A simulation sandbox to compare fixed-route, flexible-route transit, and on-demand microtransit system designs

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Abstract
With advances in emerging technologies, options for operating public transit services have broadened from conventional fixed-route service through semi-flexible service to on-demand microtransit. Nevertheless, guidelines for deciding between these services remain limited in the real implementation. An open-source simulation sandbox is developed that can compare state-of-the-practice methods for evaluating between the different types of public transit operations. For the case of the semi-flexible service, the MAST system is extended to include passenger deviations. A case study demonstrates the sandbox to evaluate and existing B63 bus route in Brooklyn, NY and compares its performance with the four other system designs spanning across the three service types for three different demand scenarios.

Keywords: public transport, semi-flexible transit, on-demand microtransit, Mobility-as-a-Service
1. Introduction

1.1. Motivation
Public transport, or mass transit, is a public service that pools travelers’ trips together to achieve economies of density and spatial scope (Jara-Díaz and Basso, 2003). Its goal is to provide quality, affordable mobility to users as a public good (Desaulniers and Hickman, 2007). The optimization of pricing and subsidies has been a conventional economic tool for policy making (Hörcher and Tirachini, 2021). But what constitutes an appropriate service for a city or region? This question has remained difficult to address due to the myriad of scenarios and needs from different communities and will only become more consequential with increasing urbanization (UN, 2018) and population increase (UN, 2017).

Solutions available to policymakers have also become more complex with the emergence of new mobility services due to innovations in information and communications technologies (ICTs) and the Internet of Things (IoT) (see Chow, 2018). What might once have “simply” involved a decision of whether to invest in a light rail line or a bus route with some flexible stops may now also include considerations of bikeshare, microtransit feeder services, or taxis, all compounded by considerations of automation and electrification (WEF, 2019), and whether to operate as public fleets or outsourced to private mobility providers.

As a result of these advances, options for public transit provision include a range of services from conventional fixed-route transit to more on-demand, door-to-door “microtransit” service, and either in isolation or in combination with other services within a “Mobility-as-a-Service” (MaaS) platform. In such a platform, multiple services are available under a common gateway or platform to support travelers (Hensher, 2017). While Via, a shared ride microtransit provider, covers a certain area in cities where it operates vehicles (Via, 2019), transit agencies like New York City MTA tend to enhance the connectivity in cities by spanning out over the region (TRAVIC, 2020).

Figure 1 provides a classification of modes available in MaaS platforms, which suggests a broad array of options for designing service solutions for travelers. A key take-away from this figure is that, despite the broad array of options, they mostly anchor around transit use. For example, active modes are often linked to public modes as part of multimodal trips (see Chow and Djavadian, 2015). The multimodality extends to shared modes as well, primarily in the first-last mile context (Ma et al., 2019a). They reinforce the importance of the public transit mode in the successful deployment of MaaS.

Figure 1. Different modes from Mobility-as-a-Service (source: Wong et al., 2020).
Despite the emergence of these technologies, deployments have varied in success. Several microtransit providers have already shut down in recent years; examples include Bridj (Woodward et al., 2017) and Ford Chariot (Korosec, 2019a). Due to these operational challenges, public agencies like the Federal Transit Administration (FTA) in the United States have sought to encourage more pilots and research through programs like the Mobility-on-Demand (MOD) Sandbox Program with many projects dealing with first- and last-mile access (GAO, 2018).

While there is an abundant new literature on these operations, there is also a long history to research in these areas, many of which preceded the advances in ICT needed to make Demand Responsive Transit (DRT) feasible. For example, even from the late 1970s, the existence of a demand density threshold over which fixed-route service operates better than DRT was well-known (Systan, 1980). More recently, Daganzo and Ouyang (2019) illustrated how the different modal services between fixed-route and on-demand may compare under simplified operating settings. They studied how the minimum door-to-door passenger travel time varies with fleet size for different levels of demand density for a stylized example. The result shows those factors affect the preference on various travel options in different ways, impacting the ranges where a mode becomes “sweet spots” regarding the travel time.

1.2. Research objective
Clearly, successful MaaS deployment depends on making full use of the spectrum of transit operations, which can range from full fixed-route services, through flexible services, to full MOD-style microtransit services serving passengers door-to-door or virtual stop to virtual stop (Hazan et al., 2019). However, which types of services work well for a certain setting? How can a practitioner evaluate the service needs of a study with regards to different types of public transit services to support a MaaS ecosystem? To effectively compare different operating strategies within one study area without the costs and risks of full deployment, a simulation-based tool is needed.

We propose such a simulation sandbox that can be adapted to any study area to compare the operations of three classes (not mutually exclusive as hybrids between them also exist) of transit operations: fixed-route transit, flexible-route transit, and on-demand transit. The term sandbox refers to a consistent framework for trying out different operating services so that their performances can be compared. The goal is to provide a transportation professional working in a public agency or a mobility provider with tools to help analyze and compare these services as societies progress toward a MaaS setting. The proposed simulation sandbox architecture is described in detail and its application is demonstrated in a case study using a common data set (a Brooklyn bus route, B63).

Accordingly, the remainder of this study consists of three main parts. Section 2 breaks down a review into each of the three categories of transit operations while Section 3 describes the proposed simulation sandbox design including each system type, key design variables, and possible scenarios which can be implemented in the proposed sandbox. Section 4 provides a case study of an open-source tool and common data set, both made publicly available, for researchers and practitioners to assess their own study areas, and Section 5 concludes.

2. Classes of public transit operation
2.1. Fixed-route transit service
Fixed-route transit service is the most rigid of the three classes of transit considered. Certain vehicle technologies within this class generally require rigid routes, e.g. metro and other railway
operations (with exceptions, e.g. Cats and Haverkamp, 2018). The trade-off of using a more rigid operational policy is that much higher passenger flow capacities can be attained and therefore is more suited for high demand density populations as shown in Vuchic (1981).

 Fixed-route transit service operational planning involves several key functions outlined by Ceder (2016): network route design, timetable development, vehicle scheduling, and crew scheduling. Network design determines the structure and service of the network, including determination of routes and stops. Timetabling cements these route-level decisions with frequencies or headways along with a public timetable. Vehicle scheduling assigns the fleet to the timetables while crew scheduling assigns drivers and other staff to the fleet operations.

 Planning routes and frequencies is on the strategic planning level, often called the line planning problem. Hasselström (1982) and van Nes et al. (1988) proposed early line planning optimization models for setting routes and frequencies jointly. Reviews of transit network design models and algorithms can be found in Guihaire and Hao (2008) and more broadly in Farahani et al. (2013).

 Line planning has been shown to be NP-Hard in computational complexity (see Schöbel and Scholl, 2006), leading to the use of route construction heuristics like Ceder and Wilson (1986). As such, line planning in practice may involve using permutations of simple structures. Fielbaum et al. (2017) explicitly tackle the problem of defining any city transit network using a parameterized network design structure. Fielbaum et al. (2016) used their network description to evaluate four different line structures: direct lines, exclusive lines, hub-and-spoke, and feeder-trunk. Hub-and-spoke and feeder-trunk lines are particularly effective in addressing first-last mile problems to serve lower demand density areas like suburbs, and are an active area being targeted for MaaS integration with transit services (Ma et al., 2019a).

 For more specific designs at a route level, continuous approximation models have been proposed to design lines (Byrne, 1975; Newell, 1979), frequency (Newell, 1971; Mohring, 1972), and stop spacing (Vuchic and Newell, 1968; Wirasinghe and Ghoneim, 1981; Tirachini, 2014). In the frequency setting methods, the optimal frequency is shown to be proportional to the square root of the demand density. Furthermore, Chen et al. (2018) incorporated short-turn strategy to the local route service design to minimize total system cost.

 Metaheuristics have also been an alternative solution to the transit route network design problem, and Iliopoulou et al. (2019) reviewed literature that applied them and categorized studies into three types: single-solution-based, population-based, and hybrid metaheuristics, following Gendreau and Potvin (2005). While single-solution-based metaheuristics repetitively improve a single candidate solution, population-based metaheuristics examine the population which consists of candidate solutions. Hybrid methods are combinations of metaheuristics from both groups.

 Meanwhile, tactical planning and operational control, lower levels of the transit route planning, involves other design elements. Timetabling, an example of tactical planning, is the task of enumerating all the times at which a vehicle will service each stop. Ceder (1987) provides a practical guide to timetabling with equal, balanced, or smooth headways based on current ridership counts, but it becomes more complex once the objective of synchronization across routes is introduced (see Bookbinder and Désilets, 1992, and Ceder et al., 2001). Emerging methods for timetabling rely on new data sources and technologies including automatic fare collection data (Yap et al., 2019) and autonomous shuttle bus service (Cao and Ceder, 2019). For operational control, vehicle holding is an example to manage bus bunching when buses arriving in bunches or platoons, increasing both average wait time and wait time variability (Bartholdi and Eisenstein, 2012). With the availability of real-time system information, the system can mitigate headway...
disturbances between vehicles by manipulating holding time or bus cruising speed (Daganzo, 2009; Daganzo and Pilachowski, 2011; Bartholdi and Eisenstein, 2012). Berribi et al. (2018) compared the schedule-, headway-, and prediction-based method of holding, and the last could balance the headway regularity and holding time the best.

2.2. Semi-flexible transit

For the more sparsely distributed demand regions, conventional fixed-route transit service is costly to serve due to the first-last mile problem. Many transportation alternatives have been proposed by researchers and transit planners to improve the degree of flexibility of fixed-route services.

The concept of demand-responsive shuttles has existed since the 1970s. As with any taxonomy, experts differ on the exact boundaries of the term. Shaheen et al. (2015, 2016a) define a term, microtransit, as a privately owned and operated shared transportation system that can offer fixed routes and schedules, as well as flexible routes and on-demand scheduling. Volinski (2019) does not limit microtransit to the private sector, defining microtransit as any shared public or private sector transportation services that offer fixed or dynamically allocated routes and schedules in response to individual or aggregate consumer demand, using smaller vehicles and capitalizing on widespread mobile GPS and internet connectivity. The key is responsiveness to demand.

For most US agencies demand-responsive transport means paratransit. As a provision of the Americans with Disabilities Act (ADA), public transportation agencies that operate fixed-route service must offer complementary ADA paratransit available everywhere within three-quarters of a mile of each fixed-route line. Because the mandate is unfunded and paratransit trips are always more costly to provide, flexible-route services are viewed as a resource drain on the prime directive of providing fixed-route service (Mulley et al., 2012).

As such, we examine flexible transit service provision across a spectrum from fully customized “on-demand microtransit” to “semi-flexible transit”. On-demand microtransit provides door-to-door or stop-to-stop service to users without a fixed route. With semi-flexible route transit, on the hand, there remains a fixed route with some stops, known as checkpoints, along with optional stops called “virtual bus stops” to allow the service to vary to some degree. Rosenbloom (1996) surveyed 40 transit agencies to find that most of them adopted flexible-route transit to remove or reduce the need to provide mandated complementary paratransit service. Koffman (2004) categorizes the semi-flexible transit service into six main types (based on flexible operating policies implemented by different transit agencies throughout North America). An overview of the semi-flexible service classification and methodological issues are presented by Errico et al. (2013).

A survey by Potts et al. (2010) reports the percentage of North American transit agencies adopting these semi-flexible systems. Among these, route deviation is the most common form of system (63.9%), whereas zone route accounts for 32.9%, request stops 30.9%, Demand-responsive connector service (DRC) 30.5%, flexible-route segments 19.5% and point deviation 16%. In addition to these, many other systems (sharing similar aspects with DRC) were explored by researchers such as check point dial-a-ride transit (Daganzo, 1984a), Feeder Transit Services (Quadrifoglio and Li, 2009), High Coverage Point-to-Point Transit System (Cortés and Jayakrishnan, 2002), Mobility Allowance Shuttle Transit (MAST) (Quadrifoglio et al., 2007), and more recently, demand adaptive paired line hybrid transit (Chen and Nie, 2017). Qiu et al. (2014) presented a flexible-route service design corresponding to “point deviation” in Koffman (2004).

According to the National Transit Database, DRT is the main provider of service in rural and sparsely populated areas (FTA, 2018). DRT is the second largest transit service type in the US
with 27,000 vehicles operating during peak service and $55M vehicle-revenue-hours. Studies found that flexible transit services show promise towards improving travel patterns in low demand areas and revealed the willingness of passengers to use such services (Yu et al., 2017; Frei et al., 2017).

2.3. On-demand microtransit

Beyond fixed-route transit and semi-flexible transit, the third category of service covered is on-demand microtransit. This is the most flexible service type that is typically most operationally costly but also can reduce user costs when user demand is sufficiently low density.

On-demand microtransit started as offline DRT to solve DARP. The first DARP models were proposed by Wilson (1967), although structural properties of tours on a plane were already being studied by the likes of Beardwood et al. (1959). The optimization of a service problem like DARP has been found to be computationally intractable (NP-hard) because it is a generalized case of a traveling salesman problem (TSP) which is also NP-hard (Papadimitriou and Steiglitz, 1977). As a result, evaluation of different routing-based service designs is often done using continuous approximation models for scalability.

Stein (1978) proved that the length of an optimal tour to serve \( n \) passengers approaches a certain bound as \( n \to \infty \). When several buses are available, the length of the tour is simply divided by the number of buses. Daganzo et al. (1977) derived a many-to-one DRT system while Daganzo (1978) examined a many-to-many DRT system under different operating policies. The latter is studied as a queueing network.

A generalization of DARP, called Integrated Dial-a-Ride (IDARP), was developed by Häll et al. (2009). The main difference between the IDARP and DARP is that the users in IDARP may change mode at transfer points and then travel a specified distance with public transport (fixed-route service). In this case, when the passenger is carried to a transfer node via dial-a-ride service, it uses the fixed-route service to travel to another transfer node where the passenger is picked up by another vehicle in the fleet of dial-a-ride vehicles and is dropped off at the respective destination. By using the existing fixed-route service, the DRT operators can reduce their operating costs. The IDARP shares similarity in many aspects with the pickup and delivery problem with transshipments and the DARP with transfers (Cortes et al., 2010; Rais et al., 2014; Masson et al., 2013). The formulation is based on a directed graph formulation of the DARP (Cordeau, 2006) with an expansion that schedules both vehicle and customer itineraries.

Real-time, or dynamic/online, routing has a long history from the late 1970s (Psaraftis, 1980; Psaraftis, 1995; Madsen et al., 1995; Agatz et al., 2011; Hosni et al., 2014). Real-time operating policies are divided into myopic and non-myopic policies: a non-myopic policy considers cumulative costs over a time horizon by using lookahead or other types of approximation.

One example variant involves constraining the service to consider hubs and single transfers. Cortés and Jayakrishnan (2002) proposed a real time routing service called “High Coverage Point to Point Transit” (HCPPT) where each transit hub is designed for a group/cluster of such cells. The design strictly eliminates more than one transfer for any passenger and significantly decreases waiting time. This work has been expanded upon by Jung and Jayakrishnan (2011). Another variant uses queueing theory to approximate the lookahead costs for making routing decisions, as exemplified by Pavone et al. (2010) (queueing network), Hyyttiä et al. (2012) (online DARP), Sayarshad and Chow (2015, 2017) (online DARP with queue tolling, and relocation).

The use of real time routing with queueing and connection with public transit network as a main line was proposed by Ma et al. (2019a), essentially an online operation of IDARP. The
algorithm was tested for travel demand in Long Island to compare the costs of operating a microtransit service versus an online IDARP service. Due to complexity of these systems, Djavadian and Chow (2017a,b) proposed an agent-based simulation to evaluate the market equilibrium of a broad range of flexible transit services through dynamics of day-to-day adjustment.

Another important development to on-demand microtransit is the consideration of “meeting points” or virtual bus stops. Instead of picking up and dropping passengers off at their stated locations, the system would consider assigning them to common meeting points that may be a few blocks away. Stiglic et al. (2015) showed that such a system can improve matching rate and lead to mileage savings.

A subset of research is conducted on feeder systems to address the “last mile problem” in transit (see Chow and Djavadian, 2015). Chang and Schonfeld (1991a,b) first compared the relative advantages of fixed-route and DRT systems as feeder services and identified a similar demand density threshold under which DRT is more cost effective. They showed that for smaller service areas, higher express speeds, lower in-vehicle times, or higher access and wait times, flexible bus system becomes more advantageous.

In 2018 alone at least 24 agencies debuted microtransit pilots, but this considerable interest still begs the question of what role these services play and how they meet the mission of transit agencies (Lazo, 2018; Schaller, 2018). In most transit agencies, the worst performing fixed-route bus lines bottom out at around 10 passenger trips per vehicle hour (Walker, 2018b). Meanwhile, the best purely demand-response systems at best achieve 4 pickups per hour. Those which exceed that level to reach up to 7 or 8 do so by basing the service on a flex route pattern or anchoring one end of the service at a transit hub.

The interaction between shared on-demand mobility services and high-capacity public transit plays an important role. In recent years, the International Transport Forum (ITF) at the Organization for Economic Co-operation and Development (OECD) has conducted several simulation studies to investigate the impact that the shared on-demand mobility services would have on replacing other forms of transport such as traffic congestion, air pollution etc. (OECD/ITF, 2015, 2016, 2017a,b,c).

2.4. Summary

Simulation-based evaluation of transit designs do exist in the literature. For example, MATSim has been used to evaluate transit designs or as part of a simulation-based transit optimization process (Kaddoura et al., 2015; Nnene et al., 2019; Manser et al., 2020; Chow et al., 2020; Ma and Chow, 2021). They have also been used in evaluating demand-responsive transit (Cich et al., 2017) and MaaS environment (Becker et al., 2020). Another recent study looked at simulations to evaluate MOD systems (Markov et al., 2021). However, these types of multi-agent simulations are designed for comprehensive travel demand modeling that captures dynamic traffic, agent activity scheduling and mode/route choice behavior. On the other hand, they do not capture detailed transit system design variables and do not have any semi-flexible route service considerations.

Strategic planning of fixed-route transit systems significantly affects the accessibility of transit services while tactical planning and operational control focuses on adjusting system elements without significantly impacting service demand. Since the purpose of a simulation sandbox is to compare users’ responses to different system operation types, it focuses on strategic planning alternatives and excludes timetabling and vehicle scheduling. Assuming that the route
structure is kept unchanged, it adjusts the number of stops and service frequency to optimize the total system cost.

Semi-flexible transit service should share some common attributes with fixed-route service. The simulation sandbox implements a design for semi-flexible transit service among the variety of examples reviewed. The modeled service should be distinguished from other system classes because of its intermediate position between those. Considering this, the MAST system proposed by Quadrifoglio is chosen as the most appropriate design due to its intermediate structure between fixed-route system and on-demand microtransit, locating some fixed stops along the base route and allowing some flexibility of route deviation within the service area.

Third, studies have investigated different aspects of on-demand microtransit, improving the system performance from the perspective of either user’s side, operators’, or both. However, some advances are hardly applicable simultaneously because it not only complicates the problems to solve but also creates conflicts between objectives. Instead, the proposed simulation sandbox incorporates a basic on-demand microtransit service without developed extensions to provide clearer comparison results among different service types, leaving the potential to attach extensions as “modules.” Based on these ideas, the direction and detailed design of the proposed sandbox are illustrated in the following section.

3. Proposed simulation sandbox design

A simulation sandbox is developed in this study to compare three alternative transit operation service system designs.

3.1. Simulation parameters

The simulation sandbox should be able to accommodate different system parameters and environmental factors and allow for consistent comparison of the three operating service system designs: a fixed-route line, a semi-flexible route service line (which we will call “flexible-route” service for simplicity), and on-demand microtransit. Parameters allow the modeler to fit the sandbox to different operations. These parameters are divided into three groups: simulation parameters that impact the simulation mechanism like time steps, system design variables like fleet size and vehicle capacity, and those reflecting operating conditions specific to the study area like passenger arrival rates, vehicle speeds, and passenger values of time.

3.1.1. Simulation parameters

The simulation is designed as a “discrete-time simulation” with a simulation length divided over discrete time steps. Each time step, vehicle states are updated according to the operating plan. For fixed-route systems, the state is simply the vehicle location and passenger assignment. For flexible-route systems, the state also includes whether a vehicle deviates to serve a virtual stop. For on-demand microtransit, the state includes the sequence of passengers being served. Passenger states are also updated: waiting to be assigned to a vehicle, walking to a stop, waiting for vehicle, on-board a vehicle, egressing from vehicle stop to the destination. The simulation length and time step can be adjusted.

To prevent users from encountering the system without vehicles fully dispatched, the simulation sets warm up time \( t_{wu} \) to locate vehicles along the route. Fixed-route and flexible-route systems require \( t_{wu} \) at least twice longer than one-way cycle time to allow the first dispatched vehicle return to departed terminal, making sure vehicles be distributed. Otherwise, it will generate
false results caused by temporal discrepancy between vehicle and passenger. For instance, a passenger should experience excessively long wait time if appearing in the region much earlier than the first vehicle arrival.

3.1.2. Scenario parameters

A common geographical boundary determines the service region and affects the system coverage and demand level. The service region is set into a rectangular shape of length \( L \) and width \( W \), which can be mapped from any shape route and corresponding catchments. The demand data should be collected within a service region according to defined boundary. Passenger arrival rate \( \lambda \), the number of passengers per unit time, defines the level of travel demand within the service region. The sandbox should accept artificially generated inputs as well.

The simulation takes inputs for weights of passenger travel time elements — in-vehicle time \( \gamma_v \), wait time \( \gamma_w \), and access time \( \gamma_a \). By multiplying them to each time element and adding them up, systems can compare the total passenger cost of candidate routes considering the unique values of time for passengers in the study area. For access time, walking speed \( v_w \) is necessary to calculate the access time, and the system sets a maximum walking distance \( \zeta_a \) to cover the effective catchment of a route or service.

Although average vehicle running speed \( v_o \) can be a feature of vehicle specification, it also can indicate congestion level that operators cannot control. For example, \( v_o \) during peak hours should be lower than that in non-peak hours in urban areas.

3.1.3. System design variables

Vehicle specification affects the capacity of the system. Vehicle capacity \( K \) limits the maximum number of passengers onboard the vehicle at any time. Fleet size \( V \) refers to the number of available vehicles in a system. Higher \( v_o \) leads to faster passenger trips and lower vehicle relocation time.

The number of stops \( S \) in a fixed-route system and checkpoints \( S_c \) in a flexible-route system are key factors that affect the accessibility and mobility of systems. For example, larger \( S \) or \( S_c \) make shorter average walking distance from/to stops, but vehicles should stop more frequently resulting in longer dwell times and stopping delay for passengers. On the other hand, the number of depots \( S_d \) determines the initial vehicle location in on-demand microtransit system and impacts the repositioning costs of idle vehicles. Average dwell time \( t_d \) defines the time that vehicles stay at stops to provide passenger with sufficient time to board or alight.

Frequency of service \( f \) in fixed-route and flexible-route services affects headway \( h \) and the average wait time, key factors of both operators’ and users’ cost. Headway \( h \) is the inverse of \( f \) \( (h = 1/f) \). One-way cycle time \( t_c \) is the required time to finish a one-way trip between two terminals. Although it is a simple sum of vehicle running and stopping time in a fixed-route system, it needs additional time budgeted for deviations in a flexible-route system.

Lastly, maximum detour time rate \( \zeta_d \) in flexible-route and on-demand microtransit systems prevents overly long routes for a passenger caused by too many insertions between origin and destination. Maximum wait time \( \zeta_w \) is an upper bound on passenger wait time, ensuring passengers being served within the time threshold. Otherwise, systems reject pickup requests to let them find other modes (they leave the system as unserved). Maximum backtracking distance \( \zeta_b \) limits the distance traveled in the opposite direction by a flexible-route service, as described in Section 3.2.2. Maximum deviation distance defines the furthest distance from the checkpoint.
route that vehicles can reach in a flexible-route system but is usually equivalent to a half of service region width. Table 1 summarizes the classification of parameters.

| Table 1. Classified parameters. |
|---------------------------------|
| **Simulation parameters**       |
| Notation | Parameter | Notation | Parameter |
| -        | Simulation length | -        | Time step |
| $t_{wu}$ | Warm up time       | -        | -         |

| Scenario parameters            |
|--------------------------------|
| Notation | Parameter | Notation | Parameter |
| $L$      | Route length | $v_o$ | Average vehicle running speed |
| $W$ | Service area width | $y_v$ | Weight for passenger in-vehicle time |
| $\lambda$ | Passenger arrival rate (passenger/unit time) | $y_w$ | Weight for passenger wait time |
| $v_w$ | Walking speed | $y_a$ | Weight for passenger access time |
| $\zeta_o$ | Maximum walking distance | - | - |

| System design parameters       |
|--------------------------------|
| Notation | Parameter | Notation | Parameter |
| $K$ | Vehicle capacity | $f$ | Service frequency |
| $V$ | Fleet size | $t_c$ | One-way cycle time |
| $S$ | Number of stops | $\zeta_d$ | Max. detour time rate |
| $S_c$ | Number of checkpoints | $\zeta_w$ | Max. wait time |
| $S_d$ | Number of depots | $\zeta_b$ | Max. backtracking distance |
| $t_d$ | Average dwell time | - | - |

3.2. Transit operation system designs

This section delineates the three system designs, distinguishes the parameters specific to each design, and makes them adjustable in the simulation sandbox.

3.2.1. Fixed-route

Fixed-route operation considers a single transit line served between two terminals located on opposite ends of the rectangle with a distance of $L$. A fleet of buses departs from one terminal and head to the other, picking up and dropping off passengers at fixed stops along the fixed route.

For a fixed-route system, the modeler may choose to manually input the design parameters or let the simulation select the service frequency and stop spacing to minimize total cost. Total system cost consisting of users’ and operator’s cost is one of the most significant factors when considering the transit service performance measures (Desaulniers and Hickman, 2007). According to Tirachini (2014), estimating the total cost $C_t$ which consists of operator cost $C_o$ and user cost $C_u$ is available using parameters as shown in Eq. (1) – (4).

$$C_t = C_o + C_u$$  \hspace{1cm} (1)

$$C_o = cf t_c$$  \hspace{1cm} (2)

$$C_u = P_a \frac{L}{2y_w S} N + P_w \frac{1}{2f} N + P_v \frac{l}{L} t_c$$  \hspace{1cm} (3)
\[ t_c = \frac{L}{v_0} + \frac{\beta N}{f} + S t_s \]  

where \( c \) ($/bus-h) is a unit bus operating cost, \( f \) (bus/h) is bus frequency, \( t_c \) (h) is the bus cycle time, \( P_a \) ($/h) is the value of access time, \( L \) (mi) is the line length, \( v_w \) (mph) is the walking speed, \( S \) is number of stops, \( N \) (passenger/h) is passenger demand, \( P_w \) is value of waiting time, \( P_v \) is value of in-vehicle time, and \( l \) (mi) is average travel distance per passenger. Eq. (1) is the total cost. Eq. (2) determines the operator cost, where \( t_c \) can be derived from Eq. (4) and \( v_0 \) (mph) is bus operating speed, \( \beta \) (sec/passenger) is average boarding and alighting time per passenger, and \( t_s \) (h) is stopping delay. Eq. (3) is the user cost divided into three components: access time, waiting time, and in-vehicle time.

Reforming Eq. (1) as a function of \( S \) and \( f \) will lead to a nonlinear relationship between total cost and the two variables. As an alternative, considering \( S \in [S_{\min}, S_{\max}] \) and \( f \in [f_{\min}, f_{\max}] \) as discrete factors, enumerated combinations \((S, f)\) are input to the Eq. (1) to find the optimal \((S^*, f^*)\) with the lowest \( C_t \). Once the optimization is externally conducted, its result can be manually input to the simulation sandbox as a different configuration.

Consequently, the simulation sandbox can consider different manually input system designs for public transit. For example, one can use a preset configuration for an existing route to evaluate its performance in the sandbox, or conversely they can determine \((S^*, f^*)\) from the optimization of the total system cost via discretized enumeration and input them as the design variables.

### 3.2.2. Flexible-route

Its operation includes some relaxations of such operational constraints as route structure, stops, or timetables and is closer to a door-to-door service. The proposed simulation sandbox adopts and extends the MAST model from Quadrifoglio et al. (2007). This system design keeps the main skeleton of a fixed route and requires vehicles to drop by a subset of stops called “checkpoints.” A vehicle can deviate from a baseline connecting checkpoints and pick up or drop off passengers at dynamically generated stops other than checkpoints.

To accommodate deviations, a timetable is built for them considering some “slack time” in addition to normal vehicle operation time. For example, if a vehicle can run between two stops in 20 minutes, a timetable can allow an additional 10 minutes and indicate the travel time difference as 30 minutes. In this case, 10 minutes are assigned to a slack time to be used for deviating the vehicle to serve users arriving at one of the virtual bus stops during its run.

The other distinctive aspect of this system is a threshold of backtracking. Since a vehicle is permitted to deviate a route at its discretion, it can also reverse direction to pick up a passenger who suddenly shows up. However, when the distance of going back becomes excessively long, passengers who are onboard or waiting for vehicle may experience delays, leading to a worse aggregated user cost. Limiting the total length of backtracking can prevent these side effects while not rejecting all trips that require routing vehicles in the opposite direction. Maximum backtracking distance indicates the upper bound of the sum of backtracking distance in any section between sequential checkpoints.

This study extends the MAST model by including access by walking. In the original version (Quadrifoglio et al., 2007), a request from a passenger is matched with an available vehicle with sufficient slack time, and its pickup and drop-off points are inserted to the route of the vehicle if both can meet within the maximum wait time. In contrast, the modified version allows passengers...
to walk to nearby temporary vehicle stops or deviated vehicle routes if they can be served within the predetermined threshold. If users cannot receive the service, they abandon the given system and search for other travel alternatives, showing different behaviors from the fixed-route system. According to preliminary analyses with specific system configuration, the extension can process 23.6-88.0% more requests compared to the original version while imposing longer weighted travel time and total vehicle mile traveled (VMT). Table 2 summarizes major differences in performance measures.

| λ  | S_c | Total ridership | Avg. weighted travel time (min) | VMT (mile) |
|----|-----|-----------------|---------------------------------|------------|
|    |     | MAST | Extended | MAST | Extended | MAST | Extended |
| 80 | 10  | 167  | 314      | 61.94 | 70.97    | 343.36 | 339.69    |
|    | 20  | 254  | 314      | 61.87 | 69.46    | 328.00 | 391.08    |
| 200| 10  | 368  | 666      | 57.73 | 73.48    | 364.03 | 349.10    |
|    | 20  | 560  | 753      | 60.94 | 69.27    | 328.36 | 419.32    |
| 400| 10  | 696  | 1192     | 58.92 | 77.09    | 387.49 | 362.93    |
|    | 20  | 1127 | 1546     | 59.99 | 71.38    | 328.12 | 423.41    |

3.2.3. On-demand microtransit

On-demand microtransit service resembles current shared ride-hailing services, which dispatch vehicles and transport passengers from their origins to their destinations. The system does not order vehicles to comply fixed routes or drop by mandatory stops. Instead, when assigning a passenger to a vehicle the system also updates the vehicle state which includes the sequence of pickup and drop-off points of passengers assigned to it. These matches determine vehicles’ trajectories, passengers’ travel experiences, and system performances.

Due to the influence of passenger assignment, it is crucial to apply the most appropriate methodology to balance the quality of the output and the instantaneousness of the reaction. This type of problems can be categorized into a vehicle routing problem with pickup and delivery (VRPPD) which has been extensively applied to the transport of the disabled and elderly, sealift and airlift of cargo and troops, and pickup and delivery for overnight carriers or urban services (Desaulniers et al., 2002).

The proposed simulation sandbox implements a simple insertion heuristic to update assigned routes for accepted passengers by searching for an updated sequence with the shortest incremental increase in travel time. The prior assigned sequence is not reassigned with each new passenger. If a route consists of n stops, for example, there are n available places where a new stop can be located, including the one after the last stop. Since the system would insert a pair of origin and destination (OD) locations with the precedence constraint that the origin cannot be located after the destination, locating the origin at the i-th place will limit the number of available places for the destination to n + 1 – i. The total number of candidate routes after insertion is shown in Eq. (5). If either origin or destination is already included in a set of stops that the route covers, the number of cases being reviewed can be reduced.

\[
\sum_{i=1}^{n} (n + 1 - i) = \frac{n(n + 1)}{2} \quad (5)
\]

In addition to finding the route with the minimum routing cost increment, the study adds a function to the insertion heuristic that can verify the existence of passengers with unacceptable
trips. If an insertion causes a significant increase of trip time of existing passengers, that combination of OD locations should be avoided. Because the main components of trip time are wait time and in-vehicle travel time, the algorithm can apply two thresholds. While the maximum wait time is an upper limit of wait time acceptable to passengers, the maximum detour time rate is the highest ratio of expected in-vehicle time after an insertion was made. If a passenger should spend additional time on a route that exceeds the product of the maximum detour time rate and direct travel time or the required time between OD if they can be directly connected without any intermediate stops, the route is considered infeasible. These constraints prohibit the system from imposing excessive cost to some passengers in the form of excessive wait and in-vehicle time. Thus, if the algorithm succeeds to suggest a route for a vehicle, that should demand the lowest routing cost among available routes which satisfies these thresholds for all passengers including a newly accepted one.

3.2.4. Summary

Figure 2 describes the conceptual visualization of simulated transit operation policies, indicating differences in vehicle routing flexibility, number of stops to be mandatorily visited, and pattern of passengers (indicated by grey triangles) accessing each system design.

Figure 2. Concepts of simulated transit operation policies.

Figure 3 briefly contrasts different flows of transit operation types. Fixed-route system does not consider the passenger rejection because it has no interactions with passengers in terms of the service feasibility. It can be assumed that passengers remain in the system until served. However, the other two systems consider possible routes through the insertion heuristics with requested pickup and drop-off points and may reject a passenger if no feasible routes exist to serve them. The flexible-route transit design allows reevaluation of passengers after they walk over to a new virtual bus stop location.

Algorithms 1 and 2 describe flexible-route and on-demand microtransit policies step by step and are implemented as shown in Figure 3. For fixed-route service, there is no dynamic policy in the simulation sandbox that needs to be implemented with an algorithm.

Algorithm 1. Extended MAST insertion with passenger walking and multiple vehicles for flexible-route policy
Input: \( L, W, T, t_d, v_0, v_w, U, V, K, h, S_c, t_c, \zeta_a, \zeta_b, \zeta_w \)
Initialization: Locate $S_c$ checkpoints evenly distributed along fixed route and define the specification of vehicle $i$ where $i \in V$ ($K$, $v_o$, initial routes in both direction (from Checkpoint 1 to $S_c$ and vice versa), travel time ($t_r = \frac{k-1}{S_{c-1} v_o}$) and slack time ($t_{slack} = \frac{t_{r_i}}{S_{c-1}} - t_t - t_{d_i}$) between checkpoints, and dispatch time ($\tau_{di} = (i-1)h$)). $i_{max} = 0$.

For $\tau = 1, 2, \ldots, T$ do
1. Dispatch vehicle $i$ when $\tau = \tau_{di}$, $i_{max} = i_{max} + 1$.
2. If there are services requests from passenger $j \in U$, go to the next step. Otherwise, go to Step 13.

For $i = 1$ to $i_{max}$ do

[Direct service]
3. Recall information of $i$ including existing route $r_i$ according to the identified direction of passenger $j$ travelling.
4. Calculate expected wait and in-vehicle time of passengers assigned to $i$, $t_{slack}$ for each section between checkpoints, and performance measure.

For $k_1 = 1$ to $|r_i|$ do
For $k_2 = k_1 + 1$ to $|r_i| + 1$ do
5. Insert $O_j$ at the $k_1$-th sequence of $r_i$ and $D_j$ at the $k_2$-th place to create $r_i'$.
6. Investigate the feasibility (the violation of $K$, $t_{slack}$, $\zeta_a$). If feasible, calculate updated expected wait and in-vehicle time of passengers and performance measure. Otherwise, try the next set of $k_1$ and $k_2$.

Next

Next

7. Choose $r_i'$ with the minimum impact on performance measure as a feasible candidate route $r_{id}$. If there is no available route, $i$ cannot directly serve $j$.

[j-walking]
8. Identify segments $r_i$ which $j$ can access from $O_i$ and egress to $D_j$ within $\zeta_a$ and determine potential intersections $s_i$ where to approach (start/mid/end point).

For $k_3 = 1$ to $|s_i|$ do
9. Create $r_i'$ by inserting $k_3$-th intersection to $r_i$, and calculate the impact on expected wait and in-vehicle time of passengers and performance measure.
10. Investigate the feasibility (the violation of $K$, $t_{slack}$, $\zeta_a$). If feasible, calculate updated expected wait and in-vehicle time of passengers and performance measure. Otherwise, try the next $k_3$.

Next

11. Choose $r_i'$ with the minimum impact on performance measure as a feasible candidate route $r_{id}$. If there is no available route, $j$ cannot be assigned to $i$.

Next

12. Assign the passenger to either $r_{id}$ or $r_{iw}$ with the minimum impact of the performance measure. Update information of $i$ and $j$. If there is no available $i$, send $j$ to the rejected passenger set $U_r$.

[Resurrection of rejected passengers $j_r$]
13. If $|U_r| > 0$, repeat from Step 3 to 12 only for $j_r$ who reach every 30 sec after their rejection except for ones added in this time step.

[Vehicle relocation]
For $i = 1$ to $|V|$ do
14. If vehicle is at a stop, determine whether a vehicle stays more or leaves a stop based on remaining dwell time. If there are additional passenger in this time step, process them.
15. If vehicle is moving, determine whether a vehicle keeps moving or arrives at a stop based on remaining distance. When arriving, process passengers waiting for being picked up or dropped off.

Next
16. If time step does not reach simulation period, go back to Step 1. Otherwise, go to Step 1.

Next
17. Aggregate simulation outputs.
a) Fixed-route transit

- Initial system condition (vehicle/service region/stop)
- Locate vehicles and update status
- Move along route
  - Arrived at stop?
    - No
    - Stay at stop
    - Yes
    - Depart

- Boarding/alighting vehicle
  - Passenger can board?
    - No
    - Update vehicle load and passenger status
    - Yes
    - Depart
  - Passenger can alight?
    - No
    - Stay at stop
    - Yes
    - Depart

- Passenger access
  - Locate passengers
    - Current location
    - Arrived at stop?
      - No
      - Walk to stop
      - Yes
      - Vehicle arrived?
        - No
        - Wait for vehicle
        - Yes

- Passenger egress
  - Locate passengers
    - Passenger location, egress time
    - Arrived at destination?
      - No
      - Walk
      - Yes

b) Flexible-route transit

- Initial system condition (vehicle/service region/stop)
- Locate vehicles and update status
  - Vehicle location, route
  - Feasible route estimation
    - Candidate routes
      - A feasible route exists?
        - No
        - Conduct insertion heuristics
          - A feasible route exists?
            - No
            - Conduct insertion heuristics
              - A feasible route exists?
                - No
                - Assign passenger to vehicle with the most feasible route
                - Yes
                - Updated vehicle route
  - Move along route
    - Arrived at stop?
      - No
      - Slide to next candidate route
      - Yes
      - Depart

- Boarding/alighting vehicle
  - Passenger can board?
    - No
    - Update vehicle load and passenger status
      - Yes
      - Stay at stop
      - Dwell time left?
        - No
        - Depart
        - Yes
      - Dwell time left?
        - No
        - Depart
        - Yes

- Passenger access
  - Locate passengers
    - Current location
    - Arrived at target stop?
      - No
      - Wait for vehicle
      - Yes
      - Vehicle arrived?
        - No
        - Wait for vehicle
        - Yes

- Passenger egress
  - Locate passengers
    - Passenger location, egress time
    - Arrived at destination?
      - No
      - Walk
      - Yes

Legend:
- Simulated actions
- Related process

User
Operator
Both
c) On-demand microtransit

Figure 3. Flow charts of transit operation systems for (a) fixed-route transit, (b) flexible-route transit (see Algorithm 1), and (c) on-demand microtransit (see Algorithm 2).

Algorithm 2. Insertion heuristic for dispatching and routing on-demand vehicles

Input: $L, W, T, t_d, v_0, v_w, U, V, K, h, S_d, \zeta_w, \zeta_d$, vehicle distribution along depot $\mu_s$

Initialization: Locate $S_d$ depots evenly distributed along fixed route and define the specification of vehicle $i$ where $i \in V$ ($K, v_0$, empty routes). The number of vehicles per depot $n_s$ is defined by the discrete distribution $\mu_s$ where $\sum_s n_s = V$.

For $\tau = 1, 2, \ldots, T$ do

1. If there are services requests from passenger $j \in U$, go to the next step. Otherwise, go to Step 8.

[Passenger assignment]

For $i = 1$ to $i_{\text{max}}$ do

2. Recall information of $i$ including existing route $r_i$.

3. Calculate expected wait and in-vehicle time of passengers assigned to $i$ and performance measure.

For $k_1 = 1$ to $|r_i|$ do

4. Insert $O_j$ at the $k_1$-th sequence of $r_i$ and $D_j$ at the $k_2$-th place to create $r_i'$. 

5. Investigate the feasibility (the violation of $K$, $\zeta_d$, and $\zeta_w$). If feasible, calculate updated expected wait and in-vehicle time of passengers and performance measure. Otherwise, try the next set of $k_1$ and $k_2$.

Next

6. Choose $r_i'$ as a feasible candidate route $r_i^*$ with the minimum impact on performance measure. If there is no available route, $i$ cannot serve $j$.

Next

7. Assign the passenger to $r_i^*$ with the minimum impact of the performance measure and update information of $i$ and $j$. If there is no available $i$, reject $j$.

[Vehicle relocation]

For $i = 1$ to $|V|$ do

8. If vehicle is staying at a stop, determine whether a vehicle stays more or leaves a stop based on remaining dwell time. If there are additional passenger in this time step, process them.
9. If vehicle is moving, determine whether a vehicle keeps moving or arrives at a stop based on remaining distance. When arriving, process passengers waiting for being picked up or dropped off.

10. If time step does not reach simulation period, go back to Step 1. Otherwise, go to Step 1.

11. Aggregate simulation outputs.

3.3. Testing system designs and scenarios

The purpose of the sandbox is to compare different transit operational system designs (fixed-route, flex-route, on-demand, and their design parameters) under different scenarios. Moreover, the sandbox can repeat multiple simulations with different design parameters. Consequently, the sandbox should conduct $N_{total}$ simulations as shown in Eq. (6), where $N_{vs,i}$ is the number of design variable sets being tested for system type $i \in \{\text{fixed route}, \text{flex – route}, \text{on – demand}\}$ and $N_{dp}$ is the number of different scenarios.

$$N_{total} = N_{dp} \sum_i N_{vs,i} \quad (6)$$

4. Common data and open-source simulation case study

An open-source simulation was developed from scratch that builds in the three types of transit operations to allow a decision-maker to compare them on any study area desired. This tool is demonstrated on a common data set in Brooklyn, NYC as a case study.

4.1. Simulation background

The Metropolitan Transportation Authority (MTA) operates 64 bus routes in Brooklyn in NYC, including Route B63, which connects northwestern and southwestern part of Brooklyn as shown in Figure 4. It has a length of 13.12 km (Stringer, 2017), covering 57 stops with 11,148 average weekday daily ridership in 2018 (MTA, 2021). The significant amount of ridership, the 16th largest among 53 routes in Brooklyn, indicates that it still provides accessibility to neighborhoods. While this route is currently operated in a fixed-route policy, we incorporate its study area into the simulation sandbox to compare performance under different demand density levels for different operating system designs.

4.2. System design and scenarios

This case study simulates three scenarios based on different demand levels with five system designs as shown in Figure 5, where the existing fixed-route operation serves as one design and an optimized fixed-route design serves as another. The existing refers to the system observed in Figure 4.
Figure 4. B63 route with vehicle locations in MTA Bus Time.

Table 3 indicates values of design variables, collected from various sources. Most are from Stringer (2017), bus route profiles in NYC. Assumed values do not have citations. Unless otherwise noted, all system designs share the value.

MATLAB, a commercial computer programming language, is used for coding the simulation. The simulation assumes several conditions and parameters of which some are introduced previously. The simulation covers four hours, equivalent to 14,400 time steps of one second.

While the peak hour passenger demand is approximately 800 passenger/hr, the case study evaluates lower demand levels 10%, 25%, and 50% of peak hour demand to observe the change of performance as the demand level varies (which can reflect less busy off-peak periods). ODs and arrival time of artificial passengers are randomly generated and evenly distributed within the area, limiting ODs not to be connectable via walking. All scenarios use the identical passenger dataset for different demand levels to prevent potential discrepancies caused by the heterogeneity of demand.

4.3. Results

Figure 6 is a sample illustration of vehicle trajectories following each system design for the same amount of time in the case study. The fixed route passes through the middle of the area, a flexible route deviates from the fixed route if there is a non-regular point, and an on-demand
microtransit service vehicle freely moves within the area. From captured trajectories and data, the simulation can aggregate some performance measures of both vehicles and passengers, total ridership, average weighted travel time, and total VMT, as shown in Tables 4–6 and Figure 7–9.

Table 3. Values of parameters

| Simulation parameters | Notation | Parameter | Simulation length | 4 hrs (14,400 secs) | Time step | 1 sec |
|-----------------------|----------|-----------|-------------------|---------------------|-----------|-------|
| t<sub>wu</sub>        | 2t<sub>c</sub> |           |                   |                     |           |       |

| Scenario parameters | Notation | Parameter | Notation | Parameter |
|---------------------|----------|-----------|----------|-----------|
| L                   | 13.12 km (Stringer, 2017) | v<sub>0</sub> | 11.41 km/h (Stringer, 2017) |
| W                   | 1.6 km | γ<sub>v</sub> | 1 (Wardman, 2004) |
| λ                   | 80<sup>a</sup>, 200<sup>b</sup>, 400<sup>c</sup> passenger/hr | γ<sub>W</sub> | 1.59 (Wardman, 2004) |
| v<sub>W</sub>       | 5 km/h | γ<sub>d</sub> | 1.79 (Wardman, 2004) |
| ζ<sub>d</sub>       | 0.8 km (Zhao et al., 2003) | - | - |

| System design parameters | Notation | Parameter | Notation | Parameter |
|--------------------------|----------|-----------|----------|-----------|
| K                        | 85<sup>d1</sup><sup>d2</sup> (MTA, 2019)/ 40<sup>e1</sup><sup>e2</sup> / 20<sup>f</sup> | f | 5<sup>d1</sup><sup>e1</sup><sup>e2</sup> (MTA, 2020b), 1.5<sup>d2</sup><sup>a</sup>, 2.4<sup>d2</sup><sup>b</sup>, 3.6<sup>d2</sup><sup>c</sup> vehicle/h |
| V                        | 15<sup>d1</sup><sup>d2</sup> / 20<sup>e1</sup><sup>e2</sup> / 40<sup>f</sup> | t<sub>c</sub> | 88<sup>d1</sup><sup>d2</sup> (Stringer, 2017) / 120 min<sup>e1</sup><sup>e2</sup> |
| S                        | 57<sup>d1</sup><sup>d2</sup> / 36<sup>d2</sup><sup>c</sup> (Stringer, 2017), 30<sup>d2</sup><sup>a</sup>, 33 | ζ<sub>d</sub> | 2<sup>f</sup> |
| S<sub>c</sub>             | 10<sup>e1</sup> / 20<sup>e2</sup> | ζ<sub>W</sub> | 12<sup>e1</sup><sup>e2</sup> / 30 min<sup>f</sup> |
| S<sub>d</sub>             | 10<sup>f</sup> | ζ<sub>b</sub> | 0.4 km<sup>e</sup> |
| t<sub>d</sub>             | 20 sec | - | - |

Note: <sup>a</sup>: scenario with 80 passenger/hr, <sup>b</sup>: scenario with 200 passenger/hr, <sup>c</sup>: scenario with 400 passenger/hr, <sup>d1</sup>: reference fixed-route system, <sup>d2</sup>: optimized fixed-route system, <sup>e1</sup>: flexible-route system with 10 checkpoints, <sup>e2</sup>: flexible-route system with 20 checkpoints, <sup>f</sup>: on-demand microtransit. Otherwise, values applied to all cases.

Figure 6. Sample of simulated vehicle trajectories.
Table 4. Simulated total ridership.

| $\lambda$ (pax/h) | Fixed (Existing) | Optimized Fixed | Flexible ($S_c=20$) | Flexible ($S_c=10$) | Microtransit |
|------------------|------------------|-----------------|---------------------|---------------------|--------------|
| 80               | 333              | 331             | 314                 | 314                 | 318          |
| 200              | 791              | 791             | 753                 | 666                 | 586          |
| 400              | 1625             | 1622            | 1546                | 1192                | 766          |

Table 5. Simulated average weighted travel time (min).

| $\lambda$ (pax/h) | Fixed (Existing) | Optimized Fixed | Flexible ($S_c=20$) | Flexible ($S_c=10$) | Microtransit |
|------------------|------------------|-----------------|---------------------|---------------------|--------------|
| 80               | 58.86            | 72.22           | 69.46               | 70.97               | 47.61        |
| 200              | 58.63            | 69.19           | 69.27               | 73.48               | 54.40        |
| 400              | 58.78            | 63.93           | 71.38               | 77.09               | 53.49        |

Table 6. Simulated total vehicle mileage (mi).

| $\lambda$ (pax/h) | Fixed (Existing) | Optimized Fixed | Flexible ($S_c=20$) | Flexible ($S_c=10$) | Microtransit |
|------------------|------------------|-----------------|---------------------|---------------------|--------------|
| 80               | 335.72           | 137.73          | 339.69              | 391.08              | 772.10       |
| 200              | 335.72           | 160.71          | 349.10              | 419.32              | 1005.31      |
| 400              | 335.72           | 228.80          | 362.93              | 423.41              | 1072.54      |

Ridership increases as demand increases for all scenarios, but increments vary. While fixed-route scenarios achieve the ridership proportional to prevailing demand, others experience some deficits, meaning that their performances are less efficient than the fixed-route designs. For flexible-route designs, fewer checkpoints may lead to lower ridership as a smaller portion of demand can reach checkpoints and vehicles require more slack time to reach more non-regular points. On-demand microtransit system only serves nearly half of the ridership of fixed-route service leaving the other half unserved. The main reason of this number drop can be the complete connection between actual origin and destination of users eliminating all access and egress trips covered by users’ feet.

Average weighted travel time is the shortest with on-demand microtransit service as it excludes passenger walking of which penalty is the highest. It also rejects user requests with estimated wait time longer than $\zeta_w$ and in-vehicle time exceeding $\zeta_d$. In flexible-route systems, the compliance of timetable with slack time can extend both wait and in-vehicle time, resulting in longer travel time than others. Numbers in on-demand microtransit and flexible-route transit increase according to the demand level because more insertions of pickup and drop-off points are made along vehicle routes. For fixed-route system, user-perceived travel time remain the same due to the unchanged number of stops and frequency keeping wait time and in-vehicle time constant. Meanwhile, the optimization of $S$ and $f$ imposes more travel cost to users since it tends to save operator’s cost when demand level is low. Decreasing weighted travel time along the demand indicates "economy of scale" in fixed-route service.

Total VMT with on-demand microtransit service is the longest among scenarios, 2.3 to 3.2 times longer than the reference fixed-route service. This may indicate the trade-off between passenger travel time and VMT and. For fixed-route system design, the optimization seems to help reduce vehicle operation significantly, which causes longer travel time for passengers due to fewer stops and less frequency. Flexible-route designs yield slightly higher VMT than fixed-route designs. The influence of demand level is significant only in on-demand microtransit service while others maintain the similar level.
5. Conclusion

This study provides a new simulation sandbox for transit operations planning considering the range of fixed-route through semi-flexible transit to on-demand microtransit. The sandbox simulates different classes of transit operation to evaluate their performances under different design variables and scenarios. For the fixed-route service, manually input design variables are allowed as well as having the sandbox optimize frequency and stop spacing. For the semi-flexible route transit operation, a new extension of the MAST design from Quadrifoglio et al. (2007) is implemented to allow passengers to reposition to another location so that they can be fit into the deviation budget of the vehicle, which increases the acceptance rate. To the best of our knowledge, such a sandbox has not been developed in the literature.
A case study is presented to demonstrate this tool using a common dataset derived from the B63 bus route in Brooklyn, NYC. Three performance measures – total ridership, average weighted travel time, and total VMT – vary along different system designs and scenarios. First, fixed-route systems show the largest total ridership while others serve fewer passengers under the same demand pattern. Although the main reason can be the availability of rejection in flexible-route and on-demand microtransit service, it also indicates how efficient the system designs in the case study are. Second, on-demand microtransit service can provide the shortest average weighted travel time among system designs despite the decreasing gap as demand level elevates. This is common in systems with any flexibility. However, optimized fixed-route systems can reduce it by providing higher frequency when facing higher demand. Lastly, total VMT is the longest in on-demand microtransit system due to the exclusion of access/egress walks of users. The measure of optimized fixed-route system increases as it operates higher frequency. Other system designs maintain the similar level of VMT regardless of demand level.

This simulation sandbox is a starting point for other efforts. A next step is to expand the simulation tool to consider multimodal trips within a MaaS setting and to evaluate the interactions between operators. Electric vehicle charging requirements can be added. The simulator can be applied en-masse to all transit lines in the U.S. to output performance metrics throughout so that relationships can be established between different local built environments, their regulatory and institutional settings, and investment levels with performance metrics under different types of operating policies.

**Availability of data and materials**
The datasets generated and/or analyzed in the study are available in:
- Simulation sandbox code: [https://github.com/BUILTNYU/FTA_TransitSystems](https://github.com/BUILTNYU/FTA_TransitSystems)
- Generated Brooklyn case study data set: [https://doi.org/10.5281/zenodo.3672151](https://doi.org/10.5281/zenodo.3672151)

**Funding**
The authors were supported by an FTA grant NY-2019-069-01-00 and the C2SMART University Transportation Center (USDOT #69A3551747124).
Acknowledgements
Help from NYU MS student Patrick Scalise in preparing the literature review and NYU Abu Dhabi student Sara Alanis Saenz in preparing the case study data are appreciated. Professor Quadrifoglio shared the insertion heuristic code for his MAST algorithm with us which is much appreciated.

References
Agatz, N. A., Erera, A. L., Savelsbergh, M. W., & Wang, X. (2011). Dynamic ride-sharing: A simulation study in metro Atlanta. Transportation Research Part B, 9(45), 1450-1464.
Bartholdi III, J. J., & Eisenstein, D. D. (2012). A self-coordinating bus route to resist bus bunching. Transportation Research Part B: Methodological, 46(4), 481-491.
Beardwood, J., Halton, J. H., & Hammersley, J. M. (1959). The shortest path through many points. In Mathematical Proceedings of the Cambridge Philosophical Society (Vol. 55, No. 4, pp. 299-327). Cambridge University Press.
Becker, H., Balac, M., Ciarì, F., & Axhausen, K. W. (2020). Assessing the welfare impacts of shared mobility and Mobility as a Service (MaaS). Transportation Research Part A 131, 228-243.
Berrebi, S. J., Hans, E., Chiabaut, N., Laval, J. A., Leclercq, L., & Watkins, K. E. (2018). Comparing bus holding methods with and without real-time predictions. Transportation Research Part C: Emerging Technologies, 87, 197-211.
Bookbinder, J. H., & Désilets, A. (1992). Transfer optimization in a transit network. Transportation Science, 26(2), 106-118.
Byrne, B. F. (1975). Public transportation line positions and headways for minimum user and system cost in a radial case. Transportation Research, 9(2-3), 97-102.
Cats, O., & Haverkamp, J. (2018). Optimal infrastructure capacity of automated on-demand rail-bound transit systems. Transportation Research Part B: Methodological, 117, 378-392.
Ceder, A. (1987). Methods for creating bus timetables. Transportation Research Part A: General, 21(1), 59-83.
Ceder, A. (2016). Public transit planning and operation: Modeling, practice and behavior. 2nd Ed. CRC press.
Ceder, A., & Wilson, N. H. (1986). Bus network design. Transportation Research Part B: Methodological, 20(4), 331-344.
Ceder, A., Golany, B., & Tal, O. (2001). Creating bus timetables with maximal synchronization. Transportation Research Part A: Policy and Practice, 35(10), 913-928.
Chang, S. K., & Schonfeld, P. M. (1991a). Optimization models for comparing conventional and subscription bus feeder services. Transportation Science, 25(4), 281-298.
Chang, S. K., & Schonfeld, P. M. (1991b). Integration of fixed-and flexible-route bus systems. Transportation Research Record, (1308), 51-57.
Chen, P. W., & Nie, Y. M. (2017). Analysis of an idealized system of demand adaptive paired-line hybrid transit. Transportation Research Part B: Methodological, 102, 38-54.
Chen, J., Liu, Z., Wang, S., & Chen, X. (2018). Continuum approximation modeling of transit network design considering local route service and short-turn strategy. Transportation Research Part E: Logistics and Transportation Review, 119, 165-188.
Chow, J. Y. J. (2018). Informed Urban Transport Systems: Classic and Emerging Mobility Methods toward Smart Cities. Elsevier.
Chow, J. Y. J., & Djavadian, S. (2015). Activity-based market equilibrium for capacitated multimodal transport systems. Transportation Research Part C: Emerging Technologies, 59, 2-18.
Chow, J. Y. J., Ozbay, K., He, B. Y., Zhou, J., Ma, Z., Lee, M., Wang, D., & Sha, D. (2020). Multi-agent simulation-based virtual test bed ecosystem: MATSim-NYC. C2SMART Final Report.
Cich, G., Knapen, L., Maciejewski, M., Bellemans, T., & Janssens, D. (2017). Modeling demand responsive transport using SARL and MATSim. Procedia Computer Science, 109, 1074-1079.
Cordeau, J. F. (2006). A branch-and-cut algorithm for the dial-a-ride problem. Operations Research, 54(3), 573-586.
Cordeau, J. F., & Laporte, G. (2003). The dial-a-ride problem (DARP): Variants, modeling issues and algorithms. Quarterly Journal of the Belgian, French and Italian Operations Research Societies, 1(2), 89-101.
Cortés, C. E., & Jayakrishnan, R. (2002). Design and operational concepts of high-coverage point-to-point transit system. Transportation Research Record, 1783(1), 178-187.
Daganzo, C. F. (1978). An approximate analytic model of many-to-many demand responsive transportation systems. *Transportation Research, 12*(5), 325-333.

Daganzo, C. F. (1984a). Checkpoint dial-a-ride systems. *Transportation Research Part B: Methodological, 18*(4-5), 315-327.

Daganzo, C. F., Hendrickson, C. T., & Wilson, N. H. M. (1977). An approximate analytic model of many-to-one demand responsive transportation systems. Proc. ISTTTT, 743-772.

Daganzo, C. F., & Ouyang, Y. (2019). A general model of demand-responsive transportation services: From taxi to ridesharing to dial-a-ride. *Transportation Research Part B: Methodological, 126*, 213-224.

Desaulniers, G., Hickman, M.D. (2007). Chapter 2 Public Transit, in: Handbooks in Operations Research and Management Science. Elsevier, pp. 69–127.

Djavadian, S. and Chow, J. Y. J. (2017a). An agent-based day-to-day adjustment process for modeling ‘Mobility as a Service’ with a two-sided flexible transport market. *Transportation Research Part B: Methodological, 104*, pp.36-57.

Djavadian, S. and Chow, J.Y.J. (2017b). Agent-based day-to-day adjustment process to evaluate dynamic flexible transport service policies. *Transportmetrica B: Transport Dynamics 5*(3), 281-306.

Errico, F., Crainic, T. G., Malucelli, F., & Nonato, M. (2013). A survey on planning semi-flexible transit systems: Methodological issues and a unifying framework. *Transportation Research Part C: Emerging Technologies, 36*, 324-338.

Farahani, R. Z., Miandoabchi, E., Szeto, W. Y., & Rashidi, H. (2013). A review of urban transportation network design problems. *European Journal of Operational Research, 229*(2), 281-302.

Fielbaum, A., Jara-Díaz, S., & Gschwender, A. (2016). Optimal public transport networks for a general urban structure. *Transportation Research Part B: Methodological, 94*, 298-313.

Fielbaum, A., Jara-Díaz, S., & Gschwender, A. (2017). A parametric description of cities for the normative analysis of transport systems. *Networks and Spatial Economics, 17*(2), 343-365.

Frei, C., Hyland, M., & Mahmassani, H. S. (2017). Flexing service schedules: Assessing the potential for demand-adaptive hybrid transit via a stated preference approach. *Transportation Research Part C: Emerging Technologies, 76*, 71-89.

FTA (2018). 2017 National Transit Summary and Trends. Federal Transit Administration, Washington, D.C., https://www.transit.dot.gov/ntd/2017-national-transit-summaries-and-trends-ntst

GAO (2018). Public transit partnerships: Additional information needed to clarify data reporting and share best practices. GAO-18-539, U.S. Government Accountability Office.

Gendreau, M., & Potvin, J. Y. (2005). Metaheuristics in combinatorial optimization. *Annals of Operations Research, 140*(1), 189-213.

Guihaire, V., & Hao, J. K. (2008). Transit network design and scheduling: A global review. *Transportation Research Part A: Policy and Practice, 42*(10), 1251-1273.

Häll, C.H., Andersson, H., Lundgren, J.T. and Värbrand, P. (2009). The integrated dial-a-ride problem. *Public Transport, 1*(1), 39-54.

Hasselström, D. (1982). Public transportation planning: A mathematical programming approach. PhD dissertation, University of Göteborg, Sweden.

Hazen, J., Lang, N., Wegscheider A.K., Fassenot, B. (2019). On-demand transit can unlock urban mobility. BCG Henderson Institute and Boston Consulting Group.

Hershler, D. A. (2017). Future bus transport contracts under a mobility as a service (MaaS) regime in the digital age: Are they likely to change?. *Transportation Research Part A: Policy and Practice, 98*, 86-96.

Hörcher, D., & Tirachini, A. (2021). A review of public transport economics. *Economics of Transportation, 25*, 100-196.

Hosni, H., Naoum-Sawaya, J., & Artail, H. (2014). The shared-taxi problem: Formulation and solution methods. *Transportation Research Part B: Methodological, 70*, 303-318.

Hyytiä, E., Penttinen, A., & Sulonen, R. (2012). Non-myopic vehicle and route selection in dynamic DARP with travel time and workload objectives. *Computers & Operations Research, 39*(12), 3021-3030.

Iliopoulos, C., Kepaptsoglou, K., & Vlahogianni, E. (2019). Metaheuristics for the transit route network design problem: a review and comparative analysis. *Public Transport, 11*(3), 487-521.

Jara-Díaz, S. R., & Basso, L. J. (2003). Transport cost functions, network expansion and economies of scope. *Transportation Research Part E: Logistics and Transportation Review, 39*(4), 271-288.

Jung, J., & Jayakrishnan, R. (2011). High-coverage point-to-point transit: study of path-based vehicle routing through multiple hubs. *Transportation Research Record, 2218*(1), 78-87.
Kaddoura, I., Kickhöfer, B., Neumann, A., & Tirachini, A. (2015). Agent-based optimisation of public transport supply and pricing: impacts of activity scheduling decisions and simulation randomness. *Transportation*, 42(6), 1039-1061.

Koffman, D. (2004). TCRP Synthesis 53: Operational Experiences with Flexible Transit Services. Federal Transit Administration.

Korosec, K. (2019a). Ford is shutting down its Chariot shuttle service. *TechCrunch*, January 10th.

Lazo, L. (2018). For public transit agencies losing riders, microtransit might be an answer. *Washington Post*, February 3.

Ma, T. Y., Rasulkhani, S., Chow, J. Y. J., & Klein, S. (2019a). A dynamic ridesharing dispatch and idle vehicle repositioning strategy with integrated transit transfers. *Transportation Research Part E: Logistics and Transportation Review*, 128, 417-442.

Ma, Z., Chow, J. Y. J. (2021). Transit Network Design with Multi-Agent Simulation to Capture Activity-Based Mode Competition. Working paper.

Madsen, O. B., Ravn, H. F., & Rygaard, J. M. (1995). A heuristic algorithm for a dial-a-ride problem with time windows, multiple capacities, and multiple objectives. *Annals of Operations Research*, 60(1), 193-208.

Manser, P., Becker, H., Hörl, S., & Axhausen, K.W. (2020). Designing a large-scale public transport network using agent-based microsimulation. *Transportation Research Part A*, 137, 1-15.

Markov, I., Guglielmetti, R., Laumanns, M., Fernández-Antolín, A. & de Souza, R. (2021). Simulation-based design and analysis of on-demand mobility services. *Transportation Research Part A*, 149, 170-205.

Masson, R., Lehuédé, F., & Péton, O. (2013). An adaptive large neighborhood search for the pickup and delivery problem with transfers. *Transportation Science*, 47(3), 344-355.

Mohring, H. (1972). Optimization and scale economies in urban bus transportation. *The American Economic Review*, 62(4), 591-604.

MTA (2019) ‘MTA announces bus service enhancements, including increased use of longer articulated buses to add capacity’. [http://www.mta.info/press-release/nyc-transit/mta-announces-bus-service-enhancements-including-increased-use-longer](http://www.mta.info/press-release/nyc-transit/mta-announces-bus-service-enhancements-including-increased-use-longer). Accessed on May 25th, 2021.

MTA (2021). 2019 Bus Ridership Tables (New York City Transit). [https://new.mta.info/document/16141](https://new.mta.info/document/16141) (accessed on Apr 28, 2021).

Mulley, C., Nelson, J., Teal, R., Wright, S., & Daniels, R. (2012). Barriers to implementing flexible transport services: An international comparison of the experiences in Australia, Europe and USA. *Research in Transportation Business & Management*, 3, 3-11.

Newell, G. F. (1971). Dispatching policies for a transportation route. *Transportation Science*, 5(1), 91-105.

Newell, G. F. (1979). Some issues relating to the optimal design of bus routes. *Transportation Science*, 13(1), 20-35.

Nnene, O.A., Joubert, J.W., & Zuidgeest, M. (2021). A dynamic ridesharing dispatch and idle vehicle repositioning strategy with integrated transit transfers. *Transportation Research Part E: Logistics and Transportation Review*, 149, 170-205.

OECD/ITF (2015), Urban Mobility System Upgrade: How shared self-driving cars could change the city traffic, International Transport Forum, Paris, [www.itf-oecd.org](http://www.itf-oecd.org).

OECD/ITF (2016), Shared Mobility: Innovation for Liveable Cities, International Transport Forum, Paris, [www.itf-oecd.org](http://www.itf-oecd.org).

OECD/ITF (2017a), Shared Mobility: Simulations for Auckland, International Transport Forum, Paris, [www.itf-oecd.org](http://www.itf-oecd.org).

OECD/ITF (2017b), Transition to Shared Mobility – How large cities can deliver inclusive transport services, International Transport Forum, Paris, [www.itf-oecd.org](http://www.itf-oecd.org).

OECD/ITF (2017c), Shared Mobility - Simulations for Helsinki, International Transport Forum, Paris, [www.itf-oecd.org](http://www.itf-oecd.org).

Papadimitriou, C. H., & Steiglitz, K. (1977). On the complexity of local search for the traveling salesman problem. *SIAM Journal on Computing*, 6(1), 76-83.

Pavone, M., Frazzoli, E., & Bullo, F. (2010). Adaptive and distributed algorithms for vehicle routing in a stochastic and dynamic environment. *IEEE Transactions on Automatic Control*, 56(6), 1259-1274.

Potts, J. F., Marshall, M. A., Crockett, E. C., & Washington, J. (2010). A guide for planning and operating flexible public transportation services. (No. 140). TCRP Report. Transportation Research Board, Washington, D.C.

Psaraftis, H. N. (1980). A dynamic programming solution to the single vehicle many-to-many immediate request dial-a-ride problem. *Transportation Science*, 14(2), 130-154.

Psaraftis, H. N. (1995). Dynamic vehicle routing: Status and prospects. *Annals of operations research*, 61(1), 143-164.
Qiu, F., Li, W., & Zhang, J. (2014). A dynamic station strategy to improve the performance of flex-route transit services. Transportation Research Part C: Emerging Technologies, 48, 229-240.

Quadrifiglio, L., Dessouky, M. M., & Palmer, K. (2007). An insertion heuristic for scheduling mobility allowance shuttle transit (MAST) services. Journal of Scheduling, 10(1), 25-40.

Quadrifiglio, L., & Li, X. (2009). A methodology to derive the critical demand density for designing and operating feeder transit services. Transportation Research Part B: Methodological, 43(10), 922-935.

Rais, A., Alvelos, F., & Carvalho, M. S. (2014). New mixed integer-programming model for the pickup-and-delivery problem with transshipment. European Journal of Operational Research, 235(3), 530-539.

Rosenbloom, S. (1996). Service routes, route deviation, and general public paratransit in urban, suburban, and rural transit systems (No. FTA-AZ-26-7000-96-1).

Sayarshad, H. R., & Chow, J. Y. J. (2015). A scalable non-myopic dynamic dial-a-ride and pricing problem. Transportation Research Part B: Methodological, 81, 539-554.

Sayarshad, H. R., & Chow, J. Y. J. (2017). Non-myopic relocation of idle mobility-on-demand vehicles as a dynamic location-allocation-queueing problem. Transportation Research Part E: Logistics and Transportation Review, 106, 60-77.

Schaller, B. (2018). The New Automobility: Lyft, Uber and the Future of American Cities.

Schöbel, A., Scholl, S. (2006). Line Planning with Minimal Traveling Time. Presented at the 5th Workshop on Algorithmic Methods and Models for Optimization of Railways (ATMOS’05), p.16.

Shaheen, S., Chan, N., Bansal, A., & Cohen, A. (2015). Shared Mobility - Definitions, Industry Developments, and Early Understanding. Transportation Sustainability Research Center.

Shaheen, S., Cohen, A., & Zohdy, I. (2016a). Shared Mobility Current Practices and Guiding Principles.

Stein, D. M. (1978). An asymptotic, probabilistic analysis of a routing problem. Mathematics of Operations Research, 3(2), 89-101.

Stiglic, M., Agatz, N., Savelsbergh, M., & Gradisar, M. (2015). The benefits of meeting points in ride-sharing systems. Transportation Research Part B: Methodological, 82, 36-53.

Systan (1980). Paratransit Handbook: A guide to paratransit system implementation. Final Report, Volume 1. UMTA-MA-06-0054-79-1. Prepared for U.S. Department of Transportation.

Tirachini, A. (2014). The economics and engineering of bus stops: Spacing, design and congestion. Transportation Research Part A: Policy and Practice, 59, 37-57.

TRAVIC (2020). Transit Visualization Client. https://tracker.geops.de/?z=12&s=1&x=-8229222.3000&y=4971106.4519&l=transport, last accessed January 23, 2020.

UN (2017). World population projected to reach 9.8 billion in 2050, and 11.2 billion in 2100. United Nations, June 21st.

UN (2018). 68% of the world population projected to live in urban areas by 2050, says UN. United Nations, May 16th.

van Nes, R., Hamerslag, R., & Immers, L. H. (1988). The design of public transport networks. Transportation Research Record 1202, 74-83.

Via (2019), Via on-demand transit system, URL https://ridewithvia.com/ (accessed 10.24.2019).

Via (2018). Getting Microtransit Right, https://www.intelligenttransport.com/transport-whitepapers/68280/whitepaper-getting-microtransit-right/.

Volinski, J. (2019). TCRP Synthesis 141: Microtransit or General Public Demand–Response Transit Services: State of the Practice. Federal Transit Administration.

Vuchic, V.R. (1981). Urban public transportation: systems and technology. Prentice-Hall, Englewood Cliffs, N.J.

Vuchic, V. R., & Newell, G. F. (1968). Rapid transit interstation spacings for minimum travel time. Transportation Science, 2(4), 303-339.

Walker, J. (2018b). Is Microtransit a Sensible Transit Investment? Hum. Transit. URL https://humantransit.org/2018/02/is-microtransit-a-sensible-transit-investment.html (accessed 5.17.19).

Wardman, M. (2004). Public transport values of time. Transport policy, 11(4), 363-377.

WEF (2019). Shared, electric and automated mobility (SEAM) governance framework: prototype for North America and Europe. World Economic Forum White Paper, July 19th, https://www.weforum.org/whitepapers/shared-electric-and-automated-mobility-seam-governance-framework-prototype-for-north-america-and-europe.

Wilson, N. H. (1967). Computer Aided Routing System (Doctoral dissertation, Massachusetts Institute of Technology, Department of Civil Engineering).

Wilson, N.H., Sussman, J., Wong, H.-K., Higonnet, T. (1971). Scheduling algorithms for a dial-a-ride system. Massachusetts Institute of Technology. Urban Systems Laboratory.

Wirasinghe, S. C., & Ghoneim, N. S. (1981). Spacing of bus-stops for many to many travel demand. Transportation Science, 15(3), 210-221.
Wong, Y. Z., Hensher, D. A., & Mulley, C. (2020). Mobility as a service (MaaS): Charting a future context. *Transportation Research Part A: Policy and Practice.*

Woodward, C., Vaccaro, A., Gans, F. (2017). Bridj, local on-demand bus service, is shutting down. *Boston Globe,* April 30th.

Yap, M., Luo, D., Cats, O., van Oort, N., & Hoogendoorn, S. (2019). Where shall we sync? Clustering passenger flows to identify urban public transport hubs and their key synchronization priorities. *Transportation Research Part C: Emerging Technologies,* 98, 433-448.

Yu, J., Lu, X., Pan, S., & Guo, C. (2017). Traveler willingness to use flexible transit services in China: Case study of Qilu Software Park. *Journal of Urban Planning and Development,* 143(2), 05016018.

Zhao, F., Chow, L. F., Li, M. T., Ubaka, I., & Gan, A. (2003). Forecasting transit walk accessibility: Regression model alternative to buffer method. *Transportation Research Record,* 1835(1), 34-41.