Theoretical studies of possible toroidal high-spin isomers in the light-mass region

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Abstract

We review our theoretical knowledge of possible toroidal high-spin isomers in the light mass region in $28 \leq A \leq 52$ obtained previously in cranked Skyrme-Hartree-Fock calculations. We report additional toroidal high-spin isomers in $^{56}$Ni with $I=114\hbar$ and $140\hbar$, which follow the same (multi-particle)-(multi-hole) systematics as other toroidal high-spin isomers. We examine the production of these exotic nuclei by fusion of various projectiles on $^{20}$Ne or $^{28}$Si as an active target in time-projection-chamber (TCP) experiments.

1 Introduction

Wheeler suggested that under appropriate conditions the nuclear fluid may assume a toroidal shape \cite{1}. If toroidal nuclei could be made, there would be a new family tree for the investigation of the nuclear species.

The rotating liquid-drop model is useful as a qualitative guide to point out essential energy balances leading to possible toroidal figures of equilibrium \cite{2}. A quantitative assessment relies on microscopic descriptions that include both the bulk properties of the nucleus and the single-particle shell effects in self-consistent mean-field theories, such as the Skyrme-Hartree-Fock (SHF) approach \cite{3}. The non-collective rotation with an angular momentum about the symmetry axis is permissible quantum mechanically for an axially symmetric toroid by making particle-hole excitations and aligning the angular momenta of the constituents along the symmetry axis \cite{4}. As a consequence, only a discrete, quantized set of total angular momentum $I=I_z$ states is allowed.
In our recent works [5], we showed by using a cranked SHF approach that toroidal high-spin isomeric states have general occurrences in $28 \leq A \leq 52$ for even-even $N=Z$ and $N \neq Z$ nuclei. Toroidal high-spin isomers have also been found theoretically in similar HF calculations in this mass region [6].

We would like to review the systematics of these nuclei and suggest ways how these nuclei may be produced.

2 Light-mass toroidal high-spin isomers

We have located the toroidal high-spin isomers at their energy minima using the cranked SHF approach [5]. We find that the ratio of the torus major radius $R$ to the torus minor radius $d$, $R/d$, increases with angular momentum and approximately linearly with the mass number while the minor radius $d$ remains essentially unchanged (see Ref. [5]). It is useful to classify these nuclei according to the $n$-particle $n$-hole nature of the isomer, relative to the toroidal nucleus at $I = 0$. One finds that all $np$-$nh$ families follow a regular well-behaved pattern as shown in Fig. 1 where we plot the total energy $E^{\text{tot}}(I)$ of the isomer $^A Z^I(I)$ as a function of $R/d$ for different toroidal isomers with various aligned angular momenta $I$.

![Figure 1](Color online.) The total energies of the isomeric toroidal states of $^{28}\text{Si}$, $^{32}\text{S}$, $^{36}\text{Ar}$, $^{40}\text{Ca}$, $^{44}\text{Ti}$, $^{48}\text{Cr}$, $^{52}\text{Fe}$, and $^{56}\text{Ni}$ and their associated angular momenta $I=I_z$ values along the symmetry axis, as a function of $R/d$. The $np$-$nh$ configurations relative to the $I=0$ states are also indicated.

We collect the properties of all known 21 toroidal high-spin isomers up to $^{52}\text{Fe}$ obtained previously in [5] in Fig. 1. These systematics predict the
The possible presence of np-nh toroidal high-spin isomers for $^{56}$Ni at $R/d \sim 6.0$. Indeed, we found energy minima for $^{56}$Ni with $I=114\hbar$ and $140\hbar$ at $R/d=6.15$ and 6.27, respectively, in subsequent cranked SHF calculations.

![Graph of excitation energies](image)

**Figure 2**: (Color online.) The excitation energies $E^* (I)$ of high-spin toroidal states of $^{28}$Si, $^{32}$S, $^{36}$Ar, $^{40}$Ca, $^{44}$Ti, $^{48}$Cr, $^{52}$Fe, and $^{56}$Ni as a function of the quadrupole moment $Q_{20}$ for different angular momenta along the symmetry axis, $I=I_z$.

In Fig. 2 the excitation energies $E^* (I)$ of even-even $N=Z$ toroidal states relative to the energy of the ground states are presented as a function of the quadrupole moment $Q_{20}$ for different $I$ values along the symmetry axis. They include those from our earlier work in Fig and the newly found $^{56}$Ni toroidal isomers. The locations of energy minima $E^*(A Z^I(I))$ representing toroidal high-spin isomeric states are indicated by star symbols.
3 Production of light-mass toroidal isomers

As the question of the mean half-life of these isomers remains unresolved, one can design experiments that could detect the existence of the exotic toroidal high-spin nuclei of Figs. 1 and 2 at appropriate energies, if the mean half-lives are longer than $\hbar/\text{MeV}$ or 200 fm/c. One way is to search for these isomers as resonances or metastable nuclei by bombarding projectile nucleus $A_p Z_p$ on an active-target nucleus $A_T Z_T$ for the production of the toroidal high-spin isomer $A Z^t(I)$ with angular momentum $I=I_z$,

$$A_p Z_p + A_T Z_T \rightarrow A Z^t(I). \quad (1)$$

The active target can be, for example, $^{20}\text{Ne}$, $^{36,38,40}\text{Ar}$, or $^{28}\text{Si}$. We shall consider the cases of $^{20}\text{Ne}$ and $^{28}\text{Si}$ as active-targets.

In recent years, TPC chambers have been used to study the nuclear spectroscopy of metastable nuclei [7, 8]. The idea is to use a chamber of noble gas under a high voltage so that the gas itself or an embedded solid layer serves as the target, and the nuclear trajectories show up as tracks. The production of a composite nucleus with a long half-life would show up as a single track with the mass and charge arising from the fusion of the projectile and target nuclei. The production of binary products indicates a two→two reaction from which one can examine the elastic and inelastic channels and study the excitation function and angular distribution to search for various meta-stable states. Previously, many metastable states formed by colliding various projectile nuclei with an active He target have been found by such a technique [7].

The cross section for producing a toroidal isomer at the correct energy and angular momentum is [9] (p. 517)

$$\sigma_{\text{res}}(E, A Z^t(I)) = \frac{4\pi}{k^2} (2I + 1) \frac{\Gamma^2/4}{[E - E_{\text{res}}(A Z^t(I))]^2 + \Gamma^2/4} B_{\text{in}} B_{\text{out}}, \quad (2)$$

where $E$ is the c.m. energy, $E_{\text{res}}(A Z^t(I))$ is the c.m. resonance energy for the toroidal high-spin isomer with spin $I$, $k$ is the c.m. momentum in the initial state, and $\Gamma$ is the full width at half maximum height of the resonance. The quantities $B_{\text{in}}$ and $B_{\text{out}}$ are the branching fractions for the resonance into the initial-state and final-state channel, respectively. Here, the width $\Gamma$ and the branching fractions $B_{\text{in}}$ and $B_{\text{out}}$ may need to be determined experimentally. The resonance energy $E_{\text{res}}$ (in the c.m. system) is given by

$$E_{\text{res}}(A Z^t(I)) = M(A Z^t(I)) - M(A_T Z_T) - M(A_p Z_p), \quad (3)$$
where \( M(A^Zt(I), M(A^TZ_T), \) and \( M(A^pZ_p) \) are nuclear masses of \( A^Zt(I), \)
\( A^TZ_T, \) and \( A^pZ_p, \) respectively. In terms of the binding energies \( B(A^Z), \)
\( B(A^TZ_T), \) and \( B(A^pZ_p), \) and excitation energy \( E^*(A^Zt(I)), \) we have

\[
E_{res}(A^Zt(I)) = E^*(A^Zt(I)) - B(A^Z) + B(A^TZ_T) + B(A^pZ_p).
\]

The resonance energy \( E_{res}(A^Zt(I))(lab) \) in the laboratory system is given by
\( E_{res}(A^Zt(I)) \times (A_p + A_T)/A_T. \) For the production of toroidal high-spin
isomers by collision on \(^{20}\text{Ne}\) or \(^{28}\text{Si}\) as an active target, the resonance
energies are shown in Fig. 3. The knowledge of the predicted values of
\( E_{res}(A^Zt(I))(lab) \) will facilitate the search of toroidal high-spin isomers.

4 Conclusions and discussion

Under (multi-particle)–(multi-hole) excitation involving large orbital angular
momentum orbitals, non-collective rotations of many light nuclei lead
to equilibrium configurations whose densities may assume the shape of a
torus. The \( np-nh \) systematics of toroidal high-spin isomers fit a regular
pattern which can be used to predict possible presence of toroidal high-spin
isomer in \(^{56}\text{Ni}\). We found additional equilibrium energy minima for \(^{56}\text{Ni}\)
with \( I=114\hbar \) and \( 140\hbar \) in subsequent cranked SHF calculations.
We examine the production of light-mass toroidal high-spin isomers by fusion of various projectile nuclei with an active target, in which the trajectories of the reaction products can be examined as a function of the collision energies. Resonance energies for the production of toroidal high-spin isomers have been calculated for $^{20}$Ne or $^{28}$Si as an active target, based on the Skyrme-Hartree-Fock energies obtained for the isomers.

The technology of building a TCP using 90% of $^{20}$Ne as its dominant ingredient has been developed by the ALICE collaboration \[10\]. The utilization of a similar TPC detector with $^{20}$Ne or $^{28}$Si as an active target for nuclear spectroscopy may prove useful for the search of toroidal isomers.

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