THREE-DIMENSIONAL SIMULATIONS OF SPHERICAL ACCRECTION FLOWS WITH SMALL-SCALE MAGNETIC FIELDS

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ABSTRACT

Spherical (nonrotating) accretion flows with small-scale magnetic fields have been investigated using three-dimensional, time-dependent MHD simulations. These simulations have been designed to model high-resolution (quasi-) steady accretion flows in a wedge computational domain that represents a small fraction of the full spherical domain. Subsonic and supersonic (superfast-magnetosonic) accretion flows have been considered. Two accretion regimes have been studied: conservative, or radiatively inefficient; and nonconservative, in which the heat released in magnetic reconnections is completely lost. The flows in both regimes are turbulent. They show flattened radial density profiles and reduction of the accretion velocities and mass accretion rates in comparison with hydrodynamic Bondi flows. In the conservative regime, the turbulence is more intensive and supported mostly by thermal convection. In the nonconservative regime, the turbulence is less intensive and supported by magnetic buoyancy and various magnetic interactions. We have concluded that steady, supersonic spherical accretion cannot develop in the presence of small-scale magnetic fields.

Subject headings: accretion, accretion disks — black hole physics — convection — MHD — turbulence

1. INTRODUCTION

Mass accretion onto a central gravitational body plays an important role in the formation and evolution of a large variety of astrophysical objects such as planets, stars, galactic nuclei, galaxies, and clusters of galaxies (see Frank et al. 1992). Depending on the amount of the angular momentum $l_0$, which the accreting mass carries at the outer boundary radius $R_{\text{out}}$, accretion flows can take either a disklike or quasi-spherical form. Flows with relatively high angular momentum, $l_0 \lesssim (G M_{\text{in}} R_{\text{out}})^{1/2}$, form centrifugally supported accretion disks (e.g., Shakura 1972). Flows with low angular momentum, $l_0 \ll (G M_{\text{in}} R_{\text{out}})^{1/2}$, can form spherical or quasi-spherical accretion flows at radii $R \gtrsim l_0^2 / GM$ in which the centrifugal force is weak and not sufficient to balance gravity (Illarionov & Sunyaev 1975). The theoretical study of spherical accretion flows is based on an analytical solution discovered several decades ago by Bondi (1952). This solution describes the idealized case of isentropic nonmagnetic accretion flows. Since that time, the theory of spherical accretion has been significantly developed so that it now includes an understanding of the role of different physical mechanisms (such as radiative cooling and heating, magnetic field dissipation, thermal and radiation transports, etc.) and more realistic inner and outer boundary conditions (for reviews, see Frank et al. 1992; Kato et al. 1998). More recent studies of spherical accretion have considered convection (Marković 1995), accretion onto a magnetic dipole (Toropin et al. 1999), magnetic diffusivity due to turbulence (Shadmehri 2004), and vorticity (Krumholz et al. 2005).

This paper continues the numerical study of radiatively inefficient spherical (nonrotating) accretion flows with magnetic fields conducted by Igumenshchev & Narayan (2002, hereafter IN02; also see Pen et al. 2003 for a similar study). IN02 have demonstrated, with the help of three-dimensional MHD simulations, that the effects of a magnetic field can significantly modify the structure of Bondi-type flows. They argued that even initially weak magnetic fields can produce dramatic changes. The main reason for these changes is the local, nonuniform release of thermal energy during the dissipation of tangled magnetic fields in reconnections. This release suppresses the inward motion of mass and results in the development of turbulence that is mainly supported by thermal convection. The dissipation of a magnetic field in reconnections is compensated by the efficient amplification of the radial field component in spherical convergent flows (Shvartsman 1971). IN02 have developed a simple analytical theory of spherical convection-dominated accretion flows (spherical CDAFs), which is similar, with regards to the basic physics involved, to the theory of rotating CDAFs (Narayan et al. 2000; Quataert & Gruzinov 2000). This theory predicts a flattened radial density profile $\rho \propto R^{-3/2}$, in contrast to the steeper density profile in asymptotic Bondi flows, $\rho \propto R^{-3/4}$ (see the Appendix, eq. [A6]). Because of this flattened profile, the mass accretion rate in spherical CDAFs is expected to be significantly lower than the Bondi mass accretion rate in flows with the equivalent outer boundary conditions:

$$\dot{M} \sim \dot{M}_{\text{Bondi}} \left( \frac{R_{\text{in}}}{R_{\text{out}}} \right),$$

where $R_{\text{in}}$ is the inner radius of the flows. These properties make the spherical CDAF solution a prominent candidate for explaining the phenomenon of dim galactic nuclei containing supermassive black holes, including our Galactic center Sgr A* (e.g., Melia 1992; Baganoff et al. 2001, 2003; Quataert 2002; Ghez et al. 2003; Ho et al. 2003; Soria et al. 2006), in which accretion disks are typically not observed. In addition, spherical CDAFs can be employed to explain the problem of missing isolated neutron stars in our Galaxy (Treves & Colpi 1991; Blaes & Madau 1993; Turolla et al. 1994; Belloni et al. 1997; Toropina et al. 2003; Perna et al. 2003) and the observed properties of isolated stellar mass black holes (e.g., Fujita et al. 1998; Agol & Kasen 2002).

Note that IN02 instead used the name “convection-dominated Bondi flows,” which we do not adopt here.
The generality of the numerical results of IN02 and Pen et al. (2003) was limited in the important aspect that no steady or quasi-steady accretion flows were obtained. These authors studied transient states of the flows, which originated because of specific initial conditions and involved moving outward shocks. These results also suffered from insufficient numerical resolution, especially in the innermost region of the flows. In addition, the assumption of an initial bipolar magnetic field in IN02 resulted in the domination of a large-scale poloidal field and donut-like density distribution near the black hole at the later evolution time (note that similar field topology was proposed by Bisnovatyi-Kogan & Ruzmaikin 1974). Unless such a large-scale poloidal field can be naturally present in some objects, other possible field configurations, such as a small-scale field with zero net magnetic flux, are also likely to be natural in spherical accretion flows. The main goal of the present study is the investigation of the role of small-scale magnetic fields in such flows.

In the numerical aspect, we overcome some of the limitations of the IN02 approach by employing a new simulation design. In this design, we assume a permanent injection of mass and magnetic field into the computational domain that allows us to obtain steady or quasi-steady accretion flows after performing long-time evolution simulations. The numerical resolution has been improved by adopting spherical coordinates and conducting simulations in the wedge computational domain, which represents a small fraction of the full spherical domain (see Fig. 1). The injected field is assumed to have a small-scale component in the form of radially extended magnetic loops with a zero net magnetic radial flux (see Fig. 2). Modifications of the simulation technique allow us to investigate two limiting energetic regimes: conservative (or radiatively inefficient) and nonconservative (in which the heat from magnetic reconnections is completely lost).

The paper is organized as follows. In § 2 we describe the simulation technique, initial and boundary conditions, and algorithm of the mass and magnetic field injection. Section 3 presents numerical results, and in § 4 we discuss these results and make final conclusions. We reproduce some analytic solutions of hydrodynamic accretion flows, including the Bondi solution, in the Appendix.

2. SIMULATION TECHNIQUE

We use the numerical method, which is similar to that used by IN02 and Igumenshchev et al. (2003). The method solves the system of ideal MHD equations (e.g., Landau & Lifshitz 1987), which describe the dynamics of non–self-gravitating plasmas in the central gravitational field. Originally, the method employed a nonconservative numerical scheme that solves the internal energy equation, which includes the reconnection heat term $Q$ (for more details see IN02):

$$\rho \frac{dT}{dt} = -P_g \nabla \cdot v + Q,$$

where $\rho$ is the density, $T$ is the specific internal energy, $P_g$ is the gas pressure, and $v$ is the velocity. Test simulations have shown that MHD solutions obtained using equation (2) conserve the total energy quite poorly because of artificially losing or gaining...
energy in numerical reconnections. In some of our test cases, the relative error of the total energy conservation was up to 10%. To solve this problem, the method has been modified by adding a conservative option. Using this option, the method solves the total energy equation

\[
\frac{\partial}{\partial t} \left( \rho \frac{v^2}{2} + \rho e + \frac{B^2}{8\pi} \right) = -\nabla \cdot \mathbf{q}
\]

(3)

instead of equation (2). Here \( \mathbf{B} \) is the magnetic induction and \( \mathbf{q} \) is the total energy flux per unit square. Note that in finite-difference MHD schemes the magnetic field is reconnected on scales in which the minimum is limited by the grid size. Typically, this grid size is much larger than the physical reconnection scales in the studied problems. Therefore, the finite-difference schemes, including our scheme, cannot accurately represent all the details in the process of reconnection. For our purposes, however, the level of accuracy provided is sufficient. This situation is somewhat analogous to the representation of shocks in finite-difference hydrodynamic schemes in which the numerical shock thickness is also limited by the grid size.

In all our simulations, we have used the ideal gas equation of state

\[
P_g = (\gamma - 1)\rho e
\]

(4)

and assumed the adiabatic index \( \gamma = 5/3 \).

This method employs the three-dimensional spherical coordinates \((R, \theta, \phi)\). The computational domain is limited by a narrow four-facet wedge located at the equatorial plane as shown in Figure 1. The domain is extended from \( R_{\text{in}} \) to \( R_{\text{out}} \) in the radial direction and over \( \theta_0 \) and \( \phi_0 \) degrees in the polar and azimuthal directions. We have assumed \( \theta_0 = \phi_0 = \pi/16 \) and the number of angular grid points \( N_\theta \times N_\phi = 30 \times 30 \). These points are uniformly spaced in both the \( \theta \)- and \( \phi \)-directions. Grid points in the radial direction are spaced logarithmically so that the three-dimensional numerical cells at any radius take an approximately cubic shape. This provides the direction-independent local spatial resolution in the simulations. The number of radial grid points \( N_R = 303 \), which corresponds to \( R_{\text{out}}/R_{\text{in}} = 10 \).

### 2.1. Boundary Conditions

We have used three different sets of boundary conditions assumed at the azimuthal, polar, and radial boundaries, respectively, of the wedge computational domain (see Fig. 1). The periodic boundary conditions for both fluid and magnetic field are applied at the azimuthal boundaries. At the polar boundaries, we use the conditions that no streamlines and magnetic lines can go inside/outside through these boundaries. This is achieved by applying reflection boundary conditions for fluid and assuming continuous tangential and reflection normal magnetic components across the polar boundaries.

At the inner \( R_{\text{in}} \) and outer \( R_{\text{out}} \) radial boundaries, we apply the absorbing boundary conditions for a fluid. This means that the mass can flow freely out of the computational domain, but no mass is allowed to return from outside. Conditions for the magnetic field are assumed considering “ghost” boundary zones located on the outside of the computational domain. We assume that these zones can contain only a radial magnetic field, whose strength is determined from the divergence-free constraint

\( \nabla \cdot \mathbf{B} = 0 \).

Numerical tests have shown that a mass with a frozen-in tangled magnetic field can freely flow out through these radial boundaries without the effects of field or mass accumulation.

Our absorbing boundary conditions at \( R_{\text{in}} \) qualitatively correctly mimic the conditions near the black hole horizon where matter is free-falling in the strong gravity field and the radial component of the magnetic field dominates the tangential component (e.g., Thorne et al. 1986). In the case of accreting stars with rigid surfaces such as white dwarfs and neutron stars, which can also have magnetospheres, the inner boundary condition will depend on the radiative efficiency of the flows. Radiatively inefficient plasma will probably form slowly accreting (\( M \ll M_{\text{Bondi}} \)) subsonic accretion flows, similar to the “tenuous continuation of the star” discussed by Bondi (1952). If plasma efficiently radiates its thermal energy near the stars’ surfaces (e.g., Shapiro & Salpeter 1975), the absorbed inner boundary conditions considered can be adequate for accretion flows far away from these surfaces.

### 2.2. Injection of Mass

To obtain steady or quasi-steady accretion flows, we permanently inject mass into the computational domain. The main problem here is to minimize the consequences of the interaction of outflows, which can originate during simulations, and the injected mass. We have employed an injection algorithm, which has some resemblance to the injection algorithm used by Igumenshchev & Abramowicz (1999) in simulations of rotating accretion flows.

We assume that the mass is steadily injected inside a thin spherical layer with radius \( R_{\text{inj}} \), which is located close to the outer absorbing boundary at \( R_{\text{out}} \). This mass is distributed uniformly over the \( \theta \)- and \( \phi \)-directions and has zero velocity. Under the action of gravity, the larger fraction of the injected mass forms an accretion flow. The smaller fraction of this mass can escape through \( R_{\text{out}} \) because of a thermal spread and interactions with outflows.

The injected mass is characterized by the specific internal energy \( \epsilon_0 \), which determines two regimes of hydrodynamic accretion: subsonic and supersonic. The critical value \( \epsilon_{\text{crit}} \approx \epsilon_{\text{vir}} \), where

\[
\epsilon_{\text{vir}} \equiv \frac{GM}{R_{\text{inj}}}
\]

(5)

separates these regimes: \( \epsilon > \epsilon_{\text{crit}} \) corresponds to subsonic flows and \( \epsilon < \epsilon_{\text{crit}} \) corresponds to supersonic flows (see the Appendix).

### 2.3. Injection of a Magnetic Field

The injected mass can also carry a magnetic field that is associated with it. The field is injected assuming that only the poloidal component \( A_\theta \) of the vector potential \( \mathbf{A} \) is nonzero in the injected mass; the other two components, \( A_R \) and \( A_\phi \), are set to zero. In this way we can produce the \( B_R \) and \( B_\phi \) components, which obey

\[
B_R = -\frac{1}{R \sin \theta} \frac{\partial A_\theta}{\partial \phi}, \quad B_\phi = \frac{1}{R} \frac{\partial}{\partial R} (RA_\theta).
\]

(6)

Three different magnetic field configurations have been used in simulations: purely (unipolar) radial, purely toroidal, and loop configurations. The former two configurations have been chosen to test our numerical method. The purely radial field is generated at the beginning of simulations assuming that

\[
A_\theta = B_\text{inj} R_{\text{inj}} \frac{R_{\text{inj}}}{R} \phi,
\]

(7)
where $B_{nj}$ is the magnetic induction at $R_{nj}$. This field remains unchanged during simulations by virtue of the conservation of the radial magnetic flux confined in the wedge computational domain.

Two other field configurations, purely toroidal and loop fields, are formed by permanently injecting the corresponding field at $R_{nj}$. This field injection is tightened to the mass injection and performed by correcting $A_0$ inside the injection layer at each time step as follows:

$$A'_0 = A_0 + \xi \Delta A,$$

where $\Delta A$ is an increment and $\xi$ is a correction factor $0 \leq \xi \leq 1$. The increment $\Delta A$ depends on the assumed field configuration. In the case of a purely toroidal field,

$$\Delta A = B_0 R_{nj},$$

where $B_0 = \sqrt{8\pi \gamma (\gamma - 1) \epsilon_0 \Delta \rho / \beta_0}$ and $\Delta \rho$ is the density of the injected mass. In the case of a loop field,

$$\Delta A = B_0 R_{nj} \frac{\sin (m \phi)}{m},$$

where the parameter $m$ determines the number of azimuthal sectors in which the radial magnetic field periodically changes direction. A combination of two such sectors forms a magnetic loop. In these simulations, we have assumed two magnetic loops in the computational domain as illustrated in Figure 2.

The correction factor $\xi$ in equation (8) is used to maintain the strength of the injected field at a given level determined in terms of the plasma $\beta \equiv P_{\parallel}/P_m$, where $P_m = B^2/8\pi$ is the magnetic pressure. We assume that

$$\xi = \min \left[ 1, \max \left( 0, \frac{\langle \beta \rangle - \beta_0}{\beta_0} \right) \right],$$

where $\beta_0$ is a parameter, which determines the field strength, and $\langle \beta \rangle$ is the $\beta$ averaged over the volume of the injection layer.

### 3. NUMERICAL RESULTS

We initiate simulations from a static, nonmagnetic, very low density medium that fills the entire computational domain. Accretion flows are created by steadily injecting the mass into this medium at $R_{nj}$. These flows generally go through an initial transient phase before relaxing to a steady or quasi-steady state. The transient phase typically takes a few tens ($\approx 10–30$) of the free-fall time, $t_{ff} \approx 0.67 R_{ejg}^{3/2}/(GM)^{1/2}$, and includes the formation and dissipation of shocks and nonlinear waves. The hydrodynamic flows and MHD flows with purely radial and toroidal fields have finally relaxed to steady states. The MHD flows with the loop field have remained time variable, or quasi-steady, even after the completion of the transient phase.

IN02 discussed in detail the role of magnetic reconnections in spherical accretion flows with $\beta \sim 1$. This role depends on the amount of reconnection heat contributed to the gas internal energy. We have considered two extreme energetic regimes, conservative and nonconservative. In the conservative regime, the dissipated magnetic energy is fully transformed into heat. Models in this regime have been calculated by employing the total energy equation (3). In the nonconservative regime, the dissipated magnetic energy is totally lost and does not produce any heat. In this regime, we employ the internal energy equation (2) in which the dissipation term $Q$ has been set to zero. Note that the term describing the artificial heat from shocks has been retained in equation (2).

The study of these conservative/nonconservative accretion regimes can be astrophysically motivated. The conservative regime can correspond to very high accretion rate flows with $\dot{M} \gg \dot{M}_{edd}$, which do not radiate much energy due to the large optical depth of the flows (e.g., Katz 1977). Here $\dot{M}_{edd} = 4\pi GM/\sigma_T m_p c$ is the Eddington accretion rate, $\sigma_T$ is the Thomson scattering cross section, and $m_p$ is the proton mass. Another option for the conservative regime is very low accretion rate flows, $\dot{M} \ll \dot{M}_{edd}$, if one assumes that the energy released in magnetic reconnections is primarily released to the ions, which cannot lose this energy efficiently via the usually considered electron-ion Coulomb collisions because of the tenuous plasma (e.g., Esin et al. 1996). There is another possibility, however, that a significant fraction of the reconnection energy will go to electrons, which can efficiently radiate this energy by a variety of mechanisms (Bisnovatyi-Kogan & Lovelace 1997, 2000; Quataert & Gruzinov 1999). The latter possibility provides an additional cooling mechanism for ions, which in some circumstances could be more efficient than the Coulomb collisions, making the low, or even very low, accretion rate flows nonconservative, or radiatively efficient.

We describe models that are either subsonic or supersonic and differ by the strength of the injected fields, amount of heat released in reconnections, and field configuration. The model parameters are listed in Table 1. For convenience, we have divided all these models into three groups: Bondi-type flows (models A1–A4), subsonic MHD flows (models B1 and B2), and supersonic MHD flows (models C1 and C2). The structure of the Bondi-type flows, i.e., flows without magnetic fields or with the fields of special topology, is similar to the structure of the Bondi (1952) solution and well approximated by nonmagnetic analytic solutions discussed in the Appendix. The structure of the latter two groups of models is significantly modified because of magnetic fields. We describe these groups separately.

#### 3.1. Bondi-Type Flows

Bondi-type flows (models A1–A4; see Table 1) have been calculated to verify our numerical method and obtain reference models. These flows are steady and laminar. Models A1–A3 represent subsonic flows, and model A4 represents a supersonic flow. Models A1 and A4 have no magnetic field and thus describe hydrodynamic flows. Model A2 has a purely toroidal but...
sufficiently subequipartition ($\beta > 1$) magnetic field, and model A3 has a uniformly ordered strong ($\beta \leq 1$) radial magnetic field. In the latter two cases, the magnetic field does not affect the structure and dynamics of the flows, and these cases are almost equivalent to model A1.

We describe the evolution and structure of subsonic Bondi-type flows using model A1 as a representative example. This model has evolved from the initial state (see §2) to a steady state through a transient phase. This phase includes the formation of temporal shocks and waves, which many times propagate inward and outward in the radial direction, reflecting from the inner and outer boundaries, before complete disappearance after $\tau \simeq 20\tau_0 - 30\tau_0$. The final flow in model A1 is steady, effectively one-dimensional, and described by an analytic solution (eqs. [A4] and [A8]). This solution suggests that model A1 represents a part of the flow located deeply inside the Bondi radius, $R_{\text{out}} = 0.0371 R_0$, and, therefore, this model can be closely approximated by an asymptotic Bondi solution (A6), which appears in the limit $R \ll R_0$. Figure 3 shows selected properties of model A1 (short-dashed lines) as functions of the radius. These properties include the distribution of the density $\rho$ (top left); gas pressure $P_g$ (middle left); relative temperature $T/T_{\infty} = \epsilon/(\text{GM}/R)$ (bottom left); Mach number $M = v/c_s$ (top right), where $c_s = (\gamma P/\rho)^{1/2}$ is the sound speed; relative radial velocity $v/\theta_R$ (middle right); and relative accretion rate (bottom right). In the latter case, the accretion rate is normalized to the mass injection rate (see §2). Because the flow is steady, the accretion rate is independent of the radius. Note that about 90% of the injected mass forms inflow. The other 10% escapes through $R_{\text{out}}$ because of a thermal spread.

For comparison, Figure 3 also shows a self-similar solution (A9) in which $M = 1$ (long-dashed lines). This solution is “boundary free” and has asymptotic Bondi profiles $\rho \propto R^{-3/2}$ and $P_g \propto R^{-5/2}$ throughout. Note that model A1 demonstrates slightly flatter density and pressure profiles, which can be explained by the deviation of $R/R_0$ from the zero limit.

Model A2 has the weak and dynamically unimportant toroidal magnetic field $\beta \gg 1$. In all other aspects, models A2 and A1 are similar. Using model A2, we have tested the ability of our numerical method to accurately model the passive transport of magnetic field. Magnetic flux conservation requires that the toroidal field near the equatorial part of a spherical flow be changed as

$$B_0 \propto (R/\theta_0)^{-1}.$$  \hfill (12)

This leads to $B_0 \propto R^{-1/2}$ and $P_{\text{m}} \propto R^{-1}$ for the self-similar velocity profile $v \propto R^{-1/2}$. Because model A2 has the steeper velocity profile, however, the actual profile of $P_{\text{m}}$ obtained in this model is accordingly flatter, and its slope is fully consistent with estimate (12).

Model A3 has a uniform radial magnetic field. This field is changed with the radius as $B_0 \propto R^{-2}$, and therefore $P_{\text{m}} \propto R^{-4}$. The strength of the field has been chosen such that it has $\beta = 30$ at $R_{\text{out}}$ and $\beta \approx 0.1$ at $R_0$. The simulations have shown no effects from the flow transition from the subequipartition to super-equipartition field regions. We see below, however, that such a transition causes significant changes in the flows with the loop field.

Note the artificial nature of the field topology assumed in model A3 that represents a small sector of the spherically symmetric monopole field. Such a field cannot have magnetic reconnections, which actually play an important role in realistic MHD flows. Accretion flows in a strong unipolar magnetic field similar to that considered in model A3, however, can occur at the magnetic poles of neutron stars and white dwarfs.

Properties of the supersonic hydrodynamic model A4 are shown in Figure 4 with short-dashed lines. The density and pressure profiles in this model demonstrate nonmonotonic behavior at the outer region and approach the asymptotic Bondi power laws $\rho \propto R^{-3/2}$ and $P_g \propto R^{-5/2}$ at the inner region (see Fig. 4, top left and middle left). The Mach number and accretion velocity (see Fig. 4, top right and middle right) show a significant increase inward from the outer boundary. All these properties of model A4 are in good agreement with analytic solution (A11), which fits model A4, assuming $R_{\text{out}}/\theta_0 = 0.071$ (see the Appendix).

3.2. Subsonic MHD Flows

Subsonic MHD flows are represented by models B1 and B2 (see Table 1) and have a dynamically important magnetic field $\beta \sim 1-10$. This field is injected into the computational domain in the form of magnetic loops stretched in the radial direction (see Fig. 2). The strength of the field is determined by the parameter $\beta_0$ (see Table 1 and eq. [11]). The total energy is conserved in model B1, whereas the energy released in magnetic reconnections is completely lost in model B2.

Simulations of models B1 and B2 have been initiated from the steady hydrodynamic model A1 by gradually increasing the strength of the injected magnetic field to provide a smooth transition from nonmagnetic to magnetic flows. Even in this case, however, models B1 and B2 have settled into their final quasi-steady states after passing through the initial transient phases. The final quasi-steady states are turbulent and characterized by random variations of all quantities on all spatial scales from the grid size to the size of the computational domain. The amplitude of these variations, however, is not large, and the time-averaged properties of the accretion flows remain unchanged.

Fig. 3.—Radial structure of the subsonic accretion flows in models A1 (short-dashed lines), B1 (solid lines), and B2 (dotted lines) and the self-similar solution (long-dashed lines; see eq. (A9)). Hydrodynamic model A1 is steady; turbulent MHD models B1 and B2 are shown in quasi-steady states. All plotted quantities in the turbulent models—the density $\rho$, gas and magnetic pressures $P_g$ and $P_m$, temperature $T$, magnetic Mach number $M_m$, accretion velocity $v$, and mass accretion rate $\dot{M}$—have been averaged over the angles $\theta$ and $\phi$ and over the time interval $\tau \simeq 3\tau_0$.  

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as discussed earlier, magnetic reconnections are very efficient in on the large timescale. The turbulence in models B1 and B2 is clearly due to the effects of the magnetic field.

Figures 5–9 illustrate the structure of conservative model B1, showing the snapshots of two-dimensional distributions of selected quantities in the equatorial cross section of the three-dimensional computational domain. Figure 5 represents the density distribution, which shows clearly recognizable small-scale (with respect to the scale $R$) density variations of the relative amplitude $\sim 2$. The lower and higher density regions typically take the form of filaments, which are predominantly extended in the radial direction. These radially extended structures can also be seen in the velocity streamlines in Figure 6. The streamlines form a complicated flow pattern consisting of radially extended narrow inflowing/outflowing streams and small-scale vortices.

The time-dependent flow pattern in model B1 demonstrates the randomly repeating events of the interchange instability. In these events, colder, denser matter accumulates above (i.e., far from the center) the region with hotter, lower density matter. With time, this denser matter begins to move down through the low-density region, forming a characteristic radially inflowing dense stream. Such a stream typically forms a “mushroom” at its head and propagates about half a radius inward from the radius of origin. At the same time, the stream carries a frozen-in magnetic loop whose field strength is amplified because of a radial convergence of the stream. The example snapshot of magnetic lines in Figure 7 shows many such magnetic loops resulting from the interchange instability. During the subsequent evolution, the inflowing radial streams are fragmented into small pieces. This fragmentation is typically triggered by reconnections of the oppositely directed magnetic lines confined in the streams. The reconnections locally release energy and produce hot low-density matter. This matter has a positive buoyancy and forms narrow low-density outflows.

Figure 8 shows a snapshot of the plasma $\beta$. The spatial distribution of $\beta$ and, accordingly, the magnetic field is highly nonuniform. The small-scale, high-$\beta$ regions, $\beta \approx 100$, cover the entire computational domain and correspond to the weak field regions, which are typically associated with reconnection regions. The large number of these regions clearly indicates the high efficiency of the reconnection dissipation. The low-$\beta$ regions, $\beta \ll 1$, or, accordingly, the strong field regions are typically elongated in the radial direction and associated with the inflowing streams. The field in such regions becomes especially strong, $\beta \ll 1$, in the innermost part of the flow, $R \approx 2R_m$, where the high-velocity inflowing streams dominate in the flow pattern.

The chaotic inward and outward turbulent motions in model B1 are only partially supported by magnetic interactions, whereas the more important support is provided by thermal convection.

As discussed earlier, magnetic reconnections are very efficient in...
the model that we considered and produce hot, low-density, narrow outflows. Sometimes several such outflows can coalesce and form a large convective bubble, which moves outward easier and faster. A representative example of such a bubble can be seen in Figure 5 as the low-density region near the outer boundary and, in Figure 9, as the corresponding increase of the specific entropy. Later in time, this particular bubble has escaped through $R_{\text{out}}$. Figure 9 also shows many other local regions of high specific entropy. These regions typically coincide with the lower density regions in Figure 5, have positive velocities, and can therefore be identified as convective bubbles. However, not all such bubbles will be able to escape through $R_{\text{out}}$. A significant part of these bubbles will be pulled inward, mixed with the inflowing cold matter, and absorbed at $R_{\text{in}}$.

To make the argument concerning the development of convection more quantitative, we have calculated a one-dimensional time- and angle-averaged distribution of the specific entropy in model B1 (see the description of the averaged procedure below). This distribution has a negative slope and therefore satisfies the Schwarzschild criterion for convection.

The nonconservative model B2 is similar in many respects to the conservative model B1. Model B2 also forms a turbulent quasi-steady flow pattern. Like in model B1, the developed nonuniformities in the density frequently take the form of narrow filaments, which are extended in the radial direction. However, the amplitude and intensity of turbulent fluctuations in model B2 are significantly reduced in comparison with model B1. As the result, the rate of reconnection dissipation is smaller in model B2 and the average magnetic field is stronger. The turbulent motions in model B2 involve different MHD effects and are mainly supported by magnetic reconnections, magnetic buoyancy, and interchange instability. Convection motions are not developed in this model because of the absence of reconnection heat. The latter explains the less efficient turbulence observed in this model.

Our MHD models are time variable, and therefore it is more practical to study the structure of these models by performing time and space averaging. In this way, we have constructed one-dimensional distributions of different quantities, averaging them over the $\theta$- and $\phi$-directions. We have also averaged the obtained spatially averaged distributions over time, assuming the time-averaged interval $\tau \simeq 34t_0$. Figure 3 shows the averaged radial profiles of selected quantities in models B1 and B2 (solid and dotted lines, respectively). A comparison of these profiles shows certain differences in all quantities except in the accretion rates (see Fig. 3, lower right). These rates are close to each other and almost independent of the radius. The latter confirms that the flows that have been considered are in quasi-steady states.

Model B2 shows a little steeper average profiles for $\rho$ and $P_0$ and a flatter profile of $\epsilon$ in the inner and middle radial ranges, $R \lesssim 5R_{\text{in}}$, in comparison with model B1. The profiles of $P_m$ almost the same slopes in both models, but the magnitude of $P_m$ is larger by a factor of $\approx 2$ in model B2 than in model B1. As discussed earlier, this difference in $P_m$ can be explained by the less efficient dissipation of magnetic field in model B2.

The presence and absence of reconnection heat resulted in different temperatures in models B1 and B2 (see Fig. 3, lower left). In agreement with naive expectations, the conservative model B1 has the larger temperature. At the same time, this model has a larger magnetic Mach number, $M_{\text{m}} = v/(c_A^2 + c_0^2)^{1/2}$, where $c_A = (B^2/(4\pi \rho))^{1/2}$ is the Alfvénic speed, and larger $\epsilon$ in the inner region (see Fig. 3, top right and middle right). The latter two facts cannot be simply understood using an analogy with Bondi-type flows in which lower $M$ and $\epsilon$ correspond to hotter flows. It seems that the stronger magnetic field is responsible for the relative reduction of these quantities in model B2.

The averaged radial structures of MHD models B1 and B2 show noticeable differences from the radial structures of hydrodynamic model A1 and self-similar solution (A9) (see Fig. 3). The MHD flows have flattened density profiles. In terms of the power-law distribution $\rho \propto R^{-\gamma}$, models B1 and B2 have $\gamma \approx 0.7$, whereas model A1 and solution (A9) have $\gamma \approx 1.3$ and 1.5, respectively (see Fig. 3, upper left). Similarly, models B1 and B2 have flattened profiles of $P_0 \propto R^{-\kappa}$ in which $\eta \approx 1.5$, whereas model A1 and solution (A9) have $\eta \approx 2.2$ and 2.5, respectively (see Fig. 3, middle left). Other differences between hydrodynamic and MHD flows include larger temperatures and reduced accretion velocities in the latter flows (see Fig. 3, lower left and middle right). The accretion rates in models B1 and B2 show an $\approx 10\%$ reduction with respect to model A1 (see Fig. 3, lower right). This is a clear indication that turbulent MHD accretion flows have reduced accretion rates.

3.3. Supersonic MHD Flows

Supersonic MHD flows are represented by models C1 and C2 (see Table 1). These models have been initiated from the supersonic hydrodynamic model A4 by injecting a loop magnetic field at $R_{\text{in}}$. Similar to the subsonic MHD models (see § 3.2), models C1 and C2 have approached quasi-steady states after passing through transient phases. However, the transient phases in these models show some differences: the temporal shocks and waves that developed are downshifted by the supersonic inflow and never reached the outer boundary $R_{\text{out}}$. In quasi-steady states, each of these models obtains a new feature—a quasi-steady shock. This shock divides the flows into two regions: the outer and inner, which have superfast- and subfast-magnetosonic
accretion velocities, respectively. In the outer region, the flows are radial and laminar. The magnetic field is dominated by the radial component; this quickly increases inward as $B \propto R^{-2}$. No significant field dissipation and reconnections have been observed in this region. In the inner region, on the other hand, reconnections become important and the flow is turbulent. The quasi-steady shocks in both models C1 and C2 can be classified as Alfvénic shocks; they change the orientation of magnetic lines but leave the field strength almost unchanged. Right after the completion of the transient phases, these shocks were located at $R \approx 2R_{in}$ in both models C1 and C2.

During the subsequent evolution, models C1 and C2 have been slowly evolved on timescales $\gg t_g$. As a result of this evolution, the inner regions in these models have been expanded, and, consequently, the Alfvénic shocks have slowly moved outward. The shock moves relatively faster in the case of the conservative model C1 and slower in the case of the nonconservative model C2. Specifically, during the evolution time $t \approx 4t_g$ counted from the end of the transient phases, the shock has propagated a distance of $\approx 6R_{in}$ in model C1 and $\approx R_{in}$ in model C2. It is worth noting that these slowly moving Alfvénic shocks do not have an analog in hydrodynamic accretion flows. In the latter flows, any shocks that have developed are nonstationary and move either inward or outward, depending on the assumed conditions, on a timescale of $\sim t_g$.\footnote{We do not consider radiative shocks here, which can be (quasi-) steady.}

Figure 10 illustrates the flow pattern in the conservative model C1, showing the velocity streamlines projected onto the equatorial plane. The Alfvénic shock is located at $R \approx 7R_{in}$ in the moment that is shown. Two flow regions can be clearly distinguished: the preshock laminar outer and postshock turbulent inner regions. Figure 11 shows magnetic lines that correspond to the flow pattern shown in Figure 10. In the preshock region, the magnetic lines are purely radial but oppositely directed in different sectors (see Fig. 2). These lines do not experience reconnections because of the purely radial flow pattern. In the postshock region, the magnetic lines are tangled because of the turbulent flow pattern and frequently reconnected.

The difference in Alfvénic shock motions in models C1 and C2 suggests that there are two different mechanisms that cause these motions. Model C1, which includes the reconnection heat, develops convection in the postshock region, similar to the convection in model B1. This convection transports the heat outward, causing a relatively fast expansion of the inner region and faster motion of the shock. This mechanism, however, is not suitable for model C2, which does not include the reconnection heat and does not develop convection. Instead, the shock motion in model C2 can be explained by magnetic buoyancy, which acts in the postshock region in a manner similar to the action of magnetic buoyancy in model B2. As a result, the magnetic field and energy are transported outward, forcing the inner region to expand and move the shock outward. The slower expansion of the postshock in model B2 is explained by the less efficient energy transport provided by magnetic buoyancy.

The radial distributions of selected quantities in models C1 and C2 are shown in Figure 4 by solid and dotted lines, respectively. These quantities have been averaged using the technique described in § 3.2 in which the time-averaged interval is chosen to be smaller, $\tau \approx 0.1t_g$, to avoid “washing out” the moving shocks were used to obtain these distributions. One can clearly see that the structure of models C1 and C2 changes quite sharply at the Alfvénic shocks, whose locations at the moments shown are $R \approx 7.5R_{in}$ and $\approx 3R_{in}$, respectively.

The outer regions in models C1 and C2 are almost identical to each other and similar to the outer part of model A4 (Fig. 4, dashed lines). This can be seen in the distributions of all quantities except for the magnetic Mach number $M_m$ (see Fig. 4, upper right). Models C1 and C2 include strong magnetic fields that significantly reduce $M_m$ (but still $M_m > 1$ before the shocks), whereas the accretion velocities remain almost the same as in hydrodynamic model A4 (see Fig. 4, middle right). Note the interesting behavior of $P_g$ and $P_m$ in these models (see Fig. 4, middle left). The values of $P_g$ before the shocks in the MHD models closely follow the corresponding value in model A4. The values of $P_m$, which are started from the subequipartition level at $t_{eq}$, $\beta = 10$, quickly exceed the equipartition level, increasing inward as $P_m \propto R^{-4}$. In the case of model C2, $\beta \approx 0.1$ just before the shock, and the energy of the flow is dominated by the kinetic and magnetic energies, which are of approximately equal magnitudes.

The inner turbulent regions in models C1 and C2 are subfast-magnetosonic, $M_m < 1$, and have Alfvénic or moderately super-Alfvénic accretion velocities, $v \approx c_A$. These regions are denser, hotter, and have lower accretion velocities than the corresponding part of model A4 (see Fig. 4). Magnetic reconnections are important here; they support turbulence and provide dissipation of the magnetic fields. This dissipation noticeably reduces the slope of $P_m$ with respect to the corresponding slope in the outer region of the flow where the dissipation is negligibly small. In the case of model C2, the power-law index $\eta$ for the magnetic pressure distribution $P_m \propto R^{-\eta}$ changes from $\eta = 4$ in the outer part to $\eta \approx 3$ in the inner part of the flow (see Fig. 4, middle left).

4. DISCUSSION AND CONCLUSIONS

We have numerically investigated quasi-steady MHD spherical accretion flows with imposed small-scale magnetic fields.
We have confirmed the previous theoretical expectation and numerical results such that the flows are turbulent and have radial structures different from the Bondi-type accretion flows (e.g., Shvartsman 1971; IN02). Turbulence in the MHD flows is developed and supported by the interchange instability, thermal convection, and various magnetic interactions, including magnetic reconnections and buoyancy. The highly nonuniform release of energy in reconnections, which is, in fact, the release of the gravitational energy converted and stored in the form of magnetic field, makes these flows so special and different from the laminar and stable Bondi-type flows (however, see Kovalenko & Eremin 1998). We have found that magnetic buoyancy, in addition to thermal convection, can play an important role in the modification of the flow structure, especially in the case of nonconservative (or radiatively efficient) flows in which convection could not develop.

The most important consequence of turbulence in our MHD models, both conservative and nonconservative, is the modification of the radial flow structure. Figure 3 demonstrates this modification. Turbulent subsonic MHD models B1 and B2 have flattened time-averaged density profiles, higher temperatures, and lower accretion velocities than their hydrodynamic counterpart model A1. These properties make models B1 and B2 more like spherical CDAFs (see IN02) than Bondi accretion flows. The theory of spherical CDAFs predicts a flattened power-law density profile $\rho \propto R^{-\sigma}$ in which $\sigma = 0.5$. Models B1 and B2 have profiles close to this but steeper, $\sigma \approx 0.7$. These steeper profiles can be explained by the influence of the inner boundary condition in our numerical models, whereas the analytic CDAF solution is boundary free. Abramowicz et al. (2002) argued that the proximity of the black hole absorbing boundary makes the inner regions of CDAFs advection dominated. In particular, they demonstrated that viscous rotating CDAFs become advection dominated inside $\sim 50$–100 gravitational radii. Our numerical models have a rather small radial range, $R_{\text{out}}/R_\infty = 10$, and, therefore, the effects of the inner boundary can be significant.

Turbulent models B1 and B2 have shown a reduction of the accretion rates in comparison with laminar model A1 (see Fig. 3, lower right). This result qualitatively confirms the predicted reduction of the accretion rate in spherical CDAFs (see eq. [1]). However, the actual reduction of the accretion rates, which is about 10%, shows a poor quantitative agreement with estimate (1). We explain this poor agreement by the limited radial range in our models, whereas estimate (1) was obtained in the limit $R_{\text{out}} \gg R_\infty$.

The conservative and nonconservative subsonic MHD models (models B1 and B2, respectively) are turbulent; however, the turbulence properties in these models are different. In the conservative model, the turbulent motions are more intensive and mainly supported by thermal convection, which makes this model similar to spherical CDAFs. The nonconservative model has less intensive turbulence, which is supported through various magnetic interactions in which magnetic buoyancy seems to be dominant. In some respects, the magnetic buoyancy acts similar to the thermal convection; it also transports (magnetic) energy outward. This transport can explain the flattened density profile in the nonconservative model, like the convection in spherical CDAFs (see IN02). However, the energy transport provided by magnetic buoyancy is less efficient and much weaker than the convection transport.

Our supersonic MHD accretion flows (models C1 and C2) have been initiated by injecting low-entropy matter into the computational domain. These flows form quasi-steady Alfvénic shocks that separate the laminar superfast-magnetosonic outer inflows and the postshock turbulent nearly Alfvénic inner inflows. We have found that the averaged flow pattern in these models is not steady on large timescales $\gg t_\text{ff}$. The postshock regions gradually expand, forcing the Alfvénic shocks to move outward. These regions are quite similar to the subsonic MHD flows in models B1 and B2, and, therefore, the gradual expansion of these regions can be explained by the outward energy flux provided by convection and/or magnetic buoyancy. We have found that the Alfvénic shock moves significantly faster in the case of model C1, which develops convection in the postshock region. The latter is consistent with our observation that the more intensive turbulence and the larger outward energy flux happens in convective model B1, rather than in nonconvective model B2.

Based on our study of the supersonic MHD models, we conclude that stationary supersonic (or superfast-magnetosonic) accretion flows cannot form in the presence of small-scale magnetic fields. This conclusion applies equally to radiatively efficient and inefficient flows. Such supersonic flows will unavoidably create shocks at the equipartition radius, and these shocks will move outward because of the action of convection and magnetic buoyancy, which develop in turbulent postshock regions of these flows. These postshock regions will continue to expand and, on large timescales, fill the entire accretion domain, causing the flows to be subsonic everywhere.

The wedge computational domain of our models has a limited opening angle (see Fig. 1), and we use specific boundary conditions in the angular directions (see § 2.1) to minimize the influence of this limited-size domain. The boundary conditions used could definitely have some effects on the properties of the simulated MHD turbulence, for example, limiting the spatial scales and affecting the motions in the vicinity of the polar sliding boundaries. Another consequence of using this domain is the inability to simulate large-scale magnetic structures, which can develop during the reverse cascade of energy from small to large spatial scales in MHD turbulence. This issue of the limited computational domain should be addressed in future work.
The accretion of mass in our MHD models is accompanied by the reconnection dissipation of the magnetic field in turbulent flows. The rate of dissipation is consistently regulated through feedback mechanisms that also regulate the intensity of the turbulence. To illustrate the dissipation process quantitatively, we consider nonconservative MHD models in which the change of turbulence. To illustrate the dissipation process quantitatively, we consider nonconservative MHD models in which the change of the time-averaged radial flux of the total energy

$$F_R = \int_{\Omega_0} R^2 q_R d\Omega$$  \hspace{1cm} (13)

directly corresponds to the amount of dissipated magnetic energy. The integral in equation (13) is taken over the solid angle of the computational domain $\Omega_0$, and the radial flux per unit square (see eq. [3]) is

$$q_R = \rho v_R \left( \frac{v^2}{2} + \epsilon + \frac{P}{\rho} + \frac{B^2}{4\pi} - \frac{GM}{R} \right) - \frac{B_R}{4\pi} (\mathbf{v} \cdot \mathbf{B})$$  \hspace{1cm} (14)

The solid lines in Figure 12 represent the radial dependence of the total energy fluxes obtained in models B2 and C2. These fluxes have been normalized to the flux $GMm/Rm$. The dashed lines in Figure 12 correspond to the flux conservation $F_R = \text{const}$. The difference between the dashed and solid lines at a given $R$ represents the amount of the energy dissipated in the radial range from $R_{\text{out}}$ to $R$. Note that the energy dissipates in the whole volume in model B2, whereas it dissipates only in the postshock region and no energy dissipates in the preshock region in model C2. The total amount of reconnection losses given in units of gravitational energy at $R_{\text{in}}$ is about 6.5% in model B2 and about 5% in model C2.

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APPENDIX

SOME ANALYTIC SOLUTIONS OF SPHERICAL HYDRODYNAMIC ACCRETION FLOWS

In this paper, we have compared our numerical models and analytic solutions of spherical hydrodynamic accretion flows. We reproduce some relevant formulae for these solutions below.

Spherical, stationary, and isentropic hydrodynamic accretion flows are described by the continuity equation

$$\dot{M} = 4\pi R^2 \rho v,$$  \hspace{1cm} (A1)

Bernoulli’s equation

$$\frac{v^2}{2} + \frac{\gamma P}{\rho - 1} - \frac{GM}{R} = \text{constant},$$  \hspace{1cm} (A2)

and the polytropic equation of state

$$P = K \rho^\gamma,$$  \hspace{1cm} (A3)

where $P$ is the gas pressure, $K$ is the polytropic constant, $\gamma$ is the polytropic index, and $\dot{M}$ is the accretion rate. Bondi (1952) solved these equations assuming that the flow is at rest and of uniform density $\rho_{\infty}$, pressure $P_{\infty}$, and sound speed $c_{\infty} = (\gamma P_{\infty}/\rho_{\infty})^{1/2}$ at infinity. The Bondi solution depends on the spatial scale $R_B = 2GM/\gamma c_{\infty}^2$, called the Bondi or accretion radius, and can be expressed in the following implicit form that gives the radial dependence of $\dot{\lambda}$:

$$\dot{\lambda}^2 \left( \frac{R_B}{R} \right)^4 \left( \frac{\rho_{\infty}}{\rho} \right)^2 + \frac{2}{\gamma - 1} \left[ \left( \frac{\rho}{\rho_{\infty}} \right)^{\gamma - 1} - 1 \right] - \frac{R_B}{R} = 0,$$  \hspace{1cm} (A4)

where $\dot{\lambda} = \dot{M}/(4\pi R_B^2 \rho_{\infty} c_{\infty})$ is the dimensionless accretion rate defined by the expression

$$\dot{\lambda} = \frac{1}{2} \left( \frac{5\gamma - 3}{2(\gamma - 1)} \right) \left( \frac{5}{4} \right)^{(3\gamma - 5)/(2(\gamma - 1))}.$$  \hspace{1cm} (A5)

In the small-radius limit, $R \ll R_B$, the Bondi solution (A4) is represented by the asymptotic power-law distributions

$$v \propto R^{-1/2},$$
$$\rho \propto R^{-3/2},$$
$$P \propto R^{-5/2}.$$

The integral in equation (13) is taken over the solid angle $\Omega_0$, and the radial flux per unit square (see eq. [3]) is
In the case of $1 \leq \gamma < 5/3$, the Bondi solution (A4) describes transonic flows in which the Mach number

$$\mathcal{M} = \lambda \left( \frac{R_B}{R} \right)^2 \left( \frac{\rho_{\infty}}{\rho} \right)^{\gamma/2}$$  \hspace{1cm} (A7)$$

is zero at infinity and monotonically increases inward, approaching the infinite value at $R = 0$.

In the subsequent discussion, we consider only the case $\gamma = 5/3$, which is known to be a special case of the Bondi solution (A4). In this case, the flow is subsonic everywhere and $\mathcal{M} = 1$ only in the limit of $R \to 0$. However, such a boundary-free solution is not practical for numerical applications, and we have modified the Bondi solution to include the finite inner boundary radius $R_{\text{in}} > 0$ in which we require $\mathcal{M} = 1$. The modified Bondi solution is described by the same equation (A4), and the same equation (A7) represents the Mach number, but a new $\lambda$ is defined as follows:

$$\lambda = \left( 1 + \frac{3 \delta}{4} \right)^{2}$$  \hspace{1cm} (A8)$$

where $\delta = R_{\text{in}}/R_B$ is a free parameter; $0 < \delta < 1$. This modified solution approximates our subsonic model A1 well, assuming $\delta = 0.00371$ (Fig. 3, short-dashed lines).

In the case of $\gamma = 5/3$, equations (A1)–(A3) allow a self-similar solution, which requires the special outer boundary condition $\rho_{\infty} = 0$ or, equivalently, that the constant on the right-hand side of equation (A2) be set to zero. This solution reads

$$v = \alpha \sqrt{\frac{GM}{R}},$$

$$\rho = \frac{\mathcal{M}}{4\pi \alpha \sqrt{GM R^{3/2}}},$$

$$P = \frac{2}{5} \left( 1 - \frac{\alpha^2}{2} \right) GM \rho / R,$$  \hspace{1cm} (A9)$$

where $\alpha$ is a free parameter; $0 < \alpha < \sqrt{2}$. Solution (A9) can be subsonic or supersonic depending on $\alpha$. This solution is characterized by the constant Mach number

$$\mathcal{M} = \alpha \sqrt{\frac{1.5}{1 - \alpha^2/2}},$$  \hspace{1cm} (A10)$$

and it is supersonic at $1/\sqrt{2} < \alpha < \sqrt{2}$.

Another kind of supersonic solution in the special case of $\gamma = 5/3$ can be constructed assuming the outer boundary at some finite radius $R_{\text{out}}$ in which we require $\mathcal{M} = 1$. This solution takes the form

$$\lambda^2 \left( \frac{R_B}{R} \right)^4 \left( \frac{\rho_{\text{out}}}{\rho} \right)^2 + 3 \left[ \left( \frac{\rho}{\rho_{\text{out}}} \right)^{2/3} - 1 \right] - \frac{R_B}{R} + \frac{1}{\delta} - 1 = 0,$$  \hspace{1cm} (A11)$$

where $\tilde{R}_B = 2GM/c_{\text{out}}^2$ and $\delta = R_{\text{out}}/\tilde{R}_B$, and we denote $\rho_{\text{out}}$ and $c_{\text{out}}$ to be the density and sound speed at $R_{\text{out}}$, respectively. Solution (A11) exists for $0 < \delta \leq \frac{1}{3}$. The dimensionless accretion rate $\tilde{\lambda} = \mathcal{M}^3 (4\pi \tilde{R}_B^2 \rho_{\text{out}} c_{\text{out}})$ can be expressed in the form

$$\tilde{\lambda} = \tilde{\delta}^2.$$  \hspace{1cm} (A12)$$

The Mach number in solution (A11)

$$\mathcal{M} = \tilde{\lambda} \left( \frac{R_B}{R} \right)^{2} \left( \frac{\rho_{\text{out}}}{\rho} \right)^{4/3}$$  \hspace{1cm} (A13)$$

monotonically increases inward and approaches the asymptotic value $\mathcal{M}_\infty$ at $R \ll R_{\text{out}}$. In the limit $\delta \ll 1$, one gets $\mathcal{M}_\infty = \tilde{\delta}^{-2/3}$. In the case of marginal $\delta = \frac{1}{3}$, one gets $\mathcal{M} = \mathcal{M}_\infty = 1$ and solution (A11) becomes equivalent to the self-similar solution (A9) in which $\alpha = 1/\sqrt{2}$. Solution (A11) approximates our supersonic model A4 well, assuming $\delta = 0.071$ (Fig. 4, short-dashed lines).

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