Introducing statistical inference to senior high school students: a textbook analysis

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Abstract. Statistical inference is a new concept for students in senior high (secondary) school in the 2013 curriculum. This study analyses how Indonesian mathematics textbooks for the 12th-grade senior high school present this concept. The method used here is content analysis, i.e., examine five books written by different authors and publishers. We find that almost all textbooks start with the explanation of random variables, followed by the binomial distribution, the normal distribution, and hypothesis testing. However, there are some differences in the depth of the materials among these textbooks. Although those textbooks explain the hypothesis testing, almost all books do not explain the sampling distribution, which is the foundation of inference. This study can be used as a guide to strengthening the explanation of statistical inference in mathematics textbooks as well as in developing statistical inference teaching strategy.

1. Introduction
The rapid growth of the computer, the internet of things (IoT), artificial intelligence, and big data begins the fourth industrial revolution for the civilizations around the world, producing data-driven societies. Following Yaqoob et al. [1], the main properties of big data can be abbreviated as 5V, namely volume, velocity, variety, integrity, and value.

As a branch of science focused on data, statistics play an essential role in a data-driven society. Statistics consist of two broad categories, known as descriptive and inferential statistics [2]. Descriptive statistics study the analysis of data that describe, show, or summarize data in a meaningful way. It is also related to exploratory research, which is done by visualizing the data. On the other hand, inferential statistics associated with the procedure for making inferences and predictions about a population based on the data sample.

In Indonesia, most of the statistical concepts were studied in the mathematics subject from elementary school until senior high school. However, almost all of the statistical competence studied at these levels is the descriptive ones. According to the 2004 competency-based curriculum and the 2006 education unit level curriculum, elementary school students learn how to draw charts and calculate simple central tendency measures such as mean, mode, and median [3]. In junior high school, they learn position measures such as quartile and solve more difficult problems related to the central tendency measure. They also understand the basic concept of probability. In the second year in senior high school, a different type of data called grouped data is introduced to the student, as well as the central tendency measure. Moreover, students also learn the concept of dispersion measure by calculating the average deviation, variance, and standard deviation [4].
Usage of the 2013 curriculum makes a meaningful change in the statistical competence in senior high school. First, the statistical competence that is previously given in the second year then separated into two parts: data visualization or diagrams for the first year, and the central tendency and dispersion measure for the subsequent year. Second, there are some additional materials for students in the Mathematics and Natural Science stream. During the second year, they also study the random sampling, the random variable, the formularization of the binomial distribution function from a random experiment, and the method for hypothesis testing [5].

The addition of statistical inference related competence in the mathematics subject is valuable progress in increasing students’ statistical literacy. Statistical literacy involves understanding and using the primary language and tools of statistics: knowing what basic statistical terms mean, understanding the use of simple statistical symbols, and recognizing and being able to interpret different representations of data [6]. Following Grant [7], it has particular relevance in the age of data science when teams from diverse backgrounds undertake complex analyses. Ben-Zvi and Garfield [8] define statistical literacy as the primary and vital skills that may be used in understanding statistical information or research result. It includes several skills, such as organizing data, constructing and displaying tables, working with different representations of data, understanding of concepts, vocabulary, and symbols, also understanding of probability as an uncertainty measure. According to the Gal (2004) [9] framework, knowing how statistical conclusions or inferences are reached is one of the base knowledge for statistical literacy.

In 2016, a meaningful change was made to the 2013 curriculum as follows: all statistical materials that studied in the first and second year of senior high school were moved into the third year [10]. It is also worthy to note that there is a slightly different way between the core competence of the additional statistics materials for students in the Mathematics and Natural Science stream, as shown in table 1.

Table 1. Core competence related to statistics materials for senior high school students in the Mathematics and Natural Science stream.

| Curriculum 2013                                      | Curriculum 2013 revised 2016                                      |
|-------------------------------------------------------|------------------------------------------------------------------|
| 3.7 Understand the concept of random sampling from a population of a group of objects or daily phenomena. | 3.5 Explain and determine the binomial probability distribution related to the binomial probability function. |
| 3.8 Understand the concept of a random variable and formulate the binomial distribution function through a random experiment. | 3.6 Explain the characteristic of normally-distributed data related to normally-distributed data. |
| 3.9 Understand (the procedure of) concluding hypothesis testing with specific criteria. | 4.5 Solve problems related to the binomial probability distribution of a random experiment and its drawing conclusion. |
| 4.6 Present and use the binomial distribution formula when predicting an event as a result of a random experiment. | 4.6 Solve problems related to a normal distribution and its drawing conclusion. |
| 4.7 Present the process and result of concluding a hypothesis testing with valid argument and procedure. |                                                   |

Setiawan [11] show that even in mathematical textbooks approved by the Indonesian Ministry of Education and Culture, there are several content differences related to the four competence in table 1. It is known that different textbooks might provide different opportunity-to-learn for students [12]. On the other hand, much research shows that statistical inference is not a simple competence. Harradine et al. [13] show that there are some difficulties in understanding statistical inference, especially related to the sampling, sampling distribution, hypothesis testing, and confidence intervals. From a survey through teachers, Koparan [14] mentions that imperfections in data representation, central tendency, and dispersion measures make students difficult when making inferences. Sotos et al. [15] show that there is much empirical evidence of students’ misconceptions of statistical inference. Haller and Krauss [16]
find that there are common misinterpretations of statistical significance that widespread even among instructors who teach statistics. All the above reason yields the need for analyzing the statistical inference-related content in mathematics textbook used by students.

Many studies have been conducted to analyze Indonesian mathematical textbooks for Senior High School [17][18][19]. However, none of them were focused on the statistical inference content for high school students. This study proposes to fulfill this gap by presents the analysis of the statistical inference component in mathematics textbooks.

2. Methods
Since the main focus of this study is to analyze the statistical inference content in textbooks, we use the content analysis method. We started by examining the general presentation of statistical content, followed by a careful examination of the presence of each statistical content [20][21]. The objects of this study are mathematics textbooks written for the XII grade students in the Mathematics and natural science stream based on 2016 revisions of the 2013 curriculum, as presented in table 2. All books studied here can be found in the bookstore and still be used by students in many senior high schools in Indonesia.

### Table 2. List of 12th-grade mathematical textbooks and their code.

| Title                                           | Publisher           | Year   | Acceptance       | Code |
|-------------------------------------------------|---------------------|--------|------------------|------|
| Buku Siswa Matematika 3                         | Bumi Aksara         | 2016   | Decree no 148/P/2016. | A    |
| Matematika untuk Siswa SMA/MA Kelas XII Peminatan MIPA | Srikandi Empat | 2016   | Decree no 148/P/2016. | B    |
| Matematika untuk Siswa SMA/MA Kelas XII Peminatan MIPA | Yrama Widya          | 2016   | Decree no 148/P/2016. | C    |
| Matematika 3                                    | Erlangga            | 2017   | -                | D    |
| PR Matematika 3 SMA/MA                          | Intan Pariwara      | 2019   | -                | E    |

The analysis was started by looking at the general presentation of statistical inference materials in the textbooks. We focus on the explanations and worked examples for each statistical competences on each mathematics textbooks.

3. Result and discussion

3.1. General overview
According to the curriculum document, the statistical inference is the latest core competence for grade XII senior high school students. Consequently, the topic is placed in the latest chapter and suggested to be given in the second semester. In general, the issue consumes between 22%-33% of the page on the mathematics textbooks for grade XII Mathematics and natural science stream.

Among these studied textbooks, there are two arrangements of statistical competence. Book A separate the statistical competences in two consecutive chapters, whereas the other books arrange them into one chapter, as shown in table 3. Each chapter in these textbooks consists of an introduction, contents (explanation, worked examples, and exercises), chapter summary, and end-chapter exercises. A guide for student reflection at the end of the chapter can be found in all textbooks except in D.

Each textbook has a different approach to introduce the student into the statistics chapter. The shortest introduction was found in textbook D, which only displays the list of basic competence and study experience, followed by a short biography about Abraham de Moivre. All other textbooks provide a concept map that contains various vital terms such as random variable, probability function, binomial distribution, normal distribution, and many more. The concept map is followed by a list of keywords, except in textbook E.

A comprehensive analysis of the concept maps finds that not all of them present harmonious relations between the central concept. For example, the map in textbook B and textbook C does not explain the
relations between the binomial distribution and normal distribution. They also contain the wrong
direction, e.g., a fork comes out from binomial distribution function to the random variable. A complete
and detailed concept map was found in textbook E, as displayed in figure 1. It can be seen that the
concept map is well organized, from the concept of random variables, which are separated into two types
of random variables (discrete and continuous), then to the specific distribution (binomial and normal).
However, this concept map still places the term ‘hypothesis testing’ improperly. There are no direct
relations between ‘the probability distribution of random variable X ~ N(0,1)’ and the hypothesis testing.

### Table 3. Organization of statistical competence in mathematics textbooks.

| A                  | B                  | C                  | D                  | E                  |
|--------------------|--------------------|--------------------|--------------------|--------------------|
| Chap. 7 Inferential| Chap. 3            | Chap. 4            | Chap. 4            | Chap. 3            |
| statistics         | Statistics and     | Binomial           | Statistics         | Binomial           |
| A. Probability     | Probability        | normal distribution and | A. Random probability |
| B. Random variable | A. Introduction    | distribution       | B. Discrete norm    |
| C. Probability     | B. Binomial        | probability        | distribution and   |
| distribution and   | probability        | A. Review the      | hypothesis testing |
| probability function| distribution      | concept of         |                |
| D. Binomial        | C. Normal          | probability and    |                |
| distribution       | distribution and   | combination.       |                |
| E. Poisson distribution| Student          | distribution       |                |
| F. Binomial hypothesis testing| distribution | distribution |                |
| C. Hypothesis testing| D. Hypothesis function | E. Drawing distribution |
| Chap. 8 Normal distribution | testing | C. Normal | B. Normal |
| A. Continuous random variable | E. Application of Statistics | distribution | |
| B. Continuous probability distribution |
| C. The continuous cumulative distribution function |
| D. Normal distribution |
| E. Hypothesis testing |

In addition to the concept map, several textbooks, namely A, B, C, and E, place a photograph and a
short paragraph to introduce the topic. In the book A, the introductory paragraph relates the topic with a hypothesis testing in scientific methods. In B, the authors mention several applications of statistics to explain the importance of the topic. Textbook C and E give a case in which the probability could be calculated using the binomial and normal distribution, respectively.

As seen in table 3, textbooks A, B, C, and D place an introductory sub-chapter before discussing the binomial random variable. This sub-chapter mainly review the basic concept of probability (found in A and C) or introduce several new terms such as 1) descriptive and inferential statistics (A); 2) variable, population, and sample (A); 3) parametric and non-parametric statistics (B); 4) types of data (B); 5) methods of data collection (B). These explanations help the student review and understand some basic concepts related to the topic.
3.2. Random variable

All mathematics textbooks used in this study explain the concept of random variables, whether located either in a separated section or under the heading of the binomial distribution. In textbook D, the number resulted from rolling dice was used as an example of a random variable and followed by the formal definition of it. Textbooks A and C use the result of flipping coins as a random experiment, followed by a list contains results of the experiment. Based on the result, they present the event and the sample space. Later, variable $X$ is introduced as the number of pictures, and the probability is calculated. Since $X$ can have many values with a probability, $X$ is defined as a random variable. However, textbook B just gives the term random variable by definition, without more explanations. Each studied textbooks provide different definitions about the random variable as follows.

A : random variable is a variable that relates to the probability of random result (sample space) of an experiment with a real value, where there is only one value for each sample space.

B : random variable is a function that is mapping every member of sample space $S$ to a real number.

C : random variable is a variable whose value depends on the result of a random experiment

D : random variable is the numerical description of the result of a random experiment

E : random variable is a variable whose value is defined on a sample space of an experiment.

Among these textbooks, the definition in textbook B is the most formal way to define the random variable, while the illustration in D still exhibits an unclear phrase numerical description. Compared to [22], the report from textbook C and E are incomplete since they do not mention that the value of the random variable must be a real number.

Following the definition, textbook A, C, D, and E define the discrete and continuous random variable along with its examples. These textbooks also explain the concept of discrete probability function in various ways. In A, C, and E, the results of a random experiment are presented, a random variable $X$ is defined, and its probability is calculated. Based on this presentation, the books define the probability function, giving an example, and show the general properties of the probability function. On the other hand, textbook D started by giving a formal definition of a probability function and did not mention the distribution function anyway. All of these four books provide a worked example on evaluating whether a function is a distribution function and calculating probability from a density function (see figure 2).
3.3. Binomial distribution

The main topic in this section should be the binomial distribution. As mentioned in the above paragraph, several textbooks explain the definition of a random variable just before presenting the definition of the binomial probability distribution. However, there are some different contents in the binomial distribution chapter/section on these mathematics textbooks, as presented in Table 4.

Table 4. Topics in a binomial distribution.

| Topics                                           | A | B | C | D | E |
|--------------------------------------------------|---|---|---|---|---|
| Binomial experiment                              | ✔ |   |   |   |   |
| The probability function of the binomial distribution |   | ✔ | ✔ | ✔ |   |
| Mean and variance of the binomial distribution     |   |   | - | - | - |
| Read the binomial coefficient table               |   |   | ✔ | - | - |
| Usage of the binomial distribution to calculate the probability |   |   | ✔ | ✔ | ✔ |
| Solving binomial distribution with the computer (Excel) |   | ✔ | - | - | - |
| The cumulative distribution function of the binomial distribution |   |   | ✔ | - | - |
| Read cumulative binomial probability table        |   |   | - | - | ✔ |
| Poisson distribution                              | ✔ |   | - | - | - |
| Binomial hypothesis testing                       |   |   | - | - | - |

Note: ✔ = present; - = absent

In general, the textbooks contain a binomial experiment, followed by the probability function of the binomial distribution, and usage of the binomial distribution to calculate the probability of an event. Other materials were found only in several books, such as the calculation of Poisson distribution and hypothesis testing. Among all studied textbooks, the explanations started with random experiments with two outcomes, such as the result of tossing the coin, the odd/even number obtained from rolling dice, the sex of a neonate, etc. Such an experiment is called Bernoulli trials [22], which are mentioned in textbooks A, C, D, and E. Except in textbook B, the books present the calculation of the probability from a repeated Bernoulli trial, which is the basic concept of the binomial distribution. With various worked examples and exercises, all of these textbooks might help the student to understand the calculation of probability using the binomial distribution manually, and perhaps, with a table or software.

Besides, textbook A explains the Poisson distribution with the connection to the binomial distribution. The probability function for Poisson distribution is given with worked example, but without a table nor guide to calculating using computer software. Related to the binomial hypothesis testing, textbook A presents the definition of the null hypothesis and alternative hypothesis, followed by the
description of type I and type II error with its example. The book also offers a binomial hypothesis testing procedure. Compared to the other textbooks, these materials can be seen as enrichment for students and can be omitted without a loss of continuation to the next section or chapter.

3.4. Normal distribution
Introducing normal distribution to high school students require a good strategy since this is a relatively new concept that for them. At first, some textbooks, such as A, D, and E, explain the probability distribution and cumulative distribution function of a continuous random variable, as well as relations between them. Moreover, textbook A and D also provide worked examples related to the calculation of probability from a probability function or distribution function, as well as evaluate whether a function is a distribution function. Textbook D gives the definition of the mean (expected value), mode, median, and variance from a continuous random variable as well as its worked example and exercise.

| Table 5. Topics in a normal distribution. |
|------------------------------------------|
| Topics                                   | A | B | C | D | E |
| Notation for normal distribution         | ✓ | ✓ | ✓ | ✓ | ✓ |
| The probability density function of normal distribution | ✓ | - | ✓ | ✓ | ✓ |
| The cumulative distribution function of normal distribution | ✓ | - | - | - | ✓ |
| Properties of the normal distribution function | ✓ | - | ✓ | ✓ | ✓ |
| Effects of changing parameters to the shape of normal distribution | - | - | ✓ | - | - |
| The standard normal distribution function | ✓ | ✓ | ✓ | ✓ | ✓ |
| Calculate the area below the standard normal density function | ✓ | ✓ | ✓ | ✓ | ✓ |
| Worked examples of calculating probability using Z transform | ✓ | ✓ | ✓ | ✓ | ✓ |
| Find the percentile/quantile of normal distribution (inverse Z) | - | - | - | ✓ | ✓ |
| Characteristic of normally distributed data | ✓ | - | - | - | ✓ |
| Normal distribution as an approximation for the binomial distribution | - | - | ✓ | - | - |
| Central limit theorem (without proof) | ✓ | - | - | - | - |
| Two-tailed hypothesis test | ✓ | ✓ | ✓ | ✓ | ✓ |
| One-tailed hypothesis test | ✓ | ✓ | - | ✓ | ✓ |
| Steps of hypothesis testing | ✓ | ✓ | ✓ | ✓ | ✓ |
| Types of error in hypothesis testing | ✓ | ✓ | - | ✓ | - |
| Z statistics for the one-sample test of mean | ✓ | ✓ | - | ✓ | ✓ |
| t statistics for the one-sample test of mean | ✓ | ✓ | ✓ | - | ✓ |
| Reading t-table | ✓ | ✓ | - | ✓ | - |
| The probability density function of (student)-t distribution | ✓ | - | - | - | - |
| Worked examples of doing hypothesis testing | ✓ | ✓ | ✓ | ✓ | ✓ |

Note: ✓ = present; - = absent; * present in previous chapter

As shown in table 5, all mathematics textbooks studied here presented the notation and probability density function of the normal distribution, the standard normal distribution, and its usage to calculate the probability using z transform. It can be seen that the depth of the content differs among these textbooks. For example, the effects of changing parameters to the shape of normal distribution and the explanation of area below the normal curve in terms of μ and σ only found in textbook C. Otherwise, the calculation of percentile or quantile of the normal density function, i.e., find k if given the probability P(X < k) while X~N(μ,σ²). It is only found in textbooks D and E. All of these materials are found only in C, as well as D and E. It should be given to the student since it will provide a complete and roughly
understanding of the normal distribution. It should be noted that the explanations of the probability density function (pdf) of normal distribution requires the concept of definite integral and the natural number. This concept is beyond senior high school mathematics based on the 2013 curriculum. An exception can be made if they are placed as an enrichment competence for mathematics in grade XI.

All studied textbooks also present the two-tailed hypothesis test along with the procedure and the worked example of it. However, not all of them give the exact meaning of the significance level by explaining the presence of errors in hypothesis testing. There are some differences in using the test statistics: some textbooks give both Z and t-test, but the others only provide one of them. It also should be noted that in textbook A, B, and E, there are no explanations why the t-test should be used when the sample size is less than 30. This lack of reasons in the population parameter, presence and type of error, usage of test statistics, choice of hypothesis, and critical region might prevent the students from understanding the whole concepts of hypothesis testing without any misconception.

It should be noted that hypothesis testing is related to the concept of the sampling distribution [22], which is not found in any of the mathematics textbook studied here. However, this concept contains the idea of variation between samples which is the main reason for the statistical inference procedure [23]. The nature and behavior of sampling variation become the foundation of any conceptual approach to statistical inference [23]. Therefore, the lack of explanations about sampling distribution in these textbooks is a serious error that should be solved if the concept of hypothesis testing will be given.

Explanations about sampling distribution is not a difficult one. For example, by guiding the student to imagine a population consists of integers, for example, 1, 1, 2, 2, 8, 8, 9, 9. The average or mean of this population is 5. Imagine two different samples taken from this population, which consists of value 1, 1, 2, 8, and 2, 8, 8, 9. The average of these samples is 3 and 9, respectively. It can be deduced that the sample estimation is not always the same as the sample population, so that we must use the sampling distribution. Using this simple example, the student may understand the relations and differences between parameters and statistics.

4. Conclusion
The present study analyses the statistical inference related content in the mathematical textbooks for the students in senior high schools. Almost all books explain the concept of discrete and continuous random variables to introduce the probability distribution. At first, the binomial distribution, along with its usage in probability calculation, has well presented. Some books provide more insight for students by introducing the use of the probability table and spreadsheet to calculate the binomial probability. However, there are some incomplete explanations about normal distribution and hypothesis testing, which might prevent the student from understanding these concepts. Last, this study does not mean that one should buy or reject a mathematics textbook. Instead, we suggest to the teacher and students to use and compare several books to get complete information about this statistical competence.

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