Dual control of spin-electromagnetic wave chaos in active ring oscillators based on artificial multiferroics

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Abstract. Properties of spin-electromagnetic wave chaos developed in active ring oscillators have been investigated. A multiferroic structure composed of yttrium iron garnet film and barium strontium titanate (BST) slab served as a nonlinear dispersive medium of the oscillator. Dual control of the fractal dimension of the chaotic signal attractor was realized by variation of the ring gain and dielectric permittivity of the BST slab.

1. Introduction
For many years, investigations of nonlinear dynamics have attracted a great research interest. Such fascinating nonlinear wave phenomena as solitons and self-modulation instability have received perhaps the most attention. They were studied for waves of different nature (see, e.g. [1-10]). In recent years, chaotic phenomena have become important due to their great potential for application in telecommunications [11-13]. In this connection, such fundamental questions as a route from stationary excitations to chaos [14-20], chaos control [18,21], synchronization of chaotic generators [22], and chaotic dynamics of solitons [19,20,23-26] are under extensive investigations especially for optical fiber ring lasers [12] and ferrite-film active rings [27].

At present, an increased research interest in magnetolectric interactions in different kind artificial multiferroic structures is evident [28-30]. Recently, nonlinear properties of spin-electromagnetic waves were begun to study [31-34]. In particular, experimental observation of dark envelope solitons and chaos was reported in [34].

The aim of this work is analysis of spin-electromagnetic wave chaos developed in active ring oscillators based on multiferroic (ferrite-ferroelectric) structure.

2. Experimental set-up
Figure 1 shows the experimental structure of the feedback active ring auto-oscillator. For the present experiments we used an artificial multiferroic waveguide made in the form of bilayer composed of a magnetic yttrium iron garnet (YIG) film strip and a ferroelectric barium strontium titanate (BST) ceramic slab. The YIG strip had lateral dimensions of 2 by 40 mm and a thickness of 5.7 µm. The BST slab had lateral dimensions of 5 by 5 mm and a thickness of 500 µm. Two microstrip antennas were placed over the YIG strip at a distance of 6.7 mm. The output antenna was connected to the input antenna through an active feedback loop consisting of a wideband microwave amplifier and a variable precision attenuator controlling the net gain coefficient G in the ring. The bilayer was magnetized in...
its plane and transverse to the wave propagation direction. This orientation of the magnetic field provided the propagation of quasi-surface spin-electromagnetic waves in the multiferroic waveguide.

The microwave excitations were propagating in the ring as spin waves, hybrid spin-electromagnetic waves, and electromagnetic waves. Thus, the electromagnetic wave travelling in the feedback loop transforms to a surface spin wave by the input antenna. Then, the excited spin wave propagates short distance in the YIG film and enters the multiferroic structure. This is the region where the YIG film is in contact with the BST slab. At the border of this region, the surface spin wave transforms into the quasi-surface spin-electromagnetic wave, which then propagates in the YIG-BST multiferroic structure. The spin-electromagnetic wave propagating through the structure reaches the YIG film where it transforms back to the spin wave and is received by the output microstrip antenna to be transformed to electromagnetic wave microwave signal. This signal is amplified in the feedback loop (see figure 1).

**Figure 1.** Block diagram of the experimental oscillator.

### 3. Experimental results and discussion

Increase of the ring gain coefficient up to $G = 0.3$ dB led to change of the active ring self-generation regime. Increased power of the microwave signal circulating in the ring led to appearance of a nonlinear interaction of hybrid waves in ferrite-ferroelectric structure. Due to the interaction the additional regularly spaced modes appeared in the frequency spectrum of the signal. In a time domain periodical amplitude modulation of signal was observed. On the figure 2(a,b) spectrum and waveform are presented. A carrier frequency was $f_0 = 7.005$ GHz. Multicomb frequency spectrum has a separation between Fourier-harmonics of $\Delta f = 3.56$ MHz. A repetition period of the corresponding nonlinear pulses was measured to be 273 ns.

For further analysis multi-dimensional phase portrait was reconstructed. Phase trajectory of corresponding regime was obtained using waveform envelope by the standard time-delay method [35] as follows:

$$x(t) = \{U(t), U(t + \tau), \ldots, U(t + (d-1)\tau)\},$$

where $\tau$ is time delay, $d$ is a phase space dimension and $U(t)$ is the amplitude of analyzed signal. In our calculation maximum dimension of phase space was set to 25. Phase portrait of the periodic regime in two-dimensional phase space is represented on figure 2(c). The form of the phase portrait was determined by an attractor, which attracts all phase trajectories in the phase space. In the case of periodic signal the trajectory represented a loop. It apparently corresponded to the attractor in the form of limit cycle. According to the work [35] this phase portrait could be used for calculation of attractor numerical parameters such as fractal dimension, Lyapunov exponents, Kolmogorov entropy etc. Fractal dimension was calculated to prove the attractor topology. Calculation was done by the
algorithm of Grassberger-Procaccia [36,37]. First correlation sums for a number of phase space dimensions were calculated according to next formula:

\[
\sum_{i \neq j}^{N} \sum \theta(\varepsilon - \|x_i - x_j\|) = C(\varepsilon) = \frac{2}{N(N-1)} \sum_{i \neq j}^{N} \sum \theta(\varepsilon - \|x_i - x_j\|),
\]

where \(x\) is the coordinate of the phase trajectory point, \(N\) is a number of point, \(\varepsilon\) is a characterization distance between points, and \(\theta\) is the Heaviside step function. Logarithmic dependence of the correlation sum on the characterization distance has a linear part. Thus

\[
C(\varepsilon) = \varepsilon^{D_c},
\]

where \(D_c\) is so-called correlation dimension of the attractor embedded into the phase space with certain dimension \(d\). On the figure 2(d) dependence of correlation dimension on phase space dimension calculated by described method is represented. Increase of the phase space dimension led to increase of the correlation dimension. The dependence has a saturation level. The saturation shows that the whole attractor is embedded into the phase space. This level is called fractal dimension \(D_f\). Fractal dimension value characterizes attractor topology. In the case of periodical signal fractal

Figure 2. Periodic signal generation regime characteristics: (a) spectrum, (b) envelope waveform, (c) 2D phase portrait, (d) dependence of correlation dimension on phase space dimension (\(G = 0.35\) dB, \(E = 0\) V/\(\mu m\)).
dimension was found to be $D_f = 1$. It proves that while self-generation of periodical modulated signal in the phase space appeared limit cycle.

Further increase of ring gain coefficient up to 0.82 dB led to enrichment of spectrum and complicacy of amplitude modulation. Calculation of fractal dimension shown that the observed regime is quasiperiodical regime with fractal dimension $D_f = 2$. In this regime in the phase space attractor in the form of 2D-torus was observed.

Values of $G > 1.1$ dB corresponded to a self-generation threshold of the signal with broadband spectrum. Each self-generated harmonic were broadened. In time domain nonperiodic signal was observed. Further increase of gain coefficient made broadened parts of the spectrum wider until broadband basement appeared in the frequency spectrum. During this process waveform modulation became more complicated. On the figure 3(a,b) spectrum and waveform corresponding to $G = 1.3$ dB are shown. The broadband spectrum is 100 MHz wide. According to described above methods phase portrait and fractal dimension were investigated. On the figure 3(c) 2D phase portrait is shown. One can see that phase trajectories are mixed. It could be caused by appearance of strange attractor, but also by influence of a noise. To investigate whether the nature of dynamic is determined or not, fractal dimension was calculated. On the figure 3(d) saturation of correlation dimension could be seen. It means that the dimension of attractor is finite, and observed dynamics is a dynamical chaos. The value of the fractal dimension for $G = 1.3$ dB were found to be $D_f = 8.5$. Characterization of a noise signal would not demonstrate saturation of correlation dimension and fractal dimension would be infinite.

![Figure 3](image_url)

**Figure 3.** Dynamical chaos generation regime characteristics (a) spectrum, (b) – envelop waveform, (c) – phase portrait, (d) - dependence of correlation dimension on phase space dimension ($G = 1.3$ dB, $E = 0$ V/µm).

For investigation of chaotic signal tunability fractal dimensions for a range of ring gain coefficients were calculated. Figure 4 represents the dependence of fractal dimension for different self-generation
regimes in the range of the ring gain coefficient from 0 dB up to 2.2 dB. One can see that increase of the ring gain coefficient leads to increase of the fractal dimension. For regular dynamic fractal dimension changed stepwise. Each step corresponded increase of fractal dimension by one. It is so-called Hopf bifurcation. Transition to dynamical chaos through sequence of Hopf bifurcation is the Ruelle-Takens scenario. In chaotic regime Increase of fractal dimension led to gradual fractal dimension increase from 4.5 till 11.5. Above $G = 2$ dB weak change of fractal dimension was observed. Constant fractal dimension indicates that complicacy of chaotic dynamic is also constant, and topology of attractor does not change.

![Figure 4](image4.png)

**Figure 4.** Dependence of fractal dimension on ring gain coefficient for $E = 0$ V/µm.

For the value of ring gain coefficient corresponding to maximum fractal dimension influence of electric bias field on chaotic signal was estimated. In the range of 0-1.6 V/µm fractal dimensions were calculated. The dependence of the fractal dimension on the bias electric field is shown on figure 5. One can see that increase of bias field leads to increase of fractal dimension. In comparison to the gain tunability influence of electric field is weaker. Fractal dimension changed from 11.5 up to 12.9.

![Figure 5](image5.png)

**Figure 5.** Dependence of fractal dimension on bias electric field ($G = 2.1$ dB).

Therefore in this work we observed and studied a dual control of the fractal dimension. In wide range the control could be done by varying of ring gain coefficient. In a narrow range the control could be done by variation of electric bias field.

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