A Reliability Estimation of a CMTSC Considering Dynamic Clamping Force

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Abstract. The reliability of coil type manual three-jaw self-centering chucks (CMTSCs) of machine tools impacts significantly on reliability and safe operations of machine tools. This paper carries out a detailed reliability estimation of a CMTSC, in which the dynamic clamping force of the chuck is considered. Initially, a mathematical model of dynamic clamping force of the CMTSC is established after analyzing forces of key components of the CMTSC. Subsequently, Taking the dynamic clamping force of the jaw as the generalized stress and the minimum clamping force specified by the national standard as the generalized strength a multi-function formula containing \( n \)-dimensional random variables is established based on the stress-strength interference model. Then, an improved first-order second-moment method is adopted to solve the formula already mentioned as a basis of that to calculate the reliability of the dynamic clamping force of the chuck. The results of this study indicate that the maximum rotating speed of the CMTSC is 1000 rpm, accordingly, the reliability of the CMTSC meets the requirements of the machining. The results of the paper have merits in reliability analysis, extreme rotational speed determination, and structural optimization of the CMTSC.

1. Introduction
Chucks are mechanical devices used to clamp the workpiece of machine tools, which impact significantly on the performance, accuracy, and reliability of machine tools. Benefit from advantages like easy-to-adjust, low price, easy centering, etc., coil type manual three-jaw self-centering chucks (CMTSCs) are extensively assembled to various machine tools such as lathes, grinders, and machining centers than other types of chucks [1].

The clamping force of CMTSCs is dynamic and decreases in terms of the increase of the rotational speed of CMTSCs. In some extreme cases, as a consequence of insufficient clamping force, workpieces that been machining will be thrown away from jaws and which gives rise to sever safety accidents. To date, experience-based methods are maturely applied by manufacturers to design CMTSCs. However, because of lacking a deep understanding of operations as well as failure features, the designed clamping force of CMTSCs tend to be too small to guarantee the safe and accuracy of the machining.

In term of published researches available about clamping force of CMTSCs, Feng et al. [2] analyzed the influence of high-speed rotary chucks and workpieces on the dynamic clamping force using multiple methods including theoretical analysis, finite element (FE) calculation, and experimental method. Wang
et al. [3] established a FE model as a substitution of the existing complicated dynamic clamping force model of CMTSCs, the feasibility of the proposed FE model was validated by experiments. Overall, mechanical properties of chucks are investigated extensively such as high-speed power chuck [4], wedge-type power chuck [5-6], and four-jaw manual chuck [7]. Such investigations mainly focused on dynamic clamping force. Zhang et al. [8] designed a wireless sensor system which could achieve real-time detection of chuck dynamic clamping force and can be used to achieve a more accurate relationship between clamping force and chuck speed. To solve the problems such as poor positioning accuracy and non-automatic processing, Liang et al. [9] proposed a fixture structure scheme with a new type of double collect chuck (DCC) and analyzed its design rationality, clamping stability, and repeat clamping positioning accuracy. Wang et al. [10] proposed a general prediction model and revealed changing laws of residual force with time for Coulomb and Johnsen-Rahbek types of bipolar electrostatic chucks. Tao et al. [11] optimized chuck/tool model, concluded that the concentrated stress in hydraulic chuck cavity corner transition region and contact pressure peak been decreased, at the same time, the reliability and stability of the high-speed hydraulic chuck were improved. Görög et al. [12] analyzed the influence of clamping forces on the roundness of turned pipes, of which a universal three-jaw chuck tightened by a torque wrench was used. The experimental results showed that roundness depends on the clamping forces implemented. Yannikov et al. [13] determined the errors of thin-walled pipe arising by three-jaw chucks, concluded that the workpiece diameter was decreased by a factor of about 0.0004. However, to date, no publications about the reliability of CMTSCs is available.

Researches on the reliability analysis based on stress-strength interference model have been presented materially. For instance, Wei et al. [14] combined the residual strength and stress-strength interference (SSI) model to analyze the reliability of the gears in a mechanical system. Based on the copula function, the correlation between the gear contact and bending failures were determined. Gao et al. [15] developed time-dependent reliability models, failure rate models, and availability models of belt driven systems based on the system dynamic equations with the dynamic stress and the material property degradation was considered. Olatubosun et al. [16] presented a coupled thermal-hydraulic reliability approach based on SSI model and applied it to the reliability analysis of a natural circulation system (NCS). Che et al. [17] predicted the dynamic reliability of shearer permanent magnet semi-direct drive gear through the modified SSI model.

Hence, this paper puts emphasis on the reliability estimation of a CMTSC. The model of the CMTSC is K11250 produced by Hohhot Zhonghuan (Group) Co., Ltd. The reliability computed is associate to the limit speed of the CMTSC and which will be a guide to achieve a safe and reliable performance of the CMTSC.

The rest of this paper is organized as follows. The mathematical model of the dynamic clamping force of the CMTSC is established in Section 2. The reliability estimation model of the CMTSC is introduced in Section 3. Case study for validating the proposed method is provided in Section 4. Section 5 demonstrates conclusions.

2. The mathematical model of dynamic clamping force of the CMTSC

The CMTSC (K11250) considered in this study is produced by Hohhot Zhonghuan (Group) Co., Ltd. see Figure 1. In detail, the CMTSC is comprised of multiple components including a wrench, a disc body, a bevel gear, a coil wire, a claw, and a gland. All components of the CMTSC work jointly to accomplish the designed function that automatic centering, clamping, and loosening workpieces. Specifically, the wrench drives directly rotation of the bevel gear and subsequently transforms the it to coil wire, the involute plane thread of the coil wire meshes with the tooth arc of the claw, so that the three claws can move in or out at the same time.
2.1. Force analysis of bevel gear

Assuming that the small bevel gear rotates evenly under the constant drive of the wrench. The force diagram of bevel gear is displayed in Figure 2. Friction between bevel gears is neglected due to the lubricating reason. Denote $M_n$ is the driving torque of the wrench, $M_{f1}$ ($M_{f2}$) is the frictional torque between bevel gear large end (small end) and disc. Moreover, $F_{n1}$ represents the normal force of the bevel gear which can be decomposed into circumferential force $F_{t1}$, radial force $F_{r1}$, and axial force $F_{a1}$. $F'_1$ is the composition of forces $F_{t1}$ and $F_{r1}$. $F_{N1}$, $F_{N2}$ are supporting forces of large and small ends of the bevel gear under the collective effect of $F_{t1}$ and $F_{r1}$. $R_1$ and $r_1$ denote indexing circle radius of large and small ends of the bevel gear, respectively. $r_{m1}$ is the indexing circle radius at the midpoint of gear tooth, $\alpha$ and $\delta_1$ are the pressure angle as well as graduated cone angle. $L_1$ and $L_2$ are axial distance from the midpoint of the pinion tooth to the point where the reaction force of the large and small ends of the bevel gear is supported.

According to Figure 2, the balanced equations can be achieved as follows.

$$F_{N1} + F_{N2} - F'_1 = 0$$  \hspace{1cm} (1)

$$F_{N2} L_2 - F_{N1} L_1 = 0$$  \hspace{1cm} (2)

$$M_n - M_{f1} - M_{f2} - F_{t1} r_{m1} = 0$$  \hspace{1cm} (3)

In which,

$$F'_1 = \sqrt{\left(F_{t1}^2 + F_{r1}^2\right)}$$  \hspace{1cm} (4)
It can be known from the force analysis of a pair of bevel gears, that:

\[ F_{11} = F_{11} \tan \alpha \cos \delta \]  \hspace{1cm} (5) \\
\[ F_{11} = F_{11} \tan \alpha \sin \delta \]  \hspace{1cm} (6)

Combine equations (1), (4), and (5), gives:

\[ F_{N1} + F_{N2} \cdot F_{11} \left(1 + \tan \alpha \cos \delta \right)^{1/2} = 0 \]  \hspace{1cm} (7)

The indexing circle radius at the midpoint of gear tooth width is:

\[ r_{m1} = R_i - \frac{b_1}{2} \tan \delta \]  \hspace{1cm} (8)

where, \( b_1 \) reflects small bevel gear width.

Based on the shaft diameter friction formula, equations (9) and (10) are obvious.

\[ M_{f1} = F_{N1} R_i f_v \]  \hspace{1cm} (9)
\[ M_{f2} = F_{N2} R_i f_v \]  \hspace{1cm} (10)

In equations (9) and (10), \( f_v \) is equivalent friction coefficient. \( f_v = \pi f_i / 2 \) and \( f_v = f_i \) if two elements of moving pair are semi-cylindrical surfaces and are in a single plane contact, respectively. \( f_i \) is the friction coefficient between bevel gear and contact surface of the disc.

Combine equations (3), (8), (9), and (10), equations (11) can be calculated as:

\[ M_{n} - \frac{\pi}{2} f_i \left(F_{N1} R_i + F_{N2} R_i \right) - F_{11} \left(R_i - \frac{b_1}{2} \tan \delta \right) = 0 \]  \hspace{1cm} (11)

Then, \( F_{11} \) can be obtained as:

\[ F_{11} = \frac{M_{n} \left(L_1 + L_2 \right)}{\frac{\pi}{2} f_i \left(R_i L_2 + r_i L_1 \right) \left(1 + \tan \alpha \cos \delta \right)^{1/2} + \left(L_1 + L_2 \right) \left(R_i - \frac{b_1}{2} \tan \delta \right)} \]  \hspace{1cm} (12)

2.2. Force analysis of coil wire

The force diagram of the coil wire is displayed in Figure 3 [18]. The large bevel gear at the lower end of the wire rotates counterclockwise because of the normal force of the small bevel gear \( F_{a2} \). Let the action point of \( F_{a2} \) be the midpoint of the gear width and which is a composition of circumferential force \( F_{c2} \), radial force \( F_{r2} \), and axial force \( F_{a2} \), and \( F_{a2} = F_{a1}, F_{a2} = F_{a1}, F_{a2} = F_{a1}, F_{a2} = F_{a1} \). The flat thread on the upper end of the wire is subject to the normal reaction force \( F_{a2} \) (a composition of radial force \( F_{r2} \) and tangential stress \( F_{t2} \)) of the three-jaw tooth arcs, and the friction force \( F_{f2} \) (a composition of radial force \( F_{r2} \) and tangential force \( F_{t2} \)) between the wire and the jaw. The wire is also affected by the frictional moment \( M_{f2} \) of the disk body, the end face of the wire generated by \( F_{a2} \), and the reaction force \( F_{f2} \) (can be deposited into radial force \( F_{r2} \) and tangential force \( F_{t2} \)) of the total support of the disc body generated by \( F_{r2} \) and \( F_{t2} \). \( L_3 \) is the distance between the meshing point of the flat thread of the wire and the jaw arc to the center of rotation of the wire, \( \rho \) represents the friction circle radius of the inner hole of the disc wire and shaft diameter of the disc body. \( r_2 \) reflects coil outer diameter. Moreover, \( r_2 \) is the coil wire inner diameter, \( r_{m2} \) denotes an indexing circle radius at the midpoint of the tooth width of the coiled bevel gear. \( \delta_2 \) and \( \theta \) are large bevel gear indexing cone angle as well as the spiral rising angle at the point where the coil wire meshes with the jaw arc, respectively.
Figure 3. The force diagram of the coil wire

Assume that the large bevel gear rotates evenly under the constant drive of the small bevel gear. Accordingly, the balance equation to the force analysis (see Figure 3) can be molded as:

\[ F_{r2} - F_{r2} = 0 \]  \hspace{1cm} (13)

\[ F_{r2} - F_{r2} = 0 \]  \hspace{1cm} (14)

\[ R_{m2} - 3F_{r32}L_{a3} - 3F_{r2}L_{a3} - F_{r32}r - M_{f2} = 0 \]  \hspace{1cm} (15)

where, \( \rho = f_2 r_2 = \pi f_2 r_3 / 2 \), \( f_2 \) represents the coefficient of friction between the inner hole of the coil wire and the contact surface of the disc body. Friction torque between the disc body and the end face of the wire \( M'_{f2} \) can be expressed as:

\[ M'_{f2} = f_3 F_{a2} r_{a2} \]  \hspace{1cm} (16)

In which, \( f_3 \) denotes the coefficient of friction between the end face of the wire and the body. The indexing circle radius at the midpoint of the tooth width of the coiled bevel gear is expressed by:

\[ r_{m2} = R_2 - \frac{b_2}{2} \tan \delta_2 \]  \hspace{1cm} (17)

where, \( b_2 \) reflects large bevel gear width. According to equations (16) and (17), equation (18) can be obtained.

\[ M_{f2} = f_3 F_{a2} \left( R_2 - \frac{b_2}{2} \tan \delta_2 \right) \]  \hspace{1cm} (18)

Considering that \( F_{r0} = f_4 F_{r32}, f_4 \) is the coefficient of friction between wire and jaw, and \( F_{r0} = F_{r2} \cos \theta, F_{r32} = F_{r32} \cos \theta, \) the following relations can be reached:

\[ F_{r2} = f_4 F_{r32} \]  \hspace{1cm} (19)

moreover,

\[ F_{r32} = F_{r32} \tan \theta \]  \hspace{1cm} (20)
According equations (13)-(15), and (18) - (21), \( F_{\gamma 32} \) can be accessed by:

\[
F_{\gamma 32} = \frac{F_{t2}}{3L_\alpha (\tan \theta + f_s)} \left[ \frac{1}{1 - f_s \tan \alpha \cos \delta} \right] \left[ \frac{1}{R_2 - \frac{b_2}{2} \tan \delta_2} \right] - \frac{\pi}{2} f_s r_2 \left[ \frac{1}{1 + \tan^2 \alpha \cos^2 \delta} \right] \right]^{1/2} \tag{22}
\]

2.3. Force analysis of the claw

The force diagram of the claw is demonstrated in Figure 4 [18]. Under the action of the wire driving force \( F_{r23} \) and the workpiece clamping reaction force \( F \), the jaw generates a counterclockwise tilting moment, which makes the jaw contact the groove of the disc body at two points \( A \) and \( B \) (the contact point is subject to positive pressure \( F_{NA} \), \( F_{NB} \) and friction force \( f_5 F_{NA} \), \( f_5 F_{NB} \)). \( f_5 \) is the friction coefficient between the claw and the groove surface of the disc body. The distance between \( A \) and \( B \) along the \( x \)-axis and \( y \)-axis are represented by \( L_4 \) and \( L_5 \), respectively. \( L_6 \) is the distance (in the \( y \)-direction) between the point the clamping reaction force \( F \) applied and point \( B \). \( L_7 \) is the distance between the point \( B \) and the line of action of the wire driving force \( F_{r23} \).

**Figure 4.** The force diagram of the claw

Set the coil wire rotates at a constant speed to drive the three-jaws to tighten the workpiece in the radial direction, and establish a rectangular coordinate system as shown in Figure 4. According to the analysis, the balanced equation of the claw can be established as:

\[
F_{r23} - F - f_5 F_{NB} - f_5 F_{NA} = 0 \tag{23}
\]

\[
F_{NB} - F_{NA} = 0 \tag{24}
\]

\[
F_{r23} L_7 + F L_6 - F_{NA} L_4 - f_5 F_{NA} L_5 = 0 \tag{25}
\]

Combine equations (23), (24), and (25), the workpiece clamping reaction force \( F \) can be expressed as:

\[
F = F_{r23} \times \left[ 1 - \frac{2f_5 (L_6 + L_7)}{2f_5 L_6 + f_5 L_5 + L_4} \right] \tag{26}
\]

Combine equations (12), (22), and (26), knowing that \( F_{\gamma 2} = F_{\gamma 1} \), the numerical relation between input torque \( M_n \) and static output clamping force \( F \) can be obtained by:

\[
F = Z_1 Z_2 Z_3 M_n \tag{27}
\]

where
\[ Z_1 = \frac{L_1 + L_2}{\pi f_1 (R_1 L_2 + r_1 L_3) \left(1 + \tan^2 \alpha \cos^2 \delta_1\right)^{1/2} + (L_1 + L_2) \left(R_1 - \frac{b_1}{2} \tan \delta_1\right)} \] (28)

\[ Z_2 = \frac{(1 - f_5 \tan \alpha \cos \delta_5) \left(R_2 - \frac{b_2}{2} \tan \delta_2\right) - \pi f_2 R_5 \left(1 + \tan^2 \alpha \cos^2 \delta_5\right)^{1/2}}{3L_3 (\tan \theta + f_4)} \] (29)

\[ Z_3 = 1 - \frac{2f_5 (L_6 + L_7)}{2f_5 L_6 + f_2 L_3 + L_4} \] (30)

2.4. The dynamic clamping force model of the CMTSC

On one hand, the clamping force of the claw decreases with the increased centrifugal force generated by high rotational speed. On the other hand, the elastic restoring force generated by the bending deformation of the jaw cancels out part of the centrifugal force and weakens the decline of the clamping force. It should be highlighted that with the change of the rotation speed and the size of the bending deformation of the jaw, the clamping force in the working state changes constantly, as a consequence the clamping force that considered to be dynamic. The force diagram of the claw under the working condition is demonstrated in Figure 5. Generally, the force of the jaw along the y-axis is unchanged while the centrifugal force generated by the rotation of the jaw along the x-axis is modeled as:

\[ F_i = m \omega^2 r = m \left(\frac{2\pi n D}{60}\right)^2 \left(\frac{D}{2} + L_y\right) \] (31)

where, \( D, L_y, m, \) and \( n \) are workpiece diameter, distance from jaw centroid to the workpiece surface, jaw mass, spindle rotational speed, respectively.

![Figure 5. The force diagram of the claw under the working condition](image)

According to force analysis of the claw in the rotating state, the balance equations of the claw can be calculated by:

\[ F_{23} - F_d - f_s F_{NB} - f_s F_{NA} - F_e = 0 \] (32)

\[ F_{NB} - F_{NA} = 0 \] (33)

\[ F_{23} L_7 + F_d L_6 + F_e L_3 = F_{NB} L_4 - f_2 F_{NA} L_3 = 0 \] (34)

where, \( L_0 \) denotes the vertical distance between the centrifugal force acting point and the B point along the y-axis direction. Accordingly, the dynamic clamping force of the workpiece can subsequently be calculated as:
\[ F_d = F_{i,23} \times \left[ 1 - \frac{2f_s(L_0 + L_r)}{2f_sL_0 + f_sL_0 + L_4} \right] - F_i \left[ 1 + \frac{2f_s(L_0 - L_0)}{2f_sL_0 + f_sL_0 + L_4} \right] \]  

Combine equations (26), (31) and (35), the following equation can be obtained.

\[ F_d = F - \eta \left( \frac{2\pi n}{60} \right)^2 \left( \frac{D}{2} + L_0 \right) \times \left[ 1 + \frac{2f_s(L_0 - L_0)}{2f_sL_0 + f_sL_0 + L_4} \right] \]

where, \( F \) represents the static clamping force. The restraint form of the claw is a cantilever beam, which is bent and deformed by gravity and centrifugal action, and its elastic restoring force and centrifugal force resist so that the centrifugal force is reduced. Hence, the dynamic clamping force should be correlated by claw stiffness \( \zeta (\zeta < 1) \) [1]. On these bases, the correction value of the dynamic clamping force is expressed as:

\[ F_d = F - \zeta \left( \frac{2\pi n}{60} \right)^2 \left( \frac{D}{2} + L_0 \right) \times \left[ 1 + \frac{2f_s(L_0 - L_0)}{2f_sL_0 + f_sL_0 + L_4} \right] \]

Substituting equation (27) into equation (37), the relationship between input torque \( M_n \) and dynamic output clamping force \( F_d \) can be obtained as:

\[ F_d = Z_1Z_2Z_3M_n - Z_4 \]

where

\[ Z_4 = \zeta \left( \frac{2\pi n}{60} \right)^2 \left( \frac{D}{2} + L_0 \right) \times \left[ 1 + \frac{2f_s(L_0 - L_0)}{2f_sL_0 + f_sL_0 + L_4} \right] \]

3. The reliability model of the CMTSC considering dynamic clamping force

3.1. Stress strength interference (SSI) model

Generally, stress represents the factors that promote structural failure, while, structural strength represents the fatigue resistance of structures. A structure fails or not depends on the relationship between structural strength and stress, that is, the structure will be failed only if the stress is absolutely larger than the structural strength, otherwise the structure would be safe [19]. In terms of the CMTSC, the generalized stress is represented by dynamic clamping force been investigated above, and the generalized strength refers to the minimum clamping force that predetermined in the related national standard. According to equation (37), The dynamic clamping force of the jaw is closely related to the structure size of the chuck, the input torque of the wrench, and the spindle speed. These parameters are random variables, so generalized stress can be regarded as a random variable. Denote the generalized stress of the jaw is \( F_d \) and whose probability density function (PDF) is \( f(F_d) \), while, the generalized strength is \( F_l \) with a PDF of \( q(F_l) \). Hence, the difference between generalized stress and the generalized strength of the jaw \( Y \) can be computed as:

\[ Y = g(F_d, F_l) = F_d - F_l \]

Specifically, the CMTSC will be safe and can accomplish the designed function if \( Y \geq 0 \), otherwise, the malfunction of the CMTSC will occur. And further, define the reliability of the CMTSC as:

\[ R = P\left[ g(F_d, F_l) \geq 0 \right] = P(F_d \geq F_l) = P(F_d - F_l \geq 0) = \int_{-\infty}^{\infty} f(F_d) \int_{-\infty}^{F_d} q(F_l) dF_l dF_d \]
3.2. Reliability calculation base on the developed FOSM

The influencing factors of generalized stress include parameters of the disk body, bevel gear, wire, jaw as well as wrench input torque and spindle speed, etc., which are n-dimensional random variables. Accordingly, Y is an n-dimensional random variable. The multivariate function Y can be constructed as:

\[ Y = g(F_d, F_l) = F_d - F_l = (Z_i Z_n Z_m Z_k - Z_4) - F_l = g(x_1, x_2, \ldots, x_n) \]  

In equation (42), \( x_i (i = 1, 2, \ldots, n) \) is random variables that affecting the reliability of the jaw which related to features like dimension parameters of disc body, bevel gear, coil wire, jaw as well as input torque of wrench, the diameter of the clamped workpiece, and spindle speed. However, the SSI cannot apply to tackle the calculation of n-dimensional random variables directly, knowing that the multivariate function Y in nonlinear, hence, a developed first-order second-moment (FSOM) technique is implemented as a basis of that to compute the reliability of the CMTSC.

Let \( x_i \) follow normal distributions with the mean value of \( \mu_i \) and standard division of \( \sigma_i \). A procedure of conducting the developed FSOM technique is shown as follows [20].

1. Assign initial values to each random variable \( x_i^* = (i = 1, 2, \ldots, n) = \mu_i \).

2. Calculate the partial differential of the functional function \( \frac{\partial g}{\partial x_i} (i = 1, 2, \ldots, n) \) at the current value of each random variable.

3. Calculate the sensitivity coefficient \( \lambda_i (i = 1, 2, \ldots, n) \) by:

\[ \lambda_i = \frac{\sigma_i \frac{\partial g}{\partial x_i}}{\left[ \sum_{i=1}^n \left( \sigma_i \frac{\partial g}{\partial x_i} \right) \right]^{\gamma/2}} \]  

4. Calculate the reliability coefficient \( \beta \) of the function’s current value by:

\[ \beta = \frac{\mu_y}{\sigma_y} = \frac{g(x_1^*, x_2^*, \ldots, x_n^*) + \sum_{i=1}^n \left( \mu_i - x_i^* \right) \frac{\partial g}{\partial x_i}}{\left[ \sum_{i=1}^n \left( \sigma_i \frac{\partial g}{\partial x_i} \right)^2 \right]^{\gamma/2}} \]  

5. Calculate the updated value of \( x_i^* (i = 1, 2, \ldots, n) \) by:

\[ x_i^* = \mu_i - \beta_i \lambda_i \sigma_i \]  

Repeat the steps (2) to (5) until the difference between two generations of calculated values \( \beta \) is acceptable that is less than the tolerance \( \varepsilon_f \) predetermined. Once the \( x^*=(x_1^*, x_2^*, \ldots, x_n^*) \) was reached, the reliability of the dynamic clamping force of the jaw can be achieved by substituting the corresponding reliability coefficient \( \beta \) value into the standard normal distribution.

4. Case study

The CMTSC (K11250) is considered in this study that produced by Hohhot Zhonghuan (Group) Co., Ltd. Wrench driving torque \( M_s \), small bevel gear large end radius \( R_1 \), small end radius \( r_1 \), width \( b_1 \), wire outer diameter \( R_2 \), wire inner diameter \( r_2 \), large bevel gear width \( b_2 \), diameter \( D \) of clamping workpiece, jaw mass \( m \), and spindle speed \( n \) are random variables, and the average and standard deviation values are shown in Table 1. The axial distance between middle point of the tooth width of the small bevel gear and reaction force of the large and small ends of the gear (\( L_1 \) and \( L_2 \)) are 22.75mm and 34.75mm, respectively. The distance from the meshing point of the flat thread of the wire and the jaw arc to the
center of rotation of the wire \((L_3)\) is 67.5mm. The distance between \(A\) and \(B\) along the \(x\)-axis \((L_4)\) is 70mm, and along \(y\)-axis \((L_5)\) is 10 mm. The distance between the action point of the clamping force \(F\) and point \(B\) along the \(y\)-direction \((L_6)\) is 25mm, and the distance from the point \(B\) to the line of action of the wire driving force \(F_{z_2}\). \((L_7)\) is 20mm. The distance from jaw centroid to the workpiece surface \((L_8)\) is 48.81mm, and the vertical distance between jaw centroid and point \(B\) \((L_9)\) is 10.4mm. The graduated cone angles of the small and large bevel gears \((\delta_1\) and \(\delta_2)\) are 11.23° and 78.77°, respectively. The pressure angle of the bevel gears \((\alpha)\) is 20°, and the helix angle \(\theta\) is 1.28°. The friction coefficients \((f_1, f_2, f_3\) and \(f_4)\) are set to be 0.1, and \(f_4\) is set to be 0.13. The claw stiffness \(\zeta\) is 0.7 [1]. The related national standard [22] determined the minimum clamping force of K11250 CMTSC is \(F_l = 37\text{KN} \). Substituting the values of the random variables and known parameters into equations (28) - (30) and (39), attain:

\[
Z_1 = \frac{57.5}{63.2959x_2 + 3.7945x_4 - 5.7083x_4} \tag{46}
\]

\[
Z_2 = \frac{0.9643x_6 - 2.4284x_3 - 0.1575x_7}{30.8496} \tag{47}
\]

\[
Z_3 = 0.8816 \tag{48}
\]

\[
Z_4 = \left(3.6907 \times 10^{-3}x_9 + 0.3603\right)x_9x_{10}^2 \times 10^{-6} \tag{49}
\]

Substituting equations (46) - (49) and values of the known parameters into equation (42), \(Y\) can be obtained as:

\[
Y = \frac{1.6432x_1\left(0.9643x_6 - 2.4284x_3 - 0.1575x_7\right)}{63.2959x_2 + 3.7945x_4 - 5.7083x_4} - \left(3.6907 \times 10^{-3}x_9 + 0.3603\right)x_9x_{10}^2 \times 10^{-6} - 37 \tag{50}
\]

### Table 1. The average and standard deviation of each random variable

| Random variable \(x_i\) | Name       | Average \(\mu_i\) | Standard deviation \(\sigma_i\) |
|------------------------|------------|------------------|-----------------------------|
| \(x_1\)                | \(M_s/\text{kN} \cdot \text{mm}\) | 320.0000         | 1.6667                      |
| \(x_2\)                | \(R_i/\text{mm}\) | 15.9720          | 0.0333                      |
| \(x_3\)                | \(r_1/\text{mm}\) | 4.9845           | 0.0030                      |
| \(x_4\)                | \(b_1/\text{mm}\) | 20.5000          | 0.0333                      |
| \(x_5\)                | \(b_2/\text{mm}\) | 9.6300           | 0.0333                      |
| \(x_6\)                | \(R_s/\text{mm}\) | 101.5000         | 0.0500                      |
| \(x_7\)                | \(r_2/\text{mm}\) | 60.0000          | 0.0018                      |
| \(x_8\)                | \(D/\text{mm}\) | 110.0000         | 0.0500                      |
| \(x_9\)                | \(m/\text{kg}\) | 0.1292           | 0.0003                      |
| \(x_{10}\)             | \(n/r \cdot \text{min}^{-1}\) | 1800.0000        | 1.0000\text{[22]}          |

The calculation steps of the developed FOSM technique is shown in Figure 6.
With the results, for the design point $x^* = (x^*_1, x^*_2, ..., x^*_n) = (319.0336, 15.9806, 4.9845, 20.4992, 9.6346, 101.4959, 60.0000, 110.0000, 0.1292, 1800.0000)$ that with $e_{\beta} = 0.001$ and reliability factor $\beta^*$ is 0.6548, it can be determined that the reliability of K11250 CMTSC is 0.7422 under the rotational speed of 1800rpm.

Moreover, according to equation (37), the clamping force of the jaw of the considered K11250 CMTSC is 37.0001KN which is proved to have a 3.6% difference with the tested value that available in [23]. The comparison results validated the correctness of the method proposed.

Similar to the above, the reliability of the K11250 CMTSC under rotational speeds of 600-2200 rpm is displayed in Figure 7.

**Figure 6.** The calculation steps of the developed FOSM.

With the results, for the design point $x^* = (x^*_1, x^*_2, ..., x^*_n) = (319.0336, 15.9806, 4.9845, 20.4992, 9.6346, 101.4959, 60.0000, 110.0000, 0.1292, 1800.0000)$ that with $e_{\beta} = 0.001$ and reliability factor $\beta^*$ is 0.6548, it can be determined that the reliability of K11250 CMTSC is 0.7422 under the rotational speed of 1800rpm. Moreover, according to equation (37), the clamping force of the jaw of the considered K11250 CMTSC is 37.0001KN which is proved to have a 3.6% difference with the tested value that available in [23]. The comparison results validated the correctness of the method proposed.

Similar to the above, the reliability of the K11250 CMTSC under rotational speeds of 600-2200 rpm is displayed in Figure 7.

**Figure 7.** The reliability of the K11250 CMTSC under various rotational speed.

With Figure 7, the reliability of the dynamic clamping force of the jaws decreases as the chuck speed increases. However, the reliability of the CMTSC is required no less than 0.95 for safe operation reason, which furtherly requires the rotational speed of the K11250 CMTSC no more than 1000rpm. Normally, the CMTSC usually is designed to refer to specialists’ experiments and which ignored the randomness of factors affecting clamping force. For instance, for the company in which the CMTSC was produced,
the limit speed of K11250 CMTSC (1800rpm) is designed as the one that when the static clamping force lost 1/3 [1]. However, according to the analysis above the predetermined limit speed (results the reliability of the CMTSC merely 0.74) of the CMTSC is too high to guarantee the safe operation.

5. Conclusions
This paper proposed a reliability estimation methodology by the combination of force analysis, FOSM, and SSI model as a basis of that the reliability of a CMTSC was carried out in which dynamic clamping force was considered. Overall serval conclusions are reached as:

1) A mathematical model of the dynamic clamping force is established and in which the several impacts like centrifugal force, the stiffness of the jaw, and the spindle at high speed are considered.

2) A SSI model for the reliability analysis of the CMTSC was developed, moreover, it aims to solve the multivariate function for n-dimensional random variables created by SSI, a developed FOSM was applied.

3) The limit speed of the rotation speed of 1000rpm is determined and which can be a meaningful substitution of the existing one that proved to be unsafe.

4) The multi-functional expression based on the SSI model can be used to further optimize the structure of the chuck to increase its limit speed and give full play to the high-speed potential of the chuck.

Acknowledgments
This research is supported by the Inner Mongolia Natural Science Foundation (No. 2018MS05059), Inner Mongolia Autonomous Region Science and Technology Planning Project (2019) and Scientific Research Foundation of Inner Mongolia University of Technology (Grant Nos.X201704).

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