A Burst of Electromagnetic Radiation from a Collapsing Magnetized Star

G. V. Lipunova

Sternberg Astronomical Institute, Universitetskii pr. 13, Moscow, 119899 Russia

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Abstract

The pattern of variations in the intensity of magnetodipole losses is studied with the relativistic effect of magnetic-field dissipation during collapse into a black hole taken into account. A burst-type solution can be obtained both for a direct collapse and for the formation of a rapidly-rotating, self-gravitating object — a spinar — using a simple model. Analytical dependences on radius describing an electromagnetic burst are derived. The time dependence of the burst shape for an infinitely distant observer and the maximum energy of relativistic particles accelerated by an electric field are numerically calculated. The objects under consideration are of particular interest because particles in their vicinity can be accelerated up to the Planck energies. Possible astrophysical applications to the theory of active galactic nuclei (AGNs) and QSOs are briefly discussed. It is shown for the first time that a spinar can be produced by a merger of neutron stars; this possibility is considered in and without connection with the formation of gamma-ray bursts.

1 Introduction

The collapse of a star with substantial angular momentum can be accompanied by the formation of a quasi-static object whose equilibrium is maintained by centrifugal forces. Such objects (spinars or magnetoids) were first considered in connection with attempts to explain the activity of QSOs and AGNs
in terms of the model of supermassive ($\sim 10^6 \div 10^9 M_\odot$) stars (Hoyle and Fowler 1963; Ozernoi 1966; Morrison 1969; Ozernoi and Usov 1973; Lipunov 1987). Ostriker (1970) and Lipunov (1983) suggested the existence of low-mass spinars with nearly solar masses.

It is well known that neither the gas pressure nor the magnetic pressure can inhibit the once started collapse, because they grow no faster than the density of gravitational energy of the collapsing star. Only fairly rapid rotation can hinder the catastrophic contraction; in this regard, a spinar, as we call it for definiteness below, is just such a rapidly-rotating object. In fact, this implies that we consider a low-entropy object. A consideration of a "hot" spinar may be the subject of a separate study.

The lifetime of "cool" spinars is determined by the characteristic time of angular-momentum loss. If a spinar possesses a strong magnetic field and there is no substantial mass outflow, then magnetodipole radiation may prove to be a major dissipative mechanism; in this case, the evolutionary parameters of the spinar can be estimated most simply (Woltjer 1971; see below).

The subsequent observations of QSOs and AGNs over a wide wavelength range, particularly in X rays and gamma rays, have provided convincing evidence for the model of a supermassive accreting black hole, and the idea that spinars exist in nature has lost its luster.

However, the question of whether spinars exist (if only temporarily) is far from being ultimately resolved, at least for several reasons, some of which have been elucidated most recently.

First, supermassive rapidly-rotating magnetized stars may be the evolutionary precursors of supermassive black holes in the nuclei of galaxies and QSOs. This scenario is considered in connection with the formation of supermassive stars at an epoch that preceded the formation of galaxies in the Universe as a result of growth of the initial perturbations after recombination (see, e.g., Loeb and Rasio 1994). Second, Ozernoi and Lipunov (1996) has recently pointed out that conditions for the generation of limiting magnetic and electric fields characteristic of the "cosmic supercollider" (Kardashev 1995), in which particles can accelerate to superhigh Planck energies, can be naturally created in the vicinity of strongly magnetized supermassive spinars; this is of fundamental importance for modern physics.

In addition, it has been noted earlier (Lipunov 1983) that a large excess of angular momentum arises during the induced collapse of an accreting white
dwarf, which is capable of halting the direct collapse with the subsequent temporary formation of a spinar of nearly solar mass.

Finally, an excess momentum can be produced by orbital motion during a merger of neutron stars, which are currently considered as a possible source of gamma-ray bursts; this is noted in this paper for the first time. The latter possibility is particularly attractive in that the presence of strong magnetic fields seems natural and that the ejected mass may be completely absent due to the high binding energy of matter in neutron stars. After the merger, the energy losses from the newly formed object — a spinar — will be governed by the mechanism of magnetodipole radiation. The intensity of magnetodipole losses, which is proportional to the fourth power of the rotation frequency, for a solar-mass spinar with $\nu = 1000$ Hz and the same magnetic flux as that of the Crab pulsar will then be $10^6$ Crab. Thus, a spinar can be observed from a distance of 60 Mpc, given that currently available radio telescopes can detect sources that are weaker than the Crab pulsar by a factor of 1000. According to the current scenarios for the evolution of binary stars, the frequency of mergers of neutron stars is of the order of one merger per $10^4$ years (Lipunov et al. 1987; Tutukov and Yungelson 1992; van den Heuvel 1994). Thus, we can observe the formation of a spinar several times per year.

The aim of this work is to study magnetodipole radiation of a spinar with the relativistic effect of magnetic-field dissipation during the formation of a black hole taken into account. We will consider magnetodipole radiation as a major factor that governs the evolution of a spinar. Our primary concern will be the evolution of magnetodipole luminosity of a spinar, or, more precisely, the evolution of the luminosity calculated using the magnetodipole formula. In this case, it should be remembered that the experience of studying radio pulsars shows that we are actually dealing with the complicated physical process of relativistic-particle acceleration with the ensuing pronounced nonthermal radiation over a wide wavelength range; at the same time, however, what is particularly remarkable is that the energy losses are well described precisely by the magnetodipole formula as before.

As a result (using a relatively simple physical model), we will obtain a complex curve for the evolution of a spinar’s luminosity in the form of a nonmonotonic burst that describes the salient features of a real astrophysical process. We will also consider the limiting case of direct collapse of a star with a magnetic field that is dipolar at a distance and finally discuss some astrophysical applications of our model.
2 A classical burst

The discovery of QSOs and AGNs in 1963 marked the beginning of an active study of black holes and their precursors. It was noted in several papers that the gravitational collapse of magnetized stars produces a burst of electromagnetic radiation. Novikov (1964) considered the direct collapse of a star with magnetodipole moment. He found the intensity of magnetodipole radiation to take the form of a monotonically increasing (formally to infinity) burst,

\[ L = \Phi^2 \frac{c}{6R_g} \left(\frac{R_g}{R}\right)^4, \]

where

\[ \Phi = \frac{\mu}{R} = const. \]

The second equality gives the law of conservation of the magnetic flux through the star, \( \mu \) is the magnetodipole moment of a spherically-homogeneous star. The stellar surface is involved in a free fall and is described by the law of motion of Newtonian form

\[ \ddot{R} = -\frac{GM}{R^2}. \]

Generally speaking, a burst will also occur during the collapse of a spinar. Hoyle and Fowler (1963), Ozernoi (1966), and Ozernoi and Usov (1973) considered supermassive spinars as the precursors of black holes in AGNs.

A decrease of a spinar’s angular momentum as a result of some dissipative processes leads to an increase of the angular rotation velocity \( \omega \) and, hence, to an increase of the kinetic rotational energy. In other words, spinars, like any gravitationally-bound object, have negative thermal capacity.

Let us write out the Newtonian system of basic equations that describe a spinar with a magnetic field characterized by the magnetodipole moment \( \mu \) (Bisnovatyi-Kogan and Blinnikov 1972; Woltjer 1971; Lipunov 1987):

\[ \omega^2 R = \frac{GM}{R^2}, \quad (1) \]

\[ \frac{\mu}{R} = const, \quad (2) \]
\[
\frac{dI\omega}{dt} = -\frac{2}{3c^3}\mu^2\omega^3, \tag{3}
\]

\[
I = AMR^2. \tag{4}
\]

Equation (1) is the condition of equilibrium for a point at the equator. As was noted above, neglect of the contribution of pressure is a simplification that can be justified by the fact that only the centrifugal force can stop the catastrophic contraction during the collapse. Equation (2) describes the law of conservation of the magnetic flux through the spinar and gives the pattern of variations in the magnetodipole moment,

\[
\mu = \mu_o \frac{R}{R_o}. \tag{5}
\]

\(I\) is the moment of inertia, \(A\) is close to unity, and equation (3) expresses the dipole nature of the losses of angular momentum \(I\omega\). The solution of this system of equations is given by

\[
R = R_o \left(1 - \frac{t}{t_k}\right)^{1/3}, \quad \omega = \omega_o \left(\frac{t_k}{t_k - t}\right)^{1/2},
\]

where \(\omega_o\) and \(R_o\) are the initial rotation frequency and radius of the spinar, respectively, and \(t_k = \frac{4c^3G}{B_2^2R_o} = 4.05 \cdot 10^8 B_{12}^{-2} R_6^{-1}\) s; \(B_o\) is the initial polar magnetic field, \(B_{12} = B_o/10^{12}\) G, and \(R_6 = R_o/10^6\) cm. Evidently, the rotation frequency of the spinar becomes infinite in a finite time \(t = t_k\); at the same time, the intensity of magnetodipole radiation also increases infinitely,

\[
L = \frac{2}{3c^3}\mu^2\omega^4 = \frac{2}{3c^3}\mu_o^2\omega_o^4 \left(\frac{t_k}{t_k - t}\right)^{4/3},
\]

where \(\mu_o = B_o R_o^3/2\) is the initial magnetodipole moment of the spinar. Note that the collapse time of a spinar increases considerably compared to that of a nonrotating object, for which it is determined by the time of a free fall of the stellar boundary:

\[
R = R_o \left(1 - \frac{t}{t_f}\right)^{2/3}, \quad t_f = \frac{2R_o^{3/2}}{3\sqrt{2GM}} \approx 4.08 \cdot 10^{-5} R_6^{3/2} M_\odot^{-1/2} s,
\]
where \( M_* \) is the mass in units of \( M_\odot \).

How does the energy of a spinar’s magnetic field changes during the collapse? Let the following condition be fulfilled at some instant in time:

\[
\frac{B_0^2}{8\pi} \frac{4\pi}{3} R^3 = k \frac{GM^2}{R},
\]

where \( k = U_{magn}/U_{grav} \). Evidently, this condition is equivalent to the condition of conservation of the magnetic flux, and, consequently, it is fulfilled throughout the entire collapse, \( k = \text{const} \approx 6.2 \cdot 10^{-13} M_*^{-2} B_0^2 R_6^4 \).

3 Magnetic field in a spherically–symmetric case in the general theory of relativity

As was shown above, the magnetodipole moment of the magnetized objects under consideration (a nonrotating star and a spinar), which linearly varies with radius, becomes zero at \( R = 0 \). Outside the scope of the classical treatment, we must take into account the fact that a black hole possesses no magnetic field that could be detected by an external observer: "a black hole has no hairs". Thus, the dependence of \( \mu \) on the radius of a magnetized star must reflect the fact that \( \mu = 0 \) at \( R = R_g \).

The behavior of a magnetic field near the gravitational radius can be elucidated by solving Maxwell’s equations in the general theory of relativity. A magnetostatic solution for a spherically-symmetric object, i.e., in the Schwarzschild metric, was found by Ginzburg and Ozernoi (1964).

In the Schwarzschild metric, Maxwell’s equations in the general theory of relativity take the form

\[
\frac{\partial F_{\theta\varphi}}{\partial r} + \frac{\partial F_{\varphi\theta}}{\partial \theta} = 0, \quad \frac{\partial}{\partial r} \left( r^2 \sin \theta F^{\varphi r} \right) + \frac{\partial}{\partial \theta} \left( r^2 \sin \theta F^{\varphi \theta} \right) = 0,
\]

given the following conditions: (a) we seek an axially-symmetric solution, and, hence, the electromagnetic-field tensor \( F_{ik} \) does not depend on \( \varphi \); (b) \( F_{\theta r} = 0 \) or \( B_{\varphi} = 0 \), i.e., we seek a magnetic-dipole field that is uniform at infinity; and (c) the current density is taken to be zero throughout, with the exception of the stellar surface.
Recall the following relations:

\[ B_r = \sqrt{g_{\theta\theta}} \sqrt{g_{\phi\phi}} F^{\theta\phi}, \quad B_\theta = \sqrt{g_{\phi\phi}} \sqrt{g_{rr}} F^{\phi r}, \quad B_\phi = \sqrt{g_{rr}} \sqrt{g_{\theta\theta}} F^{r\theta}, \]

\[ g_{\theta\theta} = r^2, \quad g_{\phi\phi} = r^2 \sin^2 \theta, \quad g_{rr} = (1 - r_g/r)^{-1}, \]

where \( g_{ik} \) is the metric tensor in the Schwarzschild metric. Considering the properties of the solution noted above, one assumes that

\[ B_r = 2 \cos \theta r^{-3} f(r) \mu \] (7)

and

\[ B_\theta = \sin \theta r^{-3} \psi(r) \mu, \] (8)

where the functions \( f(r) \) and \( \psi(r) \to 1 \) as \( r \to \infty \). The solution consists of two linearly independent parts (we will consider only the radial component):

- a uniform field \( B_o \) inside a sphere, for which

\[ f(r) = \frac{B_o r^3}{2 \mu}, \] (9)

and an external part:

\[ f(r) = -3 \left( \frac{r}{R_g} \right)^3 \left\{ \ln \left( 1 - \frac{R_g}{r} \right) + \frac{R_g}{r} + \frac{1}{2} \left( \frac{R_g}{r} \right)^2 \right\} \equiv -3 \left( \frac{r}{R_g} \right)^3 \xi \left( \frac{R_g}{r} \right). \] (10)

At infinity, we obtain the classical expression \( B_r = 2 \mu / r^3 \) for \( \theta = \pi/2 \).

Based on these expressions, we derive the dependence \( \mu(R) \), where \( R \) is the current radius of the sphere under consideration. Since the normal component of \( \mathbf{B} \) does not change at the intersection of the stellar surface, the polar field outside will be equal to the uniform field inside. Let us write this condition using (7) and (10) for \( \theta = 0 \),

\[ B^{(in)} = 2 \mu (-3) R_g^{-3} \xi (R_g / R). \] (11)

Assuming that the star is a fairly good conductor, we use the law of conservation of the magnetic flux,

\[ 2\pi \int B^{(in)} r dr = \text{const.} \]
where $r$ is the Schwarzschild coordinate inside the star. We obtain

$$B^{(in)} R^2 = B_o^{(in)} R_o^2$$  \hspace{1cm} (12)

Here, $R_o$ is some initial radius. From (11) and (12), we derive

$$\mu(R) = -\frac{1}{6} R_o^3 \left( \frac{R_o}{R} \right)^2 \frac{B_o}{\xi(R_o/R)}.$$  \hspace{1cm} (13)

or

$$\mu(R) = \mu_o \left( \frac{R_o}{R} \right)^2 \frac{\xi(R_o/R)}{\xi(R_o/R)}.$$  \hspace{1cm} (14)

It can be seen from formulas (7), (10), and (13) that the magnetic moment decreases to zero as $R$ approaches $R_g$, while the field is pressed to the star. Thus, we derived the formula that describes magnetic-field dissipation at a finite radius. Let us use it to calculate the intensity of magnetodipole losses

$$L = -\frac{2}{3} \frac{\ddot{\mu}^2}{c^3},$$

and then consider two limiting cases:

(a) for a direct spherically-symmetric collapse, the change in $\mu$ is due to a decrease in the radius alone;

(b) for a spinar — a rotating object — $\ddot{\mu}$ is determined by rotation of the magnetic-dipole vector.

In both cases, we will solve the problem in a quasi-static approximation, because we use the solution of a magnetostatic problem. For this approach, we have a simple, qualitatively adequate description of the decrease in the external magnetic field during the collapse of a magnetized object. Note that in the case of a spherically-symmetric collapse, an exact solution exists for the field, according to which the magnitude of the magnetic dipole decreases with time as $t^{-5}$ (Price 1972).

### 4 Direct collapse

In this case, we assume that the boundary of the star is involved in a free fall,

$$\frac{dR}{dt} = c \left( 1 - \frac{R_o}{R} \right) \sqrt{\frac{R_o}{R}},$$  \hspace{1cm} (15)
when \( R_g/R_o \ll 1 \), \( t \) is the time of an infinitely distant observer, because it is this observer who records the luminosity \( L \); at the same time,

\[
\frac{d^2 \mu}{dt^2} = \frac{d^2 \mu}{dR^2} \left( \frac{dR}{dt} \right)^2 + \frac{d \mu}{dR} \frac{d^2 R}{dt^2}.
\]

Using (15) and (16), we obtain

\[
L = \frac{2}{3c^3} \frac{\mu^2(x)c^4}{R_g^4} \eta^2(R_g/R)
\]

or

\[
L = L_o \left( \frac{x}{x_o} \right)^4 \frac{\xi^2(x) \eta^2(x)}{\xi^2(x_o) \eta^2(x_o)},
\]

where \( L_o \) is some luminosity at the initial time, at which \( x = x_o = R_g/R_o \). The expression for \( \eta(R_g/R) \equiv \eta(x) \) is given in the Appendix (formula (23)). Figure 1 shows that the radiation has the shape of a nonmonotonic burst. The ratio \( R/R_g \) at which the intensity reaches its maximum is always the same in the approximation \( R_o \gg R_g \) (this follows from the fact that the variables \( x \) and \( x_o \) in expression (17) are separated) and \( R_{max}/R_g \approx 2.09 \).

We can pass to the time dependence if the law of motion of the stellar surface is known,

\[
t = t_o + \frac{R_g}{c} \left\{ -\frac{2}{3y\sqrt{y}} - \frac{2}{\sqrt{y}} + \ln \frac{1 + \sqrt{y}}{1 - \sqrt{y}} \right\},
\]

\[
t_o = -\frac{R_g}{c} \left\{ -\frac{2}{3y_o\sqrt{y_o}} - \frac{2}{\sqrt{y_o}} + \ln \frac{1 + \sqrt{y_o}}{1 - \sqrt{y_o}} \right\}, \quad y_o = R_o/R_g
\]

and \( y_o \gg 1 \). Let us rewrite \( L \) with \( k = U_{magn}/U_{grav} \),

\[
L = \frac{k}{36} c^5 \frac{x^4 \eta^2(x)}{G \xi^2(x)}, \quad L_{max} = k \cdot 3.7 \cdot 10^{59} \text{ erg/s}.
\]

Note that the intensity of magnetodipole radiation for an infinitely distant observer becomes an exact zero in an infinite time, because the star contracts to the gravitational radius infinitely long for this observer.
5 A collapsing spinar

The problem of magnetodipole radiation of a collapsing spinar in the general theory of relativity differs markedly from the case of a spherically-symmetric collapse considered in the preceding section. The reason is that the spinar rotates, and, hence, the Kerr metric should be used. However, we do not know how the magnetodipole moment looks in the Kerr metric. It is not that simple to solve Maxwell’s equations in the Kerr metric. Therefore, we use the solution of a magnetostatic problem in the Schwarzschild metric, i.e., formula (13) for $\mu$ with the qualification that this formula includes the main relativistic effect: dissipation of a spinar’s magnetic field for a distant observer at a finite value of $R$.

So, let us write the system of approximate equations for a spinar,

\[
\frac{dI}{dT} = -\frac{2}{3c^3} \mu^2(R) \omega^3,
\]

\[
\mu(R) = \mu_o \left( \frac{R_o}{R} \right)^2 \xi(R_g/R_o),
\]

\[
I = AMR^2,
\]

\[
\omega^2 = \frac{MG}{R^3},
\]

where $\tau$ is the time of a local observer that is at rest with respect to a sphere of radius $R$. In the Schwarzschild metric, stable circular orbits, for which Kepler’s law has the same form as in the Newtonian mechanics, exist up to $r = 3R_g$. In the Kerr metric, stable circular orbits can exist up to $r = R_g/2$; in this case, Kepler’s law in geometrical units takes the form

\[
\Omega = \frac{\sqrt{M}}{r^{3/2} + a(R_g/2)^{3/2}},
\]

where $a \simeq 1$ is the ratio of the total angular momentum of the spinar to $M^2$ that is greater than 1. Since expression (19) does not differ much from Kepler’s classical law, we will make use of it.

The intensity of magnetodipole losses can be written as an explicit function of the spinar’s radius,

\[
L = L_o \left( \frac{R_o}{R} \right)^{10} \frac{\xi^2(R_g/R_o)}{\xi^2(R_g/R)}.
\]
where $L_o$ is some luminosity at the initial time, at which $R = R_o$.

Thus, we obtained the curve with a distinctive burst-like shape (Fig. 2a). Note that for any initial parameters of a collapsing spinar, the ratio $R/R_g$ at which the intensity reaches its maximum is the same (much as it was in the preceding section; this follows from the fact that the variables $R/R_g$ and $R_o/R_g$ in expression (20) are separated) and $R_{\text{max}}/R_g \approx 1.29$.

From the law of angular-momentum loss, we derive the law of motion $\tau = \tau(R)$ by integration; an explicit, but cumbersome form of this law is given in the Appendix (formula (24)).

In order to determine the burst shape for an infinitely distant observer, we must pass from the local time $\tau$ to the Schwarzschild time $t$ of the infinitely distant observer using the formula

$$dt = \sqrt{1 - R_g/R} d\tau.$$

For a $M = 3M_{\odot}$ spinar with a magnetic field that is standard for pulsars, the pulse width at half maximum in time is $\Delta t \approx 700$ years (Fig. 2b). Numerical simulations give

$$\Delta t \approx 5 \cdot 10^{-4} k^{-1} M_\star.$$

With $k = U_{\text{magn}}/U_{\text{grav}}$, we can write

$$L = \frac{k}{144} \frac{c^5}{G} \xi^{10}(x), \quad L_{\text{max}} = k \cdot 1.1 \cdot 10^{57} \text{ erg/s}.$$

Let us estimate the maximum energy to which particles can accelerate in the magnetosphere of a spinar using relation (6),

$$\varepsilon \sim e^{-\frac{v}{c} B R}.$$ \hfill (21a)

Assuming $v = \omega R$ and

$$\omega = \frac{\sqrt{GM}}{R^{3/2}}, \quad B = \frac{\sqrt{6kGM}}{R^2}$$

we obtain

$$\varepsilon \sim e^{\frac{\sqrt{3k}}{2\sqrt{G}} M^{3/2} R^{3/2}} = e^{\frac{\sqrt{3k}}{2\sqrt{G}} x^{3/2} k^{1/2}}.$$
\[ x = R_g/R \text{ and } k = U_{magn}/U_{grav}. \] However, the corresponding magnetic field in the vicinity of low-mass spinars for \( R \sim R_g \) and \( k \) approaching 1 will be so strong that the formation of elementary particles will begin, and, what is more, the very existence of such strong fields is called into question for such objects. At the same time, the maximum magnetic field \( (k = 1) \) for supermassive stars is considerably weaker,

\[ B_{max} = \frac{\sqrt{6c^4}}{4G^{3/2}M} \approx 10^{19} \frac{x^2}{M_\star} \text{ G}, \]

where \( M_\star \) is given in solar masses. For \( M > 2.2 \cdot 10^6 M_\odot \) B \( < 4 \cdot 10^{13} \) G, and the creation of electron–positron pairs may be neglected. Thus, particles in the vicinity of supermassive spinars can accelerate to energies that are typical of the early Universe (Ozernoi and Lipunov 1996). A similar estimate was first obtained by the magnetic field around a black hole by Kardashev (1995).

At each fixed point \( r \) in the Schwarzschild system of coordinates, the radial component of \( B \) in the vicinity of a spinar decreases as (see (7))

\[ B_r = B_o \left( \frac{R_o}{R} \right)^2 \frac{\xi(R_g/r)}{\xi(R_g/R)} \cos \theta, \]

where \( R \) is the radius of the spinar.

The accelerating potential at fixed distance then varies during the collapse in a burst-like manner (Fig. 3), \( \theta = 0 \):

\[
V = B_r(r = 2R) \frac{\omega R c}{2\sqrt{G}} = \frac{\sqrt{3c^2}}{2\sqrt{G}} x^{3/2} k^{1/2} \left( \frac{r}{R} \right)^2 \frac{\xi(R_g/r)}{\xi(R_g/R)}.
\]

These estimates follow from the first, quasi-static approximation to a real dynamical problem. A more rigorous treatment must lead to a refinement of the results. Note also that application of this approximation to a spinar seems more justified than to a direct, spherically-symmetric collapse.

6 Discussion

Ozernoi and Usov (1973) showed that the intensity of magnetodipole radiation versus radius during collapse has the shape of a burst due to the angle between the magnetic and rotation axes being reduced to zero.
However, the experience of theoretical and observational studies of pulsars gained over the last twenty years suggests (Lipunov 1992) that the energetics of rapidly-rotating magnetized stars depends only slightly on the orientation of the magnetic field with respect to the rotation axis; it is mainly determined by complex electrodynamical processes, such as the acceleration of relativistic particles, the generation of electromagnetic and Alfvén waves, that are described in order of magnitude by the magnetodipole formula (as, for example, in the Julian–Goldreich model). Therefore, the derived intensities describe the energy carried away by cosmic-ray particles and synchrotron radiation over a wide spectral range (from radio to gamma rays) rather than the low-frequency electromagnetic radiation.

Calculation of a particular shape of the spectrum of a supermassive spinar presents a separate, complicated problem. However, some distinctive features are clear even without a detailed analysis. First of all, a magnetized spinar is the most natural physical justification of the so-called phenomenological model of an oblique rotator, which is widely used to interpret the observed properties of QSOs and AGNs, especially those with strong nonthermal radio emission and jets (see, e.g., Kardashev 1995).

Of course, the evolution of a spinar must inevitably lead to the formation of a supermassive black hole, while this process can be burst-like in nature (see above). Assuming that the precursors of supermassive black holes possessed intrinsic magnetic fields and considering that it takes an infinite time for a black hole to approach its gravitational radius for an external observer (a "frozen star"), any such black hole is formally at some stage of evolution of its burst-type magnetodipole radiation. Moreover, note that the burst duration of $\sim 10^8$ years that is definitively related to the maximum luminosity of a spinar, which is taken to be $\sim 10^{46}$ erg/s (the standard luminosity of QSOs), corresponds to this luminosity for given masses (Fig. 4). These values are now considered to be typical lifetimes of QSOs. Note also that the case of a spinar in a vacuum considered in this paper by no means exhausts the whole variety of possible modes of interaction of a magnetized spinar with the surrounding medium (Lipunov 1987). The existence of at least six types of spinar (an accretor, a propeller, an ejector, a superaccretor, a superpropeller, and a superejector) holds a great variety of astrophysical manifestations of supermassive magnetized objects.

Of special, fundamental interest is the possibility of particle acceleration in the vicinity of supermassive spinars to the Planck energies or, at least, to
typical energies in the theory of great unification (10^{24} \text{ eV}).

With reference to spinars formed by a merger of neutron stars, in addition to a burst of gravitational radiation, pulsar-type electromagnetic radiation caused by orbital motion is possible even before the merger in a system of two neutron stars (Lipunov and Panchenko 1996).

The nature of gamma-ray bursts is so far unclear. It is possible that they result from mergers of neutron stars (Blinnikov et al. 1984; Paczynski 1991). The release of electromagnetic energy considered above describes a burst with a duration from fractions of a second to ten seconds, if the surface field of a spinar with \( M = 3M_\odot \) reaches \( 5 \cdot 10^{16} \div 10^{17} \text{ G} \) for \( R_o = 10 \text{ km} \) (Fig. 5).

Let us make some rough estimates for the formation of low-mass spinars by a merger of neutron stars. How and why an intermediate object that we call a spinar can form? Here, the influence of two factors is possible:

1. we do not know the exact value of the Oppenheimer–Volkov limit. In other words, the pressure of nuclear matter may play a major role in addition to rotation;
2. it is possible that excess angular momentum that inhibits a direct collapse will suffice for the formation of a spinar.

It should be noted that difficulties emerge when we consider the magnitude of the transferred momentum, at least in the first approximation. A black hole cannot have the Kerr parameter \( a \equiv K/M^2 > 1 \), where \( K \) is the total angular momentum (see, e.g., Shapiro and Teukolsky 1985). It is in this case that the formation of a spinar is implied.

Let us first estimate the rotation parameter \( a \) in the Newtonian approximation. Calculate the total angular momentum of a system of two identical neutron stars. The moment of inertia of two identical bodies about their center of symmetry is

\[
I = 2\{AmR^2 + m(d/2)^2\},
\]

where \( m \) is the mass of each star, and \( d \) is the separation between the centers of mass of the stars. We assume that the rotation of the objects around a common center of mass and around their axes is synchronous. The total angular momentum is \( K = I\Omega \), where \( K = I\Omega \) is the frequency of revolution.

Kepler’s law in geometrical units \((c = G = 1)\) takes the form

\[
\Omega = \frac{\sqrt{2m}}{d^{3/2}}.
\]
We shall consider a merger for \( d = 2R = 2\kappa R_g \). We then have

\[
a = \frac{K}{4m^2} = (A + 1)\sqrt{\frac{\kappa}{8}}.
\]

(1) \( A = 0 \) — intrinsic rotation is ignored. In this case, the condition for the formation of a spinar (the condition that a black hole is not formed) takes the form

\[
\sqrt{\frac{\kappa}{8}} > 1, \quad \kappa = R/R_g
\]

At the same time, \( \kappa \sim 2 \div 3 \) for neutron stars.

(2) For homogeneous spheres, \( A = 2/5 \) and \( a \approx \sqrt{\kappa/4} \).

(3) For flat disks, \( A = 1/2 \) and \( a \approx \sqrt{\kappa/3.5} \).

Evidently, the orbital angular momentum in the Newtonian approximation is not enough for a spinar to be formed. Let us estimate the relativistic effects.

Consider a problem that is similar to the problem of reduced mass in classical mechanics. We then have the motion of a body of mass \( \mu = m/2 \) in the effective potential of a gravitating body of mass \( M = 2m \). In the last stable circular orbit in the Schwarzschild metric, the corresponding angular momentum is \( l = 2\sqrt{3}\mu M \) (Zel’dovich and Novikov 1971). Then, we have \( K = 2\sqrt{3}m^2 \) and \( a = \sqrt{3}/2 \).

However, a more rigorous treatment of the problem using the general theory of relativity with allowance for a more or less real equation of state of matter is needed to establish what angular momentum the two neutron stars had before the merger.

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Appendix

1. In formula (17), which gives the law of motion of the surface in a free collapse,

$$\eta(x) = x(1-x)(6x(1+x) - x^2(1-3x)) + \frac{2x^9}{\xi^2(x)} + \frac{3x^6(1-x)}{\xi(x)}, \tag{23}$$

where

$$\xi(x) = \ln(1-x) + x + \frac{1}{2}x^2, \quad x = \frac{R_g}{R}.$$ 

2. Integrating (18) gives the equation of motion for the surface of a spinar in explicit form:

$$-\frac{1}{9}(x^{-9} + 1)\ln^2(1-x) + \frac{2}{9} \left\{ \sum_{k=1}^{8} \frac{1}{kx^k} + \ln \left( \frac{1-x}{x} \right) \right\} \ln(1-x) -$$

$$-\frac{2}{9} \sum_{k=1}^{8} \frac{1}{k} \left\{ \sum_{m=1}^{k} \frac{1}{mx^m} + \ln \left( \frac{1-x}{x} \right) \right\} - \frac{2}{9} \sum_{1}^{\infty} \frac{(1-x)^k}{k^2} -$$

$$- \left( \frac{x^{-8}}{4} + \frac{x^{-7}}{7} \right) \ln(1-x) - \frac{x^{-7}}{7} - \frac{x^{-6}}{6} - \frac{x^{-5}}{20} +$$

$$+ \frac{11}{28} \ln \left( \frac{1-x}{x} \right) + \frac{1}{4} \sum_{k=1}^{7} \frac{1}{kx^k} + \frac{1}{7} \sum_{k=1}^{6} \frac{1}{kx^k} =$$

$$= \frac{1}{216} \frac{c^3 R_o^4 B_o^2}{AG^2 M^3} (\tau + \tau_o), \tag{24}$$

where $x = R_g/R, \tau = 0$ at $x = R_g/R_o$.

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Figure captions

Fig. 1. The power of magnetodipole losses versus radius for a spherically-symmetric collapse (formula (17)).

Fig. 2. (a) The power of magnetodipole losses versus radius for a relativistic spinar (formula (20)); the power reaches its maximum at $R/R_g = 1.29$. (b) The power of magnetodipole losses versus time for a relativistic spinar (the magnetic flux is $\Phi = \text{const} = 10^{12} G \cdot \pi 10^{12} \text{ cm}^2$; $k \approx 7 \cdot 10^{-14}$).

Fig. 3. The accelerating electric potential (formula (22)) in the vicinity of a limiting spinar versus radius.

Fig. 4. Evolution of the luminosity of a supermassive relativistic spinar. For $R = 1.5 \cdot 10^{14} \text{ cm}$, the surface magnetic field is $B \approx 1.8 \cdot 10^4 \text{ G}$. The width at half maximum in time is $\approx 1.6 \cdot 10^8 \text{ years}$.

Fig. 5. The power of magnetodipole losses versus time for a relativistic spinar. The magnetic flux is $\Phi = \text{const} = 5 \cdot 10^{16} G \cdot \pi 10^{12} \text{ cm}^2$; $k \approx 2 \cdot 10^{-4}$. The burst width at half maximum is 8.89 s.
Fig. 1
$L_{\text{max}} = 2.6 \cdot 10^6 \text{Crab}$

$M = 3M_\odot$

$B_o = 10^{12}$ G

Fig. 2b
$\frac{U_{\text{magn}}}{U_{\text{grav}}} = 1$

$R/R_{\text{grav}}, \ R = \text{the radius of spinar}$

Fig. 3
Fig. 4

$L, \ 10^{46} \text{ erg/s}$

$M=10^8 M_\odot$

$k=10^{-11}$
$L, \ 10^{53} \ erg/s$

- $M = 3M_\odot$
- $B_0 = 5 \times 10^{16} \ G$

**Fig. 5**