Codimension two lump solutions in string field theory and tachyonic theories

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Abstract

We present some solutions for lumps in two dimensions in level-expanded string field theory, as well as in two tachyonic theories: pure tachyonic string field theory and pure $\phi^3$ theory. Much easier to handle, these theories might be used to help understanding solitonic features of string field theory. We compare lump solutions between these theories and we discuss some convergence issues.
1 Introduction

In the last few months, there has been growing evidence that level truncation is a good way of doing computations in string field theory. In particular, it allows to get very accurate results for the string field theory true vacuum, both in open bosonic string field theory and superstring field theory ([1] - [20]).

More recently, in [1], a level truncation scheme has been developed which takes non-zero momentum into account; Applied to lump solutions in one dimension, it gave numerical results for the ratio of the tension of a D-p-brane and a D-(p − 1)-brane with a precision of about 1%. In [2], de Mello Koch and Rodrigues applied this scheme to construct 2-dimensional lumps in open bosonic string field theory.

In this paper, we want to present independent results on 2-dimensional lumps. We also describe these lumps in two theories involving only the tachyon: pure tachyonic sft (string field theory in which we keep only the tachyon, including its higher derivatives), with action ([14]):

\[ S = -2\pi^2 T_{25} \int d^{26}x \left( \frac{1}{2} \partial_\mu \phi \partial^\mu \phi - \frac{1}{2} \phi^2 + \frac{1}{3} K^3 \tilde{\phi}^3 \right), \]

where \( T_{25} \) is the D-25-brane tension, \( K = 3\sqrt{3}/4 \) and \( \tilde{\phi} = K \partial_\mu \partial^\mu \phi \). And pure \( \phi^3 \) theory (the usual scalar \( \phi^3 \) theory of a tachyon, which doesn’t include higher derivatives), with action

\[ S = -2\pi^2 T_{25} \int d^{26}x \left( \frac{1}{2} \partial_\mu \phi \partial^\mu \phi - \frac{1}{2} \phi^2 + \frac{1}{3} K^3 \phi^3 \right), \] (1.2)

the only difference being that here we have \( \phi^3 \) instead of \( \tilde{\phi}^3 \).

2 Calculating the potential

We will use the notation of [1], but we will compactify two dimensions, instead of one, on a torus. Let us name \( x \) and \( y \) the compact dimensions. We impose the identifications

\[ x \sim x + 2\pi R, \]
\[ y \sim y + 2\pi R. \] (2.1)

The \( x \)- and \( y \)-momenta will be quantized:

\[ p_x = m/R, \]
\[ p_y = n/R. \] (2.2)
For each zero-momentum state $|\Phi_i\rangle$ that appears in the non-compact theory, we will have states labeled by two indices $|\Phi_{i,mn}\rangle$ with levels

$$l(\Phi_{i,mn}) = l(\Phi_i) + (m^2 + n^2)/R^2 .$$

(2.3)

By definition, when we work at level $(M, N)$, we keep fields of level $\leq M$, and terms in the potential of total level $\leq N$. In this paper, we will work in string field theory at level $(2, 4)$ and in pure tachyonic theories at arbitrary levels. Therefore all the fields we need are:

$$|T_{mn}\rangle = \begin{cases} 
c_1 \cos \left(\frac{m x}{R}\right) \cos \left(\frac{n y}{R}\right) |0\rangle , & m = n \\
1 \left(\cos \left(\frac{m x}{R}\right) \cos \left(\frac{n y}{R}\right) + \cos \left(\frac{n x}{R}\right) \cos \left(\frac{m y}{R}\right)\right) |0\rangle , & m \neq n 
\end{cases}$$

$$|U_{00}\rangle = c_{-1} |0\rangle$$

$$|V_{00}\rangle = c_1 \left(L^X_{-2} + L^Y_{-2}\right) |0\rangle$$

$$|W_{00}\rangle = c_1 L'_{-2} |0\rangle ,$$

(2.4)

where $L^X_{-2}$ and $L^Y_{-2}$ are Virasoro generators of the CFT of the compact dimensions $x$ and $y$ respectively, and $L'_{-2}$ is a Virasoro generator of the CFT of the 24-dimensional co-space. The definition of $|T_{mn}\rangle$ ensures that the solutions we will find are symmetric under $x \leftrightarrow y$ as well as under $x \to -x$ and $y \to -y$. We will thus use the string field:

$$|\vec{T}\rangle = \sum_{m \leq n} t_{mn} |T_{mn}\rangle + u_{00} |U_{00}\rangle + v_{00} |V_{00}\rangle + w_{00} |W_{00}\rangle ,$$

(2.5)

where the sum is restricted by the level truncation, the level of each field being

$$l(T_{mn}) = (m^2 + n^2)/R^2 , \quad l(U_{00}) = l(V_{00}) = l(W_{00}) = 2 .$$

(2.6)

We will not repeat here how to calculate the potential $V_{MN}(\vec{T})$ at level $(M, N)$. We refer to the literature (see for example [1], [7], [9], [10], [20]). Note however that in theories involving only the tachyon, the coefficients of the terms in the potential can be calculated straightforwardly, the only difficulties being to keep track of momentum conservation at the vertex and to figure out the combinatorial factors. We have written computer codes calculating the potentials in the following theories:
• string field theory at arbitrary radius up to level (2, 4).
• pure tachyonic sft at arbitrary radius and arbitrary level.
• pure $\phi^3$ theory at arbitrary radius and arbitrary level.

## 3 Codimension 2 lumps

### 3.1 String field theory truncated at level (2,4)

Before showing results, let us say a few words about how to find these lumps given the potential. In general, the tachyon potential has many extrema. In doing computations at high level, it might be difficult to setup the convergence on the right branch. The method we’ve used here is to go backwards: We know the approximate shape of the lump we are looking for, from it one can calculate its Fourier expansion. We can then plug these Fourier coefficients into our algorithm finding solutions of the equations of motion (by Newton’s method). If we start the numerical algorithm with a seed close enough to the solution we want, it is fairly probable that it will converge to the sought solution.

Let us present our result in string field theory at level (2,4). In order to compare our result with [2], we have computed it at $R = \sqrt{3}$. Fig.1 shows a plot of the solution. The ratio of the tension of a D-$p$-brane and a D-$(p - 2)$-brane, when divided by its expected value $(2\pi)^2$, can be approximated by (3.1):

$$r^{(2)} \equiv \frac{R^2 (2\pi^2 \mathcal{V}_{(M,N)}(\vec{T}_{\text{lump}}) - 2\pi^2 \mathcal{V}_{(M,N)}(\vec{T}_{\text{vac}}))}{(2\pi)^2 \mathcal{V}_{(M,N)}(\vec{T}_{\text{lump}})},$$

where $\vec{T}_{\text{lump}}$ is the lump solution, $\vec{T}_{\text{vac}}$ is the solution corresponding to the true vacuum at the given truncation level $(M, N)$ and $\mathcal{V}_{(M,N)}$ is the potential at level $(M, N)$. With our solution above, we find

$$r^{(2)}_{(2, 4)} = 1.13025,$$

13% away from the expected value of unity.  

To show that the size of the lump is, in the large radius limit, independent of the radius, let us compare with the solution at $R = 3$ (Fig 2).

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1I thank B. Zwiebach for this suggestion.
2In [2], the authors find 1.1378 at the same truncation level. There is a slight disagreement between these two numbers, though it might be due to round off or numerical error.
Figure 1: A lump in sft at level (2, 4) with $R = \sqrt{3}$. The plot represents $-t(x, y)$ as a function of $x$ and $y$.

To compare the sizes of this lump, we plot together their profiles $-t(x, 0)$. Fig.3 clearly shows the radius-independence of the shape of the lump.

3.2 Pure tachyonic string field theory and pure $\phi^3$ theory

We do find codimension 2 lump solutions in these theories. In fig.4, we show the profiles $-t(x, 0)$ of the lumps in the three different theories. We have taken the three lumps to be at level (2, 4) with $R = 3$. The different asymptotic values of $t(x, 0)$ show the different vev’s of the tachyon in the three theories.

The pure tachyonic theories are much more tractable for numerical computations. We have written codes giving the actions at arbitrary level. To illustrate the convergence of the level truncation scheme, we show in figs.5, 6 the lump profiles at different truncation levels.

It is interesting to see that the solution in pure $\phi^3$ theory converges much slower than in pure tachyonic sft. This is due to the fact that in pure tachyonic sft, the coefficient of
Figure 2: A lump in sft at level (2, 4) with $R = 3$. The plot represents $-t(x, y)$ as a function of $x$ and $y$.

the term $t_{m_1n_1}t_{m_2n_2}t_{m_3n_3}$ in the potential is proportional to $K^3-(m_1^2+n_1^2+m_2^2+n_2^2+m_3^2+n_3^2)/R^2$.

Since $K > 1$, at high level these terms are much less important than in pure $\phi^3$ theory where the same coefficients are proportional to $K^3$.

4 Conclusion

We do find codimension 2 lumps solutions in all three theories considered in this paper. Note that there is an apparent conflict with Derrick’s theorem which states that solitons in scalar field theory can exist only in codimension < 2. But one of the assumptions used in the proof of the theorem is that the potential must be bounded below, which is not the case in the theories considered here\(^3\). This negativity allows the existence of solutions, as shown in [13].

As it is easy to use, pure tachyonic sft is an interesting toy model of full string

\(^3\)I wish to thank B. Zwiebach for pointing this out.
Figure 3: The dashed curve is the profile \(-t(x, 0)\) of the lump solution in sft at \(R = \sqrt{3}\), the solid curve is the profile \(-t(x, 0)\) of the lump solution at \(R = 3\).

Figure 4: Lump profiles \(-t(x, 0)\) in string field theory (solid curve), pure tachyonic sft (dashed curve) and pure \(\phi^3\) theory (dotted curve) at level (2, 4) and \(R = 3\).

Field theory. We saw that it converges very fast when the truncation level is increased. Moreover we found a lump solution of approximately the same shape as in sft, this
Figure 5: Lump profiles $-t(x,0)$ in pure tachyonic sft with $R = 3$, at level (2, 4) (dashed curve), (3, 9) (dotted curve) and (10, 30) (solid curve)

Figure 6: Lump profiles $-t(x,0)$ in pure $\phi^3$ theory with $R = 3$, at level (2, 4) (big-dashed curve), (3, 9) (dotted curve), (7, 21) (small-dashed curve), and (20, 60) (solid curve)

may show that we can study other kinds of sft solitons (like intersecting branes) by
using the pure tachyonic approximation (work in progress [21]).

Finally, we saw the amusing fact that string field theory, due to the coefficients $K^3$-level in front of every term, seems to be better fit for level truncation than the simple $\phi^3$ theory which converges very slowly as the level is increased.

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