We investigate the relation between rotation periods $P_{\text{rot}}$ and photometric modulation amplitudes $R_{\text{per}}$ for ≈4000 Sun-like main-sequence stars observed by Kepler, using $P_{\text{rot}}$ and $R_{\text{per}}$ from McQuillan et al., effective temperature $T_{\text{eff}}$ from LAMOST DR6, and parallax data from Gaia EDR3. As has been suggested in previous works, we find that $P_{\text{rot}}$ scaled by the convective turnover time $\tau_c$, or the Rossby number $R_o$, serves as a good predictor of $R_{\text{per}}$: $R_{\text{per}}$ plateaus at around 1% in relative flux for $0.2 \lesssim R_o/R_{\odot} \lesssim 0.4$, and decays steeply with increasing $R_o$ for $0.4 \lesssim R_o/R_{\odot} \lesssim 0.8$, where $R_{\odot}$ denotes Ro of the Sun. In the latter regime we find $d \ln R_{\text{per}}/d \ln R_o \sim -4.5$ to $-2.5$, although the value is sensitive to detection bias against weak modulation and may depend on other parameters including $T_{\text{eff}}$ and surface metallicity. The existing X-ray and Ca II H and K flux data also show transitions at $R_o/R_{\odot} \sim 0.4$, suggesting that all these transitions share the same physical origin. We also find that the rapid decrease of $R_{\text{per}}$ with increasing $R_o$ causes rotational modulation of fainter Kepler stars with $R_o/R_{\odot} \gtrsim 0.6$ to be buried under the photometric noise. This effect sets the longest $P_{\text{rot}}$ detected in the McQuillan et al. sample as a function of $T_{\text{eff}}$ and obscures the signature of stalled spin down that has been proposed to set in around $R_o/R_{\odot} \sim 1$.

**Unified Astronomy Thesaurus concepts:** Light curves (918); Starspots (1572); Stellar activity (1580); Stellar magnetic fields (1610); Stellar rotation (1629)

### 1. Introduction

Recent studies of rotation of old Sun-like stars suggest a change in stellar activity of middle-aged main-sequence stars. There has been accumulating evidence that the rotation periods of Sun-like stars cease to scale as the square-root of age (Skumanich 1972) in the latter halves of their lives (e.g., Angus et al. 2015; van Saders et al. 2016; Hall et al. 2021; Masuda et al. 2022). A similar transition has also been noted in chromospheric activities (Metcalfe et al. 2016). These may suggest a corresponding change in the mechanism of magnetic field generation that occurs once the rotation period $P_{\text{rot}}$ becomes comparable to the convective turnover timescale $\tau_c$.

Quasi-periodic brightness modulation of stars in broadband photometry is well suited for statistical studies of rotational evolution. In particular, the light curves from the prime Kepler mission (Borucki et al. 2010) have been used to derive rotation periods of up to months for tens of thousands of Sun-like stars (e.g., Nielsen et al. 2013; Reinhold et al. 2013; McQuillan et al. 2014; García et al. 2014; Santos et al. 2019, 2021). Previous investigations of the $P_{\text{rot}}$ distribution of Kepler stars using the sample of McQuillan et al. (2014) have shown that the observed distribution is truncated roughly around the solar Rossby number $R_o = P_{\text{rot}}/\tau_c$ (van Saders et al. 2019) and also exhibits a pile-up around slightly shorter $P_{\text{rot}}$ (David et al. 2022). While these features are in qualitative agreement with the stalled spin down scenario (van Saders et al. 2019), the effects of rotational evolution and detection bias have not been clearly disentangled. In general, the longer-period tail of the $P_{\text{rot}}$ distribution is most prone to the detection bias, and so it requires a good understanding of the bias to correctly interpret the observed distribution.

In this work, we attempt to better understand the detection bias in the $P_{\text{rot}}$ sample constructed by McQuillan et al. (2014) to aid the statistical interpretation of the sample. To do so, we first investigate the generic relation between spot-modulation amplitudes and rotation periods for Sun-like main-sequence stars in the McQuillan et al. (2014) sample (as defined in Section 2), and derive a relation that predicts the modulation amplitudes given $P_{\text{rot}}$ and $T_{\text{eff}}$ (Section 3). Then we clarify how this dependence is combined with the magnitude-limited detection threshold of Kepler to sculpt the observed distribution of $P_{\text{rot}}$ as a function of $T_{\text{eff}}$ (Section 4). We also show that the general pattern derived here is consistent with the sample from Santos et al. (2019, 2021) that includes more detections of rotation periods, and that the latter catalog is subject to a different detection function. In Section 5, we discuss our finding in connection with coronal and chromospheric activity indicators and the weakened magnetic braking hypothesis, and propose a test to check the veracity of our view on the detection bias further.

### 2. The Sample

McQuillan et al. (2014) performed a homogeneous search for quasi-periodic brightness modulation associated with stellar rotation in Kepler light curves and reported detections of robust rotational modulation for ≈34,000 stars. All these stars are assigned the rotation period $P_{\text{rot}}$ as determined from the autocorrelation analysis, along with the average amplitude of variability within one period in units of parts-per-million (ppm), $R_{\text{per}}$, defined as the median of the differences between 95th and 5th percentiles of normalized flux in each rotation period cycle. The Large Sky Area Multi-Object Fiber Spectroscopic Telescope (LAMOST) project (Cui et al. 2012; Wu et al. 2014;...
Luo et al. 2015; Ren et al. 2016) provided spectroscopic parameters for \( \approx 60,000 \) Kepler stars in their sixth data release (DR6). We work on the overlap of the two samples, for which \( P_{\text{rot}}, R_{\text{per}} \), and \( T_{\text{eff}} \) have been derived homogeneously.

One concern is the presence of unresolved binaries. The contaminating flux from the secondary affects both the inferred modulation amplitude and stellar classification. Tidal interactions with a close-in companion also affect the rotation period, although such close-in companions would occur in \( \lesssim 10\% \) of Sun-like stars (Raghavan et al. 2010). The analysis may also be complicated by evolved stars whose rotation may have changed due to the evolution of internal structure rather than magnetic braking. We use the information on absolute magnitudes made available by Gaia (Gaia Collaboration et al. 2016) to remove such objects when possible.

We started with 8772 unique stars for which robust periods are detected in McQuillan et al. (2014) and LAMOST DR6 data are publicly available.\(^1\) For LAMOST stars with multi-epoch observations, the mean of \( T_{\text{eff}} \) was adopted.\(^2\) We then used the cross-match service of the Centre de Données astronomiques de Strasbourg (CDS) to find closest Gaia EDR3 sources (Gaia Collaboration et al. 2021) within 5\(^\circ\) and with \texttt{parallax}\texttt{\_}\texttt{over}\texttt{\_}\texttt{error} greater than 10. We found 8309 matches. The difference between the \( G \)-band and Kepler-band magnitudes, the latter taken from Mathur et al. (2017), has the mean of \(-0.02\) and standard deviation of 0.09, indicating correct matches. We then placed these stars on the absolute Gaia magnitude–LAMOST \( T_{\text{eff}} \) diagram, focusing on stars with 4000 K \( < T_{\text{eff}} < 6500 \) K, defined the main sequence by fitting a fifth-order polynomial and iteratively clipping \( 1\sigma \) and \( 3\sigma \) outliers below and above the sequence, respectively, and removed stars deviating by more than 0.5 magnitudes at a given \( T_{\text{eff}} \) from the derived sequence. This removes bright sources that may be either evolved stars or unresolved binaries, where the threshold of 0.5 is chosen to remove the sequence of equal-brightness binaries that are brighter than single stars by 0.75 magnitudes at a given \( T_{\text{eff}} \). This cut left us with 5022 stars. We also removed stars with multiple LAMOST measurements in the remaining sample because they shared the same \( \log T_{\text{eff}} \).

Each of the remaining sample has been removed stars deviating by more than 0.5 magnitudes at a given \( T_{\text{eff}} \), and use it for the discussion of detectability in Section 4.

We started with 8772 unique stars for which robust periods are detected in McQuillan et al. (2014) and LAMOST DR6 data are publicly available.\(^1\) For LAMOST stars with multi-epoch observations, the mean of \( T_{\text{eff}} \) was adopted.\(^2\) We then used the cross-match service of the Centre de Données astronomiques de Strasbourg (CDS) to find closest Gaia EDR3 sources (Gaia Collaboration et al. 2021) within 5\(^\circ\) and with \texttt{parallax}\texttt{\_}\texttt{over}\texttt{\_}\texttt{error} greater than 10. We found 8309 matches. The difference between the \( G \)-band and Kepler-band magnitudes, the latter taken from Mathur et al. (2017), has the mean of \(-0.02\) and standard deviation of 0.09, indicating correct matches. We then placed these stars on the absolute Gaia magnitude–LAMOST \( T_{\text{eff}} \) diagram, focusing on stars with 4000 K \( < T_{\text{eff}} < 6500 \) K, defined the main sequence by fitting a fifth-order polynomial and iteratively clipping \( 1\sigma \) and \( 3\sigma \) outliers below and above the sequence, respectively, and removed stars deviating by more than 0.5 magnitudes at a given \( T_{\text{eff}} \) from the derived sequence. This removes bright sources that may be either evolved stars or unresolved binaries, where the threshold of 0.5 is chosen to remove the sequence of equal-brightness binaries that are brighter than single stars by 0.75 magnitudes at a given \( T_{\text{eff}} \). This cut left us with 5022 stars. We also removed stars with multiple LAMOST measurements in the remaining sample because they shared the same \( \log T_{\text{eff}} \), and use it for the discussion of detectability in Section 4.

\section{3. Evolution of Modulation Amplitudes}

Here, we investigate how the photometric modulation amplitude \( R_{\text{per}} \) evolves as a function of \( P_{\text{rot}} \). As has been shown in previous works (Corsaro et al. 2021; See et al. 2021), the relation between \( R_{\text{per}} \) and \( P_{\text{rot}} \) for roughly solar-mass stars is concisely summarized in terms of the Rossby number \( \text{Ro} = P_{\text{rot}}/\tau_{c} \). We revisit such a relation for our sample stars and use it for the discussion of detectability in Section 4.

Figure 1. Absolute Gaia magnitudes and LAMOST \( T_{\text{eff}} \) of our sample stars (gray dots). Stars removed by the cuts described in Section 2 are shown with open symbols (see the legend). The tan dashed line shows the fitted main sequence (see the text).

\subsection{3.1. Spot-modulation Amplitude versus Rotation Period}

In Figures 2 and 17 in Appendix A, we show \( R_{\text{per}} \) and \( P_{\text{rot}} \) for our sample stars separated into 150 K bins ranging from 4000 to 6400 K. In Figure 2, we show two \( T_{\text{eff}} \) bins separated by \( \sim 1000 \) K to illustrate two typical behaviors: (i) \( R_{\text{per}} \) is roughly constant at shorter \( P_{\text{rot}} \) and exhibits a power-law decay at longer \( P_{\text{rot}} \) and (ii) the transition period, which we denote by \( P_{\text{break}} \), is shorter for hotter stars (bottom panel). The data for other \( T_{\text{eff}} \) fanges in Figure 17 show that the same trend holds continuously over most of the \( T_{\text{eff}} \) range, except for the coolest and hottest stars in the sample (see below).

To quantify this visual trend, we model the data with the following broken power-law function:

\begin{equation}
R_{\text{per}}(P_{\text{rot}}, \theta) = \begin{cases} 
R_{\text{break}} \left( \frac{P_{\text{rot}}}{P_{\text{break}}} \right)^{\beta_{\text{sat}}} & \text{for } P_{\text{rot}} < P_{\text{break}} \\
R_{\text{break}} \left( \frac{P_{\text{rot}}}{P_{\text{break}}} \right)^{\beta_{\text{unsat}}} & \text{for } P_{\text{rot}} > P_{\text{break}}
\end{cases}
\end{equation}

and infer \( \theta = (R_{\text{break}}, P_{\text{break}}, \beta_{\text{sat}}, \beta_{\text{unsat}}) \) for stars in each \( T_{\text{eff}} \) bin. The subscripts “sat” and “unsat” stand for saturated and unsaturated regimes, respectively, following the existing nomenclature—although both regimes here fall within the so-called “unsaturated” regimes of other activity indicators (see Section 5.1). We also model the measurement uncertainties as well as intrinsic scatters around this deterministic relation, assuming that the measured values of \( \ln R_{\text{per}}^{\text{obs}} \) and \( \ln P_{\text{rot}}^{\text{obs}} \) for each star follow independent Gaussian distributions around the model, with common standard deviations \( \sigma_{\ln R_{\text{per}}} \) and \( \sigma_{\ln P_{\text{rot}}} \). We also infer these parameters as well as the “true” value of \( P_{\text{rot}} \) of each star, bringing the total number of parameters to five plus the number of stars in each \( T_{\text{eff}} \) subsample. The likelihood function is therefore

\begin{equation}
\mathcal{L} = \prod_{j} \left[ \mathcal{N}(\ln R_{\text{per}}^{\text{obs},j}; \ln R_{\text{per}}^{\text{true},j}, \sigma_{\ln R_{\text{per}}}) \times \mathcal{N}(\ln P_{\text{rot}}^{\text{obs},j}; \ln P_{\text{rot}}^{\text{true},j}, \sigma_{\ln P_{\text{rot}}}) \right]
\end{equation}

\footnote{\text{1} We used Kepler IDs in “comment” columns in the latter catalog.}

\footnote{\text{2} Scatters of \( T_{\text{eff}} \) from multi-epoch observations are typically smaller than the \( \sim 100 \) K uncertainty estimated for dwarfs (Ren et al. 2016). The bin size in Section 3 is chosen to be larger than this latter value.
}
where \( j \) is the label for stars in each subsample, \( \{ x^j \} \) denotes the set of \( x \) in the subsample, and \( N(\nu; \mu, \sigma) \) is the Gaussian distribution for \( x \) with mean \( \mu \) and standard deviation \( \sigma \).

We consider this broken power-law model as a simple mathematical tool that is useful to quantify how the \( R_{\text{per}} - P_{\text{rot}} \) (or \( \text{Ro} \)) relation depends on \( T_{\text{eff}} \), and do not claim that this function provides the correct description of the relation (nor do we attempt to identify it). Indeed, we see a hint of more detailed structures than described by this model, as will be discussed below.

The inference was performed in a Bayesian manner. We adopt independent prior probability density functions (PDFs) for \( \theta, \sigma_{\ln P_{\text{obs}}}, \sigma_{\ln P_{\text{rot}}}, \) and \( \{ P_{\text{obs}}/P \} \) as summarized in Table 1, and infer the joint posterior PDF for these parameters given the data \( \{ R_{\text{obs}}/R \}, \{ P_{\text{obs}}/P \} \) by drawing samples from the posterior PDF using Hamiltonian Monte Carlo (Duane et al. 1987; Betancourt 2017) with No-U-Turn sampler (Hoffman & Gelman 2011) as implemented in NumPyro (Bingham et al. 2018; Phan et al. 2019).

In each subsample, we only model the stars whose rotation periods fall between their 5th and 95th percentiles. In Figures 2 and 17, those stars omitted from fitting are shown as blue open squares. They are both more sensitive to detection bias against weaker modulation in fainter stars that has not been taken into account in our model: the few rapid rotators tend to be rarer and thus fainter, and the slowest rotators tend to have smaller \( R_{\text{per}} \).

The lack of detection model may be considered as a limitation of our analysis and may introduce systematic errors in the inferred parameters; see Section 3.3 for further discussion.

The solid orange line and shaded region in Figure 2 shows the 5th and 95th percentile of the predictions by the broken power-law model in Equation 1. The inferred \( P_{\text{break}} \) (mean and 5th–95th percentile of the marginal posterior PDF) are also shown with vertical dotted lines and shades. Our model fitting locates the \( P_{\text{break}} \) that is seen visually, which decreases with increasing \( T_{\text{eff}} \). A similar pattern was not found robustly for the lowest \( T_{\text{eff}} \) bin, presumably due to a small number of data points. The break is implied, but the pattern is different for \( T_{\text{eff}} \) when the latter was shorter (longer) than 3 (30) days. The same is true for \( R_{\text{break}} \), with the corresponding thresholds being 0.5 and 2.

**Table 1: Parameters and Priors of the Model**

| Parameters | Priors |
|------------|--------|
| \( \ln P_{\text{break}} \) (ppm) | \( \mathcal{N}(\ln 10^3, 1) \) |
| \( \beta_{\text{unsat}} \) | \( \mathcal{N}(0, 5) \) |
| \( \beta_{\text{sat}} \) | \( \mathcal{N}(0, 5) \) |
| \( \sigma_{\ln P_{\text{break}}} \) | \( \mathcal{N}(1) \) |
| \( \ln P_{\text{obs}}/P_{\text{rot}} \) analysis in Section 3.1 | \( \mathcal{U}(1, 30) \) |
| \( \ln P_{\text{break}} \) (day) | \( \mathcal{U}(\ln P_{\text{obs}}/P_{\text{rot}}, \ln P_{\text{obs}}) \) |
| \( \ln P_{\text{rot}} \) (day) | \( \mathcal{U}(\ln 50, \ln 500) \) |
| \( \ln R_{\text{break}} \) | \( \mathcal{N}(0, 1) \) |
| \( \ln \text{Ro} \) | \( \mathcal{U}(0, 5) \) |

**Note.** \( \mathcal{N}(\mu, \sigma) \) is the normal distribution with mean \( \mu \) and standard deviation \( \sigma \). \( \mathcal{U}(a, b) \) is the uniform distribution between \( a \) and \( b \). \( \mathcal{N}_{\text{sat}}(\sigma) \) is the half-normal distribution with scale \( \sigma \). Parameter \( x^{\text{unsat}} \) denotes the \( \text{nth} \) percentile value of \( x \) in each subsample. The lower (upper) bound for \( P_{\text{break}} \) was set to \( P_{\text{rot}} \) (or \( \text{Ro} \)) when the latter was shorter (longer) than 3 (30) days. The same is true for \( R_{\text{break}} \), with the corresponding thresholds being 0.5 and 2.

3.2. Spot-modulation Amplitude versus Rossby Number

The exact value of \( \tau_c \) depends on how it is estimated. The formula by Noyes et al. (1984) has widely been used, which is based on the theoretical evaluation of the local turnover timescale near the bottom of the convective envelope as described in Gilman (1980) and has been calibrated to minimize the scatter in the log \( R_{\text{HK}}^{-1} - \text{Ro} \) relation. Other scales have been proposed based on up-to-date stellar models and direct inference of the thickness of convective envelopes from asteroseismology (Landin et al. 2010; See et al. 2021; Lehtinen et al. 2021;Corsaro et al. 2021). These works generally yield \( \tau_c \) values that are larger by a factor of a few than those of Noyes et al. (1984) for solar-mass
main-sequence stars and have a different dependence on $T_{\text{eff}}$. In this work, we use the formula in Cranmer & Saar (2011) based on theoretical models of Gunn et al. (1998), simply because it relates $\tau_c$ directly to $T_{\text{eff}}$, which we work on, and because it is close to the traditional scale by Noyes et al. (1984) for dwarf stars (Gunn et al. 1998) and allows for easier comparisons with other works. We find that the typical $R_{\text{per}}$--$R_o$ relation in our sample remains unchanged for the $\tau_c$ prescriptions in Lehtinen et al. (2021) and Corsaro et al. (2021). Thus the following discussion in Sections 4 and 5 is insensitive to which of these prescriptions is adopted, as long as $R_o$ is scaled adequately; see Appendix B.1 for details of these analyses.

Here, we repeat almost the same analysis as in Section 3.1, but replace $P_{\text{rot}}$ with the Rossby number $R_o = P_{\text{rot}}/\tau_c$ evaluated for each star; consequently, the model parameters $P_{\text{break}}$, $\sigma_{\ln R_{\text{rot}}}$, and $\ln P_{\text{rot}}$ in Equation 1 are replaced with $R_{\text{break}}$, $\sigma_{\ln R_o}$, and $\ln R_o$, respectively (the bottom part of Table 1). As noted above, the $\tau_c$ value was calculated using the $\tau_c$--$T_{\text{eff}}$ relation (Equation 36) given in Cranmer & Saar (2011).

Figure 4 is analogous to Figure 2; note that the $x$ axis now shows $R_o$. We see a similar broken power-law pattern as seen in the $R_{\text{per}}$--$P_{\text{rot}}$ plane, but now the break occurs at similar $R_o$ in different $T_{\text{eff}}$ bins. The same is also true in other $T_{\text{eff}}$ bins as shown in Figure 18, which is analogous to Figure 17.

Figure 5 is analogous to Figure 3, where now the break period $P_{\text{break}}$ is replaced with the break Rossby number $R_{\text{break}}$. The break location $R_{\text{break}}$ now depends much less on $T_{\text{eff}}$ than $P_{\text{break}}$ did, while the other parameters remain similar to those in Section 3.1; this is reasonable because $\tau_c$ in each narrow temperature bin is almost the same, and so the transition from $P_{\text{rot}}$ to $R_o$ shifts the whole data set by almost the same amount in the $x$ direction. That said, we also see some correlated patterns; for example, $|\beta_{\text{unsat}}|$ and $R_{\text{break}}$ may be systematically larger at higher $T_{\text{eff}}$. This could be of astrophysical origin, or it may be an artifact due to our imperfect knowledge of $\tau_c(T_{\text{eff}})$; (see Appendix B.1), or due to detection bias (see Section 3.3), or a combination of these effects. It is beyond the scope of this work to account for this possible dependence.

Motivated by the (roughly) $T_{\text{eff}}$-independent nature of the $R_{\text{per}}$--$R_o$ relation, in Figure 6 we show all stars with $T_{\text{eff}} = 4000$ K--6250 K in the $R_{\text{per}}$--$R_o$ plane and fit a single broken power-law relation $R_{\text{per}}(R_o)$ to the entire data set (orange solid line and shade). We find $R_{\text{per}} = (1.08 \pm 0.04) \times 10^4$ ppm, $R_o = 0.84 \pm 0.02$, $|\beta_{\text{unsat}}| = 0.30 \pm 0.09$, and $\beta_{\text{unsat}} = -2.6 \pm 0.1$ (mean and 90%). These values are also shown in Figure 5 with horizontal orange dashed lines, which broadly agree with the values derived in separate $T_{\text{eff}}$ bins. Here, we see that the result from the entire sample tends to be closer to those of hotter stars, simply because they are more numerous in the sample. We discuss possible systematic errors in the inferred parameters due to detection bias against weak modulation further in Section 3.3.

The kink in the $R_{\text{per}}$--$R_o$ relation has been noted in other works. The value of $R_o$ corresponding to the kink has been found to be $\sim 0.4$ in See et al. (2021), who estimated $\tau_c$ using stellar models by Amard et al. (2019); 0.23 in Corsaro et al. (2021) for their $\tau_c$ calibration using seismic stars; and 0.82 when Corsaro et al. (2021) adopted the prescription by Noyes et al. (1984). When scaled by $R_{\odot}$, all these values roughly agree with what we found, $R_o/R_{\odot} \sim 0.4$ ($R_o/R_{\odot} \approx 2$ on our scale). We also note a wiggle for stars with $R_o \sim 0.4$--0.8 on our scale (or 0.2--0.4 $R_{\odot}$), which has also been noted by See.

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3 The same appears to be the case for the analysis by See et al. (2021), according to their Figure 3.

4 The hottest bin does not follow this trend, but in Figure 17 the data at $R_o \gtrsim 1$ are visually consistent with a steeper slope, which corresponds to larger $R_{\text{break}}$. We suspect that the current result might be biased due to a larger fraction of outliers at low and high $R_o$. 

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Figure 3. The means (circles) and 90% intervals (vertical error bars) of the parameters in the $R_{\text{per}}(P_{\text{rot}})$ model (Section 3.1) in different $T_{\text{eff}}$ bins whose widths are shown as horizontal error bars.

Figure 4 is analogous to Figure 2, where the $x$ axis now shows $R_o$. We see a similar broken power-law pattern as seen in the $R_{\text{per}}$--$P_{\text{rot}}$ plane, but now the break occurs at similar $R_o$ in different $T_{\text{eff}}$ bins. The same is also true in other $T_{\text{eff}}$ bins as shown in Figure 18, which is analogous to Figure 17.

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et al. (2021) and is seen in our analyses using other \( \tau_c \) prescriptions (see Figure 19 in Appendix B.1). While we do not understand its origin, we see a hint of a similar structure in the X-ray data, suggesting that this may not be an artifact related to the calibration of \( \tau_c \). See Section 5.1 for further discussion.

3.3. On the Impact of Detection Bias

Figure 6 shows that there exists some dispersion in \( R_{\text{per}} \) at a fixed value of \( Ro \): the dispersion is inferred to be \( \approx 0.24 \) dex from our modeling in Section 3.2. Some of the dispersion may be due to differences in spin-axis inclinations and/or activity cycles. Any systematics in the adopted \( \tau_c-T_{\text{eff}} \) relation can also affect the scatter. In particular, we did not take into account its possible dependence on \([\text{Fe}/\text{H}]\). Indeed, we find that the residuals \( \Delta \log R_{\text{per}} \) of the fit in Figure 6 are correlated with the LAMOST [Fe/H], with the Pearson R coefficient being \( \approx 0.4 \) or \( \Delta \log R_{\text{per}} \approx 0.5[\text{Fe}/\text{H}] \). This is qualitatively consistent with the finding of See et al. (2021) that metal-rich stars have enhanced activities, although the dependence may be weaker than was found to be typical by these authors.

As will be discussed in Section 4, we find evidence that rotational modulation of the fainter stars has been missed due to their larger photometric noise. In the presence of such a threshold, the dispersion in the \( R_{\text{per}}-\text{Ro} \) relation—regardless of its origin—makes modulation with smaller \( R_{\text{per}} \) more likely to be missed, and thus makes the observed slope appear shallower. Correspondingly, \( \text{Ro}_{\text{break}} \) is inferred to be smaller.

To fully understand the impact of the detection bias, we need the complete knowledge of detection function as well as \( \text{Ro} \) values of all the observed stars with and without \( P_{\text{rot}} \) detections, which is impractical. Instead, here we repeat the same analysis as in Section 3.2 for the brightest stars with \( T_{\text{eff}} = 4000-6250 \) K, using only the stars with the Kepler magnitudes \( K_p < 12 \) for which detection bias appears to be minimal (see also Section 4). Due to the correlation between \( K_p \) and \( T_{\text{eff}} \), the

\[
\begin{align*}
\text{Figure 4.} & \text{ Spot-modulation amplitudes } R_{\text{per}} \text{ and Rossby numbers } \text{Ro} = P_{\text{rot}}/\tau_c \text{ for stars with } T_{\text{eff}} = 4750-4900 \text{ K (top) and with } T_{\text{eff}} = 5800-5950 \text{ K (bottom). Here, } \tau_c \text{ is based on the formula in Cranmer & Saar (2011). Gray circles: data points. Blue open squares show the ones that were not used for modeling. Orange solid line and shade: broken power-law model. Vertical orange dotted line and shade: inferred location of the break, } \text{Ro}_{\text{break}}. \text{ See Section 3.2 for details.}
\end{align*}
\]

\[
\begin{align*}
\text{Figure 5.} & \text{ The means (circles) and 90\% intervals (vertical error bars) of the parameters in the } R_{\text{per}}(\text{Ro}) \text{ model (Section 3.2) in different } T_{\text{eff}} \text{ bins whose widths are shown as horizontal error bars.}
\end{align*}
\]

\[
\begin{align*}
\text{Figure 6.} & \text{ Spot-modulation amplitudes } R_{\text{per}} \text{ and Rossby numbers } \text{Ro} = P_{\text{rot}}/\tau_c \text{ for stars with } T_{\text{eff}} = 4750-4900 \text{ K (top) and with } T_{\text{eff}} = 5800-5950 \text{ K (bottom). Here, } \tau_c \text{ is based on the formula in Cranmer & Saar (2011). Gray circles: data points. Blue open squares show the ones that were not used for modeling. Orange solid line and shade: broken power-law model. Vertical orange dotted line and shade: inferred location of the break, } \text{Ro}_{\text{break}}. \text{ See Section 3.2 for details.}
\end{align*}
\]
resulting sample is mostly limited to stars with $T_{\text{eff}} > 5500$ K. The result is shown in Figure 7; note that the data typically extend down to lower $R_{\text{per}}$ than in Figure 6. From this analysis, we find $R_{\text{break}} = 0.99^{+0.07}_{-0.08}$ and $\beta_{\text{unsat}} = -4.6 \pm 0.9$ (mean and 90% interval). These values differ by $\sim 0.1$ and $\sim 1$ from those inferred for stars with $T_{\text{eff}} \gtrsim 5500$ K (Figure 5), in the directions consistent with what we expect from the detection bias as discussed above. This should be considered as systematics unaccounted for in our analysis in Sections 3.1 and 3.2.

As will be discussed in Section 4 further, cooler Kepler stars tend to have larger apparent magnitudes than the hotter ones due to their lower intrinsic luminosities. This causes cooler dwarfs to have higher $R_{\text{per}}$ thresholds, as is evident in Figure 17 (and Figure 9). Therefore the above bias is more severe for cooler stars, and the trend we see in Figure 5 for $\beta_{\text{unsat}}$ and $R_{\text{break}}$ may in part be explained by this $T_{\text{eff}}$-dependent bias. As expected, however, the $R_{\text{per}}$–$R_{\text{o}}$ relation inferred from the brightest stars remains unchanged at $R_{\text{o}} < R_{\text{break}}$. Thus the $T_{\text{eff}}$ dependence of $R_{\text{break}}$ seems real. This could be due to smaller spot coverage fractions in cooler stars, or smaller spot contrasts, or both.

### 3.4. The Santos Sample

While this paper mainly focuses on the McQuillan et al. (2014) sample, it is useful to consult other $P_{\text{rot}}$ catalogs to better understand the applicability and limitations of the results based on this specific catalog. We thus analyze the catalog by Santos et al. (2019, 2021) that provided a larger number of $P_{\text{rot}}$ measurements than McQuillan et al. (2014). We applied the same selection as described in Section 2 to the stars in Santos et al. (2019, 2021) and found 8713 (7621) stars with $4000 < T_{\text{eff}} < 6500$ (6250) K.

Figure 8 compares the photometric modulation amplitudes and Rossby numbers of the stars in this Santos sample (orange dots) against those in the McQuillan sample (gray circles), where the latter sample is thinned by a factor of 10 to improve the visibility while showing the main trend. Because the modulation amplitudes in the Santos catalog are given using the proxy $S_{\text{ph}}$ (García et al. 2010; Mathur et al. 2014), here the values of $S_{\text{ph}}$ are scaled uniformly by 3.6, which is the median...
of $R_{\text{per}}/S_{\text{ph}}$ for stars in both samples. Despite the simpleness of the conversion, the figure shows that the distributions of amplitudes and Ro in the two samples are very similar, except that the Santos sample reports $P_{\text{rot}}$ for more stars with smaller amplitudes than in the McQuillan sample. Quantitatively, the median-filtered $R_{\text{per}}$–Ro relation in the Santos sample (thick orange line) follows more closely to the $R_{\text{per}}$–Ro model derived from the $K_p < 12$ stars in the McQuillan sample (black solid line), at least at larger Ro. This supports our argument regarding the detection bias in Section 3.3: we argued that the $R_{\text{per}}$–Ro relation based on all the stars (black dashed line) is shallower than that derived from the brightest stars (black solid line) because the former is biased against stars with weaker modulation, and here we see that the Santos catalog that is apparently less biased against stars with weaker modulation indeed follows the steeper relation.

Interestingly, the amplitude in the Santos sample appears to plateau again at $Ro > Ro_\ast$. This hints that the modulation amplitude may not keep decreasing in the same way as in $Ro < Ro_\ast$. We note, however, that a more careful assessment of the detection function is required to confirm whether this is a typical behavior or not. As will be discussed in detail below, the measured photometric amplitudes in this region are close to the photometric noise level of Kepler. It is thus conceivable that only the highest variability stars at a given Ro are seen here and/or that the measured amplitudes may be sensitive to how one corrects for the photon noise; although the latter is taken into account in the analysis of Santos et al. (2021), the authors also comment on difficulties associated with small-amplitude modulation. We leave the detailed study of the amplitude–Ro relation in this region for a future work. The following discussion is not affected by this ambiguity, because we will show that those stars mostly fall below the detection limit in the McQuillan et al. (2014) sample, anyway—unless the $R_{\text{per}}$–Ro relation turns up at larger Ro.

4. Detection Edge in the McQuillan Sample

Now we attempt to clarify how the $R_{\text{per}}$–$P_{\text{rot}}$ (Ro) relation discussed in Section 3, when combined with the detection bias, sculpts the longer-period edge of the $P_{\text{rot}}$–$T_{\text{eff}}$ distribution in the Kepler sample.

The $P_{\text{rot}}$–$T_{\text{eff}}$ distribution of Kepler stars has been known to exhibit a rather well-defined upper edge. One perplexing aspect of this upper edge is that it does not correspond to a constant variability amplitude, i.e., the lower edge of the $R_{\text{per}}$–$T_{\text{eff}}$ distribution is not flat. These features are also apparent in our sample (Figure 9).

McQuillan et al. (2014) pointed out that the upper $P_{\text{rot}}$–$T_{\text{eff}}$ edge lies roughly around a gyrochrone of the solar age. van Saders et al. (2019) pointed out that the upper edge is around $Ro \sim 2$ and discussed the possibility that this is related to detection bias. Another explanation they proposed is that the edge is due to stalled spin down: the stars stop spinning down once they reach $Ro \sim 2$ (van Saders et al. 2016) and stay around the edge. The presence of stalled spin down has now been supported by multiple studies, as mentioned in Section 1; more recently, the pile-up in the $P_{\text{rot}}$–$T_{\text{eff}}$ distribution around its upper edge (also apparent in the top panel of Figure 9) has been reported and argued to provide additional support for the stalled spin down scenario (David et al. 2022). Nevertheless, these arguments for the presence of stalled spin down do not necessarily exclude the possibility that the edge in the McQuillan et al. (2014) sample is shaped by detection bias.

4.1. Evidence for the Noise-dependent Cutoff

Here, we argue that this edge results from the detection threshold set by photometric precision of Kepler that depends on apparent magnitudes of stars in the Kepler band, $K_p$. We first note, in Figure 10, that the distribution of $R_{\text{per}}$ and $K_p$ for all the stars with a $P_{\text{rot}}$ detection in McQuillan et al. (2014) has a sharp lower edge with a positive slope, whose value at $K_p \geq 13$ is not far from the scaling for pure photon noise: $R_{\text{per}} \propto 10^{K_p/3}$. This indicates that the detectability is limited by photometric precision for those fainter stars, which comprise the majority of the sample. In the top panel of Figure 11, we show $R_{\text{per}}$ normalized by the photometric precision $\sigma_{\text{Kep}}$ for long-cadence (29.4 min) exposures of Kepler for each star.
against $T_{\text{eff}}$. Here, $\sigma_{\text{Kep}}$ was evaluated using the photometric precision estimated by the Kepler team as a function of $K_p$, which takes into account noise sources other than the photon noise and is applicable to $K_p \lesssim 12.6$. We do not use the Combined Differential Photometric Precision (CDPP; Koch et al. 2010) commonly used for evaluating noise levels relevant to planet search, because here we need to evaluate the noise that does not include intrinsic stellar variabilities. In this plane, the lower edge of the $R_{\text{per}}/\sigma_{\text{Kep}}$ distribution is flat across $T_{\text{eff}}$, again indicating that the sample is limited by photometric precision: the lower edge in the $R_{\text{per}}-T_{\text{eff}}$ distribution (Figure 9, bottom panel) is higher for cooler stars because they tend to be apparently (and intrinsically) fainter than the hotter ones. The lower edge of $R_{\text{per}}/\sigma_{\text{Kep}}$ is also flat as a function of $P_{\text{rot}}$ and $R_o$, as shown in the middle and bottom panels of Figure 11.

The histogram of $R_{\text{per}}/\sigma_{\text{Kep}}$ (Figure 12) shows a sharp cutoff at around 3 (bottom $\approx 2\%$), which we adopt as an empirical detection threshold of the sample to guide the present discussion. This value, shown as the tan horizontal dashed line, agrees visually with the lower edge of the distributions in Figure 11, and also well explains the difference between the $R_{\text{per}}$ distribution of stars with $T_{\text{eff}} = 4000-6500$ K in the McQuillan et al. (2014) sample and of stars with $K_p < 12$. We note that this threshold value is specific to the McQuillan et al. (2014) sample, as well as to the timescale for which photometric precision is defined. We suspect that this rather steep cutoff is associated with the threshold on the weight parameter $w$ that was used by McQuillan et al. (2014) to distinguish between periodic and false detections. This parameter is related to the local peak height of the autocorrelation function that would explicitly depend on the noise level. We also note again that Figure 10 shows all the...
stars with significant \( P_{\text{rot}} \) detections in McQuillan et al. (2014); thus the lower edge is not due to our sample selection.

### 4.2. Rossby Number Cutoff

Given the presence of the detection edge, the next question is what value of \( R_o \) (or \( P_{\text{rot}} \)) this edge corresponds to—and we find the detection edge should correspond to \( R_o \sim 1–2 \) in the Cranmer & Saar (2011) scale, or \( R_o \sim 0.5–1 \). This value is derived by equating the roughly \( T_{\text{eff}} \)-independent \( R_{\text{per}}(R_o) \) derived in Sections 3.2 and 3.3 with \( 3 \sigma_{\text{Kep}}(K_p) \) of each star and by solving for \( R_{\text{threshold}}(K_p) \): rotational modulation is detectable for a given star with the magnitude \( K_p \) if its \( R_o \) is lower than \( R_{\text{threshold}}(K_p) \). The distribution of \( R_{\text{threshold}}(K_p) \) computed in this way for all the stars with which rotational modulation has been searched by McQuillan et al. (2014) (i.e., stars with and without \( P_{\text{rot}} \) detection) is shown in the top panel of Figure 13. The solid histogram shows the result based on the \( R_{\text{per}}-R_o \) relation derived in Section 3.3 using the brightest \((K_p < 12)\) stars with \( T_{\text{eff}} = 4000–6250 \) K, which is likely less affected by detection bias and more representative (see also Section 3.4); the dashed histogram shows the result based on the relation derived using all the sample stars (Section 3.2), which may be more appropriate for cooler stars. Both distributions are sharply peaked around \( R_o \approx 1.2–0.6 \), which corresponds to the faintest (and hence most abundant) stars with \( K_p \sim 16 \). The peak is also narrow because of the strong \( P_{\text{rot}} \) dependence of \( R_{\text{per}} \): for \( R_{\text{per}} \sim R^{1.2–0.4}_o \), \( R_{\text{threshold}}(K_p) \) increases only by a factor of two for the \( K_p \) difference of five. By definition of \( R_{\text{threshold}} \), its normalized inverse cumulative distribution function, shown in the bottom panel of Figure 13, provides the detectability function \( p_{\text{det}}(R_o) \), the fraction of stars in the searched sample for which rotational modulation of a star with a given value of \( R_o \) would have been reported as a robust detection by McQuillan et al. (2014). We see that faint stars start to be missed at \( R_o \gtrsim 1.2 \), and that the detection becomes impossible for almost all stars at \( R_o \sim 0.5–1 \) for the \( R_{\text{per}}-R_o \) relation derived from the brightest stars (solid line), which is likely more representative than that from all stars (dashed line). This is how the combination of the rapid drop of \( R_{\text{per}} \) with increasing \( R_o \) and the roughly magnitude-limited nature of the Kepler sample imprints the detection edge around \( R_o \lesssim 2 \approx R_{\text{rot}} \); the sample becomes roughly \( R_{\text{rot}} \)-limited, and the longest detected \( P_{\text{rot}} \) increases with decreasing \( T_{\text{eff}} \) roughly as \( \tau_{\text{rot}} \). This agrees with what is observed, that the upper edge is close to a curve of \( R_o = 1.7 \) (blue dashed line in the top panel of Figure 9) on our \( \tau_{\text{rot}} \) scale. It is beyond the scope of this work to understand this value more quantitatively: the threshold depends on the unknown distribution of \( R_o \) in all the searched stars (i.e., stars with and without detected \( P_{\text{rot}} \)), as well as on exact dependence of the detectability on the signal-to-noise ratio, both of which need to be modeled. It is also very sensitive to the steepness of the \( R_{\text{per}}-R_o \) relation, as shown in Figure 13, as well as on its possible dependence on \( T_{\text{eff}} \), which is difficult to assess in the current sample (Section 3.3 and Appendix B.1).

The above argument, along with external evidence for stalled spin down, suggests that both effects discussed by van Saders et al. (2019) are important for understanding the \( P_{\text{rot}}-T_{\text{eff}} \) distribution around its upper edge. Around the time when typical Sun-like stars observed by Kepler cease to spin down at \( R_o \sim 0.5 \), their rotational modulation signals have already started to disappear under photometric noise and to be missed from the sample with \( P_{\text{rot}} \) detection. Therefore, the pile-up we see in the observed \( P_{\text{rot}}-T_{\text{eff}} \) distribution may be just a tip of the iceberg: the true pile-up may be located at longer periods but may have been capped due to the detection edge. This interpretation is qualitatively consistent with the finding of Hall et al. (2021) and Masuda et al. (2022), who worked on the \( P_{\text{rot}} \) sample much less biased against slower rotators and found that most stars in the sample are around or above the upper edge defined by stars with \( P_{\text{rot}} \) from rotational modulation (see Figure 9 of Masuda et al. 2022).
is constructed by sampling age $t$ from the uniform distribution between 0 and 10 Gyr, translating $t$ into $P_{\text{rot}} = 25 \text{ days}(t/4.6 \text{ Gyr})^{1/2}$ and to $\text{Ro} = P_{\text{rot}}/\tau_{1}(5777 \text{ K})$ using the formula in Cranmer & Saar (2011): this simulates a collection of Sun-like stars that (i) have a uniform age distribution, (ii) have the same $K_p$ distribution as the Kepler stars, and (iii) keep spinning down following Skumanich’s law during their entire main-sequence life. This is merely another toy model but provides two useful insights. First, the sharp decrease of $P_{\text{det}}$ at $\text{Ro} \geq 1$ produces a peak in the observed distribution (orange thin histogram) as long as the true Ro distribution keeps increasing across the threshold Ro, even without stalled spin down. Second, the fraction of stars with detectable rotational modulation (i.e., the mean value of $P_{\text{det}}(\text{Ro})$ in the sample) is computed to be 0.28, which is close to the observed value (McQuillan et al. 2014): a combination of the sharp detection edge and the top-heavy Ro distribution provides a reasonable explanation for why $P_{\text{rot}}$ has not been detected for the majority of solar-mass stars. This experiment suggests that stalled spin down, if real, should start operating at $\text{Ro} \geq 2$ so that a significant fraction of solar-mass stars evade detection of rotational modulation. It may also explain why the pile-up found by David et al. (2022) corresponds to a lower $\text{Ro} \sim 1.5$ than that inferred from asteroseismology or $v \sin i$ ($\text{Ro} \sim 2$). The last model discussed here might even suggest that the pile-up of Ro alone does not serve as conclusive evidence for the stalled spin down, as it shows that the observed distribution is not very sensitive to the Ro distribution above the detection edge. We note that this argument does not deny the importance of possible systematic offsets in $T_{\text{eff}}$ as discussed by David et al. (2022). Nevertheless, these toy models demonstrate the importance of considering detection bias when interpreting the observed $P_{\text{rot}}$ distribution.

### 4.4. Comparison with the Santos Sample

We saw in Section 3.4 that the $P_{\text{rot}}$ catalog by Santos et al. (2019, 2021) includes more stars with smaller modulation amplitudes than that of McQuillan et al. (2014). This suggests that the Santos sample is subject to a very different detection function. This situation is shown in Figure 15; here we reproduce Figure 12 and Figure 10 for the Santos sample, where $S_{\text{ph}}$ is used instead of $R_{\text{per}}$ and the empirical threshold in the McQuillan sample is shown in the scale of $S_{\text{ph}}$ (see Section 3.4). Although the top panel does show the decrease in the detection rate at a signal-to-noise ratio corresponding to the McQuillan edge, we do not find such a sharply defined threshold as seen in the McQuillan sample. Correspondingly, we do not see a well-defined lower edge in the amplitude–magnitude plane in the bottom panel. We also find that the number of stars “leaking” below the McQuillan threshold increases toward higher $T_{\text{eff}}$. This comparison illustrates the importance of considering detection functions in a sample-specific manner. While the Santos sample includes more period detections than the McQuillan sample, the detection function might also be more difficult to quantify.

Although it is beyond the scope of this work to fully assess the impact of detection bias in the Santos sample, the following arguments suggest that it is likely significant in the Santos sample, too, at least for nearly solar-mass stars. First, the detection fraction of $P_{\text{rot}}$ is still $\approx 30\%$ for G stars even in Santos et al. (2021). Second, the steep $R_{\text{per}}$–Ro relation implies that the longest detectable $P_{\text{rot}}$ (or largest detectable Ro) is not drastically changed by improving the detection threshold: for
5. Discussion

5.1. Comparison with Other Activity Indicators

The $R_{\text{per}}$–$R_{\text{rot}}$ relation presented in Section 3 is reminiscent of the relation known for X-ray luminosities normalized by the bolometric values $L_X/L_{\text{bol}}$ (e.g., Pizzolato et al. 2003): $L_X/L_{\text{bol}}$ plateaus at $R_{\text{rot}} \gtrsim 0.1$, and decays as $L_X/L_{\text{bol}} \sim R_{\text{rot}}^{-2.7 \pm 0.13}$ at least up to $R_{\text{rot}} \sim 2$ (Wright et al. 2011). The analysis of Wright et al. (2011) is based on the $\tau_c$ scale from Noyes et al. (1984) that is close to what we have adopted, and so the saturation of spot-modulation amplitude at $R_{\text{rot}} \sim 0.8$ occurs within the so-called unsaturated regime of $L_X/L_{\text{bol}}$, where it exhibits a power-law decay. Does this mean the X-ray activity and spot-modulation amplitude evolve differently as a function of $R_{\text{rot}}$ despite their presumably common origin?

The analysis of Wright et al. (2011) assumes a two-piece power law and has captured a transition at $R_{\text{rot}} \sim 0.1$, which therefore is insensitive to finer structures at larger $R_{\text{rot}}$. Thus, here we seek for evidence of another transition in the “unsaturated” X-ray regime. Figure 16 shows the log $L_X/L_{\text{bol}}$–$R_{\text{rot}}$ data (gray open circles) from Wright et al. (2011) along with log $R_{\text{per}}$–$R_{\text{rot}}$ data in our sample (orange dots), where the scale of $R_{\text{per}}$ is shifted arbitrarily but that of $R_{\text{rot}}$ is not. Here, we recomputed $R_{\text{rot}}$ in the Wright et al. (2011) sample using their $T_{\text{eff}}$ and the Cranmer & Saar (2011) relation so that the comparison can be made using the same $\tau_c$ scale. We see that the two data sets in fact follow the same pattern at $R_{\text{rot}} \gtrsim 0.3$ including a possible wiggle mentioned in Section 3: there is a hint of a “shoulder” in the $L_X/L_{\text{bol}}$ data beginning around $R_{\text{rot}}$, inferred from photometric modulation, which is also apparent in Figure 7 of Mittag et al. (2018). The structure is more clearly seen in the median-filtered data with the width of 0.06 dex (solid gray line), i.e., a representation that does not assume a single power-law relation in this $R_{\text{rot}}$ range. This reinforces the physical connection between surface spots and coronal X-ray emission, and suggests that the structure in the unsaturated X-ray regime is not an artifact; remember that the $R_{\text{per}}$–$R_{\text{rot}}$ relation shows a kink regardless of the prescription to compute $\tau_c$.

A break at a similar value of $R_{\text{rot}}$ in the chromospheric activities has been noted (e.g., Noyes et al. 1984; Rutten 1987; Lehtinen et al. 2021), which Lehtinen et al. (2021) attributed to a transition of dominant dynamo regimes. Although Lehtinen et al. (2021) reported a break at a lower value of $R_{\text{rot}}$ than $R_{\text{rot}}$, the location agrees with what has been inferred from $R_{\text{per}}$ and $R_{\text{rot}}^\prime$ if the common $\tau_c$ scale is adopted, and it is also confirmed by their conversion that the threshold is $R_{\text{rot}} \sim 0.91$ in the Noyes scale. The data for main-sequence stars from Lehtinen et al. (2020) and Kepler asteroseismic stars from Metcalfe et al. (2016), along with the overlapping sample from Mamajek & Hillenbrand (2008) and Mittag et al. (2018) for which both $R_{\text{per}}$ and $R_{\text{rot}}^\prime$ are readily available, are plotted with green open squares in Figure 16, which shows a kink at $\log R_{\text{rot}}' \sim -4.5$ as reported by Lehtinen et al. (2020). Here again we use the Cranmer & Saar (2011) formula to recompute $\tau_c$ for those stars, where $T_{\text{eff}}$ is estimated from $B - V$ using the table from Pecaut & Mamajek (2013). We also remark that the similarity between the chromospheric and X-ray fluxes has been noted by Mittag et al. (2018).

A transition at a similar $R_{\text{rot}}$ might also have been seen, though less clearly, in the photospheric filling factor $f_{\text{fl}}$ of the magnetic flux. Note again that here we are focusing on the region around $R_{\text{rot}} \sim 1$, rather than on the saturation similar to that in X-ray around $R_{\text{rot}} \sim 0.1$ (Reiners et al. 2009; Vidotto et al. 2014). The measurements for GKM stars presented in Cranmer & Saar (2011)—along with that for the Sun—shows that $f_{\text{fl}}$ decreases by roughly two orders of magnitudes between $R_{\text{rot}} \sim 0.2$ and 2. Although the data do not densely cover $R_{\text{rot}}$ around $R_{\text{rot}}$, seen in the spot-modulation amplitude, the empirical scaling they found, $f_{\text{fl}} \propto R_{\text{rot}}^{-2.5}$ to $f_{\text{fl}} \propto R_{\text{rot}}^{-3.4}$, is roughly in agreement with what we found for $R_{\text{per}}$ (Mittag et al. 2018) also found a hint of a similar trend in the Hα luminosity of M dwarfs studied by Newton et al. (2017). These data are not shown in Figure 16, because the transition features are visually less clear.

In summary, coronal and chromospheric fluxes (and perhaps magnetic and Hα fluxes as well) show transitions at $R_{\text{rot}}$ similar to $R_{\text{rot}} \sim 0.4$ found for the photometric modulation amplitude, thus suggesting that they share the same physical origin. We also confirmed that the same remains to be the case when the $\tau_c$ prescription from Lehtinen et al. (2021) is adopted instead; see Figure 20 in Appendix B.2. Although our sample does not constrain the $R_{\text{per}}$–$R_{\text{rot}}$ relation at $R_{\text{rot}} \lesssim 0.2$, other ground- and space-based photometry works generally show even larger spot-modulation amplitudes of up to $\sim 10\%$ for younger stars with shorter rotation periods (e.g., Hartman et al. 2009; Rebull et al. 2016; Morris 2020). Thus the evolution at lower $R_{\text{rot}}$ may also be similar to the X-ray and chromospheric
fluxes (see also, e.g., Figure 7 of Mamajek & Hillenbrand 2008, for the latter).

5.2. Implications for Weakened Magnetic Braking

van Saders et al. (2016) proposed that magnetic braking ceases at a critical Rossby number of $R_{\text{WMB}} \sim R_{\odot}$ based on a comparison between their spin evolution models and the age/rotation measurements for $\sim 20$ stars. As we saw in the previous section, the information regarding how various activity indicators evolve around $R_{\odot} \sim R_{\odot}$ is in general limited (Figure 16), but some indicators may show hints of corresponding changes. A transition at $R_{\odot} \sim R_{\odot}$ has been suggested in the chromospheric fluxes (Metcalfe et al. 2016). A small number of measurements in the Santos sample hints that $R_{\text{per}}$ may also follow a similar pattern at $R_{\odot} \gtrsim R_{\odot}$ (Section 3.4), although a more careful analysis would be required to confirm whether this is a typical behavior or not, because the detection bias is significant here (Section 4).

On the other hand, all the indicators show that the activity pattern changes in a continuous but nonmonotonic manner up to $R_{\odot}$. Thus it is also conceivable that the departure from the standard spin evolution starts earlier than $R_{\odot}$ and proceeds gradually. In particular, the decrease in $R_{\text{per}}$ at $R_{\odot} > 0.5 R_{\odot}$ may indicate that large spots suddenly start dissolving into smaller pieces. If so, this seems qualitatively consistent with a scenario in which the concentration of the magnetic fields into smaller spatial scales and the associated reduction of angular momentum loss is responsible for weakened magnetic braking (van Saders et al. 2016; Réville et al. 2015). We note, though, that the relation between the photometric light curves and spot distribution is generally very complicated (e.g., Luger et al. 2021) and that the modulation amplitude may not be readily translated into the largest spot size. More in-depth analyses of the light-curve morphology as a function of Ro may bring this hypothesis into sharper focus (e.g., Montet et al. 2017; Reinhold et al. 2019).

Regardless of whether or not the pattern in the $R_{\text{per}}$-Ro relation we derived is physically related to weakened magnetic braking, our finding has important implications for studies of weakened magnetic braking using rotation periods from photometric modulation. We presented evidence that detection bias becomes particularly important in the relevant Ro range (Section 4). The subtlety arises from the fact that the Ro corresponding to detection edge, $R_{\text{edge}}$, happens to be close to $R_{\text{WMB}}$. This is in part a coincidence, because $R_{\text{edge}}$ is determined by the photometric precision of Kepler. However, it is also true that the strong dependence makes $R_{\text{edge}}$ insensitive to photometric precision—so, if $R_{\text{break}} \sim R_{\text{WMB}}$ due to physics, it is inevitable that $R_{\text{edge}} \sim R_{\text{WMB}}$. Given $R_{\text{per}} \sim R_{\odot} ^{-4}$ is found for solar-mass stars, the photometric precision needs to be improved by an order of magnitude to push $R_{\text{edge}}$ up by a factor of two. This argument explains why it has not been easy to find the signature of weakened magnetic braking in the photometric sample, and indicates it is crucial to consider detection bias when interpreting the sample quantitatively in terms of the weakened magnetic braking scenario.

5.3. An Additional Test of the View

We argued in Section 4 that the detectability of photometric rotational modulation in Kepler stars is determined by the combination of the (roughly) $T_{\text{eff}}$-independent steep $R_{\text{per}}$-Ro relation and magnitude-dependent detection threshold. If this is correct, photometric rotational modulation should have been detected for Sun-like main-sequence stars if and only if a star is younger than a certain $T_{\text{eff}}$-dependent threshold age. In the other extreme case where the upper edge in the $P_{\text{rot}}$-$T_{\text{eff}}$ distribution is solely due to stalled spin down, by contrast, stars with photometrically detected $P_{\text{rot}}$ around the edge should contain many stars older than the age corresponding to the onset of the stalled spin down. We investigate this hypothesis in a companion paper (K. Masuda 2022, in preparation) using the isochronal age estimates for a sample of Kepler stars with and without detected rotational modulation.

The data and the code underlying this article are available through GitHub.7

Figure 16. Rossby number dependence of spot-modulation amplitude (orange dots), X-ray to bolometric luminosity $L_X/L_{bol}$ (open gray circles), and log $R_{\text{HK}}$ (open green squares). For spot-modulation amplitude and $L_X/L_{bol}$, the solid lines show the median-filtered data with the width of 0.02 and 0.06 dex, respectively.
Appendix A

The Effective Temperature Dependence of the $R_{\text{per}}$–$P_{\text{rot}}$ and $R_{\text{per}}$–$R_{\odot}$ Relations

Figures 17 and 18 show the $R_{\text{per}}$–$P_{\text{rot}}$ relations and the $R_{\text{per}}$–$R_{\odot}$ relations in different $T_{\text{eff}}$ bins, respectively.
Figure 17. Spot-modulation amplitudes $R_{\text{per}}$ and Rossby numbers $R_{\text{o}}$ for stars in each $T_{\text{eff}}$ bin. Gray circles: data points. Blue open squares show the ones that were not used for modeling. Orange solid line and shade: broken power-law model. Vertical orange dotted line and shade: inferred location of the break, $R_{\text{o, break}}$. See Section 3.2 for details.
Figure 17. (Continued.)
Figure 18. Spot-modulation amplitudes normalized by Kepler photometric precision $R_{\text{phot}}/\sigma_{\text{Kep}}$ and Rossby numbers $Ro$ for stars in each $T_{\text{eff}}$ bin. The plots show at which $Ro$ stars with different $T_{\text{eff}}$ cross the detection threshold (horizontal dashed line). See Section 4 for details.
Appendix B
Analyses Adopting Different $\tau_c$ Prescriptions

B.1. Relation between $R_{\text{per}}$ and $R_o$

We have adopted the $\tau_c$ prescription by Cranmer & Saar (2011) in the main text (see Section 3.2). For nearly solar-mass stars, the difference compared to the more recent scales is almost multiplicative and only shifts the $R_o$ axis by a factor of a few (see Figure 9 of See et al. 2021; Lehtinen et al. 2021). However, the difference becomes larger for stars much cooler or hotter than the Sun in a manner that depends on $T_{\text{eff}}$.

To check how sensitive our conclusions might be on the adopted $\tau_c$ relation, we computed $\tau_c$ for our sample stars using the prescriptions in Lehtinen et al. (2021) and Corsaro et al. (2021).
for which the necessary information is readily available. For the former, we first converted $T_{\text{eff}}$ to $B-V$, used it to compute $r_c$ in the Noyes scale, and converted it to $r_c$ from YaPSI using Equation (1) of Lehtinen et al. (2021). For the Consaro et al. prescription, we used Equation (11) of Corsaro et al. (2021); here we omitted stars with $G_{\text{BP}} - G_{\text{RP}} > 1$ for which an asteroseismic calibration was not performed. The resulting $R_{\text{per}}$--$r_c$ relations are compared to what we adopted in the main text in Figure 19. Here, we scaled $r_c$ by the solar value $r_{\odot}$ computed for each prescription so that the shape of the $R_{\text{per}}$--$r_c$ relations can be compared between different prescriptions.

Overall, the results show that $R_{\text{per}}$--$r_c$ relations based on these prescriptions are similar to the one we found using the formula in Cranmer & Saar (2011). More points exist below our standard $R_{\text{per}}$--$r_c$ relation for $r_c$ from Corsaro et al. (2021; bottom panel), which also appear to exist in their Figure 2. The presence of these points, however, does not affect our conclusion regarding the impact of detection bias: if some stars indeed fall below the $R_{\text{per}}$--$r_c$ relation we assumed, the bias simply becomes even more significant.

The difference in the scatter of $R_{\text{per}}$ at a given $r_c$, however, does indicate that the $T_{\text{eff}}$ (in)dependence of the $R_{\text{per}}$--$r_c$ relation is sensitive to the adopted $r_c$ prescription. In Section 3 we found that its shape is not very sensitive to $T_{\text{eff}}$, but this is not guaranteed in other prescriptions. This fact—in addition to the $T_{\text{eff}}$-dependent detection bias discussed in Section 3.3—makes it even more difficult to study a possible $T_{\text{eff}}$ dependence. Again, this subtlety does not alter the discussion in Section 4 because it mainly relies on an empirical fact that the typical $R_{\text{per}}$ is well predicted by $r_c$ within a certain dispersion, but it may need to be taken into account in quantitative analyses of the observed population that explicitly model the detection function.

**B.2. Comparison between Different Activity Indicators**

In Figure 20, we reproduce Figure 16 using the $r_c$ prescription in Lehtinen et al. (2021) as computed in Appendix B.1. We see transitions in all indicators around $r_c \sim 0.3$, which is $\sim 0.4 r_{\odot}$ in this prescription that gives $r_{\odot} \approx 0.8$. So, this location agrees with what we found in the
main text. We did not do the same for the Corsaro et al. (2021) prescription, because it is not applicable to most of the X-ray sample stars that are less massive than the Sun.

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Figure 20. Same as Figure 16, except that $\tau_c$ is computed for all the data sets following Lehtinen et al. (2021).

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