On the ratcheting of the VT6 alloy in a range of loading scenarios

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Abstract. A set of experimental results concerning the accumulation of inelastic strain in the VT6 titanium alloy is presented. The considered tests include cyclic stress-controlled loading with monotonically increasing stress amplitude and constant mean stress. The focus of the study is on the accurate phenomenological modelling of the complex material behaviour. Four different material models are employed to capture the phenomena of ratcheting and nonlinear kinematic hardening. Two models are of the Armstrong-Frederick type and the remaining two models are of the Ohno-Wang type. Since all the considered models contain a big number of material parameters, a sophisticated nested procedure is used for the parameter identification; the obtained material parameters are validated by additional experiments. It is shown that all the models yield nearly the same accuracy; the limitations of the phenomenological approach are highlighted and discussed.

1. Introduction

The phenomenon of ratcheting, also known as cyclic creep, consists in progressive accumulation of strain under stress-controlled cyclic loading. Ratcheting of metals is of high interest for a number of engineering applications since it may lead to a failure of a structure; the ratcheting also causes a redistribution of stresses, thus affecting the service lifetime. In this study we analyse the cyclic creep of the titanium alloy VT6 (Russian analog of Ti-6Al-4V) in the range of low cycle fatigue.

There is a big number of practical methods used to estimate the fatigue life based on the shape of the stress-strain hysteresis loops, cf. the approaches of Smith-Watson-Topper, Coffin-Manson and others \cite{2}. Therefore, an accurate prediction of the hysteresis stress-strain behaviour is important for the assessment of the fatigue strength. It is well understood that under uniaxial cyclic loading conditions, nonlinear kinematic hardening needs to be accounted for \cite{1}. For multiaxial ratcheting, however, models with yield surface distortion are needed, cf. \cite{3, 4, 8}. One class of models considered here includes the concept of Armstrong-Frederick (AF), and the other one encompasses a multi-linear hardening of Ohno-Wang type (OW). The isotropic hardening is captured by the exponential ansatz proposed by Voce. In the original versions of the models, the kinematic hardening is fully decoupled from the isotropic hardening; in extended models considered here, the isotropic hardening amplifies the Bauschinger effect.

Since the considered models contain a large number of material parameters, a sophisticated nested procedure of parameter identification is utilized. Best possible fit of the simulation results to the experimental data is shown. The identified sets of parameters are validated through additional experiments.
2. Experimental results
A preliminary experimental study of the cyclic creep of VT6 is presented in [11]. The stress-controlled loading program with the mean stress $\sigma_m$ and the maximum amplitude $\sigma_{a\,\text{max}}$ is shown in figure 1. The loading is characterized by a linearly increasing stress amplitude. In each test, a total number of 2400 cycles were accomplished; at the beginning of the cyclic part, the deformation is purely elastic. Such a set-up allows us to analyse the smooth transition of the material from the elastic to the elasto-plastic state.

![Figure 1. Stress-controlled loading program.](image)

Three different tests are considered here with a different value of the mean stress. Maximum and minimum axial strains recorded within each cycle are available for $\sigma_m = 420$ MPa, $\sigma_{a\,\text{max}} = 470$ MPa (figure 2a), $\sigma_m = 530$ MPa, $\sigma_{a\,\text{max}} = 360$ MPa (figure 2b), and $\sigma_m = 635$ MPa, $\sigma_{a\,\text{max}} = 255$ MPa (figure 2c). Note that all three tests share the same maximum achievable stress $\sigma_m + \sigma_{a\,\text{max}} = 890$ MPa.

3. Material models
Here we present small-strain versions of the material models. However, they can be generalized to large strains using the modelling steps from [5], [7]. Only uniaxial loading scenarios are considered here. For this type of loading, models of combined isotropic-kinematic hardening are fully sufficient.

Let us first specify equations which are common for all the models considered in the study. Since in the conducted experiments the temperature increase was well below 10 K, we neglect the thermal effects. The total strain tensor $\varepsilon$ is additively decomposed into the elastic part $\varepsilon_e$ and the inelastic part $\varepsilon_i$: $\varepsilon = \varepsilon_e + \varepsilon_i$. A number of rheological branches will be used: Four branches of rate-independent Maxwell material for the AF-model and four branches of the Prandtl-Reuss model for the multi-linear Ohno-Wang model. For the $k$-th branch we introduce its conservative ($\varepsilon_{ke}$) and dissipative ($\varepsilon_{ki}$) strains such that $\varepsilon_i = \varepsilon_{ke} + \varepsilon_{ki}$, $k = 1, \ldots, 4$. The stress tensor $\sigma$ is given by the Hooke law with the bulk and shear moduli $k > 0$ and $\mu > 0$, respectively; backstresses $X_k$ in the branches $k = 1, \ldots, 4$ are obtained as linear isotropic functions of $\varepsilon_{ke} = \varepsilon_i - \varepsilon_{ki}$:

$$\sigma = k\text{tr}(\varepsilon - \varepsilon_i) + 2\mu(\varepsilon - \varepsilon_i)^D,$$

$$X_k = c_k(\varepsilon_i - \varepsilon_{ki})^D, \quad k = 1, \ldots, 4,$$

![Figure 2. Experimental results on the uniaxial ratcheting of the VT6 alloy.](image)
where $A^D = A - \frac{1}{3}\text{tr}(A)I$; $c_k > 0$ is a pseudo elastic stiffness. The effective stress $\sigma^{\text{effect}}$ equals then

$$\sigma^{\text{effect}} = \sigma - \sum_{k=1}^{4} X_k.$$ 

Let $K > 0$ be the initial, uniaxial, static yield stress. Denote by $R \in \mathbb{R}$ the isotropic hardening. Then the viscous overstress $f$ and the rate of the inelastic flow $\dot{\lambda}_i = ||\dot{\varepsilon}_i||$ are computed according to

$$f := ||(\sigma^{\text{effect}})^D|| - \sqrt{\frac{2}{3}(K + R)}, \quad \lambda_i = \frac{1}{\eta} \langle f \rangle^m, \quad \langle x \rangle := \max(x, 0).$$

Here, $\eta > 0$ and $m > 0$ are material parameters governing the viscosity of Perzyna type; $f_0 = 1$ MPa. The evolution of the isotropic hardening $R$ is modelled according to the Voce rule

$$R = \gamma(s - s_d), \quad \text{where} \quad s = \sqrt{\frac{2}{3}} \lambda_i, \quad s_d = \frac{\beta}{\gamma} s R.$$ 

Here, $s$ is the accumulated inelastic arc length (Odqvist parameter), $s_d$ is its dissipative part, $\gamma \in \mathbb{R}$ and $\beta \geq 0$ are material parameters. The cases of isotropic hardening and softening correspond to $\gamma > 0$ and $\gamma < 0$, respectively. The initial state is isotropic and stress free. Therefore, the initial conditions are:

$$\varepsilon_{i|t=0} = \varepsilon_{k|t=0} = \ldots = \varepsilon_{4i|t=0} = \varepsilon_{k|t=0} = 0, \quad s|_{t=0} = s_d|_{t=0} = 0.$$ 

The global flow rule governs the evolution of $\dot{\varepsilon}_i$ according to the normality rule:

$$\dot{\varepsilon}_i = \lambda_i \frac{(\sigma^{\text{effect}})^D}{||(\sigma^{\text{effect}})^D||}.$$ 

**Models of Armstrong-Frederick type.** For the material model with the nonlinear kinematic hardening of AF-type we postulate the following flow rule on the substructural level

$$\dot{\varepsilon}_{ki} = \lambda_k \varkappa_k X_k, \quad k = 1, \ldots, 4.$$ 

Here, $\varkappa_k \geq 0$ is a material parameter governing the saturation of backstress in the $k$-th branch. Note that in the classical version of the AF-hardening, the parameter $\varkappa_k$ is constant. This setup corresponds to the so-called rate-independent (endochronic) Maxwell material. In this model, the isotropic and kinematic types of hardening are decoupled. However, in order to account for the impact of the isotropic hardening $R$ on the Bauschinger effect we can consider the following modification:

$$\dot{\varepsilon}_{ki} = \lambda_k \varkappa_k \frac{K}{K + R} X_k, \quad k = 1, \ldots, 4.$$ 

Recall that $R = R(s)$. In the modified version of the AF-hardening model, $\varkappa_k \geq 0$ is still a fixed material parameter and the multiplier $\lambda_k \varkappa_k \frac{K}{K + R}$ becomes a function of the Odqvist parameter $s$.

**Models of Ohno-Wang type.** First, we consider the multi-linear model of Ohno-Wang, which is popular in ratcheting-related applications (see Model I in [6]). Each branch is now modelled as the elastico-plastic Prandtl-Reuss body. The constitutive equations of the $k$-th branch are:

$$\dot{\varepsilon}_{ki} = \lambda_k \frac{X_k}{||X_k||}, \quad \lambda_k \geq 0, \quad ||X_k|| \leq \sqrt{\frac{7}{3} r_k}.$$
Here, $\lambda_{ki}$ is the inelastic strain rate in the $k$-th branch and $r_k > 0$ is the corresponding yield stress. As is typical for rate-independent plasticity, the following Kuhn-Tucker conditions are used:

$$\lambda_{ki} \left( ||X_k|| - \sqrt{\frac{2}{3} r_k} \right) = 0.$$ 

Again, within the classical model, the isotropic and kinematic hardening are decoupled. In the modified Ohno-Wang model we let the yield stresses of individual branches evolve according to the hardening $R$:

$$\dot{r}_k = \lambda_{ki} \frac{X_k}{||X_k||}, \quad \lambda_{ki} \geq 0, \quad ||X_k|| \leq \sqrt{\frac{2 K + R}{3} r_k}, \quad \lambda_{ki} \left( ||X_k|| - \sqrt{\frac{2 K + R}{3} r_k} \right) = 0.$$ 

Thus, for increasing $R$, the yield stress in each branch is equally increased. Effectively, this yields a more pronounced Bauschinger effect in a hardening material.

All the four material models considered in this section are thermodynamically consistent. An efficient unconditionally stable numerical procedure is proposed for the AF-model in [9]. The models are implemented as Fortran routines using fully-functional multi-axial constitutive equations. Then, the uniaxial tension test is modelled using 1000 time steps.

### 4. Nested strategy of parameter identification

In this study we neglect viscous effects by setting the viscosity to zero ($\eta \rightarrow 0, m = 1$). The elastic constants of the material are: $k = 98037$ MPa, $\mu = 37593$ MPa. At this point, each of the models contains 11 unknown parameters. For what follows it is instructive to introduce the following subsets of the remaining material parameters: $\tilde{p}_c = (c_1, c_2, c_3, c_4, \gamma, \beta)$ for all the models; $\tilde{p}_K = (K, \kappa_1, \kappa_2, \kappa_3, \kappa_4)$ for the models of AF-type; $\tilde{p}_{K} = (K, r_1, r_2, r_3, r_4)$ for the models OW-type.

Recall that each of the experiments provides maximum and minimum strains at each cycle; for 2400 cycles this makes 4800 values. For parameter identification we use only experiments with $\sigma_m = 420$ MPa and $\sigma_m = 635$ MPa. In total, 4 x 2400 = 9600 data points are available. We put these data points into the vector $\bar{E}x_p \in \mathbb{R}^{9600}$. Next, we simulate both tests using the parameter set ($\tilde{p}_c, \tilde{p}_K$). The simulation results corresponding to the vector $\bar{E}x_p$ are encapsulated within the vector of the model response $\bar{M}od(\tilde{p}_c, \tilde{p}_K)$. Further, we build the target function (error functional) $\Phi(\tilde{p}_c, \tilde{p}_K)$ which reflects the average deviation of the simulation results from the experimental data:

$$\Phi(\tilde{p}_c, \tilde{p}_K) = (\bar{E}x_p - \bar{M}od(\tilde{p}_c, \tilde{p}_K)) \cdot (\bar{E}x_p - \bar{M}od(\tilde{p}_c, \tilde{p}_K)),$$

where the dot (⋅) stands for the scalar product in $\mathbb{R}^{9600}$. Note that the experimental data are equally weighted; stable identification strategies using non-trivial weights are discussed in [10]. The standard procedure of parameter identification is based on the minimization of the error functional $\Phi(\tilde{p}_c, \tilde{p}_K)$. Unfortunately, due to the large number of material parameters, the error functional $\Phi$ exhibits numerous local minima; this leads to a poor correspondence between simulation and experiment. Therefore, a nested identification procedure is used.

First, we consider the so-called internal optimization problem. For a fixed set $\tilde{p}_K$, parameters $\tilde{p}_c$ are identified from the following partial minimization problem:

$$\bar{P}_c(\tilde{p}_K) = \operatorname{argmin}_{\tilde{p}_c} \Phi(\tilde{p}_c, \tilde{p}_K).$$

(1)

Note that $\tilde{p}_K$ is held fixed during the internal optimization (1). Next, we proceed to the external optimization:

$$\bar{p}_K = \operatorname{argmin}_{\bar{p}_K} \Phi(\bar{P}_c(\bar{p}_K), \bar{p}_K), \quad \bar{p}_c = \bar{P}_c(\bar{p}_K).$$

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In this study, both minimization problems are solved with the Nelder-Mead procedure, which is a robust gradient free method with a low probability of undesired convergence to a local minimum.

The identification results for the models of AF- and OW-type are listed in tables 1 and 2, respectively. Note that the parameters of the original models are close to the corresponding parameters of their modified counterparts; slightly inconsistent results are only observed for the isotropic hardening parameters $\gamma$ and $\beta$. The best possible fit of the AF-type models is shown in figure 3(a) and 3(c); the best attainable identification results for the OW-models are presented in figures 4(a) and 4(c). A surprising conclusion is that the modified models render virtually the same output as their original counterparts. As a validation, the cyclic test with $\sigma_m = 530$ MPa is simulated with the identified material parameters, see figures 3(b) and 4(b) for the AF- and OW-type models, respectively.

**Table 1.** Material parameters for the models of Armstrong-Frederick type

|            | conservative parameters | dissipative parameters |
|------------|-------------------------|------------------------|
|            | $\gamma$[MPa] | $c_1$[MPa] | $c_2$[MPa] | $c_3$[MPa] | $c_4$[MPa] | $\beta$[-] | $\kappa_1$[1/MPa] | $\kappa_2$[1/MPa] | $\kappa_3$[1/MPa] | $\kappa_4$[1/MPa] | $K$[MPa] |
| AF model   | 920.6       | 3106       | 7696       | 14879      | 17636      | 8.769      | 0.0209               | 0.0395             | 0.0623             | 0.0894             | 809.15    |
| mod. AF model | 786.9       | 4542       | 5739       | 15717      | 18241      | 7.220      | 0.0221               | 0.0385             | 0.0679             | 0.0860             | 807.20    |

**Table 2.** Material parameters for the models of Ohno-Wang type

|            | conservative parameters | dissipative parameters |
|------------|-------------------------|------------------------|
|            | $\gamma$[MPa] | $c_1$[MPa] | $c_2$[MPa] | $c_3$[MPa] | $c_4$[MPa] | $\beta$[-] | $r_1$[MPa] | $r_2$[MPa] | $r_3$[MPa] | $r_4$[MPa] | $K$[MPa] |
| OW model   | 5606.6       | 5568       | 6885       | 15555      | 446        | 7.366      | 18.433        | 22.888        | 23.914        | 448.64        | 794.11    |
| mod. OW model | 7515.8       | 6142       | 12042      | 16948      | 1953       | 2.981      | 18.276        | 23.430        | 24.547        | 435.92        | 778.25    |
5. Discussion and conclusion

Experimental data on ratcheting of the titanium alloy VT6 are presented. Phenomenological material models of AF- and OW-types are considered. Modified models differ from their original counterparts in the way how the isotropic hardening affects the Bauschinger effect. In order to obtain a global minimum of the error functional, a nested optimization procedure is employed in combination with the robust Nelder-Mead minimization method. As is seen from figures 3 and 4, there is a conflict between the experimental results pertaining to $\sigma_m = 420$ MPa and $\sigma_m = 635$ MPa. Indeed, all the models underestimate the ratcheting for small $\sigma_m$ and overestimate it for large $\sigma_m$. Even highly refined material models cannot provide a sufficient accuracy. The reason for the relatively poor performance of the models is as follows. In the simulations of the cyclic test with $\sigma_m = 420$ MPa and $\sigma_m = 635$ MPa, nearly the same value of the Odqvist parameter is achieved: in both tests $s$ ranges up to $4.4 \cdot 10^{-3}$. Since in both tests $s$ is the same, so is the isotropic hardening $R = R(s)$, while the material response is different. In other words, the isotropic hardening rule of type $R = R(s)$ is shown to be too restrictive. Thus, a surprising result of the study is that the well known Odqvist parameter $s$ is not sufficient for an accurate description of the isotropic hardening in the VT6 titanium alloy. There is a quest for new simple parameters which could characterize the isotropic hardening as a function of the loading history.

Acknowledgments

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