Holographic dark energy in Brans-Dicke cosmology with Granda-Oliveros cut-off

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Abstract

Motivated by the recent works of one of us [1, 2], we study the holographic dark energy in Brans-Dicke gravity with the Granda-Oliveros cut-off proposed recently in literature. We find out that when the present model is combined with Brans-Dicke field the transition from normal state where $w_D > -1$ to the phantom regime where $w_D < -1$ for the equation of state of dark energy can be more easily achieved for than when resort to the Einstein field equations is made. Furthermore, the phantom crossing is more easily achieved when the matter and the holographic dark energy undergo an exotic interaction. We also calculate some relevant cosmological parameters and their evolution.

Keywords: Dark energy; Brans-Dicke gravity; holographic principle; phantom energy

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I. INTRODUCTION

Recent cosmological observations obtained from distant supernovae SNe Ia \[^3\], cosmic microwave background radiation explorations of WMAP \[^4\], galaxy redshift surveys of SDSS \[^5\] and galactic cluster emissions of X-rays \[^6\] all convincingly indicate that the observable universe is undergoing an accelerated expansion. The data also show that this sudden change in the expansion of the universe is quite recent (\(z \approx 0.7\)) while previously the universe was in the phase of deceleration. The origin of acceleration may be caused due to exotic ‘dark energy’ (DE) in the universe. It is now the job of experimentalists to parameterize DE and for theorists to determine its origin. Although the simplest way to explain this behavior is the consideration of a cosmological constant \[^7\], the known fine-tuning problem (huge discrepancies in the theoretical and observational estimates of DE parameters particularly in its energy density) and coincidence problem (apparent constant ratio of energy densities of DE and matter) \[^8\] led to the DE paradigm. The dynamical nature of DE, at least in an effective level, can arise from a variable cosmological “constant” \[^9\], or from various fields, such is a canonical scalar field (quintessence) \[^10\], a phantom field, that is a scalar field with a negative sign of the kinetic term \[^11\], or the combination of quintessence and phantom in a unified model named quintom \[^12\]. Finally, an interesting attempt to probe the nature of DE according to some basic quantum gravitational principles is the holographic DE (HDE) paradigm \[^13\] which we shall discuss in ample detail below.

Recent studies of black hole and string theories provide an alternative solution to the DE puzzle using the holographic principle \[^14\] originally proposed by t’ Hooft \[^15\]. The principle says that the entropy of a closed system scales not by its volume but with its surface area. In other words the degrees of freedom of a spatial region reside not in the bulk but only at the boundary of the region and the number of degrees of freedom per Planck area are not greater than unity. It is widely accepted that this principle will be a part of the final quantum gravity since both string theory and loop quantum gravity incorporate this principle beautifully.

Cohen et al. \[^14\] suggested that in quantum field theory a short distance cutoff is related to a long distance cutoff due to the limit set by formation of a black hole, which results in an upper bound on the zero-point energy density. In line with this suggestion, some authors \[^13\] argued that this energy density could be viewed as the HDE density satisfying
\[ \rho_\Lambda = 3n^2M_p^2/L^2, \] where \( L \) is the size of a region which provides an IR cut-off, and the numerical constant \( 3n^2 \) is introduced for convenience. It is important to note that in the literature, various scenarios of HDE have been studied via considering different system’s IR cutoff [16]. If one takes the Hubble length as the IR cut-off \( (\rho_D \propto H^2) \) then it conveniently resolves the fine tuning problem but yields a wrong equation of state of DE \( (w_D = 0) \) which cannot drive cosmic acceleration. Moreover a different IR cut-off, a particle horizon, also yields a wrong equation of state \( (w_D > -1/3) \) of DE. Later on Granda and Oliveros [17] proposed a new cut-off based on purely dimensional grounds, by adding a term involving the first derivative of the Hubble parameter. Thus the new cut-off is similar to the Ricci scalar of the FRW metric \( \rho_D \sim \gamma H^2 + \beta \dot{H} \), where \( \gamma \) and \( \beta \) are constants of order unity. It was predicted that their values should be \( \gamma \simeq 0.93 \) and \( \beta \simeq 0.5 \), in order to be consistent with the big bang nucleosynthesis theory [17]. Other studies on the HDE have been carried out in [18].

Soon after Albert Einstein introduced his “general theory of relativity” in 1915, several attempts were made to construct alternative theories of gravity. These were intended to construct unified models of all forces. One of the most studied alternative theories was scalar-tensor theory, where the gravitational action contains, apart from the metric, a scalar field which describes part of the gravitational field. The scalar-tensor theory was invented first by Jordan [20] in the 1950’s, and then taken over by Brans and Dicke [21] some years later. Soon after the discovery of dark matter (DM) and DE, a new breed of gravities like Gauss Bonnet and f(R) gravities proposed whose Lagrangian contained several terms involving curvature tensor and scalars. Because the HDE density belongs to a dynamical cosmological constant, we need a dynamical frame to accommodate it instead of Einstein gravity. Therefore the investigation on the holographic models of DE in the framework of Brans-Dicke theory is of great importance [22]. The investigation on the holographic models of DE in the framework of Brans-Dicke cosmology, have been carried out in [23–28].

In this paper, we investigate the HDE in the Brans-Dicke gravity using the Granda-Oliveros cut-off. We follow the method of Ref. [22]. Following previous studies, we calculate the equation of state of DE and some other cosmological parameters of our interest and demonstrate that phantom crossing is possible in the present model.
II. NEW HDE IN BRANS-DICKE GRAVITY

The action of Brans-Dicke theory is

$$ S = \int d^4x \sqrt{g} \left[ \frac{1}{2} \left( \Phi R - \omega \frac{\nabla_\mu \Phi \nabla_\nu \Phi}{\Phi} \right) + L_m \right]. \quad (1) $$

The equations of motion for the metric $g_{\mu\nu}$ and the Brans-Dicke scalar field $\Phi$ are

$$ R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = \frac{1}{\Phi} T^M_{\mu\nu} + T^{BD}_{\mu\nu}, \quad (2) $$

$$ \nabla_\mu \nabla_\nu \Phi = \frac{1}{2\omega + 3} T^M_{\mu\nu}. \quad (3) $$

Here $T^M_{\mu\nu} = (2/\sqrt{g}) \delta(\sqrt{g} L_M)/\delta g^{\mu\nu}$ is the energy momentum tensor for the matter fields defined in the form of perfect fluid

$$ T^M_{\mu\nu} = (\rho_M + p_M) U_\mu U_\nu + p_M g_{\mu\nu}. \quad (4) $$

The energy momentum tensor of the Brans-Dicke scalar field is

$$ T^{BD}_{\mu\nu} = \frac{\omega}{\Phi^2} \left( \nabla_\mu \Phi \nabla_\nu \Phi - \frac{1}{2} g_{\mu\nu} \nabla_\alpha \Phi \nabla^\alpha \Phi \right) + \frac{1}{\Phi} (\nabla_\mu \nabla_\nu \Phi - g_{\mu\nu} \nabla_\alpha \nabla^\alpha \Phi). \quad (5) $$

The governing equations of the system are

$$ H^2 + \frac{k}{a^2} + H \frac{\dot{\Phi}}{\Phi} - \frac{\omega}{6} \left( \frac{\dot{\Phi}}{\Phi} \right)^2 = \frac{1}{3\Phi} (\rho_M + \rho_D), \quad (6) $$

$$ 2 \frac{\ddot{a}}{a} + H^2 + \frac{k}{a^2} + \frac{\omega}{2} \left( \frac{\dot{\Phi}}{\Phi} \right)^2 + 2H \frac{\dot{\Phi}}{\Phi} + \frac{\ddot{\Phi}}{\Phi} = -p_D. \quad (7) $$

The energy conservation equations for DE and matter are

$$ \dot{\rho}_D + 3H (1 + w_D) \rho_D = 0, \quad (8) $$

$$ \dot{\rho}_m + 3H \rho_m = 0. \quad (9) $$

We use the new HDE [17] which is modified to be consistent with the BD framework

$$ \rho_D = 3\Phi (\gamma H^2 + \beta \dot{H}), \quad (10) $$

where $\gamma$ and $\beta$ are constants. Notice that the above cut off (10) is different from the Ricci dark energy [30] and reduces to the later one if $k = 0$, $\beta = 1$ and $\gamma = 2$. We choose the ansatz

$$ \Phi(a) = \Phi_0 a(t)^\alpha, \quad \dot{\Phi} = \alpha \Phi H, \quad \ddot{\Phi} = \Phi (\alpha^2 H^2 + \alpha \dot{H}), \quad (11) $$
where in what follows we take \( \Phi_0 = 1 \). Note that the system of Eqs. (6)-(7) is not closed and we still have freedom to choose one. We shall assume that Brans-Dicke field can be described as a power law of the scale factor, \( \phi \propto a^\alpha \). In principle there is no compelling reason for this choice. However, it has been shown that for small \( \alpha \) it leads to consistent results \([24, 27]\). A case of particular interest is that when \( \alpha \) is small whereas \( \omega \) is high so that the product \( \alpha \omega \) results of order unity \([24]\). This is interesting because local astronomical experiments set a very high lower bound on \( \omega \); in particular, the Cassini experiment implies that \( \omega > 10^4 \) \([26]\). Introducing a new parameter \( x = \ln a \), the e-folding parameter. Using it we get
\[
\dot{H} = \frac{1}{2} \frac{dH^2}{dx}. \tag{12}
\]
Assuming matter evolves as \( \rho_m = \rho_{ma} a^{-3} \). Using (10), (11) and (12) in (6), we obtain
\[
\frac{dH^2}{dx} - \frac{2\delta}{\beta} H^2 = - \frac{2\rho_{ma}}{3\beta} e^{-x(\alpha+3)} + \frac{2k}{\beta} e^{-2x}, \tag{13}
\]
where
\[
\delta \equiv 1 - \gamma + \alpha \left( 1 - \frac{\alpha \omega}{6} \right). \tag{14}
\]
Equation (13) can be solved to get
\[
H^2 = \frac{2\rho_{ma}}{3(\alpha\beta + 2\delta + 3\beta)} e^{-x(\alpha+3)} + c_1 e^{2\delta x/\beta} - \frac{k}{\beta + \delta} e^{-2x}, \tag{15}
\]
where \( c_1 \) is a constant of integration. Using (10) in (8), we obtain
\[
w_D = -1 - \frac{\alpha}{3} - \frac{2\gamma H \dot{H} + \beta \ddot{H}}{3H(\gamma H^2 + \beta H)}. \tag{16}
\]
Using (15) in (16), we get the equation of state (EoS) parameter for the new HDE in the framework of the Brans-Dicke theory
\[
w_D = -1 - \frac{\alpha}{3} - \frac{\beta(\alpha+3-2\gamma)(\alpha+3)}{3(\alpha\beta + 2\delta + 3\beta)} \rho_{ma} e^{-x(\alpha+3)} + 2c_1 \left( \frac{\delta}{\beta} \right)(\gamma + \delta) e^{2\delta x/\beta} + 2 \frac{k}{\beta + \delta} (\gamma - \beta) e^{-2x} - \frac{\beta(\alpha+3-2\gamma)}{(\alpha\beta + 2\delta + 3\beta)} \rho_{ma} e^{-x(\alpha+3)} + 3c_1(\gamma + \delta) e^{2\delta x/\beta} - 3 \frac{k}{\beta + \delta} (\gamma - \beta) e^{-2x}, \tag{17}
\]
or
\[
w_D = -1 - \frac{\beta(\alpha+3-2\gamma)}{(\alpha\beta + 2\delta + 3\beta)} \rho_{ma} e^{-x(\alpha+3)} + c_1(\alpha + \frac{2\delta}{\beta})(\gamma + \delta) e^{2\delta x/\beta} - \frac{k}{\beta + \delta} (\gamma - \beta)(\alpha - 2) e^{-2x} - \frac{\beta(\alpha+3-2\gamma)}{(\alpha\beta + 2\delta + 3\beta)} \rho_{ma} e^{-x(\alpha+3)} + 3c_1(\gamma + \delta) e^{2\delta x/\beta} - 3 \frac{k}{\beta + \delta} (\gamma - \beta) e^{-2x}. \tag{18}
\]
In the limiting case \( \alpha = 0 \ (\omega \to \infty) \), the Brans-Dicke scalar field becomes trivial and Eq. (18) in the absence of matter (\( \rho_{ma} \to 0 \)) reduces to its respective expression in new HDE
model in Einstein gravity \[2\]

\[ w_D = -\frac{1}{3} \left( k \frac{(\gamma - \frac{\beta}{3+\delta})e^{-2x} - c_1(3 + \frac{24}{3})\gamma e^{2\delta x/\beta}}{\frac{k(\gamma - \frac{\beta}{3+\delta})e^{-2x} - c_1(\gamma + \delta)e^{2\delta x/\beta}}\right). \] (19)

If we compare Eq. (17) with Eq. (19) we find out that when the new HDE is combined with Brans-Dicke field the transition from normal state where \(w_D > -1\) to the phantom regime where \(w_D < -1\) for the EoS of DE \[31\] can be more easily achieved for than when resort to the Einstein field equations is made. The analysis of the properties of DE from recent observations mildly favor models with \(\omega_D\) crossing -1 in the near past \[32\].

To illustrate this result in ample detail, we investigate it for the late-time universe when \(x \to \infty\). In this case, Eq. (17) reduces to

\[ w_D = -1 - \frac{2}{3} \left( \frac{1 - \gamma}{\beta} \right) - \frac{\alpha}{3} \left[ 1 + \frac{2}{\beta} \left( 1 - \frac{\alpha \omega}{6} \right) \right], \] (20)

and Eq. (19) yields

\[ w_D = -1 - \frac{2}{3} \left( \frac{1 - \gamma}{\beta} \right). \] (21)

If we take \(\alpha \omega \approx 1\) \[24\], \(\gamma \approx 0.93\) and \(\beta \approx 0.5\) \[17\] then for the new HDE in Brans-Dicke gravity, Eq. (20) gives \(w_D = -1.09 - 1.44\alpha\) and in Einstein gravity (\(\alpha \to 0\)) from Eq. (21) we obtain \(w_D = -1.09\). Thus in the late-time universe, crossing the phantom divide line for the new HDE in Brans-Dicke gravity can be more easily achieved for than when resort to the Einstein gravity.

Using (7), the deceleration parameter can be evaluated to be

\[ q = -\frac{\ddot{a}}{aH^2} = \frac{1}{(2 + \alpha + 3\beta w_D)} \left[ (1 + \alpha)^2 + \frac{\omega}{2}\alpha^2 - \alpha + 3(\gamma - \beta)w_D + \Omega_k \right]. \] (22)

The evolution of dimensionless DE parameter is

\[ \Omega_D' = -3\gamma \left( w_D + 1 + \frac{\alpha}{3} \right) + \left( \frac{\gamma - \Omega_D}{\beta} \right) \left[ \beta(3w_D + 3 + \alpha) + 2\gamma \right] - 2\beta \left( \frac{\gamma - \Omega_D}{\beta} \right)^2. \] (23)

III. INTERACTING NEW HDE IN BRANS-DICKE GRAVITY

Next we generalize our study to the case of interacting new HDE in Brans-Dicke theory. Recent cosmological observations support the interaction between DE and DM \[33\]. Taking the interaction into account, HDE and DM do not conserve separately and enter the energy
balances 34

\[ \dot{\rho}_D + 3H\rho_D(1 + w_D) = -Q, \]  
\[ \dot{\rho}_{DM} + 3H\rho_{DM} = Q, \]  
\[ \dot{\rho}_{BM} + 3H\rho_{BM} = 0, \]

where we have assumed the BM dose not interact with DE. Here Q denotes the interaction term and we take it as

\[ Q = 3b^2H(\rho_{DM} + \rho_D), \]

where \( b^2 \) is a coupling constant. The critical energy density, \( \rho_{\text{cr}} \), and the energy density of the curvature, \( \rho_k \), are defined as

\[ \rho_{\text{cr}} = 3\Phi H^2, \quad \rho_k = \frac{3k\Phi}{a^2}. \]  

We also introduce the fractional energy densities such as

\[ \Omega_{BM} = \frac{\rho_{BM}}{\rho_{\text{cr}}} = \frac{\rho_{BM}}{3\Phi H^2}, \]  
\[ \Omega_{DM} = \frac{\rho_{DM}}{\rho_{\text{cr}}} = \frac{\rho_{DM}}{3\Phi H^2}, \]  
\[ \Omega_D = \frac{\rho_D}{\rho_{\text{cr}}} = \frac{\rho_D}{3\Phi H^2}, \]  
\[ \Omega_k = \frac{\rho_k}{\rho_{\text{cr}}} = \frac{k}{a^2 H^2}. \]

Combining Eqs. (28) and (11) with the first Friedmann equation (6), we can rewrite this equation as

\[ \rho_{\text{cr}} + \rho_k = \rho_{BM} + \rho_{DM} + \rho_D + \rho_{\Phi}, \]  

where we have defined

\[ \rho_{\Phi} \equiv \alpha H^2\Phi \left( \frac{\alpha\omega}{2} - 3 \right). \]  

Dividing Eq. (33) by \( \rho_{\text{cr}} \), it can be rewritten as

\[ \Omega_{BM} + \Omega_{DM} + \Omega_D + \Omega_{\Phi} = 1 + \Omega_k, \]

where

\[ \Omega_{\Phi} = \frac{\rho_{\Phi}}{\rho_{\text{cr}}} = \frac{\alpha}{3} \left( 3 - \frac{\alpha\omega}{2} \right). \]
Therefore we can rewrite the interaction term as

$$Q = 3b^2 H (\rho_{DM} + \rho_D) = 3b^2 H \rho_D (1 + u), \quad (37)$$

where

$$u = \frac{\rho_{DM}}{\rho_D} = -1 + \frac{1}{\Omega_D} \left[ 1 + \Omega_k - \Omega_{BM} + \frac{\alpha}{3} \left( 3 - \frac{\alpha \omega}{2} \right) \right], \quad (38)$$

is the energy density ratio of two dark components.

Following the approach of the previous section we obtain the EoS parameter of the interacting new HDE in Brans-Dicke theory as

$$w_D = -1 - \frac{\beta (\alpha + 3 - 2\gamma)}{(\alpha + \beta + 2\alpha + 3\beta)} \rho_m e^{-x(\alpha + 3)} + c_1 \left( \alpha + \frac{2\delta}{\beta} \right) (\gamma + \delta) e^{2\delta x/\beta} - \frac{k}{\beta + \delta} (\gamma - \beta)(\alpha - 2)e^{-2x}$$

$$- \frac{\Omega_D}{\Omega_D} \left[ 1 + \Omega_k - \Omega_{BM} + \frac{\alpha}{3} \left( 3 - \frac{\alpha \omega}{2} \right) \right]. \quad (39)$$

Comparing Eq. (39) with (17) shows that in the presence of interaction since the last expression in Eq. (39) has a negative contribution, hence crossing the phantom divide, i.e. $w_D < -1$, can be more easily achieved for than when the interaction between the new HDE and DM is not considered.

The deceleration parameter $q$ and the equation of motion for $\Omega_D$ are still obtained according to Eqs. (22) and (23), respectively, where $w_D$ is now given by Eq. (39).

IV. CONCLUDING REMARKS

We discuss the role of the HDE with the Granda-Oliveros cut-off in the Brans-Dicke theory. It is very interesting to investigate the role of dynamical cosmological constant (HDE) in the dynamical framework (Brans-Dicke theory). The Granda-Oliveros length scale is a natural extension of the Hubble length since the later does not resolve some DE issues such as the equation of state of DE. We have found that the new equation of state obtained above provides necessary corrections and enables a phantom crossing/divide of the state parameter. We also consider an interaction between DE and DM (ignoring the baryonic component) and found that phantom crossing is softer in this case as compared to the non-interacting case.
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