Fractional quantum Hall effect of topological surface states under a strong tilted magnetic field

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Abstract – The fractional quantum Hall effect (FQHE) of topological insulator surface state particles under a tilted strong magnetic field is theoretically studied by using the exact diagonalization method. Haldane’s pseudopotentials for the Coulomb interaction are analytically obtained. The results show that by increasing the in-plane component of the tilted magnetic field, the FQHE state at \(n = 0\) Landau level (LL) becomes more stable, while the stabilities of \(n = \pm 1\) LLs become weaker. Moreover, we find that the excitation gaps of the \(\nu = 1/3, 1/5\) and \(1/7\) FQHE states increase with increasing the tilt angle.

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The fractional quantum Hall effect (FQHE) in conventional two-dimensional electron gas (2DEG) has been studied intensively in the last 30 years [1–7] because of its rich physical properties. Recently, the unconventional sequence of FQHE in graphene, where the valley isospin combined with the usual electron spin yields fourfold degenerate Landau levels (LLs), has been observed [8–12]. Differing from the case of the conventional 2DEG, due to the Dirac nature of the electrons and the additional symmetry, the FQHE is modified and new incompressible ground states should be conjectured in graphene [13–17]. Moreover, the linear energy dispersion also modifies the inter-electron interactions, which implies a specific dependence of the ground-state energy and energy gap on the LL index [13–15]. The graphene-like integer quantum Hall effect has been observed in strained bulk HgTe [18] and the LL spectrum has also been measured in the threedimensional topological insulator (TI) material Bi\(_2\)Se\(_3\) by using scanning tunneling microscopy [19,20]. In addition, features in the Hall resistance at fractional filling factors have been speculated to be related to the FQHE of TIs [21–23]. However, the strength of the FQHE will be different from that in graphene because the LLs of the two surfaces of the TI thin film can mix with each other [23].

Furthermore, previous studies suggest that when the spin degrees of freedom and the parallel magnetic field are taken into account, spin-reversed quasiparticle (quasi-hole) excitations could be observed in certain FQHE gaps of GaAs-based quantum wells [24,25]. The quasiparticle-quasihole energy gap increases with increasing the tilt angle of the magnetic field, and the electron-hole symmetry is broken [26,27] due to the subband-LL coupling in FQHE. The contrasting behavior of the higher filling \((\nu = 5/2\) and \(7/3\)) FQHE states and the relevance of a skyrmion spin texture at \(\nu = 5/2\) associated with small Zeeman energy in wide GaAs/AlGaAs quantum wells under a tilted magnetic field have been studied in recent experiments [28,29]. With increasing the tilt angle, the \(\nu = 5/2\) FQHE state may transform into the compressible Fermi liquid state, since the spin polarization is disturbed by the tilted magnetic field.

Unambiguous characterizations of the FQHE of Dirac fermions on the TI surface in the presence of a tilted magnetic field (see fig. 1), however, is still missing in the literature. Owing to the unique spin chirality of Dirac fermions induced by intrinsic strong spin-orbit coupling in TI materials, the effect of tilted magnetic field on the FQHE on TI surface should remarkably differ from that in conventional 2DEG. Therefore, because of its importance both from basic point of interest and to the analysis of unconventional properties of TI-based FQHE, in the present paper we address the stability of the FQHE on TI under a strong tilted magnetic field by presenting a theoretical evaluation of the effective
the azimuth angle $\phi$. One can see in the following text that Haldane’s pseudopotentials rightly satisfy this symmetry condition. By employing the perturbation method and only keeping the first-order approximation of the wave function, the eigenstates of Hamiltonian (2) may be written as

$$\Psi_{n,m} = \begin{pmatrix} \Psi_{1,n,m} \\ \Psi_{2,n,m} \end{pmatrix} = \begin{pmatrix} \alpha_n \phi_{|n|-1,m} + A_n^1 \phi_{|n|,m} + A_n^2 \phi_{|n|-2,m} \\ \beta_n \phi_{|n|,m} + B_n^1 \phi_{|n|-1,m} + B_n^2 \phi_{|n|+1,m} \end{pmatrix},$$

where $\phi_{n,m}$ is the eigenstate of the 2D Hamiltonian with non-relativistic quadratic dispersion relation in the $n$-th LL with angular momentum $m$. Here,

$$\alpha_n = \begin{cases} 0, & n = 0, \\ \frac{-\text{sgn}(n) \cos \theta_n}{\sqrt{1 - \text{sgn}(n) \sin \theta_n}}, & n \neq 0, \end{cases}$$

$$\beta_n = \begin{cases} 1, & n = 0, \\ \frac{1}{\sqrt{1 - \text{sgn}(n) \sin \theta_n}}, & n \neq 0, \end{cases}$$

where $\theta_{n,m} = \tan^{-1} \left( \frac{\text{sgn}(n) \sqrt{1 - \text{sgn}(n) \sin \theta_n}}{2n+1} \right)$.

The coefficients are expressed as $A_n^1 = \frac{(\epsilon_n + b) \alpha_n}{\epsilon_n^2 - b^2 \cos^2 \theta_n}$, $A_n^2 = \frac{i \epsilon_n \alpha_n}{\epsilon_n^2 - b^2 \cos^2 \theta_n}$, $B_n^1 = \frac{(\epsilon_n - b) \alpha_n}{\epsilon_n^2 - b^2 \cos^2 \theta_n}$, and $B_n^2 = \frac{i \epsilon_n \alpha_n}{\epsilon_n^2 - b^2 \cos^2 \theta_n}$ with $b = g \mu_B B_{\perp}$, $d = g \mu BB_{z} e^{-i \phi}$, $c = \sqrt{2} \text{hef} B_{1}$.

The corresponding eigenvalues are given by $\epsilon_n = -g \mu_B B_{\perp}$, and $\epsilon_{n,m} = \text{sgn}(n) \sqrt{\left(g \mu_B B_{\perp}\right)^2 + \sqrt{2n+1} \mu_B B_{1}}$. In the limit of $B_{\perp} = 0$, the Hamiltonian (2) can be analytically solved [33, 36, 37]. Without the Zeeman term, the analytical solution is the same as that of graphene [15, 38, 39].

In order to investigate the properties of the TI surface FQHE, we first consider the Coulomb interaction $V(r) = e^2/\epsilon$ between two electrons in the $n$-th LL with relative angular momentum $m$. Haldane’s pseudopotential is given by

$$V_{\text{eff}}^{(n,m)} = \frac{1}{2} \sum_{n,m} \left| \frac{2\pi e^2}{\epsilon} \right| |f_{n,m}(q)|^2 L_m (q^2 l^2) e^{-\frac{q^2 l^2}{2}},$$

where $L_m(x)$ is the Laguerre polynomial. The form factor $F_{n}(q)$ in eq. (5) is given by

$$F_{n}(q) = \langle \Psi_{1,n,m}^{\dagger} e^{-i \eta q} | \Psi_{1,n,m} \rangle + \langle \Psi_{2,n,m}^{\dagger} e^{-i \eta q} | \Psi_{2,n,m} \rangle,$$

where $\eta =$ the cyclotron position operator. The explicit expression of $F_{n}(q)$ is derived to be written as

$$F_{n}(q) = \left[(\alpha_n^2 + |B_n^1|^2) L_{n+1} (s^2) + |B_n^2|^2 L_{n+1} (s^2) \right] + (\alpha_n^2 + |B_n^1|^2) L_{n+1} (s^2) + |A_n^2|^2 L_{n+1} (s^2)$$

$$+ \alpha_n^2 A_n^2 L_{n+1} (s^2) + \alpha_n^2 A_n^2 X_{n+1} (s^2) + A_n^2 X_{n+1} (s^2)$$

$$+ \beta_n^2 A_n^2 X_{n+1} (s^2) + \beta_n^2 A_n^2 X_{n+1} (s^2) + B_n^2 B_n^1 X_{n+1} (s^2) + B_n^2 B_n^1 X_{n+1} (s^2)$$

$$+ \alpha_n A_n^1 \beta_n A_n^1 X_{n+1} (s^2)$$

$$+ (\alpha_n A_n^1 + \beta_n A_n^1) X_{n+1} (s^2) + (\alpha_n A_n^1 + \beta_n A_n^1) X_{n+1} (s^2)$$

$$+ \epsilon_n A_n^1 + \beta_n A_n^1 X_{n+1} (s^2) e^{-\frac{q^2 l^2}{2}}.$$
in which the first four terms are expressed by Laguerre polynomials, and the remaining ten terms are linear combinations of \( X_{n\nu}(-s) \), whose definition is \( X_{n\nu}(s) = X_{n\nu}(-s) = (-is)^{\nu-n} \sum_{m=0}^{\min(n,\nu)} \frac{(-is)^{\nu-m}}{\sqrt{m!}} \), with \( s \equiv i(q_x + iq_y) / \sqrt{2} \).

When the in-plane component of the magnetic field vanishes (i.e., \( d = 0 \)), \( A_1, A_2, B_1, \) and \( B_2 \) are reduced to zero, however, the Zeeman effect induced by the perpendicular component of the external magnetic field is preserved in \( \alpha_n \) and \( \beta_n \), which results in eq. (7) shown here, that is different from eq. (7) in ref. [15] and from the result in ref. [38]. Clearly, when the Zeeman term is absent (\( g = 0 \)) in the system considered here, the form factor of eq. (7) reproduces the well-known results of previous studies [15,38,39]. A typical way to predict the stability of the FQHE is to consider only \( m = 1 \) and \( m = 3 \) pseudopotentials, since \( V_{\text{eff}}^{(m,m)} \) falls off quickly as \( m \) increases. The FQHE state may be observed when \( V_{\text{eff}}^{(n,1)} / V_{\text{eff}}^{(n,3)} \) is large enough [7]. For the purpose of studying the stability of the FQHE states under a tilted magnetic field, we also employ this crucial criterion and choose 1.5 as a critical value. In other words, when \( V_{\text{eff}}^{(n,1)} / V_{\text{eff}}^{(n,3)} \) is smaller than 1.5, we conclude the FQHE state cannot be observed.

For simplicity, let us first consider the stability of the FQHE state at \( n = 0 \) LL, whose form factor is \( F_0(q) = \left[ L_0 \left( |s|^2 \right) + \frac{d^2}{c^2} \right] L_1 \left( |s|^2 \right) e^{-2|s|^2} \). Substituting this \( F_0(q) \) into eq. (5), one easily obtains Haldane’s pseudopotential for \( n = 0 \) th LL,

\[
V_{\text{eff}}^{(0,m)} = \frac{1}{2} \sum_{q} \frac{2\pi e^2}{q} L_m \left( 2|s|^2 \right) e^{-2|s|^2} \times \left[ 1 + 2 \left| \frac{d}{c} \right|^2 \left( 1 - 2|s|^2 \right) + \left| \frac{d}{c} \right|^4 \left( 1 - |s|^2 \right)^2 \right].
\]

One can find, differing from the case under a fully perpendicular magnetic field, the form factor in a tilted magnetic field case is dependent on the in-plane component of the external magnetic field. The numerical result of \( V_{\text{eff}}^{(0,1)} / V_{\text{eff}}^{(0,3)} \) as a function of the in-plane component of the external magnetic field \( B \), is shown in fig. 2(a), where different curves correspond to different given perpendicular components of the external magnetic field \( B \). From fig. 2(a) one can see that \( V_{\text{eff}}^{(0,1)} / V_{\text{eff}}^{(0,3)} \) always increases with increasing \( B \). Based on this and on the fact that the minimum value of \( V_{\text{eff}}^{(0,1)} / V_{\text{eff}}^{(0,3)} \) is \( \approx 1.6 \) at \( B = 0 \) (larger than the critical value 1.5), we can immediately conclude that the FQHE state at \( n = 0 \) LL is more stable by increasing the in-plane component of the tilted magnetic field.

Now we turn to study the cases for the \( n = \pm 1 \) LLs, whose pseudopotentials can be written as

\[
V_{\text{eff}}^{(n=\pm 1,m)} = \frac{1}{2} \sum_{q} 2\pi e^2 \left( L_m \left( 2|s|^2 \right) e^{-2|s|^2} \times \right. \\
\left. \times \left[ \left| \alpha_n \right|^2 L_0 \left( |s|^2 \right) \right] + \left| \beta_n \right|^2 \right] \left. + 2 \left| \alpha_n \right|^2 \left( \frac{d}{c} \right)^2 \right] L_1 \left( |s|^2 \right) \right)^2 \\
+ 2 \left| \beta_n \right|^2 \left| \frac{d}{c} \right|^2 \left( L_2 \left( |s|^2 \right) \right)^2 \\
- 8 \left| \alpha_n \beta_n \right|^2 \left| \frac{d}{c} \right|^2 \left( \frac{d^2}{c^2} \right) + \frac{1}{2} \left| \alpha_n \beta_n \right|^2 \left| s \right|^4 \left( \frac{\varepsilon_n - b}{c} \right)^2 \left| \frac{d}{c} \right|^4 \\
- 2 \left| \beta_n \right|^4 \left| \frac{d}{c} \right|^4 \left( 1 - \left| \frac{d}{c} \right|^2 \right) \right].
\]

in which \( \left| \alpha_n \right|^2 \) and \( \left| \beta_n \right|^2 \) are defined by

\[
\left| \alpha_n \right|^2 = \left( 1 + \left| \frac{\alpha_n}{c} \right|^2 \right) \left| \frac{d}{c} \right|^2 \quad \text{and} \quad \left| \beta_n \right|^2 = \left( 1 + \left| \frac{\alpha_n}{c} \right|^2 \right) \left| \frac{d}{c} \right|^2,
\]

respectively. Figures 2(b) and (c) respectively plot \( V_{\text{eff}}^{(n=\pm 1,1)} / V_{\text{eff}}^{(n=\pm 1,3)} \) vs. \( B \), from which one can find that \( V_{\text{eff}}^{(n=\pm 1,1)} / V_{\text{eff}}^{(n=\pm 1,3)} \) always decrease with increasing \( B \). These results imply that Haldane’s pseudopotentials for the \( n = \pm 1 \) LLs the FQHE states become instable by
increasing the in-plane component of the tilted magnetic field, which is quite different from the case for $n = 0$ LL.

We also investigate the energy spectra of the many-body states at fractional filling $\nu = 1/3$, 1/5 and 1/7 of the LLs $n = 0$ and $\pm 1$ under a tilted magnetic field by numerically diagonalizing the many-body Hamiltonian in the spherical geometry. Figure 3 shows the energy spectra for $N = 7$ electrons at $\nu = 1/3$ FQHE states for $n = 0$ and 1 LLs with different in-plane components of the external magnetic field, $B_\parallel = 0$ (black dots) and $B_\parallel = B_\perp$ (red stars). One can clearly see that the excitation gap width at $B_\parallel = B_\perp$ is larger than that at $B_\parallel = 0$. To more clearly see how the gap width changes with the tilt angle, in fig. 4 we exhibit the gap width between the ground state and the lowest excited state as a function of the in-plane component of the magnetic field. It is obvious that by increasing the in-plane component of the magnetic field, the gap widths at $\nu = 1/3$ FQHE states for LLs $n = 0$ and $\pm 1$ become larger and larger, which is similar to the conventional 2DEG cases [40,41]. Our calculations of $\nu = 1/5$ and $1/7$ FQHE states result in the same conclusion of $\nu = 1/3$ FQHE states. Thus, we suppose that the excitation gaps for all the $1/m$ ($m = 3, 5, 7, \ldots$) FQHE states can be enlarged by the in-plane magnetic field. Although there is no experimental report of FQHE in topological insulators, the experimental methods mentioned in previous studies [25,26,29] on this issue in conventional semiconductor quantum wells should be useful to realize the findings herein.

In summary, we theoretically investigated the FQHE in TIs under a tilted strong magnetic field. The single-particle wave function was obtained by using the perturbation method. The pseudopotentials of the electron-electron interactions and the ground (excited) state energy spectra for $1/3$, $1/5$ and $1/7$ FQHE at lowest LLs were calculated within the exact diagonalization approach. We have shown that in the presence of an in-plane component of the tilted magnetic field, the FQHE state at $n = 0$ LL becomes more stable, while the stabilities of $n = \pm 1$ LLs become weaker. Moreover, we have also found that the excitation gaps of the $\nu = 1/3$, $1/5$ and $1/7$ FQHE states increase with increasing the tilt angle.

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