Lightweight MPI Communicators with Applications to Perfectly Balanced “Schizophrenic” Quicksort

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Abstract—MPI uses the concept of communicators to connect groups of processes. It provides nonblocking collective operations on communicators to overlap communication and computation. Flexible algorithms demand flexible communicators. E.g., a process can work on different subproblems within different process groups simultaneously, new process groups can be created, or the members of a process group can change. Depending on the number of communicators, the time for communicator creation can drastically increase the running time of the algorithm. Furthermore, a new communicator synchronizes all processes as communicator creation routines are blocking collective operations.

We present RBC, a communication library based on MPI, that creates range-based communicators in constant time without communication. These RBC communicators support (non)blocking point-to-point communication as well as (non)blocking collective operations. Our experiments show that the library reduces the time to create a new communicator by a factor of more than 400 whereas the running time of collective operations remains about the same. We propose “schizophrenic” quicksort, a distributed sorting algorithm that avoids any load imbalances. We improved the performance of this algorithm by a factor of 15 for moderate inputs by using RBC communicators. Finally, we discuss different approaches to bring nonblocking (local) communicator creation of lightweight (range-based) communicators into MPI.

I. INTRODUCTION

The size of supercomputers rapidly increased to petascale machines with millions of cores. The de facto standard for communication in High Performance Computing (HPC) is the Message Passing Interface (MPI). Many applications need a flexible management of process groups, e.g., to adjust the scope of parallelism for load balancing, to achieve parallelism on multiple levels, or to divide tasks into fine-grained subproblems [1] [2]. MPI uses the concept of communicators to enable multiple levels of parallelism by connecting groups of processes. MPI provides (collective) communication operations between processes of a communicator to ensure scalability, portability, and comfortable programming with a high-level interface [3], [4].

The group context of a communicator guarantees that collective communication and point-to-point communication within one communicator as well as over different communicators does not interfere. Since the introduction of MPI standard version 3.0 (MPI-3.0), communicators support both, nonblocking collective operations, and nonblocking point-to-point communication. However, creating a new communicator still remains a blocking collective operation.

Collective communicator creation has multiple disadvantages. First, as a blocking collective operation, it synchronizes all processes of the new process group. This can cause immense idle time if a process starts the communicator creation behind schedule. Moreover, as each process must create one communicator after another, worst case construction sequences can delay communicator creation or cause deadlocks if communicators overlap. Thus, the processes must agree on a schedule to create new communicators. Second, the most recent open-source implementations Open MPI 3.0 and MPICH-3.2 create an array of process IDs when the user creates a communicator. In this case, the construction time of a communicator is linear to the number of group members. Mohamad Chaarawi and Edgar Gabriel [5] integrated sparse data storages into Open MPI. Their implementation reduces the footprint of an existing communicator. But the process group is stored explicitly during the communicator construction. Finally, MPI does not provide a method to invoke nonblocking collective operations on a subset of processes. Before performing the collective operation, the user must create a communicator of the subset of processes with a blocking communicator creation routine.

The main contribution of this paper is the lightweight library RangeBasedComm (RBC) based on MPI. RBC creates new communicators, containing a process range of a parent communicator, in constant time without communication. Our range-based communicators provide (non)blocking collective operations and point-to-point communication. As RBC can not access the context ID of a message, the library does not fully support the nonblocking model of the MPI-standard. Even though we restricted the semantics of communication, the library is already applicable to many algorithms. Furthermore, we discuss possible extensions of MPI implementations to provide nonblocking communicator creation without those restrictions. We performed a large number of experiments on thousands of processes which show the advantage of lightweight communicators. Our experiments show that our library reduces the time to create a new communicator by multiple orders of magnitude whereas the running time of collective operations remains about the same.

We furthermore propose, implement, and evaluate a new load balancing approach for recursive algorithms. A straightforward approach of multi-level algorithms is to assign subtasks to disjoint subsets of processes. As a result, the algorithms
may apply expensive mechanisms in advance to reduce load imbalances between process groups. We propose the approach of schizophrenic processes for multi-level algorithms. These schizophrenic processes work on two subtasks to minimize load imbalances. Both subtasks are processed simultaneously to avoid that progress in one subtask delays progress in another subtask. Our new sorting algorithm, Schizophrenic Quicksort (SQuick), applies this approach to distributed sorting algorithms. In comparison to hypercube quicksort, SQuick runs on any number of cores, rather than being restricted to a power of two and avoids any data imbalances. Our experiments show that lightweight local communicator creation with RBC, instead of blocking communicator creation with native MPI, speeds up SQuick by multiple orders of magnitude for moderate input sizes.

**Paper Overview.** We give preliminary definitions in Section II. Then, we discuss MPI communicators and collective operations (Section III) as well as load balancing problems of massively parallel sorting algorithms (Section IV). Section V describes our RBC library in detail. We propose different interfaces and implementations of lightweight communicators in state-of-the-art MPI libraries in Section VI. Our new perfectly balanced quicksort algorithm is described and analyzed in Section VII. Section VIII gives an extensive experimental evaluation.

## II. Preliminaries

The input of sorting algorithms are $n$ elements on $p$ processes with $O(n/p)$ elements each. The output must be globally sorted, i.e., each process has elements with consecutive ranks. We also want $O(n/p)$ output elements on each process.

A common abstraction of communication in supercomputers is the (symmetric) single-ported message passing model. It takes time $\alpha + l/\beta$ to send a message of size $l$ machine words. The parameter $\alpha$ defines the startup overhead of the communication. The parameter $\beta$ defines the time to communicate a machine word. For simplicity, we assume that the size of a machine word is equivalent to the size of a data element. For example, broadcast, reduction, and prefix sums can be implemented to run in time $O(\beta l + \alpha \log p)$ [4], [6] for vectors of size $l$. We have $\alpha \gg \beta \gg 1$ where our unit is the time for executing a simple machine instruction. Most of the time, we treat $\alpha$ and $\beta$ as variables in our asymptotic analysis in order to expose effects of latency and bandwidth limitations.

For simplicity, we will assume that all elements have unique keys. This is without loss of generality in the sense that we can enforce this assumption by an appropriate tie-breaking scheme. For example, replace a key $x$ with a tuple $(x, y)$ where $y$ is the global position in the input array. With some care, this can be implemented with negligible overhead in such a way that $y$ does not have to be stored or communicated explicitly [7].

## III. Communicators and Collective Operations

Collective operations and point-to-point communication are executed in the context of a specific communicator. A communicator stores an unique context ID and a group of $p$ processes with ranks $1..p$.

The context ID guarantees that communication over one communicator as well as over different communicators does not interfere. When a process sends a message over a communicator $c$, MPI stores the context ID of that communicator in the message header. The receiver then matches the context ID of incoming messages with the context ID of $c$ to distinguish the expected message from messages send in a different context. The context ID is managed by MPI and not visible to the user.

The user can create a new communicator based on a process subset of a parent communicator. When a new communicator is created, the processes must agree on a new context ID that is not used by any process of that subset. There exist different methods to create a new context ID [4], [8]. Open MPI and MPICH-3 use the concept of context ID masks. A context ID mask is a bit vector which is used to track used context IDs. Each process holds an own mask and the masks vary between processes depending on their communicators. To find an unused context ID, the processes of the new group invoke a collective all-reduce operation on their context ID masks with MPI_BAND. Then the processes select the process ID which is represented by the least significant non-zero bit of the reduced bit vector. According to the MPI standard [9] it is theoretically possible to agree on a new context ID without communication for the case that each process of the parent communicator participates in the communicator creation. This approach would require a larger context name space. Furthermore, it is unknown whether this statement holds if just the processes of the new communicator invoke the communicator creation routine.

MPI provides two methods to create a communicator based on a process subset of a parent communicator. In the first method, the user invokes the operations `MPI_Comm_create` or `MPI_Comm_create_group`. The operation `MPI_Comm_create` is a blocking collective operation on the parent communicator whereas `MPI_Comm_create_group` is a blocking collective operation on the processes of the new communicator [1]. As a parameter, those operations expect a MPI group that stores the new group of processes. The user has two possibilities to create a MPI group – either by enumerating the ranks explicitly (`MPI_Group_incl`) or by providing a sparse representation of rank ranges (`MPI_Group_range_incl`).

The second method is to invoke the operation `MPI_Comm_split`. This operation must be called by all processes of the parent communicator. Each process passes its communicator affiliation (color) and its new rank (key). MPI groups the processes by color and creates one communicator each. When `MPI_Comm_split` is invoked, the processes create the process group collectively. Open MPI and MPICH perform an all-gather operation in $\Omega((n \log p + \beta p)$ time to route all colors and keys to each process. Then each process uses this information to calculate its group locally. The user should use the operation `MPI_Comm_split` if the processes do not have all information to generate a MPI group just locally.

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1. We use the notation $a..b$ as a shorthand for $\{a, \ldots, b\}$. 
MPI groups store the mapping from rank to process ID. E.g., MPICH-3 stores the mapping for each process explicitly in an array. Mohamad Chaarawi and Edgar Gabriel [5] implemented different storage formats into Open MPI to reduce the memory footprint of a communicator. E.g., the Range Format derives from the syntax of MPI_Group_range_incl and has constant access time and space (see also [10]). They always choose the format which minimizes memory consumption. Unfortunately, their implementation still creates an explicit mapping with \( O(n) \) space during construction time. In result, implementations of algorithms with polylogarithmic running time which split communicators are generally not possible.

MPI-3.0 extends MPI by nonblocking collective operations [11]. Torsten Hoefler and Andrew Lumsdaine [3] proposed a scheme to implement nonblocking collective operations with point-to-point communication. They create a round-based schedule in which a round executes the collective operation until a data dependency prevents further computation. The next round starts as soon as the data dependency is solved. An additional communicator avoids tag conflicts between nonblocking collective operations and point-to-point communication that the user invoked. A tag counter avoids tag conflicts between nonblocking collective operations. The tag counter must be synchronous on all processes – nonblocking collective operations on a subset of the processes would unsynchronize the tag counter.

IV. MASSIVELY PARALLEL SORTING

For sorting large inputs, there are algorithms which move the data only once. Parallel sample sort [12] is a generalization of quicksort to \( p−1 \) pivots which are chosen from a sufficiently large sample of the input. Each process partitions its local data into \( p \) pieces using the pivots and sends piece \( i \) to process \( i \). After the all-to-all exchange, each processor sorts its received pieces locally. Since every process receives \( p−1 \) pivots, sample sort can only be efficient for \( n = \Theta(p^2/\log p) \) (see [13]). Indeed, the involved constant factors can be fairly large since the all-to-all exchange implies \( p−1 \) message startups if data exchange is done directly.

Algorithms with polylogarithmic running time are practical for small \( n/p \). Hypercube quicksort [14] is a recursive algorithm on \( 2^k \) processes which performs \( k \) levels of recursion. On each level, the processes agree on a pivot and partition their data into two pieces according to the pivot. Then, the processes with small elements are routed to processes \( 1..p/2 \) and pieces with large elements are routed to processes \( p/2..p \). Finally, hypercube quicksort is executed on the left and the right group of processes.

Compromises between these two extremes – high asymptotic scalability but a logarithmic number of data exchanges versus low scalability but only a single communication – have been considered. E.g., multi-level variants of sample sort [15] agree on \( k−1 \) pivots, partition local data into \( k \) pieces, route piece \( i \) to process group \( i \) and recursively invoke sample sort on each process group.

The algorithms described above have several disadvantages. First, they partition data into buckets and assign those buckets to process groups of fixed size. In result, the workload between the process groups is not balanced or the workload is balanced but comes at the price of expensive pivot selection. Second, the user can not execute the algorithms on arbitrary numbers of processes as the algorithms partition the processes into subgroups of equal size. Finally, recursive implementations [16], [17], [18] create new communicators on each level for the sake of simplicity. This usually prohibits polylogarithmic running time.

V. NONBLOCKING COMMUNICATION ON PROCESS RANGES

We present the library RangeBasedComm (RBC) based on MPI. The key feature of the library is that RBC communicators are created in constant time without communication. A RBC communicator \( R \) is derived from a MPI communicator \( M \) and includes processes with ranks \( f..l \) in \( M \). RBC provides (non)blocking point-to-point communication operations and (non)blocking collective operations in the context of a RBC communicator. Table I gives a list of supported operations. We show example code how to use our library in Figure 1. Because the interface is identical to MPI for most operations, our library can easily be integrated to replace existing MPI code. Unless stated otherwise, the interface of our operations is identical to the interface of its equivalent in MPI. The library is implemented in the namespace RBC. Here, we give function names and class names of our library without the namespace prefix.

A. RBC Communicator

A RBC communicator \( R \) stores a MPI communicator \( M \), the rank \( f \) of its first process in \( M \), and the rank \( l \) of its last process in \( M \). The size of \( R \) is defined by \( l−f+1 \), the number of processes contained in \( R \). The RBC rank of a process in \( R \) with MPI rank \( m \) in \( M \) is defined by \( m−f \) and vice versa. Note that more complicated projections from MPI rank to RBC rank are possible, e.g., a strided range \(^2\) with a stride factor of \( s \) would contain MPI ranks \( b, b+s, b+2s, …, b+(c−b)/s \). The local operation

\[
\text{Create Comm from MPI}(\text{MPI_Comm}s, \text{Comm}+)
\]

creates a RBC communicator that contains all processes of a MPI communicator. The local operation

\[
\text{Comm create group}(\text{Comm}+, \text{Comm}+, \text{int} f, \text{int} l)
\]

\(^2\)We implemented strided ranges but explain continuous ranges here.

| Ops       | Blocking Ops | Nonblocking Ops |
|-----------|--------------|-----------------|
| Broadcast | RBC::Send    | RBC::IBcast     |
| Reduce    | RBC::Reduce  | RBC::IReducende |
| Scan      | RBC::Scan    | RBC::Iscan      |
| Gather    | RBC::Gather  | RBC::IGather    |
| Gather    | RBC::Gather  | RBC::IGather    |
| Barrier   | RBC::Barrier | RBC::IBARRIER   |
| Send      | RBC::Send    | RBC::Isend      |
| Recv      | RBC::Recv    | RBC::Irecv      |
| Barrier   | RBC::Barrier | RBC::IBARRIER   |
| Probe     | RBC::Probe   | RBC::IProbe     |
| Test      | RBC::Test    | –               |
| Wait      | RBC::Wait    | –               |
| Waitall   | RBC::Waitall | –               |
std::vector<int> v_(200);
int root = 0, f, l, rank, size;
Comm global_comm, new_comm;
Create_Comm_from_MPI(
    MPI_COMM_WORLD, &global_comm);
Comm_rank(glocal_comm, &rank);
Comm_size(global_comm, &size);
if (rank < size / 2) {
    f = 0; l = size / 2 - 1;
} else {f = size / 2; l = size - 1;}
// Local op. No synchronization.
Comm_Create(global_comm,
    first, last, &new_comm);
Request req; int flag;
Ibcast(v.data(), v.size(),
    MPI_INT, root, new_comm, &req);
while (!flag) {
    // Do something else.
    Test(&req, &flag, MPI_STATUS_IGNORE);
}

Fig. 1: Nonblocking broadcast of elements from rank 0 to ranks 0..size/2−1 and from rank size/2 to ranks size/2..size −1. Both RBC communicators are created locally without synchronizing processes.

creates a new RBC communicator that contains processes with ranks b..l of an existing RBC communicator.

We restrict the usage of tags if two communicators overlap on more than one process, meaning multiple processes are part of both communicators. All simultaneously executed communication operations have to use unique tags. This is required to distinguish between operations in different RBC communicators which base on the same MPI communicator. If at most one process is part of both communicators, the communication on both communicators does not interfere. In this case we do not restrict the usage of tags.

B. Nonblocking Communication Requests

Each nonblocking communication call returns a request of type Request. The request behaves like a smart pointer to a request of the specific nonblocking operation. When we talk about a object of type Request in the following, we mean the request object of the specific nonblocking operation. When we invoke a nonblocking communication operation, we do not guarantee that the communication is completed once the operation returns. Instead, we have to invoke the method Test on the request to check if the operation is completed or not. Test in turn calls the method test of the request with its input parameters and forwards the result of this method to the user. Our library also provides three additional methods to test nonblocking operations for completion. The operation Wait takes a request and repeatedly calls Test until the operation is completed. The operation Testall takes an array of requests, calls Test on all requests, and returns true if all operations are completed. The operation Waitall takes an array of requests and repeatedly calls Testall until all operations are completed.

RBC::Send(const void *sendbuf, int count,
    MPI_Datatype, int r, int t, RBC::Comm)
RBC::Isend(const void *sendbuf, int count,
    MPI_Datatype, int r, int t, RBC::Comm,
    RBC::Request *);
RBC::Recv(void *buffer, int count,
    MPI_Datatype, int r, int t, RBC::Comm);
RBC::Irecv(void *buffer, int count,
    MPI_Datatype, int r, int t, RBC::Comm,
    RBC::Request *);
RBC::Iprobe(int r, int t, RBC::Comm,
    MPI_Status *);
RBC::Isend(int r, int t, RBC::Comm,
    int *flag, MPI_Status *);

Fig. 2: Interface for the operations used in point-to-point communication. The parameter r is a rank and the parameter t is a tag.

C. Point-to-point Communication

RBC provides (non)blocking send and receive operations. Figure 2 gives the interface of these operations. RBC point-to-point communication routines call MPI routines internally. When the user invokes RBC with a specific tag and a RBC target rank, we call MPI with the corresponding MPI rank and the tag passed by the user.

Probing: The operation Iprobe is a nonblocking operation that tests whether a message from a source process is ready to be received. If the user invokes Iprobe with a specific rank, we invoke the operation MPI_Iprobe with the parameters provided by the user and return when MPI_Iprobe returns. If the wildcard MPI_ANY_SOURCE is used instead of a specific rank, we also invoke the operation MPI_Iprobe. However, if MPI_Iprobe returns true, we only know that any message is ready to be received. In this case, we test if the source process of that message is part of the RBC communicator. We return true if the source process is in the RBC communicator, otherwise we return false.

The operation Probe waits until a message is ready to be received. If the user calls this operation with a specific rank, we invoke the operation MPI_Probe. If the wildcard MPI_ANY_SOURCE is used instead of a specific rank, we repeatedly call the operation Iprobe with our own input parameters until the operation returns true. In both cases, we return the status that was returned by the operation MPI_Probe (specific rank) or Iprobe (wildcard).

Our implementation of Probe and Iprobe on wildcards is not straightforward. In return, our library guarantees that communication over two different RBC communicators of the same MPI communicator does not interfere if the communicators overlap on at most one process.

Sending: The operation Send is a blocking operation that sends a message to a destination process. RBC forwards the input arguments of the operation Send to the operation MPI_Send. The operation Isend is a nonblocking send operation. RBC internally calls the operation MPI_Isend with the input arguments of Isend but replaces the RBC request,
All implementations exploit binomial tree based communication, which will not be executed completely. Collective operations are held when outgoing messages of the completed collection are invoke a new nonblocking collective operation. This rule even for small input sizes \[21\]. It is easy to extend our library by communication patterns \[19\], \[20\]. The communication patterns are generic, the send buffer is required for further computations. A receive operation causes a data dependency if subsequent operations rely on data of different processes. When the user invokes a nonblocking collective operation, RBC creates a request object which contains a local state machine, executes its first state, and returns the request. To make further progress on that operation, the user has to call the operation Test (or one of the three other test operations) on the corresponding request. Test in turn calls the method test. This operation checks whether outstanding data dependencies remain. If so, we can not proceed and we return false. Otherwise, we return true if the collective operation is already finished locally or we execute the next state and return false afterwards.

VI. NONBLOCKING COLLECTIVES ON SUBGROUPS

In this section we propose four different interfaces for nonblocking communication on a subset of processes in MPI and give implementation proposals. The difficulty for all interface implementations is to agree on a distinct context ID. The first interface is invoked by processes with ranks \(f..l\) and creates a MPI communicator containing these processes:

\[
\text{int MPI_Comm_split(MPI_Comm comm, int first, int last, MPI_Comm newcomm).}
\]

The second interface provides nonblocking collective operations which accept a range of processes as an input parameter, e.g.,

\[
\text{int MPI_Ibcast(void *buffer, int count, MPI_Datatype datatype, int root, MPI_Comm comm, MPI_Request *request, int first, int last).}
\]

Suppose that the first interface is given, an implementation of the second interface is trivial as shown in Figure 3. When a MPI communicator is constructed, the processes of that communicator
must agree on a distinct context ID. We believe that it is not possible to create a distinct context ID of constant size for an arbitrary subset of processes without communication. However, for the special case of range-based process groups, as used above, a unique context ID can be defined.

**Definition 1:** Let \( P \) be a non range-based communicator with context ID \( c \) and \( P' \) a new range-based communicator that contains processes with rank \( f..l \) of \( P \). Let \( i \) be the number of previously created range-based communicators with the same parent communicator \( P \) covering processes with rank \( f..l \). Then the context ID of \( P' \) is \( (c, f, l, i) \).

This new context ID is limited to four words and can be created locally in constant time without communication. The context ID requires a larger context space than state-of-the-art open-source implementations support, e.g., MPICH-3 uses a single `int` value. Another implementation of the two former interfaces could perform a nonblocking allreduce operation on the processes \( f..l \) to determine an unused context ID of type `int`. Here, the user would either provide a tag to distinguish between multiple ongoing creation routines or MPI could exploit the tag \( (f, l, i) \) internally. State-of-the-art open-source implementations provide 32-bit tags. However, the tag \( (f, l, i) \) requires at least 64 bits – 32 bits each for \( f \) and \( l \).

The third nonblocking interface is invoked by all processes of the parent communicator and provides the same functionality as `MPI_Comm_create`:

```c
int MPI_ICcomm_create(MPI_comm comm,
                      MPI_Group group, MPI_Comm newcomm,
                      MPI_Request *request);
```

An implementation is simple as it only has to invoke `MPI_Iallreduce` to determine an unused context ID of type `int`. The last nonblocking interface is invoked by all processes of the new communicator and provides the same functionality as `MPI_Comm_create_group`:

```c
int MPI_ICcomm_create_group(MPI_comm comm,
                            MPI_Group group, int tag, MPI_Comm newcomm,
                            MPI_Request *request);
```

We suggest to use a version of `MPI_Iallreduce` implemented with a user-defined tag to determine an unused context ID of type `int`. The tag is required to avoid interferences between multiple ongoing communicator construction routines.

**VII. SCHIZOPHRENIC QUICKSORT**

We present *Schizophrenic Quicksort* (SQuick), a recursive sorting algorithm for distributed memory systems which is based on Quicksort. SQuick guarantees perfect data balance, i.e., after each level each process stores \( \lceil n/p \rceil \) or \( \lfloor n/p \rfloor \) elements. Firstly, SQuick performs a distributed phase which recursively partitions tasks into subtasks. Then a second phase sorts *base cases*. Base cases are subtasks covering only one or two processes. Figure 4 depicts one distributed level of recursion. We prove that the distributed phase of SQuick takes \( O(\alpha \log^2 p + \beta n/p \log p) \) time with probability \( 1 - O(p^{-\alpha}) \) and that the base cases are executed in \( O(\alpha + \beta n/p + n/p \log(n/p)) \) time (Lemma 1). For the sake of simplicity, we assume that \( n \) is a multiple of \( p \).

```c
int MPI_Ibcast(void *buffer, int count,
                MPI_Datatype datatype, int root,
                MPI_Comm comm, MPI_Request *request,
                int first, int last) {
    MPI_Comm range_comm;
    MPI_Comm_split(comm, first, last, range_comm);
    return MPI_Ibcast(buffer, count, datatype, root, comm, request);
}
```

**Fig. 3:** Implementation of `MPI_Ibcast` on a range of processes.

One level of recursion consists of four steps: pivot selection, data partitioning, data assignment, and data exchange. In the first step, a random element is selected and broadcasted to all processes. In the second step, the processes partition their local data into a *left partition* of elements smaller than the pivot (small elements) and a *right partition* of elements larger than or equal to the pivot (large elements). In the third step, the processes assign elements to target processes. Here, we describe a simple *greedy message assignment* algorithm. Firstly, a distributed exclusive prefix sum is performed over the number of small and large elements on each process. Let the prefix sum on process \( i \) be \( s_i \), the number of small elements on processes \( 0..i - 1 \). The number of large elements on processes \( 0..i - 1 \) is \( l_i = i \cdot n/p - s_i \). The last process broadcasts \( s_{p-1} \) and \( l_{p-1} \), the total number of small and large elements. Next, we split the processes into a left group of processes \( 0..\lfloor n/s_{p-1} \rfloor \) and a right group of processes \( \lfloor n/s_{p-1} \rfloor + 1..p - 1 \). Finally, we calculate the actual data assignment such that the small elements are assigned to the left group and the large elements are assigned to the right group. Our assignment guarantees that each process gets exactly \( n/p \) elements assigned. The first source process assigns its small elements to target process \( 0 \). Then the second source process again assigns small elements to target process \( 0 \) as long as this process has residual capacity left. If the second source process still has small elements, it assigns the remaining small elements to the next target process. We continue with the remaining source processes and also assign large elements in the same way to the right group.

Even though a process sends two messages to each group, a process may receive \( \Theta(\min(p, n/p)) \) messages in the worst case. An alternative assignment algorithm, *deterministic message assignment* [18], guarantees that each process sends at most eight messages and receives at most eight messages. Both assignment algorithms take \( O(\alpha \log p) \) time.

Once we have calculated the data assignment we start the data exchange step. Each process invokes a nonblocking send operation to its target processes and then receives messages until \( n/p \) elements have been received.

After the elements have been redistributed, the left group recursively sorts the small elements and the right group recursively sorts the large elements. If \( n/k = \lfloor n/k + 1 \rfloor \), the left and the right group are not disjoint. In this case, process
that sequential quicksort with random pivot selection runs in $O(n \log n)$ with probability $1 - \mathcal{O}(n^{-6})$. In a first step we show that SQuick performs $\mathcal{O}(\log n)$ levels of recursion until its data gets sorted locally with probability $1 - \mathcal{O}(p^{-6})$ (Lemma 1-2).

Lemma 1: A randomly selected input element passes with probability $\mathcal{O}(p^{-7})$ more than $20\log p$ levels of recursion until it is part of a base case.

Proof: We define a recursion step with global load $n$ as a successful recursion step if both subtasks of SQuick have a load of at most $7/8n$ elements. Let $e$ be an arbitrary input element. Jaja [22] has shown that random split of a sorting task with $n$ elements creates two subtasks of at most $7/8n$ elements each with probability $3/4$. From this follows that an arbitrary input element $e$ is part of a task with at most $l_e \leq (7/8)^k n$ elements after it passed $k$ successful levels of recursion. After $k = \log_{8/7}(p/2)$ successful recursion steps we have $l_e \leq (7/8)^k n = 2n/p$ elements and execute the base case.

We now show that a randomly selected input element passes at least $\log_{8/7}(p/2)$ successful recursion steps with probability $\mathcal{O}(p^{-7})$ when $20\log_{8/7} p$ recursion steps are executed. This random experiment is a Bernoulli trail as we have exactly two possible outcomes, “successful recursion step” and “non-successful recursion step”, and the probability of success is the same on each level. Let denote the random variable $X$ as the number of non-successful recursion steps after $20\log_{8/7} p$ recursions. The probability $I$

$$I = P[X > 20 \log p - \log(p/2)] \leq P[X > 19 \log p]$$

$$\leq \sum_{j > 19/20 \log p} \binom{20\log p}{j} \left( \frac{1}{4} \right)^j \left( \frac{3}{4} \right)^{20 \log p - j}$$

$$\leq \sum_{j > 19/20 \log p} \frac{20\log p}{j} \left( \frac{1}{4} \right)^j$$

$$\leq \sum_{j > 19/20 \log p} \frac{5e \log p}{19 \log p} = \sum_{j > 19/20 \log p} \left( \frac{5e}{19} \right)^j = \mathcal{O}(p^{-7}) \quad (1)$$

defines an upper bound of the probability that a randomly selected input element passes $20\log_{8/7} p$ recursion steps without
passing $\log_{8/7}(p/2)$ successful recursion levels. For the sake of simplicity, all logarithms of the equation above are to the base of $8/7$. The third “≤” uses $\binom{n}{k} \leq \left(\frac{2n}{k}\right)^k$ and the second “=” uses the geometric series.

**Lemma 2:** SQuick executes more than $\mathcal{O}(\log p)$ distributed levels of recursion with probability $\mathcal{O}(p^{-6})$.

**Proof:** Let $E = e_1, \ldots, e_p$ be the input data of SQuick in sorted order and let $L = \{e_{in/p}\}_{i=1}^p \subseteq \mathbb{N}$ be every $n/p$th element. Lemma 1 and Boole’s inequality imply that all elements $e \in L$ does not run into the base case after $20 \log_{8/7} p$ recursions with a probability of $\mathcal{O}(p \cdot p^{-7}) = \mathcal{O}(p^{-6})$.

We analyze the length of the critical path in the recursion tree of SQuick. Let $g$ be the process group to the last level of recursion (before the base case) of that path. Process group $g$ contains at least $2n/p$ elements. Those elements build a subsequence of $E$ and thus contain at least one element $e \in L$. Element $e$ participates on more than $20 \log_{8/7} p$ recursion steps with probability $\mathcal{O}(p^{-6})$. Thus SQuick executes more than $20 \log_{8/7} p$ levels of recursion with probability $\mathcal{O}(p^{-6})$.

**Theorem 1:** For arbitrary inputs, the distributed phase of SQuick takes $\mathcal{O}(\log^2 p + \beta n/p \log p)$ time with probability $1 - \mathcal{O}(p^{-6})$ and the base cases take $\mathcal{O}(\alpha + \beta n/p + n/p \log(n/p))$ time.

**Proof:** We firstly analyze a single distributed recursion level on $p$ processes. The pivot selection step takes $\mathcal{O}(\alpha \log p)$ time, the data partitioning step takes $\mathcal{O}(n/p \log p)$ time, and the data assignment step takes $\mathcal{O}(\alpha \log p)$ time. The data exchange step takes $\mathcal{O}(\alpha + \beta n/p)$ time if the deterministic message assignment algorithm is used. In this step each process sends (receives) a constant number of messages and sends (receives) exactly $n/p$ elements in total. Note that schizophrenia processes do not affect the asymptotic running time as communication of schizophrenia processes is always done simultaneously on both tasks with nonblocking operations. In total, a distributed recursion level takes $\mathcal{O}(\alpha \log p + \beta n/p)$ time. Lemma 2 implies that all distributed levels of recursion take $\mathcal{O}(\alpha \log^2 p + \beta n/p \log p)$ time with probability $1 - \mathcal{O}(p^{-6})$. All base cases are sorted in $\mathcal{O}(\alpha + \beta n/p + n/p \log(n/p))$ time. The base cases include a $\mathcal{O}(\alpha \log p + \beta n/p)$ term as base cases are usually processed by two processes.

### VIII. Experimental Results

We now present the results of our experiments. We ran all experiments with our RBC library and with MPI implementations from Intel and IBM. Our benchmarks have been executed on 32 768 cores. We divide our experiments into two sections. In Section VIII-B we present microbenchmarks of RBC operations and the corresponding MPI operations. Firstly, we present results for nonblocking collective operations executed with RBC and with its counterparts in MPI. Then we compare the running time of creating communicators on non-overlapping and overlapping sub-ranges of processes with RBC and with native MPI. Finally, we give results collective operations executed on a sub-range of processes. Section VIII-C gives running times of SQuick executed with our RBC library and with native MPI. We execute each microbenchmark five times. We perform each experiment of SQuick seven times for $n/p \leq 2^{16}$ and three times for $n/p > 2^{16}$. We report the average over all runs and use 64-bit floating point elements.

We ran our experiments at the thin node cluster of the SuperMUC (www.lrz.de/supermuc), an island-based distributed system consisting of 18 islands, each with 512 computation nodes. Each computation node has two Sandy Bridge-EP Intel Xeon E5-2680 8-core processors with a standard frequency of 2.3 GHz and 32 GByte of memory. A non-blocking topology tree connects the nodes within an island using the InfiniBand FDR10 network technology. A pruned tree connects the islands among each other with a bidirectional bisection bandwidth ratio of 4 : 1.

#### A. Implementation Details

Our implementation of SQuick uses the greedy message assignment algorithm described above. As a pivot we select the median of $\max(k_1 \log p, k_2 n/p, k_3)$ samples determined by the random sampling approach by Sanders et. al. [23]. We handle duplicates by carefully switching between the compare functions “<” and “≤” as described in [7]. Schizophrenic Quicksort and the RBC library are written in C++ and compiled with version 16.0 of the Intel icpc compiler using the optimization flags `-O3 -ipo -xHost`. For inter-process communication, we use either version 1.4 of the IBM MPI library or version 5.1.3 of the Intel MPI library. Our initial experiments with IBM MPI have shown that the bulk transfer protocol increases fluctuations in running time. We disable bulk transfer in the IBM MPI library by setting the environment variable `MP_USE_BULK_XFER=no`. Our implementation can be found at https://github.com/MichaelAxtmann/RBC.

#### B. Microbenchmarks

**Collective operations.** Figure 5 depicts the running times of the nonblocking operation `MPI_Iscan` provided by native Intel MPI and native IBM MPI and its counterpart in the RBC library executed on $2^{15}$ cores. For the sake of simplicity, we give the running times of the RBC library only on top of the IBM MPI as the running times on top of Intel MPI are almost the same. We see that all implementations need about the same amount of time for
We give running times of the noncollective operations broadcast, gather, and reduce in Figure 10 in the Appendix. The experiments show that these operations executed with RBC perform similar to their counterparts in IBM MPI and Intel MPI.

**Communicator splitting.** Figure 6 presents running times of splitting a parent communicator of \( p \) processes into one communicator containing processes \( 0..p/2 - 1 \) and one communicator containing processes \( p/2..p - 1 \). We perform this experiment with RBC communicators, MPI communicators created with MPI_Comm_split, and MPI communicators created with MPI_Comm_create_group. We invoke MPI_Comm_create_group with a MPI group created with MPI_Group_range_incl. Note that the construction of a MPI communicator is a blocking collective function. Furthermore, the function MPI_Comm_split must be invoked by all processes of the parent communicator. In contrast to that we can create a RBC communicator in constant time without communication by calling the operation RBC::Comm_create_group only using the processes of the new communicator. Our experiments show that the time to construct a RBC communicator is negligible. If MPI used a sparse representation of the process group, we could expect that the running time of MPI_Comm_create_group increases logarithmically to the number of processes. However, the running time of this operation executed with IBM MPI increases linearly with the number of processes. This supports our hypothesis that IBM MPI represents MPI groups explicitly. The running time of MPI_Comm_create_group executed with Intel MPI is disproportionately slow and outperformed by the remaining operations by multiple orders of magnitude. The implementation of MPI_Comm_split in Intel MPI and IBM MPI is slower than MPI_Comm_create_group (IBM MPI) by a factor of two for large \( p \). We expected this slowdown as a process invokes MPI_Comm_split only with its own group affiliation. Thus, this operation must construct a MPI group internally by gathering group information from each process.

**Overlapping communicators.** When a process creates multiple communicators with blocking calls, it must decide which communicator will be created first. If even multiple processes create multiple communicators at the same time, a wrong schedule results in cascaded communicator creation or even deadlocks. In this case processes will delay communicator construction until other communicators have been created. In our experiment we split a communicator of \( p \) processes into communicators containing processes \( 0..3, 3..6, 6..9 \), and so on. Note that processes \( 3, 6, 9 \) and so on create two communicators – one communicator contains processes to the left, one communicator contains processes to the right. In the **cascaded schedule**, processes which will be part of two communicators always create the left communicator first and the right communicator second. In the **alternating schedule**, every other process which will be part of two communicators creates the left communicator first, the other processes which will be part of two communicators create the right communicator first. The alternating schedule avoids cascades but can be expensive as processes which create two communicators need global view. Figure 7 gives running times of cascaded and alternating splitting with Intel MPI and the RBC library. The running time for communicator creation with our library is negligible. There is almost no difference in the running time between cascaded and alternating scheduling as both operations are executed nonblocking. However, the running time of cascaded communicator creation with Intel MPI becomes extremely slow for a large number of processes as the communicator construction of one group prohibits the communicator construction of other groups.

**Range-based collective.** The next benchmark performs the operation broadcast on a process range of a parent communicator. We performed two experiments. In the first experiment we split the parent communicator of size \( p \) into a communicator of size \( p/2 \) and invoke Ibcast on the new communicator. In the second experiment we also split the communicator once but invoke Ibcast 50 times. To split the parent communicator into a MPI communicator, we used the MPI function which performed best in the previous microbenchmark. For IBM MPI (Intel MPI) we used the operation MPI_Comm_create_group.
Running time ratios of MPI to RBC for both experiments on $2^{15}$ processes. For a moderate number $n$ of elements on the root process, $n \leq 2^{10}$, our library performs a single range-based broadcast faster than the Intel MPI (IBM MPI) library by a factor of 42.5 to 81.8 (6 to 14.7). For the same input sizes, our library performs 50 range-based broadcasts faster than the Intel MPI (IBM MPI) library by a factor of 3.2 to 7 (6 to 14.7).

For large inputs, the running times of IBM MPI converge to the running times of the RBC library. The running times of Intel MPI fluctuate for large $n$. In conclusion RBC performs nonblocking broadcast operations on a sub-range of processes whereas MPI must split the communicator with a blocking operation before a (non)blocking broadcast operation can be invoked. In addition our library outperforms its competitors for almost all inputs as the time to create the MPI sub-communicators dominates for moderate input sizes.

**C. Sorting Benchmark**

In Figure 9 we present running times of SQuick implemented with RBC, native Intel MPI, and native IBM MPI. We executed the experiments with our RBC library on top of IBM MPI as well as on top of Intel MPI. All implementations exploit an alternating splitting schedule to avoid cascaded communicator construction. In our alternating schedule every other schizophrenic process splits the left group first and the remaining schizophrenic processes split the right group first. For $n/p = 1$ no schizophrenic processes occur. For this special input, SQuick with RBC already outperforms SQuick with native MPI by a factor of 3.5 (Intel) to 16.9 (IBM). For moderate inputs, i.e., $1 < n/p \leq 2^{10}$, SQuick with RBC on top of Intel MPI outperforms SQuick with native Intel MPI by a factor of more than 1282, even though no cascades occur. For larger inputs, the running time of both implementations converge as the time of communicator construction is more and more dominated by the actual algorithm. SQuick with RBC on top of Intel MPI and SQuick with native Intel MPI are significantly slower for any input size. The increased running time is caused by immense fluctuations. We were able to avoid these fluctuations for $p \leq 4096$ by disabling dynamic connections (I_MPI_DYNAMIC_CONNECTION=no). However, this option did not work for large $p$. Still, we see that the version with RBC is up to a factor of 3.8 faster than the version with native Intel MPI for moderate inputs. Experiments with a cascaded schedule showed that the running time of SQuick with RBC remains the same. However, the versions with native MPI become even slower by multiple orders of magnitude for moderate inputs.

**IX. Conclusion and Future Work**

We have shown how practical algorithms benefit from lightweight communicators. We proposed RBC, a library which creates range-based communicators without communication in constant time. The construction of our communicators come with almost no cost while providing nonblocking collective operations as well as point-to-point communication on a sub-range of processes. This offers completely new ways to implement flexible algorithms efficiently. We validated the performance of our communication routines in microbenchmarks as well as their applicability to sorting algorithms. RBC is even interesting from a theoretical side as recursive algorithms with polylogarithmic running time can now be implemented without communicator construction which takes time linear to the number of processes.

We propose and discuss various practical interfaces and implementations to integrate nonblocking (range-based) communicator creation routines into MPI. We see these interfaces as interesting candidates for the MPI standard. Future work should include implementations of these interfaces into open-source MPI libraries to show their feasibility. We think that a nonblocking version of MPI_Comm_create_group offers the best tradeoff between practicability and efficiency.

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Fig. 10: Running times of nonblocking collective operations on $2^{15}$ cores.