Multiple electromagnetic electron positron pair production in relativistic heavy ion collisions

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Abstract

We calculate the cross sections for the production of one and more electron-positron pairs due to the strong electromagnetic fields in relativistic heavy ion collisions. Using the generating functional of fermions in an external field we derive the N-pair amplitude. Neglecting the antisymmetrisation in the final state we find that the total probability to produce N pairs is a Poisson distribution. We calculate total cross sections for the production of one pair in lowest order and also include higher-order corrections from the Poisson distribution up to third order. Furthermore we calculate cross sections for the production of up to five pairs including corrections from the Poisson distribution.

34.90.+q,12.20.-m,11.80.-m

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I. INTRODUCTION

Currently new accelerators are being built at CERN (“Large Hadron Collider”, LHC) and at Brookhaven (“Relativistic Heavy Ion Collider”, RHIC), which allow heavy ions up to lead and uranium to be accelerated to highly relativistic energies. The main motivation for this is to advance into regions of baryonic densities and temperatures in order to observe the formation of the quark gluon plasma. But this has also renewed the interest in studying pure electromagnetic processes in these collisions, especially the electron-positron-pair production in peripheral collisions, where both ions do not interact with each other.

On the one hand pair production in central collisions is a signal one wants to use in order to study the properties of the quark gluon plasma [1,2]. The pure electromagnetic pairs are a large background process to these. On the other hand these processes can also be studied already in existing accelerators (“Super Proton Synchrotron”, SPS) [3]. Recently anti-hydrogen atoms were produced at LEAR (“Low Energy Antiproton Ring”) in a similar process, where antiprotons in collisions with Xenon produce electron-positron pairs, where the positron is not produced as a free particle, but is captured in a bound state of the antiproton [4].

The theoretical treatment of this problem goes back to the beginning of QED [5]. But it was remarked only recently that calculations in lowest order perturbation theory violate unitarity [6]. In the meantime several authors have studied this problem and found that this means that the production of multiple pairs becomes important then. All found that the $N$-pair creation probability can be written approximately in the form of a Poisson distribution, which then restores unitarity [7–9]. It was shown that generally the $N$-pair amplitude can be reduced to the product of the vacuum amplitude and the antisymmetrised product of $N$ reduced one-pair amplitudes. The neglect of exchange terms in the calculation of the probability then leads to a Poisson distribution [10]. Corrections to this Poisson distribution were calculated in the case of two-pair production and were found to be small (around 1%), its importance decreasing also with higher Lorentz factors.

Whereas large impact parameter contribute to the single-pair production, the $N$-pair production with increasing $N$ is dominated mainly by small impact parameters, smaller than the Compton wavelength of the electron. Thus the “equivalent-photon-approximation” (EPA) cannot be used anymore but exact calculations of the reduced probability are needed, since the momentum of the virtual photon is not small compared to the electron mass.

In Sec. II we show how the $N$-pair production amplitude can be found using the path integral formalism. We find again the results of [10]. We then discuss in Sec. III our approach to the calculation of the cross sections for one- and multiple-pair production taken into account only peripheral collisions. Results for this are given in Sec. IV and compared with EPA. Differential cross section for the one-pair production are shown too.

II. N-PAIR PRODUCTION IN AN EXTERNAL FIELD

In the path integral formalism the generating functional for a fermion field $\psi$ in a external field $A_\mu$ can be written as [11]

$$Z[\eta, \eta, A] = N \int \mathcal{D} \psi \mathcal{D} \bar{\psi} \exp \left\{ i \int d^4 x \left[ \mathcal{L}(x) \right] \right\}$$
\[
\frac{1}{(i^2)^n} \frac{\delta}{\delta \eta(y_1)} \frac{\delta}{\delta \bar{\eta}(x_1)} \cdots \frac{\delta}{\delta \eta(y_n)} \frac{\delta}{\delta \bar{\eta}(x_n)} \left. Z[\eta, \bar{\eta}, A] \right|_{\eta = \bar{\eta} = 0}.
\]
\[ \langle f | S | i \rangle = \langle 0 | S | 0 \rangle \int d^4x_1 \ldots d^4x_N d^4y_1 \ldots d^4y_N \]
\[ \left\{ \prod_{j=1}^N \bar{u}'(s'_j)(p'_j) e^{ip'_jx_j}(i \partial_{x_j} - m) \tau(x_1, y_1, \ldots, x_N, y_N) \right\} \]
\[ \prod_{k=1}^N (-i \partial_{y_k} - m) e^{ip_ky_k} v(s_k) \].

(8)

The variations in Eq. (6) lead only to connected graphs. We extract the individual fermion lines with the help of the Wick theorem [13] in order to get

\[ \langle f | S | i \rangle = \langle 0 | S | 0 \rangle \sum_{\sigma \in S_N} \epsilon_\sigma S^R(u_1, v_{\sigma(1)}) \ldots S^R(u_N, v_{\sigma(N)}), \]

(9)

where we have summarized momentum and spin quantum numbers of electrons as \( u_i = (p'_i, s'_i) \) and of positrons as \( v_j = (p_j, s_j) \) and summed over all permutations of \( S_N \). The reduced one-pair amplitude \( S^R(u_i, v_j) \) describes a single fermion line interacting with the external field to arbitrary order. They are given by expanding the Dirac Propagator \( S_A \) and doing the integrals in Eq. (8), see also [10],

\[ S^R(u_i, v_j) = i \sum_{n=2}^\infty \int d^4z_1 \ldots d^4z_n \bar{u}'(s'_i)(p'_i) e^{ip'_iz_n} \]
\[ \times \left\{ [e A(z_n)] S_F(z_n - z_{n-1}) \ldots [e A(z_1)] \right\} \]
\[ \times e^{ip_jz_1v(s_j)(p_j)} \].

(10)

The \( N \)-pair amplitude in Eq. (9) is therefore the product of the vacuum amplitude and an antisymmetrised product of the reduced amplitudes. Due to the correlation between the produced electrons and positrons it is justified to neglect the Pauli principle in the final state, i.e., terms in the probability for \( N \) pairs coming from two different permutations are not taken into account. This leads finally to the probability to produce \( N \) pairs

\[ P(N) = \left[ \frac{P^R}{N!} \right]^N e^{-P^R}, \]

(11)

where the vacuum amplitude shows up as the exponential function. As shown in [10] the neglect of the exchange terms does not give rise to large deviations for total two-pair creation probabilities. Therefore the use of the Poisson distribution seems to be justified. With the knowledge of reduced one-pair probabilities, Eq. (11) allows the calculation of \( N \)-pair probabilities and \( N \)-pair cross sections.

III. CALCULATION OF CROSS SECTIONS

The first order that contributes to the reduced one-pair amplitude is the second order. For the creation of an electron with momentum \( p_- \) and spin \( s_- \) and a positron with momentum \( p_+ \) and spin \( s_+ \) we get the amplitude [14]
\[ S_{fi}^{R} = \bar{u}^{(s-)}(p_-)ie^2 \left\{ \int \frac{d^4p}{(2\pi)^4} \tilde{A}(p_- - p) \frac{\gamma \cdot p + m}{p^2 - m^2} \times \tilde{A}(p_+ + p) \right\} u^{(s+)}(p_+). \]  

(12)

We describe the electromagnetic potential of the two ions as a superposition of the four-potentials

\[ \tilde{A}_\mu(q) = -2\pi Z e \frac{F(q)}{q^2} \left\{ u^{(1)}_\mu e^{i\gamma Z \delta} \left( qu^{(1)} \right) + u^{(2)}_\mu e^{-i\gamma Z \delta} \left( qu^{(2)} \right) \right\}, \]

(13)

using the straight-line approximation for the trajectories of the ions. \( b \) is the impact parameter, \( u^{(1)} = \gamma(1, 0, 0, \beta) = \gamma w^{(1)} \) and \( u^{(2)} = \gamma(1, 0, 0, -\beta) = \gamma w^{(2)} \) are the four-velocities of the ions in the center-of-momentum frame. Throughout the calculations we are making use of the monopole form factor

\[ F(q) = \frac{\Lambda^2}{\Lambda^2 - q^2}, \]

(14)

with \( \Lambda = 83 \, MeV \) for \( Au \) ions, in order to describe the extended charge distribution of the ion, as this form factor can be treated analytically \(^{[13]}\). One-pair cross sections are not sensitive to the detailed choice of the form factor. The cross sections for the multiple-pair productions are more sensitive to this as they are produced mainly at smaller impact parameters.

We get the total reduced probability by summing the absolute value squared of Eq. (12) over all electron and positron spins and integrating over all momenta

\[ P^R(b) = \int \frac{d^3p_- d^3p_+}{(2\pi)^6} \frac{m^2}{E_- E_+} \sum_{s_- s_+} |S_{fi}^{R}|^2. \]

(15)

By rewriting the spin summation as a Dirac trace over \( \gamma \)-matrices we get

\[ \sum_{s_- s_+} |S_{fi}^{R}|^2 = (Z\alpha)^4 \frac{4}{\beta^2} tr \left[ \int d^2q_{1\perp} \frac{1}{q_{1\perp}^2 [q_1 - p_+ - p_-]^2} e^{iq_1 b} \right. \]

\[ \left. \times \left\{ \frac{u^{(1)}_\mu (\gamma \cdot p_+ + m) u^{(2)}_\mu}{[(q_1 - p_-)^2 - m^2]} + \frac{u^{(2)}_\mu (\gamma \cdot p_+ + m) u^{(1)}_\mu}{[(q_1 - p_+)^2 - m^2]} \right\} \frac{\gamma \cdot p_+ - m}{2m} \right. \]

\[ \times \int d^2q_{1\perp} \frac{1}{q_{1\perp}^2 [q_1 - p_+ - p_-]^2} e^{-iq_1 b} \]

\[ \left. \times \left\{ \frac{u^{(1)}_\mu (\gamma \cdot p_+ + m) u^{(2)}_\mu}{[(q_1' - p_-)^2 - m^2]} + \frac{u^{(2)}_\mu (\gamma \cdot p_+ + m) u^{(1)}_\mu}{[(q_1' - p_+)^2 - m^2]} \right\} \frac{\gamma \cdot p_+ + m}{2m} \right] \],

(16)

where \( tr \) is the usual trace over the Dirac indices. The \( \delta \) functions in the amplitude in Eq. (12) determine the longitudinal components of the integration variables. We still have to integrate over the components transverse to the velocity of the ions. We have changed these variables to the momentum of one of the photons \( q_1 = p_- - p \) and to the difference \( q = q_1' - q_1 \). This allows us to rewrite the \( b \)-dependent probability in Eq. (15) in the form
\[ P^R(b) = \int d^2q \tilde{P}^R(q)e^{iq \vec{b}}, \]  

(17)

where the Fourier transformed probability is

\[
\tilde{P}^R(q) = \int \frac{d^3p_-d^3p_+}{(2\pi)^6E_-E_+} \frac{(Z\alpha)^4}{\beta^2} \left[ \int d^2q d^2q_1 \frac{1}{N_0N_1N_3N_4} \right.
\]
\[
\left. \times \left\{ \frac{\psi^{(1)}(\vec{p}_- - \vec{q}_1 + m)}{N_{2D}} \frac{\psi^{(2)}(\vec{q}_1 - \vec{p}_+ + m)}{N_{2X}} + \frac{\psi^{(2)}(\vec{p}_+ + \vec{q}_1 + m)}{N_{5D}} \frac{\psi^{(1)}(\vec{q}_1 - \vec{p}_+ + m)}{N_{5X}} \right\} (\vec{p}_- + m) \right. 
\]
\[
\times \left\{ \frac{\psi^{(2)}(\vec{p}_- - \vec{q}_1 + m)}{N_{2D}} \frac{\psi^{(1)}(\vec{q}_1 - \vec{p}_+ + m)}{N_{2X}} + \frac{\psi^{(1)}(\vec{q}_1 - \vec{p}_+ + m)}{N_{5D}} \frac{\psi^{(2)}(\vec{q}_1 + m)}{N_{5X}} \right\} (\vec{p}_+ + m) \right],
\]  

(18)

with the propagators

\[ N_0 = \vec{q}_1^2 + m_0^2, \quad N_i = (\vec{q}_1 + \vec{k}_i)^2 + m_i^2. \]  

(19)

The momenta \( \vec{k}_i \) and parameters \( m_i^2 \) of the propagators in Eq. (18) are defined as (see also Eq. [14] in the text)

\[
\vec{k}_1 = -\vec{p}_+ - \vec{p}_-, \quad m_0^2 = -q_{1t}^2, \\
\vec{k}_{2D} = -\vec{p}_-, \quad m_{2D}^2 = m^2 - (q_{1t} - p_{-l})^2, \\
\vec{k}_{2X} = -\vec{p}_+, \quad m_{2X}^2 = m^2 - (q_{1t} - p_{+l})^2, \\
\vec{k}_3 = \vec{q}, \quad m_3^2 = m_0^2, \\
\vec{k}_4 = \vec{q} - \vec{p}_+ - \vec{p}_-, \quad m_4^2 = m_1^2, \\
\vec{k}_{5D} = \vec{q} - \vec{p}_-, \quad m_{5D}^2 = m_{2D}^2, \\
\vec{k}_{5X} = \vec{q} - \vec{p}_+ + \vec{p}_-, \quad m_{5X}^2 = m_{2A}^2.
\]  

(20)

The trace over the Dirac \( \gamma \)-matrices is done with the algebra-program FORM [16]. The advantage of calculating the Fourier transform of the total probabilities is due to the absence of oscillating terms in Eq. (17), which would otherwise prevent convergence in the Monte Carlo integrations.

From this we can get the total cross sections \( \sigma \) by integrating the probabilities (Eq. (11)) over the impact parameter

\[ \sigma(N) = 2\pi \int_{2R}^{\infty} db bP(N, b), \]  

(21)

starting with \( 2R \) (for symmetric collisions) in order to take only peripheral collisions into account.

We can use \( P(N = 1, b) \) directly as given in Eq. (17). But for large impact parameter the integrand oscillates very fast allowing not a very accurate calculation of \( P \). Therefore we choose to do the calculation for the cross section for the one-pair production by expanding the Poisson distribution of \( \sigma(N = 1) \) up to third order

\[ \sigma(N = 1) = \sigma^{(1)} - \frac{2!}{1!}\sigma^{(2)} + \frac{3!}{2!}\sigma^{(3)} - \ldots. \]  

(22)
Integrating from \( b = 0 \) on the \( \sigma^{(n)} \) can be written as

\[
\sigma^{(1)}_{b \geq 0} = \int d^2bd^2q \ e^{i\vec{q}\vec{b}} \tilde{P}^R(q) \\
= (2\pi)^2 \int d^2q \ \delta^2(q) \tilde{P}^R(q) \\
= (2\pi)^2 \tilde{P}^R(0),
\]

\[
(23a)
\]

\[
\sigma^{(2)}_{b \geq 0} = \frac{(2\pi)^3}{2!} \int dq \ [\tilde{P}^R(q)]^2,
\]

\[
(23b)
\]

\[
\sigma^{(3)}_{b \geq 0} = \frac{(2\pi)^2}{3!} \int d^2q_1d^2q_2 \tilde{P}^R(q_1)\tilde{P}^R(q_2)\tilde{P}^R(|\vec{q}_1 + \vec{q}_2|).
\]

\[
(23c)
\]

Therefore we can calculate the cross section by using the Fourier transformed \( \tilde{P}^R(q) \) alone. This leads to more accurate results for the cross section as the cross section for the one-pair production is dominated by large impact parameter, where the impact parameter dependent probability is not very accurate due to the large number of oscillations in the Fourier transform.

For the calculation of \( \sigma^{(1)} \) we have to set \( \vec{q} \) to zero in Eq. (18), i.e. \( \vec{q}_1' = \vec{q}_1 \). This leads to a number of simplifications

\[
N_3 = N_0, \ N_4 = N_1, \ N_{5D} = N_{2D}, \ N_{5X} = N_{2X}.
\]

\[
(24)
\]

The integrations over \( \vec{q}_1 \) can then be done analytically as they are standard two-dimensional Feynman integrals. The \( \vec{q}_i \)'s in the numerator only appear in connection with the scalar products \( \vec{p}_+\vec{q}_1, \vec{p}_-\vec{q}_1 \) and \( \vec{q}_1\vec{q}_1 \) and the \( \vec{k}_i \)'s in the propagators are also either \( -\vec{p}_-, -\vec{p}_+ \) or their sum. Therefore by using the relations

\[
\vec{q}_1^2 = N_0^2 - m_0^2 \\
2\vec{q}_1\vec{k}_i = N_i - N_0 - r_i,
\]

we can express the scalar products in the numerator as propagators and scalars in order to eliminate all \( \vec{q}_1 \)'s from the numerator. In the end we are left with scalar Feynman integrals with different numbers of propagator terms in the denominator. The higher integrals with quadratic propagator terms cannot be reduced to simpler integrals with the method described in [14,17]. But we solve them by differentiating the standard integrals with respect to the parameters \( m_0^2 \) and \( m_i^2 \).

The integration over the momenta of electron and positron in the final state are done with the Monte Carlo integration routine VEGAS [18]. We write the momenta in polar coordinates in the plane transverse to the velocity of the ions. The integration over one of the angles is trivial and there remains a five dimensional integral over momenta and the relative angle between the transverse electron and positron momenta. The integration boundaries were incremented until the Monte Carlo integration converged with accuracies better than 0.5%. We did not take any form factors into account for \( \tilde{P}^R(0) \)'s as the results of [14] show that the form factor is only important for small \( b \)'s, i.e., only for large \( q \)'s. At small \( q \)'s differences between calculations with and without form factors are not observable.
For the second order correction $\sigma^{(2)}$ (Eq. (23b)) we calculate the values of $\tilde{P}^{R}(q)$ for $q \neq 0$ with the method described in [14,15]. We are adding to this the value at $q = 0$, and approximate the $\tilde{P}^{R}(q)$ by cubic splines. We use a logarithmic scale for $q$ for the interpolation in order to get a better approximation for the steep increase at small $q$’s.

The calculation of the third order correction $\sigma^{(3)}$ (Eq. (23c)) follows that of $\sigma^{(2)}$.

Finally we have to correct for the fact, that we are integrating from $b = 0$ instead of $2R$. First we calculate the impact parameter dependent probability (Eq. (17)) by integrating over the angular part and get

$$P^{R}(b) = 2\pi \int dq \tilde{q} \tilde{P}^{R}(q)J_0(qb), \quad (26)$$

with the Bessel function $J_0$. Then we subtract from the $\sigma^{(n)}$ the contribution from $0 < b < 2R$ by integrating

$$\frac{d\sigma^{(n)}}{db} = 2\pi b \frac{[P^{R}(b)]^n}{n!}, \quad (27)$$

from $b = 0$ to $2R$. As $P^{R}(b)$ only decreases slowly with $b$, large impact parameters are still important for the cross section. The Bessel function in Eq. (26) oscillates fast for large $b$, which would lead to a poor accuracy for $\sigma^{(1)}$ if one would integrate over the impact parameter directly.

For increasing $N$ pair production occurs within increasingly smaller $b$’s as the external field has to supply the minimal energy of $2Nmc^2$ to create $N$ pairs, which further restricts the system. In this case the influence of the large $b$’s decreases and we can calculate the total multiple-pair cross sections exactly by using the whole Poisson distribution and integrating directly over the impact parameters

$$\sigma(N) = 2\pi \int_{2R}^{\infty} db b \frac{[P^{R}(b)]^N}{N!} e^{-P^{R}(b)}.$$ \quad (28)

IV. RESULTS AND CONCLUSIONS

Using the method described in the last section we have calculated the impact parameter dependence of the one- and multiple-pair cross section up to three pairs. In Fig. [1] we show the results of the calculation with and without a monopole form factor for a center-of-mass Lorentz factor $\gamma = 100$ and for a $^{197}Au^{79} - ^{197}Au^{79}$ collision. Also shown are the results of the equivalent photon approximation (EPA) using for the reduced one-pair probability in the Poisson distribution

$$P^{EPA}(b) = \frac{14}{9\pi^2}(Z\alpha)^4 \left(\frac{\lambda_e}{b}\right)^2 \log^2 \left(\frac{\lambda_e \gamma_{tar} \delta}{2b}\right), \quad (29)$$

with $\delta = 0.681$. $\lambda_e$ is the Compton wavelength of the electron and $\gamma_{tar} = 2\gamma^2 - 1$ is the Lorentz factor in the target system. The EPA approximation of Eq. (29) is only valid for impact parameters $\lambda_e \gamma_{tar} \delta \geq 2b \geq 2\lambda_e$. We find in agreement with earlier results [14,15]
that the EPA gives results which are much to large at small $b$. As already discussed before, the one-pair production cross section decreases only slowly with larger impact parameter in contrast to the multiple-pair cross sections.

In Fig. 2 we show the dependence of the total cross section for the one-, two- and three-pair production for a $^{208}Pb^{82} - ^{208}Pb^{82}$ collision on the Lorentz factor $\gamma$. We plot the results for the calculation with a monopole form factor using the full Poisson distribution and also using the lowest order alone. One sees that the Poisson distribution reduces the multiple-pair cross sections considerably whereas it seems to have only a small effect on the one-pair production. Also shown are the results of the EPA approximation by integrating only over impact parameters larger than the Compton wavelength. They are in fair agreement with the exact results, even though they tend to be either to large or to small systematically.

Our results for the total cross sections are summarized for the different colliders in Tables I and II. The differences between the calculations with and without a monopole form factor show the dependence on the detailed form of the charge distribution in the ion. More realistic form factors are needed in order to describe pair production with a larger $N$ more accurately. Moreover our results are in agreement with the results of [13], which were calculated only at lower $\gamma$ and with a Monte Carlo integration alone.

Recently anti-hydrogen atoms were produced in a LEAR experiment at CERN by colliding antiprotons with Xenon $^{133}$ with $\gamma_{tar} = 1.8$. With our approach we get a total cross section for the creation of a free electron-positron pair in this case of $\sigma^{(1)} = Z^2\sigma_0$, with $\sigma_0 = 14\,nb$. This is more than three orders of magnitude larger than the bound-free transition, which leads to anti-hydrogen formation.

We have applied our calculations only to electron-positron-pair production. Nevertheless the pair production of heavy leptons $\mu$ and $\tau$ gets more important with increasing energies of the ions. With our method we can calculate cross sections for this processes, too, considering only peripheral collisions. Moreover only single-pair cross sections are important here due to the larger masses.

Finally we also give differential cross sections for the energy and the angle with the beam axis of either electron or positron. We get these by integrating the first order of the one-pair cross sections in Eq. (23) with the help of the Monte Carlo integration routine sorting the results into bins. The results are shown in Figs. 3 and 4. Most electrons are produced rather close to the beam axis and also with relatively low energy of the order of a few $MeV$.

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REFERENCES

[1] R. Vogt et. al., Nucl. Phys. A 583, 693c (1995).
[2] B. Kämpfer et. al., Phys. Rev. C 52, 2704 (1995).
[3] T. Alber et. al., Phys. Rev. Lett. 75, 3814 (1995).
[4] G. Baur et. al., Phys. Lett. B 368, 251 (1996).
[5] L.D. Landau and E.M. Lifshitz, Phys. Z. Sowjet. 6, 244 (1934).
[6] C.A. Bertulani and G. Baur, Phys. Rep. 164, 300 (1987).
[7] G. Baur, Phys. Rev. A 42, 5736 (1990).
[8] M. J. Rhoades-Brown and J. Weneser, Phys. Rev. A 44, 330 (1991).
[9] C. Best, W. Greiner, and G. Soff, Phys. Rev. A 46, 261 (1992).
[10] K. Hencken, D. Trautmann, and G. Baur, Phys. Rev. A 51, 998 (1995).
[11] L.H. Ryder, Quantum Field Theory, (Cambridge University Press, 1985).
[12] H. Lehmann, K. Symanzik, and W. Zimmermann, Nuovo Cimento 1, 205 (1955).
[13] C. Itzykson and J.-B. Zuber, Quantum Field Theory, (McGraw-Hill, 1980).
[14] K. Hencken, D. Trautmann, and G. Baur, Phys. Rev. A 51, 1874 (1995).
[15] K. Hencken, D. Trautmann, and G. Baur, Phys. Rev. A 49, 1584 (1994).
[16] FORM is an algebraic calculation program by J.A.M. Vermaseren; the free version 1.0 can be found, e.g. at FTP.NIKHEF.NL
[17] W.L. van Neerven and J.A.M. Vermaseren, Phys. Lett. B 137, 241 (1984).
[18] G.P. Lepage, J. Comp. Phys. 27, 192 (1978).
[19] M.C. Güçlü et. al., Phys. Rev. A 51, 1836 (1995).
FIG. 1. The $b$ dependent differential cross sections $\frac{d\sigma(N)}{db}(b)$ for the $N$-pair production in a $^{197}Au^{79} - ^{197}Au^{79}$ collision with $\gamma = 100$ for $N = 1, 2, 3$. The solid lines show the results for the calculation with a monopole form factor, the dotted lines for a point charge and the dashed lines are the EPA results.

FIG. 2. The $\gamma$ dependent total cross sections $\sigma(N)(\gamma)$ for the $N$-pair production in a $^{208}Pb^{82} - ^{208}Pb^{82}$ collisions, for $N = 1, 2, 3$. Exact calculations are shown as solid lines, dashed lines are the results for the corresponding first orders $\sigma^{(N)}(\gamma)$ and the dotted lines are the EPA results.
FIG. 3. Differential cross sections $\frac{1}{(Z\alpha)^{4}} \frac{d\sigma^{(1)}}{dE_{-}}(E_{-})$ for the first order in Eq. (22) as a function of the energy of the electron or positron. The solid line shows the result for $\gamma = 8.916$ and the dashed line for $\gamma = 100$.

FIG. 4. Differential cross sections $\frac{1}{(Z\alpha)^{4}} \frac{d\sigma^{(1)}}{d\vartheta_{-}}(\vartheta_{-})$ as a function of the electron momenta with the beam axis. Same notations as in Fig. 3.
TABLES

with monopole form factor (kb)

| $\gamma$    | $\sigma(N = 1)$ | $\sigma(N = 2)$ | $\sigma(N = 3)$ | $\sigma(N = 4)$ | $\sigma(N = 5)$ |
|-------------|-----------------|-----------------|-----------------|-----------------|-----------------|
| 8.916 (SPS) | 2.29            | 9.89 $10^{-2}$  | 6.26 $10^{-3}$  | 4.52 $10^{-4}$  | 3.25 $10^{-5}$  |
| 100 (RHIC)  | 31.8            | 0.899           | 0.1122          | 1.827 $10^{-2}$ | 3.11 $10^{-3}$  |
| 3400 (LHC)  | 223             | 4.00            | 0.785           | 0.219           | 6.94 $10^{-2}$  |

TABLE I. Results of the calculation for the production of $N = 1, \ldots, 5$ pairs with a monopole form factor. For SPS and LHC we show total cross sections for $^{208}Pb^{82} - ^{208}Pb^{82}$ collisions and for RHIC total cross sections for $^{197}Au^{79} - ^{197}Au^{79}$ collisions.

without monopole form factor (kb)

| $\gamma$    | $\sigma(N = 1)$ | $\sigma(N = 2)$ | $\sigma(N = 3)$ | $\sigma(N = 4)$ | $\sigma(N = 5)$ |
|-------------|-----------------|-----------------|-----------------|-----------------|-----------------|
| 8.916 (SPS) | 2.28            | 0.1019          | 6.66 $10^{-3}$  | 5.08 $10^{-4}$  | 3.95 $10^{-5}$  |
| 100 (RHIC)  | 31.8            | 0.897           | 0.1136          | 1.881 $10^{-2}$ | 3.37 $10^{-3}$  |
| 3400 (LHC)  | 223             | 3.95            | 0.776           | 0.219           | 7.09 $10^{-2}$  |

TABLE II. Results for the calculations for the production of $N = 1, \ldots, 5$ pairs for a point charge. The ions used for the different $\gamma$’s are the same as in Tab. I.