High Energy FCNC search through $e\mu$ Colliders

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Abstract

We study the potential impacts of a new type of particle collider – an $e\mu$ collider – on the search for new physics beyond the Standard Model. As our first attempt for exploring its physics potential, we demonstrate that the $e\mu$ collision experiment can be highly efficient in searching for lepton-number-violating Flavor Changing Neutral Current phenomena.

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Although it is possible to explain the observed CP violation within the framework of the Standard Model (SM), it is generally believed that the amount of CP asymmetry predicted by the SM is insufficient to explain the observed non-zero baryon asymmetry in the universe, which inevitably requires a much larger extent of CP asymmetry. Consequently, it is expected that there must be new physics beyond the SM in the high energy regime, such as SUSY, GUT etc. One of the key signatures for such new physics is the lepton-number-violating Flavor Changing Neutral Current (FCNC) phenomenon. There have been numerous theoretical studies on lepton-number-violating FCNC by using new models, e.g., the generalized two Higgs doublet model, as well as by considering various collider and decay processes. Moreover, there have been experimental studies from Los Alamos, CERN etc., on low-energy reactions, $\mu^- \rightarrow e^- \gamma$, $\mu^- \rightarrow e^- e^+ e^-$. All these muon decay experiments, however, are limited within the low energy regime by the muon’s small rest mass. Therefore, even if any FCNC effects due to an unknown massive neutral particle exist, the effects would be severely suppressed due to its high virtuality.

In this paper, we show that the $e\mu$ collision, which is very similar to the above mentioned $\mu$ decay reactions, can be a powerful alternative to explore such FCNC phenomena. The $e\mu$ collision, in connection with the problem of muonium-antimuonium transitions, $\mu^+ e^- \leftrightarrow \mu^- e^+$, through doubly charged Higgs, $\Delta^{++}$, dilepton gauge boson, $X^{++}$, or flavor changing neutral scalar bosons, $H$ and $A$, has been first illustrated by Hou. By using a simple model-independent calculation, we demonstrate that the $e\mu$ collision experiment can be much more efficient than the present low-energy rare $\mu$ decay experiments and $e^+ e^-$ collision experiments, such as $\mu^- \rightarrow e^- \gamma$, $\mu \rightarrow eee$ and $e^+ e^- \rightarrow e^\pm \mu^\mp$, in searching for lepton-number-violating FCNC phenomena. In the SM, the probability of $\mu^- \rightarrow e^- \gamma$ and $\mu \rightarrow eee$ are absolutely zero, while their experimental upper limits are $4.9 \times 10^{-11}$ and $1.0 \times 10^{-12}$, respectively. The $e^+ e^-$ and $p\bar{p}$ collisions which have been most powerful tools in the high-energy particle physics experiments, or even a $\mu^+ \mu^-$ collision which is being studied for future experiments, are dominated by the SM interaction via such well-known particles as $\gamma$ (photon) or $Z^0$. On the contrary, the s-channel $e\mu$ interaction can never be mediated
by $\gamma$ or $Z^0$, and hence is very sensitive to the effects of new and unknown neutral particles.

The tree level FCNC phenomena can be detected straightforwardly in $e\mu$ collisions through

$$e^\pm \mu^\pm \rightarrow f_1 f_2, \quad f_{1,2} = \text{lepton, quark, gauge boson, supersymmetric particle etc.,}$$

i.e. through any two-body (or more-body) final state except for a few channels allowed in the SM. We explain several important advantages of the $e\mu$ collision experiment over the existing methods such as $\mu \rightarrow e\gamma$, $\mu \rightarrow eee$, and $e^+ e^- \rightarrow \mu^+ e^\mp$:

(i) $e\mu \rightarrow f_1 f_2$ vs $\mu \rightarrow e\gamma$: The advantage of $e\mu$ collision is obvious in this comparison. While the latter reaction can allow us to detect FCNC caused only through photon mediation, the $e\mu$ collision experiment enables us to detect FCNC not only through $\gamma$ or $Z$ mediation but also through the exchange of a new neutral particle, e.g. a supersymmetric Higgs, neutralino, scalar or vector GUT gauge boson, and so on.

(ii) $e\mu \rightarrow f_1 f_2$ vs $\mu \rightarrow eee$: The $e\mu$ collision experiment, where $e$ and $\mu$ can be accelerated up to very high energies, has great advantages for new physics at high-energy scales which can not be directly probed by the latter low-energy process $\mu \rightarrow eee$. In this light, the $e\mu$ collision experiment becomes much more powerful for a heavier particle mass scale. Needless to say, by lowering beam energy, we can also investigate the process equivalent to $\mu \rightarrow eee$, thus providing a cross-check for the FCNC search results obtained from low-energy $\mu$ decay experiments.

(iii) $e\mu \rightarrow f_1 f_2$ vs $e^+ e^- \rightarrow \mu^\pm e^\mp$: The two processes have an identical physics origin, if $f_{1,2} = e^+ e^-$. The $e\mu$ collision experiment has, however, a large number of channels such as $e\mu \rightarrow ee, \mu\mu, \tau\tau, \tau e, \tau\mu, u\bar{u}, d\bar{d}, s\bar{s}, c\bar{c}, b\bar{b}, t\bar{t}, u\bar{c}, u\bar{t}, c\bar{t}, d\bar{s}, s\bar{b}, c\bar{u}, ..., gg, W^+W^-, ...,$ and so on, while the $ee$ collision experiment has only three relevant processes, $ee \rightarrow \mu e$, $ee \rightarrow \tau e$ and $ee \rightarrow \tau\mu$. Moreover, once we consider the color factor $N_C = 3$, and the production of gauge bosons, supersymmetric particles or possible new scalar and vector boson pair productions, then the number of available channels becomes even larger. Another advantage of $e\mu \rightarrow f_1 f_2$ reaction over $e^+ e^- \rightarrow \mu^\pm e^\mp$ is that muons can be easily accelerated to very high energies.
without much synchrotron radiation loss because of their large mass compared to that of an electron, thus making $e\mu$ collider a much better option than the conventional $e^+e^-$ collider to reach ultra high energy regime for FCNC search.

To make a simple comparison of $e^\pm \mu^\mp \rightarrow f_1 f_2$ mode to the other cases, let us assume that the FCNC is mediated by an unknown neutral boson $X$ of mass $M_X$ and width $\Gamma_X$. When the electron mass is neglected, the decay width of $\mu^- \rightarrow e^- e^+ e^-$ is given by

$$\Gamma(\mu^- \rightarrow e^- e^+ e^-) = \frac{m_\mu^5}{2048\pi^3} \left| g_{e\mu} g_{ee} \right|^2 \frac{M_X^2}{M_X^2},$$

(1)

for a scalar boson $X$, while the decay width becomes

$$\Gamma(\mu^- \rightarrow e^- e^+ e^-) = \frac{m_\mu^5}{384\pi^3} \left| g_{e\mu} g_{ee} \right|^2 \frac{M_X^2}{M_X^2},$$

(2)

for a vector boson $X$. Here $g_{e\mu}^s (g_{ee}^s)$ and $g_{e\mu}^v (g_{ee}^v)$ are the appropriate flavor-changing (flavor-conserving) coupling strength of leptons with scalar $X$ and vector $X$, respectively. In both cases, the total decay width decreases by $1/M_X^4$, assuming that the coupling strengths are independent of $M_X$.

On the other hand, the cross-section of $e^\pm \mu^\mp \rightarrow e^+ e^-$ takes the following form (we simply chose the case $f_1 f_2 = e^+ e^-$):

- For a scalar $X$,

$$\sigma_s(e^\mp \mu^\mp \rightarrow e^+ e^-) = \frac{|g_{e\mu} g_{ee}|^2}{16\pi s} \left[ |\Pi(s)|^2 - \text{Re}[\Pi(s)] L(\xi) + 2 \left( L(\xi) - \frac{1}{\xi+2} \right) \right],$$

(3)

where $\xi = 2M_X^2/s$, and the two functions $\Pi(s)$ and $L(\xi)$ are defined as

$$\Pi(s) = \frac{s}{s - M_X^2 + i\sqrt{s}\Gamma_X}, \quad L(\xi) = 1 - \frac{\xi}{2} \log \left( \frac{\xi + 2}{\xi} \right).$$

The peak cross-section of $e^\pm \mu^\mp \rightarrow e^+ e^-$ at $s = M_X^2$ is then given by

$$\sigma_s(e^\mp \mu^\mp \rightarrow e^+ e^-)|_{s=M_X^2} = \frac{|g_{e\mu} g_{ee}|^2}{16\pi M_X^2} \left[ 1 + 2 \left( \frac{\Gamma_X}{M_X} \right)^2 \left( \frac{3}{4} - \log 2 \right) \right].$$

(4)
For a vector \( X \),

\[
\sigma_v(e^\mp \mu^\pm \rightarrow e^+ e^-) = \frac{|g_{e\mu} g_{\mu e}|^2}{32\pi s} \left[ \frac{8}{3} |\Pi(s)|^2 - \frac{1}{2} \text{Re}[\Pi(s)]A(\xi) + B(\xi) \right],
\]

where the functions \( A(\xi) \) and \( B(\xi) \) are given by

\[
A(\xi) = 2(3 + \xi) - (2 + \xi)^2 \log\left(\frac{\xi + 2}{\xi}\right),
\]

\[
B(\xi) = 2 \left[ 4 + \frac{8}{\xi} - \frac{4}{\xi + 2} - 2(\xi + 2) \log\left(\frac{\xi + 2}{\xi}\right) \right].
\]

The corresponding peak cross-section at \( s = M_X^2 \) is then

\[
\sigma_v(e^\mp \mu^\pm \rightarrow e^+ e^-)|_{s = M_X^2} = \frac{|g_{e\mu} g_{\mu e}|^2}{\pi \Gamma_X^2} \left[ \frac{1}{12} + \frac{1}{16} \left(\frac{\Gamma_X}{M_X}\right)^2 \right] (7 - 8 \log 2).
\]

Note that in a sharp contrast to \( \Gamma(\mu^- \rightarrow e^- e^+ e^-) \), which decreases as \( 1/M_X^4 \), the peak cross-section \( \sigma(e^\mp \mu^\pm \rightarrow e^+ e^-)|_{s = M_X^2} \) is simply proportional to \( 1/\Gamma_X^2 \) if \( \Gamma_X \ll M_X \). As \( \Gamma_X/M_X \) is typically \( 10^{-2} \)-\( 10^{-3} \) for a weakly decaying \( X \), it is obvious that the \( e\mu \) collision experiment can be much more efficient in the search for FCNC phenomena mediated by a heavy neutral particle than the low-energy rare decay process, \( \mu \rightarrow eee \).

In order to estimate the experimental sensitivity of \( e\mu \) collision to the FCNC phenomena, we make an assumption for the coupling strengths using the upper limit of experimental branching ratio for the \( \mu^- \rightarrow e^- e^+ e^- \) decay as a guide, \( \mathcal{BR}(\mu^- \rightarrow e^- e^+ e^-) < 1.0 \times 10^{-12} \).

For simplicity, we consider the case where \( X \) is a vector. From Eq. (2), and the total decay width of muon which is, in a good approximation, \( \Gamma_{\text{total}}(\mu) \simeq \Gamma(\mu \rightarrow e\nu\bar{\nu}) = G_F^2 m_{\mu}^5 / 192\pi^3 \), we get the branching ratio,

\[
\mathcal{BR}(\mu^- \rightarrow e^- e^+ e^-) \simeq 16 \left(\frac{\sqrt{g_{e\mu} g_{\mu e}}}{g_{ee}}\right)^4 \left(\frac{m_Z}{M_X}\right)^4.
\]

In addition, for simplicity, we assume the flavor conserving coupling \( g_{ee}^V \) to be equal to \( g_{ee}^Z \), the electroweak electron coupling to \( Z \) boson. Then the experimental upper limit gives a constraint for the coupling strength \( g_{e\mu}^V \) and the mass \( M_X \);
\[ \frac{g_{e\mu}^V}{g_{ee}^Z} < \frac{\sqrt{BR}}{4} \left( \frac{M_X}{M_Z} \right)^2. \]  

(8)

Fig. 1 shows the ratio of coupling strengths \( g_{e\mu}^V/g_{ee}^Z \) as a function of \( M_X \) for various values of \( BR(\mu^- \rightarrow e^- e^+ e^-) \): \( 10^{-12} \) (solid curve), \( 10^{-13} \) (dashed curve), and \( 10^{-15} \) (dotted curve). Since the present experimental upper limit is \( 1.0 \times 10^{-12} \) \([6]\), the region above the solid curve is experimentally excluded.

Note that the measurement of the decay width \( \Gamma(\mu^- \rightarrow e^- e^+ e^-) \) alone, even if it were measured precisely, cannot determine the coupling strength \( g_{e\mu}^V \) and \( M_X \) separately. We clearly need a high-energy collision experiment so that we can scan the energy ranges and directly determine the mass and width of the new intermediate boson \( X \). Only with the information on the mass and width, we can determine the coupling strengths.

To make a simple quantitative estimate of the required luminosity for an \( e\mu \) collider, we first choose the value of \( M_X \), and then from Eq. (8) and Fig. 1 we decide coupling strengths by assuming a branching ratio to be an order of magnitude smaller than the current experimental upper limit, \( e.g. \) for \( M_X = 500 \) GeV we get \( g_{e\mu}^S / g_{ee}^Z = 5.50 \times 10^{-6} \) and \( g_{e\mu}^V / g_{ee}^Z = 2.38 \times 10^{-6} \), which corresponds to \( BR(\mu^- \rightarrow e^- e^+ e^-) = 10^{-13} \) (\( i.e. \) \( \frac{1}{10} \) of the current experimental upper limit). The width of \( X \) is chosen to be proportional \([10]\) to the mass \( M_X \), after taking \( \Gamma_X = 20 \) GeV for \( M_X = 500 \) GeV. Fig. 2 shows the calculated cross-section as a function of \( \sqrt{s} \) for \( M_X = 500, 1000, 2000 \) GeV, with the coupling strengths determined from Fig. 1 assuming \( BR(\mu^- \rightarrow e^- e^+ e^-) = 10^{-13} \).

With those preset assumptions of coupling strengths, mass and width for a new neutral boson \( X \), the evaluated peak cross-section is found to be in the range of \( 1 \) fb for \( M_X \sim 1 \) TeV. As mentioned earlier, we emphasize again that the FCNC signals can be depicted in the \( e\mu \) collision via enormously many final-state channels. Therefore, even if we conservatively count this large number of channels as a factor of 10 increase in the summed cross-section of visible FCNC channels, we expect to observe a significant number of FCNC events on the resonance peak with \( 1 \) fb\(^{-1}\) integrated luminosity. Without a substantial background, that would be sufficient to claim an experimental evidence for FCNC, but the background issue
shall be more carefully studied with details of detector and collider design parameters.

On the other hand, Fig. 2 shows that the cross-section off the resonance peak is typically about \( \sim 10^{-2} \, \text{fb} \) for \( \sqrt{s} \sim 1 \, \text{TeV} \). Once again, if we count the multi-channel final state as a factor 10 enhancement in the total visible cross-section, this implies that an integrated luminosity of 100 \( \text{fb}^{-1} \) will be sufficient to experimentally observe quite a few FCNC signals under low-background. Note that the B-factory experiments being prepared at SLAC \(^{[1]}\) and KEK \(^{[12]}\) are aiming at a luminosity of 100 \( \text{fb}^{-1}/\text{year} \). Therefore, we conclude that if we can maintain the \( e\mu \) collision luminosity at the level of the B-factory experiments, and if the intermediate boson \( X \) has the properties that we have assumed, then we may have a very good chance to observe FCNC signals in a few years running. The chance can be much enhanced if we run the experiment at the right energy, \( i.e. \) at or near \( \sqrt{s} = M_X \).

In conclusion, we have investigated the physics potential of \( e\mu \) collision by using a simple model calculation and demonstrated that the \( e\mu \) collision experiment can offer the best laboratory to search for FCNC at high energies.

**Note added:** After we finished the first version of this manuscript, we found the similar work by Barger et al. \(^{[13]}\), which suggested to use a relatively low energy muon beam that may be available during the first stages of muon collider to probe physics.

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FIG. 1. The ratio of coupling strengths $g_{e\mu}^{V}/g_{ee}^{Z}$ as a function of $M_X$ for various values of $BR(\mu \rightarrow eee)$. In this plot, $X$ is assumed to be a vector. For a scalar $X$, the shape is very similar but the coupling strength is about twice as big. Since the experimental upper limit for the branching ratio is $10^{-12}$, the region above the solid curve is experimentally excluded.
FIG. 2. The cross-section $\sigma(e^\mp \mu^\pm \rightarrow e^\pm e^-)$ vs. $\sqrt{s}$ for $M_X = 500, 1000, 2000$ GeV: (a) for $X$ being a scalar, and (b) for a vector, respectively. The $X$ width $\Gamma_X$ is assumed to be proportional to $M_X$, and $\Gamma_X = 20$ GeV is taken for $M_X = 500$ GeV.