Some Attacks On Quantum-based Cryptographic Protocols

Hoi-Kwong Lo \textsuperscript{a}

\textit{Center for Quantum Information and Quantum Control}
\textit{Dept. of Electrical \& Computer Engineering \& Dept. of Physics}
\textit{University of Toronto, 10 King’s College Road}
\textit{Toronto, Ontario, CANADA, M5S 3G4.}

Tsz-Mei Ko \textsuperscript{b}

\textit{IBM East Fishkill}
\textit{Department of Physical Synthesis Bldg 334, Rm 2K17/424}
\textit{2070 Route 52 Hopewell Junction, NY 12533, USA}

Received (received date)
Revised (revised date)

Quantum-based cryptographic protocols are often said to enjoy security guaranteed by the fundamental laws of physics. However, even carefully designed quantum-based cryptographic schemes may be susceptible to subtle attacks that are outside the original design. As an example, we give attacks against a recently proposed “secure communication using mesoscopic coherent states”, which employs mesoscopic states, rather than single-photon states. Our attacks can be used either as a known-plaintext attack or in the case where the plaintext has not been randomized. One of our attacks requires beamsplitters and the replacement of a lossy channel by a lossless one. It is successful provided that the original loss in the channel is so big that Eve can obtain $2^k$ copies of what Bob receives, where $k$ is the length of the seed key pre-shared by Alice and Bob. In addition, substantial improvements over such an exhaustive key search attack can be made, whenever a key is reused. Furthermore, we remark that, under the same assumption of a known or non-random plaintext, Grover’s exhaustive key search attack can be applied directly to “secure communication using mesoscopic coherent states”, whenever the channel loss is more than 50 percent. Therefore, as far as information-theoretic security is concerned, optically amplified signals necessarily degrade the security of the proposed scheme, when the plaintext is known or non-random. Our attacks apply even if the mesoscopic scheme is used only for key generation with a subsequent use of the key for one-time-pad encryption. Studying those attacks can help us to better define the risk models and parameter spaces in which quantum-based cryptographic schemes can operate securely. Finally, we remark that our attacks do not affect standard protocols such as Bennett-Brassard BB84 protocol or Bennett B92 protocol, which rely on single-photon signals.

Keywords: Quantum Cryptography, Quantum Key Distribution, Unconditional Security

Communicated by: to be filled by the Editorial

\textsuperscript{a}Email: hklo@comm.utoronto.ca
\textsuperscript{b}Email: tszmei@us.ibm.com
1 Introduction

“Cryptographers do not sleep well.” In conventional cryptography, it is well-known that cryptographic protocols may fail to achieve the designed goals. It is unfortunate that those design flaws are often not fully appreciated until it is too late. A well-known example is Queen Mary of Scotland whose usage of an insecure encryption scheme to communicate with her conspirators in a plot against Queen Elizabeth led directly to her execution. Another example is the breaking of the legendary German Enigma code in World War II by the Allied code-breakers. The Allied code-breaking effort contributed significantly to the outcome of the war and involved a) a number of innovative mathematical discoveries, b) the construction of large-scale code-breaking machines (which were precursors to modern computers), c) the exploitation of a number of mistakes/weaknesses in the implementation details of the German and, finally, d) the capturing of an Enigma machine. The first lesson in cryptography is: never underestimate the effort and ingenuity that your adversaries are willing to spend on breaking your codes. Given that cryptography can be a matter of life and death, there is no surprise that cryptographers are generally paranoid.

Quantum mechanics has made a remarkable entry in cryptography. Unlike conventional cryptographic schemes that are based on computational assumptions, it is often said that quantum-based cryptographic schemes have their security guaranteed by the fundamental laws of physics. Various cryptographic protocols based on quantum mechanics have been proposed in the literature. The best-known protocol is quantum key distribution (QKD). The idea of QKD was first published by Bennett and Brassard in 1984 [1]. It took more than ten years for people to prove rigorously that QKD can, in principle, provide perfect security guaranteed by quantum mechanics [2, 3, 4, 5, 6, 7, 8]. Even in the case of imperfect devices, unconditionally secure QKD is still possible under rigorous theoretical models [9, 10]. Provided that we are willing to be conservative cryptographers, we can adjust the parameters in our experiment so that we have unconditional security proven in a precise physical model, whose validity can then be battle-tested. The battle-testing will hopefully lead to a refinement of our physical model and so on and so forth. Provided that an eavesdropper cannot break our precise physical model at the time of transmission of quantum signals, we can obtain unconditional security.

It is, however, very important to distinguish unconditional security offered by precise physical models from ad hoc security derived from handwaving arguments. Whenever the security of a protocol relies on handwaving arguments, there is serious danger that hidden loopholes may exist in a protocol. The bottom line is that it pays to make our physical model as precise and explicit as possible and it pays to battle-test our physical model in a real QKD system. Indeed, technological advances are intrinsically difficult to predict. What is not realistic today may become realistic tomorrow. Worse still, given that security agencies such as the NSA has numerous mathematicians and physicists working hard on quantum technologies and not all of their results will be published, it is rather risky to assume that ad hoc security will survive their future attacks.

Moreover, even carefully designed cryptographic schemes may be susceptible to subtle attacks that are outside the original design. It would, therefore, be interesting to study those subtle loopholes in existing schemes. In this paper, we will examine new attacks against existing quantum-based cryptographic protocols. One of our attacks is a beamsplitting at-
tack against a "secure communication using mesoscopic coherent states" [11], which employs mesoscopic signals, rather than single-photon signals. Our attack applies when the plaintext is known, or more generally, non-random. Furthermore, we remark that, under similar assumptions, Grover’s exhaustive key search attack can be applied directly to "secure communication using mesoscopic coherent states", whenever the channel loss is more than 50 percent.

2 Beamsplitting plus low loss channel attack

The idea of using mesoscopic states for quantum key distribution was first proposed by Bennett and Wiesner [12] in 1996. Recently, there has been much interest in the subject of quantum-based cryptographic schemes with coherent states. In particular, Barbosa et al. [11] have described an experimental implementation of one particular scheme which employs mesoscopic coherent states. Such a scheme can be optically amplified and can open up new avenues for secure communications at high speeds in fiber-optic or free-space channels. Just imagine transmitting quantum signals through optical amplifier and one could obtain long distance secure communication. It is, thus, important to understand the limitations of those schemes.

Let us now describe Barbosa et al.’s scheme [11] in more detail. In [11], the two users, Alice and Bob, initially share a short $k$-bit seed key, $K$. Using a (deterministic) pseudo-random number generator, each of them can expand their seed key, $K$, which is a classical string, into a longer running key, $K'$, which is also a classical $M$-ary string. Say, $K'_i = (K'_i, K'_i+M, \ldots, K'_{n-1})$ where $K'_i \in \{0, \ldots, M-1\}$. Suppose Alice would like to transmit some classical data $X = (X_0, X_1, \ldots, X_{n-1})$, which is an $n$-bit binary string, to Bob. Then, they use the running key $K'$ to encrypt the data $X$. The transmitted signal is a sequence of quantum states. In order to describe what states are transmitted, we need to introduce some standard states. More specifically, consider $M$ states $|\alpha_0(\cos \theta + i \sin \theta)\rangle$, where $\theta = 2\pi l/M$ and $0 \leq l \leq M - 1$. Note that, for each even $M$, they form $M/2$ pairs $\{l, l+M/2\}$. Each pair forms a signal set. The two signals of each pair are almost orthogonal for a large $\alpha_0$. In fact, the inner product between any two signals in a pair is $\exp(-2|\alpha_0|^2)$. Barbosa et al.’s scheme [11] considers the scenario when $M \gg \alpha_0$ so that the $M$ states are not (almost) distinguishable. Each component, say $K'_i$, of the running key is used to specify which one of the $M$ bases is used for encoding the data bit, $X_i$. Indeed, the transmitted state is of the form $|Y_0\rangle \otimes |Y_1\rangle \otimes \cdots \otimes |Y_{n-1}\rangle$ where $Y_l = \alpha_0(\cos \theta + i \sin \theta)$ where $l = K'_i$ if $X_i = 0$ and $l = K'_i + M/2$ if $X_i = 1$. If an eavesdropper has no information on the running key $K'$, then it might appear that she cannot do much better than random guessing.

Instead of considering a ciphertext only attack as originally studied in [11], we consider either a known-plaintext attack or the case where the plaintext has not been randomized. Known-plaintext attacks were employed successfully against both the Germans and the Japanese during the Second World War. We will discuss a simple theoretical attack that can break such a scheme with a known or non-random plaintext. It involves beamsplitters and the replacement of the original high loss channel by a low loss channel. Our observation is the following: By replacing a high loss channel by a low loss channel, an eavesdropper obtains many copies of the output state received by the receiver, Bob. In principle, she can use beamsplitters to split the signal, sending one copy to Bob and keeping many copies herself. Now, Eve is at liberty to perform an exhaustive key search to break the enciphering scheme, in
analogy with exhaustive key search attack in conventional cryptography. [Note that such an exhaustive search attack is possible if the plaintext is known or, more generally, not random.] Useful improvements over such an exhaustive search attack will also be presented later in this paper.

Let us begin with an exhaustive search attack. More concretely, suppose the loss of the channel is such that only a fraction \(1/(t+1)\) of the quantum signal reaches Bob. Then, by replacing a lossy channel by a lossless one and intercepting the residual signal, Eve can obtain \(t\) copies of the signal received by Bob. In the spirit of exhaustive key search, for each copy of the signal, Eve can try to decode it with a specific trial value for a seed key. In more detail, She picks a trial value \(K = K_{trial_i}\) for the seed key and runs a pseudo-random generator to obtain a running key \(K'_{trial_i}\). With such a running key in her hand, she can then measure a copy of the signal to try to determine the value of \(X\). Note that if she has guessed the value of the seed key correctly, she will most likely obtain the correct value of \(X\) from her measurement.

Our attack applies to two cases—as a known-plaintext attack or the case where the plaintext has not been randomized. Consider case one: known-plaintext attack. As in standard cryptography, we are given \(P_1, C_1 = E_k(P_1), P_2, C_2 = E_k(P_2), \ldots, P_i, C_i = E_k(P_i)\). The eavesdropper’s job is to deduce \(k\) or an algorithm that will infer \(P_{i+1}\) from \(C_{i+1} = E_k(P_{i+1})\). Known-plaintext attacks allow the eavesdropper to verify that she has guessed the correct encryption key. In case two, the plaintext, \(X\), is not known. However, we assume that it is not randomized. Suppose \(X\) is a text in English, there is a lot of redundancy in the message. Eve will be able to verify that the message is in English. On the other hand, if her trial value \(K = K_{trial_i}\) for the seed key is incorrect, then it is highly unlikely that she will find that the decoded message is a meaningful English text. In summary, she will be able to tell whether a trial seed key is correct or not simply by seeing whether the decoded message is meaningful or not. Now, suppose the number \(t\) of copies in Eve’s hand satisfies \(t \geq 2^k\) where \(k\) is the number of bit of the seed key. Then, Eve could have tried all possible combinations of the seed key. Therefore, Eve will have succeeded in breaking the security of the scheme completely. This conclusion applies to both cases (case one—known-plaintext attack and case two—where the plaintext is unknown, but not randomized).

Some remarks are in order. First, our attack may be outside the original design of the protocol. The original design of the protocol [11] seems to focus on ciphertext only attacks.

Second, our attack appears to be feasible with current or near future technology: Since installed fibers may go through many intermediate switches, they may not take the shortest path between the sender and the receiver. Therefore, it is not unrealistic to imagine that the eavesdropper may re-route the original optical path to a shorter one, thus effectively reducing the loss in the channel. Besides, low loss fibers are under active development in various laboratories in the world [13] and they give much lower loss than standard Telecom fibers through which QKD experiments are usually performed. Therefore, Eve may apply those low loss fibers for eavesdropping purposes.

For an eavesdropping attack that involves re-routing the light through a shorter path, it may lead to a different time delay in the channel. Therefore, Alice and Bob may try to defeat it by testing the transmission time. However, such a testing by Alice and Bob may be defeated by a technologically advanced Eve: Given that there have been successful "stopping
light" experiments [14, 15], we think it may not be unrealistic to believe that Eve might be able to reproduce the same transmission time by delaying the light pulses only temporarily in the near future. Therefore, detecting the transmission time may not be sufficient to defeat our attack.

Third, when Alice is sending a long message to Bob, Eve may not even need that many copies of the signal to decode it. To illustrate this point, let us consider, for simplicity the case when the plaintext is a message of the form, \( P_1P_2 \cdots P_r \) where \( P_i \) is \( s \)-bit long and \( rs = n \). In fact, we allow Alice to divide up her transmission of the message into \( r \) different time windows. For instance, \( P_1 \) might be transmitted on the first day (or hour or minute), \( P_2 \) on the second day, ..., \( P_r \) on the \( r \)-th day. We assume that Eve has no long-term quantum memory (but is able to use beamsplitters and measure mesoscopic signals). Once again, let us assume that \( P_i \) is a text in English or is a known plaintext. Suppose the channel is lossy and only a fraction \( 1/(t + 1) \) of the signal reaches Bob. Then, effectively, Eve has \( t \) copies of all the \( r \) messages, \( X_i \) in some encrypted form in different time windows. Altogether, Eve has \( rt \) encrypted messages. Now, Eve can try a different trial seed key value \( K_{\text{trial}, i} \) for each of the \( rt \) messages. Therefore, provided \( rt \geq 2^k \), Eve will succeed in an exhaustive key search and find the encryption key, \( k \). From that point on, she will be able to infer any future message \( P_{r+1} \) from its ciphertext \( E_k(P_{r+1}) \). Note that given a fixed transmission fraction \( 1/(t + 1) \), for a sufficiently large \( r \), the scheme will become insecure. This sets a fundamental limit on how many times Alice and Bob can expand a key before the scheme becomes insecure against a "beamsplitting plus exhaustive key search attack".

Fourth, Eve can substantially increase the power of an exhaustive key search attack by changing the value of her trial seed key mid-way in the decoding process. In fact, for each encrypted copy of a message, \( X_i \), she may first try a trial seed key value, \( K_{\text{trial}, i} \), on the first few bits of the ciphertext. She checks whether the decrypted message is a meaningful text in English. If it is, she will continue the decryption process for a few more bits and checks again and so on and so forth. On the other hand, whenever it becomes clear that the decrypted part of the message is not a meaningful text in English, Eve can change her trial seed key value from \( K_{\text{trial}, i} \) to another value, say \( K_{\text{trial}, j} \). She decrypts a few bits and sees if the resulting text makes sense. If it does not, she can change the trial value of the trial seed key again. The process is done until she is fairly confident that she has found the right trial seed value.

We remark that one countermeasure that Alice and Bob can use to circumvent this particular improved attack is to perform double encryption. The message \( P_i \) should be encrypted by a standard classical cipher like AES (Advanced Encryption Standard) before passing through a quantum-based encryption device.

Fifth, if Eve has a quantum computer, then she can launch a much more powerful attack. Indeed, she will be able to decode the message with high probability whenever the transmission ratio of the channel is less than 50 percent. The idea is that Eve can perform a Grover’s [16] database search on the encrypted message to see whether she has found a correct key. See [16, 17, 18] for details. Even though the ciphertext is a quantum state, we remark here that, in the context of known-plaintext attacks, such a quantum exhaustive key search method by Grover can be applied directly to [11]. Furthermore, only one plaintext-ciphertext pair will be sufficient for performing Grover’s search. Therefore, it is sufficient for Eve to have a single copy of the signal. This implies that the scheme is, in principle, insecure against a known-
plain quantum computing attack, whenever the transmission ratio is less than 50 percent. Why does a Grover’s search work? We remark that we are considering the asymptotic case where Alice and Bob are using a pseudo-random number generator to expand the seed key by a very large factor. In this case, the quantum codewords after the encoding corresponding to the various possible seed key values will asymptotically become orthogonal to each other. Therefore, there exists a measurement by Eve that will distinguish between those codewords, thus disclosing the value of the seed key.

A variant of the proposal in [11] is to use the mesoscopic quantum state for key generation only and subsequently use the generated key for one-time-pad encryption. Here, we will show that our attacks apply even to this variant. First of all, let us describe the variant. As before, Alice expands a short seed key, $K$, that she shares with Bob into a long running key, $K'$, using a classical pseudo-random number generator. Now, Alice generates a random number, $R$, and uses the mesoscopic quantum encryption method of Barbosa et al. [11] to send the encrypted version of $R$, i.e., $E_{K'}(R)$, to Bob. [Here, $E_{K'}(R)$ is a quantum state and $K'$, the long running key, is used as an encryption key.] Bob decrypts the message and recovers $R$. Note that Alice and Bob now share a common key $R$. Now, suppose Alice would like to send a message, $M$, to Bob. She uses the random number $R$ as a one-time-pad and sends $C = R \oplus M$ to Bob. Bob uses $R$ to decode the message and recover $M$. Now consider the security of this variant of a proposal in [11]. First, consider a known-plaintext attack. Eve is given a set of old plaintext together with their cipher-texts $M_1, C_1 = R_1 \oplus M_1, M_2, C_2 = R_2 \oplus M_2, \ldots , M_i, C_i = R_i \oplus M_i$. Here, the long string of random number $R$ is a concatenation of $R_1 R_2 \cdots R_i \cdots$. Note, however, that such a known plaintext attack means that Eve can obtain $R_1 = M_1 \oplus C_1$, etc, which are now the plaintext used in the mesoscopic key generation scheme. In other words, a known-plaintext attack on the variant can be reduced to a known-plaintext attack on the original protocol. Similarly, a non-random plaintext in this variant can be reduced to a non-random plaintext in the original protocol. For this reason, our attacks (based on known plaintext or non-random plaintext) apply directly to this variant of the protocol [11] too.

3 Concluding Remarks

In summary, attacks against quantum-based cryptographic schemes are discussed in this paper. Our attacks can be used to defeat a communication protocol based on mesoscopic coherent states. Such a scheme can be optically amplified. Our attack applies if a plaintext is known or, more generally, not random. We remark again that known-plaintext attacks were successfully launched against both the Germans and the Japanese in World War II. Our attack involves replacing a high loss channel by a low loss channel and using beamsplitters to steal a component of the signal. Therefore, our attack can, to some extent, be implemented with current technology. Once Eve has copies of the quantum signal, she can perform exhaustive key search on her copies to find out the value of the key. Moreover, she can improve substantially over a simple exhaustive key search by exploiting redundancies in the plaintext, unless double encryption has been performed. In fact, given any transmission ratio $1/(L+1)$ of the channel, the scheme will become insecure whenever a seed key is expanded more than $r$ times where $rt \geq 2^k$.

Furthermore, if Eve has a quantum computer, she can apply Grover’s database search
algorithm to attack the scheme. Such a quantum-computational attack will work whenever the loss of the channel is over 50 percent.

Our attacks apply not only to the original protocol in [11], but also to a variant where the protocol in [11] is used only for key generation and subsequently one-time-pad encryption is employed, using the generated key.

Our attacks show that there are fundamental limits on the amount of key expansion that can be securely achieved using mesoscopic states. In future investigations, it will be interesting to quantify this fundamental limit more precisely.

We remark that it will be interesting to look into attacks that require technologies intermediate between simple beamsplitters and a large-scale quantum computer.

As a general remark, breaking cryptographic systems is as important as building them. Our attacks are rather simple but powerful. They highlight that, just like their classical counterparts, the study of the security of quantum-based cryptographic schemes is a very slippery subject. It is important to make security assumptions as explicit as possible.

In summary, our attacks highlight the fact that even carefully designed cryptographic schemes may still be susceptible to attacks outside the original design. Understanding those attacks will thus be helpful to improving the security of those schemes, for example, by better defining the parameter space or risk models in which those schemes are secure.

Finally, we remark that our attacks do not apply to BB84[1] or other standard QKD schemes where the quantum signals are strictly microscopic in the sense that there is (on average) at most one copy of the signal available.

Acknowledgements

We thank helpful discussions with colleagues including G. Barbosa, O. Hirota, M. Koashi, D. Leung, N. Lütkenhaus, D. Mayers, T. Mor, J. Preskill and H. P. Yuen. Useful comments from an anonymous referee are gratefully acknowledged. H.-K. L. thanks CFI, CIPI, CRC program, NSERC, OIT, and PREA for financial support.

1. C. H. Bennett and G. Brassard, Quantum cryptography: Public key distribution and coin tossing, Proceedings of IEEE International Conference on Computers, Systems, and Signal Processing, IEEE, 1984, pp. 175-179.
2. D. Mayers, Journal of ACM, vol. 48, 351 (2001). Available on-line at [http://arxiv.org/abs/quant-ph/9802025]. A preliminary version in D. Mayers, Quantum key distribution and string oblivious transfer in noisy channel, Advances in Cryptology — Proceedings of Crypto' 96, Lecture Notes in Computer Science, vol. 1109, Springer-Verlag, 1996, pp. 343-357.
3. H.-K. Lo and H. F. Chau, Unconditional security of quantum key distribution over arbitrarily long distances, Science, vol. 283, (1999), pp. 2050-2056; also available at [http://xxx.lanl.gov/abs/quant-ph/9803006].
4. E. Biham, M. Boyer, P. O. Boykin, T. Mor, and V. Roychowdhury, A proof of security of quantum key distribution, Proceedings of the Thirty-Second Annual ACM Symposium on Theory of Computing, ACM Press, New York, 2000, p. 715.
5. M. Ben-Or, Simple security proof for quantum key distribution. Online presentation available at [http://www.msri.org/publications/ln/msri/2002/qip/ben-or/1/index.html].
6. P. W. Shor and J. Preskill, Simple proof of security of the BB84 quantum key distribution scheme, Phys. Rev. Lett. vol. 85, (2000), pp. 441-444.
7. M. Koashi and J. Preskill, Secure quantum key distribution with an uncharacterized source [http://xxx.lanl.gov/abs/quant-ph/0208155].
8. D. Gottesman and H.-K. Lo, Proof of security of quantum key distribution with two-way classical communications, IEEE Transactions on Information Theory, Vol. 49, No. 2, p. 457, available at http://xxx.lanl.gov/abs/quant-ph/0105121.

9. H. Inamori, N. Lütkenhaus, and D. Mayers, Unconditional Security of Practical Quantum Key Distribution, available at http://xxx.lanl.gov/abs/quant-ph/0107017.

10. D. Gottesman, H.-K. Lo, N. Lütkenhaus, and J. Preskill, Security of quantum key distribution with imperfect devices, available at http://xxx.lanl.gov/abs/quant-ph/0212066.

11. G. A. Barbosa, E. Corndorf, P. Kumar, and H. P. Yuen, Secure communication using mesoscopic coherent states, Phys. Rev. Lett. 90, 227901 (2003); available on-line at http://xxx.lanl.gov/abs/quant-ph/0212018.

12. C. H. Bennett and S. J. Wiesner, Quantum key distribution using non-orthogonal macroscopic states, US patent number 5,515,438 (1996). On-line available at www.uspto.gov.

13. See, for example, http://irfibers.rutgers.edu.

14. C. Liu, Z. Dutton, C. H. Behroozi, and L. V. Hau, Nature 409, 490-493 (2001).

15. D. F. Phillips, A. Fleischhauer, A. Mair, R. L. Walsworth, and M. D. Lukin Phys. Rev. Lett. 86, 783 (2001).

16. L. K. Grover, A fast quantum mechanical algorithm for database search, Proceedings of 28th Annual ACM Symposium on Theory of Computing, ACM Press, New York, 1996, pp. 212-219.

17. G. Brassard, Searching a Quantum Phone Book, (perspectives) Science, 31 January 1997. Available at http://us.cryptosoft.de/snews/jan97/31019700.htm.

18. P. Hoyer, M. Mosca, R. de Wolf, Quantum Search on Bounded-Error Inputs, available on-line at http://xxx.lanl.gov/abs/quant-ph/0304052.