Low Complexity Beam Searching Using Trajectory Information in Mobile Millimeter-wave Networks

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Abstract—Millimeter-wave and terahertz systems rely on beamforming/combining codebooks for finding the best beam directions during the initial access procedure. Existing approaches suffer from large codebook sizes and high beam searching overhead in the presence of mobile devices. To alleviate this problem, we suggest utilizing the similarity of the channel in adjacent locations to divide the UE trajectory into a set of separate regions and maintain a set of candidate paths for each region in a database. In this paper, we show the tradeoff between the number of regions and the signalling overhead, i.e., higher number of regions corresponds to higher signal-to-noise ratio (SNR) but also higher signalling overhead for the database. We then propose an optimization framework to find the minimum number of regions based on the trajectory of a mobile device. Using realistic ray tracing datasets, we demonstrate that the proposed method reduces the beam searching complexity and latency while providing high SNR.

Index Terms—Millimeter-wave systems, terahertz systems, beamforming codebook, beam searching complexity.

I. INTRODUCTION

Millimetre-wave (mmWave) communication is one of the important components of the fifth-generation (5G) cellular networks due to the large availability of bandwidth and underutilized spectrum [1]. For example, the current release of 5G supports several mmWave bands between 24.25 GHz and 52.6 GHz while the future releases are expected to expand up to 300 GHz (terahertz band) [2]. Due to the smaller wavelength, mmWave signals do not diffract/penetrate well through most common materials, leading to high penetration loss and blockage [1]. Moreover, a wider bandwidth translates into higher noise power at the receiver. However, small wavelength enables deployment of large arrays of antennas, which provides directional beamforming1 to compensate for the high noise power and path-loss. These directional links are highly sensitive to blockages, so finding and maintaining near-optimal beamforming directions are necessary.

Due to the high cost and power consumption of the mixed-circuit components, mmWave systems normally rely fully or partially on analog beamforming, that are based on a pre-defined codebook for both initial access and the data transmissions [3]. In the current release of 5G, beamforming directions are based on brute-force beam searching over the beam codebooks, measurements and reporting [4], [5]. In the

1Beamforming is applied by focusing the transmitted beam on a set of communication path between transmitter and receiver.

exhaustive narrow beam searching, the BS and the UE need to sweep almost all the combinations of beam pairs, which leads to a significant beam searching complexity. For instance, for a BS equipped with 32 antennas and 3-bit phase shifters, there are about $8^{32}$ beamforming vectors [3]. With such a huge search space, finding the optimal beam entails high complexity and overwhelms the useful channel time particularly during the mobility where the mmWave channels are constantly changing.

Authors in [6], [7] used the sparsity and the similarity of mmWave channels in adjacent locations and proposed beam searching methods for the stationary [6] or indoor scenarios [7]. Side-information aided or context-aware approaches use the location information of the UE to reduce the effective beam search space during the beamforming phase. In [8], [9], the BS knows the location of the UE through lower-frequency links, and the nearest BS steers towards the UE using the direct beam. However, that method is vulnerable to errors caused by blockages between the nearest BS and the UE and also the imperfect location information. The approach in [10], utilizes a database of multipath fingerprints (set of beams) of UEs at various location bins. Given the location of the UE, the BS selects the beams based on the past multipath fingerprints. However, due to the huge search space as explained before, it is cumbersome to obtain a database of best beams in all the locations in a trajectory even in an offline measurement. Further, we need high location information accuracy to build the database.

Some approaches such as [3], [11] use machine-learning tools to predict the optimal beams. However, they need an exhaustive or hierarchical search over all beam pairs between the BS and the UE for every locations during the training phase, which may increase the time of the training phase.

In this work, we utilize the channel similarity in adjacent locations in order to alleviate the beam searching complexity and design a site specific beam codebook for each BS. We consider an outdoor mobile scenario where UEs are moving along a trajectory and the UE’s locations are available. We divide the UE trajectory into non-overlapping (and probably non-uniform) regions. We assign one reference point to each region and compute the path skeleton2 in the reference point. During the run-time, the BS runs the beam searching procedure only for the reference points and over the beams

2A small set of strong beams between the BS and the UE
stored in their path skeleton, instead of exhaustive search over all potential beam codebooks in all locations. After finding the strongest beam, the BS uses it for any devices inside the region. Therefore, a higher number of reference points correspond to higher memory and signaling requirements, as a central database, co-located at the BS should frequently update and maintain the reference path skeletons, during the run-time. Hence, the reference locations should be chosen wisely to minimize the overhead of database while the final reference path skeletons are accurate enough to cover the entire trajectory of the device. To this end, we propose an optimal solution to find the reference points and defining the regions. Thus, unlike the previous works, we only run the coarse beam searching in sparse locations, not in the whole area. In our previous method in [12], [13], we started from the beginning of the trajectory, and used a greedy algorithm to find the reference points when the channel changes more than a threshold, here, the emphasis of this work lies in finding the optimal reference location and regions. Another interpretation of the regions is the maximum location error input that the algorithm can tolerate in the reference locations to keep the performance. We evaluate the performance of the proposed method based on realistic ray tracing data. Simulation results show that the proposed approach is capable of providing high SNR value while substantially reducing the beam searching complexity compared to the baselines.

The rest of the paper is organized as follows. We start with introducing the system model in Section II. In Section III, we formulate the problem. In Section IV and V, we proposed our solutions and present the numerical results, respectively. Finally, we conclude our work in Section VI.

**Notation:** Sets, vectors, matrices, random variables and their realizations are denoted by calligraphic, bold small, bold capital, capital and lower case letters, respectively. The set of complex and real numbers are denoted by \( \mathbb{C} \) and \( \mathbb{R} \), respectively. The set \( \{ 1, \ldots, n \} \), for some integer \( n \), is denoted by \( [n] \). The notation \( \mathbb{E}[X|Y] \) means the expectation of random variable \( X \) given random variable \( Y \).

**II. System and Channel Models**

We consider a downlink mmWave network comprising |\( \mathcal{B} \)| base stations (BSs) communicating. We assume each BS \( j \in \mathcal{B} \) is equipped with \( N_{\text{BS}} \) antennas and the each UE \( i \in \mathcal{U} \) is equipped with \( N_{\text{UE}} \) antennas.

We consider block fading in the channel response during a coherence interval (CI) is constant. The channel matrix \( \mathbf{H} \in \mathbb{C}^{N_{\text{UE}} \times N_{\text{BS}}} \) between BS \( j \) and UE \( i \) during a CI is [14]:

\[
H_{i,j} = \sqrt{\frac{N_{\text{BS}}N_{\text{UE}}}{L}} \sum_{\ell=1}^{L} h_{i,j,\ell} \mathbf{a}(\phi_{i,\ell}, \theta_{i,\ell})^H \mathbf{a}^H(\phi_{j,\ell}, \theta_{j,\ell}),
\]

where \( L \) is the number of paths. Each path \( \ell \) has horizontal and vertical angles of arrival (AoAs), \( \phi_{i,\ell}, \theta_{i,\ell} \), and horizontal and vertical angles of departure (AoDs), \( \phi_{j,\ell}, \theta_{j,\ell} \), respectively. \( h_{i,j,\ell} \sim \mathcal{N}(0, \beta_{i,j,\ell}) \) is the small scale fading, where \( \beta_{i,j,\ell} \) is the channel gain. Here, \( \mathbf{a}(\cdot) \) is the antenna array response. We consider uniform linear array (ULA) in both UE and BS sides. The ULA can only perform 2-D beamforming and combining at elevation angles and its array response vector is

\[
\mathbf{a}(\phi^*) = \frac{1}{\sqrt{N_x}} \left[ 1, e^{j\frac{2\pi}{\lambda} \tilde{d} \sin \phi^*}, \ldots, e^{j(N_x-1)\frac{2\pi}{\lambda} \tilde{d} \sin \phi^*} \right]^T,
\]

where \( \tilde{d} \) is the antenna spacing, \( \lambda \) is the wavelength and \( x \in \{\text{BS}, \text{UE}\} \). We consider \( \tilde{d} = \lambda/2 \). The signal to noise ratio (SNR) is defined as \( p |\mathbf{w}^H \mathbf{H} \mathbf{f}|^2 / \sigma^2 \), where \( p \) and \( \sigma^2 \) are the transmit and noise power, respectively. To maximize SNR, we can design beamforming vector \( \mathbf{f} \in \mathbb{C}^{N_{\text{BS}}} \) and combining vector \( \mathbf{w} \in \mathbb{C}^{N_{\text{UE}}} \) from maximizing \( |\mathbf{w}^H \mathbf{H} \mathbf{f}|^2 \), where \( \mathcal{F} \) and \( \mathcal{W} \) are the beamforming and combining beam codebooks, respectively.

We assume UEs are moving along a trajectory. We consider a quantized trajectory with length \( M \) and reference location indices \( x \in [M] \). For each BS \( j \in \mathcal{B} \), we divide the trajectory into \( K \) regions \( \mathcal{R}_k := \{ \alpha_{k-1}^j + 1, \ldots, \alpha_k^j \} \) for \( k \in [K] \), where \( \alpha_k^j \in [M] \) denote the location index of the end of the region \( k \) and \( 0 = \alpha_0^j \leq \alpha_1^j \leq \cdots \leq \alpha_K^j = M \). Because the problem is evaluated independently for each BS \( j \in \mathcal{B} \), we remove super-script \( j \) to make the notation simpler. Next we define the path skeleton.

**Definition 1 (path skeleton):** The path skeleton between location index \( x \) and the BS is defined as

\[
\text{PS}(x) := \left( \phi_{i,x,\ell}^{\text{BS}}, \phi_{i,x,\ell}^{\text{UE}} \right)_{\ell=1}^L
\]

where \( \phi_{i,x,\ell}^{\text{BS}} \) and \( \phi_{i,x,\ell}^{\text{UE}} \) are the AoD and AoA of the \( \ell \)-th strongest path from the BS to a UE in location index \( x \), respectively, and \( L \) is the number of paths. Note that path skeletons (PSs) are random because of the random blockages in the environment. However, they are correlated in adjacent locations [6]. Moreover, they depend heavily on second order statistics of the channel, which changes relatively slowly.

**III. Problem Formulation**

In this work, we investigate the design of mmWave codebooks that are adaptive to the surrounding environment. To this end, for any BS, we use one beam for all the locations in each region. For region \( \mathcal{R}_k \), the path skeleton of one and only one reference point \( x_k \in \mathcal{R}_k \cup \{ \alpha_{k-1} \} \) has been measured, and all the locations in the region use the strongest beam of the path skeleton of \( x_k \) as the transmission beam. You can find an illustration in Fig. 1.

Hence, for each BS, first, the reference points are determined. Then, the regions are defined based on the reference points and their measured path skeleton as

\[
\begin{align}
\text{minimize} & \quad \mathbb{E}[K] \\
\text{s.t.} & \quad \Pr \left\{ d(x,x_k) \leq \gamma \mid (\text{ps}(x_k))_{k=1}^K \right\} \leq \epsilon, \\
& \quad \forall x \in \mathcal{R}_k, \quad \forall k \in [K], \quad \forall (\text{ps}(x_k))_{k=1}^K \leq \alpha_1 \leq \cdots \leq \alpha_K
\end{align}
\]
\[\alpha_k \in [M], \quad \forall k \in [K-1] \]  
\[x_k \in R_k \cup \{\alpha_{k-1}\}, \quad \forall k \in [K], \]  
where \(d(x,y)\), for some \(x,y \in [M]\), is defined as
\[d(x,y) := \sum_{\ell=1}^{L} |a_{x,y}^{UE}(\phi_{x,y}^{UE})a_{y,x}^{UE}(\phi_{x,y}^{UE})| |a_{x,y}^{BS}(\phi_{x,y}^{BS})a_{y,x}^{BS}(\phi_{x,y}^{BS})|. \]

\[R_k = \{\alpha_{k-1} + 1, \ldots, \alpha_k\} \text{ with } \alpha_0 = 0, \alpha_K = M; \phi_{x,y}^{UE} \text{ and } \phi_{x,y}^{BS} \text{ are the AoA and AoD of the path skeleton between a } \text{UE in location index } x \text{ and the BS, respectively.} \]
In (1b), we use pre-determined parameters \(\gamma \) and \(\epsilon\) to measure if the path skeleton at reference location \(x_k\) is valid for any location \(x \in R_k\) for any possible set of path skeleton of the reference points. Higher value of \(d(\cdot, \cdot)\) means that the selected beam angles \(\phi_{x_k,\ell}\) are close to the path skeleton in location index \(x\). Small letter ps means that the path skeleton in \(x_k\) is given.

Note that constraint (1e) is equivalent to
\[x_k \in \{\alpha_{k-1}, \alpha_k\}, \quad \forall k \in [K], \]  
because if \(\alpha_{k-1} < x_k < \alpha_k\), we can divide \(R_k\) to \(R'_k\) from \(\alpha_{k-1}\) into \(x_k\) and \(R''_k\) from \(x_k+1\) to \(\alpha_k\). In this case, each new region satisfies (3) without changing the other part of the problem.

Each reference point is determined based on the path skeleton of the previous reference points. Note that the reference points are not necessarily chosen in ascending order. Hence, we define \(\tilde{x}_1, \ldots, \tilde{x}_K\) as a permutation of \(x_1, \ldots, x_K\) such that \(\tilde{x}_i\) is chosen as a reference point before \(\tilde{x}_j\) for any \(i < j\). Precisely, for choosing the \(k\)-th reference point \(\tilde{x}_k\), the set of previous reference points \(X_k := \{\tilde{x}_1, \ldots, \tilde{x}_{k-1}\} \subseteq [M]\) and their path skeletons \(\text{PS}(z)\): \(z \in X_k\) are known:
\[\{\tilde{x}_q, \text{PS}(\tilde{x}_q)\}_{q=1}^{k-1} \mapsto \tilde{x}_k \in [M], \]
\[X_{k+1} = \{\tilde{x}_k\} \cup X_k. \]

In the end, the regions \(R_k\) are determined after evaluating all the reference points:
\[(x_k)_{k=1}^{K} \mapsto (\alpha_k)_{k=1}^{K-1}. \]
Therefore, the optimal regions \(R'_k\) and the corresponding reference points \(x_k^*\), for \(k \in [K]\), are determined.

**IV. PROPOSED METHOD**

Since solving (1a) is difficult in practice, we assume a Markov property to make it easier.
**Assumption 1:** For any \(x \leq x' \leq x'' \in [M]\), we have the Markov chain
\[\text{PS}(x) \rightarrow \text{PS}(x') \rightarrow \text{PS}(x'').\]

We define blocks and their state in the following definition.
**Definition 2:** The block \(B(x_1, x_h)\), for \(0 \leq x_1 \leq x_h \leq M\), is a number of adjacent location indices \((x_1, \ldots, x_h)\). According to the knowledge of the path skeleton at the start and the end of the block, we define three different blocks:
- **Type 1:** A block whose path skeleton at the beginning and the end of the block is known.
- **Type 2:** A block whose path skeleton at the beginning of the block is known.
- **Type 3:** A block whose path skeleton at the end of the block is known.

The state of the block \(B(x_1, x_h)\) is defined as
\[S := (x_1, x_h, \text{PS}(x_1), \text{PS}(x_h)) \text{ for Type 1 blocks}, \]
\[S := (x_1, x_h, \text{PS}(x_1)) \text{ for Type 2 blocks}, \]
\[S := (x_1, x_h, \text{PS}(x_h)) \text{ for Type 3 blocks}. \]

Note that, the whole trajectory is also a block with \(x_1 = 0\) and \(x_h = M\). We generalize (1a) and define the value of a block as the minimum number of reference points in the block.
**Definition 3:** For any block \(B(x_1, x_h)\) with a given state \(s\), we define the value of the block as
\[\minimize_{\alpha_1, \ldots, \alpha_{K-1}} \mathbb{E}\{K \mid s\} \]  
s.t. \[\Pr\{d(x_k, x_h) \leq \gamma \mid (\text{PS}(x_k))_{k=1}^{K}\} \leq \epsilon, \]
\[\forall x_k \in R_k, \quad \forall k \in [K], \quad \forall (\text{PS}(x_k))_{k=1}^{K}, \]
\[\alpha_1 \leq \cdots \leq \alpha_{K-1} \]  
\[\alpha_k \in B(x_1, x_h), \quad \forall k \in [K-1] \]  
\[x_k \in \{\alpha_{k-1}, \alpha_k\}, \quad \forall k \in [K], \]  
where \(d(\cdot, \cdot)\) was defined in (2); and \(R_k = \{\alpha_{k-1} + 1, \ldots, \alpha_k\}\) with \(\alpha_0 = x_1, \alpha_K = x_h\). The reference points \(x_k\) and regions \(R_k\) are selected based on (4) and (5), respectively.

We denote the value of the block in (6a) by \(v(s)\), \(v'(s)\), or \(v''(s)\), for Type 1, Type 2, or Type 3 blocks, respectively.

The next lemma gives a recursive solution for the value of Type 1 blocks based on the value of smaller Type 1 blocks.
**Lemma 1:** Having Assumption 1, for a Type 1 block \(B = B(x_1, x_h)\) given the state \(s\), we have that
\[v(s) = 0\]
if for some \(\alpha \in B\) we have
\[\Pr\{d(x_1, x_h) \leq \gamma \mid s\} \leq \epsilon, \quad \forall x_k \in \{x_1, \ldots, \alpha\}, \]
\[\Pr\{d(x, x_h) \leq \gamma \mid s\} \leq \epsilon, \quad \forall x \in \{\alpha, \ldots, x_h\}. \]

In this case, the regions are \(R_1 = B(x_1, \alpha)\) and \(R_2 = B(\alpha, x_h)\).
Otherwise,
\[v(s) = \minimize_{x \in B(x_1, x_h)} 1 + \mathbb{E}\{v(S_1) + v(S_2) \mid s\}. \]
where $S_1$ and $S_2$ are the states of the blocks $\mathcal{B}_1 = \mathcal{B}(x_1, x)$ and $\mathcal{B}_2 = \mathcal{B}(x, x_h)$, respectively. Further, in (7), there will be no new reference point inside the block, while in (9), the minimizer will be the next reference point inside the block.

Note that $S_1$ and $S_2$ are random due to the randomness of $\text{PS}(x)$.

Proof: See Appendix A.

The next lemma gives a recursive solution for the value of a Type 2 block, based on the values of smaller Type 1 and Type 2 blocks. Similar equations are valid for Type 3 blocks, based on the values of smaller Type 1 and Type 3 blocks.

**Lemma 2:** Having Assumption 1, for a Type 2 block $\mathcal{B} = \mathcal{B}(x_1, x_h)$ given the state $s$, we have that

$$v'(s) = 0$$

if we have

$$\Pr \{d(x_1, x) \leq \gamma | s\} \leq \epsilon, \quad \forall x \in \mathcal{B}. \tag{10}$$

Otherwise,

$$\min_{x \in \mathcal{B}} 1 + \mathbb{E} [v'(S_1) + v'(S_2) | s], \tag{12}$$

where $S_1$ and $S_2$ are the states of the blocks $\mathcal{B}_1 = \mathcal{B}(x_1, x)$ (Type 1) and $\mathcal{B}_2 = \mathcal{B}(x, x_h)$ (Type 2), respectively; and $v(s)$ is the value of a Type 1 block with state $s$ (see Lemma 1).

Further, in (10), there will be no new reference point inside the block, while in (12), the minimizer will be the next reference point inside the block.

Proof: See Appendix B.

Now, we state a solution for (1a).

**Theorem 1:** Having Assumption 1, the solution of (1a) is

$$\min_{x \in \mathcal{M}} 1 + \mathbb{E} [v''(S_1) + v''(S_2)], \tag{13}$$

where $S_1$ and $S_2$ are the states of the blocks $\mathcal{B}_1 = \mathcal{B}(1, x)$ (Type 3) and $\mathcal{B}_2 = \mathcal{B}(x, M)$ (Type 2), respectively; and $v'(s)$ and $v''(s)$ are the value of a Type 2 and Type 3 blocks with state $s$, respectively (see Lemma 2). Further, the minimizer of the optimization will be the first reference point inside the trajectory. Note that $S_1$ and $S_2$ are random due to the randomness of $\text{PS}(x)$.

Proof: See Appendix C.

### Table I: Simulation parameters

| Parameter                  | Value  |
|----------------------------|--------|
| BS transmit power          | 10 dBm |
| Noise power ($\sigma^2$)   | -94 dBm|
| Carrier frequency          | 28 GHz  |
| BS antennas                | 64 x 1 ULA |
| UE antennas                | 4 x 1 ULA |
| $L$                        | 3      |
| $\epsilon$                | 0.1    |
| $\gamma$                  | 0.2    |

V. NUMERICAL RESULTS

We evaluate the performance of the proposed method in an urban environment using the ray tracing tool in MATLAB toolbox. The output of ray tracing tool is the $L$ available paths between a BS and a UE in a specif location. The ray tracing maintains spatial consistency of mmWave channels.

As depicted in Fig. 2, we extracted the building map of Kista in Stockholm city and used it as the input data for the ray tracing simulation. In our scenario, we assumed the building material is brick and the terrain material is concrete. We also assumed that the length of the trajectory is 10 m and there are 10 location indices for each trajectory. The BSs’ height is 6 m and the UEs’ is 1 m.

We consider three baselines. Baseline 1 is exhaustive beam search. In this case, the path skeleton (set of strong beams) of all the locations in the trajectory was evaluated in the pre-setup, where the BS, exhaustively, searches over all the beams in all directions, to find the path skeletons. It means the number of regions is equal to the number of locations along the trajectory. In the run-time, the BS finds the path with the largest gain, among the paths in the path skeleton.

In Baseline 2, we considered the proposed approach in our previous work [12], [13]. Here, the reference points are chosen based on a greedy algorithm starting from the beginning of the trajectory and continuing towards the end. The regions starts with a reference point and a new reference point, along the trajectory, is the one whose channel is much different in comparison with the channel of the previous reference point.

In Baseline 3 the size of the regions is fixed for all trajectories.

In order to find the solution of (13), we generated 500 trajectories randomly in the environment with the same length but in different streets and directions. For each trajectory, we considered one BS with a fixed position with respect to the trajectory (10 m above the middle of the trajectory). Hence, the location of the points of the trajectory and the BS is fixed while the environment is different.

The number of beam searches and the SNR, on average, for different methods are shown in Table II and Fig. 3, respectively. We assumed narrow beam codebook with size 128 in both the BS and the UE. As expected, the exhaustive beam search has the highest SNR because it measures all the path skeletons through the trajectory. However, it requires $10 \times 128^2$ beam searches for each trajectory. Our proposed method almost has the same SNR in comparison with Base-
lines 2. However, our method has, approximately, $3 \times 128^2$ (3 reference points) and Baseline 2 has $6 \times 128^2$ (6 reference points) beam searches, on average. However, unlike Baseline 2 that the reference points are selected from the beginning of the trajectory, in our proposed method, they can be selected with any order along the trajectory; as a result it gives more degrees of freedom. After designing the codebook based on the defined regions, in the run-time, the beam searching is performed only for reference points to find the strongest path. Hence, we have about $3 \times 9 = 27$ times beam searching, where 3 is the average number of reference points and $9 = 3 \times 3$ comes from searching over the path skeleton with $L = 3$ paths which is lower than the baselines. In Baseline 3 in order to have the same beam searching complexity as our proposed method, we choose the number of the regions equal to 3 and define regions for all the trajectories as $R_1 = \{1, 2, 3, 4\}$, $R_2 = \{5, 6\}$, and $R_3 = \{7, 8, 9, 10\}$; and the reference locations are the first location index in each region. As it is shown in Fig. 3, non-optimal selections of reference locations and regions cause high SNR fluctuations along the trajectory.

Fig. 4, represents an example of two trajectory samples and their regions. As expected, we can observe that in location indices near the BS, we have smaller regions in comparison with the beginning and the end points of the trajectory. We can interpret that the sensitivity of our method to the accuracy of the location information input increases as the size of the regions decreases.

VI. CONCLUSIONS AND FUTURE WORKS

We proposed a beam searching method that decreases the complexity by utilizing the channel similarity in adjacent locations. Simulation results verified the better performance of our method in beam searching complexity in compared to the baselines. Future directions include applying machine-learning methods for more practical scenarios where the joint distribution of the path skeletons in different locations is not available. Considering the 2-D regions, instead of 1-D trajectory is also an interesting problem to be studied.

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APPENDIX A

PROOF OF LEMMA 1

It is clear that if (8) is valid, there is no need to select any other reference point inside the block; as a result, the solution
Thus, optimization problem (15) is decomposed to two independent optimization problems and (9) follows, and the lemma is proved.

Therefore, it only remains to show (15), (16), and (17).

Proof of (15): From (6a), we obtain

\[
\mathbb{E} [K \mid s] = \mathbb{E} [\mathbb{E} [K \mid s, \text{PS}(x)] \mid s]
\]

where (a) follows from (14). Hence, we can write

\[
\begin{align*}
&\text{minimize } 1 + \mathbb{E} [\mathbb{E} [K_1 \mid s, \text{PS}(x)] + \mathbb{E} [K_2 \mid s, \text{PS}(x)] \mid s] \\
&= \text{minimize } 1 + \mathbb{E} [\mathbb{E} [K_1 \mid s, \text{PS}(x)] + \mathbb{E} [K_2 \mid s, \text{PS}(x)] \mid s] \\
&\text{subject to } \alpha_1 \leq \cdots \leq \alpha_{K-1} \\
&\alpha_k \in \mathcal{B}(x_k, \tilde{x}_1), \quad \forall k \in [K-1] \\
&\alpha_k \in \mathcal{B}(x_k, \tilde{x}_1), \quad \forall k \in \{K + 1, \ldots, K - 1\} \\
&x_k \in \{\alpha_{k-1}, \alpha_k\}, \quad \forall k \in [K],
\end{align*}
\]

Thus, optimization problem (15) is decomposed into two independent optimization problems and (9) follows, and the lemma is proved.

Appendix B

Proof of Lemma 2

The proof is similar to the proof of Lemma 1 and we only prove it for Type 2 blocks and the proof for Type 3 blocks follows the same approach.

It is clear that if (11) is valid, there is no need to select any other reference point inside the block; as a result, the solution of (6a) is 0 and the regions are \( \mathcal{B}(x_1, \alpha) \) and \( \mathcal{B}(\alpha, x_k) \).

For the case that (11) is not satisfied, it is required to have at least one reference point inside the block. The approach is exactly the same as Lemma 1, while blocks \( \mathcal{B}_1 = \mathcal{B}(x_1, \tilde{x}_1) \) and \( \mathcal{B}_2 = \mathcal{B}(\tilde{x}_1, x_k) \) are Type 1 and 2 blocks, respectively.

Hence, we can write the optimization problem (6a) as follows

\[
\begin{align*}
\text{minimize } 1 + \mathbb{E} [\mathbb{E} [K_1 \mid s, \text{PS}(x_1)], \\
&\text{subject to } \alpha_1 \leq \cdots \leq \alpha_{K-1} \\
&\alpha_k \in \mathcal{B}(x_k, \tilde{x}_1), \quad \forall k \in [K-1] \\
&\alpha_k \in \mathcal{B}(\tilde{x}_1, x_k), \quad \forall k \in [K],
\end{align*}
\]
\[
\begin{align*}
&\min_{\alpha_{K_1}, \ldots, \alpha_{K_2}} \mathbb{E} [K_2 \mid \text{PS}(\tilde{x}_1)] \mid s \\
\text{s.t.:} & \quad \Pr \left\{ d(x, x_1^{(1)}) \leq \gamma \mid \left(\text{PS}(x_1^{(1)})\right)_{k=1}^{K_1}, \text{PS}(x_i), \text{PS}(\tilde{x}_1) \right\} \\
& \quad \leq \epsilon, \quad \forall x \in \mathcal{R}_k^{(1)}, \forall k \in [K_1], \\
& \quad \forall \left(\text{PS}(x_1^{(1)})\right)_{k=1}^{K_1}, \text{PS}(x_i), \text{PS}(\tilde{x}_1) \tag{23}
\end{align*}
\]

Thus, optimization problem (22) is decomposed into two independent optimization problems and (12) follows, and the lemma is proved.

Therefore, it only remains to show (22), (23), and (24). The proof of (23) and (24) are similar to the proof of (16) and (17). For the proof of (22), from (6a), likewise the proof of (15), in the end, we obtain

\[
\begin{align*}
&\min_{\tilde{x}_1 \ldots \tilde{x}_K} \mathbb{E} [K \mid s] \\
&= \min_{\tilde{x}_1} \left[1 + \min_{\tilde{x}_1^{(1)} \ldots \tilde{x}_K^{(1)}} \mathbb{E} [K_1 \mid \text{PS}(x_i), \text{PS}(x)] \right. \\
&\quad + \min_{\tilde{x}_1^{(2)} \ldots \tilde{x}_K^{(2)}} \mathbb{E} [K_2 \mid \text{PS}(x)] \mid s \right]
\end{align*}
\]

where it follows from Assumption 1 because given \(\text{PS}(\tilde{x}_1)\), \(\left(\text{PS}(\tilde{x}_1^{(1)})\right)_{k=1}^{K_1}, \text{PS}(x_i)\) are independent of \(\left(\text{PS}(\tilde{x}_1^{(2)})\right)_{k=1}^{K_2}\).

\section*{Appendix C}

\textbf{Proof of Theorem 1}

The proof is similar to the proof of Lemmas 1 and 2. Same as Lemma 1, we have two blocks \(\mathcal{B}_1 = \mathcal{B}(1, \tilde{x}_1)\) and \(\mathcal{B}_2 = \mathcal{B}(\tilde{x}_1, M)\) where the former is Type 3 and the latter is Type 2 block. Hence, the optimization problem (1a) is transformed into

\[
\begin{align*}
&\min_{\tilde{x}_1} \left[1 + \min_{\tilde{x}_1^{(1)} \ldots \tilde{x}_K^{(1)}} \mathbb{E} [K_1 \mid \text{PS}(\tilde{x}_1)] \right. \\
&\quad + \min_{\tilde{x}_1^{(2)} \ldots \tilde{x}_K^{(2)}} \mathbb{E} [K_2 \mid \text{PS}(\tilde{x}_1)] \mid s \right] \tag{29}
\end{align*}
\]

s.t. \(\Pr \left\{ d(x, x_1^{(1)}) \leq \gamma \mid \left(\text{PS}(x_1^{(1)})\right)_{k=1}^{K_1}, \text{PS}(\tilde{x}_1) \right\} \leq \epsilon, \quad \forall x \in \mathcal{R}_k^{(1)}, \forall k \in [K_1], \)

\(\forall \left(\text{PS}(x_1^{(1)})\right)_{k=1}^{K_1}, \text{PS}(\tilde{x}_1) \tag{30}\)

\(\Pr \left\{ d(x, x_1^{(2)}) \leq \gamma \mid \left(\text{PS}(x_1^{(2)})\right)_{k=1}^{K_1}, \text{PS}(\tilde{x}_1) \right\} \leq \epsilon, \quad \forall x \in \mathcal{R}_k^{(2)}, \forall k \in [K_2], \)

\(\forall \left(\text{PS}(x_1^{(2)})\right)_{k=1}^{K_1}, \text{PS}(\tilde{x}_1) \tag{31}\)

\(x_k \in \{\alpha_{k-1}, \alpha_k\}, \quad \forall k \in \{1, \ldots, K\}, \tag{32}\)

where the proofs of objective function and the constraints follow similar to the proof of corresponding terms in Lemmas 1 and 2; and also from Assumption 1 because given \(\text{PS}(\tilde{x}_1)\), \(\left(\text{PS}(\tilde{x}_1^{(1)})\right)_{k=1}^{K_1}\) are independent of \(\left(\text{PS}(\tilde{x}_1^{(2)})\right)_{k=1}^{K_2}\).