Photo-proton emission from halo nuclei in the presence of a linearly polarized laser field

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Abstract. We investigate the emission of protons from halo nuclei which are subject to the combined electromagnetic fields of a $\gamma$-ray photon and an intense laser beam of linear polarization. Photo-proton energy spectra resulting from this laser-assisted nuclear photoeffect are calculated within an $S$-matrix approach based on the strong-field approximation. We analyze the influence of the applied $\gamma$-ray energy, laser field polarization and nuclear isotope on the proton emission spectra. The process represents an example from the field of laser-assisted photo-nuclear physics which may be explored at the upcoming Extreme Light Infrastructure.

1. Introduction

The nuclear photoeffect was discovered by Chadwick and Goldhaber in 1934. Using $\gamma$ rays from a radioactive source, they observed the photodisintegration of the deuteron into a proton and a neutron [1]. In the years after, further pioneering contributions to this new research area were made by Bothe and Gentner in Heidelberg, whose systematic experimental investigations relied on high-energy $\gamma$ rays stemming from an accelerator [2]. Since that time, studies of the nuclear photoeffect in the energy region up to a few tens of MeV have proven to be particularly important for our understanding of nuclear structure and nuclear reactions. While modern experiments on photonuclear reactions typically employ high-energy photons produced, for example, by synchrotron radiation sources, powerful lasers are nowadays emerging into an alternative tool for photonuclear studies [3]. A laboratory devoted particularly to the research area of laser-nuclear physics is currently being built as one of the pillars of the Extreme Light Infrastructure [4, 5].

The large mismatch between the typical energy scales of nuclei on the one side and laser fields on the other side generally renders the probability of inducing direct photonuclear reactions by laser radiation very small [6]. Therefore, indirect laser-nucleus interactions – mediated by secondary particles such as electrons in laser-produced plasmas – have mostly been investigated. In particular, photofission and photoneutron production arising from bremsstrahlung photons emitted by fast electrons have been observed experimentally [7, 8, 9].

Laser-assisted nuclear processes are generally less demanding than directly laser-induced nuclear processes in terms of the required laser field intensity and frequency. Such processes can already occur in the absence of any laser field but may be modified in a characteristic manner when a laser field is present. Corresponding studies have been conducted on laser-assisted internal conversion [10] and laser-assisted nuclear excitation by electron transition [6, 11]. Related studies have treated laser-triggered nuclear Coulomb excitation in atoms (see
[12, 13, 14, 15] and references therein) and laser-driven proton streaking [16]. We point out that the ongoing increase of available laser intensities and frequencies has also inspired theoreticians to investigate directly laser-induced nuclear reactions during the last few years [17, 18, 19].

Very recently, we have considered the nuclear photoeffect assisted by a laser field of circular polarization [20]. In this process, a proton is emitted from a nucleus which is exposed to the combined fields of a $\gamma$ photon and an intense laser beam (see also Fig. 1):

$$\frac{A}{2}[^{A}Xp] + \omega_{\gamma} + n\omega_{0} \rightarrow \frac{A-1}{2}[^{A-1}X] + ^{1}p.$$  

(1)

The $\gamma$ photon energy $\omega_{\gamma}$ is assumed to exceed the proton separation energy $E_{b}$, so that proton ejection can already occur without a laser field present. $\frac{A}{2}[^{A}Xp]$ denotes a nucleus with $A$ nucleons and $Z$ protons. It is taken to be a halo nucleus, consisting of a nuclear core $X$ surrounded by a rather loosely bound proton. This special structure, which some isotopes possess [21], reduces the $\gamma$ photon energy required and also facilitates the theoretical treatment of the process. Simultaneous with the $\gamma$-absorption, also a certain number $n$ of laser photons of energy $\omega_{0}$ can be emitted or absorbed during the process, due to the laser assistance. This additional photon exchange with the laser field may lead to distinct modifications of the photo-proton energy spectra. We note that a pronounced impact of an assisting laser field has also been found in the photoelectric effect in atoms (see, e.g., [22, 23, 24, 25]).

In the current contribution we extend our results from [20] to the case of a linearly polarized laser field. Besides, while in [20] we examined the process dependencies on the applied laser intensity and frequency and the $\gamma$-ray polarization, here we study the influence of the $\gamma$-photon energy, the laser polarization and the nuclear isotope. An $S$-matrix description of the process based on the strong-field approximation [26] is given in Section 2. The cross section is expressed in analytical form as an infinite sum over the number $n$ of participating laser photons. In Section 3 we discuss the various dependencies of the photo-proton spectra on the applied laser and nuclear parameters. A short summary and conclusion is given in Section 4.

Natural units with $\hbar = c = \varepsilon_{0} = 1$ are used throughout unless explicitly stated otherwise.

2. Theoretical framework

The laser-assisted nuclear photoeffect of Eq. (1) can be treated theoretically as follows. The laser field is assumed to be a linearly polarized, monochromatic wave with vector potential

$$\vec{A}_{L}(t) = A_{0} \cos (\omega_{0}t) \vec{e},$$  

(2)

![Figure 1. Scheme of the laser-assisted nuclear photoeffect. Due to the presence of the combined fields of a $\gamma$-photon and a laser beam, a proton is ejected from a halo nucleus.](image)
where $A_0$ is the amplitude. Note that, for the laser parameters under consideration, the dipole approximation applies. The vector potential of the $\gamma$ photon reads

$$\vec{A}_{\gamma}(\vec{r}, t) = \frac{2\pi}{V_{\gamma}\omega_{\gamma}} e^{i(\vec{k}_{\gamma} \cdot \vec{r} - \omega_{\gamma} t)} \vec{e},$$  

with momentum $\vec{k}_{\gamma}$, polarization vector $\vec{e}$ and normalization volume $V_{\gamma}$.

The nonrelativistic interaction Hamiltonian responsible for $\gamma$-photon absorption by the nucleus in the presence of the laser field is

$$\hat{H}_{\text{int}} = -\frac{e}{m} \left( \vec{p} - e \vec{A}_L(t) \right) \cdot \vec{A}_{\gamma}(\vec{r}, t),$$

with the proton charge $e$ and mass $m$ and the momentum operator $\vec{p} = -i\nabla$. The $S$-matrix of the process has the general form

$$S_{fi} = -i \int_{-\infty}^{\infty} \langle \Psi_f | H_{\text{int}} | \Psi_i^{(+)} \rangle \, dt.$$  

Here, $\Psi_i^{(+)}$ is a state of the full Hamiltonian $\hat{H}$, which includes the interactions of the proton with the photon fields and the nuclear potential. $\Psi_f$ denotes an unperturbed state belonging to the Hamiltonian $\hat{H} - \hat{H}_{\text{int}}$. Since neither of these states is known exactly, it is necessary to perform suitable approximations. Assuming that in the initial state the nuclear forces are dominant, we may write $\Psi_i^{(+)}$ approximately as a stationary state,

$$\Psi_i^{(+)}(\vec{r}, t) \approx \phi_0(\vec{r}) e^{iE_{fi} t},$$

which is subject solely to the nuclear potential. For the space-dependent part in Eq. (6), a Yukawa form may be used [27]

$$\phi_0(\vec{r}) = \frac{c_0}{\sqrt{4\pi}} \frac{e^{-\beta |\vec{r}|}}{\beta |\vec{r}|},$$

with $\beta = 1/(\sqrt{2} R_{\text{rms}})$ and $c_0 = \sqrt{2} \beta^{3/2}$, where $R_{\text{rms}}$ denotes the rms-radius of the proton halo. With regard to the final proton state in the continuum, we note that it will be affected only rather moderately by the nuclear core potential, provided the energy of the ejected proton is relatively high. This state is mainly affected by the laser wave and may, thus, be approximated by a Volkov state. For the linearly polarized field of Eq. (2), the latter reads (in velocity gauge)

$$\psi_p(\vec{r}, t) = \frac{e^{i\vec{p} \cdot \vec{r}}}{\sqrt{V}} \exp \left[ -i \left( \frac{\vec{p}^2}{2m} + U_p(\vec{r}) \right) t \right] f(t),$$

with the proton momentum $\vec{p}$ and a normalization volume $V$. Besides, $U_p = e^2 A_0^2/(4m)$ denotes the ponderomotive energy and

$$f(t) = e^{i(\alpha \sin \omega_0 t - \beta \sin 2\omega_0 t)}, \text{ with } \alpha = \frac{e A_0}{m \omega_0} \vec{p} \cdot \vec{e} \text{ and } \beta = \frac{e^2 A_0^2}{8m \omega_0}. $$

Within this strong-field approximation, the $S$-matrix of Eq. (5) becomes

$$S_{fi} \approx \frac{ie}{m} \sqrt{\frac{2\pi}{VV_{\gamma}\omega_{\gamma}}} \int dt \int d^3 r \ e^{-i\vec{p} \cdot \vec{r}} e^{i \left( \frac{\vec{p}^2}{2m} + U_p \right) t} f(t)^* \left( \vec{p} - e \vec{A}_L \right) \cdot \vec{e} \ e^{i(\vec{k}_{\gamma} \cdot \vec{r} - \omega_{\gamma} t)} \phi_0(\vec{r}) e^{iE_{fi} t}.$$.  


As in [20], it is also assumed here that \( A_{0r_{\text{eff}}} \ll 1 \) holds, where \( r_{\text{eff}} \) denotes the relevant radial extension of integration in Eq. (10). This condition will be fulfilled as we shall restrict our consideration to field amplitudes \( A_0 \lesssim 1 \) MeV. Since the relevant range of the space integral is limited to \( r \lesssim 10 \) fm due to the presence of the bound nuclear state, we have \( A_{0r_{\text{eff}}} \lesssim 10^{-2} \).

In order to make progress, we expand the functions \( f(t)^* \) and \( \cos (\omega_0 t) f(t)^* \), which appear in the term \( f(t)^* (\vec{p} - e \vec{A}_L) \cdot \vec{e} \) in Eq. (10), into Fourier series according to

\[
f(t)^* = \sum_{n=-\infty}^{+\infty} B_n e^{-i n \omega_0 t}, \quad \cos (\omega_0 t) f(t)^* = \sum_{n=-\infty}^{+\infty} C_n e^{-i n \omega_0 t}.
\]

The Fourier coefficients \( B_n = \tilde{J}_n(\alpha, \beta) \) and \( C_n = \frac{1}{2} \left[ \tilde{J}_{n+1}(\alpha, \beta) + \tilde{J}_{n-1}(\alpha, \beta) \right] \) can be expressed in terms of generalized Bessel functions of integer order. The latter are given by infinite sums over ordinary Bessel functions, according to [28]

\[
\tilde{J}_n(\alpha, \beta) = \sum_{\ell=-\infty}^{+\infty} J_{n-2\ell}(\alpha) J_{\ell}(\beta).
\]

As a consequence, we obtain

\[
S_{fi} \approx \frac{ie}{m} \sqrt{\frac{2\pi}{VV_{\gamma} \gamma}} \sum_{n=0}^{+\infty} \mathcal{M}_n \int d^3 \vec{r} \ e^{-i (\vec{p} - \vec{k}_\gamma) \cdot \vec{r}} \phi_0(\vec{r}) \int dt e^{i \left( \frac{e_0}{2m} + U_p + E_b - \omega_\gamma - n \omega_0 \right) t},
\]

with \( \mathcal{M}_n = \vec{e} \cdot \vec{p} B_n - e A_0 \vec{e} \cdot \vec{e} C_n \). The space and time integrations can be carried out analytically to yield

\[
S_{fi} \approx \frac{i 2 \sqrt{2\pi^2 e e_0}}{m \sqrt{VV_{\gamma} \gamma}} \sum_{n=n_0}^{+\infty} \mathcal{M}_n \ G(\vec{p} - \vec{k}_\gamma) \ \delta \left( \frac{\vec{p}^2}{2m} + U_p + E_b - \omega_\gamma - n \omega_0 \right).
\]

Here, \( n_0 \) denotes the smallest integer which is in accordance with the energy conservation condition, as dictated by the \( \delta \)-function. Besides,

\[
G(\vec{p} - \vec{k}_\gamma) = \frac{2}{\beta [\beta^2 + (\vec{p} - \vec{k}_\gamma)^2]}
\]

is the Fourier transform of the bound halo state.

From the absolute square of the \( S \)-matrix in Eq. (14) we obtain the total cross section for the laser-assisted nuclear photoeffect by integrating over the final proton momentum and by dividing out the \( \gamma \)-photon flux \( j = 1/V_{\gamma} \) and a unit time \( T \):

\[
\sigma = \frac{1}{jT} \int \frac{V \ d^3 p}{(2\pi)^3} \ |S_{fi}|^2 = \sum_{n=n_0}^{+\infty} \sigma_n.
\]

The total cross section naturally decomposes into a sum over partial cross sections \( \sigma_n \), in accordance with the structure of Eq. (14) which involves a summation over the number of exchanged laser photons. These partial cross sections read

\[
\sigma_n = \frac{e^2 q^2}{2m \omega_\gamma} \int_0^\pi \sin \theta \, d\theta \int_0^{2\pi} d\varphi p_n \ (\vec{e} \cdot \vec{p}_n B_n - e A_0 \vec{e} \cdot \vec{e} C_n)^2 \ G(\vec{p}_n - \vec{k}_\gamma)^2 \ .
\]

The absolute value of the momentum of the outgoing proton is fixed by the \( \delta \)-function to be \( p_n = [2m (\omega_\gamma + n \omega_0 - U_p - E_b)]^{1/2} \). The remaining integrals over the polar and azimuthal emission angles of the proton in Eq. (17) can be performed by numerical means.
3. Numerical results and Discussion

We have applied our theory to laser-assisted photo-proton ejection from $^8$B [21, 27, 29] and $^{26}$P [30]. These nuclear isotopes possess low proton separation energies of $E_b \approx 137$ keV and $E_b \approx 860$ keV, respectively. The rms radius of the proton halo amounts to $R_{\text{rms}} \approx 4.73$ fm in $^8$B and $R_{\text{rms}} \approx 3.11$ fm in $^{26}$P. The $\gamma$ photon is incident with a relatively high energy of a few MeV and is polarized along the $z$ axis, $\vec{\varepsilon} = \vec{e}_z$. The high $\gamma$-photon energy guarantees that the nuclear potential has only a moderate influence on the proton after ejection.

In the following we demonstrate how the properties of the laser-assisted nuclear photoeffect depend on the incident $\gamma$-photon energy, the laser polarization and the nuclear isotope.

3.1. Dependence on $\gamma$-photon energy

Figure 2 displays the partial cross sections $\sigma_n$, as a function of the number $n$ of emitted ($n < 0$) or absorbed ($n > 0$) laser photons. The laser field is assumed to have an x-ray frequency of $\omega_0 = 2$ keV at an intensity of $I = 1.0 \times 10^{23}$ W/cm$^2$ [31]. Like the $\gamma$-photon, it is polarized along the $z$ axis, $\vec{\varepsilon} = \vec{e}_z$. Note that the $\sigma_n$ distribution corresponds to the photo-proton energy spectrum since, for a given value of $n$, the proton energy amounts to $E_n = \omega_\gamma + n\omega_0 - U_p - E_b$.

As the figure illustrates, the assistance of the laser field leads to sidebands in the photo-
proton energy distribution. The sidebands are equally spaced by a laser photon energy and symmetrically surround the central line located at \( n = 0 \).

Moreover, as panels (a)-(c) of Fig. 2 show, the number of relevant sidebands grows when the \( \gamma \)-photon energy increases. The reason is that, due to the \( \hat{p} \cdot \vec{A}_L \) interaction between the proton and the laser field, larger proton momentum components along the laser polarization direction lead to enhanced proton-laser coupling. This fact is also reflected in the momentum dependence of the first argument [32] of the generalized Bessel functions in Eq. (9). When the \( \gamma \)-photon energy increases, also the emitted proton energy increases. As a consequence, also the relevant momentum component of the proton increases. Note in this regard that the proton is emitted preferentially along the polarization direction of the \( \gamma \)-photon.

The total cross sections \( \sigma = \sum_n \sigma_n \) in Figs. 2(a)-(c) amount to about 63 mb, 33 mb and 13 mb. We point out that the same values would result from our model when the laser field is switched off. I.e., while the presence of the laser modifies the energy distributions of the ejected protons, it does not affect the total ejection probability. For comparison we note that an estimate of the total cross section via the well-known Bethe-Peierls formula gives values which are somewhat smaller (by a factor \( \approx 3 \)) than the cross sections mentioned above. Inclusion of the repulsive Coulomb interaction between the ejected proton and the core nucleus into our model would lead to a reduction of the total cross sections predicted [33].

### 3.2. Influence of laser polarization

In Fig. 3 we show the distributions of partial cross sections for laser-assisted photo-proton emission from \(^8\)B by a 3-MeV \( \gamma \)-photon. Panels (a) and (b) refer to a laser field of linear polarization and circular polarization, respectively. The \( \gamma \)-photon as well as the linearly polarized laser field are polarized along the \( z \) axis, whereas the electric field vector of the circularly polarized laser wave rotates within the \( y \)-\( z \) plane. Both laser fields are assumed to have the same intensity. From the figure one can see that the linearly polarized laser field has a somewhat larger influence on the proton emission energies than the circularly polarized field. Indeed, the “plateau” in panel (a) comprising partial cross sections of similar magnitude is slightly more extended than in panel (b). This can be understood by noting that a linearly polarized field has an amplitude which is larger by a factor \( \sqrt{2} \) than an equally intense field of circular polarization.

![Figure 3](image-url)

**Figure 3.** Comparison of the distributions of partial cross sections \( \sigma_n \) for photo-proton emission from \(^8\)B assisted by a laser field of (a) linear polarization and (b) circular polarization. The laser frequency is \( \omega_0 = 2 \text{keV} \) and laser intensity \( I = 1.25 \times 10^{24} \text{W/cm}^2 \). The \( \gamma \)-photon has energy \( \omega_\gamma = 3 \text{MeV} \) and is polarized along the same axis as the linearly polarized laser field.
has. This larger amplitude leads to an enhanced proton-laser coupling, as may again be seen from the Bessel function argument $\alpha$ in Eq. (9). Note that for a circularly polarized laser wave, a very similar term enters as argument into an ordinary Bessel function $J_n$ [20].

In terms of absolute numbers, the plateau in Fig. 3(a) extends over the range $-10 \lesssim n \lesssim 10$, corresponding to an energy width of $20\omega_0 = 40\text{keV}$. The proton energies thus lie approximately between 2.84 MeV and 2.88 MeV. The energy width arising for the assisting laser field of circular polarization in Fig. 3(b) is somewhat smaller and amounts to about 30 keV. Therefore, a linearly polarized laser field is advantageous for detecting signatures of the laser-assisted nuclear photoeffect. The total cross sections are the same for both polarizations. It is worth mentioning that similar energy widths of the proton spectra would arise if an optical laser with intensity $\sim 10^{18}\text{W/cm}^2$ was used instead of the x-ray laser beam [20].

3.3. Variation of nuclear isotope and field geometry

So far, we have considered photo-proton ejection from $^8\text{B}$. Figure 4 shows distributions of partial cross sections for the laser-assisted nuclear photoeffect in $^{26}\text{P}$. (Note that they are shown on a linear scale, in contrast to Figs. 2 and 3.) Since the proton separation energy in this isotope is much higher, a $\gamma$-photon energy of 10 MeV has been assumed here in order to lift the proton high up into the continuum.

Because of the larger value of $E_b$, a proton removed from $^{26}\text{P}$ has a smaller kinetic energy than a proton from $^8\text{B}$, provided the $\gamma$-photon energy is the same. Therefore, at a laser intensity of $10^{23}\text{W/cm}^2$ the energy distribution of the photo-proton from $^{26}\text{P}$ would be slightly more narrow than the distribution in Fig. 2(c). However, when the laser intensity is enhanced, the number of sizeable sidebands in the case of $^{26}\text{P}$ grows accordingly. This is illustrated in Fig. 4(a). The total cross section amounts to 18 mb.

Besides, a comparison of panels (a) and (b) of Fig. 4 reveals that also the relative orientation of the $\gamma$-ray and laser fields has an impact on the photo-proton energy distribution. When both fields have parallel polarizations [see Fig. 4(a)], the proton can couple much more effectively to the laser field. The reason is that the proton is emitted preferably along the polarization vector of the $\gamma$-photon and, thus, has a large momentum component along this direction. When the laser vector potential points into the same direction, the $\vec{p} \cdot \vec{A}_L$ interaction is strong. This

![Figure 4](image_url)

**Figure 4.** (a) Distributions of partial cross sections $\sigma_n$ for laser-assisted photo-proton emission from $^{26}\text{P}$. The field parameters are $\omega_\gamma = 10\text{MeV}$, $\omega_0 = 2\text{keV}$ and $I = 3.13 \times 10^{23}\text{W/cm}^2$ (i.e., $A_0 = 0.5\text{MeV}$). Both the $\gamma$-photon and the laser field are polarized along the $z$ axis. (b) Same as (a) but the laser field is polarized perpendicularly to the polarization direction of the $\gamma$-photon.
leads to a relatively large number of emitted or absorbed laser photons and, accordingly, to a rather broad energy spectrum of the ejected proton. In contrast, when the polarization vectors of the $\gamma$-photon and the linearly polarized laser field are perpendicular [see Fig. 4(b)], then the proton’s momentum component along the laser field is small, so that it cannot couple strongly to the laser field. The consequence is that the energy sidebands are largely suppressed.

4. Conclusion
We have examined proton emission from halo nuclei via the laser-assisted nuclear photoeffect. In contrast to an earlier study of the same subject, the laser field was assumed to be linearly polarized. An $S$-matrix theory based on the strong-field approximation was presented which resembles the theoretical treatment of laser-assisted photo-ionization of atoms.

The assistance of the laser field was shown to substantially modify the photo-proton energy spectra where emission and absorption of laser photons during the process lead to characteristic energy sidebands. The latter were found to be more pronounced for higher energies of the incident $\gamma$-photon and for laser fields which are linearly polarized along the same axis as the $\gamma$-photon. The total cross section of the process has not been modified for the laser parameters under consideration. While nuclear isotopes with a proton halo have the advantage of low proton separation energies, we point out that similar effects may also be expected for laser-assisted photo-proton emission from ordinary nuclei.

In view of a possible experimental investigation of the laser-assisted nuclear photoeffect in the future, we note that few-MeV $\gamma$-photons can nowadays be produced efficiently via bremsstrahlung of laser-accelerated electrons [34]. Thus, future experiments on laser-assisted photo-nuclear reactions may rely on setups involving two intense laser beams, with one of them being utilized to generate high-energy bremsstrahlung photons. This might evolve into an interesting research area for the nuclear pillar of the Extreme Light Infrastructure [4, 5].

Acknowledgments
A. D. gratefully acknowledges the scholarship support from the International Max Planck Research School for Quantum Dynamics in Physics, Chemistry and Biology (IMPRS-QD).

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Brilliant x-ray beams with intensities up to $10^{18}$ W/cm$^2$ can presently be produced by free-electron lasers (see Young L et al. 2010 Nature 466 56), and a further increase of the attainable x-ray intensity is envisaged. Another potential source of high-intensity x-rays are high-order harmonics emitted from plasma surfaces (see Dromey B et al. 2009 Nature Phys. 5 146).

For the present field parameters, the first argument is relevant since $\alpha \sim 1$–10 whereas $\beta \ll 1$.  

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