Interrelationship of Isospin and Angular Momentum

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It is noted that the simple interaction in isospin variables \( a(1/4 - t(i) \cdot t(j)) \), in a single \( j \) shell calculation, can also be written with angular momentum variables. For the configuration \( (j^2)J_A \) for even \( J_A \) the isospin is one; for odd \( J_A \) it is zero. Hence the above interaction can also be written as \( a(1 - (-1)^J)/2 \). For the \( I = 0 \) state of an even-even Ti isotope with \( n \) neutrons, the hamiltonian matrix element of this interaction is \( \langle j'J' \mid H \mid jJ \rangle/a = (n + 1)\delta_{J'J} - (n + 1)\delta_{J'J + 1} \). The eigenvalues of this interaction can be found by using the isospin form of the interaction. They are \( (n + 1)a \) for \( T = |N - Z|/2 \) and zero for \( T = |N - Z|/2 + 2 \). One can apply this to some extent to obtain the number of pairs of nucleons with given total angular momentum \( J_A \) in a given Ti isotope.

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I. INTRODUCTION

In the single \( j \) shell there is a simple relationship between the isospin and angular momentum of a state of the \( (j^2) \) configuration. If the total angular momentum, \( J \), is even the isospin \( T \) is one; if \( J \) is odd \( T \) is zero. Hence for example for \( ^{42}\text{Sc} \) the \( f^2_{7/2} \) neutron-proton states with \( J = 0, 2, 4, \) and \( 6 \) will have isospin \( T = 1 \) while the \( J = 1, 3, 5 \) and \( 7 \) states will have isospin \( T = 0 \). Since the later are isosinglet states they cannot occur for a system of two protons \((^2\text{Ti})\), or of two neutrons \((^2\text{Ca})\). For these two nuclei one gets only \( J = 0, 2, 4 \) and \( 6 \) in the low lying spectrum. Of course for higher excitation one can get state of odd angular momentum from other configurations. One can easily see why the \( J = 7 \) state cannot occur for two neutrons. The \( J = 7, M = 7 \) state is \( \psi^2_j(1)\psi^2_j(2) \) \((j = f^2_{7/2})\). This state is obviously symmetric in space and spin and so cannot exist for two identical fermions, but it can exist for a neutron-proton pair.

We can obtain an interesting interrelationship between isospin and angular momentum by noting that a simple interaction can be written either with isospin variables or angular momentum variable. Consider the two particle interaction \( V = a(1/4 - t(1) \cdot t(2)) \). This is zero for \( T = 1 \) states and has a value \( a \) for \( T = 0 \) states. But we can also write this in terms of the total angular momentum of the two nucleons \( J_A \). \( V = a(1 - (-1)^J)/2 \). This vanishes for even \( J_A \) but is equal to \( a \) for odd \( J_A \). As mentioned in the previous paragraph for even \( J_A, T = 1 \) while for odd \( J_A, T = 0 \). The work in this paper is based on this interrelationship between isospin and angular momentum.

II. THE EVEN-EVEN TI ISOTOPES

Consider a Ti isotope with \( 2 f^2_{7/2} \) protons and \( n f^2_{7/2} \) neutrons. We will use the notation \( I \) for the total angular momentum of a given state of a given Ti isotope and \( J_P \) and \( J_N \) for the angular momenta of the protons and neutrons respectively. For \(^{44}\text{Ti}, ^{46}\text{Ti} \) and \(^{48}\text{Ti} \) the values of \( n \) are \( 2, 4 \) and \( 6 \) respectively. Let us get the Hamiltonian matrix and the eigenvalues with the simplified interaction \( a(1/4 - t(i) \cdot t(j)) \). We have

\[
\frac{1}{a} \sum_{i<j} V_{ij} = \frac{1}{4} \frac{A(A - 1)}{2} - \frac{1}{2} \left( \sum_i t(i) \cdot \sum_j t(j) \right) + \frac{1}{2} \sum_i t_i^2 = \frac{A(A - 1)}{8} - \frac{1}{2} T(T + 1) + \frac{3}{8} A
\]

\( a \)

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In the above we have the coefficients of fractional parentage (cfp) which isolates one neutron from the others \((E, T)\) book (p.996) which can further simplify the expression \([3]\) (see also Appendix A) as long as

\[
\langle V \rangle = (A-1)a = (n+1)a
\]

for \(T = \frac{|N-Z|}{2} = \frac{n-2}{2}\)

\[
\langle V \rangle = 0
\]

for \(T = \frac{|N-Z|}{2} + 2 = \frac{n+2}{2}\)

(2)

For the lowest isospin states of \(^{44}\text{Ti}, \(^{46}\text{Ti}\) and \(^{48}\text{Ti}\) the values are \(3a, 5a\) and \(7a\) respectively. The reason this interaction vanishes for states with \(T = \frac{|N-Z|}{2} + 2\) is that these states are double analogs of corresponding states in the calcium isotopes. For these ‘neutron only’ systems the interaction can only take place in \(T = 1\) states, but our interaction, by design, is zero for such states.

We now evaluate the Hamiltonian matrix using angular momentum variables. For a given Ti isotope the basis states are \(|(j^2)_{P(J)}n_jn_j\rangle\) where \(I\) is the total angular momentum, \(J_P\) is the angular momentum of the two protons and \(J_N\) of the \(n\) neutrons. If we specialize to \(I = 0\) state then \(J_P = J_N \equiv J\).

Let us define the two particle interaction matrix element \(E(J) = \langle (jj)J|V|(jj)J\rangle\). The general expression for the matrix element of Hamiltonian is

\[
\langle J'J'\rangle_I|H|[J'PJ'N]_I = \frac{[E(J') + E(JN)]\delta_{J'J}J'\delta{JN}J N}{2} + 2n \sum_{J_B,J_A,J_0} (j^{n-1}J_0j_0)J_B(J_0)J_N(J_0)J_A(Jj)A_0
\]

\[
\times \langle (jj)J'J'|(j_0j_0)J_B(jj)A_0E(J_A)
\]

(3)

In the above we have the coefficients of fractional parentage (cfp) which isolates one neutron from the others \((n-1)\).

The unitary \(j\) symbol is needed to combine this neutron with one of the protons in order to obtain the \(n-p\) interaction.

We now consider the simplified interaction \(a(1 - (-1)^{J_A})/2\) and we limit ourselves to \(I = 0\) states for which \(J_P = J_N = J\) and \(J'_P = J'_N = J'\). This choice of interaction the \(n-n\) and \(p-p\) interactions vanish and so do the first two terms in Eq.(3). We get

\[
\langle J'J'\rangle_{I=0}|H|[J'J]_{I=0} = 2n \sum_{J_0,J_A} (j^{n-1}J_0j_0)J_B(J_0)J_N(J_0)J_A(Jj)A_0
\]

\[
\times \langle (jj)J'J'|(j_0j_0)J_B(jj)A_0E(J_A)
\]

(4)

with \(E(J_A) = a \frac{(1 - (-1)^{J_A})}{2}\).

The unitary \(j\) symbol is related to the Wigner 9\(j\) by

\[
\langle (jj)J(J_0)J|J(J_0)J_A(jj)J_A\rangle_0 = (2J + 1)(2J_A + 1) \left\{ \begin{array}{ccc} j & j & J \\ J_0 & J & J_A \end{array} \right\}
\]

(5)

We can use the properties of 9\(j\) symbol to simplify the expression. We take them from de-Shalit and Talmi \(\text{2}\) and Talmi’s 1993 book \(\text{3}\). We show the formuia in our Appendix A. We find

\[
\langle [J'J']_0|H|[JJ]_0/a = n \delta_{J'J} - \sqrt{2J+1}(2J'+1) \sum_{J_0} (j^{n-1}J_0j_0)j^nJ
\]

\[
\times \langle [J'J']_0|j^{n-1}J_0j_0\rangle \left\{ \begin{array}{ccc} j & j & J' \\ J_0 & j & J \end{array} \right\}
\]

(6)

This would seem to be as far as we could go. However there is a recursion formula involving cfp’s in Talmi’s 1993 book (p.996) which can further simplify the expression \(\text{3}\) (see also Appendix A) as long as \(j\) is less than or equal to \(7/2\). For these \(j\) values there is only one \(J = 0\) state for a system of an even number of identical particles in the single \(j\) shell. We get finally

\[
\langle [J'J']_0|H|[JJ]_0/a = (n+1)\delta_{J'J} - (n+1)(j^nJ_j)|j^{n+1}j\rangle(j^nJ'_j)|j^{n+1}j\rangle
\]

(7)

where \(n\) is the number of neutrons in a given Ti isotope.
Note that the final expression Eq. (7) yields a separable interaction \((j^n Jj|j^{n+1}j)(j^n J'j'|j^{n+1}j)\). For \(n = 2\) we get a simple expression, from Eqs. (4) and (7),

\[
3(j^2 Jj|j^3 j)(j^2 J'j'|j^3 j) = \delta_{JJ'} + 2\sqrt{(2J + 1)(2J' + 1)} \left\{ \begin{array}{ccc}
j & j & J' \\
j & j & J
\end{array} \right\}
\]

(8)

More generally we can get the eigenvalues by comparing Eq. (7) with the expression of Eq. (2) obtained using isospin variables. For \(T = |N - Z|/2\) the expression is \((n + 1)\). But this is carried by the first term in the angular momentum expression Eq. (1). We can thus say that the eigenvalues of the separable interaction \((j^n Jj|j^{n+1}j)(j^n J'j'|j^{n+1}j)\) are zero for states with \(T = |N - Z|/2\) and one for states with \(T = |N - Z|/2 + 2\). Since the states with \(T = |N - Z|/2\) are all degenerate, all linear combinations of states with this isospin are eigenfunctions of the above interaction.

III. APPLICATION - WAVE FUNCTION OF THE TI ISOTOPES

A given wave function of a Ti isotope with total angular momentum \(I\) can be written as

\[
\psi^{\alpha I} = \sum D^{\alpha I}(J_P, J_N v)[(j^2 J_P)(j^n J_N)]_I
\]

(9)

where \(D^{\alpha I}(J_P, J_N v)\) is the probability amplitude that in a state of total angular momentum \(I\) the protons couple to angular momentum \(J_P\) and the neutrons to \(J_N\) with isospin \(v\). The \(D\)'s form a column vector representation of the wave functions. The wave functions of the lowest \(0^+\) states, with isospin \(T = |N - Z|/2\), as well as the unique \(0^+\) states with \(T = |N - Z|/2 + 2\) are given in Table I. We have changed the phases of the wave functions that are given in the technical report [1] so that they are consistent with the convention for coefficients of fractional parentage of de-Shalit and Talmi [2] and Talmi [3]. The \(T = |N - Z|/2 + 2\) wave functions are obtained with any isospin conserving interaction, as they are double analogs of corresponding states in the Calcium isotopes.

Note that for \(^{44}\text{Ti}\) the dominant parts of the \(J = 0\) ground state wave function consist of \(s\) and \(d\) couplings. That is \((J_P, J_N)\) values of \((0, 0)\) and \((2, 2)\) constitute about 95\% of \(D\) wave function. Note that for the \(J = 0\) \(T = 1\) ground state of \(^{46}\text{Ti}\) the \((4, 4 v = 4)\) admixture is larger than that of \((4, 4 v = 2)\). This is due in part to the fact that in \(^{44}\text{Ca}\) the \(J = 4\) seniority \(v = 4\) state is lower in energy by about 0.5 MeV than the \(J = 4 v = 2\) state. This can be understood by noting that the \(v = 4\) state can be formed by two "\(d\) pairs" and this is energetically favorable over the configuration of the \(v = 2\) state which must consist of a single "\(d\) pair".

If we represent the wave function of a given even-even Ti isotope by the column vector with components \(D^{\alpha I}(J_P, J_N v)\), we then get the following eigenvalue equation for \(I = 0\) state, for the Hamiltonian given by Eq. (7),

\[
(n + 1)D(J, J) - (n + 1)(j^n Jj|j^{n+1}j) \sum_{J'}(j^n J'j|j^{n+1}j)D(J', J') = \lambda D(J, J)
\]

(10)

The eigenvalue equation for the separable interaction is

\[
(j^n Jj|j^{n+1}j) \sum_{J'}(j^n J'j|j^{n+1}j)D(J', J') = \lambda' D(J, J)
\]

(11)

The eigenvalues are obtained by looking at the results for the isospin version of the interaction as given in Eq. (2). We find that \(\lambda' = 0\) for \(T = T_{\text{min}} = |N - Z|/2\) and \(\lambda' = 1\) for \(T = T_{\text{min}} + 2 = |N - Z|/2 + 2\). Thus we can write the results for the eigenfunction \(D(J, J)\) in a more compact way.

\[
\sum J D(JJ)(j^n Jj|j^{n+1}j) = 0 \quad \text{for} \quad T = T_{\text{min}}
\]

\[
= 1 \quad \text{for} \quad T = T_{\text{min}} + 2
\]

(12)

with

\[
D(JJ) = (j^n Jj|j^{n+1}j)
\]

(13)

for \(T = T_{\text{min}} + 2\). We can understand Eq. (13) better by noting that the eigenfunctions for \(T_{\text{max}} = T_{\text{min}} + 2\) are given by \(D(JJ) = (j^n J^2 J)|j^{n+2}0\rangle\), i.e., they are two particle coefficients of fractional parentage. However it has been shown by Zamick and Devi [4] in the context of the two to one relations between the spectra of even-even and corresponding even-odd nuclei that the two particle cfp is equal to one particle cfp \((j^n Jj^2 J)|j^{n+2}0\rangle = (j^n Jj|j^{n+1}j)\) and hence the one particle cfp above is equal to \(D(JJ)\), Eq. (13). The first part of Eq. (12) is just the orthogonality relation between states with \(T_{\text{min}}\) and \(T_{\text{min}} + 2\). The second part of Eq. (12) is the normalization condition for the state of \(T_{\text{min}} + 2\).
IV. APPLICATION - NUMBER OF n-p PAIRS IN THE TI ISOTOPES

The work in this section can be regarded as an extension of work done previously by Moya de Guerra et al. We shall also make a comparison with earlier work on pairs by Engel et al in which extensive results were obtained for the number of \( J = 0 \) pairs using an isovector pairing interaction. In the present work and in Ref.\(^a\) we have results for any interaction.

Using the results of previous sections we can easily get an expression for the number of pairs of particles with total angular momentum \( J_A \). This is identified as the coefficient of \( E(J_A) = ⟨(j^2)^2 J_A|V|(j^2)J_A⟩ \) in the expression for the total energy Eq.(4) between the wave function of Eq.(9). The contribution of the number of pairs from the neutron-proton interaction for an \( I = 0 \) state of Ti is

\[
\text{number of pairs} = 2n \sum_{J_0} \sum_{J} D(JJ)(j^{n-1}J_{0j}|j^nJ)\sqrt{(2J+1)(2J_A+1)} \left\{ \begin{array}{ccc} j & j & J \\ J_0 & J_A & J_A \end{array} \right\}^2
\]

We can obtain the number of even pairs by summing the expression in Eq.(14) over \( J_A \) even by inserting a factor \((1 + (-1)^{J_A})/2\); for odd pairs the factor \((1 - (-1)^{J_A})/2\). Using techniques similar to those in going from Eq.(3) to Eq.(7) we find

\[
\begin{align*}
\text{number of even pairs} &= (n - 1) + (n + 1) \left| \sum_{J} D(JJ)(j^nJ)|j^{n+1}J) \right|^2 \\
\text{number of odd pairs} &= (n + 1) - (n + 1) \left| \sum_{J} D(JJ)(j^nJ)|j^{n+1}J) \right|^2
\end{align*}
\]

The above expressions can be greatly simplified by noting the relations given by Eq.(12). Thus we find

\[
\begin{align*}
\text{number of even } n-p \text{ pairs} &= n - 1 \text{ for } T = T_{\min} \\
&= 2n \text{ for } T = T_{\min} + 2 \\
\text{number of odd } n-p \text{ pairs} &= n + 1 \text{ for } T = T_{\min} \\
&= 0 \text{ for } T = T_{\min} + 2
\end{align*}
\]

That there are no odd \( n-p \) pairs with \( T = T_{\min} + 2 \) follows from the fact that the higher isospin state is the double analog of a state in the Ca isotopes, i.e., of a system of identical particles that can only have \( T = 1 \) pairs.

We can now obtain a result for the number of pairs with \( J_A = 0 \) (in which case \( J_0 = j \)). From Eq.(14),

\[
\text{number of } n-p \text{ pairs } (J_A = 0) = \frac{2n}{(2j+1)^2} \left| \sum_{J} D(JJ)(j^{n-1}jj)|j^nJ)\sqrt{(2J+1)} \right|^2
\]

The results are, using Eq.(12) \( E(0) \),

\[
D(00) = \frac{n}{(2j+1)} \sum_{J} D(JJ)(j^{n-1}jj)|j^nJ)\sqrt{(2J+1)}
\]

for \( T = T_{\min} \), i.e., \( |D(00)|^2, |D(00)|^2/2, D(00)|^2/3 \) and \( |D(00)|^2/4 \) for \( ^{44}\text{Ti}, ^{46}\text{Ti}, ^{48}\text{Ti} \) and \( ^{50}\text{Ti} \) respectively. For \( T = T_{\min} + 2 \), we can show that the number of pairs is given by

\[
\text{number of } n-p \text{ pairs } (T = T_{\min} + 2, J_A = 0) = 2n|D(0,0)|^2 = \frac{2n(2j+1-n)}{(2j+1)(n+1)}
\]

Furthermore the values of \( |D(00)|^2 \) are respectively 1/4, 1/10 and 1/28. So the number of \( J_A = 0 \) pairs are finally 1, 4/5 and 3/7.
number of pairs = \( 2n \left\| \sum_{j} D(JJ) \sqrt{(2J+1)(2JA+1)} \binom{j}{j} \binom{j}{j} J \right\|^2 \) \( (21) \)

This follows from Eqs. (14) and (A2). But for \( n = 2 \) (\(^{44}\)Ti) (see Eq. (14)) the Hamiltonian is

\[
\langle [J']_0 | H | [JJ]_0 \rangle / a = 2\delta_{J, J'} - 2\sqrt{(2J+1)(2J'+1)} \binom{j}{j} \binom{j}{j} J \right\rangle D(JJ) = D(JAJA) \]

\( (23) \)

for \( T = 0 \) states. The sum on the left is what appears in the expression for the number of pairs (Eq. (14) or (21)) with the restriction that \( J_A \) must be even. Hence we see that for \(^{44}\)Ti in the single \( j \) shell approximation the number of neutron-proton pairs for any even \( J_A \) is \( 2 \times 2 \left| D(JAJA) \right|^2 = \left| D(JAJA) \right|^2 \). In a concurrent publication \( \frac{3}{5}, \frac{4}{5} \) this result was obtained by diagonalizing a \( 9j \) symbol. Here we show an alternate derivation.

For \(^{44}\)Ti the number of \( n-p \) pairs with even \( J_A \) is the same as the number of \( n-n \) and \( p-p \) pairs with same \( J_A \) – in each case the answer is \( |D(JAJA)|^2 \). In Ref. \( \frac{3}{5} \) our expressions are compared to the more special cases of Engel et al. They obtained the number of \( J_A = 0 \) pairs for an isovector pairing interaction and considered only states with \( T = T_{\text{min}} \). We have an alternate expression for the number of \( J_A = 0 \) pairs in the Ti isotopes which holds for any interaction - namely \( 2|D(00)|^2/n \) for \( T = T_{\text{min}} \) (Eq. (19)) and \( 2n|D(00)|^2 \) for \( T = T_{\text{min}} + 2 \) (Eq. (20)).

V. CLOSING REMARKS

The most obvious use of coefficients of fractional parentage is to derive relationships for systems of identical particles. In this work we show that there are some surprising relationships involving these cfp’s for systems of both neutrons and protons. We did so by considering the Ti isotopes in single \( j \) shell calculations. We exploited the fact that the simplified interaction \( a + bt(1) \cdot t(2) \) could be evaluated in both isospin space and angular momentum space.

We found several interesting relations in the text, e.g., Eqs. (12), (16), (18) and (19). This leads to the fact that for \( T = T_{\text{min}} \) the number of even pairs is \( (n-1) \) and the number of odd pairs is \( (n+1) \), whilst the number of \( J_A = 0 \) pairs is \( 2|D(00)|^2/n \). It is important to get relations for general interactions because, as shown in Refs. \( \frac{6}{5}, \frac{7}{5} \), the simple schematic interactions such as \( J = 0 \) \( T = 1 \) pairing or \( J = 1 \) \( T = 0 \) pairing do not yield good wave functions for systems of combined neutrons and protons. We have also applied our results to counting \( n-p \) pairs of a given angular momentum in the Ti isotopes and showed how the expressions could in some case be greatly simplified. This is of relevance to two nucleon transfer, where the motivation is to test the relative importance of say \( J = 0 \) \( T = 1 \) pairing vs \( J = 1 \) \( T = 0 \) pairing.

But our main conclusion is that there are hidden relationships to be uncovered even for complex systems involving neutrons and protons and we hope this work will stimulate others to look for such relationships.

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APPENDIX A

Useful relations used in this work (from I. Talmi \( \frac{3}{5} \)).

\[
\left\{ \begin{array}{ccc} J_3 & J_2 & J \\ J_4 & J_3 & J' \end{array} \right\} = (-1)^{j_2 + J + j_3 + J'} \sqrt{(2J+1)(2J'+1)} \left\{ \begin{array}{ccc} J_1 & J_2 & J \\ J_3 & J_4 & J' \end{array} \right\} \left\{ \begin{array}{ccc} J & J_1 & J \\ J' & J_2 & J' \end{array} \right\}
\]

\( (A1) \)
\[ \sum_{J_{13}J_{24}} (-1)^{J_1+J_4+J_{24}} (-1)^{J_2+J_3+J_{23}} (2J_{13} + 1)(2J_{24} + 1) \begin{pmatrix} J_1 & J_3 & J_{13} \\ J_2 & J_4 & J_{24} \end{pmatrix} \begin{pmatrix} J_1 & J_2 \\ J_3 & J_{23} \end{pmatrix} \begin{pmatrix} J_1 & J_3 & J_{14} \\ J_2 & J_4 & J_{23} \end{pmatrix} \\
= (-1)^{J_3+J_4+J_{24}} \begin{pmatrix} J_1 & J_3 & J_{14} \\ J_2 & J_4 & J_{23} \end{pmatrix} \begin{pmatrix} J_1 & J_2 \\ J_3 & J_{24} \end{pmatrix} (A2) \]

Cfp recursion formula (from I. Talmi [3]).

\[ n (j^{n-1} J_0 J_1 | j^n J) (j^{n-1} J_1 J_0 | j^n J) = \delta_{\alpha_0,\alpha_1} \delta_{J_1, J_0} + (n - 1) \sqrt{(2J_0 + 1)(2J_1 + 1)} \sum_{J_2} \begin{pmatrix} J_2 & J & J_1 \\ J & J & J_0 \end{pmatrix} \]
\[ \times (-1)^{J_0+J_1} (j^{n-2} J_2 J | j^n J) (j^{n-1} J_0 J_1 | j^n J) \quad (A3) \]

Values of some cfp's from de Shalit and Talmi [2]

\[ (j^{n-1} J_1 | j^n J = 0) = 1 \]
\[ (j^{n-1} J_1 | j^n J v = 2) = \sqrt{\frac{2(2j + 1 - n)}{n(2j - 1)}} \]
\[ (j^n J = 0 | j^{n+1} J) = \sqrt{\frac{(2j + 1 - n)}{(n + 1)(2j + 1)}} \quad (A4) \]
\[ (j^n J v = 2 | j^{n+1} J) = -\sqrt{\frac{2n(2J + 1)}{(n + 1)(2j + 1)(2j - 1)}} \]

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TABLE I: Wave functions of $I = 0$, $T_{\text{min}}$ and $I = 0, T_{\text{min}} + 2$ states of $^{44}$Ti, $^{46}$Ti and $^{48}$Ti, represented by column vectors $D(J, Jv)$.

|       |       | $I = 0$ | $T = 0$ | $I = 0$ | $T = 2$ |
|-------|-------|---------|---------|---------|---------|
| $^{44}$Ti | $J_P$ | $J_N$   |         |         |         |
| 0 0   | 0.7608| 0.5000  |         |         |         |
| 2 2   | 0.6090| -0.3727 |         |         |         |
| 4 4   | 0.2093| -0.5000 |         |         |         |
| 6 6   | 0.0812| -0.6009 |         |         |         |
| $^{46}$Ti | $J_P$ | $J_N$   |         |         |         |
| 0 0   | 0.8224| 0.3162  |         |         |         |
| 2 2   | 0.5420| -0.4082 |         |         |         |
| 4 4   | 0.0861| -0.5477 |         |         |         |
| 6 6   | -0.0127| -0.6583 |         |         |         |
| $^{48}$Ti | $J_P$ | $J_N$   |         |         |         |
| 0 0   | 0.9136| 0.1890  |         |         |         |
| 2 2   | 0.4058| -0.4226 |         |         |         |
| 4 4   | 0.0196| -0.5669 |         |         |         |
| 6 6   | -0.0146| -0.6814 |         |         |         |

* means $v = 4$. 

