Generalized parton distributions in the valence region from deeply virtual compton scattering

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Abstract
This work reviews the recent developments in the field of generalized parton distributions (GPDs) and deeply virtual Compton scattering in the valence region, which aim at extracting the quark structure of the nucleon. We discuss the constraints which the present generation of measurements provide on GPDs, and examine several state-of-the-art parametrizations of GPDs. Future directions in this active field are discussed.

(Some figures may appear in colour only in the online journal)

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scattering $eN \to e'X$, also called ‘deep inelastic scattering’ (DIS), and elastic scattering $eN \to e'N'$, where $e$ ($e'$) stand for an electron, or more generally a lepton, $N$ ($N'$) for a nucleon and $X$ for an undefined final state.

We recall that Hofstadter was awarded the Nobel Prize in 1961 ‘for his pioneering studies of electron scattering in atomic nuclei’ which revealed that the proton explicitly appeared as an extended object and not as a point-like particle. These measurements have shown that as the momentum transfer increases in the elastic scattering, the cross section sharply decreases compared with the electron scattering on a point-like charge. In contrast, for the inelastic scattering process it was found firstly at SLAC that the cross section at large momentum transfers does not show the sharp fall-off as elastic scattering but shows a scaling behavior. This so-called Bjorken scaling put in evidence the presence of point-like charged constituent within the nucleons, for which the 1990 Nobel prize was awarded to Friedman, Kendall and Taylor. Through Feynman’s parton model, these constituents were then identified with the ‘quarks’ introduced earlier by Gell-Mann (who was awarded the Nobel Prize in 1969 for his ‘Eightfold way’), based on theoretical symmetry considerations.

In a first approximation, electron scattering proceeds through a one-photon exchange (we will keep this approximation in the whole of this work) and is characterized by $Q^2 = -(p_e - p'_e)^2 > 0$, the squared four-momentum transferred to the nucleus by the electron. The virtuality $Q^2$ of the space-like virtual photon can be thought of as the resolution or the scale with which one probes the inner structure of the nucleon.

At sufficiently high $Q^2$, the quark structure of the nucleon can be ‘seen’ and the DIS process can be depicted by figure 1 (left panel), where the incoming lepton interacts with a single quark of the nucleon via the exchange of a virtual photon. The signature of such a point-like and elementary photon–quark process is the $Q^2$-independence of the amplitude of the process, i.e. the absence of a scale in the process. DIS results accumulated for more than 40 years show that this picture, so-called ‘scaling’, starts to be valid already at $Q^2 \approx 1$ GeV$^2$.

The complex quark and gluon structure of the nucleon, governed by the theory of strong interactions, quantum chromo-dynamics (QCD), in its non-perturbative regime is then absorbed in structure functions. This is the concept of QCD factorization where one separates a point-like, short-distance, ‘hard’ subprocess, from the complex, long-distance, ‘soft’ structure of the nucleon. The calculation of such soft matrix elements directly from the underlying theory amounts to solve QCD in its non-perturbative regime, which is still a daunting task. Ab initio calculations, by evaluating QCD numerically on a discretized space–time Euclidean lattice, are present the most promising avenue to provide predictions for some category of such non-perturbative objects. In the DIS process, the soft structure functions are the well-known unpolarized and polarized parton distribution functions (PDFs) $q(x)$ and $\Delta q(x)$, respectively. In a frame where the nucleon approaches the speed of light in a certain direction, $x$ is the longitudinal-momentum fraction carried by the quark which is struck by the virtual photon. The PDFs represent therefore the (longitudinal) momentum distribution of quarks in the nucleon.

The PDF structure functions correspond to QCD operators depending on space–time coordinates. Precisely, the PDFs are obtained as one-dimensional Fourier transforms in the light-like coordinate $y^−$ (at zero values of the other coordinates) as

$$q(x) = \frac{p^+}{4\pi} \int dy^- e^{ixyp^-} \langle \bar{q}(0)γ^+ q(y)|p⟩ |_{y^−=y^+} = 0,$$

$$\Delta q(x) = \frac{p^+}{4\pi} \int dy^- e^{ixyp^-}$$

$$\times \langle \bar{q}(0)γ^−γ^5 q(y)|pS⟩ |_{y^−=y^+} = 0, (1)$$

where $q$ is the quark field of flavor $q$, $p$ represents the initial (and final, since it is the same for DIS by virtue of the optical theorem) nucleon momentum, $x$ is the momentum fraction of the struck quark and $S$ is the longitudinal nucleon spin projection.

One uses here the light-front frame where the initial and final nucleons are collinear along the $z$-axis and the light-cone components are defined by $a^± = (a^0 ± a^3)/\sqrt{2}$. Since the space–time coordinates of the initial and final quarks are different, the operator in equation (1) is non-local, and since the momenta of the initial and final nucleons are identical, it is diagonal. This operator is illustrated in figure 1 (right panel).

The elastic $eN \to eN$ process is illustrated in figure 2 (left panel). For this process, the long-distance ‘soft’ physics
is factorized in the form factors (FFs) $F_1^q(t)$, $F_2^q(t)$, $G_\rho^q(t)$ and $G_\omega^q(t)$, where $t = (p_N - p'_N)^2 = -Q^2$. In the light-front frame, the squared momentum transfer $t$ is the conjugate variable of the impact parameter. In such a frame, the FFs reflect, via a Fourier transform, the spatial distributions of quarks in the plane transverse to the nucleon direction [1–4], reflecting, via a Fourier transform, the spatial distributions of quarks in the plane transverse to the nucleon direction.

PDFs and FFs have been measured for the last 40 years. They are therefore a richer source of nucleon structure information, which we will detail in the following subsection.

Deep exclusive scattering (DES), i.e. the exclusive electroproduction of a photon or meson on the nucleon at large $Q^2$, is illustrated on the left panel of figure 2 for the case of Deeply Virtual Compton Scattering (DVCS). The theoretical formalism and the factorization theorems associated with these processes have been laid out in \[8,9,11–14\]. The corresponding factorizing structure functions are the so-called GPDs $H^q(x, \xi, t)$, $E^q(x, \xi, t)$, $\tilde{H}^q(x, \xi, t)$ and $\tilde{E}^q(x, \xi, t)$. They correspond to the Fourier transform of the QCD non-local and non-diagonal operators which are illustrated on the right panel of figure 3:

$$\frac{P^+}{2\pi} \int dy^− e^{ix\cdot P^−} \langle \gamma^+ \gamma^5 \bar{q}_0^+(y^+, y^5) | p \rangle \bigg|_{y^+=y^5=0}$$

$$= H^q(x, \xi, t) \bar{N}(p) \gamma^+ N(p) + E^q(x, \xi, t) \bar{N}(p) \gamma^+ \gamma^5 N(p),$$

$$\frac{P^+}{2\pi} \int dy^− e^{ix\cdot P^−} \langle \gamma^+ \gamma^5 \bar{q}_0^+(y^+, y^5) | p \rangle \bigg|_{y^+=y^5=0}$$

$$= \tilde{H}^q(x, \xi, t) \bar{N}(p) \gamma^+ N(p) + \tilde{E}^q(x, \xi, t) \bar{N}(p) \gamma^+ \gamma^5 N(p),$$

where $P$ is the average nucleon four-momentum: $P = (p + p′)/2$ and $\Delta = p′ − p$, the four-momentum transfer between the final and initial nucleons. The combination of variables $x + \xi$ is the light-cone $+^-$-momentum fraction (of $P$) carried by the initial quark and the combination $x − \xi$ is the $−^-$-momentum fraction carried by the final quark going back in the nucleon. The variable $t$, the squared four-momentum transfer between the final nucleon and the initial one, is defined as $\Delta^2$. GPDs depend on additional variables compared with PDFs and FFs. They are therefore a richer source of nucleon structure information, which we will detail in the following subsection.

The QCD operators of equation (3) are ‘non-local’ since the initial and final quarks are created (or annihilated) at different space–time points and ‘non-diagonal’ since the momenta of the initial and final nucleons are different. These operators are illustrated on the right panel of figure 3.

The leading DVCS amplitude in the hard scale $Q$, the so-called twist-two amplitude, corresponds to the transition between transverse photons. The $\gamma^1_L \rightarrow \gamma_T$ transition is of order $1/Q$ and involves higher-twist (twist-3) quantities. Such quantities will not be discussed in this review which focuses on a leading-twist description of DVCS. Most studies indeed rely...
on the twist-two assumption, which allows a first interpretation of existing DVCS data as will be shown below. Moreover genuine twist-three structures are rather poorly known, and the restricted $Q^2$ range of present DVCS measurements does not allow a clean and simple separation of leading-twist and higher-twist contributions. Therefore, twist-three effects will not be discussed in this review although they are required to ensure QED gauge invariance [15–18]. More generally higher twist effects are not taken into account. In particular, we will not cover the recent results on target mass and finite-$t$ corrections to DVCS [19, 20]. Even if these new results suggest potentially large corrections to the leading-twist DVCS amplitude, they have not be included in any phenomenological study yet and their discussion is beyond the scope of this paper.

In this review, we also focus on the quark helicity conserving quantities, i.e. the operators between the quark spinors in equations (1) and (3) corresponding to $\gamma^+\gamma^5$ or $\gamma^+\gamma^5\gamma^5$ matrices. One generalization involves the use of the $\sigma^{+\nu}$ operator, allowing one to define ‘transversity’ PDFs and GPDs. We will also concentrate only on quark GPDs, as we are interested in the valence region in this work. One can also define gluonic GPDs corresponding to the operators:

$$\langle p^+|\gamma_\mu G^{\nu+}(0)\tilde{G}_{\nu+}(y)|p\rangle$$

and

$$\langle p^+|\gamma_\mu G^{\nu+}(0)\tilde{G}_{\nu+}(y)|p\rangle,$$

where $G^{\nu+}$ is the gluon field tensor and $\tilde{G}^{\nu+} = \frac{1}{2}\epsilon^{\nu\mu\rho\sigma} G_{\rho\sigma}$ its dual. Such operators are illustrated in figure 4. They will not be considered further in this review which is devoted to the study of DVCS in the valence region. At leading-twist gluon GPDs contribute to next-to-leading order (LO) in the strong coupling constant $\alpha_s$. It is commonly believed that they have a small impact in the valence region, hence justifying the LO approximation. However complete next-to-LO calculations of DVCS are available [21–29] and recent estimates [30] challenge the common view: gluon contributions may not be negligible even in the valence region at moderate energy. These results triggered an ongoing theoretical effort on the soft-collinear resummation in DVCS [31, 32]. New developments in this direction are expected in the near future but it is too early to detail them further here.

We summarize in table 1 the quark operators and the associated structure functions that we have just discussed. We refer the reader to [33–38] for complete reviews on the GPD formalism.

In equation (3), the GPDs $H$ and $E$ correspond to averages over the quark helicity. They are therefore called unpolarized GPDs. The GPDs $\tilde{H}$ and $\tilde{E}$ involve differences of quark helicities and are called polarized GPDs. At the nucleon level, $E$ and $\tilde{E}$ are associated with a flip of the nucleon spin while $H$ and $\tilde{H}$ leave it unchanged. The four GPDs therefore reflect the four independent helicity-spin combinations of the quark–nucleon system (conserving quark helicity). These are illustrated in figure 5.

Omitting the $Q^2$-dependence associated with QCD evolution equation, the GPDs depend on three independent variables: $x$, $\xi$ and $t$. $x$ varies between $-1$ and $1$ and $\xi$ in principle also between $-1$ and $1$ but, due to time reversal invariance, the range of $\xi$ is reduced between $0$ and $1$. If $|x| > \xi$, GPDs represent the probability amplitude of finding a quark (or an antiquark if $x < -\xi$) in the nucleon with a +-momentum fraction $x + \xi$ and of putting it back into the nucleon with a +-momentum fraction $x - \xi$ plus some transverse momentum ‘kick’, which is represented by $t$ (or $\Delta^2$).

The other region $-\xi < x < \xi$ implies that one ‘leg’ in figure 3 (right panel) has a positive momentum fraction (a quark) while the other one has a negative one (an antiquark). In this region, the GPDs behave like a meson distribution amplitude (DA) and can be interpreted as the probability amplitude of finding a quark–antiquark pair in the nucleon. This kind of information on $q\bar{q}$ configurations in the nucleon and, more generally, the correlations between quarks (or antiquarks) of different momenta are relatively unknown, and reveal the richness and novelty of the GPDs.

Each GPD is defined for a given quark flavor: $H^q, E^q, \tilde{H}^q, \tilde{E}^q$ ($q = u, d, s, \ldots$). $H$ and $\tilde{H}$ are a
quarks helicity and nucleon spin orientations.

\[ H(x, \xi, t) = \begin{cases} \frac{1}{2} & x > 0, \\ \frac{-1}{2} & x < 0. \end{cases} \]

\[ \tilde{H}(x, \xi, t) = \begin{cases} \Delta q(x), & x > 0, \\ \Delta \bar{q}(x), & x < 0. \end{cases} \]

The origin of these relations is the optical theorem and the symmetry of the forward Compton process, corresponding to zero four-momentum transfer, i.e. \( \xi = 0 \) and \( t = 0 \). Figure 6 illustrates this relation.

Figure 6. The optical theorem: the cross section of the DIS process is equal to the imaginary part of the forward amplitude of the (doubly virtual) Compton process.

One therefore sees that the PDFs and the FFs appear as simple limits or moments of the GPDs. Similarly to the FFs, the \( t \) variable in GPDs is the conjugate variable of the impact parameter (in the light-front frame) \([1, 39, 40]\). For \( \xi = 0 \) (where \( t = -\Delta^2 \)), one has therefore an impact-parameter version of GPDs through a Fourier integral in transverse momentum \( \Delta_\perp \):

\[ H^q(x, b_\perp) = \int \frac{d^2 \Delta_\perp}{(2\pi)^2} e^{-ib_\perp \cdot \Delta_\perp} H^q(x, 0, -\Delta^2) \]  

At \( \xi = 0 \), the GPD \( H^q(x, 0, t) \) can then be interpreted as the probability of finding a parton with longitudinal-momentum fraction \( x \) at a given transverse distance (relative to the transverse c.m.) in the nucleon. In this way, the information contained in a traditional parton distribution, as measured in DIS, and the information contained within a FF, as measured in elastic lepton–nucleon scattering, are combined and correlated in the GPD description.

The second moment of the GPDs is relevant to the nucleon spin structure. It was shown in \([9]\) that there exists a (color) gauge-invariant decomposition of the nucleon spin: \( \frac{1}{2} \) = \( J_q + J_g \), where \( J_q \) and \( J_g \) are, respectively, the total quark and gluon contributions to the nucleon total angular momentum. The second moment of the GPDs gives (F1’s sum rule):

\[ J_q = \frac{1}{2} \int_{-1}^{+1} dx \left[H^q(x, \xi, t = 0) + E^q(x, \xi, t = 0)\right]. \]  

The total quark spin contribution \( J_q \) decomposes (in a gauge-invariant way) as \( J_q = \frac{1}{2} \Delta \Sigma + L_q \) where \( 1/2 \Delta \Sigma \) and \( L_q \) are, respectively, the quark spin and quark orbital contributions to the nucleon spin. \( \Delta \Sigma \) can be measured through polarized DIS experiments, and its extracted value is shown in table 2. One sees from table 2 that the different determinations of \( \Delta \Sigma \) all point to a value in the range 20–30%.

On the other hand, for the gluons it is still an open question how to decompose the total angular momentum \( J_g \) into orbital angular momentum, \( L_g \), and gluon spin, \( \Delta g \), parts, in such a way that both can be related to observables. For a discussion on recent developments in this active field, see \([46]\) and references therein. At present, it is only known how to directly access the gluon spin contribution \( \Delta g \) in the experiment. It can be
accessed in several ways: inclusive $c\!-\!\bar{c}$ or high $p_T$ hadron pair production in polarized semi-inclusive DIS, semi-inclusive $\pi^0$, $\gamma$, jet, etc. production in polarized proton collisions and evolution of $g_1(x, Q^2)$ through global fits of polarized data. At present $\Delta g$ can only be extracted with a large uncertainty, as can be seen from table 2. While most determinations for $\Delta g$ indicate a very small value to fully explain the spin puzzle, it is clearly worthwhile to reduce the uncertainty in its extraction by further measurements.

The sum rule of equation (9) in terms of the GPDs provides a model-independent way of determining the quark orbital contribution to the nucleon spin and therefore completes the quark sector of the ‘spin-puzzle’.

Equation (9) is actually a particular case of a more general rule on the $x$ moments of GPDs. The so-called polynomiality condition states that the $x^n$ moment of GPDs must be a polynomial in $\xi$ of order $n$ (for $n$ even, corresponding to non-singlet GPDs) or $n + 1$ (for $n$ odd, corresponding to singlet GPDs), e.g. for the $H$ GPD:

\[
\text{if } n \text{ even: } \int_{-1}^{1} dx x^n H(x, \xi, t) = a_0 + a_2 \xi^2 + \ldots + a_n \xi^n, \\
\text{if } n \text{ odd: } \int_{-1}^{1} dx x^n H(x, \xi, t) = a_0 + a_2 \xi^2 + \ldots + a_{n+1} \xi^{n+1}.
\]

There are similar rules for the GPDs $E$, $\tilde{H}$ and $\tilde{E}$. For the GPD $E$, the $a_{n+1}$ coefficient is the same as for $H$ except that it has the opposite sign. For the GPDs $H$ and $E$, the maximum $\xi$ power in equation (10) for singlet GPDs is $n - 1$ (instead of $n + 1$).

We note that in equation (10) only even powers of $\xi$ appear which is a consequence of the time reversal invariance which states that $H(x, -\xi, t) = H(x, \xi, t)$.

In addition to the polynomiality constraints on GPDs, the GPDs are also constrained by positivity conditions which should be taken into account both for non-zero and zero-skewness parameter. The simplest of these conditions arises from requiring the positivity of the quark distribution in a transversely polarized nucleon. This imposes a relation between the $E$-type and $H$-type GPDs, for more details see [47].

As mentioned above, an ab initio calculation of soft matrix elements in general seems at present only practical within lattice QCD. By its nature of discretizing the theory on an Euclidean space–time lattice, lattice QCD can robustly calculate a few lowest moments of GPDs, which correspond to matrix elements of local operators. Although at present the calculations are being performed for unphysical pion masses, it is foreseeable that in the next few years calculations for such quantities at the physical point with controlled systematic uncertainties will become available. They can be used in the near future as additional constraints when confronting GPD parametrizations with experiment. We refer to the review paper of [48] for a recent review of lattice efforts in this field.

### 1.3. Generalized transverse-momentum-dependent parton distributions

The GPDs can be considered as a particular limit of generalized parton correlation functions (GPCFs). The GPCFs provide a unified framework to describe the partonic information contained in a hadron. The GPCFs parametrize the fully unintegrated off-diagonal quark–quark correlator, depending on the full four-momentum $k$ of the quark and on the four-momentum $\Delta$ which is transferred by the probe to the hadron; for a classification see [49, 50]. They have a direct connection with the Wigner distributions of the parton–hadron system [37, 51, 52], which represent the quantum mechanical analogues of the classical phase-space distributions.

When integrating the GPCFs over the light-cone energy component of the quark momentum one arrives at generalized transverse-momentum-dependent parton distributions (GTMDs) which contain the most general one-body information of partons, corresponding to the full one-quark density matrix in momentum space. These GTMDs parametrize the following general unintegrated, off-diagonal quark–quark correlator for a hadron [50]:

\[
W_{AL}^{[F]}(\Delta, \vec{k}_\perp, x; \eta) = \frac{1}{2} \int \frac{dy^- d^2 y_\perp}{(2\pi)^3} e^{i(xp^- - \vec{k}_\perp \cdot \vec{y}_\perp)} \langle p', \Lambda | \bar{\psi}(\frac{y}{2}) \gamma_V \psi(\frac{y}{2}) | p, \Lambda \rangle |_{y^- = 0},
\]

\[
\frac{1}{2} \int \frac{dy^- d^2 y_\perp}{(2\pi)^3} e^{i(xp^- - \vec{k}_\perp \cdot \vec{y}_\perp)} \langle p', \Lambda | \bar{\psi}(\frac{y}{2}) \gamma_V \psi(\frac{y}{2}) | p, \Lambda \rangle |_{y^- = 0},
\]

\[
\frac{1}{2} \int \frac{dy^- d^2 y_\perp}{(2\pi)^3} e^{i(xp^- - \vec{k}_\perp \cdot \vec{y}_\perp)} \langle p', \Lambda | \bar{\psi}(\frac{y}{2}) \gamma_V \psi(\frac{y}{2}) | p, \Lambda \rangle |_{y^- = 0},
\]

\[
\frac{1}{2} \int \frac{dy^- d^2 y_\perp}{(2\pi)^3} e^{i(xp^- - \vec{k}_\perp \cdot \vec{y}_\perp)} \langle p', \Lambda | \bar{\psi}(\frac{y}{2}) \gamma_V \psi(\frac{y}{2}) | p, \Lambda \rangle |_{y^- = 0},
\]
where the superscript $\Gamma$ stands for any element of the basis \{1, $\gamma_5$, $\gamma^\mu$, $\gamma^\mu\gamma_5$, $i\sigma^\mu\nu\gamma_5$\} in Dirac space, and $\Lambda$ ($\Lambda'$) denote the helicities of initial (final) hadron, respectively. A Wilson line $W \equiv W(-\frac{t}{2}, \frac{1}{2}|n)$ ensures the color gauge invariance of the correlator, connecting the points $-\frac{t}{2}$ and $\frac{t}{2}$ via the intermediary points $-\frac{t}{2} + \infty \cdot n$ and $\frac{t}{2} + \infty \cdot n$ by straight lines. This induces a dependence of the Wilson line on the light-cone direction $n$. Furthermore, the parameter $\eta = \text{sign}(n^0)$ gives the sign of the zeroth component of $n$, i.e. indicates whether the Wilson line is future-pointing ($\eta = +1$) or past-pointing ($\eta = -1$). Clearly, such correlators generalize the GPD correlators introduced in equation (3) by allowing the quark operator to be also non-local in the transverse direction, i.e. in addition to the GPD arguments $x$, $\xi$ and $t$, the GTMDs also depend on the quark transverse momentum $k_\perp$.

At leading twist, there are 16 complex GTMDs, which are defined in terms of the independent polarization states of quarks and hadron. In the forward limit $\Delta = 0$, they reduce to eight transverse-momentum-dependent parton distributions (TMDs) which depend on the longitudinal-momentum fraction $x$ and transverse momentum $k_\perp$ of quarks, and therefore give access to the three-dimensional picture of the hadrons in momentum space. On the other hand, the integration over $k_\perp$ of the GTMDs leads to eight GPDs which are probability amplitudes related to the off-diagonal matrix elements of the parton density matrix in the longitudinal-momentum space. After a Fourier transform of $\Lambda_\perp$ to the impact-parameter space, they provide a three-dimensional picture of the hadron in a mixed momentum-coordinate space. The common limit of TMDs and GPDs is given by the standard PDFs, related to the diagonal matrix elements of the longitudinal-momentum density matrix for different polarization states of quarks and hadron. Figure 8 illustrates how the GTMDs reduce to different parton distributions and FFs.

Although it has not been shown to date that the GTMDs can be accessed in a model-independent way in experiment, they may, however, provide useful quantities to gain insight through model calculations of hadron structure, see, e.g., [53].

2. From theory to data

Among the hard exclusive lepton production processes, the DVCS channel bears the best promises to extract the GPDs from experimental data using the leading-twist handbag diagram amplitude. In fact, in DVCS, the hard perturbative part of the handbag involves only electromagnetic vertices (figure 3) while in deeply virtual meson electroproduction (DVMP), there are strong vertices involving a gluon exchange, see figure 9. In DVMP there is another soft non-perturbative quantity in addition to the GPDs that enter the calculation: the DA of the meson which is produced. As the gluon virtuality needs to be hard to ensure the leading-twist amplitude, the endpoint behavior of the DA can potentially lead to strong power corrections to the leading-twist amplitude. In this review, we focus on the DVCS process on the proton.

Figure 8. The GTMDs reduce to different parton distributions and FFs. By integrating over the quark transverse momentum $k_\perp$, the GTMDs reduce to the GPDs, whereas the forward limit $\xi = 0, t = 0$ (i.e. $\Delta = 0$) can in turn be parametrized in terms of TMDs, which are the quantities which enter semi-inclusive DIS processes. Integrating the TMDs over $k_\perp$ gives rise to the forward parton distributions. The forward parton distributions are also obtained by taking the forward limit $\Delta = 0$ in the GPDs. Integrating the GPDs over $x$ yields the FFs. In the middle row right column figure, ‘ff’ stands for ‘fragmentation functions’ which we do not discuss here.

2.1. Compton FFs

Four independent variables are needed to describe the three-body final state reaction $e(p_e)p(p_R) \to e'(p'_e)p'(p'_R)\gamma(p_\gamma)$ at a fixed beam energy $E_e$. They are usually chosen as $Q^2$, $x_B$, $t$ and $\phi$ where $Q^2 = -(p_e - p'_e)^2$, $x_B = \frac{Q^2}{2p_\gamma q}$ (with $q$ the virtual photon four-momentum), $t = (p_R - p'_R)^2$ and $\phi$ is the azimuthal angle between the electron scattering plane and the hadronic production plane (see [54] for its explicit definition within the Trento convention). See figure 10 for an illustration of these kinematical quantities.

At leading twist, the GPDs depend on the three variables: $x$, $\xi$ and $t$ where the variable $\xi$ is related to $x_B$ by $\xi = x_B/(2 - x_B)$. In principle, GPDs depend on $Q^2$ as well. However, this dependence can be predicted and calculated through the evolution equations and does not reflect the non-perturbative structure of the nucleon. For simplicity, we will not write the dependence on $Q^2$ explicitly in the following. The variables $\xi$ and $t$ can be accessed by measuring the kinematics of the scattered electron and of the final state photon and/or proton. However, the variable $x$ is not experimentally accessible. In the calculation of the DVCS amplitude of the handbag diagram of figure 3, the variable $x$ is integrated over.
The DVCS amplitude is written as
\[
M_{\text{DVCS}} = \epsilon_\mu(q)\epsilon_\nu^*(p_\gamma)H_{\text{LO,DVCS}}^{\mu\nu}, \tag{12}
\]
where \(\epsilon_\mu(q)\) and \(\epsilon_\nu^*(p_\gamma)\) are, respectively, the polarization four-vectors of the (virtual) initial and final photons and \(H_{\text{LO,DVCS}}^{\mu\nu}\) is the DVCS amplitude.

One readily sees from equation (13) that the variable \(x\) which is a 'mute' variable is integrated over. It is also weighted by the \(\frac{1}{2m}\) factors, which originate from the propagator of the quark in the handbag diagram of figure 3 (left panel).

The DVCS amplitude contains convolution integrals of the form
\[
\int_{-1}^{+1} dx \frac{H(x, \xi, t)}{x - \xi + i\epsilon} = \mathcal{P} \int_{-1}^{+1} dx \frac{H(x, \xi, t)}{x - \xi} - i\pi H(\xi, \xi, t), \tag{14}
\]
and analogously for the GPDs \(E, \tilde{H}\) or \(\tilde{E}\). In equation (14), we have decomposed the expression into a real and an imaginary part where \(\mathcal{P}\) denotes the principal value integral. This means that the maximum information that can be extracted from the experimental data in the DVCS process at a given \((\xi, t)\) point is \(G P D(\pm\xi, \xi, t)\) or any \(\int_{-1}^{+1} dx \frac{H(x, \xi, t)}{x - \xi}\). The former is accessed when an observable sensitive to the imaginary part of the DVCS amplitude is measured, such as single beam- or target-spin observables, while the latter is accessed when an observable sensitive to the real part of the DVCS amplitude is measured, such as double beam- or target-spin observables or beam-charge sensitive observables. The unpolarized cross section is sensitive to both the real and imaginary parts of the DVCS amplitude.

There are therefore in principle eight GPD-related quantities that can be extracted from the DVCS process:
\[
H_{\text{Re}}(\xi, t) \equiv \mathcal{P} \int_{0}^{1} dx [H(x, \xi, t) - H(-x, \xi, t)] C^+(x, \xi), \tag{15}
\]
\[
E_{\text{Re}}(\xi, t) \equiv \mathcal{P} \int_{0}^{1} dx [E(x, \xi, t) - E(-x, \xi, t)] C^+(x, \xi). \tag{16}
\]
\[
\tilde{H}_{\text{Re}}(\xi, t) \equiv \mathcal{P} \int_{0}^{1} dx \left[\tilde{H}(x, \xi, t) + \tilde{H}(-x, \xi, t)\right] C^-(x, \xi), \tag{17}
\]
\[
\tilde{E}_{\text{Re}}(\xi, t) \equiv \mathcal{P} \int_{0}^{1} dx \left[\tilde{E}(x, \xi, t) + \tilde{E}(-x, \xi, t)\right] C^-(x, \xi). \tag{18}
\]
\[
H_{\text{Im}}(\xi, t) \equiv H(\xi, \xi, t) - H(-\xi, \xi, t), \tag{19}
\]
\[
E_{\text{Im}}(\xi, t) \equiv E(\xi, \xi, t) - E(-\xi, \xi, t), \tag{20}
\]
\[
\tilde{H}_{\text{Im}}(\xi, t) \equiv \tilde{H}(\xi, \xi, t) + \tilde{H}(-\xi, \xi, t), \tag{21}
\]
\[
\tilde{E}_{\text{Im}}(\xi, t) \equiv \tilde{E}(\xi, \xi, t) + \tilde{E}(-\xi, \xi, t). \tag{22}
\]
with the coefficient functions \( C^\pm \) defined as

\[
C^\pm(x, \xi) = \frac{1}{x - \xi} \pm \frac{1}{x + \xi} \tag{23}
\]

and where one has reduced the x-range of integration from \([-1, 1]\) to \([0, 1]\) in the convolutions.

The functions \( H_{Re}, \, H_{Im} \), etc. on the lhs of equations (15)–(22), which depend on the two kinematical variables \( \xi \) and \( t \), accessible in experiment, are called the Compton form factors (CFFs).\(^4\)

For further use, we also introduce the complex functions as

\[
\gamma(t) = H_{Re}(\xi, t) - i\pi H_{Im}(\xi, t), \tag{24}
\]

and analogously for the other GPDs.

In the following, we will also use the notation:

\[
\begin{align*}
H_{+}(x, \xi, t) &\equiv H(x, \xi, t) \quad - H(-x, \xi, t), \\
E_{+}(x, \xi, t) &\equiv E(x, \xi, t) \quad - E(-x, \xi, t), \\
H_{-}(x, \xi, t) &\equiv H(x, \xi, t) \quad + H(-x, \xi, t), \\
E_{-}(x, \xi, t) &\equiv E(x, \xi, t) \quad + E(-x, \xi, t),
\end{align*} \tag{25}
\]

since these are the so-called singlet (\( C = +1 \)) GPD combinations, which enter in the CFFs, and consequently in the DVCS observables.

2.2. The Bethe–Heitler process

The DVCS process is not the only amplitude contributing to the \( ep \rightarrow epy \) reaction. There is also the Bethe–Heitler (BH) process, in which the final state photon is radiated by the incoming or scattered electron and not by the nucleon itself. This is illustrated in figure 12. The BH process leads to the same final state as the DVCS process and interferes with it. Since the nucleon FFs \( F_1 \) and \( F_2 \) can be considered as well known at small \( t \), the BH process is precisely calculable theoretically. The BH cross section has the very distinct feature to sharply rise around \( \phi = 0^\circ \) and \( 180^\circ \). These are the regions where the radiated photon is emitted in the direction of the incoming electron or the scattered one. The strong enhancements when the outgoing photon is emitted in the electron–proton plane is due to singularities (for massless electrons) in the electron propagators.

In those regions, a small variation in the kinematical variables \( Q^2, x_B, t \) or \( \phi \) produces strong variations in the cross section. For instance, figure 11 shows that a variation of \( x_B \) by 1% around the particular kinematical setting \( x_B = 0.300, \, Q^2 = 2.500 \text{GeV}^2, \, t = -0.200 \text{GeV}^2, \, E_\gamma = 5.750 \text{GeV} \) induces a variation of almost 10% at very low and large \( \phi \). It should be noted that, even at \( \phi = 180^\circ \), this relative difference remains more than 5%. Experiments should therefore as much as possible quote and control the values of the kinematical variables at least at the percent level to achieve a few percent accuracy on the BH-DVCS cross sections.

\[^4\] Be aware of slightly different notations in the literature, e.g. the authors of [55] include \(-\pi\) factors in the definition of the ‘Im’ CFFs or include a minus sign in the definition of the ‘Re’ CFFs.
for unpolarized, ‘L’ for longitudinally polarized and ‘x’ or ‘y’ for a transversely polarized target. In fact, in this latter case, there are two independent polarization directions: ‘x’ is in the hadronic plane and ‘y’ is perpendicular to it (see figure 10). Furthermore, the kinematical variable $k$ is defined as $k = -t/(4m_N^2)$.

The difference of polarization sections in equations (27)–(30) is sensitive only to the BH-DVCS interference term. The CFFs arise from the DVCS process while the $F_1$ and $F_2$ FFs originate from the BH process and only products of FFs and CFFs appear, thus providing access to CFFs in a linear fashion. At LO in a $1/\rho$ expansion, only sin $\phi$ or cos $\phi$ modulations appear. One also notices in general that single spin observables are sensitive to the imaginary CFFs while double-spin observables are sensitive to the real CFFs.

In a first approximation, neglecting terms multiplied by non-trivial $\alpha$-dependence, the relations expressing the ‘p’ and ‘n’ superscripts to underline that GPDs on the proton and on the neutron are not equal. The relations expressing the ‘p’ and ‘n’ GPDs entering the DVCS amplitudes, in terms of the $u$- and $d$-quark contributions, are given for the GPD $H$ by

$$H^p(\xi, \xi, t) = \frac{4}{9} H^d(\xi, \xi, t) + \frac{1}{9} H^u(\xi, \xi, t),$$

(31)

and similarly for the GPDs $E$, $\tilde{H}$ or $\tilde{E}$.

In summary, given the number of variables on which the GPDs depend (three, omitting the $Q^2$-dependence), the convolution over $x$ in the amplitudes, the presence of the BH process with it singularities, the number of CFFs (eight at leading twist), the quark flavor decomposition, not to mention the $Q^2$ evolution and higher-twist corrections, it is clearly a non-trivial task to extract the GPDs from the experimental data and, ultimately, to map them in the three variables $x$, $\xi$, $t$. It requires a broad experimental program measuring several DVCS (or DVMP) spin observables on proton and neutron targets over large ranges in $x_B$ and $t$ (and $Q^2$).

There are several strategies to make progress in such a program. One of them is, as an intermediate step, to extract the CFFs from DVCS data for a given ($\xi$, $t$) point by fitting the $\phi$ distribution at a given beam energy. This can be carried out in an essentially model-independent way provided one has enough constraints, i.e. experimental observables, to extract the eight CFFs. As we will see in section 4, even if there are only two observables which are measured, to be fitted by eight CFFs taken as free parameters, making the problem $a$ priori largely underconstrained, some results can still be obtained due to the domaince of such few observables by one or two CFFs. However, this is only the first step of the program, since the $x$ dependence still needs to be uncovered, in principle with the help of a model with adjustable parameters. The problem can be simplified with the help of dispersion relations (DRs) which we will discuss in section 3.4. They can in principle reduce from eight to five the number of GPD quantities to be extracted. They state, in a model-independent way, that real subtraction constant (at fixed $\xi$ and $t$) which intervenes and makes the number of independent quantities to be five in total. To apply DRs, it is needed to measure data over a very wide range in $\xi$ (at fixed $t$) unless one has good reasons to truncate the integral or to extrapolate. Another strategy consists in fitting directly the experimental observables by a model which has for each GPD $H$, $E$, $\tilde{H}$ or $\tilde{E}$, a parametrization of the full $x$, $\xi$, $t$-dependence with parameters to be fitted. We will discuss these various approaches below.

Let us also mention that there is an experimental way to measure independently the $x$ and $\xi$-dependence of GPDs. The double-DVCS process consists of the DVCS process with a virtual (space-like or time-like) photon in the final state. In the case of a final time-like photon, the virtuality of this second photon can be measured and varied, thus providing an extra lever arm and allowing one to measure the GPDs for each $x$, $\xi$, $t$ values independently (though with some limitations if the final photon is time-like) [56, 57]. However, since the cross section of such a process is reduced by a factor $\alpha \approx 1/137$, and since one needs to make measurements above the vector-meson resonance region to avoid the strong vector-meson processes, the double DVCS has not revealed so far to be a practical way to access GPDs.
We now review the existing DVCS measurements, limiting ourselves to the large and intermediate $x_B$ regions.

2.4. Existing DVCS measurements

Three experiments have provided these past 10 years DVCS data which can potentially lend themselves to a GPD interpretation. These are the Hall A and CLAS experiments from JLab (with a $\approx 6$ GeV electron beam energy) and the HERMES experiment at DESY (with a $\approx 27$ GeV electron or positron beam energy).

2.4.1. JLab Hall A. The $ep \rightarrow e'p'\gamma$ reaction was measured in the JLab Hall A experiment [58] by detecting only the scattered electron in a high resolution ($\delta p \approx 10^{-4}$ for momentum) arm spectrometer and the real photon in an electromagnetic calorimeter ($\sigma_E \sqrt{E} \approx 4\%$ for energy). A cut on the missing mass of the proton which clearly stood out over a small background was used to unambiguously identify the exclusive process.

The Hall A experiment measured the four-fold beam-polarized and unpolarized differential cross sections $d\sigma/dx_BdQ^2dt\,d\phi$, i.e. without any integration over an independent variable, as a function of $\phi$, for four $-t$ values ($0.17, 0.23, 0.28$ and $0.33$) at the average kinematics: $\langle x_B \rangle = 0.36$ and $\langle Q^2 \rangle = 2.3$ GeV$^2$. The beam-polarized cross sections have also been measured at $\langle Q^2 \rangle = 1.5$ GeV$^2$ and $\langle Q^2 \rangle = 1.9$ GeV$^2$. Figure 13 shows these results. The particular shape in $\phi$ of the BH contribution in the unpolarized cross section (red curve in the upper panels of figure 13) is easily
recognizable. The difference between the red curve and the data is the contribution of the DVCS process and therefore of the GPDs.

The difference of beam-polarized cross sections, i.e. $\Delta \sigma_{LU}$, is displayed in the three lower panels of figure 13. In the LO $1/Q^2$ expansion, the amplitude of the sinusoidal is directly proportional to the combination of CFFs: $F_1H_{\text{Im}} + \xi(F_1 + F_2)\tilde{H}_{\text{Im}} - kF_2E_{\text{Im}}$ (see equation (27)). Fitting these sinusoids has thus permitted to extract the $Q^2$-dependence of this combination of CFFs at four different $t$ values. The results of these fits are presented in figure 14. At leading-twist, GPDs and CFFs are predicted to be $Q^2$ independent and the data seem to exhibit this scaling feature. Although the $Q^2$ lever arm is very limited ($\approx 1 \text{ GeV}^2$), this is a very encouraging sign that one can access the leading-twist handbag process at the JLab kinematics.

We also mention that the beam-spin asymmetry of the DVCS+BH process on the neutron $e_n \rightarrow e_n \gamma$ has been measured in an exploratory way by the JLab Hall A collaboration at one single $(x_B, Q^2, t)$ value (0.36, 1.9) as a function of $t$ [59]. Although these results are encouraging and might possibly give some first constraints on the $E_{\text{Im}}$ CFF, we decide, in this review, to focus on the proton channel. There is a variety of observables which have been measured over a wide phase space for this latter process. This should give the strongest constraints on the GPD models and fits.

2.4.2. JLab Hall B. The JLab CLAS collaboration uses a large acceptance spectrometer and has measured the DVCS process by detecting the three particles of the final state, i.e. the scattered electron, the recoil proton and the produced real photon, over a much broader phase space than in Hall A. Since CLAS has a lesser resolution ($\delta p \approx 10^{-2}$ for momentum) than the Hall A arm spectrometers, the kinematic redundancy and overconstraint due to the detection of the full final state is the best way to ensure the exclusivity of the process.

Beam-polarized and unpolarized cross section measurements are under way [60] but to this day, only beam-spin asymmetries, i.e. the ratio of $\Delta \sigma_{LU}$ to the unpolarized cross section, and longitudinally polarized target asymmetries, i.e. the ratio of $\Delta \sigma_{UL}$ to the unpolarized cross section, have been measured. These asymmetries are observables which are relatively straightforward to extract experimentally since, in a first order approximation, normalization factors such as the efficiency/acceptance of the detector and, more generally, many sources of systematic errors cancel in the ratio. Both asymmetries have a shape close to a $\sin \phi$ like equation (27) and (28) were predicting. The beam-spin asymmetry was fitted by a function of the form $a \sin \phi/(1 + c \cos \phi + d \cos 2\phi)$. Figure 15 (left panel) shows the value of this fitted asymmetry at $\phi = 90^\circ$ for the $\approx 60 (x_B, Q^2, t)$ bins covered by CLAS and which were measured with a 5.77 GeV beam.
Figure 15. DVCS-BH beam-spin asymmetry at $\phi = 90^\circ$ as a function of $t$ for different $(x_B, Q^2)$ bins, as measured by the JLab Hall B/CLAS collaboration [61] (black solid circles). The red empty triangles are the beam-spin asymmetries derived from the ratio of the beam-polarized and unpolarized cross sections of Hall A (see figure 13). The blue square point is the pioneer measurement from the CLAS collaboration [62].

The longitudinally polarized target asymmetries are displayed in figure 16. These observables show a $\sin\phi$-like shape, as predicted by theory (see equation (28)), and their $\sin\phi$ moment $A_{UL}^{\sin\phi}$ is presented in the figure. Here we extend the subscript notation of equations (27) to (30) to asymmetry moments $A$, where the superscript denotes the particular azimuthal moment considered. The use of a polarized target limited the statistics and the moments could be extracted only for three $(x_B, Q^2, t)$ bins.

2.4.3. HERMES. At higher energies, $x_B \approx 0.1$, the HERMES collaboration has carried out a measurement of all independent DVCS observables, except for cross sections: beam-spin asymmetries [64, 65], longitudinally polarized target asymmetries [66], transversally polarized target asymmetries [67, 68], beam-charge asymmetries [69–71] and all associated beam-spin/target-spin and spin/beam-charge double asymmetries. In a first stage, the HERMES experiment requested the detection of the scattered electron (or positron) and of the final real photon. Then, a cut on the missing mass of the proton was applied. The width of this missing mass peak being more than 1 GeV, a substantial background subtraction had to be performed. In a second stage, the HERMES spectrometer was completed by a recoil detector allowing the detection of the recoil proton and therefore a complete identification of the DVCS final state. The kinematics being then overconstrained, this allowed for a much cleaner selection of the exclusive reaction with a reduction of the contamination of non-DVCS events at the level of less than 1% [72]. In this ‘pure’ DVCS samples, the amplitudes of beam-spin asymmetries were actually found to be somewhat larger (by about 10% in average).

HERMES used a positron beam as well as an electron beam and the target-spin asymmetries have a different
Figure 17. Selection of ten DVCS-BH asymmetry $\phi$-moments as a function of $t$ as measured by HERMES. All moments in this figure are expected to be non-null in the leading $1/Q$ expansion but for the $A_{UL}^{\sin 2\phi}$ moment (bottom right plot).

sensitivity to the BH+DVCS amplitude according to the charge of the beam. To describe such correlated charge and beam-spin asymmetries, one therefore introduces a second index (‘I’ or ‘DVCS’) for the labeling of the asymmetries, according to (for instance for the beam-spin asymmetries):

$$A_{[LU,DVCS]} = \frac{\sigma^+ (\phi) - \sigma^- (\phi)}{\sigma^+ (\phi) + \sigma^- (\phi) + \sigma^\ast_+ (\phi) + \sigma^\ast_- (\phi)},$$

(33)

$$A_{[LU,I]} = \frac{\sigma^+ (\phi) - \sigma^- (\phi) - (\sigma^\ast_+ (\phi) - \sigma^\ast_- (\phi))}{\sigma^+ (\phi) + \sigma^- (\phi) + \sigma^\ast_+ (\phi) + \sigma^\ast_- (\phi)},$$

(34)

where the superscript represents the charge of the beam and the subscript the beam (or target) spin projection. At leading twist, only the asymmetries with an ‘I’ subscript can be sensitive to GPDs while the ones with the ‘DVCS’ subscript are null.

All DVCS azimuthal asymmetries have at leading twist a general sine, cosine or constant shape (see equations (27)–(30) for a few examples), slightly modulated by the $\phi$-dependence of the denominator. The sine, cosine and constant moments of the nine asymmetries which are expected to be non-null in the leading-twist handbag formalism are displayed in figure 17. We added as a tenth observable the $A_{UL}^{\sin 2\phi}$ moment (bottom right plot) which is expected to be power suppressed in this approximation. However the data show a two- to three-standard deviation difference from zero. If one does not consider this difference as a statistical fluctuation, it is a puzzle as it cannot be described by any leading-twist handbag calculation. In fact, HERMES extracted also the ‘DVCS-subscript’ asymmetries (see equation (34)), as well as several $\sin 2\phi$, $\cos 2\phi$ or $\cos 3\phi$ moments, which are expected to be null in the hypothesis of DVCS leading-twist dominance. We do not display here these data but they were all found to be compatible with zero within error bars. Except for this puzzling $A_{UL}^{\sin 2\phi}$ moment, this gives further support to the idea that higher-twist contributions are small at the currently finite $Q^2$ values explored, confirming the first conclusions drawn from the JLab Hall A data.

In figure 17, we display only the $t$-dependence of these moments at the average $x_B$ and $Q^2$ values of 0.09 and 2.5 GeV$^2$, respectively. The data were taken with a 27.6 GeV beam energy. HERMES also measured the $x_B$- and $Q^2$-dependences, with the other kinematic variables fixed. Also, in this figure, the data correspond to data analysis carried out without the recoil detector.

We also mention that the HERMES collaboration has measured the DVCS+BH charge, beam-spin and
longitudinally polarized target asymmetries with a deuterium target [73, 74]. Such a process is dominated by the incoherent DVCS+BH process on the proton and the results are in general shown in figure 17. We finish this section by mentioning that the unpolarized ep → epy cross section has also been measured at a much higher energy (30 < W < 120 GeV, 2 < Q^2 < GeV^2 where W is the center of mass energy of the γ^* → p system), by the H1 and ZEUS collaborations [75, 76]. At such large W (i.e. low x_b), the DVCS process is sensitive mostly to ‘gluon’ GPDs which we do not cover in this review.

3. Models of GPDs and dispersive framework for DVCS

In this section, we review a few current state-of-the-art parametrizations of GPDs. We distinguish three families of models: models based on double distributions (DDs) (VGG and GK), the dual parametrization and the Mellin–Barnes model.

3.1. DDs/Regge phenomenology: the VGG and the GK models

3.1.1. (x,ξ) dependence and DDs. DDs were originally introduced by Radyushkin [77, 78] and Muller et al [8]. They provide an elegant guideline to parametrize the (x,ξ) dependence of the GPDs which automatically satisfies the polynomiality relations (see equation (10)).

The idea of the DDs is to decorrelate the transferred longitudinal momentum (Δ) from the initial nucleon momentum P (see figure 3—right). In the light-cone frame, one introduces then the new variables α and β such that the initial quark has a longitudinal momentum βP^* + (1 + α)Δ^* (see figure 18—left), instead of (x + ξ)P^* (see figure 3—left). Since, by definition, −2ξ = (Δ/β)^2, this means that x = β + αξ. The variable α is playing the role of ξ, i.e. the fraction of longitudinal momentum of the transfer, but the difference is that α is now an absolute, i.e. it has no reference to the (average) initial nucleon momentum, unlike ξ. The link between a GPD and a DD is then only a change of variables, i.e. from (α, β) to (x, ξ), such that

\[ GPD^q(x, ξ) = \int_{-1}^{1} dβ \int_{-1+|β|}^{1-|β|} dα δ(β - ξ α) DD(α, β). \]  

(35)

One should integrate on all values/combinations of α and β which produce the (x, ξ) variables. Given that x = β + αξ, one has actually only a one-dimensional integral. The limits of the integration on the α and β variables are constrained by the fact that x has to be comprised between −1 and 1 and ξ between 0 and 1, so that one has always |x| + |α| ≤ 1. This constraint means that the integration over the variables α and β takes place over the β = x − ξα straight line ‘inside’ the rhombus defined by the equation |β| + |α| ≤ 1 (see figure 18—right).

Due to the linear relation between x and ξ imposed by the δ function, the polynomiality relation is automatically satisfied: the x^n moment of equation (35) will always produce a ξ^n power. An advantage of the DDs is that the (α, β) dependence can be more conveniently inferred than the (x, ξ)-dependence. The matrix element corresponding to figure 18—left can be written

\[ \langle p + Δ|\tilde{Ψ}_q(0)O\tilde{Ψ}_q(y)|p \rangle \]  

(36)

where we can consider two ‘extreme’ cases. When there is no longitudinal-momentum transfer brought by the photon, i.e. Δ = 0, one gets

\[ \langle p|\tilde{Ψ}_q(0)O\tilde{Ψ}_q(y)|p \rangle \bigg|_{y^*=y_+=0} \]  

(37)

and one recovers, as it should, the forward matrix element of equation (1) and which is equal to the standard inclusive PDF, the forward limit of GPDs.

However, there is now a second, new, limiting case: when P = 0 and Δ ≠ 0, which is a case that could not be considered before, since Δ was proportional to P

\[ \langle Δ|\tilde{Ψ}_q(0)O\tilde{Ψ}_q(y)|0 \rangle \bigg|_{y^*=y_+=0} \]  

(38)
This matrix element should be interpreted as the probability amplitude to find in the nucleon a $q\bar{q}$ pair which shares the momentum $\Delta$ in $(1+\alpha)$ and $(1-\alpha)$ fractions (see figure 18 with $P = 0$). Then, the idea is that the $\alpha$ functional dependence of the GPD could, in this domain, take the shape of a DA. A DA gives the probability amplitude to find in a meson $M$ a $q\bar{q}$ pair which carries $z$ and $1-z = \bar{z}$ fractions of the meson momentum $p_M$. The corresponding matrix element is the following:

$$\langle 0 | \bar{\psi}(y) \gamma_i D_F(y) \overline{\psi}(z) | p_M \rangle, \quad (39)$$

where $D_F$ can be a vector ($\gamma^\mu$) or axial ($\gamma^\mu \gamma^5$) operator according to the parity of the meson. Such a matrix element is illustrated in figure 19 and its Fourier transform reads:

$$\Phi_M(z) = \int d\eta \ e^{i(p_M \eta)} \langle 0 | \bar{\psi}(0) \gamma_i D_F(y) \overline{\psi}(z) | p_M \rangle, \quad (40)$$

A DD can therefore be considered as a ‘mixture’/’hybrid’ of a PDF and a DA, i.e. two limiting cases of a DD (respectively, $\Delta = 0$ and $P = 0$). Knowing the two limiting cases of the DD, the idea is to find a functional form of $\alpha$ and $\beta$ which smoothly interpolates between a DA and a PDF when, respectively, $\alpha \rightarrow 0$ and $\beta \rightarrow 0$. A form which fulfills these requirements and proposed by Radyushkin [77, 78] is

$$h(\beta, \alpha) = h(\beta, \alpha) \sigma(\beta), \quad (41)$$

$$h(\beta, \alpha) = \frac{\Gamma(2b+2)}{2^{2b+1} \Gamma^2(2b+1)} \left[ \frac{(1-|\beta|)^2 - \alpha^2}{(1-|\beta|)^{2b+1}} \right] \alpha^b, \quad (42)$$

where $b$ is a free parameter. It governs the amount of $\xi$ dependence of the DDs. The higher the $b$ value, the weaker the $\xi$ dependence for GPD $D_3(x, \xi, t)$. For instance, when $b \rightarrow \infty$, $h(\beta, \alpha) \rightarrow 1$ and the DDs are independent of $\xi$ and resemble a PDF. In principle, one can define a value for the valence, $b_{\text{val}}$, and another one for the sea, $b_{\text{sea}}$. Figure 20 shows $H^3(x, \xi, t)$ as a function of $x$ and $\xi$ for $t = 0$ and for $b_{\text{val}} = b_{\text{sea}} = 1$, following the DD ansatz of equations (35) and (42), based on the VGG model which will be soon discussed.

One identifies at $\xi = 0$ a standard quark density distribution, with the rise around $x = 0$ corresponding to the diverging sea contribution. The negative $x$ part is related to antiquarks. One sees the evolution with $\xi$ which tend toward the shape of an asymptotic DA.

3.1.2. The D-term. As we saw, GPDs built on the DD ansatz automatically satisfy the polynomiality rule. However, because of the $\delta(x - \beta - \xi)\alpha)$ function in equation (35), the $n$th moment of the so-defined GPDs is at most a polynomial in $\xi$ of order $n$, while the polynomiality rule allows for a term with one more power, i.e. a $\xi^{n+1}$ term. This means that the DD composition of the GPDs is not complete. The so-called D-term, denoted by $D(x/\xi, t)$, has been introduced by Polyaek and Weiss [79] to take into account this ‘missing’ power $\xi^{n+1}$. It can be decomposed in a Gegenbauer series as

$$D(z, t) = (1 - z^2) \sum_{n=0}^{\infty} d_n(t) C_n^{1/2}(z), \quad (43)$$

with $|z| \leq 1$. Since the D-term corresponds to a flavor singlet contribution, it receives the same contribution from each quark flavor. One can then define a $D$-term contribution for each quark flavor by dividing by a factor $N_f = 3$, denoting the number of light quark flavors. Furthermore, in equation (43), the FFs $d_1(t), d_3(t), \ldots$ at $t = 0$ have been estimated, in a first approach, in the chiral soliton model as [34]: $d_1 = -4, d_3 = -1.2$ et $d_5 = -0.4$.

The D-term ‘lives’ only in the $-\xi < x < \xi$ region, i.e. the quark part of the GPDs, whence the motivation to expand it on odd Gegenbauer polynomials which are the standard functions on which meson DAs are decomposed. We will come back to the D-term in section 3.4 devoted to DRs.

We are now going to describe the VGG and GK models which are both based on the DD (+ D-term) ansatz for the $(x, \xi)$-dependence and which differ essentially by the parametrization of their $t$-dependence.

3.1.3. The VGG model. The VGG model is associated with a series of publications released between 1999 and 2005 [34, 80–82]. The first version of the model was published in

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**Figure 19.** Illustration of the $\langle 0 | \bar{\psi}(y) \gamma_i D_F(y) \overline{\psi}(z) | p_M \rangle \rvert_{\gamma = 0}$ matrix element. A DA represents the probability amplitude to find a quark and antiquark of momentum fraction, respectively, an element. A DA represents the probability amplitude to find a quark momentum $p$ and antiquark fraction, respectively, in a meson $M$ of momentum $p_M$ (or, equivalently, to create from the vacuum $|0\rangle$ a meson $M$ with such a quark–antiquark pair).

**Figure 20.** The GPD $H^3(x, \xi, t)$ as a function of the longitudinal–momentum fraction $x$ and the longitudinal–momentum transfer $\xi$ at $t = 0$ according to the VGG model. One recognizes for $\xi = 0$ the typical shape of a parton distribution (with the sea quarks rising as $x$ goes to 0), the negative $x$ part being interpreted as the antiquark contribution) and as $\xi$ increases the (asymptotic) shape of a DA.
1998 but has since then continuously evolved, benefitting from and integrating the work and inputs of several other authors, as the field of GPD grew and improved. We quote here the main publications and associated steps and improvements of the VGG model over the past 10 years.

- In [80], at first a parametrization of the GPDs via a $\xi$-independent and $t$-factorized ansatz was used. In a concise notation: $H^q(x, \xi, t) = q(x) F^q(t)$, $H^q(x, \xi, t) = \Delta q(x) G^q(t)$, etc.

- In [81], the $\xi$-dependence via the DDs which was discussed in the previous subsection was introduced in the GPD parametrization. In a concise notation: $H^q(x, \xi, t) = H^{q0}(x, \xi) F^q(t)$, $H^q(x, \xi, t) = H^{q0}(x, \xi) G_A(t)$, etc.

- In [82], the Regge dependence was modified so as to satisfy the FF counting rules at large $t$. In a concise notation: $H^q(x, \xi, t) = H^{q0}(x, \xi) x^{-\alpha(t)+1} + \frac{1}{N_f} D(\frac{t}{x^2})$, $E^q(x, \xi, t) = E^{q0}(x, \xi) x^{-\alpha(t)} - \frac{1}{N_f} D(\frac{t}{x^2})$, etc.

- In [34], the 'folding in' the Regge ansatz for the $q(x)$ PDFs into the very small $x$ domain since $\xi \approx 20$ implies that for the small $x$ behavior of the polarized quark distributions are those of the axial-vector mesons).

For GPDs, i.e. the 'non-forward' PDFs, the idea is to generalize this Regge ansatz for non-zero $t$ values. Thus, the first formula which naturally suggests itself is (for the $H$ GPD and for $\xi = 0$, to simplify matter in a first stage)

$$H^q(x, 0, t) = q_\nu(x) x^{-\alpha'(t)}$$

(44)

with the assumption of a linear Regge trajectory, i.e. $\alpha(t) = \alpha_0 + \alpha' t$. $\alpha'$ is a parameter which can be strongly constrained by the sun rules linking the GPDs to the $F_1$ and $F_2$ FFs, following equation (7) and, in particular, the nucleon charge radius.

However, the ansatz of equation (44) has the shortcoming that it does not produce a correct behavior of the FFs at large $t$. At large $t$, quark counting rules dictate that $F_1(t)$ should behave as $\frac{1}{t^2}$. $F_2(t)$, which is spin flip, and is therefore suppressed, should behave as $\frac{1}{t}$. The ansatz of equation (44) does not satisfy these limits. In fact, if $q_\nu(x) \approx (1-x)^v$ for $x \to 1$, one can show that, at large $t$,

$$\int_0^1 (1-x)^v x^{-\alpha'(t)} dx \propto \frac{1}{\alpha'} |t|^{-v+1}$$

(45)

which, with $v \approx 3$, taken from phenomenology yields a $1/t^4$ asymptotic behavior for $F_1(t)$, which is at variance with the $1/t^2$ behavior seen in the $F_1$ FF data, and expected from the asymptotic behavior.

The large-$t$ power behavior of $F_1(t)$ should be governed by the large $x$ $(\to 1)$ behavior of $q(x)$. Physically, the asymptotic large-$t$ domain consists in probing the simplest configuration of the nucleon, i.e. the three-quark configuration of the nucleon wave function which corresponds to the large $x$ region. The idea is then to modify the large $x$ behavior of equation (44). It can be carried out by introducing a $(1-x)$ term in the exponent of equation (44). With

$$\int_0^1 (1-x)^v x^{-\alpha'(t)} dx \propto \frac{1}{\alpha'} |t|^{-v+1/2}$$

(46)

one has, for $v \approx 3$, a $\frac{1}{t^3}$ behavior at large $t$.

To summarize, the ansatz for the VGG GPD $H$ is therefore (for $\xi = 0$)

$$H(x, 0, t) = q_\nu(x) x^{-\alpha'(1-x)t}$$

(47)

Finally, the full $(x, \xi) - t$ correlation is introduced by 'folding in' the Regge ansatz for the $(x, \xi)$ dependence into the DD concept for the $(x, \xi)$ dependence. One then defines Regge-type DDs:

$$F^q(\beta, \alpha, t) = F^q(\beta, \alpha, 0) \beta^{-\alpha'(1-\beta)t}$$

(48)

where $F^q(\beta, \alpha, 0)$ is given by the form of equation (42).

To satisfy the polynomiality rule of equation (10) for the GPD $H$, it has been shown in [79] that a $D$-term has to
be added. The full \( (x, \xi, t) \) dependence for the GPD \( H \) in the VGG model then reads
\[
\begin{align*}
H^q(x, \xi, t) &= \int dx \, d\beta \delta(x - \beta - \xi \alpha) F^q(\beta, \alpha, t) \\
&\quad + \theta(\xi - |x|) \frac{1}{N_f} D \left( \frac{x}{\xi}, t \right),
\end{align*}
\]
with \( F^q(\beta, \alpha, t) \) defined by equation (48).

(b) Parametrization of the \( D \)-term.

The \( t \)-dependence of the \( D \)-term is unknown. The \( D \)-term, being odd in \( x \), is not at all constrained by the FF sum rules of equation (7). VGG adopts a factorized form with a dipole behavior in \( t \) with an adjustable mass scale.

(c) Parametrization of the GPD \( E \)

The parametrization of the GPD \( E^q \), corresponding to a nucleon helicity flip process, is less constrained as we do not have the DIS constraint for the \( x \)-dependence in the forward limit.

One contribution to the GPD \( E^q \) is, however, determined through the polynomiality condition, which requires that the \( D \)-term contribution is canceled in the combination \( H + E \). Therefore, it contributes with opposite sign to \( H \) and \( E \).

Similarly to equation (49) for \( H^q \), \( E^q \) is parametrized in the VGG model by adding a DD part to the \( D \)-term as
\[
E^q(x, \xi, t) = E_{DD}^q(x, \xi, t) - \theta(\xi - |x|) \frac{1}{N_f} D \left( \frac{x}{\xi}, t \right),
\]
where \( E_{DD}^q \) is the DD part.

In the forward limit, the DD part reduces to the function
\[
\int_{-1}^{+1} dx \, e^q(x) = \kappa^q,
\]
where \( \kappa^u \) and \( \kappa^d \) are the flavor combinations of the nucleon anomalous magnetic moments given by
\[
\begin{align*}
\kappa^u &= 2 \kappa^p + \kappa^n = 1.673, \\
\kappa^d &= 2 \kappa^p + 2 \kappa^n = -2.033.
\end{align*}
\]

For the \( x \)-dependence of the forward GPD \( e^q(x) \), a sum of valence and sea-quark parametrization is implemented in VGG, according to [34] as
\[
\begin{align*}
e^q(x) &= A^u u_{val}(x) + B^u \delta(x), \\
e^d(x) &= A^d d_{val}(x) + B^d \delta(x), \\
e^s(x) &= 0,
\end{align*}
\]
where the parameters \( A^u, A^d \) are related to \( J^u, J^d \) through the total angular momentum sum rule, which yields
\[
A^q = \frac{2 J^q - M^q_2}{M^q_2},
\]
and where the parameters \( B^u, B^d \) follow from the first moment sum rule equation (51) as
\[
B^u = \kappa^u - 2 A^u, \quad B^d = \kappa^d - A^d.
\]

Such parametrization allows one to use the total angular momenta carried by \( u \)- and \( d \)-quarks, \( J^u \) and \( J^d \), directly as GPD fit parameters, and can be used to see the sensitivity of hard electroproduction observables on \( J^u \) and \( J^d \), as will be shown further on in section 4.

Starting from the model for the forward distribution \( e^q(x) \), the \( \xi \)-dependence of the GPD \( E^q_{DD}(x, \xi, 0) \) is generated through a DD \( K^q(\beta, \alpha, t) \) as
\[
E^q_{DD}(x, \xi, t) = \int_{-1}^{+1} d\beta \int_{-1+|\beta|}^{1-|\beta|} d\alpha \times \delta(x - \beta - \alpha \xi) K^q(\beta, \alpha, t).
\]

The \( D \) \( K^q(\beta, \alpha, 0) \) is taken in analogy as in equation (42), by multiplying the forward distribution \( e^q(\beta) \) with the same profile function as in equation (42) as
\[
K^q(\beta, \alpha, t) = h(\beta, \alpha) e^q(\beta).
\]

The parametrization of equation (53) yields for the GPD \( E^q_{DD}(x, \xi, 0) \)
\[
E^q_{DD}(x, \xi, 0) = E^q_{DD}(x, \xi, 0) + B^q \frac{\Gamma(2b + 2)}{2^{2b+1}\Gamma^2(b + 1)} \theta(\xi - |x|) \left( 1 - \frac{x^2}{\xi^2} \right)^b,
\]
where the first (second) term originates from the valence (sea) contribution to \( e^q \), respectively, in equation (53), and where the parameter \( b \) is the power entering the profile function.

For the \( t \)-dependence of the GPD \( E \), the VGG model uses a Regge ansatz, which was constrained in [82] to provide a fit to the Pauli FF \( F_2 \). Since the large-\( t \) behavior of \( F_2(t) \) goes steeper than \( 1/t^2 \), the Drell–Yan–West relation implies a different large-\( x \) behavior of \( e^q(x) \) compared with \( q(x) \). To produce a faster decrease with \( t \), a simple ansatz is to multiply \( q_{val}(x) \) by an additional factor of the type \((1 - x)^{n_t}\) thus modifying the \( x \approx 1 \) limit which is the region driving the large-\( t \) behavior of \( F_2(t) \), as we discussed previously. This yields for the valence part
\[
K^q_{val}(\beta, \alpha, t) = h(\beta, \alpha) N_q q_{val}(\beta)(1 - \beta)^{n_t} \beta^{-\alpha' t},
\]
where the normalization constant \( N_q \) is determined from the anomalous magnetic moment, and where the Regge slope \( \alpha' \) and the parameter \( n_t \) which determines the large-\( x \) behavior of the forward GPD \( e^q(x) \), are to be determined from a fit to the nucleon Pauli FF data as. In contrast, the sea-quark cannot be constrained by the \( F_2 \) FF data. It has for simplicity been assumed to have a same \( t \)-dependence within VGG as for the valence part.

(d) Parametrization of the GPD \( \tilde{H} \).

For the \((x, \xi)\)-dependence, the GPD \( \tilde{H} \) is also based on the DD ansatz with the replacement of the unpolarized PDF
\( q(\beta) \) in equation (42) by the polarized PDF \( \Delta q(\beta) \), so as to obtain the appropriate forward limit of equation (6):

\[
\bar{F}(\beta, \alpha, 0) = h(\beta, \alpha) \Delta q(\beta).
\]

For the \( t \)-dependence, in principle, a Regge ansatz similar to the one used for the unpolarized GPDs (equation (47)) could be used. However, at this time, given the relatively few experimental constraints from DVCS on this GPD, a \( t \)-factorized ansatz has been kept

\[
\bar{H}(x, \xi, t) = \int \sigma \, d\beta \delta(x - \beta - \xi \alpha) \bar{F} \\
\times (\beta, \alpha, t) G_A(t)/G_A(0).
\]

\( (e) \) Parametrization of the GPD \( \bar{F} \).

Following the argument of [83, 84], it is parametrized by the pion exchange in the \( t \)-channel, which, due to the small pion mass, should be a major contribution:

\[
\bar{E}^{u/p} = -\bar{E}^{d/p} = \frac{1}{2} \bar{E}^{(3)}_{x-pole},
\]

\[
\bar{E}^{(3)}_{x-pole} = \theta(\xi - |x|) \, h_A(t) \frac{1}{\xi} \phi_0 \left( \frac{x}{\xi} \right)
\]

with the asymptotic DA \( \phi_0 \) is given by \( \phi_0(z) = 3/4 \) \((1 - z^2)\), and \( h_A(t) \) is the induced pseudo-scalar FF of the nucleon. The contribution of equation (62) to the \( \bar{E} \) GPD, corresponding to a meson or \( q\bar{q} \) exchange in the \( t \)-channel, lives only in the \(-\xi \leq x \leq \xi \) region and as such, contributes only to the real part of the DVCS amplitude. In summary, the VGG parametrization is based on only few inputs:

- a choice for the PDF which drives the forward limit. By choosing a PDF parametrization which takes into account the evolution equation, a \( Q^2 \)-dependence can be introduced in the GPDs,
- the parameters \( b_v \) and \( b_s \), which drive the \((x, \xi)\)-dependence and which are set to 1 by default,
- the parameters \( \alpha' \) and \( \eta_0 \) which drive the \( t \)-dependence. In [82], the fit to the proton and neutron FF data yielded \( \alpha' = 1.105 \text{ GeV}^{-2}, \eta_0 = 1.713 \) and \( \eta_0 = 0.566 \),
- the parameters \( J_u, J_d \) which control the normalization of the \( E \) GPD and which are unknown apriori.

### 3.1.4. The GK model.

The GK parametrization of the GPDs has been developed in the process of fitting the high-energy (low \( x \)) DVMP data and has been published in a series of papers [85–87]. There are numerous data available for DVMP and since the same GPDs as for DVCS enter in the DVMP handbag diagram (figure 9), strong constraints on the GPD model parameters can be derived, which are not present in VGG.

- In [85] the DVMP two-gluon exchange handbag diagram amplitude (figure 9-right) was derived for exclusive \( \phi^0 \) and \( \phi \) electroproduction on the proton, taking into account some higher-twist corrections (Sudakov suppression and transverse momenta of the quark). The authors proposed a DD-based ansatz for the gluon GPDs and compared their calculation with the HERA data.

- In [86] the two-quark exchange handbag diagram amplitude (figure 9—left) was added and a DD-based ansatz for the quark GPDs \( H^q \) and \( E^q \) was proposed, which we will describe in the following.

- In [87], exclusive \( \pi^+ \) electroproduction on the proton was investigated, which allowed one to derive a parametrization for the \( \bar{H} \) and \( \bar{E} \) GPDs (as well as for the transversity GPD \( H_T \), which we will not discuss here).

Like VGG, the GK model is based on DDs for the \((x, \xi)\)-dependence. In VGG, the \( b \) exponents in the profile function of equation (42) are usually taken as 1 but they are essentially unconstrained and left as free parameters due to the lack of constraint from the DVCS data. In GK, the \( b \) parameters are taken as 1 for valence quarks and 2 for sea quarks. This values correspond to the asymptotic behavior of quark and gluon DAs, respectively.

For the \( t \)-dependence, the GK GPD is expressed (at \( \xi = 0 \)) as its forward limit multiplied by an exponential in \( t \) with a slope depending on \( x \):

\[
\text{GPD}'(x, \xi = 0, t) = \frac{GPD(x, \xi = 0, t = 0)}{e^{p_i(x)}}
\]

with a Regge-inspired profile functional form:

\[
p_i(x) = \alpha_i' \ln 1/x + b_i.
\]

The label \( i \) stands for valence or sea-quark flavors, or gluons. Gluon GPDs are in principle taken into account in GK. This is a difference with VGG which takes into account only quark (valence and sea) GPDs. However, since this review focuses on the valence region and on the leading-twist LO domain, the gluonic degrees of freedom are not included in the following calculations.

For quark GPDs, equation (63) can be rewritten

\[
\text{GPD}'(x, t) = q(x) x^{-\alpha_1'} \, e^{b_1 t}
\]

in order to better compare with the VGG ansatz of equation (47).

The GK \( t \)-dependence is different from the one of the VGG model in that

- there is an \( x \)-independent term in the exponential (associated with the parameter \( b_1 \)),
- the \( x \)-dependence of the \( t \)-slope has an extra \((1 - x)\) factor in VGG (equation (47)).

The parameters in equations (63) and (64) are determined by the analysis of DVMP data in the kinematical region \( \xi \lesssim 0.1, Q^2 \geq 3 \text{ GeV}^2, W \geq 4 \text{ GeV} \) and \(-t \leq 0.6 \text{ GeV}^2\). The data sensitive mostly to the GPD \( H \) are available only in a restricted \( Q^2 \) range, while the existing data which have a higher sensitivity to \( E, \bar{H}, \bar{E} \) are available only in a restricted \( Q^2 \) range. Therefore, a \( Q^2 \)-dependence on the GPD \( H \) is taken into account through the \( Q^2 \) dependence of the PDF used in the DD ansatz [86, 87] (like in VGG) while it is neglected for the \( E, \bar{H} \) and \( \bar{E} \) GPDs. It is also ensured that the valence quark GPDs are in agreement with the nucleon FFs at small \( t \) and that all GPDs satisfy positivity bounds [47, 90]. We now detail the parametrization of each GPD.
(i) Parameterization of the GPD $H$.

The forward limit of the GPD $H$ is the usual unpolarized PDF. To allow an analytic evaluation of the resulting GPD, PDFs are expanded on a basis of half-integer powers of $x$:

$$H^f(x, \xi = 0, t = 0) = x^{-\alpha_H(0)}(1 - x)^{2n + 1} \sum_{j=0}^{n+1} c_{ij} x^{j/2},$$  \hspace{1cm} (66)

where $i$ represents various quark flavors. The $Q^2$-dependent expansion coefficients $c_{ij}$ have been obtained from a fit to the CTEQ6M PDFs [88] and are summarized in [89]. The parameters appearing in the profile functions (63) obey linear Regge trajectories:

$$\alpha_{Hi} = \alpha_{Hi}(0) + \alpha'_Hi t.$$  \hspace{1cm} (67)

It is assumed that $\alpha_{H_{sea}}(t) = \alpha_{H_{sea}}(0) + 1 + \delta_{Hi}$ as seen in the HERA experiments. The expression of the GPD $H$ stemming from the expansion of equation (66) is

$$H_i(x, \xi, t) = e^{b_{Hi} t} \sum_{j=0}^{n} c_{ij} H_{ij}(x, \xi, t).$$  \hspace{1cm} (68)

where integrals $H_{ij}$ are written down in [86]. The slopes $b_{Hi}$ are modeled by

$$b_{H_{val}} = 0,$$

$$b_{H_{sea}} = b_{H_{g}} = 2.58 \text{ GeV}^{-2} + 0.25 \text{ GeV}^{-2} \ln \frac{m_N^2}{Q^2 + m_N^2}.$$  \hspace{1cm} (69)

Sea-quark GPDs are further simplified [87] as

$$H_{sea}^u = H_{sea}^d = \kappa_s H_{sea}^f,$$

with

$$\kappa_s = 1 + 0.68/(1 + 0.52 \ln Q^2/Q_0^2).$$  \hspace{1cm} (70)

The flavor symmetry breaking factor $\kappa_s$ possesses a $Q^2$-dependence fitted from the CTEQ6m PDFs. The parameters in the previous equations are determined by the HERA $\rho^0$ and $\phi$ data.

(ii) Parametrization of the GPD $E$.

The constraints on $E$ come mostly from the Pauli FF data [90], through the sum rules of equation (7). A DD ansatz is also used. $E(x, \xi = 0, t = 0)$ is parametrized with a classical PDF functional form:

$$E_{val}^d(x, \xi = 0, t = 0) = \frac{n(2 - \alpha_{val} + \beta_{val}^d)}{n(1 - \alpha_{val})n(1 + \beta_{val}^d)} x^{-\alpha_{val}(0)}(1 - x)^{2n+1} \sum_{j=0}^{n+1} \kappa_{q} x^{j/2},$$  \hspace{1cm} (71)

where the ratio of $n$ functions ensures the correct normalization of $E$ at $t = 0$. The fits to the nucleon Pauli FFs performed in [90] fix the parameters specifying $E$ for valence quarks to $\beta_{val}^d = 4$ and $\beta_{val}^u = 5.6$. The trajectory $\alpha_{val}$ and slope parameter $b_{val}$ are assumed equal to the corresponding $H$ parameters.

The GK model of the gluon and sea $E$ GPDs is given in [91] following an idea of Diehl and Kugler [92]. The DD ansatz is used again and the forward limits of the gluonic and strange quark GPDs are parametrized as

$$E^f(\rho, \xi = t = 0) = N_s \rho^{-1-\beta_{Es}}(1 - \rho)^{\beta_{Es}}$$  \hspace{1cm} (72)

using $\beta_{Es} = 7$ and the same Regge trajectory as for $H$.

The sea is supposed to be flavor-symmetric. The slopes of the residues $\rho_{Es}$ are estimated as

$$\rho_{Es} = 0.9 b_{H_{g}}.$$  \hspace{1cm} (73)

The normalization $N_s$ of $E^s$ is fixed from saturating a positivity bound for a certain range of $x$ [91] (which does not allow one to fix the sign of $N_s$): $N_s = \pm 0.155$.

(iii) Parametrization of the GPD $\tilde{H}$.

The Blümlein–Böttcher results [93] are taken to describe the forward limit of $\tilde{H}$ [86, 87]. Only $\tilde{H}_{val}$ is modeled and constrained by the HERMES data [94, 95]. $\tilde{H}_{sea}$ is set to zero. In the same spirit as the modeling of GPDs $H$ and $E$, the forward limit $\tilde{H}_{val}^d(x, \xi = 0, t = 0)$ is written following a DD ansatz and in an analytical expansion, with the following profile function:

$$\tilde{H}_{val}^d(x, \xi = 0, t = 0) = \eta u A_u x^{-\alpha_{val}(0)}(1 - x)^{2} \sum_{j=0}^{n} \tilde{c}_{ij} x^{j},$$  \hspace{1cm} (74)

where $i = u, d$. The factors $\eta_u$ and $\eta_d$ guarantee the correct normalization of the first moment of $\tilde{H}_{val}$ which is known from $F$ and $D$ values and $\beta$-decay constants $\eta_u = 0.926 \pm 0.014$ and $\eta_d = -0.341 \pm 0.018$. The normalization factors $A_u$ and $A_d$ are defined by

$$A_u^{-1} = B(1 - \alpha_{\tilde{H}_u}, 4) \left[ \tilde{c}_{i0} + \tilde{c}_{i1} \frac{1 - \alpha_{\tilde{H}_u}}{5 - \alpha_{\tilde{H}_u}} \right] + \tilde{c}_{i2} \frac{(2 - \alpha_{\tilde{H}_u})(1 - \alpha_{\tilde{H}_u})}{(6 - \alpha_{\tilde{H}_u})(5 - \alpha_{\tilde{H}_u})},$$  \hspace{1cm} (75)

where $B(a, b)$ is Euler’s beta function. The coefficients $\tilde{c}$ can be found in [89].

(iv) Parametrization of the GPD $\tilde{E}$.

The GPD $\tilde{E}$ is also determined only for valence quarks. Its sea part is set to 0. As for VGG, its modeling takes into account the pion pole contribution which reads [81, 84]

$$\tilde{E}_{pole}^u = -\tilde{E}_{pole}^d = \Theta(|x| \leq \xi) \frac{F_{\pi}(t)}{4\xi} \Phi_{\pi}^{u/d} \left( \frac{x + \xi}{2\xi} \right),$$  \hspace{1cm} (76)

where $F_{\pi}$ is the pseudoscalar from factor of the nucleon. The pole contribution to the pseudoscalar FF is written as

$$F_{\pi}(t) = -m_N f_{\pi} \sqrt{2 \frac{2g_{NN} m_N F_{NN}(t)}{t - m_N^2}}.$$  \hspace{1cm} (77)

where $m_N$ denotes the mass of the pion and $g_{NN} \simeq 13.1$ is the pion–nucleon coupling constant, $f_{\pi}$ is the pion decay constant.

The pion’s DA $\Phi_{\pi}$ is taken as

$$\Phi_{\pi}(\tau) = 6\tau (1 - \tau) \left[ 1 + \alpha_2 C^{3/2}_{\pi}(2\tau - 1) \right].$$  \hspace{1cm} (78)
with $\alpha_s = 0.22$ at the initial scale $Q_0^2 = 4$ GeV$^2$. The FF of the pion–nucleon vertex $F_{\pi NN}$ is described by [87]:

$$F_{\pi NN} = \frac{\Lambda_N^2 - m_N^2}{\Lambda_N^2 - t}$$

(79)

with $\Lambda_N = 0.44$ GeV. Such a hadronic FF is not present in the VGG parametrization of $\tilde{E}$.

A non-pole contribution, which is not present in VGG, is added and modeled in the same way as $H$, $E$ and $\tilde{E}$, i.e. a functional form for the forward limit is assumed, then skewed with a profile function in a DD ansatz. Flavor independence of the Regge trajectory and the slope of the residue are assumed. The forward limit reads [87, 96]

$$\tilde{E}_{\text{val}}(x, \xi, t = 0 = N_E^0x^0(1 - x)^3).$$

(80)

The following values for the various parameters involved are $\alpha_E(0) = 0.48$, $\alpha_E' = 0.45$, $b_E = 0.9$ GeV$^{-2}$, $N_{E}^0 = 14.0$ and $N_{E}^d = 4.0$.

3.2. Dual parametrization

3.2.1. Evolution equations and conformal symmetry. By definition, conformal transformations change only the scale of the metric of Minkowski space, and in particular leave the light-cone invariant. The whole conformal group admits a particular subgroup, named collinear conformal group, which maps a given light ray onto itself. This is of special relevance for hadron structure functions since in the parton model, hadrons are viewed as a bunch of partons moving fast along a direction on the light cone. It helps classifying fields according to their collinear conformal symmetry properties. For details we refer to the review of [97].

Although QCD is not a scale invariant theory (it exhibits a spectrum of massive bound states), conformal symmetry is a symmetry of the classical theory when quarks are considered as massless. It is thus relevant for renormalization at LO since the counter terms satisfy the symmetry properties of the tree-level (classical) Lagrangian. As operators with different quantum numbers (or symmetry properties) do not mix under renormalization, conformal symmetry is a powerful tool to separate operators at LO.

In particular Gegenbauer polynomials $C_{3/2}^n$ parametrize the local conformal operators associated with the twist two matrix elements used to define PDFs or GPDs. They diagonalize the ERBL evolution equations that describe the evolution of GPDs in the inner region $-\xi < x < +\xi$ [33–38] where GPDs probe the presence of quark–antiquark pairs in the nucleon. This is the region of interest when representing GPDs as an infinite series of $t$-channel exchange resonances, as in the case of the dual model, or alternatively of the Mellin–Barnes representation. Therefore, expanding GPDs on a series of orthogonal Gegenbauer polynomials $C_{3/2}^n$ is an appealing starting point to parametrize GPDs.

3.2.2. Partial-wave expansion and CFFs. The dual parametrization of the GPDs is based on a representation of parton distributions as an infinite series of $t$-channel exchanges [98]. For the unpolarized GPDs, one defines electric ($H^E$) and magnetic ($H^M$) GPD combinations:

$$H^E(x, \xi, t) = H(x, \xi, t) + \frac{t}{4m_N^2} E(x, \xi, t),$$

(81)

$$H^M(x, \xi, t) = H(x, \xi, t) + E(x, \xi, t),$$

(82)

which are suitable for a $t$-channel partial-wave expansion, which read for the singlet combinations as [99]

$$H_0^E(x, \xi, t) = 2 \sum_{n=\text{odd}} \sum_{l=0}^{\text{even}} B_{nl}^E(t) H_{nl}(t) \theta$$

(83)

$$H_0^M(x, \xi, t) = 2 \sum_{n=\text{odd}} \sum_{l=0}^{\text{even}} B_{nl}^M(t) \theta$$

(84)

where $C_{3/2}^0(z)$ are the Gegenbauer polynomials, $P_l(z)$ are Legendre polynomials, and $B_{nl}(t)$ are generalized FFs. Note that for $H_{nl}^E$ intermediate states with $l_{PC} = 0^{++}, 2^{++}, \ldots$ contribute, whereas for $H_{nl}^M$ intermediate states with $l_{PC} = 2^{++}, 4^{++}, \ldots$ contribute.

As stated before, the expansion onto a basis of Gegenbauer polynomials allows a trivial solution of the QCD evolution equations at LO\(^5\): the $Q^2$-evolution of the generalized FFs $B_{nl}(t, Q^2)$ reads [105]

$$B_{nl}(t, Q^2) = B_{nl}(t, Q_0^2) \left( \frac{\ln Q_0^2}{\ln Q^2} \right)^{\beta_n},$$

(85)

with $\beta_n = 11 - \frac{2}{3} n_f$ and

$$\gamma_n = \frac{4}{3} \left( 3 + \frac{2}{n(n+1)} - 4(\Psi(n+1)+\gamma_E) \right),$$

(86)

where $\Psi$ denotes the digamma function and $\gamma_E$ the Euler–Mascheroni constant.

At fixed $x$ and $\xi$ the series on the right-hand side (rhs) of equations (83) and (84) are divergent: the sums $H_{nl}^E$ and $H_{nl}^M$ has a support $-\xi < x < +\xi$ while each term of the expansions has a support $-\xi < x < +\xi$. However, these formal series can be recast onto convergent Gegenbauer polynomial expansions. For example the electric singlet GPD reads [100]

$$H_0^E(x, \xi, t) = 2(1-x^2) \sum_{n=\text{odd}} A_n(\xi, t) C_{3/2}^n(x).$$

(87)

\(^5\) In fact QCD evolution equations ‘commute’ with the parametrization equation (85): the GPD at some input scale $Q_0$ has the same form as the GPD at another scale $Q$, a feature that is usually absent from the DD representation with the factorized ansatz involving the profile function $h(b, \alpha)$ (42).
The coefficients $A_n$ are defined by

$$A_n(\xi, t) = -\frac{2n + 3}{(n + 1)(n + 2)} \sum_{p=0}^{n-1} R_{np}(\xi) \frac{(p + 1)(p + 2)}{2p + 3} \times \sum_{l=0}^{p+1} B_{pl}(t) P_l\left(\frac{1}{\xi}\right).$$

Here $R_{np}(\xi)$ is a polynomial of degree $n$:

$$R_{np}(\xi) = \frac{(-1)^{n+p} \Gamma\left(\frac{3}{2} + \frac{n+p}{2}\right)}{\Gamma\left(\frac{3}{2} + \frac{1}{2} + \frac{n+p}{2}\right)} \xi^{p+2} F_1\left(\frac{p-n}{2}, \frac{3}{2} + \frac{n+p}{2}, \frac{5}{2} + p, \xi^2\right),$$

with $F_1$ the Gauss hypergeometric function. The convergent expression (87) has been used explicitly for fitting in a truncated form as explained in section 4.2.3.

The procedure to sum the formal series of equations (83) and (84) over orbital momentum $\ell$ through analytical continuation was originally outlined in [100]. We briefly discuss this in the following for the function $H_\ell(\xi)$ and for simplicity drop the superscript (E). For an analogous discussion of $H^{(M)}$, as well as the polarized GPDs $\hat{H}$ and $\hat{E}$, we refer the reader to [99].

In order to sum the formal series of equation (83), a set of generating functions $Q_{2\nu}(x, t)$ ($\nu = 0, 1, \ldots$) are introduced, whose Mellin–Barnes moments yield the generalized FFs $B_{\nu}(t)$ as [100]

$$B_{\nu, n+1-2\nu}(t) = \int_0^1 dx x^n Q_{2\nu}(x, t).$$

The functions $Q_{2\nu}(x, t)$ are forward-like because at LO, their scale dependence is given by the standard DGLAP evolution equation, so that these functions behave as usual parton distributions under QCD evolution. Furthermore, the function $Q_{2\nu}(x, t = 0)$ is directly related to the parton densities $q(x)$ measured in DIS [100]:

$$Q_{2\nu}(x, t = 0) = [q + \bar{q}] - \frac{x}{2} \int_x^1 \frac{dz}{z} [q + \bar{q}](z).$$

The usefulness of the dual parametrization originates when expanding the GPD around $\xi = 0$. The functions with higher $\nu$ are more suppressed for small values of $\xi$. An expansion with $x$ fixed to the order $\xi^{2\nu}$ involves only a finite number of functions $Q_{2\nu}(x, t)$ with $\mu \leq \nu$.

Within the dual parametrization for the GPDs, the CFFs entering hard exclusive observables can be expressed in terms of forward-like functions. For the combination of the CFF of equations (15) and (16), corresponding to the electric GPD of equation (81), this is given by [100, 101]

$$H_{\ell m} + \frac{t}{4m_N^2} E_{\ell m} = 2\pi \int_0^{t/\sqrt{\xi-1}} \frac{dx}{x} N^{(E)}(x, t) \left[\frac{1}{\sqrt{2\xi-x^2-1}}\right].$$

3.2.3. Modeling the forward functions. A number of phenomenological studies of DVCS observables have been made using the dual parametrization. Most prominently, studies involving only the forward function $Q_0$ have been made. In such a minimal model, the $x$ dependence is
parameter free as it is completely fixed by the forward parton distributions, merely leaving the \( t \) dependence of the GPD to be modeled. One typically uses a Regge motivated model to correlate the \( x \) and \( t \)-dependence of the function \( Q_0(x, t) \), analogous as it was discussed above for the DD model.

Such a minimal model was found to overpredict the data at small and intermediate \( x_B \): [104–106] found that DVCS experiments at HERA (HERMES) were overpredicted by roughly a factor of 2 (1.5), respectively. At larger values of \( x_B \), for DVCS experiments at JLab@6GeV, it was shown that the bulk effect of the DVCS beam helicity cross section difference can be understood within such a minimal dual model [108], which we show in more detail below.

To improve on the description, especially at smaller values of \( x_B \), within the dual parametrization requires to go beyond the minimal model by keeping more generating functions \( Q_2, Q_4, \ldots \), and extend to the next-to-LO accuracy. A first step to model the functions \( Q_2 \) and \( Q_4 \) has been made using a non-local chiral quark model [107] or by extracting them from experiments at HERA (HERMES) were overpredicted by roughly a factor of 2 (1.5), respectively. At larger values of \( x_B \): [104–106] found that DVCS experiments at JLab@6GeV, it was shown that the bulk effect of the DVCS beam helicity cross section difference can be understood within such a minimal dual model [108], which we show in more detail below.

For a comparison with data in the valence, we will be using a model for the LO forward-like functions \( Q_0^{(E)} \) and \( Q_0^{(M)} \) as [109]

\[
Q_0^{(E)}(x, t) = \left[ q_+(x, t) + \frac{t}{4m^2} e_+(x, t) \right],
\]

\[
Q_0^{(M)}(x, t) = \frac{1}{2} \int_x^1 \frac{dz}{z^2} \left[ q_+(z, t) + \frac{t}{4m^2} e_+(z, t) \right].
\]

For the forward GPD \( H_1(x, 0, t) \equiv q_+(x, t) \), we use a Regge type ansatz:

\[
q_+(x, t) = q_+(x) x^{-\alpha^t},
\]

with \( \alpha^t = 1.105 \text{GeV}^{-2} \) fixed from the FF sum rule [82]. For the forward GPD \( E_1(x, 0, t) \equiv e_+(x, t) \), we use an ansatz by expressing the magnetic GPD \( q_+ + e_+ \) as a Mellin convolution of \( q_+ \) with a kernel function, modeled as

\[
q_+(x, t) + e_+(x, t) = \int_x^1 \frac{dz}{z} q_+(z, t) C \left( \frac{x}{z} \right)^{\alpha} \left( 1 - \frac{x}{z} \right)^{\beta},
\]

where \( C \) is a constant, which is to be determined from the second moments of the GPDs.

The second moment of the forward parton distribution \( q_+ \) at \( t = 0 \) yields the total momentum carried by quarks and antiquarks:

\[
M_2^\alpha = \int_0^1 dx x q_+(x, 0),
\]

whereas the second moment of the magnetic GPD combination \( (q_+ + e_+) \) at \( t = 0 \) yields the total quark angular momentum:

\[
2J^\alpha = \int_0^1 dx x [q_+(x, 0) + e_+(x, 0)].
\]

Equations (100) and (101) then allow one to express the constant \( C \) as

\[
C = \frac{2J^\alpha}{M_2^\alpha} \frac{\Gamma(\alpha + \beta + 3)}{\Gamma(\alpha + 2) \Gamma(\beta + 1)}.\]

\[\text{Figure 21.} \quad \text{Parametrization for the forward function}\]

\[
(q_+^2 + e_+^2)(x, t = 0) \quad \text{for } J_2 = 0.3, \quad \text{and for different values of } \alpha, \beta :\]

\[
\alpha = 0, \beta = 0 \text{ (dashed–dotted curve)}; \quad \alpha = 1, \beta = 0 \text{ (dotted curve)}; \quad \alpha = 2, \beta = -0.5 \text{ (dashed curve).} \]

The solid curve shows the parametrization for the forward function \( q_+^2(x, t = 0) \).

3.3. Mellin–Barnes parametrization of GPDs

3.3.1. Partial-wave expansion. In this section, we will discuss the Mellin–Barnes parametrization of GPDs described in the works of [113, 114]. For simplicity, we do not write the dependence of GPDs on the momentum transfer \( t \). The method is based on making a partial-wave expansion of GPDs. It is analogous in spirit to the dual model partial-wave expansion explained in section 3.2, although both representations differ on the resummation of this expansion.

In order to recover the Mellin moments of PDFs when taking the forward limit of conformal moments of GPDs, one rescales the Gegenbauer polynomials to define the polynomials \( c_n(x, \xi) \):

\[
c_n(x, \xi) = \frac{\Gamma(\frac{1}{2} + \frac{n+1}{2}) \Gamma(n + 1)}{2^n \Gamma(\frac{n+1}{2} + \frac{3}{2})} \xi^n C_n^{(3/2)} \left( \frac{x}{\xi} \right),
\]

for any integer \( n \). Conformal moments \( F_n(\xi) \) of a GPD \( F \) \( (F = H, E, \bar{H} \text{ or } \bar{E}) \) are then defined by

\[
F_n(\xi) = \int_{-1}^{1} dx c_n(x, \xi) F(x, \xi).
\]

The conformal partial-wave expansion then reads

\[
F(x, \xi) = \sum_{n=0}^{\infty} (-1)^n p_n(x, \xi) F_n(\xi).
\]

\[\text{Orthogonality is meant in the following sense:}\]

\[
\int_{-1}^{1} dx c_n(x, \xi) p_{n'}(x, \xi) = (-1)^n \delta_{nn'},
\]

where the factor \((-1)^n\) is introduced for later convenience, precisely to write equation (108).
This is the common basis of the dual and Mellin–Barnes representations. The left-hand side (lhs) of equation (107) has support \( x \in [-1, +1] \) and the rhs has support \( x \in [-\xi, +\xi] \). Therefore, for \( |\xi| < 1 \) this sum has to be divergent and can be understood as a formal definition of conformal moments. It can be resummed by means of the Sommerfeld–Watson transform [115]:

\[
F(x, \xi) = \frac{1}{2\pi i} \int_C \frac{1}{\sin \pi j} p_j(x, \xi) F_j(\xi),
\]

where \( C \) is a contour in the complex plane enclosing all non-negative integers (which are the poles of \( j \mapsto 1/\sin \pi j \) with residues \((-1)^j/\pi\)). At this stage, it is only assumed that the analytic continuations of the functions \( p_n \) and of the moments \( F_n \) have no singularities inside the contour \( C \). Using the residue theorem one relates equation (108) to equation (107). Since the analytic continuation of a function of a discrete variable is not unique, this is a non-trivial step. A justification for it is given below.

Using such analytic continuations \( p_j \) and \( F_j \), one can deform the integration contour \( C \) so that all singularities of conformal moments lie left to a straight line parallel to the imaginary axis. If the integrand of equation (110) decreases fast enough at infinity\(^7\), one obtains

\[
F(x, \xi) = \frac{1}{2\pi i} \int_{c-i\infty}^{c+i\infty} \frac{1}{\sin \pi j} p_j(x, \xi) F_j(\xi),
\]

where the real \( c \) constant is suitably chosen\(^8\). This large-\( j \) behavior is another condition that should be fulfilled by the analytic continuations \( p_j \) and \( F_j \).

The analytic continuation of \( p_n \) can be expressed in terms of hypergeometric \( _2F_1 \) and gamma \( \Gamma \) functions:

\[
p_j(x, \xi) = \theta(\xi - |x|) \frac{1}{\xi^{j+1}} \left[ P_j \left( \frac{x}{\xi} \right) + \theta(x - \xi) \frac{1}{\xi^{j+1}} Q_j \left( \frac{x}{\xi} \right) \right],
\]

where

\[
P_j \left( \frac{x}{\xi} \right) = \frac{2^{j+1} \Gamma \left( \frac{5}{2} + j \right)}{\Gamma \left( \frac{1}{2} \right) \Gamma(1 + j)} (1 + x) _2F_1 \left( j + 1, j + 2, \frac{5}{2}, \frac{5}{2} + j, \frac{\xi + x}{2 \xi} \right),
\]

\[
Q_j \left( \frac{x}{\xi} \right) = -\frac{\sin \pi j}{\pi} _2F_1 \left( j + 1, j + 2, \frac{5}{2}, \frac{5}{2} + j, \frac{\xi + x}{2 \xi} \right).
\]

However, the explicit calculation of the analytic continuation of conformal moments for any value of \( \xi \) and any GPD model fulfilling the aforementioned conditions is an intricate mathematical question. An explicit general procedure is nevertheless described in the case \( |\xi| \leq 1 \) in [114].

\(^7\) The integrand should decrease fast enough to drop the contour at infinity. Mellin moments should also have a sub-exponential growth to guarantee the uniqueness of their analytic continuation thanks to Carlson’s theorem [116].

\(^8\) \( c \approx 0.35 \) is retained for fitting in [113].

### 3.3.2. CFFs in the Mellin–Barnes representation

To simplify the discussion we restrict ourselves to the case of the singlet GPD \( H_s \) and its associated CFF defined in equation (24). Inserting the Mellin–Barnes representation (109) for the GPD \( H \) and permuting the integrals over \( x \) and \( j \), the CFF \( H(\xi, Q^2) \) reads

\[
H(\xi, Q^2) = \frac{1}{2\pi i} \int_{c-i\infty}^{c+i\infty} \frac{1}{\xi^{j+1}} \left[ i + \tan \left( \frac{\pi j}{2} \right) \right] \times \left[ C_j^0 + \ldots \right] H_j(\xi, \mu^2),
\]

where we indicated the hard scale \( Q^2 \) explicitly. Furthermore, the coefficients \( C_j^0 \) are the conformal moments of the hard scattering kernel \( C^*_\alpha \), and the dots refer to NLO terms proportional to \( \alpha_s \). Since we will discuss LO results, we only quote the expression for \( C_0 \) for results at NLO, see [114]:

\[
C_j^0 = \frac{2^{j+1} \Gamma \left( \frac{5}{2} + j \right)}{\Gamma \left( \frac{1}{2} \right) \Gamma(3 + j)}.
\]

Evolution of GPDs can be included as well in this formalism, taking the conformal moment \( (C \otimes E)_j \) of the convolution of hard scattering \( C \) and evolution \( E \) operators.

### 3.3.3. Modeling of GPD conformal moments

In the spirit of the dual model, the conformal moments of GPDs can be viewed as the result of \( t \)-channel exchanges of resonances \( R_j \) with angular momentum \( J \), taking into account an effective \( \gamma^*\gamma R_j \) vertex \( h_j \), a propagator with an effective Regge pole \( \alpha(t) \) and a smooth profile for the DA of \( R_j \):

\[
H_j(\xi, \xi, t) = \sum_{j} H_{j,\nu} \left[ J - \alpha(t) \frac{1}{(1 - t)^{\nu}} \right] d_{0,\nu}(\xi)
\]

where

\[
Wigner’s SO(3) functions [117] are denoted \( d_{0,\nu} \) where \( \nu = 0 \) or \( \nu = \pm 1 \) depending on hadron helicities. They involve Legendre polynomials when \( \nu = 0 \) (electric GPD combination of equation (81)) and derivative of Legendre polynomials when \( |\nu| = 1 \) (magnetic GPD combination of equation (82)).

Such a modeling of conformal moments has been used in [113] to fit to unpolarized DVCS data at small \( x_B \) at LO, NLO (in the MS and CS schemes) and NNLO (in the CS scheme).

### 3.3.4. Modeling of the GPD \( H(\xi, \xi, t) \) within the quark spectator model

With a DD representation and a \( t \)-dependence inspired from a quark spectator model, the following functional form for the GPD \( H \) is used for fitting:

\[
H(\xi, \xi, t) = \frac{nr}{1 + \xi} \left( \frac{2 \xi}{1 + \xi} \right)^{\alpha(t)} \left( \frac{1 - \xi}{1 + \xi} \right)^b \frac{1}{(1 - t)^{\rho - \xi(1 + \xi)}}.
\]

where the parameters \( n, \alpha(t) \) and \( p \) are \textit{a priori} known. In the valence case these parameters are deduced from Regge \( \omega \) and \( \rho \) trajectories and PDF parametrization. In the sea case, the parametrization (116) is requested to reproduce the small \( x_B \) fits in the Mellin–Barnes representation.
LO DRs (see next section) are implemented by means of a \(t\)-dependent subtraction constant parametrized with a normalization constant \(C\) and a mass scale \(M_C\):

\[
D(t) = C \left(1 - \frac{t}{M_C^2}\right)^2. 
\tag{117}
\]

For the valence part, this leaves thus five free parameters to fit data: \(M\) (valence), \(b\) (valence), \(r\) (valence) which, respectively, control the \(t\)-dependence, the large \(x\) behavior and the skewness effect of the valence part of \(H\), and \(C\) and \(M_C\) which, respectively, control the normalization and \(t\)-dependence of the \(D\)-term. One can use an ansatz similar to equation (116) for \(\tilde{H}\) which introduces three additional parameters \(\tilde{M}\), \(\tilde{b}\) and \(\tilde{r}\). We come back to this in section 4.1.4.

3.4. DR approach to DVCS: general formalism

As has been discussed in section 2, the observables entering DVCS are the CFFs, which depend on the GPDs. The CFFs correspond to the real and imaginary parts of the DVCS amplitudes, as given by equations (15)–(22). At a fixed value \(\nu\) one can use an ansatz similar to equation (116) for \(\tilde{H}\) which introduces three additional parameters \(\tilde{M}\), \(\tilde{b}\) and \(\tilde{r}\). We come back to this in section 4.1.4.

To write down DRs, we start by introducing the kinematic (energy) variables:

\[
v = \frac{Q^2}{2m_\pi^2}, \quad v' = \frac{Q^2}{2m_\pi^2},
\]

which allow one to define amplitudes which are either even or odd in \(v\). Denoting the DVCS amplitude depending on \(v\) and \(t\) by \(\tilde{A}(v, t)\), the unpolarized DVCS amplitude is even in \(v\), i.e.

\[
\tilde{A}(v, t) = \tilde{A}(-v, t).
\tag{121}
\]

We can then write down a once-subtracted DR for the amplitude \(\tilde{A}\) (assuming one subtraction is enough to make it convergent) as

\[
\text{Re} \tilde{A}(v, t) = \tilde{A}(0, t) + \frac{v^2}{\pi} \int_0^{\infty} d\nu' \frac{\text{Im} \tilde{A}(v', t)}{\nu'^2 - v'^2 - v^2},
\]

where a subtraction has been made at \(v = 0\), and where \(v = Q^2/2m_N\) corresponds to the elastic threshold. Using equation (120), we can rewrite equation (121) in a DR in the variable \(\xi\) as

\[
\text{Re} \tilde{A}(\xi, t) = \Delta(t) + \frac{2}{\pi} \int_0^1 dx \frac{\text{Im} A(x, t)}{x(\xi^2/x^2 - 1)},
\tag{123}
\]

or equivalently

\[
\text{Re} \tilde{A}(\xi, t) = \Delta(t) - \mathcal{P} \int_0^1 dx H_s(x, x, t) \left[ \frac{1}{x - \xi} + \frac{1}{x + \xi} \right],
\tag{124}
\]

where the subtraction term (at zero energy) is denoted by \(\Delta(t)\). One notes that in contrast to the convolution integral entering the real part of the CFF in equation (119), where the GPD enters for unequal values of its first and second argument, the integrand in the DR (spectral function) corresponds to the GPD where its first and second arguments are equal. Combining equations (118) and (124) allows one to re-express the subtraction term as

\[
\Delta(t) = \mathcal{P} \int_0^1 dx \left[H_s(x, x, t) - H_s(x, x, t)\right] \times \left[\frac{1}{\xi - x} - \frac{1}{\xi + x}\right],
\tag{125}
\]

which is independent of \(\xi\). When formally taking \(\xi = 0\) in equation (125) and using time reversal invariance, \(H(x, -x, t) = H(x, x, t)\), to convert from the singlet GPD \(H_s\) to the GPD \(H\), one arrives at the sum rule

\[
\Delta(t) = -2 \int_{-1}^1 dx \frac{1}{x} \left[H(x, 0, t) - H(x, x, t)\right].
\tag{126}
\]

As pointed out in [124], since both \(H(x, 0, t)/x\) and \(H(x, x, t)/x\) are even functions of \(x\), their singularities cannot be regularized by the principle value prescription, and there
are no indications that the singularities of both functions cancel each other. However, it was shown [124] that the validity of the sum rule of equation (126) can be demonstrated by decomposing the GPD into a DD (HD) part and a D-term (HD) part. The 1/x integrals of the HD parts (plus distributions) do not contribute to the sum rule of equation (126). The D-term parts are proportional to a δ-function in x as

\[
\frac{H_D(x, 0, t)}{x} = \delta(x) \frac{1}{N_f} \int_{-1}^{1} \frac{D(z, t)}{z} \, dz,
\]

\[
\frac{H_D(x, x, t)}{x} = \delta(x) \frac{1}{N_f} \int_{-1}^{1} \frac{D(z, t)}{z(1-z)} \, dz,
\]

where \( D(z, t) \) is the D-term, see equation (43). Using equation (127), one then obtains for the sum rule of equation (126):

\[
\Delta(t) = \frac{2}{N_f} \int_{-1}^{1} \frac{D(z, t)}{1-z} \, dz.
\]

We thus observe that the subtraction term entering the DR for the DVCS amplitude \( A \) is directly proportional to the D-term FF. It can be obtained from the Gegenbauer expansion of the D-term, equation (43), as

\[
\Delta(t) = \frac{2}{N_f} \sum_{n \text{ odd}} d_n(t).
\]

In practice, one can evaluate the dispersion integral in equation (124) by the ordinary integral:

\[
\text{Re} A(x, t) = \Delta(t) - \int_{0}^{1} dx \left\{ H_\nu(x, x, t) - H_\nu(x, \xi, t) \left[ \frac{1}{x - \xi} + \frac{1}{x + \xi} \right] - H_\nu(x, \bar{\xi}, t) \ln \left( \frac{1 - \xi^2}{\xi^2} \right) \right\},
\]

which is easy to implement in a numerically stable way. Note that for the case where \( H(x, x, 0) \sim 1/x \) for \( x \to 0 \), the singularity cancels out of the integral.

For the CFF involving the GPD E, one can write down an analogous sum rule as for \( H \). In this case, the subtraction function is given by \(-\Delta(t)\), as the D-form factor drops out in the sum of \( H + E \).

Analogous to the unpolarized DVCS amplitude, which is even in \( \nu \), one can also write down a DR for the polarized DVCS amplitude, which involves the GPD \( \tilde{H} \), and which is odd in \( \nu \):

\[
\tilde{A}(v, t) = -\tilde{A}(-v, t).
\]

Assuming an unsubtracted DR for the odd amplitude, allows one to write

\[
\text{Re} \tilde{A}(v, t) = \frac{2v}{\pi} \int_{0}^{\infty} dv' \frac{\text{Im} \tilde{A}(v', t)}{v'^2 - v^2}.
\]
The black dotted curves in figure 24 show the inclusion of the $D$-term in the previous calculation. We recall that the $D$-term contributes to both $H$ and $E$ GPDs, so that this calculation also contains an $E$ contribution. The $D$-term has a significant influence on the cross section: it tends to increase the cross section around $\phi = 180^\circ$ and thus improve the agreement with the data but at the same time it reduces the cross section at low and large $\phi$ which is actually not particularly desired. The inclusion of the $D$-term in the calculation has no effect on the beam-polarized cross section (the dashed blue and black dotted curves are superposed in figure 24—bottom panel). In fact, this observable is sensitive only to the imaginary part of the DVCS amplitude (see equation (27)) while the $D$-term, which lives uniquely in the $-\xi \leq x \leq \xi$ region, contributes only to the real part.

Finally, we wanted to highlight the importance of determining precisely the kinematics of the observables, an issue that we brought up in section 2.2. The values of $x_B$ and $Q^2$ provided by [58] are, respectively, 0.36 GeV and 2.3 GeV$^2$. Since this leaves free the third digit, we explore the kinematics corresponding to $x_B = 0.365$ and $Q^2 = 2.35$ GeV$^2$, i.e. a set of extreme values yielding the rounding $x_B = 0.36$ and $Q^2 = 2.3$ GeV$^2$. The dashed blue curve in figure 24 shows the result, for the configuration with only the $H$ GPD contribution with $b_{val} = b_{sea} = 1$ and without the $D$-term (i.e. directly comparable to the solid red curves). The effect is non-negligible, decreasing the unpolarized cross section by up to 15% at the lowest $-t$ values and the beam-polarized cross sections by even a larger amount. Keeping this potential effect in mind, we continue our studies in the following with the published $x_B = 0.36$ and $Q^2 = 2.3$ GeV$^2$ kinematics.

We do not display calculations with the $\bar{E}$ GPD because its only effect on the observables of figure 24 is to increase by 1% or 2% the unpolarized cross section (it cannot contribute to the beam-polarized cross section).

Having discussed these few effects on the $H$ GPD, we are now going to focus on the effect of the $\tilde{H}$ and $E$ GPDs. In the following figures, we will keep $b_{val} = b_{sea} = 1$ for $H$. We will compare the VGG calculations with the JLab Hall A, CLAS and HERMES data. We will show systematically four sets of curves and configurations.

- Only the GPDs $H$ contribution, without the $D$-term. In the following figures, this configuration will be described by the red solid curves.
- Adding, with respect to the previous configuration, the $\tilde{H}$ contribution. This configuration will be described by the dashed red curves.
- Adding, with respect to the previous configuration, the $E$ GPD with its valence and sea contributions (see equation (53)). The values of $(J_3, J_4)$ are taken as (0.3, 0.). The $D$-term contribution to $H$ and $E$ is included. In the figures, this configuration will be described by the black dashed–dotted curves.
- Changing, with respect to the previous configuration, the values of $(J_3, J_4)$ which are taken as (0., 0.3). In the figures, this configuration will be described by the blue dashed curves.

Figure 23. Comparison of the real and imaginary parts of the CFF related to the GPD $E$ for the proton at $t=0$, excluding the $D$-FF subtraction term. The DD parametrization for $J_\alpha = 0.3$, $J_\beta = 0$, and for $b_{val} = b_{sea} = 1$ is shown by the solid red curves. The dual parametrization is shown for $J_\alpha = 0.3$, $J_\beta = 0$, and for different values of $\alpha$, $\beta$ in the model for the forward function $e_3$ : $\alpha = 0$, $\beta = 0$ (dashed–dotted black curves); $\alpha = 1$, $\beta = 0$ (dotted green curves); $\alpha = 2$, $\beta = -0.5$ (dashed blue curves).

The solid red curve in figure 24 shows the result for the BH+DVCS process when only the $H$ GPD, with $b_{val} = b_{sea} = 1$ (equation (42)) and without the $D$-term, is included. The calculation is now rather close to the data but nevertheless it does not perfectly describe the $\phi$ distribution. In the $-t=0.17$ GeV$^2$ bin, it underestimates the low and high $\phi$ data while it gives good agreement around $\phi = 180^\circ$. In the larger $-t$ bins, the situation is opposite: it gives a good agreement with the low and large $\phi$ data while it underestimates the data around $\phi = 180^\circ$. Regarding the beam-polarized cross sections (bottom panel of figure 25), we see that this configuration, with only the $H$ GPD and without the $D$-term, provides a relatively good agreement with the data for the three lowest $t$-bins. This observable is therefore largely dominated by the $H$ GPD, which was expected (see equation (27)). One notes, however, a disagreement between the data and the calculation for the largest $t$-bin. This might be a shortcoming of the VGG model in the $H$ parametrization but this might also be a sign of higher-twist effects turning in as $-t$ increases, as these calculations, we recall, have been performed at the leading-twist order.

The dashed red curve in figure 24 shows the result of the same configuration but for $b_{val} = b_{sea} = 3$. The effect is to decrease the unpolarized cross section by several percent at low $-t$ and the beam-polarized cross section by a couple of percent (in absolute value) for all $-t$ values. The effect of these parameters is therefore not dramatic.
Figure 24. Unpolarized (top row) and beam-polarized (bottom row) cross sections for the \( e^- p \rightarrow e^- p \gamma \) reaction. The solid circles are the data points from JLab/Hall A [58]. The dotted green curve is the result of the BH alone calculation. Four different configurations of the VGG model are displayed. The solid red curves are the VGG calculation with only the \( H \) GPD, without the \( D \)-term, and with \( b_{val} = b_{sea} = 1 \). The dashed red curves are the same but with \( b_{val} = b_{sea} = 3 \). The dotted black curves correspond to this latter calculation with the addition of the \( D \)-term. The blue dashed curve is the VGG calculation with only the \( H \) GPD, without the \( D \)-term, and with \( b_{val} = b_{sea} = 1 \) at the slightly shifted kinematics \( x_B = 0.365 \) and \( Q^2 = 2.35 \text{ GeV}^2 \) (compared with \( x_B = 0.36 \) and \( Q^2 = 2.3 \text{ GeV}^2 \) for the other calculations).

Figure 25 shows the results of these four calculations for the JLab Hall A unpolarized and beam-polarized cross sections. By comparing the dashed red curves (\( H+\tilde{H} \) contribution) with the solid red curves (\( H \)-only calculation), one sees that the \( \tilde{H} \) GPD has very little effect on the unpolarized cross section as the two curves are barely distinguishable in the top panel of figure 24. The effect of \( \tilde{H} \) on the beam-polarized cross section is more visible. It tends to increase by \( \approx 15\% \) the amplitude of the sine-like modulation.

One sees that the VGG \( E \) GPD has almost no influence on the beam polarized cross section as the black dashed–dotted curves and the blue dashed curves are essentially superposed on the dashed red curves in figure 25—bottom panel. The beam-polarized cross section is thus largely dominated by \( H \) with a small extra contribution of \( \tilde{H} \). Concerning the unpolarized cross section, the sizable influence of the GPD \( E \) is essentially through the \( D \)-term as the dashed–dotted black curves and dashed blue curves of figure 25 (top panel) are almost indistinguishable. In other words, there is no sensitivity of the unpolarized cross section to the \( (J_u, J_d) \) contribution of the \( E \) GPD.

In figure 26, we compare the four VGG calculations to the beam-spin asymmetries of the CLAS collaboration. We recall that these asymmetries are the ratio of the beam-polarized to the unpolarized cross sections. Since the VGG calculation with only \( H \) and with or without \( \tilde{H} \) is in general underestimating the unpolarized cross section while describing correctly the beam-polarized cross section at low \(-t\), as we saw in figure 25, it should be expected that the beam-spin asymmetry be overestimated at low \(-t\). This is indeed what we observe in figure 26. Since the addition of \( \tilde{H} \) increases the amplitude of the beam-polarized cross section (figure 25—bottom panel), the corresponding asymmetry is also amplified. At larger \(-t\) (\( > \approx 0.8 \text{ GeV}^2 \)), where the leading-twist handbag formalism is expected to be less valid, the agreement between the data and the calculation is better. However, we saw in figure 25 that, for the largest \(-t\) bin, the VGG calculation was overestimating the beam-polarized cross section so that the better agreement for the beam-spin asymmetry might simply result from the ratio of two overestimated quantities. This clearly shows the limit of comparing calculations with one single asymmetry.
Adding \( E \) in the VGG calculation (black dashed–dotted and blue dashed curves in figure 26) moves the VGG beam-spin asymmetry calculations closer to the data. This could be expected since the addition of \( E \) tends to increase the cross section around \( \phi = 90^\circ \) (see figure 25). However, some discrepancy clearly remains. One can also note that in general, as \( Q^2 \) increases, the agreement between the calculations and the data tends to improve.

We finally compare in figure 27 our four VGG calculations with the lower \( x_B \) HERMES domain. We show in this figure the nine asymmetry \( \phi \) moments which are expected to be non-null in the leading-twist handbag formalism. We added as a tenth observable the \( A_{sin 2\phi} \) moment (bottom right plot in figure 27) which is expected to be small (suppressed by powers of \( Q \)) in this approximation. However, the data show a rather large asymmetry which cannot be described by any leading-twist handbag calculation, whatever the parametrization of the GPDs.

In figure 27, we see that the main trends of the data are reproduced by the VGG model: for instance, as \(-t\) increases, the trend toward increasing negative values of \( A_C \) and \( A_{U,1} \), and \( A_{U,1} \), etc. Except for the amplitudes of \( A_C \) and \( A_{U,1} \) which are overestimated, the VGG calculation provides a good description of the amplitudes of the nine leading-twist observables. We note the particular sensitivity of \( A_C \) and \( A_{U,1} \) to the \( E \) GPD. It actually mostly comes from the \( D \)-term contribution to \( E \), since there is little difference between the dashed–dotted black and dashed blue curves (non-\( D \)-term contribution to \( E \)). However, since \( A_C \) and \( A_{U,1} \) are largely overestimated, it seems that no really reliable conclusion on \( E \) or the \( D \)-term can be extracted at the moment.

The transversally polarized target asymmetries are also expected to be particularly sensitive to \( E \) and one does see some difference between the two \((J_u, J_d)\) configurations. However, we see that these observables are actually largely dominated by \( H \) and that \( E \) comes only as a small variation around \( H \). It is also difficult under those conditions to extract a reliable information on \( E \), as long as \( H \) is not determined at a few percent accuracy.

As expected, \( A_{sin 2\phi} \) is particularly sensitive to the \( \tilde{H} \) contribution, which is necessary in order to explain the magnitude of the data. Finally, the large amplitude of the \( A_{U,1} \) moment is a puzzle.

4.1.2. The GK model versus data. We have ran the GK model in three different configurations.

- Keeping only the GPD \( H \). In the figures, this configuration will be described by the red solid curves.
- Adding with respect to the previous configuration the contribution of the \( \tilde{H} \) GPD. This configuration will be described by the dashed red curves.
- The ‘full model’, i.e. with the contributions of all four GPDs. This configuration will be described by the black dashed–dotted curves.
Figure 26. Beam-spin asymmetries at $\phi = 90^\circ$ as a function of $t$ as measured by the CLAS collaboration [61] with VGG calculations. The convention for the curves is the same as in figure 25.

Figure 28 shows the results of these three calculations for the unpolarized and beam-polarized cross sections as measured by the JLab Hall A data [58].

We observe some features very similar to the VGG calculation. The beam-polarized cross section (bottom panel of figure 28) is well described for the three lowest $-t$ bins and is mostly the result of the $H$ GPD contribution. The inclusion in the calculation of the $\tilde{H}$ GPD increases, like for VGG and in about the same proportions, the amplitude of the polarized cross section. Since the dashed–dotted black curves (‘full model’) are essentially superposed on the dashed red curves, we conclude that $E$ (and $\tilde{E}$) have basically no influence on this observable. In the largest $-t$ bin, the GK calculation tends to overestimate the data although to a lesser extent than VGG.

For the unpolarized cross section (top panel of figure 28), we also see features similar to the VGG model. The calculation with only the $H$ GPD is rather close to the data but is not fully satisfactory. There is clearly some missing strength for $\phi$ around $180^\circ$, where the handbag DVCS contribution is expected to be dominant. Adding the contribution of the other GPDs barely makes a difference. It might even slightly decrease the cross section around $\phi = 180^\circ$, worsening the situation. We recall that the GK model has no $D$-term implemented.

Figure 29 compares the CLAS beam-spin asymmetries with the GK calculations. As could be anticipated, the beam-spin asymmetries are in general overestimated at low $-t$ since the unpolarized cross sections are underestimated (top panel of figure 29) and the beam-polarized cross sections are correctly reproduced (bottom panel of figure 29). Adding the contribution of the GPDs other than $H$ tends to increase the disagreement between the calculation and the data. This can be attributed to the decrease in the unpolarized cross section that we noted in figure 28—top panel. However, like for VGG, we observe that the GK calculation provides a better agreement with the data for the largest $Q^2$ values, in which case, the inclusion of the GPDs other than $H$ does help.

Figure 30 compares the HERMES azimuthal moments with the GK calculation. Like for VGG, the general trend of the data is correctly reproduced. A notable difference is that the amplitudes of the $A_C$ and $A_C^{\cos \phi}$ moments agree much better with the data. This can in part be attributed to the absence of a $D$-term in the GK calculation as we saw that, in the VGG model, it was a major contribution, both to $H$ and $E$. We note again that the $A_{U_{y,I}}$ moments are largely dominated by the contribution of the $H$ GPD and that any reliable extraction of the comparatively small $E$ contribution requires a control of $H$ at the few percent level. Like in VGG, the strong $A_{UL}^{\sin 2\phi}$ moment is a mystery.
Figure 27. Ten azimuthal moments as a function of $-t$ as measured by the HERMES collaboration [64–71] with VGG calculations. The convention for the curves is the same as in figure 25.

4.1.3. The dual model versus data. We have ran the dual model in three different configurations:

- Keeping only the GPDs $H$, without $D$-term, i.e. for $d_1 = 0$. In the figures, this configuration will be described by the red solid curves.
- Adding, with respect to the previous configuration, the $E$ GPD with $d_1 = -4/3$ and $(J_x, J_d) = (0.3, 0.)$. The $(\alpha, \beta)$ values in the model for the function $e^s(x, t)$ (equation (99)) are taken as $(0., 0.)$. In the figures, this configuration will be described by the black dashed-dotted curves.
- Changing, with respect to the previous configuration, the values of $(\alpha, \beta)$ (equation (99)) to $(2., -0.5)$. In the figures, this configuration will be described by the blue dashed curves.

Figure 31 shows the results of these three calculations for the unpolarized and beam-polarized cross sections as measured by the JLab Hall A data [58].

The beam-polarized cross sections (bottom panel of figure 31) are very well described for the three lowest $-t$ bins. This gives support to the description of the $H_{Im}$ CFF in the dual model. One can note that there is a little influence (a few percent) of the $E$ GPD on this observable. Like for the previous models (VGG and GK), the largest $-t$ bin shows a discrepancy between the calculation and the data.

Regarding the unpolarized cross section (figure 31—top), we see that having only the GPD $H$ for the DVCS contribution is not sufficient to describe the data. We note that the GPD $H$ in the dual model brings less strength to the unpolarized cross section than in the VGG and GK models (see figures 25 and 28). Like for the VGG and GK models, adding an $E$ GPD contribution does not improve the situation: it increases the cross section around $\phi = 180^\circ$ but decreases it at low and large $\phi$.

In figure 32, we compare the three dual calculations with the beam-spin asymmetries of the CLAS collaboration. We observe the same features as with the VGG and GK calculations, i.e. the best agreement between the data and the calculation for the couple of bins with $Q^2 > 3 \text{GeV}^2$ bins but otherwise an overestimation (by 20% to 30%) of the data. This is obviously due to the underestimation of the unpolarized cross section. Adding a contribution of the $E$ GPD can bring the calculation a bit closer (or further) to the data according to the parametrization chosen but this is clearly not a main factor.
Figure 28. Unpolarized (top row) and beam-polarized (bottom row) cross sections for the $e^- p \rightarrow e^- p \gamma$ reaction. The solid circles are the data points from JLab/Hall A [58]. Three different configurations of the GK model are displayed. The solid red curves are the GK calculation with only the $H$ GPD. The dashed red curves are the GK calculation with in addition the $\tilde{H}$ GPD. The dashed–dotted black curves are the ‘full’ GK calculation, i.e. with the contribution of all GPDs. The dotted green curve is the result of the BH alone calculation.

In figure 33, we compare the three dual calculations with the azimuthal moments measured by the HERMES collaboration. We observe the same trends as for the VGG and GK models, i.e. a general good description of the trend of the data. However, in the details, the $A_C$ and $A_C^{\cos \phi}$ amplitudes are overestimated. The strong sensitivity of the $A_U^{\cos \phi}$ moment of the $E$ GPD is again noted. Here, the data clearly favor the $\alpha = 2, \beta = -0.5$ configuration for the parametrization of the $e^+(x, t)$ function.

4.1.4. The KM model versus data. The KM model is the model originally developed by Kumericki and Mueller [113] based on the Mellin–Barnes parametrization of GPDs which we discussed in section 3.3. The parameters of the KM model are determined from fitting H1/ZEUS, HERMES (with or without target polarization data) and JLab data (CLAS and with or without Hall A). In the following, we have used three versions of the code.

- KM10: in this version, the four GPDs are considered: $H$ and $\tilde{H}$ are modeled for their valence part by equation (116), the $E$ contribution is only through the $D$-term and $\tilde{E}$ by the pion pole (like in VGG and GK). There are 15 free parameters. They are $b$ (val), $r$ (val) and $M$ (val) for the valence part of $H$ (see equation (116)), three similar ones $\tilde{b}$ (val), $\tilde{r}$ (val) and $\tilde{M}$ (val) for the valence part of $\tilde{H}$, $C$ and $M_C$ for the $D$-term (see equation (117)), two for the pion pole (one for the normalization and one for the $t$-dependence) and five for the small $x$ behavior of $H$: $M$ (sea) which controls the $t$-dependence through a dipole ansatz, $s_2$ (sea), $s_2$ (gluon), $s_4$ (sea) and $s_4$ (gluon) which control the normalization and evolution flow of the sea and gluon contributions. One should, however, note that the five latter parameters are determined by the collider experiments which are then just propagated to the valence region. In other words, the JLab and HERMES data that we discuss in this review only impact the first ten parameters and the KM10 model can effectively be considered as having ten free parameters in this framework. These parameters are determined by fitting the JLab Hall A, CLAS beam-spin asymmetries and HERMES data, excluding the polarized target data. The values of the free parameters and more details in the ingredients of the model are given explicitly in [111]. In the following figures, this version will be described by the black dotted–dashed curves.

- KM10a: compared with the KM10 version, this version sets $\tilde{H}$ to zero and fixes the pion pole, which removes five free parameters and therefore reduces the total number of free parameters to five. These are determined by fitting only the HERMES (without the polarized target data) and CLAS data, i.e. excluding the polarized target data. The values of the free parameters and more details in the ingredients of the model are given explicitly in [111]. In the following figures, this version will be described by the black dotted–dashed curves.
Figure 29. Beam-spin asymmetries at $\phi = 90^\circ$ as a function of $t$ as measured by the CLAS collaboration [61] with GK calculations. The convention for the curves is the same as in figure 28.

We find results similar to those obtained with the three previous models that we discussed, i.e. that the unpolarized cross sections are underestimated while the beam-polarized cross section are approximatively well described. It is only with the addition of a $\tilde{H}$ contribution (KM10 or KMM12 models) that an agreement is obtained for the description of the unpolarized cross section. An issue is that this $\tilde{H}$ contribution is about a factor 3 larger than values given by standard parameterizations, such as in VGG or GK. In the KM model, $\tilde{H}$ could therefore be viewed as an effective GPD contribution, not clearly linked to polarized PDFs. The inclusion of the HERMES polarized target data in the fit (blue dashed curve) does not strongly affect the description of the Hall A data (one essentially notes a change of $\approx 10\%$ for the unpolarized cross section lowest $|t|$ bin).

In figure 35, we compare the results of the KM model versions with the CLAS beam-spin asymmetries. The comparison between the data and the calculations is available only on some limited range, i.e. $Q^2 > 1.5$ GeV$^2$ and $-t << Q^2$, due to the restrictions in the KM code. We note that those are actually very reasonable limits for the application of the GPD formalism in DVCS and should probably be valid for all models, not only KM. All three versions of the KM model describe relatively well the data which are expected...
Figure 30. Ten azimuthal moments as a function of $-t$ as measured by the HERMES collaboration [64–71] with GK calculations. The convention for the curves is the same as in figure 28.

since these data are included in the fit of the parameters for all three configurations. The behavior of the three model versions differ somewhat but the data do not allow one to favor one more than the others.

In figure 36, we finally compare the results of the KM model versions with the HERMES azimuthal moments. The polarized target data are described only by the KMM12 version of the code, which explicitly took those data in the fit. It achieves a relatively good description of the nine leading-twist asymmetry moments. As usual, the $A_{\sin 2\phi}$ is not explained but since it is not related to the DVCS leading-twist formalism, this should not come as a surprise.

In summary, the KM model, in particular the KMM12 version, is, to this day, the only model available on the market which achieves a relatively good description (with an overall normalized $\chi^2$ of $\approx 1.55$ for 95 data points [112]) of all currently available DVCS data in the valence region (and even beyond, at lower $x_B$ values). The challenge lied particularly in the description of the JLab Hall A data, for which the other three models that we discussed failed to give a satisfying description. Although, the price to pay has been to introduce in KM a strong $\tilde{H}$ contribution, which remains to be understood.

4.2. CFF fits

In the previous section, we have compared the JLab Hall A, CLAS and HERMES DVCS data with four models with adjustable parameters: VGG, GK, the dual model and KM. Although many general trends of the data are reproduced by these four models, none can claim to have a perfect global description of all data with fully reliable inputs (in particular, the meaning of the strong $\tilde{H}$ contribution in KM, which achieves the best $\chi^2$ fit all the data sets, remains to be understood). Specifically, the observable which is the most challenging to reproduce is the JLab Hall A unpolarized cross section. The question arises if this deficiency is due to the limitations of the models which imposes some particular functional form for the $(x, \xi, t)$-dependence of the GPDs that might be very constraining or if the data simply do not lend themselves in general to a leading-twist and LO handbag formalism.

To give some element of response to this question, we now present an alternative way to work on the data. Instead of starting from a model whose parameters, in the frame of a particular functional form, are to be fitted to the data, another
Figure 31. Unpolarized (top row) and beam-polarized (bottom row) cross sections for the $e^-p \rightarrow e^-p\gamma$ reaction. The solid circles are the data points from JLab/Hall A [58]. Three different configurations of the dual model are displayed. The solid red curves are the dual parametrization for the vector amplitude ($GPD_H$) only (without $D$-term, i.e. for $d_1 = 0$). The other curves are the dual parametrization including both $GPD_H$ and $E$, for $d_1 = -4/3$, $J_u = 0.3$, $J_d = 0$, and for different values of $\alpha, \beta$ in the model for the function $e_s(x,t)$: $\alpha = 0, \beta = 0$ (dashed–dotted black curves), and $\alpha = 2, \beta = -0.5$ (dashed blue curves). The dotted green curve is the result of the BH alone calculation.

4.2.1. ‘Brute force’ least-square minimization. The method was pioneered in 2008 in [126]. Knowing the well-established BH and DVCS amplitudes (see equation (13) for the latter process), which provides the relation between the observables and the CFFs, the procedure consists in fitting, at each $(x_B, Q^2, -t)$ experimental point, the $\phi$ distribution (or the moment) of all the available observables at this kinematic point, taking the eight CFFs as free parameters. In [126–129], actually only seven CFFs were considered: $H_{Re}$, $E_{Re}$, $\tilde{H}_{Re}$, $\tilde{E}_{Re}$, $H_{Im}$, $E_{Im}$ and $\tilde{H}_{Im}$. In this work, the eighth CFF $\tilde{E}_{Im}$ has been set to 0. The reason is that, as was seen in section 3, it is common to model the $\tilde{E}$ GPD by the pion pole, which contributes only to the real part of the DVCS amplitude. This is essentially the only model assumption in this procedure. Otherwise, the other CFFs are free to vary within a seven-dimensional hypervolume, which is only bounded by conservative limits: ±5 times the values of the VGG CFFs. We recall that some GPDs have to satisfy a certain number of normalization constraints. These are all fulfilled by the VGG CFFs. We recall that some GPDs have to satisfy a certain number of normalization constraints. These are all fulfilled by the VGG CFFs and it should be clear that ±5 times the VGG CFFs make up conservative bounds.

It is clear that fitting only one observable with seven free parameters does not converge. All data can be fitted with high quality but many combinations of the seven CFFs provide an equally good fit and no information can really be extracted on any CFFs. However, it was striking to observe in [126] that fitting simultaneously two observables, namely...
the unpolarized and the beam-polarized cross sections of Hall A, with the seven CFFs as free parameters, resulted in a convergence for two CFFs, i.e. $H_{\text{Re}}$ and $H_{\text{Im}}$. This resulted for the first time in quasi-model-independent constraints on the $H_{\text{Re}}$ and $H_{\text{Im}}$ CFFs for the Hall A kinematics. Figure 37 shows the resulting fits of the Hall A data and figure 38 (left column) the resulting $H_{\text{Re}}$ and $H_{\text{Im}}$ CFFs obtained, displayed as a function of $-t$.

The reason for the convergence of the particular $H_{\text{Re}}$ and $H_{\text{Im}}$ CFFs is, as we saw in section 4.1, that the unpolarized and the beam-polarized cross sections are largely dominated by these two CFFs (respectively). The fitting procedure allows one to determine central values for these two CFFs which are the values which minimize the $\chi^2$ between the theory and the data, and two error bars, which correspond to $\chi^2 + 1$. These error bars are asymmetric which reflects the non-linearity and undersconstrained nature of the problem (we recall that CFFs enter as bilinear combinations in a cross section). The error bars that are obtained reflect actually not the statistical accuracy of the data (which are precise at the few percent level) but the influence of the five other CFFs which are subdominant and do not converge to a particularly well-defined value within the seven-dimensional hypervolume defined previously. The error bars are therefore correlation error bars. If, guided by some theoretical considerations, one can reduce the number of CFFs entering the fit as free parameters (like it is performed for $E_{\text{Im}}$ or like it will be carried out in the next section by keeping only the $H$ GPD) or if one can reduce their range of variation into an hyperspace smaller than $\pm 5$ times the VGG CFFs as has been done here, the error bars can obviously only diminish. In this case, one has clearly to make sure that the assumptions are well founded, otherwise, errors will of course be underestimated. The present approach should be considered as a most conservative estimation of uncertainties and as minimally theory-biased.

Under these conditions, with this fitting algorithm, it was possible to determine

- as we just discussed, the $H_{\text{Im}}$ and $H_{\text{Re}}$ CFFs at $\langle x_B \rangle \approx 0.36$, and for three $t$ values, by fitting simultaneously [126] the JLab Hall A proton DVCS beam-polarized and unpolarized cross sections [58] (see figure 38—left column). For the lowest $-t$ values, the fitting procedure could not identify a central value for $H_{\text{Re}}$ with well-defined error bars and so we display only $H_{\text{Im}}$. One should also note that the fit of the largest $-t$-bin is not perfect ($\chi^2 \approx 3$) as can be seen in figure 37. Therefore, the small error bar obtained for $H_{\text{Re}}$ at $-t = 0.33 \text{GeV}^2$ might be underestimated, the meaning of an error bar on a fitting
Figure 33. Ten azimuthal moments as a function of \(-t\) as measured by the HERMES collaboration [64–71] with the dual model calculations. The convention for the curves is the same as in figure 31.

Parameter for a bad \(\chi^2\) fit being not straightforwardly interpretable.

- the \(H_{IM}\) and \(\bar{H}_{IM}\) CFFs, at \(\langle x_B\rangle \approx 0.35\) and \(\langle x_B\rangle \approx 0.25\), and for three \(t\) values, by fitting simultaneously [128] the JLab CLAS proton DVCS beam-polarized and longitudinally polarized target-spin asymmetries [61, 63] (see figure 38—center column). We recall that \(\Delta\sigma_{UL}\), and consequently \(A_{LU}\), is dominated by the \(H_{IM}\) CFF (equation (27)) and that \(\Delta\sigma_{UL}\), and consequently \(A_{UL}\), is dominated by the \(\bar{H}_{IM}\) CFF (equation (28)).

- the \(H_{IM}\), \(H_{RE}\) and \(\bar{H}_{IM}\) CFFs, at \(\langle x_B\rangle \approx 0.09\), and for four \(t\) values, by fitting simultaneously [127, 129] the series of HERMES beam-charge, beam-polarized, transversely and longitudinally polarized target-spin asymmetry moments [66–68, 70] (see figure 38—right column). In a nutshell, \(A_C\) constrains \(H_{RE}\), \(A_{LU}\) constrains \(H_{IM}\) and \(A_{UL}\) constrains \(\bar{H}_{IM}\). Unfortunately, in spite of the quasi-complete set of observables measured by HERMES, due to insufficient precision in the data, this approach did not allow one to constrain the other CFFs (while in principle, with ‘ideal’ infinitesimal resolution, they should). We recall that we showed in section 4.1 that, for instance, the \(E\) GPD was actually entering only as a relatively small variation with respect to the \(H\) contribution in most observables and that without a precise determination of \(H\), it is not surprising that no significant information on \(E\) can be extracted.

In figure 38, the results of all these fits are compiled and shown as empty squares (along with and model curves and the results of the other fitting strategy that we discuss in the remaining of this subsection). Although error bars are large and the kinematics are not exactly the same between CLAS and the Hall A, it is interesting to note in the case of \(H_{IM}\), that one extracts compatible values from fitting different observables (unpolarized and beam-polarized cross sections for Hall A and beam-spin and longitudinally polarized target asymmetries for CLAS). One can also note that in this method a better precision on the extraction of \(H_{IM}\) is achieved by fitting two asymmetries than two cross sections.

In figure 38, some general features and trends can be distinguished:

- Concerning \(H_{IM}\), it appears that, at fixed \(-t\), it increases as \(x_B\) decreases (i.e. going from JLab to HERMES...
Figure 34. Unpolarized (top row) and beam-polarized (bottom row) cross sections for the $e^-p \rightarrow e^-p\gamma$ reaction. The dotted green curve is the result of the BH alone calculation. The solid circles are the data points from JLab/Hall A [58]. Three different versions of the KM code are displayed. The red solid curves are the results of the five-free parameter model version with the Hall A data and the HERMES polarized target data excluded from the fit (version KM10a). The black dotted–dashed curves are the results of the ten-free parameter model version with the Hall A data included in the fit, but not the HERMES polarized target data (version KM10). The blue dashed curves are the results of the ten-free parameter model version with the HERMES polarized target data included in the fit (version KMM12).

It is actually possible to extract $H_{\text{Im}}$ at the quasi-common value of $-t \approx 0.28 \text{ GeV}^2$ from the JLab Hall A, CLAS and HERMES data (with a slight interpolation in some cases). We then see in figure 39, the $x_B$-dependence of $H_{\text{Im}}$ at $-t \approx 0.28 \text{ GeV}^2$. One observes the rise of this CFF as $x_B$ decreases which is similar to the rise of PDFs (due to sea quarks). We recall that $H_{\text{Im}}$ reduces to a PDF at $\xi = 0$ and $t = 0 \text{ GeV}^2$. In this figure, we also display the prediction from the VGG (solid red curve) and KM (dashed blue curve) models for $H_{\text{Im}}$ CFF at $-t = 0.28 \text{ GeV}^2$.

- Another feature concerning $H_{\text{Im}}$ is that its $t$-slope seems to increase with $x_B$ decreasing. We recall that the $t$-slope of the GPD is related to the transverse spatial densities of quarks in the nucleon via a Fourier transform (see equation (8)). This evolution of the $t$-slope with $x_B$ could then suggest that low-$x$ quarks (the ‘sea’) would extend to the periphery of the nucleon while the high-$x$ (the ‘valence’) would tend to remain at the center of the nucleon. We will come back to this discussion in section 6.

- $H_{\text{Re}}$ has a very different $t$-dependence than $H_{\text{Im}}$ both at JLab and at HERMES energies: while $H_{\text{Im}}$ decreases with $-t$ increasing, $H_{\text{Re}}$ increases (at least up to $-t \approx 0.3 \text{ GeV}^2$) and may even change sign, starting negative at small $-t$ and reaching positive values at larger $-t$. The VGG model (red solid curve) and the results of the other fitting strategy that we will discuss in the next subsection (solid squares) show or tend to show this ‘zero-crossing’ at JLab kinematics. However, the KM model does definitely not. Given the large error bars on the fitted $H_{\text{Re}}$, one cannot at this stage clearly favor or exclude any of the VGG or KM models. These two models have drastically different predictions for this CFF and more precise data on $H_{\text{Re}}$ are eagerly asked for.

Concerning $\tilde{H}_{\text{Im}}$, we note that it is in general smaller than $H_{\text{Im}}$, which can be expected for a polarized quantity compared with an unpolarized one. There is very little $x_B$ dependence. The $t$-dependence of $\tilde{H}_{\text{Im}}$ is also rather flat. The weaker $t$-dependence of $H_{\text{Im}}$ compared with $H_{\text{Re}}$ suggests that the axial charge (to which the $\tilde{H}$ GPD is related) has a narrower distribution in the nucleon than the electromagnetic charge. We remark that the slope of the axial FF (the first $x$-moment of the $\tilde{H}_{\text{Im}}$ GPD) is also well known to be flatter compared with those of the electromagnetic FFs. It is very comforting that by studying two relatively different experimental processes (DVCS and for instance $\pi$ production from which the axial FF is in general extracted), one finds similar features. One can also note that there is very little $x_B$ dependence for $\tilde{H}_{\text{Im}}$.

4.2.2. Mapping and linearization. In [112], a more elegant method has been developed. It consists in establishing a set of relations associating the DVCS observables to the CFFs. This
Figure 35. Beam-spin asymmetries at $\phi = 90^\circ$ as a function of $t$ as measured by the CLAS collaboration [61] with the KM model calculations. The convention for the curves is the same as in figure 34.

is called ‘mapping’. Given some reasonable approximations (DVCS leading-twist and LO dominance, neglect of some $1/Q^2$ terms in the analytical expressions, etc), these set of relations can be linear. Equations (27) give four examples of such relations. All others can be found in [55]. Then, if a quasi-complete set of DVCS experimental observables can be measured at a given $(x_B, Q^2, -t)$ point, one can build a system of eight linear equations with eight unknowns, i.e. the eight CFFs.

Such a system can be solved rather straightforwardly with standard matrix inversion and covariance error propagation techniques. This approach has been applied in [112] to the HERMES data. We recall that HERMES has this unique characteristic to have measured all beam-target single- and double-spin DVCS observables. The absence of cross section measurement at HERMES means, however, that these spin observables are actually under the form of asymmetries, i.e. a ratio of polarized cross sections to unpolarized cross sections of the form $\Delta \sigma/\sigma$. This has the consequence that the mapping is not fully linear and that some additional (reasonable) approximations have to be made such as the dominance of the BH squared amplitude over the DVCS squared amplitude.

With an appropriate selection (with some partial redefinition) of eight HERMES observables, the others serving as consistency checks, the authors of [112] has been able to solve the system of eight equations and extract the eight CFFs with their uncertainties. In this well-constrained approach, the uncertainties on the CFFs reflect essentially the errors of the experimental data. Figure 38 shows the results of this mapping technique for the three CFFs $H_{\text{Im}}, \tilde{H}_{\text{Im}}$ and $H_{\text{Re}}$ in the HERMES column (black circles). With the precision of the HERMES data, only the $H_{\text{Im}}$ CFFs come out to be clearly different from zero, all other CFFs being compatible with zero within error bars.

In figure 38, the agreement of the mapping technique with the ‘brute force’ least-square minimization technique discussed in the previous section is striking (we note that the least-square minimization technique was also applied in [112], with results well in agreement with those of [128]). We refer the reader to [112] for a discussion on the reason why the error bar on $H_{\text{Im}}$ for the third $t$ value (around $-0.2$ GeV$^2$) is somewhat larger than for the other $t$ values. We also note that HERMES measured the $A_{UL}$ asymmetry at four $t$ values which should therefore allow in principle to extract $\tilde{H}_{\text{Im}}$ at these four $t$ values. However, the extracted $H_{\text{Im}}$ CFF at the lowest $|t|$ point (around $-0.03$ GeV$^2$) turns out to be negative (although compatible with zero within two standard deviations, which does not discard a statistical fluctuation effect). This is both in the present mapping approach and in the least-square minimization approach discussed in the previous section. This
negative value for $\tilde{H}_{\text{Im}}$ at very low $|t|$ is a bit surprising as, unless skewness effects introduce a sign flip, it would imply a negative proton polarized PDF. We do not display it here but it is shown in [112].

4.2.3. Fitting with only $H$. One limitation of the two previous methods that we just presented is that every $(E_e, x_B, Q^2, t)$ kinematic point is taken individually and fitted independently of all others, in particular of its neighbors which have no influence on the fit. On the considered experimental data sets, it turns out that the resulting CFFs display a rather smooth behavior as a function of $-t$ and do not show oscillations. This, in a way, validates the method but nothing prevents the occurrence of oscillations when studying other measurements\(^9\). Still, one could wish to improve the procedure and, staying in an almost model-independent fitting framework, enforce some general properties such as the smoothness or continuity of the CFFs or the implementation of DRs.

\(^9\) Such an oscillating behavior is expected when studying only a single type of measurements, since the previous CFF fitting approach leads to an underconstrained problem.

One attempt to enforce the smoothness of the CFFs was made in 2009 in [125], working with the CLAS beam-spin asymmetries and the JLab Hall A unpolarized and beam-polarized cross sections and assuming the dominance of the GPD $H$. As we saw in section 4.1, this assumption is supported by all models and is expected to work at the 20% to 50% accuracy. In [125], this influence of the other GPDs was probed by fitting the data assuming that $E$, $\tilde{H}$ and $\tilde{E}$ either vanish or take their VGG values, and studying the dispersion of the fit results.

To enforce the smoothness, one imposes a generic functional form to the $H$ GPD. In [125], the singlet combination $H_s$ is parametrized in the dual model framework according to equation (83) (the GPD $E$ is neglected). The $t$-dependence of the $B_{nl}$ coefficients is parametrized as

$$B_{nl}(t, Q^2_0) = \frac{a_{nl}}{1 + b_{nl}(t - t_0)^2},$$

with $t_0 = -0.28$ GeV$^2$. Such a parametrization correlates the $x$ and $t$ dependences. The reference scale is defined by $Q^2_0 = 3$ GeV$^2$ and the $Q^2$ evolution is performed with three active quark flavors and $\Lambda = 373$ MeV.
Figure 37. Result of the fit of the unpolarized (top panel) and beam-polarized (bottom panel) cross sections of the $e^−p → e^-pγ$ reaction by the fitter code of [126] leaving seven CFFs free.

The convergent series equation (87) is truncated at some maximal value of $N_{\text{max}}$. A rough estimate of the uncertainty related to the specific choice of the truncation was obtained by comparing fit results with different values of $N_{\text{max}}$. Choosing $N_{\text{max}} = 2, 3$ or $4$ is sufficient to obtain reasonable fits to the data. Larger values of $N_{\text{max}}$ produce numerical instability with some coefficients left largely undetermined; the overall quality of the fit also becomes poor.

The fits to the Hall A and CLAS data were performed in two ways: either ‘locally’, i.e. fitting each individual kinematic bin $(E_e, x_B, Q^2, t)$ independently of the other, as described in previous section, or ‘globally’, i.e. fitting all $(E_e, x_B, Q^2, t)$ kinematic bins simultaneously. The left and right panels of figure 40 display the results for the $H_{\text{Im}}$ and $H_{\text{Re}}$ CFFs, respectively, for both methods. The results for both kinds of fits are almost always compatible, which is a good consistency check. As expected, the results of the global fits are in general smoother, due to the implementation of the functional form for $H$. This is especially true concerning $H_{\text{Re}}$; in the case of the local fits, this CFF shows large fluctuations between neighboring bins, within some cases values falling even outside the plot range. The results of the local fits suffer from large fluctuations as the fits are not constrained enough in some bins but have the advantage of being almost model-independent.

Both local and global fits give results with comparable accuracy for $H_{\text{Im}}$. While $H_{\text{Im}}$ is rather precisely extracted, $H_{\text{Re}}$ is still poorly known, which was also a conclusion of section 4.2.1.

4.2.4. Neural networks. For the sake of completeness, we finally mention the pioneering work [130] which constitutes the first attempt to extract CFFs from DVCS data with neural networks instead of traditional least-square minimization methods. We will not cover it in detail since it deals only with a subset of existing data, namely HERMES beam-spin and beam-charge asymmetries. This approach yields uncertainties in agreement with those obtained from fits with an a priori given functional form and standard statistical procedures. Being largely model-independent, the uncertainties estimated when extrapolating to the $t \to 0$ region are presumably safer. This body of results will certainly trigger new studies in the future.

5. The future

So far, the data relevant to the DVCS physics in the valence region have come from the JLab Hall A and CLAS experiments using a 6 GeV electron beam and from the HERMES experiment using a 27 GeV electron or positron beam. As described in detail above, Hall A has measured unpolarized and beam-polarized cross sections [58], CLAS beam-spin asymmetries and longitudinally target-spin asymmetries [61] and HERMES the complete set of beam-charge, beam-spin and target-spin asymmetries [64–71]. We have made use of practically all these existing data in the previous sections by comparing them with model calculations or fitting them in order to extract CFFs. These data have allowed us to show the
successes and the limits of the present GPD parametrizations and to develop the first GPD of CFF fitter codes. Putting together all the information, one can consider that the \( H_{\text{Im}} \) CFF is relatively well constrained and known at the \( \approx 15\% \) level and that some first constraints on the \( \tilde{H}_{\text{Im}} \) and \( H_{\text{Re}} \) CFFs start to appear.

Although possibly more information can still be extracted from these data, one clearly wishes to have, ideally, more observables, more precise data and a larger phase-space coverage. In the intermediate \( x_B \) region (\( \approx 0.1 \)), unfortunately not much more can be expected from HERMES since the experiment has shut down a few years ago. However, the COMPASS experiment with a 160 GeV muon beam is scheduled to have around 2016 a dedicated DVCS program with a specific recoil detector to ensure the exclusivity of the process [131]. It should then be able to explore the \( x_B \) range between 0.01 and 0.1, thus with some partial overlap with HERMES.

On a shorter time scale, a lot of new data are expected to come from JLab. The 6 GeV era has just finished in summer 2012 and many data are currently under analysis.

- In Hall A, the experiment E07-007 [132] has carried out DVCS measurements at two beam energies (6 and 4 GeV) for which it is planned to extract the unpolarized and beam-polarized cross sections. At fixed \( x_B \) and \( Q^2 \), the beam energy dependence, analog to a Rosenbluth separation,

\[ \text{Figure 38. The } H_{\text{Im}}, H_{\text{Re}} \text{ and } \tilde{H}_{\text{Im}} \text{ CFFs as a function of } -t \text{ for three different } x_B \text{ values. The empty squares show the results of the 7 CFFs free parameters fit of [126–129]. The solid circles in the HERMES column show the result of the linear mapping fit discussed in section 4.2.2 [112]. The solid squares in the JLab Hall A column show the result of the } H\text{-only CFF fit discussed in section 4.2.3 [125]. The solid red curves show the result of the VGG model (with } b_{\text{val}} = b_{\text{sea}} = 1 \text{ and without any } D\text{-term for } H \text{). The solid dashed blue curves show the results of the model-based fit of [113].} \]

- In Hall A, the experiment E07-007 [132] has carried out DVCS measurements at two beam energies (6 and 4 GeV) for which it is planned to extract the unpolarized and beam-polarized cross sections. At fixed \( x_B \) and \( Q^2 \), the beam energy dependence, analog to a Rosenbluth separation,

\[ \text{Figure 39. The } H_{\text{Im}} \text{ CFF at } -t \approx 0.28 \text{ GeV}^2 \text{ as a function of } x_B. \] The empty squares show the result of the seven CFFs free parameters fit from [126–129]. The point at \( x_B \approx 0.09 \) is from the fit of the HERMES data, the ones at \( x_B \approx 0.25 \) and 0.35 from the fit of the CLAS data and the one at \( x_B \approx 0.36 \) from the fit of the JLab Hall A data. The solid red curves show the result of the VGG model (with \( b_{\text{val}} = b_{\text{sea}} = 1 \) and without any \( D\)-term for \( H \)). The solid dashed blue curves show the results of the model-based fit of [113]. The dashed–dotted green curve shows the result of \( H_{\text{Im}} \) at \( -t = 0 \text{ GeV}^2 \), i.e. the PDF (MRST02).}
will allow one to separate the pure DVCS contribution from the BH-DVCS interference contribution. Strong constraints on the real CFFs, in particular $H_{Re}$, are expected from this measurement.

- With CLAS, the experiment E06-003 [133] has carried out DVCS measurements at $\approx 6$ GeV with a polarized beam. A first set of beam-spin asymmetries released from this experiment has already been published [61] and discussed in the previous sections but another set with double statistics is currently under analysis. Most importantly, unpolarized and beam-polarized cross sections are in the process of being extracted. A glimpse on these cross sections is available in [60]. These cross sections are expected to be less precise than the Hall A ones due to the systematic uncertainties inherent to a large acceptance detector such as CLAS. However, they will cover a much larger phase space. Strong constraints on the $H_{Re}$ and $H_{Im}$ are expected from this measurement.

- With CLAS, the experiment E05-114 [134] has carried out DVCS measurements at $\approx 6$ GeV with a longitudinally polarized proton (and neutron) target and a polarized beam. Improved $A_{UL}$ (w.r.t. [63]) and, for the first time, $A_{LL}$ measurements are thus expected soon. This will allow one to further constrain the $H_{Im}$ CFF in particular.

- With CLAS, the proposal C12-12-010 [137] aims at measuring the DVCS reaction with a transversely polarized target. This should bring strong constraints on the $E_{GPD}$ (see equation (30)).

- In addition to the DVCS experiments off the proton, the experiment E12-12-003 [138] aims at measuring DVCS on the neutron. Except for the pioneering measurement of the Hall A [59], DVCS on the neutron has never been measured. It is obviously an indispensable process to measure in order to perform a flavor separation of the GPDs.

JLab is currently in an upgrade phase and plans to deliver a 12 GeV beam around 2015. A DVCS program in Hall A and in CLAS has already been approved.

- In Hall A, the experiment E12-06-114 [135] will measure the unpolarized and beam-polarized cross sections in a new kinematical regime (smaller $x_B$ and larger $Q^2$).

- With CLAS, the experiment E12-06-119 [136] will use a polarized beam and a longitudinally polarized target to measure unpolarized cross sections and beam-spin, target-spin and double-spin asymmetries.

- With CLAS, the proposal C12-12-010 [137] aims at measuring the DVCS reaction with a transversely polarized target. This should bring strong constraints on the $E_{GPD}$ (see equation (30)).

- In addition to the DVCS experiments off the proton, the experiment E12-12-003 [138] aims at measuring DVCS on the neutron. Except for the pioneering measurement of the Hall A [59], DVCS on the neutron has never been measured. It is obviously an indispensable process to measure in order to perform a flavor separation of the GPDs.

Figure 41 compares the $(x_B, Q^2)$ domains that are explored or will be explored by JLab 12 GeV, HERMES, COMPASS and

Figure 40. $Q^2$-behavior ($1 < Q^2 < 4 \text{ GeV}^2$) of the extracted values of $H_{Im}$ and $H_{Re}$ of local fits (left) and global fit (right) on Hall B kinematics : $0.09 < -t < 0.2 \text{ GeV}^2$ (a), $0.2 < -t < 0.4 \text{ GeV}^2$ (b), $0.4 < -t < 0.6 \text{ GeV}^2$ (c), and $0.6 < -t < 1.0 \text{ GeV}^2$ (d). The error bars include both statistics and systematics. $H_{Im}$ ranges between 0 and 10, $H_{Re}$ between $-7.5$ and $+7.5$. Note the change of notational conventions with respect to equations (15) and (19) to match the results published in [125]. The black full circles correspond to $x_B = 0.125$, red squares to $x_B = 0.175$, green up triangles to $x_B = 0.250$, blue down triangles to $x_B = 0.360$ and magenta open circles to $x_B = 0.491$. 

...
H1/ZEUS regarding the DVCS and DVMP processes. This illustrates the complementarity of all these facilities, the near future belonging to the JLab 12 GeV and COMPASS facilities. In the following two sections, we show some examples of what is expected to be achieved with the JLab 12 GeV facility and in the third one a comparison of the predictions for the various models presented and discussed in sections 3 and 4 for COMPASS kinematics.

5.1. Hall A

While the CLAS12 detector will explore a wide phase-space region for the DVCS process, the DVCS program in Hall A will be to focus on some specific kinematics and make precision measurements. In terms of systematic uncertainties, we recall that the typical momentum resolution of the Hall A arm spectrometers is of the order of $10^{-4}$ (compared with $10^{-2}$ for CLAS12) and that, in terms of statistics, the luminosity that can be reached in Hall A is of the order of $10^{35}$ cm$^{-2}$ s$^{-1}$ (compared with $10^{33}$ cm$^{-2}$ s$^{-1}$ for CLAS12). In particular, before thinking of extracting GPDs or CFFs out of DVCS data, it is of the utmost importance to ensure that the `handbag’ formalism is applicable, in particular at the relatively low $Q^2$ values that can be reached at JLab. One of the signatures to be looked for is the scaling behavior of the CFFs, i.e. the property that they do not depend on $Q^2$ at leading twist.

The preliminary tests of scaling carried out at 6 GeV by the Hall A collaboration are encouraging (see figure 13) but are limited in the $Q^2$ range (between 1.4 and 2.4 GeV$^2$). Figure 42 shows the gain in the $Q^2$ range that can be obtained with the JLab 12 GeV beam energy increase. The errors on this figure were estimated for 90 days of beam time and the running conditions of the JLab Hall A proposal [135].

5.2. CLAS12

CLAS12 is expected to measure all the DVCS observables accessible with a polarized beam, a longitudinally and a transversely polarized target. Except for beam-charge observables, this makes up for a complete program. In this section, we show what can be achieved in terms of the extraction of CFFs using the technique presented in section 4.2.1. This study has been carried out in collaboration with Avakian.

For each $(x_B, Q^2, t)$ bin, the $\phi$ distributions of the various independent DVCS spin observables which are measurable with a longitudinally polarized beam and a longitudinally or/and transversely polarized proton target have been generated: $A_{UL}$, $A_{UL}$, $A_{LL}$, $A_{UL}$, $A_{UL}$, $A_{LL}$ (in addition to the unpolarized cross section). The VGG values [82] for the four GPDs $H$, $E$, $\tilde{H}$ and $\tilde{E}$ have been used to generate these distributions. Then, ep $\rightarrow$ ep$\gamma$ events generated according to the BH+DVCS cross sections have been fed into a fast Monte-Carlo simulating the CLAS12 acceptance and efficiency. Assuming 80 days of beam time for the unpolarized target run at a luminosity of $10^{35}$ mc$^{-2}$ s$^{-1}$ (from which the unpolarized cross section and $A_{UL}$ are planned to be extracted), 100 days of beam time for the longitudinally polarized target run at a luminosity of $2 \times 10^{35}$ mc$^{-2}$ s$^{-1}$ (from which $A_{UL}$ and $A_{LL}$ are planned to be extracted), 100 days of beam time for the transversely polarized target run at a luminosity of $5 \times 10^{33}$ mc$^{-2}$ s$^{-1}$ (from which $A_{UL}$ and $A_{LL}$ are planned to be extracted) and furthermore assuming 80% target polarization, one can assign (statistical) error bars to the $\phi$ distributions and then fit them. The goal is to extract the seven CFFs: $H_{Re}$, $E_{Re}$, $\tilde{H}_{Re}$, $\tilde{E}_{Re}$, $H_{Im}$, $E_{Im}$, $\tilde{H}_{Im}$. As already mentioned in section 4.2.1, $E_{Im}$ = 0 is set to 0 in this study.

The results for the seven CFFs issued from the simultaneous fitting of the $\phi$ distribution of $A_{UL}$, $A_{UL}$, $A_{UL}$, $A_{UL}$, $A_{UL}$, $A_{UL}$, $A_{UL}$ and of the unpolarized cross section with the procedure of [126–129] are displayed, for all $(x_B, Q^2, t)$ bins in figures 43 to 49. The reconstructed CFFs with error bars should be compared with the generated ones which
Figure 43. Resulting $H_{Re}$ CFF from the simultaneous fit of $A_{UL}, A_{UL}, A_{LL}, A_{UL}, A_{UL}, A_{UL}$ and of the unpolarized cross section, for each $(x_B, Q^2, t)$ bin with the fitting code of [126–129]. The extracted CFFs for which the error bar was larger than 3 were removed (study done in collaboration with Avakian).

Figure 44. Resulting $H_{Im}$ CFF from the simultaneous fit of $A_{UL}, A_{UL}, A_{UL}, A_{UL}, A_{UL}, A_{UL}$ and of the unpolarized cross section, for each $(x_B, Q^2, t)$ bin with the fitting code of [126–129]. The extracted CFFs for which the error bar was larger than 150% were removed (study done in collaboration with Avakian).
Figure 45. Resulting $\tilde{H}_{Re}$ CFF from the simultaneous fit of $A_{UL}, A_{UL}, A_{LL}, A_{UL}, A_{UL}, A_{UL}$ and of the unpolarized cross section, for each ($x_B, Q^2, t$) bin with the fitting code of [126–129]. The extracted CFFs for which the error bar was larger than 0.3 were removed (study done in collaboration with Avakian).

Figure 46. Resulting $\tilde{H}_{Im}$ CFF from the simultaneous fit of $A_{UL}, A_{UL}, A_{LL}, A_{UL}, A_{UL}, A_{UL}$ and of the unpolarized cross section, for each ($x_B, Q^2, t$) bin with the fitting code of [126–129]. The extracted CFFs for which the error bar was larger than 150% were removed (study done in collaboration with Avakian).

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are represented by the solid curve. The panels where there are blue solid curves and not CFF reconstructed means that the particular fitting code that we have used was not able to reconstruct the CFFs reliably. In figures 43 to 49, we have indeed not plotted the extracted CFFs for which the error bar was very large (see the captions of the figures was the detailed criteria). This does not mean that there are no data reconstructed in those bins. Other fitting algorithms (probably with some model-dependent inputs) can certainly make use of these data and extract some constraints on the CFFs and GPDs.

The dashed curves on the imaginary parts of the CFF plots show the corresponding values of the GPDs for zero-skewness argument, i.e. \( H(x, 0, t) \) and \( E(x, 0, t) \), according to VGG. These latter are the quantities which have a simple probability interpretation. One therefore sees from these (model-dependent) curves, showing the difference between \( H(x, x, t) \) and \( H(x, 0, t) \), the effect of the skewness. We will come back to this issue in the section 6.

With such complete experiments, which comprise all DVCS observables except for beam-charge asymmetry, we observe that the seven CFFs can be reconstructed for essentially all \((x_B, Q^2, t)\) bins with quite good precision. In particular, the measurement of the transverse target-spin observables is crucial to reconstruct the CFFs related to the GPD \( E \). The same study without such observables allows the reconstruction of the CFFs related to the GPDs \( H \) and \( H \) only, leaving the CFF \( E_{\text{Re}} \) practically unconstrained.

In sections 2.2 and 4.1 we already insisted on the required accuracy in the determination of the kinematics of observables. The BH cross section varies strongly when the outgoing photon is emitted in the direction of the incoming or scattering electron. An variation of 1% in \( x_B \) can induce a variation of 10% in the cross section. This has some important consequences regarding the CFF fitting method described above. For the CFFs to be accurately reconstructed, it is not necessary to have a complete set of observables on the same \((x_B, Q^2, t)\) bin. If this is not the case, we have to work with neighboring bins (for example with similar, but not equal \( x_B \)); this approximation will generate a systematic uncertainty on the reconstruction of CFFs that can be quite large.

5.3. COMPASS

The COMPASS experiment plans to measure the correlated beam charge-spin observables:

\[
S_{CS,U} = \sigma^{++} + \sigma^{--}, \quad D_{CS,U} = \sigma^{+-} - \sigma^{-+}
\]  

since the muons of the beam originate from pions decays which induces a correlation between the charge and the spin of the muons.

In figure 50, we show the predictions of the four models which we discussed in sections 3 and 4, i.e. the VGG, GK, dual parametrization and KM models. Calculations have been performed for \( E_{\mu} = 160 \text{ GeV}, x_B = 0.05, Q^2 = 2 \text{ GeV}^2 \) and \(-t = -0.2 \text{ GeV}^2\). All four models show relatively similar features. The differences lie in the global normalization of the

Figure 47. Resulting \( E_{\text{Re}} \), CFF from the simultaneous fit of \( A_{UL}, A_{UL}, A_{UL}, A_{UL}, A_{UL}, A_{UL}, A_{UL} \), and of the unpolarized cross section, for each \((x_B, Q^2, t)\) bin with the fitting code of [126–129]. The extracted CFFs for which the error bar was larger than 1.5 were removed (study done in collaboration with Avakian).
Figure 48. Resulting $E_{im}$ CFF from the simultaneous fit of $A_{1u}, A_{ul}, A_{ll}, A_{u}, A_{x}, A_{y}$ and of the unpolarized cross section, for each ($x_{B}, Q^2, t$) bin with the fitting code of [126–129]. The extracted CFFs for which the error bar was larger than 150% were removed (study done in collaboration with Avakian).

Figure 49. Resulting $E_{Re}$ CFF from the simultaneous fit of $A_{1u}, A_{ul}, A_{ll}, A_{u}, A_{x}, A_{y}$ and of the unpolarized cross section, for each ($x_{B}, Q^2, t$) bin with the fitting code of [126–129] (study done in collaboration with Avakian).
S_{CS,U} observable and in the behavior at the lowest and largest $\phi$ values of the $D_{CS,U}$ observable.

6. From CFFs to spatial densities

As described above, the GPDs at $\xi = 0$, e.g. $H(x, 0, t)$, are mapping out the combined probabilities in transverse position and longitudinal momentum of the quarks in the nucleon (see equation (8)). We saw in the previous section that CLAS12 will allow one to essentially extract all CFFs, with more or less precision depending on the CFF and the kinematics, over the range $0.1 \lesssim x_B, Q^2 \lesssim 0.6$ and $t_{\text{min}} \lesssim -t \lesssim 1$ GeV$^2$. In particular, if we focus on the unpolarized GPD $H$, the CFF $H_{\text{im}} = H(\xi, \xi, t) - H(\xi, \xi, t)$ can be extracted quite precisely. In the following, in this pioneering exercise, we will make the approximation of neglecting the antiquark contribution to the $H_{\text{im}}$ CFF, i.e. neglect $H(-\xi, \xi, t)$ w.r.t. $H(\xi, \xi, t)$. At CLAS kinematics, according to the GK and VGG models, $H(-\xi, \xi, t)$ is about 20% of $H_{\text{im}}$, while at HERMES kinematics, it is about 30%. This approximation being clearly set, given the uncertainties on $H(\xi, \xi, t)$, and modulo a (model-dependent) skewness correction of the form $H(\xi, 0, t)/H(\xi, \xi, t)$, one can address several questions.

- With which accuracy can one extract $H(x, b_\perp)$ from the measurement of the diagonal CFF $H(\xi, \xi, t)$?
- How does one perform such an error propagation?
- What is the model dependence of the skewness correction?

Equation (8) can be equivalently expressed, for a circularly symmetric function, through the Hankel transform:

$$H(x, b_\perp) = \int_0^\infty \frac{d\Delta_\perp}{2\pi} \Delta_\perp J_0(b_\perp \Delta_\perp) H(x, 0, -\Delta_\perp^2),$$  \hspace{1cm} (137)

where $\Delta_\perp \equiv |\Delta_\perp|$ and $J_0$ is the Bessel function of order 0.

We present here a simple numerical algorithm which addresses the error propagation in this transform. We illustrate it by focusing on $H_{\text{im}}$ and by taking one particular $(x_B, Q^2)$ bin in figure 44, e.g. $(0.1, 2.5$ GeV$^2$). Figure 51 (left panel) displays the pseudo-data and the associated errors contained in this bin. The procedure consists in smearing ‘vertically’ the seven values of $H(\xi, \xi, t)$ (which correspond to seven $t$ values) according to a Gaussian distribution with a standard deviation equal to the error bar of the point. These 7 ‘new’ points are then fitted by a function, which we take as an exponential $A e^{-B t}$ with two free parameters, the normalization $A$ and the slope $B$. At fixed $x_B$, an exponential ansatz is motivated by most GPD models (for instance VGG, GK, KM, etc which we discussed in section 3). In principle, any other fit function can be used, and can serve as a way to estimate a systematic uncertainty associated with this method. This procedure (smearing + fitting) is repeated several thousand times so that one obtains several thousand exponential functions, shown in figure 51 (middle panel).

Then, each of these several thousand exponentials is transformed through equation (137) so that one obtains a series of Hankel transforms, now as a function of $b_\perp$. This transform can be carried out analytically in the present case but any numerical method is also possible if the function fitting.
Figure 52. Distributions of the Hankel transforms $H(x = 0.053, b_\perp)$, for selected $b_\perp$ values, for the series of fit curves corresponding to the middle panel of figure 51. The mean and standard deviation (respectively the parameters P2 and P3 in the insets of each panel) of the Gaussian fit of these distributions is extracted to obtain the $b_\perp$-dependence of the spatial density.

$H(\xi, \xi, t)$ does not have a simple Hankel transform. The idea is then to look, for various values of $b_\perp$, at the dispersion of all the Hankel transforms. The spread of the Hankel transforms for a few (arbitrarily) selected $b_\perp$ values is shown in figure 52. One sees that the resulting distributions are quasi-Gaussian. One can therefore fit those distributions by a Gaussian function from which one extracts the mean and the standard deviation. The black points in figure 53 show the result of this procedure. The error bars at each $b_\perp$ value result from the propagation of the error bars of $H(\xi, \xi, t)$ displayed in figure 51. We choose to display here only seven $b_\perp$ points but the procedure can be applied to any number of $b_\perp$ points so that one can obtain a continuous distribution as a function of $b_\perp$. The interest of this method is to properly propagate the errors on $H(\xi, \xi, t)$ to its Hankel transform. In particular, it takes into account the correlations between the parameters used to fit $H(\xi, \xi, t)$ (in the present case, the normalization and the slope of the exponential).

However, in order to be able to interpret such a distribution as the transverse spatial density of the quarks at the particular value of $x = \xi = 0.053$ taken here, a correction needs to be made. So far, we have Hankel-transformed $H(\xi, \xi, t)$. However, the spatial density interpretation requires the knowledge of $H(\xi, 0, t)$. The correction to pass from one to the other has to be model-dependent since the $x$ and $\xi$-dependences cannot be measured independently with the DVCS process. A model-dependent $H(\xi, 0, t)/H(\xi, \xi, t)$ skewness correction factor should therefore be applied.

Figure 54 shows such deskewing correction factor for three models that we discussed in section 3. It is seen that below $-t = 1$ GeV$^2$, they do not differ by more than 10%. All three correspond to a correction of the order of 20% at small $t$ with a similar evolution as $-t$ grows up to 1 GeV$^2$. Such a comparison between different models can serve to estimate a systematic uncertainty in the deskewing. We note that the deskewing factor depends on $t$ in a similar way for all three models. As $|t|$ grows, the deskewing factor grows toward one (and can even go over one). This means that it flattens the slope of $H(\xi, 0, t)$ w.r.t. to the measured or extracted $H(\xi, \xi, t)$. All three models also show a similar $x_B$-
dependence between the CLAS and the HERMES kinematics, i.e. the deskewing correction is less important as $x_B$ increases.

We then apply such a deskewing factor to the pseudo-data of our selected $(x_B, Q^2)$ bin. This yields the new set of data shown in figure 51 (right panel). Then, the procedure that we just described is applied to these ‘deskewed’ data (see the red curves in figure 51, right panel). This results in the $b_\perp$ distribution given by the red points in figure 53, which can now be properly interpreted as a spatial density. We stress that we place ourselves in a leading-twist and LO framework in this whole exercise.

We have focused so far on one particular $(x_B, Q^2)$ bin in order to illustrate the method. We now process a series of $x_B$ bins in order to provide an imaging of the nucleon. We select in figure 44 the $H_{\text{Im}}$ row at $Q^2 = 2.5$ GeV$^2$ where there are seven $x_B$ bins (if $Q^2$ corrections are under control or found negligible, different $Q^2$ rows can of course be combined). Figure 55 shows the result where we now see the evolution of the transverse spatial density as a function of $x_B$. In particular, one notes that, as $x_B$ decreases the radius of the proton increases. This is a feature which was implemented in VGG and it serves as a proof of principle that, after the different steps, of computing the DVCS observables from the VGG model, generating the pseudo-data according to these, processing these through a simulation of the CLAS12 detector, fitting these pseudo-data in order to extract the $H_{\text{Im}}$ CFF and the final Hankel-transform procedure that we just discussed, one recovers the original features of the VGG model.

We finally apply this procedure to real data. We recall that $H_{\text{Im}}$ has been extracted out of the CLAS and HERMES data (see figure 38). Figures 56 and 57 show the results of the procedure. Of course, the $H_{\text{Im}}$ data have much larger error bars than the CLAS12 simulations and the exponential fits (red curves on the left panels of figures 56 and 57) have a lot of dispersion. The distribution of the Hankel transforms at fixed $b_\perp$ is then not as clean as in figure 52. In particular, the fits in figures 56 and 57 sometimes are horizontal straight lines (which correspond to exponentials with a zero slope) and this produces double peaks in the $b_\perp$ slices. Nevertheless, a dominant Gaussian-like peak always remains distinguishable from which a centroid and a standard deviation can be extracted, yielding the spatial charge densities of figure 56 (right panel), corresponding to $x_B = 0.25$, and figure 57 (right panel), corresponding to $x_B = 0.09$. Given the size of the experimental errors on the $H_{\text{Im}}$ data, we choose to ignore in this exercise the uncertainty associated with the deskewing. We also neglect at this stage the uncertainty associated with the fact that the different DVCS-BH asymmetries that are used to extract $H_{\text{Im}}$ are given at similar, but not strictly equal, kinematic $(x_B, t, Q^2)$ points. This being said, in spite of the large error bars and some uncertainties in the procedure that we ignored, we can distinguish the same features than we observed with the simulations, namely, an increase in the size of the proton with $x_B$ decreasing. The overall increase in the normalization of the density is due to the rise in the unpolarized parton distribution function as $x_B$ decreases. As a further illustration, we fitted the spatial charge densities of the right panels of figures 56 and 57 by a Gaussian function which we plot as contour plots in figure 58.

7. Conclusions and outlook

In this work, we have briefly reviewed the field of the generalized parton distributions and deeply virtual Compton...
scattering in the valence region, with emphasis on the information which can be extracted from present and forthcoming data. After briefly recalling the theoretical formalism and the properties and interests of the GPDs, we have reviewed the few existing data from the HERMES and JLab facilities. Then, we presented four widely used GPD parametrizations based on different approaches (double distributions, dual parametrization, and Mellin–Barnes representation), followed by a more general discussion of DVCS observables within a dispersion relation framework. In this work, our study was based on a leading-twist and leading-order QCD assumption. We compared the results of these different approaches with the existing data. We found that, although many features and trends of the experimental observables were correctly accounted for, no model satisfactorily describes simultaneously the full set of current HERMES and JLab data. We also presented an alternative approach of fitting the Compton form factors, which are functions of GPDs, in more or less model-independent ways. Given the few existing data, the information that can be extracted in this way is limited but nevertheless brings additional clues on the behavior of GPDs.

Putting all the model-dependent and model-independent pieces of the puzzle together, one can conclude so far that the CFF $H_{\text{Im}}$ can be considered as known in the valence region at the $\approx 15\%$ level and, to a lesser extent, the CFF $\tilde{H}_{\text{Im}}$. We have further shown that an imaging of the nucleon starts to appear from these analyses. One feature which emerges from
this analysis is that the increase in the $t$-slope of $H_{Im}$ with decreasing $x_B$ is reflecting the increasing transverse size of the nucleon as one probes partons with smaller and smaller momentum fractions (in the so-called light-front frame). This yields an image of the nucleon with a core of valence quarks surrounded by a cloud of quark-antiquarks. Another feature resulting from the observation that the CFF $\tilde{H}_{Im}$ has a flatter $t$-dependence than the $H_{Im}$ and barely varies with $x_B$ is that the axial charge of the nucleon tends to stay concentrated in the core of the nucleon.

Furthermore, we explored the future of the field, in particular by summarizing all the simulation work that has been carried out for the JLab 12 GeV facility. With the upgrade in energy and luminosity (for CLAS) of JLab, an unprecedented set of data in precision and phase-space coverage is anticipated. We showed that essentially all leading-twist CFFs will be extracted, some with better precision than others. In the final chapter of this review, we finally proposed a method to transform a CFF measurement into a spatial charge density measurement with proper error propagation. We applied this technique to the JLab 12 GeV pseudo-data as well as to the few existing data from JLab 6 GeV and HERMES.

The future of the field is bright with numerous new data in perspective from JLab 12 GeV and COMPASS. The few existing data, with their limitations in terms of precision and phase-space coverage, allow one to develop the techniques (models, fits, etc) to be employed in the future. Within this work, we showed within the leading-twist and leading-order QCD assumptions, a proof of principle to extract the GPD-related observables from the deeply virtual Compton process. Furthermore, one can anticipate further developments on the theory side in calculating the corrections required by the anticipated precision of the forthcoming data. Such a program will allow one to image the partonic structure of the nucleon. Visualizing will then deepen our understanding.

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