Applying Non-Homogeneity of Variance in The Error to The Investment Function in Iraq

Hayder Raaid Talib (1), Mahdi Ali Abdul- Husain(2), Ahmed Abd Naeem(3)

Faculty of Administration and Economics / Sumer University, Iraq
Department of Statistics/(1)
Faculty of Administration and Economics / Sumer University, Iraq
Department of Statistics/(2)
Faculty of Administration and Economics / Wasit University, Iraq
Department of Statistics/(2)

Abstract. The present study aims at studying the problem of variance in the random error in the multiple linear regression sample by using investment function in Iraq from 1980 to 2015 and testing it by using standard methods by comparing the weighted least squares method, which is considered to be one of the ways to treat the problem, with the ordinary least squares method (OLS) using the efficiency scale. As a result, it has been shown that the weighted least squares method is better than the ordinary least squares method (OLS). The major hypothesis of the present study is that random error does not have the problem of variance in the error. To achieve the aims of the present study, to validate the hypotheses, and to interpret the results, a sophisticated statistical package in this area has been used; (E views 9 program) in which chart plot test, Breusch Pajan, weighted least squares method have been used. The problem is that the dependent variable is limited because of the nature of the phenomenon, which may not lead to the validation of those assumptions. The study has adopted the standard quantative analysis method with data time series for a random sample whose size is 36 from the Iraqi economy which has shown that there is a variance in the random error in the investment function; i.e. the variance is not equal in the investment function. That has led to some recommendations that may be useful to those concerned with the present study.

1. Introduction

Building the regression model and assessing its features without checking the terms or hypotheses, when applying it, leads to statistically and economically inaccurate results. Most phenomena, including the economic ones, are one-sided; either increasing or decreasing, therefore, there is a strong relationship among the variables. That results in the non-homogeneity of variance. Thus, it is important to assess the problem and treat it through some statistical assessments.
1.1. The aims of the research
Specifically, the present study aims at:
1- Using some statistical assessments that identify or reveal the problem of non- homogeneity of the variance.
2- Using the weighted least squares method to treat the problem of non- homogeneity of variance.
3- Comparing WLS to OLS using the relative efficiency to estimate the investment function in Iraq.
4- Obtaining the best unbiased linear estimation, BLUE.
5- Obtaining the best unbiased, linear assortment, linear estimation whose variance is less than the variance of any other unbiased linear estimation.

1.2. The value of the research.
The value of the research is how to perform an assessment, obtain the best methods of unbiased linear estimation, and treat the problem of the instability of the variance in the random error using advanced software in the economical estimation; graphical interfaces. The reader of this research can notice to which extent the present study has treated this problem and provided appropriate solutions.

1.3. The hypotheses of the research.
To achieve the aims of the present study, a hypothetical scheme has been formulated to clarify the nature of the treatment of the problem of non- homogeneity of variance in the random error via weighted least squares method WLS by the weight 1; the national income; fixed prices as shown in the figure below

![Figure 1. The research schemes.](image)

1.4. The standard methods used in the analysis
The following standard methods have been used to assess the problem of Non homogeneity of variance in the error: the plot test, the rank correlation coefficient test for Spearman, and Breusch-Pajan-Godfrey's test. As far as treating the problem of non- homogeneity of variance is concerned, the weighted least squares method has been used depending the weight 132, whereas 3 represents the national income in the fixed prices to avoid the problem of non- homogeneity of variance and obtain results which correspond to the economic theory.
1.5. concept and causes of non-heterogeneity of variance in the random error; 10, 9, 8, 6, 4.

First: The concept

The ordinary least squares method assumes that the variance of the random variable is fixed. The deviations of the observed values of the affiliate variable for each point at the estimated line of the interpretive variable are equal. That is known as the equal proliferation. If this assumption is true, the variance of the values of the residual around the estimated line or the dispersion of the observed values around the estimated line will be fixed; there is one variance for all observed values around the estimated line. Estimating the variables of the sample accurately in light of the basic hypotheses basically depends on the accuracy of the hypotheses; one of which is the homogeneity.

\[
E(U_i^2) = \sigma_u^2
\]
\[
E(U_iU_j) = 0, \forall i \neq j \quad ...(1)
\]

And in case of general linier sample:

\[
E(U_{\text{liner} i}) = \sigma_u^2 \ln \quad Or \quad \sigma_{U_{\text{liner} 1}}^2 = \sigma_{U_{\text{liner} 2}}^2 = \cdots \cdots = \sigma_{U_{\text{liner} m}}^2 \quad ...(2)
\]

\[
\text{Var}(U_i) = E(U_i^2) = \sigma_i^2, i=1,2,\ldots,n \quad ...(3)
\]

\[
\sigma_1^2 \neq \sigma_2^2 \neq \cdots \neq \sigma_n^2 \quad ...(4)
\]

The above assumption may not always be correct since most of studies and researches which are based on statistical data in the form of cross-sectional data, which disperse the observations of the cross-sectional data of the dependent variable and then, it will be different for each of the independent levels. But, the variance will be as follows:

Here, arises the problem of heteroscedasticity; which consists of two words; hetero; unequal and scedasticity; distance or spreading. Changing the value of the variable of error, which increases by increasing the value of the independent variable, results in the problem of non-homogeneity of variance as in researches concerned with families for the data are persons whose incomes are variant, or researches concerned with institutions.

From the second figure, it is noticed that the change in the interpretive variable results in changing the affiliate variable, which results in changing the random variable; The value of the random variable decreases with the increase of the value of the interpretive variable regularly. Then, the relationship between the interpretive variable and the variance of the random variable is linear and adverse. Whereas, from the second figure, it is noticed that the variance of the random error increases with the increase the value of the interpretive variable regularly.

Second: Causes of the problem of non-homogeneity of variance of the random error.
The most important causes of non-homogeneity are the following:

1. The existence of a two-sided relationship among the internal variables as in the case of immediate relationships samples.

2. Using cross-sectional data instead of time-series.

3. Using partial instead of collective data; Using collective data results in the disappearance of differences among items; Some cancel the other and there is no dispersion. Whereas, in the case of partial data, the available data about individuals or individual institutions make dispersion and difference so huge.

4. The individuals' behavior for which error decreases as time passes as well as variance of error.

5. The variable of error increases with the increase of the level of income for the variance of peoples' behavior test; The variance of spending on food among families may increase with the increase of the family's income.

6. Improving data collecting methods decreases the variance in the error as accurate data decrease errors in the governmental documents which use computer for analyzing data whose less errors than those which do not use it.

Third: Results, tests, and treatment of the problem of non-homogeneity of variance in the error.

First: Results of non-homogeneity of variance in the random error: OLS method for estimating the variables of the sample is useless as the problem is there because the variables will not be the best unbiased linear estimation (BLUE) for not having the feature of the least possible variance. To avoid this problem, the sample should be reformulated. The treatment method used in such cases is the weighted least squares method.

![Figure 3. The schemes of problem of non-homogeneity of variance in the error.](image-url)
\[ Y_i = \beta_0 + \beta_1 X_{i1} + \ldots + \beta_k X_{ki} + U_i \quad \ldots (5) \]

Assuming that the variance of the error is \( \sigma_i^2 \) can be obtained as follows:

\[ \sigma_i^2 = f ( \beta_0 + \beta_1 Z_{1i} + \ldots + \beta_m Z_{mi} ) \quad \ldots (6) \]

Because \( \sigma_i^2 \) is a function of the variables which do not have something in common 'Z's thus, some or all X's may be used as Z's.

To clarify this, it is assumed that:

\[ \sigma_i^2 = \beta_0 + \beta_1 Z_{1i} + \ldots + \beta_m Z_{mi} \quad \ldots (7) \]

This means that \( \sigma_i^2 \) is a linear function for (-Z's).

\( \sigma_i^2 \) which is a fixed variance: testing the following null hypothesis and then testing that

\[ H_0 : \beta_0 = \beta_1 = \ldots = \beta_m = 0 \]

Which represents the basic idea of the (B.P.G) test using (OLS) and then estimating the residual

\( \bar{U}_1, \bar{U}_2, \ldots, \bar{U}_n \) to obtain \( \sigma^2 \) which is formulated as:

\[ \sigma^2 = \sum \bar{U}_i^2 / n \quad \ldots (8) \]

The estimated (ML) (Maximum Likelihood) of \( -\sigma^2 \).

Whereas, \( \sum \bar{U}_i^2 / n - k \) due to (OLS), then the variables are mixed \( P_i \) defined as follows

\[ P_i = \bar{U}_i^2 / \sigma^2 \quad \ldots (9) \]

Which is the sample of each residual square divided by \( \sigma^2 \). Then, a regression is made \( P_i \) to Z's as follows

\[ P_i = \beta_0 + \beta_i Z_{1i} + \ldots + \beta_m Z_{mi} + U_i \quad \ldots (10) \]

Finally, the squares shown below are obtained (S.S.R) through \( \Theta \) which is formulated as follows:

\[ \Theta = \frac{1}{2} \text{(SSR)} \quad \ldots (11) \]

Assuming that \( (U_i) \) is naturally distributed, it is noticeable that whether variance is fixed through the following testing statistics,

\[ \Theta \sim \chi^2_{m-1} \quad \ldots (12) \]

Whereas, \( \Theta \) is distributed according to Ki square (m-1), compared with the extracted value from the formula (16-2) with a value of \( \chi^2 \) whereas, \( \Theta < \chi^2 \) which leads to the refusal of null hypothesis which indicates that there is a fixed variance.

2. Plot Test \([8][10]\)

According to this method, the residual square is tested \( (\bar{U}_i^2) \) by drawing the observations of this variable as opposed to the observations of \( (\bar{Y}_i) \) estimated from the regression line. If there is no systematic pattern, errors are homogeneous as follows:
But, if errors are not homogeneous, there is a difference in the systematic pattern and there are certain patterns indicating that the variance is homogeneous as follows:

Instead of Diffusion diagram of $U_i^2$ and $Y_i$ it is possible to put $U_i^2$ opposite to one of the independent variables, especially, if the result does not show a systematic pattern between the $(X_i, U_i^2)$

3. **Spearman's Rank Correlation Test** [2][4][10]

This is the simplest and oldest test for identifying the problem of non-homogeneity of variance in the random error in the estimated sample whether simple or various. It can be used to test small and huge samples. This test is based on the absolute values of error as well as the independent values $(X_i)$ It measures the rank of correlation between $(e_i)$ and $(X_i)$ as follows:

$$-1.96 < Z^* < 1.96$$  ...(13)

the variance of error is homogeneous, which validates the null hypothesis. The above formula may be put in another way as follows:

$$r > \frac{1.96}{\sqrt{n-1}}$$  ...(14)

In case of validating (13), correlation is in 5%, which refers to the problem of non-homogeneity of variance in the random error. Otherwise, there is no problem of non-homogeneity.

In case of a correlation between an independent variable with an affiliate variable in the estimated sample, the rank correlation between $(....)$ and each independent variable is estimated and tested according to the formula (13)

This test is summarized as follows:
Identifying the statistical hypothesis to be tested as follows:

\[ H_0: \rho_s = 0 \quad \text{Vs} \quad H_1: \rho_s < 0 \quad \text{Or} \quad H_1: \rho_s > 0 \quad \text{Or} \quad H_1: \rho_s \neq 0 \]

2- Identifying the valuable level (\(a\))

3- Estimating the test statistics, (15,16,17)

\[ r_s = 1 - \frac{6 \sum_{i=1}^{n} d_i^2}{n(n^2-1)} \]  
\[ d_i = \text{Rank}|X| - \text{Rank}|e_i| \]  
\[ e = Y_i - \hat{Y}_i \]

4- Extracting critical values (\(r_s^*\)) of Spearman’s rank correlation test from the tables depending on the number of couples of variables (\(n\)) and the required valuable level (\(a\)).

5- Identifying the area of refusing of null hypothesis according to the alternative hypothesis \(H_1\).

Third; Methods of treatment of the problem of non-homogeneity of variance in the random error \([5][8][11]\).

This problem can be treated by identifying weights and then using them to change the form of the sample, which results in equal values for the area of the matrix \(\sigma^2_W I_n\). To validate this assumption, there are many ways:

1- The weighted least squares method \([10][9][8][6]\)

It is the most prominent method for treating the problem. The idea of this method is to give more weight for the least regression values on the regression line than the values whose more regression when estimating the tested relation. Hence, the weight is the reverse of the standard regression of the residual;

\[ W_i = \frac{1}{\sigma_i} \]  

The least the regression, the more the weight is and vice versa

\(W_i\) are weights which knowing them makes testing, estimation, and prediction easy. Applying (W.L.S) for identifying the sample variables requires multiplying the two sides by the roots of the weights;

\[ \sqrt{W_i Y_i} = B_0 \sqrt{W_i} + B_1 \sqrt{W_i} X_{i1} + \sqrt{W_i} U_i \]  

\[ E (\sqrt{U_i})^2 = W_i E (U_i)^2 \]

\[ E (\sqrt{U_i})^2 = \sigma_i^2 W_i = \sigma^2 \]

In case of the general linear sample, it is possible to write the sample 2-4 using the matrix in brief,

as follows:

\[
\begin{bmatrix}
\sqrt{W_1} Y_1 \\
\sqrt{W_2} Y_2 \\
\sqrt{W_n} Y_n
\end{bmatrix} =
\begin{bmatrix}
\sqrt{W_1} X_{11} & \sqrt{W_1} X_{12} & \cdots & \sqrt{W_1} X_{1k} \\
\sqrt{W_2} X_{21} & \sqrt{W_2} X_{22} & \cdots & \sqrt{W_2} X_{2k} \\
\vdots & \vdots & \ddots & \vdots \\
\sqrt{W_n} X_{n1} & \sqrt{W_n} X_{n2} & \cdots & \sqrt{W_n} X_{nk}
\end{bmatrix}
\]

this can be expressed briefly as follows:
\[ P^{-1}Y = P^{-1}XB + P^{-1}U \]  \hspace{0.5cm} \ldots (20)

\[
p^{-1} = \begin{bmatrix}
\sqrt{W_1} & 0 & 0 & \ldots & 0 \\
0 & \sqrt{W_2} & 0 & \ldots & 0 \\
0 & 0 & \sqrt{W_3} & \ldots & 0 \\
\vdots & \vdots & \vdots & \ddots & \vdots \\
0 & 0 & 0 & \ldots & \sqrt{W_n}
\end{bmatrix}
\]  \hspace{1cm} \therefore \quad p = \begin{bmatrix}
\frac{1}{\sqrt{W_1}} & 0 & 0 & \ldots & 0 \\
0 & \frac{1}{\sqrt{W_2}} & 0 & \ldots & 0 \\
0 & 0 & \frac{1}{\sqrt{W_3}} & \ldots & 0 \\
\vdots & \vdots & \vdots & \ddots & \vdots \\
0 & 0 & 0 & \ldots & \frac{1}{\sqrt{W_n}}
\end{bmatrix}
\]

\[
\therefore W = pp' = \begin{bmatrix}
\frac{1}{W_1} & 0 & 0 & \ldots & 0 \\
0 & \frac{1}{W_2} & 0 & \ldots & 0 \\
0 & 0 & \frac{1}{W_3} & \ldots & 0 \\
\vdots & \vdots & \vdots & \ddots & \vdots \\
0 & 0 & 0 & \ldots & \frac{1}{W_n}
\end{bmatrix}
\]

The above sample (20) validates the required assumptions for applying WLS:

To find the linear variance:

1) \[ E(p^{-1}U)(p^{-1}U)' = E(p^{-1}U)(U \cdot p^{-1}) = \sigma^2 I_n \]

The above result validates the two hypotheses of homogeneity of error and the auto disengagement (mutual variance). So, the basic hypotheses of the regression sample are validated and WLS in the sample (20) can be applied.

\[
U * U = (p^{-1}U)(p^{-1}U)' = (p^{-1}(Y - XB))'(p^{-1}(Y - XB))
\]

\[
U * U = (Y'(p^{-1}B'X) p^{-1})(p^{-1}Y - p^{-1}XB)
\]

\[
\frac{\partial U * U}{\partial B} = -2X'W^{-1}Y + 2X'W^{-1}XB
\]

\[
WLS = (X'W^{-1}X)^{-1}X'^{-1}Y
\]

WLS, which is the alternative of O.L.S, is based on the idea of giving the values whose least regression on the regression line more weight the values whose more regression when estimating the sample. The weight it depends is the reversed ratio of variance as in the following formula:

\[
w_i = \frac{1}{\sigma_i^2} \quad \ldots (21)
\]

To estimate \( W_i \), the following steps are followed:

Extracting the natural logarithm from the residue squares by (O.L.S) method

Making regression to \( X_i \)

From opposite logarithm \( \sigma_i^2 \)

\textit{W.L.S proving that unbaits}
∴ $E\hat{b}_{WLS} = B$

∴ $b_{WLS}$ is unbiased estimator for $\beta$

To find the matrix of mutual variance

$\therefore \text{var} - \text{Cov}(b) = \sigma^2 (X^\prime W^{-1} X)^{-1}$

$S_e^2 = \frac{1}{n-k-1} e^\prime e^*$

$S_e^2 = \frac{Y^\prime W^{-1} Y - b^\prime X^\prime W^{-1} Y}{n-k-1}$

4. Fourth Relative efficiency

It is the comparison of (W.L.S) method to (O.L.S) method

$\text{eff.}(\beta_0) = \frac{\text{Var}(\beta_0) \text{by W.L.S}}{\text{Var}(\beta_0) \text{by O.L.S}}$; \hspace{0.5cm} $\text{eff.}(\beta_1) = \frac{\text{Var}(\beta_1) \text{by W.L.S}}{\text{Var}(\beta_1) \text{by O.L.S}}$ \hspace{0.5cm} and \hspace{0.5cm} $\text{eff.}(\beta_2) = \frac{\text{Var}(\beta_2) \text{by W.L.S}}{\text{Var}(\beta_2) \text{by O.L.S}}$

If the result is:

Sound1, Both samples are relatively efficient

Less than sound 1, (W.L.S) sample is more efficient than (O.L.S) sample

More than sound 1, (O.L.S) sample is more efficient than (W.L.S) sample

5. Fifth; Tables of analyzing variance in light of the problem of non-homogeneity of variance:[4]

It is possible to analyze the hypothesis of variance in light of the problem of non-homogeneity of variance in the weights of the matrix ($P^{-1}$) Implementing the weights using the matrix results in:

$e^\prime W^{-1} e = (Y-Xb_{WLS})^\prime W^{-1}(Y-Xb_{WLS})$

$= Y^\prime W^{-1} Y - 2b^\prime_{WLS} X^\prime W^{-1} Y + b^\prime_{WLS} X^\prime W^{-1} X b_{WLS}$

$e^\prime W^{-1} e = Y^\prime W^{-1} Y - b^\prime_{WLS} X^\prime W^{-1} Y$

$Y^\prime W^{-1} Y = b^\prime_{WLS} X^\prime W^{-1} Y + e^\prime W^{-1} e \hspace{2cm} \cdots(22)$

The (22) above formula clarifies the basic sources of regressions of the positive observations of the dependent variable $Y$ in light of the hypothesis of non-homogeneity of error, whereas:

$Y^\prime W^{-1} Y$ : represents total regressions (S.S.T)

$b_{WLS}^\prime X^\prime W^{-1} Y$ : represents identified regressions (S.S.R)

$e^\prime W^{-1} e$ : represents unidentified regressions (S.S.E)

whereas, the formula of the limiting variable in light of non-homogeneity of variance is given as follows:

$R^2 = \frac{S.S.R}{S.S.T} = \frac{b_{WLS}^\prime X^\prime W^{-1} Y}{Y^\prime W^{-1} Y}$

The three basic regression sources can be expressed as follows:
\[ Y'W^{-1}YR^2 = b_{WLS}X'W^{-1}Y \]  
...(23)

From the formula (23) after alternation and rearrangement:

\[ e'W^{-1}e = Y'W^{-1}Y - Y'W^{-1}YR^2 \]

\[ e'W^{-1}e = Y'W^{-1}Y (1-R^2) \]  
...(24)

The formula (24) represents the regressions which are not limited to the limiting variable while the formula (23) represents the identified regressions in light of the limiting variable. The formula (22) represents the total regressions.

The above three basic regression sources are the corner stone of the table of analyzing variance(ANOVA) in light of the problem of non-homogeneity of variance in the error, as follows:

| Table1. Analyzing variance in light of non-homogeneity of variance in the random error. |
|---|---|---|---|
| S.o.f V | SS | D.F | MSS |
| identified regressions Due to \( X_1, X_2, \ldots, X_n \) | \( b_{WLS}X'W^{-1}Y = Y'W^{-1}YR^2 \) | \( k \) | \( \frac{Y'W^{-1}YR^2}{k} \) |
| unidentified regressions (Residual) | \( e'W^{-1}e = Y'W^{-1}Y (1-R^2) \) | \( n-k-1 \) | \( \frac{Y'W^{-1}Y(1-R^2)}{n-k-1} \) |
| Total regressions Total Variance | \( Y'W^{-1}Y = Y'W^{-1}Y \) | \( n-1 \) | \[ F_0 = \frac{Y'W^{-1}YR^2}{k} \left( \frac{Y'W^{-1}Y(1-R^2)}{n-k-1} \right) \] |
| | | | \( \because F_0 = \frac{R^2}{k} \left( \frac{(1-R^2)}{(n-k-1)} \right) \) |

Comparing the practical value to the theoretical value at freedom degree of \((n-k-1),(K)\) at level 3 as it refers to a linear relationship between the dependent variable and the independent variables and vice versa, there is a strong relationship among the variables.

6. **Estimating non-homogeneity of variance.**

In this section, the ordinary least squares method for estimating non-homogeneity of variance in the random error OLS will be discussed. In case the problem happens, it is treated through the weighted least squares method WLS to obtain the best results which correspond to the statistical theory and the economic theory, therefore, WLS has been used depending on the weight 1/ which represents the variance of the second independent variable, the national income in fixed prices to avoid the problem of non-homogeneity of variance in the random error. The methods used to test the problem: Plot test, Breusch, Pajan, and Godfrey's test, and EVIEW9 program for analyzing data and extracting results, which is an advanced statistical program that gives accurate analytical results.

7. **Describing the research sample.**

The data are some changes in the Iraqi economy from 1980-2015. The dependent variable has been used; The total change for total capital in fixed prices. The two independent variables; the
national income and the local outcome in fixed prices. Y: total capital in fixed prices; money of producing machines, instruments, building, and increasing the reserved. The local products in fixed prices (GDP); The amount of goods and services inside the country in a certain period of time whether the raw materials are possessed by the citizens or foreigners, thus, it is called "the geographical product of the country." The national income in fixed prices to avoid prices change. The national income during the two years is re estimated in fixed prices. Each year's prices are normal whether high or low. The income which is estimated in this way is called "the income in fixed prices". Each year's quantities and prices are taken.

**Table 2.** the income in fixed prices.

| View         | Pros         | Object       | Print | Name | Freeze | Estimate | Forecast | Stats | Resides |
|--------------|--------------|--------------|-------|------|--------|----------|----------|-------|---------|
|              |              |              |       |      |        |          |          |       |         |
| Dependent Variable: S1 |              |              |       |      |        |          |          |       |         |
| Method: Least Squares (Gauss-Newton / Marquardt steps) |              |              |       |      |        |          |          |       |         |
| Sample: 1980 2015 |              |              |       |      |        |          |          |       |         |
| Included observations: 36 |              |              |       |      |        |          |          |       |         |
| S1=C(1)+C(2)*S2+C(3)*S3 |              |              |       |      |        |          |          |       |         |

| Variable | Coefficient | Std. Error | t-Statistic | Prob. |
|----------|-------------|------------|-------------|-------|
| C(1)    | -1624.644   | 2567.872   | -0.632681   | 0.5345|
| C(2)    | 0.172885    | 0.142893   | 1.209890    | 0.2412|
| C(3)    | 0.157789    | 0.059761   | 2.640344    | 0.0161|

| R-squared | 0.324341    | Mean dependent var | 3405.141 |
| Adjusted R-squared | 0.253219    | S.D. dependent var | 3327.560 |
| S.E of regression | 2875.560    | Akaike info criterion | 18.89201 |
| Sum squared residue | 1.57E+08    | Schwarz criterion | 19.04078 |
| L og likelihood | -204.8121   | Hannan-Quinn criter | 18.92705 |
| F -statistic | 4.560352    | Durbin – Watson stat | 0.772670 |
| Prob(F-statistic) | 0.0244122   |                  |         |

1- Limiting variable R2. Its value is 32.4%.; The interpretation of the independent variables is 32.4%, whereas, the remaining 67.6% belongs to unexplained factors within the elements of the random error. The above sample shows that the increase of local outcome in the fixed prices is 100%, which leads to increase the total capital to 17%, and increase the national income to 100% and the fixed capital to 16%. -2(F) test. The estimated value of (F) is 4.560352, which is more than table value of (F) which is 0.772670; Independent variables effect the dependent variable.

2- (T) test: the (T (value of the independent variable of the total local outcome is (1.209890) which is greater than the table value of (T) which is (0.2412) which effects the dependent variable. The value of (T) of the independent variable of national income in fixed prices is (2.640344) which is greater than the table value (0.0161) which effects the dependent variable of total fixed capital.
Table 3. Results of testing ....... before treatment.

| View | Proc | Object | Print | Name | Freeze | Estimate | Forecast | Stats | esides |
|------|------|--------|-------|------|--------|----------|----------|-------|--------|
|      |      |        |       |      |        |          |          |       |        |
|      |      |        |       |      |        |          |          |       |        |
|      |      |        |       |      |        |          |          |       |        |
| Heteroses denticity Test: Breusch-Pagan-Godfrey |
| F-statistic | 4.274433 | Prob. F(2.19) | 0.0293 |
| Obs*R- squared | 6.826962 | Prob. Chi-Square(2) | 0.0329 |
| Scaled explained SS | 5.956107 | Prob. Chi-Square(2) | 0.0509 |

Test Equation:
Dependent Variable: RESID^2
Method: Least Squares
Sample: 1980 2015

| Variable | Coefficient | Std. Error | t-Statistic | Prob. |
|----------|-------------|------------|-------------|-------|
| 0.9031   | C           | -1075077   | 8716413     | -0.123339 |
| S2       | 27.28030    | 485.0369   | 0.056244    | 0.9557 |
| S3       | 589.2655    | 202.8525   | 2.904897    | 0.0091 |

1-The estimation results show that the test estimated value () which is (6.826962) is valuable as the hypothetical test value is (0.0329) is less than the valuable level (5%) which results in refusing the null hypothesis which indicates that random error does not suffer the problem of non- homogeneity of variance in the random error.

2-(F) test: From the above table (5), it is noticed that the estimated value of (F); (4.274433) is greater than the table value of (F) which is (0.0293) under the valuable level (5%) at a degree of freedom (2,19), which means that independent variables of national income in fixed prices and total local outcome in fixed prices effect the total dependent variable for fixed capital in fixed prices.

3-(T) test: The table shows that the (T) value of the first independent variable is (0.056244) less than the (T) table value which is (0.9557) which means that the independent variable of local outcome in fixed prices does not affect the total dependent variable of fixed capital in fixed prices. The (T) value of the second independent variable is (2.904897) which is greater than the table value (0.0091). The value of the limiting variable is 31% of changes of fixed capital. The remaining 69% belongs to uninterpreted factors within the elements of random error.

Second; Treatment of the problem of non- homogeneity of variance in random error and relative efficiency.
Table 4. Treatment by using WLS method.

| View | Proc | Object | Print | Name | Freeze | Estimate | Forecast | Stats | Resides |
|------|------|--------|-------|------|--------|----------|----------|-------|---------|
|      |      |        |       |      |        |          |          |       |         |
|      |      |        |       |      |        |          |          |       |         |
|      |      |        |       |      |        |          |          |       |         |
|      |      |        |       |      |        |          |          |       |         |
|      |      |        |       |      |        |          |          |       |         |
|      |      |        |       |      |        |          |          |       |         |
|      |      |        |       |      |        |          |          |       |         |
|      |      |        |       |      |        |          |          |       |         |
|      |      |        |       |      |        |          |          |       |         |
|      |      |        |       |      |        |          |          |       |         |
|      |      |        |       |      |        |          |          |       |         |
|      |      |        |       |      |        |          |          |       |         |
|      |      |        |       |      |        |          |          |       |         |
|      |      |        |       |      |        |          |          |       |         |
|      |      |        |       |      |        |          |          |       |         |
|      |      |        |       |      |        |          |          |       |         |
|      |      |        |       |      |        |          |          |       |         |
|      |      |        |       |      |        |          |          |       |         |
|      |      |        |       |      |        |          |          |       |         |
|      |      |        |       |      |        |          |          |       |         |
|      |      |        |       |      |        |          |          |       |         |
|      |      |        |       |      |        |          |          |       |         |
|      |      |        |       |      |        |          |          |       |         |
|      |      |        |       |      |        |          |          |       |         |

Treatment by using WLS method.

1. The limiting coefficient (R2). The value of the limiting factor is (90%); which means that the independent variables are interpreted as 90%, while the remaining 10% is due to unexplained factors within the random error component. It is clear from the estimated model that the increase in the local outcome at constant prices by 100% leads to an increase in the total fixed capital formation by 99%. The increase of the second variable of national income at constant prices by 100% leads to decrease the total capital formation at fixed prices by(-3.2%).

2. (F): (6) above shows that the estimated value of (F) is (83.52059), which is greater than the table value of (F) which is (0.574645) at a significant level (5%) and means that the independent variables have an effect on the variable of Total fixed capital formation at constant prices.

3. T-test The above table indicates that the value of t for the first independent variable is 10.40678 which is greater than the table value of t, which is 0.0000. This means that the first independent variable affects the dependent variable; The total fixed capital formation at constant prices. The value of (t) of the independent variable, the national income at constant prices, is (-0.856539), which is less than the table value..(0.4024)

Breusch-Pagan-Godfrey's Test.
To confirm that the model estimated in the weighted least squares method does not suffer a non-homogeneity of variance in random error, the test has been used as follows:

**Table 5. Breusch-Pagan-Godfrey’s Test.**

| View | Proc | Object | Print | Name | Freeze | Estimate | Forecast | Stats | Resides |
|------|------|--------|-------|------|--------|----------|----------|-------|---------|
|      |      |        |       |      | Heteroskedasticity Test: Breusch-Pagan-Godfrey |          |          |       |         |
| F- statistic | 3.052344 | Prob.F(2,19) | 0.0709 |
| Obs*R-squared | 5.349724 | Prob.Chi-square(2) | 0.0689 |
| Scaled explained SS | 4.944897 | Prob.Chi-square(2) | 0.0844 |

**Test Equation:**

Dependent Variable: WGT - RESID^2
Method: Least Squares
Sample: 1980 - 2015
Included observations: 36

| Variable | Coefficient | Std. Error | t-Statistic | Prob. |
|----------|-------------|------------|-------------|-------|
| C        | 535241.4    | 507057.8   | 1.055583    | 0.3044 |
| S2*WGT   | 194.0067    | 79.60756   | 2.437039    | 0.0248 |
| S3*WGT   | -76.48497   | 31.58840   | -2.421300   | 0.0256 |
| R-squared | 0.243169    | Mean dependent var | 1185997. |
| Adjusted R-squared | 0.163503 | S.D. dependent var | 1911095. |
| S.E of regression | 1747892. | Akaike info criterion | 31.71184 |
| Sum squared reside | 5.80E+13 | Schwarz criterion | 31.86062 |
| Log likelihood | -345.8303 | Hannan-Quinn criterion | 31.74689 |
| F-statistic | 3.052344 | Durbin-Watson stat | 1.574623 |
| Prob(F- statistic) | 0.070874 |

1- The value of the test (X^2) for the first variable (5.349724) is significant because the probability value of (0.0689) and the estimated value of the test (X2) The second variable of (4.944897) is significant because the probability value of (0.0844) (5%). This leads to the acceptance of the null hypothesis that random error does not have the problem of heterogeneity of variance in random error.

(F). The estimated F value is 3.052344, which is greater than the value of the periodic F value of 0.0709 below the 5% and the freedom level of (2,19). This means that the independent variables have an effect on the dependent variable.

(T) The above table explains that the value of t for the first independent variable is 2.437039 is greater than the tabular value of 0.0248. This means that the independent variable Gross local outcome at constant prices affects the fixed variable; fixed capital. The value of t for the second independent variable is to (-2.421300) less than the tabular value.(0.0256)

2. (%24.3)This means that the independent variables account for 24.3% of the variables in the total fixed capital formation. The remaining 774% is due to unexplained factors within the random error component.

Relative efficiency of estimation: For the sake of comparison between OLS and WLS, the table of relative efficiency (7) is taken.
Table 6. Breusch-Pagan-Godfrey'.

|               | (W.L.S)      | (O.L.S)      | MSE        | Se         | S(b₀)     | S(b₁)     | S(b₂)     | S²(b₀)     | S²(b₁)     | S²(b₂)     |
|---------------|--------------|--------------|------------|------------|-----------|-----------|-----------|------------|------------|------------|
| 1373260.547   | 8268845      | 8268845      |            |            |           |           |           |            |            |            |
| 1171.862      | 2875.56      | 2875.56      |            |            |           |           |           |            |            |            |
| 2483.424      | 2567.872     | 2567.872     |            |            |           |           |           |            |            |            |
| 0.095127      | 0.142893     | 0.142893     |            |            |           |           |           |            |            |            |
| 0.036849      | 0.059761     | 0.059761     |            |            |           |           |           |            |            |            |
| 6167394.8     | 6593966.9    | 6593966.9    |            |            |           |           |           |            |            |            |
| 0.00904917263386991 | 0.0204184 | 0.0204184 |            |            |           |           |           |            |            |            |
| 0.001357849633575406 | 0.0035714 | 0.0035714 |            |            |           |           |           |            |            |            |

Table 7. the comparison between OLS and WLS.

| Comparison    | OLS method (O.L.S) | WLS method (W.L.S) |
|---------------|---------------------|---------------------|
| MSE           | 8268845             | 1373260.547         |
| Se            | 2875.56             | 1171.862            |
| S(b₀)         | 2567.872            | 2483.424            |
| S(b₁)         | 0.142893            | 0.095127            |
| S(b₂)         | 0.059761            | 0.036849            |
| S²(b₀)        | 6593966.9           | 6167394.8           |
| S²(b₁)        | 0.0204184           | 0.00904917263386991|
| S²(b₂)        | 0.0035714           | 0.001357849633575406|

\[
\text{Eff. (β₀)} = \frac{\text{Var (β₀) by w.l.s}}{\text{Var (β₀) by o.l.s}} = \frac{6167394.764}{6593966.608} = 0.9353
\]

\[
\text{Eff. (β₁)} = \frac{\text{Var (β₁) by w.l.s}}{\text{Var (β₁) by o.l.s}} = \frac{0.00904917263386991}{0.02041840945} = 0.4432
\]

\[
\text{Eff. (β₂)} = \frac{\text{Var (β₂) by w.l.s}}{\text{Var (β₂) by o.l.s}} = \frac{0.001357849633575406}{0.003571377121} = 0.38
\]

This means that the relative efficiency of (b₀) does not exceed (94%) only, which indicates that estimation is more efficient in terms of accuracy according to (WLS). As for the second, it is clear that the relative efficiency of estimating regression is greater according to (OLS) as compared to (WLS) because the efficiency result is (44%) which is less than the sound one by (38%), which indicates that OLS is more than WLS by (1.5), which indicates that the parameters of WLS are more efficient and accurate. This means that WLS is better and more reliable than OLS.

Section three; conclusions and recommendations.

8. Conclusions

Based on the results, the conclusions have been as follows:

1. By comparing the OLS and WLS, the WLS is better than OLS due to the efficiency of the estimation, the average error squares for the capabilities, and the difference in the estimation of the parameters S2 (bj).
2. (%90) after treatment by WLS. This means that the independent variables explain 90% of the changes in the adopted variable. The remaining 10% is due to unexplained factors found in the random error.

3. The standard error of the parameters estimated in the weighted least squares (WLS) is less than the ordinary least squares (OLS) as shown in Table (7) and Table (8).

4. The graph shows that the value of the residual squares increases by increasing the estimated dependent variable.

5. In light of this, the inverse of the national income variance at constant prices (1 /) has been used as a weight to solve the problem of heterogeneity of variance in the random error.

6. The value of the average squares of errors in the manner of weighted least squares (WLS) has amounted to (1373260.547), which is less than the average squares of errors in the manner of ordinary least squares (OLS).

9. **Recommendations:**

In light of the above results, the present study recommends the following points:

1. The use of other methods in the treatment of non-homogeneity of variance, including the method of Biz and the five assumptions and the method of the general least squares (GLS) and the choice of a multiplier of LA Range (LM).

2. Studying the problem of non-homogeneity of variance in random error in the case of linear and Non-Linear models.

3. Using the methods of Biz in estimating the parameters and comparing them to the methods discussed in the present study as well as the method of greater possibility (MLE).

**References.**

[1] Ahmed Sultan, Haitham Yacoub Yousef, Hisham Faroon Abdullatif, Ibrahim Jawad Kazem (2013) "Analysis of the regression problem using the software Eviews8.1" First Edition Iraq, Baghdad.

[2] Hasnawi, Amoori Hadi and Qaisi, Bassim Shleibah (2002) "Advanced economic measurement (theory and practice)" Dunia al-Amal Library - Baghdad.

[3] Al-Issa, Nizar Saaduddin, Qatuf, Ibrahim Sulaiman (2006) "Macroeconomics Principles and Applications", First Edition, Jordan-Amman, Al-Hamid press office for Publishing and Distribution.

[4] Al-Qaisi, Basem Shalibah (1999), "The Style of Bees in Testing and Addressing the Problem of Heterogeneity in Linear Models (Comparative Study)”, Master of Statistics, Faculty of Management and Economics, University of Baghdad.

[5] Saleh, Farah Abdul Ghani Younis (2004), "Studying errors and their impact on the results of regression analysis of the variables of preterm infants," MA, Faculty of Computer Science and Mathematics, University of Mosul - Iraq.

[6] Baltagi, badi (2005) "Econometric Analysis of panel data" John wiley and sons inc. third edition.

[7] Baltagi, badi (2006) "contributions to econometric analysis" Elserier B. B.V.

[8] Greene, William H. (2003) "Econometric analysis" pearson ducation, Inc. Fifth edition.

[9] Draper, N. & Smith. H. (1981), "Applied Regression Analysis", John Wiley & Sons, New York.

[10] Park, H. (2005) "Linear regression models for panel data using SAS STATS, LIMDEP and SSPS" http://www.indiana.edu/~stat.math.

[11] Schark, M. & Remington, R.D. (2000), "Statistics with Applications to the biological and Health Sciences"; 3rd edition, Prentice Upper Saddle River, New Jersey, U.S.A.