Investigation on Multiuser Diversity in Spectrum Sharing Based Cognitive Radio Networks

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Abstract—A new form of multiuser diversity, named multiuser interference diversity, is investigated for opportunistic communications in cognitive radio (CR) networks by exploiting the mutual interference between the CR and the existing primary radio (PR) links. The multiuser diversity gain and ergodic throughput are analyzed for different types of CR networks and compared against those in the conventional networks without the PR link.

Index Terms—Cognitive radio, interference temperature, multiuser diversity, spectrum sharing.

I. INTRODUCTION

Cognitive radio (CR) is a promising technology for efficient spectrum utilization in the future wireless communication systems. The main design objective for a CR network is to maximize its throughput while providing sufficient protection to the existing primary radio (PR) network. There are two basic operation models for CRs: opportunistic spectrum access (OSA), where the CR is allowed to transmit over a frequency band only when all the PR transmissions are off, and spectrum sharing (SS), where the CR can transmit concurrently with PRs, provided that it knows how to control the resultant interference powers at PRs below a tolerable threshold [1]. On the other hand, “multiuser diversity (MD)” [2], [3], as a fundamental property of wireless networks, has been widely applied for opportunistic communications in wireless systems. The conventional form of MD is usually exploited in a wireless system with multiple independent-fading communication links by selecting one link with the best instantaneous channel condition to transmit at one time, also known as dynamic time-division multiple-access (D-TDMA).

In this letter, we consider SS-based fading CR networks and investigate for them a new form of MD, named multiuser interference diversity (MID), which is due to the CR and PR mutual interference and thus different from the conventional MD. More specifically, “transmit MID (T-MID)” is due to the fact that the maximum transmit powers of different CR transmitters vary with their independent-fading channels to a PR receiver under a given interference power constraint, while “receive MID (R-MID)” is due to the fact that the PR interference powers at different CR receivers vary with their independent-fading channels from a PR transmitter. This letter studies the MID for various types of CR networks with D-TDMA, including the multiple-access channel (MAC), broadcast channel (BC), and parallel-access channel (PAC), and analyzes the achievable multiuser diversity gain (MDG) as a function of the number of CRs, K. It is shown that due to the newly discovered MID, the MDG (as defined in this letter) for each CR network under consideration is no smaller than that in the conventional network without the PR link, which holds for arbitrary fading channel distributions and values of K. On the other hand, it is also shown that the MID results in the same asymptotic growth order over K for the CR network ergodic throughput as that due to the MD for the conventional network, as K goes to infinity.

It is noted that MD for the fading CR networks has been recently studied in, e.g., [4] and [5]. In [4], MD was analyzed for the fading CR MAC with a given K at the asymptotic regime where the ratio between the CR transmit power constraint and interference power constraint goes to infinity. In [5], MD was investigated for the fading CR PAC with the constant CR transmit power at the asymptotic region with K → ∞. In this letter, we study MD for different types of CR networks with arbitrary CR transmit and interference power constraints, and arbitrary number of CR users.

II. SYSTEM MODEL

Consider a SS-based CR network coexisting with a PR network. For the propose of exposition, only one active PR link consisting of a PR transmitter (PR-Tx) and a PR receiver (PR-Rx) is considered. All terminals in the network are assumed to be each equipped with a single antenna. We consider the block-fading (BF) model for all the channels involved and coherent communications; thus, only the fading channel power gains (amplitude squares) are of interest. It is assumed that the additive noises at all PR and CR receive terminals are independent circular symmetric complex Gaussian (CSCG) random variables each having zero mean and unit variance. We consider three types of CR networks described as follows.

- **Cognitive MAC (C-MAC)**, where K CRs, denoted by CR1, CR2, . . . , CRK, transmit independent messages to a CR base station (CR-BS). Denote \( h_k \) as the power gain of the fading channel from CRk to CR-BS, \( k = 1, . . . , K \). Similarly, \( g_k \) is defined as the fading channel from CRk to PR-Rx, \( f \) is for that from PR-Tx to PR-Rx, and \( e_k \) is for that from PR-Tx to CR-BS.
- **Cognitive BC (C-BC)**, where CR-BS sends independent messages to K CRs. Channel reciprocity is assumed between the C-MAC and C-BC; thus, the fading channel power gain from CR-BS to CRk in the C-BC is the same as \( h_k \) in the C-MAC. In addition, \( g \) is defined as the channel power gain from CR-BS to PR-Rx, and \( e \) is defined for the channel from PR-Tx to CR-BS.
- **Cognitive PAC (C-PAC)**, where K distributed CR transmitters, denoted by CR-Tx1, . . . , CR-TxK, transmit independent messages to the corresponding receivers, denoted by CR-Rx1, . . . , CR-RxK, respectively. Similarly like the C-MAC and C-BC, we denote \( h_k \) as the power gain of the
fading channel from CR-Tx to CR-Rx, \( k = 1, \ldots, K \); \( g_k \) and \( e_k \) are defined for the fading channels from CR-Tx to PR-Rx and from PR-Tx to CR-Rx, respectively.

Let \( J \) be the peak (with respect to fading states) transmit power constraint at CR-BS in the C-BC, and \( P_k \) be that at CR\(_k\) in the C-MAC or at CR-Tx in the C-PAC. It is assumed that PR-Tx transmits with a constant power \( Q \); and each CR transmit terminal protects the PR by applying the peak interference power constraint at PR-Rx, denoted by \( \Gamma \). Let \( p_k \) be the transmit power for CR \( k \) in the C-MAC or C-PAC, and \( p \) be that for CR-BS in the C-BC. Let \( g_k \) be one realization of \( g_k \) for a particular fading state (similar notations apply for the other channels). Combining both the transmit and interference power constraints, we obtain

\[
p_k \leq \min(p_k, \Gamma / g_k), \forall k, \quad p \leq \min(J, \Gamma / g).
\]

The maximum achievable receiver signal-to-noise ratio (SNR) of CR \( k \) can then be expressed as

\[
\gamma_k^{\text{(MAC)}} = \frac{h_k p_k}{1 + Q e_k}, \gamma_k^{\text{(BC)}} = \frac{h_k p}{1 + Q e_k}, \gamma_k^{\text{(PAC)}} = \frac{h_k p_k}{1 + Q e_k}
\]

for the C-MAC, C-BC, and C-PAC, respectively. Note that the noise at each CR receive terminal includes both the additive Gaussian noise and the interference from PR-Tx. We are then interested in maximizing the long-term system throughput in each CR network by adopting the following D-TDMA rule: At one particular fading state, CR \( k \) is selected for transmission if it has the largest achievable receiver SNR among all the CRs. Let \( k^* \) denote the selected user at this fading state. From (1) and (2), it then follows that

\[
k_{\text{MAC}}^* = \arg \max_{k \in \{1, \ldots, K\}} h_k \min(p_k, \Gamma / g_k)
\]

\[
k_{\text{BC}}^* = \arg \max_{k \in \{1, \ldots, K\}} h_k \min(p, \Gamma / g_k)
\]

\[
k_{\text{PAC}}^* = \arg \max_{k \in \{1, \ldots, K\}} h_k \min(p_k, \Gamma / g_k)
\]

for the C-MAC, C-BC, and C-PAC, respectively. By substituting \( k^* \) for each CR network into (2), the corresponding maximum receiver SNR over CR users is obtained as \( \gamma_{\text{MAC}}(K) \), \( \gamma_{\text{BC}}(K) \), or \( \gamma_{\text{PAC}}(K) \), each as a function of \( K \).

### III. Multiuser Interference Diversity

In this section, we study the MD for different CR networks under a set of “symmetric” assumptions, where all \( h_k \)’s are assumed to have the same distribution, so are \( g_k \)’s and \( e_k \)’s; and \( P_k = P, \forall k \). We then define \( \bar{\gamma}_{\text{MAC}}(K) \) as \( \mathbb{E}[\gamma_{\text{MAC}}(K)]/\mathbb{E}[\gamma_{\text{MAC}}(1)] \) as the MDG for the C-MAC; \( \bar{\gamma}_{\text{BC}}(K) \) and \( \bar{\gamma}_{\text{PAC}}(K) \) are similarly defined for the C-BC and C-PAC, respectively. From (3), (4), and (5), it follows that

\[
\bar{\gamma}_{\text{MAC}}(K) = \kappa_{\text{MAC}} \mathbb{E}[\max_k h_k \min(P, \Gamma / g_k)]
\]

\[
\bar{\gamma}_{\text{BC}}(K) = \kappa_{\text{BC}} \mathbb{E}[\min_k h_k / (1 + Qe_k)]
\]

\[
\bar{\gamma}_{\text{PAC}}(K) = \kappa_{\text{PAC}} \mathbb{E}[\max_k h_k \min(P, \Gamma / g_k) / (1 + Qe_k)]
\]

where

\[
\kappa_{\text{MAC}} = 1/(\mathbb{E}[h_k] \mathbb{E}[\min(P, \Gamma / g_k)])
\]

\[
\kappa_{\text{BC}} = 1/(\mathbb{E}[h_k] \mathbb{E}[1/(1 + Qe_k)])
\]

\[
\kappa_{\text{PAC}} = 1/(\mathbb{E}[h_k] \mathbb{E}[\min(P, \Gamma / g_k)] \mathbb{E}[1/(1 + Qe_k)])
\]

are constants, which can be shown independent of \( k \) due to the symmetric assumptions. In order to fairly compare the MDG in each CR network with that in the conventional network, we introduce a reference network by removing the PR link in each CR network. For some reference networks, since there is no interference power constraint nor interference from PR-Tx to CR terminals, it is easy to show that CR \( k \) with the largest \( h_k \) among all the CRs should be scheduled for transmission at each fading state due to D-TDMA. Thus, we define the MDG of the reference network as

\[
\bar{\gamma}_0(K) = \kappa_0 \mathbb{E}[\max_k h_k]
\]

where \( \kappa_0 = 1/\mathbb{E}[h_k] \) is a constant, which is independent of \( k \).

From (6)–(12), it follows that the MDGs for different CR networks all differ from the conventional MDG for the reference network. We highlight their differences as follows.

- For the C-MAC, it is observed from (6) that the MDG is obtained by taking the maximum product between \( h_k \) and \( \min(P, \Gamma / g_k) \) over all \( k \)’s, where the former also exists in the conventional MDG given in (12), while the latter is a new term due to independent \( g_k \)’s over which CR transmitters interfere with PR-Rx, thus named T-MID.
- For the C-BC, it is observed from (7) that the MDG is obtained by taking the maximum product between \( h_k \) and \( 1/(1 + Qe_k) \) over CRs, where the former term contributes to the conventional MDG, while the latter term is a new source of diversity due to independent \( e_k \)’s over which PR-Rx interferes with CR receivers, thus named R-MID.
- For the C-PAC, it follows from (8) that in addition to the conventional MD, there are combined T-MID and R-MID.

Next, we show the following theorem, which says that the MDG of each CR network for a given \( K \) is lower-bounded by that of the reference network, \( \bar{\gamma}_0(K) \); and is upper-bounded by a constant (greater than one) multiplication of \( \bar{\gamma}_0(K) \).

**Theorem 3.1:** Under the symmetric assumptions, we have

\[
\bar{\gamma}_0(K) \leq \bar{\gamma}_{\text{MAC}}(K) \leq \alpha_{\text{MAC}} \bar{\gamma}_0(K), \quad \bar{\gamma}_0(K) \leq \bar{\gamma}_{\text{BC}}(K) \leq \alpha_{\text{BC}} \bar{\gamma}_0(K), \quad \bar{\gamma}_0(K) \leq \bar{\gamma}_{\text{PAC}}(K) \leq \alpha_{\text{PAC}} \bar{\gamma}_0(K),
\]

where \( \alpha_{\text{MAC}} = P/\mathbb{E}[\min(P, \Gamma / g_k)], \alpha_{\text{BC}} = 1/\mathbb{E}[1/(1 + Qe_k)], \) and \( \alpha_{\text{PAC}} = \alpha_{\text{MAC}} \cdot \alpha_{\text{BC}} \) are all constants.

**Proof:** We only show the proof for the C-PAC case, while similar proofs can be obtained for the C-MAC and C-BC and are thus omitted here. First, we consider the lower bound on \( \bar{\gamma}_{\text{PAC}}(K) \). By denoting \( k' \) as the user with the largest \( h_k \) among all the CRs, it follows that

\[
\bar{\gamma}_{\text{PAC}}(K) \geq \kappa_{\text{PAC}} \mathbb{E}[h_{k'} \min(P, \Gamma / g_k) / (1 + Qe_k)]
\]

\[
\geq \kappa_{\text{PAC}} \mathbb{E}[h_{k'} \min(P, \Gamma / g_k)] \mathbb{E}[1 / (1 + Qe_k)]
\]

\[
= \bar{\gamma}_0(K)
\]

where \( (a) \) is due to the fact that \( k' \) is in general not the optimal \( k^* \) corresponding to the largest \( h_k \min(P, \Gamma / g_k) / (1 + Qe_k) \) in
where (a) is due to the fact that the user with the largest $h_k$ is not necessarily the one with the largest $\min(P, \Gamma/g_k)/(1 + Qe_k)$; (b) is due to independence of $h_k$ and $(g_k, e_k)$; (c) is due to the fact that $\min(P, \Gamma/g_k)/(1 + Qe_k) \leq P, \forall k$; and (d) is due to (11) and (12). Using the definitions of $\alpha_{MAC}$ and $\alpha_{BC}$ given in Theorem 3.1 it follows that $\bar{\gamma}_{PAC}(K) \leq \alpha_{PAC} \bar{\gamma}_0(K)$, where $\alpha_{PAC} = \alpha_{MAC} \cdot \alpha_{BC}$.

At last, we study the ergodic throughput of each D-TDMA based CR network and its asymptotic growth order over $K$ as $K \to \infty$. For the C-MAC, the ergodic throughput for a given $K$ is defined as $C_{MAC}(K) = \mathbb{E}[\log_2(1 + \gamma_{MAC}(K))]$; similarly, $C_{BC}(K)$, $C_{PAC}(K)$, and $C_0(K)$ are defined for the C-BC, C-PAC, and reference network, respectively. According to the extreme value theory [6, Appendix A], it is known that $C_0(K)$ behaves as $C_0(K) \sim \log_2 F(K)$ as $K \to \infty$, where $F(K)$ is given by the distribution of $\max_k h_k$ as $K \to \infty$ (e.g., for “type i” distribution of $h_k$, with unit-variance, $F(K) = \log K$ [6]). Then, from Theorem 3.1 the following corollary can be obtained (proof is omitted here due to the space limitation).

**Corollary 3.1:** Under the symmetric assumptions, as $K \to \infty$, we have $C_{MAC}(K) / \log_2 F(K) = 1$, $C_{BC}(K) / \log_2 F(K) = 1$, and $C_{PAC}(K) / \log_2 F(K) = 1$.

Corollary 3.1 says that the MID results in the same ergodic-throughput asymptotic growth order over $K$ for the CR networks as that of the conventional MD for the reference network, as $K \to \infty$, regardless of the fading distribution.

### IV. Numerical Results

We assume that all the channels involved follow the standard (unit-power) Rayleigh fading distribution. In addition, for a fair comparison of different CR networks and the reference network, we assume that $J = Q = P = \Gamma = 1$. Fig. 1 shows the ergodic throughput for different networks with $K \leq 100$, after normalizing it to the ergodic throughput with $K = 1$ in order to better examine the MDG. It is observed that the normalized ergodic throughput for different CR networks is larger than that for the reference network, thanks to the newly discover MID. It is also observed that the combined T-MID and R-MID in the C-PAC result in more substantial throughput gains than T-MID in the C-MAC or R-MID in the C-BC. Fig. 2 shows the ergodic throughput (without normalization) of different networks versus $\log_2(\log K)$ (Note that $F(K) = \log K$ in this case.) for very large values of $K$ ranging from $10^3$ to $10^6$. It is observed that all these networks have the same asymptotic growth order over $K$ for the ergodic throughput, which is in accordance with Corollary 3.1.

### V. Conclusion

This letter quantified a new form of MID for SS-based CR networks by exploiting the CR and PR mutual interference. The MDG and ergodic throughput for opportunistic communications in different types of CR networks were analyzed.

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