Structure and dynamics of weakly-bound systems: a one-dimensional model

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Abstract. A line of research has been developed to describe the structure and the dynamics of weakly-bound systems with one or more valence particles. To simplify the problem we are assuming particles moving in one dimension. Within this model one can describe, for example, one- and two-particle break-up and one- and two-particle transfer processes. In the case of reactions involving two particles one can study in particular the proper reaction mechanism in connection with the role of the pairing interaction. Exact solutions obtained by directly solving the many-body time-dependent Schroedinger equation can be compared with the results obtained with different approximation schemes (first-order, coupled channels, continuum discretization, etc). In this particular contribution we concentrate on one-particle transfer processes.

We address in this contribution the basic problem of the description of both structure and dynamics of weakly-bound systems with one or more valence particles. The problem is relatively easy with one valence particle (one-particle halo systems), but starts to be more complex with two particles (two-particle halo systems), becoming then extremely complicated with more particles. To simplify the problem, we will assume that the particles move just in a one dimension. In spite of this drastic assumption, we expect that the problem will maintain the main features and properties of the full three-dimensional case, for example for the description of pairing correlations and their connection with two-particle transfer or break-up processes.

Different aspects of the one-dimensional problem have been developed and discussed in different papers. In [1] we have discuss the problem of the excitation to the continuum (break-up) of a particle, initially moving in a single-particle orbit of a one-dimensional well, due to the action of an external perturbation (simulating the interaction with a reaction partner). The exact solution has been compared with the results obtained by expanding the wave function in a basis obtained by a discretization of the positive-energy part of the spectrum along the CDCC approach. In [2, 3] we have discussed the situation of a two-particle "borromean" system, bound due to the action of the residual pairing interaction, considering different discretization methods (HO, THO, infinite box) and studying both ground-state properties and the response functions to different operators. In [4] we have studied the break-up of such "borromean" systems due to the action of an external one-body field, pointing out the role of the residual pairing interaction in enhancing the correlated two-particle break-up in comparison with the single one-particle break-up.

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In this contribution we focus our attention on one-particle transfer processes. Our model includes a fixed well, with one particle initially situated on one of the levels of the well. This situation is assumed to be representative of a system characterized by an inert core plus a valence particle. The system is then perturbed (see figure 1) by a second moving well and we follow in time the evolution of the single-particle wave function. For simplicity the motion of the second well is assumed to have a quadratic time dependence ($x_p = x_0 + at^2$, $x_p$ being the position of the center of the well), and by choice of the distance of closest approach and of the acceleration simulates different kinematical conditions of the reaction (bombarding energy, impact parameter, etc). In the same way, the choice of the depths of the two wells (assumed to have a Woods-Saxon shape), as well as the choice of the initial single-particle state, determine the characteristics of the two colliding objects, for example the situation of weak binding in a halo-like system.

In the case depicted in figure 1 the fixed well is chosen in such a way that the single particle is moving in the only bound state (with binding energy $E_b = -3.10$ MeV), while the moving well admits two (initially empty) bound levels. For the parameters defining the trajectory we have taken $x_0 = 10$ fm and $a = 0.3$ fm/ps$^2$. The evolution of the wave function with time is illustrated in figure 2. The different frames refer to different times (the total collision time is divided in 210 steps and the corresponding time is quoted in each frame) and in each frame the upper part gives the square of the one-particle wave function while the lower frame gives the actual position of the two potentials at the same time. As apparent from the figure, when the tail of the moving well starts to overlap with the fixed well (frame c) part of the wave function enters in the moving well and then follows its movement (frames d-f). At the end of the process, by taking the overlap of the final wave function with the initial wave function and the wave functions of the single-particle states of the moving well one can determine the probability of elastic scattering and of one-particle transfer process. The presence of a node in the part of wave function inside the moving well already clearly indicates that the transfer takes place to the second single particle.
Figure 2. In all frames, corresponding to different times, the upper part gives the square of the one-particle wave function at that time, the lower part the actual position of the two potentials at the same time.

state. Note that only a negligible part of the wave function (practically undetectable in the figure) appears outside the two wells, indicating a practicably negligible break-up probability.

Figure 3. Left side: Probabilities for elastic scattering, transfer process and break-up process as a function of the difference between the energies of the single-particle states in the fixed and moving frame (Q-value). Right side: Schematic picture of the reaction process.

The simplicity of the model allows us to compare easily the results obtained by varying the parameters characterizing the different situations. As an example we can vary the depth of the moving well, so varying the energies of the corresponding bound states. We can therefore study
the transfer process as a function of the difference between the energies of the single-particle 
states in the fixed and moving frame, i.e. the Q-value associated to the transfer process (cf. the 
right part of figure 3). In the left part of figure 3 we plot the probabilities for elastic scattering, 
transfer process and (to minor extent) break-up process as a function of the Q-value. As we can 
see from the figure, the model reproduces the expected bell behavior of the transfer probability 
around the "optimal Q-value", which in our specific case of transfer of a neutral particle is close 
to zero. Of course the width of the probability distribution depends on the parameters of the 
collision (distance of closest approach and acceleration) and can therefore be modified by acting 
on these parameters.

Figure 4. Same as in figure 2, but for an initial weakly-bound single-particle state \( (E_b = -0.91 \text{ MeV}) \). The wave function is amplified in the insets, to put in evidence the continuum (break-up) part of the wave function.

We move now to the case involving weakly-bound systems. To this end we modify the depth 
of the potentials in order to obtain weakly-bound single-particle states \( (E_b = -0.91 \text{ MeV}, \text{ with } x_0=12 \text{ fm}) \). As a consequence of the weak binding the initial wave function displays a longer tail, 
with the result of larger overlaps between the two wells. The corresponding time evolution of the 
wave function is shown, at different stages of the reaction, in the frames of figure 4. The weak 
binding situation is also responsible of larger break-up probabilities. To evidence the continuum 
part of the wave function, which is distributed over a large interval, we also show in the insets 
an amplification of the wave function in a region far from the two wells.

As in the previous case of well-bound single-particle states, we can study the final probabilities 
for transfer and break-up as a function of the difference between the energies of the single-particle 
states in the fixed and moving frame, i.e. the Q-value associated to the transfer process. These 
probabilities for elastic scattering, transfer process and break-up process as a function of the 
Q-value are plotted in the left frame of figure 5. As shown by the figure, there is a large 
contribution from the break-up process, which is strongly favored by the initial weak binding of 
the particle, and which shows a smoother dependence on the value of the single-particle energies 
in the moving well. On the converse, the transfer probability displays the expected bell behavior 
around the optimal Q-value.
Figure 5. Same as in figure 3, but for an inial weakly-bound single-particle state.

Figure 6. Transfer and break-up probabilities as a function of the distance of closest approach $x_0$ between the two colliding wells. The lower frame refers to the case of a well-bound initial single-particle state, the upper frame to the case of a weakly-bound state.

As a final point in this contribution we display in figure 6 the transfer and break-up probabilities as a function of the distance of closest approach $x_0$ between the two colliding wells. The probabilities display the expected exponentially decaying behavior arising from the combination of the tail properties of the two wells and of the single-particle states. It is therefore
not surprising that, in the case of the initial weakly-bound system characterized by the long tail, the transfer probability survives to larger distances with a lower decaying rate. Note also that below a certain critical distance (approximately below 10 fm for the well-bound case and 12 fm for the weakly-bound) the transfer probability becomes so large that in a perturbative picture we will have back and forth transfer, with resulting oscillatory behavior. In this regime our model space (characterized by just one active particle) becomes clearly inappropriate.

The results shown in this contribution have been obtained by directly solving the time-dependent one-particle Schrödinger equation. The same equation can be solved within the standard time-dependent coupled-channels formalism by constructing the non-diagonal transfer formfactors and expanding the wave function into the dual basis associated with the two wells. One can in this way test the validity of the first-order approximation and the necessary truncation in the basis. This latter point is particular relevant in the case of weakly-bound systems in connection with the treatment of the positive-energy part of the spectrum and the procedures used for the discretization of the continuum.

Next step will involve the extension to the case of two-particle systems and two-particle transfer processes. In this case we will have two particles initially sitting in one single-particle state of the fixed well and one will follow the time evolution of the two-particle wave function due to action of the other moving one-body potential plus the residual pairing-like interaction between the two particles. The projections of the final wave function on the asymptotic states corresponding to two-particles in the initial well, one particle in each well, two particles in the moving well and one or two-particles in the continuum will yield the probabilities for elastic, one-particle transfer, two-particle transfer and one and two particle break-up, respectively. By switching on and off the residual pairing interaction one will determine its effect in the enhancement of the two-particle transfer probabilities with respect to the one-particle transfer. As in the case of one-particle transfer, the exact results will be compared with those obtained in the conventional coupled-channel scheme or in its lowest-order two-step perturbation limit.

References

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