UNSTEADY OUTFLOW MODELS FOR COSMOLOGICAL GAMMA-RAY BURSTS

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Abstract

The 'event' that triggers a gamma ray burst cannot last for more than a few seconds. This is, however, long compared with the dynamical timescale of a compact stellar-mass object ($\sim 10^{-3}$ seconds). Energy is assumed to be released as an outflow with high mean lorentz factor $\Gamma$. But a compact stellar-mass collapse or merger is, realistically, likely to generate a mass (or energy) flux that is unsteady on some timescales in the range $10^{-3}$ - 10 seconds. If $\Gamma$ fluctuates by a factor of $\sim 2$ around its mean value, relative motions within the outflowing material will themselves (in the comoving frame) be relativistic, and can give rise to internal shocks. For $\Gamma \sim 10^2$, the resultant dissipation occurs outside the 'photosphere' and can convert a substantial fraction of the overall outflow energy into non-thermal radiation. This suggests a mechanism for cosmological bursts that demands less extreme assumptions (in respect of $\Gamma$-values, freedom from baryonic contamination, etc) than earlier proposals.

1. Introduction

Gamma ray bursts are clearly the 'signal' of an energetic event lasting (typically) no more than a few seconds. The energy output in the gamma rays themselves, if these events are at 'cosmological' distances, would be up to $10^{51}$ ergs. Each burst would then, probably, involve the collapse of a stellar-mass object, or the coalescence of a compact binary (see Hartmann, 1993, or Paczyński, 1994, for an overall review). The main theoretical challenge is to understand how the energy can be converted, with adequate efficiency, into gamma rays with a non-thermal spectrum

A widely recognized problem is that if the rest mass energy of entrained baryons exceeded even $10^{-5}$ of the total energy, the associated opacity would trap the radiation so that it was degraded by adiabatic expansion (and thermalized) before escape, e.g. Cavallo and Rees, 1978, Paczyński, 1990. The energy would therefore be transformed into kinetic energy of bulk relativistic outflow. This problem arises if the 'event' is approximated as
an instantaneous fireball, or as an outflowing wind which is 'steady' over the entire burst duration. In earlier papers, we have discussed how kinetic energy can be reconverted into gamma rays by relativistic shocks which form when the ejecta run into external matter (either ambient interstellar matter, or a non-relativistic outflow preceding the 'event' itself). We showed that acceptable models required bulk Lorentz factors $\Gamma$ of order 1000 – still high, but allowing much more baryonic loading and opacity than would be tolerable if the energy escaped directly from a 'simple' fireball before adiabatic losses had attenuated and thermalized it.

In this paper we show that the constraints are eased still further if we adopt a less idealized picture of the 'event' itself. The production of the relativistic ejecta will be spread over a finite duration; moreover the physical conditions determining $\eta = E/Mc^2$ (or $\eta = L/\dot{M}c^2$, the ratio of radiation and magnetic energy to rest mass, which gives an upper limit for $\Gamma$) will not be steady throughout the event. Fast (higher $\eta$) ejecta can catch up with slower material that was ejected earlier: kinetic energy can then be reconverted into energetic particles (and thence into gamma-rays). This suggests a mechanism for 'cosmological' bursts that can operate for much lower values of $\eta$ (i.e. higher loading factors) than were previously believed necessary.

In these more realistic models, dissipation happens whenever internal shocks develop in the ejecta – it need not await the deceleration by sweeping up ambient external matter. Simple relativistic kinematics show, however, that the internal shocks do not develop until the ejecta have attained sufficiently large distances that the resultant radiation can escape without thermalization or adiabatic losses.

We show how variations in the terminal speed, resulting from inhomogeneous or time-varying conditions around the central object, can yield efficient production of gamma rays, and generally a complex time-structure. We outline the general kinematics in §2. We then discuss in §3 the nature of the dissipation, and the distinctive role of strong fields in 'magnetically-dominated' outflows. The relation of these ideas to gamma-ray burst phenomenology is outlined in §4.

2. Kinematics in an Unsteady Relativistic Outflow

2.1 An Illustrative Example

We postulate an outflow persisting (typically) for a few seconds. But instead of assuming this to be a steady wind we suppose that it is irregular on much shorter timescales. This may be a 'wind' from a high-$B$ newly formed pulsar (Usov 1992,94), or the debris (again probably highly magnetized) flung off from a compact binary during the complex dynamics of its coalescence into a black hole and surrounding disc (Narayan, et al., 1992, Davies, et al., 1994).

Whenever part of the ejecta 'catches up' with other material ejected earlier at a lower Lorentz factor, an internal shock forms, which dissipates the relative kinetic energy. To illustrate the basic idea, suppose that two 'blobs' of equal rest mass, but with different
Lorentz factors $\Gamma_1$ and $\Gamma_2$ (with $\Gamma_2 > \Gamma_1 \gg 1$) are ejected at times $t_1$ and $t_2$, where $t_2 - t_1 = t_{\text{var}}$. In the case of highly relativistic ejecta, the shock develops after a distance of order $ct_{\text{var}}\Gamma_1\Gamma_2$. For high Lorentz factors, therefore, the shock takes a long time to develop, even if $\Gamma \gg 1$. This is, of course, because the distance that must be caught up is (in the ‘laboratory’ frame) of order $ct_{\text{var}}$\,$\Gamma_1\Gamma_2$. For high Lorentz factors, therefore, the shock takes a long time to develop, even if $\Gamma \gg 1$. This is, of course, because the distance that must be caught up is (in the ‘laboratory’ frame) of order $ct_{\text{var}}$, but the speeds all differ from $c$ by less than $1/\Gamma_1^2$.

For example, suppose that the Lorentz factor of the outflow is, on average, 100, but varies from 50 to 200 on a timescale $t_{\text{var}}$. The velocity differences are of order $10^{-4}c$, so the distance for the shock to develop in the lab frame is $10^4ct_{\text{var}}$. The reconversion of bulk energy can nevertheless be very efficient: when the two blobs share their momentum, they move with $\Gamma_{\text{final}} = \sqrt{\Gamma_1\Gamma_2}$, so the fraction of the energy dissipated is

$$\varepsilon = (\Gamma_1 + \Gamma_2 - 2\sqrt{\Gamma_1\Gamma_2})/(\Gamma_1 + \Gamma_2) .$$

(1)

For the previous numerical example, the efficiency would be 20%. High efficiency does not, therefore, require an impact on matter at rest; all that is needed is that the relative motions in the comoving frame be relativistic –i.e. $\Gamma_2/\Gamma_1 > 2$ (c.f. Rees, 1978, for an application of this argument in a different context).

2.2 An Unsteady Wind

Suppose that the mean outflow (over the few seconds that a typical event lasts) can be characterized by a steady wind with given mean values of $L_w$ and $\eta = L_w/\dot{M}c^2$. We then envisage that the value of $\eta$ (or $L_w$) is unsteady.

The mean properties of the wind determine the average bulk Lorentz factor

$$\Gamma \sim \begin{cases} (r/r_l) , & \text{for } r \lesssim r_s ; \\ \eta , & \text{for } r \gtrsim r_s . \end{cases}$$

(2)

Here $r_l$ is the size at the base of the wind (in ‘young pulsar’ models, c.f. Usov, 1992, this would equal $cP/2\pi$, where $P$ is the period). The Lorentz factor saturates to $\Gamma \sim \eta$ at a saturation radius $r_s/r_l \sim \eta$ where the wind energy density, in radiation or in magnetic fields, drops below the baryon rest mass density in the comoving frame. For a given baryonic mass loss $\dot{M}$ the photospheric radius where the wind becomes optically thin to Thomson scattering is

$$r_{\text{ph}} = \dot{M}\kappa/(4\pi c\Gamma^2) = 1.2 \times 10^{12}L_{51}\eta_2^{-3} \text{ cm} ,$$

(3)

where $\eta_2 = (\eta/10^2)$ and $L_{51} = (L_w/10^{51}$ ergs). The above equation holds provided that $\eta$ is low enough that the wind has already reached its 'terminal' Lorentz factor at $r_{\text{ph}}$. This requires $\eta \lesssim \eta_m \sim 10^2L_{51}^{1/4}t_{\text{var}}^{-1/4}$, if one takes $r_l \sim ct_{\text{var}}$.

When a compact binary coalesces, the ejecta would obviously be very messy. The characteristic timescale at the 'base' of the wind is of order $t_{\text{dyn}} \sim 10^{-3}$ s. (The breakup
angular speed of a pulsar is of the same order). Large-amplitude variations could occur on any timescale longer than this. (Even more rapid variations are also possible, especially if there is a strong tangled magnetic field in the wind, e.g. §3).

If the value of $\eta$ at the base increases by a factor $\gtrsim 2$ over a timescale $t_{var}$, then the later ejecta will catch up and dissipate a significant fraction of their energy at some radius $r_d > r_s$ given by

$$r_d \sim ct_{var}\eta^2 \sim 3 \times 10^{14}t_{var}\eta_2^2 \text{ cm},$$

where $t_{var}$ is in seconds. Dissipation, to be most effective, must occur when the wind is optically thin – otherwise it will suffer adiabatic cooling before escaping, and be thermalized. (Outside $r_s$ and $r_{ph}$, where radiation has decoupled from the plasma, the sound speed will be far below $c$; this also guarantees that the relativistic internal motions in the comoving frame will lead to shocks in the gas). This implies the following lower limit on $\eta$:

$$\eta \gtrsim 3 \times 10^1 L_{51}^{1/5} t_{var}^{-1/5}.$$  

This simple estimate doesn’t take into account the extra pairs that may result from dissipation. We comment further on this in §3.

The physical conditions in these shocks qualitatively resemble those in the reverse shocks behind the blast waves discussed by Mészáros and Rees (1993); however the densities and magnetic fields are higher, and the cooling more efficient, because $r$ is smaller. (The internal Lorentz factors are modest, because, to first approximation, all the material is comoving outwards at more or less the same speed).

There will be variations with a range of $t_{var}$. Rapid fluctuations are dissipated at smaller $r$ than those on longer timescales. There would therefore be a dependence of the spectrum on the characteristic variability timescale. The radiation processes depend on the magnetic field strength. We discuss this next, because the field may also be dynamically important.

**3. The Role of the Magnetic Field.**

Ultra-intense magnetic fields are expected either in ‘young pulsar’ models (Usov 1992) or if the field builds up towards equipartition by differential motions (Narayan et al 1992) or a convective dynamo (Duncan and Thompson, 1992). Collapsing or coalescing neutron stars may generate fields as high as $B_i \sim 10^{16}G$. Magnetic stresses could, indeed, be dynamically dominant over the radiation: the ratio of magnetic energy to rest-mass energy at the base of the wind ($r = r_l$) would then determine the effective value of $\eta$. Even a magnetic field that was not dynamically-dominant would still be important in ensuring effective cooling by cyclotron and synchrotron emission.
If the Poynting flux provides a fraction $\alpha$ of the total luminosity $L$ at the base of the wind (at $rt_{\text{var}} \sim ct_{\text{var}}$) the magnetic field there is $B_l \sim 10^{10} \alpha^{1/2}L_{51}^{1/2}t_{\text{var}}^{-1}$ G. The comoving magnetic field at the dissipation radius (4) (which always is outside $r_s$) is

$$B_d = B_l(r_l/r_s)^2(r_s/r_d) \sim 10^4 \alpha^{1/2}L_{51}^{1/2}t_{\text{var}}^{-1}\eta_2^{-3} \text{ G}.$$  \hspace{1cm} (6)

If the electrons are accelerated in the dissipation shocks to a Lorentz factor $\gamma = 10^3\gamma_3$ the ratio of the synchrotron cooling time to the dynamic expansion time in the comoving frame is

$$\left(t_{\text{sy}}/t_{\text{ex}}\right)_d \sim 5 \times 10^{-3} \alpha^{-1}L_{51}^{-1}\gamma_3^{-1}t_{\text{var}}\eta_2^5,$$  \hspace{1cm} (7)

so a very high radiative efficiency is ensured even for $t_{\text{var}}$ as high as seconds.

It is clear from eq.(7) that a magnetic field can ensure efficient cooling even if it is not strong enough to be dynamically significant (i.e. even for $\alpha \ll 1$). If, however, the field is dynamically significant in the wind, then its stresses will certainly dominate the (pre-shock) gas pressure. Indeed, in a wind with $\alpha = 1$ the magnetosonic and Alfvén speeds may remain marginally relativistic even beyond $r_s$ if the field becomes predominantly transverse. In this extreme case, magnetic fields could inhibit shock formation unless $\eta$ varied by much more than a factor of 2. On the other hand, the presence of a dynamically-significant and non-uniform field could actually drive internal motions leading to dissipation even in a constant-$\eta$ wind. Except in the special case of an aligned dipole, the magnetic field would have reversals on a scale of order $r_l \sim cP/2\pi = 5 \times 10^6 P_{-3}$ cm. If the field inside $r_l$ has a complex (non-dipole) structure, the reversal could be even smaller. (Thompson, 1994, has discussed a detailed model where the resultant Alfvén waves are dissipated via Compton drag when the scattering optical depth is still large.)

4. Phenomenology and Discussion

An unsteady (and probably magnetized) wind or fireball has the advantage that it can accommodate a larger amount of baryon contamination than in previous models, while still producing a nonthermal gamma-ray burst via synchro-Compton radiation (as in the reverse shock of Mészáros, Laguna and Rees, 1993; c.f. also Katz, 1994) from electrons accelerated in the shock dissipation region beyond the photosphere.

If the energy were released as an 'instantaneous' fireball, or as a steady wind, and transformed into kinetic energy by adiabatic expansion, then efficient reconversion
into gamma rays occurs when (but not until) enough external matter has been swept up to decelerate it. In earlier papers, we have shown how the resultant shocks could give rise to gamma-ray bursts provided that the Lorentz factors are of order 1000; complex time-structure must then be put down to irregularities in the ambient medium.

In the present paper, we have assumed (undoubtedly more realistically) that the energy release is complex and irregular, and shown that this assumption admits the extra possibility that 'internal shocks' can dissipate a substantial fraction of the kinetic energy before the ejecta encounter the ambient medium. The compact object triggering the burst is likely to have a characteristic dynamical timescale of only about $t_{\text{dyn}} \sim 10^{-3}$ s (of order $t_{\text{var}} \sim r_t/c$ in §2). But the energy release is likely to be more prolonged, determined by, e.g., magnetic spindown, disk viscosity or neutrino diffusion timescales, depending on the model. The energy flux in the outflow, or the value of $\eta$, could then fluctuate on all timescales from $t_{\text{dyn}}$ up to the overall duration of the energy release, which could be many seconds. (Indeed, some models – e.g. rapidly spinning pulsars with complex non-dipole fields – permit irregularities on timescales even shorter than $t_{\text{dyn}}$).

In comparision with shocks involving external ambient matter, the internal shocks form at smaller radii, and in regions of higher density. The Lorentz factors (and values of $\eta$) needed in order to get efficient dissipation, and short 'observer frame' timescales, can then be somewhat lower. However, values of $\eta \gtrsim 30(L_{51}/t_{\text{var}})^{1/5}$ (eq.[5]) are still required in order to ensure that the shocks form outside the photosphere.

This implies wind mass losses $\dot{M} \lesssim 3 \times 10^{-2} L c^{-2} \sim 10^{-5} L_{51} M_{\odot}^{-1}$ s$^{-1}$. The total (isotropic) mass loss expected from an unmagnetized collapsing core or merging compact binary is of order $10^{-3} M_{\odot}^{-1}$ s$^{-1}$ (e.g. Mészáros and Rees, 1992, Woosley, 1993). However, along the rotational (or binary) axis centrifugal forces may significantly reduce the baryon losses, while in a strongly magnetized object mass loss is expected only from the open polar field lines, representing a fraction $\lesssim 10^{-2}$ of the total area, so $\dot{M} \lesssim 10^{-5} M_{\odot}$ is reasonable. (The value of $\dot{M}$ implied by our required value of $\eta$ ($\sim 10^2$) are high enough to ensure that the MHD approximation applies to any magnetic field in the wind; this contrasts with Usov’s (1992, 1994) proposal, which would require a very much lower $\dot{M}$).

The observed bursts are remarkable for their disparate and complex time-structure, and this may be a consequence of how the development of internal shocks is subject to irregularities in the outflow.

In summary, the implications of this work are

(i) The short timescales (and adequate efficiencies) do not need such high values of $\eta$ as our earlier blast wave models (which were themselves much less demanding in this respect than pre-1992 'fireball' models).

(ii) The time structure could be complex, being dependent on the time history of the lorentz factor. The dissipation associated with shorter timescales would tend to occur at smaller radii.

(iii) Since the observed gamma rays from each part of the wind are concentrated, owing to aberration, into an angle of order $1/\eta$, our discussion can be straightforwardly extended to a 'beamed' or jet-like geometry. The broad range of burst morphologies could then, at least in part, be due to the axis and the boundaries of jets having different Lorentz factors...
or internal variability.  
(iv) The energy sources in a cosmological context could be either stellar collapse (Usov, 1992, Woosley, 1993) or compact binary mergers (Paczynski , 1986, Eichler, et al. , 1989, Narayan, Paczynski and Piran, 1992).

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References

Cavallo, G. and Rees, M.J., 1978, M.N.R.A.S., 183, 359  
Davies, M.B., Benz, W., Piran, T., and Thielemann, F.K., 1994, Ap. J., in press.  
Duncan, R.C. and Thompson, C., Ap.J.(Letters), 392, L9  
Eichler, D., Livio, M., Piran, T. and Schramm, D., 1989, Nature, 340, 126  
Hartmann, D., et al., 1993, to appear in *High Energy Astrophysics*, J. Matthews, ed. (World Scientific).  
Katz, J.I., 1994, Ap.J., 422, 248  
Mészáros , P. and Rees, M.J., 1992, Ap.J., 397, 570  
Mészáros , P. and Rees, M.J., 1993, Ap.J., 405, 278  
Mészáros , P., Laguna, P. and Rees, M.J., 1993, Ap.J., 415, 181.  
Narayan, R., Paczynski, B. and Piran, T., 1992, Ap.J.(Letters), 395, L83  
Paczynski, B., 1986, Ap.J.(Lett.), 308, L43  
Paczynski, B., 1990, Ap.J., 363, 218.  
Paczynski, B., 1994, in *Proc. Huntsville Gamma-ray Burst Wkshp.*, eds. G. Fishman, K. Hurley, J. Brainerd, (AIP: New York), in press  
Rees, M.J., 1978, M.N.R.A.S., 184, 61P  
Thompson, C., 1994, M.N.R.A.S., in press  
Usov, V.V., 1992, Nature, 357, 472  
Usov, V.V., 1994, M.N.R.A.S., 267, 1035  
Woosley, S., 1993, Ap.J., 405, 273

Figure Caption

Fig. 1: Wind regimes as a function of \( \eta = L/\dot{M}c^2 \) and \( r/c_{dyn} \). The lines above which the wind becomes optically thin to scattering (due to \( \dot{M} \)) has a slope -1/3 or -1/2.
for $L$ or $\dot{M}$ constant after saturation, and is fixed before saturation. The lines of $t_{\text{var}}$ proportional to $t_{\text{dyn}}$ have a slope $1/2$. GRBs from self-consistent unsteady winds are in the triangular region lying below the line $t_{\text{var}} \sim t_{\text{dyn}}$, and above the line $\tau = 1$ representing the photosphere.
This figure "fig1-1.png" is available in "png" format from:

http://arxiv.org/ps/astro-ph/9404038v1