EVOLUTION OF PRIMORDIAL STARS POWERED BY DARK MATTER ANNIHILATION UP TO THE MAIN-SEQUENCE STAGE

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ABSTRACT

Primordial stars formed in the early universe are thought to be hosted by compact dark matter (DM) halos. If DM consists of weakly interacting massive particles (WIMPs), such stars may be powered by DM annihilation during the early phases of their evolution. We study the pre-main-sequence evolution of the primordial star using a detailed stellar evolution code under the assumption that the annihilation of adiabatically contracted WIMP DM within the star provides sufficient energy to sustain the stellar equilibrium. We follow the evolution of accreting stars using several gas mass accretion rates derived from cosmological simulations. We show that the stellar mass becomes very large, up to 900–1000 \(M_\odot\) when the star reaches the main-sequence phase for a reasonable set of model parameters such as DM particle mass and the annihilation cross section. During the dark star phase, the star expands by over a thousand solar radii, while the surface temperature remains below 10\(^4\) K. The energy generated by nuclear reactions is not dominant during this phase. We also study models with different gas mass accretion rates and the DM particle masses. All our models for different DM particle masses pass the dark star phase. The final mass of the dark stars is essentially unchanged for DM mass of \(m_\chi \lesssim 10\) GeV. Gravitational collapse of the massive dark stars will leave massive black holes with mass as large as 1000 \(M_\odot\) in the early universe.

Key words: dark matter – stars: evolution – stars: Population III

Online-only material: color figures

1. INTRODUCTION

The first stars in the universe may have contributed to early cosmic reionization and may have also enriched the intergalactic medium with heavy elements such as carbon, oxygen, and iron (see, e.g., Bromm et al. 2009 for a review). Future observations of the distant universe will exploit large ground-based and space-borne telescopes such as the James Webb Space Telescope (JWST) and Thirty Meter Telescope to answer the important questions of how and when the first stars were formed and how they affected the subsequent evolution of the universe.

Recent theoretical studies based on cosmological simulations suggest that the first stars are formed in dark matter (DM) halos with mass \(10^5–10^6 M_\odot\) at \(z \sim 20–30\) (Tegmark et al. 1997; Yoshida et al. 2003). In such a cosmological “minihalo,” the gas cools and condenses by molecular hydrogen cooling to form a star-forming gas cloud. The gravitationally unstable cloud further contracts, and finally a protostar is born inside it (Omukai & Nishi 1998; Yoshida et al. 2008). A unique characteristic of primordial star formation in the standard cold DM model is that the star and its parent gas cloud are embedded at the center of the host dark halo. Thus the formation and evolution of primordial stars may be much affected by DM dynamically.

The nature of DM still remains unknown, but the best candidates are thought to be weakly interacting massive particles (WIMPs). WIMP DM such as neutralinos must have a large self-annihilation cross-section in order to be a dominant component of DM in the present-day universe. DM annihilation produces an enormous amount of energy, essentially equal to the rest mass energy of the annihilated DM particles. If DM density can be very large in a primordial gas cloud and in a primordial star, the annihilation energy would act as an efficient heat input. DM annihilation could even supply sufficient energy to support self-gravitation of a star. This is indeed a recently proposed new stellar phase, in which stars are powered by DM annihilation energy instead of that of nuclear fusion (Spolyar et al. 2008). Hereafter we call such a star a “dark star.”

Spolyar et al. (2008) suggest that, in a collapsing primordial gas cloud, DM annihilation heating can win over the radiative cooling, effectively halting further collapse of the cloud. Natarajan et al. (2009) study the evolution of the DM density profile during the first star formation using cosmological simulations. They show that the DM density profile indeed becomes very steep, as much as expected by adiabatic contraction models.

There have been many previous studies on dark stars. While most of them assume constant stellar mass models (Iocco et al. 2008; Taoso et al. 2008; Yoon et al. 2008). Spolyar et al. (2009) and Umeda et al. (2009) study dark star evolution with gas mass accretion. By using an approximated stellar structure model, Spolyar et al. (2009) follow dark star evolution with adiabatically contracted DM (ACDM). They show that the final stellar mass becomes as large as 1000 \(M_\odot\). Umeda et al. (2009) perform detailed, dynamically self-consistent stellar evolution calculations up to gravitational core collapse. They show that the final mass can be as large as \(10^4–10^5 M_\odot\) or more, if the star captured DM efficiently. However, the DM capture rate itself is uncertain because it depends on the unknown DM–baryon scattering cross-section and also on the ambient DM density during the dark star evolution. Thus the proposed formation of intermediate massive black holes (BHs) appears to occur only in very particular cases.

An attractive feature of the dark star models is the possibility for the birth of massive stars, and for BHs as the end of such stars, in the early universe. Supermassive BHs (SMBHs) existed already at \(z \sim 6\), when the universe is less than one billion years old, but the formation mechanism of such early SMBHs...
remains unknown. Gravitational collapse of a very massive dark star might provide a solution to this problem. We explore a possibility that the first stars formed in DM halos can become very massive by being powered by DM annihilation. To this end, it is important to determine the final state of the dark star phase exactly and to determine the final stellar mass.

In this paper, we study the evolution of an accreting dark star using a detailed stellar evolution code as in Umeda et al. (2009) but with the ACDM annihilation. We calculate the density profile of ACDM using an analytical method and determine the DM annihilation rate. We then follow the evolution of a dark star which grows in mass by gas accretion. The dark star model has effectively two parameters, the gas mass accretion rate and DM particle mass. We calculate several models with different sets of parameters and investigate the effects on the final stellar mass. Especially, we clarify the effect of different mass accretion rates, which were not explicitly shown in a similar work by Spolyar et al. (2009).

The rest of the paper is organized as follows. In Section 2, we introduce our numerical calculation and explain how we implement DM annihilation in the stellar evolution. Section 3 shows the results of our dark star models and we discuss the implications in Section 4. We give our concluding remarks in Section 5.

2. METHOD

We use a stellar evolution code with gas mass accretion developed by Ohkubo et al. (2006, 2009) and Umeda et al. (2009). We implement energy generation by DM annihilation in the code in the following manner: We consider a primordial (proto)star embedded at the center of a small mass DM halo. The DM density profile, \( \rho_d \), is calculated using the analytical method of Blumenthal et al. (1986); when the primordial gas cools and collapses, DM particles are also “dragged” into the gravitational potential well. The adiabatic contraction model uses an adiabatic invariant. For spherically distributed DM and baryons with a total mass of \( M(R) \), an adiabatic invariant is

\[
M(R) R = \text{const.} \quad (1)
\]

Freese et al. (2009) compared the DM density profile calculated using this method and that calculated using a more accurate method developed by Young (1980). They conclude that the difference is not more than a factor of two.

We have performed direct numerical simulations of early structure formation to check the accuracy of this analytical method. To this end we have used the parallel N-body/smoothed particle hydrodynamics solver GADGET-2 (Springel 2005) in its version suitably adapted to follow radiative cooling processes at very high densities (Yoshida et al. 2006). We have found that the DM density calculated by Blumenthal’s method is indeed in good agreement with the result of our direct numerical simulations. Figure 1 compares the results from the analytic model and the simulations. Details will be presented elsewhere (S. Hirano et al. 2011, in preparation). We note that, although these two results agree very well, confirming the validity of the analytic model, the simulations do not resolve the DM density down to length scales of astronomical units. Because the gas density profile itself evolves in a self-similar manner (Omukai & Nishi 1998), we assume that the DM continues to contract adiabatically and the evolution of DM density can be calculated by Blumenthal’s method.

In this work, we only consider the DM density evolution based on adiabatic contraction. In principle, DM annihilation can occur even more efficiently in the star by capture. If the density of the stellar core increases enough to scatter and trap DM particles, the DM particles can rapidly sink toward the center, to self-annihilate efficiently. During the dark star phase, however, the effect of DM capture is unimportant because the star has expanded extremely and the gas density is low.

Suppose a “cloud” of DM particles contract from a radial position \( R_{\text{pre}} \) to \( R_{\text{new}} \), and the total mass distribution changes from \( M_{\text{pre}}(R) \) to \( M_{\text{new}}(R) \). Then the following relation holds during adiabatic contraction:

\[
M_{\text{new}}(R_{\text{new}}) R_{\text{new}} = M_{\text{pre}}(R_{\text{pre}}) R_{\text{pre}}. \quad (2)
\]

The “new” DM density profile can be simply calculated from the above equation. Note that, in our case, the gas distribution also changes because of gas accretion onto the central primordial protostar. We model the gas mass accretion rate in a simple parameterized form as a function of the stellar mass. We use the accretion rate calculated by cosmological simulations of Gao et al. (2007). They found a large variation of accretion rates. In particular, the accretion rates for rotationally supported disks are typically smaller than for other cases. As our fiducial model, we choose the accretion rate of their R5 run, which is well described as a power law,

\[
dM \over dT = 0.18 \times M^{-0.6} \text{ M}_\odot \text{yr}^{-1}. \quad (3)
\]

Hereafter we call this rate the “G-rate” (see Figure 2). The rate is for a cosmological halo forming from a very high-\( \sigma \) peak and is slightly larger than that adopted in a previous work (Spolyar et al. 2009). We also run models with three different accretion rates: 1.0, 0.5, and 0.2 × G-rate. The intermediate value, 0.5 × G-rate, is close to that used in Spolyar et al. (2009). We use this model to compare with their result.

The energy generation rate of DM annihilation is given by

\[
Q_{\text{DM}} = \frac{\langle \sigma v \rangle \rho^2}{m_X} \text{ GeV cm}^{-3} \text{ s}^{-1}, \quad (4)
\]
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Figure 2. Gas mass accretion rates adopted in our stellar evolution calculations. “G-rate” is the accretion rate of Equation (3). We adopt three rates: 1.0, 0.5, and 0.2 × G-rate. The other two lines are the rates adopted in Spolyar et al. (2009) for comparison. (A color version of this figure is available in the online journal.)

Figure 3. Hertzsprung–Russell (H-R) diagram for our base model. Symbols indicate the stellar mass of $M = 100$, 200, 600, 800, and 1000 $M_\odot$. The solid line is for the dark star model (“base model”), whereas the dashed line is for the standard Population III model (no-DM model of Umeda et al. 2009). Symbols indicate the times when the stellar mass is $M = 100$, 200, 600, 800, and 1000 $M_\odot$. Clearly, the dark star model moves on a very different path from that of the no-DM model, showing that the DM annihilation energy has a significant effect. In the no-DM model, the star contracts gravitationally and becomes hot quickly until the first symbol (100 $M_\odot$). The dark star, on the other hand, has essentially the same effective temperature because DM annihilation energy prevents the star from gravitational contraction. When the no-DM star reaches $\sim 100$ $M_\odot$, it expands and turns to the lower temperature side in the H-R diagram. There, the hydrogen burning becomes effective in supplying energy to stop the contraction and the star expands slightly. Subsequently, the star lands on the main-sequence phase.

3. RESULTS

3.1. Base Model

Figure 3 shows the evolution of the dark star in the Hertzsprung–Russell (H-R) diagram. The accreting star grows constantly in mass. In the H-R diagram, the star moves upward and then turns to the left (higher temperature). The solid line is for our dark star model, whereas the dashed line is for a standard Population III model (without DM annihilation energy, no-DM model; Umeda et al. 2009). Symbols indicate the times when the stellar mass is $M = 100$, 200, 600, 800, and 1000 $M_\odot$. Clearly, the dark star model moves on a very different path from that of the no-DM model, showing that the DM annihilation energy has a significant effect. In the no-DM model, the star contracts gravitationally and becomes hot quickly until the first symbol (100 $M_\odot$). The dark star, on the other hand, has essentially the same effective temperature because DM annihilation energy prevents the star from gravitational contraction. When the no-DM star reaches $\sim 100$ $M_\odot$, it expands and turns to the lower temperature side in the H-R diagram. There, the hydrogen burning becomes effective in supplying energy to stop the contraction and the star expands slightly. Subsequently, the star lands on the main-sequence phase.

Contrastingly, the dark star remains “cool” while it is powered by DM annihilation. This is because the DM annihilation energy (Equation (4)) is independent of the stellar temperature. Even if the stellar interior remains at low temperatures, DM annihilation can produce enough energy to sustain the stellar structure. As long as this condition is met, the star cannot contract while it still grows in mass by accretion. Interestingly, by the time when the dark star reaches the main sequence, the stellar mass is already $\sim 1000$ $M_\odot$, which is much larger than the no-DM case (100–200 $M_\odot$).

Figure 4 shows the evolution of the DM mass inside the star and its time derivative. The left panel shows the total DM mass inside the star, whereas the right panel shows the ratio of the rate of DM annihilation in mass to the rate of DM mass increase.

where $\langle \sigma v \rangle$ is the DM self-annihilation rate in units of cm$^3$ s$^{-1}$ and $m_\chi$ is the DM particle mass in units of GeV (e.g., Bertone et al. 2005). Note that effectively only the ratio, $\langle \sigma v \rangle/m_\chi$, determines the net energy generation rate. We vary the DM particle mass as a model parameter as $m_\chi = 1$, 10, 20, 50, 100, and 200 GeV, while keeping $\langle \sigma v \rangle$ constant. Spolyar et al. (2008) calculate the critical transition gas density above which most of the DM annihilation energy is absorbed inside the core. The critical density is smaller than the stellar density with the WIMP mass 1 GeV to 10 TeV, and thus we assume that the energy released by nuclear reactions for our dark star model, whereas the dashed line is for a standard Population III model (no-DM model of Umeda et al. 2009). Symbols indicate the times when the stellar mass is $M = 100$, 200, 600, 800, and 1000 $M_\odot$.
Figure 4. DM mass inside the star (left panel) and the DM consumption rate by annihilation (right panel) as a function of stellar mass. We normalize the DM consumption rate $dM_{\text{DM}}/dt|_{\text{annihilation}}$ by the DM mass supply by adiabatic contraction $dM_{\text{DM}}/dt|_{\text{contraction}}$. The solid line is for the dark star model (“base model”), whereas the dotted line is for the no-depletion model.

In Figure 4, we show the results for an additional model in which the depletion of DM in the star is not taken into account. Although this case might appear unrealistic, it is useful to see when DM depletion becomes substantial. The solid line represents our “base model” while the dashed line shows the result of the no-depletion case. Up to $M \sim 200–300 \, M_\odot$, there is almost no difference between the two cases. At $M > 300 \, M_\odot$, however, DM depletion becomes significant and then the total DM mass actually starts decreasing. The evolution thereafter is interesting. The DM “fuel” inside the star that sustains it runs short. The star stops expanding, the gas density increases more efficiently, and then the DM density also increases again. Then DM annihilation rapidly consumes the DM fuel inside the star. Finally, the DM annihilation energy cannot sustain the star, marking the end of the dark star phase. The star collapses and will eventually reach the main-sequence phase.

Figure 5 shows the radial profiles for various quantities of gas (left) and DM (right) when the stellar mass is $M = 200$, 800, and 1000 $M_\odot$. In all the panels, the horizontal axis shows the ratio of radial distance normalized to the stellar radius (see Table 1). Top panels show the gas and DM density profiles. The middle panels show the gas and DM enclosed mass. The bottom panels show DM annihilation rate and DM luminosity from annihilation.

(within the star) owing to adiabatic contraction:

$$\left. \frac{dM_{\text{DM}}}{dt} \right|_{\text{annihilation}} / \left. \frac{dM_{\text{DM}}}{dt} \right|_{\text{contraction}} . \quad (5)$$

In Figure 4, we show the results for an additional model in which the depletion of DM in the star is not taken into account. Although this case might appear unrealistic, it is useful to see when DM depletion becomes substantial. The solid line represents our “base model” while the dashed line shows the result of the no-depletion case. Up to $M \sim 200–300 \, M_\odot$, there is almost no difference between the two cases. At $M > 300 \, M_\odot$, however, DM depletion becomes significant and then the total DM mass actually starts decreasing. The evolution therefor is interesting. The DM “fuel” inside the star that sustains it runs short. The star stops expanding, the gas density increases more efficiently, and then the DM density also increases again. Then DM annihilation rapidly consumes the DM fuel inside the star. Finally, the DM annihilation energy cannot sustain the star, marking the end of the dark star phase. The star collapses and will eventually reach the main-sequence phase.

Figure 5 shows the radial profiles for various quantities of gas (left) and DM (right) when the stellar mass is $M = 200$ (solid lines), 800 (dashed lines), and 1000 $M_\odot$ (dotted lines), respectively. The horizontal axis is the radial distance from the stellar center divided by the stellar radius.

The top panels show the evolution of gas density and DM density. Between $M = 200$ and 800 $M_\odot$, the star has an extended
structure and the central density does not increase much. The DM density increases by adiabatic contraction but actually decreases slightly in the innermost part owing to annihilation. In the final contraction phase \((M = 800–1000 \, M_\odot)\), both the gas density and the DM density increase substantially.

The middle panels show the evolution of the enclosed mass of gas and DM. Although the gas mass profile stays roughly unchanged, the DM mass decreases dramatically during the final phase from \(M = 800\) to \(1000 \, M_\odot\). Note that the horizontal axis in the plots shows a normalized radius \(R/R_*\). The stellar radius \(R_*\) itself changes significantly over the plotted range of evolutionary stages. Because the dark star phase ends with the runaway burning of DM inside the star, the total amount of DM when \(M = 1000 \, M_\odot\) is already very small.

The bottom-left panel shows the DM annihilation rate and the bottom-right panel shows the total luminosity generated by the DM annihilation within the radius. These panels show the energy generating efficiency from DM annihilation. Again, we see that the annihilation rate is low inside the star at the final stage \(M = 1000 \, M_\odot\) and the enclosed DM luminosity at the stellar surface \((\log_{10}\text{Radius}/R_* = 0)\) is smaller than in the earlier phases. The stellar mass increases by gas accretion but the DM energy supply decreases; this causes the star to contract.

Figure 6 shows the evolution of some basic stellar quantities which characterize the dark star. We compare the results of our base model (solid lines) with those of the no-DM model (dashed lines) and the no-depletion model (dotted lines). Until the star grows to \(M \sim 200–300 \, M_\odot\), the base model and the no-depletion model appear very similar, showing again that DM depletion is negligible in the early phases. After the star grows to \(\sim 300 \, M_\odot\), we see small but appreciable differences between the base model and the no-depletion model. As has been shown in Figure 4, DM is consumed by annihilation inside the star whereas the supply by adiabatic contraction is slow owing to the small gas mass accretion rate. As the star becomes more massive, it needs more DM to produce the necessary energy to sustain gravitational equilibrium. When the DM supply becomes insufficient, this quasi-stable dark star phase cannot be sustained. This occurs at \(M \sim 600 \, M_\odot\) for the base model. The stellar properties, however, do not change immediately until up to \(M \sim 900 \, M_\odot\). At around \(M \gtrsim 900 \, M_\odot\), the DM inside the star burns out rapidly, the star begins to collapse, and the central temperature increases rapidly. Finally, the central temperature reaches \(10^7–10^8 \, \text{K}\), nuclear burning will soon start, and the star will eventually land on the main sequence.

Table 1 summarizes the basic stellar properties at several characteristic phases. Note that the elapsed time difference (the right column) is large between 800 and 1000 \(M_\odot\). The dark star phase continues for about \(\sim 0.2 \, \text{Myr}\). Throughout the run, the DM mass is very small compared with the stellar gas mass. Dark stars can be sustained by DM comprising only 0.1% of their total mass \(M_*\).

### 3.2. The Effect of Gas Mass Accretion Rate

We now explore a few parameter spaces. Our dark star model has essentially two parameters, the gas mass accretion rate and the DM particle mass. First, we examine the effect of the gas mass accretion rate. We adopt three accretion rates, 1.0, 0.5, and 0.2 \(\times\) G-rate (Figure 2). The “G-rate” is the gas accretion rate in our base model (Equation (3)). In general, less DM is attracted toward the star for smaller gas accretion rates. We consider only accretion rates smaller than the G-rate because it seems reasonable to assume that various protostellar feedback effects, such as ionizing radiation from the star, can only reduce the gas mass accretion rate.

Figure 7 shows the stellar evolution for the three models with different accretion rates in the H-R diagram. Initially, the stars...
are puffy, cool, and evolve along virtually the same path. Tables 2 and 3 show the basic stellar properties when the models with the reduced accretion rates reach the end of the stable phase where $T_{\text{eff}} = 10^4$ K, and where $T_{\text{eff}} = 10^5$ K, respectively. The latter phase is the end of our calculations. As is expected naively, the final stellar mass is smaller for lower gas accretion rates, but the period of the dark star phase is actually longer. In the reduced accretion models, the star needs a longer time to reach the same stellar mass. Then the DM density inside the star is smaller because the gas contraction is slow. Consequently, the star with a smaller gas accretion rate ends the dark star phase and begins to contract when its mass is smaller than in the base model.

Figures 8 and 9 show clearly the effect of reducing the accretion rate. There are essentially no differences among the three models when the stellar mass is small. The DM fuel inside the star increases very similarly (Figure 8, left) because the ratio of DM decrease to DM increase is very small (Figure 8, right). For the smallest accretion rate, the dark star ends when the stellar mass reaches $\sim 500 M_\odot$. Overall, the lower the accretion rate is, the smaller the DM mass is inside the star. The dark star phase lasts longer but ends at lower masses.

### 3.3. The Effect of DM Particle Mass

Finally, we study models with different DM particle masses. We adopt the base accretion rate of $1.0 \times G$-rate, which is larger

![Figure 8](image_url)

**Figure 8.** DM mass and the DM consumption rate by annihilation as a function of stellar mass for three accretion rates models, as in Figure 4. (A color version of this figure is available in the online journal.)

| Accretion Rate | $M_*$ ($M_\odot$) | $R_*$ ($R_\odot$) | $L_*$ ($L_\odot$) | $T_{\text{eff}}$ (K) | $T_{\text{cen}}$ (K) | $M_{DM}$ ($M_\odot$) | $t$ (Myr) |
|---------------|------------------|------------------|------------------|----------------------|----------------------|---------------------|-----------|
| $1.0 \times G$-rate | 821 | 1.14E3 | 1.15E7 | 1.00E4 | 1.51E6 | 0.277 | 0.160 |
| $0.5 \times G$-rate | 682 | 9.87E2 | 8.67E6 | 1.00E4 | 1.56E6 | 0.205 | 0.237 |
| $0.2 \times G$-rate | 492 | 7.87E2 | 5.55E6 | 1.00E4 | 1.63E6 | 0.125 | 0.351 |

**Notes.** Stellar properties for three accretion models when the stellar surface temperature reaches $T_{\text{eff}} = 10^4$ K. After this phase, the star begins to run out of DM fuel and begins to gravitationally contract.

| Accretion Rate | $M_*$ ($M_\odot$) | $R_*$ ($R_\odot$) | $L_*$ ($L_\odot$) | $T_{\text{eff}}$ (K) | $T_{\text{cen}}$ (K) | $M_{DM}$ ($M_\odot$) | $t$ (Myr) |
|---------------|------------------|------------------|------------------|----------------------|----------------------|---------------------|-----------|
| $1.0 \times G$-rate | 1002 | 19.27 | 3.32E7 | 1.00E5 | 1.07E8 | 2.43E-4 | 0.220 |
| $0.5 \times G$-rate | 775 | 16.55 | 2.43E7 | 1.00E5 | 1.08E8 | 1.46E-4 | 0.291 |
| $0.2 \times G$-rate | 534 | 13.44 | 1.62E7 | 1.00E5 | 1.09E8 | 7.65E-5 | 0.401 |

**Notes.** Stellar properties for three accretion models when the stellar surface temperature reaches $T_{\text{eff}} = 10^5$ K. By this phase, the star has contracted sufficiently and the density and temperature have risen enough to start hydrogen burning. At this point, the star is not supported by DM annihilation energy, so we stop the calculation.

![Figure 9](image_url)

**Figure 9.** Evolution of basic stellar quantities for dark star models with reduced accretion rates. All vertical axes are plotted on logarithmic scales. The plotted quantities are the same as in Figure 6. (A color version of this figure is available in the online journal.)
Figure 10. H-R diagram for models with variation of DM particle masses. The lines show models with DM particle masses $m_\chi = 1, 10, 20, 50, 100,$ and 200 GeV and the no-DM model (the same as in Figure 3).

(A color version of this figure is available in the online journal.)

than those adopted in Spolyar et al. (2009, see Figure 2), so our results are slightly different from theirs, especially for a small DM particle mass. We discuss this issue later in this section.

We run six models with $m_\chi = 1, 10, 20, 50, 100,$ and 200 GeV. Note that, from Equation (4), the DM annihilation energy is inversely proportional to $m_\chi$. Thus the characteristic features of dark stars appear strongly in runs with small DM masses.

Figures 10–12 show the results for the six models. For smaller DM particle masses, the DM annihilation rate is large, and thus the star expands more and the temperature and density in the star are lower. The period of the dark star phase is also prolonged for smaller $m_\chi$, and the final stellar mass gets larger than our base model.

Tables 4 and 5 list the basic properties of the stars when the surface temperature $T_{\text{eff}}$ reaches $10^4$ K and $10^5$ K, respectively. The final stellar mass $M_\ast$ differs by about a factor of two among the five models; $M_\ast = 776$–$1370$ GeV in Table 5. The overall trend is as described above, but the lightest particle model is worth describing in more detail. When the star enters the main sequence, the stellar mass in the $m_\chi = 1$ GeV model is similar to that of the 10 GeV model, although it has 10 times larger DM energy generation rate. The reason is that the DM burning rate is too large in the $m_\chi = 1$ GeV model. When the DM consumption rate exceeds the DM supply by adiabatic contraction (see the discussion in Section 3.1), the star runs out of DM fuel and begins to contract. The DM consumption becomes even more efficient then. This final “runaway” phase takes place faster in the 1 GeV model than in the 10 GeV model. Consequently, the star lands on the main sequence earlier in the 1 GeV model (see the last column of Table 5, which gives the elapsed time).

Interestingly, the $m_\chi = 1$ GeV model predicts the most luminous star, reaching $L \sim 10^7 L_\odot$ at the peak (Figure 12). The extremely high luminosity is, however, still smaller than the upper limit given by the Eddington luminosity $L_{\text{EDD}}$:

$$L_{\text{EDD}} = \frac{4\pi c GM_\ast}{\kappa},$$

where the opacity $\kappa$ determines essentially the critical luminosity. Figure 13 shows the ratio of the energy generation rate of DM annihilation (“luminosity”) to the Eddington luminosity for our five models with different DM particle masses. The ratio stays always less than unity, even for the 1 GeV model. During the dark star phase ($T_{\text{eff}} < 10,000$ K), the opacity $\kappa$ at lower temperatures is mainly contributed by the $\text{H}^-$ ion which has a small value.

Figure 11. DM mass and the DM consumption rate by annihilation for the variation of DM particle masses, plotted as in Figure 4.

(A color version of this figure is available in the online journal.)
Figure 13. We plot the ratio of DM annihilation luminosity to Eddington luminosity (left) and the opacity at the photosphere as a function of stellar mass (right) for different DM particle masses. In all the models, the luminosity ratio stays less than unity.

(A color version of this figure is available in the online journal.)

Table 4

| mχ (GeV) | M∗ (M⊙) | R∗ (R⊙) | L∗ (L⊙) | Teff (K) | Tcen (K) | MDM (M⊙) | t (Myr) |
|----------|----------|----------|----------|----------|----------|-----------|---------|
| 1        | 1349     | 1.82E3   | 2.92E7   | 1.00E4   | 1.11E7   | 0.096     | 0.354   |
| 10       | 1325     | 1.59E3   | 2.23E7   | 1.00E4   | 1.33E6   | 0.254     | 0.344   |
| 20       | 1214     | 1.51E3   | 2.02E7   | 1.00E4   | 1.42E6   | 0.297     | 0.298   |
| 50       | 1009     | 1.32E3   | 1.55E7   | 1.00E4   | 1.46E6   | 0.307     | 0.222   |
| 100      | 821      | 1.14E3   | 1.15E7   | 1.00E4   | 1.51E6   | 0.277     | 0.160   |
| 200      | 536      | 8.76E2   | 6.81E6   | 1.00E4   | 1.47E6   | 0.198     | 0.081   |

Note. Stellar properties for DM mass variation models when stellar surface temperature reaches Teff = 10⁴ K, the end of the stable dark star phase and the start of transformation into a main-sequence star.

Table 5

| mχ (GeV) | M∗ (M⊙) | R∗ (R⊙) | L∗ (L⊙) | Teff (K) | Tcen (K) | MDM (M⊙) | t (Myr) |
|----------|----------|----------|----------|----------|----------|-----------|---------|
| 1        | 1370     | 23.86    | 5.03E7   | 1.00E5   | 2.49E8   | 3.47E-6   | 0.362   |
| 10       | 1381     | 23.32    | 4.83E7   | 1.00E5   | 1.07E8   | 4.96E-5   | 0.367   |
| 20       | 1295     | 22.45    | 4.48E7   | 1.00E5   | 1.07E8   | 8.92E-5   | 0.333   |
| 50       | 1138     | 20.85    | 3.86E7   | 1.00E5   | 1.07E8   | 1.59E-4   | 0.269   |
| 100      | 1002     | 19.27    | 3.32E7   | 1.00E5   | 1.07E8   | 2.43E-4   | 0.220   |
| 200      | 776      | 18.91    | 3.16E7   | 1.00E5   | 9.95E7   | 3.88E-4   | 0.146   |

Note. Stellar properties for DM mass variation models when stellar surface temperature reaches Teff = 10⁵ K, the end of the calculation.

4. DISCUSSION

4.1. Dark Star Models

We compare our results with those of previous works. Unlike previous studies (e.g., Spolyar et al. 2009), we solve full radiative transfer and follow self-consistently the stellar structure and its evolution. A direct comparison can be made with the results of Table 2 in Spolyar et al. (2009, hereafter “SP09”), by using the result of our 0.5 × G-rate model (hereafter “05G”; see Section 3.2). The other model parameters are set to be the same, mχ = 100 GeV and \( \langle \sigma v \rangle = 3 \times 10^{-26} \text{ cm}^3 \text{ s}^{-1} \), except for the gas mass accretion rate. The gas mass accretion rates in these two models (Figure 2) are almost the same at \( M^* > 100 M_\odot \), which is the main period of the dark star phase. The results of 05G and SP09 are roughly similar; in stellar radius, luminosity, and other quantities, the differences are less than 20% at \( M^* = 106 M_\odot \) (the first line in SP09) and remain within a factor of two at later stages. During calculation, the 05G model has a slightly expanded structure and is somewhat cooler than the SP09 model. On the other hand, the difference of the central temperature is relatively large between the two models. The value in 05G becomes lower than in SP09 especially at the final stages; at \( M^* = 716, 756, \) and \( 779 M_\odot, T_{cen} = 4.0, 4.3, \) and 132 (10⁴) K in 05G whereas \( T_{cen} = 15, 78, \) and 280 (10⁴) K in SP09. Because of the lower central temperature, nuclear burning is unimportant in our calculations.

4.2. Possible Range of WIMP DM Mass

The WIMP mass significantly affects the duration of the dark star phase because the DM annihilation energy is inversely proportional to the WIMP mass \( m_\chi \), as can be seen in Equation (4). We have already shown the effect in Section 3.3 for a typical DM mass range, from 1 to 200 GeV. However, in supersymmetric particle physics models, the WIMP DM mass may be 10 TeV or even greater. To estimate the evolutionary path of dark star models with heavier WIMPs, we have run our no-depletion model for two larger WIMP masses, \( m_\chi = 1 \) and 10 TeV. Figures 14(a) and (b) show the result. At the beginning of the calculation with \( M \approx 15 M_\odot \), all the models are in the dark star phase and on
the Hayashi line in the H-R diagram. The left panel of Figure 14 shows that, at this phase, all models have \( T_{\text{eff}} \simeq 10^{3.7} \, \text{K} \). For \( m_\chi = 10 \, \text{TeV} \), the Population III star leaves the dark star phase early. Because the annihilation rate is small for a large WIMP mass, the stellar mass at the end of the dark star phase remains small. Because the annihilation rate is small for a large WIMP mass, \( m_\chi = 10 \, \text{TeV} \), the Population III star leaves the dark star phase early. The former indicates the phase where the star deviates from the Hayashi line in the H-R diagram. The left panel of Figure 14 shows that, at this phase, all models have \( T_{\text{eff}} \simeq 10^{3.7} \, \text{K} \). For \( m_\chi = 10 \, \text{TeV} \), the Population III star leaves the dark star phase early. Because the annihilation rate is small for a large WIMP mass, the stellar mass at the end of the dark star phase remains small.

All the dark star models are on the \( T_{\text{eff}} \simeq 10^{3.7} \, \text{K} \) line at first (see also Figure 10) and leave the line gradually as mass accretion continues. To quantify the evolution, we define two characteristic stellar masses, \( M_{3.8} \) and \( M_{4.0} \), the stellar masses at which the surface temperature reaches \( T_{\text{eff}} \simeq 10^{3.8} \) and \( 10^{4.0} \, \text{K} \). The former indicates the phase where the star deviates from the Hayashi line. The later indicates when a star ends the stable dark star phase. The result is \( M_{3.8} = (1145, 821, 402, 74, 26) \, M_\odot \) and \( M_{4.0} = (1349, 1325, 821, 270, 50) \, M_\odot \) for \( m_\chi = (1, 10, 100 \, \text{GeV}, \text{ and } 1, 10 \, \text{TeV}) \), respectively. We conclude that Population III stars with a large WIMP mass up to \( 10 \, \text{TeV} \) can proceed through the dark star phase, although the duration of this phase is very short. The final stellar masses are small for the dark stars with most massive WIMPs.

Note that we have used no-depletion models here for simplicity, because the DM depletion effect is small for the early stages of the dark star evolution. If we take the DM depletion into account, a dark star ends the stable phase earlier and the above two quantities \( M_{3.8} \) and \( M_{4.0} \) become smaller.

5. CONCLUSION

We have studied the pre-main-sequence evolution of dark stars by following the self-consistent stellar evolution. We have suitably modified the code to incorporate the energy generation from spherically distributed DM.

Our base model with \( m_\chi = 100 \, \text{GeV} \) and \( dM/dT = 1.0 \times 10^{-7} \, \text{G-rate} \, M_\odot \, \text{yr}^{-1} \) shows the characteristic features of the dark star phase; the large DM annihilation energy expands the star, putting the star in gravitational equilibrium. The lower temperature is one of the peculiar properties of the dark star. This stable phase continues until the energy supply from DM annihilation becomes insufficient to maintain the stable structure. Finally, the star collapses rapidly and reaches the main-sequence phase. At this point, the stellar mass has grown up to \( M \sim 900-1000 \, M_\odot \). Such features in the dark star phase are all consistent with the findings of previous works of Iocco et al. (2008) and Spolyar et al. (2009) who employed much simpler stellar models.

The dark star model has effectively two parameters. One of them is the gas mass accretion rate which determines the evolution of the gas and DM distribution. Cosmological simulations predict a variety of gas accretion rates. For a small accretion rate, the period of the dark star phase increases whereas the final stellar mass decreases to \( M \sim 500 \, M_\odot \). Note, however, that the mass is still larger than the standard Population III (no-DM) case \((100-200 \, M_\odot)\). Another parameter is the DM particle mass which determines the energy generation rate. For a small DM particle mass, the period of the dark star phase becomes longer and the final stellar mass becomes larger (see Table 5).

All models pass through the dark star phase and this phase is maintained by a little DM fuel which is less than 0.1% of the stellar mass. If the first stars in the universe have undergone such a phase, there are exotic stars which are cool and massive in the early universe.

The formation of very massive dark stars has an important implication. Such stars eventually collapse gravitationally to form massive BHs with masses as large as \( 1000 \, M_\odot \). The remnant BHs can also grow by accretion or by mergers to seed supermassive BHs. It may be possible to use the exotic nature of a dark star’s appearance, being luminous and cool, as a powerful clue to search for such stars. Our calculations show that first stars can grow to be dark stars with luminosity \( \sim \) few \( 10^7 \, L_\odot \) at most and they will not be detectable by JWST. However, Freese et al. (2010) estimate that supermassive dark stars and such very massive and bright stars (about \( 10^8-10^{11} \, L_\odot \) can be detected by JWST. Sandick et al. (2010) argue that dark star remnants might survive to the present day in the Milky Way, leaving \( \gamma \)-ray signatures from DM annihilation.

In future work, we will include the nuclear reaction to calculate the evolution of the first stars from the dark star phase to the main-sequence phase completely. It would also be interesting to include capture of DM particles. Once the stars have contracted and stop growing in mass, there will be no more DM supplied by adiabatic contraction. However, DM particles may still be captured by the star. Previous studies showed that this process can put the star back in the dark star phase again. We will explore the effect of DM capture and make a complete evolutionary model of the first stars with DM annihilation.

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