Applicability analysis of STM and SMCFT of RC beams with shearm span-depth ratios ≤ 3.0

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Abstract. The shear capacity of reinforced concrete (RC) beams is significantly affected by the shear span-depth ratio (a_v/d). The strut-and-tie model (STM) and simplified modified compression field theory (simplified MCFT) that are utilized in current design codes have both proven to be effective in the shear capacity analysis and the design of RC beams. In order to compare the validity and applicability of the two approaches within the range of a_v/d ≤ 3.0, 160 RC beams with transverse reinforcements were collected and their shear capacities were evaluated using the two models respectively. The results demonstrated that the ratio of a_v/d = 1.8 was the critical point that separated the ranges in which each approach was better able to predict the shear capacity. More specifically, it could be concluded that using the STM approach in the range a_v/d ≤ 1.8 and the simplified MCFT approach in the range 1.8 < a_v/d ≤ 3.0 respectively would result in more rational and accurate shear strength predictions.

1. Introduction

The strut-and-tie model (STM) (Figure 1) and the simplified modified compression field theory (simplified MCFT, SMCFT) utilized in current design codes have been proven to be effective in the shear capacity analysis and design for reinforced concrete (RC) beams. The STM approach was adopted by the AASHTO LRFD [1] and the ACI 318-19 [2] to design deep beams or the D-region (“deep” or “disturbed” region) of normal RC members. The STM is a simple equilibrium model which idealizes the complex flow of stresses in deep beam as axial elements in a truss model, based on the lower-bound solution of the plasticity theory [3]. On the other hand, the MCFT approach, put forward by Vecchio and Collins, considers the cracked concrete as a new material with its own stress-strain characteristics [4]. Equilibrium, compatibility, and stress-strain relationships are formulated in terms of average stresses and average strains. Since the MCFT approach takes 22 steps to complete the shear strength prediction, a simplified modified compression field theory was proposed by Bentz et al. [5] and it only takes 6 steps [6]. In addition to simplify the procedure, the method provides excellent predictions of shear strength with larger shear span-depth ratios. Thus, the simplified MCFT approach is getting more and more attention and has been adopted by CSA A23.3 [7] for shear design.

The STM approach takes the essential effect of the ratio of a_v/d on shear capacity into consideration and it is applied in the deep beams where a_v/d ≤ 3.0. However, the ratio of a_v/d was not taken into account in the derivation process of the simplified MCFT approach. Additionally, a reasonable lower limit for the shear span-depth ratio in the application of the simplified MCFT approach has not been found so far. In order to compare the validity of the STM approach and the simplified MCFT approach when they are applied to RC beams, and find out each approach’s applicable range, this paper
evaluates the shear capacities of 160 reinforced concrete beams with the ratio of \( a/v \leq 3 \) using the STM approach and the simplified MCFT approach respectively.

![Figure 1. Sketch of STM model.](image)

2. Modelling procedures

2.1. Strut-and-Tie model

2.1.1. Model setup. In the STM, a single-span deep beam consists of two inclined concrete struts which resist the compressive stress fields, and a horizontal steel tie which resists the tensile stress fields (Figure 2). The strut and tie are joined together at the nodes which transfer forces from the struts to the tie and supports [3] The STM approach is essentially a truss analogy [8]. This approach would fail in any scenario of the crushing of the concrete struts, the yielding of the ties, the failure of the nodal zones, or the anchorage failure of the reinforcement ties. For each applicable load combination, the design strength of each strut, tie, and nodal zone in a STM shall satisfy [2]

\[
F_n \leq \phi F_n^r
\]  

where \( F_n \) refers to the reaction force of the strut, tie, or nodal face due to the load; \( F_n^r \) is the nominal strength of the strut, tie or nodal face; \( \phi \) is the strength reduction factor, which is specified as 0.75 in the ACI 318-19 [2] and not taken into consideration during the calculation of the predicted shear strength in this paper.

For the tie, the nominal tensile strength is determined by the following formula:

\[
F_n^t = A_s f_y
\]

where \( A_s \) is the total area of the longitudinal reinforcements and \( f_y \) is the yield strength of the longitudinal reinforcement.

The nominal compressive strength of the strut \( F_{ns} \) is calculated by

\[
F_{ns} = A_{cs} f_{ce}
\]

where \( A_{cs} \) is the cross-sectional area of the strut, which is equal to the width of the strut multiplied by the width of beam; \( f_{ce} \) is the effective compressive strength of the concrete in the strut, which is calculated by

\[
f_{ce} = 0.85 \beta_c \beta_s f'_c
\]

where \( \beta_c \) is the strut and node confinement modification factor, \( \beta_s \) is the strut coefficient.

In order to realize the force balance, each node zone has at least 3 forces. The nodes are named according to the loads acting on the three nodal faces either CCC, CCT, or CTT, where C and T
represent the compressive and tensile force respectively [9]. The nominal compressive strength of the nodal zone, \(F_{nn}\), is calculated by

\[
F_{nn} = A_{nn} f_{ce}
\]  

(5)

where \(A_{nn}\) is the area of the nodal face, which is equal to the width of the node area multiplied by the width of the beam; \(f_{ce}\) refers to the effective compressive strength of the concrete in the nodal zone, which is calculated by

\[
f_{ce} = 0.85 \beta_n \beta_c f'_c
\]  

(6)

where \(\beta_n\) is the nodal zone coefficient; \(\beta_n=1.0\) for the nodal zone bounded by struts, bearing areas, or both; \(\beta_n=0.8\) for the nodal zone anchoring one tie; \(\beta_n=0.6\) for the nodal zone anchoring two or more ties.

### 2.1.2. Some definitions and parameters outside of ACI 318-19.

All of the above are definitions and formulas given in the ACI 318-19 [2]. However, only using these cannot predict the shear capacity of RC beams properly. Several important definitions and parameters should also be considered.

The ACI 318-19 specifies the limitation of the angle between the tie and strut as \(\theta \geq 25^\circ\) in order to prevent excessive strains from occurring in the lower longitudinal reinforcement and excessive cracks from forming in order to avoid failure of the beam [10]. According to the ACI 318-19 [2] and the ACI 445 [11], the beams are controlled by an arch load transfer mechanism which is also called a direct model or one-panel model when \(a_v/d < 1.8\) (\(a/z = 2.0\), \(z = 0.9d\), \(d = 0.9h\), and \(h = \text{depth}\)) (Figure 2). And when \(a_v/d > 1.8\), the beams follow the truss load transfer mechanism, which is also called a two-panel model (Figure 3).

### 2.2. Simplified MCFT

To predict the shear strength of RC beams using the simplified MCFT there are 6 steps [6]:

Step 1: Estimate the longitudinal strain, \(\varepsilon_x\).

Step 2: Calculate the effective crack spacing \(s_{xe}\) by

\[
s_{xe} = \frac{35s_x}{a_g + 16}
\]  

(7)

where \(s_x\) is measured as the vertical distance of the reinforcements arranged in the x direction; \(a_g\) is the largest size of aggregates in the concrete. If the concrete members contain more stirrups than the specified minimum configuration, then it can be assumed that the crack spacing is well controlled. Thus, the effective crack spacing \(s_{xe}\) can be chosen as 300 mm conservatively [6].
Step 3: Calculate the parameter $\beta$ and the average angle of the crack $\theta$ using the following equations [5]:

$$\beta = \frac{0.4}{1+1500\varepsilon_x} \times \frac{1300}{1000+\varepsilon_s}$$

(8)

$$\theta = (29 + 7000\varepsilon_x)(0.88 + \frac{s_{sw}}{2500}) \leq 75^\circ$$

(9)

Step 4: Calculate the shear stress $\nu$ according to Eq. (10):

$$\nu = f_i \cot \theta + \rho_s f_v \cot \theta$$

(10)

where $\rho_s$ is the stirrup ratio.

Step 5: Calculate the shear capacity of the beam according to

$$V = \nu b_v z$$

(11)

where $\nu$ is the shear stress of the cross section; $b_v$ is the width of the beam; $z$ is the internal lever arm of the beam. However, the shear stress is not only distributed evenly within the height of the internal lever arm, it is distributed evenly over the entire effective depth of the beam. Therefore, $z$ was replaced by the effective depth $d$ when calculating the shear capacity in this paper.

Step 6: Calculate the longitudinal strain using another formula as following where $\varepsilon_x$ is conservatively taken as half of the tensile strain of the longitudinal reinforcement, which is caused by the bending moment and shear force. According to the results of calculation in Eq. (12), return to the first step to modify the estimated value of $\varepsilon_x$ until the two values converge.

$$\varepsilon_x = 0.5 \times \left( \frac{M/z}{E_s A_s} + \frac{V}{E_s A_s} \right) = \frac{V + M/z}{2E_s A_s}$$

(12)

where $M$ is the bending moment caused by the load in the middle of the shear span, $M=V\times0.5a$; $E_s$ is the elastic modulus of the reinforcement; $A_s$ is the total area of the longitudinal reinforcements.

### 3. Data analysis and discussion

The purpose of this paper is to compare the validity and applicability of the STM approach and the simplified MCFT approach with the ratio of $a_v/d \leq 3.0$. The shear capacities of 160 collected RC beams with transverse reinforcements were evaluated by the two approaches respectively. There were 90 beams with $a_v/d < 1.8$ and 70 beams with $1.8 < a_v/d \leq 3.0$.

The rationality of the proposed critical point would be further demonstrated by using the guarantee rate ($G$) which is the extent to which the model's predicted shear strength deviates from the experimental shear strength within the required range. The equation is

$$G = \sum \left[ \frac{|V_{pre} - V_{exp}|}{V_{exp}} < A \right] / B$$

(13)

where $A$ is the extent of the deviation of the predicted shear strength from the experimental shear strength, which is either 0.8, 0.5, or 0.3 in this paper. As for $B$, it is the number of beams in the different ranges of the shear span-depth ratio. When $a_v/d < 1.8$, $B = 90$; when $a_v/d$ is between 1.8 and 3.0, $B = 70$; $B = 160$ when $a_v/d < 3.0$.

As presented in Table 1 and Figure 4, the difference between the guarantee rates in the STM approach is not very large around the recommended critical point. More specifically, the guarantee rate within the range of $a_v/d < 1.8$ is slightly higher by comparison. This is because the selected mechanism, STM approach, is more in line at $a_v/d < 1.8$ with the shear transfer mechanism in the disturbed region. However, the guarantee rate of the simplified MCFT approach varies greatly around the recommended point. It’s primarily because of this that the simplified MCFT approach seriously underestimates the shear contribution of concrete due to the arch action when the ratio of $a_v/d$ is very small. In summary, it is reasonable to choose the ratio of $a_v/d = 1.8$ as the critical point of the shear span-depth ratio between the two approaches.
Table 1. Guarantee rate and security guarantee rate.

| Shear Model | \( a_v/d \) | \( G (A=0.8) \) | \( G (A=0.5) \) | \( G (A=0.3) \) | \( G_s (A=0.8) \) | \( G_s (A=0.5) \) | \( G_s (A=0.3) \) |
|-------------|-------------|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|
| STM         | \( a_v/d < 3.0 \) | 1.000           | 0.881           | 0.563           | 0.994           | 0.875           | 0.563           |
|             | \( a_v/d < 1.8 \) | 1.000           | 0.944           | 0.611           | 1.000           | 0.944           | 0.611           |
|             | \( a_v/d > 1.8 \) | 1.000           | 0.800           | 0.500           | 0.971           | 0.771           | 0.486           |
| SMCFT       | \( a_v/d < 3.0 \) | 0.969           | 0.688           | 0.531           | 0.756           | 0.488           | 0.356           |
|             | \( a_v/d < 1.8 \) | 0.944           | 0.456           | 0.256           | 0.833           | 0.356           | 0.167           |
|             | \( a_v/d > 1.8 \) | 1.000           | 0.986           | 0.886           | 0.643           | 0.643           | 0.586           |

Figure 4. Comparison of the guarantee rates between STM and SMCFT: (a) Guarantee rate; (b) Security guarantee rate.

By statistically analyzing the shear control regions of 160 beams (Figure 5), it is found that the shear control region would shift as the shear span-depth ratio changed. As presented in Figure 5(a), the shear control region is mostly the horizontal nodal face of Node B, so using the direct model in this case will result in more conservative results. As shown in Figure 5(b), the shear control region is mostly the vertical tie, a reasonable explanation for this case is that it is not advisable to use a two-panel model when the vertical web reinforcement is very small. Then, all of the shear capacities of 160 beams were evaluated by using the direct model; the results are presented in Figure 5(c). It was found that the predicted shear strengths of beams with \( a_v/d > 1.8 \) indeed get closer to the experimental value, and the degree of dispersion gets much lower.
Figure 5. Transformation of shear control region along with various $a_v/d$ ratios: (a) Direct model ($a_v/d < 1.8$); (b) Two-panel model; (c) Direct model ($a_v/d > 1.8$).

4. Conclusion

In this paper, the shear capacities of 160 reinforced concrete beams with a ratio of $a_v/d$ less than 3.0 were evaluated using the STM approach and the simplified MCFT approach respectively. It shows that neither approach could preserve the identical accuracy with all shear span-depth ratios, but their advantage would be acquired at a given range according to the analysis. It was demonstrated that using the direct STM approach and the simplified MCFT approach in the ranges of $a_v/d \leq 1.8$ and $1.8 < a_v/d \leq 3.0$ respectively would get more accurate results. When the ratio of $a_v/d$ is very small ($a_v/d < 1.0$) and the concrete strength is small, using the direct model to predict the shear capacity would be more conservative.

Acknowledgments

The authors would like to express thanks to the National Natural Science Foundation of China (Project No: 51878415, 51678365 and 51908373) for funding this research.

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