FLUX EXPULSION AND FIELD EVOLUTION IN NEUTRON STARS

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ABSTRACT

Models for the evolution of magnetic fields of neutron stars are constructed, assuming the field is embedded in the proton superconducting core of the star. The rate of expulsion of the magnetic flux out of the core or, equivalently, the velocity of outward motion of flux-carrying proton vortices is determined from a solution of the Magnus equation of motion for these vortices. A force due to the pinning interaction between the proton vortices and the neutron-superfluid vortices, in addition to the other more conventional forces acting on the proton vortices, is also taken into account. Alternative models for the field evolution are considered based on the different possibilities discussed for the effective values of the various forces. The coupled spin and magnetic evolution of single pulsars as well as those processed in low-mass binary systems are computed for each of the models. The predicted lifetimes of active pulsars, the field strengths of the very old neutron stars, and the distribution of the magnetic fields versus orbital periods in low-mass binary pulsars are used to test the adopted field decay models. Contrary to the earlier claims, buoyancy is argued to be the dominant driving cause of flux expulsion for single as well as binary neutron stars. However, the pinning is also found to play a crucial role that is necessary to account for the observed low field binary and millisecond pulsars.

Subject headings: binaries: close — pulsars: general — stars: magnetic fields — stars: neutron

1. INTRODUCTION

The observed high rate of incidence of binary pulsars among the population of low magnetic field pulsars indicates a field decay mechanism in a neutron star that is linked with its evolution in a binary system (e.g., Phinney & Kulkarni 1994). This is also supported by the even lower field strengths of the millisecond pulsars, which are generally believed to be the end products of the binary evolution of pulsars in systems with low-mass main-sequence companions (e.g., van den Heuvel 1994). A model for the evolution of the magnetic field in a neutron star that indeed relates it to the spin evolution of the star, in an intimate way, has been proposed. The model predicts that the magnetic flux is transported out of the core, at the same rate as that of the spin-down of the star, into the crust, where it subsequently undergoes Ohmic decay (Srinivasan et al. 1990). This field decay mechanism has been already tested in a study of the spin evolution of pulsars in binary systems. The distribution of the final field strengths against the orbital periods was found to be in good agreement with that of the observed wide-orbit low-mass binary pulsars (Jahan-Miri & Bhattacharya 1994). Also, implications of the model for the evolution of single pulsars as well as those processed in massive binaries were shown to be consistent with, and provide new insights into, the existing observational data on the radio and X-ray pulsars (Jahan-Miri 1996a).

The above flux expulsion model (the so-called model of spin-down–induced flux expulsion, hereafter the SIF model) assumes that, as the star spins down, the outward moving (neutron) vortices pull the fluxoids (the proton vortices) along with them because of their interpinning. However, given the finite strength of the pinning interaction energy and the presence of resistance against the motion of both the fluxoids and the vortices, the two might be, in general, expected to move with different radial velocities, while “cutting” or “creeping” through each other. A more refined treatment of the flux expulsion, therefore, requires the dynamics of fluxoid motion to be considered explicitly, as is attempted here. A further improvement, not addressed here, would be to solve simultaneously for the dynamics of the fluxoids and the vortices, self-consistently. Our treatment is, however, justified for the steady state spin-down periods when the core superfluid spins down at a given rate that determines the radial velocity of the vortices.

In the following, we discuss the various forces that act on the fluxoids in the interior of a neutron star, including (1) a force due to their pinning interaction with the moving vortices. The other forces that we take into account are (2) viscous drag force due to magnetic scattering of electrons, (3) buoyancy force, and (4) curvature force. The velocity of the outward motion of the fluxoids can then be determined, for a given steady state spin-down rate of the star, from a solution of the Magnus equation requiring a balance of the forces acting radially on a fluxoid. The derived radial velocity of the fluxoids at the core-crust boundary would in turn determine the rate of the flux expulsion out of the core. Our approach here is analogous to that of Ding, Cheng, & Chau (1993, hereafter DC93) in their study of the field evolution of single normal pulsars (see also Muslimov & Tsygan 1985; Harvey, Ruderman, & Shaham 1986; Jones 1987, 1991). The original motivation for the present work was to investigate the results of such a treatment of the flux expulsion scenario for the magnetic evolution of binary pulsars, for which the spin-down history of the neutron star is quite different than for the single pulsars addressed in DC93. Furthermore, we have also explored other possibilities different than those assumed by DC93 about the nature and magnitude of the various forces acting on the fluxoids. The resulting alternative models for calculating the rate of flux expulsion could, in principle, probe the physics of the interior of neutron stars in more detail. Predictions of these models for the magnetic evolution of binary as well as single
pulsars are compared and tested against the available observational data.

2. DYNAMICS OF FLUXOIDS

2.1. The Scene

Superconductivity of the protons in the interior of a neutron star and the presence of a lattice of fluxoids carrying the magnetic flux in the core form the underlying premise for the field decay mechanism under study. As is also the case with the assumed superfluidity of neutrons in a neutron star, the protons are speculated to form a (type II) superconductor based on the microscopic calculations of the associated energy gap in the Bardeen-Cooper-Schrieffer theory (Ruderman 1972). The corresponding transition temperature for the proton superconductivity, though smaller than that of the neutron superfluidity in the crust, is similar to that of the neutron superfluidity in the core (see, e.g., Sauls 1989) and much larger than the expected interior temperatures for the observed pulsars (Pines & Alpar 1992). Doubts about the superconductivity of the protons have been raised, however, because the earlier calculated core coupling timescale (Alpar & Sauls 1988), for the case of superconductor protons, could not successfully accommodate the observed postglitch recovery of the Vela pulsar (Pines & Alpar 1992). This difficulty does not exist anymore since a necessary correction in the calculation of the coupling timescale has resulted in more than an order of magnitude reduction in its value and made it much smaller than the inferred observational upper limit (Jahan-Miri 1998). Moreover, the velocity relaxation timescale of the vortices for the case of normal protons in the core would be much larger than, or at the best (for the very cool neutron stars) similar to, that of superconducting protons (see, e.g., Sauls 1989). Hence normal protons may not offer any solution to the problem. On the other hand, an additional coupling mechanism due to an interpinning of the fluxoids and the vortices could result in a reduction in the velocity relaxation time of the vortices as well as in the coupling time for the superfluid bulk matter (estimates of the predicted timescales may be found in Jahan-Miri 1998). Thus superconductivity of the protons is rather favored by the above observational restriction on the core-coupling timescale.

Given a superfluid-superconductor mixture in the interior of a neutron star with the associated (neutron) vortices and (proton) fluxoids, we further assume that both families of the vortex lines form regular uniform lattices parallel to the spin and magnetic axes, respectively. A stabilizing toroidal field component might be also present in the core (Flowers & Ruderman 1977). The toroidal component, if it is assumed to be much larger ($\gg 10^{13}$ G) than the observable dipolar field, would quench the superconductivity of the protons in the core altogether. Otherwise, and if it is frozen, along with the dipolar field, within the superconducting core, it would result in a complicated distorted geometry of the fluxoids. Such distorted geometries of lines, which is neglected here for simplicity, would affect our following treatment of the flux expulsion in two ways. The effective total length of the fluxoids would be more than in the case of a regular parallel lattice assumed here, and the entanglement of the fluxoids with the neutron vortices would be realized more effectively even in the cases in which the rotation and magnetic axes of the star are nearly parallel. The case of nearly parallel axes are indeed preferred in our models (see the next paragraph); thus, the introduction of a distorted fluxoid geometry would create arguments for and against the following analysis. The rate of the Ohmic decay of the expelled flux in the crust is subject to uncertainties about the correct value of the conductivity of the crust matter as well as the geometry and transport behavior of the expelled flux due to other possible processes, namely, the Hall drift and the turbulent cascade mechanism (see, e.g., Jones 1988; Goldreich & Reisenegger 1992; Bhattacharya & Datta 1996). Different values for the effective decay timescale of the field in the crust are therefore tested in our model computations in order to bypass these complications.

Throughout, we will be considering the motion of the fluxoids only in the region close to the core-crust boundary, which also lies in the magnetic equatorial plane. The strength of the average field of the stellar core is decreased by the transport of these boundary fluxoids out of the core; the fluxoids in the interior regions are assumed to adjust their positions accordingly to maintain a uniform density. We will be referring to a cylindrical coordinate system aligned with the fluxoids in considering the radial and azimuthal directions, which coincide with their counterparts in a spherical system for the above region of interest. The radial velocities of the neutron vortices, in the same region, would, however, be colinear with that of the fluxoids only for the vortex segments located near the spin equator of the star as well. The fractional size of such a region (of coincidence of the magnetic and spin equators) would be larger, and hence our treatment of the radial velocities and forces for the fluxoids and the vortices would be more accurate, for the smaller inclination angles.

It has been suggested (Sauls 1989; Jones 1991) that, since the pinning interaction energy is independent of any displacement of a pinned neutron vortex parallel to itself, the vortices might be able to slide along the fluxoids without producing large-scale movement of the fluxoids. It is noted, however, that such a sliding might be realized only for the vortices in some parts of the spin equator, namely, for those lying at large magnetic latitudes. Moreover, this possibility does not by itself violate our following assumption of a radial reactive force acting on the fluxoids even in the regions where sliding might occur. The Magnus force on a sliding vortex would have the same magnitude and direction as for the nonsliding vortices, and its component in the direction perpendicular to the fluxoid has to be balanced in any case with a force due to the pinning, which will exert a reaction force on the fluxoid. Furthermore, since the assumed sliding cannot be realized for all of the vortices, for any assumed geometry of the lines, its partial occurrence, if at all, seems to be further questionable because it would result in an azimuthally nonuniform distribution of the vortex density.

2.2. Neutron Superfluid Vortices

In the steady state, the vortices in the core of a neutron star are expected to be corotating with the charged component of the star, including the lattice of the proton fluxoids, at a given rate $\Omega$ (Sauls 1989). However, the superfluid bulk matter has to rotate faster at a rate $\Omega_s$, maintaining a rotational lag $\omega \equiv \Omega_s - \Omega$ with its vortices, in order to follow the spinning down of the star at a rate $\dot\Omega$. The corresponding radial velocity $v_r$ of the vortex outward motion, at
The pinning force on the fluxoids. — On the other hand, if a pinning interaction is associated with an intersecting fluxoid-vortex pair, the corresponding pinning force on the vortex could, in principle, be in either (inward or outward) radial direction. The force direction is decided by the direction of the relative motion of the interacting vortex and fluxoid and is independent of the actual direction of motion of the vortices. That is, the pinning force could contribute to the viscous forces against the outward motion of the vortices if they move faster than the fluxoids. However, in the opposite case it would act as a "driving" force for the spinning down of the superfluid, being in the same direction as the outward motion of the vortices. Moreover, the viscous force due to the electron scattering is expected to be many orders of magnitudes smaller than the pinning force, even for the largest spin-down rates of interest and hence for the largest possible values of $v_n$. Typically, the pinning force, per unit length of a vortex, is expected to be $\gg 10^{12}$ dyn cm$^{-1}$, while the viscous force of the electron scattering (Alpar & Sauls 1988) is less than $10^7$ dyn cm$^{-1}$ (and the buoyancy force on the magnetized neutron vortices is even smaller than this). Therefore, for the vortices the pinning forces exerted by the fluxoids have to be balanced by the Magnus force on them.

An interesting consequence of such a balance of forces for the vortices is that the superfluid might be rotating more slowly than its vortices ($\omega < 0$) while it is spinning down. This is in contrast to the usual conditions during the spin-down phase of a superfluid, in general and particularly when pinning is present and larger values of the lag are the case. As indicated, a spinning down superfluid must be rotating faster than its vortices ($\omega > 0$), so that the viscous force (as well as the pinning force if it is present) on the outward moving vortices is balanced by an outward driving Magnus force. However, since for the above assumed case the pinning force is itself directed outward, it could as well play the role of the driving force, which in turn requires the balancing Magnus force to be directed inward. Thus, a negative rotational lag ($\omega = \Omega, -\Omega < 0$), and hence an inward Magnus force, during the spin-down phase of the superfluid in the core of a neutron star is realized when the fluxoids move faster than the vortices.

2.3. The Pinning Force on the Fluxoids

In any case, a pinning force of the same magnitude as, and in the opposite direction to, that exerted by the fluxoids will be also exerted by the vortices on the fluxoids. Considering the above discussed balance of the forces on vortices, and equating the forces communicated between the two lattices of the vortices and the fluxoids, per unit volume, the pinning force $F_n$ acting on a fluxoid, per unit length, is derived as

$$F_n = \frac{n_v n_f}{n_f} F_M \approx 2 \phi_0 R \frac{\Omega(t)\cot(t)}{B_i(t)} = 5.03 \frac{\omega_{-6}}{P_v B_i} \text{ dyn cm}^{-1},$$

where $n_v = 2\Omega_0 \kappa$ and $n_f = B_i / \phi_0$ are the number densities per unit cross section area of the vortices and the fluxoids, respectively; $\phi_0 = 2 \times 10^{-7}$ G cm$^2$ is the magnetic flux carried by a fluxoid; $B_i = 10^6 B_8$ is the strength of the core field in gauss; and $\omega_{-6}$ is the superfluid lag $\omega$ in units of $10^{-6}$ rad $s^{-1}$. Notice that the sign of $\omega$ determines the sign of $F_n$ for which, as well as for the other forces discussed below, the outward direction will be reckoned as the positive sense.

The above derivation of $F_n$ assumes that the total Magnus force acting continuously on all the vortices is communicated instantaneously to all the fluxoids. In contrast, it might be argued that in general only a small fraction of the vortices would be directly interacting with the fluxoids at any instant of time. The remaining much greater fraction of them (of the order of the ratio of an interfluxoid spacing to the size of a pinning interaction region) should reside in the interfluxoid spacings. Also, the assumed total force on the fluxoids has been divided equally among all in spite of the similar expectation that at any given time a majority of them would be located in the intervortex regions, far from any pinning site. Nevertheless, the motion of the fluxoids (as well as the vortices) is further constrained because of their mutual repulsive forces, which would require a uniform density of the lines to be maintained, in a steady state. Consequently, the fluxoids (whether in an interaction region or in a free region) are forced to move always together (on scales of, at least, an intervortex spacing), which requires that the force acting on some of them to be shared equally among all, instantaneously, as is assumed in equation (2). The same argument fails, however, for the vortices since their displacements on scales of the order of an interfluxoid spacing (which is many orders of magnitudes smaller than the intervortex spacing) is not prohibited by the above requirement of having a uniform density. On the other hand, and for the same reason, any assumed vortex that is not interacting with a fluxoid at a given time is expected to move relative to the rest of the neighboring vortices and rapidly adjust its position (within a distance of an interfluxoid spacing) until it too is located within a pinning interaction zone. That is, all the vortices would be continuously interacting with the fluxoids, which completes the justification of the above derivation of $F_n$. Hence, equation (2) is valid during a vortex-fluxoid comoving phase, which was (implicitly) assumed in the above arguments.

However, should the (radial) velocities of vortices and fluxoids be different, the above restriction on the vortex positions might not be fulfilled steadily since, in this case, they would have to continually move through the interfluxoid spacings as well. The effective instantaneous force per unit length of a fluxoid, $F_n$, during such a state would be smaller than that given in equation (2) and could be esti-
mated as the time-averaged force acting on the fluxoids (or equivalently by using the fractional number of the vortices interacting with the fluxoids at any time). Therefore, during a phase of unequal velocities of the vortices and fluxoids, one derives (in contrast to eq. [2] for a comoving phase)

\[
F_n = \frac{d_p}{d_f} \left( \frac{n_c}{n_f} F_M \right) = 2.59 \times 10^{-4} \frac{\omega - \delta}{P_j B_n^{1/2}} \text{dyn cm}^{-1},
\]

(3)

where \(d_f = 2.3 \times 10^{-7} B_n^{1/2} \text{cm} \) is the interfluxoid spacing and \(d_p \) is the effective size of a pinning interaction region around each fluxoid. A value of \(d_p = \lambda_p \) is 118 fm has been used for the assumed magnetic pinning mechanism (see below), where \(\lambda_p \) is the effective London length of the proton superconductor, being also a length scale for the spread of the magnetic field of a neutron vortex line (Sauls 1989).

The “averaged” value of \(F_n \) as given in equation (3) assumes that the velocity of a vortex while it is crossing through an interaction region is the same as in the interaction-free space between the fluxoids; equal weights have been accordingly assigned to the corresponding time periods. Realistically, however, the vortices might be expected to move much faster in the free regions than in the pinning regions because of the large difference in the effective resisting forces acting on them in the two environments (as discussed above). As a consequence, they might tend to spend most of their time within the pinning zones, which would require an almost zero weight to be assigned to the crossing times of the interfluxoid spacings. That is, their dynamics would resemble the case of the comoving phase, and \(F_n \) as in equation (2) would be applicable even when there is a relative motion between the fluxoids and the vortices.

Thus, while for a comoving phase \(F_n \) is uniquely determined by equation (2), when velocities are different one is faced with two different possibilities as given by equations (2) and (3). The two derivations represent, in fact, two extreme possibilities regarding the relative behavior of the different pinned segments of a vortex during its motion. If each pinned segment of a vortex could creep independently (over length scales of about an interfluxoid spacing), then the above argument to justify equation (2) applies to the phase of nonequivalent velocities as well. In contrast, for a vortex line of infinite rigidity the whole line moves always as a single piece and \(F_n \) as in equation (3) would be appropriate. We will test both possibilities in alternative models in order to distinguish the relevant approximation for the behavior of the vortices in the core of neutron stars.

The “critical lag.”—The magnitude of the force that could be exerted at each intersection by a vortex on a fluxoid, and vice versa, is limited by a maximum value \(f_p \) corresponding to the given strength of the pinning energy \(E_p \) and the finite length scale of the interaction \(d_p \); namely, \(E_p = f_p d_p \). The Magnus force on the vortices, which has to be balanced by the pinning force (and hence \(F_n \)), cannot therefore exceed a corresponding limit. This, in turn, implies a maximum critical lag \(\omega_{ct} \), which is determined by equating the Magnus force \(= \rho \chi \kappa R \omega \) with the maximum available pinning force \(|=f_p d_f = E_p/d_p| \), per unit length of a vortex. The pinning energy, in the magnetic interaction mechanism, arises because of the difference in the free energies of a fluxoid-vortex pair when they overlap at an intersection or are separated. For a fluxoid with an average field \(B_p \) and a vortex having an average field \(B_n \), the magnetic energy density in the overlapping case would include a term \((2/\pi) B_p B_n \), in addition to the sum of their individual contributions \((B_p^2 + B_n^2)/\pi \) when they are separated. The pinning energy, per intersection, is therefore estimated by multiplying the interaction overlap volume \(~(2\lambda_p^2 \pi/\delta^4)~\) with the above additional term of the energy density, which results in \(E_p \sim 10^{-3} \text{ergs} \) (see Jones 1991 for a more refined derivation). The critical lag \(\omega_{ct} \) may be then calculated, as indicated above, as

\[
\omega_{ct} = 1.59 \times 10^{-4} B_n^{1/2} \text{rad s}^{-1},
\]

(4)

where we have used the same parameter values as in DC93 in order to allow further comparison of the results. The critical lag is the magnitude of the lag when velocities are different: \(\omega = \omega_{ct} \) or \(\omega = -\omega_{ct} \), when the vortices move faster or slower than the fluxoids, respectively. However, during a comoving phase when the force communicated between a vortex and a fluxoid at each pinning point is less than its maximum value \(f_p \), the lag might have any value within the range \(-\omega_{ct} < \omega < \omega_{ct} \).

We note that a different estimate for the pinning energy due to the proton density perturbation gives a smaller value of \(E_p \sim 5 \times 10^{-7} \text{ergs} \). The pinning energy in this case arises because of a difference in the condensation energy between the pinned and the free configurations as a result of the change in the proton density induced by the large velocity of the neutrons close to the core of a neutron vortex (Sauls 1989). The interaction volume for this mechanism is \(~\xi_p^2 \xi_p \), and the change in the free energy density is estimated to be \(n(\Delta^2_\xi^2 E_p^2) \), where \(\Delta \) is the condensation energy gap, \(E_p \) is the Fermi energy, \(\xi \) is the coherence length, \(n \) is the number density, and the subscripts “p” and “n” refer to the protons and neutrons, respectively. The larger values of \(E_p \) in the case of the density perturbation is, however, associated with an interaction length, \(d_p = \xi_p \) which is smaller than that for the magnetic interaction, \(\lambda_p \) by about the same ratio as the inverse of the pinning energies. Thus, similar values of \(f_p \), and hence \(\omega_{ct} \), are expected for the both pinning mechanisms, as has been indicated earlier (Bhattacharya & Srinivasan 1991). Nevertheless, since the magnetic interaction depends on the angle between fluxoids and vortices, the pinning caused by the density perturbation might indeed have a dominant effect in a neutron star with a nearly parallel geometry of the vortices and fluxoids.

2.4. Other Forces on the Fluxoids

In addition to the pinning force, the fluxoids in the interior of a neutron star are also subject to the following radial forces.

The drag force.—An isolated fluxoid moving through the normal degenerate electron gas in the core of a neutron star is subject to the viscous drag force of the electrons scattering off its magnetic field. The viscous drag force, per unit length of a fluxoid, is estimated (Jones 1987) to be

\[
F_e = -\frac{3\pi n_e \rho_e^2 \phi_0^4 v_p}{64 E_p \eta_p} = -7.3 \times 10^7 v_p \text{dyn cm}^{-1},
\]

(5)

where \(v_p \) is the velocity of the outward radial motion of the fluxoids in units of centimeters per second, \(n_e = 3. \times 10^{36} \).
cm$^{-3}$ is the number density of the electrons, and $E_F = 88$ MeV is the electron Fermi energy, corresponding to a total density $\rho = 2 \times 10^{14}$ g cm$^{-3}$ and a neutron number density $n_n = 1.7 \times 10^{14}$ cm$^{-3}$ in the core.

The expression for $F_e$ in equation (5) is derived based on the assumption of independent motions for single fluxoids. However, for the typical conditions in the interior of a neutron star the lattice of fluxoids might be, as a whole (or at least as bundles consisting of not less than $10^3$ fluxoids), “frozen-in” the electron gas (Harvey et al. 1986). This is because the mean free path of the electrons turns out to be many orders of magnitudes larger than the mean distance between their successive scattering events, and also because the deflection angle at each event is very small. A treatment of the coherent electron scattering by the fluxoid lattice has been shown to indeed require an almost zero relative velocity between the electrons and the lattice (Jones 1987). Therefore, flux expulsion out of the core might be prohibited except for the presence of electron current loops across the core-crust boundary. Uncertainties about the true distribution of the magnetic flux and the correct value of the conductivity in the crust and the possibility of a mechanical failure of the solid crust due to a buildup of the magnetic stresses, however, obscure any definite conclusion to be drawn.

Moreover, there are other reasons to suspect the suggested frozen-in approximation for the fluxoids as a whole (see also DC93; Ruderman 1995) in the core of a neutron star. For example, the finite volume of the fluxoid lattice and the influence of the superconductor boundary effects on the motion of the fluxoids, which have not been included in the previous studies of the coherent scattering, could also have significant new consequences. In addition, the motion of the incompressible electron fluid in the interior of a neutron star has been argued to be divergence free (Goldreich & Reisenegger 1992). The above permitted motion of the fluxoids, along with the electrons in the frozen-in approximation, must be, therefore, of the same (divergence-free) nature. This is, however, impossible for the frozen-in approximation due to the coherent electron scattering orbits off fluxoids.

The buoyancy force.—The buoyancy force on fluxoids in a neutron star arises for reasons analogous to the case of macroscopic flux tubes in ordinary stars. Because flux tubes are in pressure equilibrium with their surroundings, the excess magnetic pressure causes a deficit in the thermal pressure, and hence in the density, of the plasma inside a flux tube, which makes the tube become buoyant. The radially outward buoyancy force $F_b$ on a fluxoid, per unit length, can be expressed as (Muslimov & Tsygan 1985; Jones 1987)

$$F_b = \left(\frac{\phi_0}{4\pi \lambda_p}\right)^2 \ln \left(\frac{\lambda_p}{\xi_p}\right) \frac{R_c}{R_p} = 0.51 \text{ dyn cm}^{-1},$$

(6)

where values of $\lambda_p = 131.5$ fm and $\lambda_p/\xi_p = \sqrt{2}$ have been used.

Harrison (1991) has raised objection against the relevance of the buoyancy force for the motion of single fluxoids in the core of neutron stars, assuming the whole lattice of the fluxoids to be frozen in the electron-proton plasma within the star. He argues that the buoyancy force would rather contribute to the gradient of the macroscopic magnetic stresses supporting the hydrostatic equilibrium of the plasma within the star, instead of acting on the fluxoids individually. His argument is not, however, applicable if a relative motion between the fluxoids and the plasma is allowed to take place, as he also makes clear (see, in particular, the last paragraph before § 3.2, p. 422, in Harrison 1991). Thus, for our model calculations, having assumed a drag force corresponding to the single fluxoid motion, we also take into account the buoyancy force on the fluxoids, for self-consistency. In this context, Ding et al. (1993) have argued that Harrison’s above objection against the buoyancy force might be irrelevant because the fluxoids’ motion could be fast enough that the conditions of hydrostatic equilibrium of the star are not satisfied during their motion. Such an argument is apparently missing the point, given that independent motions of single fluxoids has been also assumed in DC93. The decision of whether the buoyancy force acts on the fluxoid lattice as a whole, hence contributing to the hydrostatic pressure support, or else acts on each flux line individually is independent of, and would be equally effective in the, presence or absence of the equilibrium. Even if hydrostatic equilibrium within a neutron star is assumed, as is speculated in DC93, to be approached on timescales less than 1 Myr, corresponding to the largest fluxoid velocities less than $10^{-7}$ cm s$^{-1}$, the buoyancy force could be still acting on the whole lattice, if relative motion between the fluxoids and the plasma is prohibited because of the requirements of the electron scattering orbits off fluxoids.

The curvature force.—The tension of a vortex line (such as a fluxoid) implies that a curved geometry of the line would result in a restoring force, the curvature force, which tries to bring the line back to its minimum energy straight configuration. The concave directed curvature force $F_c$, per unit length, on a vortex having a tension $T$ and a curvature radius $S$ is given as $F_c = T/S$ (Harvey et al. 1986). Thus, for a fluxoid in a neutron star, having a tension $T_c = [\phi_0/(4\pi \lambda_p)]^2 \ln \left(\lambda_p/\xi_p\right)$, the magnitude of the curvature force would be

$$|F_c| = \frac{R_c}{S} \left(\frac{\phi_0}{4\pi \lambda_p}\right)^2 \ln \left(\frac{\lambda_p}{\xi_p}\right) \frac{R_c}{R_p} = \frac{R_c}{S} F_b,$$

(7)

where we have used equation (6) to express the absolute value of $F_c$ in terms of the (positive) buoyancy force. Moreover, the end points of a fluxoid, where its magnetic flux
It has been argued that the repulsive force between the fluxoids should ensure that the lattice response to a deformation is determined, to a first approximation, by their collective rigidity (Jones 1991). The force $F_e$ associated with even a piece of the lattice of a size of an intervortex spacing (including some $10^7$ flux lines) would be, in this approximation, so large that any bending of the lattice is effectively prohibited. The velocity of the fluxoids would be therefore constrained at all times by the condition $v_p \leq v_{\text{max}}$. In order to implement this latter assumption, we therefore construct models that use the following prescription for calculating $F_e$. If a value of $F_e = 0$ results in $v_p > v_{\text{max}}$ (see below), then $v_p = v_{\text{max}}$ is assumed (otherwise, we set $F_e = 0$, and the derived value for $v_p$ is used, as before), and $F_e$ is calculated from

$$F_e = -(F_n + F_b + F_s),$$

(10)

where the right-hand side is evaluated using $v_p = v_{\text{max}}$, together with $\omega = \omega_{cr}$ or $-\omega_{cr}$, whichever is the case. As indicated earlier, these latter models might be alternatively viewed as representatives of the case of coherent electron scattering.

3. THE MODELS

The steady state radial motion of a fluxoid, in the region of interest, is thus determined from the balance equation for all the radial forces acting on it per unit length, that is,

$$F_n + F_e + F_b + F_c = 0.$$  

(11)

Substituting in the above equation for the different forces from equations (2) or (3), (5), (6), and (8) or (10), respectively, it may be rewritten in the form

$$\alpha \frac{\omega - \delta}{P_s B_B} = \beta v_p, \quad \delta = 0,$$

(12)

where parameters $\alpha$, $\beta$, and $\delta (= F_n + F_s)$ are given below for the different models, and $v_p$ is the fluxoid velocity $v_p$ in units of $10^{-7} \text{ cm s}^{-1}$. Recall that $\omega_{cr}$, which is the value of $\omega$ in units of $10^{-6} \text{ rad s}^{-1}$, might have either positive or negative values, as is also the case with $\delta$ in some of the models.

This single equation includes two unknown variables $\omega$ and $v_p$ and represents the azimuthal component of the Magnus equation of motion (Sonin 1987) for the proton vortices. No radial Magnus force acts on the fluxoids; hence, the right-hand side is set to zero because of the assumed corotation of the fluxoids with the proton superconductor. There exist, however, additional restrictions on the motion of the fluxoids that can be used to fix the value of one of the variables and solve equation (12) for the other.

Namely, for a comoving state $v_p = v_n$ is given and $\omega$ can be determined. In contrast, when $v_p \neq v_n$ is unknown $\omega$ is given as $\omega = \omega_{cr}$ or $\omega = -\omega_{cr}$ for $v_p < v_n$ or $v_p > v_n$, respectively. Furthermore, inspection of equation (12) indicates that it admits one and only one of the three different solutions for the given values of $v_n, B_c, P_s$ at any time, namely,

$$\omega = \omega(v_p = v_n), \quad \text{iff} \quad -\omega_{cr} < \omega < \omega_{cr},$$

or

$$v_p = v_p(\omega = -\omega_{cr}), \quad \text{iff} \quad v_p < v_n,$$

and

$$v_p = v_p(\omega = -\omega_{cr}), \quad \text{iff} \quad v_p > v_n.$$
The rate of the flux expulsion out of the core, $\dot{B}_c = -2B_c v_p/R_s$, and the evolution of the stellar surface field $B_s$ [with a decay rate $\dot{B}_s = -(B_s - B_c)/\tau$] are hence uniquely determined from the above force-balance equation, given the spin evolution of the star, which determines (eq. [1]) the vortex velocity $v_p$ at any time.

We construct four separate models, labeled A1, A2, B1, and B2, based on the two alternative estimates discussed earlier for the pinning force $F_n$ and also for the curvature force $F_c$, by permutation. The models are summarized below, indicating to which of the four distinguishing physical assumptions they relate and giving their associated values of the parameters in the force equation (eq. [12]).

Model A1. Vortex segments creep independently; fluxoids may bend when $v_p > v_{\max}$:

$$\alpha = 5.03, \quad \delta = \begin{cases} 0.51, & \text{if } v_p < v_{\max} \\ 0.16, & \text{if } v_p \geq v_{\max} \end{cases}$$

Model A2. Vortices remain straight while moving; fluxoids may bend when $v_p > v_{\max}$:

$$\alpha = 2.587 \times 10^{-4} B_{s0}^{1/2}, \quad \delta = \text{same as for A1}.$$  

Model B1. Vortex segments creep independently; fluxoids remain straight, constrained by $v_p \leq v_{\max}$:

$$\alpha = \text{same as for A1}, \quad \delta = \begin{cases} 0.51, & \text{if } v_p < v_{\max} \\ -(F_n + F_c), & \text{if } v_p = v_{\max} \end{cases}$$

Model B2. Vortices remain straight while moving; fluxoids remain straight, constrained by $v_p \leq v_{\max}$:

$$\alpha = \text{same as for A2}, \quad \delta = \text{same as for B1}.$$  

$\beta = 7.30$ is the same for all the models.

These four models together with the model adopted by DC93 (the DCC model) will be referred to collectively as the force-balance equation (FBE) models, in contrast to the SIF model, which has a different approach for calculating the rate of flux expulsion. Table 1 compares all the six models by indicating how the forces and the fluidoid velocity are determined in each of them. The spin and magnetic evolution of the single as well as of binary pulsars are calculated according to the requirements of each of the six models, separately, and the results are discussed in the following sections.

### 4. SINGLE PULSARS

The spin evolution of a solitary pulsar driven by its electromagnetic torque has a rate $\dot{P} = 3.15 \times 10^{-3} B_s^2 / P_s$ s yr$^{-1}$, where $B_s$ is the surface field in gauss and $P_s$ is in seconds. From the instantaneous value of the spin-down rate one finds the velocity $v_n$ of the outward motion of the neutron vortices (eq. [1]). Also, the critical lag $\omega_c$ may be determined, from equation (4), for any given value of the core field strength $B_c$. The solution of equation (12) for each model, given the values of $v_n$ and $\omega_c$ at a time $t$, then determines the corresponding values of the fluidoid's outward radial velocity $v_p$ and the lag $\omega$ between the rotation rates of the vortices and the neutron superfluid in the core of the evolving solitary pulsar. The coupled evolution of the spin period and the magnetic field, in the core and at the surface, is thus followed over a period of $10^{10}$ yr in order to cover both the young and the very old neutron stars in the Galaxy.

The computed time evolution of $v_p$ and $\omega$ is shown in Figure 1, together with $v_n$ and $\omega_c$, as predicted in the A1 model. Characteristically similar results as in Figure 1 are obtained for the other FBE models as well. The evolution of the lag $\omega$, although it is not directly of interest for our present analysis of field evolution, nevertheless bears signifi-

### TABLE 1

**DIFFERENT MODELS OF FLUX EXPULSION**

| Model  | $F_n$  | $F_b$ and $F_v$  | $F_c$  | $v_p$ and $\omega$ |
|--------|--------|------------------|--------|-------------------|
| FBE:   | Eq. (2) | Eqs. (5) and (6) | Eq. (8), if $t < \tau$; $\frac{1}{2}F_p$, if $t \geq \tau$ | Eq. (12), subject to $v_p = v_n$ if $|\omega| < \omega_c$; $\omega = -\omega_c$ if $v_p < v_n$; $\omega = \omega_c$ if $v_p > v_n$ |
| DCC... | Eq. (2) | Eq. (5) and (6) | Eq. (8), if $v_p < v_{\max}$; Eq. (10), if $v_p = v_{\max}$ (eq. not permitted) | Eq. (12), subject to $v_p = v_n$ if $|\omega| < \omega_c$; $\omega = -\omega_c$ if $v_p < v_n$; $\omega = \omega_c$ if $v_p > v_n$ |
| A1..... | Eq. (2) | Eq. (5) and (6) | Eq. (8), if $v_p \geq v_{\max}$ | Eq. (12), subject to $v_p = v_n$ if $|\omega| < \omega_c$; $\omega = -\omega_c$ if $v_p < v_n$; $\omega = \omega_c$ if $v_p > v_n$ |
| B1..... | Eq. (2) | Eq. (5) and (6) | Eq. (8), if $v_p = v_{\max}$ (eq. not permitted) | Eq. (12), subject to $v_p = v_n$ if $|\omega| < \omega_c$; $\omega = -\omega_c$ if $v_p < v_n$; $\omega = \omega_c$ if $v_p > v_n$ |
| B2..... | Eq. (2) if $v_p = v_n$; Eq. (3) if $v_p \neq v_n$ | Eq. (5) and (6) | Eq. (8), if $v_p < v_{\max}$ (eq. not permitted) | Eq. (12), subject to $v_p = v_n$ if $|\omega| < \omega_c$; $\omega = -\omega_c$ if $v_p < v_n$; $\omega = \omega_c$ if $v_p > v_n$ |
| A2..... | Eq. (2) if $v_p = v_n$; Eq. (3) if $v_p \neq v_n$ | Eq. (5) and (6) | Eq. (8), if $v_p \geq v_{\max}$ | Eq. (12), subject to $v_p = v_n$ if $|\omega| < \omega_c$; $\omega = -\omega_c$ if $v_p < v_n$; $\omega = \omega_c$ if $v_p > v_n$ |
| SIF.... | $v_p = v_n$; $v_p = v_n$; $v_p = v_n$ | $v_p = v_n$; $v_p = v_n$; $v_p = v_n$ | $v_p = v_n$; $v_p = v_n$; $v_p = v_n$ | $v_p = v_n$; $v_p = v_n$; $v_p = v_n$ (assumed) |
significant consequences for an understanding of the rotational dynamics of neutron stars, to be discussed elsewhere.

The fluxoids motion in Figure 1 is seen to follow three evolutionary phases in which they move more slowly, together, and more quickly than the vortices, successively. These will be referred to as the forward creep, comoving, and reverse creep phases, respectively (we use the terminology of DC93). Transitions between these successive evolutionary phases occur because of the reduction in $v_p(\propto \Omega_s)$ as well as the increase in $P_s$, and a final comoving phase might also occur for some choices of initial conditions. Note that $\omega$ changes sign from positive to negative and remains so in the later parts of the comoving phase, as well as during the reverse creep phase. Also note that $|\omega| = \omega_{cr}$ during both the forward and reverse creeping phases.

4.1. Field Evolution

The predicted evolution of the core and surface fields for a single neutron star according to the A1 model is given in Figure 2; the other FBE models produce similar results. The two panels in Figure 2 are for two different assumed values of the decay timescale $\tau$ in the crust, where the upper panel of Figure 2 corresponds to the results in Figure 1. A substantial decrease in the core field occurs at a time $t \gtrsim 10^7$ yr, which is expected for the typical average values of $v_p \lesssim 10^{-8}$ cm s$^{-1}$ during the earlier times, because $B_c/B_s = v_p/R_s$ implies that a time period $\Delta t \sim R_s/v_p$ is needed for a major reduction in the core field to occur. However, because of the very small magnitude of $v_p$ (although $\gtrsim v_n$) and the reduced value of $B_c$ at later times, $B_c$ does not change substantially afterward. The surface field $B_s$ responds to the change in $B_c$ on the assumed decay timescale $\tau$ of the crust. The nontrivial role of the stellar crust in these field evolution models may be seen by comparing the upper panel of Figure 2 with the lower panel, where values of $\tau = 10^7$ and $10^8$ yr, respectively, have been used. A larger value of $\tau$ tends to maintain the initial $B_c$, and hence a larger $\dot{P}_s$ as well as a larger $v_n$, over a more extended period of time. Consequently, smaller final values of $B_c$ and $B_s$ are predicted for the larger assumed values of $\tau$, as is seen in Figure 2.

The upper panel of Figure 2 might seem to suggest that $B_c$ stops decaying once $B_s$ starts to decline, as DC93 also conclude. However, this is but an artifact of the assumed value of $\tau$, which happens to be close to the saturation time of the core field decay. The spurious nature of such a corre-
lation, that a decrease in the strength of the crustal field results in a halt in the decay of the core field, may be confirmed by using tentatively smaller values of $\tau \lesssim 10^6$ yr. As we have verified, in such cases the core and surface fields are seen to decay simultaneously (see Fig. 3.3 in Jahan-Miri 1996b), in contrast to the above unreal correlation. The decay of the core field, according to the present models, does not have a direct and one-to-one dependence on that of the crust; instead, they are coupled through the influence of many factors (see the preceding paragraph). This statement may be also appreciated by contrasting the predicted field evolution under distinct initial conditions for the relative fields of the core and that of the crust, $B_{\text{crust}}$. For this purpose, the results in Figure 2 with an assumed initial condition $B_{\text{crust}} \sim 0.1B_c$ may be compared with that in DC93, where $B_{\text{crust}} \sim 1.5B_c$ and $B_{\text{crust}} \sim 5 \times 10^3 B_c$ have been used (their Figs. 2 and 3). The initial crustal field in our case compares with a very late stage of evolution in DC93 at which the crustal field has been almost completely decayed. Hence, the initial core field in Figure 2 would have not decayed, from the very beginning, if the above effect were real. The above difference in the adopted initial fields might deserve a further note. For an initially uniform distribution of the magnetic flux within the star, which is assumed in DC93 as well as here, the initial condition $B_{\text{crust}} = 1.5B_c$ implies a relative radial size for the crust larger than the currently used values (e.g., Sauls 1989; Pines & Alpar 1992). It is particularly questionable that the same condition has been used in DC93 for their various choices of equation of state (EOS) that have different predictions for the relative size of the crust.

4.2. Force Analysis

The time evolution of the radial forces acting on the fluxoids is shown in Figure 3, for the A1 model (corresponding to the results in Fig. 1 and the upper panel of Fig. 2). One should note that a fluxoid, being a vortex, responds to a force by moving in a direction perpendicular to the force; a radial force does not affect its radial motion directly. However, because of the dependence on $v_n$, equation (12) (which originally describes the azimuthal motion of the fluxoids) a “driving” or a “braking” role in flux expulsion might be assigned to an outward-directed (positive) or an inward-directed (negative) force, respectively.

As is seen in Figure 3, the pinning force $F_n$ is negative (directed inward) during the reverse creep phase and also the later part of the comoving phase. Nevertheless, the major predicted flux expulsion does occur (see Fig. 1 and the upper panel of Fig. 2) during the comoving and, particularly, the reverse creep phases. This means that the dominant driving force for the flux expulsion is the buoyancy force, which is positive throughout the evolution. That the overall role of the pinning force in the field decay of solitary pulsars is more like a brake prevents the flux from being expelled too rapidly. We have further verified this conclusion, which seems to be obvious from the results in Figures 1, 2, and 3, by other tests of the model calculations in which we have tentatively set either of the two forces equal to zero. Our conclusion about the braking role of the pinning force is, however, in contradiction to that of DC93, who attribute a driving role to this force throughout their paper and make further statements that are therefore not justified. The braking role of the pinning force is also in sharp contrast with the view adopted by Srinivasan et al. (1990) about the flux expulsion mechanism, even though comparison with their (SIF) model, in this aspect, might be misleading and unjustified since their treatment does not address the dynamics.

The opposing role of the pinning force against the fluxoids outward migration at late times ($\gtrsim 10^7$ yr) is indeed true for all the FBE models adopted here. Except in that they include the pinning force, these models are, otherwise, similar to the earlier dynamical studies of flux expulsion (Muslimov & Tsygan 1985; Harvey et al. 1986) in having the buoyancy force as the driving cause of the flux expulsion, as verified above. It is for the role of the pinning force that the predicted field of a neutron star, for all of the FBE models, may never decay to very small values ($< 10^9$ G) even after very long times ($\gtrsim 10^{10}$ yr). This is a fundamental difference that is much needed to account for the low-field pulsars (to be discussed in the next section), which could not be possibly explained by the earlier flux expulsion scenarios, which neglected the pinning.

We hoped to be able to comment about the relevance of the alternative physical assumptions discussed for the FBE models by comparing their predictions. However, the predicted field evolution behaves very similarly in the different models; hence, no further conclusions may be drawn. The reason for this rather unfortunate finding may be traced to the braking role of the pinning force, $F_n$, as well as the effective period of the core field expulsion, both discussed earlier. Models A1, B1, and DCC differ from the other two (see § 3) in their prescription for calculating the magnitude of $F_n$. Because of the dependence of $F_n$ on $B_c$ and $P_n$, its predicted value for the two classes of models is markedly different only during the early short phase of forward creeping (see, also, Fig. 3.4 in Jahan-Miri 1996b). However, the major field expulsion occurs during the later prolonged phases, i.e., when $F_n$ acts as a brake and its predicted value is not much different for the different models. Hence, the distinction among the models with respect to $F_n$ is, in effect, washed out. A second distinction among the models (models A vs. B) relates to their different prescriptions for calculating the curvature force, $F_c$, which differ only when the fluid velocity is large ($\gtrsim v_{\text{max}}$). Large fluid velocities

![Fig. 3. Predicted time evolution of the various radial forces acting on fluxoids (per unit length) in a solitary neutron star, according to the A1 model and corresponding to the results in Fig. 1 and the upper panel of Fig. 2. The three curves represent the pinning force $F_n$, the drag force $F_d$, and the sum of the buoyancy and curvature forces $F_b + F_c$.](image-url)
occur only when neutron vortices also move fast; hence, large differences between the value of $F_v$ among the models arise again only during the early phase of rapid spinning down of a single pulsar. Thus, in short, for the spin-down history of single pulsars the calculated forces on fluxoids at times greater than 1 Myr are not much different for the different models treated here, resulting in similar field evolutions. The distinction among the predicted field evolutions due to the different FBE models could, however, be quite significant, in principle, as we find in the case of spin histories of neutron stars in close binaries, discussed below.

4.3. Observational Implications

*Pulsar distribution.*—The evolutionary tracks for single pulsars on the spin–magnetic field diagram as predicted by SIF and DCC (the latter being typical for all the FBE models) are plotted in Figure 4, for the different assumed initial field strengths. Points corresponding to the given ages of the neutron star are also marked along each track. As seen in Figure 4, the predicted final strength of $B_s$, for FBE models, is found to depend sensitively, and inversely, on its initial value. This is a consequence of the direct correlation between the total expelled flux and the initial value of $B_s$, which is expected because larger values of $P_s$ are achieved for larger initial $B_s$ values, as indicated earlier. However the final value of $B_s$ is insensitive (not shown in Fig. 4) to the assumed initial values of $P_s$ and $B_c$, for changes in these quantities by almost 2 orders of magnitudes. In contrast, for the SIF model the final value of $B_s$ is found to depend on the initial values of $P_s$ and $B_c$, as well as on the initial $B_s$ (Fig. 4), even though they have a direct correlation in this case. These correlations, for the SIF model, are in accord with the assumed relation $B_c/B_e = -P_s/P_s$, corresponding to $v_p = v_n$ at all times. A further point to note in Figure 4 is the power-law time behavior of the field evolution of single pulsars at late times, which is realized for the lower initial fields. This feature, which is common among the SIF and the FBE models and was also pointed out by Srinivasan et al. (1990), has new observational implications, which have been previously highlighted (Jahan-Miri 1996a).

*Very old neutron stars.*—In spite of the recent discovery of large redshifts for some $\gamma$-ray burst sources, the association of a subclass of them with a Galactic population of highly magnetic old neutron stars (see, e.g., Blandford 1992) may still be viable. While such an identification of the $\gamma$-ray bursters does not seem to be consistent with the predictions of FBEs for the field strengths of very old single neutron stars, it could be however accommodated by the SIF model. According to SIF, very old neutron stars (with ages $\sim 10^{10}$) are expected to have rather large magnetic fields in the range $2 \times 10^{10} \lesssim B_s \lesssim 2 \times 10^{11}$ G, while the FBE models predict values of $B_s \lesssim 2 \times 10^{10}$ G for such stars (see Fig. 4). The FBE-predicted upper limit for the final fields is

![Figure 4](image-url)
even smaller than above, $B_s < 3 \times 10^9$ G at age $\gtrsim 10^8$ yr, for the large initial values of $B_s \gtrsim 10^{12.5}$ G. Furthermore, while the above results from Figure 4 are for a value of $\tau = 10^7$ yr, still smaller final fields would be the case for larger values of $\tau$ (compare the upper panel of Fig. 2 with the lower).

In contrast, DC93 suggested that, for a neutron star having a magnetic axis aligned with its rotation axis, the core field, according to the DCC model, would not be expelled even on large timescales ($\gtrsim 10^7$ yr), unlike in the general case presented in Figure 4. Thus, they concluded that the earlier proposed model of Ruderman & Cheng (1988) for the burst sources being aligned neutron stars is consistent with a field evolution according to their (DCC) model. However, their conclusion has to be dismissed since it is based on the unusual assumption that the spin-down torque of a pulsar is due only to its magnetic dipole radiation. The usual and long-lived consensus that is commonly adopted also for an observational determination of the strengths of the surface fields of pulsars is that the spin-down torque, because of combined effects of dipole radiation and outflow of relativistic particles, is independent of the inclination angle (see, e.g., Manchester & Taylor 1977, pp. 176–180; Srinivasan 1989).

Nevertheless, since a discussion of the predicted field decay by the FBE models for a tentatively assumed case of little spin-down torque acting on a neutron star serves to further elucidate the nature of the models, we will pursue the discussion. Indeed, if the spin-down rate of a neutron star is assumed to be small the core field according to the FBE models would not be expelled much, as suggested by DC93. This happens mainly because a reverse creep phase does not occur in this case since the associated value of $|F_n(\omega = -\omega_c)|$ would be too large. The large value of $F_n$ follows from its inverse proportionality on $P_s$ (see eq. [2]) and the fact that the assumed small spin-down torque results in small final values of $P_s$. However, Ding et al. argued (see the last equation in DC93) that the large value of $F_n$ in this case is a consequence of the direct proportionality of $\omega_c$ on $B_c$ and of the large value of $B_c$. We argue that the dependence of $\omega_c$ on $B_c$ is irrelevant since $|F_n(\omega = -\omega_c)| \propto 1/B_c^{1/2}$, which implies a smaller $|F_n|$ for the assumed larger $B_c$. In fact the reverse creep phase starts always at a large value of $B_c$ even in the nonaligned cases in which a substantial field decay does occur, as might be expected from the above dependence of $F_n$ on $B_c$. We conclude that the FBE-predicted little flux expulsion for the “aligned” case is only because $P_s$ could retain its assumed small initial value, as the following test verifies. The two cases of field evolution presented in Figure 5 are both for an assumed very small spin-down torque (i.e., that expected due only to the dipole radiation for an inclination angle of 1°) but for two different initial values of $P_s$. Even though in both cases $B_c$ is large before the transition to the reverse creep phase, substantial flux expulsion does takes place in the case where $P_s$ is large. Note that in this latter case the reverse creep phase starts from the very beginning and persists throughout the evolution of the star. This clearly verifies the above mentioned role of $P_s$ in determining the conditions for occurrence of a reverse creep phase. In addition, the above arguments indicate a potential possibility for having very old neutron stars with strong magnetic fields.

![Graphs](image-url)
fields, in the context of the FBE models, too. This may be achieved provided the star is not spun down to large periods $\gtrsim 1$ s, independent of whether or not the surface field is aligned with the rotation axis of the star, as also happens to some extent for the low field pulsars shown in Figure 4.

Active lifetimes of pulsars.—The radioactive lifetimes (defined by the condition $B_s/P_s^2 < 0.2 \times 10^{12}$) of pulsars, calculated for the predicted spin-field evolution in the different FBE models, are shown separately in Figure 6 against the initial field strengths for the two values $\tau = 10^7$ and $10^8$ yr. Corresponding curves for the exponential field decay model, labeled “Exp.” (which assumes that the total surface field decays exponentially, with no constraint, on the given timescale $\tau$) are also included, for comparison. The predicted lifetimes by the different FBE models are very similar, as can be seen in Figure 6 (this is true also for the A2 and B2 models, which have been omitted for the clarity), again making any attempt for a selection among the five FBE models fruitless. Nevertheless, there exist a marked difference (in Fig. 6) between the results of FBEs, as well as SIF, in contrast to those of the exponential field decay model, in particular for values of $\tau \lesssim 10^7$ yr. This difference should, in principle, have testable consequences for studies of pulsar statistics to decide between the flux expulsion scenario, in general, and the exponential model, for the single pulsars (see Jahan-Miri 1996a for further discussion).

5. BINARY EVOLUTION MODELS

The spin evolution of a neutron star in a binary system with a main-sequence star is expected to be different from that of a single pulsar. The interaction of the neutron star magnetosphere with the stellar wind of the companion star could result in final large values of $P_s \sim 10^4 - 10^5$ s, in contrast to the much smaller values achieved in the case of single pulsars. Magnetic evolution of binary neutron stars as predicted by the flux expulsion models (FBEs and SIF) is therefore expected to be, in principle, quite different than that of the single pulsars. In this section we employ the FBE models, for the first time, in a study of the field evolution of neutron stars in binaries. We consider models for the orbital and spin evolution of a neutron star, of a mass $M_\odot$, born in a binary with an orbital period $P_{\text{orb}}$, corresponding to an orbital separation $a$. The binary companion of the neutron star is a main-sequence star of mass $M_2$, which loses mass in the form of a spherical uniform stellar wind at a rate $\dot{M}_2$. In the picture of flux expulsion models, the evolutions of the spin period and the magnetic field of the neutron star in such a binary would be intimately coupled. While the spin-down process would tend to reduce the field strength, the reduced field strength (together with the increased spin period) will in turn affect the rate and the direction of the spin variations. We follow this coupled evolution of the surface magnetic field and the spin period of the neutron star.
star for a time of the order of the expected main-sequence lifetime of the companion star. The orbital and spin evolution for the expected Roche lobe overflow phase are not however simulated. Nevertheless, it should be noted that the computed field evolution of the recycled pulsars is not affected by this omission of the Roche lobe phase of the binary evolution. The latter spin-up phase of a recycled pulsar would have no direct effect on the flux expulsion, and hence no consequences for the field evolution except for the general decay of the crustal field, which is accounted for in our computations. Thus our simulations are complete, within the limitations of the models, as far as the field evolution of recycled pulsars is concerned, a point that has been overlooked by some authors (Urpin, Geppert, & Konenkov 1998) in referring to the earlier cited binary evolution simulations based on the SIF model.

We assume that the first phase of the binary evolution, namely, the active-pulsar phase with a dipole spin-down (during which the neutron star spins down due only to the dipolar radiation torque on it), lasts till the ram pressure of the stellar wind overcomes the pressure of the "pulsar wind" at the accretion radius (see, e.g., Illarionov & Sunyaev 1975; Davies & Pringle 1981). During this period the stellar wind will have no dynamical effect on the neutron star. The pulsar's core magnetic field will undergo an expulsion determined by the dipole spin-down rate, according to the FBE models. In the subsequent two phases, in which the accreted wind matter interacts directly with the magnetosphere, we assume that a steady Keplerian disk is formed by the accretion flow outside the magnetosphere, with the same sense of rotation as that of the neutron star. This is the least efficient configuration for angular momentum extraction from the neutron star that we have considered, in addition to the more efficient geometries such as a spherically symmetric radial infall.

The accretion flow interacts with the magnetosphere at its characteristic boundary radius $R_{\text{mag}}$, which is defined by the condition of balance between the magnetic pressure and the ram pressure of infalling flow (Davidson & Ostriker 1973):

$$R_{\text{mag}} = 1.88 \times 10^{-12} \left( \frac{B^2}{M_{\text{acc}}} \right)^{2/7} R_\odot,$$  

where $M_{\text{acc}}$ is the rate of capture of wind matter, in units of solar masses per year, as defined in equation (4) of Jahan-Miri & Bhattacharya (1994). This interaction spins the neutron star up or down depending on the sign of the quantity $V_{\text{diff}} (= V_{\text{core}} - V_{K\text{ep}})$ evaluated at the boundary of the magnetosphere. Here, $V_{\text{core}}$ is the speed of corotation with the neutron star at a given distance from it and $V_{K\text{ep}}$ is the Keplerian speed at the same distance. In the limiting case in which the corotation velocity $V_{\text{core}}$ becomes equal to the Keplerian velocity $V_{K\text{ep}}$, the neutron star will conserve its spin period while accretion onto the star will continue. The rate $\dot{L}_s$ of transfer of angular momentum between the stellar wind and the neutron star is assumed to be equal to $M_{\text{acc}}$ times a specific angular momentum corresponding to the difference between the corotation velocity with the neutron star and the Keplerian velocity evaluated at a distance $R_{\text{mag}}$ from the neutron star. In general, then,

$$\dot{L}_s = \eta \times V_{\text{diff}} \times R_{\text{mag}} \times M_{\text{acc}} \quad (14)$$

will be used, where $\eta$ is the efficiency factor included to take into account the uncertainties due to the detailed geometry of the interaction and the actual value of the specific angular momentum carried by the accreted wind just before and after the interaction. Different rates of angular momentum transfer are thus tested by assuming different values for $\eta$ while $\eta = 1$ corresponds to the case of disk accretion with a mechanical torque acting at $R_{\text{mag}}$. Jahan-Miri (1996a) found that, in order to account for the properties of binary pulsars evolved in both massive as well as low-mass systems simultaneously and self-consistently, larger values of $\eta > 1$ were preferred. Hence, and in contrast to the adopted range of values between 0.2 and 1.0 for the efficiency factor in Jahan-Miri & Bhattacharya (1994), here we use the larger values of $\eta$; see below.

The following coupled differential equations for the time evolution of $M_n$, $a$, $P_s$, along with those for $B_0$ and $B_1$ discussed before (§ 3), are solved numerically for the various combinations of parameter values indicated below.

$$\frac{da}{dt} = 2a \left\{ L_{\text{losses}} - \frac{\dot{L}_s}{\dot{L}_s} \right\} - \frac{M_2}{M_2} \times \left[ 1 + \left( \frac{M_2}{M_n} - \frac{1}{2} \left( \frac{M_2}{M_n} - 2\beta \frac{M_n}{M} \right) \right] \right\}, \quad (15)$$

$$\frac{dM_n}{dt} = \left\{ \begin{array}{ll}
0.0 & \text{accretion phase}, \\
0.0 & \text{propeller phase},
\end{array} \right. \quad (16)$$

$$\frac{dP_s}{dt} = 3.18 \times 10^{-3} \eta \left( \frac{M_{\text{acc}}}{M_\odot \ yr^{-1}} \right) \left( \frac{R_{\text{mag}}}{\text{km}} \right) \times \left( \frac{P_s}{s} \right)^{2/3} \left( \frac{V_{\text{diff}}}{\text{km} \ s^{-1}} \right) \ s \ yr^{-1}, \quad (17)$$

where $L_{\text{losses}}$ is the orbital angular momentum, $L_{\text{losses}}$ is the rate of change in $L_{\text{losses}}$ except for the contribution due to the escaping matter from the system, which is already taken into account, $M = M_n + M_2$ is the total mass of the binary, $\alpha$ is the ratio of the mass loss rate from the system to that from the secondary, and $\beta$ is the ratio of the effective specific angular momentum of the escaping matter to that in the companion star. As indicated earlier, a spin-up phase of the neutron star would have no effect on the flux expulsion out of its core, and the core field remains constant during such a period of time. Also, note that during the active pulsar phase $P_s$ will be given by the relation indicated earlier (§ 4) for the single pulsars, instead of equation (17), which is applicable for the other two phases of magnetospheric interaction with the accreted matter; see Jahan-Miri & Bhattacharya (1994) and Jahan-Miri (1996a), for more details of the binary evolution models. The computations were repeated using different combinations of the following values of the parameters and the initial conditions, for each of the FBE models, separately:

- $P_{\text{orb}} = 2 - 600$ days;
- $\eta = 1, 10, 100$;
- $\log \tau = 7.0, 8.0, 9.0$;
- $\log M_2 = -15, -14, -13$;
- initial $P_s = 0.1, 1.0$ s;
- initial $B_0 = 3 \times 10^{12}$ G;
- initial $B_1 = 2.7 \times 10^{12}$ G.

The evolution of a neutron star in a binary with a low-mass companion ($M_2 = 1.0 \ M_\odot$) is followed for a period of $10^{10}$ yr and its final surface field is determined, in order to be
compared with the observed fields of low-mass binary and millisecond pulsars.

6. PREDICTED FIELDS OF RECYCLED PULSARS

The general features of the computed evolution for the outward velocities of the fluxoids and the vortices, as well as the rotational lag, are similar to that described earlier in the case of single pulsars. A new feature is the rapid increase in the fluxoids velocity during the reverse creep phase, which is expected because of the enhanced spinning down of the star in a close binary. The distribution of the final surface field strengths versus the initial orbital periods are plotted in Figure 7, as predicted by two of the FBE models, for the given parameter values. Figure 7 shows that the observed magnetic field strengths of the low-mass recycled pulsars and millisecond pulsars may be successfully reproduced by the FBE models. In addition, the observational data on eight low-mass binary pulsars, which are expected to have been recycled in low-mass binary systems, are also presented in Figure 7. The measured orbital periods for these systems have been corrected for the expected change in the period during a final Roche lobe overflow mass transfer phase in the binary to infer the corresponding initial values of $P_{\text{orb}}$, listed in Table 2, for use in Figure 7.

Within the uncertainties associated with the value of $M_2$, which could be also varying with time, and the other unknown parameters of the binary pulsars, the computed curves in Figure 7 seem to agree with the data points. It is also understood that while the curves in Figure 7 represent a particular choice of the values for the pulsar-binary parameters; however, the true value of each of them might have been quite different among the corresponding eight systems. Qualitatively similar agreement with the data, as in Figure 7, is obtained also for many other choices of parameter values, as well as for the other FBE models, namely, A2, B1, and B2. Nevertheless, and in contrast to the case of single pulsars, the predicted field evolution of a given binary pulsar is found to be quite different according to the different FBE models. This is promising, in the sense that it offers a potential possibility not accessible from the application of the models to single pulsars. One might hope to distinguish among the models and gain insight into the interior physics of neutron stars, based on a comparison of the predicted spin-field evolution with the observational data on the recycled systems. However, we have not been able to pinpoint any preferences among the various flux expulsion models at this stage because of the uncertainties due to the free and unknown parameters (i.e., $\eta, M_2, \tau, \text{ etc.}$) that are encountered even in a simplified treatment of the binary evolution that we have used. That is, within the indicated ranges for the parameter values, all FBE models achieve the same success in comparison to the existing observational data on the recycled pulsars.

Nevertheless, the above general success of FBE and SIF models, which has not been so far reported for any other field decay model of neutron stars, provides strong support for the flux expulsion scenario. It is further noted that the SIF model was shown previously to be consistent with the data in the case of pulsars recycled in binaries with massive companion stars (Jahan-Miri 1996a), too. Thus, judging on the overall agreement seen here between the predictions of SIF with that of FBE models, in the case of single and low-mass binary pulsars, it might be justified to generalize the success of SIF for the case of massive binaries to FBE models as well. On the other hand, the other existing field evolution model of neutron stars, which assumes the total magnetic flux to be confined to the crust, has been also shown to result in the case of binary pulsars in a decay of the field down to values similar to that of the recycled pulsar (Geppert, Urpin, & Konenkov 1996; Urpin et. al. 1998). However, the spin-orbital binary evolution has not been really followed in these studies in all its details, and instead general estimated values for the rate and duration of accretion of matter onto the neutron star have been used. More importantly, the dependence of the final fields on the orbital periods, which is shown in Figure 7 for the FBE models and is also compared with the observational data, remains to be demonstrated for the model of crustal field decay due to accretion.

As in the case of single pulsars, for neutron stars evolved in many of the binary systems that we have simulated the pinning force on fluxoids again acts as an obstacle against an otherwise more rapid and enhanced flux expulsion. Indeed, we have verified that setting $F_b + F_c = 0$ in the models, namely, having $F_n$ as the only existing driving force, results in a much smaller flux expulsion than otherwise. In contrast, if $F_n = 0$ is adopted, practically zero final field values are obtained, which further demonstrates the braking role of $F_n$. Nevertheless, the essential role played by the pinning force in the field evolution of recycled old pulsars may not be overlooked. A model that discards the pinning force and relies only on the buoyancy is obviously unable to account for any flux present in the cores of such old pulsars. In contrast, the FBE (and SIF) models not only

| TABLE 2 |
| --- |
| THE OBSERVED LOW-MASS BINARY PULSARS |
| PSR | $P_s$ (ms) | $P_{\text{orb}}$ (day) | $M_2$ ($M_\odot$) | log $B_s$ (G) | Initial $P_{\text{orb}}$ (day) |
| 0820 + 02 | 864 | 1232 | 0.2–0.4 | 11.48 | 300 |
| 1953 + 29 | 6.13 | 117 | 0.2–0.4 | 8.63 | 12 |
| 1855 + 09 | 5.36 | 12.3 | 0.2–0.4 | 8.48 | 1 |
| J1713 + 0747 | 4.57 | 67.83 | 0.3–0.5 | 8.28 | 6 |
| J2019 + 2425 | 3.93 | 76.51 | 0.3–0.5 | 8.26 | 7 |
| J1643 – 1224 | 4.6 | 147 | 0.14 | 8.6 | 24 |
| J1455 – 3330 | 8.0 | 76 | 0.3 | 8.3 | 10 |
| B1800 – 27 | 334.4 | 406.8 | 0.15 | 10.9 | 60 |

Note.—Data is from Bhattacharya & van den Heuvel 1991; Foster, Wołoszczan, & Camilo 1993; Nice, Taylor, & Fruchter 1993; Johnston et al. 1995; Lorimer et al. 1995.
account for the observed fields of the old binary and millisecond pulsars, they also predict a correlation between the final field strength of a recycled pulsar and its spin period history, by virtue of the role of the pinning force.

7. SUMMARY AND CONCLUSIONS

We have studied a scenario for the evolution of the magnetic fields of neutron stars, assuming that the magnetic flux resides in the proton superconductor core of the star and is carried by the fluxoids. We have employed a dynamical treatment for the flux expulsion trying to improve our earlier studies based on the original model, which adopted an expulsion rate equal to the spin-down rate of the star. In the present work, the rate of expulsion of the flux out of the core is determined by explicitly calculating the radial velocity of the fluxoids that are subject to various forces. We have included forces due to pinning between the fluxoids and the neutron superfluid vortices, buoyancy, scattering of electrons, and tension of the flux lines. Alternative possibilities for evaluation of these forces were considered. The predictions of the various corresponding field decay models for the evolution of single and binary pulsars were discussed, and our conclusions are further summarized below.

1. The explored flux expulsion models predict a restricted decay of the magnetic field of pulsars that also accounts for the residual fields of the very old neutron stars. The pinning force plays two opposite and essential roles in the magnetic evolution of the star. The success of the models in predicting the residual fields of the recycled binary and millisecond pulsars is due to the braking role of this force against the fluxoids outward motion. This effect also result in a long-lived slowly evolving phase, and hence an increased radioactive lifetime, for some of the single pulsars. However, the pinning force also has a positive role in driving the fluxoids out of the core, which is revealed in the predicted dependence of the final field strengths of the recycled pulsars on the binary parameters that determine the spin history of the star.

2. Flux expulsion under the influence of the buoyancy force alone, in the absence of the pinning force, leads to vanishing field strengths in old binary and millisecond pulsars; hence, it is definitely ruled out. In contrast, a (tentative) model that invokes the pinning force and neglects the buoyancy would result, in general, in larger final fields than in the presence of the both forces; however, for some choices of the parameters the final fields might be small as well.

3. The alternative dynamical models, which we have considered for the motion of fluxoids and vortices in the core of neutron stars, result in similar predicted spin–magnetic field evolutions for the single pulsars. For the binary pulsars, even though the predictions of the models are quite different in many of the cases, similar acceptable results are still obtained for each of them, albeit for the different plausible values of the parameters. Hence, the underlying assumptions in these models remain observationally equivalent. The ideas that we entertained in these models are (1) that a neutron vortex remains straight while moving versus the possibility that its pinned segments might creep independently, (2) that a fluxoid may bend and be affected by its tension versus the possibility that the collective rigidity of the lattice prevents any bending, and (3) that fluxoids are bent outward (inward) at all times smaller (larger) than the field decay timescale in the crust versus the possibility that the collective rigidity of the fluxoid lattice remains constant.

4. The role of the coherent scattering of the electrons by the fluxoid lattice does not seem to be fully understood. Flux expulsion in the presence of the coherent scattering might depend on the electron currents across the core-crust boundary. Assuming that the associated maximum radial velocity of the electrons is similar to the limiting velocity of the fluxoids in the case of collective rigidity of their lattice, we speculated that our models for the case of collective rigidity would represent the case of coherent scattering too.

5. In contrast to the dynamical models that we used to determine the time evolution of the fluxoids velocity, the original spin-down induced flux expulsion model assumes equal velocities for the fluxoids and the vortices, at all times. The close agreement between its predictions and those of
the dynamical models is largely because a substantial flux expulsion occurs during a comoving phase of the two families of the vortices, even according to the latter models. This model, which offers a much simpler approach for a calculation of the field evolution compared to that employed here, might be further justified on the dynamical grounds. The comoving phase corresponds to a less energy dissipation for the coupled system of the two families of vortices than their crossing through each other, which costs energy. It is feasible, though it still needs to be demonstrated, that the more economical comoving phase is preferred and is maintained at all times by the magnetohydrodynamics of the interior quantum liquid, considering also that the fluxoids as well as the vortices are both magnetized and are rooted in the same highly conductive medium at the bottom of the crust.

6. Finally, an effective timescale in the range $10^7$–$10^8$ yr for the decay of the magnetic field in the crust of a neutron star is suggested, based on the preferred results of the model calculations presented here.

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