Topological edge states of anyon pairs emulated in electrical circuits

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Abstract. Recent advances in two-particle topological quantum states demonstrate resilience to geometrical imperfections and hold perspectives for robust quantum computations. In this context, particles with fractional quantum statistics, the so-called anyons, attract especial attention. In particular, topological edge states of anyon pairs in one-dimensional chains of coupled cavities were recently predicted to demonstrate localization at one or another edge of the array depending on details of the quantum statistics. In this paper, we propose an equivalent electrical circuit serving as a classical emulator of such topological states. Detailed numerical studies of resonances in the circuit fully support theoretical predictions, pointing towards future experimental realizations of anyonic states analogs in electrical circuits.

1. Introduction

Anyons are quasi-particles with fractional quantum statistics in-between bosons and fermions that exist in one- and two-dimensional systems and were demonstrated experimentally with a discovery of fractional quantum Hall effect [1]. However, the direct observation of this effect requires the use of high-quality semiconductor nanostructures, extremely low temperatures, and high magnetic fields [1, 2], thus being very challenging. On the other hand, anyons offer great potential for fault-tolerant quantum computations [1]. This led to active development of various strategies for their emulation, including photonic systems [3] and flux-tube models [4]. Despite considerable progress in realizing topological states of photon pairs [5] and their emulation [6], topological states of two anyons remain uncharted. Recently, it has been theoretically predicted that one-dimensional (1D) arrays of coupled cavities supporting anyonic excitations with on-site interaction possess topological edge states with localization depending on the quantum statistics [7].

In the present paper, we develop an equivalent electrical circuit allowing for a classical emulation of such two-particle quantum systems, and perform its numerical simulations with the help of Keysight Advanced Design System software.

2. Theoretical model

We consider a 1D array of cavities with nearest-neighbor tunneling couplings $J$, on-site interaction between two anyons with energy $U$, and additional two-particle tunneling links $P$ corresponding to the simultaneous tunneling of two anyons between the neighboring cavities, as
shown in Fig. 1(a). The system is described by the following Hamiltonian (considering energy units $\hbar = 1$):

$$
\hat{H} = \omega_0 \sum_{m=1}^{N} \hat{n}_m + \sum_{m=1}^{N} U_m \hat{n}_m (\hat{n}_m - 1) - J \sum_{m=1}^{N-1} (\hat{a}^\dagger_m \hat{a}_{m+1} + \hat{a}^\dagger_{m+1} \hat{a}_m) + \\
+ \frac{P}{2} \sum_{m=1}^{(N-1)/2} (\hat{a}^\dagger_{2m-1} \hat{a}^\dagger_{2m-1} \hat{a}_{2m} + \hat{a}^\dagger_{2m} \hat{a}^\dagger_{2m-1} \hat{a}_{2m-1}),
$$

(1)

where $N$ is the number of cavities in the array, $\omega_0$ is the energy of one anyon in a cavity, $\hat{n}_m$ and $\hat{a}_m$ are the creation and annihilation operators for the anyon at the cavity $m$, and $\hat{n}_m = \hat{a}_m^\dagger \hat{a}_m$ is a particle number operator. The first three terms correspond to the Hubbard model with tunneling links $J$ and the on-site interaction energy $U_m$, while the latter term describing the simultaneous tunneling of a pair of anyons with the amplitude $P$ is similar to the model considered in Ref. [6]. For the bulk sites ($m \neq 1$ and $m \neq N$) and edges ($m = 1$ or $m = N$), the interaction energy $U_m$ equals $U$ and $U + J^2/(2U)$, respectively. As anyons obey fractional statistics, their commutation relations involve the statistical exchange angle $\theta \in [0, \pi]$:

$$
\hat{a}_l \hat{a}_k = \exp(i \theta \text{sgn}(l-k)) \hat{a}_k \hat{a}_l,
$$

(2)

As anyons obey fractional statistics, their commutation relations involve the statistical exchange angle $\theta \in [0, \pi]$. The limits of the statistical exchange angle $\theta = 0$ and $\theta = \pi$ correspond to the bosons and pseudo-fermions, respectively. The latter are characterized by the fermionic commutation relations in the case of different sites of the array ($l \neq k$), while at the same site ($l = k$) they feature bosonic commutation relations.

To find the system eigenmodes, we substitute the two-photon wave function in the form

$$
|\psi\rangle = \frac{1}{\sqrt{2}} \sum_{m,n=1}^{N} \beta_{mn} \hat{a}_m^\dagger \hat{a}_n^\dagger |0\rangle,
$$

(2)

to the eigenvalue problem $\hat{H} |\psi\rangle = E |\psi\rangle$, thus obtaining an equivalent two-dimensional (2D) tight-binding (TB) model with nearest-neighbor links $J$ and diagonal links $P$, which is shown in Fig. 1(b). However, the vertical links next to the diagonal are replaced with the statistical angle-dependent couplings $Je^{i\theta}$, which describe the statistical quantum statistics. The TB eigenenergies $\varepsilon$ are defined as $\varepsilon = E - 2\omega_0$. Then, we study such a single-particle problem of higher dimensionality numerically and obtain 2D maps of coefficients $|\beta_{mn}|^2$ characterizing the probability of detecting one anyon at the $m$-th cavity and the other one at the $n$-th cavity.

As seen from Fig. 1(c), for $\theta = 0$ there is a bandgap near energies $\varepsilon/J \approx 4$. The two bands above and below the bandgap (Fig. 1(d,f)) correspond to delocalized bound photon pairs, doublons, while the single in-gap mode represents a topological edge state localized at the first cavity, see Fig. 1(e). In the opposite case of $\theta = \pi$ shown in Fig. 1(g), the bands above and below the bandgap still represent doublons (despite the lower branch merges with the continuum of scattering states of anyons), see Fig. 1(h,j). However, the edge state localization pattern (Fig. 1(i)) changes, facilitating the presence of both anyons at the opposite edge of the cavity array.

3. Electrical circuit emulation

To support our theoretical results, we map the TB equations for the Hamiltonian Eq. (1) to a set of Kirchhoff’s rules describing voltages and currents in the electrical circuit, following the lines of paper [6]. Then, on-site voltages in the circuit $\varphi_{mn}$ represent wavefunction amplitudes $\beta_{mn}$, and frequencies of resonances $f_j$ correspond to eigenenergies $\varepsilon_j$. The resulting electric circuits shown in Fig. 2(a,c) resemble the TB model (Fig. 1(b)) up to the replacement of tunneling links $J$ and $P$ with the capacitors $C_J$ and $C_P$, respectively, and the statistical angle-dependent links.
Figure 1. (a) Design of the considered system representing a 1D array of cavities coupled via tunneling links $J$ and two-particle links $P$. (b) Equivalent 2D tight-binding model with the statistical exchange angle $\theta$. The array of five cavities is considered as an example. Coordinates of the site $(m, n)$ correspond to the location of the first and second anyons in the cavities $m$ and $n$, respectively. (c) Spectrum of eigenvalues $\varepsilon_j$ as a function of the eigenvalue number $j$ for the model with the statistical exchange angle $\theta = 0$ and the size of $9 \times 9$ sites. The parameters of the model are $J = 1$, $U = 1.5$, and $P = -0.75$. (d-f) Maps of two-particle wave function amplitudes in the case of the bosonic statistics ($\theta = 0$). (g-j) The same as (c-f), but for the model supporting topological edge states of two pseudo-fermions ($\theta = \pi$).

$Je^{i\theta}$ by the complex impedances $C_je^{i\theta}$. All nodes of the circuit are grounded with the inductors $L$, except for diagonal nodes $(m = n)$ featuring the additional capacitors $C_U + 2C_J(1 - \cos(\theta))$ corresponding to the on-site interaction. Nodes neighboring to the diagonal ones are additionally grounded by the capacitors $C_g = 2C_J$. Edge nodes are also grounded with the capacitors $C_J$ and $C_P$ to compensate for the absent neighbors. The parameters of the circuit and the tight-binding model are related as follows:

$$J = 1, \quad U = \frac{C_P + C_U}{2C_J}, \quad P = -\frac{C_P}{C_J}, \quad \varepsilon = \frac{f_0^2}{f^2} - 4, \quad f_0 = \frac{1}{2\pi\sqrt{LC_J}}. \quad (3)$$

The impedance spectroscopy for $\theta = 0$ shown in Fig. 2(b) reveals two groups of resonant peaks corresponding to doublons, and a single peak in the bandgap corresponding to the edge state, in accordance with theoretical predictions. In the case of $\theta = \pi$, however, the tunneling links $Je^{i\theta}$ are represented by the negative capacitors $-C_J$. The latter can be approximated by the inductors $L_J$ having the same absolute value of impedance as the capacitors $C_J$ at the frequency of topological edge state, Fig. 2(c). Simulated spectra for this case, shown in Fig. 2(d),
Figure 2. (a) Equivalent electrical circuit for the bosonic statistics $\theta = 0$. Capacitors $C_J$ and $C_P$ corresponding to the tunneling couplings $J$ and $P$ are shown in blue and red, respectively. (b) Numerically simulated spectra of the absolute value of impedance for the sites with coordinates $(1,1)$, $(2,2)$ and $(9,9)$ in the circuit of Panel (a) with $9 \times 9$ nodes. Insets show normalized impedance maps at the indicated frequencies. (c) The same as (a), but for the pseudo-fermionic statistics $\theta = \pi$. Inductors $L_J$, replacing the appropriate capacitors $C_J$, are shown in green. (d) The same as (b), but for the circuit with $\theta = \pi$. Grounding capacitors $C_g$ for the nodes neighboring to the diagonal ones are shown in pink.

again demonstrate the presence of two doublon bands and the in-gap edge state. Strikingly, this state localizes at the opposite corner of the circuit, corresponding to the localization of anyon pair at the opposite edge of the cavity array and facilitating the statistics-induced topological transition.

4. Conclusion
We have developed a rigorous electric circuit emulator of statistics-induced topological edge states of interacting anyons predicted previously [7]. Numerical simulations of electrical circuits fully support our theoretical findings, demonstrating the emergence of topological edge states.

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