Efficiency of the use of feedback in the stabilization of motion of the rotary hydraulic drive

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Abstract. This article reflects a study of the operation of a hydraulic drive with nonlinear characteristics of an axial piston hydraulic motor, which operates at the lower limit of the speeds of the executive bodies. A nonlinear frictional moment arises in the end valve of the hydraulic motor connected to the actuator and leads to movement with stops. The mathematical models of the dynamics of the hydraulic drive, proposed in this study, are linearized by the method of harmonic linearization and allow engineering calculations of the parameters of the modes of non-stop motion. Evaluation of the dynamic properties of the hydraulic drive, corresponding to the onset of an unstable mode, makes it possible to determine the minimum value of the constant component of the speed of the executive body.

1. Introduction

In modern technological machines equipped with a hydraulic drive, it is necessary to ensure high requirements for the accuracy of performing technological movements of the actuating elements, in particular, of the rotational type, carried out by axial rotary-piston hydraulic actuators. Oscillations in the angular rotation of the rotor of the hydraulic actuator at the lower limit of the control range can be caused by both the tightness characteristic of the hydraulic actuator [1] and the friction characteristics in it [3, 4, 5]. To reduce these oscillations, as well as to exclude modes of motion with periodic stops, you can use the feedback of the angular rotation of the rotor, correcting the flow of the pump and feedback to accelerate the rotor, adjusting the pressure in the pressure line (Figure 1 shows the scheme of the drive).

![Figure 1. Schematic diagram of hydraulic drive with correcting feedback](image)
(m³/rad) is the flow rate of the hydraulic actuator when its rotor is rotated by 1 rad; $k_{D\Omega}$ (Pa·s²/m³) is the acceleration feedback coefficient; $D$ (1/s) is the differentiation operator.

In the delivery control $Q_p$ (m³/s) of the pump the introduction of a feedback of the angular rotation $\Omega$ of the rotor of the hydraulic actuator, fluid flow rate supplying to the discharge line, in the first approximation can be represented as a linear function of the speed deviation $\Omega$ of the rotor from the specified value $\Omega_0$ (under the assumption that the feedback system is inertia-free):

$$Q_p = Q_0 + \psi_\Omega (\Omega_0 - \Omega),$$

where $Q_0$ is the fluid flow rate corresponding to the given angular rotation $\Omega_0$. Since the sign of the difference $\Omega_0 - \Omega$ can be any, then the dependence (1) should be limited to $Q_p \geq 0$, since we consider a hydraulic system with a non-reversible fluid flow supplied by a pump. Then the zero value of the feed $Q_p$ will correspond to some critical value $Q_{cr}$ of the angular rotation $\Omega$, and the equation (1) will take the form

$$Q_p = \psi_\Omega (\Omega_0 - \Omega_0) = 0.$$

The critical speed given that $Q_0 = q\Omega_0$ is determined by the dependence

$$Q_{cr} = \Omega_0 \left( \frac{q}{\psi_\Omega} + 1 \right).$$

The equations of dynamics of the hydraulic drive, taking into account the feedback on the speed and acceleration of the rotor of the hydraulic actuator, will take the form

$$\left(p_1 - k_{D\Omega}D\Omega \right)q = J\Omega + M_f + \sigma \Omega + M_0,$$

$$Q_0 = \psi_\Omega (\Omega_0 - \Omega) = q\Omega + k_1Dp_1 + \sigma_1p_1,$$

where $J$ is the moment of inertia of the rotor of the hydraulic actuator, N·m·s²/rad;

$M_0$ – is constant load, N·m;

$\sigma$ – is leakage coefficient from the tension cavity, m³/s·Pa;

$M_f$ – is the moment of dry friction forces in the hydraulic actuator, Nm;
\( \beta \) is the coefficient of damping determined by the purely viscous friction in the hydraulic actuator and the load, N m s\(^{-1}\) rad;

\( k_1 \) is the coefficient of compliance of the discharge line, taking into account the compressibility of the liquid, m\(^3\)/Pa.

The block diagram corresponding to the system of equations (5) and (6) is shown in Figure 3.

The friction in axial-piston hydraulic actuators is determined by the pressures \( p_1 \) and \( p_2 \) in the cavities, the angular rotation and the rotor coordinate \( \varphi \) [3, 4, 5]. If the rotor rotates only in one direction with an insignificant backstop \( (p_2 \approx 0) \) in the drain line and low angular rotations, then the average, independent of the angular coordinate, the friction characteristic in the first approximation can be approximated by two linear "falling" dependencies (Figure 4). For an analytical study, the characteristic with two sections must be linearized.

Let us assume that the nonlinear and linear characteristics of friction have a common point at \( \Omega = 0 \). Then

\[
\frac{\varepsilon p_1}{M_f} = \frac{\Omega}{M = 0 - \Omega}, \text{ i.e. } M_f = \varepsilon p_1 - \frac{\varepsilon p_1 \Omega}{M = 0},
\]

where \( \varepsilon \) is the coefficient of proportionality between the moment of friction in the hydraulic actuator and the pressure \( p_1 \) in the discharge cavity, N m Pa [4] or

\[
M_f = \varepsilon p_1 - \lambda \Omega,
\]

where \( \lambda = \frac{\varepsilon p_{10}}{M = 0} \), N m s\(^{-1}\) rad, \( p_{10} \) is the average pressure value \( p_1 \).

It should be taken into account that when \( M_f \geq 0 \), then in the absence of positive damping in the system, the pressure

\[
p_{10} = \frac{\lambda M = 0}{\varepsilon}.
\]

2. Problem statement

To analyze the system of equations (5) and (6), it is necessary to linearize the static characteristic of the speed feedback \( \psi_{\Omega}(\Omega) \), which is connected sequentially with a first-order linear link, which can be considered as a low-pass filter. The filtering properties of this link are determined by the coefficient of...
compliance \( k_i \). For sufficiently large values of \( k_i \), the method of harmonic linearization of the nonlinear static characteristic \( \psi_{eq}(\Omega) \) of the feedback can be used [2].

3. Theory
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For asymmetric oscillations, the equivalent angular rotation feedback coefficient (Figure 2)

\[
\psi_{\Omega e} = \psi_0^0 (A, \Omega) + b(A, \Omega)(\Omega - \Omega_0),
\]

(9)

since the output coordinate \( \psi_{\Omega e} \) does not depend on the speed of the input signal \( A \sin \omega t \), or \( \varphi \)

\[
\psi_{\Omega e} = \frac{1}{2\pi} \int_0^{2\pi} \psi_{\Omega e}(\Omega_0) + A \sin \varphi \right) d\varphi = \frac{1}{2\pi} \left[ a(2 \arcsin \left( \frac{\varphi}{n} + \pi \right) ) + \frac{Q_0}{A} \ln(n + n^2 - 1)^{-2} \right]
\]

(10)

since, \( \sin \varphi = \frac{\Omega_{cr} - \Omega}{\Omega_0} = \frac{\varphi}{n} \), where \( n = \frac{\varphi_0}{\varphi} \), \( \varphi = \omega t \).

\[
b(A, \Omega_0) = \frac{1}{\pi A_0} \left( 2 \pi A_0(\Omega_0) + A \sin \frac{\varphi}{\sin \varphi} \right) d\varphi = \frac{1}{\pi A} \left[ \frac{Q_0}{A} (\pi - 2 \arcsin \frac{1}{n}) - 2a \sqrt{1 - \frac{1}{n^2}} \right]
\]

(11)

Then the feedback coefficient will be determined as follows:

\[
\psi_{\Omega e} = \frac{1}{2\pi} \left[ a(2 \arcsin \left( \frac{1}{n} + \pi \right) ) + \frac{Q_0}{A} \ln(n + \sqrt{n^2 - 1})^{-2} \right] + \frac{1}{\pi A} \left[ \frac{Q_0}{A} (\pi - 2 \arcsin \frac{1}{n}) - 2a \sqrt{1 - \frac{1}{n^2}} \right] (\Omega - \Omega_0).
\]

(12)

For the pump feed determined by the feedback, where the expressions (1) taking into account the ratio (12) can be written as:

\[
\psi_{\Omega e} (\Omega - \Omega_0) (B + \frac{C}{A})(\Omega_0 - \Omega) = \left( \frac{M}{A^2} - \frac{N}{A} \right)(\Omega - \Omega_0)^2,
\]

(13)

where \( B = \frac{a}{2\pi} (2 \arcsin \frac{1}{n} + \pi); \, C = \frac{Q_0}{2\pi} \ln(n + \sqrt{n^2 - 1})^{-2}; \, M = \frac{Q_0}{\pi} (\pi - 2 \arcsin \frac{1}{n}); \, N = \frac{2a}{\pi} \sqrt{1 - \frac{1}{n^2}} \).

In equation (13), a new nonlinearity appears which also needs to be linearized by the method of harmonic linearization. If we accept \( \frac{M}{A^2} - \frac{N}{A} = K \), then for the components of an equivalent linear expression, we can write the following equations:

\[
\psi_0(A, \Omega_0) = K \left[ (\Omega_0 + \frac{A^2}{2})(\pi + \arcsin \frac{\Omega_0}{A}) + \frac{3}{2} \frac{Q_0}{A} \sqrt{1 - \frac{\Omega_0^2}{A^2}} \right];
\]

(14)

\[
m(A, \Omega_0) = \frac{2K}{\pi} \left[ \Omega_0 (\pi + \arcsin \frac{\Omega_0}{A}) + \frac{2A}{3} \frac{Q_0}{5A} \sqrt{1 - \frac{\Omega_0^2}{A^2}} \right].
\]

(15)

The equivalent linear expression, taking into account the relations (14) and (15), takes the form
\[
\psi_0 (\omega_0 - \omega) \approx (B + C \omega_0 - \omega) + 2K \omega_0 - K \omega_0^2 + \frac{K}{\pi} \left[ \left( \omega_0^2 + \frac{A^2}{2} \right) \left( 2 + \arcsin \frac{\omega}{A} \right) + 3 \frac{\omega_0^2}{3A} \right] - 2K \left[ \omega_0^2 + \arcsin \frac{\omega}{A} \right] \left( 1 - \frac{\omega_0^2}{A^2} \right) \left( \omega_0 - \omega \right). \tag{16}
\]

Thus, the equations of motion of a hydraulic drive taking into account the expression (7) and the relation (16) is transformed to the

\[
(p_1 - k_{D0} D \Omega) q = JD \Omega + \beta \Omega + \varepsilon p_1 - \lambda \Omega + M_0; \tag{17}
\]
\[
Q_0 - L \Omega = q \Omega + k_1 D p_1 + \delta p_1 - E, \tag{18}
\]

where \( L = (B + \frac{C}{\Omega}) \) is the linearized speed feedback coefficient \( \Omega; E = \text{const} = -4.5 \Omega_0^2 + (B + \frac{C}{\Omega_0}) \Omega_0. \)

Excluding the variable \( p_1 \), from the systems of equations (17) and (18), we obtain

\[
k_1 (k_{D0} q + J) D^2 \Omega + q \delta_1 k_{D0} + J \sigma_1 + k_1 \beta) + k_1 \lambda) D \Omega + [\sigma_1 (\beta - \lambda) + (q + L) (q - \varepsilon)] \Omega = Q_0 (q - \varepsilon) - M_0 (k_1 D + \sigma_1) + E (q - \varepsilon). \tag{19}
\]

For the stability of the second-order system, it is sufficient to establish the positivity of all the coefficients of the characteristic equation:

\[
k_1 (k_{D0} q + J) > 0; \tag{20}
\]
\[
\sigma_1 (k_{D0} q + J) + k_1 (\beta - \lambda) > 0; \tag{21}
\]
\[
\sigma_1 (\beta - \lambda) + (q + L) (q - \varepsilon) > 0. \tag{22}
\]

The inequality (20) for \( k_{D0} > 0 \) is obvious, while the relations (21) and (22) require additional analysis. Expressions (21) and (22) can be represented as

\[
\frac{(k_{D0} q + J)}{k_1} \frac{\sigma_1}{(\beta - \lambda)} \geq -1; \tag{23}
\]
\[
\sigma_1 (\beta - \lambda) \geq (q + L) (q - \varepsilon), \tag{24}
\]

The left part of the inequality (24) contains positive values, with the exception of the value \( \lambda \), due to which the left part of the expression (24) can take negative values. In this case, the system may become unstable. Solving equations (23) and (24) together, we can obtain a diagram (Figure 5, a) that characterizes its state. If you increase \( L \), and, respectively, \( n = \frac{\Phi \Omega}{q} \), then the stability zone will also increase. An increase in friction in the hydraulic actuator (an increase in \( \varepsilon \)) can cause the system to become unstable. This is the difference between the effect of friction in the hydraulic actuator, proportionally in the cavities of the hydraulic actuator, and friction in the load. Summing up the inequality (23) and (24), we get

\[
(\beta - \lambda) (1 + \frac{1}{\sigma_1} \frac{k_{D0} q + J}{k_1}) \frac{\Phi \Omega}{q} \geq (q + L) (q - \varepsilon). \tag{25}
\]

For \((\beta - \lambda) > 0 \) and \( \sigma_1 > 0 \), the relation (25) is always valid, since \( \varepsilon < q \) and the system is stable. At \((\varepsilon - \lambda) < 0 \), unstable modes of motion are possible (coefficient \( \varepsilon_1 = 0.01 \) cm\(^2\)/N-s), Figure 5, b.
4. Summary and conclusions

Thus, by adjusting the pump supply and pressure in the pressure space of the hydraulic actuator, you can control the quality of dynamic processes in the hydraulic drive. In particular, when the amplitude of self-oscillations of the angular rotation of the rotor of the hydraulic actuator increases to a value equal to or greater than the average value of this rotation, the mode of periodic motions with stops is possible [1,3].

Experimental and theoretical studies, taking into account the significant nonlinearities of the friction characteristics [5], have shown that the introduction of the considered correcting feedback leads to a decrease in the amplitude of self-oscillating processes, allowing for non-stop movement of the hydraulic drive. With a favorable combination of hydraulic drive parameters (Figure 5), it is possible to ensure the dynamic stability of the system under consideration, i.e., to exclude unstable and self-oscillating modes. The error of calculation by the method of harmonic linearization of nonlinearities does not exceed 15%.

5. References

[1] Prokofiev V N 1969 Axial-piston adjustable hydraulic drive (Moscow: Mashinostroenie) p 496
[2] Woś P and Dindorf R 2016 Modeling and Analysis of the Hydraulic Servo Drive Advances in Intelligent Systems and Computing (Electronic Materials vol 414) pp 253-262
[3] Ivanov N I, Nemirovsky I A and Syrkin V V 1975 Parametric actuation of periodic motions with stops of axial piston hydraulic actuators (Hydraulic drive and hydraulic and pneumatic control systems vol 11) pp 78-87
[4] Emelyanov R T Klimov A S, Kravtsov K S, Oleniev I B and Turyshcheva E S 2020 Improving the efficiency of a hydraulic drive with a closedloop hydraulic circuit (Journal of Physics: Conference Series vol 1515)
[5] Nemirovsky I A 1974 Nonlinear mathematical model of hydraulic drive // Technology and organization of production (Technology and organization of production vol 10) pp 53-56