QCD Critical Region and Quark Gluon Plasma from an Imaginary $\mu_B$

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We discuss the imaginary chemical potential approach to the study of QCD at nonzero temperature and density, present results for the four flavor model in the different phases and show that this method is ideally suited for a comparison between lattice data and phenomenological models.

1. QCD and a complex $\mu_B$

Results from simulations with an imaginary chemical potential can be analytically continued to a real chemical potential, thus circumventing the sign problem [1], [2], [3]. In practice, the analytical continuation is carried out along one line in the complex $\mu$ plane: first along the imaginary axes, and then along the real one. It is then meaningful to map this path in the complex $\mu^2$ plane: because of the symmetry property $Z(\mu) = Z(-\mu)$ this can be achieved without losing generality. In the complex $\mu^2$ plane the partition function is real for real values of the external parameter $\mu^2$, complex otherwise: the situation resembles that of ordinary statistical models in an external field. Hence, the analyticity of the physical observables [4] as well as that of the critical line [2] follows naturally.

The phase diagram in the temperature, (real) $\mu^2$ plane is sketched in Fig.1, where we omit the superconducting and the color flavor locked phase, which (unfortunately) play no rôle in our discussion. The region accessible to numerical simulations is the one with $\mu^2 \leq 0$: at variance with other approaches to finite density QCD which only use information at $\mu = 0$ [4], [5], [6], the imaginary chemical potential method exploits the entire halfspace. And we will also argue that there are physical questions which can be addressed without analytic continuation.

2. The Critical Line

Also in this case the consideration of the $T, \mu^2$ plane helps the analysis. Model analysis suggests the following parametrization, confirmed by numerical results:

$$(T + aT_c)(T - T_c) + k\mu^2 = 0, \quad k > 0$$

(1)

It encodes reality for real $\mu^2$, contains the physical scale $T_c$, is dimensionally consistent, gives $T(\mu = 0) = T_c$, $T(\mu \neq 0) < T_c$. We refer to Section IV of Ref. [3] for our results on the critical line in the four flavor model, and their discussion in terms of model calculations. Here it suffices to remind ourselves that the second order approx-

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Figure 2. Number density as a function of $\mu_I$: note the smooth behavior in the hadronic phase, consistent with the hadron resonance gas model, the chiral transition in the Roberge Weiss region, the rapid increase in the plasma phase, approaching a nearly free quark behavior.

imation turns out to be adequate, and that the fourth order corrections were found to be consistent with zero within errors.

3. Hadronic Phase: $T < T_c$

In this region observables are a continuous and periodic function of $\mu_I/T$, analytic continuation in the $\mu^2 > 0$ half plane is always possible, but interesting only when $\chi_q(\mu = 0, T) > 0$.

The analytic continuation of an observable $O$ is valid till $\mu < \mu_c(T)$, where $\mu_c(T)$ has to be measured independently. The value of the analytic continuation at $\mu_c$, $O(\mu_c)$ defines the discontinuity at the critical point, or, equivalently, the critical value of, say, the number density. In turns, this allows the identification of the order of the phase transition.

For observables which are even/odd in the chemical potential, $O_{e/o}$, we have considered a Fourier series, observing that the first cosine/sine terms suffice to parametrize the data [3]. This has been confirmed in [8], where this result has been interpreted within the framework of the hadron resonance gas model.

4. Roberge Weiss Regime: $T_C < T < T_E$

The analytic continuation is valid till $\mu = \infty$ but the interval accessible to the simulations at imaginary $\mu$ is small, as simulations in this area hits the chiral critical line for $\mu^2 < 0$.

The bright side of this is that the nature of the critical line can then be studied without need for analytic continuation. In Fig. 3 we show the clear correlation between the Polyakov Loop and the chiral condensate at $\mu = 0.15$. The correlation between chiral and deconfining transition persists at nonzero chemical potential.

It is also of interest to note that the non-applicability of perturbation theory in this region is almost a theorem: indeed the analytic continuation of the polynomial predicted by perturbation theory for positive $\mu^2$ would never reproduce the correct critical behavior at the second order phase transition for $\mu^2 < 0$, and it is then ruled out.

5. The QGP phase: $T_E < T$

Several analytic models have been proposed to describe the properties of this phase and, as ana-
lytic models can be obviously analytically continued in the $\mu^2 \leq 0$ half plane, imaginary chemical potential is an excellent testbed for these approaches. Here, as an example, we just contrast the data with a free field behavior $\Delta P/T^4 = 2(\mu/T)^2$, where $\Delta P = P(\mu) - P(0)$, and we have ignored the fourth order terms. In Fig. 4 we plot $\Delta P K_{L(N_t=4)}/(T^42(\mu/T)^2)$ versus $\mu/T$, where we have corrected for finite lattice effects $K_{L(N_t=4)}$ following \cite{9, 10}.

This “effective prefactor plot” is perhaps more informative than the direct quadratic fits of $\Delta P/T^4$ to $k(\mu/T)^2$, as it allows an assessment by eye of the $\mu$ dependence, if any, of the prefactor to the quadratic term.

We see that the results approach the perturbative limit, but corrections at small chemical potential are clearly visible, possibly consistent with the predictions of \cite{11}. 

6. Summary/Outlook

We have studied four flavor QCD within the imaginary chemical potential approach in a large part of the phase diagram.

We have shown that the chiral and deconfining transition remain correlated at nonzero chemical potential. We have found that the critical line is described by a polynomial, and interpreted this result in terms of simple models. We have identified and discussed three different regimes: the hadronic phase results are consistent with the hadron resonance gas model. The Roberge Weiss regime is eminently nonperturbative, and in this regime we have the possibility to study the nature of the chiral transition at nonzero chemical potential without performing any analytic continuation. The Quark Gluon Plasma phase is an ideal test bed for analytic models.

The method seems mature for quantitative studies in realistic cases, and a nice possibility is offered by a combination of this approach with other methods, for instance by using reweighting \cite{9} \cite{11} or direct calculations of derivatives \cite{10} at nonzero $\mu$ to improve the accuracy of the results at negative $\mu^2$. Finally, the study of discontinuities as sketched in Sect.3 above might offer an alternative approach to the study of the endpoints and tricritical points.

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