Magical properties of 2540 km baseline Superbeam Experiment

Sushant K. Raut, Ravi Shanker Singh∗, and S. Uma Sankar†

Department of Physics, Indian Institute of Technology Bombay, Mumbai 400076, India

(Dated: June 3, 2010)

Abstract

Lack of any information on the CP violating phase $\delta_{CP}$ weakens our ability to determine neutrino mass hierarchy. Magic baseline of 7500 km was proposed to overcome this problem. However, to obtain large enough fluxes, at this very long baseline, one needs new techniques of generating high intensity neutrino beams. In this letter, we highlight the magical properties of a 2540 km baseline. At such a baseline, using a narrow band neutrino superbeam whose flux peaks around the energy 3.5 GeV, we can determine neutrino mass hierarchy independent of the CP phase. For $\sin^2 2\theta_{13} \geq 0.05$, a very modest exposure of 10 Kiloton-years is sufficient to determine the hierarchy. For $0.02 \leq \sin^2 2\theta_{13} \leq 0.05$, an exposure of about 100 Kiloton-years is needed.

PACS numbers: 14.60.Pq,14.60.Lm,13.15.+g

Keywords: Neutrino Mass Hierarchy, Long Baseline Experiments

∗ Address after August 17, 2009: Department of Physics, Brown University, Providence, R.I., USA
† Corresponding Author
I. INTRODUCTION

Neutrino experiments in the last decade have determined a number of neutrino parameters to a good accuracy. Among the currently unknown quantities are (i) the CHOOZ mixing angle $\theta_{13}$, (ii) the sign of atmospheric mass-squared difference and (iii) the CP violating phase $\delta_{CP}$. Experiments are being designed/constructed to measure these quantities.

At present, there are three efforts to measure a non-zero value for $\theta_{13}$ using reactor neutrinos as the source. In each of these experiments, the survival probability $P(\bar{\nu}_e \rightarrow \bar{\nu}_e)$ will be measured using a pair of identical detectors, one close to the reactor and the other about a km away. The deficit in the far detector is a measure of $\sin^2 2\theta_{13}$. Since these are all disappearance experiments, they should have very low systematic uncertainties, to measure the small value of $\sin^2 2\theta_{13}$. Double-CHOOZ will start taking data soon and it will see a positive signal if $\sin^2 2\theta_{13} \geq 0.04$ [1]. Daya Bay [2] and RENO [3] are expected to improve on this measurement. Daya Bay’s final sensitivity extends up to $\sin^2 2\theta_{13} \geq 0.01$ [4].

It is possible to determine $\sin^2 2\theta_{13}$ by measuring $P(\nu_\mu \rightarrow \nu_e)$ ($P_{\mu e}$) in an accelerator experiment. T2K [5] and NO$\nu$A [6] experiments aim to do this. Even if these experiments see a positive signal, determination of $\sin^2 2\theta_{13}$ from their data will be subject to very large uncertainties because the probability $P_{\mu e}$ depends on all three unknowns mentioned above [7]. The reactor experiments, on the other hand, will give a clean measurement of $\sin^2 2\theta_{13}$ because $P(\bar{\nu}_e \rightarrow \bar{\nu}_e)$, at the relevant energies, depends only on this unknown neutrino parameter.

We label the three neutrino mass eigenstates by their respective eigenvalues $m_1$, $m_2$ and $m_3$. From the three masses, we can define two independent mass-squared differences $\Delta_{21} = m_2^2 - m_1^2$ and $\Delta_{31} = m_3^2 - m_1^2$. $\Delta_{21}$ is the mass-squared difference which drives the solar neutrino oscillations. It is known to be positive and its magnitude is much smaller than that of $\Delta_{31}$. Atmospheric neutrino oscillations are essentially driven by $\Delta_{31}$ whose magnitude is known but not the sign. If the neutrino masses follow the hierarchy $m_3 \gg m_2 > m_1$, called normal hierarchy (NH), $\Delta_{31}$ is positive. It is negative if the neutrino masses have the pattern $m_2 > m_1 \gg m_3$, called inverted hierarchy (IH). Present data allows both the possibilities. Determining the sign of $\Delta_{31}$ establishes the pattern (or hierarchy) of neutrino masses. In this letter, we propose a new scheme to realise this goal.

Neutrino propagation through dense matter, leads to an effective mass-squared term
(usually called the matter term) in their Hamiltonian \[8\]. The interference of this term with the original mass-squared differences leads to the modification of the neutrino masses and mixing angles and hence their oscillation probabilities. The change induced by the interference, of course, depends on the sign of the original mass-squared difference and hence is different for \(NH\) and \(IH\). By measuring this change in the oscillation probability \(P_{\mu e}\), induced by the matter term, we can determine the mass hierarchy.

\(P_{\mu e}\) depends on all three unknowns of neutrino physics. Hence it is very difficult to determine which of the three unknowns causes a change in \(P_{\mu e}\). Measurement of \(\theta_{13}\) in reactor neutrino experiments leads to disentanglement of one parameter. The CP violating phase \(\delta_{CP}\), in principle, can be determined by measuring the difference in oscillation probabilities of neutrinos and anti-neutrinos. However, this method can not be used to disentangle \(\delta_{CP}\) and the matter term because the matter term changes sign under CP and induces a ‘CP violation-like’ change in the probabilities. Therefore a new strategy is needed to separate these two interlinked effects.

A radical proposal was made sometime ago to disentangle \(\delta_{CP}\) from the matter term in \(P_{\mu e}\). The expression for \(P_{\mu e}\) contains three terms and only one of them is dependent on \(\delta_{CP}\). If this term can be made to vanish, by an appropriate choice of neutrino energy and baseline, then it is possible to determine neutrino hierarchy without any information on \(\delta_{CP}\). Calculation of the baseline, called \textit{magic baseline}, gives an answer \(L \approx 7500\) km \[9\] and it is independent of energy \[10\]. Having sufficient neutrino fluxes at such a large distance from the source will be very difficult with the current accelerator technology.

In this letter, we make an alternative proposal of a much shorter \textit{magical} baseline. In the original magic baseline proposal, the condition that the \(\delta_{CP}\) dependent term vanish, holds both for \(NH\) and \(IH\). This condition is quite restrictive and leads to such a large baseline. We propose an alternative condition which demands that the \(\delta_{CP}\) terms should vanish only for \(IH\). This leads to a relation between the neutrino energy \(E\) and the baseline \(L\). In addition, we also demand that \(P_{\mu e}\) should be large for \(NH\). This leads to a different condition on \(E\) and \(L\). Solving these two equations gives us the solutions \(L = 2540\) km and \(E = 3.3\) GeV. At this energy, for this baseline, \(P_{\mu e}\) is very small for \(IH\) and near the maximum for \(NH\), for any value of \(\delta_{CP}\). Thus a neutrino beam with its flux maximum at this energy can make a clean measurement of neutrino mass hierarchy, independent of \(\delta_{CP}\).
II. CALCULATION

A very good approximate expression for $P_{\mu e}$, for three flavour oscillations including matter effects, is usually given as an expansion in the small parameter $\alpha = \Delta_{21}/\Delta_{31}$. It can be written as \[11\]

$$P_{\mu e} = C_0 \frac{\sin^2((1 - \hat{A})\Delta)}{(1 - \hat{A})^2} + \alpha C_1 \frac{\sin((1 - \hat{A})\Delta)}{(1 - \hat{A})} \frac{\sin(\hat{A}\Delta)}{\hat{A}} + \alpha^2 C_2 \frac{\sin^2(\hat{A}\Delta)}{\hat{A}^2},$$

where $\Delta = (1.27\Delta_{31}L/E)$ and $\hat{A} = A/\Delta_{31}$. The matter term $A$ (in eV$^2$) = $0.76 \times 10^{-4}\rho$ (gm/cc) $E$ (GeV) eV$^2$. $\rho$ is the density of matter through which the neutrino propagates. Here $\Delta_{31}$ is given in units of eV$^2$, $L$ is in km and $E$ is in GeV. The coefficients, $C_i$ are given by

$$C_0 = \sin^2 \theta_{23} \sin^2 2\theta_{13}$$

$$C_1 = \cos \theta_{13} \sin 2\theta_{12} \sin 2\theta_{13} \sin 2\theta_{23} \cos(\Delta + \delta_{CP})$$

$$C_2 = \sin^2 2\theta_{12} \cos^2 \theta_{23}$$

We note that only $C_1$ among them depends on the phase $\delta_{CP}$.

$P_{\mu e}$ depends on all the three unknowns. Data on $P_{\mu e}$ from a single experiment leads to a degenerate set of solutions \[12\]. Data on $P_{\mu e}$ from experiments with different baselines can resolve some of these degeneracies \[13\]. Here we assume that $\theta_{13}$ will be measured in reactor neutrino experiments \[1-3\], which resolves the degeneracies involving this parameter. Our proposal in this letter makes the hierarchy-$\delta_{CP}$ degeneracy irrelevant.

In Eq. (1) the dependence on the matter term $\hat{A}$ is explicitly displayed. $\Delta_{31}$ is positive for NH and is negative for IH. $A$, on the other hand, is positive for neutrinos and is negative for anti-neutrinos. Thus, if we have only a neutrino beam, then $\hat{A}$ is positive for NH and is negative for IH. For anti-neutrino beam, the situation is reversed. In Eq. (1), the first term is the most sensitive to hierarchy but the second term provides a significant correction. As one varies $\delta_{CP}$ in its range, the change in the second term can cancel the change in the first term caused by the change in hierarchy. In other words, there exist two degenerate
sets of solutions, (NH and $\delta_{CP} = \delta_1$) and (IH and $\delta_{CP} = \delta_2$), both of which give the same value of $P_{\mu e}$ in a given experiment [12]. This leads to hierarchy-$\delta_{CP}$ degeneracy and restricts our ability to determine the hierarchy. To overcome this, it was proposed to choose a baseline and energy for which $\sin(\hat{\Delta}) = 0$, so that the second and third terms vanish. The above constraint, for the first non-trivial zero, gives the magic baseline condition $L \approx 7500$ km, independent of the energy [9]. The energy can be chosen by the condition that the oscillation probability be a maximum at this baseline. Therefore, one can indeed determine mass hierarchy at the magic baseline in a clean manner, independent of $\delta_{CP}$. However, one is now faced with the problem of obtaining large enough flux at this large a distance.

We can make $P_{\mu e}$ independent of $\delta_{CP}$ by choosing either $\sin(\hat{\Delta}) = 0$ or $\sin((1 - \hat{\Delta})\Delta) = 0$ [10]. The magic baseline uses the first condition which holds true for both NH and IH for the same $L$ and is independent of $E$. The dependence of the second condition on $L$ and $E$ is different for NH and IH. We exploit this difference by choosing $L$ and $E$ such that $\sin((1 - \hat{\Delta})\Delta) = 0$ holds for IH. This makes $P_{\mu e}$ for IH not only independent of $\delta_{CP}$ but also very small because only the $\alpha^2$ term in Eq. (1) survives. We impose the simultaneous demand that, for the same $L$ and $E$, $P_{\mu e}$ for NH should be close to maximum. This leads to a substantial difference in $P_{\mu e}$ for NH and IH and enables us to determine the neutrino mass hierarchy for all values of $\delta_{CP}$, even for relatively small values of $\theta_{13}$. The condition on $P_{\mu e}$ for IH translates into $1.27(|\Delta_{31}| + \alpha) L/E = \pi$ whereas that for NH becomes $1.27(|\Delta_{31}| - \alpha) L/E = \pi/2$. Solving the above two equations, we get $L = 2540$ km and $E = 3.3$ GeV, for $|\Delta_{31}| = 2.5 \times 10^{-3}$ eV$^2$ [14]. Note that $L$ is determined purely by $\alpha$ whereas $E$ is determined by both $\alpha$ and $\Delta_{31}$. There is an uncertainty in $L$ which is equal to the uncertainty in $\alpha$.

The large difference between NH and IH is illustrated in Fig. 1, where we have plotted $P_{\mu e}$ as a function of $E$ for $\sin^2 2\theta_{13} = 0.02$. The figure shows $P_{\mu e}$ as a band with $\delta_{CP}$ spanning the entire $0^\circ - 360^\circ$ range, for both NH and IH. As claimed above, in the neighbourhood of $E = 3.3$ GeV, $P_{\mu e}$ is about 0.002 for all $\delta_{CP}$ for IH whereas it is 6 to 18 times larger (depending on $\delta_{CP}$) in the case of NH. This large difference can be measured if there is a substantial neutrino flux at $E = 3.3$ GeV. Note that the value of $\sin^2 2\theta_{13}$ chosen here is very small and is barely above the detectable limit of the experiments under construction. But even for such small values of $\theta_{13}$, the configuration suggested here can make a distinction between NH and IH, independent of $\delta_{CP}$ and using neutrino beam only. It is worth remarking that
FIG. 1: $P_{\mu e}$ as function of $E$ for $L = 2540$ km and $\sin^2 2\theta_{13} = 0.02$. It is plotted for both NH and IH, in each case as a band. Within each band, $\delta_{CP}$ varies in the range $0^\circ - 360^\circ$.

2540 km is the distance between the Brookhaven Laboratory and the Homestake Goldmine [15]. The design for a neutrino superbeam, with flux peaking near 3.3 GeV, already exists. For example: NuMI beam with medium energy option at locations 7mr off-axis, has its peak flux at 3.5 GeV [6].

In this letter, we consider the following configuration. We assume that the $\nu_\mu$ source is located at Brookhaven which produces a NuMI-like beam with 0.8 megawatt beam power. This corresponds to $7.3 \times 10^{20}$ protons on target (POT), with energy 120 GeV, per year [6]. We assume a 300 Kton water Cerenkov detector at Homestake 2540 km away. We also assume that the orientation of the beamline is such that the detector location is 7mr off-axis. In our calculations, we take the NuMI fluxes and scale them appropriately to obtain the event numbers at this distance and for this off axis location [16]. Our signal is electron appearance in the far detector, due to $\nu_\mu \rightarrow \nu_e$ oscillations. Single $\pi^0$ events produced by neutral current interactions form a potentially huge background to this signal. The visible
energy of the neutral current events is usually much smaller than the true energy of the neutrino and hence they can be suppressed by a large factor \[15\]. The remainder of these neutral current events, together with the electron events produced by beam $\nu_e$ form the actual background. This background was estimated in a previous study to be about 1% of the unoscillated events \[15\]. We include this background in our study. We calculate the event rates for the energy range $1 - 10$ GeV, in bins of width 0.4 GeV and smear the obtained event distribution with a Gaussian probability in energy with $\sigma_E = 0.15E$. We define the statistical $\chi^2$ between the event distribution for NH and that for IH, by

$$
\chi^2_{\text{stat}} = \sum_{i=\text{bins}} \frac{(N_{i}^{TR} - N_{i}^{TE})^2}{N_{i}^{TR}}. 
$$

\(N_{i}^{TR}\) is the number of events for the true hierarchy plus the number of background events. \(N_{i}^{TE}\) is the number of events for the test hierarchy, which is the opposite of the true hierarchy, plus the number of background events. The true hierarchy can be either NH or IH and we consider both possibilities. In calculating the event distributions, the following values of neutrino parameters are used: $\Delta_{21} = 8 \times 10^{-5}$ eV$^2$, $|\Delta_{31}| = 2.5 \times 10^{-3}$ eV$^2$, $\sin^2 \theta_{12} = 0.31$ and $\sin^2 \theta_{23} = 0.5$ \[17\]. We do the calculation for various different input values of $\sin^2 2\theta_{13}$ starting from 0.1 and going down to 0.02 in steps of 0.01. $\delta_{CP}$ is varied from 0 to 360° in steps of 45°.

We also assume a 2% systematic uncertainty in the neutrino flux and a similar uncertainty in detector systematics. The systematic uncertainty in the cross section is taken to be 10\% \[6\]. These are taken into account through the method of pulls as described in \[18–20\]. In this method, the fluxes, cross sections etc are taken to be the central values in the computation of $N_{i}^{TR}$ but are allowed to deviate from their central values in the computation of $N_{i}^{TE}$. We assume that the $k^{th}$ input deviates from its central value by $\sigma_k \, \xi_k$, where $\sigma_k$ is the uncertainty in this input. Then the value of $N_{i}^{TE}$ with the changed inputs is given by

$$
N_{i}^{TE} = N_{i}^{TE}(\text{std}) + \sum_{k=1}^{n_{\text{pull}}} c_{i}^{k} \, \xi_k 
$$

where $N_{i}^{TE}(\text{std})$ is the expected number of events in bin $i$ for the test hierarchy, calculated using the central values of the fluxes, cross sections etc and $n_{\text{pull}}$ is the number of inputs which have systematic uncertainties. The $\xi_k$'s are called the pull variables and they determine the number of $\sigma$'s by which the $k^{th}$ input deviates from its central value. In eq. (6), $c_{i}^{k}$ is the change in $N_{i}^{TE}$ when the $k^{th}$ input is changed by $\sigma_k$ (i.e. by 1 standard deviation). The
uncertainties in the inputs are not very large. Therefore, in eq. (6) we consider only those changes in $N_{i}^{TE}$ which are linear in $\xi_{k}$. Thus we have a modified $\chi^2$ defined by

$$\chi^2(\xi_{k}) = \sum_{i} \left[ \frac{N_{i}^{TE}(\text{std}) + \sum_{k=1}^{n_{\text{pull}}} c_{k}^{i} \xi_{k} - N_{i}^{TR}}{N_{i}^{TR}} \right]^2 + \sum_{k=1}^{n_{\text{pull}}} \xi_{k}^2$$  \hspace{1cm} (7)

where the additional term $\xi_{k}^2$ is the penalty imposed for moving $k^{th}$ input away from its central value by $\sigma_{k} \xi_{k}$. The $\chi^2$ with pulls, which includes the effects of all theoretical and systematic uncertainties, is obtained by minimizing $\chi^2(\xi_{k})$, given in eq. (7), with respect to all the pulls $\xi_{k}$:

$$\chi_{\text{pull}}^2 = \text{Min}_{\xi_{k}} \left[ \chi^2(\xi_{k}) \right].$$  \hspace{1cm} (8)

In addition to taking the systematic uncertainties into account, we have marginalized over $|\Delta_{31}|$, $\sin^2 2\theta_{13}$, $\sin^2 2\theta_{23}$ and $\delta_{CP}$ but held $\Delta_{21}$ and $\theta_{12}$ fixed. In doing the marginalization, we assume that experimental uncertainties in $|\Delta_{31}|$ and $\sin^2 2\theta_{23}$ are what they are expected to be from T2K (about 2%) \cite{5}. It turns out that the marginalization over $|\Delta_{31}|$ has on $\chi^2$ no effect but marginalization over $\theta_{23}$ has a very significant effect, if IH happens to be the true hierarchy. We elucidate this point after discussing our results.

The dominant term in $P_{\mu e}$ is proportional to $\sin^2 2\theta_{13}$, which makes the marginalization over $\theta_{13}$ the most crucial one in hierarchy determination. In the neighbourhood of oscillation maximum, the matter effects increase this term for NH and decrease it for IH, relative to its vacuum value. In shorter baseline experiments such as NO$\nu$A, one expects to measure this increase/decrease and determine the hierarchy. However, it is possible to choose a $\theta'_{13}$, within the allowed range of $\theta_{13}$, such that $P_{\mu e}(\text{NH}, \theta_{13}) \simeq P_{\mu e}(\text{IH}, \theta'_{13})$ \cite{12}. In such a situation, marginalization over $\theta_{13}$ leads to a very small $\chi^2$. In our proposal, the condition $\sin((1 - \hat{A})\Delta) = 0$, makes $P_{\mu e}(\text{IH})$ very small, in the neighbourhood of $E = 3.3$ GeV, independent of both $\delta_{CP}$ and $\theta_{13}$. In this energy range, $P_{\mu e}(\text{NH})$ is close to oscillation maximum. It is also proportional to $\sin^2 2\theta_{13}$ and is quite large even for the very small value of $\sin^2 2\theta_{13}$, which is illustrated in Fig. 1. Therefore the oscillation pattern is very distinctive for each hierarchy and they can easily be distinguished for $\sin^2 2\theta_{13} \geq 0.02$. We also note that the marginalization over $\delta_{CP}$ ensures that this distinction between NH and IH exists even for the most unfavourable value of $\delta_{CP}$. Hence hierarchy determination is possible for the whole range of $\delta_{CP}$.

Because of its importance, marginalization over $\sin^2 2\theta_{13}$ was done in different stages. At present, we only have an upper limit, $\sin^2 2\theta_{13} \leq 0.1$. Double Chooz \cite{1} can measure this
parameter if it is $\geq 0.04$ and Daya Bay \textsuperscript{2} can measure it for values $\geq 0.01$. Therefore, for input values $\sin^2 2\theta_{13} : 0.05 - 0.1$ (that is if Double Chooz finds a positive result), we do marginalization only over this range. For input values $\sin^2 2\theta_{13} : 0.02 - 0.05$ (expecting a positive result from Daya Bay but not from Double Chooz) we do marginalization only over this restricted range.

It was mentioned above that the parameters $\Delta_{21}$ and $\theta_{12}$ are kept fixed. These parameters occur only in the second and third terms of $P_{\mu e}$. Given the smallness of $\alpha$, the third term is very small and varying $\Delta_{21}$ and $\theta_{12}$ within their ranges, changes this term by a very small amount. The second term undergoes much larger changes when $\sin 2\theta_{13}$ and $\delta_{CP}$ are varied over their ranges. The change due to $\Delta_{21}$ and $\theta_{12}$ is much smaller and can be neglected.

\section*{III. RESULTS}

In our calculations, we assumed that only the neutrino beam is used. Neutrino beams have an advantage over the anti-neutrino beams because of the larger cross section and hence larger statistics. In table 1, we list the exposure, in Kiloton-years (Kt-yr), needed to obtain a $3\sigma$ distinction between NH and IH if $\sin^2 2\theta_{13} \geq 0.05$. In this case, one requires only minimal exposure (less than 10 Kt-yr) to distinguish the two hierarchies. This is independent of whether the true hierarchy is NH or IH. For $0.05 \geq \sin^2 2\theta_{13} \geq 0.02$, the results are shown in table 2. Here the needed exposure, for $3\sigma$ distinction, sharply goes up with $\theta_{13}$ from 7 Kt-yr to 50 Kt-yr, if NH is the true hierarchy. If IH is the true hierarchy, the $3\sigma$ exposure becomes about 92 Kt-yr.

Note that the exposures, for NH being the true hierarchy, increase with decreasing values of $\sin^2 2\theta_{13}$ whereas, if IH is the true hierarchy, the exposures are more or less independent of the true value of $\theta_{13}$. This feature occurs due to the marginalization over $\theta_{13}$. The number of events in the region of peak flux of 3.5 GeV are strongly dependent on $\theta_{13}$ for NH, whereas they are independent of $\theta_{13}$ for IH, as can be seen from eq. (1). If NH is the true hierarchy, $N_i^{NH}$ is computed using the input value of $\theta_{13}$ but in computing $N_i^{IH}$ we vary $\theta_{13}$ in its marginalizing range. Since the difference between these two numbers, decreases with decreasing $\theta_{13}$, we need larger exposure to obtain the same $\chi^2_{\text{min}}$. If IH is the true hierarchy, then $N_i^{IH}$ is computed using the input value of $\theta_{13}$ and $N_i^{NH}$ is computed with varying values of $\theta_{13}$. Around 3.5 GeV, $N_i^{IH}$ is small and independent of $\theta_{13}$, whereas,
\[ \sin^2 2\theta_{13} \text{ (true)} \quad \text{Exposure(NH)} \quad \text{Exposure(IH)} \]

| \( \sin^2 2\theta_{13} \) (true) | Exposure(NH) | Exposure(IH) |
|----------------------------------|--------------|--------------|
| 0.1                              | 2.93         | 6.39         |
| 0.09                             | 3.34         | 6.04         |
| 0.08                             | 3.94         | 5.69         |
| 0.07                             | 4.77         | 5.38         |
| 0.06                             | 6.04         | 4.95         |
| 0.05                             | 8.19         | 4.55         |

TABLE I: Exposure in Kiloton-years required for 3\( \sigma \) hierarchy discrimination in the case where both Double Chooz and Daya Bay see a positive signal. Second (third) column shows the results if NH (IH) is the true hierarchy.

\[ \sin^2 2\theta_{13} \text{ (true)} \quad \text{Exposure(NH)} \quad \text{Exposure(IH)} \]

| \( \sin^2 2\theta_{13} \) (true) | Exposure(NH) | Exposure(IH) |
|----------------------------------|--------------|--------------|
| 0.05                             | 7.32         | 91.68        |
| 0.04                             | 10.69        | 86.80        |
| 0.03                             | 18.97        | 81.16        |
| 0.02                             | 49.83        | 71.50        |

TABLE II: Exposure in Kiloton-years required for 3\( \sigma \) hierarchy discrimination in the case where Daya Bay shows a positive signal, but Double Chooz does not.

\( N_i^{NH} \) takes its smallest value for the smallest of the \( \theta_{13} \) values in the marginalizing range. Therefore \( \chi^2_{\text{min}} \) and the exposure for 3\( \sigma \) distinction, have a much weaker dependence on the input value of \( \theta_{13} \), if IH is the true hierarchy. The small decrease in the exposure with decreasing values of \( \theta_{13} \) occurs due to the contribution of events in the energy range beyond 4 GeV. For this range, the difference between \( N_i^{IH} \) and \( N_i^{NH} \) increases as the input value of \( \theta_{13} \) is decreased. Thus \( \chi^2_{\text{min}} \) increases and exposure decreases.

For 0.02 \( \leq \sin^2 2\theta_{13} \leq 0.05 \), the exposure is rather large if IH is the true hierarchy. This occurs due to the marginalization over \( \theta_{23} \). Without marginalization over this parameter, the required exposure is about 50 Kt-yr which is similar to the largest exposure needed if NH is the true hierarchy. With marginalization, the IH spectrum is computed with \( \theta_{23} \) equal to the input value of \( \pi/4 \) whereas the NH spectrum is computed with different values of \( \theta_{23} \) in the allowed range. Given that the number of events in the case of IH is very small, the
difference between the IH spectrum and NH spectrum is smaller, when the NH spectrum is computed with a smaller value of $\theta_{23}$. Thus $\chi^2_{\text{min}}$ will be smaller and we need larger exposure to obtain a 3$\sigma$ discrimination.

The baseline 2540 km and the energy of peak flux 3.3 GeV were obtained based on the two conditions that $P_{\mu e}$ for IH should be independent of $\delta_{CP}$ (which makes it very small) and for NH it should be close to maximum. This leads to a large difference between the expected number of signal events for NH and IH. However, the fluxes fall off as $1/L^2$ with distance. Therefore it is imperative to check that the baseline 2540 km is the optimum distance to determine mass hierarchy for a neutrino beam with peak flux around 3.3 GeV. We generate the event distributions for NH and IH for different baselines but with the same source (NuMI-like beam with medium energy option at 7 mr off axis). The fluxes for different baselines are scaled by $1/L^2$. The input value of $\sin^2 2\theta_{13}$ is taken to be 0.02 and normal hierarchy is assumed to be the true hierarchy. The other parameter values and systematics are kept the same as in the earlier calculations. We compute the $\chi^2_{\text{min}}$ for hierarchy discrimination for the baselines 1000, 1500, 2000, 2540 and 3000 km, assuming an exposure of 150 Kt-ys. The results are shown in table 3. We note that the largest $\chi^2_{\text{min}}$, larger by a factor of 3, occurs for 2540 km baseline, showing that this indeed is the optimum distance, for hierarchy discrimination, for the given neutrino flux profile.

| Baseline (in km) | $\chi^2_{\text{min}}$ |
|-----------------|----------------------|
| 1000            | 0.38                 |
| 1500            | 0.54                 |
| 2000            | 9.92                 |
| 2540            | 26.20                |
| 3000            | 8.72                 |

TABLE III: Baseline length $L$ vs $\chi^2_{\text{min}}$ for hierarchy discrimination for $\sin^2 2\theta_{13} = 0.02$ if a NuMI-like beam with medium energy option at 7 mr off axis locations (peaking at 3.5 GeV) is used.

We have highlighted some very interesting properties of a 2540 km baseline experiment and showed, through a simple numerical calculation, that such an experiment is well capable of determining neutrino mass hierarchy. In our calculations, we have included a conservative estimate of background, which we take to be 1% of the unoscillated events. Despite such
background, the setup we discussed is capable of hierarchy discrimination for even quite small values of $\theta_{13}$. By imposing various kinematic cuts, the background can be suppressed at the cost of loss of some signal. This loss of signal can be compensated by having an increased exposure. However, the ability of the setup to determine the mass hierarchy will not be compromised because any such kinematic cut will lead to a larger signal to background ratio.

IV. CONCLUSION

In this letter we demonstrated the superior ability of a neutrino superbeam experiment with a baseline 2540 km, whose flux peaks in the energy range 3-4 GeV, to determine neutrino mass hierarchy. For $\sin^2 2\theta_{13} \geq 0.05$, a very modest exposure of $\leq 10$ Kiloton-years is sufficient to distinguish the two hierarchies at $3\sigma$ level. For $0.02 \leq \sin^2 2\theta_{13} \leq 0.05$, one needs an exposure $\leq 100$ Kiloton-years. These exposures are obtained analyzing the expected data from this superbeam set up only. If the data from this set up is analyzed in conjunction with the data from a reactor $\theta_{13}$ measurement experiment, then the required exposures are likely to be much less. The set up we assumed is not hard to realize because 2540 km is the distance from Brookhaven to Homestake and the technology for an accelerator beam, with peak flux in 3-4 GeV range, exists [6]. Such a set up, we believe, will have an excellent capability to measure not only small values of $\theta_{13}$ but $\delta_{CP}$ as well. These issues are currently being studied.

Acknowledgement Ravi Shanker Singh thanks BRNS project and Prof. Asmita Mukherjee for financial support. We thank Srubabati Goswami for discussions regarding this problem and Raj Gandhi for a critical reading of the manuscript.

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