Self-consistent quantum effects in the quark meson coupling model

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We derive the equation of state of nuclear matter including vacuum polarization effects arising from the nucleons and the sigma mesons in the quark-meson coupling model which incorporates explicitly quark degrees of freedom with quark coupled to the scalar and vector mesons. This leads to a softer equation of state for nuclear matter giving a lower value of incompressibility than would be reached without quantum effects. The in-medium nucleon and sigma meson masses are also calculated in a self-consistent manner.

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I. INTRODUCTION

Usually the frame of Quantum Hadrodynamics (QHD) [1] is the departure to the study of the nuclear many-body problem describing nucleons interacting with scalar and vector mesons. This meson field theory has quite successfully described the properties of the nuclear matter and finite nuclei using the mean field approximations for the meson fields. The vacuum polarization corrections arising from the nucleon fields as well as the meson fields have also been considered to study the nuclear matter. This is one way of obtaining a softer equation of state yielding a lower compressibility than would be reached without quantum effects.

While descriptions of the nuclear phenomena have been efficiently formulated using some hadronic degrees of freedom as in QHD, there have been interesting observations which reveals the medium modification of the internal structure of the nucleon. For this, degrees of freedom from the fundamental theory of strong interacting systems, QCD, are expected to be considered. Due to the complex structure of this theory we are lead to formulate effective models which have the main properties and symmetries of QCD as chiral symmetry and its spontaneously symmetry breaking. One of the first models put forward along these lines was the quark-meson coupling (QMC) [2,3] model. It is an extension of QHD which includes the explicit quark structure of the baryons. This model describes nuclear matter with nucleons as non-overlapping MIT bags containing quarks inside them which interact with the scalar and the vector mesons. Recently, the model has been extended to finite temperatures [4], neutron stars [5] as well as to study beyond the mean field level by the incorporation of the exchange terms [6,7]. The density dependent bag constant has also been investigated within this frame [8].

In the present work, the vacuum polarization corrections, described in ref. [9,10], are included in the quark meson coupling model taking into account the quantum fluctuations of the scalar field (as done in [11]) self-consistently as well. A similar approach is being investigated in the frame of QHD [12]. The equation of state for dense matter is derived. We organize the paper as follows: In section II, we derive the EOS for dense matter including the quantum effects arising from the nucleons and the σ-mesons within QMC model. In section III, we discuss in detail the numerical results obtained in the present work and discuss possible outlook.

II. THEORY

The details of the QMC model have been given in Ref. [2–4]. Since we now include the vacuum polarization effects, we give here a few important steps for completeness.

In this model, the nucleon in nuclear matter is assumed to be described by a static MIT bag in which quarks interact with the scalar (σ) and the vector (ω) mesons. The quark field ψ_q(\vec{r},t) inside the bag then satisfies the equation

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\[
\left[i\gamma^\mu \partial_\mu - (m^0_q - g^2_\sigma) - g^2_\omega \gamma^0\right] \psi_q(\vec{r}, t) = 0,
\]
(1)

where \(m^0_q\) is the current quark mass and \(g^2_\sigma\) and \(g^2_\omega\) are the quark couplings with the \(\sigma\) and \(\omega\) mesons.

The normalized ground state for a quark (in an \(s\)-state) in the bag, which has a radius \(R\), is given as

\[
\psi_q(\vec{r}, t) = N \exp\left(-i \frac{\epsilon_q t}{R}\right) \left(i \beta_q \hat{\sigma} \cdot \hat{r} j_1(\hat{r} x/R)\right) \frac{\chi_q}{\sqrt{4\pi}},
\]
(2)

where \(x\) is the dimensionless quark momentum and the single particle quark energy, in units of \(R^{-1}\), is

\[
\epsilon_q = \Omega_q + g^2_\omega \omega R, \quad \beta_q = \frac{\Omega_q - R m^*_q}{\Omega_q + R m^*_q},
\]
(3)

with \(\Omega_q = (x^2 + R^2 m^*_q)^{1/2}\); \(m^*_q = m^0_q - g^2_\sigma\) is the effective quark mass, \(\chi_q\) is the quark spinor and \(N\) is the normalization factor.

The boundary condition at the bag surface is given by:

\[
i\gamma \cdot \hat{n} \psi_q = \psi_q.
\]
(4)

This, for the ground state, reduces to

\[
 j_0(x) = \beta_q j_1(x)
\]
(5)

which determines the eigen frequency, \(x\), of this lowest mode in the medium. The form of the quark wavefunction in equation (3) is almost identical to that of the solution in free space. However the parameters in the expression have been substantially modified by the surrounding nuclear medium. Thus the quarks in the nucleon embedded in the nuclear medium are more relativistic than those in a free nucleon.

The energy of the nucleon bag is

\[
M^* = 3\frac{\Omega_q}{R} \frac{Z}{R} + \frac{4}{3} \pi R^3 B,
\]
(6)

where \(B\) is the bag constant and \(Z\) parametrizes the sum of the center-of-mass (c.m.) motion and the gluonic corrections. Note that this center-of-mass treatment is different from that of Jin and Jennings [8]. The bag radius \(R\) is that which minimizes the nucleon bag energy through

\[
\frac{\partial M^*}{\partial R} = 0.
\]
(7)

We now proceed to study the equation of state (EOS) for nuclear matter including the vacuum polarisation effects from nucleon and sigma mesons at zero temperature. The details of the theory have already been discussed in ref. [10]. Only a few important steps are given here. The energy density after subtracting out the pure vacuum contribution then becomes

\[
\epsilon_0 = \epsilon_{MFT} + \Delta \epsilon,
\]
(8)

with

\[
\epsilon_{MFT} = \frac{g}{(2\pi)^3} \int_{|k|<k_F} dk \left(k^2 + M^{*2}\right)^{1/2} + \frac{1}{2} m^2_\sigma \sigma_0^2 + \frac{1}{2} m^2_\omega \omega_0^2,
\]
(9)

and

\[
\Delta \epsilon = -\frac{g}{(2\pi)^3} \int dk \left[(k^2 + M^{*2})^{1/2} - (k^2 + M^2)^{1/2}\right] - \frac{g_{\sigma} \sigma_0 M}{(k^2 + M^2)^{1/2}}.
\]
(10)

The above expression for the energy density is divergent. After renormalisation by adding the counter terms [13], we have the expression for the finite renormalised energy density,

\[
\epsilon_{ren} = \epsilon_{MFT} + \Delta \epsilon_{ren},
\]
(11)
where

\[ \Delta \epsilon_{\text{ren}} = -\frac{g}{16\pi^2} \left[ M^{*4} \ln \left( \frac{M^{*}}{M} \right) + M^3(M - M^*) - \frac{7}{2} M^2(M - M^*)^2 + \frac{13}{3} M(M - M^*)^3 - \frac{25}{12} M(M - M^*)^4 \right]. \] (12)

The baryon density is given by

\[ \rho_B = \frac{g_R^3}{6\pi^2} \] (13)

In the above, \( g \) is the spin-isospin degeneracy factor which is equal to 4 for nuclear matter and to 2 for neutron matter.

Next, we consider the quantum corrections due to the scalar mesons. Including a quartic scalar self-interaction, the Hamiltonian density for the scalar mesons becomes

\[ \mathcal{H}_\sigma = \frac{1}{2} \partial_\mu \sigma \partial^\mu \sigma + \frac{1}{2} m_\sigma^2 \sigma^2 + \lambda \sigma^4, \] (14)

with \( m_\sigma \) and \( \lambda \) being the bare mass and coupling constant respectively. We calculate the expectation value of the Hamiltonian density and perform the renormalisation according to prescription of ref. [14]. The resulting gap equation for \( M_\sigma^2 \), which minimizes the energy, in terms of the renormalised parameters \( m_R^2 \) and \( \lambda_R \) can be written as

\[ M_\sigma^2 = m_R^2 + 12 \lambda_R \sigma_0^2 + 12 \lambda_R I_f(M_\sigma), \] (15)

where

\[ I_f(M_\sigma) = \frac{M_\sigma^2}{16\pi^2} \ln \left( \frac{M_\sigma^2}{\mu^2} \right). \]

Using the above equations we obtain the energy density for the \( \sigma \) in terms of \( \sigma_0 \) which is given by

\[ \epsilon_\sigma = 3 \lambda_R \left( \sigma_0^2 + \frac{m_R^2}{12 \lambda_R} \right)^2 + \frac{M_\sigma^4}{64\pi^2} \left( \ln \left( \frac{M_\sigma^2}{\mu^2} \right) - \frac{1}{2} \right) - 3 \lambda_R I_f^2 - 2 \lambda \sigma_0^4. \] (17)

Where \( \mu \) is a mass scale. The above expression is given in terms of \( \sigma \) mass \( m_R \) and \( \lambda_R \) except for the last term which is still in terms of the bare coupling constant \( \lambda \) and did not get renormalised because of the structure of the gap equation [15]. However, from the renormalisation procedure it is easy to see in this last work that when \( \lambda_R \) is kept fixed, the bare coupling \( \lambda \to 0 \). Therefore the last term in eq. (17) will be neglected in the numerical calculations.

After subtracting the vacuum contribution we obtain:

\[ \Delta \epsilon_\sigma = \epsilon_\sigma - \epsilon_\sigma(\sigma_0 = 0) \]

\[ = \frac{1}{2} m_R^2 \sigma_0^2 + 3 \lambda_R \sigma_0^4 + \frac{M_\sigma^4}{64\pi^2} \left( \ln \left( \frac{M_\sigma^2}{\mu^2} \right) - \frac{1}{2} \right) - 3 \lambda_R I_f^2 - \frac{M_\sigma^4}{64\pi^2} \left( \ln \left( \frac{M_{\sigma,0}^2}{\mu^2} \right) - \frac{1}{2} \right) + 3 \lambda_R I_{f0}^2, \] (18)

where \( M_{\sigma,0} \) and \( I_{f0} \) are the expressions given by eqs. (15) and (16) with \( \sigma_0 = 0 \).

The energy density and pressure with baryon and the sigma condensate \( \sigma_0 \) are respectively done by

\[ \epsilon_{\text{ren}} = \epsilon_{0}^{\text{finite}} + \Delta \epsilon_{\text{ren}}, \] (19)

and

\[ P = \frac{g}{3(2\pi)^3} \int_{|k| < k_F} dk \frac{k^2}{(k^2 + M^{*2})^{1/2}} + \frac{1}{2} m_\omega^2 \omega_0^2 - \Delta \epsilon_\sigma - \Delta \epsilon_{\text{ren}}, \] (20)

where

\[ \epsilon_{0}^{\text{finite}} = \frac{g}{(2\pi)^3} \int_{|k| < k_F} dk (k^2 + M^{*2})^{1/2} + \frac{1}{2} m_\omega^2 \omega_0^2 + \Delta \epsilon_\sigma \] (21)

with \( \Delta \epsilon_{\text{ren}} \) given by eq. (12) and \( \Delta \epsilon_\sigma \) by eq. (18).

The energy density from the \( \sigma \) field as given by eq. (18) is still in terms of the renormalisation scale \( \mu \) which is arbitrary. We choose this to be equal to the renormalised sigma mass \( m_R \) in doing the numerical calculations. This is because changing \( \mu \) would mean changing the quartic coupling \( \lambda_R \) and \( \lambda_R \) here enters as a parameter to be chosen to give the incompressibility for nuclear matter in the correct range. The parameters \( g_\sigma^0 \) and \( g_\omega \) are fitted so as to describe the ground-state properties of nuclear matter correctly. For a given baryon density \( \rho_B \), the energy density, the density dependent radius of the nucleon and the nucleon effective mass are calculated at zero temperature.
III. RESULTS AND DISCUSSION

We now proceed with the numerical calculations for the nuclear matter. We start fixing the bag properties in the vacuum. They are given in Table 1. We next calculate the ground state properties of the nuclear matter and fit the scalar and vector coupling constants $g^2$ and $g_σ(=3g^β_ω)$ to get the correct saturation properties for a given renormalised σ mass and coupling, $m_σ$ and $λ_σ$. The omega and sigma couplings for given $λ_σ$ are tabulated in Table II. Results from Relativistic Hartree approximation (RHA) are also shown. Little change is noted in the values of the parameters for the range of $λ_σ$ which is considered here.

Using these values, we plot the binding energy ($E_B = ϵ/ρ_B − M_N$) for nuclear matter as a function of density in Figure 1. In the same figure we also plot the results for the relativistic Hartree approximation (RHA). Clearly, including baryon and σ-meson quantum corrections leads to a softer equation of state which is further softer for a higher value of $λ_σ$. The equation of state, pressure, $P$ versus as a function of density $ϵ$ is displayed in Fig. 2 for the different cases. For comparison, the causal limit $P = ϵ$ is also shown in the figure. All the cases studied here respect the causal condition $∂P/∂ϵ ≤ 1$, so that the speed of sound remains lower than the speed of light.

In figure 3 we plot the effective nucleon mass as a function of density. At the saturation density we get $M^∗ = 0.817M$ and 0.83$M$ for $λ_σ = 3.0$ and $λ_σ = 4.5$ respectively. These values may be compared with the results of $M^∗ = 0.775M$ for normal QMC model and of 0.793$M$ with the relativistic Hartree approximation in QMC model. This influence is much higher at high nuclear densities. We can conclude that quantum effects, at the level we consider, increase the effective mass $M^∗$.

We plot the in medium effective radius of the nucleon ($R^∗$) as a function of density in figure 4. $R^∗$ is also increased with relation to the mean field approximation mainly for higher densities. It is possible to understand this result as an effect which prevents QMC from deconfinement at high densities.

In figure 5, we plot the in-medium σ-meson mass, $M_σ$, as a function of density. $M_σ$ increases with density as $λ_σ$ is positive. The higher is the coupling, higher is the increase of the sigma mass with density. This would be rather an indication of a further chiral symmetry breaking instead of its restoration [10].

To summarize, we have used a non-perturbative approach to include the quantum effects in nuclear matter in the framework of QMC. The calculations of the scalar meson quantum corrections was done here in a self-consistent manner including multi-loop effects. This leads to a softening of the equation of state. We have also calculated the effective mass of the σ-meson as modified by the quantum corrections. The effective sigma mass is seen to increase with density. The results may be suggesting that, at high densities, quantum fluctuations prevents the model from the chiral symmetry restoration as well as from deconfinement. These results will be extensively studied in another work where the coupling constant $λ_σ$ and its influence on the chiral symmetry behavior -whose order parameter can be considered to be $σ_0$- at high densities and temperatures will be shown in the frame of the model worked out here.

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| TABLE I. Parameters used in the calculation. |
|---------------------------------------------|
| M (MeV) | \( m_q \) (MeV) | \( R \) (fm) | \( B^{1/4} \) (MeV) | \( z_0 \) | \( m_R \) (MeV) | \( m_\omega \) (MeV) |
|---------|----------------|-------------|-----------------|-------|--------------|--------------|
| 939.0   | 0              | 0.6         | 211.3           | 3.987 | 550          | 783          |

TABLE II. quark-sigma, omega-nucleon couplings are used for different cases in our calculation. Effective nucleon mass, effective radius are given for different sets.

| case    | \( g^2 \) | \( g_\omega \) | \( M^*/M_N \) | \( R^* \) (fm) | \( K \) (MEV) |
|---------|-----------|-------------|--------------|--------------|-------------|
| normal QMC | 5.98575  | 8.96259    | 0.775       | 0.5961      | 290.9       |
| RHA     | 5.77097  | 8.3935     | 0.793       | 0.5967      | 272.1       |
| \( \lambda_R = 3.0 \) | 5.51202  | 7.76908    | 0.817       | 0.5975      | 256.1       |
| \( \lambda_R = 4.5 \) | 5.37293  | 7.36239    | 0.830       | 0.5978      | 244.8       |
FIG. 1. The binding energy of the nuclear matter as a function of densities. Including the quantum corrections give a softer equation of state.
FIG. 2. The pressure versus energy density of the nuclear matter.
FIG. 3. Effective baryon masses in the medium.
FIG. 4. The effective radius of the nucleon as a function of densities.
FIG. 5. In medium scalar meson mass versus baryon density.