Effect of anisotropy pores on thermohaline convective instability in micropolar ferrofluid

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Abstract: This work deals with the thermohaline convective instability on horizontal micropolar ferrofluid layer in anisotropic porous effect subjected to a transverse uniform magnetic field. The linear stability analysis is used to omit the non-linear terms on governing equations, normal mode analysis is taken to the study and free boundary conditions are applied. The critical magnetic thermal Rayleigh number $N_{SC}$ is calculated analytically for sufficient large values of $M_1$. The principle of exchange of stabilities is satisfied for micropolar ferrofluid in an anisotropic porous medium without $M_1$, $N_3$, $N_5$ and $\tau$. The sufficient conditions for the non-existent of overstability are also examined. Moreover, in this work, we tried to investigate the anisotropy effect on porous medium on the system. The parameters $N_1$ and $N_5'$ dominate the system and the large porous medium is taken into account.

Keywords: Anisotropic porous medium, thermohaline convection, micropolar ferrofluid and Brinkman model.

1 Introduction

The double diffusion related to the thermohaline convective effects were first discovered an outstanding solutions of the various investigations by Stern [1], Turner [2]-[3], Huppert and Turner [4] - [5] and Turner and Stommel [6]. The phenomena on double diffusive convective instability excellently investigated by Turner [6]. Turner and Chen [7] have taken to analyse the sugar-salt system and shadowgraph photography is made to excellent work. Veronis [8] investigated the thermohaline convective instability in fluid layer. This thermohaline convective instability has been obtained by Veronis [9] in the in the Boussinesq approximation. Huppert and Moore [10] have obtained a various results of non-linearity on double diffusive convection and then Knobloch et al. [11] used a five mode truncation given by Veronis [8] to get the solution that were in excellent qualitative agreement with the numeral results of Huppert and Moore [10]. The two illustrations of 2D non-linear double diffusive convective instability has been studied by Knobloch and Proctor [12]. In this, non-linear solution could be found analytically. The unstable mode has been studied in the most cases on thermohaline convective instability subjected to linear grandients by Baines and Gill [13] and they introduced salinity Rayleigh number. Vaidyanathan et al.
[14] examined the ferrothermohaline convection. The flow of fluid phenomena with porous behavior is an important investigation because of its natural case. The stability analysis of fluid flow in a porous behavior was taken by Wooding [16] and Lapwood [15] with the Darcy resistance. Moreover, the effect of porous on double diffusive convective instability in a fluid is of interesting study in engineering sciences. Nield and Bejan [17] gave the double diffusive convective instability in porous medium. The engrossing features of the ferrofluid is the hope of influencing flow by the magnetic field and vice-versa (Feynman et al. [19], Shliomis [20]). High quality review on the convection of ferrofluid has been done by Rosensweig [21]. Finlayson [22] opened the convective instability work on ferrofluid layer with uniform vertical magnetic field. Vaidyanathan et al. [23] studied the porous effect on Finlayson [22] using Brinkman number. Then, this analysis was taken for investigate with the anisotropy effect on Brinkman pores by Sekar et al. [24].

Eringen [25] introduced the micropolar fluids theory. This theory has been developed by Eringen [26] on thermal effect. An excellent reviews and applications of this fluids theory can be obtained in by Ariman et al. [27] and Eringen [28]. Rosensweig [21] suggested the wonderful monograph on ferrofluid in this. It is apt to take the microrotation behavior on the particles. Due to this truth, the works have been assumed by treating the ferrofluid as micropolar fluids. The Rayleigh-Bénard convection problem has been analyzed by Abraham [29] on micropolar ferrofluid layer. In this, it is allowed by uniform magnetic field and stress-free boundaries is taken. Sunil and Bharti [30] has been undertaken the porous effect in micropolar ferrofluid. The thermosolutal convection has been taken by Sharma and Sharma [31] in micropolar fluids in an existence of a porous medium. Sunil et al. [32]-[38] investigated the double diffusive convective instability in micropolar ferrofluid in an existence of pores and non-pores effects.

Our plan is prolong this work to the investigation of thermohaline convective instability in Eringen’s micropolar fluid in an existence of anisotropic porous effect and uniform magnetic field taken into account. In other words, an infinite horizontal micropolar ferrofluid layer heated from below and it is salted from above existence of anisotropic porous medium. The thickness of the fluid layer is and the temperature and salinity at the bottom and top surfaces are and respectively and are maintained.

2 Mathematical formulation of problem

The continuity equation is

\[ \nabla \cdot \mathbf{q} = 0. \quad (1) \]

The momentum equation is

\[ \rho_0 \left( \frac{\partial \mathbf{q}}{\partial t} + \mathbf{q} \cdot \nabla \right) \mathbf{q} = \rho \mathbf{g} + \nabla \cdot (\mathbf{H} \mathbf{B}) - \nabla p + 2\zeta (\nabla \times \omega) + (\zeta + \eta) \nabla^2 \mathbf{q} - \frac{1}{k} (\zeta + \eta) \mathbf{q}. \quad (2) \]

The internal angular momentum equation is

\[ \rho_0 \left( \frac{\partial \mathbf{\omega}}{\partial t} + (\mathbf{q} \cdot \nabla) \right) \mathbf{\omega} = 2\zeta (\nabla \times \mathbf{q} - 2\omega) + (\lambda' + \eta') \nabla (\nabla \cdot \mathbf{\omega}) + \mu_0 (\mathbf{M} \times \mathbf{H}) + \eta' \nabla^2 \mathbf{\omega}, \quad (3) \]

The temperature equation is

\[ \left[ \rho_0 C_{v,H} - \mu_0 \mathbf{H} \cdot \left( \frac{\partial \mathbf{M}}{\partial t} \right)_{v,H} \right] \frac{\partial T}{\partial t} + \rho_s C_s \left( \frac{\partial T}{\partial t} \right)_{v,H} + \mu_0 T \left( \frac{\partial \mathbf{M}}{\partial t} \right)_{v,H} \cdot \frac{\partial \mathbf{H}}{\partial t} = K_1 \nabla^2 T + \delta (\nabla \times \mathbf{\omega}) \cdot \nabla T + \phi, \quad (4) \]
Considering the Maxwell’s equation, one can consider that the magnetization is

\[ M = \frac{H}{\mu} M(H, T, S), \]  
(5)

The linearized magnetic equation in terms of \( H_0, T_a \) and \( S_a \) is

\[ M = M_0 + (H - H_0)\chi - (T - T_a)K + (S - S_a)K_2, \]  
(6)

The density equation is

\[ \rho = \rho_0 \left[ 1 - \alpha_t(T - T_a) + \alpha_s(S - S_a) \right], \]  
(7)

The basic state quantities obtained are

\[ \mathbf{q} = \mathbf{q}_b = (0, 0, 0), \quad T_b = T_0 - \beta_t z, \quad p = p_b(z), \quad S_b = S_0 - \beta_s z, \]  
\[ \rho(z) = \rho_0 [1 + \alpha_t \beta_t z - \alpha_s \beta_s z], \quad \mathbf{H}_b(z) = \left[ H_0 - \frac{K_1 \beta_t z}{1 + \chi} + \frac{K_2 \beta_s z}{1 + \chi} \right] \hat{k}, \]  
\[ \mathbf{M}_b(z) = \left[ M_0 + \frac{K_1 \beta_t z}{1 + \chi} - \frac{K_2 \beta_s z}{1 + \chi} \right] \hat{k}, \quad M_0 + H_0 = H_0'^{ext}, \quad \omega = \omega_b = (0, 0, 0). \]  
(8)

A small thermal disturbance is made on the system. Let us take the perturbed components of \( \mathbf{M} \) and \( \mathbf{H} \) be \( [M'_1, M'_2, M_0(z) + M'_3] \) and \( [H'_1, H'_2, (H_0(z) + H'_3)] \), respectively. The perturbed state quantities are

\[ \mathbf{q} = \mathbf{q}_b + \mathbf{q}', \quad \omega = \omega_b + \omega', \quad \rho = \rho_b + \rho', \quad p = p_b(z) + p', \quad T = T_0(z) + \theta, \]  
\[ S = S_b(z) + S', \quad \mathbf{H} = \mathbf{H}_b(z) + \mathbf{H}', \quad \mathbf{M} = \mathbf{M}_b(z) + \mathbf{M}', \]  
(9)

The equation (7) can be calculated as

\[ \rho' = \rho_0 (-\alpha_t \theta + \alpha_s S'), \]  
(10)

Then the \( x, y \) and \( z \) components of Eq. (2) become

\[ \rho_0 \frac{\partial u}{\partial t} = -\frac{\partial p}{\partial x} + \mu_0 (M_0 + H_0) \frac{\partial H'_1}{\partial x} + 2 \zeta \Omega'_1 + (\zeta + \eta) \nabla^2 u - \frac{1}{k_z} (\zeta + \eta) u, \]  
(11)

\[ \rho_0 \frac{\partial v}{\partial t} = -\frac{\partial p}{\partial y} + \mu_0 (M_0 + H_0) \frac{\partial H'_2}{\partial y} + 2 \zeta \Omega'_2 + (\zeta + \eta) \nabla^2 v - \frac{1}{k_z} (\zeta + \eta) v, \]  
(12)

\[ \rho_0 \frac{\partial w}{\partial t} = -\frac{\partial p}{\partial z} + \mu_0 (M_0 + H_0) \frac{\partial H'_3}{\partial z} - \mu_0 K \beta_t H'_3 + \frac{\mu_0 K^2 \beta_t \theta}{1 + \chi} - \frac{\mu_0 K \beta_s S'}{1 + \chi} + \mu_0 K \beta_s S'_3 + \mu_0 K \beta_s S'_3 \]  
\[ + 2 \zeta \Omega'_3 + (\zeta + \eta) \nabla^2 w - \frac{1}{k_z} (\zeta + \eta) w, \]  
(13)

Eqs. (3) can be calculated as

\[ \rho_0 I \left( \frac{\partial \Omega'_3}{\partial t} \right) = -2 \zeta (\nabla^2 w + 2 \Omega'_3) + \eta' \nabla^2 \Omega'_3, \]  
(14)

\[ \rho C_1 \frac{\partial \theta}{\partial t} - \mu_0 K T_0 \frac{\partial}{\partial t} \left( \frac{\partial \phi}{\partial z} \right) \]  
\[ = K_1 \nabla^2 \theta + \left[ \rho C_2 \beta_t - \left( \frac{\mu_0 K^2 \beta_t}{1 + \chi} \right) + \left( \frac{\mu_0 K \beta_s}{1 + \chi} \right) \right] \theta - \beta_t \Omega'_3, \]  
(15)

\[ \frac{\partial S}{\partial t} + \beta_s w = K_s \nabla^2 S', \]  
(16)

3 Normal mode analysis technique

We undertake the perturbed quantities by use of normal modes are

\[ f(x, y, z, t) = f(z, t) \exp[i k_x x + i k_y y], \]  
(17)

where \( f(z, t) \) represents \( w(z, t), \theta(z, t), \phi(z, t) \) and \( S(z, t) \).
With use of the Eqs. (11)-(13), the \( z \) component of Eq. (2) can be manipulated as
\[
\rho_0 \frac{\partial}{\partial t} \left( \frac{\partial^2}{\partial z^2} - k_0^2 \right) w
= \mu_0 K \beta_t k_0^2 \frac{\partial \phi}{\partial z} - \left( \mu_0 K^2 K_2 \beta_t \right) \frac{\partial^2}{\partial z^2} \theta + \mu_0 K_2 \beta_3 k_0^2 \phi \frac{\partial \phi}{\partial z}
+ \left( \mu_0 K \beta_3 \frac{\partial^2}{\partial z^2} \right) \theta + \left( \mu_0 K^2 K_2 \beta_3 \frac{\partial^2}{\partial z^2} \right) \phi - \rho_0 g \alpha_c k_0 \theta + \rho_0 g \alpha_s k_0^2 S
+ 2 \zeta \left( \frac{\partial^2}{\partial z^2} - k_0^2 \right) \Omega^3 + (\zeta + \eta) \left( \frac{\partial^2}{\partial z^2} - k_0^2 \right)^2 w \frac{1}{k_1} (\zeta + \eta) \frac{\partial^2 w}{\partial z^2} + \frac{k_2}{k_1} \phi (\zeta + \eta) w, \tag{18}
\]
From Eq. (3) after doing mathematical calculation, one gets
\[
\rho_0 I \frac{\partial \alpha}{\partial t} = -2 \zeta \left[ \left( \frac{\partial^2}{\partial z^2} - k_0^2 \right) w + 2 \Omega^3 ' \right] + \eta \left( \frac{\partial^2}{\partial z^2} - k_0^2 \right) \Omega^3 ', \tag{19}
\]
The modified Fourier heat conduction equation is
\[
\rho C_1 \frac{\partial \theta}{\partial t} - \mu_0 K T_0 \frac{\partial}{\partial z} \left( \frac{\partial \phi}{\partial z} \right)
= K_1 \left( \frac{\partial^2}{\partial z^2} - k_0^2 \right) \theta + \left( \rho C_2 \beta_t - \left( \mu_0 K^2 T_0 \frac{\partial^2}{\partial x^2} \right) \right) \theta - \delta \beta_t \Omega^3', \tag{20}
\]
The Salinity equation is
\[
\left( \frac{\partial S}{\partial t} \right) + \beta_3 w = K_S \left( \frac{\partial^2}{\partial z^2} - k_0^2 \right) S', \tag{21}
\]
By the investigation similar to Vaidyanathan et al. [?], we gets
\[
(1 + \chi) \frac{\partial^2 \phi}{\partial z^2} - \left[ 1 + \frac{M_0}{\mu_0} \right] k_0^2 \phi - K \frac{\partial}{\partial z} + K_2 \frac{\partial S}{\partial z} = 0, \tag{22}
\]
Then, Eqs. (18)-(22) become non-dimensional equations
\[
\frac{\partial}{\partial \tau^*} (D^2 - a^2) w^*
= R^{1/2} M_1 D \phi^* - (1 + M_1) a R^{1/2} T^* + a R^{1/2} M_5 D \phi^* + a R_S^{1/2} (1 + M_K) S^*
- a R^{1/2} M_1 M_5 T^* + a R_S^{1/2} M_5 S^* + 2 N_1 (D^2 - a^2) \Omega_3^* + (\zeta + \eta) (D^2 - a^2)^2 w^*
- (1 + N_1) \left( \frac{D^2}{k_1} - \frac{a^2}{k_2} \right) w, \tag{23}
\]
\[
N_3 \frac{\partial \Omega^3}{\partial t} = -2 N_1 \left[ (D^2 - a^2) w^* + 2 \Omega^3 ' \right] + N_3 (D^2 - a^2) \Omega_3^*, \tag{24}
\]
\[
\left[ P_r \frac{\partial \phi^*}{\partial \tau^*} + \varepsilon P_r M_2 \frac{\partial}{\partial \tau^*} (D \phi^*) \right]
= (D^2 - a^2) T^* + a R^{1/2} (1 - M_2 - M_5) w^* - a R^{1/2} N_4 \Omega_3^*, \tag{25}
\]
\[
P_r \frac{\partial S^*}{\partial \tau^*} = \tau (D^2 - a^2) S^* - a R_S^{1/2} \left( \frac{M_5}{M_0} \right) w^*, \tag{26}
\]
\[
D^2 \phi^* - M_3 a^2 \phi^* - DT^* + \frac{M_3}{\tau} \left( \frac{R}{R_S} \right)^{1/2} D S^* = 0, \tag{27}
\]

4 Linear stability theory

We consider the free boundary conditions. The boundary conditions on \( w, T, S \) and \( \Omega_3 \) are
\[
w^* = D^2 w^* = T^* = D \phi^* = S^* = \Omega_3^* = 0 \text{ at } z = \pm 1/2, \tag{28}
\]
The exact solutions satisfying above Eq. (28) are
\[
w^* = A e^{\sigma t} \cos \pi z^*, \quad T^* = B e^{\sigma t} \cos \pi z^*, \quad S^* = C e^{\sigma t} \cos \pi z^*,
D \phi^* = E e^{\sigma t} \cos \pi z^*, \quad \phi^* = \frac{E}{\pi} \sin \pi z^*, \quad \Omega_3^* = F e^{\sigma t} \cos \pi z^*, \tag{29}
\]
where $A$, $B$, $C$, $E$, and $F$ are constants.

All the partial derivatives are omitted by use of Eq. (29). Then find the result of the system of homogeneous equations in Eqs. (30)-(34). Using Eq. (29) in Eqs. (23)-(27), one get

$$
\begin{align*}
-2N_1 \left[ A + \frac{\pi^2 + a^2}{N_3} \right] 2M_1(1 + M_3)E - 2N_1 \left[ \pi^2 + a^2 \right] F = 0, \\
0 = 0
\end{align*}
$$

(30)

$$
aR^{1/2} \left( 1 - M_2 - M_2 M_5 \right) A - \left( \pi^2 + a^2 + P_r \sigma \right) B + P_r \sigma M_2 E - aR^{1/2} N_3 F = 0, \quad (32)
$$

$$
aR^{1/2} \left( \pi^2 + a^2 \right) \left[ \tau \left( \pi^2 + a^2 \right) + \sigma P_r \right] C = 0, \quad (33)
$$

$$
-R^{1/2} \pi^2 B + \pi^2 M_6 \left( \frac{M_5}{M_6} \right) \left[ + R^{1/2} \left( \pi^2 + a^2 \right) M_3 \right] E = 0, \quad (34)
$$

The determinant of co-efficients of $A$, $B$, $C$, $E$ and $F$ must vanish for the existence of non-trivial Eigen functions. Eqs. (29)-(33) have been adopted to obtain

$$
U \sigma^4 + V \sigma^2 + W \sigma^2 + X \sigma + Y = 0, \quad (35)
$$

where

$$
U = u_1 P_r P_r' P_r' h, \quad V = P_r h (l' (u_1^2 P_r^2 u_2 + 1) + u_2 u_3) + u_1 u_7 P_r', \\
W = h (u_1^2 (P_r^2 u_2 + 1) + u_2 u_3) - l' (a^2 u_4 u_8 R - u_1 (u_2^2 u_2 + u_3)) P_r - 4u_1^2 P_r^2 + u_1 \tau (l' (u_1^2 P_r^2 u_2 + 1) + u_2 u_3) + u_1 u_7 P_r' + a^2 M_6 \left[ l' P_r' R \right. \\
\left. - \pi^2 u_2 P_r u_7 u_8 - \pi^2 P_r R (M_5 / M_6), \right. \\
X = h (u_1^2 u_7 (u_2^2 P_r^2 u_2 + 1) + u_2 u_3) - l' (a^2 u_4 u_8 R - u_1 (u_2^2 u_2 + u_3)) + a^2 (hu_5 P_r' R - \pi^2 u_6 \left. P_r R (M_5 / M_6), \right. \\
\left. - h u_7 (u_2^2 u_4 u_8 R - u_1 (u_2^2 u_2 + u_3)), \right. \\
Y = a^2 u_1 u_7 M_6 \left( u_5 h R_x - u_6 \pi^2 \left( M_5 / M_6, \right) - 2u_1 N_1 \\
+ 2u_1^2 \pi^2 u_2 u_7 \tau R u_7 u_8 - u_1 u_7 \pi^2 \left( u_2^2 u_4 u_8 R - u_1 (u_2^2 u_2 + u_3) \right) + a^2 u_1 u_7 M_6 (hu_5 R_x - \pi^2 u_6 R (M_5 / M_6), \right. \\
h = \pi^2 + a^2 M_3, \quad u_1 = \pi^2 + a^2, \quad u_2 = 1 + N_1, \quad u_3 = (\epsilon \pi^2 + a^2 / k_1 \epsilon, \\
u_4 = 1 + u_6, \quad u_5 = 1 - M_4 + M_4^{-1} M_5, \quad u_6 = M_1 (1 + M_3), \\
u_7 = 4N_1 + u_1 N_3', \quad u_8 = 1 - M_2 - M_2 M_5.
$$

4.1 The case of stationary convection

For steady state, we consider $\sigma = 0$ in Eq. (35) one can examine the Eigen value $R_{sc}$ and upon using $k_2 = \epsilon k_1$, the critical thermal magnetic Rayleigh number $R_{sc}$ is obtained as

$$
R_{sc} = \frac{N_r}{D r} \quad (36)
$$

where

$$
N_r = (\pi^2 + a^2)^2 \left( 4N_1 + (\pi^2 + a^2) N_3' \right) (1 + N_1) \left( \frac{\epsilon \pi^2 + a^2}{\epsilon k_1} \right) - 4 (\pi^2 + a^2) N_3', \\
D r = a^2 (1 + M_1 (1 + M_3)) (4N_1 + (\pi^2 + a^2) (N_3' - 2N_1 N_3')).
$$
To analyze analytically the effect of $k_1$, $r$, $R_s$, $M_3$, $N_1$, $N'_3$ and $N'_5$, we obtain the behavior of $\frac{dR_{sc}}{dk_1}$, $\frac{dR_{sc}}{dR_s}$, $\frac{dR_{sc}}{dN_1}$, $\frac{dR_{sc}}{dN'_3}$ and $\frac{dR_{sc}}{dN'_5}$. Eq. (36) gives

$$\frac{dR_{sc}}{dk_1} = -\frac{(\pi^2 + a^2)(4N_1 + (\pi^2 + a^2)N'_3)(1 + N_1 N'_5)(\pi^2 + a^2)}{\epsilon k_1^2 dr},$$

(37)

$$\frac{dR_{sc}}{d\epsilon} = -\frac{a^2(\pi^2 + a^2)(4N_1 + (\pi^2 + a^2)N'_3)(1 + N_1)}{\epsilon^2 dr},$$

(38)

$$\frac{dR_{sc}}{dR_s} = -\frac{a^2(4N_1 + (\pi^2 + a^2)N'_3)(1 + M_4 + M_4 M_5^{-1})}{\tau dr},$$

(39)

This gives that the porous medium, anisotropy parameter and Salinity Rayleigh number always have a destabilizing behavior, if $L_1 > L_2$ for stationary convection. In the non-presence of coupling parameter, $N_1$, the permeability of medium, anisotropic effect and stable solute gradient have a destabilizing behavior and also the same behavior is shows for the absence of spin diffusion effect ($N'_3$).

It follows from Eq. (36) that

$$\frac{dR_{sc}}{dM_3} = -4 \frac{N_r}{\tau^2} \frac{M_3}{2} a^2 \pi^2 M_1 (1 + M_5)N_1 (1 + M_5 \tau^{-1})$$

$$- \frac{N_r}{\tau^2} (\pi^2 + a^2)(N'_3 (1 + M_5 \tau^{-1}) - 2N_1 N'_5),$$

(40)

which is negative always if

$$(1 + M_5 \tau^{-1}) > \frac{2N_1 N'_5}{N'_3} \text{ and } L_3 > L_4$$

(41)

In this, $M_3$ gives destabilizing behavior when Eq. (41) satisfies. In the non-existence of $N_1$, $R_s$, $\tau$ and $M_1$ show the destabilizing behavior on the convective system.

Eq. (36) yields that

$$\frac{dR_{sc}}{dN_1} = \frac{1}{D_T^2} \left\{ D_r \left[ (\pi^2 + a^2)(8N_1 + 4(1 + (\pi^2 + a^2)N'_3)) \left( \frac{\pi^2 + a^2}{\epsilon k_1} \right) \right] 

+ N_r \left[ \frac{a^2 \pi^2 M_1 (1 + M_5)}{\tau^2 + a^2 M_3} \frac{(4(1 + M_5 \tau^{-1}) - 2N_1')}{(\pi^2 + a^2)} \right] \right\},$$

(42)

which is positive always if

$$\frac{1}{u_1} > \frac{N'_3}{u_t} > \frac{2N'_3}{u_t} > \frac{1}{u_1} > \frac{N'_3}{u_t} > \frac{2N'_3}{u_t} > u_7 (1 + M_5) \text{ and } L_1 > L_2.$$  

(43)

which implies that

$$\frac{1}{u_1} < \frac{N'_3}{u_t} < \frac{1}{u_1} < \frac{1}{u_1} > \frac{N'_3}{u_t} > \frac{2N'_3}{u_t}$$

(44)

This gives that $N_1$ has stabilizing behavior when Eq. (43) satisfies. In the absence of $N'_3$, Eq. (42) submits that $\frac{dR_{sc}}{dN_1}$ is always positive which imply that the stabilizing effect of $N_1$. Therefore, $N'_3$ has an most important role to develop the conditions for the stabilizing behavior which gives in Eq. (43).

It can be very easily done from Eq. (36) that

$$\frac{dR_{sc}}{dN'_3} = -\frac{1}{D_T^2} \left\{ D_r \left[ \frac{a^2 (\pi^2 + a^2)(1 + M_4 + M_4 M_5^{-1})}{\tau^2 + a^2 M_3} \right] R_s M_6 \tau^{-1} \right\}$$

$$+ (\pi^2 + a^2)N_r \left[ a^2 (1 + M_1 (1 + M_5)) \frac{a^2 \pi^2 M_4 (1 + M_5)}{\tau^2 + a^2 M_3} (1 + M_5 \tau^{-1}) \right],$$

(44)
which is always negative if
\[
\frac{N'_3}{2} > N'_1 N'_5 > \frac{u_2(1 + M_3 \tau^{-1})}{2u_1}, \quad L_1 > L_2 \quad \text{and} \quad L'_1 > L'_2.
\] (45)

This gives that \(N'_3\) analyzed for destabilizing behavior when Eq. (45) satisfies. In the absence of \(k_1\) and \(\varepsilon\), Eq. (44) submits that \(\frac{dR_{sc}}{dN'_5}\) is negative always which imply that the destabilizing behavior of spin diffusion. Eq. (36) also yields
\[
\frac{dR_{sc}}{dN'_5} = \left(2\alpha^2 u_1 N_2 N_5 \frac{D\tau^2}{D\tau^2} \left(u_4 - \frac{u_0}{h}\right)\right),
\] (46)
which is always positive if
\[
u_4 > \frac{u_0}{h}.
\] (47)

This presence that \(N'_5\) is analyzed for stabilizing behavior when Eq. (47) satisfies. In the absence of \(M_3\), the convective system gets stabilizing behavior and in this moment, the convection of micropolar ferromagnetic fluid is getting more. When \(M_1\) is large, one can gets \((N_{sc} = M_1 R_{sc})\) as
\[
\frac{N_{sc}}{M_1} = \frac{N_5}{D\tau^4}
\] (48)

where
\[
D\tau_1 = a^2(1 + M_3)(4N_1 + (\pi^2 + a^2)(N'_1 - 2N_1 N'_5))
\]
\[
-\frac{a^2(1 + M_3 + \pi^2)}{(2\pi + a^2)(\pi^2 + a^2)} \left((4N_1 + (\pi^2 + a^2)N'_5)(1 + M_3 \tau^{-1}) - 2N_1 N'_5(\pi^2 + a^2))\right).
\]

4.2 Principle of exchange of stabilities

We analyze the possibility of oscillatory modes, if any, on stability problem due to the presence of magnetic numbers, porous medium, micropolar parameters, anisotropy porous medium and salinity gradient. Then, equate the imaginary part of Eq. (35), we obtain
\[
\sigma_1[(VX_1 - X_3 U)\sigma'_1 + (VX_5 + X_1 X_4 - X_2 X_3)\sigma'_1 + (X_4 X_5 - X_3 X_6)] = 0,
\] (49)

where
\[
X_1 = a^2 h P_2^I u_4 u_9 + a^2 \pi^2 u_3 (1 + P'_2) u_2 + u_3 \left((1 + P'_2) u_2 + u_3\right) - h P_2 u_1 (u_2^2 u_2 + u_3) + \tau u_1 (P'_2 - u_2 u_3) + u_1 u_2 P'_2,
\]
\[
X_2 = -4 h P_1 P'_2 N'_1 u_7^2 + h P_2 u_1 (u_2^2 (1 + P'_2) u_2 + u_3) - h P_2 u_1 (u_2^2 u_2 + u_3) + \tau u_1 (u_2^2 u_2 + u_3) + u_1 u_2 P'_2,
\]
\[
X_3 = a^2 h u_4 k_1 u_1 - a^2 h u_4 u_4 u_7 P_2^I k_1 \tau - a^2 u_6 u_7 u_9 P'_2 M_6 - a u_6 u_9 l',
\]
\[
X_4 = h u_4 [u_7 (u_2^2 (1 + P'_2 u_2 + u_3) + u_1 u_2 P'_2)]
\]
\[
+ h P_2 u_3 u_2 u_2 (u_2^2 u_2 + u_3) + a^2 h u_5 u_7 M_6 R s P'_2 + a^2 h u_3 u_2 u_2,
\]
\[
X_5 = -a^2 M_5 u_6 u_7 - a^2 \pi^2 \tau l' u_1 u_6 u_9 + 2a^2 \pi^2 \tau N_1 u_2^2 u_6
\]
\[
+ 2a^2 \pi^2 N_1 N'_5 u_2^2 u_4 - a^2 \tau u_1 u_4 u_7 u_9 - a^2 \pi^2 l'M_5 u_1 u_6,
\]
\[
X_6 = 2a^2 h M_6 R s u_1 u_5 u_7 - 4\tau u_1 + \tau u_2^2 u_7 (u_2^2 u_2 + u_3),
\]

It is very clear from Eq. (49) that \(\sigma_1\) almost either non-zero or zero which means that the modes almost either oscillatory or non-oscillatory instabilities. In the non-presence of \(N_1\), \(N'_3\), \(N'_5\), \(P_1\), \(P'_2\) and \(\tau\), we obtain the result as
\[
\sigma_1^2 ((a l'u_1 u_2)^2) = 0,
\] (50)

In Eq. (50), \((a l'u_1 u_2)^2\) is positive definite. Therefore, \(\sigma_1 = 0\), it gives that oscillatory modes are not permitted and the principle of exchange of stabilities is hold for micropolar ferrofluid layer heated from below and salted from above saturating an anisotropy
porous behavior, in the non-presence of $N_1$, $N_3, N_5$, $P_r$, $P_r'$ and $\tau$. Thus from Eq. (49), one can adjudge that the oscillatory modes are introduced because of the presence of $N_1$, $N_3$, $N_5$, $P_r$, $P_r'$ and $\tau$, which were non-existent in their non-presence.

4.3 The case of overstability

In this part, we examine the possibility that the observed instability may really be overstability. Since we desire to examine $R$ through the state of good oscillations, it is adequate to obtain the conditions for which Eq. (35) will allow solutions with real $\sigma_i$. Then, we equate real and imaginary part of Eq. (35) and removing $R$ between them, one gets

$$B_1C_1^3 + B_2C_1^2 + B_3C_1 + B_4 = 0,$$

(51)

where

$$B_1 = UX_1, \quad B_2 = X_1X_2 + UX_5, \quad B_3 = X_1X_6 + X_2X_5 + X_3X_4, \quad B_4 = X_5X_6.$$ 

It is understanding from Eq. (51), $B_1$ is positive if $I' = \frac{hP\rho u_a}{M_6}$.

Also, it is understanding from Eq. (51), $B_4$ positive if

$$N_1N_5 > \frac{|I'|}{2u_1}, \quad N_1N_5 > \frac{\pi^2\rho u_a}{\tau u_1}, \quad N_1N_5 > I'(u_1^2u_2 + u_3)/L_5,$$

$$N_1N_5 > \frac{\rho u_a}{L_5}, \quad I' > \frac{L_5}{\rho u_a}, \quad L_7 > L_8, \quad \tau > \frac{\rho u_a}{L_5} \quad \text{and} \quad \tau > a^2I'M_6R_s,$$

(52)

which implies that

$$N_1N_5 > \max \left\{ \frac{|I'|}{2u_1}, \frac{\pi^2\rho u_a}{\tau u_1}, \frac{\rho u_a}{L_5}, \frac{I'(u_1^2u_2 + u_3)/L_5}{} \right\},$$

$$I' > \max \left\{ \frac{L_5}{\rho u_a}, \frac{u_2^2u_3}{L_5^2} \right\}, \quad L_7 > L_8 \quad \text{and} \quad \tau > a^2I'M_6R_s,$$

(53)

The coefficients $B_2$ and $B_3$ of Eq. (51) are lengthy structure and it is not required in the examination of overstability.

5 Results and Discussion

The classical linear stability analysis is taken to analyse the thermohaline convective instability on micropolar ferrofluid with uniform angular velocity. The anisotropic effect is considered on porous medium. The thermal perturbation method and normal mode technique are used to get the solution. The stationary and oscillatory instabilities are obtained. In this investigation, we tried to analyse the effect of anisotropy porous on thermohaline convective instability in micropolar ferrofluid and Brinkman method is considered.

Before we analyze the various physical quantities, we first form some physical comments on these like $M_1$ is taken to be 1000 and $M_2$ is assumed to be zero, $M_3$ is ranges from 5 to 25 (Vaidyanathan et al. [18]). The porous medium $k_1$ is ranges from 0.1 to 0.9. The anisotropic porous medium $\varepsilon$ is assumed from 0.3 to 3.1 (Sekar et al.[24]) and $\tau$ is taken as 0.05 (0.02) 0.11 (Vaidyanathan et al.[18]). $R_s$ is ranges from -500 to 500 and $M_4$ and $M_6$ are taken to be 0.1 and $M_5 = 0.5$. Moreover, $N_1, N_3', N_5'$ are getting some physical comments due to the suspended particles. Assuming the Clausius-Duhem inequality, Eringen [34] given the non-negativeness of $N_1, N_3', N_5'$. It is clear that the couple stress comes into play at small values of $N_3'$. This supports the condition that $0 \leq N_1 \leq 1$ and that $N_3'$ is small positive real number and $N_5'$ has to be positive finite number (Sunil et al. [33]).

Fig. 1 shows the variation of $N_{SC}$ versus $N_1$ for various $k_1$ and anisotropic porous medium $\varepsilon$. This gives that $k_1$ and $\varepsilon$ have stabilizing effect. When the layer is taken to be following in an anisotropic porous medium, then an anisotropic porous effect $\varepsilon$ has a destabilizing behavior and this behavior gets for $k_1$ also. This is because, as anisotropy effect
and $k_1$ increases, the void space increases and the fluid flow gets on the plane will be increased clearly. Naturally, isotropic and anisotropic porous medium have a destabilizing behavior which was investigated by (Sekar et al. [35]).

Fig. 2 displays the variation of $N_{SC}$ versus $N_1$ for various $R_S$. It is obvious from the Fig. 2 that coupling parameter $N_1$ has a stabilizing behavior on the system for increasing of $R_S$ from $-500$ to $0$ and system gets high energy. But, an influence of $R_S$ (= 100 and 500) the system gets null effect. In other words, the convective system has an equilibrium state. When increasing value of salt on the fluid layer, the fluid is released to the lowest viscosity. Due to this, convection of the fluid is lead to fast.

Fig. 2 displays the variation of $N$ versus $N$ for various $R$. It is obvious from the Fig. 2 that coupling parameter $N$ has a stabilizing behavior on the system for increasing of $R$ from $-500$ to $0$ and system gets high energy. But, an influence of $R$ (= 100 and 500) the system gets null effect. In other words, the convective system has an equilibrium state. When increasing value of salt on the fluid layer, the fluid is released to the lowest viscosity. Due to this, convection of the fluid is lead to fast.

In Fig. 6, the variation of $N_{SC}$ versus $N_1'$ for different $M_3$ and $R_S = 500$ is investigated. This shows that decrease in $N_1'$ from $2$ to $4$, and shows a sudden dip up to $4$; it then increases, exempted to $M_3 = 5$. When $M_3 = 5$ and $N_1'$ increases from $2$ to $4$, $N_{SC}$ increases and then decreases. The cell shapes have peristaltic flow form when $M_3 = 5$ and $15$.

Figs. 7-11 give the variation of $N_{SC}$ with respect to $N_1'$, $k_1$, $\tau$ and $R$, respectively. Figs. 7 and 8 show that the heat induced into the fluid due to microelements is increased when $N_1'$ increases. Thus increasing of $N_1'$ gives to increase in $N_{SC}$. Hence, $N_1'$ has always a stabilizing flow for $M_3 = 5$ and $10$. Further, it is observed that Fig. 7 has exponential increase for $\epsilon = 0.3$. Where as Fig. 8 has exponential increase for all $\epsilon$.

Figs. 9 and 10 analyzed for $\tau$, $R_S = -500$ and $R_S = 500$, respectively. The nature of the stabilizing behavior is made for presence and absence of salt on the system. When $R_S = -500$, $N_{SC}$ gets a highest value also the same effect is made for $R_S = 500$, but at this moment, the system has a low energy. Fig. 11 is illustrated for $M_3 = 20$ for various $R_S$. In this situation, when $R_S$ is increasing from $-500$ to $0$, $N_{SC}$ is increased. Therefore, the system gets stabilizing flow. But, for $R_S = 100$ and $500$, the system has internal energy due to heavy salting on the system.

In Fig. 12, the variation of $N_{SC}$ versus $\epsilon$ is analyzed in existence and non-existence of coupling parameter $N_1$. It is clear that anisotropy effect $\epsilon$ has destabilizing behavior. This is indicated by a decrease in $N_{SC}$, which is given by Sekar et al. [35] in non-existence of $N_1$. In existence of $N_1$ (= 0.2), convective system gets high energy, but in non-existence of $N_1$ (= 0), the convective system gets low energy. However, $N_{SC}$ converges to zero when $\epsilon = 3.1$. Hence, the system has an equilibrium state.

In Fig. 13 gives the variation of $N_{SC}$ versus $M_3$ for different $\epsilon$ in existence and non-existence of $N_1$. It is seen from this figure that as $M_3$ increases from $5$ to $25$, $N_{SC}$ decreases. The system gets destabilized effect even with and without $N_1$. 

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From Fig. 14, the cell shape and $N_{SC}$ with respect to $\tau$, indicate that the system destabilizes as $\tau$ increases. This is indicated by decrease in $N_{SC}$ in existence and non-existence of $N_1$.

Fig. 1. Variation of $N_{sc}$ versus $N_1$ for various $k_1$ and $\varepsilon$, $N_3' = 0.2$, $N_3'' = 2$, $M_3=5$, $R_s=-500$, $\tau = 0.05$ and $\tau = 0.07$.

Fig. 2. Variation of $N_{sc}$ versus $N_1$ for various $R_s$, $k_1=0.1$, $\varepsilon = 0.3$, $N_3' = 0.2$, $N_3'' = 2$, $M_3=5$ and $\tau = 0.09$. 
Fig. 3. Variation of $N_{sc}$ versus $N_1$ for various $M_3$, $R_s = -500$, $k_1=0.1$, $\varepsilon = 0.3$, $N'_5 = 0.2$, $N'_3 = 2$, and $\tau = 0.11$.

Fig. 4. Variation of $N_{sc}$ versus $N_1$ for various $\varepsilon$ ($R_s = -500$) and $k_1=0.1$ ($R_s = -100$), $N'_5 = 0.2$, $N'_3 = 2$, $M_3=5$, and $\tau = 0.05$. 
Fig. 5. Variation of $N_{sc}$ versus $N'_3$ for various $\tau$, $\epsilon = 0.3$, $R_s = 100$, $k_1=0.1$, $N'_5 = 0.2$, $N_1 =0.2$ and $M_3=5$.

Fig. 6. Variation of $N_{sc}$ versus $N'_3$ for various $M_3$, $\epsilon = 0.3$, $R_s = 500$, $k_1=0.1$, $N'_5 = 0.2$, $N_1 =0.2$ and $\tau = 0.05$. 
Fig. 7. Variation of $N_{sc}$ versus $N_5'$ for various $\varepsilon$, $M_3$ = 5, $R_s$ = - 500, $k_1$ = 0.1, $N_3'$ = 2, $N_1$ = 0.2 and $\tau = 0.05$.

Fig. 8. Variation of $N_{sc}$ versus $N_5'$ for various $k_1$, $\varepsilon$ = 0.3, $M_3$ = 10, $R_s$ = - 500, $N_3'$ = 2, $N_1$ = 0.2 and $\tau = 0.05$. 
Fig. 9. Variation of $N_{sc}$ versus $N'_5$ for various $\tau$, $k_1=0.1$, $\varepsilon=0.3$, $M_3=15$, $R_s = -500$, $N'_3 = 2$, and $N_1=0.2$.

Fig. 10. Variation of $N_{sc}$ versus $N'_5$ for various $\tau$, $k_1=0.1$, $\varepsilon=0.3$, $M_3=15$, $R_s = 500$, $N'_3 = 2$, and $N_1=0.2$. 
Fig. 11. Variation of $N_{sc}$ versus $N_3'$ for various $R_s$, $\tau = 0.05$, $k_1=0.1$, $\varepsilon=0.3$, $M_3 = 15$, $N_3' = 2$, and $N_1 = 0.2$.

Fig. 12. Variation of $N_{sc}$ versus $\varepsilon$ for various $k_1$ ($N_i=0$ and 0.2), $R_s = -500$, $N_3' = 0.2$, $\tau = 0.05$, $M_3 = 5$ and $N_3' = 2$. 
Fig. 13. Variation of $N_{sc}$ versus $M_3$ for various $\varepsilon$ ($N_1=0$ and 0.2), $k_1=0.1$, $R_s=-500$, $N'_5=0.2$, $\tau=0.05$, $M_3=5$ and $N'_3=2$.

Fig. 14. Variation of $N_{sc}$ versus $\tau$ for various $\varepsilon$ ($N_1=0$ and 0.2), $M_3=5$, $k_1=0.1$, $R_s=-500$, $N'_5=0.2$ and $N'_3=2$.

6 Conclusion

The results of thermohaline convective instability in a micropolar ferrofluid in an anisotropic porous medium is considered. These free-free boundary conditions are most significant because we can calculate an exact solution. The critical thermal magnetic Rayleigh number for the onset of instability is depicted graphically for sufficient large values of $M_1$. The destabilizing behavior of an anisotropic porous medium, permeability of medium,
salinity Rayleigh number, non-buoyancy magnetization and ratio of mass transport to heat transport is analyzed in the presence and absence of coupling parameter $N_1$. This is shown in Figs. 12-14 and also these figures showed that the system leads to high energy when $N_1 = 0.2$. The parameters $N_1$ and $N'_3$ get stabilizing effect, which is dominating the effect of the permeability of the porous medium, anisotropic porous medium and non-buoyancy magnetization parameter which are depicted in Figs. 1-3 and Figs. 7-10. The parameter $N'_3$ is analysed for destabilizing effect which is depicted in Figs. 4-6.

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