Energy gain of heavy quarks by fluctuations in the QGP

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The collisional energy gain of a heavy quark due to chromo-electromagnetic field fluctuations in a quark-gluon plasma is investigated. The field fluctuations lead to an energy gain of the quark for all velocities. The net effect is a reduction of the collisional energy loss by 15-40% for parameters relevant at RHIC energies.

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The aim of the ongoing relativistic heavy-ion collision experiments is to explore the possible plasma phase of QCD, the so called quark-gluon plasma (QGP). High energy partons produced in initial partonic sub-processes in collisions between two heavy nuclei will lose their energy while propagating through the dense matter formed after such collisions, resulting in jet quenching. The amount of quenching depends upon the state of the fireball produced and the resulting quenching pattern may be used for identifying and investigating the plasma phase. In order to quantitatively understand medium modifications of hard parton characteristics in the final state, the energy loss of partons in the QGP has to be determined. There are two contributions to the energy loss of a parton in a QGP: one is caused by elastic collisions between the partons and the other is caused by radiative losses. It is generally believed that the radiative loss due to multiple gluon radiation (see [2, 3] for a review) dominates over the collisional one in the ultra-relativistic case. However, it has been shown recently that for realistic values of the parameters relevant for heavy-ion collisions, there is a wide range of parton energies in which the magnitude of the collisional loss is comparable to the radiative loss for heavy quark flavors as well as for light quark flavors.

Earlier estimates of the collisional energy loss in the QGP [9, 11] were obtained by treating the medium in an average manner, i.e., microscopic fluctuations were neglected. However, the QGP, being a statistical system, is characterized by omnipresent stochastic fluctuations. These fluctuations couple to external perturbations and the response of the medium to these perturbations can be expressed through suitable correlation functions of the microscopically fluctuating variables. Now, it is well known that the motion of charged particles in such an environment is stochastic in nature and resembles Brownian motion. The Fokker-Planck equation provides a natural basis for a differential characterization of such a stochastic motion. In a homogeneous and isotropic plasma, the Fokker-Planck equation can be recast in the form

$$\frac{\partial f}{\partial t} = \sum_{n=1}^{\infty} \frac{(-1)^n}{n!} \frac{\partial^n}{\partial p_1 \partial p_2 \cdots \partial p_n} \left[ D_{ij} \cdots (p, t) f (p, t) \right],$$

where $f (p, t)$ is the phase-space distribution of the test particle. $D_{ij}$ in $\mathbb{1}$ is the drag coefficient - a quantity closely related to energy loss per unit length $dE/dx$ [5, 7, 11, 12, 13]. It is to be noted that starting from $n = 2$, all terms in $\mathbb{1}$ are statistical in origin, i.e., they arise due to microscopic field fluctuations, to leading order in $\alpha_s$ [14, 15]. For the drag coefficient or energy loss, the fluctuating field as well as the polarization field contribute as will be seen later. The effect of field fluctuations on the passage of a charged particle through a non-relativistic classical plasma has been worked out by several authors [14, 15, 16, 17, 18, 19, 20] in the past. This effect leads to an energy gain of the particles and is most effective in the low velocity limit. Given the fact that the subject of the energy loss is of topical interest, it is the principal motivation of the present article to quantitatively estimate the effect of microscopic electromagnetic fluctuations on the energy loss of a heavy quark passing through an equilibrium, weakly-coupled QGP within the semiclassical approximation.

The semiclassical approach was adopted earlier to calculate the collisional energy loss of a heavy quark due to polarization effects of the medium [9, 10]. It is assumed that the energy lost by the particle per unit time is small compared to the energy of the particle itself so that the change in the velocity of the particle during the motion may be neglected, i.e., the particle moves in a straight line trajectory. The energy loss of a particle is determined by the work of the retarding forces acting on the particle in the plasma from the chromo-electric field generated by the particle itself while moving. So the energy loss of the
The electric field $\tilde{E}$ in (6) consists of the induced field $\tilde{E}$ given by (3) and a spontaneously generated microscopic field $\tilde{\xi}$, the latter being a random function of position and time.

The classical equation of motion of the particles in the electromagnetic field have the form,

$$\frac{d\vec{p}}{dt} = Q^a \vec{v} \cdot \vec{E}^\alpha \bigg|_{r = \vec{r}} ,$$

(2)

where the field is taken at the location of the particle. Within linear response theory the correlation function of the fluctuations of charge and current densities and the electromagnetic fields in the medium with space-time dispersion are completely determined in terms of the dielectric tensor of the medium. In the Abelian approximation, the total chromo-electric field $E^a$ induced in the QGP can be related to the external current of the test charge by solving Maxwell’s equations and the equation of continuity

$$\left[ \epsilon_{ij} (\omega, k) - \frac{k^2}{\omega^2} (\delta_{ij} - \frac{k_i k_j}{k^2}) \right] E_j^\alpha (\omega, k) = \frac{4\pi}{i \omega} j^\alpha_i (\omega, k) ,$$

(3)

with the color charge current $j^a_i$. It should be noted that in this approximation, the dielectric tensor $\epsilon_{ij}$ is diagonal in the color indices [21]. In an isotropic and homogeneous plasma the dielectric tensor $\epsilon_{ij}$ can be decomposed into longitudinal and transverse parts,

$$\epsilon_{ij} (\omega, k) = \epsilon_L (\omega, k) \mathbf{P}^L_{ij} + \epsilon_T (\omega, k) \mathbf{P}^T_{ij} ,$$

(4)

where, $\mathbf{P}^L_{ij} = \delta_{ij} - \frac{k_i k_j}{k^2}$ and $\mathbf{P}^T_{ij} = \delta_{ij} - \frac{k_i k_j}{k^2}$. The gauge invariant high temperature expression for the dielectric functions which is in accordance with the Abelian approximation (see below) are given by (see e.g. [21, 22])

$$\epsilon_L (\omega, k) = 1 + \frac{m_D^2}{k^2} \left[ 1 - \frac{\omega}{2k} \left( \ln \frac{\omega + k}{\omega - k} \right) - i \pi \Theta (k^2 - \omega^2) \right] ,$$

$$\epsilon_T (\omega, k) = 1 - \frac{m_D^2}{2k^2} \left[ \frac{\omega^2}{k^2} + \frac{1 - \omega^2}{2k^2} \right] \frac{\omega}{2k} \left( \ln \frac{\omega + k}{\omega - k} \right)$$

- $i \pi \Theta (k^2 - \omega^2) \right] ,$

(5)

where $m_D^2 = g^2 T^2 (1 + N_f/6)$ is the Debye mass squared. Substitution of (3) together with (4) and (5) in (2) gives the polarization loss of the moving parton [8, 10].

The previous formula for the energy loss in (2) does not take into account the field fluctuation in the plasma and the particle recoil in collisions. To accommodate these effects it is necessary to replace (2) with [12, 17],

$$\frac{dE}{dt} = \langle Q^a \vec{v} \cdot \tilde{E}^\alpha \rangle ,$$

(6)

where $\langle \cdots \rangle$ denotes the statistical averaging operation. It is to be noted that here two averaging procedures are performed: I) an ensemble average w.r.t the equilibrium density matrix and II) a time average over random fluctuations in plasma. These two operations are commuting and only after both of them are performed the average quantity takes up a smooth value [18]. In the following, we will explicitly denote the ensemble average by $\langle \cdots \rangle_\beta$ wherever required to avoid possible confusion.
\[ \times \frac{\partial}{\partial \theta_{\alpha \beta}} \left( \bar{v}_0 \cdot \hat{E}_i (\bar{r}_i (t), t) \right)_{\hat{\beta}} . \]  \tag{11}

Since the mean value of the fluctuating part of the field equals zero, \( \langle \hat{E} \rangle_{\hat{\beta}} = 0 \), \( \langle \hat{E}_i (\bar{r}_i (t), t) \rangle_{\hat{\beta}} \) equals the chromo-electric field produced by the particle itself in the plasma. The first term in \( \text{Eq. (11)} \) therefore corresponds to the usual polarization loss of the parton calculated in \( \text{Ref. [9]} \). Provided there exists a hierarchy of scales \( \text{[10]} \), it can be shown that the polarization field does not contribute to leading order in the correlations functions appearing in the second and third terms in \( \text{Eq. (11)} \) as in the case of higher order Fokker-Planck coefficients \( \text{[15, 20]} \). These terms correspond to the statistical change in the energy of the moving parton in the plasma due to the fluctuations of the chromo-electromagnetic fields as well as the velocity of the particle under the influence of this field. The second term in \( \text{Eq. (11)} \) corresponds to the statistical part of the dynamic friction due to the space-time correlation in the fluctuations in the electrical field whereas the third one corresponds to the average change in the energy of the moving particle due to the correlation between the fluctuation in the velocity of the particle and the fluctuation in the electrical field in the plasma. The temporal averaging in \( \text{Eq. (11)} \), by definition, includes many random fluctuations over the mean motion. However, the correlation function of these fluctuations are suppressed beyond their characteristic time scales. This allows us to formally extend the upper limits in time-integrations in \( \text{Eq. (11)} \) to infinity. Now within linear response theory, the power spectrum of the chromo-electromagnetic fields follows from the fluctuation-dissipation theorem and is completely determined by the dielectric functions of the medium \( \text{[16, 17]} \),

\[ \langle \hat{E}_i \hat{E}_j \rangle_{\hat{\beta}; \omega, k} = \langle \hat{E}_i \hat{E}_j \rangle_{\hat{\beta}; \omega, k} \delta_{ab} \]  \tag{12}

where,

\[ \langle \hat{E}_i \hat{E}_j \rangle_{\hat{\beta}; \omega, k} = \frac{8 \pi}{\omega} \left( \frac{\omega^2}{\omega^2 + \eta^2} - 1 \right) \left\{ v^T \frac{\Im \epsilon_T}{\epsilon_T} + v^L \frac{\Im \epsilon_L}{\epsilon_L} \right\} \]  \tag{13}

with \( \eta = k/\omega \).

Using the Fourier transform of \( \hat{E}_i \) together with \( \text{Eq. (11)} \) and \( \text{Eq. (12)} \), we obtain from \( \text{Eq. (11)} \) the energy loss of the parton due to fluctuations as (see appendix),

\[ \frac{dE}{dt} \bigg|_{\hat{\beta}} = \frac{C_F \alpha_s}{16 \pi^2 E} \int d^3k \left\{ \frac{\partial}{\partial \omega} \langle \hat{E}^2_L \rangle_{\hat{\beta}} + \langle \hat{E}^2_T \rangle_{\hat{\beta}} \right\}_{\omega = k/\hat{\beta}} , \]  \tag{14}

where \( \langle \hat{E}^2_L \rangle \) and \( \langle \hat{E}^2_T \rangle \) denote the longitudinal and transverse field fluctuations, respectively, and \( E = E_0 \) is the initial energy of the parton.

Eq. \( \text{Eq. (14)} \) can be recast as (see appendix),

\[ \frac{dE}{dt} \bigg|_{\hat{\beta}} = \frac{C_F \alpha_s}{8 \pi^2 E v^3} \int_0^{k_{\text{max}}} d\omega \coth \frac{\beta \omega}{2} F (\omega, k = \omega/v) \]  \tag{15}

where \( F (\omega, k) = 8 \pi \omega^2 \Im \epsilon_L / |\epsilon_L|^2 \) and \( G (\omega, k) = 16 \pi \Im \epsilon_T / |\epsilon_T - k^2/\omega|^2 \) and \( v_0 = v \). This result is obviously gauge invariant if we use there the semiclassical, gauge invariant expression for the dielectric functions \( \text{[9]} \).

The above expression gives the mean energy (per unit time) absorbed by a propagating particle from the heat bath. Physically, this arises from gluon absorption. Thermal absorption of gluons was also shown to reduce the radiative energy loss \( \text{[25]} \). We arrive at a somewhat similar conclusion as there, albeit in a different context. It is to be noted here that since the spectral density of field fluctuations \( \langle \hat{E}^2_L/T \rangle \) are positive for positive frequencies by definition, according to \( \text{Eq. (15)} \) the particle energy will grow due to interactions with the fluctuating fields.

\[ \text{FIG. 1: (Color online) Various contributions to the energy loss of charm quark in the QGP. The collisional energy loss (dashed line) is taken from Ref. [9].} \]

The contribution from field fluctuations to the heavy quark energy loss is shown in Fig. 1 and Fig. 2. Our choice of the parameters is \( N_f = 2, T = 250 \text{ MeV}, \alpha_s = 0.3 \), and for the charm and bottom quark masses we take 1.25 and 4.2 GeV, respectively. For the upper integration limit \( k_{\text{max}} \) we take \( \text{[20]} \),

\[ k_{\text{max}} = \min \left\{ E, \frac{2q(E + p)}{\sqrt{m^2 + 2q(E + p)}} \right\} , \]  \tag{16}

where \( q \sim T \) is the typical momentum of the thermal partons of the QGP.

In Fig. 3 and Fig. 4 we show the relative collisional energy loss of a charm and bottom quark where the effect of field fluctuations is taken into account. It is evident...
FIG. 2: (Color online) Same as Fig. 1 but for a bottom quark.

FIG. 3: (Color online) Relative importance of the fluctuation loss compared to the collisional energy loss of Ref. [9]. We take the path length to be \( L = 5 \) fm.

FIG. 4: (Color online) Same as Fig. 3 but for a bottom quark.

\[
\left. \frac{dE}{dx} \right|_{\text{fl}} = \frac{1}{v} \left. \frac{dE}{dt} \right|_{\text{fl}} \quad \text{diverges for } v \to 0.
\]

Let us summarize here the general assumptions made in this investigation:

- We consider here only the collisional energy loss due to elastic scattering.
- We consider here only heavy quarks (charm, bottom) which became of particular interest in recent experiments at RHIC [6].
- We use the semiclassical approximation which has been used to calculate the mean collisional energy loss [9]. The semiclassical approximation has been shown to be equivalent to the Hard Thermal Loop approximation which is based on the weak coupling limit \( g \to 0 \). It allows a systematic calculation of the collisional energy loss [27] and is valid in the high-temperature limit of QCD. It also corresponds to neglecting the non-Abelian terms in the QCD equations of motion (see e.g. [22]). Hence we use systematically the Abelian approximation throughout this work.
- We assume a constant momentum and temperature independent coupling constant. Recently the collisional energy loss has been reconsidered using a model to include a running temperature dependent coupling constant. Although this leads to a different functional dependence on the parameters, the energy loss for realistic situations is somewhat larger but of similar size as in the case of a constant coupling [28].
- We assume an equilibrated, isotropic and homogeneous QGP (see below).
- We assume an infinitely extended QGP. Recently it has been argued that finite size effects are negligible for the collisional energy loss [26, 29].

\[
\left. \frac{dE}{dx} \right|_{\text{fl}} = 2\pi C_F \alpha_s^2 \left( 1 + \frac{N_f}{6} \right) \frac{T^3}{E v^2} \ln \frac{1 + v}{1 - v} \times \ln \frac{k_{\text{max}}}{k_{\text{min}}} \quad (17)
\]

where \( k_{\text{min}} = m_D \) is the Debye mass. The divergence here is kinematic as \( \left. \frac{dE}{dt} \right|_{\text{fl}} \) is finite for \( v = 0 \) and therefore
Let us note here that the assumption of an equilibrium condition necessary implies isotropization in momentum space. On the other hand, matter created in non-central heavy-ion collisions is anisotropic to start with and the strong longitudinal expansion afterwards, at its own, brings an anisotropy into the system [30]. The characteristic feature of such anisotropic systems is the presence of a Weibel-type instability [31, 32, 33]. It has been argued that assuming a turbulent, weakly coupled anisotropic QGP may provide a natural explanation for the observed rapid isotropization time [34], for the small momentum space. On the other hand, matter created in non-central heavy-ion collisions is anisotropic to start with. The energy loss and hence more disagreement with single electron data. On the other hand, using the transport coefficients within pQCD energy loss calculations [41] the electron data. On the other hand, using the transport coefficients within pQCD energy loss calculations [41] the electron data.

APPENDIX A: DERIVATION OF (15)

Since the power spectrum is diagonal in color space according to [12] we can pull out the color factor from the electric fields and perform the color sum in the second and third terms in (11) easily. With $Q^aQ^\alpha = C_F\alpha_s$, where $C_F$ is the quadratic Casimir in fundamental representation, we obtain, e.g., for the second term in (11)

$$\#2 = \frac{C_F\alpha_s}{E_0} \int_0^t dt_1 \int_0^{t_1} dt_2 \left\langle \vec{E}(\vec{r}_0(t_1), t_1) \cdot \vec{E}(\vec{r}_0(t), t) \right\rangle_\beta.$$  

(A1)

Setting $t - t_1 = \tau$ and provided that there exist scales $\tau_1$ and $\tau_2$ discussed earlier, we can write (A1) as,

$$\#2 = \lim_{t \to \infty} \frac{C_F\alpha_s}{E_0} \int_0^t dt \left\langle \vec{E}(\vec{r}_0(t - \tau), t - \tau) \cdot \vec{E}(\vec{r}_0(t), t) \right\rangle_\beta.$$  

(A2)

Expressing $\vec{E}(\vec{r}_0(t), t)$’s in Fourier modes, utilizing the fact that the unperturbed orbit is a straight line trajectory $\vec{r}_0(t) = \vec{v}_0 t$, and

$$\left\langle \vec{E}_i(\vec{k}, \omega) \vec{E}_j(\vec{k}', \omega') \right\rangle_\beta = \left\langle \vec{E}_i \vec{E}_j \right\rangle_\beta \delta^3(\vec{k} + \vec{k}') = (2\pi)^4 \delta(\omega + \omega') \delta^3(\vec{k} + \vec{k}'),$$  

(A3)

we get,

$$\#2 = \frac{C_F\alpha_s}{16\pi^3 E_0} \int d^3k \left\langle \vec{E}^2 \right\rangle_{\beta; \omega = \vec{k} \cdot \vec{v}_0}.$$  

(A4)

Similarly for the third term in (11) we find,

$$\#3 = \frac{C_F\alpha_s}{E_0} \int_0^t dt_1 \int_0^{t_1} dt_2 \left\langle \sum_j \vec{E}_j(\vec{r}_0(t_2), t_2) \right\rangle_\beta \times \frac{\partial}{\partial \vec{r}_0(t_2)} \vec{E}_i(\vec{r}_0(t), t).$$  

(A5)

Interchanging the order of integration leads to,

$$\#3 = \frac{C_F\alpha_s}{E_0} \int_0^t dt_2 \int_0^{t_2} dt_1 \left\langle \sum_j \vec{E}_j(\vec{r}_0(t_2), t_2) \right\rangle_\beta \times \frac{\partial}{\partial \vec{r}_0(t_2)} \vec{E}_i(\vec{r}_0(t), t).$$  

(A6)

Using (A3) and the fact the field correlations vanish for $\omega \to \pm \infty$, we get,

$$\#3 = \frac{C_F\alpha_s}{16\pi^3 E_0} \int d^3k \omega \left\langle \vec{E}^2 \right\rangle_{\beta; \omega = \vec{k} \cdot \vec{v}_0}.$$  

(A7)

It is to be noted that in contrast to (A4) here only the longitudinal part survives. This is due to the fact that
differentiation \(w.r.t\) \(r_j\) in \(A_9\) brings down one power of \(k_j\) which, operating on field correlators \(13\), removes the transverse part. Adding \(A_4\) and \(A_7\) we get \(14\).

Integrating the first term in \(14\) with respect to the azimuthal angle yields,

\[
\#1 = \frac{CF\alpha_s}{8\pi^2E_0\tau_0} \int_0^{k_{\text{max}}} dk d\omega \frac{\partial}{\partial \omega} \langle \omega \hat{E}_L^2 \rangle_{\beta; k, \tau_0} \tag{A8}
\]

Substituting \(\omega = kv_0\eta\) the \(\eta\) integration can be performed and we get

\[
\#1 = \frac{CF\alpha_s}{8\pi^2E_0v_0} \int_0^{k_{\text{max}}} dk \left[ \langle \omega \hat{E}_L^2 \rangle_{\omega = kv_0} - \langle \omega \hat{E}_L^2 \rangle_{\omega = -kv_0} \right] \tag{A9}
\]

Now \(\langle \omega \hat{E}_L^2 \rangle\) can be written as \(n_B(\omega) f(\omega)\) where \(n_B(\omega)\) is the Bose distribution and \(f(\omega) = f(-\omega)\). Using \(n_B(\omega) = [1 + n_B(\omega)]\), and using \(\omega = kv_0\) we can write \(A9\) as;

\[
\#1 = \frac{CF\alpha_s}{8\pi^2E_0v_0} \int_0^{k_{\text{max}}} d\omega \coth \frac{\beta\omega}{2} F(\omega, k = \omega/v_0) \tag{A10}
\]

Similarly, we obtain the transverse contribution as,

\[
\#2 = \frac{CF\alpha_s}{8\pi^2E_0v_0} \int_0^{k_{\text{max}}} dk \int_0^{kv_0} d\omega \coth \frac{\beta\omega}{2} G(\omega, k) \tag{A11}
\]

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