A classical, elementary approach to the foundations of Quantum Mechanics

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(partially revised and enhanced)

Elementary particles are found in two different situations: (i) bound to metastable states of matter, for which angular momentum (AM) is quantized, and (ii) free, for which, due to their high energy-momentum and leaving aside inner AM or spin, the \( h \)-quantization step is completely harmless. Therefore, perhaps QM can be seen just as the simplest mathematical formalism able to describe ensembles of systems (and/or experiments on a system), where average AM (the magnitude of each of the three orthogonal projections of the vector \( L = r \wedge p \)) is “quantized”: all possible values are taken from a discrete set. Discreteness of that set is achieved from the complex structure of what we call the space of representation; to recover the usual spectrum with its equi-spacing in units of \( h \) a further restriction is in order: it is our taste to associate it to a choice of the linear momentum (LM) operator \( \hat{P} \), which is not completely determined (contrary to what it seems widespread belief) from its definition as the generator of the group of spatial translations. In our picture, “choosing the LM operator” means “defining the way in which LM information is coded into the wavefunction” (this wavefunction is for us nothing more than a representational device for position-momentum information, whose structure we must determine from the additional choices in the construction of the theory).

A complex representation space and a suitable LM operator \( \hat{P} \) are enough to determine everything else in the formalism, including time-evolution from the Schroedinger equation (for a yet undetermined energy operator \( \hat{H} \), generator of translations in time), without conditioning at any point nor the laws of forces neither the mechanics, which we can freely inherit from the usual Newton-Maxwell framework. For instance, Newtonian mechanics is recovered by setting the usual \( \hat{H} \equiv \hat{P}^2/(2m) \) which implements Newton’s \( \mathbf{F} = m\mathbf{a} \); Maxwell’s laws enter through the minimal coupling substitution and the addition of new degrees of freedom - spin - is naturally done by increasing the dimensionality of the state vector. Therefore, rather than a physical theory on its own right, QM would merely be a representational framework for classical probabilistic theories, when these are supplemented with an additional restriction: AM quantization (AMQ). As far as we see, Fritsche and Haugk’s approach must therefore contain an implicit choice, equivalent to our choice for \( \hat{P} \), which would explain the convergence of both approaches, at least in the mathematical foundations: see Sec. IV A.

In addition, switching from pure mathematics to a more “physical” perspective, our idea (i) finds support in very reasonable, completely classical physical arguments, if we place ourselves in the framework of Stochastic Electrodynamics (SED): there, all stationary periodic movement by a charged particle must satisfy a (radiated/absorbed) power balance that restricts the value of the average AM, on each of its projections; beyond the condition first given by Puthoff (regarding energy, a “ground state”) \[5\], the extension to an infinite spectrum \( l = 0, 1, 2, \ldots \) of possible values of AM can be achieved introducing more complex dynamics, its discreteness guaranteed by the need of a similar balance for each of them (an issue to be treated elsewhere); (ii) gives a natural explanation of the concept of “photon”, as a constraint on the observable spectrum of energy-momentum exchanges between metastable physical states, in particular its discreteness. QM would then represent a semi-static theory, transparent to all the (micro)-dynamics taking place between apparently “discrete” events (transitions in the state of the system). For instance, (the magnitude of the projections of) quantum (orbital) AM and spin - as a residual AM - would only reflect an average residual over a (classical) periodic trajectory, a fact that we regard as almost obvious given the particularity of the corresponding addition rules (in the case of AM, this would only apply to bound states, as said before). All this said, and again perhaps, many of the other “mysterious” features of QM should be interpreted as consequences of that AMQ, starting with the Heisenberg uncertainty relation (HUC)...

Besides, an explicit (and we believe NOW correct) connection with Fritsche and Haugk’s work \[1\] is also provided, in regard to the stochastic foundations of QM. A detailed proposal in relation to the nature of spin completes the picture, along with some other questions (the Pauli principle for instance), all fitting within the framework of SED (or, in fact, any other one alike, defined alone from classical electromagnetism plus a random background of fluctuating fields).

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Summary of important updates

1. Our idea of angular momentum (AM) as an “average over a cycle” only applies to SPIN - as a residual AM - or ORBITAL AM in bound states. We keep considering though that the discretization of all AM reflecting such an average due to emission/absorption balance is the (only) reason behind AM quantization in QM: it applies directly to spin and bound-states, while it merely imposes a “harmless” quantization of AM for particles out of those bound states.

2. On the “node problem” in QM and specifically on atomic orbitals:
   (i) So far to our knowledge, l > 0 fully relativistic H-orbitals do not present “forbidden regions” or “nodes” (they are washed out by the LS coupling term), therefore eliminating the problems with the “particle/ensemble-of-experiments” interpretation of wavefunctions; of course for the “ensemble-of-particles” one, that of course we also consider implicit in QM, such “node problem” has never been a threat.
   (ii) Besides, we believe the “node problem” that in general affects eigenstates in infinite potential wells does not reflect a real physical situation, and therefore neither is a valid argument against that same interpretation of wavefunctions; of course for the “ensemble-of-particles” one, that of course we also consider implicit in QM, such “node problem” has never been a threat.
   (iii) We are in debt to Dr. Fritsche for the following correction: (i) does not extend, at least, to l = 0. Indeed, in [1], the issue is addressed along different lines: pure states cannot be physically prepared (there is always some probability of transitions), hence we should not be worried about the discontinuity of l = 0 orbitals.
I. INTRODUCTION: MOTIVATION AND PREVIOUS REMARKS.

With recent new developments, both on theoretical \cite{2,3} and experimental grounds \cite{4}, suggesting the possibility of the quantum and classical worlds being reconciliated may be not so remote as many believe, perhaps it is time for someone to start looking for the intermediate steps, those pieces still missing in the puzzle. And where better to look than there, where we first abandon the safe pathway of classical physics and started taking shortcuts?

Frequently, Quantum Mechanics (QM) is introduced as a sophisticated, counterintuitive construction that has little to do with the picture of classical physics. It is our opinion that this is not the right attitude: neglecting the similarities between quantum and classical physics, we blur the vision of the (not so many at the fundamental level, as we will now try to prove) points where they do really depart ways. Of course, QM is correct, nobody doubts it. It is so correct that it has made predictions for many phenomena (say antiparticles from QED, for instance) much before they could even be experienced; the path in science was usually the opposite throughout history: let us invoke for instance the always disposable Newton’s apple. QM being correct, the question for us is why it works, why it has worked so well, what’s behind.

Sometimes, the straight line may not be the shortest distance between two points. In science, though, I do believe the straight line should always be the first route to explore.

More than that: in our opinion, it definitely should not be left unexplored.

Motivated by this conviction, we have tried to begin to formalize some ideas. Amongst them, some are not new and others may not be either, but at least to our knowledge they cannot be found anywhere else (at least so clearly exposed): that is the main reason why we are writing this paper. On the other hand, whether we commit some slight inaccuracies, any of what we write can actually lead to something tangible or whether someone thinks this all is a waste of time, sure the reader has by now become aware they are not a matter of our concern, at least here and now.

Before continuing, a necessary remark: we will be making some references to Stochastic Electrodynamics (henceforth SED) \cite{5–8,24} as a natural framework where to settle our ideas; nevertheless, conceptually, they can be considered as even previous to the technicalities of SED (no knowledge of the Zero-Point Field associated with SED is at all needed, the idea of our background of radiation being formulated, at least for our purposes here, as more abstract an element)

II. A SET OF CONJECTURES

As far as we know, Heisenberg first created “matrix mechanics” (later reformulated as a “wave mechanics” by Schroedinger, proving both to be equivalent) as a mere device to represent the atomic spectrum (the discrete lines of the H-atom); with energy quantization leading, for circular orbits, straight to angular momentum quantization (AMQ), and vice versa (see Sec. \ref{sec:V.B}), for us the discrete observational spectrum is simply telling us the following: the only observable states of matter imply quantized AM (or at least they can be modelled like that).

Departing from that simple idea, in this paper, we sketch a set of conjectures, in our view also a set of necessary steps in the natural development of physical theories. Most of them, even up to the relaxed standard that we are setting for ourselves here (only here), are far from sufficiently explained or developed: we hope this can be done in the future. Let us begin:

(i) In the first place, we show that the framework of real vectorial spaces is as natural a tool as any other to build a classical probabilistic theory where AM is not “quantized”: as a kinematic quantity (we prefer to use this term than that of “orbital”, which we will use in other contexts), its magnitude takes values from a continuum spectrum.

(ii) From (i), we suggest that QM is nothing but the simplest possible probabilistic theory that “quantizes” AM: the magnitude of each of the three orthogonal projections of AM can only take values from a discrete spectrum. For the three dimensions of ordinary space, assuming for convenience that we have a discrete spectrum of just \( n \) possible values for each of the space \( (X_i) \) and momentum \( (P_j) \) coordinates, we have now an \( n^3 \)-dimensional complex vectorial space, instead of the \( n^6 \)-dimensional real space that would correspond to point (i): when we assume continuous variables (with \( n \to \infty \)), the spectrum of each \( X_i \) and \( P_j \) becomes continuous, but not that of \( L_i \), each of the projections of the quantity \( L = X \wedge P \). In coherence with this, commutation relations of \( \hat{X} \) and \( \hat{P} \) is not for us the starting point, but just a necessary consequence in a theory that restricts the spectrum of each \( \hat{L}_i \) (projections of \( \hat{L} = \hat{X} \wedge \hat{P} \)).

(iii) As a difference with classical mechanics, in QM two \( kAM \) \( \hat{L}_1, \hat{L}_2 \) defined from different origins \( O_1, O_2 \) can be added without trauma (not always, but for instance when we are talking about AM in bound states, or AM in bound states plus spin): this leads us to consider that the projections of (orbital) quantum mechanical AM represent the average magnitude of (the projections of) a classical AM... on a closed, periodic trajectory.

(iv) Both points (ii) and (iii) seems rather natural from the point of view of Stochastic Electrodynamics (SED). Closed, stable orbits in SED, if they exist, do not keep an instantaneously constant value of the AM, because of the
interaction with the random background. But, if they do exist, they must obey a balance between radiated and absorbed power (an idea that was set forward by Boyer and Puthoff), that does quantize (restrict to a discrete spectrum), at least, the average (over the orbit) AM. Puthoff refined Boyer’s initial calculation, achieving a perfect match with the quantum law of AM quantization (AMQ): this condition only concerns, nevertheless, the “ground state” condition $n = 1$. The existence of an infinite spectrum of possible values of AM can be explained introducing more complicated dynamics, its discreteness guaranteed by a similar balance, an issue that we have been working on elsewhere. Besides, there has also been some more recent work concerning the grounds tate ofhydrogen: for instance see 9.

(v) Spin $\hat{S}$ and orbital AM $\hat{L}$ must in ultimate term correspond exactly to the same physical property, a proof of what is that they can so worrylessly added in the quantum mechanical formalism: $\hat{J} = \hat{L} + \hat{S}$. A possible explanation for the fact that spin, as a physical property, manifests itself through the algebra of group representations that present no (global $^{52}$) isomorphism with the group $SO(3)$ of rotations in ordinary three-dimensional space has recently been explored by us in Sec. VI D On the other hand (and also treated in Sec. VII D), the uncertainties and commutation relations associated to AM should be regarded, in our opinion, a consequence of how the process of measurement transforms the state of the particle. This of course refers to massive particles; regarding photons and polarization, the issue is perfectly explained by considering the ZPF background fluctuations (see side-notes of $^{53}$). We have, quite preliminarily, played with this idea in $^{46}$. We are also aware there are some relativistic difficulties with this picture of spin, that we do not consider definitive though $^{26}$.

(vi) QM describes ensembles: ensembles of particles, systems, experiments, ... Quantum AM stands, in our picture, for an average of measurements on the instantaneous values, an average performed somehow by the measurement process. But furthermore, quantum states are built to describe ensembles of systems: pure states describes ensembles where - for all members - position, momentum and angular momentum, all those three kinematic properties obey a certain relation: what we will call, rather than a “principle”, a “Heisenberg relation” (we will use the common term “Heisenberg uncertainty relation” $= $ HUC).

(vii) On the other hand, position and momentum entwined dispersion as random variables may be understood as simply that, an statistical shortage of QM, rather than a final property of nature in the spirit of widespread interpretations of the HUC. As the (unavoidable) price of quantizing AM, appears the fact that QM can only describe ensembles where those two basic properties obey some minimum mutual dispersion.

(viii) The need for mixed states is not there unless we want that HUC to hold also (individually) for certain subensembles within the total ensemble described by the quantum state, in this case given necessarily by a density matrix. I.e, the balance of radiated/absorbed power, imposed, in pure states, only on the whole ensemble of experiments, it is now also individually imposed on each member of a partition of the overall ensemble. May this be obvious consequence of mixing both concepts, “ensemble of systems” and “ensemble of experiments”?

(ix) Wavefunction collapse, maybe an old-fashioned term nowadays, is an issue that is still there, hanging around. For us, collapse is just the gain of statistical information once a measurement is performed on the system. Nothing more: once we measure something, an statistical description of (the set of) possible experiments that can be still performed on it must be formulated in terms of conditioned probabilities.

(x) The behavior of wave-functions in finite potential wells: stationary wave-functions we mean, eigenstates. For instance, that famous “tunnel effect”, is not surprising at all from the point of view of the interaction with the ZPF background: within a classical ensemble of particles, their energy given by a probability distribution, there is always a non-zero probability that some of the particles gain enough energy to scape the well, unless the walls are infinitely high, of course. An stationary state would necessarily include, then, some probability for the particles to be outside the well. Quantum “evaporation” (may this be the case of black holes too?) may be no mystery, just simple probability.

(xi) QM is, seen from a certain time scale, a static theory, smearing out a much richer dynamics that takes place between apparently “discrete’ events (transitions in the state of the system, that would correspond to metastable states - attractors - in a classical dynamical description of the system). The quantum time evolution is, in the best of cases, an average of an underlying much more complex set of phenomena, involving a higher number of degrees of freedom.

(xii) At this stage, we cannot help realize that the appearance of the “photon” ($E = h\omega$) as a constraint on the spectrum of those possible “discrete” exchanges of energy-momentum between metastable states of the systems... seems completely natural in this framework.

(xiii) As to be expected, we do not like creation and annihilation of particles: perhaps a sort of “Dirac sea” interpretation could be recovered (my bias from having studied semiconductors may have something to do here... and as long as I remember, such an interpretation of the Dirac equation is abandon simply for convenience). From our “constructive approach”, a possible solution would be to add some internal structure to the electron, for instance, adding new degrees of freedom whose modes of vibration could be excited to explain the promotion to higher energy, interacting states.

(xiv) Fermi/Bose statistics: indeed, interaction between those inner degrees of freedom, and more specifically their convergence to metastable modes, established in interaction with other particles, could give rise to a, primitive, yes, but sort of Pauli principle, whenever those
modes may be, due to dynamical considerations, mutually exclusive. We came across something like this in our work: see [29].

(xv) Quantum Information: roughly speaking, surely the gain in computational power can be explained from the fact that quantum states carry more information than classical ones; in other words, quantum states summarize the state of a much higher number of classical degrees of freedom. For instance, we can compare a quantum qubit to a classical system. A representation space $R$ whose elements are used by QM to represent the state of a physical system.

The adoption of a complex RS in fact allows a dramatic reduction of its dimensionality, compacting the way of that position and linear momentum information (the two kinematic properties that define the state of a point particle in CM) are codified inside an element of that RS, what we will call (not only in the case of QM) a state-vector (and also coherently with the usual terminology in QM, wavefunction when it does not involve spin or other additional degrees of freedom). Such a reduction comes on the other hand at the price of position and momentum density functions (DF) not being any longer free to be specified independently from one another, leading to the appearance of some unavoidable “uncertainties” inherent to the formalism, but not necessarily inherent to the underlying, complete classical theory that it might be ultimately representing.

As a last introductory remark, though at certain points the Group Theory [35] is tangentially adopted for convenience in the exposition, our developments here could also be formulated from a purely pre-group theoretical framework.

### III. A PROBABILISTIC THEORY WHERE AM IS “QUANTIZED”

#### A. Introduction: physical spaces vs. representational spaces

Both in classical and quantum physics, the state of a system can be described by an element of a certain (usually infinite dimensional) vectorial space. Our work here departs from a distinction between what we will call “physical” and “representation” configuration spaces for a classical system. A representation space $V_{rep}$ would be obtained from the physical one by means of a correspondence from each of the physical states of the system, defined in its physical space $V_{phy}$; however, this correspondence does not necessarily be one-to-one (as it is so frequently assumed implicitly in relation to QM): there might be elements of $V_{rep}$ that may not correspond to any element in $V_{phy}$ (as may be the case of some of the states in QM) or vice-versa; on the other hand, it does not even have to be faithful either, in the sense that additional hypothesis may mediate this correspondence, hypothesis that may not always be justified.

It is not infrequent to hear people talk about QM as if vectorial spaces, linear operators, eigenvalues, wavefunctions and uncertainties were something completely alien from what we would expect to encounter in a theory founded exclusively on classical mechanics (CM), relativistic or not; the truth is far from that: many of the elements present in QM would also be the most natural ones if we attempted to build a probabilistic CM. Indeed, rather than many we should say all of them, but for one: the complex nature of what we will call the “representation space” (RS) whose elements are used by QM to

#### B. Building a classical probabilistic theory: a representation space

For instance, let us think of a vector $v$ in a real vectorial space $R$, such that $\dim R = N$.

$$v = \sum_{i=1}^{N} v_i u_i. \tag{1}$$

With $u_i$ representing the $N$ different directions in a basis for $R$, we can associate the quantity $v_i^2$ with the probability of obtaining a certain outcome $\lambda_i$ when a certain measurement $M$ is performed. Imposing

$$|v|^2 = \sum_{i=1}^{N} |v_i|^2 = 1, \tag{2}$$

we guarantee the axiomatic definition of probability always holds. Now, extracting the expectation value $\langle M \rangle$ implies building a linear operator $\hat{M}$, that clearly is going to be diagonal in the basis $\{u_i\}$. For instance, let $\hat{M}$ have eigenvalues $\lambda_i$ and eigenvectors $u_i$, we would have

$$\langle M \rangle \equiv v^t \cdot \hat{M} \cdot v = \sum_{i=1}^{N} |v_i|^2 \lambda_i, \tag{3}$$

nothing but a sum of possible outcomes weighted by their probability of occurrence, i.e., an expectation value.

All the way, we have been talking about vectorial spaces over the field of real numbers: so far completely logical because, for a real system and a real measurement, we want a real outcome. On the other hand, whether the spectrum of values $\lambda_i$ is discrete (finite of not) or continuum does not really make any conceptual difference.
(as the reader surely knows, we would have to consider infinite dimensional matrix operators).

It must be remarked that, as defined $R$ is a “representation space” for the probability distribution of $\mathcal{M}$, with absolutely no relation to ordinary space. If the values that $M$ can take are given by a set (simply a set, for now not it does not have to be a vectorial space) $\mathcal{M} = \{m_k; k = 1, \ldots, n\}$, then a probability density determining
$$\rho(m_k) = P(M = m_k \in \mathcal{M}),$$
for all $k$’s is represented by a vector in $R$, with
$$\rho(m_k) = |v \cdot u_k|^2.$$ 
Each state of the physical system would correspond to a (unitary) vector in the representation space, each vector yielding a certain probability distribution for the results of measurements performed upon it.

C. Linear operators

Let us go to ordinary space, but, for the moment and to gain simplicity, let us do it to a one-dimensional one. For instance, we could represent a probability distribution for $n$ possible positions $x_l, l = 0, 1, \ldots, n$ of a particle by a real unimodular vector in an $n$-dimensional vectorial space $R^{(X)}_n$. We can now do the same for $n$ different values $p_k, k = 0, 1, \ldots, n$ of the linear momentum, in another real vectorial space $R^{(P)}_n$. It is clear now that a joint probability distribution for each pair $x_l, p_k$ would be completely determined by a (unimodular) vector $v$ in the (still real) vectorial space
$$R^{(X,P)}_n = R^{(X)}_n \otimes R^{(P)}_n,$$ 
where again, as defined, $R^{(X)}_n$ nor $R^{(P)}_n$ have nothing to do with a one-dimensional geometric space; they are representation spaces. The (squared) magnitude of a certain component of $v \in R^{(X,P)}_n$ determines the probability of a certain pair $x_l, p_k$, for the system in a state represented by the (unimodular) vector $v \in R^{(X,P)}_n$:
$$P(x_l, p_k) = |v \cdot u_{l,k}|^2,$$
where $\{u_{l,k}\}, l, k = 1, \ldots, n$ are the $n^2$ vectors of a basis of our representation space $v \in R^{(X,P)}_n$. Expectation values for position and linear momentum would be obtained through linear operators $X, P$ with $n$ possible values each, $\{x_l\}$ and $\{p_k\}$, respectively. It is evident, on the other hand, that this two operators would completely diagonalize, in a compatible way, the space of representation $R^{(X,P)}_n$. In other words,
$$[ X, P ] = 0,$$
i.e., they would commute. For three dimensions we would have now a vectorial space $R^{(X,P)}_3$ such that $\dim R^{(X,P)}_3 = n^6$, corresponding to 3 position-coordinates $X_i$, each with $n$ possible values $x_{i,l}; l = 0, 1, \ldots, n$ ($n$ possible eigenvalues for each of the corresponding operators $X_i; i = 1, 2, 3$) and other 3 momentum-coordinates $P_j$, also each with $n$ possible values $p_{j,k}; k = 0, 1, \ldots, n$ ($n$ eigenvalues for each of the corresponding operators $P_j; j = 1, 2, 3$), i.e.,
$$R^{(X,P)}_3 = \bigotimes_{i=1,2,3} R^{(X)}_1 \otimes \bigotimes_{j=1,2,3} R^{(P)}_1,$$
and
$$[ X_i, P_j ] = 0,$$
for all $i, j = 1, 2, 3$. The fact that in ordinary space both observables have a continuum spectrum ($n \rightarrow \infty$) makes no conceptual difference: in that case the representation space would be infinite-dimensional.

D. A (minimal) complex RS: dimensionality considerations

What is the consequence, then, of introducing a complex representation space? To clarify this, we keep on operating with our discrete approximation ($n$ possible values for each position or momentum coordinate), which makes no conceptual difference but it makes possible easy comparisons. The complex nature of a $n^3$-dimensional vectorial space $C^{(X,P)}_3$ introduces a freedom of phase on the directions of the $n^3$ different eigenvectors for the most general linear operator $M$, given this time by a complex $n^3$-dimensional matrix. Such “freedom” is exploited by QM to represent, in each basis vector, not only the probability of one of $n^3$ different triplets
$$(X_1, X_2, X_3) = (x_1, x_2, x_3),$$
with $x_l \in \{x_{i,l}; l = 1, \ldots, n\}$, within the configuration space of the system, but also (of course this time in a different choice of basis vectors) of another one amongst another $n^3$ different triplets
$$(P_1, P_2, P_3) = (p_1, p_2, p_3),$$
with $p_j \in \{p_{j,k}; k = 1, \ldots, n\}$. However, the incompatibility of both basis carries as a consequence the fact that the freedom is not the same as we had in the real-space (former) case: this time there is not an analogous of either (6) or (9), i.e.,
$$C^{(X,P)} \neq C^{(X)} \otimes C^{(P)}.$$
It is useful to make the comparison in terms of real dimensions (in the sense of real numbers: the number of real parameters we have to give to specify an element of the corresponding space); we have done the transition:
\[ n^6 \Rightarrow (n + n)^3, \]
or, including the overall normalization condition,
\[ n^6 - 1 \Rightarrow (n + n)^3 - 1. \]

The loss of representational capacity (the capacity of establishing an unequivocal correspondence with a real multi-variable density function) is now quite obvious, which is what we wanted to show.

In coherence with that, as it is the most common knowledge, our new “Hilbert” space \( C^{(X,P)} \) is now diagonalized by either one of the two corresponding hermitian operators \( \hat{X} \) or \( \hat{P} \), but not by the two of them simultaneously, i.e.,
\[ [\hat{X}_i, \hat{P}_j] \neq 0, \]
but instead \( [\hat{X}_i, \hat{P}_j] = i\hbar. \)

**E. Discretization of the L-spectrum**

Up to now, we have been working with discrete representational spaces, with the aim of later taking the limit to a continuum one; in this section we will follow the same procedure. In this section we will show how for a class of representational formalisms containing ordinary (non-relativistic) QM, the angular momentum operator is associated with a discrete spectrum, even in that continuum limit. For convenience, we will work in 2 dimensions and wavefunction notation. Let \( \hat{X}, \hat{Y} \) and \( \hat{P}_x, \hat{P}_y \) be the pair of position operators \( (\hat{X}_i) \) and \( (\hat{P}_i) \) that operate in a representational space for 2D physical space. These operators are completely general, yet undetermined until we specify how the information about position and momentum is actually coded in the wavefunction, but for the fact that, once \( S, Y \) are chosen, \( \hat{P}_x, \hat{P}_y \) are the generators of translations in space \( \mathbb{R}^2 \).

The commutator of \( \hat{P}_x, \hat{P}_y \) vanishes so there exists a complete set of \( N^2 \) simultaneous eigenvectors of \( \hat{P}_x, \hat{P}_y \) generating all the representation space (of dimension \( N^2 \) as well), let us call them \( \Gamma_{n,n'} \) so that, if we represent them in the position basis
\[
\begin{align*}
\hat{P}_x \cdot \Gamma_{n,n'}(x,y) &= p_{x,n} \cdot \Gamma_{n,n'}(x,y), \\
\hat{P}_y \cdot \Gamma_{n,n'}(x,y) &= p_{y,n'} \cdot \Gamma_{n,n'}(x,y),
\end{align*}
\]
for all \( x = x_1, \ldots, x_N; \ y = y_1, \ldots, y_N \) and \( n, n' \in \{1, \ldots, N\} \); i.e., they are eigenvectors of \( \hat{P}_x, \hat{P}_y \) with eigenvalues \( p_{x,n} \) and \( p_{y,n'} \), respectively. Now, we consider the eigenvalue equation for \( \hat{L}_z = \hat{X} \hat{P}_y - \hat{Y} \hat{P}_x \), i.e.,
\[
[\hat{X} \hat{P}_y - \hat{Y} \hat{P}_x] \cdot \Psi(x,y) = \lambda \Psi(x,y),
\]
for some \( \lambda \in \mathbb{C} \) and \( \Psi(x,y) \) well defined for \( x = x_1, \ldots, x_N; \ y = y_1, \ldots, y_N \), which we can always rewrite, using the basis \( \{ \Gamma_{n,n'}(x,y) \} \), as
\[
[\hat{X} \hat{P}_y - \hat{Y} \hat{P}_x - \lambda \mathbb{1}] \cdot \sum_{n,n'} c_{n,n'} \Gamma_{n,n'}(x,y) = 0,
\]
\[ x = x_1, \ldots, x_N; \ y = y_1, \ldots, y_N, \]
i.e., an \( N^2 \) square system (we have \( N^2 \) restrictions, given by the possible combinations of \( x, y \) on the independent variables \( \{c_{n,n'}\} \); each of those restrictions can equivalently be written as
\[
\sum_{n,n'} [\ x p_{y,n'} - y p_{x,n} - \lambda \ ] \cdot c_{n,n'}(x,y) = 0,
\]
\[ x = x_1, \ldots, x_N; \ y = y_1, \ldots, y_N. \]

In the most general case, \( (21) \) will be a compatible, completely determined linear system of equations, that only admits the trivial solution,
\[
c_{n,n'} = 0, \quad \forall n, n' \in \{0, 1, \ldots, N\},
\]
unless the coefficients matrix, \( A \in C_{n,n} \), with
\[
A_{n,n'} = (x p_{y,n'} - y p_{x,n}) \cdot \Gamma_{n,n'}(x,y) - \lambda,
\]
can be made to satisfy \( \det(A) = 0 \); this last is simply a polynomial equation in \( \lambda \), with grade \( N^2 \), therefore yielding \( N^2 \) possible solutions (it is no longer, however, an standard eigenvalue equation in \( \lambda \), this must be remarked and will be of use in a moment \( [37] \)):
\[
\lambda_{k,k'} = f(\{ \Gamma_{n,n'}(x_1,y_r); \ n,n', l, r = 1, \ldots, N \}),
\]
for \( k, k' \in \{1, \ldots, N\} \) and where \( f(\cdot) \) denotes some yet undetermined dependence on all (whatever they may be) momentum eigenfunctions \( \Gamma_{n,n'} \)’s. Taking this into account, it is easy to realize that the limit \( N \rightarrow \infty \) will not necessarily change things, producing an infinite set of real \( \lambda \’s \) such that \( \det(A) = 0 \), even if we add the restriction of some periodicity in the (yet undetermined) functions \( \Gamma_{n,n'}(x,y) \).

However, we should expect things to change when the convergence of the indexes \( n, n' \) is done towards a compact set, the real \( X - Y \) plane, \( \mathbb{R}^2 \); in that case, we will have a continuous \( \lambda \)-spectrum, whose real and imaginary part are determined by the \( \Gamma_{n,n'}(x,y) \)-spectrum (in this case also continuous), that we know \( [37] \) have to necessarily be periodic, and therefore:

(i) the phase of \( \lambda \)-spectrum oscillates, as each and every coefficient intervening in the equation \( \det(A) = 0 \) is multiplying one of the \( \Gamma_{n,n'} \’s \), which are periodic functions and therefore with an oscillating phase,

(ii) the continuity of the set where indexes \( x, y \) take values guarantees that our \( \lambda \)-spectrum will be indeed also
continuous in relation to $x, y$ (provided that $\Gamma_{n,n'}$’s are continuous themselves, which is a reasonable restriction, that for instance QM satisfies), and this guarantees, given the oscillation of the $\lambda$-spectrum, that there is going to be an infinite set of zero crossings for the $\lambda$-phase.

Those real $\lambda$’s are the ones that interest us here. On the other hand, in the procedure that we have used they are not actually obtained from an standard eigenvalue equation: non-zero phases in our $\lambda$-spectrum do actually correspond to zeros in the actual eigenvalue equation [39] (the actual AM operator is Hermitian, therefore all its eigenvalues must be real).

F. Further choices: QM

What we have built so far is a class of possible theories (representational formalisms for other theories indeed) to which QM belongs, as a mere instance within it; for any theory in this class, [36] is given as the infinitesimal generator of translations in space [36], but this does not fix completely the way in which momentum information, $P_i = f(X_i; i = 1, 2, 3)$, is codified in the wavefunction $\Psi(X_i; i = 1, 2, 3)$, even once that, by simply stating that $\Psi(X_i; i = 1, 2, 3)$ is a simultaneous eigenstate of $\hat{X}_i$, we have already settled that

$$\text{Prob} \left[ \hat{X} \rightarrow (X_1, X_2, X_3) \right] = |\Psi(r = \sum_i X_i \hat{x}_i)|^2, \quad (25)$$

where $\hat{A} \rightarrow A$ denotes an event where a single measurement of the observable $A$ is performed and $A$ is obtained as the outcome, and therefore

$$\Psi(r) = M(r) e^{i\theta(r, p(x))}, \quad (26)$$

i.e., the LM information can only be associated to the phase of $\Psi(r)$, an eigenstate of $\hat{X}_i$ with eigenvalue $x_i$, for $i = 1, 2, 3$.

Besides, the use of a formulation of wavefunctions in free-space does not imply loss of generality: external potentials can be included through the “minimal coupling” substitution and our development below be associated exclusively to the “physical” LM rather than the “canonical” one [38].

G. Linear momentum (LM) operator as a choice of construction

For convenience we work in a one-dimensional space, the extension to three dimensions (3D) being immediate. We seek for an operator $\hat{P}$ such that

$$(1 + i\hat{P}) dx : \psi(x, p) \rightarrow \psi(x + dx, p), \quad (27)$$

with of course $p \equiv p(x)$ to respect the fact that $\psi(x, p)$ is completely diagonalized in $x$ (or, conversely, in $p$), where

$$\psi(x) = M(x) e^{i\theta(x, p(x))}, \quad (28)$$

with $M(x)$ real and positive; it is now evident that, contrary to what it may appear, the former two conditions alone do not determine $\hat{P}$ uniquely: with

$$\int \psi^*(x) \hat{P} \psi(x) \, dx = \langle p \rangle; \quad (29)$$

there are still many possible ways of configuring $\theta(x, p)$, each one of them determining a different expression for the operator $\hat{P}$. For instance (again for convenience going to one dimension), we could choose any other linear operator

$$\hat{P}' = \sum_n a_n \frac{\partial^n}{(dx)^n}, \quad (30)$$

which would fix $\theta'(x, p)$ and which, again contrary to what it may be taken for granted, would as well serve the purpose of representing momentum information in (the phase of) $\psi(x)$: we recall we are treating $\psi(x)$ as a mere device to represent the physical information of a system (or an ensemble of them). The extension to 3D is immediate, with now

$$\big(1 + i\hat{P}_t\big) dx : \Psi(r, p) \rightarrow \Psi(r + dx, \hat{x}_i, p), \quad (31)$$

where

$$\Psi(r) = M(r) e^{i\theta(r, p(r))}. \quad (32)$$

As a clarification, a different choice to that of QM for $\hat{P}'$ for the LM operator would also discretize, as shown in Sec. III E the spectrum of $\hat{L}' = \hat{X} \wedge \hat{P}'$, but such spectrum would not be (equi)-spaced by units of $\hbar$.

H. On Schroedinger’s equation as a consequence of a choice for the LM operator

Once, from the choice $\hat{P} = -ih\nabla$, the structure of $\Psi(r)$ is fixed, the “energy operator” $\hat{H}$ can be (going again for convenience to one dimension) unambiguously defined as the generator of translations in time

$$(1 + i\hat{H}) dt : \Psi(t) \rightarrow \Psi(t + dt), \quad (33)$$

where we are obviously redefining

$$\Psi(t) \equiv \Psi(r(t), p(t)), \quad (34)$$

and from where the Schroedinger equation can be immediately obtained. The Schroedinger equation can be therefore understood as just a consequence of the choice for $\hat{P}$, itself determined by the need to assure AMQ with $\hat{L} = \hat{r} \wedge \hat{P}$.

It is important to remark that, contrary to what it may seem, up to now nothing conditions the implementation of a certain mechanics upon our representational formalism: while the former is true, it is for a yet undetermined operator $\hat{H}$, that we can for instance simply associate to

$$\hat{H} \equiv \frac{\hat{P}^2}{2m} + E_{pot}. \quad (35)$$
I. Angular momentum in QM

We have seen that the LM operator is given as the infinitesimal generator of translations in space, but this does not fix completely the way in which momentum information, even once that, by choosing a position basis \(l, m\), we already fixed \(20\). In particular, working now in the continuum, the operator

\[
P_i \equiv -i\hbar \frac{\partial}{\partial X_i},
\]

settles \(40\) the following expression for the operators \(\hat{L}_i\) (for \(\hat{L} = \hat{r} \wedge \hat{P} = \sum_i \hat{L}_i \hat{\xi}_i\), and assuming a cyclic behavior for subindexes: \(X_0 \equiv X_3, X_4 \equiv X_1\):

\[
\hat{L}_i = -i\hbar \left[ X_{i+1} \frac{\partial}{\partial X_{i-1}} - X_{i-1} \frac{\partial}{\partial X_{i+1}} \right].
\]  

(37)

In almost any textbook on QM we can find the proof that both \(\hat{L}^2 = \sum_i (\hat{L}_i)^2\) and any of the \(\hat{L}_i\)’s yield an infinite but discrete set of eigenvalues \(L^2(l)\) and \(L_i(l, m)\), respectively, with \(L(l) = h\sqrt{l(l+1)}\), \(l = 0, 1, \ldots\), and \(L_i(l, m) = mh, m = -l, \ldots, 0, \ldots, l\).

J. A conjecture

Up to now, our work here is strict mathematical analysis, whether it is at points interpreted from one perspective or another; now we will for once make a conjecture, whose reasonableness we think is more than well founded both in our previous development and the physical reasons that we will expose here:

The choice of a structure for the RS and the corresponding position and LM operators in QM were exclusively a consequence of the need to reproduce the observable AM spectrum.

This choice, which may have been an accidental (rather than a voluntary) one, is supported in obvious reasons from the physical point of view: in the classical world, the AM of any system performing a stationary (which does not necessarily mean cyclic \(41\)) trajectory in its configuration space is, on average, quantized, as forced by a necessary balance of radiated and background-absorbed power, following Puthoff’s argument \(57\) (though we should remark it works the same whether we consider the ZPF background or another more general one).

The necessary discretization imposed by the (average) power balance would apply to the following cases:

(i) spin as a residual AM:

(ii) “inner” orbital AM in bound states, for instance \(l \neq 0\) atomic orbitals, their AM adding to the total spin of the system when seen as a particle;

but of course not to AM as an orbital property of free particles: our conjecture is that, in that case, the \(h\)-quantization imposed by QM is “harmless” (i.e., quantum mechanical AM would be an approximation for free particles, but given the realistic ranges of AM in free space, this approximation is completely harmless).

On the other hand, a justification on why we include this “average” as a necessary property for “observable” is regarded by us as something almost trivial Sec. IV B at least in the former two cases (i)–(ii): it is enough to see how in QM spin and AM are added without any reference to the origin of coordinates.

At this point, one may be asking himself in what is this of any use, given the fact that it does not alter any of the quantum mechanical machinery at any point: as part of our conclusions we will give a set directions where our conjecture leads, while from the orthodox approach to QM they remain as obscure as usual.

K. Conclusions

As much as it is for QM, the representation of the state of a system by an element of a vectorial space is a completely (maybe the most) natural tool to build any other probabilistic formalism, also a completely classical one. On the other hand, the assumption of a complex (and minimal) representation space does not fix completely the mathematical structure of the formalism: there is still some freedom on how to codify position-momentum into the wavefunction. While marginal density functions for position and linear momentum are not affected by this choice in the structure of the formalism, it does however lead both to different “minimum uncertainty relations” and quantitative differences of the eigenspectrum of yet another elementary kinematic quantity, this time derived from the former: angular momentum (AM). Indeed, with strong evidence from experimental physics that the observable values of (the three orthogonal projections of) AM momentum do obey some very neat laws, that last choice on the theory is no longer a choice, but a necessary hypothesis (and remarkably the only one with real physical significance in the construction of our “representational formalism”) if we want to reproduce the well known \(h\)-equispaced spectrum typical, for instance, of the atomic orbitals. From our point of view, we think the most simple way to identify this choice is associating it with a closed expression of the LM operator which, contrary to what it may seem, is not at all fixed by its requirement to be the generator of translations in Euclidean space (at least the “canonical momentum” \(38\)). Whether what he had in mind may have had or not any relation with our approach here, we cannot help but admit that we were back in time deeply inspired by Penrose’s words: “that mysterious operator” \(42\).
Whether such a last choice is or not volunteer, or even conscious, the obvious deduction from our investigations here is that it is a sensible option to regard it as the one and only building brick of QM as a representational formalism for another underlying description of nature, this time entirely classical and possibly involving a much higher number of degrees of freedom (though this second question will not arise until we try to consider dynamical aspects of QM, and not only kinematic, which has been our goal here). In any case, once observational evidence enters in the picture, a particular formalism is singled out amongst a set of possible probabilistic theories making use of a complex representation space with a particular dimensionality (one that in a real counterpart of that space would only allow to represent either position or momentum information, not both simultaneously in the same wavefunction); this particular instance of our class of formalisms is no other than QM, exhibiting, amongst other properties, both:

(a) the commutation relations of position and LM, giving rise to the famous Heisenberg uncertainty relation (HUC);
(b) angular momentum quantization (AMQ).

Rewording what we have already said above, previous to any particularization to QM, it is our opinion that (b) should be regarded as a principle, and (a) a necessary consequence: the price (together with other related properties) of building a formalism where AM is quantized. Besides, and as far as we see, our approach here shows a perfect consistency with the ordinary mathematical framework of QM, allowing for:

(i) a seamless addition of new degrees of freedom to the theory (spin, isospin, color, flavor... whether or not some of these finally turn to be related with the more elementary ones: spin with AM, etc);
(ii) the introduction of external potentials through the “minimal coupling” substitution and the distinction of roles between “canonical” and “physical” linear momentums;
(iii) an straightforward extension to the relativistic analogue.
(iv) On the other hand, it is clear the many-particle case would account for just an escalation or the dimension of the representation space, by the tensorial product of each single space-particle subspace, just as it is done in QM. In that context, the antisymmetry requirement for fermions would have to be included as an external restriction, just the way the $h$-step AM quantization has done here: further interpretation of the Pauli principle are left for elsewhere.

IV. ADDITIONAL QUESTIONS CLOSING THE GAP TOWARDS QM

Of course the former is just an oversimplified picture and there are still many gaps to close in the transition from a purely classical to a quantum world. Clearly, one of them is the origin of spin, which we will treat later in Sec. [VI] for the moment we will concentrate in other less appealing points. One of them [IV.B] has already been referenced in the former section.

A. An apparent connection with Fritsche and Haugk’s approach

We recapitulate: here we depart from a class of representation formalisms that comprise, as one of its members, ordinary QM. This formalisms use a vector in a complex space to represent position-momentum (leaving aside for the moment other degrees of freedom) of an ensemble, well of experiments or well of systems (or well an ensemble of systems upon which one can perform an ensemble of experiments: this combination leading necessarily to density matrices, while the formalism of pure states would do for any of the former two). When we talk about “choosing” a momentum operator, what we mean is actually “choosing the way in which momentum operator is coded into that vector (the wavefunction)”. 

Until now, all of these possible representation formalisms are devoid of any physical significance, they are mere devices to represent information: the choice of neither of them conditions at all whatever “laws of forces” and “mechanics” we want to adhere to (we can simply take them from the Newton-Maxwell framework)... but for the fact that the formalism represents two physical degrees of freedom (hence necessarily real numbers) in a complex scalar, rather than in two real scalars, and this is already a restriction: a discrete spectrum for the angular momentum (only discrete for now, amongst the many possible discrete ones).

Now, once a choice for the $P$-operator is made, everything is already fixed: a choice of $\hat{P}$ has already settled completely the structure of the wavefunction $\Psi$ (going again to one dimension for convenience) as

$$\Psi(x,p) \equiv M(x)e^{ipx/\hbar},$$

with $M(x)$ real and positive, for a completely delocalized, free-particle (where $p$ is a constant), or, for the most general wavefunction as

$$\Psi(x,p) \equiv M(x)e^{-i\theta(x,p)}.$$

On the other hand, previous to the choice $\hat{P} = -i\hbar\frac{d}{dx}$ we would simply have, instead, $\Psi(x,p(x)) \equiv M(x)e^{i\theta(x,p)}$, for some yet undetermined function $\theta(x,p)$.

From our point of departure (we remark this), $\hat{P}$ is chosen so as to generate the well-known equi-squared AM spectrum inspired on which Heisenberg first created his “matrix mechanics”, as a mere representational tool. As we say, once that choice is made, all is fixed: for instance the Schroedinger equation simply states the fact that “the energy operator is the generator of time-translations”. All is fixed... but for those choices that
actually determine the “mechanics”: the former leaves a yet undetermined $H$, until we finally set $H = \hat{P}^2/2m$ which implements Newton’s $F = ma$.

Now, while for us such a choice of $\hat{P}$ is explicit, in Fritsche and Haugk’s work [11] it is an implicit one, a sort of postulate (and then of course, quantization of $L = r \wedge p$ arises as a consequence of the - time independent if I remember well - Schrödinger equation, how could it not?). Indeed, we think we know where such choice is implicitly done; following Fritsche and Haugk, velocity field is irrotational $\Rightarrow v = -\nabla \Phi$,

ok, but ... why does the phase of the “wavefunction”, $\theta(x,p)$, have to be chosen as being proportional to the velocity potential $\Phi$? I.e., we believe that

$$\theta(x,p(x)) \propto \Phi$$  \hfill (40)

is a choice of construction for the theory!

From our point of view, that is an arbitrary choice in the construction of the theory... that of course turns out to be the correct one. There could be other ways of including velocity (momentum) operator in an hypothetical pre-quantum wavefunction (once fixed that the square of the magnitude corresponds to the position probability density), but those would not yield the correct spectrum for $\hat{L}$.

**B. Classical and quantum AM**

Addition rules of quantum angular momentum (AM) and spin or “inner” orbital angular momentum playing the role of spin are something rather curious. Here we justify point (iii) from Sec. [11]

Classical and quantum AM are often treated as different concepts with only a limited relation between them. If we let aside rotations that are not (globally) isomorphic to the group $SO(3)$ of rotations in ordinary space, the main difference between the concepts of classical and quantum angular moments is that the former can not in general be added, because of their dependence on the point in respect to which they are evaluated (their respective origins). However, the average value of a classical AM on a closed, periodic trajectory is independent of the origin, as we will now see.

Consider an origin of coordinates $O_1$ and let $\mathbf{r}_1$ be the position vector defined in respect to it, as well as a second origin of coordinates $O_2$ such that the position vector is now $\mathbf{r}_2$. Obviously, the following difference

$$\mathbf{r}_{1,2} = \mathbf{r}_2 - \mathbf{r}_1 = \mathbf{O}_1 - \mathbf{O}_2,$$

is clearly a constant vector. Now, let $\Lambda$ be a closed trajectory, with a period $T$, so $\mathbf{r}_i(T) = \mathbf{r}_i(0)$, for $i = 1, 2$.

For the movement of a system of mass $m$ whose position is (simultaneously) given by $\mathbf{r}_1(t)$ and $\mathbf{r}_2(t)$, in the respective system of reference, it is easy to prove:

$$\langle \mathbf{L}_2 \rangle_\Lambda = \frac{1}{T} \int_0^T \mathbf{L}_2 \, dt = \frac{1}{T} \int_0^T \mathbf{r}_2 \wedge \mathbf{p} \, dt$$

$$= \frac{1}{T} \int_0^T (\mathbf{r}_1 + \mathbf{r}_{1,2}) \wedge \mathbf{p} \, dt$$

$$= \frac{1}{T} \int_0^T \mathbf{r}_1 \wedge \mathbf{p} \, dt + \frac{1}{T} \int_0^T m \frac{d\mathbf{r}_{1,2}}{dt} \wedge \mathbf{p} \, dt$$

$$= \langle \mathbf{L}_1 \rangle_\Lambda + \frac{1}{T} \mathbf{r}_{1,2} \wedge m (\mathbf{r}_1(T) - \mathbf{r}_1(0))$$

$$= \langle \mathbf{L}_1 \rangle_\Lambda.$$  \hfill (42)

Thus, the classical AM averaged over a closed orbit is independent of the origin and can be added up (component on component), just like sometimes happens with quantum AM.

In [14], this “spatial averaging” is justified along a different line, from the point of view of an ensemble of particles; we believe our reasoning here is complementary to theirs, for it covers the case of “ensemble of experiments” (over the same particle). It is our point of view that QM is build to combine those two approaches, as otherwise there seems to be no reason on why we would have to resort to density matrices, instead of sticking to pure states alone.

**C. The HUC and other related issues**

In coherence with the “ensemble” interpretation of QM that we advocate here (to which some experimental evidence, like the well known “tunneling effect” adds, according to us - see Sec. [11] (x) - nothing but more legitimacy), perhaps position and momentum entwined dispersion as random variables could be understood simply as an statistical limitation of the quantum formalism, rather than a final property of nature in the spirit of the HUC. The fact that QM can only describe systems where those two basic properties obey some minimum mutual dispersion appears as the price of quantizing angular momentum. Of course, we just do not accept anything far from position and momentum being legitimate, perfectly measurable if not for disturbance by the measurement process, “elements of reality” [13]: we firmly believe in localized systems, with or without some inner structure, particles!

There is an important comment to do here, recovering a lucid point made by Kracklauer [15]: while position and momentum, as “elements of reality”, lead to EPR’s conjecture of incompleteness of QM, any attempt to rule it out based on transporting the argument to the case of spin-spin entanglement is flawed from the beginning, for the simple reason that nowhere we need spin projections to be be elements of reality as well, the HUC losing significance as a proof of incompleteness of QM. However, spin-entanglement arguments still retain some as an element of collision with local realism, though we will leave this issue for Sec. [15]
Summarizing, we remark two main points:
(i) The HUC should be understood as just a mathematical limitation of QM as a physical theory: a limitation in any case consistent with the ones that all feasible measurements abide to. Following our approach, experimental measurements can only access “averages” (averages of an ensemble of systems/experiments satisfying certain equilibrium conditions: AMQ) which are subjected to the HUC.
(ii) The true significance of the HUC should be tested on things that we know should be primary elements of reality: for instance, the position and linear momentum of a particle or a set of them. Projections of spin are not elements of reality in the EPR sense.

V. APPROACHING QM FROM STOCHASTIC ELECTRODYNAMICS

A. Approaching QM as a semi-static theory

Aside we have the fact that QM is, in some way and seen from a certain time scale, a semi-static theory, tracing out a much richer dynamics that takes place between apparently “discrete” events (transitions in the state of the system). The quantum time evolution is then an average of an underlying much more complex set of phenomena, involving a higher number of (this time entirely classical) degrees of freedom. Indeed, for us the photon arises only as a convenient invention to describe the interaction of systems that - at least one of the two interacting - could only either oscillate around metastable, attracting states (states defined by some particular value of some kinematic properties, such as AM and energy; other, like linear momentum, may be completely free to take any value) or jump between two of those states. If we now add that we can only distinguish between those metastable states (so we remain oblivious to all the micro-oscillations underneath) there we have the photon and QM (or QED).

In our view, a real photon, if we could observe it as it is, would be a packet of waves more or less rich in frequencies but centered in a particular frequency that leaves wondering if it’s all just pure coincidence: the former means that to release an electron trapped in this kind of orbit, the system needs to absorb a wave-packet of energy \( \hbar \omega \) (considering only the potential energy contribution). Now, if we suppose different stable orbits of this kind exist, ranging over a discrete spectrum of energies, parameterized by a principal quantum number \( n \), transition from one to another \((n,n')\) would imply emission and absorption of wave packets involving the two “characteristic” angular frequencies \( \omega_n, \omega_{n'} \).

We believe that existence of this discrete spectrum of stable (attracting) orbits, and also of other orbits (attractors) with either null \((l = 0)\) or higher \((l > 1)\) (average) AM can be justified, as we have tried to do in another work [54]: so far to our knowledge, the Bohr model only covered orbits with non-vanishing AM.

VI. ON THE ORIGIN OF SPIN

A. Introduction

The interpretation of integer spin in the cases where it can be associated to ordinary orbital AM does not present great difficulties. A different thing happens with half-integer spin, the spin of elementary particles.
Within the framework we have settled here, together with [50], two main problems remained unsolved (at least qualitatively, in regard to quantitative accuracy everything is almost yet to start), both related to that “mysterious” physical property we call half integer spin: (i) the appearance of spin as the generator of the two-dimensional group SU(2), non-(globally)isomorphic to rotations in ordinary space; i.e., bi-valuation when one would expect, as for bosons, always three possible projections on each direction (leaving aside the relativistic limiting case of photons), (ii) the appearance of the value $\hbar/2$ when one would always expect $\hbar$, as proven from Puthoff’s landmark calculation 57.

In the following sections, a solution is proposed for both of them, at least a qualitative one. We suggest the picture of a point-like electron where spin is just a kinematic remnant of its origin from bound (for us classical, metastable) states of matter, where it performs, aside from its main trajectory, a sort of “secondary oscillation” predicted by the dynamical equations of the model we gave in 50. That remnant is a helical movement around the direction of the (average) linear momentum, for which of course the projection of AM on the transversal plane (transversal $\equiv$ perpendicular to the average LM of the trajectory) should yield, in consistency with Puthoff’s 57, either the values $\pm\hbar$ or null; however, the precession of its instantaneous AM vector (IAM) - in former versions we have used the term “instantaneous axis” which is not correct - disturbs the average performed in the measurement process (during the window measurement), opening room to reduce the observable magnitude of the projection down to $\hbar/2$, as well as eliminate the possibility of a null one (which would mean a transversal orientation of the IAM).

Completing the former picture, quantum AM commutation relations and its associated uncertainties are explained from the loss of information during of the measurement process, which necessarily reoriented the (average, observable) magnetic moment of the system, as a result of its interaction with the field lines. This model solves all geometric/kinematic difficulties to establish a classical analogue of half integer spin (Sec. VI D), but for possible relativistic difficulties (on which we also comment briefly in Sec. VI D). Moreover, our picture also seems to (qualitatively) explain the relation between spin and magnetic moment for the electron... could it be the origin of the so-called Thomas factor related to this (it really seems)? A similar picture of a helical movement is also suggested, and surely ours is inspired by, in Cavalleri’s (and collaborator’s) work [60]; we do not yet foresee if both are completely equivalent, though.

Check also [60])

B. Outline

We tackle the following problems:

(i) A suitable explanation for the AM (and spin) addition rules in QM: irrelevancy of the origin of coordinates for spin (as a residual AM) as well as orbital AM in bound states.

(ii) The appearance of spin as the generator of the bi-dimensional group SU(2), non-(globally, see [58]) isomorphic to rotations in ordinary space (in other words: bi-valuation when one would expect, as for bosons, always three possible projections on each direction, this of course leaving aside the relativistic limiting case of photons 54.

(iii) A suitable interpretation of the commutation relations (hence the uncertainties) of quantum angular momentum for massive particles (in the case of photon-polarization, the non-commutation is well explained taking into account the ZPF components in polarizing beam splitters, something we have treated elsewhere.

On the other hand, the discreteness of the AM spectrum in QM will be taken just as an external hypothesis, acting like a constraint on the (class of) models that we propose here. Nevertheless, at least for a certain “microscopic” component of the overall AM (the one coming from spin or inner orbital AM of a system of particles) such a feature could be more than justified based in the argument that any system of charges performing a periodic trajectory in its configuration space (its classical dynamical configuration space) must abide to a balance between emitted and absorbed power in the form of electromagnetic radiation. Such a balance has been in the past invoked by (for instance) Puthoff 57 to derive the AM quantization condition that we will however (and for simplicity) take as barely an external postulate here.

Question (i) will be addressed in Sec. VI C Spin will be addressed in Sec. VI D both for the case of integer [VI F] and half-integer spin [VI G], this second case involving (ii). Question (iii) is addressed in Sec. VI H. We also comment on the extension of the model to yield the rules for addition of spin [VI I], as well as include a reference to previous approaches and to the relativistic limitations of our proposal [VI J]. A reduced set of conclusions is presented in Sec. VI K

C. Angular momentum

1. Physical AM

We will assume a decomposition of the total angular momentum in two terms, a macroscopic one and a microscopic one

$$L_{(ph)} = L_{(ph)}^{mac} + L_{(ph)}^{mic}, \quad (46)$$

where $L_{(ph)}^{mac}$ would represent the component “in free space” and $L_{(ph)}^{mic}$ will yield what we know as “spin” of
a particle (including inner orbital angular momentum of its sub-particles). This may seem artificial unless we consider that there is some reason why the dynamics of our “microscopic” component differs in some way from that of the “macroscopic” one: there could be some [52], but the question will be left for elsewhere.

As a matter of fact, we should simply divide the momentum components in “free” and “spin”, because it will not be the “size” of the components that makes the difference, but their behavior (in accordance to our idea [53], the possible underlying feedback loops in the sub-structure of the particle privileging some particular dynamics over others); for lack of a better option, let us stick for the moment to that macro-micro division...

2. Measurement process: AM as an average

Classical AM averaged over a closed orbit is independent of the origin of coordinates Sec. [VI13] and can be added up (component on component), just like sometimes is usual in quantum AM (for instance AM in bound states), which suggests a trivial way of relating both concepts. In mathematical terms, we will assume the result \( L_{(ph)} \) of an AM-projection measurement upon a system with classical angular momentum \( L_{(ph)}(t) \) is always expressible as:

\[
L_{(ob)}^i = \frac{1}{\Delta T} \int_{\Delta T} \left( M \left[ L_{(ph)}(t) \right] \cdot \hat{x}_i \right) dt, \tag{47}
\]

where the (yet undetermined) function

\[
M : L_{(ph)}(t) \rightarrow L_{(ph)}'(t), \tag{48}
\]

where \( \Delta T \) is a “measurement-window”, and where \( M[\cdot] \) reflects the fact that the measurement process will in general also modify the state of the system. Later, we will invoke the loss of information in this measurement process to justify the quantum AM (and spin) commutation relations, in Sec. [VI13]

3. Spin and bound states: “inner” AM

The AM-spin (or AM - inner AM) addition rules can only be explained assuming that \( \Delta T \) equal to (or a high number of times, the deviation from the integer yielding some small deviation in the value) the period of a cyclic orbit \( \Lambda \) that the system is describing: from that assumption, the origin of coordinates is no longer relevant, therefore we will have, for the “micro” component and using spatial integration,

\[
\left[ L_{(ob)}^{\text{mic}} \right]^i = \frac{1}{L_\Lambda} \int_\Lambda \left( M \left[ L_{(ph)}^{\text{mic}}(t, r) \right] \cdot \hat{x}_i \right) dl, \tag{49}
\]

with \( dl = |dr| \) and \( L_\Lambda \) the overall length of the (cyclic) trajectory. This periodicity (or quasi-periodicity) of trajectories is typical of the inner structure of particles: atomic orbitals inside an atom, quark orbitals inside a nucleon, etc...

4. Observable AM in free particles

The “macro” component will not correspond, in general, to any sort of periodic trajectory, and therefore (47) will only account for a slight distortion of the outcome of the measurement; for this macroscopic component, the origin of coordinates is still relevant.

5. AM in Quantum Mechanics

Now, we consider QM as a mere representational formalism for an underlying more complete physical theory; this formalism would establish the correspondence [59]:

\[
QM : L_{(ob)} \rightarrow L_{(qm)}, \tag{50}
\]

where \( L_{(qm)}^i = \frac{1}{2} n \hbar \) for some integer \( n \), making use of the fact that \( L_{(ph)}^{\text{mic}} \) is discretized, as a result of the measurement process and the nature of the physical states of the system where such measurement is performed; \( L_{(ph)}^{\text{mac}} \) would not be, in principle, but we will consider that the \( \hbar \)-quantization is “harmless” within the realistic ranges for this macropscopic component. Anyway, regardless of possible deeper explanations [57], such a discretization of the quantum mechanical spectrum will be for the moment barely taken as an external constraint on our models, which does not compromise their feasibility at any point.

D. Towards a model for “spin”

In coherence with all the former, we will interpret “spin” as an effective description of an underlying residual AM, corresponding to a periodic “micro-dynamics” (what in an “average”, effective description could be associated to the inner structure of the particle - or system of them when it is interpreted as a single one) that is averaged on the measurement window.

E. Auxiliary definitions: IAM, IMM

For simplicity, we will slightly abuse notation calling instantaneous (micro) angular momentum (IAM), instead of the full angular momentum vector, just a unitary one in the direction

\[
\hat{u} \propto r \wedge p \tag{51}
\]

where \( p \) is the instantaneous linear momentum of the particle, and where the origin in relation to which \( r \) is defined will actually be, at least for our calculations here,
irrelevant (due to later integration over a “cycle”). For clarity, nevertheless, we can picture it as the position vector from the center of our cyclic trajectory, a helical one in the case of individual electrons (a helix is not exactly cyclic, we know, see precisions in [56]).

Besides, for an “effective” (in the sense of “observable”, implying an average over a time window) magnetic moment of the trajectory, we will have in turn something like

\[ \mu \propto \langle \hat{n} \rangle \equiv \frac{1}{\Delta t} \int \hat{n} dt, \tag{52} \]

where \( \Delta t \) stands for the observation window, and from where it seems justified to talk, at least just for the sake of physical intuition, about an “instantaneous magnetic moment” (IMM), proportional to \( \hat{n} \). Such quantity, we remark, would not be at all significant in the most general trajectory, due to the ambiguity in the origin definition; here, nevertheless, it is useful because it can be related (through integration, that eliminates that ambiguity) to the “observable” magnetic moment \( \mu \).

F. Integer spin

First, we will describe our picture of integer spin; then, we will do the same with half-integer spin.

1. Physical model: a spire

This is how our picture works for bosons. In the standard model, integer spin arises entirely from orbital AM (or orbital AM plus an even number of half integer spin instances), as a consequence of which it does not need to be included in the quantum formalism as an additional, mysterious extra degree of freedom (as half integer one is, through increase in the dimensionality of the state vector). It has a much more immediate physical interpretation than half-integer spin: based on the correspondence between spin and magnetic moment, we will assume we can model a boson by a spire of current: we are thinking, for instance, in an electron orbiting around a nucleus. The orientation of the (axis of the) spire (in this case also the IAM and IMM) is given by the unitary vector \( \hat{n}^{(b)} \), the ‘b’ superscript standing for “boson”.

2. Measurement process

A measurement process \( M[\cdot] \) is defined by stating the initial and final state of the system, i.e, the initial and final state of the spire axis, in this case coincident with the IAM. Indeed, let us suppose that the center of mass of the spire approaches an Stern-Gerlach (SG) device: first, the system feels the far field lines, then the close ones (both of them anti-aligned due to the fact that overall lines must be closed by the absence of scalar field sources). Then, the expectable behavior of \( \hat{n}^{(b)} \) would account for one of the following three, depending on its direction:

(i) alignment with the field lines at the core of the device (close field lines), which will lead us to a positive projection (see following sections),

(ii) anti-alignment of the axis with the field lines at the core, due to previous alignment with the far field lines, yielding a negative projection,

(iii) the axis stays more or less perpendicular to the field lines, yielding a null projection, i.e, those three possible final orientations behave like “attractors” in the (classical) configuration space of the system, modifying the initial direction of the AM (using 0, 1 subscripts to indicate “initial” and “final” state):

\[ \hat{n}^{(b)}_0 \rightarrow \hat{n}^{(b)}_1 \equiv M \left[ \hat{n}^{(b)}_0 \right] . \tag{53} \]

Our model of the measurement process is completed with a second assumption: we will suppose the direction of \( \hat{n}^{(b)}_1 \) is already settled (stabilized) during the time-window of observation.

3. An integer spin measurement

As a purely phenomenological model of the measurement process (for instance performed through a Stern-
Gerlach device \[61\) we can perhaps simply write:

\[
S(\hat{B}) = \frac{\hbar}{\Delta t} \int_{\Delta t} \hat{B} \cdot \hat{n}_{1}^{(b)} \, dt,
\]

where \(\hat{B}\) and \(\hat{n}\) are (constant) unitary vectors in the direction of the magnetic fields \(B\) (therefore fixing which projection we measure) and the symmetry axis of the spire. Indeed, following our former reasoning (or directly Puthoff’s equilibrium condition \[57\]), the magnitude of \(|S(\hat{B})| \in \{\pm \hbar, 0\}\), and \(\hat{n}\) either aligned, anti-aligned or perpendicular to \(\hat{B}\).

G. Half-integer spin

We now explore to what extent the previously formulated problems (i) and (ii) can be solved by a model of half-integer spin consisting in a helical trajectory.

1. Physical model: a helix

Of course, a helix trajectory is obviously not a “closed one“, but it is nevertheless in some way cyclic, so it also allows for (the microscopic component of) quantum AM to be defined as an “average over a cycle“ (in the way that we have done in Sec. \[56\]), at least under certain restrictions \[56\] which are enough for our purposes here. In Sec. \[56\] we have defined an “instantaneous angular momentum“ (IAM), either for a circular or a helical trajectory; in this case we denote it by \(\hat{n}^{(f)}\), the ‘f’ superscript standing for “fermion”.

2. Measurement process

We will suppose that the average \(\langle \hat{n}^{(f)} \rangle\) gets either aligned (by action of the close field lines) or anti-aligned (by action of the far field lines) to the magnetic field; the possibility that it remains perpendicular to them is made highly unlikely due to the precession of \(\hat{n}^{(f)}\). Assuming this (hypothetical, but intuitive) behavior, the coincidence with Cavalleri et al is quite remarkable; quoting from \[60\]: “What is called a “spin up” means a distribution of the spin axis in a half sphere having \(B\) as a symmetry axis“. Operating on the instantaneous values we will write (using again 0,1 subscripts to indicate “initial” and “final”):

\[
\hat{n}_{0}^{(f)}(t) \rightarrow \hat{n}_{1}^{(f)}(t) \equiv M \left[ \hat{n}_{0}^{(f)}(t) \right],
\]

where it is crucial to remark that, as a difference with the bosonic case, here \(\hat{n}_{1}^{(f)}(t)\) retains its time dependence.

3. A half-integer spin measurement

Over the observation window, this time \(\hat{n}_{0}^{(f)}(t)\) is precessing: we have an integral of a varying vector:

\[
S(\hat{B}) = \frac{\hbar}{\Delta t} \int_{\Delta t} \hat{B} \cdot \hat{n}_{1}^{(f)}(t) \, dt,
\]

Again, this is merely phenomenological model. Of course the projection of the AM vector on the transversal plane should be either \(\pm \hbar\) or null (consistently with Puthoff’s calculation \[57\]), but possibly a reduction of the observable magnitude of the projection down to \(\hbar/2\) can be explained as a consequence of the precession of \(\hat{n}_{1}^{(f)}(t)\) \[58\]. This would also explain the impossibility of a null projection on the axis of the helix (the analogous to a transversal projection in the boson case).

![FIG. 2: Fermions would be point-like charges describing a helical trajectory (their magnetic moment obtained as an “effective” description when this dynamics is traced out), for which the IAM precesses, even in the absence of external fields: the projection of the IAM \(\hat{n}\) on the transversal (here horizontal) plane would produce a circumference of radius \(\Omega\), with \(0 < \Omega < R\). As usual \(\hat{n} = R \times p\) with \(R\) taken from the point of intersection of the helix’s axis with the transversal plane (again, for our developments that choice of origin is irrelevant because of the latter averaging over a cycle (Sec. \[58\]); finally, the average vector \(\hat{n}_{0} \equiv \langle \hat{n} \rangle_{\text{cycle}}\) may or may not be reoriented during the measurement process depending on which projection is addressed (i.e., depending on the nature of the field lines created by the measuring device).

H. Quantum AM commutation relations

From our point of view the modification (reorientation) of the (average) magnetic moment during the process of measurement is the necessary element to explain the commutation relations of both spin (and of course also orbital
AM). As we have seen, this reorientation takes place both for bosons (the final state of the average momentum is approximately either aligned, anti-aligned or perpendicular to the magnetic field lines) and fermions (we have only two possibilities: either an approximate alignment or anti-alignment of the average AM vector - the IAM keeps precessing in this case), and gives a suitable explanation as to why a measurement destroys the information regarding other projections.

I. Addition of spin

Besides, it is convenient to also remark the consistency of this approach with the “combination” of half-integer spin particles giving rise to integer spin (composed) particles (at least from a first approach). We will develop this in future versions.

J. Previous approaches and relativistic considerations

It has been well known for several decades that the picture of an electron as a spinning charge sphere is not consistent from the relativistic framework. This is perhaps the main reason that led people to start looking for alternatives. In previous sections, we have made reference to the work of Cavalleri et al [60]; following [60], and for lack of our own calculations, we are aware that the helical trajectory we picture here may still collide with the relativistic c-limit for velocities; once more in analogy with [60], we do not see it as an unsolvable problem, given that customary renormalization procedures in QED, with their high-momentum transfer cut-offs, also stand for nothing different than renouncing Maxwell’s laws (and hence Lorentz invariance) at sufficiently low spatial scales.

K. Conclusions

In this section, we have shown that in classical kinematics there is room to accommodate all the main features of that mysterious, typically quantum property known as “spin”, at least where the states describe do not defy the rules of what we know as local-realism...

On the other hand, we have left dynamical aspects aside, for instance: (i) how is it possible that the electron performs an stable helical trajectory in free space? (ii) and in bound states (orbital stability)?

Aside from those questions, in future versions we will try to approach with further detail the physics underlying the measurement process $M$, instead of using a mere phenomenological model created for the occasion which is what we have done here; this of course implies a model with the (non-homogeneous) magnetic field created by the typical Stern-Gerlach device, etc... but, conceptually, we do not see where this could invalidate anything of what we have proposed here.

To conclude, the possible connections of this picture with an interpretation of the Thomas precession factor should not be ignored either: the helical movement of the electron makes necessary new relativistic corrections of the same kind thought to be necessary to modify the $H$-atom spin-orbit term so as to account for the well known additional $\frac{1}{2}$-factor, see [62].

VII. ON THE MEANING OF THE PAULI EXCLUSION PRINCIPLE

For two or more particles to be able to establish a stable energy interplay, what in QM we call a “bound state”, where “observables”, orbital degrees of freedom remain stable (or at least their averages remain stable) against a background of perturbations, inner degrees of freedom must necessarily enter in the picture, otherwise the instant excess/defect of energy would perturb irreversible the observable degrees of freedom. Bosons have those inner degrees of freedom (they have an inner structure); fermions do not, at least capable of intervening at the (energy) scale of interest. This means that when fermions enter the energy interplay of bound states, such bound state necessarily “binds” (imposes constraints on) their orbital degrees of freedom, energy-momentum, their “energy” state; bosons may belong to a bound state while keeping complete freedom for their energy-momentum. This difference has the potential to explain why energy states are mutually exclusive for fermions and not for bosons: from their most obvious nature, those energy interplays are mutually exclusive on the relevant set of degrees of freedom.

We articulate our reasoning in the following steps:

(I) On one side, systems of particles composed of at least two fermions (bosons with $s \geq 1$), possess additional degrees of freedom (elastic deformation, rotation), which allow them to resonate with the inner degrees of freedom of other particles/systems, without modifying the value of their “observable” degrees of freedom (the only ones contemplated in the mathematical formalism of QM: energy and linear momentum), hence their observable state (the information that QM provides about the system), their “energy state”.

(II) On the other, electrons do not present, at least capable or intervening at the scale of interest (see Sec. [62]), such inner degrees of freedom: as a consequence, whatever energy interplay they establish with other particles/systems (as an electron does with the nucleus in the atomic context) necessarily modifies the value of their orbital degrees of freedom, what we are calling their “observable” state.

(III) To the former two points, we add the fact that stable interplays between systems are mutually exclusive for a single energy, in respect to a given set of degrees of
freedom. This obviously forbids more than two fermions in the same state; the case of bosons is different, as inner degrees of freedom are the ones that accommodate such interplay, leaving the observable ones devoid of any constraint. For instance, in [50] we have seen that electrons interact with the nucleus in a sort of mutually excluding “modes”, with respect to the orbital degrees of freedom.

VIII. (AN APPROACH TO) THE INTERPRETATION OF OTHER TYPICALLY QUANTUM PHENOMENA

We have raw ideas on Bose/Einstein condensation (BEC), superfluidity as a result of BEC and superconductivity. See ref. [51].

IX. THE ISSUE OF NON-LOCALITY

A. Absence of conclusive experimental evidence: so far, Bell experiments prove nothing

Bell tests based on spin degrees of freedom (if photons are used, we have polarization-entanglement) can roughly be classified in three groups: the first two of them use photons (though some mixed schemes with particle-photon entanglement can also be implemented), the third involves only massive particles:

(A.1) Bell tests with limited efficiencies and implemented through data-rejection, based on the “fair sampling” (FS) assumption. A typical test of this type is testing the Clauser-Horne-Shimony-Holt inequality [11], as it was done in the second experiment of Aspect [17]. Any collision with local realism in this type of experiment is so far ruled out by the so called “efficiency loophole” (as a breaking of the FS assumption through data-rejection, as it was first acknowledged by Pearle [14]): see [32].

(A.2) Bell tests with limited efficiencies and no data-selection. A typical example of this type is testing the Clauser-Horne inequality [12], as it was done in third of Aspect’s experiments: see [16]-(ii). In principle, an “efficiency loophole” would still exist even without data-rejection [13]; much more than this we are interested in the fact that, while violations of local realism are claimed to have been obtained, these are computed on “operational inequalities” [13] involving additional hypothesis to those of the original, “genuine” version of those inequalities (the term “genuine” is due to Emilio Santos). The main of these hypothesis is the one known as “no-enhancement” (NEN): the probability of detection of a photon cannot be enhanced by the placement of any particular device (a polarizer, for instance) on its way to the detector”. This hypothesis is already implicitly included in those “operational expressions” of, for instance, the CH inequality [13]. Recently [47], we have explored in detail how the appearance of “enhancement” (ENH) makes it possible for an LHV model to violate a non-genuine Bell inequality (non-genuine: including the NEN hypothesis).

(B) Bell tests with massive particles. While devoid of the detection loophole, so far they do not meet conditions to rule out signaling between remote observers, or between the source and one of this observers. This is called the “locality loophole” or “communication loophole”; two of the main references amongst experiments performed so far are given in [18].

Recently, we had explored a different alley [16], with the final conclusion that there are experiments that rule out the kind of local models that we treated there, see [16]-(ii). This is not the case with the efficiency loophole (or what we would call, more precisely though they are very related, the “enhancement loophole” [47]): real experiments do not even approach the critical thresholds.

In regard to massive particles, we must say that, rather than the locality loophole (so far, as we have seen, still not overcome, and therefore enough to deprive them of any conclusiveness), we tend to think that, simply, in the same way as the Wigner-PDC picture suggest for photons, some quantum states (the singlet, for instance) do not correspond to real physical states: they are mere conceptual constructions, again appearing as the price for the shortcuts that the theory incorporates.

To complete the picture, the typical Franson (time-energy entanglement) is also explained as the result of wave-interference from this Wigner-PDC approach: see [20]-(ii).

FIG. 3: At least so far, experimental tests of Bell inequalities always include, implicitly or explicitly, additional assumptions: (at least for our models here) those assumptions are the ones that are violated, instead of local-realism. The diagram does not intend to be exhaustive.

B. The Wigner-PDC picture

Now, both the breach of FS from the efficiency loophole in (a.1) and ENH in (a.2) are nothing but natural consequences of the phenomena of “variable detection prob-
ability” (VDP), arising from the theoretical framework that we can call “Wigner picture of Quantum Optics based on Parametric Down Conversion” (Wigner-PDC). This approach keeps a one-to-one correspondence with an initial quantum electrodynamical model of entanglement-generation from PDC; this gives all legitimacy to the conjecture that VDP and ENH are not just inventions or pathologies of LHV models for the sake of keeping local realism alive, but important features of photon features that may have been overlooked in the frenzy to reach a conclusive proof of “non-locality”.

Of course we do not imply that QED as a whole is “local-realistic”; what we mean is that only the subset of possible states that respect local-realism do actually have a physical, preparable counterpart: in the Wigner-PDC approach, restriction to this subset is guaranteed from the point of view that the model takes the QED-vacuum state as its “input”, and this is a state that, through the so-called Wigner transform, accepts a local-realistic description. Parametric Down Conversion (PDC) [20, 21] is on the other hand a technique of entangled photon pairs generation that we can say seems to be nowadays standardized for the implementation of Bell tests and many other experiments with photons. Regarding its interpretation as a local theory, the Wigner-PDC picture still presents some gray areas (regarding the theory of detection [3], for instance) but conveniently enough, it works completely fine in regard to field correlations, reproducing the quantum predictions from a wave-like, completely locally realist framework (we are indeed working on them [3]; in any case, nothing like a dead-end alley: see [31].

When we say “natural consequences” what we mean is that the detection behavior (VDP, ENH, etc.) is not a mere physical accident, but a primary feature of the physical systems, represented by QM with an entangled photon-state: see [30]. Following this line of thought, we arrive to the following conjecture (far from new, as people like Marshall or Santos have been for long formulating similar ones): theoretical violations of Bell inequalities (the genuine versions, non-genuine ones can and indeed are experimentally violated, in [17] we show how) are simply an indicator of the non-physical character of some of the elements of the ordinary, quantum Hilbert-space. Of course, we are implying that such violations could never take place in a real experiment.

C. Towards a solution of the issue

In spite of a subset of all possible quantum states showing a “non-local” behavior (or what we would choose to call simply a non statistically interpretable one), affirmations such as “QM tells us that nature is non-local” do not hold up against, not only common sense, but neither against a serious scrutiny of decades of experimental evidence. Such evidence confirms the correctness of quantum predictions, yes, but also points in yet another direction: the need to refine our framework. The Wigner-PDC picture shows that such refinement may simply consist in the adoption of a field-theoretical description of light, within the formalism of QED. Perhaps not so needless to insist again on, we do not claim that QED as a whole is “local-realistic”, but that only the subset of possible states that respect local-realism do actually have a physical, preparable counterpart, and that the Wigner-PDC picture seems to establish this selection as a natural feature of the model.

From a different perspective, we have also recently come across a paper called “Origin and meaning of quantum nonlocality” by De La Peña et al (2011), stating that a model of a free particle immersed in the ZPF background exhibits, upon reduction of the com-
complete phase-space description (where fields in all points of space are considered) to one that models only the particle degrees of freedom, that mind-boggling non-locality. Quoting from them: “this reduction of the description from the complete phase space to the configuration space of the particle gives rise to a term representing the fluctuations of the momentum impressed by the zpf on the particle. This term is a function of the probability density \( \rho(x) \) and hence possesses a nonlocal nature, manifested even in one-particle quantum systems”. Of course, once we switch back to the complete description, that non-locality (at least for physically realizable states) disappears. This is exactly our point of arrival, only that in our case we have got there from observation of the Wigner-PDC models: there, even when quantum apparently non-local correlations keep being “observed” (see our definition of “observable” correlation in [49]), local-realism is recovered through the appearance of new observable features such as what we have called ENH and a reduced detection probability, as a particular case of VDP (see our own account in [48] and [49]).

X. SUMMARY AND OVERALL CONCLUSIONS

(still in progress)

In this paper, we have formulated a set of conjectures, all of them fitting in a common framework, and tried to provide some evidence of their reasonableness; away from further pretension, our goal was to convince that they deserve, at least, careful examination and, perhaps, some serious research. The framework that we refer to is somehow “inspired” (we do not need to invoke so much detail) the one known as Stochastic Electrodynamics, where the classical, Maxwellian picture of particle and fields is supplemented with the so-called Zero-Point Field (ZPF), a background of random field fluctuations, in equilibrium with matter.

Making a brief summary, we can (re)group our developments in the following points:

(a) The quantization (a discrete spectrum) of AM (orbital and spinorial, too) is strongly supported by obvious reasons within the framework of SED, at least in regard to the realm of molecular/atomic/nuclear/subnuclear physics, where matter seems to transit amongst a set of metastable states, probably involving many hidden classical degrees of freedom (this is necessary to justify their stability from the point of view of classical systems dynamics), that we associate with periodic (or quasi-periodic) trajectories in a classical configuration space. Moreover, (projections of) quantum AM would reflect an average over a cyclic classical orbit, a fact that strikes as almost obvious given the corresponding addition rules within the quantum formalism. From that point of view, AMQ should perhaps be considered the building block of QM, from where other features of theory should be examined, instead of a mere point of arrival as often is regarded.

(b) QM describes ensembles, well of systems well of experiments upon one system, supporting our views of quantum AM as an “average”. Combination of both approaches (“ensemble of systems” and “ensemble of experiments”) is, perhaps, the origin of the need for density matrices, beyond quantum pure states that would in principle be the obvious building block for the construction of the theory. This idea deserves much more attention, attention we will try to provide elsewhere.

(c) Transitions between those metastable states establish obvious constraints on the spectrum of energy-momentum that is exchanged: some very basic calculations support the idea that this is the origin of the “photon” with its arch-famous \( E = h \omega \) relation.

(d) As far as photon are concerned (including Bell polarization-entanglement schemes, but also Franson-like ones), the Wigner-PDC [20] picture of Quantum Optics proves that collisions with local realism are nowhere necessary: all we have to do is realize that the usual, discrete Hilbert-space formulation of the states of light is not the right one; we need a quantum formulation, yes, but a field-theoretical one (QED), where phenomena such as “variable detection probability” (VDP) and “enhancement” (ENH), or equivalently what many in the field of Quantum Information call a “limited detection efficiencies”, arise as natural features of the theory, rather than being introduced as external postulates, for instance in the form of (alleged) technological limitations of the experimental setups.

(e) Bell experiments with massive particles, on the other hand, do not close the locality loophole, but we nevertheless believe this is just a “secondary” feature: the key lies in the fact that states such as the so-called singlet may be mere conceptual constructions, devoid of physical meaning. Such an interpretation would keep a desirable symmetry with the photon case, and is in our opinion supported in the lack of evidence on the contrary (the EPR-Bohm experiment, for instance, has never been performed with massive particles).

(f) As an addition on former versions, recently we have also addressed the problem of the origin of spin, and specifically the particularities of half-integer spin, in Sec. VII We think that we have reasonably proven there that there is no ultimate barrier that prevents the inclusion of quantum spin in a purely classical framework.

The ZPF background plays, in some ways, a similar role to that of thermal noise in mechanical systems: it can be ignored unless we want to observe dynamics at a microscopic scale (so is the case, for instance, of phenomena such as spontaneous emission in the presence of a sufficiently intense external potential). There is a difference, anyway: in the case of purely mechanical systems, macro-dynamics is oblivious to thermal noise, while in electromagnetic ones, whether we choose or not to include the ZPF into the model, certain features of the macroscopical dynamics, such as typically quantum
ensemble behavior, are already in great part implicitly determined by the existence of this background, a necessary condition for some of the basic assumptions (AMQ) that shape the structure of the mathematical formalism of the theory itself (at least that is our conjecture). The origin of QM behavior in stochastic features of another underlying mechanics of purely classical nature, whether this one is implemented on the ZPF or another source of randomness, is a point on which we are not at all alone: amongst recent efforts in this directions we recommend [1].

Of course a huge number of issues are left open, many of which have already been treated in considerable detail by Fritsche and Haugk [1], whose paper we again strongly recommend. **Completing the picture, we have provided explicit (and we believe correct) connection with Fritsche and Haugk’s work [1].** In any case, definitely, non-locality is not for us a main issue (as we said, it is also present in Maxwell’s electromagnetism, and besides there are theorems forbidding the use of this feature to send superluminal information). All this said, the aim of this paper was to convince someone other than ourselves that Quantum Mechanics and classical physics do not necessarily have to be seen as so irreconcilable. Once (we believe) there is no conclusive proof of such incompatibility (see our conclusions in [19]), the former is definitely a possibility that deserves some more research.

*We do not claim all this to be entirely right, but we do claim that most of the ideas here should be regarded as necessary steps to explore, mainly for scientific hygiene and to better the understanding of what, yes, so far has worked extraordinarily well, and still does. Yes, we know, “Einstein and others already tried”.*

[1] L. Fritsche, M. Haugk. “Stochastic Foundation of Quantum Mechanics and the Origin of Particle Spin”. ArXiv, perhaps published already, too.

[2] A. Valdés-Hernández, L. de la Peña and A. M. Cetto. “Bipartite Entanglement Induced by a Common Background (Zero-Point) Radiation Field”, Foundations of Physics 41, 843-862 (2011).

[3] D. Rodríguez, “Wigner-PDC description of photon entanglement as a local theory: with and without polarizers”. ArXiv.

[4] “Can we celebrate Defeat for the Photon by Maxwell-Planck theory?”, in [http://crisisinphysics.wordpress.com/2011/08/01/can-we-celebrate-defeat-for-the-photon-by-maxwell-planck-theory-2/](http://crisisinphysics.wordpress.com/2011/08/01/can-we-celebrate-defeat-for-the-photon-by-maxwell-planck-theory-2/). Follow references there.

[5] “Stochastic Electrodynamics” considers a “Zero-Point Field” radiation background in addition to classical electromagnetism. This ZPF is consistent both with the vacuum fluctuations predicted by QM (indeed, it is parametrized by the constant \( \hbar \)) and special relativity. The basic framework was settled by T.W.Marshall in the sixties [6], and mainly T.H.Boyer [7] in the seventies. Reference [57] provides some brief but good (at the time) summary of SED’s achievements.

[6] For T.W.Marshall in SED, see: “Random Electrodynamics” in Proc. Roy. Soc. A, 276, 475-491 (1963), “Statistical electrodynamics” in Proc. Camb. Phil. Soc., 61, 537-546 (1965).

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T.W. Marshall, “Magical Photon or Real Zero-point? ” in *New Developments on Fundamental Problems in Quantum Physics*, eds. M. Ferrero and A. van der Merwe (Kluwer, Dordrecht, 1997) pages 231-245

[7] For T.H.Boyer in SED, see: “Derivation of the Blackbody Radiation Spectrum without Quantum Assumptions” in Phys. Rev., 182, No.5 (1969), “Classical Statistical Thermodynamics and Electromagnetic Zero-Point Radiation” in Phys. Rev., 186, No.5 (1969), “Random electrodynamics: The theory of classical electrodynamics with classical electromagnetic zero-point radiation” in Phys. Rev. D, 11, 790 (1975), “General connection between random electrodynamics and quantum electrodynamics for free electromagnetic fields and for dipole oscillator systems” in Phys. Rev. D, 11, 809 (1975), “Derivation of the blackbody radiation spectrum from the equivalence principle in classical physics with classical electromagnetic zero-point radiation” in Phys. Rev. D, 29, 1096 (1984).

[8] For others in SED, see for instance: “Behavior of classical particles immersed in the classical electromagnetic zero-point field”, A. Rueda, Phys. Rev. A, 23, 2020 (1980), “Radiation pressure from the vacuum: Physical interpretation of the Casimir force”, P. W. Milonni, R. J. Cook, M. E. Goggin, Phys. Rev. A, 38, 1621 (1988), “Derivation of the classical electromagnetic zero-point radiation spectrum via a classical thermodynamic operation involving van der Waals forces”, D.C.Cole, Phys. Rev. A, 42, No.4 (1990),
“Entropy and other thermodynamic properties of classical electromagnetic thermal radiation”, D.C. Cole, Phys. Rev. A, 42, No.12 (1990),
“Reinvestigation of the thermodynamics of blackbody radiation via classical physics”, D.C. Cole, Phys. Rev. A, 45, No.12 (1992),
“Stochastic electrodynamics with particle structure Part I: Zero-point induced Brownian behavior, A. Rueda, Foundations of Physics Letters, 6 No.1 (1993),
“Inertia as a zero-point-field Lorentz force”, Bernhard Haisch, Alfonso Rueda, H.E. Puthoff, Phys. Rev. A, 49, 678 (1994),
“Physics of the zero-point field: implications for inertia, gravitation and mass”, Bernhard Haisch, Alfonso Rueda, H.E. Puthoff, Speculations in Science and Technology 20, 99-114 (1997),
“Inertia as reaction of the vacuum to accelerated motion”, Bernhard Haisch, Alfonso Rueda, H.E. Puthoff, Speculations in Science and Technology 20, 99-114 (1997),
“Inertia as a zero-point-field force: Critical analysis of the Haisch-Rueda-Puthoff inertia theory”, Yefim S. Levin, Phys. Rev. A, 79, 012114 (2009).
[9] D.C.Cole, Y.Zou, “Quantum mechanical ground state of hydrogen obtained from classical electrodynamics”, Phys. Lett. A, 317, 14-20 (2003).
[10] Original Bell inequality: J.S. Bell, “On the Einstein-Podolsky-Rosen paradox”, Physics (Long Island City, N.Y.) 1, 195 (1964).
[11] CHSH inequality: J.F. Clauser, M.A. Horne, A. Shimony, and R.A. Holt, “Proposed experiment to test local hidden-variable theories”, Phys. Rev. Lett. 23, 880 (1969).
Let $A_i, A_2$ and $B_1, B_2$ be, respectively, pair of dichotomic $(\{\pm 1\})$ observables at two distant sides, we write

$$|\langle A_1 B_1 \rangle + \langle A_1 B_2 \rangle + \langle A_2 B_1 \rangle - \langle A_2 B_2 \rangle| \leq 2.$$  

As opposed to the original Bell inequality in [10], this inequality - it can be proven - remains valid for indeterministic theories (loss of information in the measurement process).

[12] CH inequality: J.F. Clauser, M.A. Horne, “Experimental consequences of objective local theories”, Phys. Rev. D 10, 526 (1974). Let $A_i, A_2$ and $B_1, B_2$ be, respectively, pair of dichotomic $(\{\pm 1\})$ observables at two distant sides, a possible formulation (there are equivalent ones) is

$$P(A_1 = B_1 = 1) + P(A_1 = B_2 = 1) + P(A_2 = B_1 = 1)$$
$$- P(A_2 = B_2 = 1) - P(A_1 = 1) - P(B_1 = 1) \leq 0, \quad (57)$$

This inequality considered, explicitly, indeterminism (although [11] includes this case as well).

[13] Operational expression used in experiments on the CH inequality (see, for instance, [17], eq. 32-33). Estimating, from the “number of counts” registered

$$P(A_i = 1, B_j = 1) \approx N(a_i, b_j)/N(\infty, \infty), \quad (58)$$
$$P(A_i = 1) \approx N(a_i, \infty)/N(\infty, \infty), \quad (59)$$
$$P(B_j = 1) \approx N(b_j, \infty)/N(\infty, \infty) \quad (60)$$

and substituting in [12] we obtain the inequality $S_{exp}(CH)$, with

$$S_{exp}(CH) = (N(a_1, b_1) - N(a_1, b_2) + N(a_2, b_1) - N(a_2, b_2)$$
$$- N(a_2, \infty) - N(\infty, b_1))/N(\infty, \infty), \quad (61)$$

where, in relation to [12], $a_i \equiv A_i = +1, b_j \equiv B_j = +1$ and $\infty$ means the photon is detected with no polarizer or device placed on its way.

On the other hand, to estimate there the marginal probabilities we are assuming no enhancement: a probability of detection cannot be enhanced by placement of a polarizer between the source and the detector.

[14] For the origin of the detection loophole from “data-rejection” see:
(i) P.M. Pearle, “Hidden-variable example based upon data rejection”, Phys. Rev. D 2, 1418 (1970). Also include a reference to E.P. Wigner: private communication to P.M. Pearle.
(ii) A. Garg and N.D. Mermin, “Detector inefficiencies in the Einstein-Podolsky-Rosen experiment”, Phys. Rev. D 35, 3831 (1987).
(iii) J. -Å. Larsson, “Necessary and sufficient detector-efficiency conditions for the Greenberger-Horne-Zeilinger paradox”, Phys. Rev. A 57, R3145 (1998).
(iv) “Bell’s inequality and detector inefficiency”, Phys. Rev. A 57, 3304 (1998). The original Bell inequality is treated, and also revisits CHSH (already treated in Garg and Mermin’s work).

[15] Detection loophole without data-rejection:
(i) P.H. Eberhard, “Background level and counter efficiencies required for a loophole-free Einstein-Podolsky-Rosen experiment”, Phys. Rev. A 47, R747 (1993).
(ii) J.-Å. Larsson and J. Semitecolos, “Strict detector-efficiency bounds for n-site Clauser-Horne inequalities”, Phys. Rev. A 63, 022117 (2001).

[16] Some of the most relevant experiments with photons:
(i) Aspect’s second experiment, testing the CHSH inequality [11], involving the fair sampling hypothesis: A. Aspect, P. Grangier, G. Roger. Phys Rev. Lett. 49 91 (1982).
(ii) Aspect’s third experiment, testing the CH inequality [12], using post-selection of observables (famous, gets rid of locality loophole) but involving the no-enhancement hypothesis: A. Aspect, J. Dalibard, G. Roger. “Experimental test of Bell’s inequalities using variables analyzers”, Phys Rev. Lett. 49 1804 (1982).
(iii) There are also proposals with time-energy entanglement, first one in: J.D. Franson, “Bell inequality for position and time”, Phys. Rev. Lett. 62, 2205 (1989), and performed, for instance, in P.R. Tapster, J.G. Rarity, P.C.M. Owens. “Violations of Bell’s inequality over 4km of optical fiber”, Phys. Rev. Lett. 73, 1923 (1994).
W. Tittel, J.Brendel, T. Herzog, H. Zbinden, N. Gisin, “Non-local two photon correlations using interferometers separated by 35 meters”, Europhys. Lett. 40, 595 (1997).
(iv) Weis’s experiment: G. Weis, T.Jennewein, C. Simon, H. Weinfurter, A. Zeilinger. “Violations of Bell’s inequality under strict Einstein locality conditions”, Phys. Rev. Lett. 81, 5039 (1998).
[17] A very nice account of photon Bell experiments by Alain Aspect himself: A. Aspect, “Bell’s theorem: the naive view of an experimentalist”, in “Quantum (Un)speakables - From Bell to Quantum Information”, ed. by R.A. Bertlmann and A. Zeilinger, Springer (2002).

[18] Bell’s tests without detection loophole:
(i) M. A. Rowe, D. Kielpinski, V. Meyer, C. A. Sackett, W. M. Itano, C. Monroe, and D. J. Wineland. “Experimental violation of a Bell’s inequality with efficient detection”, Nature (London) 409, 791 (2001).
(ii) D.N. Matsukevich, P. Maunz, D. L. Moehring, S. Olmschenk, and C. Monroe, “Bell inequality violation with two atomic qubits”, Phys. Rev. Lett. 100, 150404 (2008).

[19] Stochastic optics, previous to the connection with the Wigner picture, see:
(i) Marshall T W and Santos E 1988 Foundations of Physics 18, Number 2, 185-223.
(ii) Marshall T W and Santos E 1989 Phys. Rev. A 39, 6271-6283.

[20] Stochastic optics (SO), with Parametric Down Conversion [22] and using the Wigner picture, see:
(i) Casado A, Marshall T W and Santos E, J. Opt. Soc. Am. B 14 494 (1997),
(ii) Casado A, Fernández-Rueda A, Marshall T W, Risco-Delgado R and Santos E, Phys. Rev. A, 55 3879 (1997),
(iii) Casado A, Fernández-Rueda A, Marshall T W, Risco-Delgado R and Santos E, Phys. Rev. A, 56 2477 (1997),
(iv) Casado A, Marshall T W and Santos E J. Opt. Soc. Am. B 15 1572 (1998),
(v) Casado A, Fernández-Rueda A, Marshall T W, Martínez J, Risco-Delgado R and Santos E, Eur. Phys. J. D 11 465 (2000),
(vi) Casado A, Marshall T W, Risco-Delgado R and Santos E, Eur. Phys. J. D 13 109 (2001),
(vii) Casado A., Risco-Delgado R., y Santos E., Z. Naturforsch. 56a 178 (2001).

[21] Last - very interesting - developments in the Wigner picture of SO and PDC:
(i) “Wigner representation for experiments on quantum cryptography using two-photon polarization entanglement produced in parametric down-conversion”, Casado A, Guerra S and Plácido J, J. Phys. B: At. Mol. Opt. Phys. 41 045501 (2008),
(ii) “Partial Bell-State Analysis with Parametric down Conversion in the Wigner Function Formalism”, Casado A, Guerra S and Plácido J, Advances in Mathematical Physics, Volume 2010 (2010).

[22] Parametric Down Conversion: a pair of entangled photons is obtained by pumping a laser beam into a nonlinear crystal. Within the Wigner formalism [20], this generation can be interpreted as the non-linear mix of the laser frequency with the components of the Zero Point Field. A recent review on all this techniques is [23], for more details we refer the reader to the series of papers cited in [21].

[23] Keiichi Edamatsu. “Entangled Photons: Generation, Observation and Characterization”. Japanese Journal of Applied Physics 46, 7175 (2007).

[24] Other recent very interesting work in SED:
“Bipartite Entanglement Induced by a Common Background (Zero-Point) Radiation Field”, A. Valdés-Hernández, L. de la Peña and A. M. Cetto, Foundations of Physics Vol. 41, Number 5, 843-862.

[25] Main reference on enhancement, variable detection probability: R. Risco-Delgado. “Localidad y no localidad bajo recientes resultados teóricos y experimentales”, PHD Thesis. Universidad de Sevilla (1997).

[26] The picture of spin from a classically spinning - structured - particle collides with special relativity, this has been known for eighty years or so. Anyway, taking into account that special relativity is nothing more than a reinterpretation of Maxwell’s laws... are those QED-renormalization procedures (cut-offs on high momentum transfers) something different than renouncing (or tuning down) the laws of electromagnetism at the smallest range of distances?

[27] Nuclei, or even protons or neutrons, also have internal structure and a discrete spectrum of internal states.

[28] For a point particle of mass $m$ and charge $-q$ ($q > 0$), under the action of a electrostatic potential created by another charge $+q$, going along a circular stationary orbit, $v = \omega r$, and we have

$$m v^2 = m \omega^2 r = \frac{q^2}{4 \pi \varepsilon_0 r^2}. \tag{62}$$

and, therefore, in terms of energy,

$$E_{\text{kin}} = \frac{1}{2} m v^2 = \frac{1}{2} m \omega^2 r^2 = \frac{1}{2} \frac{q^2}{4 \pi \varepsilon_0 r} = \frac{1}{2} q V(r) = \frac{1}{2} E_{\text{pot}}, \tag{63}$$

where $V(r) \leq 0$ is the electrostatic potential.

[29] We have come across something like this in [50], though there the relevant modes are implemented on vibrations of the inner degrees of freedom of the nucleus, the movement of the electron is somehow bound to those vibrations. Two vibrations given by the same set of parameters are forgiven by the system’s dynamical equations. A sort of pedestrian Pauli principle?!

[30] To explain the enhancement of the detection probability as a result of the interposition of a polarizer between the source and the detector, we have to take into account that polarization components reflected by the polarizer must be replaced, at the entrance of the detector, by new components given by vacuum amplitudes at that point. This has been treated in detail, for circular polarizers, in T. W. Marshall and E. Santos: Found. Phys. Lett. 18, 185-223 (1988), Phys. Rev. A 39, 6271-6283 (1989), Found. Phys. Lett. 5, 573-578 (1992). For “polarizing beam splitters”, see some of the last papers in [20].

[31] To our knowledge, so far all experimental refutations of Casado et al’s work are only partial (regarding just previous - not at all definitive - attempts to interpret the detection probability expressions in consistency with local-realism) or merely apparent, as they require additional assumptions.

(i) Within the first group, we are aware of:
G. Brida, M.Genovese, M. Gramegna, C. Novero, E. Predazzi, arxiv.org/abs/quant-ph/0203048, disproving the model of detection proposed in http://arxiv.org/abs/quant-ph/0202097

The “detection” issue has recently been also treated by us in an arXiv draft titled “Wigner-PDC as a local-realistic theory”.

(ii) Within the second, some recent experimental work
We have for instance \( \eta \) in another of our papers we have worked hard to prove the parametrization implying the need for four real parameters. However, with an infinitesimal parameter orthogonal directions. For a finite transformation generated to establish a correspondence with a physical observable.

Indeed, for instance linear momentum projections are as-

dressed in the form

\[
\psi(x, p(x)) = \int \psi^* (x, p(x)) \, x \, \psi(x, p(x)) \, dx,
\]

fixes the position operator \( \hat{X} \) so that

\[
\psi(x, p(x)) = \hat{X} \psi(x, p(x)) \, dx = \int \psi^* (x, p(x)) \, x \, \psi(x, p(x)) \, dx,
\]

and settles

\[
\psi(x) = M(x) \, e^{i \theta(x, p(x)).}
\]

Of course, this is not really choice, just the way we write that the wavefunction is the position operator; there is only one real choice in the construction of the quantum mechanical wavefunction and it concerns the way linear momentum (LM) information is codified within it (or equivalently, the LM operator is not completely fixed by just specifying that is the generator of translations, see Sec. III G).

Let \( \hat{P} \) be the linear momentum (LM) operator, the introduction of an external potential in the problem can be carried out by the so-called “minimal coupling” substitution, for instance:

\[
\hat{P} \rightarrow \hat{P} - \frac{q}{c} \mathbf{A},
\]

with \( \mathbf{A} \) the electromagnetic (vectorial) potential, \( q \) the electron charge and \( c \) the speed of light; the quantity at right hand side is usually the most directly accessible and therefore sometimes interpreted as an “effective” LM of the particle (and enters Hamilton’s equations as the “canonical” momentum), while \( \hat{P} \) would play the role of the “physical one.”

What we have det(\( A \)) = 0, where in general \( A \neq A' - \lambda \mathbb{1} \)
for any \( A' \).

With \( X_i \equiv X_i \mathbb{1} \), we have

\[
\hat{L} = \hat{f} \wedge \hat{P} = \begin{vmatrix}
X_1 & X_2 & X_3 \\
\xi_1 & \xi_2 & \xi_3 \\
\hat{P}_1 & \hat{P}_2 & \hat{P}_3 \\
\end{vmatrix}.
\]

The number of independent parameters \( \alpha \) in the most general expression of a transformation is equal to the number of (independent) generators of the group. On the other hand, differential operators, as for instance the momentum operator, can be seen as operators in an infinite-dimensional space.
[41] In general we will refer to any trajectory confined to a finite region of the overall dynamical configuration space; for instance, we are thinking in the trajectory of an electron inside a particular atomic orbital. This kind of behavior is typical (and well known) from non-linear dynamics.

[42] R. Penrose, “The road to reality”.

[43] A. Einstein, B. Podolsky, N. Rosen. “Can quantum-mechanical description of physical reality be considered complete?”. Phys. Rev. 47, 777 (1935).

[44] Attila Szabo, Journal or chemical education, 46 No.10 (1969), “Contour diagrams for relativistic orbitals”.

[45] A.F. Kracklauer. “Is ‘entanglement’ always entangled?”, J. Opt. B: Quantum Semiclass. Opt. 4 S121 (2002).

[46] D. Rodríguez, “Communication loophole in a Bell-EPR-Bohm experiment: standard no-signaling may not always be enough to exclude local realism”. Arxiv.

[47] D. Rodríguez, “Detection probability enhancement as a natural feature of Local Hidden Variable models (and Quantum Mechanics too?)”. Arxiv.

[48] D. Rodríguez, “Bell tests with photon-entanglement: LHV models and critical efficiencies at the light of Wigner-PDC optics”. ArXiv.

[49] D. Rodríguez, “QM, non-locality and the vacuum fluctuations: conclusions and new directions”. ArXiv.

[50] D. Rodríguez, “Orbital stability and the quantum atomic spectrum from Stochastic Electrodynamics”. ArXiv.

[51] D. Rodríguez, “Towards an stochastic electrodynamical model of superconductivity (and any other superfluidity phenomenon involving Bose-Einstein condensation)”. In progress, check www.drb-mphysics.net or ask the author.

[52] Between SU(2) and SO(3) there is a local isomorphism. Anyway, the ambiguity can be reduced to a bi-valuated correspondence between elements of both sets: on one side, a $\theta$-rotation in real space corresponds to a $\theta/2$-rotation in the Hilbert space; on the other, each real rotation can be made to correspond simultaneously to two complex-space angular rotations: for instance, the angular interval $[0, 2\pi]$ can correspond either to $[0, \pi]$ or $[\pi, 2\pi]$ in spin space.

[53] There is a direct isomorphism between integer spin and AM. On the other hand, between SU(2) and SO(3) there is indeed also a local isomorphism, but not a global one. Anyway, the ambiguity can be reduced to a bi-valuated correspondence between elements of both sets: on one side, a $\theta$-rotation in real space corresponds to a $\theta/2$-rotation in the Hilbert space; on the other, each real rotation can be made to correspond simultaneously to two complex-space angular rotations: for instance, the angular interval $[0, 2\pi]$ can correspond either to $[0, \pi]$ or $[\pi, 2\pi]$ in spin space.

[54] I.e, the possibility that they travel at the speed of light so no co-moving reference frame of reference is possible.

[55] For instance, let us suppose substructure of the particle provides an stabilizing dynamics for the “spin” components of the momentum: this would privilege those components over the “macroscopic” ones, that would be modified by external potentials and also fluctuate due to the interaction with the background.

The interplay of the particle sub-structure at one or other scale is the element that we believe is behind the “Pauli principle”, and in general behind the distinction between Fermi and Bose statistics.

[56] As a matter of fact and in relation to a uniform helical trajectory, we must remark:

(i) it is only the projection on the normal plane to the axis of the helix that is cyclic;

(ii) in this case only the component of the AM parallel to that axis is independent of the origin of coordinates (i.e., the one that arises from the components of linear momentum in that transversal plane).

Points (i) and (ii) are in any case more than sufficient for all our developments here.

[57] H.E.Puthoff, “Ground state of hydrogen as a zero-point-fluctuation-determined state”, Phys. Rev. D, 35, No.10 (1987).

Based on the properties of the Zero Point Field, filling the vacuum, Puthoff proved that, for a trajectory (on a plane) with radius $r$ and angular frequency $\omega$, a particle of mass $m$, we have

$$m\omega r^2 = \hbar,$$

which corresponds, in quantum terms, to one of the three projections, for orbital angular momentum $l = 1$.

Once spin is understood as a micro-orbital movement, the same argument is applicable.

[58] We reason now upon Fig. 2 if we let $v_{pt}$ and $v_{av}$ be the velocity of the point particle describing the helix and the velocity of its center, respectively, for $v_{pt} >> v_{av}$, $\Omega \rightarrow 0$; for both of the same order ($v_{pt} \sim v_{av}$) what we have is $0 < \Omega < 1$. At this point, our conjecture is: integration on the observation window (due to $\Omega > 0$) has the potential to produce a decrease $J_i = \pm \hbar \rightarrow J_i = \pm \hbar/2$, also eliminating the possibility of a null (transversal) average projection. Of course, for a perfect match with the quantum values the measurement process $M[\cdot]$ should guarantee that $\Omega \rightarrow R_0$ such that $J_i \rightarrow \pm \hbar/2$ during the observation window.

I.e, departing from $\left| \frac{1}{\Delta t} \int_{\Delta t} \hat{B} \cdot \hat{n}(t) \, dt \right| \approx \frac{1}{2},$

where $\hat{B}, \hat{n}(t)$ are unitary vectors, and recall, and we wish that this is just a purely phenomenological model that does not intend to reflect the actual physical process.

[59] Wavefunctions and state vectors would merely be mathematical devices to represent (in a compact way) the state of a physical system given in terms of the usual kinematic properties (position and momentum) in classical physics. We have also developed this idea elsewhere.

[60] A review on SED (with and without spin): G. Cavalieri, F. Barbero, G. Bertazzi, E. Cesaroni, E. Tonni, L. Bosi, G. Spavieri, G.T. Gillies, Front. Phys. China, 5(1): 107-122 (2010), “A quantitative assessment of Stochastic Electrodynamics with spin (SEDS): Physical principles and novel applications”.

[61] We are aware (if there is a doubt) how the actual Stern-Gerlach measurement is performed: inhomogeneous field acting on a magnetic dipole, etc. Our model is enough for our purposes here, though, which are merely to show that SED provides space for a rather simple, phenomenological model of half integer spin.

[62] Cite paper from 93: additional velocity components (around main trajectory) introduce a correction of the
same order of the SO interaction, potentially reducing its magnitude in the well known $\frac{1}{2}$ Thomas factor. See: ...

[63] On our model of spin: dynamical stability in free space? Radiation/absorption balance fixes, as a necessary condition, the average AM of the helical trajectories... but only if they indeed can persist in time, even against the perturbations from the background, i.e., only if they are indeed stable. Power balance is only a necessary condition, not a sufficient one; sooner or later we have to face the question of how those trajectories can retain stability, in a similar fashion to what was our point of departure in our work regarding atomic orbital stability [50]. From the point of view of classical physics (SED) and the Theory of Dynamical Systems, there is only one possibility: the intervention of inner degrees of freedom in the structure of the particles. In the case of a nucleon (proton, neutron), such inner structure is obvious (whether we choose to use a more or less complicated model), and we have already made use of it in [50]; in the case of an electron, the question will stay open...