Baryon Magnetic Moments in Relativistic Quark Models

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Abstract

It is shown that the phenomenological description of the baryon magnetic moments in the quark model carries over to the Poincaré covariant extension of the model. This applies to all the three common forms of relativistic kinematics with structureless constituent currents, which are covariant under the corresponding kinematic subgroups. In instant and front form kinematics the calculated magnetic moments depend strongly on the constituent masses, while in point form kinematics the magnetic moments are fairly insensitive to both the quark masses and the wave function model. The baryon charge radii and magnetic moments are determined in the different forms of kinematics for the light-flavor, strange and charm hyperons. The wave function model is determined by a fit to the electromagnetic form factor of the proton.

1 Introduction

The prominence of the non-relativistic quark model is partly due to the simple and qualitatively satisfactory description of the masses and the magnetic moments of the ground state baryons, which it provides. Notwithstanding this satisfactory phenomenology the model lacks dynamical consistency, as the high velocity of the confined quarks a priori invalidates the non-relativistic description of the constituent quarks. This problem may be overcome by recasting the model into Poincaré covariant form. It is shown here that the satisfactory description of the baryon magnetic moments carries over to this relativistic version of the quark model at the modest price of somewhat smaller constituent masses than commonly used, provided that the baryon wave function is determined by a fit to the electromagnetic form factors of the nucleon.

The calculation of the electromagnetic observables of baryons in the relativistic quark model calls for a choice of kinematic subgroup of the group of Poincaré
transformations, a choice which is referred to as the “form of kinematics”, in addition to the choice of wave function and current operator models. The three forms of kinematics, originally outlined by Dirac [1], are commonly referred to as “instant”, “point”, and “front” form kinematics. The corresponding kinematic subgroups of the group of Poincaré transformations are the group of rotations and translations at fixed time $E(3)$, the group of Lorentz transformations $SO(1,3)$, and the symmetry group on the light cone. The present relativistic quantum mechanical approach should be distinguished from the approach based on relativistic field theory, in which classes of Feynman diagrams are summed in Bethe-Salpeter type equations, as e.g. in Ref [2].

It was recently noted that a fair description of the electromagnetic form factors of the nucleons over the empirically known range of momentum transfer may be achieved with all these three forms of relativistic kinematics with simple algebraic wave function models for the confined three-quark system [3]. The model employed structureless constituent current operators, that are covariant under transformations of the corresponding kinematic subgroup. That work was stimulated by the fairly satisfactory results obtained in point form kinematics for the electromagnetic form factors of the nucleons with a very compact wave function model in Ref. [4].

Here a calculation of the baryon magnetic moments and charge radii is carried out with the relativistic constituent quark model in all the three forms of relativistic kinematics. Both the strange and the charm as well as the recently discovered doubly charm hyperons are considered. The functional form of the spatial wave function model is taken to be similar for all three forms of kinematics, although the spatial extent of the wave function, which is required for a satisfactory description of the electromagnetic form factor of the proton, depends on the form of kinematics.

With instant and front form kinematics the calculated baryon magnetic moments are found to be very sensitive to both the wave function model and the value of the constituent mass. With point form kinematics the calculated magnetic moments of the nucleon are on the other hand found to be quite insensitive to the constituent mass. Instead there is a sensitivity to the mass of the baryon, which leads to smaller values for the calculated magnetic moments of the heavy hyperons than with the other forms of kinematics. In instant and front form kinematics both the empirical magnetic moments and the charge radii of the strange hyperons are well reproduced with the same spatial wave function, as used for the nucleons. In contrast, reproduction of the empirical charge radius of the $\Sigma^-$ hyperon would in point form demand that its matter radius be more than twice that of the nucleon. Even with such an extended wave function model the calculated magnetic moments of most of the strange hyperons remain somewhat small in point form kinematics.
In [3] it was noted that the relativistic quark model gives reasonable values for the nucleon even for zero constituent mass. By considering the hyperon magnetic moments it is shown here that, while the constituent masses of the light flavor quark masses may be taken to be very small, agreement with the experimental magnetic moments requires finite values for the strange quark mass.

This paper is organized in the following way: Section 2 contains a description of the calculation of the magnetic moments in the Poincaré covariant quark model in the three different forms of relativistic kinematics. The wave function model is described in Section 3. In Section 4 results for the magnetic moments and the charge radii of the strange hyperons are given. The corresponding results for the charm hyperons are given in Section 5. A concluding discussion of the results is given in Section 6.

2 Magnetic Moments and Charge Radii

2.1 Definition of the magnetic moment

The magnetic moment of a baryon may be defined as the value of the invariant magnetic form factor \( G_M \) at zero invariant momentum transfer. The form factor is defined as a matrix element of an appropriate component of the electromagnetic current operator, which depends on the form of kinematics. In instant and point form, the magnetic moment of a spin 1/2 baryon may thus be defined as the matrix element of the spin raising current component \( I_+ = (1/2)(I_x + iI_y) \) in the Breit frame:

\[
\mu = G_M(0) = \frac{\sqrt{1 + \eta}}{\sqrt{\eta}} \langle \frac{1}{2}, \frac{\vec{Q}}{2} | I_+ | 0 \rangle - \frac{1}{2}, \frac{\vec{Q}}{2} \rangle . \tag{1}
\]

Here \( \vec{Q} \) has been taken to be parallel to the \( z \)-axis and \( \eta \) is defined as \( \eta = Q^2/4M^2 \), where \( M \) is the baryon mass. In the case of front form kinematics the appropriate definition is on the other hand:

\[
\mu = G_M(0) = F_1(0) + F_2(0) , \tag{2}
\]

where \( F_1 \) and \( F_2 \) are the Dirac and Pauli form factors of the nucleon. These are in turn defined as the following matrix elements of the “plus” component of the current \( I^+ = I^0 + I^z \):

\[
F_1(0) = \langle \frac{1}{2} | I^+ | \frac{1}{2} \rangle , \quad F_2(0) = \frac{1}{\sqrt{\eta}} \langle -\frac{1}{2} | I^+ | \frac{1}{2} \rangle . \tag{3}
\]
In this case the momentum transfer has to be transverse to the $z$–direction: $\vec{Q} = \vec{Q}_\perp$. Without loss of generality $Q$ may be taken to lie in the direction of the $x$–axis [5].

For spin $3/2$ baryons, as the $\Delta(1232)$ and the $\Omega^-$, the corresponding definition of the magnetic moment in instant and point form kinematics is:

$$\mu = G_M(0) = \frac{\sqrt{3(1 + \eta)}}{\sqrt{\eta}} \langle \frac{3}{2}, \frac{Q}{2} | I_+(0) | \frac{1}{2}, -\frac{Q}{2} \rangle. \quad (4)$$

In front form kinematics the definition is again:

$$\mu = G_M(0) = F_1(0) + F_2(0), \quad (5)$$

but now $F_1$ and $F_2$ are obtained as [6]:

$$F_1 = \langle \frac{3}{2}, P' | I^+(0) | P, \frac{3}{2} \rangle,$$

$$F_2 - 2F_1 = \sqrt{\frac{3}{\eta}} \langle \frac{1}{2}, P' | I^+(0) | P, \frac{3}{2} \rangle. \quad (6)$$

Here $P$ and $P'$ are the initial and final total momenta.

### 2.2 Definition of the charge radii

The mean square charge radius of a baryon is defined as the derivative of the charge (electric) form factor $G_E(Q^2)$ with respect to the invariant squared momentum transfer as:

$$< r^2 > = -6 \left( \frac{dG_E(Q^2)}{dQ^2} \right)_{Q^2=0}. \quad (7)$$

For charged baryons the convention is to divide out the charge from the electric form factor in the definition of the charge radius. In the case of instant and point form kinematics $G_E$ is defined as the matrix element:

$$G_E(Q^2) = \langle \sigma, \frac{Q}{2} | I_z(0) | \sigma, -\frac{Q}{2} \rangle, \quad \sigma = \frac{1}{2}, \frac{3}{2}. \quad (8)$$

In the case of front form kinematics the appropriate definition is [5]:

$$G_E(Q^2) = F_1(Q^2) - \eta F_2(Q^2). \quad (9)$$
Here $F_1$ and $F_2$ are the Dirac form factors defined in Eq. (3) for spin 1/2 baryons and in Eq. (6) for spin 3/2 baryons.

3 The wave function model in the magnetic form factors

3.1 The wave function model

The baryon wave function models are here taken as products of a completely symmetric spatial component and $SU(N_f)F$ symmetric spin-flavor state vectors in the rest frame. The constituent quark masses appear in the current operators and in the boosts as well as in the Wigner and Melosh rotations, which connect the initial and final states to the spins and momenta that appear in the rest frame baryon wave functions. The assumption of a completely symmetric spatial wave function for all systems of three quarks implies that the mass operator, the eigenfunctions of which are the wave functions, is symmetric in the quark masses as well. Examples of mass operators of this general form, with spectra that agree with the empirical baryon spectra, are given in Ref. [7].

For the ground state baryons the spatial wave function will be taken to be a completely symmetric $S-$state. This function will be taken to have the form:

$$
\varphi_0(P) = \mathcal{N} \left( 1 + \frac{P^2}{4b^2} \right)^{-a},
$$

where $P$ is the hyperspherical momentum, $P := \sqrt{2(\mathbf{\kappa}^2 + \mathbf{q}^2)}$, where $\kappa$ and $q$ are the Jacobi coordinates for the three-quark system. In (10), $\mathcal{N}$ is a normalization constant, while the exponent $a$ and $b$ are adjustable parameters. The values of the parameters for the three different forms of relativistic kinematics are chosen as in Ref. [3], and are listed in Table 1.

Table 1
The parameter values in the ground state wave function (10) used for the three different forms of kinematics. The corresponding matter radii $r_0$ are listed in the last column.

|               | $m_q$ (MeV) | $b$ (MeV) | $a$  | $r_0$ (fm) |
|---------------|-------------|-----------|------|------------|
| instant form  | 140         | 600       | 6    | 0.63       |
| point form    | 350         | 640       | 9/4  | 0.19       |
| front form    | 250         | 500       | 4    | 0.55       |
The $SU(N_f)_F$ symmetry of the spin-flavor wave state of the baryons in the rest frame is broken by the boosts to the Breit frame, which change the spin quantization axis. Both the boosts and the current operators in addition break the flavor symmetry explicitly by the dependence on the constituent quark mass, which is different for quarks of different flavor.

The spin-flavor wave functions of the octet and decuplet baryons can be formally expressed in terms of the flavor and spin symmetry assignments as:

\[ \varphi_{SF}(\frac{1}{2}) = \frac{1}{\sqrt{2}} \left\{ \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}_{SF} + \begin{bmatrix} 1 \\ 3 \\ 2 \end{bmatrix}_{SF} \right\}, \quad (11) \]

and

\[ \varphi_{SF}(\frac{3}{2}) = \frac{1}{2} \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}_{SF}, \quad (12) \]

respectively. In the following we will make use of the following notation:

\[ |S_{MS}\rangle = \frac{1}{3} \begin{bmatrix} 1 \\ 2 \end{bmatrix}, \quad |S_{MA}\rangle = \frac{1}{2} \begin{bmatrix} 1 \\ 3 \end{bmatrix}, \quad |S_{S}\rangle = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}_{S}, \quad |S_{A}\rangle = \frac{1}{3} \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}. \quad (13) \]

In Table A.1 we give the explicit expressions for the mixed symmetry wave functions of the octet baryons.

### 3.2 Magnetic form factors in the different forms of kinematics

In instant and in point form kinematics, the magnetic form factors may be written as an integral over the spectator momenta in the Breit frame as [3]:

\[ G_M(\eta) = \int d^3p_2 d^3p_3 \varphi \left( \frac{\kappa^2 + q^2}{2b^2} \right) \varphi \left( \frac{\kappa^2 + q^2}{2b^2} \right) I_+(\vec{p}_2, \vec{p}_3). \quad (14) \]

Here the wave functions $\varphi$ have the form (10), and depend on the Jacobi momenta in the initial and final rest frames respectively, and $I_+$ is the matrix element of the spin raising component of the Wigner rotated Dirac current operator, multiplied by a Jacobian factor that takes into account the transition from the rest frame to the Breit frame. The explicit expressions for the Wigner rotated current matrix element and the Jacobian factor that applies in instant and point form kinematics are given in Ref. [3]. The Breit frame momenta $\vec{p}_i$ are related to the initial and final state momenta by Lorentz boosts, which depend on the form of kinematics.
In front form kinematics the magnetic form factors are obtained as a linear combination of the Dirac form factors of the baryons (5). The expressions for the latter take the form [3,5]:

\[
F_\alpha(Q^2) = \frac{1}{\xi_1 \xi_2 \xi_3} \int_0^1 d\xi_1 d\xi_2 d\xi_3 \frac{\delta(\xi_1 + \xi_2 + \xi_3 - 1)}{\xi_1 \xi_2 \xi_3} \int d\vec{k}_{1\perp} d\vec{k}_{2\perp} d\vec{k}_{3\perp} \delta(\vec{k}_{1\perp} + \vec{k}_{2\perp} + \vec{k}_{3\perp}) \frac{\omega_1 \omega_2 \omega_3}{M_0} \varphi^*(\xi_1, \vec{k}_{1\perp}, \xi_2, \vec{k}_{2\perp}, \xi_3, \vec{k}_{3\perp}) F_\alpha(Q^2) \varphi(\xi_1, \vec{k}_{1\perp}, \xi_2, \vec{k}_{2\perp}, \xi_3, \vec{k}_{3\perp}).
\]

(15)

Here the relations between the momentum variables of the struck (“1”) and the two spectator quarks (“2,3”) in the initial and final states are:

\[
k_{1\perp}' = k_{1\perp} + (1 - \xi_1)Q_\perp, \quad k_{i\perp}' = k_{i\perp} - \xi_i Q_\perp \quad i = 2, 3.
\]

(16)

The variables \(\xi_i\) are defined as \(\xi_i = p_i^+/P^+\), where \(P\) is the total momentum and + refers to the “plus” component in the light cone representation.

The factors \(F_\alpha(Q^2)\) involve the spin-isospin amplitudes and the effects of the Melosh rotations on both spectators and the Dirac current \(I_x\) of the struck constituent. The explicit expressions for these are given in Ref. [3]. The factor \(\omega_1 \omega_2 \omega_3/M_0\) is the Jacobian factor for front form kinematics. Here \(\omega_i\) are single constituent energies and \(M_0\) is the free mass operator.

4 Magnetic moments and charge radii of nucleons and strange hyperons

There are relations between the magnetic moments of the baryons, that arise from the spin-flavor matrix elements, which do not depend on the form of relativistic kinematics. These relations involve matrix elements of the spin part of the current density operator, which depends on the specific form of kinematics through the boosts of the quark momenta and the Wigner rotations on the quark spins. The spin-flavor matrix elements that enter in the calculation of the magnetic moments of the baryons may formally be denoted as \(\langle B | S F | B \rangle\). Here \(S\) is the spin operator, which is determined by the Dirac current of the struck constituent and the boosts (in the case of the transition moments) and the Wigner (or Melosh in the case of front form kinematics) rotations. The factor \(F\) is the flavor dependent charge operator of the struck quark:

\[
F = \left\{ \frac{\lambda_3}{2} + \frac{\lambda_8}{2\sqrt{3}} \right\}.
\]

(17)
The spin operator $S$ depends on flavor through the flavor dependence of the quark mass. Appendix A contains the explicit expressions for the spin-flavor matrix elements that appear in the calculation of both the magnetic form factors and the electric charge radii once the flavor part has been calculated using $SU(4)$ wave functions. These expressions reduce to the well-known static quark model expressions (e.g. [8]), if the spin operator is assumed to have the static form

$$S_{\text{static}} = \sum_i \frac{q_i}{2m_i} \sigma_{iz}. \quad (18)$$

The expressions in Appendix A contain matrix elements of the spin part of the baryon wave functions. These matrix elements depend on the current density operator and on the form of relativistic kinematics.

In Table 2 we give results for the magnetic moments that are obtained with equal flavor independent masses for the constituent $u$, $d$ and $s$ quarks. For the three different forms of kinematics two cases are considered: one with the quark mass equal to zero, and a second one with non-zero quark mass. The relativistic quark model differs from the non-relativistic quark model in that finite values for the magnetic moments are obtained even in the case of vanishing quark masses.

The magnetic moment values in Table 2 are of similar quality as those that are obtained with the static quark model in all forms of kinematics. The results also reveal that the calculated values are notably closer to the empirical values with finite values for the constituent masses. If however, the Dirac magnetic moment operator is employed in the naive quark model, the corresponding magnetic moments would only be about half as large as the empirical values [8]. The relativistic quark model calculation shows that it is possible to retain the qualitatively satisfactory quark model phenomenology of the baryon magnetic moments even when Dirac currents for the constituent quarks are employed.

In the case of instant form the calculated magnetic moment values are almost insensitive to the value of the constituent mass of the light flavor quark. They are, however, in notably better agreement with the empirical values when the strange quark mass is finite and $\sim 470$ MeV, as shown in Table 3.

The point form values for the hyperon magnetic moments given in Tables 2 and 3 are more sensitive to the value of the constituent mass of the light flavor quark, but clearly inferior to the instant form values for all combinations of the quark masses.

The magnetic moment values that are calculated in front form are quantitatively fairly similar to those that are calculated in instant form as shown in Tables 2 and 3. With finite quark masses they fall within 10% of the empirical
Table 2
Magnetic moments, in nuclear magnetons, calculated with flavor independent constituent masses. The constituent mass used for the $\Omega^-$ is 470 MeV. The column NR contains the static quark model values [8].

| Baryon          | NR     | Instant | Point | Front | EXP |
|-----------------|--------|---------|-------|-------|-----|
| ($m_u,m_s$)     | (360,470) | (0,0)   | (140,140) | (0,0) | (350,350) | (0,0) | (250,250) |

Octet Baryons

| Baryon          | NR     | Instant | Point | Front | EXP |
|-----------------|--------|---------|-------|-------|-----|
| $p_{(uud)}$     | 2.76   | 2.57    | 2.71  | 1.95  | 2.45 | 3.12 | 2.78 | 2.79 |
| $n_{(udd)}$     | -1.84  | -1.79   | -1.84 | -1.32 | -1.63 | -1.99 | -1.67 | -1.91 |
| $\Sigma^+_{(uus)}$ | 2.68   | 2.56    | 2.71  | 1.53  | 1.94 | 2.91 | 2.56 | 2.46 |
| $\Sigma^0_{(uds)}$ | 0.84   | 0.90    | 0.92  | 0.52  | 0.65 | 1.00 | 0.84 | ?   |
| $\Sigma^-_{(dds)}$ | -1.00  | -0.77   | -0.87 | -0.50 | -0.64 | -0.91 | -0.89 | -1.16 |
| $\Lambda^0_{(uds)}$ | -0.67  | -0.90   | -0.92 | -0.55 | -0.69 | -0.99 | -0.84 | -0.61 |
| $\Xi^-_{(uss)}$  | -1.51  | -1.79   | -1.84 | -0.94 | -1.17 | -2.00 | -1.68 | -1.25 |
| $\Xi^-_{(dss)}$  | -0.59  | -0.77   | -0.87 | -0.45 | -0.57 | -0.83 | -0.82 | -0.65 |

Decuplet Baryons

| Baryon          | NR     | Instant | Point | Front | EXP |
|-----------------|--------|---------|-------|-------|-----|
| $\Omega^-_{(sss)}$ | -2.01  | -2.18   | -1.59 | -0.93 | -1.34 | -2.09 | -1.79 | -2.019 |
| $\Delta^{++}_{(uww)}$ | 5.52   | 4.37    | 5.1   | 2.53  | 3.47 | 5.38 | 5.21 | 4.52 |

values.

The results that are obtained with different masses for the $u, d$ and the $s$ quark are shown in Table 3. It is obvious from these results that the calculated magnetic moment values are overall much closer to the empirical values, when the constituent mass of the strange quark is given a value that is $\sim$ 100 - 200 MeV larger than the constituent mass of the light flavor quarks. It also emerges that the magnetic moment values that are obtained with instant and front form kinematics are closer to the empirical values than those obtained in point form, when the spatial wave function of the hyperons is taken to be the same as that for the nucleon.

The dependence of the calculated magnetic moment of the $p, \Sigma^+, \Delta^{++}$ and the $\Omega^-$ on the value of the constituent quark mass is shown in Fig. 1. The figure reveals that in point form kinematics there is no value for the light flavor constituent mass where the calculated value would agree with the empirical value, whereas in both instant and in front form kinematics the curves cross the horizontal line, which represents the empirical value. Similar but clearer features can be seen in Fig. 2 where the mean square charge radii of the proton and the $\Sigma^-$ are depicted as functions of the light quark mass. In instant and
Table 3
Magnetic moments, in nuclear magnetons, calculated with different masses for $u, d$ and $s$ quarks.

| Baryon          | Instant | Point | Front | EXP   |
|-----------------|---------|-------|-------|-------|
| $(m_u, m_s)$    | (140,470) | (350,470) | (250,470) | (250,470) |
| Octet Baryons   |         |       |       |       |
| $p_{(uud)}$     | 2.71    | 2.57  | 2.45  | 1.95  | 2.78  | 3.12  | 2.79  |
| $n_{(udd)}$     | −1.84   | −1.80 | −1.63 | −1.32 | −1.67 | −1.99 | −1.91 |
| $\Sigma^{+}_{(uus)}$ | 2.61    | 2.56  | 1.97  | 1.69  | 2.59  | 3.47  | 2.46  |
| $\Sigma^{0}_{(uds)}$ | 0.79    | 0.78  | 0.65  | 0.56  | 0.75  | 0.94  | ?     |
| $\Sigma^{-}_{(dds)}$ | −1.03   | −1.00 | −0.67 | −0.55 | −1.08 | −1.60 | −1.16 |
| $\Lambda^{0}_{(uds)}$ | −0.55   | −0.56 | −0.68 | −0.63 | −0.55 | −0.50 | −0.61 |
| $\Xi^{0}_{(uss)}$ | −1.35   | −1.37 | −1.21 | −1.15 | −1.31 | −1.52 | −1.25 |
| $\Xi^{-}_{(dss)}$ | −0.40   | −0.38 | −0.57 | −0.55 | −0.59 | −0.74 | −0.65 |

In front form kinematics there is again a much stronger dependence on the mass of the quark than in point form.

Finally Table 4 contains the calculated values of the charge radii of the baryons with different $u, d$ and $s$ quark masses. The calculated charge radii for the hyperons are very similar in instant and front form kinematics. For the proton, neutron and the $\Sigma^-$ hyperon, for which empirical values are known, the calculated values are also close to the empirical values. In the case of the neutron, agreement with the empirical value does however require the introduction of a mixed symmetry $S-$state component into the ground state wave function, with a norm of 0.01 - 0.02.

The point form value for the mean square charge radius of the $\Sigma^-$ is considerably smaller than the empirical value. This is clearly related to the fact that in point form kinematics the magnetic moment of the $\Sigma^-$ also is only about half of the experimental value. The same conclusion applies to the case of the calculated charge radius and magnetic moment of the $\Omega^-$ hyperon. This implies that in point form kinematics, the spatial wave function of the hyperons has to differ significantly from that of the nucleons. In order to get the empirical value for the mean square charge radius of the $\Sigma^-$ hyperon in point form kinematics the parameter $b$ in the spatial wave function (10) has to be decreased from the value 640 MeV in Table 2 to 300 MeV, which corresponds to more than doubling the matter radius. This increases both the calculated mean square radii and the magnetic moments of most of the strange hyperons to agree better with both the empirical results and the results obtained with the two other forms of kinematics. This is shown in Table 5.
5 Magnetic Moments and Charge Radii of the Charm Hyperons

5.1 Charm 1 Hyperons

The $SU(3)_F$ quark model may be directly extended to $SU(4)_F$ to encompass the charm baryons. The flavor symmetry breaking is in this case large because of the large difference between the charm and light flavor constituent quark masses. The main flavor symmetry breaking appears through the quark currents and the boosts and spin rotations, which depend explicitly on the constituent mass. The flavor symmetry breaking in the mass operator may either be treated perturbatively or in the form of a flavor and spin dependent hyperfine term, which is independent of the spatial coordinates [7]. This makes it possible to employ a flavor independent spatial wave function and to use the same spatial wave function for the charm and strange hyperons.

For $SU(4)$ symmetry there are two 20plets, one of which corresponds to mixed symmetry states and the other one to the completely symmetric flavor wave.
Fig. 2. Mean square charge radius of the proton and the $\Sigma^-$ as a function of the light quark mass, $m_{u,d}$. Solid, dotted and dashed lines correspond to instant, point and front forms of kinematics. The thick line corresponds to the experimental value.

Table 4
Mean square charge radii in fm$^2$ calculated with different $u$, $d$ and $s$ quark masses. The constituent mass used for the $\Omega^-$ is 470 MeV. For the neutron the values obtained with a small mixed symmetry $-S$ state component in the wave function as in Ref. [3] are shown in brackets.

| Baryon      | Instant      | Point        | Front      | EXP          |
|-------------|--------------|--------------|------------|--------------|
| $(m_u, m_s)$| (140,470)    | (350,470)    | (250,470)  |              |

Octet Baryons

| Baryon      | Instant      | Point        | Front      |
|-------------|--------------|--------------|------------|
| $p_{(uud)}$ | 0.79         | 0.71         | 0.71       |
| $n_{(udd)}$ | 0.00 (−0.14) | −0.01 (−0.13)| −0.02 (−0.09) |
| $\Sigma^+_1(uus)$ | 1.10 | 0.47 | 0.94 |
| $\Sigma^0_{(uds)}$ | 0.19 | 0.02 | 0.13 |
| $\Sigma^-_{(dds)}$ | 0.72 | 0.41 | 0.61 |
| $\Lambda^0_{(uds)}$ | 0.19 | −0.01 | 0.12 |
| $\Xi^0_{(uss)}$ | 0.34 | 0.01 | 0.22 |
| $\Xi^-_{(dss)}$ | 0.61 | 0.35 | 0.57 |

Decuplet Baryons

| Baryon      | Instant      | Point        |
|-------------|--------------|--------------|
| $\Omega^-_{(sss)}$ | 0.54 | 0.25 | 0.51 |
| $\Delta^{++}_{(uuu)}$ | 1.64 | 0.85 | 1.47 |

states. The completely symmetric flavor states represent spin 3/2 states, while the mixed symmetry ones represent spin 1/2 states.
Table 5
Magnetic moments, in nuclear magnetons, and mean square charge radii, in fm$^2$, calculated with $m_{u,d} = 350$ MeV and $m_s = 470$ MeV using point form of relativistic kinematics. The results are obtained making use of two different values for the oscillator parameter $b$.

| Baryon | (b=640) | (b=300) | EXP |
|--------|--------|--------|-----|
| Magnetic moments | | | |
| $\Sigma^+_\langle uus\rangle$ | 1.97 | 2.20 | 2.46 |
| $\Sigma^0\langle uds\rangle$ | 0.65 | 0.71 | ? |
| $\Sigma^-\langle dds\rangle$ | -0.67 | -0.77 | -1.16 |
| $\Lambda^0\langle uds\rangle$ | -0.68 | -0.69 | -0.61 |
| $\Xi^0\langle uss\rangle$ | -1.21 | -1.31 | -1.25 |
| $\Xi^-\langle dss\rangle$ | -0.57 | -0.58 | -0.65 |
| Mean square charge radii | | | |
| $\Sigma^+_\langle uus\rangle$ | 0.47 | 0.66 | |
| $\Sigma^0\langle uds\rangle$ | 0.02 | 0.02 | |
| $\Sigma^-\langle dds\rangle$ | 0.41 | 0.64 | 0.61 [10] |
| $\Lambda^0\langle uds\rangle$ | -0.01 | 0.02 | |
| $\Xi^0\langle uss\rangle$ | 0.01 | 0.03 | |
| $\Xi^-\langle dss\rangle$ | 0.35 | 0.52 | |

The wave functions for the mixed symmetry 20plet are listed in Table A.1. The corresponding wave functions for the symmetric 20plet are readily constructed by symmetrization of the quark content of the baryon.

The spin-flavor matrix elements that enter in the calculation of the magnetic moments and charge radii of the charm hyperons may, in analogy with the case of the strange hyperons above, be written as $\langle B|SF|B'\rangle$. Here the flavor operator $F_c$ is the extension to charm of the charge operator (17):

$$F_c = \left\{ \frac{\lambda_3}{2} + \frac{\lambda_8}{2\sqrt{3}} + \frac{1}{6}(1 - \sqrt{6}\lambda_{15}) \right\}.$$

The explicit expressions for the matrix elements of the flavor operator as obtained with the wave functions in Table A.1 are given in Appendix A. These expressions reproduce the static results [11] when the spin operator is approximated by the static operator of Eq. (18). In the relativistic case the matrix elements of the quark operator depend on the form of kinematics under consideration.
Tables 6 and 7 contain the calculated values of the magnetic moments of charm baryons with mixed symmetry for the three different forms of kinematics. The results in Table 6 are given for both zero and finite values for the mass of the light quarks, while the strange and charm quark are given the constituent masses 470 MeV and 1500 MeV respectively. In instant and point form kinematics the results are insensitive to the value of the constituent mass of the light flavor quark, whereas in front form the calculated values are quite sensitive to the light quark mass. When the light flavor quarks have finite constituent masses the results are very similar for all three forms of kinematics.

To show the sensitivity of the calculated magnetic moments of the charm hyperons to the value of the constituent mass of the strange quark, the results that are obtained by setting the mass of the strange quark, along with those of the light flavor quarks, to zero are shown in Table 7. We therefore employ a finite value for the mass of the charm and zero mass for the $u, d$ and $s$ quarks. Both instant and front form values reveal notable sensitivity to the value of the strange quark mass, whereas the point form values are again very insensitive to the strange quark mass.

In the absence of empirical data on the magnetic moments of the charm hyperons, the values obtained by the static quark model with the corresponding values are given as a reference. The calculated values in instant form kinematics are very similar to the static quark model values, independently of the value of the constituent mass of the light flavor quarks. This similarity also holds in the case of front form kinematics, although only if the constituent mass of the light flavor quarks is finite.

As in the case of the strange hyperons, point form kinematics does, however, give much smaller values for the magnetic moments of the charm hyperons than both the instant and front form kinematics. The reason for this is that the large hyperon mass appears in the boost velocity in the case of point form kinematics. The larger values are expected to be the more realistic ones, however, especially in view of the fact that those values are also similar to the values given by the extension to charm of the Skyrme model [13].

Table 8 contains the calculated values for the mean square charge radii that are obtained with different masses for the light, strange and charmed quarks. The point form results again deviate considerably from the instant and front form results, which are similar to one another. In particular the charge radii obtained for the neutral hyperons in point form are usually much smaller than the those obtained with instant and front forms of kinematics. This feature is already noticeable in the results of Table 4 for the strange baryons.

In the case of the strange hyperons it was found that acceptable values of the calculated magnetic moments and mean square charge radii are obtained in
Table 6
Magnetic moments, in nuclear magnetons, of the charm hyperons calculated with different masses for \( u, d, s \) and \( c \) quarks for the three forms of kinematics. The strange mass used is 470 MeV in the three cases. The charm quark mass is taken to be 1500 MeV. Here \( M_{\Xi^{++}} = M_{\Xi^{+}} = 3.5 \text{ GeV} \) \[12\] and \( M_{\Omega^{+}} = 3.7 \text{ GeV} \). The column NR shows the corresponding values in the static quark model with these mass values and \( m_u = m_d = 350 \text{ MeV} \).

| Baryon       | Instant | Point | Front | NR  |
|--------------|---------|-------|-------|-----|
| \((m_u)\)    | 140     | 350   | 250   | 0   |

**Mixed symmetry 20plet: \((6,1)\)**

| \( \Sigma^{++}_c (uuc) \) | 2.36 | 2.39 | 1.00 | 0.90 | 2.18 | 3.07 | 2.24 |
| \( \Sigma^+_c (udc) \)   | 0.49 | 0.49 | 0.13 | 0.11 | 0.44 | 0.65 | 0.46 |
| \( \Sigma^0_c (ddc) \)   | −1.38 | −1.40 | −0.74 | −0.67 | −1.31 | −1.78 | −1.33 |
| \( \Xi^{++}_c (usc) \)   | 0.75 | 0.78 | 0.15 | 0.13 | 0.67 | 1.13 | 0.61 |
| \( \Xi^0_c (dsc) \)      | −1.12 | −1.14 | −0.73 | −0.70 | −1.24 | −1.51 | −1.18 |
| \( \Omega^0_c (ssc) \)   | −0.86 | −0.86 | −0.67 | −0.67 | −0.90 | −0.90 | −1.03 |

**Mixed symmetry 20plet: \((3,1)\)**

| \( \Xi^{++}_c (usc) \)   | 0.40 | 0.39 | 0.47 | 0.45 | 0.40 | 0.39 | 0.42 |
| \( \Xi^0_c (dsc) \)      | 0.41 | 0.42 | 0.47 | 0.46 | 0.40 | 0.39 | 0.42 |
| \( \Lambda^+_c (udc) \)  | 0.40 | 0.39 | 0.52 | 0.49 | 0.43 | 0.41 | 0.42 |

**Mixed symmetry 20plet: \((3,2)\)**

| \( \Xi^{++}_{cc} (ucc) \) | −0.10 | −0.13 | 0.30 | 0.29 | −0.04 | −0.32 | −0.04 |
| \( \Xi^+_{cc} (dcc) \)   | 0.86 | 0.87 | 0.69 | 0.68 | 0.83 | 0.94 | 0.85 |
| \( \Omega^+_{cc} (ssc) \) | 0.72 | 0.72 | 0.66 | 0.66 | 0.74 | 0.74 | 0.78 |

Point form kinematics only if the parameter \( b \) in the spatial wave function model (10) is reduced from 640 MeV to 300 MeV. This also applies to the case of the charm hyperons as shown in Tables 9 and 10, where the calculated magnetic moments and mean square charge radii are shown for both \( b = 640 \text{ MeV} \) and \( b = 300 \text{ MeV} \) with point form kinematics. The substantial reduction of the parameter \( b \) again leads values for both observables, which are much closer to the values that are obtained with instant and front form kinematics.

5.2 Doubly Charmed Hyperons

The recent discovery of doubly charmed hyperons, with rest energies at 3460, 3520 and 3780 MeV has revived the interest in the structure of those baryons.
Table 7
Magnetic moments, in nuclear magnetons, calculated with zero mass for the light quarks, \(u, d\) and \(s\) and with the charm quark mass as 1500 MeV.

| Baryon        | Instant | Point | Front |
|---------------|---------|-------|-------|
| \(\Xi^{s+}_c(usc)\) | 0.49    | 0.11  | 0.64  |
| \(\Xi^{s0}_c(dsc)\) | −1.40   | −0.67 | −1.96 |
| \(\Omega^0_c(ssc)\) | −1.40   | −0.61 | −1.77 |
| \(\Xi^{a+}_c(usc)\) | 0.39    | 0.43  | 0.36  |
| \(\Xi^{a0}_c(dsc)\) | 0.43    | 0.45  | 0.35  |
| \(\Omega^{+}_{cc}(scc)\) | 0.87    | 0.65  | 0.94  |

[12,14]. It is natural to identify the state at 3460 MeV as the \(\Xi^{+}_{cc}\), pending verification that its spin and parity assignment be \(1/2^+\). The state at 3520 MeV may most naturally be identified as the \(\Xi^{+}_{cc}\) in view of the prediction that the splitting between these states should be expected to be \(\sim 60 - 90\) MeV [7,15,16,17,18]. The state at 3780 MeV is likely to be the lowest negative parity state, in view of the prediction that this should fall around 300 MeV above the ground state [15].

The magnetic moments of the ground states of the doubly charmed hyperons may be calculated by the same methods as used above for the strange and singly charmed hyperons. In Table 6 the calculated magnetic moments of the ground state \(ucc\) (\(\Xi^{++}_{cc}\)), \(ccd\) (\(\Xi^{+}_{cc}\)) and \(csc\) (\(\Omega^{+}_{cc}\)) are given as obtained in the three different forms of kinematics. Here the mass of the \(\Xi^{++}_{cc}\) has been taken as 3.5 GeV and that of the \(\Omega^{+}_{cc}\) as 3.7 MeV. The latter value is suggested by the lattice calculation in Ref. [18]. For reference we note that the static quark model expressions for the magnetic moments (in units of nuclear magnetons) of these hyperons are:

\[
\mu(\Xi^{++}_{cc}) = \frac{8}{9} \frac{m_p}{m_c} - \frac{2}{9} \frac{m_p}{m_u}, \\
\mu(\Xi^{+}_{cc}) = \frac{8}{9} \frac{m_p}{m_c} + \frac{1}{9} \frac{m_p}{m_u}, \\
\mu(\Omega^{+}_{cc}) = \frac{8}{9} \frac{m_p}{m_c} + \frac{1}{9} \frac{m_p}{m_s}. \tag{20}
\]

Here \(m_u\), \(m_s\) and \(m_c\) are the constituent masses of the light flavor, the strange and the charm quarks respectively.
Table 8
Mean square charge radii (in fm\(^2\)) of the charm hyperons calculated with different masses for quarks \(u, d, s\) and \(c\). The masses of the strange and charm quarks, \(m_s\) and \(m_c\), are taken to be 470 MeV and 1.5 GeV respectively.

| Baryon           | Instant | Point | Front |
|------------------|---------|-------|-------|
| \(\Sigma^{++}(uuc)\) | 1.7     | 0.4   | 1.4   |
| \(\Sigma^+_c(ude)\)  | 0.5     | 0.2   | 0.4   |
| \(\Sigma^0_c(ddc)\)  | −0.7    | −0.0  | −0.6  |
| \(\Xi^{s+}(usc)\)   | 0.6     | 0.2   | 0.5   |
| \(\Xi^{s0}(dsc)\)   | −0.6    | −0.0  | −0.5  |
| \(\Omega^0_c(ssc)\) | −0.4    | −0.0  | −0.4  |
| \(\Xi^{a+}(usc)\)   | 0.6     | 0.2   | 0.5   |
| \(\Xi^{a0}(dsc)\)   | −0.6    | −0.0  | −0.5  |
| \(\Lambda^+_c(ude)\) | 0.5     | 0.2   | 0.4   |
| \(\Xi^{++}_{cc}(ucc)\) | 1.3     | 0.2   | 1.0   |
| \(\Xi^{+}_{cc}(dcc)\) | 0.1     | 0.1   | 0.0   |
| \(\Omega^{+}_{cc}(scc)\) | 0.2     | 0.1   | 0.1   |

The calculated magnetic moments of the doubly charmed hyperons are very similar in instant and in front form kinematics. They are also close to the static quark model values. In contrast the point form values differ notably from these, in particular in the case of the \(\Xi^{++}_{cc}\), for which the point form result is positive while both instant and front give a negative result for the magnetic moment.

In Table 7 the calculated result for the magnetic moment of the \(\Omega^{+}_{cc}\) is also shown for the case of zero constituent mass for the strange quark. Only the front form result shows significant dependence on the quark mass, in analogy with the case of the strange hyperons.

In Table 8 the calculated mean square charge radii of the ground state doubly charmed hyperons are listed. As in the case of the magnetic moments, the instant and front form values are similar, while the point form values are much smaller.
Table 9
Magnetic moments, in nuclear magnetons, of the charm hyperons calculated in point form with $m_{u,d} = 350 \text{ MeV}$, $m_s = 470 \text{ MeV}$ and $m_c = 1.5 \text{ GeV}$. Here $M_{\Xi^{+}_{cc}} = M_{\Xi^{0}_{cc}} = 3.5 \text{ GeV}$ [12] and $M_{\Omega^+_{cc}} = 3.7 \text{ GeV}$. The column NR shows the corresponding values in the static quark model with these mass values.

| Baryon               | (b=640) | (b=300) | NR  |
|----------------------|---------|---------|-----|
| $\Sigma^{++}_{c}(ucc)$ | 1.00    | 1.39    | 2.24|
| $\Sigma^+_c(udc)$   | 0.13    | 0.24    | 0.46|
| $\Sigma^0_c(ddc)$   | −0.74   | −0.91   | −1.33|
| $\Xi^{++}_c(usc)$   | 0.15    | 0.30    | 0.61|
| $\Xi^{0}_c(dsc)$    | −0.73   | −0.88   | −1.18|
| $\Omega^0_c(ssc)$   | −0.67   | −0.78   | −1.03|

| Baryon               | (b=640) | (b=300) | NR  |
|----------------------|---------|---------|-----|
| $\Xi^{++}_c(usc)$   | 0.47    | 0.42    | 0.42|
| $\Xi^{a0}_c(dsc)$   | 0.47    | 0.42    | 0.42|
| $\Lambda^{+}_c(udc)$| 0.52    | 0.46    | 0.42|

| Baryon               | (b=640) | (b=300) | NR  |
|----------------------|---------|---------|-----|
| $\Xi^{++}_c(ucc)$   | 0.30    | 0.17    | −0.04|
| $\Xi^{+}_{cc}(dcc)$ | 0.69    | 0.74    | 0.85|
| $\Omega^+_c(scc)$   | 0.66    | 0.69    | 0.78|

6 Discussion

The main finding above is the demonstration that the phenomenological description of baryon magnetic moments may be carried over into the Poincaré covariant quark model, and with the additional gain of a concomitant description of the charge radii, and, for the nucleons and the $\Delta(1232)$ resonance, of the electromagnetic form factors as well. In comparison with the static quark model the gain is the dynamical consistency of the calculation. Formally the calculation of the current matrix elements is considerably more cumbersome in the relativistic quark model, the results of which coincide with those of the static quark model only when $m_q/m_n \gg 1$.

Another notable finding is that if the same spatial wave function is employed the strange hyperon magnetic moments and mean square charge radii calculated in point form kinematics are considerably farther from the empirical magnetic moments than the values calculated in instant and front form kine-
Table 10
Mean square charge radii (in fm²) of the charm hyperons calculated in point form with $m_{u,d} = 350$ MeV, $m_s = 470$ MeV and $m_c = 1500$ MeV.

| Baryon          | (b=640) | (b=300) |
|-----------------|---------|---------|
| Mixed symmetry 20plet: (6,1) |         |         |
| $\Sigma^{++}(wuc)$ | 0.4     | 1.1     |
| $\Sigma^+_c(udc)$  | 0.2     | 0.5     |
| $\Sigma^0_c(ddc)$  | -0.0    | -0.1    |
| $\Xi^+_c(usc)$     | 0.2     | 0.5     |
| $\Xi^0_c(dsc)$     | -0.0    | -0.1    |
| $\Omega^0_c(ssc)$  | -0.0    | -0.0    |
| Mixed symmetry 20plet: (3,1) |         |         |
| $\Xi^{o+}_c(usc)$  | 0.2     | 0.5     |
| $\Xi^0_c(dsc)$     | -0.0    | -0.0    |
| $\Lambda^+_c(udc)$ | 0.2     | 0.6     |
| Mixed symmetry 20plet: (3,2) |         |         |
| $\Xi^{++}_{cc}(ucc)$ | 0.2     | 0.5     |
| $\Xi^+_c(dcc)$     | 0.1     | 0.2     |
| $\Omega^+_{cc}(ssc)$ | 0.1     | 0.2     |

Both instant and front form kinematics lead to very similar values for the hyperon magnetic moments, and moreover to values, which are close to the empirical values. Only if the spatial extent of the wave function of the strange hyperons is more than double that of the spatial extent of the nucleon wave function does the calculation in point form kinematics lead to values that are close to the corresponding empirical ones. A similar conclusion applies to the case of the charm hyperons, for which point form kinematics in most cases yields much smaller magnetic moment values than instant and front form unless the spatial extent of the wave function is more than double that of the nucleons. That point form kinematics differs qualitatively from the other two forms has been noted in another context in Ref. [19].

Finally the present results suggest that the apparent success of the static quark model with constituent masses in the range $300 - 400$ MeV in describing the empirical baryon magnetic moments with $\sim 10 - 15\%$ accuracy is largely an accidental consequence of the correct description of the flavor structure of current operators.

The present phenomenological approach to the baryon magnetic moments has
no direct link to QCD. There are several QCD based approaches to the ob-
servables of the baryons in the low-energy range. These are based on effective
field theory [20,21] or on the large color limit [22,23] and in both cases lead to
a description in terms of effective operators. The issue of Poincaré covariance
may, to a good approximation, be avoided in leading order in the effective
operator expansion.

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A Flavor matrix elements

Here the explicit expressions for the spin-flavor matrix elements that enter in
the evaluation of both the magnetic moments and the charge radii are given.
These matrix elements arise upon calculation of the matrix element of the
flavor operator with the wave functions of Table A.1.

A.1 Non-strange non-charm sector

The expressions for the octet baryons are,

\begin{align}
\langle p | SF | p \rangle &= \frac{1}{3} \langle S_{MA} | S | S_{MA} \rangle_u \\
\langle n | SF | n \rangle &= \frac{1}{6} \langle S_{MS} | S | S_{MS} \rangle_u - \frac{1}{6} \langle S_{MA} | S | S_{MA} \rangle_u \\
\langle \Sigma^+ | SF | \Sigma^+ \rangle &= \frac{1}{3} \langle S_{MA} | S | S_{MA} \rangle_u + \frac{1}{9} \langle S_{MS} | S | S_{MS} \rangle_u \\
&\quad - \frac{1}{9} \langle S_{MS} | S | S_{MS} \rangle_s \\
\langle \Sigma^0 | SF | \Sigma^0 \rangle &= \frac{1}{12} \langle S_{MA} | S | S_{MA} \rangle_u + \frac{1}{36} \langle S_{MS} | S | S_{MS} \rangle_u \\
&\quad - \frac{1}{9} \langle S_{MS} | S | S_{MS} \rangle_s
\end{align}
\[
\langle \Sigma^- | SF | \Sigma^- \rangle = -\frac{1}{6} \langle S_{MA} | S | S_{MA} \rangle_u - \frac{1}{18} \langle S_{MS} | S | S_{MS} \rangle_u - \frac{1}{9} \langle S_{MS} | S | S_{MS} \rangle_s
\]
\[
\langle \Lambda^0 | SF | \Lambda^0 \rangle = \frac{1}{36} \langle S_{MA} | S | S_{MA} \rangle_u + \frac{1}{12} \langle S_{MS} | S | S_{MS} \rangle_u - \frac{1}{9} \langle S_{MA} | S | S_{MA} \rangle_s
\]
\[
\langle \Xi^- | SF | \Xi^- \rangle = -\frac{1}{6} \langle S_{MA} | S | S_{MA} \rangle_s - \frac{1}{18} \langle S_{MS} | S | S_{MS} \rangle_s - \frac{1}{9} \langle S_{MS} | S | S_{MS} \rangle_u
\]
\[
\langle \Xi^0 | SF | \Xi^0 \rangle = -\frac{1}{6} \langle S_{MA} | S | S_{MA} \rangle_s - \frac{1}{18} \langle S_{MS} | S | S_{MS} \rangle_s + \frac{2}{9} \langle S_{MS} | S | S_{MS} \rangle_u
\]

For the decuplet ones,

\[
\langle \Delta^{++} | SF | \Delta^{++} \rangle = 2 \langle S_{S} | S | S_{S} \rangle_u
\]
\[
\langle \Omega^- | SF | \Omega^- \rangle = (-1) \langle S_{S} | S | S_{S} \rangle_s
\]

where for instance, \( \langle S_{MA} | S | S_{MA} \rangle_s \), stands for a matrix element between two mixed antisymmetric states of the spin operator \( S \) where the struck quark is a strange quark. Depending on the baryon the other two quarks could be \( uu \) (\( \Sigma \)) or \( us \) (\( \Xi \)).

### A.2 Strange and charm sector of the mixed symmetry 20plet

The elastic matrix elements required for the 20plet (6,1) are:

\[
\langle \Sigma^{c+} | SF | \Sigma^{c+} \rangle = \frac{1}{3} \langle S_{MA} | S | S_{MA} \rangle_u + \frac{1}{9} \langle S_{MS} | S | S_{MS} \rangle_u + \frac{2}{9} \langle S_{MS} | S | S_{MS} \rangle_c
\]
\[
\langle \Sigma^{c+} | SF | \Sigma^{c+} \rangle = \frac{1}{36} \langle S_{MS} | S | S_{MS} \rangle_u + \frac{4}{18} \langle S_{MS} | S | S_{MS} \rangle_c + \frac{1}{12} \langle S_{MA} | S | S_{MA} \rangle_u
\]
\[
\langle \Sigma^{0c} | SF | \Sigma^{0c} \rangle = -\frac{1}{18} \langle S_{MS} | S | S_{MS} \rangle_u + \frac{4}{18} \langle S_{MS} | S | S_{MS} \rangle_c - \frac{1}{6} \langle S_{MA} | S | S_{MA} \rangle_u
\]
\[
\langle \Xi_c^+ \mid SF \mid \Xi_c^+ \rangle = \frac{1}{18} \langle S_{MS} \mid S \mid S_{MS} \rangle_u - \frac{1}{36} \langle S_{MS} \mid S \mid S_{MS} \rangle_s \\
+ \frac{4}{18} \langle S_{MS} \mid S \mid S_{MS} \rangle_c + \frac{1}{6} \langle S_{MA} \mid S \mid S_{MA} \rangle_u \\
- \frac{1}{12} \langle S_{MA} \mid S \mid S_{MA} \rangle_s \\
\langle \Xi_c^0 \mid SF \mid \Xi_c^0 \rangle = -\frac{1}{36} \langle S_{MS} \mid S \mid S_{MS} \rangle_u - \frac{1}{36} \langle S_{MS} \mid S \mid S_{MS} \rangle_s \\
+ \frac{4}{18} \langle S_{MS} \mid S \mid S_{MS} \rangle_c - \frac{1}{12} \langle S_{MA} \mid S \mid S_{MA} \rangle_u \\
- \frac{1}{12} \langle S_{MA} \mid S \mid S_{MA} \rangle_s \\
\langle \Omega_c^0 \mid SF \mid \Omega_c^0 \rangle = -\frac{1}{18} \langle S_{MS} \mid S \mid S_{MS} \rangle_s + \frac{4}{18} \langle S_{MS} \mid S \mid S_{MS} \rangle_c \\
- \frac{1}{6} \langle S_{MA} \mid S \mid S_{MA} \rangle_s
\]

For the 20plet(3,1) we get,

\[
\langle \Xi_{c3}^+ \mid SF \mid \Xi_{c3}^+ \rangle = \frac{1}{6} \langle S_{MS} \mid S \mid S_{MS} \rangle_u - \frac{1}{12} \langle S_{MS} \mid S \mid S_{MS} \rangle_s \\
+ \frac{1}{18} \langle S_{MA} \mid S \mid S_{MA} \rangle_u - \frac{1}{36} \langle S_{MA} \mid S \mid S_{MA} \rangle_s \\
+ \frac{4}{18} \langle S_{MA} \mid S \mid S_{MA} \rangle_c \\
\langle \Xi_{c3}^0 \mid SF \mid \Xi_{c3}^0 \rangle = -\frac{1}{12} \langle S_{MS} \mid S \mid S_{MS} \rangle_u - \frac{1}{12} \langle S_{MS} \mid S \mid S_{MS} \rangle_s \\
- \frac{1}{36} \langle S_{MA} \mid S \mid S_{MA} \rangle_u - \frac{1}{36} \langle S_{MA} \mid S \mid S_{MA} \rangle_s \\
+ \frac{4}{18} \langle S_{MA} \mid S \mid S_{MA} \rangle_c \\
\langle \Lambda_{c3}^+ \mid SF \mid \Lambda_{c3}^+ \rangle = \frac{1}{12} \langle S_{MS} \mid S \mid S_{MS} \rangle_u + \frac{1}{36} \langle S_{MA} \mid S \mid S_{MA} \rangle_u \\
+ \frac{4}{18} \langle S_{MA} \mid S \mid S_{MA} \rangle_c
\]

For the 20plet(3,2) the matrix elements are:

\[
\langle \Xi_{cc}^{++} \mid SF \mid \Xi_{cc}^{++} \rangle = \frac{4}{18} \langle S_{MS} \mid S \mid S_{MS} \rangle_u + \frac{2}{18} \langle S_{MS} \mid S \mid S_{MS} \rangle_c \\
+ \frac{1}{3} \langle S_{MA} \mid S \mid S_{MA} \rangle_c \\
\langle \Xi_{cc}^+ \mid SF \mid \Xi_{cc}^+ \rangle = -\frac{2}{18} \langle S_{MS} \mid S \mid S_{MS} \rangle_u + \frac{2}{18} \langle S_{MS} \mid S \mid S_{MS} \rangle_c
\]
The non-zero transition matrix elements are:

\[
\langle \Sigma^+_c | S | \Lambda_c \rangle = \frac{1}{2\sqrt{12}} \langle S_{MS} | S | S_{MS} \rangle_u - \frac{1}{2\sqrt{12}} \langle S_{MA} | S | S_{MA} \rangle_u
\]

\[
\langle \Xi^+_c | S | \Xi^+_{c\bar{c}} \rangle = \frac{1}{6\sqrt{3}} \langle S_{MS} | S | S_{MS} \rangle_u + \frac{1}{12\sqrt{3}} \langle S_{MS} | S | S_{MS} \rangle_s - \frac{1}{6\sqrt{3}} \langle S_{MA} | S | S_{MA} \rangle_u - \frac{1}{12\sqrt{3}} \langle S_{MA} | S | S_{MA} \rangle_s
\]

\[
\langle \Xi^0_c | S | \Xi^0_{c\bar{c}} \rangle = -\frac{1}{12\sqrt{3}} \langle S_{MS} | S | S_{MS} \rangle_u + \frac{1}{12\sqrt{3}} \langle S_{MS} | S | S_{MS} \rangle_s + \frac{1}{12\sqrt{3}} \langle S_{MA} | S | S_{MA} \rangle_u - \frac{1}{12\sqrt{3}} \langle S_{MA} | S | S_{MA} \rangle_s.
\]

Here for example, \( \langle S_{MS} | S | S_{MS} \rangle_u \) stands for a matrix element between two mixed symmetric states of the spin operator \( S \), where the struck quark is a charm quark. Depending on the baryon the other two quarks could be for example \( uu \) (\( \Sigma^{++} \)) or \( us \) (\( \Xi^{a+} \)).
Table A.1
SU(4)\textsubscript{F} wave functions of the mixed symmetry baryons. We also include the non-strange ones for completeness.

| Baryon | \(\Phi_{MS}\) | \(\Phi_{MA}\) |
|--------|----------------|----------------|
| \(p\)  | \(\frac{1}{\sqrt{6}}[uud + udu - 2dnu]\) | \(\frac{1}{\sqrt{2}}(uud - udu)\) |
| \(n\)  | \(\frac{1}{\sqrt{6}}[ddu + dud - 2udd]\) | \(\frac{1}{\sqrt{2}}(dud - ddu)\) |
| \(\Sigma^+\) | \(\frac{1}{\sqrt{6}}[uus +usu - 2usu]\) | \(\frac{1}{\sqrt{2}}(uus - usu)\) |
| \(\Sigma^0\) | \(\frac{1}{\sqrt{6}}[\frac{udu + dus}{\sqrt{2}} + \frac{uds + dus}{\sqrt{2}} - \frac{udu + sud}{\sqrt{2}}]\) | \(\frac{1}{\sqrt{2}}[\frac{uds + dus}{\sqrt{2}} - \frac{dus + sud}{\sqrt{2}}]\) |
| \(\Sigma^-\) | \(\frac{1}{\sqrt{6}}[dds + dsd - 2sdd]\) | \(\frac{1}{\sqrt{2}}(dds - dsd)\) |
| \(\Lambda^0\) | \(\frac{1}{\sqrt{2}}[\frac{uds - dus}{\sqrt{2}} + \frac{udu - dus}{\sqrt{2}}]\) | \(\frac{1}{\sqrt{6}}[\frac{uds - dus}{\sqrt{2}} + \frac{dus - dus}{\sqrt{2}} - \frac{udu - sud}{\sqrt{2}}]\) |
| \(\Xi^-\) | \(\frac{1}{\sqrt{6}}[sds + ssd - 2sds]\) | \(\frac{1}{\sqrt{2}}(sds - ssd)\) |
| \(\Xi^0\) | \(\frac{1}{\sqrt{6}}[sus + ssu - 2uss]\) | \(\frac{1}{\sqrt{2}}(sus - ssu)\) |

Mixed symmetry 20plet: (3,1)

| \(\Lambda^+_c\) | \(\frac{1}{\sqrt{6}}[\frac{udc - dec}{\sqrt{2}} + \frac{ucd - dec}{\sqrt{2}}]\) | \(\frac{1}{\sqrt{6}}[\frac{udc - dec}{\sqrt{2}} + \frac{dec - ucd}{\sqrt{2}}]\) |
| \(\Xi^+_c\) | \(\frac{1}{\sqrt{6}}[\frac{usc - suc}{\sqrt{2}} + \frac{usc - suc}{\sqrt{2}}]\) | \(\frac{1}{\sqrt{6}}[\frac{usc - suc}{\sqrt{2}} + \frac{usc - suc}{\sqrt{2}} - \frac{usc - suc}{\sqrt{2}}]\) |
| \(\Xi^0_c\) | \(\frac{1}{\sqrt{6}}[\frac{dsc - sdc}{\sqrt{2}} + \frac{dsc - sdc}{\sqrt{2}}]\) | \(\frac{1}{\sqrt{6}}[\frac{dsc - sdc}{\sqrt{2}} + \frac{sdc - sdc}{\sqrt{2}} - \frac{dsc - sdc}{\sqrt{2}}]\) |

Mixed symmetry 20plet: (3,2)

| \(\Xi_{cc}^{++}\) | \(\frac{1}{\sqrt{6}}[ecu + cuc - 2ucc]\) | \(\frac{1}{\sqrt{2}}[ecu - cuc]\) |
| \(\Xi_{cc}^+\) | \(\frac{1}{\sqrt{6}}[ecd + cdc - 2dcd]\) | \(\frac{1}{\sqrt{2}}[ecd - cdc]\) |
| \(\Omega_{cc}^+\) | \(\frac{1}{\sqrt{6}}[ces + csc - 2scc]\) | \(\frac{1}{\sqrt{2}}[ces - csc]\) |
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