Bootstrap Inference for Panel Data Quantile Regression

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Abstract

This article develops bootstrap methods for practical statistical inference in panel data quantile regression models with fixed effects. We consider random-weighted bootstrap resampling and formally establish its validity for asymptotic inference. The bootstrap algorithm is simple to implement in practice by using a weighted quantile regression estimation for fixed effects panel data. We provide results under conditions that allow for temporal dependence of observations within individuals, thus, encompassing a large class of possible empirical applications. Monte Carlo simulations provide numerical evidence the proposed bootstrap methods have correct finite sample properties. Finally, we provide an empirical illustration using the environmental Kuznets curve.

1. Introduction

The Quantile Regression (QR) model has been widely used to capture the heterogeneous effects that covariates may have on an outcome of interest, allowing the analyst to investigate a wide variety of forms of conditional heterogeneity under weak distributional assumptions. In program evaluation studies in economics, finance, and statistics, conditional quantile methods help to analyze how a treatment or social program affects the entire outcome distribution of interest. Moreover, since the work of Matzkin (2003) and de Castro and Galvao (2019), QR has been used for empirical structural work and can provide a natural way to represent structural relationships.

Koenker (2004) introduced a general approach for estimation of QR panel models with individual specific fixed effects. Recently, there has been a growing literature on estimation and inference for panel data fixed effects (FE) QR models (FE-QR). The FE-QR model is designed to control for individual specific heterogeneity while exploring heterogeneous covariate effects, and thus provides a flexible method for the analysis of panel data models.\(^1\)

Inference in panel data quantile methods with FE has relied mainly on asymptotic approximations for computation of the variances of estimators. These variances depend on the conditional density of the innovations, which can be difficult to compute in practice. We provide simulation evidence that confidence intervals for the FE-QR estimator based on the asymptotic normal approximation can be distorted in finite samples. The bootstrap has the advantage that it does not require estimating the variance-covariance matrix, which allows one to circumvent the issue of bandwidth selection for density estimation. The use of the bootstrap as an alternative to asymptotic approximations has been informally considered, but its formal properties have not received the same amount of attention as in the cross-section or time series QR literatures. In the QR panel data context, Abrevaya and Dahl (2008), Lamarche (2010), Canay (2011), Galvao and Montes-Rojas (2015), and Graham et al. (2018) use bootstrap for constructing confidence intervals, but do not provide formal theoretical validity for the procedures, nor do they provide an explicit methodology to implement the bootstrapping for FE-QR panel data. Lamarche and Parker (2021) consider wild bootstrap for penalized QR and FE-QR estimators with panel data.

We contribute to this literature by formalizing the properties of bootstrap inference methods for QR panel data models with FE. We consider a random-weighted bootstrap (see, e.g., Rao and Zhao 1992; Præstgaard and Wellner 1993; Shao and Tu 1995) and show that it can be used to construct asymptotically valid bootstrap standard errors, confidence intervals, and hypothesis tests for the parameters of interest. We formally establish the asymptotic validity of the random-weighted bootstrap by showing distributional consistency of the bootstrap method. This result implies consistency of bootstrap percentile confidence intervals. Nevertheless, it does not necessarily imply consistency of the bootstrap variance-covariance estimation. Hence, we also prove consistency of the resampled variance-covariance matrix as an estimator for the asymptotic covariance matrix. Given the consistency of these bootstrap procedures, the practical construction of standard errors, confidence intervals,
and hypothesis tests are very simple and based on the empirical distribution of the bootstrapped regression coefficients of interest.

The proposed bootstrap uses weights that are a function of the cross-section dimension only, meaning that serial correlation in the observations will be preserved in the bootstrap data generating process. We highlight that this method is applicable to a wide variety of models and not limited to iid errors in regression models, that is, these results apply under temporal dependence of observations within individuals, and thus encompass a large class of possible empirical applications. A prominent challenge to the use of FE-QR models, and associated asymptotic methods, is that we have to deal with the incidental parameters problem even for a linear model.\(^2\) To overcome this drawback, we use recent advances in the panel QR literature (Galvao, Gu, and Volgushev 2020) and deal with the incidental parameters problem explicitly. Specifically, we show that this bootstrap is generally consistent for observations that are \(\beta\)-mixing in the time dimension, using nearly the same conditions that Kato, Galvao, and Montes-Rojas (2012) and Galvao, Gu, and Volgushev (2020) used to establish asymptotic normality of FE-QR slope coefficient estimates.

The algorithm for practical implementation of the random-weighted bootstrap method is simple. First, draw nonnegative iid random weights from an appropriate distribution satisfying mean and variance both equal to one. Second, for a given quantile of interest, fit a weighted panel FE-QR model, weighing unit \(i\)'s contribution to the objective function with the \(i\)th weight generated from the first step. These first and second steps are repeated many times. Finally, compute percentile confidence interval from the empirical distribution of bootstrapped coefficients, or calculate the variance-covariance matrix of the bootstrap coefficients to construct confidence intervals or Wald-type test statistics.

A Monte Carlo study (contained in the Supplemental Appendix) is conducted to evaluate the finite sample properties of the proposed bootstrap inference procedure. We use the FE-QR estimator, and evaluate the bootstrap procedure using an iid and a dependent case. We compute empirical coverage rates for confidence intervals using the bootstrap procedures. The results show evidence empirical coverage for confidence intervals based on the proposed bootstrap are close to the nominal size, and as expected, the results indicate that coverage improves as the sample size increases. Furthermore, this bootstrap performs well as a variance estimator under nearly the same regularity conditions as those required for bootstrap distribution consistency. Overall, the results show good performance for relatively small samples.

In an empirical example we examine the relationship between CO\(_2\) emissions and economic development. In particular, we examine the Environmental Kuznets Curve (EKC) hypothesis for a panel of 24 OECD countries and 32 non-OECD countries using a panel data QR model which allows us to account for heterogeneous effects. The EKC hypothesis describes an inverted U-shaped relationship between pollution and income per capita. The results corroborate the nature and validity of the income-pollution relationship based on the EKC hypothesis for OECD countries. They show evidence that the monotonically increasing relationship across the conditional distribution of CO\(_2\) emissions relates to the level of economic development of the country. For the non-OECD countries, although we also find empirical evidence such a income-pollution relationship, there is no evidence of validity of the EKC hypothesis.

Now we briefly review the literature related to this article. There are two different branches. First, bootstrap techniques have been used to construct confidence intervals for QR models in the cross-sectional context extensively. Buchinsky (1995) uses Monte Carlo simulation to study and compare several estimation procedures of the asymptotic covariance matrix in QR models, and the results favor the pairwise bootstrap design. Hahn (1995) shows that the construction of confidence intervals based for the QR estimators can be greatly simplified by using bootstrap. Moreover, the confidence intervals constructed by the bootstrap percentile method have asymptotically correct coverage probabilities. Bose and Chatterjee (2003) show consistency of multiplier bootstrap methods for quantile regression (among other estimators that are found by minimizing a convex objective function). Horowitz (1998) proposes the smoothed least absolute deviation (LAD) estimator and shows that the bootstrap provides asymptotic refinements for hypothesis tests and confidence intervals based on the smoothed LAD estimator. Feng, He, and Hu (2011) propose an adaptation wild residual bootstrap methods for QR. Wang and He (2007) develop inference procedures based on rank-score tests with random effects. A theoretical justification of the application of bootstrap to general semi-parametric M-estimation is provided Cheng and Huang (2010). The weighted bootstrap for QR models is considered by Hahn (1997), Bose and Chatterjee (2003), Chen and Pouzo (2009), and Belloni et al. (2019), the first three in the point-wise case and the later in the uniform context. For the bootstrap with more complex data, Hagemann (2017) develops a wild bootstrap procedure for cluster-robust inference in linear quantile regression models, and Lamarche and Parker (2021) consider a wild residual bootstrap for QR models with panel data.\(^3\) In the time series context, Fitzenberger (1998) suggested a moving block bootstrap for inference in QR models.

A second branch of the literature is related to the use of bootstrap methods for standard linear (conditional average) panel data models with FE. Kapetanios (2008) discusses bootstrap for panel data when resampling occurs in both cross sectional and time series dimensions, with strictly exogenous regressors and fixed effects, for which the incidental parameter bias does not exist. Gonçalves (2011) proved the asymptotic validity of the moving blocks bootstrap under general forms of cross sectional and time series dependence in the regression error of a panel linear regression model. More recently, Gonçalves and Kaffo (2015) propose the application of bootstrap methods for inference in linear dynamic panel data models with fixed effects. A treatment of resampling methods when \(n\) is large but \(T\) is assumed small and fixed can be found in Cameron and Trivedi\(^3\).

\(\text{2}\)The incidental parameters problem here means that as the sample size grows the number of parameters, that is, the number of individual fixed effects, increase as well. In panel QR models there is no simple transformation that removes FE and at the same time preserves the functional form of a structural equation.

\(\text{3}\)For other recent developments in resampling QR, see, for example, Chernozhukov and Fernández-Val (2005), Wang, van Keilegom, and Maidman (2018), and Gregory, Lahiri, and Nordman (2018).
A jackknife technique is applied in Hahn and Newey (2004) for a static nonlinear panel model, and in Dhaene and Jochmans (2015) for a dynamic nonlinear panel model. This article extends both of these literatures by establishing the formal properties of random-weighted bootstrap inference for panel data QR models with FE.

The rest of the article is organized as follows. Section 2 briefly describes the QR model. Section 3 describes the bootstrap methods and the inference procedures. In Section 4 we establish the asymptotic validity of the proposed methods. In Section 6, we illustrate empirical usefulness of the new approach by studying the environmental Kuznets curve. Finally, conclusions appear in Section 7. The Supplemental Appendix contains: (i) an extension of the model that allows for deterministic time trends; (ii) supplementary information about the data used in the empirical application; (iii) proofs of the theoretical results; (iv) additional Monte Carlo experiments.

2. Quantile Regression with Fixed Effects

Let $y_{it}$ be a response variable, $x_{it}$ be a $p$ dimensional vector of explanatory variables, and let $z_{it} = (1, x_{it})$. In this article we consider the following fixed effects quantile regression (FE-QR) model

$$Q_{\tau}(y_{it}|z_{it}) = \alpha_0(\tau) + x_{it}^\top \beta_0(\tau), \quad i = 1, \ldots, n, \quad t = 1, \ldots, T,$$

(1)

where $Q_{\tau}(y_{it}|z_{it})$ is the conditional $\tau$-quantile of $y_{it}$ given $z_{it}$. In model (1), $\alpha_0(\tau)$’s are intended to capture some individual-specific source of variability or unobserved heterogeneity that is not adequately controlled for by other explanatory variables. We assume that $\tau$ is fixed and for simplicity suppress the dependence on $\tau$ such as $\beta_0(\tau) = \beta_0$ and $\alpha_0(\tau) = \alpha_0$. Throughout the article, the cross-section dimension is $n$ and the time dimension is $T = T_n$ which depends on $n$, but we omit this dependence for simplicity.

Now we discuss the estimation of model (1). This approach treats each individual effect as a parameter to be estimated. The FE-QR estimates are those that solve the minimization problem

$$(\hat{\alpha}, \hat{\beta}) := \arg\min_{\alpha, \beta} \frac{1}{nT} \sum_{i=1}^n \sum_{t=1}^T \rho_\tau(y_{it} - \alpha_i - x_{it}^\top \beta),$$

(2)

where $\rho_\tau(u) := u(\tau - 1[u \leq 0])$ is the check function as in Koenker and Bassett (1978), $\beta \in \mathbb{R}^p$ is the vector of FE-QR slope coefficients and $\alpha := (\alpha_1, \ldots, \alpha_n)^\top$ is the $n \times 1$ vector of individual specific effects or intercepts. This optimization problem can be very large depending on $n$ and $T$.

The use of estimator (2) above implies that, unfortunately, even for a linear model of conditional quantiles, the resulting parameter estimates will in general be inconsistent when the number of individuals $n$ goes to infinity while the number of time periods $T$ is fixed. This is a version of the incidental parameters problem (Neyman and Scott 1948). To overcome this drawback, it has become standard in the literature to employ asymptotics for large panels (see, e.g., Koenker 2004; Kato, Galvao, and Montes-Rojas 2012; Galvao, Gu, and Volgushev 2020). In particular, it is common to derive the asymptotic properties of the estimator and associated test statistics under an assumption that $n$ and $T$ grow simultaneously.

The asymptotic properties of the FE-QR estimator have been established in the literature, for instance in Kato, Galvao, and Montes-Rojas (2012) and Galvao, Gu, and Volgushev (2020). They show consistency and asymptotic normality of the estimator under a sample size growth condition. In particular, suppose that the observations $(y_{it}, z_{it})$ are independent across units indexed by $i$ but may be serially dependent—strictly stationary and $\beta$-mixing. Let $f_{y_{it}|z}$ and $Q_{\tau}(y_{it}|z)$ be the conditional density and quantile functions of $y_{it}$ given $z$ and let

$$f_i(y|z) := f_{y_{it}|z}(y + Q_{\tau}(y_{it}|z)|z)$$

(3)

and let $f_i(y) := E[f_i(y|z_i)]$ be its marginal density function. Further define

$$g_i := E[f_i(0|z_i)]x_i^\top f_i(0),$$

(4)

$$\Gamma_n := \frac{1}{n} \sum_{i=1}^n E[f_i(0|z_i)]x_i(x_i - g_i)^\top,$$

(5)

$$V_n := \frac{1}{n} \sum_{i=1}^n \text{var}(T^{-1/2} \sum_{t=1}^T (x_{it} - g_i)(\tau - 1[y_{it} \leq \alpha_0 + x_{it}^\top \beta_0])).$$

(6)

Assume that $\Gamma_n$ is nonsingular for each $n$ and the limits $\Gamma := \lim_{n \to \infty} \Gamma_n$ and $V := \lim_{n \to \infty} V_n$ exist and are nonsingular. Then under mild regularity conditions and the rate restriction $n(\log T)^4/T \to 0$, Galvao, Gu, and Volgushev (2020) show that

$$(\sqrt{nT} (\hat{\beta} - \beta_0)) \overset{d}{\to} N (0, \Sigma),$$

(7)

where $\psi_\tau(u) := \{\tau - 1[u \leq 0]\}$ and $e_{it} = y_{it} - \alpha_0 - x_{it}^\top \beta_0$. Moreover, $\sqrt{nT} (\hat{\beta} - \beta_0) \overset{d}{\to} N (0, \Sigma)$, where

$$\Sigma := \Gamma^{-1} V \Gamma^{-1}. $$

(8)

Notice that each component of the variance-covariance matrix is a function of the quantile level $\tau$, but we suppress this dependence for simplicity.

This asymptotic result has been used to construct confidence intervals and conducting inference for $\beta_0$. One can compute an estimate of the asymptotic variance-covariance matrix $\Sigma$ and construct confidence intervals directly from it. However, the asymptotic covariance matrix of the panel QR estimator given in the literature (see, e.g., Galvao and Kato 2017) depends on the conditional density of the error term, which requires selecting a bandwidth, and it may not be easy to estimate.

Remark 1. Galvao, Gu, and Volgushev (2020) derive the asymptotic normality of the FE-QR estimator for large panels under the condition that the time dimension $T$ grows slightly faster than $n$, in particular, $n(\log T)^4/T \to 0$ as $n, T \to \infty$. Although it is strong, it significantly improved upon previous conditions in the literature. This condition may be restrictive in practice, especially for panels with large $n$ and small $T$. When $T$ is small
relative to \( n \) in finite samples, statistical inference based on the normal asymptotic distribution above, as well as the bootstrap procedure described in the next sections, may be inaccurate. In Section 5 and in the Supplemental Appendix, we use numerical simulations to investigate the finite sample properties of the bootstrap procedures for different combinations \( n \) and \( T \), and compare the results with the asymptotic inference procedure.

We note that the restriction on \( T \) (i.e., \( n/T \to 0 \)) is similar to that found in the nonlinear panel data literature (see, e.g., Arellano and Hahn 2007). There is also a literature studying nonlinear panel data models that requires a slightly weaker condition when deriving asymptotic properties of estimators, say \( n/T \to \kappa \), where \( \kappa > 0 \). Nevertheless, it is standard to rely on bias correction estimators in this literature. Several bias correction methods for the ML estimation (or more generally, \( M \)-estimation) have been proposed in the literature; see, for example, Woutersen (2002), Hahn and Newey (2004), Bester and Hansen (2009), Arellano and Bonhomme (2009), Dhaene and Jochmans (2015), and Galvao and Kato (2016). We leave such extensions to future research.

We propose to use a bootstrap procedure to facilitate statistical inference for the parameters of interest in the FE-QR model. The next section describes the proposed random-weighted bootstrap method for inference procedures.

### 3. Bootstrap

The main concern of this section is the application of weighted bootstrap procedures to the problem of constructing confidence intervals and conducting inference for the common parameters, \( \beta_0 \), of the FE-QR model in (1), based on the estimation methods described in the previous section.

Traditionally, bootstrap techniques have been successfully employed to construct confidence intervals for QR in the cross-section context. In this article we use a random-weighted bootstrap commonly used for cross-sectional resampling. This scheme consists in resampling weights from the cross-section context. In this article we use a random-weighted bootstrap procedure to facilitate statistical inference for the parameters of interest in the FE-QR model. The next section describes the proposed random-weighted bootstrap method for inference procedures.

#### 3.1. A Random-Weighted Bootstrap Method

We consider the following random-weighted bootstrap method to approximate the distribution of the panel QR estimator.

Let \( \{\omega_i, i = 1, \ldots, n\} \) be iid nonnegative random weights with mean and variance both equal to one. For each draw of \( \{\omega_i, i = 1, \ldots, n\} \), we estimate the bootstrap coefficients \( (\hat{\alpha}^*, \hat{\beta}^*) \) using a weighted FE-QR objective:

\[
(\hat{\alpha}^*, \hat{\beta}^*) = \arg \min_{\alpha, \beta} \sum_{i=1}^{n} \omega_i \sum_{t=1}^{T} \rho_{\tau}(y_{it} - \alpha_i - x_{it}^\top \beta).
\]

After conducting this procedure over many repetitions, we approximate the distribution of \( (\hat{\beta} - \beta_0) \) by that of \( (\hat{\beta}^* - \beta) \).

It should be noted that the usual pairwise bootstrap—sampling over \((i, t)\) from \( \{y_{it}, x_{it}\} \) with replacement—would only be appropriate when the data are iid. Since \( \{y_{it}, x_{it}\} \) is not iid because of the individual effects and potential serial correlation, the usual pairwise bootstrap is not directly applicable to panel data. However, the random-weighted method is applicable to a wide variety of models and not limited to iid sampling. The most closely related bootstrap to the one proposed here was that investigated by Hagemann (2017). By inspecting the score functions implied by the reweighted objective function (compare the proof of Theorem 1 with the bootstrap gradient shown on p. 448 of Hagemann 2017) it can be seen that the two bootstrap algorithms use nearly the same mechanism to generate bootstrap samples—given a sample, they differ only in the conditions put on the bootstrap weights. However, that paper deals with fixed cluster sizes and unknown dependence structure. Here we model dependence in the sample using fixed effects and as a result we rely on different asymptotic theory to show bootstrap consistency.

#### 3.2. Practical Implementation

The practical implementation of the random-weighted bootstrap method is very simple. The main algorithm for implementing the methods is as following. Take \( B \) as a large integer:

1. Draw iid nonnegative random weights \( \{\omega_i, i = 1, \ldots, n\} \) from the appropriate distribution satisfying mean and variance both equal to one.
2. For a given quantile of interest \( \tau \in (0, 1) \), fit the weighted FE-QR panel model in (9) using the sample, \( \{y_{it}, x_{it}\} \) and weights \( \omega_i \). Denote the bootstrap estimate by \( (\hat{\alpha}^*, \hat{\beta}^*) \).
3. Repeat Steps 1–2 \( B \) times.
4. Approximate the distribution of \( \sqrt{nT}(\hat{\beta} - \beta_0) \) by the empirical distribution of the \( B \) observations of \( \sqrt{nT}(\hat{\beta}^* - \beta) \).

**Percentile Confidence Interval**

Note that by choosing the number of bootstrap simulations \( B \) in the algorithm above large enough, the distribution of \( \hat{\beta} - \beta \) can be computed with any desired precision. The distribution function of \( \hat{\beta} - \beta \) can be used to estimate the distribution function of \( \beta - \beta_0 \). Specifically, suppose that the parameter of interest \( \beta \) is scalar and let \( \hat{G} \) be the cumulative distribution function of \( \hat{\beta}^* - \beta \). Then, for a given quantile level \( \tau \), we compute the \( 1 - \lambda \) percentile confidence interval for each element in the coefficient vector \( \beta_0(\tau) \) by the \( \lambda/2 \) and \( 1 - \lambda/2 \) sample percentiles of \( \hat{G} \):

\[
CL_{\lambda} = [\hat{\beta} + \hat{G}^{-1}(\lambda/2), \hat{\beta} + \hat{G}^{-1}(1 - \lambda/2)].
\]

These percentiles may be used to estimate the endpoints of a confidence interval for \( \beta_0 \). This is equivalent to using corresponding sample percentiles from the distribution of \( \beta^* \).

**Variance-Covariance Matrix Estimation**

For a fixed quantile level \( \tau \), we define the bootstrap estimate of the asymptotic covariance matrix \( \Sigma \) given bootstrap realizations \( \{\hat{\beta}^b\}_{b=1}^{B} \) as

\[
\hat{\Sigma}^* = \frac{1}{B} \sum_{b=1}^{B} (\hat{\beta}^b - \hat{\beta})(\hat{\beta}^b - \hat{\beta})^\top.
\]
The standard errors of \( \hat{\beta} \) are the square roots of the diagonal elements of \( \Sigma^* \). Given this estimated covariance matrix, testing general hypotheses \( R \beta_0 = r \) for the vector \( \beta_0 \) can be accommodated by Wald-type tests.

There are two options for the construction of confidence intervals given consistent bootstrap covariance matrix estimates. Once again, assume that \( \beta \) is scalar for simplicity. Let \( se^* \) be \( \sqrt{\Sigma^*} \) (in higher dimensions it is the square root of a diagonal element). We could use the estimated standard errors directly in the confidence interval as following

\[
C_{SE} = [\hat{\beta} - z_{\alpha/2}se^*, \hat{\beta} + z_{\alpha/2}se^*],
\]

where \( z_\alpha \) denotes the \( \alpha \)-th quantile of the standard normal distribution. Alternatively, we may compute a reference distribution of bootstrap \( t \) statistics where \( t_\alpha^* = (\hat{\beta}^* - \bar{\beta})/se^* \), and construct the following confidence interval

\[
C_{RD} = [\hat{\beta} - t_{\alpha^*}se^*, \hat{\beta} + t_{\alpha^*}se^*],
\]

where similarly, \( t_\alpha^* \) denotes the (empirical) \( \alpha \)-th quantile of the bootstrap \( t \) distribution, and \( se^* \) is the \( \sqrt{\Sigma} \) with \( \Sigma \) being the variance of \( \hat{\beta} \) estimated from the original sample (in higher dimensions it is the square root of a diagonal element). This interval suffers from the defect that we would need to estimate the standard error of \( \hat{\beta} \), which was we were hoping to avoid by using the bootstrap in this setting. However, confidence intervals of this form are included in the simulation study for comparison so we describe them here.

In the next section we formally establish the asymptotic properties of the proposed random-weighted bootstrap method and its validity for inference procedures.

4. Asymptotic Validity of the Bootstrap

After discussing basic regularity conditions, we discuss two main results. First we discuss the consistency of the bootstrap estimate of the distribution function of \( \hat{\beta} - \beta_0 \) and next, the consistency of the bootstrap estimator of the covariance of \( \hat{\beta} - \beta_0 \). Consistency of the distribution estimator does not imply that of the second moment estimator, and we discuss the way in which conditions must be strengthened to maintain second-moment estimator consistency.

4.1. Basic Assumptions

We use \( P^* \) and \( E^* \) to denote the probability measure and expected value with respect to the bootstrapped data conditional on the observations, and use \( X_n^* \xrightarrow{D} X \) to denote convergence in probability of a sequence of bootstrap statistics to a limit conditional on the observations. Recall that \( z_{\alpha} \) is a vector of dimension \( p + 1 \) and let \( Z \) denote the support of \( z_{\alpha} \). We make the following assumptions.

(A0) For each \( i \geq 1 \), the process \( \{y_{it}, x_{it} \} : t \in \mathbb{Z} \) is strictly stationary and \( \beta \)-mixing and these processes are independent across \( i \). Let \( \beta_i(j) \) denote the \( \beta \)-mixing coefficient of the process \( \{y_{it}, x_{it} \} : t \in \mathbb{Z} \). Assume that there exist constants \( b_\beta \in (0,1), C_\beta > 0 \) independent of \( i \) such that \( \sup_j \beta_i(j) \leq C_\beta b_\beta \) \( \forall j \geq 1 \).

(A1) Assume that \( \|z_{\alpha}\| \leq M < \infty \) almost surely, that \( c_\lambda \leq \lambda_{\alpha}(E[z_{\alpha}z_{\alpha}^*]) \leq \lambda_{\max}(E[z_{\alpha}z_{\alpha}^*]) \leq C_\lambda \) holds uniformly in \( i \) for some fixed constants \( c_\lambda > 0 \) and \( C_\lambda < \infty \) and that \( (\alpha_i, \beta) \) lies in a compact set for all \( i \).

(A2) The conditional distribution \( F_{\nu_i|x_i}(y|z) \) is twice differentiable w.r.t. \( y \), with the corresponding derivatives \( f_{\nu_i|x_i}(y|z) \) and \( f_{\nu_i|x_i*y}(y|z) \). Assume that

\[
\max_{i} : \sup_{y \in \mathbb{R}, z \in \mathbb{Z}} |f_{\nu_i|x_i}(y|z)| < \infty,
\]

and

\[
\max_{i} : \sup_{y \in \mathbb{R}, z \in \mathbb{Z}} |f_{\nu_i|x_i*y}(y|z)| < \infty.
\]

(A3) Denote by \( T \) an open neighborhood of \( r \). Assume that uniformly across \( i \), there exists a constant \( f_{\min} < f_{\max} \) such that

\[
0 < f_{\min} \leq \inf_{i, \eta \in \mathcal{B}} \inf_{y \in \mathbb{R}, z \in \mathbb{Z}} f_{\nu_i|x_i}(Q_{\nu_i|x_i}(\eta|z)).
\]

(B1) For each \( i = 1, \ldots, n \) and \( j > 1 \), the random vector \( (y_{it}, y_{i(t+j)}) \) has a density conditional on \( (z_{it}, z_{i(t+j)}) \) and this density is bounded uniformly across \( i, j \).

(C1) \( \{\alpha_n, i = 1, \ldots, n \} \) are iid positive random weights with mean and variance both equal to one.

Condition (A0) assumes that the data are independent across individuals, and strictly stationary within each individual. It allows for stationary \( \beta \)-mixing which is used in Kato, Galvao, and Montes-Rojas (2012), Galvao and Wang (2015) and is similar to Hahn and Kuersteiner (2011). Condition (A1) poses a boundedness condition on the norm of the regressors, which is also standard in the literature, see for instance Koenker (2004), Katô, Galvao, and Montes-Rojas (2012), and Galvao and Wang (2015). Condition (A1) also assures that the eigenvalues of \( E[z_{\alpha}z_{\alpha}^*] \) are bounded away from zero and infinity uniformly across \( i \). Similar assumptions were made in Chao, Volgushev, and Cheng (2017). Condition (A2) and (A3) impose smoothness and boundedness of the conditional distribution, the density and its derivatives. The same type of assumption has been imposed in Galvao and Wang (2015). Chao, Volgushev, and Cheng (2017) also make similar assumptions when deriving Bahadur representations for QR estimators in a setting without panel data. Condition (B1) is needed because the data are not iid and we need to impose a condition on the joint distributions; similar conditions were imposed in Kato, Galvao, and Montes-Rojas (2012) and Galvao and Wang (2015). Finally, Assumption 4.1 is a common bootstrap weight condition (see, e.g., Rao and Zhao 1992; Belloni et al. 2019).

To state the asymptotic properties of \( \hat{\beta}^* \) we make the following additional assumption.

(FD) Assume that \( \Gamma_n \) in (5) is nonsingular for each \( n \) and that \( \Gamma := \lim_{n \to \infty} \Gamma_n \) exists and is nonsingular. Further assume that \( V := \lim_{n \to \infty} V_n \) exists and is nonsingular, where \( V_n \) is defined in (6).

This assumption was also made by Kato, Galvao, and Montes-Rojas (2012) (see their condition (D3)), it involves the long run covariance matrix of the leading piece in the Bahadur representation for \( \beta \).
Finally, for one result below we assume that the bootstrap weights satisfy slightly a stronger moment condition for consistent covariance matrix estimation.

(CV) Assume that for some \(\epsilon > 0\), \(E[\omega_i^{2+\epsilon}] < \infty\).

This assumption is made to ensure uniform square integrability of the sequence of bootstrap variance estimates, which is only slightly stronger than Assumption 4.1 and ensures the convergence of bootstrap moments to finite limits (this is discussed more in the remark below Theorem 2). Since the bootstrap weights can be chosen by the researcher, this assumption is easily satisfied. The analogous moment condition for the covariates \(x_{it}\) is implied by Assumption 4.1.

4.2. Asymptotic Results

Now we formally establish the asymptotic validity of the proposed random-weighted bootstrap method. The following result show consistency of the distribution of the bootstrap estimate.

Theorem 1 (Consistency of the Bootstrap). Assume that \(n \log T / T \to 0\) and \(T = O(n^s)\) for some \(s > 1\) as \(n \to \infty\). Under Conditions (A0)–(A3), (B1), (C1), and (FD), for fixed \(\tau\), it follows that

\[
\sup_x \left[ P^* \left( \sqrt{nT} (\hat{\beta} - \beta_0) \leq x \right) - P \left( \sqrt{nT} (\hat{\beta} - \beta_0) \leq x \right) \right] \to 0.
\]

The consistency of a bootstrap confidence interval is closely related to the consistency of the bootstrap estimator of the distribution of \(\sqrt{nT} (\hat{\beta} - \beta_0)\). Theorem 1 states that the bootstrap estimator for the distribution is consistent relative to the Kolmogorov-Smirnov distance. Consistency relative to the Kolmogorov-Smirnov distance is equivalent to the requirement that, uniformly in \(x\),

\[
P \left( \sqrt{nT} (\hat{\beta} - \beta_0) \leq x \right) \to F(x),
\]

\[
P^* \left( \sqrt{nT} (\hat{\beta}^* - \beta) \leq x \right) \to F(x).
\]

It has been shown in the literature—see, for example, Galvao, Gu, and Volgushev (2020)—that \(\sqrt{nT} (\hat{\beta} - \beta_0)\) converges in distribution to a Gaussian distribution, implying that \(F\) is actually the CDF of a normal distribution with variance \(\Sigma\).

Remark 2. By making a simple alteration to the proof of Theorem 1, it can be shown that the unit-variance assumption on the bootstrap weights may be relaxed. If instead we demand that the weights are positive with mean 1 and variance \(\sigma_w^2\), then it can be shown that the distribution of \(\sqrt{nT} (\hat{\beta}^* - \beta) / \sigma_w\) is a consistent estimator of the asymptotic distribution of \(\sqrt{nT} (\hat{\beta} - \beta_0)\). This allows researchers to use a wider variety of bootstrap weights by rescaling their bootstrap coefficient realizations.

The bootstrap distributional consistency described in Theorem 1 implies the consistency of percentile confidence intervals in (10). Although Theorem 1 implies consistency of the bootstrap percentile confidence intervals, it does not necessarily imply consistency of variance-covariance estimation (Ghosh et al. 1984). Theorem 2 implies that the variance can be consistently estimated using the estimator shown in (11). The result in Theorem 2 is very useful in empirical applications since it allows one to easily estimate the variance-covariance matrix by resampling procedures without calculating each of its components separately.

Theorem 2 (Consistency of the variance-covariance matrix). Recall that \(\Sigma\) and \(\Sigma^*\) are defined in (8) and (11). Under conditions of Theorem 1 and Assumption 4.1, for fixed \(\tau\),

\[
\hat{\Sigma}^* \xrightarrow{p} \Sigma.
\]

The result of Theorem 2 is slightly stronger than that of Theorem 1. The weak convergence (conditional on the observations) of \(\sqrt{nT} (\hat{\beta}^* - \beta)\) to its limit does not imply that the sequence of bootstrap second moments converges weakly as well. One may see why by considering a Bahadur (i.e., asymptotically linear) representation that is implied by the proof of Theorem 1:

\[
\sqrt{nT} (\hat{\beta}^* - \beta) = (\hat{\Gamma}_n^*)^{-1} \sum_{i=1}^n \omega_i \sum_{t=1}^T (x_{it} - \bar{x}_{it}) \psi_t (\hat{\epsilon}_{it}) + o_P(1).
\]

This expression depends on a few terms that are defined in the proof of Theorem 1 but are inessential here. The conditions used to show distributional consistency must be strengthened for variance estimation to ensure that the covariance matrix may be consistently estimated. First, the terms in the sum must be uniformly square integrable so that this main part of the Bahadur representation converges. Second, we must also ensure that the remainder term is square integrable, which is not implied by the form of asymptotic negligibility implied by its \(o_P(1)\) characterization—although the \(o_P(1)\) characterization implies that the remainder is small with high probability, it would be a problem if with low probability, the remainder were extremely large. The moment conditions specified in Assumptions (CV) and (A1) are sufficient to ensure that both of these requirements are satisfied.

Remark 3. Trending regressors may be of concern in applications. This model accommodates trend-stationary variables, as shown in Lemma 1 in the supplemental Appendix. We leave the modeling of panel data with stochastic trends to future research.

5. Monte Carlo Simulations

In this section, we summarize the results of a simulation study to assess the finite sample performance of the proposed bootstrap inference procedures. We consider several different simulation designs, but in this section we only report on two designs and relegate the other results to the supplemental Appendix. Data is generated from the following simple version of model in (1):

\[
y_{it} = \alpha_i + \beta x_{it} + (1 + \gamma x_{it}) \epsilon_{it}, \quad i = 1, \ldots, n, \quad t = 1, \ldots, T.
\]

(14)

We let \(\alpha_i \sim \text{iid } U[0,1]\), \(x_{it} = 0.3 \alpha_i + z_{it}\) with \(z_{it} \sim \text{iid } \chi^2_1\), \(\beta = 1\) and \(\gamma = 0.2\). We use an ARMA(1,1) model for the innovation term: \(\epsilon_{it} = \rho \epsilon_{it-1} + \theta \epsilon_{it-1} + \epsilon_{it}\), with \(\rho = 0.4\) and \(\theta = 0.5\). We let \(n \in \{20, 50, 100\}\), and let \(T\) vary such that

\(n/T \in \{2, 1, 1/2, 1/4, 1/10\}\), to examine performance as \(T\) grows.
Table 1. Heteroscedastic, dependent chi square error.

| n   | T   | AT  | 1/4 | 1/2 | 3/4 |
|-----|-----|-----|-----|-----|-----|
| 20  | 10  | 99.5| 98.2| 94.4|     |
|     | 20  | 99.2| 98.7| 92.5|     |
|     | 40  | 98.8| 97.8| 93.9|     |
|     | 20  | 98.3| 96.8| 93.8|     |
|     | 20  | 98.4| 96.4| 93.9|     |
|     | 50  | 99.0| 98.1| 93.5|     |
|     | 50  | 99.6| 98.1| 94.2|     |
|     | 100 | 98.5| 96.8| 94.8|     |
|     | 200 | 98.9| 97.1| 94.8|     |
|     | 500 | 97.3| 96.5| 94.4|     |
|     | 100 | 99.0| 98.4| 94.9|     |
|     | 100 | 98.3| 98.1| 94.5|     |
|     | 200 | 98.4| 97.9| 95.6|     |
|     | 500 | 97.9| 97.1| 95.5|     |
|     | 1000| 97.3| 95.7| 94.6|     |

Table 2. Heteroscedastic, dependent normal error.

| n   | T   | AT  | 1/4 | 1/2 | 3/4 |
|-----|-----|-----|-----|-----|-----|
| 20  | 10  | 96.8| 98.6| 97.4|     |
|     | 20  | 95.4| 98.5| 96.9|     |
|     | 40  | 96.2| 97.5| 97.6|     |
|     | 80  | 96.8| 98.6| 97.3|     |
|     | 200 | 96.2| 97.4| 96.4|     |
|     | 25  | 96.5| 98.3| 96.6|     |
|     | 50  | 97.2| 97.7| 96.9|     |
|     | 100 | 97.9| 97.2| 96.0|     |
|     | 500 | 96.3| 96.8| 96.0|     |
|     | 50  | 96.9| 97.9| 97.7|     |
|     | 100 | 97.0| 97.7| 96.9|     |
|     | 200 | 96.0| 96.7| 96.6|     |
|     | 400 | 96.3| 97.5| 96.4|     |
|     | 1000| 95.5| 95.7| 94.2|     |

NOTE: Nominal 95% CIs.

compared to n. We estimate the model for the three quartiles \( \tau \in \{1/4, 1/2, 3/4\} \). The random-weighted bootstrap procedure discussed in Section 3 is used with weights that follow an exponential distribution, \( \omega_i \sim \exp(1) \), and 999 bootstrap repetitions. The results are based on 1000 simulation replications.

Tables 1 and 2 show empirical confidence interval coverage rates for nominal 95% confidence intervals for confidence intervals constructed using: the asymptotic theory (AT), the percentile interval CIP defined in (10) (RWB.p); the bootstrap variance-covariance matrix described in (12) (RWB.se), and empirical quantiles of the bootstrap reference distribution in (13) (RWB.t). Across all tables, including those in the Supplemental Appendix, we notice some patterns. Confidence intervals constructed using asymptotic theory, the AT columns here, usually have overcoverage problems, as do the RWB.t results to a lesser degree. On the other hand, the percentile and standard error CIs, RWB.p and RWB.se tend to undercover. Generally speaking, the latter two types of CI tended to be shorter, leading to undercoverage, although this tends to be smaller in degree than the overcoverage of the AT and RWB.t intervals. This is investigated in Figures 3 and 4 in the supplementary Appendix. Empirical coverage approximates nominal coverage well as \( n/T \) tend to zero. A dynamic panel model was an interesting exception: the AT/RWB.t intervals were close to nominal for the smallest values of \( n/T \).

6. Empirical Application

6.1. Environmental Kuznets Curve

This section illustrates the usefulness of the proposed methods with an empirical example. We use fixed effects (FE) panel data quantile regression (QR) methods to study the Environmental Kuznets curve. We accommodate possible heterogeneity on the effects of per capita income on the conditional distribution of environmental degradation by using FE-QR. Indeed, this heterogeneity is not revealed by conventional least squares.

There is growing interest in environmental degradation and its relationship with economic growth, the excessive use of natural resources, and climate change. The burning of fossil fuels (i.e., carbon, oil, and gases) used for the production of energy necessary for economic development continues to significantly contribute to CO2 emissions. The relationship between the economy and the environment is complex and controversial, and a number of empirical studies employ the Environmental Kuznets Curves (EKC) to shed light on this issue.
The EKC is based on the concept of the Kuznets curve, proposed by Kuznets (1955), which initially described an inverted U-shaped relationship between income inequality and income per capita. The intuition is that income inequality first increases as per capita income rises, and then starts to decrease from a certain threshold point. The notion of the Kuznets curve has been applied to the environmental economics to investigate whether the relationship between income per capita and environmental degradation follows a similar inverted U-shape relationship (see, e.g., Grossman and Krueger 1993, 1995). In this context, the EKC postulates that low income levels are directly related to the deterioration of the environment, but after a certain level of income per capita, the this relationship reverses and becomes a negative one.5 There is a relatively large more recent literature using conditional average models to estimate the EKC, see, for example, Dinda (2004), Galeotti (2007), and Kaika and Zervas (2013a, 2013b) for reviews. There is no consensus about this relationship. On the one hand, Selden and Song (1994), Grossman and Krueger (1995), List and Gallet (1999), and Stern and Common (2001), among others, find evidence in favor of inverted U-shaped relationships, at least in the case of the developed countries. On the other hand, Harbaugh, Levinson, and Wilson (2002) and Effiong and Oriabije (2018) argue that there is no evidence that this relationship is valid for a number of emission pollutants. More recently, a literature using conditional quantile models investigates the EKC hypothesis, see, for example, Flores, Flores-Lagunes, and Kapetanakis (2014), Yaduma, Kortelainen, and Wossink (2015), Allard et al. (2018), and Ike, Usman, and Sarkodie (2020).

6.2. Model

It is usual in the literature to use a log-linear model with a quadratic term of the affluence (per capita income) variable in line with the EKC hypothesis to capture possible existence of an inverted U-shaped relationship.6 The resulting panel data model is specified as follows:

\[
\ln(E_{it}) = \beta_0 + \beta_1 \ln(gdp_{it}) + \beta_2 \left(\ln(gdp_{it})\right)^2 + \beta_3 \ln(pop_{it}) + \beta_4 \ln(\text{enit}_{it}) + \alpha_i + \epsilon_{it},
\]

where \(E_{it}\) measures the environmental quality of country \(i\) at time \(t\); \(pop\) denotes the population size; \(gdp\) is the GDP per capita; and \(\text{enit}\) denotes technology which is proxied by energy intensity to capture technology’s damaging effect on the environment. The term \(\alpha_i\) captures the country-specific fixed effect that is constant over time.

The term \(\epsilon_{it}\) in (15) captures the innovation. It is usual to impose a conditional mean exogeneity to estimate the model. In this article we proposed to use a QR model and impose a zero conditional quantile restriction. Hence, we estimate the following conditional quantile function:

\[
Q_\tau[\ln(E_{it})|gdp_{it}, pop_{it}, enit_{it}] = \beta_0(\tau) + \beta_1(\tau) \ln(gdp_{it}) + \beta_2(\tau) \left(\ln(gdp_{it})\right)^2 + \beta_3(\tau) \ln(pop_{it}) + \beta_4(\tau) \ln(\text{enit}_{it}) + \alpha_i(\tau).
\]

All the variables in (16) are expressed in natural logarithms so the estimated coefficients are interpreted as elasticities. We also augment the model in (16) to include a linear time trend. The results are qualitatively similar, and collected in the supplemental Appendix.

The EKC conjecture could be investigated depending on the sign and statistical significance of the slope parameters of the income variable (gdp). For a given quantile \(\tau\), on the one hand, if \(\beta_1(\tau) > 0\) and \(\beta_2(\tau) = 0\), then the relationship income-pollution is monotonically increasing (or decreasing if \(\beta_1(\tau) < 0\) and \(\beta_2(\tau) = 0\)). On the other hand, if \(\beta_1(\tau) > 0\) and \(\beta_2(\tau) < 0\), then inverted U-shaped curve is observed for that relationship with the turning point \(E^*(\tau) = -\frac{\beta_1(\tau)}{2\beta_2(\tau)}\). See Angrist, Chernozhukov, and Fernández-Val (2006) for misspecification in quantile regression models.

6.3. Data

Our data are taken from Soberon and D’Hers (2020). To investigate the empirical relationship between wealth and pollution, we used panel datasets consisting of 24 OECD countries and 32 non-OECD countries for the period 1980 to 2016. Countries with insufficient data on CO2 emissions are dropped from the database. Using both OECD and non-OECD countries may allow to draw conclusions and compare results for both developed and developing countries. The list of countries used for the estimation is in the supplemental Appendix.

As observed in Soberon and D’Hers (2020), the data come from two main sources. Environmental degradation captured using CO2 emissions is obtained from the International Energy Statistics of the U.S. Energy Information Administration (EIA). CO2 emissions (in metric tons per capita) include burning of fossil fuels and cement manufacturing, but excludes emissions from land use such as deforestation.

The data for all other variables (population, affluence, and technology) are obtained from the World Development Indicators (WDI) of the World Bank. Population (POP) is measured as total population. Affluence, which captures economic prosperity, is measured as real GDP per capita (constant 2015 US dollars). Technology is measured using energy intensity (ENIT), which is expressed as total primary energy consumption per dollar GDP (1000 BTU per year in 2015 US dollars).

6.4. Results

The results for the coefficient estimates together with 90% confidence intervals across the quantiles are reported in Figures 1 and 2 for OECD and non-OECD, respectively.

First, consider the GDP coefficient in Figure 1. There is evidence that the coefficient is positive and statistically different from zero across all conditional quantiles. Combined with the negative coefficient estimates for GDP2, this implies the effect of GDP growth at low levels of GDP is positive. In addition, it
is decreasing across quantiles. Thus, the income-pollution relationship is monotonically decreasing across conditional quantiles of environmental quality. Recall that the dependent variable measures environmental degradation, measured by CO₂ emissions. Thus, for low quantiles of the conditional distribution environmental degradation, an increase on GDP has a relatively larger impact.

Regarding the EKC hypothesis—a positive coefficient on GDP and negative coefficient on GDP²—Figure 1 displays evidence that supports the EKC hypothesis for the OECD countries.
Note that the coefficient estimates $\hat{\beta}_1(\tau)$ are statistically positive across quantiles and $\hat{\beta}_2(\tau)$ are statistically negative across quantiles as well, hence, the evidence of inverted U-shaped curve is observed across the quantiles. Interestingly, $\hat{\beta}_1(\tau)$ is decreasing and $\hat{\beta}_2(\tau)$ increasing across quantiles, such that the ratio $\hat{\beta}_1(\tau) / 2 \hat{\beta}_2(\tau)$ is slightly increasing. That is, the shape of the curve is flatter for those countries that have higher emissions conditional on population, energy intensity and country-specific fixed effects. Hence, the turning point $E^*$ is larger at the top of the conditional distribution of environmental degradation.

The empirical estimates for the energy intensity variable are positive, statistically significant, and slightly decreasing across quantiles for the OECD countries in Figure 1. This implies that, for a given quantile, higher consumption of fossil fuels in the production process is associated with increased CO2 emissions that in turn increase pressure on environmental quality. But this coefficient is slightly larger for larger quantiles of the conditional distribution of CO2. For the population variable, the point estimates are negative across quantiles, but the confidence intervals are quite large and the estimates are statistically indistinguishable from zero.

Figure 2 displays the results for the non-OECD countries. The income-pollution is monotonically decreasing across quantiles. However, one cannot reject the hypothesis that the coefficients for GDP2 are statistically equal to zero. Hence, the EKC hypothesis is not empirically valid for non-OECD countries, and we can only observe a positive relationship between income and pollution. The estimates regarding the energy intensity variable for non-OECD countries are also positive and statistically different from zero. They are small when compared to the OECD countries in Figure 1.

The FE-QR method allows us to estimate the impacts of GDPC, population, and technology on the EKC at different quantiles of the conditional distribution of environmental degradation, as well as to investigate the empirical validity of the EKC hypothesis. Our empirical findings document the validity of the income-pollution relationship, which is diminishing along the conditional quantiles of degradation for both OECD and non-OECD countries. There is empirical evidence supporting the EKC hypothesis for OECD countries, but not for non-OECD.

7. Conclusion

This article develops bootstrap inference methods for panel data quantile regression models with fixed effects. We consider the case of randomly-weighted bootstrap and propose to construct asymptotically valid bootstrap standard errors, confidence intervals, and hypothesis tests for the parameters of interest using this resampling technique. The weighted bootstrap method has the advantage that it does not require estimating the variance-covariance matrix, which allows one to circumvent the need to select a bandwidth for conditional density estimation. The weights are only a function of the cross-section dimension such that the serial correlation in the original data can be preserved, thus, encompassing a large class of possible empirical applications. We formally establish the asymptotic validity of the randomly-weighted bootstrap by showing consistency of the bootstrap method in distribution and consistency of bootstrap covariance estimation.

The bootstrap algorithm is simple to implement in practice. Monte Carlo simulations (available in the supplemental Appendix) confirm that the proposed methods have correct finite sample properties. Numerical simulations also show evidence that confidence intervals based on the asymptotic normal approximation can be very distorted in finite samples. Instead, the proposed bootstrap greatly reduces these distortions. Finally, we provide an empirical illustration using the environmental Kuznets curve.

Supplementary Materials

qrfe_supplement.pdf: Supplemental Appendix with the results of Monte Carlo experiments, proofs and supplementary description of the data used in the environmental Kuznets curve investigation.

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