Ungauging black holes and hidden supercharges

Kiril Hristov, Stefanos Katmadas, and Valentina Pozzoli

Dipartimento di Fisica, Università di Milano-Bicocca, I-20126 Milano, Italy
Centre de Physique Théorique, École Polytechnique, CNRS, 91128 Palaiseau, France
E-mail: kiril.hristov [at] unimib.it, stefanos.katmadas [at] cpht.polytechnique.fr, valentina.pozzoli [at] cpht.polytechnique.fr

ABSTRACT: We embed the general solution for non-BPS extremal asymptotically flat static and under-rotating black holes in abelian gauged $D = 4$ $\mathcal{N} = 2$ supergravity, in the limit where the scalar potential vanishes but the gauging does not. Using this result, we show explicitly that some supersymmetries are preserved in the near horizon region of all the asymptotically flat solutions above, in the gauged theory. This reveals a deep relation between microscopic entropy counting of extremal black holes in Minkowski and BPS black holes in AdS. Finally, we discuss the relevance of this construction to the structure of asymptotically AdS$_4$ black holes, as well as the possibility of including hypermultiplets.

KEYWORDS: Supergravity theories, Black holes in string theory
1 Introduction

The interplay between the macroscopic description of black holes in supergravity and their corresponding microscopic description within string theory has been a source of important insights into the structure of the theory. In this respect, the most detailed investigations have been carried out for asymptotically flat black holes preserving some amount of supersymmetry, which provides additional control over various aspects of these systems. In particular, the microscopic counting of black hole entropy \cite{1, 2} as well as the construction of the corresponding black hole geometries \cite{3–5}, depend crucially on the presence of unbroken supercharges.

Beyond the supersymmetric sector, the non-BPS class of asymptotically flat black holes in supergravity has attracted attention, based on a deeper understanding of the first order systems underlying extremal static and under-rotating solutions (i.e. rotating black holes without an ergo-region) \cite{6–20}. While these systems are considerably more complicated than the corresponding BPS ones, they are in principle exactly solvable, since they are described by first order differential equations. In addition, various similarities to the BPS branch have been observed at the formal level, for example through the existence of a fake superpotential \cite{21–25}.

More recently, the interest in four-dimensional black holes was extended to the more general case of asymptotically anti-de Sitter (AdS) spacetimes, described as solutions to gauged supergravity theories \cite{26–32}. Although not fully exhaustive, the existing classification of black holes in AdS shows a rich variety of possibilities with both static and rotating BPS solutions, as well as new
horizon topologies. While a microscopic account of their entropy is not available yet, they provide
interesting new examples in the context of the AdS/CFT correspondence. In addition, understanding
phase transitions of extremal and thermal black holes in this class could lead to insight into the
phase structure of physically interesting field theories at strong coupling.

A priori, the above mentioned classes of black holes in Minkowski and AdS spaces are unrelated,
as they usually arise as solutions to different supergravity theories and respectively in different
string theory compactifications, when these exist. Consequently, the two systems are usually clearly
distinguished and studied by different methods, while the problems of microscopic entropy counting
for asymptotically flat and AdS black holes are viewed independently ([1, 2] vs. [33, 34]). However,
our purpose in this paper is to show that such distinction is not always present. In particular,
we show that one can embed asymptotically flat non-BPS black hole solutions in certain special
\( D = 4 \) \( \mathcal{N} = 2 \) gauged supergravity theories. Moreover, we show that the attractor geometries,
and therefore also the microscopic counting, for BPS black holes in AdS\(_4\) [28–30, 35] and asymptotically
flat extremal non-BPS black holes [36–41] fall within a common class of supersymmetric AdS\(_2\)×S\(^2\)
spaces\(^1\), or their rotating generalizations.

Let us be slightly more precise and consider the bosonic Lagrangian of abelian gauged \( \mathcal{N} = 2 \)
supergravity in four dimensions with an arbitrary number \( n_v \) of vector multiplets and no hyper-
multiplets (i.e. we consider constant gauging Fayet-Iliopoulos (FI) parameters). Such theories are
described in [45, 46] and we give more details in the following sections. For presenting the main
argument we only need to know that the bosonic part of the Lagrangian is modified with respect to
the one for the ungauged theory, \( \mathcal{L}^{\text{bos}}_0 \), by the introduction of a scalar potential term for the vector
multiplet complex scalars, \( t^i, i = 1 \ldots n_v \) as

\[
\mathcal{L}^{\text{bos}}_g = \mathcal{L}^{\text{bos}}_0 + V(t, \bar{t}) ,
\]

(1.1)

with

\[
V(t, \bar{t}) = Z_i(G) \bar{Z}^i(G) - 3 |Z(G)|^2 ,
\]

(1.2)

where \( G = \{g^I, g_I\} \) is a symplectic vector of arbitrary constant FI parameters and \( Z(G), Z_i(G) \)
denote its scalar dependent central charges. An interesting possibility arises in broad classes of
vector multiplet moduli spaces when the FI parameters are chosen in a way as to make the scalar
potential identically zero [47], without reducing the theory to the ungauged one. This requires at
least one FI parameter to be non zero, thus leading to a different supersymmetric completion of the
same bosonic Lagrangian, since \( \mathcal{L}^{\text{bos}}_g = \mathcal{L}^{\text{bos}}_0 \) when the potential \( V(t, \bar{t}) \) vanishes, but the fermionic
sector of the gauged theory still involves the vector \( G \) linearly. It is then immediately obvious that
all purely bosonic background solutions of the ungauged supergravity are also solutions of this "flat"
gauged supergravity. However, due to their different fermionic sectors, the supersymmetric vacua
of the two theories do not coincide. It is in fact easy to show that none of the BPS solutions of the
ungauged theory are supersymmetric with respect to the gauged theory and vice versa (see section
3). This situation is summarised in Figure 1.

Given the above, it is not surprising that some known non-BPS solutions in ungauged super-
gravity might be supersymmetric in these special gauged theories. Indeed, our analysis shows that
all extremal under-rotating black holes\(^2\) preserve some supersymmetry in their near horizon region.
Restricting to the static solutions, we further show that these horizon solutions are part of a larger
class of supersymmetric horizons based on FI terms, that do not a priori satisfy the flat potential

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\(^1\)This class of horizons was called magnetic AdS\(_2\)×S\(^2\) in [42] for the reason that, just like for asymptotically
magnetic AdS\(_4\) spacetimes, the fermions flip their spin and become SU(2) scalars [43, 44]. We elaborate on this more
in the following sections.

\(^2\)In what follows, we refer for simplicity to under-rotating solutions having in mind that this includes also the
static case, when the rotation vanishes.
Figure 1. The two bubbles above represent the space of BPS solutions to ungauged and abelian gauged supergravity with a flat potential, as subspaces of all bosonic solutions, common to both theories. Note the presence of two distinct $\text{AdS}_2 \times S^2$ backgrounds that are supersymmetric only within one theory. The blue line represents the BPS black hole solutions, interpolating between Minkowski space and the fully BPS $\text{AdS}_2 \times S^2$. The solutions described in this paper, represented by a red line, interpolate between Minkowski and the so called magnetic $\text{AdS}_2 \times S^2$ vacuum.

restriction and pertain to the static BPS black holes in $\text{AdS}_4$, [28–30, 35, 43]. It follows that one needs to address together the problems of microscopic entropy counting of asymptotically flat and AdS black holes in this case. We come back to this point in the concluding section of this paper, which we leave for more general discussion.

The following main sections of the paper address various aspects of the connection between solutions in gauged and ungauged theories sketched above, and are largely independent of each other. For the convenience of the reader, we give an overview of the main results presented in detail in each of these sections, as follows.

In section 2, we show that asymptotically flat extremal non-BPS black holes can be viewed as solutions to $\mathcal{N}=2$ abelian gauged supergravity, if the FI gaugings are assumed to be such that the potential is trivial. The prime example of gauged theories with an identically flat potential can be found within the interesting class of cubic prepotentials arising in the ungauged case from Calabi-Yau compactifications of string/M-theories, as first discussed in [47]. This condition is enforced by introducing a Lagrange multiplier, which allows us to write the action of the extended system as a sum of squares, similar to the $1/4$-BPS squaring in [29], while demanding that the metric is asymptotically flat, as is appropriate for a theory without a potential. The result is a first order system that is otherwise identical to the corresponding one describing asymptotically $\text{AdS}_4$ BPS solutions, except for the presence of the Lagrange multiplier, which is determined independently by its own equation of motion. We finally show that the general non-BPS solutions, in the form cast
in [20], are solutions to the system above, once a suitable regularity constraint is imposed. This includes the identification of the auxiliary very small vector appearing in that work as the vector of FI terms in the gauged theory.

In section 3, we consider the near horizon limit of our system, making use of the fact that the Lagrange multiplier above reduces to an irrelevant constant. It follows that the attractor equations for general asymptotically flat static black holes can be cast as a particular case of the attractor equations of gauged supergravity [29]. The latter are expected to belong to the family of attractors in [43] preserving four supersymmetries, which we show explicitly to be the case. We therefore obtain the result that all static non-BPS attractors in ungauged supergravity can be viewed as 1/2-BPS attractors once embedded in an abelian gauged supergravity with appropriately tuned FI terms. Finally, we generalise this result in section 3.2, where we show that the under-rotating attractors of all asymptotically flat black holes [48], again in the form described in [20], preserve two supercharges, i.e. they are 1/4-BPS. Note that this implies the presence of the same number of supercharges in the near horizon region of any particular center of a non-BPS multi-center solution.

In section 4 we consider the 1/4-BPS flow equations of [29] for gauged supergravity, without restricting the FI terms, and show that some of the structures found in asymptotically flat solutions are present in the more general case. Most importantly, the regularity constraint used to define the single center flow in [20] is shown to hold even for an unrestricted vector of FI terms. Since this constraint implies that only half of the charges can be present once a vector of gaugings is specified, we expect it to be of importance in understanding the moduli space of AdS4 solutions. In section 5 we briefly discuss the possibility of further embedding the asymptotically flat solutions above in theories with gauged hypermultiplets. In such a scenario, the additional potential induced by the hypermultiplets must also vanish, which we show to be possible in rather generic theories that result from string compactifications.

We conclude in section 6, where we comment on the implications of our results for microscopic models of black holes and on relations to recent developments in the construction of non-BPS supergravity solutions. Finally, in the appendices we present some details of our conventions, we extend the discussion of section 2 to the embedding of asymptotically flat under-rotating solutions in gauged supergravity, and we discuss an example solution in some detail for clarity.

2 Ungauging black holes

In this section, we present the essential argument of the ungauging procedure for black hole solutions and provide an explicit example by considering the static case for simplicity. The starting point is the bosonic action for abelian gauged supergravity [45, 46], which reads

\[ S_{4D} = \frac{1}{16\pi} \int_{M_4} \left( R \star 1 - 2 g_{i\bar{j}} dt^i \wedge \star d\bar{t}^\bar{j} - \frac{1}{2} F^I \wedge G_I + 2 V_g \star 1 \right), \]

and describes neutral complex scalars \( t^i \) (belonging to the \( n_v \) vector multiplets) and abelian gauge fields \( F^{I}{}_{\mu\nu} \), \( I = 0, i = 0, \ldots n_v \) (from both the gravity multiplet and the vector multiplets), all coupled to gravity\(^3\). The dual gauge fields \( G^{\mu\nu I} \) are given in terms of the field strengths and the scalar dependent period matrix \( N_{I,J} \), by

\[ G^{\mu\nu I} = N_{I,J} F^{\mu\nu J}, \]

where the expression for the period matrix will not be needed explicitly. Finally, the scalar potential \( V_g \) takes the form

\[ V_g = Z_i(G) \bar{Z}^i(G) - 3 |Z(G)|^2 = \langle G, J G \rangle - 4 |Z(G)|^2, \]

\(^3\)We refer to appendix A for some of our conventions in \( \mathcal{N}=2 \) supergravity.
where we used the definition of the scalar dependent matrix $J$ in (A.12), and the symplectic vector $G = \{g^I, g_I\}$ stands for the FI terms, which control the coupling of the vector fields. In the abelian class of gaugings we consider in this paper, these couplings occur only in the fermionic sector of the theory, through the minimal coupling of the gravitini to the gauge fields, as the kinetic term is proportional to

$$
\epsilon^{\mu\nu\rho\sigma} \bar{\psi}_\mu \gamma_\nu D_\rho \psi_\sigma \equiv \epsilon^{\mu\nu\rho\sigma} \bar{\psi}_\mu \gamma_\nu \left( \partial_\rho + \frac{i}{2} (G, A_\rho) \right) \psi_\sigma, 
$$

(2.4)

$$
\langle G, A_\mu \rangle = g_I A_{\mu I} - g^I A_{\mu I}.
$$

This coupling is in general non-local, due to the presence of the dual gauge fields $A_{\mu I}$. However, as for any vector, $G$ can always be rotated to a frame such that it is purely electric, i.e. $g^I = 0$, leading to a local coupling of the gauge fields. More generally, one can consider couplings of magnetic vectors as well, using the embedding tensor formalism [43, 49], which requires the introduction of extra auxiliary fields.

For the theories discussed in this paper however, the bosonic action is only affected through the nontrivial potential (2.3), which can be straightforwardly written in an electric/magnetic covariant way, as above. Based on this observation, we take the pragmatic view\(^4\) of using covariant versions of all quantities, keeping in mind that while the equations involving fermions strictly apply only to the electrically gauged theory, all results for the bosonic backgrounds must necessarily be covariant under electric/magnetic duality. We therefore employ covariant notation when dealing with the bosonic sector and covariantise the fermionic supersymmetry variations (see section 3), so that we do not have to choose a frame for the FI terms explicitly.

Given these definitions, we now discuss the connection of the gauged action above to the ungauged theory, at the bosonic level. As one would expect, ungauged supergravity is immediately recovered by putting $G = 0$ in the above Lagrangian. However, it turns out that this is not the most general choice if one is interested in the bosonic sector only, as the scalar potential is not positive definite, and one can find nonzero $G$ for which the potential is identically zero [47]. The appropriate FI terms are then described by a so-called very small vector, characterised by

$$
3 |Z(G)|^2 = Z_i(G) \bar{Z}^i(G).
$$

(2.5)

In the context of symmetric scalar geometries such vectors are viewed as points of the doubly critical orbit, $S$, defined as the set of vectors such that (2.5) is satisfied for any value of the scalars [50, 51]. Explicitly, they can be always brought to the frame where there is only one component, e.g.

$$
G_0 = g \{0, \delta^0\},
$$

(2.6)

but we will not impose any restriction other than (2.5). In what follows, we will be using the fact that this orbit exists for symmetric models, but the same arguments can be applied whenever (2.5) has a solution for any model. For example, (2.6) is an example solution for any cubic model, symmetric or not, and one may construct more general examples by acting with dualities\(^5\).

Given this special situation, it is natural to consider the possibility of finding asymptotically flat backgrounds in a gauged theory with a flat gauging as above. Indeed, a vector of parameters in a doubly critical orbit was recently encountered in [20, 52, 53], which considered the general under-rotating extremal black hole solutions in ungauged extended supergravity. As we now show, the presence of such a vector in asymptotically flat solutions can be seen to arise naturally by viewing the ungauged theory as a gauged theory with $G \in S$, leading to an interpretation of the auxiliary parameters introduced in [20, 52, 53] as residual FI terms.

\(^4\)A similar point of view was used in [29].

\(^5\)In fact, our treatment, as well as those of [20, 29] whose results we connect, is duality covariant, so that the form of the prepotential is not fixed.
2.1 Squaring of the action

In order to study extremal solutions in abelian gauged supergravity with a flat potential, we consider the squaring of the action for such backgrounds [4], following closely the derivation of the known flow equations of [29] for asymptotically AdS$_4$ black holes that preserve $1/4$ of the supersymmetries. The only additional ingredient we require is the introduction of a Lagrange multiplier that ensures the flatness of the potential.

As we are interested in static solutions, we consider a spherically symmetric metric ansatz of the type

$$ds^2 = -e^{2U}dt^2 + e^{-2U}(dr^2 + e^{2\psi}d\theta^2 + e^{2\psi}\sin^2\theta d\phi^2),$$

(2.7)
as well as an analogous ansatz for the gauge field strengths

$$F_{\theta\phi}^I = \frac{1}{2} p^I \sin \theta, \quad G_{I\theta\phi} = \frac{1}{2} q^I \sin \theta.$$  

(2.8)

Here, $e^U$, $e^\psi$ are two scalar functions describing the scale factor of the metric and the three-dimensional base space, and $\Gamma = \{p^I, q^I\}$ denotes the vector of electric and magnetic charges. Using these ansätze, the action (2.1) can be shown to be expressible in the form [29]

$$S_{1d} = \int dr \left\{ -\frac{1}{2} e^{2(U-\psi)} \mathcal{E}, J \mathcal{E} - e^{2\psi} \left( (Q_r + \alpha') + 2e^{-U} \text{Re}(e^{-i\alpha}W) \right)^2 
- e^{2\psi} \left[ \psi' - 2e^{-U} \text{Im}(e^{-i\alpha}W) \right]^2 - (1 + \langle G, \Gamma \rangle) 
- 2\frac{d}{dr} \left[ e^{2\psi - U} \text{Im}(e^{-i\alpha}W) + e^U \text{Re}(e^{-i\alpha}Z) \right] \right\},$$

(2.9)

where $e^{-i\alpha}$ is an arbitrary phase, we defined

$$\mathcal{E} \equiv 2e^{2\psi} (e^{-U} \text{Im}(e^{-i\alpha}V))' - e^{2(\psi-U)} J G + 4e^{-2U} \text{Re}(e^{-i\alpha}W) \text{Re}(e^{-i\alpha}V) + \Gamma,$$

(2.10)

and we introduced special notation for the central charges of $\Gamma$ and $G$ as

$$Z \equiv Z(\Gamma), \quad Z_i \equiv Z_i(\Gamma), \quad W \equiv Z(G), \quad W_i \equiv Z_i(G),$$

(2.11)

for brevity. The equations of motion following from this effective action imply the equations of motion for the scalars as well as the $tt$-component of the complete Einstein equation, whereas the remaining Einstein equations are identically satisfied upon imposing the Hamiltonian constraint

$$e^{2\psi} \psi'^2 - 1 - e^{2\psi} U'^2 - 2e^{2\psi} g_{ij} \partial_i \phi \partial_j \phi + e^{2(U-\psi)} V_{mn} + e^{2(\psi-U)} V_q = 0.$$  

(2.12)

Solutions of this system have been discussed in [28–30]. These works analysed in some detail the asymptotically AdS$_4$ solutions associated to generic values of the gaugings $G$, and we return to this case in section 4.

We now proceed to an analysis of the ungauged limit of the bosonic sector of the theory, by imposing that the vector of FI gaugings $G$ lies in the doubly critical orbit, $G \in S$, so that the potential is identically flat for any value of the scalars. Given the homogeneity of the potential in terms of $G$, we introduce a Lagrange multiplier in the original action (2.1), through a rescaling of the type

$$G \to e^\varphi G.$$  

(2.13)

Here and henceforth $\varphi$ will be treated as an independent field, whose equation of motion is exactly (2.5), enforcing the flatness of the potential. One can then write the action as a sum of squares in
a similar way as above, up to an extra term originating from the partial integration involved. The result reads

\[ S_{1d} = \int dr \left\{ -\frac{1}{2} e^{2(U-\psi)} \langle \mathcal{E}, J \mathcal{E} \rangle - e^{2\psi} \left[ (Q_r + \alpha') + 2 e^{(\varphi-U)} \text{Re}(e^{-i\alpha} W) \right]^2 \right. \\
- e^{2\psi} \left[ \varphi' - 2 e^{(\varphi-U)} \text{Im}(e^{-i\alpha} W) \right]^2 \\
\left. - \left( 1 + e^\varphi (G, \Gamma) \right) + 2 e^{2\psi} e^{(\varphi-U)} \varphi' \text{Im}(e^{-i\alpha} W) \right\}, \tag{2.14} \]

where we discarded a total derivative. We note here that \( \mathcal{E} \), originally defined in (2.10), now contains the multiplier \( e^\varphi \) due to the rescaling in (2.13) above.

Note that since the addition of the Lagrange multiplier \( e^\varphi \) in the original action leads to an ungauged theory, it is possible at this stage to proceed in solving the equations of motion by simply putting \( e^\varphi = 0 \), which is a consistent solution that eliminates all instances of the vector of gaugings. However, it is clear that it is not necessary to make this choice for this function a priori. Indeed, making instead a choice for the metric function \( e^\varphi = r \), so that the base metric in (2.7) is flat three dimensional space, one can obtain a more general squaring of the ungauged bosonic action. In this case, the kinetic term for the function \( e^\varphi \) is trivial and the action can be further rearranged into

\[ S_{1d} = \int dr \left\{ -\frac{1}{2} e^{2(U-\psi)} \langle \mathcal{E}, J \mathcal{E} \rangle - e^{2\psi} \left[ (Q_r + \alpha') + 2 e^{-U} \text{Re}(e^{-i\alpha} W) \right]^2 \right. \\
- \left[ 2 r e^{\varphi-U} \text{Im}(e^{-i\alpha} W) - (1 + \frac{1}{2} r \varphi') \right]^2 \\
\left. + r^4 e^\varphi \left( r^{-1} e^{-\varphi/2} \right)^2 - (2 + e^\varphi \langle G, \Gamma \rangle \right\}, \tag{2.15} \]

which is manifestly a sum of squares for the physical fields, along with an extra kinetic term and a Liouville-type potential for the multiplier \( \varphi \), that decouples from the rest of the action.

One can now solve the equations of motion for the physical fields by imposing that each of the squares vanishes, as

\[ \mathcal{E} = 0, \tag{2.16} \]
\[ Q_r + \alpha' = -2 e^\varphi e^{-U} \text{Re}(e^{-i\alpha} W), \tag{2.17} \]
\[ 2 r e^{-U} \text{Im}(e^{-i\alpha} W) = e^{-\varphi}(1 + \frac{1}{2} r \varphi'). \tag{2.18} \]

These equations describe the flow of the scalars and the scale factor \( e^U \), as well as fix the function \( e^\varphi \) in terms of physical fields. In addition, one still has to impose the Hamiltonian constraint (2.12) above, as well as the equation of motion for the Lagrange multiplier \( \varphi \), which reads

\[ \frac{d}{dr} \left( r^2 u' \right) - \langle G, \Gamma \rangle r^{-2} e^{-2u} = 0, \tag{2.19} \]

where we used the variable \( e^u \equiv r^{-1} e^{-\varphi/2} \) for convenience.

The flow equations above are closely related to the ones obtained in [29] for gauged supergravity, with the difference that the function \( e^\varphi \) describing the spatial part of the metric is now fixed to \( e^\varphi = r \) and that we have included the additional function \( e^\varphi \). One can decompose the scalar flow equations (2.16) in components to find

\[ U' = - r^{-2} e^U \text{Re}(e^{-i\alpha} Z) + e^\varphi e^{-U} \text{Im}(e^{-i\alpha} W), \tag{2.20} \]
\[ t'^j = - i e^{i\alpha} g^j (e^{-2\psi} \tilde{Z}_j + i e^\varphi e^{-U} W_j), \tag{2.21} \]

along with one more equation for the Kähler connection

\[ Q_r + \alpha' = - r^{-2} e^U \text{Im}(e^{-i\alpha} Z) - e^\varphi e^{-U} \text{Re}(e^{-i\alpha} W). \tag{2.22} \]
Combining the last relation with (2.17) leads to the constraint
\[ r^{-2} e^U \text{Im}(e^{-i\alpha} Z) = e^\varphi e^{-U} \text{Re}(e^{-i\alpha} W), \] (2.23)
which in the case of genuinely gauged supergravity in [29], can be shown to be equivalent to the Hamiltonian constraint (2.12). However, for the theory at hand, (2.12) is not automatically satisfied upon using (2.20)-(2.23), but takes the form
\[ (G, \Gamma) + 4 r^2 e^{-2U \varphi} \left( \text{Im}(e^{-i\alpha} W) \right)^2 = 0, \] (2.24)
which relates \( e^\varphi \) to the physical fields.

2.2 Asymptotically flat solutions

We can now look for solutions to the above system, starting with the observation that (2.19) can be solved explicitly. The general solution can be written in terms of exponentials of the type \( e^{\pm 1/r} \), which are badly singular at \( r = 0 \) and lead to unphysical results. However, this differential equation also has the particular enveloping solution
\[ e^u \equiv r^{-1} e^{-\varphi/2} = \frac{\langle \Gamma, G \rangle}{2} - \frac{1}{2} v + \frac{\langle \Gamma, G \rangle}{2} \frac{1}{r}, \] (2.25)
where \( v \) is a constant and we assumed that \( \langle \Gamma, G \rangle > 0 \), so that the distinguished harmonic function \( V \) defined above is positive definite. From (2.24), we obtain
\[ 2e^{-U} \text{Im}(e^{-i\alpha} W) = V, \] (2.26)
where the positive root was chosen by imposing (2.18). The last relation implies that the solution can be expressed in terms of harmonic functions, as shown in [20].

Indeed, the flow equations (2.20) and (2.21) with the particular solution for \( \varphi \) given by (2.25), can be straightforwardly shown to be identical to the static limit of the flow equations derived in [20] for the single centre class of asymptotically flat black holes, upon identifying the gaugings \( G \) with the auxiliary vector \( \hat{R}^* \) used to express the solution\(^6\). Note that in [20] the auxiliary vector \( \hat{R}^* \) was required to be very small by consistency of the Einstein equations for asymptotically flat black holes. Moreover, in that work it was found that regularity requires an additional constraint on the system, which can be expressed in several equivalent ways. In terms of the scalars, this constraint takes the form of the reality condition
\[ e^{-i\alpha} dt + i e^{i\alpha} d\tilde{t} - i e^{i\alpha} W_j g_{k\bar{k}} d\tilde{t} \tilde{W}_j d\tilde{t} = \tilde{W}_i \left( 1 + e^{-i\alpha} \tilde{W} \right) dU, \] (2.27)
where we defined the following shorthand expressions for convenience
\[ \tilde{W} = |W|^{-1} W, \quad \tilde{W}_a = |W|^{-1} e^a W_i. \] (2.28)
The reality condition (2.27) can be used to show the existence of a second constant very small vector throughout the flow, given by
\[ R = -4 \frac{e^{-2U}}{|V|^2 V^2} \text{Re} \left[ Y^3 \tilde{W} Y + |Y|^2 Y \tilde{W}^a D_a Y \right], \] (2.29)
\[ Y \equiv (1 + i m e^{2U}). \] (2.30)
Here, \( m \) is an arbitrary constant that is promoted to a dipole harmonic function in the rotating case (see appendix B). This vector can be shown to be mutually local with \( \Gamma, \langle R, \Gamma \rangle = 0 \), using the
\(^6\)The interested reader can find an outline of this identification in appendix B, where the full rotating single center class is considered.
flow equations above, but is nonlocal with $G$, as $\langle G, R \rangle = -4$, and in simple cases it can be viewed as the magnetic dual of $G$. Alternatively, one can derive the constraint (2.27) by demanding that the vector $R$ be constant.

Given the definitions above, the solution to the system (2.20)-(2.22) is given by

$$2 \text{Im}(e^{-U} e^{-i\alpha} V) = \mathcal{H} - 2 \frac{\langle G, \mathcal{H} \rangle}{\langle G, \mathcal{H} \rangle} R + \frac{m}{\langle G, \mathcal{H} \rangle} G,$$

where $\mathcal{H}$ are harmonic functions carrying the charges as

$$\mathcal{H} = h + \frac{\Gamma}{r},$$

and the distinguished harmonic function $\langle G, \mathcal{H} \rangle$ is fixed by (2.26) as

$$\langle G, \mathcal{H} \rangle = -V.$$

The reality constraint (2.27) can now be recast in terms of the harmonic functions describing the solution. Using the flow equations (2.20) and (2.21) to express the derivatives of the scalars in terms of the gauge fields, one can show that an equivalent form of the same reality condition can be obtained, as

$$\frac{1}{2} I_4' M(\mathcal{H}, G) = \langle G, \mathcal{H} \rangle \mathcal{H}^M - 2 \frac{\langle G, \mathcal{H} \rangle^2}{\langle G, R \rangle} R^M.$$

Here, we use the index $M, N, \ldots$ to denote both electric and magnetic components and $I_4' M$ is the derivative of the quartic invariant, defined in terms of a completely symmetric tensor $t^{MNQP}$ as

$$I_4(\mathcal{H}) \equiv \frac{1}{4!} t^{MNQP} \mathcal{H}_M \mathcal{H}_N \mathcal{H}_P \mathcal{H}_Q,$$

$$I_4' M(\mathcal{H}, G) \equiv \frac{\partial^2 I_4(\mathcal{H})}{\partial \mathcal{H}_M \partial \mathcal{H}_N} G_N = \frac{1}{2} t^{MNQP} \mathcal{H}_N \mathcal{H}_P G_Q.$$

In [20] it was shown that if $G$, and thus $R$, are very small, this constraint implies that the harmonic functions $\mathcal{H}$ lie in a Lagrangian submanifold that includes $R$. Near the horizon, one finds the same constraint for the charges, so that a particular choice of $G$ restricts the physical charges to lie in the same Lagrangian submanifold. We refer the interested reader to that work for details on the derivation of these results.

This concludes our analysis of the embedding of extremal asymptotically flat black holes in gauged supergravity for the static case. We refer to appendix B for a similar analysis in the rotating case. It turns out that the inclusion of a Lagrange multiplier in exactly the same way leads to the same equation of motion (2.19) and the same solution (2.25) as above. The result is an extension of the static embedding of this section to the most general asymptotically flat extremal under-rotating black holes, as obtained in [20]. The solution turns out to take exactly the same form as in (2.31) with the constant $m$ replaced by a dipole harmonic function describing the rotation.

### 3 BPS attractors in abelian gauged theories

As already announced in the introduction, the embedding of asymptotically flat black holes in the flat gauged theories we consider in this paper allows to show that their near-horizon geometries are in fact supersymmetric. In the previous section we saw a close similarity between the static flow equations for asymptotically flat and 1/4-BPS black holes in AdS. Below we further establish that static horizons in both Minkowski and AdS spaces in fact belong to a common 1/2 BPS
class of solutions\textsuperscript{7} already discussed in [43]. Beyond the static class, we further analyze the near-
horizon geometry of extremal under-rotating black holes [48], whose flow equations are discussed in appendix B. These turn out to preserve 1/4 of the supercharges, which completes the statement that all asymptotically flat static and under-rotating extremal black holes have BPS horizons.

In order to study supersymmetric solutions, we only need to explicitly ensure that the supersymmetry variations of the fermions vanish. All supersymmetry variations for the bosons are automatically zero by the assumption of vanishing fermions. The fermionic fields that belong to the supermultiplets appearing in the action (2.1) are the gravitini $\psi_{\mu A}$ for the gravity multiplet and the gaugini $\lambda^A$ for the vector multiplets. The corresponding supersymmetry variations are\textsuperscript{8}:

$$
\begin{align*}
\delta \psi_{\mu A} & = D_\mu \varepsilon_A - 2i X^I I_{IJ} F^{J-}_{\mu \nu} \gamma^{\nu} \epsilon_{AB} \varepsilon^B - \frac{1}{2} W \sigma^3_{AB} \gamma_{\mu} \varepsilon^B , \\
\delta \lambda^A & = -i \delta t^i \varepsilon^A - D_i X^I I_{IJ} F^{J-}_{\mu \nu} \gamma^{\nu} \epsilon_{AB} \varepsilon^B + i W^i \sigma^3_{AB} \varepsilon^B ,
\end{align*}
$$

(3.1)

where the covariant derivative $D_\mu$ reads

$$
D_\mu \varepsilon_A = (\partial_\mu - \frac{1}{4} \omega^{ab}_{\mu} \gamma_{ab} + \frac{i}{2} Q_\mu) \varepsilon_A + \frac{1}{2} \langle G, A_\mu \rangle \sigma^3_B \varepsilon^B ,
$$

(3.2)

and $W$ and $W_i$ are the central charges defined in (2.11). The symplectic product $\langle G, A_\mu \rangle$ in the standard electrically gauged supergravity just involves the electric gauge fields $A_\mu^A$ [54], but has a straightforward generalization, as shown in [43, 49]. We also used the shorthand $I_{IJ} = \text{Im} N_{IJ}$ for the period matrix. The presence of this matrix seems to spoil duality covariance on first sight, but it is possible to rewrite the relevant terms in a form convenient for our purposes, as in [55]

$$
\begin{align*}
\delta \psi_{\mu A} & = D_\mu \varepsilon_A + Z(\mathcal{F})_{\mu \nu} \gamma^{\nu} \epsilon_{AB} \varepsilon^B - \frac{1}{2} W \sigma^3_{AB} \gamma_{\mu} \varepsilon^B , \\
\delta \lambda^A & = -i \delta t^i \varepsilon^A + \frac{1}{2} \tilde{Z}(\mathcal{F})^i_{\mu \nu} \gamma^\nu \epsilon_{AB} \varepsilon^B + i W^i \sigma^3_{AB} \varepsilon^B .
\end{align*}
$$

(3.3)

Here, the central charges of the electric and magnetic field strengths are computed component-wise as in (A.6), as

$$
\begin{align*}
Z(\mathcal{F})_{\mu \nu} & = \langle \mathcal{F}_{\mu \nu}, V \rangle , \\
Z(\mathcal{F})_i_{\mu \nu} & = \langle \mathcal{F}_{\mu \nu}, D_i V \rangle ,
\end{align*}
$$

(3.4)

which are already anti-selfdual and selfdual respectively due to (A.11) and (A.10).

Note that the FI parameters $G$ above are assumed to be generic, and include the particular choice of FI parameters such that the scalar potential is identically zero. In that limit, we obtain a theory with a bosonic Lagrangian identical to ungauged supergravity, but with a different fermionic sector that involves a nonzero very small vector of FI terms explicitly. Therefore, the supersymmetry variations above are strictly valid only for gauged supergravity, even though the associated bosonic backgrounds we describe below are solutions to both gauged and ungauged supergravity. The supersymmetric solutions of the two theories however do not overlap and form two disjoint sets. This is easy to see because the ungauged supersymmetry variations are again given by (3.3) after setting $G = 0$, leading to the vanishing of $W, W^i$. Suppose now that we have a supersymmetric background solution of the ungauged theory and let us focus for simplicity on the gaugino variation. If we also want it to be a solution of the flat gauged theory, we require that it automatically satisfies $W^i = 0$, since otherwise one cannot make the variation vanish both in the gauged and in the ungauged theory. Now, using the vanishing of the scalar potential (2.3) we find

$$
|W|^2 = \frac{1}{2} W_i W^i ,
$$

---

\textsuperscript{7}This is in accordance with our results in section 2. From this point of view, the crucial factor that allows for a unified discussion is that the expression for the Lagrange multiplier reduces to a constant in the near horizon region (cf. the solution in (2.25)), thus diminishing any difference between the flat and AdS case. This is the case even for under-rotating black holes, as shown in appendix B.

\textsuperscript{8}Here we choose to orient the FI terms along direction 3 of the quaternionic moment maps, as done in [30].
which means that we also need $W = 0$ for the hypothetical BPS solution in both theories. However, it is a special geometry property that

$$
\left( \frac{\langle \mathcal{V}, G \rangle}{\langle D_{\lambda} \mathcal{V}, G \rangle} \right) = 0, \quad \Rightarrow \quad G = 0,
$$

(3.5)

since one can invert the matrix multiplying $G$ in this equation. This leads us back to the ungauged case, and we find a contradiction. Therefore every BPS solution of the ungauged theory (e.g. the asymptotic Minkowski spacetime connected to the asymptotically flat black holes) is not supersymmetric in the flat gauged theory, and vice versa (e.g. the black hole attractor geometries are BPS in the gauged theory, as shown below, but break supersymmetry in the ungauged theory) as schematically illustrated by Fig. 1.

We now move on to the explicit analysis of the supersymmetries preserved by the various horizon geometries. In doing so, we will be using a timelike Killing spinor ansatz, ensuring that once the BPS equations hold we already have supersymmetric solutions, i.e. the BPS equations together with the Maxwell equations and Bianchi identities imply the validity of the Einstein and scalar equations of motion (see [56, 57]). This is important for the discussion of backgrounds with non-constant scalars, which are the ones relevant for rotating attractors.

In section 3.1 we verify that the attractor equations obtained as a limit of the full $1/4$-BPS static solutions in AdS$_4$ in [29], do exhibit supersymmetry enhancement to 4 real supercharges. We then identify the attractor equations of static asymptotically flat non-BPS black holes of [20] as a subset of the BPS attractors in gauged supergravity, in the limit of flat gauging where the FI terms are restricted to be a very small vector. Similarly, in section 3.2 we show that the general under-rotating attractor solutions of [20] preserve $1/4$ of the supersymmetries.

3.1 Static attractors

We first concentrate on the near horizon solutions of static black holes, therefore we consider metrics of the direct product form AdS$_2 \times S^2$ with radii $v_1$ and $v_2$ of AdS$_2$ and $S^2$, respectively:

$$
ds^2 = -\frac{r^2}{v_1^2} \, dt^2 + \frac{v_1^2}{r^2} \, dr^2 + v_2^2 \left( d\theta^2 + \sin^2 \theta \, d\phi^2 \right).
$$

(3.6)

The corresponding vielbein reads

$$
e_{\mu}^a = \text{diag} \left( \frac{r}{v_1}, \frac{v_1}{r}, v_2, v_2 \sin \theta \right),
$$

(3.7)

whereas the non-vanishing components of the spin connection turn out to be

$$
\omega_{t}^{01} = -\frac{r}{v_1^2}, \quad \omega_{\phi}^{23} = \cos \theta.
$$

(3.8)

We further assume that the gauge field strengths are given in terms of the charges $\Gamma = (p^I, q_I)^T$ by

$$
F_{\mu \nu} \equiv (F_{\mu \nu}^I; G_{I \mu \nu}), \quad F_{\theta \phi}^I = \frac{1}{2} \, p^I \, \sin \theta, \quad G_{I \theta \phi} = \frac{1}{2} \, q_I \, \sin \theta,
$$

(3.9)

which are needed in the BPS equations below. The scalars are assumed to be constant everywhere, $\partial_{\mu} z = 0$, as always on the horizon of static black holes. This ansatz for gauge fields and scalars automatically solves the Maxwell equations and Bianchi identities in full analogy to the case of ungauged supergravity.

Anticipating that the near horizon geometries of the solutions described in the previous section preserve half of the supersymmetries, we need to impose a projection on the Killing spinor. This is in accordance with the fact that these solutions cannot be fully supersymmetric once we require that
not all FI terms vanish (see [58, 59] for all fully BPS solutions in $\mathcal{N}=2$ theories in 4d). Taking into account spherical symmetry, there are only two possibilities in an AdS$_2 \times$S$^2$ attractor geometry, as shown in [43]. Namely, one either has full supersymmetry, and therefore no projection is involved, or 1/2-BPS geometries satisfying the projection

$$\varepsilon_A = i\sigma_A^{AB} \gamma^{03} \varepsilon_B = \sigma_A^{AB} \gamma^{01} \varepsilon_B ,$$  

with the last equality due to the fact that spinors are chiral in the chosen conventions (these are exhaustively listed in [42, 54]). Note that a Killing spinor satisfying this projection is rather different from the standard timelike Killing spinor projection that appears in asymptotically flat 1/2-BPS solutions (shown in (3.28) below, see e.g. [3]), but is exactly the same as one of the projections appearing in asymptotically AdS$_4$ 1/4-BPS solutions (see [29, 30]).

Analysis of the BPS conditions

Now we have all the data needed to explicitly write down the supersymmetry variations of the gravitini and gaugini. To a certain extent this analysis was carried out in section 8 of [43] and will not be exhaustively repeated here. One can essentially think of the Killing spinors as separating in two - a part on AdS$_2$ and another part on S$^2$. It turns out that the AdS$_2$ part transforms in the standard way under the SO(2, 1) isometries of the AdS space, while the spherical part remains a scalar under rotations. The $t$ and $r$ components of the gravitino variation are therefore non-trivial due to the dependence of the spinor on these coordinates. We are however not directly interested in the explicit dependence, but only consider the integrability condition for a solution to exist, given by $D_t D_r \varepsilon_A = 0$ for all $A = 1, 2$. Plugging the metric and gauge field ansatz, this results in the equations

$$\frac{1}{2v_1^2} = |W|^2 + \frac{1}{v_2^2} |Z|^2 , \quad \langle G, \mathcal{F}_{tr} \rangle = 0 .$$  

(3.11)

The solution of this equation therefore ensures the vanishing of the gravitino variation on AdS$_2$. Turning to the spherical part, with the choice of Killing spinor ansatz it is easy to derive two independent equations that already follow trivially from the analysis of [30],

$$i \frac{1}{v_2} Z = -W ,$$  

(3.12)

and

$$\langle G, \Gamma \rangle = -1 ,$$  

(3.13)

which is the usual Dirac quantization condition$^9$ that seems to accompany the solutions of 'magnetic' type$^{10}$. Note that (3.12) can be used to simplify the first of (3.11), so that we can cast the above conditions in a more suggestive form for our purposes, as

$$v_1^{-2} = 4 |W|^2 , \quad v_2^2 = -i \frac{Z}{W} .$$  

(3.14)

Moving on to the gaugino variation, the condition that the scalars remain constant leaves us with only one (for each scalar) additional condition on the background solution,

$$-i Z_i = v_2^2 W_i .$$  

(3.15)

$^9$From the point of view of the flow equations derived in section 2, $\langle \Gamma, G \rangle$ can be an arbitrary non-vanishing constant. This is exactly the value of the Lagrange multiplier in (2.25) at the horizon, thus rescaling the gauging vector as $G' = (T, G)^{-1} G$ in the solution (2.31) in that limit. It follows that $\langle G', \Gamma \rangle = -1$, which is the choice made in [28–30] for the full solution and we adopt it here, dropping the primes on the gaugings, to make our notation in sections 2 and 3 consistent without any loss of generality.

$^{10}$The solution at hand, called magnetic AdS$_2 \times$S$^2$ in [42], is the near horizon geometry of asymptotically magnetic AdS$_4$ black holes [44, 60].
This concludes the general part of our analysis - it turns out that in FI gauged supergravity one can ensure that AdS$_2 \times S^2$ with radii $v_1$ and $v_2$ preserves half of the supersymmetries by satisfying equations (3.13)-(3.15) within the metric and gauge field ansatz chosen above. These equations are in agreement with the analysis of [29, 35]. Moreover, the attractors above are a realisation of the 1/2-BPS class of AdS$_2 \times S^2$ vacua of [43], which are described by the superalgebra $SU(1,1|1) \times SO(3)$, as opposed to the fully BPS AdS$_2 \times S^2$ vacua that are described by $SU(1,1|2)$.

The above equations can be written in terms of symplectic vectors (e.g. as in [29]), so that they can be directly compared to [20]. To this end, one can straightforwardly see that the condition

$$-4 \text{Im}(\bar{Z} V) = \Gamma + v_2^2 J G,$$

is equivalent to (3.12) and (3.15), while the first of (3.14) has to be used to fix the AdS$_2$ radius. Alternatively, one may write the attractor equations by solving for $V$ in terms of the charges and gaugings. Since all the above equations are invariant under Kähler transformations, we need to introduce an a priori arbitrary local phase $e^{i \alpha}$, which is defined to have unit Kähler weight. One can then combine (3.11) and (3.15) to obtain

$$2 \frac{v_2^2}{v_1} \text{Im}(e^{-i \alpha} V) = \Gamma + v_2^2 J G,$$  

while (3.12) has to be viewed as an additional constraint. Taking the inner product of (3.17) with $\Gamma + v_2^2 J G$ identifies the phase $e^{i \alpha}$ as the phase of the combination in (3.12), which drops out from that relation.

In order to show that these BPS conditions above do indeed admit solutions describing asymptotically flat black holes, one can consider the inner product of (3.16) with the gaugings, using (3.13), to show that the sphere radius is given by

$$v_2^{-2} = 2 g^{ij} W_i W_j - 2 |W|^2.$$  

Upon imposing triviality of the potential as in (2.5), the above expression and the first of (3.14) imply that $v_2 = v_1$, which is necessary for asymptotically flat black holes. Indeed, using the definition (2.29) in this special case, the generic BPS attractor equation in the form (3.17) can be written as

$$2 v_1 \text{Im}(e^{-i \alpha} V) = \Gamma + \frac{1}{2} R.$$  

These are exactly the general attractor equations for asymptotically flat black holes found in [20] for the ungauged case$^{11}$. We conclude that the near horizon region of static asymptotically flat extremal black holes can be viewed as a special case of the general attractor geometry for BPS black holes in abelian gauged supergravity, upon restricting the FI parameters to be a very small vector, thus leading to a flat potential.

In addition, when all FI parameters are set to zero, one immediately obtains the BPS attractor equations of ungauged supergravity, preserving full $\mathcal{N}=2$ supersymmetry [61–63]. This provides us with a unifying picture, since the BPS attractor equations (3.16) appear to be universal for static extremal black holes in $\mathcal{N}=2$ theories, independent of the asymptotic behavior (Minkowski or AdS) or the amount of supersymmetry preserved.

One intriguing aspect of this result is that, while in the ungauged theory ($G = 0$), the attractor equation leads to a well defined metric only when the quartic invariant of the charges, $I_4(\Gamma)$, is positive, the presence of a nontrivial $G$ does not seem to allow for a charge vector $\Gamma$ with a positive quartic invariant, i.e. in all known examples $I_4 < 0$ iff $G \neq 0$, both for asymptotically flat and

$^{11}$We remind the reader that the inner product $\langle \Gamma, G \rangle$ has to be rescaled to unity in the original reference for a proper comparison with this section.
AdS black holes. Similarly, the explicit AdS$_4$ solutions of [28–30], also have a negative quartic invariant of the charges, contrary to the intuition one might have from the asymptotically flat case. It is natural to expect that the quartic invariant of charges allowed for asymptotically AdS$_4$ BPS solutions is negative even though this is not the only quantity that controls the horizon in that case.

In view of the above, it is interesting at this point to make some comments on the potential microscopic counting of degrees of freedom, which can be now safely discussed due to the presence of supercharges on the horizon. From a microscopic string theory perspective we know that the FI parameters are usually some particular constants corresponding to topological invariants of the compactification manifolds, see [64] for a clear overview and further references. This means that one is not free to tune the value of the vector $G$. We further know that one of the electromagnetic charges is uniquely fixed by the choice of $G$, meaning that we are not free to take the large charge limit in this particular case. We then find that the black hole entropy, $S$, which is proportional to the area of the horizon, scales as $S \sim \Gamma^{3/2}$, a behavior that is in between the usual $S \sim \Gamma^2$ of 1/2 BPS asymptotically flat black holes\footnote{Note however, that the entropy of asymptotically flat 1/2-BPS black holes in five dimensions scales exactly as $\Gamma^{3/2}$, see e.g. [1].} and the $S \sim \Gamma$ case of 1/4 BPS asymptotically magnetic AdS black holes [28, 30]. This is of course not a puzzle on the supergravity side, where we know that some charges are restricted, but it provides a nontrivial check on any potential microscopic descriptions of black hole states in string theory.

### 3.2 Under-rotating attractors

We now turn to the more general case of extremal under-rotating attractors corresponding to asymptotically flat solutions [48]. These are described by a more general fibration of $S^2$ over AdS$_2$ that incorporates rotation as

$$ds^2 = -e^{2U} r^2 (dt + \omega)^2 + e^{-2U} \left( \frac{dr^2}{r^2} + d\theta^2 + \sin^2 \theta d\phi^2 \right),$$

$$e^{-2U} = -I_4(\Gamma) - j^2 \cos^2 \theta, \quad \omega = j \frac{\sin^2 \theta}{r} d\phi,$$  \hspace{1cm} (3.20)

where $j$ is the asymptotic angular momentum. It is easy to see that this metric reduces to (3.6) for $v_1^2 = v_2^2 = \sqrt{-I_4(\Gamma)}$ upon setting $j = 0$ above. We choose the vielbein

$$e^0_t = r e^U, \quad e^1_r = \frac{e^{-U}}{r}, \quad e^2_\theta = e^{-U}, \quad e^3_\phi = e^{-U} \sin \theta, \quad e^0_\phi = j e^U \sin^2 \theta,$$  \hspace{1cm} (3.21)

which leads to the following non-vanishing components of the spin connection:

$$\omega^{01} = -r e^{2U} (dt + \frac{1}{2} \omega), \quad \omega^{23} = -e^{4U} \cos \theta \langle j r dt + \tilde{v} d\phi \rangle,$$

$$\omega^{02} = j r e^{2U} \sin \theta \cos \theta \left( \frac{1}{2} r dt + \tilde{v} d\phi \right), \quad \omega^{03} = -\frac{j}{2r} e^{2U} (\sin \theta dt - r \cos \theta d\phi),$$

$$\omega^{12} = -\frac{1}{2r} j^2 e^{2U} \sin \theta \cos \theta dr, \quad \omega^{13} = \frac{1}{2} j r e^{4U} \sin \theta (dt + \omega),$$  \hspace{1cm} (3.22-3.25)

where we defined the function

$$\tilde{v} = I_4 - \frac{1}{2} j^2 (1 + \cos^2 \theta).$$  \hspace{1cm} (3.25)

The gauge fields for this class of solutions read [20]

$$F = d[\zeta r (dt + \omega) + \Gamma \sin \theta \, d\theta \wedge d\phi], \quad \zeta = -2 e^U \text{Re}[e^{-ia}\mathcal{V}] + G,$$  \hspace{1cm} (3.26)

where we used the fact that the section depends on the radial coordinate by an overall $r^{-1}$ in the near horizon region, as for the static case. We refrain from giving the full solution for the scalars at
this stage, since it will be derived from the BPS conditions below. Here we note that the physical scalars \( t^i \) only depend on the angular coordinate \( \theta \) in the near horizon region, and we give the expression for the Kähler connection

\[
Q + d\alpha = \frac{1}{2} j e^{2U} \sin \theta \, d\theta ,
\]

(3.27)

for later reference. The interested reader can find an explicit example solution to the STU model in Appendix C, both at the attractor and for the full flow.

As already mentioned above, the backgrounds we are interested in only preserve two supercharges, i.e. they are \( \frac{1}{4} \)-BPS. The fact that we now need a second projection on the Killing spinor, in addition to (3.10), can be derived directly by considering the BPS equations, e.g. the gaugino variation. We omit details of this derivation, which is straightforward, and just give the resulting additional projection

\[
\gamma^0 \epsilon_A = i e^{i\alpha} \epsilon_{AB} \epsilon^B ,
\]

(3.28)

which is the same as the one used in e.g. [3, 29, 30, 57].

As in the static case, we make use of the complex self-duality of \( F \), so that we only need to use half of its components. We therefore choose for convenience \( F_{0\hat{a}} \) and \( F_{23} \), where \( \hat{a} = 2, 3 \) is a flat index on the sphere, given by

\[
F_{23} = e^{2U} \Gamma + \zeta r (d\omega)_{23} = e^{2U} (\Gamma + 2 \zeta j \cos \theta) , \quad F_{0\hat{a}} = -\partial_{\hat{a}} \zeta .
\]

(3.29)

Since the central charges of these quantities appear in the BPS conditions, we note for clarity the following relations

\[
Z(\zeta) = -i e^{U} e^{i\alpha} + W , \quad Z_i(\zeta) = W_i ,
\]

(3.30)

\[ Z(\partial_{\hat{a}} \zeta) = -i e^{U} e^{i\alpha} [\partial_{\hat{a}} U + i(Q_{\hat{a}} + \partial_{\hat{a}} \alpha)] , \quad Z_i(\partial_{\hat{a}} \zeta) = i e^{U} e^{i\alpha} g_{i\bar{b}} \partial_{\hat{a}} \bar{\nu} ,
\]

(3.31)

which can be straightforwardly derived from (3.26) using (A.4).

**Analysis of the BPS conditions**

Given the backgrounds described above, we proceed with the analysis of the conditions for unbroken supersymmetry. This is parallel to the discussion in section 3.1, but differs in that we only analyze the supersymmetry preserved by the attractors corresponding to asymptotically flat black holes as given by (3.20) rather than derive the general conditions for \( \frac{1}{4} \)-BPS backgrounds. This is because there is at present no evidence that asymptotically AdS under-rotating black holes can be constructed and the near horizon properties of such hypothetical solutions is unclear. However, we note that there is no argument against the existence of such solutions in AdS and one can try to generalize our analysis by rescaling the sizes of the AdS\(_2\) and S\(_2\) also in the rotating case.

We now turn to the analysis, starting with the gravitino variation and imposing (3.10) and (3.28) on the spinor \( \epsilon_A \). In the conditions below, we arrange all terms with two gamma matrices in the \( 0\hat{a} \) and \( 23 \) components, in order to simplify calculations. We start from the spherical components of the variation, which can be shown to vanish if the spinor \( \epsilon_A \) does not depend on \( \phi \) and the following conditions are imposed

\[
(\partial_{\theta} + i \frac{i}{2} Q_\theta) \epsilon_A + \frac{1}{2} Z(F)_{0\hat{a}} e^{-i\alpha} \epsilon_A = 0 ,
\]

(3.32)

\[ i (G, A_{\hat{a}}) + i \omega_{-23} Z(F)_{0}^{-\hat{b}} \epsilon_{\hat{b}} = 0 ,
\]

(3.33)

\[ \frac{i}{2} \epsilon^{i\hat{a}} \omega_{\hat{a}0} e^{i\alpha} + i Z(F)_{23} + W_0 = 0 ,
\]

(3.34)

where the last relation represents a term present in both components. Using (3.20) and (3.26) for the metric and gauge fields, these are simplified as follows. The first leads to an equation that
determines the angular dependence of the spinor as
\[ 2 \partial_\theta \epsilon_A = \partial_\theta U \epsilon_A, \]  
while the second relation reduces to (3.13). Finally, the third relation boils down to
\[ Z - i e^U j \cos \theta e^{i\alpha} + (2 j \cos \theta - i e^{-2U}) W = 0, \]  
which generalises (3.12) in the rotating case.

Turning to the AdS\(_2\) part, we analyse the time component of the Killing spinor equation, which
upon assuming time independence\(^{13}\) of \(\epsilon_A\), implies the following constraints
\[ i \langle G, A_t \rangle + i \omega^{-23}_t - r e^U Z(F)_{23} e^{-i\alpha} - i r e^U W e^{-i\alpha} = 0, \]  
\[ \omega^{-0}_r e^{i\alpha} = i r e^U Z(F)^{0}_{0a}, \]
where the second equation is identically satisfied by using (3.29) and (3.31). The first relation leads to
\[ 2 e^U \Re(e^{-i\alpha} W) = e^{4U} j \cos \theta, \]
\[ e^{-U} + e^{-i\alpha} Z + 2 j \cos \theta (e^{-i\alpha} W - i e^U) + i e^{-2U} W e^{-i\alpha} = 0. \]  
Finally we consider the radial component, which leads to the constraints
\[ 2 \partial_r \epsilon_A + r^{-1} e^{-U} Z(F)_{23} e^{-i\alpha} + i r^{-1} e^{-U} W e^{i\alpha} = 0, \]
\[ \omega^{-0b}_r e^{i\alpha} = -r^{-1} e^{-U} Z(F)^{0}_{ba}. \]
These are also satisfied by using (3.29) and (3.39), for a spinor that depends on the radial coordinate according to
\[ \partial_r \epsilon_A = \frac{1}{2r} \epsilon_A, \]
where we used (3.36) and (3.39) to obtain this result. Using the last equation and (3.35), find that the spacetime dependence of the Killing spinors is given by
\[ \epsilon_A(r, \theta) = e^{U/2} \sqrt{r} \epsilon^0_A, \]
for arbitrary constant spinors \(\epsilon^0_A\) that obey the two projections (3.10) and (3.28) imposed above.

In addition, we need to consider the BPS conditions arising from the gaugino variation in (3.1), which in this case lead to
\[ e^{2U} Z_t + 2 e^{2U} j \cos \theta Z_t(\zeta) - i W_t = 0, \quad \partial_\theta t^i = i e^{i\alpha} Z_t(\partial_\theta \zeta). \]  
The second condition is identically satisfied upon using the \(\zeta\) given in (3.26), whereas the first reads
\[ e^{2U} Z_t + 2 e^{2U} j W_t \cos \theta - i W_t = 0. \]  
This concludes our analysis of the BPS conditions for rotating attractors. The value of the scalar fields at the horizon can now be cast in terms of an attractor equation generalising (3.17) to the rotating case, as
\[ 2 e^{-U} \Im \left[(1 + 2 i e^{2U} j \cos \theta) e^{-i\alpha} \mathcal{V}\right] = \Gamma + e^{-2U} J G + 2 j \cos \theta G, \]
\(^{13}\)This assumption is consistent as we eventually show that all BPS equations are satisfied and we explicitly derive the spacetime dependence of the Killing spinors, which only depend on the \(r\) and \(\theta\) coordinates.
where one still has to impose (3.36) as a constraint.

The BPS conditions above can be straightforwardly seen to be the horizon limit of the single center rotating black holes of [20], using the definition (2.29) to simplify the result as in the static case. Since these were shown to be the most general asymptotically flat extremal under-rotating black holes, we have thus shown that all under-rotating attractor solutions are 1/4-BPS (i.e. preserve two supercharges) when embedded in a gauged supergravity with a flat potential \(^{14}\). Upon taking limits of vanishing angular momentum and gaugings one finds that supersymmetry is enhanced, since the static attractors in the previous section are 1/2-BPS when \(G \neq 0\) and fully BPS when the gauging vanishes. In Table 1 we summarise the findings of this section for all static and under-rotating attractors in abelian gauged \(\mathcal{N}=2\) theories.

| Attractor | Global |
|-----------|--------|
| \(G \in \mathcal{S}, \text{ flat} \) | \(j = 0\) | 1/2 BPS | non-BPS |
| \(G \in \mathcal{S}, \text{ flat} \) | \(j \neq 0\) | 1/4 BPS | non-BPS |
| \(G \notin \mathcal{S}, \text{ AdS} \) | \(j = 0\) | 1/2 BPS | 1/4 BPS |
| \(G \notin \mathcal{S}, \text{ AdS} \) | \(j \neq 0\) | ? | ? |

Table 1. An overview of supersymmetry properties of under-rotating attractors and full solutions in abelian gauged theories without hypermultiplets, depending on whether the vector of gaugings \(G\) lies in the very small orbit \(\mathcal{S}\) or not. The '?' for the under-rotating case in AdS signify that the existence of such solutions is not certain, not only that their supersymmetry properties are not analyzed.

4 Asymptotically AdS\(_{4}\) BPS black holes

In section 2 we introduced a procedure to obtain first order equations for asymptotically flat non-supersymmetric black holes by mimicking the squaring of the action that leads to asymptotically AdS\(_{4}\) BPS black holes in an abelian gauged theory. Given the very close similarity between the equations describing the two systems, it is possible to clarify the structure of asymptotically AdS\(_{4}\) static black holes by recycling some of the objects used in the asymptotically flat case.

In this case, the appropriate form for the metric is the one in (2.7), which allows for a non-flat three dimensional base. The relevant effective action now is the one in (2.9), where no assumptions

\(^{14}\)The acute reader might notice that in the static case the mAdS\(_{2}\)×\(S^2\) superalgebra \(SU(1,1|1) \times SO(3)\) can be broken to \(SU(1,1|1) \times U(1)\) without breaking more supersymmetries. This means that one could expect the rotating attractors to also preserve half of the original supercharges. Here we explicitly showed that these attractors are 1/4 BPS by imposing (3.10) and (3.28), but this does not exclude the existence of a more general 1/2 BPS projection that also ensures the supersymmetry variations vanish. The flat rotating attractors here might also be part of a more general class of rotating attractors in gauged supergravity, such as the ones constructed in [65]. We do not pursue this subject further as our present purpose is to show that all asymptotically flat attractors are supersymmetric without focusing on the exact amount of preserved supercharges.
were made for the vector $G$. The flow equations that follow from this squaring are similar to (2.16)-(2.17), with vanishing Lagrange multiplier $\varphi$, together with an equation for the nontrivial $e^\psi$, as in [29]:

$$\mathcal{E} = 0, \quad (4.1)$$

$$\psi' = 2 e^{-U} \text{Im}(e^{-i\alpha} W), \quad (4.2)$$

$$Q_r + \alpha' = -2 e^{-U} \text{Re}(e^{-i\alpha} W), \quad (4.3)$$

and we repeat the expression for $\mathcal{E}$,

$$\mathcal{E} \equiv 2 e^{2\psi} \left( e^{-U} \text{Im}(e^{-i\alpha} V) \right)' + e^{2(\psi - U)} JG + 4 e^{2\psi - U} (Q_r + \alpha') \text{Re}(e^{-i\alpha} V) + \Gamma, \quad (4.4)$$

for the reader's convenience.

Since our goal is to show the similarities between the solutions of this system to the asymptotically flat ones, we will use an ansatz and similar definitions as in section 2.2. Here however, we use the same relations restricting the constant $m = 0$, as one can check by analysing the asymptotic fall-off of the terms in (4.4) that a nonzero $m$ spoils the asymptotic behavior of the scale factor of the metric. Thus, the role of the constant $m$ is drastically changed with respect to the asymptotically flat context, where it is 'dressed' with the Lagrange multiplier and is in fact crucial to obtain the most general static solution.

The flow equations (4.4) can be simplified by defining a vector $R$ from $G$, as in the non-BPS asymptotically flat case. Using the definition (2.29) with $m = 0$, we find

$$-|W|^2 R = JG. \quad (4.5)$$

The crucial difference with the previous situation is that here $R$ is neither constant nor small, since $G$ is not. This allows to rewrite the flow equation for the section as

$$2 e^{2\psi} \left( e^{-U} \text{Im}(e^{-i\alpha} V) \right)' - 2 e^{2\psi} |W|^2 R + 4 e^{2\psi - U} (Q_r + \alpha') \text{Re}(e^{-i\alpha} V) + \Gamma = 0. \quad (4.6)$$

It order to describe solutions, we employ the natural ansatz of [28–30], which only depends on a vector of single center harmonic functions $H$ as

$$2 e^{-U} \text{Im}(e^{-i\alpha} V) = r e^{-\psi} H, \quad (4.7)$$

and immediately leads to a vanishing Kähler connection, as

$$Q_r + \alpha' = 0. \quad (4.8)$$

Note that (4.7) reduces to the asymptotically flat solution (2.31) for $e^\psi = r$ and $m = 0$, as expected. In the more general case, equations (4.2) and (4.7) determine the function $e^\psi$ by

$$(e^\psi)' = r \langle G, H \rangle, \quad (4.9)$$

which can be easily integrated.

In order to integrate the flow equation (4.6) above, one can follow the direct approach of [28–30], that leads to explicit solutions (see the example below). Nevertheless, some intuition from the asymptotically flat case can be used, in order to simplify this process. In particular, we claim that the constraint (2.34), which we repeat here

$$\frac{1}{2} I'_r(H,G) = \langle G, H \rangle H - 2 \frac{(G, H)^2}{(G, R)} R, \quad (4.10)$$

15See (B.26) for the general case including $m$. 

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as written in the context of asymptotically flat solutions for very small vectors $G$ and $R$, is relevant also in the more general case, where these vectors are generic. Note that this might again be related to a reality constraint on the scalar flow as in (2.27), but we do not require any such assumption.

In view of the similarity in the flow equations and the fact that the ansatze in (2.31) and (4.7) are related by rescaling with a function, it is conceivable that a constraint homogeneous in all $H$, $G$ and $R$ as the one in (4.10) may indeed be common in the two cases. Using the explicit examples in [28–30], one can see that this is indeed the case, as we show below.

Example STU solution

In order to see how the constraint above is relevant, we consider the STU model, defined by the prepotential

$$ F = \frac{X^1 X^2 X^3}{X^0}, \quad (4.11) $$

as an example where fairly generic explicit solutions are known, and the expression of $R$ can be computed explicitly. Following [28–30], we choose a frame where the FI terms are

$$ G = \left( 0, g^i ; g_0, 0 \right)^T, \quad (4.12) $$

and consider a vector of single center harmonic functions

$$ H = \left( -H_0^0, 0 ; 0, H_i \right)^T, \quad (4.13) $$

where

$$ H_0^0 = \alpha_0^0 + \frac{\beta_0^0}{r}, \quad H_i = \alpha_i^0 + \frac{\beta_i}{r}. \quad (4.14) $$

The corresponding asymptotically flat solution, where the gauging is only along the $g_0$ direction is given in Appendix C. The reader can easily compare the expressions below with those in the appendix to appreciate the close similarity of the two systems.

With the above expressions one can compute from (4.7) that

$$ e^{-\psi} \text{Re}(e^{-i\alpha} V) = r e^{-\psi} e^{2U} \left( 0, \frac{1}{2} H^0 |e^{ijk}| H_j H_k ; H_1 H_2 H_3, 0 \right)^T, \quad (4.15) $$

and

$$ r^{-4} e^{4\psi} e^{-4U} = 4 H_0^0 H_1 H_2 H_3. \quad (4.16) $$

Finally, we consider a solution to (4.9), given by

$$ \langle G, H \rangle = 2, \quad e^\psi = r^2 + c, \quad (4.17) $$

where $c$ is an arbitrary constant and the first equation is a simplifying condition.

One can now impose the flow equation (4.6) using the assumptions above, to find the constraints

$$ \alpha_0^0 g_0 = \alpha^i g_i, \quad p_0 = c \alpha_0^0 - 2(\beta_0^0)^2 g_0, \quad -q_i = c \alpha_i - 2(\beta_i)^2 g_i, \quad (4.18) $$

where all equations are valid for each value of the index $i$ separately and there is no implicit sum. The explicit expression for $R$ in (4.5) then reads

$$ R^0 = g_0 (H_0^0)^2, \quad R_i = g^i (H_i)^2, \quad (4.19) $$

where again there is no implicit sum.

One can now straightforwardly evaluate the constraint (4.10) using the harmonic functions $H$ in (4.13) and the expression for $R$ in (4.19), to find that it is identically satisfied. We conclude that this constraint is also relevant for asymptotically AdS$_4$ solutions, since (similar to the asymptotically
flat case) one can invert the procedure above to find \( R \) from (4.10) rather than performing the tedious computation of the matrix \( J \) in (A.11).

Additionally, the near horizon limit of (4.10), leads to a nontrivial constraint on the charges in terms of \( G \), exactly as in the asymptotically flat non-BPS case. This is equivalent to the constraints found in [28–30] by solving the BPS flow equations in the STU model explicitly. From that point of view, (4.10) appears to be a duality covariant form of the constraints on the charges in this class of solutions, valid for other symmetric models beyond STU.

5 Extensions including hypermultiplets

Given the results of section 2 on the embedding of asymptotically flat black holes in gauged theories, it is natural to consider the possibility of extending the abelian gauged theory to include hypermultiplets. Indeed, the appearance of the vector of gaugings \( G \) multiplied by a universal function, introduced as a Lagrange multiplier, that is determined independently from the vector multiplet scalars, is a tantalising hint towards such an embedding. In this scenario, one would require the gauging of a single \( U(1) \) factor in the hypermultiplet sector, where the overall Lagrange multiplier \( e^\phi \) in (2.13) is now promoted to a dynamical field, identified with the corresponding moment map, and \( G \) is identified with the embedding tensor [43, 49]. In this section, we explore the possibilities of constructing such a theory, without explicitly considering the embedding of the known asymptotically flat solutions.

In doing so, we consider the explicit compactifications of M-theory on Calabi-Yau manifolds fibered over a circle described in [66–68], (see [64] for a recent overview). This setting is very convenient for our purposes, as it automatically leads to a flat potential for the vector multiplets (2.3), since there is only one \( U(1) \) isometry gauged, along the vector shown in (2.6), identified with the embedding tensor. The hypermultiplet scalars, \( q^u, u = 1 \ldots 4n_h \), in these models parametrise target spaces in the image of the c-map, and describe a fibration of a \( 2n_h + 2 \) dimensional space with coordinates \((a, \tilde{a}, \xi)\), where \( \xi = (\xi^A, \tilde{\xi}^A) \) is a \( 2n_h \) dimensional symplectic vector, over a special Kähler manifold of dimension \( n_h - 1 \), with coordinates arranged in a complex symplectic section \( \Omega = (Z^A, G_A) \), similar to the vector multiplets.

Within this setting, we consider the gauging along the Killing vector

\[
k_U = (U\Omega)^A \frac{\partial}{\partial Z^A} + (\bar{U}\bar{\Omega})^A \frac{\partial}{\partial \tilde{Z}^A} + (U\xi)^A \frac{\partial}{\partial \xi^A} + (\bar{U}\bar{\xi})^A \frac{\partial}{\partial \bar{\xi}^A},
\]

where \( U \) is a symplectic matrix whose explicit form can be found in [64, 68], but is not of immediate importance for what follows. This leads to the standard minimal coupling term for hypermultiplets, by replacing derivatives by covariant derivatives in the kinetic term as

\[
h_{uv} D_{\mu} q^u D^\mu q^v = h_{uv} (\partial_{\mu} q^u + k^u_0 \langle G, A_\mu \rangle) (\partial^\mu q^v + k^v_0 \langle G, A^\mu \rangle),
\]

where \( h_{uv} \) denotes the hyper-Kähler metric. The potential of the gauged theory is now given by

\[
V = V_g + V_{hyp},
\]

where \( V_g \) is a modification of (2.3) as

\[
V_g = Z_i(G) \bar{Z}^i(G) P^2 - 3 |Z(G)|^2 P^2 = \langle G, J G \rangle P^2 - 4 |Z(G)|^2 P^2,
\]

and \( P^2 \) stands for the square of the triplet of moment maps, \( \{ P^x, P^y, P^z \} \) corresponding to (5.1), given by

\[
P^2 = (P^x)^2 + (P^y)^2 + (P^z)^2.
\]
The second term in the scalar potential (5.3) arises from the hypermultiplet gauging and reads

\[ V_{\text{hyp}} = 8 h_{uv} k^u_U k^v_U |\langle G, V \rangle|^2 . \]  

(5.6)

In order for the Einstein equation to allow for asymptotically flat solutions, one must impose the condition

\[ h_{uv} k^u_U k^v_U = 0 , \]  

(5.7)

which eliminates both the potential \( V_{\text{hyp}} \) and the term quadratic in gauge fields in the scalar kinetic term. Since the vector \( G \) in (2.6) lies in the doubly critical orbit \( S \), the vector multiplet potential (5.4) vanishes identically, as usual. Note however that the vector of gaugings naturally appears multiplied by an overall function, the moment map \( P \), coming from the hypermultiplet sector. We can further simplify (5.7), using the facts that the quaternionic metric \( h_{uv} \) is positive definite and only a single \( U(1) \) is gauged, to find that\(^\text{16}\)

\[ k^u_U = 0 . \]  

(5.8)

This way the hypermultiplets condense to their supersymmetric constant values, a process described in detail in [57]. The resulting theory is again the abelian gauged theory of section 2, since the moment map \( P \), which also controls the gravitino gauging, is in general nonvanishing. Note that this is consistent despite the initial presence of a charged hypermultiplet, due to the vanishing of the quadratic term in gauge fields by (5.7), which would otherwise produce a source in the Maxwell equations.

We now show more explicitly how this can be realised in a simple example involving the universal hypermultiplet [69], which is present in all Type II/M-theory compactifications to four dimensions, and is thus included in all the target spaces in the image of the c-map described above. Moreover, the possible gaugings for this multiplet are described by particular choices for \( U \) in the Killing vector (5.1). We use the following metric for the universal hypermultiplet, parametrised by four real scalars \( \rho, \sigma, \tau \) and \( \chi \) as

\[ ds^2_{\text{hyp}} = \frac{1}{\rho^2} \left( d\rho^2 + \rho (d\chi^2 + d\tau^2) + (d\sigma + \chi d\tau)^2 \right) , \]  

(5.9)

which has eight Killing vectors (see appendix D of [42]). Now, consider the particular linear combination of Killing vectors

\[ k_c = -\tau \partial_\chi + \chi \partial_\tau + \frac{1}{2}(\tau^2 - \chi^2)\partial_\sigma - c \partial_\sigma , \]  

(5.10)

parametrised by the arbitrary constant \( c \). Now, the expression (5.7) becomes

\[ h_{uv} k^u_c k^v_c = \frac{\chi^2 + \tau^2}{\rho} + \frac{(\chi^2 + \tau^2 - 2c)^2}{4\rho^2} , \]  

(5.11)

which can vanish in two distinct situations. One is the physical minimum, corresponding to (5.8), for which

\[ \chi = \tau = 0 , \quad \rho > 0 , \quad c = 0 , \]  

(5.12)

while the second solution, given by

\[ \rho = -\frac{(\chi^2 + \tau^2 - 2c)^2}{4(\chi^2 + \tau^2)} < 0 , \]  

(5.13)

is unphysical, since the metric (5.9) is only positive definite when \( 0 < \rho < \infty \) and (5.13) implies that it is of signature \( (2,2) \) instead.

\(^{16}\)We thank Hagen Triendl for pointing out a mistake in a previous version of this paper.
The triplet of moment maps associated to the Killing vector above is given by

\[ P = \left\{ \frac{\tau}{\sqrt{|\rho|}}, \frac{\chi}{\sqrt{|\rho|}}, 1 - \frac{\chi^2 + \tau^2 - 2\epsilon}{4\rho} \right\} , \tag{5.14} \]

and reduces to \( P = \{0, 0, 1\} \) for the physical solution in (5.12), while it is a nontrivial function of \( \chi \) and \( \tau \) when pulled back on the hypersurface defined by (5.13). This is an explicit realisation of the scenario sketched above, since we have obtained a gauged theory with an everywhere vanishing scalar potential, but with nontrivial moment maps. From this standpoint, the embedding of the asymptotically flat solution of section 2 applies directly to these models, similar to examples discussed in [57].

From this simple discussion it follows that a single \( U(1) \) hypermultiplet gauging can only lead to asymptotically flat solutions with the hypermultiplet scalars fixed to constants, leading to a constant moment map. If the resulting value of the moment map is non-zero we are back in the case of FI term gauging that allows for a BPS horizon but non-BPS asymptotics at infinity. If on the other hand the moment maps vanish, one is in the ungauged case with BPS Minkowski vacuum and non-BPS horizon. Note however, that the existence of unphysical solutions of the type (5.13) may be helpful in constructing new solutions without hypermultiplets, where the remaining unfixed scalars may play the role of the unphysical Lagrange multiplier in section 2.

Finally, the interesting problem of obtaining solutions with physical charged hypermultiplets remains. Given the above, it is clear that one must gauge bigger groups (at least \( U(1)^2 \)) of the hypermultiplet isometries to construct such BPS solutions, preserving supersymmetry both at the horizon and at infinity, see e.g. [59, 70]. We can then expect that such a theory, if existent, would allow for solutions preserving two supercharges in the bulk (1/4-BPS solution), given that the attractor preserves \( \mathcal{N} = 1 \) supersymmetry, as established in section 3. Constructing such a theory seems to be a nontrivial but rather interesting task for future investigations.

6 Conclusion and outlook

In this paper we presented in some detail a novel connection between the solutions of ungauged supergravity and gauged supergravity with an identically flat potential in four dimensions. In particular, we identified the recently constructed general solution for under-rotating asymptotically flat black holes as special solutions to abelian gauged supergravity with a flat potential, where nontrivial gaugings are still present and are reflected on the solutions. As an application, we further showed explicitly that the attractor geometries of these black holes belong to the generic class of 1/2-BPS AdS\(_4\)\(\times\)S\(_2\) attractor backgrounds that pertain to (generically asymptotically AdS\(_4\)) black holes in abelian gauged theories. These results are interesting from several points of view, respectively discussed in the main sections above. In this final section, we discuss the implications of our results for possible string models of extremal black holes as well some intriguing similarities to recent results in the study of black holes in the context of supergravity.

The somewhat surprising result of obtaining hitherto hidden supercharges in the near horizon geometries of all extremal under-rotating black holes, deserves some additional attention. Firstly, the supercharges at hand only exist when appropriate FI terms are turned on for given charges, and are not present in general. This means that not all attractors characterised by a negative quartic invariant \( I_4(\Gamma) \) of the charges can be made supersymmetric simultaneously, in contrast to the ones with \( I_4(\Gamma) > 0 \), which correspond to globally BPS solutions. This situation is reminiscent of the example solutions studied recently in [71, 72], which preserve supersymmetry only when embedded in a larger theory, but appear as non-BPS in any \( \mathcal{N} = 2 \) truncation. In combination with these examples, our results show that any supercharges preserved by a solution in a higher dimensional
theory, may only be realised in more general compactifications as opposed to a naive dimensional reduction. This fits very well with the fact that asymptotically Taub-NUT non-BPS black holes in five dimensions can preserve the full $\mathcal{N}=2$ supersymmetry near their horizons [12]. Indeed, while a direct dimensional reduction along the Taub-NUT fiber breaks all supersymmetries, our results imply that the Scherk-Schwarz reduction of [68] preserves half of them. It would be interesting to develop a higher dimensional description of our alternative embedding along these lines, especially in connection to possible microscopic models.

Indeed, one of the most intriguing implications of the presence of supersymmetry for asymptotically flat under-rotating black holes is the possibility of obtaining control over the microscopic counting of the entropy, similar to globally BPS black holes. According to standard lore, one expects a dual microscopic CFT living on the worldvolume of appropriate D-branes to be the relevant description at weak coupling. Such models have been proposed in e.g. [73–77], and arguments on the extrapolation of the entropy counting for non-BPS black holes were formulated in [78, 79], based on extremality. However, our results indicate that one may be able to do better, if a supersymmetric CFT dual for extremal black holes can be found. At this point, one is tempted to conjecture that such a CFT should be a deformation of the known theories describing BPS black holes [1, 2], where half of the supercharges are broken by the presence of appropriate deformation parameters, corresponding to nonzero FI terms. Obtaining a description along these lines would be also very interesting from the point of view of black hole physics in AdS, since it would shed some light on the role of the gaugings in a microscopic setting.

A related question in this respect is the possibility of extending our embedding of asymptotically flat solutions to theories including gauged hypermultiplets. As briefly discussed in section 5, such models with identically flat potentials are possible, and exploring the various gaugings allowed is an interesting subject on its own. From a higher dimensional point of view, the particular gaugings described in [68] represent a natural choice, since they can be formulated in terms of a twisted reduction of ungauged five dimensional supergravity along a circle. A similar twist was recently used in [80] in connection to the near horizon geometry of over-rotating black holes.

Parallel to the implications on the asymptotically flat solutions, one may use the connection established here to learn more about 1/4-BPS black holes in AdS. The somehow surprising fact that the constraint on the charges defined in [20] in the case of flat gauging, is relevant in the more general setting where the gaugings are unrestricted is a hint towards a better understanding of the moduli space of these solutions. Indeed, since this constraint is relevant throughout the flow connecting two uniquely fixed vacua, the asymptotic AdS$_4$ vacuum of the theory and the BPS attractor, it may be relevant for establishing existence criteria for given charges. In addition, it would be interesting to extend our procedure to the non-extremal case, by connecting the results of [31, 32] with those of [81]. We hope to return to some of these questions in future work.

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A Conventions on $\mathcal{N}=2$ supergravity

In this paper we follow the notation and conventions of [20]. In this appendix we collect some basic definitions that are useful in the main text, referring to that paper for more details.

The vector fields naturally arrange in a symplectic vector of electric and magnetic gauge field strengths, whose integral over a sphere defines the associated electromagnetic charges as

$$F_{\mu\nu} = \left( \frac{F_{I\mu\nu}}{G_{I\mu\nu}} \right), \quad \Gamma = \left( \frac{\rho_I}{q_I} \right) = \frac{1}{2\pi} \int_{S^2} F. \quad (A.1)$$

The physical scalar fields $t^I$, which parametrize a special Kähler space of complex dimension $n_v$, appear through the so called symplectic section, $\mathcal{V}$. Choosing a basis, this section can be written in components in terms of scalars $X^I$ as

$$\mathcal{V} = (X^I), \quad F_I = \frac{\partial F}{\partial X^I}, \quad (A.2)$$

where $F$ is a holomorphic function of degree two, called the prepotential, which we will always consider to be cubic

$$F = -\frac{1}{6} c_{ijk} X^i X^j X^k X^0, \quad (A.3)$$

for completely symmetric $c_{ijk}$, $i = 1, \ldots, n_v$. The section $\mathcal{V}$ is subject to the constraints

$$\langle \bar{\mathcal{V}}, \mathcal{V} \rangle = i \langle \bar{D}_i \mathcal{V}, D_j \mathcal{V} \rangle = -i g_{ij}, \quad (A.4)$$

with all other inner products vanishing, and is uniquely determined by the physical scalar fields $t^I = \frac{X^I}{X^0}$ up to a local $U(1)$ transformation. Here, $g_{ij}$ is the Kähler metric and the Kähler covariant derivative $D_i \mathcal{V}$ contains the Kähler connection $Q_\mu$, defined through the Kähler potential as

$$Q = \text{Im}[\partial_i \mathcal{K} dt^i], \quad \mathcal{K} = -\ln \left( \frac{1}{6} c_{ijk} (t - \bar{t})^i(t - \bar{t})^j(t - \bar{t})^k \right). \quad (A.5)$$

We introduce the following notation for any symplectic vector $\Gamma$

$$Z(\Gamma) = \langle \Gamma, \mathcal{V} \rangle, \quad (A.6)$$

$$Z_i(\Gamma) = \langle \Gamma, D_i \mathcal{V} \rangle, \quad (A.7)$$

with the understanding that when an argument does not appear explicitly, the vector of charges in (A.1) should be inserted. In addition, when the argument is form valued, the operation is applied component wise. With these definitions it is possible to introduce a scalar dependent complex basis for symplectic vectors, given by $(\mathcal{V}, D_i \mathcal{V})$, so that any vector $\Gamma$ can be expanded as

$$\Gamma = 2 \text{Im}[\bar{Z}(\Gamma) \mathcal{V} + g^{ij} \bar{Z}_i(\Gamma) D_j \mathcal{V}], \quad (A.8)$$

whereas the symplectic inner product can be expressed as

$$\langle \Gamma_1, \Gamma_2 \rangle = 2 \text{Im}[Z(\Gamma_1) \bar{Z}(\Gamma_2) + g^{ij} Z_i(\Gamma_1) \bar{Z}_j(\Gamma_2)]. \quad (A.9)$$

In addition, we introduce the scalar dependent complex structure $J$, defined as

$$J \mathcal{V} = -i \mathcal{V}, \quad JD_i \mathcal{V} = i D_i \mathcal{V}, \quad (A.10)$$

which can be solved to determine $J$ in terms of the period matrix $N_{IJ}$ in (2.2), see e.g. [55] for more details. With this definition, we can express the complex self-duality of the gauge field strengths as

$$J F = -\ast F, \quad (A.11)$$

which is the duality covariant form of the relation between electric and magnetic components. Finally, we record the important relation

$$\langle \Gamma, J \Gamma \rangle = |Z(\Gamma)|^2 + g^{ij} Z_i(\Gamma) \bar{Z}_j(\Gamma) \equiv V_{\text{BH}}(\Gamma), \quad (A.12)$$

where we defined the black hole potential $V_{\text{BH}}(\Gamma)$. 

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B First order flows for rotating black holes

In this appendix we discuss the rewriting of the effective action as a sum of squares and the corresponding flow equations for stationary black holes in four dimensional abelian gauged $\mathcal{N} = 2$ supergravity. In section B.1 we present the general case, while in section B.2 we specialise to the case of flat potential to show that the general asymptotically flat under-rotating black holes are indeed solutions of the theory in this limit. We largely follow [4, 29] with respect to the method and notational conventions.

B.1 Squaring of the action

We start with a timelike reduction to three spatial dimensions using the metric ansatz:

$$ ds^2 = -e^{2U}(dt + \omega)^2 + e^{-2U}(dr^2 + e^{2\psi}d\theta^2 + e^{2\psi}\sin^2\theta d\phi^2) , $$

(B.1)

which generalises (2.7) by the addition of the angular momentum vector $\omega$. In this setting we allow for a dependence of the fields on all spatial coordinates, so that a timelike reduction is appropriate [82]. The effective three-dimensional action reads:

$$ S_{3D} = -\frac{1}{16\pi} \int dt \int d^3x \left[ -2\left( d(\psi \wedge \star d\psi) - \star 1 \right) + 2dU \wedge \star dU - \frac{1}{2} e^{4U} d\omega \wedge \star d\omega \\
+ 2g_{ij} dt^i \wedge \star d\tilde{\Omega} + (\mathcal{F}, \mathcal{F}) + e^{-2U} \langle G, \gamma G \rangle \star 1 - 8 e^{-2U} |Z(G)|^2 \star 1 \right] , $$

(B.2)

where $\mathcal{F}$ is the spatial projection of the four dimensional field strengths, $\star$ denotes the Hodge dual in three dimensions and we discarded a boundary term. The scalar dependent inner product denoted by $(,)$ is a generalisation of (A.12) in the rotating case that is explicitly given by [4]

$$ (\mathcal{A}, \mathcal{B}) = \frac{e^{2U}}{1 - w^2} \int \mathcal{A} \wedge \left[ \star (J \mathcal{B}) - \star (w \wedge J \mathcal{B}) \right] , $$

(B.3)

for any two symplectic vectors of two-forms $\mathcal{A}, \mathcal{B}$, and we define $w = e^{2U} \omega$, as a shorthand below.

In order to stay as close as possible to the static case, we treat the gauging parameters $G$ as gauge field strengths in the three dimensional base space. We then define

$$ \mathcal{G} = G \star \eta , $$

(B.4)

where $\eta$ is a one-form which we require to be invariant under the vector $\omega$, but is otherwise undetermined at this stage. The choice $\eta = dr$ is the one relevant for the static solutions.

Inspired by [29], we can use the above definitions to recombine the gauge kinetic term and the potential using the following combination ($Z(\mathcal{G}) = \langle \mathcal{G}, \mathcal{V} \rangle$)

$$ \tilde{\mathcal{F}} = \mathcal{F} - e^{-2U} (JG - w \wedge \star \mathcal{G}) + 4 e^{-2U} \Re \left[ \Re Z(e^{-i\alpha} G) e^{i\alpha} \mathcal{V} + i \Re Z(e^{-i\alpha} \star G) \wedge we^{i\alpha} \mathcal{V} \right] , $$

(B.5)

which is such that

$$ (\tilde{\mathcal{F}}, \tilde{\mathcal{F}}) = (\mathcal{F}, \mathcal{F}) + e^{-2U} \langle JG, \gamma G \rangle \star \eta \wedge \eta + 2 (\mathcal{F}, \star \mathcal{G}) + 8 \Im e^{i\alpha} Z(\mathcal{F}) \wedge \Re e^{-i\alpha} (\star \mathcal{G}) , $$

(B.6)

and $e^{i\alpha}$ is an arbitrary phase as in the static case. The scalars can be repackaged in a similar way using the standard combination [4]

$$ W = 2 \Im \star D(e^{-U} e^{-i\alpha} \mathcal{V}) - 2 \Re D(e^{U} e^{-i\alpha} \mathcal{V} \omega) , $$

(B.7)

$$ D = d + i (Q + d\alpha + \frac{i}{2} e^{2U} \star d\omega) , $$

(B.8)
which in turn is such that

\[
2 \, dU \wedge *dU - \frac{1}{2} e^{2U} \, d\omega \wedge *d\omega + 2 \, g_{ij} \, dt^i \wedge *dt^j \\
= (\mathcal{W}, \mathcal{W}) - 2 \left( Q + d\alpha + \frac{1}{2} e^{2U} *d\omega \right) \wedge d\omega + d \left[ 2w \wedge (Q + d\alpha) \right],
\]

so that the action reads

\[
S_{4D} = -\frac{1}{16\pi} \int d^4x \left[ (\mathcal{W}, \mathcal{W}) - 2 \left( Q + d\alpha + \frac{1}{2} e^{2U} *d\omega \right) \wedge d\omega \\
+ (\tilde{\mathcal{F}}, \tilde{\mathcal{F}}) - 2 (\mathcal{F}, *\mathcal{G}) - 8 e^{-2U} |Z(G)|^2 \wedge 1 \\
- 8 \left( \text{Im}(e^{-i\alpha} Z(\mathcal{F})) - w \wedge \text{Im}(e^{i\alpha} Z(*\mathcal{G})) \right) \wedge \text{Re} e^{-i\alpha} Z(*\mathcal{G}) \\
- 2 \left( d\psi \wedge *d\psi - *1 \right) \right].
\]

We now proceed to write the action as a sum of squares, making use of the further definitions

\[
\mathcal{E} = \tilde{\mathcal{F}} - \mathcal{W},
\]

\[
\text{Im}(\mathcal{E}, e^U e^{-i\alpha} \mathcal{V}) = e^U \left( \text{Im}(e^{-i\alpha} Z(\mathcal{F})) + e^{-U} \text{Re}(e^{-i\alpha} Z(\mathcal{G})) - e^{-U} w \wedge \text{Im}(e^{-i\alpha} Z(*\mathcal{G})) \right) + \frac{1}{2} dw.
\]

After some rearrangements one obtains the result

\[
S_{4D} = -\frac{1}{16\pi} \int \int_{\mathbb{R}^3} \left[ (\mathcal{E}, \mathcal{E}) - 4 \left( Q + d\alpha + 2 e^{-U} \text{Re} Z(*\mathcal{G}) + \frac{1}{2} e^{2U} *d\omega \right) \wedge \text{Im}(\mathcal{E}, e^U e^{-i\alpha} \mathcal{V}) \\
- 2 \left( [\mathcal{F} + 2 \text{Re} d(e^U e^{-i\alpha} \mathcal{V} \omega), *\mathcal{G}] - *1 \right) \\
- 2 \left( *d\psi - 2 e^{-U} \text{Im}(e^{-i\alpha} Z(\mathcal{G})) \right) \wedge (d\psi - 2 e^{-U} \text{Im}(e^{-i\alpha} Z(*\mathcal{G}))) \\
+ 4 e^{-U} e^{2\psi} \text{Im}(e^{-i\alpha} Z(G)) \, d \left( e^{-2\psi} *\eta \right) \right].
\]

Note that we added and subtracted a term in order to obtain the squaring of the third line, which leads to the additional factor $e^{-2\psi}$ in the derivative of the last line.

The last form of the action is a sum of squares, except for the terms involving the derivative of $\eta$ and $(\mathcal{F}, *\mathcal{G})$, which one should demand to be a total derivative, thus constraining the one-form $\eta$. However, since an analysis of the resulting equations of motion is outside the scope of this appendix, we restrict ourselves to the case of an identically flat potential, mentioning that the general equations have the same structure as the BPS equations of [83] and may reproduce them once $\eta$ is specified.

### B.2 Asymptotically flat solutions

Turning to the asymptotically flat case, we assume that the FI terms are given by a very small vector and we choose the one-form $\eta$ in (B.4) as

\[
\eta = e^\varphi \, dr,
\]

where we absorbed the Lagrange multiplier $\varphi$ of the static squaring in section 2, allowing it to depend on all spatial coordinates. Similar to the static case, we impose that the base space is flat, so that $e^\varphi = r$, which leads to a modified rewriting of the action as

\[
S_{4D} = -\frac{1}{16\pi} \int \int_{\mathbb{R}^3} \left[ (\mathcal{E}, \mathcal{E}) - 4 \left( Q + d\alpha + 2 e^{-U} \text{Re} Z(*\mathcal{G}) + \frac{1}{2} e^{2U} *d\omega \right) \wedge \text{Im}(\mathcal{E}, e^U e^{-i\alpha} \mathcal{V}) \\
- 2 \left( [\mathcal{F} + 2 \text{Re} d(e^U e^{-i\alpha} \mathcal{V} \omega), *\mathcal{G}] + 2 \, du \wedge *du \right) \\
- 2 \left( 2 e^{-U} \text{Im}(e^{-i\alpha} Z(\mathcal{G})) - *du \right) \wedge \left( 2 e^{-U} \text{Im}(e^{-i\alpha} Z(*\mathcal{G})) - du \right) \right].
\]
where we discarded a non-dynamical term and \( u \) is the scalar defined in (2.19). The equations of motion following from this action are solved by the relations

\[
\mathcal{E} = \mathcal{F} - W = 0, \quad (B.16)
\]
\[
Q + d\alpha + 2e^{-U} \text{Re}(e^{-i\alpha} Z(\ast \mathcal{G})) + \frac{1}{2} e^{2U} \ast d\omega = 0, \quad (B.17)
\]
\[
2 e^{-U} \text{Im}(e^{-i\alpha} Z(\ast \mathcal{G})) - du = 0, \quad (B.18)
\]
along with the equation of motion for the Lagrange multiplier, which reads

\[
d(\ast du) - dr \wedge (G, \mathcal{F} + 2 \text{Re}(e^{U} e^{-i\alpha} V \omega)) r^{-2} e^{-2u} = 0. \quad (B.19)
\]

Despite the apparent complication of the equations above, one can show that the rotating black holes of [20] are solutions to the equations above, in the following way. Firstly, we introduce the decomposition of the spatial field strengths in electromagnetic potentials and vector fields as

\[
\mathcal{F} = d(\zeta \omega) + dw, \quad (B.20)
\]
whose integrability condition implies through (B.4) and (B.14) that \( \eta \) is exact and thus \( e^\varphi \) is a total derivative with respect to the radial component. Considering a single center solution, the vector fields \( dw \) define the charges \( \Gamma \) through harmonic functions \( \mathcal{H} \), so that the equation of motion for the Lagrange multiplier takes the form

\[
d(\ast du) - r^{-2} dr \wedge (G, \Gamma) e^{-2u} \ast 1 = 0, \quad (B.22)
\]
and thus admits the same enveloping solution (2.25), which we adopt henceforth. Note that this is indeed such that \( e^\varphi \) is a total derivative as

\[
e^\varphi = \partial_r \left( \frac{1}{V} \right). \quad (B.23)
\]

We are then in a position to write the linear system to be solved in the asymptotically flat case, explicitly given by

\[
\zeta = -2 \text{Re}(e^{U} e^{-i\alpha} V) + d \left( \frac{1}{V} \right) G, \quad (B.24)
\]
\[
dw - e^{-2U} \hat{G} + 4 e^{-2U} \text{Re} Z(e^{-i\alpha} G) \text{Re} (e^{i\alpha} V) = 2 \text{Im} \ast D(e^{-U} e^{-i\alpha} V) - \frac{1}{V} d\omega G. \quad (B.25)
\]
This can be simplified using the definition (2.29)-(2.30) for the second very small vector, and the associated decomposition

\[
JG = -\frac{1}{2} e^{2U} V^2 R + M e^{2U} G + 4 M V e^{3U} \text{Re} \left( e^{-i\alpha} V \right), \quad (B.26)
\]
that follows from it, as one can compute directly. Note that here we have upgraded the constant \( m \) in (2.30) to a function \( M \), which will turn out to control the angular momentum. Multiplying the last relation with the Lagrange multiplier in (B.23) we find

\[
e^{-2U} JG = \frac{1}{2} * dV R + M * d \left( \frac{1}{V} \right) G + 4 M V * d \left( \frac{1}{V} \right) e^{U} \text{Re} \left( e^{-i\alpha} V \right), \quad (B.27)
\]
which can in turn be used in (B.25) to obtain
\[
dw - \frac{1}{2} \star dV R - M \star d\left(\frac{1}{V}\right)G + \frac{1}{V} d\omega G - 2 \Im \star d(e^{-U} e^{-i\alpha} V) \\
= -4 \left[2 e^{-2U} \Re Z(e^{-i\alpha} G) - M V \star d\left(\frac{1}{V}\right) e^U\right] \Re\left(e^{-i\alpha} V\right).
\] (B.28)

Here and henceforth we assume that the very small vector \( R \) is a constant, which will be shown to be a consistent choice at the end. Imposing that the terms proportional to the real part of the section in (B.28) cancel, leads to the constraint
\[
2 e^{-2U} \Re Z(e^{-i\alpha} G) = M V \star d\left(\frac{1}{V}\right) e^U,
\] (B.29)

which together with the additional condition
\[
\star d\omega = -dM,
\] (B.30)

that implies that both \( M \) and \( \omega \) are harmonic, results to the system of equations
\[
dw - \frac{1}{2} \star dV R - \star d\left(\frac{M}{V}\right) G = 2 \Im \star d(e^{-U} e^{-i\alpha} V).
\] (B.31)

Integrating the last equation leads to a generalisation of (2.31), given by
\[
2 \Im(e^{-U} e^{-i\alpha} V) = \mathcal{H} - \frac{1}{2} V R - \frac{M}{V} G,
\] (B.32)

where the vector fields are given by the harmonic functions \( \mathcal{H} \) as
\[
dw = \star d\mathcal{H}.
\] (B.33)

One can now compare the above to the explicit equations for the general asymptotically flat under-rotating single center solutions of [20], which turn out to be described by (B.29)-(B.31) with \( \omega \to -\omega \) and for \( R \) being a constant very small vector, as in the static case. Indeed, one could have started from the system (B.25) to establish that the scalar flow equations are the ones of [20] up to a constraint generalising (2.27), which is again equivalent to the constancy of the vector \( R \), as we did in section 2. However, we chose to present the symplectic covariant derivation of the equations both for simplicity and completeness. The analysis of [20] ensures that this is a consistent solution of the full Einstein equations, so that we do not have to consider the Hamiltonian constraint that has to be imposed on solutions to the effective action in (B.15), as in the static case (see [84] for details on this constraint).

C Example STU solution

Full solution

In this appendix we present the known rotating seed solution in a specific duality frame [14], as an example to use in the main body. For comparison, we use the STU model, in (4.11) in what follows. The charges of the solution are given by poles in the following choice of harmonic functions
\[
\mathcal{H} = (H^0, 0; 0, H_i)\bigg) ,
\] (C.1)
\[
H_i = h_i + \frac{q_i}{r}, \quad H^0 = h^0 + \frac{p^0}{r}.
\] (C.2)
In this duality frame the constant small vectors can be chosen as
\[ \hat{R} = (-4, 0, 0)^T, \quad G = (0, 0, 1, 0)^T, \] (C.3)
but we point out that the choice is not unique, see [20] for a detailed discussion. The scalar fields are given by solving (B.32), leading to the physical scalars
\[ t^i = \frac{M - ie^{-2U}}{2H^0 H_i}, \] (C.4)
as well as to the real part of the section
\[ 2e^{-U} \text{Re}(e^{-i\alpha}V) = e^{2U} \left( M H^0, -H^0 |e^{ijk}H_j H_k; -\frac{M^2}{r^2} - 2H_1 H_2 H_3, M H_i \right)^T. \] (C.5)
The metric is given by (B.1) with
\[ e^{-4U} = 4 H^0 H_1 H_2 H_3 - M^2, \quad *d\omega = -dM, \] (C.6)
where \( M \) is a dipole harmonic function
\[ M = m + \frac{j \cos \theta}{r^2}. \] (C.7)
Finally, the gauge fields are given by (B.20), with the \( \zeta \) given by (B.24) and (C.5), while the \( dw \) are given by (B.33).

**Near horizon solution**
We now take the near horizon of the solution above, which is obtained by dropping the constants in all harmonic functions. The scalars (C.4) become
\[ t^i = \frac{j \cos \theta - ie^{-2U}}{2\rho^0 q_i}, \] (C.8)
whereas the near horizon metric still given by
\[ ds^2 = -e^{-2U} r^2 (dt + \omega)^2 + e^{-2U} \left( \frac{dr^2}{r^2} + d\theta^2 + \sin^2 \theta d\phi^2 \right), \]
\[ e^{-4U} = 4 \rho^0 q_1 q_2 q_3 - j^2 \cos^2 \theta \quad \omega = j \frac{\sin^2 \theta}{r} d\phi. \] (C.9)
For convenience we give the near horizon gauge fields, which are given by (B.20) using (B.24) and (B.33) as
\[ \mathcal{F} = d[\zeta r (dt + \omega)] + \Gamma \sin \theta d\theta \wedge d\phi, \quad \zeta = -2 e^{U} \text{Re}[e^{-i\alpha}V] + \frac{1}{q_0} G, \] (C.10)
where the real part of the section follows from (C.5) by replacing harmonic functions by their poles, as above.

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