A quantum black hole universe

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Abstract

Using the result obtained in a previous paper, in which I found an upper limit on the region of particle creation in the vicinity of the event horizon of a Schwarzschild black hole, and by assuming that all the created energy will be absorbed by the black hole, a natural power law for the growth of the event horizon is deduced. This result may explain the existence of galactic black holes with very large masses. Application of this result on cosmological scale shows that if we start with a Planck-sized black hole then the natural growth of such a black hole will produce one with a density equals the present critical density of the universe. Such a black hole universe will be in the state of eternal inflation.
I. INTRODUCTION

In a previous paper [1] I considered a heuristic derivation for energy creation near the event horizon of a Schwarzschild black hole. The derivation was very general and was based on arguments from the Heisenberg uncertainty principle and the time dilation caused by the gravitational field of the black hole. The result showed that there will be an upper limit for the region in the vicinity of the event horizon within which such energy creation may take place, this was given by

\[ R < \frac{4}{3} R_s \]  

where \( R_s = 2M \) is the Schwarzschild radius.

Obviously such a proposal has nothing to do with the Hawking approach for particle creation by black holes [2]. However this kind of argument can be related to the Casimir effect and the possibility for particle creation by black holes through such mechanism.

According to the general theory of relativity the orbits of massive particles near the event horizon of a Schwarzschild black hole are unstable below the limit of \( \sqrt{3}R_s \) (see for example [3]), therefore all created particles will eventually fall into the singularity through the event horizon. The fate of all photons or any other massless particle will be the same. Therefore, we can confidently assume that all created energy within the region \( R_s < R < \frac{4}{3}R_s \) will be absorbed by the black hole. This may explain why we have no observational verification for
the Hawking radiation, despite long years of monitoring.

In this paper I am going to introduce the Casimir effect as a possible mechanism to create particles in the vicinity of the event horizon. Such phenomena has been studied by many authors \cite{4} and was extensively analyzed. In the next section I will elaborate how the Casimir system is naturally constructed and will deduce a power law for the growth of the surface area. In sec. (III) I show how a system of concentric Cauchy surfaces can be constructed by the presence of the event horizon and the barriers at distances with base of $\frac{4}{3}$. In Sec. (IV) I will try to model an inflating black hole universe based on results obtained from the finite temperature correction to the vacuum energy in an Einstein universe, where it was found that the non-zero vacuum energy renders the universe to have a non-singular start.

II. PARTICLE CREATION VERSUS CASIMIR EFFECT

In flat spacetime Casimir \cite{5} found that the vacuum fluctuations of the electromagnetic field would give rise to an attractive force between two parallel conducting flat plates, a negative energy density inversely proportional to the fourth power of the distance between the two plates is created as soon as such plates are introduced in the vacuum. This was called the Casimir effect. Application of this effect in closed spacetimes (e.g. the Einstein universe) has shown that it lead to a positive energy density (for example see ref.\cite{6}, \cite{7}). Further consideration of the problem at finite temperatures resulted in calculating the finite temperature corrections which was shown to be an important driver for the thermal development of the universe when considered as a source for the Einstein field equations\cite{8}, \cite{9}.

Following Nugayev \cite{10}, we may exchange the non-rotating black hole with two spherical conductors one just outside the event horizon but very near to it, and the other to be consider just below the upper regional limit of $\frac{4}{3}R_s$. These two surfaces will constitute concentric shells, analogous to the parallel conducting plates. Any amount of energy created through the Casimir mechanism between the two shells is assumed to be added to the total energy of the black hole, hence extending the event horizon by an amount that has to be controlled by the conditions defined for the system. This assumption seems acceptable in the light of the finding of Berezin et al. \cite{11} that particles are created in pairs of positive energy by the
black hole, where one is emitted to infinity and the other falls on the black hole causing a change of the inner structure. Then, we immediately deduce that the new event horizon will have a radius of \( \frac{4}{3} R_s \), and so the growth of the event horizon will go on. This means that

\[
R_n = \left( \frac{4}{3} \right)^n R_s, \tag{2}
\]

where \( n = 1, 2, 3, ... \).

The argument we place for considering discrete eigenvalues for the radius of the event horizon is simple and goes as follows: The first Casimir system which is composed of the event horizon and the barrier surface at \( R_1 = \frac{4}{3} R_s \) will create an amount of positive energy (the Casimir energy) once formed will be added to the total energy of the black hole by the argument of instability of orbits within the specified region. Therefore, the event horizon will be extended to a new position and the second surface of the new Casimir system (the new position of the barrier) will then be at \( R_2 = \frac{4}{3} R_1 \) and so it goes.

According to the above mechanism, the surface area of the event horizon of the black holes will grow as

\[
A_n = \left( \frac{4}{3} \right)^{2n} A_s. \tag{3}
\]

This means that the area of the event horizon is quantized. The quantization law here is much different from the law deduced by Bekenstein and Mukhanov \[12\] according to which the spectrum of the surface area of the event horizon was uniformly spaced. However, using the loop representation of quantum gravity, Barreira et.al. \[13\] have shown that the Bekenstein-Mukhanov area quantization spectrum is unrecoverable, consequently they deduce that the Bekenstein-Mukhanov result is likely to be an artefact of the ansatz used rather than a robust result.

From (2) it is clear that the expansion of the black hole will be logarithmic (i.e. inflationary). The number of folding is given by

\[
n = 8 \log \left( \frac{R_n}{R_0} \right). \tag{4}\]
III. A CONSTANT-TIME HYPERSURFACE STRUCTURE

Using the result in (2) above, we can construct a hypothetical concentric Cauchy surface structure centered at the black hole singularity. This structure is characterized by the eigenvalues of (3) and a set of time-like Killing vectors normal to the surfaces. The transition from one surface to another is associated with translation in time, and consequently generation of energy. Therefore, each surface will represent an energy level characterizing a black hole of the corresponding mass. The relative temporal separation between these surfaces is given by the basic relation

$$
\frac{t(x_1)}{t(x_2)} = \left[ \frac{g_{00}(x_1)}{g_{00}(x_2)} \right]^{-1/2}.
$$

(5)

It is clear that the temporal separations between surfaces situated near to $R_s$ are larger than those far away. If $x_2$ is a point at the asymptotically-flat region where $t(x_2) \equiv t_\infty$, then we can write

$$
t(x_n) = t_\infty [g_{00}(x_n)]^{-1/2} = t_\infty \left[ 1 - \left( \frac{3}{4} \right)^n \right]^{-1/2}.
$$

(6)

This structure represents an infinite set of hypothetical Cauchy concentric spherical shells surrounding the black hole prior to the development of the black hole’s event horizon. However, if the positive energy created by the black hole within the specified region is to be added to the black hole’s total energy, then the mass development will take the following form

$$
M_n < \left( \frac{4}{3} \right)^n M_0,
$$

where $M_0$ is the initial mass of the black hole.

Energy levels of the above structure have the following separations

$$
E_{n+1} - E_n < \frac{1}{3} \left( \frac{4}{3} \right)^n E_0,
$$

(8)

where $E_0$ is the total initial mass-energy of the black hole.
From (2) and (7) we find that the energy density of the system will develop according to the inequality

$$\rho_n > \left( \frac{3}{4} \right)^{2n} \rho_0,$$

(9)

where $\rho_0$ is the initial density.

A system developing according to (9) would resemble an Einstein static universe where the matter content of the universe is directly associated with the radius of the spacial section. This can be seen once we relate (2) and (9) from which we get

$$\frac{\rho_n}{\rho_0} \sim \frac{R_0^2}{R^2},$$

(10)

where we have used the symbol $R_0$ instead of $R_s$ as trivial substitution. This result motivates us to utilize the results obtained from calculations performed for the Einstein universe as starting parameters for an inflating black hole model for the universe.

IV. COSMOLOGICAL APPLICATIONS

If the universe was born as a singularity, then it is unknown how it has crossed its own event horizon. Although quantum effects may dissipate the creation singularity, the presently available calculations, which incorporate quantum fields into the classical curvature, do not indicate the possibility of a universe born with crossed event horizon.

The available calculations [7], [8], [9] indicates that the universe was born as a finite-sized patch with a size less than the Schwarzschild radius. This implies that the universe may have been born as a black hole and still is. This idea is not new, and there are a number of investigations that support it; for example, it was already shown long ago by Oppenheimer and Snyder [14] that the interior of the Schwarzschild solution could be described by a Friedmann universe. Moreover it was shown by Pathria [15] that our present universe may be described as an internal Schwarzschild solution if it has the critical energy density. More recent investigations [16] based on the assumption of the existence of a limiting curvature have shown that the inside of a Schwarzschild black hole can be attached to a de Sitter universe at some space-like junction which is taken to represent a short transition layer. Other scenarios in which the universe emerges from the interior of a black hole were also
proposed (see refs. [17]-[23]). On the other hand a universe that has a critical density can easily be shown to have a Schwarzschild radius of a black hole with equivalent total mass.

We will now utilize the results of the previous section to construct a model for the whole universe. The model adopts the results of previous calculations, [7] and [9], of the back-reaction of the finite temperature corrections to the vacuum energy density of the photon field in an Einstein universe. Although the Einstein universe is static, the conformal relation with the closed Robertson-Walker universe [24] allow us to consider the results as being of practical interest. In fact, the discrete spectrum provided here by the inflating black hole model can be considered as representing successions of different instantaneous states of the Einstein static universe. The results of the back-reaction calculations have shown that the thermal development of the universe covers two different regimes; the Casimir regime, which extends over a very small range of the radius but huge rise of temperature from zero to a maximum of $1.44 \times 10^{32} K$ at a radius of $5.5 \times 10^{-34} cm$. At this maximum temperature a phase transition takes place, and the system crosses-over to the Planck regime, where photons get emitted and absorbed freely exhibiting pure black-body spectrum. At this point one can identify the primordial universe with an initial energy density $\rho_i$ given by

$$\rho_i = \alpha T^4$$

$$= 3.25 \times 10^{14} \text{erg/cm}^3.$$ \hspace{1cm} (11)

However, the same calculations [9] showed that the Einstein universe exhibits the presently measured microwave background temperature of $2.73 K$ at a radius of $1.83 \times 10^{30} cm$. Consistency require us to adopt this value for the radius of the present universe. Therefore, according to (10) we get

$$\rho_n = 2.92 \times 10^{-13} \text{erg/cm}^3.$$ \hspace{1cm} (12)

This result is very close to the radiation density in the present universe, calculated in reference to the cosmic microwave background.

But one may argue that the estimated radius of the present universe (Hubble length) is $1.38 \times 10^{28} cm$ and not $1.83 \times 10^{30} cm$. Therefore using (10) again we get

$$\rho_{now}^* \approx 5.67 \times 10^{-30} \text{gm.cm}^{-3}.$$ \hspace{1cm} (13)
a figure which is very close to the critical matter density which defines a flat universe.

The above results are remarkable indeed and certainly implies some sort of self-consistency on the side of the assumptions made in this work.

V. DISCUSSION AND CONCLUSIONS

The simple approach followed in this paper led us to a new quantization law for the area of the event horizon, and consequently into a new area spectrum. The main features of the new spectrum is that it is logarithmic and macroscopic, in contrast to the Bekenstein-Mukhanov spectrum [12], which was linear and microscopic (i.e. Planck dimensional). In fact the Bekenstein-Mukhanov spectrum cannot be verified observationaly because in practice the spectrum will look continuous for macroscopic black holes; no fine structure details are measurable.

A logarithmic inflation arises naturally in our model as a result of the Casimir system assumption. Normally such a model will benefit from all the privileges of inflationary models over the standard big bang model, less their conceptual problems. This is indeed the case since it was recently shown by Easson and Brandenberger [17] that a universe born from the interior of a black hole will not posses many of the problems of the standard big bang model. In particular the horizon problem, the flatness problem and the problem of formation of structures are solved naturally. This may well be the case for a universe formed of the interior of an inflating black hole. Perhaps this is the most important result that need to be analyzed further to see if one can draw some observational consequences.

Our assumptions in this paper are strongly supported by the results we obtained for the matter and radiation densities in the present universe. One can see clearly that starting with a Planck-dimensional black hole universe, the mechanism of the quantum development of such a hypothetical universe leads to a universe having the present critical matter density, a point which is supported by the recent observational investigations of the cosmic microwave background radiation (see [26] and [27]) . Further analysis and development of this approach by investigating a Kerr or Reissner-Nördstrom black holes will be interesting, where one may expect the emergence of a different scheme for area quantization.

One more feature that we remark in the present work is that our model exhibits a discrete redshift for a time developing version. I mean that the relation in (2) if used in the context
of Hubble law for cosmic recession would imply that velocities of the galactic clusters will appear to have discrete redshift. Indeed the discrete redshift problem stands today as one observational problem that is awaiting a solution [28].

It might be of interest to point that Santilli [29] presented nine theorems which marks inconsistencies in the general relativity theory (GRT). The main point of interest in his contribution is his remark that GRT is non-canonical at the classical level and is non-unitary at the operator level. This, it seems, is the main reason behind the explicit incompatibility of GRT with quantum field theory (QFT). The need for a consistent approach may require reformulating both GRT and QFT on some new common basis, and using may be a new approach. The isogravitation suggested by Santilli may stand to be a possible alternative. However, a more general approach to quantum gravity may need to consider quantizing the gravitational field via an alternative to the standard canonical quantization scheme, possibly through a more profound mathematical approach which lead to comprehensive discreteness and more explicit coherence of the physical world.

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