Inmetro’s Quantum Hall System verification on the basis of BIPM calibration results

H R Carvalho, R E M Briones, J R B Silva
Instituto Nacional de Metrologia, Qualidade e Tecnologia – Inmetro,
Av. Nossa Senhora das Graças 50, Xerém - Duque de Caxias - RJ - Brazil

E-mail: hrcarvalho@inmetro.gov.br

Abstract. This article presents the analysis generated by the primary resistance measurement system based on the Quantum Hall Effect, currently being implemented in the Quantum Electrical Metrology Laboratory (Lameq) of the National Institute of Metrology, Quality and Technology (Inmetro). It has as reference the calibration results emitted by the Bureau International des Poids et Mesures (BIPM) for the reference resistors maintained by the Electrical Standardization Metrology Laboratory (Lampe), also of Inmetro, thus offering a preliminary parameter of the compatibility status between the primary systems of these two institutions.

1. Introduction
Since the installation of its primary resistance measurement system based on the Quantum Hall Effect, in April, 2008, Inmetro has been improving its abilities in handling the various components that integrates it, especially related to conditioning of Hall samples and the Cryogenic Current Comparator operation. Our main goals regarding this system are: the achievement of the international recognition of the electrical resistance primary standardization based on the Quantum Hall System (QHS) at Inmetro; and the insertion of this part of the scope into the Calibration and Measurement Capabilities (CMC) of the Mutual Recognition Arrangement of the International Committee for Weights and Measures (CIPM) – nominated CIPM MRA [1].

In order to reach such objectives, according to the CIPM MRA, a National Metrology Institute (NMI) must take part on international interlaboratory comparison programs (named key comparisons), regional comparison (named regional key comparisons) or other regional or bilateral comparison modalities (nominated supplementary comparisons), basic directive which allows technical competence analysis of the participants [2].

The line of conduct traced by Lameq for the international recognition of its Quantum Hall System consists in, as a preliminary step, performing an unofficial interlaboratory comparison; after this participating of the BIPM.EM-K12 (Quantum Hall resistance standards and their scaling to other resistance values) key comparison.

Until now, two unofficial interlaboratory comparisons were performed:
a) Comparison of the results generated by the quantum system with Inmetro classic resistance measurement system (maintained by Lampe);

b) Comparison of results generated by the quantum system with Inmetro standards certificates results issued by BIPM, which is the subject of this article.

2. Equipment and methodology

2.1. Inmetro Quantum Hall System Equipment
Lameq QHS is composed of:

- A cryostat manufactured by Cryogenic Ltd. which contains the quantum Hall device (a semiconductor sample composed of a GaAs/GaAlAs heterostructure) which is the electrical resistance primary standard.
- An auxiliary reservoir where $^3$He gas is stored.
- A cryogenic current comparator (CCC), assembled by National Physical Laboratory – NPL in a partnership with Cryogenic.
- The electronics pieces for data acquisition and control.

The cryostat enables reaching an approximate temperature of 0.3 K in the quantum Hall device and has a superconductive coil which allows the application of a magnetic field of up to 16 T in this device.

The CCC enables the calibration of Lameq reference measurement standards (two 100 $\Omega$ standard resistors) through its QHS. These reference standards are compared to other resistors, also through the CCC on a scale, thus composing Lameq traceability chain.

2.2. Setting up the calibration historic of the 100 $\Omega$ reference standard
In the period of 2011 and 2012, the two 100 $\Omega$ nominal value standard resistors were calibrated several times, against the quantum Hall device using the CCC. One of these resistance standards is the 100 $\Omega$ nominal value manufactured by IET Labs, model SR-102/DC, which gets the code “7D” at Lameq. In this way it was built the historical evolution of this reference standard to be used in the verification described in this article.

Each calibration during this period is the result of the mean value of various ratio measurement between this resistor and the quantum standard. The mean of these values is used to obtain the resistor value and it is registered as a measurement point in its historic. Thus it is defined the straight line that best fits these historic values through the linear regression based on the method of least squares. This way, it is obtained the resistor estimate value and the respective uncertainty for any time.

2.3. Traveling standard resistor calibration using the 100 $\Omega$ reference standard standard
The 100 $\Omega$ standard (resistor 7D), with its value traceable to Lameq quantum standard, was then used with the CCC for the 10 k$\Omega$ resistor calibration (whose Lameq code is R1) which, periodically, is also calibrated by BIPM. The value obtained in this calibration may be compared with the historic estimate of this resistor plot by means of BIPM calibration values.

3. Results

3.1. Historical evolution of 7D
The graph in figure 1 shows each calibration of 7D having the quantum Hall device as reference on the $i = 2$ plateau (squares) and on the $i = 4$ plateau (circles), thus building, the historical evolution of this resistor.
Figure 1. 7D historical evolution graph.

The linear fit (straight line in figure 1) becomes the best estimation of the 7D value for any time and is calculated by:

$$R_{7D}(\Delta t) = a \cdot \Delta t + b$$,  

(1)

where $a$ and $b$ are the angular and linear coefficients, $\Delta t = t - t_0$, $t$ is the date (horizontal axis) and $t_0$ (reference) is the beginning of 2000.

Hence, 7D resistor was characterized and may be used during R1 calibration.

3.2. BIPM calibrations of R1

Figure 2 shows all eight calibrations of R1 made at BIPM (diamonds) and a straight line that represent the best fit of these data. Each point is associated with a bar of uncertainty for $k = 2$ (a coverage probability of 95.45 %, assuming a Gaussian distribution). The mean value of this expanded uncertainty is:

$$\overline{U_{BIPM}} = \frac{1}{8} \sum_{i=1}^{8} U_{BIPM,i} = 0.033 \mu \Omega/\Omega.$$  

(2)
The value of R1 resistor at \( t = May\ 3th,\ 2013 \) (to compare with Lameq calibration made at the same date), predicted by the linear least squares fitted straight line to BIPM calibrations using equation (1) is:

\[
R_{R1,BIPM}(t) = 10000.01234 \, \Omega
\]

with an expanded combined uncertainty:

\[
U_c\left( R_{R1,BIPM}(t) \right) = 0.034 \, \mu\Omega/\Omega
\]

estimated by combination of the least squares linear regression uncertainty [3, 4] and the uncertainty of the last BIPM measurement (September, 2012).

3.3. **Lameq calibrations of R1**

Resistor 7D was used as a standard for Lameq R1 resistor calibration.

In figure 2 it is also marked this calibration value found on May 3th, 2013 by using Lameq system (circle) with its uncertainty (coverage probability is 95.45 % and \( k = 2.51 \)).

The value based on Lameq calibration at the same date, traceable to Lameq quantum Hall device, is:

\[
R_{R1,Lameq}(t) = 10000.01225 \, \Omega
\]

with associated relative expanded uncertainty:

\[
U\left( R_{R1,Lameq}(t) \right) = 0.031 \, \mu\Omega/\Omega,
\]

This uncertainty is close to each one of BIPM calibrations in the equation (2).

3.4. **Comparison between Lameq and BIPM calibrations**

The result was compared with the estimated value from BIPM periodic calibrations.
The normalized error is:

\[ En = \frac{|R_{RLameq} - R_{RBIPM}|}{\sqrt{(\Delta_{ABS}^{BIPM})^2 + (\Delta_{ABS}^{Lameq})^2}} = 0.13 \]  \hspace{1cm} (7)

where “ABS” means the absolute expanded uncertainty.

The relative difference between both values is:

\[ \frac{|R_{RLameq} - R_{RBIPM}|}{R_{RBIPM}} = 9.0 \times 10^{-9} \]  \hspace{1cm} (8)

which is a difference less than the mean of deviations of each BIPM calibrations points related to the correspondent point on the fit line.

4. Conclusion

A comparison was made between the predicted 10 kΩ resistor value for periodic calibrations performed by BIPM and the value found for this same resistor by calibration performed in Lameq traced to the quantum Hall system of this laboratory.

The difference between the two values was found to be 9.0 parts in 10^9 indicating that both the quantum Hall device and Lameq CCC have good technical and metrological performance and that could be soon approved as the new Brazilian standard of electrical resistance.

Acknowledgement

The authors gratefully acknowledge Inmetro Lampe staff, for the loan of their standard resistor (R1), which has been necessary to execute this research.

References

[1] Comité International des Poids et Mesures 1999 Mutual recognition of national measurement standards and of calibration and measurement certificates issued by national metrology institutes (Paris, French: BIPM)
[2] Comité International des Poids et Mesures 2012 Guide to the implementation of the CIPM MRA - CIPM MRA-G-01 (Paris, French: BIPM)
[3] Bussab W O and Morettin P A 2002 Estatística Básica (São Paulo, SP, Brazil: Saraiva)
[4] Working Group 1 of the Joint Committee for Guides in Metrology JCGM/WG 1 2008 Guide to the expression of uncertainty in measurement (Paris, French: BIPM)