HALO DENSITY PROFILES CONSISTENT WITH ASYMMETRIC M–B VELOCITY DISTRIBUTIONS: IMPLICATIONS ON DIRECT DARK MATTER SEARCHES

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ABSTRACT

In this paper, we obtain the weakly interacting, massive particle (WIMP) velocity distribution in our vicinity starting from spherically symmetric WIMP density profiles in a self-consistent way by employing the Eddington approach. By adding a reasonable angular-momentum-dependent term in the expression of the energy, we obtain axially symmetric WIMP velocity distributions as well. We find that some density profiles lead to approximate Maxwell–Boltzmann distributions, which are automatically defined in a finite domain, i.e., the escape velocity need not be put by hand. The role of such distributions in obtaining the direct WIMP detection rates, including the modulation, is studied in some detail and, in particular, the role of the asymmetry is explored.

Key words: dark matter – galaxies: halos

Online-only material: color figures

1. INTRODUCTION

The combined MAXIMA-1 (Hanary et al. 2000; Wu et al. 2001; Santos et al. 2002), BOOMERANG (Mauskopf et al. 2002; Mosi et al. 2002), DASI (Halverson et al. 2002), and COBE/DMR cosmic microwave background (CMB) observations (Smoot et al. 1992 (COBE Collaboration)) imply that the universe is flat (Jaffe et al. 2001), \( \Omega = 1.11 \pm 0.07 \), and that most of the matter in the universe is dark (Spergel et al. 2003), i.e., exotic. Combining the recent WMAP data (Komatsu et al. 2009) with other experiments one finds

\[
\Omega_b = 0.0456 \pm 0.0015, \quad \Omega_{\text{CDM}} = 0.228 \pm 0.013, \quad \Omega_{\Lambda} = 0.726 \pm 0.015.
\]

Since the nonexotic component cannot exceed 40% of the cold dark matter (CDM; Bennett et al. 1995), there is room for the exotic weakly interacting, massive particles (WIMPs). Supersymmetry naturally provides candidates for the dark matter constituents (Ellis & Roszkowski 1992; Goodman & Witten 1985). In the most favored scenario of supersymmetry, the lightest supersymmetric particle (LSP) can be simply described as a Majorana fermion, a linear combination of the neutral components of the gauginos and higgsinos (Goodman & Witten 1985; Bottino et al. 1997; Arnowitt & Nath 1995, 1999; Bednyakov et al. 1994). In most calculations, the neutralino is assumed to be primarily a gaugino, usually a bino. Other particle models have also been considered, like Kaluza–Klein WIMPs (see, e.g., the recent work Oikonomou et al. 2007 and references therein), sterile neutrinos (Laine & Shaposhnikov 2008), technicolor (Gudnason & Kouvaris 2006), and recently composite WIMPs (Khlopov & Kouvaris 2008; see also the recent theory review Raby et al. 2008). Even though there exists firm indirect evidence for a halo of dark matter in galaxies from the observed rotational curves, it is essential to directly detect such matter.

The possibility of such detection, however, depends on the nature of the dark matter constituents (WIMPs). Since the WIMP is expected to be very massive, \( m_X \geq 30 \text{ GeV} \), and extremely nonrelativistic with average kinetic energy \( T \approx 50 \text{ KeV} \), it can be directly detected mainly via the recoiling of a nucleus \((A, Z)\) in elastic scattering. The event rate for such a process can be computed from the following ingredients (Vergados 2007).

1. An effective Lagrangian at the elementary particle (quark) level obtained in the framework of supersymmetry as described, e.g., in Bottino et al. (1997), Bednyakov et al. (1994), and Vergados (1996).
2. A well defined procedure for transforming the amplitude obtained using the previous effective Lagrangian from the quark to the nucleon level, i.e., a quark model for the nucleon. This step is not trivial, since the obtained results depend crucially on the content of the nucleon in quarks other than up and down. This is particularly true for the scalar couplings, which are proportional to the quark masses (Drees & Nojiri 1993; Cheng 1989; Vergados 2007) as well as the isoscalar axial coupling (Ellis & Karlhner 1993; Vergados 2007).
3. Knowledge of the relevant nuclear matrix elements (Ressell et al. 1993; Ressell & Dean 1997; DiVari et al. 2000; Kosmas & Vergados 1997), obtained with as many reliable body nucleon wave functions as possible. Fortunately, in the case of the scalar coupling, which is viewed as the most important, the situation is a bit simpler, since then one needs only the nuclear form factor.
4. Knowledge of the WIMP density in our vicinity and its velocity distribution. Since the essential input here comes from the rotational curves, dark matter candidates other than the LSP (neutralino) are also characterized by similar parameters.

In the past, various velocity distributions have been considered. The one most used is the isothermal Maxwell–Boltzmann (M–B) velocity distribution with \( \langle v^2 \rangle = (3/2)\nu_0^2 \), where \( \nu_0 \) is the velocity of the Sun around the galaxy, i.e., \( 220 \text{ km s}^{-1} \). Extensions of this M–B distribution were also considered, in particular those that were axially symmetric with enhanced dispersion in...
the galactocentric direction (Drukier et al. 1986; Collar et al. 1992; Vergados 2000). In all such distributions, an upper cutoff $v_{\text{esc}} = 2.84 u_{0}$ was introduced by hand, in the range obtained by Kochanek (1996). In a different approach, Tsallis type functions, derived from simulations of dark matter densities were employed, see, e.g., recent calculations (Vergados et al. 2008) and references therein.

Nonisothermal models have also been considered. Among those one should mention the late infall of dark matter into the galaxy, i.e., caustic rings (Sikivie 1999, 1998; Vergados 2001; Green 2001; Gelmini & Gondolo 2001), dark matter orbiting the Sun (Copi et al. 1999), and Sagittarius dark matter (Green 2002).

The correct approach in our view is to consider the Eddington proposal (Eddington 1916), i.e., to obtain both the density and the velocity distribution from a mass distribution, which depends both on the velocity and the gravitational potential. Our motivation in using the Eddington (1916) approach to describing the density of dark matter is founded, of course, in his success in describing the density of stars in globular clusters. Since this approach adequately describes the distribution of stars in a globular cluster in which the main interaction is gravitational and because of its generality, we see no reason why such an approach should not be applicable to dark matter that also interacts gravitationally. It seems, therefore, not surprising that this approach has been used by Merritt (1985a) and applied to dark matter by Ullio & Kamionkowski (2001) and more recently by us (Owen & Vergados 2003; Vergados & Owen 2007).

It is the purpose of this paper to extend the previous work and obtain a dark matter velocity distribution, which need not be spherically symmetric, consistent with assumed halo matter distributions with a natural upper velocity cutoff. It will then be shown that this distribution can be approximated by an $M$–$B$ distribution with a finite domain that depends on the asymmetry parameter. The distribution obtained will be used to calculate WIMP direct detection rates including the annual modulation, as a function of the asymmetry parameter.

2. THE DARK MATTER DISTRIBUTION IN THE CONTEXT OF THE EDDINGTON APPROACH

As we have seen in Section 1, the matter distribution can be given (Vergados & Owen 2007) as follows:

$$dM = 2\pi f(\Phi(r), v_{r}, v_{t}) dxdydzv_{r}dv_{t}dv_{r},$$

where the function $f$ is the distribution function, which depends on $r$ through the potential $\Phi(r)$ and the tangential and radial velocities $v_{r}$ and $v_{t}$. We will limit ourselves in spherically symmetric systems. Then, the density of matter $\rho(r)$ satisfies the equation.

$$d\rho = 2\pi f(\Phi(|r|), v_{r}, v_{t})v_{r}dv_{t}dv_{r}.$$  (2)

The distribution is a function of the total energy.

1. The energy $E$ is $\Phi(r) + \frac{v_{t}^{2}}{2}$. Then,

$$\rho(r) = 4\pi \int \left(\Phi(r) + \frac{v_{t}^{2}}{2}\right)\frac{v^{2}}{2}d\nu = 4\pi \int_{0}^{E} f(E)\sqrt{2(E - \Phi)}dE.$$  (3)

This is an integral equation of the Abel type. It can be inverted to yield

$$f(E) = \frac{\sqrt{2}}{4\pi^{2}} \frac{d}{dE} \int_{E}^{0} \frac{d\Phi}{\sqrt{\Phi - E}} d\Phi.$$  (4)

The above equation can be rewritten as

$$f(E) = \frac{1}{2\sqrt{2\pi^2}} \int_{E}^{0} \left[ \frac{d\Phi}{\sqrt{\Phi - E}} d\Phi - \frac{1}{\sqrt{-E}} d\Phi \right]_{\Phi=0}.$$  (5)

In order to proceed, it is necessary to know the density as a function of the potential. In practice, this can be done analytically only in a few cases. This, however, is not a problem, since this function can be given parametrically by the set $(r(r), \Phi(r))$ with the position $r$ as a parameter. The potential $\Phi(r)$ for a given density $\rho(r)$ is obtained by solving Poisson’s equation.

Once the function $f(\Phi)$ is known, we can obtain the needed velocity distribution $f_{r}(\nu)$ in our vicinity ($r = r_{0}$) by writing

$$f_{r}(\nu) = f(\Phi(r)|_{\nu = r_{0}} + \frac{r_{0}^{2}}{2}) / \rho(r = r_{0}).$$  (6)

2. We suppose now that there is an additional kinetic term associated with an angular momentum (Binney & Tremain 2008), i.e., models of the Osipkov–Merritt type (Osipkov 1979; Merritt 1985a, 1985b)

$$Q = E + \frac{J^{2}}{2r_{0}^{2}} = \Phi(r) + \frac{v_{t}^{2}}{2} + \frac{|r \times \nu|^{2}}{2r_{0}^{2}} = \Phi(r) + \frac{v_{r}^{2}}{2} + \frac{v_{t}^{2}}{2} + \left(1 + \frac{r_{0}^{2}}{r^{2}}\right) \frac{v_{t}^{2}}{2},$$

where $v_{r}$ and $v_{t}$ are the radial, i.e., outward from the center of the galaxy, and the tangential components of the velocity, respectively, and $r_{0}$ is the “anisotropy radius” to be treated as a phenomenological parameter. We now have

$$\rho(r) = 2\pi \int f(\Phi(r) + \frac{v_{t}^{2}}{2} + \left(1 + \frac{r_{0}^{2}}{r_{0}^{2}}\right) \frac{v_{t}^{2}}{2})v_{r}dv_{t}dv_{r},$$

The last integral takes the form

$$\rho(r) = 4\pi \left(1 + \frac{r^{2}}{r_{0}^{2}}\right)^{-1} \int_{\Phi}^{0} f(E)\sqrt{2(E - \Phi)}dE.$$  (9)

This equation is formally the same with Equation (3) with the understanding that

$$\rho(\Phi) \rightarrow \tilde{\rho}(\Phi, r_{0}) = \rho(\Phi) \left(1 + \frac{(r(\Phi))^{2}}{r_{0}^{2}}\right).$$  (10)

$r(\Phi)$ can be obtained by inverting the equation $\Phi = \Phi(r)$. In practice, this is not needed, if, as we have mentioned above, we use $r$ as a parameter. We thus find

$$f(Q) = \frac{1}{2\sqrt{2\pi^2}} \int_{Q}^{0} \left[ \frac{d\Phi}{\sqrt{\Phi - Q}} d\Phi - \frac{1}{\sqrt{-Q}} d\Phi \right]_{\Phi=0}.$$  (11)
The velocity distribution in our vicinity becomes
\[ f_{r_c}(v) = f \left( \Phi(r)_{r=r_c} + \frac{v^2}{2} + \left( 1 + \frac{r^2}{r_0^2} \right) \frac{v^2}{2} \right) \] (12)
and is only axially symmetric. The isotropic case follows as a special case in the limit \( r_0 \to \infty \).

It should be noted that for some density profiles, such as the Navarro–Frenk–White (NFW) described in the following section, the needed derivatives of the function \( \rho(\Phi, r_0) \) with respect to the potential do not approach zero sufficiently rapidly as the potential vanishes. This can be cured by replacing \( r^2/r_0^2 \) at large distances by the substitution
\[ \frac{r^2}{r_0^2} \to \frac{\Lambda(r)}{r_0^2}, \quad r > r_c. \]
The function \( \Lambda \) must be such that it goes to zero sufficiently fast at large distances and its derivatives, up to third order, are continuous at \( r = r_c \). The precise form does not matter much, provided that \( r_c \) is of the order of \( r_0 \). In this work, the function \( \Lambda(r) \) was taken to be a third-degree polynomial times an exponential.

The characteristic feature of this approach is that the velocity distribution vanishes outside a given region specified by a cutoff velocity \( v_m \), by the positive root of the equation \( f_{r_c}(v) = 0 \). In an exact treatment \( v_m = \sqrt{2|\Phi(r_c)|} \).

3. SIMPLE REALISTIC DARK MATTER DENSITY PROFILES

There are many halo density profiles, which have been employed, see, e.g., a recent summary (Kazantzidis et al. 2006 and references therein). Among the most commonly used analytic profiles (Zhao 1996), we will consider the following.

1. The VO density profile (Vergados & Owen 2007)
\[ \rho(x) = \rho_0 \left( \frac{2[(c+1)^3]}{(c+1)^3} - \frac{1}{(c+1)^2} \right), \quad x > c \]
with \( a \) the radius of the Galaxy. The distance \( c \) is very large, so that the rotational velocity remains essentially constant with the distance from the center of the galaxy even at quite large distances. The above form was taken by the requirement that at \( x = c \) the density is continuous with a continuous derivative. This will be referred to as the VO profile. The resulting potential can be obtained by solving Poisson’s equation (Vergados & Owen 2007).

2. Another simple profile is
\[ \rho(x) = \frac{\rho_0}{x(1+x)^2}, \quad x > a \] (14)
known as the NFW distribution (Navarro et al. 1996; Ullio & Kamionkowski 2001), which has been suggested by N-body simulations. This profile provides a better description of the expected density near the center of the galaxy. It does not, however, predict the constancy of the rotational velocities at large distances.

As we have seen above, the dependence of the density on the potential is much more interesting. We thus show these functions in Figure 1.
but in addition by our previous work, in which this shape has been obtained by an appropriate limit of velocity distributions described by the radial and tangential Tsallis type functions (Vergados et al. 2008). The parameter $b$ and the allowed range of $y$, which are functions of $\beta$, are shown in Table 1.

Integrating the distribution of Equation (18) over the angles, we obtain the results shown in panel (b) of Figure 3. The agreement, with the possible exception of the portion at high velocities, is reasonable.

Table 1. Parameters Describing the M–B Distribution.

| $\beta$ | 0.5  | 0.39 | 0.31 | 0.22  | 0.14  | 0.00  |
|---------|------|------|------|-------|-------|-------|
| NFW     | $b$  | 0.358| 0.376| 0.391 | 0.396 | 0.412 | 0.416 |
| NFW     | $y_{\text{max}}$ | 1.24 | 1.25 | 1.26 | 1.27  | 1.28  | 1.29  |

Notes. We show the parameters describing the M–B distribution, which is a good fit to the velocity distribution derived from the NFW dark matter density profile via the Eddington approach. The maximum velocity $y_{\text{max}}$ is given in units of $\sqrt{\Phi_0}$. 

Figure 1. NFW density $\rho/\rho_0$ as a function of the potential $\Phi/\Phi_0$. (a) The same quantity in the case of the VO profile (b). $\Phi_0 = 4\pi G N a^2 \rho_0$. (A color version of this figure is available in the online journal.)

Figure 2. Rotational velocity due to dark matter as a function of the distance in units of $\sqrt{|\Phi_0|} = \sqrt{4\pi G N a^2 \rho_0}$. Shown on the left is the one obtained with the VO density profile, while on the right the NFW profile was employed. (A color version of this figure is available in the online journal.)

Figure 3. Velocity distribution in our vicinity, $4\pi y^2 f_v(y, \beta)$ obtained with the NFW profile as a function of the velocity on the left and its M–B approximation on the right (both in units of $\sqrt{|\Phi_0|} = 3.6 \times 10^2$ km s$^{-1}$). In the plots, the thick, dotted, long-dashed, fine continuous, short-dashed, and dot-dashed lines correspond to $\beta = 0.50, 0.39, 0.31, 0.22, 0.14, 0.00$, respectively. (A color version of this figure is available in the online journal.)
5. TRANSFORMATION INTO OUR LOCAL COORDINATES

We must now transform the above distributions from the galactic to the local coordinates

\[ f_r(\hat{y}, \xi, \beta) \rightarrow f^{\text{local}}(y, \theta, \phi, \beta, \alpha, \delta, \gamma) \]  

(19)

(here, we have switched notation into \( \hat{y} \) to make the transformation clearer, since, to be consistent with earlier notation, \( y \) is going from now on to have a different meaning). This is accomplished by the substitutions

\[ \hat{y}^2 \rightarrow X^2 + Y^2 + Z^2, \xi^2 \rightarrow \frac{X^2}{X^2 + Y^2 + Z^2}, \]  

(20)

where

\[ X = \frac{1}{\text{sc}}(y \cos \phi \sin \theta + \delta \sin \alpha), \]
\[ Y = \frac{1}{\text{sc}}(y \sin \phi \cos \theta - \delta \cos \alpha \cos \gamma), \]
\[ Z = \frac{1}{\text{sc}}(y \cos \theta + \delta \cos \alpha \sin \gamma + 1), \theta = \frac{\nu}{v_0}, \]  

(21)

where \( \text{sc} \) is the maximum allowed WIMP velocity in units of \( v_0 \). The angles \( \theta \) and \( \phi \) are the spherical coordinates defined in the usual way. Here, the polar axis has been chosen along the Sun’s direction of motion, the \( x \)-axis radially out of the galaxy, and the \( y \)-axis perpendicular to the galactic plane \((\hat{y} = \hat{z} \times \hat{x})\). \( \delta = 0.135 \) is the Earth’s rotational velocity in units of the Sun’s velocity, \( \gamma \approx \pi/6 \) is the angle between the axis of the galaxy and the axis of the ecliptic, and \( \alpha \) is the phase of the Earth \( (\alpha = 0 \text{ around June 3}) \).

6. A BRIEF DISCUSSION OF DIRECT WIMP EVENT RATES (DWER)

Even though the expressions for the event rates for WIMP detection are well known, for the reader’s convenience and to make the role of the velocity distribution more transparent, we will include the basic formulas here in our own notation (Vergados 2001, 2003, 2004, 2007)

\[ \frac{dR}{du} = \frac{\rho(0)}{m_{\chi^0}} \frac{m}{A_{nN}} \sqrt{\langle v^2 \rangle} \int \frac{|u|}{\sqrt{\langle u^2 \rangle}} f^{\text{local}}(y, \theta, \phi, \beta, \alpha, \delta, \gamma) \frac{d\sigma(u, v)}{du} d^3u, \]  

(22)

where \( A \) is the nuclear mass number, \( m_N \) is the nucleon mass, and \( m_{\chi^0} \) is the WIMP mass.

The differential cross section is given by (Vergados 2007)

\[ d\sigma(u, v) = \frac{du}{2(\mu_a \mu_b v)^2} [\tilde{\Sigma}_S F(u) + \tilde{\Sigma}_\text{spin} F_{11}(u)], \]  

(23)

where \( u \) is the energy transfer and \( Q \) in dimensionless units is given by

\[ u = \frac{Q}{Q_0}, Q_0 = [m_N Ab]^{-2} = 40A^{-4/3} \text{ MeV} \]  

(24)

with \( b \) is the nuclear (harmonic oscillator) size parameter. \( F(u) \) is the nuclear form factor and \( F_{11}(u) \) is the spin response function associated with the isovector channel.

The scalar and spin cross sections are given by

\[ \tilde{\Sigma}_S \approx \sigma_{N,\chi^0}^S \left( \frac{\mu_a(A)}{\mu_r(N)} \right)^2 A^2, \tilde{\Sigma}_\text{spin} = \frac{\mu_a(A)}{\mu_r(N)} \sigma_{N,\chi^0}^\text{spin} \]  

(25)

\( \mu_r(A) (\mu_r(N)) \) is the WIMP-nucleus (WIMP-nucleon) reduced mass, \( \sigma_{N,\chi^0}^S \) and \( \sigma_{N,\chi^0}^\text{spin} \) are, respectively, the WIMP-nucleon scalar and spin cross sections and \( \xi_{\text{spin}} \) is the nuclear spin Matrix Element (ME).

Integrating over the energy transfer \( u \), we obtain the event rate for WIMP-nucleus elastic scattering, which is given by (Vergados 2001, 2003, 2004, 2007)

\[ R = \frac{\rho(0)}{m_{\chi^0}} \frac{m}{m_N} \sqrt{\langle v^2 \rangle} \left[ f_{\text{coh}}(A, \mu_r(A)) + f_{\text{spin}}(A, \mu_r(A)) \sigma_{N,\chi^0}^\text{spin} \right] \]  

\[ (26) \]

with

\[ f_{\text{coh}}(A, \mu_r(A)) = \frac{100 \text{ GeV}}{m_{\chi^0}} \left( \frac{\mu_a(A)}{\mu_r(N)} \right)^2 A_{\text{coh}} \left( 1 + h_{\text{coh}} \cos \alpha \right), \]  

\[ (27) \]

\[ f_{\text{spin}}(A, \mu_r(A)) = \left( \frac{\mu_a(A)}{\mu_r(N)} \right)^2 f_{\text{spin}} \left( 1 + h_{\text{spin}} \cos \alpha \right). \]  

(28)

The nucleon coherent and spin cross sections are the most important particle physics parameters and \( \xi_{\text{spin}} \) is the most important nuclear physics parameter. In this work, however, we are mainly interested in the effects of the velocity distribution on the event rate. So to spare the reader the inconvenience of detailed discussions of particle and nuclear physics, we will not be concerned with the spin cross section. Thus, we essentially only need the information of the velocity distribution and the effect of the nuclear form factor. Thus, the relevant parameters are \( t_{\text{coh}} \), which deals with the time-averaged rate, and \( h_{\text{coh}} \), which deals with the modulation due to the annual motion of the Earth. They result after the folding of the nuclear form factor with the WIMP velocity distribution. More specifically

\[ t_{\text{coh}} = \int_{u_{\text{min}}}^{u_{\text{max}}} \frac{dt_{\text{coh}}}{du} du, h_{\text{coh}} = \frac{1}{t_{\text{coh}}} \int_{u_{\text{min}}}^{u_{\text{max}}} \frac{dh_{\text{coh}}}{du} du. \]  

(29)

The energy transfer \( u \) is limited from below by the detector energy threshold and from above by the maximum WIMP velocity, i.e.,

\[ u_{\text{min}} = \frac{Q_{\text{th}}}{Q_0} \leq u \leq u_{\text{max}} = \left( \frac{m_{\chi^0}}{a} \right)^2, a = \left[ \sqrt{2} \mu_a b v_0 \right]^{-1}, \]

(30)

with \( v_0 \) the Sun’s velocity. One can show that

\[ \frac{dt}{du} = \sqrt{\frac{2}{3}} a^2 F^2(u) \Psi_0(\alpha \sqrt{u}), \]
\[ \frac{dh}{du} = \sqrt{\frac{2}{3}} a^2 F^2(u) H(\alpha \sqrt{u}) \cos \alpha, \]

(31)

where the two functions \( \Psi_0(\alpha \sqrt{u}) \) and \( H(\alpha \sqrt{u}) \) are, respectively, the \( n = 0 \) (\( \alpha \) independent) and the \( \cos \alpha \) Fourier coefficients of the integral:

\[ J = \int_{\alpha \sqrt{u}}^{\alpha \sqrt{u}_0} y dy \int f^{\text{local}}(y, \theta, \phi, \beta, \alpha, \delta, \gamma) d\hat{y}. \]  

(32)
The integral $J$ contains all the relevant information on the velocity distribution. We see from this that essentially the first moment of the velocity distribution appears. Since the high-energy transfers are suppressed by the nuclear form factor $F(u)$, the above integral is not much affected by the behavior of the distribution near the escape velocity. So we expect the M–B approximation to be an adequate description of the exact distribution.

The above expressions manifestly show the essential parts. (1) The elementary cross section. This is the most important part. It depends on two ingredients: (a) the particle model which provides the amplitude at the quark level and (b) the procedure for going from the quark to the nucleon level. We will not concern ourselves with such issues here. (2) The WIMP density in our vicinity $\rho(0)$. This has been obtained in two phenomenological density profiles as described above. (3) The dependence of the rate on the cross properties of the target and the WIMP mass. (4) The quantity $t$. This is independent of the parameters of the particle model, except for the WIMP mass. It takes into account (a) the nuclear structure effects (for the coherent process the nuclear form factor), (b) The WIMP velocity distribution, (This may be obtained from a given spherical density profile via the Eddington approach as discussed above.), and (c) The energy threshold imposed by the detector. In the case of a nonzero threshold, the obtained rates depend on quenching. Such an effect will not be discussed here. The interested reader is referred to the literature (see, e.g., Vergados & Ejiri 2008; Fairbairn & Schwetz 2009). (5) The quantity $h$. This describes the modulation of the amplitude due to the Earth’s motion around the Sun. It depends on the same parameters as $t$.

The evaluation of the quantities $t$ and $h$ with the obtained asymmetric velocity distribution will be discussed below.

With the above ingredients, the number of events in time $t$ due to the scalar interaction, which leads to coherence (Tetradis et al. 2007), can be cast in the form

$$R \simeq 1.60 \times t \frac{\rho(0)}{1 \text{ y} 0.3 \text{ GeV cm}^{-3}} \times \frac{100 \text{ GeV}}{m_{\chi}} \times \frac{1 \text{ kg}}{m_{\chi}} \times \sqrt{\langle v^2 \rangle} \times \frac{\sigma^S_{N,\chi^0}}{280 \text{ km s}^{-1}} \times \frac{10^{-6} \text{ pb}}{f_{\text{coh}}(A, \mu_r(A)),}$$

where the elementary cross section $\sigma^S_{N,\chi^0}$ can be treated as a phenomenological parameter.

7. RESULTS

Since the NFW profile preceded the VO profile and is more widely known, we are going to employ the velocity distribution obtained with this profile to compute the direct WIMP detection rates.

We begin with the quantities $t$ and $h$, which are pretty independent of the particle model, but are sensitive to the WIMP velocity distribution and mass. We first consider differential quantities $dt/du$, which is proportional to the time-averaged differential rate $dR/du$, and $dh/du$, which is the relative differential modulated rate, as functions of the energy transfer $Q$. For a typical light and a typical heavy target, these quantities are shown in Figures 4–7. Two values of the WIMP mass, namely, $30$ and $100$ GeV/c$^2$ were considered. Note that in some instances the differential modulated rate changes sign. This may result in cancellations in the integrated rate yielding values smaller than expected due to the smallness of $\delta$ alone. It may also cause a shift of the maximum of the rate from June to December.

By integrating the differential rate, we obtain the parameters $t$ and $h$ as a function of the WIMP mass for a given energy threshold. Typical examples for the coherent process are shown in Figures 8 and 9 for an intermediate-heavy target ($^{127}$I) and in Figures 10 and 11 for a typical light target ($^{19}$F). Two typical threshold values of zero and $10$ keV were considered. There is no need to consider WIMPs heavier than $250$ GeV, since the shown results depend on the WIMP-nucleus reduced mass, which, for the targets employed in the experiments, does not change appreciably above this value.

8. DISCUSSION

With the above ingredients, we can now compute the total event rate. We do not know what the elementary nucleon cross section is, but following the more or less standard practice, we will assume it to be independent of the WIMP mass and equal to $10^{-6}$ pb. We will also take the WIMP density in our vicinity to have the canonical value (Yao et al. 2008) of $0.3$ GeV/c$^2$ cm$^{-3}$. Employing Equation (33) and using the values of $t$ discussed above, we find the results shown in Figure 12 for zero energy.
threshold. For even higher WIMP masses the parameter \( t \) remains essentially constant, so the total rate becomes inversely proportional to the WIMP mass (see Equation (33)). For an energy threshold value of 10 keV, we obtain the results shown in Figures 13 and 14 for \( A = 127 \) and \( A = 19 \), respectively. On these figures, we also show the effect of quenching (Vergados & Ejiri 2008; Fairbairn & Schwetz 2009) assuming an energy threshold of 10 keV. Following standard practice, these plots were drawn with a nucleon cross section of \( 10^{-5} \) pb. The recent limits extracted from the data for the CDMSII Collaboration (Ahmed et al. 2009) and the XENON10 Collaboration (Angle et al. 2008) are more than an order of magnitude smaller, the precise value depending on the assumed WIMP mass.

There is no need to show again the effects of modulation on the rate, since the above shown results are adequate (\( h \) refers to the ratio of the modulated to the time-averaged rate).

9. CONCLUSIONS

In this paper, we first derived the WIMP velocity distribution for a spherically symmetric WIMP density profile. This was done in a self-consistent way by applying the Eddington approach. The anisotropy parameter of the velocity distribution was also taken into account by incorporating angular momentum into the expression of the energy entering the phase space distribution function. Using this distribution function, we...
obtained both the differential and total rates entering direct WIMP detection. We find that the time-averaged differential rates do not sensitively depend on the anisotropy parameter \(\beta\). The obtained shape of this signal unfortunately cannot really be differentiated from that expected from most backgrounds. The differential modulation rate \(H\) shows some dependence on the anisotropy parameter, especially for light targets. We found that this differential rate changes sign at some energy transfer, which depends on the WIMP mass. Perhaps this signature may aid in discarding some season-dependent backgrounds. After that we computed the corresponding total rates. We find that the time-averaged total rates are not very sensitive to the anisotropy parameter, in agreement with earlier results obtained with distributions given in terms of Tsallis type functions (Vergados et al. 2008). The maximum rate is expected at a WIMP mass of about 75 GeV for a heavy target and 30 GeV for a light target. The modulation for a light system is always positive (maximum in June) and tends to increase with the anisotropy.
parameter. For zero threshold, it rises from $2h = 4\%$ to about $2h = 6\%$ as the anisotropy parameter increases from $\beta = 0$ to $\beta = 0.5$. Assuming a threshold of 5 keV, we get $2h = 5\%$–$8\%$. Higher modulations up to 16\% can be expected at smaller WIMP masses. For a heavy target, the modulation increases with the anisotropy parameter, but it goes through zero at a WIMP mass of about 50 GeV. We expect a positive modulation at lower WIMP masses, with a maximum of about $2h = 6\%$ and negative modulation in the case of heavier WIMPS ranging to $2|h| = 4\%$–$6\%$ depending on the anisotropy parameter.

Finally, we have seen that both the spherically symmetric and the axially symmetric velocity distributions obtained from the realistic NFW density profile can be approximated by an M–B velocity distribution in a finite domain, i.e., one in which the upper bound of the velocity (escape velocity) is not put in by hand but it comes naturally from the Eddington method. An approximation is, of course, always valid in a certain domain.
As we have discussed previously (see Section 6), we expect the approximate solution to be appropriate in calculations relevant to dark matter searches. So one may use this distribution in the future to simplify the calculations.

We did not consider in this work directional experiments, i.e., experiments in which not only the energy but the direction of the recoiling nucleus is also observed (Ahmed et al. 2003; Morgan et al. 2005). In such experiments, both the observed time-averaged rates as well as the modulation are expected to be direction dependent (Vergados & Faessler 2007). We expect that such forms of experiments are going to be much more sensitive to the form of the velocity distribution and, in particular, the asymmetry parameter.

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Figure 14. Same as in Figure 13 in the case of a light target (A = 19). The rate tends to decrease as the asymmetry parameter increases. Here, the quenched rate is about a factor of 2 smaller than that at zero threshold (see Figure 12). For the graphing, see the legend of Figure 3.