π—shifted magnetoconductance oscillations in mesoscopic superconducting-normal-heterostructures

V.N. Antonov¹, H. Takayanagi¹, F.K. Wilhelm², and A.D. Zaikin²–³

¹ NTT Basic Research Laboratories, 3-1 Morinosato-Wakamiya, Atsugi-Shi, Kanagawa 243-01, Japan
² Institut für Theoretische Festkörperphysik, Universität Karlsruhe, D-76128 Karlsruhe, Germany
³ P.N. Lebedev Physics Institute, Leninskie Prospect 53, 117924 Moscow, Russia

Interference of proximity induced superconducting correlations in mesoscopic metallic rings is sensitive to the magnetic flux Φ inside these rings. This is the reason for magnetoconductance oscillations in such systems. We detected experimentally and explained theoretically a novel effect: the phase of these oscillations can switch between 0 and π depending on the resistance of intermetallic interfaces and temperature. The effect is due to a nontrivial interplay between the proximity induced enhancement of the local conductivity and the proximity induced suppression of the density of states at low energies.

In the past few years, mesoscopic superconductor-normal metal (SN) structures have attracted lots of theoretical and experimental interest. Due to substantial progress in fabrication technology, nanoscale structures with excellent inter-metallic interfaces were fabricated, allowing to study the “coherent” dissipative conductance within the normal metal, which is increased by the proximity effect. Although it was believed that the proximity effect was already understood for many years, many surprising observations such as non-monotonic “reentrant” conductance and long-range coherent phenomena were observed experimentally and explained theoretically.

It was demonstrated that in the presence of proximity induced correlations a gap in the quasiparticle spectrum of a diffusive normal metal develops at low energies. The typical value of this gap is set by the Thouless energy $E_{Th} = D/d^2$, where $D$ is the diffusion constant and $d$ is the relevant geometric size of the N-metal. If a multiply connected SN-structure is put into a magnetic field, the proximity induced superconducting correlations may interfere destructively. As a result the proximity effect gets weakened and the correlation-induced gap in the N-metal may be lifted. This opens a possibility to control the system conductance $G$ by an externally applied magnetic flux $Φ$. The effect of the magnetic field may be twofold. It is well known that in the presence of a good metallic contact between S- and N-metals the proximity induced superconducting correlations always increase the conductance of a quasi-one-dimensional SN structure as compared to that of the N-metal. Thus in this case destruction of proximity induced correlations in the magnetic field leads to a decrease of $G$ with increasing magnetic flux for sufficiently small $Φ$. On the other hand, if one attaches tunneling electrodes to the normal metal with proximity induced correlations (e.g. like it was done in), the corresponding tunneling conductance is suppressed due to the presence of the proximity induced gap in the spectrum of the N-metal. In this case lifting the gap by applying the external flux $Φ$ will increase the conductance with increasing magnetic flux.

In the present paper, we report experiments in a multiply-connected SN-heterostructure, probed through highly transparent metallic contacts as well as through low transparent tunneling barriers. This allows us to study the two above phenomena and their interplay within the same measurement. We observe oscillations of $G$ as a function of the flux $Φ$, which can be π-shifted depending on the probes. We also develop a theoretical description of the observed phenomena.

Experiment. The key point of our work is the design of the experimental sample. The central object is a rectangular loop with opposite superconducting (Al) and normal (Ag) edges (see Fig. 1). The superconductor induces correlations into the whole sample due to the proximity effect. The strength of the proximity effect can be controlled by the flux $Φ$ through the loop, as it was discussed above. There are two probes $N−N′$ and $N′−N′$ connected to the normal edges of the loop. $N−N$ is brought into a good metallic contact to the structure, whereas $N′−N′$ contains two breaks of length $0.05−0.1μm$ near the measurement leads, which are bridged by the alu-

![FIG. 1. Scanning electron microscope picture of the structure studied; $N−N′$, $N′−N′$ and $S−S$ identify three different configurations (probes) for the resistance measurement. The schematic insets show the design of the NS interface in the loop and the breaks in the $N′−N′$ probe.](image-url)
minum strips. These breaks serve as artificial tunneling barriers which decouple the silver wire from the external reservoirs. The extra resistance of $N' - N'$ varies between 1 and 5 $\Omega$ depending on the sample. These resistive barriers play the key role in our experiment. For its best performance it is desirable to have these barriers as high as possible. Unfortunately, their resistance cannot be done very large due to technological reasons. In the fabrication process, the aluminum strips are defined simultaneously with the aluminum part of the loop. Since a strong proximity effect is needed, the contact resistance between the silver and the aluminum films should be low. To resolve the trade off between these two requirements we define different overlapping areas for the two metals in the loop and in the breaks (0.2 x 0.25 $\mu m$ versus 0.1 x 0.1 $\mu m$ respectively). This helps to keep the $SN$ interface resistance in the loop below 1$\Omega$. Details of the fabrication process can be found elsewhere.

![FIG. 2. Magnetooconductance curves of the two probes: N−N (dashed), N'−N' (triangles and solid). The curves are taken at $T = 0.3K$ (dashed and triangles) and at $T = 0.9K$ (solid). All curves are brought to zero at zero flux.](image)

The basic parameters for the silver film are as follows: the thickness is 400 $\AA$, the width of the wires 0.1$\mu m$, the sheet resistance 0.2 $\Omega$ and the phase breaking length 1$\mu m$ at 0.3K. The sample has extra leads to measure the current and the voltage also inside the loop. We identify this as S−S probe in Fig. 1. We study the magnetoconductance of the normal, $N − N$ and $N' − N'$, and superconducting, $S − S$, probes between 0.3K and 1K. A lock-in technique with a low frequency $f = 183Hz$ and a measuring current of 1$\mu A$ amplitude is used to measure the conductance. The magnetic field was produced by a superconducting solenoid and applied perpendicular to the sample plane.

The magnetoconductance curves of the two probes are presented in Fig. 2. In a small magnetic field, all curves show oscillations with $\Phi$ with a period equal to the flux quantum $\Phi_0 = \hbar c/2e$ (the effective loop area was defined by the center lines of the conductors). At low temperature the oscillations in the low resistive $N − N$ probe have a maximum at $\Phi = 0$ as predicted theoretically. An overall envelope of the curves is due to the pair-breaking effect of the field penetrating into the normal conductors. In the $N' − N'$-probe the phase of the oscillations is shifted by $\pi$ as compared to that in the $N − N$ probe. This is also in accordance with our theoretical predictions. At higher temperatures, a half-period component shows up flipping the sign of the oscillations to that of the $N − N$ probe. This effect is also not surprising: at higher $T$ the contribution of quasiparticles with energies above the gap $E \gtrsim E_{Th}$ becomes important (we estimate $E_{Th} \sim 0.1K$ in the probes). For such $E$ the density of states in the N-metal is not suppressed (on the contrary, it is even enhanced in this energy interval) and the effect of the gap becomes unimportant. Under these conditions the coherent contribution to the conductance dominates and determines the phase of oscillations. We will come back to this point within our theoretical analysis presented below.

![FIG. 3. Linear conductance as a function of temperature at different tunneling resistances within the coherent/tunneling crossover at $\phi = 0$. At $\phi = \pi$ we have $G = G_N$. The inset shows the simplified structure used for calculations. All arms 1−4 are assumed to have the same geometric parameters.](image)

Theory. Let us consider a structure which consists of an SNS-junction with extra arms and normal reservoirs attached to these arms via the tunnel barriers $R_T$ (see the inset of Fig. 3). This structure captures all essential features of our system. A finite voltage drop exists between the normal reservoirs, and the phase difference $\phi$ between the two superconducting terminals can be related to the flux through the loop by a simple gauge transformation $\phi = \pi \Phi/\Phi_0$. In order to proceed we will use the standard formalism of the quasiclassical Green-Keldysh functions in the diffusive limit (see e.g. Refs. 3). The retarded Green functions describing the spectral properties of the system can be conveniently parameterized as $G^R = \cosh \alpha$, $F^R = \sinh \alpha e^{i\chi}$. These functions obey the
Usadel equation. In the normal metal it takes the form
\[
D \partial_x^2 \alpha = -2i\epsilon \sinh \alpha - (D/2) (\partial_x \chi)^2 \sinh 2\alpha \\
\partial_x j_x = 0 \quad , \quad j_x = (\partial_x \chi) \sinh^2 \alpha. \quad (1)
\]
No supercurrent is flowing in the arms 3 and 4, therefore in these arms we have \(j_x = 0\). The equations (1) are supplemented by the boundary conditions
\[
\alpha|_S = -i\pi/2; \quad dr_T \partial_x \alpha|_N = \sinh \alpha|_N \quad (2)
\]
where \(r_T = R_T/R_N\) and \(R_N\) is the normal state resistance of the arm 3 or 4. The branching conditions at the nodal point read:
\[
\sum_{i=1}^4 \partial_i \alpha = 0; \quad \sum_{i=1}^2 \partial_i \chi = 0
\]
where \(\partial\) is the derivative in the direction of the respective branch away from the node.

The dissipative current in our system
\[
j_N = \sigma_N \int_0^\infty d\epsilon D_T(\epsilon) \nabla f_T(\epsilon) \quad (3)
\]
is determined by the local spectral conductivity \(D_T = \cosh^2 \text{Re} \alpha\) and the asymmetric component of the distribution function \(f_T(\epsilon) = f(\epsilon) + f(-\epsilon)\). Solving the kinetic equations for the side-arms, where no supercurrent is flowing, we obtain \(f_T(\epsilon)\) and the linear conductance
\[
\frac{G}{G_N} = \frac{1}{d} \int_0^\infty d\epsilon g(\epsilon) \cosh^2 (\epsilon/2T); \quad g(\epsilon) = \frac{1 + r_T}{M(\epsilon) + r_T/\nu_T(\epsilon)} \quad (4)
\]
\[
M(\epsilon) = \frac{1}{d} \int_{\text{Arm 3}} dx \cosh^2 (\text{Re} \alpha); \quad \nu_T(\epsilon) = \text{Re} (\sinh \alpha)|_{R_T} \quad (5)
\]
These results describe the interplay between two effects: conductance enhancement due to the proximity induced correlations coming from the S-metals (this effect is described by \(M(\epsilon)\)) and its suppression due to the proximity induced gap (the latter enters via the density of states \(\nu\) reduced below the gap).

**Results and discussion.** Let us consider the two opposite limits of small and large \(r_T\). For \(r_T \ll 1\) the tunnel barrier is irrelevant, and the voltage drops across the metal wire. Since the local conductivity is enhanced due to the proximity effect everywhere in the wire, its total conductance increases. Also the gap effect is softened: in this case a direct contact to a normal reservoir prevents from forming a real gap in the spectrum and only a space-dependent pseudogap in the density of states of the N-metals develops. This pseudogap vanishes in the vicinity of a normal reservoir.

At \(r_T \gg 1\) the situation is entirely different. Now the main voltage drop occurs at the tunnel barrier, and the enhancement of the local conductivity of the N-metal becomes unimportant. In this case the proximity induced gap in the density of states dominates the system behavior leading to a decrease of the system conductance at low energies. The coherent contribution to the conductance described by \(M(\epsilon)\) is relevant at higher \(\epsilon\).

The two regimes of small and large \(r_T\) as well as the crossover between them are described within our numerical analysis. At \(\phi = 0\), a typical non-monotonic "reentrant" behavior is reproduced, whereas at \(r_T = 10\) the tunneling dominates, see Fig. 3.

It can be also seen that, in the latter case, the \(G(\phi)\)-relation is highly non-sinusoidal and even has an edge around \(\phi = \pi\). This is also detected in our experiment, see Fig. 2.

The interplay between the phase-coherent and the tunneling dominated contributions to the conductance for different values of \(r_T\) and at different temperatures can be clearly observed in our results. In Fig. 3 we show the total conductance at zero flux for different values of the tunneling resistance. The conductance at \(\phi = \pi\) is temperature independent and equals to \(G_N\) in all cases. The intermediate curves illustrate that – although at low temperatures the conductance suppression due to the gap effect dominates and therefore \(G < G_N\) – at higher \(T\) we have \(G > G_N\) because for sufficiently large \(\epsilon\) the gap plays no role and the enhancement of a local conductivity due to the proximity effect turns out to be more important.

For a quantitative comparison with the experiment, we can also generalize our analysis allowing for a finite resistance \(R_{SN}\) of the SN-interface in the loop. Comparing the corresponding numerical results with our experimental data for \(G(T)\) at \(\phi = 0\), we found that the best fit is achieved at \(R_{SN} \approx 0.5\Omega\) in the loop and \(R_T \approx 2.3\Omega\).
at the barrier in the $N' - N'$ probe. These values are in reasonable agreement with the directly measured values $R_{SN} = (0.7 \pm 0.3)\Omega$ and $R_T \approx 2.5\Omega$.

![Graph showing magnetoconductance oscillations](image)

**FIG. 5.** Magnetoconductance oscillations in the $S - S$ probe of the samples with $R_{SN} \approx 4\Omega$ and $R_{SN} \approx 0.7\Omega$ (inset). The curves are brought to zero at a zero flux.

It was found in our experiment (see Fig. 4) that the normal conductance of a metal between two superconductors is also modulated by the flux and that conductance oscillations have different phases, depending on the value of $R_{SN}$ similarly to the case of the $N - N$ channel considered above. At $R_{SN} \approx 4\Omega$, we find a minimum at zero-flux and at $R_{SN} \approx 0.7\Omega$ a maximum. In the former case the proximity effect in the side arms was extremely weak and $\delta G(\Phi)$ oscillations could hardly be detected. We can also add that at low $T$ our treatment (developed for the $N - N$ channel) is not sufficient for a quantitative analysis of the charge transport in the $S - S$-channel, in which case the Josephson current between the two superconductors should also be taken into account. This current may be strongly influenced by the non-equilibrium effects. At higher $T \gg E_B$, the supercurrent is exponentially suppressed whereas the coherent contribution to the conductance decays only algebraically.

In this regime the modulation effects in the $S - S$ and $N - N$ channels are similar.

In summary, we detected a $\pi$-shift in the magnetoconductance oscillations of a multiply connected mesoscopic normal metal-superconductor structure. This shift is sensitive to the value of the resistance of intermetallic interfaces and temperature. The observed effect is explained as a result of competition between the proximity induced enhancement of the local conductivity and the proximity induced suppression of the density of states at low energies. A good agreement between our experimental and theoretical results is found.

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