A Suggestion for a Teleological Interpretation of Quantum Mechanics

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“I don’t understand you,” said Alice. “It’s dreadfully confusing!”

“That’s the effect of living backwards,” the Queen said kindly: “it always makes one a little giddy at first—”

“Living backwards!” Alice repeated in great astonishment. “I never heard of such a thing!”

“—but there’s one great advantage in it, that one’s memory works both ways.”

“I’m sure mine only works one way,” Alice remarked. “I can’t remember things before they happen.”

“It’s a poor sort of memory that only works backwards,” the Queen remarked.

[Lewis Carroll, Through the Looking-Glass]
Abstract

We suggest solving the measurement problem by postulating the existence of a special future final boundary condition for the universe. Although this is an extension of the way boundary conditions are usually chosen (in terrestrial laboratories), it is our only deviation from standard quantum mechanics. Using two state vectors, or the “two-state”, to describe completely the state of systems of interest, we analyze ideal and “weak” measurements, and show the consistency of our scheme. If the final state of a measuring device is assigned to be one of the possible outcomes of the measurement, an effective reduction is observed after an ideal measurement process. For final conditions chosen with an appropriate distribution, the predictions of standard quantum mechanics may be reconstructed, thus eliminating probability from the description of any single measurement. The interpretation attained, the Teleological Interpretation, is an ontological one; it is local and deterministic. Other special assumptions in the choice of the final boundary condition may explain certain unaccounted for phenomena, or even supply a mechanism for essential free will. In this context we believe that a new conception of time should be adopted.
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Chapter 1

Introduction

We start by presenting in Chapter 2 some of the conceptual difficulties in the foundations of quantum mechanics. They include the measurement problem, problems which arise in various popular interpretations of quantum mechanics and the problem of defining classicality. These fundamental issues are subjects of many discussions taking different approaches, and we wish to present the conceptual common ground necessary for the understanding of our work, and put it in uniform terminology.

In Chapter 3 we briefly review the time-symmetric reformulation of quantum mechanics, in which the suggested interpretation is formulated. In this formalism, two temporal boundary conditions are assumed: an initial one and a final one. Two wave-functions are evolved from these, respectively: the standard one, which we call the history vector, and a backwards evolving one, the hermitian adjoint of which we call the destiny vector. These are combined to form the two-state, constituting the complete description of any system. Final boundary conditions are usually introduced by post-selection of the measured system, taking into account only the experiments which yield a specific outcome. We wish to generalize this formulation to be applicable also to closed systems, namely to the universe. We show that the density matrix of the closed universe in standard quantum mechanics is a special case of its possible two-states in the time-symmetric formalism. From here on we work in the frame of the two state vector formalism and are interested only in the unitary evolution of two-states and in the reduced two-states of their subsystems.

In Chapter 4 we suggest choosing a special final boundary condition which solves the measurement problem. This is done by setting the final states of measuring devices as one of their classically possible pointer states, according to the measurements in which they are involved, with a probability distribution that reconstructs the predictions of standard quantum mechanics for
large ensembles of such measurements. By this, probability is eliminated from the description of any single measurement, and its specific outcome may be calculated, given its boundary conditions. We analyze ideal measurements and show how effective reduction is attained without the need of supplemental mechanisms, thus solving the measurement problem. Here we introduce the process of “two-time decoherence”, which, we show, governs the behaviour of the measuring device. We next consider non-ideal weak measurements, where a weak interaction between the measuring device and the system being measured takes place, and the outcome is the expected value of the operator dependent of the boundary conditions. Clearly these kind of measurements may be naturally described using the two-state formalism. We remark that weak values, or the strange outcomes obtained from weak observations, may be used to explain miscellaneous unaccounted for phenomena.

In Chapter 5 we discuss the validity of classical properties such as locality, causality, realism, determinism and free will, in the framework of our interpretation. This discussion is mainly philosophical, but is also an indispensable part of a complete picture of any interpretation. We argue that all properties excluding realism are valid. We further suggest a mechanism which allows essential free will within the framework of the suggested interpretation, and claim that in this context a more complex approach to the concept of time should be adopted.

Chapter 6 constitutes a discussion of ideas which relate to the interpretation suggested, and a summary of our work.
Chapter 2

Background

2.1 The Measurement Problem

Quantum theory was originally intended to describe the behaviour of microscopic particles, and indeed it predicts with great accuracy the outcomes of experiments in this regime. Furthermore, quantum theory has supplied explanations to phenomena previously unaccounted for, such as the spectrum of black body radiation. Today quantum theory is almost unanimously accepted as correct.

Nevertheless quantum theory raises considerable physical and philosophical difficulties regarding the usage of classical concepts such as locality, realism, determinism and so forth. The difficulty intensifies when we attempt to apply quantum theory to the macroscopic regime, meaning also to our measuring devices – an action that seems legitimate, for also they are made of microscopic particles. At first sight, it seems that in order to build a theoretical model which may reconstruct the empirical results, one needs to define an observer or a measuring device external to the observed quantum system, one which does not obey the normal evolution rules of quantum mechanics. This of course is not desired, because then quantum theory will be incomplete, in the sense that it will not describe all physical nature. This is known as the measurement problem.

The measurement problem can be simply demonstrated in the following manner. An experiment is performed on a spin-$\frac{1}{2}$ particle in order to find its spin component along some axis. Let the initial state of the particle be $\frac{1}{\sqrt{2}}(|\uparrow\rangle + |\downarrow\rangle)$. The initial state of the measuring device is $|R\rangle$ (device “READY”). Now assume an interaction between the particle and the measuring device takes place, such that if the device measures $|\uparrow\rangle$, it evolves into the state $|U\rangle$ (device measured “UP”), and if it measures $|\downarrow\rangle$, it evolves
into the state $|D\rangle$ (device measured “DOWN”). The evolution predicted by quantum mechanics is

$$\frac{1}{\sqrt{2}} (|\uparrow\rangle + |\downarrow\rangle) \otimes |R\rangle \rightarrow \frac{1}{\sqrt{2}} (|\uparrow\rangle \otimes |U\rangle + |\downarrow\rangle \otimes |D\rangle).$$  \hspace{1cm} (2.1)

But we know from everyday practice (assume we are experimentalists) that when measuring a spin-$\frac{1}{2}$ particle as above, we get $|\uparrow\rangle \otimes |U\rangle$ in 50% of the cases and $|\downarrow\rangle \otimes |D\rangle$ in the other 50%, not a superposition of the two as in (2.1).

In general, the experiment shows that after an ideal measurement, the quantum system is found to be in one of the eigenstates of the measured operator, correlated to the appropriate state of the measuring device. For pre-selected ensembles this happens with a probability equal to the absolute value of the projection of the initial state on the specific eigenstate, squared.

In order to explain this gap between theory and observed reality, different interpretations of quantum mechanics have been suggested, or better phrased, different interpretations of our observations. Their common goal is to provide the most complete description of physical reality possible, and to settle some of the contradictions between quantum theory and classical concepts.

### 2.2 Interpretations of Quantum Mechanics

Generally, interpretations of quantum mechanics can be divided into two main categories. The first kind explains our observations by supplementing the evolution rules of quantum mechanics with the concept of a random “collapse of the wave-function”. A reduction of the quantum superposition state to a classical state is supposed to take place at some stage of the measurement process. These interpretations are problematic because the collapse is non-local [1], the collapse mechanism, if at all specified, gives a nondeterministic outcome, and Occam’s razor, the principle of simplicity, is unsympathetic to the excessive collapse rule.

A second kind of interpretation attempts to explain experimental outcomes deterministically, without the need of a collapse. Among these are “hidden variables” interpretations and “relative state” interpretations. hidden variables interpretations, following Bohm [2], assume the existence of inaccessible variables with definite values, which determine the state after the measurement. Bell [3], in his famous inequality theorem, and recently the GHZ [4][5] argument, show that these kind of theories are inherently non-local. Also it is not clear how the empirical probabilities, as they arise in the
standard theory, are to be reconstructed. The different variations of the relative state interpretation (such as “many worlds” or “many-minds”), which are themselves interpretations of Everett’s original suggestion \[6\], settle the measurement problem by attributing different observations to the different states of consciousness correlated to them. In these interpretations there is no simple relation of the empirical probabilities to the possible outcomes. In fact it is assumed that all possible outcomes and observations coexist, so one cannot explain why he is the one observing a certain outcome. Both of these non-collapse interpretations also contain some multiplicity of entities (variables, worlds), needed for the description of the system, against the spirit of Occam’s razor.

It is worth commenting that we have discussed only ontological interpretations whereas epistemological ones exist also. The former are interpretations in the sense we have described: they attempt to explain our observations by giving a broader, presumably complete description of physical nature. The latter confine themselves to give a set of logical rules regarding our knowledge of reality, putting aside the discussion of what is really happening. We find this kind of interpretations unsatisfactory, and perhaps even opposed to the spirit of the science of physics.

In this work we suggest a new ontological interpretation, which attempts to overcome the difficulties mentioned. Namely it is local, deterministic and simple, and it reconstructs the empirical probability rules of standard quantum mechanics. The classical properties mentioned in this section will be defined more precisely and be further discussed in Chapter \[\text{4} \].

### 2.3 Classicality and Decoherence

The measurement problem, as it arises in a situation where a classical apparatus is measuring a quantum system, is tightly connected to the definition of the classicality of systems. A classical system, by definition, is a system which does not exhibit quantum-like behaviour in some sense. Generally, we recognize one definite classical basis of states, of which our classical system assumes one specific state. We do not observe superpositions among classical states, in contrast to the case of quantum states. Accordingly we may postulate that it is not possible to interfere or mix the phases between two classical states. In the case of a measurement process, it can be further shown that without a scheme of choosing the classical basis and a postulate such as above, it is not even well defined what observable is being measured \[\text{4} \]. \[\text{8} \]. This is sometimes referred to as the problem of the preferred basis.

Selection of a classical basis and the destruction of phase coherence can
be achieved in the framework of standard quantum mechanics, by the dynamical process of environment induced superselection or decoherence, first introduced by Zurek [7] [8] [9]. Here it is assumed that the to-be-classical apparatus is being constantly monitored by an environment, singling out an almost orthogonal basis. The additional degrees of freedom, now coupled to the different apparatus states in this basis, prohibit a change of this basis. Doing so will result in the destruction of any previous correlations of the apparatus with other systems, such as the system being measured. Note that such a correlation is essential for the definition of the classical basis. Due to the near orthogonality of the environment states, when tracing out the environmental degrees of freedom, one notices that the off-diagonal interference terms in the reduced density matrix are negligible. Therefore no projection onto a superposition of classical states can take place. Here it is assumed that the experimenter cannot take into account the environment itself, when performing such an experiment, since in realistic situations the exact state of at least part of the environment is unknown. Up to now we have not yet solved the measurement problem – no reduction to a single classical state has occurred, but importantly a basis for the reduction has been defined.

The environment in discussion is the dominant part of the systems “not of interest”, interacting with our apparatus. Usually these may take the form of internal parts of our macroscopic device, where the aforementioned “apparatus” is but the device’s pointer. The classical states of the preferred basis are called “preferred states” or “pointer states”. As a result of the diagonality of the density matrix, these states are insensitive to measurements performed on them, in agreement with our classical experience. By contrast, quantum states change or undergo “preparation” with each new measurement. Therefore we may relate classicality to predictability. Zurek [10] has even suggested a predictability sieve, in order to identify the classical states by the length of time that they maintain predictability. With this in mind we shall later relate different time measures to different measures of classicality of systems, where for ideal classical systems, we would like the time they remain predictable to be almost infinite or at least longer than the lifetime of the universe.

To conclude the discussion we shall rederive a simple toy model of decoherence presented in Ref. [8]. This model will be later used also in the context of our interpretation. Let us start with the initial state \( a|\uparrow\rangle + b|\downarrow\rangle \) for the quantum system and \( |R\rangle = \frac{1}{\sqrt{2}} (|U\rangle + |D\rangle) \) for the apparatus, both at time \( t_0 = 0 \). Assume an interaction Hamiltonian between them of the form

\[
H_{sa} = -g\sigma_z^{(s)} \otimes \sigma_y^{(a)},
\]

where \( g \) is a constant, \( \sigma_y \) and \( \sigma_z \) are Pauli matrices, and we write all states
in the basis of eigenstates of $\sigma_z$, ($s$) designates system and ($a$) designates apparatus. Let the interaction take place for a time

$$t_1 = \frac{\pi \hbar}{4g}. \quad (2.3)$$

The evolution will be

$$\frac{1}{\sqrt{2}} (a| \uparrow \rangle + b| \downarrow \rangle) \otimes (|U \rangle + |D \rangle) \longrightarrow a| \uparrow \rangle \otimes |U \rangle + b| \downarrow \rangle \otimes |D \rangle. \quad (2.4)$$

Next take the state obtained and couple it to an environment consisting of $N$ two-level particles with the basis $\{|u\rangle_k, |d\rangle_k\}_{k=1}^N$, in the initial state $\prod_{k=1}^N (\alpha_k |u\rangle_k + \beta_k |d\rangle_k)$. Assume an interaction Hamiltonian between the apparatus and environment of the form

$$H_{ae} = -\sigma_z^{(a)} \otimes \sum_{k=1}^N g_k \sigma_z^{(e)} \prod_{j \neq k} 1_j, \quad (2.5)$$

where $g_k$ are constants, ($e$) designates environment and 1 is the unit operator. The evolution is

$$(a| \uparrow \rangle \otimes |U \rangle + b| \downarrow \rangle \otimes |D \rangle) \prod_{k=1}^N (\alpha_k |u\rangle_k + \beta_k |d\rangle_k) \longrightarrow$$

$$\longrightarrow a| \uparrow \rangle \otimes |U \rangle \otimes |\varepsilon_U (t - t_1) \rangle + b| \downarrow \rangle \otimes |D \rangle \otimes |\varepsilon_D (t - t_1) \rangle, \quad (2.6)$$

where

$$|\varepsilon_U (t') \rangle = \prod_{k=1}^N (\alpha_k \exp (ig_k t' / \hbar) |u\rangle_k + \beta_k \exp (-ig_k t' / \hbar) |d\rangle_k), \quad (2.7)$$

$$|\varepsilon_D (t') \rangle = \prod_{k=1}^N (\alpha_k \exp (-ig_k t' / \hbar) |u\rangle_k + \beta_k \exp (ig_k t' / \hbar) |d\rangle_k) \quad (2.8)$$

are the environment states which correspond to the superselected pointer states. Tracing out the environment, the reduced system-apparatus density matrix obtained is

$$\rho_{sa}^{DEN}(t) = \text{tr}_e \rho_{sa}^{DEN}(t) = |a|^2 | \uparrow \rangle \langle \uparrow | \otimes |U \rangle \langle U | +$$

$$+ z(t - t_1) ab^* | \uparrow \rangle \langle \downarrow | \otimes |D \rangle \langle D | +$$

$$+ z^* (t - t_1) ba^* | \downarrow \rangle \langle \uparrow | \otimes |D \rangle \langle U | +$$

$$+ |b|^2 | \downarrow \rangle \langle \downarrow | \otimes |D \rangle \langle D | \quad (2.9)$$

\(^1\)Derivations may be found in Appendix A.
where
\[ z(t') = \langle \xi_U(t')|\xi_D(t') \rangle = \prod_{k=1}^{N} \left( \cos(2g_k t'/\hbar) + i \left( |\alpha_k|^2 - |\beta_k|^2 \right) \sin(2g_k t'/\hbar) \right) \]

is the correlation amplitude. The temporal average of the absolute value of this amplitude is
\[
|z(t')|^2 = \lim_{T \to \infty} T^{-1} \int_0^T |z(\tau)|^2 d\tau = 2^{-N} \prod_{k=1}^{N} \left( 1 + \left( |\alpha_k|^2 - |\beta_k|^2 \right)^2 \right).
\] (2.11)

This implies that for large enough an environment, consisting of many \(N\) particles, \(z(t)\) is negligible in average, and the diagonal terms of (2.9), which contain it as a factor, are damped out. Thus after some finite decoherence time, an effective pointer basis has been established, and neither change of basis nor projection onto a superposition of basis states are possible. The only difficulty with this toy model is that since \(z(t)\) is of the almost-periodic function family, as long as \(N\) is finite, any value of its range will recur an infinite number of times \([12]\). Thus \(z(t)\) will eventually return to assume non-negligible values causing recoherence. Only when the environmental degrees of freedom have a continuous spectrum of eigenstates, can an infinitely long recoherence time be attained.

Classical systems should have a very short decoherence time and a recoherence time longer than the lifetime of the universe. These can be achieved when the environment is large, as indeed characterizes measuring devices, which are macroscopic and therefore have large environments. Of course a more realistic model of an environment should be used. Since physical interactions are usually a function of distance, given by a potential, we would expect the pointer basis to be a position basis, so that the different states are localized. However up to now we have ignored the free Hamiltonian of the apparatus and environment, assuming that they were commutative with the interaction Hamiltonian. This is generally not true because the momentum terms of the free Hamiltonians are not commutative with the interaction potential. Thus a condition for localization is massiveness of the apparatus \([11]\), which suppresses the non-local terms. This again is true for macroscopic devices.
Chapter 3

Time Symmetric Quantum Mechanics

3.1 Background

In their famous 1964 article, ABL [13] suggested a new rule for calculating probability. In the case that a final state $\Psi_f$ is specified for the measured system, in addition to the usual choice of an initial state $\Psi_i$, the probability that an intermediate measurement of an operator $A$ yields the eigenstate $|a_k\rangle$ is

$$\text{prob}(a_k \mid \Psi_i, \Psi_f) = \frac{\text{prob}(\Psi_f(t) \mid a_k)\text{prob}(a_k \mid \Psi_i(t))}{\sum_j \text{prob}(\Psi_f(t) \mid a_j)\text{prob}(a_j \mid \Psi_i(t))} = \frac{|\langle \Psi_f(t) | a_k \rangle|^2 |\langle a_k | \Psi_i(t) \rangle|^2}{\sum_j |\langle \Psi_f(t) | a_j \rangle|^2 |\langle a_j | \Psi_i(t) \rangle|^2},$$

(3.1)

where $a_k \in \{a_j\}_j$, an eigenbasis of the measured operator $A$, and we assume that an instantaneous measurement occurs at a time $t$ intermediate of the boundary conditions, to which all wave-functions are evolved. If the measurement is not instantaneous, the initial and final wave-functions should be taken at the beginning and ending of the measurement interaction, respectively (assuming no free evolution of the wave-functions in between).

If only the initial condition is specified, (3.1) should reduce to the regular empirical probability rule:

$$\text{prob}(a_k \mid \Psi_i) = |\langle a_k | \Psi_i(t) \rangle|^2.$$  

(3.2)

Formula (3.1) is actually an application of the simple conditional probability formula to the quantum case. But a conceptual leap has been made in the recognition that boundary conditions may be chosen time-symmetrically in
contrast to the conventional asymmetric choice of an initial condition only. The gauntlet has been thrown down: why should boundary conditions be chosen with such a discrimination?

Later Aharonov et. al. introduced the concept of weak measurement and weak values \[14\] \[15\]. The idea is as follows. If one performs an isolated, weak interaction measurement, where the measured system is almost undisturbed and no reduction takes place, his apparatus will show the expected value of the measured operator, with a weighing of the appropriate probabilities. A condition for this, as will be discussed in Section \[13\] is the existence of a large enough uncertainty in the pointer’s initial state. Then the pointer’s final state will be spread around the expected value, and the final outcome observed by us, will be the result of an ideal measurement on this spread state, with the appropriate probability distribution.

When only an initial boundary condition is specified for the quantum system, or when the final condition is identical to the initial condition, one gets after interaction the expectation value

$$\langle A \rangle \equiv \langle \Psi_i | A | \Psi_i \rangle = \sum_k a_k \text{prob}(a_k | \Psi_i).$$  \hfill (3.3)

In the general case when both initial and final boundary conditions are specified the outcome is the weak value \(A_w\), which may be far from any eigenstate of the measured operator

$$A_w \equiv \frac{\langle \Psi_f | A | \Psi_i \rangle}{\langle \Psi_f | \Psi_i \rangle} \propto \sum_k a_k \sqrt{\text{prob}(a_k | \Psi_i, \Psi_f)},$$  \hfill (3.4)

where the times at which the wave-functions are taken are as above. Notice that in (3.4) the appropriate weighing of each eigenvalue is proportional to the square root of the ABL probability (3.1).

The final boundary condition may result, for example, from post-selection of the system after the interaction has taken place, which can be achieved by performing an ideal measurement, and discarding the cases with unwanted outcomes. Alternatively, some systems in nature (as, we shall suggest, the universe) may have an inherent final boundary condition, just as all systems have initial ones. Weak values may play an important role in the understanding of certain phenomena such as tunneling \[16\] or Hawking radiation from a black hole \[17\]. Later, we shall consider weak measurements in the frame of our interpretation.

In order to reduce the time-symmetric case to the pre-selected-only case, one must choose the final state identical to the initial state. This is immediately apparent when comparing the weak value formula, left equation in
with the expectation value formula, left equation in (3.3). A general-
ization to closed systems is achieved by taking the final condition as the
initial condition evolved to the final time, as will be shown in Section 3.3.
However when reducing the ABL formula (3.1) to the regular probability for-
mula (3.2), one must choose the final condition as the measured $|a_k\rangle$, which
is actually the evolved initial state, assuming it has undergone reduction to
a specific eigenstate. This discrepancy arises due to the use of a probabilistic
formula which is foreign to our unitary formalism. This choice is also a clue
for the upcoming suggestion which will later justify it.

3.2 The Two State Vector Formalism

Following Ref. [18], we wish to reformulate quantum mechanics, to be time-
symmetric, in the sense that it will take into account both initial and final
boundary conditions. The Schrödinger equation is first order in the time
derivative, therefore only one temporal boundary condition may be consist-
tently specified for a solution of the equation. Assuming both initial and final
boundary conditions exist, we must have two solutions suitable for each of
the two boundary conditions. The first is the regular wave-function evolved
forward in time from the initial condition, which we call the “history vector”
and denote by $|\Psi_{\text{HIS}}(t)\rangle$. The second is a different wave-function evolving
from the future final condition, backwards in time. We call the hermitian ad-
joint of this vector the “destiny vector”, denoted by $|\Psi_{\text{DES}}(t)\rangle$. We postulate
that the complete description of any system is given by two vectors as such.
These may be combined into operator form by defining the “two-state”

$$\rho(t) \equiv \frac{|\Psi_{\text{HIS}}(t)\rangle\langle\Psi_{\text{DES}}(t)|}{\langle\Psi_{\text{DES}}(t)|\Psi_{\text{HIS}}(t)\rangle}. \quad (3.5)$$

This, in general, is not reducible to a single wave-function. It is clear that
orthogonal boundary conditions are forbidden, therefore

$$\langle\Psi_{\text{DES}}(t)|\Psi_{\text{HIS}}(t)\rangle \neq 0, \quad (3.6)$$

which is a reasonable choice due to the fact that a final state, orthogonal
to the initial state, has probability zero for being post-selected. For a given
Hamiltonian $H(t)$, the time evolution of the two-state from time $t_1$ to $t_2$ is

$$\rho(t_2) = U(t_2,t_1)\rho(t_1)U(t_1,t_2), \quad (3.7)$$

where $U(t_2,t_1)$ is the regular evolution operator

$$U(t_2,t_1) = \exp \left( -i/\hbar \int_{t_1}^{t_2} H(\tau)d\tau \right). \quad (3.8)$$
The two-state takes the place of the density matrix in standard quantum mechanics. Any subsystem’s two-state may be obtained by taking the partial trace of all other degrees of freedom. In the current work, our formalism supports only these two operations: unitary time evolution and tracing. No other formula is allowed nor required, as we shall show in the next chapter.

3.3 The Two-State of the Universe

The Conventional approach to nonrelativistic quantum mechanics assumes that a complete description of the state of a closed system, such as the universe, is given at any time \( t \) by a wave-function

\[
|\Psi(t)\rangle = U(t, t_0)|\Psi(t_0)\rangle,
\]

(3.9)

where \( \Psi(t_0) \) is usually defined at some initial time \( t_0 = t_i \) where \( t > t_i \), hence \( \Psi(t_0) \) is the initial boundary condition \( \Psi_i(t_i) \). We shall denote this wave-function \( \Psi_i \). The density matrix associated with the state \( |\Psi(t)\rangle \) is

\[
\rho_{\text{DEN}}(t) = |\Psi_i(t)\rangle\langle \Psi_i(t)|,
\]

(3.10)

and the state of any subsystem is obtained by taking the partial trace of all other degrees of freedom.

We will show that the above description is consistent with the assumption of a special final boundary condition of the form

\[
|\Psi_f(t_f)\rangle = U(t_f, t_i)|\Psi_i(t_i)\rangle,
\]

(3.11)

at the final time \( t_f \), for a backward evolving wave-function denoted \( \Psi_f \). This will be established by writing the two-state at any intermediate time. Up to a normalization factor

\[
\rho(t) = |\Psi_i(t)\rangle\langle \Psi_f(t)| =
\]

\[
= |\Psi_i(t)\rangle\langle \Psi_f(t_f)|U(t_f, t) =
\]

\[
= |\Psi_i(t)\rangle\langle \Psi_i(t_i)|U(t_f, t_i)U(t_f, t) =
\]

\[
= |\Psi_i(t)\rangle\langle \Psi_i(t_i)|U(t_i, t) =
\]

\[
= |\Psi_i(t)\rangle\langle \Psi_i(t)| = \rho_{\text{DEN}}(t).
\]

(3.12)

Therefore if only an initial boundary condition is assumed, in all calculations two-states may by substituted by density matrices, giving the same results. In the previous section’s notation we have simply taken the destiny vector to be equal to the history vector.
Under the assumption of a deterministic evolution rule for the universe, which will be justified later, we have shown that our formalism is reducible to the conventional one, the latter being a special case of the former. Taking final boundary conditions different from (3.11), our formalism introduces a richer state structure into quantum theory. This is a generalization, to the closed universe, of the ABL suggestion, of choosing two temporal boundary conditions for the system being measured.

In the next chapter we suggest postulating a very special final boundary condition at the final time, a time which should be as late as the lifetime of the universe, whether the universe ends in a singularity, in relaxation to a steady state, or is infinite in time.
Chapter 4

Teleological Interpretation of Quantum Mechanics

4.1 The Suggestion

We are now ready to present our suggestion for a new interpretation of quantum mechanics, the Teleological Interpretation. The Webster New Collegiate Dictionary defines “Teleology” as

1 a: the study of evidences of design in nature b: a doctrine (as in vitalism) that ends are immanent in nature c: a doctrine explaining phenomena by final causes 2: the fact or character attributed to nature or natural processes of being directed toward an end or shaped by a purpose 3: the use of design or purpose as an explanation of natural phenomena.

We argue that special final conditions may exist, so that if they are taken into account, the probabilistic predictions of quantum mechanics may be explained. Setting aside for the moment the mechanism by which a proper chosen final state causes an effective reduction to the appropriate desired outcome, we wish to present the scheme by which the final states should be selected. We argue that a universe set up as follows behaves as predicted by standard quantum mechanics, such as we believe our universe does, using only unitary Schrödinger evolution.

• Choose the universe’s initial boundary condition.
• Choose the universal Hamiltonian.
• Identify the classical systems and their preferred basis.
• Identify measurement-like interactions between classical and quantum systems.

• Assign the universe’s final boundary condition as the initial one, evolved to the final time, with the following exceptions:
  
  – For classical systems select one of the preferred basis states (normalized) from the superposition, while ensuring that the probability distribution for measurements on large ensemble match the one predicted by the regular probability rules.
  
  – Set special final states to produce strange phenomena (optional).
  
  – Set special final states to match the free will of sentient beings (optional).

Assuming such boundary conditions for the universe, ideal “probabilistic” and non-ideal weak measurements will be analyzed in the following sections. Other than determining the preset distribution, the probability rules mentioned in Section 3.1 need not to be applied in any manner, on the contrary, we treat them as empirical rules and show how their predictions are reconstructed. Therefore no “projections” should be applied to the system during the measurement process. We must stress that in most situations the final classical states are not known to us prior to the completion of the measurement, and are assumed to be such or the other for the sake of the computational examples. The reader should not concern himself at this stage with questions of the amount of freedom of choice in performing different measurements. We will later examine the applicability of the concept of free will in the framework of the suggested interpretation.

### 4.2 Ideal Measurements

In this section we shall analyze the ideal measurement process, and show how effective reduction takes place. For the sake of simplicity, we stick with the decoherence toy model presented in Section 2.3. Although simple, it is important to take such a dynamic model in order to fully understand the relevant processes. There is some resemblance to the work done in Ref. [19]. Recall the state obtained in Section 2.3 after decoherence. Let us take this state as our history vector

$$|\Psi_{\text{HIS}}(t)\rangle = a|\uparrow\rangle \otimes |U\rangle \otimes |\varepsilon_U(t-t_1)\rangle + b|\downarrow\rangle \otimes |D\rangle \otimes |\varepsilon_D(t-t_1)\rangle, \quad (4.1)$$
where \( t_1 \) was some finite system-apparatus interaction time and \(|U\rangle, |D\rangle\) were superselected as the classical pointer states. Let us assume that the environment associated with these states is large enough so that the recoherence time, or the time which takes the correlation amplitude to revert to a non-negligible value, is longer than the lifetime of the universe. It is then possible to assign the final boundary condition as

\[
|\ldots\rangle \otimes |U\rangle \otimes |\varepsilon_U(t_f)\rangle,
\]

(4.2)

for example, where \(|\ldots\rangle\) are unknown systems correlated to our pointer state, and a specific state was chosen for the pointer \(|U\rangle\), from its classical basis, correlated to the adequate environment state of that time \(|\varepsilon_U(t_f)\rangle\). Such a choice is reasonable because after decoherence has taken place and before recoherence, no interference between the pointer states can take place anyway.

The quantum system, by contrast, has a certain definite state only until the next measurement made on it, which prepares it in a new state

\[
|\phi\rangle = c|\uparrow\rangle + d|\downarrow\rangle,
\]

(4.3)

which takes the role of an effective final boundary condition for the quantum system as will be soon showed. Therefore the destiny vector at a time after the measurement interaction is over is

\[
|\Psi_{\text{DES}}(t)\rangle = |\phi\rangle \otimes |U\rangle \otimes |\varepsilon_U(t - t_1)\rangle.
\]

(4.4)

The complete description of our systems is given up to a normalization factor by the two-state

\[
\rho(t) = |\Psi_{\text{HS}}(t)\rangle \langle \Psi_{\text{DES}}(t)| = \\
a|\uparrow\rangle \otimes |U\rangle \otimes |\varepsilon_U(t - t_1)\rangle \langle \phi| \otimes \langle U| \otimes \langle \varepsilon_U(t - t_1)| + \\
b|\downarrow\rangle \otimes |D\rangle \otimes |\varepsilon_D(t - t_1)\rangle \langle \phi| \otimes \langle U| \otimes \langle \varepsilon_U(t - t_1)|.
\]

(4.5)

Ignoring the environment the reduced two-state obtained is

\[
\rho_{\text{sa}}(t) = \text{tr}_e \rho(t) = a|\uparrow\rangle \langle \phi| \otimes |U\rangle \langle U| + \\
z^*(t - t_1)b|\downarrow\rangle \langle \phi| \otimes |D\rangle \langle U|.
\]

(4.6)

Refer to Section 2.3 to recall the behaviour of \(z(t)\). It is evident that after a decoherence time an effective reduction of the pointer to the state: \(|U\rangle\langle U|\) has occurred. We call this process “two-time decoherence” differing it from the regular meaning of decoherence.

While the above is true for the pointer involved in the original measurement, notice that the state of the quantum system and the state of any apparatus performing further measurements in the same basis of the quantum
system or original pointer, experience immediate effective reduction, with no decoherence time delay. This becomes evident, after tracing out the original pointer’s degree of freedom, as in the above example, the off-diagonal term which contained $|U\rangle\langle D|$ vanishes in their reduced two-states. In the “old” probabilistic nomenclature, applying the ABL rule, the original pointer has probability 1 to be found (by those correlated systems) in the reduced state, and probability 0 to be found in any other state. Therefore in a realistic experiment where a chain of measurements exists, one should expect to observe immediate reduction, in contrast to the prediction of the many worlds interpretation, for example, which states that a decoherence time until effective reduction, should always be expected. This may be an important deviation point when comparing which of the two interpretations is more applicable. It may be that the two-time decoherence does not play an important role in the measurement chain, but as we have shown, it naturally emerges from the combination of regular decoherence needed for classicality, and the two-state formalism with our special final boundary condition. Of course it remains essential that the final state is chosen from the classical pointer states superselected by regular decoherence. In the many worlds picture where each superposition term is viewed as a branching world, we have simply selected one specific branch. Such a view may help seeing why the interpretation suggested is self consistent, and why is it naturally demanded, if one does not wish to have a multitude of “worlds”.

It remains to show how the effective reduction determines the backward evolving state of the quantum system, for a previous measurement, as the process presented above shows effective reduction only after a finite positive time duration of the system-apparatus interaction. Evolving the destiny vector at time $t_1$, $(c|\uparrow\rangle + d|\downarrow\rangle) \otimes |U\rangle$, backwards to the time $t_0 = 0$ (which was chosen for convenience as the beginning of the measurement), the two-state of the quantum system and apparatus obtained is (up to normalization)

$$
\rho_{sa}(0) = ac^* |\uparrow\rangle \langle \uparrow | \otimes |R\rangle \langle R| + ad^* |\uparrow\rangle \langle \uparrow | \otimes |L\rangle \langle L| + bc^* |\downarrow\rangle \langle \downarrow | \otimes |R\rangle \langle R| + bd^* |\downarrow\rangle \langle \downarrow | \otimes |L\rangle \langle L|,
$$

where $|R\rangle = \frac{1}{\sqrt{2}} (|U\rangle + |D\rangle)$ and $|L\rangle = \frac{1}{\sqrt{2}} (|U\rangle - |D\rangle)$. Taking the partial trace on the apparatus’ degree of freedom, shows that the backward evolving vector of the quantum state is $\langle \uparrow |$ as expected from this process, in which the outcome was “UP”. This sets a final boundary condition for the quantum state in a previous measurement, in the same manner that we have taken into account the state $|\phi\rangle$ from the next measurement.

We have shown how effective reduction may take place in an ideal measurement when the final state of the classical apparatus is chosen as one of
its possible classical states after the measurement. Setting a final boundary condition for the classical states at a very late time, enables them to stay predictable as expected. Conversely, the quantum system is “prepared” at each measurement in a new state which constitutes an effective boundary conditions for both the next and previous measurement. In our analysis we have treated these as our initial and final boundary conditions for simplicity, bringing the final apparatus state from the absolute final time, under the assumption that it has not undergone any further interaction. We have thus formulated a connection between the classicality of a system and the length of time between its initial and final boundary conditions.

Our example considers only a single measurement process per measuring device. If the apparatus undergoes multiple interactions, always some initialization process of the measuring device must take place, one which is dependent on the previous measurement’s outcome. This is obvious, due to the fact that information cannot be lost but is always transferred to other systems. Hence, the information of most of the measurements’ outcomes reside in those systems’ final state.

4.3 Weak Measurements

We now wish to analyze non-ideal weak measurements, which may be naturally described using the two-state formalism. A weak measurement is one in which the precision of the measurement is low enough, so that negligible change would be induced to the measured system. In a weak measurement the measuring device gets correlated to the different eigenstates of the system, but no reduction to a specific eigenstate takes place. What is measured is an average “weak” value of the operator, which is dependent on the initial and final states of the system. Let us take a many level quantum system in a basis of the states $|a_k\rangle$ on which the operator $A$ is defined as

$$A = \sum_k a_k |a_k\rangle\langle a_k|.$$  \hspace{1cm} (4.8)

The initial state of the system is chosen to be

$$|\phi_1\rangle = \sum_k c_k |a_k\rangle,$$  \hspace{1cm} (4.9)

where $c_k$ are constants. Next we take the measuring device as a pointer, with its position $q$ described by a Gaussian-like function $Q(q)$, initially set as $Q(0)$. We let an interaction between the system and apparatus take place under the Hamiltonian

$$H = -g(t)PA,$$  \hspace{1cm} (4.10)
until the time $t_1$, where
\[ \int_0^{t_1} g(\tau) d\tau = 1, \] (4.11)
and $P$ is the operator of the momentum conjugate to $q$. The evolved state of the apparatus after the time $t_1$ is given by the law:
\[ \exp (ia_k P/h) |Q(0)\rangle = |Q(a_k)\rangle, \] (4.12)
for eigenvalues $a_k$ of $A$. Let us take the initial state of our composite system to be
\[ |\Psi_i(0)\rangle = |\phi_1\rangle \otimes |Q(0)\rangle. \] (4.13)
At the time $t = t_1$ the evolved initial state is
\[ |\Psi_i(t_1)\rangle = \sum_k c_k |a_k\rangle \otimes |Q(a_k)\rangle. \] (4.14)
At some time $t_2 > t_1$ we perform an ideal measurement on the quantum system and obtain the result
\[ |\phi_2\rangle = \sum_k c_k' |a_k\rangle, \] (4.15)
which serves as a final boundary condition for the quantum system as explained in the previous section. A calculation shows that the final composite state must then be
\[ |\Psi_f(t_2)\rangle = |\phi_2\rangle \langle \phi_2 | \otimes |\Psi_i(t_1)\rangle = |\phi_2\rangle \otimes \sum_j c_j c_j^* |Q(a_j)\rangle. \] (4.16)
The two-state at a time $t$ between $t_1$ and $t_2$ (after interaction and before post-selection, and assuming no free evolution) is given up to a normalization factor by
\[ \rho(t) = |\Psi_i(t_1)\rangle \langle \Psi_f(t_2)|, \] (4.17)
and the apparatus’ pointer will show
\[ \rho_a(t) = \text{tr}_s \rho(t) = \sum_{k,j} c_k c_k^* c_j c_j^* |Q(a_k)\rangle \langle Q(a_j)|. \] (4.18)
The condition for weakness of the measurement is that the Gaussians are wide enough so that the relation
\[ \sum_k c_k c_k^* |Q(a_k)\rangle \equiv \sum_k c_k c_k^* |Q(a')\rangle \equiv |\hat{Q}(a')\rangle, \] (4.19)
for some $a'$, holds due to their interference. Then tracing out the quantum system’s degree of freedom, the apparatus reads

$$\rho_a(t) \cong |\hat{Q}(a')\rangle\langle \hat{Q}(a')|,$$

where it is shown that $a'$ equals $A_w$, the weak value of $A$,

$$A_w = \frac{\langle \phi_1|A|\phi_2 \rangle}{\langle \phi_1|\phi_2 \rangle} = \frac{\sum_k c_k c_k^* a_k}{\sum_k c_k c_k^*},$$

by computing the weak value of the evolution operator, as follows:

$$\sum_k c_k c_k^* |Q(a_k)\rangle = \sum_k c_k c_k^* \exp (ia_k P/\hbar) |Q(0)\rangle =$$

$$= \langle \phi_2 | \exp (iAP/\hbar) | \phi_1 \rangle |Q(0)\rangle =$$

$$= \sum_{n=1}^{\infty} \frac{(iP/\hbar)^n}{n!} \langle \phi_2 | A^n | \phi_1 \rangle |Q(0)\rangle =$$

$$= \langle \phi_2 | \phi_1 \rangle \exp (iA_w P/\hbar) |Q(0)\rangle +$$

$$+ \sum_{n=2}^{\infty} \frac{(iP/\hbar)^n}{n!} (\langle \phi_2 | A^n | \phi_1 \rangle - \langle \phi_2 | A | \phi_1 \rangle^n) |Q(0)\rangle \cong$$

$$\cong \langle \phi_2 | \phi_1 \rangle \exp (iA_w P/\hbar) |Q(0)\rangle \approx |\hat{Q}(A_w)\rangle,$$

where we have applied the usual condition for weakness [14] [15], which requires that the terms with $n > 1$ in the Taylor expansion are negligible, given the choice of a wide enough initial Gaussian.

It is clear that after the post-selection takes place, the weak value emerges as a consequence of a projection onto the new quantum state. The weak value actually exist also before the post-selection, after the system-apparatus interaction is completed. This fact is usually explained by the reasoning that the order of actions, looking at the pointer and performing the post-selection, is unimportant, as the post-selection cannot affect the measuring device after the measuring interaction is over. Thus the apparatus must show the same value even before post-selection takes place. In the above two-state formulation, the weak measurement is clearly seen to arise before the post-selection, when looking at the reduced two-state of the apparatus after tracing out the quantum system.

If the weakness condition is not satisfied but the measurement is not an ideal one, we are dealing with a regime of measurement of intermediate strength. In some of these cases, the outcome of the measurement may be given by a combination of the ideal and weak mechanisms, as the outcome of an ideal measurement on a set of different weak values [18].
Measurements performed by us, on large, uncontrollable systems, may satisfy the weakness condition. Measurements of galactic properties which yield too large a spin or magnetic moment or mass, may be due to a special final boundary condition for that stellar object, which yields a weak value, far from the expected eigenvalue, calculated by theoretical means. The problem of the missing mass, a recent discovery that the universe seems too young or more generally inconsistencies in measurements of the cosmological constants by different methods [20], may as well be explained by a special final boundary condition for the universe. Another example may be the observation that the calculated number of Darwin mutations seems to be too low to explain the genetic evolution of complex life forms. Perhaps these sorts of strange phenomena may be explained by assuming the existence of special final boundary conditions for these systems, which would appear to us as new fundamental laws of our universe. Of course when dealing with everyday low-energy short-duration experiments, these should reduce to give the expected regular results [17]. It might trouble some physicists, or please others, if such a special boundary condition could be used to break the restrictions of causality.
Chapter 5

The Classical Properties

5.1 Causality, Locality and Realism

In this discussion we refer to “causality” as the impossibility of superluminal signaling or of advanced action. In the context of quantum theory this means that probability distribution of experiments’ outcomes cannot be affected by events outside of their past light cone. We use “locality” for the stronger property of complete prohibition of any action at a distance or advanced action, meaning that there may be no influence on any events outside the future light-cone, including the quantum state itself, even if it does not imply defiance of causality. The second property (and of course the first, being implied by it) should exist in order to retain consistency with the theory of relativity, which we axiomatically assume holds.

The standard quantum theory does not contradict the concept of causality in any relativistic or nonrelativistic setup [21] [22] [23]. We wish to show that causality is maintained also in the framework of the suggested interpretation. Causality requires ignorance of the future boundary state, if such exists, for knowledge of it would allow the existence of the above restricted effects. It is to be remembered that in all examples of the previous chapters, the final boundary state was assumed to take a specific state, for the sake of demonstration, and generally, it cannot be known a-priori. In fact only when an second identical measurement is performed in sequence to the preparation measurement, can the final state be known for certain (“with probability 1”). In the discussion of ideal measurements in Section 4.2, we have shown that our interpretation is consistent with the conventional formalism and therefore causal. Neither can weak measurements defy causality, because, due to the weakness condition, there is large uncertainty in the outcome of the weak measurement, and it may yield values in the proximity of a strange value,
appropriate to some final condition, also for other final conditions. In fact, an error as such will result in the majority of cases. Therefore it is not possible to deduce from the outcome of a weak measurement on the final state.

In order to demonstrate how forbidden knowledge of a final boundary state (a non-causal state of affairs by itself) could have been used for superluminal signaling, examine the following setup. We work with two spin-$\frac{1}{2}$ particles located at two far away locations. We start with the initial state $\frac{1}{\sqrt{2}}(|\uparrow\rangle_L \otimes |\uparrow\rangle_R + |\downarrow\rangle_L \otimes |\downarrow\rangle_R)$, and assume we know the final state to be $\frac{1}{\sqrt{2}}(|\uparrow\rangle_L \otimes |\uparrow\rangle_R + |\uparrow\rangle_L \otimes |\downarrow\rangle_R)$, where $L$ denotes the left particle and $R$ the right particle. An observer on the left may or may not perform a unitary rotation on his particle of the form $|\uparrow\rangle_L \rightarrow |\downarrow\rangle_L$ and $|\downarrow\rangle_L \rightarrow |\uparrow\rangle_L$, leaving the initial composite state as it is or transforming it into the state $\frac{1}{\sqrt{2}}(|\downarrow\rangle_L \otimes |\uparrow\rangle_R + |\uparrow\rangle_L \otimes |\downarrow\rangle_R)$. Now the observer on the right measures the spin of his particle, obtaining $|\downarrow\rangle_R$ or $|\uparrow\rangle_R$, according to the action or non-action of his friend on the left. In this manner, the left observer may allegedly transmit signals to the right observer in an instant. A procedure like this would be possible for many arbitrary choices of initial and final states.

The theory of relativity states that physics is simple only when analyzed locally. Therefore we would like it to be possible to analyze physics locally. Moreover non-local effects are strange to the theory of relativity as they defy Lorentz covariance. Quantum theory, due to its linear structure, has an inherent non-locality encompassed in its entangled states (a spatially separated correlated pair, for example). This property is sometimes called “wholeness” or “inseparability” of quantum mechanics. This non-locality might raise causality concerns when performing measurements on part of the entangled state. A local interpretation of quantum mechanics would have to show how could the outcomes of such a measurement be explained locally. Take for example the non-local collapse interpretation; the collapse of the wave-function is assumed to take place instantaneously in all space. It is well known that such a phenomenon cannot be used for superluminal signaling, as mentioned above. But even so, the mere concept of instantaneous state collapse is non-covariant, for even if the collapse events are simultaneous in some space-time hyper-surface, these events would not occur at the same time on any other hyper-surface when changing the frame of reference $[1]$. This is a common pitfall of collapse interpretations. On the other hand, the picture presented by the interpretation suggested by us, gives a purely local explanation for the effective state reduction.

The property of “realism” or “objectivity”, represents the classical concept that physical nature exist independently of possible observations of it. The lack of covariance of the collapse process is an example in which the
description of nature is dependent on the observer. EPR [24] define realism by the following counterfactual:

If, without in any way disturbing a system, we can predict with certainty (i.e., with probability equal to unity) the value of a physical quantity, then there exists an element of physical reality corresponding to this physical quantity.

This means that a (possibly “hidden”) variable exists, which determines the outcome of the measurement. The concern of whether this variable is actually accessible to us, goes to the discussion of free will in the following section. As mentioned before in Section 2.2 ([3],[4],[5]) it has been proved that locality and realism cannot coexist in any interpretation of quantum mechanics. This proof rests on the supplemental assumptions that no non-causal action is allowed, that asking questions of “what if” is permitted (there is “counterfactual definiteness” [25]) and that there are no predetermined “conspiratorial” dependencies between what has been measured and what has outcome. Then no local hidden variables theory can exist, and at least one of these concepts should be given up. In the many worlds interpretation counterfactual definiteness does not hold, because of the coexistence of multiple outcomes for a measurement, therefore the discussion of realism is irrelevant. Also in our interpretation, realism is irrelevant, because we have assumed predetermined final boundary conditions which determine the outcomes of measurements, knowing from the initial state and Hamiltonian what observations actually take place. Bell described such a situation as apparently separate parts of the world being deeply and conspiratorially entangled.

5.2 Determinism and Free Will

The classic property of “determinism” signifies the ability to completely deduce the state of the system at any time, only from the knowledge of its initial boundary condition and evolution rules. Sometimes this is also referred to as “causality” but we reserve the latter term for the relativistic meaning discussed above. The concept of determinism constitutes a main disagreement point between the different interpretations of the quantum theory. We have addressed deterministic and nondeterministic interpretations in Section 2.2. Deterministic interpretations strive to reach a complete description of reality with the introduction of additional rules into standard quantum mechanics, in order to make the behaviour of any system predictable in principle, as opposed to the random collapse mechanism. This is in the spirit of Einstein’s famous saying that “God does not play dice”. We claim that our suggested
interpretation is deterministic in a broader, two-time sense, where the given boundary conditions also include the final condition. With this, specific outcomes of measurements are completely accounted for, while the probabilistic structure arises due to a preset stochastic distribution in the final boundary condition, and the usual lack of knowledge of the specific final states therein. The latter is a requirement of causality as discussed before.

The discussion of free will or freedom of choice may seem awkward in the scope of a thesis in physics, but since these concepts have considerably bothered many philosophers (including the author) in their attempt to explain reality, we find the discussion relevant in the context of a theory which strives to give a complete description of reality. We make a discrimination between effective or apparent free will which means that any specific observer may effectively experience freedom of choice, and essential or real free will which is more a moral concept, of whether an individual is “really” free, in some sense, to choose his course of action.

We say that a system has an effective free will, or is a “free agent”, if at some instant a choice can be made between different possible evolutions, independently of any accessible past data, or in short, if there is freedom from the past. A theory which allows the existence of free agents must include a free-from-the-past mechanism which can supply different outcomes, as a function of something different from accessible past variables. For example, if there had existed a mechanism which generates purely random numbers, it would have served this purpose adequately. The empirical probabilistic behaviour of quantum mechanics, never mind the interpretation explaining it, would do as well as a mechanism which permits effective freedom of choice. Notice that the effective free will experienced by us, Humans, may well be due to our lack of complete knowledge of the exact past or current state of the relevant systems (intensified by chaotic processes) and it may even be to some extent a psychological illusion. By stating that a theory allows effective free will, we permit but not prove, Humans to be free agents. The possibility of effective free will is implied by, but does not necessarily imply, lack of determinism, for we have only required independence of accessible past data, while also a deterministic theory such as hidden variables can supply the necessary free-from-the-past mechanism, as the hidden variables determine the outcome of the measurement, but are themselves inaccessible. We have further broadened the concept of determinism to be time-symmetric, where for free will we do not do so, because the future data is anyway inaccessible to us, due to the restrictions of causality.

The possibility of effective freedom of choice may be demonstrated in our interpretation as follows. We analyze two large macroscopic isolated systems, such as two distant galaxies. System A containing Humans and
system B containing aliens. Assume that in system A the natural time flows toward the future as usual, while in system B it flows backwards towards the past. This may be achieved for system B by choosing a special initial boundary condition such that the entropy decreases with the usual flow of time as it is in system A; then in system B the thermodynamic arrow of time is reversed in respect to system A. Assume that both systems are classical in the sense that system B is monitoring system A without disturbing it. Now the past of system B encompasses the future state of system A. Call these data the future boundary condition of system A. As long as system B does not pass any information to system A (an action which would cause severe causal problems), system A cannot purposely change its choice to be inconsistent with (and thus spoil) system B’s memory. Now let system A be quantum mechanical. Dependence of events on the past is not obliged because of the probabilistic structure of quantum mechanics, as explained above. In our interpretation this is expressed in the unknown final boundary condition chosen for the universe. This picture perhaps suits an old saying of Our Sages in Pirkei Avot, Chapter 3, Verse 19: “All is foreseen and the choice is given”.

Moving on to essential free will; Humans have such a strong tendency to believe in their freedom of choice, that whole moral doctrines rely upon it. We believe that the individual chooses how to act (for example whether to be good or evil) and therefore he is responsible for his deeds and influences his fate. We have seen that systems may well effectively behave as having free will. But when analyzing the origin of a choice made, after discarding all past dependencies one is left only with the quantum mechanic probabilistic structure which supplies a stochastic free-from-the-past mechanism. How can such a mechanism reflect an individual’s free sober choice? Even if free from the past, effective freedom of choice is not a real choice of the individual, and the essence of the concept is lost. Actually it is quite difficult to think of any theoretical model which would allow such a thing. Now the question should be asked, whether we believe in the concept of free will strongly enough to demand such a theory. If so, we would like the deviation required from our current physical theory to be minimal. It may be that our interpretation could meet these demands. We have mentioned that effective freedom of choice is due to the probabilistic structure which is embodied in the final boundary condition in our interpretation. If we assume that for some reason the states of this boundary condition are somehow compatible with the volitional choices of Human beings, we can achieve a mechanism for essential free will. Recalling that the data in the final boundary condition are inaccessible, we are self consistent as in the example above. Alternatively one may choose to look at things in the following manner. The complete
description of any system encompasses both a history vector and a destiny vector, making the future also part of the system. Discarding its past and demanding the choice to be made by the system, one is left only with its future, to determine the choice. This might suggest that a new conception of time, should be adopted: one in which our treatment of time is more complex than the usual one-dimensional notion. Such an approach might be necessary when one attempts to describe the logical picture of events, since the cause for actions of systems comes from both their past and their future, while the effect occurs at some present time. Perhaps the words of T. S. Eliot in “Four Quartets” are suitable for this picture:

    Time past and time future
    What might have been and what has been
    Point to one end, which is always present.
Chapter 6

Conclusion

6.1 Discussion

It is interesting to compare our suggested interpretation to similar interpretations of quantum mechanics and to related philosophical ideas in general. In Section 4.2 we have already made some comparisons to the many worlds interpretation.

The idea of applying quantum mechanics to the whole universe and discussing its wave-function, and adequate boundary conditions, was introduced by Hartle and Hawking [26].

The two-state formalism may be extended to a formalism of multiple time boundaries [18]. Also, consistent history interpretations [27] [28] are based on a history of projections. As we have shown, such an approach is unnecessary in order to solve the measurement problem.

In the course of this work we have mainly used the formalism of two-states for its formal convenience, although as stated before, one may look at the complete state of the system as constituting the regular history vector, and of a destiny vector which determines the “fate” of the system. This recalls the ideas of the philosopher Henry Bergson, of systems having some internal tendency, or inner motive, “élan vital”, towards a certain destiny.

We have not yet considered the question of the role of the quantum mechanical measurement process in determining the arrow of time. We rely on decoherence as the cause of irreversible singling of the classical basis of states for classical systems, which lasts until the late end of the universe. Only then do we allow a final boundary condition which causes effective reduction. Hence we do not increase the measure of irreversibility beyond that created by the thermodynamic arrow of time (which itself is usually assumed to be a consequence of the cosmological arrow of time). Therefore it seems
that in our interpretation there is no microscopic quantum mechanical reason for irreversibility, nor does our choice of the special final boundary condition increase the irreversibility determined by the initial condition of the universe.

6.2 Summary

We have suggested a new interpretation of quantum mechanics, the *Teleological Interpretation*. We have shown that a special final boundary condition may be chosen, one which causes effective reduction consistent with the predictions of standard quantum mechanics, thus solving the measurement problem. Because the deviation taken from the conventional theory is minimal, we dare to state that by Occam’s principle, it may well be that such a final boundary condition indeed exists for our universe.

We think that the suggested interpretation may answer some of the problems and gaps left by previous interpretations of quantum mechanics. We hope that this work will be the beginning of an adequate response to the many articles discussing the time-symmetric formulation of quantum mechanics, which end with the statement, that the formulation may lead to a new interpretation of quantum mechanics.

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Appendix A

Derivation of Discussed States

The following trivial derivations are given for the completeness of this work. We use the identities:

\[
\exp (i c \sigma_z) = \begin{pmatrix} \exp (i c) & 0 \\ 0 & \exp (-i c) \end{pmatrix},
\]

\[
\exp (i c \sigma_y) = \begin{pmatrix} \cos (c) & \sin (c) \\ -\sin (c) & \cos (c) \end{pmatrix},
\]

where \(c\) is a constant, \(\sigma_y\) and \(\sigma_z\) are Pauli matrices. The designations \((s)\), \((a)\) and \((c)\) stand for system, apparatus and environment respectively. \(g\) and \(g_k\) are constants. Derivation of (2.4):

\[H_{sa} = -g \sigma_z^{(s)} \otimes \sigma_y^{(a)}.\]

\[
|\Psi_{sa}^{i}(0)\rangle = \frac{1}{\sqrt{2}} (a| \uparrow \rangle + b| \downarrow \rangle) \otimes (|U\rangle + |D\rangle).
\]

\[
|\Psi_{sa}^{i}(0 \leq t \leq t_1)\rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} \exp (i g \sigma_y^{(a)} t / \hbar) & 0 \\ 0 & \exp (-i g \sigma_y^{(a)} t / \hbar) \end{pmatrix}_{s} \begin{pmatrix} a \\ b \end{pmatrix}_{s} \otimes
\]

\[
\otimes (|U\rangle + |D\rangle) = \frac{1}{\sqrt{2}} a| \uparrow \rangle \otimes \begin{pmatrix} \cos (g t / \hbar) & \sin (g t / \hbar) \\ -\sin (g t / \hbar) & \cos (g t / \hbar) \end{pmatrix}_{a} \begin{pmatrix} 1 \\ 1 \end{pmatrix}_{a} +
\]

\[
+ \frac{1}{\sqrt{2}} b| \downarrow \rangle \otimes \begin{pmatrix} \cos (g t / \hbar) & -\sin (g t / \hbar) \\ \sin (g t / \hbar) & \cos (g t / \hbar) \end{pmatrix}_{a} \begin{pmatrix} 1 \\ 1 \end{pmatrix}_{a} =
\]

\[
= \frac{1}{\sqrt{2}} a| \uparrow \rangle \otimes ((\cos (g t / \hbar) + \sin (g t / \hbar)) |U\rangle + (\cos (g t / \hbar) - \sin (g t / \hbar)) |D\rangle) +
\]

\[
+ \frac{1}{\sqrt{2}} b| \downarrow \rangle \otimes ((\cos (g t / \hbar) - \sin (g t / \hbar)) |U\rangle + (\cos (g t / \hbar) + \sin (g t / \hbar)) |D\rangle).
\]

\[
|\Psi_{sa}^{i}(t_1 = \frac{\pi \hbar}{4 g})\rangle = a| \uparrow \rangle \otimes |U\rangle + b| \downarrow \rangle \otimes |D\rangle.
\]
Derivation of (2.6):

\[ H_{ae} = -\sigma_z^{(a)} \otimes \sum_{k=1}^{N} g_k \sigma_{z,k}^{(e)} \prod_{j \neq k} 1_j. \]

\[ |\Psi^i(t_1)\rangle = (a|\uparrow\rangle \otimes |U\rangle + b|\downarrow\rangle \otimes |D\rangle) \prod_{k=1}^{N} \otimes (\alpha_k|u\rangle_k + \beta_k|d\rangle_k). \]

\[ |\Psi^i(t_1 \leq t)\rangle = \left( a \exp \left( i \sum_{k=1}^{N} g_k \sigma_{z,k}^{(e)} \prod_{j \neq k} 1_j \right) (t - t_1)/\hbar \right) |\uparrow\rangle \otimes |U\rangle + \\
+ b \exp \left( -i \sum_{k=1}^{N} g_k \sigma_{z,k}^{(e)} \prod_{j \neq k} 1_j \right) (t - t_1)/\hbar |\downarrow\rangle \otimes |D\rangle \prod_{k=1}^{N} \otimes (\alpha_k|u\rangle_k + \beta_k|d\rangle_k) \]

\[ = a|\uparrow\rangle \otimes |U\rangle \prod_{k=1}^{N} \otimes (\alpha_k \exp (ig_k(t - t_1)/\hbar)|u\rangle_k + \beta_k \exp (-ig_k(t - t_1)/\hbar)|d\rangle_k) + \\
+ b|\downarrow\rangle \otimes |D\rangle \prod_{k=1}^{N} \otimes (\alpha_k \exp (-ig_k(t - t_1)/\hbar)|u\rangle_k + \beta_k \exp (ig_k(t - t_1)/\hbar)|d\rangle_k). \]
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\footnote{At the LANL physics archive: \url{http://xxx.lanl.gov}}