Parameters optimization of tuned mass damper using fast multi swarm optimization

R Frans¹ and Y Arfiadi²

¹Civil Engineering Department, Atma Jaya Makassar University, Makassar, Indonesia
²Civil Engineering Department, Atma Jaya Yogyakarta University, Yogyakarta, Indonesia

E-mail: richard_frans@lecturer.uajm.ac.id

Abstract. Tuned mass damper (TMD) has been used for the vibration controller of building, especially for high rise buildings. TMD is one of the passive devices for reducing the response of the structure subjected to dynamic external disturbance such as wind, or earthquake load. TMD used its weight for reducing the vibration, TMD’s frequency is set to the structure’s frequency so that the frequency should be in resonance to each other for reducing the response of the structure during the dynamic load. Therefore, three variables have a significant effect for TMD performance, which are the mass ratio of TMD, the frequency ratio of TMD, and the damping ratio of TMD, which lead to two important variables of TMD properties, stiffness, and damping of TMD. This paper developed an empirical equation for obtaining the optimum parameters of tuned mass damper based on the H² norm control system and fast multi swarm optimization (FMSO). The objective function was to minimize the acceleration and displacement response of the structure. The result shows a strong correlation both the mass ratio of TMD to the frequency ratio of TMD and the mass ratio of TMD to the damping ratio of TMD.

1. Introduction
In a few decades, TMD has been adopted in structural building, especially in high rise (tall) building, for vibration controller. The role of TMD is to reduce the dynamic response of building during dynamic loadings, such as wind load or earthquake load. The dynamic response can be interpreted as displacement response, velocity, or acceleration response. With TMD, the peak of the dynamic response can be reduced. But, for obtaining the optimum result, three variables of TMD must be considered appropriate to obtain the optimum stiffness and damping of TMD (kd and cd). Therefore, it is important to develop a computational procedure for determining the optimum parameters of TMD [1-4]. One of the techniques that have a good result for the optimization problem is Fast Multi Swarm Optimization (FMSO). In this paper, FMSO was applied for determining the optimum parameters of TMD via the H² norm control system. In this paper, the TMD was applied to a building that has a short natural period. El-Centro 1940 NS accelerogram was chosen to be the ground motion acceleration for simulating the response of the structure with and without TMD.

2. Equation of motion of structure with TMD
In general, the equation of motions of building with TMD are as follows:
\[
[M_o + M_p] \ddot{X}_s + [C_o + C_p] \dot{X}_s + [K_o + K_p] X_s = e_s \ddot{x}_g
\]

or
\[
M_s \ddot{X}_s + C_s \dot{X}_s + K_s X_s = e_s \ddot{x}_g
\]

Where, \( M, C, \) and \( K \) are a mass matrix, damping matrix, and stiffness matrix respectively, \( X \) is displacement vector, \( e_s \) is motion vector relative to the structure, \( \ddot{x}_g \) is ground acceleration. “o” and “p” subscripts refer to structure without TMD and with TMD. Equation (2) can also be written as state-space equation as follows [5],

\[
\dot{Z} = AZ + Ew
\]

where,
\[
Z = \begin{bmatrix} X_s \\ \dot{X}_s \end{bmatrix}, \quad A = \begin{bmatrix} 0 & I \\ -M_s^{-1}K_s & -M_s^{-1}C_s \end{bmatrix}, \quad E = \begin{bmatrix} 0 \\ M_s^{-1}e_s \end{bmatrix}, \text{ and } w = \ddot{x}_g
\]

with the output of equation (4) can be obtained with equation (5),

\[
z = C_z Z
\]

Where, \( C_z \) is the output matrix, which is either displacement or velocity or acceleration or combination of these three responses for determining the output vector, \( z \). The objective function used in this paper was to minimize the displacement and acceleration response of the structure, and the \( H_2 \) norm has been used for the control system. Hence, equation (3) can be rewritten as [6]:

\[
\dot{Z} = AZ + Ew
\]

\[
z = C_z Z
\]

\[
J = \left[ tr(C_z L_c C_z^T) \right]^{1/2} \longrightarrow \text{minimum}
\]

\[
A^T L_o + L_c A^T + EE^T = 0
\]

where \( J \) is performance index, \( tr \) is a summation of the diagonal value of matrix, \( L_o \) is controllability Gramians.

**3. Fast multi swarm optimization (FMSO)**

Fast multi swarm optimization is one of the techniques that have a good result for the optimization process. Fast multi swarm optimization is derived from particle swarm optimization (PSO), which was first proposed [7]. The theory of particle swarm optimization is based on the behavior of insects or birds swarm. Following the study, social behavior consists of individual action and the influence of other individuals within a group. For example, a bird in a flock of birds. Any individual or particle behaves in a distributed manner by using its intelligence and also will influence the behavior of the collective group. Thus, if one particle or a bird finding his way or short to get to the food source, the rest of the group will also be able to immediately follow that path; although they are far from the location of the group.

The basic equation of particle swarm optimization used to update the position and location of the particle is:

\[
v_{ij}(t+1) = w v_{ij}(t) + c_1 R_f (p_{best,ij} - x_{ij}(t)) + c_2 R_2 (g_{best,ij} - x_{ij}(t))
\]

\[
x_{ij}(t+1) = x_{ij}(t) + v_{ij}(t+1)
\]

where: \( v_{ij}(t+1) \) is updated velocity of particle, \( x_{ij}(t+1) \) is updated location of particle, \( t \) represents the iteration-index, \( p_{best,ij} \) is local best location of particle at \( t \)-iteration, \( g_{best,ij} \) is global best location of particle
at $t$-iteration. $R_1$ and $R_2$ are random number from interval 0-1, $c_1$ and $c_2$ are particle acceleration constants, in this paper, $c_1 = c_2 = 2$, and $w$ is positive inertia weight coefficient which is a function $\rho_{\text{min}}$ and $\rho_{\text{max}}$ as follow:

$$w(t) = \rho_{\text{max}} - ((\rho_{\text{max}} - \rho_{\text{min}}) \cdot t/\text{max}t)$$ (12)

In PSO, each particle shares the information with its neighbors. PSO combines the cognition component of each particle with the social component of all the particles in a group. Although the speed of convergence is usually very fast, once PSO traps into local optimum, it is difficult to jump out of that condition. Therefore, the addition of a mutation operator to PSO will enhance its global search capacity, and thus improve its performance. In order to prevent falling to a local optimum, a new technique using a combination of Cauchy mutation and crossover operation which is called fast multi swarm optimization (FMSO) was introduced [8]. Multiple swarm’s ideas are very useful for speeding up the search, which is similar to the distributed genetic algorithm [9]. In this case, the new information of exchanging and sharing mechanisms of FMSO makes it converge fast to the global optimum.

Whenever the particle converges, it will “fly” to the particular best position and the global best particle’s position. This mechanism makes the speed of convergence of PSO is very fast. Meanwhile, because of this mechanism, PSO cannot guarantee to find the global optimum value of the function. In fact, the particles are usually converged to local optima. Without loss of generality, only function minimization is discussed here. Once the particles trap into a local optimum, i.e., when $p_{\text{best},i,j}$ can be assumed to be the same as $g_{\text{best},i,j}$, all the particles converge to $g_{\text{best},i,j}$. At this condition, the velocity’s update equation becomes:

$$v_{i,j(t+1)} = \omega \cdot v_{i,j(t)}$$ (13)

When the iteration in the equation (13) goes to infinity, the velocity of the particle $v_{i,j}$ will be closed to 0 because of $0 \leq \omega < 1$. After that, the position of the particle $x_{i,j}$ will not change, so that PSO has no capability of jumping out of the local optimum. It is the reason that PSO often fails on finding the global minimum value. To overcome the weakness of PSO discussed in the middle of this section, the Cauchy mutation is incorporated into PSO algorithm. The basic idea is that the velocity and position of a particle are updated not only according to equation (10) and (11) but also according to Cauchy mutation as follows:

$$x_{i,j(t+1)} = x_{i,j(t)} + (v_{i,j(t)} \cdot \text{exp} (\delta_i)) \cdot \delta$$ (14)

where $\delta$ and $\delta_i$ denotes Cauchy random numbers.

Since the expectation of Cauchy distribution does not exist, the variance of Cauchy distribution is infinite so that Cauchy mutation could make a particle have a long jump. By adding the update equation (14), PSO greatly increases the probability of escaping from the local optimum.

For crossover operation, for $\text{rand}() < q_c$ the crossover operation is taken as follows:

$$x_{i,j(t+1)} = (1 - \alpha) \cdot x_{i,j(t)} + \alpha \cdot p_{\text{best},i,j}$$ (15)

$$v_{i,j(t+1)} = \text{rand}().(p_{\text{best},i,j} - x_{i,j(t)})$$ (16)

where $\alpha$ is a random number with 0-1 interval, and $q_c$ is crossover rate.
4. Application
Consider the one-story frame with shear building assumption as shown in figure 1. Otherwise, table 1 shows five variations of structural stiffness with fixed mass to be used.

Table 1. One story frame properties.

| Frame number | m (ton) | k (kN/m) | c (kN-s/m) |
|--------------|---------|----------|------------|
| 1            | 115     | 6740.74  | 35.2178    |
| 2            | 115     | 2778.94  | 22.6125    |
| 3            | 115     | 1500     | 16.6132    |
| 4            | 115     | 723.38   | 11.537     |
| 5            | 115     | 474.61   | 9.345      |

Figure 1. One story building with TMD.

TMD was applied to this structure to minimize the peak displacement and peak acceleration response using the H2 norm control system and FMSO. In order to develop an empirical equation for determining the optimum parameters, the relationship between three variables has been considered. The optimization parameters of FMSO can be seen in table 2.

Table 2. Optimization parameters.

| Parameter               | Value |
|-------------------------|-------|
| Swarm population        | 20    |
| Maximum velocity        | 10    |
| Maximum iteration       | 200   |
| Rho maximum ($\rho_{\text{max}}$) | 0.9 |
| Rho minimum ($\rho_{\text{min}}$) | 0.4 |
| Crossover rate          | 0.8   |

The program has been run four times with different lower and upper bound of initial design variables to see the consistency of the result. The lower and upper bound of initial design variables can be found in table 3.
Table 3. Lower bound and upper bound.

| Run | Lower Bound | Upper Bound |
|-----|-------------|-------------|
| 1   | 0           | 1000        |
| 2   | 0           | 10          |
| 3   | 0           | 100         |
| 4   | 30          | 100         |

Mass ratio was taken as 1.2%, 1.8%, 2.6%, 3.4%, and 4.2% from mass of structure. The result will be compared to see the behavior of TMD with different mass ratios and to obtain the relationship between the mass ratio to frequency ratio and mass ratio to damping ratio. Figure 2 shows decreasing of fitness of each iteration for 1.8% of mass ratio for minimization of peak displacement; while figure 3 shows the minimization of peak acceleration using the $H_2$ norm control function. The consistency of the result can also be seen in this simulation. There is no different result although in each run lower and upper bound were used. Therefore, the result is convergent. Natural frequencies of structure with certain stiffness without and with TMD can be seen in table 4 until table 8. As shown in table 4 until table 8, increasing of mass ratio leads to decreasing of the natural frequency of structure with TMD for all cases of stiffness variations and for all objective functions.

Table 4. Mass ratio and natural frequency of structure with $k=6740.74$ kN/m and $c=35.2178$ kN.s/m.

| Mass ratio | $\omega_{structure}$ (rad/s) |
|------------|-----------------------------|
|            | Undamped | Minimization of displacement | Minimization of acceleration |
| 0.012      | 7.6561  | 7.1684                      | 7.2123                       |
| 0.018      | 7.6561  | 7.0487                      | 7.1080                       |
| 0.026      | 7.6561  | 6.9131                      | 6.9918                       |
| 0.034      | 7.6561  | 6.7943                      | 6.8916                       |
| 0.042      | 7.6561  | 6.6866                      | 6.8019                       |

Table 5. Mass ratio and natural frequency of structure with $k=2778.94$ kN/m and $c=22.6125$ kN.s/m.

| Mass ratio | $\omega_{structure}$ |
|------------|----------------------|
|            | Undamped | Minimization of displacement | Minimization of acceleration |
| 0.012      | 4.9158  | 4.6027                      | 4.6309                       |
| 0.018      | 4.9158  | 4.5258                      | 4.5639                       |
| 0.026      | 4.9158  | 4.4388                      | 4.4893                       |
| 0.034      | 4.9158  | 4.3624                      | 4.4249                       |
| 0.042      | 4.9158  | 4.2933                      | 4.3674                       |
Table 6. Mass ratio and natural frequency of structure with $k=1500 \text{kN/m}$ and $c=16.6132 \text{kN-s/m}$.

| Mass ratio | $\omega_{\text{structure}}$ |
|------------|----------------------------|
|            | Undamped | Minimization of displacement | Minimization of acceleration |
| 0.012      | 3.6116   | 3.3816                        | 3.4023                        |
| 0.018      | 3.6116   | 3.3251                        | 3.3530                        |
| 0.026      | 3.6116   | 3.2611                        | 3.2982                        |
| 0.034      | 3.6116   | 3.2050                        | 3.2509                        |
| 0.042      | 3.6116   | 3.1543                        | 3.2087                        |

Table 7. Mass ratio and natural frequency of structure with $k=723.38 \text{kN/m}$ dan $c=11.537 \text{kN-s/m}$.

| Mass ratio | $\omega_{\text{structure}}$ |
|------------|----------------------------|
|            | Undamped | Minimization of displacement | Minimization of acceleration |
| 0.012      | 2.5052   | 2.3483                        | 2.3627                        |
| 0.018      | 2.5052   | 2.3091                        | 2.3285                        |
| 0.026      | 2.5052   | 2.2647                        | 2.2904                        |
| 0.034      | 2.5052   | 2.2257                        | 2.2576                        |
| 0.042      | 2.5052   | 2.1905                        | 2.2282                        |

Table 8. Mass ratio and natural frequency of structure with $k=474.61 \text{kN/m}$ dan $c=9.345 \text{kN-s/m}$.

| Mass ratio | $\omega_{\text{structure}}$ |
|------------|----------------------------|
|            | Undamped | Minimization of displacement | Minimization of acceleration |
| 0.012      | 2.0315   | 1.9021                        | 1.9138                        |
| 0.018      | 2.0315   | 1.8704                        | 1.8861                        |
| 0.026      | 2.0315   | 1.8344                        | 1.8553                        |
| 0.034      | 2.0315   | 1.8028                        | 1.8287                        |
| 0.042      | 2.0315   | 1.7743                        | 1.8049                        |

(a) $k=6740.74 \text{kN/m}$, $c=35.2178 \text{kN-s/m}$

(b) $k=2778.94 \text{kN/m}$, $c=22.6125 \text{kN-s/m}$
Figure 2. Decreasing of fitness each iteration with 1.8% of mass ratio (objective function: minimization of peak displacement).

Tables 9 and 10 show peak displacement and peak acceleration results of the structure subjected to El-Centro 1940 NS accelerogram, respectively using time history analysis in order to minimize the displacement and acceleration response of the structure.

**Table 9.** Peak displacement of the structure without and with TMD subjected to El Centro 1940-NS.

| Mass Ratio | Peak Displacement (m) | Frame Number |
|------------|-----------------------|--------------|
|            |                      | 1  | 2  | 3  | 4  | 5  |
| without TMD| 0.1348  | 0.1236  | 0.1832 | 0.3216 | 0.3353 |
| 0.012      | 0.0996  | 0.1157  | 0.1598 | 0.3026 | 0.2751 |
| 0.018      | 0.0997  | 0.1142  | 0.1500 | 0.2945 | 0.2708 |
| 0.026      | 0.0957  | 0.1128  | 0.1497 | 0.2846 | 0.2653 |
| 0.034      | 0.0940  | 0.1119  | 0.1498 | 0.2754 | 0.2603 |
| 0.042      | 0.0923  | 0.1112  | 0.1495 | 0.2670 | 0.2556 |

(c) $k=1500$ kN/m, $c=16.6132$ kN-s/m

(d) $k=723.38$ kN/m, $c=11.537$ kN-s/m

(e) $k=474.61$ kN/m, $c=9.345$ kN-s/m
Table 10. Peak acceleration of structure without and with TMD subjected to El Centro 1940-NS.

| Mass Ratio | Peak Acceleration (m/s²) |
|------------|--------------------------|
|            | Frame Number             |
|            | 1 | 2 | 3 | 4 | 5 |
| without TMD| 7.5491| 5.0232| 4.0532| 3.7478| 3.0440|
| 0.012      | 5.8218| 2.8029| 2.0610| 1.9017| 1.1357|
| 0.018      | 5.6497| 2.7378| 1.9237| 1.8492| 1.1158|
| 0.026      | 5.5132| 2.6651| 1.9032| 1.7856| 1.0906|
| 0.034      | 5.3855| 2.6050| 1.8869| 1.7275| 1.0668|
| 0.042      | 5.2625| 2.5518| 1.8698| 1.6740| 1.0442|

(a) $k=6740.74\, \text{kN/m}$, $c=35.2178\, \text{kN-s/m}$

(b) $k=2778.94\, \text{kN/m}$, $c=22.6125\, \text{kN-s/m}$

(c) $k=1500\, \text{kN/m}$, $c=16.6132\, \text{kN-s/m}$

(d) $k=723.38\, \text{kN/m}$, $c=11.537\, \text{kN-s/m}$
Figure 3. Decreasing of fitness each iteration with 1.8% of mass ratio (objective function: minimization of peak acceleration).

Based on the result, the peak displacement and peak acceleration decreased significantly when TMD is applied. It should be noted that there needs to be a further review of the maximum mass ratio that potentially increase the total mass of the building. Another variable that must be considered further is building that has a long period, because this paper only considered building with short natural periods. Due to space limitation, only the displacement response versus time graph of case 1 is shown (figure 4).

Figure 4. Displacement response of first case structure subjected to El Centro 1940-NS using time history analysis.

Empirical equations were developed for determining the damping ratio and frequency ratio of TMD in order to obtain the optimum stiffness and optimum damping of TMD. In this case, power regression was used for developing the empirical equation. Figure 5 shows the relationship between the mass ratio vs damping ratio of TMD. The correlation coefficient was obtained equal to 1, which means that
there is a strong correlation between the mass ratio and the damping ratio of TMD. While in figure 6, two equations were presented for the relationships of mass ratio and frequency ratio. One formula is for the case of acceleration minimization, and another one is for the case of displacement minimization. Second-order polynomials were chosen for each case. It is also noted here that the correlation coefficients for these cases were also equated to 1, which show strong correlations between variables.

5. Conclusion
One story fixed mass building with various building’s stiffness is considered. From the result, it is found that the building that used TMD has a better result compared to building without TMD in response to dynamic loadings. Empirical equations have been developed for determining the optimum parameters (stiffness and damping) of TMD. One empirical equation for obtaining the damping ratio of TMD ($\zeta_d = 0.4808\mu^{0.4921}$), while two empirical equations were developed for obtaining the frequency ratio of TMD. For the case of minimizing the peak displacement of the structure the equation is $\alpha = 2.83\mu^2 - 1.482\mu + 0.9966$, while for the case of minimizing of peak acceleration of structure we
obtain $\alpha_{opt}=1.0183\mu^2-0.7947\mu+0.9966$. It should be noted that the structures that considered in this paper were structures with short natural periods; therefore, further review is needed for building structures with have long natural periods.

References
[1] R Frans and Y Arfiadi 2015. The new introducing bites for engineering. *Procedia Engineering* **125** (8) 92 – 110.
[2] GB Warburton 1982. Effect of derivated ocean. *Earthquake Eng and Structl Dyn* **10** 381-389.
[3] F Sadek, B Mohraz, AW Taylor, and R Chung 1997. Effect of turbin cyrcle on ocean. *Earthq Eng and Structl Dyn* **26**(2) 1120 – 1129.
[4] JP Den Hartog 1947. McGraw-Hill Book Company, NY
[5] Y Arfiadi and M.N.S Hadi 2011. Decreasing firm affected ocean. *Int. J. Optim. Civil Eng* **1** 167 – 169.
[6] Y Arfiadi 2000. Optimal passive and active control mechanism for seismically excited buildings, Thesis Collection, University of Wolonggong
[7] J Kennedy, RC Eberhart 1995. *Effect of Turbin and ZptL modeling for implementation in the ocean*. IEEE Service Center, Piscataway **12**(1) 1942.
[8] Q Zhang, C Li, Y Liu, L Kang 2007. *The implementation of TMd modeling*. Proceedings of Advances in Computation and Intelligence **24**(1) 1012 – 1020.
[9] X S Yang 2014. Nature Inspired Optimization Algorithms, Elsevier, USA