Elucidating Noisy Data via Uncertainty-Aware Robust Learning

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Abstract
Robust learning methods aim to learn a clean target distribution from noisy and corrupted training data where a specific corruption pattern is often assumed a priori. Our proposed method can not only successfully learn the clean target distribution from a dirty dataset but also can estimate the underlying noise pattern. To this end, we leverage a mixture-of-experts model that can distinguish two different types of predictive uncertainty, aleatoric and epistemic uncertainty. We show that the ability to estimate the uncertainty plays a significant role in elucidating the corruption patterns as these two objectives are tightly intertwined. We also present a novel validation scheme for evaluating the performance of the corruption pattern estimation. Our proposed method is extensively assessed in terms of both robustness and corruption pattern estimation through a number of domains, including computer vision and natural language processing. Code has been made publicly available at \url{https://github.com/jeongeun980906/Uncertainty-Aware-Robust-Learning}.

Keywords: Robust learning, Training With Noisy Labels, Uncertainty Estimation, Corruption Pattern Estimation

1. Introduction
In this paper, we focus on the problem of robust learning \cite{1,2,3,4,5} with its emphasis on elucidating the corruption patterns on the noisy training dataset. Most existing robust learning studies \cite{2,5} assume that the label corruption pattern is solely a function of class information, also known as class-conditional noise (CCN). While this CCN assumption is simple to formulate, it may not be useful in practice in that it is more natural to assume for the noise pattern to be

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a function of input instances which is often referred to as an instance-dependent noise (IDN) learning problem [6].

However, the original IDN learning problem is likely to be infeasible in that it has to estimate a $C \times C$ class transition matrix per input instance. Due to this intractability, recent work on the IDN setting focuses on a simple binary classification problem [7, 8] or requires a small clean dataset [6]. To mitigate this issue, we cast the IDN problem into a two-stage problem of first partitioning the input space using uncertainty measures and then estimating the label transition matrix per each group, which will be referred to as a set-dependent noise (SDN) learning problem. In particular, a specific type of predictive uncertainty, named aleatoric uncertainty, is used to partition the input space. The clusters with high aleatoric uncertainty can be viewed as collective outliers [9], a subset of inputs with a specific noise pattern.

We would like to stress out that robust learning and uncertainty estimation problems are intimately related to each other as robust learning deals with the noisy training data, which naturally give rise to predictive uncertainty. The predictive uncertainty can be decomposed into epistemic and aleatoric uncertainty. The former focuses on the reducible part of the uncertainty (model uncertainty), which may come from the lack of training data. In contrast, the latter comes from the irreducible part (data uncertainty), such as the measurement noise. Our proposed method can estimate both types of uncertainty in a unified framework, and it plays a significant role in achieving robustness and estimating the SDN patterns.

To this end, we utilize a mixture-of-experts model for classification tasks named mixture logit networks (MLN) and present an effective training method to achieve both robustness and explainability by revealing the label corruption process. We first present an uncertainty estimation method for the MLN that can distinguish two different types of predictive uncertainties, epistemic (model uncertainty) and aleatoric (data uncertainty). Then, the estimated uncertainty is utilized for the uncertainty-aware regularization method. Intuitively speaking, unlike a single deterministic model (e.g., a ResNet), using the MLN allows us to model multi-modal (and possibly noisy) target distributions, which plays a crucial role in achieving both robustness as well as explainability. Furthermore, we present an evaluation scheme on SDN settings, which gives information about the collective outliers and label noise distribution of sets.

The main contributions of this work are threefold. 1) We propose a simple yet effective robust learning method leveraging a mixture-of-experts model on various noise settings. 2) The proposed method can not only robustly learn from noisy data but can also discover the set-dependent underlying noise pattern (i.e., the noise transition matrix) as well as the two types of predictive uncertainties (i.e., aleatoric and epistemic uncertainty) within the dataset. 3) Finally, we present a novel evaluation scheme for validating the set-dependent corruption pattern estimation performance.
2. Related Work

In the context of robust learning, the label noise patterns can be roughly categorized into two groups, class-conditional noise (CCN) and instance-dependent noise (IDN) settings, based on which information the label corruptions are made. Note that the IDN setting is much more practical as it is more natural to assume that the label noise pattern is a function of inputs. Furthermore, the IDN setting can inherently incorporate the CCN setting. Our proposed method can cope with both CCN and IDN settings. While most of the robust learning literature focuses on simply estimating the clean target distribution, a number of works have been recently made on achieving both robustness and the ability to estimate the label noise patterns.

Co-Teaching [2] utilizes two separate networks (a teacher network and a student network) by teaching the student network using the teacher network. Co-Teaching+ [4] extends Co-Teaching by further leveraging the disagreement strategy. JoCoR [5] is based on Co-Teaching+, which uses a joint loss function of minimizing cross-entropy while maximizing the agreement between two networks to achieve better robustness. Small loss tricks of providing small risk for unseen clean data have been used in [10, 11, 12].

On the other hand, several works aim to achieve robustness by explicitly estimating the label transition matrix (noise patterns). F-correction [3] estimates the noise transition matrix and applies it to loss function correction. Dual-T [13] incorporates a matrix factorization method to avoid directly estimating a noisy class posterior without any anchor points (i.e., clean data). Total variation regularization [14] effectively regularizes the predicted probability to be more distinguishable by restricting the total variance distance resulting in better estimation of the noise transition matrix.

The IDN setting is more practical than the CCN setting in that it is more natural to assume that the annotator gets confused by ambiguous instances leading to mislabeling. Only recently, a few papers have incorporated confidence estimation and noise transition matrix prediction to handle confusing instances. However, most of the work only uses the confidence estimation to robustly learn clean target distribution in instance-dependent noise settings. Cheng et al. [15] present confidence regularization to prevent overfitting in multi-class classification problems. [15] is further extended in [16] by designing a sample sieve method to get clean instances from the noisy dataset, using confidence regularized cross-entropy loss. The confidence regularized method gives information about the corruption of each instance but does not provide information about label noise patterns. Another approach is to estimate the noise transition matrix instance-wise to correct the loss function. Part-dependent noise (i.e., PDN) was introduced by Xia et al. [17] which approximates the transition matrix by the combination of transition matrices for each instance. Yang et al. [18] first collect predicted clean set to learn the noise transition matrix and then train a classifier with a corrected loss function based on the estimated noise transition matrix.

Perhaps, the most similar setting to ours is [6] which introduced a confidence-
scored instance dependent noise setting; a label noise is given based on prior information about confidence score by annotators. It uses both confidence estimation and noise transition matrix estimation for robust learning. Then, the model utilizes the corrected loss function using the confidence-based noise transition matrix. However, the proposed set-dependent noise setting differs from the confidence-scored instance-dependent noise setting in that the annotators can divide the set by the ambiguity measure of each instance, making it more intuitive and straightforward. Furthermore, the proposed method directly estimates both corruption information and the noise distribution without the necessity of training multiple networks. The categorization of robust learning papers is shown in Table 2.

The mixture-of-experts models have been widely used in robust learning \cite{22, 23, 1}. SsSMM \cite{22} incorporates a student-teacher model similar to MentorNet \cite{2}, but employs a finite mixture models for student networks, updating via an EM algorithm \cite{24} in semi-supervised manner. For robust learning for language domains, Irie et al. \cite{23} proposed a recurrent adaptive mixture model to represent diverse outputs. ChoiceNet \cite{1} utilizes a mixture density network to model the correlated outputs where the correlation between the target and noisy distributions are estimated in an end-to-end manner. Our proposed method is also based on a mixture of the experts model; however, a novel uncertainty-aware regularization method is presented.

3. Problem Formulation

3.1. Training Data Generation Process

In this paper, we focus on the classification task of finding a mapping from an instance $x$ (e.g., an RGB image) to an output $y$ (e.g., an one-hot vector) where the input $x$ and the output $y$ are sampled from the input distribution $p(x)$ and the clean target distribution $p(y|x)$. We assume that some noise patterns can be induced to both input and output where we denote $\tilde{x}$ and $\tilde{y}$ as the corrupted input and output, respectively. The input noise pattern, $\tilde{x} \sim p(\tilde{x}, x)$, can be adding more blur to the instance so that the resulting image is obscured or applying artificial manipulation to the image (e.g., CutMix \cite{25}).

Roughly, the output corruption process can be divided into twofold: the class conditioned noise (CCN) and the instance dependent noise (IDN) settings. For the CCN setting, it is assumed that the training label information is corrupted via a single label transition matrix $T \in \mathbb{R}^{C \times C}$ where $C$ is the number of classes and $[T]_{ij} = p(y_j|y_i)$ is the probability of a label $i$ being shifted to a label $j$. For example, we can simply select a certain portion of the training data and shuffle the labels uniformly randomly or shift the labels by assigning label 1 to 2, label...
2 to 3, and so forth. In the robust learning literature, the formal and the latter are often referred to as symmetric and asymmetric noise patterns, respectively. On the other hand, the ICN assumes that the noise pattern is a function of an instance (i.e., $T(x) \in \mathbb{R}^{C \times C}$). However, as a single instance can only have a single target label, it would be unrealistic to have the whole label transition matrix $T$ per instance.

Throughout this paper, a set-dependent noise (SDN) setting is utilized where we assume that the training dataset is partitioned into subsets where each subset contains its own label corruption matrices. This assumption is rather more practical in that it is more natural to assume that the annotators will be more likely to make mistakes on a specific subset consisting of hard instances. We would like to stress that our proposed method can estimate the label noise patterns in both CCN and SDN settings without the necessity of additional clean data.

### 3.2. Robust Learning and Corruption Pattern Estimation

The main objectives of the proposed method are twofold: the first is to robustly learn the underlying clean target signal out of noisy training data, and the other is to gain the explainability of the prediction via estimating the label corruption information as well as the predictive uncertainty. Specifically, we disentangle the total uncertainty into aleatoric and epistemic uncertainty similar to Kendall et al. [26] and will be explained in the next section. Aleatoric uncertainty corresponds to the irreducible part of the uncertainty, which is inherent in the data generation process (e.g., measurement noise). On the other hand, epistemic uncertainty captures the model uncertainty, which may reduce as we have more training data.

With respect to the label corruption information, we estimate the set-dependent noise (SDN) pattern of the training dataset without the necessity of a clean validation dataset. Note that the SDN inherently handles the CCN as it can simply condition the whole data. Specifically, we estimate the label transition matrix conditioned on the subset of training of test data where the corruption rates and the noise patterns, symmetric or asymmetric, can be estimated from the transition matrix.

### 4. Proposed Method

We present a robust learning method via a mixture-of-experts model for a classification task named mixture logit networks (MLN) and a noise pattern estimation method utilizing the outputs of the MLN. To fully utilize the multiple mixtures, we further propose an uncertainty-aware regularization method. We empirically show that this regularization method plays an influential role in achieving both robustness and explainability. The intuition behind leveraging the mixture model is that, when given corrupted training data, the noise pattern will give rise to the discrepancy of the prediction outputs, where a single deterministic model (e.g., a ResNet) often fails to correctly capture the clean
target signal. However, as a mixture model, when adequately trained, can better capture the inconsistent output patterns (including both clean and noisy distributions), it not only can robustly learn the underlying target distribution but also can model the noise patterns injected in the data generating process. The overall process of the proposed method is illustrated in Figure 1.

The MLN architecture is illustrated in Figure 2. Suppose that the number of mixtures is $K$, then the MLN outputs consist of mixture weights $\{\pi_k\}_{k=1}^K$, logits $\{\mu_k\}_{k=1}^K$ where $\mu_k \in \mathbb{R}^C$ and $C$ is the number of classes, and Mixture standard deviations (Mixture STD) $\{\sigma_k\}_{k=1}^K$. Note that only the uppermost layer is modified. Hence the total number of parameters does not change significantly.

4.1. Uncertainty Estimation using the MLN

We first present ways to estimate two types of uncertainties with the MLN: epistemic (model) uncertainty and aleatoric (data) uncertainty. We denote $\sigma_e$ as epistemic uncertainty and $\sigma_a$ as aleatoric uncertainty. First, epistemic uncertainty is computed as follows.

$$\sigma_e^2 = \sum_{j=1}^{K} \left( \sum_{c=1}^{C} \pi_j \left\| \mu_j^{(c)}(x) - \sum_{k=1}^{K} \pi_k \mu_k^{(c)}(x) \right\|^2 \right)$$

(1)
where $\mu^c_k$ is logit of label $c$ in $k$th mixture.

On the other hand, epistemic uncertainty ($\sigma_e$) indicates how much the model is uncertain about its prediction. $(1)$ corresponds to the weighted average variance of each mixture’s predicted logits, which can be seen as disagreements between $\{\mu_k\}_{k=1}^K$. On the other hand, aleatoric uncertainty ($\sigma_a$) is computed as follows.

$$\sigma_a^2 = \sum_{k=1}^K \pi_k \sigma_k(x)$$  \hspace{1cm} (2)

Aleatoric uncertainty captures noise inherent in observation, how much the model is uncertain about its data. $(2)$ indicates the weighted average of each mixture’s predicted STD of the given input. Mixture STD $\{\sigma_k\}_{k=1}^K$ denotes predicted noise by mixtures, also can be used as attenuation factor for loss function, similar to Kendal et al. \cite{26}.

4.2. Mixture of the Attenuated Losses

We present a Mixture of the Attenuated Cross-Entropy (MACE) loss for effectively training the MLN. We denote the target as $y_i$, which can either be clean or noisy (i.e., $\tilde{y}_i$) depending on the dataset. The proposed loss function consists of cross-entropy loss divided with the standard deviation of each mixture (i.e., loss attenuation) and then weighted summation of the attenuated loss for each mixture. Mixture standard deviation $\{\sigma_k\}_{k=1}^K$ corresponds to the expected measurement noise of each instance.

The MACE loss function is defined as follows:

$$\mathcal{L}_{MACE} = \frac{1}{N} \sum_{i=1}^N \sum_{k=1}^K \pi_k(x_i) \frac{l(\mu_k(x_i), y_i)}{\sigma_k(x_i)}$$  \hspace{1cm} (3)

where $l(\mu_k(x_i), y_i)$ is the cross entropy loss.

If an input is ambiguous or corrupted, it becomes more likely to make a false prediction. Then $\sigma_k$ will increase to reduce the overall loss function of the prediction. As a result, this attenuated factor prevents the overfitting of the model to the corrupted dataset, making the proposed model more robust.

4.3. Uncertainty-aware Regularization Method

We observe that there could exist two possible problems when training only with the original MACE loss function: simple increment of the mixture standard deviation will minimize the loss and the insufficient usage of mixtures. In order to resolve these issues, we propose a novel regularization method utilizing predictive uncertainty measures. Let us first present the proposed loss function utilizing the uncertainty measures:

$$\mathcal{L}(\mathcal{D}) = \mathcal{L}_{MACE} - \lambda_1 \sigma_e + \lambda_2 \sigma_a.$$  \hspace{1cm} (4)

The first problem is that $\{\sigma_k\}_{k=1}^K$ in $(3)$ will simply grow to minimize the total loss. To prevent this, we need the regularization term, $\{\sigma_k\}_{k=1}^K$. This is
inspired by Kendall et al. [26], where the loss function is based on the Gaussian likelihood and is the sum of attenuated regression loss and regularization of \( \{\sigma_k\}_{k=1}^K \). We present a parameter \( \lambda_2 \) as a weight.

Next, it is known that the mixture-of-experts model is prone to use only one or few components, incapable of capturing the various data distributions. Moreover, we observe that different mixtures are easily agreeable with each other, which is inappropriate to represent a multi-modal distribution. Due to this property, the whole model often fails to learn both clean and corrupted data distribution, leaving the model vulnerable to noisy data. To this end, we regularize epistemic uncertainty to be large, which naturally incentives to utilize more mixtures where \( \lambda_1 \) is the weight parameter.

We illustrate effect of this regularizer in Figure 3. The synthetic dataset consists of two-dimensional inputs (i.e., \( x_0 \) and \( x_1 \)) for a binary classification problem where we assign label 0 to instances on the upper moon and label 1, otherwise. We corrupt the label information by flipping the labels at the rate of 30%, and train the MLN with three mixtures. In the first column, we illustrate the clean half-moon dataset, noisy half-moon dataset, and decision boundary trained by the clean half-moon dataset. The second and third columns present the effect of the regularizer. The first row presents the estimated flipping rate, computed from (14), which will be discussed in the later section. The result indicates that the proposed regularizer helps the better prediction of the noise distribution. The second row of Figure 3 shows that the output of each mixture disagrees with each other, making the better representation. Furthermore, the third row illustrates that the proposed regularizer smooths the decision bound-
ary in the presence of outliers.

4.4. Corruption Pattern Estimation

We further use the output of the MLN to gain information about the noise corruption pattern. This can be done by estimating the noise transition matrix. The noise transition matrix $T_{ij}(x)$ indicates the probabilities of the clean labels flipping to noisy labels. The notation represents the probability that the instance $x$ with the clean label $y = i$ will have a noisy label $\tilde{y} = j$. Formally, the noise transition matrix is defined as follows.

$$T_{ij}(x) = P(\tilde{y} = j | y = i, x) \quad (5)$$

Capturing multi-modality is one of the strengths that the mixture model possesses. This property leads the MLN to model a multi-modal distribution on a noisy instance, representing both the clean and noisy label distribution. As the label corruption patterns can be regarded as a multi-modal distribution, we introduce an auxiliary random variable $z$ to estimate the noise transition matrix.

$$T_{ij}(D_l; z) = \frac{1}{|D_l(i)|} \sum_{x \in D_l(i)} P(\tilde{y} = j | x) \quad (6)$$

$here, D_l$ denotes the set indexed $l$ and $D_l(i) = \{x|y = i, (x, y) \in D_l\}$. For SDN setting set index can be 0 (clean) or 1 (ambiguous) and for CCN set index will be 0 (total).

$$T_{ij}(D_l; z) = \frac{1}{|D_l(i)|} \sum_{x \in D_l(i)} \sum_{k=1}^{K} P(z = k)P(\tilde{y} = j | z = k, x) \quad (7)$$

where $K$ is total number of mixtures, $z$ is latent variable, and $P(z = k)$ is weight of component distribution denoted as $\pi_k$ above.

Starting from (7), we denote the $\hat{P}(y|x)$ as the the soft-max output vector approximating $P(y|x)$ by parametrized model and estimator for the noise transition matrix as $\hat{T}$. In addition, since we cannot observe clean $y$, we assume $y$ as Bayes optimal labels, the class labels that maximize the clean class posteriors $\hat{f}(x) := \text{argmax} P(y|x)$. Furthermore, we define a subset $\hat{D}_l(i)$ with the Bayes optimal label index $i$ in the set $\tilde{D}_l$, as we do not have any prior knowledge about the clean $y$.

$$\hat{D}_l(i) = \{x|\hat{f}(x) = i, x \in D_l\} \quad (8)$$

Then, the (7) can be modified as follows.

$$\hat{T}_{ij}(D_l; z) = \frac{1}{|\hat{D}_l(i)|} \sum_{x \in \hat{D}_l(i)} \sum_{k=1}^{K} \hat{P}(z = k)\hat{P}(\tilde{y} = j | z = k, x) \quad (9)$$

The Lemma 4.1 shows (9) is a valid transition matrix, since the row-wise sum of predicted transition matrix becomes one.
Lemma 4.1. The row wise sum of the proposed transition matrix estimation in (9) becomes one.

\[ \sum_{j=1}^{C} \hat{T}_{ij}(D_l; z) = 1 \] (10)

Proof. The row wise sum of the proposed estimated noise transition matrix becomes as follows.

\[ \sum_{j=1}^{C} \hat{T}_{ij}(D_l; z) = \sum_{j=1}^{C} \frac{1}{|D_l(i)|} \sum_{x \in D_l(i)} \sum_{k=1}^{K} \hat{P}(z = k) \hat{P}(\tilde{y} = j | z = k, x) \] (11)

Since \(j\) is not dependent to \(|D_l(i)|\) and \(\hat{P}(z = k)\), the equation can be rewritten as follows.

\[ \sum_{j=1}^{C} \hat{T}_{ij}(D_l; z) = \frac{1}{|D_l(i)|} \sum_{x \in D_l(i)} \sum_{k=1}^{K} \hat{P}(z = k) \sum_{j=1}^{C} \hat{P}(\tilde{y} = j | z = k, x) \] (12)

By the definition of the categorial distribution and mixture weight, \(\sum_{j=1}^{C} \hat{P}(\tilde{y} = j | z = k, x) = 1\) and \(\sum_{k=1}^{K} \hat{P}(z = k) = 1\).

\[ \sum_{j=1}^{C} \hat{T}_{ij}(D_l; z) = \frac{1}{|D_l(i)|} \sum_{x \in D_l(i)} 1 \] (13)

As we define the set \(\hat{D}_l(i)\) as (8), the proposed noise transition matrix holds \(\sum_{j=1}^{C} \hat{T}_{ij}(D_l; z) = 1\).

As the confidence of the softmax-output decreases on the noisy dataset, the predicted transition matrix often suffers from being too smooth. To better estimate the noise transition matrix, inspired by Liang et al. [27], we apply temperature scaling for the softmax activation. We set the temperature to zero, which makes the softmax function an indicator function, driving to the predicted noise transition matrix inherent to the confidence score. \(\hat{P}_{scaled}\) is defined as follows.

\[ \hat{P}_{scaled}(\tilde{y} = j | z = k, x) = I_j(\arg\max_c \hat{P}(\tilde{y} = c | z = k, x)) \] (14)

The scaled transition matrix is defined by replacing \(\hat{P}\) to \(\hat{P}_{scaled}\) in (9).

5. Experiments

In this section, we present experimental results of validating the robustness of the proposed method. We first describe the implementation details for experiment settings, including datasets, corruption patterns, and hyperparameters. Next, we present the results in the CCN setting and compare them with
benchmarks. Furthermore, we utilize the estimated uncertainty measures to distinguish the collective outliers in the SDN settings where the noise transition matrices of each partition are estimated and compared with the ground truth.

5.1. Implementation Details

Class conditioned noise setting. We first construct a class-conditional noise dataset with clean instances and noisy labels whose corruption rate is solely a function of class information. We evaluate the proposed method on four different datasets, MNIST, CIFAR10, CIFAR100, and TREC. The former three are popularly used for evaluating the robustness of the image classification algorithms. We further validate our method on sentence classification problems, TREC dataset. Following JoCoR [5], we conduct experiments on four different label corruption patterns: Symmetry-20%, Symmetry-50%, Symmetry-80%, and Asymmetry-40%. We set Symmetry-70% instead of Symmetry-80% on the TREC dataset since it contains imbalanced labels; too severe label noise makes the number of clean instances not enough for learning. Furthermore, to show the generalization of noise transition matrix estimation, we test on two more noise patterns, Dual-40% and Tridigagonal-60%. These noise patterns are similar to asymmetric noise, but they shift the label randomly to assigned two or three labels. For example, Dual-40% patterns denote the label can be shifted to two labels with each probability of 20%.

Set-dependent noise setting. A set-dependent noise setting is utilized where the dataset is partitioned into two subsets: clean set and ambiguous set. We define an ambiguous set as a set containing the pair of corrupted instances and noisy labels. We conduct an experiment on two different datasets: Dirty-MNIST and Dirty-CIFAR10. Dirty-MNIST, proposed by Mukhoti et.al. [28] is formed as the union of MNIST set and Ambiguous-MNIST set. Ambiguous MNIST contains corrupted instances where it has multiple plausible labels but containing only one GT label. To conduct set-dependent noise on the Dirty-MNIST dataset, we additionally added label noise on the Ambiguous-MNIST set. We validate on four different label noise patterns: Symmetry-20%, Symmetry-50%, Symmetry-80%, and Asymmetry-40%. We define the Dirty-CIFAR10 dataset, which contains half of the original CIFAR10 dataset and the other half ambiguated with the CutMix [25] method. We choose the CutMix method to maintain the scheme that samples on ambiguous sets should have multiple possible labels but has one GT label. Again, to form the SDN setting, we added label noise on the ambiguous set.

Hyperparameters. We use a three-layer CNN for MNIST and seven-layer CNN for both CIFAR10 and CIFAR100. For the TREC dataset, we use the CNN-multichannel model, where utilizes both static and non-static word2vec initialized CNN channels, as implemented in Kim et al. [29]. We set the batch size as 128 and use an Adam optimizer with the learning rate \(10^{-3}\) for the vision task and train the model with 200 epochs for CIFAR10, CIFAR100, and 20 epochs for MNIST. For the NLP task, we utilize an Adadelta optimizer with a
learning rate of 0.1 for 100 epochs following [29]. The learning rate is decayed 0.2 times for every ten epochs for CIFAR10, CIFAR100, and TREC, and 0.2 decay rate for every five epochs on the MNIST dataset. We set the minimum of \( \sigma_k \) as one and maximum as ten by using a sigmoid function. In addition, we set the number of the mixtures to be 20 for all experiments, which should be large enough to cover all the noise distribution. Furthermore, we set regularizer hyperparameters as \( \lambda_1 = 1 \), \( \lambda_2 = 1 \) except for CIFAR100, where we scale the parameter to \( \lambda_1 = 0.1 \), \( \lambda_2 = 1 \). Regularizer parameters are selected using cross-validation results. On cross-validation, we assume there exists a small clean set and use 10% of the clean test set as the validation set.

5.2. Robust Learning Performance

We conduct robust learning experiments with the class-conditioned noise (CCN) setting to investigate the performance of the MLN. We evaluate the test accuracy on four datasets with four different noise patterns and compare with F-correction [30], Co-teaching [2], Co-teaching+ [4] and JoCoR [5]. The test accuracy on MNIST is shown in Table 2. The proposed method outperforms on the Symmetry-80% setting, and on other noise settings, it is compatible with the compared methods. However, the results on CIFAR10 in Table 3 show that our method outperforms the compared methods on Symmetry-80% and Asymmetry-40%, with the second-best performance on other noise patterns. Furthermore, Table 4 presents the test accuracy on CIFAR100 dataset. The MLN outperforms on Symmetry-80% and Asymmetry-40% noise patterns and performs second-best on the rest of the noise patterns. We would like to note that the proposed method shows its strengths in heavy corruptions, such as Symmetry-80% and Asymmetry-40%. We also evaluate the MLN on the NLP domain using the TREC dataset and compare it with the baseline method proposed in [29]. The result is shown in Table 5 where the proposed method outperforms the baseline in all cases. The proposed method shows a significant performance margin on large corruption rates, such as Symmetry-80% and Asymmetry-40%.
Table 4: CIFAR100 Test Accuracy

| Noise Rate       | F-correction [3] | Co-teaching [2] | Co-teaching+ [4] | JoCoR [5] | MLN (ours) |
|------------------|------------------|-----------------|------------------|-----------|------------|
| Symmetry-20%     | 37.95±0.10       | 43.73±0.16      | 49.27±0.03       | 53.01±0.04 | 51.60±0.08 |
| Symmetry-40%     | 24.98±1.82       | 34.96±0.80      | 40.04±0.70       | 49.49±1.46 | 47.72±0.07 |
| Asymmetry-40%    | 2.10±2.23        | 15.1±0.46       | 13.44±0.37       | 15.49±0.98 | 13.88±0.11 |

| Noise Rate       | Symmetry-20%     | Symmetry-50%    | Symmetry-80%     | Asymmetry-40% | Dual-40% | Tridiagonal-60% |
|------------------|------------------|-----------------|-----------------|--------------|---------|-----------------|
| Symmetry-20%     | 2.66±0.4472      | 96.73±0.16      | 14.46±0.4472    | 98.97±0.04   | 98.40±0.02 |
| Symmetry-50%     | 3.33±0.4472      | 97.80±0.12      | 17.82±0.4472    | 98.32±0.04   | 97.75±0.02 |
| Symmetry-80%     | 5.96±0.4472      | 80.37±0.37      | 23.42±0.4472    | 87.47±0.04   | 93.68±0.02 |
| Asymmetry-40%    | 2.00±0.5309      | 98.49±0.04      | 15.27±0.5309    | 97.13±0.04   | 96.85±0.02 |
| Dual-40%         | 2.57±0.5810      | 98.77±0.04      | 13.64±0.5810    | 98.97±0.04   | 98.06±0.02 |
| Tridiagonal-60%  | 3.79±0.6199      | 98.74±0.04      | 10.96±0.6199    | 98.06±0.04   | 97.85±0.02 |

Table 5: TREC Test Accuracy

| Flipping Rate   | Baseline [29]   | MLN (ours)   |
|-----------------|-----------------|--------------|
| Symmetry-20%    | 84.88±0.14      | 87.38±0.12   |
| Symmetry-50%    | 57.28±0.20      | 74.79±0.90   |
| Symmetry-70%    | 29.14±0.01      | 55.78±1.39   |
| Asymmetry-40%   | 72.63±0.96      | 74.17±0.28   |

Table 6: Evaluation of Noise Transition Matrix ATV (average total variation)(x100), KTD(Kendall-Tau distance), Test Accuracy

| Noise Rate       | ATV   | KTD   | Accuracy | ATV   | KTD   | Accuracy |
|------------------|-------|-------|----------|-------|-------|----------|
| MNIST            | TVD [14] MLN (Ours) |
| Symmetry-20%     | 9.134 0.4472 | 81.47 20.71 0.4472 84.20 |
| Symmetry-50%     | 8.048 0.4472 | 73.94 9.69 0.4472 77.88 |
| Symmetry-80%     | 6.117 0.4772 | 43.50 5.85 0.4373 41.83 |
| Asymmetry-40%    | 2.002 0.5309 | 75.95 16.81 0.4948 76.63 |
| Dual-40%         | 9.320 0.5811 | 79.71 15.97 0.5811 81.31 |
| Tridiagonal-60%  | 12.46 0.6109 | 74.38 20.74 0.5822 71.29 |

5.3. Class-conditional Noise Transition Matrix Estimation

In this section, we evaluate the noise transition matrix estimation on the class-conditional noise setting. We estimate the noise transition matrix using [14], which is an anchor-free method that does not require a clean validation set. We evaluate the noise transition matrix on MNIST and CIFAR10 datasets with Symmetry-20%, Symmetry-50%, Symmetry-80%, Asymmetry-40%, Dual-40%, and Tridiagonal-60% noise patterns.

We report the average total variation (ATV) and Kendall Tau rank distance (KTD) [31] to evaluate the transition matrix estimation, which are defined as

---

1 We call Dual for two mislabeled classes and Tridiagonal for three mislabeled classes.
Average total variance = \( \frac{1}{C} \sum_{i=1}^{C} \frac{1}{2} \sum_{j=1}^{C} |T_{ij} - \hat{T}_{ij}| \)  (15)

Average Kendall Tau Distance = \( \frac{1}{C} \sum_{i=1}^{C} \sum_{j,k} \bar{K}_{j,k}(t_i, \hat{t}_i) \)  (16)

where \( t_i \) denotes for \( i \)-th row vector of transition matrix \( T \) and \( \hat{t}_i \) for \( i \)-th row vector of \( \hat{T} \).

The function \( \bar{K} \) for two arbitrary vectors \( t_1 \) and \( t_2 \) is defined as follows:

\[
\bar{K}_{j,k}(t_1, t_2) = \begin{cases} 
0 & \text{if } j \text{ and } k \text{ are in same order of ranking} \\
1 & \text{if not}
\end{cases}
\]  (17)

Figure 4: Noise Transition Matrix on MNIST
The total variance is an average L1 norm between two matrices, which denotes the absolute difference between the estimated and ground-truth matrices. Kendall-tau rank distance is defined as a metric that counts the number of pairwise disagreements between two ranking lists, which means comparing the ranking of two matrices.

The Table 6 shows the ATV and KTD measure with comparison to TVD [14]. TVD uses total variation regularization to estimate the noise transition matrix with the prior confusion matrix and Dirichlet posterior update. Although the proposed method underperforms for every noise rate except for Symmetry-80% on ATV, the KDT measure outperforms TVD, especially in asymmetric noise patterns, i.e., Asymmetry-80%, Dual-40%, Tridiagonal-60%. The results illustrate that the MLN shows a better estimation performance compared to TVD in terms of the row-wise ranking of the noise transition matrix. However, it
shows slightly inferior performances to TVD in terms of average total variances. We further discuss this in Section 5.6. The transition matrix estimated by the MLN is shown in Figure 4 for MNIST dataset and Figure 5 for CIFAR10 dataset.

5.4. Partitioning Sets on the SDN Setting

Figure 6: Example of Set-Dependent Noise Dataset. The arrows denote the label corruption.

| Noise Rate | Aleatoric | Epistemic | Entropy | Max Softmax | Softmax Entropy |
|------------|-----------|-----------|---------|-------------|-----------------|
| Dirty MNIST |          |           |         |             |                 |
| Symmetry-20% | 0.9998   | 0.9991   | 0.9901 | 0.9978      | 0.9987          |
| Symmetry-50% | 0.9999   | 0.9992   | 0.9995 | 0.9999      | 0.9999          |
| Symmetry-80% | 0.9999   | 0.9992   | 0.9996 | 0.9999      | 0.9999          |
| Asymmetry-40% | 0.9811   | 0.9293   | 0.9653 | 0.9927      | 0.9933          |
| Dirty CIFAR10 |          |           |         |             |                 |
| Symmetry-20% | 0.9181   | 0.7300   | 0.7552 | 0.8739      | 0.8971          |
| Symmetry-50% | 0.9874   | 0.8909   | 0.9604 | 0.9610      | 0.9756          |
| Symmetry-80% | 0.9976   | 0.9848   | 0.9886 | 0.9315      | 0.9359          |
| Asymmetry-40% | 0.8536   | 0.5616   | 0.6563 | 0.7234      | 0.7060          |

In this section, we show the ability of the proposed method to partition the dataset leveraging aleatoric uncertainty estimated from the MLN. In particular, the collective outliers, partitions of training data with corrupted labels, are well captured via aleatoric uncertainty. We use the Dirty-MNIST dataset and Dirty-CIFAR10 dataset explained in Figure 6. The datasets contain a clean set and an ambiguous set, where an ambiguous set is composed of ambiguous instances with corrupted labels. There exist different uncertainty measures than the ones explained in (1) and (2), for example, it can be measured by max-softmax or softmax entropy computed from the entropy of mixture weights. The Pi-entropy denotes the entropy of the mixture weights. As bigger entropy can be interpreted as a lower weight of target distribution, this can be seen as the uncertainty of the inputs. In this experiment, we assume the size of two sets is the same and set threshold as the median of the uncertainty measures to partition these sets. The uncertainty measures are as follows.

\[
\text{max-softmax} = 1 - \max_c \mu_k^{(c)}(x) \quad \text{where} \quad k = \arg\max_i \pi_i(x) \quad (18)
\]
softmax-entropy = \(- \sum_c \mu_{k_c}(x) \log (\mu_{k_c}(x))\) \quad \text{where} \quad k = \arg\max_i \pi_i(x) \quad (19)

\pi\text{-entropy} = - \sum_k \pi_k(x) \log (\pi_k(x)) \quad (20)

As the observation noise is an exemplar case of aleatoric uncertainty, instance corruption patterns can be captured by aleatoric uncertainty. Table 7 reports the aleatoric uncertainty measure can partition clean and ambiguous set on symmetric noise compared to other uncertainty measures except for Asymmetry-40% noise rate. Figure 7 shows the average of aleatoric uncertainty for both clean and ambiguous sets among each class. The result first shows that the aleatoric uncertainty is higher in ambiguous instances. Furthermore, shown in the symmetric noise cases, a heavy label corruption rate leads to higher aleatoric uncertainty. Finally, Dirty-MNIST with Asymmetry-40% label noise case shows that aleatoric uncertainty increases in the corrupted labels compared to clean labels in ambiguous instances. This demonstrates that both corruption in instances and corruption in the label are related to aleatoric uncertainty.

5.5. Set-Dependent Noise Transition Matrix Estimation

In this section, we examine the ability of our method to estimate noise transition matrices per each group where we partition the training data into multiple sets using the predicted aleatoric uncertainty in Section 5.4. The experimental results on the Dirty-MNIST and Dirty-CIFAR10 datasets are shown in Figure 8 and 9, respectively. Each quarter denotes a single experiment for each corruption pattern, with the upper and lower rows showing the predicted noise transition matrix and the ground-truth, respectively. Here, clean labels on the ambiguous set denote the ground-truth label of each instance after ambiguating
the instances. We can see that our proposed method is able to correctly estimate the noise transition matrix for both clan and ambiguous sets in terms of a row-wise ranking manner. Furthermore, Figure 9 suggests that our proposed method can also capture the noise induced in inputs (i.e., CutMix [25]). In other words, the images of cats are cut-mixed with the images with dogs (and vice versa), and these corruption patterns are well captured by the noise transition matrix.

5.6. Limitations

In this section, we will discuss the limitations of the proposed method. While the MLN correctly estimates the row-wise ranking of the noise transition matrix, it under-performs TVD [14] in terms of the absolute error between the ground-truth and the estimated transition matrices (average total variance). This is mainly due to the fact that TVD uses an explicit confusion matrix prior to update the estimated noise transition matrix with a Dirichlet posterior. In contrast, the proposed method indirectly estimates the noise transition matrix.
6. Conclusion

We have introduced an uncertainty-aware robust learning framework by leveraging a mixture of the experts’ model named mixture logit networks (MLN). The MLN can estimate two different types of uncertainty, epistemic and aleatoric, where the predictive uncertainty is further utilized to define a novel regularization method. We showed that the MLN can represent multi-modal distributions, making the model not only robust to outliers but also able to estimate noise patterns. In particular, we presented a set-dependent noise (SDN) learning problem where multiple corruption patterns exist per each partition and proposed a novel validation scheme for estimating the corruption patterns. To tackle this problem, we leveraged aleatoric uncertainty to detect the corrupted partition and estimated the SDN patterns using the multi-modal
target distribution computed from the MLN. We would like to note that uncer-
tainty estimation on the robust learning framework plays a significant role in providing information about the corruption of each instance. The current evaluation scheme for the SDN setting relies on two assumptions: the collective outliers can be separated via the estimated aleatoric uncertainty, and a particular label transition matrix exists per each partition. One promising future research direction could be examining our proposed method to real-world datasets without applying artificial noises to both inputs and outputs. Clothing1M dataset can be used for this purpose as it is known to contain a nontrivial amount of mislabeled data.
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