Axion and neutrino physics from anomaly cancellation

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It has been recently shown that the requirement of anomaly cancellation in a (non-supersymmetric) six-dimensional version of the standard model fixes the field content to the known three generations. We discuss the phenomenological consequences of the cancellation of the local anomalies: the strong CP problem is solved and the fundamental scale of the theory is bounded by the physics of the axion. Neutrinos acquire a mass in the range suggested by atmospheric experiments.

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I. INTRODUCTION

Anomalies and their cancellation are crucial to our understanding of quantum gauge field theory. They can be used as a well-motivated selection rule in fixing the field content of a model. The authors of a recent paper [1] consider the standard model in six space-time dimensions and show that it is possible to predict the number of matter families (generations) by requiring the cancellation of a global anomaly [2]. The cancellation of the gauge anomalies in six dimensions is achieved by means of the Green-Schwarz mechanism [3]. In this paper we discuss some phenomenological consequences of the model introduced in [1].

Axions are necessarily present in the four-dimensional theory after the cancellation of the gauge anomalies in six dimensions. They provide a solution to the strong CP problem. The experimental constraints on axion couplings yield bounds on the free parameters of the model, namely, the compactification radius \( R^{-1} > 10^6 \) TeV. At the same time, neutrinos acquire a mass of the right order of magnitude by means of a see-saw mechanism with the right-handed neutrinos required by the cancellation of the gravitational anomaly in the six-dimensional theory.

We only consider the minimal version of the model with two extra dimensions in which standard model fields are allowed to propagate: the field content of the theory is then completely determined by the requirement of anomaly cancellation, and no other field, symmetry or additional extra-dimension is introduced.

II. FIELD CONTENT: ANOMALY CANCELLATION

Let us write a (non-supersymmetric) six-dimensional theory, based on the standard model gauge group \( SU(3)_c \times SU(2)_L \times U(1)_Y \): matter fields are assigned to chiral fermions (projected by \((1 \pm \Gamma_7)/2\)) in the usual representations of the gauge group \( Q, L, U, D \) and \( E \), as shown in Tab. I. A scalar doublet \( h \) must be present for the usual Higgs mechanism to take place. As explained in [1], cancellation of purely gravitational and irreducible gauge anomalies forces quark singlets to have opposite chirality with respect to doublets, and to introduce a standard-model singlet \( N \) in order to have the same number of fermions of both chiralities. Our (conventional) choice is to assign positive chirality to doublets and negative to singlets. An alternative choice would be to assign opposite chiralities to leptons and hadrons: in what follows this would turn in a change of \( O(1) \) in the couplings introduced to cancel anomalies, with minor modifications in the phenomenological consequences of the model.

| Chirality | \( U(1)_Y \) | \( SU(2)_L \) | \( SU(3)_c \) |
|----------|----------------|----------------|----------------|
| \( Q \)  | +              | \( 1/6 \)       | 2              | 3              |
| \( U \)  | −              | \( 2/3 \)       | 1              | 3              |
| \( D \)  | −              | \( -1/3 \)      | 1              | 3              |
| \( L \)  | +              | \( -1/2 \)      | 2              | 1              |
| \( E \)  | −              | \( -1 \)        | 1              | 1              |
| \( N \)  | −              | 0               | 1              | 1              |

TABLE I: Fermionic field content for each family. The six-dimensional chirality is the eigenvalue of \( \Gamma_7 \).
Cancellation of global anomalies is obtained with \( n_g = 0 \mod 3 \) copies of this matter content. In particular \( n_g = 3 \) is in agreement with the experiment \[^{[3]}\]. We are thus left with Abelian and non-Abelian reducible gauge anomalies, that would spoil unitarity unless cancelled by the Green-Schwarz mechanism. Accordingly, to recover non-Abelian gauge symmetries we must introduce two real antisymmetric tenors that would spoil unitarity unless cancelled by the Green-Schwarz mechanism. Accordingly, to recover non-Abelian reducible gauge anomalies, we must introduce two real antisymmetric tensors that would spoil unitarity unless cancelled by the Green-Schwarz mechanism.

The six-dimensional anomaly free Lagrangian can be written as:

\[
\mathcal{L}^{6D} = \mathcal{L}^{6D}_{\text{SM}} + \mathcal{L}^{6D}_{\text{GS}},
\]

where

\[
\mathcal{L}^{6D}_{\text{SM}} = \sum \bar{\psi} i \gamma^M D_M \psi - \frac{1}{4} F_{MN} F^{MN} - \frac{1}{2} \text{Tr} L_{MN} L^{MN} - \frac{1}{2} \text{Tr} V_{MN} V^{MN} + (D_M \bar{h})^i D^M h + V(\phi^i \phi) + \left( Y_{L} R \bar{h} E + Y_{R} \bar{Q} h D + Y_{c} R \bar{Q} h U + Y_{c} R \bar{L} h N + \text{h.c.} \right) + \frac{1}{2} M_N \left( N^T C^{-1} N - \bar{N} C \bar{N} \right)
\]

and

\[
\mathcal{L}^{6D}_{\text{GS}} = \frac{g^3 R^3}{16 \pi^3} \sigma F_{MN} F_{RS} F_{PQ} \epsilon^{MNRSPQ} + \frac{g^3 R^4 \Delta_L^2}{6 \pi^3} B_{MN}^L \text{Tr} \{L_{RS} L_{PQ}\} \epsilon^{MNRSPQ} + \frac{g^2 g' R^2 \Delta_L^2}{144 \pi^3} B_{MN}^H \text{Tr} \{L_{RS} L_{PQ}\} \epsilon^{MNRSPQ} - \frac{g^2 g' R^2 \Delta_L^2}{72 \pi^3} \sigma F_{MN} \text{Tr} \{L_{RS} L_{PQ}\} \epsilon^{MNRSPQ} + \frac{g^2 g' R^2}{48 \pi^3} \sigma F_{MN} \text{Tr} \{V_{RS} V_{PQ}\} \epsilon^{MNRSPQ} + \frac{g^2 g' R^2 \Delta_L^2}{96 \pi^3} B_{MN}^H \text{Tr} \{V_{RS} V_{PQ}\} \epsilon^{MNRSPQ} + \frac{g^2 g' R^2 \Delta_L^2}{48 \pi^3} B_{MN}^L \text{Tr} \{V_{RS} V_{PQ}\} \epsilon^{MNRSPQ} + \frac{g^2 g' R^2 \Delta_L^2}{48 \pi^3} B_{MN}^H \text{Tr} \{V_{RS} V_{PQ}\} \epsilon^{MNRSPQ} + \frac{1}{12} H_{MNS}^L H_{MNS}^L \epsilon^{MNRSPQ}.
\]

In Eq. (3), \( \psi \) is a generic fermionic chiral fields, \( D_M \) the covariant derivative on the associated gauge representation, \( V_{MN}, L_{MN} \) and \( F_{MN} \) are the field strength tensors of the gauge bosons \( G_M, W_M \) and \( A_M \) of \( SU(3)_c, SU(2)_L \) and \( U(1)_Y \) respectively. \( \mathcal{L}^{6D}_{\text{SM}} \) is the usual standard model Lagrangian, with Lorentz indexes in six dimensions, to which we are allowed to add a Yukawa interaction also for neutrinos (\( \bar{h} = \bar{\sigma}_2 h^* \)) and a Majorana mass term for the singlet \( N \), which behaves like a right-handed neutrino. The scalar potential \( V \) is a power series in \( \phi^i \phi \). The coefficients of the Green-Schwarz terms in Eq. (4) match the one-loop anomalous terms, computed for six space-time dimensions in \[^{[4]}\].
The Lagrangian in Eq. (4) contains the (gauge non-invariant) terms required for the cancellation of all reducible gauge anomalies, and the kinetic terms for the 2-forms $B_{MN}^{L,c}$, with:

$$H^{L,c} \equiv dB^{L,c} - \frac{1}{\Delta^2} \omega_3^{L,c},$$

(5)

and

$$\begin{align*}
\omega_3^L &= \text{Tr} \left\{ G \wedge V - \frac{1}{2} g_s R G \wedge G \wedge G \right\}, \\
\omega_3^c &= \text{Tr} \left\{ W \wedge L - \frac{1}{2} g R W \wedge W \wedge W \right\}.
\end{align*}$$

(6)

The Chern-Simons forms $\omega_3^{L,c}$ are needed to make $H_{MN}^{L,c}$ invariant, and satisfy the relations $\delta_{L,c} \omega_3^{L,c} = -d\omega_2^{L,c}$.

The presence of the scalar field $h$ can be used to cancel the $U(1)$ anomalies. The Higgs field has been decomposed in a doublet $\phi$ with vanishing hypercharge and a $SU(2)_L$ singlet $\sigma$ writing:

$$h = \phi e^{i\sigma}.$$  

(7)

Under a $U(1)_Y$ gauge transformation $\delta \sigma = \frac{1}{2} g' R \alpha$, being $\alpha$ the parameter of the transformation. In unitary gauge $\sigma = 0$, the gauge bosons acquire mass in the standard way and terms proportional to $\sigma$ in Eq. (9) vanish.

We have not written explicitly the terms that are needed to cancel mixed (gauge-gravitational) anomalies, because they are not relevant for the phenomenological discussion we are interested in. To achieve the cancellation it is enough to add for each gauge group couplings of the form

$$\omega_3 \Omega_3 + B \text{Tr} \mathcal{R} \wedge \mathcal{R},$$

(8)

with appropriate coefficients, where $B$ are the antisymmetric tensors $B^L, B^c$ and $\sigma, F$, $\Omega_3$ is the gravitational Chern-Simons form defined by $d\Omega_3 = \text{Tr} \mathcal{R} \wedge \mathcal{R}$. $\mathcal{R}$ is the Ricci tensor. For details see [8].

We denote by $R^2$ the volume of the compact extra-dimensions, so that the Newton constant is related to the fundamental scale $M_f$ of the theory by

$$M_{Pl} = R M_f^2.$$ 

(9)

By writing the dimensionfull couplings as $g R$ yields, after dimensional reduction, the (four dimensional) gauge couplings $g_3, g$ and $g'$. The only remaining parameters in the Lagrangian are the mass parameters $M_N, \Delta^L, \Delta^c$ and the couplings of the scalar potential.

### III. DIMENSIONAL REDUCTION

The two extra dimensions are assumed to be compact, and the underlying geometry flat. Chiral fermions in six dimensions correspond to Dirac fermions in four dimensions, but chirality is recovered by orbifold projection. We assume space-time to be

$$M_4 \times \frac{S^1 \times S^1}{Z_2},$$

(10)

the product of four-dimensional Minkowski and a torus with orbifold $Z_2$ which impose a symmetry under the parity transformation

$$Z_2 : (y, z) \rightarrow (-y, -z),$$

(11)

where $(y, z)$ are the coordinates on the torus $S^1 \times S^1$.

In what follows, we assume that we are allowed to work in the limit of dimensional reduction, in which the low-energy Lagrangian contains only the zero modes of the fields—while higher modes decouple because of their large masses, proportional to $2\pi/R$. The effects of this simplifying assumption should be checked at the end for consistency to make sure that the large number of these heavier states do not enhance potentially dangerous operators.

A consistent assignment of $Z_2$-parities makes it possible to have a single massless chiral field out of each $\psi$; the projection gives a factor $1/2$ in the Green-Schwarz gauge non-invariant terms in Eq. (4). The reduced Lagrangian also contains the zero modes of $h$ and of the gauge bosons, together with two anti-symmetric tensors $B^{L,c}_{\mu},$ and two pseudo-scalars $b^{L,c},$ coming, respectively, from the $\{0123\}$ and $\{56\}$ sectors, of the decomposed tensors

$$B_{MN}^{L,c} \rightarrow b^{L,c} \equiv \sqrt{3}/6 \epsilon^{\hat{M} \hat{N}} B_{\hat{M} \hat{N}}^{L,c} \quad \hat{M} \hat{N} = 5, 6.$$ 

(12)
There is no zero mode for the \( \{56\} \) part of the field strength tensors. In four dimensions an antisymmetric tensor is equivalent to a pseudo-scalar, and we redefine:

\[
\partial_\mu c^{L,c} = \frac{i}{6} \epsilon_{\mu}^{\nu \rho \sigma} H_{\nu \rho \sigma}^{L,c}.
\]

We use Greek indexes for 4-dimensional quantities.

The spectrum, after integrating out the compact dimensions, is the same as in standard model, with four additional pseudo-scalar fields \( c^L, c^c, b^L \) and \( b^c \), and the Lagrangian becomes:

\[
\mathcal{L}_{4D} = \mathcal{L}^{SM} + (Y_{\nu \bar{L}} \bar{H} N + \text{h.c.})
\]

\[
+ \frac{1}{2} M_N (N^T C^{-1} N - \bar{N} C N^T)
\]

\[
+ \frac{g^2 R^3 \Delta^2}{6 \sqrt{3\pi}} b^L \text{Tr} \bar{L} \bar{L} + \frac{g^2 g_2 R^3 \Delta^2}{24 \sqrt{3\pi}} b^c \text{Tr} L \bar{L}
\]

\[
+ \frac{g^2 g_2^2 R^3 \Delta^2}{144 \sqrt{3\pi}} b^L \text{Tr} \bar{V} \bar{V} - \frac{g^2 g_2^2 R^3 \Delta^2}{96 \sqrt{3\pi}} b^c \text{Tr} F \bar{F}
\]

\[
+ \frac{g^2 g_2^2 R^3 \Delta^2}{48 \sqrt{3\pi}} b^L \text{Tr} V \bar{V}
\]

\[
+ \frac{1}{2} \partial_\mu b^L \partial^\mu b^L + \frac{1}{2} \partial_\mu b^c \partial^\mu b^c
\]

\[
+ \frac{1}{2} \partial_\mu c^L \partial^\mu c^L + \frac{1}{2} \partial_\mu c^c \partial^\mu c^c
\]

\[
- \frac{c^L}{3 \Delta^2 R} \text{Tr} \bar{L} \bar{L} - \frac{c^c}{3 \Delta^2 R} \text{Tr} V \bar{V}.
\]

The Lagrangian \( \mathcal{L}^{SM} \) in Eq. (14) is the standard model Lagrangian (in the unitary gauge). The last terms have been obtained using the (six dimensional) Bianchi identity

\[
d H^{L,c} = - \frac{1}{\Delta^2 L,c} d \omega^{L,c},
\]

that in \( D = 4 \) gives the equation of motion for the fields \( c^L \) and \( c^c \):

\[
\partial_\mu \partial^{\mu} c^L = - \frac{1}{3 \Delta^2 R} \text{Tr} \bar{L} \bar{L},
\]

\[
\partial_\mu \partial^{\mu} c^c = - \frac{1}{3 \Delta^2 R} \text{Tr} V \bar{V}.
\]

Only three linear combinations \( \varphi_1, \varphi_2 \) and \( \varphi_3 \) of the four pseudo-scalars are coupled to gauge fields by means of axion-like terms, while the orthogonal combination gives rise to a massless free field with no phenomenological consequence.

### IV. AXIONS

After removing the decoupled scalar from the Lagrangian in Eq. (14), we obtain

\[
\mathcal{L}_{4D} = \mathcal{L}^{SM} + (Y_{\nu \bar{L}} \bar{H} N + \text{h.c.})
\]

\[
+ \frac{1}{2} M_N (N^T C^{-1} N - \bar{N} C N^T)
\]

\[
+ \frac{1}{2} \partial_\mu \varphi_1 \partial^{\mu} \varphi_1 + \frac{1}{2} \partial_\mu \varphi_2 \partial^{\mu} \varphi_2 + \frac{1}{2} \partial_\mu \varphi_3 \partial^{\mu} \varphi_3
\]

\[
+ \varphi_1 \left[ \frac{1}{F_1} F \bar{F} + \frac{1}{F_2} \text{Tr} \bar{L} \bar{L} + \frac{1}{F_4} \text{Tr} V \bar{V} \right]
\]

\[
+ \varphi_2 \left[ \frac{1}{F_2} F \bar{F} + \frac{1}{F_2} \text{Tr} V \bar{V} \right] + \varphi_3 \frac{1}{F_3} \text{Tr} V \bar{V},
\]
where the constant $F_{i}^{\mu.\nu.L}$ are functions of the coefficients in front of the scalar-gauge fields coupling terms in Eq. (14). The fields $\varphi_{i}$ have the same couplings of the Peccei-Quinn axion: they are invariant under translations but for the coupling to the gauge fields.

The axion solves the strong CP problem [9, 10, 11]: a term in the form

$$\bar{a} \frac{\alpha_s}{8\pi} a \text{Tr} V \tilde{V},$$

(18)

which is allowed by the symmetries of non-Abelian gauge theories, would induce an electric dipole moment for neutrons in conflict with experimental data unless $\bar{a} = 0$, because of instantonic effects, but the addition of a pseudo-scalar field $\varphi$ with coupling

$$\bar{\varphi} \frac{\alpha_s}{8\pi} \frac{1}{F_{\varphi}} a \text{Tr} V \tilde{V}$$

(19)

and with no tree-level potential gives dynamically

$$\frac{\langle a \rangle}{F_{\varphi}} + \bar{a} = 0.$$

(20)

In this way, no electric dipole moment is generated, and CP is restored as a good symmetry of the QCD Lagrangian.

There are experimental constraints on the axion couplings coming from the combination of cosmological, astrophysical and accelerator searches [12]. In order to perform the comparison with the experimental constraints, it is necessary to write down the low-energy effective theory in terms of photons, pions, nucleons and axions only.

In the low-energy theory, we can safely neglect interactions with $Z$ and $W$ bosons, and extract only the electromagnetic couplings

$$\begin{align*}
\{ \text{Tr} LL &= \frac{1}{2} \sin^2 \theta_W F_{\varphi e.m.} F_{\varphi e.m} + \cdots, \\
F_{\varphi} &= \cos^2 \theta_W F_{\varphi e.m.} F_{\varphi e.m} + \cdots,
\end{align*}$$

(21)

after the rotation of neutral bosons by the weak angle $\theta_W$. Accordingly, only two combinations out of the three $\varphi_{i}$ fields couple to the massless gauge fields: one to photons and gluons, the other to photons only. The former has the correct couplings and transformation properties to be identified with the Peccei-Quinn axion. Its presence is a consequence of the anomaly cancellation, and therefore of the choice of writing a six-dimensional gauge theory.

Now we turn the coupling to gluons into a coupling to quarks. This can be achieved by a chiral transformation. Then, using the methods of current algebra, we rewrite the theory in terms of pions, and eliminate quarks and gluons.

Adding the coupling of pions to photons, responsible for the decay $\pi^0 \rightarrow 2\gamma$, yields the interaction terms needed to compute all the contributions to the mass matrix of pions and axions. After all of these manipulations we can write

$$\mathcal{L}^\pi = \frac{1}{2} \partial_{\mu} \pi^0 \partial^{\mu} \pi^0 + \frac{1}{2} \partial_{\mu} a \partial^{\mu} a + \frac{1}{2} \partial_{\mu} a' \partial^{\mu} a'$$

$$- \frac{1}{2} \left( \begin{array}{cc} \pi^0 & a \\ a & a' \end{array} \right) \mathcal{M}^2 \left( \begin{array}{c} \pi^0 \\ a \\ a' \end{array} \right)$$

$$+ \left( \frac{\pi^0}{f_{\pi}} + \frac{a}{f_a} + \frac{a'}{f_{a'}} \right) \frac{\alpha_{s}}{8\pi} \left( f_{\pi e.m.} f_{\varphi e.m.} + f_{\varphi e.m.} f_{\varphi e.m.} \right) \mathcal{F}_{\varphi e.m.}$$

(22)

where

$$\mathcal{M}^2 = \Delta m_z^2 \left( \begin{array}{ccc} m_{\pi^0}/\Delta m_{\pi}^2 & f_{\pi}/f_a & k + f_{\pi}/f_{a'} \\
 f_{\pi}/f_a & (f_{\pi}/f_a)^2 & f_{\varphi}^2/(f_a f_{a'}) \\
k + f_{\pi}/f_{a'} & f_{\varphi}^2/(f_a f_{a'}) & m_{\pi^0}/\Delta m_{\pi}^2 (f_{\pi}/2m)^2 + (f_{\pi}/f_{a'})^2 \end{array} \right).$$

(23)

The parameter $m \equiv F_{3}(\alpha_s/4\pi)$ and

$$k = \frac{m_{\pi^0}}{\Delta m_{\pi}^2} m_d - m_u \frac{f_{\pi}}{2m}.$$

(24)
The masses $m_u$ and $m_d$ are those of up and down quarks, $f_\pi \simeq 93$ MeV is the pion decay constant, $m_{\pi^0}$ and $m_{\pi^+}$ are the masses of the pions, while $\Delta m_{\pi}^2 \equiv m_{\pi^0}^2 - m_{\pi^+}^2$. The decay constants are normalized in such a way that the partial decay rate of neutral pions into photons is

$$\Gamma(\pi^0 \to 2\gamma) = \frac{\alpha^2 m_\pi^3}{64 \pi^3 f_\pi^2} \simeq 7.6 \text{ eV}. \tag{25}$$

The partial diagonalization of this matrix makes it possible to identify the physical pion field and the couplings of the two remaining light pseudoscalars $a$ and $a'$. The coupling of the axions to the photon comes both from Eq. (21) and pion-axion mixing. A stringent experimental bound to consider comes from helium burning lifetimes of red giants, and imposes an upper limit to the coupling axion-photon

$$g_{a\gamma} < 10^{-10} \text{ GeV}^{-1} \tag{26}$$

with

$$\mathcal{L} = -\frac{1}{4} g_{a\gamma} a F^{\mu\nu} \tilde{F}_{\mu\nu}. \tag{27}$$

The limit of vanishing masses for axions can be used. In our case, we have that

$$g_{a\gamma} \equiv \frac{\alpha}{\pi} \sqrt{\left[\frac{1}{f_a} \left(1 + \frac{\Delta m_{\pi}^2}{m_{\pi^0}^2}\right)\right]^2 + \left[\frac{1}{f_{a'}} \left(1 + \frac{\Delta m_{\pi}^2}{m_{\pi^0}^2} + k f_{a'} \frac{\Delta m_{\pi}^2}{f_\pi m_{\pi^0}^2}\right)\right]^2}. \tag{28}$$

The coupling $g_{a\gamma}$ depends both on positive and negative powers of $\Delta_c$ and $\Delta_L$—through the parameters $m$, $f_a$ and $f_{a'}$ in Eq. (28), which, in turns, come from the couplings in Eq. (14). For a fixed value of the radius $R$, there exists a minimum of $g_{a\gamma}$ as a function of these free parameters. Taking this minimum and comparing it with the bound in Eq. (26), yields a constraint on the possible values of $R$.

A similar bound is obtained by considering the coupling of axions to nucleons

$$\mathcal{L} = -ig_{aN} \bar{N}\gamma_5 N a, \tag{29}$$

where, in our case

$$g_{aN} = \frac{g_{\Delta M_N}}{2} \sqrt{\left[\frac{1}{f_a} \frac{\Delta m_{\pi}^2}{m_{\pi^0}^2}\right]^2 + \left[\frac{1}{f_{a'}} \left(\frac{\Delta m_{\pi}^2}{m_{\pi^0}^2} + k f_{a'} \frac{\Delta m_{\pi}^2}{f_\pi m_{\pi^0}^2}\right)\right]^2}, \tag{30}$$

and $g_A$ is the axial nucleon coupling, whereas $m_N$ is the nucleon mass. Equation (31) is obtained by including only the mixing between the neutral pion and the axion.

Limits coming from supernova SN1987a impose

$$g_{aN} < 3 \times 10^{-10}. \tag{31}$$

The two bounds Eq. (26) and Eq. (31) give

$$\frac{1}{R} > 10^6 \text{ TeV}, \tag{32}$$

which, applying Eq. (3), corresponds to

$$M_f > 10^{11} \text{ TeV}. \tag{33}$$

We have thus obtained an explicit lower limit on the fundamental scale from the experimental bounds on axion couplings.
V. NEUTRINOS

Let us recall that right-handed neutrinos must be included in six dimensions in order to cancel the gravitational anomalies. The Lagrangian in Eq. (17) has two mass terms for the neutrinos:

$$\frac{1}{2} M_N \left( N^T C^{-1} N - \bar{N} C \bar{N}^T \right)$$

and the Dirac mass term induced by the Yukawa coupling after electroweak symmetry breaking. The latter has the same structure of the other fermion masses $m^D_\nu \equiv \langle \phi^0 \rangle Y_\nu = v Y_\nu / \sqrt{2}$. Together they give rise to a neutrino mass matrix

$$
\begin{pmatrix}
0 \\
0 \\
m^D_\nu & m^D_\nu & M_N
\end{pmatrix},
$$

which is of the right form for the see-saw mechanism [17]. Since there is no symmetry to protect $M_N$, the right-handed Majorana mass term, it is reasonable to assume that $M_N \sim M_f$. Accordingly the mass of the light neutrinos is given by the see-saw expression:

$$m_\nu \sim \left( \frac{m^D_\nu}{M_f} \right)^2.$$

Imposing the heaviest mass to be the one measured by atmospheric neutrino experiments, $\sqrt{\Delta m^2_{\text{atm}} \sim (0.04 \div 0.09) \text{ eV}}$ [16], we can estimate the required value for the Dirac mass term $m^D_\nu$ for the lightest allowed $M_f$: 

$$m^D_\nu \sim (65 \div 100) \text{ GeV},$$

which is consistent with the usual mass terms for fermions.

VI. HIGHER-ORDER OPERATORS

The model under consideration is non-renormalizable; it must be understood as the low-energy limit of a more fundamental theory which gives additional interactions above the cut-off scale $M_f$. These interactions give rise to operators suppressed by powers of $1/M_f$ that violate the global symmetries of the low-energy theory. However, because of the limit we obtained for $M_f$, these effects are less worrisome than in models with large extra-dimensions in which the typical scale of such operators is in the TeV range. Nevertheless, some potentially dangerous operators must be checked. In particular, operators like

$$\mathcal{L} \sim \frac{1}{M_P} QQQL,$$

could lead to too fast a proton decay unless $M_P$ is taken of the order of $10^{16}$ GeV. They are, however, excluded by the residual discrete symmetries that remain after compactification from the $SO(5,1)$ Lorentz symmetry in six dimensions [14].

Operators compatible with these discrete symmetries could, for an arbitrary phase in the coupling, lead to potentially dangerous electric dipole moments

$$\mathcal{L} = i \epsilon \frac{m_\psi}{M_d^2} \bar{\psi} \sigma^{\mu \nu} \psi F_{\mu \nu}.$$ 

Comparing $d \equiv e m_\psi / M_d^2$ with the experimental bound [18]

$$d_e < 2 \times 10^{-27} \text{ e cm},$$

we find

$$M_d^2 > 10^4 \text{ TeV}^2,$$
which is satisfied by several orders of magnitude for $M_d \sim M_f$ imposing the bounds of Eq. (32) and Eq. (33). The similar flavor violating operator

$$\mathcal{L} = i e \frac{m_\mu}{M_\mu^2} \bar{e} \sigma^{\mu\nu} \mu F_{\mu\nu},$$

(42)

would induce the decay $\mu \rightarrow e\gamma$ with the partial rate

$$\Gamma(\mu \rightarrow e\gamma) = \frac{\alpha m_\mu^5}{M_\mu^2},$$

(43)

where $m_\mu$ is the muon mass. Comparing this with the experimental constraint [1],

$$\Gamma(\mu \rightarrow e\gamma) < 4 \times 10^{-33}\,\text{TeV},$$

(44)

yields

$$M_\mu^2 > 10^5\,\text{TeV}^2.$$  

(45)

Another class of potentially dangerous corrections comes from Kaluza-Klein states. The bounds we obtain for the extra-dimensional volume justifies the approach of working with only the zero modes of the theory: the first Kaluza-Klein excitations are at a scale much larger than that experimentally relevant and they can be safely neglected in the computation of observable quantities. Those processes that take place in the standard model only at the one-loop level could be an exception to this conclusion. However, no relevant effect is expected for our value of the compactification radius (see, for instance, [19]).

VII. DISCUSSION

We have discussed the phenomenological consequences of a six-dimensional realization of the standard model.

The cancellation of a global anomaly imposes the presence of three generations [1]. Local anomaly cancellation requires that the four-dimensional spectrum contains, besides the usual fields of the standard model, right-handed neutrinos and axion fields. The axion fields solve the strong CP problem.

The fundamental scale of the theory is related to the decay constants of the axions; therefore, its value must be large enough to evade experimental bounds. The fundamental scale is thus bounded. A problem of naturalness remains because of the large scale $M_f$ of the theory: the Higgs sector requires fine-tuning in order to protect the weak scale. A dynamical explanation of the large difference between the electroweak symmetry breaking scale and the fundamental scale is required because a supersymmetric version of the model has been shown to contain irreducible anomalies [1].

Without any further assumptions, a see-saw mechanism induced by the large mass scale of the theory provides in a natural manner a neutrino mass in the range indicated by atmospheric experiments.

In order to evade the bounds on axion physics, it has been suggested to add mass terms localized at the fixed points of the orbifold for the pseudoscalars [1]. Massive pseudoscalars are not axions: they cannot be used to solve the strong CP problem, because of the lack of translational invariance. If heavy enough, they decouple from the low-energy phenomenology, and limits from axion physics do not apply. Such a term for $b$ and $b'$ is forbidden by gauge invariance of the six-dimensional theory. If it is possible to write it for $c$ and $c'$, by changing Eq. (13) and Eq. (14), still the strong CP problem is solved, thanks to the fields $b$ and $b'$. Their couplings involving only positive powers of $\Delta_L$ and $\Delta_c$, no bound can be deduced for $R$. In particular $1/R \sim \text{TeV}$ can be compatible with experiments, choosing $\Delta_L \sim \Delta_c \sim 10\,\text{GeV}$.

Nevertheless, while a softening of the hierarchy problem in the Higgs sector would be accomplished in this way, translating it into the dynamical problem of understanding the big difference between the scale of the large extra-dimensions and the fundamental scale of the theory, the lowering of this scale would require unnaturally small couplings in higher order operators and loop dominated processes, giving potentially large contributions to electroweak precision observables, flavor changing processes [13] and CP violating quantities as electric dipole moments (see Eq. (41)).

Furthermore, the good prediction for neutrino masses would be lost, and their smallness in comparison to the other fermions would once again be unnatural, in presence of TeV scale right-handed neutrinos.

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