local Operators in the Eternal Black Hole, Phys. Rev. Lett. 115 (2015) 211601 [1502.06692].
[160] A. Blommaert, T. G. Mertens and H. Verschelde, Clocks and Rods in Jackiw-Teitelboim Quantum Gravity, JHEP 09 (2019) 060 [1902.11194].
[161] P. Saad, Late Time Correlation Functions, Baby Universes, and ETH in JT Gravity, 1910.10311.

[162] P. Saad, S. H. Shenker and D. Stanford, A semiclassical ramp in SYK and in gravity, 1806.06840.

[163] P. H. Ginsparg and G. W. Moore, Lectures on 2-D gravity and 2-D string theory, in Theoretical Advanced Study Institute (TASI 92): From Black Holes and Strings to Particles, pp. 277–469, 10, 1993, hep-th/9304011.

[164] P. Di Francesco, P. H. Ginsparg and J. Zinn-Justin, 2-D Gravity and random matrices, Phys. Rept. 254 (1995) 1 [hep-th/9306153].

[165] N. Seiberg and D. Shih, Minimal string theory, Comptes Rendus Physique 6 (2005) 165 [hep-th/0409306].

[166] D. Marolf and H. Maxfield, Transcending the ensemble: baby universes, spacetime wormholes, and the order and disorder of black hole information, JHEP 08 (2020) 044 [2002.08950].

[167] P. Gao, D. L. Jafferis and A. C. Wall, Traversable Wormholes via a Double Trace Deformation, JHEP 12 (2017) 151 [1608.05687].

[168] J. Maldacena, D. Stanford and Z. Yang, Diving into traversable wormholes, Fortsch. Phys. 65 (2017) 1700034 [1704.05333].

[169] J. Maldacena and X.-L. Qi, Eternal traversable wormhole, 1804.00491.

[170] J. Maldacena, A. Milekhin and F. Popov, Traversable wormholes in four dimensions, 1807.04726.

[171] J. Maldacena and A. Milekhin, Humanly traversable wormholes, Phys. Rev. D 103 (2021) 066007 [2008.06618].

[172] Z. Fu, B. Grado-White and D. Marolf, Traversable Asymptotically Flat Wormholes with Short Transit Times, Class. Quant. Grav. 36 (2019) 245018 [1908.03273].

[173] A. R. Brown, H. Gharibyan, S. Leichenauer, H. W. Lin, S. Nezami, G. Salton et al., Quantum Gravity in the Lab: Teleportation by Size and Traversable Wormholes, 1911.06314.

[174] T. Schuster, B. Kobrin, P. Gao, I. Cong, E. T. Khabiboulline, N. M. Linke et al., Many-body quantum teleportation via operator spreading in the traversable wormhole protocol, 2102.00010.

[175] M. S. Blok, V. V. Ramasesh, T. Schuster, K. O’Brien, J. M. Kreikebaum, D. Dahlen et al., Quantum Information Scrambling on a Superconducting Qutrit Processor, Phys. Rev. X 11 (2021) 021010 [2003.03307].
[176] J. S. Cotler, G. Gur-Ari, M. Hanada, J. Polchinski, P. Saad, S. H. Shenker et al., Black Holes and Random Matrices, *JHEP* **05** (2017) 118 [1611.04650].

[177] D. Bagrets, A. Altland and A. Kamenev, Power-law out of time order correlation functions in the SYK model, *Nucl. Phys.* **B921** (2017) 727 [1702.08902].

[178] D. Stanford and E. Witten, Fermionic Localization of the Schwarzian Theory, *JHEP* **10** (2017) 008 [1703.04612].

[179] T. G. Mertens, G. J. Turiaci and H. L. Verlinde, Solving the Schwarzian via the Conformal Bootstrap, *JHEP* **08** (2017) 136 [1705.08408].

[180] A. Kitaev and S. J. Suh, Statistical mechanics of a two-dimensional black hole, *JHEP* **05** (2019) 198 [1808.07032].

[181] Z. Yang, The Quantum Gravity Dynamics of Near Extremal Black Holes, *JHEP* **05** (2019) 205 [1809.08647].

[182] L. V. Iliesiu and G. J. Turiaci, The statistical mechanics of near-extremal black holes, *JHEP* **05** (2021) 145 [2003.02860].

[183] M. Heydeman, L. V. Iliesiu, G. J. Turiaci and W. Zhao, The statistical mechanics of near-BPS black holes, *J. Phys. A* **55** (2022) 014004 [2011.01953].

[184] J. Preskill, P. Schwarz, A. D. Shapere, S. Trivedi and F. Wilczek, Limitations on the statistical description of black holes, *Mod. Phys. Lett. A* **6** (1991) 2353.

[185] S. D. Mathur, The Information paradox: A Pedagogical introduction, *Class. Quant. Grav.* **26** (2009) 224001 [0909.1038].

[186] S. L. Braunstein, S. Pirandola and K. Życzkowski, Better Late than Never: Information Retrieval from Black Holes, *Phys. Rev. Lett.* **110** (2013) 101301 [0907.1190].

[187] S. D. Mathur and D. Turton, The flaw in the firewall argument, *Nucl. Phys. B* **884** (2014) 566 [1306.5488].

[188] A. Almheiri, D. Marolf, J. Polchinski, D. Stanford and J. Sully, An Apologia for Firewalls, *JHEP* **09** (2013) 018 [1304.6483].

[189] D. Marolf and J. Polchinski, Gauge/Gravity Duality and the Black Hole Interior, *Phys. Rev. Lett.* **111** (2013) 171301 [1307.4706].

[190] A. C. Wall, The Generalized Second Law implies a Quantum Singularity Theorem, *Class. Quant. Grav.* **30** (2013) 165003 [1010.5513].

[191] G. T. Horowitz and J. M. Maldacena, The Black hole final state, *JHEP* **02** (2004) 008 [hep-th/0310281].

[192] D. Gottesman and J. Preskill, Comment on ‘The Black hole final state’, *JHEP* **03** (2004) 026 [hep-th/0311269].
[193] D. Stanford, Some open questions in quantum gravity, Talk at Gravitational Holography, KITP., January, 2020, https://online.kitp.ucsb.edu/online/qgravity20/stanford/.

[194] A. Almheiri, Comments on the final state proposal and the gravitational path integral, Talk at Recent progress in theoretical physics based on quantum information theory, YITP, March, 2020, http://www2.yukawa.kyoto-u.ac.jp/ qith2021/Almheiri.pdf.

[195] G. W. Gibbons and S. W. Hawking, Cosmological Event Horizons, Thermodynamics, and Particle Creation, Phys. Rev. D 15 (1977) 2738.

[196] R. Figari, R. Hoegh-Krohn and C. R. Nappi, Interacting Relativistic Boson Fields in the de Sitter Universe with Two Space-Time Dimensions, Commun. Math. Phys. 44 (1975) 265.

[197] W. Fischler and L. Susskind, Holography and cosmology, hep-th/9806039.

[198] R. Bousso, A Covariant entropy conjecture, JHEP 07 (1999) 004 [hep-th/9905177].

[199] R. Bousso and A. Shahbazi-Moghaddam, Island Finder and Entropy Bound, Phys. Rev. D 103 (2021) 106005 [2101.11648].

[200] J. Maldacena, G. J. Turiaci and Z. Yang, Two dimensional Nearly de Sitter gravity, JHEP 01 (2021) 139 [1904.01911].

[201] J. Cotler, K. Jensen and A. Maloney, Low-dimensional de Sitter quantum gravity, JHEP 06 (2020) 048 [1905.03780].

[202] T. Anous, J. Kruthoff and R. Mahajan, Density matrices in quantum gravity, SciPost Phys. 9 (2020) 045 [2006.17000].

[203] X. Dong, X.-L. Qi, Z. Shangnan and Z. Yang, Effective entropy of quantum fields coupled with gravity, JHEP 10 (2020) 052 [2007.02987].

[204] C. Krishnan, Critical Islands, JHEP 01 (2021) 179 [2007.06551].

[205] Y. Chen, V. Gorbenko and J. Maldacena, Bra-ket wormholes in gravitationally prepared states, 2007.16091.

[206] T. Hartman, Y. Jiang and E. Shaghoulian, Islands in cosmology, JHEP 11 (2020) 111 [2008.01022].

[207] M. Van Raamsdonk, Comments on wormholes, ensembles, and cosmology, 2008.02259.

[208] S. E. Aguilar-Gutierrez, A. Chatwin-Davies, T. Hertog, N. Pinzani-Fokeeva and B. Robinson, Islands in multiverse models, JHEP 11 (2021) 212 [2108.01278].

[209] E. Shaghoulian, The central dogma and cosmological horizons, JHEP 01 (2022) 132 [2110.13210].

[210] D. Teresi, Islands and the de Sitter entropy bound, 2112.03922.
[211] E. Shaghoulian and L. Susskind, *Entanglement in De Sitter Space*, 2201.03603.

[212] O. Lunin and S. D. Mathur, *AdS / CFT duality and the black hole information paradox*, Nucl. Phys. B 623 (2002) 342 [hep-th/0109154].

[213] H. Lin, O. Lunin and J. M. Maldacena, *Bubbling AdS space and 1/2 BPS geometries*, JHEP 10 (2004) 025 [hep-th/0409174].

[214] I. Bena, S. Giusto, R. Russo, M. Shigemori and N. P. Warner, *Habemus Superstratum! A constructive proof of the existence of superstrata*, JHEP 05 (2015) 110 [1503.01463].

[215] V. Jejjala, O. Madden, S. F. Ross and G. Titchener, *Non-supersymmetric smooth geometries and D1-D5-P bound states*, Phys. Rev. D 71 (2005) 124030 [hep-th/0504181].

[216] B. Ganchev, A. Houppe and N. Warner, *Q-Balls Meet Fuzzballs: Non-BPS Microstate Geometries*, 2107.09677.

[217] S. D. Mathur, *The Fuzzball proposal for black holes: An Elementary review*, Fortsch. Phys. 53 (2005) 793 [hep-th/0502050].

[218] K. Skenderis and M. Taylor, *The fuzzball proposal for black holes*, Phys. Rept. 467 (2008) 117 [0804.0552].

[219] B. Guo, M. R. R. Hughes, S. D. Mathur and M. Mehta, *Contrasting the fuzzball and wormhole paradigms for black holes*, 2111.05295.

[220] R. E. Prange, *The Spectral Form Factor Is Not Self-Averaging*, Phys. Rev. Lett. 78 (1997) 2280 [chao-dyn/9606010].

[221] F. Haake, H.-J. Sommers and J. Weber, *Fluctuations and ergodicity of the form factor of quantum propagators and random unitary matrices*, Journal of Physics A Mathematical General 32 (1999) 6903 [chao-dyn/9906024].

[222] J. L. Barbon and E. Rabinovici, *Geometry And Quantum Noise*, Fortsch. Phys. 62 (2014) 626 [1404.7085].

[223] D. Stanford, *More quantum noise from wormholes*, 2008.08570.

[224] P. Saad, S. H. Shenker and D. Stanford, *JT gravity as a matrix integral*, 1903.11115.

[225] D. Stanford and E. Witten, *JT Gravity and the Ensembles of Random Matrix Theory*, 1907.03363.

[226] R. Bousso and M. Tomašević, *Unitarity From a Smooth Horizon?*, Phys. Rev. D 102 (2020) 106019 [1911.06305].

[227] R. Bousso and E. Wildenhain, *Gravity/Ensemble Duality*, 2006.16289.

[228] E. Witten and S.-T. Yau, *Connectedness of the boundary in the AdS / CFT correspondence*, Adv. Theor. Math. Phys. 3 (1999) 1635 [hep-th/9910245].
[229] J. McNamara and C. Vafa, *Baby Universes, Holography, and the Swampland*, 2004.06738.

[230] A. Blommaert, T. G. Mertens and H. Verschelde, *Eigenbranes in Jackiw-Teitelboim gravity*, 1911.11603.

[231] L. Eberhardt, *Partition functions of the tensionless string*, JHEP 03 (2021) 176 [2008.07533].

[232] L. Eberhardt, *Summing over Geometries in String Theory*, 2102.12355.

[233] P. Saad, S. H. Shenker, D. Stanford and S. Yao, *Wormholes without averaging*, 2103.16754.

[234] P. Saad, S. Shenker and S. Yao, *Comments on wormholes and factorization*, 2107.13130.

[235] A. Blommaert, L. V. Iliesiu and J. Kruthoff, *Gravity factorized*, 2111.07863.

[236] A. Almheiri and H. W. Lin, *The Entanglement Wedge of Unknown Couplings*, 2111.06298.

[237] B. Mukhametzhanov, *Half-wormholes in SYK with one time point*, 2105.08207.

[238] J. Cotler and K. Jensen, *AdS$_3$ gravity and random CFT*, 2006.08648.

[239] R. Mahajan, D. Marolf and J. E. Santos, *The double cone geometry is stable to brane nucleation*, 2104.00022.

[240] A. Belin and J. de Boer, *Random Statistics of OPE Coefficients and Euclidean Wormholes*, 2006.05499.

[241] N. Afkhami-Jeddi, H. Cohn, T. Hartman and A. Tajdini, *Free partition functions and an averaged holographic duality*, 2006.04839.

[242] A. Maloney and E. Witten, *Averaging Over Narain Moduli Space*, 2006.04855.

[243] S. Collier and A. Maloney, *Wormholes and Spectral Statistics in the Narain Ensemble*, 2106.12760.

[244] N. Benjamin, C. A. Keller, H. Ooguri and I. G. Zadeh, *Narain to Narnia*, 2103.15826.

[245] V. Meruliya, S. Mukhi and P. Singh, *Poincaré Series, 3d Gravity and Averages of Rational CFT*, 2102.03136.

[246] N. Benjamin, S. Collier, A. L. Fitzpatrick, A. Maloney and E. Perlmutter, *Harmonic analysis of 2d CFT partition functions*, 2107.10744.

[247] J. J. Heckman, A. P. Turner and X. Yu, *Disorder Averaging and its UV (Dis)Contents*, 2111.06404.

[248] J. Polchinski and A. Strominger, *A Possible resolution of the black hole information puzzle*, Phys. Rev. D 50 (1994) 7403 [hep-th/9407008].
[249] D. Marolf and H. Maxfield, Observations of Hawking radiation: the Page curve and baby universes, *JHEP* **04** (2021) 272 [2010.06602].
[250] D. Marolf and H. Maxfield, The Page curve and baby universes, 2105.12211.
[251] D. Harlow and H. Ooguri, Symmetries in quantum field theory and quantum gravity, *Commun. Math. Phys.* **383** (2021) 1669 [1810.05338].
[252] C. Akers and G. Penington, Quantum minimal surfaces from quantum error correction, 2109.14618.
[253] A. May, Quantum tasks in holography, *JHEP* **10** (2019) 233 [1902.06845].
[254] A. May, G. Penington and J. Sorce, Holographic scattering requires a connected entanglement wedge, *JHEP* **08** (2020) 132 [1912.05649].
[255] V. Chandrasekaran, T. Faulkner and A. Levine, Scattering strings off quantum extremal surfaces, 2108.01093.
[256] A. Dabholkar, Strings on a cone and black hole entropy, *Nucl. Phys. B* **439** (1995) 650 [hep-th/9408098].
[257] S. He, T. Numasawa, T. Takayanagi and K. Watanabe, Notes on Entanglement Entropy in String Theory, *JHEP* **05** (2015) 106 [1412.5606].
[258] E. Witten, Open Strings On The Rindler Horizon, *JHEP* **01** (2019) 126 [1810.11912].
[259] S. A. Hartnoll and E. Mazenc, Entanglement entropy in two dimensional string theory, *Phys. Rev. Lett.* **115** (2015) 121602 [1504.07985].
[260] W. Donnelly and G. Wong, Entanglement branes in a two-dimensional string theory, *JHEP* **09** (2017) 097 [1610.01719].
[261] V. Balasubramanian and O. Parrikar, Remarks on entanglement entropy in string theory, *Phys. Rev. D* **97** (2018) 066025 [1801.03517].
[262] V. E. Hubeny, R. Pius and M. Rangamani, Topological string entanglement, *JHEP* **10** (2019) 239 [1905.09890].
[263] L. Susskind and J. Uglum, Black hole entropy in canonical quantum gravity and superstring theory, *Phys. Rev. D* **50** (1994) 2700 [hep-th/9401070].
[264] M. J. Bowick, L. Smolin and L. C. R. Wijewardhana, Role of String Excitations in the Last Stages of Black Hole Evaporation, *Phys. Rev. Lett.* **56** (1986) 424.
[265] L. Susskind, Some speculations about black hole entropy in string theory, hep-th/9309145.
[266] A. Sen, Extremal black holes and elementary string states, *Mod. Phys. Lett. A* **10** (1995) 2081 [hep-th/9504147].
[267] G. T. Horowitz and J. Polchinski, *A Correspondence principle for black holes and strings*, *Phys. Rev. D* **55** (1997) 6189 [hep-th/9612146].

[268] G. T. Horowitz and J. Polchinski, *Selfgravitating fundamental strings*, *Phys. Rev. D* **57** (1998) 2557 [hep-th/9707170].

[269] Y. Chen, J. Maldacena and E. Witten, *On the black hole/string transition*, 2109.08563.

[270] D. Marolf and A. C. Wall, *Eternal Black Holes and Superselection in AdS/CFT*, *Class. Quant. Grav.* **30** (2013) 025001 [1210.3590].

[271] D. Harlow, *Wormholes, Emergent Gauge Fields, and the Weak Gravity Conjecture*, *JHEP* **01** (2016) 122 [1510.07911].

[272] M. Guica and D. L. Jafferis, *On the construction of charged operators inside an eternal black hole*, *SciPost Phys.* **3** (2017) 016 [1511.05627].

[273] D. Harlow and D. Jafferis, *The Factorization Problem in Jackiw-Teitelboim Gravity*, 1804.01081.