Influence of communication between elements in a dynamic damper on a vibrated body behaviour

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Abstract. Body-damper dynamic system is considered. Vibration damper consist of several elements, connected with nonlinear magnetic repulsion force. The case of the absence of magnetic forces is considered. Frequency responses for both cases of elements connections are obtained.

1. Introduction

The vibration damping problem remains very relevant. The use of systems equipped with free magnets is of interest as one of the varieties of shock absorbers. The aim of this work is to study the effect of the connection between the elements in the vibration damper on the body vibrations damping. In the first case, the analytically obtained dependence of the magnetic repulsive force is used; in the second case, there are no magnetic forces of interaction between the elements.

2. Problem statement

In this work, a vibro-impact system is considered as a dynamic vibration damper, consisting of a horizontally located tube with free magnets inside. Magnets can perform longitudinal oscillations with impacts. In the cylindrical cavity of a rigid casing of mass \( m_0 \) (Figure 1.) there are \((n+2)\) identical cylindrical magnets oriented to each other with the same poles. Magnets are considered to be absolutely rigid point bodies with masses \( m \). Let us denote their coordinates counted from the left end of the cylindrical cavity as \( s_i \). Magnets with numbers \( i=0, i=n+1 \) are rigidly fixed to the body, while the remaining magnets are uniformly distributed along the cavity at the initial time. We consider the distances between them at equilibrium state to be the same and equal to \( A \). During oscillations, each magnet moves by an amount \( w_j \), and the gaps take values \( X_j \):

\[
s_i = iA + w_j, \quad i = 0,1,2,\ldots ,n+1; \quad X_j = s_j - s_{j-1} = A + w_j - w_{j-1},
\]

\[
j = 1,2,\ldots ,n, \quad s_0 \equiv 0, s_{n+1} \equiv (n+1)A, \quad w_0 \equiv 0, w_{n+1} \equiv 0
\]

Following dimensionless complexes and variables were used [1]:

\[
X = a, \quad T = \sqrt{ma/F_0}, \quad \alpha = A/X, \quad \nu = \omega T, \quad \zeta = Td/2m, \quad \xi = x/X, \quad \tau = t/T, \quad T_0 = 2\pi v^{-1}, \quad \mu = M/m
\]
This system was considered in the papers [1]. In paper [1], the equations of motion of the elements were obtained, and the laws of motion were found under external kinematic action. To assess the effectiveness of this device as a vibration damper in this work, we consider the body-vibration damper system shown in Figure 1b ($k$ – spring stiffness, $d_2$ – viscous damping coefficient, $M$ – body mass). In paper [2], the abilities of damping oscillations were estimated depending on the system settings. In this paper, we consider the influence of the nature of the interaction between free elements on the vibration damper effectiveness.

![Figure 1. a) Scheme of the used device; b) Device Installation on the damped body.](image)

### 3. Motion equations

The equations of motion are derived using the Lagrange equations:

$$\eta_i \left( \ddot{\xi}_i + \ddot{\xi}_{n+i} \right) + 2 \zeta_1 \dot{\xi}_i + \Phi_2 - \Phi_1 = 0$$

$$\cdots$$

$$\eta_i \left( \ddot{\xi}_i + \ddot{\xi}_{n+i} \right) + 2 \zeta_2 \dot{\xi}_i + \Phi_{i+1} - \Phi_i = 0, \quad i = 1, \ldots, n$$

$$\cdots$$

$$\eta_i \left( \ddot{\xi}_n + \ddot{\xi}_{2n+1} \right) + 2 \zeta_2 \dot{\xi}_n + \Phi_{n+1} - \Phi_n = 0$$

$$\ddot{\xi}_{n+i} + \sum_{i'=1}^{n} \eta_{i'} \dot{\xi}_{i'} + 2 \zeta_2 \dot{\xi}_{n+i} + \Phi_{i+1} - \Pi(\tau) = 0$$

$$\eta = m \left( \sum_{j=1}^{n} m_j + M + m_0 \right), \quad \kappa = k a / F_0, \quad \Pi_0 = P_0 / F_0$$

In this system, the variables $\ddot{\xi}_i$ ($i = 1, 2, \ldots, n$) are responsible for the relative movement of the vibration damper elements relative to the equilibrium position in the moving coordinate system associated with the damped body, and for $i = n+1$ – for the movement of the body in the global stationary coordinate system. The function $\Phi_i$ describes the interaction between $i$ and $(i-1)$ magnets.

In paper [2], the dependence of the interaction strength between magnets was analytically obtained:

$$F(z) = \frac{F_0 z}{\left(1 + z^2\right)^{5/2}}, \quad F_0 = 3 \pi B_0 h^2 / 2 \mu_0, \quad z = X / r.$$
Also there, a mathematical model was studied using the following dependence, which is close to experimentally obtained:

\[ F(z) = F_0 \left( \frac{z - h/2r}{1 + (z - h/2r)^2} \right)^3 \]

For numerical integration, the system of equations (1) is reduced to the matrix form:

\[ \mathbf{Y} = \mathbf{AY} + \mathbf{g}(\xi, \tau); \quad \mathbf{Y} = \left[ \xi, \dot{\xi} \right]^T \]

Depending on the magnitude of the external influence, vibration-impact modes are possible. To account impacts, it is necessary to introduce a variable responsible for the distance between two adjacent magnets:

\[ \rho_i = \alpha + \xi_{i-1} - \xi_{i+1}, \quad i = 1, n + 1 \]

The impact occurs when the variable \( \rho_i \) is equal to zero. Using Newton’s well-known hypothesis of the recovery coefficient \( r \) upon impact [3] we obtain the following relations for two magnets that are not boundary:

\[
\begin{align*}
\xi_{i-1}^+ &= \xi_{i-1}^- \\
\dot{\xi}_{i-1}^+ &= \left[ (m_{i-1} - rm_i) \dot{\xi}_{i-1}^- + m_i (1 + r) \dot{\xi}_i^- \right] / (m_{i-1} + m_i), \quad i = 2, \ldots, n - 1. \\
\dot{\xi}_i^+ &= \left[ m_i (1 + r) \dot{\xi}_i^- + (m_i - rm_{i-1}) \dot{\xi}_{i-1}^- \right] / (m_{i-1} + m_i)
\end{align*}
\]

For boundary magnets (the damper elements moves in a moving coordinate system, and the body moves in a global stationary coordinate system):

\[
\begin{align*}
\xi_i^+ &= \xi_i^- \\
\xi_{n+1}^+ &= \xi_{n+1}^- \\
\dot{\xi}_i^+ &= \left( m_i - rM \right) \dot{\xi}_i^- / (M + m_i), \quad i = 1, n. \\
\dot{\xi}_{n+1}^+ &= \dot{\xi}_{n+1}^- + m_i (1 + r) \dot{\xi}_i^- / (M + m_i)
\end{align*}
\]

4. Results
Realization of movement in the absence of magnetic forces is shown in Figure 2 (vibration damper elements) and Figure 3 (body movement with and without a damper), with the following system parameters:

\[ \alpha = 1.67, \mu = 25.43, \kappa = 0.0039, \Pi_0 = 0.0023, \nu = 0.0117, \nu_0 = 0.0110, r = 0.95 \]

![Figure 2. Realization of the movement of elements in the absence of magnetic repulsive forces (a – 0-5 periods of vibration; b – 27-32 periods of vibration)
Realization of body movement with and without a damper (a – 0-5 periods of oscillations; b – 20-40 periods of oscillations)

Realization of motion with analytically obtained dependence of repulsive forces is shown in Figure 4 (elements in the vibration damper) and Figure 5 (body movement with and without a damper), with the following system parameters:

\[
\begin{align*}
\alpha &= 1.67, \quad \mu = 25.43, \quad \kappa = 0.0039, \quad \Pi_0 = 0.0023, \quad \nu = 0.0117, \quad \nu_0 = 0.0110, \quad F_0 = 3098.7 \\
F(z) &= F_0 (z - h/8r) \left(1 + (z - h/8r)^2\right)^3
\end{align*}
\]

\(\text{Figure 4. Realization of the movement of elements with an analytically obtained dependence of the magnetic force (a – 0-20 periods of oscillation; b – 218-220 periods of oscillation)}\)

\(\text{Figure 5. Realization of body movement with and without a damper (a – 0-20 oscillation periods; b – 200-210 oscillation periods)}\)
Figure 6 shows the frequency response plotted for two variants of each of the absorbers (with and without magnetic forces) with two options for gaps at equilibrium state (10mm and 25mm). The following parameters are taken:

\[
\mu = 25.43, \kappa = 0.0039, \Pi_0 = 0.0023, \nu = [0.9\nu_0, 1.1\nu_0], \nu_0 = 0.0110, r = 0.95
\]

![Figure 6. Comparison the frequency response of different vibration dampers](image)

5. Conclusion
As a conclusion, the following was noted: a vibration absorber, the mathematical model of which uses relation (2), not only does not damping body vibrations, but also significantly increases the amplitude of forced vibrations. At the same time, if the force of magnetic interaction is absent or experimentally obtained data were used [1], then the applied model shows a rather high efficiency of the vibration damper.

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