Pairing and specific heat in $^{161,162}$Dy and $^{171,172}$Yb isotopes

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Abstract. The finite-temperature variation after projection BCS approach (FT-VAP) is employed to study the pairing phase-transition for the $^{161,162}$Dy and $^{171,172}$Yb isotopes. Due to the restoration of particle number conservation, the pairing gap and the specific heat calculated in the FT-VAP approach vary smoothly with the temperature, indicating a gradual transition from the superfluid to the normal phase, as expected in finite systems. FT-VAP results are compared with those obtained in the exact canonical approach as well as with experimental data.

1. Introduction
The study of possible thermal signatures of pairing correlations in atomic nuclei has been the subject of intensive works, both from a theoretical and experimental point of view. The common description of the interplay between pairing correlations and thermal effects is generally done within the Bardeen-Cooper-Schrieffer (BCS) or the Hartree-Fock-Bogoliubov (HFB) theories [1]. However, these models are not well-suited to describe accurately pairing effects in hot nuclei, because of particle number fluctuation and quasiparticle parity mixing issues. Particle-number conserving approaches are indeed necessary and recently [2, 3] it has been shown that the particle-number projected BCS approximation extended to finite temperature (FT-VAP) [4] is an appropriate tool for this kind of studies.

The influence of pairing correlations on specific heat is here analyzed for the isotopes $^{161,162}$Dy and $^{171,172}$Yb, by using the FT-VAP and the exact canonical solution [5]. The comparison between the results obtained within these two different number-conserving approaches shows the validity of FT-VAP also in realistic cases. It is shown that the FT-VAP approach, applied with a level density provided by mean field calculations and supplemented, at high energies, by...
the level density of the back-shifted Fermi gas model, can approximate reasonably well the main properties of specific heat extracted from experimental data. However, the detailed shape of the calculated specific heat is rather sensitive to the assumption made for the mean field.

2. Description of the calculations: input and procedure

The Hamiltonian used in the present calculations is

\[ \hat{H} = \sum_i \epsilon_i a_i^+ a_i + G \sum_{i,j} a_i^+ a_j^+ a_j a_i, \tag{1} \]

containing a constant pairing interaction which scatters pairs among time-reversed single-particle states \((i, i)\) and where only like-particle pairing is considered. The single-particle (s.p.) states employed in the Hamiltonian (1) are taken from mean-field models. In particular, we have used two sets of s.p. states obtained, respectively, from Skyrme-Hartree-Fock (HF) and Relativistic Mean-Field (RMF) calculations allowing, in both cases, axial symmetry deformation. The SGII and PK1 force are used, respectively. As commonly done in BCS calculations at zero temperature, pairing interaction is considered acting in a limited energy window around the Fermi level. The pairing strength \(G\) is fixed at BCS level in order to give a pairing gap approximately equal to 0.8 MeV both for protons and neutrons. Excitations outside this energy window are incorporated assuming that pairing interaction affects the excitation energies generated by the s.p. energies taken from a restricted region around the Fermi energy while the rest of s.p. spectrum

3. Results

In the top panel of Fig. 1 the pairing gap dependence with respect to the the temperature in the isotopes \(^{162}\text{Dy}\) and \(^{172}\text{Yb}\) is shown. The results correspond to the RMF s.p. spectrum. The finite-temperature BCS(FT-BCS) predicts a sharp second order type transition from the superfluid to the normal phase, at variance with what it is expected for small finite systems and predicted by the FT-VAP approach where a smooth transition from the superfluid to the normal phase is found. In the lower panel, also the pairing gaps obtained for the odd nuclei are given. As expected, at low \(T\), the gaps in odd systems are smaller than in the even systems. This difference tends to disappear as the temperature increases. For \(^{161}\text{Dy}\) and to a lesser extend for \(^{171}\text{Yb}\), we also observe a small re-entrance effect around \(T = 0.5\) MeV (resp. around 0.2-0.3 MeV).

In Fig. 2, we analyze the effect of pairing on specific heat separately for protons and neutrons for the isotopes \(^{161,162}\text{Dy}\), within both the FT-VAP approximation and the method of direct diagonalization. It can be seen that both treatments give similar results, confirming the good predictive power of the FT-VAP approximation.

How much depends the specific heat on the single-particle spectrum can be seen from Fig. 3, where are compared the results obtained with the energies provided by the Skyrme-HF and the RMF calculations, in the Yb isotopes. It can be seen that there are significant differences between the results obtained with the two mean fields. These differences are related to the distribution of the single particle levels around the Fermi energy, (see also Fig. 1 of [3]).

Finally, in Fig. 4, the specific heats for Dy and Yb extracted from the experimental level density in Refs [7, 8] are compared with the theoretical ones. The experimental heat capacities are obtained from the level density using specific assumptions. More precisely, in Refs. [7, 8] it is used the experimental level density for excitations below 8 MeV/nucleon while for higher excitation energies it is employed the level density provided by the Back Shifted Fermi Gas Model (BSFGM) (Eq. (5) of [8]). In the present calculations, we have used for all excitations energies
the level density generated by a discrete set of single-particle states. This assumption is expected to work reasonably well for low energy excitations but not for high energy excitations for which the contribution of the continuum becomes important. For this reason, a good agreement can only be obtained assuming that the free fermions gas limit is provided by the BSFGM as it was assumed in the experiment. For comparison, for even isotopes we show also the specific heats obtained in the FT-BCS approach which present the unphysical sharp transition between the superfluid and the normal phase. It can be observed that the agreement of the FT-VAP results with the experiments is reasonable good.

4. Conclusion

In the present work, the effect of pairing correlations in hot nuclei has been studied in the framework of a variation after particle-number projection BCS formalism extended to finite temperature. The calculation are done in canonical ensemble and with a Hamiltonian composed of a single-particle term, generated by Skyrme-HF or RMF calculations, and a pairing interaction of seniority type. The specific heat in the isotopes $^{161,162}$Dy and $^{171,172}$Yb have been evaluated. It is thus shown that the pairing correlations have a significant influence on the specific heat, especially for temperatures below $T = 0.5$ MeV. The comparison with the non-interacting particles.
Figure 3. (color online) Heat capacities in $^{162}$Dy and $^{161}$Dy obtained using two different sets of single-particle energies corresponding to RMF and HF mean fields. In the left (right) panels are shown the results with (without) the pairing interaction included.

Figure 4. (color online) Comparison between the calculated FT-VAP specific heats and the specific heats extracted from experimental data [7, 8]. For comparison, we show also the results obtained within the FT-BCS approximation.

systems shows clearly that the pairing correlations contribute to the S-shape behaviour of the specific heat, as noticed in the data extracted from experiment.

5. Acknowledgments
This work was supported by the French-Romanian IN2P3-IFIN agreement and by the Romanian Ministry of Education and Research through the grant Idei nr 57.

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