**Non-Minimal Higgs Inflation and non-Thermal Leptogenesis in a Supersymmetric Pati-Salam Model**

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**Abstract**

We consider a supersymmetric (SUSY) Grand Unified Theory (GUT) based on the gauge group $G_{PS} = SU(4)_C \times SU(2)_L \times SU(2)_R$, which incorporates non-minimal chaotic inflation, driven by a quartic potential associated with the Higgs fields involved in the spontaneous breaking of $G_{PS}$. The inflationary model relies on renormalizable superpotential terms and does not lead to overproduction of magnetic monopoles. It is largely independent of the one-loop radiative corrections and can become consistent with the current observational data on the inflationary observables, with the symmetry breaking scale of $G_{PS}$ assuming its SUSY value. Within our model, the strong CP and the $\mu$ problems of the minimal SUSY standard model can be resolved via a Peccei-Quinn symmetry. Moreover baryogenesis occurs via non-thermal leptogenesis realized by the out-of-equilibrium decay of the right-handed neutrinos, which are produced by the inflaton’s decay. We consider two versions of such a scenario, assuming that the inflaton can decay to the lightest or to the next-to-lightest right-handed neutrino. Both scenarios can become compatible with the constraints arising from the baryon asymmetry of the universe, the gravitino limit on the reheating temperature and the upper bound on the light neutrino masses, provided that the gravitino is somehow heavy. In the second scenario, extra restrictions from the $SU(4)_C$ GUT symmetry on the heaviest Dirac neutrino mass and the data on the atmospheric neutrino oscillations can be also met.

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1 INTRODUCTION

Non-minimal Higgs inflation (non-MHI) [1–3] – see also Ref. [4] – is an inflationary model of chaotic type which arises in the presence of a non-minimal coupling between a Higgs-inflaton field and the Ricci scalar curvature, $R$. It has been shown that non-MHI based on a quartic potential with a quadratic non-minimal coupling to gravity can be realized in both a non-supersymmetric [1] and a supersymmetric (SUSY) framework [3], provided that the coupling of the inflaton to $R$ is strong enough. In most of the existing models, the inflaton is identified with the Higgs field(s) of the Standard Model (SM) or the next-to-MSSM (Minimal SUSY SM) [3] – for non-minimal inflation driven by an inflaton other than the Higgs field see Ref. [5–9] for non-SUSY models and Ref. [10–12] for SUSY ones.

On the other hand, SUSY GUTs arise as natural extensions of Physics beyond the MSSM. Within their framework, a number of challenges – such as the $\mu$ problem, the generation of the observed baryon asymmetry of the universe (BAU) and the existence of tiny but non-zero neutrino masses – which the MSSM fails to address can be beautifully arranged. The achievement of gauge coupling unification within the MSSM suggests that the breaking of the GUT gauge symmetry group down to the SM one, $G_{SM} = SU(3)_C \times SU(2)_L \times U(1)_Y$, occurs at a scale $M_{GUT} \simeq 2 \cdot 10^{16}$ GeV through some Higgs superfields. Therefore, the latter arise naturally as candidates for driving non-MHI – for earlier attempts within non-SUSY $SU(5)$ GUT see Ref. [13]. In such a situation the GUT gauge group is already spontaneously broken during non-MHI through the non-zero values acquired by the relevant Higgs fields. Consequently, non-MHI does not lead to the production of topological defects. Moreover, the potential of non-MHI possesses a non-zero classical inclination and so, the inflationary dynamics is largely independent of the radiative corrections. As a consequence, the vacuum expectation values (v.e.vs) which the Higgs fields acquire at the end of non-MHI can be exactly equal to the values required by the unification of the MSSM gauge couplings. Finally, the predicted inflationary observables are consistent with the fitting [14] of the seven-year data of the Wilkinson Microwave Anisotropy Probe Satellite (WMAP7), baryon-acoustic-oscillations (BAO) and Hubble constant ($H_0$) data.

These features are to be contrasted with the widely adopted models of standard SUSY hybrid inflation (HI) [15], where the spontaneous breaking of the GUT gauge symmetry takes place at the end
of HI and, thus, topological defects are copiously formed [16] if they are predicted by the symmetry breaking. This is because, the standard SUSY HI is typically driven by a singlet field whereas the Higgs fields are confined to zero where the GUT symmetry is unbroken. Avoidance of cosmologically disastrous topological defects can be obtained within smooth [16, 17] or shifted [18, 19] HI by using either non-renormalizable [16, 19] or renormalizable [17, 18] superpotential terms, which generate stable inflationary trajectories with non-zero values for the Higgs fields. Some of the latter constructions, though, are much more complicated than the simplest original model [15]. In the cases of standard [15] and shifted [18, 19] HI, radiative corrections play an important role in creating the slope of the inflationary potential and the v.e.vs of the Higgs fields turn out to be mostly lower than the GUT SUSY symmetry scale, since the relevant mass scale is constrained by the normalization of the curvature perturbation [14]. Finally, all types of HI suffer from the problem of an enhanced (scalar) spectral index, $n_s$, which turns out to be, mostly, well above the current data [14]. For several proposals aiming to improve on the latter shortcoming of SUSY HI see Ref. [20–24].

In this paper we present a model of non-MHI, adopting a SUSY GUT model based on the Pati-Salam (PS) gauge group $G_{PS} = SU(4)_C \times SU(2)_L \times SU(2)_R$. Note that SUSY PS GUT models are motivated [25] from recent D-brane constructions and can also arise [26] from the standard weakly coupled heterotic string. Employing only renormalizable superpotential terms, we then show that the model naturally leads to non-MHI within SUGRA avoiding thereby the overproduction of unwanted monopoles. Also the inflationary observables turn out to lie within the current data [14]. Our model possesses a number of other interesting features too. The $\mu$-problem of the MSSM can be solved [27] via a Peccei-Quinn (PQ) symmetry, which also solves the strong CP problem, and the proton is practically stable. Light neutrinos acquire masses by the seesaw mechanism [29] and the BAU can be generated through primordial non-thermal [30] leptogenesis. We single out two cases according to whether the inflaton decays to the lightest [31] or the next-to-lightest [32] right-handed (RH) neutrino. In both cases the constraints arising from the gravitino ($\tilde{G}$) limit [33–35] on the reheating temperature and the BAU can be met provided that the masses of $\tilde{G}$ lie in the multi-TeV region. On the other hand, the second scenario gives us the opportunity to combine the calculation of BAU with the present neutrino data [36] and the prediction of $G_{PS}$ for the masses of the fermions of the third generation.

The plan of this paper is as follows. We present the basic ingredients – particle content and structure of the superpotential and the Kähler potential– of our model in Sec. 2. In Sec. 3 we describe the inflationary potential, derive the inflationary observables and confront them with observations. In Sec. 4 we outline the two scenaria of non-thermal leptogenesis, exhibit the relevant imposed constraints and restrict the parameters of our model for each scenario. Our conclusions are summarized in Sec. 5. Details concerning the derivation of the mass spectrum of the theory during non-MHI are arranged in Appendix A whereas effects of instant preheating potentially important for some values of the parameters are discussed in Appendix B. Throughout the text, we use natural units for Planck’s and Boltzmann’s constants and the speed of light ($h = c = k_B = 1$); the subscript of type $\cdot, \chi$ denotes derivation with respect to (w.r.t) the field $\chi$ (e.g., $\partial^2 / \partial \chi^2$); charge conjugation is denoted by a star and log [ln] stands for logarithm with basis 10 [e].

## 2 The Pati-Salam SUSY GUT Model

We focus on a SUSY PS GUT model described in detail in Ref. [19] – see also Ref. [37]. The representations and the transformation properties of the various superfields contained in the model under $G_{PS}$, their decomposition under $G_{SM}$, as well as their extra global charges are presented in Table 1. Recall that, in the PS GUT models, the SM hypercharge $Q_Y$ is identified as the linear combination $Q_Y = Q_{T_R^3} + Q_{(B-L)}/2$ where $Q_{T_R^3}$ is the $SU(2)_R$ charge generated by $T_R^3 = \text{diag} (1, -1)/2$ and the $Q_{(B-L)}$ is the $SU(4)_C$ charge generated by $T_C^{15} = \text{diag} (1, 1, 1, -3)/2\sqrt{6}$. Here $T_R^m$ with $m = 1, 2, 3$
are the 3 generators of \(SU(2)_R\) and \(T^a_R\) with \(a = 1, \ldots, 15\) are the 15 generators of \(SU(4)_C\) with normalizations \(\text{Tr} \left( T^a_C T^b_C \right) = \delta^{ab} / 2\) and \(\text{Tr} \left( T^a_R T^b_R \right) = \delta^{mk} / 2\), where \(\text{Tr}\) denotes trace of a matrix.

The \(i\)th generation \((i = 1, 2, 3)\) left-handed (LH) quark [lepton] superfields, \(u_{i\alpha}\) and \(d_{i\alpha}\) – where \(\alpha = 1, 2, 3\) is a color index – \([e_i \text{ and } \nu_i]\) are accommodated in a superfield \(F_i\). The LH antiquark [antilepton] superfields \(u^c_{i\alpha}\) and \(d^c_{i\alpha}\) \([e^c_i \text{ and } \nu^c_i]\) are arranged in another superfield \(F^c_i\). These can be represented as

\[
F_i = \begin{pmatrix} q_{i1} & q_{i2} & q_{i3} & l_i \end{pmatrix} \quad \text{and} \quad F^c_i = \begin{pmatrix} q^c_{i1} \\ q^c_{i2} \\ q^c_{i3} \\ l^c_i \end{pmatrix}
\]

with

\[
q_{i\alpha} = \left( d_{i\alpha} - u_{i\alpha} \right), \quad l_i = \left( e_i - \nu_i \right), \quad q^c_{i\alpha} = \left( -u^c_{i\alpha} \right) \quad \text{and} \quad l^c_i = \left( -\nu^c_i \right).
\]

The gauge symmetry \(G_{PS}\) can be spontaneously broken down to \(G_{SM}\) through the v.e.vs which the superfields

\[
H^c = \begin{pmatrix} q^c_{H1} \\ q^c_{H2} \\ q^c_{H3} \\ l^c_H \end{pmatrix} \quad \text{and} \quad \bar{H}^c = \begin{pmatrix} \bar{q}^c_{H1} \\ \bar{q}^c_{H2} \\ \bar{q}^c_{H3} \\ \bar{l}^c_H \end{pmatrix}
\]

acquire in the directions \(\nu^c_H\) and \(\bar{\nu}^c_H\). The model also contains a gauge singlet \(S\), which triggers the breaking of \(G_{PS}\), as well as an \(SU(4)_C\) 6-plet \(G\), which splits into \(g^a\) and \(\bar{g}^a\) under \(G_{SM}\) and gives [26] superheavy masses to \(d^c_{Ha}\) and \(\bar{d}^c_{Ha}\). In particular, \(G\) can be represented by an antisymmetric \(4 \times 4\) matrix as follows

\[
G = \begin{pmatrix} \varepsilon_{abc} \bar{g}^c_a & -\varepsilon^c_a \\ -\bar{g}^c_a & 0 \end{pmatrix} \Rightarrow \bar{G} = \begin{pmatrix} \varepsilon_{abc} g^c_a & \bar{g}^c_a \\ g^c_a & 0 \end{pmatrix}.
\]

Here \(\bar{G}\) is the dual tensor of \(G\), defined by \(\bar{G}^{IJ} = \varepsilon^{IJKL}G_{KL}\) which transforms under \(SU(4)_C\) as \(U^*_C \bar{G}^{IJ} U^T_C\). Also \(\varepsilon^{IJKL} = \varepsilon_{IJKL} \varepsilon^{abc} \) is the well-known antisymmetric tensor acting on the \(SU(4)_C\) \([SU(3)_C]\) indices with \(\varepsilon_{1234} = 1 \quad [\varepsilon_{123} = 1]\). In the simplest realization of this model [19, 26], the electroweak doublets \(H_u\) and \(H_d\), which couple to the up and down quark superfields respectively, are exclusively contained in the bidoublet superfield \(IH\), which can be written as

\[
IH = \begin{pmatrix} H_u & H_d \end{pmatrix} = \begin{pmatrix} H^+_u & H^0_d \\ H^0_u & H^+_d \end{pmatrix}
\]

In addition to \(G_{PS}\), the model possesses two global \(U(1)\) symmetries, namely a \(PQ\) and an \(R\) symmetry, as well as a discrete \(\mathbb{Z}_2^{\text{mp}}\) symmetry (‘matter parity’) under which \(F, F^c\) change sign. The last symmetry forbids undesirable mixings of \(F\) and \(IH\) and/or \(F^c\) and \(H^c\). The imposed \(U(1)\) \(R\) symmetry, \(U(1)_R\), guarantees the linearity of the superpotential w.r.t the singlet \(S\). Although \(S\) does not play the role of the inflaton in our case – in contrast to the case of HI – we explicitly checked that the presence of a quadratic \(S^2\) term would lead to the violation of the stability of the inflationary trajectory. Finally the \(U(1)\) \(PQ\) symmetry, \(U(1)_{PQ}\), assists us to generate the \(\mu\)-term of the MSSM. Although this goal could be easily achieved [38] by coupling \(S\) to \(IH^2\) and using the fact that \(S\), after gravity-mediated SUSY breaking, develops a v.e.v, we here prefer to follow Ref. [19, 27] imposing a \(PQ\) symmetry on the superpotential and introducing a pair of gauge singlet superfields \(P\) and \(\bar{P}\).
PQ breaking occurs at an intermediate scale through the v.e.vs of $F_i$ and $\bar{P}_i$, and the $\mu$-term is generated via a non-renormalizable coupling of $P$ and $\bar{P}$. We do not adopt here the resolution to the $\mu$-problem suggested in Ref. [38], since it introduces a renormalizable term which creates a decay channel of the inflaton which leads to a high reheating temperature in conflict with the $G$ constraint and (ii) a tachyonic instability in the $H_u - H_d$ system during non-MHI – as occurring in Ref. [11]. Lifting both shortcomings requires an unnaturally small value for the relevant coupling constant in our scenario (of order $10^{-6}$ or so), which is certainly undesirable. Following Ref. [19], we introduce into the scheme quartic (non-renormalizable) superpotential couplings of $H^c_i$ to $F^c_i$, which generate intermediate-scale masses for the $\nu_i^c$ and, thus, masses for the light neutrinos, $\nu_i$, via the seesaw mechanism. Moreover,
these couplings allow for the decay of the inflaton into RH neutrinos, $\nu^c_i$, leading to a reheating temperature consistent with the $\bar{G}$ constraint with more or less natural values of the parameters. As shown finally in Ref. [19], the proton turns out to be practically stable in this model.

The superpotential $W$ of our model splits into three parts:

$$W = W_{\text{MSSM}} + W_{\text{PQ}} + W_{\text{HPS}},$$

(2.5)

where $W_{\text{MSSM}}$ is the part of $W$ which contains the usual terms – except for the $\mu$ term – of the MSSM, supplemented by Yukawa interactions among the left-handed leptons and $\nu^c_i$:

$$W_{\text{MSSM}} = y_{ij} F_i H^c F_j^c =$$

$$= y_{ij} \left( H_d^T \varepsilon L_i e^c_j - H_u^T \varepsilon L_i \nu^c_j + H_d^T \varepsilon Q_{ia} \nu^c_{ja} - H_u^T \varepsilon Q_{ia} \nu^c_{ja} \right), \quad \text{with} \quad \varepsilon = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}. \quad (2.6)$$

Here $Q_{ia} = \begin{pmatrix} u_{ia} & d_{ia} \end{pmatrix}^T$ and $L_i = \begin{pmatrix} \nu_i \\ e_i \end{pmatrix}^T$ are the $i$-th generation $SU(2)_L$ doublet LH quark and lepton superfields respectively. Summation over repeated color and generation indices is also assumed. Obviously the model predicts Yukawa unification at $M_{GUT}$ since the third family fermion masses originate from a unique term in the underlying GUT. It is shown [39,40] that exact Yukawa unification combined with non-universities in the gaugino sector and/or the scalar sector can become consistent with a number of phenomenological and cosmological low-energy requirements. The present model can be augmented [37] with other Higgs fields so that $H_u$ and $H_d$ are not exclusively contained in $IH$ but receive subdominant contributions from other representations too. As a consequence, a moderate violation of the exact Yukawa unification can be achieved, allowing for an acceptable low-energy phenomenology, even with universal boundary conditions for the soft SUSY breaking terms. However, here we prefer to work with the simplest version of the PS model.

The second term in the right hand side (RHS) of Eq. (2.5), $W_{\text{PQ}}$, is the part of $W$ which is relevant for the spontaneous breaking of $U(1)_{\text{PQ}}$ and the generation of the $\mu$ term of the MSSM. It is given by

$$W_{\text{PQ}} = \lambda_{\text{PQ}} \frac{P_2 \bar{P}_2}{M_S} - \mu \frac{P_2^T \varepsilon}{2M_S} \text{Tr} \left( IH \varepsilon H^T \varepsilon \right), \quad (2.7)$$

where $M_S \simeq 5 \cdot 10^{17}$ GeV is the String scale. The scalar potential, which is generated by the first term in the RHS of Eq. (2.7), after gravity-mediated SUSY breaking is studied in Ref. [19,27]. For a suitable choice of parameters, the minimum lies at $|\langle P \rangle| = |\langle \bar{P} \rangle| \sim \sqrt{m_{3/2} M_S}$. Hence, the PQ symmetry breaking scale is of order $\sqrt{m_{3/2} M_S} \sim (10^{10} - 10^{11})$ GeV and the $\mu$-term of the MSSM is generated from the second term of the RHS of Eq. (2.7) as follows:

$$-\lambda_{\mu} \frac{\langle P \rangle^2}{2M_S} \text{Tr} \left( IH \varepsilon H^T \varepsilon \right) = \mu H_d^T \varepsilon H_u \Rightarrow \mu \simeq \lambda_{\mu} \frac{\langle P \rangle^2}{M_S}, \quad (2.8)$$

which is of the right magnitude if $\lambda_{\mu} \sim (0.001 - 0.01)$. Let us note that $V_{\text{PQ}}$ has an additional local minimum at $P = \bar{P} = 0$, which is separated from the global PQ minimum by a sizable potential barrier, thus preventing transitions from the trivial to the PQ vacuum. Since this situation persists at all cosmic temperatures after reheating, we are obliged to assume that, after the termination of non-MHI, the system emerges with the appropriate combination of initial conditions so that it lies [41] in the PQ vacuum.

Finally, the third term in the RHS of Eq. (2.5), $W_{\text{HPS}}$, is the part of $W$ which is relevant for non-MHI, the spontaneous breaking of $G_{\text{PS}}$ and the generation of intermediate Majorana [superheavy] masses for $\nu^c_i$ [$d^c_H$ and $d^c_H$]. It takes the form

$$W_{\text{HPS}} = \lambda S (\bar{H}^c \bar{H}^c - M_{\text{PS}}^2) + \lambda_H \bar{H}^c G \varepsilon \bar{H}^c + \lambda_R \bar{H}^c \bar{G} \varepsilon \bar{H}^c + \lambda_{\nu^c} (\bar{H}^c F_i^c)^2 \frac{M_S}{M_{\text{PS}}}, \quad (2.9)$$
where $M_{PS}$ is a superheavy mass scale related to $M_{GUT}$ – see Sec. 3.2. The parameters $\lambda$ and $M_{PS}$ can be made positive by field redefinitions. It is worth emphasizing that our inflationary model is totally tied on renormalizable superpotential terms, contrary to the model of shifted HI [19], where a non-renormalizable term added in the RHS of Eq. (2.9) plays a crucial role in the inflationary dynamics.

Suppressing henceforth the color indices, we can express $W_{HPS}$ in terms of the components of the various superfields. We find

$$W_{HPS} = \lambda S \left( \nu_H^c \nu_H^c + e_H^c e_H^c + u_H^c \bar{u}_H^c + d_H^c \bar{d}_H^c - M_{PS}^2 \right)$$

$$- 2 \lambda_H (\nu_H^c \bar{d}_H^c - e_H^c u_H^c) \bar{g} + 2 \lambda_H u_H^c \bar{d}_H^c \bar{g}^c$$

$$+ \lambda_{\nu^c} e_H^c \nu_H^c (\bar{e}_H^c \bar{d}_H^c - \nu_H^c \nu_H^c - \bar{u}_H^c u_H^c)^2 / M_S.$$

(2.10)

Let us note in passing that the combination of two [three] color-charged objects in a term involves a contraction of the color indices with the symmetric [antisymmetric] invariant tensor $\delta_{ab} \ [\varepsilon_{abc}]$, e.g., $u_H^c \bar{u}_H^c = \delta_{ab} u_H^a \bar{u}_H^b$.

According to the general recipe [3, 10], the implementation of non-MHI within SUGRA requires the adoption of a frame function, $\Omega$, related to the Kähler potential, $K$, as follows

$$\Omega = -3e^{-K/3m_P^2} = -3 + H^c H^c \frac{m_P^2}{m_H} + \frac{\bar{H}^c \bar{H}^c}{m_P^2} + \frac{\text{Tr} (G^+ G)}{m_P^2} + \frac{|S|^2}{m_P^4} - k_S|S|^4 - 3k_H \left( \bar{H}^c H^c + \text{h.c.} \right),$$

(2.11)

where the complex scalar components of the superfields $H^c, \bar{H}^c, G$ and $S$ are denoted by the same symbol and the coefficients $k_S$ and $k_H$ are taken real. It is clear from Eq. (2.11) that we adopt the standard quadratic non-minimal coupling for Higgs-inflaton, which respects the gauge and the global symmetries of the model. We also added the fifth term in the RHS of Eq. (2.11) in order to cure the tachyonic mass problem encountered in similar models [3, 10] – see Sec. 3.1. In terms of the components of the various fields, $K$ in Eq. (2.11) reads

$$K = -3m_P^2 \ln \left( 1 - \frac{\phi^\alpha \phi^\alpha}{3m_P^2} + k_S \frac{|S|^4}{m_P^4} + \frac{k_H}{2m_P^2} \left( \nu_H \nu_H + e_H^c \bar{e}_H^c + u_H^c \bar{u}_H^c + d_H^c \bar{d}_H^c + \text{h.c.} \right) \right),$$

(2.12)

with $\phi^\alpha = \nu_H^c, \bar{e}_H^c, e_H^c, \bar{e}_H^c, u_H^c, \bar{u}_H^c, d_H^c, \bar{d}_H^c, g^c, \bar{g}^c, S$ and summation over the repeated Greek indices – e.g. $\alpha$ and $\beta$ – is implied.

In the limit where $m_P$ tends to infinity, we can obtain the SUSY limit, $V_{SUSY}$, of the SUGRA potential, $V_{HF}$ – see Sec. 3. Assuming that the SM non-singlet components vanish, $V_{SUSY}$ turns out to be

$$V_{SUSY} = \lambda^2 \left( \nu_H^c \nu_H^c - M_{PS}^2 \right)^2 + \lambda^2 |S|^2 \left( |\nu_H^c|^2 + |\bar{\nu}_H^c|^2 \right).$$

(2.13)

On the other hand, assuming minimal gauge kinetic functions, the D-term scalar potential $V_{HD}$ of the PS Higgs fields takes the form

$$V_{HD} = \frac{g^2}{2} \sum_{a=1}^{15} \left( H^{c*} T^{a}_{R} \bar{H}^{c*} T^{a}_{R} - H^{c*} T^{a}_{R} \bar{H}^{c*} T^{a}_{R} - H^{c*} T^{a}_{R} H^{c*} T^{a}_{R} \right)^2 + \frac{g^2}{2} \sum_{m=1}^{3} \left( H^{c*} T^{m}_{R} \bar{H}^{c*} T^{m}_{R} - H^{c*} T^{m}_{R} \bar{H}^{c*} T^{m}_{R} H^{c*} T^{m}_{R} \right)^2,$$

(2.14)

where $g$ is the (unified) gauge coupling constant of $G_{PS}$. In terms of the components of $\bar{H}^c$ and $H^c$, $V_{HD}$ reads:

$$V_{HD} = \frac{g^2}{8} \left[ (\nu_H^c \bar{e}_H^c + \bar{\nu}_H^c e_H^c - \nu_H^c e_H^c - \bar{\nu}_H^c \bar{e}_H^c)^2 - (\nu_H^c e_H^c - \nu_H^c e_H^c + \nu_H^c \bar{e}_H^c) \right]$$

$$+ (\nu_H^c \bar{d}_H^c + \bar{\nu}_H^c \bar{d}_H^c - \nu_H^c d_H^c - \bar{\nu}_H^c \bar{d}_H^c)^2 - (\nu_H^c u_H^c - \nu_H^c u_H^c + \nu_H^c \bar{u}_H^c + \bar{\nu}_H^c \bar{u}_H^c)^2$$

$$+ \frac{3}{2} \left( |\nu_H^c|^2 - |\bar{\nu}_H^c|^2 \right)^2 + \left( |\nu_H^c|^2 - |\bar{\nu}_H^c|^2 \right)^2 + \cdots.$$

(2.15)
Here the first line includes contributions arising from the sum over the $SU(2)_{\text{R}}$ generators $T^1_{\text{R}}$ and $T^2_{\text{R}}$ in Eq. (2.14), which are the well-known Pauli matrices. The second line includes contributions from the sum over the $SU(4)_{\text{C}}$ generators $T^{a+2a}$ [$T^{a+2a}$] (a = 1, 2, 3) with 1/2 [1/2] and $-i/2$ [$i/2$] in the a4 [4a] entries respectively and zero everywhere else, and the third for $T^3_{\text{C}}$ and $T^3_{\text{R}}$. The ellipsis represents terms including exclusively the SM non-singlet directions of $H^c$ and $H^c$. Vanishing of the D-terms is achieved for $|\tilde{\nu}^c_H| = |\nu^c_H|$ with the other components of $\tilde{H}^c$ and $H^c$ frozen at zero. Restricting ourselves to the D-flat direction, from $V_{\text{SUSY}}$ in Eq. (2.13) we find that the SUSY vacuum lies at

$$\langle S \rangle \simeq 0 \quad \text{and} \quad |\langle \nu^c_H \rangle| = |\langle \tilde{\nu}^c_H \rangle| = M_{\text{PS}}.$$  

(3.16)

Therefore, $W_{\text{HPS}}$ leads to a spontaneous breaking of $G_{\text{PS}}$. The same superpotential, $W_{\text{HPS}}$, also gives rise to a stage of non-MHI as analyzed in Sec. 3. Indeed, along the D-flat direction $|\nu^c_H| = |\tilde{\nu}^c_H| \gg M_{\text{PS}}$ and $S = 0$, $V_{\text{SUSY}}$ tends to a quartic potential and so, $W_{\text{HPS}}$ can be employed in conjunction with $K$ in Eq. (2.11) for the realization of non-MHI along the lines of Ref. [10].

It should be mentioned that soft SUSY breaking and instanton effects explicitly break $U(1)_R \times U(1)_{\text{PQ}}$ to $Z_2 \times Z_6$. The latter symmetry is spontaneously broken by $\langle P \rangle$ and $\langle \bar{P} \rangle$. This would lead to a domain wall problem if the PQ transition took place after non-MHI. However, as we already mentioned above, $U(1)_{\text{PQ}}$ is assumed already broken before or during non-MHI. The final unbroken symmetry of the model is $G_{\text{SM}} \times Z^2_{\text{B}}$.

### 3 The Inflationary Scenario

We now outline the salient features of our inflationary scenario (Sec. 3.1) and then, present its predictions in Sec. 3.3, calculating a number of observable quantities introduced in Sec. 3.2.

#### 3.1 Structure of the Inflationary Action

Following the conventions of Ref. [11], we write the action of our model in the Jordan frame (JF) as follows:

$$S_{\text{HI}} = \int d^4x \sqrt{-g} \left( \frac{1}{6} m^2 \Omega_R + m^2 \Omega_{\alpha \beta} \partial_{\mu} \phi^{\alpha} \partial_{\mu} \phi^{\beta} - V_{\text{HI}} + \cdots \right),$$  

(3.1)

where $\Omega_{\alpha \beta} = \Omega_\phi \delta^{\alpha \beta}$ and $\phi^\alpha$ are identified below Eq. (2.12). The ellipsis represents terms arising from the covariant derivatives $D_\mu H^c$ and $D_\mu \bar{H}^c$, and terms that include the on-shell auxiliary axial-vector field [3,11], which turn out to be irrelevant for our analysis below. Here $V_{\text{HI}} = \Omega^2 \tilde{V}_{\text{HF}}/9 + V_{\text{HD}}$, with $\tilde{V}_{\text{HF}}$ being the Einstein frame (EF) F-term SUGRA scalar potential, which is obtained from $W_{\text{HPS}}$ in Eq. (2.10) – without the last term of the RHS – and $K$ in Eq. (2.12) by applying [42]

$$\tilde{V}_{\text{HF}} = e^{K/m^2} \left( K^{\alpha \beta} F_\alpha F_\beta - \frac{3|W_{\text{HPS}}|^2}{m^2} \right) \quad \text{and} \quad K^{\alpha \beta} = \frac{\partial^2 K}{\partial \phi^\alpha \partial \phi^\beta}.$$  

(3.2)

with $K^{\beta \alpha} K_{\alpha \beta} = \delta^\beta_\gamma$ and $F_\alpha = W_{\text{HPS}, \phi^\alpha} + K_{\phi^\alpha} W_{\text{HPS}}/m^2$.

If we parameterize the SM neutral components of $H^c$ and $\bar{H}^c$ by

$$\nu^c_H = h e^{i\theta} \cos \theta / \sqrt{2} \quad \text{and} \quad \nu^c_\bar{H} = h e^{i\bar{\theta}} \sin \theta / \sqrt{2},$$  

(3.3)

we can easily deduce from Eq. (2.15) that a D-flat direction occurs at

$$\theta = \bar{\theta} = 0, \theta_\nu = \pi/4 \quad \text{and} \quad e^c_H = e^c_{\bar{H}} = u^c_H = u^c_{\bar{H}} = d^c_H = d^c_{\bar{H}} = g^c = g^c = 0.$$  

(3.4)

Along this direction, $V_{\text{HD}}$ in Eq. (2.15) vanishes and so, $\tilde{V}_{\text{HI}} = \tilde{V}_{\text{HF}}$ takes the form

$$\tilde{V}_{\text{HIO}} = m^4 \frac{\lambda^2 (x^2_h - 4 m^2_{\text{PS}})^2}{16 f^2} \quad \text{with} \quad f = -\frac{\Omega}{3} = 1 + c_R x^2_h \quad \text{and} \quad c_R = \frac{1}{6} + \frac{K_H}{4}.$$  

(3.5)
Here $m_{PS} = M_{PS}/m_P$ and $x_h = h/m_P$. From Eq. (3.5), we can verify that for $c_R \gg 1$, $S_{HI}$ in Eq. (3.1) takes a form suitable for the realization of non-MHI: the terms in the ellipsis vanish, and more importantly $\hat{V}_{HI}$ develops a plateau since $m_{PS} \ll 1$ – see Sec. 3.2. The (almost) constant potential energy density $\hat{V}_{HI0}$ and the corresponding Hubble parameter $\hat{H}_{HI0}$ along the trajectory in Eq. (3.4) are given by

$$
\hat{V}_{HI0} = \frac{\lambda^2 h^4}{16f^2} \simeq \frac{\lambda^2 m_P^4}{16c_R^2} \quad \text{and} \quad \hat{H}_{HI0} = \frac{\hat{V}_{HI0}^{1/2}}{\sqrt{3m_P}} \simeq \frac{\lambda m_P}{4\sqrt{3c_R}}.
$$

We next proceed to check the stability of the trajectory in Eq. (3.4) w.r.t the fluctuations of the various fields. To this end, we expand them in real and imaginary parts as follows

$$
X = \frac{x_1 + ix_2}{\sqrt{2}}, \quad \tilde{X} = \frac{\bar{x}_1 + i\bar{x}_2}{\sqrt{2}} \quad \text{where} \quad X = e_H^c, u_H^c, d_H^c, g^c \quad \text{and} \quad x = e, u, d, g.
$$

respectively. Notice that the field $S$ can be rotated to the real axis via a suitable R transformation. Since along the trajectory in Eq. (3.4) the various fields, $X$ and $\tilde{X}$, are confined to zero, the radial parametrization employed in Eq. (3.3) is not convenient here. Performing a Weyl transformation as described in detail in Ref. [11], we obtain

$$
S_{HI} = \int d^4x \sqrt{-\hat{g}} \left( -\frac{1}{2} m_P^2 \hat{R} + \left( \partial_{\mu} \nu_H^c \partial_{\nu} \nu_H^c \right) \frac{M_K}{f^2} \left( \partial_{\nu} \nu_H^c \right) + \frac{1}{2f} \sum \partial_{\mu} \chi \partial^{\mu} \chi - \hat{V}_{HI} \right),
$$

where $M_K = \left( \begin{array}{cc} \kappa & \bar{\kappa} \\ \bar{\kappa} & \kappa \end{array} \right)$, $\kappa = f + \bar{\kappa}$ and $\hat{V}_{HI} = \hat{V}_{HF} + V_{HD}/f^2$. In deriving this result, we take into account that $f, \chi \ll f, h$ with $\chi = S, x_1, x_2, \bar{x}_1$ and $\bar{x}_2$, and keep only terms up to quadratic order in the fluctuations $\chi$ and their derivatives. To canonically normalize the fields $\nu_H^c$ and $\bar{\nu}_H^c$, we have to diagonalize the matrix $M_K$. This can be realized via a similarity transformation involving an orthogonal matrix $U_K$ as follows:

$$
U_K M_K U_K^T = \text{diag}(\tilde{f}, f), \quad \text{where} \quad \tilde{f} = f + 6c_R^2 x_h^2 \quad \text{and} \quad U_K = \frac{1}{\sqrt{2}} \left( \begin{array}{cc} 1 & 1 \\ -1 & 1 \end{array} \right).
$$

By inserting $I = U_K U_K^T = U_K^T U_K$ on the left and the right of $M_K$, we bring the second term of the parenthesis in the RHS of Eq. (3.8) into the form

$$
\frac{1}{2f^2} \left( f \left( \partial_{\mu} h \partial^{\mu} h + \frac{1}{2} h^2 \partial_{\mu} \theta_+ \partial^{\mu} \theta_+ \right) + f h^2 \left( \frac{1}{2} \partial_{\mu} \theta_- \partial^{\mu} \theta_- + \partial_{\mu} \theta_\nu \partial^{\mu} \theta_\nu \right) \right),
$$

along the trajectory in Eq. (3.4). Here $\theta_\pm = (\bar{\theta} \pm \theta) / \sqrt{2}$. Consequently, we can introduce the EF canically normalized fields, $\hat{h}, \hat{\theta}_+, \hat{\theta}_-, \hat{\theta}_\nu$ and $\hat{\chi}$, as follows – cf. Ref. [3, 10]

$$
\frac{\text{d} \hat{h}}{\text{d} \bar{h}} = J = \sqrt{\tilde{f}} / f = \sqrt{\frac{1}{f} + \frac{6c_R^2 x_h^2}{f^2}}, \quad \hat{\theta}_+ = \frac{J h \theta_+}{\sqrt{2}}, \quad \hat{\theta}_- = \frac{h \theta_-}{\sqrt{2f}}, \quad \hat{\theta}_\nu = \frac{h \theta_\nu}{\sqrt{f}} \quad \text{and} \quad \hat{\chi} = \frac{\chi}{\sqrt{f}}.
$$

Taking into account the approximate expressions for $\hat{h}$ – where the dot denotes derivation w.r.t the cosmic time, $\dot{t} - J$ and the slow-roll parameters $\bar{\epsilon}, \bar{\eta}$, which are displayed in Sec. 3.2, we can verify that, during a stage of slow-roll non-MHI, $\hat{\theta}_+ \simeq J h \theta_+ / \sqrt{2}$ since $J h \simeq \sqrt{6m_P}$, $\hat{\theta}_- \simeq h \theta_- / \sqrt{2f}$ and $\hat{\theta}_\nu \simeq h \theta_\nu / \sqrt{f}$ since $h / \sqrt{f} \simeq m_P / \sqrt{c_R}$, and finally $\hat{\chi} \simeq \chi / \sqrt{f}$. For the latter, the quantity
\(f / f^3/2\), involved in relating \(\hat{\chi}\) to \(\chi\), turns out to be negligibly small, since \(\dot{f} / f^3/2 = f_{,h} \dot{h} / f^3/2 = -\lambda \sqrt{e} \eta \mp m_p / 2 \sqrt{3c_R}\). Therefore the action in Eq. (3.8) takes the form

\[
S_{\text{HI}} = \int d^4x \sqrt{-\tilde{g}} \left( -\frac{1}{2} m^2_R \tilde{R} + \frac{1}{2} \tilde{g}^{\mu \nu} \sum_\phi \partial_\mu \phi \partial_\nu \phi - \tilde{V}_{\text{HI}} \right), \quad \text{with} \quad \tilde{V}_{\text{HI}} = \tilde{V}_{\text{HF}} + \frac{V_{\text{HD}}}{f^2} \tag{3.12}
\]

where \(\phi\) stands for \(h, \theta_+, \theta_-, \theta_\nu, x_1, x_2, \bar{x}_1, \bar{x}_2\) and \(S\).

Having defined the canonically normalized scalar fields, we can proceed in investigating the stability of the inflationary trajectory of Eq. (3.4). To this end, we expand \(\tilde{V}_{\text{HI}}\) in Eq. (3.12) to quadratic order in the fluctuations around the direction of Eq. (3.4), as we describe in detail in Sec. A.1. In Table 2 we list the eigenvalues of the masses-squared matrices \(M^2_{\alpha \beta} = \left( \partial^2 \tilde{V}_{\text{HI}} / \partial \chi_\alpha \partial \chi_\beta \right)\) with \(\chi_\alpha = \theta_+, \theta_-, \theta_\nu, x_1, x_2, \bar{x}_1, \bar{x}_2\) and \(S\) involved in the expansion of \(\tilde{V}_{\text{HI}}\). We arrange our findings into three groups: the SM singlet sector, \(S - \nu^c_H - \bar{\nu}^c_H\), the sector with the \(u^c_H, \bar{u}^c_H\) and the \(e^c_H, \bar{e}^c_H\) fields which are related with the broken generators of \(G_{\text{PS}}\) and the sector with the \(d^c_H, \bar{d}^c_H\) and the \(g^c, \bar{g}^c\) fields.

As we observe from the relevant eigenvalues of the mass-squared matrices, no instability – as the one found in Ref. [11] – arises in the spectrum. In particular, it is evident that \(k_S \gtrsim 1\) assists us to achieve \(m_{\phi}^2 \gtrsim 0\) – in accordance with the results of Ref. [10]. Moreover, the D-term contributions to \(m_{\theta_\nu}^2\) and \(m_{\bar{u}_-}^2\) – proportional to the gauge coupling constant \(g \approx 0.7\) – ensure the positivity of these masses squared. Finally the masses that the scalars \(\bar{d}_{1,2}\) acquire from the second and third term of the RHS of Eq. (2.9) lead to the positivity of \(m_{d_{1,2}}^2\) for \(\lambda_H\) of order unity. We have also numerically verified that the masses of the various scalars remain greater than the Hubble parameter during the last 50 – 60 e-foldings of non-MHI, and so any inflationary perturbations of the fields other than the inflaton are safely eliminated.

In Table 2 we also present the masses squared of the gauge bosons, chiral fermions and gauginos of the model along the direction of Eq. (3.4). The mass spectrum is necessary in order to calculate the one-loop radiative corrections. Let us stress here that the non-vanishing values of \(\nu^c_H\) and \(\bar{\nu}^c_H\) trigger the spontaneous breaking of \(G_{\text{PS}}\) to \(G_{\text{SM}}\). In particular we have the following pattern of symmetry breaking

\[
SU(4)_C \times SU(2)_R \rightarrow SU(3)_C \times U(1)_Y.
\]

Therefore, 9 of the 18 generators of \(SU(4)_C \times SU(2)_R\) are broken, leading to 9 Goldstone bosons which are “eaten” by the 9 gauge bosons which become massive. As a consequence, 36 degrees of freedom (d.o.f) of the spectrum before the spontaneous breaking (18 d.o.f corresponding to 8 complex scalars, \(u^c_{H\alpha}, \bar{u}^c_{H\alpha}, e^c_H\) and \(\tilde{c}^c_H\), and 2 real scalars, \(\theta\) and \(\bar{\theta}\), and 18 d.o.f corresponding to 9 massless gauge bosons, \(A^0_C - A^{14}_C, A^1_A, A^2_A\) and \(A^1_L\)) of \(G_{\text{PS}}\) are redistributed as follows: 9 d.o.f are associated with the real propagating scalars \((\theta_+, x_1-\text{ and } x_2+\text{ with } x = u^a\) and \(e\)) whereas the residual 9 d.o.f combine together with the 18 d.o.f of the initially massless gauge bosons to make massive the following combinations of them \(A^{a\pm}_C, A^{a\pm}_A\) and \(A^{1\pm}_L\) – see Sec. A.2 of Appendix A.

From Table 2 we can deduce that the numbers of bosonic and fermionic d.o.f in each sector are equal. Indeed in the \(S - \nu^c_H - \bar{\nu}^c_H\) sector, we obtain 10 bosonic d.o.f and 10 fermionic d.o.f. Note that we consider \(S\) as a complex field and we take into account the 1 d.o.f of \(h\) which is not perturbed in the expansion of Eq. (A.1). Similarly in the \(u^c_H - \bar{u}^c_H\) and \(e^c_H - \bar{e}^c_H\) sectors we obtain \(32\) (24) bosonic d.o.f and an equal number of fermionic d.o.f. Note also that the spectrum contains a massless fermion which must be present due to the spontaneous SUSY breaking caused by the tree-level potential energy density in Eq. (3.6).

The 8 Goldstone bosons, associated with the modes \(x_1+\) and \(x_2-\) with \(x = u^a\) and \(e\), are not exactly massless since \(\tilde{V}_{\text{HI},h} \neq 0\) – contrary to the situation of Ref. [19] where the direction with non vanishing...
in the contrary, the angular parametrization in Eq. (3.3) assists us to isolate the massless mode below – and no significant running of the relevant parameters occurs – contrary to the SM or next-to-MSSM non-MHI. Employing the well-known Coleman-Weinberg formula [44],

| FIELDS | MASSES SquARED | EIGENSTATES |
|--------|----------------|--------------|
| **THE $S - \nu H - \nu c$ SECTOR** | | |
| 2 real scalars | $m_{\theta}^2 = m_{\theta}^2 x_h^2 / (2 \lambda^2 (x_h^2 - 6) + 15 g^2 f) / 24 f^2$ | $\hat{\theta}^\nu$, $\hat{\theta}_+$ |
| 1 complex scalar | $m_{\psi}^2 = \lambda^2 m_{\psi}^2 x_h^2 (1 + 6 c_R) / 12 J^2 f^3 \approx 4 \tilde{H}_H^2$ | $\hat{S}$ |
| 2 gauge bosons | $m_{\pm}^2 = 5 g^2 m_{\pi}^2 x_h^2 / 8 f$, $m_\parallel^2 = 0$ | $A^\perp, A^\parallel$ |
| 4 Weyl fermions | | $\hat{\psi}_{\bar{S}\nu}\pm$, $\lambda^\pm, \hat{\psi}_{\nu}\pm$ |
| 1 Majorana fermion | | $\lambda_{\parallel}, \hat{\psi}_{\parallel}\pm$ |

| **THE $u_{Ha}^c - \bar{u}_{Ha}^c (a = 1, 2, 3)$ AND $e_{Ha}^c - \bar{e}_{Ha}^c$ SECTORS** | | |
| **2(3+1) real scalars** | $m_\parallel^2 = m_{\bar{u}_\pm}^2 x_h^2 (\lambda^2 (x_h^2 - 3) + 3 g^2 f) / 12 f^2$ | $\hat{u}_{\parallel}, \hat{u}_{\perp}$ |
| **2(3+1) gauge bosons** | $m_{\pm}^2 = m_{\psi_{\parallel}}^2 x_h^2 / 4 f$ | $\hat{e}_{\parallel}, \hat{e}_{\perp}$ |
| **4(3+1) Weyl fermions** | | $A_{\parallel}^c, A_{\perp}^c$ |

| **THE $d_{Ha}^c - \bar{d}_{Ha}^c$ AND $\bar{g}_a^c - \bar{g}_a^c (a = 1, 2, 3)$ SECTORS** | | |
| **3 · 8 real scalars** | $m_{\parallel}^2 = m_{\bar{d}_\pm}^2 x_h^2 (\lambda^2 x_h^2 + 24 \lambda^2_H f) / 24 f^2$ | $\hat{g}_1^a, \hat{g}_2^a$ |
| | $m_{\pm}^2 = m_{\psi_{\parallel}}^2 x_h^2 (\lambda^2 x_h^2 + 24 \lambda^2_H f) / 24 f^2$ | $\hat{d}_{\pm}, \hat{d}_{\parallel}, \hat{d}_{\perp}$ |
| | $m_{\pm}^2 = m_{\psi_{\parallel}}^2 x_h^2 (\lambda^2 x_h^2 - 3) + 12 \lambda^2_H f) / 12 f^2$ | $\hat{d}_{\pm}, \hat{d}_{\parallel}, \hat{d}_{\perp}$ |
| **3 · 4 Weyl fermions** | | $\hat{\bar{g}}_{\psi_d\pm}$ |
| | | $\hat{\bar{g}}_{\psi_d\pm}$ |

Table 2: The mass spectrum of our model along the inflationary trajectory of Eq. (3.4). To avoid very lengthy formulas we neglect terms proportional to $m_{\psi_{\parallel}}^2$ and we assume $\lambda_H \sim \tilde{\lambda}_H$ for the derivation of the masses of the scalars in the superfields $d_{\parallel}$ and $d_{\perp}$. The various eigenstates are defined in Sec. 3.1 and Appendix A.
we find
\[ V_{\text{rc}} = V_{S^c H} + V_{\text{d}H} + V_{\text{d}H^c} + V_{\text{d}H^c}, \]  
where the individual contributions, coming from the corresponding sectors of Table 2, are given by
\[ V_{S^c H} = \frac{1}{64\pi^2} \left( m_4^4 \ln \frac{m_2^2}{\Lambda^2} + m_4^4 \ln \frac{m_3^2}{\Lambda^2} + 2m_3^4 \ln \frac{m_2^2}{\Lambda^2} \right), \]
\[ V_{\text{d}H} = \frac{4}{64\pi^2} \left( 2m_4^4 \ln \frac{m_2^2}{\Lambda^2} + 6m_4^4 \ln \frac{m_2^2}{\Lambda^2} - 8m_4^4 \ln \frac{m_2^2}{\Lambda^2} \right), \]
\[ V_{\text{d}H^c} = \frac{3}{64\pi^2} \left( 2m_4^4 \ln \frac{m_2^2}{\Lambda^2} + 2m_4^4 \ln \frac{m_2^2}{\Lambda^2} + 2m_4^4 \ln \frac{m_2^2}{\Lambda^2} - 2m_4^4 \ln \frac{m_2^2}{\Lambda^2} \right). \]

Here \( \Lambda \) is a renormalization mass scale. Based on the action of Eq. (3.12) with \( \hat{V}_{\text{HI}} \simeq \hat{V}_{\text{HIO}} + V_{\text{rc}} \), we can proceed to the analysis of non-MHI in the EF, employing the standard slow-roll approximation \([45,46]\).

It can be shown \([47]\) that the results calculated this way are the same as if we had calculated them using the non-minimally coupled scalar field in the JF.

### 3.2 The Inflationary Observables – Requirements

Under the assumption that there is a conventional cosmological evolution (see below) after non-MHI, the model parameters can be restricted, imposing the following requirements:

3.2.1 According to the inflationary paradigm, the horizon and flatness problems of the standard Big Bag cosmology can be successfully resolved provided that the number of e-foldings, \( \tilde{N}_\ast \), that the scale \( k_\ast = 0.002/\text{Mpc} \) suffers during non-MHI takes a certain value, which depends on the details of the cosmological model. The required \( \tilde{N}_\ast \) can be easily derived \([23]\), consistently with our assumption of a conventional post-inflationary evolution. In particular, we assume that non-MHI is followed successively by the following three epochs: (i) the decaying-inflaton dominated era which lasts until the reheating temperature \( T_{\text{rh}} \), (ii) a radiation dominated epoch, with initial temperature \( T_{\text{rh}} \), which terminates at the matter-radiation equality, (iii) the matter dominated era until today. Employing standard methods \([8,23]\), we can easily derive the required \( \tilde{N}_\ast \) for our model, with the result:

\[ \tilde{N}_\ast \simeq 22.5 + 2 \ln \frac{V_{\text{HI}}(h_\ast)}{1 \text{ GeV}}^{1/4} - \frac{4}{3} \ln \frac{V_{\text{HIO}}(h_\ast)}{1 \text{ GeV}}^{1/4} + \frac{1}{3} \ln \frac{T_{\text{rh}}}{1 \text{ GeV}} + \frac{1}{2} \ln \frac{f(h_\ast)}{f(h_\ast)}. \]  

On the other hand, \( \tilde{N}_\ast \) can be calculated via the relation
\[ \tilde{N}_\ast = \frac{1}{m_2^2} \int_{h_\ast}^{h_\ast} dh \frac{\hat{V}_{\text{HI}}}{\hat{V}_{\text{HI},h}} = \frac{1}{m_2^2} \int_{h_\ast}^{h_\ast} dh J^2 \frac{\hat{V}_{\text{HI}}}{\hat{V}_{\text{HI},h}}, \]
where \( h_\ast [\tilde{h}_\ast] \) is the value of \( h [\tilde{h}] \) when \( k_\ast \), crosses the inflationary horizon. Also \( h_\ast [\tilde{h}_\ast] \) is the value of \( h [\tilde{h}] \) at the end of non-MHI determined, in the slow-roll approximation, by the condition – see e.g. Ref. \([45,46]\):

\[ \max \{ \tilde{e}(h_\ast), \tilde{e}(h_\ast) \} = 1, \]  
where
\[
\hat{\epsilon} = \frac{m_P^2}{2} \left( \frac{\hat{V}_{\text{HI},h}}{\hat{V}_{\text{HI}}} \right)^2 = \frac{m_P^2}{2J^2} \left( \frac{\hat{V}_{\text{HI},h}}{\hat{V}_{\text{HI}}} \right)^2 \approx \frac{4m_P^4}{3c_R^2 h^4} \left( 1 + 4c_R m_{PS}^2 \right)^2 / \left( 1 + 4c_R m_{PS}^2 \right) \quad (3.17a)
\]
and
\[
\hat{\eta} = m_P^2 \frac{\hat{V}_{\text{HI},hh}}{\hat{V}_{\text{HI}}} = \frac{m_P^2}{J^2} \left( \frac{\hat{V}_{\text{HI},hh}}{\hat{V}_{\text{HI}}} - \frac{\hat{V}_{\text{HI},h,J}}{\hat{V}_{\text{HI}}} \right) \approx -\frac{4m_P^2}{3c_R h^2} \left( 1 + 4c_R m_{PS}^2 \right) \quad (3.17b)
\]
are the slow-roll parameters. Here we employ Eq. (3.6) and the following approximate relations:

\[
J \approx \sqrt{6} \frac{m_P}{h}, \quad \hat{V}_{\text{HI},h} \simeq \frac{4\hat{V}_{\text{HI}}}{c_R h^3} m_P^2 \left( 1 + 4c_R m_{PS}^2 \right) \quad \text{and} \quad \hat{V}_{\text{HI},hh} \simeq -\frac{12\hat{V}_{\text{HI}}}{c_R h^3} m_P^2 \left( 1 + 4c_R m_{PS}^2 \right). \quad (3.18)
\]

The numerical computation reveals that non-MHI terminates due to the violation of the \(\hat{\epsilon}\) criterion at a value of \(h\) equal to \(h_t\), which is calculated to be

\[
\hat{\epsilon}(h_t) = 1 \Rightarrow h_t = (4/3)^{1/4} m_P \sqrt{\left( 1 + 4c_R m_{PS}^2 \right) / c_R}. \quad (3.19)
\]

Given that \(h_t \ll h_s\), we can write \(h_s\) as a function of \(N_s\) as follows

\[
\hat{N}_s \simeq \frac{3c_R}{4} \frac{h_s^2 - h_t^2}{(1 + 4c_R m_{PS}^2) m_P^2} \Rightarrow h_s = 2m_P \sqrt{\hat{N}_s \left( 1 + 4c_R m_{PS}^2 \right) / 3c_R}. \quad (3.20)
\]

3.2.2 The power spectrum \(P_R\) of the curvature perturbations generated by \(h\) at the pivot scale \(k_s\) is to be confronted with the WMAP7 data [14], i.e.

\[
P_R^{1/2} = \frac{1}{2\sqrt{3} \pi m_P^3} \left( \frac{\hat{V}_{\text{HI}}(h_s)}{\hat{V}_{\text{HI},h}(h_s)} \right)^{3/2} \approx \frac{1}{2\pi m_P^2} \sqrt{\frac{\hat{V}_{\text{HI}}(h_s)}{6\hat{\epsilon}(h_s)}} \simeq 4.93 \cdot 10^{-5}. \quad (3.21)
\]

Note that since the scalars listed in Table 2 are massive enough during non-MHI, the curvature perturbations generated by \(h\) are solely responsible for \(P_R\). Substituting Eqs. (3.17a) and (3.20) into the relation above, we obtain

\[
P_R^{1/2} \simeq \frac{\lambda h_s^2}{16 \sqrt{2} \pi m_P \left( 1 + 4c_R m_{PS}^2 \right)} \simeq \frac{\lambda \hat{N}_s}{12 \sqrt{2} \pi c_R}. \quad (3.22)
\]

Combining the last equality with Eq. (3.21), we find that \(\lambda\) is to be proportional to \(c_R\), for almost constant \(\hat{N}_s\). Indeed, we obtain

\[
\lambda \simeq 8.4 \cdot 10^{-4} \pi c_R / \hat{N}_s \Rightarrow c_R \simeq 20925 \lambda \quad \text{for} \quad \hat{N}_s \approx 55. \quad (3.23)
\]

3.2.3 The (scalar) spectral index \(n_s\), its running \(\alpha_s\), and the scalar-to-tensor ratio \(r\) must be consistent with the fitting [14] of the WMAP7, BAO and \(H_0\) data, i.e.,

(a) \(n_s = 0.968 \pm 0.024\), (b) \(-0.062 \leq a_s \leq 0.018\) and (c) \(r < 0.24\) \quad (3.24)

at 95% confidence level (c.l.). The observable quantities above can be estimated through the relations:

\[
n_s = 1 - 6\hat{\epsilon}_s + 2\hat{\eta}_s \simeq 1 - 2/N_s, \quad (3.25a)
\]
\[
\alpha_s = \frac{2}{3} \left( 4\hat{\eta}_s^2 - (n_s - 1)^2 \right) - 2\hat{\xi}_s \simeq -2\hat{\xi}_s \simeq -2/N_s^2 \quad (3.25b)
\]
and
\[
r = 16\hat{\epsilon}_s \simeq 12/N_s^2, \quad (3.25c)
\]

where \(\hat{\xi} = m_P^4 \hat{V}_{\text{HI},hh} \hat{V}_{\text{HI},hh}/\hat{V}_{\text{HI}} = m_P \sqrt{2\hat{\epsilon}} \hat{\eta}_s/J + 2\hat{\eta}\). The variables with subscript * are evaluated at \(h = h_s\) and Eqs. (3.17a) and (3.17b) have been employed.
3.2.4 The scale $M_{PS}$ can be determined by requiring that the v.e.v.s of the Higgs fields take the values dictated by the unification of the gauge couplings within the MSSM. Since the highest mass scale of the model – see Table 2 – in the SUSY vacuum, Eq. (2.16), is

$$m_{\perp 0} = \sqrt{5/2} f_0 g |\langle v_H^I \rangle| \quad \text{with} \quad f_0 = f (\langle h \rangle) = 1 + 4 c_R m_{PS}^2$$

(recall that $m_{PS} = M_{PS}/m_p$) we can identify it with the unification scale $M_{GUT}$, i.e.

$$m_{\perp} = \sqrt{5 g^2 M_{PS}^2 / f_0} = M_{GUT} \Rightarrow M_{PS} = \sqrt{2 M_{GUT} m_p} / \left(5 g^2 m_p^2 - 2 c_R M_{GUT}^2\right)^{1/2} \quad (3.27)$$

The requirement $5 g^2 m_p^2 > 2 c_R M_{GUT}^2$ sets an upper bound on $c_R$, which however can be significantly lowered if we impose the requirement of Sec. 3.2.1 – see below. When $c_R$ ranges within its allowed region, we take $M_{PS} \approx (1.81 - 2.2) \cdot 10^{16}$ GeV.

3.2.5 For the realization of non-MHI , we assume that $c_R$ takes relatively large values – see e.g. Eq. (3.8). This assumption may [2, 48] jeopardize the validity of the classical approximation, on which the analysis of the inflationary behavior is based. To avoid this inconsistency – which is rather questionable [10, 48] though – we have to check the hierarchy between the ultraviolet cut-off, $\Lambda = m_p/c_R$, of the effective theory and the inflationary scale, which is represented by $\hat{V}_{HI}(h^*_s)^1/4$ or, less restrictively, by the corresponding Hubble parameter, $\hat{H}_s = \hat{V}_{HI}(h^*_s)^1/2/\sqrt{3} m_p$. In particular, the validity of the effective theory implies [48]

(a) $\hat{V}_{HI}(h^*_s)^1/4 \leq \Lambda$ or (b) $\hat{H}_s \leq \Lambda$ for (c) $c_R \geq 1$. \quad (3.28)

3.3 Numerical Results

As can be easily seen from the relevant expressions above, the inflationary dynamics of our model depends on the following parameters:

$$\lambda, \lambda_H, \lambda_R, k_S, c_R \text{ and } T_{rh}.$$ 

Recall that we determine $M_{PS}$ via Eq. (3.27) with $g = 0.7$. Our results are essentially independent of $\lambda_H$, $\lambda_R$ and $k_S$, provided that we choose some relatively large values for these so as $m_{u^c}^2, m_{d^c}^2$ and $m_S^2$ in Table 2 are positive for $\lambda < 1$. We therefore set $\lambda_H = \lambda_R = 0.5$ and $k_S = 1$ throughout our calculation. Finally $T_{rh}$ can be calculated self-consistently in our model as a function of the inflaton mass, $m_I$, and the mass $M_{\nu^c}$ of the RH neutrino into which inflaton decays, and the unified Yukawa coupling constant $y_{33}$ – see Sec. 4.1. However the inflationary predictions depend very weakly on $T_{rh}$, because $T_{rh}$ appears in Eq. (3.15) through the one third of its logarithm, and consequently its variation upon some orders of magnitude has a minor impact on the required value of $\hat{N}_s$, which remains almost constant and close to 55.

In our numerical code, we use as input parameters $h_s, M_{\nu^c}$ and $c_R$. For every chosen $c_R \geq 1$ and $M_{\nu^c}$, we restrict $\lambda$ and $h_s$ so that the conditions Eq. (3.15) and (3.21) are satisfied. In our numerical calculations, we use the complete formulas for the slow-roll parameters and $P_{\chi^2}$ in Eqs. (3.17a), (3.17b) and (3.21) and not the approximate relations listed in Sec. 3.2 for the sake of presentation. Our results are displayed in Fig. 1, where we draw the allowed values of $c_R$ (solid line), $T_{rh}$ (dashed line) and the inflaton mass at the SUSY vacuum – see Sec. 4.1 – $m_I$ (dot-dashed line) [here (solid line) and $h_s$ (dashed line)] versus $\lambda$ (a) [or] for $M_{\nu^c} = 10^{11}$ GeV. Note that the decay of the inflaton into a RH neutrino with the mass above is kinematically permitted, for the depicted $\lambda$’s. The lower bound of the depicted lines comes from the saturation of the Eq. (3.28c). The constraint of Eq. (3.28b) is satisfied along the various curves whereas Eq. (3.28a) is valid only along the gray and light gray segments of
4 Non-Thermal Leptogenesis

In this section, we specify how the SUSY inflationary scenario makes a transition to the radiation dominated era, and give an explanation of the origin of the observed BAU consistently with the $\tilde{G}$ constraint. The main features of the post-inflationary evolution of our model are described in Sec. 4.1. In Sec. 4.2 we describe the additional constraints that we impose on our setting, and finally we delineate the allowed parameter space of our cosmological model in Sec. 4.3.

**Figure 1**: The allowed by Eqs. (3.15), (3.21), (3.28b) and (3.28c) values of $c_R$ (solid line), $T_{rh} –$ given by Eq. (4.7) – (dashed line) and $m_1$ (dot-dashed line) [$h_f$ (solid line) and $h_*$ (dashed line)] versus $\lambda$ (a) [(b)] for $k_S = 1$, $\lambda_H = \lambda_{\tilde{H}} = 0.5$, $M_{\tilde{H}^c} = 10^{11}$ GeV and $y_{33} = 0.5$. The light gray and gray segments denote values of the various quantities satisfying Eq. (3.28a) too, whereas along the light gray segments we obtain $h_* \geq m_P$.

these. Along the light gray segments, though, we obtain $h_* \geq m_P$. The latter regions of parameter space are not necessarily excluded [49], since the energy density of the inflaton remains sub-Planckian and so, corrections from quantum gravity can still be assumed to be small. As $c_R$ increases beyond $5.6 \cdot 10^3$, $4c_R M_{PS}$ becomes larger than 1, $\hat{N}_s$ derived by Eq. (3.20) starts decreasing and therefore, non-MHI fails to fulfil the relevant requirement. All in all, we obtain

$$1 \lesssim c_R \lesssim 5.6 \cdot 10^3 \quad \text{and} \quad 5 \cdot 10^{-5} \lesssim \lambda \lesssim 0.25 \quad \text{for} \quad 53.9 \lesssim \hat{N}_s \lesssim 55. \quad (3.29)$$

From Fig. 1-(a), we can verify our analytical estimation in Eq. (3.23) according to which $\lambda$ is proportional to $c_R$. On the other hand, the variation of $h_f$ and $h_*$ as a function of $c_R$ – drawn in Fig. 1-(b) – is consistent with Eqs. (3.19) and (3.20). Note that the inclusion of the term $4c_R M_{PS}^2$ in the numerators of these relations is crucial in order to obtain a reliable result for $\lambda \gtrsim 0.13$ or $c_R \gtrsim 3 \cdot 10^3$ – cf. Ref. [11]. Letting $\lambda$ or $c_R$ vary within its allowed region in Eq. (3.29), we obtain

$$0.964 \lesssim n_s \lesssim 0.965, \quad -6.5 \lesssim \frac{\alpha_s}{10^{-7}} \lesssim -6.2 \quad \text{and} \quad 4.2 \gtrsim \frac{r}{10^{-3}} \gtrsim 3.5. \quad (3.30)$$

Clearly, the predicted $\alpha_s$ and $r$ lie within the allowed ranges given in Eq. (3.24b) and Eq. (3.24c) respectively, whereas $n_s$ turns out to be impressively close to its central observationally favored value – see Eq. (3.24a) and cf. Ref. [10].
Non-MHI & non-Thermal Leptogenesis in a SUSY PS Model

When non-MHI is over, the inflaton continues to roll down towards the SUSY vacuum, Eq. (2.16). There is a brief stage of tachyonic preheating [51] which does not lead to significant particle production [52]. Soon after, as discussed in the Appendix B, the inflaton settles into a phase of damped oscillations initially around zero – where $\tilde{V}_\text{HI0}$ has a maximum – and then around one of the minima of $\tilde{V}_\text{HI0}$. Whenever the inflaton passes through zero, particle production may occur creating mostly superheavy bosons via the mechanism of instant preheating [53]. This process becomes more efficient as $\lambda$ decreases, and further numerical investigation is required in order to check the viability of our leptogenesis scenario detailed below. For this reason, we restrict to $\lambda$’s larger than $0.001$, which ensures a less frequent passage of the inflaton through zero, weakening thereby the effects from instant preheating and other parametric resonance effects – see Appendix B. Intuitively the reason is that larger $\lambda$’s require larger $c_R$’s, see Eq. (3.23), diminishing therefore $h_f$ given by Eq. (3.20), which sets the amplitude of the very first oscillations.

Nonetheless the standard perturbative approach to the inflaton decay provides a very efficient decay rate. This is to be contrasted with the SM (or next-to-MSSM) non-MHI, where the consideration of non-perturbative effects is imperative [52] in order to obtain successful reheating. Namely, at the SUSY vacuum $\nu_i^0$ and $\tilde{\nu}_i^0$ acquire the v.e. vs shown in Eq. (2.16) giving rise to the mass spectrum presented in Table 3. Note that the masses of the various scalars – contrary to the masses of the fermions and gauge bosons – are not derived from the corresponding formulas listed in Table 2 with the naive replacement $x_h = 2m_{PS}$, since terms proportional to $m_{PS}^2$ are neglected there. In Table 3 we also show the mass, $m_1$, of the (canonically normalized) inflaton $\delta h = (h - 2M_{PS})/J_0$ and the masses $M_{\tilde{\nu}_i^0}$ of the RH neutrinos, $\nu_i^0$, which play a crucial role in our leptogenesis scenario – we assume the existence of a term similar to the second one inside ln of Eq. (2.12) for $\nu_i^0$ too. From Fig. 1 we notice that $m_1$ increases with $\lambda$ – as in the case of HI, cf. Ref. [19] – only for $\lambda \lesssim 0.0013$ or $c_R \lesssim 0.3$. For larger $\lambda$'s

### Table 3: The mass spectrum of our model at the SUSY vacuum of Eq. (2.16). We use the abbreviations $(J) = J (h = 2M_{PS})$, $f_0 = 1 + 4c_R m_{PS}^2$ and $f_0 = f_0 + 24c_R m_{PS}^2 \approx J_0^2$. The various eigenstates and symbols are specified in Sec. 3.1 and Appendix A.

| Eigenstates | Masses |
|-------------|--------|
| $\delta h$ | $m_1 = \sqrt{2}M_{PS}/(J)f_0$ |
| $\hat{\theta}_\nu$ | $\sqrt{5}/2g_{PS}M_{PS}$ |
| $\hat{\theta}_+^0$ | $\sqrt{2}M_{PS}/J_0f_0$ |
| $\hat{s}$ | $\sqrt{2}M_{PS}/\sqrt{J_0}$ |
| $A_{\perp}, A_{||}$ | $\sqrt{5}/2f_0g_{PS}$ |
| $\tilde{\psi}_{SS_{\pm}}$ | $\sqrt{2}M_{PS}/\sqrt{J_0}$ |
| $\lambda_{\perp}, \tilde{\psi}_{\nu_-}$ | $\sqrt{5}/2f_0g_{PS}$ |
| $\lambda_{||}, \tilde{\psi}_{\nu_+}$ | 0 |

$\nu_i^0$ Sector

| Eigenstates | Masses |
|-------------|--------|
| $\tilde{\nu}_i^0, \tilde{\nu}_i^c$ | $M_{\tilde{\nu}_i^0} = 2\lambda_{ii^c}M_{PS}^2/M_S\sqrt{J_0}$ |

$u_+^c, \bar{u}_-^0$ and $\bar{u}_+^c$ Sectors

| Eigenstates | Masses |
|-------------|--------|
| $\hat{u}_{a1}, \hat{u}_{a2}, \hat{c}_{a1}, \hat{c}_{a2}$ | $g_{PS}/\sqrt{J_0}$ |
| $A_{a1}^0, A_{a2}^0$ | $g_{PS}/\sqrt{J_0}$ |
| $\lambda_{a1}^\pm, \psi_{a1}, \psi_{a2}$ | $g_{PS}/\sqrt{J_0}$ |
| $\lambda_{a2}^\pm, \psi_{a3}, \psi_{a4}$ | $g_{PS}/\sqrt{J_0}$ |

$d_+^c, \bar{d}_-^0$ and $\bar{d}_+^c$ Sectors

| Eigenstates | Masses |
|-------------|--------|
| $\hat{d}_{a1}, \hat{d}_{a2}$ | $2\lambda_{H}M_{PS}/\sqrt{J_0}$ |
| $\tilde{d}_{a1}, \tilde{d}_{a2}$ | $2\lambda_{H}M_{PS}/\sqrt{J_0}$ |
| $\tilde{d}_{a3}, \tilde{d}_{a4}$ | $2\lambda_{H}M_{PS}/\sqrt{J_0}$ |

$\tilde{g}_d^a, \tilde{g}_d^c$ Sectors

| Eigenstates | Masses |
|-------------|--------|
| $\hat{g}_{d1}, \hat{g}_{d2}$ | $2\lambda_H M_{PS}/\sqrt{J_0}$ |
| $\tilde{g}_{d1}, \tilde{g}_{d2}$ | $2\lambda_H M_{PS}/\sqrt{J_0}$ |
| $\tilde{g}_{d3}, \tilde{g}_{d4}$ | $2\lambda_H M_{PS}/\sqrt{J_0}$ |
\( \langle J \rangle = J(h = 2M_{PS}) \) ranges from 3 to 90 and so \( m_1 \) is kept independent of \( \lambda \) and almost constant at the level of \( 10^{13} \text{ GeV} \). Indeed, if we express \( \hat{\delta}h \) as a function of \( \delta h \) through the relation

\[
\frac{\hat{\delta}h}{\delta h} \simeq J_0 \quad \text{where} \quad J_0 = \sqrt{1 + \frac{3}{2} m_p^2 f_{\hat{h}}^2 \langle\langle h\rangle\rangle} = \sqrt{1 + 24 c_R^2 m_{PS}^2} \tag{4.1}
\]

is obtained by expanding \( J \) given in Eq. (3.11) at leading order in \( x_h \), we find

\[
m_1 \simeq \frac{\sqrt{2} \lambda M_{PS}}{f_0 J_0} \simeq \frac{\lambda m_p}{2 \sqrt{3} c_R} \simeq \frac{10^{-4} m_p}{4.2 \sqrt{3}} \simeq 3 \cdot 10^{13} \text{ GeV} \quad \text{for} \quad \lambda \gtrsim \frac{10^{-4}}{4.2 \sqrt{6} m_{PS}} \simeq 1.3 \cdot 10^{-3} \tag{4.2}
\]

where we make use of Eq. (3.23) – note that \( f_0 \simeq 1 \). Apart from some fields in the \( \nu_i^c \) sector, \( \hat{\delta}h \) is the lightest among the massive particles listed in Table 3 for \( \lambda \) given in Eq. (3.29), since \( \langle J \rangle \gg 1 \) and \( g, \lambda_H \) and \( \lambda_H \) are taken larger than \( \lambda \). Therefore perturbative decays of \( \hat{\delta}h \) into these massive particles are kinematically forbidden. For the same reason narrow parametric resonance [51] effects are absent. Also \( \hat{\delta}h \) can not decay via renormalizable interaction terms to SM particles.

The various decay channels of the inflaton are mainly determined by the Lagrangian part containing two fermions – see Eq. (A.15). The inflaton can decay into a pair of RH neutrinos (\( \hat{\nu}_i^c \)) through the following lagrangian terms:

\[
\mathcal{L}_{\nu^c} = -\lambda_{\nu^c} \frac{M_{PS}}{M_S} f_0 \left( 1 - 12 c_R m_{PS}^2 \right) \hat{\delta}h \hat{\nu}_i^c \hat{\nu}_i^c + \text{h.c.} \tag{4.3}
\]

From Eq. (4.3) we deduce that the decay of \( \hat{\delta}h \) into \( \hat{\nu}_i^c \) is induced by two lagrangian terms. The first one originates exclusively from the non-renormalizable term of Eq. (2.9) – as in the case of a similar model in Ref. [19]. The second term is a higher order decay channel due to the SUGRA lagrangian – cf. Ref. [54]. The interaction in Eq. (4.3) gives rise to the following decay width

\[
\Gamma_{\nu^c} = \frac{c_{\nu^c}^2}{64 \pi} m_1 \sqrt{1 - \frac{4 M_{PS}^2}{f_0^2 2 J_0}} \quad \text{with} \quad c_{\nu^c} = \frac{M_{\nu^c}}{M_{PS}} \frac{f_0^{3/2}}{J_0} \left( 1 - 12 c_R m_{PS}^2 \right), \tag{4.4}
\]

where \( M_{\nu^c} \) is the Majorana mass of the RH neutrino into which the inflaton can decay – see Table 3. In addition, it was [54] recently recognized that within SUGRA the inflaton can decay to the MSSM particles spontaneously – i.e., even without direct superpotential couplings – via non renormalizable interaction terms. For a typical trilinear superpotential term of the form \( W_y = y XYZ \), we obtain the effective interactions described by the langrangian part

\[
\mathcal{L}_{y} = 6 y c_R \frac{M_{PS} f_0^{3/2}}{m_p^2} \hat{\delta}h \left( \hat{X} \hat{\psi}_Y \hat{\psi}_Z + \hat{Y} \hat{\psi}_X \hat{\psi}_Z + \hat{Z} \hat{\psi}_X \hat{\psi}_Y \right) + \text{h.c.}, \tag{4.5}
\]

where \( y \) is a Yukawa coupling constant, \( \hat{\psi}_X, \hat{\psi}_Y \) and \( \hat{\psi}_Z \) are the chiral fermions associated with the superfields \( X, Y \) and \( Z \), and whose scalar components are denoted with the superfield symbol. For these scalars a term similar to the second one inside Eq. (2.12) is assumed so as to obtain canonically normalized scalars and spinors through relations similar to the last equalities in Eqs. (3.11) and (A.14) respectively. Taking into account the terms of Eq. (2.6) and the fact that the adopted SUSY GUT predicts Yukawa unification for the 3rd generation at \( M_{PS} \), we conclude that the interaction above gives rise to the following 3-body decay width

\[
\Gamma_y = \frac{14 e_y^2}{512 \pi^3} m_1^3 \simeq \frac{3 y_{33}^2}{64 \pi^2} f_0^3 \left( \frac{m_1}{m_p} \right)^2 m_1 \quad \text{where} \quad c_y = 6 y_{33} c_R \frac{M_{PS} f_0^{3/2}}{m_p^2 J_0}, \tag{4.6}
\]
with \( y_{33} \simeq (0.4 - 0.6) \) being the common Yukawa coupling constant of the third generation computed at the \( m_{1} \) scale and summation is taken over color and weak hypercharge d.o.f. in conjunction with the assumption that \( m_{1} < 2 M_{\text{PS}} \) which is valid for both inflaton-decay scenarios considered below.

Since the decay width of the produced \( \tilde{\nu}_{1} \) is much larger than \( \Gamma_{1} \) – see below – the reheating temperature, \( T_{\text{rh}} \), is exclusively determined by the inflaton decay and is given by [55]

\[
T_{\text{rh}} = \left( \frac{72}{5 \pi^{2} g_{*}} \right)^{1/4} \sqrt{\Gamma_{1} m_{\text{P}}} \quad \text{with} \quad \Gamma_{1} = \Gamma_{\nu c} + \Gamma_{1y},
\]

where \( g_{*} \) counts the effective number of relativistic degrees of freedom at temperature \( T_{\text{rh}} \). For the MSSM spectrum, \( g_{*} \simeq 228.75 \). From Fig. 1 we remark that \( T_{\text{rh}} \) does not exclusively increase with \( \lambda \), but rather follows the behavior of \( m_{1} \). For \( M_{\nu c} = 10^{11} \) GeV, we find that \( \Gamma_{1y} \) dominates over \( \Gamma_{\nu c} \) for \( \lambda \gtrsim 0.002 \) or – via Eq. (3.23) – \( c_{\mathcal{R}} \geq 50 \). For \( 0.0013 \lesssim \lambda \lesssim 0.03 \), \( T_{\text{rh}} \) remains almost constant since \( f_{0}^{3} \simeq 1 - \text{see Eq. (4.6)} \). For \( \lambda \gtrsim 0.03 \), \( f_{0}^{3} \simeq 1 + 12 c_{\mathcal{R}} m_{\text{PS}}^{2} \) starts to deviate from unity and so, \( T_{\text{rh}} \) increases as shown in Fig. 1.

If \( T_{\text{rh}} < M_{\nu c} \), the out-of-equilibrium condition [31] for the implementation of leptogenesis is automatically satisfied. Subsequently \( \tilde{\nu}_{1} \) decay into \( H_{u} \) and \( L_{i}^{*} \) via the tree-level couplings derived from the second term in the RHS of Eq. (2.6). Interference between tree-level and one-loop diagrams generates a lepton-number asymmetry \( \varepsilon_{L} \) [31], when CP conservation is violated. The resulting lepton-number asymmetry after reheating can be partially converted through sphaleron effects into baryon-number asymmetry. However, the required \( T_{\text{rh}} \) must be compatible with constraints for the \( \tilde{G} \) abundance, \( Y_{\tilde{G}} \), at the onset of nucleosynthesis (BBN). In particular, the \( \tilde{B} \) yield can be computed as

\[
(a) \quad Y_{\tilde{B}} = -0.35 Y_{L} \quad \text{with} \quad (b) \quad Y_{L} = 2 \varepsilon_{L} \frac{5 \Gamma_{\nu c} T_{\text{rh}}}{4 \Gamma_{1} m_{1}}.
\]

The numerical factor in the RHS of Eq. (4.8a) comes from the sphaleron effects, whereas the numerical factor \((5/4)\) in the RHS of Eq. (4.8b) is due to the slightly different calculation [55] of \( T_{\text{rh}} \) – cf. Ref. [31]. In the major part of our allowed parameter space – see Sec. 4.3 – \( \Gamma_{1} \simeq \Gamma_{1y} \) and so the involved branching ratio of the produced \( \tilde{\nu}_{1} \) is given by

\[
\frac{\Gamma_{\nu c}}{\Gamma_{1}} \simeq \frac{\Gamma_{\nu c}}{\Gamma_{1y}} = \frac{\pi^{2} \left( 1 - 12 c_{\mathcal{R}} m_{\text{PS}}^{2} \right)^{2} M_{\nu c}^{2} m_{1}^{2}}{72 \epsilon_{L}^{2} y_{33}^{2} m_{\text{PS}}^{4}}.
\]

For \( M_{\nu c} \simeq 10^{11} \text{–} 10^{13} \) GeV the ratio above takes adequately large values so that \( Y_{L} \) is sizable. Therefore, the presence of more than one inflaton decay channels does not invalidate the non-thermal leptogenesis scenario. On the other hand, the \( \tilde{G} \) yield at the onset of BBN is estimated to be [35]:

\[
Y_{\tilde{G}} \simeq c_{\tilde{G}} T_{\text{rh}} \quad \text{with} \quad c_{\tilde{G}} = 1.9 \cdot 10^{-22} / \text{GeV}.
\]

Let us note that non-thermal \( \tilde{G} \) production within SUGRA is [54] also possible. However, we here prefer to adopt the conservative approach based on the estimation of \( Y_{\tilde{G}} \) via Eq. (4.10) since the latter \( \tilde{G} \) production depends on the mechanism of the SUSY breaking.

Both Eqs. (4.8) and (4.10) calculate the correct values of the \( B \) and \( \tilde{G} \) abundances provided that no entropy production occurs for \( T < T_{\text{sh}} \), as we already assumed – see Sec. 3.2. Regarding the estimation of \( \varepsilon_{L} \), appearing in Eq. (4.8), we single out two cases below, according to whether the inflaton can decay into the lightest \( \tilde{\nu}_{1} \) or to the next-to-lightest \( \tilde{\nu}_{2} \) RH neutrino. Note that the decay of the inflaton to the heaviest RH neutrino \( \tilde{\nu}_{3} \) is disfavored by the kinematics and the \( \tilde{G} \) constraint.
4.1.1 Decay of the Inflaton to the Lightest RH Neutrino

In this case, we suppose that the Majorana masses of \( \nu_i^c \) are hierarchical, with \( M_{1\nu^c} \ll M_{2\nu^c}, M_{3\nu^c} \) (but with \( M_{1\nu^c} > T_{th} \)). The produced lepton-number asymmetry for a normal hierarchical mass spectrum of light neutrinos reads [31]

\[
\varepsilon_L = -\frac{3}{8\pi} \frac{m_{\nu_i} M_{1\nu^c}}{(H_u)^2} \delta_{\text{eff}}.
\]

(4.11)

Here \( |\delta_{\text{eff}}| \leq 1 \), which we treat as a free parameter, represents the magnitude of CP violation; \( m_{\nu_i} \) is the mass of heaviest light neutrino \( \nu_i \) and we take \( \langle H_u \rangle = 174 \text{ GeV} \) – adopting the large \( \tan \beta \) regime.

4.1.2 Decay of the Inflaton to the Next-to-Lightest RH Neutrino

In this case, we assume \( M_{1\nu^c} \ll M_{2\nu^c} \ll M_{3\nu^c} \) and impose the conditions \( T_{th} < M_{2\nu^c} < m_1/2 \) and \( M_{1\nu^c} > T_{th} \). The resulting lepton asymmetry is [19, 32]:

\[
\varepsilon_L = \frac{3}{8\pi} \frac{M_{2\nu^c}}{M_{3\nu^c} (H_u)^2} \left( m_{3D}^2 m_{2D}^2 - m_{3D}^2 m_{3D}^2 \right) s_\delta c_\theta \sin 2\delta,
\]

(4.12)

where \( m_{4D} \) are the Dirac masses of \( \nu_i \) – in a basis where they are diagonal and positive – \( c_\delta = \cos \theta, s_\delta = \sin \theta \), with \( \theta \) and \( \delta \) being the rotation angle and phase which diagonalize the Majorana-mass matrix, \( M_{\nu^c} \), of \( \nu_i^c \). The values of the various parameters are estimated at the leptogenesis scale. Note that since \( M_{1\nu^c} > T_{th} \), \( \varepsilon_L \) calculated by Eq. (4.12) is not washed out due to the \( i \)-inverse decays and \( \Delta L = 1 \) scatterings – the case with \( M_{1\nu^c} < T_{th} \) is treated in Ref. [56]. Also Eq. (4.12) holds provided that \( M_{2\nu^c} \ll M_{3\nu^c} \) and the decay width of \( \nu_i^c \) is much smaller than \( (M_{3\nu^c}^2 - M_{2\nu^c}^2)/M_{2\nu^c} \). Both conditions are well satisfied here – see Sec. 4.3.2.

Since recent results [36] – see, also, Ref. [57] – show that the mixing angle \( \theta_{13} \) can be taken (at 95\% c.l.) equal to zero and earlier analysis [58] of the CHOOZ experiment [59] suggests that the solar and atmospheric neutrino oscillations decouple, we are allowed to concentrate on the two heaviest families ignoring the first one. This assumption enables us to connect this leptogenesis scenario with the oscillation parameters of the \( \nu_\mu - \nu_\tau \) system. The light-neutrino mass matrix, \( m_{\nu_i} \), is given by the seesaw formula:

\[
m_{\nu} = -m_0^T M_{\nu^c}^{-1} m_{\nu_D},
\]

(4.13)

where \( m_{\nu_D} \) is the Dirac mass matrix of the \( \nu_i \). The determinant and the trace invariance of \( m_{\nu}^T m_{\nu} \) imply two constraints on the parameters involved, i.e.

\[
m_{2D}^2 m_{3D}^2 - m_{\nu_{1\mu}} m_{\nu_{1\mu}} M_{2\nu^c} M_{3\nu^c} \quad \text{and} \quad A_D s_\delta^2 + B_D s_\theta + C_D = 0
\]

(4.14)

with the coefficient of the latter equation being

\[
A_D = (m_{2D}^2 - m_{3D}^2) \left( (m_{2D}^2 - m_{3D}^2) M_{2\nu^c}^2 + (m_{2D}^2 - m_{3D}^2) M_{3\nu^c}^2 \right)
+ 2(m_{2D}^2 + m_{3D}^2) M_{2\nu^c} M_{3\nu^c} \cos 2\delta
\]

\[
B_D = 2(m_{2D}^2 - m_{3D}^2) \left( m_{2D}^2 M_{2\nu^c}^2 - m_{2D}^2 M_{3\nu^c}^2 \right)
- (m_{2D}^2 + m_{3D}^2) M_{2\nu^c} M_{3\nu^c} \cos 2\delta
\]

\[
C_D = (m_{2D}^2 M_{2\nu^c}^2 + m_{3D}^2 M_{2\nu^c}^2) - (m_{\nu_{1\mu}}^2 + m_{\nu_{1\mu}}^2) M_{2\nu^c}^2 M_{3\nu^c}^2.
\]

(4.15a, b, c)

Assuming that the Dirac mixing angle (i.e., the mixing angle in the absence of RH neutrino Majorana masses) is negligible, we can identify [32] the rotation angle which diagonalizes \( m_{\nu} \) with the physical mixing angle in the \( \nu_\mu - \nu_\tau \) lepton sector, \( \theta_{23} \), which is constrained by the present neutrino data – see below.
4.2 Imposed Constraints

The parameters of our model can be further restricted if, in addition to the inflationary requirements mentioned in Sec. 3.2 and the restriction $\lambda \geq 0.001$ which allows us to ignore effects of instant preheating – see Appendix B – we impose extra constraints arising from the post-inflationary scenario. These are the following:

4.2.1 To ensure that the inflaton decay to RH neutrinos is kinematically allowed we have to impose the constraint:

$$m_1 \geq 2M_{\tilde{G}^c} \Rightarrow M_{\tilde{G}^c} \lesssim \lambda m_1 / 4\sqrt{3}c_R \simeq 1.5 \cdot 10^{13} \text{ GeV} \quad \text{for} \quad \lambda \gtrsim 1.3 \cdot 10^{-3},$$

(4.16)

where we make use of Eq. (4.2). More specifically, we require $m_1 \geq 2M_{\tilde{G}^c} [m_1 \geq 2M_{\tilde{G}^c}]$ for the scenario described in Sec. 4.1.1 [Sec. 4.1.2].

4.2.2 In agreement with our assumption about hierarchical light neutrino masses for both inflaton-decay scenarios, the solar and atmospheric neutrino mass squared differences $\Delta m^2_{\odot}$ and $\Delta m^2_{\text{atm}}$ can be identified with the squared masses of $\nu_\mu$ and $\nu_\tau$, $m^2_{\nu_\mu}$ and $m^2_{\nu_\tau}$, respectively. Taking the central values of the former quantities [36], we set:

$$(a) \quad m_{\nu_\mu} = \sqrt{\Delta m^2_{\odot}} = 0.0087 \text{ eV} \quad \text{and} \quad (b) \quad m_{\nu_\tau} = \sqrt{\Delta m^2_{\text{atm}}} = 0.05 \text{ eV}.$$  

(4.17)

We multiply the values above by 1.12 in order to roughly approximate renormalization group effects for the evolution of these masses from the electroweak up to the leptogenesis scale – see Fig. 4 of Ref. [60]. The resulting $m_{\nu_\tau}(T_{\text{rh}})$ is low enough to ensure that the lepton asymmetry is not erased by lepton number violating $2 \rightarrow 2$ scatterings [61] at all temperatures between $T_{\text{rh}}$ and $100$ GeV. Also $\sin^2 \theta_{23}$ is to be consistent with the following $95\%$ c.l. allowed range [36]:

$$0.41 \lesssim \sin^2 \theta_{23} \lesssim 0.61.$$  

(4.18)

4.2.3 Due to the presence of $SU(4)_C$ symmetry in $G_{PS}$, $m_{3D}$ coincides with the value of the top quark mass, $m_t$, at the leptogenesis scale – see also Eq. (2.6) – i.e.

$$m_{3D}(T_{\text{rh}}) = m_t(T_{\text{rh}}).$$

(4.19)

Working in the context of the MSSM with $\tan \beta = 50$, and solving the relevant renormalization group equations, we find $m_t(T_{\text{rh}}) \simeq (120 - 126)$ GeV for the values of $T_{\text{rh}}$ encountered in our set-up.

4.2.4 The implementation of BAU via non-thermal leptogenesis dictates [14] at $95\%$ c.l.

$$Y_B = (8.74 \pm 0.42) \cdot 10^{-11} \Rightarrow 8.32 \leq Y_B / 10^{-11} \lesssim 9.16.$$  

(4.20)

4.2.5 In order to avoid spoiling the success of the BBN, an upper bound on $Y_{\tilde{G}}$ is to be imposed depending on the $\tilde{G}$ mass, $m_{\tilde{G}}$, and the dominant $\tilde{G}$ decay mode. For the conservative case where $\tilde{G}$ decays with a tiny hadronic branching ratio, we have [35]

$$Y_{\tilde{G}} \lesssim \begin{cases} 10^{-15} & \text{for} \quad m_{\tilde{G}} \simeq 0.45 \text{ TeV} \\ 10^{-14} & \text{for} \quad m_{\tilde{G}} \simeq 0.69 \text{ TeV} \\ 10^{-13} & \text{for} \quad m_{\tilde{G}} \simeq 10.6 \text{ TeV} \\ 10^{-12} & \text{for} \quad m_{\tilde{G}} \simeq 13.5 \text{ TeV}. \end{cases}$$

(4.21)

The bound above can be somehow relaxed in the case of a stable $\tilde{G}$. As we see below, this bound is achievable within our model model only for $m_{\tilde{G}} > 10$ TeV. As $m_{\tilde{G}}$ gets larger than this bound leads to the necessity of a rather fine tuned SUSY breaking mechanism such as split SUSY [63], or anomaly mediated SUSY breaking [64] where the superpartners of SM particles have masses lower than $m_{\tilde{G}}$. Using Eq. (4.10) the bounds on $Y_{\tilde{G}}$ can be translated into bounds on $T_{\text{rh}}$. Specifically we take $T_{\text{rh}} \simeq (0.53 - 5.3) \cdot 10^8$ GeV $[T_{\text{rh}} \simeq (0.53 - 5.3) \cdot 10^9$ GeV] for $Y_{\tilde{G}} \simeq (0.1 - 1) \cdot 10^{-13}$ [$Y_{\tilde{G}} \simeq (0.1 - 1) \cdot 10^{-12}$].
4 Non-Thermal Leptogenesis

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Figure 2: Allowed (shaded) regions in the \( \lambda - M_{1\tilde{\nu}^c} \) plane, for \( \lambda_H = \lambda_R = 0.5, k_S = 1 \) and \( y_{33} = 0.5 \) when the inflaton can decay into \( \tilde{\nu}_i^c \)'s. The conventions adopted for the various lines and shaded or hatched regions are also shown.

4.3 Numerical Results

Considering the constraints above in conjunction with those quoted in Sec. 3.2, we can delineate the overall allowed parameter space of our model. Recall that we set \( \lambda_H = \lambda_R = 0.5, k_S = 1 \) and \( g = 0.7 \) with \( M_{PS} \) given by Eq. (3.27). The upper [lower] bound of the used \( \lambda \)'s comes from Eq. (3.29) [the conclusions of Appendix B]. Also we fix \( y_{33} = 0.5 \) throughout our investigation. As can be deduced by Eqs. (4.8) and (4.9), for lower [larger] \( y_{33} \)'s satisfying Eq. (4.20) requires slightly [lower] larger \( M_{1\tilde{\nu}^c} \)'s. As mentioned in Sec. 4.1, we adopt two leptogenesis scenaria depending on whether \( \delta h \) can decay to \( \tilde{\nu}_1^c \) or \( \tilde{\nu}_2^c \). As we see below, in both cases, two disconnected allowed domains arise according to which of the two contributions in Eq. (4.3) dominates. The critical point \( (\lambda_c, c_{Rc}) \) is extracted from:

\[
1 - 12 c_{Rc} m_{PS}^2 = 0 \Rightarrow c_{Rc} = 1/12 m_{PS}^2 \quad \text{or} \quad \lambda_c \approx 10^{-4} / 25.2 m_{PS}^2 \approx 0.06
\]  

(4.22)

where we make use of Eq. (3.23) in the last step. From Eqs. (4.7), (4.8) and (4.9) one can deduce that for \( \lambda < \lambda_c, T_{rh} \) remains almost constant; \( \Gamma_{1\tilde{\nu}^c} / \Gamma_1 \) decreases as \( c_R \) increases and so the \( M_{1\tilde{\nu}^c} \)'s, which satisfy Eq. (4.20), increase. On the contrary, for \( \lambda > \lambda_c, \Gamma_{1\tilde{\nu}^c} / \Gamma_1 \) is independent of \( c_R \) but \( T_{rh} \) increases with \( c_R \) and so fulfilling Eq. (4.20) \( M_{1\tilde{\nu}^c} \)'s decrease.

In the following, we present the results of our investigation in each case, separately.

4.3.1. Decay of the inflaton into \( \tilde{\nu}_1^c \). In this case our cosmological setting depends on the following parameters:

\[ \lambda, c_R, M_{1\tilde{\nu}^c}, \text{ and } \delta_{eff} \]

Given our ignorance about \( \delta_{eff} \) in Eq. (4.11), we take \( \delta_{eff} = 1 \), allowing us to obtain via Eq. (4.11) the maximal [62] possible \( |\varepsilon_L| \). This choice in conjunction with the imposition of the lower bound on \( Y_B \) in Eq. (4.20) provides the most conservative restriction on our parameters.

As we explain in Sec. 3.3, \( M_{1\tilde{\nu}^c} \) does not crucially influence the inflationary observables. On the contrary, \( M_{1\tilde{\nu}^c} \) plays a key-role in simultaneously satisfying Eqs. (4.16), (4.20) and (4.21) – see Eqs. (4.8) and (4.10). For this reason we display in Fig. 2 the allowed regions by all the imposed constraints in the \( \lambda - M_{1\tilde{\nu}^c} \) plane. In the horizontally lined regions Eq. (3.28a) holds, whereas in the vertically hatched region we get \( h_s \geq m_P \). The restrictions on the parameters arising from the post-inflationary era are depicted by solid, short dotted and dot-dashed lines. Namely, the solid [dot-dashed] lines correspond to the lower [most conservative upper] bound on \( Y_B [Y_G] \) in Eq. (4.20) [Eq. (4.21)].
Since we use $|\delta_{\text{eff}}| = 1$, it is clear from Eqs. (4.8) and (4.11) that values of $M_{1\tilde{c}}$ above the solid line are compatible with the current data in Eq. (4.20) for conveniently adjusting $|\delta_{\text{eff}}| \leq 1$. However the values of $M_{1\tilde{c}}$ can be restricted by the bounds of Eq. (4.21). Specifically we obtain $Y_G \simeq (0.1 - 1) \cdot 10^{-12}$ [$Y_G \simeq (0.5 - 1) \cdot 10^{-13}$] in the [gray] dark gray area. On the other hand, the kinematical condition depicted by a short dotted line – see Eq. (4.16) – puts the upper bound on $M_{1\tilde{c}}$ in a the upper right corners of the allowed region. As anticipated above, this region has two disconnected branches. In the left [right] branch, the first [second] term in Eq. (4.3) dominates. All in all we obtain

$$0.39 \lesssim \frac{M_{1\tilde{c}}}{10^{11} \text{ GeV}} \lesssim 154 \quad \text{for} \quad 0.001 \lesssim \lambda \lesssim 0.062,$$

$$154 \gtrsim \frac{M_{1\tilde{c}}}{10^{11} \text{ GeV}} \gtrsim 4.32 \quad \text{for} \quad 0.062 \lesssim \lambda \lesssim 0.25.$$

The overall minimal [maximal] $M_{1\tilde{c}}$ can be found in the left lower [upper right] corner of the allowed region. Obviously, the maximal allowed $M_{1\tilde{c}} \simeq m_1/2$ is obtained for $0.02 \lesssim \lambda \lesssim 0.062$ and $0.062 \lesssim \lambda \lesssim 0.25$. This region gives also a lower bound on $|\delta_{\text{eff}}|$, $|\delta_{\text{eff}}| \gtrsim 1.6 \cdot 10^{-4}$.

Trying to compare, finally, the resulting allowed parameter space here in the $\lambda$ – $M_{1\tilde{c}}$ plane with the one allowed within the models of HI [15] – see Fig. 2 of Ref. [65], where the coupling constant $\kappa$ corresponds to $\lambda$ – we remark that in our case (i) higher $\lambda$’s and $M_{1\tilde{c}}$’s are allowed and (ii) there is an additional minor slice of allowed parameters exclusively due to the SUGRA induced decay channel found in Eq. (4.3).

4.3.2. Decay of the inflaton into $\tilde{\nu}_2$. In this case, our cosmological setting depends on the following input parameters:

$$\lambda, c_R, M_{2\tilde{c}}, M_{3\tilde{c}} \text{ and } \delta.$$
5 Conclusions

In our numerical program, for every \( \lambda \) and \( c_R \) consistent with the inflationary requirements of Sec. 3.2, we can resolve Eq. (4.14) w.r.t. \( m_{2D} \) and \( \theta \), if we use \( M_{2\bar{D}c} \) and \( \delta \) as input parameters – recall that \( m_{3D} \) is determined by Eq. (4.19). Diagonalizing \( m_\nu \) and employing Eqs. (4.8) and (4.12) to estimate \( Y_B \), we can restrict the parameters through Eqs. (4.18) and (4.20). In order to compare the value of \( \sin^2\theta_{23} \) extracted at the leptogenesis scale, \( T_{1h} \), with the low energy experimental result of Eq. (4.18), we solve the relevant renormalization group equations following Ref. [60]. We remark that \( \sin^2\theta_{23} \) increases by almost 8% due to these renormalization effects.

In Fig. 3 we display the allowed values of \( M_{2\bar{D}c} \) versus \( \lambda \) for various \( M_{3\bar{D}c} \) and \( \delta \)'s indicated on the left upper corner of the graph. The obtained allowed ranges of several other quantities involved are arranged in the Table below the graph. Along the displayed curves the central value of \( Y_B \) in Eq. (4.20) is achieved and we obtain \( T_{1h} \simeq (0.7 - 2.9) \cdot 10^9 \text{ GeV} \) which is translated as \( Y_G \simeq (1.3 - 5.5) \cdot 10^{-13} \) via Eq. (4.10). From our results we observe that increasing \( M_{3\bar{D}c} \) entails an increase of \( M_{2\bar{D}c} \) too whereas \(-\delta \) approaches \( \pi/4 \) – see Eq. (4.12) – and increases as \( M_{2\bar{D}c} \) drops in order Eq. (4.18) to be met. The resulting \( m_{2D} \) turns out to be in the range \((1.5 - 21.9) \text{ GeV} \) which is larger than the mass of the charm quark at \( T_{1h} \), \( m_c \simeq 0.427 \text{ GeV} \). Therefore within our scheme the \( SU(4)_C \) symmetry does not hold in the up sector of the second family.

As for the case of Sec. 4.3.1, we obtain two separate branches of allowed parameters, \( h_a \geq m_P \) for \( \lambda \leq 0.0037 \), whereas Eq. (3.28a) is satisfied for \( \lambda \geq 0.016 \) – Eq. (3.28b) is valid for the used \( \lambda \)'s, see Sec. 3.3. Finally, it is remarkable that the assumptions on \( M_{1\bar{D}c} \), \( T_{1h} < M_{1\bar{D}c} \ll M_{2\bar{D}c} \) can be fulfilled in a wide range, i.e., \( 7 \cdot 10^8 < M_{1\bar{D}c}/\text{GeV} < 10^{11} \).

5 Conclusions

In this paper we attempted to embed within a realistic GUT, based on the PS gauge group, one of the recently formulated [10] SUSY models of chaotic inflation with non-minimal coupling to gravity. We showed that the model not only supports non-MHI driven by the radial component of the Higgs field, but it also leads to the spontaneous breaking of the PS gauge group to the SM one with the GUT breaking v.e.v. identified with the SUSY GUT scale and without overproduction of monopoles. Moreover, within our model, we can resolve the strong CP and the \( \mu \) problems of the MSSM via a Peccei-Quinn symmetry breaking transition. Inflation is followed by a reheating phase, during which the inflaton can decay into the lightest or the next-to-lightest RH neutrino allowing, thereby for non-thermal leptogenesis to occur via the subsequent decay of the RH neutrinos. Although other decay channels to the SM particles via non-renormalizable interactions are also activated, we showed that, in both cases, the production of the required by the observations BAU can be reconciled with the observational constraints on the inflationary observables and the \( \tilde{G} \) abundance, provided that the (unstable) \( \tilde{G} \) masses are greater than \( 10 \text{ TeV} \). In the first inflaton-decay scenario, we restrict the lightest RH neutrino mass to values of the order \( (10^{11} - 10^{13}) \text{ GeV} \) whereas in the second scenario, extra restrictions from the light neutrino data and the \( SU(4)_C \) factor of the adopted GUT gauge group can be also met for masses of the heaviest [next-to-lightest] RH neutrino of the order of \( 10^{15} \text{ GeV} \) \((10^{11} - 10^{13}) \text{ GeV}\).

Finally, we would like to point out that, although we have restricted our discussion on the PS gauge group, non-MHI analyzed in our paper has a much wider applicability. It can be realized within other GUTs, which may (as in the case of \( G_{PS} \)) or may not lead to the formation of cosmic defects. If we adopt another GUT gauge group, the inflationary predictions and the post-inflationary evolution are expected to be quite similar to the ones discussed here with possibly different analysis of the stability of the inflationary trajectory, since different Higgs superfield representations may be involved in implementing gauge symmetry breaking to \( G_{SM} \) – see, e.g., Ref. [66] which appeared when this work was under completion.
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APPENDIX A: DERIVATION OF THE MASS SPECTRUM DURING non-MHI

In this Appendix, we describe the derivation of the mass spectrum of the model when the radial component, $h$, of the fields $\nu_H^\dagger$, $\nu_H$ slowly rolls down $\tilde{V}_{HI}$, breaking $G_{PS}$ down to $G_{SM}$. We explain the results summarized in Table 2, working exclusively in the EF. We demonstrate below the origin of the masses of the scalars (Sec. A.1), gauge bosons (Sec. A.2) and fermions (Sec. A.3).

A.1 Masses for the Scalars

Expanding $\tilde{V}_{HI}$ in Eq. (3.12) to quadratic order in the fluctuations around the trajectory in Eq. (3.4), for given $h$, we obtain

$$\tilde{V}_{HI} = \tilde{V}_{H0} + \frac{1}{2} m^2_S \bar{S}^2 + \frac{1}{2} m^2_{\theta} \bar{\theta}^2 + \frac{1}{2} \left( \hat{\theta} \hat{\theta} \right) M^2_\theta \left( \hat{\theta} \hat{\theta} \right) + \frac{1}{2} \sum_x \left( \left( \bar{x}_1 \ x_1 \right) M^2_{x1} \left( x_1 \ x_1 \right) + \left( \bar{x}_2 \ x_2 \right) M^2_{x2} \left( x_2 \ x_2 \right) \right), \quad (A.1)$$

where $x = u, e, d$ and $g$ and the decomposition of the scalar fields into real and imaginary parts is shown in Eq. (3.7). The various mass-squared matrices involved in Eq. (A.1) are found to be

$$M^2_y = \begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix} \frac{m^2_{\theta}}{2}, \quad M^2_{y1} = M^2_{y2} = \begin{pmatrix} m^2_y/2 & 0 \\ 0 & m^2_y/2 \end{pmatrix}, \quad M^2_{y1} = M^2_{y2} = \begin{pmatrix} m^2_y/2 & m^2_y/2 \\ m^2_y/2 & m^2_y/2 \end{pmatrix}, \quad (A.2a)$$

$$M^2_{y2} = \begin{pmatrix} m^2_{y1} & -m^2_{y2} \\ -m^2_{y2} & m^2_{y1} \end{pmatrix}, \quad M^2_{d1} = \begin{pmatrix} m^2_{d1} & m^2_{d2} \\ m^2_{d2} & m^2_{d3} \end{pmatrix} \quad \text{and} \quad M^2_{d2} = \begin{pmatrix} m^2_{d1} & -m^2_{d2} \\ -m^2_{d2} & m^2_{d3} \end{pmatrix} \quad (A.2b)$$

where $y = u$ and $e$ and $m^2_{\theta}$, $m^2_y$, $m^2_y$, and also $m^2_S$ are presented in Table 2. The elements of the remaining matrices above are found to be

$$m^2_{y1} = m^2_{y2} = \frac{1}{24} \left( 3g^2 f + \lambda^2 x_h^2 + 6 \frac{f}{24} \right), \quad m^2_{y2} = -m^2_{y1} \left( \lambda^2 x_h^2 + 6 \frac{f}{24} \right), \quad (A.3a)$$

$$m^2_{d1} = m^2_{d2} \left( \lambda^2 x_h^2 + 24 \lambda^2 f \right), \quad m^2_{d2} = -\lambda^2 m^2_{d1} \left( x_h^2 + 6 \right), \quad (A.3b)$$

whereas $m^2_{d3}$ is the same as $m^2_{d1}$, but with $\lambda_H$ replaced by $\lambda_H$. To simplify our formulae below, we take $\lambda_H \approx \lambda_H$. The various masses squared in Eqs. (A.2a) and (A.2b) originate mainly from $\tilde{V}_{HF}$ in Eq. (3.2). Additional contributions from $V_{HI}/f^2$ in Eq. (2.15) arise for $m^2_{\theta}$ and $m^2_{y2}$. The orthogonal matrix $U_K$, which diagonalizes $M_K$ in Eq. (3.9), diagonalizes $M^2_\theta$ too. We find the following eigenstates

$$\tilde{\theta}_\pm = \frac{1}{\sqrt{2}} \left( \hat{\theta} \pm \hat{\theta} \right) \quad (A.4)$$

with eigenvalues $m^2_{\theta}$ and $m^2_{\theta}$ = 0 respectively. The second eigenstate corresponds to the Goldstone mode absorbed by $A_\perp$ – see Sec. A.2 – via the Higgs mechanism. Upon diagonalization of $M^2_{x1}$ and $M^2_{x2} \text{ with } x = u, e$ and $d$, we obtain the eigenvalues

$$m^2_{x\pm} = m^2_{x_1} \pm m^2_{x_2} \quad (A.5)$$
which correspond to the following eigenstates respectively:
\[
\hat{x}_{1\pm} = \frac{1}{\sqrt{2}} (\hat{x}_1 \pm \hat{x}_2) \quad \text{and} \quad \hat{x}_{2\mp} = \frac{1}{\sqrt{2}} (\hat{x}_2 \mp \hat{x}_1) \quad \text{with} \quad x = u^a, e \text{ and } d. \quad (A.6)
\]
Note that \(m^2_{x_\pm} = m^2_{x_0} = \lambda^2 m_T^2 x_0^2 / 4 f^2 \ll m^2_{x_\mp} \) for \(x = u \) (3 colors) and \(x = e \) are the masses squared of the Goldstone bosons \(\hat{x}_{1\pm} \) and \(\hat{x}_{2\mp} \) which are absorbed by \(A^\pm \) (3 colors) and \(A^\mp \) – see Sec. A.2 – via the Higgs mechanism. The remaining \(m^2_{x_\pm} \)'s are listed in Table 2.

### A.2 Masses for the Gauge Bosons

Some of the gauge bosons \(A^a_C \) and \(A^{\mu}_R \) acquire masses from the lagrangian terms – cf. Ref. [39]:

\[
K_{\alpha\beta} \left( (D_\mu \bar{H}^\alpha)^\beta + (D_\nu H^\nu)^\alpha \right) = \frac{1}{2} m^2_\pm \left( \left( \sqrt{\frac{3}{2}} A^{15}_C - A^{3}_R \right) \left( \sqrt{\frac{3}{2}} A^{15}_C - A^{3}_R \right) + 2 A^{a}_C A^{\mu}_R + 2 \sum_{a=1}^3 A^{a\mu}_C A^{a\mu}_R \right),
\]

where \(m_\pm \) are given in Table 2. The action of \(D_\mu \) on \(H^\nu \) and \(\bar{H}^\alpha \) is as follows:

\[
D_\mu H^\nu = \partial_\mu H^\nu + ig \left( \sum_{a=1}^{15} T^a C^a \bar{H}^\nu + \sum_{m=1}^3 T^m R^m \bar{H}^\nu \right),
\]

\[
D_\mu \bar{H}^\alpha = \partial_\mu \bar{H}^\alpha - ig \left( \sum_{a=1}^{15} T^a C^a H^\nu + \sum_{m=1}^3 T^m R^m H^\nu \right),
\]

and we have defined the following normalized gauge fields:

\[
A^\pm_a = \frac{1}{\sqrt{2}} \left( A^{7a}_{C} \pm i A^{8a}_{C} \right) \quad \text{for} \quad a = 1, 2, 3 \quad \text{and} \quad A^a_R = \frac{1}{\sqrt{2}} \left( A^1_R \pm i A^3_R \right). \quad (A.9)
\]

The first term of the RHS of Eq. (A.7) can be written as

\[
\frac{1}{2} m^2_\pm \left( A^{15}_C - A^{3}_R \right) M_{CR} \left( A^{15}_C - A^{3}_R \right) = \frac{1}{2} m^2_\perp A^\perp A^\perp, \quad (A.10)
\]

where \(m^2_\perp = 5 m^2_\pm / 2 \) – see Table 2 – since

\[
M_{CR} = \begin{pmatrix} 3/2 & -\sqrt{3}/2 \\ -\sqrt{3}/2 & 1 \end{pmatrix} \quad \text{and} \quad U_{CR} M_{CR} U_{CR}^T = \text{diag}(5/2, 0) \quad (A.11)
\]

with

\[
U_{CR} = \begin{pmatrix} -\sqrt{3}/5 & \sqrt{2}/\sqrt{5} \\ \sqrt{2}/\sqrt{5} & -\sqrt{3}/5 \end{pmatrix} \quad \text{and} \quad \begin{pmatrix} A^\perp \\ A^\parallel \end{pmatrix} = U_{CR} \begin{pmatrix} A^{15}_C \\ A^3_R \end{pmatrix}. \quad (A.12)
\]

Therefore, from Eqs. (A.7) and (A.10), we can infer that 9 gauge bosons \( (A^\perp, A^{a\perp}_C \) and \( A^{\perp}_R \) become massive, absorbing the massless modes \(\hat{\theta}_-, \hat{\nu}_{1+}^+, \hat{\nu}_{2-}^+, \hat{\nu}_{1+}^- \) and \(\hat{\nu}_{2-}^- \), whereas \(A^\parallel\), which remains massless, can be interpreted as the \(B\) boson associated with the \(U(1)_Y\) factor of \(G_{SM}\).

### A.3 Masses for the Fermions

The lagrangian kinetic terms of the chiral fermions \(\psi_x\) with \(x = \nu, e, u, d, g, \bar{\nu}, \bar{e}, \bar{u}, \bar{d}, \bar{g}\), which are associated with the superfields \(\nu', \bar{e}', \nu'_R, \bar{d}'_R, g', \bar{\nu}', \bar{e}', \bar{d}'_R, \bar{g}'\) respectively, are

\[
K_{\alpha\beta} \bar{\psi}_\alpha \gamma^\mu D_\mu \psi^\beta = \frac{1}{f^2} \left( \bar{\psi}_\nu - \bar{\psi}_\nu \right) M_K \left( \psi_\nu \psi_\nu \right) + \sum_y \frac{1}{f} \bar{\psi}_y \gamma^\mu D_\mu \psi_y
\]

\[
= \bar{\psi}_\nu + \bar{\psi}_\nu D_\mu \psi_\nu + \bar{\psi}_\nu \gamma^\mu D_\mu \psi_\nu + \sum_y \bar{\psi}_y \gamma^\mu D_\mu \psi_y. \quad (A.13)
\]
where we have used Eq. (3.9), and the normalized spinors are defined as follows

\[ \tilde{\psi}_{\nu^+} = \frac{\sqrt{f}}{f} \psi_{\nu^+}, \quad \tilde{\psi}_{\nu^-} = \frac{1}{\sqrt{f}} (\psi_\nu \pm \psi_\nu) \quad \text{and} \quad \tilde{\psi}_y = \frac{\psi_y}{\sqrt{f}} \]  
(A.14)

with \( y = e, u, d, g, \bar{e}, \bar{u}, \bar{d}, \) and \( \bar{g}. \) In Eq. (A.13) the contraction between two Weyl spinors is suppressed; the Pauli matrices \( \sigma^\mu \) and the action of \( D_\mu \) on \( \psi^\alpha \) are specified in Ref. [3, 42].

Having defined the normalized spinors, we can proceed with the derivation of the fermion mass spectrum of our model. The masses of the chiral fermions can be found applying the formula [3, 42]:

\[ m_{\alpha\beta} = e^{K/2m_\lambda^2} \left( W_{\text{HPS}\alpha\beta} + \frac{1}{m_P} (K_{\alpha\beta} W_{\text{HPS}} + K_{\alpha} F_{\beta} + K_{\beta} F_{\alpha} - \Gamma_{\alpha\beta}^\gamma F_{\gamma}) \right) \]  
(A.15)

with \( \Gamma_{\alpha\beta}^\gamma = K^\gamma_{\alpha\beta} \partial_\alpha K_{\beta\gamma}, \) \( W_{\text{HPS}\alpha\beta} = W_{\text{HPS},\phi^\alpha,\phi^\beta}, \) \( K_{\alpha} = K_{\rho^\alpha} \) and \( F_{\alpha} \) as defined below Eq. (3.2). Upon diagonalization of the relevant mass matrix, we obtain the eigenvalues \( m_{\nu_{S\nu}}, m_{\nu_{gd}} \) and \( m_{\nu_{gd}}, \) listed in Table 2, corresponding to the following eigenstates

\[ \tilde{\psi}_{S\nu\pm} = \frac{1}{\sqrt{2}} (\tilde{\psi}_{S} \pm \tilde{\psi}_{\nu^+}), \quad \tilde{\psi}_{gd\pm} = \frac{1}{\sqrt{2}} (\tilde{\psi}_{g^d} \pm \tilde{\psi}_{\nu^d}) \quad \text{and} \quad \tilde{\psi}_{d\pm} = \frac{1}{\sqrt{2}} (\tilde{\psi}_{d} \pm \tilde{\psi}_{\nu^d}). \]  
(A.16)

We remark that \( W_{\text{HPS}} \) in Eq. (2.9) does not give rise to mass terms for fermions in the sectors \( u_\nu^c - \bar{u}_\nu^c \) and \( e_\nu^c - \bar{e}_\nu^c. \) However fermion masses also arise from the lagrangian terms

\[ -i \sqrt{2} g K_{\alpha\beta} \left( \sum_{i=1}^{15} \lambda_C^a \left( \tilde{\psi}_{H^c} T^a H^c \right)^\beta - \tilde{\psi}_{H^c} (T^a H^c)^\beta \right) + \] 
\[ \sum_{m=1}^{\lambda_R^a} \left( \tilde{\psi}_{H^c} (T^m H^c)^\beta - \tilde{\psi}_{H^c} (T^m H^c)^\beta \right) + \text{h.c.} \]  
(A.17)

where \( \lambda_C^a [\lambda_R^a], \) is the gaugino corresponding to the generator \( T_C^a \left[ T_R^a \right] \) and \( \psi_{H^c}, \psi_{H^c} \) represent the chiral fermions belonging to the superfields \( H^c, H^c \) respectively. Concentrating on \( T_C^a, T_R^a, \) we obtain

\[ -i \sqrt{2} g \left( \lambda_C^a \left( \tilde{\psi}_\nu - \tilde{\psi}_\nu \right) \left( M_K \frac{f}{f} \right) \left( -T_{15}^c H^c \right) + \left( \frac{\lambda_R^a}{\lambda_R^a} \right) \left( \tilde{\psi}_\nu - \tilde{\psi}_\nu \right) \left( \frac{f}{f} \right) \left( -T_{15}^c H^c \right) \right) + \text{h.c.} = \] 
\[ \frac{igh \tilde{\psi}_\nu - \psi_\nu}{\sqrt{2}} \left( -\frac{\lambda_R^a}{\lambda_R^a} \right) + \text{h.c.} = -i m_\perp \tilde{\psi}_{\nu^+} \lambda_\perp + \text{h.c.} \] 

with \( \lambda_\perp = U_{CR} \left( \lambda_C^a \lambda_R^a \right). \)

Therefore, we obtain a Dirac mass term between the chiral fermion \( \tilde{\psi}_{\nu^+} \) and the gaugino \(-i \lambda_\perp, \) whereas a Dirac spinor composed by the combination of \( \tilde{\psi}_{\nu^+} \) and \(-i \lambda_\perp \) remains massless and can be interpreted as the Goldstino which signals the (spontaneous) SUSY breaking along the direction of Eq. (3.4).

Similarly, focusing on the directions \( T_{C}^{8+2a} \) with \( a = 1, 2, 3 \) and \( T_{R}^{1, 2} \), we obtain the mass terms

\[ \frac{i gh}{2 \sqrt{f}} \left( \sum_{k=1}^{3} \left( \tilde{\psi}_u^a \lambda_{C}^{a+} - \tilde{\psi}_u^a \lambda_{C}^{a-} + \text{h.c.} \right) + \left( \tilde{\psi}_u^a \lambda_{C}^{-} - \tilde{\psi}_u^a \lambda_{C}^{+} + \text{h.c.} \right) \right), \]  
(A.18)

where we have defined the following combinations of gauginos

\[ \lambda_C^a = \frac{1}{\sqrt{2}} \left( \lambda_C^{a+} \pm i \lambda_C^{a-} \right) \quad \text{for} \quad a = 1, 2, 3 \quad \text{and} \quad \lambda_R^a = \frac{1}{\sqrt{2}} \left( \lambda_R^{a+} \pm i \lambda_R^{a-} \right) \]  
(A.19)

in agreement with the definition of the corresponding gauge bosons in Eq. (A.9). Therefore, the chiral fermions \( \tilde{\psi}_u^a, \tilde{\psi}_u^a \) combine with \( \lambda_C^a [\lambda_R^a] \) to form two Dirac (or four Weyl) fermions with mass \( m_\perp \) as one deduces from Eq. (A.18). This completes the derivation of the spectrum of the model along the inflationary trajectory of Eq. (3.4).
APPENDIX B: INFATON OSCILLATIONS AFTER NON-MHI

In this Appendix we discuss various (p)reheating mechanisms [51] which could become competitive with the perturbative decay of the inflaton to lighter degrees of freedom, as analyzed in Sec. 4. Indeed, in certain regions of the parameter space, the process of reheating in this theory can be quite complex. After the end of non-MHI, the inflaton develops a tachyonic mass, crosses an inflection point and enters into a phase of damped oscillations. As pointed out in Ref. [52], where a similar potential is investigated, the particle production due to tachyonic preheating is not significant because the passage of the inflaton through this region is very short. During the subsequent oscillations, perturbative production of superheavy bosons – i.e., bosons with masses at the SUSY vacuum proportional to $M_{PS}$ – is not possible, since these particles are heavier than the inflaton at the global minima of its potential as shown in Table 3. Therefore effects of narrow parametric resonance [51] are also absent. However, if the initial amplitude of the inflaton oscillations is large enough, it may pass through zero, $h = 0$, where these bosons are effectively light and can be produced through (non-perturbative) instant preheating [53], as we discuss in Sec. B.2 below. We first study the dynamics of the inflaton’s oscillations in Sec. B.1.

B.1 DYNAMICS OF THE INFATON OSCILLATIONS

The cosmological evolution of $\hat{h}$ ($h$) in the EF is governed by the equation of motion:

$$\ddot{\hat{h}} + 3\hat{H}\dot{\hat{h}} + \hat{V}_{H0,\hat{h}} = 0$$

(B.1)

where $\hat{H}$ is the Hubble parameter in the EF and $\hat{V}_{H0}$ is given in Eq. (3.5) – recall that the dot denotes derivation w.r.t. the cosmic time $t$ and $h = \text{Re}(\nu_{\hat{f}}^{\dagger} + \nu_{\hat{f}}^{\prime})/2$ along the direction of Eq. (3.4). In the LHS of Eq. (B.1), we neglect the damping term $\Gamma_{\hat{h}}\dot{\hat{h}}$ which is important only at the stage of rapid oscillations of $\hat{h}$ near one of the minima of $\hat{V}_{H0}$ [51]. Note that contrary to the case of the potential analyzed in Ref. [52], the minima of $\hat{V}_{H0}$ lie at $|h| = 2M_{PS} \gg 0$ and $\hat{V}_{H0}$ has a maximum at $h = 0$ with

$$\hat{V}_{H0}(h = 0) = \hat{V}_0 = \lambda^2 M^4_{PS}$$

(B.2)

which can not be ignored. Due to these features, the quadratic approximation to $\hat{V}_{H0}$ is not accurate enough for the description of the $h$ post-inflationary evolution.

The solution of Eq. (B.1) can be facilitated if we use as independent variable the number of e-foldings $\hat{N}$ defined by

$$\hat{N} = \ln \left( \frac{\hat{R}}{\hat{R}_i} \right) \Rightarrow \dot{\hat{N}} = \dot{\hat{H}}$$

and

$$\dot{\hat{H}} = \hat{H}'$$

(B.3)

Here the prime denotes derivation w.r.t. $\hat{N}$. $\hat{R}(t)$ is the EF scale factor and $\hat{R}_i$ is its value at the commencement of non-MHI, which turns out to be numerically irrelevant. Converting the time derivatives to derivatives w.r.t. $\hat{N}$, Eq. (B.1) is equivalent to the following system of two first order equations

$$F_h = J\hat{H}\dot{\hat{R}}^3 h' \quad \text{and} \quad J\hat{H} F_h' = -\hat{V}_{H0,\hat{h}} \hat{R}^3 \quad \text{with} \quad F_h = \dot{\hat{h}}\hat{R}^3$$

(B.4)

This system can be solved numerically by taking

$$\hat{H} = \frac{1}{\sqrt{3m_P}} \left( \frac{F_h^2}{2\hat{R}^6 + \hat{V}_{H0}} \right)^{1/2}$$

(B.5)

and imposing the initial conditions (at $\hat{N} = 0$) $h(0) = (0.5 - 2.5)m_P$ and $h'(0) = 0$. We checked that our results are pretty stable against variation of $h(0)$. 
During non-MHI we have $\hat{V} \simeq \hat{V}_{\text{HI0}}$ and the results of Eqs. (3.20) and (3.23) are well verified. Soon after the end of non-MHI, we obtain $\hat{V} \simeq \hat{V}_{\text{HI0}} e^{-3(\hat{N} - \hat{N}_f)/2}$ with $\hat{N}_f$ being the value of $\hat{N}$ at the end of non-MHI – and $h$ enters into an oscillatory phase with initial amplitude equal to $h_I$ given by Eq. (3.19). Since the value of $\hat{V}_{\text{HI0}}$ at the end of inflation, $V_{\text{HI0}}$ is larger than the value of $\hat{V}_{\text{HI0}}$ at its local maximum $h = 0$, $\hat{V}_0$, we expect that $h$ crosses zero at least once during its evolution. However, as can be deduced from Eqs. (3.19) and (3.23), lowering $\lambda$ increases $h_I$ but decreases $\hat{V}_0$. Therefore the passage of $h$ through zero is facilitated.

The intuitive results above can be established and refined through the numerical solution of Eq. (B.4), during the $h$ oscillations, depicted in the upper plots of Fig. 4. Namely in left [right] plot we present the evolution of $h$ as a function of $\hat{N} - \hat{N}_f$ for $\lambda = 0.0037$ and $c_R = 81$ $[\lambda = 0.01$ and $c_R = 235]$ In both cases, we see that $h$, decreasing slowly from its value $h_I = 0.152$ $[h_I = 0.056]$ for $\lambda = 0.0037$ $[\lambda = 0.01]$, passes from the minimum of $\hat{V}_{\text{HI0}}$ at $h = 2M_{\text{PS}}$ and then climbs up the hill of $\hat{V}_{\text{HI0}}$ at $h = 0$, falls towards the other minimum of $\hat{V}_{\text{HI0}}$ at $h = -2M_{\text{PS}}$ until it reaches a maximal value and oscillates backwards. This path is followed some times until $h$ falls finally into one of the minima of $\hat{V}_{\text{HI0}}$ at $h = -2M_{\text{PS}}$ $[h = 2M_{\text{PS}}]$ for $\lambda = 0.0037$ $[\lambda = 0.01]$ – performing damped oscillations about it. In other words, $h$ oscillates initially around the local maximum of $\hat{V}_{\text{HI0}}$ and then about one of the two SUSY vacua. The number of passages though zero increases as $\lambda$ decreases – it is equal to 4 $[12]$ for $\lambda = 0.01$ $[\lambda = 0.0037]$. Solving repetitively Eq. (B.4) we notice that $h$ ceases to cross $h = 0$ for $\lambda > 0.088$.

### B.2 Instant Preheating

Whenever $h$ crosses zero particle production may occur via instant preheating [53]. This mechanism is activated when the $h$-dependent effective masses, $m_{\text{eff}}$, of the produced particles violate the adiabaticity criterion, according to which

$$|\dot{m}_{\text{eff}}/m_{\text{eff}}^2| = |\hat{h} m'_{\text{eff}}/m_{\text{eff}}^2| \ll 1.$$  \hfill (B.6)

Here, $m_{\text{eff}}$ represents collectively the masses of superheavy bosons with masses proportional to $gM_{\text{PS}}$, $\lambda_H M_{\text{PS}}$ or $\lambda_R M_{\text{PS}}$ – see Table 2. We focus on the production of these bosons since these can subsequently decay efficiently to the light SM particles altering drastically the picture of the usual perturbative reheating. On the contrary, the scalars of the $S - \nu^c_H - \bar{\nu}^c_H$ sector with masses proportional to $\lambda M_{\text{PS}}$ have suppressed decay modes to the RH neutrinos only. Taking as an example $m_{\text{eff}} = m_{\pm}$ we
plot in the lower plots of Fig. 4 the evolution of $|\dot{m}_\pm/m_\pm^2|$ as a function of $\lambda$ for $\lambda = 0.0037$ and $c_R = 81 \ (\lambda = 0.01 \ and \ c_R = 235)$ – see left [right] plot. We observe that Eq. (B.6) is violated more frequently as $\lambda$ drops since the passages of $h$ through zero become also more frequent. From the results of our numerical treatment we find that Eq. (B.6) holds during the whole post-inflationary evolution of $h$ for $\lambda > 0.045$ whereas it fails more than 40 times for $\lambda < 0.001$.

The produced this way superheavy bosons acquire a large mass while the inflaton increases towards its maximum amplitude and start to decay into all lighter particles within almost half oscillation of the inflaton, rapidly depleting their occupation numbers. As shown in Ref. [52] an efficient transfer of energy from $h$ to the superheavy bosons requires a rather large (let say $50 - 70$) number of passages of $h$ through zero. Meanwhile effect of backreaction of the produced particles on the condensate may become significant and a more involved numerical study of the process is imperative. Trying to deliberate our leptogenesis scenario from such a complicate situation, we impose the indicative lower bound $\lambda \geq 0.001$ – which can be translated as a bound $c_R \geq 21$ via Eq. (3.23) – above which our estimations in Sec. 4 are more or less independent of the preheating effects. We hope to return to the analysis of $\lambda < 0.001$ region in the future.

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