Laser beam-induced bistability of concentration in nanofluids

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Abstract. We conduct a theoretical study of the dynamics of the concentration of nanoparticles in liquid-phase media under the influence of a laser light field. An exact solution of the nonlinear diffusion equation in the form of switching waves is found. It is shown that, in the conditions of a fixed temperature and a nonlinear medium thermal conductivity, nanofluid becomes bistable.

1. Introduction
Colloidal suspensions, or nanofluids as they are called today, are widely used in various fields of modern technology. For example, the magnetic fluid is used for polishing optical components [1], the slurry of silica particles in the liquid crystal substantially improves the characteristics of the optical drive [2]. Note also their use in chemical processes (catalysis), the creation of new drugs, lubricants and so forth. With the increasing performance of electronic devices and the development of high technologies, it is necessary to create effective cooling systems and manage large heat flows.

One of the ways to intensify heat transfer is to increase fluid thermal conductivity by adding solid particles with high thermal conductivity. Of particular interest in the creation of such suspensions are nanoparticles [3–6]. Further, as shown by recent studies [7–10], the liquid phases of the medium, in which the components such as the dispersed nanoparticles are taken from wide bandgap semiconductors and dielectrics, are very effective for a number of nonlinear optical effects. In these media, unlike in homogeneous media, the nonlinear optical response occurs due to light wave-induced changes in the refractive index and absorption coefficient, thermal diffusion phenomena due to electrostriction and particles.

At the same time, the physical mechanisms involved, in particular the optical nonlinearity and the processes of heat and mass transfer in these environments, require further study in our view.

2. Theoretical model
The aim of our work is to conduct a theoretical study of the dynamics of nanoparticle concentration in the liquid-phase medium to be subjected to laser radiation of constant intensity, taking into account the thermal conductivity of the medium depending on its concentration.

We believe that the particle size satisfies the condition: $a_0 \ll \lambda$, where $a_0$ is their linear dimension and $\lambda$ is the wavelength of light. Thus, we do not consider diffraction and scattering...
processes. We also assume that the density of the liquid and the particles are nearly the same, giving the option to exclude sedimentation processes from consideration.

Consider the liquid-phase medium with microparticles irradiated with a light beam evenly distributed over the cell intensity. As a result of the light field in the medium, temperature and concentration gradients occur, causing heat and mass transfer processes. These phenomena are described by the system of balance equations for temperature and particles [11]:

\[
C_p \rho \frac{\partial T}{\partial t} = \lambda \nabla^2 T + \alpha I_0, \tag{1}
\]

\[
\frac{\partial C}{\partial t} = D \nabla^2 C + D_T \nabla[C(1-C)\nabla T]. \tag{2}
\]

Note that in the heat equation (1), we have omitted the term responsible for the Dufour effect because of its smallness, and in the diffusion equation (2), a term corresponding to the action of gradient forces from the light field was not considered at this stage of the research. The following designations are accepted: \(\nabla\) is the Laplace operator, \(T\) is the ambient temperature, \(C = C(r,t) = \frac{m_0}{m}\) is the mass concentration of particulate matter (\(m_0\) is the mass of particles, \(m\) is the mass of the entire environment) \(C_p, \rho, \lambda\) is the constant thermal fluid, \(I_0\) is the light intensity, \(\alpha\) is the optical absorption coefficient of the medium and \(D\) and \(D_T\) are the diffusion and thermal coefficients respectively.

In such a general statement of the system, (1)–(2) are unlikely to be solved. Therefore, we make some simplifying assumptions: we consider the one-dimensional case and exclude the contribution of the convective term, which occurs in the equation (2). Then we take into account the fact that the processes establishing the temperature goes much faster diffusion. This provides the opportunity to learn the latest on the background of fixed temperature: \(\frac{\partial T}{\partial t} = 0\).

In equation (1), we assume that the thermal conductivity depends on the concentration and that this dependence has the form:

\[
\lambda(C) = \lambda_0 + \beta C = \lambda_0(1+pC), \tag{3}
\]

where \(p = \frac{\lambda_0}{\beta}\) and \(\beta\) is the proportionality factor [5, 6]. Note that this type of dependence, \(\lambda(C)\), was observed in a number of experiments [9, 10]. Given the above, the diffusion equation (2) can be written as:

\[
\frac{\partial C}{\partial t} = D \frac{\partial^2 C}{\partial x^2} - D_T \frac{\alpha I_0}{\lambda(C)} C (1-C). \tag{4}
\]

Proceeding in this equation to the dimensionless variables: \(\tau = \frac{S_T}{\lambda_0} I_0 \alpha D t, \quad y = \frac{x}{\sqrt{\frac{S_T}{\lambda_0} I_0 \alpha}}, \quad (S_T = \frac{D_T}{D} \quad \text{Soret coefficient}), \) and using the approximation \(\frac{1}{\lambda(C)} \approx \lambda_0(1+pC)\), we obtain the problem:

\[
\frac{\partial C}{\partial \tau} = \frac{\partial^2 C}{\partial y^2} - pC^3 + (1+p)C^2 - C, \tag{5}
\]

\[
0 \leq C \leq 1, \quad \infty \leq y \leq \infty, \quad 0 \leq \tau < \infty. \tag{6}
\]
Such parabolic equations with cubic nonlinearity were considered in [12, 13] for modeling dissipative media. Obviously, the zeros of the source function in equation (5): \( C_1 = 0, C_2 = \frac{1}{p} (p > 1) \), \( C_3 = 1 \) correspond to spatially homogeneous stationary states.

As is known, the kinetics strongly dissipative system depends on the stability of the stationary states. In this case, the states of \( C = C_{1,3} \) are stable (they derive from the source \( F'(C) < 0 \) and the state \( C = C_2 \) is unstable. Thus, we found that the medium is bistable. Following [12], the solution of equation (3) will be found by substituting the Cole-Hopf:

\[
C(y, \tau) = \frac{u(y, \tau)\mu}{u(y, \tau) + u_0},
\]

where \( u(y, \tau) \) is a new feature, and \( \mu \) and \( u_0 \) are permanent.

Substituting (7) into (5) and equating the coefficients of the same powers \((u + u_0)\) to zero, we obtain a system of linear equations to determine \((\mu, \tau)\):

\[
u = 3u_{yy} - (1 + p)u_y,
\]

\[
2u_{yy} - (\text{sgn} \mu)(1 + p)u_y + u_y = 0,
\]

with \( \mu = \pm\sqrt{2p} \). The characteristic equation corresponding to (9) can be written as:

\[
k [2k^2 - (\text{sgn} \mu)(1 + p)k + 1] = 0,
\]

whose roots are: \( k_0 = 0, k_1 = \frac{1}{\sqrt{2p}} \) and \( k_2 = \sqrt{\frac{p}{2}} \). Further, given the symmetry of equation (5) with respect to replacing \( y \rightarrow -y \), we restricted ourselves to a positive value \( \mu \). Therefore, the function \( u(y, \tau) \) we have:

\[
u = a_0 + a_1(\tau) \exp (k_1y) + a_2(\tau) \exp (k_2y).
\]

Substituting (11) into (9) we find:

\[a_0 = A_0, \quad a_i(\tau) = A_i \exp (\eta_i \tau), \quad \eta_i = 3k_i^2 - k_i(1 + p)\sqrt{\frac{p}{2}}, \quad i = 1, 2,
\]

where \( A_i \) are the constants determined from the initial conditions.

Thus, the exact solution of equation (5) will have the form:

\[
C(y, \tau) = \sqrt{2p} \frac{N_1k_1 \exp (k_1y + \eta_1 \tau) + N_2k_2 \exp (k_2y + \eta_2 \tau)}{1 + N_1 \exp (k_1y + \eta_1 \tau) + N_2 \exp (k_2y + \eta_2 \tau)},
\]

where \( N_i = \frac{A_i}{a_0 + z_0} \).

Note that such a solution was obtained by another method in [12].

3. Analysis of results and discussion

Obviously, the solution (13) is continuous for all values of \( y \) and \( \tau \), if \( N_i > 0 \). We note that the solution (13) describes the dynamics of a bistable system, in which a wave of switching state of the system is formed by a two-wave mechanism.

Passing from the exponent in (13) to the dimensional variables, we can obtain the expression for the wave velocities:
\[
\nu_{1,2} = \sqrt{\frac{S_f \alpha}{\lambda_0}} \left( 1 + p \right) \sqrt{\frac{p}{2} - k_{1,2}} D. 
\] (14)

Given that \( p > 1 \), as well as the expressions for \( k_{1,2} \), it is evident that this “two-phase” solution is a concentration of two plane waves moving in the same direction and \( \nu_1 > \nu_2 \). Thus, the expression (13) describes the interaction of the two waves of switching from the intermediate unstable state, with \( C = C_2 \), into the stable \( C_1 \) and \( C_3 \). Thus, the velocity of these waves (\( \nu_1 \) and \( \nu_2 \)) depends solely on the thermal conductivity (\( \lambda_0, \beta \)).

From (13) we can obtain an initial condition in which this mode is implemented:

\[
C(y,0) = \sqrt{2} \frac{N_1 k_1 \exp k_1 y + N_2 k_2 \exp k_2 y}{1 + N_1 \exp k_1 y + N_2 \exp k_2 y}. 
\] (15)

Note that, as shown in [12], the two-wave solution (13) is true for other initial conditions. Further, since \( \eta_1 > \eta_2 \), then we have at long times:

\[
C(y, \tau) = \sqrt{2} \frac{N_1 k_1 \exp (k_1 y + \eta_1 \tau)}{1 + N_1 \exp (k_1 y + \eta_1 \tau)}. 
\]

Note that if we consider the diffusion processes against a background of stationary temperature under constant thermal conductivity, equation (3) is transformed into a well-known Kolmogorov-Petrovskii-Piskunov equation [14], which, as we know, has only a single-wave solution.

Of course, our approach is to some extent a model; however, as the authors hoped, we managed to discover under which conditions the irradiated nanofluids acquire the properties of a dissipative bistable medium that can be distributed switching wave. We believe that not all of the issues have been studied exhaustively; for example, an inverse relationship between temperature and the concentration of nanoparticles has not been considered. A detailed examination of this topic will be the subject of our further research.

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