Quantum Coherence and Path-Distinguishability of Two Entangled Particles

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A modified ghost-interference experiment is theoretically analyzed, where one of the two entangled particles (particle 1) goes through a multi-slit before being detected at a fixed detector. In addition, one introduces a mechanism for finding out which of the n slits did particle 1 go through. The other particle of the entangled pair (particle 2) goes in a different direction, and is detected at a variable, spatially separated location. In coincident counting, particle 2 shows n-slit interference. It is shown that the normalized quantum coherence of particle 2, $C_2$, and the path-distinguishability of particle 1, $D_{Q1}$, are bounded by an inequality $D_{Q1} + C_2 \leq 1$. This is a kind of nonlocal duality relation, which connects the path distinguishability of one particle to the quantum coherence of the other.

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I. INTRODUCTION

Quantum coherence is a topic which has been under intense study in recent years. Coherence has always been an important issue in quantum optics [1], but with the advent of quantum information and computation there was a need for a rigorous quantitative measure of quantum coherence. Recently a measure of coherence was introduced, which is basically the sum of the absolute values of the off-diagonal elements of the density matrix of a system in a particular basis, i.e., $\sum_{i \neq j} |\rho_{ij}|$, with $\rho_{ij} = \langle i | \rho | j \rangle$ [2]. This measure can be normalized for finite dimensional Hilbert space, to give the following definition of coherence [3]

$$C = \frac{1}{n-1} \sum_{i \neq j} |\langle i | \rho | j \rangle|,$$

(1)

where $n$ is the dimensionality of the Hilbert space. This measure has proven to be particularly useful in quantifying wave-particle duality in n-path interference [3-5].

Neils Bohr’s assertion that the wave and particle natures are mutually exclusive, came to be known as the principle of complementarity, or more popularly as wave-particle duality [6]. This principle has stood its ground despite criticism and attacks over the years. It has also been given a quantitative meaning by an inequality $\sum_{i \neq j} |\rho_{ij}| \leq 1$. This is a kind of duality relation [5]. Needless to say, when we talk of path-distinguishability, we talk of the path knowledge of the same particle which contributes to the interference pattern. Thus the relation (2) is local, which might look like stating the obvious. However, in this study we propose and theoretically analyze an experiment involving pairs of entangled particles in which we relate the path information of one particle to the coherence of the other.

II. N-SLIT GHOST INTERFERENCE

We start by generalizing the well known ghost-interference experiment carried out by Strekalov et al. [9]. In the original experiment, pairs of entangled photons generated from a spontaneous parametric down conversion (SPDC) source, are separated by a polarizing beam-splitter. Photon 1 passes through a double-slit before being registered in a fixed detector D2. Photon 2 travels undisturbed before being detected by the scanning detector D2. The detectors D1 and D2 are connected to a coincidence counter. The detector D2 for photon 2, when counted in coincidence with the fixed detector D1, shows a two-slit interference pattern, although photon 2 does not pass through any double-slit. This interference, called ghost interference, has been understood to be a consequence of entanglement, and generated lot of research activity [10-18].

We look at a modified ghost interference experiment, shown in Fig. [7] where the double-slit is replaced by a n-slit. A n-slit ghost interference experiment has actually been performed by Zeilinger’s group [19]. However, here we will consider another modification of it. Entangled particle pairs, which could also more generally
be something other than photons, emerge from a source S. Particle 1 passes through a n-slit and also interacts with a path-detector. We do not assume any form of the path-detector, but just assume that it is an n-state system with states $|d_1⟩, |d_2⟩, |d_3⟩, \ldots, |d_n⟩$, which get entangled with the n paths of particle 1. This entanglement is bound to happen if the path-detector acquires the relevant information about which slit particle 1 went through. Particle 1 then travels further and is detected by a fixed detector D1. Particle 2 travels undisturbed to detector D2. Since the two particles have to be counted in coincidence, the paths travelled by both the particles, before reaching their respective detectors, are assumed to be equal. Without the path-detector, our experiment would just be a n-slit generalization of the original two-slit ghost interference experiment ⁹. It is expected to show a n-slit ghost interference for particle 2. By introducing a path-detector, we are probing if acquiring path information about particle 1, has any effect on the interference shown by particle 2.

We assume $|d_1⟩, \ldots, |d_n⟩$ to be normalized, but not necessarily orthogonal. The ultimate limit to the knowledge we can acquire as to which slit particle 1 went through, is set by how distinct the states $|d_1⟩, \ldots, |d_n⟩$ are. If $|d_1⟩, \ldots, |d_n⟩$ are orthogonal, we can in principle know with certainty which slit the particle went through. Of course in general, $|d_1⟩, \ldots, |d_n⟩$ may not be all orthogonal to each other. In such a situation, one is left with the problem of unambiguously telling which is of the states $|d_1⟩, \ldots, |d_n⟩$, is the given unknown path-detector state which the path-detector process throws up. The best bet to answer this question is using unambiguous quantum state discrimination (UQSD) ²⁰,²⁴.

Using UQSD, a path-distinguishability $D_Q$ can then be defined as the upper bound to the probability with which the states $|d_1⟩, \ldots, |d_n⟩$ can be distinguished without any error ³,²⁵. If the state $|d_k⟩$ occurs with a probability $|c_k|^2$, the path distinguishability has the following form ³,²⁵

$$D_Q = 1 - \frac{1}{n-1} \sum_{i \neq j} |c_i| |c_j| |⟨d_i|d_j⟩|.$$  

The path-distinguishability can take values between 0 and 1.

### III. GENERAL ANALYSIS

Next we formulate the n-slit ghost interference, with a path-detector, in a most general way. We assume that the particles travel in opposite directions along the x-axis. The entanglement is in the z-direction. The entangled two particle state, at the source, is given by $|Ψ(0)⟩$. We assume that after travelling for a time $t_0$, particle 1 reaches the n-slit $(vt_0 = L_2)$, and particle 2 travels a distance $L_2$ towards detector D2. The two particle state is now given by $|Ψ(t_0)⟩$.

We take into account the effect of the n-slit on the entangled state as follows. We assume that the n-slit allows the portions of the wave-function in front of the slits to pass through, and blocks the other portions. We assume that what emerge from the n-slit are localised states, whose width is approximately the width of a slit. The states of particle 1, which pass through the slits 1,2,\ldots, n, are denoted by $|φ_1⟩, |φ_2⟩, \ldots, |φ_n⟩$, respectively. The state representing the situation which particle 1 gets blocked is, say, $|χ⟩$. These n+1 states are obviously orthogonal, because they represent mutually exclusive possibilities, and form a complete set because they exhaust all the possibilities for particle 1. Thus the entangled two-particle state can be expanded in terms of these. We can thus write:

$$|Ψ(t_0)⟩ = \sum_{k=1}^{n} |φ_k⟩⟨φ_k| + |χ⟩⟨χ| \otimes I_2 |Ψ(t_0)⟩$$

$$= \sum_{k=1}^{n} |φ_k⟩⟨φ_k|Ψ(t_0)⟩ + |χ⟩⟨χ|Ψ(t_0)⟩, \quad (4)$$

where $I_2$ is the unit operator for the space of particle 2. Now, since we are only interested in those instances (through coincidence counting) where particle 1 does pass through the multi-slit, and does not get blocked, the $|χ⟩⟨χ|$ dependent term can be discarded, and one is left with the two-particle state

$$|Ψ(t_0)⟩ = \sum_{k=1}^{n} |φ_k⟩⟨φ_k|Ψ(t_0)⟩, \quad (5)$$

which needs to be normalized again. A typical term $⟨φ_k|Ψ(t_0)⟩$ represents an unnormalized state of particle 2. Normalizing it will essentially throw up a constant specific to that state:

$$⟨φ_k|Ψ(t_0)⟩ = c_k |ψ_k⟩, \quad (6)$$

where $|ψ_k⟩$ are normalized states of particle 2. Particle 1, emerging from the n-slit, interacts with a path-detector which is initially in the state $|d_0⟩$. The normalized state
of the two particles, plus the path detector, is given by

$$\ket{\Psi(t_0)} = \left( \sum_{k=1}^{n} c_k \ket{\phi_k} \ket{\psi_k} \right) \ket{d_0}. \tag{7}$$

In addition, the states of particle 1 get entangled with the n states of the which-path detector \(|d_1\rangle, \ldots, |d_n\rangle\). So, the state we get after particle 1 crosses the n-slit is:

$$\ket{\Psi} = \sum_{k=1}^{n} c_k \ket{\phi_k} \ket{\psi_k} \ket{d_k}. \tag{8}$$

Now, if one is only interested in the path detector, for all practical purposes it is in a mixed state with \(|d_k\rangle\) occurring with a probability \(|c_k|^2\), as is obvious from (8). Path distinguishability of particle 1 can then be simply written down, using \((9)\), as

$$D_{\Psi_1} = 1 - \frac{1}{n-1} \sum_{i \neq j} |c_i||c_j|\langle \langle d_i | d_j \rangle \rangle. \tag{9}$$

The above equation quantifies the amount of path knowledge about particle 1 one can obtain, given the path-detector states \(|d_k\rangle\).

After interacting with the path-detector, particle 1 travels to the fixed detector \(D1\). Particle 2 continues its travel undisturbed to reach \(D2\), and should give rise to interference. The state of the two particle just before hitting the detectors is given by

$$\ket{\Psi_s} = \sum_{k=1}^{n} c_k \ket{\phi_k} \ket{U_2} \ket{\psi_k} \ket{d_k}, \tag{10}$$

where \(U_1, U_2\) represent the time evolution operators for particle 1 and 2, respectively, from time \(t_0\) to the times the particles hit the detectors. Interference is a signature of the wave nature. It has been argued before that the wave nature of a quanton, in an interference experiment, can be quantified by its coherence \(C\), defined by \((1)\). It has also been demonstrated that it is possible to actually measure \(C\) in an interference experiment \((2)\). For particle 2, the coherence \(C\) which will quantify its wave nature, is given by

$$C_2 = \frac{1}{n-1} \sum_{i \neq j} |\langle \psi_i | U_2^\dagger \rho_i U_2 | \psi_j \rangle|, \tag{11}$$

where \(\rho_i\) is the reduced density matrix of particle 2, obtained after tracing over the states of particle and the path-detector. In writing the above, we have tacitly assumed that \(|\psi_i\rangle\) form an orthonormal set. That is actually an assumption, and will be true only if the entanglement between the two particles is good. For example, if \(|\Psi(t_0)\rangle\) is the so-called EPR state \((2)\), all \(|\psi_i\rangle\)s will be orthogonal to each other. If the \(|\psi_i\rangle\)s are not mutually orthogonal, \(C_2\) cannot be calculated using \((11)\), although interference may still arise. Let us assume that this condition is satisfied, and \(|\psi_i\rangle\)s are mutually orthogonal, and go ahead with calculating \(C_2\).

If one tries to evaluate the reduced density operator for particle 2 by using \(|\Psi\rangle\) is given by \((10)\), one gets the following result:

$$\rho_r = Tr_d \left[ \sum_k \langle \phi_k | U_1^\dagger \ket{\Psi} \bra{\psi_k} U_1 | \phi_k \rangle \right]$$

$$= \sum_k |c_k|^2 \ket{U_2} \ket{\psi_k} \bra{\psi_k} U_2^\dagger, \tag{12}$$

where \(Tr_d\) represents a trace over the path-detectors states. The above is clearly diagonal in the basis \(|U_2 \ket{\psi_k}\rangle\), and consequently yields \(C_2 = 0\). This means no interference. The reason for this apparently negative result is that the interference in particle 2 is not first order. It only occurs when the particles are detected in coincidence with a fixed \(D1\). Let us assume that particle 2 is counted only when particle 1 is found in a state \(|z_0\rangle\), which may be a state localized in position. Then the reduced density operator for particle 2 is given by

$$\rho_r = \frac{Tr_d \left[ \langle z_0 | \Psi \rangle \bra{\psi_0} \right]}{Tr \left[ \langle z_0 | \Psi \rangle \bra{\psi_0} \right]}$$

$$= \frac{\sum_{j,k} c_j c_k^* \langle z_0 | U_1 | \phi_j \rangle \langle \phi_k | U_1 | \phi_k \rangle |z_0\rangle \langle d_j | d_k \rangle |\psi_j\rangle \langle \psi_k |}{\sum_k |c_k|^2 \langle z_0 | U_1 | \phi_k \rangle^2}. \tag{13}$$

Using \((13)\) and \((11)\), \(C_2\) can be easily worked out to give

$$C_2 = \frac{1}{n-1} \sum_{i \neq j} |c_i||c_j|\langle \langle d_i | d_j \rangle \rangle |\langle z_0 | U_1 | \phi_j \rangle| |\langle z_0 | U_1 | \phi_j \rangle|$$

$$\leq \frac{1}{n-1} \sum_{i \neq j} |c_i||c_j|\langle \langle d_i | d_j \rangle \rangle, \tag{14}$$

where the inequality is saturated when all \(|z_0 \ket{U_1} \phi_j\rangle\) are equal. The above relation, together with \((9)\), results in the following inequality

$$D_{\Psi_1} + C_2 \leq 1. \tag{15}$$

This relation puts a bound on the coherence of particle 2, and the amount of path information which can be extracted for particle 1. This is a completely nonlocal effect, and is a consequence of the entanglement between the two particles.

**IV. WAVE-PACKET ANALYSIS**

The analysis in the preceding section serves to reveal the general nature of ghost interference, and provides bounds on the wave and particle natures of the two entangled particles. However, it’s applicability is restricted to the situation where \(|\psi_i\rangle\)s are mutually orthogonal.
Thus, the distance between j'th and k'th slits is...

where C is a normalization constant, and σ, Ω are certain parameters. In the limit σ, Ω → ∞ the state reduces to the so-called EPR state introduced by Einstein, Podolsky and Rosen. After performing the integration over p, (16) reduces to

\[ \Psi(z_1, z_2) = C \int_{-\infty}^{\infty} dk \ e^{-k^2/4\sigma^2} e^{-ikz_2} e^{-i(kz_2 + x_0)/4\sigma^2}, \tag{16} \]

It is straightforward to show that Ω and σ quantify the position and momentum spread of the particles in the z-direction. We would like to reemphasize that the two particles are assumed to be moving along the x-axis, in opposite directions, and the slits are in the y-z plane, each slit being parallel to the y-axis, placed at a different z position. The dynamics of the particle along the x-axis is uninteresting, and only serves to transport the particle from the source to the detectors. Consequently we will ignore this dynamics and will just assume that particle 1, moving with an average momentum p₀, reaches the n-slit at a time t₀, after traveling a distance L. A de Broglie wavelength can be associated with the particle, \( \lambda = h/p₀ \).

The state of the entangled system, after this time evolution, can be calculated using the Hamiltonian governing the time evolution, given by \( \hat{H} = \frac{p^2}{2m} + \frac{m^2 \omega^2}{2} \). After a time \( t₀ \), (17) assumes the form

\[ \Psi(z_1, z_2) = C₀ \sum_{k=1}^{n} c_k |d_k⟩e^{-(z_1-kz_0)^2/2σ^2} e^{-(z_2-kz_0)^2/2σ^2} + iθ_k, \tag{23} \]

where \( C₀ = (1/\sqrt{πσ})(\sqrt{Γ_L} + i4h₀/m)⟩^{-1/2} \), \( Γ_L, Γ_l \) being the real and imaginary parts of \( Γ \), respectively, and \( c_k^2 \) is the probability of particle 1 to emerge from the k'th slit. The constants \( c_k \) and the states \( |d_k⟩ \) are assumed to be real, as any complex phases can be absorbed in \( θ_k \). Particles travel for another time \( t \) before reaching their respective detectors. We assume that the wave-packets travel in the x-direction with a velocity \( v_0 \) such that \( \lambda = h/mv_0 \) is the de Broglie wavelength. Using this strategy, we can write \( h(t + 2t₀)/m = λD/2π, h₀/m = λL/2π \). The expression \( λD/2π \) also hold for a photon provided, one uses the wavelength of the photon for \( λ \). The state acquires the form

\[ \Psi(t) = C_t \sum_{k=1}^{n} c_k |d_k⟩e^{-\frac{(z_1-kz_0)^2}{2σ^2}} e^{-\frac{(z_2-kz_0)^2}{2σ^2}} e^{iθ_k}, \tag{24} \]

where

\[ C_t = \frac{1}{\sqrt{πσ}} e^{-(z_1-kz_0)^2/2σ^2} e^{-(z_2-kz_0)^2/2σ^2}. \tag{25} \]

In order to get simplified results, we consider the limit \( Ω ≫ σ \) and \( Ω ≫ σ \). In this limit

\[ Γ ≈ γ + 4iht₀/m, \tag{26} \]

where \( γ = ε^2 + 1/σ^2 \) and \( z_0 ≈ z₀ \).

Using (24), we can now calculate the probability of coincident detection at D1 and D2. Assuming that D1 is fixed at \( z₁ = 0 \), this probability density is given by

Using (19) and (18), the wave-function for \( |ψ_k⟩ \) can be calculated, which, after normalization, has the form

\[ ψ_k(z_2) = C_2 e^{-\frac{(z_2-kz_0)^2}{2σ^2}}, \tag{20} \]

where \( C_2 = (2/π)^{1/4}(\sqrt{Γ_R} + i4h₀/m)^{-1/2} \), \( z_0' = \frac{z_0}{4Γ_2} \), \( Γ_R, Γ_l \) being the real and imaginary parts of \( Γ \), respectively, and \( c_k^2 \) is the probability of particle 1 to emerge from the k'th slit. The constants \( c_k \) and the states \( |d_k⟩ \) are assumed to be real, as any complex phases can be absorbed in \( θ_k \). Particles travel for another time \( t \) before reaching their respective detectors. We assume that the wave-packets travel in the x-direction with a velocity \( v_0 \) such that \( \lambda = h/mv_0 \) is the de Broglie wavelength. Using this strategy, we can write \( h(t + 2t₀)/m = λD/2π, h₀/m = λL/2π \). The expression \( λD/2π \) also hold for a photon provided, one uses the wavelength of the photon for \( λ \). The state acquires the form

\[ \Psi(t) = C_t \sum_{k=1}^{n} c_k |d_k⟩e^{-\frac{(z_1-kz_0)^2}{2σ^2}} e^{-\frac{(z_2-kz_0)^2}{2σ^2}} e^{iθ_k}, \tag{24} \]

where

\[ C_t = \frac{1}{\sqrt{πσ}} e^{-(z_1-kz_0)^2/2σ^2} e^{-(z_2-kz_0)^2/2σ^2}. \tag{25} \]

In order to get simplified results, we consider the limit \( Ω ≫ σ \) and \( Ω ≫ σ \). In this limit

\[ Γ ≈ γ + 4iht₀/m, \tag{26} \]

where \( γ = ε^2 + 1/σ^2 \) and \( z_0' \approx z₀ \).

Using (24), we can now calculate the probability of coincident detection at D1 and D2. Assuming that D1 is fixed at \( z₁ = 0 \), this probability density is given by
$P(z_2) \equiv |\Psi(0, z_2, t)|^2$, which has the following form

\[
P(z_2) = |C_2|^2 \sum_{j=1}^{n} c_j^2 e^{-\frac{2\pi i z_2^2}{\lambda}G^2} + \sum_{j \neq k} c_j c_k |\langle d_j | d_k \rangle| e^{-\frac{2\pi i z_2^2}{\lambda}G^2} + \cos \left[ \frac{2\pi (k-j)z_2z_2\lambda D}{\gamma^4 \pi^2 + \lambda^2 D^2} + \theta_j - \theta_k \right].
\]

where $\alpha = e^2 + \lambda^2 L^2 / \pi^2 e^2$ and $\beta = \gamma^2 + \lambda^2 D^2 / \pi^2 \gamma^2$. Eqn. (27) represents a n-slit ghost interference pattern for particle 2, even though it has not passed through any slit. If the position of $D_2$, $z_2$ is on any primary maximum away from the one at $z_2 = 0$, $kz_2$ is negligible in its comparison. This happens basically because the $\sim e^{-\frac{2\pi i z_2^2}{\lambda}G^2}$ Gaussians $e^{-\frac{2\pi i z_2^2}{\lambda}G^2}$ are very broad because $\gamma$ is very small, and as a result $\beta$ very large. Keeping this in mind, (27) can be further simplified to:

\[
P(z_2) = |C_2|^2 e^{-\frac{2\pi i z_2^2}{\lambda}G^2} \sum_{k=1}^{n} c_k^2 e^{-\frac{2\pi z_2^2}{\lambda}G^2} + \sum_{j \neq k} c_j c_k |\langle d_j | d_k \rangle| e^{-\frac{2\pi i z_2^2}{\lambda}G^2} + \cos \left[ \frac{2\pi (k-j)z_2z_2\lambda D}{\gamma^4 \pi^2 + \lambda^2 D^2} + \theta_j - \theta_k \right].
\]

We can calculate the coherence from the interference formed by particle 2. It is has been demonstrated earlier that the coherence can be calculated from a n-slit interference pattern as $C = \frac{1}{n-1} \frac{\sum_{j} c_j^2}{\sum_{j} c_j^2}$, where $l_{max}$ is the maximum intensity at a primary maximum, and $l_{inc}$ is the intensity at the same position if $\theta_j, \theta_k$ were varying randomly. The effect of randomly varying phases $\theta_j, \theta_k$ on (28) will be that the cosine term will become zero. The quantum coherence for particle 2, from (28), is given by

\[
C_2 = \frac{1}{n-1} \sum_{j \neq k} c_j c_k |\langle d_j | d_k \rangle| e^{-\frac{2\pi i z_2^2}{\lambda}G^2} \leq \frac{1}{n-1} \sum_{j \neq k} c_j c_k |\langle d_j | d_k \rangle|.
\]

where we have used $\sum_{k=1}^{n} c_k^2 = 1$. Using the above and (9), one can write

\[
D_{Q1} + C_2 \leq 1.
\]

That is the same relation which was derived in (15), in the preceding section, from a more general analysis.

We can consider another limit which is opposite to that described by (26), namely where the entanglement is weak. This is the case where $\Omega \approx 1/\sigma$. Here $\Gamma$ can be approximated by

\[
\Gamma \approx \gamma + \frac{2\pi \hbar}{m},
\]

where $\gamma = 1/2\alpha^2$. Here $\beta (= \gamma^2 + \lambda^2 D^2 / \pi^2 \gamma^2)$ is independent of $\alpha$ and does not grow much with time. Consequently, the Gaussians $e^{-\frac{2\pi i z_2^2}{\lambda}G^2}$ in (27) are not very broad and may not overlap strongly with each other. This will lead to a reduced value of coherence $C_2$. The path distinguishability $D_{Q1}$ of particle 1, on the other hand, is unaffected by the degree of entanglement. Thus the inequality (30) remains far from saturation if the two particles are weakly entangled.

V. CONCLUSION

In conclusion, we have theoretically analyzed a modified ghost interference experiment with n-slits and a path-detector behind the multi-slit. We have shown that extracting path-knowledge about particle 1, affects the coherence of particle 2, although they are spatially separated. We have shown that a kind of non-local wave-particle duality relation applies for such a situation.

A critic might be tempted to infer that particle 1 is actually providing the path information of particle 2, and the inequality (30) is essentially a duality relation for a one particle only. There are several quantum optics experiments where entanglement has indeed been used to infer path information of a particle, by looking at its entangled partner. However, in our case, as particle 2 does not pass through any slits, its path information has no meaning. Particle 2 only shows an interference, without passing through any slits, and the path information that is obtained, is only of particle 1.

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