Estimation on afflux of bridge on plain rivers related to blockage ratio and Froude number

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Abstract. The construction of hydraulic structures such as abutments and piers in natural waterways contributes to the contraction of river channels and the upstream water level rise. The maximum increase in the water level (afflux) is considered to be a major flood risk after a new bridge is built, and the accurate estimation of it is always an important work in designing a bridge construction. However, systematic studies on the prediction of the backwater effects of piers considering the blockage ratios within 10% are rarely reported. In this paper, a 2D hydrodynamic model was numerically simulated to investigate the effect of blockage ratios in the range of 2% - 9% on the amount and location of afflux. The distribution of depth-averaged flow velocities and water level changes around bridge piers was also discussed. Results indicate that the development and attenuation of backwater effects along the river channel falls naturally into 4 regions, and as the blockage ratio gets greater, afflux increases with its location moving farther from the zone strongly influenced by the bridge piers. It can be concluded that a blockage ratio of more than 7% should be one of the key concerns to river engineers in evaluating the capacity of river channels in plains to carry flood flows. On this basis, this article presented a parametric optimization formula based on blockage ratio and Froude number to estimate afflux of rectangle bridge piers with relatively low blockage ratios on plan rivers, and to provide effective assistance in hydraulic calculation of bridge design.

1. Introduction
The construction of bridges requires placement of piers within the channel of natural waterways, which can lead to a significant obstruction to the flow and thus an increase in the water levels upstream of piers, not least for the rivers that flow through flat plains. Hydraulicians, for centuries, have been interested in determining the backwater effects of the piers, which depend chiefly on their geometric shapes [1], their positions in a stream [2], angles between the axis of the bridge and the flow direction [3], flow rate [4], and the ratio of the projected area of bridge piers normal to flow to the gross area of the channel cross section, which is the blockage ratio (α) of bridge piers [5] or the contraction ratio (Cr) of the river channel [6]. Since the contraction ratio more often than not takes into account the influence of abutments at the ends of a bridge span, the blockage ratio of the bridge piers is thus adopted in this paper to quantify the contraction of a river channel.

Modern bridges are usually designed so that the maximum increase in the water level, termed afflux, is kept to a minimum, which is considered to be a major flood risk after a bridge is newly constructed [7]. However, afflux is hard to measure in field tests because its location often varies with the flow.
discharge, so very few data from the prototype sites are available [8]. In view of the impossibility of eliminating the scale effects in physical model tests and difficulty in theoretical analysis, numerical simulation can be a perfect alternative in assessing the bridge afflux. One-dimensional (1D) numerical models, such as ISIS [9], MIKE11 [10] and HEC-RAS [11,12] have been widely used in predicting water surface profile around bridge piers. Two 1D analytical models, Afflux Estimator (AE) and Afflux Advisor (AA), were developed by Mantz and Benn [13] for the more rapid estimation of afflux rating. Though these 1D models are still generally accepted, 2D and 3D methods become increasingly popular over recent years due to their accuracy and capability in modeling the river flow through the bridge structures [14-18].

Systematic studies on reliable means of predicting the backwater effects of the bridge piers in terms of blockage ratios commonly used in real river engineering are rarely reported to date. Kingston [19] conducted a basic study of the effects of pier section size on afflux using the HEC-RAS model, and found afflux increases with the magnitude of channel obstruction by the bridge piers for all his flow scenarios. It has been universally acknowledged that larger pier section sizes lead to a higher degree of obstruction and greater head loss, and the sudden contraction of flow in turn results in greater afflux in order to overcome this head loss. However, the blockage ratios of his bridge piers ranging from 6% to 46% were divorced from reality since in normal circumstances the blockage ratio of bridge piers in rivers seldom exceeds 10% according to engineering experience [20]. The study of bridge piers backwater effects lasts for nearly a century, however, there is still less researches on bridge piers backwater effects with low blockage ratios on plan rivers, which cause inconvenience on estimating afflux on plan rivers.

Accordingly, this research was made to assess the backwater effects caused by bridge piers with low blockage ratio along the subcritical river channel, especially focus on the location and the numerical value of afflux, by means of a 2D model via Delft-3D software with a series of various blockage ratios (2% - 9%) and downstream Froude number (0.03 – 0.16) considered. This research provides some references for river engineers in evaluating the effect of bridge piers on flood-carrying capacity of river channels.

2. Materials and methods

2.1. Numerical Modeling
The 2D river channel model, as shown in figure 1, has a 0.1‰ bed slope and trapezoidal sections. Two rectangular-sectioned bridge piers in water are researched upon in detail, which were symmetrically placed with a length-width ratio of $l/b = 2.5:1$ and their longer sides parallel to the flow direction. The closest distance between each bridge pier and the dike was fixed to 8.10 m. Eight pier section sizes, with blockage ratios varying from 2 % to 9 % and their widths in the range of 1.90 m and 8.57 m, were used.

The mathematical system was solved through a set of governing equations involving a continuity equation as well as momentum equations of 2D depth-averaged flows in which, $Z$ is the water level; $H$ is the depth of water; $\bar{u}$ is depth-averaged flow velocity ($\bar{u}^2 = \bar{u}_x^2 + \bar{u}_y^2$); $C$ is Chézy's coefficient; $g$ is the gravitational acceleration; and $\nu_t$ is the coefficient of turbulent viscosity [21]. The governing equations were discretized with staggered grid using finite differential method (FDM). The alternating direction implicit method (ADI) was used as the standard time integration method. A chasing method was also employed in solving equations.
Prior to our computation, the computational domain, a 5,000m-long reach, was subdivided into structured rectangular grids. As illustrated in figure 2, finer grids near the piers, with the minimum space step falling between 0.95m and 2.30m, aided in increasing the accuracy of simulating their section sizes. A bridge is located 3,000 m downstream from the upstream boundary. The boundary conditions were prescribed as the upstream flow discharge $Q$ as $100\text{m}^3/\text{s} \leq Q \leq 500\text{m}^3/\text{s}$ and the downstream water level $Z_d = 1.51 \text{m}$. The roughness coefficient $n$ of the channel was set to 0.021 and the sidewalls were defined to be free slip wall boundaries. The time step $\Delta t$ and calculation time $T$ were set to be 1.2s and 24hr, respectively.

![Figure 2. Grids in the vicinity of bridge piers.](image)

### 2.2. Model verification

The model adopted in this paper was validated using the results experimentally obtained by El-Alfy [22]. The cross-section of El-Alfy’s channel was rectangular and his two bridge piers were rectangular sectioned and symmetrically placed. Two experimental cases of El-Alfy’s and the major controlling parameters are given in Table 1, where $W_f / W_c$ is the ratio of the total clear width of flow to the total width of the channel, and $Z_3$ refers to the water depth downstream from bridge piers that is equal to the original (undisturbed) water depth in the absence of piers in river (Section 3), as shown in figure 3. $Z_1$ and $Z_2$ are the maximum and minimum water depths around the piers (Sections 1-2). The spacing between his piers was always $W_f / 3$.

| Case | $W_f$ (m) | $W_c$ (m) | $W_f / W_c$ | $Z_3$ (m) | Q (m$^3$/s) |
|------|-----------|------------|-------------|------------|-------------|
| 1    | 11.70     | 13         | 0.90        | 0.90 ~ 1.20| 31          |
| 2    | 10.53     | 13         | 0.81        | 1.15 ~ 1.40| 31          |

Figure 4 illustrates a comparison between El-Alfy’s experimental results and our corresponding computed ones, in regard to the relationship of the dimensionless afflux $\Delta Z_1 / Z_3$ and the ratio $F_{r3} / F_{r3c}$, where $F_{r3}$ is the Froude number at Section 3 in figure 3, and $F_{r3c}$ is the corresponding value of $F_{r3}$ when the flow at Section 2 is a critical flow. $F_{r3c}$ can be determined by El-Alfy’s equation where $r$ is the residual energy ratio between Sections 2 and 3 ($r \approx 0.95$). As shown in figure 4, afflux calculated with our 2D numerical model tally with experimental results with a margin of error within 5%, which
therefore indicates that the numerical model simulates the backwater effects of bridge piers with high accuracy.

\[
\frac{W_r^2}{W_c^2} = \frac{27 r^3 Fr_{\infty}^2}{(2 + Fr_{\infty})^3}
\]  

(4)

3. Results and discussion

3.1. General Features of Bridge Piers Backwater

Conduct a study on general features of bridge piers backwater with \( Q = 300 \text{m}^3/\text{s} \). Mean water surface profiles along the river at varying blockage ratios, as exhibited in figure 5, show similar trends, where \( D \) is the distance to bridge piers.

The development and attenuation of backwater effects along the river due to bridge pier obstruction consist of four regions: (1) the backwater-rise region which characterizes the gradually varied flow conditions, (2) the backwater-drop region where there is a sudden drop due to contraction, (3) the transition region and (4) the normal-depth region where the river flow reaches its normal depth downstream of piers (dash-dot line in figure 5).

![Figure 5. Average water level changes and regions along the river at different blockage ratios (in computational domain).](image)
The depth-averaged flow velocities ($\bar{u}$) near the piers at $\alpha = 7\%$ are mapped as contour lines, as shown in figure 6a. When the upstream flow discharges into a zone strongly influenced by the piers, the velocity distribution becomes uneven and complicated. Afflux, denoted by $\Delta Z_1$, occurs at Section 1 where the elevation of water surface attains its maximum. Due to local stagnation effects, the water surface just upstream of the pier is disturbed and the river flow is abruptly slowed, as shown by the dash-dot line in figure 6b. Around the sides of the piers, rapidly varied flow develops along with rather complex flow patterns in the backwater-drop region. The water level drops where the velocity is large (solid line) and the velocity falls a bit at the flow boundary of the pier sidewall chiefly due to the drag force exerted (dashed line), as illustrated in figure 6b. At Section 2, a short distance downstream from the bridge piers, the water surface reaches its minimum height, even below the water level of an undisturbed stream. In the transition region the water level and flow velocity further downstream get back to normal (the normal-depth region) by degrees as the backwater effects of piers have largely dissipated.

Generally, as our focus moves upstream from Section 1 in figure 5, the backwater decreases practically with the same ultraslow rate at all blockage ratios, and its effect can extend for such a long way that the average water level rise at the upstream boundary is merely 10% less than afflux. Since the estimation of afflux aids in preventing the flow from overtopping river banks or even the dike crest and the floodplains during the flood events, and in evaluating the economic consequences, it is quite vital for river engineers to determine the amount and location of afflux, which will be further discussed with varied blockage ratios considered.

3.2. Effect of blockage ratios on afflux
When setting $Q = 300\text{m}^3/\text{s}$, the water level rise around two piers with $\alpha = 7\%$ is shown in figure 7 in greyscale, and both sections correspond to the ones in figure 5 where, however, only average water levels across the channel are mentioned.

At the black zone the abruptly slowed flow piles up on top of itself and increases in height, with some of its initial kinetic energy converted into a rise in its potential energy and some energy irreversibly lost to heat via turbulence. It is apparent that the water level rise is unevenly distributed at the cross-sections in the backwater-drop region.
In order to showcase the variability of backwater distribution at each cross-section in relation to the mean water level rise, the coefficient of variation \((CV)\) of backwater among computational nodes of each cross-section defined as the ratio of the standard deviation to the mean value is used. Figure 8 shows the degree of uniformity \((1-CV)\) of the backwater distribution calculated at each cross-section close to Section 1 against the distance from this cross-section to bridge piers \((D)\), for different blockage ratios. The location of afflux is also depicted on plotted curves using different types of shape. It is evident that the lower \(\alpha\) is, the shorter the distance will be between where afflux occurs and bridge piers. With a higher \(\alpha\), the water level rise will be more evenly distributed at the same cross-section at a rising rate, especially when \(\alpha > 7\%\). It is thus indicated that the location of afflux is farther from the zone strongly influenced by the bridge piers with increasing blockage ratios.

Figure 9 shows the influences of blockage ratios on the absolute afflux \(\Delta Z_1\) as well as the dimensionless afflux \(\Delta Z_1 / b\), i.e. the ratio of afflux to the pier width. Afflux increases almost linearly with \(\alpha\); however, the slope of relationship plotted between \(\Delta Z_1 / b\) and \(\alpha\) changes twice within the range of \(\alpha\) considered in this investigation, one at \(\alpha = 4\%\) and the other at \(\alpha = 7\\%\), approximately. When \(\alpha < 7\%\) there is a decrease in \(\Delta Z_1 / b\) as \(\alpha\) is raised, and afterwards \(\Delta Z_1 / b\) begins to increase, which demonstrates the effect of higher \(\alpha\) on afflux is quite strong if \(\alpha > 7\%\), notwithstanding the increasing pier width as the denominator of \(\Delta Z_1 / b\).
3.3. Estimation of afflux

One of the well-known methods to calculate afflux of straight-deck-bridge was presented by Biery and Delleur [23]. After continuous optimization and development, the generalized afflux formulair [24] can be represented by

$$\frac{\Delta Z_1}{Z_3} = m \left( \frac{Fr}{1-\alpha} \right)^{2/3} n$$

(5)

where $\Delta Z_1$ is the afflux, $Z_3$ is the normal flow depth of the unconstricted channel, $\alpha$ is the blockage ratio of bridge piers, $Fr$ is the Froude number for uniform flow in the unconstricted channel (Section 3 in Figure 5), ‘$m$’ and ‘$n$’ are undetermined parameters.

Biery and Delleur [23] recommended the value of the undetermined parameters ‘$m$’ and ‘$n$’ are respectively 0.47 and 3.39. Further study was conducted by Atabay [24] to optimize the value of ‘$m$’ and ‘$n$’ suitable for different bridge types, and the proposed value of ‘$m$’ and ‘$n$’ are 0.2457 and 2.9668 for straight-deck-bridge.

In order to improve simulation accuracy of afflux caused by bridge piers with relatively low blockage ratio in subcritical flow, afflux in 40 working conditions were simulated by setting blockage ratios $\alpha$ as $2\% \leq \alpha \leq 9\%$, and downstream Froude number $Fr_3$ as $0.03 \leq Fr_3 \leq 0.16$ (with flow rate $Q$ as $100m^3/s \leq Q \leq 500m^3/s$). According to the modeled data and test data, the formula for afflux was fitted as equation (6). By contrast, this formula has higher precision where $[Fr/(1-\alpha)]^{2/3}$ range from 0.1 to 1.

$$\frac{\Delta Z_1}{Z_3} = 0.1447 \left( \frac{Fr}{1-\alpha} \right)^{2.37}$$

(6)

Table 2. Parametric optimization formula for afflux based on Biery and Delleur method.

| Formula             | $m$    | $n$    | $R^2$ |
|---------------------|--------|--------|-------|
| Biery and Delleur   | 0.47   | 3.39   | 0.86  |
| D.Seckin            | 0.2457 | 2.9668 | 0.88  |
| Equation (6)        | 0.1447 | 2.37   | 0.93  |

Figure 10. Generalised plot of optimized fitting line of Equation (6) and other two fitting lines.
4. Conclusions
In the present work, studies were carried out upon the influence of blockage ratios on afflux as well as its location upstream of bridge piers with blockage ratios from 2% to 9%, and a function relationship among relative afflux, blockage ratio and Froude number was found with the method of parametric optimization, by means of numerical simulation with a 2D hydraulic model. Results demonstrate that the backwater effects along the river channel can be divided into four flow regions, and with higher blockage ratios, especially when $\alpha > 7\%$, afflux gets raised with its location moving away from the greatly-impacted zone. Since a larger blockage ratio may aggravate channel obstruction and lead to a higher rate of the water level rise, it can be concluded that $\alpha = 7\%$ can be a worthwhile parameter in evaluating the capacity of river channels for discharging floods. On this basis, by using the parametric optimization formula can help to quickly estimate the backwater effects caused by rectangle bridge piers with low blockage ratio on plan rivers, and provide effective assistance in hydraulic calculation of bridge design. It is recommended that more factors be taken into account in examining the effect of blockage ratio on the backwater effects, and more efforts be put into the theoretical studies of the mechanisms.

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