Improving the efficiency of the hybrid algorithm for solving inhomogeneous minimax problem

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Abstract. The article deals with the problem of inhomogeneous minimax problem solution, what is typical of scheduling theory. This problem is NP-complete, and there is no exact algorithm for it, which has a polynomial time for large-scale problems. Therefore a quick algorithm that gets approximate tables is used. A possible method for solving this problem is considered a hybrid model, representing the synthesis of two genetic algorithms models, namely models Goldberg and CGS. Goldberg’s model is viewed from multiple crossovers and most promising mutation. As it is difficult to make calculations analytically and often impossible to make it in practice, the computational experiment was carried out in this article. The results of the experiment are described in the tables, which graphically show a comparison of the hybrid model effectiveness. The comparison is based on the accuracy results obtained for two types of crossovers with the basic parameters of genetic algorithm. It is proved that the use of hybrid algorithm leads to the results which are more précised to the optimal ones, despite the deterioration in the temporary search characteristics solutions.

1 Introduction

Currently the development of methods for producing suboptimal approximate solutions to NP-complete problems in scheduling theory is relevant [1]. As the part of scheduling theory the solution of many problems are described, NP-complete problems are different from others that is why it is practically impossible to find a solution for polynomial time. For suboptimal solutions a variety of algorithms, including genetic ones are used. In terms of scheduling theory distribution problem can be formulated as follows. There is a service system consisting of N independent units \( P = \{ p_1, p_2, ..., p_n \} \) Service end enters the flow of M - a plurality of independent parallel tasks (functional operators) \( T = \{ t_1, t_2, ..., t_m \} \). \( \tau(t_i p_j) \) – service duration assignment unit \( t_i \) and defined by a matrix \( \mathbf{T} \). Devices are generally not identical, an assignment \( t_i \) can be performed on any of them. One device cannot

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simultaneously handle more than one assignment \( p_j \). It is necessary to determine a distribution of tasks on devices without interruption to make the entire set of tasks with a minimum time spending. The criterion to minimize the task execution time is the minimax criterion, which is defined as follows: \( f = \max_{1 \leq j \leq n} f_j \rightarrow \min \), where \( f_j = \sum_{t(t_i, p_j) \in T} \tau(t_i, p_j) \)

processor time shutdown \( p_j \) [2,3].

2 Materials and methods

In this article, as a basic algorithm for solving a non-uniform minimax problem, the modified Goldberg model is taken, what is a type of genetic algorithms model (hereinafter GA).

Goldberg modified model is different from a classical Holland one with the fact that the formation of a new generation uses a tournament selection, which allows to improve the algorithm results with various modification mutations by using various crossovers.

Goldberg modified model can be described as a sequence of the following steps:

Step 1. Initial generation, consisting of a predetermined number of individuals is being formed.

Step 2. Tournament selection of individuals and the use of crossover and mutation operators with a known probability of occurrence for a new generation.

Step 3. Check the conditions of the algorithm end, which usually lies in the immutability of the best solution for a given number of generations. If the test was not successful, go to the step 2.

Step 4. The best individual is chosen as the found solution [4, 5 and 6].

Graphically the functioning of Goldberg’s modified model can be represented in the Figure 1. The best individual will be selected and transferred to the next generation. The process is repeated as long as the best individual of a predetermined number of times will not be repeated in the generation.

![Graphical representation of Goldberg’s modified model](image-url)
Various modifications of mutations have been investigated from the whole spectrum which has been chosen as the most promising one, namely the simple mutation, is schematically illustrated in Figure 4:

Goldberg modified model (with the earlier found parameters) should be strengthen with the adding suitable for CGS model, which can be described by the following sequence:

In the final generation, consisting of a predetermined number of individuals, we find a suitable and memorize individual [7, 8]. If we get the best individual from the first time, the final generation is exposed to strong mutations and rerun the modified model of Goldberg. If the newly received end generation of the best individual is repeated then the number of repetitions is increased. If the number of repetitions is equal to early given ones, the resulting specimen is selected as the best solution found. If during extra runs in the final generation specimen will turn out better than the previous ones, the counter will be reset, and additional launches relaunch a modified model of Goldberg.
3 The results of study

Due to the fact that it is very difficult to solve this problem analytically or maybe it is even not possible, computational experiments to gather statistics algorithms have been made [9, 10]. Software tool was written for the realization of computing experiment in modern programming language C# in a development environment of Microsoft Visual Studio 2017.

In the course of computing experiment realization, a series of calculations were done. The study evaluated parameters such as time of search solutions, mean and minimum values obtained in the experiment. Each experiment was repeated 50 times, with 100% probability of crossover and mutation.

The experimental results with a given amount of processing devices (5) and the number of tasks equal to 53 listed in Table No1. Where the column 1 - the estimated parameters; the column 2-data obtained for single-point crossover, without the use of strong mutations; the column 3-data obtained for the two-point crossover, without the use of strong mutations; the column 4 is the number of allowable retries the best individuals with strong mutations. The results were graduated according to the average obtainable meaning.

| 1 | 2 | 3 |
|---|---|---|
| % of the individuals subjected to strong mutations | 100 | 100 | 100 | 100 | 100 |
| % of tasks is subject to severe mutations | 100 | 50 | 33 | 50 | 33 |
| % of bits subjected to strong mutation | 12.5 | 12.5 | 12.5 | 50 | 50 |

| | Single-point crossover with a strong mutation | Two-point crossover strong mutation |
|---|---|---|
| Min | 290 | 286 |
| Mid | 293.9 | 289.08 |
| Time | 28 | 27 |

200 individuals, 200 repetitions

| 4 | 5 | 6 |
|---|---|---|
| % of the individuals subjected to strong mutations | 100 | 100 | 100 |
| % of tasks is subject to severe mutations | 100 | 50 | 33 |
| % of bits subjected to strong mutation | 12.5 | 12.5 | 12.5 |

| | Single-point crossover with a strong mutation | Two-point crossover strong mutation |
|---|---|---|
| Min | 287 | 287 | 287 |
| Mid | 289.48 | 289.44 | 289.6 |
| Time | 155 | 128 | 111 |

200 individuals, 200 repetitions

| | Single-point crossover with a strong mutation | Two-point crossover strong mutation |
|---|---|---|
| Min | 287 | 286 | 286 |
| Mid | 289.28 | 289.14 | 289.24 |
| Time | 179 | 140 | 130 |

200 individuals, 200 repetitions

| | Single-point crossover with a strong mutation | Two-point crossover strong mutation |
|---|---|---|
| Min | 287 | 286 | 286 |
| Mid | 289.12 | 288.8 | 288.68 |
| Time | 209 | 156 | 143 |

200 individuals, 200 repetitions

| | Single-point crossover with a strong mutation | Two-point crossover strong mutation |
|---|---|---|
| Min | 287 | 286 | 287 |
| Mid | 288.92 | 288.38 | 289.1 |
| Time | 233 | 185 | 146 |
| Min | 287 | 286 | 286 | 286 | 288 | 287 | 287 | 286 | 286 | 286 | 286 | 286 | 286 | 286 |
|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|
| Mid | 288.98 | 288.6 | 288.62 | 289.1 | 288.74 | 289.16 | 288.42 | 287.18 | 287.62 | 287.46 | 287.26 | 287.4 | 287.24 | 287.12 |
| Time | 245 | 199 | 171 | 276 | 242 | 272 | 252 | 218 | 179 | 162 | 236 | 198 | 248 | 222 |

400 individuals; 400 repetitions

| Min | 286 | 286 | 286 | 286 | 286 | 286 | 286 | 286 | 286 | 286 | 286 | 286 | 286 | 286 |
|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|
| Mid | 288.38 | 288.54 | 288.78 | 288.42 | 288.48 | 288.72 | 288.68 | 287.02 | 287.22 | 287.42 | 287.24 | 287.3 | 286.82 | 287.3 |
| Time | 354 | 286 | 279 | 342 | 326 | 352 | 316 | 321 | 275 | 263 | 321 | 283 | 349 | 285 |

| Min | 287 | 286 | 287 | 286 | 286 | 286 | 286 | 286 | 286 | 286 | 286 | 286 | 286 | 286 |
|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|
| Mid | 288.38 | 288.28 | 288.68 | 288.44 | 288.22 | 288.44 | 288.46 | 286.86 | 287.12 | 287.2 | 286.82 | 287.14 | 287.02 | 287.24 |
| Time | 413 | 334 | 324 | 404 | 389 | 453 | 389 | 391 | 346 | 341 | 413 | 380 | 406 | 357 |

| Min | 286 | 286 | 286 | 286 | 286 | 286 | 286 | 286 | 286 | 286 | 286 | 286 | 286 | 286 |
|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|
| Mid | 287.8 | 288 | 288.3 | 288.4 | 288.24 | 288.32 | 288.2 | 286.86 | 287.12 | 287.22 | 287.02 | 287.3 | 286.76 | 287.02 |
| Time | 509 | 396 | 374 | 515 | 451 | 536 | 452 | 458 | 419 | 399 | 467 | 398 | 502 | 432 |

| Min | 286 | 286 | 286 | 286 | 286 | 286 | 286 | 286 | 286 | 286 | 286 | 286 | 286 | 286 |
|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|
| Mid | 288.08 | 288.28 | 288.18 | 287.94 | 287.9 | 287.24 | 286.88 | 287 | 287.24 | 286.62 | 286.98 | 286.72 | 286.9 |
| Time | 541 | 416 | 430 | 585 | 484 | 560 | 533 | 530 | 460 | 433 | 574 | 504 | 597 | 503 |

| Min | 286 | 286 | 286 | 286 | 286 | 286 | 286 | 286 | 286 | 286 | 286 | 286 | 286 | 286 |
|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|
| Mid | 287.68 | 288.1 | 288.28 | 288.12 | 287.98 | 288.02 | 287.64 | 286.8 | 287 | 286.98 | 286.68 | 287.04 | 286.6 | 286.84 |
| Time | 651 | 454 | 433 | 667 | 565 | 687 | 608 | 616 | 503 | 525 | 682 | 551 | 678 | 604 |

| Min | 286 | 286 | 286 | 286 | 286 | 286 | 286 | 286 | 286 | 286 | 286 | 286 | 286 | 286 |
|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|
| Mid | 287.74 | 287.86 | 287.98 | 287.96 | 287.78 | 287.84 | 287.96 | 286.52 | 287.04 | 287.24 | 286.5 | 286.92 | 286.44 | 286.9 |
| Time | 730 | 523 | 500 | 746 | 639 | 805 | 625 | 686 | 563 | 547 | 712 | 621 | 743 | 642 |

800 individuals; 800 repetitions

| Min | 286 | 286 | 286 | 286 | 286 | 286 | 286 | 286 | 286 | 286 | 286 | 286 | 286 | 286 |
|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|
| Mid | 288.88 | 287.2 | 287.94 | 287.8 | 287.72 | 287.78 | 287.58 | 287.72 | 287.62 | 287.72 | 286.8 | 286.76 | 286.5 | 286.68 | 286.56 |
| Time | 300 | 298 | 974 | 847 | 840 | 1044 | 960 | 1060 | 960 | 1075 | 986 | 970 | 1067 | 1008 | 1018 | 1024 |

| Min | 286 | 286 | 286 | 286 | 286 | 286 | 286 | 286 | 286 | 286 | 286 | 286 | 286 | 286 |
|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|
| Mid | 287.52 | 287.36 | 287.9 | 287.4 | 287.96 | 287.68 | 287.84 | 286.56 | 286.32 | 286.2 | 286.9 | 286.62 | 286.8 | 286.56 | 286.7 |
| Time | 1217 | 1047 | 972 | 1280 | 1082 | 1281 | 1108 | 1295 | 1168 | 1138 | 1349 | 1209 | 1291 | 1231 |

| Min | 286 | 286 | 286 | 286 | 286 | 286 | 286 | 286 | 286 | 286 | 286 | 286 | 286 | 286 |
|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|
| Mid | 287.56 | 287.62 | 287.84 | 287.56 | 287.46 | 287.52 | 286.6 | 286.78 | 285.78 | 285.64 | 286.86 | 286.42 | 286.78 | 286.78 |
The experimental results for a task with a given number of processing devices equal to 4 and the number of tasks equal to 101 are given in the Table No2.

**Table 2.** The results for an example with 4 devices and 101 tasks.

| 1 | 2 | 3 | Single-point crossover with a strong mutation | Two-point crossover strong mutation |
|---|---|---|---------------------------------------------|-----------------------------------|
| % of the individuals subjected to strong mutations | 4 | 100 | 100 | 100 | 100 | 100 | 100 | 100 | 100 | 100 | 100 | 100 | 100 |
| % of tasks is subject to severe mutations | | | 100 | 50 | 33 | 50 | 33 | 50 | 33 | 50 | 33 |
| % of bits subjected to strong mutation | | | 12.5 | 12.5 | 12.5 | 50 | 50 | 75 | 75 | 12.5 | 12.5 | 12.5 | 50 | 50 | 75 | 75 |

| Min | 686 | 680 | 679 | 680 | 680 | 679 | 681 | 679 | 678 | 678 | 678 | 678 | 677 | 679 | 678 |
| Mid | 696.62 | 686.88 | 684.56 | 683.94 | 683.76 | 685.74 | 685.04 | 687.06 | 685.28 | 681.7 | 680.56 | 681.16 | 681.74 | 681.48 | 681.96 | 682.02 |
| Time | 71 | 58 | 297 | 239 | 218 | 295 | 280 | 288 | 291 | 203 | 207 | 186 | 230 | 218 | 225 | 215 |
| Min | 680 | 679 | 679 | 681 | 680 | 682 | 681 | 678 | 678 | 678 | 678 | 678 | 678 | 679 | 678 | 679 | 679 |
| Mid | 684.26 | 682.5 | 682.58 | 685.74 | 684.88 | 685.76 | 684.74 | 680.08 | 680.72 | 680.94 | 681.12 | 680.8 | 682.02 | 680.4 | 680.52 | 680.02 | 680.34 | 681.04 |
| Time | 326 | 289 | 248 | 351 | 321 | 363 | 340 | 268 | 216 | 207 | 282 | 256 | 267 | |
| Min | 680 | 678 | 678 | 680 | 681 | 684 | 679 | 678 | 678 | 678 | 678 | 678 | 678 | 677 | 678 | 678 | 678 |
| Mid | 683.56 | 682.7 | 682.72 | 685 | 683.56 | 685.78 | 684.28 | 680.4 | 680.64 | 680.52 | 681.02 | 680.34 | 681.04 | 680.4 | 680.52 | 680.02 | 680.34 | 681.04 |
| Time | 384 | 344 | 294 | 392 | 409 | 447 | 410 | 310 | 251 | 260 | 321 | 310 | 335 | |
| Min | 679 | 679 | 679 | 681 | 680 | 679 | 679 | 678 | 678 | 678 | 678 | 678 | 677 | 678 | 677 | 687 | 678 |
| Mid | 683.18 | 682.16 | 682.08 | 684.58 | 683.58 | 685.42 | 684.1 | 679.92 | 680.26 | 680.48 | 681.02 | 680.56 | 681.26 | |
| Time | 425 | 368 | 324 | 478 | 448 | 537 | 448 | 368 | 310 | 270 | 366 | 348 | 382 | |
| Min | 679 | 678 | 678 | 680 | 678 | 681 | 679 | 678 | 678 | 678 | 678 | 678 | 677 | 679 | 677 | 679 | 679 |
| Mid | 682.38 | 681.72 | 681.88 | 684.76 | 682.74 | 684.5 | 683.48 | 679.84 | 679.92 | 680.32 | 680.5 | 680.56 | 680.18 | |
| Time | 502 | 398 | 350 | 489 | 474 | 525 | 535 | 427 | 356 | 308 | 422 | 390 | 415 | |
| Min | 678 | 678 | 679 | 680 | 679 | 680 | 678 | 678 | 678 | 678 | 678 | 677 | 678 | 677 | 678 | 677 | 678 |
| Mid | 682.2 | 681.48 | 681.58 | 683.78 | 682.12 | 684.6 | 682.9 | 679.72 | 679.78 | 679.92 | 680.36 | 679.8 | 680.62 | |
| Time | 558 | 421 | 367 | 565 | 583 | 588 | 544 | 430 | 359 | 339 | 474 | 461 | 519 | |

200 individuals, 200 repetitions

| Min | 682 | 678 | 678 | 678 | 678 | 678 | 678 | 677 | 677 | 677 | 677 | 677 | 677 | 677 | 677 | 677 | 677 |
| Mid | 682.04 | 681.88 | 682.72 | 681.84 | 682.64 | 682.48 | 679.28 | 679.42 | 679.46 | 679.44 | 679.04 | 679.52 | 679.4 |
### Table 3. The results for an example with 5 devices and 101 tasks.

| Time | 1 | 2 | 3 | 4 |
|------|---|---|---|---|
|      | % of the individuals subjected to strong mutations | % of tasks is subject to severe mutations | % of bin subjected to strong mutation | Single-point crossover with a strong mutation | Two-point crossover strong mutation |
| Min  | 100 | 100 | 100 | 100 | 100 | 100 | 100 | 100 | 100 | 100 | 100 | 100 |
| Mid  | 552.1 | 550.38 | 550.1 | 553.8 | 552.2 | 554.8 | 553.02 | 546.84 | 546.24 | 545.46 | 546.88 | 546.02 | 547.12 | 546.96 |
| Time | 78 | 78 | 78 | 78 | 78 | 78 | 78 | 78 | 78 | 78 | 78 | 78 | 78 |
| Min  | 545 | 543 | 543 | 544 | 546 | 546 | 546 | 543 | 541 | 541 | 543 | 542 | 542 | 542 |
| Mid  | 552.12 | 549.28 | 548.4 | 552.32 | 551.62 | 552.46 | 551.14 | 546.36 | 546.36 | 545.43 | 545.86 | 545.66 | 545.82 |
| Time | 418 | 404 | 388 | 467 | 396 | 435 | 432 | 339 | 340 | 276 | 354 | 335 | 363 | 348 |
| Min  | 546 | 543 | 543 | 544 | 546 | 545 | 546 | 542 | 540 | 541 | 542 | 542 | 542 | 543 |
| Mid  | 550.44 | 548.7 | 547.2 | 551.76 | 550.52 | 551.8 | 551.24 | 545.38 | 544.54 | 544.32 | 545.6 | 545.26 | 546.4 | 545.56 |
| Time | 495 | 469 | 440 | 522 | 471 | 491 | 465 | 410 | 379 | 351 | 452 | 398 | 382 | 422 |
| Min  | 544 | 543 | 543 | 546 | 546 | 546 | 542 | 542 | 540 | 541 | 542 | 542 | 542 | 543 |
| Mid  | 550.12 | 547.9 | 546.98 | 551.26 | 550.22 | 551.54 | 550.64 | 544.48 | 544.41 | 543.86 | 543.76 | 544.64 | 545.8 | 545.46 |
| Time | 560 | 541 | 483 | 571 | 532 | 578 | 570 | 477 | 425 | 410 | 476 | 447 | 469 | 425 |
| Min  | 546 | 542 | 543 | 546 | 546 | 544 | 546 | 541 | 540 | 540 | 542 | 542 | 542 | 542 |
| Mid  | 550.56 | 547.78 | 545.92 | 550.92 | 549.68 | 551.36 | 549.56 | 545.04 | 543.62 | 543.98 | 545.26 | 544.98 | 545.88 | 544.82 |
4 Discussion and conclusion

Thus, analyzing the results shown in the Tables 1-3, we can make the following conclusions:

Basic parameters of the genetic algorithm (number of individuals and the number of the best solutions repetition) affect the quality of the solutions by using a modified Goldberg model.

Using a two-point crossover gives the best results in the overwhelming number of experiments.

In the overwhelming number of experiments a hybrid algorithm gives better results.

Results of the hybrid algorithm depend on the number of repetitions. The more repetitions, the closer to the optimum results the average and the best solutions.

The number of repetitions degrades the temporal characteristics of the hybrid algorithm.

The number of repetitions over five rarely gives improvement with a serious deterioration of temporal characteristics.

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