Electronic Liquid Crystal Phases of a Doped Mott Insulator

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The character of the ground state of an antiferromagnetic insulator is fundamentally altered upon addition of even a small amount of charge. The added charges agglomerate along domain walls at which the spin correlations, which may or may not remain long-ranged, suffer a $\pi$ phase shift. In two dimensions, these domain walls are “stripes” which are either insulating, conducting, i.e. metallic rivers with their own low energy degrees of freedom. However, quasi one-dimensional metals typically undergo a transition to an insulating ordered charge density wave (CDW) state at low temperatures. Here it is shown that such a transition is eliminated if the zero-point energy of transverse stripe fluctuations is sufficiently large in comparison to the CDW coupling between stripes. As a consequence, there exist novel, liquid-crystalline low-temperature phases – an electron smectic, with crystalline order in one direction, but liquid-like correlations in the other, and an electron nematic with orientational order but no long-range positional order. These phases, which constitute new states of matter, can be either high temperature superconductors or two-dimensional anisotropic “metallic” non-Fermi liquids. Evidence for the new phases may already have been obtained by neutron scattering experiments in the cuprate superconductor, La$_{1.6-x}$Nd$_x$Sr$_{2}$CuO$_4$.

A single metallic stripe is a prototypical example of the one-dimensional electron gas in an active environment about which much is known. In the absence of coupling between the stripes, this system is generally “quantum critical”, i.e. it exhibits power law correlations and has an infinite correlation length at temperature $T = 0$. Frequently it has a gap in its spin excitation spectrum, $\Delta_s$, so the only low-energy degrees of freedom are the CDW and the dual superconducting fluctuations whose susceptibilities diverge as $T \to 0$, as

$$\chi_{CDW} \sim \Delta_s T^{-(2-K_c)}, \quad \chi_{SC} \sim \Delta_s T^{-(2-1/K_c)}$$

where $K_c$ is a non-universal critical exponent which depends on the magnitude and sign of the interactions and satisfies $0 < K_c < 1$ for repulsive interactions.

Direct evidence of stripe correlations in the cuprate superconductors, themselves doped antiferromagnets, has been obtained over the past few years from neutron scattering experiments. It is thus reasonable to ask whether these stripes are relevant for the mechanism of high temperature superconductivity. Typically, in conventional superconductors, the superconducting gap and transition temperature, $T_c$, are small because they involve pairing of charged particles, i.e. electrons. However, in one dimension, because of the remarkable fact that the low energy excitations are independent spin and charge collective modes, superconducting pairing involves only the (neutral) spin degrees of freedom, and hence the gap can be large. This would be a good starting point for a mechanism of high temperature superconductivity except for the fact that higher dimensional interactions typically lead to CDW order rather than superconductivity because the CDW susceptibility is the more divergent (see Eq. (1)) and moreover the Coulomb interaction between charge density fluctuations on nearby stripes (which promotes CDW order) is typically larger than the inter-stripe Josephson coupling (which promotes superconducting order). On the other hand, as we show below, transverse stripe fluctuations have an important effect on the competition between superconducting and CDW order. Of course, such fluctuations are relatively unimportant in conventional quasi one-dimensional solids because of the large mass of the constituent molecules. But the zero-point energy $\hbar \omega$ of the transverse stripe fluctuations becomes a significant energy scale in a stripe phase of a doped antiferromagnet which arises from a purely electronic correlation effect.

Our principal new conclusions are that, to all orders of perturbation theory in powers of the Coulomb interaction $V$ between stripes: (i) The phase locking between the CDW fluctuations on neighbouring stripes is entirely eliminated by the transverse fluctuations, leaving the charge motion liquid-like along the stripe; in other words, there exists a stable zero temperature liquid crystalline (quantum smectic) phase. (ii) The Josephson coupling $J$ between stripes is greatly enhanced by the same transverse fluctuations. For $K_c > 1/2$, or more generally for large enough $J$, the ground state in this phase is always globally superconducting. Because of the broken rotational symmetry of this phase, all that can be said about the symmetry of the superconducting state is that it is singlet; its symmetry is necessarily a mixture of “$s$-wave” and “$d$-wave”. Conversely, for $K_c < 1/2$ and small enough $J$, the ground state is “metallic” in the sense that it has gapless charge excitations unrelated to any broken symmetry. While such quantum critical phases are common in one dimension where they are called “Luttinger liquids,” we believe this is the first theoretically well justified example in two dimensions.

We begin with a simple model of a two-dimensional array of stripes along the $x$ direction. In the ordered state, we can safely ignore dislocations and overlaps. This allows us to introduce a coordinate system
in which points on the stripes are labelled by a stripe number, \( j \), and by a position \( x \) along the stripe direction. Then the stripe configuration is described by the transverse displacement in the \( y \)-direction, \( Y_j(x) \), of the \( j \)th stripe at position \( x \). (See Fig.1.) Because of the spin gap, the only other low energy degrees of freedom involve fluctuations of the charge density, \( \rho_j(x) \), on each stripe,

\[
\rho_j(x) = \tilde{\rho} + \tilde{\rho}_0 \cos[\sqrt{2\pi} \phi_j + 2k_F L_j(x)],
\]

\[
L_j(x) = \int_0^x dx' \sqrt{1 + (\partial_x Y_j)^2} + L_j(0)
\]

where \( \phi_j(x) \) defines the phase of the CDW with wave vector equal to twice the Fermi wave vector \( k_F \) and \( L_j(x) \) is the arc-length along stripe \( j \). The quantum dynamics of this system is equivalent to a theory of the longitudinal \( (\phi_j) \) and transverse \( (Y_j) \) vibrations of coupled elastic strings. Technically, this defines the fixed point Hamiltonian for the smectic phase. (A substrate potential would merely introduce periodic structures along the stripes, which are forced out of phase by the transverse fluctuations of the charge density, \( \rho_j(x) \), on each stripe, and hence the transverse modes would be gapped.

![Fig. 1. Schematic representation of a smectic stripe phase.](image)

The coloured circles represent periodic structures along the stripes, which are forced out of phase by the transverse fluctuations.

The coupling between the CDW’s on neighbouring stripes is of the form

\[
H_c = \sum_j \int dx V(\Delta_j Y) \cos[\sqrt{2\pi}(\Delta_j \phi) - 2k_F(\Delta_j L)]
\]

plus higher harmonics. Here \( L_j \) is the arc-length defined in Eq. (3), \( \Delta_j F \equiv F_{j+1} - F_j \), and the function \( V[\Delta_j Y] \) reflects the fact that CDW’s on adjacent stripes are more strongly coupled where the stripes are close together than where they are far apart. When this coupling is strong, it will drive the system into a fully-crystalline state. Finally, there is a term in the Hamiltonian representing the Josephson tunnelling of (superconducting) pairs of electrons between stripes. The tunnelling matrix element

\[
\mathcal{J}(Y) \approx \mathcal{J}_0 \exp[\alpha Y]
\]

depends roughly exponentially on the local spacing of the stripes. The fact that superconductivity is a \( k = 0 \) order implies that the Josephson coupling does not depend on the arc length \( L_j \), and hence it is not affected by the geometry of the stripes.

With this background, it is possible to state our central point that, to all orders in perturbation theory in powers of \( V \), all terms that are not invariant under the transformation \( \phi_j(x, \tau) \to \phi_j(x, \tau) + \delta_j \) for arbitrary \( \delta_j \) are non-vanishing only near the “surface,” so in the thermodynamic limit there is no locking of the phase of the CDW fluctuations on neighbouring stripes. (Technically, this proves that the fixed point Hamiltonian is perturbatively stable.) The physical origin of this effect is easily understood. The difference in arc lengths, \( \Delta_j L = L_{j+1}(x) - L_j(x) \), is a sum of contributions of random sign, and more or less independently distributed along the distance \( |x| \). For this reason, \( \Delta_j L \) (and the dephasing) grow with increasing \( |x| \) as in a random walk, i.e. \( \Delta_j L^2 \sim D |x| \), where \( D \) is a quantum diffusion constant.

This result may be obtained formally by integrating out the stripe fluctuations \( Y \) perturbatively in powers of \( V \) and, subsequently, \( \mathcal{J} \). To first order in \( V \), the effective interaction between the CDW’s on neighbouring stripes, \( V^{(1)}(\phi_1 - \phi_2) \), is given by the expression

\[
V^{(1)} = \langle V(\Delta Y) \cos[\sqrt{2\pi}(\Delta \phi) - 2k_F(\Delta L)] \rangle,
\]

where \( < > \) implies averaging over transverse stripe fluctuations. To lowest order in a cumulant expansion

\[
V^{(1)} = \tilde{V} \cos[\sqrt{2\pi}(\Delta \phi)],
\]

\[
\tilde{V} = \langle V(\Delta Y) \rangle \exp\{-(2k_F^2)(|\Delta L|^2)\}
\]

\[
\approx \langle V(\Delta Y) \rangle \exp\{- (2k_F^2 D)/|x|\}.
\]

This expression, which can readily be extended to higher order in perturbation theory and higher order in the cumulant expansion, captures the essential general point of physics— that the coupling between CDW’s vanishes very rapidly except in a region of width \( \sim (2k_F^2 D)^{-1} \) at the ends of the stripes, and hence can be ignored in the thermodynamic limit.

It is interesting to note that the quantum problem may be reformulated as a classical theory in space-time to bring out the close analogy with a well established phase of conventional liquid crystals, the three-dimensional hexatic smectic B phase. In the space-time representation, the world sheets of the stripes can be regarded as classical fluctuating membranes and the CDW fields are analogous to a two-dimensional hexatic phase living on the membrane. Despite the fact that the power-law order in the plane of each “membrane” is modulated only in the space direction, whereas the classical hexatic has a triangular lattice form, this analogy assures us that we have not omitted any important interactions from our analysis.
The effect of Josephson coupling between stripes may be analysed in the same way. To first order in $\mathcal{J}$, the effective action is proportional to

$$<\mathcal{J}＞\approx \mathcal{J}_0 \exp \left\{ (\alpha^2/2) <[\Delta_j Y]^2 > \right\}. \quad (8)$$

Notice that the superconducting coupling is strongly enhanced by the transverse stripe fluctuations. (Of course, there is a similar enhancement of the CDW coupling, $V$, but it is overwhelmed by the dephasing effect.) Physically, this enhancement reflects the fact that the mean value of $\mathcal{J}$ is dominated by regions where neighbouring stripes come close together so that $<\mathcal{J}>$ is very much larger than the median. From Eq. (1) it can be seen that, when $K_c > 1/2$, the pair susceptibility on an individual stripe diverges as $T \rightarrow 0$ and hence for non-zero $\mathcal{J}$, the smectic phase is always globally superconducting below a finite (Kosterlitz-Thouless) ordering temperature, $T_c \sim (\Delta \mathcal{J})^{K_c/(2K_c-1)}$, while for $K_c < 1/2$ and $<\mathcal{J}>$ sufficiently small, the system remains a (quantum critical) non-Fermi liquid all the way to $T = 0$.

To complete the physical picture of the quantum smectic, we construct a global phase diagram, shown schematically in Fig. 2, by considering the possible zero and finite temperature phase transitions from the smectic state to states with other symmetries. This can be done, to a large extent, on the basis of general considerations of symmetry and by analogy with the phase diagram of conventional liquid crystals, and the argument relies on nothing more than the existence (and electronic character) of the quantum smectic phase.

**FIG. 2.** Schematic phase diagram for $K_c > 1/2$. Here, $T$ is the temperature, and $\hbar \omega$ is a measure of the magnitude of the transverse zero-point fluctuations of the stripes. Thin lines represent continuous transitions and the thick line is a first order transition. The dashed line is the superconducting $T_c$. The symbols “B”, “C”, and “T” label, respectively, bicritical, quantum-critical, and tetracritical points. Depending on microscopic details, the positions of $C_1$ and $C_2$ could be interchanged.

First consider the $T = 0$ axis of the figure, in which the phases are studied as a function of $\hbar \omega$:

i) To the left, as the system becomes progressively more “classical”, i.e. for $\hbar \omega/V$ small enough, it is clear that there is a phase transition to a crystalline state, in which the CDW order on neighbouring stripes phase locks, the transverse stripe fluctuations become the phonons of a fully-ordered crystal, superconducting order is destroyed, and the system becomes globally insulating. This transition is typically first order.

ii) To the right, as the system becomes more quantum and, in particular, when the rms magnitude of the transverse fluctuations of the stripes becomes comparable to their spacing, we expect a $T = 0$ transition to a quantum nematic phase in which there is no broken translational symmetry, but lattice rotational symmetry is spontaneously broken, i.e. there are oriented but positionally disordered stripes. We generally expect this transition to be continuous, as shown; this implies that, in the case $K_c > 1/2$, the superconducting order must continue across the smectic to nematic phase boundary, and in the case $K_c < 1/2$, the Luttinger liquid behavior must similarly persist across the phase boundary.

iii) At still larger $\hbar \omega/V$, there must be a transition to an isotropic phase. Landau theory suggests that the nematic to isotropic transition should be continuous in two spatial dimensions, although it is first order in three.

iv) For the case $K_c > 1/2$, there are two possible scenarios for the termination of the high temperature superconducting order with increasing $\hbar \omega$: If the nematic region of the phase diagram is narrow, so that significant local stripe correlations survive into the isotropic phase, then one can imagine that the superconducting state survives until some larger value of $\hbar \omega$, as shown in the figure; in this case, the superconducting state will have a pure symmetry (“s” or “d”) where it extends into the isotropic phase. Otherwise, the high temperature superconducting phase could terminate at a critical point within the nematic phase. In either case, beyond this point, the ground state is an anisotropic Fermi liquid (similar to a conventional metal) or, if there remain sufficient residual interactions, a low temperature superconductor. By the same logic, when the smectic phase has a Luttinger liquid rather than a superconducting ground state ($K_c < 1/2$), there must be an additional zero temperature phase transition (in place of $C_1$) in either the nematic or isotropic regions of the phase diagram, beyond which the system becomes a Fermi liquid.

It is straightforward to extend this picture to $T \neq 0$. In isotropic two dimensional systems, at low temperature, the long-range stripe positional order gives way to power-law order, although there is true, long-range orientational (nematic) order. Ultimately, at high enough temperature, there must be a transition to an isotropic (symmetric) phase. While there is the logical possibility of a direct, first order smectic to isotropic phase transition, we consider it more likely, as at zero temperature, that the low temperature order is destroyed in a sequence of two transitions: first, a dislocation unbinding transition to a nematic phase with short-range positional and...
power-law orientational order, and then a second transition to the isotropic state.

Modulo the choices among possible scenarios described above, the topology of the phase diagram is constrained to be that shown in Fig. 2 for the case $K_x > 1/2$. We have shown the superconducting $T_c$ rising with $\hbar \omega$ through the smectic and nematic phases, reflecting the enhancement of the Josephson coupling, $J$, by transverse stripe fluctuations; we show it dropping at larger $\hbar \omega$ following the isotropic to nematic phase boundary, since we expect that the stripes lose their local integrity far into the isotropic phase. A further subtlety is that both the crystalline and smectic regions are actually a series of commensurate phases at $T = 0$, and a complicated pattern of commensurate and incommensurate phases for $T \neq 0$. While the commensurate smectic has true positional long-range order, the incommensurate smectic will have only power-law order.

We conclude the theoretical discussion with a few remarks. Crystals do not have the full rotational symmetry of free space, so the stripes will tend to align along a symmetry axis of the underlying crystal. Then the smectic will acquire a finite transverse elastic modulus $\kappa_i$, rather than the anomalous “splay elastic” constant $\kappa_s$ of a classical smectic. Similarly, the incommensurate smectic and nematic phases will have long-range order rather than power law behaviour. A two-fold symmetric crystal field is symmetry breaking, and changes the nematic to isotropic phase transition into a crossover; if the field is small, the crossover remains sharp. A four-fold or six-fold rotational symmetry will preserve this transition, but change it to the Ising or three-state-Potts universality class, respectively. The analysis of this paper can be applied to systems with low energy spin degrees of freedom by considering the most general model of the one-dimensional electron gas with or without a charge gap. Also, it is intuitively clear that our analysis is relevant for sufficiently anisotropic systems, since strong three dimensional effects tend to make the stripes more rigid and in consequence will help CDW formation.

What does this have to do with the cuprate superconductors? Tranquada et al. have observed static peaks in the spin and charge structure factors of La$_{1.6-x}$Nd$_{0.4}$Sr$_x$CuO$_4$, corresponding to incommensurate stripe order, with the stripes along the CuO direction. In this material, each CuO$_2$ plane has a two-fold symmetry axis that rotates through 90° from plane to plane to give a tetragonal structure with two inequivalent CuO$_2$ planes. The peaks have a finite width, which is consistent with a nematic stripe phase in an orienting potential. However, the power-law stripe order of a smectic phase provides a possible alternative explanation of the observations. We feel that it is the two-fold lattice potential that drives the material either into or close to the smectic phase, and freezes the dynamics. In our opinion, these experiments constitute strong evidence that La$_{1.6-x}$Nd$_{0.4}$Sr$_x$CuO$_4$ has a low temperature electronic liquid crystal phase. In La$_{2-x}$Sr$_x$CuO$_4$ there are similar incommensurate peaks in the magnetic neutron scattering factor at about the same position in $k$-space, but they are inelastic i.e. there are dynamically-fluctuating analogues of the stripe phases seen in La$_{1.6-x}$Nd$_{0.4}$Sr$_x$CuO$_4$. In this case the structure is orthorhombic, and two-fold lattice potential is, itself, dynamical. Neutron scattering experiments on underdoped YBa$_2$Cu$_3$O$_{7-\delta}$, which is orthorhombic, also have found dynamical incommensurate peaks corresponding to dynamical stripes parallel to the diagonal of the CuO$_2$ unit cell. (Note that while, in general, the zero-point kinetic energy of fermions increases with increasing density, the relationship between $\hbar \omega$ and the concentration of doped holes may be complicated in these materials.)

In underdoped high temperature superconductors, two crossover lines have been identified at which there is a rather rapid change in certain physical properties, but apparently no phase transition. The upper crossover is marked by a drop in the magnetic susceptibility, and the development of a pseudogap in the $c$-axis optical conductivity with a transfer of spectral weight to high frequencies. Both of these features could be signatures of an isotropic-nematic transition, rounded by a two-fold orienting field. It has been suggested by several authors that the unusual normal-state above the upper crossover temperature is controlled by a zero-temperature quantum critical point; we note that the end of the isotropic-nematic phase boundary, $C_2$ in Fig. 2, is just such a point.

In this paper we have focussed on doped antiferromagnetic insulators. However other doped insulators or semiconductors could have liquid crystal phases, especially if the band structure is anisotropic. One interesting possibility is that, at moderately low density, a Wigner crystal may be replaced by a liquid crystal.

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quences for the high temperature superconductors.

Examples of such problems include the one dimensional electron gas interacting with phonons, an array of Kondo or other dynamical impurities, and a spin chain, which may or may not be gapped.

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It is straightforward to extend the present analysis to the case in which there is transverse magnetic order using Landau thory of coupled order parameters, as in Zachar, O., Kivelson, S. A., and Emery, V. J., Phys. Rev. B, in the press.

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