Unbound exotic nuclei studied by transfer to the continuum reactions

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Abstract

In this paper we show that the theory of transfer reactions from bound to continuum states is well suited to extract structure information from data obtained by performing “spectroscopy in the continuum”. The low energy unbound states of nuclei such as \(^{10}\)Li and \(^{5}\)He can be analyzed and the neutron-core interaction, necessary to describe the corresponding borromean nuclei \(^{11}\)Li and \(^{6}\)He can be determined in a semi-phenomenological way. An application to the study of \(^{10}\)Li is then discussed and it is shown that the scattering length for s-states at threshold can be obtained from the ratio of experimental and theoretical cross sections. The scattering single particle states of the system \(n+^{9}\)Li are obtained in a potential model. The corresponding S-matrix is used to calculate the transfer cross section as a function of the neutron continuum energy with respect to \(^{9}\)Li. Three different reactions are calculated \(^{9}\)Li\((d,p)^{10}\)Li, \(^{9}\)Li\((^{9}\text{Be},^{8}\text{Be})^{10}\)Li, \(^{9}\)Li\((^{13}\text{C},^{12}\text{C})^{10}\)Li, to check the sensitivity of the results to the target used and in particular to the transfer matching conditions. Thus the sensitivity of the structure information extracted from experimental data on the reaction mechanism is assessed.
1 Introduction

This paper deals with the application of the “transfer to the continuum method”, very well understood for normally bound nuclei [1], to light unbound nuclei which recently have attracted much attention [2]–[5] in connection with exotic and halo nuclei. Halo nuclei are very complicated systems to describe. In particular the accuracy of reaction theories used to extract structure information is a key issue of the field. From the structure point of view, simple semi-phenomenological models have been proposed which exhibit the properties of those nuclei in terms of one (or two) single nucleon wave functions and which make also easy the calculation of cross sections for various reactions initiated by such projectiles [6]. One-neutron halo nuclei can be described in a two-body model as a core plus one neutron. All the complexity of the many-body problem, when two-body correlations are important, can be put in an effective one-body potential between the extra neutron and the core, that is the Hartree-Fock potential plus the contribution due to the particle-core vibration couplings. This contribution which is small in normal nuclei is so strong in nuclei such as $^{11}$Be or $^{10}$Li for example that it might be responsible for an inversion of $1/2^+$ and $1/2^-$ states [7, 8]. It also induces a strong modification of the wave functions which become mixtures of one-single nucleon state and more complicated ones formed of a single nucleon coupled with core vibrations. As a consequence the one-nucleon spectroscopic factors are smaller than one. They in turn can be extracted from one-neutron removal cross sections if one has a good description of the reaction. Then the comparison between theoretical and measured spectroscopic factors constitutes a strong test of the model.

On the other hand two-neutron halo nuclei such as $^6$He, $^{11}$Li have a two-neutron halo due to the properties of the single extra neutrons which are unbound in the field of the core, the two neutron pair being weakly bound due to the neutron-neutron pairing force. In a three-body model those nuclei are described as a core plus two neutrons. The properties of core plus one neutron system are essential and the model relies on the knowledge of angular momentum and parity as well as energies and corresponding neutron-core effective potential, therefore spectroscopic strength for neutron resonances in the field of the core. Again these information can be directly obtained from the analysis of one-neutron breakup or transfer cross sections.

The two borromean nuclei that have been studied more extensively so far are $^6$He [9], and $^{11}$Li [10]–[14]. The two neutron halo is build on a core
which in some cases is itself a radioactive nucleus (i.e. $^9$Li which is the core of $^{11}$Li). They are “borromean” since the corresponding (A-1) nuclei are unbound. Thus $^5$He, $^{10}$Li, as well as $^{13}$Be and $^{16}$B exist only as neutron plus core resonance states and it takes an extra neutron and its paring energy to finally bind $^{11}$Li and $^6$He. However the two neutron separation energy is typically very small (0.3MeV in $^{11}$Li).

The study of unbound systems showing resonances very close to particle threshold is giving rise to the very interesting field of research that can be defined as “spectroscopy in the continuum” [15] and some of the most recent applications have been discussed in Refs. [2]-[5]. Ideally one would like to study the neutron elastic scattering at very low energies on the ”core” nuclei. This is however not feasible at the moment as many such cores, like $^9$Li, $^{12}$Be or $^{15}$B are themselves unstable and therefore they cannot be used as targets. Other indirect methods instead have been used so far, mainly aiming at the determination of the energy and angular momentum of the continuum states. This information should help fixing the parameters and form of the neutron-core interaction. We remind the reader that the problem of a consistent treatment of the nucleon-nucleus interaction yielding at the same time bound and unbound states has already been studied for normal nuclei [16,17] and it would be extremely interesting to see how generalizations of such approaches could be obtained from studies of exotic nuclei.

The reactions used so far to study unbound nuclei can be grouped as: projectile breakup following which the neutron-core coincidences have been recorded and the neutron energy spectrum relative to the core has been reconstructed [15], [18]-[21]; multiparticle transfer reactions [22, 23] or just one proton [2] transfer. In a few other cases the neutron transfer from a deuteron [4] or $^{9}$Be target [4,5], both having low neutron separation energy, has been induced and the neutron has undergone a final state interaction with the projectile of, for example $^9$Li. In this way the $^{10}$Li resonances have been populated in what can be defined a “transfer to the continuum reaction” [24]-[29]. Thus the neutron-core interaction could be determined in a way which is somehow close in spirit to the determination of the optical potential from the elastic scattering of normal nuclei.

In both the projectile fragmentation or the transfer method the neutron-core interaction that one is trying to determine appears in the reaction as a ”final state” interaction and therefore reliable information on its form and on the values of its parameters can be extracted only if the primary reaction is perfectly under control from the point of view of the reaction theory. In this
paper we argue and show that among the several methods discussed above to perform spectroscopy in the continuum, the neutron transfer method looks very promising since the reaction theory exists and has been already tested in many cases \cite{24-30}. This has been possible thanks to very accurate and systematic studies of transfer to the continuum reactions in normally bound nuclei \cite{1,31-36}. We anticipate here that one of the characteristics of the theory is to allow the consistent use of a different neutron-core interaction at each neutron-core energy. This is of basic importance for nuclei such as $^{10}$Li which have two low lying continuum states with $l=0$ and $l=1$, within an interval of about 0.5 MeV from threshold, which can be reproduced only by using two very different potentials. The energy and state dependence of most of the effective nucleon-nucleus interactions is still a challenge in Nuclear Physics studies. We proceed then to the next session where the basic formalism for the transfer to the continuum theory is given. Then in Sec. 3 the properties of $^{10}$Li are resumed. Sec. 4 contains the discussion of the results and finally some conclusions and outlook are given in Sec. 5.

2 Transfer to the continuum theory

A full description of the treatment of the scattering equation for a nucleus which decays by single neutron breakup following its interaction with another nucleus, can be found in Refs. \cite{24,25,37}. There it was shown that within the semiclassical approach for the projectile-target relative motion, the cross section differential in $\varepsilon_f$, the final, continuum, neutron energy is

$$\frac{d\sigma}{d\varepsilon_f} = C^2 S \int_0^\infty dB_c \frac{dP_c(b_c)}{d\varepsilon_f} P_{ct}(b_c), \quad (1)$$

(see Eq. (2.3) of \cite{29}) and $C^2 S$ is the spectroscopic factor for the initial single particle orbital.

The core survival probability $P_{ct}(b_c) = |S_{ct}|^2$ \cite{29} in Eq. (1) takes into account the peripheral nature of the reaction and naturally excludes the possibility of large overlaps between projectile and target. $P_{ct}$ is defined in terms of a S-matrix function of the core-target distance of closest approach $b_c$. A simple parameterization is $P_{ct}(b_c) = e^{-\ln 2 \exp((R_s-b_c)/a)}$, where the strong absorption radius $R_s \approx 1.4(A_p^{1/3} + A_t^{1/3})\text{fm}$ is defined as the distance of closest approach for a trajectory that is 50% absorbed from the elastic channel and $a = 0.6\text{fm}$ is a diffuseness parameter.
Therefore according to [24] the matrix element in the amplitude for a transition from a nucleon bound state $\psi_i$ in the projectile to a final continuum state $\psi_f$

$$A_{fi} = \frac{1}{i\hbar} \int_{-\infty}^{\infty} dt <\psi_f(t)|V(r)|\psi_i(t)>, \quad (2)$$
can be reduced to an overlap integral between the asymptotic parts of the wave functions for the initial and final state. Here $V$ is the interaction responsible for the neutron transition to the continuum. In the case of a light exotic nucleus interacting with another light nucleus $V(r)$ is just the neutron-target optical potential $V(r)=U(r)+iW(r)$, and the differential probability with respect to the neutron energy can be written as

$$\frac{dP}{d\varepsilon_f} = \frac{1}{8\pi^3 h^2 k_f^2} \sum_{m_i} |A_{fi}|^2 \approx \frac{4\pi}{2k_f^2} \sum_{j_f} (|1-\bar{S}_{j_f}|^2 + 1 - |\bar{S}_{j_f}|^2)(2j_f + 1)(1 + F_{l_f,i,j_f,j_i})B_{l_f,i}, \quad (3)$$

where $A_{fi}$ is given by Eq.(2) and we have averaged over the neutron initial state.

Equation (3) has a very transparent structure which makes it suitable to describe the kind of reactions we are interested in this paper. In fact the term

$$\sigma_{nN}(\varepsilon_f) = \frac{4\pi}{2k_f^2} \sum_{j_f} (|1-\bar{S}_{j_f}|^2 + 1 - |\bar{S}_{j_f}|^2)(2j_f + 1) \quad (4)$$
gives the neutron-nucleus free particle cross section. $\bar{S}_{j_f}$ is the neutron-nucleus optical model S-matrix, which is calculated for each nucleon final energy in the continuum with an energy dependent optical model. The two terms $|1-\bar{S}_{j_f}|^2$ and $1 - |\bar{S}_{j_f}|^2$ represent the shape elastic scattering and the absorption respectively. For the cases described in this paper only the shape elastic term will contribute, since we will discuss scattering states below the first core excited state and therefore we will use a real optical potential.

The term

$$\mathcal{F} = (1 + F_{l_f,i,j_f,j_i})B_{l_f,i}, \quad (5)$$

represents what in the theory of final state interactions [38] has been called the enhancement factor. $F_{l_f,i,j_f,j_i}$ is an $l$ to $j$ recoupling factor between the
angular momenta of the neutron in the initial and final states. It is also energy dependent and reflects the spin matching conditions well known for transfer between bound states [39]-[42]. It is important to notice that our theory takes properly into account not only the angular momentum dependence of the final continuum states but also their spin. The term
\[
B_{l_f,l_i} = \frac{1}{4\pi} \left[ \frac{k_f}{mv^2} \right] |C_i|^2 e^{-2\eta b_c} \frac{1}{2\eta b_c} M_{l_f,l_i},
\]
contains the matching conditions between the initial and final neutron energies and the relative motion energy per particle \( \frac{mv^2}{2} \) at the distance of closest approach. \( \eta = \sqrt{k_1^2 + \gamma_i^2} \) is the transverse component of the neutron momentum which is conserved in the neutron transition, \( \gamma_i = \frac{\sqrt{-2m\epsilon_i}}{\hbar} \) and \( k_f = \frac{\sqrt{2m\epsilon_f}}{\hbar} \) are the neutron momenta in the initial and final states and \( k_1 = \frac{\epsilon_f - \epsilon_i - mv^2/2}{\hbar} \) is the parallel component of the neutron momentum in the initial state. Also \( b_c \) is the core-target impact parameter, \( C_i \) is the initial state asymptotic normalization constant and \( M_{l_f,l_i} \) is a factor depending on the angular parts of the initial and final wave functions [26, 29].

An important characteristic of the present formalism is that the transfer probability Eq. (3) contains the factor \( 1/k_f^2 \) which corresponds to the inverse of the neutron entrance channel flux. It was noticed in Ref. [43] that if a “transfer to the continuum” formalism does not contain such factor then the model cross sections will always vanish at zero energy, which is unphysical. Our calculated cross section instead will have in the case of a virtual state of exactly zero energy and \( l = 0 \) a divergent-like behavior at zero energy, in accordance to experimental data and to the physical expectations for a s-state at threshold. It should be also noticed that in the term \( B_{l_f,l_i} \), there is a modulating factor \( \frac{k_f}{mv^2} \approx \frac{v_f}{v} \) which takes into account the matching between the projectile velocity at the distance of closest approach \( v \) and the neutron final velocity in the continuum \( v_f \).

A particularly interesting case is when the final continuum energy approaches zero. Then only the \( l = 0 \) partial wave contributes and using \( \frac{1}{4}|1 - \tilde{S}_0|^2 = \sin^2 \delta_0 \), Eq. (11) becomes very simple if we write it as differential in the final neutron momentum, in particular if the neutron initial state is also \( l = 0 \) and we assume unit spectroscopic factor. In that case the spin coupling factor \( (1 + F_{l_f,l_i,j_f,j_i}) \) and the \( M_{l_f,l_i} \) factor are independent of energy such that the differential cross section finally reads:
\[
\frac{d\sigma}{dk_f} = (\sin \delta_0)^2 |C_i|^2 \left[ \frac{\hbar}{mv} \right]^2 \int_0^\infty db_c \frac{e^{-2\eta b_c}}{\eta b_c} e^{-\ln 2 \exp[(R_s-b_c)/a]} .
\]

If the LHS of the previous equation is measured experimentally, then \((\sin \delta_0)^2\) can be obtained by doing the ratio between \(d\sigma_{exp}/dk_f\) and the remaining terms in the RHS of Eq. \((7)\), in the limit of zero energy. In fact the above equation is well behaved, because the only dependence on the neutron energy is contained in the term \(\frac{e^{-2\eta b_c}}{\eta b_c}\), where \(\eta\) goes to a constant in the limit of zero energy. Finally the scattering length can be obtained from \(a_s = -\lim_{k\to0} \frac{\tan \delta_k}{k}\).

It is interesting to note the similarity between Eq. \((7)\) and the corresponding formula of the theory of transfer between bound states
\[
\sigma(\varepsilon_f) = \frac{\pi}{2} |C_i C_f|^2 \left[ \frac{\hbar}{mv} \right]^2 \int_0^\infty db_c \frac{e^{-2\eta b_c}}{\eta b_c} e^{-\ln 2 \exp[(R_s-b_c)/a]}
\]
as discussed in [24] where it was shown that the term \((\sin \delta_0)^2\) after integrating over the final continuum energy, plays the same role as the asymptotic normalization constant of the final bound state \(C_f^2\).

3 Application to \(^{10}\text{Li}\) structure

Since the link between reaction theory and structure model is made by the optical potential determining the S-matrix in Eq. \((3)\), once that the theory has fitted position and shape of the continuum n-nucleus energy distribution, what can be deduced are the parameters of a model potential. Therefore we are now going to use such a model to describe the properties of \(^{10}\text{Li}\). \(^{10}\text{Li}\) is unbound and in its low energy continuum four states (two spin doublets) are expected to be present due to the coupling with the 3/2\(^{-}\) p-state of the extra proton in the \(^9\text{Li}\) core. The states with a total spin of 1\(^{-}\) or 2\(^{-}\) would be due to the coupling with a neutron in a s-state, while coupling with the p-state would give 1\(^{+}\) or 2\(^{+}\). There is already a rich literature on the subject both from the experimental [4] as well as from the theoretical point of view [10]-[11]. In particular the best evidences are in favor of \(^{10}\text{Li}\) having a 1\(^{-}\) ground state due to an s-virtual state close to the threshold. Recently a proton pickup experiment \(d(^{11}\text{Be},^3\text{He})^{10}\text{Li}\) [2] has definitely confirmed the earlier hypothesis that the ground state of \(^{10}\text{Li}\) is the 2s virtual state and that the 1p\(_{1/2}\) orbit gives an excited state.
To describe the valence neutron in $^{10}$Li we assume that the single neutron hamiltonian with respect to $^{9}$Li has the form

$$ h = t + U $$  \hspace{1cm} (9)$$

where $t$ is the kinetic energy and

$$ U(r) = V_{WS} + \delta V $$  \hspace{1cm} (10)$$

is the real part of the neutron-core interaction. $V_{WS}$ is the usual Woods-Saxon potential plus spin-orbit

$$ V_{WS}(r) = \frac{V_0}{1 + e^{(r-R)/a}} - \left( \frac{\hbar}{m_\pi c} \right)^2 \frac{V_{so}}{ar} \frac{e^{(r-R)/a}}{(1 + e^{(r-R)/a})^2} \mathbf{l} \cdot \mathbf{\sigma} $$  \hspace{1cm} (11)$$

and $\delta V$ is a correction which originates from particle-vibration couplings. They are important for low energy states but can be neglected at higher energies. If Bohr and Mottelson collective model is used for the transition
Table 1: Woods-Saxon and spin-orbit potential parameters.

| $V_0$ (MeV) | $r_0$ (fm) | $\alpha$ (fm) | $V_{so}$ (MeV) | $a_{so}$ (fm) |
|-------------|------------|--------------|----------------|-------------|
| -39.83      | 1.27       | 0.75         | 7.07           | 0.75        |

amplitudes between zero and one phonon states, calculation of such couplings suggest the following form \[13\]:

$$\delta V(r) = 16\alpha e^{2(r-R)/a}/(1 + e^{(r-R)/a})^4$$

(12)

where $R \approx r_0A^{1/3}$. The parameters of $V_{WS}$ for the n-$^9$Li interaction used in this paper are those given in Table 1. In Table 2 we give the scattering lengths and energy obtained for the 2s and 1p$_{1/2}$ states, with different values of the strength $\alpha$.

4 Results and discussion

It would be therefore interesting and important if an experiment could determine the energies of the unbound $^{10}$Li states such that the interaction parameters could be deduced. Two $^9$Li($d,p)^{10}$Li experiments have recently been performed. One at MSU at 20 A.MeV \[4\] and the other at the CERN REX-ISOLDE facility at 2 A.MeV \[5\]. For such transfer to the continuum reactions the theory underlined in Sec. 2 is very accurate. It should be noticed that the theory has usually been applied to projectile breakup reactions, in order to study single particle excitations in the target. Here it will be applied to single neutron target breakup leading to excitations of the n-projectile continuum.

In order to study the sensitivity of the results on the target, and therefore on the spin selection rules for transfer and on the energies assumed for the s and p states, we have calculated the reaction $^9$Li($X, X-1)^{10}$Li at 2 A.MeV for three targets d, $^9$Be, $^{13}$C. The $^{13}$C target has been chosen because in such a case the neutron transfer from the p$_{1/2}$ initial state to the p$_{1/2}$ final state in $^{10}$Li will be a non spin flip transition $j_i = l_i - 1/2 \rightarrow j_f = l_f - 1/2$ while the transfer to the s$_{1/2}$ state would be a spin-flip transition which as it is well
Table 2: Scattering length of the s-state, energy and width of unbound p-state and strength parameter for the $\delta V$ potential.

| $\varepsilon_{\text{res}}$ (MeV) | $\Gamma$ (MeV) | $a_s$ (fm) | $\alpha$ (MeV) |
|----------------------------------|----------------|------------|----------------|
| $2s_{1/2}$                       | 323            | -12.5      | -17.20         |
| $1p_{1/2}$                       | 0.595          | 0.48       | 3.3            |

Known are enhanced at low incident energy [39]-[42]. $^{14}$N would also be a good target, having a valence neutron in a $p_{1/2}$ state, but the absolute cross sections would be smaller as the separation energy is larger (10.55 MeV) than in $^{13}$C. It would however provide good matching conditions at higher beam energies ($E_{\text{inc}} \approx 10$ A.MeV). For the other two cases, the initial state is a $s_{1/2}$ in the deuteron and a $p_{3/2}$ in $^9$Be thus in both cases $j_i = l_i + 1/2$. Then the transfer to the $2s$ state is a non spin-flip transition which is hindered, while the transfer to the $p_{1/2}$ is enhanced at low incident energy. The initial state parameters are given in Table 3. For each initial state a unit spectroscopic factor was assumed.

Table 3: Targets and initial state parameters of the bound neutron.

| Target | $d$ | $^9$Be | $^{13}$C |
|--------|-----|--------|---------|
| $\varepsilon_i$ (MeV) | -2.22 | -1.66 | -4.95 |
| $l_i$ | 0   | 1      | 1       |
| $j_i$ | 1/2 | 3/2    | 1/2     |
| $C_i(fm^{-1/2})$ | 0.95 | 0.68  | 1.88    |

We show in Fig. 2 the neutron energy spectrum relative to $^9$Li obtained with the interaction and single particle energies of Tables 1 and 2. We define as the resonance energy of the p-state the energy at which $\delta j_f = \pi/2$. This is also the energy at which $|1 - \bar{S}_{j_f}|^2$ in Eq. (3) gets its maximum value as it can be seen in Fig. 3. The results of Fig. 2 show that the peak of the cross section for transfer to the p-state will determine without ambiguity the position of the p-state in a target independent way. The measured width instead would depend on the reaction mechanism, but the ”true” resonance width can however be obtained from the phase shift energy variation near
Figure 2: Neutron-$^9$Li relative energy spectra for transfer to the s and p continuum states in $^{10}$Li given in Table 2. Dotdashed lines are absolute cross sections for transfer from a deuteron target, dashed lines from a $^9$Be target, and short dashed line from a $^{13}$C target. The Be and C cross section have been renormalized to the deuteron cross sections by the factors indicated on the figure. The solid line is the transfer cross section from the C target to the second s-state given in Table 2.

resonance, given by the well known formula $d\delta_j/d\varepsilon|_{\varepsilon_{res}} = 2/\Gamma$, once that the resonance energy is fixed. Using this formula we obtained, in the case of the p-state in $^{10}$Li, the value $\Gamma = 0.48 MeV$ given in Table 2. From Fig. 2 one can see that the target dependence would influence the extracted width by about 10-15%. It is important to notice that in the approach of this paper the line shape is determined by the energy dependence of the phase shift and S-matrix and eventually it could be influenced by an energy dependence of the potential parameters. Fig. 3 shows indeed the energy dependence of $|1 - \tilde{S}_{j_f}|$ for $l=1$. Therefore there is no need to introduce any a priori form for the resonance shape and width.

For the s final state we see that there is a larger probability of population in the spin-flip reaction initiated by the carbon target. An important question
is whether a measure of the line-shape (or spectral function) and absolute value of the cross section will determine the characteristics of the state, and therefore the interaction, also in this case. We have already shown in Sec.2 that in principle it should be possible. However in order to elucidate better this difficult question we first remind the reader some of the peculiarities of the low energy scattering of neutral particles in the $l = 0$ partial wave. It is well known that because of the absence of the centrifugal barrier the energy and width of an s-state are difficult to define. Therefore we will in the following study the energy dependence of the phase shift in various potentials and determine for each case the values of the scattering length. The potential parameters are those of Table 1 and Table 4. Fig. 4 shows the behavior, as a function of the neutron momentum, of $\tan \delta_0$ (dotdashed curve) for the potential (2) of Table 4, of the cross section (full curve) calculated in the case of a deuteron target and of the factor $e^{-2\eta_b c/\eta_b}$ (dashed curve) from Eq.(7). The latter has a very smooth behavior and therefore it is easy to see that $(\sin \delta_0)^2$ and then $|a_s|$, could be determined from the ratio between the experimental cross section and the remaining part of the RHS of Eq.(7).

Table 4: Strengths of the s-state potential in Eq.(10) and corresponding scattering lengths. Labels in the first column identify the corresponding curves in Fig. 5.

| $V_0$ (MeV) | $\alpha$ (MeV) | $a_s$ (fm) |
|------------|---------------|-----------|
| (1) -39.83 | -4.0          | -2.4      |
| (2)  -10.0 | -17.2         | 17.2      |
| (3)  -12.2 | -318          | -318      |
| (4)  -13.5 | 45.1          | 45.1      |
| (5)  -15.0 | 21.4          | 21.4      |
| (6) -42.80 | -13.3         | 12.9      |

The sensitivity of the results for transfer to an s-state, on the potential assumed, is illustrated by Figs. 5a, 5b, 5c, 5d. In Fig. 5a the $l = 0$ phase shift is plotted as a function of the continuum energy. There are several potentials which give a similar behavior of the phase shift but different scattering lengths, (cf. Table 4) and in particular a different line shape for the transfer cross section from a deuteron target to an s-state, as shown in Figs. 5c, 5d. The curves from bottom to top in Fig. 5a, correspond to calculations in the potentials of Table 4 in increasing order of depth. Therefore the dashed and
solid lines in Fig. 5c correspond to unbound states with negative scattering lengths, while the long dotdashed line corresponds to a virtual state with a large scattering length consistent with infinity and therefore of zero energy. Then the other three, short dashed, dotted and short dashdotted curves, cases (4), (5) and (6) of Table 4 respectively, correspond to weakly bound states close to threshold. Our results for the phase shifts and scattering length are consistent with those of the thesis of S. Pita [2] and with the well known theory of low energy scattering of neutral particles in s-wave. We have indeed in Fig. 5a that for unbound states the phase shift is zero at zero energy, then increases up to a maximum value and then decreases again. Because it never increases going through the value $\pi/2$ when the energy increases, as instead it might happen for $l > 0$ states, then the states corresponding to cases (1),(2) of Table 4 cannot be defined as resonances, even though they give rise to an enhancement of the cross section (see Ref. [38], Eq.(4.235) and following discussion). Furthermore they do not give rise to singularities in the scattering amplitude on the physical sheet of the complex energy plane. For
each of them instead, the scattering amplitude has a pole at negative energy \( \varepsilon_f = -|\varepsilon_{(1,2)}| \) on the un-physical sheet. These poles represent bound states close to threshold which give the same free particle scattering cross section as the unbound states, namely \( \sigma = 4\pi/(k_f^2 + \kappa^2) \) where \( \kappa^2 = 2m|\varepsilon|/\hbar^2 \) (see Ref. [44], Eq. (133.8) and following discussion). Therefore cases (1), (2) are broad states with a width of 1-2MeV. In case (3) instead the phase shift value is very close to \( \pi/2 \) at zero energy corresponding to a virtual state. In fact the S-matrix gets its maximum value of \( |1 - \bar{S}| = 2 \) in Fig. 5b. Cases (4), (5) and (6) are from potentials which barely bind states very close to threshold. The phase shift approaches the value \( \pi \) at zero energy and the cross sections shown in Fig. 5d are a typical example of how weakly bound states can affect scattering at low energy.

On the other hand, what is clear is that because of the sharp rise towards zero of the factor \( 1/k_f \) and of the less fast decreasing of the \( |1 - \bar{S}|^2 \) term
Figure 5: Phase shift (a), shape elastic factor $|1 - \bar{S}|$ (b) and cross section (c,d) as a function of the neutron continuum energy for an s-state and a deuteron target. Figure (c) contains the results for unbound states with negative scattering length, while figure (d) for bound states with positive scattering length. Labels on the curves identify the corresponding potentials in Table 4.

of Eq. (3) for $l = 0$, shown in Fig. 5b, the peak of the s-state transfer cross section would always be “downshifted” with respect to the maximum of $|1 - \bar{S}|$, furthermore a maximum for this term always exists irrespective of the fact that the s-state at threshold is bound or unbound. The absolute value of the corresponding transfer cross sections in Fig. 5c, 5d increases and has the typical divergent-like behavior in correspondence to cases (3) and (4) of Table 4. Then for the more bound states (dotted and dotdashed line), the transfer cross section decreases again. One such state is obtained decreasing both the depth of the Woods-Saxon and of the surface term and it corresponds to the smallest positive scattering length in Table 4 ($a_s = 12.9 \text{fm}$). The cross section that one would measure in the continuum, shown in Fig. 5d is just a
reflection of the fact that the wave function for a weakly bound s-state has a long tail and thus some of the transfer strength is in the continuum. In fact, in the region over which the matrix element in Eq. (2) is different from zero, the behavior of bound and unbound state wave functions with energies very close to threshold, is almost the same, due to the very large wave length.

Therefore although it would seem quite hard to search experimentally for the energy and “nature” of weakly bound or just unbound s-states in exotic nuclei we hope to have shown that the absolute value of the cross section right at threshold together with the line shape should determine the scattering length of the state. It appears that in the case of $^{10}$Li states with scattering lengths larger than $|a_s| \sim 20$ fm would all lead to a divergent-like behavior of $\sigma(\varepsilon_f)$ when $\varepsilon_f \rightarrow 0$. The absolute value of $a_s$ can be determined from the experimental spectrum as discussed in relation to Eq.(7) and then the parameters of the n-$^{9}$Li interaction will be fixed as well. Those are the so called virtual states. One should also be aware that, as shown in Fig.5d, resonant-like structures seen in the low energy continuum could be an indication of weakly bound s-states as well as of unbound s-states. In order to disentangle these two situations one would obviously need complementary measurements. If the s-state is expected to be the ground state, then the mass measurement of the nucleus will determine whether it is bound or unbound. In the specific case of $^{10}$Li we know indeed that it is unbound. In other cases one could use different targets and/or different incident energies to study the variation of the maximum of the structures and thus deduce the energy of the final state from the matching conditions with the initial state.

Finally as the neutron scattering will happen in all partial waves, if there is an unbound or virtual s-state the corresponding cross section would seat on top of a background due to scattering on all partial waves, as one goes away from threshold. The behavior of such a background would be different for different potentials and therefore a whole calculation with all relevant partial waves, as contained in our formula Eq.(3) and a comparison with good resolution data, should help extracting the correct n-core interaction. On the other hand it is important to stress that in the case discussed in this work there is no spreading width of the single particle states since the n-$^{9}$Li interaction is real at such low energies. In fact the first excited state of $^{9}$Li is at $E^* = 2.7$ MeV. For “normal” nuclei instead the single particle resonances appear at higher excitation energies (approximately 4-6MeV), and for higher l-values ($l=6-10$). Then it was shown in Ref.[28] that the spreading width is much larger than the escape width due to the influence of the imaginary
Finally we conclude that if a transfer to the continuum experiment could measure with sufficient accuracy (energy resolution) the line-shapes or energy distribution functions for the s and p-states in $^{10}\text{Li}$ our theory would be able to fix accurately the energy of the p-state and the scattering length of the s-state. Those in turn could be used to test microscopic models of the n-$^9\text{Li}$ interaction. The integral of the energy distribution will determine the total spectroscopic strength of the state. From our results it appears that such an integral would depend on the neutron initial state in the target in a way which is however perfectly under control in the theory, since it is all contained in the B-term given by Eq.(6). Thus the spectroscopic strength of the state would be determined by the comparison between measured and calculated values of the whole energy distribution.

5 Conclusions and future challenges

In this paper we have argued that, apart from the experimental difficulties, the transfer to the continuum method is well suited to study unbound systems such as $^{10}\text{Li}$ which are the building blocks of borromean nuclei.

There is a very well tested theory to study such reactions, which allows to determine energy distributions for population of unbound states in absolute value. Provided the same information is available from the experimental point of view, the theory would allow the determination of the scattering length of s-states and the "resonance" energy of unbound single particle states, the associated $l$ and $j$ and the total strength. Those studies would eventually be used to determine the neutron-core interaction.

The great advantage of our method is that the basic ingredient of the theory is the S-matrix describing the neutron-nucleus scattering. It can be calculated with an energy dependent potential which can incorporate consistently certain peculiarities of unbound nuclei such as $^{10}\text{Li}$, whose continuum energy 0-0.5MeV range, for example, contains at least two states with $l=0,1$ obtainable only with two very different potential wells.

Furthermore the spin-orbit interaction can also be included so that at any energy the contribution from all states with given $l$ and $j$ can be obtained. This is very useful because not only the excitation of states of fixed angular momentum can be studied, but also the background due to the presence of all other possible angular momentum states can be calculated and in this way
the strength of just one single particle state can be obtained unambiguously
from data which would contain the contribution from all angular momenta.
The theory has the correct behavior when the continuum energy approaches
threshold such that the contribution from virtual states can be distinguished
from that from weakly bound or unbound states.

In this work we have calculated neutron transition probabilities for going
from an initial bound state in a nucleus to a scattering state including final
state interaction with another nucleus. Our way of describing the final state
interaction in the continuum is through an optical model S-matrix. A similar
approach could be applied to the treatment of inelastic projectile excitations
in which, following its interaction with the target, a neutron goes from a
bound to an unbound state with final state interaction in the same nucleus.
This is the process which creates $^{10}\text{Li}$ in the final state in the projectile-
breakup-type of experiments [15]. By using such a procedure a very accurate
time of two neutron breakup could be obtained, incorporating properly the
two step mechanism implicit in the formation of a neutron-core resonance
state in reactions like $^{11}\text{Li} + \text{X} \rightarrow ^{10}\text{Li}^* + \text{n} \rightarrow ^9\text{Li}+2\text{n}$ [3].

In fact $^{11}\text{Li}$ breakup and other 2n breakup reactions have often been
treated as a process in which the two neutrons are emitted simultaneously in
a single breakup process. This in principle could be improved by considering
the second neutron which decays in flight from a resonant state, as seen for
$^6\text{He}$, by a breakup form factor different than that of the first neutron and by
taking into account explicitly the sequential nature of the process.

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