Exploiting an Oracle that Reports AUC/2AFC Scores in Machine Learning Contests

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Abstract

In machine learning contests such as the Large Scale Visual Recognition Challenge [1] and the KDD Cup [2], contestants can submit candidate solutions and receive from an oracle (typically the organizers of the competition) the accuracy of their guesses compared to (a subset of) the ground-truth labels. A commonly used accuracy metric for binary classification tasks is the Area Under the Receiver Operating Characteristics Curve (AUC), which is equivalent to the probability of correct response in a 2-Alternative Forced Choice (2AFC) task [3] in which the classifier must distinguish one positively labeled example from one negatively labeled example. In this paper we illustrate how knowledge of the 2AFC score \(c\) of a set of guesses constrains the set of possible labelings that the dataset can have. We then show that the worst-case number of possible binary labelings of \(n\) examples corresponding to any particular 2AFC score \(c\) w.r.t. some candidate solution is \(O(2^n)\).

1 Computing the 2AFC Score

Let \(y_1, \ldots, y_n\) represent the ground-truth binary labels of a dataset of \(n\) examples and let \(\hat{y}_1, \ldots, \hat{y}_n\) represent the contestant’s real-valued guesses for the labels. To simplify our analysis in this paper, we assume that there are no ties, i.e., each \(\hat{y}_i\) is unique. In this case, the 2AFC score for the real-valued guesses w.r.t. the true binary labels can be computed as:

\[
2\text{AFC} = \frac{1}{|\mathcal{Y}^-||\mathcal{Y}^+|} \sum_{i \in \mathcal{Y}^-} \sum_{j \in \mathcal{Y}^+} I[\hat{y}_i < \hat{y}_j]
\]

where \(\mathcal{Y}^- = \{i : y_i = 0\}\) is the index set of the negatively labeled examples and \(\mathcal{Y}^+ = \{j : y_j = 1\}\) is the index set of the positively labeled examples. As is evident in Equation 1, all that matters to the 2AFC is the relative ordering of the \(\hat{y}_i\), not their exact values.

2 Exploiting the 2AFC Score – A Simple Example

Consider a tiny dataset containing 4 examples with ground-truth binary labels \(y_1, y_2, y_3, y_4\). Suppose that a contestant in a machine learning contest generates real-valued guesses \(\hat{y}_1 = 0.5\), \(\hat{y}_2 = 0.6\), \(\hat{y}_3 = 0.9\), and \(\hat{y}_4 = 0.4\), which he/she submits to the oracle. If the oracle replies that this candidate solution has a 2AFC of 0.75, then the contestant can conclude with complete certainty that the true solution is \(y_1 = 1\), \(y_2 = 0\), \(y_3 = 1\), and \(y_4 = 0\) because this is the only labeling satisfying Equation 1. The contestant can then submit these “guesses” as his/her next submission and receive a perfect score.

This toy example raises the question: for a dataset of size \(n\) and some candidate real-valued solution \(\hat{y}_1, \ldots, \hat{y}_n\), how many possible ground-truth binary labelings (mappings from \(\{1, \ldots, n\} \rightarrow \{0, 1\}\)) are there that generate a 2AFC score of \(c\)? Is the number of such labelings small enough to enable some simple method of cheating?

3 The Number of Solutions with a Particular 2AFC Score

To simplify the analysis, we assume that the size of the dataset is a multiple of 4, i.e., \(n = 4m\) for some integer \(m\). We first show that we can construct \(O(2^m)\) possible ground-truth labelings for which a candidate solution \(\{\hat{y}_i\}\) obtains a 2AFC of \(c = 0.5\).

Without loss of generality, suppose that the indices are arranged such that the \(\hat{y}_i\) are sorted, i.e., \(\hat{y}_1 < \ldots < \hat{y}_{4m}\). Since the 2AFC is invariant under monotonic transformations of the real-valued guesses, we can represent each \(\hat{y}_i\) simply by its index \(i\).
Now we can construct one labeling in the following way:

| Guess $y_i$ | 1 | ... | $m$ | $m+1$ | ... | $2m$ | $2m+1$ | ... | $3m$ | $3m+1$ | ... | $4m$ |
|-------------|---|-----|----|------|-----|-----|-------|-----|-----|-------|-----|-----|
| Label $y_i$ | 0 | ... | 0  | 1    | ... | 1    | 1     | ... | 1    | 0     | ... | 0    |

Note that this labeling is symmetric in the sense that $y_i = l \iff y_{4m-i+1} = l$. The 2AFC for any symmetric labeling is 0.5 because for any index pair $(i, j)$ for which $y_i = 0$ and $y_j = 1$ and for which $i < j$ (and hence $y_i < y_j$), we can find exactly one other pair $(i', j') = (4m - i + 1, 4m - j + 1)$ for which $y_{i'} = 0$ and $y_{j'} = 1$ and for which $i' > j'$. Since the pair $(i, j)$ is correctly classified whereas the pair $(i', j')$ is incorrectly classified, and since overall half the pairs consisting of one positively labeled example and one negatively labeled example are classified correctly, the 2AFC is 0.5.

Given the fact that any symmetric labeling has a 2AFC of 0.5, we can construct more labelings similar to the example above simply by varying the positions of the positively labeled indices (as long as we preserve symmetry). Since the dataset contains $n = 4m$ examples, then each “half” contains $2m$ examples, and we can vary the positions of $m$ positively labeled examples within each half. There are $\binom{2m}{m}$ such constructions, each of which gives a 2AFC of 0.5. It can be shown using Stirling’s approximation that $\binom{2m}{m} = O(2^m)$, and then $O(2^m) = O(2^{n/4}) = O(2^n)$. Since for any dataset containing $n$ examples with binary labels there are $2^n$ possible labelings, we conclude that the worst-case number of possible labelings for any 2AFC $c$ is $O(2^n)$.

References

[1] ILSVRC Organizers. [http://www.image-net.org/challenges/LSVRC/announcement-June-2-2015](http://www.image-net.org/challenges/LSVRC/announcement-June-2-2015)

[2] KDD CUP 2015 Organizers. [https://www.kddcup2015.com/submission-rules.html](https://www.kddcup2015.com/submission-rules.html)

[3] C. Tyler and C.-C. Chen. Signal detection theory in the 2AFC paradigm: attention, channel uncertainty and probability summation. *Vision Research*, 40(22):3121–3144.