Polarized Foreground from Thermal Dust Emission

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Abstract. In this review, we intend to present the current knowledge of the polarized emission from thermal dust in our Galaxy. We show different methods to estimate the spatial distribution statistics of this emission in the lack of any data from the diffuse ISM, and compare it to the expected CMB polarized signal. We finally show how this contaminant could be efficiently removed from CMB maps using multi-frequency observations.

1. Introduction

The polarization of the cosmic microwave background was the subject of extensive theoretical studies in the last three years (Seljak 1997, Seljak & Zaldarriaga 1997, Zaldarriaga & Seljak 1997, Kamionkowski et al. 1997a, Kamionkowski et al. 1997b, Kamionkowski & Kosowsky 1998). Indeed, it should provide us with additional information on the cosmological parameters, in particular the reionization optical depth (Zaldarriaga 1997) and the tensor-related parameters $n_T$ (spectral index of tensor perturbations), and $T/S$ (tensor to scalar amplitudes ratio, see Kamionkowski et al. 1997b, Seljak & Zaldarriaga 1997).

Although the existence of polarization in the CMB has first been predicted long ago in the context of an anisotropic universe (Rees 1968), there are so far only upper limits on the polarization level of the cosmic microwave background (Netterfield et al. 1995), which in the case of an isotropic universe is expected to be at best 10% of the anisotropy level (Bond & Efstathiou 1984). The late revival of interest in the polarized part of the CMB is motivated by the fact that the sensitivities of future satellite missions like MAP and PLANCK are comparable to the expected level of polarization in the CMB.

While this is enough to raise the interest of theoreticians in the matter, the issue of the measurability of such a signal in a given instrumental configuration needs further investigation. In particular, one needs to study the level of contamination of this polarized signal by other astrophysical sources of emission as a function of both frequency and spatial scale.
This paper intends to give the present status of our knowledge concerning a Galactic foreground polarized emission: the polarized emission coming from the thermal emission of aligned dust grains.

The polarization of the stellar light in absorption has been discovered a long time ago (Hiltner 1949, Hall 1949). This polarization has been interpreted as a selective absorption of the stellar light by dust grains aligned respectively to magnetic field. Currently UV, optical, near-infrared and far-infrared polarimetry is one of the major sources of information about magnetic field structure.

The very same polarization is the impediment for the cosmic microwave studies, as it shall contribute to the polarized signal to be measured by MAP and PLANK. In this review we estimate the level of contamination of polarized signal by galactic dust.

In what follows we estimate the intrinsic polarized emissivity of dust at submillimeter wavelengths (section 2), discuss the angular distribution of polarized dust emission (section 3) and compare the dust polarized spectrum with that of CMB (section 4). We discuss intended work in section 5 and summarize our results in section 6.

2. Intrinsic polarized emissivity of Galactic dust at submillimeter wavelengths

2.1. Constraints on the grains shape and sizes

As we will see in this section, the polarized emissivity of the grains depends on their shape, more specifically on their asphericity. There have been therefore several attempts to constrain their shape using optical (Kim & Martin 1995 and references therein) and infrared data (Hildebrand & Dragovan 1995, Lee & Draine 1985 and references therein). The idea is to constrain the shape of the grains by fitting the observed polarization (in absorption) to extinction ratios of stellar light in some frequency range. These ratios, in the UV and optical wavelengths, are very sensitive to the size distribution of the grains (because the size of the grains and the wavelengths observed are comparable), and therefore are mainly used to constrain it, but they nevertheless tend to show that, assuming spheroidal shape for the grains, oblate grains are somewhat preferred to prolate grains.

On the other hand, given the dielectric properties of the grain materials in the infrared, it is possible to constrain the shape of the grains using the same ratios, but in a frequency range covering some absorption features. This is what has been done around the $3.1\mu m$ $H_2O$ ice feature, and around the $9.7\mu m$ silicate feature (see Hildebrand & Dragovan 1995, Lee & Draine 1985). At those wavelengths, the polarization curves are less sensitive to the size distribution of the grains, provided that the maximal size of the distribution remains much smaller than the wavelengths of interest. Indeed, we are then in the conditions where the dipole approximation
for the computation of polarized cross-sections is valid (cf Draine & Lee 1984).

Hildebrand & Dragovan (1995), analyzing the polarization in absorption around the 9.7 $\mu$m silicate feature, found that this curve was best fitted by oblate grains with an axis-ratio of (2 : 3); this is in agreement with the other studies of Kim & Martin (1995) and Lee & Draine (1985) who both inferred that the correct shape of the grains should be oblate, with an axis ratio of (1 : 2). This axis ratio, inferred from near-infrared and optical spectro-polarimetry, predicts a polarization degree in emission of 35% for perfectly aligned grains on a uniform magnetic field in the plane of the sky. Of course, such an idealized situation is not likely to occur, and different depolarization mechanisms will diminish the observed degree of polarization in the FIR wavelengths. Such mechanisms are discussed in the next section.

One could argue that the grains responsible for the near-infrared and optical dichroic absorption could be different from those responsible for the FIR polarized emission (see Goodman 1995). This indeed may be true for molecular clouds, where cold grains far from emission sources may not be aligned (Lazarian, Goodman & Myers 1997, Hildebrand et al. 1999). In diffuse medium, however, there are both theoretical and observational reasons to believe that adsorbing and emitting grains are indeed the same.

First, suppose that a grain population emits a polarized thermal radiation in the FIR, if we want this population not to contribute to the polarization by absorption in the optical or near-infrared region, the size of those grains should be very big ($\gtrsim 10 \mu$m), and they would then induce a lot of absorption in the near-infrared, which is not seen (unless we grossly overestimate the absorption properties of the grain materials). Another reason why such hypothesis does not seem plausible is the measured metallicity of the diffuse interstellar medium, indeed the interstellar abundance of Si provides us with barely enough grain material to build the grain mass distributions necessary to fit the optical data (Kim et al. 1994, Grevesse & Anders 1989).

There are also observational reasons to believe that the same grains are responsible for the polarization by absorption in the near infra-red and the polarized emission in the FIR. Indeed, one of the simplest tests of this ansatz is the expected orthogonality of the direction of polarization between the two wavelengths regimes (see below). This orthogonality was verified by Hildebrand et al. 1984 in Orion (see also Hildebrand 1988). A good match of the expected maximum polarization in emission predicted from the observed polarization to extinction ratio in the near-infrared also supports the idea that the grains are the same.

Indeed, if we consider an optically thin medium, we can show that the polarization degree in emission $P_{em}$ and polarization degree in absorption $P_{abs}$ are related via a simple relation

$$P_{em}(\lambda) = -P_{abs}(\lambda) / \tau(\lambda)$$

(1)
(see Hildebrand 1988). From the observed polarization to extinction ratio at $\lambda_2 = 2.2\mu m$ (Jones et al. 1992) and using the dielectric functions of Draine & Lee (1984) for a mixture of “astronomical silicate” and graphite grains to compute $P_{em}(\lambda_1)/P_{em}(\lambda_2)$ Hildebrand & Dragovan (1995) obtained, using eq.(1), the maximum polarization one could expect at $\lambda_1 = 100\mu m$. They find a maximum polarization of the order of 9%, which corresponds to the maximum polarization observed by Hildebrand et al. (1995) in the Orion nebulae.

It seems then that the same grains are responsible for the polarization of starlight in the near-infrared and for the FIR polarized emission. We will now expose the different depolarization mechanisms responsible for the discrepancy between the observed maximum polarization (9%) and the theoretical prediction of 35%.

2.2. Polarized emissivity of the grains - Sources of depolarization

As shown in Greenberg (1968), the dichroism of the medium (defined as the observed polarization cross-section can be written as:

$$C_x - C_y = C_{pol}RF\cos^2(\zeta)$$

where $(x, y)$ defines the plane of the sky, the regular magnetic field being in the $(y, z)$ plane, making an angle $\zeta$ with the $y$ axis (see Fig...). The radiation is experiencing different depolarisation effects, as it travels towards the observer:

- $R = 3/2(\langle\cos^2(\beta)\rangle - 1/3)$ is the so-called “Rayleigh reduction factor”, which gives the amount of depolarisation arising from the imperfect alignment of dust grains on the magnetic field lines, $\beta$ being the precession angle of the grain (for a review on dust grain alignment mechanisms, see e.g. Lazarian et al. 1997).

- $F = 3/2(\langle\cos^2(\theta)\rangle - 1/3)$ is the reduction factor coming from a turbulent random component of the Galactic magnetic field. In this case, $\theta$ is the angle between the random and the regular component of the field.

- Finally, the $\cos^2(\zeta)$ factor arises from a projection effect.

As stressed by Goodman (1996), the observed polarization flux in absorption depends on the direction of the magnetic field, and its fluctuations, while the extinction does not$^1$. Thus, there is a large scatter in the ob-

$^1$There is in fact a small difference of extinction for a population of perfectly aligned grains and for another population of the same grains but unaligned. This difference could account for an error on the determination of $\tau$ no bigger than 10%, which is less than the usual observational errors on $P_{abs}/\tau$ anyway (see Lee & Draine 1985).
Figure 1. The geometry of observations. Angle $\zeta$ is between the magnetic field and the $XY$-plane. After Roberge & Lazarian 1999.

Observed values of $P_{\text{abs}}/\tau$, which implies a scatter in the corresponding $P_{\text{em}}$ in the far infrared (for an illustration see Fig. 5a of Hildebrand 1996).

The maximum observed value of $P_{\text{em}}$ (around 10% in Orion) has been interpreted by Hildebrand & Dragovan (1995) to correspond to lines of sight where the magnetic field lies in the plane of the sky. The discrepancy between 10% polarization observed and the predicted $\sim 35\%$ expected for the given grain geometry was interpreted by Hildebrand & Dragovan (1995) as a consequence of limited efficiency of Gold alignment calculated in Lazarian (1994). We, however, would tend to believe that the fluctuations of magnetic field direction are mostly responsible for the discrepancy, while the alignment is close to being complete (see below). For instance, polarization measurements from BIMA that resolve smaller scale structure that those by Hildebrand’s group reveal higher degrees of polarization (Goodman 1999). In the diffuse gas the structure of magnetic field should be more ordered and we expect to observe polarization higher than 10%.

We claim below that the grains responsible for the FIR emission in dark clouds constitute just a subset of the total grain population, namely, a subset of grains that are close to the stars and are aligned because of this. The rest of the grains within dark clouds may not be aligned and therefore the observed optical and near-infrared polarization does not follow extinction for large optical depths (Goodman 1995). This reduction of the grain alignment efficiency is nevertheless a characteristic of high col-
umn density regions, and is not likely to appear in the diffuse ISM. Thus for our modeling of the polarized emission of dust in the “infrared cirrus” at high galactic latitudes (which are more representative of the foreground emission for CMB polarization measurements), we will assume that the grain alignment efficiency is perfect.

We should finally wonder about the wavelength dependence of the polarization degree in emission in the FIR and submillimeter wavelengths. According to the theory, because the wavelengths considered are much greater than the maximum size of the grains, this polarization degree should be fairly independent of wavelength (see Draine & Lee 1984). This has been confirmed by multifrequency observations (ranging from 1300\(\mu\)m to 100\(\mu\)m, see Leach et al. 1991, Schleuning et al. 1996, Hildebrand et al. 1995).

### 2.3. Grain alignment and polarization

Grain alignment is a problem of half a century standing with the first observations dated as far back as 1949 (Hiltner 1949, Hall 1949) and first theories put forward in 1951 (Spitzer & Tukey 1951, Davis & Greenstein 1951, Gold 1951). Intensive recent research have partially uncovered the mystery of how grains can be aligned and we briefly summarize below what is known of grain alignment.

Various mechanism of alignment (see table 1 in Lazarian, Goodman & Myers 1997, henceforth LGM97) can be broadly separated in three categories: paramagnetic, mechanical and via radiative torques. The latter seem to dominate in diffuse ISM (Draine & Weingartner 1996, 1997), while mechanical alignment is important in particular regions of outflows, MHD turbulence (Lazarian 1994, 1997) and also in the regions of supersonic ambipolar diffusion (Roberge et al. 1995).

The role of paramagnetic effects is not clear so far. Jones and Spitzer (1967) have shown that grains can be well aligned via paramagnetic dissipation if they contain ferromagnetic or superparamagnetic inclusions. This idea was further supported by Mathis (1986), who showed that such grain can provide a proper wavelength dependence for polarization and also by Martin (1995) and Goodman & Whittet (1996) who observed that GEMs\(^2\) (glass with embedded metal) and other strongly magnetic inclusions found in primitive meteorites and in the interstellar dust particles caught in Earth atmosphere can render dust the required magnetic response.

Work by Lazarian & Draine (1997) made the case of paramagnetic alignment stronger: they showed that grains can be nearly perfectly aligned even when they have properties of ordinary paramagnetic materials, provided that grains rotate via \(H_2\) torques discovered by Purcell (1979). How-

\(^2\)A recent study by Draine & Lazarian (1999a) has shown that not more than 5% of iron can be in the form of metal as otherwise emissivity at 90 GHz would be stronger than observed.
ever, further research showed that Purcell’s torques can be suppressed by thermal flips of interstellar grains (Lazarian & Draine 1999 a,b) and the resulting alignment may be small. The thermal flipping should be enhanced for grains with superparamagnetic and ferromagnetic inclusions and therefore these grains are likely to rotate thermally (Lazarian & Draine 1999a) which entail the decrease of alignment for “supermagnetic” grains.

Radiative torques unlike Purcell torques are not fixed in the grain body frame. Therefore they are less sensitive to averaging arising from thermal flipping. Therefore these torques are likely to be the principal mechanism for grain spin-up and grain alignment in diffuse ISM. Unfortunately, we do not have proper theoretical understanding of the alignment via radiative torques. The existing numerical simulations disregard grain flipping and therefore cannot provide the quantitative answer. However, it seems plausible that the degree of radiative alignment should be high, e.g. of the order $\sim 80\%$.

The picture of grain alignment in diffuse media is simpler than in molecular clouds, where grains in cold dense environments can come to thermal equilibrium with gas and randomize. According to LGM97 the latter process is responsible for marginal polarizing power of grains in dark clouds observed by Goodman (1995). Recently Hildebrand et al. (1999) confirmed this prediction of LGM97, observing that cold grains are not aligned, while hot grains are aligned.

3. Angular distribution of polarized dust emission

3.1. The problem

Because the grains align themselves on the magnetic field, and polarize the light (absorption or emission) by dichroism, it is clearly tempting to try to infer the distribution of the underlying magnetic field from polarization maps. The problem is however far from being trivial. As we saw, the ratio of polarization to extinction is sensitive to a large number of factors, which include the random changes of the turbulent part of the magnetic field, as well as possible changes of the polarization efficiency of grains along the line of sight, and from one line of sight to the other. However, observations seem to show in a robust way that the polarization efficiency is decreased only in the denser parts of the interstellar objects (where $A_V \gg 1$), and is maximal in the diffuse interstellar medium (LGM97), so we will concentrate mainly on the distribution of magnetic field flux.

The problem has been first studied by Chandrasekhar & Fermi (1953) in the case of wave-like fluctuations of the magnetic field (describing the case of Alfvénic turbulence). They used the distribution of polarization position angles to infer the amplitude of the mean magnetic field. This approach was further elaborated by Zweibel (1990) in the case of a collection of clouds threaded on the magnetic field lines. The value of the magnetic
field amplitude computed by this method is in good agreement with values inferred from completely different methods, like Zeeman splitting of the HI line (see Heiles 1987), and provides additional indication that the polarization patterns are indeed related to the underlying magnetic field distribution.

Jokipii & Parker (1969) addressed the statistics of magnetic field in a more general way. They concluded that in order for cosmic rays to escape the Galactic disk in about 1 Myr, the correlation length of the magnetic field lines had to be of the order of 100 pc in the diffuse ISM. This method has been extended by Myers & Goodman (1990), who inferred from the observation of the distribution of polarization position angles in dark clouds the relative amplitudes of the random and regular components of the magnetic field, as well as the amplitude and direction of the mean field (with the addition of Zeeman splitting measurements in the same regions, see Heiles 1987, 1988). They obtained a rough estimate for the number of magnetic field correlation lengths along the line of sight.

A unifying approach was adopted by Jones, Klebbe & Dickey (1992). They describe the magnetic field fluctuations in a statistical way, but relate its correlation length to the optical depth variations along the line of sight, thereby stressing the coupling between the magnetic field and the underlying density field. They generalized the earlier studies by the introduction of a correlation function of the magnetic field position angle along the line of sight, as a function of optical depth. They obtained an estimate of the typical correlation length of the magnetic field of the order of $\Delta \tau_0^K = 0.1$.

The idea of relating the magnetic field directions distribution to the underlying density field seems physically well motivated. Unfortunately, it is difficult to quantify the correlation between interstellar density and magnetic field. Some insight could be obtained via numerical simulations of compressible MHD. The results available thus far deal unfortunately either with 2D turbulence (Vazquez-Semadeni, Passot & Pouquet 1995; Passot, Vazquez-Semadeni & Pouquet 1995; Vazquez-Semadeni, Passot & Pouquet 1996) or with 3D turbulence (Padoan & Nordlund 1998, Jones, Ostriker & Gammie 1998), but with insufficient resolution to obtain a detailed statistical picture. These simulations, however, do indicate a correlation of magnetic field and the underlying gas density.

The kinds of statistical approaches described above may be useful in addressing the problem of predicting the polarized foreground from dust in the diffuse ISM since the CMB will also be analyzed in term of its statistical properties. To be more practical, we would like to compare not only the one-point PDF of both emissions, but also their two-point functions, or equivalently their spatial power spectra. This problem, to be solved, requires not only a knowledge of the dust density and magnetic field distribution, but also the knowledge of their cross-correlation which seems to be non-vanishing, as shown by numerical simulations.
Unfortunately, only some derived statistical properties (like projected distributions) are available through the observations. A fair amount of modeling will then be necessary to guess what the power spectra of the dust polarized emission could look like. We will present different possible ways to estimate its properties in the following.

3.2. Getting some ideas about the dust polarized power spectra

A first (and rather crude) model to infer the statistical properties of dust polarized emission has been discussed in Prunet et al. (1998, henceforth PSBM98). The fundamental ideas behind their method are the following:

- The dust emissivity (as seen by IRAS/DIRBE maps at 100µm) is strongly correlated to the underlying gas density (as seen by the HI 21cm line maps of the Galaxy, see Hartmann & Burton 1995), at least for column densities smaller than $5 \times 10^{20} \text{cm}^{-2}$ (Boulanger et al. 1996).

- The HI maps contain an additional information, which is the distribution of HI densities in velocity space along each line of sight (measured through the Doppler shift of the HI emission lines), and therefore contain some information about the statistical properties of the 3D distribution of HI gas.

PSBM98 used the galaxy rotation curve to map the HI density field from velocity space to real space, and then assumed three different possibilities for the correlation of the magnetic field lines distribution with respect to the underlying HI density distribution:

1. The magnetic field lines follow the filamentary HI structures (i.e. are locally aligned with the density least-gradient direction).

2. The magnetic field lines are randomly chosen in the plane perpendicular to the local direction of the filaments (this could be representative of helicoidal field lines wrapped around filaments).

3. The magnetic field is constant throughout the observed patch (as a comparative case).

Those prescriptions, together with the arguments provided in sections 2.1 and 2.2 allowed PSBM98 to infer the spatial power spectra of the Stokes parameters of the dust polarized emission, as well as their cross-correlation power spectra with unpolarized emission (hereafter referred to as temperature emission). However, the first observable quantity predicted by this method is the distribution of polarization degree in emission, which can be compared to the measured histograms of the same quantity toward dense objects (see Hildebrand 1996 and references therein). This histogram is shown in Fig. 2. We can see immediately that the uniform magnetic field
Figure 2. Histogram of the polarization degree of dust emission in three different cases for the correlation magnetic field-density. Solid lines correspond to the case where field lines are aligned with the gas filaments, dotted lines correspond to random field lines perpendicular to the filaments, and dashed lines to a constant magnetic field with an angle $\zeta = \pi/2$ from the plane of the sky. After PSBM98.

The histograms shown in Fig. 2 are supposedly representative of the diffuse ISM where the grain alignment is expected to be nearly perfect. Although they peak at slightly higher values of polarization (in the two realistic cases) compared to the histograms observed in dark clouds, they can not reproduce this factor of 4. This discrepancy may have numerous reasons. First, this value of 4 for the reduction of polarization in dark clouds, was interpreted by Hildebrand & Dragovan (1995) entirely as a reduction of the grain alignment efficiency, under the assumption that the maximum observed polarization level was representative of a single magnetic field line in the plane of the sky; or if the field had already experienced one or two reversals along the line of sight, this factor would have to be lowered. Another possible explanation comes from the naive mapping used in the model to go from velocity space to real space. Indeed, the presence of the turbulent velocities can artificially stretch the structures along the line of sight, thereby artificially reducing the amount of polarization in the case where magnetic field lines are aligned with the filaments.
Finally, the (almost) random magnetic field case is giving too low results, as expected since it does not reproduce the smooth magnetic field component observed in the galactic disc (called “uniform” or “regular” magnetic field). The latter is necessary to reproduce the observed polarization to extinction ratios in the near infra-red (see Jones, Klebbe & Dickey 1992), as well as the position angles distribution (see Myers & Goodman 1991). We can then consider the constant magnetic field case and the random magnetic field case as limiting cases for this study. We will see in what follows that they should give some upper and lower bounds on the slope of the power spectrum of polarization, as well as a rough normalization.

The power spectra of the Stokes parameter Q are shown in Fig. 3 for the three different assumptions for the magnetic field lines. The case with constant magnetic field has, as expected, the same power-law dependence as the unpolarized emission \( C(\ell) \propto \ell^{-3} \), see Gautier et al. 1992). What is more interesting is the fact that the two other power spectra are both flatter; indeed, even if the correlation length of the magnetic field is similar to the density’s one, the modulation of the density field by the magnetic field orientation results always in a flatter power spectrum. Then we can conclude that the slope of the real power spectrum of the polarized dust emission should lie somewhere between those given by the constant and random magnetic field cases.

Another quantity of interest for the measurement of polarization in the CMB is the cross-correlation between the polarized and unpolarized emission (e.g. Zaldarriaga & Seljak 1997). The cross-correlation power spectra are given in Fig. 3 for the dust, and for the three assumptions concerning the magnetic field. We can see from this figure that this spectrum

Figure 3.  **Left Panel:** Power spectra of the Stokes parameter Q in the three cases for the magnetic field: dotted line for field lines along gas structures, dashed line for random field lines perpendicular to gas structures, and dot-dashed line for the constant field case. The power spectrum of the unpolarized emission is shown for comparison (solid line).  **Right panel:** Power spectra of the cross-correlation between the temperature and Stokes parameter Q, same line styles. Figure after PSBM98.
is expected to have more or less the same value around the degree scale, which is of interest for CMB polarization measurements (see next section). For a mathematical definition of the power spectra, see PSBM98.

This model has several advantages and drawbacks. The main advantage of this model is to put upper and lower limits on the slope and normalization of the different polarized power spectra expected for diffuse dust emission, which will be of interest for a comparison to the predicted CMB polarized power spectra. From this point of view it gives a rough first idea of what we should be dealing with when processing CMB polarized data. However, it does not give a precise picture of what should be the polarization patterns of dust emission in any localized region. Moreover, it could be that the predicted statistical observables themselves may be flawed by the presence of the turbulent motions that spoil any attempt to do a precise mapping of HI density from velocity space to real space. It appears that only in the case of short wavelength dominated density spectra can we hope to measure the statistical properties of the gas density field from HI data (see Lazarian & Pogosyan 1999). We will then give some guidelines for alternative methods to derive the polarized spectra of dust emission, taking into account the turbulent velocity component of HI, or relying on 2D data only.

4. Comparison with CMB power spectra

Finally, we would like to compare the amplitude of the different polarized power spectra of the dust emission from the diffuse ISM to the predicted amplitudes of the same quantities for the CMB, as inferred by theoretical computations (Seljak & Zaldarriaga 1996). It is most convenient to redefine new variables of polarization (Zaldarriaga & Seljak 1997), called “electric” and “magnetic” modes of polarization ($E, B$) in analogy with the transformation properties of the electro-magnetic field under parity transformations. These modes are particularly well adapted to describe CMB polarization, as scalar perturbations induce only electric modes of polarization (Seljak & Zaldarriaga 1997, see also Kamionkowski et al. 1997a,b for a similar analysis in terms of tensor spherical harmonics). As we dealt with relatively small patches on the sky in the analysis performed on the HI maps (see PSBM98), we can use the definition of those modes in the small-angle (or flat space) approximation (Seljak 1997):

\[
C_E(\ell) = \frac{1}{N} \sum_{i=1}^{N} \left| Q(\vec{\ell}) \cos(2\phi_{\vec{\ell}}) + U(\vec{\ell}) \sin(2\phi_{\vec{\ell}}) \right|^2
\]

\[
C_B(\ell) = \frac{1}{N} \sum_{i=1}^{N} \left| -Q(\vec{\ell}) \sin(2\phi_{\vec{\ell}}) + U(\vec{\ell}) \cos(2\phi_{\vec{\ell}}) \right|^2
\]

\[
C_{TE}(\ell) = \frac{1}{N} \sum_{i=1}^{N} \left[ (Q(\vec{\ell})T^*(\vec{\ell}) + Q^*(\vec{\ell})T(\vec{\ell})) \cos(2\phi_{\vec{\ell}}) \right]
\]
Figure 4. The power spectra of the E modes of polarization are shown for dust at 143 GHz (dashed line) and 217 GHz (dot-dashed line). The scalar-induced (solid line) and tensor-induced (dotted line) power spectra of CMB E-modes are also shown. The CMB temperature power spectrum (thick solid line) is shown for comparison. 143 and 217 GHz are the central frequencies of the two most sensitive polarized channels of PLANCK.

\[ \left( \frac{1}{1+1} \right) C_{\ell} \frac{1}{2 \pi} \left( \langle U(\ell) T^*(\ell) + U^*(\ell) T(\ell) \rangle \sin(2\varphi_\ell) \right) \]  

(5)

The different power spectra for CMB and dust are shown in figures 4 and 5 in the case where the magnetic field lines are assumed to follow the directions of the gaseous filaments. The CMB power spectra have been computed for a tilted CDM model with scalar spectral index \( n_S = 0.9 \), tensor index \( n_T = n_S - 1 \) and a tensor to scalar temperature quadrupole ratio of \( T/S = 7n_T \), using the Boltzmann code CMBFAST (Seljak & Zaldarriaga 1996). We can draw some interesting conclusions from those figures. If we keep in mind the uncertainties discussed before concerning the dust polarized power spectra, we can conclude that the E modes power spectra of CMB should be measurable at least for scales smaller than the degree (i.e. in the Doppler peaks region). Concerning the temperature-polarization cross-correlation, we expect, as shown in figure 5, that the correlation will be stronger for the CMB signal than for the dust signal, making the CMB signal accessible to measurements up to even larger scales (\(~ 5^\circ\)). However, the B-modes of CMB polarization are of the same level as their contaminants, and we cannot at this stage conclude about the possibility of their measurement.

We discussed earlier the fact that the degree of polarization of the dust thermal emission should be fairly independent of wavelengths. Then, if we learn about the spectral dependence of dust emissivity on wavelength by
Figure 5. Same as Fig. 4, but for the B modes of polarization.

Figure 6. Same as Fig. 4, but for the T-E correlation.
observing denser regions of the ISM, we should be able to predict with a
good accuracy the spectral behavior of the polarized dust from the diffuse
ISM (this is subject to the fact that the denser regions observed to mea-
sure the emissivity of dust at different wavelengths are close in composition
and grain temperatures to the very diffuse ISM; we therefore refer to the
former as intermediate density regions.) We can then use this information,
combined with the polarized maps observed at higher frequencies where
the dust emission is dominant over all other processes (including CMB),
to clean the low frequency polarized maps where we seek the CMB sig-
нал. This multi-frequency filtering of the data can be implemented in a
lot of different ways, and has been applied to CMB anisotropy data by
Kogut et al. (1996) for the first time on the COBE DMR data. Since
then, different techniques have been developed, which can be grouped in
two broad categories: Wiener filtering techniques (Bouchet et al. 1996,
Tegmark & Efstathiou 1996), and Maximum Entropy Methods (Maisinger
et al. 1997, Hobson et al. 1998). Bouchet et al. (1999) generalized the
Wiener filtering method to take into account polarized CMB data. The
resulting errors on the polarized power spectra, after removal of the fore-
ground emissions (which were in this article Galactic dust and synchrotron
polarized emissions) are shown in figures 1, 8 and 9 for different instru-
mental configurations. We have to be careful in reading those figures,
as a logarithmic smoothing has been performed; the covariances shown
here are for band-power estimates of the power spectra, not for individ-
ual modes. They give nevertheless a good idea of what accuracy should
be reached by the upcoming satellite missions in the observation of CMB
polarization. For instance, relatively broad band features in the spectra of
E modes and cross-correlation should be observed with a good accuracy.
The B modes power spectrum is, as expected from the weakness of the
signal, much less constrained. However, even a poor determination of this
signal is important for constraining tensor-induced cosmological param-
ters (Prunet et al. 1998a,1998b). We should also notice that the amount of
gravitational waves could be much higher for a given class of inflationary
models (Lesgourgues et al. 1998), thus making the signal of B modes easier
to measure.

5. Discussion

5.1. Various approaches

Out results sensitively depend on the assumed angular distribution of the
polarized dust emission. The model used so far has its flaws. It is therefore
important to test other approaches.

A full statistical approach: As shown by Lazarian & Pogosyan (1999),
the gas underlying density distribution in the diffuse ISM should be mea-
Figure 7. Band-power estimates of the $1 - \sigma$ error on the measurement of the E-modes power spectrum, for the PLANCK-HFI (dotted line), PLANCK-LFI (short-dashed line) and MAP (long-dashed line) configurations. The bands have constant logarithmic width $\Delta \ell/\ell = 0.2$.

Figure 8. Same as Fig. 7, but for the T-E cross-correlation.
surable in the HI velocity space data cubes (Hartmann & Burton 1995) if it is short-wave dominated. With some assumption about the correlation of the field lines with the density, inspired by numerical simulations and/or theoretical considerations of MHD turbulence, one could infer the resulting angular distribution of polarized dust assuming perfect, or non perfect alignment. This method, under the above assumption concerning the magnetic field-density correlation, would at least be correct statistically, which is the primary goal of the exercise. The measurement of polarization statistics by upcoming satellite missions would then be a mean to check the validity of theories of MHD compressible turbulence!

A pragmatic approach: Whether or not the underlying statistics of HI is short- or long-wave dominated the density distribution integrated over 21 cm emission line reveal HI density structure. To find the statistics of the 3D underlying density one may use the inversion technique in Lazarian (1995) or use the observe 2D structure to infer the expected projected pattern of magnetic field. If magnetic field direction fluctuates along the line of sight this method should provide an overestimate of the expected variations of polarization.

Using synchrotron: Synchrotron emissivity depends both on the distribution of magnetic field and the distribution of relativistic cosmic electrons. The latter are distributed more smoothly than the former and this allows...
to infer the small-scale statistics from variations of synchrotron intensity (see Lazarian 1992). Potentially synchrotron is a very valuable tool for studying the statistics of magnetic fields. Having this statistics and 2D statistics of HI emissivity it seems possible to evaluate the expected polarized emissivity.

5.2. Polarization at lower frequencies

So far we have discussed the polarization at frequencies larger than 100 GHz. At lower frequencies dust contributes to microwave emissivity via magneto-dipole emission and rotational emission from ultra-small grains (Draine & Lazarian 1999b) and both types of emission are partially polarized. The magneto-dipole emission is most prominent when the grain materials are strongly magnetic, e.g. ferro- or ferrimagnetic. However, even for ordinary paramagnetic grains magneto-dipole emission dominates the ordinary electro-dipole emission for frequencies less than 30 GHz. The emission from single dipole ferro or ferrimagnetic grains is strongly polarized (up to 40%) and shows characteristic frequency dependence signatures (Draine & Lazarian 1999b).

Emission from ultra-small grains emanates due to grain rotation. Small grains are likely to have dipole moment and therefore the emission can be called rotational emission in analogy with the corresponding emission from molecular transitions. If small grains are aligned their emission is polarized. Calculations in Lazarian & Draine (1999c) show that polarization degrees up to a few percent is possible for rotational emission at frequencies less than 40 GHz.

6. Conclusions

Our results can be summarized as follows:

1. Polarized radiation from dust can appreciably interfere with the attempts to measure polarization of cosmological origin. The signal from tensor modes is expected to be completely masked by dust. However, with the expected independence of the dust polarization degree on wavelength, we can hope to remove this contaminant from CMB maps with reasonable efficiency, provided that we have polarized measurements of the same regions at higher frequencies where the dust emission is dominant.

2. The degree of dust polarization is expected to vary with scale with the tangling of magnetic field limiting the achievable polarization.

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3 Variations of synchrotron polarization is another channel of valuable information.

4 With iron and nickel constituting up to 30 percent of “silicate” grains, these properties do not look unrealistic.
3. The expected polarization from dust differs in the range of high (> 100 GHz) and low (< 100 GHz) frequencies. The emission at high frequencies is of electro-dipole nature and therefore can be treated using the accepted technique. The low frequency emission is either of magneto-dipole origin or from rotating ultra-small grains and the expected polarization depends on various factors, e.g. on whether grains consist of single magnetic domains or have multi-domain structure.

4. Multi-frequency modeling of the galactic polarization on the basis of synchrotron, HI and infrared polarization maps should allow high-precision study of cosmological polarization.

References

Bond, J. R., Efstathiou, G. 1984, ApJ, 285, L45
Bouchet, F. R., Gispert, R., Puget, J.-L. 1996, in “Unveiling the Cosmic Infrared Background”, AIP Conf. Prog. 348, E. Dwek ed., Baltimore.
Bouchet, F. R., Prunet, S., Sethi, S. K. 1999, MNRAS, 302, 663
Boulanger, F., Abergel, A., Bernard, J.-P., Burton, W. B., Désert, F.-X., Hartmann, D., Lagache G., Puget J.-L., 1996, A&A, 312, 256
Chandrasekhar, S., Fermi, E. 1953, ApJ, 118, 113
Davis, L. Jr., Greenstein, J. L. 1951, ApJ, 114, 206
Draine, B.T., Lazarian 1999a, ApJ, in press
Draine, B.T., Lazarian 1999b, this volume
Draine, B. T., Lee, H. M. 1984, ApJ, 285, 89
Draine, B. T., & Weingartner, J.C. 1996, ApJ, 470, 551
Draine, B. T., & Weingartner, J.C. 1997, ApJ, 480, 633
Gautier, T. N. III, Boulanger, F., Pérault, M., Puget, J.-L. 1992, AJ, 103, 4
Greenberg, J. M. 1968, in “Stars and Stellar Systems”, vol 7, p221, eds. B. M. Middlehurst & L. H. Aller, University of Chicago Press
Grevesse, N., Anders, E. 1989, AIP COnt. Proc. 183, eds. R. G. Lerner (New York: AIP), 1
Gold, T, 1951, Nature, 169, 322
Goodman, A.A. 1999, ApJ, in preparation
Goodman, A.A. 1995, in ASP Conf. Ser. 73, From Gas to Stars to Dust, ed. J. Davidson, E. Erickson,& M. Haas (San Francisco: ASP), 45
Goodman, A.A, & Whittet, D.C.B. 1995, ApJ, 455, L181
Hall, J. S. 1949, Science, 109, 166
Hartmann, D., Burton, W. B. 1995, Atlas of Galactic HI emission, Cambridge University Press
Heiles, C. 1987, in Interstellar Processes, eds. D. J. Hollenbach & H. A. Thronson (Dordrecht: Reidel), p171
Hildebrand, R. H. 1988, QJRAS, 29, 327
R. H. Hildebrand, J. L., Dotson, C. D. Dowell, D. A. Schleuning, & Vaillancourt 1999, ApJ 516, 000
Hildebrand, R. H., Dotson, J. L., Dowell, C. D., Platt, S. R., Schleuning, D. A., Davidson, J. A., Novak, G. 1995, in Airbone Astronomy Symposium on the Galactic Ecosystem: “From Gas to Stars to Dust”, eds. M. R. Haas, J. A. Davidson & E. F. Erickson (ASP, San Francisco), p97
Hildebrand, R. H. 1996, in “Polarimetry of the Interstellar Medium”, ASP Conf. Series vol. 97, p254
Hildebrand, R. H., Dragovan, M. 1995, ApJ, 450, 663
Hildebrand, R. H., Dragovan, M., Novak, G. 1984, ApJ, 284, L5
Hiltner, W. A., Science, 109, 165
Hobson, M. P., Jones, A. W., Lasenby, A. N., Bouchet, F. R. 1998, MNRAS, 300, 1
Jokipii, J. R., Parker, E. N. 1969, ApJ, 155, 799
Jones, T. J., Klebe, K., Dickey, J. M. 1992, ApJ, 389, 602
Kamionkowski, M., Kosowsky, A., Stebbins, A. 1997a, Phys.Rev.D, 55, 7368
Kamionkowski, M., Kosowsky, A., Stebbins, A. 1997b, Phys.Rev.Lett, 78, 2058
Kamionkowski, M., Kosowsky, A. 1998, Phys.Rev.D, 57, 685
Kim, S. H., Martin, P. G. 1995, ApJ, 444, 293
Kim, S. H., Martin, P. G., Hendry, P. D. 1994, ApJ, 422, 164
Kogut, A., Banday, A. J., Bennett, C. L., Gorski, K. M., Hinshaw, G., Smoot, G. F., Wright, E. I. 1996, ApJ, 464, L5
Lazarian, A. 1992, Astron. and Astroph. Transactions, 3, 33
Lazarian, A. 1994, MNRAS, 268, 713
Lazarian, A. 1995, A&A, 293, 508
Lazarian, A. 1997, ApJ, 338, 902
Lazarian, A., Draine, B.T. 1999a, ApJ, in press
Lazarian, A., Draine, B.T. 1999b, ApJ, submitted
Lazarian, A., Draine, B.T. 1999c, ApJ, in preparation
Lazarian, A., Goodman, A. A., Myers, P. C. 1997, ApJ, 490, 273
Lee, H. M., Draine, B. T. 1985, ApJ, 290, 211
Lesgourgues, J., Prunet, S., Polarski, D. 1998, accepted in MNRAS

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Maisinger, K., Hobson, M. P., Lasenby, A. N. 1997, MNRAS, 290, 313
Myers, P. C., Goodman, A. A. 1991, ApJ, 373, 509
Netterfield, C. B., Jarosik, N., Page, L., Wilkinson, D., Wollack, E. 1995, ApJ, 445, L69
Padoan, P., Nordlund, A. A. 1998, astro-ph/9901288
Passot, T., Vazquez-Semadeni, E., Pouquet, A. 1995, ApJ, 455, 536
Prunet, S., Sethi, S. K., Bouchet, F. R., Miville-Deschênes, M.-A. 1998, A&A, 339, 187 (PSBM98)
Prunet, S., Sethi, S. K., Bouchet, F. R. 1998a, in “Wide field surveys in cosmology”, Proc. of the XIV IAP Coll., S. Colombi, Y. Mellier, B. Raban eds., Editions Frontières, p305
Prunet, S., Sethi, S. K., Bouchet, F. R., 1998b, in preparation
Rees, M. J. 1968, ApJ, 153, L1
Roberge, W.G., Hanany, S., & Messinger, D.W. 1995, ApJ, 453, 238
Roberge, W.G., & Lazarian 1999, MNRAS, in press
Schleuning, D. A., Dowell, C. D., Platt, S. R. 1996, in “Polarimetry of the Interstellar Medium”, ASP Conf. Ser. 97, ed W. G. Roberge & D. C. B. Whittet (ASP: New York), p285
Seljak, U. 1997, ApJ, 482, 6
Seljak, U., Zaldarriaga, M. 1997, Phys.Rev.Lett, 78, 2054
Spitzer, L., & Tukey, J.W. 1951, ApJ, 114, 187
Stone, J. M., Ostriker, Ostriker, E. C., Gammie, C. F. 1998, ApJ, 508, 99
Tegmark, M., Efstathiou, G. 1996, MNRAS, 281, 1297
Vazquez-Semadeni, E., Passot, E., Pouquet, A. 1995, ApJ, 441, 702
Vazquez-Semadeni, E., Passot, E., Pouquet, A. 1996, ApJ, 473, 881
Zaldarriaga, M. 1997, Phys.Rev.D, 55, 1822
Zaldarriaga, M., Seljak, U. 1997, Phys.Rev.D, 55, 1830
Zweibel, E. 1990, ApJ, 362, 545