Fraction Magnitude Knowledge: Students Strategy in Estimating Fractions

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Abstract. Understanding concepts of fraction magnitude is necessary and inevitable steps of mastering fractions. One way to assess understanding of fraction magnitude is through magnitude estimation. Every student has different strategies for solving mathematics problem. These strategies show how deep students understanding, especially in estimating fractions. This study examined students’ strategies in estimating fractions based on fraction magnitude knowledge. The sample was the students of VIIB class of a school in Surabaya, 20 boys and 8 girls. All students had to complete a diagnostic’s test which was the fraction estimation test. There were 9 students with the high ability in estimating fractions, 8 students with medium ability in estimating fraction, and 11 students with low ability in estimating fraction. Three volunteer students with high ability in estimating fraction who had different and interesting strategies were chosen as our respondents. The semi-structured interview was utilized to investigate the process, strategy and students’ conceptual of fraction magnitudes. The strategy used by the first and the third students are the concept of equivalence fraction and convert fraction into decimal. Moreover, the strategy used by the second student is applying the concept of part-whole, but he was only fluent in estimating unit fraction. For further study, there is need to categorize the logical stage of students’ strategy in estimating fractions.

1. Introduction
Fractions have been used not only in mathematics but also in everyday life. In addition, understanding fractions are important to success in various professions such as sciences and technology. Fraction is one of the topics that must be mastered by students of 7th grades [1]. Understanding the concept of fractions and how to use them are fundamental to the development of mathematics and higher concepts in mathematics [2-6].

The concept of fractions is difficult for students in Indonesia, America and Australia. The main source of the difficulty is because students do not know about the fraction magnitudes [2, 6, 7, 8]. There are some errors of 10 and 11 years old students in Belgium in understanding the concept of fractions [2]. The error indicates that the students do not understand the relation between the numerator and the denominator, or the students do not have a good ability about the fraction magnitudes. An understanding of fraction magnitude is a necessary and unavoidable stage of understanding about fractions [6]. Difficulties to understand the fractions can affect students’
opportunity to learn advanced topics in mathematics [2, 4]. Therefore, it is very important for students to have the understanding of fractions correctly [2].

Understanding of fraction magnitudes can be assessed through the ability to compare fractions, the ability to estimate fractions on a number line, and the ability to represent fractions [9-11]. The magnitude comparison task and the number line estimation task both assess central component of mathematical competence [12]. Number line estimation assess children representation of fraction magnitude. Making placements on a number line is one form of basic numerical magnitude [13].

Students are said to understand fractions when achieving conceptual and procedural understanding [2, 4]. Conceptual understanding of fractions is the knowledge of a concept, nature, and how it is derived [14]. This understanding includes fractions consisting of numerators and denominators, its magnitude is determined by numerator-denominator relation, and that these magnitudes can be ordered on a number line [4]. While procedural understanding is the ability to perform a series of actions to solve the problem [2, 15].

Students use strategy in solving problems, including estimating fractions on a number line. The strategy chosen depends on the characteristics of the questions and the learners. Therefore not only different students use different strategies on the same problem, but also one student can use different strategies on similar problems or same problems at different times. The students’ strategies use will change over time. Students with greater experience and knowledge used more adaptive and efficient strategies [16].

This research will be studied strategy used by students in fraction magnitudes that is estimating fractions on the number line which analyzed based on conceptual and procedural capability. Students with the high score will be chosen based on the interesting and different strategy used.

2. The Number Line Fraction Estimation
A fraction is a number that can be represent by an ordered pair of whole numbers \( \frac{a}{b} \) (or \( a/b \)), where \( b \neq 0 \) [17, 18]. Fraction also can be defined as part of whole [19]. Fraction can be represented on a number line. Fraction \( \frac{1}{b} \) can be located on a number line by defining a number line with the interval 0 to 1 as a whole and partitioning into \( b \) equal parts, each part is of length \( \frac{1}{b} \) and the number \( \frac{1}{b} \) is located at the right endpoint of the part with its left endpoint at 0.

The number line also can be used in estimating fraction. Estimating fraction on a number line is used to involve students in demonstrating their understanding of the concept of fraction as a part of whole. Besides, it also demonstrating their understanding about fraction magnitude [9, 11]. In estimating fractions, students are engaged in multiple steps in estimating a location for a given fraction by using strategies, such as identifying a benchmark fraction (example \( \frac{1}{2} \)) and comparing it to the given fraction to figure out which side of the number line the given fraction belongs to (example: less than \( \frac{1}{2} \) or greater than \( \frac{1}{2} \)) [13].

3. Method
This research is conducted on 7th grade students of a school in Surabaya, selected the students of VIIB class. This class consists of 20 boys and 8 girls, chosen purposively from three available classes. All students have completed a test namely fraction estimation task for 30 minutes. There were five fractions including unit and non-unit fractions that have been checked and estimated in the line number by students which are \( \frac{1}{6}, \frac{5}{7}, \frac{7}{12}, \frac{84}{216}, \) and \( \frac{110}{312} \). The test results were categorized into high, medium, and low ability in estimating fraction based on procedural and conceptual understanding. Three students with high ability in estimating fraction, different, and interesting strategies were chosen as subjects to analyze deeply about their abilities. The semi-structured interview was conducted on three subjects for 20 minutes to examine students’ conceptual understanding deeply, process and
strategy used. Table 1 shows the indicators of conceptual and procedural understanding in estimating fractions.

Table 1. Indicators of Conceptual and Procedural Understanding in Estimating Fractions.

| Understanding the problem | Procedural Understanding | Conceptual Understanding |
|--------------------------|--------------------------|--------------------------|
| Devising a plan          | Determine the procedure and strategy used to answer the problem | Explain the argument of using procedure and strategy |
| Carrying out the plan    | Carry out the procedure and strategy that has been determined to solve the problem | |
| Looking back             | Recheck the consistency of the use of the procedure and strategy at each stage of problem solving | |

4. Results
The fraction estimation task was analyzed based on procedural and conceptual understanding then categorized into high, medium, and low ability. Based on the analysis results there were 9 students with the high ability in estimating fractions, 8 students with medium ability in estimating fraction, and 11 students with low ability in estimating fractions. Therefore most students had low ability in estimating fractions. The difficulties were they did not understand the problem, they were not got the similar problem before and they did not understand the concept of fraction. From 9 students with the high ability in estimating fraction, there were three students who had the different and interesting strategy in estimating fraction and will be chosen as subjects. The first student is students with the higher score, the second and third students are students with the second and third rank of the scores.

4.1. The Strategy Used by Student 1

Student 1 estimated and checked correctly four from five fractions in the number line. From his answered, he used the concept of the decimal. Figure 1 shows the resulting test of student 1.
From the resulting test, he converted the fraction into decimal. He used that strategy fluently. It can be seen from the process he was written in the answer sheet. He used division concept to convert the fraction into decimal. It means he had a better procedural understanding. But for making it sure, the above answer was justified with the result of the interview as follows.

Q : Could you tell me what is the information on the problem as you know?
S1 : There are a number line and fractions
Q : What did you do with these fractions?
S1 : I’ll check them and estimate the location in the number line
Q : Could you tell me what did you know from this number line?
S1 : The number line is between 0 to 1, it was divided into 10 parts. And these parts are decimals

From the result of the interview, student 1 was able to mention and explain the information in the problem. He understood the instruction, he stated that he will check and estimate all known fraction in the number line. He also understood the number line is divided into 10 parts, therefore the value of these parts must be between 0 to 1 or must be decimal.

The strategy used by student 1 is converted the fraction into the decimal. It was justified with the result of the interview as follow

Q : How did you check and estimate them?
S1 : I used the concept of the decimal. For example $\frac{7}{121}$, because 121 is more than 100 so I used 1000 for the denominator. So 121 is multiplied by a number and I get 1000, suppose this number is $a$ and I multiplied 7 with $a$. Just like finding the equivalent fraction.

From the interview’s result, he used the concept of the decimal and equivalent fraction fluently. He also understood the argument of the strategy used. Therefore, student 1 satisfied all the indicator of procedural and conceptual understanding.

4.2. The Strategy Used by Student 2

Student 2 has been answered the question in the answer sheet but it was not clear enough. Figure 2 shows the answer of student 2 in the fraction estimation task.

![Figure 2](image)

Figure 2. The Answer of Student 2 in Fraction Estimation Task

From the resulting test, it can not be analyzed because student 2 just wrote the few information. Therefore it needed interview for deep analysis. A part of the interview with student 2.

Q : Could you mention the information in the problem?
S2 : There are fractions in the number line, and I will check and estimate them in the number line
Q : How did you estimate and check them?
S2 : I just imagined the number line as a cake, and it will be divided into some parts. The number line is between 0 to 1. I divided 1 by 2 becomes $\frac{1}{2}$ and divided $\frac{1}{2}$ by 2 becomes $\frac{1}{4}$ until I got the fraction that I will estimate.
Q : So, what did your answer in the answer sheet means?
S2 : I mean between 0 and \( \frac{1}{4} \) is \( \frac{5}{7} \), but I can not estimate fraction that has big numerator and denominator.

From the result of the interview, student 2 understand what are known and asked in the problem. He stated that the instruction is to check the known fraction and estimate the location in the number line. The strategy used by student 2 is the concept of part-whole. He imagines a number line as a whole and divided it into 2 parts, it becomes \( \frac{1}{2} \), and \( \frac{1}{2} \) is divided into 2 part becomes \( \frac{1}{4} \), and so on. He divided a whole becomes 2 smaller parts until he gets the fraction that will be estimated. It means he knows the concept and strategy used to estimate fraction. But he had difficulty in procedural understanding. He could not apply the procedure of that concept. It can be seen in the fraction \( \frac{5}{7} \), he stated that the location of \( \frac{5}{7} \) is between 0 and \( \frac{1}{4} \). He also could not apply that concept in the fraction with large numerator and denominator.

4.3. The Strategy Used by Student 3

Student 3 used converting the fraction into the decimal to estimate location of the fraction in the number line. In the answer sheet, he estimated four fractions from five fractions fluently. Figure 3 shows the resulting test of student 3.

![Figure 3. The Answer of Student 3 in Fraction Estimation Task](image)

Based on the resulting test, the strategy used by student 3 is convert fraction into decimal. He used division to convert them. He divided the numerator by the denominator and used rounding concept. The above answer was justified with the result of the interview as follows.

Q : Did you know what are known and asked about the problem?
S3 : Fraction that has been located in the number line. And the instruction is to check and to estimate the location of the fraction in the number line
Q : What strategy did you use?
S3 : I represent fractions into decimals.
Q : How did you do?
S3 : Divided the numerator by the denominator
Q : Why you used that strategy?
S3 : I don’t know, I just applied what my teacher did

Student 3 understood the information in the problem. He explained that he has to check the location of fractions that have been put in the number line and estimated them. He used the same strategy as student 1 but he did not find the equivalent fraction. He divided the numerator by the denominator to represent the fraction into the decimal. He did that strategy fluently but he did not know the concept of that strategy. He just applied the procedure without knowing the concept.
5. Discussion
Students use different strategies to estimate the fractions on the number line. Student with greater understanding including knowledge and experience of the fraction magnitudes, the students will use an efficient and adaptive strategy [16]. This is contrary to the results of research. The first student has a good procedural and conceptual understanding but the student uses an inefficient strategy. The strategy used in estimating fractions on a number line takes a lot of time. But by using the strategy, it can be seen that the student understands the concepts related to fractions such as equivalent fractions and decimal. The first student use inefficient strategy because he has no knowledge about other strategies.

Students are said to understand the fraction magnitudes when they have conceptual and procedural understanding in estimating fractions [2, 4]. The second student has only a conceptual understanding. So based on the theory, the second student has not understood the fraction magnitudes [2, 4]. However, the second student uses a more efficient strategy than the first student. The strategy used is the basic concept of fractions (part-whole). This is because the student experience is lacking in using the strategy, therefore he did not use the procedure fluently.

6. Conclusion
Based on the result, we conclude that the strategy used by the first and the third students are the concept of equivalence fraction and convert the fraction into the decimal. The third student faced difficulty in conceptual understanding. He just applied the procedure without knowing the concept. Moreover, the strategy used by the second student is applying the concept of part-whole, but he was only fluent in estimating unit fraction. For using the strategy students’ experience and knowledge are needed.

7. Acknowledgements
This research reported herein was supported by Junior High School of LabSchool Universitas Negeri Surabaya. The opinions expressed herein do not necessarily reflect the position, policy, or endorsement of the supporting agency.

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