Collective flows of light particles in the Au+Au collision at intermediate energies

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Abstract

The Skyrme potential energy density functional is introduced into the Ultrarelativistic Quantum Molecular Dynamics (UrQMD) model and the updated version is applied to studying the directed and elliptic flows of light particles (protons, neutrons, deuterons, tritons, $^3$He and $^4$He) in $^{197}$Au+$^{197}$Au collisions at beam energies 150, 250 and 400 MeV/nucleon. The results are compared with the recent FOPI experimental data. It is found that the yields and collective flows of light particles can be described quite well. The influence of the equation of state (EoS), medium-modified nucleon-nucleon elastic cross sections (NNECS) and cluster recognition criteria on the directed and elliptic flows is studied in detail. It is found that the flows of light particles are sensitive to the medium-modified NNECS, but not sensitive to the isospin dependent cluster recognition criteria. It seems difficult, however, even with the new data and calculations, to obtain a more accurate constraint on the nuclear incompressibility $K_0$ than the interval 200-260 MeV.

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I. MOTIVATION

The equation of state (EoS) of nuclear matter and the nucleon-nucleon cross sections (NNCS) in the nuclear medium are hot topics in nuclear physics since a long time [1]. Heavy ion collisions (HICs) provide a unique opportunity to study these subjects in the laboratories around the world. It has been always difficult, however, to directly extract the information on the EoS and NNCS from the measured quantities of HIC experiments because of the complexity of the collision process and the restriction of the experimental data to the asymptotic configurations recorded by the detectors. Microscopic transport theory has, therefore, been a valuable tool for simulating the dynamical process of HICs, so as to link the experimental observables to both the nuclear EoS and the in-medium NNCS [2].

The collective flow is a common phenomenon of HICs, first discovered at the Bevalac in 1984 (see Ref. [3] and references therein). The directed flow (also named in-plane or sidwards flow) and the elliptic flow (also named out-of-plane flow) are two lower-order components of the flow which have been widely used for studying HICs in a large range of beam energies varying from tens of MeV up to several TeV per nucleon. Newly measured experimental data of flows were usually compared with corresponding theoretical results, calculated with the most recent updated theoretical transport models, in order to obtain further insight into the properties of the EoS and the in-medium NNCS. A large effort has been devoted to constraining the stiffness of the EoS of isospin symmetric nuclear matter, with the result that it is most likely soft with an incompressibility $K_0$ of about $230\pm30$ MeV [4]. Up to now, however, the stiffness of the EoS of isospin asymmetric nuclear matter (“$K_{asy}$”), especially at high densities, is still not well constrained and the medium modified NNCS have not been well understood either. Thus both, more precise experimental data and self-consistent theoretical models are still called for.

One of the interesting phenomena, already known from early-stage flow-related experiments [5–7], is the dependence of the directed and elliptic flows on the particle species. The flow effect is larger for composite particles than for protons. With the subsequent large number of experimental (see, e.g., Refs. 8–12) and theoretical (see, e.g., Refs. 13–18) endeavors, the presence of this effect was confirmed by observing the increase of flow with the particle mass more precisely, even though the definitions of flow and the interpretations were somewhat different in the respective studies. Recently, by using the large acceptance appa-
ratus FOPI at the Schwerionen-Synchrotron (SIS) at GSI, a large amount of directed and elliptic flow data for light charged particles (protons, deuterons, tritons, $^3$He and $^4$He) from intermediate energy HICs have been made available [19, 20]. Moreover, flows are presented differentially in the FOPI data [20] in the form of both rapidity and transverse momentum distributions. Therefore, new opportunities have been opened up which will allow us to discuss the following questions:

1. Is it possible to reduce the uncertainty of $K_0$ of the EoS by comparing a large number of two-dimensional flow data with model calculations?
2. Is it now possible to extract more information on the medium modifications of NNCS?
3. How do different cluster recognition criteria affect the flows of light particles? This latter question arises because the newly developed isospin-dependent cluster recognition method has been reported to affect the production of light particles [21].

The paper is arranged as follows. In the next section the new version of the UrQMD transport model with the Skyrme potential energy density functional is presented. In Section III, results of collective flows of light particles from $^{197}$Au+$^{197}$Au reactions at beam energies 150, 250 and 400 MeV/nucleon are shown. Finally, a summary and outlook is given in Section IV.

II. URQMD MODEL UPDATES

The UrQMD model [22–25] has been widely and successfully used to study pp, pA, and AA collisions within a large energy range from Bevalac and SIS up to the AGS, SPS, RHIC, and LHC. At lower energies, the UrQMD model is based on principles analogous to the quantum molecular dynamics model (QMD) [26] in which each nucleon is represented by a Gaussian wave packet in phase space. The centroids $r_i$ and $p_i$ of a nucleon $i$ in the coordinate and momentum spaces are propagated according to Hamilton’s equations of motion:

$$\dot{r_i} = \frac{\partial H}{\partial p_i}, \quad \text{and} \quad \dot{p_i} = -\frac{\partial H}{\partial r_i}. \quad (1)$$

The Hamiltonian $H$ consists of the kinetic energy $T$ and the effective two-body interaction potential energy $U$,

$$H = T + U \quad (2)$$
with
\[ T = \sum_i (E_i - m_i) = \sum_i (\sqrt{m_i^2 + p_i^2} - m_i), \tag{3} \]
and
\[ U = U_\rho + U_{md} + U_{coul} \tag{4} \]
where \( U_{coul} \) is the Coulomb energy, while the nuclear interaction potential energy terms \( U_\rho \) and \( U_{md} \) can be written as
\[ U_{\rho,md} = \int u_{\rho,md} d\mathbf{r}. \tag{5} \]

In the current new version of the UrQMD model, the form of the momentum dependent term \( u_{\rho,md} \) is taken from the QMD model \[26\] while the Skyrme potential energy density functional \( u_\rho \) is introduced in the same manner as in the improved quantum molecular dynamics (ImQMD) model \[27, 28\] in which
\[ u_\rho = \frac{\alpha}{2} \rho^2 + \frac{\beta}{\eta + 1} \frac{\rho^{\eta+1}}{\rho_0^\eta} + \frac{g_{sur}}{2\rho_0} (\nabla \rho)^2 + \frac{g_{sur,iso}}{2\rho_0} [\nabla (\rho_n - \rho_p)]^2 + (A\rho^2 + B\rho^{\eta+1} + C\rho^{8/3})\delta^2 + g_{\rho\tau} \frac{\rho^{8/3}}{\rho_0^{5/3}}. \tag{6} \]
Here \( \delta = (\rho_n - \rho_p)/(\rho_n + \rho_p) \) is the isospin asymmetry defined through the neutron \( (\rho_n) \) and proton \( (\rho_p) \) densities with \( \rho = \rho_n + \rho_p \). The parameters \( \alpha, \beta, \eta, g_{sur}, \) and \( g_{sur,iso} \) are related to the Skyrme parameters via \( \alpha/2 = \frac{3}{8}t_0\rho_0, \beta/(\eta + 1) = \frac{1}{16}t_3\rho_0^\eta, g_{sur}/2 = \frac{1}{64}(9t_1 - 5t_2 - 4x_2t_2)\rho_0, \) and \( g_{sur,iso}/2 = -\frac{1}{64}(3t_1(2x_1 + 1) + t_2(2x_2 + 1))\rho_0 \). The parameters \( A, B, \) and \( C \) in the volume symmetry energy term of Eq. [6] are given by \( A = -\frac{9}{4}(x_0 + 1/2), B = -\frac{9}{24}(x_3 + 1/2), \) and \( C = -\frac{1}{24}(\frac{3x_1^2}{2})^{2/3}\Theta_{sym} \) where \( \Theta_{sym} = 3t_1x_1 - t_2(4 + 5x_2) \). The last term reads \( g_{\rho\tau} = \frac{3}{80}(3t_1 + (5 + 4x_2)t_2)(\frac{3x_1^2}{2})^{2/3}\rho_0^{5/3} \). The coefficients \( t_0, t_1, t_2, t_3 \) and \( x_0, x_1, x_2, x_3 \) are the well-known parameters of the Skyrme force.

In this work, we choose three sets of the Skyrme force, SkP \[4, 29\], SV-mas08 \[4, 30\], and SkA \[4, 31\] for incompressibility values \( K_0 \) varying within 230±30MeV. The main saturation properties of each set are listed in Table I which shows that the saturation density \( (\rho_0) \), the saturation energy \( (E_0) \), and the symmetry energy \( (S_0) \) at \( \rho_0 \) are close to their commonly accepted values, 0.16 fm\(^{-3}\), −16 MeV, and 32 MeV, respectively. The other three parameters, the slope \( L \) of the symmetry energy, the symmetry incompressibility \( K_{asy} \), and the effective mass ratio \( m^*/m \) at \( \rho_0 \), are also found within their known regions of uncertainty.
TABLE I. Saturation properties of three Skyrme parametrizations used in this work.

|                | SkP[4, 29] | SV-mas08[4, 30] | SkA[4, 31] |
|----------------|------------|-----------------|------------|
| $\rho_0$ (fm$^{-3}$) | 0.163      | 0.160           | 0.155      |
| $E_0$ (MeV)   | -15.95     | -15.90          | -15.99     |
| $S(\rho_0)$ (MeV) | 30.00      | 30.00           | 32.91      |
| $L$(MeV)      | 19.68      | 40.15           | 74.62      |
| $K_{asy}$(MeV) | -266.60    | -172.38         | -78.46     |
| $m^*/m$       | 1.00       | 0.80            | 0.61       |
| $K_0$(MeV)    | 201        | 233             | 263        |

Concerning the NNCS, it is known that it will be modified by the nuclear medium, according to the QHD theory (see, e.g., Refs. [32–39]). However, the details of this modification are still not clear. In this work, as done previously [24, 40], the in-medium nucleon-nucleon elastic cross sections (NNECS) are treated to be factorized as the product of a medium correction factor $F$ and the free cross sections. For the inelastic channels, we still use the experimental free-space cross sections which will not have a significant influence on results studied in this work. The total nucleon-nucleon binary scattering cross sections can thus be expressed as

$$\sigma_{tot}^* = \sigma_{in} + \sigma_{el}^* = \sigma_{in} + F(\rho, p)\sigma_{el}$$

(7)

with

$$F(\rho, p) = \begin{cases} 
  f_0 & p_{NN} > 1\text{GeV}/c \\
  \frac{f_0 \left(F_0 - f_0\right)}{1 + \left(p_{NN}/p_0\right)^\kappa} + f_0 & p_{NN} \leq 1\text{GeV}/c
\end{cases}$$

(8)

where $p_{NN}$ denotes the relative momentum of two colliding nucleons. Here $\sigma_{el}$ and $\sigma_{in}$ are the nucleon-nucleon elastic and inelastic cross sections in free space, respectively, with the proton-neutron cross sections being considered as different from the proton-proton and neutron-neutron cross sections in accordance with experimental data. The factor $F_\rho$ in Eq. 8 can be expressed as

$$F_\rho = \lambda + (1 - \lambda)\exp\left[-\frac{\rho}{\zeta \rho_0}\right],$$

(9)

which is also illustrated in Fig. 1(a). In this work, $\zeta=1/3$ and $\lambda=1/6$ are adopted which corresponds to the parametrization FU3 in Ref. [24]. The three parameters $f_0$, $p_0$ and $\kappa$ in Eq. 8 can be varied in order to obtain various momentum dependences of $F(\rho, p)$.
TABLE II. The parameter sets FP1, FP2, FP3, FP4 and FP5 used for describing the momentum dependence of $F(u,p)$.

| Set | $f_0$ | $p_0$ [GeV c$^{-1}$] | $\kappa$ |
|-----|-------|----------------------|---------|
| FP1 | 1     | 0.425                | 5       |
| FP2 | 1     | 0.225                | 3       |
| FP3 | 1     | 0.625                | 8       |
| FP4 | 1     | 0.3                  | 8       |
| FP5 | 1     | 0.34                 | 12      |

We select several parameter sets for this work which are shown in Table II. The corresponding $F(\rho,p)$ functions at $\rho = 2\rho_0$ are illustrated in Fig. 1(b). The parameterizations FP1, FP2, and FP3 were investigated and used in our previous works \cite{24, 41, 42}. Specifically, the parameter set FU3FP1 was used to investigate HICs around the balance energy where the experimental data can be reproduced quite well with this set. Here, we further introduce the FP4 and FP5 sets which lie roughly between FP1 and FP2. This will permit more accurate tests of the momentum dependence of the in-medium NNCS by taking advantage of the large number of new FOPI data for directed and elliptic flows of light charged particles. FP4 and FP5 differ mainly within $p = 0.2 - 0.4$ GeV/c and the largest difference is within the narrow region $p = 0.25 - 0.35$ GeV/c. The treatment of the Pauli blocking effect is the same as that in Ref. \cite{24}.

The UrQMD transport program stops at 250 fm/c at which time a phase-space coalescence mode \cite{43} is used to construct clusters. Usually, the minimum spanning tree (MST) algorithm is used. Recently, an isospin-dependent MST (iso-MST) method was introduced by Zhang et al. \cite{21}. Accordingly, in this work we will apply the two methods of fragment recognition. The relative distance and momentum parameters $R_0$ and $P_0$ are set to $R_0^{nn} = R_0^{np} = R_0^{pp} = 3.2$ fm for MST and $R_0^{nn} = R_0^{np} = 4.5$ fm and $R_0^{pp} = 3.2$ fm for iso-MST and $P_0 = 0.25$ GeV/c for both.
FIG. 1. (Color online) (a) The medium correction factor $F_\rho$ obtained with the parameterization FU3 and (b) the momentum dependence with the four options FP1, FP2, FP4, and FP5 given in Table II for FU3 at $\rho = 2\rho_0$.

III. OBSERVABLES AND CALCULATIONS

About 300 thousand events of $^{197}$Au+$^{197}$Au collisions for each of the beam energies $E_{\text{lab}}=150$, 250, and 400 MeV/nucleon are simulated randomly within the impact parameter region 0-7.5 fm. As in Ref. [20], the centrality is characterized by the reduced impact parameter $b_0$ defined as $b_0 = b/b_{\text{max}}$, taking $b_{\text{max}} = 1.15(A_1^{1/3} + A_T^{1/3})$ fm = 13.4 fm for $^{197}$Au+$^{197}$Au. At each beam energy, the calculations are divided into 4 groups according to $b_0$: $b_0 < 0.15$, $0.15 < b_0 < 0.25$, $0.25 < b_0 < 0.45$, and $0.45 < b_0 < 0.55$ ($b_{\text{max}} \cdot 0.55 = 7.4$ fm). Five options of the UrQMD model differing in the treatment of the mean-field potential (EoS), the medium modified NNCS and the cluster recognition method are adopted and listed in Table III. Clearly, the options UrQMD-I, UrQMD-IV, and UrQMD-V are for testing the influence of the mean field potential, the options UrQMD-III and UrQMD-IV are for testing the in-medium NNCS, and, UrQMD-II and UrQMD-IV are for testing the influence of the cluster recognition method.

As a general test of the model, we first calculated fragment spectra as a function of atomic number $Z$ for central $^{197}$Au+$^{197}$Au collisions at beam energies $E_{\text{lab}}=150$, 250, and 400 MeV/nucleon. It is found that results obtained by the five UrQMD options listed
TABLE III. Five options of the UrQMD transport model differing in the treatments of the potential terms (EoS), of the medium-modified NNCS, and of the cluster recognition method.

| Set       | EoS      | NNCS   | Cluster recognition |
|-----------|----------|--------|---------------------|
| UrQMD-I   | SkP      | FU3FP4 | iso-MST             |
| UrQMD-II  | SV-mas08 | FU3FP4 | MST                 |
| UrQMD-III | SV-mas08 | FU3FP5 | iso-MST             |
| UrQMD-IV  | SV-mas08 | FU3FP4 | iso-MST             |
| UrQMD-V   | SkA      | FU3FP4 | iso-MST             |

in Table [III] are in agreement with experimental data and the difference among them are relatively small. Since the aim of this work is to explore whether more accurate constraints to the whole dynamic process of HICs can be obtained by comparing with the new flow data of the FOPI collaboration, we will not present results on the fragment spectrum in this paper.

It is known that one of the most important observables to constrain the stiffness of EoS of nuclear matter, especially at supra-normal densities, is the collective flow in HICs at intermediate energies. Using the same parameterization as in Ref. [20], we have

$$\frac{dN}{u_tdu_tdyd\phi} = v_0[1 + 2v_1 \cos(\phi) + 2v_2 \cos(2\phi)],$$

(10)

in which the directed and elliptic flow parameters $v_1$ and $v_2$ can be written as:

$$v_1 \equiv \langle \cos(\phi) \rangle = \langle \frac{p_x}{p_t} \rangle; v_2 \equiv \langle \cos(2\phi) \rangle = \langle \frac{p_x^2 - p_y^2}{p_t^2} \rangle.$$

(11)

Here $\phi$ is the azimuthal angle of the emitted particle with respect to the reaction plane, and $p_t = \sqrt{p_x^2 + p_y^2}$ is the transverse momentum of emitted particles. The angle brackets in Eq. [11] denote an average over all considered particles from all events. The $v_1$ and $v_2$ have complex multi-dimensional dependences. For a certain reaction with fixed reaction system, beam energy, and impact parameter, they are functions of $u_t$ and rapidity $y$. Here $u_t = \beta_r \gamma$ is the transverse component of the four-velocity $u=(\gamma, \beta \gamma)$. We use the scaled units $u_{t0} \equiv u_t/u_{1cm}$ and $y_0 \equiv y/y_{1cm}$ as done in [20], and the subscript $1cm$ denotes the incident projectile in the center-of-mass system.
We first investigate how the condition $u_{t0} > 0.8$, applied by FOPI to their data, influences the directed flow of different particles. In Fig. 2, the $p_x/A$ vs. $p_y/A$ contour plots for emitted protons, deuterons, and $A = 3$ clusters (considering $^3H$ and $^3He$ results) are shown without the $u_{t0}$ cut in the upper and with the cut $u_{t0} > 0.8$ in the lower panels. The interval of forward rapidities $0.4 < y_0 < 0.6$ is selected, so that more particles have positive $p_x$. The solid lines represent the averaged $\langle p_x/A \rangle$ values for each $p_y/A$ bin and the numerical values in the upper left corners of the panels are the averages over all considered $p_y/A$ bins. It is apparent from the upper plots (a) – (c) that the $\langle p_x/A \rangle$ values of protons, deuterons, and $A = 3$ clusters are the same when the $u_{t0}$ cut is not taken into account. When the cut $u_{t0} > 0.8$ is applied, however, shown in the lower plots (d) – (f), the $\langle p_x/A \rangle$ value increases.
with increasing particle mass. This shows that the expected collective proportionality to the particle mass is observed when all particles are included and suggests that the phenomenon of an additional increase of the flow effect with the particle mass is strongly correlated with whether a transverse momentum cut is applied or not.

![Graph](image)

**FIG. 3.** (Color online) Rapidity distribution of (a) the flow parameter $v_1$ of protons under various centralities and (b) flow parameter $v_1$ for protons, deuterons, $A = 3$ clusters, and alpha particles from $^{197}$Au+$^{197}$Au collisions at 250 MeV/nucleon with $0.25 < b_0 < 0.45$, as calculated with the UrQMD-IV option (open symbols). The cut $u_{t0} > 0.8$ is chosen. The lines are fits to the calculation results (see text), while the corresponding experimental data from Ref. [20] are given by the solid symbols.

Now, let us look at the collective flow as a function of rapidity when a $u_{t0}$ cut is applied. Fig. 3 shows the directed flow $v_1$ of protons under different centralities (open symbols) in plot (a), and $v_1$ of protons, deuterons, $A = 3$ and $\alpha$ particles (open symbols) with the centrality $0.25 < b_0 < 0.45$ in (b) as a function of $y_0$. The UrQMD-IV is adopted for calculations, the reaction conditions in Fig 3 (b) are chosen to be the same as the FOPI experimental data (solid symbols) of Ref. [20]. The solid curves in the figure are fits to calculation results assuming $v_1(y_0) = v_{11} \cdot y_0 + v_{13} \cdot y_0^3 + c$ in the range of $-1.1 < y_0 < 1.1$. The fit also
provides the slope value \( v_{11} \) of \( v_1 \) at \( y_0 = 0 \) which will be discussed later. In Fig. 3 (b), it is found that our calculated results for all particles considered are in agreement with the experimental data in the whole rapidity region.

![Rapidity distribution of the flow parameter \( v_2 \) of protons for various centralities (a) and the \( v_2 \) of protons, deuterons, and alpha particles for the impact-parameter bin \( 0.25 < b_0 < 0.45 \) for \(^{197}\text{Au} + ^{197}\text{Au} \) collisions at 400 MeV/nucleon. Calculations with UrQMD-IV are shown with open symbols while the FOPI data, taken from Ref. [20], are shown by solid symbols. The lines are fits to the calculated results (see text).](image)

The elliptic flow \( v_2 \) of light particles is also calculated and compared with FOPI data from Ref. [20]. In Fig. 4 the results of calculations with UrQMD-IV and the FOPI data from \(^{197}\text{Au} + ^{197}\text{Au} \) collisions at 400 MeV/nucleon are represented by the open and solid symbols, respectively. In the left panel, the elliptic flow parameter \( v_2 \) of protons as a function of \( y_0 \) is shown for three centralities, while the \( v_2 \) for different particles, i.e., protons, deuterons and alpha particles, is given in the right panel (for semi-central collisions and with the less restrictive \( u_{t0} \) cut applied by FOPI at the higher energy). The figure shows that the FOPI \( v_2 \) flow data, within a large centrality region and for several particles, can also be quite well described with the updated UrQMD transport model. Further, with the fit \( v_2(y_0) = v_{20} + v_{22} \cdot y_0^2 + v_{24} \cdot y_0^4 \) to the calculation, the elliptic flow at mid-rapidity, \( v_{20} \), can be obtained.

In order to show why the sets FP4 and FP5 have been introduced in addition to FP1 and
FIG. 5. (Color online) (a) The $v_{11}$ and (b) the $v_{20}$ values for light particles up to mass number $A=4$ calculated with FP1, FP2, FP4 and FP5 (lines with symbols) while other inputs are the same as those in the UrQMD-IV set. The $^{197}\text{Au}^{+} \cdot ^{197}\text{Au}$ collision at the beam energy $250\text{ MeV/nucleon}$ with $0.25 < b_0 < 0.45$ is considered as an example. The FOPI experimental data (stars) are taken from Ref. [20].

FP2 used previously for testing the momentum dependence of the in-medium NNCS, we display in Fig. 5 (a) the $v_{11}$ and in Fig. 5 (b) the $v_{20}$ values for light particles calculated with the four sets FP1, FP2, FP4, and FP5. Other inputs are the same as those in the UrQMD-IV set. Firstly, one sees clearly that calculation results with FP4 and FP5 are well separated. It means that the directed and elliptic flows of light particles are very sensitive to the exact momentum dependence of in-medium NNCS within a narrow region of $p = 0.2 - 0.4\text{ GeV/c}$. Secondly, the $v_{11}$ of light particles calculated with FP2 and FP4 and the $v_{20}$ calculated with FP1 and FP5 are very close to each other, respectively. Remembering that there is a large difference between FP2 and FP4 at the low momentum part and between FP1 and FP5 at high momenta (see Fig. 1), we may conclude that the directed flow of light particles is not sensitive to the low momentum part while the elliptic flow is not sensitive to the high momentum part of the momentum dependent NNECS. The figure finally also shows that the calculations with FP4 can best reproduce the experimental data.

Besides the medium modification on NNECS, also the influence of the mean field and of the cluster recognition method on flows is further investigated. In Fig. 6 the $v_{11}$ and $v_{20}$
FIG. 6. (Color online) $v_{11}$ [(a) and (b)] and $v_{20}$ [(c) and (d)] for light particles from semi-central $(0.25 < b_0 < 0.45)\ ^{197}\text{Au}+^{197}\text{Au}$ collisions at $E_{\text{lab}} = 150$ (left) and 400 MeV/nucleon (right). The calculations performed with five UrQMD parameter sets are distinguished by different lines as indicated. The FOPI experimental data from Ref. [20] are shown by stars.

values obtained from calculations with different UrQMD sets for light particles from semi-central $^{197}\text{Au}+^{197}\text{Au}$ collisions at two beam energies, 150 (left) and 400 MeV/nucleon (right), are compared with the FOPI data. More specifically, the mean field effect is examined in Fig. 6(a) and (c) where calculations with UrQMD-I, UrQMD-III, UrQMD-IV and UrQMD-V sets
are shown, while the cluster recognition effect is tested in Fig. 6 (b) and (d) with calculations using the UrQMD-II, UrQMD-III and UrQMD-IV sets. One immediately sees that, for both $v_{11}$ and $v_{20}$, calculations with UrQMD-I, UrQMD-II, UrQMD-IV and UrQMD-V are grouped together, while absolute values obtained with UrQMD-III are apparently smaller, especially for composite particles. The main reason is that, with FP5, the reduction of the in-medium cross section is stronger in the UrQMD-III case. Flows of composite particles at intermediate energies are, apparently, very useful to test the behavior of the momentum dependence of in-medium NNECS, especially in the momentum region $p=0.2-0.4$ GeV/c.

Secondly, although the absolute values of $v_{11}$ and $v_{20}$ are still seen to increase gradually with the increasing incompressibility $K_0$ of the EoS, by examining calculations going from UrQMD-I, UrQMD-IV, to UrQMD-V sets, the differences between them are too small to extract a more accurate $K_0$ value than $230\pm30$ MeV from the present calculations and experimental data. We note that, in order to obtain an improved flow data set of light fragments for $^{197}\text{Au}+^{197}\text{Au}$ collisions and to extend the study of the density dependent symmetry energy to other systems, a new experiment (S394) was recently carried out at the GSI laboratory by the ASY-EOS collaboration [44]. It is certainly hopeful for us to further reduce the uncertainties in both $K_0$ and $K_{\text{asy}}$ with the help of the new experiment.

Finally, from Fig. 6 (b) and (d) one finds that, no matter which flow parameter is chosen, the difference between results calculated with UrQMD-II and with UrQMD-IV is also very small. It indicates that the different cluster recognition methods MST and iso-MST have only a weak effect on the flow parameters. This is understandable since it is known that collective flows of light particles are mainly produced at the earlier stages of HICs and weakly affected by the final-state interactions. Generally speaking, we can conclude that the new FOPI flow data can be reproduced by the UrQMD model calculations when the FU3FP4 medium modification of NNECS is adopted, with the only exception of $\alpha$ particle flow which is underestimated and should be further studied.

In order to see more clearly effects of the mean field potential, the in-medium NNCS, and the cluster recognition method on flows, the calculated parameters $v_1$ of directed and $v_2$ of elliptic flow are shown as a function of $u_{t0}$ in Fig. 7. For this purpose, we compare results of protons and $A=3$ clusters obtained with UrQMD-I and UrQMD-V in (a) and (d), with UrQMD-III and UrQMD-IV in (b) and (e), and with UrQMD-II and UrQMD-IV in (c) and (f), respectively. As an example, $^{197}\text{Au}+^{197}\text{Au}$ collisions at the beam energy 250
FIG. 7. (Color online) Parameters $v_1$ of directed flow (upper panels) and $v_2$ of elliptic flow (lower panels) for protons and $A = 3$ clusters as a function of $u_{t0}$. Calculations are obtained with UrQMD-I and UrQMD-V in (a) and (d), UrQMD-III and UrQMD-IV in (b) and (e), and UrQMD-II and UrQMD-IV in (c) and (f), respectively. The $^{197}$Au+$^{197}$Au collision at the beam energy 250 MeV/nucleon with $0.25 < b_0 < 0.45$ is considered as an example. The rapidity cuts $0.4 < y_0 < 0.8$ and $|y_0| < 0.4$ are chosen for $v_1$ and $v_2$, respectively.

MeV/nucleon with $0.25 < b_0 < 0.45$ are chosen. One sees a significant effect on both flow parameters only in the case of the comparison of calculations with UrQMD-III to UrQMD-IV shown in (b) and (e), especially for $A = 3$ clusters. This situation is quite similar to that shown in Fig. 6. Further, it is found that at about $0.5 < u_{t0} < 1.0$ the effect of medium modified NNCS on flows of $A = 3$ clusters is enlarged while the other two effects are reduced so that one may be able to more cleanly determine the medium modifications of NNCS in this momentum region.

We finally show in Fig. 8 the $u_{t0}$ dependence of calculated directed (left panels) and
FIG. 8. (Color online) The $u_{t0}$ dependence of parameters $v_1$ of directed (left) and $v_2$ of elliptic flows (right) of light charged particles from semi-central $(0.25 < b_0 < 0.45)$ $^{197}$Au+$^{197}$Au collisions at beam energies 150 [(a) and (b)], 250 [(c) and (d)], and 400 MeV/nucleon [(e) and (f)]. The rapidity cuts $0.4 < y_0 < 0.8$ and $|y_0| < 0.4$ are chosen for $v_1$ and $v_2$, respectively. Calculated results with UrQMD-IV are represented by different lines as indicated, the FOPI experimental data from Ref. [20] are shown by solid symbols.

elliptic flows (right panels) of light charged particles at beam energies 150, 250, and 400 MeV/nucleon (lines). The reaction system and chosen rapidity cuts are the same as for the experimental data taken from Ref. [20] and shown by the full symbols. It is firstly observed that calculations with the UrQMD-IV set reproduce the $v_1$ and $v_2$ data reasonably well with some exceptions. Although the experimental data of directed flow of $\alpha$ particles can not be well described by the model, the relatively large flow effect is clearly exhibited in plots (a), (c) and (e) of Fig. 8. Secondly, calculation results for absolute $v_1$ and $v_2$ values of protons are slightly larger than the FOPI data, which is similar to the simulation results shown in Ref. [20] where the IQMD model was used. A possible reason is that the QMD-like model
calculations produce more free protons and neutrons than observed in the experiment. Some of the free nucleons might thus actually belong to fragments. Since the flow effect is larger for fragments than for emitted nucleons, the calculated flows of free protons are consequently overestimated. Thirdly, as for flows of deuterons and $A = 3$ clusters, it is seen that the comparison of UrQMD-IV calculations with the experimental data is fairly good in the range $0.5 < u_{t0} < 1.0$. In view of the result shown in Fig. 7, it is highly advantageous to investigate the detailed behavior of the medium corrected NNCS in this momentum region. Finally, when $u_{t0}$ is larger than about 1.0, the deviation of the calculated $v_1$ from the data starts to increase in some of the particle cases. But, on the other hand, the yields of these particles are quite small in these $u_{t0}$ and $y_0$ regions and the contribution to the final $v_{11}$ value is thus very limited. One has indeed seen the successful description of the $u_t$-integrated data by the UrQMD-IV set shown in Fig. 8. Nevertheless, it is certainly possible that the internal magnetic fields [45] and non-central forces as, e.g., the tensor force and spin-orbit coupling [46, 47] might influence the freeze-out mode of HICs, especially for non-central collisions at large momenta and rapidities, topics which should be further studied in the transport theory.

IV. SUMMARY AND OUTLOOK

In summary, we have studied the directed and elliptic flows of light particles in $^{197}$Au+$^{197}$Au collisions at beam energies 150, 250 and 400 MeV/nucleon by using the updated UrQMD model in which the Skyrme potential energy density functional is introduced. After the detailed study of the influence of equation of state (EoS), medium-modified nucleon-nucleon elastic cross section (NNECS) and cluster recognition criteria on flows, the three questions asked in the introduction can be answered: (1) it is difficult to get a more exact value of the incompressibility from the present flow data than $K_0 = 230 \pm 30$ MeV, (2) the different choices of medium-modified NNECS exhibit a significant influence on the light particle flows and, particularly, on the flows of light composite particles; (3) the influence of the cluster recognition method on cluster flows is weak. The version of UrQMD-IV, comprising the SV-mas08 force with a corresponding incompressibility $K_0 = 234$ MeV, the FU3FP4 medium-modified NNECS and the iso-MST cluster recognition method, describes the directed and elliptic flows of light particles as functions of both rapidity and transverse momentum rather
Theoretically, the spin-orbit coupling term in the Skyrme interactions will be further put into the UrQMD transport model after incorporating the spin degree of freedom and its contribution to flows, especially at large rapidities and/or transverse momenta, for intermediate energy HICs can then be identified. Together with the forthcoming new flow data of light particles measured by the ASY-EOS collaboration at GSI, we hope to further reduce the uncertainties in both $K_0$ and $K_{asy}$ of the isospin-dependent EoS within the present framework of UrQMD in the near future.

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