Unsupervised Anomaly Detection on Temporal Multiway Data

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Abstract—Temporal anomaly detection looks for irregularities over space-time. Unsupervised temporal models employed thus far typically work on sequences of feature vectors, and much less on temporal multiway data. We focus our investigation on two-way data, in which a data matrix is observed at each time step. Leveraging recent advances in matrix-native recurrent neural networks, we investigated strategies for data arrangement and unsupervised training for temporal multiway anomaly detection. These include compressing-decompressing, encoding-predicting, and temporal data differencing. We conducted a comprehensive suite of experiments to evaluate model behaviors under various settings on synthetic data, moving digits, and ECG recordings. We found interesting phenomena not previously reported. These include the capacity of the compact matrix LSTM to compress noisy data near perfectly, making the strategy of compressing-decompressing data ill-suited for anomaly detection under the noise. Also, long sequence of vectors can be addressed directly by matrix models that allow very long context and multiple step prediction. Overall, the encoding-predicting strategy works very well for the matrix LSTMs in the conducted experiments, thanks to its compactness and better fit to the data dynamics.

I. INTRODUCTION

Unsupervised detection of unusual temporal signals that deviate from the norms is vital for intelligent agents. In the most common setting, data is represented as a sequence of 1D feature vectors, thus lacking expressiveness over the data sequences whose elements are multiway (2D or more). For examples, electroencephalography (EEG) spectrums are sequences of channel-frequency matrices; and a video can be represented as a sequence of clips, each of which can be summarised using a covariance matrix (e.g., see [6]). Classical methods require flattening these matrices into vectors, thus breaking the internal structure of the data. This results in demand for more data, bigger models and higher computational burden to compensate for the loss. But a bigger model would be ineffective in unsupervised anomaly detection because irregularity can sneak in as a form of overfitting, thus reducing the discriminative power. A better way is to seek a compact model that is native to multiway data.

We investigate the use of matrix recurrent neural networks (RNNs) for unsupervised anomaly detection for temporal two-way data [3], as inspired by the recent successes of RNNs on temporal one-way domains [4], [9], [19]. In particular, Matrix Long Short-Term Memory (matLSTM) is chosen for its high memory capacity, compactness, and ease of training. matLSTM maintains a dynamic memory matrix of past seen data, i.e., it compresses the variable-size history tensor into a fixed-size matrix. This suggests we may use the compression loss as a measure of abnormality. More concretely, if a data sequence is compressible by the learnt model, we can reconstruct the sequence with little loss. Further, as matLSTM is also able to capture long-term dependencies between distant input matrices, we can predict the future data after seeing a sufficiently long history. This suggests prediction loss can be used as a measure of abnormality.

Our investigation reveals interesting phenomena not previously reported, to the best of our knowledge. While using reconstruction loss is popular in autoencoder-based methods, the presence of the high-capacity memory in compact matLSTM enables the model to memorise the data noise in the test sequences. This is different from the usual problem of overfitting in large models, where the noise is memorised for the training data only. Thus a small reconstruction loss implies neither better model fit nor normality under noisy conditions. The predictive model, fortunately, does not suffer from this drawback.

Second, with prediction-based anomaly detection, we can operate on sequences of vectors that are much longer than before. A standard vector LSTM would have a hard time learning from \( T \geq 100 \) steps in the past and predicting \( N \geq 10 \) steps ahead. This is because the long history makes gradient flow much more difficult due to the nonlinearity of the RNN system, and the far future would quickly accumulate prediction errors to the point that normality cannot be judged upon. But this is not an issue for matLSTM as we can stack \( N \) vectors into a matrix, thereby operating on a history of \( \left\lfloor \frac{T}{N} \right\rfloor \) length and one step prediction, while still capturing the temporal relationship between the vectors through matrix operations.

The main contributions of this work can be summarised as follows:

- We investigate the applications of matrix recurrent neural networks for unsupervised anomaly detection for temporal multiway data.
- Two anomaly detection settings (reconstruction and prediction) are examined, and the empirical results on synthetic data, moving digits and electrocardiogram (ECG) readings are reported.

II. RELATED WORK

Anomaly detection (AD) in sequential data has been studied widely in the literature, with learning methods ranging from supervised, semi-supervised to unsupervised [1]. In real-world settings where data has no label, supervised and semi-supervised methods are thus inapplicable. Therefore, one has to
resort to unsupervised approaches, which assume the majority of instances are normal and a set of descriptors are learnt to represent their distribution. The instances which are under-represented by those descriptors are deemed anomalous. Conventionally, shallow anomaly detectors such as the One-Class SVM (OC-SVM) or Support Vector Data Description (SVDD) are used [2]. However, these methods require substantial efforts for manual feature engineering, making them inefficient to work on high-dimensional data. In contrast, deep learning approaches, such as deep autoencoders (AE) [8] and their variants can extract compact features from high-dimensional inputs and leverage those for AD.

Coupling AE with sequential models such as RNNs is one of the common strategies to detect anomalies in sequential data [13, 14, 15]. In [13], the authors utilised the LSTM encoder-decoder framework to compress and decompress the input sequence. Since the model is exposed to normal samples during training, it will yield high reconstruction error upon observing anomalous samples. Other than reconstruction-based methods, prediction-based methods are also proposed for this task. In [14], an LSTM decoder is tasked to predict a segment of audio signals given the previous ones. While normal signals are predictable, the novel ones are not well-predicted. Although two strategies work in certain cases, the model can be forced to memorise the input or favour information from recent observations over older ones. In [15], a composite model was adopted to alleviate this behaviour. The reconstruction branch aims to reconstruct the first half of the sequence, while the prediction branch aims to predict the future observations. Hence, the learnt representation by the encoder must store information for both goals, which regularise the model and improve its generalisation ability.

Similar to the above approaches, our work falls into the unsupervised and self-supervised learning branches where we use the encoder-decoder/reconstruction [10, 13] and sequence-to-sequence/prediction architectures [14]. However, while their methods operate on (sequence of) vector inputs, we target matrix structured data that are permutation-invariant [3, 5]. This is different from outlier detection in images [17] and videos [15, 16] where the translation-invariant assumption in these data no longer holds for permutation-invariant data.

Closely related to our work is [3], in which matrix-structured LSTM was proposed, but for anomaly detection. Matrix-structured latent variable model has been studied in [18], where a matrix normal distribution was used for both the prior and posterior in the latent space. However, although the latent is a matrix, this work used the encoder and decoder that required vectorising its input and output. It was shown that vectorised inputs and outputs will break the permutation invariant properties [7]. In contrast, our models maintain the matrix structure in the hidden states as well as using matrix network for the mappings, thus is more compact.

III. METHODS

Temporal two-way data is a sequence \( X_{1:T} = (X_1, X_2, \ldots, X_T) \) where the input at each step \( X_t \in \mathbb{R}^{n_r \times n_c} \) is a matrix representing data observed at time \( t \). Without loss of generality, let \( n_r \geq n_c \). We focus on the setting where the matrix rows (or columns) can be permuted without changing the matrix’s key characteristics. This makes the setting applicable to a wider range of application scenarios.

Standard recurrent neural networks (RNN, LSTM and GRU) assume vectorised data, and thus have space complexity of \( \mathcal{O}(n_r n_c k) \), where \( k \) is the size of the hidden layer. For a typical model with rich representation, \( k \) is in the order of \( n_r n_c \), i.e., resulting in the space complexity of \( \mathcal{O}(n_r^2 n_c^2) \). The model size explodes for large \( n_r \) and \( n_c \). Hence, a more compact modelling is needed.

A. Matrix LSTM

Matrix LSTM (matLSTM) [3] is an extension of the LSTM designed to effectively deal with sequences of matrices. Like LSTM, the matLSTM maintains a short-term memory that summarises the historical data. However, unlike LSTM, the matLSTM uses matrices to natively represent input \( X_t \), neural activations in hidden state \( H_t \) and working memory \( C_t \). matLSTM compresses the data tensor \( X_{1:t} \) into \( C_t \). It is also highly compact as the number of parameters typically scales linearly with \( n_r^2 \).

Let us define the following operation:

\[
\text{mat}(X, H; \theta) = U_{xh}^T X V_{zh} + U_{hh}^T H V_{hh} + B
\]

where \( U_{xh} \in \mathbb{R}^{k_r \times n_r}, V_{zh} \in \mathbb{R}^{n_r \times k_c}, U_{hh} \in \mathbb{R}^{k_c \times k_c}, V_{hh} \in \mathbb{R}^{k_c \times k_c}, B \in \mathbb{R}^{k_c \times k_c} \) are free parameters. Upon seeing a new input, the memory is refreshed using

\[
C_t = F_t \odot C_{t-1} + I_t \odot \hat{C}_t
\]

where \( \hat{C}_t = \text{tanh}(\text{mat}(X_t, H_{t-1}; \theta_t)) \), and \( I_t, F_t \in (0, 1) \) are input and forget gates. The state is estimated using \( H_t = O_t \odot \hat{C}_t \), where \( O_t \in (0, 1) \) is output gate. The gates are computed as \( I_t = \sigma(\text{mat}(X_t, H_t; \theta_i)) \), \( F_t = \sigma(\text{mat}(X_t, H_t; \theta_f)) \) and \( O_t = \sigma(\text{mat}(X_t, H_t; \theta_o)) \), respectively, for \( \sigma(\cdot) \) is element-wise logistic function.

Prediction at time \( t \) takes \( H_t \) as input and compute a matrix feedforward net:

\[
\hat{Y}_t = \text{matnet}(H_t)
\]

whose basic fully connected layers assume the form: \( Z_{l+1} = f(U^T Z_{l} V + B^T) \) for element-wise nonlinear transformation \( f \).

Training proceeds by minimising a loss function. For example, for continuous outputs, we may use the quadratic loss:

\[
L_{\text{mse}} = \frac{1}{2} \sum_{t=1}^T \| Y_{t} - \hat{Y}_{t} \|_F^2,
\]

For binary outputs, the cross-entropy loss is applied:

\[
L_{\text{cross}} = -\frac{1}{T} \sum_{t=1}^T \left[ Y_{t} \odot \log(\hat{Y}_{t}) + (1 - Y_{t}) \odot \log \left(1 - \sigma(\hat{Y}_{t})\right)\right]
\]
B. Lossy sequence compression

As the memory matrix $C_T$ at time step $T$ of the matLSTM is a lossy compression of the data $X_{1:T}$, we can use the reconstruction loss as a measure how regular the sequence is, similar to the case of vector sequence \cite{13}. That is because an abnormal sequence does not exhibit the regularities, it is hardly compressible, and thus its reconstruction error is expected to be higher than the error in the normal cases.

![Figure 1: (Matrix) LSTM AutoEncoder model](image1)

The model thus has two components: an encoder matLSTM which compresses $X_{1:T}$ into $C_{T_{enc}}$ (and $H_{T_{enc}}$) by reading one matrix at a time; and a decoder matLSTM decompresses the memory into $\hat{X}_{T+1:2T}$ by predicting one matrix at a time. The decoding takes the initial state $H_{T_{enc}}$ and proceeds backward, starting from the last element, until the first. At each step $t = T + 1, T + 2, ..., 2T$, it predicts an output matrix $\hat{X}_t$. See Fig. 1 for a graphical illustration. We denote the two models employing this strategy as LSTM-AutoEncoder and matLSTM-AutoEncoder.

The anomaly score $e_X$ is computed as the mean reconstruction error $\frac{1}{T} \sum_{t=1}^{T} \| X_t - \hat{X}_{2T-t+1} \|_F^2$ for continuous data, and mean cross-entropy for binary data.

C. Predicting the unrolling of sequence

An alternative to the autoencoder method relies on the premise that if a sequence is regular (i.e., normal), the history may contain sufficient information to predict several steps ahead, as the temporal regularities unrolled over time. This is arguably based on a stronger assumption than the compress-decompress strategy in the autoencoder, because we cannot rely on the working memory $C_T$ of the current past only but also the statistical regularities embedded in model parameters.

![Figure 2: (Matrix) LSTM Encoder-Predictor predictive model](image2)

More formally, given a past sub-sequence $X_{1:T_e}$, we want to predict the future sub-sequence $X_{T_e+1:T}$ using $P \left( X_{T_e+1:T} \mid X_{1:T_e} \right)$. At time $t \leq T_e$, the encoder reads the past matrices into memory. At time $t > T_e$, the decoder predicts future matrices, one at a time. Fig. 2 illustrates the encoder-predictor architecture. We denote the two models employing this strategy as LSTM-Encoder-Predictor and matLSTM-Encoder-Predictor.

The anomaly score $e_X$ is computed as the mean prediction error $\frac{1}{T} \sum_{t=T_e+1}^{T} \| X_t - \hat{X}_t \|_F^2$ for continuous data, and mean cross-entropy for binary data.

D. Stacking LSTM layers

Raw data may be too noisy to provide informative outliers signals. This necessitates data denoising or abstraction. We propose to abstract data by using the lower LSTM states as input for the higher LSTM. That is, the stack of LSTMs is trained on a layerwise manner, starting from the bottom to the top. At each layer, we compute an outlier score, using methods in Sec. III-B and Sec. III-C. How to combine the scores across layers remains open. For simplicity, we use the score at the top layer.

E. Dynamics of changes

If $X_t$ is an image, one may argue that since the loss and anomaly score are based on pixel intensities, the models focus too much on the appearance of the digits, and less on its dynamics. Thus the detection of dynamic abnormality may be less effective as a result. To test whether it is the case, we also study the dynamics of changes, that is, instead of studying the original sequence $\{ X_t \mid 1 \leq t \leq T \}$, we study the differences between time step, that is, $\{ \Delta X_t = X_{t+1} - X_t \mid 1 \leq t < T \}$.

IV. Experiments and Results

We experimentally validate our proposed strategies of using matLSTM for unsupervised temporal two-way anomaly detection on three datasets: (1) synthetic sequences of binary matrices, (2) sequences of moving digits and (3) ECG recordings.

Experimental setup: In all experiments, we compare the performances among LSTM, matLSTM and 3D-CNN models. The 3D-CNN models can handle spatio-temporal input in which two of the dimensions are for space and the remaining dimension is for time. We adopt two variations of the 3D-CNN to match the LSTM counterparts. The first, named 3D-ConvAE, follows compress-decompress strategy is the second, named 3D-Conv-Predictor, encodes the past observations and predict future observations. For predictive models, we empirically use the first half of the input sequence as context to predict the second half. For LSTM models, which require vector inputs, we flatten the matrices into vectors. For LSTM and matLSTM models, in both compression and prediction strategies, we use a conditional decoder which, at each decoding step, uses the decoded output at the previous timestep as its input. We also experimented with unconditional decoder but found that it was empirically worse, thus we only report results for the conditional decoder here. For LSTM and matLSTM models, we investigate the ones with a single hidden layer and two
hidden layers, c.f. Fig. 1 and Fig. 2. We use the Adam optimiser with a learning rate of $3 \times 10^{-3}$, minibatch size 64, and a maximum of 100 training epochs or when the model converges. Sequence-level Area Under ROC Curve (ROC-AUC) and F1 performance measures are used throughout the experiments. We repeat all experiments 3 times and report the mean and standard deviation of measures on left-out test sets.

A. Synthetic data

In this section, we conduct three different ablation studies on a synthetic dataset: (1) noise-free, (2) noisy, and (3) long movement pattern. In each study, the data are sequences of binary matrices generated as follows:

$$X_t = \left[ X_{t,j}^i \sim \text{Bern}(0.5) \right]_{i=1:n_r, j=1:n_c}$$

where circshift is the circular shift of the matrix to the right by $c \sim \text{Uniform}[1,5]$ columns, $n_r = 10$ and $n_c = 10$ are the number of rows and columns respectively, and $T = 20$ is the sequence length. The abnormal data sequences are created similarly but instead of the right-shifting pattern, we use a random permutation of columns. Each of the train and test set consists of 5,000 sequences with 5% outlier. We randomly take 20% of the train set for validation. For simplicity, we only examine the single-layer models in this setting.

Noise-free data: Fig. 3a compares the performance of matLSTM to LSTM on the test set. As shown, performance improves with more model capacity, but the matLSTMs improve much faster, suggesting matLSTMs can capture the multi-way structure better than LSTMs. In this setting, both compression (Sec. III-B) and prediction strategies (Sec. III-C) work well for matLSTM models while for vector models, the prediction strategy is more robust than the compression strategy.

Noisy data: In this setting, we randomly set 20% of the pixels to zero at each timestep but keep the moving pattern the same as the noise-free cases. The results are shown in Fig. 3b. With noisy inputs, the performance of vector models drop, while matrix models retain their near-perfect performance.

Longer movement pattern: In this setting, we investigate the changes in performances of matLSTM vs. LSTM when the moving pattern is longer. We make the data matrices horizontally longer by increasing $n_c = 100$. Table I shows that both vector LSTM methods require more number of free parameters than matLSTM to characterise the moving patterns.

| Models                        | #params | AUC    |
|-------------------------------|---------|--------|
| 1-layer LSTM-AutoEncoder      | 472k    | 94.7 ± 0.2 |
| 2-layer LSTM-AutoEncoder      | 505k    | 93.9 ± 0.7 |
| 1-layer LSTM-Encoder-Predictor| 472k    | 95.3 ± 0.4 |
| 2-layer LSTM-Encoder-Predictor| 505k    | 87.1 ± 0.5 |
| 1-layer matLSTM-AutoEncoder   | 104k    | 100.0 ± 0.0 |
| 2-layer matLSTM-AutoEncoder   | 177k    | 100.0 ± 0.0 |
| 1-layer matLSTM-Encoder-Predictor| 104k   | 100.0 ± 0.0 |
| 2-layer matLSTM-Encoder-Predictor| 177k   | 100.0 ± 0.0 |

Table I: LSTM vs. matLSTM in longer horizontal movement pattern, matrix size 10 x 100, on synthetic data.

B. Moving permuted digits

For ease of visualisation, we generate moving digits from the MNIST dataset. MNIST dataset contains 60,000 images in the train set and 10,000 images in the test set. For normal sequences, we first select a random slope between $[0, 2\pi]$ and move the digit along a straight line; for abnormal sequences, we also select a random slope between $[0, 2\pi]$ but the digit moves along a curved line. A curved line has a slightly different moving pattern compared to a straight line, therefore we want to investigate if all models can discriminate between two moving patterns. We use all of the 60,000 digits in the MNIST train set to form the train set of moving digit data, and use all the digits in MNIST test set to generate test set of moving digit data. The train set contains only normal sequences and the test set has 5% outlier.

We treat the images as matrices of pixels, whose rows or columns are permutable, ignoring the strict grid structure which typically warrants CNN models. Our goal is not to compete against CNN-based techniques, but to expose regularities found in 2D motions. To simulate the permutation-invariant scenario, in each sequence, we permute the rows and columns for each image following the same permutation matrix.

The cross-entropy loss is used as the pixel intensities fall within the range $[0, 1]$. For each model, the hidden size of LSTM unit is chosen so that the number of parameters increases from a small number up to approximately 500K parameters.

1) Noise-free data: Fig. 4a compares the AUC against the number of parameters for the single-layer models. Again we observe that matrix neural network can capture movement pattern and predict future frames well, yielding high performance using much fewer parameters than vector counterparts.

We visualise reconstructed frames of the autoencoder models for a normal and abnormal sequence in Fig. 5a and 5b respectively. We choose the LSTM-AutoEncoder models which have the highest number of parameters from Fig. 4a. In Fig. 5b, LSTM-AutoEncoder fails to reconstruct the moving pattern of the sequence (the digit lies lower in the first few frames), as they try to reconstruct the moving pattern in a straight line. In contrast, matLSTM-AutoEncoder reconstructs the sequence well, thus making it unable to discriminate between two patterns and yields random performance in Fig. 4a.

Similarly, we show that matLSTM model is able to encode and predict digit movement well, also produce sharper images compared to the vector counterpart, in Fig. 5c and Fig. 5d. Since matLSTM is able to encode the past movement well, its prediction will deviate from ground truth if the movement pattern is anomalous, hence giving high prediction error. In Fig. 5d, the digit is predicted to keep moving downward, while ground truth frames show that the digit moves further to the right.

Adding outlier to training data: We add 5% of outlier into the training data, in the same way as the testing data, to reflect a real situation where our training data is not lean but contaminated with unknown outliers by a small proportion. Table III shows the results for all models, including 3D-CNN.
and two-layer LSTM models. With the presence of outlier, the performances of both vector models drop.

| Models               | #Params | AUC  |
|----------------------|---------|------|
| 3D-ConvAE            | 140k    | 59.6 ± 0.1 |
| 3D-Conv-Predictor    | 66k     | 81.9 ± 0.4 |
| 1-layer LSTM-AutoEncoder | 331k | 61.6 ± 0.7 |
| 2-layer LSTM-AutoEncoder | 357k | 65.9 ± 0.4 |
| 1-layer LSTM-Encoder-Predictor | 331k | 76.8 ± 0.1 |
| 2-layer LSTM-Encoder-Predictor | 357k | 75.3 ± 0.5 |
| 1-layer matLSTM-AutoEncoder | 52k  | 64.9 ± 2.7 |
| 2-layer matLSTM-AutoEncoder | 101k | 66.7 ± 0.6 |
| 1-layer matLSTM-Encoder-Predictor | 52k  | 83.4 ± 1.7 |
| 2-layer matLSTM-Encoder-Predictor | 101k | 78.8 ± 2.2 |

Table II: Moving permuted digits: Models performance (AUC) with outliers in training data.

**Dynamics only:** In this experiment, we run single-layer models that take the temporal difference $\Delta X_t$ as input, as described in Section III-E. The results are shown in Fig. 7. While the matLSTM-Encoder-Predictor model stills perform well, LSTM-Encoder-Predictor model suffers a loss in performance. This suggests that while using temporal difference as inputs may suffice given a suitable model (e.g., the predictive matLSTM in this case), appearance-based data can be used to detect irregularities in trajectory dynamics.

2) **Noisy pixels:** From the noise-free data, we add salt and pepper noise to each frame in the sequence with a probability of 0.1 for each type of noise. Performance vs model size for single-layer models is shown in Fig. 8. All models require more parameters to reach high performance compared to the noise-free counterparts. In fact, except for the prediction strategy in matLSTM, other models struggle to learn regularities at all. The matLSTM-AutoEncoder does not improve with more parameters. This suggests that it may encode noise into its
To examine the model behaviours in more detail, we visualise reconstructed frames of the autoencoder models for normal and abnormal sequences in Fig. 6a and Fig. 6b respectively. The models with highest number of parameters from Fig. 4b are chosen for evaluation. The figures further confirm that the matLSTM-AutoEncoder actually memorises the noise as well as the signals, and thus cancels out its discriminative capability. However, this is not the case for the prediction strategy, as shown in Fig. 6c and Fig. 6d. This is expected because the noise is random, and there is no structure to be learnt, and thus future prediction is still smooth.

3) Noisy trajectories: A new dataset is created by randomly shifting the digits from the original trajectories within the 3-pixel margin. In both of the train and test set, 5% of the sequences are abnormal. Each sequence is then permuted using the same procedure as in the previous sections. The performance of all models are reported in Table III. We tune the number of free parameters independently for each model using the validation set. Again the matLSTM-AutoEncoder suffers greatly from the noise, suggesting that it does not suit for this task. The predictive counterpart, however, performs very well, and matLSTM-Encoder-Predictor achieves similar performance compared to LSTM-Encoder-Predictor, using much fewer number of free parameters. The two-layer predictive models experience overfitting, thus their performances are worse than single-layer models.
samples, with the R-peak values given in expert annotation. Each heartbeat therefore corresponds to one second in data recordings.

In this experiment, we remove the unknown class and consider heartbeat in one of three classes V, S, F as anomalous instances. We randomly select 20% of subjects for testing and the remaining subjects are used for training. We perform 4-fold cross validation to optimise the hyperparameters. The final AUC and F1 measures are calculated from 3 different runs.

**Matrix construction:** The common practice is that each heartbeat is classified into one of the predefined classes. We consider a different setting in which we consider multiple consecutive heartbeats as one unit. The units are extracted from original data using a sliding-window strategy. Each unit is labelled abnormal if one of the heartbeats is abnormal. Thus the data will become more balanced, as seen in Table V. The detection may be more sensitive as a result, allowing a way of screening before the doctor has a detailed investigation into the suspicious units. Another motivation is that for heartbeat prediction models, predicting multiple beats at once may be easier than predicting a sequence of beats due to their local dependencies.

The matrices are then constructed by using one heartbeat per row. In essence, we are modelling the dependencies between wave signals within a beat, and across multiple beats. This construction allows using longer contexts. For example, when \( N = 10 \) beats are grouped, a sequential model of 20 steps will account for 200 beats, which is much more difficult to handle by conventional beat-based models.

**Evaluation result:** Table VI compares the performances of LSTM and matLSTM in predicting 5 heartbeats ahead. For LSTM models, we adopt two different types of inputs, the first type uses one heartbeat as the observation at each timestep and the second type uses flattened vector of 5 heartbeats as the observation at each timestep. For matLSTM model, a group of 5 heartbeats can be represented by a matrix, as denoted above. From Table VI, we observe that matLSTM yields the best performance, compared to LSTM models. For LSTM models using one heartbeat as input at each step, the results suggest that using longer context helps improve the performance.

Table VII reports the performances of models under various context length and group size. We use one heartbeat as input at each timestep for LSTM model since it gives better performance than using a flattened vector of multiple heartbeats. For fair comparisons, LSTM and matLSTM are compared using the same context length. In every case, matrix models show better performance than vector LSTM models and 3D-CNN models.

### C. ECG anomalies

We use MIT-BIH Arrhythmia dataset\(^1\) which contains 48 half-hour recordings of two-channel ECG signals, obtained from 47 subjects. The recordings are digitised at 360 samples per second. According to [11], each heartbeat is classified into one of five classes and detail statistics is shown in Table IV.

- **Normal**
- **Fusion**
- **Supraventricular premature beat**
- **Premature ventricular contraction**
- **Unknown**

![Figure 7: AUC vs. number of parameters for difference of moving permuted digits.](image)

Table III: AUC for moving permuted digit data with noisy trajectories.

| Models                | #Params | AUC   |
|-----------------------|---------|-------|
| 3D-Conv-AE            | 140k    | 55.3 ± 0.0 |
| 3D-Conv-Predictor    | 66k     | 74.9 ± 0.3 |
| 1-layer LSTM-AutoEncoder | 331k | 60.3 ± 0.7 |
| 2-layer LSTM-AutoEncoder | 357k | 67.5 ± 0.1 |
| 1-layer LSTM-Encoder-Predictor | 331k | 82.9 ± 0.7 |
| 2-layer LSTM-Encoder-Predictor | 357k | 82.6 ± 0.2 |
| 1-layer matLSTM-AutoEncoder | 52k  | 53.1 ± 0.9 |
| 2-layer matLSTM-AutoEncoder | 101k | 52.2 ± 0.2 |
| 1-layer matLSTM-Encoder-Predictor | 52k  | 85.1 ± 1.2 |
| 2-layer matLSTM-Encoder-Predictor | 101k | 82.1 ± 0.2 |

**Table IV: Heartbeats statistics, classes are divided according to [11].**

We perform data preprocessing steps similar to those in [11], [12]. First, we manually pick 38 subjects whose recordings have both MLII and V1 channels and contain no paced beats. For each univariate signal, the raw ECG signal is detrended by first fitting a 6-order polynomial and then subtracting it from the signal. Following this, a 6-order Butterworth bandpass filter with 5Hz and 15Hz range is applied. Finally, filtered signals are normalised individually by using Z-score normalisation. Each heartbeat is then represented by a window of length 360

\(^1\)https://physionet.org/content/mitdb/1.0.0/
Table VI: ECG: Performance of different models for predicting 5 heartbeats. \( T_c \): past context length. All models use the encoder-predictor version. LSTM-flat denotes LSTM model using flattened vectors as inputs.

| Models          | \( T_c \) | \#Params | AUC (%) | F1 (%) |
|-----------------|----------|---------|--------|--------|
| LSTM            | 10       | 386k    | 90.7 ± 0.3 | 71.2 ± 0.6 |
| LSTM            | 45       | 822k    | 91.2 ± 0.3 | 71.9 ± 0.9 |
| LSTM-flat       | 9        | 727k    | 87.8 ± 0.2 | 69.9 ± 0.2 |
| matLSTM         | 9        | 257k    | 92.5 ± 0.1 | 72.8 ± 0.2 |

Table VII: ECG: Changes in performance with different prediction length. \( T_c \): past context length. LSTM denotes LSTM-Encoder-Predictor, matLSTM denotes matLSTM-Encoder-Predictor.

| Predict 5 heartbeats ahead | \( T_c \) | \#Params | AUC (%) | F1 (%) |
|----------------------------|----------|---------|--------|--------|
| 3D-Conv-Predictor          | 9        | 81k     | 91.7 ± 0.1 | 71.4 ± 0.7 |
| 1-layer LSTM               | 45       | 822k    | 91.2 ± 0.3 | 71.9 ± 0.9 |
| 2-layer LSTM               | 45       | 984k    | 90.9 ± 0.2 | 71.9 ± 0.4 |
| 1-layer matLSTM            | 9        | 257k    | 92.5 ± 0.1 | 72.8 ± 0.2 |
| 2-layer matLSTM            | 9        | 343k    | 92.5 ± 0.1 | 72.9 ± 0.4 |

| Predict 10 heartbeats ahead | \( T_c \) | \#Params | AUC (%) | F1 (%) |
|----------------------------|----------|---------|--------|--------|
| 3D-Conv-Predictor          | 9        | 106k    | 90.9 ± 0.1 | 72.9 ± 0.2 |
| 1-layer LSTM               | 90       | 1,010k  | 89.2 ± 0.1 | 75.1 ± 0.2 |
| 2-layer LSTM               | 90       | 1,243k  | 89.1 ± 0.2 | 75.2 ± 0.4 |
| 1-layer matLSTM            | 9        | 263k    | 91.4 ± 0.1 | 75.0 ± 0.2 |
| 2-layer matLSTM            | 9        | 350k    | 91.3 ± 0.1 | 74.7 ± 0.2 |

| Predict 20 heartbeats ahead | \( T_c \) | \#Params | AUC (%) | F1 (%) |
|----------------------------|----------|---------|--------|--------|
| 3D-Conv-Predictor          | 9        | 143k    | 90.4 ± 0.1 | 78.7 ± 0.2 |
| 1-layer LSTM               | 180      | 1,388k  | 87.5 ± 0.4 | 77.8 ± 0.2 |
| 2-layer LSTM               | 180      | 1,670k  | 87.0 ± 0.4 | 77.5 ± 0.7 |
| 1-layer matLSTM            | 9        | 276k    | 90.8 ± 0.1 | 79.7 ± 0.2 |
| 2-layer matLSTM            | 9        | 362k    | 90.9 ± 0.1 | 79.9 ± 0.1 |

V. CONCLUSION

We have studied the task of unsupervised anomaly detection on temporal multiway data. Unlike well-studied spatio-temporal data that exhibit translation-invariance in time and space, multiway data are usually permutation-invariant in each of the modes. We investigated the power and behaviours of matrix recurrent neural networks for the task. Models were evaluated using a comprehensive suite of experiments designed to expose model behaviours on synthetic sequences, moving digits, and ECG recordings. Overall we empirically found that matrix LSTMs, configured to predict the future subsequences, are highly suitable for the problem. The models require a far less number of parameters compared to the vector counterparts while capturing the temporal regularities and predicting future well. The autoencoder configuration of the matrix LSTMs, however, is not suitable for noisy data because of its high memory capacity to compress the entire input sequence including the noise. We also discovered a nice unintended consequence of matrix RNNs: we can model accurately a very long sequence of vectors just by rearranging data blocks into matrices.

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