Effect of load history on shear strength of reinforced concrete I-Beams

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Abstract. A series of 32 reinforced concrete I-beams was tested to investigate the effect of existing inclined cracks on shear strength. Testing of beams consisted of two stages. At the preliminary stage, load large enough to cause a formation of a developed crack pattern in the web was applied with the primary shear span-depth ratio. After that, the load with breaking shear span-depth ratio was applied until failure of the specimen. The load applied during the preliminary loading stage was maintained with a full or partial value by a lever, or removed. The test variables include shear span, shear reinforcement ratio and angle of stirrups inclination. Pre-cracking was found to decrease the ultimate shear strength of beams by a maximum of 16 percent depending on the angle of stirrups. A corresponding reduction factor is suggested to the formula of the Russian code for the design of bridges. The finite element simulation of beams was fulfilled by using the program OpenSees. The webs of beams were simulated with a 2D element built on the Cyclic Softened Membrane Model to take into account the deterioration of concrete due to the previous loading. The accuracy of the proposed FE model was verified with the aforementioned tests and the results of several other studies.

1. Introduction
This paper addresses a specific case of shear in reinforced concrete I-beams, namely the shear capacity considering the influence of loading history. Changing the loading scheme is a common practice during different kinds of reconstruction. The action of the shear force causes the emergence of inclined cracks oriented in the direction of principal tensile stresses. The cracks that form due to service loads under one shear span might influence the behavior of a specimen after applying the load to the new shear span. To study the possible influence of this effect, 32 reinforced concrete I-beams were tested under two-stage loading with changing the shear span. In addition to the shear span-depth ratio \(a/h_0\), the concrete strength \(R_b\), shear reinforcement ratio \(\mu_{sw}\) and angle of inclination of stirrups \(\alpha\) were varied. For further convenience, let us define the primary shear span-depth ratio \((a/h_0)_1\) to be the shear span-depth ratio under which the preliminary cracks are formed. The shear span-depth ratio under which the specimen is tested to failure will be accordingly called the breaking shear span-depth ratio \((a/h_0)_2\).
During the last four decades, a large amount of research was conducted in the attempt to solve the problem of shear in reinforced concrete and prestressed concrete elements. As a result, a lot of experimental data has been obtained and numerous effective models were suggested [1]. However, due to the complexity of the shear problem, there is still a significant amount of argument around its certain aspects [2-5]. As opposed to bending design provisions for shear in building codes not only vary in their formulations but also are based on different assumptions [1, 6, 7]. The reason for this is that a lot of parameters need to be considered during the shear analysis. Semi empirical methods of building codes are often not considered to be cross-functional and efficient [8].

Considerable progress in the subject was made with the development of softened truss models (MCFT [9], RA-STM [10], FA-STM [11], CSMM [12, 13], etc.) which recognize cracked reinforced concrete as a new material that has its own stress-strain characteristics. These rational models satisfy the three fundamental principles of the mechanics of materials: stress equilibrium, strain compatibility and the constitutive laws of materials. One of the crucial advantages of softened truss models over discrete models is that former can be effectively incorporated in finite element simulation [14, 15]. In such analyses, a whole reinforced concrete structure can be visualized as an assembly of plane stress (membrane) or shell elements [16, 17].

Rotating crack models (CFT, RA-STM) assume that the direction of cracks is inclined at an angle that follows the direction of principal compressive stress in the concrete. For this reason effect of loading history on the shear strength of I-beams cannot be assessed with these models. In fixed crack models, inclined cracks in membrane elements are assumed to form and stay in the direction of applied principal stresses that caused their emergence. Therefore, fixed crack models can be utilized for analysis of the shear behaviour of I-beams with existing cracks.

2. Experimental program

2.1. Test specimens

All 32 beams had an effective depth \( h_0 = 260 \) mm, web width \( b = 30 \) mm and the total length of 2.34 m, whereas the span length was equal to 2.26 m. The cross-section of the beams is presented in Figure 1. Normal strength concrete was selected with the values of compressive strength \( R_0 \) in the range of 24.8-34.7 MPa. Cold-drawn deformed wire with diameters 3 mm and 5 mm at a 40 mm spacing was used as stirrups, providing shear reinforcement ratio \( \mu_{sh} = 0.59-1.64\% \). Stirrups were set either vertical or inclined at the angle \( \alpha = 45^\circ \) with respect to the longitudinal axis of the specimen. Four mild steel bars with diameter 18 mm were used as longitudinal reinforcement in all beams. To prevent anchorage slip, the ends of the bars were welded with a lateral steel plate. The values of the variables for the specimens along with the values of \((a/h_0)_1\), \((a/h_0)_2\) and failure loads \(Q_{fail}\) are presented in Table 1.

2.2. Loading scheme and instrumentation

Testing of the specimens was carried out in two stages. At the preliminary stage, load \( Q_1 \) was applied with the primary shear span-depth ratio \((a/h_0)_1\) up to the level approximately corresponding to 70% of the shear capacity. This load was enough for the formation of a developed crack pattern in the web of the beam. Afterwards, the load with breaking shear span-depth ratio \((a/h_0)_2\) was applied until failure of the specimen. The load applied during the preliminary loading stage was maintained with a full or partial value, or entirely removed.

Additionally, the reference beams with similar characteristics were tested in which the shear span was not changed. In Table 1 the specimens are divided into groups according to the breaking shear span-depth ratio \((a/h_0)_2\).

In some tests, the presented loading scheme required applying the constant load with \((a/h_0)_1\) along with the increasing load with \((a/h_0)_2\). This loading scheme was implemented with a specially manufactured lever facility (figure 2) which allows constant load up to 60 kN at each support. The increasing load was created by a hydraulic jack embedded in a self-bearing frame and transmitted to the specimen at two points via a crossbeam.
Electrical-resistance strain gauges were attached to the surface of the web to measure the local compression strains of the concrete. The angle of gauge direction was selected so it would coincide with the direction of forming cracks as closely as possible. After each loading step, cracks were marked as they were developing, and their widths were measured with a hand-held microscope. For the convenience of further analysis, cracks were marked with two different colors corresponding to two stages of testing.

3. Web-crushing shear strength formula
Normalized shear strength $Q_{test}/(R_bh_0)$ of tested beams is given in table 1. Its variation becomes smaller with the increase of the breaking shear span. In tests with $(a/h_0)_2 = 3.2$ loading history doesn’t affect the strength of beams. Pre-existing cracks also were not found to influence the strength of beams with vertical stirrups tested with medium breaking shear span $(a/h_0)_2 = 2.5$. Cracks formed on preliminary stage led to a reduction of shear strength up to 10% in tests with $(a/h_0)_2 = 1.6$ and 8% in two series of beams with $(a/h_0)_2 = 2.5$ and $\gamma=45^\circ$. Most significant 19% reduction of shear strength was observed in these series in tests with the least primary shear span-depth ratio $(a/h_0)_1 = 1.6$. 
Table 1. Test variables and failure loads.

| Beam | $Q_1$, kN | $Q_i$ stays $(a/h_0)_1$ $(a/h_0)_2$ | $a$, deg | $R_{cr}$, MPa | $Q_{cr}$, kN | $Q_{cr}/(R_p h_0)$ | $Q_{cr}/Q_{SPS5}$ | $Q_{cr}/Q_{SPS3}$ | $Q_{cr}/Q_{mod}$ | $Q_{cr}/Q_{DS}$ |
|------|-----------|--------------------------------------|---------|-------------|-------------|-----------------|--------------------|--------------------|----------------|----------------|
| 1    | -         | 1.6 90 0.59 31.9 88.7 0.357 1.50 |         |             |             |                 | 1.19 0.95 0.97    |        |
| 2    | 55.22     | 2.5 1.6 90 0.59 26.9 68.1 0.325 1.27 |         |             |             |                 | 1.08 0.85 0.86    |        |
| 3    | 50.32     | 3.2 1.6 90 0.59 33.0 91.6 0.356 1.52 |         |             |             |                 | 1.19 1.02 0.99    |        |
| 4    | 23.86     | 2.5 1.6 90 0.59 25.8 75.0 0.372 1.43 |         |             |             |                 | 1.24 0.96 0.87    |        |
| 5    | 21.90     | 3.2 1.6 90 0.59 26.1 65.7 0.323 1.25 |         |             |             |                 | 1.08 0.83 0.86    |        |
| 6    | 39.05     | 2.5 1.6 90 0.59 29.8 81.8 0.352 1.43 |         |             |             |                 | 1.17 0.96 0.93    |        |
| 7    | 42.48     | 3.2 1.6 90 0.59 31.6 96.5 0.392 1.64 |         |             |             |                 | 1.31 1.10 1.14    |        |
| 8    | -         | 2.5 90 0.59 34.7 66.0 0.244 1.07 |         |             |             |                 | 0.81 1.04 0.91    |        |
| 9    | 60.12     | 1.6 2.5 90 0.59 30.8 68.9 0.287 1.19 |         |             |             |                 | 0.96 1.16 0.99    |        |
| 10   | 35.62     | 3.2 2.5 90 0.59 31.9 71.9 0.289 1.21 |         |             |             |                 | 0.96 1.18 1.03    |        |
| 11   | 50.52     | 1.6 2.5 90 0.59 29.4 84.8 0.370 1.50 |         |             |             |                 | 1.23 0.95 0.96    |        |
| 12   | 42.48     | 3.2 2.5 90 0.59 27.1 68.0 0.322 1.26 |         |             |             |                 | 1.07 1.23 1.09    |        |
| 13   | -         | 2.5 45 0.83 29.1 86.6 0.381 1.21 |         |             |             |                 | 1.27 1.02 1.03    |        |
| 14   | 74.82     | 1.6 2.5 45 0.83 29.0 64.0 0.283 0.90 |         |             |             |                 | 0.94 0.83 0.86    |        |
| 15   | 38.56     | 3.2 2.5 45 0.83 29.1 80.7 0.355 1.13 |         |             |             |                 | 1.18 1.04 0.97    |        |
| 16   | -         | 2.5 45 0.83 31.9 85.6 0.344 1.14 |         |             |             |                 | 1.15 0.96 1.00    |        |
| 17   | 69.92     | 1.6 2.5 45 0.83 31.4 73.8 0.302 0.99 |         |             |             |                 | 1.01 0.91 0.86    |        |
| 18   | 65.02     | 3.2 2.5 45 0.83 30.5 93.4 0.393 1.28 |         |             |             |                 | 1.31 1.18 1.11    |        |
| 19   | -         | 3.2 90 0.59 28.0 52.1 0.239 0.95 |         |             |             |                 | 0.80 0.93 0.90    |        |
| 20   | 55.22     | 1.6 3.2 90 0.59 28.2 56.2 0.256 1.02 |         |             |             |                 | 0.85 1.00 0.99    |        |
| 21   | 50.32     | 2.5 3.2 90 0.59 27.7 53.4 0.247 0.98 |         |             |             |                 | 0.82 0.96 0.93    |        |
| 22   | 50.32     | 1.6 3.2 90 0.59 28.9 81.4 0.361 1.45 |         |             |             |                 | 1.20 0.92 0.98    |        |
| 23   | 48.90     | 2.5 3.2 90 0.59 24.8 48.9 0.253 0.96 |         |             |             |                 | 0.84 0.94 0.85    |        |
| 24   | -         | 3.2 90 1.64 26.9 61.9 0.249 1.04 |         |             |             |                 | 0.83 1.03 1.00    |        |
| 25   | -         | 3.2 90 0.59 31.9 59.6 0.248 1.03 |         |             |             |                 | 0.83 1.01 1.03    |        |
| 26   | 65.02     | 1.6 3.2 90 0.59 30.8 59.6 0.245 1.02 |         |             |             |                 | 0.82 1.00 1.03    |        |
| 27   | 40.52     | 2.5 3.2 90 0.59 31.2 62.9 0.300 0.94 |         |             |             |                 | 1.00 0.92 0.98    |        |
| 28   | -         | 3.2 90 1.64 30.8 78.1 0.325 1.07 |         |             |             |                 | 1.08 1.05 1.11    |        |
| 29   | 65.02     | 1.6 3.2 90 1.64 32.4 82.7 0.327 1.10 |         |             |             |                 | 1.09 1.08 1.12    |        |
| 30   | 69.92     | 2.5 3.2 90 1.64 33.0 84.1 0.327 1.11 |         |             |             |                 | 1.09 1.09 1.14    |        |

Mean value for reference beams 1.11 1.02 0.99 0.99
Coefficient of variation for reference beams 0.161 0.182 0.053 0.069
Mean value for all beams 1.19 1.05 1.00 0.982
Coefficient of variation for all beams 0.172 0.159 0.100 0.091

In design formula, the influence of the loading history can be taken into account by introducing a reduction factor $\gamma_h$ to the nominal design web-crushing strength of beam without initial shear cracks. The latter can be calculated by modified formula of SP 35.13330.2011

$$Q_{mod} = k \varphi_{w1} \varphi_{b1} \varphi_{at} R_p b h_0$$  \hspace{1cm} (1)

where

$$\varphi_{at} = 1 + \eta \cdot n_1 \cdot \mu_{sw}$$  \hspace{1cm} (2)

$\eta = 5$ for beams with vertical stirrups;
$\eta = 15$ for beams with stirrups inclined at $a=45^\circ$;

$$n_1 = \frac{E_s}{E_b};$$  \hspace{1cm} (3)
\[ \varphi_{b1} = 1 - 0.01R_b \]  
\[ \varphi_{a1} = \frac{2.5h_0}{a} \geq 1 \]  

Compared to the formula of SP 35.13330.2011 equation (1) is complemented with the factor \( \varphi_{a1} \) to take into account the influence of shear span-depth ratio. Besides that, the value of parameter \( \eta \) for inclined stirrups was increased from 10 to 15. On the basis of statistical analysis of our experimental data factor \( \gamma_h \) was found to be equal to 0.95 and 0.91 for beams with vertical and inclined stirrups respectively with overall constant \( k = 0.305 \). Table 1 gives the ratios of \( Q_{\text{test}} \) to the values of shear strength calculated using methods of SP 35.13330.2011 \( (Q_{\text{SP35}}) \) and SP 63.13330.2013 \( (Q_{\text{SP63}}) \) and proposed equation (1). Factor \( \gamma_h \) was used in equation (1) for beams with \( (a/h_0) = 1.6 \) and beams with inclined stirrups and \( (a/h_0) = 2.5 \) which strength was most significantly affected by loading history. It can be seen that for tested beams mean value of \( Q_{\text{test}} / Q_{\text{mod}} \) is 1.0 with a coefficient of variation being approximately 1.5 less as compared to \( Q_{\text{test}} / Q_{\text{SP35}} \) and \( Q_{\text{test}} / Q_{\text{SP63}} \).

For practical feasibility, overall constant \( k \) in equation (1) can be reasonably rounded to 0.3. Considering relatively large reduction of strength in certain tests with the loading history value of \( \gamma_h \) is recommended to be conservatively taken equal to 0.9 and 0.85 in cases of vertical and inclined shear reinforcement respectively.

4. **Nonlinear FE modeling of two-stage loading of beams**

Design equations like (1) are simple and convenient to use. Their reliability is reasonably acceptable only in the experimentally confirmed range. Therefore, it is essential to develop more versatile tools such as FE method for numerical analysis of structures.

Modelling of the performed experiments was fulfilled in object-oriented FE framework OpenSees [18]. The web of the beams (figure 3) was modeled with quadrilateral elements with an assigned material class RC Plane Stress [19]. Performance of this class is based on a Cyclic

![Figure 3. FE model for the tested beams](image-url)
Softened Membrane Model for reinforced concrete plane stress condition. The flanges of the beams, which resist the bending moment, were modeled with nonlinear Beam Column elements having fiber section [20]. For both flanges uniaxial Kent-Scott-Park concrete material and uniaxial bilinear steel material were used.

For all tested beams, analytical load-displacement curves were obtained using aforementioned model. As an example, the analytical load-displacement curves for beams 13, 14 and 15 are shown in figure 4. The peak loads on these curves are the analytical value of shear strength $Q_{OS}$. Values of $Q_{OS}/Q_{test}$ listed in table 1 demonstrate the good ability of the model in predicting the shear strength in both single- and two-stage loadings. Mean value $Q_{OS}/Q_{test}$ is 0.982 with a coefficient of variation 0.091. In 60 % of results difference in the values of $Q_{OS}$ and $Q_{test}$ is less than 10 %.

![Analytical load-displacement curves for beams 13, 14 and 15](image)

**Figure 4.** Analytical load-displacement curves for beams 13, 14 and 15

The model was verified on the tests of reinforced concrete I-beams with web-crushing failure found in the literature [21, 22, 23, 24]. Considered tests comprise a sufficiently wide range of shear reinforcement ratio $\mu_{sw} = 0.78 \pm 3.19 \%$, concrete strength $R_c = 16 \pm 40$ MPa, sections of different sizes, loadings with concentrated and distributed loads (table 2). Calculated and test values of shear strength are close to each other with the mean value and coefficient of variation for $Q_{OS}/Q_{test}$ equal to 1.07 and 0.06 respectively.

**Table 2.** Calculated values of shear strength for beams found in the literature

| Author   | Beam    | $b$, cm | $h_0$, cm | $R_c$, MPa | $\mu_{sw}$, % | $a/h_0$ | $Q_{test}$, kN | $Q_{OS}$, kN | $Q_{test}/Q_{OS}$ |
|----------|---------|---------|-----------|-------------|---------------|---------|----------------|--------------|------------------|
| Aliev [21] | BT-II-1(a) | 4.2    | 33.0  | 16      | 1.75        | 2       | 102.5          | 91.2         | 1.12             |
|          | BT-II-1(b) | 4.3    | 33.0  | 16      | 1.36        | 2       | 95             | 86.2         | 1.10             |
|          | BO-II-3   | 4.5    | 33.0  | 18      | 1.63        | 2       | 106.9          | 93.8         | 1.14             |
| Baloyan [22] | B-11/4/1/2A | 2.8 | 26     | 20.5  | 1.75        | 2.5     | 51.3           | 51.9         | 0.99             |
|          | B1-4/3/4/14B | 2.8 | 26     | 39.9  | 0.78        | -       | 91.2           | 79.3         | 1.15             |
| Rangan [23] | I-1     | 7.4    | 56.3  | 34.3  | 2.72        | 2.5     | 453.1          | 411.1        | 1.10             |
|          | I-2     | 7.4    | 56.3  | 28.4  | 1.53        | 2.5     | 371            | 378          | 0.98             |
|          | I-3     | 6.3    | 56.3  | 29.3  | 3.19        | 2.5     | 369.1          | 365.5        | 1.01             |
|          | I-4     | 6.4    | 56.3  | 33.5  | 1.77        | 2.5     | 416            | 367.9        | 1.13             |
| Abdullaev [24] | BD-I-3  | 5.1    | 32.2  | 22.2  | 1.48        | 3       | 118.1          | 113.6        | 1.04             |
|          | BD-I-3x | 5.2    | 32    | 20.8  | 1.46        | 3       | 114.9          | 112.5        | 1.02             |
5. Conclusions
1. Thirty-two reinforced concrete I-beams were tested for failure to study their shear behavior in cases of two-stage loadings with changing shear span. Experimental variables were the values of primary and breaking shear span-depth ratios, concrete strength, shear reinforcement ratios and angles of stirrups inclination.
2. Shear strength of beams with pre-cracked web decreases by a maximum of 10% for beams with vertical stirrups and approximately by 16% in case of inclined stirrups. In calculations by design formulas effect of the previous loading can be considered by introducing reduction factor $\gamma_h$ to the nominal design web-crushing strength of beam without initial shear cracks. Recommended values of the $\gamma_h$ are 0.9 and 0.85 for beams with vertical and inclined stirrups respectively. For calculation of the nominal web-crushing strength, a modified formula of SP 35.13330.2011 is suggested.
3. Nonlinear FE modeling in OpenSees is shown to be an effective tool for predicting the shear strength of beams under two-stage loading with changing shear span. The behavior of the web in the model is defined by the constitutive laws of CSMM. The accuracy of the model was validated by analysis of beams tested by the authors and beams with web-crushing failure found in the literature.

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