New compound fractional sliding mode control and super-twisting control of a MEMS gyroscope

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Abstract—In this research we propose a new compound Fractional Order Sliding Mode Controller (FOSMC) and Super-Twisting Controller (STC) to control a MEMS gyroscope. A new sliding mode surface has been defined to design the proposed new sliding mode controller. The main advantages of a FOSMC is its high tracking performance and robustness against external perturbation, but it is susceptible to chattering. By augmenting a STC with a FOSMC, the chattering phenomenon is eliminated, singularity problem is solved and systems robustness has significantly improved. Simulation results validate the effectiveness of the proposed control approach.

Index Terms—Chattering reduction, Compound control, Fractional sliding mode control, MEMS gyroscope, Super-twisting control.

I. INTRODUCTION

MICRO electro-mechanical system (MEMS) gyroscope are usually used as sensors for measuring angular velocity in stabilization applications with close loop control. The performance of the MEMS gyroscope is impacted by mismatch in the frequency of oscillation between the two vibrating axes resulting from the effect of external disturbances and time varying model parameters [1]. Performance can be improved by designing a closed loop controller to negate the effects of external disturbances and time varying model parameters. In this paper we develop a novel the Fractional-Order Sliding Mode Controller (FOSMC) combined with a Super-Twisting Controller (STC) to control a MEMS gyroscope.

Yang and Liu numerically considered the FOSMC for a new hyperchaotic structure [4]. Gao and Liao presented integral sliding mode control to enhance robustness of FOSMC [5]. Balochian used variable structure control for an individual polytopic system with fractional order operator. A specific feedback law is considered by proposing a sliding surface with fractional order operator [6]. Rabah et al. guaranteed asymptotic stability of fractional systems, by provided a novel technique of FOSMC [7]. Shah and Mehta described Thiran’s delay estimation scheme in order to compensate the controller for actuator fractional delay. This considered the real time networked medium and packet loss situation [8]. Sun and Mah applied a fractional integral sliding mode control for tracking control of a linear motor in order to achieve high convergence precision.. Experimental results validated that the proposed control law has high tracking performance in comparison with conventional sliding mode control [9]. Wang et al. proposed a new fractional order nonsingular terminal sliding mode control. The proposed controller due to the fractional order nonsingular terminal sliding mode controller and fast terminal sliding mode controller, guarantees fast convergence and high tracking performance [10]. Aghababa presented a new fractional hierarchical terminal sliding mode surface, where finite time convergence to the origin was demonstrated. A robust sliding mode switching control method was designed to guarantee the fractional Lyapunov stability [11]. Wang et al. implemented a novel sliding mode controller for an active vehicle suspension system to suppress the effect of external noise [12]. Based on aforementioned researches, FOSMC can be used as a strong control method in different systems [13-15], but its main drawback is creating chattering phenomenon. Guruganesh et al. designed a control method for Micro Aerial vehicle using the second sliding mode control and super-twisting control [16]. Jeong et al. designed a robust super-twisting sliding mode control that guarantees high tracking trajectory of a robotic system. In order to satisfy the properties of a conventional sliding mode control, a super-twisting sliding mode surface is designed for obtaining the transient and steady-state time performances of the position of robotic manipulator [17]. Chuei et al. described a super-twisting observer based repetitive control, which overcome aperiodic disturbances [18]. Zargham and Mazinan applied super-twisting sliding mode control method to control of wind turbine system. Conventional sliding mode control is not guaranteed to maintain the closed loop performance against external perturbations. As a result of this drawback, a STC technique used for rapid response and high accuracy in chattering reduction [19]. Zhao et al., proposed a non-singular terminal sliding mode control based on STC method [20]. Lu and Xia addressed a new adaptive super-twisting algorithm for
control of rigid spacecraft. The applied controller is anti-
singularity and anti-chattering when encountered with external
disturbances [21]. Evangelista et al. used modified STC to
improve robustness of the system in the presence of external
perturbations acting on a wind turbine shaft [22]. Becerra et al.
proposed a STC which guarantees continuous control inputs
and enhance robustness properties [23]. Salgado et al.
troduced a discrete time super-twisting algorithm to solve the
problems of control and state estimation [24]. As a result of
considered studies, STC can be utilized as a strong tool in
control systems. It includes some advantages such as improves
robustness of control systems, removes controller singularity,
and eliminates chattering phenomenon. In this research a novel
FOSMC is proposed, which is robust against external
perturbations and shows high tracking performance. The main
drawbacks of FOSMC is creating chattering phenomenon. By
using STC, it continuously calculates an error value and
applies a correction value to the system. This process will
eliminate chattering phenomenon and improve the robustness
of the system.

In next section of the paper, dynamic modeling of a MEMS
gyroscope is presented. In Section 3, the FOSMC is described.
In Section 4, compound FOSMC+STC has been delineated.
Section 5 presents simulation results. Finally the paper ends
with the conclusion and the contributions of the research work
in the section 6.

II. DYNAMIC OF MEMS GYROSCOPE

A z-axis MEMS gyroscope is shown in Figure 1. The
conventional MEMS vibratory gyroscope consists of a proof
mass (m) suspended by springs, where x and y are the
coordinates of the proof mass with respect to the gyro frame in
a Cartesian coordinate system, sensing mechanisms, and an
electrostatics actuation for forcing an oscillatory motion and
velocity of the proof mass and sensing the position. $\Omega_x, \Omega_y, \Omega_z$ are
the angular rate components along each axis of the gyro frame.
The frame that the proof mass is mounted moves with a
constant velocity and the gyroscope rotates at a slowly
changing angular velocity $\Omega$. The centrifugal forces
$m\Omega_x^2 x$ and $m\Omega_y^2 y$ are supposed to be negligible because of
small displacements. The Coriolis force is generated in a
direction perpendicular to the drive and rotational axes [25].
The dynamics of gyroscope according to assumptions
presented above becomes as follows:

$$m\ddot{x} + d_{xx}\dot{x} + d_{xy}\dot{y} + k_{xx}x + k_{xy}y = u_x^* + 2m\Omega_z^*y$$

$$m\ddot{y} + d_{yy}\dot{y} + d_{xy}\dot{x} + k_{yy}y + k_{xy}x = u_y^* - 2m\Omega_z^*x$$

The origin for x, y coordinates is at the center of the proof
mass without force employed. Fabrication imperfections is
supposed to be helping basically to the asymmetric spring and
damping terms, $k_{xx}$ and $d_{xy}$ respectively.

The x and y axes spring and damping terms $k_{xx}, k_{yy}, d_{xx}$ and $d_{yy}$ are often recognized, but may have
small unknown variations from their nominal values [1, 25].

The mass of the proof mass m can be obtained exactly.

The positive direction of the control force is same as the x-y
direction.

Dividing gyroscope dynamics (1) and (2) by the reference
mass results in the following vector forms as:

$$\ddot{q} + \frac{D_s}{m}\dot{q} + \frac{K_a}{m}\dot{q} = \frac{u}{m} - 2\Omega^*q^*$$

where

$$q^* = \begin{bmatrix} x^* \\ y^* \end{bmatrix}, \quad u = \begin{bmatrix} u_x^* \\ u_y^* \end{bmatrix}, \quad \Omega^* = \begin{bmatrix} 0 \\ -\Omega_z^* \end{bmatrix}$$

$$D_s = \begin{bmatrix} d_{xx} & d_{xy} \\ d_{xy} & d_{yy} \end{bmatrix}, \quad K_a = \begin{bmatrix} k_{xx} & k_{xy} \\ k_{xy} & k_{yy} \end{bmatrix}$$

The final form of the nondimensional equation of motion as
follows:

$$\ddot{q} + \frac{D_s}{m\omega_0^2}\dot{q} + \frac{K_a}{m\omega_0^2}\dot{q} = \frac{u}{m\omega_0^2\Omega_z^*} - 2\Omega^*q^*$$

We determine a set of new parameters as follows:

$$q = \frac{q^*}{\omega_0}, \quad \dot{d}_{xy} = \frac{d_{xy}}{m\omega_0^2}, \quad \Omega_z = \frac{\Omega_z^*}{\omega_0}$$

$$u = \frac{u_x^*}{m\omega_0^2\Omega_z^*}, \quad u_y = \frac{u_y^*}{m\omega_0^2\Omega_z^*}$$

$$\omega_x = \sqrt{\frac{k_{xx}}{m\omega_0^2}}, \quad \omega_y = \sqrt{\frac{k_{yy}}{m\omega_0^2}}, \quad \omega_z = \sqrt{\frac{k_{xy}}{m\omega_0^2}}$$

As a result, the nondimensional representation of (1) and (2)
written as follows:

$$\ddot{q} + D\dot{q} + Kq = u - 2\Omega\dot{q}$$

where
\[
q = \begin{bmatrix} x \\ y \end{bmatrix}, \quad u = \begin{bmatrix} u_x \\ u_y \end{bmatrix}, \quad \Omega = \begin{bmatrix} 0 & -\Omega_z \\ \Omega_z & 0 \end{bmatrix}
\]

\[
D = \begin{bmatrix} d_{xx} & d_{xy} \\ d_{xy} & d_{yy} \end{bmatrix}, \quad K_b = \begin{bmatrix} \omega_s^2 & \omega_{xy} \\ \omega_{xy} & \omega_s^2 \end{bmatrix}
\]

Clearly, Eq. (8) can be rearranged as:
\[
\dot{q} = -(D + \Omega)q - K_b q + u + E
\]

Where \( E \) is external disturbance. From Equation (9), the dynamic equations for a MEMS gyroscope become as:
\[
\ddot{q} = -M\ddot{q} - Nq + u + E
\]

where \( M = (D + 2\Omega) \) and \( N = K_b \).

### III. NEW FRACTIONAL SLIDING MODE CONTROL

Selecting fractional sliding mode surface is the main part of FOSMC design. If a fractional sliding mode surface choose correctly, the best performance will be obtained. The new fractional sliding mode surface can be selected as follows:

\[
s(t) = \dot{e}(t) + \alpha D^\mu e(t) + \beta D^{\mu-\alpha} e(t) + \gamma \int_0^t e(\tau)^m d\tau
\]

Where \( r, m, \alpha, \beta, \) and \( \gamma \) are positive constant and \( D \) is fractional order operator \((D=\frac{d}{dt}, \text{and } \mu>2)\). Fractional order operator type is Grunwald-Letnikov. The tracking error can be shown as:

\[
e(t) = \dot{q} - q_d
\]

The derivative of fractional sliding mode surface is:

\[
\dot{s}(t) = \ddot{e}(t) + \alpha D^\mu e(t) + \beta D^{\mu-\alpha} e(t) + \gamma e(t)^m
\]

\[
= \ddot{q} - \ddot{q}_d + \alpha D^\mu e(t) + \beta D^{\mu-\alpha} e(t) + \gamma e(t)^m
\]

\[
= -M\ddot{q} - Nq + u + E - \ddot{q}_d + \alpha D^\mu e(t)
\]

\[
+ \beta D^{\mu-\alpha} e(t) + \gamma e(t)^m
\]

Equivalent control can be obtained by setting \( \dot{s}(t) = 0 \):

\[
u_{eq}(t) = M\ddot{q} + Nq - E - \ddot{q}_d - \alpha D^\mu e(t)
\]

\[
- \beta D^{\mu-\alpha} e(t) - \gamma e(t)^m
\]

The FOSMC can be shown as:

\[
u_{FOSMC}(t) = u_{eq}(t) + u(t) = M\ddot{q} + Nq - E + \ddot{q}_d
\]

\[
- \alpha D^\mu e(t) - \beta D^{\mu-\alpha} e(t) - \gamma e(t)^m - K_s s
\]

The equivalent control cannot compensate external perturbation and unmodel dynamic uncertainties. A reaching control law can be designed in order to remove those problems as \( u_r(t) \), which can be defined as:

\[
u_s(t) = -K_s s
\]

where \( K_s \) is positive constant.

Lyapunov theory can be taken into considerations as a strong tool for stability proving. Considering the following Lyapunov function candidate \( V \) which is continuous and nonnegative [26-28].

\[
V = \frac{1}{2} s^T s
\]

The time derivative of \( V \) yields:

\[
\dot{V} = s^T \dot{s} = -M\ddot{q} - Nq + u(t) + E - \ddot{q}_d
\]

\[
+ \alpha D^\mu e(t) + \beta D^{\mu-\alpha} e(t) + \gamma e(t)^m
\]

By substituting Eq. (15) into Eq. (18), generates:

\[
\dot{V} = s^T \dot{s} = s^T (-M\ddot{q} - Nq + M\ddot{q} + Nq - E + \ddot{q}_d
\]

\[
- \alpha D^\mu e(t) - \beta D^{\mu-\alpha} e(t) - \gamma e(t)^m
\]

\[
- \beta D^{\mu-\alpha} e(t) + \gamma e(t)^m
\]

Simplify Eq. (19) results in:

\[
\dot{V} = s^T (-K_s s)
\]

Therefore, Eq. (20) can be expressed as:

\[
\dot{V} = -K_s s^2
\]

The Eq. (21) shows that \( \dot{V} < 0 \), which expressed that the proposed control law is stable.

### IV. NEW COMPOUND CONTROL SYSTEM

FOSMC is a strong tool which is able to enhance robustness of control system and improve tracking performance. It’s main drawbacks is creating chattering phenomenon. However, STC can be used in control systems as a powerful control approach, which its advantages can be enumerated as enhance robustness of the system, improve trajectory tracking, removing singularity problem, and eliminate chattering phenomenon.

By combining both FOSMC and STC, a new control method will be obtained which benefits both controller properties. The compound control law can be defined as:

\[
u(t) = u_{FOSMC}(t) + u_{STC}(t)
\]

where \( u_{STC}(t) \) is

\[
u_{STC}(t) = -k_1 \left| \frac{r}{2} \right| \int_0^t \frac{r}{2} \left| \text{sign}(e^m) - k_2 \int_0^t \text{sign}(e^m) d\tau \right|
\]

where \( k_1, k_2, r, \) and \( m \) are positive constants. The stability proving of the proposed control law can be arranged by substituting Eq. (22) into Eq. (18) as:

\[
\dot{V} = s^T \dot{s} = s^T (-M\ddot{q} + u_{FOSMC}(t) + u_{STC}(t)
\]

\[
+ E - \ddot{q}_d + \alpha D^\mu e(t) + \beta D^{\mu-\alpha} e(t) + \gamma e(t)^m
\]

Eq. (24) can be expressed as:
Fig. 2. Position tracking of x-axis and y-axis.

\[
\dot{V} = s^T \dot{s} = s^T (-M \ddot{q} - N \dot{q} + \dot{M}q + Nq - E + \ddot{\alpha} - \alpha D e(t) - \beta D^{\mu-1} e(t) \\
- \gamma e(t)^m - K_s s - k_1 \left| e^m \right|^\frac{1}{2} \text{sign}(e^m) \\
- \int_0^r \text{sign}(e^m) d\tau + E - \ddot{\alpha} + \alpha D^{\mu} e(t) + \beta D^{\mu-1} e(t) + \gamma e(t)^m \\
-k_2 \int_0^r \text{sign}(e^m) d\tau
\]

where \( K_s \) is positive and \( \alpha > 0 \), the \( \dot{V} < 0 \) will be obtained.

V. SIMULATION RESULTS

Selection of proposed controller parameters (\( \alpha, \beta, \gamma, K_s, \mu, r, m, k_1, \) and \( k_2 \)) is the most important part of the controller design procedure. If parameters are chosen inappropriately, the proposed control method cannot guarantees the desired performance such as trajectory tracking, robustness, stability, and chattering elimination.

The controller parameters are chosen based on designer’s experiences and trial-error process. Simulation results have shown that the parameters are selected appropriately.

Parameters of the fractional order sliding mode surface are selected as \( \alpha = \text{diag}(40,40), \beta = \text{diag}(50,50), \gamma = \text{diag}(60,60), K_s = \text{diag}(10,10), \mu = 2.5, r = 1.5 \) and \( m = 1.25 \).

The STC parameters are chosen as \( k_1 = \text{diag}(20,20) \) and \( k_2 = \text{diag}(20,20) \). The desired motion trajectory is determined by \( q_{d1} = \sin(4.17t) \) and \( q_{d2} = 1.2\sin(5.11t) \).

The initial values of the system are selected as \( q_1(0) = 0.5, q_2(0) = 0.5, q_1(0) = 0 \) and \( q_2(0) = 0 \).

The parameters of the MEMS gyroscope are selected as:
The unknown angular velocity is assumed as\( \omega = \frac{d}{dt} \). The unknown control input \( u \) is obtained as \( u = \frac{d}{dt} \omega \). The unknown angular velocity \( \omega \) is estimated using an Extended State Observer (ESO) as \( \hat{\omega} = \frac{1}{J} \frac{d}{dt} \hat{\theta} \), where \( J \) is the moment of inertia of the gyroscope.

The conventional natural frequency of each axis of a MEMS gyroscope is in the KHZ range, so, choose the \( \omega_0 \) as 1KHZ. It is suitable to choose \( 1\mu \)m as the reference length \( q_0 \) when the displacement range of the MEMS gyroscope in each axis is sub-micrometer level. The unknown angular velocity is assumed \( \Omega_\omega = 100 \text{ rad/s} \). Therefore, the non-dimensional values of the MEMS gyroscope parameters are chosen as:

\[
\omega_x^2 = 355.3, \quad \omega_y^2 = 532.9, \quad \omega_{xy} = 70.99, \quad d_{xx} = 0.01, \quad d_{xy} = 0.01, \quad d_{yy} = 0.002, \quad \Omega_x = 0.1
\]

Figure 2 shows position tracking of x-axis and y-axis under FOSMC and FOSMC+STC. It can be seen clearly that tracking performance under proposed controllers is consistent with desired tracking for the MEMS gyroscope. Figure 3 illustrates tracking error of x-axis and y-axis under FOSMC and proposed control. FOSMC creates chattering phenomenon, which by using STC, chattering phenomenon reduced. In addition, FOSMC+STC has lower maximum overshoot and undershoot in comparison with FOSMC. Figure 4 shows velocity of x-axis and y-axis under FOSMC and proposed control law.

VI. CONCLUSION

In this study, we proposed a novel FOSMC+STC law for control of a MEMS gyroscope. A new FOSMC is applied for control of x-axis and y-axis of a MEMS gyroscope. It has high tracking performance, but its main drawbacks was creating chattering phenomenon. In order to solve this problem, a STC is proposed in parallel with FOSMC, which continuously claculates an error value and applies a correction value. Simulated results demonstrate that the developed STC significantly reduce the chattering phenomenon. In addition, by using STC, maximum overshoot and undershoot reduce and trajectory tracking performance improved. Simulation results thus validated the effectiveness of the proposed control strategy.

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