Time-Windowed Contiguous Hotspot Queries

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Abstract

A hotspot of a moving entity is a region in which it spends a significant amount of time. Given the location of a moving object through a certain time interval, i.e., its trajectory, our goal is to find its hotspots. We consider axis-parallel square hotspots of fixed side length, which contain the longest contiguous portion of the trajectory. Gudmundsson, van Kreveld, and Staals (2013) presented an algorithm to find a hotspot of a trajectory in $O(n \log n)$, in which $n$ is the number of vertices of the trajectory. We present an algorithm for answering time-windowed hotspot queries, to find a hotspot in any given time interval. The algorithm has an approximation factor of $1/2$ and answers each query with the time complexity $O(\log^2 n)$. The time complexity of the preprocessing step of the algorithm is $O(n)$. When the query contains the whole trajectory, it implies an $O(n)$ algorithm for finding approximate contiguous hotspots.

Keywords: Trajectory, Hotspot, Geometric algorithms, Time-windowed queries

1 Introduction

The identification of hotspots or stay points is an important preprocessing step for analyzing trajectories (Zheng mentions several interesting applications [1]) and the recent growth of trajectory data sets is increasing the emphasis on efficient trajectory analysis algorithms, hotspot identification algorithms not excepted.

Several heuristics have been proposed for identifying hotspots (e.g., [2] [3] [4]). Gudmundsson et al. paved the way for devising accurate geometric algorithms for identifying hotspots by formalizing the definition of hotspots

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and posing several questions about their identification [5]. One of the problems they introduce is that of finding a hotspot, i.e. a placement of an axis-aligned square of fixed side length, which maximizes the time the entity spends inside it without leaving it. In other words, the square should contain a contiguous sub-trajectory with the maximum duration. They also present an algorithm for this problem with the time complexity $O(n \log n)$. Their algorithm relies on the fact that there exists a hotspot with at least one trajectory vertex on its boundary. It finds the hotspot by testing the squares, on one of whose boundaries lies a vertex of the sub-trajectory.

We study the problem of answering time-windowed hotspot queries. Each query specifies a time interval and the goal is finding a hotspot for the sub-trajectory of this interval (e.g. for finding the stay point of a bird in August or finding the hotspot of a mobile device per day, week, and month). Time-windowed geometric queries have recently attracted much attention. Bannister et al. introduce a framework for answering time-windowed queries and present algorithms for answering such queries for three problems including convex hull, for which they present an algorithm that answers each query in poly-logarithmic time and performs the preprocessing in $O(n \log n)$ time [6]. For the time-windowed closest pair of points problem, Chan and Pratt present a quadtree-based algorithm in the word-RAM model [7]. For decision problems, Bokal et al. present time-windowed algorithms for deciding hereditary properties (i.e., if a sequence has the property, all of its subsequences do as well) [8]. Chan and Prat improve Bokal et al.’s results by reducing time-windowed decision problems to range successor problem and using dynamic data structures [9].

In this paper we present an approximation algorithm for answering time-windowed hotspot queries. It answers each query with the time complexity $O(\log^2 n)$ and with an approximation ratio of $1/2$. The space complexity of the algorithm and the time complexity of its preprocessing step is $O(n)$. This also implies a $1/2$-approximation algorithm for finding contiguous hotspots of whole trajectories in $O(n)$ time. The low complexity of this algorithm, its practicable data structures, and small approximation factor makes this algorithm suitable for large real world trajectory data sets.

The rest of this paper is organized as follows. In Section 2 we describe the notation used in this paper and in Section 3 we present our algorithm. Finally, in Section 4 we conclude this paper and mention possible directions for further studies.
2 Preliminaries and Notation

Let $T$ be a polygonal trajectory, which describes the location of a moving entity through a specific time interval. We use $T(t)$ to denote the entity’s location at time $t$ in trajectory $T$. In polygonal trajectories, the location of the entity is recorded as different points in time; these we call the vertices of the trajectory. These vertices are linearly interpolated to decide the location of the entity between two contiguous vertices. The sub-trajectory that connects any two contiguous vertices of the trajectory are its edges.

For each vertex $v$ of trajectory $T$, let $\text{loc}(v)$ denote its location and $\text{tstamp}(v)$ denote its time-stamp. With slight abuse of notation, we use vertices and time-stamps interchangeably. Thus, $u < v$ for vertices $u$ and $v$ means $\text{tstamp}(u) < \text{tstamp}(v)$. For two vertices or time-stamps $u$ and $v$ of trajectory $T$, $T_{uv}$ denotes the sub-trajectory from $u$ to $v$.

For a square $r$ of fixed side length $s$, the weight of $r$ with respect to trajectory $T$ is the maximum duration of a contiguous sub-trajectory of $T$ contained in $r$. A hotspot of trajectory $T$ is an axis-parallel square of fixed side length $s$ with the maximum weight. Every square discussed in this paper has fixed side length $s$ and is axis-parallel. We may not mention these constraints explicitly hereafter.

3 An Algorithm for Answering Time-Windowed Queries

To improve the readability, we first present the algorithm with the assumption that queries start and end with the time-stamp of a trajectory vertex and prove its approximation factor. We then discuss how to construct the data structures required in the algorithm. Finally, we handle general queries that may start or end at a time different from the time-stamps of all trajectory vertices and show that this preserves the approximation factor.

3.1 The Main Algorithm

For any trajectory vertex $v$, we define $\text{hotend}_-(v)$ to denote the start of a sub-trajectory of $T$ ending at $v$ with the maximum duration that can be contained in a square, $\text{hotsquare}_-(v)$ to denote one such square, and $\text{hotdur}_-(v)$ to denote its duration. Similarly, $\text{hotend}_+(v)$ denotes the end of a sub-trajectory of $T$ starting at $v$ with the maximum duration that can be contained in a square, $\text{hotsquare}_+(v)$ denotes one such square, and
hotdur\(_+\)(v)\) denotes its duration. The implementation of these functions is described in Section 3.2.

We also define hotdur\(_-\)(u, v) as the maximum value of hotdur\(_-\)(w) for all vertices like w in \(T_{uv}\), hotsquare\(_-\)(u, v) as its corresponding square, and hotend\(_-\)(u, v) as its corresponding starting vertex, in which u and v are two trajectory vertices. We define hotdur\(_+\)(u, v), hotsquare\(_+\)(u, v), and hotend\(_+\)(u, v) similarly. The implementation of these functions are also explained in Section 3.2.

We now describe the algorithm for answering query \((u, v)\), to find an approximate hotspot of sub-trajectory \(T_{uv}\). As mentioned before, here we assume that both u and v are trajectory vertices.

1. If \(uv\) is an edge of the trajectory, it is trivial to find its hotspot by considering the largest portion of the edge \(uv\) that can fit in a square.

2. Let \(w\) be a trajectory vertex between \(u\) and \(v\) such that the number of vertices in sub-trajectories \(T_{uw}\) and \(T_{wv}\) differ by at most one. A binary search between the vertices of \(T_{uv}\) can find \(w\). Note that the duration of sub-trajectories \(T_{uw}\) and \(T_{wv}\) may differ greatly.

3. Let \(u'\) be hotend\(_-\)(w, v) and let \(v'\) be hotend\(_+\)(u, w). There are three cases to consider.

   (a) If \(u' < u\) and \(v < v'\), hotsquare\(_-\)(w, v) contains \(T_{uw}\) and hotsquare\(_+\)(u, w) contains \(T_{wv}\); let \(r\) be hotsquare\(_-\)(w, v) if the duration of \(T_{uw}\) is greater than that of \(T_{wv}\) and hotsquare\(_+\)(u, w) otherwise. Given that the duration of \(T_{uw}\) is the sum of the durations of \(T_{uw}\) and \(T_{wv}\), the weight of \(r\) is at least half of the weight of the hotspot of \(T_{uw}\).

   (b) Now suppose \(u < u'\) and \(v' < v\). Any hotspot of \(T_{uw}\) either starts at a vertex of \(T_{uw}\) or ends at a vertex of \(T_{wv}\). Consider the first case: if a hotspot of \(T_{uw}\) starts at a vertex of \(T_{uw}\), its weight should be equal to hotdur\(_+\)(u, w). For the second case, we can similarly argue that the weight of a hotspot that ends at a vertex of \(T_{wv}\) is hotdur\(_-\)(w, v). Therefore, we can return either hotsquare\(_+\)(u, w) or hotsquare\(_-\)(w, v) based on the relative values of hotdur\(_+\)(u, w) and hotdur\(_-\)(w, v).

   (c) The remaining case is when \(u' < u\) and \(v' < v\) (the case when \(u < u'\) and \(v' < v\) is similar and is omitted for brevity). Again, any hotspot of \(T_{uw}\) either starts at a vertex of \(T_{uw}\) or ends at
a vertex of $T_{uw}$. If a hotspot of $T_{uw}$ starts at a vertex of $T_{uw}$, its weight equals hotdur$^+(u, w)$. Otherwise, the hotspot should start and end at two vertices of $T_{uw}$. We perform this algorithm recursively for the interval $(w, v)$ to find the approximate hotspot $r$ for this interval. We can return either hotend$^+(u, w)$ or $r$ based on their weight.

The time complexity and approximation factor of this algorithm is shown in Theorem 3.1.

**Theorem 3.1.** For each query $(u, v)$, the algorithm presented in this section finds a square, whose weight is at least half of the weight of the hotspot of $T_{uw}$, with the time complexity $O(\log^2 n)$.

**Proof.** The correctness of the algorithm is explained in its description. For the approximation ratio, consider the tree formed by the recursive invocations of the algorithm for a query. The only step of the algorithm that returns an approximate result is step 3.a, which appears only once and as a leaf in this tree. Therefore the weight of the square returned by the algorithm is at least half of the weight of a hotspot.

For the time complexity, note that the algorithm is invoked recursively only once in step 3.c and for a query containing half as many points as the original query. In each invocation, hotend and hotdur functions, the time complexity of both of which is $O(1)$, are called a constant number of times. Therefore, the time complexity of answering a query containing $m$ vertices is $T(m) \leq T(m/2) + O(\log m)$ (the $O(\log m)$ term is for finding $w$ in step 2). Solving this recurrence yields $T(n) = O(\log^2 n)$ as required.

### 3.2 Preprocessing

We next show how to implement functions hotend, hotdur, and hotsquare for single vertices. The multi-vertex version of these functions can be implemented using Range Minimum Query (RMQ) data structures. RMQ data structures support finding the minimum (or the maximum) of any contiguous interval of a sequence. Using Cartesian trees, RMQ can be implemented with linear space and $O(1)$ query complexity (for details, consult [10]).

In the following algorithm, we use MinQueue data structure, supporting the following operations: Insert for inserting an item, Remove for removing the oldest item, and Min for finding the item with the minimum value in the queue (MaxQueue data structure is similar with a Max operation instead).
There exists a clever implementation of MinQueue (and MaxQueue) using two stacks, in which the time complexity of all three operations is $O(1)$.

We now describe how to compute hotdur_−(v), hotend_−(v) and hotsquare_−(v) can be computed in parallel but are omitted for brevity. Also note that hotdur_+(v), hotend_+(v), and hotsquare_+(v) can be implemented similarly by negating the time-stamps of the vertices. Suppose $\text{minx}$ and $\text{miny}$ are instances of MinQueue and $\text{maxx}$ and $\text{maxy}$ are instances MaxQueue and are initially empty. The following steps are performed for every vertex of $T$ ordered by their time-stamps.

1. Define $(x, y)$ as $\text{loc}(v)$. Insert $v$ with the value $x$ into $\text{minx}$ and $\text{maxx}$.
   Insert $v$ with the value $y$ into $\text{miny}$ and $\text{maxy}$.

2. Repeatedly remove the oldest items from each of the four queues, until both $\text{Max}(\text{maxx}) - \text{Min}(\text{minx})$ and $\text{Max}(\text{maxy}) - \text{Min}(\text{miny})$ are at most $s$. Defining $u$ as the oldest item in any of the queues, the sub-trajectory $T_{uv}$ is the longest that ends at vertex $v$, starts at a vertex of $T$, and can be contained in a square.

3. Let $u'$ be the vertex before $u$ in $T$ (the last vertex removed from the queues). Based on the condition in the previous step, $T_{u'v}$ cannot be contained in a square but $T_{uv}$ can be. To find hotend_−(v), we need to find $p$, the earliest point on the edge $u'u$ such that $T_{pu}$ can be contained in a square. To do so, consider the four ways of aligning a corner of a square with a corresponding corner of the bounding box of $T_{uv}$, and choose the one that contains the longest portion of $u'u$.

**Theorem 3.2.** The time and space complexity of the preprocessing step of the algorithm of Section 3.1 is $O(n)$.

**Proof.** All steps of the preprocessing perform $O(1)$ computation for each vertex except step 2, which may extract many items from the queues. However, since only $n$ items are inserted into each queue and each item can be removed at most once, $O(n)$ items are removed from the queues during the whole algorithm. Therefore, the time complexity of the algorithm is $O(n)$. Given that Cartesian tree-based RMQ implementation for the multi-vertex version of the functions has linear space and time complexity, the time and space complexity of the preprocessing is thus likewise linear.

### 3.3 Handling General Queries

We now show how to handle queries whose start or end does not coincide with the time-stamp of a trajectory vertex. Consider the query $(x, y)$, in
which the time-stamps $x$ or $y$ (or both) are different from the time-stamps of the vertices of $T$. Let $u$ be the first vertex at or after $x$ and $v$ be the first vertex at or before $y$ (these can be found using binary search in $O(\log n)$).

If the query is totally contained in a trajectory edge, a hotspot can be found trivially by finding a square that contains the longest portion of the edge. Otherwise, suppose square $r$ is a hotspot of $T_{xy}$ and let $x'$ and $y'$ denote the start and end of a sub-trajectory of $T_{xy}$ contained in $r$ with the maximum duration. There are two cases to consider. If $u \leq x'$ and $y' \leq v$, the algorithm discussed in Section 3.1 finds an approximate hotspot with an approximation factor of $1/2$. Otherwise, suppose $x' < u$ (handling the case $v < y'$ is similar and is omitted). Suppose $T_{x'y'}$ contains $u$ (otherwise we can move $r$ towards $u$ without changing its weight, since the $r$ is on a single edge). Clearly, the weight of $r$ equals the sum of the durations of $T_{x'u}$ and $T_{uv}$; the former is at most $\text{hotdur}_-(u)$ and the latter is at most $\text{hotdur}_+(u)$. Therefore, the weight of either $\text{hotsquare}_-(u)$ or $\text{hotsquare}_+(u)$ in respect to $T_{xy}$ is at least half of the weight of $r$.

4 Concluding Remarks

The algorithm presented in this paper is very fast, even for finding an approximate hotspot of the whole trajectory. The following is a corollary of the algorithm presented in Section 3 (obtained by querying the whole trajectory after preprocessing).

**Corollary 4.1.** An approximate contiguous hotspot of a trajectory can be found with the time complexity $O(n)$ and an approximation ratio of $1/2$.

Several related problems seem interesting for further investigation. It may be possible to include the side length of the hotspot $s$ in the query by returning a sequence from the functions introduced in Section 3.1 an algorithm to answer these extended queries would be very interesting. The approximation ratio, although very good, may be improved. This looks very important, especially from a practical point of view, for querying large trajectory data sets. Also, it seem interesting to answer time-windowed queries for non-contiguous hotspots (see [5] for more information).

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