The Potential Fate of Local Model Building

Christoph Lüdeling
bctp and PI, University of Bonn

Perimeter Institute
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CL, Hans Peter Nilles, Claudia Christine Stephan
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Motivation and Outline

- F-Theory Model Building: Generalisation of type IIB intersecting branes
- Usually, consider local models: Focus on brane stack or points within the stack and decouple bulk of the compactification manifold
- Advantage: Simple, physics basically fixed by symmetry
- Obvious question: Existence of global completion
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• GUT models need to address proton stability

• Dimension-four proton decay: Forbidden by matter parity or variants – should be defined locally

• Dimension-five proton decay: Use zero mode assignment, i.e. additional $U(1)$ symmetries present in the setup
Contents

1 F-Theory GUT Model Building

2 Local $SU(5)$ GUTs in F-Theory

3 Matter Parity and Proton Stability in Local Models

4 Semilocal Embedding

5 Conclusion
1. Find realistic particle physics models in string theory:
   - Gauge group (standard model or GUT)
   - Matter content (chiral spectrum, doublet-triplet splitting, absence of light exotics)
   - Proton stability
   - Fermion masses and mixings
   - Spontaneously broken $\mathcal{N} = 1$ SUSY in four dimensions

2. Look for imprints of string theory in low-energy physics:
   - Mediation schemes, patterns of soft masses
   - Exotics below GUT/Planck scale
   - Thresholds, gauge coupling unification
Promising paths:

- $E_8 \times E_8$ heterotic string on orbifolds or smooth Calabi–Yaus
  - Global models, i.e. full compactification space is specified
  - Gauge fields live in bulk, matter in bulk or on lower-dimensional subspaces
- Type II theories with intersecting branes $\rightsquigarrow$ F-theory
  - Mostly local models, i.e. focus on branes and “decouple” bulk
  - Gauge fields on branes, matter on intersections of branes

General features:

- Exceptional symmetry groups (though not as gauge groups in four dimensions)
- Nontrivial pattern of gauge and matter fields localised on different subspaces of compactification space
Cartoon: Intersecting Branes in Perturbative Type IIB

Transverse D7 brane

Stack of 5 D7 branes
Cartoon: Intersecting Branes in Perturbative Type IIB

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Transverse D7 brane

Stack-stack strings: $U(5)$ gauge field
Cartoon: Intersecting Branes in Perturbative Type IIB

Intersection: 6 Branes $\sim U(6)$

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stack-stack strings: $U(5)$ gauge field
Cartoon: Intersecting Branes in Perturbative Type IIB

Intersection: 6 Branes \( \sim U(6) \)

Stack of 5 D7 branes

Transverse D7 brane

stack-stack strings: 

\( U(5) \) gauge field

stack-brane strings: 

localised 5 matter
Intersecting Branes

- Matter localised at intersection ("matter curve") where symmetry is enhanced – representations can be inferred from decomposition of adjoint of higher group: 

\[
G \rightarrow G_1 \times G_2
\]

\[
\text{adj}(G) \rightarrow \text{adj}(G_1) \oplus \text{adj}(G_1) \oplus \bigoplus \text{matter reps } R_i
\]

- Stacks of D7 branes and their intersections: \(U(N)\) gauge groups, bifundamental matter
- Include O7 planes: Realise \(SO(2N)\) gauge groups and two-index antisymmetric representations, e.g. \(10\) of \(SU(5)\)
- Matter curves six-dimensional – matter still in hypermultiplets, 4D zero modes determined by fluxes
- Triple intersection of matter curves: Yukawa couplings via triple adjoint interaction

\[
(\text{adj}(G))^3 \supset R_1 R_2 R_3 + \cdots
\]
Problems

- Possible symmetry groups: $U(N)$, $SO(2N)$ and $Sp(2N)$
- Matter representations: Bifundamental, two-index antisymmetric
- For $SU(5)$ GUTs: diagonal $U(1) \subset U(5)$ forbids top quark Yukawa coupling
- For $SO(10)$ GUTs: no spinors available
Problems

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- For $SO(10)$ GUTs: no spinors available
- Both require local $E_6$ enhancement:
  
  $SU(5)$ Yukawas:  \[ (78)^3 \supset 10 \cdot 10 \cdot 5 \]
  
  $SO(10)$ Spinors:  \[ 78 \rightarrow 45 + 1 + 16 + \overline{16} \]

- Type IIB string theory has more general $(p, q)$ branes – cannot be treated perturbatively
- Nicely realised in F-theory
F-Theory: Axiodilaton Monodromy

Type II\(B\) contains complex scalar field: axiodilaton

\[ \tau = C_0 + i e^{-\phi} \]

When encircling a 7-brane, \(\tau\) undergoes \(SL(2, \mathbb{Z})\) monodromy transformation

\[ \tau \longrightarrow \frac{a\tau + b}{c\tau + d} \]

E.g. for a single D7 brane at \(z = 0\),

\[ \tau \longrightarrow \tau + 1 \quad \Rightarrow \quad \tau \sim \ln z \]

\(\Rightarrow\) at brane positions, \(\tau\) diverges
Key idea of F-Theory: $SL(2, \mathbb{Z})$ is also symmetry of torus complex structure $\leadsto$ describe variation of $\tau$ by auxiliary torus over every point of compactification space $B_3$: elliptic fibration
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F-Theory: Extra Torus

Key idea of F-Theory: $SL(2, \mathbb{Z})$ is also symmetry of torus complex structure $\leadsto$ describe variation of $\tau$ by auxiliary torus over every point of compactification space $B_3$: elliptic fibration

[Vafa 96]
F-Theory: Tate Model

Elliptic fibration: Torus over complex three-dimensional base, described by Weierstraß model in Tate form

\[ y^2 = x^3 + a_5 xy + a_4 x^2 + a_3 y + a_2 x + a_0 \]

\( x, y \in \mathbb{C}, \quad a_k \text{ functions on the base } B_3 \)

Brane positions ⇔ torus degenerates ⇔ Discriminant vanishes:

\[ \Delta = \text{polynomial in the } a_k = 0 \]

Type of brane (stack), i.e. gauge symmetry, determined by vanishing orders of the \( a_k \) and \( \Delta \) (cf. ADE classification of singularities) [Kodaira '60s]

Intersections with other branes ⇔ Local symmetry enhancement ⇔ Locally \( \Delta \) and \( a_k \) vanish to higher order: Matter curves, Yukawa points
For F-Theory GUTs, different degrees of locality:

- **Global** model: Specify full compactification space (CY fourfold):
  Includes all branes, fluxes, obeys consistency conditions, can stabilise moduli etc.
  [Blumenhagen et al.; Grimm et al.; Marsano et al.; ...]

- **Semilocal** model: Focus on the GUT surface (brane stack) $S$ and matter curves within $S$: Decouples bulk of compactification space, certain consistency conditions included
  [Hayashi et al.; Donagi, Wijnholt; Grimm, Weigand; Marsano et al.; Dudas, Palti; CL, Nilles, Stephan; ...]

- **Local** model: Consider only points within $S$ where matter curves intersect and interactions are localised: Simple, hope for predictivity. Certain questions cannot be answered, and actual existence of global completion is not guaranteed.
  [Donagi, Wijnholt; Heckman, Vafa et al.; Watari et al.; ...]
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Local $SU(5)$ GUT

To engineer $SU(5)$ GUT, take brane position locally given by coordinate $w = 0$ and choose Tate model appropriately:

$$y^2 = x^3 + a_5 xy + a_4 x^2 + a_3 y + a_2 x + a_0$$

$a_k$: functions on base
Local $SU(5)$ GUT

To engineer $SU(5)$ GUT, take brane position locally given by coordinate $w = 0$ and choose Tate model appropriately:

$$y^2 = x^3 + b_5 \, xy + b_4 w \, x^2 + b_3 w^2 \, y + b_2 w^4 \, x + b_0 w^5$$

$a_k$: functions on base
$b_k$: functions on brane stack
Local $SU(5)$ GUT

To engineer $SU(5)$ GUT, take brane position locally given by coordinate $w = 0$ and choose Tate model appropriately:

$$y^2 = x^3 + b_5 x y + b_4 w x^2 + b_3 w^2 y + b_2 w^4 x + b_0 w^5$$

$a_k$: functions on base
$b_k$: functions on brane stack

Discriminant becomes

$$\Delta = w^5 \left( b_5^4 P + w b_5^2 (8b_4 P + b_5 R) + \mathcal{O}(w^2) \right)$$

$P, R$: polynomials in the $b_k$
Local $SU(5)$ GUT

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$a_k$: functions on base

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Discriminant becomes

$$\Delta = w^5 \left( b_5^4 P + w b_5^2 (8 b_4 P + b_5 R) + O(w^2) \right)$$

$P$, $R$: polynomials in the $b_k$

Locally, $SU(5)$ is enhanced

- to $SU(6)$: $P = 0 \Rightarrow$ localised 5
- to $SO(10)$: $b_5 = 0 \Rightarrow$ localised 10
Yukawa Couplings

- Tate model for $SU(5)$ GUT localised at $w = 0$:

$$y^2 = x^3 + b_5 xy + b_4 w x^2 + b_3 w^2 y + b_2 w^4 x + b_0 w^5$$

- Yukawa couplings require $SO(12)$ and $E_6$ enhancements:

$$(66)^3 \supset [5_{H_d} 5_M 10_M] \Rightarrow b_5 = b_3 = 0$$

$$(78)^3 \supset [5_{H_u} 10_M 10_M] \Rightarrow b_5 = b_4 = 0$$

For both, $\Delta$ vanishes to order 8

- Matter spectrum and Yukawa couplings can be engineered in F-theory
Point of $E_8$

- Need $E_6$ and $SO(12)$ enhancements for up- and down-type Yukawas
- CKM matrix: Favourably, these points coincide (no geometric suppression of quark mixing) $\sim E_7$
- For PMNS matrix: Further enhancement to $E_8$ (but we do not consider neutrinos in the following)
- Hence: One single Yukawa “point of $E_8$”, all interactions localised here

 Allows for higher interaction terms – Froggatt–Nielsen type masses using GUT singlets

- Simple and potentially predictive: All (superpotential) interactions determined by group theory, geometric data can be largely ignored
• $SU(5)$ GUT, variously enhanced (potentially) up to $E_8$
  $\leftrightarrow E_8$ gauge theory variously broken, generically to $SU(5)$

• 8D super-Yang–Mills theory contains adjoint scalar field
  $\rightsquigarrow E_8$-breaking Higgs

• Actually: rank-preserving breaking

  $E_8 \longrightarrow (SU(5) \times SU(5)_\perp) \longrightarrow SU(5) \times U(1)^4$

• Extra $U(1)$’s generically massive in F-Theory by geometric Stueckelberg effect, but this cannot be analysed in local model – $U(1)$’s remain as *global selection rules* [Grimm, Weigand]

• Higgs field varies over $S$ – matter curves now visible as vanishing loci of Higgs eigenvalues
Type IIB interpretation: Higgs as Brane Splitter

Adjoint Higgs field – parameterises brane motion:

\[
\langle \Phi \rangle \sim \begin{pmatrix} 1 \\ -1 \end{pmatrix}
\]

\[
\langle \Phi \rangle \sim \begin{pmatrix} z \\ -z \end{pmatrix}
\]

\(\langle \Phi \rangle \neq 0\): Masses for W bosons – correspond to strings between the branes.
Symmetry is (partially) restored locally where (parts of) \(\langle \Phi \rangle = 0\)
$E_8$ Higgs

$$E_8 \rightarrow SU(5) \times SU(5)_{\perp}$$

$$248 \rightarrow (24, 1) \oplus (1, 24) \oplus [(10, 5) \oplus (5, \overline{10}) \oplus \text{c.c.}]$$

Higgs $\Phi \sim \begin{pmatrix} t_1 \\ t_2 \\ t_3 \\ t_4 \\ t_5 \end{pmatrix} \in (1, 24) , \quad \sum_i t_i = 0$

Connection to Tate model: Deformed $E_8$ singularity,

$$y^2 = x^3 + b_0 w^5 \quad \rightarrow \quad y^2 = x^2 + b_0 \prod (w - t_i)$$

$\bowtie$ the $b_k$ are symmetric polynomials in the $t_i$ of order $k$, no $b_1$ because of tracelessness
Matter Curves

$t_i$ are eigenvalues in the 5 of $SU(5)\subset$, i.e.

$$\Phi e_i = t_i e_i$$

$\sim 10$ of $SU(5)\subset$ spanned by $e_i \wedge e_j, i \neq j$, with eigenvalue $t_i + t_j$

Representations of $SU(5) \times SU(5)\subset$ appear as $(10, 5) \oplus (5, \overline{10})$

$\sim$ in terms of $SU(5)$ reps, matter curves are given by

$$t_i = 0 \quad \text{localised 10}$$

$$-t_i - t_j = 0 \quad \text{localised 5}$$

$$t_i - t_j = 0 \quad \text{localised 1}$$

$t_i$ double as charges: For gauge-invariant terms, $t_i$ must sum to zero (possibly using $\sum_i t_i = 0$) – realises $U(1)^4 \subset SU(5)\subset$ selection rules
Monodromy

- The $b_k$ in the Tate model are symmetric polynomials in the $t_i$ \Rightarrow Invariant under permutations of the $t_i$
- Interpretation: Self-intersection, locally distinct-looking branes are the same

\[
5_{H_u} 10_{\text{top}} 10_{\text{top}} \sim \{t_1, t_2\}, \quad 5_{H_u} \sim -t_1 - t_2
\]
- Heavy top requires coupling $5_{H_u} 10_{\text{top}} 10_{\text{top}}$
  \sim (at least) $\mathbb{Z}_2$ monodromy $t_1 \leftrightarrow t_2$
- Fixes top and up-type Higgs curve: $10_{\text{top}} \sim \{t_1, t_2\}, \quad 5_{H_u} \sim -t_1 - t_2$
- Reduces $SU(5)_{\perp}$ to lower rank
$SU(5)$ Breaking

Need to break $SU(5) \rightarrow G_{SM}$ and remove $X$, $Y$ bosons and Higgs triplets from spectrum
Discrete Wilson lines and adjoint Higgses not available for $S$ a del Pezzo surface!

$\sim$ Break $SU(5)$ by hypercharge flux

$$F_Y \sim \begin{pmatrix} 2 & 2 & \phantom{2} \\ 2 & 2 & \phantom{2} \\ \phantom{2} & \phantom{2} & -3 \end{pmatrix}$$

$F_Y$ must be globally trivial to preserve hypercharge – mechanism not available in heterotic models!
Superpotential Couplings

Good couplings: Quark and lepton masses, weak-scale $\mu$ term

$$W_{\text{good}} = \mu 5_{H_u} \bar{5}_{H_d} + Y_u 5_{H_u} 10_M 10_M + Y_d \bar{5}_{H_d} \bar{5}_M 10_M$$
Superpotential Couplings

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Bad couplings: Baryon and lepton number violating operators

$$W_{\text{bad}} = \beta 5_{H_u} \bar{5}_M + \lambda \bar{5}_M \bar{5}_M 10_M$$

$$+ W^1 10_M 10_M 10_M \bar{5}_M + W^2 10_M 10_M 10_M \bar{5}_{H_d}$$

$$+ W^3 \bar{5}_M \bar{5}_M 5_{H_u} 5_{H_u} + W^4 \bar{5}_M \bar{5}_{H_d} 5_{H_u} 5_{H_u}$$

$$K_{\text{bad}} = K^1 10_M 10_M 5_M + K^2 \bar{5}_{H_u} \bar{5}_{H_u} 10_M$$

Coefficients can contain singlet VEVs, suppressed by $M_{\text{GUT}}$ [Conlon, Palti]
Superpotential Couplings

Good couplings: Quark and lepton masses, weak-scale $\mu$ term

$$W_{\text{good}} = \mu \, 5_{H_u} \bar{5}_{H_d} + Y_u \, 5_{H_u} \, 10_M \, 10_M + Y_d \, \bar{5}_{H_d} \, \bar{5}_M \, 10_M$$

Bad couplings: Baryon and lepton number violating operators

$$W_{\text{bad}} = \beta \, 5_{H_u} \, \bar{5}_M + \lambda \bar{5}_M \, \bar{5}_M \, 10_M$$
$$+ W^1 \, 10_M \, 10_M \, 10_M \, \bar{5}_M + W^2 \, 10_M \, 10_M \, 10_M \, \bar{5}_{H_d}$$
$$+ W^3 \, \bar{5}_M \, \bar{5}_M \, 5_{H_u} \, 5_{H_u} + W^4 \, \bar{5}_M \, \bar{5}_{H_d} \, 5_{H_u} \, 5_{H_u}$$

$$K_{\text{bad}} = K^1 \, 10_M \, 10_M \, 5_M + K^2 \, \bar{5}_{H_u} \, \bar{5}_{H_u} \, 10_M$$

Coefficients can contain singlet VEVs, suppressed by $M_{GUT}$ [Conlon, Palti]

Some terms related by interchange $\bar{5}_{H_d} \leftrightarrow \bar{5}_M$
Various discrete symmetries for proton stability — compatibility with $SU(5)$ singles out $\mathbb{Z}_2$ “matter parity”:

\[
\begin{array}{c|cc|}
 & 5_{H_u}, \bar{5}_{H_d} & 10_M, \bar{5}_M \\
\hline
P_M & +1 & -1 \\
\end{array}
\]

Forbids all baryon and lepton number violating operators except

\[ W^1 10_M 10_M 10_M \bar{5}_M \quad \text{and} \quad W^3 5_M 5_M 5_{H_u} 5_{H_u} \]

$W^3$ (Weinberg operator), can be tolerated if suppression scale high enough (but will not be generated, so ignore from now on).

$W^1 \supset QQQL, \bar{u}\bar{u}\bar{d}\bar{e}$ extremely constrained — forbid this by clever choice of matter curves (i.e. $U(1)$s)
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Model Requirements

For the local model we require

- \( P_M \) defined locally
- heavy top quark (i.e. rank-one up-type Yukawa matrix at tree level)
- No dim-5 proton decay (the \( W^1 \) operator forbidden at all orders)
- Masses for all quarks and leptons after switching on VEVs (down-type Yukawa matrix can be rank-zero or one, but not rank-two)
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- Masses for all quarks and leptons after switching on VEVs
  (down-type Yukawa matrix can be rank-zero or one, but not rank-two)

Local model building freedom: Freely choose

- Monodromy (at least $\mathbb{Z}_2$)
- Assignment of matter and Higgs zero modes to curves
- Singlet VEVs (for matter parity even singlets)
- Assume: Allowed terms generated with order-one coefficients
Matter Parity

Define $\mathbb{Z}_2$ matter parity in terms of the $t_i$ (i.e. as subgroup of $SU(5)_\perp$):

$$P_M = (-1)^{c_i t_i}, \quad c_i = 0, 1 \quad \text{(defined mod 2)}$$

- Monodromy $t_1 \leftrightarrow t_2$ requires $c_1 = c_2 = 1$ so $10_{\text{top}}$ is odd
- Up-type masses always allowed once gauge invariant
- Down-type masses give constraint:

$$\begin{align*}
\mathbf{5}_{H_d} & \quad \mathbf{5}_M & \quad 10_M \\
\text{charge} & \quad t_i + t_j & \quad t_k + t_l & \quad t_m
\end{align*}$$
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$$\begin{array}{cccc}
\bar{5}_{H_d} & \bar{5}_M & 10_M \\
\text{charge} & t_i + t_j & t_k + t_l & t_m \\
c_i t_i & 0/2 & 1 & 1
\end{array}$$

Gauge invariant iff all $t_i$ distinct – can only be matter parity even if even number of $c_i = 1$ (singlets have charge $t_i - t_j$, so don’t change the argument)

- Note: $W^1$ operator has same charge structure
Two Possibilities

Hence, two possible definitions of matter parity:

Case I: \( P_M = (-1)^{t_1+t_2+t_3+t_4} \)

Case II: \( P_M = (-1)^{t_1+t_2} \)

Now analyse matter, Higgs and VEV assignment for both cases: \( 10_{\text{top}} \) and \( 5_{H_u} \) already fixed, need to distribute remaining matter and \( 5_{H_d} \) according to their matter parity

Main restriction: Forbid \( W^1 \), but allow down-type Yukawas
### Case I: Matter and VEV Assignment

#### Matter 10 Curves

| 10₁ | \( t_{1,2} \) | - | top |
| 10₂ | \( t_3 \) | - |
| 10₃ | \( t_4 \) | - |

#### Matter 5 Curves

| 5₃ | \( -t_{1,2} - t_5 \) | - |
| 5₅ | \( -t_3 - t_5 \) | - |
| 5₆ | \( -t_4 - t_5 \) | - |

#### Even Singlet Curves

| 1₉ | \( \pm (t_{1,2} - t_3) \) | + |
| 1₂ | \( \pm (t_{1,2} - t_4) \) | + |
| 1₄ | \( \pm (t_3 - t_4) \) | + |
| 1₇ | \( t_1 - t_2 \) | + |
Case I: Matter and VEV Assignment

| Matter 10 Curves |  |
|------------------|---|
| 10_1             | $t_{1,2}$ | top |
| 10_2             | $t_3$     |   |
| 10_3             | $t_4$     |   |

| Matter 5 Curves  |  |
|------------------|---|
| 5_3              | $-t_{1,2} - t_5$ |   |
| 5_5              | $-t_3 - t_5$     |   |
| 5_6              | $-t_4 - t_5$     |   |

| Even Singlet Curves |  |
|---------------------|---|
| 1_1                 | $\pm(t_{1,2} - t_3)$ |   |
| 1_2                 | $\pm(t_{1,2} - t_4)$ |   |
| 1_4                 | $\pm(t_3 - t_4)$     |   |
| 1_7                 | $t_1 - t_2$          |   |

- $W^1$ without singlets:
  - $10_1 10_1 10_2 \overline{5}_6$
  - $10_1 10_1 10_3 \overline{5}_5$
  - $10_1 10_2 10_3 \overline{5}_3$
Case I: Matter and VEV Assignment

**Matter 10 Curves**

| 10_1 | t_{1,2} |  | top |
|------|---------|---|-----|
| 10_2 | t_3     |  | no matter |
| 10_3 | t_4     |  | matter |

**Matter 5 Curves**

| 5_3 | -t_{1,2} - t_5 |  | matter |
|-----|-----------------|---|--------|
| 5_5 | -t_3 - t_5     |  | no matter |
| 5_6 | -t_4 - t_5     |  | matter |

**Even Singlet Curves**

| 1_1 | ±(t_{1,2} - t_3) |  | + |
|-----|-----------------|---|---|
| 1_2 | ±(t_{1,2} - t_4) |  | + |
| 1_4 | ±(t_3 - t_4)   |  | + |
| 1_7 | t_1 - t_2     |  | + |

- $W^1$ without singlets:
  - $10_1 10_1 10_2 5_6$,  
  - $10_1 10_1 10_3 5_5$,  
  - $10_1 10_2 10_3 5_3$

  $\rightsquigarrow$ no matter on $10_2, 5_5$
### Case I: Matter and VEV Assignment

#### Matter 10 Curves

| 10₁  | t₁,₂  | - | top       |
|------|-------|---|-----------|
| 10₂  | t₃    | - | no matter |
| 10₃  | t₄    | - | matter    |

#### Matter 5 Curves

| 5₃   | -t₁,₂ - t₅ | - | matter |
|------|-------------|---|--------|
| 5₅   | -t₃ - t₅   | - | no matter |
| 5₆   | -t₄ - t₅   | - | matter |

#### Even Singlet Curves

| 1₁   | ±(t₁,₂ - t₃) | + |
|------|--------------|---|
| 1₂   | ±(t₁,₂ - t₄) | + |
| 1₄   | ±(t₃ - t₄)  | + |
| 1₇   | t₁ - t₂     | + |

- $W^1$ without singlets:

$$10_110_110_2\bar{5}_6,$$

$$10_110_110_3\bar{5}_5,$$

$$10_110_210_3\bar{5}_3$$

$\rightsquigarrow$ no matter on $10_2$, $5_5$

- $W^1$ with singlets:

  e.g. $10_110_110_3\bar{5}_61_4,$

  $$10_110_110_3\bar{5}_31_1$$
Case I: Matter and VEV Assignment

| Matter 10 Curves |
|------------------|
| 10\(_1\) | \(t_{1,2}\) | top |
| 10\(_2\) | \(t_3\) | no matter |
| 10\(_3\) | \(t_4\) | matter |

| Matter 5 Curves |
|------------------|
| 5\(_3\) | \(-t_{1,2} - t_5\) | matter |
| 5\(_5\) | \(-t_3 - t_5\) | no matter |
| 5\(_6\) | \(-t_4 - t_5\) | matter |

| Even Singlet Curves |
|---------------------|
| 1\(_1\) | \(\pm (t_{1,2} - t_3)\) | no VEV |
| 1\(_2\) | \(\pm (t_{1,2} - t_4)\) | VEV |
| 1\(_4\) | \(\pm (t_3 - t_4)\) | no VEV |
| 1\(_7\) | \(t_1 - t_2\) | VEV |

- \(W^1\) without singlets:
  \[10_1 10_1 10_2 \overline{5}_6,\]
  \[10_1 10_1 10_3 \overline{5}_5,\]
  \[10_1 10_2 10_3 \overline{5}_3\]
  \(\leadsto\) no matter on 10\(_2\), 5\(_5\)

- \(W^1\) with singlets:
  e.g. \[10_1 10_1 10_3 \overline{5}_6 1_4,\]
  \[10_1 10_1 10_3 \overline{5}_3 1_1\]
  \(\leadsto\) no VEVs for 1\(_1\), 1\(_4\) (because of \(t_3\))
### Case I: Down-Type Higgs

| Higgs-like 5 Curves | Down-type Yukawas |
|---------------------|--------------------|
| \(\overline{5}_{H_u}\) | \(-t_1 - t_2\) |
| \(\overline{5}_1\) | \(-t_{1,2} - t_3\) |
| \(\overline{5}_2\) | \(-t_{1,2} - t_4\) |
| \(\overline{5}_4\) | \(-t_3 - t_4\) |
### Case I: Down-Type Higgs

| Higgs-like $\mathbf{5}$ Curves | Down-type Yukawas |
|-------------------------------|-------------------|
| $\overline{\mathbf{5}}_{H_u}$ | $-t_1 - t_2$  |
| $\overline{\mathbf{5}}_1$     | $-t_{1,2} - t_3$ |
| $\overline{\mathbf{5}}_2$     | $-t_{1,2} - t_4$ |
| $\overline{\mathbf{5}}_4$     | $-t_3 - t_4$     |

- Down-type Higgs needs a factor of $t_3$ to allow for Yukawa couplings (at any order)

- No masses at tree level or with singlets
### Case I: Down-Type Higgs

| Higgs-like $\mathbf{5}$ Curves | Down-type Yukawas |
|---------------------------------|-------------------|
| $\mathbf{5}_{H_u}$ $\dashrightarrow -t_1 - t_2$ | No masses at tree level or with singlets |
| $\mathbf{5}_1$ $\dashrightarrow -t_{1,2} - t_3$ | either rank-two Yukawa matrix, or no up-type masses with singlets |
| $\mathbf{5}_2$ $\dashrightarrow -t_{1,2} - t_4$ | No masses at tree level or with singlets |
| $\mathbf{5}_4$ $\dashrightarrow -t_3 - t_4$ | |

- Down-type Higgs needs a factor of $t_3$ to allow for Yukawa couplings (at any order)
- Down-type Yukawa should not be rank-two
### Case I: Down-Type Higgs

| Higgs-like $\mathbf{5}$ Curves | Down-type Yukawas |
|-------------------------------|------------------|
| $\overline{5}_{H_u}$ $\leftrightarrow -t_1 - t_2$ | No masses at tree level or with singlets $\mu$ term |
| $\overline{5}_1$ $\leftrightarrow -t_{1,2} - t_3$ | either rank-two Yukawa matrix, or no up-type masses with singlets |
| $\overline{5}_2$ $\leftrightarrow -t_{1,2} - t_4$ | No masses at tree level or with singlets |
| $\overline{5}_4$ $\leftrightarrow -t_3 - t_4$ | |

- Down-type Higgs needs a factor of $t_3$ to allow for Yukawa couplings (at any order)
- Down-type Yukawa should not be rank-two
- String-scale $\mu$ term for both Higgses on one curve
Case I: Down-Type Higgs

| Higgs-like $\mathbf{5}$ Curves | Down-type Yukawas |
|---------------------------------|-------------------|
| $\overline{5}_{H_u}$ $\not\!\!\not\!t$ $-t_1 - t_2$ | No masses at tree level or with singlets $\mu$ term |
| $\overline{5}_1$ $\not\!\!\not\!t$ $-t_{1,2} - t_3$ | either rank-two Yukawa matrix, or no up-type masses with singlets |
| $\overline{5}_2$ $\not\!\!\not\!t$ $-t_{1,2} - t_4$ | No masses at tree level or with singlets |
| $\overline{5}_4$ $\checkmark$ $-t_3 - t_4$ | Rank-one Yukawa matrix, bottom quark heavy |

- Down-type Higgs needs a factor of $t_3$ to allow for Yukawa couplings (at any order)
- Down-type Yukawa should not be rank-two
- String-scale $\mu$ term for both Higgses on one curve
- $\overline{5}_4 = \overline{5}_{H_d}$ is unique choice, tree-level coupling $\overline{5}_{H_d} \mathbf{10}_{\text{top}} \overline{5}_3$
Case I: Yukawas and CKM

- Example Assignment: Third generation on $10_1$ and $\bar{5}_3$, light generations on $10_3$ and $\bar{5}_6$
- Higgses: $\bar{5}_{H_u}$ and $\bar{5}_4$, only $\langle 1_2 \rangle \sim \epsilon$ required at first order
- Ignore $1_7$ and $\mathcal{O}(1)$ coefficients
- Yukawa matrices (schematically):

$$
\begin{align*}
Y^u & \sim Y^d \sim \begin{pmatrix}
\epsilon^2 & \epsilon^2 & \epsilon \\
\epsilon^2 & \epsilon^2 & \epsilon \\
\epsilon & \epsilon & 1
\end{pmatrix} \\

V_{\text{CKM}} & \sim \begin{pmatrix}
1 & 1 & \epsilon \\
1 & 1 & \epsilon \\
\epsilon & \epsilon & 1
\end{pmatrix}
\end{align*}
$$

- CKM matrix:
- Masses and mixings possible though not a great fit
- Degeneracy because three generations come from two curves
Case II

\[ P_M = (-1)^{t_1+t_2} \]

\[ \sim \] split \( t \)'s into \( t_{\text{odd}} = \{t_1, t_2\} \) and \( t_{\text{even}} = \{t_3, t_4, t_5\} \)

- Symmetric setup, possible monodromy acting on \( t_{\text{even}} \)
- \( 10_{\text{top}} \) is the unique matter 10 curve
- Down-type Higgs unique (up to relabeling)
- Matter-parity even singlets do not mix \( t_{\text{odd}} \) and \( t_{\text{even}} \)
- \( W^1 \) operator cannot be generated: Charge 4\( t_{\text{odd}} + t_{\text{even}} \) cannot be compensated by matter-parity even singlets
- Three possible matter \( 5 \) curves (charges \( t_{\text{odd}} - t_{\text{even}} \)): model building choice
- Different choices of singlet VEVs possible, achieve masses and mixing
Already locally, rather constrained model: Only two possible definitions of matter parity

In both cases, assignments of matter and Higgses is unique or strongly constrained

Restrictions mainly from forbidding $W^1$ while allowing for down-type masses

$W^3$ operator (neutrino masses) is not generated in any case

Masses for all matter fields and CKM mixing possible

Involves choices of zero modes and VEVs by hand – these cannot be calculated in the local framework
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Now *semilocal* picture: Consider GUT surface $S$ and fluxes, using spectral cover approach

Two types of fluxes (actually, both merge to $G_4$ flux in F-theory):

- $U(1) \subset SU(5)_\perp$ fluxes on matter curves (from the transverse branes): Determines (chiral) 4D zero modes for full GUT multiplets, free parameters up to anomaly cancellation requirements

- Hypercharge flux on $S$ (globally trivial so hypercharge stays unbroken): Breaks $SU(5)$, restrictions to matter curves splits $SU(5)$

Aim: Find relations between homology classes of matter curves $\rightsquigarrow$ relation between flux restrictions and multiplet splittings
Spectral Cover

Spectral cover: Five-fold cover of $S$ in projective threefold

$$\mathbb{P}(K_S \oplus \mathcal{O}_S)$$

with homogeneous coordinates $U : V$ given by spectral equation for $\Phi$. Because of $\mathbb{Z}_2$ monodromy, spectral equation must factorise:

$$0 = b_0 U^5 + b_2 U^3 V^2 + b_3 U^2 V^3 + b_4 U V^4 + b_5 V^5$$

$$= (a_1 V^2 + a_2 U V + a_3 U^2) (a_4 V + a_7 U) (a_5 V + a_8 U) (a_6 V + a_9 U)$$

$b_k$ are sections in certain line bundles on $S \Rightarrow$ line bundles for the $a_i$ \Rightarrow homology classes of matter curves

Impose triviality of hypercharge flux $\leadsto$ solution contains three arbitrary line bundles

Involves particular solution of $b_1 = 0$ constraint – might not be most general one?
Fluxes and Zero Modes

$U(1)$ fluxes $M_5$, $M_{10}$: Free up to consistency conditions

Hypercharge flux on matter curves: $N_Y = F_Y \cdot (\text{homology class})$.

For curve with flux numbers $M$ and $N_Y$, zero modes given by

\[
\begin{align*}
10 & \quad (3, 2) : M_{10} \\
(\overline{3}, 1) & : M_{10} - N_Y \\
(1, 1) & : M_{10} + N_Y
\end{align*}
\]

\[
\begin{align*}
5 & \quad (3, 1) : M_5 \\
(1, 2) & : M_5 + N_Y
\end{align*}
\]

Hypercharge flux must be globally trivial, hence no net "$SU(5)$ breaking chirality":

\[
0 = F_Y \cdot c_1 = F_Y \cdot \eta \quad \implies \quad \sum_{5} N_Y = \sum_{10} N_Y = 0
\]
### 10 Curves

|    | $M$                          | $N_Y$   |
|----|------------------------------|---------|
| $10_1$ | $- (M_{5_1} + M_{5_2} + M_{5_3})$     | $- \tilde{N}$ |
| $10_2$ | $M_{10_2}$                  | $N_7$  |
| $10_3$ | $M_{10_3}$                  | $N_8$  |
| $10_4$ | $M_{10_4}$                  | $N_9$  |

### 5 Curves

|    | $M$                          | $N_Y$   |
|----|------------------------------|---------|
| $5_{H_u}$ | $M_{5_{H_u}}$            | $\tilde{N}$  |
| $5_1$ | $M_{5_1}$                  | $- \tilde{N}$ |
| $5_2$ | $M_{5_2}$                  | $- \tilde{N}$ |
| $5_3$ | $M_{5_3}$                  | $- \tilde{N}$ |
| $5_4$ | $M_{5_4}$                  | $N_7 + N_8$ |
| $5_5$ | $M_{5_5}$                  | $N_7 + N_9$ |
| $5_6$ | $M_{5_6}$                  | $N_8 + N_9$ |

- Three free parameters $N_{7,8,9}$ for the hypercharge flux, corresponding to three unspecified line bundles
- $\tilde{N} = N_7 + N_8 + N_9$
- Split some 5 curves $\Rightarrow$ split some 10 curves

[Marsano et al.; Dudas, Palti]
Case I: Matter Sector is Fine

- Doublet-triplet splitting for Higgses ($\tilde{N} \neq 0$) inevitably splits $\mathbf{10}_{\text{top}}$ and at least one more $\mathbf{10}$ curve (and at least one matter $\mathbf{5}$ curve).
- However, splitting of matter multiplets is OK as long as there are three generations of zero modes in the end, i.e. other $\mathbf{10}$ curve must have “opposite” split (hence cannot have three generations from one matter curve).
- Matter on $\mathbf{10}_1$, $\mathbf{10}_3$, $\mathbf{5}_3$ and $\mathbf{5}_6$, so to have full net generations, we require

$$N_7 = N_9 = 0 \quad \Rightarrow \quad \text{only } N_8 \text{ left free}$$

- No exotics from $\mathbf{10}$’s and remaining matter-like $\mathbf{5}$ curve can be satisfied by choosing appropriate $M$’s.

◇ Satisfactory matter sector can be engineered easily
Case I: Higgs Sector is not Fine

- Higgs sector:

|       | \((3, 1)\)          | \((1, 2)\)          |
|-------|---------------------|---------------------|
| \(5_{Hu}\) | \(M_{5_{Hu}}\)    | \(M_{5_{Hu}} + N_8\) |
| \(5_1\)    | \(M_{5_1}\)        | \(M_{5_1} - N_8\)   |
| \(5_2\)    | \(M_{5_2}\)        | \(M_{5_2} - N_8\)   |
| \(5_4\)    | \(M_{5_4}\)        | \(M_{5_4} + N_8\)   |

- We can pairwise decouple unwanted triplets from \(5_{Hu}\) and \(5_2\), and from \(5_1\) and \(5_4\) by coupling to VEV for \(1_2\)

- However:

\[ \#(\text{doublets from } 5_{Hu}, 5_2) = \#(\text{triplets from } 5_{Hu}, 5_2) \]

- Problem persists even when allowing exotics from the matter sector
- Separately, down-type Higgs on \(5_4\) cannot be realised
Case II: Again not Fine

- Only one matter $10$ curve, split with same parameter as up-type Higgs
- Higgs triplets cannot be decoupled by even matter parity singlets
- No exotic matter $\Rightarrow$ no doublet-triplet splitting
- Ignoring matter sector: Still cannot achieve pair of Higgs doublets
Case II: Again not Fine

• Only one matter $\mathbf{10}$ curve, split with same parameter as up-type Higgs
• Higgs triplets cannot be decoupled by even matter parity singlets
• No exotic matter $\Rightarrow$ no doublet-triplet splitting
• Ignoring matter sector: Still cannot achieve pair of Higgs doublets

Upshot: In both cases, proper doublet-triplet splitting in Higgs sector does not work, even when allowing for exotics from the matter sector – both models cannot be realised already in semilocal setup!
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- Goal: Use locally defined matter parity and additional $U(1)$s to ensure proton stability
- Local model is already very constrained: Two cases only
- In semilocal embedding, doublet-triplet splitting cannot be realised in either case

Models fail first step towards realisation
Conclusions

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Models fail first step towards realisation

- Predictivity of local point in question – Crucial model features required to have nonlocal origin?
- GUT breaking by hypercharge flux seems too restrictive, also problems with exotics

[Marsano et al.; Dudas, Palti]
Conclusions

- Analysed F-Theory GUT in local and semilocal approach
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- ⇒ Models fail first step towards realisation
- Predictivity of local point in question – Crucial model features required to have nonlocal origin?
- GUT breaking by hypercharge flux seems too restrictive, also problems with exotics
- Possible loopholes: Localised matter might be more subtle
  - Non-diagonal Higgs fields ("T-Branes", "Gluing Morphisms")
  - Relation to higher symmetry groups

[Marsano et al.; Dudas, Palti]
[Cecotti et al.; Donagi, Wijnholt]
[Esole, Yau; Marsano, Schäfer-Nameki]