A natural framework for bi-large neutrino mixing

Stuart Raby

Department of Physics, The Ohio State University, 174 W. 18th Ave, Columbus, OH 43210, USA
Email: raby@pacific.mps.ohio-state.edu

In this talk I describe a natural framework for bi-large neutrino mixing within the context of two models – 1) a simple generalization of the MSSM and 2) an SO(10) model. Our starting point is the Frampton, Glashow, Yanagida [FGY] neutrino mass ansatz which can easily accomodate bi-large neutrino mixing. The main point of FGY, however, is to obtain a theory of neutrino masses with only one possible CP violating angle. They argue that the sign of the baryon asymmetry of the universe (assuming leptogenesis) is then correlated with CP asymmetries possibly observable in accelerator experiments. Unfortunately, there is a fly in the ointment. It was later shown by Raidal and Strumia [RS] that there is a sign ambiguity which frustrates the above correlation. We note that the Raidal-Strumia ambiguity is resolved in our models.

1. Neutrinos: Masses and Mixing Angles

Let us first summarize the present values of neutrino masses and mixing angles obtained by fitting atmospheric, solar, reactor and accelerator neutrino oscillation data. The atmospheric and solar neutrino masses and mixing angles are given below

- $\Delta m^2_{\text{atm}} = |m^2_3 - m^2_2| \approx 3 \times 10^{-3} \text{eV}^2$
  $\sin 2\theta_{\text{atm}} \approx 1$

- $\Delta m^2_{\text{sol}} = |m^2_2 - m^2_1| \approx 7 \times 10^{-5} \text{eV}^2$
  $\sin 2\theta_{\text{sol}} \leq 1$

with an approximate mixing matrix given by

$$
\begin{pmatrix}
\nu_e \\
\nu_\mu \\
\nu_\tau
\end{pmatrix} \approx 
\begin{pmatrix}
c_{\text{sol}}/\sqrt{2} & s_{\text{sol}}/\sqrt{2} & 0 \\
-s_{\text{sol}}/\sqrt{2} & c_{\text{sol}}/\sqrt{2} & 1/\sqrt{2} \\
-s_{\text{sol}}/\sqrt{2} & c_{\text{sol}}/\sqrt{2} & -1/\sqrt{2}
\end{pmatrix}
\begin{pmatrix}
\nu_1 \\
\nu_2 \\
\nu_3
\end{pmatrix}
$$

For this talk, I will assume three light neutrinos. Recall, the observed small neutrino masses can naturally be generated via the See-Saw mechanism. The $3 \times 3$ neutrino mass matrix is given by

$$
\sim m^2_\nu M^{-1}_N m_\nu,
$$

where $m_\nu$ ($M_N$) is the Dirac RL (Majorana RR) mass matrix. Of course, the main problem with neutrino mixing angles is the fact that they are significantly larger than CKM mixing. These large mixing angles may be obtained from one (or a combination) of the following sources – $m_\nu$, $M_N$; or $m_e$ (the charged lepton mass matrix). In the models presented below, the origin of large neutrino mixing angles is in $m_\nu$.

1.1. Frampton-Glashow-Yanagida ansatz

Consider now the FGY ansatz given by

$$
\mathcal{L} = (\nu_e a + \nu_\mu a' e^{-i\phi/2}) N_1 + (\nu_\mu b + \nu_\tau b') N_2 + \frac{1}{2} (M_1 N_1^2 + M_2 N_2^2)
$$

$$
\equiv \nu D^T N + \frac{1}{3} N M_N N
$$

where $N_{1,2}$ are the two right-handed (sterile) neutrinos and

$$
D^T = \begin{pmatrix}
a & 0 & a' e^{-i\phi/2} \\
0 & b
\end{pmatrix}
$$

This neutrino mass matrix ansatz is expressed in the lepton flavor eigenbasis. The dimensionful parameters $a$, $a'$, $b$, $b'$ are chosen to satisfy the relations $b \approx b'$ for maximal atmospheric mixing angle and $a \sim a'$ for a large, but not maximal, solar mixing angle. In addition, due to the zeros in the mass matrix ansatz, there is only one
non-vanishing CP violating angle, $\phi$. Upon integrating out the heavy sterile neutrinos ($N$), we obtain the $3 \times 3$ FGY light neutrino mass matrix given by

$$\mathcal{M}_{FGY} = D^T M_N^{-1} D$$

The neutrino mass eigenvalues and small mixing angle are given by

$$m_{\nu_1} \approx 2b^2/M_2 \approx 0.05 \text{ eV} = \sqrt{\Delta m^2_{\text{atm}}}$$

$$m_{\nu_2} \approx 2a^2/M_1 \approx 8.4 \times 10^{-3} \text{ eV} = \sqrt{\Delta m^2_{\text{sol}}}$$

$$m_{\nu_3} = 0; \theta_{13} \sim m_{\nu_2}/(\sqrt{2} m_{\nu_3})$$

1.2. Raidal and Strumia analysis

A detailed $\chi^2$ analysis of the FGY ansatz including atmospheric and solar neutrino oscillation data was performed by Raidal and Strumia \[2\]. Their best fit then makes predictions for $\theta_{13} = 0.078 \pm 0.015$, observable in planned long baseline experiments, and for the effective electron neutrino mass measured in neutrinoless double beta decay, $m_{\nu_ee} = 2.6 \pm 0.4 \text{ meV}$, which is unobservable in any planned experiment.

In addition, RS perform a detailed analysis of leptogenesis with FGY. They find two successful solutions providing an acceptable cosmological baryon asymmetry [this is the RS ambiguity]. One with

$$M_1 \ll M_2; \; M_1 \approx 10^{11} \text{ GeV}/|\sin \phi| \; \text{and} \; \phi < 0. \quad (3)$$

In this case, the prediction for CP violating neutrino oscillation is given by

$$P(\nu_e \rightarrow \nu_\mu) < P(\nu_\mu \rightarrow \nu_e) \equiv P(\bar{\nu}_e \rightarrow \bar{\nu}_\mu).$$

In a supersymmetric generalization of FGY, they predict the lepton number flavor violating branching ratios

$$B(\mu \rightarrow e\gamma) \approx 2 \times 10^{-13};$$

$$B(\tau \rightarrow \mu\gamma) \geq 3 \times 10^{-12}$$

where $r \approx (\tan \beta/10)^2 (150 \text{ GeV}/m_{\text{SU}(2)})^2$.

The other with

$$M_1 \gg M_2; \; M_2 \approx 10^{12} \text{ GeV}/|\sin \phi| \; \text{and} \; \phi > 0. \quad (4)$$

they find

$$P(\nu_e \rightarrow \nu_\mu) > P(\nu_\mu \rightarrow \nu_e) \equiv P(\bar{\nu}_e \rightarrow \bar{\nu}_\mu)$$

and

$$B(\tau \rightarrow \mu\gamma) \approx 7 \times 10^{-11};$$

$$B(\mu \rightarrow e\gamma) \geq 10^{-11}.$$

2. “Natural” FGY texture in SUSY with $[\text{SU}(2) \times \text{U}(1)]_{FS}$

We now obtain the FGY texture in a supersymmetric theory with an $[\text{SU}(2) \times \text{U}(1)]_{FS}$ family symmetry protecting the zeros \[3\]. This is a theory of leptons with the 3 families of electroweak doublets given by $l_i = \left(\nu_i, e_i\right)$. The first two families transform as a doublet under the SU(2) family symmetry with $L_a = l_a, \; a = 1, 2$, and then $l_3$ is an SU(2) singlet. The superpotential for the neutrino mass sector is given by (we will discuss the charged lepton mass sector next)

$$W = \frac{H_u}{M} \left( L_a \phi^a N_1 + L_a \tilde{\phi}^a N_2 + l_3 \omega N_2 \right) + \frac{1}{2} \left( S_i N_1^2 + S_2 N_2^2 \right) \quad (5)$$

where $M$ is a new scale satisfying $M \gg M_Z$. The familon fields $\phi_a, \tilde{\phi}_a, \omega$ are assumed to get the following vacuum expectation values [VEVs] breaking the family symmetry and generating neutrino masses. We have, in complete generality, $\langle \phi \rangle = \left(\sqrt{\langle \phi_1 \rangle}, \sqrt{\langle \phi_2 \rangle}\right)$ and $\langle \tilde{\phi} \rangle = \left(0, \sqrt{\langle \tilde{\phi}_2 \rangle}\right)$. In addition the two right-handed neutrinos $N_i, \; i = 1, 2$ obtain Majorana masses when the SU(2) singlet familon fields get VEVs given by $\langle S_i \rangle = M_i, \; i = 1, 2$.

The family symmetry [with specific U(1) charges for all the fields, as discussed in more detail in the paper \[3\] is sufficient to make this the most general superpotential consistent with the symmetry. Hence the zeros of the FGY ansatz are obtained, without fine tuning. When the Higgs doublet $H_u$ gets a VEV at the weak scale given by $\langle H_u \rangle = \left(0, v \sin \beta/\sqrt{2}\right)$ we finally obtain the FGY neutrino mass matrix with the parameters $a, \; a', \; b, \; b'$ given below.

$$a = v \sin \beta \frac{\langle \phi_1 \rangle}{\sqrt{2}M}, \quad a' = -i e^{i\phi/2} = v \sin \beta \frac{\langle \tilde{\phi}_2 \rangle}{\sqrt{2}M},$$

$$b = v \sin \beta \frac{\langle \phi_2 \rangle}{\sqrt{2}M}, \quad b' = v \sin \beta \frac{\langle \omega \rangle}{\sqrt{2}M}. \quad (6)$$
Note, however, unlike FGY we must now consider the charged lepton mass matrix which is also constrained by the family symmetry. Until we do this, we cannot be certain that we have obtained the FGY ansatz in the lepton flavor basis.

3. Charged lepton masses

Consider the superpotential for charged leptons given by

$$W_{ch. \text{leptons}} = \frac{H_d}{M} (L_a \phi^a \tilde{e}_1 + L_a \tilde{\phi}^a \tilde{e}_2 + l_3 (\omega \tilde{e}_2 + \tilde{\omega} \tilde{e}_3))$$  \hspace{1cm} (7)

where the left-handed anti-leptons $\tilde{e}_i$, $i = 1, 2, 3$ are SU(2) singlets. When the Higgs $H_d$ gets a VEV given by $\langle H_d \rangle = \left( \begin{array}{cc} v \cos \beta / \sqrt{2} \\ 0 \end{array} \right)$ we obtain the charged lepton $3 \times 3$ mass matrix below

$$m_l = \left( \begin{array}{ccc} \tilde{a} & \tilde{a}' & e^{-i\phi/2} & 0 \\ 0 & \tilde{b} & 0 & \tilde{b}' \\ 0 & 0 & \tilde{c} \end{array} \right)$$ \hspace{1cm} (8)

with

$$\tilde{a} = v \cos \beta \frac{\langle \phi^1 \rangle}{\sqrt{2} M}, \quad \tilde{a}' e^{-i\phi/2} = v \cos \beta \frac{\langle \phi^2 \rangle}{\sqrt{2} M},$$

$$\tilde{b} = v \cos \beta \frac{\langle \tilde{\phi}^2 \rangle}{\sqrt{2} M}, \quad \tilde{b}' = v \cos \beta \frac{\langle \omega \rangle}{\sqrt{2} M},$$

$$\tilde{c} = v \cos \beta \frac{\langle \tilde{\omega} \rangle}{\sqrt{2} M}$$

satisfying $\tilde{a}$, $\tilde{a}' \ll \tilde{b}$, $\tilde{b}' \ll \tilde{c}$. Note the parameters $\tilde{a}$, $\tilde{a}'$, $\tilde{b}$, $\tilde{b}'$ are, up to order one coefficients, the same as $a$, $a'$, $b$, $b'$ appearing in the neutrino mass matrix.

The charged lepton mass eigenvalues are approximately given by $m_e \approx \tilde{a}$, $m_\mu \approx \tilde{b}$, $m_\tau \approx \tilde{c}$ and the charged lepton mass matrix is diagonalized by the bi-unitary transformation $m_l^{diagonal} = U_e^T m_l U_e$ with the unitary matrix defining the left-handed mass eigenstates satisfying $U_e \approx Diag(1, e^{i\phi/2}, -e^{i\phi/2})$. Using this matrix we finally obtain the neutrino mass matrix in the flavor basis. We find

$$M = U_e^T \left[ D^T M_N^{-1} D \right] U_e \approx M_{FGY}$$ \hspace{1cm} (9)

with

$$D^T = \left( \begin{array}{ccc} a & 0 & 0 \\ a' e^{-i\phi/2} & b & 0 \\ 0 & 0 & b' \end{array} \right).$$

Note, the ratios $|a/a'|$ and $|b/b'|$ can be adjusted to accommodate bi-large neutrino mixing.

As a bonus we now see that the ratio of Majorana neutrino masses $M_1/M_2$ is fixed. We find $(m_\tau/m_\mu)^2 \approx (a/b)^2 \approx (a'/b')^2 \approx (M_1/M_2)(m_{\nu_2}/m_{\nu_3})$. Thus $(M_1/M_2) \sim 10^{-4}$ and the RS ambiguity is resolved!

3.1. Related issues

Note, an SU(2) family symmetry in SUSY theories is desirable for completely different reasons. It has been shown that it can ameliorate the SUSY flavor problem \[^4\]. For example, prior to SU(2) symmetry breaking the first and second generation sleptons are degenerate. Hence the off diagonal smuon-selectron mass term necessary for processes such as $\mu \rightarrow e\gamma$, seen in the figure below, are suppressed.

Finally, at the moment we have an effective higher dimensional field theory with new scales $M \sim M_1 \sim M_2 \gg M_2$. Recall that successful leptogenesis requires $M_1, M_2 > 10^{11}$ GeV. Hence it is reasonable to expect that $M \sim M_{GUT} \approx 3 \times 10^{16}$ GeV. Consider the following virtues of SUSY GUTs. We have

- $M_2 << M_{GUT}$ “Naturally”
- Explains Charge Quantization
- Predicts Gauge Coupling Unification*
- Predicts Yukawa Coupling Unification
- + Family Symmetry \implies Hierarchy of Fermion Masses and Protects against large flavor violation
- Neutrino Masses via See - Saw scale $\sim 10^{-3} - 10^{-2}$, $M_G \sim M_G^2/M_{Pl}$
- LSP – Dark Matter Candidate, and
- Baryogenesis via Leptogenesis

With all of these virtues it is worth considering embedding our generalized MSSM into an SO(10) SUSY GUT. We consider the following SO(10) SUSY GUT with an $[SU(2) \times U(1)^n]_{FS}$ proposed initially in [5] and analyzed in great detail in [6].

4. SO(10) SUSY GUT × $[SU(2) \times U(1)^n]_{FS}$

The superpotential for the charged fermion sector, including the heavy Froggatt-Nielsen [FG] states $\{ \chi^a, \bar{\chi}_a \}$, familon fields $\{ \phi^a, \bar{\phi}^a \}$ and SO(10) adjoint $\{ 45 \}$, is given by

$$W \supset 16_3 10_H 16_3 + 16_a 10_H \chi^a$$

$$+ \chi_a (M_X \chi^a + 45 \frac{\bar{\phi}^a}{M} 16_3$$

$$+ 45 \frac{\phi^a \bar{\phi}^b}{M} 16_b)$$

where we have the three families in $16_a (a,b = 1, 2, 16);$ and the Higgs doublets in $10_H$.

In addition, the FG mass $M_X$ necessarily includes SO(10) breaking VEVs with $M_X = M(1 + \alpha X + \beta Y)$. $X, Y$ are the SO(10) breaking VEVs in the adjoint representation of SO(10) with $X$ corresponding to the $U(1)$ in SO(10) which preserves $SU(5)$, and $Y$ the standard weak hypercharge. $\alpha, \beta$ are arbitrary parameters.

Upon integrating out the FG fields we obtain the effective fermion mass operators in the figure. Finally upon giving the familon fields VEVs we obtain the effective Yukawa couplings below.

$$Y_u = \begin{pmatrix}
0 & \epsilon' & \rho & -\epsilon \xi \\
-\epsilon' & \epsilon & \xi & -\epsilon \\
\epsilon & \xi & \epsilon & 1
\end{pmatrix} \lambda$$

$$Y_d = \begin{pmatrix}
0 & \epsilon' & -\epsilon \xi \sigma \\
-\epsilon' & \epsilon & -\epsilon \sigma & \epsilon \\
\epsilon & \xi & \epsilon & 1
\end{pmatrix} \lambda$$

$$Y_e = \begin{pmatrix}
0 & -\epsilon' & 3 \epsilon \xi \\
\epsilon' & 3 \epsilon & 3 \epsilon \xi & -3 \epsilon \xi \sigma & -3 \epsilon \sigma & 1
\end{pmatrix} \lambda$$

with

$$\xi = \langle \phi^1 \rangle / \langle \phi^2 \rangle; \quad \bar{\epsilon} \propto (\langle \bar{\phi}^2 \rangle / M)^2; \quad \epsilon \propto \langle \phi^2 \rangle / M; \quad \epsilon' \sim \langle A^2 \rangle / M;$$

$$\sigma = \frac{1 + \alpha}{1 - 3\alpha}; \quad \rho \sim \beta \ll \alpha.$$  

The small parameters are given by $\epsilon, \bar{\epsilon}, \epsilon'$; all other parameters are numbers of order one. All these parameters are fit to the low energy data via a detailed $\chi^2$ analysis.

4.1. Features of the model

The model has the following nice features. The family hierarchy is obtained via the hierarchy of family symmetry breaking $SU_2 \times U_1 \rightarrow U_1 \rightarrow$ nothing where the first (second) breaking is due to the familon VEVs determining the small ratios $\epsilon, \bar{\epsilon}$ ($\epsilon'$). As a result the 3rd family is much heavier than the 2nd family; is then heavier than the 1st family.

We incorporate the following mass patterns – 1) an approximate Georgi - Jarlskog mechanism “naturally” with the familon VEV $\langle 45 \rangle = (B - L)M_G$ giving $m_\nu \sim \frac{1}{3} m_\nu$ and $m_d \sim 3 m_e$; 2) third generation Yukawa unification with $\lambda_t = \lambda_b =$
\( \lambda \tau = \lambda \nu, = \lambda \odot M_G \) and 3) \( \beta \ll \alpha \sim 1 \) leads to \( m_u < m_d \) even though \( m_1 \gg m_b \).

We “naturally” satisfy gauge coupling unification and the \( SU_2 \) family symmetry suppresses flavor violation such as \( \mu \rightarrow e \gamma \).

The model has 10 Yukawa parameters (6 real coefficients and 4 phases) to fit 13 fermion masses and mixing angles. Varying these 10 Yukawa parameters and mixing angles. Varying these 10 Yukawa parameters (three gauge (\( \alpha_G, M_G, \epsilon_3 \)) and 7 soft SUSY breaking parameters) at \( M_Z \) we use the two (one) loop RG equations for dimensionless (dimensionful) variables to obtain the \( \chi^2 \) function at \( M_Z \). All observables at \( M_Z \) are evaluated including one loop threshold corrections. The result of the \( \chi^2 \) analysis, using the code of T. Blazek (see Refs. [7,6]), is given in the Table. The fit is quite good. In particular there are only 10 Yukawa parameters with 16 independent fermion mass and mixing angle observables.

4.2. Bi-large neutrino mixing in 
\( SO_{10} \times [SU_2 \times U_1']_{FS} \) model

Let us now consider neutrino masses in this model. The model has three right-handed neutrinos contained in the fields 16\( _a, 16_3 \). Moreover the Dirac neutrino Yukawa matrix is given by

\[
Y_\nu = \begin{pmatrix}
0 & -c_\omega \, c_\xi & \frac{3}{2} \, c_\omega \, c_\xi \\
-c_\omega \, c_\xi & 3 \, c_\omega \, c_\xi & \frac{3}{2} \, c_\omega \, c_\xi \\
-3 \, c_\xi & -3 \, c_\xi & 1
\end{pmatrix} \lambda \quad (12)
\]

with \( \omega = 2 \sigma/(2 \sigma - 1) \). This then gives the Dirac LR neutrino mass matrix \(-m_\nu = Y_\nu \frac{v_\nu}{\sqrt{2}} \sin \beta \).

We now add three SO(10) singlet fields \( N_i, i = 1, 2, 3 \) and couple them to neutrinos via the superpotential

\[
W_{\text{neutrino}} = \frac{\mu}{\lambda^2} \left( N_1 \tilde{\phi}^a \, 16_3 + N_2 \phi^a \, 16_3 \right)
+ \frac{\mu}{\lambda^2} \left( N_3 \, \theta \, 16_3 \right)
+ \frac{1}{2} \left( S_1 \, N_1^2 + S_2 \, N_2^2 \right). \quad (13)
\]

The field \( \text{16} \) (and another field \( 16 \)) are assumed to obtain VEVs in the right-handed neutrino direction. In fact, these VEVs are necessary (along with the 45 VEVs) to break SO(10) to the standard model. Given \( \text{16} = V, \langle S_i \rangle = M_i, i = 1, 2 \) and the familon VEVs we obtain the effective neutrino mass terms given by

\[
W_{\nu}^{eff} = \nu \, m_\nu \, \tilde{\nu} + \tilde{\nu} \, V \, N + \frac{1}{2} \, N \, M \, N \quad (14)
\]

where

\[
(V^T)^{-1} = \begin{pmatrix}
-1/(\langle \phi^2 \rangle) & \xi & 0 \\
1/(\langle \phi^2 \rangle) & 0 & 0 \\
0 & 0 & 1/(\theta)
\end{pmatrix} \quad (15)
\]

Once the heavy \( \tilde{\nu}, \, N \) fields are integrated out near the GUT scale, we obtain the \( 3 \times 3 \) Majorana neutrino mass matrix (in the flavor basis)

\[
\mathcal{M}_N = U^T_\nu \left[ m_\nu \, (V^T)^{-1} \, M_N \, V^{-1} \, m^T_\nu \right] \, U_\nu. \quad (15)
\]

Note, defining the \( 3 \times 2 \) Dirac neutrino mass matrix

\[
D^T \equiv m_\nu \, (V^T)^{-1} \, M_N \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} = \begin{pmatrix} a & 0 \\ a' & b \end{pmatrix}
\]

and the \( 2 \times 2 \) right-handed neutrino mass matrix

\[
\tilde{M}_N \equiv \begin{pmatrix} M_1 & 0 \\ 0 & M_2 \end{pmatrix},
\]

we finally obtain the light neutrino mass matrix

\[
\mathcal{M} = U^T_\nu \left[ D^T \, \tilde{M}_N^{-1} \, D \right] \, U_\nu. \quad (16)
\]

Hence we obtain the Frampton, Glashow & Yanagida ansatz without any fine-tuning. In particular, the zeros are exact and fixed by the family symmetry. This may be seen by the intermediate calculation below.

We have

\[
Y_\nu \, (V^T)^{-1} \quad (17)
\]

\[
\sim \begin{pmatrix}
0 & -c_\omega \, c_\xi & \frac{3}{2} \, c_\omega \, c_\xi \\
-c_\omega \, c_\xi & 3 \, c_\omega \, c_\xi & \frac{3}{2} \, c_\omega \, c_\xi \\
-3 \, c_\xi & -3 \, c_\xi & 1
\end{pmatrix}
\times \begin{pmatrix}
-1/(\langle \phi^2 \rangle) & \xi & 0 \\
1/(\langle \phi^2 \rangle) & 0 & 0 \\
0 & 0 & 1/(\theta)
\end{pmatrix}
\]

\[
\sim \begin{pmatrix} a & 0 & X \\ a' & b & X \\ 0 & b' & X \end{pmatrix}
\]

In addition, bi-large mixing requires significant constraints on the parameters \( b, \, b', \, a, \, a' \). Maximal atmospheric neutrino oscillation requires \( b \approx b' \), whereas large mixing for solar neutrino oscillations requires \( a \) of order \( a' \). On the other hand, the ratios \( |b/b'| \) and \( |a/a'| \) are, now, also
Table 1
\(\chi^2\) analysis including 5 precision electroweak and 16 fermion mass and mixing angle observables plus the branching ratio for the process \(b \to s\gamma\).

| Observable | Data(\(\sigma\)) (masses in GeV) | Theory | Pull |
|------------|---------------------------------|--------|------|
| \(M_Z\)   | 91.188 (0.091)                  | 91.21  | <0.50 |
| \(M_W\)   | 80.419 (0.080)                  | 80.40  | <0.50 |
| \(G_\mu \cdot 10^5\) | 1.1664 (0.0012)          | 1.166  | <0.50 |
| \(\alpha_{EM}^{-1}\) | 137.04 (0.14)              | 137.0  | <0.50 |
| \(\alpha_{\nu}(M_2)\) | 0.11720 (0.002)             | 0.1139 | 2.65  |
| \(M_t\)   | 174.30 (5.1)                    | 171.3  | <0.50 |
| \(m_b(M_b)\) | 4.220 (0.09)                    | 4.377  | 3.04  |
| \(M_b - M_c\) | 3.400 (0.2)                    | 3.430  | <0.50 |
| \(m_c(m_c)\) | 1.3000 (0.15)                   | 1.212  | <0.50 |
| \(m_s\)   | 0.089 (0.011)                   | 0.100  | 1.01  |
| \(m_d/m_s\) | 0.050 (0.015)                   | 0.0751 | 2.80  |
| \(Q^{-2}\) | 0.00203 (0.00020)               | 0.0020 | <0.50 |
| \(M_\tau\) | 1.777 (0.0018)                  | 1.777  | <0.50 |
| \(M_\mu\) | 0.10566 (0.00011)              | .1057  | <0.50 |
| \(M_e \cdot 10^3\) | 0.5110 (0.00051)            | 0.5110 | <0.50 |
| \(V_{us}\) | 0.2230 (0.0040)                | 0.2213 | <0.50 |
| \(V_{cb}\) | 0.04020 (0.0019)               | 0.0391 | <0.50 |
| \(V_{ub}/V_{cb}\) | 0.0860 (0.008)              | 0.0850 | <0.50 |
| \(V_{td}\) | 0.00820 (0.00082)              | 0.00846| <0.50 |
| \(\epsilon_K\) | 0.00228 (0.00023)              | 0.00233| <0.50 |
| \(\sin 2\beta\) | 0.7270 (0.036)                | 0.6898 | 1.07  |
| \(\beta(b \to s\gamma)\) \(10^3\) | 3.340 (0.38)                   | 3.433  | <0.50 |
| TOTAL \(\chi^2\) | 12.16                           |        |      |

constrained by charged fermion masses and mixing angles. Without performing a joint \(\chi^2\) analysis (including both charged fermions and neutrinos) we can still make the following observations. The fit to charged fermion masses naturally gives \(\epsilon' \sim \epsilon\). Hence given
\[
b \equiv \epsilon' \omega \lambda (M_2/\phi^1) \frac{M_\nu \epsilon \sin \beta}{\sqrt{2}}
\]
\[
b' \equiv -3 \epsilon \xi \sigma \lambda (M_2/\phi^1) \frac{M_\nu \epsilon \sin \beta}{\sqrt{2}}.
\]
we find \(|b/b'| = (\epsilon' \omega)/(3 \epsilon \xi \sigma) \approx 1\). With regards \(a, a'\) we have \(\epsilon' \xi^{-1} \sim \epsilon\). Moreover
\[
a \equiv -\epsilon' \omega \lambda (M_1/\phi^2) \frac{M_\nu \epsilon \sin \beta}{\sqrt{2}}
\]
\[
a' \equiv (-\epsilon' \xi^{-1} + 3 \epsilon) \omega \lambda (M_1/\phi^2) \frac{M_\nu \epsilon \sin \beta}{\sqrt{2}}
\]
gives \(|a/a'| = \epsilon'/(-\epsilon' \xi^{-1} + 3 \epsilon)\) which apparently requires fine-tuning of order 1 in 10 to get \(a \sim a'\). Clearly in order to test this model, we must perform an extended \(\chi^2\) analysis including both charged fermions and neutrinos. Note, there are only two new parameters in the neutrino sector which can be fit to the solar and atmospheric neutrino mass squared differences.

Finally we find \(m_{\nu_2}/m_{\nu_3} \approx (m_e/m_\mu) (M_1/M_2) \epsilon\) and thus \(M_1/M_2 \sim 10^3\). Unfortunately, leptogenesis is more complicated in this model since we must first identify the heavy neutrino mass eigenstates and their lepton/Higgs couplings. Moreover there are, in principle, more CP violating phases.

4.3. Summary

The Frampton-Glashow-Yanagida neutrino mass matrix ansatz has several nice features. Since it has only one CP violating angle it can, in
principle, correlate the sign of the matter-antimatter asymmetry with CP violating neutrino oscillations. Unfortunately, the Raidal-Strumia analysis uncovered an ambiguity which allows either sign of the CP violating angle for successful leptogenesis, depending on whether the right-handed neutrino masses satisfies $M_1 \ll M_2$ or $M_1 \gg M_2$. In addition the FGY ansatz provides a natural framework for bi-large neutrino mixing.

In this talk we obtained the FGY ansatz in a supersymmetric model with an $[\text{SU}(2) \times \text{U}(1)]_{FS}$ family symmetry. This had two advantages, 1) the FGY ansatz is fixed by the family symmetry and 2) it resolves the RS ambiguity. This is because the same family symmetry acting on neutrinos also constrains the charged fermion masses. As a result, the ratio of right-handed neutrino masses is fixed by the ratio of $\delta m^2$ for atmospheric and solar neutrinos and the ratio of charged fermion masses. It is also important to note that we are only able to obtain the normal hierarchy for neutrino masses.

The second model presented in this talk was a SUSY SO(10) model with an $[\text{SU}(2) \times \text{U}(1)^n]_{FS}$ family symmetry. This model is significantly more constrained since it relates all fermion masses and mixing angles. There are only two new arbitrary parameters in the neutrino sector which are fixed by the neutrino mass differences. Nevertheless, we “naturally” obtain the FGY ansatz for neutrinos (in a theory which initially starts with 3 right-handed neutrinos and 3 SO(10) singlet neutrinos). We find a large neutrino mixing angle for atmospheric neutrino oscillations “naturally.” On the other hand, an extended $\chi^2$ analysis must be performed to see if we can obtain a large solar mixing angle. If a good fit is obtained for neutrino masses and mixing, it will then be very interesting to analyze CP violation and leptogenesis in this model.

Acknowledgements

This work is partially supported by DOE/ER/01545-857.

REFERENCES

1. P. H. Frampton, S. L. Glashow and T. Yanagida, Phys. Lett. B 548, 119 (2002) [arXiv:hep-ph/0208157].

2. M. Raidal and A. Strumia, Phys. Lett. B 553, 72 (2003) [arXiv:hep-ph/0210021].

3. S. Raby, Phys. Lett. B 561, 119 (2003) [arXiv:hep-ph/0302027].

4. M. Dine, R. G. Leigh and A. Kagan, Phys. Rev. D 48, 4269 (1993) [arXiv:hep-ph/9304299]; A. Pomarol and D. Tommasini, Nucl. Phys. B 466, 3 (1996) [arXiv:hep-ph/9507462]; R. Barbieri, G. R. Dvali and L. J. Hall, Phys. Lett. B 377, 76 (1996) [arXiv:hep-ph/9512388].

5. R. Barbieri, L. J. Hall, S. Raby and A. Romanino, Nucl. Phys. B 493, 3 (1997) [arXiv:hep-ph/9610449].

6. T. Blazek, S. Raby and K. Tobe, Phys. Rev. D 60, 113001 (1999) [arXiv:hep-ph/9903340]; T. Blazek, S. Raby and K. Tobe, Phys. Rev. D 62, 055001 (2000) [arXiv:hep-ph/9912482].

7. T. Blazek, M. Carena, S. Raby and C. E. M. Wagner, Phys. Rev. D 56, 6919 (1997) [arXiv:hep-ph/9611217].