Static polarizability of two-dimensional hole gases

Thomas Kernreiter\textsuperscript{1}, Michele Governale\textsuperscript{1,4} and Ulrich Zülicke\textsuperscript{2,3,4}

\textsuperscript{1} School of Chemical and Physical Sciences and MacDiarmid Institute for Advanced Materials and Nanotechnology, Victoria University of Wellington, PO Box 600, Wellington 6140, New Zealand
\textsuperscript{2} Institute of Fundamental Sciences and MacDiarmid Institute for Advanced Materials and Nanotechnology, Massey University, Manawatu Campus, Private Bag 11 222, Palmerston North 4442, New Zealand
\textsuperscript{3} Centre for Theoretical Chemistry and Physics, Massey University, Albany Campus, Private Bag 102904, North Shore MSC, Auckland 0745, New Zealand
\textsuperscript{4} E-mail: thomas.kernreiter@vuw.ac.nz, michele.governale@vuw.ac.nz and u.zuelicke@massey.ac.nz

New Journal of Physics \textbf{12} (2010) 093002 (11pp)
Received 21 May 2010
Published 1 September 2010
Online at http://www.njp.org/
doi:10.1088/1367-2630/12/9/093002

Abstract. We have calculated the density–density (Lindhard) response function of a homogeneous two-dimensional (2D) hole gas in the static ($\omega = 0$) limit. The bulk valence-band structure comprising heavy-hole (HH) and light-hole (LH) states is modeled using Luttinger’s $k \cdot p$ approach within the axial approximation. We elucidate how, in contrast to the case of conduction electrons, the Lindhard function of 2D holes exhibits unique features associated with (i) the confinement-induced HH–LH energy splitting and (ii) the HH–LH mixing arising from the charge carriers’ in-plane motion. Implications for the dielectric response and related physical observables are discussed.

\textsuperscript{4} Authors to whom any correspondence should be addressed.
1. Introduction

The density–density response function is a very fundamental materials characteristics, as it
determines a host of thermodynamic and transport properties in condensed-matter systems [1].
It has been discussed extensively within the paradigmatic model of the homogeneous
electron gas [2] and studied for low-dimensional conductors realized in semiconductor
heterostructures [3]. More recently, the static and dynamic response properties of two-
dimensional (2D) conduction-electron systems with spin–orbit coupling have been investigated
in considerable detail [4]–[9]. This surge of interest arose partly because of important
ramifications for possible spintronics applications [10, 11]. In contrast, very few studies have
considered how the peculiar electronic properties of a typical semiconductor’s valence band [12]
affect the polarizability and other many-body response functions of p-type semiconductor
materials, and these existing works [13]–[15] have focused on bulk (3D) systems. As
high-quality 2D hole gases have recently become available for experimental study, in both
modulation-doped [16]–[20] and accumulation-layer [21] heterostructures, a detailed theoretical
analysis of their many-body response properties is warranted. Here, we provide such a study and
show how the intricate interplay between quantum confinement and strong spin–orbit-coupled
dynamics in the valence band [22] has a profound effect on the static polarizability.

Charge carriers from the conduction and valence bands of typical semiconductors exhibit
profoundly different spin properties. Conduction electrons are quite ordinary in that they are
spin-1/2 particles carrying a fixed intrinsic magnetic dipole moment, like free electrons in
vacuum. Holes are different; they have an intrinsic spin-3/2 degree of freedom because the
valence-band states are strongly modified by spin–orbit coupling [12]. As a result, the orbital
dynamics of holes in a bulk sample also depends on the magnitude of projection for their spin
parallel to the direction of motion. States with spin-3/2 projection quantum number \( m_J = \pm 3/2 \)
\((\pm 1/2)\) are called heavy holes, HHs (light holes, LHs), because their band-energy dispersion
has a smaller (larger) curvature. When holes are confined in a 2D heterostructure, the quantum-
well growth direction is the natural spin-quantization axis (taken to be the \( z \)-direction in the
following), and the difference in effective masses translates into an energy splitting between the
HH and LH subband edges corresponding to the same transverse orbital bound state [23]. As
the in-plane motion couples HH and LH states, 2D holes with finite wave vector \( \mathbf{k}_\parallel = (k_x, k_y) \)
are no longer of purely HH or LH type [22, 23]. While the HH–LH (subband-edge) splitting
is easily accounted for and usually included in theoretical analyses, the HH–LH mixing has
sometimes been ignored. It may be tempting to make such a simplification, given that the
density of typical 2D hole gases is quite often low enough that only the lowest (HH-like)
subband is occupied. However, detailed analysis shows that this approach is too crude for most
relevant situations [22]. Even only qualitatively accurate predictions basically always require
the inclusion of HH–LH mixing alongside the HH–LH splitting. As we will show below, the
density–density response of 2D hole gases is strongly affected by HH–LH mixing, i.e. it is not
simply the sum of the response functions of independent 2D (HH and LH) gases.

This paper is organized as follows. In section 2, we introduce our model for the uppermost
valence band of typical semiconductors, which is based on the Luttinger–Hamiltonian [24, 25]
in axial approximation [22, 26, 27]. The definition and basic calculational details for the
density–density (Lindhard) response function are given in section 3, including analytical results
pertaining to the 2D hole gas in certain limits. We present plots of the numerically determined
static polarizability in section 4 and discuss basic features. Our conclusions are given in
section 5.

2. Luttinger-model description of a two-dimensional (2D) hole system

The Luttinger model [25] provides a useful description of the uppermost valence band of typical
semiconductors in situations where its couplings to the conduction band and split-off valence
band are irrelevant. We adopt this model here to investigate how the many-body physics of
2D holes is affected by their peculiar spin-3/2 properties. In principle, more extended [27, 28]
multiband Hamiltonians could be employed to improve the accuracy of quantitative predictions.
However, to illustrate the qualitatively new features exhibited by 2D hole gases in contrast
to their conduction-electron counterparts, the Luttinger-model description is adequate. The
particular geometry for our case of interest suggests using, as our starting point, the Luttinger-
model Hamiltonian \( H_L \) in axial approximation [22, 26, 27]:

\[
\begin{align*}
H_L &= H_0 + H_1 + H_2, \\
H_0 &= -\frac{\hbar^2}{2m_0} \left[ \gamma_1 \left( k_x^2 + k_y^2 \right) + \tilde{\gamma}_1 \left( k_x^2 - 2k_z^2 \right) \left( \hat{J}_z - \frac{5}{4} \hat{I} \right) \right], \\
H_1 &= \frac{\hbar^2}{m_0} \sqrt{2} \tilde{\gamma}_2 \left( \{k_z, k_+\}\{\hat{J}_z, \hat{J}_-\} + \{k_z, k_-\}\{\hat{J}_z, \hat{J}_+\} \right), \\
H_2 &= \frac{\hbar^2}{2m_0} \tilde{\gamma}_3 \left( k_x^2 \hat{J}_x^2 + k_y^2 \hat{J}_y^2 \right).
\end{align*}
\]

Cartesian components of the spin-3/2 matrix vector are denoted by \( \hat{J}_x, \hat{J}_y, \hat{J}_z \), and we used the abbreviations \( k_{\pm} = k_x \pm ik_y, \hat{J}_{\pm} = (\hat{J}_x \pm i\hat{J}_y)/\sqrt{2} \) and \( \{A, B\} = (AB + BA)/2 \). The constants
\( \gamma_1 \) and \( \tilde{\gamma}_j \) are material-dependent band-structure parameters [29]. Note that the \( \tilde{\gamma}_j \) also depend on
the quantum-well growth direction; their explicit expressions in terms of the standard Luttinger
parameters [25, 29] \( \gamma_2 \) and \( \gamma_3 \) can be found in table C.10 of [22].

The dynamics of holes confined in a 2D quantum well is modeled by the Hamiltonian \( H_L + V(z) \). In the following, we assume the external potential \( V(z) \) to be a hard-wall confinement
of width \( d \) and consider only its lowest size-quantized orbital bound state. An effective
Hamiltonian describing the 2D hole gas is then obtained from (1a) by replacing \( k_z \rightarrow \langle k_z \rangle = 0 \).
and \( k_z^2 \rightarrow \langle k_z^2 \rangle = (\pi / d)^2 \). Introducing the energy scale \( E_0 = \pi^2 \hbar^2 \gamma_1 / (2m_0 d^2) \) and measuring wave vectors in units of \( \pi / d \), the 2D hole-gas Hamiltonian is given by

\[
H_{L}^{(2D)} = H_0^{(2D)} + H_{\text{mix}}, \tag{2a}
\]

\[
H_0^{(2D)} = -E_0 \left\{ 1 - 2\tilde{\gamma} \left( \bar{k}_z^2 - \frac{5}{4} \hat{i} \right) + \left[ 1 + \tilde{\gamma} \left( \bar{k}_z^2 - \frac{5}{4} \hat{i} \right) \right] \right\} \tilde{k}_z^2, \tag{2b}
\]

\[
H_{\text{mix}} = E_0 \alpha \bar{\gamma} \left( \bar{k}_x^2 \bar{J}_-^2 + \bar{k}_y^2 \bar{J}_+^2 \right). \tag{2c}
\]

Here, \( \bar{k}_{x,y} = k_{x,y} d / \pi \), \( \tilde{\gamma} = \gamma_1 / \gamma_1 \) and \( \alpha = \gamma_3 / \gamma_1 \). We are using the parameterization in terms of \( \tilde{\gamma} \) and \( \alpha \) to be able to separately discuss the effects of HH–LH splitting, which is embodied in \( H_0^{(2D)} \), and HH–LH mixing arising from \( H_{\text{mix}} \).

Diagonalizing \( H_{L}^{(2D)} \) from (2a) yields in-plane dispersion relations \( E_j(k_i) = -E_0 \varepsilon_{k_i}^{(j)} \) with \( j = 1, \ldots, 4 \) and

\[
\varepsilon_{k_i}^{(j)} = 1 + \bar{k}_x^2 + \bar{k}_y^2 + \sigma_j \tilde{\gamma} \sqrt{(\bar{k}_x^2 + \bar{k}_y^2 - 2)^2 + 3\alpha^2(\bar{k}_x^2 + \bar{k}_y^2)^2}. \tag{3}
\]

Here, \( \sigma_1 = \sigma_2 = -\sigma_3 = -\sigma_4 = 1 \). Using result (3), we obtain the two dimensionless Fermi wave vectors

\[
\bar{k}_{F_{1,2}} = \left[ \frac{\varepsilon_F - 1 - 2\tilde{\gamma}^2 \mp \tilde{\gamma} \sqrt{(\varepsilon_F - 3)^2 + 3\alpha^2(\varepsilon_F - 1)^2 - 4\tilde{\gamma}^2}}{1 - \tilde{\gamma}^2(1 + 3\alpha^2)} \right]^{1/2} \tag{4}
\]

in terms of the dimensionless Fermi energy \( \varepsilon_F = -E_F / E_0 \). The 2D hole sheet density \( n_{2D} \) is related to the dimensionless Fermi wave vectors according to

\[
n_{2D} = \left( \frac{\pi}{d} \right)^2 \left( \frac{T (\varepsilon_F - [1 + 2\tilde{\gamma}]) + \bar{k}_{F_2} \Theta(\varepsilon_F - [1 - 2\tilde{\gamma}])}{2\pi} \right), \tag{5}
\]

where \( \Theta(x) \) denotes the Heaviside step function.

The eigenvectors \( |\chi_{k_i}^{(j)} \rangle \) corresponding to eigenvalues \( E_j(k_i) \) of \( H_{L}^{(2D)} \) can be straightforwardly determined. For \( j = 1, 2 \) and in the basis representation where \( \hat{J}_z \) is diagonal, we find

\[
|\chi_{k_i}^{(1)} \rangle = \begin{pmatrix} 0 \\ \frac{(s-t)(\bar{k}_x - i\bar{k}_y)^2}{\bar{k}_x^2 \sqrt{(s-t)^2 + 3\alpha^2 \bar{k}_y^2}} \\ 0 \\ \frac{\sqrt{3} \bar{k}_x^2}{\sqrt{(s-t)^2 + 3\alpha^2 \bar{k}_y^2}} \end{pmatrix}, \quad |\chi_{k_i}^{(2)} \rangle = \begin{pmatrix} 0 \\ \frac{(-s-t)(\bar{k}_x - i\bar{k}_y)^2}{\bar{k}_x^2 \sqrt{(s-t)^2 + 3\alpha^2 \bar{k}_y^2}} \\ 0 \\ \frac{\sqrt{3} \bar{k}_x^2}{\sqrt{(s-t)^2 + 3\alpha^2 \bar{k}_y^2}} \end{pmatrix}, \tag{6}
\]

with \( s \equiv \bar{k}_x^2 - 2 \) and \( t \equiv \sqrt{s^2 + 3\alpha^2 \bar{k}_y^4} \). The remaining eigenvalues \( |\chi_{k_i}^{(3)} \rangle \) and \( |\chi_{k_i}^{(4)} \rangle \) are obtained by changing \( t \rightarrow -t \) in \( |\chi_{k_i}^{(1)} \rangle \) and \( |\chi_{k_i}^{(2)} \rangle \), respectively. As the scalar products \( \langle \chi_{k_i}^{(j)} | \chi_{k_i+q}^{(l)} \rangle \) enter in the calculation of the Lindhard function, we briefly discuss their relevant properties. The moduli \( |\langle \chi_{k_i}^{(j)} | \chi_{k_i+q}^{(l)} \rangle| \) are found to be equal for all \( j = 1, \ldots, 4 \). Also, \( |\langle \chi_{k_i}^{(1)} | \chi_{k_i+q}^{(l)} \rangle| \) are pairwise the same for \( (j, l) = (1, 3) \) and \( (3, 1) \), and \( (j, l) = (2, 4) \) and \( (4, 2) \). These relations can be verified using the explicit form of the column vectors in (6) together with the fact that the eigenvectors satisfy orthonormality relations.
3. Lindhard function of a 2D hole gas: general expression and special cases

The general definition [2] of the Lindhard function, specialized to a 2D hole system, reads

\[
\chi(\omega, q) = \lim_{\delta \to 0} \sum_{j,l=1}^{4} \int \frac{d^2k_j}{(2\pi)^2} |\langle \chi_{k_j}^{(j)} | \chi_{k_j+q}^{(l)} \rangle|^2 \frac{n_F[E_j(k_j)] - n_F[E_j(k_j+q)]}{\hbar \omega + i\delta + E_j(k_j) - E_j(k_j+q)},
\]  

(7)

with \(n_F(E)\) denoting the Fermi–Dirac distribution function. The expression given in (7) can be simplified by using a polar-coordinate representation where \(k_x = k_\perp \cos \phi\) and \(k_y = k_\perp \sin \phi\) and performing a change of variables in the terms involving \(\phi\). Because the Luttinger Hamiltonian in axial approximation exhibits rotational invariance of in-plane hole motion, the Lindhard function depends on wave vector \(q\) only via its (dimensionless) magnitude \(\tilde{q}\). Also, within our effective 2D description, \(\chi(\omega, q)\) is independent of the quantum-well width \(d\) and inversely proportional to \(\gamma_1\).

We use a notation where \(\tilde{K}_{F_1} = K_{F_1} \equiv \tilde{k}_{F_1}, \quad \tilde{K}_{F_2} = K_{F_2} \equiv \tilde{k}_{F_2}, \quad \tilde{K}_{F_3} = K_{F_3} \equiv \tilde{k}_{F_3}\) and \(\tilde{\omega} = \hbar \omega / E_0\). Note that the Luttinger Hamiltonian in axial approximation exhibits rotational invariance of in-plane hole motion, the Lindhard function depends on wave vector \(q\) only via its (dimensionless) magnitude \(\tilde{q}\). Also, within our effective 2D description, \(\chi(\omega, q)\) is independent of the quantum-well width \(d\) and inversely proportional to \(\gamma_1\).

In the following, we consider the static limit, which is obtained by setting \(\tilde{\omega} = 0\). Specializing further to certain limiting situations, we can find analytical expressions for the Lindhard function. For example, for the case of vanishing HH–LH mixing obtained by letting \(\alpha \to 0\), the matrix \(\langle \chi_{k_j}(j) | \chi_{k_{j+q}}^{(l)} \rangle\) of modulus-squared scalar products reduces to the unity matrix, and the simple analytical expression

\[
\tilde{\chi}(0, \tilde{q})|_{\alpha=0} = \frac{-1}{2\pi \tilde{q}} \sum_{j=1}^{2} \left[ \frac{\Theta(\tilde{k}_{F_j})}{1 - \sigma_j \tilde{y}} \left[ \tilde{q} - \sqrt{\tilde{q}^2 - 4\tilde{k}_{F_j}^2} \right] \Theta \left( \frac{\tilde{q}}{2k_{F_j}} - 1 \right) \right]
\]  

(9)

is found, where \(\sigma_1 = -\sigma_2 = 1\). Inspection of the result (9) shows that, with only HH–LH splitting included, the static Lindhard function comprises two separate HH and LH contributions, each being the standard 2D-electron-gas expression [2] with Fermi wave vector and effective mass adjusted to the respective HH and LH values. On the other hand, taking the limit \(\tilde{q} \to 0\) in (8), the matrix of modulus-squared spinor overlaps again becomes the unity matrix, and we find an analytical result for the (dimensionless) density of states at the Fermi energy,

\[
\lim_{\tilde{q} \to 0} \tilde{\chi}(0, \tilde{q}) = \frac{-1}{2\pi} \sum_{j=1}^{2} \Theta(\tilde{k}_{F_j}) \left[ 1 - \sigma_j \tilde{y} \frac{2 - \tilde{k}_{F_j}^2 (1 + 3\alpha^2)}{\sqrt{(2 - \tilde{k}_{F_j}^2)^2 + 3\alpha^2 \tilde{k}_{F_j}^4}} \right]^{-1}.
\]  

(10)

Thus, we see that one effect of HH–LH mixing is to introduce an energy (and concomitant density) dependence into the density of states of 2D holes.
4. Static polarizability of 2D holes: numerical method and results

With analytical expressions unavailable for the Lindhard function (8) in the more general case with both $\mathbf{q}$ and $\alpha$ finite, we have to resort to numerical calculations to investigate in greater detail how HH–LH mixing affects the static polarizability $\tilde{\chi}(0, \tilde{\mathbf{q}})$. Note that the latter is an entirely real-valued function. The procedure for its numerical calculation is explained in the following subsection, and our results are given thereafter.

4.1. Brief outline of the calculational method

For $\tilde{\omega} = 0$, the integrand of (8) has poles whenever the energy difference in the denominator vanishes. These poles are regularized by the parameter $\delta$, which needs to be set to zero after performing the integrations. We calculate these integrals numerically, taking special care in the regions close to the values of the integration variables corresponding to a vanishing denominator. To identify the pole structure of the Lindhard function, we write the inverse of the energy difference as

$$
\left(\varepsilon_{k||}^{(j)} - \varepsilon_{k||+\mathbf{q}}^{(l)}\right)^{-1} = (\delta_{j,1} + \delta_{j,2})(\delta_{l,1} + \delta_{l,2}) \frac{a_1 - b}{a_1^2 - b^2} + (\delta_{j,1} + \delta_{j,2})(\delta_{l,3} + \delta_{l,4}) \frac{a_1 + b}{a_1^2 - b^2} + (\delta_{j,3} + \delta_{j,4})(\delta_{l,1} + \delta_{l,2}) \frac{a_2 - b}{a_2^2 - b^2} + (\delta_{j,3} + \delta_{j,4})(\delta_{l,3} + \delta_{l,4}) \frac{a_2 + b}{a_2^2 - b^2},
$$

where $\delta_{j,l}$ denotes Kronecker’s delta symbol. The quantities appearing in (11) are

$$
a_{1,2} = \mp \sqrt{4 - 4k_{||}^2 + (1 + 3\alpha^2)k_4^2 + \tilde{q}^2 + 2k_{||}\tilde{q} \cos \phi},
$$

$$
b = \sqrt{(-2 + 2k_{||}^2 + \tilde{q}^2 + 2k_{||}\tilde{q} \cos \phi)^2 + 3\alpha^2(k_4^2 + \tilde{q}^2 + 2k_{||}\tilde{q} \cos \phi)^2}.
$$

The denominators in (11) can be written as

$$
\frac{1}{a_{1,2}^2 - b^2} = \frac{1}{4k_{||}^2\tilde{q}^2[1 - (1 + 3\alpha^2)\tilde{\gamma}^2]} \frac{1}{X_{1,2} - Y_{1,2}} \left(\frac{1}{\cos \phi - X_{1,2}} - \frac{1}{\cos \phi - Y_{1,2}}\right),
$$

with the positions of the poles given by

$$
X_{1,2} = -\frac{\tilde{q}}{2k_{||}},
$$

$$
Y_{1,2} = \pm \frac{2\tilde{\gamma}^2 - 4 - 4k_{||}^2 + (1 + 3\alpha^2)k_4^2 - \tilde{q}^2 + \tilde{\gamma}^2(2 + 6\alpha^2)k_4^2 - 4 + \tilde{q}^2 + 3\alpha^2\tilde{q}^2}{2k_{||}\tilde{q}^2[1 - (1 + 3\alpha^2)\tilde{\gamma}^2]}.
$$

As can be seen from (13), poles are encountered in the integration over $\phi$ when $|X_{1,2}|, |Y_{1,2}| \leq 1$. We have employed a Cauchy principle-value integration to regularize the Lindhard function in the vicinity of the poles specified in (14).

4.2. Results for model parameters applying to a [001] quantum well in GaAs

High-quality 2D hole gases have recently been fabricated from [001]-grown GaAs heterostructures [16, 17, 20]. To obtain results applicable to these systems, we use the appropriate model parameters $\tilde{\gamma} = 0.31$ and $\alpha = 1.2$. The results for this configuration are presented below. For
Figure 1. (a) Normalized Lindhard function $\chi(q)$ for the static limit and (b) the quantity $\Delta \chi(q; \alpha)$ that measures the impact of HH–LH mixing (see text), plotted as a function of wave-vector magnitude $q$. The blue curve is for $\gamma = 0.31$ and $\alpha = 1.2$, which are parameter values applying to a [001]-grown heterostructure in GaAs. For comparison, the results are also shown for $\alpha = 1$ (red curve), 0.5 (magenta curve) and 0 (green curve). For all cases, a value of $\bar{n} = 0.0608$ for the dimensionless 2D hole density was used.

To avoid cluttering our notation, we suppress the zero-frequency argument in the formal expression of the static Lindhard function from now on: $\chi(0, q) \equiv \chi(q)$. In figure 1(a), we plot $\chi(q)/\chi(0)$ as a function of $q/\sqrt{8\pi n}$ for different values of the parameter $\alpha$ that quantifies the HH–LH mixing. It is apparent that a finite $\alpha$ leads to a significant suppression of $\chi(q)$ below the constant-plateau value usually associated with 2D systems \cite{2} for $q < \sqrt{8\pi n}$. To make the impact of HH–LH mixing quantitatively explicit, we define the variable

$$
\Delta \chi(q; \alpha) = 1 - \frac{\chi(q)}{\chi(0)} \left[ \frac{\tilde{\chi}(q)}{\tilde{\chi}(0)} \right]^{-1},
$$

where $\tilde{\chi}(q)$ is the analytical result \cite{9} obtained for the limit $\alpha = 0$, but with Fermi wave vectors adjusted to coincide with those found in the case of the finite $\alpha$ under consideration. Thus, the function $\Delta \chi(q; \alpha)$ measures the relative change exhibited in the normalized static polarizability that is due to a finite $\alpha$ but goes beyond a simple renormalization of Fermi wave vectors\footnote{In the low-density limit considered here, there is only one Fermi wave vector whose magnitude is the same for all values of $\alpha$, and $\tilde{\chi}(q)$ actually coincides with $\tilde{\chi}(q)|_{\alpha=0}$. However, as we will see further below, $\tilde{\chi}(q) \neq \tilde{\chi}(q)|_{\alpha=0}$ in the more general case when both the HH and LH subbands arising from the lowest 2D orbital bound state are occupied and, thus, two Fermi wave vectors exist.}. In figure 1(b), we show $\Delta \chi(q; \alpha)$ for $\alpha = 1$ (red curve) and $\alpha = 1.2$ (blue curve). It shows a strong variation as a function of $q/\sqrt{8\pi n}$ and reaches the 10% level.

Comparison and to clearly show the impact of HH–LH mixing, we also show the results for the static polarizability when $\alpha = 1, 0.5$ and 0. In all these cases, we limit ourselves to the low-density regime where only the highest, HH-like, 2D subband is occupied. The reason for this precaution is the fact that, within our model using a hard-wall confinement, the spectrum of all other than the highest subband poorly matches that of the real GaAs sample. To be specific, we choose $\bar{n} = 0.0608$. Recalling that the 2D-hole sheet density is related to the dimensionless density by $n = (\pi/d)^2 \bar{n}$, this value corresponds to a density of $n = 1.5 \times 10^{15} \text{m}^{-2}$ in a 20 nm quantum well.

To avoid cluttering our notation, we suppress the zero-frequency argument in the formal expression of the static Lindhard function from now on: $\chi(0, q) \equiv \chi(q)$. In figure 1(a), we plot $\chi(q)/\chi(0)$ as a function of $q/\sqrt{8\pi n}$ for different values of the parameter $\alpha$ that quantifies the HH–LH mixing. It is apparent that a finite $\alpha$ leads to a significant suppression of $\chi(q)$ below the constant-plateau value usually associated with 2D systems \cite{2} for $q < \sqrt{8\pi n}$. To make the impact of HH–LH mixing quantitatively explicit, we define the variable

$$
\Delta \chi(q; \alpha) = 1 - \frac{\chi(q)}{\chi(0)} \left[ \frac{\tilde{\chi}(q)}{\tilde{\chi}(0)} \right]^{-1},
$$

where $\tilde{\chi}(q)$ is the analytical result \cite{9} obtained for the limit $\alpha = 0$, but with Fermi wave vectors adjusted to coincide with those found in the case of the finite $\alpha$ under consideration. Thus, the function $\Delta \chi(q; \alpha)$ measures the relative change exhibited in the normalized static polarizability that is due to a finite $\alpha$ but goes beyond a simple renormalization of Fermi wave vectors\footnote{In the low-density limit considered here, there is only one Fermi wave vector whose magnitude is the same for all values of $\alpha$, and $\tilde{\chi}(q)$ actually coincides with $\tilde{\chi}(q)|_{\alpha=0}$. However, as we will see further below, $\tilde{\chi}(q) \neq \tilde{\chi}(q)|_{\alpha=0}$ in the more general case when both the HH and LH subbands arising from the lowest 2D orbital bound state are occupied and, thus, two Fermi wave vectors exist.}. In figure 1(b), we show $\Delta \chi(q; \alpha)$ for $\alpha = 1$ (red curve) and $\alpha = 1.2$ (blue curve). It shows a strong variation as a function of $q/\sqrt{8\pi n}$ and reaches the 10% level.
4.3. Results for a model semiconductor: high- and low-density regimes

For high-enough 2D densities, holes will occupy both the HH-like and LH-like subbands arising from the lowest-energy orbital bound state in the quantum well. It can be expected that this high-density regime is qualitatively different from the situation at low density where only the highest (HH-like) 2D subband is occupied. To treat the case of high density consistently within our adopted model, it needs to be ensured that the LH-like subband arising from the lowest-energy orbital bound state is still higher in energy than the HH-like subband associated with the next orbital-bound-state level. For a hard-wall confinement considered here, a system with $\tilde{\gamma} = 0.2$ satisfies that condition. Although this value does not directly correspond to a specific semiconductor material, we use it to illustrate the generically different impact of HH–LH mixing in the low- and high-density regimes, respectively.

To provide a clear benchmark for comparing high- and low-density regimes, we start by presenting the result for the low-density case ($\tilde{n} = 0.0608$, same value as used in the calculations for figure 1) in figure 2. The obtained curves look qualitatively similar to those found for the low-density regime in GaAs (different $\tilde{\gamma}$, shown in figure 1), but the quantitative level of suppression below the plateau value obtained in the limit of vanishing $\alpha$ is different here.

As an illustration of the high-density regime, we present the results for $\tilde{n} = 0.4055$, which correspond to $n = 10^{16}$ m$^{-2}$ in a 20 nm quantum well. The normalized Lindhard function for this case is plotted in figure 3(a). Note that, for the three values of $\alpha$ for which results are presented, the two Fermi wave vectors are different, see (4). As a result, the sharp features arising in the Lindhard function from poles at $-q/2k_{F_i}$ appear at different values of $q$ in each curve. In contrast to the low-density case, a plateau is again exhibited in the static Lindhard function (for $q \leq 2k_{F_i}$). To illustrate the effects due to HH–LH mixing beyond a simple renormalization of the two Fermi wave vectors, we also show (as the green curve) the analytical result (9) for the Lindhard function in the limit $\alpha = 0$, but with values for the Fermi wave vectors taken from the case $\alpha = 1$. The latter result corresponds to that expected for two independent 2D hole gases with different Fermi wave vectors. Thus, any deviation between the red and green curves is entirely due to the mixed HH–LH character of 2D hole states. Figure 3(b) shows the corresponding difference function $\Delta \chi(q; \alpha)$ for $\alpha = 1$. Again, HH–LH mixing is seen to cause differences around the 10% level.
5. Summary and conclusions

We have calculated the density–density response function of a homogeneous 2D hole gas in the static limit, based on the Luttinger-model description of the uppermost valence band within the axial approximation. While this approach neglects the warping of energy dispersions (and, hence, Fermi surfaces) for the holes’ in-plane motion due to the cubic crystal symmetry, it already captures essential new features arising from the peculiar valence-band properties. We furthermore focused only on the lowest orbital bound state in a symmetric quantum well defined by a hard-wall confinement. HH–LH splitting gives rise to the existence of two energetically separated 2D hole subbands, one (at higher energy) mostly HH-like and the other of mostly LH character. However, except for states with zero in-plane kinetic energy, HH and LH amplitudes are mixed, and we have elucidated how this mixing gives rise to marked changes in the shape and magnitude of the static density response function. New analytical results are derived for the limit $q \to 0$, but the case with finite $q$ and HH–LH mixing included could only be treated numerically.

For both practical and theoretical reasons, it makes sense to distinguish two basic situations. One corresponds to the low-density regime where only the highest (mostly HH-like) 2D subband is occupied. In this limit, our model can be expected to describe real semiconductor heterostructures quite accurately, even quantitatively, if adequate band-structure parameters are used as input to our calculations. As it turns out, even though holes are present only in the HH-like 2D subband, HH–LH mixing affects the static density response. In particular, the response function is suppressed below the plateau exhibited by the standard 2D-electron-gas result [2], as illustrated in figures 1(a) and 2. In the high-density regime, where both the HH-like and LH-like 2D subbands associated with the lowest-energy quantum-well bound state are occupied, the density response differs from that expected for two independent 2D
hole gases. Thus, HH–LH mixing is shown to influence the Lindhard function beyond a trivial renormalization of Fermi-wave-vector magnitudes. We have defined, and calculated, the quantity \( \Delta \chi (q; \alpha) \) to make the nontrivial effects arising from HH–LH mixing quantitatively explicit. For the parameters considered, relative changes of the order of 10% are seen.

In this work, we employed the approximation of hard-wall (infinite-height) quantum-well barriers. In the more realistic case of finite-height barriers, the HH and LH bound-state wave functions will both penetrate into the barrier regions. Due to HH–LH splitting, the range of penetration will be higher for LH states, i.e. the effective quantum-well width will be larger for LHs. This subtle difference between HH and LH bound states scales with the parameter \( \bar{\gamma} \) and can be expected to result in corrections of order \( \alpha \bar{\gamma}^2 \) to effects due to HH–LH mixing.

The qualitative and quantitative impact of HH–LH mixing can be expected to affect the physical properties of 2D hole gases in an important way and, thus, render their behavior quite different from that exhibited by 2D conduction-electron systems. Examples for physical observables that are affected include the shape and range of Friedel oscillations exhibited by 2D hole gases in response to impurity charges present, e.g. in the doping layer of modulation-doped heterostructures. HH–LH mixing should then also influence the 2D-hole-mediated Ruderman–Kittel–Kasuya–Yosida (RKKY) interaction between magnetic impurities. A detailed investigation of this effect could shed new light on how to tailor the ferromagnetic properties of 2D diluted-magnetic-semiconductor heterostructures [30]–[34].

References

[1] Ziman J M 1972 Theory of Solids 2nd edn (Cambridge: Cambridge University Press)
[2] Giuliani G and Vignale G 2005 Quantum Theory of the Electron Liquid (Cambridge: Cambridge University Press)
[3] Ando T, Fowler A B and Stern F 1982 Rev. Mod. Phys. 54 437
[4] Chen G H and Raikh M E 1999 Phys. Rev. B 59 5090
[5] Wang X F 2005 Phys. Rev. B 72 085317
[6] Farid A K and Mishchenko E G 2006 Phys. Rev. Lett. 97 096604
[7] Pletyukhov M and Gritsev V 2006 Phys. Rev. B 74 045307
[8] Badalyan S M, Matos-Abiague A, Vignale G and Fabian J 2009 Phys. Rev. B 79 205305
[9] Ambrosetti A, Pederiva F, Lipparrini E and Gandolfi S 2009 Phys. Rev. B 80 125306
[10] Zutic I, Fabian J and Sarma S D 2004 Rev. Mod. Phys. 76 323
[11] Awschalom D D and Flatté M E 2006 Nature Phys. 3 153
[12] Yu P Y and Cardona M 1999 Fundamentals of Semiconductors 2nd edn (Berlin: Springer)
[13] Schliemann J 2006 Phys. Rev. B 74 045214
[14] Stanescu T D and Galitski V 2006 Phys. Rev. B 74 205331
[15] Schliemann J 2010 The dielectric function of the semiconductor hole gas arXiv:1003.4820
[16] Grbić B, Ellenberger C, Ihn T, Ensslin K, Reuter D and Wieck A D 2004 Appl. Phys. Lett. 85 2277
[17] Manfra M J, Pfeiffer L N, West K W, de Picciotto R and Baldwin K W 2005 Appl. Phys. Lett. 86 162106
[18] Fischer F, Schuh D, Bichler M, Abstreiter G, Grayson M and Neumaier K 2005 Appl. Phys. Lett. 86 192106
[19] Schmult S, Gerl C, Wurstbauer U, Mitzkus C and Wegscheider W 2005 Appl. Phys. Lett. 86 202105
[20] Gerl C, Schmult S, Tranitz H P, Mitzkus C and Wegscheider W 2005 Appl. Phys. Lett. 86 252105
[21] Clarke W R, Micloich A P, Hamilton A R, Simmons M Y, Muraki K and Hirayama Y 2006 J. Appl. Phys. 99 023707
[22] Winkler R 2003 Spin–Orbit Coupling Effects in Two-Dimensional Electron and Hole Systems (Berlin: Springer)
[23] Bastard G, Brum J A and Ferreira R 1991 Solid State Physics vol 44 ed H Ehrenreich and D Turnbull (San Diego, CA: Academic) pp 229–415
[24] Luttinger J M and Kohn W 1955 Phys. Rev. 97 869
[25] Luttinger J M 1956 Phys. Rev. 102 1030
[26] Suzuki K and Hensel J C 1974 Phys. Rev. B 9 4184
[27] Trebin H R, Rössler U and Ranvau R 1979 Phys. Rev. B 20 686
[28] Mayer H and Rössler U 1991 Phys. Rev. B 44 9048
[29] Vurgaftman I, Meyer J R and Ram-Mohan L R 2001 J. Appl. Phys. 89 5815
[30] Haury A, Wasiela A, Arnoult A, Cibert J, Tatarenko S, Dietl T and Merle d’Aubigné Y 1997 Phys. Rev. Lett. 79 511
[31] Wojtowicz T, Lim W L, Liu X, Dobrowolska M, Furdy na J K, Yu K M, Walukiewicz W, Vurgaftman I and Meyer J R 2003 Appl. Phys. Lett. 83 4220
[32] Nazmul A M, Sugahara S and Tanaka M 2003 Phys. Rev. B 67 241308
[33] Nazmul A M, Amemiya T, Shuto Y, Sugahara S and Tanaka M 2005 Phys. Rev. Lett. 95 017201
[34] Wurstbauer U and Wegscheider W 2009 Phys. Rev. B 79 155444