Predictions for $B \to \tau \bar{\mu} + \mu \bar{\tau}$

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The observation of $B \to \tau \bar{\mu} + \mu \bar{\tau}$ at present experiments would be a clear sign of new physics. In this paper we calculate this process in an extended higgs sector framework where the decay is mediated by the exchange of spin zero particle with flavour changing neutral current couplings. If we identify the scalar with the newly discovered state at LHC with a mass $\sim 125$ GeV then we find that, after imposing all experimental constraints, the $BR(B_s \to \tau \bar{\mu} + \mu \bar{\tau})$ can be as high $\sim 10^{-6}$ and $BR(B_d \to \tau \bar{\mu} + \mu \bar{\tau})$ can be as high as $\sim 10^{-7}$. We also calculate this process in minimal supersymmetric standard model and find the $BR(B_s \to \tau \bar{\mu} + \mu \bar{\tau})$ is typically of order $\sim 10^{-9}$. 

I. INTRODUCTION

Flavor changing neutral current (FCNC) events are rare in the Standard Model (SM) both in the quark and the lepton sectors. These processes can be affected by new physics (NP). FCNC involving the third family quark and leptons are interesting as their larger masses make them more susceptible to new physics effects in various extensions of the SM. As an example, in certain versions of the two Higgs doublet models (2HDM) the couplings of the new Higgs bosons are proportional to the masses and so new physics effects are more pronounced for the heavier generations. Moreover, the constraints on new physics involving, specially the third generation leptons and quarks, are somewhat weaker allowing for larger new physics effects. Hence, in the down quark sector there is intense interest in the search for NP in FCNC $b \to s$ and $b \to d$ transitions in $B_{d,s}$ mixing, $b \to s(d)q \bar{q}$ and $b \to s(d)\ell^+\ell^-$ decays. Many measurements in the $B$ sector have constrained NP effects though slight discrepancies from SM predictions still remain in several decays $^1$. In the up quark sector one can look for NP is the processes $t \to c(u)$ transitions $^2$.

In the lepton sector FCNC decays are severely suppressed and lepton number for each family is conserved in the SM. Therefore, FCNC effects and lepton flavor violation in this sector are sensitive probes of NP. The presence of neutrino masses and mixing suggest NP in the lepton sector which in turn could enhance FCNC effects and lepton flavor violation. It is interesting that measurements involving the tau lepton show some discrepancies from SM expectations. The branching ratio of $B \to \tau\nu_\tau$ shows some tension with the SM predictions $^3$ though more recent measurement are consistent with the SM $^4$. This could indicate NP $^5$, possibly coming from an extended scalar or gauge sector. There is also a seeming violation of universality in the tau lepton coupling to the $W$ suggested by the LEP II data which could indicate new physics associated with the third generation lepton $^6$.

More recently, the BaBar collaboration with their full data sample of an integrated luminosity 426 fb$^{-1}$ has reported the measurements of the quantities $^7$

$$R(D) = \frac{BR(\bar{B} \to D\tau^-\bar{\nu}_\tau)}{BR(\bar{B} \to D\ell^-\bar{\nu}_\ell)} = 0.440 \pm 0.058 \pm 0.042,$$

$$R(D^*) = \frac{BR(\bar{B} \to D^*\tau^-\bar{\nu}_\tau)}{BR(\bar{B} \to D^*\ell^-\bar{\nu}_\ell)} = 0.332 \pm 0.024 \pm 0.018.$$  

The SM predictions for $R(D)$ and $R(D^*)$ are $^8$$^9$

$$R(D) = 0.297 \pm 0.017,$$

$$R(D^*) = 0.252 \pm 0.003,$$  

which deviate from the BaBar measurements by $2\sigma$ and $2.7\sigma$ respectively. The BaBar collaboration themselves reported a $3.4\sigma$ deviation from SM when the two measurements of Eq. $^1$ are taken together. This result, if confirmed, could indicate NP involving new charged scalar or vector boson states $^10$. Models with new charged scalars or charged vector bosons also include neutral particles that can cause FCNC effects.

In this paper we will focus on the process $B_{s,d} \to \ell_i\ell_j$. This involves two FCNC interactions- one in the quark and one in the lepton sector. Typically one believes that this process will be very small even with new physics and in many NP models this is indeed true. This is because in these NP models this FCNC process arises in loops and is suppressed. In this paper we will focus on the decay $B \to \tau\bar{\mu} + \mu\bar{\tau}$ and make predictions for its rate. This decay was considered in earlier papers in various extensions of the SM $^11$. In this work we will calculate the process in two scenarios representing two extension of the SM.

In the first scenario we will consider this process mediated by a scalar which we will call $X$ in a model with an extended higgs sector. In this model we will assume that there are scalar particles that have FCNC couplings and can mediate the decay $B \to \tau\mu$. We will identify the lowest mass such state with the $X$ particle. It is true that in this model there can be other heavier neutral states that can also mediate the decay $B \to \tau\mu$. When multiple scalar states contribute to the decay there will be more parameters that cannot be all constrained from
the data resulting in loss of predictive power. To regain some predictive power, our assumption will be that the
dominant contribution to the decay comes from the lowest lying state $X$. We expect that the contributions
to the decay from the higher mass states will in general not lower our estimated branching ratios significantly
unless there are strong cancellations among the various amplitudes. We will assume that there are no strong
cancellations among the various contributions.

We will identify $X$ with the newly discovered particle, with mass $\sim 125$ GeV, observed at LHC \cite{12, 13}
with supporting evidence for its existence from Fermilab \cite{14}. In this case $B \rightarrow \tau \bar{\mu} + \mu \bar{\tau}$ is generated at tree level
due to the presence of $bqX$ and $\tau \mu X$ couplings. The coupling $bqX$ can be constrained by $B_{d,s}$ mixing while the
$\tau \mu X$ couplings can be constrained from rare $\tau$ decays. Recently in Ref. \cite{12, 16} these coupling were considered
and it was noticed that the $\tau \mu X$ coupling could be of similar size as the SM higgs to $\tau \tau$ coupling. Since $B_{s,d}$ are
pseudoscalars, the flavor changing coupling of $X$ to the $B_{s,d}$ mesons must contain a pseudoscalar component of
the form $(\bar{q}\gamma_5 b)X$ where $q = s, d$.

The spin parity of the $X$ particle is not yet known though various strategies that allow for the spin and
parity determination are being actively pursued \cite{17}. In this scenario the branching ratio for $B \rightarrow \tau \bar{\mu} + \mu \bar{\tau}$ can
be significant and can potentially be observed at LHCb. Generally, final states with $\tau$ leptons are difficult to
measure at LHCb, however because of an associated $\mu$ in the final state this process may be somewhat easier to
observe than $B_{d,s} \rightarrow \tau^+ \tau^-$. We would like to point that the decay can also be mediated through the exchange
of a spin one particle at tree level. Since we will be identifying the mediator with the newly discovered state at
LHC we will not consider the mediator to be a spin one particle. Furthermore, as can be seen from Eq. \ref{7} the
matrix elements with scalar currents, associated with a scalar mediator, are enhanced relative to vector currents
associated with a spin one mediator.

In the second scenarios we will consider a popular extension of the SM- the minimal supersymmetric SM
(MSSM). Here the FCNC couplings arise via loops and so our expectation is that the branching ratio for $B \rightarrow \tau \bar{\mu} + \mu \bar{\tau}$ will be tiny. The main point here is that if this process were to be observed at the LHCb in the near future this would indicate the presence of a rather non-traditional NP with appreciable level FCNC
couplings in the lepton sector and possibly a light spin zero mediator.

The paper is organized in the following manner. In the next section we describe the general effective Hamiltonian
for $b \rightarrow s l_i^- l_j^+$ decays. In the following section we consider the decay $B \rightarrow \tau \bar{\mu} + \mu \bar{\tau}$ in an extended higgs
sector model with FCNC higgs couplings. This is followed by a calculation of $B \rightarrow \tau \bar{\mu} + \mu \bar{\tau}$ in MSSM. Finally
we summarize our results and present our conclusions.

II. EFFECTIVE HAMILTONIAN

In this section we discuss the effective Hamiltonian for $b \rightarrow q(=d, s) l_i^- l_j^+$ transitions. We will focus on the
$b \rightarrow s l_i^- l_j^+$ Hamiltonian as the $b \rightarrow d l_i^- l_j^+$ Hamiltonian can be obtained from it with the obvious replacements.
The effective Hamiltonian for the quark-level transition $b \rightarrow s l_i^- l_j^+$ (l = e, $\mu, \tau$) in the SM is mainly given by

$$
H_{\text{eff}}^{SM} = -\frac{4G_F}{\sqrt{2}} V_{ts} V_{tb} \left\{ \sum_{i=1}^{6} C_i(\mu) O_i(\mu) + C_7 \left( \frac{e}{16\pi^2} \right) \left[ \bar{s} \gamma_\mu (m_s P_L + m_b P_R) b \right] F^{\mu\nu} 
+ C_9 \left( \frac{\alpha_{em}}{4\pi} \right) \left( \bar{s} \gamma_\mu P_L b \right) L_{ij}^{\mu} + C_{10} \left( \frac{\alpha_{em}}{4\pi} \right) \left( \bar{s} \gamma_\mu P_L b \right) L_{ij}^{5\mu} \right\},
$$

(3)

where $L_{ij}^{\mu} = \bar{l}_i \gamma_\mu l_j$, $L_{ij}^{5\mu} = \bar{l}_i \gamma_\mu \gamma_5 l_j$, and $P_{L,R} = (1 \mp \gamma_5)/2$. The operators $O_i$ ($i = 1, ..6$) correspond to the $P_i$
in \cite{18}, and $m_b = m_b(\mu)$ is the running $b$-quark mass in the $\overline{\text{MS}}$ scheme. We use the SM Wilson coefficients as
given in \cite{19}.

The total effective Hamiltonian for $b \rightarrow s l_i^- l_j^+$ in the presence of new physics operators with all the possible
Lorentz structure excluding tensor once can be expressed as \cite{20}

$$
H_{\text{eff}}(b \rightarrow s l_i^- l_j^+) = H_{\text{eff}}^{SM} + H_{\text{eff}}^{V} + H_{\text{eff}}^{SP},
$$

(4)
where $H_{\text{eff}}^{SM}$ is given by Eq. (3), and the NP contributions are

\[ H_{\text{eff}}^{VA} = -N_F \left\{ R_V (\bar{s} \gamma^\mu P_L b) L_{ij} + R_A (\bar{s} \gamma^\mu P_L b) L_{ij}^5 + R'_V (\bar{s} \gamma^\mu P_R b) L_{ij}^5 + R'_A (\bar{s} \gamma^\mu P_R b) L_{ij}^5 \right\}, \] (5)

\[ H_{\text{eff}}^{SP} = -N_F \left\{ R_S (\bar{s} P_R b) L_{ij} + R_P (\bar{s} P_R b) L_{ij}^5 + R'_S (\bar{s} P_R b) L_{ij} + R'_P (\bar{s} P_R b) L_{ij}^5 \right\}, \] (6)

where $N_F = \frac{4G_F}{\sqrt{2}} \alpha/m_W V_{ts} V_{tb}$, $L_{ij} = \bar{l}_i l_j$, and $L_{ij}^5 = \bar{l}_i \gamma_5 l_j$. In the above expressions, $R_i$, $R'_i (i = V, A, S, P)$ are the NP effective couplings which are in general complex.

In terms of all the Wilson coefficients, the branching ratio for the decay $B_s \rightarrow l_i^- l_j^+$ can be obtained from

\[ BR(B_s \rightarrow l_i^- l_j^+) = \frac{\tau_{B_s} f_{B_s}^2 G_F m_{B_s} \alpha^2_{em} |V_{tb} V_{ts}|^2}{64\pi^3} \sqrt{\left[ 1 - \left( \frac{m_i + m_j}{m_{B_s}} \right)^2 \right]} \left[ 1 - \left( \frac{m_i - m_j}{m_{B_s}} \right)^2 \right] \left\{ f_V (m_i - m_j) + F_S^2 + \left[ 1 - \left( \frac{m_i - m_j}{m_{B_s}} \right)^2 \right] |F_A (m_i + m_j) + F_P|^2 \right\}, \] (7)

where

\[ F_V = C_9 + R_V - R'_V, \]
\[ F_A = C_{10} + R_A - R'_A, \]
\[ F_S = r_\chi (R_S - R'_S), \]
\[ F_P = r_\chi (R_P - R'_P), \] (8)

with $r_\chi = \frac{m_{b\mu}^2}{m_{b\mu}^2 + m_{\chi}^2}$. The Wilson coefficient $F_V$ does not contribute in the lepton flavor conserving decays $B_s \rightarrow \mu^- \mu^+$ and $B_s \rightarrow \tau^- \tau^+$. Within the SM, $F_A = C_{10}$ only contributes to $B_s \rightarrow \mu^- \mu^+$ and $B_s \rightarrow \tau^- \tau^+$. The lepton flavor violating decay $B_s \rightarrow \tau \mu$ is not allowed in the SM. Thus, the Wilson coefficients $C_9$ and $C_{10}$ do not contribute to this decay. Next, we look at two models of NP that contribute to $B_{s,d} \rightarrow l_i^- l_j^+$ decays.

### III. FLAVOR VIOLATING HIGGS DECAYS

In this section we consider the model where a neutral scalar boson $X$ with mass $\sim 125$ GeV and flavor-violating couplings [15, 16] mediates the $B_{s,d} \rightarrow l_i^- l_j^+$ decays. In this model, the effective Lagrangian which describes the possible flavor-violating couplings of $X$ to SM fermion pairs in the mass basis is, [15, 16]:

\[ \mathcal{L}_Y = -m_i f_{L} f_{R} - Y_{ij} (\bar{f}_{L} f_{R}) X + h.c. + \ldots, \] (9)

where ellipses denote nonrenormalizable couplings involving more than on Higgs field operator. $f_L = q_L$, $l_L$ are SU(2)$_L$ doublets, $f_R = u_R d_R, \nu_R, l_R$ are the weak singlets. The indices run over generations and fermion flavors with summation implicitly understood. In the SM the Higgs coupling are diagonal, $Y_{ij} = (m_i/v)\delta_{ij}$ with $v = 246$ GeV, but in general NP models the structure of the $Y_{ij}$ can be different. The couplings in Eq. (9) will also lead to the decay $b \rightarrow s l_i^- l_j^+$ at tree level, mediated by a virtual $X$. The lagrangian gives

\[ R_{S(P)} = \frac{1}{N_F} \frac{Y_{sb} (Y_{\mu\tau} + Y^{*\mu\tau})}{4m_X^2}, \]
\[ R'_{S(P)} = \frac{1}{N_F} \frac{Y^{*sb} (Y_{\mu\tau} - Y^{*\mu\tau})}{4m_X^2}, \]
\[ R_{V(A)} = R'_{V(A)} = 0. \] (10)

The branching ratio for the $B_s \rightarrow \mu^- \tau^+$ decay can be obtained from Eq. (7) as,

\[ BR(B_s \rightarrow \mu^- \tau^+) = \frac{\tau_{B_s} f_{B_s}^2 m_{B_s}^5}{512\pi (m_b (\mu + m_\mu))} \sqrt{\left[ 1 - \left( \frac{m_\tau + m_\mu}{m_{B_s}} \right)^2 \right]} \left[ 1 - \left( \frac{m_\tau - m_\mu}{m_{B_s}} \right)^2 \right] \frac{Y_{sb} - Y^{*sb}}{m_X} \left\{ 1 - \left( \frac{m_\tau + m_\mu}{m_{B_s}} \right)^2 \right\} |Y_{\mu\tau} + Y^{*\mu\tau}|^2 + \left[ 1 - \left( \frac{m_\tau - m_\mu}{m_{B_s}} \right)^2 \right] |Y_{\mu\tau} - Y^{*\mu\tau}|^2. \] (11)
Also, the branching ratio for the $B_s \to l^- l^+(l = \mu, \tau)$ decays is,

$$BR(B_s \to l^- l^+) = \frac{T_{B_s} f_{B_s}^2 G_F^2 m_{B_s} \alpha^2 |V_{tb} V_{ts}^\ast|}{64 \pi^3} \sqrt{1 - \frac{4 m_{l}^2}{m_{B_s}^2}} \left(1 - \frac{4 m_{l}^2}{m_{B_s}^2} \right) \left[1 - \frac{Re[Y_{ll}](Y_{sb} - Y_{bs})}{2 m_{X}^2 N_F (m_{b} + m_{s})} \right]^2 + \left(\frac{2 m_{l}}{m_{B_s}}\right)^2 |C_{10}|^2 ,$$

(12)

where we assume $Y_{ll}$ is real and set it to its SM value $Y_{ll} = m_l/v$.

Flavor violating Higgs coupling in Eq. (9) can generate flavor changing neutral currents at tree level. The couplings $Y_{sb}$ and $Y_{bs}$ are constrained by the mass difference $\Delta M_s$ in the $B_s - \overline{B_s}$ mixing. The $\Delta B = 2$ weak Hamiltonian for this process can be found in the Ref. [16]. The theoretical expression for $\Delta M_s$ is given in Ref.[21]. It is found that one can reproduce the measured value of $\Delta M_s = (17.719 \pm 0.043) ps^{-1}$ [22] if the upper bound on the flavor changing couplings is

$$|Y_{sb} - Y_{bs}^\ast| \sim 10^{-3} .$$

(13)

We perform a fit to constrain the leptonic couplings $Y_{\mu \tau}$ and $Y_{\tau \mu}$. The bound on these couplings are obtained from the decay rates of $\tau \to \mu \gamma$, $\tau \to 3\mu$, and the magnetic ($\delta \alpha_\mu$) and electric dipole ($d_\mu$) moments. The theoretical expressions for $\Gamma(\tau \to \mu \gamma)$, $\Gamma(\tau \to 3\mu)$, $\Delta \alpha_\mu = \alpha^{exp} - \alpha^{SM}_\mu$, and $d_\mu$ are taken from [16]. The experimental bounds for these observables are taken from [23 24]:

$$BR(\tau \to \mu \gamma) < 4.4 \times 10^{-8}$$

$$BR(\tau \to 3\mu) < 2.1 \times 10^{-8}$$

$$\Delta \alpha_\mu = (2.87 \pm 0.63 \pm 0.49) \times 10^{-9}$$

$$-10 \times 10^{-20} e cm < d_\mu < 8 \times 10^{-20} e cm .$$

(14)

In the fit, $BR(\tau \to \mu \gamma)$, $BR(\tau \to 3\mu)$, $\Delta \alpha_\mu$ and $d_\mu$ are constrained by their experimental results in Eq. (13) and $\Delta \alpha_\mu$ is varied within $1\sigma$ errors. As shown in Fig. 1 (right panel), $\Delta \alpha_\mu$ provides a strong bound on the couplings $Y_{\mu \tau}$ and $Y_{\tau \mu}$, and in particular requires $|Y_{\mu \tau} + Y_{\tau \mu}^\ast| \geq 0.09$ . It is found that one can satisfy the above four experimental bounds if $|Y_{\mu \tau}| < 0.064 , |Y_{\tau \mu}| < 0.061$ for $m_X = 125$ GeV. The fit results allow us to estimate the decay rate $BR(B_s \to \mu \tau) = BR(B_s \to \mu^- \tau^+) + BR(B_s \to \tau^- \mu^+)$. The variations of the decay rates $BR(B_s \to \mu \tau)$ with $|Y_{sb} - Y_{bs}|$ and $|Y_{\mu \tau} + Y_{\tau \mu}^\ast|$ are shown in Fig. 1. Our result predict that the $BR(B_s \to \mu \tau)$ can be as large as $\sim 6 \times 10^{-6}$ for $m_X = 125$ GeV and $f_{B_s} = 0.229 \pm 0.006$ GeV [26].

In this model, we obtain the branching ratio $BR(B_s \to \mu^- \mu^+) < 4.3 \times 10^{-9}$ using $Y_{\mu \mu} = m_{\mu}/v$ with $v = 246$ GeV, $m_X = 125$ GeV and $f_{B_s} = 0.229 \pm 0.006$ GeV. The corresponding SM prediction is $\approx 3.17 \times 10^{-9}$. Our results for $BR(B_s \to \mu^- \mu^+)$ is consistent with the current upper limit $BR(B_s \to \mu^- \mu^+) \equiv [1.1 - 6.4] \times 10^{-9}$ at 95% C.L. in Ref. [27]. Also, we obtain the branching ratio $BR(B_s \to \tau^- \tau^+) < 8.1 \times 10^{-7}$ using $Y_{\tau \tau} = m_{\tau}/v$ and $m_X = 125$ GeV. The values of this branching ratio in the SM is $\approx 6.81 \times 10^{-7}$. The latest LHCb-measurement of $\Gamma_d/\Gamma_s$ implies a limit of $BR(B_s \to \tau^- \tau^+) < 3\%$ [28]. In passing we note that the large width difference in the $B_s$ meson system can changes these results by $\sim \mathcal{O}(10\%)$ as pointed out in [30 32].

Finally, in this model we obtain the tau longitudinal polarization fraction for the decays $B_s \to \mu^- \tau^+$:

$$P_L = \frac{BR_{B_s \to \mu^- \tau^+}[\lambda_\tau = 1/2] - BR_{B_s \to \mu^- \tau^+}[\lambda_\tau = -1/2]}{BR_{B_s \to \mu^- \tau^+}[\lambda_\tau = 1/2] + BR_{B_s \to \mu^- \tau^+}[\lambda_\tau = -1/2]} = \sqrt{\left[1 - \left(\frac{m_\tau + m_\mu}{2 m_{B_s}}\right)^2\right] \left[1 - \left(\frac{m_\tau - m_\mu}{2 m_{B_s}}\right)^2\right]} \times \left(2|Y_{\mu \tau}|^2 - |Y_{\tau \mu}^\ast|^2\right)/\left[\left[1 - \left(\frac{m_\tau + m_\mu}{2 m_{B_s}}\right)^2\right]|Y_{\mu \tau} + Y_{\tau \mu}^\ast|^2 + \left[1 - \left(\frac{m_\tau - m_\mu}{2 m_{B_s}}\right)^2\right]|Y_{\mu \tau} - Y_{\tau \mu}^\ast|^2\right].$$

(15)

The result indicates $P_L$ does not depend on the $b \to s$ couplings. Fig. 4 shows the dependency of $P_L$ on the quantity $(|Y_{\mu \tau}|^2 - |Y_{\tau \mu}^\ast|^2)$. 
FIG. 1. The variation of $\text{BR}(B_s \rightarrow \mu^- \tau^+ + \mu^+ \tau^-)$ with the couplings $|Y_{sb} - Y_{bs}^*|$ and $|Y_{\mu\tau} - Y_{\tau\mu}^*|$ for $m_X = 125 \text{ GeV}$ and $f_{B_s} = 0.229 \pm 0.006 \text{ GeV}$. Scatter points are allowed by $\Delta M_s^{\text{exp}}$ and the experimental bounds in Eq. (14).

The couplings $Y_{db}$ and $Y_{bd}$ for the decay $\text{BR}(B \rightarrow \mu \tau) = \text{BR}(B \rightarrow \mu^- \tau^+) + \text{BR}(B \rightarrow \tau^- \mu^+)$ are similarly constrained by the mass difference $\Delta M_d = (0.507 \pm 0.004) \text{ ps}^{-1}$ [29]. The measured $\Delta M_d$ can be reproduced if the upper bound on the flavor changing couplings is $|Y_{db} - Y_{bd}^*| \sim 10^{-4}$. Using this result, we find the decay rate $\text{BR}(B \rightarrow \mu \tau) < 2 \times 10^{-7}$.

IV. SUPERSYMMETRY

The supersymmetry contributions to the process $B_s \rightarrow \mu^+ \tau^-$ are given by the one-loop box diagrams, shown in Fig 3, where charginos and neutralinos are exchanged. The interaction of charginos, neutralinos can be written as

$$
\mathcal{L} = 2 \sum_{j=1}^{\tilde{c}} \bar{\tilde{c}}^j \left( C_{\ell j ij}^{\ell j} P_L + C_{\ell j ij}^{\ell j} P_R \right) d^I + \tilde{\nu}^j \left( C_{\nu j ij}^{\nu j} P_L + C_{\nu j ij}^{\nu j} P_R \right) l^I + \text{h.c.}
$$

$$
+ \sum_{k=1}^{4} \tilde{\chi}^k \left( \bar{\tilde{d}}^i \left( Z_{ik}^{D} P_L + Z_{ik}^{D} P_R \right) d^I + \tilde{\nu}^i \left( Z_{ik}^{L} P_L + Z_{ik}^{L} P_R \right) l^I \right) + \text{h.c.,}
$$

(16)
where the mixing matrices in the super-CKM basis are given by

\[
G_{IJij}^{UL} = U_L^{Ii} \delta_{ij}, \quad G_{IJij}^{UR} = U_R^{Ii} \delta_{ij}, \quad G_{IJij}^{L} = U_R^{Ii} U_L^{Jj}, \quad G_{IJij}^{R} = U_L^{Ii} U_R^{Jj}.
\]

where \( I, J = 1, 3 \) and \( i = 1, 6 \). The matrices \( \Gamma \)'s are the matrices that diagonalize the squark and slepton mass matrices, while \( U \) and \( V \) are the unitary matrices that diagonalize the charged mass matrices. Finally, \( N \) is the matrix that diagonalizes the neutralino mass matrix.

### A. Chargino Contribution

The general expression for the amplitude of chargino contribution to \( b \to s l^+ l^- \) is given by

\[
A_{\tilde{\chi}^\pm} = -\frac{i}{16\pi^2} \left( m_{\tilde{\chi}^+} m_{\tilde{\chi}^0} B(m_{\tilde{\chi}^+}, m_{\tilde{\chi}^0}, m_\tau, m_\nu) \right) \left[ \frac{1}{2} G_{L}^{L} G_{L}^{R} G_{L}^{U} G_{L}^{U} (\widetilde{\sigma} P_L)(\widetilde{\sigma} P_R) \right] (\tau \mu) \\
+ \frac{1}{2} G_{L}^{L} G_{L}^{R} G_{L}^{U} G_{L}^{U} (\widetilde{\sigma} P_R)(\widetilde{\sigma} P_L) \right] (\tau \mu) \\
+ 2 G_{L}^{L} G_{L}^{R} G_{L}^{U} G_{L}^{U} (\widetilde{\sigma} P_R)(\widetilde{\sigma} P_L) \right] (\tau \mu) \\
+ 2 G_{L}^{L} G_{L}^{R} G_{L}^{U} G_{L}^{U} (\widetilde{\sigma} P_R)(\widetilde{\sigma} P_L) \right] (\tau \mu) \\
+ 2 G_{L}^{L} G_{L}^{R} G_{L}^{U} G_{L}^{U} (\widetilde{\sigma} P_R)(\widetilde{\sigma} P_L) \right] (\tau \mu) \\
+ 2 G_{L}^{L} G_{L}^{R} G_{L}^{U} G_{L}^{U} (\widetilde{\sigma} P_R)(\widetilde{\sigma} P_L) \right] (\tau \mu) \\
+ 2 G_{L}^{L} G_{L}^{R} G_{L}^{U} G_{L}^{U} (\widetilde{\sigma} P_R)(\widetilde{\sigma} P_L) \right] (\tau \mu) \\
+ 2 G_{L}^{L} G_{L}^{R} G_{L}^{U} G_{L}^{U} (\widetilde{\sigma} P_R)(\widetilde{\sigma} P_L) \right] (\tau \mu) \\
+ 2 G_{L}^{L} G_{L}^{R} G_{L}^{U} G_{L}^{U} (\widetilde{\sigma} P_R)(\widetilde{\sigma} P_L) \right] (\tau \mu) \\
+ 2 G_{L}^{L} G_{L}^{R} G_{L}^{U} G_{L}^{U} (\widetilde{\sigma} P_R)(\widetilde{\sigma} P_L) \right] (\tau \mu)
\]

Therefore, one can write this amplitude as

\[
A_{\tilde{\chi}^\pm} = A_{\tilde{\chi}^\pm}^{S-P} + A_{\tilde{\chi}^\pm}^{V-A},
\]

with

\[
A_{\tilde{\chi}^\pm}^{S-P} = -\frac{i}{16\pi^2} \left( (C_{SRR} + C_{SRL})(\widetilde{\sigma} P_R)(\tau \mu) + (C_{SLL} + C_{SLR})(\widetilde{\sigma} P_L)(\tau \mu) \\
+ (C_{SRR} - C_{SRL})(\widetilde{\sigma} P_R)(\tau \gamma^5 \mu) + (C_{SLL} - C_{SLR})(\widetilde{\sigma} P_L)(\tau \gamma^5 \mu) \right).
\]
and

$$A_{\chi^\pm}^{V-A} = -\frac{i}{16\pi^2} \left( (C_{VL} + C_{VLR})(\bar{\tau}_\mu P_L b)(\tau_\gamma \mu) + (C_{VR} + C_{VRL})(\bar{\tau}_\mu P_R b)(\tau_\gamma \mu) \
+ (C_{LR} - C_{VLL} +)(\bar{\tau}_\mu P_L b)(\tau_\gamma \gamma^5 \mu) + (C_{RR} - C_{VRL})(\bar{\tau}_\mu P_R b)(\tau_\gamma \gamma^5 \mu) \right)$$ (26)

where

$$C_{SLL} = \frac{g^4}{2m_W^2} V_{tb}^{CKM} V_{ts}^{CKM} V_{V_{1}U_{12}^{*}}^{\bar{\nu}} S_{33}^{33} \left[ \frac{m}{\sqrt{2m_W \sin \beta}} U_{k2}^{*} V_{k2}^{*} \Gamma_{3i}^{U_{LR}} \Gamma_{3i}^{U_{LR}} \right] 
\times \frac{m_s m_t m_{\chi^+_i} m_{\chi^+_i}}{4 \cos^2 \beta} B(m_{\chi^+_i}, m_{\chi^+_i}, m_{\tilde{t}_i}, m_{\tilde{\nu}_i}) \tag{27}$$

$$C_{SRR} = \frac{g^4}{2m_W^2} V_{tb}^{CKM} V_{ts}^{CKM} V_{k2}^{*} U_{k2}^{*} \left[ U_{k2}^{*} V_{k2}^{*} \frac{m}{\sqrt{2m_W \sin \beta}} U_{12} \right] 
\times \frac{m_s m_t m_{\chi^+_i} m_{\chi^+_i}}{4 \cos^2 \beta} B(m_{\chi^+_i}, m_{\chi^+_i}, m_{\tilde{t}_i}, m_{\tilde{\nu}_i}) \tag{28}$$

$$C_{SLR} = \frac{g^4}{2m_W^2} V_{tb}^{CKM} V_{ts}^{CKM} \left[ \Gamma_{3i}^{U_{LR}} \Gamma_{3i}^{U_{LR}} \left( U_{k2}^{*} V_{k2}^{*} \frac{m}{\sqrt{2m_W \sin \beta}} U_{12} \right) \right] 
\times \frac{m_s m_t m_{\chi^+_i} m_{\chi^+_i}}{4 \cos^2 \beta} S(m_{\chi^+_i}, m_{\chi^+_i}, m_{\tilde{t}_i}, m_{\tilde{\nu}_i}) \tag{29}$$

$$C_{SRL} = \frac{g^4}{2m_W^2} V_{tb}^{CKM} V_{ts}^{CKM} \left[ \frac{m}{\sqrt{2m_W \sin \beta}} \left( U_{k2}^{*} V_{k2}^{*} \frac{m}{\sqrt{2m_W \sin \beta}} U_{12} \right) \right] 
\times \frac{m_s m_t m_{\chi^+_i} m_{\chi^+_i}}{4 \cos^2 \beta} S(m_{\chi^+_i}, m_{\chi^+_i}, m_{\tilde{t}_i}, m_{\tilde{\nu}_i}) \tag{30}$$

and

$$C_{VL} = \frac{g^4}{2m_W^2} V_{tb}^{CKM} V_{ts}^{CKM} U_{12}^{*} S_{33}^{33} \left[ \frac{m}{\sqrt{2m_W \sin \beta}} U_{k2}^{*} V_{k2}^{*} \Gamma_{3i}^{U_{LR}} \Gamma_{3i}^{U_{LR}} \right] 
\times \frac{m_s m_t m_{\chi^+_i} m_{\chi^+_i}}{8 \cos^2 \beta} S(m_{\chi^+_i}, m_{\chi^+_i}, m_{\tilde{t}_i}, m_{\tilde{\nu}_i}) \tag{31}$$

$$C_{VR} = \frac{g^4}{2m_W^2} V_{tb}^{CKM} V_{ts}^{CKM} U_{k2}^{*} S_{33}^{33} \left[ \frac{m}{\sqrt{2m_W \sin \beta}} U_{12} \right] 
\times \frac{m_s m_t m_{\chi^+_i} m_{\chi^+_i}}{8 \cos^2 \beta} S(m_{\chi^+_i}, m_{\chi^+_i}, m_{\tilde{t}_i}, m_{\tilde{\nu}_i}) \tag{32}$$

$$C_{VL} = \frac{g^4}{2m_W^2} V_{tb}^{CKM} V_{ts}^{CKM} \left[ \frac{m}{\sqrt{2m_W \sin \beta}} U_{12} \right] 
\times \frac{m_s m_t m_{\chi^+_i} m_{\chi^+_i}}{8 \cos^2 \beta} B(m_{\chi^+_i}, m_{\chi^+_i}, m_{\tilde{t}_i}, m_{\tilde{\nu}_i}) \tag{33}$$

$$C_{VL} = \frac{g^4}{2m_W^2} V_{tb}^{CKM} V_{ts}^{CKM} \left[ U_{k2}^{*} V_{k2}^{*} \frac{m}{\sqrt{2m_W \sin \beta}} U_{12} \right] 
\times \frac{m_s m_t m_{\chi^+_i} m_{\chi^+_i}}{8 \cos^2 \beta} B(m_{\chi^+_i}, m_{\chi^+_i}, m_{\tilde{t}_i}, m_{\tilde{\nu}_i}) \tag{34}$$

$$C_{VR} = \frac{g^4}{2m_W^2} V_{tb}^{CKM} V_{ts}^{CKM} \left[ \Gamma_{3i}^{U_{LR}} \Gamma_{3i}^{U_{LR}} \left( U_{k2}^{*} V_{k2}^{*} \frac{m}{\sqrt{2m_W \sin \beta}} U_{12} \right) \right] 
\times \frac{m_s m_t m_{\chi^+_i} m_{\chi^+_i}}{8 \cos^2 \beta} S(m_{\chi^+_i}, m_{\chi^+_i}, m_{\tilde{t}_i}, m_{\tilde{\nu}_i}) \tag{35}$$

Therefore, from Eqs. (3) and (9) one finds

$$R_{V,A} = \frac{i}{16\pi^2 N_F} (C_{VLR} \pm C_{VLL}) \tag{36}$$

$$R_{V,A} = \frac{i}{16\pi^2 N_F} (C_{VRR} \pm C_{VRL}) \tag{37}$$

$$R_{S,P} = \frac{i}{16\pi^2 N_F} (C_{SRR} \pm C_{SRL}) \tag{38}$$

$$R_{S,P} = \frac{i}{16\pi^2 N_F} (C_{SRR} \pm C_{SRL}) \tag{39}$$
These co-efficients can be used in Eq. 7 to calculate the branching ratio for $B_s \to \mu^+\tau^-$. The loop functions $B(m_{\tilde{\chi}_k^+}, m_{\tilde{\chi}_l^+}, m_{\tilde{\nu}_i}, m_{\tilde{\nu}_j})$ and $S(m_{\tilde{\chi}_k^+}, m_{\tilde{\chi}_l^+}, m_{\tilde{\nu}_i}, m_{\tilde{\nu}_j})$ are given by

$$B(m_{\tilde{\chi}_k^+}, m_{\tilde{\chi}_l^+}, m_{\tilde{\nu}_i}, m_{\tilde{\nu}_j}) = \frac{m_{\tilde{\nu}_j}^2}{(m_{\tilde{\nu}_j} - m_{\tilde{\nu}_i})(m_{\tilde{\chi}_k} - m_{\tilde{\chi}_l})(m_{\tilde{\chi}_k} - m_{\tilde{\chi}_l})} \ln \frac{m_{\tilde{\nu}_j}^2}{m_{\tilde{\chi}_k}^2}$$

$$+ \frac{m_{\tilde{\nu}_i}^2}{(m_{\tilde{\nu}_i} - m_{\tilde{\nu}_j})(m_{\tilde{\chi}_l} - m_{\tilde{\chi}_k})(m_{\tilde{\chi}_l} - m_{\tilde{\chi}_k})} \ln \frac{m_{\tilde{\nu}_i}^2}{m_{\tilde{\chi}_l}^2}$$

$$+ \frac{m_{\tilde{\chi}_k}^2}{(m_{\tilde{\chi}_k} - m_{\tilde{\nu}_i})(m_{\tilde{\chi}_l} - m_{\tilde{\nu}_j})(m_{\tilde{\chi}_l} - m_{\tilde{\nu}_j})} \ln \frac{m_{\tilde{\chi}_k}^2}{m_{\tilde{\nu}_j}^2},$$

(40)

and

$$S(m_{\tilde{\chi}_k^+}, m_{\tilde{\chi}_l^+}, m_{\tilde{\nu}_i}, m_{\tilde{\nu}_j}) = \frac{m_{\tilde{\nu}_i}^2}{(m_{\tilde{\nu}_i} - m_{\tilde{\nu}_j})(m_{\tilde{\chi}_l} - m_{\tilde{\chi}_k})(m_{\tilde{\chi}_l} - m_{\tilde{\chi}_k})} \ln \frac{m_{\tilde{\nu}_i}^2}{m_{\tilde{\chi}_l}^2}$$

$$+ \frac{m_{\tilde{\nu}_j}^2}{(m_{\tilde{\nu}_j} - m_{\tilde{\nu}_i})(m_{\tilde{\chi}_k} - m_{\tilde{\chi}_l})(m_{\tilde{\chi}_k} - m_{\tilde{\chi}_l})} \ln \frac{m_{\tilde{\nu}_j}^2}{m_{\tilde{\chi}_l}^2}$$

$$+ \frac{m_{\tilde{\chi}_l}^2}{(m_{\tilde{\chi}_l} - m_{\tilde{\nu}_i})(m_{\tilde{\chi}_k} - m_{\tilde{\nu}_j})(m_{\tilde{\chi}_k} - m_{\tilde{\nu}_j})} \ln \frac{m_{\tilde{\chi}_l}^2}{m_{\tilde{\nu}_j}^2},$$

(41)

where $i, j, k, l = 1, 2$ denote the heavy and light sparticles (stop-quark, sneutrino and chargino). It is worth mentioning that the above expression for the chargino amplitude is consistent with the amplitude reported in Ref. [24] and also with the expressions given in the second paper in Ref. [11]. We have expressed it in the above form to be close to the new physics amplitudes discussed in Ref. [20] and adopted in the previous section. Finally, in deriving these expression, the following hadronic matrix elements have been assumed:

$$\langle 0 | \overline{\psi} \gamma^5 b | B_s(p) \rangle = \frac{i}{2} f_{B_s} \frac{m_{B_s}^2}{m_b + m_s} \langle 0 | \overline{\psi} b | B_s(p) \rangle = 0,$$  

(42)

$$\langle 0 | \overline{\psi} \gamma_{\mu} \gamma^5 b | B_s(p) \rangle = \frac{i}{2} p_\mu f_{B_s} \langle 0 | \overline{\psi} b | B_s(p) \rangle = 0,$$  

(43)

where $p_\mu$ is the momentum of the $B_s$ meson of mass $B_s$.

**B. Neutalino Contribution**

Similar to the chargino contribution, we can write the amplitude of $b \to s \mu^+\tau^-$ through the neutralino exchange as follows:

$$A_{\chi^0} = -\frac{i}{16\pi^2} \left( m_{\tilde{\chi}_l^0} m_{\tilde{\chi}_k^0} B(m_{\tilde{\chi}_k^0}, m_{\tilde{\chi}_l^0}, m_{\tilde{\nu}_i}, m_{\tilde{\nu}_j}) \right) \left( \frac{1}{2} Z^L_{3j} \overline{Z}^R_{3k} \overline{Z}^D_{3l} \overline{Z}^D_{2l} (\overline{\psi} P_L b)(\overline{\tau} P_L \mu) \right)$$

$$+ \frac{1}{2} Z^L_{3j} \overline{Z}^R_{3k} \overline{Z}^D_{3l} \overline{Z}^D_{2l} (\overline{\psi} P_R b)(\overline{\tau} P_R \mu) + \frac{1}{4} Z^L_{3j} \overline{Z}^R_{3k} \overline{Z}^D_{3l} \overline{Z}^D_{2l} (\overline{\psi} \gamma_{\mu} P_R b)(\overline{\tau} \mu P_L \mu)$$

$$+ \frac{1}{4} Z^L_{3j} \overline{Z}^R_{3k} \overline{Z}^D_{3l} \overline{Z}^D_{2l} (\overline{\psi} \gamma_{\mu} P_R b)(\overline{\tau} \mu P_L \mu)$$

$$+ \frac{1}{4} S(m_{\tilde{\chi}_k^+}, m_{\tilde{\chi}_l^+}, m_{\tilde{\nu}_i}, m_{\tilde{\nu}_j}) \left( Z^L_{3j} \overline{Z}^R_{3k} \overline{Z}^D_{3l} \overline{Z}^D_{2l} (\overline{\psi} \gamma_{\mu} P_R b)(\overline{\tau} \mu P_R \mu) \right)$$

$$+ \frac{1}{4} Z^L_{3j} \overline{Z}^R_{3k} \overline{Z}^D_{3l} \overline{Z}^D_{2l} (\overline{\psi} \gamma_{\mu} P_R b)(\overline{\tau} \mu P_L \mu)$$

$$+ \frac{1}{4} Z^L_{3j} \overline{Z}^R_{3k} \overline{Z}^D_{3l} \overline{Z}^D_{2l} (\overline{\psi} \gamma_{\mu} P_R b)(\overline{\tau} \mu P_L \mu)$$

$$+ \frac{1}{4} Z^L_{3j} \overline{Z}^R_{3k} \overline{Z}^D_{3l} \overline{Z}^D_{2l} (\overline{\psi} \gamma_{\mu} P_R b)(\overline{\tau} \mu P_L \mu)$$

(44)
Therefore, one can write the neutralino amplitude as

\[ A_\chi^0 = A_{\chi^0}^{S-P} + A_{\chi^0}^{V-A}, \]  

with

\[ A_{\chi^0}^{S-P} = -\frac{i}{16\pi^2} \left[ (N_{SR} + N_{SL})(\bar{\tau}P_R b)\tau\mu + (N_{SL} + N_{SR})(\bar{\tau}P_L b)\tau\mu \right. \]

\[ + (N_{SR} - N_{SL})(\bar{\tau}P_R b)\tau\gamma^5\mu + (N_{SL} - N_{SR})(\bar{\tau}P_L b)\tau\gamma^5\mu \right], \]

and

\[ A_{\chi^0}^{V-A} = -\frac{i}{16\pi^2} \left[ (N_{VL} + N_{VR})(\bar{\tau}\gamma^\mu P_L b)\tau\gamma^\mu\mu + (N_{VR} + N_{VL})(\bar{\tau}\gamma^\mu P_R b)(\tau\gamma^5\mu) \right. \]

\[ + (N_{VR} - N_{VL})(\bar{\tau}\gamma^\mu P_L b)(\tau\gamma^5\mu) + (N_{VR} - N_{VL})(\bar{\tau}\gamma^\mu P_R b)(\tau\gamma^5\mu) \right], \]

where

\[ N_{SLL} = \frac{g^4}{2m_W^2} \left[ \left( (N_{s2} + \tan \theta_W N_{i1})\Gamma^{L_j L_j}_{2j} - N_{s3} \frac{\Gamma^{L_j L_j}_{3j}}{m_W \cos \beta} \right) \left( 2 \tan \theta_W N_{i1}^{L_j L_j} \right. \right. \]

\[ \left. + N_{s3}^{L_j L_j} \frac{m_t}{m_W \cos \beta} \right) \left( (N_{s2} + \frac{1}{2} \tan \theta_W N_{i1}^{L_j L_j}) \Gamma^{D_j D_j}_{3j} + N_{s3}^{L_j L_j} \frac{m_b}{m_W \cos \beta} \right) \]

\[ \times \left( \frac{2}{3} \tan \theta_W N_{i1}^{L_j L_j} \Gamma^{D_j D_j}_{2j} + N_{s3}^{L_j L_j} \frac{m_s}{m_W \cos \beta} \right) \right] \frac{m^2 W m_{\chi^0} m_{\chi^0}}{8} B(m_{\chi^0}, m_{\chi^0}, m_b, m_{\chi^0}), \]

\[ N_{SRR} = \frac{g^4}{2m_W^2} \left[ \left( (N_{s2} + \tan \theta_W N_{i1})\Gamma^{L_j L_j}_{2j} - N_{s3} \frac{\Gamma^{L_j L_j}_{3j}}{m_W \cos \beta} \right) \left( 2 \tan \theta_W N_{i1}^{L_j L_j} \right. \right. \]

\[ \left. + N_{s3}^{L_j L_j} \frac{m_t}{m_W \cos \beta} \right) \left( \frac{2}{3} \tan \theta_W N_{i1}^{L_j L_j} \Gamma^{D_j D_j}_{3j} + N_{s3}^{L_j L_j} \frac{m_b}{m_W \cos \beta} \right) \]

\[ \times \left( (N_{s2} + \frac{1}{2} \tan \theta_W N_{i1}^{L_j L_j}) \Gamma^{D_j D_j}_{2j} + N_{s3}^{L_j L_j} \frac{m_s}{m_W \cos \beta} \right) \right] \frac{m^2 W m_{\chi^0} m_{\chi^0}}{8} S(m_{\chi^0}, m_{\chi^0}, m_b, m_{\chi^0}), \]

\[ N_{SLR} = \frac{g^4}{2m_W^2} \left[ \left( (N_{s2} + \tan \theta_W N_{i1})\Gamma^{L_j L_j}_{2j} - N_{s3} \frac{\Gamma^{L_j L_j}_{3j}}{m_W \cos \beta} \right) \left( 2 \tan \theta_W N_{i1}^{L_j L_j} \right. \right. \]

\[ \left. + N_{s3}^{L_j L_j} \frac{m_t}{m_W \cos \beta} \right) \left( \frac{2}{3} \tan \theta_W N_{i1}^{L_j L_j} \Gamma^{D_j D_j}_{3j} + N_{s3}^{L_j L_j} \frac{m_b}{m_W \cos \beta} \right) \]

\[ \times \left( (N_{s2} + \frac{1}{2} \tan \theta_W N_{i1}^{L_j L_j}) \Gamma^{D_j D_j}_{2j} + N_{s3}^{L_j L_j} \frac{m_s}{m_W \cos \beta} \right) \right] \frac{m^2 W m_{\chi^0} m_{\chi^0}}{8} S(m_{\chi^0}, m_{\chi^0}, m_b, m_{\chi^0}), \]

and

\[ N_{VLL} = \frac{g^4}{2m_W^2} \left[ \left( (N_{s2} + \tan \theta_W N_{i1})\Gamma^{L_j L_j}_{2j} - N_{s3} \frac{\Gamma^{L_j L_j}_{3j}}{m_W \cos \beta} \right) \left( 2 \tan \theta_W N_{i1}^{L_j L_j} \right. \right. \]

\[ + N_{s3}^{L_j L_j} \frac{m_t}{m_W \cos \beta} \left( \frac{2}{3} \tan \theta_W N_{i1}^{L_j L_j} \Gamma^{D_j D_j}_{3j} + N_{s3}^{L_j L_j} \frac{m_b}{m_W \cos \beta} \right) \]

\[ \times \left( (N_{s2} + \frac{1}{2} \tan \theta_W N_{i1}^{L_j L_j}) \Gamma^{D_j D_j}_{2j} + N_{s3}^{L_j L_j} \frac{m_s}{m_W \cos \beta} \right) \right] \frac{m^2 W m_{\chi^0} m_{\chi^0}}{8} S(m_{\chi^0}, m_{\chi^0}, m_b, m_{\chi^0}), \]
\[ N_{VRR} = \frac{g^4}{2m_W^2} \left[ \left( (N_{k2} + \tan \theta_W N_{k1}) \frac{\Gamma_{3j}}{\Gamma_{3j}} - N_{k3} \frac{\Gamma_{3j}}{\Gamma_{3j}} \frac{m_\tau}{m_W \cos \beta} \right) \left( 2 \tan \theta_W N_{k1} \Gamma_{2j} \frac{m_\mu}{m_W \cos \beta} \right) \right] = \frac{m_W^2}{16} S(m_{\chi^0_k}, m_{\chi^0_i}, m_{\tilde{b}_i}, m_{\tilde{t}_j}), \quad (53) \]

\[ N_{VLR} = \frac{g^4}{2m_W^2} \left[ \left( (N_{l2} + \frac{1}{2} \tan \theta_W N_{l1}) \frac{\Gamma_{3j}}{\Gamma_{3j}} - N_{l3} \frac{\Gamma_{3j}}{\Gamma_{3j}} \frac{m_\tau}{m_W \cos \beta} \right) \left( 2 \tan \theta_W N_{l1} \Gamma_{2i} \frac{m_\mu}{m_W \cos \beta} \right) \right] = \frac{m_W^2}{16} B(m_{\chi^0_k}, m_{\chi^0_i}, m_{\tilde{b}_i}, m_{\tilde{t}_j}). \quad (54) \]

\[ N_{VRL} = \frac{g^4}{2m_W^2} \left[ \left( (N_{k2} + \frac{1}{2} \tan \theta_W N_{k1}) \frac{\Gamma_{2j}}{\Gamma_{2j}} + N_{k3} \frac{\Gamma_{2j}}{\Gamma_{2j}} \frac{m_\tau}{m_W \cos \beta} \right) \left( 2 \tan \theta_W N_{k1} \Gamma_{3i} \frac{m_\mu}{m_W \cos \beta} \right) \right] = \frac{m_W^2}{16} B(m_{\chi^0_k}, m_{\chi^0_i}, m_{\tilde{b}_i}, m_{\tilde{t}_j}). \quad (55) \]

Thus, one finds

\[ A^{S-P}_{\chi^0} = \frac{i}{2} f_B N_F \left[ \frac{\Gamma_{CKM}}{\Gamma_{ts}} \right] \left( \frac{m_{\tilde{B}_s}}{m_b + m_s} (R_{0S} - R_{0}\gamma_\mu) + \frac{m_{\tilde{B}_s}}{m_b + m_s} (R_{0P} - R_{\tau^\prime}) \right) \quad (56) \]

and

\[ A^{V-A}_{\chi^0} = \frac{i}{2} f_B N_F \left[ \frac{\Gamma_{CKM}}{\Gamma_{ts}} \right] \left( (R_{0V} - R_{0}\gamma_\mu) + (R_{0A} - R_{\tau^\prime}) \right) \quad (57) \]

where

\[ R_{0V,A} = \frac{i}{16\pi^2} \left[ \frac{\Gamma_{CKM}}{\Gamma_{ts}} \right] N_F \quad (58) \]

\[ R_{0V,A}' = \frac{i}{16\pi^2} \left[ \frac{\Gamma_{CKM}}{\Gamma_{ts}} \right] N_F \quad (59) \]

\[ R_{0S,P} = \frac{i}{16\pi^2} \left[ \frac{\Gamma_{CKM}}{\Gamma_{ts}} \right] N_F \quad (60) \]

\[ R_{0S,P}' = \frac{i}{16\pi^2} \left[ \frac{\Gamma_{CKM}}{\Gamma_{ts}} \right] N_F \quad (61) \]

In this case, these co-efficients can be used in Eq. 17 to calculate the branching ratio for \( B_s \to \mu^+ \tau^- \).

C. Numerical Analysis

We can estimate the typical prediction of SUSY models to the branching ration of the \( B_s \to \tau \bar{\mu} + \mu \bar{\tau} \) process. As usual one can easily show that the neutralino contribution is typically one or two order of magnitude less than the light chargino effect. Therefore, here we will neglect the neutralino diagrams and focus on the dominate chargino contribution. As explicitly shown above the chargino amplitude depends on stops and tau-sneutrinos masses and mixing. Therefore a light stop and light tau-sneutrinos may significantly enhance the chargino contribution to \( BR(B_s \to \tau \bar{\mu} + \mu \bar{\tau}) \).
In MSSM, the stop mass matrix is given by

$$M_t^2 = \begin{pmatrix} m_Q^2 + m_{\tilde{t}}^2 + \frac{4}{3}M_Z^2 \cos 2\beta (3 - 4 \sin^2 \theta_W) & m_t (A_t - \mu \cot \beta) \\ m_t (A_t - \mu \cot \beta) & m_t^2 + \frac{4}{3}M_Z^2 \cos 2\beta \sin^2 \theta_W \end{pmatrix},$$

(62)

where $m_Q^2$ and $m_{\tilde{t}}^2$ are the low energy values of the soft SUSY breaking scalar masses. Due to large off-diagonal elements, the diagonalization of the stop mass-squared matrix leads to the physical eigenstates with one light and one heavy stops. In addition, with no right-handed neutrinos in MSSM the sneutrino mass matrix is given by

$$M_\nu^2 = M_L^2 + \frac{1}{2}M_Z^2 \cos^2 \beta,$$

(63)

where $M_L$ is the soft SUSY scalar masses of the sleptons. It is also a part of the left-left sector of the charged slepton mass-squared matrix. However, it is worth noting that the experimental limits of lepton flavor violations, like $\mu \to e\gamma$ impose stringent constraints on the off-diagonal elements of $M_L^2$ between the first and second generations, while the mixing between first or the second and third generations are much less constrained [35].

In Fig. 4 we present the $BR(B_s \to \tau \bar{\mu} + \mu \bar{\tau})$ as function of $\tan \beta$. We assume that the lightest and heaviest stop masses are of order 200 GeV and 800 GeV respectively. We consider $100 \text{ GeV} \leq M_2 \leq 1 \text{ TeV}$ and take $\mu$ between 250 GeV and 1 TeV. The lightest sneutrino mass is assumed to be of order 200 GeV and the sneutrino mixing parameters are of order $O(0.1)$. As can be seen from this figure, the branching ratio $BR(B_s \to \tau \bar{\mu} + \mu \bar{\tau})$ can be enhanced up to $10^{-7}$ for large $\tan \beta$ and light charginos and sneutrinos. Also if we have semi-degenerate masses for chargino, stops, and sneutrino, then a kind of a resonance in the branching ratio may be obtained (as can be seen from the loop functions: $B(m_{\tilde{\chi}_1^\pm}, m_{\tilde{\chi}_1^0}, m_{\tilde{\nu}_i})$ and $S(m_{\tilde{\nu}_1}, m_{\tilde{\nu}_2}, m_{\tilde{\nu}_i}, m_{\tilde{\nu}_j})$). In this case the branching ratio can reach $10^{-6}$, however, we think this is a fine tuning therefore we assume that the mass difference between these particles $> 50$ GeV at least. From this analysis one can conclude that the typical SUSY predictions for this branching ratio of $B_s \to \tau \bar{\mu} + \mu \bar{\tau}$ is of order $10^{-8} - 10^{-9}$.

Finally, it is worth noting that in the second paper of Ref. [11], it was emphasized that in the supersymmetric seesaw extensions of the SM, the two loop double penguin diagrams may be relevant at a very large $\tan \beta$ (of order 60) even with flavor universal slepton masses. In this case, it was shown that the double penguins mediated by Higgs bosons may induce a branching ratio of order $B_s \to \mu^+\tau^- \sim 10^{-7} \times (\tan \beta/60)^8 \times (100 \text{ GeV}/m_A)^4$, which may give dominant results at very large $\tan \beta$ and low $m_A$.

V. SUMMARY

In this paper we analyzed the rare decay $B \to \tau \bar{\mu} + \mu \bar{\tau}$ in two possible extensions of the SM. In particular, we considered the decay in a model with an extended higgs sector, where the decay is mediated by a light scalar
which is identified with the recently observed state at LHC. In the second example, the one loop contribution to this process in MSSM was computed. We found that in the former scenario the branching ratio $B_s \rightarrow \tau \bar{\mu} + \mu \bar{\tau}$ can be as high as $10^{-6}$ ( $B_d \rightarrow \tau \bar{\mu} + \mu \bar{\tau}$ can be as high as $\sim 10^{-7}$), while in the later scenario the $BR(B_s \rightarrow \tau \bar{\mu} + \mu \bar{\tau})$ could be $\sim 10^{-8}$. Therefore, one concludes that probing this lepton flavor violating process at the LHCb would be a significant hint for a non-traditional new physics beyond the SM with large FCNC couplings.

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