Experimental validation of depolarizing Mueller matrix model via \textit{ex vivo} colon samples

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Abstract. Experimental validation of a previously reported depolarizing Mueller matrix model is crucial to understanding the light-tissue interactions and the morphological alterations originating in malignant tissue zones in terms of their optical properties. This manuscript is a continuation of a previously reported case study, where a theoretical model was introduced with an additional Monte Carlo simulation. The main aim of this study is to empirically validate both theory and modelling. Once extracted from an experimental Mueller matrix via symmetric decomposition, the polarimetric quantities were compared with the corresponding parameters previously determined from polarized Monte Carlo simulations. Our results were obtained from an \textit{ex vivo} colon sample and indicate a potential capability to provide supplementary polarimetric data to physicians and the gold standard histopathology analysis.

1. Introduction
Polarimetry has established its place as a metrology tool for material characterization by providing information about the inner structure of the test samples under examination.[1, 2, 3] Although highly efficient for homogeneous and isotropic samples, when probing anisotropic and heterogeneous structures an ambiguity in results’ interpretation may arise, followed by a lack of repeatability. Extensive data were already amassed in the numerous studies of turbid media by polarimetry. It allowed physicians to apply this optical technique to supplement regular medical diagnostics [4, 5, 6], in order to lessen invasiveness and avoid toxicity of contrast agents and hazardous impact of ionizing radiation. Thus, tissue polarimetry relies on a fairly simple diagnostic approach by probing tissue under examination with known input polarization state of light and tracing changes in the output polarization state altered by a tissue sample, whereafter the transmitted, reflected and/or scattered photons with modified polarization states carry the diagnostic information. However, extracting in a straightforward manner the diagnostic information is not simple and trivial task. Different polarimetric techniques can be implemented, such as Stokes[7, 8, 9], Stokes-Mueller matrix analysis[10, 11] with additional combination of matrix decomposition techniques[12, 13]. The latter is required due to the intrinsic complex structure of turbid media associated with the non-linear dependence of the Mueller matrix elements on the tissue optical properties under examination[14]. A phenomenological and theoretical model has been introduced earlier[15], which satisfactorily describes both phase and amplitude anisotropy, alongside light depolarization in turbid media, originating from multiple scattering events. Subsequently, in [15] a Monte Carlo simulation was presented, taking into account all physical restrains of the experimental conditions. Additionally, reference polarimetric parameters were evaluated all of which were related to the matrix elements from the model used. The indirect measurement of Mueller matrix (i.e. its reconstruction from the set of the Stokes parameters
measured with Stokes polarimeter) has been described in details in [16]. The current manuscript is focused entirely on the comparison between the simulated and the experimentally measured matrices from an ex vivo colon sample that contains healthy and malignant zones. The results are briefly discussed in Section 4.

2. Materials and methods
Whereas Stokes-Mueller algebra is well described in detail [17], the experimental setup is essentially the same as that in [15]. The specimen selected for the polarimetric examination and differentiation of the tumor type is described in [15]. It represents a 1-mm thick ex vivo human colon tissue with both healthy and tumor zones. The histology analysis performed in the Queen Joanna-ISUL University Hospital in Sofia (collaboration with IE-BAS under local Ethical Committee consent #286/2012) has confirmed that the malignant zone of the specimen represents the G2-adenocarcinoma (T2, N0, M0). The diagnosis of a pathologist was used as the ground truth to assess the accuracy of optical polarimetric diagnostics. Prior to any optical measurements, the tissue specimen has been kept in formalin. For the sake of brevity, both the Stokes-Mueller algebra and schematic representation of the optical system are omitted from description and can be found in the references provided above.

3. Mueller matrix model

\[
M_{\Delta} = \text{diag}(1, d_1, d_2, d_3); M_R = \begin{pmatrix}
1 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 \\
0 & 0 & c(\varphi) & s(\varphi) \\
0 & 0 & -s(\varphi) & c(\varphi)
\end{pmatrix}; M_D = \begin{pmatrix}
1 & 0 & 0 & 0 \\
D & 1 & 0 & 0 \\
0 & 0 & \sqrt{1-D^2} & 0 \\
0 & 0 & 0 & \sqrt{1-D^2}
\end{pmatrix},
\]

(1)

where \(M_{\Delta}\) is a diagonal depolarizer with depolarization coefficients \(d_i\), \(M_R\) is a horizontal linear retarder with the retardance \(\varphi\) and \(M_D\) is a horizontal linear diattenuator with the diattenuation \(D\), \(c \equiv \cos\) and \(s \equiv \sin\). \(M_R\) and \(M_D\) are defined for the particular cases of linear retardation and diattenuation, respectively, but the symmetric decomposition described by Eq. 2 can be applied for the elliptical retarders and diattenuators. Any physically realizable Mueller matrix [18] can be decomposed symmetrically by placing the depolarizer between two diattenuators and retarders.

\[
M_s = M_{R_i}M_{D_2}M_{D_1}M_{D_1}M_{R_i}
\]

(2)

Substitution of the matrices from Eq. 1 into Eq. 2 yields:

\[
M = \begin{pmatrix}
1 + d_1 D^2 & D(1 + d_1) & 0 & 0 \\
D(1 + d_1) & D^2 + d_1 & 0 & 0 \\
0 & 0 & (1 - D^2)(d_2 c^2(\varphi) - d_3 s^2(\varphi)) & c(\varphi)s(\varphi)(1 - D^2)(d_2 + d_3) \\
0 & 0 & -c(\varphi)s(\varphi)(1 - D^2)(d_2 + d_3) & (1 - D^2)(d_1 c^2(\varphi) - d_2 s^2(\varphi))
\end{pmatrix}
\]

(3)

4. Results
Each subscript in the following matrix notations refers to the origin of the corresponding Mueller matrix: sim - Monte Carlo simulation, H/T - Healthy/Tumor colon tissue.

4.1. Monte Carlo simulation

\[
M_{\text{sim}} = \begin{pmatrix}
1 & -0.201 & 0.000 & -0.001 \\
-0.029 & 0.584 & 0.000 & -0.001 \\
0.000 & 0.000 & -0.584 & -0.027 \\
0.000 & 0.000 & 0.020 & -0.691
\end{pmatrix}
\]

(4)
4.2. Colon Sample

\[
M_H = \begin{bmatrix}
1 & -0.065 & 0.028 & 0.041 \\
-0.055 & 0.110 & 0.009 & 0.007 \\
0.009 & 0.021 & -0.074 & 0.006 \\
0.019 & 0.001 & -0.001 & -0.006
\end{bmatrix}
\]  

\[
M_T = \begin{bmatrix}
1 & -0.051 & -0.002 & 0.048 \\
-0.067 & 0.100 & 0.001 & -0.006 \\
0.001 & 0.011 & -0.063 & 0.006 \\
-0.003 & 0.001 & -0.006 & -0.021
\end{bmatrix}
\]  

(5) (6)

5. Discussion

As can been seen from Eq.4 through Eq.6, all matrices follow the form of the matrix in Eq.3. One should notice that in our simulations, a mono-disperse spatially uniform distribution of the spherical scatterers with fixed refractive index was assumed within the homogeneous host medium, again with akin index of refraction, whereas this assumption corresponds to a very simplified optical model of a biological tissue. Therefore, the modeling results demonstrate 2 x 2 zero-th off-diagonal blocks as predicted by theory, while those elements slightly differ from zero in the experimental matrices from the biological specimen. Before applying the symmetric decomposition algorithm, all matrices were tested positively for physical realizability. In compliance to this condition, all polarimetric parameters can be extracted from the Mueller matrices that represent the “building blocks’ of the symmetric decomposition. Its evaluation is presented in the following table, where the net depolarization coefficient \( \Delta \) can be calculated from \[19\]:

\[
\Delta = 1 - \frac{|d_1| + |d_2| + |d_3|}{3}, \quad 0 \leq \Delta \leq 1,
\]  

(7)

Table 1: Comparison of polarimetric parameters

| Pol.par. | MC     | H     | T     |
|----------|--------|-------|-------|
| \(D_1\) [a.u.] | 0.007  | 0.078 | 0.066 |
| \(D_2\) [a.u.] | 0.026  | 0.055 | 0.063 |
| \(\varphi_1\) [deg] | 0.85   | 5.78  | 4.71  |
| \(\varphi_2\) [deg] | 2.94   | 1.21  | 4.01  |
| \(d_1\) [a.u.] | 0.584  | 0.110 | 0.098 |
| \(d_2\) [a.u.] | -0.585 | -0.075| -0.063|
| \(d_3\) [a.u.] | -0.692 | -0.007| -0.022|
| \(\Delta\) [a.u.] | 0.379  | 0.936 | 0.939 |

It can be seen from the Table 1, that both the diattenuation and retardance parameters \(D_{1,2} \& \varphi_{1,2}\) calculated from the corresponding Mueller matrices \((M_{D_{1,2}} \& M_{R_{1,2}})\) are comparable for the tumor tissue zone, but differ significantly for the healthy tissue zone (i.e. tumor tissue is more isotropic in terms of both absorption and retardation). Although the net depolarization coefficients are similar for both tissue zones, taking into account the absolute values of \(d_i\) one can easily observe them in the same order of magnitude for the tumor zone, while differing by an order of magnitude for the healthy zone. The latter relation can be attributed to an alteration in tissue anisotropy by tumors that destroy the finely ordered fabric of a healthy tissue. Nevertheless, for both tissue zones \(d_3\) has the lowest value, contributing to the increased circular
depolarization compared to linear one, thus giving a rise to better applicability of circular polarization for healthy versus tumor tissue discrimination, with respect to the current experimental geometry of the optical setup and the scattering regime, respectively. By measuring the parameters describing the optical properties of the colon sample (i.e. absorption/scattering coefficients, anisotropy factor and index of refraction) and then subsequently refining the optical model of tissue used in polarized Monte Carlo simulations, one could be able to reproduce more precisely the polarimetric parameters from the physical measurements. Despite this inevitable recursive factor, the simulation remains an adequate and powerful validation of the theoretical model.

6. Conclusion
In this work, a validation of previously reported depolarizing Mueller matrix model was done by means of physical measurements and corresponding polarized Monte Carlo simulations. The latter unambiguously justifies the theoretically predicted form of Mueller matrices with 2 x 2 zero-th off-diagonal blocks. The application of the Mueller matrix decomposition algorithm for all Mueller matrices allows us to extract valuable information on the inner structure of a sample under investigation. For example, one can better differentiate between healthy and malignant tissue zones. To sum up, our results from both the current and prior studies [15], suggest that Stokes and Mueller matrix polarimetry may be employed as a complementary technique for supporting physicians diagnostics. By increasing the number of measurements and samples used, eventually a detailed and profound statistical analysis needs to be carried out for acquiring more reference polarimetric parameters for screening depolarization properties of tissue specimens with different health conditions. Nevertheless, one should take into account the advantages and limitations of polarimetry [20] as such knowledge can significantly enhance results interpretation, reduce measurements errors and etc. An accurate categorization of the available polarimetric instrumentation should be established, alongside the experimental geometry used. Both aspects are crucial to define the scattering regime (ballistic or diffuse) and the proper selection of the comprising optical elements. The latter could determine the spectral region for measurements, as liquid crystal-based polarizing elements are efficient in the visible range, but under-perform in the UV range [21]. Thus, it becomes very important whether the polarization information is to be obtained as a function of the wavelength or extracted either from scanning or imaging. Additionally, by knowing the optical properties of a given sample and its inner structure one can make approximate estimation of the size of scatterers (Mie or Rayleigh regime), which influences the choice of the input polarization used [22]. Finally, one should be alert that when probing such complex, heterogeneous structures both elastic and inelastic scattering modes occur. The former is sensitive to the morphological alterations, while the latter, however beyond the scope of the current study, can evaluate biochemical modifications in tissues [23]. In combination, both can provide anisotropy information about the tissue zones investigated and can enrich even further the diagnostic support to physicians [24, 25].

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Conflict of Interests & Financial Disclosure
Neither financial, nor any conflict of interests are declared by the Authors of this manuscript.
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