Characterising dynamic non-linearity in floating wind turbines

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Abstract. Fully coupled aero-hydro-control-elastic codes are being developed to cope with the new modelling challenges presented by floating wind turbines, but there is also a place for more efficient methods of analysis. One option is linearisation and analysis in the frequency domain. For this to be an effective method, the non-linearities in the system must be well understood. The present study focuses on understanding the dynamic response of the rotor to the overall platform motion, as would arise from wave loading, by using a simple model of a floating wind turbine with a rigid tower and flexible rotor (represented by hinged rigid blades). First, an equation of motion of the blade is derived and an approximate solution for the blade response is found using the perturbation method. Secondly, the full non-linear solution is found by time-domain simulation. The response is found to be linear at lower platform pitching frequencies, becoming non-linear at higher frequencies, with the approximate solution giving good results for weakly non-linear behaviour. Higher rotor speeds have a stabilising effect on the response. In the context of typical floating turbine parameters, it is concluded that the blade flapwise response is likely to be linear.

1. Introduction
Interest is growing in floating wind turbines as a way to access a large worldwide deep-water wind energy resource [1]. Placing a wind turbine on a floating platform, though, introduces new modelling challenges compared with existing fixed-base turbines and existing floating platforms. In particular, coupling between the wind turbine rotor, the platform motion, and the mooring-line dynamics is introduced, and larger platform dimensions and motions cause more complex hydrodynamic loading.

Fully coupled aero-hydro-control-elastic codes are being developed to cope with these challenges [2], but the computational cost of such codes can be high. There is a place for more efficient methods of analysis. One option is linearisation and analysis in the frequency domain; while this is widely used in offshore engineering, it is less popular in wind turbine analysis. For this to be an effective method, the non-linearities present in the system must be well understood. The present study focuses on understanding the dynamic response of the rotor to the overall platform motion in pitch, as would arise from wave loading. Together with a similar understanding of the other parts of the system, this can inform the introduction of more advanced linearisation schemes which will improve the accuracy and applicability of such models.

This study uses a simple model of a floating wind turbine with a rigid tower and flexible rotor (represented by hinged rigid blades). In the first part, an equation of motion of the blade is derived and an approximate solution for the blade response is found using the perturbation
method. Secondly, the full non-linear solution is found by time-domain simulation, and the results are compared.

2. Platform pitch model

2.1. Description of model
The model is shown in Figure 1. The tower flexibility and mass are neglected. The rotor can rotate about the tower top, and is made up of three blades. The flexibility of the blades is modelled by rotational hinges and springs at the blade root; the blades themselves are rigid. The blade properties and tower height match the OC3-Hywind model [3]. There are three degrees of freedom which are labelled in Figure 1. Only inertial and gravity loads are considered.

2.2. Parameter ranges
Pitching motion of the platform has been considered at a range of frequencies and amplitudes, for several rotor speeds, listed in Table 1. Since the aim is to uncover non-linear behaviour, it is appropriate to cover a larger range of conditions than might be expected in reality. Published data on pitch natural frequencies of floating turbines has been collected in Table 2, which is consistent with the range of frequencies considered here. Real platform pitch amplitudes and rotor speeds are also expected to lie within the ranges in Table 1.

| Parameter                  | Values       |
|----------------------------|--------------|
| Rotor speeds               | 0, 10, 20, 30 rpm |
| Platform pitch frequency   | 0.1, 0.2, 0.3, 0.4, 0.5, 0.6 Hz |
| Platform pitch amplitude   | 5, 10, 15, 20, 25, 30 deg |
3. Analytical solution

3.1. Equation of motion

In terms of the unit vectors and angles shown in Figure 1, the velocity of a point on the blade at radius \( r \) is given by

\[
\dot{r} = \dot{\phi} (h + r \cos \alpha \cos \theta) \hat{i} - r \dot{\phi} \sin \alpha \hat{k} - r \dot{\theta} \cos \alpha \hat{b} + r \ddot{m} \tag{1}
\]

From this, the kinetic energy of a blade can be found as

\[
T = (M h^2 + 2 h l_1 \cos \alpha \cos \theta + l_2 \cos^2 \alpha \cos^2 \theta + l_2 \sin^2 \alpha) \dot{\phi}^2 + l_2 \cos^2 \alpha \dot{\theta}^2 + l_2 \dot{\alpha}^2 + 2 \ddot{\alpha} (h l_1 \cos \alpha + l_2 \cos \theta) + l_2 \ddot{\theta} \sin 2 \alpha \sin \theta \tag{2}
\]

where \( M \) is the blade’s mass, and \( l_1 \) and \( l_2 \) are its first and second moments of mass.

The vertical height of a point on the blade is

\[
\frac{1}{2} k_{\text{flap}} \dot{\alpha}^2 + M g h \cos \phi + l_1 g \cos \alpha \cos \phi \cos \theta - \sin \alpha \sin \phi \tag{3}
\]

This analysis aims to investigate the rotor dynamic response to platform motion, so the motion of \( \phi \) and \( \theta \) will be prescribed. The platform will pitch harmonically at frequency \( \omega \), and the rotor will rotate at constant speed \( \Omega \):

\[
\phi = A \sin \omega t \tag{4a}
\]

\[
\theta = \Omega t \tag{4b}
\]

The equation of motion for the blade flap motion, \( \alpha \), is derived from Lagrange’s equation. The equation is derived without approximation, to ensure that all effects are included, but to simplify the rest of the analysis several assumptions are now introduced. First, the blade flap angle is assumed small, such that \( \sin \alpha \approx \alpha \) and \( \cos \alpha \approx 1 \), which is justified as \( \alpha \) is representing the elastic deflection of the blade. Second, the potential energy (3) contains sinusoidal functions of the platform pitch angle, \( \phi \), which is itself sinusoidal. \( \sin \phi \) and \( \cos \phi \) can be expanded as infinite harmonic series, but for reasonable amplitudes \( A \), the higher terms are small and \( \sin \phi \approx \phi \) and \( \cos \phi \approx 1 \). With these assumptions, the equation of motion becomes

\[
\ddot{\alpha} + (\omega_n^2 + \Omega^2 - \lambda g \cos \theta) \alpha = \lambda g \phi + 2 \phi \Omega \sin \theta - \dot{\phi} (\lambda h + \cos \theta) - \frac{\alpha \dot{\phi}^2}{2} (2 \lambda h \cos \theta + \cos 2 \theta - 1) \tag{5}
\]

where \( \lambda = l_1 / l_2 \) is the ratio of blade moments of mass, and \( \omega_n^2 = k_{\text{flap}} / l_2 \) is the natural frequency of the blade.

The left hand side of equation (5) has the same form as the Mathieu equation [12],

\[
\ddot{w} + (\delta + \epsilon \cos z) w = 0.
\]

This equation is known to have regions of instability, dependent on the parameters \( \delta \) and \( \epsilon \). In equation (5), the maximum of the ratio \( \epsilon / \delta \) is \( \lambda g / \omega_n^2 \), when the rotor speed \( \Omega \) is zero. For the OC3-Hywind blade this is about 0.01, and so the parametric forcing term will be neglected in the following analysis. If softer blades were used this effect might be worth investigating further.

| Table 2. Published floating platform pitch frequencies |
|-----------------------------------------------------|
| **Type**          | **Frequency range (Hz)** | **Reference** |
|-------------------|--------------------------|---------------|
| Tension-leg platforms | 0.22 – 0.63              | [4–6]         |
| Semi-submersibles             | 0.16                     | [7]           |
| Barges                | 0.072 – 0.086            | [8, 9]        |
| Spar buoys            | 0.024 – 0.035            | [10, 11]      |
3.2. Perturbation method

The equation of motion (5), neglecting the parametric excitation term, can be written as
\[
\ddot{\alpha} + (\omega_n^2 + \Omega^2) \alpha = A f(t) + A^2 g(t) \alpha \tag{6}
\]

This equation is non-linear and cannot be solved directly, but an approximate solution can be found by the method of perturbation [12]. First, the blade flap angle \(\alpha\) is expanded as a power series in terms of the parameter \(A\), the amplitude of the platform pitch motion:
\[
\alpha(t) = \alpha_0(t) + A \alpha_1(t) + A^2 \alpha_2(t) + A^3 \alpha_3(t) + \ldots \tag{7}
\]

Substituting (7) into (6) and then equating each power of \(A\) gives a series of equations which can be solved in turn:
\[
\begin{align*}
\ddot{\alpha}_0 + (\omega_n^2 + \Omega^2) \alpha_0 &= 0 \tag{8a} \\
\ddot{\alpha}_1 + (\omega_n^2 + \Omega^2) \alpha_1 &= f(t) \tag{8b} \\
\ddot{\alpha}_2 + (\omega_n^2 + \Omega^2) \alpha_2 &= \alpha_0 g(t) \tag{8c} \\
\ddot{\alpha}_3 + (\omega_n^2 + \Omega^2) \alpha_3 &= \alpha_1 g(t) \tag{8d} \\
&\vdots
\end{align*}
\]

The approximate solutions \(\alpha_i(t)\) are found in the Appendix, equations (A.1)–(A.4). The even terms vanish, so the total approximate response to third order is
\[
\alpha(t) = A \alpha_1(t) + A^3 \alpha_3(t) \tag{9}
\]

4. Simulation results

Time-domain simulations of the same simple model have been run using a custom multibody dynamics code. This code has been designed for modelling floating wind turbines, and accounts for the full non-linear rigid-body dynamics of this model. It has been verified against other codes’ results.

Simulations were run for 200s, with a small amount of damping to remove the transient free vibration. A discrete Fourier transform was used to identify the transfer functions from the input platform pitch motion to the blade flap response.

5. Results

Transfer functions found from numerical simulation and from the approximate solution (9) are shown in Figure 2 for various platform pitch frequencies and rotor speeds. At low frequencies (left of figure), the peaks of the transfer functions are coincident, showing that the behaviour is linear. At higher frequencies (right of figure), the transfer functions for different inputs are no longer identical, implying that the blade flap response has become non-linear.

The transfer functions are dominated by up to three peaks, which occur at the platform pitch frequency \(\omega\) and the two sidebands \(\omega \pm \Omega\). The results are shown more clearly by plotting the amplitudes of these peaks separately (Figures 3–5).

In each case, the response depends linearly on the platform motion at lower frequencies of pitch motion (left of figures). At higher frequencies, the response is less than linear (right of figures). In addition, it can be seen that the response at higher rotor speeds is more linear, probably due to the centrifugal stiffening effect.

The third-order solution (9) matches the simulated results quite well under weakly non-linear conditions. Adding more terms to the perturbation series may give better results for the larger-amplitude motion.
In the context of the frequency ranges listed in Table 2, it can be seen that the platform pitch motion of most types of floating platform is low enough to fall within the linear region of the response. Although tension-leg platforms have higher natural frequencies, the amplitude of their motion should be less.

6. Conclusions
This study has shown that, for realistic platform pitch frequencies and amplitudes, the flapwise blade response of a floating wind turbine will depend linearly on the amplitude of the platform pitch motion. Realistic platform natural frequencies have been estimated based on the range of values found in the literature. Outside this range, the response does become non-linear.

The response has been approximated by a perturbation solution of the non-linear equation of motion governing the blade flap motion. This predicts the main peaks in the transfer function quite well. Further terms in the perturbation solution may give an improved result.

A weak parametric excitation was identified in the blade flap equation of motion. If softer blades were used it could be worth exploring this effect further.

Work is underway to extend this study, by checking the results against a more complete model including flexible blades and tower, and by considering the response to other platform motions.

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Figure 3. Amplitude of $\omega$ peak, against platform motion amplitude.

Figure 4. Amplitude of $\omega + \Omega$ peak, against platform motion amplitude.
Figure 5. Amplitude of $\omega - \Omega$ peak, against platform motion amplitude. Note that scale of this figure is smaller than Figures 3 and 4.

References
[1] Henderson A R and Witcher D 2010 Wind Engineering 34 1–16
[2] Cordle A and Jonkman J M 2011 Proceedings of the International Offshore and Polar Engineering Conference (Maui, Hawaii) pp 367–374
[3] Jonkman J M 2010 Definition of the Floating System for Phase IV of OC3 Tech. Rep. NREL/TP-500-47535 (Golden: National Renewable Energy Laboratory)
[4] Lee K H 2005 Responses of Floating Wind Turbines to Wind and Wave Excitation Ph.D. thesis (MIT)
[5] Withee J E 2004 Fully Coupled Dynamic Analysis of a Floating Wind Turbine System Ph.D. thesis (MIT)
[6] Matha D 2009 Model Development and Loads Analysis of an Offshore Wind Turbine on a Tension Leg Platform, with a Comparison to Other Floating Turbine Concepts Ph.D. thesis (Boulder: University of Colorado)
[7] Larsen T J, Kallesøe B S and Hansen H F 2011 Proceedings of the International Offshore and Polar Engineering Conference (Maui, Hawaii) pp 391–398
[8] Wayman E N, Sclavounos P D, Butterfield S, Jonkman J M and Musial W 2006 Offshore Technology Conference (Houston, Texas)
[9] Jonkman J M 2007 Dynamics Modeling and Loads Analysis of an Offshore Floating Wind Turbine Tech. Rep. NREL/TP-500-41958 (Golden: National Renewable Energy Laboratory)
[10] Larsen T J and Hanson T D 2007 Journal of Physics: Conference Series 75 012073
[11] Karimirad M and Moan T 2012 Marine Structures
[12] Stoker J J 1950 Nonlinear Vibrations (New York: Interscience Publishers Ltd)
Appendix A. Solutions to perturbation equations

Zeroth-order solution

The zeroth-order equation (8a) describes free vibration of the blade at a frequency $p \sqrt{\omega_0^2 + \Omega^2}$, that is, $a_0 = C \cos pt$. However, in the presence of damping (which has not been shown explicitly) this free vibration will disappear and so in steady-state

$$a_0 = 0$$

(A.1)

First-order solution

The first-order equation (8b) expands to

$$\ddot{a}_1 + (\omega_0^2 + \Omega^2) a_1 = \lambda (g + h \omega^2) \sin \omega t + \frac{\omega (\omega + 2\Omega)}{2} \sin(\omega + \Omega)t + \frac{\omega (\omega - 2\Omega)}{2} \sin(\omega - \Omega)t$$

for which the forced vibration solution is

$$a_1 = K_0^{(1)} \sin \omega t + K_+^{(1)} \sin(\omega + \Omega)t + K_-^{(1)} \sin(\omega - \Omega)t$$

(A.2)

where

$$K_0^{(1)} = \frac{\lambda (g + h \omega^2)}{\omega_0^2 + \Omega^2 - \omega^2} \quad K_+^{(1)} = \left(\frac{\omega}{2}\right) \frac{\omega + 2\Omega}{\omega_0^2 - \omega^2 - 2\omega\Omega} \quad K_-^{(1)} = \left(\frac{\omega}{2}\right) \frac{\omega - 2\Omega}{\omega_0^2 - \omega^2 + 2\omega\Omega}$$

Second-order solution

Substituting the zeroth-order solution (A.1) into the second-order equation (8c) shows that

$$\ddot{a}_2 = 0$$

(A.3)

Third-order solution

Substituting the first-order solution (A.2) into the third-order equation (8d) gives

$$\ddot{a}_3 + (\omega_0^2 + \Omega^2) a_3 = -\left(\frac{\omega}{2}\right)^2 \left[K_0^{(1)} \sin \omega t + K_+^{(1)} \sin(\omega + \Omega)t + K_-^{(1)} \sin(\omega - \Omega)t\right] \times$$

$$\times \left[ \lambda h (\cos(\omega + \Omega)t + \cos(\omega - \Omega)t) + \frac{1}{2} (\cos(2\omega + 2\Omega)t + \cos(2\omega - 2\Omega)t) \right. + 2\lambda h \cos \Omega t + \cos 2\Omega t - \cos 2\omega t - 1 \right]$$

The solution for $a_3$ includes terms at many harmonic combinations of $\omega$ and $\Omega$. However, as the final solution is dominated by the harmonics at $\omega, \omega + \Omega$ and $\omega - \Omega$, only these amplitudes are given here. This partial solution is given by

$$a_3 = K_0^{(3)} \sin \omega t + K_+^{(3)} \sin(\omega + \Omega)t + K_-^{(3)} \sin(\omega - \Omega)t + \ldots$$

(A.4)

where

$$K_0^{(3)} = -\left(\frac{\omega}{2}\right)^2 \left[\frac{3}{2} K_0^{(1)} + \frac{\lambda h}{2} \left( K_+^{(1)} + K_-^{(1)} \right) \right] \left[ \omega_0^2 + \Omega^2 - \omega^2 \right]^{-1}$$

$$K_+^{(3)} = -\left(\frac{\omega}{2}\right)^2 \left[\frac{\lambda h}{2} K_0^{(1)} + \frac{3}{4} K_-^{(1)} + K_+^{(1)} \right] \left[ \omega_0^2 + \omega^2 - 2\omega\Omega \right]^{-1}$$

$$K_-^{(3)} = -\left(\frac{\omega}{2}\right)^2 \left[\frac{\lambda h}{2} K_0^{(1)} + K_+^{(1)} + \frac{3}{4} K_+^{(1)} \right] \left[ \omega_0^2 + \omega^2 + 2\omega\Omega \right]^{-1}$$