Anderson Localization in the Sky
and Cosmological Magnetic Field

Akio HOSOYA and Shiho KOBAYASHI

Department of Physics, Tokyo Institute of Technology
Oh-Okayama, Meguro-ku, Tokyo 152, Japan

Abstract
We discuss the Anderson localization of electromagnetic fields in the fluctuating plasma induced by the gravitational density perturbation before the recombination time of the Universe. Randomly distributed localized coherent electromagnetic fields emerge in the thermal equilibrium before the recombination time. We argue that the localized coherent electric fields eventually produce cosmological magnetic fields after the decoupling time.

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\[1\text{E-mail address: ahosoya@th.phys.titech.ac.jp}\]
\[2\text{E-mail address: kobayasi@th.phys.titech.ac.jp}\]
1 Introduction

It has long been suspected that the presently observed galactic and cluster magnetic fields \(10^{-6}\, \text{gauss}\) \[1\] (and possibly the intercluster magnetic field \(\approx 10^{-9}\, \text{gauss}\), existence of which is under disputes \[1\] \[2\] \[3\].) has a cosmological origin. There are two competing theories for the galactic magnetic field: the primeval scenario and the dynamo theory. The former explains the galactic magnetic field as a consequence of amplification of a seed magnetic field by protogalaxy collapse. On the other hand, in the more popular galactic dynamo theory \[4\] \[5\], the magnetic field has been exponentially amplified from a tiny seed magnetic field by differential galactic rotation and turbulence. One of the weak points of the primeval magnetic field scenario has been the lack of its viable explanation.

Nevertheless, an alternative theory of density irregularities on the basis of the primeval magnetic field \[6\] \[7\] is attractive, because it can explain the magnitude of the density perturbation and the galactic magnetic field simultaneously. Recently a direct estimate of magnetic fields of young galaxies \((z \approx 2)\) has been performed by measuring the Faraday rotation of quasar spectra through the Lyman-\(\alpha\) absorption systems to give \(10^{\text{-6}}\, \text{gauss}\) \[8\] \[9\]. This large magnetic field of the galaxy in the early stage may revive interest in the primeval scenario. Further, even for the galactic dynamo theory we need a seed magnetic field, however small it is.

In this paper we would like to point out that before the recombination time the cosmic plasma is strongly fluctuating at large scales due to the gravitational density perturbation. The plasma density fluctuation plays a role of randomly distributed scatterers for the electromagnetic fields and make it impossible for the electromagnetic fields to propagate if the wave length is longer than the elastic mean free path of electromagnetic wave by the scatterers. The localized modes are thermally excited and the electric field is more or less aligned in the localization region. The localized electric field induces a coherent current and then the magnetic field \(\approx 10^{-26}\, \text{gauss}\) at the time of decoupling of photons from matter.

The localization phenomenon by random fields was first proposed by Anderson \[10\] to explain the behavior of the electric resistivity of metals in the presence of impurities and the theories were developed by himself and his collaborators \[11\] \[12\]. The localization phenomena are observed in various physical systems, e.g. in the propagation of light waves \[13\] and of sound waves \[14\]. Probably the first paper which studied the possible localization of electromagnetic field in plasma is the one by Escande and Souillard \[15\]. A preliminary experiment was performed for the longitudinal electron plasma wave by Doveil et al. \[16\]. (We shall adopt the localization theory in plasma \[17\] to the cosmological plasma in the expanding Universe.)

Roughly speaking, the Anderson localization phenomenon occurs because of the destructive interference of the incident wave and the scattered waves from
the randomly distributed sources. It is known that the localization occurs if the strength of the randomness is larger than a critical value. For a sufficiently large random potential, only the modes of the wave length shorter than the mean free path can propagate, while the other modes are localized.

We are going to argue that this localization phenomenon should occur in the matter dominated era in the standard cosmology. The resultant localized magnetic field may explain the origin of the galactic magnetic field and possibly a structure formation in the early stage [6][7].

This paper is organized as follows. After briefly reviewing the electromagnetism in the cosmological plasma in §2, we will derive the basic equation for the Anderson localization of electromagnetic fields in the standard big bang model of the Universe in §3. In §4 and §5, we will estimate the mobility edge, the localization length and the number of localized states. We will describe a scenario which tells how the localized coherent electromagnetic fields emerge before the recombination time in §6 and give a rough estimate of the cosmological magnetic field at the decoupling time. The final section §7 is devoted to summary and discussions.

2 The Plasma Equations in the Friedmann-Robertson-Walker Universe

Here we will first review the plasma equations in the Friedmann-Robertson-Walker background.

Let us recall that the relativistic expression for the Maxwell equation is given by

$$F^\alpha\beta |\beta = j^\alpha,$$  \hspace{1cm} (1)

where the electromagnetic field strength $F_{\alpha\beta} = \partial_\alpha A_\beta - \partial_\beta A_\alpha$ with $A_\alpha$ being the vector potential. Here the current $j^\alpha = enu^\alpha$ with $u^\alpha$ being the four velocity of charged particles (electrons) with $u_\alpha u^\alpha = -1$ and $n$ the density. The velocity $u^\alpha$ satisfies the Lorentz force equation:

$$mu^\alpha u_\lambda |\alpha = F_{\mu\alpha} eu^\alpha.$$  \hspace{1cm} (2)

For simplicity, consider the spatially flat Friedmann-Robertson-Walker Universe with the metric

$$ds^2 = a(\lambda)^2[-d\lambda^2 + dx^2].$$  \hspace{1cm} (3)

Here $\lambda$ is the conformal time defined by $\lambda = \int_0^t \frac{dt}{a}$. We assume that the charged particles are non-relativistic. So $u^\alpha = (a^{-1}, v^i)$ with $v^i$ being small compared with unity. Then the Maxwell equation reads

$$\eta^{\alpha\beta} \partial_\beta F_{i\alpha} = ena^2 v_i.$$  \hspace{1cm} (4)
with $\eta^{\alpha\beta}$ being the Lorentzian metric and the particle equation is approximately given by

$$m \frac{d v_i}{d \lambda} \approx e F_{i0},$$

(5)

where $m$ is the electron mass and $e$ is its charge. From the conservation of the current $j^\mu|_{\mu} = 0$, we see that $na^3$ is approximately constant in our non-relativistic case before the recombination time.

### 3 Random Potential by Fluctuating Plasma Density

Combining the equations (4) and (5), we can derive a general relativistic equation for the vector potential $A$ in terms of the conformal comoving coordinates $(\lambda, x)$ by

$$-\frac{\partial^2 A}{\partial \lambda^2} + \frac{\partial^2 A}{\partial x^2} = \frac{e^2 n a^2}{m} A.$$

(6)

Split the plasma density $n$ into the homogeneous part $\bar{n}$ and the space dependent random part $\delta n$;

$$n = \bar{n} + \delta n.$$  

(7)

Then the right hand side of the plasma equation becomes

$$[\omega_{rec}^2 a + \frac{\omega_{rec}^2}{a} \frac{\delta n}{\bar{n}}] A,$$

(8)

where $\omega_{rec} = \sqrt{\frac{e^2 n_{rec} m}{m_e}}$.

Here we have used the conservation law $\bar{n} = n_{rec}/a^3$ before the recombination time, with the scale factor $a$ normalized at the recombination time. $n_{rec}$ is the electron density (therefore the baryon density) at the recombination time.

The first term in the bracket is spatially constant but a decreasing function of the conformal time $\lambda$. The second term represents the fluctuating random environment induced by the baryonic density perturbation $\delta \rho_b/\rho_b$ which is equal to $\delta n/\bar{n}$ in our non-relativistic case. Knowing the standard theory of linear perturbation in cosmology we see that $\delta n/\bar{n} \propto a$ in the matter dominated era, so that the second term is independent of the conformal time in that era. We write the random potential as $V(x) \equiv \frac{\omega_{rec}^2}{a} \frac{\delta n}{\bar{n}}$, which is a function of comoving coordinates $x$ only. So far we have discussed the period after the equal time and before the recombination time and found that the random potential induced by the density perturbation is constant in time. As we shall see this is important for the stability of the Anderson localization. After the recombination time, the density of free electrons sharply decreases so that the random potential changes as rapidly as the other terms in the Maxwell equations so that the localization
breaks down after the recombination time. We will discuss this localization and de-localization phenomena in more detail in §6.

For the matter dominated universe, the scale factor is given by 

$$a(t) = \left(\frac{t}{t_{\text{rec}}}\right)^{2/3} = \left(\frac{\lambda}{M_{\text{rec}}}\right)^2$$

with $$\lambda = \int \frac{dt}{a(t)} = t_{\text{rec}}^{2/3}v_{\text{rec}}^{1/3}$$. Putting them all together we reduce the Maxwell equations to the following wave equation as

$$\frac{\partial^2 A}{\partial \lambda^2} + \frac{\partial^2 A}{\partial x^2} = |Q^2/\lambda^2 + V(x)|A.$$  \hspace{1cm} (9)

Here $$Q^2 = 9t_{\text{rec}}^2 \omega_{\text{rec}}^2$$.

We expand the vector potential $$A$$ in terms of the complete orthonormal set $$\{\psi_n\}$$ as

$$A = \sum a_n f_n(\lambda) \psi_n(x) + \text{c.c.}$$  \hspace{1cm} (10)

with $$a_n$$ being arbitrary coefficients. Here $$\psi_n(x)$$ is a normalized solution of the eigenvalue equation:

$$[-\Delta + V(x)] \psi_n(x) = E_n^2 \psi_n(x).$$ \hspace{1cm} (11)

The wave equation reduces to

$$\frac{d^2 f_n}{d\lambda^2} + \frac{Q^2}{\lambda^2} f_n + E_n^2 f_n$$

$$\approx \frac{d^2 f_n}{d\lambda^2} + \frac{Q^2}{\lambda^2} f_n = 0,$$ \hspace{1cm} (12)

with the normalization condition: $$i(f_n^* \partial_\lambda f_n - f_n \partial_\lambda f_n^*) = 1$$.

Localization means that a number of modes $$\{\psi_n, n \leq n_0\}$$ are bound states exponentially falling off at infinity. The marginal state $$n_0$$ corresponds to the so-called mobility edge as we shall explain in the next section.

### 4 Mobility Edge and Mean Free Path

Throughout this paper we only consider the adiabatic baryonic density perturbation for simplicity, the spectrum of which is roughly like the picture below. To simplify our discussion we assume that the spectrum is sharply peaked around the Jeans length $$l_c$$ at the equal time \[^3\] and the distribution of perturbation is Gaussian. To understand the Anderson localization in the Universe before the recombination time, we have to evaluate the mobility edge of the wave number, which is roughly the inverse of the mean free path $$l_{mf}$$ of the electromagnetic wave by the random potential. Consider a Schrödinger equation for the spatial part of the electromagnetic field:

$$[-\Delta + V(x)] \psi = E^2 \psi,$$ \hspace{1cm} (13)

[^3]: We are fully aware that inclusion of the "scale invariant spectrum" for scales larger than $$l_c$$ produces a hierarchy of structures. We would like to postpone the investigation in this direction to future works.
Figure 1: The spectrum of the adiabatic baryonic perturbation

with $x$ being the comoving coordinates. Recall that the random potential $V(x) = \omega_p^2 \delta_n^2$ does not depend on the conformal time in the matter-dominated era. The characteristic length scale $l_c$ of the density perturbation is given by the Jeans length at the equal time $t_{eq}$, which is roughly the same as the horizon scale at that time. After that time the length scale $l_c$ develops as $\propto a$. This gives $l_c \approx 10^{23} \text{cm}$ at the recombination time, while the horizon is $\approx 2 \times 10^{23} \text{cm}$.

Then the mean free path is given by

$$l_{mf} = \frac{1}{n_c \sigma},$$

where $\sigma$ is the elastic scattering cross section of the electromagnetic wave by a single potential and $n_c \approx 3 \times 10^{-3}$ is the density of "impurities". The elastic cross section is estimated roughly as $\approx 4\pi l_c^2$ by the standard wave mechanics, because the "energy" is low and is roughly given by $E^2 \approx 1$, while the potential is effectively gigantic $V \times l_c^2 \approx \omega_p^2 \delta_n^2 l_c^2 \approx 5 \times 10^{32}$. (Here we have taken $\delta_n \approx 10^{-5}$ because the assumed baryonic perturbation can be comparable to the temperature fluctuation of CMB observed by COBE [17]. However, the localization length and other results in the present work are insensitive to the magnitude of the density perturbation.) Therefore the localization should occur,

4 The equal time $t_{eq}$ is the time when the matter and radiation energy density are equal. This value depends on the value of the present baryonic density. We take $\frac{\Omega(t_{eq})}{\Omega(t_{eq})} = \left(\frac{t_{eq}}{t_{re}}\right)^{2/3} = 6$ as a typical value for $\Omega_0 = 0.2$. 

5
since the critical value of $Vl_c^2$ will be of order one. Then the mean free path is given by

$$l_{mf} \approx l_c/3,$$

and the mobility edge is $E^* \approx \frac{12\pi}{c}$, which is consistent with our low energy picture.

5 Localization Length

Fortunately in our present case, the randomness is huge so that an estimate of the localization length is available [18]. We have

$$\xi = l_c/\log(|V|/E^2),$$

(16)

where $l_c$ is the minimum length scale in which range the potential can be considered as a constant (the same notation as the one by Escande and Souillard [15]). For the adiabatic baryonic perturbation, it will be reasonable to take the Jeans length at the equal time $t_{eq}$ as the small scale cut-off $l_c$, which is $\approx 10^{23} cm$ at the recombination time. In our present case, $|V|/E^2 \approx 5 \times 10^{32}$ so that $\xi \approx 10^{21} cm$ at the recombination time.

It is easy to estimate the number of localized states $N$ by equating it with the number of states below the mobility edge $E^* = 2\pi l_{mf}^{-1}$. Namely,

$$\frac{V}{(2\pi)^3} \times \int^{E^*} d^3k = N.$$

(17)
We find that the density of the localized states is given by

\[ N/V = \frac{8\pi}{3} l_m^{-3}. \]  

(18)

There will be around 2000 lumps of electromagnetic fields inside the horizon at the recombination time. We will not discuss its direct observational implications in the present work, because there are uncertainties in the estimate of the mobility edge so that we should not take the predicted number of the lumps too seriously.

6 Cosmological Magnetic Field

The localized modes in the previous sections are excited in the thermal equilibrium after the equal time and before the recombination time. We estimate the magnitude of the thermally excited localized electric fields. The localized electric field \( E_{\text{loc}} \) is expanded as

\[ E_{\text{loc}} = - \sum_{n=0}^{n_0} a_n \hat{f}_n(\lambda) \psi_n(x) + \text{c.c.} \]  

(19)

so that the thermal average of \( E_{\text{loc}}^2 \) is

\[ \overline{E_{\text{loc}}^2} = \sum_{n=0}^{n_0} \frac{\overline{a_n^* a_n}}{\frac{Q}{\lambda T}} |\hat{f}_n|^2 |\psi_n|^2 \]  

(20)

Since \( Q/\lambda T << 1 \), \( |\hat{f}_n|^2 \approx Q/\lambda \) and \( |\psi_n|^2 \approx \xi^{-3} \), we obtain

\[ \overline{E_{\text{loc}}^2} \approx T/\xi^3, \]  

(21)

in a localization region of the size \( \xi \). Here we have taken into account the contribution from a single localized mode only, because the peaks of the other localized modes are randomly distributed and will be located outside of the region under consideration. The estimate of \( \overline{E_{\text{loc}}^2} \) can also be derived by the equal partition law. Note that the propagating modes \( (n > n_0) \) contribute to the Stefan-Boltzmann law.

At this stage we realize that approximately \( T/\omega_p \) photons occupies a localized state. As far as \( T/\omega_p >> 1 \), a coherent localized state of the size \( \xi \) is a good picture so that we can approximately describe it by a classical field:

\[ E_{\text{loc}} = - \sum_{n=0}^{n_0} \sqrt{T/\omega_p} \hat{f}_n(\lambda) e_n(x) \psi_n(x) + \text{c.c.}, \]  

(22)

where \( e_n(x) \) is the polarization vector, which is perpendicular to the gradient of the wave function \( \psi_n(x) \). Therefore, in a localization domain of the size \( \xi \)
Figure 3: The diffusion of the localized electromagnetic field

in the cosmic plasma, we have aligned electric and magnetic fields, which are perpendicular to each other and oscillate almost at the plasma frequency.

One might worry about the damping of the magnetic structure of the size $\xi$ by photon diffusion. However, the diffusion does not occur to the localized modes. This is one of the virtues of the Anderson localization mechanism.

Since the localization is a delicate interference phenomenon, the random potential $V(x)$ should vary slowly so that the motion of wave packet is not disturbed \[13\]. This condition is met before the recombination time, which we have discussed so far. After that time we have to take into account the recombination process in our equations for the electromagnetic fields. The number of free electrons sharply decreases, and the random potential changes in time.

After recombination time the packets of the localized coherent electromagnetic fields begin to diffuse. We will see the time evolution of the packet by solving the Vlasov equation for the electron distribution and the Maxwell equations for the electromagnetic fields. For our weak field we can linearize the Vlasov equation as

$$\frac{\partial f_1}{\partial t} + v \cdot \frac{\partial f_1}{\partial x} + \frac{e}{m} E \cdot \frac{\partial f_0}{\partial v} = 0,$$

(23)

where $f_1(t, x, v)$ is a small correction to the equilibrium Maxwell distribution function $f_0(v)$. We assume that the initial distribution function is the Maxwell distribution around the coherent electron velocity $u = \frac{e}{m\omega_p} E$,

$$f(t, x, v) = f_0(v - u(x)) \quad t < t_{\text{rec}}.$$

(24)

To solve the transport equation and the Maxwell equations, the standard method is to use the Laplace transformation,

$$\tilde{f}_1(\omega, v) = \int_0^\infty dt e^{i\omega t} f_1(t, v).$$

(25)
In the Laplace transformed space we can easily obtain an electric current density \( \tilde{j} = e \int d^3v \tilde{f}_1 \), and then evaluate the Laplace inverse transform. Since the high frequency parts of the current density are averaged out, we evaluate only a part which originates from “the \( \omega = \mathbf{k} \mathbf{v} \) pole” with \( \mathbf{k} \) being the wave vector in the Fourier transformed space (see §34 in [19] for the detailed calculation). The result is

\[
\tilde{j}(t, x) \approx -\frac{e^2}{m} \int d^3v f_0(v) E(0, x - vt).
\] (26)

In order to roughly estimate the total current \( I \), we may simply suppose that the localized electromagnetic field has a Gaussian form \( E = (T/\xi^3)^{1/2} \exp[-(x/\xi)^2] \) at the recombination time. The induced magnetic \( B \) is then given by

\[
B \approx \frac{I}{2\pi \xi} \approx \frac{nr_0 \xi}{2\omega_p} \left( \frac{T}{\xi^3} \right)^{1/2} \approx 10^{-26} \text{ gauss},
\] (27)

at the decoupling time with \( r_0 \) being the classical radius of electron.

This linear picture holds until the \( \mathbf{v} \times \mathbf{B} \) term becomes important, and after then the magnetohydrodynamics stage will set in. Since the localized region has a gigantic conductance, the magnetic field created at the decoupling era stays and will not dissipate to the Joule heat.

7 Conclusion and Discussions

In this paper we presented a new possible scenario which is intended to explain the origin of cosmological magnetic field.

The baryonic density perturbation plays a role of random potential of the size \( 10^{23} \text{ cm} \) to the electromagnetic fields and triggers the Anderson localization of the electric field before the recombination time. A large number of low energy photons occupy the localized states in the thermal equilibrium. The resultant coherent localized electric field is aligned in the localization region. The localized electric field then induces a coherent current and the magnetic field \( \approx 10^{-26} \text{ gauss} \) at the decoupling time.

Roughly, we expect 2000 magnetic lumps of the size \( 10^{21} \text{ cm} \) inside the horizon \( 2 \times 10^{23} \text{ cm} \) at that time. If the primeval magnetic field \( (10^{-26} \text{ gauss}) \) at the decoupling time is efficiently amplified by dynamo or other mechanism in the later stage, it may provide an explanation of the seed magnetic field in the Universe. However, it is beyond the scope of the present paper to work out the evolution of the magnetic fields in the long history of the Universe after the decoupling time. It is possible to argue that the resultant magnetic field traps sufficient number of protons so that the gravitational instability eventually creates a galaxy at the localization region.

In the present work, we assumed that the large scale density perturbation is primordial and its magnitude is chosen to be around \( \delta n \approx 10^{-5} \) so as to be
consistent with the COBE data on the anisotropy of CMB. In this case, the large scale structure is primordial and the galaxy scale one is its consequence by the Anderson localization. As remarked in the text, we only took into account a single scale \( l_c \) for the primordial perturbation for simplicity. The contribution from the larger scale to the localization phenomenon is interesting in the sense that it may account for the hierarchy of cosmological structures.

It is well known that a density perturbation of a scale larger than the Jeans length \( l_J = \frac{c_s}{\sqrt{\rho G}} \) gravitationally collapses against the thermal pressure gradient and therefore the perturbation grows. An analogous phenomenon exists for the gravitational collapse of localized magnetic fields. Correspondingly the magnetic Jeans length is obtained by replacing the sound velocity \( c_s \) in the expression of \( l_J \) by the Alfvén velocity \( c_a = \frac{B}{\rho \sqrt{\mu}} \).

\[
l_{mJ} = \frac{B}{\rho \sqrt{G}}
\]  

(For more satisfactory derivation, see Kim et al.\[7\].) Just after recombination time, \( \rho \approx 10^{-20} \text{g/cm}^3 \), we obtain

\[
l_{mJ} \approx \frac{B}{\text{gauss}} \times 10^{26} \text{cm},
\]  

which is comfortably less than the previously estimated localization length \( 10^{21} \text{cm} \), if \( B \) is less than the present galactic value \( 10^{-6} \text{ gauss} \). Therefore the gravity exceeds the magnetic pressure so that the magnetic lumps will not blow up after the decoupling time.

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Figure captions

• **Figure 1.** The spectrum of the adiabatic baryonic perturbation. The dashed and solid lines represent the initial scale invariant spectrum and the spectrum at the recombination time, respectively. $l_c$ is the Jeans length at the equal time.

• **Figure 2.** The diagram shows the time evolution of some characteristic lengths, the Hubble horizon, the Jeans length at equal time $l_c$, the mean free path $l_{mf}$, and the localization length $\xi$.

• **Figure 3.** The diffusion of the localized electromagnetic field. The upper figure represents the localized field before the recombination time, while the lower one represents the diffusion process after that time.