LevelHeaded: Making Worst-Case Optimal Joins Work in the Common Case

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ABSTRACT

Pipelines combining SQL-style business intelligence (BI) queries and linear algebra (LA) are becoming increasingly common in industry. As a result, there is a growing need to unify these workloads in a single framework. Unfortunately, existing solutions either sacrifice the inherent benefits of exclusivity using a relational database (e.g., logical and physical independence) or incur orders of magnitude performance gaps compared to specialized engines (or both). In this work, we study applying a new type of query processing architecture to standard BI and LA benchmarks. To do this, we present a new in-memory query processing engine called LevelHeaded. LevelHeaded uses worst-case optimal joins as its core execution mechanism for both BI and LA queries. With LevelHeaded, we show how crucial optimizations for BI and LA queries can be captured in a worst-case optimal query architecture. Using these optimizations, LevelHeaded outperforms other relational database engines (LogicBlox, MonetDB, and HyPer) by orders of magnitude on standard LA benchmarks, while performing on average within 31% of the best-of-breed BI (HyPer) and LA (Intel MKL) solutions on their own benchmarks. Our results show that such a single query processing architecture is capable of delivering competitive performance on both BI and LA queries.

1. INTRODUCTION

The efficient processing of classic SQL-style workloads is no longer enough; machine learning algorithms are being adopted at an explosive rate. In fact, Intel projects that by 2020 the hardware cycles dedicated to machine learning tasks will grow by 12x, resulting in more servers running this than any other workload [14]. As a result, there is a growing need for query processing engines that are efficient on (1) the SQL-style queries at the core of most business intelligence workloads and (2) the linear algebra operations at the core of most machine learning algorithms. In this work, we explore whether a new query processing architecture is capable of delivering competitive performance in both cases.

An increasingly popular workflow combines business intelligence (BI) and linear algebra (LA) queries by executing SQL queries in a relational warehouse as a means to extract feature sets for machine learning models. Unsurprisingly, these SQL queries are similar to standard BI workloads: the data is de-normalized (via joins), filtered, and aggregated to form a single feature set [26, 27]. Still, BI queries are best processed in a relational database management system (RDBMS) and LA queries are best processed in a LA package. As a result, there has been a flurry of activity around building systems capable of unifying both BI and LA querying [20, 12, 17, 21, 25, 29]. At a high-level, existing approaches fall into one of three classes:

- **Exclusively using a relational engine.** There are many inherent advantages to exclusively using a RDBMS to process both BI and LA queries. Simplifying extract-transform-load (ETL), increasing usability, and leveraging well-known optimizations are just a few [29]. Although it is known that LA queries can be expressed using joins and aggregations, executing these queries via the pairwise join algorithms in standard RDBMSs is orders of magnitude slower than using a LA package (see Section 6). Thus, others [29] have shown that a RDBMS must be modified to compete on LA queries.

- **Extending a linear algebra package.** Linear algebra packages, like BLAS [11] or LAPACK [7], provide high-performance through low-level procedural interfaces and therefore lack the ability for high-level querying. To address this, array databases with high-level querying, like SciDB [12], have been proposed. Unfortunately, array databases are highly specialized and are not designed for general BI querying. As a result, support for SQL-style BI querying [13] has recently been combined with the LA support found in popular packages like Spark’s MLlib [31] and Scikit-learn [35]. Still, these solutions lack many of the inherent benefits of a RDBMS, like a sophisticated (shared-memory) query optimizer or efficient data structures (e.g. indexes) for query execution, and therefore achieve suboptimal performance on BI workloads (see Section 7).

- **Combining a relational engine with a linear algebra package.** To preserve the benefits of using a RDBMS on BI queries, while also avoiding their pairwise join algorithms on LA queries, others (e.g. Oracle’s UT_NLA [2] and MonetDB/NumPy [30]) have integrated a RDBMS with a LA package. Still, these approaches just tack on an external LA package to a RDBMS—they do not fully integrate it. Therefore, users are
Therefore, existing approaches either (1) sacrifice the inherent benefits of using only an RDBMS to process both classes of queries, (2) are unable to process both classes of queries, or (3) incur orders of magnitude performance gaps relative to the best single-class approach.

In this work we study an alternative approach to building a RDBMS for both BI and LA querying. In particular, we study using worst-case optimal join (WCOJ) algorithms as the mechanism to unify these query workloads. To do this, we present a new in-memory query processing engine called LevelHeaded. LevelHeaded uses a novel WCOJ query architecture to optimize, plan, and execute both BI and LA queries. In contrast to previous WCOJ engines, LevelHeaded is designed for and evaluated on more than just graph queries. As such, LevelHeaded is the first WCOJ engine to present an evaluation on both BI and LA queries. In contrast to other query engines, LevelHeaded competes with both LA packages and RDBMSs on their own benchmarks (see Figure 1).

However, designing a new query processing engine that is efficient on both BI and LA queries is a challenging task. The recently proposed WCOJs at the core of this new query processing architecture are most effective on graph queries where they have an asymptotic advantage over traditional pairwise join algorithms. In contrast, pairwise join algorithms are well-understood, and have the advantage of 45+ years of proven constant factor optimizations for BI workloads. Further, LA queries, which also have the benefit of decades of optimizations, are typically not a good match for the relational model. Therefore, it is not at all obvious whether they are good match for this new type of relational query processing architecture.

Despite these challenges, we found that the unification of three new techniques in a WCOJ architecture could enable it to deliver competitive performance on both BI and LA queries. The techniques we leverage are (1) a new mechanism to translate general SQL queries to WCOJ query plans, (2) a new cost-based query optimizer for WCOJ’s, and (3) a simple storage engine optimizer for GROUP BY. It was not at all obvious how to identify and unify these techniques in a WCOJ framework and, as such, we are the first to do so.

The three core techniques of LevelHeaded in more detail are:

- **SQL to GHDs: Pushing Down Selections and Attribute Elimination.** Neither the theoretical literature nor the query compilation techniques for WCOJs maps directly to all SQL features. In LevelHeaded we implement a practical extension of these techniques that enables us to capture more general query workloads as well as the classic query optimizations of pushing down selections and attribute elimination. Besides providing up to 4x speedup on BI queries, a core artifact of our attribute elimination implementation is that it enables LevelHeaded to target BLAS packages on dense LA queries at little to no execution cost. This is because it is challenging to outperform BLAS packages, like Intel MKL on sparse data and is usually not possible on dense data. Therefore, LevelHeaded leverages attribute elimination to opaquely call Intel MKL on dense LA queries while executing sparse LA queries as pure aggregate-join queries (entirely in LevelHeaded).

- **Cost-Based Optimizer: Attribute Ordering.** WCOJ query optimizers need to select an attribute order in a similar manner to how traditional query optimizers select a join order. With LevelHeaded, we present a cost-based optimizer to select a WCOJ attribute order for the first time. We highlight that our optimizer follows heuristics that are different from what conventional wisdom from pairwise join optimizers suggests (i.e., highest cardinality first) and describe how to leverage these heuristics to provide an accurate cost-estimate for a WCOJ algorithm. Using this cost-estimate, we validate that LevelHeaded’s cost-based optimizer selects attribute orders than can be up to 8815x faster than attribute orders that previous WCOJs might select.

- **Group By Tradeoffs: Battling Skew on Group By.** Finally, we study the classic tradeoffs around executing GROUP BYs in the presence of skew. Although these tradeoffs are well understood for BI queries, they have yet to be applied to this new style of query processing engine and are also essential for LA queries (see Figure 2). We describe LevelHeaded’s GROUP BY optimizer that exploits these tradeoffs and show that it provides up to a 875x and 185x speedup over a naive implementation on BI and LA queries respectively.

We evaluate LevelHeaded on standard BI and LA benchmarks: seven TPC-H queries and four (two sparse, two dense) LA kernels. These benchmarks are de-facto standards for relational query processing and LA engines. Thus, each engine we compare to is designed to process one of these benchmarks efficiently using specific optimizations that enable high-performance on one type of benchmark, but not necessarily the other. Therefore, although these engines are the state-of-the-art solutions within a benchmark, they are unable to remain competitive across benchmarks. For example, Hyper delivers high performance on BI queries, but is not competitive on most LA workloads; similarly, Intel’s Math Kernel Library (MKL) delivers high performance on LA queries, but does not provide support for BI querying. In contrast, LevelHeaded is designed to be generic, maintaining efficiency across the queries in both benchmarks.

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1 Intel MKL often gets peak hardware efficiency on dense LA.
2 The TPC-H queries are run without the ORDER BY clause.
Contribution Summary. This paper introduces the LevelHeaded engine and demonstrates that its novel architecture can compete on standard BI and LA benchmarks. We show that LevelHeaded outperforms other relational engines by at least an order of magnitude on LA queries, while remaining on average within 31% of best-of-the-breed solutions on both BI and LA queries.

Our contributions and outline are as follows.

- In Section 2, we describe the LevelHeaded architecture. In particular, we describe how LevelHeaded’s query and data model, which is different from that of previous WCOJ engines, preserves the theoretical benefits of a WCOJ architecture and enables it to efficiently process BI and LA queries.

- In Sections 3 to 5, we present the core (logical and physical) optimizations that we unify in a WCOJ query architecture for the first time. We show that these optimizations provide up to a three orders of magnitude performance speedup and are necessary to compete on BI and LA queries. In more detail, we present the classic query optimizations of pushing down selections and attribute elimination in Section 3, a cost-based optimizer for selecting an attribute order for a WCOJ algorithm in Section 4, and, finally, a simple query execution optimizer for GROUP BY’s in Section 5.

- In Section 6, we show that LevelHeaded can outperform other relational engines by an order of magnitude on standard BI and LA benchmarks while remaining on average within 31% of a best-of-breed solution within each benchmark. For the first time, this evaluation validates that a WCOJ architecture can compete on BI and LA workloads. We argue that the inherent benefits of such a unified (relational) design has the potential to outweigh its minor performance overhead.

We believe LevelHeaded represents a first step in unifying relational algebra and machine learning in a single engine. As such, we extend LevelHeaded in Section 7 to explore some of the potential benefits of this approach on a full application. We show here that LevelHeaded is up to an order of magnitude faster than the popular (unified) solutions of MonetDB/Scikit-learn, Pandas/Scikit-learn, and Spark on a workload that combines SQL-style querying and a machine learning algorithm. We hope LevelHeaded adds to the debate surrounding the design of unified querying systems.

1.1 Related Work

LevelHeaded extends previous work in worst-case optimal join processing (EmptyHeaded and LogicBlox), relational data processing, and linear algebra processing.

EmptyHeaded. LevelHeaded is designed around many of the techniques presented in the EmptyHeaded engine, but with many fundamental differences. First, EmptyHeaded was only evaluated in the graph and resource description framework (RDF) domains. As such, the EmptyHeaded design is unable to support BI workloads that contain many attributes. The limitations in the EmptyHeaded design stem from its storage model and query model which was specifically designed for graph and RDF workloads. In addition, the EmptyHeaded design only supported equality selections and GROUP BY’s on indexed attributes. Finally, the design of EmptyHeaded was optimized for only sparse data and achieved suboptimal performance on workloads with dense data. LevelHeaded is designed to fix these flaws by leveraging new techniques (Sections 2.1, 2.2, and 3 to 5) that eliminate these limitations, while preserving the theoretical benefits of such a design.

LogicBlox. LogicBlox is a full featured commercial database engine built around similar worst case optimal join and query compilation algorithms. Still, a systematic evaluation of the LogicBlox engine on BI and LA workloads is yet to be presented. From our conversations with LogicBlox, we learned that they often avoid using a WCOJ algorithm on these workloads in favor of more traditional approaches to join processing. In contrast, LevelHeaded always uses a WCOJ algorithm (even on queries not approaching the worst-case). Further, LogicBlox uses a query optimizer that has similar benefits to LevelHeaded’s (see Section 3), but does so using custom algorithms.

In contrast, LevelHeaded uses a generalization of these algorithms that maps to well-known techniques. Finally, LogicBlox stores its data in a manner that resembles a row store, whereas LevelHeaded resembles column store.

Relational Processing. A significant amount of work has focused on bringing LA to relational data processing engines. Some have suggested treating LA objects as first class citizens in a column store. Others, such as Oracle’s UTL_NLA and MonetDB with embedded Python allows users to call LA packages through user defined functions. Still, the relational optimizers in these approaches do not see, and therefore are incapable of optimizing, the LA routines. Even worse, these packages are cumbersome to use and place significant burden on the user to make low-level library calls. Finally, the SimSQL project suggests that relational engines can be modified in straightforward ways to accommodate LA workloads. Our goals are similar to the SimSQL, but explored with different mechanics. SimSQL studied the necessary modifications for a classic database architecture to support LA queries and was only evaluated on LA queries. In LevelHeaded, we evaluate a new, theoretically advanced, query processing architecture on both BI and LA workloads. Additionally, SimSQL was evaluated in the distributed setting and therefore compared to higher-level baselines. LevelHeaded is evaluated in a shared memory setting against Intel MKL and HyPer. Other high performance in-memory databases, like HyPer, focused on classic OLTP and OLAP workloads and were not designed with other workloads in mind.

Linear Algebra Processing. Researchers have long studied how to implement high-performance LA kernels. Intel MKL represents the culmination of this work on Intel CPUs. Unsurprisingly, researchers have shown that it requires tedious low-level code optimizations to come near the performance of a package such as Intel MKL. As a result, processing these queries in a traditional RDBMS (using relational operators) is at least an order of magnitude slower than using such packages (see Section 4). In response, researchers have released array databases, like SciDB and TileDB, which provide high-level LA querying, often by wrapping lower-level libraries. In contrast, our goal is not to design an entirely different and specialized engine for these workloads, but rather to design a single (relational) engine that processes multiple classes of queries efficiently.
2. LEVELHEADED ARCHITECTURE

In this section we overview the data model, storage engine, query compiler, and join algorithm at the core of the LevelHeaded architecture. This overview presents the preliminaries necessary to understand the optimizations presented in Sections 3 to 5. LevelHeaded ingests structured data from delimited files on disk and has a Python frontend that accepts Pandas dataframes. The query language is a subset of the SQL 2008 syntax with some standard limitations that we detail in Section 2.2. The SQL queries are translated to generalized hypertree decompositions (GHDs) which we describe in Section 2.2. Finally, code is generated from the selected GHD using a WCOJ algorithm which we describe in Section 2.4. The entire process is shown in Figure 2.

2.1 Data Model

The LevelHeaded data model is relational with some minor restrictions. A core aspect of LevelHeaded’s data model is that attributes are classified as either keys or annotations via a user-defined schema. Keys in LevelHeaded correspond to primary or foreign keys and are the only attributes which can partake in a join. Keys cannot be aggregated. Annotations are all other attributes and can be aggregated. Both keys and annotations support filter predicates and GROUP BY operations. In its current implementation, LevelHeaded supports equality (=) filters on keys and range (>,<,=) filters on annotations. This represents the existing implementation, not fundamental restrictions. LevelHeaded’s current implementation supports attributes with types of int, long, float, double, and string. In many regards LevelHeaded is similar to a key-value store combined with the relational model. LevelHeaded is not unique in this regard, and commercial databases like Google’s Mesa [20] and Spanner [14] follow a similar (if not identical) model.

2.2 Storage Engine

LevelHeaded’s data model is tightly coupled with how it stores relations. All key attributes from a relation are stored in a trie, which serves as the only physical index in LevelHeaded. In the trie, each level is composed of sets of dictionary encoded (unsigned integer) values. As is standard [4], LevelHeaded stores dense sets using a bitset and sparse sets using unsigned integers. Annotations are stored in separate buffers attached to the trie. LevelHeaded supports multiple annotations, and each can be reached from any level of the trie (core differences from EmptyHeaded). An example LevelHeaded trie is shown in Figure 3 where the dimensions i and j are keys and v is an annotation. LevelHeaded tries can be thought of as an index on key attributes or a materialized view of the input table.

2.3 Query Compiler

We briefly overview the theoretical foundation of LevelHeaded’s query compiler by describing the essential details.

Representing Queries as Hypergraphs. Like EmptyHeaded [4], a query is represented using a hypergraph \( H = (V,E) \), where \( V \) is a set of vertices (attributes) and each hyperedge \( e \in E \) (relation) is a subset of \( V \). A join query is represented as a subgraph of \( H \). Example hypergraphs are shown in Figure 4. In Section 3, we describe how SQL is translated to hypergraphs in more detail.

Generalized Hypertree Decompositions. The LevelHeaded query compiler uses generalized hypertree decompositions (GHDs) [19] to represent query plans. It is useful to think of GHDs as an analog of relational algebra for a WCOJ algorithm. Formally, given a hypergraph \( H = (V,H) \), a GHD is a tree \( T = (V_T,E_T) \) and a mapping \( \chi : V_T \rightarrow 2^V \) that associates each node of the tree with a subset of vertices in the hypergraph. The asymptotic runtime of a plan is bound by the fractional hypertree width (FHW) of the GHD [23] and, as is standard [4], LevelHeaded chooses a GHD with the lowest FHW to select a query plan that matches the best worst-case guarantee.

Capturing Aggregate-Join Queries. Aggregate-join queries are common in both BI and LA workloads. To capture aggregate-join queries, LevelHeaded uses the AJAR framework [23]. Despite that others [6] have argued that custom algorithms are necessary to capture such queries, AJAR extends the theoretical results of GHDs to queries with aggregations. AJAR does this by associating each tuple in a one-to-one mapping with an annotation. Aggregated annotations are members of a commutative semiring, which is equipped with product and sum operators that satisfy a set of properties (identity and annihilation, associativity, commutativity, and distributivity). Therefore, when relations are joined, the annotations on each relation are multiplied together to form the annotation on the result. Aggregations are expressed by an aggregation ordering \( \alpha = (\alpha_1,\oplus_1), (\alpha_2,\oplus_2),... \) of attributes and operators.

Using AJAR, LevelHeaded picks a query plan with the best worst-case guarantee by going through three phases. First, it breaks the input query into characteristic hypergraphs, which are subgraphs of the input that can be decomposed to optimal GHDs. Second, for each characteristic hypergraph all possible decompositions are enumerated, and a decomposition that minimizes FHW is chosen. Finally, the chosen GHDs are combined to form an optimal GHD. To avoid unnecessary intermediate results, LevelHeaded compresses all final GHDs with a FHW of 1 into a single node, as the query plans here are always equivalent to running just a WCOJ algorithm. The LevelHeaded GHDs for TPC-H query 5 and matrix multiplication are shown in Figure 4.

Figure 2: System overview with matrix multiplication input.

Figure 3: Storage of a matrix in a LevelHeaded trie.

CREATE TABLE matrix
(i INT NOT NULL,
j INT NOT NULL,
v DECIMAL NOT NULL,
PRIMARY KEY (i,j));

| i | j | v |
|---|---|---|
| 0 | 1 | 0.1 |
| 0 | 5 | 0.2 |
| 3 | 2 | 0.3 |
| 3 | 4 | 0.4 |

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2.4 Join Algorithm

The generic WCOJ algorithm shown in Algorithm 1 is the computational kernel for all queries in the LevelHeaded engine. This algorithm is used in each node of the GHD-based query plan. The generic WCOJ algorithm can be asymptotically better than any pairwise join algorithm. Intuitively, the generic WCOJ algorithm joins attributes in a multiway fashion as opposed to the traditional approach of joining relations in a pairwise fashion. This means that queries in LevelHeaded are processed one attribute equivalence class at a time, and the core operation to process an equivalence class is a set intersection. Selecting the order that these attributes are processed in is the focus of Section 3. An unrolling of this algorithm on TPC-H query 5 and matrix multiplication is shown in Figure 4.

3. SQL TO GHDS

The query compilation techniques presented in Section 2.3 are able to capture a wide range of domains, including linear algebra, message passing, and graph queries. However, most of this work has been theoretical and none of the current literature demonstrates how to capture general SQL-style queries in such a framework. In this section we show how to extend this theoretical work to more complex queries. In particular we describe how LevelHeaded translates generic SQL queries which EmptyHeaded could not) to hypergraphs in Section 3.1. Using this translation, we show how the well-known optimization of attribute elimination is captured both logically and physically in LevelHeaded. In Section 3.2 we describe the manner in which LevelHeaded selects a GHD and optimizes it by pushing down selections. We argue that with these extensions LevelHeaded represents the first practical implementation of these techniques capable of capturing general SQL workloads.

3.1 SQL to Hypergraph Translation

We demonstrate how certain features of SQL, such as complex expressions inside aggregation functions, can be translated to operations on annotated relations using a series of simple rules to construct query hypergraphs. The rules LevelHeaded uses to translate a SQL query to a hypergraph \( H = (V,E) \) and an attribute tuple is:

1. The set \( V \) of vertices in the hypergraph contains all of the key columns in the SQL query. The set of hyperedges \( E \) is each relation in the SQL query. All attributes that appear in an equi-join condition are mapped to the same attribute in \( V \).
2. All key attributes that do not appear in the output of the query must be in the aggregation ordering \( \alpha \).
3. If only the columns of a single relation appear inside of an aggregation function, the expression inside of the aggregation function is the annotation of that relation. If none of the relation’s columns appear inside of an aggregation function, the relation’s annotation is the identity element. If the inner expression of an aggregation function touches multiple relations, those
The rules above do not capture annotations which are not aggregated, so these annotations are added to a meta data container \( M \) that associates them to the hyperedge (relation) from which they originate.

**Example 3.1.** Consider how these rules capture TPC-H query 5 from Figure 4 in a LevelHeaded query hypergraph. By Rule 1, the equality join in this query is captured in the set of vertices (\( V \)) and hyperedges (\( E \)) shown in the hypergraph in Figure 4. The columns \( c_{\text{custkey}} \) and \( o_{\text{custkey}} \) must be mapped to the same vertex in “\( \text{custkey} \)” \( \in V \). Similarly, the columns \( l_{\text{orderkey}} \) and \( o_{\text{orderkey}} \) are mapped to the vertex “orderkey”, the columns \( l_{\text{suppkey}} \) and \( s_{\text{suppkey}} \) are mapped to the vertex “suppkey”, the columns \( c_{\text{nationkey}}, s_{\text{nationkey}}, \text{and } n_{\text{nationkey}} \) are mapped to the vertex “nationkey”, the columns \( n_{\text{regionkey}} \) and \( r_{\text{regionkey}} \) are mapped to the vertex “regionkey”.

By Rule 2, a valid aggregation ordering is:

\[
\alpha = [\text{regionkey}, \text{nationkey}, \text{suppkey}, \text{custkey}, \text{orderkey}]
\]

with the aggregation operator \( \Sigma \) (the order is irrelevant here).

To apply Rule 3, consider the expression inside the aggregation function, \( l_{\text{extendedprice}} * (1 - l_{\text{discount}}) \). Only columns on the lineitem table are involved in this expression, so the annotations on the lineitem table will be this expression for each tuple. The orders and customer tables do not have any columns in aggregation expressions, so they are annotated with the identity element.

By Rule 4, the hypergraph does not capture the attributes \( n_{\text{name}}, o_{\text{orderdate}}, \) or \( r_{\text{name}} \) but our metadata container \( M \) here is the following: \{\( n_{\text{name}} \leftrightarrow \text{nation}, r_{\text{name}} \leftrightarrow \text{region}, o_{\text{orderdate}} \leftrightarrow \text{orders}\}\).

**Attribute Elimination.** The rules above only add the attributes that are used in the query to the hypergraph. Although this elimination of unused attributes is obvious, ensuring this physically in the LevelHeaded trie data structure was non-trivial. To do this we ensured that any number of levels from the trie can be used during query execution. This means that annotations can be reached individually from any level of the trie. Further, the annotations are all stored in individual data buffers (like a column store) to ensure that they can be loaded in isolation. These fundamental differences with EmptyHeaded, enable LevelHeaded to support attribute elimination both logically and physically. This is essential on dense LA kernels because it enables LevelHeaded to call BLAS routines by storing a single dense annotation in flat (BLAS compatible) buffer.

### 3.2 GHD Optimization

After applying the rules in Section 3.1 a GHD is selected using the process described in Section 2.3. Still, LevelHeaded needs a way to select among multiple GHDs that the theory cannot distinguish. In this section we explain how LevelHeaded adapts the theoretical definition of GHDs to both select and produce practical query plans.

**Choosing Among GHDs with the same FHW.** For many queries, multiple GHDs have the same FHW. Therefore, a practical implementation must also include principled methods to choose between query plans with the same worst-case guarantee. Fortunately, there are three intuitive characteristics of GHD-based query plans that makes this choice relatively simple (and cheap). The first is that the smaller a GHD is (in terms of number of nodes and height), the quicker it can be executed (less generated code). The second is that fewer intermediate results (shared vertices between nodes) results in faster execution time. Finally, the lower selection constraints appear in a query plan corresponds to how early work is eliminated in the query plan. Therefore, LevelHeaded uses the following order of heuristics to choose between GHDs with the same FHW:

1. Minimize \(|V_T|\) (number of nodes in the tree).
2. Minimize the depth (longest path from root to leaf).
3. Minimize the number of shared vertices between nodes.
4. Maximize the depth of selections.

Although most of the queries in this paper are single-node GHDs, on the two node TPC-H query 5, using these rules to select a GHD results in a 3x performance advantage over a GHD (with the same FHW) that violates the rules above. We explain how selections are pushed down below joins in LevelHeaded GHDs next.

**Pushing Down Selections Below Joins.** Our mechanism of selecting a GHD ensures that selections appear as low as possible in a GHD, but we still need a mechanism to push down selections down below joins in our chosen GHD. Motivated by how EmptyHeaded does this [4], the LevelHeaded query compiler pushes down selections below joins by:

1. Taking as input an optimal (with respect to FHW) GHD \( T = (V_T, E_T) \) and a set of selections \( \sigma_{a_i} \) applied to attributes \( a_i \).
2. For each \( \sigma_{a_i} \): Let \( e_i \) be the edge that contains the vertex derived from \( a_i \) or that has the meta data \( M \) associated with \( a_i \). Let \( t_i \) be the GHD node in \( V_T \) associated with \( e_i \). If \( t_i \) contains more than one hyperedge, create a new GHD node \( t_i' \) that contains only \( e_i \), and make \( t_i' \) a child of \( t_i \).

This means that LevelHeaded pushes down selections by creating new GHD nodes with only the selection constraints under the original GHD nodes. On TPC-H query 5 this results in the GHD shown in Figure 4 which executes 1.8x faster than the GHD without this optimization.

### 4. COST-BASED OPTIMIZER

After a GHD-based query plan is produced using the process described in Sections 3.1 and 2.3, LevelHeaded needs to select an attribute order for the WCOJ algorithm. Similar to the classic query optimization problem of selecting a join order [15], WCOJ attribute ordering can result in orders of magnitude performance differences on the same query. Unfortunately, the known techniques for estimating the cost of join orders are designed for Selinger-style [10] query optimizers using pairwise join algorithms—not a GHD-based query optimizer with a WCOJ algorithm. In this section we present the first cost-based optimizer for the generic WCOJ algorithm and validate the crucial observations it uses to derive its cost estimate. We show that this optimizer selects attribute orders that provide up to a three orders of magnitude speedup over ones that EmptyHeaded could select.
Optimizer Overview. LevelHeaded’s cost-based optimizer selects a key attribute order for each node in a GHD-based query plan. As is standard \cite{4}, LevelHeaded requires that materialized key attributes appear before those that are projected away (with one important exception described in Section 4.1.2) and that materialized attributes always adhere to some global ordering (e.g. if attribute ‘a’ is before attribute ‘b’ in one GHD node order, then ‘a’ must be before ‘b’ in all GHD orders). To assign an attribute order to each GHD node, LevelHeaded’s cost-based optimizer: traverses the GHD in a top-down fashion, considers all attribute orders\footnote{We remind the reader that the number of attributes considered here is only the number of key (joined) attributes inside of a GHD node which is typically small.} adhering to the previously described criteria at each node, and selects the attribute order with the lowest cost estimate.

For each order, a cost estimate is computed based on two characteristics of the generic WCOJ algorithm: (1) the algorithm processes one attribute at a time and (2) the bottleneck of the generic WCOJ algorithm is set intersection operations. In this section, we describe how to derive a simple cost estimate, called $\text{icost}$

\begin{align*}
\text{Cost} = \sum_{i = 0}^{\mid V \mid} (\text{icost}(v_i) \times \text{weight}(v_i))
\end{align*}

The remainder of this section discusses how the $\text{icost}$s (Section 4.1.1) and weights (Section 4.2) are derived.

4.1 Intersection Cost

The bottleneck of the generic WCOJ algorithm is set intersection operations. In this section, we describe how to derive a simple cost estimate, called $\text{icost}$, for the set intersections in the generic WCOJ algorithm.

4.1.1 Cost Metric

Recall that the sets in LevelHeaded tries are stored using an unsigned integer layout (uint) if they are sparse and a bitset layout (bs) if they are dense, a design inherited from EmptyHeaded. Thus, the intersection algorithm used is different depending on the data layout of the sets. These different layouts have a large impact on the set intersection performance, even with similar cardinality sets. For example, Figure 5a shows that a $\text{bs} \cap \text{bs}$ is roughly 50x faster than a $\text{uint} \cap \text{uint}$ with the same cardinality sets. Therefore, LevelHeaded uses the results from Figure 5a to assign the following $\text{icosts}$:

$$\text{icost}(\text{bs} \cap \text{bs}) = 1, \text{icost}(\text{bs} \cap \text{uint}) = 10, \text{icost}(\text{uint} \cap \text{uint}) = 50$$

Unfortunately, it is too expensive for the query compiler to check (or track) the layout of each set during query compilation—set layouts are chosen dynamically during data ingestion and a single trie can have millions of sets. To address this LevelHeaded uses Crucial Observation 4.1.

\text{Crucial Observation 4.1.} The sets in the first level of a trie are typically dense and therefore represented as a bitset. The sets of any other level of a trie are typically sparse (unless the relation is completely dense) and therefore represented using the unsigned integer layout.

\text{Empirical Validation:} Consider a trie for the TPC-H lineitem relation where the trie levels correspond to the key attributes $\{\text{l_orderkey}, \text{l_suppkey}, \text{l_partkey}, \text{l_linenumber}\}$ in this order. At scale factor 10, each level of this trie has the following number of uint and bs sets:

- 1st level(\text{l_orderkey}) = \{0 uint sets, 1 bs set\}
- 2nd level(\text{l_suppkey}) = \{14999914 uint sets, 86 bs sets\}
- 3rd level(\text{l_partkey}) = \{59984817 uint sets, 0 bs sets\}
- 4th level(\text{l_linenumber}) = \{59986042 uint sets, 0 bs sets\}

Thus, given a key attribute order $\{v_0, \ldots, v_{\mid V \mid}\}$ (where each $v_i \in V$), the LevelHeaded optimizer assigns an $\text{icost}$ to each $v_i$ in order of appearance, using the following method which leverages Crucial Observation 4.1.

- For each edge $e_j$ with node $v_i$, assign $l(e_j)$ (where $l=layout$), to either uint or bs. As a reminder, edges are relations and vertices are attributes. Thus, for each relation this assignment guesses one data layout for all of the relation’s $v_i$ sets. If $e_j$ has been assigned with a previous vertex $v_k$ where $k < i$, $l(e_j) = \text{uint}$ (not the first trie level), otherwise $l(e_j) = \text{bs}$.
- Compute the cost of intersecting the $v_i$ attribute from each edge (relation) $e_j$. For a vertex with two edges, the pairwise $\text{icost}$ is used. For a vertex with $N$ edges, where $N > 2$, the $\text{icost}$ is the sum of pairwise $\text{icost}$s where the bs sets are always processed first. For example, when $N = 3$ and $l(e_0) \leq l(e_1) \leq l(e_2)$ where bs $< \text{uint}$, $\text{icost} = \text{icost}(l(e_0) \cap l(e_1)) + \text{icost}(l(e_1) \cap l(e_2))$. Note, $\text{uint} = l(\text{bs} \cap \text{uint})$. 

\begin{table}[h]
\centering
\begin{tabular}{|c|c|c|}
\hline
Lineitem & Cost & Time (ms) \\
\hline
\text{lineitem} & 10 & 5.2ms \\
\text{lineitem} & 50 & 48ms \\
\text{lineitem} & 100 & 8.4ms \\
\hline
\end{tabular}
\caption{Cost estimation experiments.}
\end{table}

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{figure5.png}
\caption{Cost estimation experiments.}
\end{figure}
Example 4.1. Consider the attribute order \[/orderkey, custkey, nationkey, suppkey]\ for one of the GHD nodes in TPC-H query 5 (see Figure 4). The orderkey vertex is assigned an \(i\text{cost}\) of 1 as it is classified with \([\text{bs} \cap \text{bs}]\) intersections. The custkey vertex is assigned an \(i\text{cost}\) of 10, classified with \([\text{uint} \cap \text{uint}]\) intersections. The nationkey vertex is assigned an \(i\text{cost}\) of 11, classified with \([\text{bs} \cap \text{bs} \cap \text{uint}]\) intersections. Finally, the suppkey vertex is assigned an \(i\text{cost}\) of 50, classified with \([\text{uint} \cap \text{uint}]\) intersections.

Finally, in the special case of a completely dense relation, the LevelHeaded optimizer assigns an \(i\text{cost}\) of 0 because no set intersection is necessary in this case. This is essential to estimate the cost of LA queries properly.

4.1.2 Relaxing the Materialized Attributes First Rule

An interesting aspect of the intersection cost metric is that the cheapest key attribute order could have materialized key attributes come after those which are projected away. To support such key attribute orders, the execution engine must be able to combine children (in the trie) of projected away key attributes using a set union or \(\text{GROUP BY}\) (to materialize the result sets). Unfortunately, it is difficult to design an efficient data structure to union more than one level of a trie (materialized key attribute) in parallel (e.g., use a massive 2-dimensional buffer or incur expensive merging costs). Therefore, EmptyHeaded kept its design simple and never considered relaxing the rule that materialized attributes must appear before those which are projected away. In LevelHeaded we relax this rule by allowing 1-attribute unions (see Section 3) on keys when it can lower the \(i\text{cost}\).

Within a GHD node, LevelHeaded relaxes the materialized attributes first rule under the following conditions:

1. The last attribute is projected away.
2. The second to last attribute is materialized.
3. The \(i\text{cost}\) is improved by swapping the two attributes.

These conditions ensure that 1-attribute union will only be introduced when the \(i\text{cost}\) can be lowered.

Example 4.2. Consider the sparse matrix multiplication query and its unrolling of the generic WCOJ algorithm for an attribute order of \([i, j, k]\) shown in Figure 4. This attribute order has a cost 50 \([\text{uint} \cap \text{uint}]\) assigned to the \(k\) attribute.

Now consider an attribute order \([i, k, j]\). Here a cost 10 \([\text{uint} \cap \text{bs}]\) is assigned to \(k\) and the unrolling of the generic WCOJ algorithm (where \(j\) denotes accesses to keys and \(i\) denotes accesses to annotations) is the following:

\[
\begin{align*}
\text{for } i \in \pi_1 M_1 & \quad \text{do} \\
& \quad s_j \leftarrow \emptyset \\
\text{for } k \in \pi_k M_1(i) \cap \pi_k M_2 & \quad \text{do} \\
& \quad \text{for } j \in \pi_j M_2(k) \quad \text{do} \\
& \quad \quad s_j[j] + \pi_{\text{col}} M_1[i, k] \times \pi_{\text{col}} M_2[k, j] \\
\text{out}(i) & \leftarrow s_j
\end{align*}
\]

This lower cost attribute order recovers the same loop ordering as Intel MKL on sparse matrix multiplication and its bottleneck is the \(\ast\) operation that unions \(j\) values and sums \(\text{uint}\) values. This is in contrast to the standard bottleneck operation of a set intersection as in the \([i, j, k]\) order. In Section 5 we discuss the tradeoffs around implementing this union add or \(\text{GROUP BY}\) operation. Figure 5 shows that this order is essential to run sparse matrix multiplication as a join query without running out of memory.

4.2 Weights

Like classic query optimizers, LevelHeaded also tracks the cardinality of each relation as this influences the \(i\text{costs}\) for the generic WCOJ algorithm. Figure 5 shows the unsurprising fact that larger cardinality sets result in longer intersection times. To take this into account when computing a cost estimate LevelHeaded assigns weights to each vertex using Crucial Observation 4.2 which directly contradicts conventional wisdom from pairwise optimizers:

**Crucial Observation 4.2.** The highest cardinality attributes should be processed first in the generic WCOJ algorithm. This enables these attributes to partake in fewer intersections (outermost loops) and ensures that they are a higher trie levels (more likely \(\text{bs}\)'s with lower \(i\text{costs}\)).

**Empirical Validation:** We show in Figure 5c that the generic WCOJ can run over 70x faster on TPC-H query 5 when the high cardinality orderkey attribute is first in the attribute order instead of last.

LevelHeaded’s goal when assigning weights is to follow Crucial Observation 4.2 by assigning high cardinality attributes heavier weights so that they appear earlier in the attribute order. To do this, LevelHeaded assigns a cardinality score to each queried relation and uses this to weight to each attribute. We describe this in more detail next.

**Score.** LevelHeaded maintains a cardinality score for each relation in a query which is just the relation’s cardinality relative to the highest cardinality relation in the query. The score (out of 100) for a relation \(r_i\) is:

\[
\text{score} = \text{ceiling} \left( \frac{|r_i|}{r_{\text{heavy}}} \times 100 \right)
\]

where \(r_{\text{heavy}}\) is the highest cardinality relation in the query.

**Weight.** To assign a weight to each vertex LevelHeaded uses the highest score edge (or relation) with the vertex when a high selectivity (equality) constraint is present, otherwise LevelHeaded takes the lowest score edge (or relation). The intuition for using the highest score edge (or relation) with a high selectivity constraint is that this relation represents the amount of work that could be filtered (or eliminated) at this vertex (or attribute). The intuition for otherwise taking the lowest score edge (or relation) is that the output cardinality of an intersection is at most the size of the smallest set.

Example 4.3. Consider TPC-H Q5 at scale factor 10. The cardinality score for each relation here is:

\[
\begin{align*}
\text{score(linetem)} &= 100, \text{score(orders)} = 26, \text{score(customer)} = 3, \\
\text{score(region)} &= 1, \text{score(supplier)} = 1, \text{score(nation)} = 1
\end{align*}
\]

The weight for each vertex is \(\text{region}\) is equality selected:

\[
\begin{align*}
\text{weight(orderkey)} &= \text{min}(26, 100), \text{weight(custkey)} = \text{min}(3, 26) \\
\text{weight(suppkey)} &= \text{min}(1, 100), \text{weight(nationkey)} = \text{min}(1, 1, 3) \\
\text{weight(regionkey)} &= \max(1, 1)
\end{align*}
\]

These weights are then used to derive the cost estimates shown in Figure 5c.

---

\(^3\)We remind the reader that later (or after) in the attribute order corresponds to a lower level of the tries.
5. GROUP BY TRADEOFFS

Designing a skew-resistant GROUP BY operator requires careful consideration of the tradeoffs associated with its implementation. In this section we present these tradeoffs in the context of LevelHeaded. Although many GROUP BYs are automatically captured in LevelHeaded’s tries, two classes of GROUP BYs (that were not supported in EmptyHeaded) require an additional implementation in LevelHeaded: (1) a GROUP BY to union keys in the attribute orders from Section 4.1.2 and (2) a GROUP BY on annotations. In this section we present the tradeoffs around two implementations for each type of GROUP BY and describe a simple optimizer that automatically exploits these tradeoffs to select the implementation used during execution. In Section 6 we show that this GROUP BY optimizer provides up to a 875x and 185x speedup over using a single GROUP BY implementation on BI and LA queries respectively.

GROUP BY Key. An important artifact of the key attribute orders from Section 3 is that a projected away attribute can appear before a materialized attribute in LevelHeaded. In this case a GROUP BY on a key attribute is needed to union the result. LevelHeaded selects between two implementations for this: (1) a hash map that upserts (key, value) pairs as they are encountered and (2) a bitset that unions (OR’s) keys and a dense array to hold the values. Figure 6a shows that the performance of these implementations depends greatly on the density of the output key attribute set. The bitset performs better when the output density is high and the hash map performs better when the output density is low. To predict the output set’s density LevelHeaded leverages a simple observation: the output density is correlated in a 1-1 manner with the density of projected away attribute (set being looped over). As such, LevelHeaded uses the density of the projected away attribute to predict the output set’s density and select the implementation used.

GROUP BY Annotation. LevelHeaded supports SQL queries with GROUP BY operations on annotations or on both annotations and keys. To support this in parallel, LevelHeaded selects between two implementations of a parallel hash map: (1) libcuckoo and (2) a per-thread instance of the C++ standard library unordered map. In the context of a GROUP BY, the crucial operation for these hash maps is an upsert and we found that each implementation worked best under different conditions. Figures 6b and 6c show that when the output key size is small (many collisions), the per-thread implementation outperforms libcuckoo by up to 4x. In contrast, libcuckoo outperforms the per-thread approach by an order of magnitude when the output size is large (few collisions). Unfortunately, predicting output cardinalities is a difficult problem, so LevelHeaded leverages a general observation to avoid this when selecting between these two approaches: libcuckoo is at worst close (<2x) to the performance of the per-thread approach when the hash-map key tuple is wide (>3 values), and the per-thread approach is at worst close (<2x) to the performance of libcuckoo when the hash map key is small (<3 values). This trend is shown in Figures 6b and 6c. Thus, the LevelHeaded optimizer selects a per-thread hash map when the hash map key is a tuple of three elements or less and libcuckoo otherwise.

6. EXPERIMENTS

We compare LevelHeaded to state-of-the-art relational database management engines and LA packages on standard BI and LA benchmark queries. We show that LevelHeaded is able to compete within 2.5x of these engines, while sometimes outperforming them, and that the techniques from in Sections 4 to 6 can provide up to a three orders of magnitude speedup. This validates that a WCOJ architecture is a practical solution for both BI and LA queries.

6.1 Setup

We describe the experimental setup for all experiments.

Environment. LevelHeaded is a shared memory engine that runs and is evaluated on a single node server. As such, we ran all experiments on a single machine with a total of 56 cores on four Intel Xeon E7-4850 v3 CPUs and 1 TB of RAM. For all engines, we chose buffer and heap sizes that were at least an order of magnitude larger than the dataset to avoid garbage collection and swapping data out to disk.

Relational Comparisons. We compare to HyPer, MonetDB, and LogicBlox on all queries to highlight the performance of other relational databases. Unlike LevelHeaded, these engines are unable to compete within an order of magnitude of the best approaches on BI and LA queries. We compare to HyPer v0.5.0 as HyPer is a state-of-the-art in-memory RDBMS design. We also compare to the MonetDB Dec2016-SP5 release. MonetDB is a popular columnar
Table 1: Runtime for the best performing engine ("Baseline") and relative runtime for comparison engines. '-' indicates that the engine did not provide support for the query. ‘t/o’ indicates the system timed out and ran for over 30 minutes. ‘oom’ indicates the system ran out of memory.

| Query | Data | Baseline | LevelHeaded | Intel MKL | HyPer | MonetDB | LogicBlox |
|-------|------|----------|-------------|-----------|-------|---------|-----------|
| Q1    | SF 1 | 12ms     | 1.79x       | -         | 1x    | 30.59x  | 74.17x    |
| Q2    | SF 10| 84ms     | 1.73x       | -         | 1x    | 17.86x  | 23.45x    |
| Q3    | HF 100| 608ms  | 1.78x       | -         | 1x    | 80.43x  | 26.12x    |
| Q4    | SF 1 | 29ms     | 1.11x       | -         | 1x    | 5.56x   | 48.28x    |
| Q5    | SF 10| 111ms    | 1.45x       | -         | 1x    | 9.88x   | 32.50x    |
| Q6    | HF 100| 963ms  | 1.01x       | -         | 1x    | 9.76x   | 10.99x    |
| Q7    | SF 1 | 19ms     | 1.49x       | -         | 1x    | 6.04x   | 199x      |
| Q8    | SF 10| 92ms     | 1.40x       | -         | 1x    | 4.84x   | 55.33x    |
| Q9    | HF 100| 867ms  | 1.21x       | -         | 1x    | 4.04x   | 21.33x    |
| Q10   | SF 1 | 5ms      | 1.73x       | -         | 1x    | 12.27x  | 270x      |
| Q11   | SF 10| 34ms     | 1.50x       | -         | 1x    | 6.65x   | 101x      |
| Q12   | HF 100| 283ms  | 1.61x       | -         | 1x    | 7.42x   | 73.43x    |
| Q13   | SF 1 | 16ms     | 1x          | 2.78x     | 7.98x | 72.77x  |           |
| Q14   | SF 10| 45ms     | 1.74x       | -         | 1x    | 15.16x  | 73.78x    |
| Q15   | HF 100| 1.06ms | 1.88x       | -         | 1x    | 21.55x  | 25.02x    |
| Q16   | SF 1 | 115ms    | 1x          | 4.05x     | 4.14x | 57.84x  |           |
| Q17   | HF 100| 1020ms | 1x          | 5.71x     | 5.19x | 21.78x  |           |
| Q18   | SF 1 | 32ms     | 1.36x       | -         | 1x    | 5.88x   | 31.56x    |
| Q19   | SF 10| 196ms    | 1.26x       | -         | 1x    | 6.12x   | 18.06x    |
| Q20   | HF 100| 869ms  | 1.78x       | -         | 1x    | 9.9x    | 7.79x     |
| TPC-H | Harbor | 2.66ms | 1x          | 2.89x     | 10.81x| 30.8x   | 89.74x    |
| Linear Algebra | SMV | HV15R | 68.01ms | 2.43x | 1x | 25.82x | 40.72x |
| Linear Algebra | NLP | 240 | 114.97ms | 1.49x | 1x | 17.23x | 53.94x |
| Linear Algebra | SMM | 1109ms | 1.63x | 1x | 13.10x | 27.27x |
| Linear Algebra | HV15R | 18.79s | 1.35x | 1x | oom | t/o | 48.11x |
| Linear Algebra | NLP | 240 | 1.92s | 2.44x | 1x | 4.91x | 78.70x |
| Linear Algebra | SMM | 1109ms | 1.63x | 1x | 13.10x | 27.27x |
| Linear Algebra | 12288 | 13.5ms | 1x | 5.78x | 88.89x | 330x |
| Linear Algebra | 16384 | 23.45ms | 1x | 18.13x | 51.18x | 587x |
| Linear Algebra | 8192 | 2.76s | 1.02x | 1x | oom | t/o | 1/t |
| Linear Algebra | 16384 | 9.29s | 1.01x | 1x | oom | t/o | 1/t |

Table 1: Runtime for the best performing engine ("Baseline") and relative runtime for comparison engines. '-' indicates that the engine did not provide support for the query. ‘t/o’ indicates the system timed out and ran for over 30 minutes. ‘oom’ indicates the system ran out of memory.

store database engine and is a widely used baseline. Finally, we compare to LogicBlox v4.4.5 as LogicBlox is the first general purpose commercial engine to provide similar worst-case optimal join guarantees. Our setup of LogicBlox was aided by a LogicBlox engineer. Because of this, we know that LogicBlox does not use a WCOJ algorithm for many of the join queries in the TPC-H benchmark. HyPer, MonetDB, and LogicBlox are full-featured commercial database engines that LevelHeaded does not.

Linear Algebra Package Comparison. We use Intel MKL v121.3.0.109 as the specialized linear algebra baseline. This is the best baseline for LA performance on Intel CPUs (as we use in this paper). Others have shown that it takes considerable effort and tedious low-level optimizations to approach the performance of such libraries on LA queries.

Omitted Comparisons. We omit a comparison to array databases like SciDB as these engines call BLAS or LAPACK libraries (like Intel MKL) on the LA queries that we present in this paper. Therefore, Intel MKL was the proper baseline for our LA comparisons. Additionally, SciDB is not designed to process TPC-H queries.

Metrics. For end-to-end performance, we measure the wall-clock time for each system to complete each query. We repeat each measurement seven times, eliminate the lowest and the highest value, and report the average. This measurement excludes the time used for outputting the result, data statistics collection, and index creation for all engines. We omit the data loading and query compilation for all systems except for LogicBlox and MonetDB where this is not possible. The query compilation time for these queries, especially at large scale factors, is negligible to the execution time. Still, we found that LevelHeaded’s unoptimized query compilation process performed within 2x of HyPer’s. To minimize unavoidable differences with disk-based engines (LogicBlox and MonetDB) we place each database in the tmpfs in-memory file system and collect hot runs back-to-back. Between measurements for the in-memory engines (HyPer and Intel MKL), we wipe the caches and re-load the data to avoid the use of intermediate results.

6.2 Experimental Results

We show that LevelHeaded can compete within 2x of HyPer on seven TPC-H queries and within 2.5x of Intel MKL on four LA queries, while outperforming MonetDB and LogicBlox by up to two orders of magnitude.

6.2.1 Business Intelligence

On seven queries from the TPC-H benchmark we show that LevelHeaded can compete within 2x of HyPer for seven TPC-H queries and within 2.5x of Intel MKL for four LA queries, while outperforming MonetDB and LogicBlox by up to two orders of magnitude.

Datasets. We run the TPC-H queries at scale factors 1, 10, and 100. We stopped at TPC-H 100 as in-memory engines, such as HyPer, often use 2-3x more memory than the size of the input database during loading—therefore approaching the memory limit of our machine. For reference, on TPC-H query 1 HyPer uses 161GB of memory whereas LevelHeaded uses 25GB (only loading the data it needs from disk).

Queries. We choose TPC-H queries 1, 3, 5, 6, 8, 9 and 10 to benchmark, as these queries exercise the core operations
of BI querying and also containing interesting join patterns (except 1 and 6). TPC-H queries 1 and 6 do not contain a join and demonstrate that although LevelHeaded is designed for join queries, it can also compete on scan queries.

Discussion. In Table 1 we show that LevelHeaded can outperform MonetDB by up to 80x and LogicBlox by up to 270x while remaining within 1.88x of the highly optimized HyPer database. Unsurprisingly, the queries where LevelHeaded is the farthest off the performance of the HyPer engine is TPC-H queries 1 and 8. Here the output cardinality is small and the runtime is dominated by the GROUP BY operation. As described in Section 5 LevelHeaded leverages existing hash map implementations to perform these GROUP BYs, and a custom built (NUMA aware) hash map would likely close this performance gap. On queries 3 and 9, where the output cardinality is larger (and closer to worst-case), LevelHeaded is able to compete within 11% of HyPer and sometimes outperforms it. In LevelHeaded, we also tested additional optimizations like indexing annotations with selection constraints and pipelining intermediate results between GHD nodes. We found that these optimizations could provide up to a 2x performance increase on queries like TPC-H query 8, but did not consistently outweigh their added complexity.

6.2.2 Linear Algebra

We show that LevelHeaded can compete within 2.5x of Intel MKL while outperforming HyPer and LogicBlox more than 18x and 587x respectively on LA benchmarks.

Datasets. We evaluate linear algebra queries on three dense matrices and three sparse matrices. The first sparse matrix dataset we use is the Harbor dataset, which is a 3D CFD model of the Charleston Harbor [15]. The Harbor dataset is a sparse matrix that contains 46,835 rows and columns and 2,329,092 nonzeros. The second sparse matrix dataset we use is the HV15R dataset, which is a CFD matrix of a 3D engine fan [15]. The HV15R matrix contains 2,017,169 rows and columns and 283,073,458 nonzeros and is a large, non-graph, sparse matrix. The final sparse matrix dataset we use is the nlpkt240 dataset with 29,993,600 rows and columns and 401,232,976 nonzeros [39]. The nlpkt240 dataset is a symmetric indefinite KKT matrix. For dense matrices, we use synthetic matrices with dimensions of 8192x8192 (8192), 12288x12288 (12288), and 16384x16384 (16384).

Queries. We run matrix dense vector multiplication and matrix multiplication queries on both sparse (SMV,SMM) and dense (DMV,DMM) matrices. These queries were chosen because they are simple to express using joins and aggregations in SQL and are the core operations for modern machine learning algorithms. Further, Intel MKL is specifically designed to process these queries and, as a result, achieves the largest speedups over using a RDBMS here. Thus, these queries represent the most challenging LA baseline queries possible. For both SMM and DMM we multiply the matrix by itself, as is standard for benchmarking.

Discussion. Table 1 shows that LevelHeaded is able to compete within 2.44x of Intel MKL on both sparse and dense LA queries. On dense data, LevelHeaded uses the attribute elimination optimization from Section 3 to store dense annotations in single buffers that are BLAS compatible and code generates to Intel MKL. Still, MKL produces only the output annotation, not the key values, so LevelHeaded incurs a minor performance penalty (<2%) for producing the key values. On sparse data, LevelHeaded is able to compete with MKL when executing these LA queries as pure aggregate-join queries. To do this, the attribute order and GROUP BY optimizations from Sections 3 and 5 were essential. Although we tested more sophisticated optimizations, like cache blocking (by adding additional levels to the trie), we found that the performance benefit of these optimizations did not consistently outweigh their added complexity. In contrast, other relational designs fail flat on these LA queries. Namely, HyPer usually runs out of memory on the matrix multiplication query and, on the queries which it does complete, is often an order of magnitude slower than Intel MKL. Similarly, LogicBlox and MonetDB are at least an order of magnitude slower than Intel MKL on these queries.

6.3 Micro-Benchmarking Results

We break down the performance impact of each optimization presented in Sections 4 to 6.

Attribute Elimination. Tables 2 and 3 show that attribute elimination can enable up to a 4.82x performance advantage on the TPC-H queries and up to a 500x performance advantage on dense LA queries. Attribute elimination is crucial on most on TPC-H queries, as these queries typically touch a small number of attributes from schemas with many attributes. Unsurprisingly, Table 2 shows that attribute elimination provides the largest benefit on the scan TPC-H queries (1 and 6) because it allows LevelHeaded to scan less data. On dense LA queries, LevelHeaded calls Intel MKL with little overhead because attribute elimination enables us to store each dense annotation in a BLAS acceptable buffer. As Table 4 shows, this yields up to a 500x speedup over processing these queries purely in LevelHeaded.

Selections. As shown in Table 2, pushing down selections can provide up to 2.67x performance advantage on the TPC-H queries we benchmark. Although pushing down selections is generally useful, on TPC-H Q10 it actually hurts execution time. This is because this optimization adds additional GHD nodes (intermediate results) to our query plans (some-
thing pipelining would fix). Still, the performance impact is small on this query (<12%) and on average this optimization provided a 90% performance gain across TPCH queries.

**Attribute Order.** As shown in Tables 2 and 3, the cost-based attribute ordering optimizer presented in Section 4 can enable up to a 8815x performance advantage on TPCH queries and enables LevelHeaded to run sparse matrix multiplication as a join query without running out of memory. Tables 2 and 6 show the difference between the best-cost and the worst-cost attribute orders. The most interesting queries here are TPCH query 5 and TPCH query 8. On TPCH query 5, it is essential that the high cardinality orderkey attribute appears first. On TPCH query 8, it is essential that the partkey attribute, which was connected to an equality selection, appears first. The process of assigning weights to the intersection costs in Section 4.2 ensures that orders satisfying these constraints are chosen. Finally, the cost-based attribute ordering optimizer is also crucial on sparse matrix multiplication. Here it is essential that the lower cost attribute order, with a projected away attribute before one that is materialized, is selected. This order not only prevents a high cost intersection, but eliminates the computation and materialization of annotation values do not participate in the output (due to sparsity).

**GROUP BY.** Finally, we show in Tables 2 and 6 that LevelHeaded’s GROUP BY optimizers provide up to a 875x performance advantage on TPCH queries and up to a 185x performance advantage on LA queries. The GROUP BY annotation optimizer provides a 875x speedup on TPCH query 1 because GROUP BY is the bottleneck operation in this query and our optimizer properly selects to use a per-thread hash map instead of libscuckoo here. The GROUP BY key optimizer provides a 185x speedup on SMM with the nlp240 dataset because it correctly predicts that the many of the output key attribute sets are sparse. As such LevelHeaded’s optimizer chooses to use a standard hash map to produce these sparse sets because using a bitset is highly inefficient (due to the amount of memory it wastes). This optimizer making the opposite choice provides a 3x performance advantage on SMM with the HV15R dataset.

7. EXTENSION

We extend LevelHeaded to show that such a unified query processing architecture could enable faster end-to-end applications. To do this, we add the ability for LevelHeaded to process workloads that combine SQL queries and full machine learning algorithms (as described in [3]). With this, we show on a full-fledged application that LevelHeaded can be an order of magnitude faster than the popular solutions of Spark v2.0.0, MonetDB Dec2016/Sk litter-v0.17.1, and Pandas v0.18.1/Scikit-learn v0.17.1 (using the same experimental setting as Section 6).

**Application.** We run a voter classification application [37] that joins and filters two tables to create a single feature set which is then used to train a logistic regression model for five iterations. This application is a pipeline workload that consists of three pipeline phases: (1) a SQL-processing phase, (2) a feature engineering phase where categorical variables are encoded, and (3) a machine learning phase. The dataset [37] consists of two tables: (1) one with information such as gender and age for 7,503,555 voters and (2) one with information about the 2,751 precincts that the voters were registered in.

**Performance.** Figure 7 shows that LevelHeaded outperforms Spark, MonetDB, and Pandas on the voter classification application by up to an order of magnitude. This is largely due to LevelHeaded’s optimized shared-memory SQL processing and ability to minimize data transformations between the SQL and training phase. To expand on the cost of data transformations a bit further, in Table 4 we show the cost of converting from a column store to the compressed sparse row format used by most sparse library packages. This transformation is not necessary in LevelHeaded as it always uses a single, trie-based data structure. As a result, Table 3 shows that up to 41 SVM queries can be run in LevelHeaded in the time that it takes for a column store to convert the data to a BLAS compatible format. Similarly, on the voter classification application LevelHeaded avoids expensive data transformations (in the encoding phase) by using its trie-based data structure for all phases.

8. CONCLUSIONS

This paper introduced the LevelHeaded engine and demonstrated that a query architecture built around WCQJs can compete on standard BI and LA benchmarks. We showed that LevelHeaded outperforms other relational engines by at least an order of magnitude on LA queries, while remaining on average within 31% of best-of-the-breed solutions on BI and LA benchmark queries. Our results are promising and suggest that such a query architecture could serve as the foundation for future unified query engines.

Acknowledgments: We thank LogicBlox for their helpful conversations and assistance in setting up our comparisons.
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