Heat transfer features in a PCM-based system under the time-dependent thermal load

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Abstract. Present study is devoted to a numerical analysis of heat transfer processes inside a complex radiator system based on phase change materials (PCM) under the influence of unsteady heat generation local element and external convective cooling. The numerical solution has been implemented by the finite difference method using the enthalpy-based approach to solve the energy equation. As a result of the solution, temperature distributions and liquid flow structures for different time moments, as well as integral characteristics of the melting process as a function of time, have been obtained. The effect of the external Newtonian cooling on the melting intensity and heat transfer patterns has been analyzed.

1. Introduction
Materials with "solid-liquid" phase transitions have proven themselves in systems of transportation, storage and removal of energy in technological processes [1-3]. A good example is an application of paraffins in electronic devices with high peak thermal loads. High heat fluxes lead to intensive melting of the material and energy absorption. The PCM-filled radiators are effective for the confined spaces where it is not possible to provide a permanent active cooling system.

The study of heat transfer processes in PCM is complicated by the presence of a moving boundary, whose position is sometimes difficult to track. It should also be noted that paraffins and fatty acids usually have low thermal conductivity, as a result of which the movement of the interphase boundary is largely determined by convective heat transfer. An experimental study of lauric acid melting in a rectangular enclosure under the conditions of different gravitational acceleration orientation has been carried out in [4]. The experiments have been performed in a closed system containing a metal profile and filled with PCM. The material melting process occurs while maintaining a constant temperature inside the metal profile due to the heat transfer fluid flow with temperatures $T = 55^\circ$C, $T = 60^\circ$C and $T = 70^\circ$C. It has been shown that natural convection plays an important role in the melting process and changing the angle of inclination of the cavity can double the melting time and reduce the heat transfer coefficient by 2 times.

To intensify the heat transfer in PCM the finned profiles with high thermal conductivity are usually used [4-7]. The solar photovoltaic panel with the PCM-based system has been numerically analyzed in [7]. Aluminum container with and without interior fins filled with RT 25 HC having the melting temperature of 26.6ºC has been considered. It has been shown that increasing the number of ribs and their elongation contribute to a decrease in the temperature of PV panel.

In this paper, the main attention has been paid to the influence of the external Newtonian cooling of PCM-based heat sink on the material melting process and local heater temperature.
2. Physical and mathematical model

In this study, the two-dimensional conjugate problem of paraffin melting inside a closed copper radiator in the presence of a local heater with a time-dependent volumetric heat generation has been considered (see figure 1). The outer vertical and upper horizontal walls of the cavity have been cooled by external fluid; the ambient fluid temperature is constant. The external surface of the bottom wall is considered to be thermally insulated. At the initial time moment, the ambient fluid temperature coincides with the initial temperature of the system under consideration. The properties of the materials are considered to be constant. Paraffin melt is assumed to be the viscous and heat-conducting liquid where the Boussinesq approximation is valid.

![Figure 1. Considered cavity.](image)

Equations (1) and (2) describe the law of energy conservation inside the melt and solid material, respectively. The behavior of the interface is described by the Stefan condition \( \frac{ds}{dt} = -k \frac{\partial T}{L_v \frac{\partial n}{\partial t}} \) and the condition where the boundary temperature is equal to the melting point. The heat conduction equations describe also the heat transfer inside the radiator and energy source.

\[
\frac{\partial h}{\partial t} + U \frac{\partial h}{\partial x} + V \frac{\partial h}{\partial y} = k \left( \frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} \right) \quad (1)
\]

\[
\frac{\partial h}{\partial t} = k_i \left( \frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} \right). \quad (2)
\]

To describe the natural convection in the melt region, the Oberbeck–Boussinesq equations using the non-primitive variables such as stream function \( \psi (u = \partial \psi / \partial y, \ v = -\partial \psi / \partial x) \) and vorticity \( \omega (\omega = \partial v / \partial x - \partial u / \partial y) \) have been utilized. Natural convection equations taking into account the phase transitions are solved in dimensionless form using the following relations: \( X = x / H \), \( Y = y / H \), \( U = u / V_0 \), \( V = v / V_0 \), \( V_0 = \sqrt{g \beta \Delta T H} \), \( \tau = H / V_0 \), \( \Theta = (T - T_m) / (T_h - T_m) \), \( \Psi = \psi / V_0 H \), \( \Omega = \omega H / V_0 \). These transformed non-dimensional governing equations can be written as follows:
\[
\frac{\partial^2 \Psi}{\partial x^2} + \frac{\partial^2 \Psi}{\partial y^2} = -\Omega, \tag{3}
\]
\[
\frac{\partial \Omega}{\partial \tau} + U \frac{\partial \Omega}{\partial x} + V \frac{\partial \Omega}{\partial y} = \sqrt{\frac{Pr}{Ra}} \nabla^2 \Omega + \frac{\partial \Theta}{\partial x}, \tag{4}
\]
\[
\zeta(\varphi) \left[ \frac{\partial \Theta}{\partial \tau} + U \frac{\partial \Theta}{\partial x} + V \frac{\partial \Theta}{\partial y} \right] + \text{Stef} \left[ \frac{\partial \varphi}{\partial \tau} + U \frac{\partial \varphi}{\partial x} + V \frac{\partial \varphi}{\partial y} \right] = \frac{\xi(\varphi)}{\sqrt{Ra \cdot Pr}} \nabla^2 \Theta. \tag{5}
\]
\[
\frac{\partial \Theta}{\partial \tau} = \frac{\alpha_s/\alpha_i}{\sqrt{Ra \cdot Pr}} \left( \nabla^2 \Theta + Os \left(1 - \sin(\pi f)\right)\right) \tag{6}
\]
\[
\frac{\partial \Theta}{\partial \tau} = \frac{\alpha_s/\alpha_i}{\sqrt{Ra \cdot Pr}} \nabla^2 \Theta \tag{7}
\]

Here the function \( \varphi \) and functions \( \zeta \) and \( \xi \) are:

\[
\varphi = \begin{cases} 
0, & T < T_m - \eta \\
\frac{T - (T_m - \eta)}{2\eta}, & T_m - \eta \leq T \leq T_m + \eta \\
1, & T > T_m + \eta 
\end{cases}
\]

\[
\zeta(\varphi) = \left( \frac{\rho c}{\rho c_i} \right)_s + \varphi \left( 1 - \left( \frac{\rho c}{\rho c_i} \right)_l \right), \quad \xi(\varphi) = \frac{k_s}{k_j} + \varphi \left( 1 - \frac{k_s}{k_j} \right).
\]

In equations (3)–(7) the dimensionless parameters are the Prandtl number \( Pr = \nu \rho / \alpha_i \), Rayleigh number \( Ra = g\beta \Delta T H^3 / (\nu \alpha_i) \), Stefan number \( Ste = L_f / (c_1 \Delta T) \), Biot number \( Bi = h_m H / k \), Ostrogradsky number \( Os = qH^2 / (k_h \Delta T) \). Also the following notation has been adopted, namely, \( x, y, X, Y \) are dimensional and dimensionless Cartesian coordinates, respectively; \( t \) and \( \tau \) are dimensional and dimensionless time, respectively; \( g \) is gravitational acceleration; \( \nu \) is kinematic viscosity; \( \beta \) is thermal expansion coefficient; \( \rho \) is density; \( u, v \) and \( U, V \) are dimensional and non-dimensional velocity components, respectively; \( p \) is pressure; \( T \) and \( \Theta \) are dimensional and dimensionless temperature, respectively; \( T_m \) is melting temperature; \( h \) is enthalpy, \( k \) is thermal conductivity, \( c \) is heat capacity; and the considered subscripts are \( s \) for the solid paraffin, \( l \) is for the melt, \( cp \) is for the copper profile and \( hs \) is for the heat source.

At the initial time, paraffin was in a solid state and had a temperature that was below than the melting point \( \Theta = \Theta_0, \nabla = 0 \). On all solid boundaries, including the phase transition boundary, the no-slip boundary conditions were established: \( \Psi = 0 \) and \( \Omega = -\nabla^2 \Psi \). For the temperature thermal insulation conditions on the bottom walls we had \( \partial \Theta / \partial n = 0 \), and Newtonian cooling was considered for the vertical and upper horizontal walls like \( \partial \Theta / \partial n = -Bi \left( \Theta - \Theta_n \right) \).

The system of dimensionless equations (3)-(7) has been solved on the basis of the finite difference method. The difference elliptic equation for the stream function has been solved by the successive over relaxation method. The energy and vorticity equations have been solved on the basis of the locally one-dimensional Samarsky scheme. The algorithm has been tested using the experimental data [8, 9] and a good agreement has been obtained.
3. Results and discussions
As a result of calculations, the local characteristics of heat and mass transfer, as well as the time dependences of the melt volume fraction of the paraffin and the average temperature of the heater have been obtained for different values of the Biot number. Numerical modeling was carried out with the following values of dimensionless parameters, namely, $Pr = 48.36$, $Ra = 4.03 \cdot 10^6$, $Ste = 1.84$, $Os = 0.845$ and Biot number was varied in the range $1.27 \leq Bi \leq 10.16$.

Figure 2 shows the isotherms for two cases corresponding to $Bi = 1.27$ and $Bi = 5.08$. At the initial melting stage, the phase change line moves more uniformly. High density of isotherms near the inner surface of the radiator is due to the lower thermal conductivity of the PCM. With an increase in the melt volume, convective flows that contribute to the heating of the upper part of the area filled with paraffin appear. It can be seen that the temperature grows much faster with a smaller value of $Bi$, the higher temperature gradients are observed, therefore, natural convection develops more intensively in the case of $Bi = 1.27$ due to less intensive cooling from the external walls. It should be also noted that in the case of $Bi = 5.08$, the temperature distributions in the copper profile are more uniform.

![Figure 2. Temperature fields for $Os = 0.0845, f = 400$

As one would expect, at low $Bi$ numbers, the complete melting of paraffin occurs faster; it is also worth noting that, in the case of $Os = 0.0845$, the melting time increases by 3.5 times with a growth of the $Bi$ number from 1.27 to 5.08, while in the case of high Ostrogradsky number ($Os = 0.338$), the melting time increases by only 26% at $Bi = 10.16$ and 12% at $Bi = 5.08$ in comparison with the results for $Bi = 1.27$.

With increasing temperature inside the heater, heat transfer with the environment is intensified, thus, the heat removal to the environment approaches heat generation from the source and with time the thermal regime of the system acquires a periodic nature. A growth of the Biot number characterizes the less intensive rise of the average heater temperature. So, at $\tau = 4000$, the average heater temperature is $\Theta_{avg} = 3.05$, $\Theta_{avg} = 1.81$ and $\Theta_{avg} = 1.157$ for the cases of $Bi = 1.27$, $Bi = 5.08$ and $Bi = 10.16$, respectively, while for $\tau = 6000$ one can find $\Theta_{avg} = 3.97$, $\Theta_{avg} = 1.76$ and $\Theta_{avg} = 0.89$ for the cases of $Bi = 1.27$, $Bi = 5.08$ and $Bi = 10.16$, respectively.
4. Conclusions
The influence of intensity of external convective air cooling on the thermal processes occurring inside the PCM-based heat sink heated from a local element of time-dependent volumetric heat generation has been studied. It has been shown that at low Biot numbers, the melting process occurs faster, where more intensive melting convection occurs. The temperature in the source at the melting stage, especially at the initial stage, does not strongly depend on the intensity of air convective cooling. However, after half-melting of the material with increasing temperature of the profile, the effect of external cooling increases, since the temperature gradient on the external surface of the system rises. As a result of intensification of convective heat exchange with the environment the thermal characteristics of the system switch to the cyclic mode in accordance with the changes in the heater power. An increase in the Biot number (from 1.57 to 5.08) contributes to a decrease in the source temperature by more than 55%, and the following growth of Bi till 10.16 leads to the temperature reduction by more than 77%, which is observed at time $\tau = 6000$. The melting rate also decreases, at high Ostrogradsky numbers ($O_s = 0.338$), namely, the melting time increases by 26%, while at low Ostrogradsky numbers ($O_s = 0.0845$), the melting time increases several times with a growth of the Biot number from 1.27 to 10.18.
Acknowledgments
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