Single-shot realization of nonadiabatic holonomic gates with a superconducting Xmon qutrit

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Abstract

Nonadiabatic holonomic quantum computation has received increasing attention due to its robustness against control errors and high-speed realization. The original protocol of nonadiabatic holonomic one-qubit gates has been experimentally demonstrated with a superconducting transmon qutrit. However, it requires two noncommuting gates to complete an arbitrary one-qubit gate, doubling the exposure time of the gate to error sources and thus leaving the gate vulnerable to environment-induced decoherence. Single-shot protocol has been subsequently proposed to realize an arbitrary one-qubit nonadiabatic holonomic gate. In this paper, a single-shot protocol of nonadiabatic holonomic gates is experimentally demonstrated by using a superconducting Xmon qutrit, with all the single-qubit Clifford gates carried out by a single-shot implementation. Characterized by quantum process tomography and randomized benchmarking, the single-shot gates reach a fidelity exceeding 99%.

1. Introduction

Circuit-based quantum computation requires a universal set of quantum gates, that include arbitrary one-qubit gates and a nontrivial two-qubit gate. Geometric quantum computation is a noteworthy approach to implementing the universal quantum gates by using geometric phases [1, 2] or their nonAbelian counterpart, the holonomies [3, 4]. Geometric phases are dependent on evolution paths but independent of evolution details, enabling a build-in resilience to certain noises and control errors. Early geometric quantum computation [5–7] was based on adiabatic geometric phases [1, 3]. However, due to the long operation time required for an adiabatic process, those quantum gates are vulnerable to environment-induced decoherence. To overcome this difficulty, nonadiabatic geometric quantum computation [8, 9] based on nonadiabatic Abelian geometric phases [3] and nonadiabatic holonomic quantum computation [10, 11] based on nonadiabatic nonAbelian geometric phases [4] have been proposed. Nonadiabatic holonomic quantum computation retains the merits of both robustness against control errors as well as high-speed realization, and is increasingly utilized in practical applications [12–33].

The original protocol [10, 11] of nonadiabatic holonomic one-qubit gates has been experimentally demonstrated with a superconducting circuit [21], nuclear magnetic resonance [22], and nitrogen-vacancy centers in diamond [23, 24]. Based on the original protocol, a single-step operation can only rotate the quantum state about an arbitrary axis, with a fixed angle of $\pi$. An arbitrary one-qubit gate then requires two sequential steps, doubling the exposure time of the gate to error sources. A single-shot protocol of nonadiabatic holonomic gates has been further proposed, in which the quantum state can be rotated about an arbitrary axis with a variable angle [16, 17]. Recently, the single-shot protocol was investigated using nitrogen-vacancy centers in
diamond [25, 26] and nuclear magnetic resonance [27]. In a different approach, a single-loop protocol of holonomic gates was also put forward [19] and experimentally realized [30]. In this paper, the names of single-shot and single-loop protocols follow the clarified definition in [34]. A characteristic feature to distinguish two protocols is that the single-shot protocol is realized with off-resonant pulses [16, 17] but the single-loop protocol is realized with resonant pulses [19].

A superconducting circuit provides an appealing scalable platform for implementing nonadiabatic holonomic quantum computation. As a solid state system, the integrated circuit can be easily scaled up to a multi-qubit system, with each qubit controlled by individual lines. The superconducting Xmon is a high quality qubit with relatively long coherence time, the design of which also balances the coherence, the connectivity, and the fast control [35–37]. In this paper, a single-shot protocol of nonadiabatic holonomic one-qubit gates is demonstrated experimentally with a three-level superconducting Xmon qutrit.

An arbitrary single-qubit gate can be realized using the single-shot protocol. Here a group of single-qubit Clifford gates are used as an example. Excluding the identity operation, the group is classified to $\pi$-rotation, $\pi/2$-rotation, and $2\pi/3$-rotation Clifford gates. The $\pi$-rotation gates are simultaneously driven by two resonant microwave pulses as in the original protocol [10, 11], while the other gates are driven by two off-resonant pulses following the single-shot protocol [16, 17]. In the previous proposals, a square-shaped off-resonant pulse has been applied due to experimental restrictions, whose performance is sensitive to the time jitter. In this work, two off-resonant pulses are applied with a time-dependent and flexibly-shaped detuning to implement the single-shot protocol. The high fidelity single-shot gates are demonstrated experimentally, with the fidelity characterized by both quantum process tomography (QPT) [38] and randomized benchmarking (RB) [39–41].

2. Results

2.1. Protocol

We first explain how nonadiabatic holonomic gates arise [10, 11]. A quantum system is described by a $N$-dimensional state space and exposed to the Hamiltonian $H(t)$. Then it is assumed that there is a time-dependent $L$-dimensional subspace $S(t) = \text{Span}\{|\psi(t)\rangle\}_{k=1}^{L}$, where $|\psi(t)\rangle$ satisfies the Schrödinger equation

$$i|\dot{\psi}(t)\rangle = H(t)|\psi(t)\rangle, \quad S(0)$$

is taken as the computational space. The unitary transformation acting on $S(0)$ is a nonadiabatic holonomic gate if the following requirements are satisfied:

$$(i) \sum_{k=1}^{L}|\psi_{k}(\tau)\rangle\langle \psi_{k}(\tau)| = \sum_{k=1}^{L}|\psi_{k}(0)\rangle\langle \psi_{k}(0)|,$$

$$(ii) \langle \psi_{k}(t)|H(t)|\psi_{l}(t)\rangle = 0, \quad k, l = 1, 2, \ldots, L.$$  \hspace{1cm} (1)

Here, condition (i) entails that $S(t)$ undergoes a cyclic evolution and condition (ii) ensures a parallel transport with vanishing dynamical phases.

In this experiment, a single-shot protocol of nonadiabatic holonomic gate [16, 17] is realized. Consider a three-level superconducting Xmon consisting of three lowest levels $|0\rangle$, $|e\rangle$ and $|1\rangle$ with ladder configuration, as shown in figure 1(a). The states $|0\rangle$ and $|1\rangle$ are taken as the qubit computational basis, while the state $|e\rangle$ acts as an auxiliary state, with $\omega_{01}$ and $\omega_{12}$ being energy differences between neighboring states. The transitions $|0\rangle \leftrightarrow |e\rangle$ and $|e\rangle \leftrightarrow |1\rangle$ are facilitated by two microwave pulses with pump frequency $\omega_{p}(t)$ and stocks frequency $\omega_{s}(t)$ [42]. In this work, the two microwave drives are off-resonant with detunings $\Delta_{p}(t)$ and $\Delta_{s}(t)$, where $\Delta_{p}(t) = \omega_{0e} - \omega_{p}(t)$ and $\Delta_{s}(t) = \omega_{1e} - \omega_{s}(t)$. In the two-photon resonance condition $\omega_{p}(t) + \omega_{s}(t) = \omega_{0e} + \omega_{1e}$, the Hamiltonian of the Xmon in the double-rotating frame and by using rotating wave approximation, reads

$$H(t) = \Delta_{p}(t)|e\rangle\langle e| + \frac{1}{2}[\Omega_{p}(t)|e\rangle\langle 0| + \Omega_{s}(t)|e\rangle\langle 1| + \text{h.c.}],$$  \hspace{1cm} (2)

where $\Omega_{p}(t)$ and $\Omega_{s}(t)$ are time-dependent envelopes and h.c. represents the Hermitian conjugate terms.

To realize the nonadiabatic holonomic gates, the parameters in equation (2) are taken as

$$\Delta_{p}(t) = \Omega(t) \sin \alpha,$$

$$\Omega_{p}(t) = \Omega(t) \cos \alpha \cos \frac{\theta}{2},$$

$$\Omega_{s}(t) = \Omega(t) \cos \alpha \sin \frac{\theta}{2} e^{-i\varphi},$$  \hspace{1cm} (3)

where $\Omega(t)$ is time-dependent, and $\alpha$, $\theta$ and $\varphi$ are time-independent constants. Accordingly, the Hamiltonian in equation (2) can be expressed as
\[
\mathcal{H}(t) = \Omega(t) \sin \alpha |e \rangle \langle e| + \frac{1}{2} \Omega(t) \cos \alpha (|e \rangle \langle b| + |b \rangle \langle e|),
\]

with the bright state \( |b \rangle = \cos \frac{\theta}{2} |0 \rangle + \sin \frac{\theta}{2} e^{i\varphi} |1 \rangle \). Here, a dark state is orthogonal to the bright state, as \( |d \rangle = \sin \frac{\theta}{2} e^{i\varphi} |0 \rangle - \cos \frac{\theta}{2} |1 \rangle \). If the evolution period \( \tau \) is taken to satisfy

\[
\int_0^\tau \frac{\Omega(t)}{2} dt = \pi,
\]

the evolution operator can be obtained as

\[
U(\tau) = |d \rangle \langle d| + e^{-i\tau(1+\sin \alpha)} |b \rangle \langle b| + e^{-i\tau(1+\sin \alpha)} |e \rangle \langle e|.
\]

Consequently, an arbitrary one-qubit gate can be obtained by projecting the evolution operator onto the computational subspace, denoted by

\[
U_\gamma(\tau) = e^{-i\gamma \sigma_1/2},
\]

where \( \gamma = \pi(1+\sin \alpha) \), \( n = (\sin \theta \cos \varphi, \sin \theta \sin \varphi, \cos \theta) \), and \( \sigma = (\sigma_x, \sigma_y, \sigma_z) \) with \( \sigma_x, \sigma_y, \sigma_z \) being the Pauli operators acting on computational basis \( |0 \rangle \) and \( |1 \rangle \). Here, an unimportant global phase is neglected.

Next, \( U_\gamma(\tau) \) is demonstrated to be a nonadiabatic holonomic gate. First, condition (i) is satisfied as \( U(\tau) |0 \rangle \langle 0| + |1 \rangle \langle 1| U^\dagger(\tau) = |0 \rangle \langle 0| + |1 \rangle \langle 1| \). Second, with the aid of the commutation relation \( [\mathcal{H}(t), U(t)] = 0 \), it can be verified that condition (ii) is satisfied as \( \langle \Phi(t) | \mathcal{H}(t) | \Phi(t) \rangle = (i\hbar) U^\dagger(t) \mathcal{H}(t) U(t) | j \rangle \langle j| = 0 \), where \( \langle \Phi(t) | j \rangle = U(t) | i \rangle \) with \( U(t) = e^{-i \int_0^\tau \mathcal{H}(t') dt'} \) and \( i\hbar = 0, 1 \). Therefore, \( U_\gamma(\tau) \) is a nonadiabatic holonomic gate.

In the original protocol of nonadiabatic holonomic gates, the three-level system is controlled by two resonant pulses with \( \Delta_p(t) = 0 \), which is equivalent to a fixed rotation angle of \( \gamma = \pi \). In the single-shot protocol, the two off-resonant pulses are applied with a nonzero and time-dependent detuning. An arbitrary rotation angle \( \gamma \) can be obtained. In other words, an arbitrary one-qubit gate can be realized in a single-shot implementation. The original protocol is a specific case of the current single-shot protocol.

### 2.2. Single-qubit Clifford group

An arbitrary single-qubit gate can be implemented using the single-shot nonadiabatic holonomic protocol by choosing specific gate parameters \( \alpha, \theta \) and \( \varphi \). To demonstrate the arbitrariness of the protocol, single-qubit gates...
in the Clifford group (Clifford gates) are implemented. For a single qubit, the Clifford group consists of 24 rotations preserving quantum states along vertices of an octahedron in the Bloch sphere. The rotation axes are lines connecting the origin of the Bloch sphere and a face center, vertex or midpoint of an edge of the octahedron, as shown in figure 1(c). Classified by the rotation angle, the single-qubit Clifford gates can be divided into four sets: identity, π-rotation, π/2-rotation and 2π/3-rotation. For the set of π-rotation, there are nine single-qubit gates in the Clifford group, corresponding to the red and blue axes in figure 1(c). This set of gates can be realized by the original nonadiabatic holonomic protocol, and implemented with a single step of resonant pump and stocks drives. However, for the sets of π/2- (blue axes) and 2π/3- (green axes) rotations, the remaining 14 gates must be realized by combining two π-rotation gates in the original nonadiabatic holonomic protocol. With the single-shot protocol, these two sets can be implemented with a single step by the off-resonant pump and stocks drives.

2.3. Experimental parameters

The superconducting Xmon used in this work is an aluminum-based circuit operated at approximately 10 mK in a cryogen-free dilution refrigerator. The Xmon sample was fabricated on a silicon substrate using a standard nano-fabrication method. Four arms of the Xmon cross were connected to different lines for separate functions of coupling, control and readout. The lowest three energy levels of the Xmon were then utilized in the single-shot protocol. A readout resonator coupling the Xmon and a readout line were used for a dispersive measurement of the quantum state of the three-level qutrit. A 1 μs long microwave signal was sent to the sample with a frequency of $\omega = 6.56 \text{ GHz}$. After interacting with the readout resonator, the signal was amplified by a Josephson parametric amplifier [43, 44] and a high electron mobility transistor. The signal was further digitalized and demodulated using an analog to digital converter for high fidelity measurement. By heralding the ground state $|0\rangle$ [45], the readout fidelity for the lowest three levels was $F_0 = 99.5\%$, $F_e = 92.3\%$ and $F_i = 89.5\%$, respectively. For a single Xmon in this experiment, the lowest three levels are used as $|0\rangle$, $|e\rangle$, and $|1\rangle$ in the single-shot protocol. The relevant transition frequencies were $\omega_{0e}/2\pi = 4.849 \text{ GHz}$ and $\omega_{1e}/2\pi = 4.597 \text{ GHz}$, and the nonlinearity was $\eta = (\omega_{1e} - \omega_{0e})/2\pi = -252 \text{ MHz}$. The coherence of the qubit was characterized by energy relaxation time $T_1^0 = 25.3 \mu s$, $T_1^e = 12.8 \mu s$, and the pure dephasing time $T_\phi^0 = 28.1 \mu s$, $T_\phi^e = 13.4 \mu s$ was measured using a Ramsey interference experiment.

In the single-shot nonadiabatic holonomic protocol, the rotation angle $\gamma$ is determined by the time-independent parameter $\alpha$. For the single-qubit Clifford gates, the set of $\gamma = \pi$, π/2- and 2π/3- rotations can be assigned with $\alpha = 0, -\pi/6$ and $-\arcsin(1/3)$, respectively. The rotation axis for each gate is determined by two other time-independent parameters $\theta$ and $\phi$. The same time-dependent pulse envelope $\Omega(t)$ is shared for all the gates regardless of the different gate parameters $\beta, \theta$ and $\phi$. In principle, the form of $\Omega(t)$ can be arbitrary as long as it satisfies the constraint of equation (5). Due to general experimental restrictions, a square-shaped pulse was employed in the original proposal [10]. However, this requires a precise control of the starting and ending time, and the time jitter of a nonzero ending of the pulse may induce a population leakage. In this experiment, the form of $\Omega(t)$ is designed as $\Omega(t) = \Omega_0 \sin^2(\pi t/\tau)$, with the gate time $\tau$ set at 100 ns and the constraint of equation (5) as $\Omega_0 \tau = 4 \pi$. For this specified form of $\Omega(t)$ rather than using the square-shaped pulse, the microwave drive pulse is turned on and off smoothly with $\Omega(0) = \Omega(\tau) = 0$, and the population leakage is suppressed. Flexibly-shaped detunings can be induced by changing the frequency of the driving pulses, as details in appendix B. Following this process, all single-qubit Clifford gates are realized using the nonadiabatic holonomic single-shot protocol.

2.4. π-rotation Clifford gates

In the Clifford group, holonomic gates with π-rotation were firstly implemented. In this unique case, the single-shot protocol reverts back to the original protocol. Accordingly, in the experiment, two resonant microwave pulses simultaneously drove the $|0\rangle \leftrightarrow |e\rangle$ and $|e\rangle \leftrightarrow |1\rangle$ level transitions with the pump pulse $\Omega_p(t) = \Omega(t) \cos \theta/2$ and the drive pulse $\Omega_d(t) = \Omega(t) \sin \theta/2$, respectively. The gate parameter $\alpha = 0$ (i.e. $\Delta_\beta(t) = 0$), yielded the rotation angle $\gamma = \pi$. The rotation axis of the quantum gates was then determined by choosing specific parameters $\theta$ and $\phi$. To completely characterize the holonomic gates, we performed a QPT involving all three basis $|0\rangle$, $|e\rangle$, $|1\rangle$ [46], which is quantified by a reconstructed $\chi$-matrix. As shown in figure 1(b), for the three-level QPT, 16 different initial states $|\rho\rangle$ were prepared by sequentially applying the identity $I$, the π/2-rotation $X/2$, $Y/2$ and the π-rotation $X$ pulses to $|0\rangle \leftrightarrow |e\rangle$ and $|e\rangle \leftrightarrow |1\rangle$ transitions. Initialized states were then followed by a specified holonomic quantum gate in the Clifford group. Finally, a state measurement was performed with full quantum state tomography. The output state $|\rho\rangle$ was then extracted using the maximum likelihood estimation method. The process matrix $\chi$ was reconstructed from the input and output states by numerically solving the equation $|\rho\rangle = \sum_{m,n} \chi_{mn} E_m |\rho\rangle E_n$ [38]. The full set of nine orthogonal basis operators $E_m$ was chosen as $\{ I_{x1}, X_{01}, Y_{01}, Z_{01}, X_{0e}, Y_{0e}, X_{e1}, Y_{e1}, I_e \}$ [21, 46], in which the first four operators...
The population leakage of Hadamard gate, the \( \frac{c}{2} \)-rotation gate was demonstrated by setting \( \chi_{ab} \), as a reference. For the Hadamard gate, the fidelity of QPT \( F(H) \) reaches 99.2\%, and the error of the process is less than 1\%. In figure 2(c), the fidelity for all \( \pi \)-rotation gates in single-qubit Clifford group are provided with specific gate parameters \( \theta \) and \( \varphi \). The operations of gates \( C_i \) shown in this and following figures are provided in appendix A. The fidelities for all gates are above 99\%, with an average fidelity 99.3\%.

2.5. \( \pi / 2 \)-rotation Clifford gates

Advancing beyond the \( \pi \)-rotation gates, the single-shot protocol was followed and holonomic gates with \( \pi / 2 \)-rotation in the Clifford group were implemented. To obtain a rotation angle \( \gamma = \pi / 2 \), the off-resonant pump pulse and stocks pulse were simultaneously applied with the parameter \( \alpha \) set as \(-\pi / 6\) and the detuning of \( \Delta \beta(t) = -\Omega(t) / 2 \). Different \( \pi / 2 \)-rotation Clifford gates are specified by their corresponding rotation axes, which determine the parameters \( \theta \) and \( \varphi \).

As an example of \( \pi / 2 \)-rotation gates, the \( X \)-rotation gate was implemented. To obtain a rotation angle \( \gamma = \pi / 2 \), the off-resonant pump pulse and stocks pulse were simultaneously applied with the parameter \( \alpha \) set as \(-\pi / 6\) and the detuning of \( \Delta \beta(t) = -\Omega(t) / 2 \). Different \( \pi / 2 \)-rotation Clifford gates are specified by their corresponding rotation axes, which determine the parameters \( \theta \) and \( \varphi \).

As an example of the \( \pi \)-rotation gate, a Hadamard gate \( (H) \) was experimentally carried out with the setting parameters \( \alpha = 0 \), \( \theta = \pi / 4 \) and \( \varphi = 0 \). The real and imaginary parts of \( \chi \)-matrix for the Hadamard gate are provided in figures 2(a) and (b), respectively. The dominant elements in the subspace \( \{ 0 \} \), \( \{ 1 \} \) is \( X_0 \) and \( Z_0 \), and the imaginary part of \( \chi \)-matrix is close to zero, which are both consistent with the theoretical expectation. Moreover, the element \( \chi_{i,0} \) is close to one, which illustrates that the auxiliary state was almost unaffected during the holonomic gate operation, as described in the single-shot protocol. The population leakage of Hadamard gate is described by the trace of the reduced \( \chi \)-matrix, \( Tr(\chi) \), which is about 0.96. The main leakage error arises from the imperfection of the microwave signal and the limited nonlinearity. Following the definition of fidelity [22, 47, 48]

\[
F = \frac{|Tr(\chi_{a,b} \chi_{a,b}^\dagger)|}{\sqrt{Tr(\chi_{a,b} \chi_{a,b}^\dagger) Tr(\chi_{a,b} \chi_{a,b}^\dagger)}},
\]

the fidelity of the process matrix can be calculated using the ideal process matrix \( \chi_{a,b} \) as a reference. For the Hadamard gate, the fidelity of QPT \( F(H) \) reaches 99.2\%, and the error of the process is less than 1\%.

\[
F = \frac{|Tr(\chi_{a,b} \chi_{a,b}^\dagger)|}{\sqrt{Tr(\chi_{a,b} \chi_{a,b}^\dagger) Tr(\chi_{a,b} \chi_{a,b}^\dagger)}},
\]

The fidelities for all \( \pi \)-rotation gates in single-qubit Clifford group are provided with specific gate parameters \( \theta \) and \( \varphi \). The operations of gates \( C_i \) shown in this and following figures are provided in appendix A. The fidelities for all gates are above 99\%, with an average fidelity 99.3\%.

Figure 2. QPT result and the fidelities. (a) Real parts of process matrix \( \chi \) for \( \pi \)-rotation. (b) Imaginary parts of process matrix \( \chi \) for \( \pi \)-rotation. (c) Fidelities for all \( \pi \)-rotation gates. The dashed line represents a fidelity of 99\%.

2.5. \( \pi / 2 \)-rotation Clifford gates

Advancing beyond the \( \pi \)-rotation gates, the single-shot protocol was followed and holonomic gates with \( \pi / 2 \)-rotation in the Clifford group were implemented. To obtain a rotation angle \( \gamma = \pi / 2 \), the off-resonant pump pulse and stocks pulse were simultaneously applied with the parameter \( \alpha \) set as \(-\pi / 6\) and the detuning of \( \Delta \beta(t) = -\Omega(t) / 2 \). Different \( \pi / 2 \)-rotation Clifford gates are specified by their corresponding rotation axes, which determine the parameters \( \theta \) and \( \varphi \).

As an example of the \( \pi \)-rotation gate, a Hadamard gate \( (H) \) was experimentally carried out with the setting parameters \( \alpha = 0 \), \( \theta = \pi / 4 \) and \( \varphi = 0 \). The real and imaginary parts of \( \chi \)-matrix for the Hadamard gate are provided in figures 2(a) and (b), respectively. The dominant elements in the subspace \( \{ 0 \} \), \( \{ 1 \} \) is \( X_0 \) and \( Z_0 \), and the imaginary part of \( \chi \)-matrix is close to zero, which are both consistent with the theoretical expectation. Moreover, the element \( \chi_{i,0} \) is close to one, which illustrates that the auxiliary state was almost unaffected during the holonomic gate operation, as described in the single-shot protocol. The population leakage of Hadamard gate is described by the trace of the reduced \( \chi \)-matrix, \( Tr(\chi) \), which is about 0.96. The main leakage error arises from the imperfection of the microwave signal and the limited nonlinearity. Following the definition of fidelity [22, 47, 48]
The experimental result shows that all fidelities are above 99% with an average fidelity of 99.2%. According to the original protocol of holonomic gates, a $\frac{\pi}{2}$-rotation gate can only be realized by sequentially combining two $\pi$-rotation gates. The fidelity of such a gate can be roughly estimated to be about 98%, with a similar fidelity of $\pi$-rotation gate used in the current experiment. With the single-shot protocol, the arbitrary gate can be implemented within a single step. By shortening the gate operation time, the accumulated environment-induced error is reduced in a combined operation, and the corresponding fidelity is increased.

### 2.6. $2\pi/3$-rotation Clifford gates
To complete the whole Clifford group of single-qubit gates, eight $2\pi/3$-rotation gates were realized using the single-shot protocol. The axes of these gates are the lines connecting the origin and face centers of the octahedron, as shown in figure 1(c). For all $2\pi/3$-rotation gates, $\alpha = -\arcsin(1/3)$, leading to a rotation angle $\gamma = 2\pi/3$. The detuning satisfies $\Delta_p(t) = -\Omega(t)/3$, and the other two parameters $\theta$ and $\varphi$ were determined by the corresponding rotation axis for each gate.

For example, gate $C_8$ is an operation of $2\pi/3$-rotation about the axis $n = \frac{1}{\sqrt{3}}(1, 1, 1)$, which determines the parameters $\theta = \arccos(1/3)$ and $\varphi = \pi/4$. According to equation (7), the gate operation in the computation subspace for $C_8$ reads as:

$$U = \frac{1}{2} \begin{bmatrix} 1 & -i & -1 & -i \\ -i & 1 & i & 1 \\ -1 & i & 1 & i \\ i & -1 & i & 1 \end{bmatrix} = \frac{1}{2}(I - i\sigma_x - i\sigma_y - i\sigma_z).$$

(9)

The $\chi$-matrix for the gate $C_8$ is provided in figures 4(a) and (b). The real part of $\chi$-matrix has a similar value in the $X, Y, Z$ components, corresponding well with the theoretic expectation in equation (9). The QPT fidelity for gate $C_8$ is approximate to $F(C_8) = 99.2\%$, which is comparable to previous holonomic gates with other rotation angles. The population leakage characterized by the quantity $\text{Tr}(\tilde{\chi})$ is about 98%. In figure 4(c), the fidelities for all Clifford gates with a $2\pi/3$-rotation are presented, in which all the fidelities are larger than 99%, with an average fidelity reaching 99.1%.

### 2.7. Randomized benchmarking
RB is another systematic method to extract the quantum gate fidelity. The gate fidelity in the RB measurement is separately quantified by excluding errors in state preparation and measurement [36, 39–41]. The Clifford-based RB measurement was implemented to obtain the holonomic gate fidelity. Twenty-four Clifford holonomic gates were used in the RB experiment, with each gate individually implemented by choosing specific gate parameters $\alpha, \theta$, and $\varphi$.

A reference RB experiment was first performed. As shown in the pulse sequence in figure 5(a), the qubit was initially prepared at $|0\rangle$ state, then a sequence of $m$ Clifford gates were randomly chosen to drive the qubit. As the
Clifford group is a closed set, a recovery gate can be defined and finally applied to reverse the operation of Clifford gates. The remaining population of the initial state is measured afterwards. After repeating this random operation sequence \( k \) (=50 in our experiment) times, the average result of the remaining population as a function of \( m \) was obtained, which is also called a sequence fidelity. As shown in figure 5(b), the sequence fidelity can be fitted using the function \( [40] \):

\[
F = Ap^m + B,
\]

where \( F \) is the sequence fidelity, \( p \) is a depolarizing parameter, and the parameters \( A \) and \( B \) absorb the error in state preparation and measurement. The average error \( r \) over the randomized Clifford gates is given by \( r = (1 - p)/2 \). From the fitting of the reference RB measurement, the depolarized parameter \( p_{\text{ref}} = 0.986 \) is obtained, which yields an average error \( r_{\text{ref}} = 0.007 \), or an average RB fidelity 99.3%.
The reference RB experiment can only give an average gate fidelity over the Clifford gates. For a specific gate, interleaved RB experiment can be applied to determine the gate fidelity \([\mathcal{F}]\). The pulse sequence for the interleaved RB experiment is provided in figure 5(a). At each step, the qubit was driven by a combination of a randomly selected Clifford gate and the target holonomic gate. After a reversed recovery gate was applied, the remaining population or the sequence fidelity was measured as a function of the number of steps \(m\). Similarly, the sequence fidelity was fitted by the same function as in the reference RB experiment, leading to a new depolarized parameter \(P_{\text{gate}}\).

The specific gate fidelity is given by \(F_{\text{gate}} = 1 - (1 - P_{\text{gate}}/P_{\text{det}})/2\). The experimental results for seven gates are shown in figure 5(b). The chosen gates were the idle gate \(I\), two \(\pi\)-rotation gates, \(X\) and \(H\), two \(\pi/2\)-rotation gates \(X/2\) and \(Z/2\), and two \(2\pi/3\)-rotation gates \(C_6\) (rotation axis \(n = \sqrt[3]{(1, 1, 1)}\)) and \(C_4\) (rotation axis \(n = \sqrt[3]{(1, 1, 1)}\)). According to the interleaved RB experiment, the fidelities for \(X\), \(H\), \(X/2\), \(Z/2\), \(C_6\) and \(C_4\) are 99.2\%, 99.0\%, 99.3\%, 99.4\%, 99.0\% and 99.3\%, respectively. After benchmarking all the Clifford gates, RB fidelities for all the Clifford holonomic gates are obtained, each of which is larger than 99.0%.

3. Discussion

In this paper, QPT and RB experiments were applied to verify that all the single-qubit Clifford gates can be implemented by the single-shot protocol with high fidelity. Off-resonant and time-dependent detuning is a key step of the protocol, and were applied in a superconducting Xmon. Experiments confirmed that all the fidelities for single-qubit gates are higher than 99.0\%. It is also suggested that the main error arises from decoherence and energy relaxation, which is confirmed by the fidelity of the idle gate, \(F = 99.1\%\). Similar fidelities for \(\pi\)-rotation gates and gates with other rotation angles were also obtained, illustrating that the control errors are similar for resonant and off-resonant pulses.

The single-qubit gates used in the experiment are compatible with the previously proposed two-qubit nonadiabatic holonomic gate. By combining two-qubit gates with the current results, a universal nonadiabatic holonomic quantum computation can be obtained in the future.

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Appendix A. Single-qubit Clifford gates

In this experiment, 24 single-qubit Clifford gates were implemented using the single-shot protocol. To simplify the notation, \(C_i, (i = 0, 1, \ldots, 23)\) were used as the names of the Clifford gates with the rotation axes and rotation angles shown in table A1.

Appendix B. Realization of time dependent detuning

Implementation of a time-dependent detuning is the key step of the single-shot protocol. A detailed explanation is presented as follows. The superconducting Xmon is driven by two microwave tones with pump frequency \(\omega_p(t)\) and stocks frequency \(\omega_s(t)\). The microwave drives can be denoted as

\[
\text{drive}_{\{p, s\}}(t) = \Omega_{\{p, s\}}(t) \cos[\phi_{\{p, s\}}(t)],
\]

(\ref{eq:drive})

in which \(\phi_{\{p, s\}}(t)\) is the phase of the microwave drive. The instantaneous frequency is the rate of phase accumulation, \(\omega_{\{p, s\}}(t) = \frac{d}{dt}[\phi_{\{p, s\}}(t)]\), which leads

\[
\phi_{\{p, s\}}(t) = \int_0^t \omega_{\{p, s\}}(\tau) d\tau.
\]

(\ref{eq:phi})

In the experiments conducted in this study, the frequency \(\omega_{\{p, s\}}(t)\) can be calculated after the form of \(\Omega(t)\) and the parameter \(\alpha\) were selected. Then corresponding microwave drive was then produced by mixing two low-frequency quadratures (I and Q) with the local oscillator signal \([36]\) to realize the time-dependent detuning.
Table A1. The 23 single-qubit Cliffords with the name $C_i$ and the rotation axis $i$ excluding the identity operation $I$ ($C_0$). Here $X/2$ denotes a $\pi/2$ rotation over the $X$ axis with unitary $R_X(\pi/2) = \exp(-i\pi/4)$.

| Gate name | Rotation axis (θ) |
|-----------|-------------------|
| $\pi$-rotation | |
| $C_1$ (X) | (1, 0, 0) |
| $C_2$ (Y) | (0, 1, 0) |
| $C_3$ (Z) | (0, 0, 1) |
| $C_{18}$ | $\frac{1}{2}(1, 0, -1)$ |
| $C_{19}$ (H) | (1, 0, 1) |
| $C_{20}$ | $\frac{1}{2}(0, 1, 1)$ |
| $C_{21}$ | $\frac{1}{2}(0, 1, -1)$ |
| $C_{22}$ | $\frac{1}{2}(1, 1, 0)$ |
| $C_{23}$ | $\frac{1}{2}(1, -1, 0)$ |
| $2\pi/3$-rotation | |
| $C_4$ | $\frac{1}{2}(1, 1, -1)$ |
| $C_5$ | $\frac{1}{2}(1, -1, 1)$ |
| $C_6$ | $\frac{1}{2}(-1, 1, 1)$ |
| $C_7$ | $\frac{1}{2}(-1, -1, -1)$ |
| $C_8$ | $\frac{1}{2}(1, -1, 1)$ |
| $C_9$ | $\frac{1}{2}(-1, -1, 1)$ |
| $C_{10}$ | $\frac{1}{2}(1, 1, -1)$ |
| $C_{11}$ | $\frac{1}{2}(-1, -1, 1)$ |
| $\pi/2$-rotation | |
| $C_{12}$ (X/2) | (1, 0, 0) |
| $C_{13}$ (−X/2) | (−1, 0, 0) |
| $C_{14}$ (Y/2) | (0, 1, 0) |
| $C_{15}$ (−Y/2) | (0, −1, 0) |
| $C_{16}$ (Z/2) | (0, 0, 1) |
| $C_{17}$ (−Z/2) | (0, 0, −1) |

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