Špenko, Špela; Van den Bergh, Michel; Bell, Jason P.

On the noncommutative Bondal-Orlov conjecture for some toric varieties. (English)

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Summary: We show that all toric noncommutative crepant resolutions (NCCRs) of affine GIT quotients
of “weakly symmetric” unimodular torus representations are derived equivalent. This yields evidence for
a non-commutative extension of a well known conjecture by Bondal and Orlov stating that all crepant
resolutions of a Gorenstein singularity are derived equivalent. We prove our result by showing that all
toric NCCRs of the affine GIT quotient are derived equivalent to a fixed Deligne-Mumford GIT quotient
stack associated to a generic character of the torus. This extends a result by Halpern-Leistner and Sam
which showed that such GIT quotient stacks are a geometric incarnation of a family of specific toric
NCCRs constructed earlier by the authors.

MSC:

14F08 Derived categories of sheaves, dg categories, and related constructions in algebraic geometry
14M25 Toric varieties, Newton polyhedra, Okounkov bodies
14A22 Noncommutative algebraic geometry
18G80 Derived categories, triangulated categories

Full Text: DOI arXiv

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