CRACK DETECTION IN REINFORCED CONCRETE BEAM STRUCTURES BASED ON THE HIGHEST MODE SHAPES SUBJECTED TO INCREMENTAL LOADS

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ABSTRACT: This paper presents the use of Frequency Response Functions (FRFs) to determine damage index and crack damage in reinforced concrete (RC) beam structures using vibration signals. Three methods consisting of Static Residual Strength Index (SRSI), Intermediate Load Damage Index (ILDI), and Modified Flexure Damage Index (MFDI) were adopted in this research. Impact tests on simply supported RC beams were conducted to measure vibration signals on the beams by recording the curvature mode shapes during the experimental testing. The ICATS software was carried out to capture the Frequency Response Functions (FRFs) data at each load step. Cracks occurred on the beam due to the applied load, reducing its natural frequency, indicating an initial stage of the damage having occurred. The midspan vertical deflection of the beam results in its mode shape changes and curvatures increased. The mode shape curvature square difference was used to determine the extent of damage and location of damage indicating the beam residual strength. A numerical algorithm of the finite difference method was performed using the FRFs data to calculate the different FRFs for undamaged and damaged beam conditions based on the mode shape curvature square (MSCS) method. The damage index and crack detection based on the numerical computation were determined by subtracting the MSCS between undamaged to damaged beams. The resulting accuracy of the damage index used to define the level of damage and damage location was absolutely achieved by comparing the numerical and observed experimental results.

Keywords: Crack detection, Mode shape, Frequency response function, and Vibration testing.

1. INTRODUCTION

Propagated cracks in reinforced concrete have been indicated as one of the initial factors affecting the failure of concrete structures. With regard to maintaining the safety of the structure, cracks should be initially detected [1, 2]. The last few years, there have been a couple of evaluation methods of structural failure that can be categorized into a conventional and dynamic method. The vibration method has been widely used in many countries over the world [1, 3]. A method of structural health monitoring is commonly implemented nowadays utilizing the vibration test to detect damage of structure at an initial stage [4-6].

In a real structure, the incremental load often makes the RC beam experiencing cracks either due to flexural, shear, and/or both stresses. The beam structure is one of the essential components in the building construction, but it usually becomes the primary component that resists the main load on a building or bridge depending on its structure geometry [7]. Cracked beams subjected to the applied load have reduced natural frequency, that indicates the initial damage process. Furthermore, the deflected beam results in its mode shape changes and curvature increases. In general, the decline in the value of natural frequencies in a structure decreases in the value of stiffness in the structure as well [1-2, 4-6]. Whilst, the natural frequency is efficiently measured by high accuracy to detect the damage [8].

Research that detects damage by using natural frequency on beam has been undertaken by many researchers, their difference depends only on the use of different analysis methods. Some methods that have been applied among others are modal curvature [8-10], time-frequency distribution [11, 12], frequency response functions [13-19], and damage index method [20-24].

This paper presents a research program focusing on crack damage detection in RC beams subjected to dynamic and static incremental loads using natural frequency measurements. The adopted methods of analysis were the Static Residual Strength Index (SRSI), the Intermediate Load Damage Index (ILDI) and the Modified Flexure Damage Index (MFDI).

2. THEORETICAL REVIEW

A common formula used for natural angular
frequencies \( (\omega_n) \) of a simply supported beam with uniform cross-section has been widely published and is now available in the literature [23]; the standard notations are described in detail as follows:

\[
\omega_n = \sqrt{\frac{n^2 \pi^2 EI}{L^4}} \sqrt{\frac{m}{M}} \tag{1}
\]

where \( n \) is the order of the mode shape, \( m \) is the mass per unit length, \( L \) is the beam length, \( E \) is Young's modulus, and \( I \) is the moment of inertia. Engineering beam theory provides for curvature:

\[
\frac{1}{EI} = \frac{\kappa}{M} \tag{2}
\]

where \( M \) is the bending moment at the cross-section considered and \( \kappa \) is the curvature of the beam's axis. Combining Eq. (1) and Eq. (2) gives:

\[
\kappa = \frac{n^4 \pi^4 M}{mL^4} \frac{1}{\omega_n^2} \tag{3}
\]

Eq. (3) can be derived for a linear prismatic beam, but it is not exact for a non-linear non-prismatic beam. It presents a relationship between the curvature and the natural frequencies. In general, the curvature can always be defined based on the beam deflection by:

\[
\kappa = \left[ \frac{\partial^2 v}{\partial x^2} \right]^{-1/2} \left[ 1 + \left( \frac{\partial^2 v}{\partial x^2} \right)^2 \right]^{-3/2} \tag{4}
\]

The last term in Eq. (4) can be estimated utilizing a central difference formula as follows:

\[
\kappa = \frac{v_{i+1} - 2v_i + v_{i-1}}{l^2} \tag{5}
\]

where \( l \) is the grid length of the measuring grid (or the element size of the finite element in a numerical solution) and \( v_i \) is the deflection of the beam at the cross-section considered.

In this research, \( P_{\text{max}} \) is a maximum load assigned when the measured displacement increases dramatically while the load nearly does not increase at all, the beam is then assumed totally (100\%) damaged. Based on the intermediate loads between \( P_{cr} \) and \( P_{\text{max}} \), the damage index is well-defined as:

\[
DI = \frac{P - P_{cr}}{P_{\text{max}} - P_{cr}} \tag{6}
\]

Eq. (6) represents the definition of \( DI \) that is extensively recognized but is arbitrary as \( P_{cr} \) relates to the onset of obvious crack when some damage may have already inflicted.

Eq. (3) shows that for the damaged beam, the change in \( \omega_n \) is intimately related to \( \kappa \). This is the basis to define a modified flexure damage index (MFDI) to indicate the extent of the damage:

\[
MFDI = \frac{(1/\omega^2) - (1/\omega_{cr}^2)}{(1/\omega_{\text{max}}^2) - (1/\omega_{cr}^2)} \tag{7}
\]

where \( \omega \), \( \omega_{cr} \), and \( \omega_{\text{max}} \) are the natural frequencies of the damaged beam, natural frequency at the first crack, and the natural frequency at the maximum crack, respectively. Table 1 presents a classification list of damage index and damage level.

| Damage Index (DI) | Damage Level (DL) | Description |
|-------------------|-------------------|-------------|
| 0.00 – 0.25       | 1                 | Low Damage  |
| 0.25 – 0.50       | 2                 | Moderate Damage |
| 0.50 – 1.00       | 3                 | High Damage |

3. EXPERIMENTAL SETUP

Four specimens consisted of simply supported RC beams with a cross-section of 100x150 (mm²), and the beam length of 1200 mm. Each specimen had a different variation of stirrups and was subjected to static and dynamic loads. Two-point static loads were positioned at the middle of the beam span. The natural frequency at each loading stage was recorded using an accelerometer tool. Fig. 1 depicts the RC beams having a variety of shear reinforcements. Selected beam specimens reported in this paper were given the following designation 1B, 2A, 3B and 4B.

Basically, all beam specimens except beam 1B were provided shear reinforcements with a diameter of 6 mm and varying spacing of 150, 100, and 60 mm for beams 2A, 3B, and 4B, respectively. The stirrups were applied to start from the supports to the load installation position.

Fig. 2 depicts an experimental setup for the static testing presenting a simply supported beam that has been discretized in the size of 50x50 mm², i.e. 25 nodes available. The load position was set at the middle of the span using two-point loads. The dial gauge was mounted on the tensile fiber of the specimen right in the middle of the beam span to measure vertical deflection that occurred.

Fig. 3 illustrates all devices used in this research for the dynamic tests. The ICATs software was used to read required frequency data that were initially supplied from the result of laboratory measurement. Whilst Fig. 4 presents an example of the analysis process for the beam 3B specimen that was carried out from the software. Based on the recorded data, the beam damage was successfully detected correlating to the relationship between applied load and vertical deflection of each beam.
Fig. 1 Shear reinforcement detail of RC beam specimens

Fig. 2 Experimental setup
Four selected RC beams having different shear reinforcement ratios are presented in this paper (Fig. 1). Each beam is sequentially tested under static load (Fig. 2) and dynamic load (Fig. 3). A series of tests were carried out to measure the important data needed to determine the flexural and the shear strengths, the crack propagation, the crack detection, and the crack location of the beam, respectively. Each step consisted of increasing the static load, the vertical deflection at the middle of the beam span is recorded. Similar action in the dynamic load has been conducted to measure the frequency response function (FRF). In addition to the dynamic load, the hammer’s vibration is used to hit at predetermined nodes on the upper side of the beam. During the experimental testing, the ICATS software was used to capture the FRF data at each node. Fig. 4 presents the load-displacement response for all beam specimens producing different applied static load vs displacement curves.

The lowest beam strength occurred in beam 1B, where the shear reinforcement was not provided at all, producing the maximum load of 22.8 kN, which was correlated to the maximum vertical deflection of 6.67 mm. The other three specimens consisting of beams 2A, 3B, and 4B resulted in higher strengths achieving the maximum loads of 47.2, 49.8, and 53.4 kN, respectively. Based on the load-displacement response, a displacement ductility factor for each beam specimen was determined by computing the ratio between the maximum displacement and the displacement at first yield. Furthermore, the displacement ductility gradually increased as the shear reinforcement distance reduced, where the maximum vertical deflections achieved were 7.76, 13.1, and 11.7 mm for the following beams 2A, 3B, and 4B, respectively. In line with previous researches [1-3], less shear reinforcement in the RC beam generally produces significant strength reduction, in withstanding shear stresses. In contrast, the beams with higher shear reinforcement ratio give better strength and displacement ductility [4-6] as demonstrated by the beams 3B and 4B.

4. RESULTS AND DISCUSSION

The beam 4B is the best specimen that was able to obtain the highest load compared to the other test specimens. The beam 1B only resisted the strength of about 42.69% of the beam 4B. This phenomenon
indicates that the shear reinforcement greatly affects the performance of the reinforced concrete beam [15]. Meanwhile, the maximum value of deflection occurred in the beam 3B; which had shear reinforcement at spacing of 100 mm. Compared to the beam 1B, the 3B specimen is much better in receiving the applied load and able to resist a larger deflection. In another case, beam 2A trending closely to beam 3B in supporting the higher applied load, however, it has a lower displacement ductility. Overall, the beam 4B produced the highest strength compared to the other two beams (2A and 3B), but the displacement ductility is approximately the same as the 3B beam.

As an example of running the ICATS software for the beam 3B, the highest mode shape was achieved at the 3rd mode which correlated to the maximum applied load of 49.85 kN and the frequency of 485.234 (Hz) where the accelerometer was positioned at node 16. Typical results of this stage of the ICATS process using the data that was recorded from the experimental test of the beam 3B are briefly presented in Fig. 5, where the straight line characteristic of both Real and Imaginary Parts can be seen clearly. It is shown that both these straight-line slopes, \( m_R \), \( m \), and \( m_I \), are simple functions of \( \Omega \). The frequency of each specimen was measured from each specified node in every increment of the static load until the beam failure occurred. In this research, each specimen was captured for its frequency before and after applying the loads progressively. It was noted [19] that the higher frequencies were found at the initial stage of unloaded condition then the frequency decreased significantly after beam failure was achieved. Table 2 presents a frequency measurement at the extreme of applied loads, i.e., unloaded and maximum load at each beam specimen. Giving this condition and referring to Fig. 4, the decrease in flexural strength of reinforced concrete beams seems to be related to the decrease in frequency and along with the level of damage. The smallest level of damage on reinforced concrete beams occurred in beam 4B and this corresponds to the shear reinforcement ratio installed. In contrast, the beam 1B provided the smallest percentage of frequency reduction.

| Specimen (beam) | Unloaded (Hz) | Maximum load (Hz) | Reduction percentage |
|-----------------|--------------|-------------------|---------------------|
| 1B              | 783.106      | 592.748           | 24.31               |
| 2A              | 764.402      | 537.234           | 29.72               |
| 3B              | 709.608      | 485.234           | 31.62               |
| 4B              | 806.499      | 591.576           | 26.65               |

Fig. 5 Line-fit for the 3rd mode shape (the highest) of the beam 3B.

Complete frequency measurement of the beam specimens is depicted in Fig. 6 demonstrating the flexural strength of the beam significantly reduces frequency and propagates crack damage. When the frequency ratio for all specimens is set to 1.0, the applied load ratio then equals to zero. In other words, the variety of frequency ratio in each beam is directly affected by the applied load and provides different values of its frequency.

After reaching the value of frequency and maximum load, the collected data were then analyzed by using the proposed methods of Statics Residual Strength Index (SRSI), Intermediate Loads Damage Index (ILDI) and Modified Flexure Damage Index (MFDI) to define the value of damage index referring to Table 1. Based on the results, the level of damage on each beam specimen is absolutely identified referring to Eraky et al. [20] and Labib et al. [21].

Hence these three proposed methods [23] were adopted to determine the damage index of each beam specimen. Referring to Eq. (6), the ILDI method can be used to compute the damage index based on the static applied loads, i.e. the intermediate loads between \( P_{cr} \) and \( P_{max} \). Similarly, the SRSI method determines the damage index based on residual strength index subjected to static loads, where the stiffness \( k = P/\delta \), is a ratio
between applied load \((P)\) and displacement at midspan \((\delta)\). The stiffness is measured at each load step and the percentage of static residual strength (SRS) can be calculated by subtracting 100 percent with the ratio of \(k\) divided by the summation of \(k\) times 100 percent. The damage index is then calculated by subtracting 100 percent with the value of SRS. The damage index based on the MFDI method refers to Eq. (7) by taking account of the measured natural frequency.

![Fig. 6 Typical frequency and applied load relationship.](image)

Fig. 7 presents a typical frequency and damage index relationship demonstrating the capability of the three proposed methods for all specimens. A negative sign of the damage index resulted from both the ILDI and MFDI methods indicating the beam has not yet started cracking. In contrast, the SRSI does not produce the negative sign of the damage indexes at all. The beam starts to crack after the load is applied, and the cracks will propagate as the load gradually increases. In general, the graphs of the relationship between the frequency and damage index presented in Fig. 7 show the variation of high and low frequencies generated from the SRSI and MFDI methods, while the ILDI method produces the frequency in the middle between the two methods. All categories of damage level at each beam specimen (Table 1) have been demonstrated according to the strength level, and the achievement is greatly influenced by the frequency, ie. the greater the load received the smaller the frequency, as shown in Table 2. It can be concluded that the level of damage of the beam correlates directly with the beam strength in withstanding the load it receives.

Fig. 7 has also illustrated a process of detecting crack where the highest mode shapes of each beam specimen occurred. Furthermore, the hinge mode in each beam does not always occur in a certain node, but it changes depending on its strength condition in resisting the load applied. In addition to the crack damage detection, the first crack and the damage were differently identified at every specimen, and the applied loads at the first crack as well as the final stage of damage were randomly experienced. According to Fadillawaty [24], these phenomena were dependent on the beam strength.

Fig. 8 presents a typical example of a crack pattern due to the static loads imposed gradually on each specimen. The crack pattern on every specimen was identical to the first crack occurred vertically at the middle span. Based on this crack pattern, it could be categorized that the specimen suffered flexural damage at the early stage due to a static load. When the load was increased gradually, the crack propagation spread out vertically and diagonally along the beam length representing as flexural and shear damage. The flexural and shear damage could be detected progressively following the width and length of the beam cracks. As with the load values marked in Fig. 8, the numbers indicate the increase in the load imposed along with the width and length of the crack development. This study presents a novelty of detecting crack damage in reinforced concrete beams and the highest mode shapes subjected to load increases give accurate results. The development of cracks due to load increases in the static testing may be accurately predicted according to the highest mode shapes based on dynamic testing. This research method is potentially extended for future works on a variety of structural elements.

5. CONCLUSIONS

Based on the results and brief discussion, the conclusions can be drawn as follows:

a. The load increases on each specimen correlate directly to the natural frequency degradation along with the increases in beam damage, which is indicated by the increase in damage index.

b. The level of damage of the beam correlates directly with the beam strength in withstanding the load it receives.

c. Detecting crack damage in reinforced concrete beams based on the highest mode shapes subjected to load increases gives accurate results, utilizing any of the three proposed methods.

d. Both static and dynamic testing produces precise results in detecting crack damage in reinforced concrete beams having the same longitudinal rebar and different confinement ratios.

e. Based on the research findings, it is strongly recommended that the advanced research topic in detecting crack damage location in other reinforced concrete structures in resisting the seismic vibrations is essential in future research.
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7. REFERENCES

[1] Zhong S., and Oyadiji S. O. Crack Detection in Simply Supported Beams using Stationary Wavelet Transform of Modal Data. Structural Control and Health Monitoring, Vol 18, 2011, pp. 169-190.

[2] Neild S. A., Williams M. S., and McFadden P. D., Nonlinear Vibration Characteristics of Damaged Concrete Beams. Journal of Structural Engineering. Vol. 129, Issue 2, 2003, pp. 260-268.

[3] Gonzalez J. V., De-la-Colina J., and Gonzalez-Perez C. A., Experiments for Seismic Damage Detection of an RC Frame Using Ambient and Forced Vibration Records. Structural Control and Health Monitoring, 2014.
[4] Yang Y., Liu H., Mosalam K. M., and Huang S., An Improved Direct Stiffness Calculation Method for Damage Detection of Beam Structures. Structural Control and Health Monitoring, 2012.

[5] Waeytens J., Rosic B., Charbonnel P. E., Merliot E., Siegert D., Chapeleau X., Vidal R., Corvec V. L., and Cottincau L. M., Model Updating Techniques for Damage Detection in Concrete Beam using Optical Fiber Strain Measurement Device, Engineering Structures, 2016.

[6] Zhao Y., Noori M., Altabey W. A., Beheshti-Aval S. B., Mode Shape-based Damage Identification for a Reinforced Concrete Beam using Wavelet Coefficient Differences and Multiresolution Analysis, Structural Control Health Monitoring, Vol 25, pp. 1-41, 2018.

[7] Prayuda H., Syandy R. R., Soebandono B., Maulana T. I., and Cahyati M. D., Numerical Study on Beam-Column Connection of Cantilever Precast Concrete Beam with Asymmetric Shape Under Static Load. The 4th International Conference on Rehabilitation and Maintenance in Civil Engineering, MATEC Web of Conference, Vol 195, 2018.

[8] Ciambella J., and Vestroni F., The Use of Modal Curvatures for Damage Localization in Beam-Type Structures. Journal of Sound and Vibration, Vol 340, 2015, pp. 126-137.

[9] Dessi D., and Camerlengo G., Damage Identification Techniques via Modal Curvature Analysis: Overview and Comparison. Mechanical Systems and Signal Processing, 2014.

[10] Sun Z., Nagayama T., Nishio M., and Fujino Y., Investigation on a Curvature-Based Damage Detection Method using Displacement under Moving Vehicle. Structural Control of Health Monitoring, 2018.

[11] Ahmadi H. R., Daneshjoo F., and Khaji N., New Damage Indices and Algorithm Based on Square Time-Frequency Distribution for Damage Detection in Concrete Piers of Railroad Bridges, Structural Control Health Monitoring, 2014.

[12] Gillich G. R., and Praisch Z., Modal Identification and Damage Detection in Beam-like Structures using The Power Spectrum and Time-Frequency Analysis. Signal Processing, 2013.

[13] Owolabi G. M., Swamidas A. S. J., and Seshadri R., Crack Detection in Beams using Changes in Frequencies and Amplitudes of Frequency Response Functions, Journal of Sound and Vibration, Vol 265, 2003, pp. 1-22.

[14] Esfandiari A., Rahai A., Sanaye M., and Nejad F. B., Model Updating of Concrete Beam with Extensive Distributed Damage using Experimental Frequency Response Function. Journal of Bridge Engineering, 2016.

[15] Kim J. T., Ryu Y. S., Cho H. M., and Stubbs N., Damage Identification in Beam-Type Structures: Frequency Based vs. Mode Shape Based Method. Engineering Structures, Vol 25, 2003, pp. 57-67.

[16] He W. Y., and Ren W. X., Structural Damage Detection using a Parked Vehicle Induced Frequency Variation, Engineering Structures, Vol 170, 2018, pp. 34-41.

[17] Ercolani G. D., Felix D. H., and Ortega N. F. Crack Detection in Prestressed Concrete Structures by Measuring Their Natural Frequencies. Journal of Civil Structural Health Monitoring, 2018.

[18] Naito H., and Bolander J. E., Damage Detection Method for RC Members using Local Vibration Testing. Engineering Structures, Vol 178, 2019, pp. 361-374.

[19] Mohan V., Varivallal S., Kesavan K., Arumsundaram B., Ahmed A. K. F., and Ravisankar K., Studies on Damage Detection using Frequency Change Correlation Approach for Health Assessment. 1st International Conference on Structural Integrity, Procedia Engineering, Vol 86, 2014, pp. 503-510.

[20] Eraky A., Anwar A. M., Saad A., and Adbo A., Damage Detection of Flexural Structural Systems using Damage Index Method-Experimental Approach. Alexandria Engineering Journal, 2015.

[21] Labib A., Kennedy A., and Featherston C., Free Vibration Analysis of Beams and Frame with Multiple Cracks for Damage Detection. Journal of Sound and Vibration, 2014.

[22] Gao Y., Du Y., Jiuyi Y., and Zhao W., Research on Cracking of Reinforced Concrete Beam and Its Influence on Natural Frequency by Expanded Distinct Element Method. Journal of Aerospace Engineering, 2016.

[23] Ewins D. J., Modal Testing: Theory, Practice and Application, Second Edition, Research Studies Press LTD, Baldock, Hertfordshire, England, 2000.

[24] Fadillawaty S., Determination of Damage Location in Reinforced Concrete Beams Using Mode Shape Curvature Square (MSCS) Method, Applied Mechanics and Materials Vol. 845, pp 140-147, 2016.

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