Fluctuation Study of the Specific Heat of Mg\textsuperscript{11}B\textsubscript{2}

Tuson Park and M. B. Salamon

Department of Physics and Material Research Laboratory, University of Illinois at Urbana-Champaign, IL 61801, USA

C. U. Jung, Min-Seok Park, Kyunghee Kim, and Sung-Ik Lee

National Creative Research Initiative Center for Superconductivity and Department of Physics, Pohang University of Science and Technology, Pohang 790-784, Republic of Korea

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The specific heat of polycrystalline Mg\textsuperscript{11}B\textsubscript{2} has been measured with high resolution ac calorimetry from 5 to 45 K at constant magnetic fields. The excess specific heat above T\textsubscript{c} is discussed in terms of Gaussian fluctuations and suggests that Mg\textsuperscript{11}B\textsubscript{2} is a bulk superconductor with Ginzburg-Landau coherence length $\xi_0 = 26$ Å. The transition-width broadening in field is treated in terms of lowest-Landau-level (LLL) fluctuations. That analysis requires that $\xi_0 = 20$ Å. The underestimate of the coherence length in field, along with deviations from 3D LLL predictions, suggest that there is an influence from the anisotropy of B\textsubscript{12} between the c-axis and the a-b plane.

Experimental observations of thermodynamic fluctuations in the specific heat have been limited in low-T\textsubscript{c} superconductors because the long coherence lengths make the excess specific heat very small compared to the mean-field term. By contrast, the high transition temperatures and small coherence lengths of cuprate superconductors lead to significant fluctuation effects. In the recently discovered superconductor Mg\textsuperscript{11}B\textsubscript{2}, the coherence length and superconducting transition temperature lie between these extremes, suggesting that fluctuation effects will be observable and lead to further information on the superconducting coherence length. Indeed, the excess magnetoconductance of Mg\textsuperscript{11}B\textsubscript{2} was reported recently and discussed in terms of fluctuation effects.

Here we report the specific heat of Mg\textsuperscript{11}B\textsubscript{2} from 5 K to 45 K at several magnetic fields. Using high resolution ac calorimetry, we could study the superconducting transition region in detail. At zero-field, the excess specific heat is treated in terms of 3D Gaussian fluctuations and in field, the broadening and shift of the transition is analyzed in terms of lowest-Landau-level (LLL) fluctuations.

Polycrystalline Mg\textsuperscript{11}B\textsubscript{2} was prepared at $T = 950$ °C and $\rho = 3$ GPa from a stoichiometric mixture of Mg and B\textsubscript{12} isotope using a high-pressure synthesis method. Since the sample was synthesized at high pressure, there has been no additional annealing. Details of the synthesis can be found elsewhere.

Measurements of the heat capacity were based on an ac-calorimetric technique. A long cylindrical sample was cut into a disk by a diamond saw and then was sanded to a thin rectangular shape whose dimensions are approximately $1.1 \times 1.5 \times 0.1$ mm$^3$; its mass is 375 µg. The front face of the prepared sample was coated with colloidal graphite suspension (DAG) thinned with isopropyl alcohol to prevent a possible change of the optical absorption properties of the sample with temperature. The sample was weakly coupled to the heat bath through helium gas and suspending thermocouple wires. As a heating source, we used square-wave modulated laser pulse. The oscillating heat input incurred a steady temperature offset (or dc offset) from the heat bath with an oscillating temperature superposed. The ac part was kept less than 1/10 of the dc offset and was then converted to heat capacity by the relationship $C \propto 1/T_{ac}$. The heat capacity obtained was converted to a specific heat by using a literature value above the superconducting temperature. The frequency of the periodic heating was chosen so that the ac temperature was inversely proportional to the frequency and, therefore, to the heat capacity; 23 Hz was used in this experiment. The ac and dc temperatures were measured by type E thermocouple, which were varnished on the back face of the sample using a minute amount of GE7031 diluted with a solvent of methanol and toluene. The GE varnish typically amounts to less than 1 % of the sample mass. Since the field induced er-

![Graph](image-url)

FIG. 1: Temperature dependence of $\Delta C$ at zero field. The dash-dotted line is a BCS fit with $\Delta C_{exp}/\gamma_nT_c = 0.7$. The star represents a 3D Gaussian fluctuation model above $T_c$ of 38.4 K. Inset: The temperature dependence of the specific heat at 0 and 7 T from 5 to 45 K.
or of type E thermocouple is less than 1 % at 40 K in 8 tesla, we will neglect the field dependence of the addenda contribution (DAG, GE-varnish, and type E thermocouple) and treat the field dependence as due only to the sample.

The inset in Fig.1 shows the temperature dependence of the specific heat at zero and 7 Tesla from 5 to 45 K. The main graph is a plot of \( \Delta C_{\text{exp}} \) vs temperature at zero field. Here \( \Delta C_{\text{exp}} \) is the measured difference between the mixed- and normal-state specific heats. A 7-Tesla data set was used as a reference state above 20 K because it shows no observable transition in that range. The subtraction was executed without any smoothing of the 7-T data. The dash-dotted line is a BCS fit with the ratio of \( \Delta C_{\text{exp}}/\gamma_nT_c \) being variable. The normal electronic coefficient \( \gamma_n \) was set to be 2.6 mJ/mol K from the literature and \( T_c \) of 38.4 K was determined from scaling discussed below. The best fit showed that the ratio is 0.7, which is much smaller than the weak coupling BCS value of 1.43. Since the ratio is generally larger for strong coupling superconductors, the small value does not tell us anything about its coupling strength. Recently, there has been a plethora of experimental and theoretical evidence which supports two-gap features in MgB2, which can explain the non-BCS jump magnitude with some success. However, we cannot rule out such other scenarios as an anisotropic gap structure. For a system in which fluctuation effects are pronounced, the experimentally determined transition temperature is lower than the mean-field critical temperature \( (T_{c}^{mf}) \) because fluctuations drive the system into the normal state even below \( T_{c}^{mf} \). It is unlikely, however, that this can explain the large deviation from the BCS value.

Above the transition temperature, there is an excess specific heat tail apparent in Fig. 1. Thouless and subsequently Aslamazov and Larkin showed that Gaussian fluctuations arise above \( T_c \) and predicted that \( C_{\text{fl}} = C^+ t^{(2-d/2)} \) with \( C^+ = (k_B/8\pi)\xi_{GL}(0)^{-3} \), where \( t = T/T_c \), \( d \) is the dimensionality, and \( \xi_{GL}(0) \), the T=0 K Ginzburg-Landau coherence length. Figure 2 shows the temperature dependence of the excess specific heat on a log-log scale. The data follow a power law with an exponent of -0.5 and \( C^+ = 0.66 \) mJ/mol K. The exponent indicates that Mg\(^{11}\)B\(_2\) is a 3D superconductor and the substitution of C\(^+\) into the above formula gives \( \xi_{GL}(0) = 26 \pm 1 \) A.

When a magnetic field is applied, the specific heat broadens. Figure 3 shows the temperature dependence of \( \Delta C/T \) at several magnetic fields. The ratio of the transition temperature shift to the transition width broadening in field is unique in that it is not as large as in low-\( T_c \) superconductors nor as small as in high-\( T_c \) materials. Its intermediate behavior is related to the fact that the coherence length and the superconducting transition temperature of Mg\(^{11}\)B\(_2\) are intermediate between low-\( T_c \) and high-\( T_c \) superconductors. Lee and Shenoy studied fluctuation phenomena in the presence of a magnetic field, arguing that bulk superconductors exhibit a field-induced effective change to one-dimensional behavior in the vicinity of the transition temperature \( T_c(B) \). In a uniform magnetic field, the fluctuating Cooper pairs move in quantized Landau orbits and, close to upper critical field \( (B_{c2}) \), the lowest Landau level dominates the contribution to the excess specific heat. So, a bulk superconductor behaves like an array of one-dimensional rods parallel to the field. Thouless extended the idea above and below \( T_c \) and suggested a scaling parameter for the fluctuation specific heat that is valid throughout the transition region:

\[
\frac{\Delta C_{\text{exp}}}{\Delta C_{mf}} = g\left(\frac{t}{\tau}\right),
\]

where \( \Delta C_{\text{exp}} \) is the measured difference between the mixed- and normal-state specific heats, \( \Delta C_{mf} \) is a 3-D Gaussian fluctuation term.
where $t$ is the reduced temperature and $\tau$ is a field dependent dimensionless broadening parameter. The functional form $g(y)$ is model dependent. When a Hartree-like approximation is used to examine the fluctuation effects of the quartic term in the free energy functional, it results in a simple form:

$$g(y) = (1 + x(y))^{-1}, \quad (2)$$

$$y = x^{2/3}(1 - 2/x), \quad (3)$$

where $y = t/\tau(B)$.

In Fig. 4, the ratio of $\Delta C_{\text{exp}}/\Delta C_{mft}$ in the transition region was plotted as $t/\tau(B)$, where $\Delta C_{\text{exp}}(B) = C(B) - C(0)$ and $\Delta C_{mft}$ was determined as in the classic work by Farrant and Gough by fitting the low temperature side of $\Delta C_{\text{exp}}(B)$ as in Figs 1 and 3 and extrapolating linearly above $T_c(B)$. A sketch is shown in the inset of Fig. 4. The scaling parameters $\tau(B)$ and $T_c$ were chosen to make the data collapse onto the Hartree-like approximation (solid-squares). The values of $\tau(B)$ and $T_c$ are listed in Fig. 4. The temperature dependence of the upper critical field $T_c(B)$ is plotted in Fig. 5, and shows positive curvature close to $T_c(B = 0)$. A simple empirical formula, $B_{c2}(T) = B_{c2}(0)[1 - (T/T_c)^2][1 - a(T/T_c)^2]$, was used to describe the curvature, in which $a$ is a fitting parameter that is 0 and 0.3 for two-fluid model and for WHH model respectively. The best fit, solid line in Fig. 5, was produced with $B_{c2}(0) = 15.4$ tesla and $a = 0.8$. Positive curvature near $T_c(B = 0)$ was also observed in non-magnetic rare-earth nickel borocarbides $R$Ni$_2$Ba$_2$Cu$_3$O$_{7-\delta}$ ($Lu, Y$) and could be explained by the dispersion of the Fermi velocity using an effective twoband model.

$B_{c2}(T)$ is shown. The solid line is a simple empirical formula, $B_{c2}(0)[1 - (T/T_c)^2][1 - a(T/T_c)^2]$, with $a = 0.8$ and $B_{c2}(0) = 15.4$ T. The dotted line is a linear fit with $B_{c2}(0) = 16$ T and $dB_{c2}/dT = -0.44 T/K$.

$\Delta C_{\text{exp}}/\Delta C_{mft}$ vs $t/\tau$ is shown, where $\Delta C_{\text{exp}} = C(B) - C(0)$ and $\Delta C_{mft}$ are defined in the inset. Here $t$ is reduced temperature and $\tau$ is a field dependent dimensionless broadening parameter. Solid squares are from a Hartree-like approximation. Other field data were scaled so that they were collapsed onto the Hartree result. The obtained parameters ($T_c$, $\tau$) are tabulated inside the figure at each magnetic field. The inset sketches the definition of $\Delta C_{mft}(T, B)$.

$R$Ni$_2$Ba$_2$Cu$_3$O$_{7-\delta}$($Lu, Y$) and could be explained by the dispersion of the Fermi velocity using an effective two-band model.

The broadening parameter $\tau(B)$ consists of a field dependent part ($\tau_B$) and a field independent part ($\tau_{in}$). We postulate that they are independent of each other and add in quadrature, such that the total broadening is $\tau^2 = \tau_B^2 + \tau_{in}^2$. The field independent part was obtained in zero field and accounts for sample inhomogeneity and zero-field fluctuation effects while the field dependent part is due solely to field-induced fluctuations. The field dependence of the broadening parameter is given by:

$$\tau_B = \left( \frac{B}{B_W} \right)^{1/2}, \quad (4)$$

$$B_W = \left( \frac{\Delta C}{k_B/8\pi^2} \right) B_S, \quad (5)$$

where $\alpha = 2 - (d - 2)/2$ ($d = 3/2$ for a bulk superconductor) and $B_S = -T_c(dB_{c2}/dT)_{T_c}$ in the mean-field scheme. The exponent $d - 2$ indicates a dimensional crossover from $d$-dimension to $d$–2 behavior. The shift field $B_S$ is a characteristic field that sets the scale of the shift of the transition temperature while $B_W$ sets the scale of the width broadening of the transition region. In a standard superconductor, the ratio $B_W/B_S$ is very large ($\sim 10^4$), and the transition is shifted much more rapidly in field than it is broadened. In high temperature superconductors, such as YBCO, the broadening is as large as the shift ($B_S \sim B_W$), which is an indication that a mean-field approach based on a perturbation expansion might not be proper and that fluctuations should be treated in the context of critical phenomena.
In Mg$^{11}$B$_2$, the ratio is in the order of 10$^2$, which is in between those two extremes. This feature seems consistent with other properties that show aspects of both conventional and high T$_c$ superconductors. In order to study the anomalous broadening in field, we plot $\tau_B$ vs $(B/\Delta C)$ on a log-log scale in Fig. 6. The solid line represents a lowest-Landau-level (LLL) fluctuation: $\tau_B = \beta(B/\Delta C)^{1/3}$ with $\alpha = 3/2$ and $\beta = 2.7 (A/m \cdot K)^{2/3}$.

Before making any further conclusions, we discuss some of the assumptions that we made in the analysis. In the zero-field analysis, sample inhomogeneity has been neglected. Boron 11 isotope Mg$^{11}$B$_2$ has a T$_c$ of 39 K while the excess specific heat extends well above 40 K. We might expect inhomogeneity effects to complicate the fluctuation analysis below the transition temperature but above T$_c$, where our analysis is concerned, the effect will be negligible. However, when nonzero field is applied, sample inhomogeneity must be considered because the analysis is of the field dependent behavior of the transition region. Sample inhomogeneity could produce an additional broadening through the Ginzburg-Landau parameter $\kappa$, and hence H$_{c2}$. Information on H$_{c2}$ slopes at different parts of the sample with different T$_c$’s would be needed to account for the additional broadening correctly. To be more precise, the H$_{c2}$ slope in the T$_c = 39$ K part of the sample and that in the, e.g., T$_c = 38$ K part would be needed. In our in-field analysis, we assumed that the slopes of H$_{c2}$ at different parts of the sample are same or if they are different, the difference is small, which leads to field independent inhomogeneity effect. It is necessary to study high quality single crystals with different T$_c$’s to better understand sample inhomogeneity effects on transition-width broadening.

In summary, the zero-field specific heat was discussed in the context of BCS theory plus 3D Gaussian fluctuations. The analysis indicates that Mg$^{11}$B$_2$ is a bulk superconductor and its coherence length is about 26 Å. In-field specific heat was treated in terms of lowest-Landau-level fluctuations. That analysis requires that $\xi_0 = 20$ Å. The in-field analysis could be complicated due to the effect of anisotropy in a polycrystalline sample. The anisotropy of B$_{c2}$ between ab-plane and c-axis directions can lead to a field dependent broadening due to the T$_c$ distribution arising from the randomly oriented grains of the present sample. This in turn leads to an underestimation of the G-L coherence length. In order to understand the influence of anisotropy, we can assume that the transition broadening arises solely from B$_{c2}$ anisotropy and calculate the required anisotropy in our experimental temperature range. The reported anisotropy value of $\sim 3$ from single crystal measurements is much smaller than the ratio 6 required to explain the broadening. From this consideration, we conclude that the anisotropy alone cannot explain the whole broadening and therefore that fluctuation effects should be considered in explaining the anomalous broadening.

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