ABSTRACT

We provide a detailed description of the Fortran code CPsuperH, a newly-developed computational package that calculates the mass spectrum and decay widths of the neutral and charged Higgs bosons in the Minimal Supersymmetric Standard Model with explicit CP violation. The program is based on recent renormalization-group-improved diagrammatic calculations that include dominant higher-order logarithmic and threshold corrections, $b$-quark Yukawa-coupling resummation effects and Higgs-boson pole-mass shifts. The code CPsuperH is self-contained (with all subroutines included), is easy and fast to run, and is organized to allow further theoretical developments to be easily implemented*. The fact that the masses and couplings of the charged and neutral Higgs bosons are computed at a similar high-precision level makes it an attractive tool for Tevatron, LHC and LC studies, also in the CP-conserving case.

*The program may be obtained from [http://theory.ph.man.ac.uk/~jslee/CPsuperH.html](http://theory.ph.man.ac.uk/~jslee/CPsuperH.html)
1 Introduction

The quest for the still-elusive Higgs boson [1], the missing cornerstone of the renormalizable Standard Model (SM), has become not only more pressing, after the completion of the LEP experimental programme, but also more exciting in light of the upcoming experiments at the upgraded Tevatron collider and the Large Hadron Collider (LHC). Indeed, direct searches for possible realizations of the mechanism of spontaneous electroweak symmetry breaking within and beyond the SM are expected to dominate the scene of particle-physics phenomenology in the present and next decades.

One of the most theoretically appealing realizations of the Higgs mechanism for mass generation is provided by Supersymmetry (SUSY). The minimal supersymmetric extension of the SM (MSSM) has a number of interesting field–theoretic and phenomenological properties, if SUSY is softly broken such that superparticles acquire masses not greatly exceeding 1 TeV. Specifically, within the MSSM, the gauge hierarchy can be made technically natural [2,3]. Unlike the SM, the MSSM exhibits quantitatively reliable gauge-coupling unification at the energy scale of the order of $10^{16}$ GeV [4]. Furthermore, the MSSM provides a successful mechanism for cosmological baryogenesis via a strongly first-order electroweak phase transition [5, 6], and provides viable candidates for cold dark matter [7, 8].

The MSSM makes a crucial and definite prediction for future high-energy experiments, that can be directly tested at the Tevatron and/or the LHC. It guarantees the existence of (at least) one light neutral Higgs boson with mass less than about 135 GeV [9]. This rather strict upper bound on the lightest Higgs boson mass is in accord with global analyses of the electroweak precision data, which point towards a relatively light SM Higgs boson, with $M_{H_{\text{SM}}} \lesssim 211$ GeV at the 95 % confidence level [10]. Furthermore, because of the decoupling properties of heavy superpartners, the MSSM predictions for the electroweak precision observables can easily be made consistent with all the experimental data [11].

Recently, a new important phenomenological feature of the MSSM Higgs sector has been observed. It has been realized that loop effects mediated dominantly by third-generation squarks may lead to sizeable violations of the tree-level CP invariance of the MSSM Higgs potential, giving rise to significant Higgs scalar-pseudoscalar transitions [12], in particular. As a consequence, the three neutral Higgs mass eigenstates $H_{1,2,3}$, labelled in order of increasing mass such that $M_{H_1} \leq M_{H_2} \leq M_{H_3}$, have no definite CP parities, but become mixtures of CP-even and CP-odd states. Much work has been devoted to studying in greater detail this radiative Higgs–sector CP violation in the framework of the MSSM [13,14,15,16,17,18,19,20,21,23,22]. In the MSSM with explicit CP violation, the couplings of the Higgs bosons to the SM gauge bosons and fermions, to their supersymmetric partners and to the Higgs bosons themselves may be considerably modified from those predicted in the CP-conserving case. Consequently, radiative CP violation in the MSSM Higgs sector can significantly affect the production rates and decay branching fractions of
the Higgs bosons. In particular, the drastic modification of the couplings of the $Z$ boson to the two lighter Higgs bosons $H_1$ and $H_2$ might enable a relatively light Higgs boson with a mass $M_{H_1}$ even less than about 70 GeV to have escaped detection at LEP 2 \cite{16, 22}. The upgraded Tevatron collider and the LHC will be able to cover a large fraction of the MSSM parameter space, including the challenging regions with a light Higgs boson without definite CP parity \cite{24, 22, 25}. Furthermore, complementary and more accurate explorations of the CP-noninvariant MSSM Higgs sector can be carried out using high-luminosity $e^+e^-$ \cite{26} and/or $\gamma\gamma$ \cite{27} colliders. In addition, a complete determination of the CP properties of the neutral Higgs bosons is possible at muon colliders by exploiting polarized muon beams \cite{28}.

It is obvious that a systematic study of Higgs phenomenology in the MSSM with explicit CP violation would be greatly facilitated by an appropriate computational tool. For this purpose, we have developed the Fortran program CPsuperH, a new self-contained computational package, which calculates the mass spectrum and the decay widths of the neutral and charged Higgs bosons in the MSSM with explicit CP violation\footnote{We note in passing that a Fortran code called HDECAY has already been developed for the calculation of Higgs boson decays in the CP–invariant version of the MSSM \cite{29}.}. It calculates the neutral Higgs-boson masses $M_{H_{1,2,3}}$ and the corresponding $3 \times 3$ Higgs-boson mixing matrix $O$, employing the renormalization-group- (RG-)improved diagrammatic approach of \cite{23}. We include the leading two-loop QCD logarithmic corrections as well as the leading two-loop logarithmic corrections induced by the top- and bottom-quark Yukawa couplings \cite{30}. We also include the leading one-loop logarithmic corrections due to gaugino and higgsino quantum effects \cite{31}. Moreover, we implement the potentially large two–loop non–logarithmic corrections originating from one–loop threshold effects on the top- and bottom-quark Yukawa couplings, associated with the decoupling of the third-generation squarks \cite{32, 16, 15}. Finally, the RG-improved diagrammatic calculation takes account of mass shifts determined by the positions of the poles in the corresponding Higgs-boson propagators. These Higgs–boson pole-mass shifts for the lightest Higgs boson are small, of the order of a few GeV. However, the mass shifts for the heavier Higgs bosons, $H_2$ and $H_3$, can be much larger and of the order of several tens of GeV \cite{23}, especially if their masses $M_{H_2}$ and $M_{H_3}$ happen to be close to thresholds for the on-shell production of squark pairs. Finally, we note that the computation of all the Higgs-boson decay widths by the code CPsuperH relies on the extensive analytic results for the decay widths presented in \cite{18}.

After this introductory discussion, the next section summarizes all the topics of Higgs phenomenology that can be studied with the code CPsuperH. In Section 3 we describe the execution procedure of the Fortran code and present examples of input and output files from a test run. Finally, we summarize the essential features of the code and provide an outlook for further developments of CPsuperH in Section 4.
2 Higgs Phenomenology in the MSSM with Explicit CP Violation

In the presence of nontrivial CP-violating phases for the higgsino mass parameter $\mu$ and the soft SUSY-breaking parameters in the MSSM, the couplings of the Higgs bosons to the gauge bosons, fermions and sfermions, as well as those to the Higgs bosons themselves, are strongly modified. In order to investigate these modifications and their phenomenological implications, we begin by stating our conventions for the mixing matrices of the neutral Higgs bosons and SUSY particles, and then we present all the relevant Higgs interactions with the MSSM particles to be used subsequently for calculating the masses, total decay widths and decay branching fractions of the neutral and charged Higgs bosons.

2.1 Conventions

In this subsection, we give our conventions for the mixing matrices of neutral Higgs bosons, charginos, neutralinos and third-generation sfermions.

- **Neutral Higgs bosons:** In the presence of nontrivial CP-violating phases of the soft supersymmetry-breaking parameters, most relevantly in the third-generation sfermion sector, the three neutral Higgs bosons all mix together via radiative corrections:

$$O^T M_H^2 O = \text{diag}(M_{H_1}^2, M_{H_2}^2, M_{H_3}^2)$$

with $M_{H_1} < M_{H_2} < M_{H_3}$. We refer to [16, 23] for the details of the calculations of the mass-squared matrix $M_H^2$, the diagonalization matrix $O$ and the pole masses of the Higgs bosons.

- **Charginos:** In SUSY theories, the spin–1/2 partners of the $W^\pm$ gauge bosons and the charged Higgs bosons, $W^\pm$ and $H^\pm$, mix to form chargino mass eigenstates. We adopt the convention $\tilde{H}_{L(R)} = \tilde{H}_{1(2)}$, where the subscripts 1 and 2 are associated with the Higgs supermultiplets leading to the tree-level mass generation of the down- and up-type quarks, respectively. The chargino mass matrix in the $(\tilde{W}^-, \tilde{H}^-)$ basis

$$\mathcal{M}_C = \begin{pmatrix} M_2 & \sqrt{2} M_W c_\beta \\ \sqrt{2} M_W s_\beta & \mu \end{pmatrix}$$

is diagonalized by two different unitary matrices $C_R \mathcal{M}_C C_L^d = \text{diag}\{m_{\tilde{\chi}_1^\pm}, m_{\tilde{\chi}_2^\pm}\}$, where $m_{\tilde{\chi}_1^\pm} \leq m_{\tilde{\chi}_2^\pm}$. The chargino mixing matrices $(C_L)_{ia}$ and $(C_R)_{ia}$ relate the electroweak eigenstates to the mass eigenstates, via

$$\tilde{\chi}_{aL} = (C_L)_{ia} \tilde{\chi}_{iL}, \quad \tilde{\chi}_{aL} = (\tilde{W}^-, \tilde{H}^-)_L^T, \quad \tilde{\chi}_{aR} = (C_R)_{ia} \tilde{\chi}_{iR}, \quad \tilde{\chi}_{aR} = (\tilde{W}^-, \tilde{H}^-)_R^T.$$

(3)
We use the following abbreviations throughout this paper: $s_\beta \equiv \sin \beta$, $c_\beta \equiv \cos \beta$, $t_\beta = \tan \beta$, $s_{2\beta} \equiv \sin 2\beta$, $c_{2\beta} \equiv \cos 2\beta$, $s_W \equiv \sin \theta_W$, $c_W \equiv \cos \theta_W$, etc.

- **Neutralinos**: The neutralino mass matrix in the $(\tilde{B}, \tilde{W}^3, \tilde{H}^0_1, \tilde{H}^0_2)$ basis is given by

$$
\mathcal{M}_N = \begin{pmatrix}
    M_1 & 0 & -M_Z c_\beta s_W & M_Z s_\beta s_W \\
    0 & M_2 & M_Z c_\beta c_W & -M_Z s_\beta c_W \\
    -M_Z c_\beta s_W & M_Z c_\beta c_W & 0 & -\mu \\
    M_Z s_\beta s_W & -M_Z s_\beta c_W & -\mu & 0
\end{pmatrix}.
$$

(4)

This neutralino mass matrix is diagonalized by a unitary matrix $N$: $N^* \mathcal{M}_N N^T = \text{diag}(m_{\chi_1^0}, m_{\chi_2^0}, m_{\chi_3^0}, m_{\chi_4^0})$ with $m_{\chi_1^0} \leq m_{\chi_2^0} \leq m_{\chi_3^0} \leq m_{\chi_4^0}$. The neutralino mixing matrix $N_{i\alpha}$ relates the electroweak eigenstates to the mass eigenstates via

$$
(\tilde{B}, \tilde{W}^3, \tilde{H}^0_1, \tilde{H}^0_2)_\alpha = N_{i\alpha} (\tilde{\chi}_1^0, \tilde{\chi}_2^0, \tilde{\chi}_3^0, \tilde{\chi}_4^0)_i.
$$

(5)

- **Stops, sbottoms, staus and tau sneutrino**: The stop and sbottom mass matrices may conveniently be written in the $(\tilde{q}_L, \tilde{q}_R)$ basis as

$$
\tilde{M}_q^2 = \begin{pmatrix}
    M_{Q_3}^2 + m_\tilde{q}^2 + c_{2\beta} M_Z^2 (T_z^q - Q_q s_W^2) & h_q v_q (A_q^* - \mu R_q)/\sqrt{2} \\
    h_q v_q (A_q - \mu^* R_q)/\sqrt{2} & M_{R_3}^2 + m_\tilde{q}^2 + c_{2\beta} M_Z^2 Q_q s_W^2
\end{pmatrix}.
$$

(6)

with $q = t, b$, $R = U, D$, $T_z^q = -T_z^b = 1/2$, $Q_t = 2/3$, $Q_b = -1/3$, $v_b = v_1$, $v_t = v_2$, $R_b = \tan \beta = v_2/v_1$, $R_t = \cot \beta$, and $h_q$ is the Yukawa coupling of the quark $q$. On the other hand, the stau mass matrix is written in the $(\tilde{\tau}_L, \tilde{\tau}_R)$ basis as

$$
\tilde{M}_\tau^2 = \begin{pmatrix}
    M_{L_3}^2 + m_\tilde{\tau}^2 + c_{2\beta} M_Z^2 (s_W^2 - 1/2) & h_{\tau} v_{\tau} (A_{\tau}^* - \mu \tan \beta)/\sqrt{2} \\
    h_{\tau} v_{\tau} (A_{\tau} - \mu^* \tan \beta)/\sqrt{2} & M_{E_3}^2 + m_{\tilde{\tau}}^2 - c_{2\beta} M_Z^2 s_W^2
\end{pmatrix}.
$$

(7)

and the mass of the tau sneutrino $\tilde{\nu}_\tau$ is simply $m_\tilde{\nu}_\tau = \sqrt{M_{E_3}^2 + 1/2 c_{2\beta} M_Z^2}$, as it has no right-handed counterpart in the MSSM. The $2 \times 2$ sfermion mass matrix $\tilde{M}_f^2$ for $f = t, b$ and $\tau$ is diagonalized by a unitary matrix $U_f$: $U_f^\dagger \tilde{M}_f^2 U_f = \text{diag}(m_{\tilde{f}_1}^2, m_{\tilde{f}_2}^2)$ with $m_{\tilde{f}_1}^2 \leq m_{\tilde{f}_2}^2$. The mixing matrix $U_f$ relates the electroweak eigenstates $\tilde{f}_{L,R}$ to the mass eigenstates $\tilde{f}_{1,2}$, via

$$
(\tilde{f}_{L,R})_\alpha = U_{\alpha i} (\tilde{f}_1, \tilde{f}_2)_i.
$$

(8)

We parameterize the mixing matrices as follows:

$$
U_f = \begin{pmatrix}
    \cos \theta_f & -\sin \theta_f e^{-i\phi_f} \\
    \sin \theta_f e^{+i\phi_f} & \cos \theta_f
\end{pmatrix},
$$

(9)

and we calculate numerically the mixing angle $\theta_f$ and phase $\phi_f$ in the ranges between $-\pi/2$ and $\pi/2$, so that $\cos \theta_f \geq 0$ and $\cos \phi_f \geq 0$. 


2.2 Higgs–Boson Interactions

In this subsection, we list all the Higgs interactions with gauge bosons, SM fermions, squarks, sleptons, charginos, and neutralinos. We also present all the trilinear and quartic Higgs–boson self–couplings.

- Interactions of Higgs bosons with gauge bosons: The interactions of the Higgs bosons with the gauge bosons \( Z \) and \( W^\pm \) are described by the three interaction Lagrangians:

\[
L_{HVV} = g M_W \left( W^+_\mu W^-\mu + \frac{1}{2c_W^2} Z_\mu Z^\mu \right) \sum_i g_{h_{iVV}} H_i ,
\]

\[
L_{HHZ} = g^4 c_W \sum_{i,j} g_{h_{iHj}} Z^\mu (H_i i \partial_\mu H_j) ,
\]

\[
L_{HHW^\pm} = -g^2 \sum_i g_{h_{iH}^+W^-} W^{-\mu}(H_i i \partial_\mu H^+) + h.c. ,
\]

where \( g = e / \sin \theta_W \) is the SU(2)\(_L\) gauge-coupling constant, and the couplings \( g_{h_{iVV}} \), \( g_{h_{iHj}} \) and \( g_{h_{iH}^+W^-} \) are given in terms of the neutral Higgs-boson mixing matrix \( O \) by (note that \( \det(O) = \pm 1 \) for any orthogonal matrix \( O \)):

\[
g_{h_{iVV}} = c_\beta O_{\phi_1i} + s_\beta O_{\phi_2i} ,
\]

\[
g_{h_{iHj}} = \text{sign}[\det(O)] \varepsilon_{ijk} g_{h_{kVV}} ,
\]

\[
g_{h_{iH}^+W^-} = c_\beta O_{\phi_2i} - s_\beta O_{\phi_1i} - iO_{ai} ,
\]

leading to the following sum rules:

\[
\sum_{i=1}^3 g_{h_{iVV}}^2 = 1 \quad \text{and} \quad g_{h_{iVV}}^2 + |g_{h_{iH}^+W^-}|^2 = 1 \quad \text{for each } i .
\]

- Higgs–quark–antiquark and Higgs–lepton-antilepton interactions: The effective Lagrangian governing the interactions of the neutral Higgs bosons with quarks and charged leptons is

\[
L_{H_i\bar{f}f} = - \sum_{f=u,d,l} \frac{g_{m_f}}{2M_W} \sum_{i=1}^3 H_i \bar{f} \left( g_{h_{i\bar{f}f}}^S + ig_{h_{i\bar{f}f}}^P \gamma_5 \right) f .
\]

At the tree level, \( (g_S^S, g_P^S) = (O_{\phi_1i}/c_\beta, -O_{ai} \tan \beta) \) and \( (g_S^P, g_P^P) = (O_{\phi_2i}/s_\beta, -O_{ai} \cot \beta) \) for \( f = (l, d) \) and \( f = u \), respectively. In the case of third-generation quarks, the program \texttt{CPsuperH} computes the finite threshold corrections induced by the exchanges of gluinos and charginos. As described in Appendix A, we include the all–orders resummation \[33, 34, 22\] of the leading powers of \( \tan \beta \), as required for a meaningful perturbative expansion. Correspondingly, in the presence of CP violation, the
effective couplings of the charged Higgs boson to quarks and leptons in the weak-interaction basis are described by the interaction Lagrangian:

\[
\mathcal{L}_{H^\pm f_i f_i} = \frac{g}{\sqrt{2} M_W} \sum_{(f_i, f_i) = (u, d), (e, \nu)} H^+ \bar{f}_i \gamma^\lambda (m_{f_i} g_{H_+ f_i f_i}^L P_L + m_{f_i} g_{H_+ f_i f_i}^R P_R) f_i + \text{h.c.,}
\]

where \( P_{L/R} \equiv (1 \mp \gamma_5)/2 \). At the tree level, \( g^L = \cot \beta \) and \( g^R = \tan \beta \) with \( m_\nu = 0 \). The loop–induced threshold corrections to the \( g_{H_+ ud} \) couplings are presented in Appendix A.

- Higgs–sfermion–sfermion interactions: The Higgs–sfermion–sfermion interactions can be written in terms of the sfermion mass eigenstates as

\[
\mathcal{L}_{H f \bar{f}} = v \sum_{f = u, d} g_{H f \bar{f}} \bar{f}_k (H_i f_j^* \bar{f}_k),
\]

where

\[
v g_{H f \bar{f}} = \left( \Gamma^{f \bar{f}} \right)_{\beta \gamma} O_{\alpha i} U_{\beta j}^* U_{\gamma k},
\]

with \( \alpha = (\phi_1, \phi_2, a) = (1, 2, 3), \beta, \gamma = L, R, i = (H_1, H_2, H_3) = (1, 2, 3) \) and \( j, k = 1, 2 \). Likewise, the charged Higgs-boson interactions with up– and down–type sfermions are given by

\[
\mathcal{L}_{H^\pm f \bar{f}} = v g_{H^+ f \bar{f}} (H^+ f_j^* \bar{f}_k) + \text{h.c.,}
\]

where

\[
v g_{H^+ f \bar{f}} = \left( \Gamma^{H^+ f \bar{f}} \right)_{\beta \gamma} U_{\beta j}^* U_{\gamma k}.\]

The expressions for the couplings \( \Gamma^{f \bar{f}} \) and \( \Gamma^{H^+ f \bar{f}} \) to third–generation sfermions are presented in Appendix B. As shown in [34], our iterative treatment of the threshold corrections that are enhanced at large \( \tan \beta \) ensures that the corresponding corrections to the Higgs-boson couplings to squarks are also resummed correspondingly.

- Interactions of neutral Higgs bosons and charginos: These are described by the following Lagrangian:

\[
\mathcal{L}_{H^0 \chi^+ \chi^-} = -\frac{g}{\sqrt{2}} \sum_{i, j, k} H_k \bar{\chi}_i \left( g_{H_k \chi_i^\dagger \chi_j}^S + i \gamma_5 g_{H_k \chi_i^\dagger \chi_j}^P \right) \chi_j^-,
\]

\[
g_{H_k \chi_i^\dagger \chi_j}^S = \frac{1}{2} \left\{ \left[ (C_R)_{i1} (C_L)_{j2}^* G_{k}^{\phi_1} + (C_R)_{i2} (C_L)_{j1}^* G_{k}^{\phi_2} \right] + [i \leftrightarrow j]^* \right\},
\]

\[
g_{H_k \chi_i^\dagger \chi_j}^P = \frac{i}{2} \left\{ \left[ (C_R)_{i1} (C_L)_{j2}^* G_{k}^{\phi_1} + (C_R)_{i2} (C_L)_{j1}^* G_{k}^{\phi_2} \right] - [i \leftrightarrow j]^* \right\},
\]

where \( G_{k}^{\phi_1} = (O_{\phi_1} - i s_\beta O_{ak}), G_{k}^{\phi_2} = (O_{\phi_2} - i c_\beta O_{ak}), i, j = 1, 2, \) and \( k = 1, 2 \).
• Interactions of neutral Higgs bosons and neutralinos: These are described by the following Lagrangian:

\[
\mathcal{L}_{H^0\tilde{\chi}_i^0\tilde{\chi}_j^0} = -\frac{g}{2} \sum_{i,j,k} H_k \tilde{\chi}_i^0 \left( g_{H_k \tilde{\chi}_i^0 \tilde{\chi}_j^0}^S + i\gamma_5 g_{H_k \tilde{\chi}_i^0 \tilde{\chi}_j^0}^P \right) \tilde{\chi}_j^0 : \]

\[
g_{H_k \tilde{\chi}_i^0 \tilde{\chi}_j^0}^S = \frac{1}{2} \Re[(N^*_j - t_W N^*_j)(N^*_i G_k^{\phi_1} - N^*_i G_k^{\phi_2}) + (i \leftrightarrow j)]
\]

\[
g_{H_k \tilde{\chi}_i^0 \tilde{\chi}_j^0}^P = -\frac{1}{2} \Im[(N^*_j - t_W N^*_j)(N^*_i G_k^{\phi_1} - N^*_i G_k^{\phi_2}) + (i \leftrightarrow j)], \tag{20}
\]

where \(i, j = 1-4\) for the four neutralino states and \(k = 1-3\) for the three neutral Higgs bosons.

• Interactions of charged Higgs bosons, charginos and neutralinos: These are described by the following Lagrangian:

\[
\mathcal{L}_{H^+\tilde{\chi}_i^0\tilde{\chi}_j^0} = -\frac{g}{\sqrt{2}} \sum_{i,j} H^+ \tilde{\chi}_i^0 \left( g_{H^+\tilde{\chi}_i^0\tilde{\chi}_j^0}^S + i\gamma_5 g_{H^+\tilde{\chi}_i^0\tilde{\chi}_j^0}^P \right) \tilde{\chi}_j^0 + \text{h.c.} : \]

\[
g_{H^+ \tilde{\chi}_i^0 \tilde{\chi}_j^0}^S = \frac{1}{2} \left\{ s_\beta \left[ \sqrt{2} N^*_i (C_L)_{j1} - (N^*_j + t_W N^*_j) (C_L)_{j2} \right] 
+ c_\beta \left[ \sqrt{2} N^*_i (C_R)_{j1} + (N^*_j + t_W N^*_j) (C_R)_{j2} \right] \right\}, \tag{21}
\]

\[
g_{H^+ \tilde{\chi}_i^0 \tilde{\chi}_j^0}^P = \frac{i}{2} \left\{ s_\beta \left[ \sqrt{2} N^*_i (C_L)_{j1} - (N^*_j + t_W N^*_j) (C_L)_{j2} \right] 
- c_\beta \left[ \sqrt{2} N^*_i (C_R)_{j1} + (N^*_j + t_W N^*_j) (C_R)_{j2} \right] \right\}.
\]

• Trilinear and quartic Higgs-boson self-couplings \[18\text{,}\] \[22\]: The effective trilinear and quartic Higgs self–couplings can be cast into the form

\[
\mathcal{L}_{3H} = v \sum_{i \geq j \geq k = 1}^3 g_{H_i H_j H_k} H_i H_j H_k + v \sum_{i = 1}^3 g_{H_i H^+ H^-} H_i H^+ H^- , \tag{22}
\]

\[
\mathcal{L}_{4H} = \sum_{i \geq j \geq k \geq l = 1}^3 g_{H_i H_j H_k H_l} H_i H_j H_k H_l + \sum_{i \geq j = 1}^3 g_{H_i H_j H^+ H^-} H_i H_j H^+ H^- 
+ g_{H^+ H^- H^+ H^-} (H^+ H^-)^2, \tag{23}
\]

where

\[
g_{H_i H_j H_k} = \sum_{\alpha \leq \beta \leq \gamma = 1}^3 \{O_{\alpha \beta \gamma} O_{\beta \gamma \delta} O_{\gamma \delta \alpha} \} g_{\alpha \beta \gamma}, \quad g_{H_i H^+ H^-} = \sum_{\alpha = 1}^3 O_{\alpha \beta} g_{\alpha H^+ H^-} , \tag{24}
\]

\[
g_{H_i H_j H_k H_l} = \sum_{\alpha \leq \beta \leq \gamma \leq \delta = 1}^3 \{O_{\alpha \beta \gamma} O_{\beta \gamma \delta} O_{\gamma \delta \alpha} \} g_{\alpha \beta \gamma} ,
\]

\[
g_{H_i H_j H^+ H^-} = \sum_{\alpha \leq \beta = 1}^3 \{O_{\alpha \beta} O_{\beta 3} \} g_{\alpha \beta H^+ H^-} . \tag{25}
\]
In the above equations (24) and (25), the expressions within the curly brackets \{\cdots\} need to be symmetrized with respect to the indices \(i, j, k, l\) and divided by the corresponding symmetry factors in cases where two or more indices are the same. For example, \(\{O_{\alpha i}O_{\beta j}O_{\gamma k}\}\) can explicitly be evaluated as follows:

\[
\{O_{\alpha i}O_{\beta j}O_{\gamma k}\} \equiv \frac{1}{N_S} \left( O_{\alpha i}O_{\beta j}O_{\gamma k} + O_{\alpha i}O_{\gamma j}O_{\beta k} + O_{\alpha j}O_{\beta i}O_{\gamma k} + O_{\alpha j}O_{\gamma i}O_{\beta k} + O_{\alpha k}O_{\beta i}O_{\gamma j} + O_{\alpha k}O_{\beta j}O_{\gamma i} \right),
\]

with \(N_S = 6\) when \(i = j = k\), \(N_S = 1\) when \((i, j, k) = (3, 2, 1)\), and \(N_S = 2\) in all the other cases. We present the couplings \(g_{\alpha\beta\gamma}\), \(g_{\alpha H^+H^-}\), \(g_{\alpha\beta\gamma\delta}\), and \(g_{\alpha H^+H^-}\) of the Higgs weak eigenstates in Appendix C.

### 2.3 Neutral and Charged Higgs Boson Decays

In this subsection, we calculate all the two-body decay widths of the Higgs bosons. We consider the decays of the Higgs bosons into pairs of leptons, quarks, charginos, neutralinos, massive gauge bosons, Higgs bosons, squarks, sleptons, photons and gluons as well as into a massive gauge boson and a Higgs boson. For the decay modes involving more than one massive gauge boson, three-body decays are also considered [18].

- \(H_i, H^+ \rightarrow ff'\): First, let us consider the decays into a pair of fermions. Without loss of generality, the Lagrangian describing the interactions of the Higgs bosons with two fermions can be written as

\[
\mathcal{L}_{Hff'} = -\sum_{i,j,k} g_{lf}H_i\bar{f}_k\left(g_{H^+f_j}^S + ig_{H^+f_j}^P \gamma_5\right)f_j - \sum_{i,j,k} g_{lf'}H^+_{f'_k}\left(g_{H^+f'_j}^S + ig_{H^+f'_j}^P \gamma_5\right)f'_j + \text{h.c.},
\]

where \(f^{(l)}\) stands for a lepton, a quark, a chargino, or a neutralino, and the tree-level couplings \(g_f, g_{ff'}\) and \(g_{S,P}\) are given in Table I. In terms of these generic couplings, the width for a decay into two Dirac fermions is given by

\[
\Gamma_D = N_C \frac{g_{lf(l')}^2 M_H}{8\pi} \frac{\lambda^{1/2}(1, \kappa_j, \kappa_k)}{\lambda_j - \lambda_k} \left[ (1 - \kappa_j - \kappa_k)(|g_S|^2 + |g_P|^2) - 2\sqrt{\kappa_j \kappa_k} (|g_S|^2 - |g_P|^2) \right],
\]

where \(\lambda_j \equiv m_{f_j}^2/M_H^2\) and \(\lambda(1, x, y) \equiv (1 - x - y)^2 - 4xy\). We note that \(\Gamma_D\) becomes

\[
N_C \frac{g_{lf(l')}^2 M_H \beta_k}{8\pi} (\beta_k^2 |g_S|^2 + |g_P|^2)\]

when \(\kappa_j = \kappa_k = \kappa\), where \(\beta_k \equiv \sqrt{1 - 4\kappa}\). The colour factor \(N_C = 3\) for quarks and 1 for leptons, charginos, and neutralinos. The decay widths into two Majorana fermions are given by

\[
\Gamma_M = \left( \frac{4}{1 + \delta_{jk}} \right) \Gamma_D,
\]

\[
29
\]
where $\delta_{jk} = 1$ for identical Majorana fermions.

Table 1: The couplings $g_f$, $g_{ff'}$ and $g^{S,P}$ in Eq. (27) at the tree level.

| Decay Mode | $g_f$ | $g^S$ | $g^P$ |
|------------|-------|-------|-------|
| $H_i \rightarrow l^+l^-$ | $\frac{g_{m_l}}{2m_{W}}$ | $O_{\phi_1i}/c_{\beta}$ | $-(s_{\beta}/c_{\beta})O_{ai}$ |
| $H_i \rightarrow d\bar{d}$ | $\frac{g_{m_d}}{2m_{W}}$ | $O_{\phi_1i}/c_{\beta}$ | $-(s_{\beta}/c_{\beta})O_{ai}$ |
| $H_i \rightarrow u\bar{u}$ | $\frac{g_{m_u}}{2m_{W}}$ | $O_{\phi_1i}/s_{\beta}$ | $-(c_{\beta}/s_{\beta})O_{ai}$ |
| $H_i \rightarrow \tilde{\chi}^0_j \tilde{\chi}^0_k$ | $g/2$ | $g^{S}_{H_i}\tilde{\chi}^0_j\tilde{\chi}^0_k$ | $g^{P}_{H_i}\tilde{\chi}^0_j\tilde{\chi}^0_k$ |
| $H_i \rightarrow \tilde{\chi}^-_j \tilde{\chi}^+_k$ | $g/\sqrt{2}$ | $g^{S}_{H_i}\tilde{\chi}^-_j\tilde{\chi}^+_k$ | $g^{P}_{H_i}\tilde{\chi}^-_j\tilde{\chi}^+_k$ |

For the couplings $g_{H_{bb}}$, $g_{H_{bt}}$ and $g_{H^+tb}$, the finite loop-induced threshold corrections due to the exchanges of gluinos and charginos can be included by taking $\text{IFLAG}_H(10)=0$ (the default setting) in the code CPsuperH, as explained in Sec. 3.

For $H_i \rightarrow q\bar{q}$, the leading-order QCD correction is taken into account by applying the enhancement factor $K_i^q \equiv 1 + 5.67 \frac{\alpha_s(M_{H_i}^2)}{\pi}$ to the decay width given above. We take the running fermion masses at the scale $m_i^\text{pole}$ as reference values. The effect on the couplings of the running of the quark masses from the top–quark pole mass scale to the Higgs–boson mass scale is also considered in calculating the corresponding decay widths, as

$$\Gamma(H_i \rightarrow q\bar{q}) = K_i^q \left( \frac{m_q(M_{H_i})}{m_q(m_i^\text{pole})} \right)^2 \Gamma_D(H_i \rightarrow q\bar{q}) ,$$

where $m_q(m_i^\text{pole})$ is used in $g_f$, but $m_q(M_{H_i})$ is used in $\kappa_f$, when calculating $\Gamma_D$. Likewise, running $q$ and $q'$ quark masses are used when computing $\Gamma(H^+ \rightarrow q\bar{q}')$, while the dominant one-loop QCD corrections have been included by factors very similar to $K_i^q$. Finite quark-mass and higher-order QCD effects will depend, to some extent, on the CP-violating parameters of the MSSM, and require an independent study. In the present version of the code, we only include the leading-order QCD effects which remain unaffected by CP violation. Also, we do not include flavour-violating decays of the neutral Higgs bosons. We plan to implement such refinements in future versions of CPsuperH.
The width for decay into two massive gauge bosons is given by
\[
\Gamma = \frac{G_F g_{H_{ii},\gamma\eta}}{16\sqrt{2}\pi} \delta_{ii}(1 - 4\kappa_{ii} + 12\kappa_{ii}^2),
\]
where \(G_F/\sqrt{2} = g^2/8M_W^2\), \(\kappa_{ii} = M_W^2/M_H^2\), \(\beta_{ii} = \sqrt{1 - 4\kappa_{ii}}\), and \(\delta_{ii} = M_W^4/(c_W M_Z)^4 = 1\). The three-body decay width \(\Gamma(H_i \rightarrow VV^*)\) is also calculated, using [13]:
\[
\Gamma(H_i \rightarrow VV^*) = \frac{G_F g_{H_{ii},\gamma\eta} M_{H_i}^3}{16\sqrt{2}\pi} \int_0^{(\sqrt{\omega_i^2 - \omega_j^2})^2} dx \frac{\epsilon_V \lambda^{1/2}(\omega_i, x, 1)[\lambda(\omega_i, x, 1) + 12x]}{\omega_i^3 \pi[(x - 1)^2 + \epsilon_V^2]},
\]
where \(\omega_i = M_{H_i}^2/M_V^2 = 1/\kappa_{ii}\), \(\delta_{VV^*} = 2\), and \(\epsilon_V = \Gamma_V/M_V\). We note that
\[
\lambda^{1/2}(\omega_i, 1, 1)[\lambda(\omega_i, 1, 1) + 12]/\omega_i^3 = \beta_{ii}(1 - 4\kappa_{ii} + 12\kappa_{ii}^2).
\]

The decay width of a heavier Higgs boson into a lighter Higgs boson and a massive gauge boson is given by
\[
\Gamma = \frac{G_F M_{H_i}^3 |g_{H_{ii},\gamma\eta}|^2}{8\sqrt{2}\pi} \lambda^{3/2}(1, \kappa_j, \kappa_V),
\]
where \((M_H, G) = (M_{H_i}, g_{H_{ii},\gamma\eta})\) or \((M_{H^+}, g_{H_{ii},\gamma\eta})\), \(\kappa_j = M_{H_i}^2/M_{H_j}^2\), and \(\kappa_V = M_V^2/M_{H_i}^2\). The three-body decay widths \(\Gamma(H_i \rightarrow H_j Z^*)\) and \(\Gamma(H^+ \rightarrow H_j W^+)\) are given [13] by:
\[
\Gamma(H_i \rightarrow H_j Z^*) = \frac{G_F M_{H_i}^3 |g_{H_{ii},\gamma\eta}|^2}{8\sqrt{2}\pi} \int_0^{(\sqrt{\omega_i^2 - \omega_j^2})^2} dx \frac{\epsilon_Z \lambda^{3/2}(\omega_i, \omega_j, x)}{\omega_i^3 \pi[(x - 1)^2 + \epsilon_Z^2]},
\]
with \(\omega_i = M_{H_i}^2/M_Z^2\), and similarly
\[
\Gamma(H^+ \rightarrow H_j W^+) = \frac{G_F M_{H^+}^3 |g_{H_{iH^+},\gamma\eta}|^2}{8\sqrt{2}\pi} \int_0^{(\sqrt{\omega_{\pm}^2 - \omega_j^2})^2} dx \frac{\epsilon_W \lambda^{3/2}(\omega_{\pm}, \omega_j, x)}{\omega_j^3 \pi[(x - 1)^2 + \epsilon_W^2]},
\]
with \(\omega_j = M_{H_j}^2/M_W^2\) and \(\omega_{\pm} = M_{H_{iH^+}}^2/M_W^2\).

The decay widths into two scalar particles can be written as
\[
\Gamma = N_F \frac{v^2 |G|^2}{16\pi M_{H_i}} \lambda^{1/2}(1, \kappa_j, \kappa_k),
\]
where \(N_F, G = (1 + \delta_{jk}, g_{H_{ii},\gamma\eta}), (N_C, g_{H_{iH^+},\gamma\eta})\) or \((N_C, g_{H_{iH^+},\gamma\eta})\), and \(\kappa_j = M_{H_i}^2/M_{H_j}^2\).

\[\text{We note that the couplings } g_{H_{ii},\gamma\eta} \text{ are defined via the Lagrangian [22]. The } H_i H_j H_j \text{ vertices for } i > j \text{ therefore contain an extra factor of 2.}\]
The QCD correction to the width $\Gamma(H_i \to \gamma\gamma)$ is included in the large loop-mass limit by multiplying the rescaling $J_1^0$ and $J_2^0$ factors to the quark and squark contributions to $S_1^\gamma$, respectively. The rescaling factors in the large loop-mass limit are given by 

$$
J_1^0 = 1 - \frac{\alpha_s(M_{H_i}^2)}{\pi}, \quad J_2^0 = 1 + \frac{8\alpha_s(M_{H_i}^2)}{3\pi}.
$$
• $H_i \rightarrow gg$: The amplitude for the decay process $H_i \rightarrow gg$ ($i = 1, 2, 3$) can be written as

$$\mathcal{M}_{ggH_i} = -\frac{\alpha_s M^2_{H_i}}{4\pi v} \delta^{ab} \left\{ S^g_i(M_{H_i}) (\epsilon^*_1 \cdot \epsilon^*_2) - P^g_i(M_{H_i}) \frac{2}{M^2_{H_i}} (\epsilon^*_1 \epsilon^*_2 k_1 k_2) \right\}, \quad (43)$$

where $a$ and $b$ ($a, b = 1$ to 8) are indices of the eight SU(3) generators in the adjoint representation, and $k_{1,2}$ and $\epsilon_{1,2}$ are the four-momenta and wave vectors of the two gluons, respectively. The scalar and pseudoscalar form factors, retaining only the dominant contributions from third-generation (s)quarks, are given by

$$S^g_i(M_{H_i}) = \sum_{f=b,t} g_f g_{H_i,ff} \frac{v}{m_f} F_{sf}(\tau_{if}) - \sum_{f_1,f_2,b_1,b_2} g_{H_i,f_1 f_2 b_1 b_2} \frac{v^2}{4 m^2_{f_1}} F_0(\tau_{i f_1}),$$

$$P^g_i(M_{H_i}) = \sum_{f=b,t} g_f g_{H_i,ff} \frac{v}{m_f} F_{pf}(\tau_{if}). \quad (44)$$

The decay width of the process $H_i \rightarrow gg$ is then given by

$$\Gamma(H_i \rightarrow gg) = \frac{M^3_{H_i}}{32\pi^3 v^2} \left[ K^g_H |S^g_i(M_{H_i})|^2 + K^g_A |P^g_i(M_{H_i})|^2 \right], \quad (45)$$

where $K^g_{H,A}$ are QCD loop enhancement factors that include the leading-order QCD corrections. In the heavy-quark limit, the factors $K^g_{H,A}$ are given by \cite{35}

$$K^g_H = 1 + \frac{\alpha_s(M^2_{H_i})}{\pi} \left( \frac{95}{4} - \frac{7}{6} N_F \right),$$

$$K^g_A = 1 + \frac{\alpha_s(M^2_{H_i})}{\pi} \left( \frac{97}{4} - \frac{7}{6} N_F \right), \quad (46)$$

where $N_F$ is the number of quark flavours lighter than the $H_i$ boson. Away from isolated regions of the parameter space \cite{35}, the above $K^g_{H,A}$ factors capture the main bulk of the NLO corrections with an accuracy at the 10% level.

### 3 The Structure of CPsuperH

The program CPsuperH is self-contained with all necessary subroutines included. The Fortran code CPsuperH uses three input arrays for reading the input parameters and five output arrays for generating the Higgs couplings, decay widths and branching fractions. The three input arrays are named

\texttt{SMPARA\_H(IP)}, \texttt{SSPARA\_H(IP)}, \texttt{IFLAG\_H(NFLAG)}.

\footnote{We ignore the small difference between the $K$-factors of the quark and squark loop contributions to $S^g_i$.}
Among the five output arrays, the arrays for generating the Higgs couplings are named

\[ \text{NHC}_H(NC, IH), \quad \text{SHC}_H(NC), \quad \text{CHC}_H(NC), \]

and the other two output arrays for the decay widths and branching fractions are named

\[ \text{GAMBRN}(IM, IWB, IH), \quad \text{GAMBCR}(IM, IWB). \]

The code \text{CPSuperH} contains also arrays for the masses and mixing matrices of the Higgs bosons, stops, sbottoms, staus, charginos and neutralinos as explained below.

### 3.1 Input Arrays

In this subsection, we describe the details of the input arrays.

- **SMPARA\_H(IP)**: This is the array for the SM input parameters. In the current version, we are dealing with 15 inputs as shown in Table 2 below but these can easily be extended by changing \text{NSMIN} in \text{cpsuperh.f}. This array is filled from the file \text{run}.

| IP | Parameter          | IP | Parameter          | IP | Parameter          |
|----|--------------------|----|--------------------|----|--------------------|
| 1  | \(\alpha_{em}(M_Z)\) | 6  | \(m_\mu\)         | 11 | \(m_a(m_t^\text{pole})\) | 16 | ... |
| 2  | \(\alpha_s(M_Z)\)  | 7  | \(m_\tau\)        | 12 | \(m_c(m_t^\text{pole})\) | 17 | ... |
| 3  | \(M_Z\)            | 8  | \(m_d(m_t^\text{pole})\) | 13 | \(m_t^\text{pole}\) | 18 | ... |
| 4  | \(\sin^2 \theta_W\) | 9  | \(m_s(m_t^\text{pole})\) | 14 | \(\Gamma_W\) | 19 | ... |
| 5  | \(m_e\)            | 10 | \(m_b(m_t^\text{pole})\) | 15 | \(\Gamma_Z\) | 20 | ... |

- **SSPARA\_H(IP)**: This array is for the SUSY input parameters. In the current version, we are dealing with 21 inputs as shown in Table 3 below but these can easily be extended as well by changing \text{NSSIN} in \text{cpsuperh.f}. This array is also filled from the file \text{run}.

- **IFLAG\_H(NFLAG)**: This NFLAG–dimensional array controls \text{CPSuperH}. This flag array is used for printing options, calculating options, integer input parameters, error messages, etc. The default value for every flag is zero. This array also can be filled from the file \text{run}. Only a part of IFLAG\_H is being used presently by the code:
  - IFLAG\_H(1)=1: Print out the input parameters.
  - IFLAG\_H(2)=1: Print out the masses and mixing matrix of the Higgs bosons.
Table 3: The contents of $\text{SSPARA}_H(\text{IP})$. 

| IP | Parameter | IP | Parameter | IP | Parameter | IP | Parameter |
|----|-----------|----|-----------|----|-----------|----|-----------|
| 1  | $\tan \beta$ | 6  | $\Phi_1$ | 11 | $m_{Q_3}$ | 16 | $|A_t|$ |
| 2  | $M_{H^\pm}^{\text{pole}}$ | 7  | $|M_2|$ | 12 | $m_{U_3}$ | 17 | $\Phi_{A_t}$ |
| 3  | $|\mu|$ | 8  | $\Phi_2$ | 13 | $m_{D_3}$ | 18 | $|A_\mu|$ |
| 4  | $\Phi_\mu$ | 9  | $|M_3|$ | 14 | $m_{L_3}$ | 19 | $\Phi_{A_b}$ |
| 5  | $|M_1|$ | 10 | $\Phi_3$ | 15 | $m_{E_3}$ | 20 | $A_r$ |

- $\text{IFLAG}_H(3)=1$: Print out the masses and mixing matrices of the stops, sbottoms, tau sneutrino and staus.
- $\text{IFLAG}_H(4)=1$: Print out the masses and mixing matrices of the charginos and neutralinos.
- $\text{IFLAG}_H(5)=IX$: Print out the Higgs-boson couplings. The couplings of $H_1$, $H_2$, $H_3$ and $H^\pm$ to two particles will be printed for $IX=1$, 2, 3, and 4, respectively, and the Higgs–boson self-couplings will be printed for $IX=5$. All these couplings can be printed out altogether by taking $IX=6$.
- $\text{IFLAG}_H(6)=IX$: Print out the decay widths and branching ratios. The decay widths and branching ratios of $H_1$, $H_2$, $H_3$, and $H^\pm$ will be printed for $IX=1$, 2, 3, and 4, respectively. $IX=5$ is for printing out all the decay widths and branching ratios of the neutral and charged Higgs bosons.
- $\text{IFLAG}_H(10)=1$: Do not include the finite threshold corrections to the top- and bottom-quark Yukawa couplings due to the exchanges of gluinos and charginos.
- $\text{IFLAG}_H(11)=1$: Use the effective potential masses for Higgs bosons instead of their pole masses.
- $\text{IFLAG}_H(20)= ISMN$. The index $ISMN$ is used for $GAMBRN$, which is an array for the neutral Higgs decay widths, and its default value is $ISMN = 50$. In general, $(ISMN-1)$ is the maximal number of different decay modes of the neutral Higgs bosons into SM particles. The value of $ISMN$ may be changed in order to incorporate additional rare decay modes. The index $ISMN$ is reserved for the subtotal decay width and branching fraction of the decays into the SM particles in the output array $GAMBRN$ (see below).
- $\text{IFLAG}_H(21)=ISUSYN$. Similarly to $ISMN$, the index $ISUSYN$ is used for $GAMBRN$, and its default value is $ISUSYN = 50$, with $(ISUSYN-1)$ being the maximal number of different decay modes of the neutral Higgs bosons into SUSY particles. The index $ISMN+ISUSYN$ is reserved for the subtotal decay width and branching fraction of the decays into the SUSY particles, while the index $ISMN+ISUSYN+1$
is used for the total decay width, considering decays into both the SM and SUSY particles in the output array **GAMBRN** (see below).

- **IFLAG\_H(22)= ISMC**. The index **ISMC** is used for **GAMBRC**, which is an array for the charged-Higgs decay width, and its default value is **ISMC = 25**. In general, **(ISMC-1)** is the maximal number of different decay modes of the charged Higgs bosons into SM particles. The index **ISMC** is reserved for the subtotal decay width and branching fraction of the decays into the SM particles in the output array **GAMBRC** (see below).

- **IFLAG\_H(23)=ISUSYC**. Similarly to **ISMC**, the index **ISUSYC** for **GAMBRC**, with **(ISUSYC-1)** being the maximal number of different decay modes of the charged Higgs bosons into SUSY particles. The index **ISMC+ISUSYC** is reserved for the subtotal decay width and branching fraction of the decays into the SUSY particles, while the index **ISMC+ISUSYC+1** is used for the total decay width considering decays both into the SM and SUSY particles in the output array **GAMBRC** (see below).

In Appendix D, we list all the parameter common blocks filled or calculated from **SMPARA\_H** and **SSPARA\_H**.

### 3.2 Output Arrays

In this subsection, we give detailed descriptions of the output arrays. Some of the entries of **IFLAG\_H** are reserved for various error messages. This feature might be helpful when using **CPsuperH** to scan many parameter points:

- **IFLAG\_H(50)=1**: This is an error message that appears when a stop or sbottom squared mass is negative.

- **IFLAG\_H(51)=1**: This is an error message that appears when the Higgs–boson mass matrix contains a complex or negative eigenvalue.

- **IFLAG\_H(52)=1**: This is an error message that appears when the diagonalization of the Higgs mass matrix is not successful.

- **IFLAG\_H(53)=1**: This is a warning message that appears when the second–step improvement in the calculations of the pole masses is needed.

- **IFLAG\_H(54)=1**: This is an error message that appears when the iteration resumming the threshold corrections is not convergent.

- **IFLAG\_H(55)=1**: This is an error message that appears when the Yukawa coupling has a non–perturbative value: \(|h_t| \) or \(|h_b| > 2\).
• **IFLAG\_H(56)=1**: This is an error message that appears when a tau sneutrino or a stau squared mass is negative.

The main numerical output is stored in the following arrays:

• **NHC\_H(NC,IH)**: This is an array for the IH-th neutral Higgs boson \(H_{IH}\) couplings to two particles with index NC. Currently, this array is filled up to NC = 93 as shown in Table [I]

• **SHC\_H(NC)**: This array is for the self-couplings of Higgs bosons. Currently, this array is filled up to NC = 35, as shown in Table [I]

• **CHC\_H(NC)**: This array is for the couplings of the charged Higgs boson to two particles. Currently, this array is filled up to NC = 48, as shown in Table [I]

• **GAMBRN(IM,IWB,IH)**: This output array is for the decay width in GeV (IWB=1) and branching fraction (IWB=2,3) of the decay mode specified by the index IM of the neutral Higgs bosons \(H_{IH}\). The value IWB=2 is for the branching fraction taking into account the decays only into SM particles, and IWB=3 for that taking account both the SM and SUSY decays. By default, the code takes ISMN = ISUSYN = 50. All the decay modes considered are listed in Table [I] for specific IM. In particular, GAMBRN(IM=ISMN+ISUSYN+1,IWB=1,IH) is the total decay width of the neutral Higgs boson \(H_{IH}\) and GAMBRN(IM=ISMN,IWB=1,IH) and GAMBRN(IM=ISMN+ISUSYN,IWB=1,IH) are the subtotal decay widths into SM particles and into SUSY particles, respectively. Therefore, we have the following relations for the branching fractions IWB=2,3:

\[
GAMBRN(ISMN, 2, IH) = GAMBRN(ISMN + ISUSYN + 1, 3, IH) = 1
\]

and

\[
GAMBRN(ISMN, 3, IH) \leq 1, \quad GAMBRN(ISMN + ISUSYN + 1, 2, IH) \geq 1.
\]

• **GAMBRC(IM,IWB)**: This array is for the decay width in GeV (IWB=1) and branching fraction (IWB=2,3) of the decay mode number IM of the charged Higgs boson. The convention for IWB is the same as that for GAMBRN. In the code, ISMC = ISUSYC = 25 is taken. The decay modes considered are shown in Table [I] In particular, GAMBRC(IM=ISMC+ISUSYC+1,IWB=1) is the total decay width of the charged Higgs boson and GAMBRC(IM=ISMC,IWB=1) and GAMBRC(IM=ISMC+ISUSYC,IWB=1) are subtotal decay widths into SM and into SUSY particles, respectively. Similarly to the case of the neutral Higgs bosons, we have the relations

\[
GAMBRC(ISMC, 2) = GAMBRC(ISMC + ISUSYC + 1, 3) = 1
\]
\[ \text{GAMBRC}(\text{ISMC}, 3) \leq 1, \quad \text{GAMBRC}(\text{ISMC} + \text{ISUSY}C + 1, 2) \geq 1. \]

- The code CPsuperH contains output arrays for the masses and mixing matrices of the neutral Higgs bosons, the sfermions, the charginos, and the neutralinos, named as follows:
  
  - \text{HMASS\_H(3)}: The masses of the three neutral Higgs bosons, \( M_{H_i} \).
  - \text{OMIX\_H(3, 3)}: The \( 3 \times 3 \) Higgs mixing matrix, \( O_{\alpha i} \).
  - \text{STMASS\_H(2)}: The masses of the stops, \( m_{\tilde{t}_i} \).
  - \text{STMIX\_H(2, 2)}: The mixing matrix of the stops, \( U^L_{\alpha i} \).
  - \text{SBMASS\_H(2)}: The masses of the sbottoms, \( m_{\tilde{b}_i} \).
  - \text{SBMIX\_H(2, 2)}: The mixing matrix of the sbottoms, \( U^R_{\alpha i} \).
  - \text{STAUMASS\_H(2)}: The masses of the staus, \( m_{\tilde{\tau}_i} \).
  - \text{STAUMIX\_H(2, 2)}: The mixing matrix of the staus, \( U^\tau_{\alpha i} \).
  - \text{SNU3MASS\_H}: The mass of the tau sneutrino, \( m_{\tilde{\nu}_\tau} \).
  - \text{MC\_H(2)}: The masses of the charginos, \( m_{\tilde{\chi}^\pm_i} \).
  - \text{UL\_H(2, 2)}: The mixing matrix of the left–handed charginos, \( (C_L)_{\alpha i} \).
  - \text{UR\_H(2, 2)}: The mixing matrix of the right–handed charginos, \( (C_R)_{\alpha i} \).
  - \text{MN\_H(4)}: The masses of the neutralinos, \( m_{\tilde{\chi}^0_i} \).
  - \text{N\_H(4, 4)}: The mixing matrix of the neutralinos, \( N_{\alpha i} \).

### 3.3 How to Run CPsuperH

The package CPsuperH consists of two text, five Fortran, and three shell–script files. The main features of the files are as follows:

- **Text files:**
  
  - The file \text{ARRAY} shows all the arrays described in the previous two subsections.
  - The file \text{COMMON} lists the parameter common blocks, as described in Appendix D.

- **Fortran files:**
  
  - \text{cpsuperh.f} fills all the arrays in \text{ARRAY} from the shell–script file \text{run} by calling the following four Fortran files.
fillpara.f fills the common blocks in COMMON from SMPARA_H and SSPARA_H.

fillhiggs.f fills the arrays for the masses and the mixing matrix of the neutral Higgs bosons, HMASS_H and OMIX_H.

fillcoupl.f fills the arrays for the masses and the mixing matrices of the stops, the sbottoms, the charginos, and the neutralinos as well as the couplings arrays NHCH_H, SHCH_H, and CHCH_H.

fillgambr.f fills the arrays GAMBRN and GAMBRC.

Shell–script files:

- makelib creates the library file libcpsuperh.a from the four Fortran files of fillpara.f, fillhiggs.f, fillcoupl.f, and fillgambr.f.

- compit creates the execution file cpsuperh.exe by compiling cpsuperh.f, linked with the library libcpsuperh.a.

- run supplies cpsuperh.f with the input values for SMPARA_H and SSPARA_H and part of IFLAG_H, and then shows the results by running cpsuperh.exe. The example presented in the present work is based on the so–called CPX scenario with $\Phi_A = \Phi_B = \Phi_C = 90^\circ$, $M_{SUSY} = 500$ GeV, $\tan\beta = 5$ and $M_{H^\pm} = 300$ GeV. Details may be found by inspecting the file. We note that, in the example, only IFLAG_H(1) = 1 is turned on initially. The user will have to edit run to choose new sets of parameters. The original version of run provides ample explanations of the various input parameters.

It is straightforward to run the code CPsuperH. Type './makelib' and './compit' followed by './run':

```
Run CPsuperH: ./makelib → ./compit → ./run
```

and then one can see some outputs depending on the values of IFLAG_H(1 − 6).

In Appendix E, we show some sample outputs from a CPsuperH test run based on the CPX scenario. All the values for the input parameters used in the test run (IFLAG_H(1) = 1), the masses and mixing matrix of the neutral Higgs bosons (IFLAG_H(2) = 1), the masses and mixing matrices of the charginos and neutralinos (IFLAG_H(4) = 1), the lightest Higgs boson couplings (IFLAG_H(5) = 1), and the decay width and branching fractions of the lightest Higgs boson (IFLAG_H(6) = 1) are generated by taking the IFLAG_H values given in the parentheses.

In order to check whether the code CPsuperH generates numerical outputs consistent with those provided by the code HDECAY in the CP–invariant case, we run the code CPsuperH in the ‘maximal mixing’ scenario: $|A_t| = \sqrt{6}M_{SUSY}$ with the common SUSY
scale $M_{\text{SUSY}} = M_Q = M_U = M_D = M_E = 1$ TeV and $|\mu| = 100$ GeV, by setting all the CP phases to zeros, but not including the threshold corrections. Fig. 1 shows the branching ratios and total decay widths of the MSSM Higgs bosons as functions of the Higgs boson masses. Although most of the parameter space presented in Fig. 1 is already ruled out by Higgs searches at LEP, we use it to present an effective comparison of the results of the CPsuperH code with those of the HDECAY code in the small-tan $\beta$ regime. We find that the CPsuperH results are indeed consistent with those obtained by the code HDECAY. There are very few visible discrepancies, for example in $B(H_1 \to gg)$ for $M_{H_1} \lesssim 80$ GeV, which may be due in part to the improved calculation of the Higgs-boson mass spectrum and the mixing matrix in CPsuperH.

We also show in Figs. 2 and 3 the branching ratios, $\text{GAMBRN}(\text{IM}, 2, \text{IH})$, and total decay widths, $\text{GAMBRN}(\text{ISMN}, 1, \text{IH})$ and $\text{GAMBRC}(\text{ISMC}, 1, \text{IH})$, found in the CPX scenario, which are consistent with the results previously reported in [18]. Finally, in Fig. 4 we illustrate the strong phase dependence of the Higgs boson decay widths into charginos and neutralinos. The left frame is for the lightest neutral Higgs boson, which for the given choice of parameters can only decay into two lightest neutralinos, i.e. $\text{GAMBRN}(\text{ISMN+ISUSYN}, 3, 1) = \text{GAMBRN}(\text{ISMN}+1, 3, 1)$. The right frame shows the branching ratios into superparticles of the heavier neutral Higgs bosons, $\text{GAMBRN}(\text{ISMN+ISUSYN}, 3, 2)$ and $\text{GAMBRN}(\text{ISMN+ISUSYN}, 3, 3)$, as well as the charged Higgs boson, $\text{GAMBRC}(\text{ISMC+ISUSYC}, 3)$, which only receive contributions from chargino/neutralino final states. The results of this figure are in close agreement with ref. [18].

4 Summary and Outlook

We have presented a detailed description of the Fortran code CPsuperH, a new computational package for studying Higgs phenomenology in the MSSM with explicit CP violation. Based on recent RG-improved diagrammatic calculations [23], the program CPsuperH computes the neutral and charged Higgs-boson masses as well as the $3 \times 3$ neutral Higgs-boson mixing matrix $O$ in the presence of CP violation in the MSSM Higgs sector. Although the dominant one- and two-loop contributions to the Higgs-boson self-energies are incorporated, there are still finite but subdominant two-loop contributions that may cause shifts of 3–4 GeV [32, 36] in the lightest Higgs-boson mass. These subdominant contributions can be estimated [37] to be of comparable size with the dominant three-loop effects. Because of the current lack of a detailed three-loop calculation, the subdominant two–loop contributions are not included in the present version of the code CPsuperH.

In addition to the Higgs mass spectrum, the program CPsuperH computes all the couplings and the decay widths of the neutral Higgs bosons $H_{1,2,3}$ and the charged Higgs boson $H^+$, incorporating the most important quantum corrections [18, 24]. In particular,
the leading-order QCD corrections are included for the Higgs decays into photons, gluons and most hadronic channels, so that (with the possible exception of squark final states) the theoretical uncertainties in these decay modes are kept small.

The Fortran code \texttt{CPsuperH} provides several options that can be selected in the input files for generating a variety of outputs for Higgs boson masses, Higgs decays and their respective branching fractions. Its structure offers the possibility of extending the list of input data to include flavour textures of off-diagonal trilinear and/or soft-squark masses, as these arise in some predictive schemes of soft SUSY breaking, e.g., in minimal supergravity models or in models with gauge and anomaly mediation. The code \texttt{CPsuperH} allows for straightforward extensions such as the additions of possible lepton- or quark-flavour-violating decays of the Higgs bosons. Another possible extension is the inclusion of loop-induced absorptive phases, which allows to generate CP–odd rate asymmetries in Higgs boson decays \cite{38}.

Radiative Higgs-sector CP violation in the MSSM has a wealth of implications for many different areas in particle-physics phenomenology. CP-violating phenomena mediated by Higgs-boson exchanges may manifest themselves in a number of low-energy observables such as the electron, neutron and muon electric dipole moments \cite{39,40,41,42}. They may also affect flavour-changing neutral-current processes and CP asymmetries involving \( K \) and \( B \) mesons \cite{43,33,44}. Moreover, CP-violating Higgs effects may influence the annihilation rates of cosmic relics and hence the abundance of dark matter in the Universe \cite{8}. Finally, an accurate determination of the Higgs spectrum in the presence of CP violation is crucial for testing the viability of electroweak baryogenesis in the MSSM.

To conclude, the Fortran code \texttt{CPsuperH} can be used as a powerful and efficient computational tool in quantitatively understanding these various phenomenological subjects, which are inter-related within the framework of the MSSM with explicit CP violation. Even in the CP-conserving case, \texttt{CPsuperH} is unique in computing the neutral and charged Higgs-boson couplings and masses with equally high level of precision, and should be therefore a useful tool for the study of MSSM Higgs phenomenology at present and future colliders.

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A Threshold Corrections

The exchanges of gluinos and charginos give finite loop–induced threshold corrections to the Yukawa couplings $h_{u,d}$, with the structure

$$h_d = \frac{\sqrt{2} m_d}{v \cos \beta} \left( \frac{1}{1 + (\delta h_d/h_d) + (\Delta h_d/h_d) \tan \beta} \right),$$

$$h_u = \frac{\sqrt{2} m_u}{v \sin \beta} \left( \frac{1}{1 + (\delta h_u/h_u) + (\Delta h_u/h_u) \cot \beta} \right),$$

(A.1)

modifying the couplings of the neutral Higgs–boson mass eigenstate $H_i$ to the scalar and pseudoscalar fermion bilinears as follows:

$$g^S_{H_i \bar{d} d} = \text{Re} \left( \frac{1}{1 + \kappa_d \tan \beta} \right) \frac{O_{\phi_1 i}}{\cos \beta} + \text{Re} \left( \frac{\kappa_d}{1 + \kappa_d \tan \beta} \right) \frac{O_{\phi_2 i}}{\cos \beta} + \text{Im} \left[ \frac{\kappa_d (\tan^2 \beta + 1)}{1 + \kappa_d \tan \beta} \right] O_{ai},$$

$$g^P_{H_i \bar{d} d} = -\text{Re} \left( \frac{\tan \beta - \kappa_d}{1 + \kappa_d \tan \beta} \right) O_{ai} + \text{Im} \left( \frac{\kappa_d \tan \beta}{1 + \kappa_d \tan \beta} \right) \frac{O_{\phi_1 i}}{\cos \beta} - \text{Im} \left( \frac{\kappa_d}{1 + \kappa_d \tan \beta} \right) \frac{O_{\phi_2 i}}{\cos \beta},$$

(A.2)

$$g^S_{H_i \bar{u} u} = \text{Re} \left( \frac{1}{1 + \kappa_u \cot \beta} \right) \frac{O_{\phi_2 i}}{\sin \beta} + \text{Re} \left( \frac{\kappa_u}{1 + \kappa_u \cot \beta} \right) \frac{O_{\phi_1 i}}{\sin \beta} + \text{Im} \left[ \frac{\kappa_u (\cot^2 \beta + 1)}{1 + \kappa_u \cot \beta} \right] O_{ai},$$

$$g^P_{H_i \bar{u} u} = -\text{Re} \left( \frac{\cot \beta - \kappa_u}{1 + \kappa_u \cot \beta} \right) O_{ai} + \text{Im} \left( \frac{\kappa_u \cot \beta}{1 + \kappa_u \cot \beta} \right) \frac{O_{\phi_1 i}}{\sin \beta} - \text{Im} \left( \frac{\kappa_u}{1 + \kappa_u \cot \beta} \right) \frac{O_{\phi_2 i}}{\sin \beta}.$$  

(A.3)

In the above equations, we have used the abbreviation

$$\kappa_q = \frac{(\Delta h_q/h_q)}{1 + (\delta h_q/h_q)},$$

(A.4)

for $q = u, d$. Detailed expressions for the threshold contributions $(\delta h_q/h_q)$ and $(\Delta h_q/h_q)$ can be found in [22].

For the couplings of the charged Higgs boson to quarks, we have

$$g^L_{H^+ \bar{d} d} = \frac{\cot \beta (1 + \rho_u) - \bar{\kappa}_u}{1 + \kappa_u \cot \beta},$$

$$g^R_{H^+ \bar{d} d} = \frac{\tan \beta (1 + \rho_d^*) - \bar{\kappa}_d^*}{1 + \kappa_d^* \tan \beta}.$$  

(A.5)
The quantities \( \kappa_{u,d} \) are given in (A.4), whilst the quantities \( \tilde{\kappa}_{u,d} \) and \( \rho_{u,d} \) in (A.5) and (A.6) are defined as follows:

\[
\tilde{\kappa}_q = \frac{\Delta h_q/h_q}{1 + (\Delta h_q/h_q)}, \quad \rho_q = \frac{\delta h_q/h_q - (\delta h_q/h_q)}{1 + (\Delta h_q/h_q)}.
\]

The expressions for the additional threshold contributions \( (\delta h_q/h_q) \) and \( (\Delta h_q/h_q) \) can also be found in [22].

\section{B \ Higgs–Boson Couplings to Third Generation Sfermions}

Here we present the Higgs–sfermion–sfermion couplings in the weak-interaction basis. The couplings \( \Gamma^{\alpha \bar{f} f} \) are given in the \( (\bar{f}_L, \bar{f}_R) \) basis by

\[
\begin{align*}
\Gamma_{ab \bar{t} \bar{t}} &= \frac{1}{\sqrt{2}} \begin{pmatrix} 0 & i h_t^*(s_\beta A^*_b + c_\beta \mu) \\ -i h_t(s_\beta A_b + c_\beta \mu^*) & 0 \end{pmatrix}, \\
\Gamma_{\phi_1 \bar{t} \bar{t}} &= \left( -|h_t|^2 v c_\beta + \frac{1}{3} \left( g^2 + \frac{1}{3} g'^2 \right) v c_\beta \right) \begin{pmatrix} 1 & -\frac{1}{\sqrt{2}} h_t^* A^*_b \\ -\frac{1}{\sqrt{2}} h_t A_b & -|h_t|^2 v c_\beta + \frac{1}{3} g'^2 v c_\beta \end{pmatrix}, \\
\Gamma_{\phi_2 \bar{t} \bar{t}} &= \left( -\frac{1}{4} \left( g^2 - \frac{1}{3} g'^2 \right) v s_\beta \right) \begin{pmatrix} \frac{1}{\sqrt{2}} h_t^* \mu^* & -\frac{1}{3} g'^2 v c_\beta \\ -\frac{1}{\sqrt{2}} h_t \mu & -|h_t|^2 v s_\beta + \frac{1}{3} g'^2 v s_\beta \end{pmatrix}, \\
\Gamma_{\alpha \bar{t} \bar{t}} &= \frac{1}{\sqrt{2}} \begin{pmatrix} 0 & i h_t^*(s_\beta A^*_r + c_\beta \mu) \\ -i h_t(s_\beta A_r + c_\beta \mu^*) & 0 \end{pmatrix}, \\
\Gamma_{\phi_1 \bar{t} \bar{t}}^* &= \left( -|h_t|^2 v s_\beta + \frac{1}{4} \left( g^2 - \frac{1}{3} g'^2 \right) v s_\beta \right) \begin{pmatrix} \frac{1}{\sqrt{2}} h_t^* A^*_r & -\frac{1}{\sqrt{2}} h_t A_r \\ -\frac{1}{\sqrt{2}} h_t^* A^*_r & -|h_t|^2 v s_\beta + \frac{1}{3} g'^2 v s_\beta \end{pmatrix}, \\
\Gamma_{\phi_2 \bar{t} \bar{t}}^* &= \left( -\frac{1}{4} \left( g^2 - g'^2 \right) v s_\beta \right) \begin{pmatrix} \frac{1}{\sqrt{2}} h_t^* \mu^* & -\frac{1}{3} g'^2 v s_\beta \\ -\frac{1}{\sqrt{2}} h_t \mu & -|h_t|^2 v s_\beta + \frac{1}{3} g'^2 v s_\beta \end{pmatrix}, \\
\Gamma_{\alpha \bar{t} \bar{t}}^* &= 0, \quad \Gamma_{\phi_1 \bar{t} \bar{t}}^* = \frac{1}{4} \left( g^2 + g'^2 \right) v c_\beta, \quad \Gamma_{\phi_2 \bar{t} \bar{t}}^* = \frac{1}{4} \left( g^2 + g'^2 \right) v s_\beta. \quad (B.1)
\end{align*}
\]

The coupling \( \Gamma^{H^+ \tilde{d}} \) is given in the LR basis by

\[
\Gamma^{H^+ \tilde{d}} = \begin{pmatrix} \frac{1}{\sqrt{2}} (|h_u|^2 + |h_d|^2 - g^2) v s_\beta c_\beta & h_d^* (s_\beta A^*_d + c_\beta \mu) \\ h_u (c_\beta A_u + s_\beta \mu^*) & \frac{1}{\sqrt{2}} h_u h_d^* \nu \end{pmatrix}
\]
The quartic couplings for the neutral Higgs bosons \([22]\) are

\[
\Gamma^{H^+\tilde{\nu}_L} = \frac{1}{\sqrt{2}} (|h_\tau|^2 - g^2) v s_\beta c_\beta , \quad \Gamma^{H^+\tilde{\nu}_R} = h_\tau^* (s_\beta A_\tau^* + c_\beta \mu)
\]  

(B.2)

C  Higgs–Boson Self–Coup\ls

In the following, we list all the effective trilinear and quartic Higgs–boson self–couplings of the Higgs weak eigenstates, which can be expressed in terms of the conventional quartic couplings \(\lambda_{1,2,\ldots,7}\) of the Higgs potential [13] obtained in an expansion of the effective Higgs potential up to operators of dimension 4. The trilinear couplings of the neutral Higgs bosons [13] are given by

\[
\begin{align*}
g_{\phi_1\phi_1\phi_1} &= c_\beta \lambda_1 + \frac{1}{2} s_\beta \Re \lambda_6 , \\
g_{\phi_1\phi_1\phi_2} &= s_\beta \lambda_{34} + s_\beta \Re \lambda_5 + \frac{3}{2} c_\beta \Re \lambda_6 , \\
g_{\phi_1\phi_2\phi_2} &= c_\beta \lambda_{34} + c_\beta \Re \lambda_5 + \frac{3}{2} s_\beta \Re \lambda_7 , \\
g_{\phi_2\phi_2\phi_2} &= s_\beta \lambda_2 + \frac{1}{2} c_\beta \Re \lambda_7 , \\
g_{\phi_1\phi_{1\alpha}} &= -s_\beta c_\beta \Im \lambda_5 - \frac{1}{2} (1 + 2 c_\beta^2) \Im \lambda_6 , \\
g_{\phi_1\phi_{2\alpha}} &= -2 \Im \lambda_5 - s_\beta c_\beta \Im (\lambda_6 + \lambda_7) , \\
g_{\phi_{2\alpha}^2} &= -s_\beta c_\beta \Im \lambda_5 - \frac{1}{2} (1 + 2 s_\beta^2) \Im \lambda_7 , \\
g_{\phi_{1\alpha}^2} &= s_\beta^2 c_\beta \lambda_1 + c_\beta^3 \lambda_{34} - c_\beta (1 + s_\beta^2) \Re \lambda_5 + \frac{1}{2} s_\beta (s_\beta^2 - 2 c_\beta^2) \Re \lambda_6 + \frac{1}{2} s_\beta c_\beta^2 \Re \lambda_7 , \\
g_{\phi_{2\alpha}^2} &= s_\beta^2 c_\beta \lambda_2 + s_\beta^3 \lambda_{34} - s_\beta (1 + c_\beta^2) \Re \lambda_5 + \frac{1}{2} s_\beta^2 c_\beta \Re \lambda_6 + \frac{1}{2} c_\beta (c_\beta^2 - 2 s_\beta^2) \Re \lambda_7 , \\
g_{\alpha\alpha} &= s_\beta c_\beta \Im \lambda_5 - \frac{1}{2} s_\beta \Im \lambda_6 - \frac{1}{2} c_\beta \Im \lambda_7 .
\end{align*}
\]

with \(\lambda_{34} = \frac{1}{2} (\lambda_3 + \lambda_4)\). The effective couplings \(g_{\alpha H^+H^-}\ [13]\) read:

\[
\begin{align*}
g_{\phi_1 H^+H^-} &= 2 s_\beta^2 c_\beta \lambda_1 + c_\beta^3 \lambda_3 - s_\beta^2 c_\beta \lambda_4 - 2 s_\beta^2 c_\beta \Re \lambda_5 + s_\beta (s_\beta^2 - 2 c_\beta^2) \Re \lambda_6 + s_\beta c_\beta^2 \Re \lambda_7 , \\
g_{\phi_2 H^+H^-} &= 2 s_\beta^2 c_\beta \lambda_2 + s_\beta^3 \lambda_3 - s_\beta c_\beta^2 \lambda_4 - 2 s_\beta^2 c_\beta \Re \lambda_5 + s_\beta^2 c_\beta \Re \lambda_6 + c_\beta (c_\beta^2 - 2 s_\beta^2) \Re \lambda_7 , \\
g_{\alpha H^+H^-} &= 2 s_\beta c_\beta \Im \lambda_5 - s_\beta \Im \lambda_6 - c_\beta \Im \lambda_7 .
\end{align*}
\]

(C.2)

The quartic couplings for the neutral Higgs bosons [22] are

\[
\begin{align*}
g_{\phi_1\phi_1\phi_1\phi_1} &= \frac{1}{4} \lambda_1 , \\
g_{\phi_1\phi_1\phi_1\phi_2} &= \frac{1}{2} \Re \lambda_6 , \\
g_{\phi_1\phi_1\phi_2\phi_2} &= \frac{1}{2} \lambda_{34} + \frac{1}{2} \Re \lambda_5 ,
\end{align*}
\]

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with the quartic coupling of the charged Higgs bosons being given by

\[ g_{\phi_1\phi_2\phi_2} = \frac{1}{2} \Re \lambda_7, \quad g_{\phi_2\phi_2\phi_2} = \frac{1}{4} \lambda_2, \]
\[ g_{\phi_1\phi_1\phi_1} = -\frac{1}{2} c_\beta \Im \lambda_6, \quad g_{\phi_1\phi_1\phi_2} = -c_\beta \Im \lambda_5 - \frac{1}{2} s_\beta \Im \lambda_6, \]
\[ g_{\phi_1\phi_2\phi_2} = -s_\beta \Im \lambda_5 - \frac{1}{2} c_\beta \Im \lambda_7, \quad g_{\phi_2\phi_2\phi_2} = -\frac{1}{2} s_\beta \Im \lambda_7, \]
\[ g_{\phi_1\phi_1\phi_3} = \frac{1}{2} s_\beta^2 \lambda_1 + \frac{1}{2} c_\beta \lambda_5 - \frac{1}{2} c_\beta \Re \lambda_5 - \frac{1}{2} s_\beta c_\beta \Re \lambda_6, \]
\[ g_{\phi_1\phi_2\phi_3} = -2 s_\beta c_\beta \Re \lambda_5 + \frac{1}{2} s_\beta^2 \Re \lambda_6 + \frac{1}{2} c_\beta \Re \lambda_7, \]
\[ g_{\phi_2\phi_2\phi_3} = \frac{1}{2} c_\beta^2 \lambda_2 + \frac{1}{2} s_\beta^2 \lambda_4 - \frac{1}{2} s_\beta^2 \Re \lambda_5 - \frac{1}{2} s_\beta c_\beta \Re \lambda_7, \]
\[ g_{\phi_1\phi_3\phi_3} = s_\beta c_\beta \Im \lambda_5 - \frac{1}{2} s_\beta^2 c_\beta \Im \lambda_6 - \frac{1}{2} c_\beta^3 \Im \lambda_7, \]
\[ g_{\phi_2\phi_3\phi_3} = s_\beta^3 c_\beta \Im \lambda_5 - \frac{1}{2} s_\beta^3 \Im \lambda_6 - \frac{1}{2} s_\beta c_\beta^2 \Im \lambda_7, \]
\[ g_{\phi_3\phi_3\phi_3} = \frac{1}{4} g_{H^+H^-H^+H^-}, \quad (C.3) \]

Finally, the remaining quartic couplings involving the charged Higgs boson pairs, \( g_{\alpha\beta H^+H^-} \), are given by

\[ g_{\phi_1\phi_1H^+H^-} = s_\beta^4 \lambda_1 + c_\beta^4 \lambda_2 + s_\beta^2 c_\beta^2 (\lambda_4 + \lambda_3) + 2 s_\beta^2 c_\beta^2 \Re \lambda_5 - 2 s_\beta^3 c_\beta \Re \lambda_6 - 2 s_\beta^3 c_\beta \Re \lambda_7. \quad (C.4) \]

\[ g_{\phi_2\phi_2H^+H^-} = c_\beta^4 \lambda_1 + \frac{1}{2} c_\beta^2 \lambda_3 - s_\beta c_\beta \Re \lambda_6, \]
\[ g_{\phi_1\phi_2H^+H^-} = -s_\beta c_\beta \lambda_4 - 2 s_\beta c_\beta \Re \lambda_5 + s_\beta^2 \Re \lambda_6 + c_\beta^2 \Re \lambda_7, \]
\[ g_{\phi_2\phi_2H^+H^-} = c_\beta^2 \lambda_2 + \frac{1}{2} s_\beta^2 \lambda_3 - s_\beta c_\beta \Re \lambda_7, \]
\[ g_{\phi_1aH^+H^-} = 2 s_\beta c_\beta \Im \lambda_5 - s_\beta^2 c_\beta \Im \lambda_6 - c_\beta^3 \Im \lambda_7, \]
\[ g_{\phi_2aH^+H^-} = 2 s_\beta^2 c_\beta \Im \lambda_5 - s_\beta^3 \Im \lambda_6 - s_\beta c_\beta^2 \Im \lambda_7, \]
\[ g_{aaH^+H^-} = g_{H^+H^-H^+H^-}. \quad (C.5) \]

## D Common Blocks

Here we list three common blocks for the SM and SUSY parameters, which are filled from two input arrays SMPARA_H and SSPARA_H.

- /HC_SMPARA/: This common block is for the SM parameters.
\[ AEM_H = \alpha_{em}(M_Z) \]
\[ SW_H = \sin \theta_W \]
\[ M_{TAU_H} = m_{\tau} \text{ [GeV]} \]
\[ MBMT_H = m_b(m_t^{pole}) \text{ [GeV]} \]
\[ M_{POLE_H} = m_t(m_t^{pole}) \text{ [GeV]} \]
\[ EEM_H = e = (4\pi\alpha_{em}(M_Z))^{1/2} \]
\[ TW_H = \tan \theta_W \]
\[ GP_H = g' = e/\cos \theta_W \]
\[ MTMT_H = m_t(m_t^{pole}) \]

\[ \alpha_s(m_t^{pole}) = \frac{\alpha_s(M_Z)}{1 + \beta_{n_f} \frac{\alpha_s(M_Z)}{4\pi} \ln(m_t^{pole}/M_Z^2)} \text{ with } \beta_{n_f} = 11 - \frac{2}{3} n_f, \]
\[ G_F = \frac{\sqrt{2}g^2}{8M_W^2}, \]
\[ m_t(m_t^{pole}) = \frac{m_t^{pole}}{1 + \frac{4\alpha_s(m_t^{pole})}{3\pi}}. \]

• /HC_RSUSYPARA/: This is for the real SUSY parameters.

\[ TB_H = \tan \beta \quad CB_H = \cos \beta \quad SB_H = \sin \beta \quad M_{Q3_H} = M_{Q3} \]
\[ M_{U3_H} = M_{U3} \quad M_{D3_H} = M_{D3} \quad M_{L3_H} = M_{L3} \quad M_{E3_H} = M_{E3} \]

• /HC_CSUSYPARA/: This is for the complex SUSY parameters.

\[ COMPLEX*16 \quad MU_H, M1_H, M2_H, M3_H, AT_H, AB_H, ATAU_H \]
\[ \text{COM} \quad MU_H, M1_H, M2_H, M3_H, AT_H, AB_H, ATAU_H \]
\[ MU_H = \mu \text{ [GeV]} \quad M1_H = M_1 \text{ [GeV]} \quad M2_H = M_2 \text{ [GeV]} \quad M3_H = M_3 \text{ [GeV]} \]
\[ AT_H = A_t \text{ [GeV]} \quad AB_H = A_b \text{ [GeV]} \quad ATAU_H = A_{\tau} \text{ [GeV]} \]

E Sample Outputs

Here we show the results of a test run of the code CPsuperH for the CPX scenario of MSSM Higgs-sector CP violation.
• IFLAG_H(1) = 1: The list of the SM and SUSY input parameters

Standard Model Parameters in /HC_SMPARA/

AME_H = 0.7812E-02 : alpha_em(MZ)
ASMZ_H = 0.1172E+00 : alpha_s(MZ)
MZ_H = 0.9119E+02 : Z boson mass in GeV
SW_H = 0.4608E+00 : sinTheta_W
ME_H = 0.5000E-03 : electron mass in GeV
MMU_H = 0.1065E+00 : muon mass in GeV
MTAU_H = 0.1777E+01 : tau mass in GeV
MDMT_H = 0.6000E-02 : d-quark mass at M_t^pole in GeV
MSMT_H = 0.1150E+00 : s-quark mass at M_t^pole in GeV
MBMT_H = 0.3000E+01 : b-quark mass at M_t^pole in GeV
MCMT_H = 0.6200E+00 : c-quark mass at M_t^pole in GeV
MTPOLE_H = 0.1750E+03 : t-quark pole mass in GeV
GAMW_H = 0.2118E+01 : Gam_W in GeV
GAMZ_H = 0.2495E+01 : Gam_Z in GeV
EEM_H = 0.3133E+00 : e = (4*pi*alpha_em)^1/2
ASMT_H = 0.1072E+00 : alpha_s(M_t^pole)
CW_H = 0.8768E+00 : cosTheta_W
TW_H = 0.5483E+00 : tanTheta_W
MW_H = 0.7996E+02 : W boson mass MW = MZ*CW
GW_H = 0.6517E+00 : SU(2) gauge coupling gw = e/s_W
GP_H = 0.3573E+00 : U(1)_Y gauge coupling gp = e/c_W
V_H = 0.2454E+00 : V = 2 MW / gw
GF_H = 0.1174E-04 : GF = sqrt(2)*gw^2/8 MW^2 in GeV^-2
MTMT_H = 0.1674E+03 : t-quark mass at M_t^pole in GeV

Real SUSY Parameters in /HC_RSUSYPARA/

TB_H = 0.5000E+01 : tan(beta)
CB_H = 0.1961E+00 : cos(beta)
SB_H = 0.9806E+00 : sin(beta)
MQ3_H = 0.5000E+03 : M_tildeQ_3 in GeV
MU3_H = 0.5000E+03 : M_tildeU_3 in GeV
MD3_H = 0.5000E+03 : M_tildeD_3 in GeV
ML3_H = 0.5000E+03 : M_tildeL_3 in GeV
ME3_H = 0.5000E+03 : M_tildeE_3 in GeV
Complex SUSY Parameters in /HC_CSUSYPARA/

| Parameter | Value | Description |
|-----------|-------|-------------|
| MU_H | 0.2000E+04 | Magnitude of MU parameter in GeV |
| M1_H | 0.5000E+02 | Magnitude of M1 parameter in GeV |
| M2_H | 0.1000E+03 | Magnitude of M2 parameter in GeV |
| M3_H | 0.1000E+04 | Magnitude of M3 parameter in GeV |
| AT_H | 0.1000E+04 | Magnitude of AT parameter in GeV |
| AB_H | 0.1000E+04 | Magnitude of AB parameter in GeV |
| ATAU_H | 0.1000E+04 | Magnitude of ATAU parameter in GeV |
| ARG(MU_H) | 0.0000E+00 | Argument of MU parameter in Degree |
| ARG(M1_H) | 0.0000E+00 | Argument of M1 parameter in Degree |
| ARG(M2_H) | 0.0000E+00 | Argument of M2 parameter in Degree |
| ARG(M3_H) | 0.9000E+02 | Argument of M3 parameter in Degree |
| ARG(AT_H) | 0.9000E+02 | Argument of AT parameter in Degree |
| ARG(AB_H) | 0.9000E+02 | Argument of AB parameter in Degree |
| ARG(ATAU_H) | 0.9000E+02 | Argument of ATAU parameter in Degree |

Charged Higgs boson pole mass : 0.3000E+03 GeV

- **IFLAG_H(2) = 1**: The masses and mixing matrix of the neutral Higgs bosons

Masses and Mixing Matrix of Higgs bosons:

\[
\begin{bmatrix}
H1 \text{ Pole Mass} & = & 0.1188E+03 \text{ GeV} \\
H2 \text{ Pole Mass} & = & 0.2703E+03 \text{ GeV} \\
H3 \text{ Pole Mass} & = & 0.2981E+03 \text{ GeV} \\
\end{bmatrix}
\]

Charged Higgs Pole Mass = 0.3000E+03 GeV [SSPARA_H(2)]

\[
\begin{bmatrix}
\phi_1 \end{bmatrix} / 0.2451E+00 -0.3373E+00 -0.9089E+00 \\
O(IA,IH) = \begin{bmatrix}
\phi_2 \\
\end{bmatrix} | 0.9694E+00 0.7532E-01 0.2335E+00 |
\]

- **IFLAG_H(4) = 1**: The masses and mixing matrices of the charginos and neutralinos

Chargino Masses and Mixing Matrices:

\[
\begin{bmatrix}
\end{bmatrix}
\]
MC\_H(I), UL\_H(I,A), and UR\_H(I,A)

| MC1       | MC2       |
|----------|----------|
| 0.9861E+02 GeV | 0.2003E+04 GeV |

UL\_H =
\[
\begin{pmatrix}
0.9984E+00 & 0.0000E+00 \\
0.5603E-01 & 0.0000E+00
\end{pmatrix} \begin{pmatrix}
-0.5603E-01 & 0.0000E+00 \\
0.9984E+00 & 0.0000E+00
\end{pmatrix}
\]

UR\_H =
\[
\begin{pmatrix}
0.9999E+00 & 0.0000E+00 \\
0.1385E-01 & 0.5460E-09
\end{pmatrix} \begin{pmatrix}
0.1385E-01 & 0.0000E+00 \\
0.9999E+00 & 0.0000E+00
\end{pmatrix}
\]

Neutralino Masses MN\_H(I) and Mixing Matrix N\_H(I,A)

| MN1       | MN2       |
|----------|----------|
| 0.4960E+02 GeV | 0.9862E+02 GeV |
| MN3       | MN4       |
| 0.2001E+04 GeV | 0.2003E+04 GeV |

\[
N\_H(1,1) = (0.9996E+00, 0.0000E+00) \\
N\_H(1,2) = (-0.1462E-01, 0.0000E+00) \\
N\_H(1,3) = (0.2218E-01, 0.0000E+00) \\
N\_H(1,4) = (-0.4962E-02, 0.0000E+00)
\]

\[
N\_H(2,1) = (-0.1553E-01, 0.0000E+00) \\
N\_H(2,2) = (-0.9991E+00, 0.0000E+00) \\
N\_H(2,3) = (0.3931E-01, 0.0000E+00) \\
N\_H(2,4) = (-0.9704E-02, 0.0000E+00)
\]

\[
N\_H(3,1) = (0.0000E+00, -0.1186E-01) \\
N\_H(3,2) = (0.0000E+00, 0.2112E-01) \\
N\_H(3,3) = (0.0000E+00, 0.7066E+00) \\
N\_H(3,4) = (0.0000E+00, 0.7072E+00)
\]

\[
N\_H(4,1) = (0.1867E-01, 0.0000E+00) \\
N\_H(4,2) = (-0.3494E-01, 0.0000E+00) \\
N\_H(4,3) = (-0.7062E+00, 0.0000E+00) \\
N\_H(4,4) = (0.7069E+00, 0.0000E+00)
\]

- IFLAG\_H(5) = 1: The couplings of the lightest Higgs boson
The Lightest Higgs H₁ Couplings : NHC_H(NC,1)

| H₁ | Couplings | Coupling | Value |
|----|-----------|----------|-------|
| e  | e e       | GF       | (0.2038E-05,0.0000E+00) |
|    |           | GS       | (0.1250E+01,0.0000E+00) |
|    |           | GP       | (0.5148E-01,0.0000E+00) |
| μ  | μ μ       | GF       | (0.4340E-03,0.0000E+00) |
|    |           | GS       | (0.1250E+01,0.0000E+00) |
|    |           | GP       | (0.5148E-01,0.0000E+00) |
| τ  | τ τ       | GF       | (0.7242E-02,0.0000E+00) |
|    |           | GS       | (0.1250E+01,0.0000E+00) |
|    |           | GP       | (0.5148E-01,0.0000E+00) |
| d  | d d       | GF       | (0.2445E-04,0.0000E+00) |
|    |           | GS       | (0.1250E+01,0.0000E+00) |
|    |           | GP       | (0.5148E-01,0.0000E+00) |
| s  | s s       | GF       | (0.4687E-03,0.0000E+00) |
|    |           | GS       | (0.1250E+01,0.0000E+00) |
|    |           | GP       | (0.5148E-01,0.0000E+00) |
| b  | b b       | GF       | (0.1223E-01,0.0000E+00) |
|    |           | GS       | (0.1246E+01,0.0000E+00) |
|    |           | GP       | (-.1741E-01,0.0000E+00) |
| u  | u u       | GF       | (0.1223E-04,0.0000E+00) |
|    |           | GS       | (0.9886E+00,0.0000E+00) |
|    |           | GP       | (0.2059E-02,0.0000E+00) |
| c  | c c       | GF       | (0.2527E-02,0.0000E+00) |
|    |           | GS       | (0.9886E+00,0.0000E+00) |
|    |           | GP       | (0.2059E-02,0.0000E+00) |
| t  | t t       | GF       | (0.6821E+00,0.0000E+00) |
|    |           | GS       | (0.9892E+00,0.0000E+00) |
|    |           | GP       | (0.4501E-02,0.0000E+00) |
| N1 | N1 N1     | GF       | (0.3258E+00,0.0000E+00) |
|    |           | GS       | (-.5767E-02,0.0000E+00) |
|    |           | GP       | (.1317E-03,0.0000E+00) |
| N2 | N2 N2     | GF       | (0.3258E+00,0.0000E+00) |
|    |           | GS       | (-.1886E-01,0.0000E+00) |
|    |           | GP       | (0.4125E-03,0.0000E+00) |
| N3 | N3 N3     | GF       | (0.3258E+00,0.0000E+00) |
|    |           | GS       | (0.1415E-01,0.0000E+00) |
|    |           | GP       | (0.1576E-03,0.0000E+00) |
| N4 | N4 N4     | GF       | (0.3258E+00,0.0000E+00) |
|   |   |   | GS=(0.3878E-01,0.0000E+00) | GP=(-.3866E-03,0.0000E+00) |
|---|---|---|--------------------------|--------------------------|
| H1 N1 N2 | [NC=40] | GF=(0.3258E+00,0.0000E+00) | GS=(-.1043E-01,0.0000E+00) | GP=(0.2331E-03,0.0000E+00) |
| H1 N1 N3 | [NC=43] | GF=(0.3258E+00,0.0000E+00) | GS=(-.1602E-02,0.0000E+00) | GP=(0.1443E+00,0.0000E+00) |
| H1 N1 N4 | [NC=46] | GF=(0.3258E+00,0.0000E+00) | GS=(0.2413E+00,0.0000E+00) | GP=(-.2402E-02,0.0000E+00) |
| H1 N2 N3 | [NC=49] | GF=(0.3258E+00,0.0000E+00) | GS=(-.2820E-02,0.0000E+00) | GP=(0.2540E+00,0.0000E+00) |
| H1 N2 N4 | [NC=52] | GF=(0.3258E+00,0.0000E+00) | GS=(0.4247E+00,0.0000E+00) | GP=(-.4228E-02,0.0000E+00) |
| H1 N3 N4 | [NC=55] | GF=(0.3258E+00,0.0000E+00) | GS=(-.2470E-03,0.0000E+00) | GP=(-.2789E-03,0.0000E+00) |
| H1 C1+ C1− | [NC=58] | GF=(0.4608E+00,0.0000E+00) | GS=(.2714E-01,0.0000E+00) | GP=(.5936E-03,-.2135E-18) |
| H1 C1+ C2− | [NC=61] | GF=(0.4608E+00,0.0000E+00) | GS=(.6058E+00,.4035E-02) | GP=(.6042E-02,-.3618E+00) |
| H1 C2+ C1− | [NC=64] | GF=(0.4608E+00,0.0000E+00) | GS=(.6058E+00,.4035E-02) | GP=(.6042E-02,0.3618E+00) |
| H1 C2+ C2− | [NC=67] | GF=(0.4608E+00,0.0000E+00) | GS=(.5771E-01,0.0000E+00) | GP=(.2527E-03,.7454E-19) |
| H1 V V | [NC=70] | GF=(0.9987E+00,0.0000E+00) | GS=(0.2184E+01,-.3755E-16) | GP=(0.2447E+00,-.1144E+00) |
| H1 ST1* ST1 | [NC=71] | GF=(0.2447E+00,-.1144E+00) | GS=(.2447E+00,.1144E+00) | GP=(.2447E+00,.1144E+00) |
| H1 ST2* ST1 | [NC=73] | GF=(.3987E+01,-.4869E-17) | GS=(.4330E+00,.1506E-18) | GP=(.2070E-01,.8020E-02) |
| H1 SB1* SB1 | [NC=75] | GF=(.4330E+00,.1506E-18) | GS=(.2070E-01,.8020E-02) | GP=(.2070E-01,.8020E-02) |
H1 SB2* SB2  [NC=78]: G =(-.5584E+00,0.1683E-18)
H1 STA1* STA1  [NC=79]: G =(0.2300E+00,-.1144E-17)
H1 STA1* STA2  [NC=80]: G =(0.1602E-02,0.6836E-02)
H1 STA2* STA1  [NC=81]: G =(0.1602E-02,-.6836E-02)
H1 STA2* STA2  [NC=82]: G =(-.3549E+00,-.1785E-17)
H1 SNU3* SNU3  [NC=83]: G =(0.1246E+00,0.0000E+00)
H1 glue glue  [NC=84]: S =(0.5827E+00,0.3665E-01)
[NC=85]: P =(0.5195E-02,-.5135E-03)
H1 CH+ CH-  [NC=86]: G =(-.4047E-01,0.0000E+00)
H1 CH+ W-  [NC=87]: G =(-.5024E-01,0.1030E-01)
H1 photon photon  [NC=88]: S =(-.6557E+01,0.2443E-01)
[NC=89]: P =(-.1385E-01,-.3423E-03)
H1 glue glue (M=0)  [NC=90]: S =(-.1427E+01,0.1915E-17)
[NC=91]: P =(-.1291E-01,0.0000E+00)
H1 photon photon(M=0)  [NC=92]: S =(-.4878E+01,0.5235E-17)
[NC=93]: P =(-.1726E-02,-.4811E-18)

- **IFLAG_H(6) = 1**: The decay width and branching fractions of the lightest Higgs boson

**Neutral Higgs Boson Decays with ISMN = 50 : ISUSYN = 50**

| DECAY MODE | [ IM] | WIDTH[GeV] | BR[SM] | BR[TOTAL] |
|------------|------|------------|--------|-----------|
| H1 -> e e  | [ 1] | 0.3070E-10 | 0.5766E-08 | 0.5760E-08 |
| H1 -> mu mu | [ 2] | 0.1393E-05 | 0.2616E-03 | 0.2613E-03 |
| H1 -> tau tau | [ 3] | 0.3873E-03 | 0.7274E-01 | 0.7266E-01 |
| H1 -> d d  | [ 4] | 0.1686E-07 | 0.3166E-05 | 0.3163E-05 |
| H1 -> s s  | [ 5] | 0.6193E-05 | 0.1163E-02 | 0.1162E-02 |
| H1 -> b b  | [ 6] | 0.4163E-02 | 0.7820E+00 | 0.7812E+00 |
| H1 -> u u  | [ 7] | 0.2632E-08 | 0.4944E-06 | 0.4939E-06 |
| H1 -> c c  | [ 8] | 0.1124E-03 | 0.2111E-01 | 0.2109E-01 |
| H1 -> t t  | [ 9] | 0.0000E+00 | 0.0000E+00 | 0.0000E+00 |
| H1 -> W W  | [10] | 0.4106E-03 | 0.7711E-01 | 0.7703E-01 |
| H1 -> Z Z  | [11] | 0.3303E-04 | 0.6203E-02 | 0.6197E-02 |
| H1 -> H1 Z | [12] | 0.0000E+00 | 0.0000E+00 | 0.0000E+00 |
| H1 -> H2 Z | [13] | 0.0000E+00 | 0.0000E+00 | 0.0000E+00 |
| H1 -> H1 H1 | [14] | 0.0000E+00 | 0.0000E+00 | 0.0000E+00 |
| H1 -> H1 H2 | [15] | 0.0000E+00 | 0.0000E+00 | 0.0000E+00 |
| Source | Target | Result 1 | Result 2 | Result 3 |
|--------|--------|----------|----------|----------|
| H1 -> H2 | H2     | 0.0000E+00 | 0.0000E+00 | 0.0000E+00 |
| H1 -> ph  | ph     | 0.9427E-05 | 0.1771E-02 | 0.1769E-02 |
| H1 -> gl  | gl     | 0.2004E-03 | 0.3763E-01 | 0.3760E-01 |
| H1 TOTAL(SM) | 50 | 0.5324E-02 | 0.1000E+01 | 0.9990E+00 |
| H1 -> N1 | N1     | 0.5557E-05 | 0.1044E-02 | 0.1043E-02 |
| H1 -> N1 | N2     | 0.0000E+00 | 0.0000E+00 | 0.0000E+00 |
| H1 TOTAL(SUSY) | 100 | 0.5557E-05 | 0.1044E-02 | 0.1043E-02 |

*Note: WIDTH=GAMBRN(IM,1,1), BR[SM] =GAMBRN(IM,2,1) and BR[TOTAL]=GAMBRN(IM,3,1)*

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Table 4: The couplings of the $\text{IH}$-th neutral Higgs boson to two particles specified with the index $\text{NC}, \text{NHC}_\text{H}(\text{NC}, \text{IH})$. For the definitions of the couplings to two fermions, $g_f$, $g^S$, and $g^P$, see Eq. (27) and Table I.

| NC   | Coupling | NC   | Coupling | NC   | Coupling | NC   | Coupling |
|------|----------|------|----------|------|----------|------|----------|
| 1    | $g_e$    | 26   | $g_{H_{16} t t}$ | 51   | $g^P_{H_{16} \tilde{\chi}_1^0 \chi_2^0}$, $g^P_{H_{16} \tilde{\chi}_2^0 \chi_1^0}$ |
| 2    | $g^S_{H_{16} e^{-}}$ | 27   | $g^P_{H_{16} t t}$ | 52   | $g_{\tilde{\chi}^0}$ |
| 3    | $g^P_{H_{16} e^{-}}$ | 28   | $g^0_{\tilde{\chi}^0}$ | 53   | $g^S_{H_{16} \tilde{\chi}_1^0 \chi_2^0}$, $g^S_{H_{16} \tilde{\chi}_2^0 \chi_1^0}$ |
| 4    | $g_\mu$  | 29   | $g^S_{H_{16} \tilde{\chi}_1^0 \chi_2^0}$ | 54   | $g^P_{H_{16} \tilde{\chi}_1^0 \chi_2^0}$, $g^P_{H_{16} \tilde{\chi}_2^0 \chi_1^0}$ |
| 5    | $g^P_{H_{16} \mu^{-}}$ | 30   | $g^P_{H_{16} \tilde{\chi}_1^0 \chi_2^0}$ | 55   | $g_{\tilde{\chi}^0}$ |
| 6    | $g^P_{H_{16} \mu^{-}}$ | 31   | $g_{\tilde{\chi}^0}$ | 56   | $g^S_{H_{16} \tilde{\chi}_1^0 \chi_2^0}$, $g^S_{H_{16} \tilde{\chi}_2^0 \chi_1^0}$ |
| 7    | $g_\tau$ | 32   | $g^S_{H_{16} \tilde{\chi}_1^0 \chi_2^0}$ | 57   | $g^S_{H_{16} \tilde{\chi}_1^0 \chi_2^0}$, $g^S_{H_{16} \tilde{\chi}_2^0 \chi_1^0}$ |
| 8    | $g^P_{H_{16} \tau^{-}}$ | 33   | $g^P_{H_{16} \tilde{\chi}_1^0 \chi_2^0}$ | 58   | $g_{\tilde{\chi}^0}$ |
| 9    | $g^P_{H_{16} \tau^{-}}$ | 34   | $g_{\tilde{\chi}^0}$ | 59   | $g^S_{H_{16} \tilde{\chi}_1^0 \chi_2^0}$ |
| 10   | $g_d$    | 35   | $g^S_{H_{16} \tilde{\chi}_1^0 \chi_2^0}$ | 60   | $g^P_{H_{16} \tilde{\chi}_1^0 \chi_2^0}$ |
| 11   | $g^S_{H_{16} d d}$ | 36   | $g^P_{H_{16} \tilde{\chi}_1^0 \chi_2^0}$ | 61   | $g_{\tilde{\chi}^0}$ |
| 12   | $g^P_{H_{16} d d}$ | 37   | $g^0_{\tilde{\chi}^0}$ | 62   | $g^S_{H_{16} \tilde{\chi}_1^0 \chi_2^0}$ |
| 13   | $g_s$    | 38   | $g^S_{H_{16} \tilde{\chi}_1^0 \chi_2^0}$ | 63   | $g^P_{H_{16} \tilde{\chi}_1^0 \chi_2^0}$ |
| 14   | $g^S_{H_{16} s s}$ | 39   | $g^P_{H_{16} \tilde{\chi}_1^0 \chi_2^0}$ | 64   | $g_{\tilde{\chi}^0}$ |
| 15   | $g^P_{H_{16} s s}$ | 40   | $g^0_{\tilde{\chi}^0}$ | 65   | $g^S_{H_{16} \tilde{\chi}_1^0 \chi_2^0}$ |
| 16   | $g_b$    | 41   | $g^S_{H_{16} \tilde{\chi}_1^0 \chi_2^0}$, $g^S_{H_{16} \tilde{\chi}_2^0 \chi_1^0}$ | 66   | $g^P_{H_{16} \tilde{\chi}_1^0 \chi_2^0}$ |
| 17   | $g^S_{H_{16} b b}$ | 42   | $g^P_{H_{16} \tilde{\chi}_1^0 \chi_2^0}$, $g^P_{H_{16} \tilde{\chi}_2^0 \chi_1^0}$ | 67   | $g_{\tilde{\chi}^0}$ |
| 18   | $g^P_{H_{16} b b}$ | 43   | $g^0_{\tilde{\chi}^0}$ | 68   | $g^S_{H_{16} \tilde{\chi}_1^0 \chi_2^0}$ |
| 19   | $g_u$    | 44   | $g^S_{H_{16} \tilde{\chi}_1^0 \chi_2^0}$, $g^S_{H_{16} \tilde{\chi}_2^0 \chi_1^0}$ | 69   | $g^P_{H_{16} \tilde{\chi}_1^0 \chi_2^0}$ |
| 20   | $g^S_{H_{16} u u}$ | 45   | $g^P_{H_{16} \tilde{\chi}_1^0 \chi_2^0}$, $g^P_{H_{16} \tilde{\chi}_2^0 \chi_1^0}$ | 70   | $g_{H_{16} v v}$ |
| 21   | $g^P_{H_{16} u u}$ | 46   | $g^0_{\tilde{\chi}^0}$ | 71   | $g_{H_{16} \tilde{\tau}_1^0}$ |
| 22   | $g_c$    | 47   | $g^S_{H_{16} \tilde{\chi}_1^0 \chi_2^0}$, $g^S_{H_{16} \tilde{\chi}_2^0 \chi_1^0}$ | 72   | $g_{H_{16} \tilde{\tau}_1^0}$ |
| 23   | $g^S_{H_{16} c c}$ | 48   | $g^P_{H_{16} \tilde{\chi}_1^0 \chi_2^0}$, $g^P_{H_{16} \tilde{\chi}_2^0 \chi_1^0}$ | 73   | $g_{H_{16} \tilde{\tau}_1^0}$ |
| 24   | $g^P_{H_{16} c c}$ | 49   | $g^0_{\tilde{\chi}^0}$ | 74   | $g_{H_{16} \tilde{\tau}_1^0}$ |
| 25   | $g_t$    | 50   | $g^S_{H_{16} \tilde{\chi}_1^0 \chi_2^0}$, $g^S_{H_{16} \tilde{\chi}_2^0 \chi_1^0}$ | 75   | $g_{H_{16} \tilde{\tau}_1^0}$ |

Index: $g$, $g^S$, $g^P$.
Table 5: The trilinear (\(\text{NC}=1-13\)) and quartic (\(\text{NC}=14-35\)) Higgs-boson self-couplings, \(\text{SHC}_H(\text{NC})\). We note that \(\text{SHC}_H(10+\text{IH})=\text{NHCH}(86,\text{IH})\) for \(\text{IH}=1-3\).

| NC | Coupling | NC | Coupling | NC | Coupling | NC | Coupling |
|----|----------|----|----------|----|----------|----|----------|
| 1  | \(g_{H_H_3H_3}\) | 11 | \(g_{H_H_3H_3}\) | 21 | \(g_{H_H_2H_2H_1}\) | 31 | \(g_{H_H_1H_1H_1}\) |
| 2  | \(g_{H_H_3H_2}\) | 12 | \(g_{H_H_3H_2}\) | 22 | \(g_{H_H_2H_2H_1}\) | 32 | \(g_{H_H_2H_2H_1}\) |
| 3  | \(g_{H_H_3H_1}\) | 13 | \(g_{H_H_3H_1}\) | 23 | \(g_{H_H_1H_1H_1}\) | 33 | \(g_{H_H_1H_1H_1}\) |
| 4  | \(g_{H_H_2H_2}\) | 14 | \(g_{H_H_2H_2}\) | 24 | \(g_{H_H_2H_2H_2}\) | 34 | \(g_{H_H_1H_1H_1}\) |
| 5  | \(g_{H_H_2H_1}\) | 15 | \(g_{H_H_2H_1}\) | 25 | \(g_{H_H_2H_2H_1}\) | 35 | \(g_{H_H_1H_1H_1}\) |
| 6  | \(g_{H_H_2H_1}\) | 16 | \(g_{H_H_2H_1}\) | 26 | \(g_{H_H_2H_2H_1}\) | 36 | \(g_{H_H_1H_1H_1}\) |
| 7  | \(g_{H_H_1H_1}\) | 17 | \(g_{H_H_1H_1}\) | 27 | \(g_{H_H_1H_1H_1}\) | 37 | \(g_{H_H_1H_1H_1}\) |
| 8  | \(g_{H_H_1H_1}\) | 18 | \(g_{H_H_1H_1}\) | 28 | \(g_{H_H_1H_1H_1}\) | 38 | \(g_{H_H_1H_1H_1}\) |
| 9  | \(g_{H_H_1H_1}\) | 19 | \(g_{H_H_1H_1}\) | 29 | \(g_{H_H_1H_1H_1}\) | 39 | \(g_{H_H_1H_1H_1}\) |
| 10 | \(g_{H_H_1H_1}\) | 20 | \(g_{H_H_1H_1H_1}\) | 30 | \(g_{H_H_1H_1H_1}\) | 40 | \(g_{H_H_1H_1H_1}\) |

Table 6: The charged Higgs-boson couplings to two particles specified with the index NC, \(\text{CHC}_H(\text{NC})\). For the definitions of the couplings to two fermions, \(g_{fp}, g_s\), and \(g_P\), see Eq. (27) and Table 7.

| NC | Coupling | NC | Coupling | NC | Coupling | NC | Coupling | NC | Coupling | NC | Coupling |
|----|----------|----|----------|----|----------|----|----------|----|----------|----|----------|
| 1  | \(g_{\nu_e}\) | 11 | \(g^{S}_{H+\bar{u}d}\) | 21 | \(g^{P}_{H+\chi_{1}^{0}\chi_{1}^{0}}\) | 31 | \(g^{S}_{\chi_{1}^{0}\chi_{1}^{0}}\) | 41 | \(g^{S}_{H+\chi_{4}^{0}\chi_{4}^{0}}\) |
| 2  | \(g^{S}_{H+\nu_e}\) | 12 | \(g^{P}_{H+\bar{u}d}\) | 22 | \(g^{S}_{\chi_{1}^{0}\chi_{1}^{0}}\) | 32 | \(g^{P}_{H+\chi_{4}^{0}\chi_{4}^{0}}\) | 42 | \(g^{P}_{H+\chi_{4}^{0}\chi_{4}^{0}}\) |
| 3  | \(g^{S}_{H+\nu_e}\) | 13 | \(g_{es}\) | 23 | \(g^{S}_{H+\chi_{1}^{0}\chi_{2}^{0}}\) | 33 | \(g^{P}_{H+\chi_{4}^{0}\chi_{4}^{0}}\) | 43 | \(g^{P}_{H+\tau_{1}b_{1}}\) |
| 4  | \(g^{S}_{H+\nu_{\mu}}\) | 14 | \(g^{S}_{H+\bar{e}s}\) | 24 | \(g^{P}_{H+\chi_{4}^{0}\chi_{2}^{0}}\) | 34 | \(g^{S}_{H+\chi_{4}^{0}\chi_{4}^{0}}\) | 44 | \(g^{P}_{H+\tau_{1}b_{1}}\) |
| 5  | \(g^{S}_{H+\nu_{\mu}}\) | 15 | \(g^{P}_{H+\bar{e}s}\) | 25 | \(g^{S}_{H+\chi_{4}^{0}\chi_{4}^{0}}\) | 35 | \(g^{S}_{H+\chi_{4}^{0}\chi_{4}^{0}}\) | 45 | \(g^{S}_{H+\tau_{1}b_{1}}\) |
| 6  | \(g^{P}_{H+\nu_{\mu}}\) | 16 | \(g_{tb}\) | 26 | \(g^{S}_{H+\chi_{4}^{0}\chi_{4}^{0}}\) | 36 | \(g^{S}_{H+\chi_{4}^{0}\chi_{4}^{0}}\) | 46 | \(g^{S}_{H+\chi_{4}^{0}\chi_{4}^{0}}\) |
| 7  | \(g_{\tau_{\tau}}\) | 17 | \(g^{S}_{H+\bar{t}b}\) | 27 | \(g^{S}_{H+\chi_{4}^{0}\chi_{4}^{0}}\) | 37 | \(g^{S}_{H+\chi_{4}^{0}\chi_{4}^{0}}\) | 47 | \(g^{S}_{H+\chi_{4}^{0}\chi_{4}^{0}}\) |
| 8  | \(g^{S}_{H+\nu_{\tau}}\) | 18 | \(g^{P}_{H+\bar{t}b}\) | 28 | \(g^{S}_{H+\chi_{4}^{0}\chi_{4}^{0}}\) | 38 | \(g^{S}_{H+\chi_{4}^{0}\chi_{4}^{0}}\) | 48 | \(g^{S}_{H+\chi_{4}^{0}\chi_{4}^{0}}\) |
| 9  | \(g^{S}_{H+\nu_{\tau}}\) | 19 | \(g_{\bar{\chi}_{1}^{0}\chi_{1}^{0}}\) | 29 | \(g^{S}_{H+\chi_{4}^{0}\chi_{4}^{0}}\) | 39 | \(g^{S}_{H+\chi_{4}^{0}\chi_{4}^{0}}\) | 49 | \(g^{S}_{H+\chi_{4}^{0}\chi_{4}^{0}}\) |
| 10 | \(g_{ud}\) | 20 | \(g^{S}_{H+\chi_{4}^{0}\chi_{4}^{0}}\) | 30 | \(g^{S}_{H+\chi_{4}^{0}\chi_{4}^{0}}\) | 40 | \(g^{S}_{H+\chi_{4}^{0}\chi_{4}^{0}}\) | 50 | \(g^{S}_{H+\chi_{4}^{0}\chi_{4}^{0}}\) |
Table 7: The decay mode with the index IM of the neutral Higgs bosons $H_{IH}$ used in \textsc{GAMBRN}(IM, IWB, IH). When IWB=1, IM=ISMN+ISUSYN+1 is for the total decay width of the neutral Higgs boson $H_{IH}$ and IM=ISMN and IM=ISMN+ISUSYN for the subtotal decay widths into SM particles (\textsc{GAMSM}) and into SUSY particles (\textsc{GAMSUSY}), respectively. Note that the current version of \textsc{CPSuperH} does not compute (loop–induced) absorptive phases, i.e., it currently returns equal decay widths into CP–conjugate final states.

| IM  | Decay Mode | IM  | Decay Mode | IM  | Decay Mode |
|-----|------------|-----|------------|-----|------------|
| 1   | $H_{IH} \to e\bar{e}$ | 11  | $H_{IH} \to ZZ$ | ..  | ..  |
| 2   | $H_{IH} \to \mu\bar{\mu}$ | 12  | $H_{IH} \to H_1 Z$ | ..  | ..  |
| 3   | $H_{IH} \to \tau\bar{\tau}$ | 13  | $H_{IH} \to H_2 Z$ | ..  | ..  |
| 4   | $H_{IH} \to d\bar{d}$ | 14  | $H_{IH} \to H_1 H_1$ | ..  | ..  |
| 5   | $H_{IH} \to s\bar{s}$ | 15  | $H_{IH} \to H_1 H_2$ | ..  | ..  |
| 6   | $H_{IH} \to b\bar{b}$ | 16  | $H_{IH} \to H_2 H_2$ | ..  | ..  |
| 7   | $H_{IH} \to u\bar{u}$ | 17  | $H_{IH} \to \gamma\gamma$ | ..  | ..  |
| 8   | $H_{IH} \to c\bar{c}$ | 18  | $H_{IH} \to g g$ | ..  | ..  |
| 9   | $H_{IH} \to t\bar{t}$ | ..  | ..  | ..  | ..  |
| 10  | $H_{IH} \to WW$ | ..  | ..  | ISMN | GAMS |

| IM  | Decay Mode  | IM  | Decay Mode  | IM  | Decay Mode  |
|-----|-------------|-----|-------------|-----|-------------|
| ISMN+1 | $H_{IH} \to \chi_1^0 \chi_1^0$ | ISMN+11 | $H_{IH} \to \chi_2^+ \chi_2^-$ | ISMN+21 | $H_{IH} \to b_2^+ b_1^-$ |
| ISMN+2 | $H_{IH} \to \chi_1^+ \chi_1^-$ | ISMN+12 | $H_{IH} \to \chi_1^+ \chi_2^-$ | ISMN+22 | $H_{IH} \to b_2^+ b_2^-$ |
| ISMN+3 | $H_{IH} \to \chi_2^0 \chi_2^0$ | ISMN+13 | $H_{IH} \to \chi_2^+ \chi_1^-$ | ISMN+23 | $H_{IH} \to \tilde{t}_1^+ \tilde{t}_1^-$ |
| ISMN+4 | $H_{IH} \to \chi_1^+ \chi_1^-$ | ISMN+14 | $H_{IH} \to \chi_2^+ \chi_2^-$ | ISMN+24 | $H_{IH} \to \tilde{t}_1^+ \tilde{t}_2^-$ |
| ISMN+5 | $H_{IH} \to \chi_2^0 \chi_2^0$ | ISMN+15 | $H_{IH} \to \tilde{t}_1^+ \tilde{t}_1^-$ | ISMN+25 | $H_{IH} \to \tilde{t}_2^+ \tilde{t}_1^-$ |
| ISMN+6 | $H_{IH} \to \chi_2^0 \chi_3^0$ | ISMN+16 | $H_{IH} \to \tilde{t}_1^+ \tilde{t}_2^-$ | ISMN+26 | $H_{IH} \to \tilde{t}_2^+ \tilde{t}_2^-$ |
| ISMN+7 | $H_{IH} \to \chi_2^0 \chi_4^0$ | ISMN+17 | $H_{IH} \to \tilde{t}_2^+ \tilde{t}_1^-$ | ISMN+27 | $H_{IH} \to \tilde{\nu}_1^+ \tilde{\nu}_1^-$ |
| ISMN+8 | $H_{IH} \to \chi_3^0 \chi_3^0$ | ISMN+18 | $H_{IH} \to \tilde{t}_2^+ \tilde{t}_2^-$ | ..  | ..  |
| ISMN+9 | $H_{IH} \to \chi_3^0 \chi_4^0$ | ISMN+19 | $H_{IH} \to b_1^+ b_1^-$ | ISMN+ISUSYN | GAMS |
| ISMN+10 | $H_{IH} \to \chi_4^0 \chi_4^0$ | ISMN+20 | $H_{IH} \to b_1^+ b_2^-$ | ISMN+ISUSYN | GAMS+GAMS |

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Table 8: The decay mode with the index IM of the charged Higgs boson used in GAMBRC(IM,IWB). When IWB=1, IM=ISMC+ISUSYC+1 is for the total decay width of the charged Higgs boson and IM=ISMC and IM=ISMC+ISUSYC for the subtotal decay widths into SM particles (GAMSM) and into SUSY particles (GAMSUSY), respectively.

| IM | Decay Mode | IM | Decay Mode | IM | Decay Mode |
|----|------------|----|------------|----|------------|
| 1  | $H^+ \rightarrow \bar{e}\nu$ | .. | ...... | ISMC+9 | $H^+ \rightarrow t_1b_1^*$ |
| 2  | $H^+ \rightarrow \bar{\mu}\nu$ | ISMC | GAMSM | ISMC+10 | $H^+ \rightarrow \bar{t}_1\bar{b}_2^*$ |
| 3  | $H^+ \rightarrow \bar{\tau}\nu$ | ISMC+1 | $H^+ \rightarrow \tilde{\chi}_0^0\tilde{\chi}_1^+$ | ISMC+11 | $H^+ \rightarrow \bar{t}_2b_1^*$ |
| 4  | $H^+ \rightarrow u\bar{d}$ | ISMC+2 | $H^+ \rightarrow \tilde{\chi}_0^0\tilde{\chi}_1^+$ | ISMC+12 | $H^+ \rightarrow \bar{t}_2\bar{b}_2^*$ |
| 5  | $H^+ \rightarrow c\bar{s}$ | ISMC+3 | $H^+ \rightarrow \tilde{\chi}_0^0\tilde{\chi}_1^+$ | ISMC+13 | $H^+ \rightarrow \bar{\nu}_r\tilde{\tau}_1^*$ |
| 6  | $H^+ \rightarrow t\bar{b}$ | ISMC+4 | $H^+ \rightarrow \tilde{\chi}_0^0\tilde{\chi}_1^+$ | ISMC+14 | $H^+ \rightarrow \bar{\nu}_r\tilde{\tau}_2^*$ |
| 7  | $H^+ \rightarrow H_1W^+$ | ISMC+5 | $H^+ \rightarrow \tilde{\chi}_0^0\tilde{\chi}_2^+$ | .. | ...... |
| 8  | $H^+ \rightarrow H_2W^+$ | ISMC+6 | $H^+ \rightarrow \tilde{\chi}_0^0\tilde{\chi}_2^+$ | .. | ...... |
| .. | ...... | ISMC+7 | $H^+ \rightarrow \tilde{\chi}_0^0\tilde{\chi}_2^+$ | ISMC+ISUSYC | GAMSUSY |
| .. | ...... | ISMC+8 | $H^+ \rightarrow \tilde{\chi}_0^0\tilde{\chi}_2^+$ | ISMC+ISUSYC+1 | GAMSM+GAMSUSY |
Figure 1: The branching ratios and total decay widths of the MSSM Higgs bosons, taking into account only the decays into SM particles. All the CP phases are set to zero and $\tan \beta = 1.5$ is taken. For the comparison with hDECAY, the threshold corrections are not included and we assume the ‘maximal mixing’ scenario: $|A_{t,b,\tau}| = \sqrt{6} M_{\text{SUSY}}$ with the common SUSY scale $M_{\text{SUSY}} = M_{\tilde{Q}_3} = M_{\tilde{U}_3} = M_{\tilde{D}_3} = M_{\tilde{L}_3} = M_{\tilde{E}_3} = 1 \ TeV$ and $|\mu| = 100 \ GeV$. The results are consistent with those from the code hDECAY.
Figure 2: The branching ratios and total decay widths of the MSSM Higgs bosons, taking into account only the decays into SM particles. All the CP phases are set to zero and $\tan \beta = 4$ is taken. We assume the CPX scenario: $|M_3| = 1 \text{ TeV}$, $|\mu| = 4M_{\text{SUSY}}$ and $|A_{t,b,\tau}| = 2M_{\text{SUSY}}$ with the common SUSY scale $M_{\text{SUSY}} = M_{\tilde{Q}_3} = M_{\tilde{U}_3} = M_{\tilde{D}_3} = M_{\tilde{L}_3} = M_{\tilde{E}_3} = 0.5 \text{ TeV}$. 
Figure 3: The same as in Fig. 2 but with non-trivial CP phases: $\Phi_{A_t} = \Phi_{A_b} = \Phi_{A_e} = \Phi_3 = 90^\circ$. 
Figure 4: The dependence of the branching ratios of Higgs bosons into superparticles on the phase $\phi_\mu$, for $\tan \beta = 5$, $M_{H^\pm}$ = 0.3 TeV, $M_{\tilde{Q}_3} = M_{\tilde{U}_3} = M_{\tilde{D}_3} = M_{\tilde{E}_3} = 0.5$ TeV, $|\mu| = 250$ GeV, $|M_1| = 50$ GeV, $|M_2| = 150$ GeV, $|M_3| = 0.5$ TeV, $|A_t| = |A_b| = |A_\tau| = 1.2$ TeV, $\Phi_2 = \Phi_3 = 0$, and $\Phi_{A_t} = \Phi_{A_b} = \Phi_{A_\tau} = -\Phi_\mu$. The left frame shows results for $H_1$ for several choices of $\Phi_1$; in this case the only contributing final state is $\tilde{\chi}_1^0 \tilde{\chi}_1^0$. The right frame shows results for the heavier Higgs bosons, where the solid (dotted) lines are for $\Phi_1 = 0$ (180°); in this case heavier neutralinos and charginos contribute, but no sfermions.