Physical vacuum behind Higgs mechanism in the Abelian-gauge interaction

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(Dated: August 2, 2022)

The concept of spontaneous symmetry breaking is reconsidered from a new viewpoint. We examined massless fermions and antifermions coupled to the Abelian-gauge field. These fermions are formulated in the path-integral, and the space-time explanation on the raison d’etre of antiparticles by Feynman is applied to them. It is shown that the relationship between massless fermions and antifermions depends on the observer, and as a state with the lowest-possible energy to all observers, physical vacuum that realizes spontaneous symmetry breaking is derived. The fermion mass arises from the condensed field-energy of the gauge field, and the gauge-boson mass arises from the condensed kinetic-energy of the massless Fermi field. The Higgs-like boson appears when a local excitation propagates in this physical vacuum, the mass of which is obtained as the excitation energy. With the effective coupling of this Higgs-like mode to gauge bosons, the total cross section of a reaction through this Higgs-like mode is calculated. Renormalizability of this model is discussed using the inductive method. Since the Higgs Lagrangian is not assumed, the divergence we must renormalize is only the logarithmic divergence, not the quadratic one.

I. INTRODUCTION

The Englert-Brout-Higgs (EBH) mechanism holds the key to understanding the electroweak interaction. Recently, a Higgs-like particle, which has a relatively small mass ($m_H = 125$ GeV), is found. On the nature of the Higgs particle, it has been intensely debated whether it is an elementary particle or a composite one made of other constituents of the system. However, whether the Higgs particle is elementary or composite, it is no doubt that the Glashow-Weinberg-Salam (GWS) model using the EBH model (from now, we abbreviate it as Higgs model) is a simple and successful model with which almost no experimental results conflict up to now.

In this paper, we explore the physical mechanism lying behind the phenomenological aspect of the Higgs model. To avoid the complications due to non-Abelian gauge invariance, we consider as a warmup example an Abelian-gauge interaction up to now.

Let $\phi(x)$ be the Higgs field in the non-chiral manner. The basic Lagrangian density $L_0(x)$ is

$$L_0(x) = -\frac{1}{4} F^{\mu\nu} F_{\mu\nu} + \bar{\phi} (i\gamma^{\mu}\gamma_5) \phi,$$

where $F_{\mu\nu} = \partial_{\mu} B_{\nu} - \partial_{\nu} B_{\mu}$. The operator of massless fermion field is written as

$$\varphi(x) = \frac{1}{\sqrt{V}} \sum_{p,s} \frac{1}{\sqrt{2E_p}} \times \left[ a^s(p) u^s(p) e^{-ipx} + b^s((-p)) \bar{v}^s((-p)) e^{-ipx} \right],$$

$$\bar{\varphi}(x) = \frac{1}{\sqrt{V}} \sum_{p,s} \frac{1}{\sqrt{2E_p}} \times \left[ a^{s\dagger}(p) \bar{u}^s(p) e^{ipx} + b^{s\dagger}(-p) \bar{v}^s(-p) e^{ipx} \right],$$

where the four-component spinors are normalized as

$$\bar{u}^s(p) u^s(p) = -\bar{v}^s(p) v^s(p) = 2\epsilon_p \delta^{sx},$$

$$\bar{u}^s(p) v^s(-p) = -\bar{v}^s(-p) u^s(p) = 0.$$}

The creation and annihilation operators with no dimension obey the anti-commutation rules

$$[a^s(p), a^{s\dagger}(p')] = [b^s(p), b^{s\dagger}(p')] = \delta_{ss'} \delta^{3}(p - p').$$

In the Higgs model, the mass of gauge boson is derived from the following Lagrangian density

$$L_h(x) = [((i\partial_\mu + gB_\mu)(v_h + h))^2 + \mu^2 |v_h + h|^2 - \lambda |v_h + h|^4],$$

Furthermore, the Yukawa interaction ($m_f/v_h) \bar{\varphi} \varphi h$) between the Higgs field $h$ and the massless fermion particle $\varphi$ is introduced. After the Higgs field condenses in the vacuum $|0 h 0| = v_h$, a mass $m_f$ arises in the fermion. The purpose of such a procedure is to give a mass to the fermion in the gauge-invariant manner when it is chirally coupled to the gauge field.

The recent experiments show that the amplitude of the Higgs decay to quarks are proportional to the quark mass $m_f$. This result agrees with the prediction by the Yukawa coupling in the Higgs model. Despite these success, we have not yet understood the meaning behind it.

The Higgs model is a powerful theory in the accessible energy region now. If we regard the Higgs particle as an elementary one, however, this model possesses a strange feature. The Higgs potential $-\mu^2 |h|^2$ plays a double role, as the potential energy to stabilize the vacuum condensate, and as the mass term of the Higgs particle. If we regard it as a phenomenology, it is the simplest assumption. But if we regard it as the fundamental theory, it is questionable that the large-scale property such as the stabilization of the vacuum, and the one-particle property such as the mass of the Higgs boson, are expressed...
by the same term. Furthermore, $\lambda |v_L + h|^4$ is necessary to stabilize the symmetry-broken vacuum, and therefore the quadratic divergence appears inevitably in the perturbation calculation. Hence, the fine tuning to cancel this divergence is necessary. This situation requires us to obtain a deeper understanding of it.

In this paper, using the Abelian-gauge system $L_0(x)$, we derive spontaneous symmetry breaking without using $L_0(x)$, and the Yukawa coupling. In the real world, all fermions we know have their own masses, including neutrinos. Hence, the massless fermion in $L_0(x)$, in which symmetry breaking has not yet occurred, is not a real particle. In dealing with $L_0(x)$, we must reconsider some assumptions usually made for the massive particle.

1. There is no reason to expect that such massless fermions have the properties that the physical law does not necessarily require, such as the excess of fermions over antifermions in the real world, even though the massless fermion after symmetry breaking have these properties.

2. The vacuum is electrically neutral. Hence, when the massless fermion becomes a massive one by symmetry breaking, the massless antifermion must be equally involved in this process.

3. The process of spontaneous symmetry breaking does not depend on the specific inertial-system. Rather, it must be formulated as if it is looked from the spacetime viewpoint, because it is related to the definition of antiparticle.

These views can be formulated in a simple way when we use the path-integral formalism.

In Section 2, using the path-integral formalism, the physical vacuum of massless fermion is derived from the requirement of relativity and superposition principle. In Section 3, the generation of mass of the massless fermion in this physical vacuum is explained. In Section 4, the generation of mass of the gauge boson in the physical vacuum is explained. In Section 5, we derive the Higgs-like boson as a local excitation propagating in the physical vacuum, the mass of which is calculated as an excitation energy. In Section 6, we perform one-loop perturbation calculation for the effective coupling of the Higgs-like mode to the gauge field $B_\mu$, and the self interaction of this mode. In Section 7, as an example of the reaction through this Higgs-like mode, we calculate the total cross section of the pair-annihilation of massive fermion and antifermion to massive gauge bosons. In Section 8, renormalizability of this model is examined. In Section 9, we briefly discuss some generalization of the present model to the electroweak interaction, which will be explained in details in the next paper.

II. A SPACE-TIME VIEW OF SPONTANEOUS SYMMETRY BREAKING

II.1. Possible paths and antiparticle

The mechanism of spontaneous symmetry breaking will be explained in this paper using the path-integral picture. The path-integral framework will be used in an intuitive rather than a formal way. (In what follows, there is no need to regard $\varphi(x)$ in $L_0(x)$ as a Grassmann number.) In this formalism, the propagation of particle is given by a weighted average over all possible classical paths. The classical orbit and an example of possible paths in space-time are shown in Figure 1. The only for the $x$-axis in space. The classical orbit is represented by a solid straight line $AC$, and other possible paths are illustrated as thin solid curves connecting $A$ and $C$. The weight associated for a given set of paths include a factor $\exp \left( i \int_{t_a}^{t_c} \mathcal{L}[\dot{x}(t), x(t)] dt \right) \equiv \exp(iS)$. The total amplitude for traveling from $A$ to $C$ is

$$\int_A^C Dx(t) \exp \left( i \int_{t_a}^{t_c} \mathcal{L}[\dot{x}(t), x(t)] dt \right) \equiv \int_A^C Dx(t) \exp(iS)$$

This total amplitude satisfies relativistic invariance. The one-point-dotted lines represent the light cones relative to $A$, $B_1$ and $B_2$, respectively. The classical orbit from $A$ to $C$ must remain within the absolute future relative to $A$ satisfying $(ct)^2 - x^2 > 0$. However, the possible paths from $A$ to $C$ in $\exp(iS)$ include not only the timelike path, but also the spacelike path, as illustrated for example by the path $AB_1C$ or $AB_2C$. Since the total amplitude in Eq. 8 must be zero outside of the light cone relative to $A$, the variable $x(t)$ in the path integral cannot be confined to the timelike paths. Rather, by cancellation of timelike and spacelike paths, zero outside of the light cone is realized.

When the particle travels at a large velocity, that is, when the de Broglie wavelength of the particle is small in comparison with the characteristic length of a given problem, $\exp(iS)$ coming from the spacelike path will vary periodically with high frequency about zero, and it will practically vanish in the path integral. Only the timelike paths near the classical orbit $AC$ contribute to $\exp(iS)$. However, when the particle travels from $A$ to $C$ at a small velocity and its de Broglie wavelength is large, it implies that the fermion travels in a narrow space, and the interference between possible paths has a significant effect on $\exp(iS)$. In such a case, a comparatively large variation of path from the classical orbit, even if it includes the spacelike path, produces only a small variation in $\exp(iS)$. Hence, in addition to the timelike paths, the spacelike paths will make non negligible contribution to total amplitude.

When we view the propagation of particle in Figure 1 from other inertial systems, how does it look to us? To study it, we consider a path $AB_1$ in Figure 2. A particle with a negative charge and momentum $p$ departs from $x_1$.
FIG. 1. The motion of particle from $A$ to $C$ in space-time. The solid straight line represents a classical orbit. Various thin solid curves represent various possible paths from $A$ to $C$ contributing to the action $S$ in the path integral. The one-point-dotted lines represent the light cones relative to $A$, $B_1$ and $B_2$, respectively.

at $t_1$ and arrives at $x_2$ at $t_2 (> t_1)$ in an inertial system represented by an orthogonal coordinate $(x, t)$. When we observe this motion from another inertial system moving along the $x$-axis at a relative velocity $v$ to the first system, it follows the Lorentz transformation in an oblique and inclined to the $x$-axis at a relative velocity $v/c$ to $B$. The time difference $t_2 - t_1$ between $A$ and $B$ are Lorentz transformed to

$$t_2' - t_1' = \frac{1}{\sqrt{1 - (v/c)^2}} \left[ t_2 - t_1 - \frac{v}{c^2}(x_2 - x_1) \right].$$  \hspace{1cm} (9)

When the particle travels along the timelike path as in Figure 2(a), we will not observe strange phenomenon. One of the remarkable feature of the Lorentz transformation is that when the particle travels along the spacelike path as in Figure 2(b), the order of events is not left invariant. When the velocity $v$ of an observer is sufficiently large relative to the initial frame, or when the spacelike interval between two events is sufficiently large as $c(t_2 - t_1) < (v/c)(x_2 - x_1)$ in Eq. (9), the order of two events separated by the spacelike interval is reversed as $t_2' < t_1'$. A natural interpretation of this observation is that an antiparticle with an opposite charge moves in the opposite direction, that is, from $B$ to $A$. Extending the idea of Stueckelberg on this feature [11], Feynman made an intuitive explanation for the raison d’etre of antiparticle using the virtual processes [12].

II.2. Massless fermion and antifermion

In Figure 2 when the massless fermion travels at a small velocity from $A$ to $C$, not only the timelike but also the spacelike path such as the path $AB_1C$ makes a significant effect on $\exp(iS)$. When we view such a path from another inertial system, how does it look to us? When the spacelike interval between $A$ and $B_1$ is large enough to satisfy $c(t_2 - t_1) < (v/c)(x_2 - x_1)$ (such a $B_1$ always exists near $A$ in Figure 2), two different interpretations of the phenomenon at $A$ are possible. At the starting point $A$ of the timelike path $AC$ in Figure 2 we find the annihilation of massless fermion. and at the end point $A$ of the spacelike path $AB_1$, we find the creation of massless antifermion. As for the path from $A$ to $C$ via $B_2$, the interval between $B_2$ and $C$ is spacelike as well. Hence, at the end point $C$ of the timelike path $AC$, we find the creation of massless fermion, and at the starting point $C$ of the spacelike path $B_2C$, we find the annihilation of massless antifermion. When the massless fermion travels at a large velocity, only the timelike paths near the classical orbit $AC$ contribute to $\exp(iS)$ in the path integral. Hence, the weight of these two interpretations depends on the relative momentum between the observer and the object.

In the real world after symmetry breaking, the influence of antifermion is not so large, because the massive antifermion such as positrons appears only in particular phenomena in the real world. But, when it is before symmetry breaking, the massless fermion and antifermion must be equally dealt with, and these two interpretations at $A$ or $C$ in Figure 2 have equal validity.

To formulate such a relationship between massless fermion and antifermion at $A$ in Figure 2 a new annihilation operator $A^s(p)$ which is a superposition of $a^s(p)$ and $b^s(-p)$

$$A^s(p) = \cos \theta p a^s(p) + \sin \theta p b^s(-p),$$  \hspace{1cm} (10)

is introduced. The ratio $\theta p$ in Eq. (10) that determines the relative importance of antifermion depends on the rel-
ative momentum $p$ between the object and the observer. $A^s(p)$ is reduced to $a^s(p)$ at $p \to \infty$ limit, and therefore $\theta p$ in Eq. (11) must approach zero at $p \to \infty$. Similar interpretation is possible at C in Figure 1 as well. Two different interpretations: the annihilation $b^s(−p)$ of massless antifermion at C, and the creation $a^s(p)$ of massless fermion at C are both possible. (The spin direction of $a^s(p)$ is opposite to that of $b^s(−p)$ with the same $s$. Hence, a similar argument holds for the flow of spin as well.) Hence, a new annihilation operator $B^s(−p)$

$$B^s(−p) = \cos θ pb^s(−p) − \sin θ pa^s(−p),$$

is introduced, which is orthogonal to $A^s(−p)$. (Equations (10) and (11) have the same form as the Bogoliubov transformation in superconductivity, but they have different physical meaning.)

II.3. Physical vacuum of massless fermion and antifermion

The vacuum is a state with the lowest-possible energy to all observers. For the massive object, the free vacuum $|0⟩$ is the starting point of all models. However, for the massless fermion in which symmetry breaking has not yet occurred, the free vacuum $|0⟩$ satisfying $a^s(p)|0⟩ = b^s(−p)|0⟩ = 0$ is a state regarded as a vacuum only by an observer who sees the massless particle moving at a large velocity. For all observers on different inertial-systems, we define a new state $|0⟩$ which satisfies the condition of vacuum of the new annihilation operator $A^s(p)$ and $B^s(−p)$, so that it satisfies $A^s(p)|0⟩ = B^s(−p)|0⟩ = 0$ to all observers. We call such a $|0⟩$ physical vacuum of massless fermion and antifermion.

The explicit form of $|0⟩$ can be guessed as follows. When the momentum of the fermion is large, $\exp(iS)$ is determined, in effect, only by the timelike paths, and $\cos θ p → 1$ is expected at $p → ∞$ in Eqs. (10) and (11). This implies that the physical vacuum becomes the free vacuum, and $|0⟩$ includes $\cos θ p|0⟩$. On the other hand, when $p → 0$, various possible paths extending into the wider region in space-time contribute to the total amplitude. At the point A or C in Figure 1 massless fermion and antifermion are both possible in the interpretation of $A^s(p)$ and $B^s(−p)$. Hence, $\cos θ p$ is expected at $p = 0$ in Eqs. (10) and (11). This suggests that $|0⟩$ includes $\sin θ pb^s(−p)a^s(−p)|0⟩$. The simplest possible form of $|0⟩$ is a superposition of these two limits for each $p$

$$|0⟩ = \prod_{p,s} [\cos θ p + \sin θ p b^s(−p)a^s(−p)] |0⟩,$$

and applying the following expansion

$$e^{-iG} e^{iG} = F + [-iG, F] + \frac{1}{2!} [-iG, [-iG, F]] + \cdots, \quad (14)$$

to the operators $a^s(p)$ and $b^s(−p)$ for $F$, and Eq. (13) for $G$, we rewrite Eqs. (10) and (11) in the following compact form

$$A^s(p) = e^{-iG} a^s(p)e^{iG}, \quad (15)$$
$$B^s(−p) = e^{-iG} b^s(−p)e^{iG}. \quad (16)$$

The vacuum $|0⟩$ in $A^s(p)|0⟩ = B^s(−p)|0⟩ = 0$ is expressed as

$$|0⟩ = e^{-iG}|0⟩ = \exp \left( \sum_{p,s} \theta p [b^s(−p)a^s(−p) − a^s(−p)b^s(−p)] \right) |0⟩,$$

$$= \prod_{p,s} \left[ \sum_n \frac{1}{n!} \theta^n p [b^s(−p)a^s(−p) − a^s(−p)b^s(−p)] \right] |0⟩.$$

(17)

Since massless fermions and antifermions obey Fermi statistics, only a single particle can occupy each state on the hyperboloid ($p^2 = 0$) set in $|0⟩$ at each point in space-time, and we obtain for each $p$

$$\sum_n \frac{\theta^n}{n!} (b^s a^s − ab^s)a^s |0⟩ = |0⟩ + \theta b^s a^s |0⟩ − \frac{\theta^2}{2!} b^s a^s b^s a^s |0⟩ − \frac{\theta^3}{3!} b^s a^s b^s a^s b^s a^s |0⟩ + \cdots. \quad (18)$$

In this expansion, $\cos θ p$ appears from the sum of even-order terms of $θ$, and $\sin θ p$ appears from the sum of odd-order terms, and then Eq. (12) is yielded.

(1) This $|0⟩$ has the same structure as that of Nambu and Jona-Lasinio on analogy with superconductivity [5], but this $|0⟩$ does not come from the attractive interaction. There is the following reason for deriving $|0⟩$ kinematically. In superconductivity, the BCS ground state is expressed using the relative momentum between the electron and the center-of-mass of the crystal [17]. If a naive analogy between the BCS ground state and the physical vacuum was made, the center-of-mass of the crystal would be replaced by the center-of-mass of the universe. Such an absolute rest-frame is not appropriate to the relativistic model, and the vacuum $|0⟩$ must be derived using the relative motion of the particle to the observer.

(2) When the operators in $L_0(x)$ act on $|0⟩$, the massless fermion and antifermion annihilate to a gauge boson in $B^μ$, and this massless boson annihilates to another massless fermion-antifermion pair. There is no threshold energy in such a $s$-channel process between massless objects. Hence, this $|0⟩$ is an equilibrium state between the massless fermion-antifermion pairs and gauge bosons.
III. MASSIVE FERMIONS

The physical vacuum in Eq. (20) is a consequence of general properties. However, the specific form of $\sin \theta p$ is determined by the interaction in $L_0(x)$. In this physical vacuum $|\tilde{0}\rangle$, $\varphi(x)$ is coupled not only to $B_\mu$, but also to the gauge field included in $\tilde{\beta}$ in Eq. (19). Hence, the condensed field-energy $\tilde{\beta}$ acts on the massless fermion as if it is a mean field as follows

$$-\frac{1}{4}F_{\mu\nu}F_{\mu\nu} + \varphi(x) \left( i\partial + g \left[ \tilde{\beta} + B_\mu \gamma^\mu \right] \right) \varphi(x), \quad (21)$$

This mean field $\tilde{\beta}$ is an average over many degrees of freedom, and therefore it does not easily change through the creation or annihilation of each fermion. Hence, we approximate $\tilde{\beta}$ as a constant $\langle \beta \rangle$. The substitution of Eqs. (2) and (3) into Eq. (21) yields

$$\frac{1}{2} \sum_{p,s} [\alpha^s(p) a^s(p) + b^s(-p) b^s(-p)] + \frac{1}{2} g(\tilde{\beta}) \sum_{p,s} [\alpha^s(p) b^s(-p) + b^s(-p) a^s(p)], \quad (22)$$

plus $-\frac{1}{4}F_{\mu\nu}F_{\mu\nu}$ and the coupling term to $B_\mu$. By inverting Eqs. (10) and (11), we find

$$a^s(p) = \cos \theta p A^s(p) - \sin \theta p B^s(-p), \quad (23)$$

$$b^s(-p) = \cos \theta p B^s(-p) + \sin \theta p A^s(p). \quad (24)$$

Using these $a^s(p)$ and $b^s(-p)$ in Eq. (22), we obtain

$$\frac{1}{2} \sum_{p,s} [\epsilon_p (\cos^2 \theta p - \sin^2 \theta p) - 2g(\tilde{\beta}) \cos \theta p \sin \theta p]$$

$$\times [A^s(p) A^s(p) + B^s(-p) B^s(-p)]$$

$$+ \frac{1}{2} \sum_{p,s} [2\epsilon_p \cos \theta p \sin \theta p + g(\tilde{\beta}) (\cos^2 \theta p - \sin^2 \theta p)]$$

$$\times [A^s(p) B^s(-p) + B^s(-p) A^s(p)]. \quad (25)$$

The stability of the physical vacuum $|\tilde{0}\rangle$ requires that the coefficient of $A^s(p) B^s(-p) + B^s(-p) A^s(p)$ vanishes, with the result that

$$\cos \theta p = \frac{1}{2} \left( 1 + \frac{\epsilon_p}{\sqrt{\epsilon_p^2 + (g(\tilde{\beta}))^2}} \right), \quad (26)$$

$$\sin^2 \theta p = \frac{1}{2} \left( 1 - \frac{\epsilon_p}{\sqrt{\epsilon_p^2 + (g(\tilde{\beta}))^2}} \right), \quad (27)$$

which satisfies $\cos \theta p \to 1$ at $p \to \infty$, and $\cos \theta p = \sin \theta p$ at $p = 0$ as expected. Substitution of Eqs. (26) and (27) into the first term of Eq. (25) yields

$$\frac{1}{V} \sum_{p,s} \sqrt{\epsilon_p^2 + (g(\tilde{\beta}))^2} [A^s(p) A^s(p) + B^s(-p) B^s(-p)]. \quad (28)$$

The mass $m_f$ of the real fermion is defined by the condensed field-energy of the massless $B_\mu$

$$m_f = g(\tilde{\beta}) = g(|\tilde{0}\rangle) \int T^{00}(x) d^3x |\tilde{0}\rangle. \quad (29)$$

The above derivation is irrelevant to whether the effective interaction between massless fermions $\varphi(x)$ is an attractive or a repulsive one $\tilde{\beta}$.

The factors represented by Eqs. (26) and (27) has the following meaning.

(1) When the momentum of the massive particle increases, it becomes difficult to distinguish the massive particle from the massless one. The factor $\sin \theta p$ in Eq. (27) is a decreasing function of the momentum $p$. Hence, $A^s(p)$ and $B^s(-p)$ in Eqs. (10) and (11) changes to $a^s(p)$ and $b^s(-p)$ as $p \to \infty$, which is consistent to the explanation using Figure 1.

(2) When the momentum of particle is large, its vacuum approaches $|\tilde{0}\rangle$ in the physical vacuum. When its momentum is small, its vacuum approaches $(1/\sqrt{2})(1 + b^s \bar{a}^s)|\tilde{0}\rangle$. The physical vacuum $|\tilde{0}\rangle$ with Eqs. (26) and (27) is consistent to this picture.

With the transition of vacuum from $|0\rangle$ to $|\tilde{0}\rangle$, new four-component spinors $\bar{U}^s(p)$ and $V^s(p)$ satisfying

$$\bar{U}^s(p) U^s(p) = -V^s(p) V^s(p) = 2 \sqrt{\epsilon_p^2 + m_f^2} \delta^s. \quad (30)$$
are defined. Hence, new field operators $\psi(x)$ and $\bar{\psi}(x)$ are defined as

$$
\psi(x) = \frac{1}{\sqrt{V}} \sum_{p,s} \frac{1}{\sqrt{2E_p}} e^{-ipx} A^s(p) \bar{U}^s(p),
$$

$$
\bar{\psi}(x) = \frac{1}{\sqrt{V}} \sum_{p,s} \frac{1}{\sqrt{2E_p}} e^{ipx} B^s(-p) V^s(-p),
$$

where $A^s(p)$ and $B^s(-p)$ are the massless gauge boson fields.

The relations between the old $u(p), v(p)$ and the new $U(p), V(p)$ are given by

$$
\bar{U}^T(p) u^T(p) = \bar{V}^T(p) v^T(p) = \cos \theta_p,
$$

$$
\bar{U}^T(p) v^T(p) = -\bar{V}^T(p) u^T(p) = \sin \theta_p.
$$

In place of $L_0(x)$, a Lagrangian density expressed by $\bar{\psi}(x)$ is needed. We obtain such a $L_1(x)$ as

$$
L_1(x) = \frac{1}{4} F_{\mu\nu} F^{\mu\nu} + \bar{\psi}(i\partial_\mu + gB_\mu)\gamma^\mu \psi - m_f \bar{\psi}\psi.
$$

In place of $A^s$ and $B^s$ in $\Phi(x), A^s \Phi$ and $B^s \Phi$ in $\psi(x)$ are directly coupled to $B_\mu$, hence, $g\bar{\psi}B_\mu\gamma^\mu \psi$ appears. This $L_1(x)$ has the same form as that of QED, and the physical vacuum $|0\rangle$ exists as if it is the free vacuum of massive fermion $\psi(x)$. However, this Lagrangian density includes qualitatively different phenomena, such as the massive gauge boson and Higgs-like mode.

**IV. MASSIVE GAUGE Boson**

In the Higgs model, the mass of gauge boson is derived from $((i\partial_\mu + gB_\mu)(i\partial_\mu + h))$ in $L_0(x)$. In the present model, instead of such a phenomenological coupling, we derive the massive gauge boson from $L_0(x)$ acting on the physical vacuum $|0\rangle$.

In $L_0(x)$, the interaction of the massless gauge boson in $B_\mu(x)$ with the massless fermion $\phi(x)$ is as follows

$$
\mathcal{H}_I(x) = g\bar{\phi}(x)\gamma^\mu \phi(x)B_\mu(x) = g j^\mu(x) B_\mu(x).
$$

Since the physical vacuum is not a simple system, the response of $|0\rangle$ to $B_\mu$ causes a non-linear behavior of $B_\mu$. Owing to this $\mathcal{H}_I(x)$, the kinematic term $L_0^{\text{kin}}(x) = \bar{\phi}(x)i\partial_\mu + gB_\mu)\gamma^\mu \phi(x)$ itself changes its form as follows. Let us consider a perturbation expansion of $L_0^{\text{kin}}(x)$ in powers of $g$ in $\mathcal{H}_I(x)$

$$
\langle 0 | \int d^4x L_0^{\text{kin}}(x)|0\rangle
$$

$$
= \langle 0 | T \left[ \int d^4x L_0^{\text{kin}}(x) \right] |0\rangle
$$

$$
= \langle 0 | T \left[ \int d^4x j^\mu(x) B_\mu(x) \right] |0\rangle
$$

$$
= \langle 0 | T \left[ \int d^4x j^\mu(x) B_\mu(x) \right] |0\rangle
$$

$$
+ \langle 0 | T \left[ \int d^4x L_0^{\text{kin}}(x) i g \int d^4x J^\nu(x) B_\nu(x) \right] |0\rangle
$$

$$
+ \cdots,
$$

where $|0\rangle_p$ in the first line represents a vacuum $|0\rangle$ perturbed by $\mathcal{H}_I(x)$. The structure of the physical vacuum comes, not from $\mathcal{H}_I(x)$, but from the general properties, and therefore the perturbation of $|0\rangle$ by $\mathcal{H}_I(x)$ is not a double counting.

**IV.1. Mass of the gauge boson**

In the last line of Eq. (37), the second-order term of $B_\mu$ is the following two-point-correlation function

$$
\langle 0 | T \left[ \int d^4x_1 \mathcal{H}_I(x_1) \int d^4x_2 \mathcal{H}_I(x_2) \right] |0\rangle.
$$

This correlation implies that a massless fermion-antifermion pair $\bar{\phi}(x_1)\gamma^\mu \phi(x_1)$ annihilates to $B_\mu$ through Eq. (21), and this $B_\mu$ in turn annihilates to another massless fermion-antifermion pairs $\bar{\phi}(x_2)\gamma^\mu \phi(x_2)$. As a result, the correlation between $\bar{\phi}(x_1)\gamma^\mu \phi(x_1)$ and $\bar{\phi}(x_2)\gamma^\mu \phi(x_2)$ appears in Eq. (38) when $\mu = \nu$. To obtain the coefficient of $B_\mu(x_1)B_\mu(x_2)$, we transform Eq. (38) as follows

$$
g^2 \langle 0 | T \left[ \int d^4x_1 j^\mu(x_1) d^2x_1 \int d^4x_2 j^\mu(x_2) d^2x_2 \right] |0\rangle
$$

$$
	imes B_\mu(x_1)B_\mu(x_2) d^2x_1 d^2x_2
$$

$$
= g^2 \langle 0 | T \left[ \int \partial_\mu j^\mu(x_1) d^4x_1 \int \partial_\nu j^\nu(x_2) d^4x_2 \right] |0\rangle
$$

$$
	imes B_\mu(x_1)B_\mu(x_2) d^2x_1 d^2x_2.
$$

These $x_1$ and $x_2$ are microscopically separated points in space-time, and if we observe this phenomenon from a far distant point, it looks like a local phenomenon at $X = (x_1 + x_2)/2$, and the relative motion corresponding to $x_2 - x_1$ is not directly observed, but contained in the coefficient corresponding to a mass. To such an observer, it is appropriate to assume $x_1 \to X$ and $x_2 \to X$ in $B_\mu(x)$, and the gauge boson behaves as a mass boson as

$$
m_B^2 = \int B_\mu(X) B_\mu(X) d^4X.
$$

The mass $m_B$ is given by the following two-point-correlation function

$$
m_B = g \left[ \langle 0 | T \left( \frac{1}{2} \int \partial_\mu j^\mu(x_1) d^4x_1 \int \partial_\nu j^\nu(x_2) d^4x_2 \right) |0\rangle \right]^{1/2}.
$$

where $\frac{1}{2}$ comes from the time-ordered product in Eq. (39). Since the physical vacuum $|0\rangle$ is filled with the kinetic energy of massless fermion, the right-hand side of Eq. (41)
IV.2. Goldstone mode

The Goldstone mode comes from the phase factor of the condensate. In the Higgs model, the phase $\theta$ is assumed in the vacuum condensate $|\nu_h\rangle = \exp(i\theta)$. The present model gives it a concrete meaning as the phase $\alpha(x)$ in the physical vacuum $|\bar{0}\rangle$.

(1) In the last line of Eq. (57), there is a first-order term of $B_\nu(x_2)$. Let us partially integrate this term over $x_1$ in $\bar{\phi}(x_1)i\partial^\mu\gamma_\mu\phi(x_1)$ included in $L_0^{kin}(x_1)$, in which $\phi(x_1)$ vanishes at $x_1 \to \infty$. The physical vacuum $|\bar{0}\rangle$ in Eq. (20) has an explicit $x$-dependence in the phase $\alpha(x)$. As a result of partial integration, we obtain a term including $i\partial^\mu\bar{\phi}(x_1)\gamma_\mu\phi(x_1)$, and that including $\partial\alpha(\bar{0})$. The latter term is given by

$$g(\bar{0})T\left[\int d^4x_1\bar{j}_\mu(x_1)\int d^2x_2j_\nu(x_2)B_\nu(x_2)\right]\partial_\mu(\bar{0})$$

$$+g\partial_\mu(\bar{0})T\left[\int d^4x_1\bar{j}_\mu(x_1)\int d^2x_2j_\nu(x_2)B_\nu(x_2)\right]|\bar{0}\rangle.$$  

(42)

in which $\partial_\mu(\bar{0})$ is a product of $\partial_\mu\alpha(x)$ and $\prod_{p,A}\sin\theta p e^{i\alpha(x)}h^A(-p)\gamma^A(p)|\bar{0}\rangle$.

To an observer at a distant point in space-time, it is appropriate to rewrite $d^4x_1d^4x_2$ using the variable $X = (x_1 + x_2)/2$. Hence, we obtain $d^4x_1d^4x_2 \to \partial^\mu d^4x_1\partial^\nu d^4x_2 \to \partial^\mu\gamma^\nu\partial^\mu\gamma^\nu X$. With the definition of $m_B$ in Eq. (11), we transform Eq. (12) in $\mu = \nu$ as

$$\frac{2i}{g}m_B^2 \int B_\mu(X)\partial^\mu\alpha(X)d^4X = \equiv m_B \int B_\mu(X)\partial^\mu\phi(x)d^4X,$$

where the Goldstone mode $\phi(X)$ is defined as $\phi(X) = 2ig^{-1}m_B\alpha(X)$. Since the global rotation of phase requires no energy, the propagator of the Goldstone mode is given by

$$F(q^2) = \int \frac{d^4X}{(2\pi)^4} \langle \bar{0}|T[\phi(X)\phi(0)]|\bar{0}\rangle e^{iqx} = \frac{i}{q^2}.$$  

(44)

The Fourier transform of Eqs. (40) and (43) are given by $m_B^2B^\mu(q)B_\mu(q)$ and $m_Bq^\mu\phi(q)B_\mu(q)$, respectively. Regarding the latter as a perturbation to the former, its second-order perturbation is obtained

$$B^\mu(q)[im_B^2g^{\mu\nu} - m_Bq^\mu F(q^2)m_Bq^\nu] B_\nu(q)$$

$$= im_B^2 \left( g^{\mu\nu} - \frac{q^\mu q^\nu}{q^2} \right) B^\mu(q)B^\nu(q).$$  

(45)

Adding this term to the Fourier transform of $-\frac{i}{4}F^{\mu\nu}F_{\mu\nu}$, we obtain

$$iB^\mu(q)[q^2 - m_B^2] \left( g^{\mu\nu} - \frac{q^\mu q^\nu}{q^2} \right) B^\nu(q).$$  

(46)

The inversion of this matrix yields

$$D^{\mu\nu}(q) = \frac{-i}{q^2 - m_B^2} \left( g^{\mu\nu} - \frac{q^\mu q^\nu}{q^2} \right)$$

$$\equiv iD(q^2) \left( g^{\mu\nu} - \frac{q^\mu q^\nu}{q^2} \right),$$  

(47)

which is the propagator of the massive gauge boson in the Landau gauge [10]. The massless gauge boson becomes massive by eating the phase of the condensed fermion-antifermion pairs in the physical vacuum.

(2) The Goldstone mode directly couples to fermions. There is a zeroth-order term of $B_\mu$ in the third line of Eq. (57). We partially integrate it over $x_1$, and obtain a term including $i\partial^\mu\bar{\phi}(x_1)\gamma_\mu\phi(x_1)$, and that including $\partial\alpha(\bar{0})$. The latter term is given by

$$i(\bar{0}) \int d^4x_1j_\mu(x_1)\partial_\mu(\bar{0}) + i\partial_\mu(\bar{0}) \int d^4x_1j_\mu(x_1)|\bar{0}\rangle.$$  

(48)

Expressing $\partial_\mu(\bar{0})$ in terms of $\partial_\mu\alpha(x)$, and with the definition $\phi(x) = 2ig^{-1}m_B\alpha(x)$, we rewrite it as

$$\frac{g}{m_B}(\bar{0}) \int d^4x_1\gamma^\mu\phi(x)\partial_\mu(\bar{0})\phi(x)|\bar{0}\rangle.$$  

(49)

As a result, an additional term $gm_B^{-1}\partial_\mu\phi(x)\bar{\phi}(x)\gamma^\mu\phi(x)$ appears. This coupling to fermions is different from $g(m_1/m_B)\phi(x)\bar{\phi}(x)\phi(x)$ in the Higgs model, because it is not derived from the Yukawa coupling $(m_f/v_h)|\psi_i\rangle + h(x) + i\phi(x)|\bar{\phi}(x)\phi(x)$, but from the response of the physical vacuum.

In place of $L_1(x)$, a new Lagrangian density describing the massive gauge boson is needed. We obtain such a $L_2(x)$ including Eqs. (10), (13) and (19)

$$L_2(x) = -\frac{1}{4}F^{\mu\nu}F_{\mu\nu} + m_B^2B^\mu B_\mu + m_B\partial_\mu\phi B_\mu$$

$$+ \frac{g}{m_B}\bar{\psi}\gamma^\mu\psi\partial_\mu\phi + \bar{\psi}(i\partial_\mu + gB_\mu)\gamma^\mu\psi - m_f\bar{\psi}\psi.$$  

(50)

V. HIGGS-LIKE BOSON AS A LOCAL EXCITATION PROPAGATING IN VACUUM

The coupling of the massless fermions $\bar{\phi}(x)$ to the mean field $\tilde{\beta}$ in Eq. (21) is accompanied by the creation of a local excitation $\beta(x)$ from $\tilde{\beta}$ through the annihilation of these fermion and antifermion at $x$. This excitation then annihilate to another massless fermion-antifermion pair at $x'$. Hence, it propagates through a chain of pair creations and annihilations of massless fermion-antifermion
pairs as illustrated in Figure 3. Since this mode is a local excitation from the physical vacuum, it plays a similar role to the Higgs particle that causes a deviation from the minimum point of the Higgs potential, and therefore we can call it a Higgs-like excitation mode \( H_β(x) \). The excitation does not have any specific direction in space, and therefore it is appropriate to regard \( H_β(x) \) as a scalar field.

(1) The static field \( β \) determines the fermion mass as \( m_f = g(β) \), and therefore the propagating field \( H_β(x) \) of the local excitation \( β(x) \) is also coupled to the massless fermion with a strength proportional to \( m_f \).

(2) For the coupling of \( ϕ \) to \( H_β(x) \), a dimensionless coupling constant \( m_f/M \) is necessary, where \( M \) is a constant with a dimension of mass, and we assume

\[
\mathcal{V}_I(x) = \frac{m_f}{M} [\bar{ϕ}_L(x)ϕ_R(x)H_β(x) + \bar{ϕ}_R(x)ϕ_L(x)H_β^*(x)].
\]  

This \( \mathcal{V}_I(x) \) is not directly derived from \( L_0(x) \), but derived physically. (At first sight, this \( \mathcal{V}_I(x) \) looks like the Yukawa coupling, but it contributes neither to the mass generation of the fermion nor to its coupling to the Goldstone mode.) The reason why the mass of the Higgs particle has been an unknown parameter in the electroweak model is that it is not a quantity which we can guess using symmetry principle, but a result of the many-body phenomenon.

Figure 4 shows the interaction between the massless fermions by the exchange of the Higgs-like mode as

\[
\bar{ϕ}(k)ϕ(k + q) \left( \frac{m_f}{M} \right) \frac{1}{q^2[1 - J(q^2)]} \frac{m_f}{M} \bar{ϕ}(p)ϕ(p + q),
\]  

where \( J(q^2) \) represents the creation and annihilation of the fermion-antifermion pair

\[
q^2J(q^2) = \frac{m_f}{M} \int_0^Λ \frac{d^4p}{(2π)^4}\text{Tr} \left[ \frac{i}{p - m_f} \frac{i}{p + q - m_f} \right].
\]  

This \( J(q^2) \) has following features.

(1) Although the fermion \( ϕ(x) \) in the bubble diagram is massless, its excitation from \( |0⟩ \) can be simply written by the massive fermion operator \( ψ(x) \). The resulting \( J(q^2) \) has a similar form to the vacuum polarization in QED, but an important difference is that \( γμ \) and \( γ^ν \) are absent in the trace.

(2) \( Λ \) in Eq. (53) is not a cutoff for the Pauli–Villars regularization of the divergent integral, but an upper end of energy-momentum of massless fermion-antifermion pairs in the excitation. We can calculate Eq. (53) as if \( Λ \) is such a cutoff for regularization. But, since the upper end \( Λ \) is a dynamical variable, it is evaluated, not as \( p_F^2 \) in the Euclidian space, but as \( p^2 = (ip_0)^2 - p^2 = -p_F^2 \) in the Minkowski space. Hence, after a 4-momentum integration, the square of upper end appears as \( -Λ^2 \) in the result.

Using the ordinary rule for diagrams, we obtain

\[
J(q^2) = \frac{1}{4π^2} \frac{m_f}{M} \times \int_0^1 dx \left[ x(x - 1) + \frac{m_f^2}{q^2} \right] \ln \left( \frac{x(1 - x)Λ^2 + m_f^2}{x(x - 1)q^2 + m_f^2} \right).
\]  

A peculiar feature of this \( J(q^2) \) is that \( m_f^2/q^2 \) appears in the integrand. With this \( J(q^2) \), the propagator of the Higgs-like excitation mode \( H_β(x) \) is given by

\[
\int \frac{d^4x}{(2π)^4} ⟨0|TH_β(x)H_β(0)|0⟩e^{iqx} = \frac{1}{q^2[1 - J(q^2)]}. \tag{55}
\]

Figure 4 shows \( 1/[q^2(1 - J(q^2))] \) with parameters \( m_f/M = 0.64 \) and \( Λ/m_f = 800 \) as an example. A single pole at \( q^2 = 0.42 \) appears as if it is represented by

\[
\frac{1}{q^2 - m_H^2}, \tag{56}
\]

with \( m_H = 0.42m_f \). The reason for the pole to appear is the existence of \( m_f^2/q^2 \) in the integrand of \( J(q^2) \) in Eq. (54). For this \( m_f^2/q^2 \) to appear, the absence of \( γ \) matrix in the interaction energy density \( \mathcal{H}_I(x) \) in Eq. (51).
plays an important role. If there is $\gamma^\mu$ and $\gamma^\nu$ in the trace of Eq. (53), such a $m_f^2/q^2$ disappears, and then the pole structure is not yielded in Eq. (55) [17].

The Higgs-like boson's mass $m_H$ is strongly depends on the parameter $M$ and $\Lambda$. Figure 5 shows $m_H/m_f$, obtained by solving $J(m_H^2) = 1$, as a function of $m_f$ and $\Lambda/m_f$. As these variables increase, it takes much energy $m_H$ to excite the Higgs-like collective mode.

This Higgs-like mode is unstable with respect to the decay into fermion-antifermion pairs at $q^2 > (2m_f)^2$. In Eq. (54), the logarithmic function includes $x(x-1)q^2 + m_f^2$ in the denominator, in which $x = x(1)$ is at most $-1/4$ at $x = 1/2$. Hence, $x(x-1) + m_f^2/q^2$ becomes negative at $q^2 > (2m_f)^2$, which leads to an imaginary energy in Eq. (54). For any fixed $q^2$ at $q^2 > (2m_f)^2$, the $x$-value that can contribute to the imaginary energy in Eq. (54) satisfies $x(x-1)q^2 + m_f^2 < 0$, which lies in a region between the points $x = 1/2 \pm \frac{1}{2}\delta$, where $\delta = \sqrt{1-4m_f^2/q^2}$.

Using $Im[-X \pm i\epsilon] = \pm \pi$, and $y = x - \frac{1}{2}$, we obtain the imaginary part

\[ - \frac{1}{4\pi^2 M} m_f^2 (\mp \pi) \int_{(1-\delta)/2}^{(1+\delta)/2} dx \left[ x(x-1) + \frac{m_f^2}{q^2} \right] \]

\[ = \pm \frac{1}{4\pi^2 M} m_f \int_{-\delta/2}^{\delta/2} dy \left[ (y^2 - 1) + \frac{m_f^2}{q^2} \right]. \]  

Finally, we obtain the propagator of $H_\beta(x)$ at $q^2 > (2m_f)^2$

\[ \int \frac{d^4x}{(2\pi)^4} \langle 0 | TH_\beta(x)H_\beta(0) | 0 \rangle e^{iqx} \]

\[ = \frac{1}{q^2 - m_H^2 \pm i \frac{m_f}{24\pi M} \sqrt{1 - \frac{4m_f^2}{q^2}(q^2 - 4m_f^2)}}. \]  

in which $m_H^2$ is used for the real part of the logarithm function in Eq. (54). The imaginary part of the self energy increases with increasing $q^2$, and finally the excitation mode becomes unstable. For the electroweak interaction, however, we know $m_W = 125$ GeV $< 2m_f = 346$ GeV. Since $m_H < 2m_f$, the structure of the pole-mass around $q^2 = m_H^2$ is not affected by the onset of damping at $q^2 > (2m_f)^2$.

In place of $L_2(x)$, the final Lagrangian density including the Higgs-like excitation mode is needed. We obtain such a $L_3(x)$ as

\[ L_3(x) = -\frac{1}{4} F^{\mu\nu} F_{\mu\nu} + m_f^2 B^\mu B_\mu + m_B B_\mu \partial_\mu \phi \]

\[ + \bar{\psi} (i\partial_\mu + g B_\mu) \gamma^\mu \psi - m_f \bar{\psi} \psi \]

\[ + \frac{g}{m_B} \bar{\psi} \gamma^\mu \psi \partial_\mu \phi \]

\[ + |\partial_\mu H_\beta|^2 - m_H^2 H_\beta^\dagger H_\beta + \left( \frac{m_f}{M} \right) \bar{\psi} \psi H_\beta. \]  

VI. EFFECTIVE COUPLING OF THE HIGGS-LIKE MODE

Recently the decay of the Higgs-like particle to two gauge bosons $W^+ + W^-$ are observed [1][2]. In the Higgs model applied to $B_H$, the coupling responsible for such a decay is simply derived from $\langle (i\partial_\nu + g B_\nu)(v_h + h) \rangle^2$. As a result, the following direct coupling

\[ g^2 v_h g^{\mu\nu} \times B_\mu(p) B_\nu(k) h(p + k), \]  

appears in the tree processes. Furthermore, by minimizing the Higgs potential $-\mu^2 |v_h + h|^2 + \lambda |v_h + h|^4$ with respect to $v_h$, the direct self-coupling of the Higgs particle

\[ \lambda v_h \times h(p) h(k) h(p + k), \]  

appear at $v_h = \sqrt{\mu^2/2\lambda}$. The quadratic divergence comes from Eqs. (60) and (61).

In the present model, the effective coupling of the Higgs-like mode $H_\beta(x)$ comes from one-loop processes illustrated in Figure 6. As a warmup example, we derive such an effective coupling in the case of $U(1)$ gauge field, and use it in the calculation of the cross section in Section 7. The result in Section 6 and 7 is obtained using the computing algorithm Package-X [18]. The future precise measurement of the electroweak interaction will determine whether the deviation from the simple Higgs model exists or not.
VI.1. Coupling to massive gauge bosons

The effective coupling term responsible for the \( H_\beta \) decay into gauge bosons in Figure 6(a) is composed of \(- (m_f/M) \bar{\psi} \psi H_\beta \) and \( \bar{\psi} B_\mu \gamma^\mu \psi \) in Eq. (59). In coordinate space, such a coupling takes a form

\[
g^2 \frac{m_f}{M} \times B^\mu(x_1)B^\nu(x_2)H_\beta(x_3)
\]

\[
\times \langle \bar{0} | T \left[ \int j_\mu(x_1) d^4x_1 \int j_\nu(x_2) d^4x_2 \int \bar{\varphi}(x_3) \varphi(x_3) d^4x_3 \right] | 0 \rangle.
\]

(A) When this coupling is viewed from a distant point in space-time, it looks like a local phenomenon at \( x = (x_1 + x_2 + x_3)/3 \) as

\[
g^2 \frac{m_f}{M} \int B^\mu(X)B^\nu(X)H_\beta(X) d^4X
\]

\[
\times \langle \bar{0} | T \left[ \int j_\mu(x_1) \int j_\nu(x_2) \int t(x_3) d^4x_1 d^4x_2 d^4x_3 \right] | 0 \rangle,
\]

where \( t(x) = \bar{\varphi}(x) \bar{\psi}(x) \). The coefficient of \( B^\mu B^\nu H_\beta \) is a three-point correlation function with a dimension of mass. Since the physical vacuum \( | 0 \rangle \) is filled with massless fermion-antifermion pairs, the coefficient of \( g_{\mu\nu}B^\mu B^\nu H_\beta \) in Eq. (63) has a finite value \( V_h \)

\[
V_h = \langle \bar{0} | T \left[ \int j_\mu(x_1) \int j_\nu(x_2) \int t(x_3) d^4x_1 d^4x_2 d^4x_3 \right] | 0 \rangle,
\]

because of \( \prod_{p,s} \sin \theta_p e^{i(\alpha(x)b^+(p)\psi^s(p)\bar{\psi}(p))} | 0 \rangle \) in \( | 0 \rangle \). This \( V_h \) plays the role of \( v_h \) in Eq. (60).

(B) When this coupling is viewed in high resolution, anomalous momentum-dependent coupling is observed. We consider an amplitude \( M[H_\beta(p+k) \to B_\mu(p)B_\nu(k)] \) in Figure 6(a)

\[
g^2 iM[H_\beta(p+k) \to B_\mu(p)B_\nu(k)]
\]

\[
\propto (-ig)^2 \left( -i \frac{m_f}{M} \right)
\]

\[
\times \int \frac{d^4q}{(2\pi)^4} tr \left[ \gamma^\mu \frac{i}{q - p - m_f} \gamma^\nu \frac{i}{q - m_f} \right]
\]

\[
+ (p \leftrightarrow k, \mu \leftrightarrow \nu).
\]

(65)

Following the standard procedure, the analytic result is obtained as follows. The general form of the interaction between \( B_\mu \) and \( H_\beta \)

\[
g^2 \left( \frac{m_f}{M} \right) [V_h g^{\mu\nu} + m_f \Gamma_{BBH}^{\mu\nu}(q, p, k, m_B, m_f)]
\]

\[
\times B_\mu(p)B_\nu(k)H_\beta(p + k),
\]

has the following anomalous momentum-dependence

\[
\Gamma_{BBH}^{\mu\nu}(q, p, k, m_B, m_f)
\]

\[
= F_1(q^2, p^2, k^2, m_f) \frac{p^\mu k^\nu + p^\nu k^\mu}{m_B^2}
\]

\[
+ F_3(q^2, p^2, k^2, m_f) \frac{p^\mu p^\nu + q^\mu q^\nu}{m_B^2}.
\]

(67)

In these \( F_i \), the divergences coming from the triangle-loop integral are cancelled to each other in Eq. (65).

(a) We obtain the leading anomalous couplings \( F_1(q^2, p^2, k^2, m_f) \) in Eq. (67)

\[
F_1(q^2, p^2, k^2, m_f) = \frac{8}{(4\pi)^2} \left[ 1 - \frac{p^2}{2 m_f} \right] - \frac{2}{(4\pi)^2} \left[ p^2 - 4p^2 f \left( \frac{p}{2 m_f} \right) + (p \leftrightarrow k) \right] - \frac{2}{(4\pi)^2} \left[ q^2 - 4 q^2 f \left( \frac{q}{2 m_f} \right) + (p \leftrightarrow k) \right]
\]

\[
- C_0(p^2, q^2, m_f, m_f, m_f, m_f)
\]

(68)

where

(1) The function \( f(x) \)

\[
f(x) = \sqrt{1 - \frac{1}{x^2}} \ln \left[ 1 - 2x^2 + 2x^2 \sqrt{1 - \frac{1}{x^2}} \right],
\]

(69)

is illustrated in Figure 7

(2) \( C_0(p^2, q^2, m_f^2, m_f^2, m_f^2, m_f^2) \) is the scalar \( C_0 \) function in the Passarino-Veltman integrals \( 21, 22, \)

\[
C_0(p^2, q^2, m_f^2, m_f^2, m_f^2)
\]

\[
= \frac{2}{q^2} A \left( \frac{q}{2m_f} \right) + \left( \frac{1}{m_f^2 q^2} + \frac{1}{q^2} A \left( \frac{q}{2m_f} \right) \right) p^2 + \cdots,
\]

(70)
for $x < 1$, and
\[
A(x) = \frac{1}{4} \left( \ln \left[ 1 - 2x^2 + 2x^2 \sqrt{1 - \frac{1}{x^2}} \right] - i\pi \right)^2,
\]
(72)
for $x > 1$, which is illustrated in Figure 8. Since $f(x) \rightarrow -2$ and $\arcsin x \rightarrow x$ as $x \rightarrow 0$, $F_1(0, 0, 0, m_f) = 0$ is obtained in Eq. (65).

(b) For the decay $H_\beta(p + k) \rightarrow B_\mu(p)B_\nu(k)$ illustrated in Figure 8(a), we numerically calculate the anomalous effective coupling at $p^2 = k^2 = m_B^2$. Figure 9 shows the overall $q^2$-dependence of $F_1(q^2, m_B^2, m_B^2, m_f)$: its real part $ReF_1$ (solid curve), and its imaginary part $ImF_1$ (one-point-dotted curve), in which $m_B = 80$ GeV, $m_H = 123$ GeV, and $m_f = 160$ GeV are used. The real part $ReF_1(q^2, m_B^2, m_B^2, m_f)$ increases with increasing $\sqrt{q^2}$ to $2m_f$, and decreases at $\sqrt{q^2} > 2m_f$, then changing its sign. The amplitude of imaginary part $ImF_1(q^2, m_B^2, m_B^2, m_f)$ remains zero at $\sqrt{q^2} < 2m_f$, but gradually increases at $\sqrt{q^2} > 2m_f$.

(c) In the present model, other effective anomalous couplings $F_2(q^2, p^2, k^2, m_f)$ and $F_3(q^2, p^2, k^2, m_f)$ inevitably appears. (See Appendix). Figure 10 shows $ReF_1$, $ReF_2$ and $ReF_3$ at $\sqrt{q^2} < 2m_f$, in the case of same $m_f$, $m_B$, $m_H$ as in Figure 9. $ReF_2$ and $ReF_3$ are much smaller than $ReF_1$. The imaginary parts $ImF_1$, $ImF_2$ and $ImF_3$ are zero in $\sqrt{q^2} < 2m_f$. 

FIG. 7. Schematic view of $f(x)$. $ReF$ and $ImF$ are represented by a solid, and a one-point-dotted curve, respectively.

FIG. 8. Schematic view of $A(x)$. $ReA$ and $ImA$ are represented by a solid, and a one-point-dotted curve, respectively.
Hence, \( F \) is renormalized one as \( \lambda \nu \) which corresponds to \( q^2 = m_H^2 \). We set a condition that \( q^2 = m_H^2 \) such as \( q^2 \). The effective self-coupling of the Higgs-like mode is created by the one-loop process illustrated in Figure 11(b). In coordinate space, this coupling is expressed by another three-point correlation \( \bar{V}_h \)

\[
\bar{V}_h = (0|T \left[ \int \bar{\psi}(x_1) \int \bar{\psi}(x_2) \int t(x_3)d^3x_1d^3x_2 \right]|0),
\]

where \( \bar{\psi}(x) = \bar{\varphi}(x) \varphi(x) \). The anomalous momentum-dependent self-coupling is obtained by the following amplitude

\[
i\mathcal{M}[H_{\beta}(p + k) \rightarrow H_{\beta}(p)H_{\beta}(k)] \propto (-i \frac{m_f}{M})^3 \int \frac{d^4q}{(2\pi)^4} \operatorname{tr} \left[ \bar{\psi} - \bar{\psi} - m_f \bar{\psi} - \bar{\psi} - m_f \right] \left[ \bar{\psi} - \bar{\psi} - m_f \right] \]

\[
+ (p \leftrightarrow k). \tag{74}
\]

Hence, the effective self-coupling of the Higgs-like mode has a form such as

\[
\left( \frac{m_f}{M} \right)^3 |\bar{V}_h + m_f F_H(q^2, p^2, k^2, m_f)| H_\beta(p)H_\beta(k)H_\beta(p + k), \tag{75}
\]

which corresponds to \( \lambda \nu \) in Eq. (63). This \( (m_f/M)^3 \bar{V}_h \) no longer plays the role of stabilizing the symmetry-broken vacuum as in the Higgs potential in \( L_3(x) \).

Following the standard procedure, we calculate the anomalous self-coupling, and find that the divergence \( 1/\epsilon \) coming from the triangular-loop integrals does not cancel each other in Eq. (74). We set a condition that \( F_H(q^2, p^2, k^2, m_f) \) is zero at the on-shell level, and has a finite value only at the off-shell level \( q^2 > m_H^2 \). Hence, \( F_H(q^2, p^2, k^2, m_f) \) must vanish at \( q^2 = m_H^2 \). (For the divergence that is independent of momentum, it is to be renormalized to \( \bar{V}_h \).

VI.2. Self coupling

As an example of the reaction including the production and decay of the Higgs-like mode, we consider a s-channel process: fermion \( (p_1) \) + antifermion \( (\bar{p}_1) \) \( \rightarrow H_\beta(q) \rightarrow B_\mu(p) + B_\mu(k) \) illustrated in Figure 12. This reaction occurs together with the background reactions, such as the tree process through the \( t \)- and \( u \)-channel exchanges of fermion. The cross section of such a tree process is a slowly and monotonously decreasing function of the center-of-mass energy \( E_{cm} \). In the total cross section, the effective coupling of the Higgs-like mode will appear above this almost constant background cross section. In view of Figure 9 and 10 we use only \( F_1(q^2, m_B^2, m_B^2, m_f) \) as the first approximation of the total effective coupling.

We obtain the amplitude of the process in Figure 12

\[
i\mathcal{M} = \frac{m_f}{M} \bar{V}(k_1)U(p_1)q^2 m_f M 
\times \left[ V_h + m_f \left[ \Re F_1(q^2) + i \Im F_1(q^2) \right] \right] e^{\mu}(p) e_{\mu}(k) 
\]

\[
q^2 - m_B^2 \pm i \frac{m_f}{24 \pi} M \sqrt{1 - 4 m_B^2 q^2 (q^2 - 4 m_f^2)}, \tag{77}
\]

where \( e^{\mu}(p) \) is one of polarization vectors of the massive gauge boson, satisfying \( e^{\mu}(p) e_{\mu}(p) = -1 \) for longitudinal and transverse polarizations.

Since the incident fermion and antifermion are lighter than the particles in the triangle loop, the former are assumed to be massless. Using \( \sum_s U(p_1)U(p_1) = p_1 \) and \( \sum_s V(k_1) \bar{V}(k_1) = k_1 \), we obtain the squared amplitude

\[
\frac{1}{4} \sum_{s, s'} |\mathcal{M}|^2 = g^4 \left( \frac{m_f}{M} \right)^4 \frac{12(p_1 \cdot k_1)}{(q^2 - m_B^2)^2 + \frac{1}{24 \pi} \frac{m_f}{M} \sqrt{1 - 4 m_B^2 q^2 (q^2 - 4 m_f^2)}}.
\]
FIG. 12. Fermion-antifermion pair annihilation through the intermediate Higgs-like mode to gauge boson pair.

The Higgs-like collective mode in the intermediate state $V_k E$ increases below $E_{cm}$. This $E_{cm}$-dependence mainly comes from $Re F_1$ and $Im F_1$. It is not affected by the damping of the Higgs-like mode $H_\beta$, because the effect of damping is weakened by the small factor $(1/24\pi)^2$ in Eq. (78). The gradual increase and decrease of the cross section around $E_{cm} = 2.3 m_f$ is a key feature for the experimental confirmation of the physical vacuum, which is absent in $L_h(x)$.

VIII. RENORMALIZABILITY

In the Higgs model, symmetry breaking is caused simply by changing the sign of the coefficient $\mu^2$ in the Higgs potential $-\mu^2 |v_h + h|^2 + \lambda |v_h + h|^4$. Renormalizability of the Higgs model is systematically proved using the generating functional with this Higgs potential [23]. (1) In the symmetric vacuum, this generating functional is expanded in powers of the Higgs field $h(x)$. In the symmetry-broken vacuum, it is expanded in powers of $h(x) - v_h$. The algebraic relationship between these two types of expansion can be obtained, which assures that the renormalizability of the theory in the symmetric vacuum can be transferred to that in the symmetry-broken one. (2) When the Bogoliubov-Parasuk-Hepp-Zimmermann (BPHZ) method is applied to the Ward-Takahashi identities derived from this generating functional, renormalizability is systematically proved without reference to the symmetric model [24].

In the present model using $L_\lambda(x)$ in Eq. (59), however, the Higgs potential is not assumed. Hence, we can not immediately apply this systematic method. Rather, we go back to the inductive proof of renormalizability originated by Dyson in QED [25]. Let us consider the following Green’s functions: $S(p)$ for the massive fermion $\psi$,

$$[S(p)]^{-1} = -\gamma \cdot p + m_f - \Sigma^*(p),$$

$D(p^2)$ for the massive gauge boson $B_\mu$ in Eq. (77),

$$[D(p^2)]^{-1} = -p^2 + m_B^2 - \Pi^*(p^2),$$

and $G(p^2)$ for the Higgs-like mode $H_\beta$

$$[G(p^2)]^{-1} = -p^2 + m_H^2 - \Pi^*_H(p^2).$$

The self energy $\Sigma^*(p)$ of fermions satisfies the following Dyson equations illustrated in Figure 13:

$$i\Sigma^*(p) = \frac{g^2}{(2\pi)^4} \int d^4 k \gamma_\nu S(p-k) \Gamma_\nu (p-k) D^{\mu\nu}(k)$$

$$+ \frac{1}{(2\pi)^4} \left( \frac{m_f}{M} \right)^2 \int d^4 k S(p-k) \tilde{\Gamma}_H(p-k) G(k)$$

$$= i\Sigma_1(p) + i\Sigma_2(p)$$

FIG. 13. The rate of total cross section $\sigma(E_{cm})$ to $\sigma(2m_f)$ in the fermion-antifermion annihilation to gauge boson pair through the intermediate Higgs-like collective mode, in the case of $m_B = 80$ GeV, $m_f = 160$ GeV, $m_H = 123$ GeV and $V_h/m_f = 0.02$.

In the center-of-mass frame where $p_1 = (E_{cm}/2, p_1)$ and $k_1 = (E_{cm}/2, -p_1)$, we can use $q^2 = (p_1 + k_1)^2 = E_{cm}^2$ and $p_1 \cdot k_1 = E_{cm}^2/2$. The total cross section $\sigma(E_{cm})$ is given by

$$\sigma(E_{cm}) = \frac{\pi}{E_{cm}^2} \sqrt{1 - \frac{4m_B^2}{E_{cm}^2}} \sum_{s,s'} |M|^2.$$
where the vertex $\Gamma_\mu$ illustrated by a white triangle is the vertex function between $B_\mu$ and $\psi$, and another vertex $\tilde{\Gamma}_H$ illustrated by a black triangle is the vertex function between $H_\beta$ and $\psi$. Correspondingly, the self energy $\Sigma(p)$ of fermion is composed of $\Sigma_1(p)$ due to the coupling to $B_\mu$, and $\Sigma_2(p)$ due to the coupling to $H_\beta$.

Similarly, the vacuum polarization $\Pi^*_H(p^2)$ of the massive gauge boson satisfies

$$i\Pi^*_H(p^2) = \frac{g^2}{(2\pi)^4} \int d^4k tr[S(p + k)\gamma_\mu S(k)\Gamma_\nu(k, p + k)],$$

and the vacuum polarization $\Pi^*_H(p^2)$ of the Higgs-like mode satisfies

$$i\Pi^*_H(p^2) = \frac{g^2}{(2\pi)^4} \int d^4k tr[S(p + k)\gamma_\mu S(k)\tilde{\Gamma}_H(k, p + k)].$$

**VIII.1. The vertex $\Gamma_\mu(p, p')$ between $B_\mu$ and $\psi$**

For the vertex $\Gamma_\mu(p, p') = \gamma_\mu + \Lambda^*_\mu(p, p')$ between $B_\mu$ and $\psi$ in Eq.\((83)\), its proper vertex $\Lambda^*_\mu(p, p')$ satisfies the following Dyson equation as illustrated in Figure 15(a)

$$\Lambda^*_\mu(p, p') = \frac{g^2}{(2\pi)^4} \int d^4k \Gamma_\mu(k, p + k) S(p + k) D^\rho(\rho) \tilde{\Gamma}_H(k, p + k)$$

$$+ \frac{1}{(2\pi)^2} \left( \frac{m_f}{M} \right)^2 \int d^4k \tilde{\Gamma}_H(k, p + k) S(p + k) G(k)$$

$$\times \tilde{\Gamma}_H(k, p + k) S(-p' + k) \Gamma_\mu(p + k, -p' + k)$$

$$+ \cdots.$$ (86)

This $\Lambda^*_\mu(p, p')$ is related to the fermion self-energy $\Sigma^*_\mu(p)$ (due to the coupling to $B_\mu$) as

$$\frac{\partial}{\partial p^2} \Sigma^*_\mu(p) = \Lambda^*_\mu(p, p).$$ (87)

Similarly, if we consider a proper vertex $\Lambda_H(p, p')$ for $\tilde{\Gamma}_H(p, p') = 1 + \Lambda_H(p, p')$ between $H_\beta$ and $\psi$ in Eq.\((83)\), it satisfies the Dyson equation illustrated in Figure 15(b). The other fermion self-energy $\Sigma_2(p)$ (due to the coupling to $H_\beta$) is related to this $\Lambda_H$ as

$$\frac{\partial}{\partial p^2} \Sigma_2(p) = \Lambda_H(p, p).$$ (88)

With these $\Lambda_\mu(p, p)$ and $\Lambda_H(p, p)$, we obtain the Green functions of fermions

$$[S(p)]^{-1} = -\gamma \cdot p + m_f - \int_{p^0}^p dk^\mu \Lambda_\mu(k, k') - \int_{p^0}^p dk \Lambda_H(k, k).$$ (89)

We will introduce the following renormalization-constants $Z_i$ ($i = 1 \sim 5$)

$$S(p) = Z_2 S^{ren}(p),$$ (90)

$$D_{\mu\nu}(p^2) = Z_3 D^{ren}_{\mu\nu}(p^2),$$ (91)

$$G(p^2) = Z_5 G^{ren}(p^2),$$ (92)

$$\Gamma_\mu(p, p') = Z_1^{-1} \Gamma^{ren}_\mu(p, p'),$$ (93)

$$\tilde{\Gamma}_H(p, p') = Z_4^{-1} \tilde{\Gamma}^{ren}_H(p, p').$$ (94)

Let us estimate the divergence appearing successively in the perturbation expansion in Eq.\((86)\).
In the expansion of $\Lambda_\mu(p,p')$, we focus on $\Lambda_\mu$ of the order of $g^{2j}$ obtained by $j$ times of iteration. These $\Lambda_\mu$ contain $2j\,S(p)$, $j\,D(p^2)$, and $2j\,\Gamma_\mu(p,p')$, in addition to the original $\Gamma_\mu$. When $S(p)$, $D(p^2)$, $\Gamma_\mu$ and $\hat{\Gamma}_H$ are replaced by their counterparts in Eqs. (90), $\Lambda_\mu$ is renormalized as

$$
\Lambda_\mu(p,p') = (Z_4^{-1}Z_2Z_5^{1/2})^2 Z_1^{-1}\Lambda_{\text{ren}}^\mu(p,p'),
$$

(95)

Next, we focus on each $\Lambda_\mu$ of the order of $(m_f/M)^2$. In addition to the original $\Gamma_\mu$, they contain $2j\,S(p)$, $j\,G(p^2)$, and $2j\,\hat{\Gamma}_H(p,p')$. Hence, this $\Lambda_\mu(p,p')$ is renormalized as

$$
\Lambda_\mu(p,p') = (Z_4^{-1}Z_2Z_5^{1/2})^2 Z_1^{-1}\Lambda_{\text{ren}}^\mu(p,p'),
$$

(96)

If $g$ is replaced by

$$
g^{\text{ren}} = Z_4^{-1}Z_2Z_3^{1/2}g,
$$

(97)

and $M$ by

$$
M_{\text{ren}}^{-1} = Z_4^{-1}Z_2Z_3^{1/2}M^{-1},
$$

(98)

the total proper vertex is renormalized as

$$
\Lambda_\mu(p,p') = Z_4^{-1}\Lambda_{\text{ren}}^\mu(p,p',g^{\text{ren}},M_{\text{ren}}),
$$

(99)

For the higher-order terms with various combination of $g^2$ and $(m_f/M)^2$, similar renormalization is possible.

VIII.2. The vertex $\hat{\Gamma}_H(p,p')$ between $H_\beta$ and $\psi$

For the vertex $\hat{\Gamma}_H(p,p') = 1 + \Lambda_\mu(p,p')$ between $H_\beta$ and $\psi$ in Figure 15(b), the proper vertex $\Lambda_\mu(p,p')$ has a similar structure to $\Lambda_\mu$ if $\Gamma_\mu$ for the external $B_\mu$ is replaced by $\hat{\Gamma}_H$ for the external $H_\beta$. Hence, using Eqs. (97) and (98), $\hat{\Lambda}(p,p')$ is renormalized as

$$
\hat{\Lambda}(p,p') = Z_4^{-1}\Lambda_{\text{ren}}^\mu(p,p',g^{\text{ren}},M_{\text{ren}}),
$$

(100)

and $Z_1 = Z_4$. Since Eqs. (87) and (88) also hold for the renormalized quantities, $Z_2 = Z_1 = Z_4$ is obtained. Using Eqs. (99) and Eq. (100) in (89), we obtain the total renormalized fermion self-energy $\Sigma(p) = \Sigma_1(p) + \Sigma_2(p)$, and the renormalized $S^{\text{ren}}(p)$.

VIII.3. The vacuum polarization $\Pi(p)$ of the massive gauge field $B_\mu$

For the vacuum polarization $\Pi^\ast(p)$ of $B_\mu$ in Eq. (61), the proper vertex $\Delta_\mu(p,p')$ satisfying

$$
\frac{\partial}{\partial p_\mu} \Pi^\ast(p) = \Delta_\mu(p,p),
$$

(101)

is defined. This $\Delta_\mu(p,p')$ is useful, because the Green’s function $D(p^2)$ of $B_\mu$ is expressed as

$$
[D(p^2)]^{-1} = -p^2 + m_B - \int_{p_0}^p dk^\mu \Delta_\mu(k,k).
$$

(102)

This $\Delta_\mu(p,p')$ satisfies the following Dyson equation illustrated in Figure 16:

\[\Delta_\mu(p,p') = \frac{g^2}{(2\pi)^2} \int d^4k \Gamma_\nu(p+k) S(p+k) S(k) \times \Gamma_\nu(k,k+p') S(-p'+k) \Gamma_\mu(p+k,-p' + k) + \frac{g^4}{(2\pi)^8} \int d^4k d^4l \Gamma_\nu(p+k) S(p+k) S(k) \times \Gamma_\nu(k,k') S(k,k') D^{\sigma\tau} (k-k') S(k') \times \Gamma_\nu(k',-p'+k') S(k'-p') \Gamma_\mu(p+k',-p+k') + \frac{1}{(2\pi)^2} \frac{g^2}{M} \int d^4k \Gamma_\nu(p+k) S(p+k) \times \Gamma_\nu(k,k') G(k-k') S(k') \times S(k'-p') \Gamma_\mu(p+k',-p+k') + \cdots .\]

(103)

Only few terms are written in the above expansion, but it extends to the higher-order terms of $g^2$ and $(m_f/M)^2$.

1. Let us consider $\Delta_\mu$ of the order of $g^{2j}$ in the above expansion. They contain $(2j+1)\,S(p)$, $(j-1)\,D_{\mu\nu}(p^2)$, and $(2j+1)\,\Gamma_\mu(p,p')$.

$$
\Delta_\mu(p,p') = Z_2^{2j+1} Z_3^{-1} Z_1^{-(2j+1)} \Delta_{\text{ren}}^\mu(p,p'),
$$

(104)

Hence, if we replace $g$ by $g^{\text{ren}}$ defined in Eq. (97), each $\Delta_\mu$ with $g^{2j}$ is renormalized.

2. Similarly, for $\Delta_\mu$ with the coefficient $g^{j}(m_f/M)^{j}$, it contains $(i+j+1)\,S(p)$, $(i/2-1)D_{\mu\nu}(p^2)$, $(i/2-1)\,\Gamma_\mu(p,p)$, and $j\,\hat{\Gamma}_H(p,p')$.

$$
\Delta_\mu(p,p') = Z_2^{i+j+1} Z_3^{i/2-1} Z_5^{-(i+1)} Z_1^{-i} Z_4^{-j} \Delta_{\text{ren}}^\mu(p,p'),
$$

(105)
Hence, if we replace \( g \) by \( g^{\text{ren}} \) in Eq.\((77)\), and \( M \) by \( M^{\text{ren}} \) in Eq.\((78)\), each \( \Delta_\mu \) with \( g'(m_f/M)^2 \) is renormalized.

(3) As a result, the total proper vertex in Figure\(16\) is renormalized as

\[
\Delta_\mu(p,p') = Z_\mu^{-1}\Delta_\mu^{\text{ren}}(p,p',g^{\text{ren}},M^{\text{ren}}). \tag{106}
\]

Using Eq.\((106)\) in Eq.\((102)\), we obtain the renormalized \( D^{\text{ren}}(p^2) \).

For the higher terms, similar renormalization is possible.

**VIII.4. The vacuum polarization \( \Pi_H(p) \) of the Higgs-like mode \( H_\beta \)**

For the vacuum polarization \( \Pi_H(p) \) of the Higgs-like mode \( H_\beta \) in Eq.\((52)\), the proper vertex \( \Delta_H(p,p') \) satisfying

\[
\frac{\partial}{\partial p} \Pi_H(p) = \Delta_H(p,p), \tag{107}
\]

is defined. This \( \Delta_H(p,p) \) is useful, because the Green’s function \( G(p^2) \) of the Higgs-like mode \( H_\beta \) is expressed as

\[
[G(p^2)]^{-1} = -p^2 + m_f^2 - \int_0^p dk \Delta_H(k,k). \tag{108}
\]

The proper vertex \( \Delta_H(p,p') \) satisfies the Dyson equation shown in Figure\(17\). If \( \Gamma_H \) (black triangle) is replaced by \( \Gamma_\mu \) (white triangle), and \( \Gamma_\mu \) by \( \Gamma_H \), it shows a similar structure to Figure\(16\) for \( \Delta_\mu \). Hence, in analogy with Eq.\((106)\), the proper vertex is renormalized as

\[
\Delta_H(p,p') = Z_H^{-1}\Delta_H^{\text{ren}}(p,p',g^{\text{ren}},M^{\text{ren}}). \tag{109}
\]

where \( Z_\mu = Z_3 \). Using Eq.\((109)\) in Eq.\((108)\), we obtain the renormalized \( G^{\text{ren}}(p^2) \).

**VIII.5. The renormalized masses of \( \psi, B_\mu \) and \( H_\beta \)**

The renormalized mass of the fermion \( \psi \) is a solution of \( [S(p)]^{-1} = 0 \), in which \( \Lambda^{\text{ren}}_\mu(p,p',g^{\text{ren}},M^{\text{ren}}) \) and \( \Lambda^{\text{ren}}_H(p,p',g^{\text{ren}},M^{\text{ren}}) \) are used in Eq.\((59)\).

Similarly, the renormalized masses of \( B_\mu \) and \( H_\beta \) are solutions of \( [D(p^2)]^{-1} = 0 \) in Eq.\((102)\), and \( [G(p^2)]^{-1} = 0 \) in Eq.\((108)\), respectively. (When such masses are calculated, \( \Delta^{\text{ren}}_\mu(p,p',g^{\text{ren}},M^{\text{ren}}) \) and \( \Delta^{\text{ren}}_H(p,p',g^{\text{ren}},M^{\text{ren}}) \) must be used, respectively.)

In view of Eqs.\((59) \sim (52)\), we obtain the first approximation of such renormalized masses as follows

\[
m_f^{\text{ren}} = m_f - \Sigma^{\text{ren}}(m_f), \tag{110}
\]

\[
(m_B^{\text{ren}})^2 = m_B^2 - \Pi^{\text{ren}}(m_B^2), \tag{111}
\]

\[
(m_H^{\text{ren}})^2 = m_H^2 - \Pi_H^{\text{ren}}(m_H^2), \tag{112}
\]

in which \( \Sigma^{\text{ren}}, \Pi^{\text{ren}} \) and \( \Pi_H^{\text{ren}} \) are self energy and vacuum polarizations in the right-hand side of Eqs.\((59)\), \((102)\) and \((108)\) using the renormalized quantities.

Since the Higgs Lagrangian density \( L_h(x) \) does not exist in our model, all renormalization constants \( Z_i \) \((i = 1 \sim 5)\) are determined so as to absorb only the logarithmic divergence, not the quadratic one.

**IX. DISCUSSION**

**IX.1. Implications for the Higgs Lagrangian**

The power of the Higgs Lagrangian density \( L_h(x) \) in providing experimental predictions comes from its simple structure. Many quantities are derived from a single quantity \( v_h \), such as the gauge boson’s mass in \( m_B = g v_h \), fermion’s mass in \( (m_f/m_h) \bar{\phi} \phi h \), and the coupling of Higgs boson to gauge boson pair in \( g v_h g^\mu\nu B_\mu B_\nu h \). If \( L_h(x) \) is replaced by a more microscopic description, these \( v_h \)’s appearing in the different quantities may not have the same value. The precise measurement of the properties of the Higgs-like particle on this point will have a crucial importance for future development.

In the Higgs model, there are three parameters: two coefficients \( \mu \) and \( \lambda \) in the Higgs potential \( -\mu^2|\phi|^2 + \lambda |\phi|^4 \), and the fermion masses \( m_f \). In the present model, there are six parameters: four type of condensed energies in the physical vacuum, (a) the two-point correlation in the physical vacuum leading to \( m_B \) in Eq.\((41)\), (b) the field energy of massless gauge boson, leading to \( m_f \) in Eq.\((29)\), (c) the three-point correlations in the physical vacuum leading to \( V_h \) and \( \bar{V}_h \) in Eqs.\((63)\) and \((73)\). In addition to them, (d) the upper end of energy-momentum \( \Lambda \) of fermions involved in the excitation in Eq.\((53)\), and (e) \( M \) in the coupling of the Higgs-like mode to fermions \( m_f/M \) in Eq.\((51)\).

When the precise measurement of the Higgs-like particle is performed, the above degree of freedom will turn out to be important.
IX.2. Extension to the electroweak interaction

When we apply the present model to the electroweak interaction, it is appropriate to begin with the third generation, in which fermions with large masses, such as the top and bottom quarks (plus τ lepton and τ neutrino), are included. Such a Lagrangian density without the top and bottom quarks (plus lepton and neutrino) is

\[ L_0(x) = -\frac{1}{4} B^{\mu\nu} B_{\mu\nu} - \frac{1}{4} W^{a\mu\nu} W_{\mu\nu}^a + \bar{q}(i\partial_\mu + g_\tau W^\mu_\tau + g' Y Q B_\mu) \gamma^\mu q + \bar{l}(i\partial_\mu + g_\tau W^\mu_\tau + g' Y L B_\mu) \gamma^\mu l + \bar{r}(i\partial_\mu + g_\tau W^\mu_\tau + g' Y R B_\mu) \gamma^\mu r + \bar{\nu}(i\partial_\mu + g_\nu W^\mu_\nu + g' Y R B_\mu) \gamma^\mu \nu, \]

where

\[ B^{\mu\nu} = \partial_\mu B_\nu - \partial_\nu B_\mu, \]
\[ W^{a\mu\nu} = \partial_\mu W^a_\nu - \partial_\nu W^a_\mu - g f^{abc} W^b_\mu W^c_\nu, \]

and massless fermions (top and bottom quarks, τ lepton and τ neutrino) make up left-handed doublets \( q \) and \( l \), and right-handed doublets \( r \) and \( \nu \).

For the quarks in the third generation, the physical vacuum in Eq. (20) is generalized so that it reflects \( SU(2) \times U(1) \) symmetry of the top and bottom quarks as

\[ |\bar{0}\rangle = e^{i \tau_3 a_3(x)} \times \left( \prod_{p,s} \left( \cos \theta_W^p + \sin \theta_W^p e^{i \alpha_3(x)} b^s_1 \begin{pmatrix} -p \end{pmatrix} a^s_1 \begin{pmatrix} p \end{pmatrix} \right) |0_r\rangle \right) \]
\[ \times \left( \prod_{p,s} \left( \cos \theta_W^p + \sin \theta_W^p e^{i \alpha_3(x)} b^s_1 \begin{pmatrix} -p \end{pmatrix} a^s_1 \begin{pmatrix} p \end{pmatrix} \right) |0_r\rangle \right) \]

where \( a(p) \) and \( b(-p) \) are reorganized to \( W^+_a, Z_\mu \) and \( A_\mu \). When \( L_h(x) \) in Eq. (6) is extended to include \( W^a_\mu \) and \( B_\mu \), the couplings of \( B_\mu \) to \( W^1_\mu \) and \( W^2_\mu \) inevitably appear. In order to eliminate such couplings, the vacuum condensate of Higgs particle is phenomenologically assumed to have a structure \( 0 \langle h | 0 \rangle = (0, v_h) \). The extension of the present model has a possibility of deriving the vanishing of \( B_\mu W^a_\mu \) and \( B_\mu W^2_\mu \), not from \( (0, v_h) \), but from the dynamical reason.

When the argument in Section 3 is extended across different generations, a microscopic explanation of CKM matrix is expected. If \( a(p) \) and \( b(-p) \) in the second term of Eq. (22) represent the down and strange quarks respectively, an orthogonal transformation giving rise to the mixing of different generations is introduced for the state coupled to the gauge field, and the Cabibbo angle is defined as an angle of such a transformation.

In the electroweak version of the present model, two types of condensed kinetic energy of massless quarks in Eqs. (118) and (119) are assumed, and therefore the definition of Weinberg angle \( \theta_W \), which determines the mixing of \( W^a_\mu \) and \( B_\mu \), will be slightly changed.

In the electroweak version of the present model, the effective coupling of the Higgs-like mode to \( A_\mu \), \( Z_\mu \) and \( W^a_\mu \), and the existence of the anomalous coupling in it [31]. In the Higgs model, the coupling of the Higgs field \( h \) to \( A_\mu \), \( Z_\mu \) and \( W^a_\mu \) comes from the common \( v_h \). In the electroweak version of the present model, the effective coupling of the Higgs-like mode to \( A_\mu \), \( Z_\mu \) and \( W^a_\mu \) have a variety of strength and \( q^2 \)-dependence. The anomalous effective coupling such as Eq. (67) is worth precise measurements.

The present model predicts the total cross section \( \sigma(q^2) \) of the fermion-antifermion annihilation to gauge boson pair, in a different way from the Higgs model. In addition to the rise at \( q^2 = (2m_p)^2 \), the gradual increase and decrease of \( \sigma(q^2) \) around \( q^2 = (2m_f)^2 \) is expected. When the present model is generalized to the electroweak interaction, the Higgs model and the present model will give different predictions in the intermediate- and high-energy processes. The precise measurements of properties of the recently discovered Higgs-like particle...
are expected in future experiment.

Appendix A: The anomalous coupling of Higgs-like mode

In the anomalous effective coupling of the Higgs-like collective mode to gauge boson, \( F_2(q^2, p^2, k^2, m_f) \) and \( F_3(q^2, p^2, k^2, m_f) \) are included in Eq. (67). Following the standard procedure, we obtain such \( F_2 \) and \( F_3 \) as

\[
F_2(q^2, p^2, k^2, m_f) = -\frac{16m_B^2}{(4\pi)^2} \left[ \frac{q^2 - 3(p^2 + k^2)q^2 + 8p^2k^2}{q^2(q^2 - 4p^2)(q^2 - 4k^2)} \right] \\
\times f \left( \frac{q}{2m_f} \right) \\
+ \frac{16m_B^2}{(4\pi)^2} \left[ \frac{q^2 + 2p^2}{q^2(q^2 - 4p^2)(q^2 - 4k^2)} \right] f \left( \frac{p}{2m_f} \right) \\
+ \frac{16m_B^2}{(4\pi)^2} \left[ \frac{q^2 - 4p^2}{q^2(q^2 - 4p^2)(q^2 - 4k^2)} \right] f \left( \frac{p}{2m_f} \right) \\
+ (p \leftrightarrow k)
\]

\[
F_3(q^2, p^2, k^2, m_f) = \frac{16m_B^2}{(4\pi)^2} \left[ \frac{(p^2 + k^2)q^2 - 8p^2k^2}{q^2(q^2 - 4p^2)(q^2 - 4k^2)} \right] \\
+ \frac{32m_B^2}{(4\pi)^2} \left[ \frac{q^2(q^2 - 4p^2)^2(q^2 - 4k^2)}{q^2(q^2 - 4p^2)(q^2 - 4k^2)} \right] f \left( \frac{q}{2m_f} \right) \\
+ \frac{32m_B^2}{(4\pi)^2} \left[ \frac{(q^2 - p^2)^2}{q^2(q^2 - 4p^2)(q^2 - 4k^2)} \right] f \left( \frac{p}{2m_f} \right) \\
+ \frac{8}{(4\pi)^2} \left[ \frac{q^2 + 4m_f^2}{q^2(q^2 - 4p^2)(q^2 - 4k^2)} \right] f \left( \frac{p}{2m_f} \right) \\
+ (p \leftrightarrow k)
\]

1. F. Englert and R. Brout, Phys.Rev.Lett 13, 321 (1964).
2. P.W. Higgs, Phys.Lett 12, 132 (1964).
3. T.W.B. Kibble, Phys.Rev. 155, 1554 (1967).
4. ATLAS Collaboration, Observation of a new particle in the search for the Standard Model Higgs boson with the ATLAS detector at the LHC, Phys.Lett.B 716, 1 (2012).
5. CMS Collaboration, Observation of a new boson at a mass of 125 GeV with the CMS experiment at the LHC, Phys.Lett.B 716, 30 (2012).
6. S.L. Glashow, Nucl.Phys. 22, 579 (1961).
7. S. Weinberg, Phys.Rev.Lett.B 19, 1264 (1967).
8. A. Salam, in Elementary Particle Theory edited by N. Svartholm, (Almqvist and Wiksell, Stockholm, 1968) 367.
9. Y. Nambu and G. Jona-Lasinio, Phys.Rev. 122, 345 (1961).
10. J. Bardeen, L.N. Cooper, and J.R. Schrieffer, Phys.Rev. 106, 162 (1957).
11. E.C.G. Stueckelberg, Helv.Phys.Acta 14, 588 (1941).
12. R.P. Feynman, Phys.Rev. 76, 749 (1949). As a review, R.P. Feynman, The reason for antiparticles, in Elementary Particles and the Law of Physics, edited by R. MacKenzie and P. Doust, (Cambridge, 1987) 1.
13. The direction of spin represented by \( a^s(p) \) is opposite to that by \( b^s(\cdot) \) for the same \( s \). Hence, a similar argument holds for the flow of spin as well.
14. Equations (10) and (11) have the same form as the Bogoliubov transformation in superconductivity, but they have different physical meaning.
15. If the gap equation is regarded as the starting point of the formalism as in [9], the attractive interaction is necessary for the gap equation to have a nontrivial solution such as massive fermions. The gap equation is not necessary in the present model.
16. In Eq. (43), the interaction \( m_B \phi \phi B_\mu \) is derived, not from \( (i\partial_\mu + g B_\mu)(v_n + b + i\phi) \) as in the Higgs model, but from the physical vacuum \( |\bar{0}\rangle \). By assuming the gauge-fixed Lagrangian \( L_2 - \frac{1}{2} G^2 \) where \( G = (1/\sqrt{2})[\partial_\mu B_\mu - \xi \phi \phi] \), the propagator of gauge boson with other gauge fixing \( (\xi \neq 0) \) can be obtained using the standard method. K. Fujikawa, B.W. Lee and I. Sanda, Phys.Rev. D 6, 2923 (1972).
17. In the model by Nambu and Jona-Lasinio [9], a similar bubble diagram is considered for the Goldstone mode coming from the violation of the \( \gamma \)-invariance, but their interaction between the bubbles is a chiral one including \( \gamma_5 \). From the gap equation with such a chiral interaction, a massless mode corresponding to the meson is derived. In the present model, the pairing interaction leading to the gap equation does not exist in the Lagrangian density \( L_0(x) \). Hence, it takes an energy to excite the system, hence making its excitation massive.
18. H.H. Patel, Comput.Phys.Commun. 197, 276 (2015).
19. The triangle loop in Figure 6(a) is composed of massless fermions, but their excitation from \( |\bar{0}\rangle \) is simply written by the massive excitation \( \psi(x) \) as in Eq. (49).
20. In the GWS model of the electroweak interaction, the triangular-loop coupling similar to \( F_1(q^2, p^2, k^2, m_f) \) causes the decay of the Higgs particle into photon pairs. A.I. Vainshtein, M.B. Voloshin, V.I. Zakharov and M.A. Shifman, Sov.J.Nucl.Phys 30, 711 (1979).
21. G. Passarino and M.J.G. Veltman, Nucl.Phys.B 160, 151 (1979).
22. D. Bardin and G. Passarino, The standard model in the making, (Oxford, 1999).
23. E.S. Abers and B.W. Lee, Gauge theories, Phys.Rep. 9 (1973) 1.
24. B.W. Lee, Phys.Rev. D5, 823 (1972).
25. F.I. Dyson, Phys.Rev. 75, 1736 (1949).
26. The Goldstone mode \( \phi(x) \) also couples to fermion \( \psi(x) \) in \( L_3(x) \) of Eq. (59). But, when the propagator \( D^{\mu\nu}(q) \) in
Eq. (47) is decomposed as
\[
D_{\mu\nu}(q) = \frac{-i}{q^2 - m_B^2} \left( g^{\mu\nu} - \frac{q^\mu q^\nu}{m_B^2} \right) + \frac{-i}{q^2} q^\mu q^\nu, 
\]
(A3)
the renormalization of the fermion self-energy \( \Sigma(p) \) due to the propagator of \( \phi(x) \) is cancelled by that due to the second term of this \( D_{\mu\nu}(q) \). Hence, only the renormalization of \( \Sigma(p) \) due to \( D_{\mu\nu}(q) \) is considered.

[27] J.C. Ward, Proc. Phys. Soc. A 64, 54 (1951).

[28] Since one of \((2j + 1) \Gamma_\mu(p,p')\) comes from the differential of \( S^{-1}(p) \), \( Z_{2j}^{-(2j+1)} \) in the first line of Eq. (104) must be replaced by \( Z_{1}^{-(2j)}Z_{2}^{-1} \). For the perturbation expansion of \( \Pi_H \) in Figure 17 as well, the same situation occurs for \((2j + 1) \Gamma_H(p,p')\).

[29] For the order of \( g^2(m_f/M)^2 \) in Eq. (103) where \( i = 2 \) in \( g'(m_f/M)' \), \( D_{\mu\nu} \) is not explicitly written. But, for the higher-order terms in the expansion, \((i/2 - 1)D_{\mu\nu}\) must be added. Hence, \( Z_{2j}^{i/2-1} \) appears in the first line of Eq. (105). As in [28], since one of \((i + 1) \Gamma_\mu(p,p')\) comes from the differential of \( S^{-1}(p) \), \( Z_{1}^{-(i+1)} \) must be replaced by \( Z_{1}^{-1}Z_{2}^{-1} \) in the second line.

[30] CMS Collaboration, Constraints on anomalous Higgs boson couplings using production and decay information in the four-lepton final state, Phys. Lett. B 775, 1 (2017): arXiv:1707.00541 [hep-ex].