Fano resonances can provide two criteria to distinguish Majorana bound states from other candidates in experiments

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There are still debates on whether the observed zero energy peak in the experiment by Stevan et al. [Science 346, 602(2014)] reveals the existence of the long pursuing Majorana bound states (MBS). We propose that, by mounting two scanning tunneling microscopic tips on top of the topological superconducting chain and measure the transmission spectrum between these two metallic tips, there are two kinds of characteristics on the spectrum that are caused by MBS uniquely. One is symmetric peaks with respect to zero energy and the other is 4π period caused by a nearby Josephson junction. The former refers to the fact that MBS are composited by Majorana fermions which distributed in the particle and hole subspaces equally. The latter is based on the well known 4π period of Josephson effect in topological superconductor. We think such two characteristics can be used as criteria to distinguish MBS from other candidates, such as impurities, Kondo effect and traditional Andreev bound states.

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I. INTRODUCTION

Fano resonances (FR)1 are a universal interference phenomenon when waves coherently transfer through systems with two kinds of paths. One is a media with continuous energy spectrum and the other is scattering centers with discrete eigen-energies. When the energy of an incoming wave is near those of discrete lev-

ers, the total transmission fluctuates rapidly and forms continuous energy spectrum and the other is scatter-
dering centers with discrete eigen-energies. When the en-

ergy of an incoming wave is near those of discrete lev-

ers, the total transmission fluctuates rapidly and forms an asymmetric Fano line shape because the inter-

erence causes both resonant and anti-resonant tunnelings2. FR and their line shapes are good experimental di-

agnosis on finding the nature of transport paths2. The applications include: tunneling measurement of Kondo scattering on single magnetic atom attached on gold surface3, Raman spectrum of heavily doped semiconductors4,5 and high-Tc superconductors6, electron transport of quantum dots embedded in Aharonov-Bohm ring7,8 and carbon nanotubes9,10, conductance of quasi-one-dimensional quantum wires with donor impurities or quantum dots11,12, and so on. External parameters can also modify FR. For instance, the external bias can change the asymmetric absorption spectrum of the Wannier-Stark transitions in semiconductor superlattices13,14. Besides these, dynamical FR takes place when the Rabi oscillations and coherent phonons interfere, in which the Rabi oscillations with broadened linewidth play the role of continuous spectrum15. Because FR provides an accurate method to measure the linewidth of discrete levels, it was also proposed to link FR with the measurement of phase decoherence in mesoscopic system16.

In this paper, we propose to setup a feasible ex-

periment and use FR to answer the debated question on whether the experimentally observed zero energy modes are associated with the Majorana bound states (MBS). MBS are Majorana fermions in condensed matter physics, which are their own antiparticles17,18. Recently, these long-pursuing quasiparticles are claimed to be observed experimentally in various systems, including nanowire-superconductor junctions21,22, InSb/Nb junction23, chains of magnetic atoms on a surface of superconductor24, and vortex in Bi2Te3/NbSe2 heterostructure25. However, alternative explanations based on impurities, Kondo effect or Andreev bound states (ABS) on a ferromagnetic wire have been proposed26-31. These criticisms mostly stem from the fact that the experiments only measure the current through a scanning tunneling microscopy (STM) tip, or through the heterostructure. The observed zero bias peaks only reveal a finite local density of states at zero energy. But such peaks can not be associated with MBS uniquely.

Our proposal is based on the fact that MBS at the zero energy are the quantum states composited by half fermions. As a result, their weights in the particle and hole subspaces in the Nambu representation are balanced. But for other Bogoliubov quasi-particles, their weights in the particle subspace are |u|2 = 1(1 + ξ/E) and |v|2 = 1(1 − ξ/E) in the hole subspace, where E = √2ξ2 + Δ2 is the eigen-energy. These weights in the two subspaces are not equal in general. As we will show, FR can identify whether such balance is broken or not. By this way, it is easy to distinguish the effect of MBS from impurities or Kondo effect. But such criterion may not be sufficient to distinguish the MBS from the near-zero-energy ABS, as the latter may also have nearly balanced weights in the two subspaces. We then propose another criterion which is based on the well known 4π period of the current through a Josephson junction in the topological superconductor27,28. We will show that although finite size effect induced anti-crossing destroys
the 4π period in the energy spectrum, FR can still reveal this double period. In such a way, we may finally distinguish MBS from a traditional ABS in experiments. At the end of this paragraph, we want to emphasize that this proposal is mostly suitable to be applied in the system of magnetic atoms on superconductor and we have not posed any new experimental challenges.

II. DISTINGUISHING MBS WITH FANO RESONANCES

We propose to place two metallic STM tips on top of the left end of a topological superconducting (TS) chain possessing MBS. When the bias voltage between the STM tips is smaller than the superconducting gap of the underlying topological superconductor, FR should be observed on the transmission spectrum between the two STM tips. Here MBS are taken as the discrete levels near the zero energy and the metallic STM tips provide the necessary continuous energy spectrum. The superconductor beneath the TS chain is separated into two parts so that a Josephson junction is formed in the above TS chain. The phase of superconducting order in the two parts can be varied temporally by an external voltage.

![Diagram of proposed experimental setup](image)

FIG. 1. A sketch of the proposed experimental setup. The blue thick line is the TS chain placed on top of superconductor (SC). The two parts of superconductor are different by a phase, Φ, in the order parameter Δ. Such phase can be induced by an external voltage or a magnetic flux not shown here explicitly. An ABS whose energy is proportional to cos(Φ/2) is created in the TS chain at the Josephson junction (JJ). Two metallic STM tips are placed on top of the left end of the TS chain. In experiments, one measures the transmission spectrum between these tips.

As Fig. I shows, we consider a TS chain coupled with two STM tips. The Hamiltonian reads

\[ H = H_K + H_L + H_R + H_{hyb}, \]

where we use the spinless Kitaev model on discrete lattice to simulate the TS chain:

\[
H_K = \sum_{i=1}^{N} \mu c_i^\dagger c_i + \sum_{i=1}^{N/2} (t c_i^\dagger c_{i+1} + \Delta c_i^\dagger c_{i+1} + h.c.) + \sum_{i=N/2}^{N-1} (d_i^\dagger c_{i+1} + \Delta e^{i\Phi} c_i^\dagger c_{i+1} + h.c.). \tag{2}
\]

Here, \( N \) is the total number of sites, \( \mu \) is on-site energy, \( t \) is the strength of hopping between the nearest neighboring sites, and \( \Delta \) is the strength of the p-wave superconducting pairing. \( \Phi \) is the phase difference inherited from that of the beneath superconductors. \( c_i^\dagger \) and \( c_i \) are fermionic creation and annihilation operators.

The spinless Kitaev model is plausible to catch the major physics of a TS chain, as among the two spin degree of freedom, only one effectively contributes to the topological nontrivial bands and the other has already been gapped out by, typically spin-orbital interaction. Our setup of TS chain is similar to those proposals for the measurements of the 4π periodic Josephson current. But for magnetic atoms on top of superconductor, such longitudinal current measurement is hard to perform in experiments. We also ignore the decoration terms, such as gate or smaller hopping across the junction, as these terms will not alter the spectrum of the ABS qualitatively, that is \( E \sim \pm \cos(\Phi/2) \), at the junction.

\[
H_{L(R)} = \sum_{i=0}^{\infty} (d_i^\dagger d_{i+1} + h.c.) \quad \tag{3}
\]

are the two semi-infinite metallic chains stand for the two STM tips. \( 2t_0 \) is their band width and \( d_i^\dagger \) and \( d_i \) are the creation and annihilation operators on these tips. For the sake of clarity, we only take one channel in each chain. Introducing multi-channels in each tip will induce nothing but increase of the total conductance by a factor.

The Hamiltonian that hybridizes the TS chain and the two STM tips reads

\[
H_{hyb} = t_{int}(d_i^\dagger d_0 + h.c.) + \sum_{\alpha=L,R} \sum_{i=0}^{M} (J_{\alpha} c_i^\dagger d_{i,\alpha} + h.c.). \tag{4}
\]

Here the first term represents the hopping between the two STM tips and the second one stands for the hopping between the TS chain and the STM tips. \( M \) is the range out of which such hoppings are ignored.

In Fig. 2(a), we present the spectrum of an isolated TS chain. Such chain is within TS phase so that there is an MBS at the zero energy (although finite size effect lifts the energy a little from exact zero). Beside MBS, the fractional Josephson effect in TS chain induces an ABS whose energy is varying with \( \Phi \) as \( \cos(\Phi/2) \). This is why the ABS state at the Josephson junction is approaching...
zero energy at $\Phi = \pi$, instead of $\pi/2$ in traditional superconductors. Around zero energy, finite size effect hybridizes it with MBS and induces the small anti-crossing there.

We then mount the two metallic semi-infinite chains on top of an end of the TS chain. The electric transmission through these metallic chains is characterized by the differential conductance $G$ (in the units of $\mu$) shown in Figs. 2(b), (c) and (d). Here the parameters are taken as $t_0 = 1$, $t_{int} = 0.1$, $J_i = 0.1$ and $M = 5$. The conductance $G$ is calculated by the BTK method, which is explained in details in Appendix.

Figures 2(c) and (d) are for the cases with $\Phi = 0.5\pi$ and $\pi$, respectively. In Fig. 2(c), the broadened central peaks are caused by the resonant transmission though the MBS at the zero energy. We notice that there is a sharp dip at the zero energy that splits the center peak into two peaks. Such splitting can be understood as the finite-size caused level splitting for MBS. We also notice that the heights of the two peaks are identical and their shapes are near-symmetric with respect to the zero energy. After comparing this FR with that caused by an impurity state, we will show that such near symmetric peaks around zero energy cannot appear in the impurity case. There are also small, sharp peaks decorated on the board peak at $|E| \sim 0.13$. These small peaks are caused by the resonant tunneling through ABS at the nearby Josephson junction. One notices that the heights of these two peaks are different. This asymmetry stems from the fact that the eigenstate of ABS is asymmetric in particle and hole space. In Fig. 2(d), $\Phi = \pi$ and the anti-crossing takes place rightly. So the two broad peaks are pushed away from zero energy and the remaining sharp peak near zero energy is caused by the ABS.

In Fig. 2(b) we show how these peaks evolve with $\Phi$. Figures 2(c) and (d) are the two slides taken from it at $\Phi = \pi/2$ and $\pi$, respectively.

To explicitly show the unique characteristics of FR of MBS, we plot what kind of resonant peaks they will look like when the scattering center is made from impurities. Such system can be described by a similar Hamiltonian as Eq. 4 with $H_K$ replaced with

$$H_{imp} = \varepsilon_0 c_0^\dagger c_0 + \varepsilon_1 \cos(\Phi) c_1^\dagger c_1 + t_1 c_0^\dagger c_1 + \Delta_1 c_0^\dagger c_1. \tag{5}$$

Here we simplify the scattering center with only two effective states and ignore all other states out of the gap. $c_1$ and $c_2$ are the annihilation operators on these states respectively. State 1 is representing the impurity state at the end of the chain accidentally having energy $\varepsilon_0 \sim 0$ and state 2 stands for the possible ABS at the junction. The energy of ABS should be a periodic function of $\Phi$, which is written down as $\varepsilon_1 \sim \cos(\Phi)$. The strength of hopping between the two states is $t_1$. For the sake of clarity, we take $\Delta_1 = 0$ in Fig. 3. Such approximation runs to an extreme that in the Nambu representation, the states above the zero energy are particle-like with $u = 1$ and those states below the zero energy are hole-like with $v = 1$. We take such extreme to illustrate that the transmission in our setup is only contributed by the particle subspace in the Nambu representation.

In Fig. 3(a), we plot the energy spectrum of the isolated scattering center. Nambu representation is taken although $\Delta_1 = 0$. In Fig. 3(c) and (d) we plot the differential conductance through the metallic leads when $\Phi = 0$ and $\pi/2$ respectively. We find that only the particle-part of the impurity states contribute to the conductance, while their imaging states in the hole-subspace are invisible in the transmission spectrum. This is easy to be understood in such an extreme case. As the superconducting order $\Delta_1$ is zero, Nambu representation will introduce an artificial trivial duplication. These FR are well understood when we give up the Nambu representation and only consider the model within the particle subspace.

Such extreme model indicates that only the particle part of the impurity states will contribute to the FR. This

![Figure 2](image1.png)

![Figure 3](image2.png)
is further confirmed when we switch on $\Delta_1$. In Fig. 4 we show the differential conductance with $\Delta_1 = 0.1$. The peaks at $E > 0$ and $E < 0$ are highly asymmetric. This is because except MBS, other states in superconducting gap, are generally unbalanced in the particle and hole subspaces.

In Fig. 4, Transmission spectrum with respect to $\Phi$ and energy $E$. Other parameters are the same as those in Fig. 3 except $\Delta_1 = 0.1$.

After compare the FR of the two models, we conclude that the FR can provide two criteria for the verification of MBS in experiments. One is the symmetric broadened peaks around zero energy when $\Phi$ is around $0$. So such criterion is still when the Josephson junction is eliminated. It can exclude the possible impurity effect or the Kondo effect solidly from the candidates. But such criterion may not distinguish the MBS from the near-zero-energy ABS, because both of them have almost equal weights in the two subspaces.

In this case, we need the second criterion, which must engage a Josephson junction as we have shown in Fig. 1. A traditional ABS must evolve with $\Phi$ periodically, while it because $4\pi$ for MBS. As a result, we can exam when and how much time the FR peaks cross (or anti-cross) the zero energy as varying $\Phi$ from 0 to $2\pi$. As shown in Fig. 2(b), in topological superconductor, the cross (or anti-cross) takes place at $\Phi = \pi$ and only takes place one time in a period $\Phi = [0, 2\pi]$. While in traditional superconductor with ABS, as shown in Fig. 3(b), such cross (or anti-cross) takes place at $\Phi = \frac{\pi}{2}$ and $\frac{3\pi}{2}$, and appears twice in one period.

With these two criteria, we can undoubly exclude the impurity states, the Kondo effect and the ABS from the candidates of the possible explanations. They may help us go a little further on the route to MBS. Although the compared model shown in Fig. 4 seems tricky and crude, it catches the major physics we are facing with. The range of energy we are interested in is within the superconducting gap. This allows us simplify the scattering center with only two states. We do approximate these two states as regular fermion states, while the in-gap states should be Bogoliubov states with both components in the particle and hole subspaces. But as soon as their energies are not fixed at zero, their components in these two subspaces are unbalanced. As a result, our first criterion about the absence of the symmetry of Fano peaks with respect to zero energy is not depended on the approximation. One can further confirms that in Fig. 2(d), as the heights of the two sharp peaks caused by the ABS are not equal.

### III. CONCLUSIONS

We have proposed an experiment-friendly setup to verify the existence of MBS in experiments. Unlike the previous proposals which require a relative long chain to eliminate the finite size effect, this proposal takes benefits from the finite size effect so that only a shorter TS chain is required. By measuring the transmission spectrum through STM tips mounted on top of TS chain, one can distinguish MBS from impurities, kondo effect. One can also watch the fractional Josephson effect from the transmission spectrum after introducing a Josephson junction in the chain. Under these judgments, the experimentalists may undoubtly claim the observation of MBS in experiments.

After completing this paper, we also notice the experimental and theoretical works on the transport characteristics of a TS chain with only one STM superconducting tip on top. It seems that MBS will also induce balance peaks at the edge of tip’s gap in that case.

### Appendix A: BTK method

BTK method is used to calculate the scattering matrix through the metal-superconductor-metal interfaces. As only one channel is taken in each metallic lead, we can write down the incoming wave, the reflected wave and the transmitted wave in the Nambu representation as

$$\psi_{in} = \left( \begin{array}{c} 1 \\ 0 \end{array} \right) e^{ik_p x} \tag{A1}$$

$$\psi_{refl} = A \left( \begin{array}{c} 1 \\ 0 \end{array} \right) e^{ik_h x} + B \left( \begin{array}{c} 0 \\ 1 \end{array} \right) e^{-ik_p x} \tag{A2}$$

$$\psi_{trans} = C \left( \begin{array}{c} 0 \\ 1 \end{array} \right) e^{-ik_h x} + D \left( \begin{array}{c} 1 \\ 0 \end{array} \right) e^{ik_p x}, \tag{A3}$$

where $k_p$ and $k_h$ is the wave vector of particle and hole, with their group velocities pointing to the correct directions during this scattering process. Here $2t_0 \cos(k_p) = -2t_0 \cos(k_h) = E$. The total wave-functions are $\psi_{in} + \psi_{refl}$ in the left lead and $\psi_{trans}$ in the right lead. We then solve the Schrödinger equation to find these parameters $A$, $B$, $C$ and $D$. The transmission spectrum is calculated by

$$G = \frac{e^2}{h} (1 - |B|^2 + |A|^2). \tag{A4}$$
