Correction to: An Extension of Raşa’s Conjecture to $q$-Monotone Functions

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There is an inadvertent mistake in the statement of Theorem 3.2 and its proof. The authors are indebted to Ioan Raşa for pointing out the mistake in the statement of Theorem 3.2, which has led us to revise the proof.

The theorem should be

**Theorem 3.2.** Let $n, q \in \mathbb{N}$. If $f$ defined on $[0, \infty)$ and such that $|f(x)| \leq C(1 + x)^\gamma$, for some $C, \gamma > 0$, is a $q$-monotone function there, then for all $x, y \in [0, \infty)$,

\[
\text{sgn}(x - y)^q \sum_{\nu_1, \ldots, \nu_q = 0}^{\infty} \sum_{j=0}^{q} (-1)^{q-j} \binom{q}{j} \left( \prod_{i=1}^{j} m_{n, \nu_i}(x) \right) \left( \prod_{i=j+1}^{q} m_{n, \nu_i}(y) \right) \int_{0}^{1} f \left( \frac{\nu_1 + \cdots + \nu_q + \alpha t}{n + \alpha} \right) dt \geq 0.
\]

**Proof of Theorem 3.2.** Let $n, m, q \in \mathbb{N}$, and denote $\nu := (\nu_1, \ldots, \nu_q) \in (\mathbb{N}_0)^q$. By (3.2) it follows that

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\[
\sum_{|\nu|=m} \prod_{i=1}^q m_{n,\nu_i}(x_i) = \frac{1}{m!} \sum_{|\nu|=m} \binom{m}{\nu} \prod_{i=1}^q \left( \frac{\partial}{\partial z} \right)^{\nu_i} (1-x_i z)^{-n} \bigg|_{z=-1} = \frac{1}{m!} \left( \frac{\partial}{\partial z} \right)^m \prod_{i=1}^q (1-x_i z)^{-n} \bigg|_{z=-1}. \tag{3.4}
\]

Hence, as is done in (2.2), for \(0 \leq x, y < \infty\) and any sequence \((a_k)_{k=0}^{\infty}\), we have

\[
\sum_{|\nu| \geq 0} \sum_{j=0}^q (-1)^{q-j} \binom{q}{j} \left( \prod_{i=1}^j m_{n,\nu_i}(x) \right) \left( \prod_{i=j+1}^q m_{n,\nu_i}(y) \right) a_{|\nu|} = \sum_{m=0}^\infty a_m \frac{1}{m!} \left[ \left( \frac{\partial}{\partial z} \right)^m [(1-xz)^{-n} - (1-zy)^{-n}]^q \right] \bigg|_{z=-1} =: I. \tag{3.5}
\]

Let

\[
h(z) := ((1-xz)^{-n} - (1-zy)^{-n})^q.
\]

Then

\[
h(z) = ((1-xz)^{-1} - (1-zy)^{-1})^q \left( \frac{(1-xz)^{-n} - (1-zy)^{-n}}{(1-xz)^{-1} - (1-zy)^{-1}} \right)^q = \left( \frac{1-zy}{1-xz} \right)^q \left( \sum_{m=0}^{n-1} (1-xz)^{-m}(1-zy)^{-(n-1-m)} \right)^q = (x-y)^q z^q g(z).
\]

Note that \(\left( \frac{\partial}{\partial z} \right)^j (1-uz)^{-m} \bigg|_{z=-1} = j! m_{m,j}(u) \geq 0\), for \(j, m = 0, 1, 2, \ldots\), and \(0 \leq u < \infty\). Hence

\[
g^{(k)}(-1) \geq 0, \quad k \geq 0. \tag{3.6}
\]

Finally, we may rewrite the last line in (3.5), as we did in (3.3),

\[
I = \sum_{m=0}^\infty a_m \frac{1}{m!} \left[ \left( \frac{\partial}{\partial z} \right)^m [(1-xz)^{-n} - (1-zy)^{-n}]^q \right] \bigg|_{z=-1} = (x-y)^q \sum_{m=0}^\infty a_m \frac{1}{m!} \frac{d^m}{dz^m} (z^q g(z)) \bigg|_{z=-1} = (x-y)^q \sum_{m=0}^\infty \frac{1}{m!} g^{(m)}(-1) \Delta^q a_m,
\]

and Theorem 3.2 follows by taking \(a_m := \int_0^1 f \left( \frac{m+\alpha t}{n+\alpha} \right) dt\). \(\square\)
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Reference
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