Invited Comment

Are inertial forces ever of significance in cricket, golf and other sports?

Garry Robinson\(^1\) and Ian Robinson\(^2,3\)

\(^1\)School of Physical, Environmental and Mathematical Sciences, The University of New South Wales, Australian Defence Force Academy, Canberra ACT 2600, Australia
\(^2\)Centre for International Research on Education Systems, Victoria University, Queen Street Campus, PO Box 14428, Melbourne VIC 8001, Australia

E-mail: g.robinson@adfa.edu.au and robinson.ian.503@gmail.com

Received 30 December 2016, revised 9 February 2017
Accepted for publication 28 February 2017
Published 30 March 2017

Abstract

In previous papers we investigated the motion of a spherical projectile rotating about an arbitrary axis, subject to a drag force, a lift or Magnus force, and in the presence of a wind. The aim was to determine the motion of balls used in sporting games, primarily cricket. Newton’s laws of motion apply in an inertial (unaccelerated) coordinate system, but the rotating Earth is not an inertial system. In such a non-inertial system two additional forces are present, the Coriolis force which produces a side-ways movement, and the centrifugal force. Generally these two inertial forces produce noticeable effects only on the large scale, when either the time of travel and/or the path length is large. In this paper we have added both of these forces to the equations of motion. In addition, we have also included a ground friction force in order to simulate a ball rolling over a surface or, more generally, a body moving through a resistive medium such as water. Here we quantitatively investigate the magnitude and direction of the effect of the inertial forces in a number of cases. It is found that, as expected, the effects of the inertial forces are generally small for ball games. In cricket the side-ways movement is \(\leq 1\) cm for a throw from the boundary and \(\leq 1\) mm for a slow bowler’s delivery, and for a long drive in golf it is \(\leq 10\) cm. In lawn bowls the side-ways movement can be \(\sim 2.8\) cm, which may be significant, given the nature of this sport. The inertial forces are also potentially of relevance in sporting events not employing spherical projectiles. For example, in Olympic rowing we find that the side-ways movement can be more than 40 m for a 2 km race if it is not compensated for, and nearly 20 m for a 4 min mile event in athletics. The effect is also of significance in events such as swimming and horse racing. The importance of this is that athletes may not be aware of the effect and, in the case of rowing for example, may attribute it to side-ways currents, winds, or a deficiency in their rowing style. As a further complication, the magnitude of the side-ways movement is latitude dependent and its direction is hemisphere dependent, being to the right in the northern hemisphere and to the left in the southern hemisphere.

\(^3\) Author to whom any correspondence should be addressed.

Original content from this work may be used under the terms of the Creative Commons Attribution 3.0 licence. Any further distribution of this work must maintain attribution to the author(s) and the title of the work, journal citation and DOI.
Keywords: projectile motion, trajectories, Coriolis force, centrifugal force, cricket, golf, sporting games

(Some figures may appear in colour only in the online journal)

1. Introduction

Over the years many authors have studied the trajectories followed by rotating spherical projectiles as used in ball games, dating back to at least the time of Sir Isaac Newton (1672) and his famous observation and explanation of the curved path of a spinning tennis ball. The history of the subject has included phases involving the use of so-called ballistic tables to compute trajectories, attempts to find approximate analytic solutions to simplified cases and, most recently, the use of numerical solutions to complicated cases. Some recent works include, amongst others, those of Baker (2013) on cricket, Erlichson (1983) on golf, Nathan (2008) on baseball, Štěpánek (1988) on tennis, Bray and Kerwin (2003) on football (soccer) and Cross (1998) on lawn bowling. Important reviews on the aerodynamics of the motion of balls used in sporting games in general have been presented by Mehta (1988) and more recently by Goff (2013), and Mehta (2005, 2014) has reviewed the subject of cricket ball motion in particular. A number of books on the subject of the physics of ball games and the determination of the trajectories of balls involved, at varying degrees of specialisation, have also been produced. These include the works of Daish (1972), Hart and Croft (1988), de Mestre (1990), Jorgensen (1999) and Adair (2002).

In the study of projectile motion in ball games the projectile is usually considered to be subject to, at least in the first instance, three forces. These three forces are gravity, which acts vertically downwards, a drag force which opposes the motion and is directed opposite to the instantaneous velocity vector and, if the projectile is spinning, a lift or Magnus force. This latter force acts perpendicular to both the instantaneous velocity vector and the instantaneous projectile spin vector, and thus produces a ‘side-ways’ force. If there is an asymmetry in the shape of the ball due, for example, to the way the ball is manufactured and the presence of a seam or seams, or to selective polishing of one side of the ball, there can also be an additional side-ways force. Examples of this occur with baseballs and cricket balls, in the latter case the term ‘swing’ of the ball is used to describe possible side-ways movement due to the asymmetry of the ball. If the ball is in contact with the ground, such as in lawn bowls or croquet, there will also be an additional resistive friction force opposing the motion. Using the forces listed above and subject to the relevant initial conditions, Newton’s laws of motion may be used to obtain the trajectory of the projectile. In all but the simplest cases no analytic solution exists for the motion and the equations must be solved numerically.

In an inertial (unaccelerated) frame of reference Newton’s laws of motion do apply. However, in an accelerated frame, such as a rotating Merry-go-round or on the rotating Earth, unless they are modified Newton’s laws do not apply. This modification involves the introduction of extra forces, called inertial forces, which are sometimes referred to, perhaps misleadingly, as ‘fictitious forces’ or ‘pseudo forces’. If the accelerated frame is in fact a rotating system, these extra forces are the centrifugal force and the Coriolis force (Coriolis 1835), forces which exist only in the rotating (non-inertial) frame and which are unavoidable bi-products of the rotation. The centrifugal force always acts radially outwards from the axis of rotation and is easily felt while standing stationary on a rotating Merry-go-round. The Coriolis force, which is present only if the body concerned is moving in the rotating frame, is perpendicular to both the axis of rotation of the rotating frame and the velocity vector, and thus acts in a side-ways direction. It would be felt on a Merry-go-round if, for example, a person were to attempt to walk radially inwards towards the axis of rotation. In this case they would feel a side-ways force perpendicular to their velocity vector and in the same sense of direction as the velocity due to the rotational velocity of the Merry-go-round (i.e., to the right if the Merry-go-round was rotating counter-clockwise).

Some informative animations of the motion of a ball on the surface of a rotating turntable, and hence subject to both the Coriolis and centrifugal forces, may be found in the Wikipedia. Such animations clearly show the difference between the motion as viewed from the inertial frame and the rotating frame. Many other papers exist on the motion of a ball on a rotating turntable, especially in the American Journal of Physics (e.g., Sokirko et al 1994, Weckesser 1997, Rodriguez 1998, Jensen 2012).

In order to preserve Newton’s second law in the rotating frame, the Coriolis force, \( \mathbf{F}_{\text{Cor}} \), and the centrifugal force, \( \mathbf{F}_{\text{Cent}} \), must be added to the resultant external force, \( \mathbf{F} \). With this modification and for constant mass, \( m \), Newton’s second law in the rotating frame may be written as

\[
\mathbf{F} + \mathbf{F}_{\text{Cor}} + \mathbf{F}_{\text{Cent}} = m\mathbf{a}_{\text{rot}},
\]

where \( \mathbf{F}_{\text{rot}} \) and \( \mathbf{a}_{\text{rot}} \) are, respectively, the effective force and the (actual) acceleration as measured in the rotating frame.

The equations governing the Coriolis and centrifugal forces may be found in many intermediate or advanced books on mechanics and dynamics (e.g., French 1971, Goldberg 1950, McCuskey 1959). In general on, or in the near vicinity of the surface of the rotating Earth, the Coriolis force is given by

\[
\mathbf{F}_{\text{Cor}} = -2m\mathbf{\Omega} \times \mathbf{v},
\]

where \( \mathbf{\Omega} \) is the angular velocity of the rotating Earth.

\footnote{The centrifugal force is equal in magnitude to the centripetal force which acts radially inwards towards the axis of rotation. The centripetal force is a ‘real force’, present in the inertial frame, and is responsible for the inwardly directed centripetal acceleration. In the case of the rotating Earth the centripetal force is provided by a component of the gravitational force.}

\footnote{https://en.wikipedia.org/wiki/Coriolis-force}

\footnote{All symbols used in this paper are defined in appendix A.}
and the centrifugal force is given by

$$\mathbf{F}_{\text{cent}} = -m\mathbf{\Omega} \times (\mathbf{\Omega} \times \mathbf{v}).$$  \hspace{1cm} (3)

In these equations $\mathbf{\Omega}$ is the angular velocity of the Earth (of magnitude $\Omega \approx 7.29 \times 10^{-5}$ rad s$^{-1}$), $\mathbf{v}$ is the velocity of the body relative to the Earth’s surface (i.e., as measured in the rotating reference frame)$^7$ and we take $\mathbf{r}$ to be the radius vector from the Earth’s centre to the point considered (the radius of the Earth, $R_E \approx 6.37 \times 10^6$ m). The vector $\mathbf{F}$ can in fact be from any point on the Earth’s axis of rotation to the point considered.

From equation (2) the maximum value of the Coriolis acceleration, $2\Omega v$, is about $4.4 \times 10^{-4}$ m s$^{-2}$ (0.0045% of $g$) for $v = 3$ m s$^{-1}$ (10.8 km h$^{-1}$), which may seem negligible. However, the resultant displacement after 10 s is about 2.2 cm, which may not be negligible. Neglecting the oblateness of the Earth, from equation (3) and using $r = R_E$, the centrifugal acceleration has a maximum value of $\Omega^2 R_E \approx 3.4 \times 10^{-2}$ m s$^{-2}$ (0.35% of $g$) at the Earth’s surface at the equator, which is certainly not negligible. However, the effect of the centrifugal acceleration is somewhat more difficult to assess than the Coriolis acceleration, since it is only the component relative to the origin of the chosen coordinate system which is relevant to this study. This will be discussed in detail in section 4.2 below.

The Coriolis force is important in influencing large scale atmospheric winds and air movement (see e.g., Scorer 1997). In particular it plays a fundamental role in causing most cyclones in the northern hemisphere to rotate in a counterclockwise direction and most cyclones in the southern hemisphere to rotate in a clockwise direction. It also causes, amongst other things, the preferential erosion of the right banks (as viewed looking down-stream) of rivers in the northern hemisphere and the left banks in the southern hemisphere. In human activities the Coriolis force becomes of increasing importance as the distance travelled and/or the time taken for a journey increases. For example, a projectile dispatched horizontally to the south from the north pole and travelling with a constant speed of 1500 km h$^{-1}$ would, after travelling 1000 km, suffer an enormous side-ways movement (to the west) of approximately 175 km. The Coriolis force is clearly of great importance in navigation in aircraft flights.

The question arises as to whether the Coriolis and centrifugal forces are of any significance in sporting activities such as ball games, or in the movement of athletes across sporting fields where the distances travelled and times taken are generally small. Even if the size of the effect is apparently not significant, perhaps there are subtle effects that remain unnoticed by the athletes but which could possibly cause a degradation in their performance? In any event it is of value to explore the role of the inertial forces from both the phenomenological and pedagogical point of view.

It should be noted that there is a clear distinction between the motion of a body which is not in contact with the ground, such as a ball travelling through the air, and other sporting events such as a person running, swimming or rowing a boat in which there is contact with the ground or water and hence a frictional force is in operation. Clearly the size of the frictional force is of paramount importance since, if large enough, conceivably it may totally negate any side-ways movement due to the Coriolis force.

In this paper we investigate the effect of the Coriolis and centrifugal forces in a variety of sporting activities. We approach the problem by adding the two inertial forces together with the frictional force to the equations of motion presented in our earlier paper published in this journal (Robinson and Robinson 2013). The present paper is structured as follows. In section 2 we summarise some properties of the Coriolis force and in section 3 we present some simple considerations regarding the size of the effect in various sporting activities. In section 4 we document the equations of motion including the Coriolis force (section 4.1), the centrifugal force (section 4.2) and the friction force (section 4.3). In section 5 we list some checks that have been made of the numerical solution including cases involving northerly motion (section 5.1), easterly motion (section 5.2), a body falling under gravity (section 5.3) and a body projected vertically upwards (section 5.4). In section 6 we present the results for some model trajectories, specifically for a cricket ball thrown from the boundary (section 6.1), a slow bowler’s delivery in cricket (section 6.2), a golf drive from the tee (section 6.3), a lawn bowl (section 6.4) and an Olympic rowing event (section 6.5). Finally, in section 7 we present a summary of this work, list the principal conclusions and provide some suggestions for possible extensions to this work.

As with some of our previous papers in Physica Scripta, we do not pretend this work to be ground-breaking research. We have, however, endeavored to make this paper as rigorous as possible and with enough background information so that it will be accessible to readers with a wide range of experiences. In particular, we have attempted to present the equations of motion in such a way that they can easily be adapted (and perhaps modified and extended) by others.

2. Properties of the Coriolis force

We will employ a right-handed coordinate system in the plane of the Earth’s surface with the $x$-axis pointing towards the east, the $y$-axis pointing towards the north and the $z$-axis pointing vertically upwards, as shown in figure 1. This is essentially the same coordinate system as employed in figure 1–19 of McCuskey (1959). Considering equation (2) and figure 1 and the directions of the velocity vector, $\mathbf{v}$, and the Earth’s angular velocity vector, $\mathbf{\Omega}$, at point $P$ in this figure, we summarise some well known facts regarding the Coriolis force. These can best be seen by considering the
Also gives rise to a horizontal force. The magnitude of this horizontal force is latitude dependent but may be either upwards or downwards, depending on the actual direction of the Coriolis force. The direction of this force is independent of the direction of the Coriolis force.

(iii) Except at the poles, and in the two specific cases when \( \dot{v} \) is directed to the north or the south at other latitudes, the horizontal component of \( \dot{v} \) also gives rise to a vertical force. The direction of this force is not hemisphere dependent but may be either upwards or downwards, depending on the actual direction of \( \dot{v} \).

(iv) Except at the poles, the vertical component of \( \dot{v} \) gives rise to a horizontal force. The magnitude of this horizontal force is latitude dependent, and in both hemispheres its direction is to the west while rising and to the east while falling.

A table showing the direction of the Coriolis force for various directions of the velocity vector is given in appendix B.

3. Some simple considerations

Before going into detailed considerations, we will present some simple illustrative examples regarding the Coriolis force in sport. Considering figure 1 and applying equation (2) in the case of a body moving towards the north with velocity of \( v_y \) there will be a side-ways acceleration given by

\[
\alpha_{Cor} = \frac{d^2v_y}{dt^2} = \frac{d^2v_y}{d\lambda^2} = 2\Omega v_y \sin \lambda.
\]

Note this means that, for a body moving towards the north, in the northern hemisphere the Coriolis acceleration is in the positive \( x \) direction (i.e., to the east) and in the southern hemisphere the acceleration is in the negative \( x \) direction (i.e., to the west). If we integrate equation (4) twice with the following assumptions: (i) \( \lambda \) and \( v_y \) are constant and (ii) at time \( t = 0 \), \( x = 0 \) and \( v_y = 0 \), we arrive at

\[
x = \Omega v_y t^2 \sin \lambda = \Omega \frac{v_y^2}{v_y} \sin \lambda = \Omega v_y t \sin \lambda.
\]

The last expression in this equation reveals that the side-ways movement is proportional to both the distance travelled, \( y \), and the time taken, \( t \). Clearly equation (5) applies only for small distances travelled since \( \lambda \) is considered constant and, in addition, neglects the fact that once the body starts to move in the \( x \) direction this will introduce an additional contribution to the Coriolis acceleration\(^8\). This additional acceleration will have both horizontal and vertical components and these will, themselves, lead to further terms in the Coriolis acceleration. However, these effects will be small enough to neglect at this point although we will consider them later on in the paper. We also note that it is easily shown that equation (5) also gives the magnitude of the side-ways movement for any horizontal direction of the velocity vector. In the northern hemisphere the body is always deflected to the right and in the southern hemisphere to the left by the same amount as that given by equation (5). However, if the direction of the initial velocity is other than in the north–south direction, there will also be a movement in the vertical (\( z \)) direction.

We now apply equation (5) to some simple cases in sport in order to estimate the size of the side-ways movement. Table 1 shows the results of these considerations. The entries

---

\(^8\) To illustrate the validity of the assumption of \( \lambda \) being constant, consider the example given in section 1 of a projectile dispatched horizontally from the north pole and travelling with a constant speed of 1500 km h\(^{-1}\). After travelling 1000 km to the south, the side-ways movement to the west, calculated from the differential linear distance through which the Earth rotates in time \( t \), \( \Delta R_{eq} \cos \lambda_{initial} = \cos \lambda_{final} \Delta R_{eq} \), is 174.24 km and, from equation (5) with the assumption that \( \lambda \) remains constant at 90°, is \( |v| = 174.96 \) km (in equation (5) both \( v_y \) and \( \lambda \) are negative in this case). The difference is only about 0.4% even though the distance of 1000 km corresponds to a latitude change of almost 9°.
in the table have been divided into two classes, projectile type cases in which there is no contact with the ground, and cases where there is contact with the ground or water for which there is a frictional resistance force present. In the projectile type cases it may be seen that the side-ways motion, \( x \), due to the Coriolis force as obtained from equation (5) for some representative sporting and other events. The results shown are for a constant horizontal velocity of \( v_x \) and a latitude of \( \lambda = +45^\circ \).

Table 1. Table showing the amount of side-ways movement, \( x \), due to the Coriolis force as obtained from equation (5) for some representative sporting and other events. The results shown are for a constant horizontal velocity of \( v_x \) and a latitude of \( \lambda = +45^\circ \).a

| Sporting event                  | \( y \) (m) | \( v_x \) (km h\(^{-1}\)) | \( v_y \) (m s\(^{-1}\)) | \( t \) (s) | \( x \) (cm) |
|---------------------------------|-------------|---------------------------|---------------------------|------------|-------------|
| **Projectile type cases**       |             |                           |                           |            |             |
| Cricket: slow bowler’s delivery  | 18          | 80                        | 22.2                      | 0.81       | 0.0752      |
| Archery                         | 65          | 300                       | 83.3                      | 0.78       | 0.261       |
| Cricket: throw from boundary     | 80          | 160                       | 44.4                      | 1.80       | 0.742       |
| Football: 60 m kick             | 60          | 54                        | 15                        | 4.00       | 1.237       |
| Golf: drive from the tee         | 250         | 300                       | 83.3                      | 3.00       | 3.866       |
| Rifle shot                      | 500         | 1200                      | 333                       | 1.50       | 3.866       |
| World War 1 Naval gun           | 24 000      | 2696                      | 749                       | 32.05      | 39.65 m     |
| **Ground or water contact type cases** |      |                           |                           |            |             |
| Lawn bowls                      | 35          | 9.0                       | 2.5                       | 14.0       | 2.53        |
| Athletics: 100 m Olympic event   | 100         | 36                        | 10                        | 10.0       | 5.15        |
| Swimming: 50 m Olympic event     | 50          | 6.0                       | 1.67                      | 30.0       | 7.33        |
| Horse racing: straight 1200 m sprint | 1200      | 64                        | 17.78                     | 67.5       | 4.18 m      |
| Athletics: 4 min mile           | 1609        | 24.1                      | 6.70                      | 240        | 19.91 m     |
| Rowing: 2 km Olympic event      | 2000        | 18                        | 5                         | 400        | 41.24 m     |

| a The parameters are as follows: \( y \) is the distance travelled in the \( y \) direction, \( v_x \) is the velocity in the \( y \) direction, \( t \) is the time of flight and \( x \) is the amount of side-ways movement. The table is intended for preliminary illustrative purposes only and clearly many other factors need to be taken into account, two examples being friction with the contact medium and the shape of the trajectory. In general the side-ways movement is to the \( r \) right in the northern hemisphere (i.e., in the positive \( x \) direction) and to the \( r \) left in the southern hemisphere (i.e., in the negative \( x \) direction).

| b These five examples are treated in detail in section 6.

| c In these cases friction due to ground/water contact is large and will oppose the motion and the side-ways movement.

| d In the case of lawn bowls, the bowl is bowled so as to stop near the ‘kitty’ or ‘jack’ at the end of the green, and so the assumption of constant velocity is clearly totally unrealistic and friction with the ground must be included. The length of the green can be anywhere between about 30 and 40 m, and so we have arbitrarily adopted a length of 35 m and a run time for the green of 14 s.

4. The equations of motion

The approach that we have adopted in determining the equations of motion is to start with the three basic equations for projectile motion presented in Robinson and Robinson (2013) (equations (14)–(16) in that paper). Some additional results for these equations applied to cricket were presented in Robinson and Robinson (2015, 2016b). These three equations represent the \( x \), \( y \) and \( z \) components of the motion and, aside from gravity, they include an atmospheric drag force, a Magnus or lift force due to spin of the ball about an arbitrary axis, and the presence of an arbitrary wind. These three
The Coriolis force

In this and subsequent sections we assume that the latitude, \( \lambda \), is constant (see the discussion in section 3). However, if it were deemed necessary to take into account the variation of latitude, one could replace \( \lambda \) in the following analysis with \( \lambda + y/RE \), which holds to a good approximation (\( \lambda \) in radians; see figure 1).

For the chosen coordinate system, as shown in figure 1, the components of the Earth’s angular velocity vector are, \( \Omega_x = 0 \), \( \Omega_y = \Omega \cos \lambda \) and \( \Omega_z = \Omega \sin \lambda \). Thus, using \( \vec{v} = \hat{v}_x + \hat{v}_y + \hat{v}_z \), expanding the cross product for the Coriolis force in equation (2) yields

\[
\vec{F}_{\text{Cor}} = -2m\Omega \times \vec{v} = -2m \left[ \hat{v}_x (\Omega_y v_y - \Omega_z v_z) + \hat{v}_y (\Omega_x v_x - \Omega_z v_z) + \hat{v}_z (\Omega_z v_x - \Omega_y v_y) \right].
\]

Hence the three components of the Coriolis force are

\[
F_{x,\text{Cor}} = -2m\Omega (v_x \cos \lambda - v_y \sin \lambda),
\]

\[
F_{y,\text{Cor}} = -2m\Omega v_x \sin \lambda, \quad \text{and}
\]

\[
F_{z,\text{Cor}} = +2m\Omega v_x \cos \lambda.
\]

These three terms may be added to the right-hand sides of equations (6)–(8) respectively.

We note in passing that for the velocity vector, \( \vec{v} \), in the the y–z plane and with the particular direction shown in figure 1, from equation (2) the Coriolis force acts into the plane of the paper in this figure (i.e., in the easterly or positive x direction). However, as the \( z \) (vertical) component of \( \vec{v} \) is increased, there will come a point where the elevation angle of \( \vec{v} \) is greater than that of the Earth’s angular velocity vector, \( \Omega \), and the Coriolis force will reverse in direction and act out of the plane of the paper (i.e., in the westerly or negative x direction).

4.2. The centrifugal force

Reference to figure 1 reveals that, since \( \vec{r} = \vec{R}_E + \vec{r}_O \), the centrifugal force at point P as given by equation (3) may be simplified as follows

\[
\vec{F}_{\text{cent}} = -m\Omega \times (\Omega \times \vec{r}) = -m [\Omega \times (\Omega \times \vec{R}_E) + \Omega \times (\Omega \times \vec{r}_O)].
\]

The first term in the expansion on the right-hand side of equation (13) involving the radius of the Earth, \( \vec{R}_E \), is the centrifugal force at the origin of the coordinate system fixed on the Earth’s surface, \( \Omega' \), and acts radially away from the spin axis of the Earth. It has two effects. Firstly, it reduces the effective gravity in a latitude dependent way such that it has zero effect at the poles and a maximum effect at the equator. Secondly, it alters the direction of the vertical, as defined by the direction of a ‘plumb-line’. In fact in practice the vertical is usually defined by the direction of a plumb-line which necessarily includes the effect of the centrifugal force. Although this term is not small, in view of the above discussion and because we wish to find the motion of a body relative to the origin \( \Omega' \), it need not be considered.

The last term on the right-hand side of equation (13) involving \( \vec{r}_O \) represents the centrifugal force at point P relative to the origin at \( \Omega' \), \( \vec{F}_{\text{cent}} \). For values of \( \rho_0 \) relevant to this paper, and since \( \Omega \approx 7.29 \times 10^{-5} \text{ rad s}^{-1} \), the corresponding centrifugal acceleration, \( \Omega \times (\Omega \times \vec{r}_O) \), is \( \sim 10^{-8} \text{ m s}^{-2} \). Thus this component of the centrifugal force is, in general, small compared to the Coriolis force, but for completeness we have included it in the model trajectory computations.

---

9 Equivalently, the direction of the vertical may be taken as the normal to the surface of a liquid in equilibrium. The deviation of the plumb-line vertical from the direction of the radius vector from the centre of the Earth is zero at the poles and at the equator, and has its maximum value of about 6 min of arc at a latitude of \( \pm 45^\circ \). As was mentioned in section 1, neglecting the oblateness of the Earth the centrifugal acceleration is about 0.35% of \( g \) at the equator.
Since \( \mathbf{r}_0' = \mathbf{i}x + \mathbf{j}y + \mathbf{k}z \), the vector triple product may be expanded to yield

\[
\mathbf{F}_{\mathbf{r}}' = -m(\mathbf{\Omega} \times (\mathbf{\Omega} \times \mathbf{r}_0'))
\]

\[
= -m[-\mathbf{\Omega}^2 x + \mathbf{j}(\mathbf{\Omega}_2 z - \mathbf{\Omega}_3 y) + \mathbf{k}(\mathbf{\Omega}_3(-\mathbf{\Omega}_2 z + \mathbf{\Omega}_3 y))],
\]

(14)

where \( \mathbf{\Omega} = (\mathbf{\Omega}_1^2 + \mathbf{\Omega}_2^2 + \mathbf{\Omega}_3^2)^{1/2} \).

Hence the three components of the centrifugal force, relative to the origin at \( \mathbf{O}' \), are

\[
F_{\mathbf{x},\mathbf{r}}' = m\mathbf{\Omega}^2\mathbf{x},
\]

(15)

\[
F_{\mathbf{y},\mathbf{r}}' = m\mathbf{\Omega}^2\sin(\mathbf{z}\cos\mathbf{\lambda} + \mathbf{y}\sin\mathbf{\lambda}),
\]

and

(16)

\[
F_{\mathbf{z},\mathbf{r}}' = m\mathbf{\Omega}^2\cos(\mathbf{z}\cos\mathbf{\lambda} - \mathbf{y}\sin\mathbf{\lambda}).
\]

(17)

Again, as for the Coriolis force, these three terms may be added to the right-hand sides of equations (6)–(8) respectively.

4.3. The ground friction force in sports like lawn bowls

In lawn bowls the aim is to bowl a ball (called a 'bowl') along a green of about 35 m in total length and for it to come to rest as close as possible to a smaller ball called the 'kitty' or 'jack'. Bowls are constructed so that they incorporate a 'bias', essentially a weight on one side of the bowl, so that the bowl follows a curved path across the green (for further information see Cross 1998, and for an advanced mathematical treatment see Brearley and Bolt 1958 and Brearley 1961). Aside from the bias being an essential part of the game, amongst other things it enables the skilled player to avoid other player’s bowls that may lie in front of the kitty. We will not delve further into the complexities of the effects of the bias and for the purposes of this paper we will neglect it altogether, since the aim here is to determine the size of the side-ways deviation due to the Coriolis force. The deviation due to the Coriolis force will always be present and will be superimposed on the much larger side-ways movement due to the bias.

In lawn bowls it is usually considered desirable for the bowl to roll smoothly right from the start and to avoid sliding or skidding across the green. To accomplish this the bowl is bowed with a forward and upward motion of the hand, the aim being to apply just the correct amount of top-spin to the bowl to match the forward motion and thus to avoid skidding. If the bowl rolls smoothly across the green a rolling friction force, \( F_r \), acts on the bowl in the opposite direction to the velocity vector and thus opposes the bowl’s forward motion. The actual value of the rolling friction force may vary from day to day for a given green depending on the condition of the green (e.g., how dry the green is, the length of the grass etc). The size of the rolling friction force clearly influences the speed that the bowl must be bowed at and the so-called ‘runtime’ of the green—how long it takes for the bowl to travel the length of the green and come to rest. For basic information on friction forces, see e.g., Halliday et al (2011), and for an excellent discussion of the forces existing when a bowl rolls over a surface, designed for readers with a wide range of background knowledge and sporting interests, see Daish (1972).

We assume that the friction force acts in a direction opposite to the instantaneous velocity vector, \( \mathbf{\hat{v}} \), and has a magnitude \( F_r = \mu_r mg \), where \( \mu_r \) is the coefficient of rolling friction. The coefficient of rolling friction is in fact not simply related to any other coefficient of friction (see Cross 1998 and other references therein). Since the unit vector in the direction of \( \mathbf{\hat{v}} \) is given by \( \mathbf{\hat{v}} = \mathbf{\hat{v}}(v_x^2 + v_y^2 + v_z^2)^{1/2} \), the friction force can be written as

\[
\mathbf{f}_r = -\mu_r mg\mathbf{\hat{v}} = -\mu_r mg\mathbf{i}\mathbf{v}_x + \mathbf{j}\mathbf{v}_y + \mathbf{k}\mathbf{v}_z.
\]

(18)

Hence the three components of the friction force are

\[
F_{x,f} = -\mu_r mg\frac{v_x}{(v_x^2 + v_y^2 + v_z^2)^{1/2}},
\]

(19)

\[
F_{y,f} = -\mu_r mg\frac{v_y}{(v_x^2 + v_y^2 + v_z^2)^{1/2}},
\]

(20)

\[
F_{z,f} = -\mu_r mg\frac{v_z}{(v_x^2 + v_y^2 + v_z^2)^{1/2}}.
\]

(21)

The above three terms may be added to the right-hand sides of equations (6)–(8) respectively. We note that for a ball rolling on a horizontal surface \( v_z \) will be zero, but we include it here for completeness.

When a body moves through water, it is subject to a drag force dependent on the velocity, and possibly also a frictional component. For the purposes of this study, at least as a first attempt, a frictional resistance force of the type above may be adaptable to sporting events involving motion through water, such as rowing or even swimming, particularly if the velocity is constant.

5. Some checks of the numerical solution

The equations of motion given in section 4 were solved numerically using a fourth order Runge–Kutta method invoked in an EXCEL spreadsheet, as employed by Robinson and Robinson (2015, 2016a, 2016b). We note in passing that Spathopoulos (2009) has also used the spreadsheet approach for finding the trajectory of spinning footballs (soccer balls) in flight. With any numerical procedure it is always possible to introduce errors. We have therefore carried out numerous checks, both qualitative and quantitative, although this does not guarantee that either the equations of motion or the numerical procedure are both error free. Here we list four of the checks, each being made using analytic solutions which are available in simple cases.

5.1. Northerly motion and table 1 results

Firstly, we have compared the numerical results with the side-ways movements, \( x \), listed in table 1. The results in this table were obtained using the analytic equation (5), which applies for a northerly motion in the absence of gravity, and gives rise

\[
W = W + W + W - W
\]

(22)

\[
W = W - W + W, \quad W = W + W, \quad W = W - W,
\]

(23)

\[
W = W + W - W
\]

(24)
to a side-ways movement to the east (in the northern hemisphere). In the numerical procedure this was accomplished by setting all forces, with the exception of the Coriolis, to zero, and using the parameter values listed in table 1 to find the side-ways movement. In all cases the analytical and numerical results were in close agreement.

5.2. Easterly motion

Secondly, we considered the case of a body moving to the east with a velocity of \( v_x \), in the absence of gravity. In this case the Coriolis force gives rise to a side-ways movement consisting of a southerly component (for the northern hemisphere) and a vertically upwards component (see figure 1 and appendix B). To a very good approximation these may be shown to be given, respectively, by

\[
y = -\Omega v_x t^2 \sin \lambda = -\frac{\Omega v_x^2}{v_y} \sin \lambda = -\frac{\Omega}{v_y} \sin \lambda, \quad \text{and} \quad (22)
\]

\[
z = \Omega v_x t^2 \cos \lambda = \frac{\Omega v_x^2}{v_y} \cos \lambda = \Omega t \cos \lambda. \quad (23)
\]

We note in passing that the southerly deflection by equation (22) is identical in magnitude to the easterly deflection arising from the Coriolis force for movement to the north, as given by equation (5).

The numerical solution has been checked against the analytic solution represented by equations (22) and (23). The two solutions were found to agree to typically 1 part in \( 10^8 \) or better for a range of values of \( v_x, x, t \) and \( \lambda \) throughout the entire trajectory appropriate to each case.

5.3. Body falling under gravity

As a third check we considered the case of a body falling under gravity from a height \( h \) to height \( z \), with the only additional force being the Coriolis force (i.e., the drag, lift, friction and centrifugal forces were all set to zero). The analytic solution for the side-ways movement in this case is, to a very good approximation, easily shown to be (see e.g., French 1971, p 526)

\[
x = \frac{2\sqrt{2}}{3} \frac{\Omega \cos \lambda}{g^{1/2}} (h - z)^{3/2}. \quad (24)
\]

As a numerical example, if \( h = 23 \) m, \( z = 0 \) and \( \lambda = 42^\circ \) N, equation (24) gives \( x = 0.179971 \) cm.\(^{10}\) The deflection is to the east (in fact this is the case irrespective of the latitude, except at the poles; see figure 1 and appendix B) and, with an integration step interval of about 0.002 s, the numerical result is in agreement with this value to six significant figures. In addition, however, because of this small easterly motion there will also be a southerly deflection due to the Coriolis force, and the analytic solution for this is, again to a very good approximation, easily shown to be

\[
y = -\frac{2\sqrt{2}}{3} \frac{\Omega \cos \lambda}{g^{1/2}} (h - z)^2. \quad (25)
\]

For the example above, at \( z = 0 \) equation (25) yields a southerly deflection of \( y = -9.5099 \times 10^{-6} \) cm. The numerical result for this southerly deflection is in agreement with the analytic solution to five significant figures.

It is interesting to note that the centrifugal force in this case also yields a southerly deflection (but not an easterly deflection), and it may be shown that a very good approximate analytic solution for this deflection, relative to the origin at \( O' \) (see figure 1), is

\[
y = -\frac{\Omega^2 \sin \lambda \cos \lambda}{g} \left[ h(h - z) - \frac{1}{6}(h - z)^2 \right]. \quad (26)
\]

Comparison with equation (25) reveals that at ground impact when \( z = 0 \) this is identical to the southerly deflection due to the Coriolis force, except for the multiplying constant which is exactly 25% larger in the case of the centrifugal force. For the above example equation (26) gives a value of \( y = -1.188742 \times 10^{-5} \) cm at \( z = 0 \). The numerical solution, obtained by setting the Coriolis force to zero and reinstating the centrifugal force, is in agreement to seven significant figures. The combined effect of the southerly deflection due to the Coriolis and centrifugal forces is thus about \( y = -2.1397 \times 10^{-5} \) cm.

5.4. Body projected vertically upwards against gravity

As a fourth check we considered the case of a body projected vertically upwards with only the gravitational and Coriolis forces acting. An approximate analytic solution for the side-ways deflection may be shown to be

\[
x = -\frac{2\sqrt{2}}{3} \frac{\Omega \cos \lambda}{g^{1/2}} \left[ 2h^{3/2} + 3h(h - z)^{1/2} \pm (h - z)^{3/2} \right], \quad (27)
\]

where the upper signs apply to the upward part of the trajectory and the lower signs apply to the downward part.

At the maximum height when \( z = h \) equation (27) becomes

\[
x = -\frac{4\sqrt{2}}{3} \frac{\Omega \cos \lambda}{g^{1/2}} h^{3/2}, \quad (28)
\]

and on ground impact when \( z = 0 \) it becomes

\[
x = -\frac{8\sqrt{2}}{3} \frac{\Omega \cos \lambda}{g^{1/2}} h^{3/2}. \quad (29)
\]

\(^{10}\) The reason for choosing this example is that Hall (1903a, 1903b), of ‘Hall effect’ fame, carried out an experiment in Cambridge, Massachusetts (latitude 42° N) where he allowed objects to fall through a height of 23 m and measured their side-ways deflection on impact with the ground. The measurements were clearly very difficult to perform, but from the results of almost 1000 trials he found, on average, an easterly deflection of 0.149 cm and a southerly deflection of 0.0045 cm. The apparent southerly deflection attracted considerable attention in the 1980s in a discussion which played out in the pages of the American Journal of Physics (French 1984, Reddiguus 1984, Stirling 1984, Desloge 1985, 1989, Belorizky and Sivardière 1987). It appears that the explanation is related to how one defines the vertical and the fact that, owing to the oblateness of the Earth, the upper part of a long plumb-line is at a very slightly different latitude to the lower part. In fact even if the Earth were spherical, a short plumb-line at the top of the fall would be at a slight angle to one at the bottom.
The side-ways movement arising from the Coriolis force for a cricket ball thrown in a northerly direction, from the boundary to the central wicket, for a range of latitudes, from the equator ($\lambda = 0^\circ$) to the north pole ($\lambda = 90^\circ$). In (a) the trajectories are shown from the side and in (b) from the perspective of the person who has thrown the ball (i.e., looking to the north, down the y-axis). The initial parameter values for the trajectories are listed in table 2. The ball is thrown without spin and is subject to gravity, a drag force with a constant drag coefficient of $C_D = 0.45$, and the inertial forces. In all cases the range and travel times are about 83.2 m and 2.34 s respectively. The maximum side-ways movement occurs, as expected, at the pole and is about 1.28 cm.

**Table 2.** Table showing the initial values of the parameters used for the trajectories shown in figures 2–7 together with representative values for the amount of side-ways movement, $x$, due to the Coriolis force. The results shown are for a latitude of $\lambda = +45^\circ$.

| Sporting event                  | Figure number | \(y\) (m) | \(v(0)\) (km h\(^{-1}\)) | \(v(0)\) (m s\(^{-1}\)) | \(\phi(0)\) (degrees) | \(z(0)\) (m) | \(\omega_z\) (rad s\(^{-1}\)) | \(t\) (s) | \(x\) (cm) |
|---------------------------------|---------------|------------|---------------------------|------------------------|----------------------|------------|-----------------------------|--------|----------|
| Cricket: throw from boundary    | 2 and 3       | 83.2       | 180                       | 50.0                   | 14                   | 1.5        | —                           | 2.34   | 0.84     |
| Cricket: slow bowler’s delivery | 4 and 5       | 17.92      | 79.2                      | 22.0                   | 5                    | 2.0        | —                           | 0.872  | 0.082    |
| Golf: drive from the tee        | 6 and 7       | 254.83     | 300                       | 83.3                   | 11.6                 | 0          | 314.16                      | 6.895  | 6.3      |

*The parameters are as follows: $y$ is the distance travelled in the $y$ direction to ground impact, $v(0)$, $\phi(0)$ and $z(0)$ are, respectively, the velocity, elevation angle and height of projection, $\omega_z$ is the angular velocity of the ball (positive values of $\omega_z$ correspond to underspin), $t$ is the time of flight and $x$ is the amount of side-ways movement on impact with the ground due to the Coriolis force. The mass and cross-sectional area of the cricket ball and golf ball are taken, respectively, to be $m = 1.59 \times 10^{-1}$ kg and $4.59 \times 10^{-2}$ kg, and $A = 4.07 \times 10^{-3}$ m\(^2\) and $1.43 \times 10^{-3}$ m\(^2\). Note that values shown for $x$ in this table are intended to be indicative only because the trajectories presented in figures 2–7 include a wide range of latitudes and a variety of projection directions.

For the rising part of the trajectory the Coriolis force acts to the west and the body moves to the west. For the falling part the Coriolis force acts to the east, opposing the acquired westerly motion. On ground impact the body in fact moves vertically downwards with the net westerly deflection on impact being twice that at the maximum height.

The numerical solution has been checked against the analytic solution represented by equations (27)–(29) and has been found to agree to typically 1 part in $\sim 10^9$ or better throughout the entire trajectory and for a range of values of $\lambda$ and $h$.

### 6. Model trajectories

In this section we present some results for the trajectories, as influenced by the Coriolis and centrifugal forces, for selected sports including cricket, golf, lawn bowls and Olympic rowing. In the case of lawn bowls and Olympic rowing the situation is complicated by the fact that, as noted above, a ground friction-like force acts and certainly needs to be included. For the results presented in this section the effects of the centrifugal force have been found to be small in general compared to those of the Coriolis force. Hence the following discussion will focus primarily on the motions produced by the Coriolis force. However, the centrifugal force may not necessarily always produce negligible effects (see, for example, the discussion in section 5.3 related to equation (26)) and so in the interests of completeness it is desirable to include it in the analysis.

#### 6.1. Cricket ball thrown from the boundary

Figure 2 shows the trajectories for a cricket ball thrown in a northerly direction in the northern hemisphere from the boundary to the central wicket, the trajectories being for a range of latitudes from the equator to the north pole. The initial parameter values used to calculate these trajectories are listed in table 2 together with those values appropriate to some of the trajectories considered later in the paper. Also included in table 2 is an indicative value for the side-ways
movement due to the Coriolis force for a latitude of \( \lambda = 45^\circ \). Panels (a) and (b) of figure 2 are, respectively, for the trajectories shown from the side and from the perspective of the person who has thrown the ball. The trajectories include the effects of gravity, a drag force with a constant value of the drag coefficient \( C_D = 0.45 \), but with no spin since the spin of the ball would cloud the issue of the effects of the inertial forces. The calculated trajectories include both the Coriolis and centrifugal forces. However, in practice the numerical results show that the centrifugal force has no noticeable effect on any of the trajectories, as was foreshadowed in section 4.2 and above.

The maximum side-ways movement shown in figure 2 is about 1.28 cm. To keep this number in perspective we have run extra models (not included here) with various side-winds but without the Coriolis force present. We find that a very small horizontal easterly side-wind of \( W_h = 2.06 \text{ cm s}^{-1} \) (0.074 km h\(^{-1}\)) is required to produce the deflection of \( x = 1.28 \text{ cm} \). A wind of 10 cm s\(^{-1}\) (0.36 km h\(^{-1}\)) produces a deflection of about 6.2 cm and a wind of 1 m s\(^{-1}\) (3.6 km h\(^{-1}\)) produces a deflection of about 0.62 m. Thus a very small side-ways breeze would completely mask the effects of the Coriolis force.

In figure 3, panel (a), the trajectories of figure 2 are shown as viewed from above. For comparison purposes, in panels (b)–(d) we have included similar trajectories but for balls thrown to the south, east and west respectively in the northern hemisphere. These choices of trajectory directions were employed because they represent a ball thrown towards the central wicket from all four quarters of the cricket field. It can be seen that, for the ball thrown to the north, the Coriolis force causes the ball to move to the east (i.e., to the right) except for the initial phases of the motion for the ball projected at the equator. In this latter case the initial deviation to the west is due to the vertical component of the ball’s velocity. Reference to figure 1 and equation (2) reveals that the vertical component of the velocity will give rise to a Coriolis force to the west as the ball ascends and to the east as it descends. A similar effect can be seen for the ball thrown to the south. On the other hand, for the balls thrown to the east and west, at the equator there is no side-ways deflection in the horizontal plane at all. For these situations, although the
velocity vector, $\vec{v}$, and the vector representing the Earth’s rotation, $\Omega$, are always perpendicular, any deflection is in the vertical $x$–$z$ plane. In fact for any given latitude the major cause of the horizontal side-ways displacement of the ball is, in general, due to the horizontal component of the velocity. The horizontal component of the velocity produces the same deflection whether the ball is projected to the north, south, east or west, or in fact for any other horizontal direction of projection, and the direction of this deflection is always to the right in the northern hemisphere. The vertical motion of the ball contributes to the horizontal side-ways movement to a lesser extent in general.

As can be seen from figures 2 and 3, for the northerly thrown ball the maximum side-ways deflection is about 1.28 cm at the poles and about 0.84 cm at a latitude of $\lambda = 45^\circ$, which it could be argued is negligible from the practical point of view. While this may be true, it should be remembered that in going from the northern to the southern hemisphere, the deflection shifts from the right to the left and effectively doubles the effect. This will be discussed in section 6.2 below in connection with a slow bowler’s delivery in cricket.

As a point of interest we list the initial values of the components of the centrifugal force, relative to the origin at $O'$, and the Coriolis force for the four trajectories originating at the equator in figure 3. For all four trajectories the centrifugal force components are $F(0)_x,0',\text{cent} = F(0)_y,0',\text{cent} = 0$ and $F(0)_z,0',\text{cent} = 1.27 \times 10^{-9}\text{N}$. For the Coriolis force, for all four trajectories, $F(0)_x,\text{Cor} = -2.81 \times 10^{-4}\text{N}$ and $F(0)_y,\text{Cor} = 0$. For the northerly and southerly trajectories $F(0)_z,\text{Cor} = 0$, for the easterly trajectory $F(0)_z,\text{Cor} = 1.13 \times 10^{-3}\text{N}$ and for the westerly trajectory $F(0)_z,\text{Cor} = -1.13 \times 10^{-3}\text{N}$. Thus, the Coriolis force is many orders of magnitude greater than the centrifugal force, as measured relative to the origin at $O'$.

6.2. Slow bowler’s delivery in cricket

Figure 4 is a similar diagram to figure 2 but for the case of a slow bowler’s delivery in cricket, the initial parameter values for the trajectories being listed in table 2. The ball is bowled in a northerly direction, the trajectories being for the northern hemisphere and for a range of latitudes from the equator to the north pole. The same general conclusions regarding the ball’s motion as those for the cricket ball thrown from the boundary as displayed in figure 2 apply. However, primarily because of the shorter distance of travel of the spin bowler’s delivery until impact with ground (~17.92 m compared to ~83.2 m), and the shorter duration of the flight (~0.872 s compared to ~2.34 s), the maximum deflection is smaller, being about 1.11 mm at the pole. At the more usual latitude of $45^\circ$ the deflection is about 0.82 mm. From the practical point of view, even given that the direction of the deflection reverses in the southern hemisphere (see below) giving a difference of about 1.64 mm between latitudes of $+45^\circ$ and $-45^\circ$, it is hard to see that this deflection would be noticeable, even by the best of bowlers. Although a faster ball will move less than a slower ball, all other things being equal, it should be remembered that the Coriolis deflection is a systematic and (largely) unavoidable effect.

In order to minimise the effect of loss of visibility due to the position of the Sun, cricket pitches are usually aligned in the north–south direction. Therefore, rather than consider trajectories originating from all four points of the compass, as was done in figure 3 for the cricket ball thrown from the boundary, we have chosen to present trajectories only for balls bowled from the north and the south. However, we have also included results for the southern hemisphere as well as the northern hemisphere, and these trajectories are shown in figure 5. It can be seen from panels (a) and (b) of figure 5 that for the northern hemisphere the deflection is, in general, always to the right, irrespective of the end of the pitch from...
which the ball is delivered. That is, the direction of movement in the air due to the Coriolis force is the same as the direction of movement off the pitch for a right-hand bowler’s off-spin delivery to a right-hand batsman (for terminology used in cricket and some relevant explanatory diagrams, see e.g., Robinson and Robinson 2016b). As can be seen from panels (c) and (d) of figure 5, the direction of movement in general reverses in the southern hemisphere and is to the left, that is in the direction of a right-hand bowler’s leg-spin delivery to a right-hand batsman.

The shape of the trajectories at the equator in figure 5 perhaps need further explanation. From panels (a) and (c) it may be seen that at the equator \( \lambda = 0^\circ \) the two relevant trajectories are identical, as they must be, and the deflection is to the west while rising and to the east while falling. Similarly, an examination of panels (b) and (d) also reveals that the two deflections are identical, again as they must be, and are also to the west while rising and to the east while falling. This deflection is due to the vertical component of the ball’s velocity vector, \( \vec{v} \), since the horizontal component of \( \vec{v} \) is either parallel or anti-parallel to the Earth’s angular velocity vector, \( \vec{\Omega} \), and thus produces no Coriolis force (see figure 1 and equation (2)). It is generally true that, due to the Coriolis force, the vertical component of \( \vec{v} \) always produces a westerly displacement while rising and an easterly displacement while falling, irrespective of the location on the Earth’s surface and (of course) the actual direction of \( \vec{v} \) in the horizontal plane. However, this displacement is generally masked by the larger effect of the horizontal component of \( \vec{v} \), as can be seen from the trajectories at all latitudes other than the equator in figure 5.

6.3. Golf drive from the tee

A golf drive from the tee is, from the point of view of the trajectory, broadly similar to a cricket ball thrown from the boundary as discussed in section 6.1, except that the distances travelled and flight times are greater. However, in order to be realistic we have introduced an amount of under-spin, as occurs in a practical situation. This will produce true lift in the upwards vertical direction and in fact may increase the range...
of the drive. Pure under-spin does not produce any side-ways motion and hence will not mask the effects of the centrifugal and Coriolis forces.

Figure 6 shows the trajectories for a golf ball hit from the tee in a northerly direction in the northern hemisphere, and this may be compared with the cricket ball trajectories of figure 2. The initial parameter values for the trajectories are listed in table 2. Panels (a) and (b) of figure 6 are, respectively, for the trajectories shown from the side and from the perspective of the person who has played the golf drive. The velocity for a golf tee shot is normally above the so-called critical velocity at which the drag coefficient, \( C_D \), first drops dramatically and then slowly rises (see the extensive data of Smits and Smith 1994). While the actual value of \( C_D \) is not critical to this work, we have used a function for \( C_D \) which varies from about 0.267 at the initial (maximum) velocity of 300 km h\(^{-1}\) down to about 0.250 for the minimum velocity of about 100 km h\(^{-1}\). This minimum velocity occurs at a time of about \( t = 5.038 \) s compared with the total flight time until ground impact of about 6.895 s with a velocity of about 110 km h\(^{-1}\). The lift coefficient, \( C_L \), like the drag coefficient, is not critically important to our analysis, and for the purpose of this paper we have assumed it to follow an exponential rise with angular velocity, \( \omega \), as given by Robinson and Robinson (2013) (see figure 5 and equation (10) in that paper). It may be seen from figure 6(a) that the initial part of the trajectory is slightly concave upwards, as a result of the under-spin of 3000 rpm (314.16 rad s\(^{-1}\)) applied to the ball. That is, the gradient of the trajectory initially increases, as is well known to occur in golf shots played from the tee.

For the curves shown in figure 6(b) the golf tee shot experiences a maximum deflection of about 10.2 cm at the poles due to the Coriolis force compared to about 1.28 cm for the cricket ball throw of figure 2(b). This increase, approaching a factor of ten, is clearly due to the increased range and time of travel of the golf tee shot. At the more usual latitude of 45°, the deflection of the golf tee shot is about 6.3 cm. For comparison, extra model results not included here show that a side-wind of about \( W_x = 2.90 \) cm s\(^{-1}\) (0.104 km h\(^{-1}\)) gives a deflection of 10.2 cm, and side-winds of 10 cm s\(^{-1}\) (0.36 km h\(^{-1}\)) and 1 m s\(^{-1}\) (3.6 km h\(^{-1}\)) give deflections of 35.3 cm and 3.53 m respectively. While the maximum deflection of 10.2 cm due to the Coriolis force is small compared to that due to even small winds, bearing in mind that such a deflection is hemisphere, latitude and direction dependent in various ways, perhaps golfers at the highest level should at least be aware of it.

Figure 7 panel (a) shows the trajectories of figure 6 as viewed from above. For comparison purposes, in panels (b)–(d) of figure 7 we have included similar trajectories but for golf tee shots hit to the south, east and west respectively. These diagrams may be compared with those shown in figure 3 for a cricket ball thrown from the boundary, and it can be seen that all curves in figure 7 are very similar to the respective curves in figure 3. Hence in general terms the discussion regarding cricket balls thrown from the boundary also applies to golf tee shots, except for the magnitude of the effect. However, we note in particular the comparison between the trajectories at a latitude of 45° degrees for the north and south struck balls of figures 7(a) and (b) respectively. The maximum deflection for the north struck ball is about 6.3 cm (to the east) while the maximum deflection for the south struck ball is increased to about 8.2 cm (to the west). The reason for the increase is the contribution to the horizontal deflection by the vertical component of the velocity. The rising part of the trajectory always gives rise to a deflection to the west and the falling part to east, with the net result of the vertical motion for the specific trajectory shown here being a deflection to the west. This can most clearly be seen from the trajectories for a latitude of 0° in figures 7(a) and (b).
For shorter shots to the green (e.g., pitch shots) where any side-ways movement is more critical, the Coriolis force will of course have less of an effect. For example, for a latitude of 45°, for a golf shot hit to the east at an elevation angle of projection of \( \phi(0) = 11.60° \), representative trajectory calculations (not included here) yielded ranges and

Figure 7. Panel (a) shows the trajectories of figure 6, as viewed from above, the curves depicting the side-ways movement due to the Coriolis force for a golf tee shot for a range of latitudes. Panels (b)–(d) show similar trajectories to those in (a) but for the tee shot directed to the south, east and west respectively. Note that similarly styled lines in each panel are for the same latitude and thus may be directly compared. For a given latitude the sizes of the deflections are similar, although certainly not identical, the discrepancies being due to differing impacts of the vertical component of the ball’s velocity. The curves are very similar in shape to those for a cricket ball thrown from the boundary as shown in figure 3, although the deflections in the present case are about a factor of 10 larger.

Figure 8. The side-ways movement arising from the Coriolis force for a lawn bowl. Panel (a) is for a fixed latitude of \( \lambda = 45° \) and different velocities of projection (to the north) and (b) is for a fixed velocity of projection of \( v_y(0) = 4.5 \text{ m s}^{-1} \) and different latitudes. Note that the heavy continuous curve in the centre of each panel, corresponding to \( \lambda = 45° \) and \( v_y(0) = 4.5 \text{ m s}^{-1} \), is the only curve in common in both (a) and (b), and the remaining curves in each panel may not be directly compared with each other. In the context of lawn bowls, the maximum deflection of about 2.81 cm for a latitude of \( \lambda = 45° \) and a velocity of \( v_y(0) = 3.0 \text{ m s}^{-1} \), as shown in (a), is significant (see table 3).

For shorter shots to the green (e.g., pitch shots) where any side-ways movement is more critical, the Coriolis force will of course have less of an effect. For example, for a
corresponding side-ways shifts due to the Coriolis force as follows: 25 m (0.145 cm), 50 m (0.440 cm), 75 m (0.865 cm), 100 m (1.41 cm), 125 m (2.07 cm), 150 m (2.84 cm) and 175 m (3.72 cm). It should be emphasised that these values are indicative only and clearly depend on the exact trajectory, and particularly on the travel times. For example, the side-ways movement for a 50 m pitch shot can range from the value listed above of 0.440 cm for \( \phi(0) = 11.60^\circ \) up to perhaps 0.875 cm for a very lofted shot with \( \phi(0) = 45^\circ \).

6.4. Lawn bowls

In section 3 and table 1 we presented a result for the side-ways deflection due to the Coriolis force for a lawn bowl travelling a distance of 35 m with a constant speed of \( v_x = 2.5 \) m s\(^{-1}\) at a latitude of \( \lambda = +45^\circ \), the calculated deflection being \( x = 2.53 \) cm. This was in the absence of any ground friction force and hence represents an unrealistic situation since the bowl will not come to rest. As discussed in section 4.3, the friction force acts to bring the bowl to rest and will also oppose any side-ways motion due to the Coriolis force. Here we present model trajectories for the realistic case of a finite friction force.

Figure 8 shows the amount of side-ways movement, due to the Coriolis force in the presence of the ground friction force, for various initial velocities of projection, \( v(0) \), and various northern hemisphere latitudes, \( \lambda \), of the bowl used in lawn bowling, and table 3 lists the results in tabular form. The bowl is delivered in the positive \( y \) (northerly) direction and, as noted in section 4.3, the effects of the bias will be ignored since its inclusion would mask all other effects. In panel (a) of figure 8 the latitude is kept constant at \( 45^\circ \) and the projection velocity is varied, while in panel (b) the projection velocity is kept constant at \( 4.5 \) m s\(^{-1}\) and the latitude is varied. In all cases both the atmospheric drag force and the Magnus (lift) force are assumed to be negligible compared to the ground friction force. This means that the mass and diameter of the bowl do not enter into the analysis. For each individual velocity the coefficient of rolling friction, \( \mu_f \), has been adjusted so that the bowl comes to rest at the end of the green, taken to be 35 m in length, and \( \mu_f \) is assumed to remain constant until the bowl comes to rest.

An examination of table 3 reveals that values determined for \( \mu_f \) are quite small, in the range from 0.013 to 0.052. However, they are not inconsistent with those values quoted by Cross (1998), who notes that it is found experimentally that on most bowling greens \( \mu_f \sim 0.032 \), independent of the mass or speed of the bowl, but can vary from 0.025 on a fast green up to 0.038 on a slow green. It may be seen that for \( \lambda = +45^\circ \) the maximum amount of side-ways movement occurs, as expected, for the smallest velocity of projection of 3.0 m s\(^{-1}\) with the not insignificant value of 2.81 cm, and varies down to 1.40 cm for the highest velocity of 6.0 m s\(^{-1}\). These values may be compared with the deflection of 2.53 cm shown in table 1 noted above, this value being for a constant velocity of 2.5 m s\(^{-1}\) and derived on the basis of a simple analysis. For a fixed velocity of projection of 4.5 m s\(^{-1}\), table 3 shows that the side-ways movement varies from zero at the equator (\( \lambda = 0^\circ \)), as expected, to a maximum of about 2.65 cm at the pole (\( \lambda = 90^\circ \)).

While the values in table 3 are for the direction of projection in the positive \( y \) direction, the same numerical values would be obtained for any direction of projection for a given latitude. In the northern hemisphere the direction of movement is, in general, towards the right and in the southern hemisphere it is to the left. Thus, in the northern hemisphere, for example, if the bowler were to change ends, the direction

| Figure number | \( \lambda \) (degrees) | \( v(0) \) (km h\(^{-1}\)) | \( v_y(0) \) (m s\(^{-1}\)) | \( \mu_f \) | \( t \) (s) | \( x \) (cm) |
|---------------|-----------------|-----------------|-----------------|--------------|--------|--------|
| 8(a)          | 45              | 10.8            | 3.0             | 0.01312      | 23.32  | 2.81 |
| "            | 25              | 12.6            | 3.5             | 0.01786      | 20.00  | 2.41 |
| "            | 35              | 14.4            | 4.0             | 0.02334      | 17.48  | 2.10 |
| "            | 45              | 16.2            | 4.5             | 0.02952      | 15.56  | 1.87 |
| "            | 55              | 18.0            | 5.0             | 0.03643      | 14.00  | 1.69 |
| "            | 65              | 19.8            | 5.5             | 0.04410      | 12.72  | 1.53 |
| "            | 75              | 21.6            | 6.0             | 0.05245      | 11.68  | 1.40 |
| 8(b)          | 0               | 16.2            | 4.5             | 0.02952      | 15.56  | 0.00 |
| "            | 15              | "               | "               | "            | "      | 0.68 |
| "            | 30              | "               | "               | "            | "      | 1.32 |
| "            | 45              | "               | "               | "            | "      | 1.87 |
| "            | 60              | "               | "               | "            | "      | 2.29 |
| "            | 75              | "               | "               | "            | "      | 2.56 |
| "            | 90              | "               | "               | "            | "      | 2.65 |

The parameters are as follows: \( \lambda \) is the latitude, \( v_x(0) \) is the velocity of projection to the north, \( \mu_f \) is the relevant coefficient of rolling friction necessary to bring the bowl to rest at a distance of \( y = 35 \) m, the values shown being found by trial and error, \( t \) is the run-time of the bowl to the end of the green and \( x \) is the amount of side-ways movement to the east.

Table 3. Side-ways movement, \( x \), of a lawn bowl due to the Coriolis force for various velocities of projection and latitudes as shown in figure 8.

### Table 3.

| Figure number | \( \lambda \) (degrees) | \( v(0) \) (km h\(^{-1}\)) | \( v_y(0) \) (m s\(^{-1}\)) | \( \mu_f \) | \( t \) (s) | \( x \) (cm) |
|---------------|-----------------|-----------------|-----------------|--------------|--------|--------|
| 8(a)          | 45              | 10.8            | 3.0             | 0.01312      | 23.32  | 2.81 |
| "            | 25              | 12.6            | 3.5             | 0.01786      | 20.00  | 2.41 |
| "            | 35              | 14.4            | 4.0             | 0.02334      | 17.48  | 2.10 |
| "            | 45              | 16.2            | 4.5             | 0.02952      | 15.56  | 1.87 |
| "            | 55              | 18.0            | 5.0             | 0.03643      | 14.00  | 1.69 |
| "            | 65              | 19.8            | 5.5             | 0.04410      | 12.72  | 1.53 |
| "            | 75              | 21.6            | 6.0             | 0.05245      | 11.68  | 1.40 |
| 8(b)          | 0               | 16.2            | 4.5             | 0.02952      | 15.56  | 0.00 |
| "            | 15              | "               | "               | "            | "      | 0.68 |
| "            | 30              | "               | "               | "            | "      | 1.32 |
| "            | 45              | "               | "               | "            | "      | 1.87 |
| "            | 60              | "               | "               | "            | "      | 2.29 |
| "            | 75              | "               | "               | "            | "      | 2.56 |
| "            | 90              | "               | "               | "            | "      | 2.65 |
of the movement due to the Coriolis force will always be to the bowler’s right. An experimental way of testing these results might be for a very good player to aim a bowl, manufactured without bias, directly at the kitty from either end of the green, without trying to compensate for any apparent side-ways movement, and then to statistically analyse the results of many trials. Of course other factors such as the slope of the green might play a role, although the best greens are usually prepared to an extremely high degree of precision, often with the use of lasers to ensure that they are as close as possible to being horizontal. In an attempt to minimize such effects, trials could be made at a number of different greens. It may be that some lawn bowlers are aware of a systematic side-ways movement to the right (in the northern hemisphere), largely masked by the effect of the bias, but are unaware that it might be a real effect and not due to their particular bowling idiosyncrasies.

6.5. Olympic rowing

In section 1 it was noted that it is well known that the Coriolis force can cause preferential erosion of the right-hand bank (when looking downstream) of rivers in the northern hemisphere and the left bank in the southern hemisphere. Thus it will cause any floating (or submerged) objects to move in these side-ways directions unless the side-ways motion is resisted.

In the context of this paper the case of a rowing (sculling) event differs from that of lawn bowls in that, rather than the resistance force slowing the bowl down to zero speed, the rower maintains the speed throughout the event. For simplicity we will assume that this speed remains constant. To take a specific example, we consider a rower travelling north in the northern hemisphere over a 2 km course at a velocity of \( v = 5 \text{ m s}^{-1} \) \((18 \text{ km h}^{-1})\) giving a total time for the event of \( t = 400 \) s. We have approached the problem as follows.

We assume that there is a resistance force, \( F_{r} \), due to the water which opposes the motion and that this force acts in the opposite direction to the instantaneous velocity vector and is constant in magnitude, as was the case with lawn bowls. Because of its direction this force will (i) slow the rower down and (ii) resist any side-ways movement due to the Coriolis force, the Coriolis force acting towards the east in this case (i.e., in the positive \( x \) direction). We then assume that there is a ‘propulsion’ force provided by the rower which has two components, one in the direction of intended travel (in this case to the north or in the positive \( y \) direction, \( F_{x,p} \); see figure 1) and a second which opposes the effect of the Coriolis force (in this case to the west or in the negative \( x \) direction, \( F_{x,p} \)). It is also important to note that if the side-ways propulsion force \( F_{x,p} \) is not set to zero, the direction of the friction force \( F_{r} \) will vary slightly as it is in the opposite direction to the instantaneous velocity vector, which will no longer remain in the positive \( y \) direction. This is of course incorporated in the equations of motion and the numerical solution, and the conclusions of this section are not compromised by this effect.

It can be seen from figure 9 and table 4 that, without the compensating efforts of the rower, the side-ways movements are large, up to 40 m in the positive \( x \) direction. For a fixed latitude a number of important points are evident from figure 9(a) and table 4. Firstly, and perhaps most importantly, the larger the friction resistance force the smaller the side-ways movement. Qualitatively this is as expected, since the friction force is in the opposite direction to the instantaneous velocity vector and therefore has a component opposing the side-ways movement, that is in the negative \( x \) direction. If the friction force were large enough there would be no side-ways movement at all. (For the trajectories of figure 9(a), for this to actually occur the numerical computations indicate that \( F_{r} \) must be very large indeed, \( \sim 1.5 \text{ mg} \).)

A second point of importance is that for a fixed latitude table 4 indicates that the actual magnitude of the side-ways propulsion force \( F_{x,p} \) is in fact constant and independent of the magnitude of the resistance force. At first sight this would appear to be questionable, since any resistance force reduces the side-ways motion, as can be seen from figure 9(a). However, for the south to north path considered here, if an adequate side-ways compensating force is applied by the rower at all times, beginning from \( t = 0 \), movement will be directly north in the positive \( y \) direction at all times. Since the resistance force is always in the opposite direction to the instantaneous velocity vector, the resistance force can then make no contribution whatsoever to the reduction of the side-ways motion. If, for a fixed value of the resistance force, the side-ways compensating force were to be increased from zero, the instantaneous velocity vector would be directed increasingly more towards the north. Thus, the component of the resistance force opposing the side-ways motion would be reduced in direct proportion to the increase in the side-ways compensating force.
that the heavy continuous curve in the centre of each panel, corresponding to time for the event is both up the force the smaller the component of the propulsion force again this is to be expected since the smaller the resistance energy for forward propulsion, a matter of some significance.

Finally, an examination of table 4 reveals that the smaller the resistance force the larger the ratio of the lost energy necessary to compensate for the Coriolis force to the useful energy for forward propulsion, a matter of some significance. again this is to be expected since the smaller the resistance force the smaller the component of the propulsion force necessary to provide forward movement, \( F_{c,p} \), but the side-ways compensating force, \( F_{s,p} \), remains constant. For a latitude of 45° the ratio of the side-ways to forward propulsion forces ranges from about 0.5% for the maximum resistance force used up to about 7.5% for the minimum resistance force. These values may be compared with the maximum side-ways movement of \( \sim 40 \text{ m} \) if no side-ways compensating force is applied, which is about 2% of the total distance travelled of 2000 m. From a practical point of view clearly the smaller the resistance force the better. However, one then has to be
reconciled to the necessity of devoting relatively more of the total energy expended in providing a sufficient side-ways force to compensate for the Coriolis force.

The side-ways shift of 41.24 m for the zero friction case of table 4 is identical (to four significant figures) to the relevant entry in table 1, the latter being obtained on the basis of simple considerations. This is as expected since the two cases are for a constant velocity of 5 m s\(^{-1}\) and a distance travelled of 2000 m, and equation (5), which was used to generate the entry in table 1, is a very good approximation to the true situation.

7. Summary and conclusions

To the equations of motion for a rotating spherical projectile subject to gravity, a drag force and a lift force in the presence of an arbitrary wind, we have added three extra terms representing (i) the Coriolis force, (ii) the centrifugal force, and (iii) a friction force. The first two of these terms are the inertial forces present on the rotating Earth, and the third is a resistive force which is present if there is contact with the ground or another medium such as water. We have applied the equations to representative sporting cases, specifically cricket, golf, lawn bowls and Olympic rowing. The principal conclusions of this work are as follows.

(i) The simple estimates of the side-ways movement due to the Coriolis force, as presented in table 1, yield results broadly consistent with the detailed trajectory calculations in section 6 for four of the five cases in common. The one exception is the golf tee stroke, where table 1 gives a side-ways movement of \(\sim 3.9\) cm whereas the detailed calculation in section 6.3 yields \(\sim 6.3\) cm for the trajectory most similar to that in table 1 (see point (iv) below). The difference is attributable primarily to the much longer flight time of 6.9 s in the trajectory calculation compared to 3.0 s for the table 1 results.

(ii) For a cricket ball thrown from the boundary, a distance taken to be about 80 m, the side-ways movement due to the Coriolis force for the trajectories calculated is about 1.28 cm at the poles, and about 0.84 cm at a latitude of \(\lambda = 45^\circ\) (see figures 2 and 3). The deflection is primarily to the right in the northern hemisphere and to the left in the southern hemisphere. For practical purposes the size of the deflection is probably not significant, but the reversal in going from one hemisphere to the other may be worthy of consideration.

(iii) For a slow bowler’s delivery in cricket the deflection due to the Coriolis force is also primarily to the right in the northern hemisphere and to the left in the southern hemisphere (see figures 4 and 5). This means that in the northern hemisphere it acts in the same direction as the movement off the pitch of a right-hand off-spinner’s delivery to a right-hand batsman. In the southern hemisphere the direction of the movement is reversed and is in the direction of a right-hand leg-spinner’s delivery to a right-hand batsman. However, the size of the movement being \(\sim 1\) mm at most is likely to be completely masked by other effects.

(iv) In golf, for a typical long drive from the tee of about 250 m, at a latitude of 45\(^\circ\) the Coriolis force driven side-ways deflection is about 6.3 cm (see figures 6 and 7). The directions of the deflections are identical to those for the cricket ball throw shown in figures 2 and 3, although for the golf drive they are approaching an order of magnitude larger in size. For a 50 m pitch shot to the green, depending on the loft of the stroke the deflection is in the range 0.440–0.875 cm, perhaps enough for the best professionals to take into account, or at least be aware of.

(v) In the case of lawn bowls, at a latitude of 45\(^\circ\) the side-ways deflection due to the Coriolis force is found to be up to about 2.8 cm (see figure 8 and table 3). In the context of the game, where the aim is for the bowl to come to rest as close as possible to the kitty, we believe that this is significant, and players should at least be aware of the possibility of an un-explained and systematic side-ways movement.

(vi) In an Olympic 2 km rowing (sculling) event, at a latitude of 45\(^\circ\) the side-ways movement due to the Coriolis force can be more than 40 m over the course of the race if the effect is not compensated for (see figure 9 and table 4). In order to correct or nullify this side-ways movement, a compensating force must be applied by the rower, a compensating force which the rower may not be aware of. The required force, may be up to \(\sim 7.5\%\) of the forward propulsion force provided by the rower. While this is unavoidable, if the rower is made aware of this side-ways force it could possibly lead to an improvement in performance, particularly given that the force’s magnitude is latitude dependent and its direction is hemisphere dependent.

Clearly there are many opportunities for future work in this field, particularly in sports where a resistive friction-like drag force is in play. For example, in the field of Olympic rowing one could obtain real data for the resistive drag force, based on the actual water-drag experienced by boats used in this sport. Such data could very easily be incorporated into the equations of motion documented in this paper. Also, in general any sport in which either long distances of travel and/or large times are involved is worthy of investigation for the importance or otherwise of the inertial forces.

Acknowledgments

Again we thank the editor-in-chief of Physica Scripta, Dr Suzanne Lidström, for her enthusiastic and on-going encouragement, and especially for her efforts in attempting to make our work more useful to potential readers. We also thank an anonymous associate editor of Physica Scripta for earlier helpful suggestions, and two anonymous referees for their valuable constructive comments which have led to the clarification of a number of points.
### Table A1. Table of symbols used in this paper (see also figure 1).

| Symbol | Meaning |
|--------|---------|
| A      | Cross-sectional area of ball, for cricket taken to be $4.07 \times 10^{-3}$ m$^2$, for golf taken to be $1.43 \times 10^{-3}$ m$^2$ |
| $a_{rot}$ | Ball’s acceleration as measured in the rotating (Earth) frame |
| $a_x, a_y, a_z; \dot{x}, \dot{y}, \dot{z}$ | $x, y, z$ components of the ball’s acceleration as measured in the rotating (Earth) frame |
| $a_{x,\text{Cut}}, a_{y,\text{Cut}}, a_{z,\text{Cut}}$ | $x, y, z$ components of the Coriolis acceleration |
| $C_D$ | Drag coefficient (dimensionless) |
| $C_L$ | Lift coefficient (dimensionless) |
| $F_{\text{cent}}$ | Centrifugal force, $F_{\text{cent}} = -m \Omega \times (\Omega \times \ddot{r})$; symbols as below |
| $F_{\text{rot}}$ | Centrifugal force measured relative to the origin at $O'$ |
| $F_{x,\text{Cut}}, F_{y,\text{Cut}}, F_{z,\text{Cut}}, F_{x,\text{cent}}, F_{y,\text{cent}}, F_{z,\text{cent}}$ | $x, y, z$ components of $F_{\text{cent}}$ |
| $F_{\text{Cor}}$ | Coriolis force, $F_{\text{Cor}} = -2m \Omega \times \dot{\omega}$; symbols as below |
| $F_{\text{Cut}}$ | Magnitude and $x, y, z$ components of the Coriolis force |
| $F_{x}, F_{y}, F_{z}$ | Friction force |
| $F_{\text{tot}}$ | Magnitude and $x, y, z$ components of the friction force |
| $F_{\text{tot}}$ | Resultant external force |
| $\ddot{r}$ | Effective force as measured in the rotating (Earth) frame |
| $F_{\text{tot}}$ | Propulsion force in $x$ and $y$ directions respectively (used in connection with rowing) |
| $\ddot{g}$ | Acceleration due to gravity vector, magnitude $g = 9.8$ m s$^{-2}$ |
| $h$ | Height from which a body is dropped |
| $i, j, k$ | Unit vectors in the $x, y$ and $z$ directions respectively |
| $m$ | Mass of ball, for cricket taken to be $1.59 \times 10^{-1}$ kg, for golf taken to be $4.59 \times 10^{-2}$ kg |
| $R_E$, $R_E$ | Radius vector from the Earth’s centre to the origin on the Earth’s surface, $O'$, magnitude $R_E$, taken to be $6.37 \times 10^6$ m |
| $\ddot{r}$ | Radius vector from the Earth’s centre to the point of observation, $P$ |
| $r_{O'}$ | Position vector from $O'$ to the point of observation, $P$, $\ddot{r}_{E} + \ddot{r}_{O'} = \ddot{r}$ |
| $t$ | Time |
| $\ddot{v}$ | Ball’s velocity relative to the Earth’s surface (i.e., as measured in the rotating reference frame); in the absence of a wind this is equal to the velocity relative to the air |
| $v; v_x, v_y, v_z$ | Magnitude and $x, y, z$ components of $\ddot{v}$ |
| $v(0); v_x(0), v_y(0), v_z(0)$ | Magnitude and components of initial velocity of ball |
| $\ddot{v}$ | Unit vector in the direction of $\ddot{v}$ |
| $\ddot{v} - \dot{\omega} = \ddot{\omega}$ | Ball’s velocity relative to the air, magnitude, $\ddot{V}$ ($\dot{\omega}$ as below) |
| $\dot{\omega}$ | Wind velocity and $x, y, z$ components, relative to the Earth’s surface, normally taken to be zero in this paper |
| $x, y, z$ | Position coordinates of ball, $x$ and $y$ in the horizontal plane, with $x$ to the east and $y$ to the north, $z$ vertically upwards |
| $x(0), y(0), z(0)$ | Initial position of ball |
| $\lambda$ | Latitude of the point of observation ($-90^\circ \leq \lambda \leq 90^\circ$) |
| $\lambda_{\text{initial}}, \lambda_{\text{final}}$ | Initial and final latitudes of observation |
| $\mu_t$ | Coefficient of rolling friction |
| $\rho$ | Density of air, taken to be $1.22$ kg m$^{-3}$ |
| $\phi$ | Elevation angle of the trajectory, measured positive above the horizontal $(x-y)$ plane ($-180^\circ \leq \phi \leq 180^\circ$) |
| $\phi(0)$ | Initial ball elevation angle of projection, measured as for $\phi$ |
| $\ddot{\omega}$ | Ball’s angular velocity (spin) vector |
| $\omega; \omega_x, \omega_y, \omega_z$ | Magnitude and $x, y, z$ components of $\ddot{\omega}$ (rad s$^{-1}$) |
| $\Omega$ | Angular velocity (spin) vector of the Earth |
| $\Omega_x, \Omega_y, \Omega_z$ | Magnitude and $x, y, z$ components of $\Omega$, magnitude taken to be $7.29 \times 10^{-5}$ rad s$^{-1}$ (based on a rotation period relative to the fixed stars of 23 h 56 min 4.099 s (see e.g., Hopkins 1976)) |

---

**Notes:**
- $\ddot{r}$ represents the acceleration due to gravity vector, magnitude $g = 9.8$ m s$^{-2}$
- $R_E$ is the radius of the Earth
- $\ddot{V}$ represents the wind velocity
- $\phi$ is the elevation angle of the trajectory
- $\mu_t$ is the coefficient of rolling friction
- $\rho$ is the density of air
- $\Omega$ is the angular velocity of the Earth
- $f$ is the friction force
- $C_D$ and $C_L$ are the drag and lift coefficients, respectively
- $F_{\text{rot}}$ and $F_{\text{cent}}$ are the centrifugal force and centripetal force, respectively
- $F_{\text{Cor}}$ is the Coriolis force
- $v_x, v_y, v_z$ are the components of the initial velocity of the ball
- $\ddot{v}$ is the acceleration of the ball
- $\ddot{r}$ is the acceleration of the observer with respect to the Earth's center
- $\ddot{V}$ is the wind velocity
- $\lambda$ is the latitude of the point of observation
- $\mu_t$ is the coefficient of rolling friction
Table B1. Table showing the direction of the Coriolis force, \( \vec{F}_{\text{Cor}} = -2 m\vec{\Omega} \times \vec{v} \), for various directions of the velocity vector, \( \vec{v} \) (see also figure 1, equation (2) and section 2).\(^a\)

| Hemisphere | Velocity Direction | \( v_x \) | \( v_y \) | \( v_z \) | \( F_{x,\text{Cor}} \) | \( F_{y,\text{Cor}} \) | \( F_{z,\text{Cor}} \) | Force Direction | Limiting Latitudes |
|------------|--------------------|----------|----------|----------|----------------|----------------|----------------|-----------------|------------------|
| Northern N | 0 +ve 0 +ve 0    | E,R      |          |          | \( \vec{F}_{\text{Cor}} = 0 \) | \( \vec{F}_{\text{Cor}} \) max |                |                 |
| Southern N | 0 +ve 0 +ve 0    |          |          |          |                      |                 | \( F_{x,\text{Cor}} = 0 \) | \( F_{y,\text{Cor}} \) max | \( F_{z,\text{Cor}} = 0 \) |
| Northern S | 0 −ve 0 −ve 0    | W,R      |          |          | \( \vec{F}_{\text{Cor}} \) max | \( \vec{F}_{\text{Cor}} = 0 \) |                |                 |
| Southern S | 0 −ve 0 −ve 0    |          |          |          |                      |                 | \( F_{x,\text{Cor}} = 0 \) | \( F_{y,\text{Cor}} \) max | \( F_{z,\text{Cor}} = 0 \) |
| Northern E | +ve 0 0 0        | S,VU,R   |          |          | \( \vec{F}_{\text{Cor}} = 0 \) | \( \vec{F}_{\text{Cor}} \) max |                |                 |
| Southern E | +ve 0 0 0        |          |          |          |                      |                 | \( F_{x,\text{Cor}} = 0 \) | \( F_{y,\text{Cor}} \) max | \( F_{z,\text{Cor}} = 0 \) |

\(^a\) The meanings of the symbols are as follows: \( m \) (mass of the body considered), \( \vec{\Omega} \) (angular velocity vector of the Earth), N (north), S (south), E (east), W (west), VU (vertically upwards), VD (vertically downwards), R (the deflection is to the right in the horizontal plane) and L (the deflection is to the left in the horizontal plane). For example, under ‘Force Direction’, S,VU,R means that the Coriolis force has a southerly component, a vertically upwards component, and the deflection is to the right in the horizontal plane relative to the velocity vector. Note that in the northern hemisphere the deflection is to the right and in the southern hemisphere it is to the left.

References

Adair R K 2002 The Physics of Baseball 3rd edn (New York: Harper Collins)

Baker C J 2013 A unified framework for the prediction of cricket ball trajectories in spin and swing bowling Proc. Inst. Mech. Eng. P 227 31–81

Belorizky E and Sivardière J 1987 Comments on the horizontal deflection of a falling object Am. J. Phys. 55 1103–4

Bray K and Kerwin D G 2003 Modelling the flight of a soccer ball in a direct free kick J. Sports Sci. 21 75–85

Brearley M N 1961 The motion of a biased bowl with perturbing projection conditions Math. Proc. Camb. Phil. Soc. 57 131–51

Brearley M N and Bolt B A 1958 The dynamics of a bowl Q. J. Mech. Appl. Math. 11 351–63

Coriolis G G 1835 Mémoire sur les équations du mouvement relatif des systèmes de corps J. de l’école Polytech. 15 142–54

Cross R 1998 The trajectory of a ball in lawn bowls Am. J. Phys. 66 735–8

Daish C B 1972 The Physics of Ball Games (London: The English Universities Press) pp 73–90, 147–67

de Mestre N 1990 The Mathematics of Projectiles in Sport (Cambridge: Cambridge University Press)

Desloge E A 1985 Horizontal deflection of a falling object Am. J. Phys. 53 581–2

Desloge E A 1989 Further comments on the horizontal deflection of a falling object Am. J. Phys. 57 282–4

Erichson H 1983 Maximum projectile range with drag and lift, with particular application to golf Am. J. Phys. 51 357–62

French A P 1971 Newtonian Mechanics (New York: W W Norton) pp 507–30

French A P 1984 The deflection of falling objects Am. J. Phys. 52 199

Goff J E 2013 A review of recent research into aerodynamics of sport projectiles Sports Eng. 16 137–54

Goldberg H 1950 Classical Mechanics (Reading, MA: Addison-Wesley) pp 132–40

Hall E H 1903a Do falling bodies move south?: I. Historical Am. J. Phys. Rev. 17 179–90

Hall E H 1903b Do falling bodies move south?: II. Methods and results of the author’s work Phys. Rev. 17 245–55

Halliday D, Resnick R and Walker J 2011 Fundamentals of Physics 9th edn (New York: Wiley) pp 116–21, 278–80

Hart D and Croft T 1988 Modelling with Projectiles (Chichester: Ellis Horwood)

Hopkins J 1976 Glossary of Astronomy and Astrophysics (Chicago, IL: University of Chicago Press) p 140

Jensen J H 2012 Five ways of deriving the equation of motion for rolling bodies Am. J. Phys. 80 1073–7

Jorgensen T P 1999 The Physics of Golf 2nd edn (New York: Springer)

McCuskey S W 1959 An Introduction to Advanced Dynamics (Reading, MA: Addison-Wesley) pp 28–38

Mehta R D 1985 Aerodynamics of sports balls Annu. Rev. Fluid Mech. 17 151–89

Mehta R D 2005 An overview of cricket ball swing Sports Eng. 8 181–92
Mehta R D 2014 Fluid mechanics of cricket ball swing Proc. 19th
Australasian Fluid Mechanics Conf. (Melbourne, Australia,
8–11 December 2014) pp 1–8
Nathan A M 2008 The effect of spin on the flight of a baseball Am. J.
Phys. 76 119–24
Newton I 1672 New theory about light and colours Phil. Trans. R.
Soc. 80 3075–87 (reprinted 1993 Am. J. Phys. 61 108–12)
Reddingius E 1984 Comment on “The eastward deflection of a
falling object” Am. J. Phys. 52 562–3
Robinson G and Robinson I 2013 The motion of an arbitrarily
rotating spherical projectile and its application to ball games
Phys. Scr. 88 018101
Robinson G and Robinson I 2015 The effect of spin in swing bowling
in cricket: model trajectories for spin alone Phys. Scr. 90 028004
Robinson G and Robinson I 2016a Radar speed gun true velocity
measurements of sports-balls in flight: application to tennis
Phys. Scr. 91 023008
Robinson G and Robinson I 2016b Spin-bowling in cricket re-
visited: model trajectories for various spin-vector angles Phys.
Scr. 91 083009
Rodriguez L 1998 Comment on “A ball rolling on a freely spinning
turntable” by Warren Weckesser [Am. J. Phys. 65 (8), 736–738
(1997)] Am. J. Phys. 66 927
Scorer R S 1997 Dynamics of Meteorology and Climate (Chichester:
Wiley-Praxis) pp 108–14
Smits A J and Smith D R 1994 A new aerodynamic model of a golf
ball in flight Science and Golf II: Proc. World Scientific
Congress of Golf ed A J Cochran and M R Farrally (London:
E and F N Spon) pp 340–7
Sokirko A V, Belopolskii A A, Matytsyn A V and Kossakowski D A
1994 Behavior of a ball on the surface of a rotating disk Am. J.
Phys. 62 151–6
Spathopoulos V M 2009 A spreadsheet model for soccer ball flight
mechanics simulation Comput. Appl. Eng. Educ. 19
508–13
Štěpánek A 1988 The aerodynamics of tennis balls—the topspin lob
Am. J. Phys. 56 138–42
Stirling D R 1984 Reply to “Comment on ‘The eastward deflection
of a falling object’” Am. J. Phys. 52 563
Weckesser W 1997 A ball rolling on a freely spinning turntable Am.
J. Phys. 65 736–8