Transport coefficients of heavy quarkonia comparing with heavy quark coefficients

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(Dated: April 11, 2022)

Abstract

We revisit the transport coefficients of heavy quarkonia moving in high-temperature QCD plasmas. The thermal width and mass shift for heavy quarkonia are closely related to the momentum diffusion coefficient and its dispersive counterpart for heavy quarks, respectively. For quarkonium at rest in plasmas the longitudinal gluon part of the color-singlet self-energy diagram is sufficient to determine the leading-order thermal width, whereas the momentum dependence is obtained from the transverse gluon channel. Using the quarkonium-gluon effective vertex based on the dipole interaction of color charges, we discuss the damping rate, the effective rest and kinetic mass shifts of slowly moving quarkonia and compare with the corresponding coefficients of heavy quarks.
I. INTRODUCTION

The suppression of quarkonium production has been predicted to indicate the formation of quark-gluon plasmas in relativistic heavy-ion collisions [1]. In a hot medium, color-screening prevents heavy quark-antiquark pairs from binding and bound states are dissociated by medium interactions. The dynamics of heavy quarkonia are characterized by transport coefficients such as the thermal width and mass shift. These two coefficients correspond to thermal corrections to the imaginary and real parts of the static potential for a heavy quark-antiquark pair, respectively [2, 3].

In high-temperature plasmas, dissociation of a bound state such as Υ(1S) is described mostly by inelastic parton scattering [4]. At leading order in a weak coupling expansion, the thermal width is related to the momentum diffusion coefficient of heavy quark [5, 6]. Medium interactions of heavy quarks are described by the diffusion coefficient and its dispersive counterpart [7]. In this work, we investigate the transport coefficients of heavy quarkonia, especially bound states with finite spatial momentum with respect to plasmas. Although the transport coefficients for heavy quarks and quarkonia can be extracted from lattice QCD computations, current lattice data have large uncertainties and are not practical for measuring the momentum dependence of the heavy quark-antiquark potential. Furthermore, it would be useful to identify the main partonic processes contributing to the transport coefficients.

In previous studies, the momentum dependence of the thermal width has been considered phenomenologically in a moving medium because most theoretical investigations have been carried out in the rest frame of a bound state. We have computed the thermal width by calculating the dissociation cross section in the quarkonium rest frame and then convoluting parton distribution functions given by \( f(K) = (e^{K \cdot V / T} \pm 1)^{-1} \), where \( V = \frac{1}{\sqrt{1 - v^2}}(1, v) \) [8]. An effective field theory approach has also been employed to calculate the thermal width [9]. In the kinematical regime \( T \gg \frac{1}{T} \gg m_D \gg E \), they have taken into account dynamic screening [10, 11] (which depends on velocity) but ignored the energy transfer between a bound state and plasma. The energy transfer can be neglected at leading order for quarkonium at rest, but both energy and momentum transfer are required to evaluate the thermal width of bound states moving in quark-gluon plasmas.

To understand dynamic properties of heavy particles in hot QCD plasmas, there have
been many investigations and they need to be brought together to study transport processes of heavy quarkonia. The goal of this paper is to compute the transport coefficients for heavy quarkonium depending on its velocity and to explain the difference from the heavy quark coefficients which have been investigated in literature. Our analysis is based on the leading quarkonium-gluon interaction [12] which has been derived from the Bethe-Salpeter amplitude using the dipole interaction of color charges [13]. In Section II, we calculate the velocity dependence of the thermal width of $O(g^4 T^3 r^2)$ through inelastic parton scattering. In Section III, the rest and kinetic mass shifts for slowly moving quarkonia are determined at leading order $O(g^4 T^3 r^2)$ and next-to-leading order $O(g^5 T^3 r^2)$. In Section IV, we revisit the damping rate of nonrelativistic quarkonium by using the spectral density derived from a Schrödinger-type equation. Finally, we summarize our result in Section V. The details on the quarkonium-gluon effective interaction are presented in Appendix A.

II. THERMAL WIDTH

In high-temperature QCD plasmas, quarkonium dissociation occurs through inelastic parton scattering ($\Upsilon + p \rightarrow b + \bar{b} + p$, $p = g, q, \bar{q}$) exchanging spacelike gluons. The cross section calculated in hard-thermal-loop (HTL) perturbation theory has been shown to agree with the result obtained by potential nonrelativistic QCD (pNRQCD) in the kinematical regime $\frac{1}{r} \gg T \gg m_D \gg E$ (where $r$ is the distance between $b$ and $\bar{b}$, and $E$ is the binding energy) [4]. The leading-order dissociation is described by taking the dipole interaction of color charges (which is valid at short $r$) and ignoring the interaction between $b$ and $\bar{b}$ after breakup in the large $N_c$ limit [13]. In the same approach, we consider the thermal width for moving quarkonium and compare with the momentum diffusion coefficient of heavy quark.

A bound state of heavy quarks and its interaction with partons can be described by the Bethe-Salpeter amplitude in the rest frame of quarkonium (see Appendix A) [12]. Using the effective vertex Eq. (A4), quarkonium dissociation through inelastic parton scattering has been calculated [4]:

$$|\mathcal{M}|^2 = C g^4 m^2 m_\Upsilon |\nabla \psi(p)|^2 \left| k^2 \frac{k^2_{10}}{(k^2 + m^2_D)^2} \left\{ \begin{array}{c} (1 + \cos \theta_{k_1k_2}) (q, \bar{q}) \\ (1 + \cos^2 \theta_{k_1k_2}) (g) \end{array} \right. \right|,$$

where $C = 16 \frac{(N_c^2 - 1) N_f}{N_c}$, $16 (N_c^2 - 1)$ for $(q, \bar{q})$, $(g)$, respectively, $\psi(p)$ is the normalized wave function for a bound state with the relative momentum, $p = \frac{1}{2} (p_1 - p_2)$, and $k = k_1 - k_2$ is
the momentum transfer. The matrix elements are similar to those for heavy quark diffusion, 
t-channel gluon exchange \((b + p \rightarrow b + p)\) weighted by the squared momentum transfer \([14]\).
The thermal width of quarkonium is obtained by the following phase space integrations \([4]\):
\[
\Gamma = \frac{1}{2 d_T q^0} \int \frac{d^3 k_1}{(2\pi)^3} \int \frac{d^3 k_2}{(2\pi)^3} \int \frac{d^3 p_1}{(2\pi)^3} \int \frac{d^3 p_2}{(2\pi)^3}
\times (2\pi)^4 \delta^4(Q + K_1 - K_2 - P_1 - P_2) |\mathcal{M}|^2 n(k_1) [1 \pm n(k_2)], \tag{2}
\]
where \(d_T\) is the degeneracy factor of quarkonium. For quarkonium at rest, the interaction
rate is proportional to the momentum diffusion coefficient of heavy quark: \(\Gamma^{1S} = 3a_0^2 \kappa\) for a
Coulombic bound state \([5, 6]\). Note that Eq. (1) involves only the longitudinal part of the
effective gluon propagator as the leading contribution. Since the transverse gluon propagator
couples with the energy transfer rather than the momentum transfer, the thermal width of
quarkonium with finite momentum will be different from the momentum diffusion coefficient
of heavy quark.

To consider the relative motion between quarkonium and thermal medium, we use
\(\cos \theta_{kk_1} = \frac{\omega}{k} + \frac{k^2 - \omega^2}{2kk_1}\). Then the square bracket in Eq. (1) becomes
\[
\left[\frac{k^2}{2} [4k_1(k_1 - \omega) - (k^2 - \omega^2)] D_L(K)^2 + \frac{\omega^2}{2k^2} [(2k_1 - \omega)^2 + k^2](k^2 - \omega^2) |D_T(K)|^2 \right] \tag{3}
\]
at leading order. Due to the long-range interactions mediated by gluon, infrared divergences
arise and HTL resummation is necessary. In Coulomb gauge, the effective propagators for
soft gluon are \([15]\)
\[
D_L(K) = \frac{1}{k^2 + \Pi_L(K)}, \quad D_T(K) = -\frac{1}{k^2 - \omega^2 + \Pi_T(K)}, \tag{4}
\]
with HTL corrections \([10, 16]\)
\[
\Pi_L(K) = m_D^2 \left[1 - \frac{\omega}{2k} \left(\ln \frac{k + \omega}{k - \omega} - i\pi\right)\right],
\Pi_T(K) = m_D^2 \left[\frac{\omega^2}{2k^2} + \frac{\omega(k^2 - \omega^2)}{4k^3} \left(\ln \frac{k + \omega}{k - \omega} - i\pi\right)\right]. \tag{5}
\]
The leading-log thermal width is determined by a method similar to that developed for
calculating the screening effect \([17–19]\). The effective propagators screen the static Coulomb
interaction, eliminating infrared divergences. Although the purely static magnetic interaction
is not screened, there is dynamic screening at nonzero frequency \([10]\): this dynamical
FIG. 1. The velocity dependence in a leading-log approximation: comparing the thermal width of quarkonium and the momentum diffusion coefficient of heavy quark.

screening cuts off infrared divergences if divergences are only logarithmic. After angular integrations, the thermal width is given by

$$\Gamma(v) = \frac{1}{16(2\pi)^5 a_0^2} \int dk_1 \int dk \int_{-v_k}^{v_k} d\omega \int dp \frac{p^2}{P_{10}^2} |M|^2 n(k_1)[1 \pm n(k_1 - \omega)].$$  (6)

For a Coulombic bound state, we have

$$\langle r^2 \rangle = \int \frac{d^3p}{(2\pi)^3} |\nabla \psi_1 S(p)|^2 = 3a_0^2$$

and

$$\Gamma^{1S}(v) = \frac{3Cg^4a_0^2}{128\pi^3 a_T^2} \int dk_1 k_1^2 n(k_1)[1 \pm n(k_1)]$$

$$\times \int dk \int_{-v_k}^{v_k} d\omega \left[ k^2|D_L(K)|^2 + \frac{\omega^2(k^2 - \omega^2)}{k^2}|D_T(K)|^2 \right],$$

$$\simeq \left( \frac{N_c^2 - 1}{8\pi a_T v} \right) \left( \frac{N_f}{2N_c} \right) \ln \frac{1 + v}{1 - v} \ln \frac{T}{m_D}. \tag{7}$$

We notice that the velocity-dependence comes from the transverse gluon part: in Eq. (3) $O(v^2)$ terms from the longitudinal part have been ignored compared to the leading-log term.

The momentum diffusion coefficient of heavy quark has been computed in a leading-log approximation [14]

$$\kappa(v) = \frac{C_F g^4 T^3}{12\pi} \left( \frac{N_f}{2} + N_c \right) \left[ 1 - \frac{1 - v^2}{6v} \ln \frac{1 + v}{1 - v} \right] \ln \left( \frac{T}{m_D} \right). \tag{8}$$

In Fig. 1, we present the velocity dependence of the thermal width, comparing with that of the heavy quark diffusion coefficient. The width increases with quarkonium momentum, in qualitative agreement with the previous investigation in Ref. [8]. For small $v$, $\frac{\Gamma(v)}{\Gamma(v=0)} =$
FIG. 2. The self-energy diagram for heavy quark-antiquark color-singlet with the effective gluon propagator.

\[ \frac{\kappa(v)}{\kappa(v=0)} = 1 + \frac{v^2}{3} + O(v^4) \]: the thermal width of moving quarkonium agrees with the momentum diffusion coefficient of heavy quark (multiplied by \( \langle r^2 \rangle \)) up to \( O(v^2) \).

Dynamics of heavy quarks and quarkonia is characterized by dissipative transport coefficients and their dispersive counterparts. To determine the former such as heavy quark diffusion and damping rate, we consider the interaction rate of heavy particles. The interaction rate is proportional to the imaginary part of the self-energy and is related to the spectral density of soft gluon [18, 20]. On the other hand, dispersive coefficients such as mass shift corresponds to the real part of the gluon self-energy. Indeed, the thermal width and mass shift of quarkonia are given by the imaginary and real parts of the color-singlet self-energy (see Fig. 2) [3, 6].

We reexpress the thermal width in terms of the spectral density of soft gluon, \( \rho_{L,T}(K) = -\frac{1}{\pi} \text{Im} D_{L,T}(\omega + i\epsilon, k) \):

\[
\Gamma^{1S}(v) = \frac{3\pi(N_c^2 - 1)g^2a_0^2}{N_c d_T} \int \frac{d^3k}{(2\pi)^3} \int d\omega [1 + n_B(\omega)] \delta(\omega - k \cdot v) \\
\times \left[ k^2 \rho_L(K) + 2\omega^2 \rho_T(K) \right], \quad (9)
\]

where quark and gluon contributions correspond to \( m_{D_{(q,g)}}^2 = \frac{2\alpha_g e^2}{\pi} \int \frac{d^q k}{(2\pi)^q} n(k_1)[1 \pm n(k_1)] \) in HTL corrections. For \( \omega \ll T \), the bosonic thermal distribution is expanded as \( 1 + n_B(\omega) = \frac{T}{\omega} + \frac{1}{2} + \cdots \). Since the spectral density is odd in \( \omega \), the first term, \( \frac{T}{\omega} \), contributes to the thermal width. This limit is consistent with the soft momentum transfer in the thermal width in Eq. (7), and \( \Gamma^{1S} \) for small \( v \) will be reproduced later when discussing the damping rate. Replacing the spectral density by the corresponding real part in Eq. (9), the second term, \( \frac{1}{2} \), produces the thermal mass shift which will be evaluated in the next section.
III. MASS SHIFT

The thermal width $\Gamma \sim g^4T^3a_0^2$ is determined through inelastic parton scattering with soft momentum transfer by using HTL resummed perturbation theory. The thermal mass shift is considered in the same way, but the leading contribution $\delta m_{\text{LO}}^{1S} = \frac{1}{4\pi}\zeta(3) \left( N_f + \frac{4}{3}N_c \right) g^4T^3a_0^2$ comes from momenta larger than $\mathcal{O}(gT)$ scale [3]. It turns out that soft momentum transfer yields $\delta m_{\text{NLO}}^{1S} \sim g^5T^3a_0^2$ which is an $\mathcal{O}(g)$ higher term: the higher order corrections for the heavy quark diffusion coefficient are known to be significant at realistic values of the strong coupling constant [21]. The mass shift provides thermal corrections to the dispersion relation of heavy particles, $E_p = m_{\text{rest}} + \frac{p^2}{2m_{\text{kin}}}$, where $m_{\text{rest}}$ and $m_{\text{kin}}$ are the rest mass and the kinetic mass, respectively [22]. For $m \gg T$, the leading corrections to the mass-shell seem to be independent of the spatial momentum. In this section, we discuss the rest mass shift (which has been computed in Ref. [3]) and the kinetic mass shifts for slowly moving quarkonia, focusing on smaller momentum transfer than incoming parton momentum.

The dispersive counterpart of Eq. (9) is given by

$$\delta m_{\text{NLO}}^{1S}(v) = \frac{3(N_c^2 - 1)g^2a_0^2}{2N_c d_T} \int \frac{d^3 \kappa}{(2\pi)^3} \int d\omega \frac{1}{2} \delta(\omega - \kappa \cdot v) \times \text{Re} \left[ \frac{k^2}{k^2 + \Pi_L(K)} - \frac{2\omega^2}{k^2 - \omega^2 + \Pi_T(K)} \right]. \quad (10)$$

For small $v$, we expand the longitudinal gluon propagator:

$$\delta m_{\text{NLO}}^{1S}(v) = \frac{3(N_c^2 - 1)g^2a_0^2}{2N_c d_T} \int \frac{d^3 \kappa}{(2\pi)^3} \int d\omega \frac{1}{2} \delta(\omega - \kappa \cdot v) \times k^2 \left[ \frac{1}{k^2 + m_D^2} + \frac{m_D^2\omega^2}{(k^2 + m_D^2)^3} + \frac{(4 - \pi^2)m_D^4\omega^2}{4k^2(k^2 + m_D^2)^3} + \cdots \right], \quad (11)$$

which has been evaluated in dimensional regularization. Subtracting from the integrand the corresponding unresummed propagator $(\frac{1}{k^2 - \omega^2 + \Pi_T(K)} - \frac{1}{k^2 - \omega^2})$, we can show that the transverse gluon contributes to $\mathcal{O}(v^4)$. The first and second terms in the last line of Eq. (11) correspond to the rest mass shift and the kinetic mass shift, respectively. We note that the effective kinetic mass shift is not equal to the rest mass shift. Similarly, the dispersion relation of heavy quark has been discussed in Ref. [22]:

$$\delta m_{\text{HQ}} = -\frac{C_F g^2 m_D}{8\pi} + \frac{1}{2} \left( 1 - \frac{\pi^2}{16} \right) \frac{C_F g^2 m_D}{24\pi} v^2. \quad (12)$$
In comparison to heavy quark $\delta m_{HQ} \sim g^2 T$, the thermal mass shift of quarkonium is suppressed by a factor $\sim (m_D a_0)^2$ in the case of soft momentum transfer.

The leading-order mass shift of moving quarkonia is

$$\delta m_{1S}^{1S}(v) = -\frac{3(N_c^2 - 1)g^2 a_0^2}{2N_c N_f} \int \frac{d^3 k}{(2\pi)^3} \int d\omega \frac{1}{2} \delta(\omega - k \cdot v) \left[ \Re \Pi_L(K) \frac{k^2}{k^2 - \omega^2} + 2\omega^2 \Re \Pi_T \right].$$

(13)

Here, we expand the gluon self-energy [23–25] in $\omega$:

$$\Pi_L(K) = \frac{g^2 N_f}{2\pi^2} \int_0^\infty dk_1 k_1 n_F(k_1) \left[ 2 + \frac{(2k_1/k) - k/2k_1}{k-2k_1} \ln k + 2k_1 \right. $$

$$- \frac{\omega^2}{k^2} \left( \frac{2(k^2 - 8k_1^2)}{k^2 - 4k_1^2} - \frac{k}{2k_1} \ln \frac{k + 2k_1}{k - 2k_1} \right) + \cdots $$

$$+ \frac{g^2 N_c}{8\pi^2} \int_0^\infty dk_1 k_1 n_B(k_1) \left[ 8 - \frac{4k^2}{k^2} + \frac{8k_1}{k_1} \left( \frac{4k}{k_1} - \frac{4k}{k_1} + \frac{k_1^3}{k_1^3} \right) \ln \frac{k + 2k_1}{k - 2k_1} \right. $$

$$- \frac{\omega^2}{k^2} \left( \frac{3k^4 - 8k^4 k_1^2 - 112k^2 k_1^4 + 256k_1^6}{k_1^2(k^2 - 4k_1^2)^2} + \frac{5k(k^2 - k_1^2)^2}{k^4} \ln \frac{k + k_1}{k - k_1} \right) $$

$$- \left. \frac{k(5k^4 - 12k^2 k_1^2 + 24k_1^4)}{4k_1^5} \ln \frac{k + 2k_1}{k - 2k_1} \right) + \cdots ,$$

(14)

where the first and second integrals correspond to quark and gluon contributions, respectively. The $O(\omega^0)$ and $O(\omega^2)$ terms yield the rest mass shift and the relative correction, respectively, while the transverse part is subleading. If the momentum transfer is smaller than the incoming parton momentum, we have

$$\Re \Pi_L(K) \sim \frac{2g^2 N_f}{\pi^2} \int_0^\infty dk_1 k_1 n_F(k_1) \left( 1 - \frac{\omega^2}{k^2} \right) $$

$$+ \frac{2g^2 N_c}{\pi^2} \int_0^\infty dk_1 k_1 n_B(k_1) \left( 1 - \frac{\omega^2}{k^2} \right) + \mathcal{O} \left( \frac{k^2}{k_1^2}, \omega^4 \right),$$

(15)

and the thermal mass shift is estimated to be $\delta m_{\text{rest}}(1 - \frac{\omega^2}{k^2})$. However, we need to use the resummed propagators when $k \sim gT$.

IV. DAMPING RATE

The static potential for a heavy quark-antiquark pair has been computed at high temperature, $T \gtrsim \frac{1}{\tau} \sim mg^2$ [2, 3]. Employing the potential, the quarkonium contribution to the spectral function of the electromagnetic current has been numerically computed [26, 27]. Especially for $T \gg \frac{1}{\tau} \sim m_D$, the real part $\mathcal{O}(g^2 m_D)$ of the potential is smaller than the imaginary part $\mathcal{O}(g^2 T)$ (whose $m_D r \ll 1$ expansion yields $\Gamma \sim g^2 T(m_D r)^2$ discussed in
Section II), so the heavy quark-antiquark pair is expected to decay before forming a bound state [3]. It has been noticed that the real part correction coincides with the free energy of static heavy quark-antiquark [28, 29] (or twice the rest mass shift of heavy quark), while the imaginary part approaches twice the heavy quark damping rate [30, 31] (with minus sign) in the large $r$ limit [3, 32]. The heavy quark propagator can be approximated by its nonrelativistic particle pole:

$$D_b \left( P \pm \frac{Q + K}{2} \right) \rightarrow \pm \frac{i(1 \pm V)}{2(\pm p_0 + \frac{\omega - k \cdot v}{2} + E - \frac{p^2}{2m} + i\Gamma_b)},$$

(16)

where $V = \frac{1}{\sqrt{1-v^2}}(1, v)$ and $\Gamma_b$ is the heavy quark width. For a heavy quark-antiquark pair with the effective vertex $V^\mu\nu_{\text{eff}}(K)$ in Fig. 2,

$$\int \frac{dp^0}{2\pi} D_b \left( P + \frac{Q + K}{2} \right) V^\mu\nu_{\text{eff}}(K) D_b \left( P - \frac{Q + K}{2} \right) V^{\rho\sigma\dagger}_{\text{eff}}(K) \simeq \frac{1 + V}{2} V^\mu_{\text{eff}}(K) - \frac{1 - V}{2} V^{\rho\sigma\dagger}_{\text{eff}}(K) \frac{i}{\omega - k \cdot v - E - \frac{p^2}{2m} + i\Gamma_b},$$

(17)

where the width of the quarkonium state is twice that of heavy quark [33].

In this work, we are interested in the kinematical regime where $\frac{1}{\epsilon} \gg T$ and $m_D \gg E$ are satisfied so that a dipole approximation is appropriate and quarkonium dissociation can be parallel to heavy quark diffusion. In Eq. (17), without $\Gamma_b$ we get $\delta(\omega - k \cdot v)$ which appears throughout the paper. We have calculated the thermal width and mass shift of quarkonium moving in quark-gluon plasmas in the previous two sections. They provide the thermal corrections, $\delta m - i\Gamma$, to the singlet static potential [3, 6]. In the following, we employ the complex potential to estimate the spectral density of nonrelativistic quarkonium near the threshold $\omega \sim 2m + \frac{q^2}{4m}$. The result is then used to compute the quarkonium damping rate which is comparable with the thermal width.

As discussed in Refs. [2, 26, 27], a heavy quark current-current correlator $C^>(X) = \langle \hat{J}^\mu(X)\hat{J}^\mu(0) \rangle$, where $\hat{J}^\mu(X) = \hat{\psi}(X)\gamma^\mu\hat{\psi}(X)$, satisfies a Schrödinger-type equation with a complex potential:

$$\left[ i\frac{\partial}{\partial t} - \left( 2m + \frac{q^2}{4m} - \frac{\nabla^2}{m} + V(r) \right) \right] C^>(t, r) = 0. \tag{18}$$

Although one can solve the equation numerically, we consider a simple case in the static limit: $\text{Re} V(r) \approx -\frac{q^2C_F}{4\pi r} e^{-m_D r}$ (plus the thermal mass shift) and $\int d^3r \text{Im} V(r) = -\frac{\Gamma}{2}$ at short distance $r$ (we assume that only the real part depends on $r$). Since the potential is
independent of time, \( C^>(t, r) \propto e^{-i(E_q - i\frac{\Gamma}{2})t} \), where \( E_q = 2m + \frac{q^2}{4m} - E \). The radial part is given by a bound state for a Debye-screened Coulombic potential,

\[
\left[ \frac{\nabla^2}{m} - \text{Re} V(r) \right] \psi(r) = E\psi(r) .
\] (19)

In the limit \( r \to 0 \), we take a Fourier transform with respect to time:

\[
\rho(\omega) \propto \int_0^\infty dt \, e^{-\frac{\Gamma}{2}t}[\cos(E_q t) \cos(\omega t) + \sin(E_q t) \sin(\omega t)] = \frac{\frac{\Gamma}{2}}{(\omega - E_q)^2 + \frac{\Gamma^2}{4}} . \tag{20}
\]

Retaining the leading terms in the real and \((r\text{-integrated})\) imaginary parts of the potential, we obtain a Breit-Wigner form of the spectral density for a spherically symmetric wave function: the imaginary part of the potential provides a finite width of a bound state.

The damping rate is given by the imaginary part of the self-energy at the pole (which has an imaginary part) in the propagator, while on the mass-shell half the interaction rate is expected [34]. Since the spectral density has been estimated in Eq. (20), we follow Ref. [31] for heavy quark to compute the damping rate of nonrelativistic quarkonium. After integrating a wave function and averaging over polarization,

\[
\frac{\Gamma}{2} = \frac{4g^2a_0^2}{3} \int \frac{d^3 k}{(2\pi)^3} \int \frac{d\omega}{\omega} \int d\omega_\Upsilon \frac{\frac{\Gamma}{2}}{[(\omega_\Upsilon - E_{q-k})^2 + \frac{\Gamma^2}{4}]} \times \left[ k^2 \rho_L(K) + 2\omega^2 \rho_T(K) \right] \delta(\omega + \omega_\Upsilon - E_q) . \tag{21}
\]

We redefine \( \omega_\Upsilon \to \omega_\Upsilon + E_{q-k} \) and \( E_{q-k} \approx m_\Upsilon + \frac{q^2}{2m_\Upsilon} - k \cdot v \). Then the delta function becomes \( \delta(\omega + \omega_\Upsilon - k \cdot v) \). Since \( \omega_\Upsilon \sim g^4T^3a_0^3 \) is much smaller than the scale of \( \omega \) or \( vk \), we can perform the integrations over \( \omega_\Upsilon \) and \( \omega \) separately: \( \int_{-vk}^{vk} d\omega \frac{\frac{\Gamma}{2}}{\omega^2 + \frac{\Gamma^2}{4}} = 2 \arctan \left( \frac{2vk}{\Gamma} \right) \).

Finally, approximating the spectral densities in the limit \( \omega \to 0 \),

\[
\rho_L(K) \simeq -\text{Im} \frac{1}{\pi \left[ k^2 + m_D^2(1 + i\frac{\pi\omega}{2k}) \right]} ,
\]

\[
\rho_T(K) \simeq \text{Im} \frac{1}{\pi \left( k^2 - i \frac{m_D^2\pi\omega}{4k} \right)} , \tag{22}
\]

we have

\[
\frac{\Gamma}{2} \simeq \frac{g^2a_0^2}{3\pi} \left( 1 + \frac{v^2}{3} \right) m_D^2 T \ln \frac{T}{m_D} . \tag{23}
\]

This result agrees with the thermal width, Eq. (7), for small \( v \).

Due to the quarkonium-gluon effective vertex involving momentum and energy transfer, the long-range interactions in both longitudinal and transverse gluon parts are screened by
the Debye mass in the leading-log damping rate. This is in contrast to the heavy quark damping rate where the transverse part is in need of an infrared regulator such as the magnetic mass ($\sim g^2 T$) or the imaginary part of the pole (that is, the damping rate itself) [31, 35]. The damping rate of slow heavy quark has been computed for $g^2 \ll v \ll 1$: 

$$\gamma_{HQ} = \frac{g^2 C_F T}{8\pi} \left( 1 + \frac{v}{2} \ln \frac{\pi m_D^2 v^3}{4\gamma_{HQ}^2} \right),$$

where $\gamma_{HQ}$ inside the logarithm is replaced by its value at $v = 0$ [31].

V. SUMMARY

We have presented the velocity dependence of the thermal width and mass shift for heavy quarkonia slowly moving in hot QCD plasmas. In the kinematical regime where $\frac{1}{r} \gg T$ and $m_D \gg E$, the longitudinal gluon contribution is same for both the thermal width and the momentum diffusion coefficient of heavy quarks, but the transverse gluon part is different. For nonrelativistic quarkonia, we have discussed the thermal width in relation to the damping rate which is proportional to the imaginary part of the color-singlet self-energy. We have also discussed the thermal corrections to the dispersion relation, the effective rest mass shift and its correction (kinetic mass shift) for small momentum transfer. On the other hand, at a higher temperature regime $T \gg \frac{1}{r}$ a heavy quark-antiquark pair is likely to decay before forming a bound state. The real and imaginary part of the static potential approach the mass shift and the damping rate, respectively, of heavy quark and antiquark in the large $r$ limit.

For a slowly moving bound state, the thermal width agrees with the momentum diffusion coefficient of heavy quark up to $O(v^2)$. However, the thermal width of nonrelativistic quarkonium is same as neither the heavy quark diffusion coefficient nor the damping rate of heavy quark-antiquark pair in the short $r$ limit. The relative correction to the rest mass shift for quarkonium is also different from that for heavy quark. In studying quarkonium dynamics, the transport coefficients of heavy quarks and quarkonia might be relevant in different kinematical regimes. Eventually the nonperturbative determination of the transport coefficients is desirable, while our perturbative analysis of the velocity dependence can supplement current lattice QCD computations.
Appendix A: Quarkonium-gluon interaction

In this appendix, we provide the details on the leading-order quarkonium-gluon interaction which has been derived with the Bethe-Salpeter amplitude [12] through dipole interaction [13]. The Bethe-Salpeter equation for a bound state is [36]

\[
\Gamma_{\text{BS}}^\mu(P_1, -P_2) = \frac{ig^2 C_F}{(2\pi)^4} \int \frac{d^4K}{(2\pi)^4} V(K) \gamma^\nu D_0(P_1 + K) \Gamma_{\text{BS}}^\nu(P_1 + K, -P_2 + K) D_0(-P_2 + K) \gamma_\nu, \quad (A1)
\]

where \( V(K) \simeq -\frac{1}{k^2} \) after computing a dominant residue. For \( q, p_1, 2 \gg k \) and in the rest frame of quarkonium, the Bethe-Salpeter amplitude reduces to

\[
\Gamma_{\text{BS}}^\mu \left( P + \frac{Q}{2}, P - \frac{Q}{2} \right) = \sqrt{\frac{m_\Upsilon}{N_c}} \left( E + \frac{p^2}{m} \right) \psi(p) \frac{1 + \gamma^0}{2} \gamma^i \delta^{\mu i} \frac{1 - \gamma^0}{2}, \quad (A2)
\]

and Eq. (A1) becomes the nonrelativistic Schrödinger equation for a Coulombic bound state,

\[
\left( E + \frac{p^2}{m} \right) \psi(p) = g^2 C_F \int \frac{d^3k}{(2\pi)^3} V(k) \psi(p + k). \quad (A3)
\]

As in Eq. (16), the leading expression in the plasma rest frame is expected with the replacement, \( 1 \pm \gamma^0 \rightarrow 1 \pm \gamma \).

Inelastic parton scattering is a dominant dissociation process of a bound state like \( \Upsilon(1S) \) which survives in high-temperature plasmas, whereas at low temperature near the phase transition gluo-dissociation (\( \Upsilon + g \rightarrow b + \bar{b} \)) becomes effective [4]. Using Eq. (A2), the scattering amplitude of gluon absorption has been obtained as follows [12]:

\[
M_{\text{gluo}}^{\mu\nu} = -g \sqrt{\frac{m_\Upsilon}{N_c}} \left[ k \cdot \frac{\partial \psi(p)}{\partial p} \delta^{\mu 0} + k_0 \frac{\partial \psi(p)}{\partial p^i} \delta^{\mu i} \right] \delta^{\nu j} \bar{u}(P_1) \frac{1 + \gamma^0}{2} \gamma^j \frac{1 - \gamma^0}{2} T^a v(P_2),
\]

\[
\equiv \bar{u}(P_1)V_{\text{eff}}^{\mu\nu}(K) v(P_2), \quad (A4)
\]

where a quarkonium-gluon effective vertex \( V_{\text{eff}}^{\mu\nu}(K) \) can be defined. The effective vertex is similar to the pNRQCD vertex [37], with \( \frac{\partial \psi(p)}{\partial p} \) corresponding to \( r \) multiplied by a bound state.

The thermal width by gluo-dissociation is

\[
\Gamma_{\text{gluo}} = \frac{1}{2d_T q^0} \int \frac{d^3k}{(2\pi)^3} k^0 \int \frac{d^3p_1}{(2\pi)^3} p_{10} \int \frac{d^3p_2}{(2\pi)^3} p_{20} \times \frac{4}{(2\pi)^4} \delta^4(Q + K - P_1 - P_2) |M_{\text{gluo}}^{\mu\nu}(k)|^2, \quad (A5)
\]
which corresponds to a different imaginary part of the heavy quark-antiquark potential from inelastic parton scattering [3]. Gluo-dissociation is dominant in the kinematical regime \( \frac{1}{r} \gg T \gg E \gg m_D \) [5] and is not directly related to the momentum diffusion coefficient of heavy quark. In contrast to inelastic parton scattering, the thermal width by gluodissociation decreases with \( v \) [8].

ACKNOWLEDGMENTS

This research was supported by Basic Science Research Program through the National Research Foundation of Korea (NRF) funded by the Ministry of Education (No. 2021R1I1A1A01054927).

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