ABSTRACT

In this paper, we introduce a novel algorithm that can dramatically reduce the number of antenna elements needed to accurately predict the direction of arrival (DOA) for multiple input multiple output (MIMO) radar. The new proposed algorithm predicts the received signal of a large antenna setup using reduced number of antenna by using coupled dictionary learning. Hence, this enables the MIMO radar to resolve more paths, which could not be resolved by the fewer antennas. Specifically, we overcome the problem of inaccurate DOA estimation due to a small virtual array setup. For example, we can use dictionary learning to predict 100 virtual array elements using only 25. To evaluate our algorithm, we used multiple signal classification (MUSIC) as a DOA estimation technique to estimate the DOA for non coherent multiple targets. The results show that using the predicted received signal, the proposed algorithm could resolve all the targets in the scene, which could not been resolved using only the received signal from the reduced antenna setup.

Index Terms—Sparse signal processing, dictionary learning, DOA estimation, MIMO radar, MUSIC

1. INTRODUCTION

Super resolution DOA estimation is a vital research area for many applications like radars, sonars and wireless communications. However, DOA estimation algorithms faces a lot of challenges that affect its precision, i.e. the existence of very close targets, and low signal to noise ratio (SNR). This can be solved by increasing aperture size and the number of antennas, which enhances the DOA algorithm resolution [1]. However, this increases the system complexity and computational cost. In [2], the authors suggested a sparse non uniform array with low number of transmit / receive antennas placed random on a large aperture, which may be not suitable for some applications. In contrast, the main contribution of this paper is proposing a novel approach that offers a super resolution DOA estimation, using only a subset of uniform linear array without the need to increase the array aperture size. Since our algorithm predicts the received signal of a filled array with high number of antennas given a received signal of the only a subset of this array on a smaller aperture, offering super resolution DOA estimation. This is applied on MIMO radar, which is known to combine the degrees of freedom of the transmitter and receiver in a collaborative manner using the concept of virtual array. We use MUSIC to estimate the DOA of the targets using the predicted signal, which could successively resolve targets that were originally un-resolvable using the few antenna array elements.

Our signal prediction method is based on data-driven coupled dictionary learning, where the dictionary is defined based on sparse approximation. The sparse approximation is approximating a given signal as linear combination of few basis functions (atoms) and collection of these basis functions called a dictionary [3]. For some signals predefined dictionaries such as wavelet, curvelet and short-time Fourier transforms suitable for sparse approximation. However, learning a dictionary from training data would result better sparse representation [3], [4], [5]. Sparse approximation and dictionary learning has been used for many applications such as signal separation [6], missing data estimation and super resolution imaging [7], [8], [9]. In this paper, we train a model which learns a coupled dictionary pair which has a common sparse representation for the received signals of both high and reduced antenna setup. Afterwards, these learned dictionaries will be used to predict the received signal for high antenna array setup using only a subset of this array in reality.

2. SYSTEM MODEL

2.1. MIMO Radar Signal Model

In this section, we describe a general signal model for the MIMO radar. Consider a co-located MIMO radar system with $M$ transmit antennas, each transmitting a train of $P$ non overlapping pulses, and $N$ receive antennas. The transmit and receiver antennas are placed in uniform linear array (ULA) with $\frac{\lambda}{2}$ spacing between each antenna. We assume that there

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are $K$ targets in the radar scene, each has direction of arrival (DOA) at angle $\theta_k$, with radar cross section (RCS) modeled by $\alpha_{k,p} \in \mathbb{C}$. Moreover, each antenna $m$ transmits narrow-band signal denoted by $s_m(t)$, where all $M$ transmitted signals are assumed to be perfectly orthogonal such that

$$f = \left\{ \begin{array}{ll} 1 & m = q \\ 0 & m \neq q \end{array} \right. \quad (1)$$

where $()^*$ denotes a conjugate operator. Hence, the signal received at target $k$ after transmitting $p$th pulse is given by $\sum_{m=1}^{M} \alpha_m^T(\theta_k) s_m(t-pT)$. Here, $T$ is the pulse repetition interval, and $\alpha_m^T(\theta_k)$ is the steering vector towards target $k$ and defined as $\alpha_m^T(\theta_k) = [1, e^{jkd\sin \theta_k}, \ldots, e^{jkd(M-1)\sin \theta_k}]^T$, where $k = \frac{2\pi}{\lambda}$ and $\lambda$ is the operating wavelength. Let us define $r(t) \in \mathbb{C}^N$ as the received signal, which is given by

$$r(t) = \sum_{k=1}^{K} \sum_{p=1}^{P} \alpha_{k,p} v_r(\theta_k) a_r^T(\theta_k)s(t-pT) + n(t), \quad (2)$$

where $s(t) = [s_1(t), \ldots, s_M(t)]$, and $a_r(\theta_k)$ is the receive steering vector for target $k$. Here, $n(t) \in \mathbb{C}^N$ is Gaussian noise with variance $\sigma^2$. Here, we assume that the target RCS is fixed during pulse interval $T$ and changes independently from each pulse to another, following Swerling model II [10]. Afterwards, at each receive antenna, the received signal is cross correlated with filters matched to the transmitted waveforms, such that

$$y_p = \int_T r(t)s(t-pT)^H dt,$$

$$= \sum_{k=1}^{K} \sum_{p=1}^{P} \alpha_{k,p} v_r(\theta_k) a_r^T(\theta_k) + \int_T n(t)s(t-pT)^H dt.$$

Furthermore, we can define received signal $Y \in \mathbb{C}^{MN \times P}$ as

$$Y = A(\theta)X + N. \quad (3)$$

Here, $Y = [\text{vec}(y_1), \ldots, \text{vec}(y_P)]$, hence $A(\theta)$ contains the virtual array steering vector $v(\theta_k) = \alpha_k(\theta_k) \otimes a_r(\theta_k)$, such that $A(\theta) = \{v(\theta_1), \ldots, v(\theta_K)\}$. The RCS of $K$ targets is defined in $K \times P$ matrix $X$, where $X = [x_1, \ldots, x_P]$ and $x_p = [\alpha_{1,p}, \ldots, \alpha_{K,p}]^T$. To estimate the DOA, the covariance matrix of the received signal is used in (2,3) which is given by

$$R = \mathbb{E}[YY^H] = A(\theta) \mathbb{E}[XX^H]A^H(\theta) + \sigma^2 I. \quad (4)$$

In the next section, we discuss the proposed signal prediction method in detail.

2.2. Coupled Dictionary learning for MIMO radar

In this section, we discuss the proposed dictionary learning based signal prediction model for the MIMO radar setup. We consider two antenna setups as high and reduced based on the number of antennas. Here, reduced antenna setup is subset of the high antenna setup. For reduced and high antenna setups, the number of transmit and receive antennas are given by $M_l$, $N_l$, $M_h$, and $N_h$, respectively. Now, suppose that, the received signal (in (3)) for reduced and high antenna setups are given as $Y_l$ and $Y_h$, respectively. Note that, the received signals ($Y_l$, $Y_h$) are complex matrices. Therefore, in dictionary learning and signal prediction we treat real and imaginary components separately. We assume that there exist a coupled dictionary pair $(D_l, D_h)$ which has the same sparse representation for both $Y_l$ and $Y_h$. The dimension of each dictionary is $M_lN_l \times L$ and $M_hN_h \times L$, respectively. Thus, the received data can be decomposed as

$$Y_l = D_l W + N_l,$$

$$Y_h = D_h W + N_h. \quad (5)$$

Here, $W$ is $L \times P$ the sparse coefficient matrix which is column-wise sparse and $i$-th column of the matrix $W$ denoted by $w_i$. The $N_l$, $N_h$ are the noise matrices, modeled as zero mean random Gaussian noise with covariance matrix of $\sigma^2 I$. Based on (5), the coupled dictionary learning problem can be formulated as

$$\{ \hat{W}, \hat{D}_l, \hat{D}_h \} = \arg \min_{W,D_l,D_h} \left\| Y_l - D_l W \right\|_F^2 \quad (6)$$

$$\text{s.t.} \quad \|w_i\|_0 \leq s, \forall i,$$

Here, $s$ is the sparsity constraint. The optimization problem in (6) is non-convex and thus challenging. By relaxing the $l_0$-norm with the $l_1$-norm and alternative minimization, this problem can be solved numerically [3], [11]. Let $\tilde{Y} = [Y_l, Y_h]$ and $D = [D_l, D_h]^T$, then (6) can be written as

$$\{ \hat{W}, \hat{D} \} = \arg \min_{W,D} \left\| \tilde{Y} - DW \right\|_F^2 \quad (7)$$

$$\text{s.t.} \quad \|w_i\|_1 \leq s, \forall i.$$

This problem in (7) is referred as dictionary learning [12]. In this paper, we choose the online dictionary learning (ODL) [11] approach to solve (7) due to its computational efficiency and accuracy.

2.2.1. Training

In the training stage, we used $P$ number of pulses to generate the training data ($\hat{Y}_l$, $\hat{Y}_h$) from each setup. Then, the dictionary learning is performed to learn the dictionary pair $(D_l, D_h)$. As a pre-processing step we normalize $\hat{Y}_l$ and $\hat{Y}_h$ column-wise with respect to $\hat{Y}_l$ to bring the data to a common scale. This improves the prediction accuracy. Also, we subtract the column-wise mean from the signal, to make the signal to have zero mean column-wise. This is done to avoid the dictionary learning process to be ill-conditioned [12].
2.2.2. Prediction

After learning the $D_l$ and $D_h$ in training stage, for a given received signal of the reduced antenna setup $\hat{Y}_l$, the received signal of higher antenna configuration $\hat{Y}_h$ is predicted using the algorithm\[1\] In algorithm\[1\] least absolute shrinkage and selection operator (LASSO) approach \[13\] is used to calculate the sparse coefficients. Here, $\lambda$ is a regularization parameter. Its value is selected from a predefined uniform range which provides the lowest reconstruction error $\left(\|\hat{Y}_l - D_lW\|_2^2\right)$. 

Algorithm 1: Signal prediction using coupled dictionary learning

**Input:**
- Received signal of reduced antenna setup $\hat{Y}_l$
- Dictionaries from training phase $D_l$, $D_h$

**Initialization:**
- Data pre-processing
  - $\phi = \text{column-wise mean of } \hat{Y}_l$
  - $\Phi_l = \text{reshape } \phi \text{ to have same dimension of } \hat{Y}_l$
  - $\hat{Y}_l = \hat{Y}_l - \Phi_l$
  - $\hat{Y}_l = \text{column-wise normalization of } \hat{Y}_l$
- Calculate the sparse coefficient matrix using $\hat{Y}_l$
  - $\{\hat{W}\} = \arg\min_W \|\hat{Y}_l - D_lW\|_2^2 + \lambda\|W\|_1$.

**Initial prediction:** $\hat{Y}_h = D_h\hat{W}$.

**repeat**
- Joint sparse coefficient update:
  - $D = \begin{bmatrix} D_l \\ D_h \end{bmatrix}$, $\hat{Y} = \begin{bmatrix} \hat{Y}_l \\ \hat{Y}_h \end{bmatrix}$
  - $\{\hat{W}\} = \arg\min_W \|\hat{Y} - DW\|_2^2 + \lambda\|W\|_1$.
- Prediction: $\hat{Y}_h = D_h\hat{W}$.
  - until convergence
  - $\hat{Y}_h = \text{column-wise de-normalization of } \hat{Y}_h$ w.r.t $\hat{Y}_l$.
  - $\Phi_h = \text{reshape } \phi \text{ to have same dimension of } \hat{Y}_h$
  - $\hat{Y}_h = \hat{Y}_h + \Phi_h$.

**return** Predicted signal $= \hat{Y}_h$.

2.3. DOA Estimation using MUSIC

In order to evaluate our prediction algorithm, we applied MUSIC on the received signal in \[1\], to estimate the DOA for all the targets. The resolution of MUSIC depends on the number of array elements. Thus, with reduced antenna configuration, it might not be able to resolve all the paths reflected from the targets. Hence, we use MUSIC to evaluate the ability of the predicted receive signal to resolve targets, which could not be resolved in reduced antenna setup. MUSIC depends on the orthogonality of the target eigenvector to the noise eigenvector \[14\] as the covariance matrix in \[1\], can be written as

$$\mathbf{R} = \mathbf{U}_x\Lambda_x\mathbf{U}_x^H + \mathbf{U}_n\Lambda_n\mathbf{U}_n^H,$$

where $\mathbf{U}_n$ is the noise eigenvector, and $\Lambda_n = \sigma^2\mathbf{I}$. Hence, the expression of the MUSIC spectrum is given by $P(\theta) = (\mathbf{A}^H(\theta)\mathbf{U}_n\mathbf{U}_n^H\mathbf{A}(\theta))^{-1}$. 

3. SIMULATION SETUP AND RESULTS

We perform numerical simulations to evaluate the performance of the signal prediction and DOA estimation. To evaluate the performance of the prediction, the mean square error (MSE) is used with $\text{MSE} = \frac{1}{M_hN_hNP} \|\hat{Y}_h - \tilde{Y}_h\|_2^2$. Here, $\tilde{Y}_h$ is the actual received signal of higher antenna configuration. In our simulations, the size of the dictionary ($L$) and $\lambda$ are set to be 512 and 0.01, respectively. In the training phase (in \[2.2.1\]) we consider number of iterations as 300. Also, we consider that $N_l = M_l$ and $M_h = N_h$.

Fig. 1: MSE of the prediction for two and four targets

3.1. Effect of number of antennas to the prediction

In this simulation, we aim at investigating the effect of the number of antennas of the antenna array to the signal prediction for fixed number of targets. For this simulation, we consider two and four target scenarios. In the training and testing phase, we consider that the angles of the targets are randomly distributed in the range 0 to 90 degree. Here, we increase $M_l$ from 3 to 12 with the step size of 1 and $M_h$ is set to twice $M_l$. During the training phase, a dictionary pair corresponding to each antenna configuration is learned. In the prediction stage, the received signal corresponding to the higher antenna setup is predicted using the received signal of the reduced antenna setup using the algorithm\[1\]. In the training phase $P$ is set as 45000. In this simulation our aim is to find the accuracy of the prediction, therefore for a single trial in testing phase $P$ is set as 5000. Here, to evaluate the effect of data normalization, we perform the simulation with and without data normalization. The MSE of the prediction for two and four targets averaged on 100 trials are shown in Fig. 1. The data normalization significantly improves the prediction error. This is due to the fact that normalization brings the data to a common scale. Based on the MSE, it can be seen that the MSE of the prediction...
decreases as the number of antenna elements increases. Next, we evaluate the DOA estimation using the predicted signal.

3.2. DOA estimation using predicted data

For this simulation, we consider two and four targets scenarios. In the two target scenario, we consider two targets at angles 20 and 25. Here, \( N_l \) and \( N_h \) are set to be 5 and 10, respectively. Then the corresponding virtual array (VA) sizes are 25 and 100, respectively. In the four target scenario, we consider targets at angles 20 to 35 with step of 5. For this case, \( N_l \) and \( N_h \) are set to be 10 and 16, respectively. Then, the corresponding virtual array sizes are 100 and 256, respectively. Here, for all simulations we consider \( \sigma^2 \) as 1. Estimation of DOA using less number snapshots is important in practical applications. Therefore, we set \( P=100 \). The DOA estimation for two and four targets for a single trial are shown in Fig. 2 and 3, respectively. Based on the results, it can be seen that the reduced antenna setup is not able to identify the correct number of targets. However, it can be seen that the predicted signal is able to identify correct number of targets. This result shows that with the aid of dictionary learning based signal prediction, it is possible to use lower number of antenna elements to detect more targets.

In Fig. 4, we explore the performance of the DOA estimation using independent 200 trials for two targets. Based on the DOA estimation, it can be seen that the reduced antenna setup is able to identify only one target. However, it can be seen that predicted signal is able to identify two targets most of the time.

4. CONCLUSION

In this work, we propose a coupled dictionary learning framework for MIMO radar data prediction. The key idea of this paper is to learn a coupled dictionary pair, which has the same sparse representation for the received signals of both reduced and high antenna setup. In general, higher number of antennas improves the DOA estimation of targets. However, it increases complexity and computational cost. Thus, by utilizing the learned dictionaries, our algorithm can predict the received signal of the high antenna array setup given received signal of only a subset of this array. Hence, we showed that DOA estimation is improved while utilizing low number of antennas resolving very close targets, while using few antennas.
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