Analysis on the microstructure of sands

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Abstract: The macroscopic behavior of sands is closely related with its microstructure. Starting from the microstructure of sands, using micromechanics and integrated with statistically homogenization, considering intergranular contact force and branch vector, the paper focuses on the expressions for macroscopic stress; Integrated with the fabric characteristics of granular material, the second-order fabric tensor F is derived, and the relationship between stress-force-fabric is established. It shows that the macroscopic stress of sands is closely directed with inherent and stress-induced anisotropy, and the tendency between shear strength and fabric is same which lay the foundation for the following constitutive relationship for sands.

1. Introduction

The macroscopic mechanical behavior of sand is closely related to its microstructure, such as the geometric arrangement of interacting particles. In connection with the modern mechanics framework, the stress dilatancy can be thought of as an internal constraint imposed by discrete particles (Goddard & Didwania, 1998[1]). There is no doubt that the change in particle arrangement, ie, the organization will have a large impact on the mechanical properties of the sand.

Reviewing the previous soil sample tests for soil-like structures, in order to obtain different initial densities and configurations, different preparation methods were used for the experimental soil samples: wet vibrating method, water sedimentation method, dry deposition method (for details, see Zlatovic & Ishihara, 1997[2]) and so on. In addition to using the above method to obtain soil-like structures with different degrees of anisotropy, the soil layer deposition is considered by changing the angle of the base plane, that is the normal direction of the base plane and the angle of the maximum principal stress acting on the soil sample. The effect of the resulting fabric orientation was studied (Oda 1972a[3], Nakata et al 1998[4]), and it was also possible to obtain( Yamada & Ishihara (1979)[5], Uthayakumar(1996)[6] ) by fixing the initial fabric and changing the direction of the maximum principal stress acting on the sample. The effect of the resulting fabric orientation was studied (Oda 1972a[3], Nakata et al 1998[4]), and it was also possible to obtain( Yamada & Ishihara (1979)[5], Uthayakumar(1996)[6] ) by fixing the initial fabric and changing the direction of the maximum principal stress (Oda , 1972b[7]). Oda and Nakayama (1989)[8] clearly pointed out that in order to discuss the anisotropy of soils of granular materials, it is necessary to consider the influences such as contact normals and pore shapes. Throughout the literature, second order or higher order is usually adopted, and the even-ordered tensor defines the particle contact normal and the branch vector (the connection vector of the particle center) (Satake, 1978[9]; Nemat and Mehrabadi, 1983[10]), and Zhong Xiaoxiong[11] analyzes the microscopic of the granular material. The constitutive model based on microstructure is proposed and proposed. Combined with a large number of experimental studies, in recent years, great progress has been made in the research on the theory of anamorphic shape of...
granular materials.

In this paper, the anisotropy of sand is considered mainly based on the basic concept of micromechanics, such as considering the direction of the contact normal, the structure characteristics, etc., in order to combine the micromechanical content and incorporate the constitutive model to establish the macroscopic stress-strain relationship of sands.

2. Macroscopic stress expression of granular materials

Considering that the particulate material is composed of randomly arranged particles, the particles in contact with each other are filled by pores. In order to analyze the macroscopic properties and state variables of random structural materials, it is assumed that the granular materials are homogeneous, and it is assumed that the average stress is obtained on the basis of uniform deformation of the particle boundaries. Therefore, the average stress is written \( \langle \sigma \rangle \), brackets \( \langle \cdot \rangle \) represents the total average of the physical quantities associated with the particulate material.

2.1 Average stress

Using the micromechanical theory and the averaging principle\(^{[12]}\), the total average stress of the granular material is obtained by finding the average stress on the volume \( V \), and therefore, the average stress at one point \( \sigma \) can be expressed in the form of indicators:

\[
\langle \sigma \rangle = \frac{1}{V} \int_V \sigma \, dV = \frac{1}{V} \int_V \sigma_0 \, dV + \frac{1}{V} \sum_{V} \sigma_i \, dV
\]

(1)

Here \( V \) refers to the volume occupied by the pores, \( V \) for particle volume, \( n \) is the total number of particles contained in volume \( v \). The first item in the above formula is for pores containing air, so the first integral is 0. Using the Gaussian divergence theorem for the whole \( V \), you can get:

\[
\int_V \sigma_0 \, dV = \int_V \delta_0 \sigma_0 \, dV = \int_V x_i \sigma_i \, dV
\]

(2)

The above equation, according to the principle of static balance of stationary objects, satisfies:

\( \sigma_{i,j} = 0 \), average stress \( \langle \sigma_i \rangle \) can be written to:

\[
\langle \sigma_i \rangle = \frac{1}{V} \sum_{V} \sigma_i \, dV = \frac{1}{V} \sum_{S} x_i \sigma_i n_i \, dS
\]

(3)

Here \( S \) is the boundary surface of closed particle volume \( V \). Noticed \( (x_i \sigma_i n_i) \, dS \) as the force acting on the grain boundary \( f \), then the above formula becomes:

\[
\langle \sigma_i \rangle = \frac{1}{V} \sum_{f} x_i f_i
\]

(4)

Here \( n_i \) indicating the number of contact points of individual particles, as can be seen from the above equation, \( x_i f_i \) product is not exchangeable, stress tensor \( \sigma_i \) is not symmetrical.
As shown in Figure 1, the particles are in contact with the surrounding particles, \( x_j \) for two particle contact points, it can be expressed as: \( x_j = x_j^c + r_j \), \( x_j^c \) the position of the center of gravity for the particles, \( r_j \) a vector that connects the center of gravity of the particle with the point of contact. Thus, the above formula can be expressed as:

\[
\langle \sigma \rangle = \frac{1}{V} \sum_{i=1}^{n} \sum_{j=1}^{n} x_i f_{ij} = \frac{1}{V} \sum_{i=1}^{n} \sum_{j=1}^{n} x_j^c f_{ij} + \frac{1}{V} \sum_{i=1}^{n} \sum_{j=1}^{n} r_j f_{ij}.
\]  

(5)

Since each particle interacts locally with other particles, then \( \sum f_i = 0 \). Then the first item disappears. And the second item contains two particles in contact with each other, \( (\alpha, \beta) \) the interaction between them, so the above formula can be rewritten as:

\[
\langle \sigma \rangle = \frac{1}{V} \sum_{\alpha} \sum_{\beta} (r_j^{\alpha\beta} f_{ij}^{\alpha\beta} + r_j^{\beta\alpha} f_{ij}^{\beta\alpha}).
\]  

(6)

Figure 2 Schematic Illustration of branch vector

Superscript here \( \alpha\beta \) representing particles \( \alpha, \beta \) interaction. Branch vector \( l_{ij}^{\alpha\beta} \) connects the center of gravity of the two particles in contact with each other, as shown in Figure 2, then: \( l_{ij}^{\alpha\beta} = r_{ij}^{\alpha\beta} - r_{ij}^{\beta\alpha} \). The internal structure can be defined by this vector, noting that each particle satisfies the static balance, the force and the reaction force a \( r_{ij} \) equal in magnitude, and the direction is opposite, at each contact point, \( f_{ij}^{\alpha\beta} = -f_{ij}^{\beta\alpha} \), then the above formula is changed to:

\[
\langle \sigma \rangle = \frac{1}{V} \sum_{\alpha} \sum_{\beta} f_{ij}^{\alpha\beta} (r_{ij}^{\alpha\beta} - r_{ij}^{\beta\alpha}) = \frac{1}{V} \sum_{\alpha} \sum_{\beta} f_{ij}^{\alpha\beta} l_{ij}^{\alpha\beta}.
\]  

(7)

For the total number of particles \( n \) and the number of contacts \( N \), The double summation can be the total number of contacts of the unit body \( N \), the single summation representation:

\[
\langle \sigma \rangle = \frac{1}{V} \sum_{i=1}^{N} f_i l_i.
\]  

(8)

The above formula can be written as:

\[
\langle \sigma \rangle = \frac{1}{V} \sum_{i=1}^{N} f_i \otimes l_i.
\]  

(9)

The above formula reflects the microscopic definition of the average stress tensor, that is, the average stress can be expressed by the weighted average of the interparticle contact force and the sum of the product tensors of the branches. The intergranular contact force is the microscopic reaction caused by the macroscopic average stress, and the branch vector reflects the internal structure of the granular material.

2.2. Fabric tensor

Using the above formula, we can establish the equilibrium equation for the microscopic local contact force \( f_i \) and macroscopic average stress \( \langle \sigma \rangle \), namely:
Here vector \( A'_c \) needed to be further deduced. Substituting the above formula into the average stress \( \langle \sigma \rangle \) formula, getting:

\[
\langle \sigma \rangle = \frac{1}{V} \sum_c f'_c I'_c = \frac{1}{V} \sum_c \langle \sigma \rangle A'_c I'_c
\]

\[
= \langle \sigma \rangle \frac{1}{V} \sum_c A'_c I'_c
\]

Therefore, according to the above formula, all contact points \( c \) satisfy:

\[
\frac{1}{V} \sum_c A'_c I'_c = \delta_{ij}
\]

And assumed that a connection is established between the local second order tensor \( A_{lm} \) and branches vector \( f'^c \), then \( A'_c \) can be derived:

\[
A'_c = A_{lm} f'^c
\]

Use formula (10)(11)(12), local contact force \( f'_c \) for:

\[
f'_c = \langle \sigma \rangle A_{lm} f'^c
\]

\[
\langle \sigma \rangle = \frac{1}{V} \sum_c f'_c I'_c = \frac{1}{V} \sum_c \langle \sigma \rangle A_{lm} f'_c I'_c
\]

\[
= \langle \sigma \rangle A_{lm} \frac{1}{V} \sum_c f'_c I'_c
\]

\[
A_{lm} \frac{1}{V} \sum_c f'_c I'_c = \delta_{ij}
\]

And fabric tensor \( F_{mj} \) can be obtained:

\[
F_{mj} = A^{-1}_{lm} = \frac{1}{V} \sum_c f'_c I'_c \text{ or } F = A^{-1} = \frac{1}{V} \sum L' \otimes L'
\]

Integrated formula (14) and (17), local contact force \( f'_c \) can be expressed as:

\[
f'_c = \langle \sigma \rangle \left( \frac{1}{V} \sum_c f'_c I'_c \right)^{-1} I'_c = \overline{\sigma}_{lm} f'_c
\]

or \( f' = \langle \sigma \rangle : F \cdot L = \overline{\sigma} \cdot L' \)

\[
\langle \sigma \rangle = \frac{1}{V} \sum_{i=1}^{N} f'_c I'_c = \langle \sigma \rangle A_{lm} \frac{1}{V} \sum_{i=1}^{N} f'_c I'_c = \overline{\sigma}_{lm} F_{mj}
\]

It can be seen from the above formula that the local contact force can be regarded as mapping of a dummy stress on the branch vector \( L' \), and average stress \( \langle \sigma \rangle \) is the dyadic product of imaginary stress \( \overline{\sigma}_{lm} \) with fabric tensor \( F_{mj} \).

3. Establishment of stress-force-composition relationship

Due to the random arrangement of granular materials, the contact force and contact vector of each contact point are different. Therefore, when studying uniform granules (the size is much larger than the particle size), statistical averaging is often used, and local variations are ignored. Assuming the number the particle contact normal in \( (n, n + \Delta n) \) is \( M(n) \), Setting up \( p(n) = \lim(M(n) / \Delta n) / N_c \) as the
directional distribution probability function of particle contact direction $\mathbf{n}$, for granular materials, Average branch vector $\mathbf{L}(\mathbf{n})$ in contact normals $(\mathbf{n} + \Delta \mathbf{n})$ and average contact force $\mathbf{f}(\mathbf{n})$ are statistically unrelated variables, the above equation can be rewritten as:

$$
\langle \sigma \rangle = \frac{1}{V} \sum_n M(\mathbf{n}) \mathbf{f}(\mathbf{n}) \otimes \mathbf{L}(\mathbf{n}) \\
= \frac{N}{V} \sum_n p(\mathbf{n}) \mathbf{f}(\mathbf{n}) \otimes \mathbf{L}(\mathbf{n}) d\mathbf{n} \\
= \frac{N}{V} \int p(\mathbf{n}) \mathbf{f}(\mathbf{n}) \otimes \mathbf{L}(\mathbf{n}) d\mathbf{n}
$$

(20)

3.1 Fabric feature

The internal geometrical features of the granules are the structural features of the granules. Studies have shown[13] that among the geometric features of particle alignment, the most important factors affecting mechanical properties include particle radius, number of contacts, direction distribution of branch vectors, and contact normals, shape and distribution of pores. According to the integral expression of the stress of the above formula, the influence of the fabric is described as the distribution probability through the contact direction. $p(\mathbf{n})$ and the average branch vector $\mathbf{L}_c(\mathbf{n})$ as a function of the contacted normal direction. Micromechanical analysis, in order to simplify the calculation, is considered in terms of spherical particles, so for the arrangement of spherical particles, the direction distribution of the branch vectors and the direction of the contact normals coincide. In fact, for non-spherical particles, the average branch vector is basically the same as the average contact force and can be decomposed into normal and tangential components which are not considered in this paper. Here we mainly consider the influence of the direction distribution of the contact normal.

3.2. Directional distribution of particle contact $P(n)$:

At the same time, $\mathbf{n}$, $-\mathbf{n}$ indicates the direction of normal contact, we use second-order tensor here $\mathbf{d}$ ($\mathbf{d} = \mathbf{I} + \varphi$ ) (Here $\varphi$ is a second-order tensor, reflecting the arrangement characteristics of the particles) approximated along the normal direction $\mathbf{n}$. Contact probability distribution $p(\mathbf{n})^{[14][15][16]}$ is:

$$
p(\mathbf{n}) = 1/4\pi d : (\mathbf{n} \otimes \mathbf{n}) = 1/4\pi (\mathbf{I} + \varphi) : (\mathbf{n} \otimes \mathbf{n}), \quad \varphi^\prime \text{ is a second order tensor representing the anisotropy coefficient the of particle contact normal. In case } \varphi^\prime = 0, \text{ indicating that the distribution is direction-independent and corresponds to an isotropic state. Tensor } \varphi, \text{ the extent to which the contact normal direction distribution deviates from the reference isotropic state (all contact directions are equally probabilistic) is determined. The probability distribution is known from the simulated data. } p(\mathbf{n}), \text{ the normal anisotropy tensor coefficient can be measured by calculating the following "structural tensor" } F_{ij} \text{ and get: }
$$

$$
F_{ij} = \int p(\mathbf{n}) n_i n_j d\mathbf{n} = \frac{1}{3} \delta_{ij} + \frac{2}{15} \varphi^\prime_{ij}
$$

(21)

The above formula shows $\varphi^\prime_{ij}$ is proportional to fabric tensor $F_{ij}$. And $F_{ij}$ can be derived through calculating the discrete data of the particle assembly, that is, taking the average value $F_{ij} = (\sum n_i n_j) / N_{ij}$ for all $N_{ij}$ points of the particle contact.

Usually the mechanical properties of the soil are transversely isotropic (primary anisotropy), so:
\[ \varphi^c = \begin{pmatrix} a & 0 & 0 \\ 0 & -\frac{a}{2} & 0 \\ 0 & 0 & -\frac{a}{2} \end{pmatrix} \]  

(22)

Here \( a \) is the second-order tensor coefficient, and the degree of anisotropy of the reaction particle contact distribution.

3.3. Average intergranular contact force

In the following analysis, the vector \( \vec{f}(n) \) is composed of normal component of the contact plane \( \vec{f}^n \) and tangential component \( \vec{f}^t \), so the average contact force can be written as:

\[ \vec{f}(n) = \vec{f}^n(n) + \vec{f}^t(n) \]  

(23)

Assuming that the particles are spherical particles, the branch vector direction coincides with the normal line of the particle contact, and the length is \( 2R \), \( R \) is the average radius of the particles, i.e.,

\[ L^0 = |L^0|n = 2Rn \]  

(24)

Thus, formula 20 can be rewritten:

\[ \langle \sigma \rangle = \frac{N}{V} \int p(n)[\vec{f}^n(n) + \vec{f}^t(n)] \otimes [2Rn]/n \]

\[ = \frac{2RN}{V} \int p(n)[\vec{f}^n(n) + \vec{f}^t(n)] \otimes n/n \]  

(25)

**Average normal contact force:** \( \vec{f}^n(n) = \vec{f}(n) \cdot n \). According to the theory of granular material mechanics, the contact force comes from the contact between the particles, and its size is controlled by the contact stiffness. Generally, the normal and tangential contact stiffness of the interparticle contact are different, if the intergranular contact force and the intergranular displacement is known, and the stress-strain relationship of the particulate material can be derived.

From the definition of average normal contact force, \( \vec{f}^n(n) = \vec{f}^n(-n) \), so the distribution normality of the contact normal, the average normal contact force can be expressed as\cite{12}:

\[ \vec{f}^n(n) = \vec{f}^n[(1 + \varphi^n) : (n \otimes n)n] \]  

(26)

Here \( \vec{f}^n \) corresponding to the average normal contact force scalar value on all contact surfaces in the isotropic state, satisfying \((1/4\pi)\int f^n(n_i)dn_i = \vec{f}^n(1 + \varphi^n/3) = \vec{f}^n \), \( \varphi^n \) an anisotropic coefficient tensor representing the average normal contact force, describing the offset average contact force relative to the isotropic state \( \varphi^n = 0 \). In an isotropic state, \( \vec{f}^n(n) = \vec{f}^n \), \( \vec{f}^n \) representing average contact force.

Average tangential contact force: \( \vec{f}^t(n) = \vec{f}(n) \cdot \vec{n} \), \( \vec{f}^t(n) = -\vec{f}'(-n) \). If there is no isotropic part, the average tangential contact force can be expressed as:

\[ \vec{f}^t(n) = \vec{f}'[\varphi \cdot n - \varphi': (n \otimes n)n] \]  

(27)

Here \( \varphi \) indicating the anisotropy coefficient tensor for average tangential contact force.
3.4. Stress-force-composition relationship

Substituting the previous deduced section $\rho(n), f^s(n), f^t(n)$ into formula 20, then it can be rewritten as:

$$
\langle \sigma \rangle = \frac{N}{V} \int \frac{1}{4\pi} d : (n \otimes n) \left\{ \tilde{f}^0 \left[ (I + \varphi^0) : (n \otimes n) \right] 
+ f^t \left[ \varphi \cdot n - \varphi^t : (n \otimes n) \right] \right\} \otimes [2Rn] d\ln
$$

$$
= \frac{N}{V} \int \frac{1}{4\pi} d(n)[2Rn] \left\{ f^0_{ij} \right\} \otimes [2Rn] d(n) d\ln
$$

$$
= D_{ij} \sigma_{ij}^0
$$

The above formula is decomposed as follows:

$$
D_{ij} \sigma_{ij}^0 = \frac{1}{3} \int \frac{1}{5} d_{mn} J_{mnp} + \frac{1}{3} d_{mn} \varphi_{ij} \ell_{mnp}
$$

$$
+ \frac{1}{7} d_{mn} J_{ijkl,(\varphi^0 \varphi^0)} \sigma_{ij}
$$

$$
= \frac{1}{3} \int \frac{1}{5} d_{mn} \left( \delta_{ij} + \frac{2}{5} \varphi^0 \varphi^0 + \frac{3}{5} \varphi^0 \varphi^0 + \frac{2}{35} (7 \varphi^0 \varphi^0 \varphi^0) \right) \sigma_{ij}
$$

$$
= (F_{ij} + F_{ij}) \sigma_{ij} = F_{ij} (\delta_{ij} + F_{ij} F_{ij} \sigma_{ij})
$$

$$
= F_{ij} \sigma_{ij}
$$

$$
F_{ij} = \frac{1}{3} \delta_{ij} + \frac{2}{15} \varphi^0
$$

$$
\sigma_{ij} = (\delta_{ij} + F_{ij} F_{ij} \sigma_{ij})
$$

$$
F_{ij} = \frac{1}{3} \left( \frac{2}{5} \varphi^0 + \frac{3}{5} \varphi^0 \varphi^0 + \frac{2}{35} (7 \varphi^0 \varphi^0 \varphi^0) \right)
$$

$$
d(n) = d_{mn} n_m n_n \quad m,n = 1,2,3
$$

$$
f^s_j(n) = f^0 \left[ (\varphi^0_j n_j + (\varphi^0_i - \varphi^0_j) n_i n_j) \right], \quad i,j = 1,2,3
$$

$$
\sigma^0 = \frac{2RN. f^0}{V}
$$

$$
\varphi^0_{ij} = [\varphi^0_i - \varphi^0_j]
$$

(29)
\[
\frac{1}{4\pi} \int n_i n_j dn = \frac{1}{3} \delta_{ij}
\]
\[
\frac{1}{4\pi} \int n_i n_j n_k n_l dn = \frac{1}{5} J_{mnij}
\]
\[
J_{mnij} = \frac{1}{3} (\delta_{mi} \delta_{nj} + \delta_{mj} \delta_{ni} + \delta_{ni} \delta_{mj})
\]
\[
\frac{1}{4\pi} \int n_i n_j n_k n_l n_m dn = \frac{1}{7} J_{ijklmn}
\]
\[
J_{ijklmn} = \frac{1}{5} (\delta_{ij} J_{klmn} + \delta_{ik} J_{jlmn} + \delta_{il} J_{jkmn} + \delta_{jk} J_{ilmn} + \delta_{jl} J_{ikmn})
\]

Here, \(\langle \sigma \rangle\) for the macroscopic average stress tensor, the first term on the right side of the equation is the constitutive tensor of the granule contact normal \(F(F_0 = 1/3 \delta_{ij} + 2/15 \phi_{ij}^c)\). For undisturbed soil, due to the influence of particle shape and historical deposition \(\phi^c\), the original anisotropy causes the anisotropy of the contact normal, so the stress-strain relationship of the soil is firstly affected by the initial state. Under the action of shearing force, the direction of contact normal direction, the direction of normal contact force, and the direction of tangential contact force changes with the direction of principal stress, that is, stress-induced anisotropy occurs, and a second imaginary stress \(\overline{\sigma} = (\delta_{ij} + F_0^c \sigma_{ij})\) is formed between the interior of the granular body which varies with the change of the particle contact normal, normal force, and tangential force, and is determined by the normal direction of the particle contact and the direction distribution of the contact force.

3.5. Relationship between shear strength and fabric

Calculating the average stress and its shear stress by Equation 26, It can be obtained:

\[
\rho^e = \sigma_{ij} / 3 = \sigma_{ij} (1 + 2/15 \phi_{ij}^c)
\]
\[
s_{ij} = \sigma_{ij} - \rho^e
\]
\[
= \frac{2}{5} \sigma_{ij} [\phi_{ij}^c + \phi_{ij}^c + 3/2 \phi_{ij}^c]
\]
\[
+ \frac{2}{35} \sigma_{ij} [7 \phi_{ij}^{\text{w}} + 4 \phi_{ij}^{\text{m}}]
\]
\[
q = \sqrt{3J_2} = \frac{3}{\sqrt{2}} s_{ij} s_{ij}
\]

We introduce new tensors here:

1. \(\phi_{ij}^c = \phi_{ik}^c \delta_{kj}\)
2. \(\phi_{ij}^c = \phi_{ij}^c \phi_{ij}^c\)
3. \(\phi_{ij}^{w} = \phi_{ij}^{w} - (\phi_{ik}^{w} / 3) \delta_{ij}\)
4. \(\phi_{ij}^{m} = (\phi_{ij}^{w} + \phi_{ij}^{c} / 2 - (\phi_{ik}^{w} / 3) \delta_{ij}\)

For the convenience of calculation, converted to the triaxial space, you can get:

\[
q = \sqrt{3J_2} = \frac{3}{\sqrt{2}} s_{ij} s_{ij}
\]
\[
= \frac{2}{5} \left[ \sigma_{ij} (\phi_{ij}^c - \phi_{ij}^c) + (\phi_{ij}^c - \phi_{ij}^c) + \left( \frac{3}{2} \phi_{ij}^c - \phi_{ij}^c \right) \right]
\]
\[
\frac{q}{\sigma_{ij}} = \frac{2}{5} \left[ (\phi_{ij}^c - \phi_{ij}^c) + (\phi_{ij}^c - \phi_{ij}^c) + \left( \frac{3}{2} \phi_{ij}^c - \phi_{ij}^c \right) \right]
\]

From above formula, It can be seen that in the triaxial stress space, the internal friction angle of the particles \(\phi\) and the right side of the equation \((2/5) \sigma_{ij} ((\phi_{ij}^c - \phi_{ij}^c) + (\phi_{ij}^c - \phi_{ij}^c) + (3/2) \phi_{ij}^c - \phi_{ij}^c))\) related, that is to
say, the tendency of bias stress and fabric change is the same.

4. Conclusion
Starting from the microstructure of granular materials, this paper firstly derives the expression of macroscopic average stress of granular materials by using the averaging theory, and derives the expressions of microscopic contact force and average stress, and introduces the tensor $f$ of the fabric which can be found. The change of stress is directly related to the change of fabric. Combining the structural characteristics of the granular material, the relationship between macroscopic stress-microscopic contact force-composition is established, and the shear strength and structure are obtained. The relationship of change lays the foundation for the establishment of the subsequent constitutive relationship of sands.

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