On the mixing rules for astrophysical inhomogeneous grains

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Accepted 2004 October 25. Received 2004 September 16; in original form 2004 March 16

ABSTRACT

We present the computation of effective refractive coefficients for inhomogeneous two-component grains with three kinds of inclusions with $m_{\text{incl}} = 3.0 + 4.0i$, $2.0 + 1.0i$, $2.5 + 0.0001i$ and a matrix with $m_m = 1.33 + 0.01i$ for 11 volume fractions of inclusions from 0 to 50 per cent and wavelengths $\lambda = 0.5$, 1.0, 2.0 and 5.0 $\mu$m. The coefficients of extinction for these grains have been computed using a discrete dipole approximation. Computation of the extinction by the same method for grains composed of a matrix material with randomly embedded inclusions has been carried out for different volume fractions of inclusions. A comparison of extinction coefficients obtained for both models of grain material allows one to choose the best mixing rule for a mixture. In cases of inclusions with $m_{\text{incl}} = 2.0 + 1.0i$ and $2.5 + 0.0001i$ the best fit for the whole wavelength range and volume fractions of inclusions from 0 to 50 per cent is not very significant but the best fit has been obtained for the Hanai rule. For volume fractions of inclusions from 0 to 15 per cent a very good fit has been obtained for the whole wavelength range for Rayleigh and Maxwell-Garnett mixing rules.

Key words: methods: numerical – dust, extinction – ISM: general – ISM: molecules.

1 INTRODUCTION

The primary goal of the research we present here has been to use computational electrodynamics methods to choose the best mixing rule for different materials (dielectric, semiconductor and metal) of inclusions in a dielectric matrix. Light scattering computations for composite particles can be performed using the discrete dipole approximation (DDA). Vaidya et al. (2001) applied this method to calculate extinction and scattering efficiencies for a silicate sphere with embedded graphite inclusions and evaluated the interstellar extinction. Andersen et al. (2003) tested the programs DDSCAT by Draine & Flatau (2000a,b) and MARCODES by Markel (1998). The program by Markel (1998) is much faster than that by Draine & Flatau (2000a,b), which is very time consuming; however, it is meant to be used mainly for calculating the extinction by sparse clusters, whereas for compact clusters the accuracy of this method is very low. For this reason the DDA methods are not used for mass studies of interior circumstellar extinction. This is why many researchers still consider various models of grains such as the host material in which other materials are embedded (for example, Mathis & Whiffen 1989 or Maron 1989) or core–mantle (Jones 1988) and multilayer spherical particles (Voshchinnikov & Mathis 1999). Besides, the use of a particular model compared with observations can provide information concerning the inhomogeneous grains. Perrin & Lamy (1990) computed the cross-section of extinction using the 1\textsuperscript{st} DDA method assuming a cubic lattice with the elementary cell being treated as an inclusion of 10 Å for a spherical grain of radius 100 Å and the 2\textsuperscript{nd} Mie theory for a grain with the same radius. They considered two mixtures: first a matrix of silicate with a complex refractive index as proposed by Draine (1985) and inclusions of thaolin (an organic dielectric with the imaginary part of the refractive index $k < 0.001$) and in the second case, adesite containing inclusions of water ice. Both cases concern only a mixture of dielectrics for two values of the inclusion volume fraction in the infrared region. In their paper the effective optical constants applied to the Mie theory were obtained from the most popular rules used in astrophysics: those of Maxwell-Garnett and Bruggeman. Perrin & Lamy (1990) compared cross-sections of extinction obtained by both methods and stated that the application of effective-medium theories (EMT) to the problem of light scattering by inhomogeneous grains is not straightforward. Other authors (e.g. Chylek et al. 2000) compared the DDA and EMT methods but only for a very limited range of volume fractions of inclusions, wavelengths and materials.

2 MIXING RULES AND EFFECTIVE PERMITTIVITY

We limited our study to grains without electric charge, magnetic susceptibility and considered only two component mixtures where inclusions were monodisperse.
In our investigation we tested six mixing rules, not only the most popular ones in astrophysics such as those of Maxwell-Garnett and Bruggeman. The mixing rules considered are presented below.

The Maxwell-Garnett (1904) mixing rule was originally applied to metal particle encapsulated in an insulating matrix and for small filling factors, now it is used without restriction. The following equation is known as the Maxwell-Garnett formula (Bohren & Huffman 1983):

$$\varepsilon = \varepsilon_m + 3f \varepsilon_m \frac{\varepsilon_i - \varepsilon_m}{\varepsilon_i + 2\varepsilon_m - f(\varepsilon_i - \varepsilon_m)}.$$  

(1)

For composite media consisting of spherical particles of both basic components without the requirement of complete encapsulation Bruggeman (1935) has established the following equation (Bohren & Huffman 1983):

$$f \frac{\varepsilon_i - \varepsilon}{\varepsilon_i + 2\varepsilon} + (1 - f) \frac{\varepsilon_m - \varepsilon}{\varepsilon_m + 2\varepsilon} = 0.$$  

(2)

The Bruggeman equation is symmetrical, i.e. $\varepsilon_i$ and $\varepsilon_m$ can be exchanged. A similar equation, but not for spherical inclusions, was derived by Landauer (1952) for the conductivity of metallic mixtures.

For spherical inclusions and randomly oriented ellipsoids Landau & Lifszic (1960) and later, in a different way, Looyenga (1965) found the following relation:

$$\varepsilon^{1/3} = f \varepsilon_i^{1/3} + (1 - f)\varepsilon_m^{1/3}.$$  

(3)

The Hanai–Bruggeman equation was originally derived by Bruggeman (1935) for a composite dielectric consisting of a host material with spherical inclusions. Hanai in 1961 modified the Bruggeman equation and generalized it for complex dielectric permittivities. The Hanai–Bruggeman equation is given as follows (equation 3.58 in Beek 1967):

$$\frac{\varepsilon_i - \varepsilon}{\varepsilon_i - \varepsilon_m} \left( \frac{\varepsilon_m}{\varepsilon} \right)^{1/3} = 1 - f.$$  

(4)

Another mixing law for homogeneous and isotropic systems has been obtained by Lichtenecker (1926). The logarithmic mixing rule for a mixture of two components has the form

$$\log \varepsilon = f \log \varepsilon_i + (1 - f) \log \varepsilon_m.$$  

(5)

Meredith & Tobias (1960) reconsidered Rayleigh’s derivation for a cubic array of spheres $\varepsilon_i$ enclosed in a medium $\varepsilon_m$ and obtained an equation that appears to be more satisfactory at high values of $f$. The following equation describes the improved Rayleigh mixing rule (Beek 1967):

$$\varepsilon = \varepsilon_m \left( \frac{2\varepsilon_m + \varepsilon_i}{\varepsilon_i - \varepsilon_m} + 2f - 1.227 \frac{2\varepsilon_m + \varepsilon_i}{4\varepsilon_m + 3\varepsilon_i} f^{7/3} \right.$$ 

$$- 6.399 \left( \frac{\varepsilon_i - \varepsilon_m}{4\varepsilon_m + 3\varepsilon_i} f^{10/3} \right) \times \left( 2\varepsilon_m + \varepsilon_i - f - 1.227 \frac{2\varepsilon_m + \varepsilon_i}{4\varepsilon_m + 3\varepsilon_i} f^{7/3} \right.$$ 

$$- 2.718 \left( \frac{\varepsilon_i - \varepsilon_m}{4\varepsilon_m + 3\varepsilon_i} f^{10/3} \right)^{-1}. \right)$$  

(6)

In all formulae $f$ is the inclusion volume fraction and $\varepsilon_m$ and $\varepsilon_i$ are the complex dielectric permittivity of a matrix and the inclusion, respectively. $\varepsilon$ (without a subscript) is the complex dielectric permittivity of the mixture. Generally, the applicability of all mixing rules is restricted to low concentrations of inclusions. In this paper we have extended the search for the best mixing rule up to a 50 per cent inclusion concentration.

### Details of Calculations

We examined three different cases for the refractive index of inclusions, $m_{incl} = 3.0 + 4.0i, 2.0 + 1.0i, 2.5 + 0.0001i$. The chosen refractive indices for inclusions refer to those of materials examined by Draine & Goodman (1993). In all cases, the matrix was dielectric with refractive index $m_m = 1.33 + 0.01i$. Because the refractive indices are not additive quantities they have been changed into a complex permittivity relative to free space using the following formulae:

$$\varepsilon' = n^2 - k^2$$  

(7)

and

$$\varepsilon'' = 2nk,$$  

(8)

where $n$ and $k$ are the real and imaginary parts of the complex refractive index, and $\varepsilon'$ and $\varepsilon''$ are real and imaginary parts of the complex permittivity. Next, we have calculated the complex permittivities for the mixture based on the above mixing rules. The computation has been carried out for volume fractions in the range from 0 to 50 per cent with a 5 per cent step. The complex permittivities thus obtained have been changed into complex refractive indices using the following relations:

$$n = \sqrt{\varepsilon'' + \varepsilon'^2 + \varepsilon'}$$  

(9)

and

$$k = \frac{\sqrt{\varepsilon'' + \varepsilon'^2 + \varepsilon'}}{2}.$$  

(10)

The extinction coefficients have been computed using the discrete dipole approximation, first proposed by Purcell & Pennypacker (1973). DDA methods divide the particle into numerous polarizable volume elements. The induced dipole polarizations in these cubes are determined self-consistently, then properties such as the extinction cross-section are determined in terms of the induced polarization. In our study, we used the same approach as Draine & Flatau (1994) to compute the efficiency factor for extinction. The refraction indices for each kind of mixture and all considered mixing rules were calculated using the FORTRAN routine DSCAT.5A10 given by Draine & Flatau (2000a).

The efficiency factors for extinction $Q_{j,l,p}^{hom}$ for homogeneous spherical grains (each element of the DDA array has the same refractive index calculated for the mixture) have been computed for the following wavelengths: $\lambda = 0.5, 1.0, 2.0, 5.0 \mu m$ and grains with radius $r = 0.15 \mu m$ containing 1791 dipoles. The location of dipoles in the array for such a pseudospherical particle has been obtained using the routine CALLTARGET, also made available by Draine & Flatau (2000a). The subscript $p$ in the symbol $Q_{j,l,p}^{hom}$ corresponds to the appropriate mixing rule, $j$, the volume fraction of inclusion and the subscript $l$ corresponds to the wavelength. The efficiency factors determined for the extinction have been compared using those computed with the exact Mie theory. It has been found that the differences between the two methods are very small, which is similar to the results of Draine & Goodman (1993).

We have applied a mixture of DDA elements with different refractive indices, while the ratio of the number of DDA elements with the inclusion refractive index to all elements in the grain is equal to the inclusion volume fraction.
The assumed size of the inclusion equals the volume of the DDA element (the size of inclusion cannot be smaller than the volume element considered in the DDA approximation). Draine (1985) considered how the surface granularity affects the accuracy of the calculations. In our study the validity criteria considered by Draine (1985) are satisfied and furthermore the influence of surface granularity is reduced because it is almost the same in both cases: for elements with a mixture of materials and for a random mixture of elements with two kinds of materials. Similarly, the departure from sphericity for both pseudospheres is the same and does not influence the difference of extinction in both cases. The arrangement of inclusions (DDA elements) is random. In order to obtain a random number of DDA elements in the discrete dipole arrays we have used the random number generator ‘research randomizer’ available at http://www.randomizer.org. We have generated 10 series of numbers corresponding to the given volume fractions of inclusions out of all 1791 dipoles. The numbers generated have been sorted into ascending order. The location of dipoles in the array for the spherical particle obtained from the routine CALLTARGET.F was the same for homogenous grains with the only difference being that for the randomly generated numbers the DDA elements had the refractive index of inclusions and for the others the refractive index of a matrix. Fig. 1 presents the dependence of the efficiency factors for extinction of the volume fraction for inclusions with regular and random locations. One can see from this figure that the geometrical structure of the inhomogeneous grain plays a very important role. The authors of the considered mixing rules had assumed a statistical distribution of inclusions. Therefore, in our calculations random distributions have been used. Because there is a scattering of results for different random distributions, the efficiency factors for extinction have been calculated for 10 different distributions and then averaged. The dependence curves of extinction on inclusion volume fractions have been interpolated by fifth degree polynomials. For grains with radii of r = 0.15 µm, 10 values of efficiency factors for extinction Q_{\text{eff}}^\text{rand} depending on random location have been calculated for each of the four wavelengths, λ = 0.5, 1.0, 2.0 and 5.0 µm, using the computer program DDCAT5A10. The subscript i in the symbol Q_{\text{eff}}^\text{rand} corresponds to the number of the random location for inclusions, j, the inclusion volume fraction and the subscript l corresponds to the wavelength. Next, the mean extinction was calculated as

\[ Q_{\text{eff}}^\text{rand} = \frac{1}{10} \sum_{i=1}^{10} Q_{i,j,l}^\text{rand} \]  

(11)

4 RESULTS AND DISCUSSION

Figs 2–13 below show the dependence of the mean values for the efficiency factors for extinction Q_{ij,l}^\text{rand} and Q_{ij,\text{homog}} for the inclusion volume fractions for the given mixing rule and wavelength, and the best-fitted curves.

The best fit of extinction for homogeneous grains consisting of material of averaged refractive index calculated from the mixing rule and extinction of grains consisting of DDA elements of two materials (matrix and inclusions) with different refractive indices (random) was obtained in the following way.

(i) For the given mixing rule (subscript p) and the given wavelength (subscript l), \( \chi_{ij,p}^l \) was calculated from

\[ \chi_{ij,p}^l = \frac{1}{10} \sum_{j=1}^{10} \left( \frac{Q_{ij,j,\text{p}}^l - Q_{ij,j,\text{homog}}}{{\sigma_{ij,j,\text{p}}}} \right)^2, \]  

(12)

where the summation is performed for volume fractions of inclusions j, Q_{ij,j,\text{p}}^l is the averaged extinction coefficient for randomly located inclusions, Q_{ij,j,\text{homog}} is the extinction coefficient for homogeneous grains for a given mixing rule. Because the values of Q_{ij,\text{p}}^l for different random locations of inclusions (i) were not treated as equivalent to measured values, therefore \( \sigma_{ij,j,\text{p}} \) is not the standard

Figure 1. Example of different values of extinction coefficients for different random inclusion locations with respect to inclusion volume fractions (dots and solid line best fit) and the dependence of extinction on the volume fraction for the regular location of inclusions (stars and dashed line), both calculated using the DDA. The inclusion refractive index is \( m_{\text{inc}} = 3.0 + 4.0i \).
Table 1. Goodness of fit for each mixing rule and each wavelength $\Delta_{l,p} - m_{\text{inc}} = 3.0 + 0.4i, \sigma = 10\%$ of $Q_{\text{ext}}^{\text{rand}}$.

| Mixing rule  | $\Delta_{l,p}(0.5\,\mu m)$ | $\Delta_{l,p}(1.0\,\mu m)$ | $\Delta_{l,p}(2.0\,\mu m)$ | $\Delta_{l,p}(5.0\,\mu m)$ | $\Delta_p$ |
|--------------|-----------------|-----------------|-----------------|-----------------|-----------|
| Bruggeman    | 0.616E+00       | 0.643E-40       | 0.000E+00       | 0.000E+00       | 0.000E+00 |
| Hanai        | 0.399E+00       | 0.162E-06       | 0.911E-02       | 0.449E-01       | 0.231E-06 |
| Lichteneker  | 0.996E+00       | 0.750E-22       | 0.000E+00       | 0.000E+00       | 0.000E+00 |
| Looyenga     | 0.121E+00       | 0.000E+00       | 0.000E+00       | 0.000E+00       | 0.000E+00 |
| Maxwell-Garnett | 0.240E-01 | 0.595E-01       | 0.535E-21       | 0.616E-29       | 0.000E+00 |
| Rayleigh     | 0.132E+00       | 0.433E-01       | 0.120E-11       | 0.123E-18       | 0.129E-26 |

Table 2. Goodness of fit for each mixing rule and each wavelength $\Delta_{l,p} - m_{\text{inc}} = 3.0 + 0.4i, \sigma = 10\%$ of $Q_{\text{ext}}^{\text{rand}}$.

| Mixing rule  | $\Delta_{l,p}(0.5\,\mu m)$ | $\Delta_{l,p}(1.0\,\mu m)$ | $\Delta_{l,p}(2.0\,\mu m)$ | $\Delta_{l,p}(5.0\,\mu m)$ | $\Delta_p$ |
|--------------|-----------------|-----------------|-----------------|-----------------|-----------|
| Bruggeman    | 0.826E+00       | 0.383E-12       | 0.518E-34       | 0.000E+00       | 0.000E+00 |
| Hanai        | 0.592E+00       | 0.182E-02       | 0.184E-05       | 0.597E-09       | 0.950E-13 |
| Lichteneker  | 0.999E+00       | 0.158E-37       | 0.000E+00       | 0.000E+00       | 0.000E+00 |
| Looyenga     | 0.157E-09       | 0.000E+00       | 0.000E+00       | 0.000E+00       | 0.000E+00 |
| Maxwell-Garnett | 0.100E+00 | 0.997E+00       | 0.599E+00       | 0.399E+00       | 0.994E+00 |
| Rayleigh     | 0.100E+00       | 0.997E+00       | 0.624E+00       | 0.423E+00       | 0.996E+00 |

Table 3. Goodness of fit for each mixing rule and each wavelength $\Delta_{l,p} - m_{\text{inc}} = 2.0 + 1.0i, \sigma = 2.5\%$ of $Q_{\text{ext}}^{\text{rand}}$.

| Mixing rule  | $\Delta_{l,p}(0.5\,\mu m)$ | $\Delta_{l,p}(1.0\,\mu m)$ | $\Delta_{l,p}(2.0\,\mu m)$ | $\Delta_{l,p}(5.0\,\mu m)$ | $\Delta_p$ |
|--------------|-----------------|-----------------|-----------------|-----------------|-----------|
| Bruggeman    | 0.929E+00       | 0.245E-26       | 0.000E+00       | 0.000E+00       | 0.000E+00 |
| Hanai        | 0.643E+00       | 0.216E-15       | 0.420E-38       | 0.000E+00       | 0.000E+00 |
| Lichteneker  | 0.256E+00       | 0.237E+00       | 0.987E+00       | 0.999E+00       | 0.896E+00 |
| Looyenga     | 0.988E+00       | 0.000E+00       | 0.000E+00       | 0.000E+00       | 0.000E+00 |
| Maxwell-Garnett | 0.693E-01 | 0.103E-05       | 0.946E-11       | 0.450E-13       | 0.317E-26 |
| Rayleigh     | 0.172E+00       | 0.158E-07       | 0.131E-14       | 0.327E-17       | 0.736E-35 |

Table 4. Goodness of fit for each mixing rule and each wavelength $\Delta_{l,p} - m_{\text{inc}} = 2.5 + 0.001\,i, \sigma = 2.5\%$ of $Q_{\text{ext}}^{\text{rand}}$.

| Mixing rule  | $\Delta_{l,p}(0.5\,\mu m)$ | $\Delta_{l,p}(1.0\,\mu m)$ | $\Delta_{l,p}(2.0\,\mu m)$ | $\Delta_{l,p}(5.0\,\mu m)$ | $\Delta_p$ |
|--------------|-----------------|-----------------|-----------------|-----------------|-----------|
| Bruggeman    | 0.259E-06       | 0.371E-19       | 0.483E-03       | 0.342E-05       | 0.143E-29 |
| Hanai        | 0.177E+00       | 0.645E-04       | 0.277E-01       | 0.999E+00       | 0.138E-02 |
| Lichteneker  | 0.682E+00       | 0.991E+00       | 0.100E+01       | 0.367E-01       | 0.884E+00 |
| Looyenga     | 0.217E-20       | 0.455E-32       | 0.110E-03       | 0.966E-12       | 0.000E+00 |
| Maxwell-Garnett | 0.995E+00 | 0.822E+00       | 0.307E+00       | 0.263E-10       | 0.730E-05 |
| Rayleigh     | 0.998E+00       | 0.375E+00       | 0.132E+00       | 0.397E-03       | 0.240E-01 |

deviation, $\sigma_{l,p}$ is the fraction $\alpha$ of the averaged extinction coefficients $Q_{l,j}^{\text{rand}}$: $\sigma_{l,p} = \alpha Q_{l,j}^{\text{rand}}$. (13)

The calculations of $\chi^2_{l,j}$ have been made for $\alpha = 0.005, 0.01, 0.025, 0.05$ and 0.1. The distribution of $Q_{l,j}^{\text{rand}}$ values has been assumed to be normal.

(ii) The goodness of fit $\Delta_{l,p}$ has been calculated using the incomplete gamma function $\Delta_{l,p} = \text{gamma}(0.5\nu, 0.5\chi^2_{l,j})$ described in Press et al. (1992), where $\nu = N - M$ is the number of degrees of freedom, $N$ is the number of points in the curve (number of inclusion volume fractions) and $M$ is an adjustable parameter ($M = l$ is the number of mixing rules for calculating $\chi^2_{l,j}$). The obtained values of fitting coefficients $\Delta_{l,p}$ for each mixing rule and each wavelength for $\alpha = 0.025$ are presented in Tables 1 and 2, and for the metallic inclusions $\alpha = 0.1$ in Tables 3 and 4 because for smaller values of $\alpha$ the goodness-of-fit coefficients were very small. For the mixture of given materials determined by a mixing rule, different fitting coefficients have been obtained for each wavelength. For different wavelength ranges different mixing rules may be used based on the given values of $\Delta_{l,p}$.

(iii) In order to choose the best mixing rule over the whole range of wavelengths considered, the value

$$\chi^2_{P} = \sum_{i=1}^{4} \chi^2_{l,p}$$ (14)

has been calculated and the procedure described in (ii) carried out leading to the obtained goodness-of-fit coefficient $\Delta_p$. © 2005 RAS, MNRAS 357, 873–880
Figure 2. The dependence of the efficiency factors for extinction on the inclusion volume fraction for different mixing rules and wavelengths. The refractive index of the matrix is $1.33 + 0.01i$ and for the inclusion $m_{\text{inc}} = 3.0 + 4.0i$, wavelength $\lambda = 0.5 \, \mu\text{m}$. The solid curve ‘random’ shows the best fit for 10 extinction dependences for randomly distributed DDA elements with a refractive index corresponding to an inclusion among elements with the refractive index of a matrix. The grain radius in all cases is $0.15 \, \mu\text{m}$.

Figure 3. Same as in Fig. 2 but for $\lambda = 1.0 \, \mu\text{m}$.

Figure 4. Same as in Fig. 2 but for $\lambda = 2.0 \, \mu\text{m}$.

Figure 5. Same as in Fig. 2 but for $\lambda = 5.0 \, \mu\text{m}$.
Figure 6. Same as in Fig. 2 but for $m_{\text{inc}} = 2.0 + 1.0 \, \text{i}$ and $\lambda = 0.5 \, \text{\mu m}$.

Figure 8. Same as in Fig. 2 but for $m_{\text{inc}} = 2.0 + 1.0 \, \text{i}$ and $\lambda = 2.0 \, \text{\mu m}$.

Figure 7. Same as in Fig. 2 but for $m_{\text{inc}} = 2.0 + 1.0 \, \text{i}$ and $\lambda = 1.0 \, \text{\mu m}$.

Figure 9. Same as in Fig. 2 but for $m_{\text{inc}} = 2.0 + 1.0 \, \text{i}$ and $\lambda = 5.0 \, \text{\mu m}$.
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Figure 10. Same as in Fig. 2 but for $m_{\text{inc}} = 2.5 + 0.0001i$ and $\lambda = 0.5$ μm.

Figure 11. Same as in Fig. 2 but for $m_{\text{inc}} = 2.5 + 0.0001i$ and $\lambda = 1.0$ μm.

Figure 12. Same as in Fig. 2 but for $m_{\text{inc}} = 2.5 + 0.0001i$ and $\lambda = 2.0$ μm.

Figure 13. Same as in Fig. 2 but for $m_{\text{inc}} = 2.5 + 0.0001i$ and $\lambda = 5.0$ μm.
In summary, we have examined three different materials as inclusions in a matrix of \( m_m = 1.33 + 0.01 \, \mathrm{i} \).

1. \( m_{\text{inc}} = 3.0 + 4.0 \, \mathrm{i} \) (Figs 2–5).

   (i) The values of the goodness-of-fit coefficients for \( \alpha = 0.025 \) and 0.05 are too small to be considered for an analysis of the fit, therefore Table 1 shows the values of \( \Delta_{\lambda p} \) and \( \Delta_p \) for \( \alpha = 0.1 \).

   (ii) The goodness of fit of the extinction curve for each mixing rule depends strongly on the wavelength. For \( \lambda = 0.5 \, \mu\text{m} \) the best fit has been obtained for the Lichtenecker rule. However, for the whole range of wavelengths the Hanai rule gives the best fit for this material. It is necessary to point out that the value of the fitting coefficient \( \Delta_p \) is very small. It may be necessary to investigate other mixing rules in order to obtain a better fit.

   (iii) For a high inclusion volume factor they may be placed so close that, on one hand, they may create inclusions of much bigger sizes and therefore more vulnerable to skin effect, and on the other hand they may exceed the percolation threshold. Therefore, we have considered the fit for the inclusion volume factor in the range from 0 to 15 per cent. The results are presented in Table 2, from which we point out that for the whole wavelength range the best fit is obtained for Rayleigh and Maxwell-Garnett mixing rules.

2. \( m_{\text{inc}} = 2.0 + 1.0 \, \mathrm{i} \) (Figs 6–9).

   (i) The smallest value of \( \alpha \) that gives goodness-of-fit coefficients suitable for further analysis is 0.025, and for this value the coefficients \( \Delta_{\lambda p} \) and \( \Delta_p \) have been calculated.

   (ii) For \( \lambda = 0.5 \, \mu\text{m} \) the best fit has been obtained for the Looyenga mixing rule (Fig. 6). However, for the whole range of wavelengths considered, the Lichtenecker rule gives the best fit (Table 3).

3. \( m_{\text{inc}} = 2.5 + 0.0001 \, \mathrm{i} \) (Figs 10–13).

   (i) The smallest value of \( \alpha \) that gives goodness-of-fit coefficients suitable for further analysis is 0.025, and for this value the coefficients \( \Delta_{\lambda p} \) and \( \Delta_p \) have been calculated.

   (ii) For \( \lambda = 0.5 \, \mu\text{m} \) the best fit has been obtained for the Rayleigh and Maxwell-Garnett mixing rules. However, for the whole range of wavelengths considered the Lichtenecker rule gives the best fit (Table 4).

Considering the above results it is possible that different mixing rules should be applied for the same mixture of materials for different wavelengths, especially with higher inclusion volume fractions. In spite of their deficiencies, such as simplifying idealizations, while deriving them or relying only upon experiments, the mixing rules are able to provide important information concerning inhomogeneous materials. In astrophysics the models of Maxwell-Garnett and Bruggeman are widely used because they are justified by theory. They are based on the different topology of inclusions. Other mixing rules, less theoretically justified, are often neglected, although they show better agreement with experimental data. However, Zakri, Laurent & Vauclin (1998), on the basis of an effective medium theory, found physical grounds for the Lichtenecker rule which, as shown by the calculations, give better fits for inclusions with refractive indices of \( m_{\text{inc}} = 2.0 + 1.0 \, \mathrm{i} \) and 2.5 + 0.0001 \, \mathrm{i} in the whole considered wavelength range and for \( m_{\text{inc}} = 3.0 + 4.0 \, \mathrm{i} \) in the short-wavelength region.

ACKNOWLEDGMENTS

The authors thank the referee for very useful and constructive comments that improved the paper.

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