Linking the fate of massive black hole binaries to the active galactic nuclei luminosity function

M. Dotti,1,2 A. Merloni3 and C. Montuori4

1Università degli Studi di Milano-Bicocca, Piazza della Scienza 3, I-20126 Milano, Italy
2INFN, Sezione di Milano-Bicocca, Piazza della Scienza 3, I-20126 Milano, Italy
3Max Planck Institut für Extraterrestrische Physik, Giessenbachstrasse 1, D-85748 Garching bei München, Germany
4Università degli Studi dell’Insubria, Via Valleggio 11, I-22100 Como, Italy

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ABSTRACT
Massive black hole binaries are naturally predicted in the context of the hierarchical model of structure formation. The binaries that manage to lose most of their angular momentum can coalesce to form a single remnant. In the last stages of this process, the holes undergo an extremely loud phase of gravitational wave emission, possibly detectable by current and future probes. The theoretical effort towards obtaining a coherent physical picture of the binary path down to coalescence is still underway. In this paper, for the first time, we take advantage of observational studies of active galactic nuclei evolution to constrain the efficiency of gas-driven binary decay. Under conservative assumptions we find that gas accretion towards the nuclear black holes can efficiently lead binaries of any mass forming at high redshift ($z > 2$) to coalescence within the current time. The observed ‘downsizing’ trend of the accreting black hole luminosity function further implies that the gas inflow is sufficient to drive light black holes down to coalescence, even if they bind in binaries at lower redshifts, down to $z \approx 0.5$ for binaries of $\sim 10^7 \, M_{\odot}$, and $z \approx 0.2$ for binaries of $\sim 10^9 \, M_{\odot}$. This has strong implications for the detection rates of coalescing black hole binaries of future space-based gravitational wave experiments.

Key words: black hole physics – gravitational waves – galaxies: active – galaxies: interactions – galaxies: nuclei.

1 INTRODUCTION
Massive black hole (MBH) pairs are expected to form during galaxy mergers (Begelman, Blandford & Rees 1980, hereafter BBR). If the nuclei of the two merging galaxies manage to survive against the tidal forces in play for long enough (e.g. Callegari et al. 2009; Van Wassenhove et al. 2014), dynamical friction can efficiently bring the two MBHs to the centre of the galactic remnant, forcing them to bind in an MBH binary (BHB).

Starting from the binary formation, dynamical friction becomes less and less efficient (BBR) and other dynamical processes are needed to further evolve the binary. In particular, the interaction with single stars and with nuclear gas have been thoroughly investigated (see Dotti, Sesana & Decarli 2012, for a recent review). The sole effect of gravitational wave emission forces the two MBHs to coalesce within the Hubble time if any physical process manages to shrink the semimajor axes of the BHB down to

$$a_{\text{GW}} \approx 2 \times 10^{-3} f(e)^{1/4} q^{1/4} \left( \frac{M}{10^9 \, M_{\odot}} \right)^{3/4} \text{pc},$$

(1)

where $M = M_1 + M_2$ is the total mass of the binary, $q = M_2 / M_1$ is its mass ratio, and $f(e) = [1 + (73/24)e^2 + (37/96)e^4][1 - e^2]^{-3/2}$ is a function of the binary eccentricity $e$ (Peters 1964). The assessment of how effective the various processes are to evolve the binary down to $a_{\text{GW}}$ is usually referred to as the ‘last parsec’ problem.

Attempts to determine the fate of BHBs (whether they manage to reach $a_{\text{GW}}$ and to coalesce or they remain bound in double systems forever) have been made first considering gas-poor environments, where BHB dynamics is assumed to be driven by three-body interactions with single stars. In principle, only stars whose orbits intersect the BHB can efficiently interact with it. In an extended stellar system, however, only a small fraction of the phase space (the so called binary ‘loss cone’) is populated by such orbits. Stars interacting with the BHB remove energy and angular momentum from the binary, getting ejected from the loss cone. The binary evolution time-scale is hence related to the rate at which new stars are fed into the loss cone (e.g. Makino & Funato 2004). Physical mechanisms able to

* E-mail: massimo.dotti@mib.infn.it
efficiently refuel the loss cone are required in order for the binary to coalesce in less than a Hubble time. Possible mechanisms that have been proposed so far are: the presence of massive perturbers (such as giant molecular clouds; Perets & Alexander 2008), deviations from central symmetry (e.g. Khan, Just & Merritt 2011; Pretot et al. 2011; Guandalris & Merritt 2012, but see also Vasiliev, Antonini & Merritt 2014) and gravitational potential evolving with time (e.g. Vasiliev, Antonini & Merritt 2014).

Similarly, the effect of the interaction between BHBs and nuclear gas has been explored analytically as well as numerically (see Dotti et al. 2012, for an up to date review). While the full details of the gas/binary interaction are still under debate, mainly due to the complexity of the system, a clear issue remains to be addressed. Similarly to the stellar-driven case, the migration time-scale of the BHB primarily depends on how much gas is able to spiral towards the MBHs and interact with them, instead of e.g. turn into stars (e.g. Lodato et al. 2009). This problem is remarkably similar to the fuelling problem of active galactic nuclei (AGN), i.e. how gas manages to lose most of its angular momentum in order to sustain the observed nuclear activity.

Differently from the stellar driven case, however, observational studies of the AGN population allow us to constrain the properties of the gas flowing on to a MBH (in particular its mass accretion rate). Decades of multiwavelength surveys of accreting MBHs have provided a relatively robust picture of the AGN luminosity function evolution (see e.g. Hasinger, Miyaji & Schmidt 2005; Hopkins, Richards & Herquist 2007; Buchner et al. 2015). Coupling such evolution with the observationally determined MBH mass function via a continuity equation (Cavaliere, Morrison & Wood 1971; Small & Blandford 1992) allows us to further infer the evolution of the nuclear inflow rates as a function of MBH mass and redshift (e.g. Merloni & Heinz 2008; Shankar et al. 2013).

In this work we will assume that the fuelling of BHBs is consistent with that of single MBHs in the same mass and redshift interval. In this way we can estimate the incremental reservoir of angular momentum that a BHB can interact with during its cosmic evolution, constraining the binary fate, at least in a statistical fashion, directly from observations.

2 MODEL

2.1 Gas-driven BHB dynamics

To get an order of magnitude estimate of the binary coalescence time-scale, we propose a very simple zeroth-order model for the interaction between a BHB and a circumbinary accretion disc. We assume that the BHB is surrounded by an axisymmetric, geometrically thin accretion disc corotating with the BHB. Under this assumption, the gas inflow is expected to be halted by the binary, whose gravitational torque acts as a dam, at a separation \( r_{\text{gap}} \approx 2a \) (e.g. Artymowicz & Lubow 1994), where \( a \) is the binary semimajor axis. At this radius the gravitational torque between the binary and the disc is perfectly balanced by the torques that determine the large scale \( (r \gg a) \) radial gas inflow. Then, we can write the variation of the binary orbital angular momentum magnitude \( (L_{\text{BHB}}) \) as:

\[
dL_{\text{BHB}} = -dL_{\text{gas}} = -\dot{m} \, dt \sqrt{GMr_{\text{gap}}},
\]

where \( M \) is the total binary mass, and \( \dot{m} \) is the accretion rate within the disc. Considering the definition of \( L_{\text{BHB}} = \mu \sqrt{GMa} \), where \( \mu \) is the binary reduced mass, from equation (2) we derive:

\[
\frac{\mu}{2} \frac{da}{dt} \approx -\dot{m} \, dt.
\]

where \( \mu \) does not evolve in time consistently with the assumption that the binary interacts with the disc only through the gravitational torques, that stop the gas inflow preventing the binary components to accrete.

As a note of caution, we stress that the circumbinary discs could be, in principle, counter-rotating with respect to the BHBs they orbit (Nixon, King & Pringle 2011b). In this configuration the gas interacts with the BHB at \( \approx a \), instead of \( \sim 2a \) (Nixon et al. 2011a), and the specific angular momentum transfer per unit time is uncertain, depending on how strongly the secondary MBH is able to perturb the gas inflow. As an example, if all the gas passing through \( a \) bounds to the secondary MBH, the binary angular momentum diminishes by two times the angular momentum carried by the gas. In this scenario, the BHB evolves on the same time-scale regardless of the BHB-disc relative orientation. Moreover, Rödig & Sesana (2014) demonstrated that BHBs embedded in self-gravitating retrograde discs may secularly tilt their orbital plane towards a coplanar prograde equilibrium configuration. For these reasons, we will focus our investigation on the prograde case in the following.

As a first order of magnitude estimate of the coalescence time-scale, we can make the simplifying (although quite common) assumption of an Eddington limited accretion event and integrate equation (3). Further assuming a fixed radiative efficiency \( (\epsilon = 0.075, \text{see Section 2.2}) \), we obtain

\[
\Delta t_{\text{BHB}} \sim \frac{a_i}{a_e} \frac{\mu \epsilon c^2}{2 \sqrt{2} L_{\text{Edd}}} \sim 10^7 \frac{q}{(1+q)^{\frac{3}{2}}} \ln \left( \frac{a_i}{a_e} \right) \text{yr},
\]

where \( a_i \) and \( a_e \) define the binary separation range where the MBH-gas interaction drives the binary orbital decay.

Equation (4) shows that, in order to coalesce, the binary has to interact with an amount of matter of the order of its reduced mass, with only a weak dependence on the exact ratio between the initial and final separation. A conservative estimate for this ratio can be obtained setting \( a_e \) equal to the radius at which the two MBHs bind in a binary

\[
a_e \sim \frac{GM}{2a^2} \approx 0.5 \left( \frac{M}{10^4 M_\odot} \right)^{1/2} \text{pc},
\]

where we have assumed the \( M-\sigma_\star \) relation (Gültekin et al. 2009). The final separation can be conservatively estimated as \( a_e \sim 6 \times 10^{-5}(M/10^4 M_\odot)^{1/2} \text{pc} \), in order for the BHB to coalesce due to the emission of gravitational waves in \( \sim 10^7 \text{yr} \). Under these assumptions \( a_i/a_e \propto M^{-1/4} \) and \( \ln (a_i/a_e) < 9 \) for a binary mass \( > 10^6 M_\odot \).

2.2 Gas inflow on to BHBs: observational constraints

We can now, for the first time, try to relax any a priori assumption on the accretion rate in the circumbinary disc (such as the Eddington limit used in equation 4), assuming an observational driven prescription for the evolution of \( \dot{m} \) as a function of MBH and redshift. In particular, we adopt the average accretion rates obtained assuming that the MBH evolution is governed by a continuity equation, where the MBH mass function at any given time can be used to predict that at any other time, provided the distribution of accretion
rates as a function of BH mass is known. The continuity equation can be written as

$$\frac{\partial \psi(m, t)}{\partial t} + \frac{\partial}{\partial m} \left( \psi(m, t) \langle M(m, t) \rangle \right) = 0,$$

where $m = \log M$ ($M$ is the BH mass in solar units), $\psi(m, t)$ is the MBH mass function at time $t$ and $\langle M(m, t) \rangle$ is the average accretion rate of an MBH of mass $M$ at time $t$. The average accretion rate can be defined through a ‘fuelling’ function, $F(m, m, t)$, describing the distribution of accretion rates for objects of mass $M$ at time $t$: $\langle M(M, z) \rangle = \int M F(m, m, z) \, dm$. Such a fuelling function is not known a priori, and observational determinations thereof have been able so far to probe robustly only the extremes of the overall population. However, the fuelling function can be derived by inverting the integral equation that relates the luminosity function ($\phi$) of the AGN population with its mass function. Indeed we can write

$$\phi(\ell, t) = \int F(\ell - \zeta, m, t) \psi(m, t) \, dm,$$

where we have called $\ell = \log L_{bol}$ and $\zeta = \log (\epsilon c^2)$, with $\epsilon$ the radiative efficiency. This is assumed to be constant and its average value can be estimated by means of the Soltan argument (Soltan 1982), which relates the mass density of remnants MBH in the local Universe with the integrated amount of accreted gas during the AGN phases, as identified by the luminosity function.

Gilfanov & Merloni (2014) reviewed the most recent assessments of the Soltan argument. Adopting as a starting point the bolometric AGN luminosity function of Hopkins et al. (2007), the estimate of the (mass-weighted) average radiative efficiency, ($\epsilon$), can be expressed as

$$\frac{\langle \epsilon \rangle}{\Gamma - \langle \epsilon \rangle} \approx 0.075 \left[ \xi_{0} (1 - \xi_{\ell} - \xi_{CT} + \xi_{lost}) \right]^{-1},$$

where $\xi_{0} = \rho_{BH, z=0}/4.2 \times 10^{5} M_{\odot} Mpc^{-3}$ is the local ($z = 0$) MBH mass density in units of $4.2 \times 10^{5} M_{\odot} Mpc^{-3}$ (Marconi et al. 2004); $\xi_{\ell}$ is the mass density of BHs at the highest redshift probed by the bolometric luminosity function, $z \approx 6$, in units of the local one, and encapsulates our uncertainty on the process of BH formation and seeding in protogalactic nuclei; $\xi_{CT}$ is the fraction of the MBH mass density (relative to the local one) grown in heavily obscured, Compton-thick AGN; finally, $\xi_{lost}$ is the fraction of BH mass contained in ‘wandering’ objects, that have been ejected from a galaxy nucleus, for example, in the aftermath of a merging event because of the anisotropy in the emission of gravitational waves (e.g. Lousto & Zlochower 2013, and references therein). More recent estimates of the fraction of MBH mass density accumulated in heavily obscured, Compton-thick AGN (Buchner et al. 2015) suggest that $\xi_{CT} \approx 0.35$. Neglecting $\xi_{\ell}$ in equation (8) (i.e. assuming a negligibly small seed BH mass density), the average radiative efficiency will vary approximately between 0.075 and 0.1 for $0 < \xi_{lost} < 0.3$. Therefore in the following we will use the results obtained by performing a numerical inversion of equation (7), based on a minimization scheme that used the Hopkins et al. (2007) AGN bolometric luminosity function as a constraint, and assuming a fixed radiative efficiency in the range $0.075 < \epsilon < 0.1$. The average Eddington ratios $\bar{\epsilon}_{BHB}$ (bolometric luminosity normalized to the Eddington limit) and accretion rates obtained in this way are shown as a function of redshift in the left-hand and right-hand panels of Fig. 1, respectively. We note that increasing the radiative efficiency value implies a decrease in the average accretion rates, especially for higher MBH masses at higher redshifts where the MBH evolution is relatively more important. This is consistent with the adopted calculation scheme where the AGN luminosity function is assumed as a constraint to derive the accretion rates estimates.

3 RESULTS

The observational constraints on the average value of $\dot{m}$ (function of $M$ and $z$) discussed in Section 2.2 allow us to numerically integrate equation (3) to determine the migration time-scale $\Delta t_{BHB}$ for any BHB. We can further translate $\Delta t_{BHB}$ into an estimate of the minimum redshift $z_{BHB}$ at which a BHB of mass $M$ must form in order to coalesce within a given redshift $z_{coal}$. The results of the numerical integration are shown in Fig. 2 for a binary mass ratio $q = 0.1$ (lower panel) and $q = 0.3$ (upper panel).
2. Binaries forming at $q = 0.3$ (right curve). Thin dotted lines mark the three values assumed for $q_{\text{coal}}$ with $L$ within the redshift interval considered in this analysis. As expected, decreasing $f$ the evolution of every binary slows down, but the general trends discussed while commenting the $f = 1$ cases remain valid. BHBs with total mass $M \sim 10^7 M_\odot$ at $z > 2$ are particularly affected, because of the redshift at which their typical Eddington ratio peaks (see Fig. 1). Still, these BHBs manage to coalesce between $z_{\text{coal}} \approx 1-2$, as well as their more massive counterparts.

4 CONCLUSIONS

We estimated the gas driven orbital decay of BHBs from the instant at which they bind in a binary down to their final coalescence. For the first time we propose an observationally driven approach, that has the advantage of not being affected neither by any assumption on the (largely unknown) feeding process driving the accretion, nor by the fraction of the gas inflow that turns into stars at large scales before interacting with the BHB.

Our investigation proves that (1) high-redshift BHBs of any mass coalesce on a very short time; (2) low-mass BHBs ($M \lesssim 10^7 M_\odot$) formed at low redshift manage to merge anyway within $z = 0$, since their accretion history peaks at lower redshifts. These findings are particularly relevant since the coalescence of low-mass BHBs is one of the sources of gravitational waves detectable by future space-based gravitational wave interferometers, such as the mission concept eLISA (Amaro Seoane 2013).

We have worked under very conservative assumptions.

(i) We assumed that binaries in the late stages of galaxy mergers are fuelled as much as MBHs in comparable isolated galaxies, without assuming any merger driven boost in the accretion. The merger process itself is considered, however, an efficient reshuffler of the gas angular momentum at galactic scales, driving efficiently gas inflows all the way down to the two MBHs, as confirmed by observations (e.g. Kennicutt & Keel 1984; Keel et al. 1985; Alonso et al. 2007; Ellison et al. 2011; Koss et al. 2011; Silverman et al. 2011; Satyapal et al. 2014) as well as by a wealth of numerical works.
performed on different kind of mergers (e.g. Di Matteo, Springel & Hernquist 2005; Johansson, Burkert & Naab 2009; Hopkins & Quataert 2010; Callegari et al. 2011; Van Wassenhove et al. 2012; Capelo et al. 2015).

(ii) We have assumed that all the gas accretion on to MBHs is radiatively efficient, and that the mass accretion rate at few gravitational radii ($R_g$, where basically all the luminosity is emitted) is equal to the that at thousands of $R_g$, where the gas interacts with the secondary MBH. We stress that a significant fraction of the accretion flow, however, could be ejected in the form of fast outflows, as often found in numerical simulations (e.g. Proga 2003; Narayan et al. 2012).

Under our conservative assumptions, gas-driven migration of high-mass ($M \gtrsim 10^8 M_\odot$) BHBs formed at low redshift could be inefficient. Such binaries are of particular interest, being the only ones observable through pulsar timing (Hobbs et al. 2010, and references therein). The morphological and dynamical characteristic of their hosts suggest, however, that interactions with stars could play a significant role in the binaries shrinking. The hosts of very massive MBHs often show triaxial profiles (e.g. Faber et al. 1997; Kormendy et al. 2009, and references therein). The lack of spherical and axial symmetry in the potential of the hosts allows the single stars to modify substantially their angular momentum components. Stars can hence refill the loss cone of the binaries at rates significantly higher than those expected in spherical systems, leading BHBs to a fast coalescence (see Vasiliev 2014, for a recent discussion).

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\[1 \text{In spherical potentials the collisional refilling of the binary loss cone can lead binaries to coalescence within } 10^{10} \text{ yr only if the total mass of the binary is } M \lesssim 10^8 M_\odot; \text{ see e.g. section 8.3 in Merritt (2013).}\]