The epidemic of Tuberculosis on vaccinated population

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Abstract. Tuberculosis is an infectious disease which has caused a large number of mortality in Indonesia. This disease is caused by Mycobacterium tuberculosis. Besides affecting lung, this disease also affects other organs such as lymph gland, intestine, kidneys, uterus, bone, and brain. This article discusses the epidemic of tuberculosis through employing the SEIR model. Here, the population is divided into four compartments which are susceptible, exposed, infected and recovered. The susceptible population is further grouped into two which are vaccinated group and unvaccinated group. The behavior of the epidemic is investigated through analysing the equilibrium of the model. The result shows that administering vaccine to the susceptible population contributes to the reduction of the tuberculosis epidemic rate.

1. Introduction

Tuberculosis (TB) is a contagious disease which brings on death in developing country [1]. TB disease is caused by Mycobacterium tuberculosis bacteria. Generally, this disease affects lung, called as pulmonary TB, and other organs such as pleura, brain’s membrane, pericardium, lymph glands, bone, joint, kidney, ureter, genital, etc. Tuberculosis affecting organs other than lung is called as extrapulmonary TB. A patient suffers from pulmonary and extrapulmonary TB is classified as pulmonary TB [2].

The most dominant factor for TB disease is direct contact to the TB sufferer living in the same house [3]. TB spreads through air contaminated by Mycobacterium tuberculosis bacteria. The air is inhaled and then affect the lungs. The common symptom of pulmonary TB is productive cough, combined with the systematic symptoms such as cold, night sweat, and weight loss [4]. According to World Health Organization (WHO) 2015, Indonesia ranks second in the number of TB cases after India [5].

In Indonesia, the death rate caused by Mycobacterium tuberculosis is high enough so that the government prompts to vaccinate once when the baby is 2 months old [6]. Vaccine is suspended germ
or virus which has been weakened and used to treat or prevent an epidemic. Vaccine which is used for TB disease is BCG (Basilillus Calmette Guerin). In 1973, BCG vaccine was part of the immunization program [7]. Although there is a way to decrease the susceptibility to the infection by vaccination, early diagnosis and medical treatment are priorities in controlling it.

The phenomenon of TB disease spread can be observed through a mathematical model. A mathematical model can predict and control the TB disease problem in the future. The threat of the TB disease which is hard to control requires an effort to study the spread pattern. Mathematically, the TB disease spread can be modeled in several types, which are epidemic models of SIR, SIRS, SEI, and SEIR.

Fredlina et al. [1] constructed a SIR type model of TB epidemic and conducted a simulation by using fourth order Runge-Kutta method. The result shows that the TB spread can be restrained from an epidemic by sending down the spreading rate and increasing the recovery rate. A way to decrease the spreading rate is by keeping at a distance of the TB-infected individual from the susceptible population, while to increase the recovery rate, a maximal treatment needs to be conducted. Side et al. [8] also developed two models of TB spread, which are SIR and SEIR. By implementing Lyapunov function method, it is obtained that the disease will disappear if the basic reproduction number is less than or equal to one, and vice versa.

Komsiyah [10] and Porwal et al. [11] modified the SIR model [9] by adding the rate of vaccination. Komsiyah considered two models which are models with and without vaccination, then the results of the models are compared. Meanwhile, Porwal et al. only considered a model with one vaccination parameter. From the result, it can be concluded that the vaccination can effectively inhibit the spreading of TB [10, 11]. Previously, Porwal and Badshah [12] created a SIRS type model with an addition of vaccinated compartment. Thus, this model consists of four compartments which are susceptible, infected, recovered, and vaccinated. In the model, recovered individuals can turn to become susceptible individuals, so can the vaccinated individuals. Immunity from the vaccination process is assumed impermanent which means that it will lose over time and the individual can return to be susceptible to the disease [12].

Rafflesia [6] developed a TB spread model in SEI type by considering the vaccination effect. Administering vaccine to the susceptible individuals can affect the TB spreading, which is by vaccine administration, the TB spreading does not happen [6]. The article discusses the spreading of TB in SEIR type model which consists recovered individuals. In this model, it is assumed that vaccine is administered to the susceptible individuals. Through this process, the susceptible individuals are divided into two groups, vaccinated and unvaccinated susceptible individuals. Furthermore, the model is solved by using the equilibrium theory, with some steps which are determining the disease-free equilibrium, analyzing the system stability near the equilibrium, finding the solution of the model, and analyzing how far the vaccination effect to the spreading of TB.

2. The Mathematical Model

The model construction of the TB spread discussed in this article is showed in figure 1. Figure 1 presents the TB spread model considering the vaccination process and is an extent of the model presented in [6] by adding the recovery element. The recovery is not a permanent state such that if there is interaction with an infected individual then the recovered individual can be infected by TB again.
Figure 1. Compartments of TB disease spread SEIR model with vaccination.

By considering the vaccination process, the susceptible model is divided into two groups which are unvaccinated susceptible group \(S_u\) and vaccinated susceptible groups \(S_v\). \(E\) is the group of the latent, \(I\) is the group of the infected, \(R\) is the group of the recovered, \(N\) is the total population, \(c\) is vaccination rate, \(\beta_u\) is infection rate of the unvaccinated susceptible group, \(\beta_v\) is infection rate of the vaccinated susceptible group, \(q\) is the infection probability of the latent, \(\varepsilon\) is the rate of change of the latent become infected, \(\nu_1\) is recovery rate of the infected, \(\nu_2\) is re-infected rate, \(\alpha\) is birth rate, \(\pi\) is natural death rate, and \(\mu_T\) is death rate caused by the tuberculosis disease. It is assumed that the population is closed and the recovery state is not permanent. The vaccine is administered only to the individual who is never infected by TB. Thus, the mathematical model of the TB disease spread reads

\[
\begin{align*}
\frac{dS_u}{dt} &= (1 - c)\pi N - \beta_u \frac{I}{N} S_u - \mu S_u, \\
\frac{dS_v}{dt} &= c\pi N - \beta_v \frac{I}{N} S_v - \mu S_v, \\
\frac{dE}{dt} &= q\beta_u \frac{I}{N} S_u + \beta_v \frac{I}{N} S_v - \nu_1 E - \varepsilon E - \mu E + \alpha \frac{I}{N} R, \\
\frac{dI}{dt} &= (1 - q)\beta_u \frac{I}{N} S_u + \varepsilon E - \nu_2 I - \mu I - \mu_T I, \\
\frac{dR}{dt} &= \nu_1 E + \nu_2 I - \mu R - \alpha \frac{I}{N} R, \\
N &= S_u + S_v + E + I + R.
\end{align*}
\]

The values of the parameters are given in the table 1.

| Notation | Value (per year) | Value interval |
|----------|-----------------|---------------|
| \(C\)    | 0.025, 0.045 dan 0.9 | \(0 < c < 1\) |
| \(\pi\)  | 0.02695         | \(0 < \pi < 1\) |
| \(\beta_u\) | 0.35         | \(0 < \beta_u < 1\) |
| \(\beta_v\) | 0.35         | \(0 < \beta_v < 1\) |
| \(Q\)    | 0.5            | \(0 < q < 1\) |
| \(\varepsilon\) | 0.0003      | \(0 < \varepsilon < 1\) |
| \(\nu_1\) | 0.25          | \(0 < \nu_1 < 1\) |
| \(\nu_2\) | 0.25          | \(0 < \nu_2 < 1\) |
| \(\alpha\) | 0.3          | \(0 < \alpha < 1\) |
| \(\mu\)  | 0.01           | \(0 < \mu < 1\) |
| \(\mu_T\) | 0.3           | \(0 < \mu_T < 1\) |
The parameter’s values are modified values implemented in the [9]. By assuming that the initial value of the susceptible is zero, it is obtained that the initial values for unvaccinated susceptible, the latent, infected, and recovered individuals are 95704, 13670, and 1950, and zero people respectively.

To know the number of individuals in each compartment in equilibrium state, the system (1) is set at constant over time. Hence, it is obtained the disease-free equilibrium point as follow,

\[ E_0 = (S_0, S_0, E, I, R) = \left( \frac{(1-c)\pi N}{\mu}, \frac{c\pi N}{\mu}, 0, 0, 0 \right). \]  

The values \( E = 0, I = 0 \) and \( R = 0 \) describe that at a certain time the population will be at equilibrium state if no individual is sick, such that there no individual that can be infected and is recovered; all individuals are only susceptible to the disease.

Furthermore, the system behaviour near the equilibrium point is analyzed by determining its stability. The stability can be established by finding the eigen values of the Jacobian matrix (J) of the linearized model. By substituting the parameter’s values and the equilibrium point to the Jacobian matrix, it gives real negative eigen values for every given vaccination rate. Therefore, based on the stability criteria, this equilibrium point is asymptotically stable.

3. Results and discussion

Figure 2 below presents the number of the individuals in the vaccinated susceptible and latent groups over time with vaccination rate 0.025, 0.045 and 0.9.

![Graph of the number of the individuals in (a) vaccinated susceptible and (b) latent groups.](image)

The graphs in figure 2 display that there is change in the number of individuals in the vaccinated susceptible and latent groups for each given vaccination rate.

Vaccination effects an increase in the number of individuals in the vaccinated susceptible group. During the time \( t = 10 \) years until \( t = 40 \) years, the average percentage of the increase at each 10 years is 74.19% for vaccination rate 0.025 and the number of the individuals become 2135 people. Meanwhile, it is 73.52% at vaccination rate 0.045 and the number of the individuals becomes 3878 people. Also, it is 59.38% at vaccination rate 0.9 and the number of the individuals becomes 86783 people.

Moreover, the vaccination affects a decrease in the number of latent individuals. For the first 40 years, the average percentage of the decrease at each 10 years is 69.18% for vaccination rate 0.025 and the number of the individuals becomes 111 people. For vaccination rate 0.045, it is 71.50% and the number of the individuals becomes 83 people. For vaccination rate 0.9, it is 90.61% and the number of the individuals becomes 1 person.

Figure 3 is graphs illustrating the number of the individuals in the infected and recovered groups for vaccination rate 0.025, 0.045 and 0.9.
The graphs in figure 3 display that there is change in the number of individuals in the infected and recovered groups for each given vaccination rate.

The vaccination effects a decrease in the number of infected individuals. For the first 40 years, the average percentage of the decrease at each 10 years is 62.89% for vaccination rate 0.025 and the number of the individuals becomes 37 people. For vaccination rate 0.045, it is 66.35% and the number of the individuals becomes 25 people. Meanwhile, at vaccination rate 0.9, for the first 10 years the average percentage of the decrease is 99.33% then after 18 years there is no more infected individual.

The vaccination results that the individuals in latent and infected groups recover from the disease such that the number of the recovered group increases although the recovery in the infected group is not all affected by the vaccination. The number of individuals in the recovered group highly depends on the number of the individuals coming from the latent and infected groups. If the number of latent and infected individuals is small then the number of individuals moving to the recovered group is small. The number of individuals in the recovered group is initially zero. By the coming individuals recovered from TB, whether from latent or infected groups, a year after there will be a number of individuals in the recovered group.

At vaccination rate 0.025, the number of individuals in the recovered group keeps increasing for the first 23 years. When t = 1 year the group consists 3560 people and when t = 23 years it consists 21364 people. When t = 24 years, the number of the individuals starts decreasing and will approach the equilibrium point. It happens because the number of the coming-in individuals are less that the number of coming-out individuals.

At vaccination rate 0.045, the number of individuals in the recovered group keeps increasing for the first 22 years. When t = 1 year the group consists 3560 people and when t = 22 years it consists 20827 people. When t = 23 years, the number of the individuals starts decreasing approaching the equilibrium point.

At vaccination rate 0.9, the number of individuals in the recovered group keeps increasing for the first 14 years. When t = 1 year the group consists 3542 people and when t = 14 years it consists 15229 people. When t = 15 years, the number of the individuals starts decreasing approaching the equilibrium point.

From figure 2 and 3, it can be observed that the higher the vaccination rate, the higher the increase in the number of vaccinated susceptible individuals and the decrease in the number of infected individuals. This results explains that the higher the vaccination rate, the faster the curing effort of the TB epidemic.

4. Conclusion

According to the analysis which was explained, the equilibrium point of the disease-free state $E_0 = (S_0, S_0, E, I, R) = \left( \frac{(1-c)\pi N}{\mu}, \frac{c\pi N}{\mu}, 0, 0, 0 \right)$ is asymptotically stable which means that at a time the TB
The epidemic will vanish from the population. The vaccination rate highly affects the spread of the TB disease; the higher the vaccination rate, the lower the spread of TB epidemic.

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