Z boson production via Pb-Pb collisions at $\sqrt{s_{pp}}=5.02$ TeV

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Abstract

We estimate the production of $Z^i$ bosons, with $i$ the component of a $Z$ vector boson via Pb-Pb collisions using previous work on $J/\Psi$, $\Psi(2S)$ production in p-p and A-A collisions, with the new aspect being the creation of $Z^i$ bosons via quark interactions.

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1 Introduction

This an extension of our recent work on heavy quark state production via Xe-Xe collisions at $\sqrt{s_{pp}}=5.44$ TeV[1] and heavy quark state production in Pb-Pb collisions at $\sqrt{s_{pp}}=5.02$ TeV[2]. More than three decades ago W and Z bosons were observed at CERN via proton-antiproton experiments at $\sqrt{s_{pp}}=540$ GeV[3]. CMS experiments on electroweak boson production via relativistic heavy ion collisions (RHIC) are related to our present research[4]. $J/\Psi+Z$ boson production in p-p collisions was recently estimated[5].

Note that since the Z boson is a vector, with angular momentum $J=1$, it has three components. Our present estimate of the production of $Z^i$ bosons, with $i$ the component of the vector Z boson, via Pb-Pb collisions is motivated by the fact that Z bosons have little interaction with the nuclear medium, and by ALICE experiments that measured Z bosons production in Pb-Pb collisions[6] and in p-Pb collisions[7] at $\sqrt{s_{NN}}=5.02$ TeV. The estimate of Z boson production via Pb-Pb collisions make use of Ref[8], which was based estimates of heavy quark state production in p-p collisions[9]. Note that when the final calculation and results are presented in Secs 3, 4 the momentum $p^i \rightarrow p_Z$, the momentum of the Z boson produced by Pb-Pb collisions at $\sqrt{s_{pp}}=5.02$ TeV, and $Z^i \rightarrow Z$, a Z boson.

For $\Psi(2S)$ production we use the mixed hybrid theory for the $\Psi(2S)$ state. It was shown[10] that the $\Psi(2S)$ state is approximately 50%-50% mixture of a standard charmonium and hybrid charmonium state:

$$|\Psi(2S) > \simeq -0.7 |c\bar{c}(2S) > + \sqrt{1-0.5} |c\bar{c}g(2S) > , \quad (1)$$

while the $J/\Psi$ is essentially a standard $q\bar{q}$ state $|J/\Psi(1S) > \simeq |c\bar{c}(1S) >$, which we use in our estimate of $Z^i$ boson production via $\Psi(2S) \rightarrow J/\Psi(1S) + Z^i$. Having a hybrid component, $c\bar{c}g$, is important for Z boson production from $\Psi(2S)$ decay as the active gluon component of $\Psi(2S)$ produces a Z boson, as shown in Figure 2 (Section 2).
2 \textit{J/Ψ + Z production in Pb-Pb collisions with } \sqrt{s_{pp}}\textit{ 5.02 TeV}

The cross section for the production of a heavy quark state \( \Phi \) with helicity \( \lambda = 0 \) (for unpolarized collisions\([9]\)) in the color octet model\([10, 11, 12]\) in Pb-Pb collisions is given by\([8]\)

\[
\sigma_{pp \rightarrow \Phi} = R_{PbPb}^{E} N_{bin}^{PbPb} \sigma_{pp \rightarrow \Phi},
\]

(2)

with\([2]\) \( R_{PbPb}^{E} N_{bin}^{PbPb} \simeq 130 \). The cross section for \( pp \rightarrow \Psi(2S) \rightarrow J/\Psi(1S) + Z^i \) in terms of \( f_q \)\([14, 9]\), the quark distribution function, is

\[
\sigma_{pp \rightarrow \Psi(2S) \rightarrow J/\Psi(1S) + Z^i} = f_q(\bar{x}(y), 2m)f_q(a/\bar{x}(y), 2m)\sigma_{\Psi(2S) \rightarrow J/\Psi(1S) + Z^i},
\]

(3)

where \( y=\text{rapidity}, \ a = 4m^2/s = 3.6 \times 10^{-7} \) and \( \bar{x}(y) = 1.058 x(y) \). We take \( y = 0 \) in the present work, so \( \bar{x}(0) \simeq 6.4 \times 10^{-4} \)

For \( \sqrt{s}=5.02 \) TeV The quark distribution functions \( f_q \)\([14, 9]\) are

\[
f_q(\bar{x}(0), 2m) \simeq 82.37 - 63582.36 x(0) \simeq 41.6
\]

\[
f_q(a/\bar{x}(0), 2m) \simeq 82.37 - \frac{a}{\bar{x}(0)} \simeq 82.4 .
\]

(4)

Therefore from Eqs(2,3,4)

\[
\sigma_{PbPb \rightarrow \Psi(2S) \rightarrow J/\Psi(1S) + Z^i} \simeq 4.46 \times 10^5 \sigma_{pp \rightarrow \Psi(2S) \rightarrow J/\Psi(1S) + Z^i}.
\]

(5)

We use the notation

\[
\sigma_{pp \rightarrow \Psi(2S) \rightarrow J/\Psi(1S) + Z^i} = \sigma_{HHZ}(p) = g^{\mu\nu}(\Pi_{HZ}^{\mu\nu}(p) + \Pi_{HHZ}^{\mu\nu}(p)),
\]

(6)

with \( \Pi_{HZ}^{\mu\nu}(p), \Pi_{HHZ}^{\mu\nu}(p) \) defined below.

The normal \( |c\bar{c}(2S)\rangle \) component of \( \Psi(2S) \) decaying to \( |c\bar{c}(1S)\rangle \) with \( Z^i \) production via quark-Z coupling shown in Figure 1.

\[
\begin{array}{c}
\text{c} \\
\uparrow \\
|c\bar{c}\ 2S\rangle \\
\text{p-k} \\
\downarrow \\
\text{-c} \\
\end{array}
\]

\[
\begin{array}{c}
\text{-------------Z}^i \\
\end{array}
\]

\[
\begin{array}{c}
\text{|c\bar{c}\ 1S\rangle} \\
\downarrow \\
\text{-------------} \\
\end{array}
\]

Figure 1: \( \Psi(2S) \), standard component, to \( J/\Psi(1S) + Z^i \)
In Figure 2 $Z^i$ production with the hybrid $|c\bar{c}g(2S)^> component of $\Psi(2S)$ it is shown.

Figure 2: $\Psi(2S)$, hybrid component, to $J/PSi(1S) + Z^i$

Figure 3 shows the coupling processes needed for Figure 1 and Figure 2.

Figure 3: (a) gluon-quark coupling, (b) $c-Z$ coupling

In Figure 3 (a) the operator giving the gluon sigma coupling is

$$S^G_{\kappa\delta}(k) = \left[\sigma_{\kappa\delta}, S(k)\right] = \sigma_{\kappa\delta} S(k) + S(k)\sigma_{\kappa\delta} ,$$

(7)

with $\sigma_{\kappa\delta} = i(\gamma_{\kappa}\gamma_{\delta} - g^{\kappa\delta})$ and $G^{\kappa\delta}$ is the gluon field.

In Figure 3 (b), with $Z^i$ the $i$ component of the vector $Z$ boson and defining $g_c \equiv g^V_c \simeq 0.25$ [17], the $ccZ^i$ coupling is [16]

$$S_{Z^i} = \gamma^a (g_c - g^A_c \gamma^5)$$

(8)

As shown in subsection 3.2 the $\gamma^i\gamma^5$ term does not contribute to $\sigma_{HHz}(p)$, so we define $g^A_c = g_c$. 

3
3 \( \Psi(2S') \) decay to \( J/\Psi + Z^i \)

In this section we estimate the decay of the \( |\Psi(2S) > \) decay to \( |J/\Psi(1S) > + Z^i \) for both the standard and hybrid components of \( |\Psi(2S) > \) as shown in Figures 1 and 2.

3.1 \( \Psi(2S') \) decay to \( J/\Psi + Z^i \) via standard component of \( \Psi(2S) \)

The correlator corresponding to Figure 1 is

\[
\Pi_{HZ}^{\mu\nu}(p) = \sum_{ab} g_{ab}^2 \int \frac{d^4 k}{(2\pi)^4} Tr[S(k)\gamma^\mu S_Z S(p-k)\gamma^\nu],
\]

where the quark propagator \( S(k) = (\not k + M)/(k^2 - M^2) \), \( M \) is the mass of a charm quark \( (M_c) \), \( k = \sum \mu k^\mu \gamma^\mu \), and \( g_{ab}^2 = 4\pi\alpha_s \simeq 1.14 \). Since \( Tr[S(k)\gamma^\mu S_Z S(p-k)\gamma^\nu] \) is independent of color \( \sum_{ab} = 3 \).

Thus the correlator for \( \Psi(2S)_{normal} \) decay to \( J/\Psi(1S) + Z \) is

\[
\Pi_{HZ}^{\mu\nu}(p) = 3g_{c}^2 \int \frac{d^4 k}{(2\pi)^4} Tr[S(k)\gamma^\mu\gamma^i(1 - \gamma^5)S(p-k)\gamma^\nu].
\]

The trace in  Eq (10) is

\[
Tr[S(k)\gamma^\mu\gamma^i(1 - \gamma^5)S(p-k)\gamma^\nu] = \frac{Tr[(k + M)\gamma^\mu\gamma^i(1 - \gamma^5)((\not k - k) + M)\gamma^\nu]}{(k^2 - M^2)((k-p)^2 - M^2)}.
\]

Using the fact that the trace of an odd number of \( \gamma \)s vanish and \( Tr[\gamma^5\gamma^\alpha\gamma^\beta\gamma^\delta\gamma^\lambda] = -4i\epsilon^{\alpha\beta\delta\lambda} \)

\[
Tr[(k + M)\gamma^\mu\gamma^i(1 - \gamma^5)((\not k - k) + M)\gamma^\nu] = 4M[p_\mu g^{i\nu} + p_\nu g^{i\mu} - p_\mu g^{\nu\mu} + i\epsilon^{i\mu\alpha\nu} + 2k_\mu g^{i\nu} - 2k_i g^{\mu\nu} - 2ik_\alpha \epsilon^{i\mu\nu\alpha}] .
\]

Therefore

\[
\Pi_{HZ}^{\mu\nu}(p) = 12g_{c}^2 M \int \frac{d^4 k}{(2\pi)^4} \left( p_\mu g^{i\nu} + p_\nu g^{i\mu} - p_\mu g^{\nu\mu} + i\epsilon^{i\mu\alpha\nu} + 2k_\mu g^{i\nu} - 2k_i g^{\mu\nu} - 2ik_\alpha \epsilon^{i\mu\nu\alpha} \right).
\]

Using

\[
\int \frac{d^4 k}{(2\pi)^4} \left( \frac{1}{(k^2 - M^2)((k-p)^2 - M^2)} \right) = \frac{(2M^2 - p^2/2)}{(4\pi)^2} I_0(p),
\]

\[
\int \frac{d^4 k}{(2\pi)^4} \left( \frac{k^\mu}{(k^2 - M^2)((k-p)^2 - M^2)} \right) = \frac{p^\mu(2M^2 - p^2/2)}{(4\pi)^2} I_1(p),
\]

one finds

\[
\Pi_{HZ}^{\mu\nu}(p) = AM \left( \frac{2M^2 - p^2/2}{(4\pi)^2} \right) \left( (p_\mu g^{i\nu} - p_\mu g^{i\nu} + i\epsilon^{i\mu\alpha\nu})I_0(p) + 2p_\mu g^{i\nu} - 2p_\nu g^{i\mu} - 2ip_\alpha \epsilon^{i\mu\nu\alpha}I_1(p) \right)
\]

\[
A = 12g_{c}^2 M ,
\]

with

\[
I_0(p) = \int_0^1 d\alpha \frac{1}{p^2(\alpha - \alpha^2) - M^2}, \quad I_1(p) = \int_0^1 d\alpha \frac{\alpha}{p^2(\alpha - \alpha^2) - M^2}.
\]
3.2 $\Psi(2S)$ decay to $J/\Psi + Z'$ via hybrid component of $\Psi(2S)$

The two-point correlator for the hybrid $\Psi(2S)$-J/$\Psi$, corresponding to Figure 2 without the gluon-Z or quark-Z coupling is\cite{13} (see Eq.(18))

$$
\Pi_{HH}^{\mu\nu}(p) = \frac{3g^2}{4} \int \frac{d^4k}{(2\pi)^4} Tr \left[ [\sigma_{\delta\lambda} S(k)] + \gamma_\lambda S(p - k) \gamma_\mu \right] Tr[G^{\gamma\lambda}(0) G^{\alpha\delta}(0)].
$$

(17)

The correlator $\Pi_{HHq\bar{q}Z}^{\mu\nu}$, obtained from Figure 2 is

$$
\Pi_{HHZ}^{\mu\nu}(p) = \frac{3g^2}{4} \int \frac{d^4k}{(2\pi)^4} Tr \left[ [\sigma_{\delta\lambda} S(k)] + \gamma_\lambda S_Z.S(p - k) \gamma_\mu \right] Tr[G^{\gamma\alpha}(0) G^{\rho\delta}(0)].
$$

(18)

Note that (with $< G^2 > = .476 \text{ GeV}^2$\cite{15})

$$
Tr[G^{\gamma\lambda}(0) G^{\alpha\delta}(0)] = (2\pi)^4 \frac{12}{96} < G^2 > (g^{\nu\gamma} g^{\lambda\delta} - g^{\nu\delta} g^{\lambda\gamma})
$$

(19)

$$
[\sigma_{\delta\lambda} S(k)]_+ = i[-2g^{\alpha\delta}(k + M) + 2M\gamma^\gamma \gamma^\delta + k_\alpha(\gamma^\gamma \gamma^\delta + \gamma^\alpha \gamma^\gamma \gamma^\delta)]/(k^2 - M^2).
$$

(20)

Therefore,

$$
\Pi_{HHZ}^{\mu\nu}(p) = B \int \frac{d^4k}{(2\pi)^4} \frac{i}{(k^2 - M^2)^2}(g^{\nu\gamma} g^{\lambda\delta} - g^{\nu\delta} g^{\lambda\gamma}) Tr A^{11}
$$

$$
M^2 B = \frac{3g^2}{4}(2\pi)^4 \frac{12}{96} < G^2 > \approx 25.87 \text{ GeV}^2 \text{ with }
$$

(21)

$$
Tr A^{11} = Tr [(k_\alpha(\gamma^\gamma \gamma^\alpha + \gamma^\gamma \gamma^\alpha \gamma^\gamma) \gamma^\alpha(1 - \gamma_5)\gamma^\lambda((\not{\gamma} - \not{k}) + M)\gamma^\mu
$$

$$
-2g^{\alpha\delta}(k + M) + 2M\gamma^\gamma \gamma^\delta + k_\alpha(\gamma^\gamma \gamma^\delta + \gamma^\alpha \gamma^\gamma \gamma^\delta)]/(k^2 - M^2).}
$$

(22)

Note that $(g^{\nu\gamma} g^{\lambda\delta} - g^{\nu\delta} g^{\lambda\gamma}) g^{\alpha\delta} = 0$, so the $g^{\alpha\delta}$ term in Eq.(22) vanishes. Therefore from Eq.(22)

$$
(g^{\nu\gamma} g^{\lambda\delta} - g^{\nu\delta} g^{\lambda\gamma}) Tr A^{11} = 2Tr \left[ [(k_\alpha(\gamma^\gamma \gamma^\alpha + \gamma^\gamma \gamma^\alpha \gamma^\lambda) \gamma^\gamma(1 - \gamma_5)\gamma^\lambda + 4M(\gamma^\nu - \gamma^\lambda)\gamma^\alpha(1 - \gamma_5)]
$$

$$
((\not{\gamma} - \not{k}) + M)\gamma^\mu).
$$

(23)

Since $Tr[\text{odd number of } \gamma \text{ s}] = 0$,

$$
(g^{\nu\gamma} g^{\lambda\delta} - g^{\nu\delta} g^{\lambda\gamma}) Tr A^{11} = 2M Tr [(k_\alpha(\gamma^\gamma \gamma^\alpha + \gamma^\gamma \gamma^\lambda) \gamma^\gamma(1 - \gamma_5)\gamma^\lambda \gamma^\mu + 4(p_\beta - k_\beta)(\gamma^\nu - \gamma^\lambda)\gamma^\alpha(1 - \gamma_5)\gamma^\beta \gamma^\mu)]
$$

(24)

As in Eq.(12), using $\epsilon^{\alpha\beta\gamma\lambda} = 0$ one obtains for the $4 - \gamma$ terms

$$
2M Tr [-2k_\alpha g^{\lambda\gamma} \gamma^\alpha(1 - \gamma_5)\gamma^\lambda \gamma^\mu + 4(p_\beta - k_\beta)(\gamma^\nu - \gamma^\lambda)\gamma^\alpha(1 - \gamma_5)\gamma^\beta \gamma^\mu] =
$$

$$
16M[-(k_\gamma g^{\nu\mu} + k_\mu g^{\alpha\nu} - k_\nu g^{\alpha\mu}) + ik_{\lambda} \epsilon^{\alpha\mu\nu\rho} + 2[(p_\mu - k_\mu) g^{\alpha\nu} + (p_\nu - k_\nu) g^{\alpha\mu} - (p_\nu - k_\nu) g^{\alpha\mu} + (p_\nu - k_\nu) g^{\alpha\mu} + 2i(p_\beta - k_\beta)(\epsilon^{\alpha\beta\gamma\lambda} - \epsilon^{\alpha\beta\gamma\lambda})]
$$

(25)

For the $6 - \gamma$ terms in Eq.(24)

$$
Tr[\gamma^\alpha \gamma^\beta \gamma^\gamma \gamma^\lambda \gamma^\mu \gamma^\nu] = 4(g^{\alpha\beta} Tr[\gamma^\delta \gamma^\lambda \gamma^\gamma \gamma^\mu \gamma^\nu] + g^{\alpha\delta} Tr[\gamma^\beta \gamma^\lambda \gamma^\gamma \gamma^\mu \gamma^\nu] + g^{\alpha\lambda} Tr[\gamma^\beta \gamma^\gamma \gamma^\gamma \gamma^\nu] +
$$

$$
g^{\alpha\mu} Tr[\gamma^\beta \gamma^\gamma \gamma^\gamma \gamma^\nu] + g^{\alpha\nu} Tr[\gamma^\beta \gamma^\gamma \gamma^\gamma \gamma^\nu])
$$

(26)

$$
Tr[\gamma^\alpha \gamma^\beta \gamma^\gamma \gamma^\lambda \gamma^\mu \gamma^\nu \gamma_5] = -16i(g^{\alpha\beta} \epsilon^{\delta\lambda\mu\nu} + g^{\alpha\delta} \epsilon^{\beta\lambda\mu\nu} + g^{\alpha\lambda} \epsilon^{\beta\delta\mu\nu} + ...).
$$

(27)
From Eqs(24,26,27) and using $\epsilon^{\beta\gamma\mu\nu} = 0$, the 6-\(\gamma\) terms in Eq(23) are
\[
2MT \{ k_{\alpha}(\gamma^\nu \gamma^\lambda \gamma^\alpha + \gamma^\alpha \gamma^\nu \gamma^\lambda) (1 - \gamma_5) \gamma^\beta \gamma^\mu \} = 4M(3k_i g^{\mu\nu} + k_\mu g^{i\nu} + k_\nu g^{i\mu}) .
\] (28)

Defining $\Pi_{\mu\nu}^Z(p) = \Pi_{\mu\nu}^{HHZ+}(p) + \Pi_{\mu\nu}^{HHZ-}(p)$, with $p_Z$ the Z boson momentum, from Eqs(15,23,24,25,28,14)
\[
\Pi_{\mu\nu}^Z(p) = M^2 - p^2/2 \left( \frac{\pi^2}{4} \right)^2 \left( A(p_\mu g^{i\nu} - p_\nu g^{i\mu} + ip_\alpha \epsilon^{i\mu\nu}) + B(2p_\mu g^{i\nu} - 2p_Z g^{\mu\nu} - 2ip_\alpha \epsilon^{i\mu\nu}) 
- 32B(p_Z g^{\mu\nu} + p_\mu g^{i\nu} - p_\nu g^{i\mu} - p_Z g^{\mu\nu} - p_\mu g^{i\nu} + p_\nu g^{i\mu}) + ip_\alpha \epsilon^{i\mu\nu} \right) 
- 2(p_\mu g^{i\nu} + p_Z g^{i\mu} - p_\nu g^{i\mu} + p_\mu g^{i\nu} - p_Z g^{\mu\nu} + p_\nu g^{i\mu}) I_0(p) + (A(2p_\mu g^{i\nu} - 2p_Z g^{\mu\nu} - 2ip_\alpha \epsilon^{i\mu\nu}) 
- 32B(p_Z g^{\mu\nu} + p_\mu g^{i\nu} - p_\nu g^{i\mu} - p_Z g^{\mu\nu} - p_\mu g^{i\nu} + p_\nu g^{i\mu}) + ip_\alpha \epsilon^{i\mu\nu} \right) I_1(p) .
\] (29)

From Eqs(6,21), taking the $\mu$ sum with $g^{\mu\nu}$
\[
\sigma_{HHZ}(p) = 4.6 \times 10^5 M^2 - p^2/2 \left( \frac{\pi^2}{4} \right)^2 Bp_Z(I_0(p) + I_1(p)) 
= 7.4 \times 10^4 GeV^2 \times p_Z \frac{2M^2 - p^2/2}{M}(I_0(p) + I_1(p)) ,
\] (30)

with the Z boson momentum $p_Z \simeq 1 - 3$ MeV, so $p_z \ll p$ as $p \simeq 2 - 3$ GeV in our calculation.

4 Calculation of $\sigma_{HHZ}(p)$ for $p \simeq M_c \simeq 1.27$ GeV

Carrying out the integrals for $I_0(p)$, $I_1(p)$ shown in Eq(14) one obtains from Eq(30) the values of $\sigma_{HHZ}(p)$, with $p_Z = 1, 2, 3$ MeV = 0.001, 0.002, 0.003 GeV, shown in the figure below, with $\sigma_{HHZ}(p)$ the Z boson production cross section via Pb-Pb collisions with the proton-proton energy=5.02 TeV.

![Figure 4: $\sigma_{HHZ}(p)$ with $p_Z = (a)1$, (b)2, (c) 3 MeV](image)
5 Conclusions

Using the relationship between the cross section $\sigma_{PbPb \rightarrow \Psi(2S) \rightarrow J/\Psi(1S)+Z}$ and $\sigma_{pp \rightarrow \Psi(2S) \rightarrow J/\Psi(1S)+Z}$ shown in Eq.(2), and $\Psi(2S)$ decay to $J/\Psi + Z$ for both the standard and hybrid components of $\Psi(2S)$, the cross section $\sigma_{HHZ}(p) \equiv \sigma_{PbPb \rightarrow \Psi(2S) \rightarrow J/\Psi(1S)+Z}$ was estimated for $\sqrt{s_{pp}}=5.02$ TeV and the Z boson momentum $p_Z = 1, 2,$ and 3 MeV, as shown in the figure. This should be useful for the experimental measurement of Z boson production via Pb-Pb collisions at $\sqrt{s_{pp}}=5.02$ TeV, although for simplicity we assumed that the rapidity=y=0, while current experiments$^{[6]}$ measure Z boson production via Pb-Pb collisions at $\sqrt{s_{pp}}=5.02$ TeV at large rapidities.

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