Tuning the dipolar interaction in quantum gases

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We have studied the tunability of the interaction between permanent dipoles in Bose-Einstein condensates. Based on time-dependent control of the anisotropy of the dipolar interaction, we show that even the very weak magnetic dipole coupling in alkali gases can be used to excite collective modes. Furthermore, we discuss how the effective dipolar coupling in a Bose-Einstein condensate can be tuned from positive to negative values and even switched off completely by fast rotation of the orientation of the dipoles.

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In the Bose-Einstein condensates (BECs) created thus far [1] atoms interact essentially only at very short distance. At low temperatures, the interaction between these neutral atoms can be typically characterized by a single parameter, the s-wave scattering length. The magnitude and sign of the s-wave scattering length can be changed by tuning the external magnetic fields in the vicinity of a so-called Feshbach resonance [2]. This technique has opened new possibilities for the study and manipulation of BECs. In particular, the value of the s-wave scattering length can be varied in real time, allowing, for example, the observation of collapsing and exploding BECs [3].

In recent years an increasing interest is devoted to dipole-dipole interactions in ultracold gases [4–12]. Dipolar interactions would largely enrich the variety of phenomena to be observed due to their long-range and anisotropic character. If atomic dipole moments are sufficiently large, the resulting dipole-dipole forces may influence, or even completely change the properties of Bose gases [4,5,11], the conditions for BCS transition in Fermi gases [4,7], or the phase diagram for quantum phase transitions in ultracold dipolar gases confined in optical lattices [8]. The interplay of short-range scattering and long-range interaction may give rise to phenomena like ferromagnetic order and spin waves [3]. Moreover the dipolar particles are considered to be promising candidates for the implementation of fast and robust quantum-computing schemes [14].

In this Letter, we consider the case of permanent (magnetic) dipoles. For alkali atoms with a magnetic moment $m$ of $\mu_B$, where $\mu_B$ is a Bohr magneton, the long range part of the magnetic dipole-dipole interaction is generally neglected since it is very small compared to the s-wave pseudo-potential. Here, we show that even in alkali BECs this dipolar interaction can be made visible if the dipole-dipole interaction is modulated by rotating the atomic dipoles. For appropriate rotation frequencies, a coupling to elementary excitations can be achieved leading to a dynamic growth of the amplitude of the excitation. In particular, we will show how the dipolar coupling in an alkali condensate can be used to excite the quadrupole mode and how it is possible to extract quantitative information about the ratio of the dipolar coupling and the usual s-wave coupling.

In ensembles of atoms with a larger magnetic moment, such as chromium where $m = 6\mu_B$, the magnetic dipole-dipole interaction energy may become comparable in size with the s-wave contribution to the mean-field energy. Therefore, even the static properties of a condensate such as the aspect ratio are significantly altered. Here, we show how the effective strength of the interaction and thus its effect on the static properties of a condensate can be varied by fast rotation of the atomic dipoles. In the case of a dipolar interaction which is stronger than the s-wave interaction only certain combinations of trap geometry and orientation of the dipoles lead to a stable (or metastable) condensate [4,14]. By tuning the dipolar interaction unstable condensates can thus be transformed into stable ones and vice versa. In this paper, we study the interaction between permanent magnetic dipoles. However, the methods we propose can also be adapted for particles with electric dipole moment such as heteronuclear molecules. In this case the dipolar interaction is expected to dominate over the s-wave interaction and thus the tuning of the interaction might be necessary to reach stability.

Manipulating the magnetic dipolar coupling in a BEC—The long-range part of the interaction between two magnetic dipole moments $\mathbf{m}_1$ and $\mathbf{m}_2$ of atoms located at $\mathbf{r}_1$ and $\mathbf{r}_2$ takes the following dependence

\[
U_{dd}(\mathbf{r}) = -\frac{\mu_0}{4\pi} \frac{3 (\mathbf{m}_1 \cdot \hat{\mathbf{r}}) (\mathbf{m}_2 \cdot \hat{\mathbf{r}}) - (\mathbf{m}_1 \cdot \mathbf{m}_2)}{r^3} \tag{1}
\]

as a function of the interatomic distance $\mathbf{r} = \mathbf{r}_2 - \mathbf{r}_1$, where $\hat{\mathbf{r}} = \mathbf{r}/r$ and $\mu_0$ is the magnetic permeability of the vacuum. We consider the case of a general time-dependent homogeneous magnetic field

\[
\mathbf{B}(t) = B \mathbf{e}(t), \tag{2}
\]

slowly-varying with respect to the Larmor frequency $\omega_{\text{Larmor}} = mB/\hbar$, where $m = |\mathbf{m}_1| = |\mathbf{m}_2|$ for identical atoms. A spin-polarized atomic ensembles will adiabatically follow the external magnetic field $B(t)$ and the resulting interatomic energy becomes

\[
U_{dd}(\mathbf{r}, t) = -\frac{\mu_0 m^2}{4\pi} \frac{3 (\mathbf{e}(t) \cdot \hat{\mathbf{r}})^2 - 1}{r^3} \tag{3}
\]
In the Thomas-Fermi limit it is useful to define a dimensionless dipolar interaction mean-field potential generated by the (time dependent) dipole length which we assume to be positive. \( \Phi \) and is thus
time-dependent.

Our analysis is based on the mean-field approach in the Thomas-Fermi limit already used in the context of dipolar Bose gases \[1,2\]. Such a description can be accomplished through the hydrodynamic equations for superfluids \[1\]:

\[
\begin{align*}
\frac{\partial n}{\partial t} &= -\nabla (n \mathbf{v}) , \\
\frac{\partial \mathbf{v}}{\partial t} &= -\nabla \left( \frac{v^2}{2} + \frac{\delta \mu}{M} \right) ,
\end{align*}
\]

where \( n \) is the density, \( \mathbf{v} \) the velocity field and \( \delta \mu \) the difference between the mean-field plus external fields and the chemical potential \( \mu \) given by

\[
\delta \mu = gn + \Phi_{dd} + \frac{M \omega_n^2 r^2}{2} - \mu .
\]

Here \( M \) is the atomic mass, \( M \omega_n^2 r^2/2 \) is an isotropic harmonic potential which could be created using optical dipole forces, \( g = 4 \pi \hbar^2 a/M \) and \( a \) is the s-wave scattering length which we assume to be positive. \( \Phi_{dd}(\mathbf{r}, t) \) is the mean-field potential generated by the (time dependent) dipolar interaction

\[
\Phi_{dd}(\mathbf{r}, t) = \int d\mathbf{r}' U_{dd}(\mathbf{r} - \mathbf{r}', t) n(\mathbf{r}') .
\]

In the Thomas-Fermi limit it is useful to define a dimensionless quantity

\[
\varepsilon_{dd} = \frac{\mu_0 n_0^2 M}{12\pi \hbar^4 a} ,
\]

which is a measure of the strength of the dipole-dipole interaction relative to the s-wave scattering energy. Modifications in the ground state condensate density can only be expected if \( \varepsilon_{dd} \) is appreciably different from zero.

For chromium the s-wave scattering length \( a_{Cr} \) is not known. If we assume that \( a_{Cr} \) is equal to the sodium scattering length (\( a = 2.8 \) nm) the dipolar strength parameter becomes \( \varepsilon_{dd}^{Cr} = 0.29 \) (\( m = 6 \mu_B \)). Therefore, the dipolar force is expected to lead to visible modification in the density of a chromium BEC. Differently, in both rubidium and sodium the magnetic dipolar energy is rather small compared to the mean-field s-wave scattering energy. The values of \( \varepsilon_{dd} \) are \( \varepsilon_{dd}^{Na} = 0.0064 \) and \( \varepsilon_{dd}^{Na} = 0.0035 \), in the doubly spin-polarized ground-state, where \( m = \mu_B \). For heteronuclear molecules with a permanent electric dipole moment of one Debye the corresponding dipolar coupling will be increased by approximatively the inverse square of the fine structure constant. This implies that \( \varepsilon_{dd} \) is on the order of \( 10^2 \).

Probing the quadrupole modes in sodium and rubidium—The time modulation of the dipolar coupling \( \Phi_{dd}(\mathbf{r}, t) \) can be used to resonantly excite a quadrupole mode even for sodium and rubidium condensates. For simplicity we restrict ourselves to the case of a condensate in the Thomas-Fermi regime confined in an isotropic harmonic trap with frequency \( \omega_0 \) (for example \( \omega_0/2\pi = 100 \) Hz). Because \( \varepsilon_{dd} \) is small the dipole-dipole mean-field potential \( \Phi_{dd} \) can be calculated using the symmetric equilibrium density distribution \( n_0 = M \omega_0^2 (R^2 - r^2)/2g \), where \( R \) is its radius

\[
\Phi_{dd}(\mathbf{r}, t) = -\frac{\varepsilon_{dd} M \omega_0^2}{5} \left[ 3 (\mathbf{e}(t) \cdot \mathbf{r})^2 - 1 \right] / r^2 ; \quad r < R ,
\]

\[
\Phi_{dd}(\mathbf{r}, t) = -\frac{\varepsilon_{dd} M \omega_0^2}{5} \left[ 3 (\mathbf{e}(t) \cdot \mathbf{r})^2 - 1 \right] / r^3 ; \quad r > R .
\]

The potential outside the condensate region (\( r > R \)) corresponds exactly to the field generated by \( N \) dipoles located in the center of the condensate. Differently, inside the condensate (\( r < R \)) the mean-field potential has an anisotropic but harmonic dependence.

If the interaction is driven on a resonance frequency the evolution of the condensate can be modified even for very small dipolar coupling. Consider a magnetic field rotating in the plane \( x - y \) as \( \mathbf{B}(t) = Be(t) \), where

\[
\mathbf{e}(t) = \cos(\Omega t) \mathbf{x} + \sin(\Omega t) \mathbf{y} .
\]

The resulting dipolar interatomic energy \( \Phi_{dd}(\mathbf{r}, t) \) and its corresponding contribution to the mean-field potential \( \Phi_{dd}(\mathbf{r}, t) \), will have a ‘rotating’ part \( \Phi_{dd}(\mathbf{r}, t) \). For small dipolar coupling within alkali gases the non oscillating part of \( \Phi_{dd}(\mathbf{r}, t) \) can be neglected and the time-dependent part will resonantly couple to the ‘transversal’ quadrupole mode when \( \Omega \approx \omega_0/\sqrt{2} \) (see Ref. \[12\]). After linearization of the equations of motion \( \Phi_{dd}(\mathbf{r}, t) \) and \( \Phi_{dd}(\mathbf{r}, t) \) we obtain

\[
\frac{\partial^2 \delta n}{\partial t^2} = \nabla \left( \frac{gn_0 M}{M} \nabla (\delta n) \right) + \nabla \left( \frac{n_0 M}{M} \nabla (\Phi_{dd}) \right) ,
\]

where \( \delta n = n - n_0 \) is the density displacement around the equilibrium density \( n_0 \). The time-dependent part of \( \Phi_{dd} \) corresponds in Eq. \( \Phi_{dd}(\mathbf{r}, t) \) to the rotation of an anisotropic harmonic trap with deformation

\[
\varepsilon = \frac{\omega_X^2 - \omega_Y^2}{\omega_X^2 + \omega_Y^2} = \frac{3}{10} \varepsilon_{dd} ,
\]

where \( \omega_X \) and \( \omega_Y \) are the maximum and minimum values of the rotating trap frequencies. Neglecting the time-independent part of \( \Phi_{dd} \) the density will evolve as

\[
\delta n = -\frac{3 \varepsilon_{dd} M \omega_0^2}{10 g} r^2 \sin^2 \theta \sin(2\phi - 2\Omega t) \Omega t ,
\]

where \( \phi \) is the azimuthal angle (\( r < R \)). It is noteworthy that the rotating part \( \Phi_{dd}(\mathbf{r}, t) \) couples only the first quadrupole mode in the Thomas-Fermi limit \( \Phi_{dd}(\mathbf{r}, t) \). From the study of the evolution of the condensate deformation

\[
\alpha = \frac{(x^2 - y^2)}{(x^2 + y^2)} = \frac{3}{5} \varepsilon_{dd} \Omega t \sin(2\Omega t)
\]

we can extract information about the ratio of the dipolar coupling versus the standard mean-field energy. If the
The non-linearity of the mean-field is expected to give a significant correction for larger $Q.$ The extension of the dipolar interaction from attractive to repulsive. At a particular angle along the $z$-direction and a fast rotating field $\Omega$—Let us now apply this tunability to the stability diagram of dipolar BECs. We consider the case in which the dipolar coupling is larger than the s-wave pseudopotential ($\epsilon_{dd} \gtrsim 1$). Here the tunability can stabilize strongly interacting dipolar gases like heteronuclear molecules which otherwise exhibit instability over a wide range of parameters. This regime, considered also in Ref. [4,6], may be achieved in a cloud of magnetic dipolar atoms, by making use of a Feshbach resonance to reduce the s-wave scattering length.

In a spin polarized cloud the stability diagram as well as the condensate anisotropy can be studied in the Thomas-Fermi limit as function of $\epsilon_{dd}$ (see Ref. [4]). When $\epsilon_{dd} \geq 1$ the cloud may be unstable against collapse, i.e. can be always possible to find a density configuration of arbitrary large negative energy. The stability as $\epsilon_{dd}$ and the trap anisotropy $l = \omega_z/\omega_r$ are varied can be investigated in analogy to Ref. [4].

Within the Thomas-Fermi approximation, the stability analysis can be made using a scaling variational approach [4]. Assuming a density for the condensate of the form

$$n(x, y, z) = \frac{1}{\lambda_r^2 \lambda_z} n_0 \left( \frac{x}{\lambda_r}, \frac{y}{\lambda_r}, \frac{z}{\lambda_z} \right),$$

where $\lambda_r$ and $\lambda_z$ are scaling variational parameters and $n_0$ is a spherically symmetric density distribution (for instance the usual Thomas-Fermi inverted parabola). The condensate anisotropy $\lambda_r/\lambda_z$ can be found by minimiz-
ing the expectation value of the total energy $H_{\text{tot}} = H_{\text{ho}} + H_{\text{int}}$ where

$$H_{\text{ho}} = c_1 N m \left( 2\omega_z^2 \lambda_z^2 + \omega_l^2 \lambda_l^2 \right)$$  \hspace{1cm} (19)$$

is the harmonic potential energy and

$$H_{\text{int}} = c_2 \frac{N^2 g}{\lambda_l \lambda_z} \left[ 1 - \epsilon_{dd} \left( \frac{3 \cos^2 \varphi - 1}{2} \right) f \left( \lambda_r / \lambda_z \right) \right]$$  \hspace{1cm} (20)$$

the mean-field interaction energy, where $c_1$ and $c_2$ are numerical constants \[20\] and

$$f(\kappa) = \frac{1 + 2\kappa^2}{1 - \kappa^2} - \frac{3\kappa^2 \text{tanh}^{-1} \sqrt{1 - \kappa^2}}{(1 - \kappa^2)^{3/2}}$$  \hspace{1cm} (21)$$

derives from an angular integration.

The stability can be varied for a fixed value of $\epsilon_{dd}$ and the trap anisotropy $l$ just varying one easy experimental parameter, namely the angle $\varphi$. In Fig. 3 we show one example of how the stability phase diagram for $\epsilon_{dd} = 4$ can be varied as function of the angle $\varphi$. For very large $\epsilon_{dd}$ the stable region in the $\varphi$-$l$ plane becomes a thin region around the magic angle line.

**Conclusion**—We have shown that the anisotropy of the dipole-dipole interaction of a polarized gas can be used to tune the strength of the dipolar coupling and to excite collective excitations. Tuning the dipolar coupling will become an important tool for designing atomic quantum gases with novel properties and dynamical aspects. This approach in optically trapped Bose condensed gases will become an important tool for designing atomic quantum gases with novel properties and dynamical aspects.

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13. One can define an expression analogous to Eq. \[1\] for 'permanent' electric dipoles as $\epsilon_{dd} = \frac{\epsilon_{dd}}{12\pi c_0^2 d^2}$ where $d$ is the electric permanent dipole moment and $a$ a scattering length.
14. The angular dependence of (9) and (10) $\left[ 3(\mathbf{e}(t) \cdot \mathbf{r})^2 - 1 \right] e$ for $e$ given by (11) can be rewritten in terms of spherical harmonics as $12 \sqrt{2\pi} / 15 R [2 \sin(2\pi/5) - \sqrt{4\pi}/5 \sin(2\pi/5)]$. Because of the $\varphi^2$ dependence in $H_{\varphi}$ only the first quadrupole mode in the Thomas-Fermi limit (with $n=0$ number of radial nodes in the density displacement $\delta n$) \[13\] is resonantly excited.
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20. $c_1$ and $c_2$ depend only on $n_0$; because $c_1$ and $c_2$ can be reabsorbed in a global re-scaling of the energy and variational parameters, the study of the condensate anisotropy and stability does not depend on $n_0$. 

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**FIG. 3.** Stability phase diagram in the $\varphi$-$l$ plane \((l = \omega_z / \omega_l)\) obtained in the Thomas-Fermi limit \((N \to \infty)\) and with $\epsilon_{dd} = 4$. Condensate are always stable in the vicinity of the magic angle $\varphi_M = 54.7^\circ$. 

[Diagram showing the stability phase diagram in the $\varphi$-$l$ plane]