The Unification of Four Fundamental Interactions

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In this paper, we first incorporate the weak interaction into the theory of General Nonlocality [J. Math. Phys. 49, 033513 (2008)] by finding an appropriate metric for it. Accordingly, we suggest the theoretical frame of General Nonlocality as the candidate theory of grand unification. In this unifying scenario, the essential role of photon field is stressed.

I. INTRODUCTION

It has been decades since scientists commenced their efforts to unify the four fundamental interactions. The efforts focused mainly on how to quantize the gravity. In that context some new theories such as String theory, Quantum Loop Gravity and Non-Commutative Geometry have been developed. In contrast, as we know, no attempt has been practiced in the opposite direction: to unify the quantum theory under the frame of General Relativity. Now such an opposite scheme is feasible. Since we find our previous theory of General Nonlocality [1] can also include the weak interaction as a part.

The remainder of the paper is arranged as follows. in section 2, we first present the detailed analysis on where the fermion mass should come from –the problem unresolved in our previous manuscript[1], and subsequently the weak interaction is incorporated into the theory of General Nonlocality in section 3. Finally, we suggest the theoretical frame of General Nonlocality as universal law of fundamental forces.

II. THE ORIGIN OF MASS OF MATERIAL PARTICLES

In the restored Dirac equation (8.12) in Ref. [1], the required mass term in Eq. (8.5) is missing. And we have tried to remedy this by two methods there, but they are not satisfactory. One method is to add a mass term to the right hand side of Eq. (8.9) directly, then Eq. (8.2) would have a nonzero term on its right hand side too. That obviously contradicts the hypothesis that motion equation is just the geodetic line. The other method is to accept the term $i \int A_\nu \partial_\lambda \partial^\nu \psi d\xi^\lambda$ as the mass term. However, on one hand this term is path-dependent (and thus nonlocal) if the integration is not over a closed loop; on the other hand, even if the closed loop is performed, one notes that the coupling of $A_\nu$ and $\psi$ would give the mass value no more than $m/\sqrt{137}$ ($m$ is the electron mass). So this method is also infeasible. The origin of fermion mass is still a problem.

The mass problem arises after the projection from geodetic equation (8.2) to its space-time representation (8.12) via the replacement $d \rightarrow \gamma_\mu \partial^\mu$. To respect the hypothesis that motion equation is the geodetic line we may not make any alteration to the starting point Eq. (8.2). Also, the phrase "plane wave" appears in the introduction of section VIII–A should not be confused with the conventional term in quantum mechanics. In former case the "plane wave" means the local plane wave $d\psi$, after the electron wave is observed by another fermion locally in a particular complex-frame (such frame is assumed to always exist). So putting a plane wave $e^{ik \cdot x - i\omega t}$ (in the common sense of quantum mechanics) into Eq. (8.2) to check the mass term is inappropriate.

Obviously, the replacement $d \rightarrow \gamma_\mu \partial^\mu$ is invariant under Lorentz transformation. However, if the same Lorentz group is viewed as the structure group obeyed by local plane wave $d\psi$, then the parallel displacement as well as the motion equation (8.2) may not exist any longer, since the existence of the particular frame can not be guaranteed under the mere transformations of complex representation of Lorentz group $SL(2, \mathbb{C})$–$D(1/2,1/2)$. Therefore, the transformation of Eq. (8.2) has nothing to do with the mass term either.

Summarizing the above analyses, the mass problem must lie in the replacement $d \rightarrow \gamma_\mu \partial^\mu$ and the form of plane wave we substitute in Eq. (8.12). Therefore the mass origin has to do with real space instead of complex space. In complex space, the mass differences become trivial for material particles. So, the origin of the fermion mass may lie in the more delicate replacement than $d \rightarrow \gamma_\mu \partial^\mu$. If we refine the replacement by

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\[
\begin{align*}
d^+ & \rightarrow \gamma_\mu \partial^\mu - im \\
d & \rightarrow \gamma_\mu \partial^\mu + im \\
d d & \rightarrow d^+ d
\end{align*}
\]
then we obtain the satisfactory dominating terms \((\Box + m^2)\psi\), but in the second term of Eq. (8.9) several other redundant terms would appear. Even if these redundant terms are not the troublesome, we still face the fact of applying consistently the same replacement to the derivation of the boson field equations, which undoubtedly will ruin the previous formalism of the obtained field equation as well as the perfect understanding of the boson mass origin.

So, the refinement of the replacement is also limited.

Now let’s return to physics. It can be noticed that fermion masses always accompany the appearance of charges, with the exceptions such as the neutrino and neutron, but the neutrino has almost no mass and neutron has a relatively larger magnetic moment. So mostly, if a fermion of spin-1/2 has nonzero mass, it must be nontrivially relevant to the electromagnetism. In formalism concerning special relativity, this relevance is best expressed by the relationship between \(\partial = \gamma_\mu \partial^\mu\) and \(m\). So, we pose the hypothesis that the appearance of the term \(\gamma_\mu \partial^\mu\) is always accompanied by the mass term, both before the wave function \(\psi\). The further understanding of this judgement roots in the fact that in micro-world, the fermion mass is detected almost completely via electromagnetic interaction, in contrast to our knowledge that in our everyday life mostly we evaluate masses with the aids of gravitation. Based on this understanding of the operator \(\gamma_\mu \partial^\mu\), we add a mass term \(-m^2 \psi\) at will to the right hand of the equation (8.9) once we begin to project it to space-time, as a part of projection.

By ruling out several possibilities of adding mass term, finally we are subject to a physical manner. So far, all the terms in quadratic form of Dirac equation [1] can be restored from our geodetic equation.

The understanding of origination of boson mass in the paper [1] is consistent and perfect.

### III. THE METRIC FOR THE WEAK INTERACTION

Naturally, the method of nonlocality [1] is expected to describe the weak interaction too. Here we use the known rules—the definition of two sides of the boundary of physical region—to carry out what the metric matrix should be for the weak interaction. In the attempt we use the criteria (11.6)

\[
|A_{\beta\beta}| = 1 - A_0^2 + \vec{A}_0^2 = \begin{cases} 
0 & \text{bound states} \\
1 & \text{asymptotically–free states}
\end{cases}
\]

and approximation form (11.14a)

\[
(A_{\alpha\beta}^{\text{nk}}) = \Gamma^0 \otimes I_{2\times2} + A_\mu \Gamma^1 \gamma_\mu \otimes \tilde{A}_a \tau^a, \ \tau^a \text{are the Pauli matrices}
\]

in Ref. [1] as the starting point (In this paper these two equations denoted as Eq. (1) and Eq. (2) respectively.). The \(\Gamma^0\) in the above equation is the initial metric matrix we are searching for. To make the metric matrix satisfy the boundary condition Eq. (1), we have to search for the appropriate form of \(\Gamma^0\) in order to include matrix factor \(\gamma_\mu \gamma^5\) in the \(\Gamma^1\) of Eq. (2). In what follows we list the possibilities and give the discussions.

1. First we examine the possibility that in the absence of interaction, if the (initial) metric matrix form is \(\gamma_0 \gamma_5\), then after a period of interaction the matrix evolves into \(\gamma_0 \gamma_5 + \gamma_0 \gamma_\mu \gamma_5 A^\mu\), i.e. the interaction vertex \(\bar{\psi} \gamma_5 \psi \rightarrow \bar{\psi} \gamma_\mu \gamma_5 \psi A^\mu\) (apart from a coupling constant). The explicit form of the metric matrix is

\[
(g_0 + g_0 \gamma_\mu \gamma_5 A^\mu)g_5 = \left( \begin{array}{cc} -\vec{\sigma} \cdot \vec{A} & 1 + A_0 \\ -1 - A_0 & -\vec{\sigma} \cdot \vec{A} \end{array} \right),
\]

which leads to the determinant

\[
\det(g_0 \gamma_5 + g_0 \gamma_\mu \gamma_5 A^\mu) = \vec{A}^2 + 1 - A_0^2,
\]

which is just the form of electrodynamics, meeting the requirements of both strong interaction side (bound state) and weak interaction side (asymptotically–free state). The choice of \(g_0 \gamma_5\) is a promising candidate for \(\Gamma^0\). Whereas
the actual vertex of weak interaction has the form like $\bar{\psi}\gamma_\mu(1 \pm \gamma_5)\psi A^\mu$, we should combine this vertex with initial metric $\gamma_0 \gamma_5$.

2. Examine the possibility $\bar{\psi}\gamma_5\psi \rightarrow \bar{\psi}\gamma_\mu(1 \pm \gamma_5)\psi A^\mu$: the metric matrix is

$$\gamma_0\gamma_5 + \gamma_0\gamma_\mu A^\mu(1 \pm \gamma_5) = \begin{pmatrix} X & 1 \pm X \\ -1 \pm X & X \end{pmatrix},$$  \hspace{1cm} (5)

where $X = A_0 \pm \vec{\sigma} \cdot \vec{A}$. The corresponding determinant yields

$$\det(\begin{pmatrix} X & 1 \pm X \\ -1 \pm X & X \end{pmatrix}) = 1,$$  \hspace{1cm} (6)

which meets the requirement of asymptotically-free side only. As well known, the weak interaction gives nonsupport of bound states. So the Eq. (5) and the initial metric form $\gamma_0 \gamma_5$ are just the perfect ones.

The Eq. (6) automatically holds regardless of the details of operator $X$, so we cannot infer any further conclusions from it, even if substituting the concrete form of $\vec{A}$

$$\vec{A} = A_\alpha \tau^\alpha.$$  \hspace{1cm} (7)

The other determinants such as $\det(\gamma_0 + \gamma_0\gamma_\mu(1 \pm \gamma_5)A^\mu)$ and $\det(\gamma_0(1 \pm \gamma_5) + \gamma_0\gamma_\mu(1 \pm \gamma_5)A^\mu)$ are also examined, but they present no required boundary properties.

Additionally, extension of the group from $SU(2)$ to $U(2)$ is still necessary while considering the curving effect of weak interaction.

IV. UNIVERSAL LAW FOR FUNDAMENTAL FORCES

So far, we have included all of the three microscopic interactions in the theoretical frame of General Nonlocality. Additionally, in view of the well-known puzzles of conservation law of energy-momentum tensor in General Relativity– which is understandable if the energy-momentum tensor is also nonlocal, we can put forward the hypothesis that the dynamics for all of the fundamental forces should be described with the common motion equation (geodetic line) and field equation ($R = 0$) in geometrical manner, which automatically induces the Nonlocality.

The history of the development of description of dynamics, after Newton equation, have underwent several stages, as follows

- Hamiltonian $\rightarrow$ Lagrangian $\rightarrow$ Action $\rightarrow$ Geometry,

each of the succeeding one is more general than its preceding one. The Lagrangian is able to include the covariant form of special relativity, and the action form can incorporate the gauge fixing condition naturally and make the quantization process more fluent. As for geometry, we hope it can circumvent the renormalization processes, as well as provide other nonperturbative methods.

The curvings (Geometries) corresponding to the four fundamental interactions are all relevant to effects of photon. It is well known that the curvature of space-time (in General Relativity) has been confirmed by the curving effect of light. But for the microscopic interactions, one may not be aware of how does the photon field curve in complex space–unitary space. We assert here that the unifying picture of electrodynamics, weak interaction (the weak doublets), and quark dynamics (color triplet) may be achieved by regarding the curving of photon’s field, just as implied in Ref. [1]:

The photon exists as a pure energy form, which can decay into lepton-antilepton pair $\rightarrow$ proton-antiproton pair $\rightarrow$ quark-antiquark pair. The processes successively take place with the increase of the photon energy. The remarkable feature of the successive processes is that the lepton-antilepton is in the $U(1, \mathcal{C})$ region of the gauge fields, and proton-antiproton is in $U(2, \mathcal{C})$ hadron region [forms the weak doublet together with neutron], and quark antiquark in $U(3, \mathcal{C})$ color region. This feature leads to the ansätz that the photon field can be curved into the $U(n, \mathcal{C})$-space with its energy increasing. So, we can add another curving to the curvings of solely in $GL(n, \mathcal{C})$-space and/or only in $U(n, \mathcal{C})$-space. Now different sections of complex spaces can be linked by the photon-field curving: $U(1, \mathcal{C}) \rightarrow U(2, \mathcal{C}) \rightarrow U(3, \mathcal{C})$. In formalism, this kind of curving can be interpreted by generalizing the meaning of Eq. (7.3) in Ref. [1], with the indices $a, c$ being not confined to one unitary group, but able to change continuously among $U(1, \mathcal{C}), U(2, \mathcal{C})$ and $U(3, \mathcal{C})$. 
Possibly, the dynamics of General Nonlocality also governs other realms of nature, such as Thermodynamics or Hydrodynamics, if only we find the appropriate space, as well as the metric forms and the structure groups for the space. Then using geodetic line and the field equation $R = 0$ can mean all.

[1] H. J. Wang, General Nonlocality in Quantum Fields, J. Math. Phys. 49, 033513 (2008), also at arXiv: quant-ph/0512191.