Interaction of temperature fields in an unlimited array of soil during the selection of low-grade heat by U-shaped probes of a geothermal heat pump units

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Abstract. The methodology and results of mathematical modeling of the unsteady process of selecting low-grade soil heat with a vertical U-shaped geothermal probe placed in a well of heat pump units (HPU), the free space of which is filled with a heat-conducting filler, are presented. The developed mathematical model is based on the joint use of two classical analytical methods: a source-drain adapted to the unsteady process of heat extraction from an unlimited soil mass, and a superposition method that allows one to quantify the effect of the interaction of the temperature fields of the downpipe and the riser pipe laid at a small distance from each other in one well. As a result, the main calculated dependencies underlying the developed mathematical model on which the computational experiment was performed were obtained analytically. The results of a computational experiment, performed on the example of a well with a vertical U-shaped soil probe, designed to select low-grade heat from an unlimited array of dry sand, are presented in graphs. An analysis of the results made it possible to identify the main factors affecting the intensity of heat extraction, justify the conditions for the effective operation of U-shaped probes in the wells of the soil contour of heat pump units, and quantitatively determine the maximum allowable temperature increment of the heated coolant according to the condition of efficient use of the heat transfer surface.

1. Introduction

Geothermal HPUs used today in Europe, the USA and other countries to cover the heat loads of stand-alone facilities remote from district heating sources show high energy and economic efficiency.

In Russia, the practical implementation of such installations is still at an initial stage for a number of reasons, the main of which is a sharply continental climate and much lower background soil temperature compared to European countries and the United States. In such climatic conditions, the need arises for the installation of longer soil contours, which significantly increases capital investments in earthwork, contributing to a decrease in the overall economic efficiency of the use of HPU. Therefore, issues related to the scientific justification and development of a methodology for calculating the intensity of the processes of selecting low-grade heat from soil are today particularly relevant for our country and are of practical importance for determining the area of efficient use of geothermal heat-generating units as alternative heat sources.

The selection of low-grade heat from an unlimited array of soil during the operation of geothermal heat pump units with vertical U-shaped probes is a rather complex, multifactorial non-stationary process. Therefore, despite the fact that in the special scientific and technical literature, domestic [1–7]
and foreign periodicals [8–12] and Internet sources there is a fairly large number of publications on topics related to geothermal heat pumps, the theory of vertical Soil probes in HPU wells have not been practically developed to date. There is also no official methodology for calculating earth loops with vertical U-shaped probes.

Research objective. Development of a mathematical model of unsteady selection of low potential soil heat by vertical U-shaped heat exchangers (soil probes) of geothermal heat pump units (HPU), taking into account the effect of the interaction of temperature fields arising around the downcomer and lift pipe, laid together in a common well, analysis of factors affecting the rate of heat perception, and obtaining the basic calculated dependencies.

Problem description. The layout of the downpipe and the riser pipe in the HPU well with a U-shaped soil probe is shown in figure 1. From this diagram it can be seen that both pipes having the same diameter \( d_{\text{out}} \) are eccentrically located with an offset \( S \) in different directions relative to the axis of the well. As is known from practice, the distance \( b \) between the pipes in most cases does not exceed 0.1–0.2 m with a well radius of 0.1–0.15 m. The depth of the wells can reach 100 meters or more.

![Figure 1. Design scheme of a geothermal well HPU with a U-shaped probe.](image)

The low-temperature coolant passing inside the probe is heated from the surrounding soil. With a significant length of the U-shaped loop, the difference between the temperatures of the coolant in the lower and lifting pipes in the upper part of the well can reach significant values.

At the same time, due to the intense internal flow of heat through the well filler from a warmer (return) to a less heated (direct) coolant entering the downpipe directly from the HPU condenser, heating of the heat carrier at the end of the riser can stop, and under certain conditions it can even start to cool.

It cannot be considered effective. To eliminate the inefficient use of the heat exchange surface in each case, it is necessary to carry out a calculation justification of the maximum allowable temperature difference between the coolant at the inlet and outlet of the U-shaped probe. The development of a methodology for calculating the rationale for the maximum allowable temperature increment during heating of the coolant in the soil circuits of HPU is one of the objectives of this study.

Another objective is the analytical justification for the occurrence of the effect of the interaction of temperature fields that arise around the downhole and lift pipes laid in one well at a close distance from each other. As well as identifying the main significant factors with obtaining quantitative indicators of a general decrease in the heat perception intensity during the annular interaction, depending on the structural characteristics of the probes, the complex of thermophysical parameters and specific operating conditions.
2. Theory

To evaluate the thermal resistance of the TNU well shown in figure 1, we conventionally imagine it consisting of two parts, each of which individually will correspond to the elementary case of an eccentric arrangement of a single pipe in the well, shown in figure 2. Moreover, to assess the thermal resistance of the unequal layer of the filler of the well with a single pipe, the well-known formula [13] can be used, obtained for the case of an eccentric arrangement of a single pipe in thermal insulation, which, taking into account the accepted designations, has the form:

\[
R_{\text{sol}} = \frac{1}{2\pi\lambda_{\text{sol}}} \ln \left( \frac{\sqrt{r_{\text{well}}^2 + 0.5d_{\text{pipe}}^2} - S^2}{\sqrt{r_{\text{well}}^2 - 0.5d_{\text{pipe}}^2} - S^2} + \frac{\sqrt{r_{\text{well}}^2 + 0.5d_{\text{pipe}}^2} - S^2}{\sqrt{r_{\text{well}}^2 - 0.5d_{\text{pipe}}^2} - S^2} \right),
\]

where \(R_{\text{sol}}\) – linear thermal resistance of an unequal annular layer of heat-conducting filler filling the annulus of the well, (m·°C)/W; \(S\) – axial displacement of the axis of the pipe, relative to the center of the well, m; \(d_{\text{pipe}}\) – pipe outer diameter, m; \(r_{\text{well}}\) – radius of the external heat-receiving cylindrical surface of the well, m; \(\lambda_{\text{sol}}\) – thermal conductivity of the filler, W/(m·°C).

As a result of mathematical transformations, formula (1) can be reduced to a dimensionless form:

\[
R_{\text{sol}} = \frac{1}{2\pi\lambda_{\text{sol}}} \ln \left( \frac{\sqrt{(D^* + 1)^2} - (s^*)^2}{\sqrt{(D^* - 1)^2} - (s^*)^2} + \frac{\sqrt{(D^* + 1)^2} - (s^*)^2}{\sqrt{(D^* - 1)^2} - (s^*)^2} \right),
\]

where \(D^*\) and \(s^*\) – dimensionless geometric characteristics: relative borehole diameter, and eccentricity, respectively determined by the expressions:

\[
D^* = \frac{d_{\text{pipe}}}{r_{\text{well}}},
\]

\[
s^* = \frac{S}{S_{\text{max}}},
\]

where \(S_{\text{max}}\) – maximum possible displacement of the axis of the pipe from the center of the well, m.

The expressions for determining the limiting numerical values of \(S_{\text{max}}\) and \(S_{\text{min}}\), as well as the corresponding calculated intervals for the variation of the relative indices \(D^*\) and \(s^*\) for two typical options for placing pipes in the well according to the diagrams in figure 1 and figure 2 are presented in table 1.
Table 1. The geometric characteristics of the well according to the schemes of figure 1 and figure 2.

| Typical options         | Axial displacement limits | Intervals of variation of dimensionless parameters |
|-------------------------|---------------------------|--------------------------------------------------|
|                         | Maximum | Minimum | $D^*$ | $s^*$ |
| 1. U-shaped umbrella in the well, figure 1. | $S_{\text{max}} = r_{\text{well}} - 0.5d_{\text{pipe}}$ | $S_{\text{min}} = 0.5 \cdot d_{\text{pipe}}$ | $2 \leq D^* \leq \infty$ | $\frac{1}{D^*-1} \leq s^* \leq 1$ |
| 2. Single pipe in the borehole, figure 2 | $S_{\text{min}} = 0$ | $1 \leq D^* \leq \infty$ | $0 \leq s^* \leq 1$ |

An analysis of expression (2) shows that for any given value of $\lambda_{\text{sol}} = \text{const}$, the maximum value of $R_{\text{sol}}^{m}$ will be achieved if there is no displacement of the pipe axis relative to the center of the well, that is, when $s^* = 0$. In this particular case, expression (2) takes the simplest view:

$$R_{\text{sol}}^{m} = \frac{1}{2\pi\alpha_{\text{sol}}} \ln(D^*) \rightarrow \text{when } s^* = 0. \quad (5)$$

Using expressions (2) and (5), it is possible to obtain a dependence for quantitatively assessing the effect of the axial displacement of pipes on the thermal resistance of an unequal annular layer of well filler in the form:

$$k_s = \frac{R_{\text{sol}}}{R_{\text{sol}}^{m}} = \frac{1}{2\pi\alpha_{\text{sol}}} \ln\left(\frac{\sqrt{(D^* + 1)^2 / (D^* - 1)^2} - (s^*)^2 + \sqrt{1 - (s^*)^2}}{\sqrt{(D^* + 1)^2 / (D^* - 1)^2} - (s^*)^2 - \sqrt{1 - (s^*)^2}}\right) / \ln(D^*). \quad (6)$$

where $k_s$ – coefficient of influence of eccentricity on the thermal resistance of the well filler.

With a known value of $k_s$, previously determined by expression (6) taking into account the numerical values of $D^*$ and $s^*$, the thermal resistance of the unequal layer of the heat-conducting filler of the well can be calculated by the usual formula, additionally containing the coefficient of influence of eccentricity:

$$R_{\text{sol}} = k_s \frac{d_{\text{well}}}{d_{\text{pipe}}} = \frac{k_s}{2\pi\alpha_{\text{sol}}} \ln(D^*). \quad (7)$$

Taking into account (7), the basic linear resistance to heat transfer, $(\text{m} \cdot ^\circ\text{C})/W$, in the section from the outer cylindrical surface of the well to the coolant flowing through each of the two eccentrically located pipes of the U-shaped soil probe, will be:

$$R_{\text{well}}^{b} = \frac{1}{\pi \cdot d_{\text{pipe}} \cdot \alpha} + \frac{1}{2\pi \cdot \lambda_{\text{mat}}} \ln\left(\frac{d_{\text{pipe}}}{d_{\text{out}}}\right) + \frac{k_s}{2\pi\alpha_{\text{sol}}} \ln(D^*), \quad (8)$$

where $d_{\text{pipe}}$ – internal diameter of the heat-receiving pipe, m; $\alpha$ – heat transfer coefficient from the inner surface of the pipe to the coolant, $W/(\text{m}^2 \cdot ^\circ\text{C})$; $\lambda_{\text{mat}}$ – thermal conductivity of the pipe material, $W/(\text{m} \cdot ^\circ\text{C})$.

The basic heat transfer resistance of the well, defined by expression (8), allows us to individually evaluate the heat fluxes perceived by the coolant in the lowering and lifting pipes of the U-shaped probe, with known temperature differences between the soil and the coolant in each of these pipes.

However, due to the fact that the lowering and lifting pipes in the well are located at an insignificant distance, in addition to heat transfer resistances, an additional factor affecting the formation of specific heat fluxes should be considered the effect of the annular interaction of temperature fields.
To take into account the effect of the annular interaction of temperature fields, we use the well-known analytical method of superposition [14, 15, 16], that is, the superposition of temperature fields. The main principle of the superposition method is that if there are several heat sources or sinks in the array, their temperature fields overlap each other, and the resulting temperature field is determined by adding the temperature differences created by each individual heat source (sink) in the areas between the selected array point and centers of each source (runoff).

According to this principle, when laying two pipes of a U-shaped heat exchanger in one well, the calculated temperature differences between the external heat-absorbing surface of the well and the low-temperature coolant in each of the pipes should be determined by adding the basic temperature differences determined for each single pipe of this well individually. In the future, the term “basic” will mean that any parameter belongs to the basic version, that is: a well of a given diameter with one eccentrically located pipe of a given size for a given value of axial displacement $S=0.5b$.

In particular, the basic values of the specific heat flux perceived by the lowering and lifting pipes of the U-shaped probe, $q_{1\text{bas}}$ and $q_{2\text{bas}}$, W/m, are determined separately:

$$q_{1\text{bas}} = \frac{t_1 - t_x}{R_{\text{well}}}, \quad (9)$$

where $t_1$ – coolant temperature in the downpipe, °C; $t_x$ – average temperature of the heat of the surface of the well (conditional isotherm), °C; $R_{\text{well}}$ – the basic value of the linear thermal resistance of the well, defined by the expression (8), m·°C/W.

$$q_{2\text{bas}} = \frac{t_2 - t_x}{R_{\text{well}}}, \quad (10)$$

where $t_2$ – coolant temperature in the return pipe of the probe, °C.

In this case, the specific heat fluxes $q_{1\text{bas}}$ and $q_{2\text{bas}}$, defined by expressions (9) and (10), can be considered quantitative characteristics of the intensity of conventional heat sinks acting individually in the well.

On the other hand, the unlimited soil mass surrounding the borehole can be considered a kind of infinite source of heat, as a result of which it acts against the heat-absorbing surface of the borehole at any given time determined by the estimated Fourier number from all sides in the radial direction (i.e., towards the center of the borehole) specific heat flux is supplied with instantaneous intensity $q_{gr}$, W/m:

$$q_{gr} = \frac{t_{gr} - t_x}{R_{gr(Fo)}}, \quad (11)$$

where $t_{gr}$ – constant background temperature of the soil mass outside the well coverage area, °C; $R_{gr(Fo)}$ – instantaneous value of the linear resistance of thermal conductivity of the soil mass in the well coverage area, m·°C/W, defined by expression:

$$R_{gr(Fo)} = \frac{1}{4\pi \cdot \lambda_{gr}} Ei\left(\frac{1}{4 \cdot Fo_{p}}\right) - R_{\text{bas}}^{\text{well}}, \quad (12)$$

where $\lambda_{gr}$ – soil thermal conductivity, W/(m·°C); $Ei$ – special function integral exponent [17, 18]; $Fo_{p}$ – estimated Fourier number, defined as:

$$Fo_{p} = Fo + Fo^*, \quad (13)$$

where Fo and Fo* – respectively, the actual and initial Fourier numbers, determined for the well by expressions (14) and (15):

$$Fo = \frac{a\tau}{r_{\text{well}}^2}. \quad (14)$$
where $\tau$ – the actual value of the current time, counted from the start of the well, s; $R_{\text{well}}$ – radius of the external heat-absorbing surface of the well, m; $a$ – thermal diffusivity of the surrounding soil, m$^2$/s.

The initial Fourier number can be defined as:

$$
\begin{align*}
Fo^* &= 0.44527 \cdot \exp \left[12,5664 \left( \frac{R_{\text{well}}^2}{a} \right) \right] - \left\{ 5,71 + 737,5 \left[ \ln \left( \frac{1}{R_{\text{well}}^2 a} \right) \right] \right\} - 0,25; \quad \text{when} \quad R_{\text{well}}^2 a < 1 \\
Fo^* &= 0.44527 \cdot \exp \left[12,5664 \left( \frac{R_{\text{well}}^2}{a} \right) \right] \quad \text{when} \quad R_{\text{well}}^2 a \geq 1
\end{align*}
$$

(15)

An analysis of expression (12) shows that as heat is taken from the soil layers adjacent to the well, over time (which will be determined by increasing values of the calculated Fourier numbers), the instantaneous values of soil thermal conductivity, determined by expression (12), should increase. In this case, the intensity of heat supply to the well should decrease. Therefore, all further considerations will be carried out in relation to the instantaneous values of the parameters of heat perception, varying in time.

The temperature field that arises in the well of a TNU with a U-shaped soil probe and an unlimited soil mass surrounding this well is formed as a result of the interaction of the temperature fields of three elements: a downcomer pipe with a coolant temperature $t_1$, °C, a lift pipe with a coolant temperature $t_2 > t_1$, °C, and unlimited soil massif with background temperature $t_{gr} > t_2 > t_1$, °C. At the same time, the parameters of the resulting temperature field at the external boundary of the well, according to the main position of the superposition method [14, 15, 16], can be determined by adding the temperature fields of all the listed heat sources and sinks.

Having solved equations (9), (10) and (11) with respect to the corresponding temperature differences, we individually determine the instantaneous temperature differences between the well surface $t_x$ and the temperatures of the heat source and sinks:

$$
\begin{align*}
& t_x - t_1 = q_1^{\text{bas}} \cdot R_{\text{well}}^2 \\
& t_x - t_2 = q_2^{\text{bas}} \cdot R_{\text{well}}^2 \\
& t_x - t_{gr} = -q_{gr} \cdot R_{gr(Fo)}^2
\end{align*}
$$

(16, 17, 18)

Based on the heat balance, the instantaneous intensity of the conditional heat source (unlimited soil mass) at each moment of time should be equal to the sum of the intensities of the conditional heat sinks:

$$
q_{gr} = q_1^{\text{bas}} + \Delta q_{2,1} + q_2^{\text{bas}} - \Delta q_{2,1} = q_1^{\text{bas}} + q_2^{\text{bas}}
$$

(19)

where $\Delta q_{2,1}$ – the part of the heat flux, W/m, additionally perceived by the coolant of the lowering pipe of the U-shaped soil probe through the heat-conducting filler from the jointly installed lifting pipe in the presence of a temperature difference between the coolants passing through these pipes.

Since the redistribution of heat fluxes between the pipes occurs due to heat transfer inside the well, we can assume that the $\Delta q_{2,1}$ values included in the balance equation (20) do not affect the external heat transfer between the well and the soil, since they are reduced, having different signs.

As a result of separately summing the right and left sides of equations (16), (17) and (18) after the transformations, taking into account the substitution of (19) in (18), we obtain:

$$
3 t_x - (t_1 + t_2 + t_{gr}) = \left( q_1^{\text{bas}} + q_2^{\text{bas}} \right) \left( R_{\text{well}}^2 - R_{gr(Fo)}^2 \right)
$$

(20)

Using expressions (9) and (10), we can express the sum of the basic intensities of heat sinks in terms of temperature and the basic resistance to heat transfer of the well:
\[ q_{1}^{\text{bas}} + q_{2}^{\text{bas}} = \frac{2t_{x} - (t_{1} + t_{2})}{R_{\text{well}}} . \] (21)

Substituting (21) into (20) and solving the obtained equation with respect to the temperature of the heat-absorbing surface of the well, after transformations we obtain the calculation formula for determining the instantaneous value of the calculated temperature of the heat-receiving surface of the well in the following form:

\[ t_{x} = \frac{R_{\text{well}}^{\text{bas}} \cdot t_{\text{gr}} + (t_{1} + t_{2})R_{\text{gr}(Fo)}}{R_{\text{well}}^{\text{bas}} + 2R_{\text{gr}(Fo)}} . \] (22)

where \( t_{x} \) – average (balance) temperature, °C, conditionally isothermal heat-absorbing surface of the well at a time determined by the estimated Fourier number at which the value of \( R_{\text{gr}} \) was determined by the expression (12).

Further, substituting \( t_{x} \), defined by expression (22) in (9) and (10), we obtain, after transformations, taking into account the internal redistribution of heat fluxes, the following formulas for determining the calculated specific heat inflows to each of the two pipes of a U-shaped soil probe, W/m that take into account the effect of the interaction of temperature fields:

– for a lowering pipe:

\[ q_{1}^{n} = \frac{(t_{\text{gr}} - t_{1})}{R_{\text{well}}^{\text{bas}} + 2R_{\text{gr}(Fo)}} \left[ I + \frac{R_{\text{gr}(Fo)}}{R_{\text{well}}^{\text{bas}}} \frac{(t_{2} - t_{1})}{t_{\text{gr}} - t_{1}} \right] + \Delta q_{2,1} , \] (23)

– for lifting pipe:

\[ q_{2}^{n} = \frac{(t_{\text{gr}} - t_{2})}{R_{\text{well}}^{\text{bas}} + 2R_{\text{gr}(Fo)}} \left[ I - \frac{R_{\text{gr}(Fo)}}{R_{\text{well}}^{\text{bas}}} \frac{(t_{2} - t_{1})}{t_{\text{gr}} - t_{2}} \right] - \Delta q_{2,1} , \] (24)

The total specific heat gain from the soil to the well is determined by adding the values obtained from expressions (23) and (24). In this case, the resulting expression for determining the total calculated heat input to the well \( q_{\text{well}}^{p} \), W/m, after the transformations will take the form:

\[ q_{\text{well}}^{p} = \frac{t_{\text{gr}} - 0.5(t_{1} + t_{2})}{0.5R_{\text{well}}^{\text{bas}} + R_{\text{gr}(Fo)}} = \frac{t_{\text{gr}} - t_{12}^{av}}{0.5R_{\text{well}}^{\text{bas}} + R_{\text{gr}(Fo)}} , \] (25)

where \( t_{12}^{av} \) – arithmetic mean temperature of the coolants in the lowering and lifting pipes at the design point of the U-shaped probe, °C.

The formula for the quantitative assessment of the instantaneous calculated values of the linear coefficient of heat transfer from the soil to the external heat-absorbing surface of the well \( K_{\text{well}}^{p} \), W/(m·°C), can be obtained by dividing both parts of expression (25) by the determining temperature difference:

\[ K_{\text{well}}^{p} = \frac{q_{\text{well}}^{p}}{(t_{\text{gr}} - t_{12}^{av})} = \frac{1}{0.5R_{\text{well}}^{\text{bas}} + R_{\text{gr}(Fo)}} . \] (26)

The index “p” in all the formulas obtained means that the corresponding parameters belong to the design variant, which takes into account the effect of the annular interaction of temperature fields inside the soil mass surrounding the well of HPU.

In this case, the basic value of the instantaneous linear coefficient of heat transfer from the soil to the well, \( K_{\text{well}}^{\text{bas}} \), W/(m·°C), not taking into account the effect of the interaction of temperature fields, at the same temperatures of coolants and soil is determined by the expression:
\[ K_{\text{well}}^{\text{bas}} = \frac{q_{\text{sum}}}{t_{\text{out}} - t_{1,2}^{\text{bas}}} = \frac{2}{R_{\text{well}} + R_{\text{gr(Fo)}}}. \]  

After dividing (26) by (27) term, we obtain the following expression after the transformations, which is convenient for quantitatively assessing the decrease in the intensity of the overall heat perception of the well, caused by the interaction of temperature fields that intensifies over time as heat is taken from the soil layers adjacent to the well:

\[ \psi = \frac{K_{\text{well}}^{p}}{K_{\text{well}}^{\text{bas}}} = 1 - \frac{R_{\text{gr(Fo)}}}{R_{\text{well}} + 2R_{\text{gr(Fo)}}}, \]  

where \( \psi \) – an indicator of a decrease in the overall heat perception of the well calculated relative to the base case, which quantitatively determines the enhancement of the effect of annular interaction of temperature fields over time during the selection of geothermal heat by a U-shaped soil probe.

Values of \( \Delta q_{2,1} \), occurring in equations (24) and (25), can we determine how:

\[ \Delta q_{2,1} = \frac{t_2 - t_1}{R_{\text{adj}}}, \]  

where \( R_{\text{adj}} \) – internal specific linear thermal resistance of the well, \((\text{m}^\circ \text{C})/\text{W}\), which prevents the flow of heat through the heat-conducting filler from the warmer return heat medium passing through the riser pipe to the less heated coolant of the lowering pipe of a U-shaped soil probe, calculated by the expression:

\[ R_{\text{adj}} = \varphi \frac{b}{\rho_{\text{out}}} \left[ 2R^* + \frac{1}{2\alpha_{\text{soil}}} \ln \left( \varphi \frac{b}{\rho_{\text{out}}} - 1 \right) \right], \]  

where \( \varphi \) – coefficient taking into account the shape and thickness of the filler layer between the pipes:

\[ \varphi = \frac{1}{6} \left( \frac{\rho_{\text{out}}}{b} \right)^2 \left[ \left( \frac{b}{\rho_{\text{out}}} + \frac{1}{\rho_{\text{out}}} \right)^3 - \left( \frac{b}{\rho_{\text{out}}} - 1 \right)^3 \right]. \]  

Using the obtained formulas (23) and (24) taking into account expression (29), we can obtain the following formulas for determining the instantaneous calculated values of the linear heat transfer coefficients of each individual pipe of a U-shaped probe:

\[ K_j^{p} = \frac{q_j^{p}}{t_{\text{gr}} - t_1} = \frac{I}{R_{\text{well}} + 2R_{\text{gr(Fo)}}} \left[ 1 + \frac{R_{\text{gr(Fo)}}}{R_{\text{well}}} \left( \frac{t_2 - t_1}{t_{\text{gr}} - t_1} \right) \right] + \frac{I}{R_{\text{adj}}} \left( \frac{t_2 - t_1}{t_{\text{gr}} - t_1} \right); \]  

\[ K_2^{p} = \frac{q_2^{p}}{t_{\text{gr}} - t_2} = \frac{I}{R_{\text{well}} + 2R_{\text{gr(Fo)}}} \left[ 1 - \frac{R_{\text{gr(Fo)}}}{R_{\text{well}}} \left( \frac{t_2 - t_1}{t_{\text{gr}} - t_1} \right) \right] - \frac{I}{R_{\text{adj}}} \left( \frac{t_2 - t_1}{t_{\text{gr}} - t_1} \right). \]  

In this case, the basic values of the linear heat transfer coefficients can be determined at the same temperatures of coolants and soil without taking into account the interaction of temperature fields, as:

\[ K_j^{\text{bas}} = \frac{q_j^{\text{bas}}}{t_{\text{gr}} - t_1} = \frac{I}{R_{\text{well}} + R_{\text{gr}}}; \]  

\[ K_2^{\text{bas}} = \frac{q_2^{\text{bas}}}{t_{\text{gr}} - t_2} = \frac{I}{R_{\text{well}} + R_{\text{gr}}}. \]
After dividing (32) by (34) and (33) by (35) terminately, after transformations with allowance for (28), we can obtain the following expressions convenient for quantifying the change in the heat perception of each pipe of a U-shaped soil probe:

\[
\Psi_1 = \frac{K_2}{K_{bas}} = \Psi_2 \left[ 1 + \frac{R_{wp(Fo)}}{R_{well}^0 \left( t_2 - t_1 \right)} + \frac{R_{well}^0 + R_{gr(Fo)}}{R_{dq}} \left( t_2 - t_1 \right) t_{gr} - t_1 \right];
\]

\[
\Psi_2 = \frac{K_2}{K_{gas}} = \Psi_2 \left[ 1 - \frac{R_{wp(Fo)}}{R_{can}^0 \left( t_2 - t_1 \right)} - \frac{R_{can}^0 + R_{wp(Fo)}}{R_{dq}} \left( t_2 - t_1 \right) t_{wp} - t_2 \right].
\]

where \( \Psi_1 \) and \( \Psi_2 \) – accordingly, indicators of changes in the heat perception rate of the lowering and lifting pipe of the U-shaped soil probe due to the effect of the interaction of temperature fields; \( \Psi \) - an indicator of the overall decrease in heat perception of a well with a U-shaped soil probe as a result of the effect of the interaction of temperature fields.

3. Analysis and discussion of the results

Let us consider three characteristic particular cases of the operation of U-shaped soil probes.

**First case** corresponds to the initial moment of time: \( \tau = 0 \rightarrow R_w = 0 \). In this case, \( \Psi = 1 \), and the temperature at the well boundary, according to (28), is equal to the background soil temperature: \( t_w = t_{gr} \). Therefore, at the initial time, the effect of the interaction of temperature fields in the adjacent soil is absent. However, in the well itself, due to the temperature difference between the coolant in the pipes, redistribution of perceived heat fluxes can occur, as a result of which the thermal perception of the downpipe can increase and the downpipe's heat decrease relative to the corresponding basic options due to heat transfer through the array of heat-conducting filler. Therefore, the expressions (36) and (37) in this case will take the form:

\[
\Psi_1 = 1 + \frac{R_{well}^0}{R_{dq}} \left( t_2 - t_1 \right) t_{gr} - t_1 ;
\]

\[
\Psi_2 = 1 - \frac{R_{can}^0}{R_{dq}} \left( t_2 - t_1 \right) t_{wp} - t_2.
\]

**Second case** (limited) corresponds to an infinitely long time of well operation: \( \tau = \infty \rightarrow R_w = \infty \). In this case, \( \Psi = 0.5; \\Psi_1 = \infty; \\Psi_2 = -\infty \). Therefore, the total thermal perception of the probe in this case can be no more than half of the base. In this case, the temperature at the well boundary, according to (22), should be set at the level of the arithmetic mean temperature of the coolants: \( t_w = 0.5(t_1 + t_2) \). This means that with an infinitely long time of continuous operation of the well, the effect of the interaction of temperature fields in the soil should be manifested to the maximum extent, reducing the total heat perception of the probe by half relative to the base case.

In this case, the lowering pipe of the probe will absorb heat from the external boundary of the well due to the positive temperature difference \( t_2 - t_1 > 0 \), and the lifting pipe, on the contrary, will transfer part of its heat back to the ground due to the negative temperature difference \( t_2 - t_1 < 0 \). The other part of the heat will be given to the lift pipe by the coolant of the lower pipe due to the positive temperature difference between the heat carriers in these pipes: \( t_2 - t_1 > 0 \). Therefore, the coolant in the tail of the riser, where the temperature difference \( t_2 - t_1 \) is especially large, will be cooled instead of heating. The mode in which the coolant is cooled in the tail of the riser pipe is irrational, since this reduces the efficiency of using the heat exchange surface of the soil probe.

**Third case** corresponds to the condition \( \Psi_2 = 0 \), which determines the transitional mode of operation of the lifting pipe (from heat perception to heat transfer). Consideration of this case allows us to justify the ratio of determining parameters at which heating can stop and cooling of the coolant in
the upper part of the lifting pipe of the U-shaped soil probe can begin. As a result of substituting the value \( \varphi_2 = 0 \) in (37) and solving the obtained equation with allowance for (28), the following relation was obtained, which has a dimensionless form:

\[
\Theta_{tr} = \left( \frac{t_2 - t_1}{t_{gr} - t_1} \right) \int \left( \frac{2 \frac{R_{ekv}}{R_{well}} - 1}{\frac{R_{ekv}}{R_{well}} + \frac{R_{ekv}}{R_{well}}} \right) \]

where \( \Theta_{tr} \) – dimensionless criterion for the occurrence of a transitional regime with zero heat perception of the lifting pipe; \( (t_2 - t_1)_{tr} \) – the difference between the temperatures of the coolant in the lifting and lowering pipes of the U-shaped soil probe, °C, corresponding to the transition conditions; \( R_{well}^{bas} \) – basic linear resistance to heat transfer in the area from the outer cylindrical surface of the well to the coolant flowing through the pipes, \((\text{m}^2\cdot\text{°C})/\text{W})\), defined by expression (8); \( R_{ekv} \) – instantaneous equivalent linear heat transfer resistance, \((\text{m}^2\cdot\text{°C})/\text{W})\), determined depending on the calculated Fourier number, as:

\[
R_{ekv} = \frac{1}{4\pi \cdot \lambda_{gr}} Ei \left( \frac{1}{4 \cdot F_{op}} \right).
\]

The resulting equation (40) establishes the dependence of the dimensionless criterion \( \Theta_{tr} \) on two dimensionless parameters, which can be denoted by:

\[
X = \frac{R_{ekv}}{R_{well}^{bas}}; \quad Y = \frac{R_{well}^{bas}}{R_{ekv}}.
\]

Graphs illustrating the nature of the obtained dependence (40) are shown in figure 3.

An analysis of expression (41) shows that with an increase in the time of heat extraction (for \( F_{op} \to \infty \)), \( R_{ekv} \) should increase. At the same time, according to (42), the value of parameter X should increase, since the basic thermal resistance of the well, defined in each case by expression (8), has a constant value \( (R_{well}^{bas} = const) \), determined by the geometry and design features of the probe and the well.
Figure 3. Dependencies of the dimensionless transition criterion on the defining dimensionless parameters, X and Y, defined by expressions (42).

An analysis of the graphs in figure 3 shows that with an increase in X, the value of the dimensionless transition criterion should decrease $\Theta_{gr}$.

With the known numerical value $\Theta_{gr}$ calculated according to expression (40) or found from the graph in figure 3, it is easy to determine the maximum permissible increment in the temperature of the heat carrier heated by a U-shaped soil probe by the condition of efficient use of the heat-receiving surface:

$$\left(t_2 - t_1\right)_{tr} = \Theta_{tr} \cdot \left(t_{gr} - t_1\right). \quad (43)$$

Example. Let us determine the maximum permissible increase in the temperature of the coolant 200 days after the start of the operation of the well under the condition of efficient use of the heat-sensing surface of the U-shaped soil probe with the following initial data.

Soil type dry sand with thermophysical characteristics: $c_{gr}=820/(\text{kg} \cdot \text{°C})$; $\rho_{gr}=2634 \text{ kg/m}^3$; $\lambda_{gr}=1.1 \text{ W/(m} \cdot \text{°C})$; $d_{gr}=5.0929 \cdot 10^7 \text{ m}^2/\text{s}$. Thermal conductivity of the well filler $\lambda_{sol}=1.5 \text{ W/(m} \cdot \text{°C})$. XLPE pipes: $\lambda_{pipe}=0.38 \text{ W/(m} \cdot \text{°C})$; $d_{out} = 0.032 \text{ m}$; $d_{in} = 0.026 \text{ m}$; annulus, $b = 0.15 \text{ m}$. Well radius, $r_{well}=0.15 \text{ m}$. The heat transfer coefficient from the inner surface of the pipe to the coolant $\alpha = 370 \text{ W/(m}^2 \cdot \text{°C})$. Background soil temperature at an infinite distance from the well $t_{gr}=8 \text{ °C}$. Initial coolant temperature $t_1=0 \text{ °C}$.

Calculation results:

1. Dimensionless well geometry:
   - relative borehole diameter: $D^* = 2r_{well}/d_{out} = 0.3/0.032 = 9.375$;
   - eccentricity indicator: $s^* = S/S_{max} = 0.075/0.134 = 0.5597$;
   - the coefficient of influence of eccentricity on the thermal resistance of the filler in the expression (6): $k_s = 0.869$.
   - coefficient taking into account the shape and thickness of the filler layer between the pipes according to the expression (31): $\varphi = 1.0038$.

2. Thermal resistance:
   - linear thermal resistance of an unequal annular heat-conducting layer according to the expression (7): $R_{sol} = 0.2065 (\text{m} \cdot \text{°C})/\text{W}$;
   - basic linear resistance to heat transfer in the area from the external surface of the well to the coolant in the expression (8): $R_{well}^{bas} = 0.3293 (\text{mm} \cdot \text{°C})/\text{W}$;
   - borehole linear thermal resistivity by expression (30): $R_{s\alpha} = 4, 4378 (\text{m} \cdot \text{°C})/\text{W}$.

3. Dimensionless well Life:
   - initial Fourier number at $R_{well}^{bas} \cdot \lambda_{gr} = 0.3622$ by expression (15) : $F_{o}^* = 41.94$;
   - actual Fourier number for the duration of the operation of the well at 200 days. ($\tau = 17306480$) by expression (14): $F_{o} = 391.7$;
   - calculated Fourier number by expression (16): $F_{o} = 433.6$.

4. Equivalent linear heat transfer resistance
   - by expression (41) $R_{\Theta} = 0.4978$.

5. Dimensionless parameters by expression (42): $X = 1.512$; $Y=0.0742$.

6. Dimensionless transition rate by expression (40) $\Theta_{tr} = 0.6$

7. The maximum allowable increase in the temperature of the coolant under the condition of efficient use of the heat-absorbing surface of a U-shaped soil probe according to the expression (43):

$$\Delta t_{2-1} = \left(t_2 - t_1\right)_{tr} = 4.8 \text{ °C}.$$
the U-shaped probe $t_1=0°C$, the temperature at the probe outlet should not exceed $t_2=4.8°C$. At a temperature of $t_2>4.8°C$, further heating of the coolant in the upper part of the probe’s lifting pipe will become impossible, which will indicate the inefficient use of the heat-absorbing surface.

Graphs illustrating changes in $\Psi_1$, $\Psi_2$, and $\Psi$, as well as linear heat transfer coefficients $K_1$, $K_2$, and $K_{well}$ and specific heat inflows $q_1$, $q_2$, $q_{well}$ to each pipe and to the entire well with a U-shaped soil probe during the heating period, calculated according to the developed method, based on the initial data of the considered example at a constant value $\Delta t_{2-1}^{\text{max}} = 4.8 °C$, are presented in figure 4–6.

![Figure 4](image-url)

**Figure 4.** Dependences of indicators of changes in heat perception intensity on the time of heat extraction taking into account the effect of the interaction of temperature fields (under the conditions of the considered example).

![Figure 5](image-url)

**Figure 5.** Dependences of linear coefficients of heat transfer to pipes and a well with a U-shaped soil probe on the time of heat extraction (under the conditions of the considered example).
Figure 6. Dependences of specific heat inflows to pipes and a well with a U-shaped soil probe on the time of heat extraction (under the conditions of the considered example).

4. Conclusions
A mathematical model and an applied technique for the engineering calculation of unsteady heat transfer processes during the selection of low-grade heat from an unlimited soil array have been developed, taking into account the interaction of temperature fields that are formed as a result of joint laying in the wells of a low-pressure and low-pressure pipe of a vertical U-shaped soil probe.

The main calculated dependences were obtained for evaluating the indicators of changes in the heat perception intensity for each of the pipes and for the entire probe as a whole, as well as the dependences for quantifying the actual heat transfer coefficients and specific heat inflows when operating wells with U-shaped soil probes.

A calculation methodology has been developed for the maximum allowable temperature increment during heating of the coolant in the soil circuits of HPU with U-shaped soil probes.

All the obtained dependences are analytical and can be considered as a theoretical basis for a rigorous mathematical description of the processes of unsteady heat transfer in the development of applied methods and programs for the engineering calculation of soil contours of HPU with vertical U-shaped soil probes.

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