Factored Adaptation for Non-Stationary Reinforcement Learning

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Abstract

Dealing with non-stationarity in environments (i.e., transition dynamics) and objectives (i.e., reward functions) is a challenging problem that is crucial in real-world applications of reinforcement learning (RL). Most existing approaches only focus on families of stationary MDPs, in which the non-stationarity is episodic, i.e., the change is only possible across episodes. The few works that do consider non-stationarity without a specific boundary, i.e., also allow for changes within an episode, model the changes monolithically in a single shared embedding vector. In this paper, we propose Factored Adaptation for Non-Stationary RL (FANS-RL), a factored adaption approach that explicitly learns the individual latent change factors affecting the transition dynamics and reward functions. FANS-RL learns jointly the structure of a factored MDP and a factored representation of the time-varying change factors, as well as the specific state components that they affect, via a factored non-stationary variational autoencoder. Through this general framework, we can consider general non-stationary scenarios with different changing function types and changing frequency. Experimental results demonstrate that FANS-RL outperforms existing approaches in terms of rewards, compactness of the latent state representation and robustness to varying degrees of non-stationarity.

1. Introduction

Reinforcement learning (RL) (Sutton & Barto, 2018) has achieved impressive performances in various tasks, including playing video games (Mnih et al., 2015), robotics (Levine et al., 2016), healthcare (Gottesman et al., 2019), and recommendation systems (Zou et al., 2019). In standard RL settings, the environments and objectives are assumed to be stationary over the lifetime of agents. However, in the real-world deployment, the stationarity assumption hardly holds due to the changes in dynamics (i.e., state transitions) or objectives (i.e., reward functions) both across different episodes and within each episode. Imagine a service robot aiming at picking and placing objects. The ground friction might change continuously within an episode due to the usage or abruptly at discrete timepoints when moving across different surfaces. The objectives might also vary for different objects with different shapes or weights. In this setting, an RL algorithm should therefore learn a policy that can deal with all these sources of non-stationarity.

Learning a good and stable policy under non-stationary environments is a long-standing challenge in RL (Dulac-Arnold et al., 2021). Recently, several works adapted Meta-RL methods to learn sequences of non-stationary tasks (Al-Shedivat et al., 2018; Poiani et al., 2021; Zintgraf et al., 2021). However, the continuous adaptation of MAML (Finn et al., 2017) by Al-Shedivat et al. (2018) does not explicitly model temporal changing components, while TRIO (Poiani et al., 2021) and VariBAD (Zintgraf et al., 2021) need to meta-train the model on a set of non-stationary tasks, which might not be accessible in real-world applications. Using a different approach, LILAC (Xie et al., 2021) and ZeUS (Sodhani et al., 2021) leverage latent variable models to directly model the change factors in the environment in a shared embedding space. To model non-stationarity, these works consider families of MDPs indexed by a latent parameter, which makes it non-trivial to model continuously varying environments, e.g., changes within an episode.

In this paper, we propose Factored Adaptation for Non-Stationary RL (FANS-RL), a framework that learns a factored representation to adapt to temporal changes, including discrete and continuous changes, and that can be combined with standard RL algorithms. Instead of considering each state, action and change vector monolithically, we learn a factored representation that can exploit the inherent structure between the components of the vectors. We formalize our setting as a Factored Non-stationary MDP (FN-MDP), which combines a Factored-MDP (Boutilier et al., 2000; Kearns & Koller, 1999; Osband & Van Roy, 2014) with latent change factors that evolve in time following a Markov process. FN-MDP can factorize the state transition and reward function, since they only rely on a subset of states,
actions and latent change factors, as well as the latent change factors dynamics. As opposed to other approaches for non-stationarity, e.g., the Dynamic Parameter MDPs proposed by Xie et al. (2021), FN-MDPs do not model a family of MDPs, but instead naturally include the dynamics of latent change factors across time analogously to the dynamics of the state components.

We build upon the AdaRL framework (Huang et al., 2022), with which we propose a general framework that can be applied to transition function for each dimension \( k \). The executed action and \( s \) are the only parameters that depend on the domain \( k \). The change factor \( \theta^k \in \mathbb{R}^p \) is the only parameter that depends on the domain \( k \) in Eq. 1 and it encodes any change across domains in the dynamics. The binary mask \( c^k \in \{0, 1\}^p \) represents which of the \( \theta^k \) components influence the \( s, t \). Finally, \( c^k \) is an i.i.d. random noise. Similarly, the reward function is modeled as:

\[
    r_t = h(e^{ar} \odot s_{t-1}, e^{ar} \odot a_{t-1}, \theta^k, c^k_i) \tag{2}
\]

where \( e^{ar} \in \{0, 1\}^d \), \( e^{ar} \in \{0, 1\}^m \), and \( e^{ar} \) is an i.i.d. random noise. The change factor \( \theta^k \in \mathbb{R}^p \) is the only parameter that depends on the domain \( k \) in Eq. 2 and it encodes any change in the reward function. In this simplified setting, the binary masks \( c^k \) can be seen as indicators of edges in a Dynamic Bayesian Network (DBN). Under Markov and faithfulness assumptions, i.e., assuming the conditional independences in the data and d-separations in the true underlying graph coincide, the edges in the graph can be uniquely identified. This means one can learn the true causal graph representing jointly all of the environments, even if the change parameters are latent.

In the general AdaRL framework, the representation is learned via a combination of a state prediction network (to estimate the various \( f_i \)) and a reward prediction network (to estimate \( h \)). All binary masks \( c^k \) and change factors \( \theta_k \) are learnable parameters. All change factors \( \theta_k \) are assumed to be constant in each domain \( k \). If the inputs are pixels, another encoder is added to infer the symbolic states, forming a Multi-model Structured Sequential Variational Auto-Encoder (MiSS-VAE).

In general, not all of the dimensions of the learned state and change factor vectors are useful in policy learning. Huang et al. (2022) show that the only dimensions of the state and change factors that are useful for policy learning are those that affect eventually the reward. Interpreting the learned representation as a DBN, one can describe compact representations as having a directed path to a reward:

\[
    s_{i,t} \in s_{min} \iff s_{i,t} \rightarrow \ldots \rightarrow r_{t+\tau} \text{ for } \tau \geq 1
\]

\[
    \theta_i \in \theta_{min} \iff \theta_i \rightarrow \ldots \rightarrow r_{t+\tau} \text{ for } \tau \geq 1
\]

The optimal policy across domains is then learned using these compact representations \( a_t = \pi^* (s_{t, min}, \theta^k_{min}) \).

### 3. Factored Non-stationary MDPs

An MDP is incapable of capturing non-stationarity, where the transition dynamics and reward functions are changing due to an exogenous process. Thus, we propose Factored Adaptation for Non-Stationary Reinforcement Learning (FANS-RL), a general non-stationary RL approach that fully captures the non-stationarity of dynamics and reward functions, allowing for continuous and discrete changing functions, both within-episode and across-episode. Our main contributions can be summarized as:

- **We formalize a unified framework that can handle different non-stationary settings, including discrete and continuous changes, both within and across episodes.**
- **We introduce Factored Adaptation for Non-Stationary RL (FANS-RL), a general non-stationary RL approach that learns a factored representation for adapting to changes in dynamics and reward functions.**
- **We evaluate FANS-RL on simulated benchmarks for continuous control and robotic manipulation tasks and show it outperforms the state of the art on rewards, compactness of the latent space representation and robustness to varying degrees of non-stationarity.**

### 2. Background

Our work extends the factored representation for fast policy adaptation across domains introduced in AdaRL (Huang et al., 2022), which we summarize here. While Huang et al. (2022) propose a general framework that can be applied to both MDPs and POMDPs, in this work we focus on MDPs, so we only present the simplified version of AdaRL for MDPs. The simplified AdaRL setting considers \( n \) source domains and \( n' \) target domains. The state at time \( t \) is represented as \( s_t = (s_{1,t}, \ldots, s_{n,t})^T \in \mathbb{R}^d \), while \( a_t \in \mathcal{A}^m \) is the executed action and \( r_t \in \mathcal{R} \) is the reward signal. The generative process of the environment in the \( k \)-th domain with \( k = 1, \ldots, n + n' \) can be described in terms of the transition function for each dimension \( i = 1, \ldots, d \) of \( s_t \) as:

\[
    s_{i,t} = f_i (c^{as}_{i} \odot s_{i-1}, c^{as}_{i} \odot a_{i-1}, c^{ars}_{i} \odot \theta^k_i, c^k_{i}, c^k_{i,t}) \tag{1}
\]
Non-stationary Markov Decision Processes (FN-MDPs), an augmented form of a factored MDP (Boutilier et al., 2000; Kearns & Koller, 1999; Osband & Van Roy, 2014) with latent change factors that evolve over time following a Markov process. Since the change factors are latent, FN-MDPs are partially observed, but if we fix the change factors, we get an MDP, similarly to contextual MDP (Hallak et al., 2015).

**Definition 1 (FN-MDP).** A Factored Non-stationary Markov Decision Process (FN-MDP) is defined by tuple $\langle S, A, \Theta, \Theta^r, \gamma, G, \mathbb{P}_s, \mathbb{P}_\theta, \mathbb{P}_\theta^r \rangle$, where $S$ is the state space, $A$ is the action space, $\Theta$ is space of the change factors for transition dynamics, $\Theta^r$ is the space of the change factors for the reward and $\gamma$ is the discount factor.

We assume $G$ is a Dynamic Bayesian Network over $\{s_{1,t}, \ldots, s_{d,t}, a_{1,t}, \ldots, a_{m,t}, r_t, \theta^s_{1,t}, \ldots, \theta^s_{p,t}, \theta^r_{1,t}, \ldots, \theta^r_{q,t}\}$. We define the factored state transition distribution $\mathbb{P}_s$ as:

$$
\mathbb{P}_s(s_{t+1} | s_t, a_{t-1}, \theta^s_t) = \prod_{i=1}^{d} \mathbb{P}_s(s_{i,t} | pa(s_{i,t}))
$$

where $pa(s_{i,t})$ denotes the causal parents of $s_{i,t}$ in $G$, which are a subset of the dimensions of $s_{t-1}$, $a_{t-1}$ and $\theta^s_t$. We also assume a given initial state distribution $\mathbb{P}_s(s_0)$. Similarly, we can define the reward function $\mathbb{R}$ as a function of only the parents of $r_t$ in $G$, i.e., $\mathbb{R}(s_t, a_t, \theta^r_t) = \mathbb{R}(pa(r_t))$, where $pa(r_t)$ are a subset of dimensions of $s_t$, $a_t$, and $\theta^r_t$. We define the factored latent change factors transition distributions $\mathbb{P}_\theta^s$ and $\mathbb{P}_\theta^r$ as:

$$
\mathbb{P}_\theta^s(\theta^s_t | \theta^s_{t-1}) = \prod_{j=1}^{p} \mathbb{P}_\theta^s(\theta^s_{j,t} | pa(\theta^s_{j,t}))
$$

$$
\mathbb{P}_\theta^r(\theta^r_t | \theta^r_{t-1}) = \prod_{k=1}^{q} \mathbb{P}_\theta^r(\theta^r_{k,t} | pa(\theta^r_{k,t}))
$$

where $pa(\theta^s_{j,t})$ are a subset of the dimensions of $\theta^s_{j,t-1}$, while $pa(\theta^r_{k,t})$ are a subset of dimensions of $\theta^r_{k,t-1}$. We additionally assume given initial distributions $\mathbb{P}_\theta^s(\theta^s_0)$ and $\mathbb{P}_\theta^r(\theta^r_0)$.

We show an example Dynamic Bayesian network $G$ representing the graph of a Factored Non-stationary MDP in Fig. 1. Since we are interested in learning the graphical structure of the FN-MDP, as well as identifying the values of the latent change factors from data, describe a generative process of an environment that would lead to an FN-MDP. We adapt the generative process from Eq. 1 & 2 and introduce two additional equations to model the dynamics of the latent change factors:

$$
\begin{align*}
    s_{i,t} & = f_i(\epsilon^{a,s}_{i,t} \odot s_{i,t-1}, \epsilon^{a,s}_{i,t-1} \odot a_{i,t-1}, \epsilon^{a,r}_{i,t} \odot \theta^s_{i,t}, \theta^r_{i,t}), \\
    r_t & = h(\epsilon^{a,r} \odot s_t, \epsilon^{a,r} \odot a_t, \theta^s_t, \theta^r_t, \epsilon^r_t), \\
    \theta^s_{j,t} & = g^s(\epsilon^{a,s} \odot \theta^s_{j,t-1}, \epsilon^s_{j,t}), \\
    \theta^r_{k,t} & = g^r(\epsilon^{a,r} \odot \theta^r_{k,t-1}, \epsilon^r_{k,t})
\end{align*}
$$

where $f$, $h$, $g^s$, and $g^r$ are all non-linear functions. We assume the binary masks $c^a$ are stationary across timesteps and so are the $\epsilon^{a,s}$, $\epsilon^{a,r}$, $\epsilon^s$, and $\epsilon^r$, the i.i.d. random noises. Note that although $c^a$, $c^r$, and $\epsilon$ are stationary, we allow the change of graph structure and noise distributions, whose changes are captured by $\theta$ instead. We group the binary masks in transition distributions in the matrix $C^{a,s} := [c^{a,s}_{i,j}]_{i,j=1}^d$, $C^{a,r} := [c^{a,r}_{i,j}]_{i,j=1}^d$, $C^{a,s} := [c^{a,s}_{i,j}]_{i,j=1}^d$. Similarly, we also group the binary vectors in the dynamics of the latent change factors in matrices $C^{\theta^s} := [c^{\theta^s}_{j} | j=1]^p$ and $C^{\theta^r} := [c^{\theta^r}_{k} | k=1]^q$.

A general form of non-stationarity. In our formulation, latent change factors $\theta^s$ and $\theta^r$ follow a Markov process based on the nonlinear functions $g^s$ and $g^r$. This assumption allows us to consider different types of changes by varying the form of $g^s$ and $g^r$, generalizing the approaches in literature. We can also model multiple changes happening concurrently in the dynamics and in the reward, including different types, e.g. a continuous $g^s$ and a piecewise-constant $g^r$.

**Continuous changes.** If $g^s$ and $g^r$ are continuous, then they can model smooth changes in the environment, including within and across episodes. While the equations in Eq. 3 allow us to model within-episode changes, i.e. changes that can happen only before $t = H$ where $H$ is the horizon, we also want to model across-episode changes. We then use a separate time index $\tilde{t}$ that models the agent’s lifetime. Ini-
We first assume that we can observe the change factors and we assume the change points only occur at the end of each environments. We cannot identify the graph with the previous result, since the latent factors are not constant in our setting. We first assume that we can observe the change factors and show that we can then learn the true causal graph $\mathcal{G}$.

**Proposition 1** (Identifiability with observed change factors). Suppose the generative process follows Eq. (3) and all change factors $\theta^t_s$ and $\theta^t_r$ are observed, i.e., Eq. (3) is an MDP. Under the Markov and faithfulness assumptions, all the binary masks $C^{\theta^t_s}$ are identifiable.

We provide the proof in Appendix A. If we do not observe the change factors $\theta^t_s$ and $\theta^t_r$, we cannot identify their dimensions. On the other hand, even if the latent factors are not observed, we can still identify a partial graph structure over the state variables $s_t$, reward variable $r_t$, and action variable $a_t$. Moreover, we can also identify which dimensions in $s_t$ have changes, i.e., we can identify $C^{\theta^t_s}$. This result in the following proposition (proof in Appendix A):

**Proposition 2** (Partial Identifiability with latent change factors). Suppose the generative process follows Eq. (3) and the change factors $\theta^t_s$ and $\theta^t_r$ are unobserved. Under the Markov and faithfulness assumptions, the binary masks $C^{\theta^t_s}$, $C^{\theta^t_r}$, $C^{\alpha^s}$, and $C^{\alpha^r}$ are identifiable. Moreover, we can identify which state dimensions are affected by $\theta^t_s$ and whether the reward function changes.

This means that even in the most general case, we can learn most of the true causal graph $\mathcal{G}$ in a FN-MDP, with the exception of the transition structure of the latent change factors. In the following, we show a variational autoencoder setup to learn the generative process in FN-MDPs.

### 4. Factored Adaptation in Non-Stationary RL

In this section, we introduce the FANS-RL framework, in which the estimation of Factored Non-stationary MDP and policy learning can happen concurrently in an online mode.

#### 4.1. Learning the generative process in FN-MDPs

We propose a Factored Nonstationary variational autoencoder (FN-VAE) to learn the FN-MDP system described by Eq. 3. We jointly learn the structural relationships, state transition function, reward function, and transition function of latent change factors. An overview of FN-VAE is provided in Fig. 2 and its learning procedure is described in Alg. 1. In particular, FN-VAE contains four components: two change factor (CF) inference networks that reconstruct the current latent change factors, two change factor (CF) dynamics networks that model their dynamics with an LSTM (Hochreiter & Schmidhuber, 1997), two transition decoders that reconstruct the state dynamics at the current time $t$ and predict one step further at $t + 1$, and two reward decoders that reconstruct the reward at $t$ and predict the future reward at $t + 1$. All these components, except the change factor inference network, use the learned binary masks $C^{\theta^t_s}$. In the following, we give a detailed description of the four components, and we denote $\tau_{0:t} = (s_0, a_0, r_0, s_1, a_1, r_1, \ldots, s_t, a_t, r_t)$.

**CF inference networks (blue boxes in Fig. 2).** The two inference models for latent change factors $q_{\phi^{\tau}}(\theta^t_s | s_t, a_t)$ and $q_{\phi^{\tau}}(\theta^t_r | s_t, a_t, r_t)$ are parameterised by $\phi^{\alpha^s}$ and $\phi^{\alpha^r}$, respectively. To model the time-dependence of $\theta^t_s$ and $\theta^t_r$, we use LSTMs (Hochreiter & Schmidhuber, 1997) as inference networks. At timestep $t$, the dynamics change factor LSTM infers $q_{\phi^{\tau}}(\theta^t_s | s_t, a_t, r_t, h^t_{t-1})$, where $h^t_{t-1} \in \mathbb{R}^l$ is the hidden state in the LSTM. Thus we can obtain $\mu_{\phi^{\tau}}(\tau_{0:t})$ and $\sigma_{\phi^{\tau}}^2(\tau_{0:t})$ using $q_{\phi^{\tau}}$, and sample the latent changing factor $\theta^t_s \sim N(\mu_{\phi^{\tau}}(\tau_{0:t}), \sigma_{\phi^{\tau}}^2(\tau_{0:t}))$. Similarly, the reward change factor LSTM infers $q_{\phi^{\tau}}(\theta^t_r | s_t, a_t, r_t, h^t_{t-1})$. 

![Figure 2. The architecture of FN-VAE, explained in Section 4.1. Best viewed in color.](image-url)
where \(h_{t-1} \in \mathbb{R}^L\) is the hidden state, such that we can sample \(\theta_t^* \sim \mathcal{N}(\mu(\theta_t^*; \tau_{0:t}), \sigma^2(\theta_t^*; \tau_{0:t}))\).

**CF dynamics network (orange boxes in Fig. 2).** We model the dynamics of latent change factors with \(p_{\gamma_t}(\theta_{t+1} \mid \theta_t^*; C^{\gamma-t})\) and \(p_{\gamma_t}(\theta_{t+1} \mid \theta_t^*; C^{\theta-t})\). To ensure the Markovianity of \(\theta_t^*\) and \(\theta_t\), we define a loss \(L_{KL}\) that helps minimize the KL-divergence between \(q_{\phi}\) and \(p_{\gamma_t}\).

\[
L_{KL} = \sum_{t=2}^{T} KL(q_{\phi}^{(t)}(\theta_t^* \mid \theta_{t-1}^*, \tau_{0:t})) || p_{\gamma_t}(\theta_t^* \mid \theta_{t-1}^*, C^{\theta-t})
\]
\[
+ KL(q_{\phi}^{(t)}(\theta_t^* \mid \theta_{t-1}^*, \tau_{0:t})) || p_{\gamma_t}(\theta_t^* \mid \theta_{t-1}^*, C^{\gamma-t})
\]

If we additionally assume that the change between \(\theta_t^*\) and \(\theta_{t+1}^*\) is smooth, we can also add a smoothness constraint \(L_{\text{smooth}}\). We provide a modified smooth loss also for discrete changes in Appendix C.1.

\[
L_{\text{smooth}} = \sum_{t=2}^{T} ||\theta_t^* - \theta_{t-1}^*||_1 + ||\theta_t^* - \theta_{t-1}^*||_1
\] (5)

**Transition decoders (purple boxes in Fig. 2).** We learn an approximation of the transition dynamics in Eq. 3 by learn a reconstruction, parameterized by \(\alpha_1\), and a prediction encoder, parametrized by \(\alpha_2\). To simplify the formulas, we define \(C^{-s} := (C^{\alpha-s}, C^{\beta-s}, C^{\gamma-s})\). At timestep \(t\), the reconstruction encoder \(p_{\alpha_1}(s_{t} \mid s_{t-1}, \alpha_{t-1}, \theta_t^*; C^{-s})\) reconstructs the state from current state \(s_t\) with sampled \(\theta_t^*\). The one-step prediction encoder \(p_{\alpha_2}(s_{t+1} \mid s_t, \alpha_t, \theta_t^*)\) instead tries to approximate the next state \(s_{t+1}\). The corresponding loss functions are given below:

\[
L_{\text{rec-dyn}} = \sum_{t=1}^{T-2} \mathbb{E}_{\theta_t^* \sim q_{\phi}} \log p_{\alpha_1}(s_t \mid s_{t-1}, \alpha_{t-1}, \theta_t^*; C^{-s})
\]

\[
L_{\text{pred-dyn}} = \sum_{t=1}^{T-2} \mathbb{E}_{\theta_t^* \sim q_{\phi}} \log p_{\alpha_2}(s_{t+1} \mid s_t, \alpha_t, \theta_t^*)
\] (6)

**Reward decoders (green boxes in Fig. 2).** Similar to the transition decoders, we use a reconstruction encoder \(p_{\beta_1}(r_t \mid s_t, \alpha_t, \theta_t^*; C^{\alpha+r}, C^{\gamma+r})\), parameterized by \(\beta_1\), and a one-step prediction encoder \(p_{\beta_2}(r_{t+1} \mid s_{t+1}, \alpha_{t+1}, \theta_t^*)\), parameterized by \(\beta_2\), to approximate the reward function, with the loss functions given below:

\[
L_{\text{rec-rw}} = \sum_{t=1}^{T-2} \mathbb{E}_{\theta_t^* \sim q_{\phi}} \log p_{\beta_1}(r_t \mid s_t, \alpha_t, \theta_t^*; C^{\alpha+r}, C^{\gamma+r})
\]

\[
L_{\text{pred-rw}} = \sum_{t=1}^{T-2} \mathbb{E}_{\theta_t^* \sim q_{\phi}} \log p_{\beta_2}(r_{t+1} \mid s_{t+1}, \alpha_{t+1}, \theta_t^*)
\] (7)

**Sparsity loss.** We encourage sparsity in the binary masks \(C^{\gamma-r}\) to improve identifiability, by following loss with

**Algorithm 1 Learning FN-MDPs using FN-VAE.**

**Input:** Trajectories \(\tau\), FN-VAE parameters \(\phi = (\phi^*, \phi^r)\), \(\alpha = (\alpha_1, \alpha_2)\), \(\beta = (\beta_1, \beta_2)\), \(\gamma\); Mask matrices \(G = (C^{\gamma-r})\), Boolean update \(G\), Learning rates \(\lambda_\phi, \lambda_\gamma, \lambda_\alpha, \lambda_\beta, \lambda_G\). Length of collected rollouts \(k\); Number of training epochs \(E\).

**Output:** \(\phi, \alpha_1, \alpha_2, \beta_1, \beta_2, \gamma\)

for \(i = 1, 2, \ldots, E\) do

Randomly sample a batch of trajectories \(\tau_{0:k}\) in \(\tau\)

# Infer the latent change factors

for \(j = s, r\) do

Infer \(\mu_{\phi_j}(\tau_{0:k})\) and \(\sigma^2_{\phi_j}(\tau_{0:k})\) using \(q_{\phi_j}\)

Infer \(\mu_{\gamma_j}(\theta_{0:k})\) and \(\sigma^2_{\gamma_j}(\theta_{0:k})\) using \(p_{\gamma_j}\)

Sample \(\theta_{0:k}^* \sim \mathcal{N}(\mu_{\phi_j}(\tau_{0:k}), \sigma^2_{\phi_j}(\tau_{0:k}))\)

end for

Reconstruct and predict \(\hat{s}_{0:k}, \hat{s}_{1:k}, \hat{r}_{0:k}, \hat{r}_{1:k}\) using \(p_{\alpha_1}, p_{\beta_1}, p_{\beta_2}\)

# Update the FN-VAE model

\(\phi \leftarrow \phi - \lambda_\phi \nabla_{\phi} L_{\text{VAE}}\)

\(\gamma \leftarrow \gamma - \lambda_\gamma \nabla_{\gamma} (L_{\text{KL}} + L_{\text{smooth}})\)

\(\alpha \leftarrow \alpha - \lambda_\alpha \nabla_{\alpha} (L_{\text{rec-dyn}} + L_{\text{pred-dyn}})\)

\(\beta \leftarrow \beta - \lambda_\beta \nabla_{\beta} (L_{\text{rec-rw}} + L_{\text{pred-rw}})\)

if update \(G\) then

\(G \leftarrow G - \lambda_G \nabla_G (L_{\text{rec-dyn}} + L_{\text{rec-rw}} + L_{\text{KL}} + L_{\text{spare}})\)

end if

end for

end for

**Learning from raw pixels.** In some applications, the state variables may not be directly observed, but instead we observe the image pixels. Our framework can be easily extended to cover such a case, by adding another encoder \(\phi^r\) to learn the latent state variables from high-dimensional images, similar to other works learning latent states from pixels for RL. (Ha & Schmidhuber, 2018; Zhao et al., 2020; Huang et al., 2022). We describe the reconstruction and prediction loss of this component in Appendix C.2.

**4.2. Online model estimation and policy optimization**

We propose a factored adaptation algorithm for non-stationary RL that interleaves model estimation and policy
Algorithm 2 Factored Adaptation for non-stationary RL

1: Init: Env; VAE parameters: $\phi = (\phi^s, \phi^r)$, $\alpha = (\alpha_1, \alpha_2)$, $\beta = (\beta_1, \beta_2)$, $\gamma$; Mask matrices: $C^\to$; Policy parameters: $\psi$; replay buffer: $D$; Number of episodes: $N$; Episode horizon: $H$.
2: Output: $\phi$, $\alpha_1$, $\alpha_2$, $\beta_1$, $\beta_2$, $\gamma$, $\psi$
3: # Model initialization
4: Collect multiple trajectories $\tau = \{t_{0:k}^1, t_{0:k}^2, \ldots\}$ with policy $\pi_\psi$ from Env;
5: Learn an initial VAE model on $\tau$ (Alg. 1);
6: Identify the compact representations $s^{\text{min}}$ and change factors $\theta^{\text{min}}$ based on $C^\to$;
7: # Model estimation & policy learning
8: for $n = 0, \ldots, N - 1$ do
9: for $t = 0, \ldots, H - 1$ do
10: Observe $s_t$ from Env;
11: # Estimating latent change factors
12: if $t = 0$ then
13: for $j = s, r$ do
14: Infer $\mu_j(\theta^{\text{old}}_{t})$ and $\sigma^2_j(\theta^{\text{old}}_{t})$ via $p_{\gamma_j}$
15: Sample $\theta^0_t \sim \mathcal{N}(\mu_j(\theta^{\text{old}}_{t}), \sigma^2_j(\theta^{\text{old}}_{t}))$
16: end for
17: else
18: for $j = s, r$ do
19: Infer $\mu_j(\theta^{t-1}_{t})$ and $\sigma^2_j(\theta^{t-1}_{t})$ via $p_{\gamma_j}$
20: Sample $\theta^t_t \sim \mathcal{N}(\mu_j(\theta^{t-1}_{t}), \sigma^2_j(\theta^{t-1}_{t}))$
21: end for
22: end if
23: if $t = H - 1$ then
24: $\theta^0_{t} \leftarrow \theta^t_t$ and $\theta^{\text{old}}_{t} \leftarrow \theta^t_t$;
25: end if
26: Generate $a_t \sim \pi_\psi(a_t \mid s^{\text{min}}_t, \theta^{\text{min}}_t)$
27: Receive $r_{n,t}$ from Env
28: Add $(s_t, a_t, r_t, \theta^t_t, \theta^t_t)$ to replay buffer $D$;
29: Extract a trajectory with length $k$ from $D$;
30: Learn VAE (Alg. 1) with updateG=False;
31: Sample a batch of data from $D$
32: Update policy network parameters $\psi$
33: end for
34: end for

Continuous changes. In Alg. 2 we describe our framework in case of continuous changes that can span across episodes. We start by collecting a few trajectories $\tau$ and then learn our initial FN-VAE (Line 4-5). We can use the graphical structure of the initial FN-VAE to identify the compact representations (Line 6). During the online model estimation and policy learning stage, we estimate the latent change factors $\theta^0_t$ and $\theta^{\text{old}}_t$ using the CF dynamics networks $\gamma^s$ and $\gamma^r$ (Line 11-24). Since we assume the dynamics of the change factors are smooth across episodes, at time $t = 0$ we will use the last timestep $(H - 1)$ of the previous episode as a prior on the change factors (Line 12-16). Otherwise we will estimate the change factors using their values in the previous timestep $t - 1$. We use the estimated latent factors $\theta_t$ and observed state $s_t$ to generate $a_t$ using $\pi_\psi$ (Line 26). We receive a reward $r_t$ and add $(s_t, a_t, r_t, \theta^t_t, \theta^t_t)$ to the replay buffer (Line 27-28). We now update our estimation of the FN-VAE with Alg. 1, but we keep the graph $G$ fixed. (Line 29). Finally, we sample a batch of trajectories in the replay buffer and update the policy network $\psi$.

Discrete changes. Since we assume discrete changes happen at specific timestep $\tilde{t} = (\tilde{t}_1, \ldots, \tilde{t}_M)$, we can easily modify Alg. 2 for discrete changes, both within-episode and across-episode, by changing Lines 11-25. We provide the pseudocode in Appendix C.1.

5. Evaluation

We evaluate our approach on four representative tasks of well-established benchmarks, Half-Cheetah-V3 from MuJoCo (Todorov et al., 2012; Brockman et al., 2016), Sawyer-Reach and Sawyer-Peg from Sawyer (Yu et al., 2020; Zhao et al., 2020), and Minitaur (Tan et al., 2018). We modified these tasks to test several non-stationary RL scenarios with continuous and discrete change functions.

Halfcheetah-v3 in MuJoCo. In this task, the agent is moving forward using the joint legs and the objective is to achieve the target velocity $v^g$. The reward signal is $r_t = -\|v^g - v^o_t\|_2 - 0.05\|a_t\|_2$, where $v^o$ and $a_t$ are the agent’s velocity and action, respectively, at timestep $t$. The horizon in each episode is 150. We consider both the changes in the dynamics (change of the wind forces $w^f$) and reward functions (change of target velocity $v^g$). The change function in the dynamics $f^w$ can be either continuous or discrete, and discrete changes can happen both within and across episodes. Similarly to LILAC (Xie et al., 2021), we choose a sine function, but besides allowing the change at specified intervals, we also allow it to change smoothly. The change in the reward function $v^g$ is not generally stable in the continuous case, so we only consider the discrete and
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Figure 3. Average rewards across 10 runs on (a)-(d) Half-Cheetah-V3, (e) Sawyer-Reaching and (f) Minitaur. We only indicate the averaged highest result of all times for oracle and SAC. Detailed information: (a) Discrete across-episode changes on $f_w$: (b) Discrete within-episode changes on $f_w$: (c) Continuous changes on $f_w$: (d) Discrete across-episode changes on $s^g$ and $v^g$ concurrently; (e) Discrete across-episode changes on $s^g$: (f) Discrete across-episode changes on $m$ and $s_{t,v}$ concurrently.

We also design a non-stationary scenario where dynamic and reward functions change concurrently. We report all equations for $g^r$ and $g^s$ in Appendix B.1.

**Sawyer.** We consider two robotic manipulation tasks, Sawyer-Reaching and Sawyer-Peg. In Sawyer-Reaching, the Sawyer arm is trained to reach a target position $s^g_t$. The reward $r_t$ is the difference between the current position $s_t$ and the target position $r_t = -\|s_t - s^g_t\|_2^2$. In this task, we cannot directly modify the dynamics in the simulator, so consider a reward-varying scenario, where the target location changes across each episode following a periodic function. In Sawyer-Peg, the robot arm is trained to insert a peg into a designed target location $s^g$. The reward function is $r_t = I(\|s_t - s^g\|_2^2 \leq 0.05)$. In this task, we evaluate our method on raw pixels, in order to compare with similar approaches, e.g., ZeUS (Sodhani et al., 2021), CADM (Lee et al., 2020), Hyperdynamics (Xian et al., 2021), and Meld (Zhao et al., 2020), based on their reported results. Following (Sodhani et al., 2021), we consider discrete across-episode changes, where the target location can change in each episode, and is randomly sampled from a small interval. We describe the details in Appendix B.2.

**Minitaur.** A minitaur robot is a simulated quadruped robot with eight direct-drive actuators. The minitaur is trained to move at the target speed $s_{t,v}$. The reward is $r_t = 0.3 - |0.3 - s_{t,v}| - 0.01 \cdot \|a_t - 2a_{t-1} + a_{t-2}\|_1$. We modify (1) the mass $m$ (dynamics) and (2) target speed $s_{t,v}$ (reward) of the minitaur. We consider continuous, discrete across-episode and discrete within-episode changes for the dynamics, and across-episode changes for the reward. We describe the setting in Appendix B.3.

**Baselines.** We compare our approach with two representative meta-RL approaches for non-stationary RL, TRIO (Poiani et al., 2021) and VariBAD (Zintgraf et al., 2021), as well as with two representative task embedding approaches, LILAC (Xie et al., 2021) and ZeUS (Sodhani et al., 2021). Moreover, we also compare with a method for stationary RL, SAC (Haarnoja et al., 2018), which will be considered as a lower-bound, and compare with an oracle agent that can obtain the full information of non-stationarity (e.g., the wind forces) and conduct policy learning based on these observed change factors, which will be our upper-bound. For all baselines, we use SAC for policy learning. We compare with the baselines on average rewards and the compactness of the latent space in varying degrees of non-stationarity. Finally, we conduct ablation studies on each
Rewards

ZeUS (final)
ZeUS (best)

Figure 4. Average rewards across 10 runs on Sawyer-Peg (raw pixels) with across-episode changes.

6. Related work

Learning a good and stable policy under non-stationary environments is a long-standing challenge in RL (Dulac-Arnold et al., 2021). Early works (Da Silva et al., 2006; Sutton et al., 2007) can only detect changes that have already happened, instead of anticipating potential changes. Recently, several methods have been developed that learn to anticipate changes in non-stationary deep RL.

A first line of research adapts Meta-RL methods to optimize learning performances across a sequence of non-stationary tasks. In particular, Al-Shedivat et al. (2018) extend the model-agnostic meta learning (MAML) (Finn et al., 2017) framework into the non-stationary settings. However, this work does not explicitly model the temporal changing components, which can degrade the performance during the testing phase if the temporal changes differ from those in the trained tasks. Instead, TRIO (Poiani et al., 2021) and VariBAD (Zintgraf et al., 2021) track the non-stationarity via inferring the evolution of latent parameters, which can capture the temporal change factors during the meta-testing phase. However, these methods have to meta-train the model on a set of non-stationary tasks, which may not be accessible in real-world applications.

Another line of research directly learns the latent representation to capture the non-stationary components. In particular, LILAC (Xie et al., 2021) and ZeUS (Sodhani et al., 2021) leverage latent variable models to directly model the change factors in environments. Our approach fits in this line of work; however, as opposed to LILAC and ZeUS, which model all temporal changes using a shared embedding space, we use a more efficient, factored representation. Moreover, these works consider families of MDPs indexed by a latent parameter, which makes it non-trivial to model continuously varying environments, e.g., changes within an episode.

7. Conclusions

In this paper, we introduced Factored Adaptation for Non-Stationary RL, a framework that learns a factored representation to adapt to non-stationarity that can be combined with any standard RL algorithm. We first formalized our problem as a Factored Non-stationary MDP (FN-MDP), augmenting a factored MDP with latent change factors evolving in time as a Markov process. FN-MDPs do not model a family of...
MDPs, but instead naturally include the dynamics of latent change factors analogously to the dynamics of the state components. This allows us to capture different types of non-stationarity, e.g., continuous and discrete changes, both within each episode and across different episodes. In order to learn FN-MDPs we proposed FN-VAE, a factored non-stationary variational autoencoder, which we then combined in Factored Adaptation for Non-Stationary RL (FANS-RL), an online model estimation and policy evaluation approach. We evaluated FANS-RL on simulated benchmarks for continuous control and robotic manipulation tasks, including with raw pixel inputs, and show it outperforms the state of the art on rewards, compactness of the latent space representation and robustness to varying degrees of non-stationarity.

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A. Proofs

**Proposition 3** (Identifiability with observed change factors). Suppose all the change factors $\theta^s_t$ and $\theta^r_t$ are observed, i.e., Eq. (3) is an MDP. Under the Markov and faithfulness assumptions, all the binary masks $C^s - \cdot$ are identifiable.

**Proof.** We construct the graph in the Factored Non-stationary MDP as a dynamic Bayesian network (DBN) $G$ over the variables $V = \{s_{1,t-1}, \ldots, s_{d,t-1}, s_{1,t}, \ldots, s_{d,t}, a_t, r_t, \theta^s_{1,t}, \ldots, \theta^s_{p,t}, \theta^r_{1,t}, \ldots, \theta^r_{q,t}\}$. We rewrite the time index of $r_t$ as $r_{t+1}$, so the DBN is Markov, stationary (since we have observed the change factors) and there are no instantaneous relations. In this setting, identifiability of the graph $G$ is trivial (Peters et al., 2013).

**Proposition 4** (Partial Identifiability with latent change factors). Suppose the generative process follows Eq. (3) and the change factors $\theta^s_t$ and $\theta^r_t$ are unobserved. Under the Markov and faithfulness assumptions, the binary masks $C^{s-a}, C^{a-s}, C^{s-r}$ and $C^{a-r}$. Moreover, we can identify which state dimensions are affected by $\theta^s_t$ and if the reward function changes.

**Proof.** In this case, if the MDP is non-stationary, we assume that we can represent the latent change factors as a smooth function of the observed time index $t$. We use the time index $t$ as a surrogate variable to characterize the unobserved change factors.

We denote the variable set in the system by $V$, with $V = \{s_{1,t-1}, \ldots, s_{d,t-1}, s_{1,t}, \ldots, s_{d,t}, a_t, r_t\}$, and the variables form a dynamic Bayesian network $G$. Note that in our particular problem, according to the generative environment model in Eq. (3), the possible edges in $G$ are only those from $s_{i,t-1} \rightarrow s_{j,t}$ to $s_{i,t}$, from $a_{t-1}$ to $s_{j,t}$, from $s_{i,t-1} \rightarrow s_{i,t}$, and from $a_{t-1}$ to $r_t$. We further include the time index $t$ into the system to characterize the unobserved change factors.

It has been shown that under the Markov condition and faithfulness assumption, for every $V_i, V_j \in V$, $V_i$ and $V_j$ are not adjacent in $G$ if and only if they are independent conditional on some subset of $\{V_l \neq i, l \neq j\}$ (Huang et al., 2020). Thus, we can asymptotically identify the correct graph skeleton over $V$. Moreover, since we assume a dynamic Bayesian network, there the direction of an edge between a variable at time $t$ to one at time $t + 1$ is fixed. Therefore, the structural matrices $C^{s-a}, C^{a-s}, C^{s-r}$ and $C^{a-r}$, which encode parts of the graph $G$ over $V$, are identifiable.

Since $t$ inherits all of the children of the latent change factors in $G$, we can further show that if $s_{i,t} \perp \perp t | s_{i,t-1}, a_{t-1}$ in $G$, then none of the latent change factor dimensions $\theta^{s,t}_{j,t}$ affect $s_{i,t}$, i.e., $s_{i,t} \perp \perp t | s_{i,t-1}, a_{t-1} \iff \theta^{s,t}_{i,j} = 0$. Under the same principle, if $r_t \perp \perp t | s_{i,t}, a_t$, then the reward is stationary.

B. Details on experimental designs

B.1. MuJoCo

We modify the Half-Cheetah environment into a variety of non-stationary settings. Details are given as below.

**Changes on dynamics.** We change the wind forces $f^w$ in the environment. We consider the changing functions can be both continuous and discrete.

- Continuous changes: $f^w_t = 10 + 10 \sin(0.005 \cdot t)$, where $t$ is the timestep index.
- Discrete changes: (1) Across-episode: $f^w = 10 + 10 \sin(0.5 \cdot i)$, where $i$ is the episode index; (2) Within-episode: $f^w = 10 + 10 \sin(0.4 \cdot \lfloor t/10 \rfloor)$, where $t$ is the timestep index.

**Changes on reward functions.** To make the rewards be non-stationary, we change the target speed $v_g$ in each episode. To make the learning process stable, we only consider the discrete changes and the change points are located at the beginning of each episode. The changing function is $v_g = 1.5 + 1.5 \sin(0.2 \cdot i)$, where $i$ denotes the episode index.

**Changes on both dynamics and rewards.** We consider a more general but challenging scenario, where the changes on dynamics and rewards can happen concurrently during the lifetime of the agents. We change the wind forces and target.
speed at the beginning of each episode. At episode $i$, the dynamics and reward functions are:

\[
\begin{align*}
    f_w &= 10 + 10 \sin(w \cdot i) \\
    v_g &= 1.5 + 1.5 \sin(w \cdot i)
\end{align*}
\]

Here, $w$ is the non-stationary degree. We consider multiple values of $w$ in our experiments. In Fig. 3 (d), $w = 0.5$. In Fig. 5 (b), $w$ is the value of non-stationary degree.

B.2. Sawyer benchmarks

We change the target location in Sawyer reaching task. The target location $s_g^t$ is given as below:

\[
s_g^t = \begin{bmatrix}
    0.1 \cdot \| \cos(0.2 \cdot i) \|
    \\
    0.1 \cdot \sin(0.5 \cdot i)
    \\
    0.2
\end{bmatrix}
\]

where $i$ is the episode index. For Sawyer-Peg task, the target location $s_g$ changes at each episode. The parameters in each dimension of $s_g$ is randomly sampled at episode $i$ as below:

- $x_{\text{range}}_1$: $(0.44, 0.45)$;
- $x_{\text{range}}_2$: $(0.6, 0.61)$;
- $y_{\text{range}}_1$: $(-0.08, -0.07)$;
- $y_{\text{range}}_2$: $(0.07, 0.08)$;

B.3. Minitaur benchmarks

We consider both the changes on dynamics and reward functions.

Changes on dynamics. We change the mass of taur $m$ in the environment. Specifically, we consider both the continuous and discrete changes.

- Continuous changes: $m_t = 1.0 + 0.75 \sin(0.005 \cdot t)$
- Discrete and within-episode changes: $m_t = 1.0 + 0.75 \sin(0.3 \cdot \lfloor t/20 \rfloor)$

Changes on both dynamics and reward functions. We also consider a case where both the dynamics and reward functions change at the beginning of each episode. We change the target speed of minitaur to introduce the non-stationarity of reward functions. The change functions are given below:

\[
\begin{align*}
    m_i &= 1.0 + 0.5 \sin(0.5 \cdot i) \\
    s_v &= 0.3 + 0.2 \sin(0.5 \cdot i)
\end{align*}
\]

B.4. Full results

Fig. 6 and 7 give the full results on average rewards over 10 runs versus timesteps in Half-Cheetah and Minitaur experiments. Table B.4 shows the average final rewards over 10 runs for all experiments. Fig. 9 gives average rewards on different benchmarks with varying numbers of latent features with all evaluated approaches. Fig. 8 demonstrates the rewards on Half-Cheetah with different non-stationary degrees on multi-factor changing scenario.
Figure 6. The average rewards across timesteps in Half-Cheetah experiments. (a) Discrete (across-episode) changes on wind forces; (b) Discrete (within-episode) changes on wind forces; (c) Continuous changes on wind forces; (d) Discrete (across-episode) changes on target speed; (e) Discrete (across-episode) changes on wind forces and target speed concurrently.

Figure 7. The average rewards across timesteps in Minitaur experiments. (a) Continuous changes on the mass; (b) Discrete (across-episode) changes on the target speed; (c) Discrete (across-episode) changes on mass and target speed concurrently.
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Table 1. Averaged final rewards of different methods on Half-Cheetah, Sawyer-Reaching, and minitaur benchmarks with a variety of non-stationary settings. The best non-oracle results w.r.t. the mean are marked in bold. “•” indicates the baseline for which the improvements of our approach are statistically significant (via Wilcoxon signed-rank test at 5% significance level).

|                  | Oracle  | SAC     | LILAC   | TRIO    | VariBAD | Ours     |
|------------------|---------|---------|---------|---------|---------|----------|
| Half-Cheetah: A-EP (D) | -24.4 ±16.2 | -113.4 ±28.5 | -70.1 ±27.7 | -76.0 ±47.3 | -75.5 ±41.6 | -32.6 ±25.0 |
| Half-Cheetah: A-EP (R) | -10.9 ±20.1 | -131.5 ±16.9 | -60.1 ±21.7 | -53.1 ±20.6 | -61.0 ±33.3 | -38.7 ±33.3 |
| Half-Cheetah: W-EP (D) | -48.2 ±41.6 | -107.5 ±20.6 | -72.9 ±29.3 | -84.4 ±21.7 | -65.1 ±20.1 | -54.0 ±23.0 |
| Half-Cheetah: CONT (D) | -12.3 ±27.7 | -112.0 ±16.9 | -58.4 ±22.3 | -45.0 ±31.1 | -53.0 ±38.6 | -24.8 ±21.1 |
| Minitaur: CONT (D) | 6.4 ±3.9   | -52.5 ±9.1  | -34.0 ±8.2  | -28.1 ±2.9  | -31.3 ±4.3  | -9.7 ±2.5  |
| Minitaur: W-EP (D) | 44.9 ±5.8  | -9.6 ±5.5   | 8.5 ±14.9   | -0.8 ±4.7   | 5.4 ±14.1   | 20.2 ±11.9 |
| Minitaur: A-EP (R+D) | 43.0 ±4.7  | -8.7 ±5.4   | 3.8 ±12.9   | 5.8 ±9.7    | 21.5 ±9.7   | 40.2 ±5.3  |

Figure 8. Averaged final rewards on 10 runs on Half-Cheetah with different non-stationary degrees on across-episode and multi-factor changes.
Figure 9. Average rewards on different benchmarks with different number of latent features. (a) Half-Cheetah experiments with discrete (across-episode) changes on wind forces; (b) Half-Cheetah experiments with discrete (within-episode) changes on wind forces; (c) Half-Cheetah experiments with continuous changes on wind forces; (d) Half-Cheetah experiments with discrete (across-episode) changes on target speed; (e) Half-Cheetah experiments with discrete (across-episode) changes on wind forces and target speed concurrently; (f) Sawyer-Reaching experiment with discrete (across-episode) changes on target locations; (g) Minitaur experiments with continuous changes on the mass; (h) Minitaur experiments with discrete (across-episode) changes on the target speed; (i) Minitaur experiments with discrete (across-episode) changes on mass and target speed concurrently.

B.5. Ablation studies on FANS-RL

To verify the effectiveness of each component in our proposed framework, we consider the following ablation studies:

- Without smoothness loss ($L_{\text{smooth}}$);
- Without structural relationships ($C^*$);
- Without compact representations ($s^{\min}, \theta^{\min}$).

As shown in Fig. 10, all the studied components benefit the performance. Furthermore, FANS-RL can still outperform the strong baselines even without some of the components.
Figure 10. Ablation studies on different components in FANS-RL on (a) Half-Cheetah experiment; (b) Sawyer experiment; and (c) Minitaur experiments. CONT, A-, W-EP indicate continuous, across-episode, and within-episode changes, respectively. (D) and (R) represent changes on dynamics and reward functions, respectively. Best viewed in color.

C. Details on the factored adaptation framework

C.1. The framework dealing with discrete and across-episode changes

Alg. 3 gives the extended framework for handling both across- and within-episode changes in non-stationary RL, respectively. The major difference between Alg. 3 and Alg. 2 is that we only infer \( \theta \) using via CF dynamics networks at change points. Furthermore, we also adjust the objective functions of FN-V AE to fit the discrete changes. At timestep \( t \) in episode \( n \), where \( t_m \leq (n-1) \cdot H + t < t_{m+1} \), we have:

- Prediction and reconstruction losses:

\[
\mathcal{L}_{\text{rec-dyn}} = \sum_{t=1}^{T-2} \mathbb{E}_{\theta_{r_{m}} \sim \pi_{q}} \log p_{\alpha_1}(s_{t}, s_{t-1}, a_{t-1}, \theta_{r_{m}}; C^{s-s})
\]

\[
\mathcal{L}_{\text{pred-dyn}} = \sum_{t=1}^{T-2} \mathbb{E}_{\theta_{r_{m}} \sim \pi_{q}} \log p_{\alpha_2}(s_{t}, a_{t}, \theta_{r_{m}})
\]

\[
\mathcal{L}_{\text{rec-rw}} = \sum_{t=1}^{T-2} \mathbb{E}_{\theta_{r_{m}} \sim \pi_{q}} \log p_{\beta_1}(r_{t}|s_{t}, a_{t}, \theta_{r_{m}}; c^{s-r}, c^{a-r})
\]

\[
\mathcal{L}_{\text{pred-rw}} = \sum_{t=1}^{T-2} \mathbb{E}_{\theta_{r_{m}} \sim \pi_{q}} \log p_{\beta_2}(r_{t}|s_{t+1}, a_{t+1}, \theta_{r_{m}})
\]
Algorithm 3 Factored Adaptation for non-stationary RL (discrete changes.)

1: \textbf{Init}: Env; VAE parameters: $\phi = (\phi^a, \phi^r)$, $\alpha = (\alpha_1, \alpha_2)$, $\beta = (\beta_1, \beta_2)$, $\gamma$; Mask matrices: $C^{\alpha \beta}$; Policy parameters: $\psi$; replay buffer: $D$; Number of episodes: $N$; Episode horizon: $H$; Change index $t = \{t_1, \ldots, t_M\}$; $m = 0$.

2: \textbf{Output}: $\phi, \alpha_1, \alpha_2, \beta_1, \beta_2, \gamma, \psi$

3: \# Model initialization

4: Collect multiple trajectories $\tau = \{\tau_{0:k}, \tau_{0:k}, \ldots\}$ with policy $\pi_\psi$ from Env;

5: Learn an initial VAE model on $\tau$ (Alg. 1)

6: Identify the compact representations $s^{min}$ and change factors $\theta^{min}$ based on $C^{\alpha \beta}$.

7: \# Model estimation & policy learning

8: for $n = 0, \ldots, N - 1$ do

9: for $t = 0, \ldots, H - 1$ do

10: Observe $s_t$ from Env;

11: if $n \cdot (H - 1) + t \in t$ then

12: $m \leftarrow m + 1$

13: for $j = s, r$ do

14: Infer $\mu_j(\theta_{t_{m-1}})$ and $\sigma^2_j(\theta_{t_{m-1}})$ via $p_{\gamma_j}$

15: Sample $\theta_{t_{m}} \sim N(\mu_j(\theta_{t_{m-1}}), \sigma^2_j(\theta_{t_{m-1}}))$

16: end for

17: if $\#$ Estimating latent change factors

18: Generate $\alpha_t \sim p_\psi(\alpha_t | s_t^{min}, \theta_{t_{m}})$

19: Receive $r_{n,t}$ from Env

20: Add $(s_t, \alpha_t, r_t, \theta_{t_{m}}, \theta_{t_{m}})$ to replay buffer $D$

21: Extract a trajectory with length $k$ from $D$

22: Learn VAE (Alg. 1) with updateG=False

23: Sample a batch of data from $D$

24: Update policy network parameters $\psi$

25: end if

26: end for

27: end for

- **KL loss:**

\[
\mathcal{L}_{KL} = \sum_{t=2}^{T} KL(q_{\phi^r}(\theta^r_{t_{m}}, \theta^s_{t_{m-1}}, \tau_{0:t}) || p_{\gamma^r}(\theta^r_{t_{m}}, \theta^s_{t_{m-1}}; C^{\theta^r \theta^s})) \]

\[
+ KL(q_{\phi^s}(\theta^s_{t_{m}}, \theta^r_{t_{m-1}}, \tau_{0:t}) || p_{\gamma^s}(\theta^s_{t_{m}}, \theta^r_{t_{m-1}}; C^{\theta^s \theta^r})) \] (11)

- **Sparsity loss:**

\[
\mathcal{L}_{\text{sparsity}} = w_1\|C^{\theta^s \theta^s}\|_1 + w_2\|C^{\theta^r \theta^s}\|_1 + w_3\|C^{\theta^s \theta^r}\|_1 + w_4\|C^{\theta^r \theta^r}\|_1 \]

\[
+ w_5\|C^{\theta^r \theta^s}\|_1 + w_7\|C^{\theta^s \theta^r}\|_1 \] (12)

- **Smoothness loss:**

\[
\mathcal{L}_{\text{smooth}} = \sum_{t=2}^{T} (\|\theta^r_{t_{m}} - \theta^r_{t_{m-1}}\|_1 + \|\theta^s_{t_{m}} - \theta^s_{t_{m-1}}\|_1) \]

The total loss $\mathcal{L}_{\text{vae}} = k_1\mathcal{L}_{\text{rec}} + k_2\mathcal{L}_{\text{pred}} - k_3\mathcal{L}_{\text{KL}} - k_4\mathcal{L}_{\text{sparsity}} - k_5\mathcal{L}_{\text{smooth}}$, where $k_1, k_2, k_3, k_4, k_5$ are adjustable hyper-parameters to balance the objective functions.

C.2. The framework dealing with raw pixels

We augment the generative process in Eq. 3 with the generative process of observation.

\[
o_t = u_t(C^{s \alpha}_t \odot s_t, C^{r}_t), \]

(14)

where $u$ is a non-linear function and $i = 1, \ldots, d$. $C^{s \alpha}_t := [c^{s \alpha}_i]_{i=1}^{d}$ is an i.i.d. random noise. To learn the $u_t$, we model the states as the latent variables in FN-VAE. Fig. 11 gives the modified FN-VAE dealing with raw pixels, where the states
are also in the latent space. Different from the original FN-VAE, we incorporate state inference networks and state dynamics networks. Moreover, we reconstruct and predict the current and future observations using the observation decoder. Detailed objective functions are given below.\(^2\)

- Prediction and reconstruction losses

\[
\mathcal{L}_{\text{rec-obs}} = \sum_{t=1}^{T-2} \mathbb{E}_{s_t \sim q_{s|o}} \log p_{a_1}(o_t|s_t; c^{s-o})
\]

\[
\mathcal{L}_{\text{pred-obs}} = \sum_{t=1}^{T-2} \mathbb{E}_{s_t \sim q_{s|o}} \log p_{a_2}(o_{t+1}|s_t, \theta_{t_m}^s)
\]

\[
\mathcal{L}_{\text{rec-cw}} = \sum_{t=1}^{T-2} \mathbb{E}_{(\theta_{t_m}^s \sim q_d \theta_{t-1}^s \sim q_d)} \log p_{b_1}(r_t|s_t, a_t, \theta_{t_m}^s; c^{s-r}, c^{s+r})
\]

\[
\mathcal{L}_{\text{pred-cw}} = \sum_{t=1}^{T-2} \mathbb{E}_{(\theta_{t_m}^s \sim q_d \theta_{t-1}^s \sim q_d)} \log p_{b_2}(r_{t+1}|s_t, a_{t+1}, \theta_{t_m}^s)
\]

- KL loss

\[
\mathcal{L}_{\text{KL}} = \sum_{t=2}^{T} \text{KL}(q_{r^s}(\theta_{t_{m-1}}^{s^2}|\theta_{t_{m-1}}^{s^1}, \tau_{0:t}^r)) || p_{r^s}(\theta_{t_{m-1}}^{s^2}|\theta_{t_{m-1}}^{s^1}, C^{s^2-s^1})
\]

\[
+ \text{KL}(q_{r^s}(\theta_{t_{m-1}}^{s^2}|\theta_{t_{m-1}}^{s^1}, \tau_{0:t}^r)) || p_{r^s}(\theta_{t_{m-1}}^{s^2} C^{s^2-s^1})
\]

\[
+ \text{KL}(q_{r^s}(s_t|\tau_{0:t}^s, \theta_{t_{m-1}}^{s^2})) || p_{r^s}(s_t|s_{t-1}, a_{t-1}, \theta_{t_{m-1}}^{s^2}, C^{s-s^2}, C^{s^2-s^1}, C^{s^2-s^1})
\]

where \(\tau_{0:t} = \{o_0, r_0, o_1, r_1, \ldots, o_t, r_t\}\).

\(^2\text{Here we give the example of handling the discrete changes.}\)
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- Sparsity loss

\[
L_{\text{sparse}} = w_1 \| C_{s^*s} \|_1 + w_2 \| C_{a^*s} \|_1 + w_3 \| C_{\theta^*s} \|_1 + w_4 \| e_{s^*} \|_1 + w_5 \| e_{a^*} \|_1 + w_6 \| C_{\theta^*s} \|_1 + w_7 \| C_{\theta^*r} \|_1 + w_8 \| C_{s^*o} \|_1
\]  
(18)

- Smooth loss

\[
L_{\text{smooth}} = \sum_{t=2}^{T} \left( \| \theta_{s_{tm}} - \theta_{s_{tm-1}} \|_1 + \| \theta_{r_{tm}} - \theta_{r_{tm-1}} \|_1 \right)
\]  
(19)

C.3. Hyper-parameter selection

C.3.1. Factored model estimation

**Input with symbolic states.** In Half-Cheetah, Sawyer-Reaching, and Minitaur the symbolic states are observable. For the CF dynamic networks, we use 2-layer fully connected networks. The number of neurons is 512. For CF inference networks, we use 2-layer fully connected networks, where the number of neurons is 256. The initial learning rates for all losses are set to be 0.1 with a decay rate 0.99. The batch size is 256 and the length of time steps is equal to the horizon in each task. The number of RNN cells is 256. The decoder networks are 2-layer fully connected networks. The number of neurons is 512. We use the weighting approaches in (Kendall et al., 2018) to adaptively balance the weights among the objective functions.

**Input with raw pixels.** In Sawyer-Peg, we directly learn and adapt in non-stationary environments with raw pixels observed. Different from other experiments, we use the architecture described in Fig. 11. At timestep \( t \), we stack 4 frames as the input \( o_t \). A 5-layer convolutional networks is used to extract the features of the trajectories of observations and rewards. The layers have 32, 64, 128, 256, and 256 filters. And the corresponding filter sizes are 5, 3, 3, 3, 4. The observation decoders are the transpose of the convolutional networks. Then the extracted features are used as the input of LSTM networks in state inference networks. The state inference networks and state dynamic networks share the same architectures with the CF inference and dynamics networks, respectively. We use the same CF inference networks, CF dynamics networks, and reward decoders with those in cases with symbolic states as input. The number of latent features is 40.

**Model initialization.** Table 2, 3, and 4 provide the settings of learning the model initialization.

| CONT (D) | A-EP (D) | W-EP (D) | A-EP (R) | A-EP (R+D) |
|----------|----------|----------|----------|------------|
| # trajectories | 500 | 20 | 20 | 20 | 100 |
| # steps in each epoch | 50 | 50 | 50 | 50 | 50 |
| # epoches | 10 | 100 | 100 | 100 | 100 |

*Table 2. The selected hyper-parameters for model estimation in Half-Cheetah experiment.*

| Sawyer-Reaching | Sawyer-Peg |
|-----------------|------------|
| # trajectories | 500 | 20 |
| # steps in each epoch | 150 | 40 |
| # epoches | 10 | 100 |

*Table 3. The selected hyper-parameters for model estimation in Sawyer experiments.***
|                   | CONT (D) | W-EP (D) | A-EP (R+D) |
|-------------------|----------|----------|------------|
| # trajectories    | 500      | 50       | 80         |
| # steps in each epoch | 100      | 100      | 100        |
| # epoches         | 10       | 50       | 100        |

Table 4. The selected hyper-parameters for model estimation in Minitaur experiments.

C.3.2. POLICY LEARNING

In the Half-Cheetah, Sawyer-Reaching, and Minitaur experiments, we follow the learning rates selection for policy networks in (Xie et al., 2021). In Sawyer-Peg, for both actor and critic networks, we use 2-layer fully-connected networks. The number of neurons is 256. For all experiments, we use standard Gaussian to initialize the parameters of policy networks. The learning rate is $3 \times 10^{-4}$. The relay buffer capacity is 50,000. The number of batch size is 256.