Analytical model of the deformation response of bedding slopes to excavation on the basis of Mindlin’s strain solution

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Abstract. To reveal the deformation law and mechanism of bedding slopes under excavation unloading, an unloading rebound model of slope deformation is established on the basis of Mindlin’s strain solution, and a bedding-slip model of slope deformation is deduced on the basis of the Kalhaway constitutive model. Combining the two models, this study presents a computational model for the deformation response of bedding slope excavation. This model can reflect the mechanism of excavation unloading and the characteristics of rock mass structure. The reliability of the model is verified by comparing the calculated results of the analytical model with the experimental results in the laboratory. On this basis, the influences of excavation angle, excavation depth, and stratum thickness are analyzed by using several calculation examples. The calculation results show that the deformation induced by excavation increases with the increase in excavation angle or depth and decreases with the increase in stratum thickness. The excavation response of bedding slopes is mainly affected by the effect of unloading and the slip of the structural plane. Moreover, the unloading effect is controlled by the amount of excavation, and the rebound deformation of slopes is approximately linearly correlated with excavation volume. Bedding slip is affected by many factors because the increase in excavation angle or the amount of rock stratum leads to the increase in slip deformation. The proposed model can provide a basis for the deformation mechanism of bedding slopes under excavation.

1. Introduction
Slope instability and deformation are affected by many factors, such as excavation unloading. Thus, in recent decades, scholars worldwide have conducted substantial research to reveal the law and mechanism of slope deformation induced by excavation.
Li Ming (2011) conducted centrifugal model tests for slope excavation and studied the time effect of slope deformation under different excavation conditions [1]. Cai Guojun (2008) used the bottom friction test method to establish a geological physical model of a slope and studied the deformation response of the slope under different excavation conditions [2]. On the basis of a comprehensive analysis of engineering geological conditions, an in-situ test, and the deformation monitoring results of the Longtan Project, Leng Xianlun (2004) established a geological generalization model and a slope excavation model [3]. Wang Zaiquan (1997) established the displacement transfer function of slope deformation law and preliminarily explored the use of such function to study the deformation law and stability of a slope on the basis of deformation monitoring data obtained during the excavation of the high slope of the Geheyan Hydropower Station [4]. Zhu Jiliang (2010) revealed the basic law and special performance of the deformation response of such slope through numerous geological investigations [5]. Xiao Keqiang (2007) studied the slope stability and deformation variation rule of soft-rock high slopes during excavation and rainfall through a geomechanical model test and analyzed the influence mechanism of the retaining structure on slope deformation [6]. Feng Jun (2005) used the geomechanical model to conduct an excavation failure test on a bedding rock slope and proposed a method to determine the scope of excavation relaxation zones [7]. Tang Hongmei (2011) discussed the variation rules of internal displacement and stress in slopes through a similarity model test [8]. Zhao Hua (2018) studied the evolution process of the tipping deformation of rock slopes by using graded excavation to simulate the downward action of valleys in accordance with the physical model of the slope [9]. Based on viscoelastic theory, Ding Xiuli (1995) used the finite element
method to calculate and analyze the excavation deformation response of the high slope of the Three Gorges Shiplock [10]. Zhang Xiangdong (2006) used ADINA to conduct a numerical simulation of slope excavation deformation and analyzed the influence of different construction schemes on slope deformation [11]. Meng Guo-tao (2007) conducted a three-dimensional discontinuous numerical simulation of antidip layered slopes and studied the stability and deformation response of such slopes under the action of excavation and reinforcement [12]. Zhang Yuyang (2017) used a discrete element program to simulate and verify the boundary conditions and failure modes of the rock slope of the Lungu Hydropower Station along the Yalong River [13]. Li Yi (2018) combined deformation monitoring data and numerical simulation to predict the deformation caused by excavation [14].

In view of rock slopes, especially high and steep ones, physical model tests and numerical calculation methods are mainly used to study the deformation response of slopes to excavation. Only a few studies have used mechanical analytical models to investigate the rock excavation deformation law. Model tests have the problems, such as long cycle and high cost. For numerical calculation models, the modeling and calculation processes are relatively complex. Considering the complexity and diversity of slopes, this study focuses on bedding slopes to understand the major contradictions. To reveal the deformation law and mechanism of bedding slopes under excavation, an unloading rebound model of slope deformation is established on the basis of Mindlin’s strain solution, and a bedding-slip model of slope deformation is deduced on the basis of the Kalhaway constitutive model. Combining the two models, this study presents a computational model for the deformation response of bedding slope excavation. This model can reflect the mechanism of excavation unloading and the characteristics of rock mass structure. The deformation characteristics of bedding slopes can be analyzed reasonably on the basis of the formation mechanism.

2. Setup of the calculation model

Under the action of excavation unloading, bedding slopes exhibit deformation, including material and structural deformation. The material deformation of slopes induced by excavation mainly refers to unloading rebound deformation. After the excavation of bedding slopes, a free surface is formed, resulting in a decrease in the antisliding force. By contrast, structural deformation refers to the sliding deformation of stratified rock mass along the bedding. The deformation of bedding slopes induced by excavation can be characterized by the following equation:

\[ u = u_m + u_s, \]

where \( u_m \) represents the unloading rebound deformation of the rock mass material, and \( u_s \) represents the sliding deformation of the rock mass structure.

The failure of bedding slopes mostly occurs in the interlayer structural plane. The material of the rock stratum can be assumed to be in the elastic deformation state before slope failure, and the material of the structural plane meets the nonlinear deformation condition. When calculating the unloaded rebound deformation of a slope, the influence of the structural plane is not considered due to the small proportion of structural surface material in the entire rock mass. Under the assumption that the rock mass is a continuous, homogeneous, and isotropic elastomer, the unloaded rebound model of the slope excavation is established on the basis of Mindlin’s strain solution to calculate \( u_m \). When calculating the structural deformation of bedding slope, the sliding calculation model based on the Kalhaway constitutive model is established to calculate \( u_s \). The calculation methods of the excavation unloading rebound and bedding-slip models are respectively introduced below.

2.1. Excavation unloading rebound model based on Mindlin’s strain solution

2.1.1 Mindlin’s strain solution

Mindlin’s theory (1936) states that when a horizontal concentrated force \( P \) is acted on the elastic semi-infinite body at depth \( c \) (as shown in Fig. 1) [15], the lateral and vertical deformations generated at point M at where the depth is \( z \) are respectively

\[ u_{lh} = P_1, \]

\[ u_{sh} = P_2. \]
In the formula, $I_{vh}$ and $I_{hv}$ are the respective vertical and lateral deformations generated by the unit’s horizontal concentrated load at point M and are also known as the influence coefficients of the vertical and lateral deformations. $I_{vh}$ and $I_{hv}$ are respectively

$$I_{vh}(x, y, z, c) = \frac{x}{16\pi G(1-v)} \left( \frac{z-c}{R_1} + \frac{3}{R_2} + \frac{6c(z+c)}{R_2^3} \right),$$

$$I_{hv}(x, y, z, c) = \frac{1}{16\pi G(1-v)} \left( \frac{3-4v}{R_1} + \frac{x^2}{R_2^2} + \frac{8v(1-v)}{R_2} + \frac{6c(z+c)}{R_2^3} \right) + \frac{4(1-v)(1-2v)}{R_2(R_2 + z + c)},$$

where $G$ is the shear modulus, $v$ is Poisson’s ratio, and $c$ is the depth of the concentration point.

Similarly, when a vertical concentrated force $Q$ is acted on the elastic semi-infinite body at depth $c$, the vertical and lateral deformations of point M at distance $z$ where the depth is $z$ are respectively

$$u_v = QI_{vh},$$

$$u_h = QI_{hv},$$

where $I_{vh}$ and $I_{hv}$ are the respective vertical and lateral deformation generated by the unit’s vertical concentrated force at point M.

$$I_{vh}(x, y, z, c) = \frac{1}{16\pi G(1-v)} \left( \frac{z-c}{R_1} + \frac{8(1-v)^2 - (3-4v)}{R_2} + \frac{6cz(z+c)}{R_2^3} + \frac{3z(3-4v)(z+c)^2}{R_2^2} \right),$$

$$I_{hv}(x, y, z, c) = \frac{\sqrt{x^2 + y^2}}{16\pi G(1-v)} \left( \frac{z-c}{R_1} + \frac{3(3-4v)}{R_2} \right) - \frac{4(1-v)(1-2v)}{R_2(R_2 + z + c)} + \frac{6cz(z+c)}{R_2^3}.$$

2.1.2 Mechanical analysis of slope under excavation

Assume that the original slope angle is equal to the dip angle $\alpha$, and the angle of excavation surface is greater than the dip angle. Before the excavation of slope rock mass, the excavation surface is in a state of three-direction stress. The horizontal force on the excavation surface is analyzed, as shown in Fig. 2. The excavation slope ratio is $1:n$, and the dead weight of the rock mass produces two equilibrium forces ($P_1$ and $P_2$) with opposite directions and equal magnitude, that is, the horizontal lateral pressure of the rock mass. We also have equal and opposite forces in the vertical directions $Q_1$ and $Q_2$. The excavation of slope is equivalent to the unloading of internal forces $P_2$ and $Q_2$, and the mountain experiences the corresponding rebound deformation.
The influence of the structural plane is not considered when calculating the unloading rebound deformation of slope materials. According to the above assumption, the rock mass material is in elastic state; thus, the initial stress state of rock mass satisfies the Kinnick hypothesis (2010) [16].

![Sketch of the mechanical analysis of the excavation slope without considering the influence of the structural plane](image)

**Fig. 2.** Sketch of the mechanical analysis of the excavation slope without considering the influence of the structural plane

The vertical stress of the rock mass is calculated in accordance with dead weight stress. When analyzing the vertical forces on the excavation surface, the dead weight of the rock mass produces two vertical balance forces \( Q_1 \) and \( Q_2 \) of equal magnitude and opposite direction, whose strength is as follows:

\[
Q_1 = Q_2 = \gamma z_0.
\]  

(12)

The horizontal stress of the rock mass is calculated in accordance with the Kinnick formula. When analyzing the horizontal force on the excavation surface, the horizontal rock pressure strength can be calculated as follows:

\[
P_1 = P_2 = K_0 \gamma z_0.
\]  

(13)

where \( K_0 \) is the horizontal stress coefficient, \( \gamma \) is the rock density, \( z_0 \) is the vertical (gravitational) depth of the action point, and \( \nu \) is the rock’s Poisson’s ratio.

2.1.3 Unloading rebound model based on Mindlin’s strain solution

First, the basic coordinate system \((x_j, y_j, z_j)\) is established by taking the natural slope line as the ground line of the elastic semi-infinite body as follows:

The Cartesian coordinate system (Fig. 3) is established by taking the apex of the slope excavation line as the origin, the inclination line of the natural slope as the \( x_j \) axis, and the line pointing into the slope perpendicular to the \( x_j \) axis as the \( z_j \) axis. In this coordinate system, the \( x_j \) and \( z_j \) axes are set as horizontal and vertical, respectively. In accordance with Formulas (12) and (13), the strength of the lateral and vertical forces on the excavation surface in this coordinate system can be deduced and calculated as follows:

\[
P_1' = \gamma z(K_1 \tan \alpha),
\]  

(15)

\[
Q_1' = \gamma z(1 + K_0 \tan \alpha),
\]  

(16)

where \( z \) is the \( z_j \) coordinate value of the action point in the basic coordinate system.

After the unloading of internal forces \( P_1' \) and \( Q_1' \) caused by slope excavation, the mountain correspondingly produces rebound deformation, which can be expressed by the displacement of the original mountain generated by the action of \( P_1' \) and \( Q_1' \), which are respectively equal to \( P_1 \) and \( Q_1 \) whose directions are opposite. The following part adopts Mindlin’s theory for modeling.

The action point of concentrated force is located directly below the origin when calculating using Mindlin’s formula; thus, an auxiliary coordinate system \((x_c, y_c, z_c)\) is established to facilitate the correct use of Mindlin’s formula. The auxiliary coordinate system is established as follows:
Fig. 3. Sketch of the mechanical analysis of the excavation slope under the basic coordinate system

With \( M(x_c, y_c, z_c) \) regarded as the coordinate of the action point of the concentrated force, an auxiliary coordinate system (Fig. 3) is established by taking \((x_c, y_c, 0)\) as the origin, the inclination line of natural slope as the \( x_f \) axis, and the line pointing into the slope perpendicular to the \( x_f \) axis as the \( z_f \) axis. Then, the auxiliary coordinate transformation formula of any point \((x_j, y_j, z_j)\) in the basic coordinate system is as follows:

\[
\begin{align*}
&x_f = x_j - x_c \\
y_f = y_j - y_c \\
z_f = z_j
\end{align*}
\]  

In the basic coordinate system, the coordinates of any point in the excavation surface or in the slope body are assumed to be \( N(x_0, y_0, z_0) \), and the coordinates of any point of concentrated force in the excavation surface are assumed to be \( M(x_c, y_c, z_c) \). After the coordinate transformation, the coordinate of point \( N \) is \((x_0 - x_c, y_0 - y_c, z_0)\), and the coordinate of point \( M \) is \((0, 0, z_c)\) in the auxiliary coordinate system. Then, in accordance with the formulas of Mindlin's strain solution and under the action of concentrated force \((P, Q)\) at any point \( M(x_c, y_c, z_c) \) in the excavation surface, the lateral and vertical deformation of \( N(x_0, y_0, z_0) \) at any point in the slope excavation surface or the slope body can be calculated by the following formula:

\[
\begin{align*}
&u_{lh} = P I_{lh}(x_0 - x_c, y_0 - y_c, z_0, z_c) \\
&u_{sh} = P I_{sh}(x_0 - x_c, y_0 - y_c, z_0, z_c) \\
&u_{lm} = Q I_{lm}(x_0 - x_c, y_0 - y_c, z_0, z_c) \\
&u_{sm} = Q I_{sm}(x_0 - x_c, y_0 - y_c, z_0, z_c)
\end{align*}
\]  

The lateral and vertical deformations of any point \( N(x_0, y_0, z_0) \) in the slope excavation surface or in the slope body under the action of horizontal and vertical forces can be obtained by a two-dimensional integration of Formula (18) in a region of the excavation surface \( A(0 \leq z \leq H \cos \alpha, -L/2 \leq y \leq L/2) \) and the summation based on principle of stress superposition.

\[
\begin{align*}
U_h &= U_{h'} + U_{h''} = \int_A (P I_{lh}(x_0 - x_c, y_0 - y_c, z_0, z_c) + Q I_{lm}(x_0 - x_c, y_0 - y_c, z_0, z_c))dA \\
U_v &= U_{v'} + U_{v''} = \int_A (P I_{sh}(x_0 - x_c, y_0 - y_c, z_0, z_c) + Q I_{sm}(x_0 - x_c, y_0 - y_c, z_0, z_c))dA
\end{align*}
\]  

Integration region \( A \) is related to the width and depth of excavation, and it can be calculated by the following formula:
\[
A = \begin{cases} 
  x = \frac{z}{\tan(\beta - \alpha)} \\
  \frac{L}{2} \leq y \leq \frac{L}{2} \\
  0 \leq z \leq H \cos \alpha 
\end{cases}, \quad (21)
\]

where \(L\) is the excavation width of the slope, and \(H\) is the excavation depth of the slope.

2.1.4 Solution method for the model

Given the complex form of Formulas (19) and (20), the integral function cannot be given directly, and the rebound deformation caused by slope unloading is calculated by using the numerical integral method. First, the lateral and vertical forces in the excavation surface are discretized, and the distributed load over each small area can be equivalent to a concentrated force acting within the semi-infinite body. Then, by summing the displacement caused by the concentrated force in the excavation surface, the displacement of any point in the excavation surface or slope body can be obtained. The mechanical model (taking the lateral force as an example) is shown in Fig. 4.

Then, the calculation formulas of the lateral deformation \(u_{mh}(x_0, y_0, z_0)\) and vertical deformation \(u_{mv}(x_0, y_0, z_0)\) of any point \(N(x_0, y_0, z_0)\) in the slope excavation surface or the slope body are as follows:

\[
\begin{align*}
  u_{mh}(x_0, y_0, z_0) &= \sum_{i=0}^{N} \sum_{j=0}^{M} (I_{mij}P_{ij} + I_{mij}Q_{ij}), \quad (22) \\
  u_{mv}(x_0, y_0, z_0) &= \sum_{i=0}^{N} \sum_{j=0}^{M} (I_{vij}P_{ij} + I_{vij}Q_{ij}), \quad (23)
\end{align*}
\]

where \(P_{ij}', Q_{ij}'\) are the respective lateral and vertical concentrated forces at any node; \(i\) is the serial number of the nodes along the inclination of the excavation surface; \(j\) is the serial number of the nodes along the strike of the excavation surface; and \(I_{mij}, I_{vij}, I_{mij}, I_{vij}\) are the corresponding influence coefficients of the lateral and vertical deformations in the auxiliary coordinate system, respectively.

Fig. 4. Sketch of the equivalent horizontal uniform load

2.2. Bedding-slip model based on the Kalhaway constitutive model

2.2.1 Influence range of slope excavation

The influence range of slope excavation is first calculated before calculating the sliding deformation of the stratified rock mass. Based on Mindlin's strain solution, the distribution of rebound deformation \(u\) at different points on different layers after slope excavation can be calculated, as shown in Fig. 5. In the figure, the rebound deformation of each point on the bedding line \(BB''\) gradually decreases from point \(B\) to \(B''\). It has been very small at point \(B'\) and will remain unchanged. Therefore, \(B'\) can be considered the boundary point of slope deformation induced by excavation in the direction of \(BB''\); that is, as a result of excavation, the displacement and stress of each point on \(BB''\) changes, whereas the displacement and stress of each point on \(B'B''\) are basically unaffected. Similarly, the boundary points \(C', D',\) and \(E',\) and \(E''\) on \(CC'', DD'', EE''\), and other layers can be obtained, and the boundary line affected by excavation can be obtained by connecting these points. The scope of \(A'B'C'D'E'DCBA\) in the slope body is affected by excavation, and this area is the influence range of excavation.
2.2.2 Mechanical analysis of bedding slope rock mass

In the calculation model, the strata dip angle is assumed to be $\alpha$, which is equal to the angle of the original slope, and the angle of excavation surface is $\beta$. The strata numbering from the top of the slope to the foot are $1, 2, \ldots, n_y$, where $n_y$ is the total number of rock strata. The bulk density of each rock stratum is $\gamma$, the thickness of each rock stratum is $d$, and the shear strength parameters ($f$ and $c$) of all the bedding surfaces are the same.

To calculate the bedding-slip deformation within the influence range of excavation, the shear stress $\tau_i$ and $\sigma_i$ after and before excavation should be calculated, respectively. When calculating the bedding-slip deformation, the shear force on the bedding surface should be equal to the sliding force generated by the upper rock. Considering that the thickness of the rock stratum is small relative to the length of the rock stratum, the section of the upper stratum is visible as a rectangle. Then, the shear stress $\tau_i$ and normal stress $\sigma_i$ at the $i$th bedding surface after excavation can be calculated in accordance with the following formulas:

$$\tau_i = yid\sin\alpha,$$

$$\sigma_i = yid\cos\alpha,$$

where $\gamma$ is the bulk density of the rock mass, and $d$ is the thickness of the rock strata.

The section of the rock stratum is assumed to be rectangular, that is, assuming that the excavation surface of each stratum is vertical. The strength of the horizontal rock pressure on the excavation surface can be calculated in accordance with Formula (13). Then, the shear stress $\tau_2$ and normal stress $\sigma_2$ at the $i$th bedding surface should be calculated in accordance with the following formula:

$$\tau_2 = \tau_i - \frac{1}{l_{i}} \int_{l_{i}}^{\gamma K_{a}zdz},$$

$$\sigma_2 = \sigma_i + \frac{1}{l_{i}} \int_{l_{i}}^{\gamma K_{a}\tan\alpha zdz},$$

where $l_{i}$ is the critical calculating length of the bedding surface determined on the basis of the scope of the excavation influence area.

2.2.3 Calculation of bedding-slip deformation

Through numerous experiments, Kallhaway (2014) found that the curve describing the relationship between $\tau$ and $\Delta u$ can be fitted with a hyperbolic function before the shear stress reaches the peak strength of the structural plane. He proposed the following equation:

$$\Delta u = \frac{\tau m'}{1 - \tau n'},$$

where $m'$ and $n'$ are the hyperbola shape coefficients, $m' = 1/K_{si}$, $n' = 1/\tau_{ult}$, $K_{si}$ is the initial shear stiffness, $\tau_{ult}$ is the limit shear strength, and $\tau_{ult}$ is calculated in accordance with the More–Coulomb criterion. $K_{si}$ (Xu H F 2000, Wang He 2014, Bandis 1983) is determined in accordance with the following formula:

$$K_{si} = K \gamma \sigma^\gamma,$$

where $K_j$ is the degree of shear stiffness (20 MPa/mm is adopted (Xu H F, 2000) in this study), $n_j$ is the shear stiffness index (0.7 is adopted in this study), and $\sigma_j$ is the normal stress on the structural plane.
The Kalhaway constitutive equation is proposed on the basis of the test data of small-sized rock blocks; the material deformation of the specimen during the test can be ignored relative to the deformation of the structural plane, and the deformation obtained from the test can be considered the deformation of the structural plane. Therefore, the Kalhaway constitutive equation can be used to describe the deformation of the structural plane of rock mass.

The shear deformation of the structural surface at the bottom of the ith stratum in the area affected by excavation is calculated as follows:

By substituting Equations (24), (25), (26), and (27) into Equation (30), the shear deformation $u_{si}$ of the structural plane caused by excavation can be obtained as follows:

$$u_{si} = u_{si1} - u_{si2},$$

where $u_{si1}$ and $u_{si2}$ are the shear deformation of the structural plane after and before excavation, respectively.

Suppose the slope has $j$ strata in total, then the sliding deformation $u_i$ of the ith stratum caused by excavation is

$$u_i = \sum_{k=1}^{j} u_{sk},$$

2.3. Model of the deformation response of bedding slopes

By combining the unloading rebound model and the bedding-slip model, the model of deformation response of bedding slope can be obtained. In accordance with Formula (1), the slope deformation in the bedding direction $u_h$ and the deformation in vertical stratum direction $u_v$ can be obtained by accumulating the rebound and bedding-slip deformations.

$$u_h = u_{mh} + u_s,$$

$$u_v = u_{mv},$$

where $u_{mh}$ is the unloading rebound deformation in the bedding direction, $u_{mv}$ is the unloading rebound deformation in the vertical stratum direction, and $u_s$ is the bedding-slip deformation.

Then, the total deformation caused by excavation can be calculated as follows:

$$u = \sqrt{u_h^2 + u_v^2}.$$  

Formulas (34) and (35) show that the proportion of unloading rebound deformation and bedding-slip deformation in the total deformation can be clearly identified by the model of the deformation response of bedding slopes, thus indicating that the application of this model can reasonably explain the formation mechanism of the deformation of the bedding slope caused by excavation. Analyzing the deformation response of bedding slopes to excavation from the perspective of formation mechanism can help identify slope stability accurately; moreover, a scientific plan for slope reinforcement and early warning is beneficial.

3. Experimental verification

This section takes the data of the model test in Feng J (2005) as an example to illustrate the validity of the model established [17]. Feng J (2005) performed several laboratory model tests for the excavation deformation of bedding slopes. The geometry of model 9 is shown in Fig. 6 [18]. The angle of excavation surface $\beta$ is 60°, the dip angle $\alpha$ is 30°, the excavation depth $H$ is 27.71 cm, and the excavation width is 54 cm. After the establishment of the basic coordinate system mentioned above, the coordinates of points 1, 6, and 11 are N1(8.072, 0, 4), N6(1.144, 0, 8), and N11(5.785, 0, 12), respectively. The geometry of model 5 is shown in Fig. 7. The angle of excavation surface $\beta$ is 60°, the dip angle $\alpha$ is 25°, and the excavation depth $H$ is 24.16 cm. After the establishment of the basic coordinate system mentioned above, the coordinates of points 1, 6, and 11 are N1(−4.287, 0, 4), N6(1.425, 0, 8), and N11(7.138, 0, 12), respectively. The physical and mechanical parameters of similar materials are shown in Tables 1 and 2.
The calculation results of the total deformation of key points after excavation are shown in Table 3. As shown in the table, the calculated values of deformation at each measuring point of models 9 and 5 are close to the experimental values, indicating that the established analytical model for the excavation deformation response of bedding slopes on the basis of Mindlin’s strain solution can well simulate bedding slope deformation caused by excavation.

Table 3 Comparison between model test and analytic calculation (unit: mm)

| comparison of results | Model 9- Strata dip angle 30° Measuring point | Model 5- Strata dip angle 25° Measuring point |
|-----------------------|---------------------------------------------|---------------------------------------------|
| Calculated value test results | 0.33 0.38 0.44 | 0.13 0.14 0.19 |
|                          | 0.33 0.38 0.45 | 0.10 0.14 0.19 |
4. Analysis of the influencing factors of slope deformation

The excavation deformation of the slope under different conditions is calculated by the established analytical model. With model 9 taken as the basic model of slope calculation, the measuring points near the slope surface are selected for analysis.

The coordinates of the measuring points are (−12,0,0), (1.856,0,8), (12.713,0,16), respectively. The influence of factors, such as excavation angle, excavation depth, and stratum thickness, on the excavation deformation of bedding slopes is studied below.

4.1. Excavation angle

Excavation angle is an important factor affecting the stability of slope deformation. With other factors unchanged, different excavation angles are selected to calculate the deformation of measuring points at the top, middle, and foot of the slope after excavation. The calculation results are shown in Fig. 8. As shown in the figure, when the excavation angle is 40°, the deformation of the measuring point at the top of the slope is 0.161 mm, and that at the foot of the slope is 0.253 mm. When the excavation angle is 65°, the deformation of the measuring point at the top of the slope is 0.348 mm, which is 1.16 times higher than that at 40°. When the excavation angle is 65°, the deformation at the measuring point at the foot of the slope is 0.531 mm, which is 1.10 times higher than that at 40°.

The results show that the displacement of each measuring point increases gradually when the angle of excavation slope increases. With the increase in excavation slope, the increase rate of deformation caused by the same increase in excavation angle gradually slows down because when the excavation angle is large, the amount of unloading caused by the same increase in excavation angle gradually decreases. In this model, the unloaded rebound deformation accounts for a larger proportion of the total deformation than the sliding deformation of the structural plane. Moreover, the total deformation at the measuring point is mainly controlled by the unloaded rebound deformation; thus, the deformation at the foot of the slope is larger than that at the top of the slope.

![Fig. 8. Comparison of the deformation response of the bedding slope under different excavation angles](image)

4.2. Excavation depth

Excavation depth directly determines the excavation strength (unloading volume). Different excavation depths are selected to calculate the deformation of the measuring point at the top, middle, and foot of the slope after excavation, and the influence of different excavation strength on the slope deformation is analyzed. The calculation results are shown in Fig. 9. As shown in the figure, when the excavation depth is 27.71 cm, the deformation of the measuring point at the top of the slope is 0.317 mm, and that at the foot of the slope is 0.49 mm. When the excavation depth is 33 cm, the deformation of the measuring point at the top of the slope is 0.428 mm, which increased by 35% compared with 27.71 cm. When the excavation depth is 33 cm, the deformation of the measuring point at the foot of the slope is 0.645 mm, which increased by 31.6% compared with 27.71 cm. The results show that when the excavation depth increases, the displacement of each measuring point considerably increases; moreover, a good linear relation is found between the deformation of measuring point and the excavation depth. Excavation depth is directly related to unloading volume, and the deformation increment of the measuring point can be approximately considered to be linearly related to the excavation volume. In addition, under the same condition of excavation depth, the
deformation of the measuring point at the foot of the slope is the greatest, followed by that at the middle of the slope; that at the top of the slope has the least deformation.

![Comparison of the deformation response of the bedding slope under different excavation depths](image)

**Fig. 9.** Comparison of the deformation response of the bedding slope under different excavation depths

4.3. **Stratum thickness**

For the same slope model, the number of strata is different when the stratum thickness is different, that is, the rock mass structure characteristics are different. Different stratum thicknesses are selected to calculate the deformation of the measuring point at the top, middle, and foot of slope after excavation, and the influence of different stratum thicknesses on slope deformation is analyzed. The calculation results are shown in Fig. 10. As shown in the figure, when the thickness of the rock stratum increases from 0.5 cm to 8 cm, the deformation of the measuring point at the top of the slope decreases from 0.328 mm to 0.309 mm, and the deformation of the measuring point at the middle of slope decreases from 0.413 to 0.406. The results show that the deformation of the measuring point decreases with the increase in the thickness of the rock stratum because when the thickness of the rock stratum increases, the amount of excavation unloading and the form of the excavated slope and the rebound deformation remain the same. However, the exposed stratum on the excavation surface is reduced; thus, the structural deformation caused by bedding sliding decreases. In addition, the deformation of the measured point does not change with the thickness of the rock stratum because the rock strata at the foot of the slope are not exposed to free face. The analysis results reveal that when the thickness of the rock stratum changes, the change of deformation at the measuring point is determined by the bedding slip, which reflects the change of the structural characteristics of the rock mass. When the thickness of the rock stratum is small and many strata are exposed on the excavation surface, the deformation caused by excavation becomes considerable.

![Comparison of the deformation response of the bedding slope under different rock thicknesses](image)

**Fig. 10.** Comparison of the deformation response of the bedding slope under different rock thicknesses

5. **Conclusion**

An unloading rebound model of slope deformation is established on the basis of Mindlin’s strain solution, and a bedding-slip model of slope deformation is deduced on the basis of the Kalhaway constitutive model. By combining the two models, a computational model for the deformation response of bedding slope excavation is presented. This model can reflect the mechanism of excavation unloading and the
characteristics of rock mass structure. The validity of the model is verified on the basis of laboratory test data. The results show that the calculated values of the model are close to the experimental values, indicating that the analytical model for the deformation response of bedding slope on the basis of Mindlin’s strain solution can well simulate the deformation process of bedding slope caused by excavation. The model can be used to identify the proportion of unloading rebound and bedding-slip deformations in the total deformation and thus reasonably explain the formation mechanism of the deformation of bedding slope caused by excavation. The model can also help identify slope stability accurately and develop a scientific plan for slope reinforcement and early warning.

On this basis, the influences of excavation angle, excavation depth, and stratum thickness are analyzed by using several calculation examples. The results show that the displacement of each measuring point increases with the increase in excavation angle. When the excavation depth increases, the displacement of each measuring point increases linearly. When the thickness of rock stratum increases, the deformation of measuring point decreases with the decrease in the number of exposed structural surfaces. The deformation of bedding slope caused by excavation is controlled by unloading and bedding sliding. The excavation volume directly affects the unloading rebound deformation, and the excavation volume is approximately linearly related to the rebound deformation. Therefore, when the excavation volume increases due to the increase in excavation angle or excavation depth, the slope rebound deformation increases remarkably. Bedding slip is affected by many factors, and the increase in excavation angle or the number of rock strata exposed on the excavation surface leads to the increase in slip deformation. The quantitative calculation results can be used for reference by engineers during analysis and design.

In addition, this study suggests that the main failure mode of bedding slope is bedding slip. The rock material is assumed to be in the stage of linear elastic deformation before slope failure, and the material of structural surface meets the nonlinear deformation condition. The calculation model of unloading rebound deformation is based on linear elastic theory, and the calculation model of slip deformation is based on the nonlinear deformation constitutive model. The method proposed in this study has some limitations and must be verified by additional experimental data and numerical results. In the future, we will combine additional experimental data and numerical results to optimize the proposed calculation model. We will also use the model in complex cases to investigate some significance problems.

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