Exploring the Phase Structure and the Dynamics of QCD

RIKEN Lunch Seminar
12/04/14 BNL

Nils Strodthoff, ITP Heidelberg
Outline

QCD Phase Structure
- QCD phase structure from functional approaches
- Quenched QCD in the vacuum
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- Quenched QCD in the vacuum

Dynamics I Spectral functions
- Spectral functions from a Euclidean framework
- Mesonic spectral functions in simple models
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QCD Phase Structure
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- Quenched QCD in the vacuum

Dynamics I Spectral functions
- Spectral functions from a Euclidean framework
- Mesonic spectral functions in simple models

Dynamics II Transport Coefficients
- Kubo formula with expansion in full propagators/vertices
- Transport coefficients in YM and QCD
QCD Phase Structure

- Mitter, Pawlowski, arXiv:1411.7978
The QCD phase diagram?

- Fukushima, Hatsuda Rept.Prog.Phys. 74 (2011) 014001
- A. Andronic et al Nucl.Phys. A837 (2010)
- adapted from GSI
The QCD phase diagram?

- Fukushima, Hatsuda Rept.Prog.Phys. 74 (2011) 014001
- A. Andronic et al Nucl.Phys. A837 (2010)

Phase structure at large chemical potentials largely unknown due to sign problem in lattice QCD...

- adapted from GSI
Continuum perspective
...using functional approaches

Functional relations between off-shell Green’s functions
Continuum perspective
...using functional approaches

Functional relations between off-shell Green’s functions

e.g. Dyson-Schwinger equation for quark propagator

\[
\begin{align*}
    \quad & -1 = -1 - \\
    \quad & \quad -1
\end{align*}
\]
Continuum perspective
...using functional approaches

Functional relations between off-shell Green’s functions

e.g. Dyson-Schwinger equation for quark propagator

-1 = -1

✓ Easy access to mechanisms:
  • Chiral symmetry breaking
  • Confinement

✓ Complementary to the lattice
✓ No sign problem
✓ Effective models incorporated
Functional Approaches: finite T

PQM model, Nf=2+1, FRG

Herbst, Mitter, Pawlowski, Schaefer, Stiele
Phys.Lett. B731 (2014) 248-256

Matter+Glue system, Nf=2, FRG

Braun, Haas, Marhauser, Pawlowski
PRL 106 (2011) 022002

Quark propagator, Nf=2+1, DSE

Fischer, Luecker, Welzbacher
Phys.Rev. D90 (2014) 034022
Functional Approaches: finite T & μ

PQM model, Nf=2, FRG

Herbst, Pawlowski, Schaefer Phys.Lett. B696 (2011)

Quark propagator, Nf=2+1, DSE

Fischer, Fister, Luecker, Pawlowski Phys.Lett. B732 (2014) 273-277
Functional Approaches: finite T & μ

PQM model, Nf=2, FRG

Quark propagator, Nf=2+1, DSE

But: so far all require additional phenomenological input

PQM-model: UV parameters, glue input (Polyakov-loop potential)
DSE calculation: vertex models e.g. for the quark-gluon vertex

Herbst, Pawlowski, Schaefer Phys.Lett. B696 (2011)

Fischer, Fister, Luecker, Pawlowski Phys.Lett. B732 (2014) 273-277
fQCD Collaboration (J. Braun, L. Fister, T. K. Herbst, M. Mitter, J. M. Pawlowski, F. Rennecke and N. Strodthoff)

- Mitter, Pawlowski, NSt arXiv:1411.7978
- Braun, Fister, Pawlowski, Rennecke arXiv:1412.1045
Finite $\mu$ requires fluctuations to be **quantitatively** under control.

Mismatches in fluctuation scales lead to large systematic errors at finite $\mu$. 

- Helmboldt, Pawlowski, NSt arXiv:1409.8414
fQCD Collaboration

Mitter, Pawlowski, NSt arXiv:1411.7978
Braun, Fister, Pawlowski, Rennecke arXiv:1412.1045

✓ Finite $\mu$ requires fluctuations to be quantitatively under control

✓ Mismatches in fluctuation scales lead to large systematic errors at finite $\mu$
  ➢ Helmboldt, Pawlowski, NSt arXiv:1409.8414

✓ No phenomenological input (vertex models, running couplings...)

✓ Input parameters only the fundamental parameters of QCD

$\alpha_s (20 \text{ GeV})$ strong running coupling
$M_q (20 \text{ GeV}) \approx 1-2 \text{ MeV}$ current quark mass

at large perturbative momenta.
Finite $\mu$ requires fluctuations to be **quantitatively** under control

Mismatches in fluctuation scales lead to large systematic errors at finite $\mu$

No phenomenological input (vertex models, running couplings...)

Input parameters only the fundamental parameters of QCD

$\alpha_s(20 \text{ GeV})$ **strong running coupling**

$M_q(20 \text{ GeV}) \approx 1$-$2 \text{ MeV} \text{ current quark mass}$

at large perturbative momenta.

Quantitative FRG approach towards the investigation of the phase diagram and the hadron spectrum
Functional RG for QCD

- Spirit of **Wilson RG**: Calculate full quantum effective action $\Gamma$ by integrating fluctuations with momentum $k$

\[ k \to 0 \quad \Gamma \quad k \to \Lambda_{UV} \]
Functional RG for QCD

- Spirit of **Wilson RG**: Calculate full quantum effective action $\Gamma$ by integrating fluctuations with momentum $k$

\[ k \to 0 \quad \Gamma_k \quad \Gamma \quad S \quad k \to \Lambda_{UV} \]

Functional Renormalization Group (FRG)

\[ \partial_t \Gamma_k[\phi] = \frac{1}{2} \]

Free energy/Grand potential

- gluon
- ghost
- quark
- hadrons
Functional RG for QCD

- Spirit of **Wilson RG**: Calculate full quantum effective action $\Gamma$ by integrating fluctuations with momentum $k$

$$ k \rightarrow 0 \quad \Gamma_k \quad k \rightarrow \Lambda_{UV} $$

**Functional Renormalization Group (FRG)**

$$ \partial_t \Gamma_k[\phi] = \frac{1}{2} \text{gluon} - \text{ghost} - \text{quark} + \frac{1}{2} \text{hadrons} $$

**Free energy/Grand potential**

**Dynamical hadronization**

- Gies, Wetterich Phys.Rev. D65 (2002) 065001

Store resonant 4-Fermi structures in terms of effective mesonic interactions
Truncation

Vertex expansion

FRG Yang-Mills results
Truncation

Vertex expansion

FRG Yang-Mills results

mom. dep. classical tensor structure

mom. dep. classical tensor structure
Truncation

Vertex expansion

FRG Yang-Mills results

mom. dep. classical tensor structure

mom. dep. classical tensor structure

full mom. dep.
Truncation

Vertex expansion

FRG Yang-Mills results

mom. dep. classical tensor structure

mom. dep. classical tensor structure

full mom. dep.
all tensor structures
Truncation

Vertex expansion

-1
FRG Yang-Mills results

mom. dep.
classical tensor structure

mom. dep.
classical tensor structure

-1
full mom. dep.
all tensor structures

mom. dep.
STI-consistent dressing
Truncation

Vertex expansion

-1
FRG Yang-Mills results

mom. dep.
classical tensor structure

mom. dep.
classical tensor structure

-1
full mom. dep.
all tensor structures

mom. dep.

STI-consistent dressing

Fierz-complete basis at $p = 0$ and mom. dep.
Truncation

Vertex expansion

-1
FRG Yang-Mills results

-1
mom. dep.
classical tensor structure

mom. dep.
classical tensor structure

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Fierz-complete basis
at $p = 0$ and mom. dep.

-1
mom. dep.

mom. dep.
Truncation

Vertex expansion

-1

FRG Yang-Mills results

mom. dep. classical tensor structure

mom. dep. classical tensor structure

full mom. dep. all tensor structures

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mom. dep.

mom. dep.
Truncation

Vertex expansion

-1
FRG Yang-Mills results

mom. dep.
classical tensor structure

mom. dep.
classical tensor structure

-1
full mom. dep.
all tensor structures

mom. dep.
STI-consistent dressing

Fierz-complete basis at \( p = 0 \) and mom. dep.

mom. dep.

mom. dep.

full effective potential
Truncation

Vertex expansion

- FRG Yang-Mills results
- Full momentum dependence
- Fierz-complete basis at $p = 0$ and momentum dependence

Computer-algebraic generation of equations using DoFun

- Huber, Braun Comput. Phys. Commun. 183 (2012) 1290-1320
Propagators (T=0)

Quenched gluon propagator (input)

\[ \Gamma_{A^2}^{\mu\nu}(p) = Z_A(p)p^2 \Pi_T^{\mu\nu}(p) \]

- Bowman et al. Phys. Rev. D70, 034509 (2004)
- Sternbeck et al. PoS LAT2006, 076 (2006)
- Fischer, Maas, Pawlowski Annals Phys. 324, 2408 (2009)
- Fister, Pawlowski in prep.
Propagators (T=0)

Quenched gluon propagator (input)

\[ \Gamma^{\mu\nu}_{A^2}(p) = Z_A(p)p^2 \Pi^\mu\nu_T(p) \]

Quark propagator

\[ \Gamma_{\bar{q}q}(p) = Z_q(p)(i\not{p} + M_q(p)) \]

Very good agreement with (quenched) lattice results!

- Bowman et al. Phys. Rev. D70, 034509 (2004)
- Sternbeck et al. PoS LAT2006, 076 (2006)
- Fischer, Maas, Pawlowski Annals Phys. 324, 2408 (2009)
- Fister, Pawlowski in prep.

- Bowman et al. Phys. Rev. D71, 054507 (2005),
- Mitter, Pawlowski, NSt arXiv:1411.7978
Chiral symmetry breaking

\[ \beta\text{-function:} \]

\[ k \partial_k \hat{\lambda}_\psi = (d - 2) \hat{\lambda}_\psi - a \hat{\lambda}^2_\psi - b \hat{\lambda}_\psi g^2 - c g^4 \]

- Review: Braun J.Phys. **G39** (2012) 033001
Chiral symmetry breaking

\[ k \partial_k \hat{\lambda}_\psi = (d - 2) \hat{\lambda}_\psi - a \hat{\lambda}^2 \psi - b \hat{\lambda}_\psi g^2 - c g^4 \]

- reflects gluon mass gap
- area above the critical value decides

Review: Braun J.Phys. G39 (2012) 033001
Effective model perspective

- Independence of initial scale and initial condition
Effective model perspective

- Independence of initial scale and initial condition
- only requirement: decoupling of gluons
- **Low-energy models completely fixed by QCD flow**
**Outlook**

**Unquenching**: first qualitative results available

- Braun, Fister, Pawlowski, Rennecke arXiv:1412.1045
Outlook

Unquenching: first qualitative results available

Shopping list

✓ Quantitative results in the vacuum
☐ Full unquenching
☐ Quantitative investigations in the vacuum (YM vertices, 4-Fermi)
☐ Transition to low-energy effective models
☐ Finite temperature
☐ Finite Density (important: role of baryonic/diquark d.o.f.)

Braun, Fister, Pawlowski, Rennecke arXiv:1412.1045
Dynamics I
Spectral Functions

- Kamikado, NSt, von Smekal, Wambach Eur.Phys.J. C74 (2014) 2806
- Tripolt, NSt, von Smekal, Wambach Phys.Rev. D89 (2014) 034010
- Helmboldt, Pawlowski, NSt arXiv:1409.8414
Spectral Functions

Real-time observable from Euclidean framework

\[ \Gamma^{(2)}_R (\omega, \vec{p}) = - \lim_{\epsilon \to 0} \Gamma^{(2)}_E (-i(\omega + i\epsilon), \vec{p}) \]

\[ \rho(\omega, \vec{p}) = \frac{\text{Im} \Gamma^{(2)}_R (\omega, \vec{p})}{\text{Im} \Gamma^{(2)}_R (\omega, \vec{p})^2 + \text{Re} \Gamma^{(2)}_R (\omega, \vec{p})^2} \]

requires analytical continuation from Euclidean to Minkowski signature numerically hard or even ill-posed problem
Spectral Functions

Real-time observable from Euclidean framework

\[
\Gamma_R^{(2)}(\omega, \vec{p}) = -\lim_{\epsilon \to 0} \Gamma_E^{(2)}\left(-i(\omega + i\epsilon), \vec{p}\right)
\]

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\rho(\omega, \vec{p}) = \frac{\text{Im} \Gamma_R^{(2)}(\omega, \vec{p})}{\text{Im} \Gamma_R^{(2)}(\omega, \vec{p})^2 + \text{Re} \Gamma_R^{(2)}(\omega, \vec{p})^2}
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requires analytical continuation from Euclidean to Minkowski signature numerically hard or even ill-posed problem

Popular approaches (based on Euclidean data)
- Maximum Entropy Method (MEM)
- Padé Approximants
Spectral Functions

Real-time observable from Euclidean framework

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\]

requires analytical continuation from Euclidean to Minkowski signature numerically hard or even ill-posed problem

Popular approaches (based on Euclidean data)
• Maximum Entropy Method (MEM)
• Padé Approximants

Alternative: analytic continuation on the level of the functional equation

- Kamikado, NSt, von Smekal, Wambach Eur.Phys.J. C74 (2014) 2806
- Floerchinger JHEP 1205 (2012) 021
- Strauss, Fischer, Kellermann PRL 109 (2012) 252001
Analytical continuation

- Kamikado, NSt, von Smekal, Wambach Eur.Phys.J. C74 (2014) 2806
• Compute flow equation for Euclidean 2-point function perform analytically for 3d regulator function $R = \bar{p}^2 r(\bar{p}^2)$
• Compute flow equation for Euclidean 2-point function
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• Perform analytical continuation in ext. momentum
  \( p_0 \rightarrow -i(\omega + i\epsilon) \)

Kamikado, NSt, von Smekal, Wambach Eur.Phys.J. C74 (2014) 2806
Analytical continuation

- Compute flow equation for Euclidean 2-point function
  perform analytically for 3d regulator function $R = \bar{p}^2 r(\bar{p}^2)$

- Perform analytical continuation in ext. momentum
  $p_0 \rightarrow -i(\omega + i\epsilon)$

- Ensure correct continuation
  $n_B/F(E + i p_0) \rightarrow n_B/F(E)$

Kamikado, NSt, von Smekal, Wambach Eur.Phys.J. C74 (2014) 2806
Analytical continuation

- Compute flow equation for Euclidean 2-point function perform analytically for 3d regulator function
  \[ R = \bar{p}^2 r(\bar{p}^2) \]

- Perform analytical continuation in ext. momentum
  \[ p_0 \rightarrow -i(\omega + i\epsilon) \]

- Ensure correct continuation
  \[ n_B/F(E + ip_0) \rightarrow n_B/F(E') \]

- For small but finite \( \epsilon \) compute real and imaginary part of
  \[ -\Gamma^{(2)}_E(\omega + i\epsilon, \bar{p}) \]
Analytical continuation

- Compute flow equation for Euclidean 2-point function
  perform analytically for 3d regulator function \( R = \tilde{p}^2 r(\tilde{p}^2) \)

- Perform analytical continuation in ext. momentum
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- Ensure correct continuation
  \( n_B/F(E + ip_0) \rightarrow n_B/F(E') \)

- For small but finite \( \epsilon \) compute real and imaginary part of
  \( -\Gamma^{(2)}_E(-i(\omega + i\epsilon), \vec{p}) \)

**Test cases**: simple bosonic/ Yukawa models
Mesonic Spectral Functions

O(N) model (T=0)

- Kamikado, NSt, von Smekal, Wambach
  Eur.Phys.J. **C74** (2014) 2806
Mesonic Spectral Functions

O(N) model (T=0)

- Kamikado, NSt, von Smekal, Wambach
  Eur.Phys.J. C74 (2014) 2806

Quark-meson (Yukawa) model

- Tripolt, NSt, von Smekal, Wambach
  Phys.Rev. D89 (2014) 034010
QM Model at $T>0$

Tripolt, NSt, von Smekal, Wambach Phys.Rev. D89 (2014) 034010
QM Model at $T>0$

Tripolt, NSt, von Smekal, Wambach Phys.Rev. D89 (2014) 034010
QM Model at $T>0$
QM Model at T>0

Tripolt, NSt, von Smekal, Wambach Phys.Rev. D89 (2014) 034010
Outlook
Outlook

Generalization towards a fully numerical procedure

First step: Euclidean momenta (via an iterative procedure)
- Helmboldt, Pawlowski, NSt arXiv:1409.8414

| step | $m_{\text{cur}}$ [MeV] | $m_{\text{pol}}$ [MeV] | $\sigma_{\text{min}}$ [MeV] |
|------|-----------------------|------------------------|-----------------------------|
| 0    | 412.8                 | 412.8                  | 16.8                        |
| 1    | 144.8                 | 142 ± 2                | 83.5                        |
| 2    | 136.4                 | 135 ± 2                | 91.8                        |
| 3    | 135.1                 | 134 ± 2                | 93.1                        |
| 4    | 134.9                 | 133 ± 2                | 93.2                        |
| 5    | 134.9                 | 133 ± 2                | 93.2                        |
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**Second step:** Minkowski external momenta
- Pawlowski, NSt in prep.
Outlook

Generalization towards a fully numerical procedure

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- Helmboldt, Pawlowski, NSt arXiv:1409.8414

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Second step: Minkowski external momenta
- Pawlowski, NSt in prep.

Shopping list
- Continuation procedure set-up
- Tested in simple models
- Generalization towards a fully numerical procedure
- Quark & gluon spectral functions
- Vector meson spectral functions
- Charmonium spectral functions
Dynamics II
Transport Coefficients

- Christiansen, Haas, Pawlowski, NSt arXiv:1411.7986
Transport Coefficients

- Evolution of the hot plasma well-described by hydrodynamics
- Extract viscosity from $v_2$
- Transport coefficients as important microscopic input
Transport Coefficients

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Bass, Heinz, Hirano, Shen Phys. Rev. Lett. 106 (2011) 192301
Transport Coefficients

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Bass, Heinz, Hirano, Shen Phys. Rev. Lett. 106 (2011) 192301

Kubo formula

$$\eta = \lim_{\omega \to 0} \frac{1}{20} \frac{\rho_{\pi \pi}(\omega, \vec{0})}{\omega}$$

Require

$$\rho_{\pi \pi}(\omega, \vec{p}) = \int x e^{-i\omega x_0 + i\vec{p} \cdot \vec{x}} \langle [\pi_{ij}(x), \pi_{ij}(0)] \rangle$$
Computing EM Correlators

DSE-like expansion formula

\[ \langle \pi_{i,j}[\hat{A}]\pi_{i,j}[\hat{A}] \rangle = \pi_{i,j} [G_{A\phi_k} \frac{\delta}{\delta \phi_k} + A] \pi_{i,j} [G_{A\phi_k} \frac{\delta}{\delta \phi_k} + A] \]

Pawlowski Annals Phys. 322 (2007) 2831-2915
Computing EM Correlators

DSE-like expansion formula
- Pawlowski Annals Phys. 322 (2007) 2831-2915

\[
\langle \pi_{i,j} [\hat{A}] \pi_{i,j} [\hat{A}] \rangle = \pi_{i,j} [G_{A\phi_k} \frac{\delta}{\delta \phi_k} + A] \pi_{i,j} [G_{A\phi_k} \frac{\delta}{\delta \phi_k} + A]
\]

**Finite number** of diagrams involving **full** propagators/vertices

- All diagrams to 2-loop order
Computing EM Correlators

DSE-like expansion formula

\[
\langle \pi_{ij}[\hat{A}]\pi_{ij}[\hat{A}] \rangle = \pi_{ij}[G_{A\phi_k} \frac{\delta}{\delta \phi_k} + A]\pi_{ij}[G_{A\phi_k} \frac{\delta}{\delta \phi_k} + A]
\]

Finite number of diagrams involving full propagators/vertices

Input: **gluon spectral function** from Euclidean FRG data using MEM

- Haas, Fister, Pawlowski Phys. Rev. D90 091501 (2014)
$\eta/s$ in Yang-Mills Theory

$T_{\text{min}} = 1.26 \, T_c$
Value $= 0.14$

- Result
- Fit
- GRG/HTL
- Lattice
- KSS

Christiansen, Haas, Pawlowski, NSt\, arXiv:1411.7986
$\eta/s$ in Yang-Mills Theory

Direct sum:

$$\frac{\eta}{s}(T) = \frac{a}{\alpha_s^\gamma} + \frac{b}{(T/T_c)\delta}$$

High $T$: consistent with HTL-resummed pert. theory supporting quasiparticle picture

Small $T$: algebraic decay glueball resonance gas

$T_{\text{min}} = 1.26 T_c$

Value = 0.14

Christiansen, Haas, Pawlowski, NSt arXiv:1411.7986
More 2-Loop

- Consistent with 1-loop around $T_c$
- Dominant contribution from Maki-Thompson and Eight at large $T$
From YM to QCD in three simple steps

1. Replace $\alpha_s$; impose equality at $T_c$
   \[ \alpha_{s_{\text{YM}}}^{N_f=0} \big|_{T_c} = \alpha_{s_{\text{QCD}}}^{N_f=3} \big|_{T_c} \]

2. Genuine quark contributions to $\eta$ and $s$

3. Replace GRG by HRG
   ➤ Demir, Bass PRL 102 (2009) 172302
From YM to QCD in three simple steps

1. Replace $\alpha_s$; impose equality at $T_c$
   \[ \alpha_s^{N_f=0}|_{T_c} = \alpha_s^{N_f=3}|_{T_c} \]

2. Genuine quark contributions to $\eta$ and $s$

3. Replace GRG by HRG

Demir, Bass PRL 102 (2009) 172302

\[ T_{\text{min}} = 1.3 \ T_c \]
Value: 0.17
Outlook

Quark contributions

Require quark and gluon spectral functions in QCD
Outlook

Quark contributions

Require quark and gluon spectral functions in QCD

Shopping List

✓ Formalism set-up
✓ Quantitative results for $\eta/s$ in YM
☐ Bulk viscosity
☐ Relaxation times
☐ Application to non-relativistic systems e.g. ultracold atoms
Summary

- **QCD phase structure**
  - towards a quantitative continuum approach to QCD
  - ✔ Quantitative grip on fluctuation physics in the vacuum
  - ❏ finite temperature and density
Summary

• QCD phase structure
towards a quantitative continuum approach to QCD
  ✓ Quantitative grip on fluctuation physics in the vacuum
  ❑ finite temperature and density

• Spectral Functions
  new approach to analytical continuation problem
  ✓ tested in simple models (O(N), QM model)
  ❑ quark & gluon spectral functions, vector mesons, charmonium
Summary

• **QCD phase structure**
  towards a quantitative continuum approach to QCD
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  ❑ quark & gluon spectral functions, vector mesons, charmonium

• **Transport Coefficients**
  from loop expansion involving full propagators and vertices
  ✓ Global quantitative prediction for $\eta/s$ in YM theory
  ❑ Full QCD, bulk viscosity, relaxation times
Summary

• **QCD phase structure**
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  from loop expansion involving full propagators and vertices
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Thank you for your attention!
Backup
More Matter system?

- Mitter, Pawlowski, NSt arXiv:1411.7978
• Take into account all 8 tensor structures of the trans. projected vertex
• 5,7 most important (non-classical) tensor structures in the symmetric phase
• **but keep in mind gauge-invariance**

\[
\begin{align*}
[T^{(1)}_{\bar{q}Aq}]^\mu(p, q) &= \gamma^\mu, \\
[T^{(2)}_{\bar{q}Aq}]^\mu(p, q) &= -i(p - q)^\mu, \\
[T^{(3)}_{\bar{q}Aq}]^\mu(p, q) &= -i(\bar{q} - q)\gamma^\mu, \\
[T^{(4)}_{\bar{q}Aq}]^\mu(p, q) &= i(\bar{q} + q)\gamma^\mu, \\
[T^{(5)}_{\bar{q}Aq}]^\mu(p, q) &= (\bar{q} + q)(p - q)^\mu, \\
[T^{(6)}_{\bar{q}Aq}]^\mu(p, q) &= -(\bar{q} - q)(p - q)^\mu, \\
[T^{(7)}_{\bar{q}Aq}]^\mu(p, q) &= \frac{1}{2} [\bar{q}, q] \gamma^\mu, \\
[T^{(8)}_{\bar{q}Aq}]^\mu(p, q) &= -\frac{1}{2} [\bar{q}, q](p - q)^\mu,
\end{align*}
\]
STI-consistent expansion

Setting up a sensible truncation scheme:
• Use an expansion in terms of gauge-invariant operators
• Construct from combinations of covariant derivatives
STI-consistent expansion

Setting up a sensible truncation scheme:

• **Use an expansion in terms of gauge-invariant operators**
• **Construct from combinations of covariant derivatives**

• e.g. for the dominant (non-classical) structure in the chirally symmetric regime: gauge invariant operator

\[
\text{i} \sqrt{4\pi \alpha_s} \bar{q} \gamma_5 \gamma_\mu \epsilon_{\mu \nu \rho \sigma} \{ F_{\nu \rho}, D_\sigma \} q
\]

gives rise to contribution proportional to

\[
\frac{1}{2} T^{(5)}_{\bar{q} A q} + T^{(7)}_{\bar{q} A q}
\]
STI-consistent expansion

Setting up a sensible truncation scheme:
• Use an expansion in terms of gauge-invariant operators
• Construct from combinations of covariant derivatives

• e.g. for the dominant (non-classical) structure in the chirally symmetric regime: gauge invariant operator

$$i\sqrt{4\pi \alpha_s} \bar{q} \gamma_5 \gamma_\mu \epsilon_{\mu \nu \rho \sigma} \{F_{\nu \rho}, D_\sigma \} q$$

gives rise to contribution proportional to

$$\frac{1}{2} \mathcal{T}^{(5)}_{\bar{q} A q} + \mathcal{T}^{(7)}_{\bar{q} A q}$$

• Associated non-classical vertices are quantitatively important
4-Fermi Interactions

(a) Renormalisation group scale dependence of dimensionless four-fermi interactions, see App. B.2.c and bosonised $\sigma$-$\pi$ channel. Grey: respects chiral symmetry, blue: breaks $U(1)_A$, red: breaks $SU(2)_A$, magenta: breaks $U(2)_A$. 
Bosonizing the $\sigma$-$\pi$ channel only is sufficient to remove divergence
In the vacuum: other channels not quantitatively relevant
Euclidean Iteration

- Helmboldt, Pawlowski, NSt arXiv:1409.8414
Why momentum dependence?

Quantitative precision
Why momentum dependence?

Quantitative precision

• QCD perspective on low-energy effective models:
  ✓ **UV parameters fixed** by QCD flows
    ➢ Talks by L. Fister, M. Mitter, J. Pawlowski, F. Rennecke
  ✓ Increase in predictive power
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• Models have to be treated **quantitatively**
  ✓ Full effective potential (grid or fixed Taylor expansion)
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• **Benchmark** of popular **truncation schemes** (LPA and LPA’)

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• **Benchmark** of popular **truncation schemes** (LPA and LPA’)

• Momentum dependence crucial for critical physics
Euclidean Iteration I

Momentum dependence of 2-point functions in an iterative procedure

Example: mesonic propagators in a quark meson model

\[
\partial_t U_k = \frac{1}{2} \quad - \quad \text{Diagram}
\]

\[
\partial_t \Delta \Gamma_k^{(2)}(p^2) = \begin{bmatrix}
- \frac{1}{2} \quad - \quad \text{Diagram}
\end{bmatrix}
\]

\[
- 2 \quad - \quad \text{Diagram} \quad - \quad \text{Diagram}
\]

\[
- p \rightarrow 0
\]
Euclidean Iteration I

Momentum dependence of 2-point functions in an iterative procedure
Example: mesonic propagators in a quark meson model

\[ \partial_t U_k = \frac{1}{2} \]

\[ \partial_t \Delta \Gamma_k^{(2)}(p^2) = \]

\[ -2 - \begin{matrix} q \times \overline{p} - \overline{p} \times q \\ p + q \end{matrix} - \begin{matrix} q \times \overline{p} - \overline{p} \times q \\ p + q \end{matrix} - \frac{1}{2} \]

momentum-independent vertices from eff. potential
Euclidean Iteration I

Momentum dependence of 2-point functions in an iterative procedure

Example: mesonic propagators in a quark meson model

\[ \partial_t U_k = \frac{1}{2} \]

\[ \partial_t \Delta \Gamma^{(2)}_k (p^2) = \]

**Iteration procedure**

\[ \Gamma_{cl,k}^{(2)} \rightarrow U_k \rightarrow \Gamma_k^{(2)} \rightarrow U_k \rightarrow \Gamma_k^{(2)} \rightarrow \cdots \]

#0

#1

#2
Euclidean Iteration II

- **Numerically inexpensive upgrade** for existing Euclidean calculations
- Here: **Quark-meson model at finite T; fixed ren. Yukawa coupling**
- **4d exponential** regulator function
Euclidean Iteration II

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- Here: **Quark-meson model at finite T; fixed ren. Yukawa coupling**
- **4d exponential** regulator function

**Convergence properties:**

| step | $m_{\text{cur}}$ [MeV] | $m_{\text{pol}}$ [MeV] | $\sigma_{\text{min}}$ [MeV] |
|------|------------------------|--------------------------|-----------------------------|
| 0    | 412.8                  | 412.8                    | 16.8                        |
| 1    | 144.8                  | 142 ± 2                  | 83.5                        |
| 2    | 136.4                  | 135 ± 2                  | 91.8                        |
| 3    | 135.1                  | 134 ± 2                  | 93.1                        |
| 4    | 134.9                  | 133 ± 2                  | 93.2                        |
| 5    | 134.9                  | 133 ± 2                  | 93.2                        |
Mass Definitions

Renormalized 2-point function:

\[ \bar{\Gamma}^{(2)}(p_0, \bar{p}^2) = \Gamma^{(2)}(p_0, p^2) / \bar{Z} \]
Renormalized 2-point function:

\[ \Gamma^{(2)}(p_0, \vec{p}^2) = \frac{\Gamma^{(2)}(p_0, \vec{p}^2)}{\tilde{Z}} \]

Pole mass:

\[ \Gamma^{(2)}(im_{pol}, 0) = 0 \]

Temporal screening:

\[ T \sum_{p_0} \Gamma^{(2)}(p_0, 0)^{-1} e^{ip_0 t} \sim e^{-m_{pol}|t|} \]
Mass Definitions

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**Screening mass:** \[ \bar{\Gamma}^{(2)}(0, -m_{scr}^2) = 0 \]

**Spatial screening:** \[ \int d^3p \ \Gamma^{(2)}(0, p^2)^{-1} e^{ipx} \sim e^{-m_{scr}|x|} \]
### Mass Definitions

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**Curvature mass:**

\[ \bar{\Gamma}^{(2)}(0, 0) = m_{cur}^2 \]

No physical observable; dependent on renormalization procedure, parameterization of the propagator.
Mass Definitions

Renormalized 2-point function:
\[ \tilde{\Gamma}^{(2)}(p_0, \vec{p}^2) = \Gamma^{(2)}(p_0, \vec{p}^2) / \tilde{Z} \]

**Pole mass:**
\[ \tilde{\Gamma}^{(2)}(im_{pol}, 0) = 0 \]

**Temporal screening:**
\[ T \sum_{p_0} \Gamma^{(2)}(p_0, 0)^{-1} e^{ip_0 t} \sim e^{-m_{pol} |t|} \]

**Screening mass:**
\[ \tilde{\Gamma}^{(2)}(0, -m_{scr}^2) = 0 \]

**Spatial screening:**
\[ \int d^3 p \Gamma^{(2)}(0, \vec{p}^2)^{-1} e^{i\vec{p}\vec{x}} \sim e^{-m_{scr} |x|} \]

**Curvature mass:**
\[ \tilde{\Gamma}^{(2)}(0, 0) = m_{cur}^2 \]
No physical observable; dependent on renormalization procedure, parameterization of the propagator

**Onset mass:** Silver Blaze property links mass to critical chemical potential; coincides with pole mass
Physics Results

From converged iteration

![Graph showing the relationship between $m_\pi$ and $T$ with lines for $m_{cur}$, $m_{pol}$, and $m_{scr}$]
Physics Results

From converged iteration

$T=0$

by O(4) invariance

$m_{pol} = m_{scr}$

$m_{pol} \approx m_{cur}$

$m_{cur}^2 = \frac{Z_{\parallel}(p_0=im_{pol}, \hat{p}^2=0)}{Z} m_{pol}^2$
Physics Results

From converged iteration

\[ m_{\text{cur}} \]
\[ m_{\text{pol}} \]
\[ m_{\text{scr}} \]

\[ T = 0: \]

\[ m_{\text{pol}} = m_{\text{scr}} \] by O(4) invariance

\[ m_{\text{pol}} \approx m_{\text{cur}} : m_{\text{cur}}^2 = \frac{Z_{\|}(p_0 = i m_{\text{pol}}, \vec{p}^2 = 0)}{Z} m_{\text{pol}}^2 \]

\[ T > 0: \]

\[ \frac{m_{\text{pol}}^2}{m_{\text{scr}}^2} = \frac{Z_{\perp}(p_0 = 0, \vec{p}^2 = -m_{\text{scr}}^2)}{Z_{\|}(i m_{\text{pol}}, \vec{p}^2 = 0)} \]
LPA: Mismatches of Fluctuation Scales

More than an academic exercise...

\[ m_{\text{cur}} \approx m_{\text{pole}} = m_{\text{ons}} \]

\[
\left[ \frac{\mu_c}{T_c} \right]_{\text{full}} / \left[ \frac{\mu_c}{T_c} \right]_{\text{LPA}} \approx \left[ \frac{m_{\text{cur}}}{m_{\text{ons}}} \right]_{\text{LPA}} \approx 1.4
\]
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in the full calculation
but large deviations in LPA

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quantum fluctuations

density fluctuations

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LPA: Mismatches of Fluctuation Scales

More than an academic exercise...

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Quantum fluctuations
Density fluctuations

Rough estimate:

\[
\left[ \frac{\mu_c}{T_c} \right]_{\text{full}} / \left[ \frac{\mu_c}{T_c} \right]_{\text{LPA}} \approx \left[ \frac{m_{\text{cur}}}{m_{\text{ons}}} \right]_{\text{LPA}} \approx 1.4
\]

- mismatch of fluctuation scales
  \[ \Rightarrow \] large systematic errors at finite \( \mu \) (curvature, CEP)
- resolved by including momentum dependence
Comparison: Fixed UV

QCD perspective

- LPA with these initial conditions => no $\chi$SB
- Full calculation and LPA' in quantitative agreement
LPA’ Comparison

- **LPA’** includes only a **scale-dependent Z**
- Very good approximation to the full calculation (**deviation < 3 %**)
- Upgrade: calculate momentum dependence on LPA’ solution (1 step)
Reasonably good agreement at $\mu=0$ (in terms of relative scales)
But in LPA still **large systematic error at finite $\mu$**
Spectral Functions

- Kamikado, NSt, von Smekal, Wambach; Eur.Phys.J. C74 (2014) 2806
- Tripolt, NSt, von Smekal, Wambach; Phys.Rev. D89 (2014) 034010
QM Model at $\mu > 0$
QM Model at $\mu > 0$

$\mu=200$ MeV

$[\Lambda^2_{U,V}]$

$0 \quad 100 \quad 200 \quad 300 \quad 400 \quad 500 \quad 600 \quad 700 \quad \omega$ [MeV]

$1: \sigma^* \rightarrow \sigma \sigma$

$2: \sigma^* \rightarrow \pi \pi$

$3: \sigma^* \rightarrow \bar{\psi} \psi$

$4: \pi^* \rightarrow \sigma \pi$

$5: \pi^* \pi \rightarrow \sigma$

$6: \pi^* \rightarrow \bar{\psi} \psi$
QM Model at $\mu > 0$
QM Model at $\mu > 0$
QM Model at $\mu > 0$

\[
\frac{\Lambda_{\text{UV}}^2}{\Gamma}
\]

\[
\mu=400\text{ MeV}
\]

1: $\sigma^* \rightarrow \sigma \sigma$
2: $\sigma^* \rightarrow \pi \pi$
3: $\sigma^* \rightarrow \bar{\psi} \psi$
4: $\pi^* \rightarrow \sigma \pi$
5: $\pi^* \pi \rightarrow \sigma$
6: $\pi^* \rightarrow \bar{\psi} \psi$

Graph showing values of $\rho_\pi$ and $\rho_\sigma$.
Going beyond...

So far: 3d regulator function  \( R = \vec{p}^2 r(\vec{p}^2) \)
Going beyond...

So far: 3d regulator function \( R = \tilde{p}^2 r(\tilde{p}^2) \)

**Generalization:**
- Either regulators which allow to perform Matsubara sums analytically
  - Floerchinger; JHEP 1205 (2012) 021
Going beyond…

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- Either regulators which allow to perform Matsubara sums analytically
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- Or **fully numerical procedure**- perform Matsubara sum numerically
  - Require: analytical regulator function for complex momenta
  - for free: finite chemical potential
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Generalization:

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  - Require: analytical regulator function for complex momenta
  - for free: finite chemical potential

Shopping list:

- Proper regulator for complex external momenta
- Suitable for numerical applications
- Analytical functions
- Analytical structure of regularized propagator: as few poles as possible
  - Require pole procedures to obtain the correct real-time result
4d Spectral Functions

\[
\text{4d N=2 exponential regulator, } \epsilon=0.1 \text{ MeV}
\]

\[
\begin{align*}
\rho_\pi & \\
\rho_\sigma &
\end{align*}
\]

Preliminary

Pawłowski, NSt [in prep.]