Radiative Corrections
in the Presence of
Majorana Neutrinos *

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Abstract

We have studied the radiative corrections from the fourth generation leptons in the context of the see-saw mechanism. We have estimated numerically the differential cross section for the process \((e^+e^- \rightarrow W^+W^-)\) at one-loop level, and found in the cross section the threshold behaviors for Majorana neutrino productions.

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1 Introduction

Recent measurements at LEP and SLC do not necessarily rule out the existence of the fourth generation. What they have established is that the number of neutrino species whose masses are lighter than 45 GeV is to be three. When we consider the fourth generation as a replication of the well-known generations, the following remarkable feature comes out; the first three generations have nearly massless neutrinos, while the fourth one has a very heavy neutrino.

In order to explain the lightness of the observed neutrinos, we usually introduce the see-saw mechanism [1], including right-handed neutrinos into the first three generations. In this mechanism, neutrinos acquire a Dirac mass $m_D$ and a right-handed Majorana mass $M_R$, which are reduced to two mass eigenvalues $M_1$ and $M_2$ through left-right mixing. We expect that the Dirac masses of neutrinos generated by the vacuum expectation value of ordinary Higgs bosons are comparable to masses of their charged partners. On the other hand, the Majorana mass originates from “beyond the standard model”. Therefore this $M_R$ should be much greater than Dirac masses $m_D$, so that we are able to identify eigenvalues $M_1$ with observed small masses of neutrinos.

We can also apply this see-saw mechanism to a heavier generation [2], which leads to an interesting result as follows: The LEP experiments suggest that the mass $M_\nu$ of the fourth neutrino is greater than 45 GeV. This $M_\nu$ is identified with the smaller mass eigenvalue $M_1$, where the smaller and larger eigenvalues, $M_1$ and $M_2$, are related to the Majorana and Dirac masses through the relations, $M_2 - M_1 = M_R$ and $M_1 M_2 = m_D^2$. Although $m_D$ is large, it is bounded above because of the requirement of triviality; $m_D \sim m_W$. Consequently, it turns out that the Majorana mass $M_R$ cannot be so large, and both mass eigenvalues $M_1$ and $M_2$ are of order $m_W$, i.e., the see-saw is almost balancing in the heavy neutrino sector. In this case, the existence of the Majorana neutrinos affects various weak processes through the large left-right mixing. Thus the contributions from heavy Majorana neutrinos must be taken into account when analyzing the next decade experiments whose energy scales are close to $M_R$.

Here we concentrate on the radiative corrections from these heavy neutrinos in the process $e^+e^- \to W^+W^-$ which is the most important process in the LEP II experiments. In the processes $e^+e^- \to f\bar{f}$ observed at the current colliders (LEP, SLC), the effects of heavy neutrinos appear only in the corrections to the self energies of gauge bosons [3, 4]. Contributions to $S, T$ parameters [4, 5], which only self energies are converted into, have already calculated and turned out to be negative when $M_1/m_D \sim 0.4$ [3, 4]. We will pay
attention to this Majorana mass region, because precision measurements favor negative values for $S$ and $T$ \[\text{[8]}\]. In the W pair production process, we should take into account radiative corrections not only on self energies but also trilinear gauge vertices due to the gauge invariance.

In this talk, we study the one-loop corrections from the fourth generation leptons in above mentioned LEPII process in the context of see-saw mechanism with $M_R$ of order $10^2$ GeV. We calculate the differential cross section numerically and analyze the threshold behavior for the various types of neutrino production.

2 Models and amplitudes at one-loop level

We consider the fourth generation leptons having no mixing with other generations. Our notation is almost the same as that of ref.\[\text{[6, 7]}\]. To make this talk self-contained, however, we shall exhibit the notational convention explicitly.

We assume the following mass term for neutrinos

$\mathcal{L}_{\text{mass}} = -\frac{1}{2}(\nu_L \nu^c_R) \begin{pmatrix} 0 & m_D \\ m_D & M_R \end{pmatrix} \begin{pmatrix} \nu_L \\ \nu_R \end{pmatrix} + \text{h.c.},$

where $\nu^c = C \nu^T$ is the charge-conjugated state of $\nu$. Then we diagonalize the mass matrix and perform the chiral transformation so as to get positive mass eigenvalues, $M_1$ and $M_2$. The result is given by

$\mathcal{L}_{\text{mass}} = \frac{1}{2} (N_1 N_2) \begin{pmatrix} M_1 & 0 \\ 0 & M_2 \end{pmatrix} \begin{pmatrix} N_1 \\ N_2 \end{pmatrix},$

where

$\begin{pmatrix} N_1 \\ N_2 \end{pmatrix} = \begin{pmatrix} i \gamma_5 c_\theta & -i \gamma_5 s_\theta \\ s_\theta & c_\theta \end{pmatrix} \begin{pmatrix} \nu_L + \nu^c_L \\ \nu_R + \nu^c_R \end{pmatrix},$

$M_{1,2} = \frac{1}{2} (\sqrt{M_R^2 + 4m_D^2} + M_R),$

$\tan \theta = \frac{M_1}{m_D} = \left(\frac{M_2}{m_D}\right)^{-1}.$

As described in Sec.1, $\tan \theta$ is $\sim O(0.1)$ for the fourth generation.
Now the gauge interactions of the fourth generation leptons can be written in terms of mass-eigenstates,

\[ \mathcal{L}_{CC} = \frac{g}{\sqrt{2}} \left[ W_+^\mu \{ -(i c_\theta N_1 + s_\theta N_2) \gamma_\mu \frac{1 - \gamma_5}{2} E \} + W_-^\mu \{ E \gamma_\mu \frac{1 - \gamma_5}{2} (i c_\theta N_1 + s_\theta N_2) \} \right], \]

\[ \mathcal{L}_{NC} = -e A^\mu \bar{E} \gamma_\mu E + \frac{g}{c_W} Z^\mu \{ \left( -\frac{1}{4} + s_W^2 \right) \gamma_\mu + \frac{1}{4} \gamma_\mu \gamma_5 \} \bar{E} - \frac{1}{4} c_\theta^2 \bar{N}_1 \gamma_\mu \gamma_5 N_1 - \frac{1}{4} s_\theta^2 \bar{N}_2 \gamma_\mu \gamma_5 N_2 + \frac{i}{2} s_\theta c_\theta \bar{N}_2 \gamma_\mu N_1, \] (1)

where \( E \) refers to charged lepton field. We note that the Z coupling to neutrinos with the same masses is axial vector, while it becomes a vector coupling when mixing the neutrinos with different masses.

In the \( e^+ e^- \rightarrow W^+ W^- \) reaction, one-loop corrections carried out using eq.(1) appear both in the self energies and the trilinear vertices of gauge bosons. The places in which radiative corrections are necessary are depicted as shaded blobs in fig.1, where the internal line of neutrino in the t-channel exchange diagram is free from corrections. We denote the self energies as \( \Pi_{AA}, \Pi_{AZ}, \Pi_{ZZ}, \Pi_{WW} \) and the vertex corrections as \( \Gamma_{\mu \alpha \beta}^{\mu \alpha \beta}, \Gamma_{\mu \alpha \beta}^{\mu \alpha \beta} \),

\[ \Pi_{AA} = e_0^2 \Pi_{QQ}, \quad \Pi_{AZ} = \frac{e_0^2}{s_0 c_0} (\Pi_{3Q} - s_0^2 \Pi_{QQ}), \]

\[ \Pi_{ZZ} = (\frac{e_0}{s_0 c_0})^2 (\Pi_{33} - 2 s_0^2 \Pi_{3Q} + s_0^4 \Pi_{QQ}), \quad \Pi_{WW} = (\frac{e_0^2}{s_0})^2 \Pi_{11}, \]

\[ \Gamma_{\mu \alpha \beta}^{\mu \alpha \beta} = e_0 g_0^2 \Sigma_{Q11}^{\mu \alpha \beta}, \quad \Gamma_{\mu \alpha \beta}^{\mu \alpha \beta} = \frac{e_0}{s_0 c_0} g_0^2 (\Sigma_{311}^{\mu \alpha \beta} - s_0^2 \Sigma_{Q11}^{\mu \alpha \beta}), \]

where \( s_0, c_0 \) are defined by \( s_0 = e_0 / g_0 \) and \( 1 \sim 3 \), \( Q \) refer to vector indices of \( SU(2) \) and \( U(1) \)-charge, respectively. Possible diagrams corresponding to \( \Pi \)'s and \( \Gamma \)'s are given in fig.2. Generally speaking, radiative corrections are called “oblique” ones if they appear only on gauge bosons and not directly on light fermions. The diagrams in fig.2 belong to such oblique corrections.

A renormalization program for oblique corrections is proposed by Kennedy and Lynn on the concept of effective lagrangian. This scheme performed in the processes \( e^+ e^- \rightarrow f f \), containing only self energies, is familiar to us. In these processes, we can shuffle all the appearing self energies into six bare parameters, and define the corresponding running parameters; the coupling constant \( (e_*(g_*)) \), the Weinberg angle \( (s_*) \), the W
and Z masses \( (M_{W^*}, M_{Z^*}) \) and the wave function renormalization constants of W and Z \( (Z_{W^*}, Z_{Z^*}) \). Thus the amplitude for \( e^+e^- \rightarrow f f \) at one-loop level has the same form as the one at the tree level, except that all bare parameters are replaced by the corresponding “starred” parameters. On the other hand, in the process of \( e^+e^- \rightarrow W^+W^- \), which we are considering here, additional corrections appear in the vertices. The extension of the formalism of Kennedy and Lynn to this process is given by Ahn et al. [9]. In this regard, divergences in the vertex corrections can also be removed without modifying the definitions of the “starred” parameters. The vertex functions at the one-loop level, however, have additional Lorentz structures not found at tree level. We can formally write the s-channel amplitude for the on-shell W’s as

\[
\mathcal{M} = -\frac{ie^2}{P^2} Z_{W^*} \bar{v} \gamma_\mu u \Gamma^{\mu\alpha\beta} \varepsilon^*_\alpha(q) \varepsilon^*_\beta(\bar{q}),
\]

where \( P, q, \bar{q} \) are momenta as depicted in fig.1, \( u \) and \( v \) are spinors of electron and positron, and \( \varepsilon_\alpha(q), \varepsilon_\beta(\bar{q}) \) refer to polarization-vectors of \( W^\pm \), respectively. The vertex function \( \Gamma^{\mu\alpha\beta} \) combining A and Z vertices is expressed in terms of canonical Lorentz structures parametrized by Hagiwara et al.,

\[
\Gamma^{\mu\alpha\beta} = \sum_{i=1}^7 (Q f_i^A + \frac{I_3 - s^2}{z^2} \frac{P^2}{P^2 - m^2} f_i^Z) T^{\mu\alpha\beta}_i,
\]

\[
T^{\mu\alpha\beta}_1 = (q - \bar{q})^\mu g^{\alpha\beta}, \quad T^{\mu\alpha\beta}_2 = \frac{1}{m^2_W} (q - \bar{q})^\mu P^\alpha P^\beta,
\]

\[
T^{\mu\alpha\beta}_3 = P^\alpha g^{\mu\beta} - P^\beta g^{\mu\alpha}, \quad T^{\mu\alpha\beta}_4 = i(P^\alpha g^{\mu\beta} + P^\beta g^{\mu\alpha}),
\]

\[
T^{\mu\alpha\beta}_5 = \epsilon^{\mu\alpha\beta\rho}(q - \bar{q})_\rho, \quad T^{\mu\alpha\beta}_6 = i\epsilon^{\mu\alpha\beta\rho} P_\rho,
\]

\[
T^{\mu\alpha\beta}_7 = i m_W \frac{1}{m^2_W} (q - \bar{q})^\mu \epsilon^{\alpha\beta\rho\sigma} P_\rho (q - \bar{q})_\sigma.
\]

Here, \( Q \) and \( I_3 \) are the charge and the isospin of e\(^-\), respectively. Form factors \( f_i \) include \( \Delta f_{i(V)} \) and \( \Delta f_{i(S)} \) which come from vertex corrections and self energies, respectively.

\[
f_i^{V=\Delta, Z} = f_i^{V\text{tree}} + \Delta f_i^{V\text{tree}} + \Delta f_i^{V\text{V}},
\]

\[
f_i^{A\text{tree}} = f_i^{Z\text{tree}} = \frac{1}{2} f_{3\text{tree}}^{A} = \frac{1}{2} f_{3\text{tree}}^{Z} = 1,
\]

\[
f_i^{A\text{tree}} = f_i^{Z\text{tree}} = 0 \quad (i = \text{others}),
\]

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\[
\Delta f^A_{i(S)} = \frac{1}{2} \Delta f^A_{3(S)} = g_s^2 \frac{\Pi_{3Q}}{P^2}, \\
\Delta f^Z_{i(S)} = \frac{1}{2} \Delta f^Z_{3(S)} = g_s^2 \frac{\Pi_{3Q}}{P^2} + \frac{M_{4s}^2 - m_Z^2}{P^2 - m_Z^2}, \\
\Delta f^A_{i(V)} = g_s^2 \Sigma_{Q11}, \\
\Delta f^Z_{i(V)} = \frac{g_s^2}{c_s^2} (\Sigma_{311} - s_s^2 \Sigma_{Q11}),
\]

(3)

where

\[
M_{Zs}^2(P^2) - m_Z^2 = \left( \frac{e_s^*}{s_s c_s} \right)^2 \left[ \{ \Pi_{33}(P^2) - \Pi_{33}(m_Z) \} - \{ \Pi_{3Q}(P^2) - \Pi_{3Q}(m_Z) \} \right] \\
+ m_Z^2 (c_s^* - s_s^*) \left[ \frac{\Pi_{3Q}(P^2)}{P^2} - \frac{\Pi_{3Q}(m_Z)}{P^2} \right] \\
+ m_Z^2 s_s^4 \left[ \frac{\Pi_{QQ}(P^2)}{P^2} - \frac{\Pi_{QQ}(m_Z)}{P^2} \right].
\]

Above definition of \( M_{Zs} \) is able to set \( Z_{Zs} \) to be unity. Remaining factor \( Z_{Ws} \) in eq.(2) is

\[
Z_{Ws}(P^2) = 1 - g_s^2 \left\{ \frac{\Pi_{3Q}(P^2)}{P^2} - \Pi_{11}(m_W) \right\}
\]

This \( Z_{Ws} \) is common to \( s \)- and \( t \)-channels.

3 Results

Using dimensional regularization, we calculate the one-loop corrections in eq.(3) from the leptons in the fourth generation. First, we present the expressions for the self energies, focusing on the threshold behaviors.

\[
16\pi^2 \Pi_{33} = \\
\bar{\epsilon}^{-1} \left\{ -\frac{1}{3} P^2 + \frac{1}{2} M_{E}^2 + (c_\theta^2 M_1^2 + s_\theta^4 M_2^2) + \frac{1}{2} (M_1 - M_2)^2 \right\} \\
- \frac{2}{3} M_{E}^2 - \frac{1}{3} \left( c_\theta^2 M_1 + s_\theta^4 M_2 \right)^2 + \frac{25}{6} c_\theta^2 \left( M_1 - M_2 \right)^2 \\
- \frac{1}{2} M_{E}^2 (\ln \frac{M_{E}^2}{\mu^2} - 2) - c_\theta^2 M_1^2 (\ln \frac{M_1^2}{\mu^2} - 2) - s_\theta^4 M_2^2 (\ln \frac{M_2^2}{\mu^2} - 2) \\
- \frac{s_\theta^2 c_\theta^2}{4} (M_1^2 - M_2^2) (\ln \frac{M_1^2}{\mu^2} + \ln \frac{M_2^2}{\mu^2}) \\
- P^2 \left\{ \begin{array}{c}
\frac{5}{9} \left( 1 - \frac{1}{3} \int \left( \frac{M_{E}^2}{\mu^2} + c_\theta \int \frac{M_1^2}{\mu^2} + s_\theta^4 \int \frac{M_2^2}{\mu^2} \right) + s_\theta^2 c_\theta \int \left( \frac{M_1^2}{\mu^2} + \frac{M_2^2}{\mu^2} \right) \right) \\
\end{array} \right\}
\]

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+ \frac{s_6^2 c_6^2}{6} \frac{1}{P^2} (M_1^2 - M_2^2) \{(M_1^2 - M_2^2) + 3M_1 M_2 \ln \frac{M_1^2}{M_2^2}\}
- \frac{s_6^2 c_6^2}{12} (P^2)^2 (M_1^2 - M_2^2)^3 \ln \frac{M_1^2}{M_2^2}
- \frac{1}{12} (P^2 \beta_{EE}^2 + 3M_2^2 \beta_{EE} \ln \frac{-P^2 + 2M_1^2 + P^2 \beta_{EE}}{-P^2 + 2M_1^2 - P^2 \beta_{EE}}
- \frac{c^4}{12} P^2 \beta_{N_1N_1}^3 \ln \frac{-P^2 + 2M_1^2 + P^2 \beta_{N_1N_1}}{-P^2 + 2M_1^2 - P^2 \beta_{N_1N_1}}
- \frac{s^4}{12} P^2 \beta_{N_2N_2}^3 \ln \frac{-P^2 + 2M_2^2 + P^2 \beta_{N_2N_2}}{-P^2 + 2M_2^2 - P^2 \beta_{N_2N_2}}
- \frac{s_2^2 c_2^2}{12} (2P^2 - (M_1^2 + M_2^2 - 6M_1 M_2) - \frac{(M_1^2 - M_2^2)^2}{P^2})
	imes \beta_{N_1N_2} \ln \frac{-P^2 + M_1^2 + M_2^2 + P^2 \beta_{N_1N_2}}{-P^2 + M_1^2 + M_2^2 - P^2 \beta_{N_1N_2}}, \tag{4}

where \(\tilde{c}^{-1} \equiv c^{-1} - \gamma + \ln 2\pi \) (\(\gamma\); Euler’s constant), \(\mu\) is the reference point, and \(\beta_{ij}\) are defined by,

\[
\beta_{ij} = \left[1 - \frac{(m_i + m_j)^2}{P^2}\right]^{1/2}.
\]

In eq. (4), \(\beta_{ij} \ln[(-P^2 + m_i^2 + m_j^2 + P^2 \beta_{ij})/(-P^2 + m_i^2 + m_j^2 - P^2 \beta_{ij})]\) express the singularities at \(P^2 = (m_i + m_j)^2\), giving the threshold behavior for the production of particles with masses \(m_i\) and \(m_j\).

We note that the threshold behaviors for \(N_1s‘\) (\(N_2’s\)) pair production and \(N_1-N_2\) production are different. The former is as strong as for the Dirac fermion productions \((\sim \beta)\). The latter is as mild as for the scalar productions \((\sim \beta^3)\). The similar expressions for \(\Pi_{QQ}, \Pi_{3Q}, \Pi_{11}\) are also obtained. For more details, see ref. [10].

Next, we consider the vertex corrections. In our case, there are four different kinds of Lorentz structures \(T_1, T_2, T_3\) and \(T_5\) [11, 12], because of the \(CP\) invariance.

\[
\sum_{i=1}^7 \Delta f_{i(V)}^\gamma T_i \sim (\tilde{c}^{-1} - 1)(T_1 + 2T_3) + \frac{1}{6}(T_1 + 3T_3 + T_5)
+ \sum_{i=1,2,3,5} \Delta f_{i(V)}^{\gamma(\text{finite})} T_i, \tag{5}
\]

where Lorentz indices are omitted. The divergence in the first term in eq. (5) cancels that existing in the self energy \(\Delta f_{1,3}^Z\) in eq. (3). The Gauge anomalies in the second term should cancel themselves if we consider the full fourth generation. We perform the
estimation of finite part of the form factors, $\Delta f_1^{Z(\text{finite})}(i = 1, 2, 3, 5)$. For example, $\Delta f_1^{Z(\text{finite})}$ in eq.(5) is obtained explicitly as follows,

\[
(4\pi)^2 \frac{c_0^2}{y_s} \Delta f_1^{Z(\text{finite})} =
\]

\[
\frac{35}{36} c_0^2 - \frac{1}{3} \left( \frac{1}{2} - s_s^2 \right) \ln \frac{M_E^2}{\mu^2} - \frac{1}{6} c_0^2 \ln \frac{M_1^2}{\mu^2} - \frac{1}{6} s_0^2 \ln \frac{M_2^2}{\mu^2}
\]

\[
- \frac{1}{P^2} \{ \left( \frac{1}{2} - s_s^2 \right) M_E^2 + \frac{1}{3} (c_0^2 M_1^2 + s_0^2 M_2^2) + \frac{1}{4} s_0^2 c_0^2 (M_2^2 - M_1^2) \ln \frac{M_2^2}{M_1^2} \}
\]

\[
+ \frac{s_0^2 c_0^2}{(P^2)^2} \left( \frac{1}{3} (M_2^2 - M_1^2)^2 + \frac{1}{4} (M_2^2 - M_1^2) (M_2^2 + M_1^2) \ln \frac{M_2^2}{M_1^2} \right)
\]

\[
- \frac{s_0^2 c_0^2}{6} \frac{1}{(P^2)^3} (M_2^2 - M_1^2)^3 \ln \frac{M_2^2}{M_1^2}
\]

\[
- \frac{1 - 2 c_0^2}{12} \frac{P^2 - M_2^2}{P^2 \beta_{EE} \ln \frac{P^2 + 2M_2^2}{P^2 + 2M_2^2 - P^2 \beta_{EE}}}
\]

\[
+ \frac{c_0^2}{12} \frac{P^2 - M_1^2}{P^2 \beta_{N_1 N_1} \ln \frac{P^2 + 2M_2^2 + P^2 \beta_{N_1 N_1}}{P^2 + 2M_1^2 - P^2 \beta_{N_1 N_1}}}
\]

\[
+ \frac{s_0^2}{12} \frac{P^2 - M_2^2}{P^2 \beta_{N_2 N_2} \ln \frac{P^2 + 2M_2^2 + P^2 \beta_{N_2 N_2}}{P^2 + 2M_2^2 - P^2 \beta_{N_2 N_2}}}
\]

\[
+ \frac{s_0^2 c_0^2}{12} \frac{2(P^2)^2 - P^2(M_1^2 + M_2^2) + 2(M_1^2 - M_2^2)^2}{P^2 \times \beta_{N_1 N_2} \ln \frac{P^2 + M_1^2 + M_2^2 + P^2 \beta_{N_1 N_2}}{P^2 + M_2^2 + M_2^2 - P^2 \beta_{N_1 N_2}}}
\]

\[
+ c_0^2 \int dx dy \left( \frac{1}{3} - s_s^2 \right) N(x, y; M_E, M_E, M_1) - \frac{1}{8} s_0^2 M_E^2 (x + y)
\]

\[
D(x, y; M_E, M_E, M_1)
\]

\[
+ s_0^2 \int dx dy \left( \frac{1}{3} - s_s^2 \right) N(x, y; M_E, M_E, M_2) - \frac{1}{8} s_0^2 M_E^2 (x + y)
\]

\[
D(x, y; M_E, M_E, M_2)
\]

\[
+ \frac{1}{2} c_0^4 \int dx dy \frac{N(x, y; M_1, M_1, M_E) - \frac{1}{2} M_1^2 (x + y)}{D(x, y; M_1, M_1, M_E)}
\]

\[
+ \frac{1}{2} s_0^4 \int dx dy \frac{N(x, y; M_2, M_2, M_E) - \frac{1}{2} M_2^2 (x + y)}{D(x, y; M_2, M_2, M_E)}
\]

\[
+ \frac{1}{2} s_0^2 c_0^2 \int dx dy \frac{N(x, y; M_1, M_2, M_E) + \frac{1}{4} M_1 M_2 (x + y)}{D(x, y; M_1, M_2, M_E)}.
\]

where

\[
N(x, y; m_a, m_b, m_c) = -\frac{1}{2} m_W^2 (x^3 - x^2 y - x y^2 + y^3 - x^2 - y^2 + x + y)
\]
$$D(x,y; m_a, m_b, m_c) = -m_W^2(x+y)(1-x-y) - P^2 xy$$
$$+ m_a^2 x + m_b^2 y + m_c^2(1-x-y).$$

The difference of threshold behaviors between $N_1-N_1$ ($N_2-N_2$) and $N_1-N_2$ productions are contained in the 5 ~ 14 lines of eq.(6). To clarify this difference and analyze observability of the neutrinos’ radiative corrections in our model, we need the numerical calculations. We get the expressions of the remaining form factors, similarly. The form factors $\Delta f_{1(V)}^A, \Delta f_{3(V)}^A, \Delta f_{3(V)}^Z$ have also divergences which cancel in the similar way as in the case of $\Delta f_{1(V)}^Z$, while $\Delta f_{2(V)}^A, \Delta f_{5(V)}^A$ are finite. Detailed expressions for $\Delta f_i$ will be given in ref.

Now, we present the numerical results of the differential cross section for $e^+e^- \rightarrow W^+W^-$. We use numerical method by Fujimoto et al. [13] to perform double Feynman parameter integrals in $\Delta f_i(V)$. In this analysis, we take the following values at $m_Z$ for fixing the parameters,

$$\frac{4\pi}{e^2_*} = 128.0, \quad s^2_* = 0.223,$$

$$m_Z = 93.00 \text{ GeV}, \quad m_W = 81.97 \text{ GeV},$$

where we neglect the influence of the running of $e_*, s_*$. We also neglect gauge anomalies. The differential cross section $d\sigma/d\cos\theta$ for $e^+e^- \rightarrow W^+_L W^-_L$ at scattering angle $\theta = \frac{\pi}{2}$ are shown in fig.3. In the process for W’s polarized longitudinally, we expect that the radiative corrections from heavy particles will be enhanced [7]. The dashed curve shows $d\sigma/d\cos\theta$ at tree level, and the solid curve at one-loop level assuming $M_D = 500 \text{ GeV}$ and $\tan\theta = 0.4$; S and T parameters are negative for this region. In comparison, the dotted curve is shown, assuming vanishing Majorana mass, i.e., this is the Dirac neutrino limit of $M_D = 200$ GeV. Here, these cross sections are expressed in units of the point cross section $1R = 4\pi\alpha^2/3s$ where $s = P^2$.

In Dirac neutrino case, we can see peaks at the pair production thresholds of charged leptons and neutrinos ($\sqrt{s} = 400, 1000$ GeV), in the similar manner as in the heavy quark case [7]. Heavy leptons, however, bring small effects because of having no color factor 3. Especially, the effect of Dirac neutrinos is very small, since they have no couplings with photons. For Majorana neutrinos, there is a quite small peak only at $N_1-N_2$ threshold ($\sqrt{s} = M_1 + M_2 = 1450$ GeV) because neutrino effects are suppressed due to the small couplings including mixing angle in eq.(1). We can see no other peaks.
except for the charged leptons’ pair production threshold. As mentioned above, there exists the difference of threshold behaviors in self energies, depending on the neutrino types. Numerical calculations including vertex corrections also suggest that threshold behaviors are scalar-like for $N_1$’s ($N_2$’s) pair production, but Dirac fermion-like for $N_1$-$N_2$ production. This is caused by the difference in the coupling types between $ZN_1N_1$ ($ZN_2N_2$) and $ZN_1N_2$, which can be seen in eq.(1).

4 Conclusion

In this talk, we have investigated the one-loop corrections from the heavy Majorana neutrinos in the differential cross section for the process $e^+e^- \rightarrow W^+W^-$, which will be seen in the near future at LEPII. We have adopted the formalism of Kennedy and Lynn as a renormalization procedure, using “starred” parameters. The differential cross section has been estimated numerically, and we have found that there is no visible peak at the threshold of producing a pair of light neutrinos ($N_1$-$N_1$) or a pair of heavy neutrinos ($N_2$-$N_2$). We have recognized that there exists a small peak at the threshold of producing a pair of light and heavy neutrinos ($N_1$-$N_2$). Then the effect from the Majorana neutrinos on the differential cross section at the scattering angle $\theta = \frac{\pi}{2}$ is quite small at the LEP region. Therefore in order to investigate further the role of Majorana neutrinos in the weak processes, we need to study the angular distribution of the differential cross section, electric (magnetic) moment of $W$ \cite{12, 14}, or some other quantities which are sensitive to the new physics.

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