Effects of $\Lambda$ hyperons on the deformations of even-even nuclei

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The deformations of multi-$\Lambda$ hypernuclei corresponding to even-even core nuclei ranging from $^{8}$Be to $^{40}$Ca with 2, 4, 6, and 8 hyperons are studied in the framework of the deformed Skyrme-Hartree-Fock approach. It is found that the deformations are reduced when adding 2 or 8 $\Lambda$ hyperons, but enhanced when adding 4 or 6 $\Lambda$ hyperons. These differences are attributed to the fact that $\Lambda$ hyperons are filled gradually into the three deformed $p$ orbits, of which the $[110]1/2^-$ orbit is prolately deformed and the degenerate $[101]1/2^-$ and $[101]3/2^-$ orbits are oblately deformed.

I. INTRODUCTION

Since the first discovery of a hypernucleus in cosmic rays [1], the study of hypernucliei has become one of the most interesting topics in nuclear physics from both experimental and theoretical sides [2–8]. In particular, $\Lambda$ hypernucliei have been studied by many experiments and related theoretical analyses. Due to the limitation of current experimental conditions, the experimental data of hypernucliei are mainly for single-$\Lambda$ nuclei [4, 6, 8, 15–21]. Of particular interest is the fact that the addition of $\Lambda$ hyperons can lead to the appearance of the so-called impurity effect [2, 7], since the hyperon(s) can enter deeply into the center of a hypernucleus regardless of the restriction of the Pauli exclusion principle, and thus can be used as a good probe to study the nuclear environment. The impurity effects of single-$\Lambda$ hypernucliei have been investigated extensively in the past decades, such as the shrinkage effect [22–24], the modification of the drip lines [25, 26], and the modification of the deformation [27–33].

Since the experimental discovery of double-$\Lambda$ hypernucliei [15–21], several kinds of nuclear models have been extended to study the $\Lambda\Lambda$ hypernucliei sector. For example, the microscopic cluster model was used to study the $\Lambda$ binding energies of light hypernucliei and reproduced well the observation of the ground state of $^{11}_{\Lambda\Lambda}$Be [34, 35]. The Faddeev calculations with the Nijmegen soft-core potential NSC97 described well the binding energies of light $\Lambda\Lambda$ hypernucliei [36]. The shell-model calculation showed how the $\Lambda N$ spin-dependent interaction terms influence the $\Lambda\Lambda$ hypernucliei across the nuclear $p$ shell [37]. The beyond-mean-field approach was applied to study the evolution of nuclear deformation in $\Lambda\Lambda$ hypernucliei and the hyperon impurity effect in hypernucliei with shape co-existence [38, 39]. Extensive research of binding energies and deformation effects has been carried out by the self-consistent mean-field model on the shape of hypernucliei [26, 27, 29, 40–44].

Very recently, the impurity effects of multi-$\Lambda$ hyperons on the deformations in the hyperisotope chains $^{8+n}_{\Lambda\Lambda}$Be ($n = 2, 4$), $^{20+n}_{\Lambda\Lambda}$Ne ($n = 2, 4, 8$), and $^{28+n}_{\Lambda\Lambda}$Si ($n = 2, 4, 8$) have been studied using the relativistic mean field (RMF) theory in Ref. [24]. It was pointed out that in the Ne hyperisotopes, the deformation is slightly reduced by the additional $\Lambda$ hyperons, whereas it is significantly reduced or even disappears in the Si hyperisotopes. Studies on multi-$\Lambda$ hyperisotopes have theoretical significance, although the corresponding experiments are currently unfeasible. First of all, the impurity effects in multi-$\Lambda$ hyperisotopes are evidently stronger than those of single-$\Lambda$ ones. Moreover, a multi-$\Lambda$ system can provide important information on the $\Lambda\Lambda$ interaction, and the effects of the core nucleus on the $\Lambda$ hyperons can be studied.

The aforementioned work in Ref. [24] studied only a few nuclei and did not address the impurity effects caused by 6 $\Lambda$ hyperons. Therefore, further studies on the impurity effect of $n = 2, 4, 6, 8$ $\Lambda$ hyperons on the properties of even-even nuclei ranging from $^{8}$Be to $^{40}$Ca are carried out in this work. In contrast to the RMF model adopted in Ref. [24], we will employ the deformed Skyrme-Hartree-Fock (SHF) approach [45–47], which is one of the widely used models for hypernucliei [26, 43, 48].

II. FORMALISM

In the framework of the SHF approach, the energy of a hypernucleus is given by an energy-density functional,

$$E = \int d^3r \varepsilon(\mathbf{r}), \quad \varepsilon = \varepsilon_{NN} + \varepsilon_{NN} + \varepsilon_{\Lambda\Lambda},$$

(1)

where $\varepsilon_{NN}$, $\varepsilon_{NN}$, and $\varepsilon_{\Lambda\Lambda}$ account for the nucleon-nucleon interaction, the hyperon-nucleon interaction, and the hyperon-hyperon interaction, respectively. The energy-density functional depends on the one-body densities $\rho_q$, kinetic densities

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\[ \tau_q, \] and spin-orbit currents \( J_q, \)
\[ \left[ \rho_q, \tau_q, J_q \right] = \sum_{i=1}^{N_q} n_q^i \left[ |\phi_q^i|^2, |\nabla \phi_q^i|^2, i\phi_q^i (\nabla \phi_q^i \times \sigma) / \hbar \right], \] (2)
where \( \phi_q^i (i = 1, N_q) \) are the self-consistently calculated single-particle (s.p.) wave functions of the \( N_q \) occupied states for the species \( q = n, p, \Lambda \) in a hypernucleus. They satisfy the Schrödinger equation, obtained by the minimization of the total energy functional (1) according to the variational principle, as
\[ \nabla \cdot \frac{1}{2m_q^i(r)} \nabla V_q(r) + iW_q(r) \cdot (\nabla \times \sigma) \phi_q^i(r) = \epsilon_q \phi_q^i(r) \] (3)
in which \( W_q(r) \) is the spin-orbit interaction part for the nucleons as given in Refs. [45, 49]. The central mean fields \( V_q(r) \), corrected by the effective-mass terms following the procedure described in [40, 42, 50] are
\[ V_N = V_{N}^{\text{SHF}} + \frac{\partial \epsilon_{N\Lambda}}{\partial \rho_N} m_{\Lambda} \left( \epsilon_{N\Lambda} - \frac{\hbar^2}{2m_{\Lambda}} \right) - \frac{3}{2} \left( \frac{\rho_N}{m_{\Lambda}} \right)^{2/3}, \] (4)
\[ V_{\Lambda} = \frac{\partial \epsilon_{N\Lambda}}{\partial \rho_N} \left( \frac{m_{\Lambda}}{m_{\Lambda}(\rho_N)} - 1 \right) \left( \frac{3\rho_N}{2m_{\Lambda}} \right)^{2/3}. \] (5)

The occupation probabilities \( n_q^i \) (for nucleons only) in Eq. (2) are calculated by taking into account pairing interactions within a BCS approximation. In this work, the pairing interaction is taken as a density-dependent \( \delta \) interaction [52],
\[ V_q(r_1, r_2) = V_0 \left[ 1 - \frac{\rho_N((r_1 + r_2)/2)}{0.16 \text{ fm}^{-3}} \right] \delta(r_1 - r_2). \] (9)

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The parameters \( \epsilon_1, \ldots, \epsilon_6 \) in Eq. (6) and the \( \Lambda \) effective-mass parameters \( \mu_1 \) were determined in Brueckner-Hartree-Fock calculations of hypernuclear bulk matter with the Nijmegen potential NSC97f [41, 42], while the empirical expression involving the parameter \( \epsilon_7 \) in Eq. (7) has been proposed by fitting the bond energy of \( ^6_{\Lambda\Lambda}\text{He} \) in Ref. [50]. All parameters are listed in Table I. This procedure gives a good description of the binding energies of single- and double-\( \Lambda \) hypernuclei [40, 42, 50].

| Parameter | Value   |
|-----------|---------|
| \( \epsilon_1 \) | 384     |
| \( \epsilon_2 \) | 1473    |
| \( \epsilon_3 \) | 1933    |
| \( \epsilon_4 \) | 635     |
| \( \epsilon_5 \) | 1829    |
| \( \epsilon_6 \) | 4100    |
| \( \epsilon_7 \) | 33.25   |
| \( \mu_1 \) | 0.93    |
| \( \mu_2 \) | 2.19    |
| \( \mu_3 \) | 3.89    |

### Table I. Parameters of the NSC97f+EmpC functionals [41, 50] of energy density and \( \Lambda \) effective mass, Eqs. (6,7,8), used in this work.
FIG. 2. Potential energy surfaces as functions of quadrupole deformation $\beta_2$ calculated by the self-consistent deformed SHF method for even-even nuclei ranging from $^8$Be to $^{40}$Ca and their corresponding multi-$\Lambda$ ($n_\Lambda = 2, 4, 6, 8$) hypernuclei. Energies are normalized with respect to the binding energy of the absolute minimum for a given isotope. Positive (negative) values of $\beta_2$ correspond to prolate (oblate) deformation.

III. RESULTS

Due to the lack of experimental data of multi-$\Lambda$ hypernuclei, we compare in Fig. 1 the average $\Lambda$ binding energy of multi-hyperon hypernuclei, $\langle B_\Lambda \rangle \equiv B_{n\Lambda}/n$, with that of experimental single-$s_\Lambda$ hypernuclei, double-$s_\Lambda\Lambda$ hypernuclei, and single-$p_\Lambda$ hypernuclei. Both theoretical and experimental results show that $\langle B_\Lambda \rangle$ decreases with $A^{-2/3}$. Due to the weak $\Lambda\Lambda$ interaction, the $\langle B_\Lambda \rangle$ values of double-$s_{\Lambda\Lambda}$ hypernuclei are very close to those of single-$s_\Lambda$ hypernuclei. For a given isotope, $\langle B_\Lambda \rangle$ decreases with increasing hyperon number, since the higher s.p. orbits are being filled. As a consequence, the $\langle B_\Lambda \rangle$ of 8-$\Lambda$ hypernuclei are close to those of the single-$p_\Lambda$ hypernuclei. These comparisons are rather qualitative, and experimental binding energies of multi-$\Lambda$ hypernuclei are necessary to do a strict evaluation of the current theoretical calculations. Nevertheless, as the energies of single- and double-$\Lambda$ hypernuclei are reasonably well reproduced, we continue the analysis for other quantities based on the current model.

The impurity effect of additional hyperons in single-$\Lambda$ or double-$\Lambda$ hypernuclei is usually reflected by the shape shrinkage or deformation reduction of the nuclear core. To study the impurity effect of multi-$\Lambda$ systems, we show in Fig. 2 the calculated potential energy surfaces as functions of the quadrupole deformation $\beta_2$ for even-even nuclei ranging from $^8$Be to $^{40}$Ca and their corresponding multi-$\Lambda$ ($n_\Lambda = 2, 4, 6, 8$) hypernuclei. All energies are normalized with respect to the binding energy of the absolute minimum for a given isotope. Apart from $^{12}$C, $^{32}$S, $^{36}$Ar, and the doubly magic nuclei $^{16}_{n\Lambda}$O and $^{40}_{n\Lambda}$Ca ($n = 0, 2, 4, 6, 8$), all other (hyper)nuclei are well deformed. $^{12+}_{n\Lambda}$C ($n = 2, 4, 6$), $^{28+}_{n\Lambda}$Si ($n = 0, 2, 4, 6, 8$), and $^{42}_{n\Lambda}$Ar are oblately deformed, while the others are prolately deformed.

One observes that the impurity effects become stronger with more hyperons involved, but the dependence is not regular. For 2$\Lambda$ and 8$\Lambda$ hypernuclei, the impurity effect gives similar results of deformation reduction as in the case of single-$\Lambda$ hypernuclei. This observation is the same as that obtained by
the RMF model in Ref. [24]. However, for 4$\Lambda$ and 6$\Lambda$ hypernuclei, the opposite impurity effects can be seen, namely the deformations of the hypernuclei become larger than those of the core nuclei. The energy differences between the prolate and oblate local minima in Ne, Mg, and Si isotopes are smaller than 2 MeV, which characterize them as typical nuclei with shape-coexistence phenomenon [55]. With the addition of hyperons, not only these nuclei retain their shape coexistence, but also other hypernuclei, such as C, O, S, and Ar, can develop the shape-coexistence phenomenon.

In order to achieve a microscopic understanding of the behavior of $\langle B_\Lambda \rangle$ in Fig. 1 and the impurity effects of multi $\Lambda$'s on the deformation in Fig. 2, we take $^{48}$Ca as example, and show in Fig. 3 the s.p. energies of $\Lambda$ hyperons as a function of $\beta_2$, and in Fig. 4 the density distributions at $\beta_2 = 0$ as functions of $r$ ($z = 0$) and $z$ ($r = 0$) for the occupied $s$ and $p$ $\Lambda$ s.p. orbits. Note that the $z$ axis is the symmetry axis.

Fig. 3 shows that the [000]$1/2^+$ s orbit is the lowest $\Lambda$ s.p. energy level with a spherical density distribution concentrated at the center, as seen in Fig. 4. As two hyperons can occupy this level, and their mutual interaction is small, the $\langle B_\Lambda \rangle$ values of double-$\Lambda$ hypernuclei are very close to those of single-$\Lambda$ hypernuclei.

Regarding the three negative-parity $p$ states, Fig. 3 shows that the [101]$1/2^-$ and [101]$3/2^-$ orbits are degenerate as the spin-orbit interaction is neglected in the $\Lambda\Lambda$ channel. Their s.p. energies are lower than those of the [110]$1/2^-$ orbit on the oblate side, but higher on the prolate side. Therefore a partial filling of the $p$ states (4$\Lambda$ and 6$\Lambda$ hypernuclei) allows to lower the total energy by increasing the magnitude of the deformation, whereas a complete filling (8$\Lambda$ hypernuclei) does not exhibit this feature. As shown in Fig. 4, the [110]$1/2^-$ orbit is prolate with zero density at $z = 0$, while the degenerate [101]$1/2^-$ and [101]$3/2^-$ orbits are both oblate with zero densities at $r = 0$. When the 8 hyperons occupy fully the three $p$ orbits, their density distribution becomes spherical.

A more detailed visualization of the effects of hyperons on the deformation of hyperisotopes is given in Fig. 5, which shows the $\Lambda$ density distribution in the $(r, z)$ plane. We choose the prolate Ne, the oblate Si, and the spherical Ca hyperisotopes as examples. It can be seen that the density distribution of double-$\Lambda$ hypernuclei changes in accordance with the deformation of the core nuclei, since the additional double-$\Lambda$ occupy the spherical [000]$1/2^+$ orbital.

When 4 $\Lambda$ hyperons are filled in, the shape of the first $p$ orbit occupied by the hyperons is the same as that of the core nuclei. For example, the hyperons of $^{24}$Ne with prolate deformation first fill into the [110]$1/2^-$ orbit, which is also prolate, and then gradually fill into the [101]$1/2^-$ and [101]$3/2^-$ orbits, which are oblate. Thus the deformation of $^{24}$Ne reaches the largest value due to the maximal distribution of 4 $\Lambda$ hyperons in the prolate orbit. When the hyperons begin to fill into the oblate orbits, a reduction of the deformation occurs in $^{26}$Ne. Finally, the spherical distribution of 8$\Lambda$'s renders also the core nucleus more spherical.

The hyperons in $^{28}$Si hyperisotopes with oblate core nucleus first fill into the degenerate oblate orbits [101]$1/2^-$ and [101]$3/2^-$. Therefore, the deformation increase can last up to 6$\Lambda$ hypernuclei, when the deformation reaches the maximum. Then hyperons will fill into the prolate [110]$1/2^-$ orbit, and cause a reduction of the deformation. This also explains the different trends of deformation of prolate and oblate hyperisotopes with the increasing hyperon number as shown in Fig. 2.

However, for spherical core nuclei, such as $^{16}$O and $^{40}$Ca, the 4$\Lambda$ hypernuclei have no preference for oblate or prolate orbits; therefore their deformation trends have the characteristics of both oblate and prolate hypernuclei. As a consequence, they show a more or less soft potential energy surface around the spherical shape, in particular in the $^{16}$O hypernuclei as shown in Fig. 2.
IV. SUMMARY

In summary, we study within the deformed SHF formalism the impurity effects of $\Lambda$ hyperons on the deformation of even-even nuclei ranging from $^8$Be to $^{40}$Ca, employing an effective $\Lambda N$ interaction that reproduces well the experimental binding energies of single-$\Lambda$ hypernuclei.

The effects of $\Lambda$ hyperons on the deformation of the core nuclei are studied in detail. Those deformations are generally reduced by adding 2 or 8 $\Lambda$ hyperons, while they are enhanced by adding 4 or 6. These behaviors are interpreted in a microscopic manner by analyzing the $\Lambda$ s.p. orbits and the density distributions for the occupied ones. It is demonstrated that the order of filling the hyperons into the $p$ orbits is determined by the shape of the core nucleus. When the core nucleus is oblate, the hyperons are filled first into the degenerate oblate $[101]1/2^-$ and $[101]3/2^-$ orbits, so that the deformation of the core nucleus increases to the oblate side, and can reach a maximum when 6 hyperons are added. When the core nucleus is prolate, the hyperons are filled first into the prolate $[110]1/2^-$ orbit, which leads to an increase of the prolate deformation of the core nucleus, and the deformation reaches a maximum when 4 hyperons are added. When the core nucleus is spherical, hyperons have no preferences for the shape of the $p$ orbits and result in a soft potential energy surface.

Future experimental data of multi-$\Lambda$ hypernuclei are necessary to examine these effects and refine the assumptions and ingredients of the model calculations presented here.

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