Leptogenesis, neutrinoless double beta decay and terrestrial $CP$ violation

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Leptogenesis in left–right symmetric theories is studied. The usual see–saw mechanism is modified by the presence of a left–handed Higgs triplet. A simple connection between the properties of the light left–handed and heavy right–handed neutrinos is found. Predictions of this scenario for neutrinoless double beta decay and terrestrial $CP$ violation in long–baseline experiments are given. These observables can in principle distinguish different realizations of the model.

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1. Introduction

One of the problems waiting to be solved in particle physics and cosmology is the explanation of the baryon asymmetry of the universe. Since Standard Model baryogenesis fails to produce a sufficient baryon asymmetry, other, new physics approaches are being followed. Among them towers out leptogenesis [1] as one of the most popular. Heavy right–handed Majorana neutrinos violate $CP$ and lepton number during their out–of–equilibrium decay, thereby — when sphalerons [2] convert the lepton asymmetry in a baryon asymmetry — fulfilling all of Sakharov’s three conditions [3]. The impressive evidence for non–vanishing neutrino masses opens now the possibility to study this new physics problem on a broader phenomenological basis. Typical models build to explain the neutrino mass and mixing scheme predict also heavy right–handed Majorana neutrinos, mostly due to some see–saw [4] mechanism. It is now a fruitful question to ask if a given

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model for neutrino masses also explains the baryon asymmetry via the leptogenesis mechanism. A number of groups have studied this within their respective approach [5].

As the name already indicates, left–right (LR) symmetric theories represent a natural way to connect the light left–handed with the heavy right–handed neutrino sector. In [7] the relationship of both sectors and the impact on leptogenesis was analyzed. An observable effect of the relation between neutrino oscillation and leptogenesis was then proposed in [8]. The three yet unknown phases in the left–handed neutrino mass matrix govern the magnitude of the effective neutrino mass measured in neutrinoless double beta decay and the size of terrestrial CP violating effects in long–baseline experiments. Many models explain the baryon asymmetry as well as the light mass and mixing scheme. Predictions of other observables are then very helpful to rule out or confirm models. The relationship of terrestrial CP violation and leptogenesis was also analyzed in [9].

The paper is organized as follows: In Section 2 the connection of leptogenesis and neutrino oscillation in left–right symmetric theories is given and the results on the baryon asymmetry are presented. The connection to terrestrial CP violation is made in Section 3 and the conclusions are drawn in Section 4.

2. Neutrino oscillation and leptogenesis in left–right symmetric theories

In LR symmetric theories the see–saw formula reads

\[ m_\nu = m_L - \tilde{m}_D M_R^{-1} \tilde{m}_D^T, \]  

(1)

where \( m_L \) and \( M_R \) are Majorana mass matrices generated by Higgs triplets and \( \tilde{m}_D \) is a Dirac mass matrix. The matrix \( m_\nu \) is further diagonalized by

\[ U_L^T m_\nu U_L = \text{diag}(m_1, m_2, m_3), \]  

(2)

where \( m_i \) are the light neutrino masses. The symmetric matrix \( M_R \) also appears in the Lagrangian

\[ -\mathcal{L}_Y = \bar{l}_i L \frac{\Phi}{v} \tilde{m}_{Dij} N^i_{Rj} + \frac{1}{2} \bar{N}^i_{Rj} M_{Rij} N^i_{Rj} + \text{h.c.} \]  

(3)

with \( l_i L \) the leptonic doublet and \( v \simeq 174 \text{ GeV} \) the vacuum expectation value (vev) of the Higgs doublet \( \Phi \). Diagonalizing \( M_R \) brings us to the physical basis

\[ U_R^T M_R U_R = \text{diag}(M_1, M_2, M_3). \]  

(4)
The asymmetry is caused by the interference of tree level with one–loop corrections for the decays of the lightest Majorana, $N_1 \rightarrow \Phi l^c$ and $N_1 \rightarrow \Phi^{\dagger} l$:

$$\varepsilon = \frac{\Gamma(N_1 \rightarrow \Phi l^c) - \Gamma(N_1 \rightarrow \Phi^{\dagger} l)}{\Gamma(N_1 \rightarrow \Phi l^c) + \Gamma(N_1 \rightarrow \Phi^{\dagger} l)}$$

$$= \frac{1}{8 \pi v^2} \frac{1}{(m_D^2 m_{D})_{11}} \sum_{j=2,3} \text{Im}(m_D^j m_{D})_{1j}^2 f(M_j^2/M_1^2).$$

The function $f$ includes terms from vertex and self–energy contributions:

$$f(x) = \sqrt{x} \left( 1 + \frac{1}{1-x} - (1+x) \ln \left( \frac{1+x}{x} \right) \right) \simeq - \frac{3}{2} \sqrt{x}. \quad (6)$$

The approximation holds for $x \gg 1$.

In our approach, the left–right symmetry [10] plays an important role. It relates the unitary matrices $U_L$ and $U_R$ to each other since the triplet induced Majorana mass matrices in Eq. (1) have the same coupling matrix $f$ in generation space:

$$m_L = f v_L \quad \text{and} \quad M_R = f v_R. \quad (7)$$

The numbers $v_{L,R}$ are the vevs of the left– and right–handed Higgs triplets, whose existence is needed to maintain the left–right symmetry. They receive their vevs at the minimum of the potential, producing at the same time masses for the gauge bosons. In general [10], this results in

$$v_L v_R \simeq \gamma v^2, \quad (8)$$

where the constant $\gamma$ is a model dependent parameter of $O(1)$. Inserting this equation as well as Eq. (7) in (1) yields

$$m_\nu = v_L \left( f - \tilde{m}_D f^{-1} \tilde{m}_D^T \right). \quad (9)$$

If one compares the relative magnitude of the two contributions in Eq. (1), denoting the largest mass in the Dirac matrix with $m$, one finds that

$$\frac{|\tilde{m}_D M_R^{-1} \tilde{m}_D^T|}{|m_L|} \simeq \frac{m^2/v_R}{v_L} \simeq \frac{m^2}{\gamma v^2}. \quad (10)$$

Here, we only used Eq. (8) and assumed that the matrix elements of $f$ and $f^{-1}$ are of the same order of magnitude. It is seen that this ratio is of order one only for the top quark mass, i.e. if one identifies the Dirac mass matrix with the up quark mass matrix.
We finally specify the order of magnitude of $v_{L,R}$. The scale of $m_\nu = v_L f$ is $10^{-2} \ldots 10^{-3}$ eV, which — for not too small $f$ — is only compatible with $v_L v_R \simeq \gamma v^2$ for $v_R \simeq 10^{14} \ldots 10^{15}$ GeV. This means that $v_R$ is close to the grand unification scale and $v_L$ is of the order of the neutrino masses, which is expected since $m_L$ is the dominating contribution to $m_\nu$. In the following, $v_R = 10^{15}$ GeV and $\gamma = 1$ is assumed.

From the decay asymmetry $\varepsilon$ the baryon asymmetry $Y_B$ is obtained by

$$Y_B = c \kappa \frac{\varepsilon}{g^*},$$

(11)

where $c \simeq -0.55$ is the fraction of the lepton asymmetry converted to a baryon asymmetry via sphaleron processes [11], $\kappa$ a suppression factor due to lepton–number violating wash–out processes (see [6] for an improved fit) and $g^* \simeq 110$ the number of massless degrees of freedom at the time of the decay. Experimentally, the preferred range for the asymmetry is [12] $Y_B \simeq (0.1 \ldots 1) \cdot 10^{-10}$.

The strategy goes as follows: In Eq. (9) one inserts the solar solution, i.e. the small angle (SMA), large angle (LMA) or quasi–vacuum (QVO) solution, see e.g. [13]. The light neutrino masses $m_i$ are obtained by assuming the hierarchical scheme. The Dirac mass matrix $\tilde{m}_D$ can be expected to be an up (down) quark or lepton mass matrix, denoted $m_{up}$, $m_{down}$ and $m_{lep}$, respectively. Eq. (9) is then solved for $f = M_R/v_R$ and $M_R$ is diagonalized to obtain the baryon asymmetry via Eqs. (5,11).

Performing a random scan of the allowed oscillation parameters and the three phases, it is found that if $\tilde{m}_D$ is a down quark or lepton mass matrix, $m_1$ should not be too small, i.e. larger than $10^{-6}$ eV. The LMA solution gives in more cases the correct baryon asymmetry and is thus slightly favored over SMA and QVO. If $\tilde{m}_D$ is an up quark mass matrix, fine tuning of the parameters is required. Due to the large hierarchy of the quark and lepton masses, it is sufficient to use a mass matrix which has just the heaviest mass as the (33) entry. Fig. 1 shows $Y_B$ in case of $\tilde{m}_D = m_{lep}$.

If we identify $\tilde{m}_D$ with the down quark or charged lepton mass matrix, then the ratio in Eq. (10) is always much smaller than one, so that the second term in Eq. (9) can be neglected and it follows [8]

$$f \simeq \frac{1}{v_L} m_\nu.$$  

(12)

Therefore, with the help of Eqs. (2,4,7), one arrives at a very simple connection between the left– and right–handed neutrino sectors:

$$U_R = U_L \text{ and } M_i = m_i \frac{v_R}{v_L}.$$  

(13)
Fig. 1. Baryon asymmetry as a function of $s_3$ for $\tilde{m}_D = m_{\text{lep}}$ and all three solar solutions. We chose $3\alpha = 4\beta = 6\delta = \pi$, $\Delta m^2_\odot = 3.2 \cdot 10^{-3}$ eV$^2$ and $\tan^2 \theta_1 = 1$. For the solar solutions, we took $\Delta m^2_\odot = 5 \cdot 10^{-6}$ eV$^2$ and $\tan^2 \theta_1 = 5 \cdot 10^{-4}$ for SMA, $\Delta m^2_\odot = 5 \cdot 10^{-5}$ eV$^2$ and $\tan^2 \theta_1 = 1$ for LMA and $\Delta m^2_\odot = 10^{-8}$ eV$^2$ and $\tan^2 \theta_1 = 1$ for QVO. The smallest mass state is $m_1 = 10^{-5}$ eV for QVO and $m_1 = 10^{-4}$ eV for SMA as well as LMA.

The striking property is that the light neutrino masses are proportional to the heavy ones. Analytical estimates for the baryon asymmetry can now be performed. We work with a convenient parametrisation of $U_L$,

$$U_L = U_{\text{CKM}} \cdot P = U_{\text{CKM}} \text{diag}(1, e^{i\alpha}, e^{i(\beta+\delta)})$$

$$= \begin{pmatrix} c_1c_3 & s_1c_3 & s_3e^{-i\delta} \\ -s_1c_2 - c_1s_2s_3e^{i\delta} & c_1c_2 - s_1s_2s_3e^{i\delta} & s_2c_3 \\ s_1s_2 - c_1c_2s_3e^{i\delta} & -c_1s_2 - s_1c_2s_3e^{i\delta} & c_2c_3 \end{pmatrix} \text{diag}(1, e^{i\alpha}, e^{i(\beta+\delta)}),$$

where $c_i = \cos \theta_i$, $s_i = \sin \theta_i$ and the diagonal matrix $P$ contains the additional two Majorana phases $\alpha$ and $\beta$. Assuming maximal atmospheric and solar mixing, $c_1^2 = c_2^2 = 1/2$, and taking only the leading order in $s_3$, one
finds for the LMA and QVO solutions [8]

\[
Y_B \cdot 10^{10} \lesssim 4.1 \left( \frac{m_\text{GeV}}{\text{eV}} \right)^2 \left\{ \left( s_{2\alpha} + 4 s_3 c_\delta c_{2\alpha} \right) \frac{m_1}{\Delta m^2_\odot} \right. \\
+ 2 \left( s_{2(\beta+\delta)} - 2 s_3 s_{2\beta+\delta} \right) \frac{m_1}{\Delta m^2_A} \right\},
\]

(15)

where \( c_\delta = \cos \delta \), \( s_{2\alpha} = \sin 2\alpha \) and so on. The solar (atmospheric) \( \Delta m^2 \) is denoted \( \Delta m^2_\odot \) (\( \Delta m^2_A \)). It is seen explicitly that \( Y_B \) vanishes if \( CP \) conservation holds, \( i.e. \) if all phases are zero or \( \pi \). The asymmetry is proportional to the square of the heaviest entry in \( \tilde{m}_D \), \( i.e. \) the tau or bottom quark mass. Furthermore, \( Y_B \) is proportional to the lightest neutrino mass eigenstate \( m_1 \), which can be used to set a lower limit on it, it is of the order \( 10^{-7} \) to \( 10^{-8} \) eV.

If \( \tilde{m}_D = m_{up} \) then \( m_\nu \) receives a contribution from the conventional see–saw term \( \tilde{m}_D M_R^{-1} \tilde{m}_D^T \) and the proportionality on \( m_1 \) vanishes, see [7] for details.

### 3. Terrestrial CP violation

The remaining unknowns in this approach are the three \( CP \) violating phases in the mixing matrix \( U_L \) and the size of the smallest mass eigenstate \( m_1 \). Within the parametrisation Eq. (14) the phases \( \alpha \) and \( \beta \) govern the magnitude of neutrinoless double beta decay. The third phase \( \delta \) is responsible for \( CP \) violating effects in oscillation experiments.

The latest SuperKamiokande [14] and first SNO [15] data favor LMA over the other solar solutions. This is good news since leptonic \( CP \) violation in long–baseline experiments can only be measured if nature has chosen LMA. Effects of \( CP \) violation are proportional to the rephasing invariant determinant \( J_{CP} \) [16], which shows up \( e.g. \) in the difference of the \( CP \) conjugated oscillation probabilities

\[
P(\nu_e \to \nu_\mu) - P(\bar{\nu}_e \to \bar{\nu}_\mu) \propto J_{CP} = \frac{1}{8} \sin^2 \theta_1 \sin 2 \theta_2 \sin 2 \theta_3 \cos \theta_3 \sin \delta \\
\leq \frac{1}{4} \sin \theta_3 (1 - \sin^2 \theta_3).
\]

(16)

In addition, the higher \( \Delta m^2_\odot \) is, the higher are the prospects for detecting the \( CP \) violation [17], though the details depend on the experimental facilities.

In the hierarchical mass scheme, LMA also provides the highest Majorana mass for the electron neutrino, which can be measured through neutrinoless double beta decay (0\( \nu \beta \beta \)). It is defined as

\[
\langle m \rangle = \sum_i U_{Lei}^2 m_i
\]

(17)
and due to the complex matrix elements $U_{Lai}$ there is the possibility of cancellation [18] of terms in Eq. (17).

The quantities $\langle m \rangle$ and $J_{CP}$ are observables, which are depending on the $CP$ violating phases which also govern the lepton asymmetry. It is therefore interesting to ask if the parameters that produce a satisfying $Y_B$ also deliver sizable $\langle m \rangle$ and/or $J_{CP}$. To study this, a random scan of the allowed variables of the LMA solution was performed. The highest fraction of parameter sets providing sufficient $Y_B$ occurs for high $m_1$ and a “low” Dirac mass matrix, i.e. $\tilde{m}_D$ should be a lepton (43 %) or down quark (23 %) mass matrix. It is interesting to note that in the most simple realization of LR models $\tilde{m}_D$ is the charged lepton mass matrix. For lower $m_1$ or $\tilde{m}_D = m_{up}$ the fraction of parameters producing a correct asymmetry decreases to less than 5 %. As mentioned, basically no $m_1$ dependence exists for $\tilde{m}_D = m_{up}$. Approximately all the parameter sets providing a correct asymmetry also produce $\langle m \rangle$ bigger than $2 \cdot 10^{-3}$ eV, the lowest limit achievable by the GENIUS project [19]. For $m_1 = 10^{-3}$ eV, about 4 % of the parameter sets give $\langle m \rangle$ bigger than 0.01 eV. Fig. 2 shows the distribution of events in the $\langle m \rangle$–$\sin^2 \theta_3$ plane. The difference for different cases is easily seen.

![Fig. 2. Distribution of events in the $\langle m \rangle$–$\sin^2 \theta_3$ plane for the LMA solution, different $m_1$ and $\tilde{m}_D$.](image-url)
Regarding CP violation, a criterion for observability might be $J_{CP} \geq 10^{-4}$ and $\Delta m^2_{\odot} \geq 10^{-4}$ eV$^2$. Approximately half of the events that give sufficient $Y_B$ also fulfill these constraints. Therefore, again high $m_1$ and $\tilde{m}_D = m_{\text{down}}$ or $m_{\text{lep}}$ are required to expect measurable CP violation. Fig. 3 shows the distribution of $J_{CP}$ against $\sin^2 \theta_3$ and $\Delta m^2_{\odot}$, respectively. Again, the difference is easily seen. The case $\tilde{m}_D = m_{\text{up}}$ favors low $\Delta m^2_{\odot}$.

4. Conclusions

Leptogenesis in left–right symmetric models is studied. A simple formula for $Y_B$ can be derived, expressing the baryon asymmetry in terms of oscillation parameters and CP violating phases. In many cases a sufficient baryon asymmetry is produced and the LMA solution is favored. Many models in this scenario as well as other approaches fulfill these constraints. In search for an additional criterion we therefore apply our model also to $0\nu\beta\beta$ and terrestrial CP violating effects in long–baseline experiments. In order to expect a sizable signal in $0\nu\beta\beta$ and measurable CP violating effects in long–baseline experiments, $m_1$ of order $10^{-3}$ eV is required, and $\tilde{m}_D$ should be a lepton or perhaps a down quark mass matrix. The low energy observables $J_{CP}$ and $\langle m \rangle$ can in principle be used to distinguish these possibilities and could also be used to distinguish other leptogenesis models.
Baryon number and $CP$ violation are necessary conditions for the generation of a baryon asymmetry. Since $Y_B$ gets converted from a lepton asymmetry, lepton number violation is required. Thus, $0\nu\beta\beta$ and terrestrial $CP$ violation provide a possibility to check two of Sakharov’s conditions at low energy. Furthermore, given that in many models the heavy right–handed neutrinos may not be observable at realistic collider energies, $0\nu\beta\beta$ and terrestrial $CP$ violation could be useful to validate leptogenesis.

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