Analyses of pp, Cu–Cu, Au–Au and Pb–Pb Collisions by Tsallis-Pareto Type Function at RHIC and LHC Energies

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Abstract: The parameters revealing the collective behavior of hadronic matter extracted from the transverse momentum spectra of π+, π−, K+, K−, p, p0, N, Λ, Σ or Ξ, Σ+ or Ω or Ω+ or Ω− produced in the most central and most peripheral gold–gold (Au–Au), copper–copper (Cu–Cu) and lead–lead (Pb–Pb) collisions at 62.4 GeV, 200 GeV and 2760 GeV, respectively, are reported. In addition to studying the nucleus–nucleus (AA) collisions, we analyzed the particles mentioned above produced in pp collisions at the same center of mass energies (62.4 GeV, 200 GeV and 2760 GeV) to compare with the most peripheral AA collisions. We used the Tsallis–Pareto type function to extract the effective temperature from the transverse momentum spectra of the particles. The effective temperature is slightly larger in a central collision than in a peripheral collision and is mass-dependent. The mean transverse momentum and the multiplicity parameter (N0) are extracted and have the same result as the effective temperature. All three extracted parameters in pp collisions are closer to the peripheral AA collisions at the same center of mass energy, revealing that the extracted parameters have the same thermodynamic nature. Furthermore, we report that the mean transverse momentum in the Pb–Pb collision is larger than that of the Au–Au and Cu–Cu collisions. At the same time, the latter two are nearly equal, which shows their comparatively strong dependence on energy and weak dependence on the size of the system. The multiplicity parameter, N0 in central AA, depends on the interacting system’s size and is larger for the bigger system.

Keywords: identified; strange; effective temperature; mass-dependent; transverse momentum spectra; mean transverse momentum

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1. Introduction

The study of identified and strange particles in high-energy collisions is fundamental. The former allows the disquisition of the particle production mechanisms in a superhot and dense nuclear matter and explores the features of the quark–gluon plasma (QGP). On the other hand, the strange particles are a magnificent probe to identify the phase boundary of the onset of the deconfinement. The transverse momentum (pT) spectra of identified particles are one of the pillars of significant discoveries in high-energy physics [1–4]. According to [5,6], the shape of the spectra is sensitive to the dynamic of nucleus–nucleus collisions and may be used to get the radial flow and the temperature at freeze-out. Additionally, the hadrons with strange content are argued to have smaller hadronic interaction cross-sections and may decouple earlier from the system compared to nonstrange particles [1,5,6]. In this way, the hadrons with strange content would carry direct information from the collisions at
an earlier stage without dilution due to hadronic scattering at a late stage. The particles with a lower cross-section interaction are supposed to freeze out early [7,8]. However, other studies [9,10] claim that the decoupling of the particles depends on their mass such that massive particles decouple early from the system. In other works [11-16], the decoupling scenarios of different particles are different, including the single decoupling scenario in which all particles are decoupled at the same time, the double-decoupling scenario in which strange and nonstrange particles are decoupled separately, and each particle’s multiple decoupling scenarios for decoupling from the system, respectively. This is an open question up to now in the community. There are two types of freeze-out/decoupling after the fireball expansion. The system cools down as it expands, and the quarks and gluons become reconfined and hadronized. Two other transitions, chemical freeze-out and kinetic freeze-out, happen along the way. The former occurrence is very close to the phase transitions line. It is marked by the system’s temperature becoming low enough for inelastic interactions between the particles to stop. The net yield of all the particles gets fixed at this point. For some time, the particles still experience the elastic collision, and this elastic collision stops when the system expands enough, at the kinetic freeze-out. The particles’ interactions end at this stage and their transverse momentum spectra ($p_T$) get fixed. It should be noted that the freeze-out scenarios discussed above refer to kinetic decoupling, and we keep the focus of the present work on kinetic decoupling because we are studying the final state particles.

Indeed, the system evolution undergoes several stages, corresponding to different temperatures, as seen from the above discussion. The initial collision is the first stage of the system evolution, which corresponds to the initial temperature and describes the system’s characteristics at the initial stage. There is also another kind of temperature called the effective temperature, and it occurs just before the kinetic freeze-out temperature, which includes the flow effect. The details of these temperatures can be found in [17-21]. The present work is focused on the effective temperature, and we shall extract it from the transverse momentum spectra of the particles in different collisions.

The particles’ transverse momentum ($p_T$) spectra are essential because they give the particulars about [22] the transverse excitation degree and dynamic expansion of the collision system. This paper studied the identified and strange particles in $Au-Au$, $Cu-Cu$ and $Pb-Pb$ collisions at 62.4, 400 and 2760 GeV. We also analyzed the identical particles in $pp$ collisions at the exact center of mass energy to compare the results of $AA$ collisions with $pp$ collisions.

The remainder of the paper consists of the method and formalism in Section 2, followed by the results and discussion in Section 3. In Section 4, we summarize our main observations and conclusions.

2. The Method and Formalism

It is believed that a few emission sources are formed in high-energy collisions according to the multithermal source model [23-26]. For nuclear fragments and for the other produced particles (such as identified, strange and charmed particles) from the target and projectile in nucleus–nucleus collisions, the sources for the former may be nucleon or nucleon clusters. In contrast, the seeds for the latter may be the participant quarks or gluons, although the contributors $c + \bar{c}$ can be from the gluon fusion. Different statistics such as Fermi–Dirac, Bose–Einstein, Boltzmann–Gibbs and Tsallis statistics can describe the properties of sources. The above statistics have relations with each other because they may result in similar or different distributions while describing the spectra of the produced hadrons.

The Boltzmann–Gibbs statistic describes the transverse momentum spectra of the particles in a narrow $p_t$ range, while the Tsallis statistic describes a wider $p_T$ range, although it is derived from the former [27-29]. In fact, the Boltzmann–Gibbs statistic is a special case of the Tsallis distribution in which entropy $q = 1$. For the parameterization of the final state hadrons, the Tsallis distribution is widely used in high-energy collisions from
lower to higher energies (such as from a few GeV to 13 TeV). The form of the Tsallis distribution [30–36] is expressed as

\[ \frac{E d^3N}{d^3p} = \frac{1}{2\pi p_T dp_T dy} \frac{dN}{dy} \frac{(n-1)(n-2)}{2\pi n T [nT + m_0(n-2)]} \times (1 + \frac{m_T - m_0}{n T})^{-n} \]  

(1)

where \( E \) denotes the energy, and \( p, N \) and \( y \) are the momentum, number of particles and rapidity, respectively. \( m_T \) is the transverse mass and can be represented as \( m_T = \sqrt{p_T^2 + m_0^2} \) [37–42], and \( m_0 \) is the rest mass of the particle. \( T \) is the effective temperature, and \( n \) is the power index, particularly \( n = 1/(q - 1) \), where \( q \) describes the degree of equilibrium. The emission source is more equilibrated if \( q(n) \) is closer to 1 (it has a larger value).

Nonextensive thermodynamics is a new method for studying the heavy-ion collisions at relativistic energy. The Tsallis–Pareto function [42–47] can be used for fitting transverse momentum \( (p_T) \) spectra in low as well as in intermediate regions, especially in the hadronization process, and demonstrates an impressive relation among hadrons. The \( p_T \) distribution of the Tsallis–Pareto function can be expressed as

\[ f_1(p_T) = \frac{1}{N} \frac{dN}{dp_T} = A \frac{(n-1)(n-2)}{nT} \times (1 + \frac{m_T - m_0}{n T})^{-n} \]  

(2)

The present work is a continuation of our work published in [16,48–56] using different statistical fit functions to extract parameters relevant to the collective properties of the hadronic medium.

3. Results and Discussion

The transverse momentum spectra \( (p_T) \) of \( \pi^+, \pi^-, K^+, K^-, p, \bar{p}, K_0^0, \Lambda, \Xi \) or \( \bar{\Xi}^+ \) and \( \Omega \) or \( \bar{\Omega}^+ \) or \( \Omega + \bar{\Omega} \) produced in the most central and peripheral nucleus–nucleus collisions are displayed in Figure 1. Panels (a) and (b) show the \( p_T \) spectra of the nonstrange and strange particles in \( Au – Au \) collisions at \( \sqrt{s_{NN}} = 62.4 \text{ GeV} \), while panels (c) and (d) show the \( m_T \) spectra of these mentioned particles in \( Cu – Cu \) collisions at \( \sqrt{s_{NN}} = 200 \text{ GeV} \). Panels (e) and (f) represent the \( p_T \) spectra of nonstrange and strange particles in \( Pb–Pb \) collisions at \( \sqrt{s_{NN}} = 2760 \text{ GeV} \). The rapidity for \( \pi^+, \pi^-, K^+, K^-, p \) and \( \bar{p} \) in panels (a) and (b) is \( |y| < 0.1 \). Similarly, for \( K_0^0, \Lambda, \bar{\Lambda}, \bar{\Xi}^+, \bar{\Xi}^- \) and \( \Omega, |y| < 0.1 \). Similarly, the rapidity for all the particles, as mentioned earlier in panels (c)-(f) is \( |y| < 0.5 \). The symbols are used to display the experimental data from the BRAHMS Collaboration [57], STAR Collaboration [29,58,59] and ALICE Collaboration [60,61], and the curves over the data are our fit results by using the Tsallis–Pareto type function. It can be seen that Equation (2) fits the data approximately well. Different symbols represent different particles. The filled and open symbols show the positive and negative charged particles.

Figure 2 is similar to Figure 1, but it shows the transverse momentum spectra of the particles in \( p–p \) collisions at 62.4, 200 and 2760 GeV in panels (a)-(c), respectively. Panel (a) presents the transverse momentum spectra of \( \pi^+, \pi^-, K^+, K^-, p \) and \( \bar{p} \), while panel (b) presents the transverse momentum spectra of \( \pi^+, \pi^-, K^+, K^-, p, \bar{p}, K_0^0, \Lambda, \bar{\Lambda}, \bar{\Xi}, \Xi \) and \( \Omega + \bar{\Omega} \), and panel (c) displays the transverse momentum spectra of \( \pi^+, \pi^-, K^+, K^-, p \) and \( \bar{p} \). Different symbols represent different particles. The symbols are used to display the experimental data from the PHENIX Collaboration [37], STAR Collaboration [29,38,62] and CMS Collaboration [42], and the curves over the data are our fit results by Equation (2). The filled and open symbols show the positive and negative charged particles, respectively. The related extracted parameters, along with \( \chi^2 \) and degree of freedom (DOF) are listed in Table 1. One can see that Equation (2) fits the data well in \( pp \) collisions at 62.4, 200 and 2760 GeV at the RHIC and LHC.
Figure 1. Transverse momentum spectra of nonstrange and strange particles produced in Au–Au, Cu–Cu and Pb–Pb collisions at 62.4 GeV in panels (a,b), at 200 GeV in panels (c,d) and at 2760 GeV in panels (e,f), respectively. The symbols represent the experimental data measured by the BRAHMS Collaboration [57], STAR Collaboration [29,58,59] and ALICE Collaboration [60,61]. The lines are the fits from Equation (2).
Figure 2. Transverse momentum spectra of identified and strange particles in \( pp \) collisions at (a) 62.4, (b) 200 and (c) 2760 GeV. The symbols represent the experimental data measured by the PHENIX Collaboration [37], STAR Collaboration [29,38,62] and CMS Collaboration [42]. The lines are the fits from Equation (2).

Figure 3 shows the result of the dependence of \( T \) on \( m_0 \) and centrality. Different symbols are used to represent different collisions. Filled and empty symbols show the central and peripheral collisions, respectively, and the blue-colored star symbols denote the \( pp \) collisions. The symbols from left to right show the mass dependence of the parameters. One can see that \( T \) is slightly larger in the central collisions compared to the peripheral collisions because there is a large number of participants in the former, which makes the reaction very intense, and thus more energy is stored in the former. These results validate our recent results [16,18,48,49]. In addition, \( T \) in \( pp \) collisions are also shown, which is slightly lower than or approximately equal to that in peripheral \( AA \) collisions at the same center of mass energy. We also note that \( T \) in \( AA \) and \( pp \) collisions increases with increasing \( m_0 \), which shows a differential freeze-out scenario that validates our previous results [17,20,53,54,56]. \( T \) is the temperature which includes the contribution of both the kinetic freeze-out temperature and radial flow; therefore, the freeze-out refers to the kinetic freeze-out. Different \( T \)s for different particles indicate that the scenario of the decoupling of the particles is a multiple-kinetic-decoupling scenario. In the present work, different collision systems with different center of mass energies were considered to check the system size and energy dependence of \( T \). Still, we did not report any specific dependence of \( T \) on either.
Figure 3. Dependence of the effective temperature on centrality and mass of the particles in nucleus–nucleus and pp collisions at (a) 62.4 GeV (b) 200 GeV (c) 2760 GeV and (d) (62.4, 200, and 2760) GeV energies.

Figure 4 is similar to Figure 3, but it displays the dependence of the mean transverse momentum \(<p_T>\) on \(m_0\) and centrality. We note that \(<p_T>\) is slightly larger in a central collision than in peripheral and pp collisions. This is because more energy is transported in the system in central collisions than in the latter two. \(<p_T>\) in pp collisions is close to that in peripheral AA collisions at the same center of mass energy. \(<p_T>\) also depends on \(m_0\). The heavier the particle, the larger the \(<p_T>\). We can see that \(<p_T>\) is larger in \(Pb-Pb\) collisions than in \(Au-Au\) and \(Cu-Cu\) collisions, which shows that \(<p_T>\) depends on the size of the system, but this dependence is weak because the values of \(T\) in the \(Au-Au\) and \(Cu-Cu\) collisions are approximately close to each other due to the different collision energies of the two systems. The \(Au-Au\) system is approximately three times larger than the \(Cu-Cu\) system, but its collision energy is approximately three times lower than that of \(Cu-Cu\) collisions, and this may increase the energy dependence of \(<p_T>\), which becomes more prominent in pp collisions because we can see that \(<p_T>\) is larger at 2760 GeV than at 200 GeV, and \(<p_T>\) at the latter is larger than at 62.4 GeV.
Figure 4. Dependence of the mean transverse momentum on centrality and mass of the particles in nucleus–nucleus and \( pp \) collisions at (a) 62.4 GeV (b) 200 GeV (c) 2760 GeV and (d) (62.4, 200, and 2760) GeV.

Figure 5 is similar to Figure 3, but it shows the dependence of \( N_0 \) on \( m_0 \) and centrality. \( N_0 \) is the multiplicity parameter, not only the normalization constant. It can be seen that \( N_0 \) is slightly larger in central AA collisions than in peripheral AA collisions as well as \( pp \) collisions, since the central collision systems are larger and more violent than the latter two collisions, which results in an enormous multiplicity. In most cases, it is also observed that \( N_0 \) in \( pp \) collisions is close to the peripheral AA collisions at the exact center of mass energy. \( N_0 \) is reported to be mass-dependent. The heavier the particles, the smaller the multiplicity. However, \( N_0 \)'s dependence on the size of the system in central AA collisions can be seen. The larger the size of the system is, the larger the \( N_0 \).

Figure 6 is similar to Figure 5, but it represents the result for the entropy parameter \( n \). As discussed in the second section, \( n \) measures the degree of equilibrium of the system. The larger the value of \( n \), the closer the system will be to an equilibrium state. Figure 6 highlights that \( n \) is higher in most cases in central collisions than in peripheral collisions and \( pp \) collisions, which means that the central collision system equilibrates quickly.
Figure 5. Dependence of the mean $N_0$ on centrality and mass of the particles in nucleus–nucleus and $pp$ collisions at (a) 62.4 GeV (b) 200 GeV (c) 2760 GeV.

Figure 6. Dependence of the mean $n$ on centrality and mass of the particles in nucleus–nucleus and $pp$ collisions at (a) 62.4 GeV (b) 200 GeV (c) 2760 GeV.
Before going to the conclusion section, we would like to point out that the central collision has a more significant $T$ and $n$. The central collision system approaches the equilibrium state quickly compared to the peripheral and $pp$ collisions. However, in peripheral collisions, the system has a lower $T$ and $n$, away from the equilibrium state.

Table 1. Values of free parameters $T$ and $n$, normalization constant ($N_0$), mean transverse momentum ($<p_T>$), $\chi^2$ and degree of freedom (dof) corresponding to the curves in Figures 1 and 2.

| Figure 1a | Collab. | Centrality | Particle | Factor | $T$ (GeV) | $n$ | $<p_T>$ | $N_0$ | $\chi^2$ | dof |
|-----------|---------|------------|----------|--------|----------|----|---------|-------|--------|-----|
| Figure 1a | STAR    | 62.4       | $\pi^+$  | 0.0000005 | 0.183 ± 0.004 | 36 ± 2 | 0.420 ± 0.013 | 21,980 ± 14 | 26 ± 7 |
|           |         |            | $\pi^-$  | 0.0000001 | 0.185 ± 0.003 | 36 ± 1 | 0.424 ± 0.013 | 21,980 ± 22 | 25 ± 7 |
|           |         |            | $K^+$    | 0.001    | 0.275 ± 0.006 | 36 ± 2 | 0.708 ± 0.021 | 4100 ± 13 | 1 ± 7  |
|           |         |            | $K^-$    | 0.0005   | 0.271 ± 0.003 | 36 ± 1 | 0.700 ± 0.021 | 3410 ± 32 | 12 ± 7 |
|           |         |            | $\rho$   | 0.0005   | 0.501 ± 0.008 | 30 ± 0.5 | 1.316 ± 0.039 | 1790 ± 14 | 165 ± 13 |
|           |         |            | $\Lambda$| 0.05     | 0.277 ± 0.006 | 64 ± 2 | 0.863 ± 0.026 | 1996 ± 14 | 103 ± 9 |
|           |         |            | $\Sigma^-$| 5.000    | 0.286 ± 0.006 | 70 ± 4 | 0.931 ± 0.028 | 1009 ± 13 | 84 ± 9  |
|           |         |            | $\Sigma^+$| 2.000    | 0.290 ± 0.003 | 70 ± 0.1 | 0.940 ± 0.028 | 124 ± 3 | 39 ± 8 |
|           |         |            | $\Omega^0$| 0.005    | 0.273 ± 0.006 | 50 ± 2 | 0.693 ± 0.021 | 2800 ± 12 | 154 ± 12 |
| 0–20%     | STAR    | 500        | $\Omega^+$| 200.000  | 0.299 ± 0.006 | 57 ± 2 | 0.104 ± 0.031 | 25 ± 0.3 | 1 ± 2  |
|           |         |            |          |         |          |       |         |       |        |     |
| 40–60%    | STAR    | 500        | $\Omega^+$| 200.000  | 0.299 ± 0.006 | 57 ± 2 | 0.104 ± 0.031 | 25 ± 0.3 | 1 ± 2  |
|           |         |            |          |         |          |       |         |       |        |     |

| Figure 1c | BRAHMS | 0–10%      | $\pi^+$  | 0.01     | 0.211 ± 0.005 | 50 ± 2 | 0.469 ± 0.014 | 2370 ± 21.00 | 34          | 11  |
|           | BRAHMS | 200        | $\pi^-$  | 0.001    | 0.213 ± 0.005 | 51 ± 3 | 0.473 ± 0.014 | 2380 ± 18.00 | 34          | 11  |
|           |         |            | $K^+$    | 0.05     | 0.200 ± 0.004 | 37 ± 2 | 0.552 ± 0.017 | 697 ± 5.0   | 32          | 7   |
|           |         |            | $K^-$    | 0.001    | 0.200 ± 0.007 | 37 ± 2 | 0.552 ± 0.017 | 89 ± 6.0    | 10          | 7   |
|           |         |            | $\rho$   | 0.0005   | 0.211 ± 0.005 | 60 ± 3 | 0.681 ± 0.020 | 45 ± 2.0    | 14          | 13  |
|           |         |            | $\Lambda$| 0.2      | 0.255 ± 0.006 | 69 ± 4 | 0.815 ± 0.024 | 60 ± 5.0    | 27          | 8   |
|           |         |            | $\Lambda$| 0.1      | 0.242 ± 0.004 | 74 ± 3 | 0.787 ± 0.024 | 40 ± 2.0    | 20          | 7   |
|           |         |            | $\Sigma^-$| 10.00    | 0.267 ± 0.005 | 43 ± 1 | 0.902 ± 0.027 | 3 ± 0.0     | 7       | 6   |
|           |         |            | $\Omega^+$| 500.000  | 0.280 ± 0.006 | 40 ± 3 | 1.014 ± 0.030 | 2 ± 0.0     | 7       | 1   |
|           |         |            |          |         |          |       |         |       |        |     |
|           |         |            | $\Omega^+$| 100.000  | 0.280 ± 0.004 | 36 ± 1 | 1.018 ± 0.031 | 2 ± 0.0     | 10       | 1   |

| Figure 1d | BRAHMS | 50–70%     | $\pi^+$  | 0.01     | 0.187 ± 0.003 | 26 ± 0 | 0.440 ± 0.013 | 299 ± 7.0   | 24          | 11  |
|           | BRAHMS | 50–70%     | $\pi^-$  | 0.001    | 0.187 ± 0.004 | 28 ± 0 | 0.437 ± 0.013 | 320 ± 8.0   | 11          | 11  |
|           |         |            | $K^+$    | 0.01     | 0.207 ± 0.005 | 39 ± 1 | 0.565 ± 0.017 | 45 ± 2.0    | 14          | 7   |
|           |         |            | $K^-$    | 0.001    | 0.207 ± 0.004 | 39 ± 2 | 0.565 ± 0.017 | 34 ± 1.0    | 7          | 8   |
|           |         |            | $\rho$   | 0.5      | 0.220 ± 0.005 | 47 ± 2 | 0.705 ± 0.021 | 80 ± 6.0    | 28          | 11  |
|           |         |            | $\omega$ | 0.5      | 0.206 ± 0.005 | 47 ± 2 | 0.674 ± 0.020 | 17 ± 0.0    | 19          | 8   |
|           |         |            | $\omega$ | 0.5      | 0.217 ± 0.004 | 13 ± 0 | 0.652 ± 0.020 | 199 ± 6.0   | 40          | 15  |
|           |         |            | $\omega$ | 10.00    | 0.275 ± 0.007 | 25 ± 1 | 0.892 ± 0.027 | 82 ± 8.0    | 52          | 16  |
|           |         |            |          |         |          |       |         |       |        |     |
| 40–60%    | BRAHMS | 50,000,000 | $\Omega^+$| 50,000,000 | 0.328 ± 0.005 | 33 ± 1 | 1.131 ± 0.034 | 2 ± 0.0     | 15          | 2   |
|           |         |            |          |         |          |       |         |       |        |     |

Table 1. Values of free parameters $T$ and $n$, normalization constant ($N_0$), mean transverse momentum ($<p_T>$), $\chi^2$ and degree of freedom (dof) corresponding to the curves in Figures 1 and 2.
4. Conclusions

The main observations and conclusions are summarized here.

(a) The transverse momentum spectra of identified and strange particles were analyzed in Au-Au, Cu-Cu and Pb-Pb collisions at 62.4 GeV, 200 GeV and 2760 GeV, respectively, by the Tsallis–Pareto type function, and the effective temperature and mean transverse momentum were extracted. We also analyzed the pp collisions at 62.4 GeV, 200 GeV and 2760 GeV to check the nature of the extracted parameters in the peripheral AA collisions and pp collisions at the exact center of mass energy.

(b) The effective temperature (T) was more prominent in a central collision than in a peripheral collision because many hadrons were involved in the reaction, which transferred more energy in the central collision systems. T in peripheral collisions was closer to that of pp collisions at the exact center of mass energy, which showed that the two systems had similar thermodynamic properties.
(c) The mean transverse momentum was more significant in central collisions than in peripheral collisions due to substantial momentum transfer. In peripheral collisions, it was close to that of the \( pp \) collisions.

(d) Both the effective temperature and mean transverse momentum were mass-dependent and increased with mass. The increase of \( T \) with \( m_0 \) was consistent with the multiple kinetic freeze-out scenarios.

(e) \( <p_T> \) was larger in \( Pb-Pb \) collisions than in \( Au-Au \) and \( Cu-Cu \) collisions, and in the latter two cases, the values were close to each other, which showed a weak dependence on the size of the system and comparatively strong dependence on the collision’s energy because it increased with the increase of energy in \( pp \) collisions.

(f) The multiplicity parameter \( N_0 \) was slightly larger in central \( AA \) collisions than in peripheral \( AA \) collisions. In peripheral collisions, it was close to that in \( pp \) collisions at the exact center of mass energy. In addition, \( N_0 \) was mass-dependent and was higher for lighter particles. \( N_0 \) in central \( AA \) collisions depended on the size of the interacting system; larger sizes of the interacting system yielded higher values of the \( N_0 \).

(g) The entropy parameter \( n \) was larger in a central collision, rendering the system to an equilibrium state more quickly compared to the peripheral collisions.

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