New Concepts in Particle Physics from Solution of an Old Problem

Bert Schroer
Institut für Theoretische Physik
FU-Berlin, Arnimallee 14, 14195 Berlin, Germany
presently: CBPF, Rua Dr. Xavier Sigaud, 22290-180 Rio de Janeiro, Brazil
schroer@cbpf.br

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Abstract

Recent ideas on modular localization in local quantum physics are used to clarify the relation between on- and off-shell quantities in particle physics; in particular the relation between on-shell crossing symmetry and off-shell Einstein causality. Among the collateral results of this new nonperturbative approach are profound relations between crossing symmetry of particle physics and Hawking-Unruh like thermal aspects (KMS property, entropy attached to horizons) of quantum matter behind causal horizons, aspects which hitherto were exclusively related with Killing horizons in curved spacetime rather than with localization aspects in Minkowski space particle physics. The scope of this modular framework is amazingly wide and ranges from providing a conceptual basis for the d=1+1 bootstrap-formfactor program for factorizable d=1+1 models to a decomposition theory of QFT’s in terms of a finite collection of unitarily equivalent chiral conformal theories placed a specified relative position within a common Hilbert space (in d=1+1 a holographic relation and in higher dimensions more like a scanning). The new framework gives a spacetime interpretation to the Zamolodchikov algebra and explains its thermal aspects.

1 Introduction

Theoretical physicists, contrary to mathematicians, rarely return to their old unsolved problems; often they replace them by new inventions. The content of the present article on some new concepts in particle physics does not follow this pattern. The old problems it addresses and partially solves are those of the relation between off-shell and on-shell quantities (or between fields-particles)
and in particular of crossing symmetry in local quantum physics (LQP)\(^1\). A more restricted form of on-shell crossing symmetry also led to the invention of the dual model and string theory, a line of development which we will not follow except some remarks in the last section.

The most prominent of on-shell quantities is the S-matrix of a local QFT, whereas fields and more general operators are “off-shell”. The derivation of on-shell quantities from LQP through the use of the rigorous LSZ scattering theory was one of the high points of the QFT of the 60\(^{\text{th}}\)s. In the opposite direction the problem (“The Inverse Problem of QFT”) lay dormant for a long time. Recently the adaptation of the Tomita-Takesaki modular theory to wedge-localized algebras suggested new methods to construct unique off-shell local operator algebras from the scattering data in a quite interesting and novel way\(^2\)[5]. Thus the inverse problem of QFT has a much better status than in QM. To bring this into the open requires the introduction of a wealth of new concepts relevant to particle physics, while maintaining all the principles of QFT.

In this paper we will have to study a new kind of operators which, as a result of their weak semiinfinite (wedge-like) localization and their close relation to the S-matrix, are to be considered as on-shell. This on-shell operators are essential for our new approach which avoids pointlike fields at the beginning and rather starts with on-shell generators of wedge-localized algebra which encode the structure of the S-matrix. Off-shell compactly localized operators and local field generators are then obtained via intersections of wedge algebras. Here and in the sequel the word localization region always stand for the causal completion of a spacetime region; these are typically the regions which one obtains by intersecting wedges.

Besides these two extremes there are intermediate possibilities where on-shell and off-shell aspects appear together. The most prominent and useful mixed objects are bilinear forms on scattering vector states i.e. matrix elements of local operators \(A\) (either pointlike fields or bounded operators localized in smaller than wedge regions) taken between incoming and outgoing multiparticle scattering states (in terms of Feynman graphs, one leg is off-shell).

\[
\text{out} \langle q_1, \ldots q_{n-1}, q_n | A | p_n, p_{n-1}, \ldots p_1 \text{in} \rangle \quad (1)
\]

which we will call (generalized) formfactors, following the standard terminology of \(d=1+1\) factorizing models. These objects fulfil the important crossing symmetry which acts on the on-shell momenta.

\[
\text{out} \langle q_1, \ldots q_{n-1}, q_n | A | p_n, p_{n-1}, \ldots p_1 \text{in} \rangle = \text{out} \langle -\bar{p}_1, q_1, \ldots q_{n-1}, q_n | A | p_n, p_{n-1}, \ldots p_2 \text{in} \rangle \quad (2)
\]

\(^1\)We will often use the name “local quantum physics” (LQP) instead of QFT\(^2\), if we have in mind the physical principles of QFT implemented by different concepts than those of the various quantization formalisms (canonical, quantization via path integrals etc.) which most of the readers are familiar with from the various textbooks. To the extend that the reader does not automatically identify QFT with those formalisms, he may without danger of misunderstandings continue to use the good old name QFT.
where the analytic continuation \( p \rightarrow -p \) is carried out in the rapidity parametriza-
tion by an \( i\pi \)-shift: \( \theta \rightarrow \theta + i\pi \), and the bar denotes the antiparticle. The difficul-
ties in physical interpretation of this relation (about which rigorous information
outside of perturbation theory of sufficient generality is scarce) reflects the lack
of its conceptual understanding. It is in a way deeper than the TCP-symmetry,
a symmetry derived from causality which among other things requires the exis-
tence of an antiparticle to each particle. In fact the crossing transformation is a
kind of individual TCP-transformation which Effects only one particle at a time
within the multiparticle incoming ket configuration and carries it to the outgo-
ing bra configuration as an antiparticle. In spite of its name it is not a quantum
theoretical (Wigner) symmetry, since that crossing process involves an on-shell
analytic continuation \( p \rightarrow -p \). Together with vacuum polarization it belongs to
the most characteristic aspects of QFT. Although its physical meaning in terms
of the basic principles of LQP remained vague, most physicists liked to view it
as a kind of on-shell imprint of Einstein causality, the latter being an off-shell
concept. One of the results of the new conceptual framework presented here
is an interpretation of the crossing property in terms of "wedge localization"
and the ensuing thermal Hawking-Unruh properties. They are usually associ-
ated exclusively with black hole quantum physics, but in fact turn out to be
general properties of any local quantum description including particle physics
in Minkowski space. Although in constructive terms the control in passing from
the on-shell S-matrix-dominated aspects to the off-shell local quantum physics
remains a formidable problem, it is easy to see that a local theory is (if it exists
at all) uniquely determined in terms of its on-shell "shadow".

The S-matrix whose matrix elements result from the previous formula for \( A
= 1 \), is the observable of particle physics par excellence; it is totally intrinsic
and independent of any field coordinatizations, although in the LSZ theory
it is calculated from specific fields. Strictly speaking in high energy physics
only (inclusive) cross sections and not amplitudes are directly measured; a fact
which is especially important if interactions between zero mass particles leads
to infrared problems.

The reason why most theoretical methods in particle physics do not aim
directly at the S-matrix is that most of our physical intuition about causality
and charge flows in spacetime is based on (off-shell) local fields or local observ-
able. The new on-shell wedge-algebra generators introduced in this paper are
somewhat hidden and in particular are not obtainable by Lagrangian or more
generally by any kind of quantization approach. Although their role in general
QFT is only at the beginning of being understood, there is already a very good
spacetime comprehension in the class of factorizing \( d=1+1 \) models [22].

The old problems on which there has been significant recent progress can be
compressed in terms of the following questions

- Does a physically admissible S-matrix fulfilling unitarity, crossing sym-

\footnote{Any approach which leads to explicate solution and not just to formal representations as
e.g. euclidean functional integral representations would of cause present all properties. But
the Lagrangian approach only achieves the latter.}
metry and certain analytic properties (needed in its formulation), have an underlying unique local QFT? This one may call the inverse problem of QFT associated with scattering. It is a problem of principle interest to take notice not only of the well-known fact that fields and local observable lead to scattering, but that also local equivalence classes of fields or nets of local observables are in turn determined by particle scattering data.

- Is there a constructive procedure in which, similar to the \( d=1+1 \) bootstrap-formfactor program for factorizing \( d=1+1 \) models (which in fact reappears as a special case), the S-matrix and the generalized formfactors enter as important constructive elements in order to obtain off-shell objects as fields or local observables? In particular can one formulate such a constructive approach in a conceptually intrinsic manner i.e. without any quantization parallelism to classical field theory and without the use of field coordinizations and short-distance divergence problems? This could be of tremendous practical importance.

The progress obtained on both questions by the modular method be presented in the sequel.

In order to understand better what is meant by the word "old" in the title of this paper, it is very instructive to pause and take stock of some past achievements and failures in an S-matrix approach to particle physics. Already as far back as 1946 Heisenberg \[8\] proposed to do particle physics in a pure S-matrix setting in order to avoid the at that time nonsensical aspects of short distance divergencies in QFT. His requirements of unitarity, Poincaré invariance and some aspects of cluster decomposition properties turned out to be much too general in order to be useful. A second attempt with the full backing of renormalized perturbation theory was launched in the early 60\[9\]. Part of the motivation was similar to the previous one. Although renormalization theory meanwhile allowed to extract a class of perturbatively finite QFTs, the formally infinite intermediate steps and the not entirely natural but rather technical looking division into renormalizable/nonrenormalizable models nourished the hope that those discomforting features would disappear in a pure S-matrix approach \[12\]. The second more pragmatic motivation was the idea that the dispersion theoretical research of the 50\[13\] could be extended into a computational scheme for strong interactions within an S-matrix setting. Since the physical principles of the S-matrix approach as any physical principles cannot be dealt with directly but rather require an operator or functional formalism, several ideas which could not be motivated through QFT (in fact they are not true in perturbative QFT) had to be added. The most prominent ones were on-shell spectral representations as the Mandelstam representation and a strengthened form of crossing called duality. The first one served to specify analyticity domains and the second implemented the purely phenomenological idea of saturating the coexistence of charges in the different crossed channels already by disregarding cuts and only taking one-particle poles into account ("Nuclear Democracy", "Reggeization").

\[3\]That an S-matrix cannot determine individual fields had been known since the late 50\[13\].
I believe that these added requirements which made heavy use of analyticity contributed to the failure of the program. It did not even achieve to reproduce those perturbative S-matrices which via LSZ scattering theory were obtained from the Feynman perturbation theory of time-ordered functions. What was left over from this program got swept aside by the ascending gauge theory at the beginning of the 70\textsuperscript{ies}.

For the purpose of a good understanding of the content of this paper it is very helpful to localize the cause of the failure of the S-matrix bootstrap program. I think I will be in agreement with most of my colleagues who followed these developments or later read about them that free-floating (and often ill-defined) analyticity requirements are too fine instruments in order to harmonize with physical intuition. Only analytic properties which appear directly in the formulation of physical concepts are useful for the construction of theories. This is best illustrated by two examples. The x-space analyticity of correlation functions in QFT which was discovered by Wightman \cite{5} is equivalent to the spectral-, covariance- and locality- properties of the operator theory. On the other hand the dispersion relations, even if restricted to the simplest case of forward scattering, involve analyticity properties which arise from a quite complicated interplay between the off-shell causality of retarded functions with on-shell spectrum properties \cite{6}. Such non-constructive analytic properties are still useful for experimentally verifying particular consequences of causality and they also have their merits in the study of possible nonperturbative high energy bounds on cross sections, but they have no natural role in an actual construction.

Only a very few people took notice of the fact that the bootstrap program finally worked in the more limited context of d=1=1 integrable models; it was too far away from a “theory of everything” which was on the minds of the ambitious protagonists of the 4-dim. bootstrap which finally ended in failure. The modest 2-dim. program led to a nice classification of families of factorizing elastic S-matrices (thus showing that the idea of the bootstrap being a TOE was incorrect) and it also set the path for the construction of associated QFT models via a formfactor program. A side result of the S-matrix research in d=1+1 was the discovery of an on-shell perturbation theory which, if specialized to on-shell tree graphs without particle creation allowed to show the absence of creation for on-shell one-loop approximation \cite{10}. Apparently the extension to multi-loops was never elaborated in sufficient generality; some partial results can be found in \cite{11}. The very existence of these formulae shows that a finite on-shell approach which avoids the characteristic off-shell short distance problems of QFT is more than just a nice dream.

The present line of research directly takes off where the original program failed. It removes the unfortunate TOE ideology\cite{5} from the S-matrix boot-

\footnote{The absence of particle creation is not an issue which is evident on the level of tree graphs since it only happens on-shell. Properties which are only valid on-shell are too subtle to be seen by inspection of Feynman diagrams.}

\footnote{“Theories of everything” seem to be also the favorite past time of post-S-matrix physicist. The underlying idea that certain principles allow for only one solution usually originate in connection with nonlinear structures for which initially no solution is known.}
strap and incorporates the latter with the help of modular theory into the QFT structure of wedge algebras. This return into QFT is based on the fact that the S-matrix has in addition to the large time scattering interpretation (well-known from the LSZ theory) another little known aspect namely it is the relative modular invariant between the wedge algebra of incoming free fields and that of the actual interacting Heisenberg operators. Whereas the scattering aspect also applies to QM, the modular role is totally characteristic for local quantum physics. Having established a direct modular relation between the S-matrix and the wedge algebra, the old S-matrix formalism becomes enriched with new physical concepts and mathematical tools. In particular the thermal aspects of the wedge algebra (the Hawking-Unruh temperature of matter behind a Rindler horizon) gets inexorably tied up with the crossing symmetry of particle physics. A pivotal role in the linkage of the S-matrix with the wedge algebras is played by special wedge-localized operators which applied to the vacuum create one-particle states without the usual associated cloud of particle-antiparticles well known from the vacuum polarization phenomenon. These PFGs (polarization-free generators) cannot exist for spacetime localization regions whose causal completion is smaller than a wedge, unless the theory has no interactions, in other words the wedge region is the smallest causally complete region for which PFGs are compatible with the presence of interactions.

The new framework shares with the old S-matrix bootstrap program (and with string theory) the absence of any ultraviolet problems since it uses no coordinatizations in terms of pointlike fields. Whether a theory exists or not is not decided by the short distance singularities of some field coordinates in terms of which the Lagrangian quantization happened to be done, but rather depends on the nontriviality of the intersection structure of wedge algebras. If intersections representing double cone algebras contain more operators than just multiples of the identity, the theory is nontrivial in the sense that possesses a nontrivial net for small localization regions and not just for wedge regions. This avoidance of particular pointlike fields and their short-distance problems was the main dream and the raison d’être of the S-matrix bootstrap. It is fully realized in the new approach by the use of the field-coordinate independent algebraic formulation of QFT (AQFT). The intention of the S-matrix protagonists to abandon fields was reasonable, but unfortunately they thought that they also should abandon the principle of locality.

The philosophy that the S-matrix has nothing to do with localization and locality was also not quite right, as the relation to wedge-localized algebras shows. Since from the net of wedge algebras one can get the algebras of compact regions by intersections, all of local quantum physics is determined by the S-matrix. An AQFT for a given S-matrix turns out to be uniquely determined and it is believed that if S is admissible in the old sense (unitary, crossing symmetric+associated analyticity), the net of wedge algebras also exist and leads to the required nontrivial intersections. For d=1+1 factorizing S-matrices the formfactor program goes a long way towards proving this conjecture.

After having outlined our physical motivation and the position of the new concepts with respect to older ideas, we now briefly mention our main mathe-
mathematical tool which will be used for problems of (quantum) localization: (Tomita’s) modular theory of von Neumann algebras\(^6\). These concepts, which for the first time clarified the on/off-shell relation and in particular the spacetime interpretation of on-shell crossing symmetry, were not available at the time of the S-matrix bootstrap of the 60\(^{th}\)s. In a seminal paper \(^{13}\) the connection of wedge localized algebras with modular theory was for the first time established. The present approach may be considered as the inverse of the Bisognano-Wichmann Theorem. Instead of extracting a deep mathematical property from the QFT of wedge algebras, we are using this property together with the modular role of the S-matrix for the construction of QFTs via wedge algebras. The main new mathematical tool is briefly described in an appendix, a more detailed account can be found in \(^2\).

The ideas of AQFT used in this paper are not as much known as their importance would suggest. Perhaps this is due to the fact that most particle physicist consider QFT as a basically settled issue with only some nasty technical problems remaining. We will demonstrate in this paper that such a view is quite premature and unrealistic.

We have organized this paper as follows. The next section reviews and illustrates the field-coordinate-free approach for theories without interactions and for interacting d=1+1 factorizing model. In the latter case the Zamolodchikov-Faddeev algebra emerges in a natural way without having been put in \(^{22}\), and the hitherto formal Z-F operators for the first time acquire a spacetime interpretation in connection with the new PFG generators of wedge algebras. The presentation of these polarization-free wedge generators is extended to systems which are not factorizing (i.e. to theories with on-shell particle creation) in section 3.

After a brief introduction to the AQFT framework in section 4, the fifth section treats the light ray/front restriction and algebraic holography in terms of associated chiral conformal field theories. This connection is again a deep result of modular theory, more specifically of modular inclusions and intersections. There we also discuss the problem of inverting such maps (the “blow up” property) in such a way that the original theory becomes reconstructed from a finite number of copies of one abstract chiral theory whose relative position in one Hilbert space has to be carefully chosen. In colloquial terms this is like scanning a higher dimensional massive theory by one chiral theory in different positions and we will refer to it as “chiral scanning”. Since a finite number of relatively positioned chiral theories seems to be easier understandable than one higher dimensional massive theory, the chiral scanning is in addition to the wedge algebra method and the use of PFGs explained before a second potential constructive idea based on modular theory. The mathematical technology used in this section 5 is one of the most powerful which AQFT presently is able to offer namely the theory of modular inclusions and intersections \(^{14}\).\(^{2}\).

\(^6\)Special aspects (the thermal KMS characterization) of Tomita’s mathematical modular theory were discovered by physicists in connection with quantum statistics of Fermi/Bose systems formulated directly in the infinite volume limit when the Gibbs formulation breaks down.\(^1\).
In the same section 5 we also take up the problem of associating entropy for localized matter. In view of the fact that modular localization leads to thermal aspects which show up in the appearance of a Hawking-Unruh temperature it is natural to ask for a concept of “localization entropy” and in case it exists whether it gives a quantum version of the Bekenstein area law for the area of the causal horizon of the localization region.

The last section finally tries to compare our approach with string theory. This is on the one hand natural since both have similar historical roots, but difficult from a conceptual viewpoint because string theory despite all its mathematical formalism has never developed beyond a collection of recipes towards formulating underlying principles. Whereas our approach has strong ties with the older S-matrix bootstrap program (minus the wrong TOE philosophy) which only used properties abstracted from QFT, string theory via the dual model has added many ad hoc inventions which did not originate through the intrinsic logic of S-matrix theory and QFT and are not asked for by any known principle of particle physics. This is in my view the reason why string theory despite all its semantic changes has remained a collection of computational prescriptions without the guidance of a conceptual framework. Since basic physical issues as locality and localizations, algebras versus states etc. should be discussed on the level of physical principles and not by looking at formalism and computational recipes, our compression in the last section unavoidably remains somewhat vague and superficial.

This presentation is in part a survey of published material [22][24][7][51][44][31] as well as of new results, in particular the presentation of localization-entropy in section 4. For the convenience of the reader we attached an two part appendix the first part containing standard facts about modular theory and the second proving that in interacting theories there are no polarization-free generators for subwedge spacetime regions.

2 Systems without Interactions and Factorizing Models

In trying to bring readers with a good knowledge of standard QFT in contact with some new (and old) concepts in algebraic QFT (AQFT) without sending him back with a load of homework, I face a tricky problem. Let us for the time-being put aside the intrinsic logic which would ask for a systematic presentation of the general framework, and let us instead try to maneuver in a more less ad hoc (occasionally even muddled) way.

In a pedestrian approach the problem of constructing nets of interaction free systems from Wigner’s one particle theory may serve as a nice pedagogical exercise for the new ideas. The reader who is not familiar with Wigner’s representation theoretical method to describe particle spaces is referred to [21][1]. Since Wigner’s representation theory restricted to positive energy representations was the first totally intrinsic relativistic quantum theory without any quantization...
parallelism to classical particle theory, it is reasonable to expect in general that if we find the right concepts, we should be able to avoid covariant pointlike fields altogether in favor of a more intrinsic way to implement the causality/locality principle. In that case the local fields should be similar to coordinations of local observables in analogy to the use of coordinates in modern differential geometry. This viewpoint is indeed consistent and essential in the present context\[22\][23].

Let us first understand how the free field algebras are directly abstracted from the Wigner theory. By using a spatial variant of Tomita’s theory for the wedge situation i.e. by defining a kind of antilinear involutive “pre-Tomita” operator $s$ on the Wigner representation space (without a von Neumann algebra), one obtains a real closed subspace $H_R(W)$ of the Wigner space $H$ of complex multi-component momentum space wave functions as a $+1$ eigenspace of the Tomita-like quantum mechanical operator $s$ in $H$. Here $W$ denotes the x-t wedge $x > |t|$ and $s$ is defined to be the product of the $i\pi$-continued x-t Lorentz boost $\delta_x^2$ (obtained by the functional calculus associated with the spectral theory of the boost operator $\delta_x^{i\tau} := U(\Lambda_{x,\tau}(\pi\tau)), \pi\tau = \text{rapidity}$) multiplied with the one-particle version of the (antiunitary) $j$-reflection in the x-t plane

$$s = j\delta_x^2$$

$$s\psi = \psi, \psi \in H_R(W)$$

For the definition of the antiunitary Tomita involution $j$ which represents the x-t reflection in case of antiparticles $\neq$ particles one needs to extend the Wigner representation to the direct sum of particle/antiparticle spaces; a process well-known in the Wigner theory if one wants to include the disconnected Poincaré transformations. Since the x-t reflection commutes with the x-t boost $\delta_x^2$ and is antiunitary, it formally inverts the unbounded $\delta$ i.e. $j\delta = \delta^{-1}j$ which is formally the analytically continuing boost at the imaginary value $t = -i$. As a result of this commutation relation the unbounded antilinear operator $s$ is involutive on its domain of definition $s^2 \subset 1$. These unusual properties, which are not met anywhere else in QM, encodes geometric localization properties within abstract operator domains. They also preempt the relativistic locality properties of QFT which Wigner looked for in vain.\[15\]. The opposite localization i.e. $H_{R}(W_{opp})$ turns out to correspond to the symplectic (or real orthogonal) complement of $H_{R}(W)$ in $H$ i.e. $\text{Im}(\psi, H_{R}(W)) = 0 \Leftrightarrow \psi \in H_{R}(W_{opp})$. One furthermore finds the following properties for the subspaces called “standardness”

$$H_{R}(W) + iH_{R}(W) \text{ is dense in } H$$

$$H_{R}(W) \cap iH_{R}(W) = \{0\}$$

Having arrived at the wedge localization spaces, one may construct localization spaces for smaller spacetime regions by forming intersections over all wedges

\[7\]Apart from a rotation around the x-axis by an angle $\pi$, this is the famous TCP operator restricted to the one-particle/antiparticle subspace.
which contain this region

$$H_R(O) = \bigcap_{W \supset O} H_R(W) \quad (5)$$

These spaces are again standard and have their own premodular objects $\delta, j$ and $s$, but this time their action cannot be described in terms of spacetime diffeomorphism. Note also that the modular formalism characterizes the localization of subspaces, but is not able to distinguish individual elements in that subspace. There is a good physical reason for that, because as soon as one tries to do that, one is forced to leave the unique Wigner $(m, s)$ representation framework and pick a particular covariant representation by selecting one specific intertwiner among the infinite set of $u$ and $v$ intertwiners which link the unique Wigner $(m, s)$ representation to the countably infinite many covariant possibilities.

In this way one would then pass to the framework of covariant fields explained and presented in the first volume of Weinberg’s book. The description of a concrete element in $H_R(W)$ or $H_R(O)$ then depends on the choice of covariant formalism. A selection by e.g. invoking Euler equations and the existence of a Lagrangian formalism may be convenient for doing particular perturbative computations or as a mnemonic device for classifying polynomial interaction densities, but is not demanded as an intrinsic attribute of physics.

The way to avoid nonunique covariant fields is to pass from Wigner subspaces directly to von Neumann subalgebras of the algebra of all operators in Fock space $B(\mathcal{H}_{Fock})$, i.e. the transition from real subspaces to von Neumann subalgebras in Fock space is well known. With the help of the Weyl (or CAR in case of Fermions) functor $Weyl(\cdot)$ one defines the local von Neumann algebras

$$\mathcal{A}(W) := alg \{Weyl(f) | f \in H_R(W)\} \quad (6)$$

a process which is sometimes misleadingly called “second quantization”. These Weyl generators have the formal appearance

$$Weyl(f) = e^{\alpha(f)}$$

$$a(f) = \sum_{s_3=-s} \int (a^*(p, s_3)f_{s_3}(p) + h.c.) \frac{d^3p}{2\omega}$$

i.e. unlike the covariant fields they are independent of the nonunique intertwiners and depend solely on the unique Wigner data. An analogue statement holds for the halfinteger spin case for which the CAR functor maps the Wigner wave function into the fermionic generators of von Neumann subalgebras. The statistics is already preempted by the premodular theory on Wigner space.

The local net $\mathcal{A}(O)$ may be obtained in two ways, either one first constructs the

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8The causal approach permits the transformation of a polynomial interaction from one coordinatization to any other whereas a formalism using classical actions involving free field Lagrangians $\mathcal{L}_0$ is restricted to the use of Euler-Lagrange field coordinatizations.
spaces $H_R(\mathcal{O})$ via (5) and then applies the Weyl functor, or one first constructs the net of wedge algebras (6) and then intersects the algebras.

If we would have taken the conventional route via interwiners and local fields as in [21], then we would have been forced to use Borchers construction of equivalence classes in order to see that the different free fields associated with the $(m,s)$ representation with the same momentum space creation and annihilation operators in Fock space are just different generators of the same coherent families of local algebras i.e. yield the same net. This would be analogous to working with particular coordinates in differential geometry and then proving at the end that the objects of interests are invariant and therefore independent of coordinates.

The implementation of interactions in the framework of nets requires a radical re-thinking of the formalism, even if we are only interested in perturbative aspects. The use of the above method for the Wigner one-particle representation and the subsequent introduction of interactions will inevitably force us to reintroduce field coordinates in order to define what we mean by perturbative interactions. In order to avoid the standard approach we therefore have to find a way to introduce interactions directly into wedge-localized multiparticle spaces or wedge algebras.

In order to get a clue of how we can avoid the use of pointlike fields in interacting situations, let us first ask this question in a more limited context. It is well-known that there exists a special class of theories in $d=1+1$ in which the $S$-matrix commutes with the incoming particle number

$$[S_{sc}, N_{in}] = 0 \quad (8)$$

and factorizes on multi-particle in-states [16] [17] [18]. For this reason these theories are often referred to as factorizing or integrable (since this leads to an infinite number of conservation laws) models. For those one finds that not only the old bootstrap program can be carried through, but the application of the so-called formfactor-program allows to compute even the fields in the sense of bilinear forms between in and out states [20] [21]. Let us ignore those bootstrap-formfactor recipes and try find a modular access to these models by implementing the idea of a relativistic particle pair interaction with a most naive Ansatz (assuming for simplicity a situation of selfconjugate particles) which formally generalizes the standard creation/annihilation operators. Using rapidities instead of momenta we require

$$Z(\theta)Z(\theta') = S(\theta - \theta')Z(\theta')Z(\theta) \quad (9)$$

$$Z(\theta)Z^*(\theta') = S^{-1}(\theta - \theta')Z^*(\theta')Z(\theta) + \delta(\theta - \theta')$$

with the star-structure determining the remaining commutation relations and the unitarity of $S$ with $S^{-1}(\theta) = S(\theta) = S(-\theta)$ etc. Together with $Z(\theta)\Omega = 0$

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9The class of covariant free fields belonging to the same $(m,s)$ is a linear subclass of the full equivalence class which comprises all Wick-polynomials. In the analogy with coordinates in differential geometry this subclass corresponds to linear coordinate transformations.
we can express all $Z$ correlation functions in terms of $S$’s and the computation of correlation functions proceeds as for free fields namely by commuting the annihilation operators $Z$ to the right vacuum e.g.

\[
(\Omega, Z(\theta_4)Z(\theta_4)Z^*(\theta_2)Z^*(\theta_1)) = S(\theta_4 - \theta_3)\delta(\theta_4 - \theta_1)\delta(\theta_3 - \theta_4) + \delta(\theta_4 - \theta_2)\delta(\theta_3 - \theta_4)
\]

Although we use the preemptive notation “$Z$" which refers to the Zamolodchikov-Faddeev algebra\(^{10}\), there are for the time being no requirements on the coefficients which go beyond those following from the structure of a distributive *-algebra, i.e. crossing symmetry is not required but will be result from wedge localization.

One easily sees that instead of postulating commutation relations we could also have started from the following formula which represents $Z^*Z^*\Omega$ state vectors in terms of corresponding free field terms

\[
Z^*(\theta_2)Z^*(\theta_1)\Omega = \frac{1}{\sqrt{2}}(\chi_{21}a^*(\theta_2)a^*(\theta_1)\Omega + \chi_{12}S(\theta_2 - \theta_1)a^*(\theta_1)a^*(\theta_2)\Omega)
\]

Here the symbol $\chi_{P(1),...P(n)}$ denotes the characteristic function of the region $\theta_{P(1)} > ... > \theta_{P(n)}$. It is easy to see that the inner product agrees with (10); one only has to use the identity

\[
\{\chi_{12}S(\theta_2 - \theta_1) + \chi_{21}\bar{S}(\theta_3 - \theta_4)\} \delta(\theta_3 - \theta_1)\delta(\theta_4 - \theta_2)
\]

In fact if we had started with a more general two-particle interaction Ansatz by allowing the structure of the second equation in (9) to be different say $S^{-1} \rightarrow T$, the consistency with (11) would immediately force us to return to $T = S^{-1}$.

The formula for the 4-point function suggests the possibility to replace the algebraic Ansatz by the following formula for multi-$Z^*$ state vectors

\[
Z^*(\theta_n)...Z^*(\theta_1)\Omega = \sum_{\text{perm}} \chi_{P(n),...P(1)}(\prod_{\text{transp}} S)a^*(\theta_{P(n)})...a^*(\theta_{P(1)})\Omega
\]

where the product of S-factors in the bracket contains one S for each transposition which expresses the two-body nature of the interaction. The associativity of the $Z$’s i.e. the Yang-Baxter relation for matrix-valued $S$’s insures the consistency of the formula. We call $\theta_{P(1)} > ... > \theta_{P(n)}$ the natural order of the multi-$Z^*$ state vector. From the state characterization (13) one can derive the algebraic definition (8).

With these algebraic prerequisites out of the way, let us now return to physics and investigate spacetime localization properties of the following hermitian op-

\(^{10}\)The missing delta-function contribution in Zamolodchikov's original proposal\(^ {27}\) was later added by Faddeev.
where \( C \) is a path consisting of the upper/lower rim of a \( i\pi \)-strip with the real \( \theta \)-axis being the upper boundary. Whereas the on-shell value of the Fourier transform \( \hat{f}(\theta) \) of \( \hat{f} \) is analytic in this strip, the last relation is a notation (since operators by themselves are never analytic in spacetime labels!) which however inside expectation values becomes coherent with meromorphic properties. If we take instead of \( Z^\# \) free creation/annihilation operators, the corresponding formula \( \int_C a(\theta) \hat{f}(\theta) \) represents wedge-localized smeared free fields. Formally we may write in analogy to free fields

\[
F(x) = \frac{1}{\sqrt{2\pi}} \int (e^{-ipx} Z(\theta) + h.c.) d\theta
\]

but we should be aware that the argument \( x \) is not related to a pointlike localization in the sense of causality since on-shell fields are local iff they are bona fide free fields i.e. iff the \( Z's \) reduce to the standard creation/annihilation operators (see appendix).

In the following we will prove that the operators \( F(\hat{f}) \) with \( Z^\# \) fulfilling (13) are localized in the wedge \( x > |t| \) if and only if the \( \ast \)-algebra can be extended to a Zamolodchikov-Faddeev algebra i.e. iff the coefficients \( S \) are crossing symmetric including the crossing symmetric bootstrap pole structure. Following our previously introduced terminology [22], we will use the name polarization-free generators (PFG) for localized operators in interacting QFT whose one time application to the vacuum vector results in a one particle state vector. It is well-known [19] that PFGs with smaller than wedge localization regions (e.g. double cones, spacelike cones) can only exist in theories without interactions i.e. \( \psi_{f_1...f_n} = \psi_{f_1...f_n}^{out} \). For the convenience of the reader we present the argument in an appendix. PFGs however always exist in regions in interacting theories if the localization region is a wedge or bigger [32]. The argument is based on modular theory and will be recollected in the next section.

We want to show that the above \( F \) can be converted indeed into bona fide PFGs and a for a proof we have to check the KMS property for the F-correlation functions with the modular generator being the infinitesimal boost \( K \). This property is a prerequisite for any wedge-localized algebra in a Wightman QFT [13]. The KMS property is well-known from statistical mechanics and is the substitute for Gibbs formula which for many quantum systems becomes meaningless in the thermodynamic limit. In the present context its thermal aspects has been
discussed in [22]. The desired KMS-property for the wedge reads
\[
\langle F(\hat{f}_n)\ldots F(\hat{f}_1) \rangle = \langle F(\hat{f}_{n-1})\ldots F(\hat{f}_2)F(\hat{f}_{2\pi i}) \rangle
\]
where the superscript \(2\pi i\) indicates the imaginary rapidity translation from the lower to the upper rim of the KMS strip.

A rather straightforward calculation based on the previously explained rules for the Zs yields the following result

Theorem 1 ([22],[51]) the KMS-thermal aspect of the wedge algebra generated by the PFGs is equivalent to the crossing symmetry of the S-matrix

\[
\mathcal{A}(W) := \text{alg} \left\{ F(\hat{f}); \text{supp}\hat{f} \in W \right\} \Leftrightarrow S(\theta) = S(i\pi - \theta)
\]

Furthermore the possible crossing symmetric poles in the physical strip of S will be converted into intermediate composite particle states in the GNS Hilbert space associated with the state defined by the correlations on the \(\mathcal{A}(W)\)-algebra. The latter commutes with its geometric opposite \(\mathcal{A}(W^{\text{opp}})\) in case of \(\mathcal{A}(W^{\text{opp}}) = \mathcal{A}(W)^{\prime} = \text{AdJ}A(W)\). A sufficient condition for this is the existence of a parity transformation whose action on \(\mathcal{A}(W)\) equals the commutant \(\mathcal{A}(W)^{\prime}\).

Since the Fs are unbounded operators with (particle number) N-bounds which are the same as for free fields, the algebra generated by them is to be understood in the sense that they are affiliated with that von Neumann algebra which they generate.

We recall the proof for the 4-point function of \(F^{\prime}\)'s which may be obtained as the scalar product of two-particle state vectors (\(c.t.\) denotes the F-contraction terms)

\[
F(\hat{f}_2)F(\hat{f}_1)\Omega = \int \int \hat{f}_2(\theta_2 - i\pi)\hat{f}_1(\theta_1 - i\pi)Z^*(\theta_1)Z^*(\theta_2)\Omega + c.t.
\]

\[
= \int \int \hat{f}_2(\theta_2 - i\pi)\hat{f}_1(\theta_1 - i\pi)\left\{ \chi_{12}a^*(\theta_1)a^*(\theta_2)\Omega + \chi_{21}S(\theta_2 - \theta_1)a^*(\theta_2)a^*(\theta_1)\Omega \right\} + c\Omega
\]

and the analogous formula for the bra-vector. The formula needs some explanation. The symbol \(\chi\) with the permutation subscript denotes as before the characteristic function associated with the permuted rapidity order. The order for the free creation operators \(a^*\) is governed by particle statistics. For each transposition starting from the natural order (13), one obtains an \(S\) factor. The Yang-Baxter relation assures that the various ways of doing this

\[\text{\textsuperscript{11}}\] The notation has used the statistics in order to bring the product of incoming fields \(a_{in}\) into the natural order say 1...n. The ordering of the \(Z\)’s encodes the \(\theta\)-ordering and not the particle statistics. It is connected with the different boundary values of state vectors and expectation values in \(\theta\)-space in approaching the physical boundary from the analytic region. This is analogous to the association of the n! n-point x-space correlation functions with different boundary values of one analytic “master function” in the Wightman theory.

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are consistent. For the inner product the $S$-dependent terms are. Finally the terms proportional to the vacuum are contraction terms corresponding to the $\delta$-function in (11). For the $S$-dependent terms in the inner product we obtain

$$
\int \int f_4(\theta_2)f_3(\theta_1)\{\chi_{21}S(\theta_2 - \theta_1) + \chi_{12}\tilde{S}(\theta_1 - \theta_2)\}f_2(-i\pi)f_1(\theta_1 - i\pi)d\theta_1d\theta_2
$$

$$
= \int \int f_4(\theta_2)f_4(\theta_1)S(\theta_2 - \theta_1)f_2(-i\pi)f_1(\theta_1 - i\pi)d\theta_1d\theta_2
$$

(19)

The analogous computation for KMS crossed term in (24) gives

$$
\int \int f_2(\theta_1)f_2(\theta_2)S(\theta_1 - \theta_2)f_1(-i\pi)f_1(-i\pi + 2\pi i)d\theta_1d\theta_2
$$

(20)

This formula makes only sense if the $F(f)$ operators are restricted in such a way that the $2\pi i$ translation on them is well-defined, i.e. for wave functions $f$ which are analytic in a strip of size $2\pi i$. It is well-known that the KMS condition does not hold on all operators of the algebra but rather on a dense set of suitably defined analytic elements [24]. The $S$-independent terms which we have not written down are identical to terms in the 4-point function of free fields. They separately satisfy the KMS property. What remains is to show the identity of (19) and (20). This is done by $\theta_2 \rightarrow \theta_2 - i\pi$ contour-shift in (20) without picking up terms from infinity. Using the denseness of the wave functions one finally obtains

$$
S(\theta_2 - \theta_1) = S(\theta_1 - \theta_2 + i\pi)
$$

(21)

which is the famous crossing symmetry or the $z \leftrightarrow -z$ reflection symmetry around the point $z_0 = \frac{1}{2}i\pi$. For non self-conjugate situation the crossed particles are antiparticles and the $S$ on the right hand side has to be modified accordingly.

In physical terms we may say that the wedge structure of factorizing models is that of a kind of relativistic quantum mechanic. This continues to be true if the crossing symmetric $S$-matrix has poles in the physical strip. In that case the above contour shift would violate the KMS property unless one modifies the multi-$Z^*$ state vector formula (13) by the inclusion of bound states. For the case $n=2$ (11) this means

$$
Z^\ast(\theta_2)Z^\ast(\theta_1)\Omega = (Z^\ast(\theta_2)Z^\ast(\theta_1)\Omega)^{\text{scat}} + |
\theta, b\rangle \langle \theta, b|Z^\ast(\theta - i\theta_b)Z^\ast(\theta + i\theta_b)|\Omega
$$

(22)

The bracket with the superscript scat denotes the previous contribution (11), whereas the second line denotes the bound state contribution. The validity of the KMS property demands the presence of this term and determines the coefficient; here $\theta_b$ is the imaginary rapidity related to the bound state mass. For a detailed treatment which includes the bound state problem, we refer to a forthcoming paper. We emphasize again that it is the representation of the $F$-correlations in terms of the $S$-matrix and the KMS property of these correlation
functions, which via the GNS construction converts the poles in the (possibly matrix-valued) function S into the extension of the Fock space of the a’s by additional free field operators. In this way the poles in numerical functions are converted into the enlargement of Fock space in such a way that a few Z’s can describe many more particles. One may call the Z to be “fundamental” and the introduce new Z₀ and F’₀’s; the latter will however be operators which are already associated with the original F-algebra. What needs an extension is the wedge algebra of incoming fields. It is very important to note that this apparent quantum mechanical picture is converted into LQP with vacuum polarization as soon as we e.g. go to double cone localization; this will be shown in the sequel. The extension of the above proof beyond 4-point functions is left to the reader.

With this theorem relating wedge localization via the thermal KMS property to crossing symmetry, we have achieved the main goal of this section: to show that the Zamolodchikov-Faddeev algebra which consists of \(\mathcal{Z}\) together with the crossing symmetry of its structure function has a deep spacetime interpretation and an associated thermal KMS aspect. In fact the simplest PFGs which fulfill conservation of real particle number and have only elastic scattering (possible in \(d=1+1\)) are precisely the Z-F algebra operators! In a moment we will see that these models have the full interacting vacuum structure (virtual particle nonconservation) with respect to operators from smaller localization regions (e.g. double cones), i.e. we are dealing with a genuine interacting field theory (and not some relativistic quantum mechanics).

The KMS computation can be immediately extended to “formfactors” i.e. mixed correlation functions containing in addition to Fs one generic operator \(A \in \mathcal{A}(W)\) so that the previous calculation results from the specialization \(A = 1\). This is so because the connected parts of the mixed correlation function is related to the various \((n, m)\) formfactors \(\mathcal{F}\) obtained by the different ways of distributing \(n+m\) particles in and out states. These formfactors are described by different boundary values of one analytic master function which is in turn related to the various forward/backward on shell values which appear in one mixed A-F correlation function. We may start from the correlation function with one A to the left and say n Fs to the right and write the KMS condition as

\[
\left< AF(\hat{f}_n)\ldots F(\hat{f}_2)F(\hat{f}_1) \right> = \left< F(\hat{f}_1^{2\pi i})AF(\hat{f}_n)\ldots F(\hat{f}_2) \right> \tag{23}
\]

The n-fold application of the Fs to the vacuum on the left hand side creates besides an n-particle term involving \(n\) operators \(Z^*\) to the vacuum (or KMS reference state vector) \(\Omega\) also contributions from a lower number of \(Z^*\)’s together with \(Z-Z^*\) contractions. As with free fields, the n-particle contribution can be isolated by Wick-ordering:

\[
\left< A : F(\hat{f}_n)\ldots F(\hat{f}_2)F(\hat{f}_1) : \right> = \left< F(\hat{f}_1^{2\pi i})A : F(\hat{f}_n)\ldots F(\hat{f}_2) : \right> \tag{24}
\]

\(^{12}\)Note that as a result of the commutation relation \(\mathcal{F}\), the change of order within the Wick-ordered products will produce rapidity dependent factors.
Rewritten in terms of $A$-formfactors the n-particle scattering contribution (using the denseness of the $f(\theta)$) reads as

$$\langle \Omega, AZ^*(\theta_n)\ldots Z^*(\theta_2)Z^*(\theta_1-2\pi i)\Omega \rangle$$

(25)

$$= \langle \Omega, Z(\theta_1+i\pi)AZ^*(\theta_n)\ldots Z^*(\theta_2)\Omega \rangle$$

$$= \langle Z^*(\theta_1-i\pi)\Omega, AZ^*(\theta_n)\ldots Z^*(\theta_2)Z^*(\theta)\Omega \rangle$$

Here the notation suffers from the usual sloppiness of physicists notation: the analytic continuation by $2\pi i$ refers to the correlation function and not to the operators. For the natural order of rapidities $\theta_n > \ldots > \theta_1$ this yields the following crossing relation

$$\langle \Omega, Aa_{\text{out}}^*(\theta_1)\ldots a_{\text{out}}^*(\theta_2)\Omega, Aa_{\text{in}}^*(\theta_n)\ldots a_{\text{in}}^*(\theta_2)\Omega \rangle$$

(26)

$$= \langle a_{\text{out}}^*(\theta_1)\Omega, Aa_{\text{in}}^*(\theta_n)\ldots a_{\text{in}}^*(\theta_2)\Omega \rangle$$

The out scattering notation on the bra-vectors becomes only relevant upon iteration of the KMS condition since the bra $Z'$s have the opposite natural order. The above KMS relation (24) contains additional information about bound states and scattering states with a lower number of particles. The generalization to the case of antiparticles $\neq$ particles is straightforward. More generally we see that the connected part of the mixed matrix elements

$$\langle a_{\text{out}}^*(\theta_1)\ldots a_{\text{out}}^*(\theta_2)\Omega, Aa_{\text{in}}^*(\theta_n)\ldots a_{\text{in}}^*(\theta_k-1)\Omega \rangle$$

(27)

is related to $\langle \Omega, AZ^*(\theta_n)\ldots Z^*(\theta_2)Z^*(\theta_1)\Omega \rangle$ by analytic continuation which a posteriori justifies the use of the name formfactors in connection with the mixed A-F correlation functions.

The upshot of this is that such an $A$ must be of the form

$$A = \sum \frac{1}{n!} \int_C \ldots \int_C a_n(\theta_1, \ldots \theta_n) : Z(\theta_1)\ldots Z(\theta_n) :$$

(28)

where the $a_n$ have a simple relation to the various formfactors of $A$ (including bound states) whose different in-out distributions of momenta correspond to the different contributions to the integral from the upper/lower rim of the strip bounded by $C$, which are related by crossing. The transcription of the $a_n$ coefficient functions into physical formfactors (27) complicates the notation, since in the presence of bound states there is a larger number of Fock space particle creation operators than PFG wedge generators $F$. It is comforting to know that the wedge generators, despite their lack of vacuum polarization clouds, nevertheless contain the full (bound state) particle content. The wedge algebra structure for factorizing models is like a relativistic QM, but as soon as one sharpens the localization beyond wedge localization, the field theoretic vacuum structure will destroy this simple picture and replace it with the appearance of the characteristic virtual particle structure which separates local quantum physics from quantum mechanics.

In order to see by what mechanism the quantum mechanical picture is lost in the next step of localization, let us consider the construction of the double
cone algebras as a relative commutants of shifted wedge (shifted by $a$ inside the standard wedge)

$$\mathcal{A}(C_a) : = \mathcal{A}(W_a)' \cap \mathcal{A}(W)$$

For $A \in \mathcal{A}(C_a) \subset \mathcal{A}(W)$ and $F_a(\hat{f}_i) \in \mathcal{A}(W_a) \subset \mathcal{A}(W)$ the KMS condition for the W-localization reads as before, except that whenever a $F_a(\hat{f}_i)$ is crossed to the left side of $A$, we may commute it back to the right side since $[\mathcal{A}(C_a), F_a(\hat{f}_i)] = 0$. The resulting relations are e.g.

$$\langle AF_a(\hat{f}_i) : F_a(\hat{f}_n)\ldots F_a(\hat{f}_2) : \rangle = \langle A : F_a(\hat{f}_n)\ldots F_a(\hat{f}_2)F_a(\hat{f}_2^{\pi i}) : \rangle$$

Note that the $F_a(\hat{f}_i)$ in the first line is outside the Wick-ordering. Since it does neither act on the bra nor the ket vacuum, it contains both frequency parts. The creation part can be combined with the other F$s under one common Wick-ordering whereas the annihilation part via contraction with one of the Wick-ordered F$s will give an expectation value of one $A$ with $(n-2)$ F$s. Using the density of the F$s and going to rapidity space we obtain (24) the so-called kinematical pole relation

$$\text{Res}_{\theta_{12}=\pi} \langle AZ^*(\theta_a)\ldots Z^*(\theta_2)Z^*(\theta_1) \rangle = 2tC_{12}\langle AZ^*(\theta_a)\ldots Z^*(\theta_3) \rangle (1 - S_{1n}...S_{13})$$

Here the product of two-particle S-matrices results from commuting the $Z(\theta_1)$ to the right so that it stands to the left of $Z^*(\theta_2)$, whereas the charge conjugation matrix $C$ only appears if we relax our assumption of self-conjugacy.

I believe that the general issue of the shape of polarization clouds in terms of their asymptotic (say incoming) particle content is intimately related to the structure of the yet unknown modular automorphisms which exist for each spacetime region.

This relation appears for the first time in Smirnov’s axiomatic approach [29] as one of his recipes; more recently it was derived as a consequence of the LSZ formalism adapted to the factorizing model situation [30]. In the present approach it has an apparently very different origin: it is together with the Z-F algebra structure a consequence of the wedge localization of the generators $F(\hat{f})$ and the sharpened double cone locality [29] of $A$. The existence problem for the QFT associated with an admissable S-matrix (unitary, crossing symmetric, correct physical residua at one-particle poles) of a factorizing theory is the non-triviality of the relative commutant algebra i.e. $\mathcal{A}(C_a) \neq C \cdot 1$. Intuitively the operators in double cone algebras are expected to behave similar to pointlike fields applied to the vacuum; namely one expects the full interacting polarization cloud structure. For the case at hand this is in fact a consequence of the
above kinematical pole formula since this leads to a recursion which for non-trivial two-particle S-matrices is inconsistent with a finite number of terms in (28). Only if the bracket containing the S-products vanishes, the operator $A$ is a composite of a free field.

The determination of a relative commutant or an intersection of wedge algebras even in the context of factorizing models is not an easy matter. We expect that the use of the following “holographic” structure significantly simplifies this problem. We first perform a lightlike translation of the wedge into itself by letting it slide along the upper light ray by the amount given by the lightlike vector $a_+$. We obtain an inclusion of algebras and an associated relative commutant

$$\mathcal{A}(W_{a_+}) \subset \mathcal{A}(W)$$

$$\mathcal{A}(W_{a_+})' \cap \mathcal{A}(W)$$

The intuitive picture is that the relative commutant lives on the $a_+$ interval of the upper/lower light ray, since this is the only region inside $W$ which is spacelike to the interior of the respective shifted wedges. This relative commutant subalgebra is a light ray part of the above double cone algebra, and it has an easier mathematical structure. One only has to take a generic operator in the wedge algebra which formally can be written as a power series (28) in the generators and find those operators which commute with the shifted $F$s

$$[A, U(e_+)^* F(f) U(e_+)] = 0$$

Since the shifted $F$s are linear expressions in the $Z$s, the $n^{th}$ order polynomial contribution to the commutator comes from only two adjacent terms in $A$ namely from $a_{n+1}$ and $a_{n-1}$ which correspond to the annihilation/creation term in $F$. The size of the shift gives rise to a Paley-Wiener behavior in imaginary direction, whereas the relation between $a_{n+1}$ and $a_{n-1}$ is identical to (31), so we do not learn anything new beyond what was already observed with the KMS technique (30). However as will be explained in section 5, the net obtained from the algebra

$$\mathcal{A}_\pm := \cup a_\pm \mathcal{A}(C_{a_\pm})$$

is a chiral conformal net on the respective subspace $H_\pm = \mathcal{A}_\pm \Omega$. If our initial algebra were $d=1+1$ conformal, the total space would factorize $H = H_+ \otimes H_-$ = $(\mathcal{A}_+ \otimes \mathcal{A}_-) \Omega$, and we would recover the well-known fact that two-dimensional local theories factorize into the two light ray theories. If the theory is massive, we expect $H = \mathcal{A}_+ \Omega$ i.e. the Hilbert space obtained from one horizon already contains all state vectors. This would correspond to the difference in classical propagation of characteristic massless versus massive data in $d=1+1$. There it is known that although for the massless case one needs the characteristic data on the two light rays, the massive case requires only one light ray. In fact there exists a rigorous proof that this classical behavior carries over to free quantum fields: with the exception of $m=0$ massless theories, in all other cases (including light-front data for higher dimensional $m=0$ situations) the vacuum is cyclic
with respect to one light front \( H = \mathcal{A}_\pm \Omega \). The proof is representation theoretic and holds for all cases except the \( d=1+1 \) massless case. Hence in the case of interaction free algebras the holographic light front reduction which, has \( d-1 \) dimensions, always fulfills for \( d>2 \) the Reeh-Schlieder property, where for \( d=1+1 \) only massive theories obey holographic cyclicity. In order to recover the wedge algebra with its net structure from the holographic restriction, one needs the opposite light ray translation with \( U(a^-) \) i.e. \( \mathcal{A}(W) = \cup_{a^-<0} \text{Ad}U(a^-)\mathcal{A}_+ \). For the nontriviality of the net associated with \( \mathcal{A}(W) \) it is sufficient to show that the associated chiral conformal theory is nontrivial. In order to achieve this, one has to convert the bilinear forms (28) in the \( Z \)-basis which fulfil the recursion relation into genuine operators on the one-dimensional light ray. This is outside the scope of this paper.

Hence the modular approach leads to a dichotomy of real particle creation (absent in factorizing models) in the PFGs and in the aspect of wedge localization, versus the full QFT virtual particle structure of the vacuum\(^{13}\) for the more sharply localized operators. In some sense the wedge is the best compromise between the particle/field point of view. In this and only in this sense the particle-field dualism (as a generalization of the particle-wave dualism of QM) applies to QFT. Since it is left invariant by an appropriate L-boost, the algebra contains enough operators in order to resolve at least the vacuum and one-particle states (which cannot be resolved from the remaining states in any algebra with a smaller localization).

In the next section we will argue that these properties are not freaks of factorizing models, whereas in a later section we will reveal the less pedestrian aspects of light cone subalgebras and holography. As we have argued on the basis of the previous pedestrian approach, the holography aspect will be important in the modular construction of QFT’s, because it delegates certain properties of a rather complicated theory to those of (in general several) simpler theories.

It is worthwhile to highlight two aspects which already are visible from this pedestrian considerations. One is the notion of “quantum localization” in terms of algebraic intersections as compared to the more classical localization in terms of test function smearing of pointlike fields. As mentioned already, the wedge localization of the PFGs cannot be improved by choosing smaller supports of test functions inside the wedge; the only possibility is to intersect algebras. In that case the old generators become useless e.g. in the description of the double cone algebras; the latter has new generators. Related to this is that the short distance behavior loses its dominating (and somewhat threatening) role.

If one does not use field-coordinatizations, it is not even clear what one means by “the (good or bad) short distance behavior of a theory”. Short distance behavior of what object? There is no short distance problem of PFGs, since they have some “natural cutoff” (to the extend that the use of such words which are filled with preassigned old meaning are reasonable in the new context). Intersection of algebras does not give rise to short distance problems in

\(^{13}\) The deeper understanding of the virtual vacuum structure (or the particle content of say state vectors obtained by application of a double cone localized operator to the vacuum) is presumably hidden in the modular groups of double cone algebras.
the standard sense of this word. An explicit construction of pointlike field coordinates from algebraic nets is presently only available for chiral conformal theories [34]. It produces fields of arbitrary high operator dimension, and as a result of its group theoretical techniques it also does not suffer from short distance problems. The absence of short distance problems in the modular localization approach seems to be of an entirely different nature than statements about the absence of ultraviolet problems in string theory.

The results in this section should be viewed as an extension of the Wigner theory into the realm of interactions for a special class of models.

### 3 PFGs in Presence of Real Particle Creation

For models with real particle creation it is not immediately clear how to construct PFGs. In order to get some clue we first look at d=1+1 theories which do not have any transversal extension to wedges. Furthermore we assume that there is only one kind of particle which corresponds to the previous assumption concerning the absence of poles in the two-particle S-matrix for factorizing models. Modular theory always assures the existence of PFGs [12]. In fact for every modular localized state vector

\[ S\psi = \psi \quad (35) \]

\[ S = J \Delta^{\frac{1}{2}} \]

There exists a (generally unbounded) operator \( G \) associated with the von Neumann \( \mathcal{A} \) algebra in standard position such that

\[ G\Omega = \psi \quad (36) \]

\[ G^*\Omega = S\psi \quad (37) \]

For the case at hand \( \mathcal{A} = \mathcal{A}(W), \Omega = |0\rangle = \text{vacuum} \), there exists a dense subspace of explicitly constructible (see below) wedge-localized state vectors \( \mathcal{H}_R^{\text{Fock}} + i\mathcal{H}_R^{\text{Fock}} \) which possess affiliates \( G_s \). Since \( H_R(W) \subset \mathcal{H}_R^{\text{Fock}} \) there exists an algebra affiliated operator \( F(f) \) for each vector \( f \) in \( H_R(W) + iH_R(W) \) with

\[ F(f)\Omega = |f\rangle = 1 - \text{particle state} \quad (38) \]

\[ F(f)^*\Omega = S|f\rangle \]

Although the existence of PFGs outside of factorizing models poses no problems, the presence of particle creation prevents them to have amenable algebraic properties. The interpretation in the form of

\[ F(f) = \int F(x)f(x)d^2x \quad (39) \]

\[ F(x) = \int (Z(\theta)e^{-ipx} + \bar{Z}^*(\theta)e^{ipx})d\theta \quad (40) \]
where $F(x)$ is a tempered operator-valued distribution on a dense translation invariant domain which holds in the factorizing case is not compatible with particle creation because it leads to relative commutation relations of $F$ with the incoming/outgoing free field

$$
\begin{align*}
\left[Z^\#(\theta), a^\#_{in}(\theta')\right] &= 0, \ \theta < \theta' \\
\left[Z^\#(\theta), a^\#_{out}(\theta')\right] &= 0, \ \theta > \theta'
\end{align*}
$$

(41)

and similarly for the antiparticle operators $\bar{Z}^\#$. Therefore we will first try to see how far we can get with localized states.

Again we specialize to the self-conjugate case $\bar{Z} = Z$ and the absence of bound states. From the previous discussion we take the idea that we should look for a relation between the ordering of rapidities and the action of the scattering operator. We fix the state vector $\psi(\theta_n, ..., \theta_1)$ for the natural $\theta$-order to be an incoming $n$-particle state as we did for the previous particle conserving situation. The totally mirrored order should then be a vector obtained by applying the full $S$-matrix to the incoming $n$-particle vector.

But what should we do for the remaining permutations? We should end up with a prescription which for factorizing systems agrees with the previous formalism. For two $fs$ there is no problem; the formula looks as before (11), except that the application of the $S$-operator to the two-particle in-vector has components to all $n$-particle multiparticle vectors for $n \geq 2$, i.e. the rapidities are labels which are not related to the incoming particle content of the state vector

$$
\psi_2(f_2, f_1) \sim \int_C (\chi_{21} a(\theta_2) a(\theta_1) + \chi_{21} S a(\theta_1) a(\theta_2)) |0\rangle f_2(\theta_2) f_1(\theta_1) d\theta_2 d\theta_1
$$

$$
\langle 0| a(\theta_n) ... a(\theta_3) S a^*(\theta_1) a^*(\theta_2) |0\rangle \neq 0, \ n \geq 4
$$

(42)

The check of the localization equation $J \Delta^\dagger \psi_2 = \psi_2$ with $J = J_0 S$ (again we omitted the subscript $\text{scat}$ from the $S$-matrix and we used the previous notation where the integration path $C$ includes creation as well as annihilation contributions there will be also a contraction term. The inner product between two $\psi_2$ comes out to be

$$
\begin{align*}
\int f_1(\theta'_1) \bar{f}_2(\theta'_2) \langle 0| a(\theta'_1) a(\theta'_2) a^*(\theta_2) a^*(\theta_1) |0\rangle f_2(\theta_2) f_1(\theta_1) + c.t. \\
+ \int \bar{f}_1(\theta'_1) f_2(\theta'_2) \langle 0| a(\theta'_2) a(\theta'_1) (S \chi_{21} + S^* \chi_{12}) a^*(\theta_2) a^*(\theta_1) |0\rangle f_2(\theta_2) f_1(\theta_1)
\end{align*}
$$

(43)

where c.t. denote the contraction term contributions coming from an annihilation part in $C$ and we have disregarded problems of overall normalizations and the integration is done over all $\theta$ and $\theta'$. Using the unitarity property of $S$ and
the boundary property $\tilde{f}(i\pi - \theta) = f(\theta)$, the last term can be written without the ordering $\chi$s as

$$
\tilde{f}_1(\theta_1') \tilde{f}_2(\theta_2') \langle 0 | a(\theta_2') a(\theta_1') S a^\ast(\theta_2) a^\ast(\theta_1) | 0 \rangle \ f_2(\theta_2) f_1(\theta_1) 
$$

and has the same form as for the previous factorizing case if one replaces (20).

In order to establish the KMS property we have to write this inner product as

$$
(\psi_2', \psi_2) = (\psi_1', \psi_3)
$$

where $\psi_1$ is a one-particle vector. So we have to figure out how permutations beyond the natural order and its mirror image are represented on tensor product factors of incoming state vectors. Some thinking reveals that subsequent applications of $S$-matrices on tensor factors of the $n$-particle tensor product vectors only makes sense for nonoverlapping situations. The action of the $S$-matrix on one tensor factor is associated with the mirror perturbation of that tensor factor $12...k \to k...21$ since intuitively speaking one only obtains the full $k$-particle scattering if the incoming velocities (or rapidities) are such that all particles meet kinematically which only happens if the order of incoming velocities is the mirrored natural order. Mathematically we should write each permutation as the nonoverlapping product of “mirror permutations” The smallest mirror permutations are transpositions of adjacent factors. An example for an overlapping product is the product of two such transpositions which have one element in common e.g. $123 \to 132 \to 312$; there is no meaning in terms of a subsequent tensor $S$-matrix action. However the composition $123 \to 213 \to 312$ has a meaningful $S$-matrix counterpart: namely $S \cdot S_{12}a^\ast(\theta_1) a^\ast(\theta_2) a^\ast(\theta_3) \Omega$ where $S_{12}$ leaves the third tensor factor unchanged i.e. is the Fock space vector $\langle S a^\ast(\theta_1) a^\ast(\theta_2) \Omega \rangle \otimes a^\ast(\theta_3) \Omega$ on which the subsequent action of $S$ (which corresponds to the mirror permutation of all 3 objects) is well defined. In general if one mirror permutation is completely inside a larger one the scattering correspondence which is consistent with the tensor product structure of Fock space. On the other hand for overlapping products of mirror permutations the association to scattering data becomes meaningless, where overlapping means that part of each mirror permutation is outside of the other. Fortunately, as it is easy to see, there is precisely one representation in terms of nonoverlapping mirror permutations. This leads to a unique representation of multi $f$ labeled state vectors in terms of scattering data. On the other hand if we were to write each mirror permutation as a product of (necessarily overlapping) transpositions, we loose the uniqueness and we then need the Yang-Baxter structure in order to maintain consistency; in this case we return to the modular setting of factorizing models in the previous section.

Let us elaborate this in a pedestrian fashion by writing explicit formulas for $n=3$. The state vector is a sum of $3! = 6$ terms
\[ \psi_3(f_3, f_2, f_1) \simeq \int \int \int_C \{ \chi_{321} a(\theta_3) a(\theta_2) a(\theta_1) + \chi_{312} S_{21} a^*(\theta_3) a^*(\theta_2) a^*(\theta_1) \\
+ \chi_{231} S_{32} a(\theta_3) a(\theta_2) a(\theta_1) + \chi_{123} S_{321} a(\theta_3) a(\theta_2) a(\theta_1) \\
+ \chi_{132} S_{321} S_{23} a^*(\theta_3) a(\theta_2) a(\theta_1) + \chi_{213} S_{321} S_{12} a(\theta_3) a(\theta_2) a(\theta_1) \} \mid 0 \rangle \bar{f}_3(\theta_3) \bar{f}_2(\theta_2) \bar{f}_1(\theta_1) \]

Here \( \chi \) denotes again the characteristic function of the respective \( \theta \)-orders and \( S \), acts on the respective tensor factor with the remaining particle being a spectator. As before one checks that this vector fulfills the modular localization equation \( \mathcal{S} J_0 \Delta ^{\frac{1}{2}} \psi_3 = \psi_3 \); the Tomita operator acting on \( \psi_3 \) just re-shuffles the six terms. As in the two-particle case, this action creates a vector with a complicated incoming particle content having components to all particle numbers. The last two terms correspond to nested mirror permutations and, as will be seen below, results in the appearance of “nondiagonal inclusive processes” terms in the \( (\psi'_3, \psi_3) \) inner product which generalize the diagonal inclusive processes \( \chi \) which result from the summation over final states in cross sections.

As an example we write down the integrand of one of those nondiagonal inclusive terms

\[ \langle 0 \mid a(\theta'_1) a(\theta'_2) a(\theta'_3) S \cdot S_{12} a^*(\theta_3) a^*(\theta_2) a^*(\theta_1) \mid 0 \rangle \quad (46) \]

In a graphical scattering representation particle 1 and 2 would scatter first and produce arbitrarily many (subject to the conservation laws for the total energy-momentum) particles which together with the third incoming particle (which hitherto was only a spectator) enter an additional scattering process of which only the 3-particle outgoing component is separated out by the matrix element in \( \chi \). The dot means summation over all admissible intermediate states and could be represented by e.g. a heavy line in the graphical representation in order to distinguish it from the one-particle lines. We will not write down the 6 contribution to the inner product coming from the creation part and the remaining ones involving the annihilation parts from the path \( C \).

Our main interest is the study of \( (\psi'_3, \psi_3) \)

\[ (\psi'_3, \psi_3) \quad (47) \]

\[ = \int_C \langle 0 \mid a(\theta'_1) \{ 1 + S_{12} \chi_{123} + S \cdot S_{12} \chi_{213} \} a(\theta_3) a(\theta_2) a(\theta_1) \mid 0 \rangle \bar{f}'_1(\theta'_1) \bar{f}_3(\theta_3) \bar{f}_2(\theta_2) \bar{f}_1(\theta_1) \]

\[ = \int \langle 0 \mid a(\theta'_1) \{ 1 + S \} a^*(\theta_2) a^*(\theta_1) \mid 0 \rangle \bar{f}'_1(\theta'_1) \bar{f}_3(\theta_3) \bar{f}_2(\theta_2) \bar{f}_1(\theta_1) \]

The last line is (apart from a \( a(\theta_2) - a^*(\theta_1) \) contraction term) the only nonvanishing contribution. Here the S-factor in front of the \( S_{12} \) has been transferred as \( S^* \) onto the left hand one-particle vector whereupon it acts as the identity.
Renaming $f_3(\theta_3) \rightarrow f'(\theta'_2)$ we obtain the result (43). We now apply the KMS property for inner products of modular subspaces

$$
(\psi', \psi) = (\psi, \Delta \psi')
$$

for the case at hand $\psi' = \psi'_1$, $\psi = \psi_3$ the particle interpretation of this KMS relation for modular vectors is precisely the crossing symmetry relation. For the more general case antiparticles $\neq$ particles one has to work in the dense complex subspace $\mathcal{H}_{R}(W) + i\mathcal{H}_{R}(W)$ (which is a complete Hilbert space in its own right in the thermal topology [22]. The conversion of the KMS property for the inner product of the modular localized state vectors with n-1 f labels with a one-particles vector containing one $f$ contains the crossing information for scattering of $n_{in} + n_{out} = n$ particles.

The crucial question is whether these inner products can also be used in order to define correlation functions of PFG n-point functions

$$
(0 | F(f_1)...F(f_n) | 0)
$$

for the lowest nontrivial case of 4-point function we already checked one such condition [13]. I have carried out other consistency checks and do not think that the prerequisites of [22] leading back to particle conservation can be derived from these correlation functions. The question if and how these would be correlation functions are related to the perturbative on-shell S-matrix representations mentioned in the introduction is particularly interesting I hope to return to the issue of the form of PFG-correlation functions in a more complete and systematic way in a future paper.

For non-factorizing theories the interest in the modular localization approach is (besides the improvement in the understanding the structure of interacting QFT) the possible existence of an on-shell perturbation theory of local nets avoiding the use of the non-intrinsic field coordinatizations. This is a revival of the perturbative version of the old dream to construct S-matrices just using crossing symmetry in addition to unitarity and no pointlike fields. The old S-matrix bootstrap program admittedly did not get far, but now we perhaps can formulate a similar but structurally richer problem as a perturbative approach to correlation functions of the on-shell PFGs. Modular theory has given us a lot of insight and nobody nowadays would try to cleanse the Einstein causality and locality concepts from the stage as it was done in the 60's. To the contrary, the local off-shell observable algebras would be in the center of interest and the avoidance of quantization would have entirely pragmatic reasons. In particular the sharpening of localization beyond wedges is done by algebraic intersections of wedge algebras rather than by cut-off or test function manipulations on field coordinates.

The successful d=1+1 bootstrap-formfactor program of the previous section for factorizing models yields S-matrices and formfactors which for models with a continuous coupling are analytic around $g=0$. A good illustration is the Sine-Gordon theory [30]. The more local off-shell quantities however (i.e. pointlike
field operators or operators from algebras belonging to bounded regions) are radically different since they involve virtual particle polarization clouds which formally may be represented by infinite series in the on shell $F$s similar to the factorizing $d=1+1$ case of the previous section. The analytic status of these quantities (i.e. localized operators and their correlation functions) is presently not known; it may well turn out that they are only Borel summable or (in the general non-factorizable case) worse. The on-shell/off-shell dichotomy of the modular approach for the first time allows to determine more precisely if the cause of the possible breakdown of analyticity at $g=0$ are the polarization clouds.

A solution of these problems, even if limited to some new kind of perturbation theory (perturbation theory of wedge algebras and their intersections) should also shed some light on the question of how to handle theories involving higher spin particles, which in the standard off-shell causal perturbation theory lead to short distance non-renormalizability. A very good illustration of what I mean is the causal perturbation of massive spin=1 vectormesons. Here the coupling of covariant fields obtained by covariantizing the Wigner particle representation theory in the sense of the previous section will not be renormalizable in the sense of short distance power counting. In the standard perturbative approach the indefinite metric ghosts are used to lower the operator dimension of the interaction densities (free field polynomials) $W(x)$, which as a result of the free vectormeson dimension $\dim A_\mu = 2$, are at least 5, down to the value 4 permitted by the renormalization requirements in a $d=1+3$ causal perturbative approach \[2\]. Since the ghosts are removed at the end, the situation is akin to a catalyzer in chemistry: they do not appear in the original question and are absent in the final result (without leaving any intrinsic trace behind). In theoretical physics the presence of such catalyzers should be understood as indicating that the theory wants to be analyzed on a deeper level of local quantum physics i.e. further away from quantization and quasiclassics. Indeed in the present on-shell modular approach the short distance operator reason for introducing such ghosts would not be there and the remaining question is again whether the modular program allows for a perturbative analytically managable formulation.

4 The AQFT Framework

After our pedestrian presentation of the wedge algebra approach, it is time to be more systematic and precise. For noninteracting free system the conversion of the rather simple spatial nets of real subspaces of the Wigner space of momentum space $(m,s)$ wave functions into a interaction-free net in Fock space produces with the following three properties which continue to hold in the presence of interactions. They have been explained in many articles \[1\] and in a textbook \[1\], and my main task here is to adapt them to the problems of this paper.
1. A net of local (C*- or von Neumann) operator algebras indexed by classical spacetime regions $O$

$$\mathcal{O} \to \mathcal{A}(\mathcal{O})$$

Without loss of generality the regions $\mathcal{O}$ may be restricted to the Poincaré covariant family of general double cones and the range of this map may be described in terms of concrete operator algebras in Hilbert space for which the vacuum representation $\pi_0$ may be taken i.e. $\mathcal{A}(\mathcal{O}) \equiv \pi_0(\mathcal{A}(\mathcal{O}))$. The geometrical and physical coherence properties as isotony: $\mathcal{A}(\mathcal{O}_1) \subset \mathcal{A}(\mathcal{O}_2)$ for $\mathcal{O}_1 \subset \mathcal{O}_2$ and Einstein causality: $\mathcal{A}(\mathcal{O}') \subset \mathcal{A}(\mathcal{O})'$ are then evident coherence requirements. Here we use the standard notation of AQFT: the dash superscript on the region denotes the causal disjoint and on the von Neumann algebra it stands for the commutant within $B(H)$ where $H$ is the ambient Hilbert space (here the representation space of the vacuum representation). Einstein causality can be interpreted as an a priori knowledge about some with $\mathcal{A}(\mathcal{O})$ commensurable observables in the sense of von Neumann. This causality property suggests the question if complete knowledge about commensurability $\mathcal{A}(\mathcal{O}') = \mathcal{A}(\mathcal{O})'$ is possible. It turns out that this is indeed the generic behavior of vacuum nets called Haag duality. The cases of violation of this duality are of particular interest since they can be related to a very fundamental intrinsic characterization of spontaneous symmetry breaking, thus vastly generalizing the Nambu-Goldstone mechanism which was abstracted from quantization [1].

2. Poincaré covariance and spectral properties.

$$g \in \mathcal{P} \to \alpha_g$$ automorphism

$$\alpha_g(\mathcal{A}(\mathcal{O})) = \mathcal{A}(\mathcal{O})$$

is unitarily implements in the vacuum representation

$$U(g)\mathcal{A}U^*(g) = \alpha_g(A)$$

$$A \in \mathcal{A}(\mathcal{O})$$

The unitaries for the translations have energy-momentum generators which fulfil the relativistic spectrum (positive energy) condition, symbolically $\text{spec}U(a) \in \mathcal{V}^+$ (the closed forward light cone)

3. The phase space structure of local quantum physics or the “nuclearity property”.

Remark 2 The precise formulation of the third property is somewhat involved and will be presented after the following remarks on the first two structural properties. Since in the formulation of the net one may work without loss of generality with von Neumann algebras [1], the first question is what type in the Murray-von Neumann-Connes-Haagerup classification occurs. There is a very
precise answer for wedges (which may be considered as double cones at infinity). As a result of the existence of a one-sided translation into a wedge as well as of the split property below, the wedge algebras $\mathcal{A}(W)$ turn out to be a hyperfinite factor of type $\text{III}_1$. This implies in particular that the algebra has properties which take it far away from the structure of QM (factors of type $\text{I}_{\infty}$). Such algebras do not have pure states or minimal projectors, rather all faithful states on such algebras are thermal i.e. obey the KMS condition. This makes them similar to states appearing in CST with bifurcated horizons as in Hawking-Unruh situations however with modular flows instead of Killing flows (but more “quantum”, i.e. without the classical geometric Killing vector aspects of horizons). The modular flow near the boundary of e.g. double cone regions become asymptotically geometric and Killing-like.

The nuclearity requirement results from the idea that there should be a local quantum physical counterpart of the phase space properties of QM in a box. The famous finite number of degrees of freedom law per unit cell of QM phase space results from limiting the discrete box spectrum by a cut-off in energy. As first suggested by Haag and Swieca [1], the corresponding LQP counterpart, based on the causally closed double cone analogue of the quantization box in Schrödinger QM, points into the direction of a “weakly” infinite number; according to their estimates this set of state vectors was compact in Hilbert space. Subsequent refinements of techniques revealed that this set is slightly smaller namely “nuclear” [1], and exact calculations with interaction-free theories demonstrated that the phase space situation also cannot be better than nuclear.

The best way to understand this issue is to follow the motivating footsteps of Haag and Swieca. They, as many other physicists at that time (and as some contemporary philosophers [38]), were attracted by the intriguing consequences of the so-called Reeh-Schlieder property of QFT

$$\mathcal{P}(\mathcal{O})\Omega = H, \text{ cyclicity of } \Omega$$
$$A \in \mathcal{P}(\mathcal{O}), A\Omega = 0 \implies A = 0 \text{ i.e. } \Omega \text{ separating}$$

(50)

which either holds for the polynomial algebras of fields (which are affiliated to the von Neumann algebras which they generate) or for operator algebras $\mathcal{A}(\mathcal{O})$. The first property, namely the denseness of states created from the vacuum by operators from arbitrarily small localization regions (e.g. a state describing “a particle behind the moon” and a charge compensating antiparticle in some other far away region can be approximated inside a laboratory of arbitrary small size and duration) is totally unexpected from the global viewpoint of general QT. In the algebraic formulation this can be shown to be dual to the second one (in the sense of passing to the commutant), in which case the cyclicity

\[\text{This weird aspect should not be held against QFT but rather be taken as indicating that localization by a piece of hardware in a laboratory is also limited by an arbitrary large but finite energy, i.e. is a “phase space localization” (see subsequent discussion). In QM one obtains genuine localized subspaces without energy limitations.}\]
passes to the separating property of $\Omega$ with respect to $A(O')$. Referring to
its use, the separating property is often called the field-state relation. The
mathematical terminology is to say that the pair ($A(O),\Omega$) is “standard”.
The large enough commutant required by the latter property is guarantied by
causality (the existence of a nontrivial causal disjoint $O'$) and thus shows that
causality is again responsible for this unexpected denseness property.

Of course the claim that somebody causally separated from us may provide
us nevertheless with a dense set of states is somewhat perplexing especially if
one compares it with the tensor factorization properties of good old Schrödinger
QM with respect to an inside/outside separation via a quantization box.

If the naive interpretation of cyclicity/separability in the Reeh-Schlieder the-
orem leaves us with a feeling of science fiction (and for this reason as already
mentioned justifiably also has attracted attention in philosophical quarters), the
challenge for a theoretical physicist is to find an argument why, for all practical
purposes, the situation nevertheless remains similar to QM. This amounts to
the fruitful question which vectors among the dense set of state vectors can
be really produced with a controllable expenditure (of energy); a problem from
which Haag and Swieca started their investigation. In QM this question was not
that interesting, since the localization at a given time via support properties of
wave functions leads to a tensor product factorization of inside/outside so that
the inside state vectors are evidently never dense in the whole space and the
“particle behind the moon paradox” does not occur.

Later we will see that most of the very important physical and geometrical
informations are encoded into features of dense domains, in fact the aforemen-
tioned modular theory is explaining this deep relation between operator domains
of the Tomita $S$ and spacetime geometry. As mentioned before the individuaility
of the various $S$-operators is only the difference in domains, since all of them
act as $SA\Omega = A^*\Omega, A \in A(O)$

For the case at hand the reconciliation of the paradoxical aspect [35] of
the Reeh-Schlieder theorem with common sense has led to the discovery of
the physical relevance of localization with respect to phase space in LQP, i.e.
the understanding of the size of degrees of freedom in the set: (notation $H = \int EdP_E$)

\[ P_E A(O)\Omega \text{ is compact} \quad (51) \]
\[ P_E A(O)\Omega \text{ or } e^{-\beta H} A(O)\Omega \text{ is nuclear} \quad (52) \]

The first property was introduced way back by Haag and Swieca [1] whereas
the second more refined statement (and similar nuclearity statements involving
modular operators of local regions instead of the global hamiltonian) which is
saturated by QFT (i.e. cannot be improved) and easier to be used, is a later
result of Buchholz and Wichmann [40]. It should be emphasized that the LQP
degrees of freedom counting of Haag-Swieca, which gives an infinite but still
compact (and even nuclear) set of phase-space localized states, is different from
the QM finiteness of degrees of freedom per phase used in some contemporary
entropy calculations.
The map \( \mathcal{A}(\mathcal{O}) \to e^{-\beta H} \mathcal{A}(\mathcal{O})\Omega \) is only nuclear if the mass spectrum of LQP is not too accumulative in finite mass intervals; in particular, infinite towers of equal mass particles are excluded (which then would cause the strange appearance of a maximal “Hagedorn” temperature). The nuclearity assures that a QFT, which was given in terms of its vacuum representation, also exists in a thermal state. An associated nuclearity index turns out to be the counterpart of the quantum mechanical Gibbs partition function \([41]\) and behaves in an entirely analogous way.

The peculiarities of the above degrees-of freedom-counting are very much related to one of the oldest “exotic” and at the same time characteristic aspects of QFT, namely vacuum polarization. As first observed by Heisenberg, the partial charge:

\[
Q_V = \int_V j_0(x)d^3x = \infty
\]

(53)
diverges as a result of uncontrolled vacuum particle/antiparticle fluctuations near the boundary. For the free field current it is easy to see that a better definition involving test functions, which smoothens the behavior near the boundary and takes into account the fact that the current is a 4-dim. distribution which has no restriction to equal times, leads to a finite expression.

\[
Q_R = \int j_0(x)f(x_0)g\left(\frac{x}{R}\right)d^3x
\]

(54)

where \( f \) and \( g \) are test functions of compact support with \( \int f(x_0)dx_0 = 1 \) and \( g(\vec{x}) = 1 \) for \( |\vec{x}| < 1 \) and \( g(\vec{x}) = 0 \) \( |\vec{x}| > 1 + \delta \). The vectors \( Q_R\Omega \) only converge weakly for \( R \to \infty \) on a dense domain. Their norms however diverge as \([33]\)

\[
(Q_R\Omega, Q_R\Omega) \leq const \cdot R^{s-1} \sim area
\]

(55)
The surface-layer character of this vacuum polarization is reflected in this area behavior together with the original divergence \([33]\) for fixed \( R \) and \( \delta \to 0 \).

The algebraic counterpart is the so-called “split property”, namely the statement \([33]\) that if one leaves between say the double cone (the inside of a “relativistic box”) observable algebra \( \mathcal{A}(\mathcal{O}) \) and its causal disjoint (its relativistic outside) \( \mathcal{A}(\mathcal{O}') \) a “collar” (geometrical picture of the relative commutant) \( \mathcal{O}_1' \cap \mathcal{O} \), i.e.

\[
\mathcal{A}(\mathcal{O}) \subset \mathcal{A}(\mathcal{O}_1), \quad \mathcal{O} \ll \mathcal{O}_1, \text{ properly}
\]

then it is possible to construct in a canonical way a type I tensor factor \( \mathcal{N} \) which extends in a “fuzzy” manner into the collar \( \mathcal{A}(\mathcal{O}') \cap \mathcal{A}(\mathcal{O}_1) \) i.e. \( \mathcal{A}(\mathcal{O}) \subset \mathcal{N} \subset \mathcal{A}(\mathcal{O}_1) \). With respect to \( \mathcal{N} \) the Hilbert space factorizes i.e. as in QM; there are states with no fluctuations (or no entanglement) for the “smoothened” operators in \( \mathcal{N} \). Whereas the original vacuum will be entangled from the box point of view, there also exists a disentangled product vacuum on \( \mathcal{N} \). The algebraic analogue
of a smoothening of the boundary by a test function is the construction of a factorization of the vacuum with respect to a suitably constructed type I factor algebra which uses the above collar extension of \( \mathcal{A}(O) \). It turns out that there is a canonical, i.e. mathematically distinguished factorization, which lends itself to define a natural “localizing map” \( \Phi \) and which has given valuable insight into an intrinsic LQP version of Noether’s theorem \([1]\), i.e. one which does not rely on quantizing classical Noether currents. It is this “split inclusion” which allows to bring back the familiar structure of pure states, tensor product factorization, entanglement and all the other properties at the heart of standard quantum theory and the measurement process. However despite all the efforts to return to structures known from QM, the original vacuum retains its thermal (entanglement) properties with respect to all localized algebras, even with respect to the “fuzzy”-localized \( \mathcal{N} \).

Let us collect in the following some useful mathematical definitions and formulas for “standard split inclusions” \([42]\)

**Definition 3** An inclusion \( \Lambda = (\mathcal{A}, \mathcal{B}, \Omega) \) of factors is called standard split if the collar \( \mathcal{A}' \cap \mathcal{B} \) as well as \( \mathcal{A}, \mathcal{B} \) together with \( \Omega \) are standard in the previous sense, and if in addition it is possible to place a type I\(_\infty\) factor \( \mathcal{N} \) between \( \mathcal{A} \) and \( \mathcal{B} \).

In this situation there exists a canonical isomorphism of \( \mathcal{A} \vee \mathcal{B}' \) to the tensor product \( \mathcal{A} \bar{\otimes} \mathcal{B}' \) which is implemented by a unitary \( U(\Lambda) : H_\Lambda \rightarrow H_1 \bar{\otimes} H_2 \) (the “localizing map”) with

\[
U(\Lambda)(AB')U^*(\Lambda) = A \bar{\otimes} B'
\]

\( A \in \mathcal{A}, \ B' \in \mathcal{B}' \)

This map permits to define a canonical intermediate type I factor \( \mathcal{N}_\Lambda \) (which may differ from the \( \mathcal{N} \) in the definition)

\[
\mathcal{N}_\Lambda := U^*(\Lambda)B(H_1) \otimes 1)U(\Lambda) \subset \mathcal{B} \subset B(H_\Lambda)
\]

It is possible to give an explicit formula for this canonical intermediate algebra in terms of the modular conjugation \( J = U^*(\Lambda)J_\Lambda \otimes J_\mathcal{B}U(\Lambda) \) of the collar algebra \((\mathcal{A}' \cap \mathcal{B}, \Omega)\) \([42]\)

\[
\mathcal{N}_\Lambda = \mathcal{A} \vee JAJ = \mathcal{B} \wedge JBJ
\]

The tensor product representation gives the following equivalent tensor product representation formulae for the various algebras

\[
\mathcal{A} \sim \mathcal{A} \otimes 1
\]
\[
\mathcal{B}' \sim 1 \otimes \mathcal{B}'
\]
\[
\mathcal{N}_\Lambda \sim B(H_\Lambda) \otimes 1
\]
As explained in [42], the uniqueness of \( U(\Lambda) \) and \( N_\Lambda \) is achieved with the help of the “natural cones” \( P_{\Omega}(A \vee B') \) and \( P_{\Omega \otimes \Omega}(A \otimes B') \). These are cones in Hilbert space whose position in \( H_\Lambda \) together with their facial subcone structures pre-empt the full algebra structure on a spatial level. The corresponding marvelous theorem of Connes [47] goes far beyond the previously mentioned state vector/field relation.

Returning to our physical problem, we have succeeded to find the right analogue of the QM box. Contrary to the causally closed local type III algebras with their sharp light cone boundaries (“quantum horizons”), the “fuzzy box” type I factor \( N_\Lambda \) permits all the structures we know from QM: pure states, inside/outside tensor factorization, (dis)entanglement etc. with one exception: the vacuum is highly entangled in the tensor product description; the modular group of the state \( \omega \mid_{A \bar{\otimes} B'} \) represented in the tensor product natural cone \( P_{\Omega \otimes \Omega}(A \bar{\otimes} B') \) is not the tensor product of the modular groups of \( A \) and \( B' \), whereas the modular conjugation \( J \) acts on the tensor product cone as \( J \circ \omega \mid_{A \bar{\otimes} B'} \) (since the restriction \( \omega \mid_{A \bar{\otimes} B'} \) is faithful). Note also that the restriction of the product state \( \omega \otimes \omega \) to \( B \) or \( B' \) is not faithful resp. cyclic on the corresponding vectors and therefore the application of those algebras to the representative vectors \( \eta \omega \otimes \omega \) yields nontrivial projectors (e.g. \( P_{\Lambda} = U^*(\Lambda)B(H_1) \otimes 1U(\Lambda) \)).

Since the fuzzy box algebra \( N_\Lambda \) is of quantum mechanical type I, we are allowed to use the usual trace formalism based on the density matrix description, i.e. the vacuum state can be written as a density matrix \( \rho_{\Omega} \) on \( N_\Lambda \) which leads to a well-defined von Neumann entropy

\[
(\Omega, A\Omega) = \operatorname{tr} \rho_{\Lambda} A, \quad A \in \mathcal{A}
\]
\[
S(\rho_{\Lambda}) = -\operatorname{tr} \rho_{\Lambda} \log \rho_{\Lambda}
\]

but this is not sufficient to determine \( \rho_{\Lambda} \) which is needed for the von Neumann entropy of the fuzzy box \( S(\rho_{\Lambda}) \). If we would be able to compute the unitary representer \( \Delta_{\omega_1,\omega_2} \) of the modular group of the pair \( (N_\Lambda, \Omega) \) then we know also \( \rho_{\Lambda} \) since the modular operator of a type I factor is known to be related to an unnormalized density matrix \( \tilde{\rho}_{\Lambda} \) with \( \rho_{\Lambda} = \frac{1}{\operatorname{tr} \tilde{\rho}_{\Lambda}} \tilde{\rho}_{\Lambda} \) through the tensor product formula on \( H_1 \bar{\otimes} H_2 \)

\[
\Delta = \tilde{\rho}_{\Lambda} \bar{\otimes} \tilde{\rho}_{\Lambda}^{-1}
\]

Actually there are several intuitively equivalent definitions of localization entropy [48]. Among those the most convenient one seems to be the relative entropy of the vacuum \( \omega \) with respect to the split vacuum \( \omega \times \omega \). The relative entropy of a von Neumann algebra \( \mathcal{M} \) of one faithful state \( \omega_1 \) with respect to \( \omega_2 \) uses the relative modular operator [1] \( \Delta_{\omega_1,\omega_2} \)

\[
S(\omega_1|\omega_2)_M = -\langle \log \Delta_{\omega_1,\omega_2} \rangle
\]
Kosaki was able to convert this (in the most general setting) into a variational formula

\[ S(\omega_1|\omega_2)_M = \sup \int_0^1 \left[ \frac{\omega(1)}{1+t} - \omega_1(y^*(t)y(t)) - \frac{1}{t} \omega_2(x^*(t)x(t)) \frac{dt}{t} \right] \]

(65)

where in our case \( \omega_1 = \omega \times \omega \), \( \omega_2 = \omega \), \( M = A \vee B' \).

Despite the very clear conceptual setting of this split entropy, it is difficult to obtain good estimates for this entropy, not to mention exact calculations. As for the above partial charges (55) one expects a surface behavior, the quantum version of the Bekenstein-Hawking area law. An existing estimate shows that its increase for e.g. double cones is weaker than the spatial volume [48]. The most accessible situation for entropy calculations seems to be conformal QFT.

It seems that for double cones in conformal theories one can use the geometric aspects of the situation and do an explicit calculation. This still needs to be carried out, but an outline of the strategy can be found in [7].

Ideas about localization entropy are quite inaccessible in perturbation theory because they require an intrinsic description in terms of a net of algebras, whereas for perturbation theory no description without the use of field-coordinatizations is known. This has led to speculative remarks in the literature claiming the necessity of new degrees of freedom for the understanding of the area law.

5 Modular Inclusions and Intersections, Holography

One of the oldest alternative proposals for canonical (equal time) quantizations is the so called light ray or light front (or \( p \to \infty \) frame) quantization. The trouble with it is that it apparently inherits some of the short distance diseases from the canonical quantization. The latter is known to only make sense for superrenormalizable interactions but not for strictly renormalizable ones, which lead to infinite multiplicative renormalizations. Let us ignore this for a moment and look at some additional problems of light cone quantization which canonical equal time quantization does not have. This is the apparent loss of the connection with local QFT; in fact in none of the papers on light cone quantization it is spelled out how to return to a local QFT. The problem of light front restricted free fields was rigorously studied in [43], but in the interacting case the reconstruction of the local theory from that on the light cone (which may be called the holographic reconstruction) is a serious problem indeed.

Our modular inclusion techniques in section 2 suggested that for massive (and massless for \( d \neq 1 + 1 \)) theories the wedge algebra and the chiral light front algebra are identical

\[ A(W) = A(\mathbb{R}^+) \]

(66)
Since this is a consistent property which is fulfilled by all known quantum field theoretic models, we will be focus our interests on theories which obey this “characteristic shadow” property and leave open the question whether this property is a consequence of the standard physical requirements of AQFT. We already mentioned in the same section that the chiral algebra really should be thought of as the “transversally unresolved light front algebra”. But since the use of a light front notation like $A(R^d_{>1})$ could suggest the wrong idea that one deals with a full light front net, we prefer the light ray notation since it does not count any localizationwise unresolved dimensions. If we just refer to the global algebras and not to their local (sub)net structure, then all three objects are equal and there could be no confusion.

The rigorous construction of a chiral net for $A(R^d_{>1})$ indicated in the third section will now be presented in more detail within its natural setting of modular inclusions.\[^{15}\]

One first defines an abstract modular inclusion in the setting of von Neumann algebras. There are several types of inclusions which have received mathematical attention.\[^{15}\] An inclusion of two factors $N \subset M$ is called (+ halfsided) modular if the modular group $\Delta^it_M$ for $t < 0$ transforms $N$ into itself (compression of $N$)

$$Ad\Delta^it_M N \subset N$$

We assume that $\cup t Ad\Delta^it_M N$ is dense in $M$ (or that $\cap t \Delta^it_M N = \mathbb{C} \cdot 1$). This means in particular that the two modular groups $\Delta^it_M$ and $\Delta^it_N$ generate a two parametric group of (translations, dilations) in which the translations have positive energy.\[^{15}\] Let us now look at the relative commutant as done e.g. in the appendix of 43.

Let $(N \subset M, \Omega)$ be modular with nontrivial relative commutant. Then look at the subspace generated by relative commutant $H_{\text{red}} \equiv (N' \cap M)\Omega \subset H$. The modular groups to $N'$ and $M$ leave invariant this subspace: $\Delta^it_M, t < 0$ maps $N'$ into itself by the inclusion being modular. Look at the orthogonal complement of $H_{\text{red}}$ in $H$. This orthogonal complement is mapped into itself by $\Delta^it_M$ for positive $t$. Let $\psi$ be in that subspace, then

$$\langle \psi, \Delta^it_M (N' \cap M)\Omega \rangle = 0 \text{ for } t > 0.$$  \hspace{1cm} (68)

Analyticity in $t$ then gives the vanishing for all $t$.

Due to Takesaki’s theorem,\[^{3}\] we can restrict $M$ to $H_{\text{red}}$ using a conditional expectation to this subspace defined in terms of the projector $P$ onto $H_{\text{red}}$. Then

$$E(N' \cap M) \subset \underline{M\{N' \cap M\}H} = E(M)$$ \hspace{1cm} (69)

$$E(\cdot) = P \cdot P$$ \hspace{1cm} (70)

is a modular inclusion on the subspace $H_{\text{red}}$. $N'$ also restricts to that subspace and this restriction is obviously in the relative commutant of $E(N' \cap$\[^{15}\]In addition to the split inclusion used in the previous section, there are the famous V. Jones inclusions whose characteristic property is the existence of conditional expectations. Their domain in particle physics is in the area of charge fusion and internal symmetry.
Moreover using arguments as above it is easy to see that the restriction is cyclic with respect to $\Omega$ on this subspace. Therefore we arrive at a reduced modular “standard inclusion”

$$E(N) \subset E(M, \Omega)$$

(71)

Standard modular inclusions are known to be isomorphic to chiral conformal field theories \[36\].

This theorem and its extension to modular intersections leads to a wealth of physical applications in QFT, in particular in connection with “hidden symmetries” symmetries which are of purely modular origin and have no interpretation in terms of quantized Noether currents \[24\] [44]. The modular techniques unravel new structures which are not visible in terms of field coordinatizations. Holography and problems of degrees of freedom counting (phase space in LQP) as well as the issue of localization entropy are other examples.

Let us briefly return to applications for $d=1+1$ massive theories. It is clear that in this case we should use the two modular inclusions which are obtained by sliding the (right hand) wedge into itself along the upper/lower light ray horizon. Hence we chose $M = A(W)$ and $N = A(W_{a_+})$ or $N = A(W_{a_-})$ where $W_{a\pm}$ denote the two upper/lower light like translated wedges $W_{a\pm} \subset W$. As explained in section 2 following (\[31\]) and mentioned above, we do not expect the appearance of a nontrivial subspace (i.e. we expect $P = 1$) in the action of the relative commutants onto the vacuum

$$A(I(0, a_{\pm})) = A(W_{a_{\pm}})' \cap A(W)$$

(72)

where the notation indicates that the localization of $A(I(0, a_{\pm}))$ is thought of as the piece of the upper/lower light ray interval between the origin and the endpoint $a_{\pm}$.

From the standardness of the inclusion one obtains according to the previous discussion an associated conformal net on the line, with the following formula for the chiral conformal algebra on the half line

$$A_{\pm}(R_+) \equiv \bigcup_{t \geq 0} Ad_{U(t)} \left( A(I(0, a_{\pm})) \right) \subseteq A(W),$$

(73)

We expect the equality sign to hold

$$A_{\pm}(R_+) = A(W)$$

but our argument was tied to the existence of PFGs since as a result of their mass-shell structure

$$F(\hat{f}) = \int Z(\theta)f(\theta)d\theta = F_{res}(\hat{f}_{res})$$

(75)

where the notation $res$ indicates the corresponding generators in light ray theory which are identical in rapidity space and only differ in their x-space appearance.
This is a significant strengthening of the cyclicity property $\overline{\mathcal{A}_\pm(R_>)\Omega} = \mathcal{A}(W)\Omega$ for the characteristic data on one light ray. The argument is word for word the same in higher spacetime dimensions, since the appearance of transversal components (which have no influence on the localization) in addition to $\theta$ do not modify the argument. One would think that the inference of PFG generators can even be disposed of and the equality should follow from the standard causal shadow property of QFT in the form

$$\mathcal{A}(W) = \mathcal{A}(R_\alpha^>) \tag{76}$$

where $R_\alpha^>$ is a spacelike positive halfline with inclination $\alpha$ with respect to the x-axis. The idea is that if this relation would remain continuous for $R_\alpha^>$ approaching the light ray ($\alpha = 45^\circ$) which then leads to the desired equality. We believe that the relation $\mathcal{A}(W) = \mathcal{A}(R_\alpha^>)$ for massive theories, which will be called “characteristic shadow property”, is a general consequence of the standard causal shadow property (the identity of $\mathcal{A}(O) = \mathcal{A}(O''\mathrm{c})$ where $O''\mathrm{c}$ is the causal completion of the convex spacelike region $O$) in any spacetime dimension.

Theories with the characteristic shadow property are the objects of the light ray folklore. The present conceptually more concise approach explains why the light ray quantization in the presence of interactions is basically nonlocal which significantly restricts its unqualified physical use. The reason is that although the halfline algebra is equal to the wedge algebra (since all rays of forward light cone propagation which pass through the upper/lower half light ray $R_>$ have passes or will pass through $W$), the locality on the light ray cannot be propagated into the wedge (the strips inside the wedge subtended from an interval $I$ on the light ray by the action of the opposite light ray translation are for massive theories not outside the propagation region of the complement of $I$). Only for the halfline itself one obtains a 2-dimensional shadow region namely the wedge region. If one uses both light cones then it is possible to reconstruct a causal $d=1+1$ net by intersections. This construction uses the two-dimensional translation group on the wedge and the ensuing double cone relative commutants. Note that in order to achieve this with parity reflected halflines of light rays, one needs the relative position of the two halfline light ray algebras relative to each other in the common space $H$. In fact one shifted right light ray chiral algebra together with its parity reflected image is equal to the union of two opposite spacelike separated wedge algebras. The reflected light ray algebra may also be replaced by the algebra on the left hand extension of the original light ray since both create the same left wedge algebra. However the natural net structure of that algebra is very nonlocal with respect to that of the parity reflected one. This prevents its use in the construction of the 2-dim. net from shifts and geometric intersections on one light ray. An algebra localized in an interval on one light ray corresponds to a completely spread out algebra on the other ray. The modular group of each light ray interval is geometric. This agrees with the qualitative behavior one expects for the modular group of the double cone in a massive theory near the causal horizon. Note that the relative nonlocality of the chiral conformal theories is also necessary in order
to be consistent with a massive situation. The chiral conformal field theory contains the standard light ray translation with a gapless spectrum. However, this spectrum is not the physical one since in that chiral theory there exists yet another nonlocally acting translation and it is the spectrum of the product of the two generators $P_+ P_-$ which gives the physical mass. Hence chiral conformal theories constitute a multipurpose tool in LQP. This is why they can serve as “holographic” pieces for the construction of massive $d=1+1$ theories. So with just one light ray and two translations, one acting locally and the other nonlocally, one ray one can already reconstruct the full $d=1+1$ net. Later we will see that this is enough to understand the localization entropy which turns out to have the surface behavior first observed in the context of classical localization behind black hole horizons by Bekenstein.

Because of the transversal extension, the holography in terms of one-dimensional chiral conformal theories is more complicated for higher dimension. There one needs a family of chiral conformal theories which is obtained from “modular intersections”. Rather than associating the chiral conformal theory with a light ray, it is more appropriate to associate it with the transverse space of the wedge which contains the light ray i.e. with the light front. A family of light front algebras is obtained by applying $L$-boosts to the standard wedge $W$ which tilts $W$ around one of its defining light rays, so that the transversal degeneracy of the modular inclusion is partially destroyed (in $d=1+2$ it would be completely destroyed). In this way one obtains a fan-like ordered family of wedges corresponding to a family of chiral conformal theories whose relative position within the original Hilbert space contains all the informations which are necessary in order to reconstruct the original (massive) theory. A detailed and rigorous account of this construction will be given in a future paper. Here we will only mention some analogies to the above light ray situation. The process of tilting by applying a family of boost transformations which leave the common light ray invariant is described by unitary transformations of one chiral conformal theory into another. Each single one, according to the higher dimensional characteristic shadow property, is equal to a wedge algebra. Knowing the position of a finite number of such chiral conformal theories with respect to each other (the number increases with increasing spacetime dimensions), determines the relative position of a finite number of wedge algebras ($\simeq$ chiral conformal QFTs) which according to the previous remarks is sufficient to reconstruct the original net (the blow-up property in [44]). As previously mentioned, in the $d=1+1$ case the second light ray can be thought of as obtained from the first one by a unitary parity reflection (assuming that the theory is parity invariant). All the finitely many chiral conformal field theories are unitarily equivalent (either by parity or by $L$-boosts); the important physical information is contained in their relative position within the same Hilbert space. The terminology “scanning by a finite family of chiral conformal theories” is perhaps more appropriate for this construction of higher dimensional theories [51][44].

It has been shown elsewhere [51] that the modular inclusion for two wedges gives rise to two reflected eight-parametric subgroups of the 10-parametric Poincaré group which contain a two parametric transversal Galilean subgroup of the type
found by formal light front quantization arguments [50]. All these considerations show the primordial role of the chiral conformal QFT as a building block for the higher dimensional QFTs.

There is another much more special kind of holography in which an isomorphism of a massive QFT in d+1 dimensions to a conformal d-dimensional theory is in the focus of interest. This isomorphism appears in Rehren’s solution [20] of Maldacena’s conjecture about the existence of a holographic relation of quantum matter in a (d+1)-dimensional Anti de Sitter spacetime with that in a d-dimensional conformal QFT. This type of holography has not been observed outside the anti de Sitter spacetime and since it is an isomorphism to a conformal theory, the degrees of freedom are not really reduced in the sense of ’t Hooft [45], as it was the case in the previous holography via light ray reduction. The Maldacena-Witten holography is apparently of importance within the development of string theory, in fact the protagonists believe that it contains information about a possible message about quantum gravity of string theory. Within the present AQFT setting its main interest is that it requires the field-coordinatization free point of view in its strongest form: whereas in most problems of QFT there exist appropriate field coordinatizations which often facilitate calculations, the M-W isomorphism defined in rigorous terms by Rehren is not pointlike and has no description in terms of fields outside its algebraic version. In contradistinction to the light ray holography which happens at the causality horizon (light front boundary) of modular localization (or its classical Killing counterpart in case of black holes) the AdS holography takes place at the boundary at infinity.

A very simple presentation in the spirit of Rehren’s approach which takes into account the covering of the relevant spaces can be found in [52].

6 Comparison with String Theory

As mentioned in the introduction, historically string theory originated from the attempt to understand and implement the issue of crossing symmetry of the S-matrix. Without the intervention of QFT it was difficult to combine unitarity and crossing symmetry into a manageable formalism. It came as somewhat of a surprise that by assuming an additional stronger form of crossing called ”duality” one actually could obtain the dual model formalism. Duality was an idea of entirely phenomenological origin which consisted in the hypothesis that crossing can already hold if one only restricts ones attention to (reggeized) one-particle states (“particle democracy”). There was no theoretical support from QFT, nevertheless the very appealing form of duality by Veneziano led eventually into string theory. But whereas the content of QFT can be separated from the perturbative formalism and cast into totally intrinsic form which is strongly related with its underlying principles, string theory leaves a lot to be desired on conceptual aspects and remained a collection of prescriptions. In particular string theorists have not been able to successfully address the issue of locality of operators and localization of states which are absolutely crucial propertie on
which any particle physics theory stands and falls and which are even indispensable for the physical interpretation of its formalism. The formal basis of string theory is a kind of momentum space “engineering” rather than a conceptual spacetime analysis. The latter remained within the realm of quasiclassical physics using geometrical pictures with some fluctuation caused fuzziness i.e. pictures which in the setting of quantum theory fall behind Heisenberg’s dictum that positions and momenta are not properties of the electron but are characteristics of the events involving interactions with a measurement apparatus which causes the factualization of potentialities. Related to this is the fact that the word scattering theory has an entirely different meaning in both areas. Whereas in QFT it is an asymptotic relation to free fields for whose derivation spacelike locality is absolutely essential, in string theory its use in the sense of the $0/\infty$ behavior of the analytically continued source space conformal field theory in the complex plane has nothing to do with any standard scattering concept of physical particles. Whereas all important ideas in QFT have been tested outside quasiclassical or perturbative settings at least in $d=1+1$ interacting theories, this is not the case in string theory. For example the Klein-Kaluza mechanism for the conversion of spacetime into inner symmetry which is a (semi) classical idea has never been tested in a full QFT. Since the physical origin of internal symmetries is closely related to particle/field statistics there is some subtle problem with the Klein-Kaluza mechanism in QFT away from the quasiclassical pictures of functional integrals.

Another problematic point is the intrinsic meaning of “stringyness” in form of an infinite tower of particles with an oscillator-like mass spectrum. As long as mass spectra do not accumulate (by increase of multiplicities) too densely, they are compatible with the phase space structure of QFT and lead to reasonable thermal behavior, i.e. the pathological situation of a finite Hagedorn-temperature can presumably also be avoided in string theory. But it is not known to me how one can distinguish an infinite collection of resonances i.e. poles in the second Riemann sheet (since presumably in string theory most of the particles in the tower are unstable through higher order interactions as it would be the case in Feynman theory). I do not know of any theorem in QFT which forbids such a resonance situation and therefore I do not understand the meaning of stringyness. Extended objects can also exist in QFT build on perfect local observables; in fact the superselection theory even demands in some cases the existence of noncompactly localized objects which intertwine between inequivalent representations of perfectly local observable algebras, examples are the carriers of braid group statistics in $d=1+2$ dimensions are necessarily extended along semiinfinite spacelike strings. So it is very questionable if there exists an intrinsic meaning of stringyness.

The relation of string theory with the wedge-localization approach to QFT presented in this paper goes only via the common historical root of the S-matrix.
theory of the 60ies and basically consists in the claim that both theories are ultraviolet finite. In fact the on-shell nature of wedge algebras as exemplified by the modular wedge-localization equation (35) provides a field theoretic link for the S-matrix bootstrap and transports the ultraviolet finiteness of the latter into QFT. Although this finiteness is shared with string theory, the cause of it is very different. Whereas in string theory this finiteness results from the extension of a string as an indecomposable state of matter, the modular approach to QFT is ultraviolet-finite in a much more radical and at the same time much more conservative way. The radical aspect is that by not using the inevitably light-cone singular field coordinatizations in the actual construction but rather a net of algebras, the objects to which the the bad short distance behaviour and the ultraviolet divergencies is attached have disappeared from the scene. They may be constructed at the end as local generators of the already constructed spacetime-indexed nets of algebras, but there they cannot do no harm anymore. The conservative aspect is that by taking this approach which requires a radically changed formalism, one remains in total harmony with the causality-, spectral-, and degrees of freedom principles which underlie QFT. The short distance behavior of the field approach is substituted by the nontriviality of intersections of algebras. This approach has already been tested in the bootstrap-formfactor constructions of d=1+1 factorizable models. In d=1+3 one expects that its perturbative version reproduces the renormalizable field theories and in addition reveals whether the frontiers of the standard approach (renormalizable/nonrenormalizable) which appear in a purely formal way (power counting in auxiliary objects) are really the the intrinsic formalism-independent frontiers defined by the physical principles of QFT. Massive gauge theories analyzed from the slightly physical point of view of selfinteracting massive vectormesons nourish the suspicion that the intrinsic frontiers may be wider than those set by the standard perturbative power counting for interaction polynomials.

Both the modular wedge localization approach as well as string theory attribute a basic significance to chiral conformal theory, and both know the notion of holography. But the use and the physical interpretation of these concepts is quite different. Whereas in AQFT chiral conformal theories are the building blocks of holographic images of higher dimensional theories and therefore are positioned in the same Minkowski space, string theory places the chiral conformal data into an auxiliary source space and identifies the physical space as the target space of the fields in which they take their values. Related to this is in fact the notorious difficulty of defining a string field theory, a problem which is presumably related to the difficulty in separating the intrinsic conceptual content of string theory from its procedural prescriptions.

On the other hand the modular approach has all the hallmarks of a conceptually based intrinsic formulation of local particle physics which makes it a candidate for an extension into the realm of interactions of the Wigner’s

\[17\] This means that the string of string theory is not an extended object in an otherwise local theory as. e.g. a Mandelstam string in gauge theory.
representation theory of free particles which was the first totally intrinsic (independent of quantization) approach to relativistic quantum theory.

Note added: Meanwhile there has been substantial progress on an issue, which although dating back to the QFT of the 70's, received new attention through the string theorist’s observation about a correspondence of QFT in anti De Sitter space and conformal QFT. The methods of AQFT have turned out to be very powerful in unravelling some real time quantum physical aspect of this isomorphism and have led to substantial progress on higher dimensional conformal QFT (B. Schroer hep-th/0005010 and 0005134). The role of string theory as a search machine of such unexpected connections versus the conceptual power of AQFT to analyse and understand such observations on a profound and useful level for particle physics makes an interesting continuation of the theme of the last section in which the reader may be interested in.

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A Appendices on Modular Theory

A.1 Some Facts about Modular Theory

Definition 5 A von Neumann algebra \( \mathcal{A} \) (weakly closed operator sub-algebra of the full algebra \( B(H) \) on a Hilbert space \( H \)) is in “standard” position with respect to a vector \( \Omega \in H \), denoted as \( (\mathcal{A}, \Omega) \), if \( \Omega \) is a cyclic \( (\mathcal{A} \Omega = H) \) and separating \( (\mathcal{A} \Omega = 0, A \in \mathcal{A} \) iff \( A = 0) \) vector for \( \mathcal{A} \). In this situation Tomita defines the following involutive antilinear but unbounded operator (the Tomita involution \( S \))

\[
SA\Omega := A^*\Omega
\]

(77)

where the star operation is the hermitian conjugate in operator algebras. Its closability property (as physicists we will use the same notation for the closure) is the prerequisite for the polar decomposition

\[
S = J\Delta^\frac{1}{2}
\]

(78)

where the angular part \( J \) (the modular involution) is antiunitary with \( J^2 = 1 \) and \( \Delta \) is unbounded positive and therefore leads to a unitary group \( \Delta^it \).

Theorem 6 (Tomita 1965, with significant improvements from Takesaki): The modular involution maps \( \mathcal{A} \) onto its von Neumann commutant \( \mathcal{A}' \) in \( H \):

\[
AdJ \cdot \mathcal{A} = \mathcal{A}'
\]

(79)
The unitary $\Delta^it$ defines a “modular” automorphism group by

$$Ad\Delta^it \cdot \mathcal{A} = \mathcal{A}$$  \hspace{1cm} (80)

(analogy to a dynamical law for the algebra).

More details and references to the proof can be found in [1]. Actually physicists have independently discovered some important properties of modular theory which later were incorporated by mathematicians into the Tomita-Takesaki theory. In fact Haag, Hugenholtz and Winnink[1] observed that the KMS property which Kubo, Martin and Schwinger just used as a computational trick in order to avoid the calculation of traces in quantum statistical mechanics took on a fundamental conceptual role if one works directly in the thermodynamic limit of infinitely extended systems. a closely related independent discovery in their pursuit of physical-conceptual problems in quantum statistical mechanics which arise if one works directly in the thermodynamic limit [1]. As it is well known, the Gibbs representation formula

$$\langle AV \rangle^{(V)}_\beta = \frac{t^e e^{-H(V)} AV}{t^e e^{-H(V)}}$$ \hspace{1cm} (81)

ceases to make sense\footnote{In a box the bounded below hamiltonian acquires a discrete spectrum and $e^{-\beta H}$ is of trace \textit{class} ($\Omega_\beta = e^{-\frac{1}{2}\beta H}$ is H.S.), a property which is lost in the infinite volume limit.} for infinite volume open systems and the algebra changes its Murray von Neumann type. Whereas in the quantization box it was type I, the open system algebra becomes type III$_1$ and the Gibbs formula passes to the KMS condition which is a cyclic relation for thermal correlation functions [1]. In the 70’s Haag and collaborators were able to derive the KMS condition directly from stability properties under local deformations and Pusz and Woronowicz found a direct link to the second law of thermodynamics [1]. These profound results were recently used for the derivation of thermal properties of quantum matter in an anti-de Sitter spacetime [1].

The relation of modular theory with the Einstein causality of observables and locality of fields in QFT was made around 1975 in a series of papers by Bisognano and Wichmann [1]. Specializing to wedge algebras $\mathcal{A}(W)$ generated by Wightman fields, they proved the following theorem

\textbf{Theorem 7} The Tomita modular theory for the wedge algebra and the vacuum state vector $(\mathcal{A}(W), \Omega)$ yields the following physical identifications

$$\Delta^it = U(\Lambda_W(2\pi t))$$ \hspace{1cm} (82)

$$J = TCP \cdot U(R_x(\pi))$$

Here $\Lambda_W(\chi)$ denotes the boost ($\chi$ is the x-space rapidity) which leaves the wedge $W$ invariant. If we choose the standard $t-x$ wedge, then the rotation...
which aligns the TCP with Tomita’s $J$ is a rotation around the x-axis by an angle $\pi$.

Now I come to my own contributions which are of a more recent vintage [22]. They result from the desire to invert the Bisognano-Wichmann theorem i.e. to use Tomita’s modular theory for the actual construction (and classification) of (a net of) wedge algebras belonging to interacting theories with the final goal to intersect wedge algebras in order to obtain a net of compactly localized double cone algebras. For the arguments which show that the particle physics properties, in particular the scattering matrix and formfactors of distinguished fields (conserved currents) can be abstracted from the net observables, I refer to [1] [41] [37]. If desired, the nets can also be coordinatized by more traditional pointlike fields and a rigorous derivation for chiral nets can be found in [34]. For the derivation of LSZ scattering theory one makes the assumption of the existence of a mass gap. With this one immediately realizes that, whereas the connected part of the Poincaré group is the same as that of the free incoming theory, the disconnected part containing time reversals, in particular the modular involution $J$ for the wedge carry the full interaction

$$\Delta_{W}^{it} = \Delta_{W,in}^{it} = e^{-itKt}$$

Here $J_{W,in}$ refers to the Tomita involution (or TCP reflection) of the wedge algebra generated by the incoming free field. If the theory is not asymptotically complete (i.e. the vacuum is not cyclic with respect to the incoming fields) these relations have to be modified, but here we discard such pathologies for which no physical illustration exists. Since we do not want to temper with historically grown notations, we have added a subscript to the S-matrix $S_{sc}$ in order to distinguish it where necessary from Tomita’s $S$. The modular “Hamiltonian” $K$ defined in the first equation (the boost generator= Hamiltonian of a particular uniformly accelerated Unruh observer) has always symmetric instead of one-sided spectrum.

The last relation (83) is nothing but the TCP-transformation law of the S-matrix rewritten in terms of modular objects associated to the wedge algebra. The above role of the S-matrix as a kind of relative modular invariant of the wedge algebra (relative to the free one) is totally characteristic for local quantum physics and has no counterpart in quantum mechanics.

### A.2 Absence of PFGs for Sub-Wedge Regions in Theories with Interactions

**Theorem 8** In interacting theories there exist no PFGs localized in subwedge regions. The wedge region is the smallest spacetime region for which PFGs in the presence of interactions are possible.

For the proof let us first assume that the spacetime localization region $O$ of the would be PFGs is compact, e.g. a double cone. Let $\phi$ be an operator

\[^{19}\text{The proof is similar to that of the Jost-Schroer theorem in [1] and to that in [4].}\]
which is affiliated with $A(\mathcal{O})$ which means that on the domain of definition it commutes with all operators from the commutant $A'(\mathcal{O})$. The PFG property of $\phi$ means ($\phi^\#$ stands for either $\phi$ or $\phi^*$)

$$\begin{align*}
\phi^\#(x)\Omega &= \text{one - particle vector} \\
\phi^\#(x) &= U(x)\phi^\#U^*(x)
\end{align*}$$

(84)

without any admixture of additional polarization contribution from higher particle configurations. As a result the vector satisfies the free field equation in $x$. On the other hand we have that $[\phi^\#(x), \phi^\#(y)] = 0$ for sufficiently large spacelike separations. Let us now look at matrix-elements

$$\langle \psi_2 | \phi^\#(x) | \psi_1 \rangle$$

(85)

with say $\psi_2 \in \text{domain}(\phi^\#)$ and choose $\psi_1$ from the dense set of state vectors which are localized in some region spacelike relative to $\text{loc}(\phi^\#(x))$. This is done by applying spacelike separated operators onto the vacuum $|\psi_1\rangle = A |0\rangle$. Since $\phi^\#(x)$ commute with such operators we obtain

$$\langle \psi_2 | \phi^\#(x) | \psi_1 \rangle = \langle \psi_2 | \phi^\#(x)A | \Omega \rangle$$

(86)

$$= \langle \psi_2 | A\phi^\#(x) | \Omega \rangle$$

i.e. $\phi^\#(x)$ fulfills the free field equation on a dense set of states in its domain. Since all affiliated operators are closable, the operator itself fulfills the free field equation. If we succeed to prove in addition that the commutator with itself is a c-number

$$[\phi^*(x), \phi(y)] = c(x - y)1$$

(87)

then we would have achieved our goal since it would follow that $\phi^\#(x)$ is a linear expression in terms of the particle creation and annihilation operator which contradicts the presence of an interaction. But this last step follows almost literally the argument in the derivation of the Jost-Schroer theorem [5], the fact that the present $\phi$ has no well defines L-covariance does not matter. In the first step one shows that

$$[\phi^*(x), \phi(y)] |\Omega\rangle = c(x - y) |\Omega\rangle$$

(88)

which requires the creation×creation contribution to vanish i.e. $[\phi^*(x), \phi^*(y)] |\Omega\rangle = 0$. For this one uses causality and the separate analyticity in $x$ and $y$ which follows from the forward mass-shell support property. The generalization from a relation on the vacuum to a relation on a dense set of states is as before.
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