The photon Green’s function for bounded media: Splitting property and nonequilibrium radiation laws

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Abstract. The presence of a medium boundary has been a major obstacle for the theoretical description of the propagation, emission and absorption of light due to the loss of translational invariance. We present a nonequilibrium photon Green’s function theory that is valid for bounded (i.e., spatially inhomogeneous) media systems and also yields an energy flow law which can be seen as a generalization of the Kirchhoff and Planck laws to nonequilibrium. With the help of this law, we discuss mechanisms of emission and optical signatures of quantum condensates. An important finding is that the $D^c$ components of the photon GF, which describe field-field fluctuations, decompose universally into two parts related to medium kinetics and external light sources. Thanks to their specific structure, the propagation of arbitrary (even nonclassical) light can be analyzed straightforwardly. These properties are used to demonstrate the energy flux and scattering of squeezed light incident on the medium.

1. The photon Green’s function

1.1. Definition

The photon Green’s function (PGF) [1; 2] is a nonequilibrium (real-time) Green’s function for the vector potential operator $\hat{A}$ of the electromagnetic field in Coulomb gauge, which obeys the Maxwell equation

$$\Box \hat{A} = -\mu_0 \left( \hat{j}_{\text{ind}} + \hat{j}_{\text{ext}} \right)$$

with an operator-valued induced (bound) current $\hat{j}_{\text{ind}}$ and a c-number external (free) current $\hat{j}_{\text{ext}}$.

The PGF $D$ is defined by functional variation of Maxwell’s equation for the effective field $A_{\text{eff}} = \langle \hat{A} \rangle$ [1; 2] on the Keldysh double time contour [3; 4]:

$$D(1,2) = -\frac{1}{\mu_0} \frac{\delta A_{\text{eff}}(1)}{\delta \hat{j}_{\text{ext}}(2)} (2)$$

Here, $1$ denotes variables $(r_1, t_1)$ and $t_1$ a time on the contour. The polarization $P$

$$P(1,2) = -\frac{\mu_0}{\delta A_{\text{eff}}(2)} \frac{\delta j_{\text{ind}}(1)}{\delta A_{\text{eff}}(2)} (3)$$

represents the effects of the medium on the electromagnetic field and plays the role of a “photon self-energy” in the Dyson equation

$$\left( D^{-1}_0(1,2) - P(1,2) \right) D(2,1') = \delta_T(1,1'), (4)$$
where $\delta_T$ is the transverse delta function [5]. For notational simplicity, we restrict the discussion in this paper to isotropic media, in which $D$ is essentially reduced to a scalar function. The generalization to anisotropic media with tensor-valued $D$ is straightforward and has been presented in former articles referenced here.

1.2. The PGF and classical electrodynamics

As all Keldysh GF’s do, $D$ contains four different physical functions which obey a set of identity relations. With the above definitions, the “greater” and “less” components of $D$ give the fluctuations of the electromagnetic field,

$$D^>(1,2) = D^<(2,1) = \frac{1}{i\hbar\mu_0} \left( \langle \hat{A}(1)\hat{A}(2) \rangle - A_{\text{eff}}(1)A_{\text{eff}}(2) \right),$$

while the “retarded” Green’s function solves the classical wave propagation problem:

$$A_{\text{eff}}(1) = -\mu_0 D^{\text{ret}}(1,2)j_{\text{ext}}(2).$$

The electromagnetic properties of the medium here are contained in the retarded polarization function $P^{\text{ret}}$, which is related to the induced current and to the susceptibility $\chi$,

$$j_{\text{ind}}(1) = \frac{\partial}{\partial t_1} P(1) = -\frac{1}{\mu_0} P^{\text{ret}}(1,2)A_{\text{eff}}(2),$$

$$P^{\text{ret}}(1,2) = -\frac{1}{c^2} \frac{\partial^2 \chi(1,2)}{\partial t_1 \partial t_2}. \tag{8}$$

Note that $P$ should not be confused with the polarization field $P$ appearing in the macroscopic Maxwell equations, and that the linear approximation was used. Thus, it is implied that fields are weak, and $P^{\text{ret}}$ gives the linear response of the medium.

2. Bounded media problems

2.1. Spatial inhomogeneity

In the PGF approach shown here, the susceptibility $\chi$ describes fully and exactly the electromagnetic properties of the medium, for which appropriate approximations have to be employed in calculations. In nonequilibrium and bounded media systems, $\chi$ is inherently a nonlocal function: It is non-local in space (or spatially inhomogeneous) due to the finite size of the dipoles that create the polarization field in response to the exciting outer field, and it is non-local in time if the system is in nonequilibrium [6].

Only in bulk media in (quasi-)equilibrium, a dispersion relation for the wave vector of the field propagating through the system can be obtained, because due to translational invariance, the dependency reduces to differences of variables only:

$$\chi(\mathbf{r},\mathbf{r}',t,t') \rightarrow \chi(\mathbf{r}-\mathbf{r}',t-t') \rightarrow \chi(\mathbf{q},\omega). \tag{9}$$

As a result of such a Fourier-transformed susceptibility function, one obtains multiple branches of solutions for the dispersion relation, which are known as polariton solutions in semiconductor physics,

$$q^2(\omega) = \frac{\omega^2}{c^2} (1 + \chi(\mathbf{q},\omega)) \rightarrow q_i(\omega). \tag{10}$$

If further the long-wavelength limit is taken ($\mathbf{q} \rightarrow 0$, since optical wavelengths are much larger than atoms), one arrives at a dielectric function reduced to frequency dispersion only, $\varepsilon(\omega) = 1 + \chi(\omega)$. However, there exist a number of effects associated with medium boundaries and surfaces which cannot be represented by such a simple form [6]. On the other hand, the non-locality of the polarization function vastly complicates exact manipulations in the PGF theory.
2.2. Vacuum polarization

In contrast to bulk systems, bounded media systems have free space or vacuum regions, where the polarization tends to zero ($P \rightarrow 0$). In these regions, the form of $D_{\text{ret}}$ can be readily derived in the Fourier domain as

$$D_{\text{ret}}^0(q, \omega) = \frac{\epsilon^2}{(\omega + i\epsilon)^2 - c^2 q^2} = D_{\text{adv}}^0(q, \omega). \quad (11)$$

The addition of an infinitesimal imaginary displacement ($\epsilon \rightarrow 0$) is needed to ensure causality.

With regard to the optical theorem, the analogue of the dissipation-fluctuation theorem in particle GF theory,

$$D = D_{\text{ret}}(1, 2) + D_{\text{adv}}(4, 2),$$

it appears necessary to add an infinitesimal contribution to $P$ in order to ensure the validity of the theorem in the vacuum limit $P \rightarrow 0, D_{\text{ret}} \rightarrow D_{\text{ret}}^0 [1; 7]$: \hspace{1cm} (12)

$$P^\ast(\omega) \rightarrow P^\ast(\omega) + i\epsilon \Theta(\pm \omega) \frac{2\omega}{c^2} \quad (13).$$

This contribution, sometimes referred to as “vacuum polarization”, could be justified as a homogeneous solution $D_{\text{h}}^0$ to the optical theorem, and it was known to play a decisive role for steady-state emission and absorption [1; 7] and also for the energy flow law [8; 9] to be discussed later in this article. Its interpretation, however, was not satisfying and its handling rather difficult, requiring to exploit subtle cancellation effects between infinitesimal and diverging factors.

3. The splitting property

An elegant derivation and interpretation of the “vacuum polarization” in spatially inhomogeneous systems is found with the help of the Langreth rules [10], which can be used to extract the Keldysh components of PGF products, e.g.,

$$(FG)^\ast = F_{\text{ret}} G^\ast + F^\ast G_{\text{adv}}, \quad (FG)_{\text{ret/adv}} = F_{\text{ret/adv}} G_{\text{ret/adv}}. \quad (14)$$

Applying these rules to the integral form of the Dyson equation on the Keldysh contour [Eq. (4)],

$$D(1, 2) = D_0(1, 2) + D_0(1, 3) P(3, 4) D(4, 2), \quad (16)$$

leads after some rearrangements to a splitting of the field-field fluctuations into two contributions [11; 12]. After introduction of the inverse retarded dielectric function,

$$\varepsilon_{\text{T}}^{-1, \text{ret}} = \delta + D_{\text{ret}}^0 P_{\text{ret}}, \quad (17)$$

they can be written as

$$D^\ast = D_{\text{med}}^\ast + D_{\text{vac}}^\ast \quad \left\{ \begin{array}{l}
D_{\text{med}}^\ast = D_{\text{ret}}^0 P_{\text{ret}} D_{\text{adv}} \\
D_{\text{vac}}^\ast = \varepsilon_{\text{T}}^{-1, \text{ret}} D_0^\ast \varepsilon_{\text{T}}^{-1, \text{adv}} = D_{\text{h}}^\ast. \end{array} \right. \quad (18)$$
Comparison with Eq. (12) shows that $D_{\text{med}}^\Xi$, the *medium-induced* contribution, is known from the bulk-matter optical theorem. It is driven by the dissipation in the medium, or more specifically, from electronic recombination or generation processes described by the rates $P^\Xi$. Second, with $D_{\text{vac}}^\Xi$, called the *vacuum-induced* contribution, obviously the explicit form of the homogeneous solution $D_h^\Xi$ is found, whose sources are now clearly identified as freely evolving fluctuations $D_0^\Xi$, while medium processes do not play a role \cite{11; 12}.

Following from bare rearrangements of the Dyson equation, the splitting of the fluctuations in a bounded media system appears as a universal property of the PGF. It is valid regardless of the electromagnetic properties or geometrical shape of the medium, and spatial as well as temporal inhomogeneity are fully considered without further assumptions.

Let us have a closer look on the structure of the vacuum-induced fluctuations $D_{\text{vac}}^\Xi$: Their sources, the fluctuations $D_0^\Xi$, are *free* PGFs defined in terms of the freely evolving vector potential $\hat{A}_0$ according to Eq. (5), i.e., they are pure vacuum solutions of the Maxwell equations. These free fluctuations are renormalized in $D_{\text{vac}}^\Xi$ by the inverse of the retarded dielectric function $\varepsilon^{-1}_{\text{ret}}$, which describes the propagation of an incident classical free wave,

$$\Box A_{\text{ext}} = -\mu_0 j_{\text{ext}},$$

perturbed by the presence of the medium and resulting in the effective field

$$A_{\text{eff}} = \varepsilon^{-1}_{\text{ret}} A_{\text{ext}}. \tag{20}$$

Hence, the vacuum-induced fluctuations $D_{\text{vac}}^\Xi$ can be attributed to light coming from external sources ($j_{\text{ext}}$), in other words, incident light. Their propagation in presence of the medium can always be traced back to classical wave propagation, no matter what the properties of the incident light are.

It is to be noted that the splitting property does not mean a spatial decomposition of the system, since all PGFs are defined globally. Rather, it allows for a discrimination of the light fluctuations by their source together with a clear interpretation.

4. Propagation of quantized light
4.1. Mode expansion and quantized light

In the discussion above, no assumptions have been made on the properties of the incident light. Thanks to the specific structure of $D_{\text{vac}}^\Xi$, even the propagation of non-classical quantized light can be readily analyzed. If the freely evolving vector potential operator in the definition of $D_0^\Xi$ is expanded into normal modes according to

$$\hat{A}_0(r) = \sum_{\lambda q} \sqrt{\frac{\hbar}{2\varepsilon_0 c q V}} e_{\lambda q}(\hat{a}_{\lambda q} e^{iqr} + \hat{a}_{\lambda q}^* e^{-iqr}), \tag{21}$$

where $\hat{a}_{\lambda q}$ is a photon annihilation operator for mode vector $q$ and polarization direction $\lambda$, one obtains a general expression with expectation values of photon operator correlations in the normal form $\langle \hat{a}^+ \hat{a} \rangle - \langle \hat{a}^+ \rangle \langle \hat{a} \rangle$ and in the anomalous form $\langle \hat{a} \hat{a} \rangle - \langle \hat{a} \rangle \langle \hat{a} \rangle$ \cite{11; 13}. Their values depend on the quantum state of the light, for example

$$\langle \hat{a}^+ \hat{a} \rangle = \langle 0 | S_{\xi}^+ \hat{a}^+ \hat{a} S_{\xi} | 0 \rangle \tag{22}$$

is to be evaluated for incident light prepared to the squeezed vacuum state \cite{14}. This is demonstrated systematically for various states in Ref. \cite{13}. We will see later in Eq. (29) that
the normal and anomalous terms are associated with stationary and instationary energy fluxes, respectively.

Furthermore, the normal-mode expansion reveals that $D_0^\xi$ and, hence, $D_{\text{vac}}^\xi$, split further according to [11; 13]

$$D_0^\xi = D_{0,sp}^\xi + D_{0,\text{stim}}^\xi$$  \hspace{1cm} (23)

into a ubiquitous fixed contribution from the spontaneous ground-state fluctuations,

$$D_{0,sp}^\xi(1, 2) = \sum_{\lambda q} \frac{c}{21V q} \mathbf{F}(1) \otimes \mathbf{F}^*(2)$$  \hspace{1cm} (24)

with free plane waves $\mathbf{F}_{\lambda q}(r, t) = e^{i\lambda q \cdot r} \exp[imr - i\omega t]$, and a contribution $D_{0,\text{stim}}$ from external stimulation by a light source. For example, for incident light prepared to a Fock state with mode coefficient is linked in an obvious way to absorptivity and reflectivity [9]:

$$D_{0,\text{stim}}(1, 2) = \sum_{\lambda q} \frac{c}{21V q} n_{\lambda q}^F \mathbf{F}_{\lambda q}(1) \otimes \mathbf{F}_{\lambda q}^*(2).$$  \hspace{1cm} (25)

Just as the splitting into medium- and vacuum-induced contributions, the splitting into ground-state fluctuations and stimulation is very useful to analyze the various contributions separately. For both $D_{0,sp}^\xi$ and $D_{0,\text{stim}}^\xi$, the renormalization by the medium is formally simple. The free normal waves just have to be replaced with effective normal waves thanks to Eq. (20) [11]:

$$D_{\text{vac}}^\xi = D_0^\xi(\mathbf{F}_{\lambda q} \rightarrow \mathbf{A}_{\lambda q}).$$  \hspace{1cm} (26)

The PGF spectral function is defined as the difference between the “greater” and the “less” component of the corresponding PGF. The vacuum spectral function $D_0^\xi$ stays a fixed universal function as it should since $D_{0,\text{stim}}^\xi$ is the same for both components $D_0^\xi$. The vacuum-induced spectral function $D_{\text{vac}}$ appears as a fixed function for a given media system, i.e., it depends only on $\varepsilon_T^{\text{ret}}$ [11]:

$$D_{\text{vac}}(1, 2) = \sum_{\lambda q} \frac{c}{21V q} [\mathbf{A}_{\lambda q}(1) \otimes \mathbf{A}_{\lambda q}^*(2) - \mathbf{A}_{\lambda q}(1)^* \otimes \mathbf{A}_{\lambda q}(2)].$$  \hspace{1cm} (27)

4.2. Energy flow of quantized light

The energy flux vector (Poynting vector) $\mathbf{S} = \frac{1}{\mu_0} \left(\mathbf{E} \times \mathbf{B}\right)_{\text{sym}}$ [1; 15] can also be expressed by PGFs. The splitting properties (18), (23) then translate directly to $\mathbf{S}$:

$$\mathbf{S} = \mathbf{S}_{sp} + \mathbf{S}_{\text{stim}} + \mathbf{S}_{\text{med}}.$$  \hspace{1cm} (28)

The quantity $\mathbf{S}_{\text{stim}}$ is just the energy flux caused by an external light source. It is equivalent to the normally-ordered energy flux $\langle : \mathbf{S} : \rangle$ that is measured by photodetectors, e.g., in absorption-transmission experiments [13].

We can now derive, e.g., an exact relation for the signal of nonclassical light transmitted through arbitrary media slabs [11; 13], which yields the form

$$S_{\text{stim}}^t = \frac{\hbar c^2}{V} \sum_{\lambda q} q_1 |t_{\lambda q}|^2 \left(\langle \hat{a}^+ \hat{a} \rangle - |\langle \hat{a} \hat{a} \rangle| \cos [2q_1 x - 2\omega t + \varphi]\right).$$  \hspace{1cm} (29)

Both stationary and instationary parts of the signal are modulated by the classical transmission coefficient $t_{\lambda q}$, which appears here due to the fact that the renormalization in $D_{\text{vac}}^\xi$ is governed by $\varepsilon_T^{\text{ret}}$ which represents classical optical properties [Eq. (20)]. For slabs, the transmission coefficient is linked in an obvious way to absorptivity and reflectivity [9]: $a_{\lambda q} = 1 - |r_{\lambda q}|^2 - |t_{\lambda q}|^2$.

Thus, scattering of light by media is always classical, independent of the quantum state of light.
4.3. Comparison to other theories

This result arises from a quantum-kinetically exact treatment of the vector potential operator in the PGF. It is valid for arbitrarily dispersive and absorptive nonequilibrium media and fully considers the spatial inhomogeneity of bounded media problems. In contrast, recent well-developed quantum-optical approaches (compare, e.g., [14; 16; 17]) still must resort to an effective description of the medium properties, and represent ground-state fluctuations phenomenologically by noise currents, for which only recently a physical description has been established [18]. However, the present approach confirms the results of these effective theories and generalizes them to spatial inhomogeneity, nonequilibrium and oblique incidence.

5. Nonequilibrium energy flow

Further analytic evaluation of the spatially inhomogeneous nonequilibrium PGF seems out of reach without further assumptions. Let us thus leave complete universality, and consider the energy flow in a slab in the steady state according to Fig. 1.

![Figure 1. Slab geometry: TE-polarized light is incident in the x direction on a slab with thickness L and infinite extension in the y-z plane.](image)

The energy conservation law (Poynting’s theorem) \( \frac{\partial U}{\partial t} + \text{div} \, S = -jE \) [15] in this geometry reduces to

\[
S(x = -L/2) - S(x = L/2) = \int dx \, j(x)E(x) . \tag{30}
\]

The dissipation \( W = jE \) can be expressed by PGFs [1; 9]

\[
W(1) = \frac{i}{\hbar} \frac{\partial}{\partial t_2} \Theta(t_3 - t_1) \left[ P^> (1, 3) D^< (3, 2) - P^< (1, 3) D^> (3, 2) \right] \big|_{2 \rightarrow 1} \tag{31}
\]

and decomposes into \( W_{\text{med}} \) and \( W_{\text{vac}} \) according to the splitting of \( D^\Xi \), Eq. (18). The further evaluation of these terms is far from trivial and has been presented in Refs. [8; 9]. One can show that \( W_{\text{med}} \) vanishes in steady-state slab geometry, reflecting that, e.g., any emission from a medium process must be re-absorbed in order to keep the steady state. The vacuum-induced fluctuations then are fully responsible for any energy flow in the system, and one arrives at an energy flow law [8; 9],

\[
s(q_\|, \omega) = [b(q_\|, \omega) - n(q_\|, \omega)] a(q_\|, \omega) . \tag{32}
\]

The spectrally resolved energy flux \( s \) leaving the slab surfaces is composed of an emission contribution, \( s_e = ba \), and an absorption contribution, \( s_a = -na \), and both are proportional to the classical absorptivity \( a \). The prefactors \( b \) and \( n \) are distributions of internal and external excitations. Application of the splitting property and mode expansion on this theory leads to deeper insight on their properties and the interplay of light, matter and ground-state fluctuations.
5.1. Internal and external excitations in the energy flux

The ratio of the renormalized incident fluctuations and their fixed spectral function [Eq. (27)] gives, in the sense of a Kadanoff-Baym ansatz [8; 9], the distribution of photons in the incident light, which is also referred to as external optical excitations:

\[ n^\text{\#} (\omega, q_\parallel) = \frac{D^\text{\#}_{\text{vac}}}{D_{\text{vac}}} \]  

(33)

Indeed, from mode expansion follows

\[ n^\text{\#} = n^\text{\#}_{\text{sp}} + n_{\text{stim}} \]

(34)

and in a Fock state \( n_{\text{stim}} = n^F \). For other quantum states, \( n_{\text{stim}} \) will represent an effective mode population. Due to the definition of the spectral function, \( n^> = 1 + n^< \), and \( n^< = n \) is the distribution entering the above energy flow law, Eq. (32). In analogy to that, the distribution of internal optical excitations \( b^\text{\#} (\omega, q_\parallel) \) is introduced as the ratio of (global) recombination to the spectral function of the polarization [8; 9]. We will assume here that a similar decomposition \( b^\text{\#} = b^\text{\#}_{\text{sp}} + b_{\text{stim}} \) holds.

Re-inserting the decomposed distributions into the energy flow law leads to

\[ s = 2 (n^>_{\text{sp}} b_{\text{stim}} - b^>_{\text{sp}} n_{\text{stim}}) a \]  

(35)

a balance of absorption of incident light \( (D^>_{\text{0}} \rightarrow n_{\text{stim}}) \) and emission due to medium processes \( (P^\# \rightarrow b_{\text{stim}}) \). Both contributions are coupled to the ground-state fluctuations of the opposite party. This expression reduces further to

\[ s = (b_{\text{stim}} - n_{\text{stim}}) a \]  

(36)

and results in an effectively classical, quasi-thermal picture of emission and absorption. The contributions of the ground-state fluctuations cancel out, but ignoring them in the calculations would lead to vanishing energy flow. Hence, they are indispensable for emission and absorption to occur. This result includes spontaneous and stimulated emission as well as light amplification \( (a < 0) \) and can also be seen as a generalization of the Planck and Kirchhoff laws to distributions in nonequilibrium [9].

5.2. Quasi-equilibrium behavior

A strong theoretical requirement arises from the Kubo–Martin–Schwinger condition [19; 20] for the polarization rates \( P^\# \): All internal (medium) excitation states must (optically) behave bosonic, regardless of the underlying fermionic particle system. In quasi-equilibrium, the distribution \( b \) will take the form of the Bose distribution

\[ b \rightarrow b(\mu, T) = \frac{1}{\exp \left[ \frac{1}{k_B T} (h\omega - \mu) \right] - 1} \]  

(37)

and in full equilibrium, the Planck distribution \( (\mu = 0) \) is reached as a limiting case.

For the example of polaritons in the electron-hole plasma of semiconductors, the chemical potential \( \mu \) in the Bose distribution gives the excitation (carrier) density, \( n_c (\mu, T) \). The energy \( h\omega = \mu \) marks the frequency at which a crossover from amplifying \( (a (\omega) < 0) \) to absorbing \( (a (\omega) > 0) \) behavior occurs. On this basis, the emission of a semiconductor in quasi-equilibrium shall be analyzed in the next section [9; 21].
5.3. Application example: Semiconductor emission in quasi-equilibrium

In Figure 2, the emission spectrum $s_e(\omega) = b(\omega, \mu, T) \cdot a(\omega)$ perpendicular to the surface ($q_\parallel = 0$) is depicted for a zinc selenide (ZnSe) slab at high excitation. The according susceptibility exhibits a strong gain region ($\text{Im} \chi(\omega) < 0$) below the crossover energy $\mu = \hbar \omega = 2807$ meV. The absorptivity in this region, while still following the Fabry–Perot interference conditions marked by dotted vertical lines, exceeds the values in the absorbing range by orders of magnitude. The singularity of the Bose function at the crossover keeps $s_e$ positive and shifts the weight of the Fabry–Perot resonances closer to the crossover, which is an effect of the degeneracy in the distribution of medium excitations.

6. Signatures of quantum condensation

If the temperature approaches absolute zero, the quasi-equilibrium emission will vanish in the absorbing region and reflect the gain in the amplifying region due to the behavior of the Bose distribution:

\[ b(\hbar \omega > \mu, T \to 0 \text{ K}) \to 0 \quad b(\hbar \omega < \mu, T \to 0 \text{ K}) \to -1. \]  

(38)

At such low temperatures, effects of quantum condensation need to be considered. In electron-hole plasmas, quantum condensation manifests itself in the Bose–Einstein condensation of excitons at low density or as Cooper-like electron-hole pairs at higher densities, with a smooth crossover regime between both states (Fig. 3). With a recent particle Green function approach to the problem [23; 24], an anomalous contribution $P_{\text{cond}}$ is predicted to appear in the polarization $P^z$ in presence of a condensate. This contribution does not affect the coherent absorption, but in the emission, an additional (background broadened) $\delta$ peak is expected at $\hbar \omega = \mu$,

\[ s_{\text{cond}}(\omega, q_\parallel) \propto \delta_{q_\parallel, 0} \cdot \delta(\hbar \omega - \mu) P_{\text{cond}} \int_{\text{slab}} dx |A(x)|^2, \]  

(39)

which increases for $T \to 0$, whereas the normal emission disappears [8; 9]. Its strength, given by $P_{\text{cond}}$, is illustrated in Fig. 4 for various temperatures. This additional peak can serve as

Figure 2. Absorptivity $a$ and emission $s_e = ba$ in a 1500 nm ZnSe slab as a function of frequency relative to the chemical potential $\mu = 2807$ meV. The susceptibility $\chi$ [21; 22] was calculated for an excitation density of $n_e = 2 \times 10^{17}$ cm$^{-3}$ at $T = 77$ K.
Figure 3. Phase diagram: Condensed and non-condensed phase in the electron-hole plasma of a model semiconductor, from [23; 24]. The phase boundary follows the ideal Bose condensation condition, then the condensate develops smoothly from a Bose–Einstein condensate to a BCS-like electron-hole pair state due to the breakup of the excitons (Mott effect).

Figure 4. Strength of the condensate contribution to the polarization function at different temperatures as a function of the chemical potential of the electrons. Calculated according to the model from [23]. The crossover region around $\mu = 0$ corresponds to the Mott density.

the optical signature of a quantum condensate, since the regular emission just at this frequency vanishes ($a(\hbar \omega = \mu) \equiv 0$) [8; 9].

7. Summary
The description of radiation in bounded media with the help of the nonequilibrium photon Green’s function was presented. Thanks to the universal splitting property of the field fluctuations represented by $D^2$ and the specific structure of the vacuum-induced contribution, light can be discriminated by its source in this approach. The splitting into contributions from ground-state fluctuations of the vacuum field, incident light, and electronic processes in the medium translates to the energy flux vector and to the dissipation. Incident quantized light can be readily described. Regardless of its state, it is predicted to propagate classically through a medium. The results are valid for arbitrarily dispersive and absorptive media of any geometrical shape. The spatial inhomogeneity inherent to bounded media problems is fully considered.

Further analysis of energy flux and dissipation in the steady-state slab geometry leads to an energy flow law valid for nonequilibrium excitations of the outside (incident light) and the medium. Application of the splitting properties to the excitation distributions shows the influence of the ground-state fluctuations on emission and absorption. Finally, the energy flow law provides an effectively classical picture. Both emission and absorption are governed by the classical absorptivity, and the contributions of the ground-state fluctuations on the resulting energy flux cancel out.

For demonstration, the quasi-equilibrium emission of an highly excited semiconductor is discussed. In combination with a general Green function approach to quantum condensation in the electron-hole plasma, predictions can be made for the optical signature of, e.g., a Bose–Einstein condensate of excitons.
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