WHY IS THE ZEL’DOVICH APPROXIMATION SO ACCURATE?

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ABSTRACT

Why does the Zel’dovich approximation (ZA) work well to describe gravitational collapse in the universe? This problem is examined by focusing on its dependence on the dimensionality of the collapse. The ZA is known to be exact for a one-dimensional collapse. We show that the ZA becomes progressively more accurate in the order of three-, two-, and one-dimensional collapses. Furthermore, using models for spheroidal collapse, we show that the ZA remains accurate in all collapses, which become progressively lower dimensional with the passage of time. That is, the ZA is accurate because the essence of the gravitational collapse is incorporated in the ZA.

Subject headings: cosmology: theory — galaxies: clusters: general — large-scale structure of universe

1. INTRODUCTION

For gravitational collapse in the universe, Zel’dovich (1970) proposed an approximation, the Zel’dovich approximation (ZA). The ZA is a Lagrangian approximation based on the motion of a fluid element. If its initial position is \( q \) in comoving coordinates, the Eulerian position \( x(q, t) \) at time \( t \) is

\[
x(q, t) = q + \Psi(q, t).
\]

The displacement \( \Psi(q, t) \) satisfies the Lagrangian equation of motion (Peacock 1999, § 15.2):

\[
\frac{\partial^2 \Psi}{\partial t^2} + 2H \frac{\partial \Psi}{\partial t} = -\nabla \Phi.
\]

Here \( H(t) \) is the Hubble parameter. The gravitational potential \( \Phi(x, t) \) is obtained from the density contrast \( \delta(x, t) = \det \left( \frac{\partial x_i}{\partial q_j} \right)^{-1} - 1 \) and the density parameter \( \Omega(t) \) via the Poisson equation

\[
\nabla^2 \Phi = \frac{3}{2} H^2 \Omega \delta.
\]

From the first-order perturbation to equations (2) and (3), Zel’dovich (1970; see also Shandarin & Zel’dovich 1989; Sahni & Coles 1995) assumed that the displacement \( \Psi \) scales with the linear growth factor \( D(t) \):

\[
\Psi = -\frac{2D}{3H^2 \Omega \infty} \nabla q \Phi_{\text{in}}.
\]

Hence, the displacement \( \Psi \) is assumed to satisfy the following equation:

\[
\frac{\partial^2 \Psi}{\partial t^2} + 2H \frac{\partial \Psi}{\partial t} = \frac{3}{2} H^2 \Omega \Psi.
\]

Equation (4) or (6) provides an approximation for the gravitational collapse, until the shell crossing where fluid elements from different initial positions reach the same position in Eulerian coordinates. The approximation applies to any cosmological model.

The ZA is used for various topics in observational cosmology, e.g., the construction of the initial conditions in an N-body simulation (Doroshkevich et al. 1980; Bertschinger 1998), the estimation of the initial conditions of the universe from observations of galaxies in the present universe (Dekel 1994), and the analytical study of the pairwise peculiar velocities of galaxies (Yoshisato et al. 2003). The ZA is also used to develop the adhesion approximation (Gurbatov et al. 1989), the truncated ZA (Coles et al. 1993), and the post-, post-post-, and Padé-ZAs (Bernardeau 1994; Buchert 1994; Munshi et al. 1994; Bouchet et al. 1995; Catelan 1995; Matsubara et al. 1998). Thus, the ZA plays an important role.

Despite its simplicity, the ZA is accurate even in the quasilinear regime, provided that the shell crossing is unimportant as at early times or at large scales. This accuracy has been demonstrated in statistical comparison with an N-body simulation (Coles et al. 1993; Kolman et al. 1994) and with a high-order Eulerian perturbation theory (Bernardeau et al. 1994; Munshi & Starobinsky 1994).

Why is the ZA so accurate? The usual explanation invokes the fact that the Lagrangian description is intrinsically nonlinear in the density field (Catelan 1995; Bernardeau et al. 2002). The density contrast \( \delta \) is obtained from the displacement \( \Psi \). Even if the displacement is small and lies in the linear regime, the
corresponding density contrast could be large and lie in the nonlinear regime. However, the ZA's accuracy depends on the shape of the overdense region. We believe that this dependence is the most essential (Yoshisato et al. 1998; Peacock 1999, § 15.8). The past comparison of the ZA with an exact solution was made only for a spherical collapse (Munshi et al. 1994; Sahni & Shandarin 1996). Yoshisato et al. (1998) made the comparison for spheroidal collapses, which are more usual situations in the real universe. It was found that for a given value of the density contrast, the ZA is more accurate in a lower dimensional collapse.3

Ze’l’dovich (1970) noticed that in the ZA equation (6), the acceleration term \(-\nabla_x \Phi\) in the true equation of motion (2) has been replaced by the linear term for the displacement \(3H^2 \Omega \Psi /2\). This approximate gravitational acceleration is identical to the true gravitational acceleration in a one-dimensional collapse (Buchert 1989), for which the ZA is exact (Doroshkevich et al. 1973).

Here we analytically show that the ZA’s gravitational acceleration approaches the true gravitational acceleration in the order of three-, two-, then one-dimensional collapses (§ 2). We also numerically show that any spheroidal collapse becomes progressively lower dimensional with the passage of time. The ZA’s gravitational acceleration remains close to the true gravitational acceleration (§ 3). Then we discuss why the ZA is accurate (§ 4).

2. DIMENSIONAL DEPENDENCE OF EQUATION OF MOTION

The Lagrangian equations of motion (2) are derived for one-dimensional planar, two-dimensional cylindrical, and three-dimensional spherical collapses. We compare these equations with the ZA equation (6).

2.1. One-dimensional Planar Collapse

When the collapse is planar, the gravitational potential \(\Phi(x, t)\) is obtained using Green’s function \(3H^2 \Omega |x|/4\) for the one-dimensional Poisson equation (3):

\[
\Phi = \frac{3}{4} H^2 \Omega \int_{-\infty}^{\infty} \delta(x', t)|x - x'| dx'.
\]

(7)

The gravitational acceleration of the fluid element at position \(x\) is

\[
-\frac{\partial \Phi}{\partial x} = -\frac{3}{4} H^2 \Omega \left[ \int_{-\infty}^{x} \delta(x', t) dx' - \int_{x}^{\infty} \delta(x', t) dx' \right].
\]

(8)

This gravitational acceleration is proportional to the difference of the integrated density contrast between both sides of position \(x\). The reason is that in a one-dimensional collapse, the gravity is independent of the distance. We rewrite equation (8) with the displacement of the fluid element \(\Psi = x - q\). Here \(q\) is the initial position. Until the shell crossing, mass conservation leads to

\[
\int_{-\infty}^{x} 1 + \delta(x', t) dx' = \int_{-\infty}^{q} dx',
\]

\[
\int_{x}^{\infty} 1 + \delta(x', t) dx' = \int_{q}^{\infty} dx'.
\]

(9)

Here we set the initial density contrast to zero. It follows from equation (9) that the integrated density contrast is proportional to the displacement \(\Psi\):

\[
\int_{-\infty}^{x} \delta(x', t) dx' = \int_{-\infty}^{q} dx' = q - x = -\Psi,
\]

\[
\int_{x}^{\infty} \delta(x', t) dx' = \int_{q}^{\infty} dx' = x - q = \Psi.
\]

(10)

The gravitational acceleration is

\[
-\frac{\partial \Phi}{\partial x} = \frac{3}{2} H^2 \Omega \Psi.
\]

(11)

Thus, in one-dimensional planar collapse, the true gravitational acceleration is identical to the ZA’s gravitational acceleration.

2.2. Two-dimensional Cylindrical Collapse

When the collapse is cylindrical, we use Green’s function \(3H^2 \Omega \ln |x|/4\pi r\) for the two-dimensional Poisson equation (3). The gravitational acceleration of the fluid element at position \(x\) is

\[
-\frac{\partial \Phi}{\partial |x|} = -\frac{3}{2} H^2 \Omega \frac{1}{|x|} \int_{0}^{|x|} \delta(x', t)r' dr'.
\]

(12)

This acceleration is proportional to \(|x|^{-1}\) times the density contrast integrated over the radius \(|x|\), in accordance with Gauss’s theorem (Binney & Tremaine 1987, § 2.0). The gravity in a two-dimensional collapse is proportional to the inverse of the distance. We rewrite equation (12) with the initial position of the fluid element \(q\). Until the shell crossing, mass conservation leads to

\[
\int_{0}^{|x|} \left[ 1 + \delta(r', t) \right] r' dr' = \int_{0}^{q} r' dr'.
\]

(13)

Then, as in § 2.1, the gravitational acceleration is

\[
-\frac{\partial \Phi}{\partial |x|} = \frac{3}{4} H^2 \Omega \frac{|x|^2 - |q|^2}{|x|}.
\]

(14)

Since the displacement \(\Psi = |x| - |q|\), this true gravitational acceleration differs from the ZA’s gravitational acceleration \(3H^2 \Omega \Psi /2\) by a factor \((|x| + |q|)/2|x|\).

2.3. Three-dimensional Spherical Collapse

When the collapse is spherical, we use Green’s function \(-3H^2 \Omega /8\pi r^2\) for the three-dimensional Poisson equation (3). The gravitational acceleration of the fluid element at position \(x\) is

\[
-\frac{\partial \Phi}{\partial |x|} = -\frac{3}{2} H^2 \Omega \frac{1}{|x|^2} \int_{0}^{|x|} \delta(x', t)r'^2 dr'.
\]

(15)

This acceleration is proportional to \(|x|^{-2}\) times the density contrast integrated over the radius \(|x|\), in accordance with Gauss’s theorem (Binney & Tremaine 1987, § 2.0). The gravity in a three-dimensional collapse is proportional to the inverse of the distance squared. We rewrite equation (15) with the initial position of the fluid element \(q\). Until the shell crossing, mass conservation leads to

\[
\int_{0}^{|x|} \left[ 1 + \delta(r', t) \right] r'^2 dr' = \int_{0}^{q} r'^2 dr'.
\]

(16)

3 Throughout this paper, as in Yoshisato et al. (1998), we use the dimension to characterize the shape of the collapse, which is assumed to occur in a three-dimensional space. The one-, two-, and three-dimensional collapses correspond to planar, cylindrical, and spherical collapses, respectively.
Then, as in §§ 2.1 and 2.2, the gravitational acceleration is

\[- \frac{\partial \Phi}{\partial |x|} = \frac{1}{2} H^2 \Omega \frac{|x|^3 - |q|^3}{|x|^2}. \tag{17}\]

Since the displacement \( \Psi \) is \( |x| - |q| \), this true gravitational acceleration differs from the ZA’s gravitational acceleration \( 3H^2 \Omega \Psi /2 \) by a factor \((|x|^2 + |x||q| + |q|^2) / 3|x|^2 \).

### 2.4. Dimensional Dependence

Summarizing the equations of motion for one-dimensional planar, two-dimensional cylindrical, and three-dimensional spherical collapses (eqs. [11], [14], and [17]), we have

\[ \frac{\partial^2 \Psi}{\partial t^2} + 2H \frac{\partial \Psi}{\partial t} = \frac{3}{2} H^2 \Omega \frac{|x|^n - |q|^n}{n}, \tag{18}\]

with the displacement \( \Psi = |x| - |q| \) and the dimension \( n = 1, 2, \) or 3. The factor \( 1 / |x|^n \) reflects the dependence of the gravity on the distance \( |x| \). The factor \( (|x|^n - |q|^n) / n \) reflects the change of the integrated density contrast in the course of the gravitational collapse.

We compare equation (18) with the ZA equation (6), in order to study the ZA’s accuracy. When the displacement is small, these two equations are almost identical. The ZA is accurate. Even when the displacement is large, in a lower dimensional collapse, the gravity \( 1 / |x|^n \) is more independent of the distance \( |x| \), and the change of the integrated density contrast \( (|x|^n - |q|^n) / n \) is closer to the displacement \( |x| - |q| \). The ZA becomes progressively more accurate in the order of three-, two-, and one-dimensional collapses. For a one-dimensional collapse, the ZA equation (6) is exact. These results are independent of the functional forms of \( H(t) \) and \( \Omega(t) \) and thus apply to any cosmological model.

### 3. SPHEROIDAL COLLAPSE

During a real gravitational collapse, the overdense region changes its shape. The effect of this change on the ZA’s accuracy, especially in the gravitational acceleration, is studied by using the gravitational collapse of a homogeneous spheroid as an idealized model (see also Yoshisato et al. 1998).

When pressure is negligible, the gravitational collapse increases the ellipticity of the spheroid (Lin et al. 1965). While an initially oblate spheroid forms a planelike structure, an initially prolate spheroid forms a linelike structure. This is because the gradient of the gravitational potential generated by the spheroid is larger along the minor axis of the spheroid than along the major axis. The gravitational acceleration is stronger and the gravitational collapse is more significant along the minor axis (Binney 1977). Since the collapse becomes progressively lower dimensional with the passage of time, the ZA is expected to remain accurate.

#### 3.1. Equation of Motion

The density contrast in the presence of a homogeneous ellipsoid is determined by the half-lengths of its principal axes \( \alpha_i(t) \):

\[ \delta(x, t) = \delta_c(t) \Theta \left[ 1 - \frac{x_1^2}{\alpha_1^2(t)} - \frac{x_2^2}{\alpha_2^2(t)} - \frac{x_3^2}{\alpha_3^2(t)} \right]. \tag{19}\]

Here \( \Theta(x) \) is a step function: \( \Theta(x) = 1 \) for \( x \geq 0 \) and 0 for \( x < 0 \). The density contrast within the ellipsoid \( \delta_c(t) \) is

\[ \delta_c(t) = (1 + \delta_m) \frac{\alpha_1 \alpha_2 \alpha_3}{\alpha_1(t) \alpha_2(t) \alpha_3(t)} - 1. \tag{20}\]

We adopt spheroidal symmetry, \( \alpha_1(t) = \alpha_2(t) = \alpha_3(t) \). Then the Poisson equation (3) yields the gravitational potential \( \Phi(x, t) \) as

\[ \Phi = \frac{3}{8} H^2 \Omega \delta_c \sum_{\ell=1}^{3} A_{\ell} \alpha_{\ell}^2, \tag{21}\]

with

\[ A_2(t) = A_3(t) = \frac{2}{3} [1 + h(t)], \quad A_3(t) = \frac{2}{3} [1 - 2h(t)]. \tag{22}\]

For an oblate case \( (\alpha_1 = \alpha_2 > \alpha_3) \), we have

\[ h(t) = \frac{3}{4} \frac{1 - \frac{\tilde{e}^2}{e^2}}{\tilde{e}^3} \ln \left( \frac{1 + \tilde{e}}{1 - \tilde{e}} \right) + \frac{3 - 2\tilde{e}^2}{2\tilde{e}^2}, \tag{23}\]

with the ellipticity

\[ e(t) = \left[ 1 - \frac{\alpha_3(t) \alpha_2(t)^2}{\alpha_1(t)^3} \right]^{1/2}. \tag{24}\]

For a prolate case \( (\alpha_1 = \alpha_2 < \alpha_3) \), we have

\[ h(t) = \frac{3}{4} \frac{1 - \frac{\tilde{e}^2}{e^2}}{\tilde{e}^3} \ln \left( \frac{1 - \tilde{e}}{1 + \tilde{e}} \right) + \frac{3 - 2\tilde{e}^2}{2\tilde{e}^2}, \tag{25}\]

with the ellipticity

\[ \tilde{e}(t) = \left[ 1 - \frac{\alpha_3(t) \alpha_2(t)^2}{\alpha_1(t)^3} \right]^{1/2}. \tag{26}\]

These equations are derived in Binney & Tremaine (1987, § 2.3). We consequently obtain the equation of motion (2) for the half-lengths \( \alpha_i(t) \),

\[ \frac{d^2 \alpha_i}{dt^2} + 2H \frac{d\alpha_i}{dt} = -\frac{3}{4} H^2 \Omega \delta_c A_i \alpha_i. \tag{27}\]

The evolution of the half-lengths \( \alpha_i(t) \) describes the evolution of the homogeneous spheroid.

#### 3.2. ZA

Equations (4) and (21) yield the equation for the evolution of the half-lengths \( \alpha_i(t) \) in the ZA:

\[ \alpha_i - \alpha_{i,\infty} = -\frac{1}{2} D\delta_{i,\infty} A_i \alpha_{i,\infty}. \tag{28}\]

The ZA equation of motion (6) is

\[ \frac{d^2 \alpha_i}{dt^2} + 2H \frac{d\alpha_i}{dt} = \frac{3}{2} H^2 \Omega (\alpha_i - \alpha_{i,\infty}). \tag{29}\]

The factor \( A_i \alpha_{i,\infty} \) is larger along the minor axis of the spheroid than that along the major axis. Hence, also in the ZA, gravitational

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*The text contains typographical errors: Eqs. (33) and (36) in Yoshisato et al. (1998) should correspond to our eqs. (20) and (25).*
collapse is more significant along the minor axis. The ellipticity $e(t)$ or $\dot{e}(t)$ accordingly increases.

### 3.3. ZA versus True Solution

Equation (27) is numerically solved to obtain the evolution of the homogeneous spheroid. For simplicity, we adopt the Einstein–de Sitter model ($\Omega = 1$ and $H = 2/3t$), but the results could qualitatively apply to any cosmological model. The initial axis ratio $a_1/a_3$ or $a_3/a_1$ is 2, 4, or 8. The initial density contrast is $\delta_{e,0} = D_{e,0}$. The initial velocity $v_{e,0}$ is determined using the ZA (eq. [28]). We also obtain the evolution in the ZA.

Table 1 shows the scale factor at the moment of complete collapse $a_{col}$. It is evident that in a lower dimensional collapse, the ZA provides a more accurate result.

Figures 1 and 2 show the results for oblate collapses ($a_1 = a_2 > a_3$) and for prolate collapses ($a_1 = a_2 < a_3$), respectively, as a function of the normalized scale factor $a/a_{col}$ (solid lines). For reference, the solutions for the one-dimensional planar, two-dimensional cylindrical, and three-dimensional spherical collapses are also shown (dotted lines).

### Table 1: Scale Factor $a_{col}$ at the Moment of Complete Collapse

| Initial Shape                | True  | ZA    |
|-----------------------------|-------|-------|
| Three-dimensional spherical | 1.686 | 3     |
| Two-dimensional cylindrical | 1.466 | 2     |
| One-dimensional planar      | 1     | 1     |

Oblate:
- $a_1/a_3 = 2$: 1.412, 1.897
- $a_1/a_3 = 4$: 1.216, 1.421
- $a_1/a_3 = 8$: 1.110, 1.204

Prolate:
- $a_3/a_1 = 2$: 1.577, 2.420
- $a_3/a_1 = 4$: 1.510, 2.163
- $a_3/a_1 = 8$: 1.480, 2.059

Fig. 1.—Oblate spheroidal collapse. (a) Half-lengths $a_1$ and $a_3$ in the true solution. (b) The ZA gravitational acceleration along the minor axis $3H^2\Omega(a_1 - a_3)/2$ in the ZA. (c) Density contrast $\delta_e$ in the ZA. The half-lengths are normalized by their initial values. The ZA's gravitational acceleration and density contrast in the ZA are respectively normalized by the true gravitational acceleration $-3H^2\Omega\Delta a_1/4$ and density contrast in the true solution. The abscissa is the scale factor $a$ normalized by its value at the moment of complete collapse $a_{col}$ in the true solution. The dotted lines denote the one- and three-dimensional collapses.

Fig. 2.—Same as Fig. 1 but for prolate spheroidal collapse. The ZA's gravitational acceleration is $3H^2\Omega(a_1 - a_3)/2$, and the true gravitational acceleration is $-3H^2\Omega\Delta a_1/4$ along the minor axis. The dotted lines denote the one-, two-, and three-dimensional collapses.
The abscissa is the density contrast \( \rho_1 / \rho_{1,\text{in}} \) / \( \rho_1 / \rho_{1,\text{in}} \) spheroidal collapse. These are the same as Figs. 1 and 2 by the true gravitational acceleration. (a) Oblate spheroidal collapse. (b) Prolate spheroidal collapse. These are the same as Figs. 1b and 2b, respectively, but the abscissa is the density contrast \( \delta \), in the true solution.

First, we discuss the evolution of the half-lengths (Figs. 1a and 2a). The collapse is rapid along the minor axis, while it is slow along the major axis. In the oblate cases (Fig. 1a), the collapse asymptotically becomes one-dimensional. The collapse along the minor axis is intermediate between the one- and three-dimensional collapses. If the initial axis ratio \( \alpha_{1,\text{in}} / \alpha_{3,\text{in}} \) is large, the collapse is close to the one-dimensional collapse. In the prolate cases (Fig. 2a), the collapse asymptotically becomes two-dimensional. The collapse along the minor axis is intermediate between the two- and three-dimensional collapses. If the initial axis ratio \( \alpha_{3,\text{in}} / \alpha_{1,\text{in}} \) is large, the collapse is close to the two-dimensional collapse.

Second, for the gravitational acceleration along the minor axis, we compare the ZA with the true numerical solution (Figs. 1b and 2b). In the oblate cases (Fig. 1b), the ZA’s accuracy is intermediate between those in the one- and three-dimensional collapses. If the initial axis ratio is large, the ZA’s accuracy is close to that in the one-dimensional collapse. The ZA asymptotically becomes one-dimensional with the passage of time. That is, the ZA is accurate because of the essence of the gravitational collapse is incorporated in the ZA.

Third, for the density contrast, we compare the ZA with the true numerical solution (Figs. 1c and 2c). In the oblate cases (Fig. 1c), the ZA’s accuracy is intermediate between those in the one- and three-dimensional collapses. If the initial axis ratio is large, the ZA’s accuracy is close to that in the one-dimensional collapse. In the prolate cases (Fig. 2c), the ZA’s accuracy is intermediate between those in the two- and three-dimensional collapses. If the initial axis ratio is large, the ZA’s accuracy is close to that in the two-dimensional collapse. The ZA is less accurate in the density contrast than in the gravitational acceleration at the late stage of the evolution. This is because complete collapse occurs earlier in the true numerical solution than in the ZA (Table 1).

4. DISCUSSION

The ZA is the linear approximation to replace \(-\nabla, \Phi / 3H^2 \Omega \Psi / 2\) for the gravitational acceleration in the Lagrangian equation of motion (eqs. [2] and [6]). This approximation becomes progressively more accurate in the order of three-, two-, and one-dimensional collapses (eq. [18]). This is because in a lower dimensional collapse, the gravity 1/|x| is more independent of the distance |x|, and the change of the integrated density contrast \(|x| - |q| / n\) is closer to the displacement \( \Psi = |x| - |q| \). For the one-dimensional collapse, the ZA’s gravitational acceleration is exact.

Spheroidal collapse preferentially proceeds along the minor axis, i.e., the direction of the strongest gravitational acceleration (Figs. 1 and 2). While an oblate spheroid asymptotically undergoes a one-dimensional collapse, a prolate spheroid asymptotically undergoes a two-dimensional collapse. The dimension of the collapse decreases in the course of the collapse. This is well reproduced in the ZA. The ZA’s gravitational acceleration remains close to the true gravitational acceleration.

During structure formation in the real universe, an overdense region preferentially collapses along the direction of the strongest gravitational acceleration. The collapse asymptotically becomes one-dimensional as in the case of oblate spheroidal collapse. Since, at a later stage, the overdense region could collapse along the other directions, the final collapse is not necessarily planelike but could be point- or linelike (Arnold et al. 1982; see also Sahni & Coles 1995; Kurokawa et al. 2001). The ZA is not accurate, because of the shell crossing, after the complete collapse along the direction of the strongest gravitational acceleration.

5 Since, at a later stage, the overdense region could collapse along the other directions, the final structure is not necessarily planelike but could be point- or linelike (Arnold et al. 1982; see also Sahni & Coles 1995; Kurokawa et al. 2001). The ZA is not accurate, because of the shell crossing, after the complete collapse along the direction of the strongest gravitational acceleration.
dimensional collapse (eq. [18]). The ZA should be more accurate in a higher dimensional collapse, according to this usual explanation. However, in practice (Figs. 1–3; see also Yoshisato et al. 1998), the ZA is more accurate in a lower dimensional collapse.

The ZA has long been known to be exact for a one-dimensional collapse (Doroshkevich et al. 1973). However, with this property alone, it is impossible to explain the ZA’s accuracy. This is because, in general, gravitational collapse is not exactly one-dimensional. Our study has clarified that, even if the gravitational collapse is not exactly one-dimensional, the ZA is sufficiently accurate. Thus, we have successfully explained why the ZA is accurate.

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