Age constraints on the Agegraphic Dark Energy Model

Yi Zhang\(^{1,2,*}\), Hui Li\(^{1,2,†}\), Xing Wu\(^{3,‡}\), Hao Wei\(^{4,§}\), Rong-Gen Cai\(^{1,¶}\)

\(^1\)Institute of Theoretical Physics, Chinese Academy of Sciences P.O. Box 2735, Beijing 100080, China
\(^2\)Graduate University of the Chinese Academy of Sciences, Beijing 100049, China
\(^3\)Department of Astronomy, Beijing Normal University, Beijing 100875, China
\(^4\)Department of Physics and Tsinghua Center for Astrophysics, Tsinghua University, Beijing 100084, China

We investigate the age constraint on the agegraphic dark energy model by using two old galaxies (LBDS 53W091 and LBDS 53W069) and the old high redshift quasar APM 08279 + 5255. We find that the agegraphic dark energy model can easily accommodate LBDS 53W091 and LBDS 53W069. To accommodate APM 08279 + 5255, one can take the reduced Hubble parameter as large as \(h = 0.64\), when the fraction matter energy density \(\Omega_m \approx 0.22\).

1. INTRODUCTION

The observational data strongly suggest that the universe is accelerating today\(^{1,2,3,4,5,6,7,8}\); as a consequence, the study of dark energy has been one of the most active topics in modern cosmology. The simplest candidate for dark energy is the famous cosmological constant which, however, is plagued with the so-called “cosmological constant problem” and “coincidence problem” \(^9\). Some dark energy models have been proposed with the dynamical scalar field(s), and some others by means of plausible quantum gravity arguments; the former refers to the well-known quintessence \(^{10,11}\), phantom \(^{12,13,14}\), k-essence \(^{15}\) and quintom models \(^{16,17,18,19}\), hessence \(^{20}\), etc, while the latter contains holography dark energy \(^{21}\), for instance.

As an important next step, these theoretical models have to be confronted by observational data. Indeed, a lot of observational constraints on these models have been carried out by using observational data, for example, from SNe, CMB, large scale structure (LSS), etc. Recently a kind of new constraints has attracted a lot of attention: the age of some old high redshift objects (OHROs) as a constraint on the cosmological model. The basic idea is that these OHROs can not be older than the universe itself. In the literatures, one usually uses the age of three OHROs to constrain some theoretical models: two of them are old galaxies (LBDS 53W091 and LBDS 53W069) and the other is a high redshift quasar (APM 08279 + 5255). The relevant data are listed in Table I, where \(z\) is the redshift with

\begin{center}
\begin{tabular}{ccc}
Name & Redshift & Age \\
LBDS 53W091 & \(z = 1.55\) & 3.5 Gyr \\
LBDS 53W069 & \(z = 1.43\) & 4.0 Gyr \\
APM 08279+5255 & \(z = 3.91\) & 2.0-3.0 Gyr or 2.1 Gyr \\
\end{tabular}
\end{center}

the definition \(z = a^{-1} - 1\) (assuming today’s scale factor \(a_0 = 1\) ). Those three OHROs are used extensively in the group of the old objects in our universe as today’s observation mentioned. LBDS 53W091 \(^{22,23}\) was 3.5 Gyr old at \(z = 1.55\). LBDS 53W069 \(^{24}\) was 4.0 Gyr at \(z = 1.43\). The age of APM 08279+5255 is in debate: one method \(^{25,26}\) shows its age between 2.0-3.0 Gyr at \(z = 3.91\), while the other method \(^{38}\) shows that it was 2.1 Gyr old at the same redshift. We use \(T = 2.0\)Gyr as the age of the APM 08279+5255\((z = 3.91)\). Because the \(T = 2.0\)Gyr is the lowest

\(*\) Email: zhangyi@itp.ac.cn
\(†\) Email: lihui@itp.ac.cn
\(‡\) Email: wxxwxxw@mail.bnu.edu.cn
\(§\) Email: haowei@mail.tsinghua.edu.cn
\(¶\) Email: cairg@itp.ac.cn
limit of the age of APM 08279+5255 \((z = 3.91)\), it is the loosest constraint of APM 08279+5255\((z = 3.91)\) on the age of the universe.

It turns out that the constraints on the age of the universe are not easy to satisfy. Taking the matter-dominated flat FRW universe, for example, its age \(T\) reads:

\[
T = \frac{2}{3} H_0^{-1} (1 + z)^{-3/2},
\]

where \(H\) is the Hubble parameter and the subscript “0” denotes today’s value of the corresponding quantity at redshift \(z = 0\). We define a dimensionless parameter \(h\) for convenience with \(H_0 = 100h \, km \cdot s^{-1} \cdot Mpc^{-1}\). According to (1.1), the flat matter-dominated FRW model can be ruled out unless \(h < 0.48\). Not only does the flat matter dominated FRW model have the problem of being compatible with the observational age-redshift relation of OHROs, but also the closed FRW matter dominated model. The age problem becomes even more serious when we consider the age of the universe at high redshift. The age problem is one of the reasons that we need an accelerated expansion of the universe today. When some dark energy component is introduced to the universe, it is shown that most dark energy models can only accommodate data of LBDS 53W091 \((z = 1.55)\) and LBDS 53W069 \((z = 1.43)\), but unfortunately cannot be compatible with APM 08279+5255\((z = 3.91)\). The list is long: the dark energy models with different EoS parameterizations \cite{27, 28}, the generalized Chaplygin gas \cite{29}, the \(\Lambda(t)CDM\) model \cite{30}, the model-independent EoS of dark energy \cite{31}, the scalar-tensor quintessence \cite{32}, the \(f(R)\) model \cite{33}, the DGP braneworld model \cite{34, 35}, the power-law parameterized quintessence model \cite{36}, holographic dark energy \cite{37}, and so on. In particular, the most famous and WMAP most favored model \(\Lambda CD\)M is also included in the list \cite{38, 39, 40}. This gives rise to the so-called age crisis in dark energy cosmology.

More recently, a new dark energy model, named agegraphic dark energy, has been proposed \cite{41}, which takes into account the uncertainty relation of quantum mechanics together with the gravitational effect in general relativity. One has the so called Károlyházy relation \(\delta t = \beta t^2 p t^{1/3}\) \cite{42}, and energy density of spacetime fluctuations \cite{43, 44, 45}

\[
\rho_q \sim \frac{1}{t_p^2 t^2},
\]

\(\beta\) is a numerical factor of order one, and \(t_p\) is the Planck time. The agegraphic dark energy model assumes that the observed dark energy comes from the spacetime and matter field fluctuations in the universe. The dark energy has the form (1.2) and \(t\) is identified with the age \(T\) of the universe \cite{41}

\[
\rho_q = \frac{3 n^2 M_{pl}^2}{T^2},
\]

where \(M_{pl}\) is the reduced Planck mass and the constant \(n^2\) has been introduced for representing some unknown theoretical uncertainties. Both in the radiation-dominated and matter-dominated epochs, the energy density of the agegraphic dark energy just scales as \(\rho_q \sim t^{-2}\) tracking the dominant energy component. Moreover, the model can also make the late-time acceleration. For the further development of the model, see \cite{46, 47, 48}.

In this paper we will use those three ORHOs to constrain the agegraphic dark energy model and see whether resulting constraints are compatible with constraints coming from other observational data. In the next section we introduce the method used to constrain the model. In Sect. 3 we will give the result on the age problem in the agegraphic dark energy. Sec. 4 will be devoted to the conclusion.

2. THE METHOD

A. The age of the universe versus redshift

We consider a spatially flat FRW universe which contains the agegraphic dark energy and the pressureless (dark and dust) matter. The Friedmann equation is given by

\[
3M_{pl}^2 H^2 = \rho_m + \rho_q,
\]

\[(2.1)\]
where \( \rho_m \) and \( \rho_q \) are the energy density of the pressureless matter and the agegraphic dark energy respectively. We assume there is no direct coupling between them and each of them satisfies the continuity equation separately,

\[
\dot{\rho}_m = -3H \rho_m, \tag{2.2}
\]

\[
\dot{\rho}_q = -3H(\rho_q + p_q). \tag{2.3}
\]

Taking derivative of Eq. (1.3) with respect to \( t \), we obtain

\[
\dot{\rho}_q = -2H \rho_q \sqrt{\Omega_q}, \tag{2.4}
\]

where \( \Omega_q = \rho_q / (3M^2_{pl}H^2) \) is the fractional energy density of the dark energy component. One can also define \( \Omega_m = \rho_m / (3M^2_{pl}H^2) \) as the fractional energy density of the pressureless matter. The Friedmann equation then can be rewritten as

\[
\Omega_q + \Omega_m = 1. \tag{2.5}
\]

Furthermore, taking derivative of the above formula with respect to \( z \) and considering the relationship \( dz = -H(1 + z)dt \), we reach

\[
\frac{d\Omega_q}{dz} = -(1 + z)^{-1} \Omega_q (1 - \Omega_q) \left( 3 - \frac{2}{n} \sqrt{\Omega_q} \right). \tag{2.6}
\]

This is the evolution equation which encodes the main information of the FRW cosmology. Given an initial \( \Omega_q \) at certain \( z \), we may evaluate \( \Omega_q \) at any specific redshift in terms of the model parameter \( n \).

FIG. 1: The three solid lines are, from left to right, contours \( T_z(3.91) = T_{z_{obj}}(3.91) \), \( T_z(1.43) = T_{z_{obj}}(1.43) \) and \( T_z(1.55) = T_{z_{obj}}(1.55) \). Only using the age constraint, it is obvious that the allowed parameter pairs \((\Omega_{m0}, n)\) should lie in the left common region of these three contours, as is indicated by the arrows. For a cross-check procedure with other observations, the WMAP3 bound \( \Omega_{m0} = 0.268 \pm 0.018 \) \([3]\) is indicated by two short-dashed lines and the model-independent cluster estimate \( \Omega_{m0} = 0.3 \pm 0.1 \) \([49]\) is indicated by two long-dashed lines. Here, we have used the reduced Hubble constant \( h = 0.68 \).

The age of our universe at redshift \( z \) is given by

\[
T(z) = \int_0^a \frac{da}{aH} = \int_z^\infty \frac{d\tilde{z}}{(1 + \tilde{z})H(\tilde{z})}. \tag{2.7}
\]
$T(z)$ means the age of the universe at a certain redshift $z$. It is convenient to introduce a dimensionless age parameter

$$T_z(z) = H_0 T(z) = \int_{z}^{\infty} \frac{d\tilde{z}}{(1 + \tilde{z})E(\tilde{z})},$$  \hspace{1cm} (2.8)$$

where $E(z) \equiv H(z)/H_0$. Considering the evolution of matter, $\rho_m = \rho_{m0}(1 + z)^{-3}$, we can easily get:

$$E(z) = \left[ \frac{\Omega_{m0}(1 + z)^3}{1 - \Omega_q} \right]^{1/2},$$  \hspace{1cm} (2.9)$$

So if $\Omega_{m0}$ is fixed, the value of $\Omega_q$ is fixed by Eq.(2.5). We can get the values of $\Omega_q$ by Eq.(2.6), $E(z)$ by Eq.(2.9), $T_z(z)$ by Eq.(2.8) at any redshift with only a model parameter $n$.

At any redshift, the age of our universe should not be smaller than the age of the OHROs, namely

$$T_z(z) \geq T_{\text{obj}} = H_0 T_{\text{obj}}$$  \hspace{1cm} (2.10)$$

where $T_{\text{obj}}$ is the age of the corresponding OHROs. If we fixed $T_z(z) = T_{\text{obj}}$, from the analysis before, we can get the value of $n$ with the initial $\Omega_{m0}$ fixed. Here we can not forget the parameter $H_0$ in the equation $T_{\text{obj}} = H_0 T_{\text{obj}}$ which should also be fixed in advance. We discuss the choice of the parameters in the next subsection.

Let's make a short summary. If we had the value of $H_0$, we know the value of $T_{\text{obj}}$ by Eq.(2.10). And given an initial $\Omega_{m0}$, it is equivalent to give an initial $\Omega_q$, then we can know $\Omega_q$ by Eq.(2.6), $E(z)$ by Eq.(2.9), and $T_z(z)$ by Eq.(2.8) with the model parameter $n$. Using the relation $T_z(z) = T_{\text{obj}}$, we can get a model parameter $n$. If we let $\Omega_{m0}$ run in a range, there is a curve showing the relation between the different $\Omega_{m0}$ and its corresponding minimal estimation of $n$.

**B. The choice of the parameter**

There are various observational methods such as SN Ia, CMB, LSS, which could give out many data on the cosmological parameters. We will use the scope of some cosmological parameters provided by them to test the age problem in the agegraphic dark energy model with respect to three OHROs listed in Table.
FIG. 3: The same as in Fig. 1 except for $h = 0.623$.

As stated in the previous subsection, one should first choose an $\Omega_q$ or $\Omega_m$ as an initial condition. The loosest $\Omega_m$ value is $\Omega_m = 0.3 \pm 0.1$ from model-independent cluster estimation [49]. A tighter WMAP3 bound is $\Omega_m = 0.268 \pm 0.018$ [5] with the combined constraint from the latest SNe Ia, galaxy clustering and CMB anisotropy.

The choice of Hubble constant is not a direct one. In the literatures, the reduced Hubble constant $h = 0.72 \pm 0.08$ of Freedman et al. [50] has been used extensively. This Hubble constant seems too high to explain away the age problem. And many authors also argue for a lower Hubble constant, for instance, $h = 0.68 \pm 0.07$ at 2$\sigma$ confidence level in [51]. In the past few years, it has been also argued that there exits systematic bias in the result of Freedman et al. [50]. Sandage and collaborators advocate a lower Hubble constant in a series of works [52, 53, 54, 55, 56], and their final result reads $h = 0.623 \pm 0.063$ [56]. We will take the parameter values of $h = 0.68$ (the center value in [51]), $h = 0.64$ (the lower limit of $h = 0.72 \pm 0.08$), $h = 0.623$ (the center value after eliminating the alleged systematic bias), $h = 0.59$ (a useful mediate value), $h = 0.56$ (the lower limit of $h = 0.623 \pm 0.063$) in our analysis in turn.

3. RESULTS

We show the numerical results in Fig. 3 for different Hubble parameters. From Eq. (2.9), the lower $\Omega_m$, the lower $E(z)$ is, and then from Eq. (2.8), the larger $T_z(z)$ is, so the allowed parameter space must be constrained to the left by the three curves $T_z(z) > T_{z, obj}$. The leftmost curve $T_z(3.91) > T_{z, obj}(3.91)$ gives the most stringent bound and gives consequently the allowed parameter space, which corresponds to the common left regions of these three contours, as the arrows indicate. We see from the figures that when $h$ decreases from 0.68 to 0.56, the allowed parameter space expands horizontally to the right and begins to cover the regions which are required by cluster estimation of $\Omega_m = 0.3 \pm 0.1$ and WMAP3 $\Omega_m = 0.268 \pm 0.018$, respectively. The constraints given by LBDS 53W091 ($z = 1.55$) and LBDS 53W069 ($z = 1.43$) can be easily satisfied in the agegraphic dark energy model, as many other dark energy models. The most difficult one to accommodate is APM 08279+5255. In Table II we list the exact result for APM 08279+5255 curves in Fig. 3.

If $\Omega_m = 0.3 \pm 0.1$, Fig. 2 shows that one can take the Hubble parameter as large as $h = 0.64$ in our model. When $h = 0.59$, the parameter scope coming from WMAP3 begins to be reached, while $h = 0.56$ makes the allowed region
cover nearly the whole interval $\Omega_{m0} = 0.268 \pm 0.018$. We therefore conclude that our model alleviate the age crisis.

We can make a simple comparison among some dark energy models. The WMAP most favored $\Lambda CDM$ model can not accommodate APM 08279+5255 ($z = 3.91$) unless the Hubble parameter is taken as low as $h = 0.58$ [38], and $\Lambda(t)CDM$ model considered in [30] can not change this conclusion. In Table III, we list some results for $\Lambda(t)CDM$ model, $\Lambda CDM/DGP$ model, holographic dark energy model and agegraphic dark energy model, in order to accommodate the APM 08279+5255.
TABLE II: The range of fraction matter energy density, which can accommodate the old quasar APM 08279+5255, with different Hubble parameter.

| $h$ | $\Omega_{m0}(0.2, 0.4)$ | $\Omega_{m0}(0.22, 0.4)$ | $\Omega_{m0}(0.25, 0.286)$ |
|-----|------------------------|------------------------|------------------------|
| 0.56 | Ok                     | Ok                     | Ok                     |
| 0.59 | Ok                     | Ok                     | Ok                     |
| 0.623 | Ok                     | Ok                     | No                     |
| 0.64 | Ok                     | Marginal               | No                     |
| 0.68 | No                     | No                     | No                     |

TABLE III: A rough comparison of some dark energy models

| Name                          | $\Omega_{m0}$ | $h$  | $z$        |
|-------------------------------|---------------|------|-----------|
| $\Lambda(t)\text{CDM}[30]$   | 0.2           | 0.64 | > 5.11    |
| Holography DE [37]            | ~ 0.2         | 0.64 | 3.91      |
| $\Lambda\text{CDM}/\text{DGP}$ [38] | 0.23         | 0.58 | 3.91      |
| Agegraphic DE                 | ~ 0.22        | 0.64 | 3.91      |

According to $[30]$, even when the present energy density of matter takes a value as low as $\Omega_{m0} = 0.2$ and $h = 0.64$, one still finds $z > 5.11$, which is clearly incompatible with the observation $z = 3.91$. When $h = 0.64$, $z = 3.91$, one could have $\Omega_{m0} \sim 0.2 [37]$, in the holography dark energy, while in the agegraphic dark energy model, the same parameter evaluation tells us $\Omega_{m0} \sim 0.22$.

In addition, let us mention here three features of the behavior on the solid lines $T_z(z) = T_{z\text{obj}}$ in the figures. (1) at the bottom of the curves with low $\Omega_{m0}$ and small $n$, the curves have an oscillation behavior. (2) at the middle of the curves, as $\Omega_{m0}$ increases, $n$ increases, and the curves in $(n - \Omega_{m0})$ plane, go from the left-bottom to the right-top. (3) at the top of the curves, $n$ increases fast but $\Omega_{m0}$ nearly keeps unchanged.

We may give a possible explanation for the oscillation behavior from the state of parameter (EoS) $w$ of the universe. It reads

$$w = \frac{p_m + p_q}{\rho_m + \rho_q} = \Omega_q \left( -1 + \frac{2\sqrt{\Omega_q}}{3n} \right).$$

Taking derivative of Eq.(3.1) with respect to $\sqrt{\Omega_q}$, one has

$$\frac{dw}{d\sqrt{\Omega_q}} = -2\sqrt{\Omega_q}(1 - \sqrt{\Omega_q}/n).$$

We can find that, as $\Omega_q$ increases from zero, $w$ monotonously decreases first and then increases, therefore the transition could happen at $\sqrt{\Omega_q} = n$ if $n < 1$. That is, if $n$ was too low while $\Omega_{m0}$ was low too, one value of $n$ may correspond to two $\Omega_q$ or $\Omega_{m0}$ equivalently; as a consequence, the oscillation of the curves at the left-bottom of $(n - \Omega_{m0})$ parameter space appears. This argument may be illustrated in Fig. 6. In fact, the region with the oscillation behavior should be regarded as unphysical one since in order to have an accelerated expansion for the universe, the model requires $n > 1 [41]$.

We see from the figures that $n$ increases very quickly when $\Omega_{m0}$ gets large. Finally $n$ becomes insensitive to $\Omega_{m0}$. Thus we can hardly get the upper limit of the parameter $n$ since the curve is too straight to leave from the allowed parameter region as the parameter $n$ increases. However the contour given by $T_z(z) = T_{z\text{obj}}$ is enough to give the age constraint we need. We can know from figures whether the resulting range of $\Omega_{m0}$ is in the range given by other observational constraints. In the figures the WMAP3 bound $\Omega_{m0} = 0.268 \pm 0.018 [3]$ is indicated by two short-dashed lines and the model-independent cluster estimation $\Omega_{m0} = 0.3 \pm 0.1 [49]$ is indicated by two long-dashed lines.
FIG. 6: The relationship between $w$ and $\sqrt{\Omega_q}$ when $n = 0.85$, one value of $w$ may correspond to two different values of $\Omega_q$, and then one value of $n$ may result in two different values of $\Omega_q$.

4. CONCLUSION

The relationship between the red-shift $z$ and the cosmic age $t$ contains a lot of information about the evolution of the universe. In this paper we have investigated the age constraint on the agegraphic dark energy model by using the observational data for two old galaxies (LBDS 53W091 with $z = 1.55$ and LBDS 53W069 with $z = 1.43$) and an old high redshift quasar (APM 08279 + 5255 with $z = 3.91$). As most dark energy models, the agegraphic dark energy model can easily accommodate these two old galaxies. In order to accommodate the old quasar, one has to take a little lower Hubble parameter $h = 0.64$ when $\Omega_{m0} \approx 0.22$. Although the behavior looks slightly better than some other dark energy models, the age crisis is still there, unless the current Hubble parameter has indeed a lower value than the best fitting value of WMAP3, for example, $h = 0.59$ as advocated by Sandage and collaborators.

Finally we stress here that the numerical results are obtained through integrating the equation (2.6) by imposing an initial condition, for example, $\Omega_q = 0.73$ at redshift $z = 0$. As stressed in [41], the equation (2.6) not only holds for the form $T = \frac{n}{H\sqrt{\Omega_q}}$, but also for another form, $T' = T + \delta = \frac{n}{H\sqrt{\Omega_q}}$. And the constant $\delta$ can be obtained by $\delta = \frac{n}{H\sqrt{\Omega_q}} - \int_0^z \frac{da}{H}$ at $z = \infty$. The integration from $z = 0$ to $z = \infty$ does not guarantee the constant $\delta$ vanishes. As a result, the energy density could have a form $\rho_q = \frac{3n^2M_p^2}{(T+\delta)^2}$ in a general case. In addition, it would be of great interest to study the age constraint on the new model of agegraphic dark energy [48], where a conformal time scale is introduced to the energy density (1.3), instead of the cosmic age.

ACKNOWLEDGMENTS

We are grateful to Bin Hu for useful discussions. We also thank Fu-qiang Xu and Ding Ma for kind help. The work was supported in part by a grant from Chinese Academy of Sciences (No. KJCX3-SYW-N2), and by NSFC under grants No. 10325525, No. 10525060 and No. 90403029.

[1] A. G. Riess et al. [Supernova Search Team Collaboration], Astron. J. 116, 1009 (1998) [astro-ph/9805201]; S. Perlmutter et al. [Supernova Cosmology Project Collaboration], Astrophys. J. 517, 565 (1999) [astro-ph/9812133]; J. L. Tonry et al. [Supernova Search Team Collaboration], Astrophys. J. 594, 1 (2003) [astro-ph/0305008]; R. A. Knop et al. [Supernova Cosmology Project Collaboration], Astrophys. J. 598, 102 (2003) [astro-ph/0309368]; A. G. Riess et al. [Supernova Search Team Collaboration], Astrophys. J. 607, 665 (2004) [astro-ph/0402512].
[2] A. G. Riess et al. [Supernova Search Team Collaboration], astro-ph/0611572
The numerical data of the full sample are available at
http://braeburn.pha.jhu.edu/~ariess/R06 or upon request to ariess@stsci.edu

[3] P. Astier et al. [SNLS Collaboration], Astron. Astrophys. 447, 31 (2006) astro-ph/0510447;
J. D. Neill et al. [SNLS Collaboration], astro-ph/0605148.

[4] C. L. Bennett et al. [WMAP Collaboration], Astrophys. J. Suppl. 148, 1 (2003) astro-ph/0302207;
D. N. Spergel et al. [WMAP Collaboration], Astrophys. J. Suppl. 148 175 (2003) astro-ph/0302209.

[5] D. N. Spergel et al. [WMAP Collaboration], astro-ph/0603449;
L. Page et al. [WMAP Collaboration], astro-ph/0603450;
G. Hinshaw et al. [WMAP Collaboration], astro-ph/0603451;
N. Jarosik et al. [WMAP Collaboration], astro-ph/0603452.

[6] M. Tegmark et al. [SDSS Collaboration], Phys. Rev. D 69, 103501 (2004) astro-ph/0310723;
M. Tegmark et al. [SDSS Collaboration], Astrophys. J. 606, 702 (2004) astro-ph/0310725;
U. Seljak et al., Phys. Rev. D 71, 103515 (2005) astro-ph/0407372;
J. K. Adelman-McCarthy et al. [SDSS Collaboration], Astrophys. J. Suppl. 162, 38 (2006) astro-ph/0507711;
K. Abazajian et al. [SDSS Collaboration], astro-ph/0410239 astro-ph/0403325 astro-ph/0305492;
M. Tegmark et al. [SDSS Collaboration], Phys. Rev. D 74, 123507 (2006) astro-ph/0608632.

[7] S. W. Allen, R. W. Schmidt, H. Ebeling, A. C. Fabian and L. van Speybroeck, Mon. Not. Roy. Astron. Soc. 353, 457 (2004) astro-ph/0405340;
S. W. Allen, D. A. Rapetti, R. W. Schmidt, H. Ebeling, G. Morris and A. C. Fabian, arXiv:0706.0033 [astro-ph].

[8] W. M. Wood-Vasey et al. [ESSENCE Collaboration], astro-ph/0701041;
G. Miknaitis et al. [ESSENCE Collaboration], astro-ph/0701043.

[9] P. J. E. Peebles and B. Ratra, Rev. Mod. Phys. 75, 559 (2003) astro-ph/0207347;
T. Padmanabhan, Phys. Rept. 380, 235 (2003) hep-th/0212290;
S. M. Carroll, astro-ph/0310342;
R. Bean, S. Carroll and M. Trodden, astro-ph/0510059;
V. Sahni and A. A. Starobinsky, Int. J. Mod. Phys. D 9, 373 (2000) astro-ph/9904398;
S. M. Carroll, Living Rev. Rel. 4, 1 (2001) astro-ph/00004075;
T. Padmanabhan, Curr. Sci. 88, 1057 (2000) astro-ph/0411044;
S. Weinberg, Rev. Mod. Phys. 61, 1 (1989);
S. Nobbenhuis, Found. Phys. 36, 613 (2006) gr-qc/0411093;
E. J. Copeland, M. Sami and S. Tsujikawa, Int. J. Mod. Phys. D 15, 1753 (2006) hep-th/0603057;
A. Albrecht et al., astro-ph/0609591;
R. Trotta and R. Bower, astro-ph/0607066;
M. Kamionkowski, arXiv:0706.2986 [astro-ph];
B. Ratra and M. S. Vogeley, arXiv:0706.1565 [astro-ph];
E. V. Linder, arXiv:0705.4102 [astro-ph].

[10] R. R. Caldwell, R. Dave and P. J. Steinhardt, Phys. Rev. Lett. 80, 1582 (1998) astro-ph/9708069;
C. Wetterich, Nucl. Phys. B 302, 668 (1988);
P. J. E. Peebles and B. Ratra, Astrophys. J. 325, L17 (1988);
B. Ratra and P. J. E. Peebles, Phys. Rev. D 37, 3406 (1988).

[11] P. J. Steinhardt, L. M. Wang and I. Zlatev, Phys. Rev. D 59, 123504 (1999) astro-ph/9812313;
I. Zlatev and P. J. Steinhardt, Phys. Lett. B 459, 570 (1999) astro-ph/9906481;
R. R. Caldwell, Phys. Lett. B 545, 23 (2002) astro-ph/9908168.

[12] R. R. Caldwell, Phys. Rev. Lett. 91, 071301 (2003) astro-ph/0302506;
S. M. Carroll, M. Hoffman and M. Trodden, Phys. Rev. D 68, 023509 (2003) astro-ph/0301273;
J. M. Cline, S. Y. Jeon and G. D. Moore, Phys. Rev. D 70, 043543 (2004) hep-ph/0311312.

[13] C. Armendariz-Picon, V. Mukhanov and P. J. Steinhardt, Phys. Rev. Lett. 85, 4438 (2000) astro-ph/0004134;
C. Armendariz-Picon, V. Mukhanov and P. J. Steinhardt, Phys. Rev. D 63, 103510 (2001) astro-ph/0006373;
T. Chiba, T. Okabe and M. Yamaguchi, Phys. Rev. D 62, 023511 (2000) astro-ph/9912463;
M. Malquarti, E. J. Copeland and A. R. Liddle, Phys. Rev. D 68, 023512 (2003) astro-ph/0304277;
M. Malquarti, E. J. Copeland, A. R. Liddle and M. Trodden, Phys. Rev. D 67, 123503 (2003) astro-ph/0302279;
H. Wei and R. G. Cai, Phys. Rev. D 71, 043504 (2005) hep-th/0412045.
[16] B. Feng, X. L. Wang and X. M. Zhang, Phys. Lett. B 607, 35 (2005) [astro-ph/0404224].
[17] Z. K. Guo, Y. S. Piao, X. M. Zhang and Y. Z. Zhang, Phys. Lett. B 608, 177 (2005) [astro-ph/0410654].
[18] H. Wei and R. G. Cai, Phys. Lett. B 634, 9 (2006) [astro-ph/0512018].
Z. K. Guo, Y. S. Piao, X. M. Zhang and Y. Z. Zhang, Phys. Rev. D 74, 127304 (2006) [astro-ph/0608165].
X. F. Zhang, H. Li, Y. S. Piao and X. M. Zhang, Mod. Phys. Lett. A 21, 231 (2006) [astro-ph/0501652]. M. Z. Li, B. Feng and X. M. Zhang, JCAP 0512, 002 (2005) [hep-ph/0503268].
X. F. Zhang and T. T. Qiu, Phys. Lett. B 642, 187 (2006) [astro-ph/0603824].
Y. F. Cai, H. Li, Y. S. Piao and X. M. Zhang, Phys. Lett. B 646, 141 (2007) [gr-qc/0609039].
Y. F. Cai, M. Z. Li, J. X. Lu, Y. S. Piao and X. M. Zhang, Phys. Lett. B 651, 1 (2007) [hep-th/0701016].
Y. F. Cai, T. T. Qiu, Y. S. Piao and X. M. Zhang, arXiv:0704.1090 [gr-qc].
[19] R. Lazkoz and G. Leon, Phys. Lett. B 638, 303 (2006) [astro-ph/0602590].
R. Lazkoz, G. Leon and I. Quiros, Phys. Lett. B 649, 103 (2007) [astro-ph/0701353].
M. Alimohammadi, arXiv:0706.1360 [gr-qc].
[20] H. Wei, R. G. Cai and D. F. Zeng, Class. Quant. Grav. 22, 3189 (2005) [hep-th/0501160].
H. Wei and R. G. Cai, Phys. Rev. D 72, 123507 (2005) [astro-ph/0509328].
M. Alimohammadi and H. Mohseni Sadjadi, Phys. Rev. D 73, 083527 (2006) [hep-th/0602268].
W. Zhao and Y. Zhang, Phys. Rev. D 73, 123509 (2006) [astro-ph/0604460].
H. Wei, N. N. Tang and S. N. Zhang, Phys. Rev. D 75, 043009 (2007) [astro-ph/0612746].
H. Wei and S. N. Zhang, arXiv:0705.4002 [gr-qc].
W. Zhao, arXiv:0706.2211 [astro-ph].
[21] M. Li, Phys. Lett. B 603, 1 (2004) [hep-th/0403127].
[22] J. Dunlop et al., Nature 381, 581 (1996).
[23] H. Spinrad et al., Astrophys. J. 484, 581 (1999).
[24] J. Dunlop, in The Most Distant Radio Galaxies, edited by H. J. A. Rottgering, P. Best and M. D. Lehnert, Kluwer, Dordrecht (1999), p. 71.
[25] G. Hasinger, N. Schartel and S. Komossa, Astrophys. J. 573, L77 (2002) [astro-ph/0207005].
S. Komossa and G. Hasinger, astro-ph/0207321.
[26] J. A. S. Lima and J. S. Alcaniz, Mon. Not. Roy. Astron. Soc. 317, 893 (2000) [astro-ph/0005441].
[27] D. Jain and A. Dev, Phys. Lett. B 633, 436 (2006) [astro-ph/0509212].
[28] J. S. Alcaniz, D. Jain and A. Dev, Phys. Rev. D 67, 043514 (2003) [astro-ph/0210476].
[29] J. S. Alcaniz, D. Jain and A. Dev, Phys. Rev. D 67, 043514 (2003) [astro-ph/0210476].
[30] J. V. Cunha and R. C. Santos, Int. J. Mod. Phys. D 13, 1321 (2004) [astro-ph/0402169].
[31] J. F. Jesus, astro-ph/0603142.
[32] M. A. Dantas, J. S. Alcaniz, D. Jain and A. Dev, Astron. Astrophys. 467, 421 (2007) [astro-ph/0607060].
S. Capozziello, P. K. S. Dunsby, E. Piedipalumbo and C. Rubano, arXiv:0706.2615 [astro-ph].
[33] M. S. Movahed, S. Baghram and S. Rahvar, arXiv:0705.0889 [astro-ph].
[34] M. S. Movahed, M. Farhang and S. Rahvar, astro-ph/0701339.
M. S. Movahed and S. Ghassemi, arXiv:0705.3894 [astro-ph].
[35] N. Pires, Z. H. Zhu and J. S. Alcaniz, Phys. Rev. D 73, 123530 (2006) [astro-ph/0606689].
S. Rahvar and M. S. Movahed, Phys. Rev. D 75, 023512 (2007) [astro-ph/0604206].
[36] H. Wei and S. N. Zhang, arXiv:0707.2129 [astro-ph].
[37] A. Friaca, J. Alcaniz and J. A. S. Lima, Mon. Not. Roy. Astron. Soc. 362, 1295 (2005) [astro-ph/0504031].
[38] J. S. Alcaniz, J. A. S. Lima and J. V. Cunha, Mon. Not. Roy. Astron. Soc. 340, L39 (2003) [astro-ph/0301226].
[39] J. S. Alcaniz and J. A. S. Lima, Astrophys. J. 521, L87 (1999) [astro-ph/9902298].
[40] R. G. Cai, arXiv:0707.4049 [hep-th].
[41] F. Károlyházy, Nuovo.Cim. A42, 390 (1966); F. Károlyházy, A. Frenkel and B. Lukács, In Physics as natural Philosophy (Eds. A. Shimony and H. Feschbach, MIT Press, Cambridge, MA, 1982); F. Károlyházy, A. Frenkel and B. Lukács, In Quantum Concepts in Space and Time (Eds. R. Penrose and C.J. Isham, Clarendon Press, Oxford, 1986).
[42] M. Maziajashvili, arXiv:gr-qc/0612110. M. Maziajashvili, arXiv:0705.0924 [gr-qc].
[43] N. Sasakura, Prog. Theor. Phys. 102, 169 (1999) arXiv:hep-th/9903146.
[44] Y. J. Ng and H. Van Dam, Mod. Phys. Lett. A 9, 335 (1994); W. A. Christiansen, Y. J. Ng and H. van Dam, Phys. Rev. Lett. 96, 051301 (2006) arXiv:gr-qc/0508121. Y. J. Ng, arXiv:gr-qc/0703090.
[45] H. Wei and R. G. Cai, arXiv:0707.4052 [hep-th]; H. Wei and R. G. Cai, arXiv:0707.4526 [physics.gen-ph].
[46] X. Wu, Y. Zhang, H. Li, R. G. Cai and Z. H. Zhu, arXiv:0708.0349 [astro-ph].
[48] H. Wei and R. G. Cai, arXiv:0708.0884 [astro-ph].

[49] R. Carlberg, H. K. C. Yee, E. Ellingson, R. Abraham, P. Gravel, S. Morris and C. J. Pritchet, Astrophys. J. 462, 32 (1996) [astro-ph/9509034].

A. Dekel, D. Burstein and S. White, in Critical Dialogues in Cosmology, edited by N. Turok, World Scientific, Singapore (1997).

[50] W. L. Freedman et al., Astrophys. J. 553, 47 (2001) [astro-ph/0012376].

[51] J. R. I. Gott, M. S. Vogeley, S. Podariu and B. Ratra, Astrophys. J. 549, 1 (2001) [astro-ph/0006103];

G. Chen, J. R. I. Gott and B. Ratra, Publ. Astron. Soc. Pac. 115, 1269 (2003) [astro-ph/0308099].

[52] G. A. Tammann, A. Sandage and B. Reindl, Astron. Astrophys. 404, 423 (2003) [astro-ph/0303378].

[53] A. Sandage, G. A. Tammann and B. Reindl, Astron. Astrophys. 424, 43 (2004) [astro-ph/0402424].

[54] B. Reindl, G. A. Tammann, A. Sandage and A. Saha, Astrophys. J. 624, 532 (2005) [astro-ph/0501664].

[55] A. Saha, F. Thim, G. A. Tammann, B. Reindl, and A. Sandage Astrophys. J. Suppl. 165, 108 (2006) [astro-ph/0602572].

[56] A. Sandage, G. A. Tammann, A. Saha, B. Reindl, F. D. Macchetto and N. Panagia, Astrophys. J. 653, 843 (2006) [astro-ph/0603647].