Non-Magnetic Solid Body in Ferrofluid Containers: Wall Effects

A S Ivanov * and C A Khokhryakova
Institute of Continuous Media Mechanics UrB RAS, Ac. Korolev Str., 1, Perm, Russia
E-mail: *lesnichiy@icmm.ru

Abstract. Experimental and numerical investigations of a magnetic ponderomotive force acting on non-magnetic body immersed in ferrofluid were carried out. The study was performed on a non-magnetic sphere in a cylindrical container filled with the magnetic fluid and magnetized by a uniform magnetic field. The symmetry of the experimental setup allowed the simplification of the problem: 2D axisymmetric numerical simulation corresponded to the 1D experimental measurements. The magnetic ponderomotive force turned out to be non-monotonic: it has two extrema and one or three zero values depending on the geometrical parameters of the container and magnitude of the applied field intensity. It was shown that wall effects are crucial for this problem, because the ponderomotive force in the vicinity of the container’s bottom (or top) may change its sign (direction). This behaviour can be described only in the framework of the inductive approach, which takes into account all demagnetizing fields generated by the ferrofluid container. On the contrary, the simplified non-inductive approach is unable to explain the magnetic force behaviour, especially the wall effects.

1. Introduction
Floating of solids in magnetic fluids (ferrofluids) is a very interesting fundamental and applied problem [1, 2]. Interest in the subject resulted from the opportunity to control the magnetic fluid properties with the help of the magnetic field, and, ultimately, to manipulate the bodies immersed in magnetic fluid. This potential of magnetic fluids meets the challenge of controlled drug delivery, separation of solid granular media, sensors design, etc. Promising prospects for the magnetic fluids application for these problems draw the researches’ attention in the late 1960s, as long ago as the first samples of magnetic fluids appeared.

It is interesting to note that the large majority of scientific researches in 1960-1990s were of technical nature [2] by virtue of the technological development, while since 2000s the focus of investigations moved to soft matter and biomedical applications (i.e., drug delivery, cell separation, hyperthermia, etc.). Yet the problem of rigid bodies floatation [3] reduces to the particular cases of: (1) levitation of a permanent magnet in magnetic fluid (being important for ferrofluid sensors), and (2) floating of non-magnetic solid grains in the electromagnet gap filled with ferrofluid (ferrofluid separator, sealer). These special cases have the following common features.

Firstly, permanent magnets and electromagnets, used by industrial manufacturers, produce/apply a strong magnetic field upon the magnetic fluid to an extent of its full magnetizing saturation \(M \approx M_s, M \ll H_0\), magnetic susceptibility \(\chi \approx 0\) and magnetic permeability \(\mu = 1 + \chi \approx 1\). In this case the magnetic field in ferrofluid \(H\) barely differ from the applied field \((H = H_0 + h \approx H_0)\).

Secondly, this strong magnetic field is significantly non-uniform \((\nabla H_0 \sim 10^6 \text{ A/m}^2)\). Therefore, the ponderomotive force on the magnetic fluid volume (and hence on the floating bodies) is essentially independent of the magnetic field produced by the fluid itself (the demagnetizing field of the ferrofluid \(h\)), but is determined by the gradient of the applied field.
These options considerably simplify the analysis of the problem by introducing the so-called non-inductive approach. This approach implies the complete neglecting of the ferrofluid’s magnetic field as it is resulted from the inequation $H_0 >> h$. The non-inductive approach has well-known limitations [4]. Thus, a good agreement between the exact solution and the predictions of the non-inductive approach is observed only at $\chi < 0.25$. At $\chi = 1.5$ the difference between the accurate and the non-inductive solutions reaches the magnitude of 30 % and monotonically increases with increasing $\chi$. It means that the non-inductive approach does not work in case of small and moderate magnetic fields ($H_0$ up to 10 kA/m).

Another limitation of the non-inductive approach is when the external magnetic field is uniform. In this case the ferrofluid demagnetizing field is low ($H_0 >> h$). Given that $h$ remains actually the only reason for the magnetic ponderomotive force, one cannot neglect it any more, because in this case the force $F$ would lose all reasonable grounds.

To sum it up, this work is devoted to the magnetic ponderomotive force acting on non-magnetic bodies, immersed into containers filled with magnetic fluid, magnetized by external uniform magnetic field. The study is carried out in the framework of the inductive approach, which assumes that all demagnetizing fields generated by the container with magnetic fluid, are taken into account.

2. Investigation methods

The study was carried out experimentally and numerically. Both investigation methods used the same test object, shown in Fig. 1, for adequate comparison of obtained results. The experimental setup was generally similar to the classic experiment proposed by R. E. Rosensweig [3]. The test object was the non-magnetic sphere 1 (with radius $R_b$) placed inside the vertical cylindrical container 2 (with internal radius $R_c$) flat-bottomed and topped. The system was magnetized in the vertical uniform magnetic field $H_0$, and additional demagnetizing fields were generated by both the ferrofluid container and the sphere. The applied field induced additional magnetic ponderomotive force acting on the sphere that changed the weight of the sphere – the force, which can be measured in experiment and calculated in numerical simulations. Let us now discuss some specific details concerning experiment and numerical methods.

The non-magnetic sphere was a hollow glass shell filled with melted dia- or paramagnetic metal (lead or tin). The container was filled with magnetic fluid of the «magnetite – oleic acid – kerosene» type prepared by the standard chemical precipitation method. The sphere was suspended on a thin nichrome wire passing through a small hole made in the top of the container beforehand. Another end of the wire was attached to the measuring device (analytical balance 3). The container was placed on a non-magnetic platform, movable along the vertical axis inside a solenoid, generating uniform magnetic field $H_0$.

In the absence of the external applied field the weight of the sphere was defined by gravity and Archimedes buoyancy force and did not depend on the position of the sphere inside the container. When the external field was switched on, the weight of the sphere changed by the value of the magnetic ponderomotive force $F$ being a function of the field intensity $H$ and the solid body vertical displacement $z$ from the center of the container. The cylindrical symmetry of the problem allowed us to measure only...
vertical z-component of the ponderomotive force $F_z$, because the suspension (of about 40 cm long) was centered manually with high accuracy in the absence of the applied field. The symmetric demagnetizing fields generated the radial force component (in the x-y plane), that squeezed the sphere and automatically aligned the z-axis. The ponderomotive force, acting on the test solid sphere being offset by the distance $z$ from the container’s center, was determined as the difference between two weight measurements in the zero and non-zero applied field.

As it was mentioned earlier, in case of the applied field $H_0$ the resultant magnetic ponderomotive force $F(z)$ was totally defined by demagnetizing fields, which in its turn were totally defined by the demagnetizing factor of the container [5]. Thus, the geometry of the problem plays the key role in this study, what was predicted earlier [6] by numerical simulations for the case of a model ferrofluid with linear magnetization law. There are three geometrical parameters of the problem, as shown in the Fig. 1: $R_c$, $R_s$ and $d$. Parameters are chosen taking into account the following aspects simultaneously. Firstly, there are at least two main formulations of the problem: the case of a flat horizontal layer ($R_s \gg R_c$, $R_s > d$) and the case of maximum demagnetizing fields generated by container ($R_s > R_c$, $R_s \approx d$). The values of $R_c$, $R_s$, $d$ parameters were limited by real possibilities of laboratory technique. Thus, $R_c$ is limited by the internal diameter of the solenoid. The container’s height $2d$ is limited by the finite area inside solenoid, where the magnetic field is uniform. And the size $R_s$ is limited by the analytical balance accuracy: it is impossible to measure the changes of weight with high accuracy, if the object is too light.

Numerical simulation of the problem was performed with the help of free software package FEMM (Finite Element Method Magnetics), developed by Dr. Meeker (see e.g. [7]). The problem (Fig. 1) is axisymmetric and magnetostatic: the sphere moves along the z-axis, but the calculations and laboratory measurements are performed for the static case. The simulation model matches the laboratory experiment in details: the geometrical parameters and magnetic system configuration were the same, the magnetic fluid magnetization law $M(H)$ was measured experimentally (Fig. 2) and included in the FEMM library of materials. The fluid-magnetic buoyancy force was calculated according to the known formula [1–3]:

$$
\vec{F} = -\mu_0 \oint_S \left( \frac{H}{\mu} d\vec{H} + \frac{M_n^2}{2} \right) \hat{n} dS
$$

(1)

where $\mu_0 = 4\pi \times 10^{-7}$ H/m is the magnetic constant, $S$ is the surface of the sphere, $n$ is the unit vector normal to the surface $S$ at some point with coordinates $(z; R_s^2 - z^2)^{1/2}$, $H$ is the magnetic field intensity at that point, $M_n = \vec{H} \hat{n}$ is the normal component of ferrofluid magnetization. The first term in equation (1) is the ferrohydrodynamic pressure described by the common Bernoulli equation and acting in any arbitrary point inside the magnetic fluid. The second term stands for the magnetic pressure drop that exists only in the vicinity of the interface between two media.

To calculate the force (1) one needs to solve the system of Maxwell equations

$$
\nabla \vec{B} = 0; \ \nabla \times \vec{H} = \vec{j}
$$

(2)

with standard boundary conditions for the magnetic flux density $\vec{B}$ and field intensity $\vec{H}$ vectors at the interface between two media

$$
(\vec{B}_1 - \vec{B}_2) \hat{n}_{1,2} = 0; \ (\vec{H}_1 - \vec{H}_2) \times \hat{n}_{1,2} = \vec{j}_S.
$$

(3)

The vector $\vec{j}$ in (2), (3) is the electric current flux density ($\vec{j} = 0$ everywhere except for the solenoid current-conducting wires), $\vec{j}_S$ denotes the surface current, $\hat{n}_{1,2}$ is the unit normal vector. The known solution of (2) is formulated for the vector potential $\vec{A}$ ($\vec{B} = \nabla \times \vec{A}$) as follows:

$$
\nabla \times \left( \frac{1}{\mu_0 \mu(B)} \nabla \times \vec{A} \right) = \vec{j},
$$

(4)

where $\mu(B)$ is the magnetic permeability being a non-linear field function for magnetic fluids. After the magnetostatic axisymmetric problem (4), (2) is solved numerically, it is possible to calculate the force (1) and compare the result with experimental measurements.
3. Ponderomotive force: wall effects

The magnetic ponderomotive force, acting on a non-magnetic sphere immersed in a magnetic fluid container, demonstrates a complex non-monotonous behavior with two maximums [8] due to the competition of two opposite repulsion forces: the force generated by the demagnetizing field of the container on the one hand and the repulsion force between the sphere and the container wall which is the interface between magnetic and non-magnetic media on the other hand.

The first force is explained by the demagnetizing field of the container. It is well-known, that the field intensity $H$ inside a ferrofluid ellipsoidal container in the uniform magnetic field $H_0$ is uniform and equals to

$$H = \frac{H_0}{\left(1 + \kappa \chi\right)} \tag{5}$$

where $\kappa$ is the demagnetizing factor of ellipsoid. E.g., for a high cylinder (when $d$ approaches infinity) $\kappa = 0$ if $H_0$ is parallel to $z$-axis; $\kappa = 0.5$ if $H_0$ is perpendicular to $z$-axis. In case of a finite cylinder the demagnetizing factor [9] is not constant along the $z$-axis, and thus the field $H$ inside the container is non-uniform. The field gradient is directed towards the center of the container and the magnetic ponderomotive force, acting on the ferrofluid, is pointed in the same direction. Thus, the ferrofluid pulls the non-magnetic body out of the center to the bottom (or top).

The second force repulses the non-magnetic sphere from the bottom (or top) boundary of the container and can be interpreted as a repulsion force between two magnetic dipoles (the body with its mirror image [6]). This force is produced due to the demagnetizing fields of the non-magnetic body. The explanation in terms of the field line concept is the following: the magnetic field lines in the vicinity of the non-magnetic body are pushed out of it and concentrate in the outer magnetic fluid. When the body comes close to the non-magnetic wall, the field lines resemble in the ferrofluid and thus prevent themselves (and the sphere) from being pushed out into outer non-magnetic space around the container. So, this force tends to keep the non-magnetic body in the center of the container.

4. Results and discussion

The combination of two magnetic ponderomotive forces gives the resultant complex force $F(z)$ with two extrema shown in the Fig. 3 (the curves are symmetric with respect to the origin). As one can see, the force magnitude in case of a narrow container $F \approx 5.2$ mN (Fig. 3, A) is several times larger than in the case of a wide container $F \approx 1.3$ mN (Fig. 3, B), because the demagnetizing gradient fields inside the narrow container are significantly stronger. The Fig. 3 demonstrates that the geometry of the problem is
essential: it affects not only quantitative, but also the qualitative behavior of the force $F(z)$. In case of relatively small gradient demagnetizing fields (Fig. 3, B) the force may have three zero values (near the bottom, at the center and at the top of the container). This happens when the demagnetizing field of the sphere exceeds the demagnetizing field of the container. This phenomenon (switching the sign) is a top and bottom wall effect, and could be observed only at very small distances from the container’s bottom or top ($\approx 0.2$ mm in our experiments and simulations). The present work describes the detailed measurements and simulations of the ponderomotive force, acting on a non-magnetic body in the vicinity of the container’s bottom or top.

Figure 3. The magnetic ponderomotive force $F(z)$ acting on a non-magnetic sphere ($R_s = 7.5$ mm) in a narrow (A) and wide (B) containers ($R_c$ equals to 13.3 mm and 30.5 mm respectively). For convenience, only a one half of each curve is shown. The applied uniform field is $H_0 = 20$ kA/m for both plots. Points – the experiment, and solid lines – FEMM simulations according to equation (1)

We performed a series of measurements and simulations of the small (0.35 mm) area near the bottom for the case of a wide container (Fig. 3, B). The results are presented in Fig. 4. As one can see, the force $F(z)$ behavior is rather complicated because of the field dependence $F=F(z, H_0)$. In case of a linear magnetization law $M(H)$ this behavior is monotonous: the stronger applied field $H_0$ induces stronger ponderomotive force $F$ (when the sphere is in contact with the container’s bottom).

Figure 4. Non-monotonous field dependence of force $F(z, H_0)$ acting on a non-magnetic sphere in a wide ferrofluid container. The applied field $H_0$, kA/m: 7.7 (1); 10.1 (2); 12.6 (3); 15.0 (4); 17.5 (5); 20 (6)
5. Conclusion
We carried out a comprehensive experimental and numerical investigation of a magnetic ponderomotive force acting on non-magnetic bodies immersed in containers with ferrofluid. The study was performed with a suitable test-object: a non-magnetic sphere in a cylindrical container magnetized by an applied uniform magnetic field. Due to the symmetry of the test-object our numerical simulation problem was 2D axisymmetric and experimental measurements were actually 1D. The uniform applied field made it possible to observe the following interesting and fundamental effects. The ponderomotive force is non-monotinous: it has two extrema and one or three zero values depending on the geometrical parameters of the container and applied field intensity. It was shown that wall effects are crucial for this problem, because the ponderomotive force in the vicinity of the container’s bottom may change the sign (change the direction). Real magnetic fluids exhibit non-linear behavior of the ponderomotive force. This effect can be described only in the framework of the inductive approach, which takes into account all demagnetizing fields generated by the ferrofluid container. On the contrary, the simplified non-inductive approach is unable to explain this force behavior, especially the wall effects.

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