Analysis of a multi-loop temperature stabilization system for compact rubidium standard of frequency and time

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Abstract. A possible design of a thermal stabilization system for compact atomic clock based on coherent population trapping is analyzed. The feasibility of independent control of the semiconductor laser’s characteristics, such as wavelength and optical power, is considered. An algorithm is suggested for choosing the performance parameters of various control circuits, eliminating the occurrence of self-oscillation mode.

1. Introduction
Development of methods for precise frequency and time measurements was crucial for progress in science and technology. The researches devoted to standards of time are closely connected with achievements in fundamental physics and global telecommunication networks. Improvement of frequency standards is important for understanding many physical phenomena and searching for new approaches in technological development.

At the same time, simultaneously with the creation of extremely accurate and stable quantum clock, the work on designing the compact ultra-precise atomic clocks is progressing. To meet the needs of technology, Miniature Atomic Clock (MAC) on \(^{87}\text{Rb}\) atoms with relative long-term stability better than \(10^{-11}\) are being developed [1-5] and are already commercially available [6,7].

2. A method for stabilizing the probe laser
Miniature Atomic Clock based on the effect of coherent population trapping (CPT) contains a number of elements requiring precise temperature maintenance. A generalized scheme of the MAC including the adjustment elements is shown in Figure 1, taken from [8]. An atomic cell with rubidium vapor is thermally stabilized and used as a quantum discriminator for RF generator. The wavelength of the semiconductor laser can be adjusted by heating. A separate loop is used to stabilize the reference crystal oscillator. The external thermostat regulates the temperature of the MAC case.

Advances in miniaturization of the rubidium frequency and time standard are associated with the use of semiconductor lasers for generating optical emission used to stabilize the radio-frequency signal with CPT effect in rubidium vapor. The appearance of high speed single-mode semiconductor lasers with a vertical structure of VCSEL allowed creating a compact version of MAC in the form factor of...
conventional high stable crystal oscillators. The long-term stability of this atomic clock is several orders higher than the highest achieved for quartz oscillators.

The basic problem concerning thermal stabilization of the separate units of MAC is taking into account the mutual influence of the different control systems on each other. In this case, only part of the systems operates using the thermal sensors data, while the others rely on the optical effects and receive the control signal from the quantum discriminator.

![Figure 1. The generalized scheme of one of MAC options](image)

For example, to adjust the laser wavelength, an effect of the optical absorption of light in the rubidium vapor cell is used. To fine tune to the absorption line, the frequency of the laser emission is changed by several hundred MHz with a frequency of tens of kHz, by modulating the current through the VCSEL. The signal received by the photodetector is detected by synchronous detector and enters the control circuit of the Peltier element that changes the average temperature of the laser diode. The dependence of the wavelength $\lambda$ on the current $I$ and temperature $T$ in the first-order approximation is linear and can be described by equation (1) with typical values $k_i = 0.6 \text{ nm mA}^{-1}$, $k_t = 0.06 \text{ nm K}^{-1}$:

$$\lambda = \lambda_0 + k_i(I - I_0) + k_t(T - T_0)$$

(1)

This type of dependence holds for modulation frequency of the laser current much lower than 3.4 GHz, at which the laser light is modulated to bind to the CPT resonance. Linear relationship between the wavelength of the laser and the current and temperature is due to thermal expansion of the laser cavity caused by its heating.

Thus, laser power in a first-order approximation is linearly dependent on the current and temperature. This relationship is shown in equation (2) where the coefficients have the following values: $\eta_s = 0.3 \text{ mW mA}^{-1}$, $\eta_t = 0.01 \text{ mW K}^{-1}$.

$$P = P_0 + \eta_s(I - I_0) - \eta_t(T - T_0)$$

(2)

Solving equations (1) and (2) can yield the conditions under which the simultaneous variation of temperature and current leaves unchanged the wavelength or power of the laser. To stabilize the wavelength when the current changes, it is necessary to have $\Delta T = (k_i^{-1} \cdot k_t^{-1}) \Delta I$, and to stabilize the power when the temperature changes, the correction current is required: $\Delta I = (\eta_t^{-1} \cdot \eta_s^{-1}) \Delta T$. Both these conditions, apparently, are not equivalent since $(\eta_t^{-1} \cdot \eta_s^{-1})(k_i^{-1} \cdot k_t^{-1}) \neq 1$, and it is generally impossible to stabilize the power and the wavelength of the laser diode simultaneously. However, the very possibility of such modulation of solely amplitude or frequency is important and can be used to optimize the algorithm of the MAC.
3. Analysis of a multi-loop system of thermal stabilization

When creating MAC, different variants of thermal stabilization of the cell and laser are used. Examples of such circuits, leading to long-term stability of various MACs, are shown in Figure 2, taken from [9].

![Figure 2. Experimental setup for chip scale atomic clock parameter control.](image)

Therm – thermistor; Temp. – temperature control; Current – current control; PD – photodiode; VCSEL – vertical-cavity surface-emitting laser, solid lines – dc signals; dashed lines – phase detection signals. (a) Conventional setup. (b) Laser frequency is controlled by temperature feedback. (c) In addition to (b), cell temperature is controlled by atomic absorption. (d) In addition to (c), laser intensity is controlled by use of a photodetector.

A fairly full review of the basic approaches to the design of MAC is given in [10-12]. As temperature sensors in such devices thermostors are traditionally used, having a high sensitivity to temperature change. The exponential resistance versus temperature dependence makes these sensors greatly nonlinear and complicates creating analog thermoregulation systems with optimal speed and accuracy. The resistance of the thermistor and hence its sensitivity changes by several times (usually falls from 10 kOhm to 2 kOhm) while MAC reaches its operating conditions, that is becomes heated from 20°C to 60°C times. Such a significant change of the thermal control system parameters requires separate modes of operation for the first turning on and then for consequent holding the operating parameters. The most advisable in such cases is to make a digital temperature control system capable of adapting to the conditions of work and algorithmically suppressing the oscillations emerging in the system.

The fastest channel is the current control channel of the laser and its time constant in terms of heating the crystal is typically less than a few microseconds. Next in terms of speed of response is the Peltier element that regulates average temperature of the laser. The time constant of this element integrated in the VCSEL structure is a few tenths of a second. Then there are two systems with approximately the same operating speed – the base of heating, where a Peltier element with a laser are located, and an optical cell with rubidium vapor. Performance of these heaters varies from tens to hundreds of seconds, depending on the dimensions of the elements used. The most inertial system of the MAC is a general thermal stabilization circuit of casing as a whole. Here, using a thermal insulation system of the MAC from the environment, characteristic time of the transition process can reach several thousand seconds. Crystal oscillator, essential in any MAC, has its own, sometimes double-loop, thermal stabilization system, and in this case operates as independent system, disturbing the thermal equilibrium of the MAC stabilization system. Actually, it is a simple source of heat, whose power depends on the ambient temperature of the crystal oscillator.

Using existing data, following approach to the implementation of sustainable multi-loop temperature control system for MAC can be proposed. Based on a clear hierarchy of the response times of different sub-systems, the method of frequency division of signals can be used, and the whole
system can be designed so that the effect of low-frequency control systems on the higher-speed ones occurs in the plateau region of the amplitude-frequency response characteristic of the higher-speed systems. In general, the system of equations describing the operation of a multi-loop interconnected temperature control system can be determined by the equation (3):

\[ T_i = b_i P_i + q_i T_i^0 + \sum_{j=1}^{k} a_{ij} (T_j - T_i), \]

where \( T_i \) – deviation of temperature of the \( i \)-th element from the case temperature, \( P_i \) – power of the \( i \)-th heater, \( q_i \) – coefficient characterizing the thermal conductivity of the \( i \)-th element to the casing of the device, \( b_i \) – coefficient characterizing specific heat of the \( i \)-th element, \( a_{ij} \) – coefficients describing mutual influence of various elements on each other’s system.

This system of equations must be supplemented with the equations describing the behavior of specific thermal regulators which in the simplest cases are represented by the PID regulators used to control the power of the heater [13]. Purpose of the PID controller is to maintain the setpoint \( T_0 \) of a certain quantity \( T \) by changing another quantity \( u \). The value of \( T_0 \) is a predetermined value, and the difference \( e = (T_0 - T) \) is a regulation error. Controller output is given by equation (4):

\[ u(t) = P + I + D = K_p e(t) + K_i \int_0^t e(\tau) d\tau + K_d \frac{de}{dt} \]  

where \( K_p \), \( K_i \), \( K_d \) – respectively, the proportional, integral and derivative gain components of the controller.

Introducing the PID controllers into consideration allows optimizing the transition process for each of the control channels and ensuring its sustainability. In this case the behavior of the individual control loop is described by the differential equation of the second order, and the analysis of its stability does not represent any particular problems. In simple systems, each PID controller operates independently of the other elements of the system, and the mutual influence is manifested in the physical layer through the terms of the form \( \sum_{i=1}^{k} a_{ij} (T_j - T_i) \). However, in the case of discussed multi-loop control system, this approach is not the productive one. When creating digital temperature control system, it is more rational to use all information available to the controller to manage the system as a whole. However, to properly configure such a system one needs detailed information about all the relationships, and the only opportunity to gather it is during the testing of the finished product. Considering the strict requirements on Allan dispersion and fluctuations range of the reference frequency signal generated by the MAC, it is possible to propose the following temperature control circuit for basic operational components of atomic clock.

Using LED current (and thus its temperature) modulation, the optical wavelength tuning is carried out, and the signal of absorption line capture is formed. The data from this very fast system of regulation is used to govern the Peltier element controlling the average temperature of the laser. The time constant of the optical wavelength capture system may be of the order of 1–10 ms, which is significantly less than the Peltier element response time of 0.1–1 s. Tenfold margin of the cutoff frequency corresponding to the frequency response ensures the stability of the control loop.

The temperature of the substrate, where the Peltier element regulating the wavelength of the laser is situated, and the temperature of the optical cell with rubidium vapor can be the same in the working MAC version, but in any case, the information about the required temperature data values is taken from the thermistors and used in the control circuit with a characteristic speed 10–100 seconds. The proximity of time constants of these control objects indicates the inexpediency of organizing a parallel operation of these circuits. Therefore, based on the data of both thermistors (when entering the operating mode) and using only the temperature of the cells for the fine tuning, this unit can be made single, and a distributed heater can be used to reduce the possibility of temperature fluctuations in the quantum discriminator. This control loop based on the Peltier element and the loop of cells are well spaced in their typical response times (more than 10 times) and do not interfere with each other.

The circuit involved in the maintenance of the working temperature of the MAC casing in this case performs a purely subsidiary function, accelerating the system entering the operating mode and
facilitating the work of the internal systems of fine thermal stabilization. Its main purpose is to reduce the external temperature drops, reaching tens of degrees, down to a value not exceeding a degree Celsius.

4. Conclusion
The analysis shows the importance of performance of the control channels included in the MAC and indicates the need to measure the response of the system as a whole and its individual components, in particular, to the work of each individual regulatory system. Since the performance between parts of a multi-loop control system differs by tens and hundreds of times, the sustainability of the whole system can be ensured by the control of the specific parameters of the PID controllers.

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