Using dynamic geometry software Cabri 3D for teaching analytic geometry

Temel Kösa\textsuperscript{a}, Fatih Karakuş\textsuperscript{a} *

\textsuperscript{a}Fatih Faculty of Education, Karadeniz Technical University, Trabzon, 61335, Türkiye

Received October 29, 2009; revised December 7, 2009; accepted January 15, 2010

Abstract

In this study, we investigated that whether Cabri 3D can be a useful tool for teaching analytic geometry of space or not. For this purpose, we focused on solving some traditional spatial analytic problems by using Cabri 3D. Experiments were conducted with 24 prospective high school teachers who had a mathematics degree and were attending the postgraduate teacher training course. Results of the study show that Cabri 3D is potentially very useful software for learning and teaching spatial analytic geometry. Another important result of the study is that Cabri 3D facilitates understanding by visualizing.

© 2010 Elsevier Ltd. All rights reserved.

Keywords: Analytic geometry; dynamic geometry software; Cabri 3D; visualization; problem solving

1. Introduction

Using dynamic geometry software (DGS) in geometry classes is increasing more and more nowadays. However, DGS use in solid geometry and analytic geometry of space has been neglected. One of the main reasons of this, two-dimensional Euclidean geometry is still more popular than three-dimensional geometry. Though, students have often had difficulties to visualize three-dimensional figures.

Geometry can be considered as the origin of visualization in mathematics but, if we examine the papers or books published in the last years dealing with visualization in mathematics education, we find many of them focusing on the teaching or learning of calculus (i.e. advanced mathematical thinking), quite a lot on (pre-)algebra and number systems, some on plane geometry and only a few focusing on space geometry (Gutiérrez, 1996). Although visualization is an important element for teaching geometry, in especial space geometry, there are not effective tools for the teaching these topics, yet.

There are a lot of powerful tools such as Logo (Clements & Sarama, 1997; Papert, 1993), Cabri Geometry (Laborde, 2000), Geometer’s Sketchpad (Jackiw, 1995) and GeoGebra (Hohenwarter, 2005) that have been designed to facilitate the learning of geometry. It can be said that these software have brought revolutionist innovations to geometry education and it can be asserted that it is the most important step in geometry instruction since Euclid. But, these software operate within 2D environments. The Dynamic Geometry Software Cabri 3D for
exploring three-dimensional geometry was launched in 2004. It promises to revolutionize the computer-assisted visualization and reasoning in 3D geometry, similar to the earlier dynamic geometry software (DGS) for plane geometry. (Güven & Kösa, 2008). Cabri 3D allows the user to construct and manipulate solid geometry objects in three dimensions via a 2D interface. By using Cabri 3D, three-dimensional objects such as prism, pyramids, cylinder and cone can be constructed, rotated and seen from a certain aspect on the screen and also prisms can be opened on the screen. Prisms and half plane can be intersected and thus, new three-dimensional objects may be formed. It is a practical tool for solid analytic geometry, too. Lines, vectors, planes and conics can be constructed on the screen and seen from different view point by rotating screen with a simple dragging. Vectorial operations such as vector sum, cross product and dot product can be performed. Coordinates of a point or vector, equations of a line, plane or sphere in space can be represented on the screen.

Research studies shows that many students have difficulties to imagine spatially the analytic geometric task in Euclidian 3D space (E^3). Blackboard drawings or handmade transparencies, mainly of questionable quality of perception, are no basis for developing an adequate spatial-geometric understanding in working on tasks of spatial analytic geometry – which doesn’t exclude that students can solve those tasks algorithmically without spatial understanding (Schumann, 2003). For helping the development of good concept images of three-dimensional geometric objects, educators have some possible aids: models, manipulatives and diagrams. The recent availability of 3D dynamic software, like Cabri3D (Bainville & Laborde, 2004) gives a potentially important new tool for developing visual education for solid geometry (Accascina & Rogara, 2006). In this study, we investigated that whether Cabri3D can be a useful tool for teaching analytic geometry of space and received prospective mathematic teachers' opinion.

2. Method

2.1. Sample

Experiments were conducted 24 prospective high school teachers who had a mathematics degree and were attending the postgraduate teacher training course at Fatih Faculty of Education, in Karadeniz Technical University. All of the students had studied 2D and 3D Analytic Geometry and taken a course named Using Computers in Mathematics Education at least one semester. All students were already acquainted with Cabri 2D from the previous year.

2.2. Procedure and intervention

We designed a small course for the students to learn what ideas they have while they are using Cabri 3D to learn some concepts and relations related with solid analytic geometry. Experiments were conducted in three distinct hours each in a computer laboratory. Two students shared a PC and we used a projector linked to the instructor’s PC. All students were familiar with Cabri 2D but they have not used Cabri 3D, yet. Thus, we introduced Cabri 3D in the first meeting. In the second and third meeting, students studied with worksheets. Worksheets aimed to gain some concepts such as lines and planes in space were developed by researches. The schedule of the course activities is shown in Table 1.

| Meeting | Course content |
|---------|----------------|
| 1st     | Learning about Cabri 3D |
|         | Forming basic geometric objects and constructing basic elements for analytic geometry of space |
| 2nd     | Constructing a line with a point on it and its director vector in space and finding its equation |
| 3rd     | Constructing a line with two points and finding its equation |
|         | Parallel and perpendicular lines in space |
|         | Forming a plane and obtaining its general equation |
3. Results (Finding)

We prepared worksheets before the study. Worksheets include 3D problems in analytic geometry. After completing the solution of problems we required students to discuss each other and formulate the general form of the statement. An exemplary problem that students studied on was given below.

We gave to students a point and a director vector in space and asked them to find the equation of the line shown in figure 1.

Students used Coord. & equation(s) features of Cabri 3D to obtain the equation of line shown in figure 2 and then followed the instructions on worksheet how they can gain the equation of the line analytically.
First, they selected an arbitrary point (B) on the line and constructed $\overrightarrow{OA}$ and $\overrightarrow{AB}$ vectors shown in figure 3. Then, they expressed the $\overrightarrow{OB}$ vector as sum of $\overrightarrow{OA}$ and $\overrightarrow{AB}$ vectors.

\[
\overrightarrow{OB} = \overrightarrow{OA} + \overrightarrow{AB}
\]

\[
\overrightarrow{OB} = \overrightarrow{OA} + \lambda \cdot d
\]

\[
x = -3 - 1.6\lambda \quad y = -4 + 1.8\lambda \quad z = 1 + 1.4\lambda
\]

Second, students wrote the elements of vector separately as shown above and then wrote the general form of the equation as below.

The equation of the line was \[
\frac{x+3}{-1.6} = \frac{y+4}{1.8} = \frac{z-1}{1.4} = \lambda
\]

Third, we asked students to verify the equation whether it is right or wrong by taking a random point on the line and using calculator of Cabri 3D. All students found same $\lambda$ value. Other problems were on worksheets were completed by students and after meetings we all together discussed about the implementations.

4. Discussion, conclusion and recommendation

In the literature, several studies show that using dynamic geometry software facilitates the visualization (Güven & Kösa, 2008; Jackiw, 2003; Oldknow & Tetlow, 2008). Visualization is very important while studying on geometry, especially 3D geometry. It can be difficult for students to visualize spatial figures in their mind. The dynamic nature of dynamic geometry software provides students to learn geometric concepts and to explore and visualize geometric relationships easily.

The present study aimed to determine whether the three-dimensional computer supported activities designed by dynamic geometry software Cabri 3D for analytic geometry of space can help to students a better understanding and have a positive attitude or not. Results of the study show that Cabri3D has an important potential to teach analytic geometry of space. Besides, prospective mathematics teachers liked Cabri3D and found it very useful for teaching 3D geometry since students always found it very hard to study. They also stated that Cabri3D facilitates to visualize. All students were willing to use Cabri 3D to teach analytic geometry to their students in order to make them to discover themselves and make a better understanding for them.
In this study, we studied on a small part of analytic geometry with Cabri 3D. Further researchers can investigate other topics for analytic geometry with Cabri 3D and whether Cabri 3D can bring out misconceptions or not.

References

Accascina, G. & Rogora, E. (2006). Using Cabri 3D diagrams for teaching geometry, *International Journal for Technology in Mathematics Education*, 13(1), 11-22.

Bainville, E. & Laborde, J. M. (2004) *Cabri3D, v.1.0.3 Cabrilog*, http://www.cabri.com

Clements, D. H. & Sarama, J. (1997). Research on Logo: A decade of progress. In C. D. Maddux & D. L. Johnson (Eds.), *Logo: A retrospective* (pp. 9-46). New York: Haworth Press.

Gutiérrez, A. (1996). Visualization in 3-dimensional geometry: In search of a framework, in L. Puig & A. Gutierrez (eds.) *Proceedings of The 20th Conference of The International Group for The Psychology of Mathematics Education* (vol. 1, pp. 3-19). Valencia: Universidad de Valencia.

Güven, B. & Kösa, T. (2008). The effect of dynamic geometry software on student mathematics teachers’ spatial visualization skills. The Turkish Online Journal of Educational Technology, 7(4), 100-107.

Hohenwarter, M. & Fuchs, K. (2005). Combination of dynamic geometry, Algebra and Calculus in the software system GeoGebra. *Computer Algebra Systems and Dynamic Geometry Systems in Mathematics Teaching Conference*. Pecs, Hungary.

Jackiw, N. (1995). *The Geometer's Sketchpad, v3.0*. Berkeley, CA: Key Curriculum Press.

Jackiw, N. (2003). Visualizing complex functions with the Geometer’s Sketchpad. *Proceedings of The 6th International Conference on Technology in Mathematics Teaching*. (pp.291-299). Volos, Greece: University of Thessaly.

Laborde, C. (2000). Dynamic geometry environments as a source of rich learning contexts for the complex activity of proving. *Educational Studies in Mathematics*, 44(1/2), 151-156.

Oldknow, A. & Tetlow, L. (2008). Using dynamic geometry software to encourage 3D visualization and modelling. *The Electronic Journal of Mathematics and Technology*, 2(1), 54-61.

Papert, S. (1993). *Mindstorms: Children, computers, and powerful ideas* (2nd ed.). New York: Harvester Wheatsheaf.

Schumann, H. (2003). Computer aided treatment of 3D problems in analytic geometry. *The International Journal on Mathematics Education*, 35(1), 7-13.