Fermion mixing in an $S_3$ model with three Higgs doublets

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Abstract. We present a brief overview of a unified treatment of masses and mixings of quarks and leptons, as a realization of an $S_3$-flavour permutational symmetry in an extension of the Standard Model. In order to leave $S_3$ as an exact flavour symmetry two extra Higgs fields, $SU(2)_L$ doublets, are added. In this model, the mass matrices of the fermions are reparametrized in terms of their eigenvalues allowing us to derive exact, explicit analytical expressions for the mixing matrices, $V_{CKM}$ and $U_{PMNS}$, as functions of the quark and lepton masses, respectively. In both the quark and lepton sectors a $\chi^2$ fit of the theoretical expressions to the experimentally determined values of the mixing angles yields results in excellent agreement with the most recent experimental data on quarks and leptons.

1. Introduction

Family or flavour models offer a way of expressing mixing angles in terms of fermion masses [1–4] in good agreement with phenomenology. Here, we consider fermion mixing in an extension of the Standard Model (SM) with an $S_3$ family symmetry group and three Higgs fields, electroweak doublets. We include also three right-handed neutrino fields, to implement the seesaw mechanism. In this way, all fermionic and Higgs fields, including the right-handed neutrino fields, have three species and transform under the family symmetry group as the three dimensional representation $1 \oplus 2$ of the permutational group $S_3$. The $S_3$ family symmetry strongly constrains the number of free parameters in the fermionic mass matrices and allows us to obtain exact mathematical relations among fermion masses and mixing angles. In section 2, we introduce the $S_3$ symmetry and briefly comment on its interesting features as a family symmetry. In section 3, we discuss and introduce the construction of this model, which we will denote as $S_3$-3H. In section 4, we confront the model with the most up to date experimental data. In section 5, we summarize and conclude with some remarks.

2. $S_3$ as a symmetry of fermion families

The permutational symmetry group of three objects, $S_3$, is the smallest non-Abelian finite group. The group $S_3$ has three irreducible representations (irreps): a doublet $2$, and two singlets, $1_S$.
and $1_A$, symmetric and antisymmetric, respectively. The Kronecker products of irreducible representations may be decomposed as a direct sum of irreducible representations

$$1_S \otimes 1_S = 1_S, \quad 1_A \otimes 1_A = 1_S, \quad 1_A \otimes 1_S = 1_A,$$

$$1_S \otimes 2 = 2, \quad 1_A \otimes 2 = 2, \quad \text{and} \quad 2 \otimes 2 = 1_A \oplus 1_S \oplus 2.$$  \hfill (1)

In the SM all gauge interactions are invariant under the permutation of the family index in the fermion fields. If we extend this invariance to the Yukawa sector we are led to introduce two more Higgs fields, $SU(2)_L$ doublets, such that all Yukawa interactions are made invariant under the permutation of the family index in both the fermion and the Higgs boson fields.

In Figure 1 the mass ratios, $\tilde{m}_i^f = m^f_i / m^f_3$, are shown. There, a clear pattern can be observed, which suggests that the structure obtained in the decomposition of the fermion mass matrix is a direct sum of irreducible representations of $S_3$, namely $3 = 2 \oplus 1$, and is a good representation of the fermion mass hierarchy.

![Figure 1](image.png)

**Figure 1.** The mass ratios appear naturally ordered, from left to right, from the first family to the third family. An underlying symmetry structure, $2 \oplus 1$, can be clearly seen from here.

### 3. An $S_3$-invariant Extension of the Standard Model ($S_3$-3H)

Models which have $S_3$ as family symmetry group have been discussed for a long time (see for instance [4] and references therein).

#### 3.1. Assignments between $S_3$ irreps and families

We assign the first two families, $f_I(L,R)$ and $f_{II}(L,R)$, to the doublet representation, $2$, and the third family, $f_{III}(L,R)$, to the symmetric singlet representation, $1_S$, of $S_3$

$$\begin{pmatrix} f_{I}(L,R) \\ f_{II}(L,R) \end{pmatrix} \sim 2; \quad f_{III}(L,R) \sim 1_S,$$

respectively. Here $I, II$ or $III$ is the family index of a left or right-handed fermion field $f_{I}(L,R)$, and $f_{I}(L,R)$ may represent any SM fermion field. We make a similar assignment in the Higgs sector

$$H_D \equiv \begin{pmatrix} H_1 \\ H_2 \end{pmatrix} \sim 2; \quad H_S \sim 1_S,$$  \hfill (2)

where $H_i$ ($i = 1, 2, S$) represents a Higgs $SU(2)_L$ doublet field.
3.2. A weak-basis transformation

After electroweak symmetry breaking, in the symmetry adapted basis, the generic $S_3$-invariant mass matrix form for Dirac fermions is

$$
\mathcal{M}^f_{S_3} = \begin{pmatrix}
\sqrt{Y^f_2} v_S + Y^f_3 w_1 & Y^f_3 w_1 & \sqrt{2} Y^f_5 w_1 \\
Y^f_3 w_1 & \sqrt{2} Y^f_2 v_S - Y^f_3 w_2 & \sqrt{2} Y^f_5 w_2 \\
\sqrt{2} Y^f_3 w_1 & \sqrt{2} Y^f_5 w_2 & 2 Y^f_1 v_S
\end{pmatrix},
$$

(3)

where the $Y^f_i$ are complex Yukawa couplings and $w_1$, $w_2$, and $v_S$ are the real vacuum expectation values of the $S_3$ doublet components and the $S_3$ symmetric singlet representation, respectively. In this model, we consider the case where the vacuum expectation values of the $S_3$ doublet are the same, $w_1 = w_2$. This relation implies that in the minimum of the Higgs potential there is an accidental $S_3'$ symmetry [5,6].

It has been already noticed that a suitable weak-basis transformation of the mass matrices may help to reduce the number of free parameters by introducing a set of zeroes in their entries [7,8]. In our case, a $\pi/4$ rotation in the $S_3$ doublet subspace is enough to introduce one texture zeroes in the generic $S_3$-invariant Dirac fermion mass matrix

$$
\mathcal{M}^f_{\text{Hier}} = \begin{pmatrix}
|\mu^f_1| - |\mu^f_2| & |\mu^f_2| & 0 \\
|\mu^f_2| & |\mu^f_1| + |\mu^f_2| & \mu^f_8 \\
0 & \mu^f_8 & |\mu^f_3|
\end{pmatrix},
$$

(4)

where $\mu^f_1 \equiv \sqrt{2} Y^f_2 v_S$, $\mu^f_2 \equiv Y^f_3 w_1$, $\mu^f_3 \equiv 2 Y^f_1 v_S$, and $\mu^f_8 \equiv 2 Y^f_2 w_1$, and we have assumed Hermiticity, which in this case implies a particular value for the phase of the complex Yukawa couplings.

It has been already noticed [9] that the matrix form of Eq. (4) reproduces the four zero Fritzsch-like texture [1,10–13] which, along with the Nearest-Neighbour Interaction (NNI) [14,15] form of the mass matrix, give a good description of the phenomenology of quark and lepton masses and their mixing.

The computation simplifies if we notice that the mass matrix $\mathcal{M}^f_{\text{Hier}}$ can be expressed as $\overline{\mathcal{M}}^f_{\text{Hier}} + 1 \Delta_f$, where $\Delta_f = |\mu^f_1| - |\mu^f_2|$. Notice that the same unitary matrix diagonalizes both the original mass matrix and the shifted one. The expression for the shifted mass matrix is

$$
\overline{\mathcal{M}}^f_{\text{Hier}} = \begin{pmatrix}
0 & |\mu^f_2| & 0 \\
|\mu^f_2| & 2 |\mu^f_2| & \mu^f_8 \\
0 & \mu^f_8 & |\mu^f_3| - \Delta_f
\end{pmatrix},
$$

(5)

4. The quark sector

In the following section we give analytical expressions for the CKM mixing matrix elements as explicit functions of quark mass ratios. For the purpose of illustrating the relations among mixing parameters and mass ratios, we take an approximation where we neglect the ratio of the shift $\Delta_f$ over the heaviest (third generation) mass, which is a very small number. The complete analysis of the model, taking into account the full expression, will be presented elsewhere. In this way it is possible to express the elements of the quark mixing matrix $V^{th}_{CKM}$

$$
V^{th}_{CKM} = \begin{pmatrix}
V^{th}_{ud} & V^{th}_{us} & V^{th}_{ub} \\
V^{th}_{cd} & V^{th}_{cs} & V^{th}_{cb} \\
V^{th}_{td} & V^{th}_{ts} & V^{th}_{tb}
\end{pmatrix},
$$

(6)
as an explicit function of the quark mass ratios [16–18]

\[
V_{ud}^{\text{th}} = \sqrt{\frac{m_u m_d}{D_{1u} D_{1d}}} + \sqrt{\frac{m_u m_d}{D_{1u} D_{1d}}} \left( \sqrt{\frac{1}{D_{1u} D_{1d}}} (1 - \delta_d) \xi_{1}^{u,d} + \sqrt{\delta_d (1 - \delta_d)} \xi_{2}^{u,d} \right) e^{i\phi_1},
\]

\[
V_{us}^{\text{th}} = -\sqrt{\frac{m_u m_d}{D_{1u} D_{1d}}} + \sqrt{\frac{m_u m_d}{D_{1u} D_{1d}}} \left( \sqrt{\frac{1}{D_{1u} D_{1d}}} (1 - \delta_d) \xi_{1}^{u,d} + \sqrt{\delta_d (1 - \delta_d)} \xi_{2}^{u,d} \right) e^{i\phi_1},
\]

\[
V_{ub}^{\text{th}} = \frac{\sqrt{m_u m_d \delta_d}}{D_{2u} D_{2d}} + \sqrt{\frac{m_u m_d}{D_{2u} D_{2d}}} \left( \sqrt{\frac{1}{D_{2u} D_{2d}}} (1 - \delta_d) \xi_{1}^{u,d} - \sqrt{\delta_d (1 - \delta_d)} \xi_{2}^{u,d} \right) e^{i\phi_1},
\]

\[
V_{cb}^{\text{th}} = \frac{\sqrt{m_u m_d \delta_d}}{D_{2u} D_{2d}} + \sqrt{\frac{m_u m_d}{D_{2u} D_{2d}}} \left( \sqrt{\frac{1}{D_{2u} D_{2d}}} (1 - \delta_d) \xi_{1}^{u,d} - \sqrt{\delta_d (1 - \delta_d)} \xi_{2}^{u,d} \right) e^{i\phi_1},
\]

\[
V_{td}^{\text{th}} = \frac{\sqrt{m_u m_d \delta_d}}{D_{3u} D_{3d}} + \sqrt{\frac{m_u m_d}{D_{3u} D_{3d}}} \left( \sqrt{\frac{1}{D_{3u} D_{3d}}} (1 - \delta_d) \xi_{1}^{u,d} - \sqrt{\delta_d (1 - \delta_d)} \xi_{2}^{u,d} \right) e^{i\phi_1},
\]

\[
V_{ts}^{\text{th}} = \frac{\sqrt{m_u m_d \delta_d}}{D_{3u} D_{3d}} + \sqrt{\frac{m_u m_d}{D_{3u} D_{3d}}} \left( \sqrt{\frac{1}{D_{3u} D_{3d}}} (1 - \delta_d) \xi_{1}^{u,d} - \sqrt{\delta_d (1 - \delta_d)} \xi_{2}^{u,d} \right) e^{i\phi_1},
\]

\[
V_{tb}^{\text{th}} = \sqrt{\frac{m_u m_c m_d}{D_{4u} D_{4d}}} + \sqrt{\frac{m_u m_c m_d}{D_{4u} D_{4d}}} \left( \sqrt{\frac{1}{D_{4u} D_{4d}}} (1 - \delta_d) \xi_{1}^{u,d} + \sqrt{\delta_d (1 - \delta_d)} \xi_{2}^{u,d} \right) e^{i\phi_1},
\]

with

\[
\xi_{1}^{u,d} = 1 - \delta_u d - \delta_u d, \quad \xi_{2}^{u,d} = 1 + \tilde{m}_{c,s} - \delta_u d,
\]

\[
D_{1(u,d)} = (1 - \delta_u d)(\tilde{m}_{u,d} + \tilde{m}_{c,s})(1 - \tilde{m}_{u,d}), \quad D_{2(u,d)} = (1 - \delta_u d)(\tilde{m}_{u,d} + \tilde{m}_{c,s})(1 + \tilde{m}_{c,s}),
\]

\[
D_{3(u,d)} = (1 - \delta_u d)(1 - \tilde{m}_{u,d})(1 + \tilde{m}_{c,s}).
\]

In this notation \( \tilde{m}_u = m_u / m_t \), \( \tilde{m}_c = |m_c| / m_t \), \( \tilde{m}_d = m_d / m_b \) and \( \tilde{m}_s = |m_s| / m_b \).

4.1. \( \chi^2 \) analysis for the quark sector

For the \( \chi^2 \) fit we proceed as follows. We construct the \( \chi^2 \) function as

\[
\chi^2 = \frac{(V_{ud}^{\text{th}} - V_{ud})^2}{\sigma_{V_{ud}}^2} + \frac{(V_{us}^{\text{th}} - V_{us})^2}{\sigma_{V_{us}}^2} + \frac{(V_{ub}^{\text{th}} - V_{ub})^2}{\sigma_{V_{ub}}^2} + \frac{(J_{\text{th}} - J)^2}{\sigma_{J}^2},
\]

where the quantities with super-index “th” are the complete expressions for the CKM elements, as given by the \( S_3 \) model, and those without, are the experimental quantities along with their uncertainty,

\[
\begin{align*}
2011: & \quad V_{ud} = 0.97428 \pm 0.00015, \quad V_{us} = 0.2253 \pm 0.007, \\
& \quad V_{ub} = 0.00347 \pm 0.00014, \quad J = (2.91 \pm 0.155) \times 10^{-5},
\end{align*}
\]

\[
\begin{align*}
2012: & \quad V_{ud} = 0.97427 \pm 0.00015, \quad V_{us} = 0.2253 \pm 0.007, \\
& \quad V_{ub} = 0.00351 \pm 0.00015, \quad J = (2.96 \pm 0.18) \times 10^{-5}.
\end{align*}
\]
We present a comparison of fits using the available data known before July 2012 (labelled as 2011) and the up-to-date data presented by the PDG, given the sizeable change in the uncertainty of the mass of the strange quark. Since we assume unitarity of the CKM mixing matrix, we need to fit just to four observables. The theoretical expressions of the parameters of the CKM elements are given in terms of the mass ratios, \( m_i \), hence the minimization of the defined \( \chi^2 \) is a function of \( m_i \), and the parameters \( \delta_u \), \( \delta_d \), and \( \cos \phi_i \). That means, that as a result of the minimization, there is a best fit value for each of those quantities, where \( \chi^2 \) reaches its minimum. For the minimization procedure we used MINUIT. We allow the mass ratios \( m_i \) to take values within their 3\( \sigma \) range, as given in Table 1, whereas \( \delta_u \) and \( \delta_d \), vary freely. Minimization with MINUIT starts with a a seed whose value is close to the minimum of the parameters being considered. Therefore, if the range of variation is large, it is difficult to find a best fit point. On the other hand, to check for global minima, one should remove the limits of the “free” parameters. If we perform the fit leaving completely free the values for \( m_i \), the quality of the fit decreases, and most importantly, it turns out that \( m_u \) is of order 10\(^{-3} \). In Figure 2, we present the \( \chi^2 \) value as a function of \( m_d \), where we have taken \( \phi_1 = \pi/2 \). This is not the most general case but it represents an interesting one\(^1\). For this case the best fit points (BFP) of the parameters \( \delta_d \) and \( \delta_u \) are respectively \( 6.05 \times 10^{-2} \) and \( 4.09 \times 10^{-2} \). The BFP of the mass ratios \( m_u \), \( m_c \), \( m_d \) and \( m_s \) are respectively \( (1.73 \pm 0.75) \times 10^{-5}, (3.46 \pm 0.43) \times 10^{-5}, (1.12 \pm 0.007) \times 10^{-3} \), and \( (2.32 \pm 0.84) \times 10^{-2} \). The plot of Figure 2 corresponds to the exact result of the minimization, which allows to use values of the mass ratios \( m_i \) with more precision than the corresponding precision of the experimental/lattice determinations. This explains the change in value of \( \chi^2 \) in the plot, from \( 3.4 \times 10^{-4} \) up to \( 7.4 \times 10^{-1} \), which is the resulting value when we restrict the values of the BFP of mass ratios to have a precision equal to that of experimental/lattice determinations.

5. The leptonic sector

A similar analysis to the quark sector can be performed in the leptonic one, where a \( Z_2 \) symmetry is also introduced [19–22]. In this case, the masses of the left-handed Majorana neutrinos, \( M_{\nu L} \), are generated by the type I seesaw mechanism. We take the mass matrix of the right-handed neutrinos to be real and diagonal but non-degenerate \( \mathbf{M}_{\nu R} = \text{diag}(M_1, M_2, M_3) \), hence the mass matrix \( \mathbf{M}_{\nu L} \) takes the form [22]

\[
\mathbf{M}_{\nu L} = \begin{pmatrix}
\frac{2(\mu^2)}{M} & \frac{2\lambda(\mu^2)}{M} & \frac{2\nu^2 \mu^2}{M} \\
\frac{2\lambda(\mu^2)}{M} & \frac{2(\nu^2)}{M} & \frac{2\mu^2 \nu^2 \lambda}{M} \\
\frac{2\nu^2 \mu^2}{M} & \frac{2\mu^2 \nu^2 \lambda}{M} & \frac{2(\nu^2)}{M} + \frac{(\nu^2)^2}{M}
\end{pmatrix}, \quad \lambda = \left( \frac{M_2-M_1}{M_1+M_2} \right), \quad \text{and} \quad \mathbf{M} = 2 \frac{M_1 M_2}{M_2+M_1}.
\]

\(^1\) The results of the most general case will be presented elsewhere.
Figure 2. Results of the $\chi^2$ fit for the quark sector as a function of $\tilde{m}_d$. We have made a fit using only an average of the theoretical determination of $m_s$, $m_s(2\text{GeV}) = 0.101 \pm 0.011$, in order to assess the impact of the reduction in the uncertainty of $m_s$. The black vertical lines represent the $\tilde{m}_s$ allowed experimental $1\sigma$ region and its central value.

Since we assumed the right-handed neutrino mass matrix $\mathbf{M}_{\nu_R}$ to be real, the complex symmetric neutrino mass matrix $\mathbf{M}_{\nu_L}$ has one Dirac phase and two Majorana phases which may be factored out of $\mathbf{M}_{\nu_L}$ as

$$
\mathbf{M}_{\nu_L} = \mathbf{Q} \mathbf{U}_{\frac{\pi}{2}} \left( \mu_0 \mathbf{I}_{3 \times 3} + \widehat{\mathbf{M}} \right) \mathbf{U}_{\frac{\pi}{2}}^\dagger \mathbf{Q},
$$

(13)

where $\mathbf{Q} = e^{i\theta_2} \text{diag} \{ 1, 1, e^{i\delta_\nu} \}$ with $\delta_\nu = \phi_1 - \phi_2 = \arg \{ \mu'_3 \} - \arg \{ \mu'_5 \}$,

$$
\mathbf{U}_{\frac{\pi}{2}} = \begin{pmatrix}
\frac{1}{\sqrt{2}} & 0 & \frac{1}{\sqrt{2}} \\
\frac{1}{\sqrt{2}} & 0 & \frac{1}{\sqrt{2}} \\
0 & 1 & 0
\end{pmatrix}, \quad \mu_0 = \frac{2 |\mu'_2|^2}{|\mathbf{M}|} (1 - |\lambda|), \quad \text{and} \quad \widehat{\mathbf{M}} = \begin{pmatrix}
0 & A & 0 \\
A & B & C \\
0 & C & 2d
\end{pmatrix},
$$

(14)

with $A = \sqrt{2} |\mu'_2||\mu'_5| (1 - |\lambda|)$, $B = \frac{2 |\mu'_2|^2 + |\mu'_5|^2}{|\mathbf{M}|} - \frac{2 |\mu'_5|^2}{|\mathbf{M}|} (1 - |\lambda|)$, $C = \sqrt{2} \frac{|\mu'_2||\mu'_5|}{|\mathbf{M}|} (1 + |\lambda|)$ and $d = \frac{2 |\lambda||\mu'_5|^2}{|\mathbf{M}|}$. As mentioned before, the diagonalization of $\mathbf{M}_{\nu_L}$ is reduced to the diagonalization of the real symmetric matrix $\widehat{\mathbf{M}}$ with two texture zeroes of class I [23]. Hence, the matrix $\mathbf{M}_{\nu_L}$ is diagonalized by a unitary matrix

$$
\mathbf{U}_\nu = \mathbf{Q'} \mathbf{U}_{\frac{\pi}{2}} \mathbf{O}^{N[i]}_{\nu_L}.
$$

(15)

As in the case of the charged leptons, the matrices $\mathbf{M}_{\nu_L}$ and $\mathbf{U}_\nu$ can be reparametrized in terms of the neutrino masses, following the procedure outlined in refs. [17, 18, 23, 24]. Then, the mass matrix $\mathbf{M}_{\nu_L}$ for a normal [inverted] hierarchy in the mass spectrum takes the form

$$
\mathbf{M}_{\nu_L}^{N[i]} = \begin{pmatrix}
\mu_0 + d & d & \frac{1}{\sqrt{2}} \left( C^{N[i]} + A^{N[i]} \right) \\
d & \mu_0 + d & \frac{1}{\sqrt{2}} \left( C^{N[i]} - A^{N[i]} \right) \\
\frac{1}{\sqrt{2}} \left( C^{N[i]} + A^{N[i]} \right) & \frac{1}{\sqrt{2}} \left( C^{N[i]} - A^{N[i]} \right) & m_{\nu_1} + m_{\nu_2} + m_{\nu_3} - 2 (\mu_0 + d)
\end{pmatrix},
$$

(16)
with $C^{N[1]} = \sqrt{\frac{(2d+\mu_0-m_{\nu_1})(2d+\mu_0-m_{\nu_2})}{2d}}$ and $A^{N[1]} = \sqrt{\frac{(m_{\nu_2}^2 - \mu_0)(m_{\nu_3}^2 - \mu_0)(\mu_0 - m_{\nu_1}^2)}{2d}}$.

The allowed values for the parameters $\mu_0$ and $2d + \mu_0$ are in the following ranges: $m_{\nu_{2[3]}} > \mu_0 > m_{\nu_{3[2]}}$ and $m_{\nu_{3[2]}} > 2d + \mu_0 > m_{\nu_{2[1]}}$.

### 5.1. The reactor mixing angle

The theoretical expression for the lepton mixing angles as functions of the lepton mass ratios is readily obtained when the theoretical expressions for the modulii of the entries in the PMNS mixing matrix are substituted for $|\langle (V_{PMNS}^\text{th})_{ij}\rangle|$. In a first preliminary analysis for the reactor mixing angle $\theta_{13}^l$ and for an inverted neutrino mass hierarchy $(m_{\nu_2} > m_{\nu_1} > m_{\nu_3})$ we obtain

$$\sin^2 \theta_{13}^l \approx \frac{(\mu_0 + 2d - m_{\nu_3}) (\mu_0 - m_{\nu_3})}{(m_{\nu_1} - m_{\nu_3}) (m_{\nu_2} - m_{\nu_3})}.$$  

(17)

Now, with the following values for the neutrino masses $m_{\nu_2} = 0.056$ eV, $m_{\nu_1} = 0.053$ eV and $m_{\nu_3} = 0.048$ eV, and the parameter values $\delta_1 = \pi/2$, $\mu_0 = 0.049$ eV and $d = 8 \times 10^{-5}$ eV, we get $\sin^2 \theta_{13}^l \approx 0.029 \rightarrow \theta_{13}^l \approx 9.8^\circ$, in good agreement with the most recent global fits to neutrino oscillation data [25, 26]. A complete analysis, from a $\chi^2$ fit of the exact theoretical expressions for the modulii of the entries of the lepton mixing matrix of the $|\langle (V_{PMNS}^\text{th})_{ij}\rangle|$ to the experimental values will be considered elsewhere.

### Conclusions

The introduction of the $S_3$ family symmetry in the Standard Model was motivated by the need to reduce the number of free parameters in the theory, and was guided by the analysis of the phenomenology of masses and mixings in the quark and lepton sectors. In order to preserve the $S_3$ family symmetry, two more Higgs doublets have to be introduced in addition to the one of the SM, and thus the concept of flavour is extended to the Higgs sector of the theory. We identified the conditions under which the four-zero Fritzsch-like texture mass matrices are obtained. This allowed us to (i) reduce drastically the number of parameters needed to describe the extended model with three Higgs SU(2)$_L$ doublets, (ii) to find an exact parameterization of the mass matrices in terms of their eigenvalues, and (iii), to compute the mixing angles in both the quark and lepton sectors as functions of mass ratios. We also derived exact formulas for the mixing angles of the CKM and PMNS matrices in terms of quark and lepton mass ratios, which we found to be in excellent agreement with all data on quark and neutrino masses and their mixing.

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