Pontecorvo neutrino-antineutrino oscillations: theory and experimental limits

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Abstract

We study Pontecorvo neutrino-antineutrino oscillations both in vacuum and in matter within a field theoretic approach, showing that this phenomenon can occur only if neutrinos have a Dirac-Majorana mass term. We find that matter effects suppress these oscillations and cannot explain the solar neutrino problem. On the contrary, a vacuum neutrino-antineutrino oscillations solution to this problem exists. We analyze this solution and available data from laboratory experiments giving stringent limits on $\nu_e$ and $\nu_\mu$ Majorana masses.

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1 Introduction

One of the most striking problems in Elementary Particle Physics is that of neutrino mass. In the Standard Model by Glashow, Weinberg and Salam [1] neutrino are massless, but there is no physical explanation for this fact. On the contrary, in the framework of Grand Unified Theories massive neutrinos appear to be more natural [2] and, moreover, when quantum gravity effects are taken into account, a non vanishing neutrino mass term naturally arises [3].

Furthermore, massive neutrinos are also required in Cosmology and Astrophysics to explain the large scale structure and hydrogen ionization in the universe (see [4] and ref. therein) and peculiar velocities of pulsar [5]. From the experimental point of view, indications of a non-vanishing neutrino mass came from the so called “solar neutrino problem” [6] and atmospheric neutrino flux anomaly [7].

Actually, for example, the most simple and natural explanation for the solar $\nu_e$ flux deficit is given in term of neutrino flavour oscillations in the solar medium [8], according to the MSW mechanism [9], occurring only if neutrinos are massive.

From a phenomenological point of view, the problem of neutrino mass is strictly related to lepton number conservation (both individual ($L_e, L_\mu, L_\tau$) and total ($L$)), and the Dirac or Majorana nature of neutrinos themselves. In fact, if neutrinos are massless, the conservation of all lepton numbers ($L_e, L_\mu, L_\tau, L$) is allowed, according to the Glashow-Weinberg-Salam theory, due to the invariance of the electroweak lagrangian for an arbitrary (global) phase transformation of the matter field. Instead, if neutrino have a non-vanishing mass and are of the Dirac type (particle states different from the antiparticle ones), so that to the e.w. lagrangian the following mass term is added:

$$\mathcal{L}^D_m = - \sum_{l,l'} \nu_{l'R} M^D_{l'l} \nu_{lL} + h.c.$$ \hspace{1cm} (1)

with $l,l' = e, \mu, \tau$ and $M^D$, in general, a complex non diagonal 3x3 matrix, individual lepton number is no longer conserved ($L^D_m$ is not invariant for $\nu_l \rightarrow e^{i\alpha} \nu_l$), while the total lepton number is still conserved ($L^D_m$ is invariant for $\nu_l \rightarrow e^{i\alpha} \nu_l$). However, if neutrinos are massive Majorana particles (particle states coincide with antiparticles ones), so that the mass term is

$$\mathcal{L}^M_m = - \frac{1}{2} \sum_{l,l'} \bar{\nu}_{l'R} M^M_{l'l} \nu_{lL} + h.c.$$ \hspace{1cm} (2)

with $M^M$, in general, a non symmetric 3x3 matrix, no lepton number is conserved ($L^M_m$ is not invariant for every (global) phase transformation). Note that, while the term (2) does not require additional neutrino states besides those present in the Standard Model, the term (1) involves right-handed neutrino states (and related antiparticles) not present in the S.M. but predicted in many GUTs (see for example [2]).

The phenomenological consequences of eq. (1) or (2) are very intriguing.

For instance, if $M^D$ and $M^M$ are non diagonal (in analogy to what happens with the quark mass matrix), neutrino flavour oscillations are predicted [11]. But, furthermore, many other processes involving charged lepton, which violate lepton number conservation,
are allowed. For example, with Dirac neutrinos, the decays \( \mu \to e\gamma, \mu \to 3e, \tau \to e\pi^0 \) and the conversion \( \mu^- + Ti \to e^- + Ti \), can be realized, while with Majorana neutrinos neutrinoless double beta decay \( (Z,A) \to (Z+2,A) + 2e^- \) and reaction \( \mu^- + Ti \to e^+ + Ca \) can also occur.

More in general, one can consider a lagrangian mass term whith both Dirac and Majorana terms

\[
-L_{DM}^{\nu} = \sum_{l,l'} \bar{\nu}_{lR} M_{l'l}^D \nu_{lL} + \frac{1}{2} \sum_{l,l'} \bar{\nu}_{lR}^c M_{l'l}^1 \nu_{lL} + \frac{1}{2} \sum_{l,l'} \bar{\nu}_{lR} M_{l'l}^2 \nu_{lR} + h.c. \tag{3}
\]

involving \( \nu_L \) and \( \nu_R^c \), as well as \( \nu_R \) and \( \nu_L^c \) predicted in many GUTs. This scenario, from a theoretical point of view, allows to give a very small mass to neutrinos in a very natural way through the so-called “see-saw” mechanism [11]. The mass eigenstates coming from (3) are in general Majorana states, and the phenomenology previous described in the discussion of eq. (2) applies in this case as well. However, as we will show in this paper, another interesting phenomenon (also violating the total lepton number) is now possible: neutrino-antineutrino oscillations. Using field-theoretic methods, in section II we study neutrino-antineutrino oscillations in vacuum, redériving the formula for the survival probability first introduced by Pontecorvo [12]. In section III we shall extend the results found to keep into account matter effects for neutrinos propagating in a medium. In section IV we analyze the available experimental data and give limits on the \( \nu_e \) Majorana mass. Finally the conclusions.

2 Neutrino-antineutrino oscillations in vacuum

We add the Dirac-Majorana mass term (3) for neutrinos to the standard electroweak lagrangian. Since here we are interested only to neutrino-antineutrino oscillations, let us assume that the mass matrices appearing in (3) are diagonal and concentrate our attention on only one flavour at a time. So the propagation of a neutrino in vacuum is described by the lagrangian

\[
L = \bar{\nu} \gamma^\mu k \nu + \bar{\nu}^c \gamma^\mu k \nu^c - \frac{1}{2} m_D \left( \bar{\nu} \nu + \bar{\nu}^c \nu^c \right) - \frac{1}{2} m_M \left( \bar{\nu} \nu^c + \bar{\nu}^c \nu \right) \tag{4}
\]

where \( k_\mu = (\omega,k) \) is the neutrino 4-momentum and \( m_D, m_M \) are respectively the Dirac and Majorana mass for the given flavour. The neutrino equations of motion are then:

\[
\begin{align*}
\bar{k} \nu - \frac{1}{2} m_D \nu - \frac{1}{2} m_M \nu^c &= 0 \tag{5} \\
\bar{k} \nu^c - \frac{1}{2} m_D \nu^c - \frac{1}{2} m_M \nu &= 0 \tag{6}
\end{align*}
\]

equivalent to

\[
\begin{align*}
(k - m_+) \left( \nu + \nu^c \right) &= 0 \tag{7} \\
(k - m_-) \left( -\nu + \nu^c \right) &= 0 \tag{8}
\end{align*}
\]
where
\[ m_\pm = \frac{1}{2} (m_D \pm m_M) \]  
(9)

From (8), (8) it is easy to recognize that the mass eigenstates are Majorana states, \((\nu + \nu^C)\) and \((-\nu + \nu^C)\), with \(C = +1\) and \(C = -1\) respectively.

Let us now adopt the chiral Weyl base for the Dirac gamma matrices and define:
\[
\begin{align*}
\nu &= \left( \begin{array}{c} \nu_R \\ \nu_L \end{array} \right) \\
\nu^C &= \left( \begin{array}{c} \nu^C_L \\ \nu^C_R \end{array} \right)
\end{align*}
\]  
(10)

We introduce the helicity eigenstates
\[
\sigma \cdot k \phi_\lambda = \lambda \phi_\lambda
\]  
(11)

with \(\lambda = \pm 1\), and write
\[
\begin{align*}
\nu_R &= x \phi_\lambda \\
\nu_L &= y \phi_\lambda \\
\nu^C_L &= z \phi_\lambda \\
\nu^C_R &= w \phi_\lambda
\end{align*}
\]  
(12)

the equations of motion (8), (8) imply:
\[
\begin{align*}
\begin{pmatrix} \omega + \lambda k & -m_+ \\ -m_+ & \omega - \lambda k \end{pmatrix} & \begin{pmatrix} n_1 \\ n_2 \end{pmatrix} = 0 \\
\begin{pmatrix} \omega + \lambda k & -m_- \\ -m_- & \omega - \lambda k \end{pmatrix} & \begin{pmatrix} \tilde{n}_1 \\ \tilde{n}_2 \end{pmatrix} = 0
\end{align*}
\]  
(14)

where
\[
\begin{align*}
n_1 &= y + w \\
n_2 &= x + z \\
\tilde{n}_1 &= -y + w \\
\tilde{n}_2 &= -x + z
\end{align*}
\]  
(16)

From (14) (for 13) it suffices the replacements \(m_+ \to m_-\) and \(n_1, n_2 \to \tilde{n}_1, \tilde{n}_2\) one gets:
\[
\begin{align*}
\left( \omega^2 - k^2 - m_+^2 \right) n_1 &= 0 \\
n_2 &= \frac{\omega + \lambda k}{m_+} n_1
\end{align*}
\]  
(18)

so that the dispersion relations (in vacuum) for the \(C = \pm 1\) Majorana states are
\[ \omega^2 - k^2 = m_\pm^2 \]  
(20)

The time evolution is described by:
\[
\begin{align*}
| n_{1,2}(t) > &= e^{-i\omega_{1,2} t} | n_{1,2}(0) > \\
| \tilde{n}_{1,2}(t) > &= e^{-i\tilde{\omega}_{1,2} t} | \tilde{n}_{1,2}(0) >
\end{align*}
\]  
(21)

(22)
In the ultrarelativistic limit

\[
\omega_1 \simeq k + \frac{m_1^2}{2k} \quad (23)
\]
\[
\tilde{\omega}_1 \simeq k + \frac{m_1^2}{2k} \quad (24)
\]

From (21), (22) and (16) we may obtain, the time evolution of the left-handed weak-interacting neutrino:

\[
|y(t)\rangle = \frac{1}{2} \left( e^{-i\omega_1 t} + e^{-i\tilde{\omega}_1 t} \right) |y(0)\rangle + \frac{1}{2} \left( e^{-i\omega_1 t} - e^{-i\tilde{\omega}_1 t} \right) |w(0)\rangle \quad (25)
\]

If at \( t = 0 \) we produce (through a weak-interaction process) a left-handed neutrino, at later times there is a non-vanishing probability of detecting (again through a weak-interaction process) the corresponding antiparticle, i.e. a right-handed antineutrino. The survival probability of the initial \( \nu_L \), \( P(\nu_L \rightarrow \nu_L) = | < y(0) | y(t) > |^2 \), is given by

\[
P(\nu_L \rightarrow \nu_L) = \frac{1}{2} \left( 1 + \cos (\omega_1 - \tilde{\omega}_1) t \right) = \\
\simeq 1 - \sin^2 \frac{m_D m_M}{4k} t \quad (26)
\]

and exhibits a typical oscillatory behaviour. The formula (26) was first introduced by Pontecorvo [12] in analogy to what happens in the \( K^0 - \bar{K}^0 \) system and, as we have shown, can be derived for neutrinos with both Dirac and Majorana mass terms. From (26) we emphasize that both \( m_D \) and \( m_M \) must be non-vanishing for neutrino-antineutrino oscillations to occur.

### 3 Neutrino-antineutrino oscillations in matter

When neutrinos propagate in a medium, their self-energy, acquired through the interaction with the particles in the medium, must be taken into account in writing the equations of motion; substantially, this is done with the replacements of the mass terms with the self-energy terms, as explained for example in [13]. For non-magnetized media (the extension to media with a magnetic field is straightforward following [13]), at first order in the Fermi coupling constant \( G_F \), the self-energies are given by [13, 14]

\[
\Sigma'_{\nu_L} = b_L \mu \frac{1 - \gamma_5}{2} \quad (27)
\]
\[
\Sigma'_{\nu_R} = 0 \quad (28)
\]
\[
\Sigma'_{\bar{\nu}_R} = -b_L \mu \frac{1 + \gamma_5}{2} \quad (29)
\]
\[
\Sigma'_{\bar{\nu}_L} = 0 \quad (30)
\]
where \( u_\mu \) is the medium 4-velocity (we will consider the medium rest frame, \( u_\mu = (1, 0) \)) and

\[
-b_L = \sqrt{2} G_F \left( N_e - \frac{1}{2} N_n \right)
\]  
(31)

for the electron flavour, while

\[
-b_L = \frac{G_F}{\sqrt{2}} N_n
\]  
(32)

for the \( \mu \) and \( \tau \) flavours, \( N_e \) and \( N_n \) being the electron and neutron number density of the medium respectively. The equations of motions are:

\[
(k - m_+) \nu_R + (k - m_+ + b_L \lambda) \nu_L + (k - m_+) \nu_R^C +
(k - m_+ - b_L \lambda) \nu_L^C = 0
\]  
(33)

\[
-(k - m_-) \nu_R - (k - m_- + b_L \mu) \nu_L + (k - m_-) \nu_R^C +
(k - m_- - b_L \mu) \nu_L^C = 0
\]  
(34)

and, explicitly, in the chiral Weyl base,

\[
\begin{pmatrix}
-\omega + b_L + \sigma \cdot k \\
\omega - \sigma \cdot k
\end{pmatrix}
\begin{pmatrix}
\nu_R \\
\nu_L
\end{pmatrix}
\]

\[
+ \begin{pmatrix}
-\omega + b_L + \sigma \cdot k \\
\omega - \sigma \cdot k
\end{pmatrix}
\begin{pmatrix}
\nu_R^C \\
\nu_L^C
\end{pmatrix}
\]

\[
= 0
\]  
(35)

\[
\begin{pmatrix}
\omega + \lambda k \\
\omega - \lambda k
\end{pmatrix}
\begin{pmatrix}
n_1 \\
\tilde{n}_1
\end{pmatrix}
\]

\[
- m_+ n_2 - b_L \tilde{n}_1 = 0
\]
(37)

\[
- m_+ n_1 + (\omega - \lambda k) n_2 = 0
\]
(38)

\[
(\omega + \lambda k) \tilde{n}_1 - m_- \tilde{n}_2 - b_L n_1 = 0
\]
(39)

\[
- m_- \tilde{n}_1 + (\omega - \lambda k) \tilde{n}_2 = 0
\]
(40)

Introducing now, as above, the helicity eigenstates, in the (well verified) approximation that the interaction with the medium does not change neutrino helicity, the equations (33), (34) take the form

\[
\begin{align*}
(\omega + \lambda k) n_1 - m_+ n_2 - b_L \tilde{n}_1 &= 0 \\
- m_+ n_1 + (\omega - \lambda k) n_2 &= 0 \\
(\omega + \lambda k) \tilde{n}_1 - m_- \tilde{n}_2 - b_L n_1 &= 0 \\
- m_- \tilde{n}_1 + (\omega - \lambda k) \tilde{n}_2 &= 0
\end{align*}
\]  
(37-40)

With simple algebra we can write these equations as

\[
\begin{align*}
\left( \omega^2 - k^2 - m_+^2 \right) n_2 &= b_L \frac{m_+}{m_-} \left( \omega - \lambda k \right) \tilde{n}_2 \\
\left( \omega^2 - k^2 - m_-^2 \right) \tilde{n}_2 &= b_L \frac{m_-}{m_+} \left( \omega - \lambda k \right) n_2 \\
n_1 &= \frac{\omega - \lambda k}{m_+} n_2 \\
\tilde{n}_1 &= \frac{\omega - \lambda k}{m_-} \tilde{n}_2
\end{align*}
\]  
(41-44)
These equations admit non trivial solutions only if
\[
(\omega^2 - k^2 - m_1^2) (\omega^2 - k^2 - m_-^2) = b_L^2 (\omega - \lambda k)^2
\] (45)

Eq. (45) is the dispersion relation for Dirac-Majorana neutrinos propagating in matter.

Let us now focus on \(\lambda = -1\) case; in the ultrarelativistic limit, the dispersion relation (45) can be viewed as the eigenvalue equation for the "effective hamiltonian"
\[
H_{\text{eff}} = k + \frac{1}{2k} \begin{pmatrix} m_1^2 & 0 \\ 0 & m_-^2 \end{pmatrix} + \begin{pmatrix} 0 & b_L \\ b_L & 0 \end{pmatrix}
\] (46)
in the base \(\begin{pmatrix} n_1 \\ \tilde{n}_1 \end{pmatrix}\).

The time evolution of this states is then obtained by the Schrödinger equation
\[
i \frac{d}{dt} \begin{pmatrix} n_1 \\ \tilde{n}_1 \end{pmatrix} = H_{\text{eff}} \begin{pmatrix} n_1 \\ \tilde{n}_1 \end{pmatrix}
\] (47)

For the sake of simplicity, we will consider a constant density medium; then the hamiltonian in (47) can be diagonalized by a matrix
\[
U = \begin{pmatrix} \cos \theta_m & \sin \theta_m \\ -\sin \theta_m & \cos \theta_m \end{pmatrix}
\] (48)
(neglecting CP violating effects) depending on a constant mixing angle between \(n_1\) and \(\tilde{n}_1\). The eigenstates are then
\[
\begin{pmatrix} n_{1m} \\ \tilde{n}_{1m} \end{pmatrix} = \begin{pmatrix} \cos \theta_m & -\sin \theta_m \\ \sin \theta_m & \cos \theta_m \end{pmatrix} \begin{pmatrix} n_1 \\ \tilde{n}_1 \end{pmatrix}
\] (49)
and the matrix \(U^T H_{\text{eff}} U\) becomes diagonal when the mixing angle is given by
\[
\tan 2\theta_m = -\frac{4b_L k}{m_D m_M}
\] (50)

The time evolution of the physical matter eigenstates is
\[
| n_{1m}(t) > = e^{-i\omega_- t} | n_{1m}(0) > \\
| \tilde{n}_{1m}(t) > = e^{-i\tilde{\omega}_+ t} | \tilde{n}_{1m}(0) >
\] (51)
(52)
and the eigenvalues are:
\[
\omega_{\pm} = k + \frac{m_D^2 + m_M^2}{8k} \pm \frac{1}{2} \sqrt\left(\left\frac{m_D m_M}{2k}\right^2 + 4b_L^2\right)
\] (53)
From the time dependence of the weak-interaction eigenstate $\nu_L$,

$$|y(t)\rangle = \frac{1}{2} \left( (e^{-i\omega_- t} + e^{-i\omega_+ t}) + \sin 2\theta_m \left( e^{-i\omega_- t} - e^{-i\omega_+ t} \right) \right) |y(0)\rangle +$$

$$+ \frac{1}{2} \cos 2\theta_m \left( e^{-i\omega_- t} - e^{-i\omega_+ t} \right) |w(0)\rangle$$

we can finally obtain the expression for the survival probability for an initial produced $\nu_L$:

$$P(\nu_L \to \nu_L) = 1 - \cos^2 2\theta_m \sin^2 \frac{\omega_+ - \omega_-}{2} t =$$

$$\simeq 1 - \left( 1 + \left( \frac{4b_L k}{m_D m_M} \right)^2 \right)^{-1} \sin^2 \left( \sqrt{\left( \frac{m_D m_M}{2k} \right)^2 + 4b_L^2 t} \right)$$

We immediately note, from (55), a completely different behaviour of neutrino-antineutrino oscillations in matter with respect to the case of flavour oscillations. In fact, matter neutrino-antineutrino oscillations have both an amplitude and a period (and then an oscillation length) smaller than those corresponding to vacuum oscillations. In other words, opposite to what happens for flavour oscillations, neutrino-antineutrino oscillations in matter are always suppressed with respect to the vacuum case; furthermore, no resonance condition (analogous to the MSW effect [9]) can occur.

### 4 Limits on the Majorana neutrino mass

The fact that neutrino-antineutrino oscillations in matter are suppressed with respect to the vacuum makes this phenomenon pratically non-interesting for astrophysical scopes. In particular, there cannot be a matter solutions in terms of neutrino-antineutrino oscillations to the solar neutrino problem [4]. This can be easily seen from the following arguments. The amplitude $\cos^2 2\theta_m$ in (55) is lower for more energetic neutrinos (such as those produced by the $Be^7$ and $B^8$ reactions in the sun [13]) than for less energetic ones (such as those from the p-p chain). So one predicts a smaller reduction for $Be^7$ and $B^8$ neutrinos than for p-p neutrinos. This is just the opposite of what is needed to explain the experimental results [16], so the matter effects on neutrino-antineutrino oscillations cannot help to solve the solar neutrino problem.

On the contrary, the analysis of the experimental data on solar neutrino [3] shows that a vacuum neutrino-antineutrino oscillations solution is indeed possible with the following value of the product of Dirac and Majorana $\nu_e$ masses:

$$m_D m_M \simeq 6 \times 10^{-11} \text{eV}^2$$

Note that this solution is equivalent to the vacuum flavour oscillation solution with maximal (flavour) mixing angle ($\sin^2 2\theta \simeq 1$) and $\Delta m^2 = m_{\nu_2}^2 - m_{\nu_1}^2 \simeq 6 \times 10^{-11} \text{eV}^2$ [17].
However, the best probe for neutrino-antineutrino oscillations (in vacuum) is with laboratory experiments. We have analyzed the current data on disappearance laboratory experiments for giving limits on \(m_D \cdot m_M\). The most stringent limit for the electron flavour comes from the experiment at the Krasnoyarsk reactor \[18\]; with the formula (26) for the survival probability we have obtained

\[
m_{\nu_e}^D \cdot m_{\nu_e}^M \leq 7.5 \times 10^{-3} \text{eV}^2 \quad (90\% \text{C.L.}) \quad (57)
\]

For the muon flavour, experiments allow both low and large values for \(m_{\nu_\mu}^D \cdot m_{\nu_\mu}^M\) (see \[19\]); from the Particle Data Group analysis \[19\] we deduce

\[
m_{\nu_\mu}^D \cdot m_{\nu_\mu}^M \leq 0.23 \text{eV}^2 \quad (58)
\]

or

\[
m_{\nu_\mu}^D \cdot m_{\nu_\mu}^M \geq 1500 \text{eV}^2 \quad (59)
\]

Given the form (26) for the survival probability, from the experiments one can only deduce limits on the product of the Dirac times the Majorana mass. However, if the neutrino Dirac mass terms are generated by the same Higgs doublet giving mass to the charged fermion (as happens in G.U.T.), it is natural to assume that \(m_D\) is of the same order of magnitude of the (Dirac) mass of the charged fermion associated to the given neutrino in the same supemultiplet (for example, in SO(10) \(\nu_e, \nu_\mu, \nu_\tau\) are linked to the up-quarks \(u, c, t\)). Assuming then \(m_{\nu_\mu}^D \approx O(100 \div 1000 \text{MeV})\), from (58), (59) one can deduce the following indicative limits on \(\nu_e\) and \(\nu_\mu\) Majorana masses:

\[
m_{\nu_e}^M \leq 10^{-8} \text{eV} \quad (60)
\]

and

\[
m_{\nu_\mu}^M \leq 10^{-9} \text{eV} \quad (61)
\]

or

\[
m_{\nu_\mu}^M \geq 10^{-6} \text{eV} \quad (62)
\]

Note that the limit for \(m_{\nu_\mu}^M\) is smaller by several orders of magnitude than the ones obtained from neutrinoless double \(\beta\) decay (some eV) \[20\].

5 Conclusions

In this paper we have analyzed the theory of Pontecorvo neutrino-antineutrino oscillations in vacuum and in matter, showing that they can occur only if neutrinos are Majorana particles with both Dirac and Majorana masses non-vanishing. With a field-theoretic approach we have rederived the Pontecorvo formula (eq. (26)) for vacuum oscillations and then generalized this formula to matter oscillations, finding that neutrino-antineutrino transitions in matter are suppressed with respect to vacuum ones.
Furthermore, we have shown that for solar neutrinos, the matter effects are not in agreement with the experimental data, so that matter Pontecorvo oscillations does not explain the solar neutrino problem. However, a vacuum solution to this problem in terms of neutrino-antineutrino transitions indeed exists, and we give the corresponding range of the product of $\nu_e$ Dirac and Majorana mass.

The existing limits from laboratory experiments for this product for electron and muon flavours have also been analyzed, leading to limits on $\nu_e$ Majorana mass (in the framework of Grand Unified Theories) several orders of magnitude smaller than the one found from the experiments on neutrinoless double beta decay.

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References

[1] S.L.Glashow, *Nucl. Phys.* **22** (1961) 579
S.Weinberg, *Phys. Rev. Lett.* **19** (1967) 1264
A.Salam, “Weak and Electromagnetic interactions”, in Svartholm N., *Elementary Particle Theory*, Stockholm, Almquist and Wiksell, 1968

[2] M.F.Abud, F.Buccella, L.Rosa and A.Sciarrino, *Zeit. für Phys.* **C 44** (1989) 589;
F.Buccella and L.Rosa, *Nucl. Phys B (Proc. Suppl.) 28A*, (1992) 168;
R.N. Mohapatra and M.K. Parida, *Phys. Rev. D* **47** (1993) 264.

[3] R.Barbieri, J.Ellis and M.K.Gaillard, *Phys. Lett. B* **90** (1980) 249.

[4] A.Yu.Smirnov, preprint [hep-ph/9611465].

[5] A.Kusenko and G.Segre, *Phys. Rev. Lett.* **77** (1996) 4872.

[6] R.Davis Jr., Proc. of the 23rd ICRC, Calgary, Canada (1993), *Prog. in Nucl. and Part. Phys.*, **32** (1994);
K.Hirata *et al.*, *Phys. Rev. D* **44** (1991) 2241;
A.Abazov *et al.*, *Phys. Rev. Lett.* **67** (1991) 3332;
P.Anselmann *et al.*, *Phys. Lett.*, **B 327** (1994) 377.

[7] Y.Fukuda *et al.*, *Phys. Lett.*, **B 335** (1994) 237;
R.Becker-Szendy *et al.*, *Phys. Rev. D* **46** (1992) 3720.

[8] S.T.Petcov and A.Yu.Smirnov, *Phys. Lett. B322* (1987) 109;
S. Esposito, *Mod. Phys. Lett., A* **8** (1993) 1557;
N.Hata and P.Langacker, *Phys. Rev., D 50* (1994) 632.
[9] L. Wolfenstein, *Phys. Rev.*, **D 17** (1978) 2369;  
S.P. Mikheyev and A.Yu. Smirnov, *Il Nuovo Cimento*, **9 C** (1986) 17;  
*Sov. J. Nucl. Phys.*, **42** (1986) 913; *Sov. Phys. Usp.*, **30** (1987) 759

[10] V. Gribov and B. Pontecorvo, *Phys. Lett B* **28** (1969) 493;  
S. Bilenky and B. Pontecorvo, *Phys. Rep.* **41** (1978) 225.

[11] T. Yanagida, in “Proc. of Workshop on Unified Theory and Barion Number of the Universe” (KEK, Japan), 1979,  
M. Gell-Mann, P. Ramond and R. Slansky, in *Supergravity* (eds. P. Van Nieuwenhuizen, D. Z. Freedman), 315 (1979, Amsterdam, North Holland).

[12] B. Pontecorvo, *Sov. Phys. JETP* **VI** (1958) 429.

[13] S. Esposito and G. Capone, *Zeit. für Physik C* **70** (1996) 55, hep-ph/9511417.

[14] S. Esposito, *Il Nuovo Cimento* **111** (1996) 1449, hep-ph/9607371.

[15] J. N. Bahcall, *Neutrino Astrophysics*, (Cambridge University Press, 1989).

[16] J. N. Bahcall, invited talk at the symposium “The Inconstant Sun”, Naples, March 18, 1996; astro-ph/9606161.

[17] J. N. Bahcall and P. L. Krastev, *Phys. Rev. D* **53** (1996) 4211;  
Z. G. Berezhiani and A. Rossi, *Phys. Lett. B* **367** (1996) 219.

[18] G. S. Vidyakin *et al.*, *JETP Letters* **59** (1994) 390.

[19] R. M. Barnett *et al.*, *Phys. Rev. D* **54** (1996) 1.

[20] M. K. Moe, *Nucl. Phys. B (Proc. Suppl.)* **38** (1995) 36.