Novel Phenomena in Dilute Electron Systems in Two Dimensions

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For the past two decades, all two-dimensional systems of electrons were believed to be insulating in the limit of zero temperature. We review recent experiments that provide evidence for an unexpected transition to a conducting phase at very low electron densities. The nature of this phase is not understood, and is currently the focus of intense theoretical and experimental attention.

BACKGROUND

A two-dimensional (2D) system of electrons or “holes” (a hole, or missing electron, behaves like a positively charged electron) is one in which the positions of the electrons and their motion are restricted to a plane. Physical realizations can be found in very thin films, sometimes at the surface of bulk materials, in “quantum well” systems such as GaAs/AlGaAs that are specifically engineered for this purpose, and in the silicon metal-oxide-semiconductor field-effect transistors described below. Two-dimensional electron systems have been studied for nearly forty years \(^{[1]}\), and have yielded a number of important discoveries of physical phenomena that directly reflect the quantum mechanical nature of our world. These include the integer Quantum Hall Effect (QHE), which reflects the quantization of electron states by a magnetic field, and the fractional Quantum Hall Effect, which is a manifestation of the quantum mechanics of many electrons acting together in a magnetic field to yield curious effects like fractional (rather than whole) electron charges \(^{[2]}\).

For nearly two decades it was believed that in the absence of an external magnetic field \((H = 0)\) all two-dimensional systems of electrons are insulators in the limit of zero temperature. The true nature of the conduction was expected to be revealed only at sufficiently low temperatures; in materials such as highly conducting thin films, this was thought to require unattainably low temperatures in the mKelvin range. Based on a scaling theory for non-interacting electrons \(^{[3]}\), these expectations were further supported by theoretical work for weakly interacting electrons \(^{[4]}\).

Confirmation that two-dimensional systems of electrons are insulators in zero field was provided by a beautiful series of experiments in thin metallic films \(^{[5]}\) and silicon metal-oxide-semiconductor field-effect transistors \(^{[6]}\), where the conductivity was shown to display weak logarithmic corrections leading to infinite resistivity in the limit of zero temperature. It was therefore quite surprising when recent experiments in silicon metal-oxide-semiconductor field-effect transistors suggested that a transition from insulating to conducting behavior occurs with increasing electron density at a very low critical density, \(n_c \sim 10^{11} \text{ cm}^{-2} \) \(^{[7]}\). These experiments were performed on unusually high quality samples, allowing measurements at considerably lower electron densities than had been possible in the past. First viewed with considerable skepticism, the finding was soon confirmed for silicon metal-oxide-semiconductor field-effect transistors fabricated in other laboratories \(^{[8]}\), and then for other materials, including p-type SiGe structures \(^{[9]}\), p-type GaAs/AlGaAs heterostructures \(^{[10]}\), and n-type AlAs \(^{[11]}\) and GaAs/AlGaAs heterostructures \(^{[12]}\).

It was soon realized that the low electron (and hole) densities at which these observations were made correspond to a regime where the energy of the repulsive Coulomb interactions between the electrons exceeds the Fermi energy (roughly, their kinetic energy of motion) by an order of magnitude or more. For example, at an electron density \(n_s = 10^{11} \text{ cm}^{-2} \) in silicon metal-oxide-semiconductor field-effect transistors, the Coulomb repulsion energy, \(U_c = e^2/(\pi n_s)^{1/2} \), is about 10 meV while the Fermi energy, \(E_F = \pi n_s \hbar^2/2m^* \), is only 0.55 meV. (Here \(e\) is the electronic charge, \(\epsilon\) is the dielectric constant, and \(m^*\) is the effective mass of the electron.) Rather than being a small perturbation, as has been generally assumed in theoretical work done to date, interactions instead provide the dominant energy in these very dilute systems.

EXPERIMENTS

The inset to Fig. 1(a) shows a schematic diagram of the band structure of a silicon metal-oxide-semiconductor field-effect transistor consisting of a thin-film metallic gate deposited on an oxide layer adjacent to lightly p-doped silicon, which serves as a source of electrons. A voltage applied between the gate and the oxide-silicon interface causes the conduction and valence bands to bend, as shown in the diagram, creating a potential minimum which traps electrons in a two-dimensional layer perpendicular to the plane of the page. The magnitude of the applied voltage determines the degree of band-bending and thus the depth of the potential well, allowing continuous control of the number of electrons trapped in the two-dimensional system at the interface.

For a very high-mobility (low disorder) silicon metal-oxide-semiconductor field-effect transistor, the resistivity is shown at several fixed temperatures as a function of electron density in Fig. 1(a). There is a well defined crossing at a “critical” electron density, \(n_c\), below which the
resistivity increases as the temperature is decreased, and above which the reverse is true. This can be seen more clearly in Fig. 1(b) where the resistivity is plotted as a function of temperature for various fixed electron densities. A resistivity that increases with decreasing temperature generally signals an approach to infinite resistance at $T = 0$, that is, to insulating behavior; a resistivity that decreases as the temperature is lowered is characteristic of a metal if the resistivity tends to a finite value, or a superconductor or perfect conductor if the resistivity tends to zero. The crossing point of Fig. 1(a) thus signals a transition from insulating behavior below $n_s < n_c$ to conducting behavior at higher densities ($n_s > n_c$). Similar behavior obtains in other materials at critical densities determined by material parameters such as effective masses and dielectric constants. The value of the resistivity at the transition (the “critical resistivity” $\rho_c$) in all systems remains on the order of $h/e^2$, the quantum unit of resistivity.

The electrons’ spins play a crucial role in these low-density materials, as demonstrated by their dramatic response to a magnetic field applied parallel to the plane of the two-dimensional system. We note that an in-plane magnetic field couples only to the electron spins and does not affect their orbital motion. The parallel-field magnetoresistance is shown for a silicon metal-oxide-semiconductor field-effect transistor in Fig. 2 for electron densities spanning the critical density $n_c$ at a temperature of 0.3 K. The resistivity increases by more than an order of magnitude with increasing field, saturating to a new value in fields above 2 or 3 Tesla above which the spins are presumably fully aligned. The total change in resistance is larger at lower temperatures and for higher mobility samples, exceeding two or three order of magnitude in some cases. Although first thought to be associated only with the suppression of the conducting phase, the fact that very similar magnetoresistance is found for electron densities above and below the zero-field critical density indicates that this is a more general feature of dilute two-dimensional electron systems.

**SOME OPEN QUESTIONS**

Strongly interacting systems of electrons in two dimensions are currently the focus of intense interest, eliciting a spate of theoretical attempts to account for the presence and nature of the unexpected conducting phase. Most postulate esoteric new states of matter, such as a low-density conducting phase first considered by Finkelshtein, a perfect metallic state, non-Fermi liquid behavior, and several types of superconductivity. A number of relatively more mundane suggestions have been advanced that attribute the unusual behavior seen in Figs. 1 and 2 to effects that are essentially classical in nature. These include a vapor/gas separation in the electron system, temperature- and field-dependent filling and emptying of charge traps unavoidably introduced during device fabrication at the oxide-silicon interface, and temperature-dependent screening associated with such charged traps. Although some may strongly advocate a particular view, all would agree that no consensus has been reached.

A great deal more experimental information will be required before the behavior of these systems is understood. Information will surely be obtained in the near future from NMR, tunneling studies, optical investigations, and other techniques. One crucially important question that needs to be resolved by experiment is the ultimate fate of the resistivity in the conducting phase in the limit of zero temperature. Data in all 2D systems showing the unusual metal-insulator transition indicate that, following the rapid (roughly exponential) decrease with decreasing temperature shown in Fig. 1(b), the resistivity levels off to a constant, or at most weakly temperature-dependent, value. The temperature at which this leveling off occurs decreases, however, as the transition is approached. The question is whether the resistivity of dilute two-dimensional systems tends to a finite value or zero in the zero-temperature limit as the transition is approached. If the resistivity remains finite, this would rule out superconductivity or perfect conductivity. The question may then revert to whether localization of the electrons reasserts itself at very low temperatures, yielding an insulator as originally expected. There are well-known experimental difficulties associated with cooling the electron system to the same temperature as the lattice and bath (that is, the temperature measured by the thermometer), and these experiments will require great skill, care and patience.

An equally important issue is the magnetic response of the electron system. Superconductors expel magnetic flux and are strongly diamagnetic, while Finkelshtein’s low-density phase would give a strongly paramagnetic signal. There are very few electrons in a low-density, millimeter-sized, 100Å-thick layer, and measurements of the magnetization will be exceedingly difficult.

In closing, we address a crucial question regarding the nature of the apparent, unexpected zero-field metal-insulator transition: do these experiments signal the presence of unanticipated phases and new phenomena in strongly interacting two-dimensional electron systems, or can the observations be explained by invoking classical effects such as recharging of traps in the oxide or temperature-dependent screening? Some recent experiments suggest the former. A Princeton-Weizmann collaboration has demonstrated that the magnetic field-induced phase transition between integer Quantum Hall Liquid and insulator (the QHE-I transition) evolves smoothly and continuously to the metal-insulator transition in zero magnetic field discussed in this paper, raising the possibility that the two transitions are closely related. This conjecture is supported by the strong similarity between the temperature dependence of the resis-
itivity in zero magnetic field and in the Quantum Hall Liquid phase. Additional insight can be obtained from a comparison of the "critical" resistivity, $\rho_c$, at the zero-field metal-insulator transition and the critical resistivity, $\rho_{QHE-I}$, at the QHE-I transition measured for the same sample. Fig. 3(a) shows values of the zero-field metal-insulator transition and the critical resistivity in zero magnetic field and in the Quantum Hall Liquid phase. Additional insight can be obtained from a comparison of the "critical" resistivity, $\rho_c$, at the zero-field metal-insulator transition and the critical resistivity, $\rho_{QHE-I}$, at the QHE-I transition measured for the same sample. Fig. 3(a) shows values of the zero-field critical resistivity, $\rho_c$, for a number of samples of different 2D electron and hole systems; $\rho_c$ varies by an order of magnitude, between approximately $10^4$ and $10^5$ Ohm, and exhibits no apparent systematic behavior. In contrast, Fig. 3(b) shows that the ratio $\rho_c/\rho_{QHE-I}$ is close to unity when measured on the same sample for three different materials. Since the QHE-I transition is clearly a quantum phase transition, this suggests that the zero-field metal-insulator transition is a quantum phase transition as well. The intriguing relationship between critical resistivities for these two transitions shown for only a few very few samples in Fig. 3(b) clearly needs further confirmation. Future work will surely resolve whether, and what, exciting and unanticipated physics is required to account for the puzzling and fascinating recent observations in two dimensions.

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FIGURE CAPTIONS

Figure 1:
(a): Resistivity as a function of electron density for the two-dimensional system of electrons in a high-mobility silicon metal-oxide-semiconductor field-effect transistor. The different curves correspond to different temperatures. Note that at low densities the resistivity increases with decreasing temperature (insulating behavior), while the reverse is true for higher densities (conducting behavior). The inset shows a schematic diagram of the electron bands to illustrate how a two-dimensional layer is obtained (see text).
(b): Resistivity as a function of temperature for the two-dimensional system of electrons in a silicon MOSFET. Different curves are for different electron densities.

Figure 2:
For different electron densities, the resistivity at 0.3 Kelvin is plotted as a function of magnetic field applied parallel to the plane of the two-dimensional system of electrons in a silicon MOSFET. The top three curves are insulating while the lower curves are conducting in the absence of a magnetic field. The response to parallel field is qualitatively the same in the two phases, varying continuously across the transition.

Figure 3:
(a): The critical resistivity, $\rho_c$, which separates the conducting and insulating phases in zero magnetic field is shown for several 2D systems for which the transition occurs at different electron (or hole) densities, shown along the x-axis. Although the critical resistivity is of the order of the quantum unit of resistivity, $h/e^2 \approx 26$ kOhm, it varies by about a factor of 10.

(b): For several materials, measurements of $\rho_c$ and $\rho_{QHE-I}$ on the same sample yield ratios $\rho_c/\rho_{QHE-I}$ that are near unity. Here $\rho_c$ is the critical resistivity separating the conducting and insulating phases in the absence of magnetic field and $\rho_{QHE-I}$ is the critical resistivity at the transition from the Quantum Hall Liquid to the insulator in finite magnetic field. Data were obtained for p-GaAs/AlGaAs heterostructures from Refs. [11,12,25], for n-GaAs/AlGaAs from Ref. [14], and for p-SiGe from Refs. [10,27].
Fig. 1(a)

![Graph showing resistivity vs electron density](image)

- Resistivity $\rho$ ($10^4$ Ohm)
- Electron density $n_s$ ($10^{11}$ cm$^2$)

Graph labels:
- $0.3$ K
- $0.4$ K
- $0.5$ K
- $0.6$ K
- $0.7$ K

Inset showing Al SiO$_2$ Si with conductance and valence bands.
Fig. 1(b)

Graph showing the relationship between resistivity $\rho$ (Ohm) and temperature $T$ (K). The resistivity values are labeled for different temperature ranges:
- $0.86 \times 10^{11}$ cm$^{-2}$
- 0.88
- 0.90
- 0.93
- 0.95
- 0.99
- 1.10

The x-axis represents temperature $T$ (K) ranging from 0 to 2, and the y-axis represents the resistivity $\rho$ in Ohm, ranging from $10^3$ to $10^6$. The graph shows how resistivity changes with temperature for different values.
Fig. 2

![Graph showing the relationship between magnetic field $H_{\parallel}$ (Tesla) and resistivity $\rho$ (Ohm).]
Fig. 3

(a) $\rho_c$ (Ohm)

(b) $\rho_c / \rho_{QHE-I}$

carrier density (cm$^{-2}$)