Corrigendum: Linear response formula for piecewise expanding unimodal maps

2008 Nonlinearity 21 677–711

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Received 5 May 2012, in final form 11 May 2012
Published 22 June 2012
Online at stacks.iop.org/Non/25/2203
Recommended by C Liverani
Mathematics Subject Classification: 37E05, 37F15, 28C05

1. Theorem 7.1

The last claim of theorem 7.1 should read: ‘If the postcritical orbit of $f_0$ is dense in $[c_2, c_1]$ then there exist $\varphi \in C^\infty(I)$ and a sequence $t_n \to 0$ so that $c$ is not periodic under any $f_{t_n}$, and so that we have

$$
\lim_{n \to \infty} \left| t_n^{-1} \left( \int \varphi \rho_{t_n} - \int \varphi \rho_0 \, dx \right) \right| \to \infty.$$

\textbf{Proof if the orbit of $c$ is dense.} We have $E_0 = \mathcal{F}_0 = \text{id}$ and, using (94), we can consider $\mathcal{P}_t$ as in the case when the orbit is infinite but not dense.

Let $x_0$ be the fixed point of $f$ which lies in the interior of $I$, and assume that $\varphi(x_0) = 1$ and $\int \varphi \, d\mu = 0$. Since the postcritical orbit is dense, for any $\delta > 0$ there exists $j_0(\delta) \geq 1$ so that $d(c_j, x_0) < \delta$. Clearly, $\lim_{\delta \to 0} j_0 = \infty$. Put $\Lambda_f = \sup |f'|$. If $\delta \Lambda_f^m \leq 1/2$ for some large $m$ then for all $j_0 \leq n \leq j_0 + m$ we have

$$
\sum_{k=0}^{n-j_0} |c_{j_0+k} - f^k(x_0)| \leq \delta \sum_{k=0}^{n-j_0} \Lambda_f^k \leq \frac{\delta \Lambda_f^{n-j_0}}{1-1/\Lambda_f}.
$$

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we can choose the sequence $t_n$ and thus $M(t_n)$ and this defines a sequence, denoted $(\phi'(x))$

$$
\sup |\phi' - \sum_{k=1}^{j_0-1} \phi(c_k)|.
$$

(\star)

Let $t_n \to 0$ be a sequence of non periodic parameters and let $M(t_n)$ be defined by (92). Now,

$$
\sum_{k=1}^{M(t_n)} \frac{X(c_j)}{(f_{j-1}^j)'(c_1)} = \mathcal{J}(f, X) \sum_{k=1}^{M(t_n)} \phi(c_k) - \sum_{j=k+1}^{M(t_n)} \sum_{j=k+1}^{M(t_n)} \frac{X(c_j)}{(f_{j-1}^j)'(c_1)}
$$

and

$$
\sum_{k=1}^{M(t_n)} \phi(c_k) \sum_{j=k+1}^{\infty} \frac{X(c_j)}{(f_{j-1}^j)'(c_1)} \leq \sup |\mathcal{J}(f, X)| \sum_{j=k+1}^{M(t_n)} \frac{(\inf |f'|)^{-1} \sup |\phi|}{1 - 1/\inf |f'|^2}.
$$

Thus, for arbitrarily large $n$, recalling also $|C_n| \leq \hat{C}$ from the previous cases,

$$
\sum_{k=1}^{M(t_n)} \frac{s_{1,j_n}}{(f_{j-1}^j)'(c_1)} \int \phi \frac{H_{c_1,j_n} - H_{c_1}}{t_n} dx \geq C_n + \sum_{k=1}^{M(t_n)} \phi(c_k) \sum_{j=k+1}^{M(t_n)} \frac{X(c_j)}{(f_{j-1}^j)'(c_1)}
$$

Assume now for a contradiction that for any sequence $t_n \to 0$ as above we have $|\int \phi d\mu_{t_n}| \leq A|t_n|$ for some $A < \infty$ and all large enough $n$. Then, for all large enough $n$, we would have

$$
\sum_{k=1}^{M(t_n)} \phi(c_k) \leq A + \hat{C}(f, \phi) \mathcal{J}(f, X) := D.
$$

(\star\star\star)

To end the proof we shall find sequences $t_n$ so that the above estimate gives a contradiction. (Note that $\phi$ cannot be a coboundary since $\phi(x_0) = 1$.)

For $m \geq 1$, let $\delta(m) > 0$ be so that $\delta \Lambda_m^\nu < 1/2$. Next, take $j_0 = j_0(\delta(m)) \geq 1$ so that $d(c_{j_0}, x_0) < \delta$. Then, letting $J(m) \geq 1$ be minimum for the property $j_0(\delta(m) + J(m)) - 1 > j_0(\delta(m)) + m$, we have

$$
j_0(\delta(m)) - 1 < j_0(\delta(m)) + m < j_0(\delta(m) + J(m)) - 1 < \ldots,
$$

and this defines a sequence, denoted $L(n)$, so that $L(n) \to \infty$ as $n \to \infty$. We claim that we can choose the sequence $t_n \to 0$ of nonperiodic parameters so that for all large enough $n$ we have $M(t_n) = L(n)$. Indeed, by the definition of $M(t)$, and since $\inf |f| > 1$, there is a sequence $0 < \tau_L < \tau_{L-1}$, $L \geq 1$, with $\tau_L \to 0$ as $L \to \infty$, so that for any $t \in [\tau_L, \tau_{L-1})$ we have $M(t) = L$. Thus, since the set of non periodic parameters is dense (see [1, corollary 4.1, item A]), there is a sequence of non periodic parameters $t_n \to 0$ so that $M(t_n) = L(n)$.
Then, recalling (⋆),
\[
\left| \sum_{k=1}^{j_0(\delta(m))} \varphi(c_k) \right| \geq m - \frac{1}{2(1 - 1/\Lambda_f)} \sup |\varphi'| - \left| \sum_{k=1}^{j_0(\delta(m)) - 1} \varphi(c_k) \right|
\]
\[
\geq m - \frac{1}{2(1 - 1/\Lambda_f)} \sup |\varphi'| - D.
\]
The rightmost lower bound clearly diverges as \( m \to \infty \), giving the desired contradiction with (⋆⋆).

\[ \square \]

2. Typographical errors

We use this opportunity to correct two minor typographical errors:

In the proof of theorem 7.1, if the orbit of \( c \) is infinite but not dense, ‘We now consider the first term in (95)’ on p 706, line 6, should read ‘We now consider the second term in (95).’

In the beginning of section 3.3, functions in \( \tilde{BV} \) are supported in \((−∞, b]\), not \([a, b]\).

Reference

[1] Baladi V and Smania D 2009 Smooth deformations of piecewise expanding unimodal maps Discrete Contin. Dyn. Syst. Ser. A 23 685–703