General structure of a gauge boson propagator and pressure of deconfined QCD matter in a weakly magnetized medium

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Abstract. We have systematically constructed the general structure of the gauge boson self-energy and the effective propagator in presence of a nontrivial background like hot magnetized material medium. Based on this as well as the general structure of fermion propagator in weakly magnetized medium we have calculated pressure of deconfined QCD matter within HTL approximation.

1 Introduction

Quark Gluon Plasma is a thermalized color deconfined state of nuclear matter in the regime of Quantum Chromo Dynamics (QCD) under extreme conditions such as very high temperature and/or density. For the past couple of decades, different high energy Heavy-Ion-Collisions (HIC) experiments are under way, e.g., RHIC @ BNL, LHC @ CERN and upcoming FAIR @ GSI, to study this novel state of QCD matter. In recent years the noncentral HIC is also being studied, where a very strong magnetic field can be created in the direction perpendicular to the reaction plane due to the spectator particles that are not participating in the collisions [1,2,3]. Also some studies have showed that the strong magnetic field generated during the noncentral HIC is time dependent and rapidly decreases with time [4,5]. At the time of the noncentral HIC, the value of the created magnetic field \( B \) is very high compared to the temperature \( T (T^2 < q_f B) \) where \( q_f \) is the absolute charge of the quark with flavor \( f \) associated with the system, whereas after few \( fm/c \), the magnetic field is shown to decrease to a very low value \( (q_f B < T^2) \). In this regime one usually works in the weak magnetic field approximation.

The presence of an external anisotropic field in the medium calls for the appropriate modification of the present theoretical tools to investigate various
properties of QGP and a numerous activity is in progress. The EoS is a generic quantity and of phenomenological importance for studying the hot and dense QCD matter, QGP, created in HIC.

2 General structure of gauge boson propagator

Finite temperature breaks the boost symmetry of a system whereas magnetic field or anisotropy breaks the rotational symmetry. We consider the momentum of gluon as \( P_\mu = (p_0, p_1, 0, p_3) \). We work in the rest frame of the heat bath, i.e., \( u^\mu = (1, 0, 0, 0) \) and represent the background magnetic field as \( n_\mu \equiv \frac{1}{2B} \epsilon_{\mu
u\rho\lambda} u^\nu F^{\rho\lambda} = \frac{1}{2} u^\nu \tilde{F}_{\mu\nu} = (0, 0, 0, 1) \). The general structure of the gauge boson self energy in presence of magnetic field can be written as [6]

\[
\Pi^{\mu\nu} = bB^{\mu\nu} + cR^{\mu\nu} + dQ^{\mu\nu} + aN^{\mu\nu},
\]

(1)

where the form factors \( b, c, d \) and \( a \) can be calculated as

\[
b = B^{\mu\nu} \Pi_{\mu\nu}, \quad c = R^{\mu\nu} \Pi_{\mu\nu}, \quad d = Q^{\mu\nu} \Pi_{\mu\nu}, \quad a = \frac{1}{2} N^{\mu\nu} \Pi_{\mu\nu}.
\]

(2)

Using Dyson-Schwinger equation, one can write the general structure of gluon propagator as

\[
D^{\mu\nu} = \frac{\xi P_{\mu} P_{\nu}}{P^4} + \frac{(P^2 - d)B^{\mu\nu}}{(P^2 - b)(P^2 - d) - a^2} + \frac{R^{\mu\nu}}{P^2 - c} + \frac{(P^2 - b)Q^{\mu\nu}}{(P^2 - b)(P^2 - d) - a^2} + \frac{aN_{\mu\nu}}{(P^2 - b)(P^2 - d) - a^2}.
\]

(3)

It is found from the poles of Eq. (3) that the gluon in hot magnetized medium has three dispersive modes which are given as

\[
(P^2 - c) = 0, \quad (P^2 - b)(P^2 - d) - a^2 = (P^2 - \omega^+_n)(P^2 - \omega^-_n) = 0,
\]

(4)

(5)

where \( \omega^+_n = \frac{b+d+\sqrt{(b-d)^2+4a^2}}{2} \) and \( \omega^-_n = \frac{b+d-\sqrt{(b-d)^2+4a^2}}{2} \).

We consider small magnetic field approximation and calculate all the quantities up to \( \mathcal{O}((eB)^2) \). Within this approximation Eq. (5) becomes

\[
(P^2 - b)(P^2 - d) = 0.
\]

(6)

The form factors \( b, c \) and \( d \) are calculated [6] from Eq. (2) using HTL approximation.

3 Free energy and pressure in weak Field approximation

The total one-loop free energy of deconfined QCD matter in a weakly magnetized hot medium reads as [7]

\[
F = F_q + F_g + F_0 + \Delta \mathcal{E}_0,
\]

(7)
General structure of a gauge boson propagator and pressure of

\[ \Lambda = 2\pi T, \mu = 0 \text{ MeV} \]

\[ |eB| = 0 \]

\[ |eB| = m_\pi \]

\[ \frac{1}{2} \]

\[ |eB| = m_\pi \]

\[ \frac{1}{2} \]

\[ |eB| = 3m_\pi \]

\[ \frac{1}{2} \]

\[ T \text{ [GeV]} \]

\[ \frac{P}{P_{\text{ideal}}} \]

\[ T \text{ [GeV]} \]

\[ \frac{P}{P_{\text{ideal}}} \]

\[ \mu = 300 \text{ MeV} \]

\[ |eB| = 0 \]

\[ |eB| = m_\pi \]

\[ \frac{1}{2} \]

\[ |eB| = m_\pi \]

\[ \frac{1}{2} \]

\[ |eB| = 3m_\pi \]

\[ \frac{1}{2} \]

\[ T \text{ [GeV]} \]

\[ \frac{P}{P_{\text{ideal}}} \]

\[ T \text{ [GeV]} \]

\[ \frac{P}{P_{\text{ideal}}} \]

Fig. 1: Variation of the scaled one-loop pressure with temperature for \( N_f = 3 \) with \( \mu = 0 \) (left panel) for \( \mu = 300 \text{ MeV} \) (right panel) in presence of weak magnetic field. Renormalization scales are chosen as \( \Lambda_g = 2\pi T \) for gluon and \( \Lambda_q = 2\pi \sqrt{T^2 + \mu^2/\pi^2} \) for quark.

where \( F_q, F_g \) are quark and gluon free energy in weak magnetized medium which are calculated [7] using the form factors corresponding to quark [8] and gluon self energy [6]. \( F_0 = \frac{1}{2} B^2 \) is the tree level contribution due to the constant magnetic field and the \( \Delta \xi_0 \) is the HTL counter term given as

\[ \Delta \xi_0 = \frac{d_A}{128\pi^2\epsilon} m_D^4, \]

with \( d_A = N_c^2 - 1 \), \( N_c \) is the number of color in fundamental representation and \( m_D \) is the Debye screening mass in HTL approximation. The divergences present in the total free energy are removed by redefining the magnetic field in \( F_0 \) and by adding counter terms [7].

The pressure of the deconfined QCD matter in weakly magnetized medium is given by

\[ P(T, \mu, B, A) = -F(T, \mu, B, A), \]

where \( A \) is the renormalization scale.

4 Results

The variation of scaled pressure with temperature is shown in Fig. 1 for \( \mu = 0 \) and \( \mu = 300 \text{ MeV} \). It can be seen from the figure that the magnetic field dependence of the scaled pressure decreases with temperature because temperature is the dominant scale in the weak field approximation (\( |eB| < T^2 \)). At high temperature, all the plots for different magnetic field asymptotically reach the one-loop HTL pressure. The scaled pressure is plotted with magnetic field strength in
Fig. 2: Variation of the scaled one-loop pressure with magnetic field for \( N_f = 3 \) with \( \mu = 0 \) (left panel) and \( \mu = 300 \) MeV (right panel) for \( T = (0.3, 0.4, 0.6 \) and \( 1) \) GeV.

Fig. 2. We note from Fig. 2 that the slopes of the plots decrease with increase of temperature reflecting the reduced magnetic field dependence.

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