Universal texture of quark and lepton mass matrices and a discrete symmetry $Z_3$

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(Received 27 September 2002; published 19 November 2002)

Recent neutrino data have been favorable to a nearly bimaximal mixing, which suggests a simple form of the neutrino mass matrix. Stimulated by this matrix form, the possibility that all the mass matrices of quarks and leptons have the same form as in neutrinos is investigated. The mass matrix form is constrained by a discrete symmetry $Z_3$ and a permutation symmetry $S_2$. The model, of course, leads to a nearly bimaximal mixing for the neutrino sector, while, for the quark sectors, it can lead to reasonable values of the CKM mixing matrix and masses.

DOI: 10.1103/PhysRevD.66.093006 PACS number(s): 12.15.Ff, 11.30.Hv, 14.60.Pq

I. INTRODUCTION

Recent neutrino oscillation experiments [1] have highly suggested a nearly bimaximal mixing ($\sin^2 2\theta_{13} \approx 1$, $\sin^2 2\theta_{12} \approx 1$) together with a small ratio $R = \Delta m_{21}^2/\Delta m_{32}^2 \approx 10^{-2}$. This can be explained by assuming a neutrino mass matrix form [2–7] with a permutation symmetry between second and third generations. We think that quarks and leptons should be unified. It is therefore interesting to investigate the possibility that all the mass matrices of the quarks and leptons have the same matrix form, which leads to a nearly bimaximal mixing and $U_{13}=0$ in the neutrino sector, against the conventional picture that the mass matrix forms in the quark sectors will take somewhat different structures from those in the lepton sectors. In the present paper, we will assume that the mass matrix form is invariant under a discrete symmetry $Z_3$ and a permutation symmetry $S_2$.

Phenomenologically, our mass matrices $M_u$, $M_d$, $M_e$, and $M_e$ [mass matrices of up quarks ($u$, $c$, $t$), down quarks ($d$, $s$, $b$), neutrinos ($\nu_e$, $\nu_\mu$, $\nu_\tau$) and charged leptons ($e$, $\mu$, $\tau$), respectively] are given as follows:

$$M_f = P_{Lf} \hat{M}_f P_{Rf},$$

with

$$\hat{M}_f = \begin{pmatrix} 0 & A_f & A_f \\ A_f & B_f & C_f \\ A_f & C_f & B_f \end{pmatrix} (f = u, d, \nu, e),$$

where $P_{Lf}$ and $P_{Rf}$ are the diagonal phase matrices and $A_f$, $B_f$, and $C_f$ are real parameters. Namely the components are different in $\hat{M}_f$, but their mutual relations are the same. This structure of mass matrix was previously suggested and used for the neutrino mass matrix in Refs. [2–7], using the basis where the charged-lepton mass matrix is diagonal, motivated by the experimental finding of maximal $\nu_\mu - \nu_\tau$ mixing [1]. In this paper, we consider that this structure is fundamental for both quarks and leptons, although it was speculated from the neutrino sector. Therefore, we assume that all the mass matrices have this structure.

Let us look at the universal characters of the model. Hereafter, for brevity, we will omit the flavor index. The eigenmasses $m_i$ of Eq. (1.2) are given by

$$m_1 = \frac{1}{2} \left( B + C - \sqrt{8A^2 + (B+C)^2} \right),$$

$$m_2 = \frac{1}{2} \left( B + C + \sqrt{8A^2 + (B+C)^2} \right),$$

$$m_3 = B - C.$$

The texture’s components of $\hat{M}$ are expressed in terms of eigenmasses $m_i$ as

$$A = \sqrt{\frac{m_2 m_1}{2}},$$

$$B = \frac{1}{2} m_3 \left( 1 + \frac{m_2 - m_1}{m_3} \right),$$

$$C = -\frac{1}{2} m_3 \left( 1 - \frac{m_2 - m_1}{m_3} \right).$$

That is, $\hat{M}$ is diagonalized by an orthogonal matrix $O$ as

$$O^T \hat{M} O = \begin{pmatrix} -m_1 & 0 & 0 \\ 0 & m_2 & 0 \\ 0 & 0 & m_3 \end{pmatrix}.$$
with

\[
O = \begin{pmatrix}
  c & s & 0 \\
  -s & c & 1 \\
  -s & c & 1
\end{pmatrix},
\]

(1.8)

Here \( c \) and \( s \) are defined by

\[
c = \sqrt{\frac{m_2}{m_2 + m_1}}, \quad s = \sqrt{\frac{m_1}{m_2 + m_1}}.
\]

(1.9)

It should be noted that the elements of \( O \) are independent of \( m_3 \) because of the above structure of \( \hat{M} \).

The zeros in this mass matrix are constrained by the discrete symmetry that is discussed in the next section, defined at a unification scale (the scale does not always mean “grand unification”). This discrete symmetry is broken below \( \mu = M_R \), at which the right-handed neutrinos acquire heavy Majorana masses, as we discuss in Sec. IV. Therefore, the matrix form (1.1) will, in general, be changed by renormalization group equation (RGE) effects. Nevertheless, we would like to emphasize that we can use the expression (1.1) with (1.2) for the predictions of the physical quantities in the low-energy region. This will be discussed in the Appendix.

This article is organized as follows. In Sec. II we discuss the symmetry property of our model. Our model is realized when we consider two Higgs doublets in each up-type and down-type quark (lepton) mass matrices. The quark mixing matrix in the present model is argued in Sec. III. In Sec. IV, the lepton mixing matrix is analyzed. Section V is devoted to a summary.

II. Z3 SYMMETRY AND MASS MATRIX FORM

We assume a permutation symmetry between second and third generations, except for the phase factors. However, the condition \( \hat{M}_{(j)1} = 0 \) cannot be derived from such a symmetry. Therefore, in addition to the \( 2 \rightarrow 3 \) symmetry, we assume a discrete symmetry \( Z_3 \), under which symmetry the quark and lepton fields \( \psi_L \), which belong to 10\(_L\), \( \tilde{5}_L \) and 1\(_L\) of SU(5) (1\(_L\) = \( \nu_R^c \)), are transformed as

\[
\psi_{1L} \rightarrow \psi_{1L}, \\
\psi_{2L} \rightarrow \omega \psi_{2L}, \\
\psi_{3L} \rightarrow \omega^2 \psi_{3L},
\]

(2.1)

where \( \omega^3 = +1 \). [Although we use a terminology of SU(5), at present, we do not consider the SU(5) grand unification.]

Then, the bilinear terms \( \tilde{q}_L u_R, \tilde{q}_L d_R, \tilde{\ell}_L v_R, \tilde{\ell}_L e_R \) and \( \tilde{v}_R^c v_R, [v_R = (\nu_R)^c = C \nu_R^c \text{ and } \tilde{v}_R = (\nu_R)] \) are transformed as follows:

\[
\begin{pmatrix}
  1 & \omega^2 & \omega^2 \\
  \omega^2 & \omega & \omega \\
  \omega & \omega & \omega
\end{pmatrix},
\]

(2.2)

where

\[
q_L = \begin{pmatrix} u_L \\ d_L \end{pmatrix}, \quad \ell_L = \begin{pmatrix} \nu_L \\ e_L \end{pmatrix}.
\]

(2.3)

Therefore, if we assume two SU(2) doublet Higgs scalars \( H_1 \) and \( H_2 \), which are transformed as

\[
H_1 \rightarrow \omega H_1, \quad H_2 \rightarrow \omega^2 H_2,
\]

(2.4)

the Yukawa interactions are given as follows:

\[
H_{\text{int}} = \sum_{A=1,2} (Y_{(1)}^u \tilde{q}_L \tilde{H}_A u_R + Y_{(2)}^u \tilde{q}_L \tilde{H}_A d_R) + \sum_{A=1,2} (Y_{(1)}^e \tilde{\ell}_L \tilde{H}_A e_R + Y_{(2)}^e \tilde{\ell}_L \tilde{H}_A e_R) + (Y_{(1)}^R \tilde{v}_R^c \tilde{\nu}_R H + Y_{(2)}^R \tilde{v}_R^c \tilde{\nu}_R H) + \text{H.c.,}
\]

(2.5)

where

\[
H_A = \begin{pmatrix} H_A^+ \\ \tilde{H}_A \end{pmatrix}, \quad \tilde{H}_A = \begin{pmatrix} \tilde{H}_A^+ \\ -H_A \end{pmatrix},
\]

(2.6)

so that

\[
Y_{(1)}^u, Y_{(2)}^u, Y_{(1)}^e, Y_{(2)}^e, Y_{(2)}^R = \begin{pmatrix} 0 & 0 & 0 \\ 0 & * & * \end{pmatrix}, \quad Y_{(1)}^e, Y_{(2)}^e, Y_{(1)}^R, Y_{(2)}^R = \begin{pmatrix} 0 & * & * \\ * & 0 & 0 \end{pmatrix}.
\]

(2.7)

In Eq. (2.7), the symbol * denotes nonzero quantities. Here, in order to give heavy Majorana masses of the right-handed neutrinos \( \nu_R \), we have assumed an SU(2) singlet Higgs scalar \( \Phi^0 \), which is transformed as \( H_1 \).

In the present model, the phase difference \( \arg(Y_{(1)}^f + Y_{(2)}^f)_{11} - \arg(Y_{(1)}^f + Y_{(2)}^f)_{11} \) plays an essential role. Therefore, for the permutation symmetry \( S_3 \), we set the following assumption: the permutation symmetry can be applied to only the special basis that all Yukawa coupling constants are real. (Of course, for the \( Z_3 \) symmetry, such an assumption is not required.) We consider that the phase factors are caused by an additional mechanism after the requirement of the permutation symmetry \( S_3 \) (after the manifestation of the linear combination \( Y_{(1)} + Y_{(2)} \)). In the present paper, we consider that although the \( Z_3 \) symmetry is rigorously defined for the fields by Eq. (2.1), the permutation symmetry \( S_2 \) is a rather
phenomenological one (i.e., ansatz) for the mass matrix shape. Then, under this $S_2$ symmetry, the general forms of $Y_f = Y_f(1) + Y_f(2)$ are given by

$$Y_f = P_L^1 Y_f^1 P_R^f,$$

$$(2.8)$$

We have already assumed that $\psi_f = (v e_L, d_L^*, u_L, u_R^*, e_R^*, v_R^*)$ have the same transformation (2.1) under the discrete symmetry $Z_3$, so that $\bar{\psi}_f \psi_f$ are transformed as Eq. (2.2). From this analogy, we assume that the phase matrices $P_L^f$ and $P_R^f$ come from the replacement $\psi_f \rightarrow \psi_f \bar{Y}_f P_R^f$, i.e.,

$$\bar{\psi}_f Y_f \psi_f^c - \bar{\psi}_f Y_f^1 P_f^1 \psi_f^c.$$  

(2.9)

However, differently from the transformation (2.1), we do not assume in Eq. (2.9) that all the phase matrices $P_f$ are identical, but we assume that they are flavor dependent. This explains the assumption

$$\delta_{Li}^f = -\delta_{Li}^f = \delta_i^f,$$  

(10.10)

in the expression (2.8). [However, this assumption (10.10) is not essential for the numerical predictions in the present paper, because the predictions of the physical quantities depend on only the phases $\delta_{Li}^f$.]

Since the present model has two Higgs doublets horizontally, in general, flavor-changing neutral currents (FCNCs) are caused by the exchange of Higgs scalars. However, this FCNC problem is a common subject to be overcome not only in the present model but also in most models with two Higgs doublets. The conventional mass matrix models based on a grand unified theory (GUT) scenario cannot give realistic mass matrices without assuming more than two Higgs scalars [8]. Besides, if we admit that two such scalars remain until the low energy scale, the well-known beautiful coincidence of the gauge coupling constants at $\mu \sim 10^{16}$ GeV will be spoiled. Although the present model is not based on a GUT scenario, as are the conventional mass matrix models, for the FCNC problem, we optimistically consider that only one component of the linear combinations among those Higgs scalars survives at the low energy scale $\mu = m_Z$, while the other component is decoupled at $\mu < M_X$ [9]. The study of the renormalization group (RGE) effects given in the Appendix will be based on such an “effective” one-Higgs scalar scenario.

### III. QUARK MIXING MATRIX

The quark mass matrices

$$M_f = P^1_f M_f^1 P^1_f (f = u, d),$$  

(3.1)

are diagonalized by the bunitary transformation

$$D_f = U_L^f M_f U_R^f,$$  

(3.2)

where $U_L^f = P^1 f U_f$, $U_R^f = P_f U_f$, and $O_d (O_u)$ is given by Eq. (1.8). Then, the Cabibbo-Kobayashi-Maskawa (CKM) [10] quark mixing matrix ($V$) is given by

$$V = U^f_L U_d = O^f_f U_f^a D_f$$

(3.3)

where $\rho$ and $\sigma$ are defined by

$$\rho = \frac{1}{2} (e^{i\delta_1} + e^{i\delta_2}) = \cos \frac{\delta_1 - \delta_2}{2} \exp \left( -i \frac{\delta_1 + \delta_2}{2} \right),$$

(3.4)

$$\sigma = \frac{1}{2} (e^{i\delta_1} - e^{i\delta_2}) = \sin \frac{\delta_1 - \delta_2}{2} \exp \left( i \frac{\delta_1 + \delta_2}{2} + \frac{\pi}{2} \right).$$

(3.5)

Here we have set $p = p_u p_d = \text{diag}(e^{i \delta_1}, e^{i \delta_2}, e^{i \delta_3})$, and we have taken $\delta_3 = 0$ without loss of generality. Then, the explicit magnitudes of the components of $V$ are expressed as

$$|V_{cb}| = |\sigma| c_\rho = \frac{\delta_3 - \delta_2}{\sqrt{1 + m_u/m_c}},$$

(3.6)

$$|V_{ub}| = |\sigma| s_\rho = \frac{\delta_3 - \delta_2}{\sqrt{1 + m_u/m_c}} \sqrt{\frac{m_u}{m_c}},$$

(3.7)

$$|V_{ts}| = |\sigma| c_\rho = \frac{\delta_3 - \delta_2}{\sqrt{1 + m_d/m_s}},$$

(3.8)

$$|V_{td}| = |\sigma| s_\rho = \frac{\delta_3 - \delta_2}{\sqrt{1 + m_d/m_s}} \sqrt{\frac{m_d}{m_s}},$$

(3.9)

$$|V_{us}| = c_{\rho} s_\rho = 1 - \rho s_{\delta_1} c_{\delta_2}$$

$$= \sqrt{\frac{m_c}{m_c + m_u}} \sqrt{\frac{m_u}{m_d + m_d}} \times \left[ 1 - 2 \cos \frac{\delta_1 - \delta_2}{2} \cos \frac{\delta_1 + \delta_2}{2} \sqrt{\frac{m_m}{m_m}} \right].$$

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The heavy-quark-mass-independent predictions (3.13) and (3.14) have first been derived from a special ansatz for quark mixings by Branco and Lavoura [13], and later, a similar formulation has also been given by Fritzsch and Xing [14]. For example, the CKM matrix $V$ is given by the form \[ V = R_{12}(\theta_u)R_{23}(\theta_Q, \phi_Q)R_{13}^\dagger(\theta_d) \] in the Fritzsch-Xing ansatz, and their rotation $R_{23}(\theta_Q, \phi_Q)$ with a phase $\phi_Q$ corresponds to $R_{23}(\pi/4)P_3P_4R_{23}^\dagger(\pi/4)$ in the present model, because the present rotation given in Eq. (1.8) is expressed as $O_f = R_{23}(\pi/4)R_{13}(\theta)$.

Next let us fix the parameters $\delta_3$ and $\delta_2$. When we use the expressions (3.6)–(3.11) at $\mu = m_Z$, the parameters $\delta_2$ and $\delta_3$ do not mean the phases that are evolved from those at $\mu = M_X$. Hereafter, we use the parameters $\delta_2$ and $\delta_3$ as phenomenological parameters that approximately satisfy the relations (3.6)–(3.11) at $\mu = m_Z$. In order to fix the value of $\delta_3 - \delta_2$, we use the relation (3.6), which leads to

$$\sin \frac{\delta_3 - \delta_2}{2} = \sqrt{1 + \frac{m_u}{m_c}}|V_{cb}|_{\exp} = 0.0401 \pm 0.0018,$$

$$\delta_3 - \delta_2 = 4.59^\circ \pm 0.21^\circ.$$  

Then, we obtain

$$|V_{ub}| = \sqrt{\frac{m_d}{m_c}}|V_{cb}|_{\exp} = 0.00234 \pm 0.00028,$$

$$|V_{ts}| = \sqrt{\frac{m_d}{m_s}}|V_{cb}|_{\exp} = 0.0391 \pm 0.0018,$$

$$|V_{td}| = \sqrt{\frac{m_d}{m_s}}|V_{cb}|_{\exp} = 0.00880 \pm 0.00094,$$

which are consistent with the present experimental data. Therefore, the value (3.17) is acceptable as reasonable. Then, by using the value (3.17) and the expression (3.10), we can obtain the remaining parameter $(\delta_3 + \delta_2)$:

$$\delta_3 + \delta_2 = 93^\circ \pm 22^\circ \text{ or } -80^\circ \pm 22^\circ.$$  

Since $\sin(\delta_3 - \delta_2)/2 = 0.04$ and $\cos(\delta_3 + \delta_2)/2 = 0.2$, the present model also predicts the following approximated relations:

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so that we obtain
\[ |V_{us}| = c_{us} s_d |1 - \rho c_{us} c_d| = \sqrt{\frac{m_d}{m_s}}, \]  
(3.22)
\[ |V_{cd}| = c_{us} s_d |\rho - \frac{s_u c_d}{c_u s_d}| = \sqrt{\frac{m_d}{m_s}}. \]  
(3.23)

On the other hand, in the standard expression of Eq. (3.11), we obtain
\[ \rho = \frac{|s_{ud}|}{|s_{cd}|}. \]  
(3.24)

Using the rephasing of the up-type and down-type quarks, Eq. (3.3) is changed to the standard representation of the CKM quark mixing matrix
\[ V_{us} = (e^{i \alpha_u} e^{i \alpha_d} e^{i \alpha_s}) V R_{us}, \]
\[ V_{cd} = (e^{i \alpha_c} e^{i \alpha_d} e^{i \alpha_s}) V R_{cd}. \]

Here, \( \alpha_u \) comes from the rephasing in the quark fields to make the choice of phase convention. The \( CP \)-violating phase \( \delta \) in the representation (3.25) is expressed with the expression \( V \) in Eq. (3.3) by
\[ \delta = \arg \left[ \frac{|V_{12} V_{22}^\dagger|}{\sqrt{|V_{12}|^2 + |V_{13}|^2}} \right]. \]  
(3.26)
so that we obtain
\[ \delta = \pm (80^\circ \pm 22^\circ). \]  
(3.27)
It is interesting that nearly maximal \( |\sin \delta| \) is realized in the present model.

The rephasing invariant Jarlskog parameter \( J \) [15] is defined by \( J = \text{Im}(V_{us} V_{cd}^\dagger V_{ub} V_{cb}) \). In the present model with Eqs. (3.6)–(3.11), the parameter \( J \) is given by
\[ J = |\sigma|^2 |c_{us} c_d| \frac{\delta_1 + \delta_2}{2}, \]
\[ = \left| \frac{V_{ub}}{|V_{ub}|^2 + |V_{ub}|^2} \right| \sin \delta. \]  
(3.28)

Using the relation \( |V_{ud}| \approx |V_{cb}| |V_{us}| \) in Eq. (3.24), and the experimental findings \( |V_{us}|^2 \approx |V_{cb}|^2 \approx |V_{ub}|^2, \) \( |V_{ts}| \approx |V_{cb}|, \) and \( |V_{tb}| \approx 1, \) we obtain
\[ J \approx \left| \frac{V_{ub}}{|V_{ub}|^2 + |V_{ub}|^2} \right| |V_{us}| \sin \delta. \]  
(3.29)

On the other hand, in the standard expression of \( V \), Eq. (3.25), \( J \) is given by
\[ J = \left( \frac{V_{ub}}{|V_{ub}|^2 + |V_{ub}|^2} \right) \sin \delta. \]

Comparing Eq. (3.29) with Eq. (3.30), we obtain
\[ \sin \delta = \sin \frac{\delta_1 + \delta_2}{2}. \]  
(3.31)

By using the numerical results (3.17)–(3.21), we obtain
\[ |J| = (1.91 \pm 0.38) \times 10^{-5}. \]  
(3.32)

IV. LEPTON MIXING MATRIX

Let us discuss the lepton sectors. We assume that the neutrino masses are generated via the seesaw mechanism [16]:
\[ M_\nu = M_D M_R^{-1} M_D^T. \]  
(4.1)

Here \( M_D \) and \( M_R \) are the Dirac neutrino and the right-handed Majorana neutrino mass matrices, which are defined by \( \bar{\nu}_L M_D v_R \) and \( \bar{\nu}_R M_R v_R \), respectively. Since \( M_D = P_D^\dagger \tilde{M}_D P_D \) and \( M_R = P_R^\dagger \tilde{M}_R P_R \), according to the assumption (2.9), we obtain
\[ 
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\]
Here and hereafter, $m_1$, $m_2$ and $m_3$ denote neutrino masses unless they are specifically mentioned. In the last expression, we have used the fact\(^1\) that the product of $AB^{-1}A$ of the matrices $A$ and $B$ with the texture (1.1) with Eq. (1.2) again becomes a matrix with the texture (1.1) with (1.2).

On the other hand, the charged lepton matrix $M_e$ is given by

$$M_e = P_e\begin{pmatrix} 0 & \sqrt{\frac{m_\mu m_e}{2}} & \sqrt{\frac{m_\mu m_e}{2}} \\ \sqrt{\frac{m_\mu m_e}{2}} & \frac{1}{2} m_e \left(1 + \frac{m_\mu - m_e}{m_\tau} \right) & -\frac{1}{2} m_e \left(1 - \frac{m_\mu - m_e}{m_\tau} \right) \\ -\sqrt{\frac{m_\mu m_e}{2}} & \frac{1}{2} m_e \left(1 - \frac{m_\mu - m_e}{m_\tau} \right) & \frac{1}{2} m_e \left(1 + \frac{m_\mu - m_e}{m_\tau} \right) \end{pmatrix} P_e^\dagger, \quad (4.3)$$

where $m_e$, $m_\mu$, and $m_\tau$ are charged lepton masses.

Those mass matrices $M_e$ and $M_\mu$ are diagonalized as $(P_e^\dagger O_e)^\dagger M_e (P_e O_e) = D_e$ and $(P_\mu^\dagger O_\mu)^\dagger M_\mu (P_\mu O_\mu) = D_\mu$, respectively, where

$$O_e = \begin{pmatrix} c_\epsilon & s_\epsilon & 0 \\ -\frac{s_\epsilon}{\sqrt{2}} & \frac{c_\epsilon}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ -\frac{c_\epsilon}{\sqrt{2}} & -\frac{s_\epsilon}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{pmatrix} \quad \text{and} \quad O_\mu = \begin{pmatrix} c_\nu & s_\nu & 0 \\ -\frac{s_\nu}{\sqrt{2}} & \frac{c_\nu}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ -\frac{c_\nu}{\sqrt{2}} & -\frac{s_\nu}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{pmatrix} \quad \text{with} \quad \delta_{\nu_1} = 0 \quad \text{without loss of generality.}$$

(4.4)

The explicit forms of absolute magnitudes of the components of $U$ are given by expressions similar to Eqs. (3.4)–(3.12), where $[V_{ij}]$, $(m_\mu, m_e, m_\tau)$, and $(m_d, m_s, m_b)$ are replaced by $[U_{ij}]$, $(m_1, m_2, m_3)$, and $(m_{\mu e}, m_{\mu \tau}, m_{\tau})$, respectively. It should again be noted that the elements of $U$ are independent of $m_\tau$ and $m_3$. The independent parameters of the unitary matrix $U$ are $\theta_e = \tan^{-1}(m_\mu/m_\tau)$, $\theta_\nu = \tan^{-1}(m_1/m_2)$, $\delta_{\nu_3}$, and $\delta_{\tau_2}$. Among them, $\theta_e$ is given by charged-lepton masses of the first and second generations. Therefore, the model has the three adjustable parameters $\delta_{\nu_3}$, $\delta_{\tau_2}$, and $m_1/m_2$ to reproduce the experimental values [11].

Let us estimate the values $\theta_\nu$, $\delta_{\nu_3}$ and $\delta_{\tau_2}$ by fitting the experimental data. In the following discussions we consider the normal mass hierarchy $\Delta m^2_3 = m^2_3-m^2_2 > 0$ for the neutrino mass. The case of the inverse mass hierarchy $\Delta m^2_{32} < 0$ is quite similar to it. It follows from the CHOOZ [19], solar [20], and atmospheric neutrino experiments [1] that

$$|U_{13}|^2 \exp < 0.03. \quad (4.6)$$

From the global analysis of the SNO solar neutrino experiment [20].

\(^1\)The seesaw invariant texture form was discussed systematically in [17].
\[
\Delta m_{12}^2 = m_2^2 - m_1^2 = \Delta m_{\text{sol}}^2 = 5.0 \times 10^{-5} \text{ eV}^2, \quad (4.7)
\]
\[
\tan^2 \theta_{12} = \tan^2 \theta_{\text{sol}} = 0.34, \quad (4.8)
\]
with \(\chi^2_{\text{min}}/\text{DOF}=57.07/2\), for the large mixing angle (LMA) Mikheyev-Smirnov-Wolfenstein (MSW) solution. From the atmospheric neutrino experiment [1], we also have
\[
\Delta m_{23}^2 = m_3^2 - m_2^2 = \Delta m_{\text{atm}}^2 = 2.5 \times 10^{-3} \text{ eV}^2, \quad (4.9)
\]
\[
\sin^2 2\theta_{23} = \sin^2 2\theta_{\text{atm}} = 1.0, \quad (4.10)
\]

with \(\chi^2_{\text{min}}/\text{DOF}=163.2/170\).

Independently of the parameters \(\delta_{\nu_3}\) and \(\delta_{\nu_2}\), the model predicts the following two ratios:
\[
\frac{|U_{13}|}{|U_{23}|} = \frac{s_\nu}{\cos \theta} = \frac{\sqrt{m_\mu}}{m_\mu} \sqrt{\frac{0.048}{103}} = 0.0688, \quad (4.11)
\]
\[
\frac{|U_{31}|}{|U_{32}|} = \frac{s_\nu}{\cos \theta} = \frac{m_1}{m_2}, \quad (4.12)
\]

Here we have used the running charged-lepton mass at \(\mu = m_Z\) [12]: \(m_\nu(m_Z) = 0.48684727 \pm 0.00000014\) MeV, and \(m_\mu(m_Z) = 102.75138 \pm 0.00033\) MeV. The neutrino mixing angle \(\theta_{\text{atm}}\) under the constraint \(|\Delta m_{23}^2| \gg |\Delta m_{12}^2|\) is given by
\[
\sin^2 2\theta_{\text{atm}} = 4|\langle U \rangle |^2 = 4|\rho_{\nu_1}|^2 |\rho_{\nu_2}|^2 \cos^2 \theta = \sin^2 2\theta_{\nu_3 - \nu_2} \sqrt{\frac{m_\mu}{m_\mu + m_e}}, \quad (4.13)
\]

The observed fact \(\sin^2 2\theta_{\text{atm}} = 1.0\) highly suggests \(\delta_{\nu_3 - \nu_2} = \pi/2\). Hereafter, for simplicity, we take
\[
\delta_{\nu_3 - \nu_2} = \frac{\pi}{2}. \quad (4.14)
\]

Under the constraint (4.13), the model predicts
\[
U_{\text{std}} = \begin{pmatrix}
S_{\nu_1 \nu_2} S_{\nu_3 \nu_12} & C_{\nu_1 \nu_2} S_{\nu_3 \nu_12} e^{i\delta_{\nu_3 - \nu_2}} & S_{\nu_3} S_{\nu_12} e^{i\delta_{\nu_3}} \\
-S_{\nu_2} S_{\nu_3} & C_{\nu_3} S_{\nu_12} & C_{\nu_3} \nu_12 e^{i\delta_{\nu_3}} \\
S_{\nu_2} & -C_{\nu_2} S_{\nu_3} & S_{\nu_3} S_{\nu_12}
\end{pmatrix}, \quad (4.20)
\]
as
\[
U_{\text{std}} = \text{diag}(e^{ia_1^\nu}, e^{ia_2^\nu}, e^{ia_3^\nu}) U \text{ diag}(e^{\pm i\pi/2}, 1, 1). \quad (4.21)
\]

Here, \(a_i^\nu\) comes from the rephasing in the charged lepton fields to make the choice of phase convention, and the specific phase \(\pm \pi/2\) is added on the right-hand side of \(U\) in order to change the neutrino eigenmass \(m_1\) to a positive quantity. Similarly to the quark sector, the \(CP\)-violating phase \(\delta_{\nu_3}\) in the representation (4.20) is expressed as
Hence, we can also predict the averaged neutrino mass $m_\nu$ from the observed value of $\tan^2 \theta_{13}$ [22, 23]. The value of Eq. (5) is highly sensitive to the value of $(\delta_{\alpha_3} - \delta_{\alpha_2})/2$, which is unknown at present, because the values $s_{\alpha_3}/c_{\alpha_3} = \sqrt{m_1/m_2} = 0.070$ and $m_1/m_3 = 0.088$ are in the same order. For $(\delta_{\alpha_3} + \delta_{\alpha_2})/2 = 0$, $\pi/2$ and $\pi$, we obtain the numerical results $\langle m_\nu \rangle = 0.00018$ eV, 0.00049 eV and 0.00069 eV, respectively. However, these values should not be taken strictly because the value $m_1/m_2$ is also sensitive to the observed value of $\tan^2 \theta_{13}$. In any case, the predicted value of $\langle m_\nu \rangle$ will be less than the order of $10^{-5}$ eV.

The rephasing-invariant parameter $J$ in the lepton sector is defined by

$$J = |\sigma_\mu|^2 |\rho_\mu| c_{\alpha_3} s_{\alpha_3} c_{\alpha_2} s_{\alpha_2} \sin \frac{\delta_{\alpha_3} + \delta_{\alpha_2}}{2}$$

which is explicitly given by

$$J = \frac{|U_{13}| |U_{31}| |U_{32}| |U_{33}|}{|U_{23}|} \frac{\delta_{\alpha_3} + \delta_{\alpha_2}}{2}$$

The upper bound is described in terms of the ratio $m_1/m_2$, so that we obtain

$$J \leq 0.019.$$ (4.27)

It should be noted that if we again assume the maximal CP violation in the lepton sector, the magnitude of the rephasing invariant $|J|$ can be considerably larger than in the quark sector, $|J_{\text{quark}}| \approx 2 \times 10^{-5}$.

V. CONCLUSION

In conclusion, stimulated by recent neutrino data, which suggest a nearly bimaximal mixing, we have investigated the possibility that all the mass matrices of quarks and leptons have the same texture as the neutrino mass matrix. We have assumed that the mass matrix form is constrained by a discrete symmetry $Z_3$ and a permutation symmetry $S_2$, i.e. that the texture is given by the form (1.1) with Eq. (1.2). The most important feature of the present model is that the textures (1.1), (1.2) are practically applicable to the predictions at the low energy scale (the electroweak scale), although we assume that the textures are exactly given at a unification scale.

It is well known that the matrix form (1.1) leads to a bimaximal mixing in the neutrino sector. In the present model, the mixing angle $\theta_{12}'$ between the first and second generations is given by

$$\tan \theta_{12}' = \sqrt{m_1/m_2},$$ (5.1)

where $m_1'$ and $m_2'$ are the first and second generation fermion masses. This leads to a large mixing in the lepton mixing matrix (MNSP matrix) $U$ with $m_1 \sim m_2$ (neglecting $\tan \theta_{12}' = \sqrt{m_1/m_2}$ in the charge-lepton sector), and it also leads to the famous formula [24] $|V_{ud}| = \sqrt{m_1/m_2}$ in the quark mixing matrix (CKM matrix) $V$ (neglecting $\tan \theta_{12}' = \sqrt{m_1/m_2}$ in the up-quark sector). In the present model the mixing angle $\theta_{23}'$ between the second and third generation is fixed as $\theta_{23}' = \pi/4$. However, the (2,3) component of the quark mixing matrix $V$ (and also the lepton mixing matrix $U$) is highly dependent on the phase difference $\delta_3 - \delta_2$, as follows:

$$V_{23} = \frac{1}{\sqrt{1 + m_1/m_2}} \frac{\delta_3 - \delta_2}{2},$$ (5.2)
where $\delta_i = \delta^u_i - \delta^d_i$. Replacing the arguments by their lepton counterparts, we have the same form for $U_{23}$. We have understood the observed values $V_{23}$ and $U_{23}$ by taking $(\delta^u - \delta^d)/2$ as a small value for the quark sectors and as $\pi/2$ for the lepton sectors, respectively. As predictions, which are independent of such phase parameters, there are two relations

$$|V_{ub}/V_{cb}| = \sqrt{m_u/m_c}, \quad |V_{td}/V_{ts}| = \sqrt{m_d/m_s}$$

and the similar relations for $U$). The relations (5.3) are in agreement with experiments. The relation $|U_{13}/U_{23}| = \sqrt{m_s/m_d}$ in the lepton sectors leads to $|U_{13}|^2 = m_s^2/2m_d = 0.0024$ if we accept $\sin^22\theta_{\text{ atm}} = 1.0$. This value will be testable in the near future.

Since, in the present model, each mass matrix $M_f$ (i.e. the Yukawa coupling $Y_f$) takes different values of $A_f$, $B_f$, and so on, the present model cannot be embedded into a GUT scenario. In spite of such a demerit, however, it is worthwhile noting that it can give a unified description of quark and lepton mass matrices with the same texture.

**ACKNOWLEDGMENTS**

One of the authors (Y.K.) wishes to acknowledge the hospitality of the Theory group at CERN, where this work was completed. Y.K. also thanks Z. Z. Xing for informing him of many helpful references.

**APPENDIX**

The mass matrix texture (1.1) with Eq. (1.2), which is defined at the unification energy scale $\mu = M_X$, is applicable to the phenomenology at the electroweak scale $\mu = m_Z$. In the present appendix, we demonstrate this for the quark mass matrices $M_u$ and $M_d$.

It is well known [25] that the energy scale dependences $R(A) = A(\mu)/A(M_X)$ for observable quantities $A$ approximately satisfy the relations $R(|V_{ub}|) = R(|V_{cb}|) = R(|V_{td}|) = R(|V_{ts}|) = R(m_d/m_s) = R(m_s/m_d)$, and that the ratios $R(|V_{ud}|) = R(U_{1d}) = R(U_{1d}) = R(m_d/m_s) = R(m_s/m_d)$ are approximately constant. This is caused by the fact that the Yukawa coupling constant $y_i$ of the top quark is extremely large with respect to other coupling constants. The above relations on $R$ are well explained by the approximation $y_t^2 \approx y_b^2, y_e^2$. Therefore, we will also use approximation below.

The one-loop RGE for the Yukawa coupling constants $Y_f (f = u, d)$ has the form

$$\frac{dY_f}{dt} = \frac{1}{16\pi^2} \left( C_f \gamma_1 + C_{ff} Y_f' Y_f + C_{ff'} Y_f' Y_{f'} \right) Y_f,$$

where $f' = d$ ($f' = u$) for $f = u$ ($f = d$), and the coefficients $C_f$, $C_{ff}$ and $C_{ff'}$ are energy scale dependent factors which are calculated from the one-loop Feynman diagrams. We start from the Yukawa coupling constants $Y_f(M_X)$, corresponding to the mass matrix form (1.1) [with Eq. (1.2)].

Since the matrix $Y_u Y_u^\dagger$ is approximately given by

$$Y_u(M_X) Y_u^\dagger(M_X) \approx \frac{m_t^2}{v_u^2} \begin{pmatrix} 0 & 0 & 0 \\ 0 & 1 - e^{+i\delta_u} & 0 \\ 0 & 0 & 1 \end{pmatrix},$$

where $\delta_u = \delta^u_1 - \delta^u_2$, $v_u / \sqrt{2} = (H_u^0)$, and we have used the relations (1.6) and the approximation $y_t^2 \approx y_b^2, y_e^2$, the up-quark Yukawa coupling constant $Y_u(\mu)$ in the neighborhood of $\mu = M_X$ is given by the form

$$Y_u(\mu) = r_u(\mu) \begin{pmatrix} 0 & 0 & 0 \\ 0 & 1 - e^{+i\delta_u} & 0 \\ 0 & 0 & 1 \end{pmatrix} Y_u(M_X) \left[ 1 + e_u(\mu) \begin{pmatrix} 0 & 0 & 0 \\ 0 & 1 - e^{+i\delta_u} & 0 \\ 0 & 0 & 1 \end{pmatrix} \right]$$

$$= \frac{r_u(\mu)}{v_u / \sqrt{2}} \begin{pmatrix} 0 & A_u e^{-i\delta^u_2} & A_u e^{-i\delta^u_3} \\ A_u(1 + e_u - e_u) e^{-i\delta^u_2} & B_u(1 + e_u - e_u) e^{-2i\delta^u_2} & C_u(1 + e_u - e_u) e^{-i(\delta^u_2 + \delta^u_3)} \\ A_u(1 + e_u - e_u) e^{-i\delta^u_3} & C_u(1 + e_u - e_u) e^{-i(\delta^u_2 + \delta^u_3)} & B_u(1 + e_u - e_u) e^{-2i\delta^u_3} \end{pmatrix}.$$

(3)

Although this form is one in $\mu = M_X$, since the texture keeps the same form under the small change of energy scale, as a result, the texture of $Y_u(\mu)$ given by Eq. (3) holds at any energy scale $\mu$. Therefore, we can obtain the expression (1.1) at an arbitrary energy scale $\mu$. [The demonstration (3) has been done for the case $P_R = P_L$ mentioned in Eq. (2.10). However, the conclusion does not depend on this choice.]

On the other hand, the evolution of the down-quark Yukawa coupling constant $Y_d(\mu)$ is somewhat complicated. By a way similar to Eq. (3), we obtain
where \( \delta_u - \delta_d = (\delta_u^0 - \delta_d^0) - (\delta_u^d - \delta_d^d) = \delta_1 - \delta_2 \). Note that the part that is sandwiched between \( P^\dagger_d \) and \( P^d \) includes imaginary parts and those phase factors cannot be removed by an additional phase matrix \( P_d(\mu) \) into the form \( P^\dagger_d(\mu) Y_d(\mu) P^d(\mu) \).

However, the quantity that has the physical meaning is \( Y_d Y^\dagger_d \). When we define

\[
\xi e^{-i\alpha} = 1 + \epsilon_d (1 - e^{-i(\delta_u - \delta_d)}), \quad \eta e^{+i\beta} = 1 + \epsilon_d (1 + e^{i(\delta_u - \delta_d)}),
\]

we obtain

\[
Y_d(\mu) Y^\dagger_d(\mu) = \frac{r^2_d(\mu)}{v_d^2/2} P^\dagger_d P \left( \begin{array}{ccc} A^2_d & A_d(B_d + C_d) & A_d(B_d + C_d) \\ A_d(B_d + C_d) & A^2_d + (B_d^2 + C_d^2) \eta^2 & A^2_d e^{2i(\alpha + \beta)} + 2B_d C_d \eta^2 \\ A_d(B_d + C_d) & A^2_d e^{2i(\alpha - \beta)} + 2B_d C_d \eta^2 & A^2_d + (B_d^2 + C_d^2) \eta^2 \end{array} \right) P^\dagger_d P,
\]

where

\[
P_\beta = \text{diag}(1, e^{i\beta}, e^{-i\beta}),
\]

so that we can obtain a real matrix for the part which is sandwiched by the phase matrix \( P_d P_\beta \) under the approximation \( A^2_d [B_d C_d] = 0 \). This means that we can practically write

\[
\hat{Y}_d(\mu) = \frac{r_d(\mu)}{v_d \sqrt{2}} \left( \begin{array}{ccc} 0 & A_d & A_d \\ A_d & B_d \eta & C_d \eta \\ A_d & C_d \eta & B_d \eta \end{array} \right),
\]

with

\[
P^\dagger_d(\mu) = \text{diag}(1, e^{-i(\delta_u^d - \beta)}, e^{-i(\delta_u^d + \beta)}),
\]

at an arbitrary energy scale \( \mu \). It should be noted that the changes of the phases \( \delta_u^d \rightarrow \delta_d^d - \beta \) and \( \delta_u^d \rightarrow \delta_u^d + \beta \) do not come from the evolution of the phases \( \delta_u^d(\mu) \) and \( \delta_u^d(\mu) \), but they are brought effectively by absorbing the unfactorizable phase parts in \( Y_d(\mu) \). Thus, we can again use the texture (1.1) at an arbitrary energy scale \( \mu \) from a practical point of view.

In the Yukawa coupling constants \( Y_e \) and \( Y_\nu \) of the leptons, the RGE effects are not so large as in the quark sectors. In the charged lepton sector, since \( m^2_e \gg m^2_\mu \gg m^2_\tau \), we can again demonstrate that the expression (1.1) is applicable at an arbitrary energy scale in a way similar to the quark sectors. For the neutrino Yukawa coupling constant \( Y_\nu(\mu) \), the evolution equation is different from Eq. (A1). We must use the RGE for the seesaw operator \([26]\). However, the calculation and result are essentially the same as those in \( Y_d(\mu), Y_e(\mu) \) and \( Y_\nu(\mu) \), because \( m^2_\tau \gg m^2_\mu \gg m^2_e \) in the present model.

Finally, we would like to add that these conclusions on the evolution of the mass matrices \( M_f (f = u, d, e, \nu) \) are exactly confirmed by numerical study, without approximation.

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