Experimental and Theoretical Study on the Optimal Tilt Angle of Photovoltaic Panels

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Abstract
As a result of global warming people are paying more attention to the use of natural energy, such as solar power. As one of the main forms of solar energy, photovoltaic (PV) power generators have been developed rapidly in the past few years. Proper installation, especially the tilt angle, directly affects the system's output. Determination of the optimal tilt angle of a solar cell module depends on the solar radiation characteristics, season, and reflectivity in the local area.

This paper evaluates the performance of four small PV modules at different tilt angles, and analyzes the relationship of solar radiation power output with tilt angle by using actual measurement data at Kitakyushu city. A calculation method for optimal tilt angle is then presented using horizontal and diffuse radiation. The method has been verified through comparison with the experimental data. In addition, the sensitivity of optimal tilt angle to radiation rate, reflection rate, solar declination and latitude have been studied through parameter analysis.

Keywords: solar energy use; optimal tilt angle; theoretical calculation; parameter analysis

1. Introduction
The reduction of greenhouse gases is an important measure in the prevention of global warming. In order to prevent global warming, an important measure is to reduce the greenhouse gas emissions. Therefore, the search for a substitute for fossil energy sources has increased the interest in PV panels as a long-term, inexhaustible, environmentally friendly and reliable energy technology. The selection and proper installation of PV modules directly affects system output. It has been found that for every location on earth with specific radiation characteristics, there is an optimal tilt angle for the best solar energy reception. In Japan, solar radiation has been observed at more than 60 places by the Japan Meteorological Agency, and the expanded AMeDAS Weather Data can supply the calculated radiation data for 842 places in Japan. Some theoretical research has been carried out regarding the optimal tilt angle using these data [1,2,3, and 4]. However, little research has been done on experimental and parameter studies concerning the optimal tilt angle of PV panels.

This paper evaluates the performance of four small modules at different tilt angles and fixed orientation in order to define the optimal tilt angle for Kitakyushu.

Solar radiation data and PV power output have been measured in each season and their relationships with the tilt angle have been investigated. Based on the experimental data, the optimal tilt angle has been calculated. Furthermore, according to total solar radiation and diffused solar radiation, a theoretical calculation method of optimal tilt angle has been presented. The calculated values have been verified by comparison with the measurement values. In addition, influential factors regarding the optimal tilt angle of the PV module have been analyzed by using parameter analysis.

2. Experiment Outline
As shown in Fig.1., four PV modules facing towards the south with various tilt angles of 0°, 15°, 35° and 55° have been installed on the roof of the Building-
Shinkoukan at Kyushu Kyoritsu University. The PV power output and solar radiation data have been measured for each PV module. In addition, a measurement of level surface diffused radiation and ground reflected radiation has been performed. The size of each PV module is 0.4m×0.188m, with an area of about 0.0752m², and consists of 34 solar cells, with an area of 0.0333795m².

In this paper, the data used for analysis is from December 1, 2002 to November 31, 2003. On January 29, the radiation meters failed due to strong winds. Although the meters for the 0° and 15° PV modules were reactivated from February 1, those for 35° and 55° PV modules did not work until February 28 because of breakage. However, the 0° PV module was reset to 75° on the evening of February 27, and resumed operation on the evening of March 31. Furthermore, because of a power blackout, the measurement was stopped from March 29 to 31. From August 7 to 9 the measurement was again temporarily stopped due to a typhoon, and resumed from the 10th. From the above statements, it can be found that the measurement days add up to 348, of which 336 days are efficacious. The measurement was completely stopped for 12 days.

3. Experimental Results

3.1 Radiation characteristics

Fig.2. shows the total monthly solar radiation, as well as the direct and diffused solar radiation. As shown in this figure, the peak and the lowest periods of total solar radiation are May and December, with a value of 519.38 MJ/m² per month and 185.27 MJ/m² per month respectively, while the average value is 371.29 MJ/m² per month. From the figure, it can be found that the total summer solar irradiance is relatively low for the cool summer of 2003.

Fig.3. shows the radiation rate for the whole year. The radiation rate is defined as the ratio of daily total solar radiation at the ground to the total solar radiation in outer space, which illustrates the percentage of total outer space solar radiation arriving at the ground. Kishida[5] has used the radiation rate as an evaluation index for the prediction of weather. According to his research, the results can be summarized as follows: fine and clear weather (radiation rate> 0.6) for 69 days; fine weather with temporary cloud (0.59> radiation rate> 0.5) for 68 days; cloudy weather with temporary sunshine (0.49> radiation rate> 0.34) for 78 days; cloudy days with temporary rain (0.33> radiation rate > 0.18) for 60 days and rainy days with temporary cloud (0.17> radiation rate) for 64 days.

Fig.4. shows the relationship of radiation rate with the rate of direct and diffused solar radiation and illustrates that the direct and diffused solar radiations have an approximate linear relationship to the radiation rate, with a correlation coefficient of 0.9124. Knowing the radiation rate, direct and diffused radiation can be
predicted by the following expressions.

\[a_{dir} = 1.1876 a_{Irr} - 0.0395\]
\[a_{dif} = 1 - a_{dir}\]  

(1)

where \(a_{dir}\) is the value of direct radiation rate
\(a_{dif}\) is the value of diffused radiation rate
\(a_{Irr}\) is the value of radiation rate

Total solar radiation has been measured by many weather stations, and the outer space solar radiation can be obtained by theoretical calculation. By using the above expressions, the direct and diffused solar radiation can be predicted.

Fig. 5 shows the relationship between total horizontal radiation and reflected radiation, which has been measured from 9:00 to 17:00 by pointing the radiation meter towards the ground. The slope value 0.4564 can be described as the average reflection rate of the ground, which is comparatively high because of the white-coated roof.

3.2 Radiation intensity and tilt angle

The monthly total solar radiations received by PV modules of different tilt angles are shown in Fig.6. The tilt angle at which the PV modules receive the largest radiation was 55° from December to February, 35° between March and April, 15° in May, 0° between June and July, 15° in August, 35° in September and 55° in October and November, respectively.

3.3 Power output and tilt angle

The monthly power output of PV modules with various tilt angles has a similar tendency as the radiation intensity shown in Fig.6. However, the value of power output is about 1/10 of radiation because of the low conversion efficiency.

3.4 Radiation intensity and power output

Fig. 7 illustrates the relationship between daily power output and radiation intensity. As shown in this figure, the power output per solar cell area has an approximate linear relationship with the radiation intensity, which can be described by the following expression.

\[E_{p} = 0.1073 I_{p} - 0.357\]

Where,
\(E_{p}\) is the daily power output of PV module with the tilt angle of \(\beta\), MJ/m²·Day
\(I_{p}\) is the daily total solar radiation with the tilt angle of \(\beta\), MJ/m²·Day

3.5 Conversion efficiency

Fig. 8 shows the module conversion efficiency at different radiation intensities and weather conditions with a tilt angle of 15°. The largest daily average conversion efficiency of the PV system is 10% in this experiment, and the convention efficiency increases with the radiation intensity. Moreover, weather condition also has an influence on the conversion efficiency of the PV system. Higher conversion efficiency of the PV system can be obtained with the clearness of the weather. The regression lines for various weather conditions are shown in Fig.8. From this figure, it can be found that the regression line has a mild inclination for fine weather and a sharp one for cloudy weather. That is to say, the conversion efficiency of the PV system falls more rapidly than the decrease of slope radiation in bad weather conditions. The modules with other tilt angles also have a similar tendency.

3.6 Optimal tilt angle

In order to decide the optimal tilt angle, the following expression has been deduced from the
experimental data.

\[ E_{0\beta} = a \beta^2 + b \beta + c \] (2)

According to the above expression, the optimal tilt angle can be obtained by the following expression.

\[ \beta = - \frac{b}{2a} \] (3)

The result is shown in Fig. 9. Conforming to the custom, the result is displayed from January to December. According to the reference [4], the optimal tilt angle of NEDO obtained from solar radiation is displayed in Fig.9. for comparison. At the same time, the monthly average angle of direct radiation at noon is also displayed.

Generally, the profile of optimal tilt angle gained from the experiment has a similar trend to both the value of NEDO and the direct radiation angle. The validity of the experiment has been verified.

According to the solar radiation, it can be found that the optimal tilt angle of June is 2.2°. Taking June as the center, the optimal tilt angle increases for the months either before or after it. The largest value is 60.4° and the optimal tilt angle for the whole year is about 25°.

A similar profile has been obtained for the PV system. The smallest optimal tilt angle is 6.7° in June, and the largest 69.8°. The optimal tilt angle for the whole year is around 32°.

The reason for the difference in optimal tilt angles obtained according to radiation and power output is that not only radiation intensity, but also the cell temperature and other factors affect the power output, although the influence of the latter is relatively small.

In the ideal state, the tilt angle should meet with the direct radiation angle of south, in order to receive the most radiation. However, at the same time, reflected radiation and diffused radiation also arrive at the peak point, so the tilt angle is always a little different from the direct radiation angle.

The optimal tilt angle of NEDO has been calculated according to the solar radiation, with the albedo of 0.2 in calculating the reflected radiation. In this research, the average albedo is about 0.45. It can be considered that the optimal tilt angle of this research is larger than that of NEDO regarding the influence of the reflected radiation. However, since the reflected solar radiation received by the module's surface is very small because of the high solar declination in June and July (June is 79.18° and July is 77.40°), the experiment value of the optimal tilt angle is close to that of NEDO. Therefore, a further study should be conducted concerning the influence of albedo on the optimal tilt angle.

4. Calculation of Solar Radiation on a Sloped Surface with the Total Level Solar Radiation

4.1 Direct solar radiation on a sloped surface

The direct radiation of a slope can be calculated with the following expression, by using the spherical trigonometry, which has been called \( R_b \) model [4] by the Meteorological Society of Japan.

\[ H_{0\beta} = (H - H_d) R_b \] (4)

Where,

\[ H_{0\beta} \] is the direct radiation of the slope
\[ H \] is the daily total solar radiation on a level surface
\[ H_d \] is the daily-diffused irradiance on a level surface

\[ R_b = \frac{X \times B_1 + Y \times B_2 + Z \times B_3}{W} \] (5)

If the slope is towards the south, then

\[ X = \sin(\phi - \beta) \sin \delta \]
\[ Y = \cos(\phi - \beta) \cos \delta \]
\[ Z = 0 \]

\[ W = 2(\cos \phi \cos \delta \sin \omega_s + \frac{\pi}{180} \omega_s \sin \phi \sin \delta) \]
\[ B_1 = 2(\frac{\pi}{180}) \omega_s, \quad B_2 = 2 \sin \omega_s, \quad B_3 = 0 \]

Where,

\( \phi \) is the latitude
\( \delta \) is the solar declination
\( \beta \) is the tilt angle of the slope
\( \omega_s \) is the arrival angle on a level surface (°)
\( \omega_{ss} \) is the arrival angle on a sloped surface (°)
\( \omega_{ss} = \min \{\cos^{-1}[-\tan(\phi - \beta) \tan \delta], \omega_s\} \)

Therefore,

\[ R_b = \frac{(\cos(\phi - \beta) \cos \delta \sin \omega_s + \frac{\pi}{180} \omega_s \sin(\phi - \beta) \sin \delta)}{(\cos \phi \cos \delta \sin \omega_s + \frac{\pi}{180} \omega_s \sin \phi \sin \delta)} \]

4.2 Diffused solar radiation on a sloped surface

The diffused solar radiation of slope is calculated by the following expression.

\[ H_{0\beta} = H_d \left(1 + \frac{\cos \beta}{2}\right) \] (6)

Where,

\( H_d \) is daily-diffused irradiance on a level surface

4.3 Reflective solar radiation on a sloped surface

In the calculation of reflective solar radiation, the
following isotropic reflective model \(^4\) has been used.

\[
H_{\beta} = \rho H \left( 1 - \frac{\cos \beta}{2} \right)
\]  

(7)

Where,

\(\rho\) is the albedo (radiation reflection rate) of the ground.

### 4.4 Total solar radiation on a sloped surface

The total solar radiation of a slope can be calculated by the following expression.

\[H_{\beta} = H_{\beta 0} + H_{\beta 0} + H_{\beta 0}\]

(8)

The monthly and yearly solar radiation can be calculated by the integration of expression (8).

### 5. Comparison of Calculation Value and Experimental Value

Fig.10. shows the comparison between calculation value and experimental value. From this figure, it can be found that there is little difference between the measured and calculated value of total radiation of the sloped surface. The value is calculated from the total level of solar radiation and diffused radiation. Furthermore, as shown in Fig.10., the correlation coefficient is very high with a value of 0.9942. It can be considered that the calculation method is reliable.

### 6. Parameter Analysis

#### 6.1 Tilt angle, reflection rate and radiation of a slope

The influence of the tilt angle and reflection rate on the solar radiation received on a sloped surface is shown in Fig.11. It can be found that the influence of the reflection rate became smaller as the tilt angle decreased. When the reflection rate of the ground changes between 0.2 and 0.8, the maximum radiation of the slope on August 17 and December 11 was 7.32 MJ/m\(^2\) per day and 3.44 MJ/m\(^2\) per day, respectively.

In summer (August 17), furthermore, the solar radiation decreases with an increase of the tilt angle. However, in winter (December 11), the situation is reversed, a rise of the tilt angle leads to an increase of solar radiation until an angle of about 75\(^\circ\), where the radiation reaches the maximal value.

#### 6.2 Radiation rate and optimal tilt angle

The influence of the radiation rate on optimal tilt angle is shown in Fig.12. In this paper, the reflective rate of the ground is assumed to be 0.2 during the calculation.

As shown in the figure, an increase of the radiation rate leads to an increase of the optimal tilt angle. That is to say, the optimal tilt angle on fine days is larger than that on cloudy days. Furthermore, it can be found that the optimal value is more sensitive in winter than in summer. For example, when the radiation rate changes from 0.1 to 0.7, the change of optimal tilt angle is 7.7\(^\circ\) on August 17, and 43.2\(^\circ\) on December 11.

#### 6.3 Ground reflective rate and optimal tilt angle

In the calculation of the reflection rate, the weather has been assumed to be clear, fine, (\(a_{irr} = 0.65\)) and cloudy (\(a_{irr} = 0.30\)). The calculation result is shown in Fig.13. It can be found that the optimal tilt angle increases with the reflection rate. This is because the
The calculated result of optimal tilt angles on representative days is shown in Fig. 14. From January to summer, the optimal angle becomes smaller and from the summer to December, it becomes larger, while a minimum value is reached in the summer. Furthermore, it is found that the level is best on a cloudy day in summer.

In addition, as shown in Fig. 15., it can be found that the solar declination has an approximate linear relationship with the optimal tilt angle and changes remarkably with different weather conditions. The change range of the optimal tilt angle is about $60^\circ$ on fine days and about $48^\circ$ on cloudy days.

6.5 Latitude and optimal tilt angle

Fig. 16. shows the relationship between latitude and optimal tilt angle, which is almost linear. The higher latitude leads to a larger optimal tilt angle. For example, the optimal tilt angle is larger in Hokkaido (latitude $45^\circ$) than that in Kyusyu, (latitude $35^\circ$). The change is more obvious on a fine day than a cloudy day.

7. Conclusion

In this paper, four PV panels and radiation meters facing towards the south, with various tilt angles of 0°, 15°, 35°, and 55° were installed in Kitakyushu. By analyzing the measured data, the relationship between the tilt angles, solar radiation and power output were investigated. Based on the results, it can be found that the direct solar radiation and diffused solar radiation have a strong relationship with the radiation rate, and we can use this rate to predict the direct solar radiation and diffused solar radiation.

The power output of the PV module is approximately proportionate to the solar radiation received at the module surface. Moreover, the conversion efficiency of the PV system has a linear relationship with the radiation rate. In addition, conversion efficiency is greatly influenced by the weather conditions; fine weather will lead to higher efficiency.

The optimal tilt angle obtained in the experiment has the same tendency as that of NEDO and the direct radiation angle of south, which verifies the
According to this research, the optimal tilt angle for the whole year is from 25° to 32° in Kitakyushu.

A theoretical calculation method of slope radiation from level total solar radiation was presented, and the calculation values were verified by comparison with the measurement values. Influential factors such as diffused solar radiation, radiation rate, and reflection rate of the ground, solar declination and latitude were analyzed by using parameter analysis. The result can be summarized as follows: The diffused solar radiation has a big influence on the optimal tilt angle; a high reflection rate leads to a larger change tendency of the optimal tilt angle; the relationship between solar declination and optimal tilt angle is almost linear; and the higher latitude causes the optimal tilt angle to increase.

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