Developing spatial reasoning activities within geometry learning

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Abstract. Spatial ability is a capability that is used in everyday life as well as one of the keys to be successful in the fields such as science, technology, engineering and mathematics (STEM). Numbers of studies support the idea that spatial ability can be developed through training because of the malleability of spatial thinking. The spatial thinking was explored to demonstrate spatial thinking’s malleability and its importance in STEM education [10]. Even though the opinions on the result of spatial ability improvement are diverse, research indicates that spatial training can help to improve spatial ability when the appropriate material is given [11]. To date, even though only in a small number of studies, the enhanced spatial ability has been shown to improve students’ performance in STEM subjects. Six studies have been identified for its finding to improve STEM achievement through spatial training [12]. Also, a recent study from [13] found that embedded spatial reasoning into an existing elementary mathematics curriculum showed promising results that both spatial ability and mathematics performance improve after spatial training.

1. Introduction
Spatial ability can be described as the ability to mentally understand, manipulate, reorganise and interpret visual relationship [1]. Some research has revealed the positive correlation between spatial ability and mathematics performance [2,3]. Those who perform better in spatial tasks tend to have a better performance on a test of mathematical ability [4,5]. To date, some countries have recognised the importance of developing spatial ability during the school years such as Turkey, the United States and Australia [6,7,8,9]. Students from those countries did not perform in the top 10 countries with the highest mathematics performance in international tests such as Programme for International Student Assessment (PISA) and Trends in International Mathematics and Science Study (TIMSS). Nevertheless, these students performed far better than students from many other countries including Indonesia.

Numbers of studies support the idea that spatial ability can be developed through training because of the malleability of spatial thinking. The spatial thinking was explored to demonstrate spatial thinking’s malleability and its importance in STEM education [10]. Even though the opinions on the result of spatial ability improvement are diverse, research indicates that spatial training can help to improve spatial ability when the appropriate material is given [11]. To date, even though only in a small number of studies, the enhanced spatial ability has been shown to improve students’ performance in STEM subjects. Six studies have been identified for its finding to improve STEM achievement through spatial training [12]. Also, a recent study from [13] found that embedded spatial reasoning into an existing elementary mathematics curriculum showed promising results that both spatial ability and mathematics performance improve after spatial training.
The study from [13] gives the possibilities that students in higher level will have the support for their mathematics learning through spatial reasoning intervention as well. This present study, therefore, investigated the effect of spatial reasoning activities to support high school student’s mathematics learning, particularly geometry. Considering that spatial ability is an essential factor in geometry achievement and geometry problem solving [14], this study analyses grade 8 students’ learning process on the subchapter of a flat surface 3D shape (i.e., prism). For achieving the aim of this study, spatial activities embedded within mathematics classrooms will be designed which involves several spatial activities involving spatial visualisation and spatial orientation.

2. Methods

2.1. Participants
This developmental research involved two classes of grade 8 in one of the female boarding school in West Lombok, West Nusa Tenggara, Indonesia. The school covered mainly middle-low socio-economic demography. The two classes that participated were chosen among six classes based on the availability of the teachers to join the spatial training before conducting the classroom implementation. In total, there are 57 students with the mean age in years = 13.56 and one mathematics teacher who participate in this study.

2.2 Data collection and analysis
Data was collected from the implementation of lesson design during classroom intervention through classroom observation, video recording, interviews, and students’ worksheets then analysed in a retrospective analysis. Classroom intervention consisted of spatial activities embedded within mathematics classroom particularly in a subchapter of surface area and volume of a prism. The classroom activities were videotaped, and the author transcribed it herself. The interview was aimed to get more understanding of students thinking about specific activities which cannot be explained only by watching the video recording. Students’ works were also utilised to analyse the way students employ their understanding on the discussion of surface area and perimeter of a prism.

Table 1. Lesson design.

| Lesson | Activity | Spatial Construct | Mathematics strand | General Capabilities |
|--------|----------|-------------------|--------------------|----------------------|
| A      | Form a prism from connected cube blocks: 1. Visualize on adding up some cubes on a prism 2. Draw some different possible prisms that can be formed from 4 cubes (isometric drawing) 3. Game: form a prism | Spatial Visualisation & Spatial Orientation | - Define the surface area and volume of a prism and how to find it  - The ability to form a prism from cube blocks  - The ability to represent/visualise 3D object into the 2D isometric drawing | - Use the spatial visualisation and the spatial orientation skills in decoding and encoding the information  - Use the spatial visualisation skill in constructing a prism from the unit cubes. |
| B      | Form some prism with the same surface area but different volume and vice versa: 1. Form different possible prisms with the same volume but with the different surface area (isometric drawing) 2. Form different possible prisms with the same surface area but with the different volume (isometric drawing) 3. Find the surface area and volume of a prism | Spatial Visualisation | - Define the surface area and volume of a prism  - Different prism can have the same volume but the different surface area and vice versa | Use the spatial visualisation skill in constructing the prism |
2.3 Lesson design
This lesson design is included in the designing stage of developmental research. The lesson was designed collaboratively with the participating teacher in a series of lesson plan where spatial activities are embedded in mathematics lessons. It is expected that, while learning mathematics, students will also develop their spatial reasoning through the activities designed. The activities that were designed will also help students to understand the mathematics lesson better. The brief description of the lessons is presented in table 1.

3. Results and discussion
3.1. Results
The results of this study are presented in the retrospective analysis of students learning process during the lesson. The finding shows that the spatial activities within mathematics lesson help students to improve their spatial understanding as well as their understanding of surface area and volume of a prism.

In the first activity of lesson A, the teacher shows a model made from 4 x 3 x 3 unit cubes (see figure 1) then asked the student to name the model as well as to find the surface area and volume of it. Prior the lesson A, the students have learned about the characteristics of the prism as well as the surface area of it and volume of 3D shape has been learned in brief during their elementary years.

Some students mention the model as cuboid and some as a prism, and many of them use their gesture to find the surface area and the volume of the prism since the teacher only hold the model and did not hand it to students. In this activity, students started using their spatial orientation when they were counting the surface area; it was shown from the gesture of student’s palm as it open downward, upward, left-right, front and back or when they count the volume, the finger keeps pointing to the prism while counting.

On the second activity, the teacher asked the students to close their eyes and imagine if they were asked to place three more cubes on the top left of the prism, where will it be? In this activity, since the students are asked to close their eyes, then their option was visualising the model in their mind and where to place the additional three cubes. Students then orientate themselves as the teacher position as one of the students’ explanation that she placed the three cubes on her right side because her right side was the left side of the teacher. In this activity, the teacher gave enough time for students to orientate and visualise then predict where to locate the cubes. In the end, the class checked together whether the placing was right.

Teachers then asked the students whether the new shape was still named as a prism (see figure 2). This question provoked students to think back on the definition of the prism. The teacher gave an uncommon form of a prism so that students would not glue on the visualisation of the typical form of a prism with the regular base. The students knew that they need to find at least one pair of parallel and congruence side and it is not necessarily on the top or bottom part of the shape.

Figure 1. A model of a prism.

Figure 2. Two models of the prism.
Along with the activity in the first part of the lesson, after forming the new prism, the teacher also asked the students whether the surface area and volume of the prism remained the same. The class agreed that the surface area and volume were changing. Students might use the sense that adding up three cubes also change the surface area and volume.

In the next activity, students were asked to form a prism from 4 cube blocks, as many different prisms as possible. The students did not practically have four cube blocks in their hands, but they had it in mind then drawing all the possible arrangement on isometric paper. The example of students’ drawing is shown in figure 3.

Figure 3. Examples of students work on the isometric papers.

The idea that students did not use real unit cubes was that they were employing their spatial thinking while drawing a different kind of prism on isometric paper. First, they visualise the arrangement of those four cubes then predicted it on isometric paper so that it will be seen as the stacks they have in mind. In the end, they checked all their design by using real cube blocks. Unfortunately, there is no further data on how actually student thought while drawing, but from the interview, we find that student found difficulties in drawing on the paper and how to connect between dots. “To visualise or imagine the arrangement of 4 cubes were easy but to draw it on a paper was hard”, said one of the students.

The next activity is a game of cubes arrangement (see figure 4). In this game students alternately instructed each other to form a prism from 6 unit cubes by first wrote down the instruction. This activity might not directly correlate to the concept of area and perimeter, but it built the students awareness of spatial orientation while receiving the instruction, despite using their spatial visualisation skill while writing the instruction. Also, the student used the concept of the prism to make sure that the shape they formed was a prism, not just merely arranging different forms of 4 unit cubes. For the one who gave instruction, she/he must think of the steps in building the blocks so that her/his friend who receive the instruction could understand it easily. On the other hands, the one who received the instruction should be able to orientate he/herself as the person who gives the instruction.

Figure 4. A game of cubes arrangement.

The following activity was arranging 6 unit cubes to form different prism with the same volume but with the different surface area and vice versa. The same as previous activity, students, did the arrangement in mind and drew it on isometric paper. The idea of this activity was getting students to understand the concept of area and volume better, as well as deepening their spatial visualisation through this activity.

The teacher gave the initial prism made from 6 unit cubes (see figure 5) then asked the students to form different prism from that model. The new prism should have the same volume as the initial prism but with the different surface area and vice versa. Some example of students’ drawing is shown in figure 6.
Figure 5. The initial form of prism given by the teacher as a model.

Figure 6. Examples of students work on the isometric papers

In these activities, students thought in two ways, first was when students start thinking about the different arrangement so that the surface area would be different but the volume remained the same. The second was when the students tried to add or reduce the number of the cube so that the prism would have different volume but the surface area remained the same. The students were mainly used spatial visualisation in this activity.

In the last activity, students were given a picture of 3D shape without dimension, as shown in figure 7.

Figure 7. A composite 3D shape.

The teacher asked whether the shape of the picture is a prism. Students, in fact, could explain that the shape is a prism by giving the justification that the shape has a pair of parallel congruent sides in its front part and back part. The student could also explain how to find the surface area of the shape, one of the students explain during the interview that to find the total surface area she needed to add all side planes of the prism which all were in rectangle form except the front and the back side of the solids. When she was asked how to find the area of the front and back sides, she said she would divide the shape into rectangle and square as shown in figure 8.

Figure 8. Illustration in splitting the front side of the 3D shape

The student also explained that he needs to divide the shape into two prisms as shown in figure 9. Students explain that she needed to add up the volume of prism 1 and prism 2 by using the formula of
length \times width \times height for each prism.

Figure 9. Illustration in splitting the front side of the 3D shape.

In this activity, the student explained clearly every step on how to calculate the surface area and volume of the prism. Since there is no dimension given, then students were urged to use their spatial skills in describing the steps. Such as mentioning all parts of the sides planes and how to do the partition for the front and back sides of the solid as well as the partition of the solids to find the volume of it.

3.2. Discussion

Mathematics problem solving, often, requires students to reason about spatial information [12]. In this instance, for learning surface area and volume of a 3D shape, students require to be able to identify which parts of the 3D shape that was called as the surface and which part is the volume before they know the formula of that measurement. The series of activities in this study was designed to promote the spatial thinking of students in various ways to support their understanding of surface and volume of the 3D shape. The activities involved can be grouped into three main activities namely manipulating the unit cubes, drawing on isometric paper, and at the end, there was a review regarding the surface area, volume, and a prism shape. All the activities designed gave a chance for students to visualise, use their spatial reasoning as well as construct a geometric modelling to solve a problem. It is in line with one of the principles and standards for school mathematics from National Council of Teachers of Mathematics [15] where students are expected to learn mathematics with understanding, actively building new knowledge from experience and previous knowledge.

The first group of activities (Lesson A), manipulating unit cubes into a different type of prisms, let the students use a lot of spatial visualisation and somewhat spatial orientation. Despite sharpening students’ spatial reasoning, this activity also deepens students’ mathematical understanding. Through the activities of forming a prism from 4 unit cubes and a game of forming a prism, the students were expected to keep on repeating the idea of a prism so that they will also know the shapes which are not a prism. Meanwhile, from the activity of forming a prism from 6 unit cubes let the students understand the concept of surface area and volume. Students also get the idea that two prisms with the same volume do not necessarily have the same surface area and vice versa.

The second group of activities (Lesson B), drawing on isometric paper, was a novel activity for all students and they found it somewhat challenging at the beginning. Besides the reasons for novelty, the challenge most probably came from the urgency of using spatial visualisation and or spatial orientation while drawing on a paper. In this activity, students need to use spatial visualisation frequently as well as spatial orientation. Students need to think about how to connect the dots so that the shape they draw was the same as the shape they have in mind. The ability to visualise the shape, know the whole part of the shape and how it was constructed help students to get the concept of surface area and volume of the 3D shape.

In the closing activities, the students were given a picture of a composite figure without dimensions. In this activity, students were provoked to not glued into the formula of surface area and volume but more to the exploration on how they give their reasoning on finding the surface area and volume of the shape when the shape is not a regular prism. In this activity, students recalled back their knowledge about prism to make sure that the shape they saw was a prism. Regarding the counting of surface area, the students have the idea that side planes need to be added together, but for the front side and back side, they need to think on how to count
it. Students use their mental cutting of the front part so that it comes into square and rectangle. This activity is known as partitioning and [16] described partitioning as one of the five basic concepts in learning to measure area other than unit iteration, conservation, structuring the array and linear measurement.

4. Conclusion

The series of activities in this study was designed to promote the spatial thinking of students in various ways to support their understanding of surface and volume of the 3D shape. All the activities designed gave a chance for students to visualise, use their spatial reasoning as well as construct a geometric modelling to solve a problem. Current study contributes to the possibility to embed spatial activities within the mathematics classroom through various activities related to the topic given (i.e., geometry). Because this study only gave examples on geometry topic, further study is needed to see the possibilities of other mathematics topics to be embedded with spatial activities.

5. References

[1] Tartre L A 1990 Spatial orientation skill and mathematical problem solving. *Journal for Research in Mathematics Education* 21 216
[2] Cheng Y and Mix K S 2014. Spatial training improves children's mathematics ability *Journal of Cognition and Development* 15 2
[3] Pittalis M and Christou C 2010 Types of reasoning in 3D geometry thinking and their relation to spatial ability *Educ. Stud Math.*, 75 191
[4] Holmes J, Adams J W and Hamilton C J 2008 The relationship between visiospatial sketchpad capacity and children's mathematical skills *European Journal of Cognitive Psychology* 20 272
[5] Rasmussen C and Bisanz J 2005 Representation and working memory in early arithmetic *Journal of experimental child psychology* 91 137
[6] Cakmak S, Isiksal M and Koc Y 2014. Investigating the effect of origami-based instruction on elementary students’ spatial skills and perceptions *The Journal of Educational Research* 107 59
[7] National Academy of Science 2012 *Learning to Think Spatially: GIS as a Support System in the K-12 Curriculum* (Washington DC: National Academy of Science)
[8] Foundation N S 2010 *Preparing the next generation of stem innovators: Identifying and developing our nation's human capital* (National Science Foundation)
[9] Australian Curriculum, Assessment, and Reporting Authority 2010 *Australian F-10 Curriculum* Retrieved from https://www.australiancurriculum.edu.au/f-10-curriculum/general-capabilities/numeracy/
[10] Miller D I, Newcombe N S and Uttal D H 2013 Exploring and Enhancing Spatial Thinking: Links to Achievement in Science, Technology, Engineering, and Mathematics *Current Directions in Psychological Science* 22 367
[11] Olkun S 2003 Making connections: Improving engineering drawing activities *International Journal of Mathematics Teaching and Learning*, 3 1
[12] Stieff M and Uttal D 2015 How much can spatial training improve STEM achievement? *Educational Psychology Review* 27 607
[13] Lowrie T, Logan T and Ramful A 2017 Visiospatial training improves elementary students’ mathematics performance *British Journal of Educational Psychology*
[14] Battista M T 2007 The development of geometric and spatial thinking *Second handbook of research on mathematics teaching and learning* 2 843
[15] National Council of Teachers of Mathematics 2000 *Principles and standards for school mathematics* (Reston: NCTM)
[16] Clements D H and Stephan M 2004. Measurement in Pre-K to grade 2 mathematics *Engaging young children in mathematics: Standards for early childhood mathematics education* 299-317