MILP model for integrated balancing and sequencing mixed-model two-sided assembly line with variable launching interval and assignment restrictions

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Abstract. This research explores the Mixed-Model Two-Sided Assembly Line (MMTSAL). There are two interrelated problems in MMTSAL which are line balancing and model sequencing. In previous studies, many researchers considered these problems separately and only few studied them simultaneously for one-sided line. However in this study, these two problems are solved simultaneously to obtain more efficient solution. The Mixed Integer Linear Programming (MILP) model with objectives of minimizing total utility work and idle time is generated by considering variable launching interval and assignment restriction constraint. The problem is analysed using small-size test cases to validate the integrated model. Throughout this paper, numerical experiment was conducted by using General Algebraic Modelling System (GAMS) with the solver CPLEX. Experimental results indicate that integrating the problems of model sequencing and line balancing help to minimise the proposed objectives function.

1. Introduction

Mixed-Model Two-Sided Assembly Line (MMTSAL) is designed for producing high volume intermixed product’s model. Along this line, successional mated stations consist of left and right workstation that are placed facing each other and are connected by steadily moving conveyer belt. The product units to be assembled move along the line and visit each station. Within limited length of workstations, operators are placed which each is responsible for each workstation to perform a non-overlapping tasks without interfering with one another [1]. The tasks to be performed in MMTSAL have restrictions on the operation directions. That is, some tasks may be performed on a specific side of the line whether it is left or right side, while others may be performed at either side of the line [2]. Therefore, the tasks are classified into three types which are left-side tasks (L), right-side tasks (R) and either-side tasks (E). MMTSAL consists of two different problems that are line balancing problem and model sequencing problem. Line balancing is the problem of assigning tasks to workstations without violating the precedence constraint and other restrictions, while model sequencing is a problem of determining a production sequence of models.

At the beginning stage of studies regarding line balancing and model sequencing, many researches have worked on it in separate studies. Only few researches studied integrating problems of mixed model assembly line (MMAL) and solved them simultaneously such as Kim and Kim [3], and Mosadegh et al.
To model and solve both problems, the characteristics of MMTSAL is taken into account where Kim et al. [5], Kim et al. [6], Ozcan and Toklu [2], and Simaria and Vilarinho [7] introduced assignment restriction of sequence dependence finishing time, positional constraint, zoning constraint and synchronous constraint respectively. They model the balancing of two-sided assembly line (TSAL) and solved for one model only.

In this paper, a MILP model of (MMTSAL) is developed to solve simultaneously the integration of line balancing and model sequencing problems. The variable launching interval and assignment restrictions are considered in the model. A variable launching interval is the launch intervals of products that is launched down on the line and it is used because it can augments the flexibility of operating the line. The launching interval is very important to be determine accurately because it can influence the efficiency of the system. The objective functions for the model is to minimize the total utility works and idle time. To the best of our knowledge, the MILP model of MMTSAL for minimizing aforementioned objectives with the consideration of variable launching interval and assignment restriction in this research is the first in the literature. The MILP is solved using General Algebraic Modelling System (GAMS) with the solver CPLEX.

2. Problem Description
Line balancing and model sequencing problems of MMTSAL are solved simultaneously with the objective of minimizing the total utility works and idle time. Total utility works occur when operators cannot complete the tasks within the boundary of a station. It is a reaction on imminent work overload [8]. To describe how the utility works and the idle time occur, an example is illustrated in Figure 1. In the figure, assume that the conveyor speed is at constant and the launching interval of each model is vary. Operators begin operating first task from left and move downstream within their allowable work area. If the tasks is not completed by the time it reaches the boundary, operators need to return to their starting position to start operating next task and additional operator or utility worker helps to complete the unfinished task. The operators moving time from end boundary to the next starting position is ignored. If the next task has not yet entered the operator’s allowable work area, they need to wait for it and this waiting time is known as idle time. After finishing one cycle, the operators must be at the beginning of the station which cause uncompleted tasks and it is taken into account as utility work. In MMTSAL, interference phenomenon are unavoidable and can cause idle time. It is also known as sequence dependant finish time and it is one of the problems that may occur during assigning tasks to workstation. It happens because some workstations need to wait for a predecessor task to be completed at the opposite side of the line before starting to operate a new task.

Figure 1. An example of model sequence and task assignment in MMTSAL.
In order to solve model sequencing problem, model mix in a production cycle or also known as Minimum Part Set (MPS) is manufactured. MPS is represented by \( d_m = D_m/h \) where \( d_m = (d_1, d_2, ..., d_m) \) is the demand vector of each model and \( D_m = (D_1, D_2, ..., D_m) \) is the planning horizon or demands. While \( h \) is the greatest common divisor that will be obtained from the demands. To explain the problem, assume three models namely A, B and C with their demands of 200, 100, and 200 respectively. Therefore from the demands given, \( h = 100 \) and the demand vector become \( d_m = (2, 1, 2) \). Hence, the model sequence obtained is A, C, B, A, C. To distinguish the task assignment, the sequence is numbered because the task assignment might differ even for the same models.

When assigning task to workstation, an assignment restrictions are taken into account. The assignment restrictions considered in the modeling are:

- **Positive zoning constraint:** Because of some tasks may require the same tools or fixture, positive zoning constraint will ensure set of tasks to be assigned together in the same workstation. While, if some tasks require different equipment, negative zoning constraint can ensure set of tasks to be assigned in different workstations. This constraint is introduced by Ozcan and Toklu [2].
- **Positional constraint:** This constraint is introduced by Kim et al. [6] where some tasks must be assigned to a predetermined station because these tasks required to be performed at specific equipment.
- **Synchronous constraint:** This constraint enforces some tasks to be assigned at the opposite stations, so that these tasks can be performed simultaneously. This constraint is introduced by Simaria and Vilarinho [7].

### 3. Mathematical Formulation

The modeling developed in this paper is a modified model originated from Mosadegh et al. [4]. The characteristic of MMTSAL is added to the model. The following notations are used to develop the model.

#### NOTATIONS

**Indices**

-\( i, h, p, r \) Task
-\( j, g \) Mate station
-\( m \) Product model
-\( s \) Sequence

**Parameters**

-\( I \) Set of task; \( I = \{1,2,...,i,..nt\} \)
-\( S \) Set of model sequence with \( S = \{1,2,...t,..ns\} \)
-\( A_L \) Set of tasks which should be performed at left-side stations; \( A_L \subset i \)
-\( A_R \) Set of tasks which may be performed at either-side of a stations; \( A_R \subset i \)
-\( P(i) \) Set of all predecessor of task \( i \)
-\( S(i) \) Set of immediate successor of task \( i \)
-\( t_m \) Completion time of task \( i \) for model \( m \)
-\( CZ \) Set of compatible tasks for positive zoning; \( CZ = \{(i,h),...(p,r)\} \)

**Symbols**

-\( k, f \) Side of the line
-\( k, f = 1 \) indicates a left-side station
-\( k, f = 2 \) indicates a right-side station
-\( (j,k) \) Station of mate-station \( j \) and its operation direction is \( k \)
-\( J \) Set of mate stations \( J = \{1,2,...i,..ntms\} \)
-\( M \) Set of product models with \( M = \{1,2,...,i,..ntm\} \)
-\( A_h \) Set of tasks which should be performed at right-side stations; \( A_h \subset i \)
-\( P(i) \) Set of immediate predecessor of task \( i \)
-\( P_0 \) Set of task that have no immediate predecessor
-\( P_0 = \{i \in J | P(i) = \emptyset \} \)
-\( S_n(i) \) Set of all successor of task \( i \)
-\( PZ \) Set of tasks and predetermined stations for positional constraint;
-\( PZ = \{(r,(g,f))...(i(j,k))\} \)
Set of incompatible tasks for negative zoning: 
\[ IZ = \{(i,h), \ldots, (p,r)\} \]

Length of mate-station \( j \) 
\[ L_j \]

Speed of conveyor movement 
\[ \nu \]

Set of tasks whose operation directions are opposite to operation direction of task \( i \); 
\[ C(i) = \left\{ \begin{array}{ll}
A_i & \text{if } i \in A_g \\
A_p & \text{if } i \in A_k \\
\emptyset & \text{if } i \notin A_g
\end{array} \right. \]

Set of indicating the preferred operation direction of task \( i \); 
\[ K(i) = \begin{cases}
1 & \text{if } i \in A_g \\
2 & \text{if } i \in A_k \\
\{1,2\} & \text{if } i \notin A_g
\end{cases} \]

Decision Variables
- \( X_{im} \), 1, if task \( i \) of model \( m \) is assigned to station \( (j,k) \); 0, otherwise
- \( Y_{ms} \), 1, if model \( m \) is performed at sequence \( s \); 0, otherwise
- Utility work that occurred at sequence \( s \) of station \( (j,k) \); 0, otherwise
- Ending position of operator for last model at station \( (j,k) \)
- Finish time of task \( i \) of model \( m \)
- Launching interval at sequence \( s \)

Indicator Variables
- \( N_{ip} \), 1, if task \( i \) is assigned earlier than task \( p \) in the same station; 0, otherwise.

The mathematical model of MMTSAL for minimizing total utility works and idle time is as follows:

Minimize 
\[ Z = \sum_{j \in J, k \in K} \left( \sum_{s \in S} (W_{ijk} + ID_{ijk}) + EP_{jk} \right) \]  

Subject to:
1. 
\[ \left( \sum_{i \in I} t_{im} X_{ijk} \right) v_e - L_j \leq W_{ijk} \quad \forall j \in J, k \in K(i), \forall m \in M, \forall s \in S \]  
2. 
\[ \left( \sum_{i \in I} t_{im} X_{ijk} - W_{ijk} - a_{s+1} + ID_{(s+1)jk} \right) v_e + \left( Y_{ms} + Y_{(s+1)\psi} - 2 \right) \psi \leq P_{(s+1)jk} \]  
3. 
\[ \sum_{i \in I} t_{im} X_{ijk} - W_{ijk} - a_{s+1} \right) v_e + (Y_{ms} - 1) \psi \leq EP_{jk} \]  
4. 
\[ \sum_{i \in I} t_{im} X_{ijk} - W_{ijk} - a_{s+1} \right) v_e + (Y_{ms} - 1) \psi \leq ID_{ijk} \]  
5. 
\[ \sum_{i \in I} t_{im} X_{ijk} - W_{ijk} - a_{s+1} \right) v_e + \left( Y_{ms} + Y_{(s+1)\psi} - 2 \right) \psi \leq P_{(s+1)jk} \]  
6. 
\[ \sum_{i \in I} t_{im} X_{ijk} - W_{ijk} - a_{s+1} \right) v_e + \left( Y_{ms} + Y_{(s+1)\psi} - 2 \right) \psi \leq P_{(s+1)jk} \]  
7. 
\[ \sum_{i \in I} t_{im} X_{ijk} - W_{ijk} - a_{s+1} \right) v_e + \left( Y_{ms} + Y_{(s+1)\psi} - 2 \right) \psi \leq P_{(s+1)jk} \]  
8. 
\[ \sum_{i \in I} t_{im} X_{ijk} - W_{ijk} - a_{s+1} \right) v_e + \left( Y_{ms} + Y_{(s+1)\psi} - 2 \right) \psi \leq P_{(s+1)jk} \]
\[ \sum_{j \in J} \sum_{k \in K(h)} X_{slk} = 1 \quad \forall i \in I, \forall s \in S \quad (9) \]

\[ \sum_{j} X_{sij} = 1 \quad \forall i \in A_L, \forall s \in S \quad (10) \]

\[ \sum_{j} X_{sij} = 1 \quad \forall i \in A_R, \forall s \in S \quad (11) \]

\[ \sum_{g \in J} gX_{slg} \leq \sum_{j \in J} \sum_{k \in k(i)} gX_{slk} \quad \forall i \in I - P_0, h \in P(i), \forall s \in S \quad (12) \]

\[ t'_{im} - t'_{ln} + \psi \left( 1 - \sum_{k \in K(h)} X_{slk} \right) + \psi \left( 1 - \sum_{k \in k(i)} X_{sij} \right) \geq t_{im} \quad (13) \]

\[ \forall i \in I - P_0, \forall j \in J, h \in P(i), \forall s \in S \]

\[ t'_{pm} - t'_{ln} + \psi \left( 1 - X_{sij} \right) + \psi \left( 1 - X_{slk} \right) + \psi \left( 1 - N_{sp} \right) \geq t_{pm} \quad (14) \]

\[ \forall i \in I - P_0, \forall m \in M, \forall j \in J, p \in \{ r \mid r \in I - \left( P_a(i) \cup S_s(i) \cup C(i) \right) \} \text{ and } i < r \}

\[ k \in K(i) \cap K(p), \forall s \in S \]

\[ t'_{im} - t'_{pm} + \psi \left( 1 - X_{sij} \right) + \psi \left( 1 - X_{slk} \right) + \psi N_{sp} \geq t_{im} \quad (15) \]

\[ \forall i \in I - P_0, \forall m \in M, \forall j \in J, p \in \{ r \mid r \in I - \left( P_a(i) \cup S_s(i) \cup C(i) \right) \} \text{ and } i < r \}

\[ k \in K(i) \cap K(p), \forall s \in S \]

\[ X_{slk} - X_{sij} = 0 \quad \forall (i, h) \in CZ, \forall j \in J, k \in K(i) \cap K(h), \forall s \in S \quad (16) \]

\[ X_{slk} + X_{sij} \leq 1 \quad \forall (i, h) \in IZ, \forall j \in J, k \in K(i) \cap K(h), \forall s \in S \quad (17) \]

\[ X_{sij} = 1 \quad \forall \{ r, (g, f) \} \in PZ, \forall s \in S \quad (18) \]

\[ X_{slk} - X_{sij} = 0 \quad \forall (i, h) \in SZ, \forall j \in J, k \in K(i), f \in K(h), \forall s \in S \quad (19) \]

\[ t'_{im} - t'_{ln} - t_{mn} + t_{im} = 0 \quad \forall (i, h) \in SZ, \forall m \in M \quad (20) \]

\[ t'_{im} \geq t_{im} \quad \forall i \in I, \forall m \in M \quad (21) \]

\[ X_{sij} = 0.1 \quad \forall i \in I, \forall j \in J, k \in K(i), \forall s \in S \quad (22) \]

\[ Y_{im} = 0.1 \quad \forall m \in M, \forall s \in S \quad (23) \]

\[ N_{sp} = 0.1 \quad \forall i \in I, p \in \{ r \mid r \in I - \left( P_a(i) \cup S_s(i) \cup C(i) \right) \} \text{ and } i < r \} \quad (24) \]

The objective in (1) is to minimize the total utility works and idle time for one cycle. In constraint (2), the utility work of each sequence at each side of stations is computed. Constraint (3) is related to the new starting position of operator after finishing each model. Constraint (4) calculates the value of ending position of uncompleted task of the last model in a cycle which also acts as utility work. The idle time is computed in constraint (5). Constraint (6) connects the value of \( P_{nijk} \) to the value of \( Y_{ma} \). Constraint (7) guarantees that the demand of each model in MPS cycle is satisfied. Constraint (8) ensures that exactly one model is assigned to each position in a sequence. Constraint (9) is the occurrence constraint which a task is only assigned to one workstation. Constraint (10)-(11) enforce the tasks with specific operation direction to be assigned to the appropriate side of station. Constraint (12) is the precedence constraint which means that task can only be assigned to station if all of its predecessors are finished. Constraint (13)-(15) are introduced by Kim et al. [5] which are related to the sequence dependence finishing time. Constraint (13) are applied for a pair task such that task \( h \) is the immediate predecessor of task \( i \), then both tasks are assigned to the same station \( j \). This represents that, operator can start working on task \( i \) immediately after task \( h \) is finished. Constraint (14) and (15) are applied to
two tasks that do not have precedence relations such that both tasks $i$ and $p$ are assigned to the same station $j$. If task $i$ is assigned earlier than task $p$ in the same station, then constraint (14) is active. Otherwise, if task $p$ is assigned earlier than task $i$ in the same station, then constraint (15) is active. Constraint (16)-(17) is the positive and negative zoning constraint respectively [2]. Constraint (18) is positional constraint which ensures task $r$ to be assigned to a predetermined station $(g,f)$ [12]. Constraint (19)-(20) are the synchronous constraint that ensure task $h$ and $i$ to be assigned to station directly facing each other with the same starting time for all models [13]. Constraint (21) ensures the finishing time of task $i$ for product model $m$ must be greater than or equal to the completion time of task $i$ for model $m$. Constraint (22)-(24) are the integrality constraints which restate the definition of variables.

4. Experimental Result

The model presented in previous section is solved using General Algebraic Modelling System (GAMS) with the solver CPLEX for a small size data. The data is shown in table 1. For this data, nine tasks are assigned to two stations and the assignment restrictions are determined. Different lengths of workstations are used, so that a variety of variable launching intervals with the value of objective function can be obtained for analyzing. The feasible solutions of the integrating problems of MMTSAL obtained are shown in Table 2 and the optimal schedule of tasks assignment with model sequence for station length of two meter is shown in Figure 2. As shown in the figure, the idle time and utility works are occurred at sequence 2 and 3. From the solution obtained, big value of launching interval gives small value of total utility works and idle time which means that operators have much time to finish operating their task. While, if the launch interval is small, it means that operators have less time to operate their current task and will cause unfinished work.

| No. of tasks | Immediate predecessor | Task time (min) | Assignment restrictions |
|--------------|-----------------------|-----------------|-------------------------|
|              |                       | Model A         | Model B                 |
| 1            | -                     | 2               | 0                       |
| 2            | -                     | 3               | 1                       |
| 3            | -                     | 0               | 1                       |
| 4            | 1                     | 3               | 0                       |
| 5            | 2                     | 1               | 3                       |
| 6            | 2,3                   | 1               | 1                       |
| 7            | 4,5                   | 2               | 2                       |
| 8            | 5                     | 0               | 3                       |
| 9            | 6                     | 1               | 1                       |

| No. of station | Zoning constraint | Positional constraint | Synchronous constraint |
|----------------|-------------------|-----------------------|------------------------|
| 2              | Positive Zoning   | (1,(1,1)),           | (8,9)                  |
|                | (4,8)             | (4,(2,1)),           |
|                | Negative Zoning   | (6,(2,1))            |                        |
|                |                   | (3,5)                |

| Stations length, $L_j$ (m) | Launching interval, $a_i$ (min) | Objective function, $Z$ (min) | Model Sequence |
|---------------------------|-------------------------------|-------------------------------|----------------|
| 2                         | S_1-0, S_2-4, S_3-4          | 7                             | A_1A_2B_1      |
| 1.5                       | S_1-0, S_2-4, S_3-3          | 10                            | A_1B_2A_2      |
| 1                         | S_1-0, S_2-3, S_3-2          | 17                            | A_1B_2A_2      |

Table 1. A small size data for solving simultaneously the MMTSAL.

Table 2. The feasible solution.
5. Conclusion

This paper deals with solving simultaneously the integration problems of line balancing and model sequencing. A MILP on mixed-model two-sided assembly line (MMTSAL) is presented. The mathematical modeling developed considers the objective function of minimizing the total utility works and idle time with the variable launching interval and assignment restriction. The results obtained showed that the model is feasible and solution is optimal. The line balancing and model sequencing problems in MMTSAL are known to be NP-hard problems because the solution space is limited when solving them using exact solution. Hence, it is recommended for future research that is to solve the MMTSAL by using fast and effective algorithms.

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