Mathematical model comparing of the multi-level economics systems

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Abstract. The mathematical model (scheme) of a multi-level comparison of the economic system, characterized by the system of indices, is worked out. In the mathematical model of the multi-level comparison of the economic systems, the indicators of peer review and forecasting of the economic system under consideration can be used. The model can take into account the uncertainty in the estimated values of the parameters or expert estimations. The model uses the multi-criteria approach based on the Pareto solutions.

1. Introduction

The presented report is a logical continuation of the previous papers of the authors and is devoted to analysis of multi-level economic system [1, 2]. In the analysis of the CES it is necessary to have a mathematical scheme to compare several CES or more options for the development of one of CES and to answer the question - which one is more efficient and reliable. These mathematical models are considered also for complex technical systems (CTS) [3] to determine their technical proficiency and to compare them with other similar CTS [3-5]. As in the previous papers [1, 2], in this report we use MLS, which is characterized by a big number of criteria [5-11].

2. The mathematical scheme clotting particular indices in integrated indicators for the multi-level economic systems

We consider \( i = 1, \ldots, n \) the CES, each of which is characterized on the lower level by \( j = 1, \ldots, m \) the particular indices (the more value of indices, the more effective CES), and on every levels by the integral indicators \( R_{ij}, j = 1, \ldots, m_1, \ldots, R_{ijk}, j = 1, \ldots, m_M \), respectively.

Thus, each integral index of the level consists of the index of the previous level.

Let \( R_{ijk}, j = 1, \ldots, m_1, k = 1, \ldots, n_{ij}, \ldots, R_{ijlk}, j = 1, \ldots, m_M, k = 1, \ldots, n_{ij} \) be the particular initial indicators which are included in the \( j \)-th integral index of the first level and in the \( j \)-th integral index of the \( M \)-th level, respectively.

To solve the problems of the forecasting of the particular indicators, the methods of the metric analysis [12-15] can be used.

There are the data values on the lower level:
$$R_{ijk}, k = 1, \ldots, n_j, i = 1, \ldots, n, j = 1, \ldots, m_1,$$

$$R_{ijk}$$ is the value of the $j$-th particular index of the $i$-th CES.

Similar to paper [2], experts consider the priority of indicators of each level, with the help of which the partial indicators are reduced to integral indicators of the first level. Then, similarly to the paper [2], the integral indicators of the previous level are sequentially rolled up to the integral indicators of the subsequent level up to the last $M$-th level.

For example, for a three-level economic system $R_{ijk}, j = 1, \ldots, m_1, k = 1, \ldots, n_j$ and $R_{2jk}, j = 1, \ldots, m_2, k = 1, \ldots, n_{2j}$ are the particular first-level indicators that are included in the $j$-th integral index of the second level, and in the $j$-th integral index of the third level, respectively (see [1, 2]).

To solve the problems of the forecasting of the particular first-level indicators, the methods of the metric analysis [12-15] can be used.

There are the data baseline values the first level

$$\{ R_{ijk}, k = 1, \ldots, n_j, i = 1, \ldots, n, j = 1, \ldots, m_1, \}$$

are the priority factors of the particular index of the $i$-th CES.

Let:

1$^{\circ}$. $V_{1,jkl}, \ 0 < V_{1,jkl} < 1, \sum_{k=1}^{n_i} V_{1,jkl} = 1, \ j = 1, \ldots, m_2$

are the priority factors of particular indices of the $l$-th expert to $j$-th of the integral index of the second level, $l = 1, L$.

2$^{\circ}$. $V_{2,jkl}, \ 0 < V_{2,jkl} < 1, \sum_{k=1}^{n_i} V_{2,jkl} = 1, \ j = 1, \ldots, m_3$

are the priority factors of integral indicators of the second level of the $l$-th expert to $j$-th of the integral index of the third level, $l = 1, L$.

3$^{\circ}$. $V_{3,jl}, \ 0 < V_{3,jl} < 1, \sum_{j=1}^{m_j} V_{3,jl} = 1$

are the priority factors of integral indicators in the third level of the $l$-th expert.

4$^{\circ}$. $\omega, \ l = 1, \ldots, L, \ 0 < \omega < 1, \sum_{l=1}^{L} \omega_l = 1$

are the boost factors of experts.

Then we perform the following operations.

1) We produce the normalization of the values of particular indices

$$R_{ijk}^{II} = \frac{R_{ijk}}{R_{ijk, MAX}}, \ i = 1, \ldots, n_j; \ j = 1, \ldots, m_1; \ k = 1, \ldots, n_j,$$

where $R_{ijk, MAX} = \max \{ R_{ijk}, i = 1, \ldots, n_j \}$.

2) We calculate the coefficients of the priority of private indicators for each group of the integral index of the second level:

$$V_{1,jkl} = \sum_{l=1}^{L} \omega_l V_{1,jkl}, \ j = 1, \ldots, m_2, k = 1, \ldots, n_{2j}$$

3) We calculate the normalized value of the evaluation of integrated indicators of the second
level: \[ R_{2jk}^H = \sum_{k=1}^{n_j} V_{1jk} \cdot R_{ijk}^H, \quad j = 1,\ldots,m_2, \quad k = 1,\ldots,n_{2j} \] \tag{8}

4) We calculate the coefficients of the priority of integrated indicators of the second level for each group of the integral index of the third level:
\[ V_{2jk} = \sum_{l=1}^{k} \omega_l \cdot V_{2jkl}, \quad j = 1,\ldots,m_3, \quad k = 1,\ldots,n_{2j} \] \tag{9}

5) We calculate the normalized values of the evaluations of integrated indicators in the third level:
\[ R_{3j}^H = \sum_{k=1}^{n_j} V_{2jk} \cdot R_{2jk}^H, \quad j = 1,\ldots,m_2 \] \tag{10}

6) We calculate the coefficient priority for the integral indicators in the third level:
\[ V_{3j} = \sum_{l=1}^{k} \omega_l \cdot V_{3jkl}, \quad j = 1,\ldots,m_3 \] \tag{11}

7) We calculate the normalized value of the complex criteria for each CES:
\[ R_{ij}^H = \sum_{j=1}^{n_j} V_{3j} \cdot R_{3j}^H, \quad i = 1,\ldots,n \] \tag{12}

8) We produce a ranking of SES on the values of complex criteria:
\[ R_{1}^H \geq R_{2}^H \geq \ldots \geq R_{n}^H \] \tag{13}

CES with number \( i = (1) \) is the best CES by the integrated efficacy.

We use fuzzy numbers \([16, 17]\) to account for uncertainties in expert estimates of the indicators. In this case, each SES is characterized by two complex indicators: a complex indicator of efficiency and a complex index of uncertainty.

3. The mathematical definition of the uncertainty of the multi-level economic system integrated indicators

In the case of the performance of expert evaluations and values to account for uncertainties in the values of particular indicators we use the triangular fuzzy numbers \([16, 17]\). It is necessary to produce the normalization (6) for each of three numbers \((R_{ijk \min}^H, R_{ijk}^*, R_{ijk \max}^H)\), i.e. go to the normalized values
\[ (R_{ijk \min}^H, R_{ijk}^*, R_{ijk \max}^H) \] \tag{14}

where
\[ R_{ijk \min}^H = \frac{R_{ijk \min}^H}{R_{jk \max}} \cdot R_{jk}^*, R_{ijk}^* = \frac{R_{ijk \max}^H}{R_{jk \max}} \cdot R_{jk}^*, R_{ijk \max}^H = \max \{R_{ijk}^*, l = 1,\ldots,L, i = 1,\ldots,n \} \] \tag{15}

Then the normalized score of each of the integral indicator for the \( i \)-th CES will be presented by the triangular fuzzy number
\[ (R_{ij \min}^H, R_{ij}^*, R_{ij \max}^H) \] \tag{16}

where (see the formula (5), (7))
\[ R_{ij \min}^H = \sum_{k=1}^{n_j} V_{jk} \cdot R_{ijk \min}^H, \quad R_{ij}^* = \sum_{k=1}^{n_j} V_{jk} \cdot R_{ijk}^*, \quad R_{ij \max}^H = \sum_{k=1}^{n_j} V_{jk} \cdot R_{ijk \max}^H, \quad i = 1,\ldots,n \] \tag{17}

\[ R_{ijk \min}^H = \sum_{l=1}^{L} \omega_l \cdot R_{ijk \min}^H, \quad R_{ijk}^* = \sum_{l=1}^{L} \omega_l \cdot R_{ijk}^*, \quad R_{ijk \max}^H = \sum_{l=1}^{L} \omega_l \cdot R_{ijk \max}^H, \quad j = 1,\ldots,m, \quad k = 1,\ldots,n_j \] \tag{18}
We calculate the normalized value of the integrated indicator of efficiency for each CES presented in the form of a triangular fuzzy number

\[ (R_i^{H}, R_i^{H^*}, R_i^{r_{max}}), \ i = 1, \ldots, n \]

where

\[ R_i^{H_{min}} = \sum_{j=1}^{m_i} W_j \cdot R_j^{H_{min}}, \quad R_i^{H^*} = \sum_{j=1}^{m_i} W_j \cdot R_j^{H^*}, \quad R_i^{r_{max}} = \sum_{j=1}^{m_i} W_j \cdot R_j^{r_{max}}. \]

4. Choosing the best CES as the solutions Pareto of two-criteria problem: an integrated efficiency criterion and an integrated criterion of uncertainty

In the case of the triangular fuzzy numbers to describe uncertainties in the values of indicators of two complex criteria for each SES the complex criterion of efficiency is defined by equality

\[ K_{i_{eff}} = R_i^{H^*}, \quad i = 1, \ldots, n, \]

the complex criterion equality uncertainty is defined by equality

\[ K_{i_{unc}} = R_i^{r_{max}} - R_i^{r_{min}}, \quad i = 1, \ldots, n. \]

The final selection of the best CES is carried out among the Pareto solutions with respect to two criteria

\[ K_{i_{eff}} - \text{max}, \]
\[ K_{i_{unc}} - \text{min}. \]

On the plane \((K_{i_{eff}}, K_{i_{unc}})\) the Pareto points form a "south-eastern" part of the boundary of the set of points of criterions \(K_{i_{eff}}\) and \(K_{i_{unc}}\).

Conclusion

The paper presents the mathematical model for the comparing of the multi-level economic CES or the comparison of several variants of the same CES which is characterized by the criteria system. This mathematical comparison circuit may use the design or expert assessment indicators and predict some or all of the considered indicators of CES. The model provides the account of uncertainties in the estimated values and expert assessment indicators or forecast values. The model uses a multi-criteria approach to the possibility of taking into account both quantitative and uncertainty indicators. The final decision is taken among the Pareto solutions of the multi-criteria problem with two integral criteria: the complex criterion of efficiency and complex criterion of risk for integral indicators of the final level. The mathematical model presented in this report can be used by offices and enterprises of various branches [18, 19].

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