Functional observability and target state estimation in large-scale networks

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Observing the internal states of a network system via measurement and/or estimation is fundamental for the prediction, control, and identification of large-scale complex systems, such as power grids, neuronal networks, and food webs. High dimensionality, however, poses physical and costs constraints on sensor placement, limiting our ability to make a network observable. Noting that often only a relatively small number of state variables are essential for control, intervention, and monitoring purposes in large-scale networks, we propose a graph-based theory of \textit{functional observability} \cite{1}. A system is functionally observable when a targeted subset of state variables can be reconstructed from the available measurements, and our results establish conditions under which this is possible for large-scale networks. Figure 1A provides an illustrative example of a network system which, although not completely observable, is functionally observable with respect to the considered target node. Based on the developed theory, we further design two highly-scalable algorithms to: (i) place a minimal set of sensors to ensure the network functional observability and (ii) design the corresponding functional observer (estimator) with minimum computational cost. Figure 1B shows that the number of sensor nodes required to make a system functionally observable decreases substantially for a smaller number of target nodes in different complex network models and real-world datasets. Our methods are applied to cyberattack detection in power grids and the monitoring of the COVID-19 pandemic, demonstrating that the proposed functional observability approach can achieve accurate estimation with substantially fewer resources.

\textbf{Keywords:} network dynamics \cdot observability \cdot network control \cdot sensor placement

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\begin{figure}[h]
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\includegraphics[width=\textwidth]{figure1.png}
\caption{(A) Structural functional observability analysis of the inference graph of a dynamical system, where nodes represent the state variables and edges represent the interaction patterns between state variables. Sensor and target nodes are highlighted in blue and red, respectively. (B) Minimum sensor placement in complex network models (left) and empirical networks (right).}
\end{figure}