Scalable and Robust Local Community Detection via Adaptive Subgraph Extraction and Diffusions

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ABSTRACT
Local community detection, the problem of identifying a set of relevant nodes nearby a small set of input seed nodes, is an important graph primitive with a wealth of applications and research activity. Recent approaches include using local spectral information, graph diffusions, and random walks to determine a community from input seeds. As networks grow to billions of nodes and exhibit diverse structures, it is important that community detection algorithms are not only efficient, but also robust to different structural features.

Toward this goal, we explore pre-processing techniques and modifications to existing local methods aimed at improving the scalability and robustness of algorithms related to community detection. Experiments show that our modifications improve both speed and quality of existing methods for locating ground truth communities, and are more robust across graphs and communities of varying sizes, densities, and diameters. Our subgraph extraction method uses adaptively selected PageRank parameters to improve on the recall and runtime of a walk-based pre-processing technique of Li et al. [Li et al., 2015b] for extracting subgraphs before searching for a community. We then use this technique to enable the first scalable implementation of the recent Local Fiedler method of Mahoney et al. [Mahoney et al., 2012]. Our experimental evaluation shows our pre-processed version of Local Fiedler, as well as our novel simplification of the LEMON community detection framework of Li et al. [Li et al., 2015b], offer significant speedups over their predecessors and obtain cluster quality competitive with the state of the art.

CCS Concepts
• Mathematics of computing → Graph algorithms;

Keywords
community detection; semi-supervised learning; graph algorithms; local algorithms

1. INTRODUCTION

We consider the general problem of identifying a community around a given seed node or nodes of interest. That is, given an input node (or nodes) in the graph, we consider the goal of trying to find a cluster of which the node is a member.

An evolving research direction in graph data mining is to develop community detection algorithms that scale to extremely large graphs, e.g., algorithms that rely on local computations involving only nodes relatively close to the given seeds. For example, recent work on local graph diffusion methods have shown promise that one can find clusters in a localized way, i.e. without looking at most of the graph. The general framework operates by first computing a diffusion vector, then returning as the detected community the set of nodes that have largest mass. These diffusion procedures can be viewed as propagating large probability values from the labeled nodes (“seeds”) to the remaining unlabeled nodes, which is the key ingredient in many local graph diffusion algorithms [Andersen and Lang, 2006, Andersen et al., 2006b, Avron and Horesh, 2015, Gleich and Mahoney, 2015, Kloster and Gleich, 2014, Li et al., 2015a,b, Spielman and Teng, 2008]. A useful local diffusion process is one that effectively propagates probabilities to the nodes that are most relevant to the given seeds, without mixing to the entire graph.

To combat this, we present a pre-processing technique that is robust in the face of such varied characteristics. The technique uses PageRank with adaptive parameters to consistently extract a high recall subgraph of medium size. Comprehensive experiments show that our PageRank-based technique attains the best recall among algorithms considered, especially on networks in which communities have larger diameter. This improves over the \( k \)-walk approach used in [Li et al., 2015b], which fails to expand to much of the larger-diameter communities.

The subgraph extraction method enables efficient use of more sophisticated methods on the subgraph without losing
large portions of valuable graph regions. Our hope is this
technique will combine with other community detection and
other semi-supervised learning algorithms. As a simple case
study, we show our technique enables the first efficient use of
the recently proposed locally-biased Fiedler vector method
(MOV) [Mahoney et al., 2012] on large datasets such as social
networks from the SNAP repository Leskovec and Krevl
2014.

Finally, we make modifications to the local spectral method
of Li et al. Li et al., 2015b to produce a much simpler and
faster method that still attains competitive cluster qual-
ity. We demonstrate significant improvement on 11 different
real-world network spanning various domains of network
application (see Table 5).

We summarize our main contributions as follow:
1. We propose and systematically evaluate different sub-
graph extraction techniques for improving the per-
formance of any local graph analysis algorithms. We show
our proposed modification improves recall.
2. We make substantial simplifications to the framework
of Li et al. to yield a simpler, yet much faster imple-
mentation.
3. We present experiments on 11 datasets spanning various
domains of network applications and observe how the
performance of subgraph extraction algorithms relate to
certain community and graph properties like diameter
and edge-density, leading to a better understanding of
what kind of algorithm is best to employ on different
categories of networks.
4. We investigate the novel problem of seed set augmenta-
tion and evaluate common community detection tools
for obtaining a small, high-precision set of seed nodes.

We make our experimental codes available in the spirit of re-
producible research: https://github.com/kkloste/lemon-seq.

2. RELATED WORK

Much recent work has studied network and community
properties and algorithmic approaches related to finding
communities from seed sets. One study identified common
structural properties of ground truth communities, like sepa-
ratedness and internal cohesion [Yang and Leskovec, 2012],
while another observed that characteristics such as conduc-
tance and diameter can be seen as classification features that
can distinguish between ground truth communities and the
outputs of a variety of community detection algorithms [Abra-
hao et al., 2012]. Kloumann and Kleinberg [Kloumann and
Kleinberg, 2014] studied the impact of seed set characteristics
on the quality of clusters produced by personalized PageRank
diffusions. While it is widely known that social networks
exhibit small and even shrinking diameter [Leskovec et al.,
2005], it has also been found that spectral methods produce
clusters that have small diameter [Leskovec et al., 2009]. A
related study found that egonets (node neighborhoods, which
by definition have a diameter of at most 2) can be good clusters,
in the sense that they can have low conductance [Gleich
and Seshadhri, 2012].

From the algorithmic side, a swath of new methods have
appeared for identifying local communities without having
to look at the entire network structure. Methods for locating
local network community structure from a given seed set
have included diffusions, Monte Carlo methods, and spectral
approaches. Recent diffusion vectors include random-walk
vectors [Andersen and Lang, 2006] Spielman and Teng, 2008,
the personalized PageRank vector [Andersen et al., 2006b],
Jeh and Widom, 2003b heat kernel vector [Chung, 2007],
Kloster and Gleich, 2014, and the time-dependent PageRank
vector [Avron and Horesh, 2015], [Gleich and Ross, 2014].
(We briefly review the vectors most relevant to our study
in Section 4). Local spectral-based approaches include a
locally-biased version of the Fiedler vector method
Mahoney et al., 2012, as well as the recent LEMON vector
He et al., 2016.

3. PRELIMINARIES

Here we fix our notation and review recent approaches
related to local community detection. Let $G = (V, E)$ be a
graph with $n = |V|$ nodes and $m = |E|$ edges. We assume
that the graph is unweighted and undirected, which is com-
monly assumed in the context of community detection. We
denote the adjacency matrix associated with the graph $G$ by
$A$, with entries $a_{ij} = 1$ if nodes $i$ and $j$ are connected
and $a_{ij} = 0$ otherwise. Let $D$ be the diagonal matrix of node
degrees where $D_{ii} = d(v_i)$, and $P = (D^{-1} A)^T = AD^{-1}$ be
the random walk transition matrix, and note that in our
notation it is column stochastic. Lastly, $L = D − A$ is the
combinatorial Laplacian.

We assume vectors are column vectors. For a fixed node
$j$ we denote by $e_j$ a standard basis vector with a 1 in the
$j$th entry, and $e$ denotes the vector of all 1s. We consider
applications that seek information about a small set of input
nodes which we call “seed” nodes. For a node set $S$, $e_S$
denotes the indicator vector of the set $S$, i.e. $e_S$ is all 0s
except with 1s in entries corresponding to nodes in $S$.

Proposed definitions and properties of communities vary
widely, though a often-given intuition for a community is
“a set of nodes with high internal connectivity, and rela-
tively lower external connectivity”. A related and commonly-
adopted metric for evaluating how much a node set “resem-
bles” a community is conductance, which is defined as follows.
Given a set of nodes $C \subseteq V$, the conductance of $C$ is

$$\phi(C) = \frac{\text{cut}(C, \bar{C})}{\min\{\text{vol}(C), \text{vol}(\bar{C})\}},$$

where $\bar{C} = V \setminus C$ consists of all nodes not in $C$, $\text{cut}(X, Y)$
denotes the number of edges between the node sets $X$ and
$Y$, and $\text{vol}(X) = \sum_{v \in X} d(v)$, i.e. the “edge volume” of the
node set $X$. Conceptually, $\phi(C)$ is the probability that a
random walk of length one will escape $C$, given that we start
from an edge-endpoint chosen uniformly at random inside $C$.

Diffusion vectors. A graph diffusion is a probability
vector of the form

$$f = \sum_{k=0}^{\infty} c_k P^k p_0,$$

where the coefficients $c_k$ are any values such that $c_k \geq 0$,
$\sum_{k=0}^{\infty} c_k = 1$, and $p_0$ is the initial distribution of probability
across the nodes defined by

$$p_0 = \begin{cases} d(v)/\sum_{v \in S} d(v) & \text{if } v \in S \\ 0 & \text{otherwise} \end{cases}$$
Some approaches also use the weighting \( p_0 = e_S / |S| \) in place of the above degree-weighted normalization. We call the terms \( c_k \) the diffusion coefficients. A particular choice of diffusion coefficients is essentially an assignment of weights to random walks of different lengths, and hence a way to emphasize nodes at specific walk depths.

**Sweep procedure.** Once the estimation of a diffusion from a seed set \( S \) is obtained, one can produce a small conductance community via the so-called sweep procedure. A sweep over a vector involves sorting the nodes in descending order according to the entries in the vector, and computing the conductance of each prefix of the sorted list. The set found to have smallest conductance is then returned by the sweep procedure as the detected community for the given seed set.

4. LOCAL DIFFUSION METHODS

In this section we describe several recent methods for local community detection, which we select for their widespread attention in the literature and because of the diversity in characteristics of the clusters that they identify. We give a brief overview of the local graph diffusions including random walks, personalized PageRank (PPR) \cite{Andersen06a}, heat kernel (HK) \cite{Chung09}, and local spectral methods MOV \cite{Mahoney12} and LEMON \cite{Li15b}.

### 4.1 Overview of algorithms considered

**The \( k \)-walk diffusion vector.** The \( k \)-walk diffusion vector is a graph diffusion in which a single diffusion coefficient in the formulation \( \sum_{i=0}^{\infty} \frac{t^k}{k!} P^k p_0 \) is 1, i.e., \( c_k = 1 \), and \( c_j = 0 \) for all other \( j \neq k \). In other words, \( f = P^k p_0 \) for some \( k \in \mathbb{N} \).

The personalized PageRank diffusion vector. For a fixed \( \alpha \in (0, 1) \), the personalized PageRank vector can be defined as

\[
PPR = (1 - \alpha) \sum_{k=0}^{\infty} \alpha^k P^k p_0.  \tag{3}
\]

The personalized PageRank vector can be interpreted as the stationary distribution of a random walk with restart. The diffusion was proposed in \cite{Jeh03a}, but we use the algorithm presented in \cite{Andersen06b}.

The heat kernel diffusion vector. The heat kernel diffusion replaces the weights \( c_k = (1 - \alpha) \alpha^k \) with \( e^{-t}(t^k/k!) \):

\[
HK = e^{-t} \left( \sum_{k=0}^{\infty} \frac{t^k}{k!} P^k \right) p_0.  \tag{4}
\]

In contrast with PPR, the heat kernel diffusion models the spread of heat across a graph starting from a seed set. The diffusion was proposed and analyzed in \cite{Chung07}, but we use the algorithm presented in \cite{Kloster14}.

Both the PPR and HK algorithms that we study in this paper were designed to be local: even though the diffusion vectors themselves are global (i.e. totally dense on a connected graph), the algorithms used here are approximations that explore only a small portion of the graph. Both approximation algorithms have theoretical guarantees that limit the amount of work they perform before converging (and, hence, limit the size of the output cluster) by a small constant that depends only on the accuracy parameter \( \varepsilon \).

#### The MOV (locally-biased Fiedler) vector.

The MOV vector, first introduced in \cite{Mahoney12}, offers a way to bias the standard global Fiedler vector so that it identifies a good conductance cut localized near an input set of nodes. Formally, the MOV vector is the solution \( x \) to

\[
\arg\min_{x} x^T L x \quad \text{s.t.} \quad x^T D x = 1 \quad \text{and} \quad (x^T D s)^2 \geq \kappa, \tag{5}
\]

where \( \kappa \in (0, 1) \) is an input parameter that controls the extent to which the solution \( s \) is localized into the input seed set \( S \) represented by the specially constructed seed vector \( e_S \). In contrast with the local algorithms for the above diffusions, the seed vector here is not a sparse indicator vector \( e_S \) but instead the following dense vector

\[
s = \sqrt{\frac{\text{vol}(S)\text{vol}(\bar{S})}{\text{vol}(\text{vol}(S))}} \left( \frac{1}{\text{vol}(S)} e_S - \frac{1}{\text{vol}(\bar{S})} e_S \right). \tag{6}
\]

Once the MOV vector \( x \) is computed, a sweep over \( x \) obtains a good conductance cut near the seeds \( S \). Although the MOV vector yields a localized set, the method itself is global in that computing \( x \) requires reading the whole graph due to the density of the vector \( s \) and the global nature of the objective function \( 5 \). This prevents the MOV algorithm from scaling to larger datasets; we address this concern later by using a subgraph extraction technique in pre-processing and running MOV on a much smaller subgraph (Section 5).

**The LEMON (local spectral subspace) method.** The LEMON method \cite{Li15b} iteratively grows a community from input seeds using a diffusion that is computed over only a small subspace of walk vectors, such that the diffusion maximizes overlap with the iterative seed set. This subspace has been referred to as a local spectral subspace, which is simply a partial Krylov subspace. The local spectral subspace is formed from the input seed vector \( e_S \) and a specially normalized form of the adjacency matrix defined as

\[
\tilde{A} \equiv \frac{1}{2} (D + I)^{-1/2} (A + I)(D + I)^{-1/2}. \tag{7}
\]

Given an input set \( S \) and input parameters \( k \) and \( l \), form the local spectral subspace matrix

\[
V_{k,l} = \tilde{A}^k e_S, \ldots, \tilde{A}^{k+l-1} e_S. \tag{8}
\]

Then the LEMON algorithm solves for the following \( \ell_1 \)-norm optimization problem:

\[
\min_{y} ||V_{k,l} y||_1 \quad \text{s.t.} \quad V_{k,l} y \geq 0, \quad (V_{k,l} y)(S) \geq 1. \tag{9}
\]

where the objective function is a regularized term with sparsity penalty. Once a solution \( y \) is found, then \( x = V_{k,l} y \) is the LEMON diffusion vector we are looking for.

5. DATASETS
Table 1: Summary of the ground truth community properties. From left to right, the columns indicate: dataset name, abbreviated name used in later figures, number of nodes, number of edges, number of communities analyzed (we restricted analysis to communities with 10 or more nodes), average community size, average within-community degree of community members, average fraction of community member’s edges that stay within the community, and average diameter of communities.

| data     | abbrv. | |V|  | |E|  | number | |C|  | dC | dC/d | diameter |
|----------|--------|-------------|------|-------------|------|-------------|------|-------------|-------|--------|--------|---------|---------|
| citeseer | cite   | 2,110       | 3,668| 7           | 207  | 2.9         | 0.85 | 14.3        |
| cora     | cora   | 2,485       | 5,069| 8           | 273  | 3.7         | 0.88 | 11.8        |
| senate   | sen    | 1,884       | 16,662| 110         | 82   | 12.1        | 0.58 | 4.4         |
| usps-3nn | us3    | 9,298       | 21,256| 10          | 918  | 4.4         | 0.96 | 16.8        |
| usps-10nn| us10   | 9,298       | 68,381| 10          | 925  | 13.7        | 0.92 | 9.5         |
| amazon   | amaz   | 334,863     | 925,872| 2,110       | 25   | 6.3         | 0.96 | 3.9         |
| dblp     | dblp   | 317,080     | 1,049,866| 1684       | 53   | 6.0         | 0.74 | 3.5         |
| friendster| fri    | 65,608,366  | 1,806,067,135| 3,704 | 61       | 21.0     | 0.23 | 2.9         |
| livejournal | lj    | 3,997,962  | 34,681,189| 3,442       | 38   | 22.7        | 0.68 | 2.5         |
| orkut    | ork    | 3,072,441   | 117,185,083| 4,371       | 236  | 29.9        | 0.31 | 4.7         |
| youtube  | yout   | 1,134,890   | 2,987,624| 1,266       | 47   | 3.4         | 0.24 | 4.4         |

We summarize the dataset properties in Table 1. We include the ground truth community datasets from the SNAP collection, the co-purchasing networks amazon and co-authorship graph dblp, as well as the social networks friendster, livejournal, orkut, and youtube [Yang and Leskovec, 2012, Mislove et al., 2007]. These datasets are widely used as ground truth for evaluating community detection algorithms as they have thousands of annotated ground truth communities; we use the top 5,000 ground truth communities supplied by the SNAP collection for each dataset.

Additionally, we use citeseer and cora, citation networks with categories as communities [Sen et al., 2008], and senate, a 3-NN (nearest-neighbor) network of all US senators in the first 110 congresses, where edges connect senators with similar voting patterns and communities are taken to be individual congresses; data made available by [Jeub et al., 2015] and processed by [Gleich and Kloster, 2016]. Finally, we include two versions of the widely used USPS hand-written digits dataset, where nodes represent images of hand-written digits, edges are determined via a 3-NN (nearest-neighbor) network of all US senators in the first 110 congresses, where edges connect senators with similar voting patterns and communities are taken to be individual congresses; data made available by [Jeub et al., 2015] and processed by [Gleich and Kloster, 2016]. Additionally, we use citeseer and cora, citation networks with categories as communities [Sen et al., 2008], and senate, a 3-NN (nearest-neighbor) network of all US senators in the first 110 congresses, where edges connect senators with similar voting patterns and communities are taken to be individual congresses; data made available by [Jeub et al., 2015] and processed by [Gleich and Kloster, 2016].

Processing the datasets. We emphasize that many of the ground truth communities have very few nodes, in the SNAP collection in particular. Furthermore, some of the k-NN network communities are disconnected (if we look at the subgraph induced by the set of community members), even though the full networks are themselves connected. In light of this, to make the community characteristics more meaningful, we preprocess all datasets as follows. All datasets are first made undirected and unweighted, then the largest connected component (LCC) is extracted. For each dataset, we restrict each community to just its members that lie inside this LCC. Any community that is not connected as an induced subgraph we separate into its connected components, and each such connected component we count as a separate community. Finally, we consider only communities that have at least 10 nodes.

Computing details. All experiments were performed using a Dual CPU system with Intel Xeon E5-2670 processors (2.6 GHz, 8 cores) with 16 cores total and 256 GB of RAM. All algorithms were implemented in Matlab and Matlab’s C++ MEX-interface. For timing purposes, all algorithms were run in serial, with the exception of the MOV algorithm (without subgraph extraction), which used 12 cores in parallel when performing Matlab backslash solves.

6. SUBGRAPH EXTRACTION WITH GOOD RECALL

In this section, we study the task of identifying a large set T of nodes with high recall, from a single seed node. Given a set of ground truth nodes C, the recall of a proposed cluster T is defined by recall(T) = |C ∩ T|/|C|, i.e. this is the “fraction of the truth that we obtain”. We use recall to measure how well subgraph extraction methods perform in capturing the full ground truth community.

Baselines. We carry out the experiment as follows. Following the subgraph extraction method of Li et al. [Li et al., 2015b], we perform a k-step walk from the seed node by computing A^k eS, where S is a seed node from a given community C. We then take the largest 3,000 entries in the vector, and extract the subgraph corresponding to those nodes. We call the set of nodes in this subgraph T, and record the recall of T with respect to the ground truth community C. For comparison, in addition to using k = 3 as in [Li et al., 2015b], we also perform the k-walk subgraph extraction for values k = 2 and 4.

We also consider the personalized PageRank diffusion. Specifically, we use the “push” implementation of PPR in [Anderson et al., 2006a] to carry out the same procedure as for the k-walk methods: first compute the diffusion vector starting from the seed node, then extract a subgraph using the top ranked 3,000 nodes from the diffusion vector. For PPR, we used α = 0.99, a common setting in community detection, and an accuracy of ε = 10^-4. It is common to normalize diffusion vectors by dividing each
node’s diffusion score by the node’s degree. We performed this normalization on all diffusion vectors in this experiment.

The results presented are computed as follows. For each dataset, for each community, run each algorithm seeded on each individual community member as seed and average those results for that community. Then, average these community scores over all communities to obtain the results for that dataset. The errorbars indicate the standard upper and lower semi-deviations.

Adaptive PPR subgraph extraction. In addition to PPR, which uses fixed parameter values throughout, we propose a new variant called Adaptive PPR (or ‘PPR-d’), which chooses $\varepsilon$ and $\alpha$ so that the expected size of the output equals the target subgraph size 3,000 (or $n/5$ if $n < 3000$). This can be accomplished by setting the desired edge-volume to equal (desired number of nodes) $\times$ (estimated average degree of community). For PPR this amounts to setting $1/(\varepsilon(1-\alpha))$ equal to the desired edge volume. This is because the PageRank-based algorithm produces a cluster with edge volume roughly equal to $O(1/(\varepsilon(1-\alpha)))$ [Andersen et al., 2006a]. The advantage of PPR-d over PPR is that PPR-d is able to carve out larger chunks in graphs regardless of the density of the graph. For all these methods, if the subgraph reached by the walk (or diffusion) is smaller than 3,000 nodes, then simply use the full set of nodes as the extracted subgraph.

Next, we note that PPR-d achieves very high recall on most datasets, in part owing to the large clusters it identifies (visible in Figure[1], middle plot). The datasets where PPR-d performs the worst are orkut and friendster – this poor performance is likely due to the smaller size of clusters that PPR-d is identifying on those datasets. This smaller PageRank-cluster size is caused by the larger edge-density in those datasets: this is because for a fixed accuracy parameter $\varepsilon$, the PPR produces a cluster with edge volume roughly equal to $O(1/(\varepsilon(1-\alpha)))$, as noted above. From Table[1] we can see that friendster and orkut have some of the most edge-dense communities of all datasets. Interestingly, livejournal, which has very edge-dense communities, does not exhibit this behavior, and PPR-d is able to attain large clusters, and good recall, here. This could be because of the larger within-community edge-ratio of livejournal, compared to the other dense datasets.

Conclusion. We conclude that, overall, the PPR-d subgraph extraction technique attains the best recall, especially on networks with larger diameter where the $k$-walk approach used in Li et al. [2015b] fails to expand to much of the large-diameter community. Furthermore, the results suggest

![Figure 1](https://example.com/figure1.png)

Figure 1: (Top) Each column gives the average recall attained by various pre-processing techniques intended for extracting a good recall set of nodes near a seed node. (Middle) The average size of the cluster returned by each method. (Bottom) The adaptive PageRank parameters cause a slight increase in runtime, but both PPR versions are much faster on larger graphs where even a length-3 walk can involve touching a huge number of nodes.
that, regardless of which subgraph extraction technique is used, the parameters (length of $k$-walks, or accuracy $\varepsilon$ in diffusions) should be tuned to the specific properties of the communities being sought, i.e. diameter and edge density, when possible. We discuss this in more detail in Section [10].

**Theoretical analysis.** Here we argue from a theoretical perspective why we should expect subgraph extraction to improve the performance of local graph diffusions in community detection. Provided with a seed node $s$, suppose the subgraph extraction algorithm obtains a subgraph $G_s = (V_s, E_s)$ with high recall $(1 - \varepsilon)$ so that we have $|C| = (1 - \varepsilon)|V_s \cap C|$. Consider a $k$-step random walk on the subgraph $G_s$, starting from $s$. At step 1, the amount of probability leaving $C$ can be bounded by

$$\Delta = |\mathcal{C}| = \sum_{v \in \mathcal{C}} \mathbf{P}(s, v) / \sum_{v \in \mathcal{V}} \mathbf{P}(s, v),$$

which entry of $\phi$ gives the probability of reaching $C$ in a single step. From the above, it follows that

$$\mathbf{P}^0 \mathbf{p_s}^T \mathbf{e}_C \leq \frac{1}{|\mathcal{C}|} \frac{\text{cut}(C, V_s)}{\text{cut}(C, \mathcal{V})},$$

where $\text{cut}(C, \mathcal{V})$ is the escaping probability bounded on the original graph $G$.

If we assume subgraph size $|V_s| \ll |\mathcal{V}|$ then the size ratio is $\Delta = |V_s| / |\mathcal{V}| \ll 1$. The number of possible edges connecting to nodes outside the community $C$ has been approximately reduced by a factor of $\Delta^2$. Therefore,

$$\mathbf{P}^0 \mathbf{p_s}^T \mathbf{e}_C \leq \frac{1}{|\mathcal{C}|} \frac{\text{cut}(C, V_s)}{\text{cut}(C, \mathcal{V})} \approx \Delta^2 \frac{\text{cut}(C, V_s)}{\text{cut}(C, \mathcal{V})},$$

which entry of $\phi(C)$ is the escaping probability bounded on the original graph $G$.

7. **SEED SET AUGMENTATION WITH GOOD PRECISION**

For many semi-supervised learning tasks it can suffice to produce just a few new groundtruth nodes with high precision, rather than an entire community. Given the difficulty of identifying a whole community with high quality, in this section we explore whether the algorithms we consider in this paper are reliable for the task of seed set augmentation.

Formally, we address the problem of identifying a small set $T$ of groundtruth nodes with high precision, given a single seed node. Given a set of groundtruth nodes $\mathcal{C}$, precision of the set $T$ is defined by precision$(T) = |\mathcal{C} \cap T| / |T|$, i.e. this is the “fraction of our guesses that are correct”. Different applications might use different values of $\tau$ in precision$(T) = |\mathcal{C} \cap T| / |T|$. We fix $\tau = 3$.

Figure 2 (top) displays the precision of the PageRank and heat kernel diffusions alongside the same $k$-step random walks as above. For all methods, the given diffusion vector is computed from a single seed, each node’s diffusion value is then divided by the node’s degree, the $\tau = 3$ nodes with largest normalized score are then taken from the vector to form $T$, and finally the precision of the set $T$ is computed.

For each algorithm and each dataset, we compute the precision obtained using each community member as the seed and average those results to obtain the score for that community; then we average the scores for each community to produce the score for the dataset. The error bars give the upper and lower semi-deviations taken over the community scores. The runtimes displayed in Figure 2 (bottom) were computed the same way (note the log scale).

Figure 2 (top) shows that the diffusion methods offer noticeably superior precision in locating new ground truth nodes; but all of the methods fail on the datasets with worst (lowest) within-community edge-ratio (i.e. $\text{lj}$, $\text{senate}$, and $\text{youtube}$).

We later see that these same datasets (i.e. with low within-community edge-ratio) have the lowest F1-score for all algorithms. This precision experiment gives another window into why these datasets are so problematic: even the nodes that the diffusions rank as most relevant to the seed are wrong quite often when communities have low within-community edge-ratio.

8. **OUR SIMPLER LEMON**

In addition to the heat kernel and PageRank diffusions and the MOV and LEMON vectors that we discussed above, we introduce here our own novel improvements to the LEMON method (Algorithm 8).

To better understand the relationship of LEMON to diffusions we ran an experiment to determine which walk vector $\mathbf{A}^0 \mathbf{p}_0$ the LEMON procedure most heavily weighted when solving the minimization problem in (14). More concretely, if $x = V_{k, i}y$ is the iterative LEMON vector, which entry of $y$ is largest in magnitude – that is, which walk vector $\mathbf{A}^0 \mathbf{p}_0$ has the largest coefficient $y_i$?

To attempt to address this question, we randomly selected 100 nodes in each small dataset, and 1,000 nodes in each of dblp, lj, and youtube, and computed LEMON vectors as $\mathbf{A}^0 \mathbf{p}_0$.

\[1\] It is defined by the average fraction of edges of a community member that stay within the community.

\[2\] Because the walk methods ran prohibitively slow on the densest datasets, we do not include results here for friendster and orkut.
follows. For each randomly selected node, we generated a set of 10 seeds using the largest ranked nodes from a local heat kernel diffusion; then we computed the LEMON vector using that seed set and recording the coefficient $y$ such that $x = Vbyssy$. We found that the vector $y$ computed in each instance placed at least 90% of the weight on the walk vector $\tilde{A}^s e_S$ in over 95% of trials, for all but three datasets. For the remaining three datasets (cora, senate, and youtube) this occurred in over 88% of the trials.

With this in mind, we designed a modification of LEMON to be faster by avoiding the expensive optimization problem in the original algorithm that computes $x = Vbyssy$. In contrast with the original LEMON, our method, which we call LEMONeasy, uses our adaptive subgraph extraction technique, and always uses $\tilde{A}^s e_S$ as the iterative set-augmentation vector. Finally, we simplify the LEMON stop criterion by removing their conductance-related auto-termination criterion, and instead simply perform 10 iterations of seed-set augmentation, with a final sweep to determine the output cluster. The sweep uses an ordering on the nodes determined by the order in which nodes are added to the seed set, rather than simply by the values of the final iterative LEMON vector.

This framework is much simpler to implement than the original LEMON (since it no longer relies on solving a convex optimization problem), but uses the same key ideas of subgraph extraction and iterative set augmentation. Our experiments in Section 7 confirm that simple diffusion vectors are reasonably good at identifying a small number of ground truth nodes with good precision; our LEMONeasy leverages this idea to iteratively build up a set of nodes with high precision.

Algorithm 1 LEMONeasy($G, s, r, f$)

Input: graph $G$, seed node $s$, augment steps $r$, augment size $f$
1: Initialize $p_0$.
2: Extract subgraph $G_s = (V_s, E_s)$ starting from $s$, using method in Section 6.
3: Initialize stack $S_0 = [s]$.
4: For $j = 0$ to $r$, push onto $S_j$ the top $j : r$ nodes of the vector $x = \tilde{A}^s e_{S_j}$; largest $z_{i_1}$ first.
5: Perform sweep over the nodes in the stack $S_r$ plus the remaining nodes of $G_s$, ordered by $\tilde{A}^s e_{S_r}$.
Output: The best conductance set found by the sweep.

9. RECOVERING GROUND TRUTH COMMUNITIES

Here we evaluate the algorithms’ ability to identify a ground truth community given a single seed from that community. All performance results reported in this section are computed as follows. For each dataset, for each community, use each community member as an individual seed and run each algorithm from it; then average the performance over each seed to obtain a score for the community. Finally, compute the average results over all communities to obtain the score displayed for each dataset. In this manner, the results we report here reflect the expected performance of each algorithm if a seed node were chosen uniformly at random from a community of nontrivial size that was chosen uniformly at random. Error bars in the plots indicate the standard upper and lower semi-deviations from the average.

Effect of subgraph extraction.

To understand the effect that our subgraph extraction technique has on the different algorithms, we display in Figure 3 the performance of HK, PPR, and MOV both with and without subgraph extraction. The top plot shows that, on the smaller datasets, subgraph extraction noticeably improves the cluster quality of PPR without substantially affecting HK and MOV in most cases. We emphasize that “MOV” indicates the algorithm ran on 12 processors in parallel, whereas “MOVs” operated in serial.

In contrast, Figure 3 (bottom) shows that HK and PPR without subgraph extraction performs better than HKs and PPRs. We did not compute MOV on the larger datasets without using subgraph extraction for comparison because, as a global method, it was prohibitively slow. To explain this discrepancy (improvements on small graphs, but not on large), we look at the size and conductance of the clusters returned before and after subgraph extraction, in Figure 3.

A plausible explanation is that a set of good conductance can be missed if that set has worse conductance when computed with respect to the extracted subgraph. Without subgraph extraction, PageRank was clumping together two or more clusters into one, even better conductance cluster (as suggested by both the conductance and size plots); but with subgraph extraction, that large set no longer had good conductance, and PageRank could instead identify the true community better using conductance. To combat this effect, during the sweep step of the algorithm, the conductance could be computed with respect to the full graph, instead of the subgraph.

![Figure 3: Effect of subgraph extraction on the F1 score attained by baseline codes (HK, PPR, MOV).](image-url)
Figure 4: Subgraph extraction decreases the size (top plot) and increases the conductance (bottom plot) of the clusters output by the baseline codes (HK, PPR), often substantially. (An “s” in the name indicates subgraph-extraction is used.) This suggests that the difference in F1 performance of HK and PPR depicted in Figure 3 is caused by the subgraph extraction technique cutting out so much of the whole graph that the underlying good conductance cut is no longer good conductance, and the corresponding cluster is obscured.

We remark that subgraph extraction is primarily intended to enable the use of more sophisticated, global algorithms like MOV and our method LEMONeasy. Figure 5 shows that MOV, even running on 12 processors in parallel, is still slower on the medium-sized datasets than MOVs running in serial.

Because of how slow MOV runs on large datasets, we did not carry out thorough experiments to demonstrate the speed-up yielded by our MOVs on the largest sets. But, to give an idea, we ran our algorithm for MOV on one randomly selected community (13 nodes) in dblp; the average runtime to compute MOV seeded on a single member of the community was over 4,100 seconds, using 12 processors in parallel. In contrast, MOVs averaged 0.78 seconds across all communities (running on a single processor). (Because MOV is a global algorithm, this runtime should not vary much across different seeds or different communities.)

Ground truth recovery performance Finally, Figure 6 compares the performance of our novel modified versions of previous algorithms (LEMONeasy and MOVs), with baseline approaches. Our LEMONeasy attains a two orders of magnitude speed up compared to the original proposal by Li et al. [Li et al., 2015b], and obtains cluster quality competitive with all baseline codes. The running time of our improved version MOVs is competitive even with the local methods on larger datasets (bottom plot).

10. DISCUSSION

Characteristics of datasets. First we remark on variety of characteristics across the 11 datasets: the graph size and density and the average community diameter, size, and “average fraction of community member’s edges that stay inside the community”.

For example, in Section 6 we find that the walk-based methods for subgraph extraction perform much better on the social networks than on the collaboration and k-NN graphs – we explain this by noting that the social networks have very small diameter, enabling a small walk to stay within the community, whereas the other networks have larger diameter and so a short walk does not reach a large chunk of each community.

Also, we notice that the average community size ranges from 23 (Amazon) to almost 1,000 (in the USPS graphs). The study of Li et al. [Li et al., 2015b] showed that using loose lower and upper bounds on the average size of a community can improve the quality of clusters output by an algorithm. In the absence of such knowledge, an algorithm must be robust to such wide variation in size to be able maintain quality performance.

Next, we observe that the difference in graph construction parameters for the 3-NN and 10-NN versions of the USPS graph lead to some significant differences in community structure. The denser 10-NN version has communities with significantly lower diameter; we see that this leads to significant differences in the recall of k-walk based subgraph
weight on a small set of short-length walks, which can cause the diffusion to identify smaller clusters. See Section 4.1 for details of the parameters $\alpha$ and $t$.

Properties of algorithmic clusters. We also discuss here how the different diffusion algorithms that we consider respond to the dataset characteristics discussed above. We begin by discussing the traits of the clusters identified by the methods. As an example of how different methods find clusters with different characteristics, we point to two recent studies that found that different sets of diffusion coefficients can be selected to control the size of the output clusters, to better match the size of the communities expected to be found in a given network [Avron and Horesh 2013; Kloster and Gleich 2014]. Figure 2 reused from Kloster and Gleich 2014 shows that the heat kernel diffusion puts more weight on shorter walks, and so emphasizes smaller communities than PageRank does.

In this paper, however, we have considered not only the size but also the conductance of the output clusters, and how the different local methods are affected by the diameter and density of the underlying graph.

Community density, graph construction, and tuning diffusion parameters. If PageRank (or heat kernel) uses fixed parameter settings then the clusters identified will be smaller on denser datasets. This might sound counter-intuitive (i.e. “shouldn’t a denser graph spread a diffusion more quickly?”), but this fits in with the theoretical properties of the algorithms as follows. The papers that introduced both algorithms give theoretical bounds on the amount of work performed: PageRank touches no more than $O(1/\varepsilon(1 - \alpha))$ edges and heat kernel roughly $O(e^{\varepsilon t}/\varepsilon)$ edges. Thus, for fixed parameter settings, these two diffusions can touch no more than a fixed constant number of edges. On denser graphs, there will be fewer total nodes attached to a set of edges of constant size, and so clusters with fewer nodes will be output.

This fact can have an interesting consequence when a graph is constructed from a single dataset in different ways. As a case study, we consider two graphs constructed from the same USPS handwritten digits dataset. We find that MOV and PageRank spread to larger portions of the graph, and so consider a larger number of possible sweep-cuts — thus, MOV and PageRank find the lowest conductance sets out of any algorithm, but also potentially the largest sets. At the same time, the tendency of these algorithms to explore such large chunks of the graph means they tend to require more time than the $k$-step walk or heat kernel methods.

On the other hand, $k$-step random walks, PageRank and heat kernel diffusion vectors have non-zero weights on paths of potentially excessive length ($k > 15$), thereby diffusing probabilities to a larger chunk of the graph and encouraging large communities. In contrast, using $k$-step random walks can miss out on potentially important portions of the graph if $k$ is not large enough (e.g., small relative to the average community diameter).

11. CONCLUSIONS

Our results suggest that community and graph characteristics must be taken into account when selecting algorithms for semi-supervised learning tasks, as well as parameters. In particular, the details of a graph’s construction (e.g., 3-nearest-neighbor vs 10-NN), when known, should lead to differences in algorithm choice and parameter selection. Our experiments with PageRank-based subgraph extraction show that adapting parameters can lead to consistent performance across varied networks, and that subgraph extraction can enable rapid application of more sophisticated techniques that would otherwise be intractable.

We also observed that the recent, successful LEMON method for local community detection, despite some algorithmic sophistication, effectively uses a simple walk vector to expand a seed set. We used this insight to present a greatly simplified algorithm that iteratively grows a seed set with a simple walk vector. Our experiments on the precision of different methods found that diffusions outperformed walk vectors; in the future, we plan to explore the effect of using such diffusions in the LEMON framework instead of the lower precision walk vector.

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