Nonexistence of time-reversibility in statistical physics

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Abstract: Contrary to the customary thought prevailing for long, the time reversibility associated with beam-to-beam collisions does not really exist. Related facts and consequences are presented. The discussion, though involving simple mathematics and physics only, is well-related to the foundation of statistical theory.

1 Introduction

More than one hundred years ago, the debate concerning time reversibility arose in a confusing way: Boltzmann derived his kinetic equation from the time reversibility of mechanics while the equation itself was of time irreversibility[1]. Even today, though a long time has passed and countless papers in the literature have revealed a variety of aspects of the issue, paradoxical things still bother some of us[2].

Here, it will be shown that the real problem of Boltzmann’s theory is related not to the time irreversibility assumed by it, but to the time reversibility assumed by it. To make the topic more intriguing and more profound, the investigation will manifest that any attempt to formulate the ‘true distribution function’ will fail in the ultimate sense.

2 Particle-to-particle and beam-to-beam collisions

Before entering the detailed discussion, it is essential to establish distinction between particle-to-particle collisions and beam-to-beam collisions.

A particle-to-particle collision involves two individual particles. The time reversal symmetry of it has been fully elucidated in classical mechanics and it says that if the collision expressed by \((v_1, v_2) \rightarrow (v'_1, v'_2)\) is physically possible, where \(v_1, v_2\) are respectively the velocities of the two particles
before the collision and \( \mathbf{v}'_1, \mathbf{v}'_2 \) after the collision, then the inverse collision expressed by \((-\mathbf{v}'_1, -\mathbf{v}'_2) \rightarrow (-\mathbf{v}_1, -\mathbf{v}_2)\) is also physically possible. This kind of time reversibility is not truly relevant to the subject herein and we shall not discuss it too much in this paper.

Beam-to-beam collisions, involving particle densities or distribution functions, are of great significance to statistical mechanics. For instance, Boltzmann’s theory treats \( f(\mathbf{v})d\mathbf{v} \) as a beam and derives its formulation on the premise that certain types of time-reversibility are there.

However, we happen to realize that no time-reversibility of any form can be defined in the context of Boltzmann’s theory. This conclusion is surprising, seems very imprudent and directly contradicts what has been embedded in our mind. In view of such strong resistance, it is felt that a very clear and very detailed discussion should be given. In this section the subject will be studied intuitively and in the next two sections mathematical investigations will be presented.

![Figure 1](image)

**Figure 1:** A candidate for time reversibility of beam-to-beam collision: (a) the original collisions; and (b) the inverse collisions.

Take a look at Fig. 1. Fig. 1a shows a process in which two beams with two definite velocities collide and the produced particles diverge in space. Fig. 1b illustrates the inverse process, in which converging beams collide and the produced particles form two definite beams. In no need of detailed discussion, we surely know that the first process makes sense in statistical mechanics while the second one does not.
3 No time-reversibility in terms of cross sections

In the textbook treatment, the time reversibility concerning beam-to-beam collisions is expressed in terms of cross sections[1]:

\[ \sigma(v_1, v_2 \rightarrow v'_1, v'_2) = \sigma(v'_1, v'_2 \rightarrow v_1, v_2), \]  

(1)

where \( \sigma(v_1, v_2 \rightarrow v'_1, v'_2) \) is defined in such a way that

\[ N = \sigma(v_1, v_2 \rightarrow v'_1, v'_2)dv'_1dv'_2 \]  

(2)

represents the number of type-1 particles emerging, after collisions, between \( v'_1 \) and \( v'_1 + dv'_1 \) per unit incident flux and unit time, while the type-2 particle emerges between \( v'_2 \) and \( v'_2 + dv'_2 \).

\[ \sigma(v_1, v_2 \rightarrow v'_1, v'_2) = \sigma(v'_1, v'_2 \rightarrow v_1, v_2), \]  

Figure 2: Constraints imposed by the energy and momentum conservation laws.

Unfortunately, the cross section in (1) and (2) is ill-defined. Notice that the energy and momentum conservation laws state that (assuming every particle to have the same mass for simplicity)

\[ c = c' \quad \text{and} \quad |u| = |u'| \equiv u, \]  

(3)

where \( 2c = v_1 + v_2, 2c' = v'_1 + v'_2, 2u = v_2 - v_1 \) and \( 2u' = v'_2 - v'_1 \). Fig. 2a shows how \( v_1 \) and \( v_2 \) determine \( c \) and \( u \), while Fig. 2b shows how \( c \) and \( u \equiv |u| \) form four constraint constants on \( v'_1 \) and \( v'_2 \). Notably, \( v'_1 \), as well as \( v'_2 \), falls on the spherical shell \( S \) of radius \( u \) in the velocity space, which will be called the accessible shell. With these constraints in mind, two
problems associated with the definition (2) will surface by themselves. The first is that, after $d\mathbf{v}'_1$ is specified, specifying $d\mathbf{v}'_2$ in the definition is a work overdone (since $\mathbf{v}'_1$ and $\mathbf{v}'_2$ are not independent of each other). The second is that the cross section should be defined in reference to surface elements rather than to volume elements.

To see the second problem aforementioned more vividly, let’s consider $d\mathbf{v}'_1$ shown in Fig. 3a, which is cube-shaped with equal sides $l$. If we let $\rho$ denote the area density of particles on the accessible shell caused by unit flux of type-1 particles, the number of type-1 particles found in $d\mathbf{v}'_1$ can be expressed as $N \approx \rho l^2$. Then, the cross section defined by (2) is equal to, with $d\mathbf{v}'_2$ omitted,

$$\sigma = \frac{N}{d\mathbf{v}'_1} = \frac{\rho l^2}{l^3} = \frac{\rho}{l}, \quad (4)$$

which depends on $l$ and tends to infinity as the cube becomes smaller and smaller. Nevertheless, if $d\mathbf{v}'_1$ is chosen to be a slim box in Fig. 3b and the box becomes slimmer and slimmer, then $\sigma$ tends to zero; if $d\mathbf{v}'_1$ is chosen to be a short box in Fig. 3c and the box becomes shorter and shorter, $\sigma$ tends to infinity again. These representative examples inform us that the cross section defined by (2), and thus the time reversibility expressed by (1), does not mean anything.

![Diagram](image)

Figure 3: In defining the cross section, the velocity volume elements $d\mathbf{v}'_1$ may take on different shapes.

As a matter of fact, the above problem can be examined more briefly. For any definite velocities $\mathbf{v}_1$ and $\mathbf{v}_2$, the six components of $\mathbf{v}'_1$ and $\mathbf{v}'_2$ are under four constraint equations imposed by the energy-momentum conservation laws. This literally means that the free space of $\mathbf{v}'_1$ and $\mathbf{v}'_2$ is just two-dimensional and there are no enough degrees of freedom allowing us to define a cross section in reference to six-dimensional volume elements.
4 No time-reversibility in terms of velocity volumes

Another form of time reversibility is simultaneously employed in such textbooks[1]:

\[ dv_1 dv_2 = dv'_1 dv'_2. \]  \hspace{1cm} (5)

It should be noted that there is a conceptual conflict between (1) and (5).
In connection with (1), when incident particles have two definite velocities
the velocities of scattered particles are allowed to distribute over almost
the entire velocity space; whereas, in connection with (5), an infinitesimal
velocity range of incident particles strictly corresponds to another infinitesimal
velocity range of scattered particles. Nevertheless, for purposes of this paper,
we shall leave this conflict alone.

The mathematical proof of (5) goes as follows. First,

\[ dv_1 dv_2 = \|J\| dc du \quad \text{with} \quad \|J\| = \left\| \frac{\partial (v_1, v_2)}{\partial (c, u)} \right\|. \] \hspace{1cm} (6)

Then,

\[ dv'_1 dv'_2 = \|J'\| dc' du' \quad \text{with} \quad \|J'\| = \left\| \frac{\partial (v'_1, v'_2)}{\partial (c', u')} \right\|. \] \hspace{1cm} (7)

In view of that

\[ \|J\| = \|J'\|, \quad c \equiv c' \quad du = du', \] \hspace{1cm} (8)

we obtain (5).

Unfortunately again, the formulation given above also involves errors. All
equations in (6), (7) and (8) hold except \( du = du' \). Referring to Fig. 2, we
find that when \( u \) is a definite vector, \( u' \) distributes over a spherical shell,
pointing in any direction. This means that if \( du = u^2 dud\Omega_u \) is an infinitely
thin line-shaped volume element (say, \( d\Omega_u \) is infinitesimal while \( du \) finite),
the corresponding volume element \( du' \) will be a spherical shell with finite
thickness. It is then obvious that the two elements are not equal in volume.

The issue can be analyzed more economically in terms of variable trans-
formation. If we identify \( v_{1x}, v_{1y}, v_{1z} \) and \( v_{2x}, v_{2y}, v_{2z} \) as six variables and
identify \( v'_{1x}, v'_{1y}, v'_{1z} \) and \( v'_{2x}, v'_{2y}, v'_{2z} \) as six new variables, then we have

\[ dv_1 dv_2 = \|\tilde{J}\| dv'_1 dv'_2. \] \hspace{1cm} (9)
What has been proven by equations (6), (7) and (8) is nothing but $\|\hat{J}\| = 1$. However, in order for (9) to make sense, there must exist six independent equations connecting those variables. In our case, we have four equations only. That is to say, the ‘variable transformation’ is incomplete and expression (9) is not truly legitimate.

It is instructive to look at the issues discussed in the last and this sections in a unifying way. If there were no single constraint equation, defining the cross section could make sense; if there were six independent constraint equations, defining the Jacobian would be meaningful. Since there are four and only four constraint equations, neither the cross section nor the Jacobian can be defined.

5 Formulation of beam-to-beam collisions

It is now rather clear that the concepts and methodologies of Boltzmann’s theory are in need of reconsideration.

For instance, one of the major steps in deriving Boltzmann’s equation is to identify $f(v_1')dv_1'$ and $f(v_2')dv_2'$ as two definite beams and then to determine how many beam-1 particles will emerge between $v_1$ and $v_1 + dv_1$ due to collisions of the two beams. As has been shown, this context leads to nothing but an absurd result: the number of such emerging particles actually depends on the size and shape of $dv_1$, varying drastically from zero to infinity.

In what follows, we shall propose a new context to do the job. Surprisingly, the formulation will reveal some of deep-rooted properties of statistical mechanics.

To involve less details, we adopt the following assumptions: (i) The zeroth-order, collisionless, distribution function of the gas is completely known. (ii) Each particles, though belonging to the same species, is still distinguishable (which is possible in terms of classical mechanics). (iii) No particle collides twice or more. (General treatments can be accomplished along this line[3].)

Referring to Fig. 4, we consider that two typical beams, denoted by $f_1^{(0)}(v_1')dv_1'$ and $f_2^{(0)}(v_2')dv_2'$, collide with each other, and suppose that a particle detector has been placed somewhere in the region. Let $\Delta S$ be the entry area of the detector and $\Delta N_1$ be the number of the beam-1 particles entering the detector within the velocity range $\Delta v_1 v_1^2 \Delta \Omega_1$ during $dt$. Since
any beam of the system can be regarded as the first beam, or the second beam, aforementioned, the total distribution function due to collisions is, at the detector entry,

\[ f_1^{(1)}(t, r, v_1, \Delta S, \Delta v_1, \Delta \Omega_1) \approx \frac{\sum_1 \sum_2 \Delta N_1}{(\Delta S v_1 dt)(\Delta v_1^2 \Delta \Omega_1)}, \quad (10) \]

in which \( r \) is the representative position of \( \Delta S \) and \( v_1 \) is the representative velocity of \( \Delta v_1^2 \Delta \Omega_1 \). According to the customary thought, when \( \Delta S, \Delta v_1 \) and \( \Delta \Omega_1 \) shrink to zero simultaneously, expression (10) stands for the ‘true distribution function’ there; for reasons to be clear a bit later, we shall, in the following formulation, assume that \( \Delta S \) and \( \Delta v_1 \) are infinitely small while \( \Delta \Omega_1 \) is kept fixed and finite (though rather small). The spatial region \( -\Delta \Omega_1 \), that has been shaded in Fig. 4 and ‘opposite’ to the velocity solid-angle range \( \Delta \Omega_1 \) in (10), will be called the effective cone. It is intuitively obvious that the particles that collide somewhere in the effective cone and move, after the collision, toward the detector entry along their free trajectories will contribute to \( \Delta N_1 \). (Even if such particles are allowed to collide again, some of them will still arrive at the detector freely, which means the concept of effective cone holds its significance rather generally.)

![Diagram](Figure 4: Two beams collide and a particle detector is placed in the region.)

Observing the colliding beams in the center-of-mass reference frame, we find that the number of collisions in a volume element \( dr' \), which is located inside the effective cone \( -\Delta \Omega_1 \), can be represented by, as in Boltzmann’s theory,

\[ [dr' f_1^{(0)}(v'_1)dv'_1][f_2^{(0)}(v'_2)dv'_2][2u\sigma_c(u', u) d\Omega_c dt], \quad (11) \]

where \( \Omega_c \) is the solid angle between \( u' \) and \( u \), and \( \sigma_c(u', u) \) is the cross section in the center-of-mass frame. By integrating (27) over the effective cone and
taking account of all the particles that are registered by the detector, the right side of (10) becomes

\[
\int_{-\Delta \Omega_1} dr' \int_{\Delta v_1 \Delta \Omega_1} dv_1 \int d\Omega_c \int d\Omega_c' \int d\Omega_c'' \frac{2u_2 \sigma_c(u', u)f_1^{(0)}(v_1')f_2^{(0)}(v_2')}{(|r - r'|^2 \Delta \Omega_0 v_1)(v_1^2 \Delta v_1 \Delta \Omega_1)},
\] (12)

where \(\Delta \Omega_0\) is the solid-angle range formed by a representative point in \(dr'\) (as the apex) and the detector entry area \(\Delta S\) (as the base). In view of that \(\Delta S\) is truly small and \(\Delta \Omega_1\) is fixed and finite by our assumption, we know that \(\Delta \Omega_0 \ll \Delta \Omega_1\) and every particle starting its free journey from the effective cone and entering the detector can be treated as one emerging within \(\Delta \Omega_1\). Then, we can rewrite expression (12) as, with help of the variable transformation from \((v_1', v_2')\) to \((c', u')\),

\[
\int_{-\Delta \Omega_1} dr' \int_{\Delta v_1 \Delta \Omega_0} d\Omega_c \int d\Omega_c' \int d\Omega_c'' \frac{2u_2 \sigma_c(u', u)f_1^{(0)}(c' - u')f_2^{(0)}(c' + u')}{(|r - r'|^2 \Delta \Omega_0 v_1)(v_1^2 \Delta v_1 \Delta \Omega_1)},
\] (13)

in which \(\|\vec{J}\| \equiv \partial(v_1', v_2')/\partial(c', u')\) and the subindex \(\Delta v_1 \Delta \Omega_1\) has been replaced by \(\Delta v_1 \Delta \Omega_0\). In view of the energy-momentum conservation laws, we arrive at

\[
\int_{-\Delta \Omega_1} dr' \int d\Omega_c' \int d\Omega_u' \int_{\Delta v_1 \Delta \Omega_0} d\Omega_c \int du \
\cdot \|\vec{J}\| \frac{2u_2 \sigma_c(u', u)f_1^{(0)}(c' - u')f_2^{(0)}(c' + u')}{(|r - r'|^2 \Delta \Omega_0 v_1)(v_1^2 \Delta v_1 \Delta \Omega_1)},
\] (14)

\[\text{Figure 5: The relation between the velocity element } v_1^2 \Delta v_1 \Delta \Omega_0 \text{ and the velocity element } u^2 du d\Omega_c.\]
By examining the situation shown in Fig. 5, which is drawn in the velocity space, the following relation can be found out:

\[
\int_{\Delta v_1} u^2 dud\Omega \cdots \approx v_1^2 \Delta v_1 \Delta\Omega_0 \cdots \quad (15)
\]

Therefore, the distribution function due to collisions is equal to

\[
f^{(1)}_1(t, r, v_1, \Delta\Omega_1) = \frac{1}{v_1 \Delta\Omega_1} \int_{-\Delta\Omega_1} dr' \int dc' \int d\Omega_{u'} \frac{2||\tilde{J}||\sigma_c(u', u)f^{(0)}_1(c' - u')f^{(0)}_2(c' + u')}{u|r - r'|^2},
\]

(16)
in which \(u\) is determined by \(u = c - v_1\) (\(v_1\) is in principle along the direction of \(r - r'\)) and \(u'\) by \(u\) and \(\Omega_{u'}\). It should be noted that \(f^{(1)}_1\) in (16) differs from that in (10) in the sense that \(\Delta S\) and \(\Delta v_1\) cease to be arguments and \(v_1\) takes the place of \(v_1\).

If the zeroth-order distribution functions depend on time and space, the replacement

\[
f^{(0)}_1(c' - u')f^{(0)}_2(c' + u') \rightarrow f^{(0)}_1(t', r', c' - u')f^{(0)}_2(t', r', c' + u')
\]

(17)
needs to be taken, where \(t' = t - |r - r'|/v_1\) stands for the time delay.

Since \(\Delta\Omega_1\) has been set finite, expression (16) is nothing but the distribution function averaged over \(\Delta\Omega_1\), which is not good enough according to the standard theory. At first glance, if we let \(\Delta\Omega_1\) become smaller and smaller, expression (16) will finally represent the true distribution function there. However, the discussion after (12) has shown that if \(\Delta\Omega_1\) shrinks to zero the definition of the effective cone will collapse and thus the entire formalism will no longer be valid.

A careful inspection tells us that, if we assumed \(\Delta S\) to be finite and \(\Delta\Omega_1\) to be infinitesimal at the starting point, a different average distribution function, averaged over \(\Delta S\), would be obtained. The fact that no distribution function can be determined if \(\Delta\Omega_1\) and \(\Delta S\) simultaneously take on their infinitesimal values suggests that even in classical statistical mechanics there also exists an uncertainty principle. The connection between this uncertainty principle and the uncertainty principle in quantum mechanics remains to be seen.
In view of that the integration in (16) is carried out over an effective region defined by free trajectories of particles, this methodology has been named as a path-integral approach[3]. As (16) has partially shown, anything taking place in the effective region can make direct impact along ‘free’ paths and any macroscopic structures, continuous or not, will help shape microscopic structures elsewhere along ‘free’ paths. Concepts like the aforementioned, though appearing to be foreign from the viewpoint of differential approach, are in harmony with what takes place in realistic gases.

6 Summary

It has been shown that, when we concern ourselves with beam-to-beam collisions, part of the system’s information has, knowingly or not, been disregarded, and the information loss is characterized by the fact that we have four and only four constraint equations (rather than six). Since statistical mechanics deals with beam-to-beam collisions, the time-reversibility related to each individual collision becomes irrelevant from the very beginning and will not reemerge at any later stage.

By proposing a new context in which what can be really measured in an experiment is of central interest, beam-to-beam collisions have been reformulated. The new formulation reveals that only distribution functions averaged over certain finite ranges make good physical sense.

Apparently, this paper raises many difficult questions related to the very foundation of statistical physics. Reference papers can be found in the regular and e-print literature[2, 3, 4].

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Appendix A. An alternative cross section

As revealed in Sect. 3, the original cross section $\sigma(v_1, v_2 \rightarrow v'_1, v'_2)$ defined by (2) cannot be interpreted as a function of the usual kind due to the existence of the energy-momentum conservation laws. However, one may still wish to
define cross sections in which the time reversibility related to a collision of two particles plays a certain role. Interestingly, this can be done and doing so will help us to find out what kind of problems Boltzmann’s equation really has.

If we adopt the definition of (2) and, at the same time, take the energy-momentum conservation laws into account, we are led to a cross section of delta-function type

$$\sigma(v_1, v_2 \to v'_1, v'_2) = \eta(v_1, v_2 \to v'_1, v'_2) \cdot \delta \left( \sqrt{(v'_1)^2 + (v'_2)^2} - \sqrt{(v_1)^2 + (v_2)^2} \right) \cdot \delta^3 [(v'_1 + v'_2) - (v_1 + v_2)].$$

(18)

To work out the physical meaning of $\eta$, we integrate the above expression over a finite but small volume element $\Delta v'_1 \Delta v'_2$

$$\int_{\Delta v'_1 \Delta v'_2} \eta(v_1, v_2 \to v'_1, v'_2) \cdot \delta \left( \sqrt{(v'_1)^2 + (v'_2)^2} - \sqrt{(v_1)^2 + (v_2)^2} \right) \cdot \delta^3 [(v'_1 + v'_2) - (v_1 + v_2)] d v'_1 d v'_2.$$

(19)

As shown in Sect. 3, these delta-functions define an accessible shell, denoted by $S$ there, and the integration becomes

$$\eta(v_1, v_2 \to v'_1, v'_2) \Delta S,$$

(20)

where $\Delta S$ is enclosed by $\Delta v'_1 \Delta v'_2$ (actually by one of $\Delta v'_1$ and $\Delta v'_2$ since the two are not independent of each other). By comparing this with the cross section defined in the center-of-mass system $\sigma_c(u, u') d \Omega$, it is clear that

$$\eta(v_1, v_2 \to v'_1, v'_2) = \sigma_c(u, u')/u^2.$$

(21)

The above formalism has illustrated that the introduced delta-functions make good sense as long as they are integrated over an adequate volume (finite or infinitely large). Similarly,

$$\sigma(v'_1, v'_2 \to v_1, v_2) = \eta(v'_1, v'_2 \to v_1, v_2) \cdot \delta \left( \sqrt{(v_1)^2 + (v_2)^2} - \sqrt{(v'_1)^2 + (v'_2)^2} \right) \cdot \delta^3 [(v_1 + v_2) - (v'_1 + v'_2)].$$

(22)

With help of (21), we see that

$$\eta(v_1, v_2 \to v'_1, v'_2) = \eta(v'_1, v'_2 \to v_1, v_2).$$

(23)
This reflects the fact that all collisions, including the original collision and
the inverse collision, are of head-on type in the center-of-mass system.

It is now in order to find out whether or not the newly defined cross section
is relevant to Boltzmann’s equation. We first examine the particles leaving
d\textbf{r}d\textbf{v}_1 during \textit{dt} because of collisions. Following the textbook treatment, the
number of collisions is represented by

\[ [2uf(v_1)d\textbf{v}_1][f(v_2)d\textbf{r}d\textbf{v}_2]\sigma(v_1, v_2 \rightarrow v'_1, v'_2)dt. \]  \hspace{1cm} (24)

Integrating it over \textit{v}'_1, \textit{v}'_2 and \textit{v}_2 yields, by virtue of (18), (20) and (21),

\[ dt\text{d}\textbf{r}\text{d}\textbf{v}_1 \int 2uf(v_1)f(v_2)\sigma_c(u, u')d\textbf{v}_2d\Omega. \]  \hspace{1cm} (25)

Dividing it by \textit{dt}\text{d}\textbf{r}\text{d}\textbf{v}_1, we obtain the collision number per unit time and
unit phase volume

\[ \int 2uf(v_1)f(v_2)\sigma_c(u, u')d\textbf{v}_2d\Omega. \]  \hspace{1cm} (26)

If we adopt the assumption that the above number is identical to the number
of particles leaving the unit phase volume per unit time because of collisions
(though a different conclusion is offered in Ref. 2), we find that the above
derivation is entirely consistent with that in the standard approach.

Then, we examine the particles entering \textit{d}\textbf{r}\Delta\textbf{v}_1 during \textit{dt} because of col-
lisions (\Delta\textbf{v}_1 has been set finite for a reason that will be clarified). To make
our discussion a bit simpler, it is assumed that there are only two incident beams

\[ f(v'_1)\Delta v'_1 \text{ and } f(v'_2)\Delta v'_2. \]  \hspace{1cm} (27)

Again, following the standard approach, we know that the collision number
caused by the two beams within \textit{d}\textbf{r} during \textit{dt} is

\[ [2uf(v'_1)\Delta v'_1][f(v'_2)d\textbf{r}\Delta v'_2]\sigma(v'_1, v'_2 \rightarrow v_1, v_2)dt. \]  \hspace{1cm} (28)

At this point, a sharp question arises. Can we identify these particles as those entering \textit{d}\textbf{r}\Delta\textbf{v}_1 during \textit{dt}? The answer is apparently a negative one. As one thing, only a small fraction of the scattering particles will enter \Delta\textbf{v}_1, and the following integration needs to be done:

\[ \int_{\Delta v_1 \Delta v_2} 2uf(v'_1)\Delta v'_1 \cdot f(v'_2)d\textbf{r}\Delta v'_2 \cdot \sigma(v'_1, v'_2 \rightarrow v_1, v_2)dt d\textbf{v}_1 d\textbf{v}_2. \]  \hspace{1cm} (29)
in which $\Delta v_1$ has to be finite, since the integrand contains delta-functions, while $\Delta v_2$ can be infinitely large. If we are interested in knowing the number of the entering particles per unit phase volume and unit time, we are supposed to evaluate

$$\lim_{dt dr \Delta v_1 \to 0} \frac{\Delta N}{dt dr \Delta v_1},$$

(30)

where $\Delta N$ is nothing but expression (29). Unfortunately, there are problems. As one thing, it has just been pointed out that $\Delta v_1$ has to be finite. As another, expression (4) and the accompanying discussion have shown that the limit expressed by (30) does not exist.

The above discussion, verifiable with numerical computation, suggests that (i) The methodology of determining particles entering a phase volume element must be very different from that of determining particles leaving a phase volume element, namely there is no symmetry. (ii) Taking limits $dt \to 0$, $dr \to 0$ and $dv_1 \to 0$, simultaneously or not, can create unexpected problems. (iii) In order to formulate the collisional dynamics, new concepts and new methodologies are in need.

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