Observing a Quantum Measurement

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Abstract

With the example of a Stern–Gerlach measurement on a spin-1/2 atom, we show that a superposition of both paths may be observed compatibly with properties attributed to state collapse—for example, the singleness (or mutual exclusivity) of outcomes. This is done by inserting a quantum two-state system (an ancilla) in each path, capable of responding to the passage of the atom, and thus acting as a virtual detector. We then consider real measurements on the compound system of atomic spin and two ancillae. Nondestructive measurements of a set of compatible joint observables can be performed, one for a superposition and others for collapse properties. A novel perspective is given as to why, within unitary quantum theory, ordinary measurements are blind to such superpositions. Implications for the theory of measurement are discussed.

Keywords Measurement · Nondestructive · Entanglement · Irreversibility

1 Introduction

Opinions differ on whether or not there is a quantum measurement problem, and if so, exactly what it is [1, 2]. A more focused question regards the collapse of the state vector in projective measurements, as formalized by von Neumann [3]. By collapse we mean the observed process described by one of the textbook postulates, which states (in its simplest form).¹

¹ There are more general statements of this postulate, extending beyond orthogonal measurements and ideal measurement conditions. See the comprehensive discussion in [4].
Measurement Postulate: A measurement of the observable A yields one of its eigenvalues, \( a_n \), and, in an ideal measurement \[5\],\(^2\) places the measured object in the corresponding eigenstate, \( \phi_n \) (where \( A\phi_n = a_n\phi_n \)).

Included with this, or stated as a separate postulate, is the Born rule probability of this outcome, \( |\langle \phi_n | \psi \rangle|^2 \), where \( \psi \) is the initial state of the object being measured. All interpretations must agree on the above as a statement of fact, but they disagree on its status—i.e., is it independent of the other postulates, which stipulate unitary evolution in the appropriate Hilbert space, or is it derivable from them?

Regarding attitudes on this more focused issue, it seems reasonable to identify a small number of broad categories. Here is a grouping into three: The most conservative takes collapse as axiomatic—that is, it cannot be derived from the other axioms—suggesting that the collapse process itself is not subject to quantum analysis \[6, 7\].\(^3\) This position is consistent with most textbooks written over more than the last half century, which list collapse among the axioms. It is intended to include those who apply quantum theory according to these textbook axioms, without (however) adopting any particular interpretation philosophically. It also includes the epistemic and information-based approaches \[8\],\(^4\) whose intellectual roots extend back to the Copenhagen interpretation \[9, 10\].\(^5\) We refer to this general position as Standard Quantum Theory (SQT), interpreted broadly.

A contrasting position is that the collapse phenomenon, as observed, is in fact derived from the other axioms, following unitary evolution of an appropriate closed system which includes the apparatus and the relevant environment \[11\],\(^6\) as well as the object of study. And indeed unitary evolution describes what we see, but it also describes what we do not see—namely, that all branches of the state vector (representing all possible measurement outcomes) survive the measurement process. This is nevertheless consistent because it also predicts that an observer can be aware of only one such outcome.\(^7\) We shall refer to this position as unitary quantum theory (UQT). It includes the Many Worlds Interpretation \[12, 13\],\(^8\) which asserts that the unobservable branches are just as real as the branch we experience, but it is broader. It includes orthodox decoherence theory \[14, 15\] and variations \[16, 17\], whose

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\(^2\) An ideal measurement, as assumed in \[3\] and defined and demonstrated in \[5\], is projective.

\(^3\) Weinberg \[7\] offers an excellent discussion on interpretations of quantum theory. See pp. 81–82 on the Copenhagen Interpretation.

\(^4\) For a broad classification of interpretations, see \[8, p. 138\].

\(^5\) Aage Petersen quotes Bohr: “It is wrong to think that the task of physics is to find out how nature is. Physics concerns what we can say about nature...”, in \[9\].

\(^6\) In principle, the environment could be the rest of the universe, but Zurek discusses the more realistic notion of that part most responsible for decoherence, defined within the appropriate closed system (see \[11, p. 1520\]). In this work we will regard the relevant environment to be just the internal degrees of freedom of the individual detectors.

\(^7\) The observer’s blindness follows from the singleness of measurement outcomes on any given branch, and the incoherence of different branches. We take this up in Sect. 3, but also see \[7, pp. 86–88\] for a more conventional argument on the role of environmental decoherence.

\(^8\) Weinberg \[7, pp. 83–84\] provides a brief description of the Many-Worlds Interpretation and a comparison with the Copenhagen Interpretation.
practitioners represent a variety of interpretations, and other operational approaches which assert independence from interpretations [4, 18], while assuming unitarity.

A third position holds that the unobserved branches are removed from the theory by a mechanism of yet unknown origin, which takes effect in sufficiently large systems, and which is, in principle, subject to quantum analysis. The mechanism is represented by adding a nonlinear stochastic term to the Hamiltonian, whose effect is to remove all but a single branch [19–21]. This approach, in effect, replaces the collapse postulate with an expansion of the dynamics postulate beyond its otherwise unitary and deterministic character. This has consequences, which are measurable in principle, but to date undetected. Predicted effects are difficult to separate from decoherence and other random influences. There are proposals to utilize molecular interferometry and optomechanical phenomena, as well as particle diffusion [22], and it is hoped that over the next decade or two, definitive tests will be possible [23]. We refer to this general position as objective collapse theory (OCT). It has fewer adherents than the other two [2], but it provides an important alternative.

In this paper we will make several points about the measurement process, mostly interpretation independent, although the blindness of ordinary measurements to superpositions calls for specific justification within the UQT approach. For all points it will be useful to distinguish two stages of the measurement process. First comes the reversible premeasurement stage, where the object of interest becomes entangled (unitarily) with an ancillary system in the apparatus (in our case, two paths). Second is the detection stage, where the ancillary system transfers its entanglement to the detector system (in our case, two detectors), which then act irreversibly and record a result. Perhaps surprisingly, two signature collapse properties are established in the premeasurement stage, as properties of the object/ancillary system, and survive through the detection stage as correlations among the object and the two detectors. These are implicit in the postulate stated above: (i) singleness (or mutual exclusivity of outcomes $a_n$); and (ii) projection (the correlations between detector readings and the post-measurement spin state). Note that the singleness of outcomes implies randomness. Randomness is not a property of the premeasurement state—it only shows up at the detection stage as a result of local (“which path”) measurement, which breaks the entanglement while preserving the correlations. There is a third property of the premeasurement state (not a collapse property!), namely (iii) superposition (the state vector is a superposition of distinct collapse scenarios). In the ancilla model, this property can be detected, but in an ordinary apparatus it cannot be, so that its survival in the state vector is open to interpretation.

The ancilla model expands the premeasurement stage by adding a physical realization (qubits) to the ancillary system. The three properties are represented by Hermitian operators in the Hilbert space of the object/ancilla system. The operators commute, and all three properties are observable at the detection stage. In an ordinary apparatus without the ancillae, we will show that all three properties are again present in the premeasurement stage, but that only the two collapse properties are observable at the detection stage.

Interpretations differ on the reason behind this blindness of an ordinary apparatus to superpositions. In two of the approaches outlined above, only one branch of the state vector survives the detection stage—in SQT this is axiomatic; in OCT it is by
construction of the model interaction. In UQT, on the other hand, the superposition extends to the detectors and persists through the detection stage. We will offer a physically intuitive explanation why it is nonetheless undetectable, prompted by comparison with the ancilla model. This will provide a useful perspective on a more conventional explanation in decoherence theory (Sect. 3 and Appendix A).

In the next section we introduce the Stern–Gerlach measurement model with ancilla qubits as virtual detectors, and we then show how this atom/ancilla system may be “observed” in a real experiment. This observation demonstrates the compatibility of the two collapse properties with the superposition property for this system. In Section III, we compare the analogous measurements made with two ordinary detectors. We show that the collapse properties are identical to those of the ancilla model, while the superposition that persists in the UQT approach is now undetectable. We discuss the reason for this blindness and compare with the decoherence perspective. Results are summarized in Sect. 4.

2 A Model Measurement with Virtual Detectors

Consider the compound system consisting of a spin-1/2 atom and two quantum two-state systems (ancillae, \(A_\uparrow\) and \(A_\downarrow\)) serving as virtual detectors in the Stern–Gerlach interferometer pictured in Fig. 1 [24]. Each ancilla interacts locally with the atom, and it makes a transition from its 0 to its 1 state if and only if the atom passes through it. The interaction is spin-independent, preserving the spin state of the atom on its path. We assume that the process is reversible, so that the ancilla by itself does not perform a measurement—hence we call it a virtual detector.

Let us trace the evolution of entanglement as the atom passes through the device from points 1–4. The atom enters the picture at time \(t_1\) with the spatial wavefunction \(\phi(\mathbf{r}, t_1)\), in an arbitrary pure spin state, \((\alpha, \beta),\)

\[
|\psi(t_1)\rangle = \phi(\mathbf{r}, t_1) \left( \alpha |\uparrow\rangle_s + \beta |\downarrow\rangle_s \right) |0\rangle_{A_\uparrow} |0\rangle_{A_\downarrow},
\]

with ancillae in their 0 states. By the time \(t_2\), the Stern–Gerlach magnetic field gradient has separated the spin components into two ideally nonoverlapping paths,

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Margalit et al. [24] report the first experimental realization.
entangling the atom’s spatial and spin degrees of freedom. By \( t_3 \), the atom has passed through an ancilla, \( A \uparrow \) or \( A \downarrow \), conditioned on its path, so that

\[
\ket{\psi(t_3)} = \alpha \phi_1(\mathbf{r}, t_3)|\uparrow\rangle_s |1\rangle_{A\uparrow} |0\rangle_{A\downarrow} + \beta \phi_4(\mathbf{r}, t_3)|\downarrow\rangle_s |0\rangle_{A\uparrow} |1\rangle_{A\downarrow}.
\]

Finally, a reversed magnetic field gradient brings the two paths back together at \( t_4 \). Assuming that there is no net phase difference between the paths, the result is

\[
\ket{\psi(t_4)} = \phi(\mathbf{r}, t_4) \left( \alpha |\uparrow\rangle_s |1\rangle_{A\uparrow} |0\rangle_{A\downarrow} + \beta |\downarrow\rangle_s |0\rangle_{A\uparrow} |1\rangle_{A\downarrow} \right).
\]

The last step allows us to ignore the spatial part and study the remaining entanglement between the spin and the two ancillae. We further simplify by setting \( \alpha = \beta = 1/\sqrt{2} \), leaving the three-qubit state,

\[
\ket{\psi(t_4)} \rightarrow \frac{1}{\sqrt{2}} \left( |110\rangle + |001\rangle \right),
\]

where the spin states (\( \uparrow, \downarrow \)) are relabeled as (1, 0), and the ordering of the indices identifies with (spin, \( A\uparrow, A\downarrow \)).

This is a Greenumber-Horne-Zeilinger (GHZ) state [25, 26], an entangled state of three qubits. It was first realized experimentally in 1999 [27, 28] as a polarization state of three photons. Analogous states (and their generalizations to more than three particles) have been produced and documented in other systems—for example, trapped ions [29], superconducting circuits [30, 31], and Rydberg atoms [32]. The original goal was to demonstrate non-locality; more practical goals involve quantum error correction [33] and quantum communication [34], and in general the manipulation of entanglement.

Let us discuss the observables that characterize the state, and then the question of how to measure them. The three-qubit system lives in a Hilbert space of dimension eight, and \( \ket{\psi(t_4)} \) is an eigenstate of three tensor product operators, whose eigenvalues determine it completely. The choice is not unique; the most revealing

\[\text{(2)}\]

\[\text{(3)}\]

\[\text{(4)}\]

\[\text{(5)}\]

Nonlocality refers, in this context, to the failure of local hidden variables theories to duplicate the quantum correlations.
in the present context is $ZZI$, $ZIZ$, and $XXX$. Recalling the definitions of the individual Pauli matrices $Z$, $X$, and $Y$, as given in Table 1, one may confirm that $|\psi(t_4)\rangle$ is indeed the (unique) simultaneous eigenstate of the three tensor products with the eigenvalues quoted in Table 2.

Each observable may be characterized by a statement about its physical meaning, with eigenvalue $(\pm 1)$ giving the truth value [35]. With GHZ entanglement, all such statements concern either two-particle or three-particle correlations, and none concerns a property of an individual particle. The combination of statements appearing in Table 2 is an example. These statements may appear contradictory, because the product of the first two tells us that $IZZ = -1$, whose clear physical meaning is that the atom can be found on one and only one path (the “singleness” property, or mutual exclusivity). On the other hand, the definiteness of $XXX$ implies that the state vector (5) is a superposition of two classically inconsistent scenarios. We will explain in detail why the singleness and superposition statements are not contradictory. But first we must describe the measurements, made with real (irreversible) detecting devices [36].

We will describe two modes (called local and joint) which differ in the acquisition of local information.

### 2.1 Local Measurements

We begin with the simpler and more typical mode of GHZ experiments [27, 28], in which one measures the local factors and multiplies the results together to

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**Table 2** Tensor product operators, eigenvalues and physical interpretations

| (Spin, $A \uparrow, A \downarrow$) | Eigenvalue | Meaning |
|-------------------------------|-----------|---------|
| $ZZI$                         | +1        | Atom takes upper path if and only if spin is up |
| $ZIZ$                         | −1        | Atom takes lower path if and only if spin is down |
| $XXX$                         | +1        | State is invariant under simultaneous flips (0 ↔ 1) |

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**Fig. 2** Local measurements of $Z$ factors (a), and $X$ factors (b). In both parts, $M$ denotes standard read-out devices, and $M_z$ ($M_x$) denote the measurement of the atom’s spin component $Z_s$ ($X_s$) with a “downstream” Stern–Gerlach apparatus. Wavy lines denote the atom’s wave packet at $t_4$. 

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11 An ordinary (real) detector records an outcome for an observer, as noted by J. A. Wheeler in [36]: “No phenomenon is a physical phenomenon until it is an observed phenomenon...”
obtain the eigenvalue of the desired joint observable. First consider the local \( Z \) factors (Fig. 2a), each of which can take the value 1 or \(-1\). We measure the ancilla factors, \( Z_{A↑} \) and \( Z_{A↓} \), with a standard readout device\(^{12}\) for each ancilla qubit. We measure the atomic spin component, \( Z_{s} \), with another “downstream” Stern–Gerlach device, similarly oriented, with a single-atom detector placed in the \( \uparrow \) path to register the arrival (or not) of the atom and thus record the value of \( Z_{s} \). The results are as follows: The outcome of each local measurement is random because no local observable has \( \Psi(t) \sim |110\rangle + |001\rangle \) as an eigenstate, but the product of any two \( Z \) factors is definite, as shown in the top two panels in Table 3. These are collapse properties, and they are properties of the state \( \Psi(t) \) prior to measurement. They characterize the observed collapse phenomenon that we would see in an ordinary measurement without the ancillae. We emphasize that \( IZZ = -1 \) represents the singleness, or mutual exclusivity of measurement outcomes, while \( ZZI = 1 \) and \( ZIZ = -1 \) represent the projection property—the perfect correlation between detector readouts and the (repeated) value of the atomic spin downstream.

It is notable that the two-way correlations above persist in the face of local measurements (\( ZII, IZI, \) and \( IIZ \)), whose random outcomes indicate that the measurement changed the state of the system. The specific “which path” information thus obtained is not a property of \( \Psi(t) \); it enters only at the detection stage. Its randomness, a feature shared with ordinary measurements, is a necessary consequence of the singleness property.

Now consider the measurements of the individual \( X \) factors (Fig. 2b). Clearly, to measure \( X_{s} \), we simply reorient the “downstream” Stern–Gerlach system. To measure \( X_{A↑} \) and \( X_{A↓} \), noting that the readout devices are keyed to the ancillas’ \( Z \)-bases, we apply a Hadamard transformation \((H)\)\(^{13}\) to each ancilla before the readout. The readout value (\( \pm 1 \)) then indicates in which linear combination of \( Z \)-basis states (\( |1\rangle \pm |0\rangle \)) each ancilla has been “found.” The results of these \( X \)-measurements are the following: The individual outcomes are again random, but the product of all three is always +1. The definiteness of the product indicates a coherent superposition, and its positivity confirms the sign in (5). Note that this measurement distinguishes the pure entangled state from a mixed state of the same two components.

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\(^{12}\) A qubit readout device faithfully records pure \( Z \) eigenstates, \( Z = \pm 1 \).

\(^{13}\) The Hadamard transformation takes \( X \)-basis states onto \( Z \)-basis states and vice-versa, e.g., \( H|k\rangle_{x} = |k\rangle_{z} \).
The mixed state would duplicate the \( Z \)-measurement results but reveal itself through random outcomes for the product \( XXX \).

To better fill out the picture of how the same state, \( \Psi(t_f) \), can accommodate both a collapse scenario and a superposition, consider the local measurement of the spin component \( X_s \), which “finds” the atom to have taken both paths. The outcome is random: When it is \((+1)\), then the product \( X_{A_1}X_{A_\perp} \) must also be \((+1)\). Knowing the compatible product \( Z_{A_1}Z_{A_\perp} = -1 \) (Table 3), it is easy to see that the state of the two ancillae is the Bell state,

\[
|\Psi\rangle_{AA} \rightarrow \frac{1}{\sqrt{2}}(|10\rangle + |01\rangle)_{AA}.
\]  

(6)

On the other hand, when \( X_s \) is \((-1)\), then \( X_{A_1}X_{A_\perp} \) must also be \((-1)\), and this, combined again with \( Z_{A_1}Z_{A_\perp} = -1 \), indicates another Bell state,

\[
|\Psi\rangle_{AA} \rightarrow \frac{1}{\sqrt{2}}(|10\rangle - |01\rangle)_{AA}.
\]  

(7)

Each of these Bell states represents perfect anticorrelations between the ancilla’s \( Z \)-values, which are individually random. This shows how the ancillae are able to respect the singleness property, while (in correlation with the atomic spin) enabling the coherent superposition of both paths.

It may be worth noting that the observation of either superposition state is just another form of apparent collapse—instead of finding “which path,” one is finding “which superposition of paths.”

So far, we have appealed to local measurements to verify the joint eigenvalues which define \( \Psi(t_f) \). But since these measurements change the state, each new set of them requires another identically prepared state of the system. One can circumvent this need and achieve a more direct demonstration of compatibility, as follows.

### 2.2 Joint Measurements and Compatibility

One can measure compatible joint observables nondestructively by refusing to acquire local information. For the singleness property \( IZZ \), one couples \( A_1 \) and \( A_\perp \) to an added ancilla qubit \( B \) through CNOT gates (Fig. 3a), so that the state of \( B \) changes if and only if \( Z_{A_1} \) and \( Z_{A_\perp} \) are opposite. Thus, the readout will show that
IZZ = −1 without revealing the value of either $Z_{A↑}$ or $Z_{A↓}$. Similar measurements hold for the projection property ($ZZI$ or $ZIZ$), by moving one of the CNOT connections to ancilla $C$, located on the (↑) leg of the downstream Stern–Gerlach device. To measure $XXX$, one couples three ancillae ($A↑$, $A↓$, and $C$) to $B$ through Hadamard and CNOT gates (Fig. 3b). The readout will show that $XXX = 1$, and this, together with the above, establishes the superposition property.

While these joint measurements provide the (logically) most direct demonstration of compatibility, the local measurements have the virtue of demonstrating the randomness characteristic of ordinary measurements. So the ancilla model, subjected to local $Z$ measurements only, is reduced to an ordinary apparatus. But the $X$ measurements give it access to the superposition property, which is not accessible to ordinary measurements.

It should be noted in passing that the ancilla setup in either form can be realized starting with a photon propagating through a Mach-Zender interferometer and two ancillae, one associated with each arm. This is an extension of the so-called delayed choice quantum eraser (DCQE), which employs a single ancilla and is capable of realizing Wheeler’s proposed delayed choice experiment [36] (the choice between “which path” and a superposition of paths), as closely realized experimentally by Jacques et al. [37]. A different realization, and a comprehensive discussion of DCQE setups, is provided in [38]. With a single ancilla, one has a two qubit system and is able to detect the superposition and projection properties ($XX$ and $ZZ$). Superposition ($XX$) is manifested by interference as a function of the difference in path lengths, and the action of the ancilla $X$ factor corresponds to quantum erasure of “which path” information. But this system cannot detect singleness as the third independent observable. With two ancillae, on the other hand, one has a three qubit system which accommodates this observable. Furthermore, the two-ancillae system forms a parallel with an ordinary apparatus which employs two detectors. We show below that the singleness property must hold here as a correlation between the detectors.

3 Apparati with Real Detectors

Reconsider the Stern–Gerlach setup of Fig. 1 with ancillae replaced by real detectors, $D↑$ and $D↓$. These act like the ancillae together with their readout devices: Like the ancillae, they transmit the atom without changing its spin state (this is in keeping with the von Neumann measurement formalism [3] and the stated postulate). And, like the readout devices, they record (irreversibly) the passage (or not) of the atom.

In this section, first, we show that the collapse properties of ordinary apparati are equivalent to those of the ancilla model, in the sense that the singleness and projection properties are represented by observables and established at the premeasurement stage, while the randomness of “which path” information enters only at the detection stage. Then second, we discuss the superposition property, which survives the detection stage in UQT but is nonetheless undetectable. We offer a physical explanation why this is the case.
3.1 Equivalence of Collapse Properties

Recall that the ancilla-based premeasurement state, $\Psi(t_4)$, exhibits nonrandom collapse properties represented by joint observables: (i) the singleness property, by $Z_A^\uparrow Z_A^\downarrow = -1$, and (ii) the projection property, by $Z_s Z_A^\uparrow = +1$ and $Z_s Z_A^\downarrow = -1$. To show that these properties are similarly (pre)established in ordinary measurements, note that the ancilla operators, $Z_A^\uparrow$ and $Z_A^\downarrow$, act as stand-ins for path occupation variables. And we can define such variables for the ordinary setup as $Z_P^\uparrow = 2P^\uparrow - 1$ and $Z_P^\downarrow = 2P^\downarrow - 1$, where $P_k$ is a projector onto that subvolume of the path $k = (\uparrow, \downarrow)$ occupied by the spatial wave packet $\psi_k(\mathbf{r}, t_2)$ at time $t_2$. So $Z_P^k \to \pm 1$ tells us whether or not the atom will enter the detector $D_k$. The path occupation states analogous to the ancilla states may be written as $|1\rangle_p^s |0\rangle_p^\uparrow$ and $|0\rangle_p^s |1\rangle_p^\uparrow$. These convey all of the necessary information about the spatial wave packets $\psi_\uparrow(t_2)$ and $\psi_\downarrow(t_2)$, so that $|\psi(t_2)\rangle$ may be written as

$$|\psi(t_2)\rangle = \frac{1}{\sqrt{2}} \left( |1\rangle_s |1\rangle_p^\uparrow |0\rangle_p^\downarrow + |0\rangle_s |0\rangle_p^\uparrow |1\rangle_p^\downarrow \right),$$

which is analogous to (4) of the ancilla system prior to readout. This is clearly an eigenstate of $Z_p^\uparrow Z_p^\downarrow$, $Z_s Z_p^\uparrow$, and $Z_s Z_p^\downarrow$, with eigenvalues -1, 1, and -1, respectively, thus establishing the singleness and projection properties in the premeasurement state. These properties arise here from the entanglement of just the spin and the path, with path occupation alone playing the ancillary role.

By the time $t_3$, the path occupation variables have mediated correlations between the atom’s spin and the detectors—alogous to the readout state in the ancilla case. The main difference is that here, a detector readout follows closely the passage of the atom—one cannot delay this readout as was done with the ancilla system. But the ancilla system can duplicate this situation by moving the ancilla readouts to an earlier time, say $t_3$, ahead of the atomic spin remeasurement (at some $t > t_4$). The time ordering has no effect on the final result.\(^\text{14}\)

As an aside on the projection property—although this property is axiomatic in SQT, it is nonetheless conditioned on the above assumption that a detector transmits the atom without changing its spin state (see Footnote 2). Clearly this assumption fails for detectors which work by absorbing the atom. In this case the projection property still holds, but it takes the form of a correlation between the reading of a detector and the angular momentum imparted to it. Only one detector can receive the impulse (the singleness property), and that impulse ($\pm \hbar$) is delivered to $D_\uparrow$ or $D_\downarrow$, respectively, depending on which detector reads 1.

3.2 Blindness to Superpositions

While the collapse properties are common to the two systems, the superposition property is not, being detectable in the ancilla system but not in the ordinary system.

\(^{14}\) It is interesting to note that a spacelike separation of local measurement events is possible, in principle. This rules out causal connections between them.
So now we focus on the blindness of ordinary apparati to superpositions of the two collapse scenarios, and the special challenge it presents to the UQT approach. While all approaches outlined at the beginning of the paper must agree on the facts as stated in the measurement postulate, they differ fundamentally on the physics behind these facts. In the SQT and OCT approaches, because the detectors are macroscopic, one of the terms existing in the premeasurement state is removed from the theory at detection. In SQT the removal is axiomatic; in OCT it is dynamical, a result of the nonlinear stochastic model. The evolution is nonunitary in both cases. In the UQT approach, on the other hand, the evolution including the detectors is unitary, and therefore, the state of the atom/detectors system must reflect the superposition in (8). This approach is viable, as we have said, only if the superposition of distinct detector states is undetectable, so that the detector outputs are perceived to be random. Decoherence theory shows that this is the case (see Appendix A), but the ancilla analogy suggests a more fundamental explanation involving irreversibility [39].

In UQT, the state of the atom/detectors system, at time \( t_4 \), has the same GHZ form as (8) (or, for that matter, (5) of the ancilla system). The crucial difference is that the detector states involve microscopic internal degrees of freedom,\(^{15}\) whose states are labeled by \( \mu \) and \( \mu' \) in addition to the necessary readout variables, 0 and 1 respectively. So (8) becomes

\[
|\psi(t_4)\rangle = \frac{1}{\sqrt{2}} \left( |1\rangle_s |1,\mu\rangle_{D^1} |0,\mu\rangle_{D^1} + |0\rangle_s |0,\mu\rangle_{D^1} |1,\mu\rangle_{D^1} \right), \tag{9}
\]

where we have dropped the common spatial factor at \( t_4 \) as done in (5). One can imagine (0) to be a metastable configuration of a detector, which would make a transition to a final stable configuration (1) if triggered by the passage of the atom. Equation (9) represents just a single element of an ensemble in which each initial state \( \mu \) of the (0) configuration evolves unitarily into the state \( \mu' \) of the (1) configuration.\(^{16}\) The corresponding density matrix is written in Appendix A. In the text, for clarity, we shall continue to refer to state vectors.

Again we ask—how can one observe a superposition involving both paths? One must measure, among other things, the spin component \( X_s \). Since this measurement by itself produces random outcomes, one must measure a correlation of which \( \Psi(t_4) \) is an eigenstate, of which the simplest is \( XXX \). There are three other options, such as \( XYY \), but these offer nothing further. So, supposing that the \( X_s \) measurement produces the outcome +1, we must then show that the product of detector \( X \) values is also +1. Given that the product of detector \( Z \) values is −1, this would demonstrate that the detectors are in the Bell state analogous to (6):

\[^{15}\text{The internal degrees of freedom of the individual detectors are sufficient to produce decoherence (see Appendix A), and hence they comprise the relevant environment (see footnote 6). External degrees of freedom could contribute, but not decisively. And they certainly do not mediate interactions between the detectors.}\]

\[^{16}\text{To define } \mu' \text{ more precisely, it is the microscopic state of the (1) configuration at the time when the outcome (1) is recorded. This is the time when the measurement is completed (see footnote 11).}\]
A detector operator $X_k$ connects its 0 and 1 states, that is, $|1, \mu^\prime\rangle_{D_k} = X_{D_k}|0, \mu\rangle_{D_k}$ and $|0, \mu\rangle_{D_k} = X_{D_k}|1, \mu^\prime\rangle_{D_k}$. Since the first of these represents the natural evolution of the detector, $|1, \mu^\prime\rangle = U(t_3, t_2)|0, \mu\rangle$, the $X$ operators must be

$$X_{D_k} = P_k(1)U_k(t_3, t_2)P_k(0) + P_k(0)U_k^{-1}(t_3, t_2)P_k(1),$$

where $P_k(i)$ are projection operators onto the $i = 0$ or 1 configurations of detector $D_k$. Now $U_k(t_3, t_2)$ represents the time evolution of a complex many-body system, and while this is reversible in principle, it is not reversible thermodynamically [40]. That is, we do not have control over the microscopic degrees of freedom required to implement $U_k^{-1}(t_3, t_2)$. So the detector operators $X_k$ are not accessible to us, and without them we cannot detect a superposition of states in the 0 and 1 configurations of $D_k$—and we cannot access XXX, which would demonstrate a Schrödinger cat-like superposition of the two collapse scenarios of the spin/detectors system. In short, we only have access to the detector variables $Z_k$ which the detectors record, and in these there can be no evidence for the existence of the superposition which (in UQT) continues to exist in the state vector. A different but related argument on thermodynamic irreversibility in measurement was given by Peres [41].

The demonstration of blindness changes very little with detectors which absorb the atom rather than transmitting it, as is essentially the case in the original Stern–Gerlach experiment [42]. This case is discussed in Appendix B.

The above arguments suggest that our inability to detect superpositions of detector output states is a manifestation of the second law of thermodynamics. One can imagine a quantum Maxwell Demon who possesses the microscopic control that we lack, and is capable of detecting superpositions which are invisible to us. Thus, the quantum measurement process, by its construction, employs the thermodynamic arrow of time. There is no need to invoke a different “measurement” arrow.

### 3.3 A Decoherence Perspective

A decoherence argument for blindness involves the concept of the pointer; Brasil and deCastro [43] define “pointer states (as) eigenstates of the observable of the measuring apparatus that represents the possible positions of the display pointer of the equipment.” The concept was introduced in the present context by Zeh [44] and developed by Zurek [11, 45], who argued that interactions with the environment select the pointer’s preferred basis (the states we observe). In our system these states are associated with the pair of detectors; they are denoted by $|1\rangle_{D_1}|0\rangle_{D_1}$ and $|0\rangle_{D_1}|1\rangle_{D_1}$ (or in shorthand, $|10\rangle_{DD}$ and $|01\rangle_{DD}$), by simply dropping the environmental variables $\mu$ and $\mu^\prime$.17

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17 Batalhao et al. [40] show experimentally how thermodynamic irreversibility arises in an isolated quantum system whose microscopic dynamics is reversible.
An argument specific to our system is written out in Appendix A. In brief, one traces over the environmental degrees of freedom ($\mu$ and $\mu'$) within each detector, and derives the reduced density matrix of the spin/pointer system, which is diagonal in the basis $|110\rangle_{DD}$ and $|001\rangle_{DD}$, where the second and third indices refer to the pointer. In the case of detectors which absorb the atom, this basis reduces to just that of the pointer, $|10\rangle_{DD}$ and $|01\rangle_{DD}$. In either case, blindness to superpositions is represented by the diagonality of the $(2 \times 2)$ density matrix.

There are interesting consequences when the pointer is realized within the system of two detectors. Environmental interactions determine the preferred basis of an individual detector, $|1\rangle_{Dk}$ and $|0\rangle_{Dk}$ (corresponding to atom/no atom, or $Z_{Dk} = \pm 1$). But they do not determine the preferred basis of the full pointer associated with the pair of detectors, because they do not exclude the possibility of $|11\rangle_{DD}$ and $|00\rangle_{DD}$—exclusion resulting from the singleness property. In fact, the environment is not responsible for the singleness or the projection property, or implicitly, for the choice of which spin component is measured—all of which are established in premeasurement. So, while the environment is crucial for enforcing the blindness to superpositions, its role (whether internal or expanded to include the external) is limited to the proper functioning of the individual detectors.

This separation between the premeasurement stage (governed by reversible unitary evolution), and the detection stage (where the environment enters bringing practical irreversibility), is the defining characteristic of controlled von Neumann-type measurements. The roles of these stages are in some sense complementary: While blindness to superpositions may be seen as an emergent classical property induced by interactions with the environment, the singleness and projection properties are quantum entanglement properties, represented by observables and manifested in correlations between noninteracting macroscopic objects.

## 4 Conclusions

We studied an ancilla-aided Stern–Gerlach experiment allowing delayed-choice measurements on the three-qubit system of atomic spin and two ancillae acting as virtual detectors. We first considered local measurements, and showed that one choice ($Z_i$) reproduces the collapse properties of ordinary (unaided) measurements, while another ($X_i$) demonstrates a superposition of the two collapse-like scenarios, involving both paths. Both choices require repeated measurements on identically-prepared states of the system, since local measurements destroy the state.

So secondly, we showed that nondestructive measurements can be made by avoiding the acquisition of local information. Thus relinquishing only the specific “which path” information, one can still measure a complete set of commuting joint observables—these represent the superposition property and the two (nonrandom) collapse properties, namely the singleness and projection properties.

In Sect. 3 we applied the above ideas to an ordinary apparatus. First, we showed that the collapse properties occur with the same status as in the ancilla model: The singleness and projection properties are represented by operators and established at the premeasurement stage, while the randomness of local measurement outcomes enters
only at the detection stage, as a consequence of the singleness property. The various approaches mentioned in the introduction—the standard, unitary, and objective collapse approaches—agree on the observed collapse phenomenon itself, but they differ on its unobservable underpinnings—the existence/nonexistence of unobserved branches in the state vector—and the nature of the observed randomness of outcomes (subjective or objective?). It is possible, but far from certain, that future experiments alone will resolve these differences.

The viability of the UQT approach rests upon the invisibility of the alternate (unobserved) branches in the state vector. The ancilla system points to a simple explanation: Detectors are irreversible, and this makes the required complementary local observables \( (X_{D\uparrow} \text{ and } X_{D\downarrow}) \) inaccessible. The absence of a known fundamental mechanism of irreversibility acting in typical measurements suggests that the irreversibility is thermodynamic, so that (at least within UQT), the observed collapse phenomenon is a manifestation of the second law.

**Appendix A: Density Matrix of Spin-Detectors System**

Here we write out the density matrix of the spin/detectors system, and we show how the trace over the unobserved states of the detectors’ internal degrees of freedom yields the appropriate reduced density matrix, which expresses the blindness of the apparatus to superpositions of output states.

The initial mixed state of the spin/detectors system, assuming probabilities \( p_{\mu_1} \) and \( p_{\mu_i} \) for the microstates, \( |0, \mu_{\uparrow}\rangle_{D_1} \) and \( |0, \mu_{\downarrow}\rangle_{D_i} \) of the two detectors, is

\[
\rho(t_1) = \sum_{\mu_1, \mu_i} p_{\mu_1} p_{\mu_i} \left( \alpha |1\rangle_s + \beta |0\rangle_s \right) |0, \mu_{\uparrow}\rangle_{D_1} |0, \mu_{\downarrow}\rangle_{D_i} \\
\times \left( \alpha^* \langle 1 |_s + \beta^* \langle 0 |_s \right) \langle 0, \mu_{\uparrow}\rangle_{D_1} \langle 0, \mu_{\downarrow}\rangle_{D_i}.
\]

After the atom passes through the detectors and the paths are recombined at \( t_4 \), this becomes

\[
\rho(t_4) = \sum_{\mu_1, \mu_i} p_{\mu_1} p_{\mu_i} \left( \alpha |1\rangle_s |1, \mu'_{\uparrow}\rangle_{D_1} |0, \mu_{\downarrow}\rangle_{D_i} + \beta |0\rangle_s |0, \mu_{\uparrow}\rangle_{D_1} |1, \mu'_{\downarrow}\rangle_{D_i} \right) \\
\times \left( \alpha^* \langle 1 |_s |1, \mu'_{\uparrow}\rangle_{D_1} \langle 0, \mu_{\downarrow}\rangle_{D_i} + \beta^* \langle 0 |_s \langle 0, \mu_{\uparrow}\rangle_{D_1} |1, \mu'_{\downarrow}\rangle_{D_i} \right).
\]

Since we only read the detectors’ outputs and do not monitor the microscopic degrees of freedom (considered as the “environment” \( E \)), we trace over the latter to define the reduced density matrix describing the state of the spin and the detector displays, which is called the spin/pointer system [43]. To be more precise, each detector consists of its own pointer (with readout states 0 and 1), and its own environment (with associated states \( \mu \) and \( \mu' \), respectively). The trace consists of independent traces over the environments within each detector, i.e., \( \rho'(t_4) \equiv \text{Tr}_{E_{\uparrow}, E_{\downarrow}} \rho(t_4) \).
To evaluate each of these, it is convenient to sum over \( \mu \) (i.e., \( \sum_{\mu} \langle \mu | \cdots | \mu \rangle \)) in those terms where the pointer state 0 appears, and over \( \mu' \) where it does not. The latter choice is legitimate because the two sets are related unitarily. It is straightforward then to show that

\[
\rho'(t_3) = |\alpha|^2 |1\rangle_3 \langle 1|_{D_i} |0\rangle_{D_i} \langle 1|_{D_j} \langle 0|_{D_i} \\
+ \alpha \beta^* |1\rangle_3 \langle 1|_{D_i} |0\rangle_{D_i} \langle 0|_{D_j} \langle 1|_{D_i} \sum_{\mu, \mu'} p_{\mu, \mu'} \langle \mu|_{D_i} \langle \mu'|_{D_i} \langle \mu|_{D_j} \langle \mu'|_{D_j} \\
+ \alpha^* \beta |0\rangle_3 |0\rangle_{D_i} |1\rangle_{D_i} \langle 1|_{D_j} \langle 0|_{D_i} \langle 0|_{D_j} \sum_{\mu, \mu'} p_{\mu, \mu'} \langle \mu|_{D_i} \langle \mu'|_{D_i} \langle \mu|_{D_j} \langle \mu'|_{D_j} \\
+ |\beta|^2 |0\rangle_3 |0\rangle_{D_i} |1\rangle_{D_i} \langle 0|_{D_i} \langle 0|_{D_j} \langle 1|_{D_i} 
\]

(14)

The environmental sums in the second and third terms essentially vanish (they are undetectably small) because the inner product factors, none greater than unity in magnitude, have random phases, in contrast with analogous factors \((\langle \mu|\mu|\mu'|\mu'\rangle = 1)\) which appeared in the first and fourth terms and summed to unity. Thus \( \rho' \) is diagonal in the spin-pointer basis, which consists of \(|1\rangle_3 |1\rangle_{D_i} |0\rangle_{D_i} \) and \(|0\rangle_3 |0\rangle_{D_i} |1\rangle_{D_j} \). The surviving singleness and projection correlations result from the entanglement generated between (12), (13) by the passage of the atom. In fact it should be noted that, except for the remaining summations in the off-diagonal terms, (14) is equivalent in form to the density matrix of the spin-ancilla system (see (4) in Sect. 2). Thus, the environmental factors in (14) neatly summarize how the superposition of outcomes becomes undetectable with real detectors.

**Appendix B: Detectors that Absorb**

In the original Stern–Gerlach experiment [42], silver atoms were directed at a glass plate and formed two separated deposits, with segments of the glass acting as the two detectors. Imagining ideally a pair of absorbing single-atom detectors, their state at time \( t_3 \) could still be written as in (10), but the 1 states now represent the absorbed atom as well as excitations created by the absorption event. Natural evolution produces these states from the 0 states of the detectors multiplied by the corresponding path occupation states \(|1\rangle_{D_k} \) of the atom:

\[
|1, \mu'\rangle_{D_k} = U(t_3, t_2) |0, \mu\rangle_{D_k} |1\rangle_{p_k}.
\]

So the \( X_{D_k} \) operator analogous to (11) is

\[
X_{D_k} = P_k(1) U_k(t_3, t_2) |1\rangle_{p_k} P_k(0) |1\rangle_{p_k} + |1\rangle_{p_k} P_k(0) |1\rangle_{p_k} U_k^{-1}(t_3, t_2) P_k(1),
\]

(15)

and the subsequent blindness argument is unchanged.

The decoherence approach of Appendix A is similarly adapted: Since the \(|\mu'|\) states include the absorbed atom, the inner product factors in (14) are replaced by \(|1\rangle_{p_k} \langle \mu| \langle \mu'| \) or its complex conjugate. The set \{\( \mu' \)\} is not complete because it refers to more particles than \{\( \mu \)\}, but it includes all states generated unitarily from \{\( \mu \)\} and the incident atom.
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