**Abstract:** This paper proposes a distributed parameter-based voltage stability index (DPVSI) for assessment of voltage stability of the power system. Proposed index incorporates the effects of distributed parameters of transmission line to predict voltage collapse. Critical loading of different lines is determined by raising active/reactive loading of different lines till the index reaches its critical value. Different contingencies are also ranked on the basis of index. Reactive power margin of the system is measured by mapping the index value in terms of reactive power margin of the system. The proposed index is investigated on IEEE 30-bus and 118-bus test systems to prove its potential.

**Keywords:** Voltage stability index; voltage collapse; contingency analysis; line outage; distributed parameters

**Public Interest Statement**

The continuous increase in energy demand leads existing power systems to operate under a stressed condition. The stressed condition may create a low voltage/voltage collapse in many parts of power systems which is known as voltage instability. The online assessment of voltage stability state of the power system plays a vital role in the reliable operation as well as uninterrupted supply. Therefore, this research work proposes a voltage stability index which assesses the voltage stability of power systems. The proposed index identifies the critical lines and buses which are threat to voltage security of the system. Planners can also use the index to find reactive power margin available in the line. The proposed index is rigorously tested for various possible conditions on IEEE 30 and 118-bus test systems. It is observed that the proposed index is capable to indicate the voltage stability state accurately for all possible operating conditions.
1. Introduction
In the past few decades, the demand for electric power is rapidly increasing which yields increased complexity in the interconnected power system. Therefore, it is very arduous to plan, analyze and operate the interconnected system (Taylor). As the power system operates under stressed conditions, it becomes more prone to voltage stability issues (Kundur, Balu, & Lauby, 1994). The voltage instability problem has got more attention during past decades due to the proliferate blackouts that occurred in some of the countries (Ajjarapu, 2007; Momoh, 1991). The problem related to voltage instability in a power system is one of the influential criteria in power system planning and operation (Wang et al., 2014; Amjad & Velayati, 2011). The voltage instability arises due to insufficient reactive power support. This can be overcome considerably by proper allocation of adequate reactive power sources at appropriate locations (Kessel & Glavitsch, 1986). To achieve this, ingenious and vigorous approach is required. The voltage stability analysis can be broadly classified as into dynamic and static analysis. The static voltage stability methods depend mainly on the steady-state model in the analysis, such as power flow model or a linearized dynamic model described by the steady-state operation. The dynamic analysis implies the use of a model, characterized by nonlinear differential and algebraic equations, which include generators dynamics, tap changing transformers etc. through transient stability simulations (Prada & Santos, 1999; Reis & Maciel-Barbosa, 2006; Wu, Zhang, & Chen, 2009).

During the past few decades, researchers have devised various approaches to analyzing voltage stability. These can be bifurcated into two broad categories namely graphical approach and analytical approach. The graphical approach contains P-V curves and Q-V curves (Guimares, Fernandez, & Ocariz, 2011; Mohn & Zambroni-De-Souza, 2006). These approaches are time-consuming as it requires repeated application of load flow, whereas the analytical approach is based upon calculation of voltage stability indices like jacobian matrix singularity indices (Araposthatis, Sastry, & Varaiya, 1981; Gao, Morison, & Kundur, 1992; Lof, Smed, Andersson, & Hill, 1992), thevenin's equivalent-based index (An, Zhou, Yu, & Zhang, 2006; Smon, Verbic, & Gubina, 2006), active and reactive voltage stability index (P-VSI and Q-VSI) (Jamian, Musa, Mustafa, Mokhlis, & Adamu, 2014), short-circuit capacity-based voltage stability index (SCC-VSI) (Xu et al., 2016), simplified voltage stability index (SVSI) (Perez-Londono, Rodriguez, & Olivar, 2014), voltage collapse proximity indicator (VCPI) (Chebbo, Irving, & Sterling, 1992; Moghavvemi & Faruque, 1998), voltage instability predictor (VIP) (Julian et al., 2000), L-index (Kessel & Glavitsch, 1986), extended L-index (Wang et al., 2013), improved voltage stability index (Lj) (Hongjie, Xiaodan, & Yixin, 2005), line stability index (Lm) (Moghavvemi & Omar, 1998), line stability factor (LOP) (Mohamed, Jasmon, & Yusoff, 1989), fast voltage stability index (FVSI) (Musirin & Rahman, 2002), line collapse proximity index (LCPI) (Tiwari, Niazi, & Gupta, 2012), line porosity coefficient index (POR) (Halliçević & Softić, 2016), novel line stability index (NLSI) (Yazdanpanah-Goharizzi & Asghari, 2007), equivalent node voltage collapse index (ENVCI) (Wang, Li, & Lu, 2009) and voltage stability index (LVSI) (Shan-Hang). These indices use the elements of the bus admittance matrix and system variables such as bus voltages, bus angles and complex power flow through transmission lines.

From the literature survey, it has been observed that most of the voltage stability indices are based upon the approximate model of transmission line where line capacitance and resistance have been neglected. In Tiwari et al. (2012), the transmission line is considered as equivalent pi-model where line parameters were assumed as lumped parameters. Prediction of the state of a system based on these approaches was erroneous due to the assumptions considered during development of these indices. A transmission system can be best represented by distributed parameters model. No efforts have being made so far to develop a voltage stability index considering distributed parameters of a transmission system. It is also observed from Tiwari et al. (2012) that voltage stability of line is affected by relative direction of active and relative power flow in the line. Therefore, effect of the relative direction of active and reactive power flow must be considered in developing an effective index. Moreover, the accurate loading margins can’t be estimated by Moghavvemi and Omar (1998) and Musirin and Rahman (2002), due to negligence of various parameters as discussed above. Considering these limitations and shortcomings of the
existing indices, a new index is proposed to accurately predict the proximity of the actual system’s operating point to the point of instability. Proposed index has following features:

- Distributed parameters of transmission system are accounted.
- Line resistance and shunt admittance are accounted.
- Incorporates the effect of the relative direction of active and reactive power in the line.
- Estimation of available reactive loading margin.
- Assessment of maximum loadability of the system.
- Identification of weak lines and nodes.
- Online measurement of voltage stability.
- Assessment of viability of new lines from voltage stability point of view at the planning stage.
- Indication of local as well as global voltage stability.

The effectiveness of the proposed index has been tested on IEEE 30-bus and 118-bus system under various operating conditions and compared with existing voltage stability indices.

In the following sections, some of the most common voltage stability indices have been briefly discussed to appreciate the proposed distributed voltage stability index (DPVSI).

2. Various voltage stability indices
Some of the established indices are $L_{mn}$, FVSI and POR (Halilčević & Softić, 2016; Moghavvemi & Omar, 1998; Musirin & Rahman, 2002). A brief discussion of these indices is presented below.

2.1. Fast voltage stability index
The fast voltage stability index named FVSI has been defined by Musirin and Rahman (2002). It is formulated from two bus representation of a system as shown in Figure 1. The FVSI is defined as

$$FVSI = \frac{4Z^2Q_j}{V^2X} \leq 1$$

(1)

where $Z$ is the line impedance, $X$ is the line reactance, $P_j$ and $Q_j$ are the active and reactive powers at the receiving end, respectively, $V_i$ is the bus voltage at the sending end. FVSI of a line near to unity indicates that the line is close to its stability limit. The shortcoming of this index is that when $Q_j/0$, it indicates the respective line and thus, the system is always secure at that particular condition.

2.2. Line stability index
Moghavvemi and Omar derived the line stability index known as $L_{mn}$, with the line flow data. The line stability index, as derived by Moghavvemi and Omar (1998), can be represented as

$$L_{mn} = \frac{4Q_jX}{V_j \sin(\theta - \delta)} \leq 1$$

(2)

Figure 1. 2-Bus power system model.
where $\theta$ is the impedance angle of line and $\delta = \delta_i - \delta_j$ is the phase difference of sending and receiving end bus voltages, while all other variables are same as explained in Section 2.1. To maintain the voltage stability, the value of $L_{mn}$ should be less than unity. This index also suffers from the same problem as FVSI.

2.3. Line porosity coefficient index

The line’s porosity coefficient index namely $POR$ for voltage stability assessment of power system has been developed in Halilčević and Softić (2016). Mathematically, $POR$ can be expressed as

$$POR = \frac{X^6}{X_{line}} > 1$$ (3)

where $X^6$ is the reactance of porosity and $X_{line}$ is the transmission line reactance. To maintain the voltage stability, the value of $POR$ should be greater than unity.

3. Proposed DPVSI

The indices proposed by Moghavvemi and Omar (1998), Mohamed et al., (1989), and Musirin and Rahman (2002) used the model of a medium transmission line. The line charging reactance and shunt conductance in deriving equations of indices are neglected and the parameters are assumed to be lumped by Moghavvemi and Omar (1998), Mohamed et al. (1989) and Musirin and Rahman (2002). However, the line charging reactance may support voltage stability of the system. In the existing indices (Moghavvemi & Omar, 1998; Mohamed et al., 1989; Musirin & Rahman, 2002), the effect of distributed parameters was also neglected. As a result, the methods which utilize approximate model may not provide a precise prediction of voltage stability. Consequently, a distributed parameter based voltage stability index (DPVSI) is proposed based upon distributed parameters of transmission line, considering the effect of the relative direction of real and reactive power flow through the transmission line of the system. An exact model of long length transmission line is usually described by two-port equivalent pi-model. Therefore, the proposed index is derived using distributed parameters of pi-model of a long length transmission line. The pi-model of a long length transmission line of a two bus system is shown in Figure 2. The voltage and current relationship can be expressed as

$$\bar{V}_s = \cosh(\gamma l)\bar{V}_r + \bar{Z}_0\sinh(\gamma l)\bar{I}_r$$ (4)

$$\bar{I}_s = \frac{\sinh(\gamma l)}{\bar{Z}_0}\bar{V}_r + \cosh(\gamma l)\bar{I}_r$$ (5)

where $\bar{Z}_0$ and $\gamma$ are known as the characteristic impedance and propagation constant, respectively, and they can be expressed as

$$\bar{Z}_0 = \sqrt{\frac{Z}{Y}}$$ (6)

Figure 2. Typical one-line diagram of long transmission line.
\[ \bar{\gamma} = \sqrt{Z\bar{Y}} \]  
(7)

where \( Z \) and \( \bar{Y} \) denote the impedance and line charging admittance of line, respectively. The current at the receiving end of the line is expressed as

\[ \bar{I}_r = \frac{P_r - jQ_r}{V_r} \]  
(8)

where \( P_r \) and \( V_r \) are the active power and voltage at the receiving end, respectively, while other variables are same as defined above. The sending end voltage \( V_s \) of the line can be written from Equation (4) as

\[ V_s/\delta_s = \cosh(a_l + j\beta_l)V_r/\delta_r + (Z_{0x} + jZ_{0y})\sinh(a_l + j\beta_l)I_r/\delta_r \]  
(9)

where \( \sqrt{Z_{0x}^2 + Z_{0y}^2} \) and \( \sqrt{a_l^2 + \beta_l^2} \) are magnitudes while \( \tan^{-1} \frac{Z_{0y}}{Z_{0x}} \) and \( \tan^{-1} \frac{\beta_l}{a_l} \) are phase angles of parameters \( Z_0 \) and \( \beta_l \), respectively. Substituting the value of \( I_r \) obtained from Equation (8) into Equation (9), we get

\[ V_s/\delta_s = \cosh(a_l + j\beta_l)V_r/\delta_r + (Z_{0x} + jZ_{0y})\sinh(a_l + j\beta_l)\left(\frac{P_r - jQ_r}{V_r} - \delta_r\right) \]  
(10)

Rearranging Equation (10) yields

\[ V_s V_r/\delta = \cosh(a_l + j\beta_l)\left(V_r^2 + (Z_{0x} + jZ_{0y})\sinh(a_l + j\beta_l)(P_r - jQ_r)\right) \]  
(11)

where \( \delta = (\delta_s - \delta_r) \). Separating Equation (11) into real and imaginary parts, we obtain the following quadratic equation from the real part.

\[ V_r^2 - \left[ \frac{V_s\cos\delta}{\cosh(\beta_l)\cosh(a_l)} \right] V_r + [H_l\tanh(a_l) - H_l^r\tan(\beta_l)] = 0 \]  
(12)

where

\[ H_l' = P_rZ_{0x} + Q_rZ_{0y} \]

\[ H_l^r = P_rZ_{0y} - Q_rZ_{0x} \]

\[ V_r = V_s\sqrt{2V_r^2 - 4[H_l\tanh(a_l) - H_l^r\tan(\beta_l)]} \]  
(13)

To find real and nonzero values of \( V_r \), Equation (13) must have real and nonzero roots, which can be obtained by setting the discriminant of Equation (13) greater than 0, i.e.

\[ \left[ \frac{V_s\cos\delta}{\cosh(\beta_l)\cosh(a_l)} \right]^2 - 4[H_l\tanh(a_l) - H_l^r\tan(\beta_l)] > 0 \]  
(14)

On the basis of Equation (14), it is concluded that the following condition must be satisfied to avoid voltage collapse of the system:

\[ \frac{2[H_l\cos^2(\beta_l)\sinh(2a_l) - H_l^r\cosh^2(a_l)\sin(2\beta_l)]}{(V_s\cos\delta)^2} \leq 1 \]  
(15)

Distributed parameter based voltage stability index (DPVSI) for line “sr” can be described as

\[ DPVSI_{sr} = \frac{2}{(V_s\cos\delta)^2} \left| H_l\cos^2(\beta_l)\sinh(2a_l) - H_l^r\cosh^2(a_l)\sin(2\beta_l) \right| \]  
(16)
To maintain the voltage stability of the system, the proposed index must be less than unity, i.e.
\[
DPVSI_{sr} < 1; \forall (s, r) \in \{1, 2, \ldots, nt\} \& s \neq r
\]
(17)
where \(nt\) is equal to total number of transmission lines. The proposed index \(DPVSI_{sr}\) has indeterminate form when the shunt admittance is 0. The term \(\frac{\sinh(2\sqrt{Z\cos\delta})}{\sqrt{Y}}\) will be equal to unity. Therefore, the \(DPVSI_{sr}\) can be modified as given below:
\[
DPVSI_{sr}|_{Y=0} = \frac{4Z}{(V_0\cos\delta)^2} \left[ H^+ \cos\theta - \cos^3 \left( \sqrt{ZY\sin\frac{\theta}{Z}} \right) - H^- \sin\theta \cos^3 \left( \sqrt{ZY\cos\frac{\theta}{Z}} \right) \right]
\]
(18)
where
\[
\theta^+ = \angle Z + \angle Y
\]
\[
\theta^- = \angle Z - \angle Y
\]
\[
H^+ = P_r \cos\frac{\theta}{Z} + Q_r \sin\frac{\theta}{Z}
\]
\[
H^- = P_r \sin\frac{\theta}{Z} - Q_r \cos\frac{\theta}{Z}
\]
\[
DPVSI = \frac{(\max_{s\neq r} (DPVSI_{sr}))}{nt}
\]
(19)
\(DPVSI_{sr}\) of all the transmission lines are computed simultaneously and voltage stability state of different lines is measured on the basis of the index value. If a transmission line is at the brink of voltage instability, then the value of \(DPVSI_{sr}\) of the line will be close to unity. The highest value of \(DPVSI_{sr}\) is denoted as \(DPVSI\) which shows global voltage stability state of the system. The proposed index also considers the effect of relative direction of active and reactive power flow in a line. If \(P_r\) and \(Q_r\) flow in the same direction index value is high, conversely index value is low if active and reactive power flow in opposite direction. This proves that voltage stability condition of a line is a function of both magnitude and relative direction of real and reactive powers. Moreover, \(DPVSI_{sr}\) is based upon distributed parameters of a long transmission line which improves the accuracy in measurement of voltage stability. In Equation (16), the parameters \(\alpha_l, \beta_l, Z_{0x}\) and \(Z_{0y}\) are known for a given transmission network and \(P_r, Q_r\) and \(\delta\) can be obtained online. Therefore, the proposed index is also a valuable tool for online voltage stability monitoring and prediction. The effectiveness of the proposed index has been investigated in the next section.

4. Test results and discussion
The IEEE 30-bus and IEEE 118-bus systems are considered with the different loading combination and different topology (i.e. contingency analysis) to appraise the potential of the proposed index. A MATLAB code is developed to perform the test and tolerance \(10^{-12}\) is considered for mismatch of power. The results are also compared with other indices \(L_{mn}, FVSI\) and \(POR\) (Halilčević & Softić, 2016; Moghavvemi & Omar, 1998; Musirin & Rahman, 2002) to prove its feasibility. The transmission lines whose \(DPVSI\) index are found too close to unity are denoted as critical lines.

4.1. IEEE 30-bus
As mentioned, the IEEE 30-bus test system (Power system test archive-UWEE) has been considered to examine different voltage stability indices under different loading pattern such as

- Base case loading
- Single load change
- Multiple load change
4.1.1. Base case loading
This loading denotes that at all the nodes, the loads are fixed at base load values. The voltage stability indices $L_{mn}$, $FVSI$ and $POR$ are evaluated and compared with proposed index $DPVSI$. Lines with smaller values of $DPVSI$ are expected to have adequate stability margins, whereas larger values of $DPVSI$ indicate that lines are critical and further any addition of load may lead to voltage instability. The test results are presented in Table 1 for base case loading of IEEE 30-bus test system. The observation of Table 1 reveals that for some lines, the indices (Moghavvemi & Omar, 1998; Musirin & Rahman, 2002) offer lower values in comparison to proposed index for branches 1-3, 6-2, 6-7 and 25-27 and for branches 6-4 and 25-24, other indices (Moghavvemi & Omar, 1998; Musirin & Rahman, 2002) offer higher values in comparison to proposed index. The dissimilarity between the values of proposed index and the indices $L_{mn}$ and $FVSI$ is due to the fact that index $L_{mn}$ and $FVSI$ have neglected the effect of distributed parameters, line resistance and relative direction of active and reactive power flows. Equation (16) reveals that the value of proposed index is higher if the direction of active and reactive power flows is same. On the other hand, if the direction is different, then the value of the index is low. It is also observed that from Table 1, when the resistances and shunt capacitances of branches (like 6-9, 6-10) are 0, then the value of indices $L_{mn}$ and $DPVSI_{fr}$ is consistent with each other because the direction of active power flow relative to reactive power flow has negligible effect. The results obtained by proposed index ($DPVSI$) are in agreement with the another voltage stability assessment index $POR$. Therefore, the proposed index works as an unerring tool for prediction of voltage collapse under all circumstances.

4.1.2. Single load change
In this case, the load has been changed at one particular node at a time and all other nodes are fixed at their base values. Various combinations are considered to accomplish this (Moghavvemi & Omar, 1998):

- Single load change with real load only
- Single load change with reactive load only
- Single load change with real and reactive load

4.2. Single load change with real load only
An exhaustive analysis is performed at all nodes and only two representative cases at nodes 6 and 27 are discussed. The voltage stability indices (Halilčević & Softić, 2016; Moghavvemi & Omar, 1998; Musirin & Rahman, 2002) and proposed index values are calculated and presented in Table 2. A heavy real load of value 6.487 p.u. at node 6 increases the index $DPVSI$ to 0.8404. It shows that the transmission line 7-5 is the most critical line. The line 1-2 is the second most critical line. The results are confirmed by other indices but $L_{mn}$ and $FVSI$ in some cases cross the unity. On the particular

| (Line) from-to | $L_{mn}$ (Moghavvemi & Omar, 1998) | $FVSI$ (Musirin & Rahman, 2002) | $POR$ (Halilčević & Softić, 2016) | Proposed index ($DPVSI_{fr}$) |
|---------------|-----------------------------------|----------------------------------|-----------------------------------|-----------------------------|
| 1-3           | 0.042173                          | 0.038565                         | 3.18302                           | 0.183055                    |
| 6-2           | 0.007918                          | 0.007310                         | 4.76506                           | 0.134563                    |
| 6-4           | 0.031247                          | 0.030677                         | 22.5914                           | 0.004254                    |
| 25-24         | 0.026763                          | 0.026979                         | 42.8403                           | 0.010695                    |
| 6-7           | 0.004654                          | 0.004557                         | 17.2050                           | 0.034613                    |
| 6-9           | 0.072628                          | 0.072424                         | 12.2029                           | 0.072628                    |
| 6-10          | 0.013675                          | 0.013586                         | 5.94850                           | 0.013675                    |
| 25-27         | 0.008401                          | 0.008479                         | 60.0046                           | 0.027305                    |
| 28-8          | 0.020420                          | 0.020433                         | 62.5243                           | 0.019876                    |
Table 2. Single load change with real load only (30-bus)

| Node loading (p.u.) | (Line) from-to | \(l_{mn}\) (Moghavvemi & Omar, 1998) | FVSI (Musirin & Rahman, 2002) | POR (Halilčević & Softić, 2016) | Proposed index | Most stressed |
|---------------------|----------------|-------------------------------------|--------------------------------|----------------------------------|----------------|---------------|
|                     |                |                  |                                |                                  | (DPVSI\(_{mn}\)) | DPVSI | Line | Node |
| \(P = 6.487 \text{ at node 6}\) | 7-5            | 0.5902            | 0.4950                          | 1.3359                           | 0.8404         | 0.8404 | 7-5 | 5    |
|                     | 1-2            | 3.4247            | 1.2451                          | 1.9011                           | 0.7739         |        |      |      |
|                     | 4-6            | 1.5040            | 1.0839                          | 2.7166                           | 0.7492         |        |      |      |
|                     | 2-5            | 2.0928            | 1.0211                          | 1.6931                           | 0.7354         |        |      |      |
|                     | 6-8            | 0.7333            | 0.6916                          | 54.776                           | 0.6152         |        |      |      |
|                     | 6-9            | 0.5607            | 0.5597                          | 64.420                           | 0.5607         |        |      |      |
|                     | 2-4            | 0.2202            | 0.1001                          | 71.439                           | 0.5453         |        |      |      |
| \(P = 1.134 \text{ at node 27}\) | 1-3            | 0.3577            | 0.3005                          | 1.4164                           | 0.5779         | 0.5779 | 1-3 | 3    |
|                     | 25-24          | 0.0476            | 0.0332                          | 2.4204                           | 0.5128         |        |      |      |
|                     | 30-27          | 0.2132            | 0.1899                          | 2.9828                           | 0.4182         |        |      |      |
|                     | 24-22          | 0.2210            | 0.1988                          | 3.4234                           | 0.4133         |        |      |      |
|                     | 27-25          | 0.0152            | 0.0275                          | 3.6022                           | 0.4062         |        |      |      |
|                     | 27-28          | 0.3937            | 0.2482                          | 3.9463                           | 0.3937         |        |      |      |
|                     | 2-8            | 0.1225            | 0.1033                          | 4.0901                           | 0.3885         |        |      |      |
|                     | 28-8           | 0.3460            | 0.3431                          | 9.0174                           | 0.3611         |        |      |      |
occasion some lines namely 7-5, the indices $L_{mn}$ and $FVSI$ show very less value of the index. Therefore, indices $L_{mn}$ and $FVSI$ show pessimistic results. The index $POR$ also shows the transmission line 7-5 as a most stressed line. For the heavy real load of value 1.134 p.u. at node 27 makes the line 1-3 most stressed and the results are confirmed by all indices. From the test results, it may be concluded that indices (Moghavvemi & Omar, 1998; Musirin & Rahman, 2002) don’t provide actual assessment of voltage stability near the voltage collapse in certain cases. But the proposed index provides promising information regarding the voltage stability status of a transmission line.

4.3. Single load change with reactive load only
In this pattern of loadability, the reactive load at a single node is increased at a time. The value of voltage stability indices increases with the increase in reactive loading. The maximum reactive loading point at any selected node is identified as the reactive load for which $DPVSI$ is close to unity. The load flow solution will diverge for any further augmentation in loading. The results of simulation are presented in Table 3. A heavy reactive load of value 10.434 p.u. at node 6 increases the index $DPVSI$ to 0.9821. It shows that line 2-6 is the most critical line. The criticality of line 2-6 is confirmed by $L_{mn}$. For the line 8-6, the indices $L_{mn}$ and $FVSI$ cross unity which denotes that voltage collapse had occurred. Therefore, these indices present the erroneous prediction of voltage instability. In some cases, a difference is observed in the identification of most critical line by proposed index and other indices. This is due to fact that these indices are based upon approximation and therefore, the indices values are not correct representation of actual voltage stability of line. Studies of results reveal that the proposed index provide promising information of system state during heavy reactive loading.

4.4. Single load change with real and reactive load
In an actual electric power system, the real and reactive load changes simultaneously. In this case, the real and reactive load at a particular node are varied in same proportion. The loads at all other nodes are fixed at base values. As shown in Table 4 when both real and reactive loads are augmented gradually at node 6 up to a level of very close to voltage collapse, then it is observed that the line 5-2 is the most critical line with an index value of 0.9230. The indices $L_{mn}$ and $FVSI$ show very less value in this case. The index $POR$ also indicates the transmission line 5-2 as a most critical line but the value 0.8587 (for stable system $POR > 1$) indicates that the system has already been collapsed, but actually the system is still voltage stable. Therefore, the index $POR$ is also not able to represent voltage stability of the system accurately in some cases. Therefore, these indices fail to provide accurate results. Similarly, for a load $P = Q = 0.524$ p.u. at node 27, the line 28-27 is the most critical line. Thus, the proposed index provides promising information regarding the voltage stability status of a transmission line.

4.3.1. Multiple load change
A practical electric power system consists of hundreds of nodes and loads changes simultaneously at several nodes. To prove the suitability of the proposed index, the loads are varied uniformly at all nodes up to the level of voltage instability. For the load level of 3.2 times of system’s real load, the line 4-12 is the most stressed line as shown in Table 5. It is observed from Table 6 that the line 10-9 is the most critical line for load level of 5.7 times of system’s reactive load. Results of simulation for combined active and reactive loading are shown in Table 7. For the load level of 2.73 times of both real and reactive system’s load, the line 29-27 is the most critical line. Index $L_{mn}$ and $FVSI$ are not able to detect the voltage collapse under this condition as their values are much less than one. The results are also supported by index $POR$. Therefore, the proposed index is a versatile index which is able to detect the voltage unstable condition under all type of conditions.

Figures 3–5 display the variation of the voltage stability indices namely $L_{mn}$, $FVSI$, $POR$ and $DPVSI$ of the weakest node with an increase in loading factor ($\lambda$) (w.r.t. only real or only reactive or both real and reactive power loading for the constant power load model) in the IEEE 30-bus test system. It can be easily observed that when the loading is increased, the value of index $POR$ declines, while the
| Node loading (p.u.) | (Line) from- | \( I_{mn} \) (Moghavvemi & Omar, 1998) | \( FVSI \) (Musirin & Rahman, 2002) | \( POR \) (Haličević & Softić, 2016) | Proposed index | Most critical |
|-------------------|--------------|---------------------------------|---------------------------------|---------------------------------|---------------|--------------|
| Q = 1.0434 at node 6 | 2-6 | 0.9538 | 0.7700 | 1.1747 | 0.9821 | 0.9821 | 2-6 | 6 |
| | 8-6 | 1.3357 | 1.0711 | 47.126 | 0.9680 | | | |
| | 8-28 | 0.9293 | 1.0002 | 1.0954 | 0.8842 | | | |
| | 4-2 | 0.8082 | 0.7263 | 1.3092 | 0.8555 | | | |
| | 6-9 | 0.7904 | 0.7790 | 24.044 | 0.7904 | | | |
| | 5-7 | 0.8045 | 0.8280 | 1.3784 | 0.7829 | | | |
| | 6-28 | 0.9534 | 0.8925 | 71.598 | 0.7692 | | | |
| Q = 2.207 at node 10 | 6-10 | 0.9993 | 0.9908 | 0.9943 | 0.9993 | 0.9993 | 6-10 | 10 |
| | 17-16 | 0.9479 | 0.9724 | 1.1999 | 0.9098 | | | |
| | 9-6 | 0.9005 | 0.8982 | 1.0943 | 0.9005 | | | |
| | 16-12 | 0.8797 | 0.8970 | 1.3296 | 0.8502 | | | |
| | 10-9 | 0.7585 | 0.7571 | 1.3117 | 0.7585 | | | |
| | 27-28 | 0.7043 | 0.6910 | 1.2311 | 0.7043 | | | |
| | 12-4 | 0.6816 | 0.6544 | 1.1322 | 0.6816 | | | |
| Q = 0.7129 at node 27 | 28-27 | 0.9998 | 0.9960 | 0.9973 | 0.9998 | 0.9998 | 28-27 | 27 |
| | 27-25 | 0.7845 | 0.8128 | 1.5407 | 0.7281 | | | |
| | 24-25 | 0.6734 | 0.6987 | 1.8398 | 0.6380 | | | |
| | 30-27 | 0.3057 | 0.2639 | 2.2683 | 0.5558 | | | |
| | 24-22 | 0.3668 | 0.3660 | 3.5034 | 0.3705 | | | |
| | 1-3 | 0.2334 | 0.2129 | 2.2292 | 0.3691 | | | |
| | 27-30 | 0.1361 | 0.1175 | 3.4573 | 0.3507 | | | |
Table 4. Single load change with real and reactive load (118-bus)

| Node loading (p.u.) | (Line) from-to | \( l_{mn} \) (Moghavvemi & Omar, 1998) | FVSI (Musirin & Rahman, 2002) | POR (Halilčević & Softić, 2016) | Proposed index | Most critical |
|---------------------|---------------|----------------------------------------|-------------------------------|--------------------------------|----------------|---------------|
|                     |               | \( P = Q = 4.864 \) at node 6        |                               |                               | (DPVSI)         |                |
|                     | 5-2           | 0.3501                                 | 0.1930                        | 0.8587                        | 0.9230         | 0.9230        | 2             |
|                     | 6-10          | 0.8614                                 | 0.8555                        | 21.400                        | 0.8614         |               |
|                     | 8-6           | 1.0454                                 | 0.9351                        | 119.78                        | 0.8344         |               |
|                     | 2-4           | 0.3832                                 | 0.2228                        | 1.3753                        | 0.7402         |               |
|                     | 8-28          | 0.7743                                 | 0.8157                        | 1.3473                        | 0.7226         |               |
|                     | 6-7           | 0.5247                                 | 0.5549                        | 6.3756                        | 0.7181         |               |
|                     | 7-5           | 0.5185                                 | 0.4610                        | 1.8496                        | 0.6270         |               |
|                     | 6-9           | 0.5843                                 | 0.5824                        | 68.156                        | 0.5843         |               |
|                     | \( P = Q = 0.524 \) at node 27 |                               |                               |                               | (DPVSI)         |                |
|                     | 28-27         | 0.9722                                 | 0.8594                        | 0.9547                        | 0.9722         | 0.9722        | 28-27         | 27            |
|                     | 27-25         | 0.6440                                 | 0.5875                        | 1.6291                        | 0.7868         |               |
|                     | 24-25         | 0.5607                                 | 0.4984                        | 1.9913                        | 0.6701         |               |
|                     | 30-27         | 0.3646                                 | 0.3093                        | 1.9895                        | 0.6375         |               |
|                     | 3-1           | 0.2807                                 | 0.2475                        | 1.7913                        | 0.4539         |               |
|                     | 24-22         | 0.3353                                 | 0.3202                        | 3.3341                        | 0.4176         |               |
|                     | 27-29         | 0.1984                                 | 0.1821                        | 3.5581                        | 0.3532         |               |
|                     | 2-6           | 0.1503                                 | 0.1335                        | 2.5335                        | 0.3426         |               |
values of indices $L_{mn}$, FVSI and DPVSI rise, and at the bifurcation point, indices approach to unity value. Figure 6 shows the voltage profile of the entire network for different types of loading pattern.

4.5. IEEE 118-bus

To prove the effectiveness of the proposed index, the tests are also performed on a larger IEEE 118-bus test system for different operating conditions. The results of simulation are presented below.

| Table 5. Multiple load change with real load only (30-bus) |
|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|
| Loading multiplier (p.u.) | (Line) from-to | $L_{mn}$ (Moghavvemi & Omar, 1998) | FVSI (Musirin & Rahman, 2002) | POR (Halilčević & Softić, 2016) | Proposed index | Most stressed |
|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|
| $\lambda = 3.2$ | 4-12 | 0.7857 | 0.6172 | 1.2195 | 0.7857 | 4-12 | 12 |
| | 29-27 | 0.4389 | 0.3615 | 1.6978 | 0.7497 | | |
| | 27-30 | 0.0868 | 0.0573 | 2.0948 | 0.5720 | | |
| | 10-6 | 0.5351 | 0.4600 | 0.9609 | 0.5351 | | |
| | 8-28 | 0.5078 | 0.5178 | 2.1262 | 0.4809 | | |
| | 30-29 | 0.1901 | 0.1621 | 2.5931 | 0.4689 | | |
| | 6-8 | 0.5948 | 0.5631 | 47.010 | 0.4623 | | |

| Table 6. Multiple load change with reactive load only (30-bus) |
|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|
| Loading multiplier (p.u.) | (Line) from-to | $L_{mn}$ (Moghavvemi & Omar, 1998) | FVSI (Musirin & Rahman, 2002) | POR (Halilčević & Softić, 2016) | Proposed index | Most stressed |
|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|
| $\lambda = 5.7$ | 10-9 | 0.9318 | 0.9290 | 1.0556 | 0.9318 | 10-9 | 9 |
| | 12-4 | 0.9124 | 0.8815 | 1.4378 | 0.9124 | | |
| | 28-27 | 0.8486 | 0.8324 | 1.1320 | 0.8486 | | |
| | 6-10 | 0.7704 | 0.7564 | 1.2282 | 0.7704 | | |
| | 25-26 | 0.7693 | 0.8445 | 1.6853 | 0.6914 | | |
| | 27-29 | 0.6534 | 0.6405 | 1.8005 | 0.6857 | | |
| | 12-15 | 0.6986 | 0.7045 | 1.7069 | 0.6848 | | |

| Table 7. Multiple load change with real and reactive load (30-bus) |
|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|
| Loading multiplier (p.u.) | (Line) from-to | $L_{mn}$ (Moghavvemi & Omar, 1998) | FVSI (Musirin & Rahman, 2002) | POR (Halilčević & Softić, 2016) | Proposed index | Most critical |
|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|
| $\lambda = 2.73$ | 29-27 | 0.7355 | 0.6135 | 1.3019 | 0.9986 | 29-27 | 27 |
| | 6-2 | 0.4646 | 0.3026 | 2.0043 | 0.9821 | | |
| | 4-2 | 0.6077 | 0.4762 | 1.1046 | 0.9164 | | |
| | 10-6 | 0.9140 | 0.8132 | 1.3530 | 0.9140 | | |
| | 27-30 | 0.3217 | 0.2132 | 1.8339 | 0.7030 | | |
| | 2-5 | 1.8444 | 0.9487 | 1.7225 | 0.6378 | | |
| | 30-29 | 0.3503 | 0.2961 | 2.0029 | 0.6599 | | |
4.5.1. Base case loading

The results of the base case are illustrated in Table 8. It is observed from Table 8 that the highest value of DPVSI is 0.406332 for the line 27-25. The indices $L_{mn}$ (Moghavvemi & Omar, 1998) and FVSI (Musirin & Rahman, 2002) show higher values as compared to proposed index DPVSI for the lines connected between nodes 80-99, 92-100, 100-101 etc., where active and reactive powers flow in opposite direction. The direction of power flows is not considered in these indices and also neglects the effect of distributed parameters. The proposed index value is almost consistent with values of other indices (Moghavvemi & Omar, 1998; Musirin & Rahman, 2002) for lines 37-38, 59-63 etc., where lines have 0 value of resistance and shunt admittance. The results obtained by proposed index are supported by the index POR.

4.5.2. Single load change

In this case, the load has been changed at one particular node at a time and all other loads are fixed at their base values. Literature reveals that the voltage stability is very sensitive to the flow of reactive power (Kessel & Glavitsch, 1986; Momoh, 1991; Taylor; Tiwari et al., 2012).
Therefore, the results for single load change with the reactive load only is illustrated in Table 9. The reactive load at node 28 is increased gradually up to a level of 4.128 p.u. where the value of proposed index DPVSI reaches to 0.9987 for line 27-28. Therefore, the transmission line 27-28 is the most critical line, the value of index POR for this case is 1.0219. Therefore, the results are supported by the index POR. For the same loading, both the indices \( L_{mn} \) and FVSI exceed unity. The load flow solution diverged for further addition of load at node 28, which confirms that the loading is critical and very close to voltage collapse point. Near unity value of proposed index at the points confirms that it is able to detect the maximum loading point. It is also observed that line 29-28 connected with the same node 28 is also close to instability. The same results are obtained for other critically loaded buses where the proposed index approaches to unity while indices \( L_{mn} \) and FVSI exceed unity. Therefore, indices \( L_{mn} \) and FVSI do not provide accurate estimation of voltage stability. The difference in values of DPVSI is on account of distributed parameters modeling approach. In some cases, a difference is observed in the identification of most critical line by proposed index and other indices as shown in Table 9. This is due to fact that these indices are based upon approximation and therefore, the indices values are not correct representation of actual voltage stability of line. It is observed that the proposed index provides consistent results in all the cases.
4.5.3. Multiple load change

In this case, the total load at each bus of a system is increased up to the level of voltage collapse. As shown in Table 10 for a loading factor 1.97, the index $DPVSI$ of line 38-37 reaches at 0.9877, which shows that this line is in the critical state. The result is also confirmed by other indices.

4.6. Line reactive power margin estimation

The proposed index is also capable of measuring the reactive power margin of a line. Margin of reactive power can be measured by multiplying $(1 - DPVSI)$ with maximum reactive loading. Maximum reactive loading is the permissible loading at which index reaches near unity value. Case study for IEEE 30-bus test system is presented below.

IEEE 30-bus: To estimate the line reactive power margin, randomly line 27-28 is chosen. At the node 27, reactive load is slowly varied till maximum reactive loading. The actual reactive power margin at any point is the difference between the maximum reactive load and the reactive load at that point. The maximum possible reactive load at bus 27 is 0.72 p.u. The estimated reactive power margin is the multiplication of maximum reactive load and $(1 - DPVSI)$. The results are presented in Table 11. From Table 11, it is observed that for reactive load 0.5 p.u., the actual reactive power margin is 0.22 p.u. The estimated margin is 0.25 p.u. which is nearly equal to actual reactive loading margin available.

| (Line) from-to | $l_{mn}$ (Moghavvemi & Omar, 1998) | $FVSI$ (Musirin & Rahman, 2002) | $POR$ (Halličević & Sofić, 2016) | Proposed index ($DPVSI_{sr}$) |
|---------------|----------------------------------|-------------------------------|-------------------------------|------------------------------|
| 3-5           | 0.059389                         | 0.061063                      | 7.322066                      | 0.131312                     |
| 23-32         | 0.021461                         | 0.020150                      | 5.124336                      | 0.120560                     |
| 27-25         | 0.246559                         | 0.216315                      | 1.671721                      | 0.406332                     |
| 26-30         | 0.132346                         | 0.123515                      | 2.371836                      | 0.184164                     |
| 37-38         | 0.166235                         | 0.165271                      | 3.832773                      | 0.166235                     |
| 65-38         | 0.259556                         | 0.261474                      | 7.088009                      | 0.285923                     |
| 54-49         | 0.176056                         | 0.164677                      | 3.005861                      | 0.285942                     |
| 59-63         | 0.107970                         | 0.107604                      | 5.414449                      | 0.107970                     |
| 65-66         | 0.103402                         | 0.103402                      | 9.672023                      | 0.103402                     |
| 77-80         | 0.056490                         | 0.058577                      | 8.867690                      | 0.142908                     |
| 80-99         | 0.179022                         | 0.183594                      | 4.303939                      | 0.124007                     |
| 92-100        | 0.228067                         | 0.213611                      | 7.471049                      | 0.111095                     |
| 100-101       | 0.132684                         | 0.134510                      | 6.791953                      | 0.105017                     |
| 105-107       | 0.079521                         | 0.078233                      | 9.389853                      | 0.106616                     |
| 101-102       | 0.055375                         | 0.056515                      | 7.623743                      | 0.009725                     |
| 98-100        | 0.093496                         | 0.096204                      | 4.965965                      | 0.025887                     |
| 95-96         | 0.079335                         | 0.078095                      | 61.94438                      | 0.047679                     |
| 94-96         | 0.082556                         | 0.079340                      | 13.44302                      | 0.007748                     |
| 93-94         | 0.087608                         | 0.084473                      | 15.19913                      | 0.018233                     |
| 86-85         | 0.040747                         | 0.040172                      | 34.93888                      | 0.012463                     |
| 84-83         | 0.137418                         | 0.146118                      | 5.338816                      | 0.012638                     |
| 83-82         | 0.064680                         | 0.066084                      | 9.726264                      | 0.020119                     |
| 77-82         | 0.174639                         | 0.182169                      | 4.652905                      | 0.088191                     |
| 44-45         | 0.024169                         | 0.024409                      | 19.09432                      | 0.003487                     |
| 20-21         | 0.024605                         | 0.024850                      | 16.66352                      | 0.003640                     |
Table 9. Single load change with reactive load only (118-bus)

| Node loading (p.u.) | (Line) from-to | $L_{\text{mv}}$ (Moghavvemi & Omar, 1998) | FVSI (Musirin & Rahman, 2002) | POR (Haličević & Softić, 2016) | Proposed index | Most critical/stressed |
|---------------------|---------------|---------------------------------|-----------------------------|---------------------------------|----------------|------------------------|
|                     |               |                                 |                             |                                 | (DPVSI$_{sr}$) | (DPVSI)               |
|                     |               |                                 |                             |                                 | Line           | Node                  |
| Q = 4.128 at node 28| 27-28         | 1.0007                          | 1.0256                      | 1.0219                           | 0.9987         | 0.9987                 | 27-28                | 28          |
|                     | 29-28         | 1.1055                          | 1.0287                      | 93.728                           | 0.9735         |                        |                      |             |
|                     | 27-25         | 0.5883                          | 0.5003                      | 1.1121                           | 0.7509         |                        |                      |             |
|                     | 29-31         | 0.7618                          | 0.7517                      | 160.59                           | 0.7222         |                        |                      |             |
|                     | 76-77         | 0.1433                          | 0.1515                      | 4.7472                           | 0.3047         |                        |                      |             |
|                     | 49-54         | 0.1851                          | 0.1732                      | 3.3427                           | 0.2713         |                        |                      |             |
|                     | 65-38         | 0.2200                          | 0.2204                      | 4.1726                           | 0.2673         |                        |                      |             |
| Q = 1.956 at node 44| 43-44         | 0.9446                          | 0.9810                      | 1.0818                           | 0.9189         | 0.9189                 | 43-44                | 44          |
|                     | 45-44         | 0.9327                          | 0.9140                      | 47.901                           | 0.8891         |                        |                      |             |
|                     | 49-45         | 0.7189                          | 0.7442                      | 8.6502                           | 0.7952         |                        |                      |             |
|                     | 46-45         | 0.7264                          | 0.7256                      | 32.856                           | 0.7214         |                        |                      |             |
|                     | 43-34         | 0.6778                          | 0.6759                      | 1.5411                           | 0.6766         |                        |                      |             |
|                     | 27-25         | 0.2468                          | 0.2164                      | 1.6672                           | 0.4072         |                        |                      |             |
| Q = 8.688 at node 94| 100-94        | 1.1953                          | 1.0700                      | 24.028                           | 0.9984         | 0.9984                 | 100-94               | 94          |
|                     | 92-94         | 1.0010                          | 1.0003                      | 1.0679                           | 0.9964         |                        |                      |             |
|                     | 96-97         | 0.9273                          | 0.9250                      | 168.12                           | 0.9183         |                        |                      |             |
|                     | 97-80         | 0.8614                          | 0.8579                      | 1.1913                           | 0.8662         |                        |                      |             |
|                     | 94-96         | 1.0437                          | 0.9423                      | 333.24                           | 0.8421         |                        |                      |             |
|                     | 96-80         | 0.8367                          | 0.8354                      | 1.2298                           | 0.8322         |                        |                      |             |
|                     | 94-93         | 0.8252                          | 0.8290                      | 1.2941                           | 0.8216         |                        |                      |             |

Source: Singh & Tiwari, Cogent Engineering (2018), 5: 1515573. https://doi.org/10.1080/23311916.2018.1515573
IEEE 118-bus: To verify consistency of results, proposed index is also tested on IEEE 118-bus test system. Here to estimate the line reactive power margin, randomly line 117-12 is chosen. At the node 117, reactive load is slowly varied until power flow diverge and continuously line 117-12 is monitored. The maximum possible reactive load at bus 117 is 1.53 p.u. The results are presented in Table 12 and it is verified that proposed index is capable to measure reactive power margin.

### Table 10. Multiple load change with real and reactive load (118-bus)

| Loading multiplier (p.u.) | (Line) from-to | \( L_{\text{act}} \) (Moghavvemi & Omar, 1998) | FVSI (Musirin & Rahman, 2002) | POR (Halilčević & Softić, 2016) | Proposed index | Most critical
|--------------------------|----------------|-----------------------------------------|-------------------------------|---------------------------------|----------------|----------------|
| \( \lambda = 1.97 \)     | 38-37          | 0.9877                                  | 0.9533                        | 4.7294                         | 0.9877         | 0.9877 37-37 37 |
|                          | 45-46          | 0.5171                                  | 0.5511                        | 4.3262                         | 0.7645         | 0.7645 37-37 37 |
|                          | 49-66          | 0.2326                                  | 0.2326                        | 1.3989                         | 0.6346         | 0.6346 37-37 37 |
|                          | 27-77          | 0.3253                                  | 0.2407                        | 1.0681                         | 0.6190         | 0.6190 37-37 37 |
|                          | 75-77          | 0.7207                                  | 0.6890                        | 180.40                         | 0.5991         | 0.5991 37-37 37 |
|                          | 30-26          | 0.4938                                  | 0.4379                        | 1.1490                         | 0.5566         | 0.5566 37-37 37 |
|                          | 96-80          | 0.3980                                  | 0.3501                        | 1.4535                         | 0.5494         | 0.5494 37-37 37 |

### Table 11. Actual versus estimated reactive power margin of line 27-28 when reactive load at bus 27 is varied (30-bus)

\( Q_{\text{max}} = 0.72 \) p.u.

| Reactive load at bus 27 (p.u.) (\( A \)) | Proposed index (DPVSRI) (\( B \)) | Actual reactive power margin of line 28-27 (p.u.) = \( Q_{\text{max}} - A \) | Estimated reactive power margin of line 28-27 (p.u.) = \( (1 - B) \times Q_{\text{max}} \) |
|-----------------------------------------|-----------------------------------|------------------------------------------|-------------------------------------------|
| 0.2                                     | 0.2800                            | 0.52                                     | 0.52                                      |
| 0.3                                     | 0.3912                            | 0.42                                     | 0.44                                      |
| 0.4                                     | 0.5106                            | 0.32                                     | 0.35                                      |
| 0.5                                     | 0.6447                            | 0.22                                     | 0.25                                      |
| 0.7                                     | 0.9590                            | 0.02                                     | 0.02                                      |

4.7. Contingency analysis

To appraise the suitability of proposed DPVSRI index to predict the voltage collapse during line outage of the system, single line contingency analysis is performed under base load by removing one line at a time (Devaraj, Preetha-Roselyn, & Uma-Rani, 2007). The proposed index DPVSRI and other indices are calculated from the solution of power flow. The critical lines are identified on the basis of DPVSRI value, whose outage may instigate the voltage collapse. The outages of a line from the power system may overload other lines. The most stressed line can be identified by the maximum value of DPVSRI. The results of contingency analysis of IEEE 30-bus test system at the base case are presented in Table 13. An outage of line 6-7 leads to the highest value of DPVSRI equal to 0.4000 for branch 5-2. The results are also confirmed by the index POR. The line 5-2 has significant value of series impedance and shunt admittance; therefore, the proposed index is higher than other indices (Moghavvemi & Omar, 1998; Musirin & Rahman, 2002) for branch 5-2. The indices (Moghavvemi & Omar, 1998; Musirin & Rahman, 2002) don’t count the relative direction effect and distributed parameters effect; therefore, they have shown inaccurate value. Conversely, the proposed index provides accurate results under all the conditions.
5. Conclusions

In the proposed work, a new voltage stability index, DPVSI, has been developed for voltage stability assessment. The novelty of index is that it is based on distributed parameters of the transmission line which is a meticulous representation of power system leading to precise indicator of voltage stability state. Stressed condition of a line can be monitored and reactive loading margin can be calculated online using proposed index. The work also contributes to assess local and global voltage stability of the power system. In this work, index value of individual line is indicative of local voltage stability while the maximum index value reflects the global voltage stability. Application potential of the work is explored on IEEE 30-bus and 118-bus test systems.

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