Testing Effective String Models of Black Holes with Fixed Scalars

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Abstract

We solve the problem of mixing between the fixed scalar and metric fluctuations. First, we derive the decoupled fixed scalar equation for the four-dimensional black hole with two different charges. We proceed to the five-dimensional black hole with different electric (1-brane) and magnetic (5-brane) charges, and derive two decoupled equations satisfied by appropriate mixtures of the original fixed scalar fields. The resulting greybody factors are proportional to those that follow from coupling to dimension (2,2) operators on the effective string. In general, however, the string action also contains couplings to chiral operators of dimension (1,3) and (3,1), which cause disagreements with the semiclassical absorption cross-sections. Implications of this for the effective string models are discussed.
1. Introduction

Effective string models of $D = 5$ black holes with three $U(1)$ charges \[1, 2, 3, 4\] and of $D = 4$ black holes with four $U(1)$ charges \[5, 6\] are being actively explored in the current literature. In the $D = 5$ case the effective string models the dynamics of the intersection of D-branes \[7\], while in the $D = 4$ case -- that of triply intersecting 5-branes of M-theory \[8\]. The initial success of the models was in reproducing the Bekenstein-Hawking entropy of black holes \[1, 3, 4, 9, 10, 8\], but more recently the emphasis has shifted to more dynamical comparisons -- those involving emission and absorption rates of massless quanta. For minimally coupled scalar fields such calculations were carried out in \[3, 11, 12, 13, 14, 15, 16, 17, 18, 19\]. Remarkably, it was found that the energy-dependence of the semiclassical absorption cross-sections (the so-called greybody factors) are correctly reproduced by effective string calculations at sufficiently low energies \[15, 16\]. This success has been attributed to the validity of the moduli space approximation \[20\].

An important issue is whether the effective string continues to be a good description beyond this regime. A good test for this is provided by the fixed scalars \[21, 22\], whose non-minimal couplings to the gauge fields render their greybody factors different from those of the minimally coupled scalars \[23, 24\]. In \[24\] the effective string explanation of the new greybody factors was traced to the fact that the leading coupling of fixed scalars is to operators of dimension higher than $(1, 1)$. One of the $D = 5$ fixed scalars, related to the volume of the internal $T^4$ over which the 5-branes are wrapped, and called $\nu$ in \[24\], was found to couple to an operator of dimension $(2, 2)$. The subsequent string calculation of the absorption cross-section yielded precise agreement with the semi-classical greybody factor \[24\].

However, an important technical obstacle, which arises in the classical supergravity, put a restriction on the range of comparisons that could be carried out in \[24\]. For general 1-brane and 5-brane charges, $Q$ and $P$, the fluctuations of the two fixed scalar fields, $\nu$ and $\lambda$, mix with each other and also with the fluctuations of the metric. For this reason, the comparison carried out in \[24\] was limited to the simplest case of $P = Q$, where only $\lambda$ mixes with gravity while $\nu$ is unmixed. In this paper we overcome this obstacle and disentangle the fixed scalar equations for $P \neq Q$. The resulting pair of equations are remarkably simple and are very similar to the fixed scalar equation derived in \[24\]. In fact, the greybody factors that follow from them are both proportional to the greybody factor calculated in \[24\]. This turns out to disagree with the effective string action derived in \[24\]. Even for $P = Q$ the $\lambda$ greybody factor is not in agreement, while for $P \neq Q$ neither greybody factor appears to agree. The disagreement is caused by the appearance of chiral operators with dimensions $(3, 1)$ and $(1, 3)$ in the effective string action.

\[1\] Another test is to compare the absorption of minimally coupled scalars in higher partial waves, which appears to agree up to normalization factors \[25, 26\].
The organization of the paper is as follows. In section 2 we discuss the simplest situation where a fixed scalar arises: the $D = 4$ example, which was studied in [23] for equal charges. We show how to decouple the fixed scalar fluctuations from gravity even for unequal charges and derive the resulting equation. In section 3 we proceed to the more complicated $D = 5$ example, whose advantage is that it can be directly compared with the effective string. We derive two decoupled equations for appropriate mixtures of the original fields $\nu$ and $\lambda$. Comparison of the resulting greybody factors with those that follow from the effective string calculations is presented in section 4. We conclude in section 5.

2. The $D = 4$ case

First, we consider the simpler case of the extremal black hole in $D = 4$ with two $U(1)$ charges and one fixed scalar (the dilaton) [27,23]. The action to which this black hole is a solution is

$$S = \int d^4x \sqrt{-g} [R - 2(\partial_\mu \phi)^2 - e^{-2\phi} F_{\mu\nu}^2 - e^{2\phi} G_{\mu\nu}^2] .$$

The resulting equations of motion are:

$$\partial_\mu (\sqrt{-g} e^{-2\phi} F^{\mu\nu} ) = \partial_\mu (\sqrt{-g} e^{2\phi} G^{\mu\nu} ) = 0 ,$$

$$ (\partial_\mu \phi)^2 + \frac{1}{2} e^{-2\phi} F^2 - \frac{1}{2} e^{2\phi} G^2 = 0 ,$$

$$R_{\mu\nu} + 2\partial_\mu \phi \partial_\nu \phi + e^{-2\phi} (2F_{\mu\lambda} F_{\nu\delta} g^{\lambda\delta} - \frac{1}{2} g_{\mu\nu} F^2) + e^{2\phi} (2G_{\mu\lambda} G_{\nu\delta} g^{\lambda\delta} - \frac{1}{2} g_{\mu\nu} G^2) = 0 .$$

We are looking for spherically symmetric perturbations, so we will take the metric to be of the form

$$ds^2 = -e^{2A} dt^2 + e^{2B} dr^2 + r^2 e^{-2U} d\Omega_2^2 ,$$

where $A$, $B$ and $U$ depend on $r$ and $t$ only. The gauge invariance present in the problem will later allow us to specify the precise form of the function $U$.

Since we are interested in solutions with fixed charges, we first solve for the $U(1)$ fields. From (1) we have

$$\partial_r (r^2 e^{A+B-2U-2\phi} F^{rt}) = \partial_r (r^2 e^{A+B-2U+2\phi} G^{rt}) = 0 .$$

Let the $F$-field carry charge $Q$ and the $G$-field carry charge $P$. Then we get

$$F^{rt} = \frac{Q e^{-A-B+2U+2\phi}}{r^2} , \quad F^2 = \frac{-2Q^2}{r^4} e^{4U+4\phi} ,$$

and

$$G^{rt} = \frac{P e^{-A-B+2U-2\phi}}{r^2} , \quad G^2 = \frac{-2P^2}{r^4} e^{4U-4\phi} .$$
Substituting (3) and (4) into (2) we obtain
\[-\partial_t^2 \phi + \frac{1}{\sqrt{-g}} \partial_r (\sqrt{-g} \partial_r \phi) - \frac{Q^2}{r^4} e^{4U+2\phi} + \frac{P^2}{r^4} e^{4U-2\phi} = 0. \tag{7}\]

We are interested in deriving the fluctuation equation for \(\phi\) around the static black hole solution. This solution is [27]:
\[ds_0^2 = -e^{2U} dt^2 + e^{-2U} (dr^2 + r^2 d\Omega^2), \tag{8}\]
\[e^{-2U} = H_1 H_2, \quad e^{2\phi_0} = \frac{H_2}{H_1}, \tag{9}\]
\[F = \frac{1}{\sqrt{2}} dH_1^{-1} \wedge dt, \quad G = \frac{1}{\sqrt{2}} dH_2^{-1} \wedge dt, \]
\[H_1 = 1 + \frac{\sqrt{2} Q}{r}, \quad H_2 = 1 + \frac{\sqrt{2} P}{r}. \tag{10}\]

We now let both the metric and \(\phi\) fluctuate, taking the metric to be of the form (4),
\[A = U(r) + \delta A(r,t), \quad B = -U(r) + \delta B(r,t), \quad \phi = \phi_0(r) + \delta \phi(r,t).\]

That is, we keep the angular part of the metric fixed, which we can achieve by a gauge transformation. Note that the fluctuations are functions of \(r\) and \(t\) only. For the \(\phi\) field this means that we consider only the \(l = 0\) partial wave. At low frequencies, this gives the dominant contribution to the absorption cross-section.

We will solve the equations of motion to first order in the fluctuations. First, since we are keeping the charges fixed, the expressions for the \(U(1)\) fields are as above. We now turn to the gravity equations. The ‘\(rt\)’ component of the Ricci tensor is
\[R_{rt} = -2r^{-1} \dot{B} (1 - rU'), \]
and consequently the ‘\(rt\)’ equation is
\[-2r^{-1} \dot{B} (1 - rU') + 2\dot{\phi} \phi' = 0. \]

Taking the variation, and remembering that \(\phi_0\) is time-independent, we obtain
\[\delta \dot{B} = \frac{r \phi_0'}{1 - rU'} \delta \dot{\phi}. \]

This may be integrated to give
\[\delta B = \frac{r \phi_0'}{1 - rU'} \delta \phi. \tag{11}\]
Next, we use the angular Einstein equation (the ‘θθ’ and ‘φφ’ components yield the same equation):

\[-1 - re^{-2U-2B}[(B' + U' - A')(1 - rU') + U' + rU'' - r^{-1}(1 - rU')^2] - \frac{1}{2}e^{-2\phi}g_{\theta\theta}F^2 - \frac{1}{2}e^{2\phi}g_{\theta\theta}G^2 = 0.\]

Inserting the expressions for the fields and taking the variation, we obtain

\[\delta A' - \delta B' = \frac{-2\delta B}{r(1 - rU')} (r^2U'' + 2rU' - 1) - \frac{2e^{2U}}{r^3(1 - rU')} (Q^2e^{2\phi_0} - P^2e^{-2\phi_0}) \delta \phi .\]  

Equations (11) and (12) will be sufficient to decouple the fixed scalar fluctuations from the gravity fluctuations.

We now turn to the fixed scalar equation (7). Taking the variation, and considering fluctuations of the form \(e^{i\omega t}\delta \phi(r)\), we get

\[r^{-2}\partial_r(r^2\partial_r\delta \phi) + \omega^2e^{-4U}\delta \phi - 2\delta Br^{-2}\partial_r(r^2\partial_r\phi_0) - \phi_0'(\delta B' - \delta A') - \left(\frac{2Q^2}{r^4}e^{2U+2\phi_0} + \frac{2P^2}{r^4}e^{2U-2\phi_0}\right) \delta \phi = 0.\]

Substituting for \(\delta A' - \delta B'\) from (12) and for \(\delta B\) from (11), as well as for \(U\) and \(\phi_0\) from (9), we find that the dilaton fluctuations obey the following simple equation:

\[
\left[ r^{-2}\partial_r r^2\partial_r + \omega^2(H_1H_2)^2 - \frac{4(P + Q)^2}{r^2(\sqrt{2}P + \sqrt{2}Q + 2r)^2} \right] \delta \phi = 0. \tag{13}
\]

This is essentially the same equation as that obtained in [23] for the special case \(P = Q\), but with the charge \(P\) in the potential replaced by the average of the two charges, \((P + Q)/2\). We see that the unmixing of the gravitational fluctuations for unequal charges results in the same type of equations as found for equal charges, but with new parameters. Remarkably, this phenomenon occurs also for the \(D = 5\) black hole which we discuss next.

3. The \(D = 5\) case

In this section we address our main goal: decoupling the fixed scalar fluctuations for the five-dimensional black hole with three unequal charges. The action to which this black hole is a solution is [24]

\[S = \frac{1}{2\kappa_5^2} \int d^5x \sqrt{-g} \left[ R - \frac{4}{3}(\partial_\mu \lambda)^2 - 4(\partial_\mu \nu)^2 - \frac{1}{4}e^{\frac{2}{3}\lambda}F^{(K)}_{\mu\nu} - \frac{1}{4}e^{-\frac{2}{3}\lambda+4\nu}F_{\mu\nu}^2 - \frac{1}{4}e^{-\frac{2}{3}\lambda-4\nu}H_{\mu\nu}^2 \right]. \tag{14}\]
We omit the dilaton \( \phi \), which in this case is a minimally coupled scalar, since it can be set to 0 in what follows.

We will now proceed in precise analogy with the four-dimensional case, with the only differences lying in technical details. We take the metric to be of the form

\[
ds_5^2 = -e^{2A}dt^2 + e^{2B}dr^2 + r^2 e^{-2U}d\Omega_3^2 ,
\]

where \( A, B, \) and \( U \) are functions of \( r \) and \( t \) only. The equations obtained by varying with respect to the \( U(1) \) fields are:

\[
\partial_{\mu}(\sqrt{-g}e^{\frac{8}{3}\lambda}F^{(K)\mu\nu}) = \partial_{\mu}(\sqrt{-g}e^{-\frac{4}{3}\lambda+4\nu}F^{\mu\nu}) = \partial_{\mu}(\sqrt{-g}e^{-\frac{4}{3}\lambda-4\nu}H^{\mu\nu}) = 0 .
\]

The \( U(1) \) fields will carry fixed charges \( Q_K, Q \) and \( P \). Solving (14) we get

\[
F^{(K)rt} = \frac{2QK}{r^3}e^{-A-B+3U-\frac{4}{3}\lambda} , \quad F^{(K)2} = -\frac{8Q^2}{r^6}e^{6U-4\frac{8}{3}\lambda} ,
\]

\[
F^{rt} = \frac{2Q}{r^3}e^{-A-B+3U+\frac{4}{3}\lambda-4\nu} , \quad F^2 = -\frac{8Q^2}{r^6}e^{6U+\frac{4}{3}\lambda-8\nu} ,
\]

\[
H^{rt} = \frac{2P}{r^3}e^{-A-B+3U+\frac{4}{3}\lambda+4\nu} , \quad H^2 = -\frac{8P^2}{r^6}e^{6U+\frac{4}{3}\lambda+8\nu} .
\]

By varying (14) with respect to the metric we get the following equations of motion,

\[
R_{\mu\nu} + \frac{4}{3}\partial_\mu\lambda\partial_\nu\lambda - 4\partial_\mu\nu\partial_\nu\lambda + e^{\frac{8}{3}\lambda}\left(\frac{1}{2}F^{(K)\mu\nu}F^{(K)}_{\nu\delta} - \frac{1}{8}g_{\mu\nu}F^{(K)^2}\right) + e^{-\frac{4}{3}\lambda+4\nu}\left(\frac{1}{2}F_{\mu\lambda}F_{\nu\delta} - \frac{1}{8}g_{\mu\nu}F^2\right)
\]

\[
+ e^{-\frac{4}{3}\lambda-4\nu}\left(\frac{1}{2}H_{\mu\lambda}H_{\nu\delta} - \frac{1}{8}g_{\mu\nu}H^2\right) = 0 .
\]

The equations for the fixed scalars are, after inserting the metric and fields from (15), (17), (18) and (14),

\[
\partial_t\left(\frac{8}{3}r^3e^{-3U+B-A\lambda}\right) - \partial_r\left(\frac{8}{3}r^3e^{-3U+A-B\lambda}\right) -
\]

\[
-2r^{-3}e^{A+B+3U}\left[\frac{8}{3}Q^2Ke^{-\frac{8}{3}\lambda} - \frac{4}{3}P^2e^{\frac{4}{3}\lambda+4\nu} - \frac{4}{3}Q^2e^{\frac{4}{3}\lambda-4\nu}\right] = 0 \quad (21)
\]

and

\[
\partial_t(8r^3e^{-3U+B-A\nu}) - \partial_r(8r^3e^{-3U+A-B\nu}) -
\]

\[
-2r^{-3}e^{A+B+3U}\left[-4P^2e^{\frac{4}{3}\lambda+4\nu} + 4Q^2e^{\frac{4}{3}\lambda-4\nu}\right] = 0 . \quad (22)
\]

We are interested in deriving the fixed scalar fluctuation equations around the static solution given in (20), (21) and (22),

\[
e^{-2U} = (H_\bar{Q}KH_{\bar{H}Q})^{1/3} , \quad B_0 = -U - \frac{1}{2}\ln h , \quad A_0 = 2U + \frac{1}{2}\ln h ,
\]
\begin{equation}
e^{2\lambda_0} = H_q^* (H_q H_p)^{-1/2}, \quad e^{4\nu_0} = H_q H_p^{-1}, \tag{23}
\end{equation}

where

\begin{align*}
h &= 1 - \frac{r_0^2}{r^2}, \quad H_q = 1 + \frac{\hat{q}}{r^2}, \quad \hat{q} = \sqrt{q^2 + \frac{r_0^4}{4} - \frac{r_0^2}{2}}, \quad q = Q_K, Q, P.
\end{align*}

Here \( r_0 \) is the radius of the horizon, i.e. the parameter governing the non-extremality of the solution.

We now let the metric and the fixed scalars vary, keeping the angular part of the metric fixed. Thus we have

\begin{align*}
A &= A_0 + \delta A, \quad B = B_0 + \delta B, \quad \lambda = \lambda_0 + \delta \lambda, \quad \nu = \nu_0 + \delta \nu,
\end{align*}

but \( U \) is kept fixed. Again, we can do this because of the gauge freedom. We allow the fluctuations to depend on \( t \) and \( r \) only, since for sufficiently low frequencies the \( l = 0 \) partial wave will give the dominant contribution to the absorption cross-section.

To decouple the fixed scalar fluctuations from the metric fluctuations, we look, as before, at the \('rt'\) and the angular Einstein equations. The \('rt'\) component of the Ricci tensor \( R_{\mu\nu} \) is

\begin{equation}
R_{rt} = -3r^{-1}(1 - rU')\dot{B}.
\end{equation}

The corresponding equation of motion is found from (20) to be

\begin{equation}
-3r^{-1}(1 - rU')\dot{B} + \frac{4}{3} \lambda' \dot{\lambda} + 4 \nu' \dot{\nu} = 0.
\end{equation}

Taking the variation, we find

\begin{equation}
\delta \dot{B} = \frac{r}{3(1 - rU')} \left( \frac{4}{3} \lambda'_0 \delta \lambda + 4 \nu'_0 \delta \nu \right).
\end{equation}

This is integrated to give

\begin{equation}
\delta B = \frac{r}{3(1 - rU')} \left( \frac{4}{3} \lambda'_0 \delta \lambda + 4 \nu'_0 \delta \nu \right). \tag{24}
\end{equation}

\( \delta \lambda' \)From (20) the angular Einstein equation is found to be

\begin{align*}
-2 - e^{-2U - 2B} r [ (B' + U' - A')(1 - rU') + U' + r U'' - 2r^{-1}(1 - rU')^2 ] + \\
+ \frac{2e^{4U}}{3r^4} [ Q_K e^{-\frac{8}{3} \lambda} + Q^2 e^{\frac{4}{3} \lambda - 4 \nu} + p^2 e^{\frac{4}{3} \lambda + 4 \nu} ] = 0.
\end{align*}

Taking the variation, we find

\begin{align*}
\delta A' - \delta B' &= -\frac{2\delta B}{r(1 - rU')} \left[ r^2 U'' + 3r U' - 2 - \frac{r h'(1 - rU')}{h} \right] + \\
&\quad + \frac{2e^{4U}}{3r^4} [ Q_K e^{-\frac{8}{3} \lambda} + Q^2 e^{\frac{4}{3} \lambda - 4 \nu} + p^2 e^{\frac{4}{3} \lambda + 4 \nu} ] = 0.
\end{align*}
\[ + \frac{2h^{-1}e^{4U}}{3r^5(1-rU')} \left[ \frac{8Q^2_K}{3} e^{-\frac{4}{3}\lambda_0} \delta \lambda - Q^2 e^{\frac{4}{3}\lambda_0 - 4\nu_0} \left( \frac{4}{3} \delta \lambda - 4 \delta \nu \right) - P^2 e^{\frac{4}{3}\lambda_0 + 4\nu_0} \left( \frac{4}{3} \delta \lambda + 4 \delta \nu \right) \right] . \]

Again, the relations (24) and (25) will suffice to decouple the fixed scalar fluctuations from the gravity fluctuations. Taking the variations of the fixed scalar equations (21) and (22) with frequency \( \omega \), and using (24), (25) and (23), we get the following two coupled equations

\[ [r^{-3} \partial_r h r^3 \partial_r + \omega^2 h^{-1} H_{\tilde{Q}K} H_{\tilde{Q}H} + f_1(r)] \delta \tilde{\lambda} + \sqrt{3} f_2(r) \delta \nu = 0 \]  

(26)

and

\[ [r^{-3} \partial_r h r^3 \partial_r + \omega^2 h^{-1} H_{\tilde{Q}K} H_{\tilde{Q}H} + f_3(r)] \delta \nu + \sqrt{3} f_2(r) \delta \tilde{\lambda} = 0 , \]  

(27)

where we have defined

\[ \delta \lambda = \sqrt{3} \delta \tilde{\lambda} , \]

so that the kinetic terms for \( \delta \nu \) and \( \delta \tilde{\lambda} \) have the same normalization in the action. The functions entering the fixed scalar equations have the following form,

\[
 f_1(r) = - \frac{8}{r^2 [P \hat{Q} + \hat{P} \hat{Q}_K + \hat{Q} \hat{Q}_K + 2(\hat{P} + \hat{Q} + \hat{Q}_K) r^2 + 3 r^4]} \times \left[ \hat{P}^2 \hat{Q}^2 + \hat{P}^2 \hat{Q}_K^2 + \hat{Q}^2 \hat{Q}_K^2 + 2 \hat{P} \hat{Q} \hat{Q}_K + 2 \hat{P} \hat{Q}_K \hat{Q} + 2 \hat{Q} \hat{Q}_K \hat{P} + 2 \hat{Q}_K \hat{P} \hat{Q} + 2 \hat{Q}_K \hat{P} \hat{Q}_K + 2 \hat{Q}_K \hat{Q}_K \hat{P} + \hat{Q}_K \hat{Q}_K \hat{P} \right] ,
\]

\[
 f_2(r) = \frac{8(\hat{Q} - \hat{P})}{r^2 [P \hat{Q} + \hat{P} \hat{Q}_K + \hat{Q} \hat{Q}_K + 2(\hat{P} + \hat{Q} + \hat{Q}_K) r^2 + 3 r^4]} \times \left[ - \frac{1}{2} r_0^2 (\hat{P} \hat{Q} + \hat{P} \hat{Q}_K + \hat{Q} \hat{Q}_K) \right] ,
\]

\[
 f_3(r) = - \frac{8}{r^2 [P \hat{Q} + \hat{P} \hat{Q}_K + \hat{Q} \hat{Q}_K + 2(\hat{P} + \hat{Q} + \hat{Q}_K) r^2 + 3 r^4]} \times \left[ \hat{P}^2 \hat{Q}^2 + \hat{P}^2 \hat{Q}_K^2 + \hat{Q}^2 \hat{Q}_K^2 + 2 \hat{P} \hat{Q} \hat{Q}_K + 2 \hat{P} \hat{Q}_K \hat{Q} + 2 \hat{Q} \hat{Q}_K \hat{P} + 2 \hat{Q}_K \hat{P} \hat{Q} + 2 \hat{Q}_K \hat{P} \hat{Q}_K + 2 \hat{Q}_K \hat{Q}_K \hat{P} + \hat{Q}_K \hat{Q}_K \hat{P} \right] ,
\]

Compared to the \( D = 4 \) case we now encounter the additional difficulty that the two fixed scalars couple to each other. Luckily, however, the fixed scalar equations (26) and (27) may be decoupled by a position-independent rotation of the fields,

\[ \delta \tilde{\lambda} = (\cos \alpha) \phi_+ + (\sin \alpha) \phi_- , \]

\[\tan \alpha = \frac{r_0}{r} \]
\[ \delta \nu = -(\sin \alpha) \phi_+ + (\cos \alpha) \phi_-, \]

where the rotation angle satisfies

\[ \tan \alpha - \frac{1}{\tan \alpha} = \frac{2}{\sqrt{3}} \frac{\hat{P} + \hat{Q} - 2\hat{Q}_K}{\hat{Q} - \hat{P}}. \]

(28)

Solving this quadratic equation, we find that

\[ \tan \alpha = \frac{1}{\sqrt{3}} \frac{\hat{P} + \hat{Q} - 2\hat{Q}_K \pm 2\sqrt{\hat{P}^2 + \hat{Q}^2 + \hat{Q}_K^2 - \hat{P}\hat{Q} - \hat{P}\hat{Q}_K - \hat{Q}\hat{Q}_K}}{\hat{Q} - \hat{P}}, \]

which implies that

\[ \cos^2 \alpha = \frac{1}{2} \pm \frac{1}{4} \frac{\hat{P} + \hat{Q} - 2\hat{Q}_K}{\sqrt{\hat{P}^2 + \hat{Q}^2 + \hat{Q}_K^2 - \hat{P}\hat{Q} - \hat{P}\hat{Q}_K - \hat{Q}\hat{Q}_K}}. \]

Using this result, we find that \( \phi_\pm \) satisfy the following simple equations,

\[ \left[ r^{-3} \partial_r h r^2 \partial_r + \omega^2 h^{-1} H_{\hat{Q}_K} \hat{H} \hat{H} - \frac{8Q_\pm^2}{r^2 (r^2 + Q_\pm)^2} \left( 1 + \frac{r_0^2}{Q_\pm} \right) \right] \phi_\pm = 0, \]

(29)

where we have defined

\[ Q_\pm = \frac{1}{3} (\hat{P} + \hat{Q} + \hat{Q}_K \mp \sqrt{\hat{P}^2 + \hat{Q}^2 + \hat{Q}_K^2 - \hat{P}\hat{Q} - \hat{P}\hat{Q}_K - \hat{Q}\hat{Q}_K}). \]

Note that these equations are manifestly symmetric under interchange of any pair of the charges, i.e. U-duality invariant. This is a nice consistency check on our results.\footnote{This check was suggested by A. Tseytlin.}

Calculation of the absorption cross-sections from (29) in the "dilute gas regime" \( (r_0^2, Q_K \ll P, Q) \) is analogous to that presented in [24], and we find that the greybody factors are proportional to the \( \nu \) greybody factor found for \( Q = P \). The coefficient of proportionality is a function of \( P \) and \( Q \). The absorption cross-sections for \( \phi_\pm \) are

\[ \sigma_\pm = \frac{9\pi^3 P^3 Q^3}{64(P + Q + \sqrt{P^2 - PQ + Q^2})^2} \frac{\omega (e^{\pi T_H} - 1)}{(e^{\pi T_L} - 1)(e^{\pi T_R} - 1)} (\omega^2 + 16\pi^2 T_L^2)(\omega^2 + 16\pi^2 T_R^2), \]

(30)

where \( T_H \) is the Hawking temperature, while \( T_L \) and \( T_R \), which are determined by \( r_0 \) and the charges [15], play the role of the left- and right-moving temperatures on the effective string. In the next section we compare (30) with the results of the effective string model.
4. Comparison of Greybody factors

The greybody factors one finds from the two equations (29) in general disagree with the predictions of the effective string action derived in [24]. This even happens for \( Q = P \) where there is no mixing between \( \nu \) and \( \lambda \). In [24] agreement was found for the scalar field \( \nu \). However, now that we have derived the equation for \( \lambda \), we will see that for this scalar there is no agreement.

Setting \( P = Q \) in (30), we find that the classical absorption cross-section for \( \lambda \) is

\[
\sigma_{\text{abs}}(\omega) = \frac{9\pi^3 P^4}{64} \frac{\omega \left( e^{\frac{\omega}{T_L}} - 1 \right) \left( e^{\frac{\omega}{T_R}} - 1 \right)}{\left( e^{\frac{\omega}{T_L}} - 1 \right) \left( e^{\frac{\omega}{T_R}} - 1 \right) \left( \omega^2 + 16\pi^2 T_L^2 \right) \left( \omega^2 + 16\pi^2 T_R^2 \right) } , \tag{31}
\]

On the effective string side, the \( \lambda \)-coupling is [24]

\[
-\frac{T_{\text{eff}}}{8} \lambda \left[ \partial_+ X \partial_- (\partial_+ X)^2 + (\partial_- X)^2 + (\partial_+ X)(\partial_- X)^2 \right] \tag{32}
\]

plus the fermionic terms required by supersymmetry (\( T_{\text{eff}} \) is the effective string tension). The last term is an operator of dimension \( (2, 2) \) which also couples to \( \nu \). Its effects were studied in [24], and the resulting contribution to the cross-section is

\[
\sigma_1(\omega) = \frac{\pi P^2}{1024 T_{\text{eff}}^2} \frac{\omega \left( e^{\frac{\omega}{T_L}} - 1 \right) \left( e^{\frac{\omega}{T_R}} - 1 \right)}{\left( e^{\frac{\omega}{T_L}} - 1 \right) \left( e^{\frac{\omega}{T_R}} - 1 \right) \left( \omega^2 + 16\pi^2 T_L^2 \right) \left( \omega^2 + 16\pi^2 T_R^2 \right) } , \tag{33}
\]

which is proportional to (31). However, there are additional contributions to the cross-section arising from the first two operators in (32) which have dimensions \( (3, 1) \) and \( (1, 3) \). These operators give rise to processes involving 3 left-movers and 1 right-mover or 3 right-movers and 1 left-mover.

Let us consider first the processes with 3 left-movers and 1 right-mover. Using the methods of [24] we find that their contribution to the absorption rate is

\[
\sim \frac{\kappa_5^2 L_{\text{eff}}}{T_{\text{eff}}^2} \frac{1}{1 - e^{-\frac{\omega}{T_R}}} \int_{-\infty}^{\infty} dp_1 dp_2 dp_3 \delta \left( p_1 + p_2 + p_3 - \frac{\omega}{2} \right) \frac{p_1}{1 - e^{-\frac{p_1}{T_L}}} \frac{p_2}{1 - e^{-\frac{p_2}{T_L}}} \frac{p_3}{1 - e^{-\frac{p_3}{T_L}}} \\
\sim \frac{\kappa_5^2 L_{\text{eff}}}{T_{\text{eff}}^2} \frac{\omega}{\left( 1 - e^{-\frac{\omega}{T_L}} \right) \left( 1 - e^{-\frac{\omega}{T_R}} \right)} \left( \omega^2 + 16\pi^2 T_L^2 \right) \left( \omega^2 + 32\pi^2 T_L^2 \right) . \tag{34}
\]

Processes with 3 left-movers and 1 right-mover make a contribution with \( T_L \) interchanged with \( T_R \). Converting the rate to the absorption cross-section using detailed balance, we find the following additional contribution on the effective string side,

\[
\sigma_2(\omega) \sim \frac{\kappa_5^2 L_{\text{eff}}}{T_{\text{eff}}^2} \frac{\omega \left( e^{\frac{\omega}{T_R}} - 1 \right) \left( e^{\frac{\omega}{T_H}} - 1 \right)}{\left( e^{\frac{\omega}{T_R}} - 1 \right) \left( e^{\frac{\omega}{T_H}} - 1 \right) \left( \omega^2 + 16\pi^2 T_L^2 \right) \left( \omega^2 + 32\pi^2 T_L^2 \right) + \left( \omega^2 + 16\pi^2 T_R^2 \right) \left( \omega^2 + 32\pi^2 T_R^2 \right) } .
\]
Thus, there is no agreement for the $\lambda$ greybody factors. At extremality (for $T_R = 0$) $\sigma_2$ dominates over $\sigma_1$ for small $\omega$. This is because $\sigma_1 \sim \omega^2$ while

$$\sigma_2 \sim \frac{\kappa_5^2 L_{\text{eff}} \tau_5}{T_{\text{eff}}^2}.$$ 

This behavior is in marked disagreement with the fact that $\sigma_{\text{class}} \sim \omega^2$.

We have shown that there is some disagreement between the semiclassical and the effective string cross-sections even for $P = Q$. This could be traced to the presence of dimension (1, 3) and (3, 1) operators in the $\lambda$-coupling, which are coming from the $h_{55}$ part of $\lambda$. Even more mysterious from the effective string point of view is the mixing between $\lambda$ and $\nu$ induced by $P \neq Q$. If one takes the lagrangian derived in [24] at face value, then both these mixtures now have coupling to dimension (1, 3) and (3, 1) operators, which implies disagreement of the greybody factors for both of them.

5. Conclusions

Let us summarize our results. The form of the semiclassical greybody greybody factors suggests that both $\nu$ and $\lambda$ couple to dimension (2, 2) operators on the effective string. However, the fact that $\lambda$ contains $h_{55}$ implies that dimension (1, 3) and (3, 1) operators are also present in the coupling. One possibility of restoring agreement between the supergravity and the effective string is by finding an overlooked mixing with yet another scalar field. In fact, a surprising new mixing was recently found for fields which couple to effective string operators of dimension (1, 2) and (2, 1) [29]. However, we have not been able to find a scalar that mixes with $\nu$ and $\lambda$, and in general our calculations exhibit a marked disagreement between the semiclassical and the effective string greybody factors for these fixed scalars.

We may attempt a different approach: rather than try to derive the string action, as was done in [24], we could simply guess the terms that reproduce the semiclassical greybody factors [30]. Although this type of modeling is not predictive, we indeed find that a coupling of the form

$$\int d^2 \sigma (c_+(P, Q) \phi_+ + c_-(P, Q) \phi_-) T_{++} T_{--},$$

(35)

where $T_{\alpha\beta}$ is the energy-momentum tensor, can lead to agreement provided that the functions $c_{\pm}(P, Q)$ are appropriately chosen. It seems difficult, however, to explain the peculiar form of these functions.

We have shown that the fixed scalars pose a challenge for the effective string models of black holes. It will be interesting to see whether there is a way out of this difficulty.

6. Acknowledgements

We are grateful to A. Tseytlin for constructive suggestions and for many valuable discussions. This work was supported in part by DOE grant DE-FG02-91ER40671, the NSF Presidential Young Investigator Award PHY-9157482, and the James S. McDonnell Foundation grant No. 91-48.
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