Radiation via Tunneling in the Charged BTZ Black Hole

by

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ABSTRACT

It is often convenient to view Hawking (black hole) radiation as a process of quantum tunneling. Within this framework, Kraus and Wilczek (KW) have initiated an analytical treatment of black hole emission. Notably, their methodology incorporates the effects of a dynamical black hole geometry. In the current paper, the KW formalism is applied to the case of a charged BTZ black hole. In the context of this interesting model, we are able to demonstrate a non-thermal spectrum, with the usual Hawking result being reproduced at zeroth order in frequency. Considerable attention is then given to the examination of near-extremal thermodynamics.
1 Introduction

Roughly three decades ago, Bekenstein and Hawking established the well-known analogy that exists between black hole mechanics and thermodynamic systems. This intriguing relationship, which appears to have even deeper physical significance, has since been the subject of a “countless” number of investigations. However, in spite of all this attention, black hole thermodynamics continues to have several unresolved issues. For example, the microscopic origin of the Bekenstein-Hawking entropy is currently a prominent open question. (See Ref.[3] for a review and references.)

Although better understood than the origin of entropy, the mechanism of black hole radiance remains shrouded in some degree of mystery. To review, it was Hawking who first demonstrated that black holes radiate (via quantum effects) such that the emission spectrum (at infinity) is essentially thermal. It was this remarkable discovery that gave physical credibility to the thermodynamic analogy; but at a cost, as this result has dramatic implications regarding our understanding of quantum evolution. More specifically, the evaporation of a black hole in this manner implies that a pure state (the original matter that forms the black hole) can evolve into a mixed state (the thermal spectrum at infinity). Such an evolution is a violation of the fundamental principles of quantum theory, as these prescribe a unitary time evolution of basis states. This contradictory nature of black hole radiation has been often labelled as the “information loss paradox”.

The above paradox can perhaps be attributed to the semi-classical nature of such investigations, as a conspicuously absent quantum theory of gravity remains a formidable obstacle. However, there is another fundamental issue that must necessarily be dealt with; namely, energy conservation. It seems clear that a radiating black hole should be losing mass (which is directly related to the temperature), but this dynamical effect is often neglected in formal treatments.

To further explore the issue of dynamics, it proves convenient to adapt the viewpoint (commonly accepted, but often overlooked) that Hawking radiation is really a quantum tunneling process. According to this scenario, a pair of particles is spontaneously created just inside of the black hole horizon. One of the particles then “tunnels” out to the opposite side, where it emerges with positive energy. Meanwhile, the negative-energy “partner” remains behind and effectively lowers the mass of the black hole.
In a program of study that was initiated by Kraus and Wilczek (KW) [3, 4, 8] (since developed and generalized in Refs. [9, 10, 11, 12, 13, 14, 15]), this tunneling picture was the foundation for a dynamical treatment of black hole radiance. In a nutshell, KW considered the effects of a positive-energy matter shell (i.e., “s-wave”) propagating outwards through the horizon of a spherically symmetric black hole. The pertinent point of their work was the incorporation of a dynamical description of the black hole background. More specifically, the background geometry is allowed to fluctuate and thus support a black hole of varying mass. (That is, the “self-gravitation” of the radiation is taken into account.) Such formalism leads to the enforcement of energy conservation in a natural way. In particular, KW allow the black hole to lose mass while radiating, but maintain a constant energy for the total system.

Another salient point of the KW method was their choice of boundary conditions on the quantum matter fields. These are effectively enforced via the choice of coordinates that have been used to foliate the spacetime. In their analysis, KW implemented so-called “Painleve” coordinates that are not only time independent and regular at the horizon, but for which time reversal is manifestly asymmetric (unlike the often-studied Kruskal coordinates [18]). That is, the coordinates are stationary but not static. This gauge choice seems to be particularly appropriate for describing the geometry of a slowly evaporating black hole.

Let us summarize some important findings of the KW analysis, which will be exploited later in the paper. Firstly, a canonical Hamiltonian formulation (in the previously discussed framework) yields the remarkably simple result for the total action of the system:

$$I = \int d\tau \left[ \frac{dr}{d\tau} p_r + p_\tau \right].$$

(1)

Here, $r$ and $\tau$ are respectively the radial and temporal coordinate in the Painleve gauge, while $p_\mu$ represents the conjugate momentum of coordinate $\mu$. Secondly, a semi-classical (WKB) approximation leads to the following

1Also of interest is Ref. [16], where the tunneling picture has been applied via a different methodology.

2The coordinate system in question was first proposed by Painleve in 1921 [17].

3It should be kept in mind that such an approximation treats the radiating matter as
expression for the emission rate ($\Gamma$):

\[- \frac{1}{2} \ln(\Gamma) \approx \text{Im}\mathcal{I},\]

where only the first term in Eq.(1) contributes to the imaginary part of the action. With these expressions, it can be shown that the spectrum of black hole radiation is not strictly thermal; rather, it contains a frequency-dependent “greybody” factor. (Although this greybody factor is well known \cite{4}, it is often neglected in the relevant literature.)

The purpose of the current paper is to examine the thermodynamics of a charged BTZ black hole via a method based on the KW formalism. The BTZ theory refers to special solutions of 2+1-dimensional anti-de Sitter gravity having all of the properties of black holes. (These solutions were first identified by Banados, Teitelboim and Zanelli \cite{19}. ) Charged BTZ black holes are simply the analogous solutions in 2+1-dimensional AdS-Maxwell gravity \cite{20,21}. Although a “toy” model in some respect, the BTZ black hole has stirred significant interest by virtue of its connections with certain string theories \cite{22} and its role in microscopic entropy calculations \cite{23}. Furthermore, the BTZ model has proven to be an especially useful “laboratory” for studying quantum-corrected thermodynamics \cite{24,25,26,27,28,29}.

This paper is organized as follows. In Section 2, we begin by introducing the model of interest and the solution corresponding to a charged BTZ black hole. This is followed by a semi-classical calculation of the black hole emission rate from which the spectrum can be directly extrapolated. In Section 3, we elaborate on our results; particularly, in the context of near-extremal black holes. It is worth noting that the KW formalism has particular importance near the extremal limit, where even the smallest changes in the black hole mass can significantly deform the background geometry. Section 4 ends with a brief summary and concluding remarks.
2 Analysis

The model of interest, 2+1-dimensional AdS-Maxwell gravity, can be described by the following gravitational action:

\[
I_G = \frac{1}{4} \int d^3x \sqrt{-g} \left[ \frac{1}{4\pi G} (R - 2\Lambda) - F^{\mu\nu} F_{\mu\nu} \right].
\] (3)

Here, \( G \) is the 3-dimensional Newton constant (with dimensions of inverse mass) and \( \Lambda = -l^{-2} \) is the negative cosmological constant. Note that we are assuming vanishing rotation for the sake of simplicity.

A static, charged black hole solution has been found for the above action \([13, 20, 21]\). This can be expressed as follows:

\[
ds^2 = -N^2(r) dt^2 + N^{-2}(r) dr^2 + r^2 d\phi^2,
\] (4)

where:

\[
N^2(r) = \frac{r^2}{l^2} - 8GM - Q^2 \ln \left( \frac{r^2}{l^2} \right).
\] (5)

Here, \( M \) is the generalized ADM mass \([30]\) of the charged BTZ black hole and \( Q \) is a dimensionless parameter that represents the charge.

It will often prove convenient to re-express Eq.(5) in the following form:

\[
N^2(r) = \frac{r^2}{l^2} - \frac{r_+^2}{l^2} - Q^2 \ln \left( \frac{r^2}{r_+^2} \right),
\] (6)

where \( r_+ \) (the black hole horizon) is the outermost value of \( r \) for which \( N^2(r+) = 0 \). Typically, there will exist some second value \( r_- \leq r_+ \) such that \( N^2(r) = 0 \) as well.

To obtain the condition of extremality, \( r_- = r_+ \), let us consider the Hawking temperature \( (T_H) \) as determined by the surface gravity \( (\kappa) \) at the horizon \([31]\):

\[
T_H = \frac{\kappa}{2\pi} = \frac{1}{4\pi} \frac{dN^2}{dr} \bigg|_{r=r_+} = \frac{r_+}{2\pi l^2} \left( 1 - \frac{l^2 Q^2}{r_+^2} \right).
\] (7)

It follows that the black hole becomes extremal when \( T_H \) vanishes or at:

\[
(r_+)^2_{ext} = l^2 Q^2.
\] (8)
With black hole emission in mind, we now begin a semi-classical calculation that is based on the Kraus-Wilczek treatment \cite{6, 7, 8}. (Also see Refs.\cite{9}-\cite{14}.) First, it is appropriate to re-express the metric in a form that is manifestly stationary (but not static) and regular at the horizon. These “Painleve-like” coordinates \cite{17} can be obtained with the following redefinition of the time coordinate:

$$d\tau = dt - \frac{1 - N^2(r)}{N^2(r)} dr.$$  \hspace{1cm} (9)

Substituting into Eq.(4), we now have:

$$ds^2 = -N^2(r)d\tau^2 + dr^2 + 2\sqrt{1 - N^2(r)}d\tau dr + r^2d\phi^2.$$ \hspace{1cm} (10)

It is useful to evaluate the radial, null geodesics. Under these conditions \((d\phi = ds^2 = 0)\), Eq.(10) reduces to:

$$0 = -N^2(r) + \dot{r}^2 + 2\sqrt{1 - N^2(r)}\dot{r},$$ \hspace{1cm} (11)

where \(\dot{r} = dr/d\tau\). Solving for \(\dot{r}\), we find:

$$\dot{r} = \pm 1 - \sqrt{1 - N^2(r)},$$ \hspace{1cm} (12)

where the +/- sign can be identified with outgoing/incoming radial motion.

Next, we consider a self-gravitating shell of positive energy \((\omega)\) radiating outwards through the black hole horizon. For simplicity, we assume a shell having zero rest mass, zero charge and symmetry with respect to the angular coordinate. Our viewpoint will be that the total mass of the system stays fixed, while the black hole mass varies according to \(M \rightarrow M - \omega\). It then follows that the shell of energy travels along geodesics which are described by:

$$\dot{r} = 1 - \sqrt{1 - N^2(r, M - \omega)}.$$ \hspace{1cm} (13)

In our analysis, the primary interest is the semi-classical emission rate of this shell-charged BTZ system. That is (cf. Eq.(2)):

$$\Gamma(\omega) = e^{-\omega/T(\omega)} \approx e^{-2mT},$$ \hspace{1cm} (14)

where \(T(\omega)\) is the temperature at “frequency” \(\omega\).
For an positive-energy “s-wave” propagating outwards, KW have shown that the imaginary part of the total action can be expressed as

$$\text{Im} \mathcal{I} = \text{Im} \int d\tau \dot{r} p_r = \text{Im} \int_{r_{\text{in}}}^{r_{\text{out}}} \int_0^{p_r} dp_r' \, dr,$$

(15)

where $p_r$ is the canonical momentum (conjugate to $r$) and the total action includes the gravitational action ($\mathcal{I}_G$) along with the action for the shell.

At this point, it is useful to apply Hamilton’s equation: $\dot{r} = dH/dp_r = d(M - \omega)/dp_r$. Hence, Eq.(15) can be re-written in the following manner:

$$\text{Im} \mathcal{I} = \text{Im} \int_{r_{\text{in}}}^{r_{\text{out}}} \int_0^\omega \frac{-d\omega' dr}{r(r, M - \omega')} = \text{Im} \int_{r_+(M)}^{r_+(M-\omega)} \int_0^\omega \frac{-d\omega' dr}{1 - \sqrt{1 - \frac{r^2}{l^2} + 8G [M - \omega'] + Q^2 \ln \left(\frac{r^2}{l^2}\right)}},$$

(16)

where we have applied Eqs.(13) in attaining the lower line.

The integration over $\omega'$ can be readily done as a contour integral. Significantly to this calculation, the correct sign is obtained via the requirement that $\omega \rightarrow \omega - i\delta$ (where $\delta > 0$). This choice ensures that the positive-energy solution ($\sim e^{-i\omega \tau}$) decays in time.

For the explicit evaluation of this integral, let us first change variables, $\mu = 1 - \frac{r^2}{l^2} + 8G [M - \omega] + Q^2 \ln \left(\frac{r^2}{l^2}\right)$, to obtain the following form:

$$\text{Im} \mathcal{I} = \text{Im} \int_{r_+(M)}^{r_+(M-\omega)} \int_{\mu(0)}^{\mu(\omega)} \frac{d\mu'}{1 - \sqrt{\mu^2} 8G}.$$

(17)

Given that the above condition (on $\omega$) implies $\mu \rightarrow \mu + i\tilde{\delta}$ (where $\tilde{\delta} > 0$) and that $\mu(\omega) < \mu(0)$, it is appropriate to integrate clockwise in the upper half of the complex-$\mu'$ plane. This process yields:

$$\text{Im} \mathcal{I} = -\frac{\pi}{4G} \int_{r_+(M)}^{r_+(M-\omega)} dr = \frac{\pi}{4G} [r_+(M) - r_+(M - \omega)].$$

(18)

Although the original KW analysis was for a spherically symmetric system in an asymptotically flat spacetime, this formalism has since been generalized for AdS spacetimes with an arbitrary number of dimensions.
We will now proceed to evaluate the quantity $r_+(M - \omega)$ via the following argument. First, there must exist some real function $\eta = \eta(\omega)$ such that $r_+^2(M) \to r_+^2(M) - \eta^2$ as $M \to M - \omega$. A comparison of Eqs.(5,6) then yields the relation:

$$
\frac{r_+^2(M) - \eta^2}{l^2} - 8G[M - \omega] - Q^2\ln \left(\frac{r_+^2(M) - \eta^2}{l^2}\right) = 0. \quad (19)
$$

Eliminating the zeroth-order terms, we have:

$$
8G\omega = \frac{\eta^2}{l^2} + Q^2\ln \left(1 - \frac{\eta^2}{r_+^2(M)}\right). \quad (20)
$$

For a sufficiently large black hole (as is appropriate for a semi-classical analysis), it follows that $\eta^2 << r_+^2(M)$. Hence, we can expand the above logarithm to obtain:

$$
\eta^2 \approx \frac{8Gl^2r_+^2(M)}{r_+^2(M) - l^2Q^2\omega}. \quad (21)
$$

Recalling Eq.(8) for the extremal limit ($r_+^2(M) = l^2Q^2$), we have an apparent breakdown in the formalism when this limit is approached. This is not surprising, as black hole geometries are known to be altered dramatically in this limiting case [32, 33]. To examine this issue more carefully, let us consider the next term in the logarithmic expansion. This addition results in a quadratic expression for $\eta^2$, which can be solved to yield:

$$
\eta^2 = \frac{r_+^2(M) - l^2Q^2}{l^2Q^2} \pm \sqrt{\frac{[r_+^2(M) - l^2Q^2]^2}{l^4Q^4} - \frac{16Gr_+^2}{Q^2\omega}}. \quad (22)
$$

Since $\omega$ is presumed to be positive, the square root tends to an imaginary quantity in the extremal limit (unless $\omega = 0$). Thus, the extremal breakdown in the formalism appears to persist. The problem may be linked to the prior assumption of $\eta^2 << r_+^2(M)$. It is possible that this is not a valid constraint when probing the extremal condition. We conjecture, however, that it will always be possible to choose a sufficiently large enough black hole so that this condition is valid even in a near-extremal regime. In this case, it follows from Eq.(22) that $\omega$ (and, hence, $\eta^2$) goes rapidly to zero as the extremal limit is approached. The obvious implication is that a (sufficiently large) black hole will cease to radiate as it approaches extremality (which also follows from the
third law of thermodynamics). This point will be elaborated on in Section 3; until then, we focus on black holes far from extremality.

Incorporating the result of the above analysis \((21)\) into Eq.\((18)\), we obtain:

\[
ImI \approx \frac{\pi}{4G} \left[ r_+(M) - \sqrt{r_+^2(M) - \frac{8Gl^2r_+^2(M)}{r_+^2(M) - l^2Q^2}} \right]. \tag{23}
\]

The square root can be expanded to yield:

\[
ImI \approx \omega \left[ \frac{\pi l^2r_+(M)}{r_+^2(M) - l^2Q^2} - \frac{2\pi Gl^4r_+(M)}{[r_+^2(M) - l^2Q^2]^2} \omega + \ldots \right], \tag{24}
\]

where “…” represents the higher-order (in \(\omega\)) corrections.

By recalling from Eq.\((14)\) that \(2ImI \approx \omega/T(\omega)\), we are now able to deduce the black hole temperature at any given frequency:

\[
T(\omega) \approx \frac{r_+^2(M) - l^2Q^2}{2\pi l^2r_+(M)} \left[ 1 + \frac{2Gl^2}{r_+^2(M) - l^2Q^2} \omega + \ldots \right], \tag{25}
\]

where the higher-order terms \((\ldots)\) are essentially an expansion in powers of \(\omega/(r_+^2(M) - l^2Q^2)\). Notably, the zeroth-order terms reproduce the expected value for the Hawking temperature; cf. Eq.\((7)\). However, the higher-order quantum corrections, which are clearly non-vanishing, lead to a frequency-dependent “greybody” factor. That is, the emission spectrum deviates from that of a pure black body. Strictly speaking, the black hole emits non-thermal radiation!

A useful check on this formalism follows from the first law of thermodynamics. Consider that, during black hole emission, the expected change in entropy is given by \(\Delta S = \Delta M/T = -\omega/T\). That is (cf. Eq.\((14)\)):

\[
\Delta S = -2ImI. \tag{26}
\]

An inspection of Eq.\((18)\) thus yields:

\[
S(M) = \frac{\pi r_+(M)}{2G} + \text{constant}. \tag{27}
\]

For a vanishing constant, this is just the Bekenstein-Hawking area law \([1,2]\) of \(S = A_+/4G\) (generalized to a 3-dimensional black hole).
3 Discussion

It is instructive to consider the following observation: the black hole emission rate, $\Gamma \approx e^{-2ImI}$, is a measurable and (hence) real quantity. This restricts the square root in Eq. (23) to be a real quantity as well. Thus, naively, it follows that:

$$r_+^2(M) - (r_+^2)_{\text{ext}} \geq 8G\ell^2\omega.$$  \hfill (28)

We say “naively” because of the formal breakdown that occurred in the extremal limit. (See the prior section.) However, our conjecture that constrains the horizon geometry of sufficiently large black holes, $\eta^2 << r_+^2(M)$, effectively implies the same condition. (Again, see last section.) So, at least on the basis of these arguments, radiation past extremality is impossible; thus demonstrating a natural enforcement of the third law of thermodynamics.

To put it another way: if the black hole ceases to radiate in the extremal limit, then the classical result of vanishing extremal temperature remains essentially valid, and (hence) the emission rate vanishes exponentially as this limit is approached. That is, there is zero possibility of a black hole decaying into a naked singularity!

Although naked singularities are censored against, one might still wonder if a state of absolute extremality can be achieved. This is a difficult dilemma to resolve, one way or the other. However, studies elsewhere in the literature have argued that extremal and non-extremal black holes are qualitatively distinct objects \[32, 33, 34\]. Such arguments are primarily based on the topological differences that exist between extremal and non-extremal spacetimes. Moreover, these differences seem to imply that a non-extremal black hole would not be able to continuously deform into an extremal one (and vice versa)\[^5\]. Also of note, this viewpoint has been substantiated by a recent investigation into the physical spectra of charged black holes \[36\]: generically, extremal black holes can not be achieved (at the quantum level) due to vacuum fluctuations in the horizon.

For the sake of argument, let us accept the above conjecture and also accept that a black hole can not come arbitrarily close to an extremal state\[^6\].
Furthermore, let us assume that the black hole “freezes” at a point suitably far from extremality. In this case (and with charge regarded as a fixed quantity), the prior formalism can be shown to yield lower bounds on the black hole temperature and entropy. We demonstrate this as follows.

First, if the smallest allowed quanta of energy is taken to be $\epsilon$, then Eq. (28) leads to the following near-extremal (“ne”) limit for the horizon radius:

$$ (r_+)^2_{\text{ne}} \approx l^2 Q^2 + 8 G l^2 \epsilon. \quad (29) $$

By substituting this relation into Eq. (25), we can obtain a near-extremal bound on the temperature.

To first order in $\epsilon$, the following is found:

$$ T_{\text{ne}} \approx \frac{4 G}{\pi l \sqrt{Q^2}} \epsilon. \quad (30) $$

Similarly, we can obtain a near-extremal bound on the entropy:

$$ S_{\text{ne}} \approx S_{\text{ext}} + \frac{2 \pi}{\sqrt{Q^2}} \epsilon, \quad (31) $$

where $S_{\text{ext}} = \pi \sqrt{l^2 Q^2}/2G$.

Let us take note of a related work that has recently been carried out by Vagenas. The viewpoint of this study was the existence of a well-defined extremal limit. On this basis, Vagenas has proposed that extremal thermodynamics are calculated by: (i) fixing $\omega$ such that the condition of extremality ($r_– = r_+$) is satisfied for $M – \omega$ (rather than $M$) and then (ii) using this extremal constraint to eliminate $\omega$ from the thermodynamic expressions.

If we fix $\omega$ (or, equivalently, $\eta$) in an analogous fashion, then the following extremal constraint can be obtained:

$$ \eta^2_{\text{ext}} = (r_+)^2_{\text{ext}} - l^2 Q^2, \quad (32) $$

where $(r_+)^{\text{ext}}$ is now the “revised” extremal horizon. Substituting this constraining relation into Eq. (19), we find:

$$ \omega_{\text{ext}} = M + \frac{Q^2}{8G} \left[ \ln \left( \frac{Q^2}{l^2} \right) - 1 \right]. \quad (33) $$

---

7 One might expect such quanta to be roughly on the order of the Planck mass.

8 It is interesting to note that a similar bound was arrived at by a much different rationalization in Ref. [37].

9 Vagenas applied the KW program to a “string-inspired” (charged) dilaton theory [14] and a rotating (but not charged) BTZ model [15].
This result implies that \( \omega_{ext} \) will be at least on the order of \( M \), which directly contradicts our previous observation (applicable to sufficiently large black holes): \( \omega \to 0 \) as the extremal limit is approached.

The contradictory behavior of the extremal limiting case may be a consequence of an apparent pathology in the charged BTZ black hole. (This pathology is described in Section 4.) However, this extremal breakdown may rather be a manifestation of the third law of thermodynamics; that is, an extremal limit of a non-extremal calculation may actually be erroneous procedure. In the latter case, Eqs.(30,31) can be regarded, at least in some schematic sense, as the correct near-extremal bounds on the temperature and entropy.

4 Conclusion

In the preceding paper, we have considered the thermal emission from a charged (non-rotating) BTZ black hole. Our analysis followed the viewpoint that Hawking radiation is due to a tunneling process and was appropriately based on the methodology of Kraus and Wilczek \([6, 7, 8]\). The pertinent point of this approach is that black hole radiance is a dynamical mechanism for which energy conservation must be enforced.

A rigorous application of the prescribed method allowed us to verify the “standard” results: the Hawking temperature (at zeroth order in frequency) and the Bekenstein-Hawking entropy \([1, 2, 4]\). Furthermore, the black hole temperature was found to have frequency-dependent corrections (i.e., a greybody factor), which means that the emission spectrum actually deviates from thermality. Often, the relevant literature conveniently depicts the black hole spectrum as being perfectly thermal, although it is well known that this is not actually the case \([1]\).

In general, one can evaluate such greybody factors (at a semi-classical level) by solving the appropriate Klein-Gordon equation on a fixed black hole background \([38]\). Notably, such a calculation has been considered for a rotating BTZ black hole \([39, 40]\). Although it would be interesting to compare these results with our derivations, this seems impractical given the complexity of their expressions and the lack of compatibility with the charged BTZ scenario.

We also considered the case of black hole extremality and found that
naked singularities are forbidden by this formalism in a natural way. Using
the premise that extremal and non-extremal black holes are qualitatively
distinct entities [32, 33] (and other conjectural considerations), we were able
to evaluate “near-extremal” limits to the black hole temperature and entropy.
Meanwhile, the alternate viewpoint (of a well-defined extremal limit) gave
rise to an apparent contradiction.

Before concluding, a couple of points are in order. Firstly, it has been
suggested that the charged BTZ black hole is a somewhat pathological model
[20]. The reasoning is as follows: (i) it exists for arbitrarily negative values of
mass and (ii) there is no upper bound on the electric charge. (Such behavior
is contrary to, for instance, that of the Reissner-Nordstrom black hole.) We
point out, however, that a charged black hole will tend to discharge by prefer-
entially emitting charged particles of the same sign as itself [41]. Hence,
black hole emission may serve as a natural mechanism for effectively sup-
pressing the charge (relative to the mass). Perhaps, such a mechanism would
be analogous to that which censors naked singularities. This question could
possibly be addressed in a more realistic (but more complex) study.

Secondly, we again remind the reader that the preceding study was a
semi-classical analysis. In fact, the formal methods of Kraus and Wilczek
are akin to a WKB approximation, meaning that the radiation should be
treated as point particles. Such an approximation can only be valid in a
regime of sufficiently large black holes. If we are to properly address the
thermodynamics of microscopic black holes, then a better understanding of
physics at the Planck scale is a necessary prerequisite.

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