CP Violation in $B_s$ Mixing 

in the SUSY SU(5) GUT with Right-handed Neutrinos

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Abstract

It is recently announced by the Utfit collaboration that the CP phase of the $B_s$ mixing amplitude, $\phi_{B_s}$, deviates more than $3\sigma$ from the standard-model prediction. In this paper we discuss how large correction to $\phi_{B_s}$ is possible in the supersymmetric SU(5) ground unified model (SUSY SU(5) GUT) with right-handed neutrinos. Here, we assume the supergravity-like boundary condition for the SUSY-breaking terms. We found that the 95% probability region derived by the Utfit collaboration is marginal in this model.
In the supersymmetric standard model (SUSY SM), the SUSY-breaking terms are introduced, and they are sources of flavor and/or CP violation. We may get insights to physics affecting the SUSY-breaking terms beyond the SUSY SM by studies of flavor and/or CP violating phenomena in the hadronic and leptonic sectors.

In the supersymmetric ground unified models (SUSY GUTs), new flavor-violating interactions are introduced, and rich flavor-violating structure in sfermion mass matrices may be generated by the GUT interactions. When the SUSY-breaking terms in the SUSY SM are originated from dynamics above the GUT scale, the GUT interactions affect them radiatively [1]. In the SUSY GUTs, quarks and leptons are embedded in common GUT multiplets. Thus, one of tests of the SUSY GUTs is studies of correlations among flavor-violating phenomena in the leptonic and hadronic sectors.

Recently, it is announced by the Utfit collaboration that the phase of the $B_s$ mixing amplitude deviates more than $3\sigma$ from the SM prediction [2]. They parametrized the new-physics contribution to the $B_s$ mixing as

$$C_{B_s}e^{2i\phi_{B_s}} = \frac{\langle B_s | H^{\text{full}}_{\text{eff}} | \bar{B}_s \rangle}{\langle B_s | H^{\text{SM}}_{\text{eff}} | \bar{B}_s \rangle},$$

and performed the model-independent analysis using the available observables of the $B_s$ system. They found that the 68% (95%) probability regions of $\phi_{B_s}$ and $C_{B_s}$ are given as

$$\phi_{B_s} = \begin{cases} [-25.5^\circ, -14.3^\circ] \quad ([-30.45^\circ, -9.29^\circ]) \\ [-73.1^\circ, -63.3^\circ] \quad ([-78.45^\circ, -58.2^\circ]) \end{cases},$$

$$C_{B_s} = \begin{cases} [0.78, 1.36] \quad ([0.62, 1.93]) \end{cases}.$$

The phase $\phi_{B_s}$ deviates from zero at $3.7\sigma$. The further studies are necessary so that the deviation is confirmed, since their result comes from a combination of various observables. They also used the SU(3) symmetry in order to evaluate unknown strong phases. However, their result encourages us to reanalyze the new-physics contribution to various flavor-changing neutral current (FCNC) processes.

In this paper we discuss how large deviation of the phase $\phi_{B_s}$ is possible in the SUSY SU(5) GUT with right-handed neutrinos. Here, we assume the supergravity-like (or CMSSM-like) boundary condition for the SUSY-breaking terms. The large mixing angle observed in the atmospheric neutrino experiments suggests that the sizable deviations
from the SM may appear in the bottom-strange quark transitions in the SUSY GUTs. The neutrino Yukawa interaction radiatively generates the bottom-strange component in the right-handed down-squark mass matrix \[3\]. On the other hand, the contribution to the bottom-strange quark transition is constrained from the upperbound on \(Br(\tau \rightarrow \mu \gamma)\) in the model \[4\]. The neutrino Yukawa interaction also induces the lepton-flavor violating terms in the left-handed slepton mass matrix so that lepton-flavor violating processes are predicted \[5\], and \(Br(\tau \rightarrow \mu \gamma)\) is also enhanced by the large atmospheric neutrino-mixing angle \[6\]. We evaluate \(\phi_{B_s}\) in the model, and it is found that it can deviate at most \(\sim (8^\circ - 9^\circ)\) from zero under the experimental constraint. Thus, it is difficult to get such a large deviation in \(\phi_{B_s}\) from the SM as in the 68\% probability regions derived by the Utfit collaboration in Eq. (2), while one of the 95\% probability region is marginal.

The SUSY contribution to the phase in the \(B_s\) mixing is also evaluated by Refs. \[7, 8, 9, 10\] in a framework of the SUSY GUTs. In Ref. \[7\] the constraints on the mass insertion parameters for the right-handed squarks are derived from the lepton-flavor violating processes using the GUT relation among the squark and slepton mass matrices at the GUT scale. The authors in Refs \[8, 9\] evaluate the phase in the \(B_s\) mixing in the SUSY GUTs, though they do not include the Higgs boson mass bound in their analysis. The analysis in Ref. \[10\] is similar to ours, though they do not evaluate the maximum value of the phase in the \(B_s\) mixing in the model.

First, we briefly explain the SUSY SU(5) GUT with right-handed neutrinos. In the model, quarks and leptons in the SUSY SM are \(10\)- and \(5^\star\)-dimensional multiplets, while the right-handed neutrinos are singlets. The Yukawa couplings for quarks and leptons and the Majorana mass terms for the right-handed neutrinos in this model are given as

\[
W = \frac{1}{4} f^u_{ij} \Psi_i \Psi_j H + \sqrt{2} f^d_{ij} \Psi_i \Psi_j H + f^\nu_{ij} \Phi_i \overline{N}_j H + M_{ij} \overline{N}_i \overline{N}_j, \tag{3}
\]

where \(\Psi\) and \(\Phi\) are the \(10\)- and \(5^\star\)-dimensional multiplets, respectively, and \(\overline{N}\) is the right-handed neutrinos. \(H (\overline{H})\) is \(5\) (\(5^\star\))- dimensional Higgs multiplets. After removing the unphysical degrees of freedom, the Yukawa coupling constants in Eq. (3) are given as follows,

\[
f^u_{ij} = V_{ki} f^u_{fk} e^{i \varphi_{uk}} V_{kj}, \quad f^d_{ij} = f_{dj} \delta_{ij}, \quad f^\nu_{ij} = e^{i \varphi_{\nu d}} U^*_{ij} f_{\nu j}, \tag{4}
\]

where \(\sum_i \varphi_{fi} = 0\) \((f = u, d)\). The unitary matrix \(V\) is the CKM matrix in the extension of
the SM to the SUSY SU(5) GUT. When the Majorana mass matrix for the right-handed neutrinos $M$ is diagonal in the above basis, $U$ is the the MNS matrix (with the Majorana phases), since the left-handed neutrino mass matrix is

$$(m_\nu)_{ij} = (f^{\nu}(M^{-1})f^{\nu T})_{ij}\langle H_f \rangle^2.$$  \hfill (5)

Here, $H_f$ is a doublet Higgs in $H$. In the following, we assume for simplicity that $M$ is diagonal, $M = \text{diag}(M_{N_1}, M_{N_2}, M_{N_3})$.

The colored-Higgs multiplets $H_c$ and $\overline{H}_c$ are introduced in $H$ and $\overline{H}$ as SU(5) partners of the Higgs doublets in the SUSY SM, $H_f$ and $\overline{H}_f$, respectively, and they have new flavor-violating interactions given by Eq. (3). When the SUSY-breaking terms are generated above the GUT scale, the colored-Higgs interactions give corrections to the sfermion mass matrices. When the supergravity-like boundary condition is assumed, the SUSY-breaking scalar masses and the trilinear SUSY-breaking terms at the reduced Planck scale ($M_G$) are given universally by $m_0$ and $A_0$, respectively. In this case, the off-diagonal terms ($i \neq j$) in the sfermion mass matrices at low energy are approximated as

$$m_{Q}^{2}_{ij} \approx -\frac{2}{(4\pi)^2}V_{ki}^{*}f^{2}_{uk}V_{kj}(3m_0^2 + A_0^2)(3\log \frac{M_G}{M_{GUT}} + \log \frac{M_{GUT}}{M_{SUSY}}),$$

$$m_{U}^{2}_{ij} \approx -\frac{4}{(4\pi)^2}e^{i\varphi_{x}}V_{ik}f_{uk}^{2}V_{jk}^{*}e^{-i\varphi_{x}}(3m_0^2 + A_0^2)\log \frac{M_G}{M_G},$$

$$m_{D}^{2}_{ij} \approx -\frac{2}{(4\pi)^2}U_{ik}^{*}f^{2}_{uk}U_{jk}^{*}e^{-i\varphi_{d}}(3m_0^2 + A_0^2)\log \frac{M_G}{M_G},$$

$$m_{L}^{2}_{ij} \approx -\frac{2}{(4\pi)^2}U_{ik}f_{uk}U_{jk}^{*}(3m_0^2 + A_0^2)\log \frac{M_G}{M_{N_k}},$$

$$m_{E}^{2}_{ij} \approx -\frac{6}{(4\pi)^2}e^{-i\varphi_{d}}V_{k_{i}}^{*}f_{u_{k}}^{2}V_{k_{j}}^{*}e^{i\varphi_{d}}(3m_0^2 + A_0^2)\log \frac{M_G}{M_{GUT}},$$

where $(m_{Q}^{2})$ $(m_{U}^{2})$, $(m_{D}^{2})$ is left-handed (right-handed) squark mass matrix(es) and $(m_{L}^{2})$ $(m_{E}^{2})$ is for left-handed (right-handed) sleptons. Here, $M_{GUT}$ and $M_{SUSY}$ are the GUT scale and the SUSY-breaking scale in the SUSY SM, respectively. While the off-diagonal terms in the left-handed squark and slepton mass matrices are generated in the SUSY SM with the right-handed neutrinos, those for right-handed squarks and sleptons are generated by the colored-Higgs interactions.

As mentioned above, the bottom-strange quark and $\tau-\mu$ transitions are correlated in the SUSY SU(5) GUT with right-handed neutrinos. This is because a following simple
relation is valid from Eq. (6),

\[
\frac{(m^2_D)_{23}}{(m^2_L)_{23}} \simeq e^{-i(\varphi_d^2 - \varphi_d^3)} \frac{\log \frac{M_G}{M_{GUT}}}{\log \frac{M_G}{M_{N_3}}},
\]

under an assumption, \( f^2_{\nu e} \gg f^2_{\nu \mu} \gg f^2_{\nu e} \). The corrections to this relation are possible, when the the right-handed neutrino mass matrix is not diagonal. However, it is shown in Ref. [4] that this relation is approximately valid when the right-handed neutrino masses are hierarchical. The hierarchical spectrum is welcome from a phenomenological viewpoint, since the muon-neutrino Yukawa coupling is so small that \( Br(\mu \to e\gamma) \) is suppressed below the experimental bound. Here, we also assume \( U_{e3} \lesssim 0.01 \) in order to suppress \( Br(\mu \to e\gamma) \). In such a case the large corrections to \((m^2_D)_{23}\) and \((m^2_L)_{23}\) by the colored-Higgs interactions may be allowed.

Now we evaluate how large deviation of the phase \( \phi_{B_s} \) in the \( B_s \) mixing is possible in the SUSY SU(5) GUT with right-handed neutrinos. The input parameters for the SUSY breaking parameters are \( m_0, A_0, M_{1/2} \) and \( \tan \beta \). Here, \( M_{1/2} \) is the gaugino mass at \( M_G \). We scan following parameter regions,

\[
0 < m_0, M_{1/2} < 3 \text{ TeV},
A_0/m_0 = -2, 0, 2,
\tan \beta = 5, 10, 30.
\]

For the neutrino sector, we take \( m_{\nu e} = 5 \times 10^{-2} \text{ eV}, U_{23} = 1/\sqrt{2}, \) and \( M_{N_3} = 6 \times 10^{14} \text{ GeV}. \)

We evaluate the SUSY mass spectrum and the interactions by numerically solving the renormalization-group equations for the SUSY breaking parameters in the SUSY GUT and the SUSY SM (with right-handed neutrinos).

In the following, we give constraints on the above parameter space from the experimental bounds on the Higgs boson mass \( m_h \) in addition to \( Br(\tau \to \mu\gamma) \). The SUSY contribution to the \( B_s \) mixing is dominated by the box diagrams including gluino exchanges when both left- and right-handed down-type squarks have flavor-violating mass terms [11]. Thus, the SUSY correction is less sensitive to \( \tan \beta \). On the other hand, \( Br(\tau \to \mu\gamma) \) is proportional to \( \tan^2 \beta \) since it is generated by the effective dipole operators. The constraint on \((m^2_D)_{23}\) is more severe for larger \( \tan \beta \) due to the null results in
Figure 1: Scatter plots on plane of $Br(\tau \to \mu \gamma)$ and $m_h$ for $\tan \beta = 5, 10, 30$. Red, green, and blue points are for $A_0/m_0 = -2, 0, +2$. Vertical lines are the upperbounds on $Br(\tau \to \mu \gamma)$ derived in BaBar and Belle, and the combined one.

the searches for $\tau \to \mu \gamma$. The larger correction to the $B_s$ mixing is possible when $\tan \beta$ is not large. The size is, however, limited by the Higgs boson mass bound in the case. The Higgs boson mass bound pushes up sfermion masses including top squarks, especially for small $\tan \beta$.

First, we show scatter plots on a plane of $Br(\tau \to \mu \gamma)$ and $m_h$ in Fig. 1 in order to show that the large and small $\tan \beta$ regions are constrained from $Br(\tau \to \mu \gamma)$ and $m_h$, respectively, in the SUSY SU(5) GUT with large neutrino Yukawa interaction. The current upperbound on $Br(\tau \to \mu \gamma)$ is derived as $6.8 \times 10^{-8}$ [12] and $4.5 \times 10^{-8}$ by BaBar and Belle, respectively, [13]. The combined upperbound of them is $1.6 \times 10^{-8}$ [14]. The SM Higgs boson mass lowerbound is 114.4 GeV [15]. In the following we impose $m_h > 111.4$ GeV [16], taking theoretical uncertainties in the Higgs boson mass evaluation. The constraint from $Br(b \to s \gamma)$ is also included in our analysis, though we found that it is not significant compared with $Br(\tau \to \mu \gamma)$. 
Figure 2: Scatter plots on plane of $\phi_{B_s}$ and $Br(\tau \rightarrow \mu \gamma)$. Here, we imposed the Higgs boson mass bound. Others are the same as in Fig. 1. Horizontal lines show 68% and 95% probability regions of $\phi_{B_s}$ derived by the Utfit collaboration.

It is found from Fig. 1 that the prediction for $Br(\tau \rightarrow \mu \gamma)$ is larger and the model is strongly constrained by the experimental upperbound when $\tan \beta$ is larger. On the other hand, when $\tan \beta = 5$, the Higgs boson mass bound also gives a severe constraint on the model. When $A_0$ has a negative value, the $A_t$ term in the stop mass matrix is sizable so that the Higgs boson becomes heavier. Then, larger value of $A_0$ with negative sign is favored from the Higgs boson mass bound.

Now we show the correlation between $\phi_{B_s}$ and $Br(\tau \rightarrow \mu \gamma)$ in Fig. 2. In the figure we imposed the Higgs mass constraint. The deviation of $\phi_{B_s}$ from zero is limited by $Br(\tau \rightarrow \mu \gamma)$, especially when $\tan \beta = 30$. When $\tan \beta = 5$, the constraint from $Br(\tau \rightarrow \mu \gamma)$ is not strong, however, the size of $\phi_{B_s}$ is limited. While heavier SUSY particle mass spectrum gives heavier Higgs boson mass, it suppresses the SUSY contribution to the FCNC processes. The largest deviation of $\phi_{B_s}$ from zero is $\phi_{B_s} \simeq -(8^\circ - 9^\circ)$ when $\tan \beta = 10$. Thus, it is found that one of the 95% probability region derived by the Utfit
collaboration in Eq. (2) is marginal.

Now we check the consistency with other observables. First, the mass difference of the $B_s$ mesons normalized by the SM prediction, $C_{B_s}(\equiv \Delta M_s/\Delta M_s|_{SM})$, is shown in Fig. 3. Here, we take $\tan \beta = 5$ and 10. While $\Delta M_s$ is precisely measured by CDF [17], the hadronic uncertainties in the theoretical prediction are large. The constraint on $C_{B_s}$ is given in Eq. (2). It is found that this constraint is not significant in Fig. 3. Next, the CP asymmetry in $B_d \to \phi K_s$, which is induced by the $b$-$s$ penguin diagrams, is also shown in Fig. 3. The correction to the process can be as large as $\sim 30\%$. The experimental and theoretical uncertainties [18, 19] in the $b$-$s$ penguin processes are still large, so that we could not give a rigid constraint on the parameters from it.

When the flavor-violating mass terms for the right-handed squarks are non-vanishing, the hadronic EDMs are generated at one-loop [20] and two-loop levels [21]. The non-zero $(m^2_D)_{23}$ generates the strange-quark chromoelectric dipole moment, which contributes to the hadronic EDMs [22, 23]. When evaluating the neutron EDM using the formula in Ref. [23], it can be as large as $\sim 10^{-25} e \text{ cm}$ even in the region allowed by the $Br(\tau \to \mu \gamma)$ and $m_h$. (See Fig. 3) The current experimental bound is $2.9 \times 10^{-26} e \text{ cm}$ [24]. Thus, the sizable deviation of $\phi_{B_s}$ from the SM prediction is possible if the neutron EDM suffers from the hadronic uncertainties or it is suppressed due to an accidental cancellation.

In the above discussion we took $M_{N_3} = 6 \times 10^{14}$ GeV for the right-handed tau neutrino mass. When $M_{N_3}$ is larger, the neutrino Yukawa coupling is larger with the left-handed tau neutrino mass fixed. However, the scalar mass squareds for the 5*-dimensional multiplets may become negative above the GUT scale. In the case, the flavor-violating mass terms of the 5*-dimensional multiplets rather become smaller so that the deviation of $\phi_s$ is not enhanced. In fact, we found it difficult to get larger deviation of $\phi_{B_s}$ by raising $M_{N_3}$. In Fig. 4, $\phi_{B_s}$ and $Br(\tau \to \mu \gamma)$ are shown for several values of $M_{N_3}$. The input parameters for the SUSY breaking parameters are taken here as

\begin{align*}
0 < m_0, \ M_{1/2} < 3 \text{ TeV}, \\
-3 < A_0/m_0 < +3, \\
5 < \tan \beta < 40 ,
\end{align*}

which are wider than in Eq. (8).
In summary, we discussed how large correction in $\phi_B$ is possible in the SUSY SU(5) GUT with right-handed neutrinos. Here, we assumed the supergravity-like boundary condition for the SUSY-breaking terms. We found that the 95% probability region derived by the UTfit collaboration is marginal in this model.

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Figure 3: The mass difference of the $B_s$ mesons normalized by the SM prediction ($C_{B_s}$), the CP asymmetry in $B_d \to \phi K_s$ ($S_{\phi K_s}$), and the neutron EDM induced by the strange CEDM ($d_n$) are shown for $\tan \beta = 5$ and 10. The experimental bounds on them are shown by the horizontal lines in the figures. Others are the same as in Fig. 2.
Figure 4: Scatter plots on plane of $\phi_{B_s}$ and $Br(\tau \rightarrow \mu \gamma)$ for $M_{N_3} = 2$(red), 4(green), 6(blue), 8(purple)$\times 10^{14}$ GeV. Here, we imposed the Higgs boson mass bound. The SUSY breaking parameters are given from Eq. (9).