Measuring galaxy biasing with gravitational weak lensing

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Abstract. Gravitational weak lensing by large scale structures is viewed as a tool to probe the bias relation between the mass and the light distributions. It is explained how a particular statistic can be used to deproject the 2D mass distribution observed by weak lensing, in order to probe the bias at a given redshift and a given scale. Dependence with the cosmology is written, and observational issues are pointed out, like the importance of the redshift distribution of the galaxies. A signal to noise analysis shows that the scale dependence of the bias can be measured with a rather modest lensing survey size.

1 Weak lensing and galaxy catalogues

Gravitational weak lensing by large scale structures is known to be a promising approach to probe the mass distribution in the Universe. Gravitational lensing by cluster of galaxies already gave new insights about the cluster mass and their 2D mass distribution whereas observation of lensing by large scale structure is only at its beginning ([7]). Its potential scientific impact on the knowledge of the cosmological parameters and the mass power spectrum pioneered by [2], [3], [5], [8], has been developed by many authors (see [1], [4], [6], [14]). One may also be interested in comparing the mass maps reconstructed from weak lensing and the galaxy catalogues (i.e. the mass and the light distributions) in order to probe the so-called bias relation. Galaxy catalogues (via high order statistic) and velocity fields being the standard ways to probe the bias, a method able to see directly the mass, like gravitational lensing, may play a role.

In the following some clues about the measurement of bias using gravitational lensing will be exposed, most of them are detailed in [12].

Biasing theory has known recently a raise of interest because of the growing evidences that it is not trivial. However, in order to give some insights on the method, local linear biasing relation is used as a basis along this paper, and the extension to non trivial biasing is let for a forthcoming paper [13].

The basic idea is to probe the bias by measuring the correlation $C$ between the projected density contrast of a set of foreground galaxies and the shear of background galaxies. The difficulty with the gravitational lensing is that it gives the projected mass along the line of sight, whereas the 3D mass field is required in order to constrain the bias, in particular if we wish to investigate its redshift evolution. Fortunately the $M_{ap}$ statistic introduced by [11] gives
the possibility to deproject the observed mass distribution. To understand how it is possible, let us write the correlation function $C$ explicitly \[10\],

$$C = \langle M_{\text{ap}}(\theta_c) N(\theta_c) \rangle = 3\pi \Omega b \int dw \frac{p_t(w) g(w)}{a(w) f_K(w)} \int ds s P \left( \frac{s}{f_K(w)}, w \right) I^2(s \theta_c).$$  

\[1\]

$M_{\text{ap}}(\theta_c)$ is the convergence smoothed with a compensated filter of size $\theta_c$, and $N(\theta_c)$ is the projected density contrast of the foreground galaxies smoothed with the same filter. $b$ is the linear bias parameter, $w(z)$ is the comoving distance to a redshift $z$, $f_K$ is the comoving angular diameter distance, and $P$ is the time-evolving 3-D mass power spectrum. $\Omega$ is the density parameter, and $a$ is the cosmic expansion factor. The function $g(w) = \int w^{(\infty)} \text{d}w' p_b(w') f_K(w' - w) / f_K(w')$ depends on the redshift distribution of the sources $p_b(w)$ and $p_f(w)$ is the redshift distribution of the foreground galaxies. $I(s \theta_c)$ is the Fourier transform of the compensated filter, which is very localised in the Fourier space around the angular scale $\theta_c$. If in addition a narrow redshift distribution is used for the foreground galaxies then the integrals in Eq. \[3\] are almost reduced to a point in the $(w, s)$ plane, which is equivalent to a deprojection (a single angular scale $s$ is observed at a given redshift $w_f$).

## 2 Probe of the bias

A useful estimator to probe the bias, independent on the power spectrum normalisation is,

$$R_{\theta_c} = \frac{\langle M_{\text{ap}}(\theta_c) N(\theta_c) \rangle}{\langle N^2(\theta_c) \rangle},$$  

\[2\]

where $\langle N^2(\theta_c) \rangle$ is the dispersion of the smoothed projected density contrast of the foreground galaxies,

$$\langle N^2(\theta_c) \rangle = 2\pi b^2 \int dw \frac{p_t^2(w)}{f_K^2(w)} \int ds s P \left( \frac{s}{f_K(w)}, w \right) I^2(s \theta_c).$$  

\[3\]

In the case of linear theory, for a power law power spectrum and for a narrow foreground redshift distribution (located around $w_f$) it is easy to show from Eq.\[1\] and Eq.\[3\] that Eq.\[2\] can be approximated by,

$$R_{\theta_c} \simeq \frac{3 \Omega}{2 b} g(w_f) f_K(w_f) p_f(w_f),$$  

\[4\]

which is a number depending only on the cosmological parameters, the biasing factor $b$ and the foreground and background redshift distributions. It does not depend on the power spectrum index and the smoothing angle $\theta_c$. It turns out that this is still true for a general power spectrum even in the non-linear regime. The reason for this is that if the redshift integration in Eq.\[1\] and Eq.\[3\] are narrow enough, a general power spectrum can be approximated
Figure 1:
The left plot shows the value of $R_{\theta_c}$ and the right plot the ratio $R = R_{\theta_c}/R_{1'}$. Different linestyle correspond to different cosmological models as indicated on the left plot, thin lines are for the linear regime and thick lines for the non-linear regime. CDM-like power spectrum has been used.

Locally as a power law because of the narrow function $I(s\theta_c)$. Locally here means in the $(w, s)$ space. An illustration of this is given if figure 1 where $R_{\theta_c}$ and $\mathcal{R} = R_{\theta_c}/R_{1'}$ are plotted versus the smoothing angle $\theta_c$ for different cosmological models, and with the narrow foreground redshift distribution $p_f(z) \propto z^5 \exp\left(-\frac{z}{0.4}\right)^6$ well localised around $z_f = 0.4$. The power spectrum evolution has been extended into the non-linear regime via the Peacock & Dodds formula in [9]. Figure 1 shows that $R_{\theta_c}$ is has a constant value to a percent accuracy over the scale range [1', 60'] which only depends on the cosmological parameters, the bias (at the redshift of the foregrounds) and the redshift distributions whatever the power spectrum. Calculations with various power spectra shows that $R_{\theta_c}$ changes by less than 3%. The $R_{\theta_c}$ values fitted to the $\Omega$ dependence (assuming a zero cosmological constant) gives,

$$R_{\theta_c} \simeq 0.162 \frac{\Omega^{0.42}}{b},$$

where the proportionality constant and the exponent both depend on the redshift distributions according to Eq. (3). Conjointly used with the velocity fields or any independent determination of $\Omega$, Eq. (5) allows to determine the bias at the redshift of the foreground galaxies (here roughly 0.4) and at the scale $\theta_c$ from weak lensing data provided that the redshift distributions are known. Preliminary investigations show that only the mean and the variance of these distributions are important even if they are skewed.

The quantity $\mathcal{R}$ defined above gives a direct estimate of the scale dependence of the bias, independent on the cosmology, the power spectrum and

1The background redshift distribution is $p_b(z) \propto z^5 \exp(-z^{1.5})$
the redshift distributions. Strictly speaking it gives the ratio of the fourier components of the bias at two different scales provided that the bias can be expressed as a convolution, \( \delta_g(k) = b(k)\delta(k) \) (linear non-local non-stochastic biasing scheme). A signal to noise analysis shows that assuming that \( \langle N^2(\theta_c) \rangle \) is perfectly known (from the future big galaxy surveys for instance), a lensing survey of \( 5 \times 5 \) degrees (with 4 hours exposure depth) should be able to detect a bias scale variation of 20% or more in the \([1',10']\) scale range.

3 Conclusion

Gravitational lensing can probe the bias relation between mass and light distributions. The simultaneous use of a narrow foreground redshift distribution and a compensated filter permits to deproject the observed mass distribution and to recover the redshift and scale dependence of the bias using Eq.(5) and the estimator \( R \). Redshift distortion effects are not a problem in this method since the foreground redshift range is large enough do dilute these effects, and on the other hand, it is small enough to assume that a given angular scale corresponds to the physical scale at which the bias is probed. Of course, the statistical bias properties are assumed to remain unchanged within the foreground galaxies redshift interval.

Obviously the biasing scheme adopted in this paper is too simple and should be extended to non-linear, stochastic and non-local biasing. One can hope to use weak lensing jointly with galaxy catalogues and velocity fields to separate these features and try to build a consistent bias picture.

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