Two Scheduling Models, One Project: Are Models Applicable in Case of Real Projects?

Miklós Hajdu*

Ybl Miklós Faculty of Architecture and Civil Engineering SZIU, Budapest H-1146, Hungary

Abstract

Modern project management has developed numerous techniques based on mathematical models in order to be able to plan the projects’ processes in time, their costs and resources. Even though a significant majority of the users do not know that, there are hypotheses behind every technique. These hypotheses help simplify the problems to such an extent that they can be handled by mathematical tools. In this paper, two mathematical models are examined, compared; then the results are analyzed. The first model is the cost optimization model, which can be applied to determine the optimal direct cost corresponding to a given project duration. With the help of the other model the expected distribution of the project duration is determined, assuming that the distribution of the activity durations is stochastic. Both models are adapted in Precedence Diagramming networks, consequently, PDM/cost and PDM/PERT expressions are used in the paper. A PDM network plan made by the contractor of a construction project is developed further into a PDM/cost and a PDM/PERT model in a case study described in the paper. Then calculations are performed according to the models. Finally, consequences are drawn, and attempts are made to find common points of interpretation of the two models.

Keywords: Precedence diagramming; precedence-PERT; time-cost trade-off

1. Introduction

In this paper, two very well-known scheduling models are examined and compared. One is the original time-cost trade-off model developed by Kelley and Walker (Kelley & Walker 1959, 1961, 1989), known as CPM - to avoid ambiguity, hereinafter referred to as CPM/cost -; the second is the PERT that was
developed by Malcolm et al. in 1957 (Malcolm et al., 1959). A schedule of a highway bridge project serves as a case study throughout the paper. Precedence Diagramming Method is the scheduling technique behind the baseline plan of the case study. Firstly, cost optimization model is applied to define the lowest direct cost curve within a given interval. Secondly, the PERT model is used to determine the distribution of the project duration. Both the cost optimization and PERT model are originally defined in an activity-on-arrow structure, but here Precedence Diagramming (Roy, 1959; Fondahl, 1961) is used as a network that describes the project’s logic. Application of these models in Precedence Diagramming is quite new in project management; therefore the introduction of these models is inevitable.

2. CPM: the first time-cost trade-off model

Critical Path Method was developed by Kelley and Walker in 1957. An acyclic directed graph with one start and one finish node was used to describe the logic among activities. Activities of the projects were represented by arrows, events were represented by nodes. Events were used to describe the logic between activities (An event occurs when all the preceding activities finish, and succeeding activities can start after that). According to the original hypothesis, the duration of an activity can be shortened compared to the normal duration, but only until a certain point, which is called crash duration. However shortening an activity affects (increase) the project cost. The so-called normal cost belongs to the normal durations, while the cost belonging to the crash duration is called crash cost. Crash cost is greater than or equal to the normal cost, the change of the cost between the normal and crash durations is linear. This is shown in Figure 1.

![Fig. 1. Time-cost trade-off hypothesis for activities in CPM](image)

Changing the activity durations within their upper and lower bounds will result in different project durations and different project costs even for the same project duration. The set of possible solutions can be seen in Figure 2.

The goal of the original CPM model was to define the lower envelop of that set in Figure 2, that is to determine the least direct cost solution within the interval defined by the normal and crash durations.

Over the decades dozens of sophisticated embellishments were developed to introduce new, sometimes faster, or easier algorithms (Fulkerson, 1961; Hindelang & Muth, 1971; Klafszky, 1969), or to make the CPM more general. In spite of all these developments CPM has slowly lost its significance due to the following four reasons:

- The activity-on-arrow structure is inconvenient to use. Even in case of medium-sized projects depicting a proper activity-on-arrow diagram is almost impossible based on a list of preceding activities (Fondahl, 1961).
Modeling real life projects with their difficult internal logic is difficult with CPM, because CPM can only handle FS0 relations (An activity can start when all the preceding activities have been finished.). Collecting reliable data on crash durations and crash cost is a demanding job, very often with results of high uncertainty. The model can only handle time-cost trade-offs. Other trade-offs like time vs. quality or time vs. risk, or quality vs. risk are still unknown, the model does not give any information on what we have to pay in terms of risk, quality etc. when we shorten a project.

As to the first two notes, some recent research results offer some help. Today’s project planning practice uses solely the Precedence Diagramming Method (PDM) for planning, and scheduling. This technique is much more flexible due to its enhanced modeling capabilities. Minimal and maximal relations give the opportunity to create a model of the project that is closer to reality. The application of the time-cost trade-off model in PDM is a work of Hajdu (Hajdu, 1993, 1997). Over the years all those improvements to CPM/cost have become available to PDM/cost, as well (Mályusz, 2004; Hajdu, 1996).

However, the problem mentioned last seems to be a challenging one. Since the development of the project management’s golden triangle, we know that at least one more trade-off (quality) should be examined when making a time-cost trade-off, and now we also know that many more dimensions of a project exist. When performing a simple time-cost trade-off, probably all of the above-mentioned dimensions of the project will be affected, however we do not have models that can handle these connections. Recently Babu and Suresh (Babu & Suresh, 1996) started to investigate the quality, time, cost dimensions together, but their results need to be investigated, and further developed for the purpose of practical use.

3. PERT: impacts of risks on project duration.

PERT was developed in 1957 (Malcolm et al., 1959) in order to help the control of the Polaris missile program. The structure of PERT was the same as that of CPM: an acyclic directed activity-on-arrow graph with one start and one finish node. The main difference compared to CPM was that in PERT activities were introduced as stochastic variables. The following hypotheses were assumed regarding activities and activity durations:

- Activity durations follow a so-called PERT-Beta distribution (1), where $\Gamma(x)$ is the gamma function, $B(\alpha, \beta)$ is the beta function, and both $\alpha$ and $\beta$ are greater than or equal to zero. If both $\alpha$ and $\beta$ are greater than 1, the function is called PERT-beta function.
\[ f(x) = \frac{1}{B(\alpha, \beta)} x^{\alpha-1} (1-x)^{\beta-1} = \frac{\Gamma(\alpha+\beta)}{\Gamma(\alpha) \Gamma(\beta)} x^{\alpha-1} (1-x)^{\beta-1}, \quad x \in [0,1] \]  

1. Activities and their durations are independent of each other.
2. The probability density function of the activity duration can be estimated by its optimistic (\(a\)), pessimistic (\(b\)) and most probably duration (\(m\)). (Clark, 1962)
3. The mean, that is the expected value (\(\mu\)), and the variance (\(\sigma^2\)) can be calculated according to (2) and (3) (Malcolm et al., 1959)
   \[ \mu = (a+4m+b)/6 \]  
   (2)
   \[ \sigma^2 = (b-a)^2/36 \]  
   (3)

The following hypotheses were assumed regarding the project duration and its distribution:

- Project duration follows a normal distribution.
- The expected value of the density function of the project duration can be determined as a result of a time analysis performed based on the expected activity times. (Theoretically the shortest (optimistic) project duration is the one that is calculated with the optimistic activity durations, the longest (pessimistic) project duration is the one calculated with pessimistic activity durations.)
- The variance of the project duration - according to the central limit theorem of probability theory - can be defined as the sum of the activity variances, as it is shown in (4)
   \[ \sigma^2_{PD} = \sum_{i \in CP} \sigma_i^2 = \sum_{i \in CP} \left( \frac{(b-a)^2}{6} \right) \]  
   (4)

From the beginning PERT has been criticized about the followings:

- the inaccuracy of the three-point estimation ,
- the independence of activities,
- the optimistic approximation of the expected project duration, and the distribution of it.

The last point is the most important in terms of our further examinations. According to the original PERT assumptions, the project duration - due to the central limit theorem - follows a normal distribution. The expected duration is derived from a time analysis performed based on the activity expected values. It was soon realized (Clark, 1961) that PERT usually gives a smaller expected duration compared to that of the Monte Carlo simulation. According to Trietsch and Baker, the difference is called the “Jensen gap” according to (Trietsch & Baker, 2012). The reason for this phenomenon is that PERT assumes only one critical path in a plan, but the reality is that sometimes various separate or combined critical paths exist in a project network. Better estimation of the expected duration can be found in different works (Dodin, 1985; Yao & Chu, 2007). An even more serious problem with PERT is that the original model does not handle activity calendars. However, activity calendars are extensively used when planning real-life projects, especially in the construction industry. Hajdu (Hajdu, 2012) presents some sample projects and shows that the distribution of project duration is far from normal when different activity calendars are applied. Hajdu also argues that this simplification (i.e. omitting activity calendars from the original PERT model) distorts the real distribution of the project duration to such an extent that the original PERT
assumptions ought not to be used. The following example is used to justify this statement.

![Sample PERT network](image)

**Fig. 3. Sample PERT network**

A sample project is given according to Figure 3. The distribution of the activity durations is symmetric. All activities have the same most probable duration (40 days) and the same optimistic and pessimistic durations (20 days, 60 days). The density function is close to uniform, with zero probability for 20 and 60 days. The distribution of the project duration - when calendars are not used - can be seen in Figure 4. The upper row shows the results when the original PERT assumptions are used. The bottom row and the density function of the project duration are the results of a Monte Carlo simulation after 1.5 million iterations. The results of the simulation definitely support the PERT theory.

![Distribution of the project duration: No activity calendars in use](image)

**Fig. 4. Distribution of the project duration: No activity calendars in use**

In Figure 5, the probability function of the project can be seen when Act. 2 cannot work from day 80 to day 125. The distribution is far from normal. PERT - similarly to the original CPM model - has also lost its significance over the decades, due to the same reasons as we experienced with CPM.

These were the following:

- The activity-on-arrow structure is inconvenient to use.
- Modeling real-life projects with their difficult internal logic is difficult with PERT, because CPM can only handle FS0 relations.
- Defining the activity distributions is a demanding job, very often with results of high uncertainty.
- The model shows some kind of time and risk dependencies. However, no information is given as to how these affect the cost, quality and other dimensions of the project.

To overcome the first two problems one solution could be the application of the PERT model in Precedence Diagramming. PDM offers an easier and better way for modeling the project’s internal dependencies. This model can be called Precedence-PERT, which is introduced in Hajdu’s work. The last note draws attention to the fact that even though PERT indicates the result of a risk vs. project duration trade-off, no information is provided on how this phenomenon may affect other dimensions of the project.
In the following, a case study is presented, where both time-cost trade-off and PERT models are applied on the same construction project. The original schedule has been prepared by using Precedence Diagramming Method, therefore the time-cost trade-off is used in Precedence Diagramming, and the PERT model is applied in Precedence Diagramming as well.

Fig. 5. Distribution of the project duration: Activity calendars in use.

4. The case study

The project chosen for testing is the construction of a bridge over the river Danube in Budapest, Hungary, with the connecting roads at the two ends. The project is an important part of the program which aims to make a ring around Budapest in order to reduce the amount of - mainly - international transit traffic in the capital. The project started in February, 2006 and was planned to finish is September, 2008. The project finished according to the deadline set in the baseline plan.

The total length of the bridge is 1862 m. Structurally it is composed of five parts with the following bridge spans:

- left quayside (Pest) inundation area bridge: 37m + 2*33m + 45 m
- main Danube-branch bridge: 145m + 300m + 145m
- “Szentendrei” island inundation area bridge: 42m + 11* 47m
- “Szentendrei” Danube-branch bridge 94m + 144m + 94m
- right quayside (Buda) inundation area bridge: 43m + 3*44m + 43m

The contracted fee was € 260 million. The client was the National Infrastructure Development Ltd. (NID), which was responsible for managing all the government financed infrastructural projects (road and railway). The contractors’ consortium was formed by Hídépító Co. and Strabag Hungary Ltd.

NID had and still has very strict requirements regarding project planning and monitoring, and demands that all the contracted partners fulfill their regulations in these fields. The reason for this is that NID manages not only the project but a whole portfolio; therefore projects have to be managed and handled in the project management system in a uniform way. This includes:

- The methodology of developing the schedule of quantities (It is prepared by the designers and serves as the basis of the bids, cost monitoring and control, surveys etc.)
- The methodology of preparing the baseline plan (A baseline plan generally consists of 1000 activities, but, in some cases, there can be more than 5000 activities.)
- The methodology of monitoring the performance in time, which is carried out monthly.
- The methodology of handling claims and paid/unpaid extra works.
• The methodology of monthly actualization of the schedule.

These regulations of NID have been in use since the year 2000, so the contractors had enough time to learn how to fulfill these requirements. The baseline plan which was the basis of our work was made in 2006, and was comprised of 1316 activities and 1420 logical relationships. The schedule of quantities consisted of 1750 items. The WBS (Work Breakdown Structure) was developed automatically from the schedule of quantities according to the standardized rules of NID. In the baseline plan there was one critical path with the length of 938 days. The plan was developed in ProjectDirector 4.0, a scheduling application used by NID and the contractors, which can handle maximal-type precedence relationships. Four different calendars were used in the network.

4.1. Least cost scheduling

Least cost scheduling requires a normal time with the related normal cost, and a crash time with the related crash cost to be defined in case of each activity, so the most important task during the preparation for least cost scheduling was the definition of these data.

When determining the activity durations and activity costs, we followed the principle that the activity durations and costs in the baseline plan served as the normal durations and normal costs of the activities. Therefore, during the least cost scheduling, our task was to define the crash duration and crash cost for each activity. For this two methods were applied either detailed investigation of an activity or estimation, based on expert opinion. In the course of the estimation several meetings with the chief engineer responsible for the construction were held in order to determine crash durations and crash costs. The result of the estimation was that activity durations could be reduced to 60-80 percent of the original, that is of the normal duration, which resulted in an average 25 percent (10-40 percent) increment in cost. This kind of detailed investigation was carried out for only 10 activities. For the rest of the activities we applied a similar principle: crash durations were set to the 70% of normal ones, with an associated 30% of cost increment.

The result of the analysis can be summarized as below. There were 140 breakpoints in the cost curve. The minimum project duration decreased to 912 days from 1121 days. The increment of the project direct cost in this interval was € 2 403 949. It means that a more than 200-day shortening of the project duration was possible; and it cost less than 1% of the contracted fee. The results of the calculations are shown in Figure 6.

![Cost increment vs. Project duration](image)
4.2. Precedence PERT

PERT was applied in case of this schedule in 2012 after Precedence PERT technique (the application of the PERT model in Precedence Diagramming) had been developed. In the model the original activity durations served as the most probable duration; and the crash durations of the time-cost trade-off model as the optimistic durations. However, the least cost scheduling model could not provide data that could serve as the pessimistic durations. This problem was solved by the assumption that pessimistic durations could be defined as 50% longer than the original (normal duration in the cost optimization model) activity durations. The algorithm that generated the distribution of the project duration was based on Monte Carlo simulation. The results can be seen in Figure 7. Firstly, it can be stated that the distribution does definitely not follow a normal distribution. The calculated expected duration is 1083 days, which is less than the expected duration determined by using the original PERT assumptions.

The reason for this distribution is the use of different activity calendars in the project, and the anomalous behavior of critical activities in Precedence Diagramming, which were first described by Weist (Weist, 1981).

Fig. 7. Case study: distribution of project durations

5. Conclusions and further recommendations

Two mathematical models have been applied to the same project, the schedule of which was made by using the Precedence Diagramming Method. Common points are very hard to find at first sight. The only similarity is that the project duration calculated based on the normal durations of the least cost scheduling model (PDM/cost) and the expected project duration determined by PERT estimation for the Precedence PERT model are the same (1121 days).

If we accept that both models show a different dimension of the same project (time-cost and time-risk trade-off), then we have to accept that the optimistic activity durations of the PERT model are equal to the crash durations of the cost optimization model, and that the shortening of an activity increases the risk of not finishing the activity within the shortened duration. Models are not going to be adequate, in spite of all the beautiful developments, if researchers do not make an effort to investigate the connections among all the possible trade-offs.
References

Babu, A. J. G. & Suresh, N. (1996). Project management with time, cost, and quality considerations. *European Journal Operations Research*, 88(2), pp. 320-327.

Clark, C. E. (1961). The greatest of a finite set of random variables. *Operations Research*, 9(2), pp. 145-162.

Clark, C. E. (1962). The PERT model for the distribution of an activity time. *Operations Research*, 10(3), pp. 405-406.

Dodin, B. M. (1985). Bounding the project completion time distribution in PERT networks. *Operations Research*, 33(4), pp. 862-881.

Fondahl, J. W. (1961). A Non-Computer Approach to the Critical Path Method for the Construction Industry. (1st ed.). Department of Civil Engineering, Stanford University, USA

Fulkerson, R. D. (1961). A network flow computation for project cost curves. *Management Science*, 7(2), pp. 167-168.

Hajdu., M. (1993). An algorithm for Solving the PDM Time -Cost Trade Off Problem. *Periodica Politechnica* 37/3, pp. 231-247.

Hajdu, M. (1996). PDM Time Cost Trade Off: Activities Are Splittable or Non-Spittable. *Optimization*, 38, pp. 155-171.

Hajdu, M. (1997). *Network Scheduling Techniques for Construction Project Management*. Kluwer Academic Publisher, ISBN 0-7923-4309-3.

Hajdu, M. (2012). Impacts of risks on project scheduling: Can optimistic project duration be greater than the pessimistic one? *AUTCON special issue on CC2012* (accepted for publication).

Hindelang, T. J., & Muth, J. F. (1979). A Dynamic Programming Algorithm for Decision CPM Networks. *Operations Research*, 27(2), pp. 225-241.

Kelley, J. E. Jr, & Walker, M. R. (1959). Critical Path Planning and Scheduling. *Proceedings of the Eastern Joint Computer Conference*, pp. 160-173.

Kelley, J. E. Jr, & Walker, M. R. (1961). Critical Path Planning and Scheduling: Mathematical Basis. *Operations Research*, 9(3), pp. 296-320.

Kelley, J. E. Jr, & Walker, M. R. (1989). The Origins of CPM, a Personal History. *pmNetwork* 3/2

Klafszky, E. (1969). *Hálózati folyamok*, János Bólyai Mathematical Society, Budapest

Malcolm, D. G., Roseboom, J. H., Clark, C. E., & Fazar, W. (1959). Applications of a Technique for research and development Program Evaulation. *Operations research*, 7(5), pp. 646-669.
Mályusz, L. (2004). A költségtervezési „time-cost trade-off” feladat általánosítása és megoldása, *Alkalmazott Matematikai Lapok*, 21, pp. 365-377.

Roy, B. (1959). Théorie des graphes: Contribution de la théorie des graphes à l’étude de certains problèmes linearise *Comptes rendus des Séances de l’Académie des Sciences*, pp. 2437-2449.

Trietsch, D., & Baker, K. R. (2012). PERT 21: Fitting PERT/CPM for use in the 21st century. *International Journal of Project Management*, 30(4), pp. 490-502.

Weist, J. D. (1981). Precedence Diagramming Method: Some Unusual Characteristics and Their Implications for Project Management. *Journal of Operations Research*, 1(3), pp. 121-136.

Yao, M., & Chu, W. (2007). A new approximation algorithm for obtaining the probability distribution function for project completion time. *Computers and Mathematics with Applications*, 54(2), pp. 282-295.