Magnetoconductance of the Corbino disk in graphene: chiral tunneling and quantum interference in the bilayer case

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Abstract
Quantum transport through an impurity-free Corbino disk in bilayer graphene is investigated analytically, using the mode-matching method to give an effective Dirac equation, in the presence of uniform magnetic fields. Similarly as in the monolayer case (see Rycerz 2010 Phys. Rev. B 81 121404; Katsnelson 2010 Europhys. Lett. 89 17001), conductance at the Dirac point shows oscillations with the flux piercing the disk area \(\Phi_D\) characterized by the period \(\Phi_0 = 2(h/e) \ln(R_o/R_i)\), where \(R_o (R_i)\) is the outer (inner) disk radius. The oscillation magnitude depends either on the radii ratio or on the physical disk size, with the condition for maximal oscillations being \(R_o / R_i \sim [R_i t_\perp / (2\hbar v_F)]^{4/p}\) (for \(R_o / R_i \gg 1\)), where \(t_\perp\) is the interlayer hopping integral, \(v_F\) is the Fermi velocity in graphene, and \(p\) is an even integer. Odd-integer values of \(p\) correspond to vanishing oscillations for the normal Corbino setup, or to oscillation frequency doubling for the Andreev–Corbino setup. At higher Landau levels, magnetoconductance behaves almost identically in the monolayer and bilayer cases. A brief comparison with the Corbino disk in a two-dimensional electron gas is also provided in order to illustrate the role of chiral tunneling in graphene.

Keywords: graphene, graphene bilayer, magnetoconductance, Corbino geometry, quantum transport

(Some figures may appear in colour only in the online journal)

1. Introduction

The potential of bilayer graphene (BLG) for carbon-based electronics rests on the possibility for controlling its transport properties using external electromagnetic fields employing the mechanisms that have no analogs in monolayer graphene (MLG) or in semiconducting heterostructures containing two-dimensional electron gas (2DEG) (for a recent review of the topic, see [3]). BLG with an \(AB\) stacking order can be converted from a semimetal to a narrow gap semiconductor by applying a perpendicular electrostatic field [4–8]. This is possible, because: (i) the interlayer hopings break the sublattice symmetry in a single layer, leading to the formation of two parabolic chiral bands touching themselves at the so-called Dirac points [4]\(^1\), and (ii) the perpendicular electric field further breaks the inversion symmetry, opening a gap between conduction and valence bands. Several experiments on dual-gated devices in ultraclean BLG have pursued the possibility of exploiting such a field-tunable energy gap [9–15]. Yan and Fuhrer [11] used the Corbino geometry, proposed over a century ago, to measure the magnetoresistance without generating the Hall voltage (for a historical introduction, see [16]). In such a geometry (see figure 1), the current is passed through a disk-shaped sample surrounded on both its exterior and interior sides with metallic leads, which suppress the influence of boundary modes [17] on various dynamical properties of nanosystems in both BLG and MLG [18–21].

\(^1\) Skew-interlayer hopings may lead to the appearance of secondary Dirac points affecting the transport properties of unbiased samples, but a gap still opens when applying the electrostatic field; see [3].
From a more fundamental point of view, several relativistic quantum effects, observed for MLG and resulting from the chiral nature of effective quasiparticles, are predicted to manifest themselves in BLG in slightly modified versions, mainly due to the presence of a new characteristic length scale for low-energy excitations [22]

\[ l_\perp = h v_F / l_\perp \approx \xi_0, \]  

(1)

where \( v_F \approx 10^6 \text{ m s}^{-1} \) is the energy-independent Fermi velocity in MLG, \( l_\perp \approx 0.4 \text{ eV} \) is the nearest-neighbor interlayer hopping integral, and \( \xi_0 = 0.142 \text{ nm} \) is a C–C bond length. For instance, the universal ballistic conductivity of MLG \( \sigma_0 = (4/\pi) e^2 / h \), characterizing the so-called pseudodiffusive transport regime [23–27], is replaced by the length-dependent value \( \sigma(L) \), which varies from \( \sigma_0 \) to \( 3\sigma_0 \) per layer [28–30], with the upper limit approached for the system size \( L \to \infty \). In the quantum-Hall regime, the zero-energy Landau level (LL) shows the eightfold degeneracy for BLG (instead of the fourfold degeneracy for MLG), which can be lifted by manipulating the external electromagnetic fields, partly due to electron–electron interactions [10, 13, 20]. Also, the quantum interference in graphene Aharonov–Bohm rings (for a review, see [31]) may result in different oscillation patterns appearing for MLG and BLG [32–34].

An intriguing quantum-interference phenomenon was predicted theoretically for impurity-free Corbino disks in MLG [1, 2, 35–38]. In brief, periodic (approximately sinusoidal) magnetoconductance oscillations are followed, for an undoped sample, by similar oscillations of the shot-noise power [2] and the third charge-transfer cumulant [35, 38]. The effect has a direct analog for strain-induced pseudomagnetic fields [36], allowing consideration of a fully mesoscopic spintronics [44].

Most remarkably, the disk conductance averaged over a single period restores the pseudodiffusive value [27]

\[ G_{\text{MLG}}^{\text{diff}} = \frac{2\pi \sigma_0}{\ln (R_o/R_i)}, \]  

(2)

Analogous behavior is predicted for higher charge-transfer cumulants\(^2\), showing that the effect is another manifestation of the chiral nature of Dirac fermions in graphene. For these reasons, we have coined the term of quantum-relativistic Corbino effect (QRCE).

In this paper, magnetoconductance of the Corbino disk in BLG is discussed in analytical terms, starting from the four-band effective Hamiltonian [4] and employing the Landauer–Büttiker formalism [45] for the linear-response regime. The

\(^2\) In the linear-response regime, the average Fano factor quantifying the shot-noise power \( F = J_{\text{shot}} / 13 \), and the average R-factor quantifying the third charge-transfer cumulant \( R = \kappa_{\text{diff}} = 1/15 \). For a discussion of a finite-voltage situation, see [38].

\[ H = \begin{pmatrix} V/2 & \pi & t_\perp & 0 \\ \pi^\dagger & V/2 & 0 & 0 \\ t_\perp & 0 & -V/2 & \pi^\dagger \\ 0 & 0 & \pi & -V/2 \end{pmatrix} + U(r) \mathbb{I}, \]  

(3)

where \( V \) is the electrostatic bias between the layers, \( \pi = \pi_x + i\pi_y \) and \( \pi^\dagger = \pi_x - i\pi_y \), with \( \pi / v_F = (-i\hbar \partial_j + eA_j) \) being a component of the gauge-invariant momentum operator \((j = 1, 2)\). The electron charge is \(-e\), the potential energy term \( U(r) \mathbb{I} \) depends only on \( r = \sqrt{x^2 + y^2} \) (with \( \mathbb{I} \) the identity matrix), and the remaining symbols are the same.
as in equation (1). We choose the symmetric gauge $A \equiv (A_x, A_y) = (B/2)(−y, x)$, with the uniform magnetic field $B \neq 0$ in the disk area ($R_1 < r < R_0$) and $B = 0$ otherwise. The inner and outer contacts are modeled with heavily doped BLG areas; that is, we set the potential energy profile in equation (3) as follows

$$U(r) = \begin{cases} \infty & \text{if } r < R_1 \text{ or } r > R_0, \\ 0 & \text{if } R_1 < r < R_0, \end{cases}$$ (4)

and focus on the limit of $|U_{\infty}| \rightarrow \infty$. In order to obtain the Hamiltonian for the other valley ($K'$), it is sufficient to substitute $V → − V$ and $\pi → − \pi$ in equation (3).

Since our model system possesses a cylindrical symmetry, the Hamiltonian (3) commutes with the total angular-momentum operator [46]

$$J_z = -i\hbar \partial_\phi + \frac{\hbar}{2} \begin{pmatrix} \sigma_0 & 0 \\ 0 & -\sigma_0 \end{pmatrix} + \frac{\hbar}{2} \begin{pmatrix} -\sigma_z & 0 \\ 0 & \sigma_z \end{pmatrix},$$ (5)

where $\sigma_0$ is the $2 \times 2$ identity matrix, $\sigma_z$ is one of the Pauli matrices, and we have used the polar coordinates $(r, \phi)$. In turn, the wavefunctions can be written as products of angular and radial parts (the so-called envelope wavefunctions), namely

$$\psi(r, \phi) = e^{im\phi} \begin{pmatrix} \phi_1(r) \\ i^m\phi_2(r) \\ \phi_3(r) \\ i^{m+1}\phi_4(r) \end{pmatrix},$$ (6)

where $m = 0, \pm 1, \pm 2, \ldots$ Notice that the angular momentum quantum number in the BLG case is an integer $m$, in contrast to the half-odd integer the $j$ in MLG case [47].

### 2.2. The contact regions

For the contact regions ($r < R_1$ or $r > R_0$), we have $B = 0$ and thus the four-band Dirac equation $H\psi = E\psi$, with $H$ given by equation (3) and $E$ being the Fermi energy, can be written as

$$
\begin{pmatrix}
\tilde{\varepsilon} + \Delta & -l^{-1} \\
-k_- & \tilde{\varepsilon} - \Delta \\
l^{-1} & \tilde{\varepsilon} + \Delta \\
0 & k_+
\end{pmatrix}
\begin{pmatrix}
\phi_1(r) \\
\phi_2(r) \\
\phi_3(r) \\
\phi_4(r)
\end{pmatrix} = 0,$$ (7)

where $\tilde{\varepsilon} = (E - U_{\infty})/(\hbar v_F)$, $\kappa_\mp = i e^{\pm i\phi} (\partial_r \pm i r^{-1} \partial_\phi)$ and $\Delta = -V/(2h v_F)$. Substituting $\psi(r, \phi)$ (6) into equation (7) and decoupling the equation for $\phi^\pm_1(r)$ one gets

$$
\left(\frac{\partial^2}{\partial r} + \frac{1}{r} \partial_r - \frac{m^2}{r^2} + \eta_\pm\right)\phi^\pm_1(r) = 0,$$ (8)

where $\eta_\pm = (\Delta^2 + \tilde{\varepsilon}^2) \pm \sqrt{\tilde{\varepsilon}^2 (4\Delta^2 + 1/l_B^2) - \Delta^2 l_B^2}$. Next, using the differential relations following from equation (7), one can obtain the remaining components of the wavefunction $\phi^\pm(r) = [\phi^\pm_1(r), \phi^\pm_2(r), \phi^\pm_3(r), \phi^\pm_4(r)]^T$, which are given explicitly in appendix A.

### 2.3. The disk area

For the disk area ($R_1 < r < R_0$), we have $B \neq 0$ and it is convenient to define the dimensionless variable $\rho = r/l_B$, with the magnetic length $l_B = \sqrt{\hbar/|eB|}$. In turn, equation (7) is replaced by

$$
\begin{pmatrix}
\varepsilon + \delta & \xi_+ & -t & 0 \\
\xi_- & \varepsilon - \delta & 0 & 0 \\
t & 0 & \varepsilon - \delta & \xi_- \\
0 & 0 & -\xi_+ & \varepsilon + \delta
\end{pmatrix}
\begin{pmatrix}
\psi(\rho, \phi) \\
\psi(\rho, \phi)
\end{pmatrix} = 0,$$ (9)

where $t = l_B/l_\perp$, $\varepsilon = E l_B/(\hbar v_F)$, $\delta = -V l_B/(2l_v F)$ and $\xi_\pm = i \exp(\pm i\phi) (\partial_\rho \pm i \rho^{-1} \partial_\phi) \pm \rho/2$. Eliminating the angle-dependent part of the wavefunction, we obtain

$$
\left(\frac{\partial^2}{\partial \rho} + \frac{1}{\rho} \partial_\rho - \frac{\rho^2}{4} - \frac{m^2}{\rho^2} - m + 1 + \gamma_\pm\right)\phi^\pm(\rho) = 0,$$ (10)

where $\gamma_\pm = (\delta^2 + \tilde{\varepsilon}^2) \pm \sqrt{\tilde{\varepsilon}^2 (4\Delta^2 + t^2) - \delta^2 t^2}$. The complete solution of equation (9) is presented in appendix A. It can be shown that the normalization condition for the wavefunction leads to the energies of LLs [8]

$$\gamma_\pm = n + \frac{|m| + m + 1}{2},$$ (11)

where $n = 0, 1, 2, \ldots$

### 2.4. Reflection and transmission coefficients

Next, we consider the scattering problem for the radial wave functions, assuming that the initial wave is incoming from the inner lead. The solutions of equation (7) for the inner and outer leads can be presented as follows

$$\phi^\pm_1(r) = \phi^\pm_1(r) + r^\pm r^\pm_\rho \phi_\rho(\rho) + r^\pm_\rho \phi_\rho(\rho),$$ (12)

$$\phi^\pm_\rho(r) = r^\pm r^\pm_\rho \phi_\rho(\rho) + r^\pm_\rho \phi_\rho(\rho),$$ (13)

where $\phi^\pm_1(r)$ and $\phi_\rho(\rho)$ denote the wavefunctions propagating from $r = 0$ and $r = \infty$ (respectively) and carrying the unit current. In analogy, a general solution of equation (9) for the disk area corresponds to the linear combination of four eigenspinors, namely

$$\phi^\pm(r) = \sum_{\mu = 1}^4 \alpha^\pm_\mu \phi_\mu(r),$$ (14)

where $[\alpha^\pm_\mu]_{\mu = 1, \ldots, 4}$ are arbitrary complex coefficients. Matching the wavefunctions $\phi^\pm_1(r)$ (12) and $\phi^\pm_\rho(r)$ (14) at $r = R_1$, as well as $\phi^\pm_1(r)$ (14) and $\phi^\pm_\rho(r)$ (13) at $r = R_0$, we obtain the reflection and transmission coefficients corresponding to the $K$ valley and the angular momentum quantum number $m$, which can be arranged in $2 \times 2$ matrices, namely

$$\begin{pmatrix}
r^\pm_1(r) & r^\pm_\rho \\
r^\pm_\rho & r^\pm_1(r)
\end{pmatrix}, \quad \begin{pmatrix}
t^\pm_1(r) & t^\pm_\rho \\
t^\pm_\rho & t^\pm_1(r)
\end{pmatrix}.$$ (15)
The remaining details of the mode-matching procedure are given in appendix B.

It is worth mentioning here that skew-interlayer hoppings [3], neglected in the Hamiltonian $H$ (3), are predicted theoretically to enhance, typically by a factor of 3, the zero-magnetic field conductivity of large bilayer samples at the Dirac point $[3,28,48]$. The experimental values reported by [12] are close, but noticeably smaller than the theoretical prediction, which can be attributed to several factors, including the finite system size [30]. Nevertheless, it is also shown in [30] that the conductance of finite bilayer samples (with length $L \lesssim 100$ nm) becomes insensitive to skew-interlayer hoppings for high magnetic fields $B \gtrsim 5$ T, and thus the scattering approach constituted by the four-band Hamiltonian (3) is sufficient to discuss basic magnetotransport characteristics of nanoscale devices in BLG.

3. Quantum relativistic Corbino effect in BLG

In this section, we present our main results concerning the magnetococonductance of the Corbino disk in BLG. In the linear-response regime, the conductance is given by the Landauer–Büttiker formula [49]

$$G = 2e^2/g_0 \text{Tr} \, T,$$

where $g_0 = e^2/h$ is the conductance quantum, $2g_{\nu}(o)$ is the spin (valley) degeneracy, $T = i\Gamma \, t$ and $t$ is a block-diagonal matrix with each block given by the second equality in equation (15). The Zeeman splitting is neglected for clarity. As the first step, we have also assumed the unbiased sample case ($V = 0$), for which the twofold valley degeneracy occurs. The $V \neq 0$ case is discussed separately later in this section.

3.1. Magnetoconductance at the Dirac point

For $E = V = 0$ and $|U_{\infty}| \to \infty$, transmission eigenvalues can be found analytically, and read

$$T_{m^\pm} = \frac{1}{\cosh^2 \left[(\Lambda \, m \pm A + \Phi_D/\Phi_0)\right]},$$

where $\Phi_D = \pi (R_o^2 - R_i^2)$ is the flux piercing the disk area, $\Lambda = \ln (R_o/R_i)$ and $\Phi_0 = 2(h/e) \, L$. The parameter

$$A = -\frac{\ln \left(\sqrt{\gamma^2 - 1}\right)}{2\Lambda},$$

with $\gamma = \cosh(L) + \Lambda \sinh(L)$ and $\Lambda = (R_o^2 - R_i^2)/(4L^2)$, takes values from the range $1/2 < A < \infty$. Summing over the normal modes labeled by integer $m$, one immediately finds that $G$ (16) shows periodic oscillations as a function of $\Phi_D$, with a period equal to $\Phi_0$ (see figure 2), closely resembling the magnetococonductance behavior predicted for the Corbino disk in MLG [1, 2]. However, for any fixed $\Phi_D$, equation (17) describes the two transmission maxima separated by a distance of $2A \, h$ in the angular-momentum space. In turn, the corresponding contributions to the magnetococonductance may interfere constructively or destructively with each other.

Figure 2. Conductance of different graphene-based Corbino devices with inner radius $R_i = 50 \text{ nm}$ as a function of the magnetic field. (a) Magnetoconductance oscillations in mono- and bilayer disks at the Dirac point for the two values of the radii ratio, for which the oscillation magnitude is close to the maximal ($R_o/R_i = 5$) and to the minimal values ($R_o/R_i = 6.2$) in the bilayer case. (b) Magnetoconductance of bilayer disks in normal Corbino and Andreev–Corbino (NS) setups. Notice the oscillation frequency doubling for the Andreev–Corbino setup and $R_o/R_i = 6.2$.

The nature of the interference depends both on the sample size and on the interlayer hopping integral $t_{\perp}$.

For a clear overview of the effect, we represent $G$ following from equations (16) and (17) by a Fourier series

$$G = \frac{16g_0}{\mathcal{L}} + \sum_{q=1}^{\infty} G_q \cos \left(\frac{2\pi q \, \Phi_D}{\Phi_0}\right),$$

where

$$G_q = \frac{32\pi^2 q \, g_0 \cos (2\pi q \, A)}{\mathcal{L} \sinh (\pi^2 q / \mathcal{L})} \equiv 2(-)^q G_q^{\text{MLG}} \cos (2\pi q \, A),$$

$q = 1, 2, 3, \ldots$. (20)

The constant term in equation (19), $16g_0/\mathcal{L} = 2G_0^{\text{MLG}}$ (see equation (2)), gives the average conductance, which is simply twice as large as in the monolayer case [1, 2]. Such a sum rule does not generically apply to the Fourier amplitudes $G_q$, which are related to the corresponding amplitudes for MLG ($G_q^{\text{MLG}}$) via the second equality in equation (20). A special case of $G_q = 2G_0^{\text{MLG}}$ occurs for $A = 1/2$. For sufficiently large

$$3 \text{ In the limit } t_{\perp} \to 0, \text{ corresponding to the separation of two layers, we have } l_{\perp} \to \infty \text{ and } A \to 1/2, \text{ leading to the same conductance per layer as in the monolayer case.}$$
systems, we have $|G_1| \gg |G_2| \ldots$, and it is possible to find approximate conditions for maximal and minimal oscillation magnitudes $\Delta G \equiv G_{\text{max}} - G_{\text{min}} \simeq 2|G_1|$, namely

$$pL \simeq 4 \ln \left( \frac{R_i}{2l} \right),$$

where $p$ are even (odd) integers for maximal (minimal) oscillation magnitudes. As illustrated in figure 3, the parameter values following from equation (21) for odd $p$ (white dashed lines) coincide with the actual regions where the oscillation magnitude vanishes (black areas), provided that $R_o/R_i \gtrsim 3$ and $R_i/l \gtrsim 10$.

For the sake of completeness, we discuss now the magnetoconductance in the Andreev–Corbino setup, in which the disk-shaped sample is attached to one normal and one superconducting lead. In such a configuration, the conductance is given by [50]

$$G^{\text{NS}} = 2g_0 \sum_{m=\infty}^{\infty} \frac{1}{1 + \cosh^2 \left[ \frac{(m + A)}{2} \right]}.$$  

For a BLG disk at the Dirac point, this leads to

$$G^{\text{NS}} = 8g_0 \sum_{m=\infty}^{\infty} \frac{1}{\cosh^2 \left[ \frac{(m + A)}{2} \right]}.$$  

where we have defined $m = m + \Phi_D/\Phi_0$. $G^{\text{NS}}$ (23) can be represented by a Fourier series of the form given by equation (19) with the same average conductance ($2G^{\text{MLG}}_{\text{diff}}$), and the amplitudes $G_q$ (20) replaced by

$$G_q^{\text{NS}} = \frac{16\pi^2 q g_0 \cos \left( \pi q A \right)}{L^2 \sinh \left[ \pi^2 q^2 / (2L) \right]}.$$  

Strictly speaking, the scaling rule earlier found for the disk in MLG, namely $G_q^{\text{MLG,NS}}(L) = 2G_q^{\text{MLG}}(2L)$ [1], does not apply in the bilayer case due to the interlayer coupling manifesting itself via the $A$-dependent factor in equation (24). However, we still have $G_q^{\text{NS}}/G_q \to 1$ for $R_o/R_i \to \infty$ (and arbitrary $q$). Also, magnetoconductance oscillations for bilayer disks with moderate radii ratios are noticeably amplified in the Andreev–Corbino setup compared to the normal Corbino setup (see figures 2(a) and (b)).

The approximate conditions for maximal and minimal oscillations, given by equation (21), are essentially valid for both the normal Corbino and the Andreev–Corbino setups. The relation $|G_1^{\text{NS}}| \gg |G_2^{\text{NS}}| \gg |G_3^{\text{NS}}| \gg \ldots$ is satisfied, for moderate radii ratios, near the oscillation maxima (even $p$ in equation (21)), whereas close to the minima one typically gets $G_1^{\text{NS}} \approx 0$ and $G_2^{\text{NS}} \gg G_3^{\text{NS}} \gg \ldots$, leading to the visible oscillation frequency doubling (see figure 2(b)). In the normal Corbino setup, with the radii fixed at $R_i = 50l$ and $R_o = 6.2 R_i$, the magnetoconductance is almost constant (yellow dotted line). On the other hand, if one of the leads is superconducting, the frequency of conductance oscillations is doubled in comparison to $\Phi_0$ (green dashed line).

### 3.2. Finite-doping effects

We now extend our analysis to situations where the Fermi energy is close but not precisely adjusted to the Dirac point, keeping the zero bias between the layers ($E \neq 0$ and $V = 0$). (Hereinafter, the normal Corbino setup is considered.) The corresponding magnetoconductance spectra are presented in figure 4. In the monolayer case, the disk conductance at weak dopings follows the zero-doping curve for first few oscillation periods, and then starts to decrease rapidly with increasing field [1] (see blue dashed lines in all panels of figure 4). For BLG (see red solid lines) we have a relatively wide crossover field interval, separating the oscillating and the field-suppressed conductance ranges. Typically, the conductance in the crossover interval does not decay monotonically with the field. Instead, a well-defined magnetoconductance peak appears, with $G \simeq G^{\text{diff}}$ near the maximum. Below, we link these features to the presence—in the vicinity of the Dirac point—of the two independent transmission channels for any angular momentum quantum number $m$, characterized by the transmission probabilities, which are numerically close to $T_l/\Phi_0$ (17).

The contribution to the disk conductance originating from evanescent waves, for either MLG or BLG close to the Dirac point, can be roughly estimated by

$$\sum_{l, \text{eva}} T_l \sim \left( \frac{R_i}{R_o} \right)^{2l_{\text{max}}} \quad \text{(for } R_i \ll R_o),$$

where $l_{\text{max}}$ denotes the angular momentum corresponding to the maximal transmission at $E = 0$, namely $l_{\text{max}} = \eta A - \Phi_D/\Phi_0$, where $\eta = 0$ for MLG or $\eta = \pm 1$ for BLG. The contribution from the propagating waves appearing for $E \neq 0$ is of the order of

$$\sum_{l, \text{pro}} T_l \sim (k_0 R_i)^2 \quad \text{(for } k_0 R_i \ll 1),$$

where we have defined the wavevector $k_0 = |E|/\hbar v_F$. Quasiperiodic magnetoconductance oscillations can be
observed as long as $\sum_{l,eva}T_l \geq \sum_{l,pro}T_l$, directly leading to the limits for magnetic fluxes

$$|\Phi_D| \lesssim \Phi_D^{\text{max},\eta} = \frac{2\hbar}{e} \left[ \eta AC - \ln \left( k_0 R_i \right) \right].$$

(27)

The values of $\Phi_D^{\text{max},\eta}$, for $\eta = 0, \pm 1$, are also depicted in figure 4 (see vertical lines), showing that the flux range defined as $\Phi_D^{\text{max},-1} \leq \Phi_D \leq \Phi_D^{\text{max},+1}$ coincides with the crossover field interval for a BLG disk with $R_i = 50l_\perp$, $R_o/R_i = 8$ and $|E| \leq 10^{-6}$ eV. For larger $R_o$, such a coincidence can also be observed at higher $E$, provided that $\Phi_D^{\text{max},-1} \gtrsim 2\Phi_0$.

3.3. The biased sample case ($V \neq 0$)

We focus now on the effect of a nonzero electrostatic bias between the layers in the normal Corbino geometry. The corresponding magnetoconductance spectra for the two selected radii ratios $R_o/R_i = 5$ and $R_o/R_i = 6.2$ (with $R_i = 50l_\perp$) are presented in figure 5, where we have fixed the Fermi energy at $E = V/2 = 0.1$ eV. The disk conductance first shows rather irregular behavior with increasing field, varying in the range $0 < G \lesssim G_{\text{MLG}}^{\text{diff}}$ (the corresponding magnetoconductance spectra for $E = V = 0$, and for undoped MLG disks, are also shown in figure 5). For $\Phi_D \gtrsim 10\Phi_0$, periodic oscillations are restored, but the average conductance is $4g_0/L = G_{\text{MLG}}^{\text{diff}}/2$. Also, the oscillation magnitude $\Delta G = G_{\text{BLG}}^{\text{max}}$. (Notice that we have selected the disk radii such that $G_{\text{BLG}}^{\text{max}}$ is close to the maximal and to the minimal value in the $E = V = 0$ case, see green dashed lines.) These features can be attributed to the splittings of layer and valley degeneracies of the lowest LL in the presence of a band gap and magnetic field (see [30]).

Also for higher LLs, the disk conductance oscillates periodically with $\Phi_D$, qualitatively reproducing the behavior predicted for the monolayer case in [1]. This is because finite doping eliminated the level degeneracy associated with the two layers, even in the absence of the electrostatic bias ($V = 0$). For $V \neq 0$, the valley degeneracy no longer applies, and
the conductance further drops by a factor of 2. A complete overview of different transport regimes on the field-doping parameter plane is given in section 4, where we compare (in a quantitative manner) the magnetococonductance of the Corbino disks in BLG and in 2DEG.

4. Magnetococonductance of the Corbino disk in 2DEG

For both BLG and 2DEG systems, parabolic bands appear in the low-energy dispersion relation, and the effective masses are in the range \( m_*/m_e = 10^{-2} \) to \( 10^{-1} \) (where \( m_e \) denotes the free electron mass). Therefore, a detailed comparison of the magnetic field effects described in section 3, with analogous effects for the Corbino disk in 2DEG, is desired to identify the role of chiral tunneling of Dirac fermions in BLG. Below, we extend the mode-matching analysis presented in [27] for the nonzero field situation.

The effective Schrödinger equation for electrons in a 2DEG system reads

\[
\left[ \frac{1}{2m_e} \left( \frac{\hbar}{i} \nabla + eA \right)^2 + U(r) \right] \psi = E\psi, \tag{28}
\]

where \( \psi(r) \) is the complex-scalar wavefunction, the vector potential \( A \) is same as in equation (3) and the Zeeman term is neglected again. The electrostatic potential energy \( U(r) \) is still given by equation (4), but we no longer assume infinite doping in the leads, as the mismatch in Fermi velocities results in zero transmission in such a system [51–53]. Instead, \( U_\infty \) can be adjusted such that \( \pi R_i \sqrt{m_*(E - U_\infty)/\hbar^2} \gtrsim 10 \), entering the multimode leads regime, in which the conductance only weakly depends on \( U_\infty \).

Since the Hamiltonian in equation (28) commutes with the orbital momentum operator \( L_z = -i\hbar \partial_\phi \), we choose wavefunctions of the form \( \psi(r, \phi) = \varphi(r) \exp(i\phi) \), with \( l \) integer. This brings us to solving the effective one-dimensional scattering problem, with the Schrödinger equation

\[
\left[ -\frac{\partial^2}{\partial r^2} + \frac{l^2}{r^2} + \frac{r^2}{4l_B^2} \right] \varphi(r) = \zeta \varphi(r), \tag{29}
\]

where \( \zeta_l = 2m_*(E - U(r))/\hbar^2 - l/l_B^2 \). For the contact regions we have \( l_B^{-1} = 0 \), and the solutions are given by the Hankel functions [54], namely

\[
\varphi_i^{(i)}(r) = H_i^{(1)}(K r) + r_l H_i^{(2)}(K r),
\]
\[
\varphi_i^{(o)}(r) = r_l H_{l+1}^{(2)}(K r), \tag{30}
\]

where \( K = \sqrt{2m_*(E - U_\infty)/\hbar^2} \), \( r_l \) (\( t_l \)) is the reflection (transmission) coefficient, and we have assumed scattering from the inner lead.

We further notice that the early theoretical works on the Corbino disks in 2DEG [51, 52] utilized a particular form of the effective radial potential, leading to conductance quantization. The quantization steps are smeared out when employing the direct mode-matching technique (see [27]), leading to yet another surprise when discussing the applicability of the adiabatic expansion to scattering problems in physics [53].

Figure 6. Conductance as a function of doping and magnetic field for the Corbino disks in unbiased BLG (left) and in 2DEG (right). The radii are fixed at \( R_i = 25 l_\perp \simeq 40 \) nm and \( R_o = 4R \) for both cases. Black dashed lines mark the condition for cyclotron diameters \( 2\ell = R_o - R_i \). White dotted lines depict the energy levels given by equation (33).

For the disk area, we get

\[
\varphi_i^{(d)}(r) = (C_l/r) W_{\Omega_l,1/2} \left( -\frac{r^2}{2R_o^2} \right) + (D_l/r) W_{-\Omega_l,1/2} \left( \frac{r^2}{2R_o^2} \right), \tag{31}
\]

where \( \Omega_l = (l - k^2l_B^2)/2 \) with \( k = \sqrt{2m_*/\hbar^2} \), \( W_{\pm\Omega_l,1/2}(x) \) is the Whittaker function [55], and \( C_l, D_l \) are arbitrary constants. In particular, imposing the normalization of \( \varphi_i^{(d)} \), one can obtain the well-known energy quantization

\[
E_n = \hbar \omega_c (n + 1/2), \tag{32}
\]

with \( \omega_c = eB/\hbar \) and \( n = 0, 1, 2, \ldots \). For the open system studied here, the normalization condition for wavefunctions does not apply, but the LL energies \( E_n \) (32) coincide with the transmission maxima of \( T_l = |t_l|^2 \).

Carrying out the mode-matching procedure for each value of \( l \) separately (see appendix C for the details), we get the Landauer–Büttiker conductance \( G = 2e^2 \sum_l T_l \) for arbitrary dopings and magnetic fields. For the numerical analysis, we set the effective mass to be the same as in GaAs systems \( m_* = 0.067m_e \), the inner radius is \( R_i = 25 l_\perp \simeq 40 \) nm and the doping on the leads is such that \( E - U_\infty = 0.4 \) eV.

The results are displayed in figures 6 and 7. Both for BLG and 2DEG disks (see figure 6) we observe, at low magnetic fields, well-defined conductance maxima corresponding to the quantum-dot energy levels

\[
E_q = \begin{cases} 
\frac{1}{2} \left[ -l_\perp + \sqrt{l_\perp^2 + \left( \frac{\hbar m_e}{eB} \right)^2 q^2} \right] 
\hbar^2 q^2 / (8m_\star L) 
\end{cases} \text{ for BLG}, \tag{33}
\]

for 2DEG,

with \( L \equiv R_o - R_l \) and \( q \) an integer. These maxima gradually evolve, with increasing field, towards narrow peaks corresponding to the resonances with LLs, at energies given by equation (11) for BLG or equation (32) for 2DEG. Away from the maxima, some background conductance \( G \gtrsim g_0 \).
appears when the cyclotronic diameter \(2r_C \gtrsim L\). (Otherwise, \(G \ll G_0\).) In turn, the ballistic and the quantum-tunneling transport regimes can be identified for both the systems considered.

The key difference in charge transport via Corbino disks in BLG and in 2DEG appears in the quantum-tunneling regime, and is visualized in figure 7. For BLG, the conductance at the local maximum corresponding to the resonance with the \(n\)-th LL, for two different values of the magnetic field \((B = 2.5 \text{ and } 5 \text{T})\), is visualized. The inset shows the conductance at the resonance with the \(-\)th LL, for two different values of the magnetic field \((B = 2.5 \text{ and } 5 \text{T})\).

5 The system parameters \(R_i = 25L_1\) and \(R_o = 4R_i\) are adjusted such that the oscillation magnitude at the Dirac point \(\Delta G^{\text{MLG}} / G^{\text{MLG}} \approx 0\). At higher LLs, the relative magnitude is given by \(\Delta G^{\text{MLG}} / G^{\text{MLG}} \approx 0.05\). Defining the range in which the peak conductance may vary with increasing field, namely: \([G_{\text{max}} - G_{\text{diff}}] \lesssim \Delta G^{\text{MLG}} / 2\).

Figure 7. Conductance as a function of doping at fixed \(B = 5 \text{T}\). The parameters are the same as in figure 6. The inset shows the maximal conductance at the resonance with the \(n\)-th LL, for two different values of the magnetic field \((B = 2.5 \text{ and } 5 \text{T})\).

5 The system parameters \(R_i = 25L_1\) and \(R_o = 4R_i\) are adjusted such that the oscillation magnitude at the Dirac point \(\Delta G^{\text{MLG}} / G^{\text{MLG}} \approx 0\). At higher LLs, the relative magnitude is given by \(\Delta G^{\text{MLG}} / G^{\text{MLG}} \approx 0.05\). Defining the range in which the peak conductance may vary with increasing field, namely: \([G_{\text{max}} - G_{\text{diff}}] \lesssim \Delta G^{\text{MLG}} / 2\).

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The interference is maximally destructive, leading to the approximately field-independent conductance (twice as large as the pseudodiffusive value for the disk setup in a monolayer [27]) for moderate radii ratios \(R_o/R_i \lesssim 10\) in the normal Corbino setup, or to the oscillation frequency doubling for the Andreev–Corbino setup. We notice that the effect that we described offers, at least in principle, an independent way of determining the basic microscopic parameters of bilayer graphene.

Quite remarkably, the energy-gap opening from applying the external electrostatic bias affects the transport properties of the Corbino disk in bilayer graphene in a rather unexpected manner. New features, mentioned above and absent in the monolayer, appear for unbiased disks at the Dirac point, whereas the gap opening essentially reduces the variety of magnetotransport behavior to that earlier described for monolayer disks. This observation seems particularly significant, as some experimental works showed that the energy gap may also appear spontaneously, due to electron–electron interactions, for bilayer samples close to the charge-neutrality point [13, 14]. It must be noticed, however, that the results of other conductance measurements [12] coincide with theoretical predictions for an unbiased bilayer, leaving an ambiguity concerning the role of interactions in the system.

The effects of disorder [3], lattice defects [56] or magnetic impurities [57, 58], which may modify the transport properties of graphene-based devices, are beyond the scope of this paper as we have focused on perfectly clean ballistic systems. Certain features of the results, including the fact that unit transmission appears periodically (for consecutive normal modes) with increasing field, and that the oscillation period is proportional to the ratio of fundamental constants \(h/e\), allow us to believe that symmetry-protected quantum channels [59] would lead to magnetoconductance oscillations appearing in a more general situation as well.

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### Appendix A. Wavefunctions

In this appendix, we present the wavefunctions of a charge carrier in bilayer graphene, having the form of eigenspinors of the total angular-momentum operator $J_z$ (5), and thus adjusted for a scattering problem with cylindrical symmetry. The cases of zero and nonzero magnetic fields, relevant for the leads and the sample area in the system of figure 1, are discussed separately.

#### A.1. Zero magnetic field

Four linearly independent solutions of the Dirac equation $H \psi = E \psi$ with the Hamiltonian given by equation (3), corresponding to the angular-momentum quantum number $m$, have forms of envelope wavefunctions given by equation (6). For $B = 0$, the radial parts of these functions can be written as

$$
\phi_{m}^{\pm}(r) = \begin{pmatrix}
\frac{H_{-m}^{(1)}(s_{\pm} r)}{s_{\pm}(\epsilon^{2} - \eta^{2})}, \\
s_{\pm}(\epsilon^{2} - \eta^{2}) \frac{H_{m-1}^{(1)}(\epsilon_{\pm} \rho)}{s_{\pm}(\epsilon^{2} - \eta^{2})
\end{pmatrix}
$$

(A1)

for the waves propagating from $r = 0$ (the index $\pm$ refers to the two subbands), or

$$
\phi_{m}^{\pm}(r) = \begin{pmatrix}
\frac{H_{m}^{(2)}(s_{\pm} r)}{s_{\pm}(\epsilon^{2} - \eta^{2})}, \\
s_{\pm}(\epsilon^{2} - \eta^{2}) \frac{H_{-m+1}^{(2)}(\epsilon_{\pm} \rho)}{s_{\pm}(\epsilon^{2} - \eta^{2})
\end{pmatrix}
$$

(A2)

for the waves propagating from $r = \infty$, with $s_{\pm} = \sqrt{\eta^{2} + \epsilon^{2}}$, $\eta_{\pm} = (\Delta^{2} + \tilde{\epsilon}^{2})^{1/2} \sqrt{2} (4 \Delta^{2} + 1/\tilde{L}_{\perp}^{2} - \tilde{\Delta}^{2} - 1/\tilde{L}_{\perp}^{2})$, $\epsilon_{\pm} = \tilde{\epsilon} \mp \Delta$, $H_{m}^{(j)}(x)$ being the Hankel function of the first (second) kind [54], and the remaining symbols are the same as in equation (7). For $\tilde{\epsilon} \rightarrow \infty$, the asymptotic forms of the radial wavefunctions are

$$
\phi_{m}^{\pm}(r) \simeq \sqrt{\frac{2}{\pi e r}} \exp \left[ i \left( \tilde{\epsilon} r - \frac{1}{2} \frac{\pi m - 1}{4} \right) \right] \begin{pmatrix}
\frac{1}{\mp 1} \\
\frac{1}{\pm 1}
\end{pmatrix}
$$

(A3)

and

$$
\phi_{m}^{\pm}(r) \simeq \sqrt{\frac{2}{\pi e r}} \exp \left[ -i \left( \tilde{\epsilon} r - \frac{1}{2} \frac{\pi m - 1}{4} \right) \right] \begin{pmatrix}
\frac{1}{\mp 1} \\
\frac{1}{\pm 1}
\end{pmatrix}
$$

(A4)

6 We use $H_{m}^{(1)}(\rho) \approx \sqrt{2/(\pi \rho)} \exp[i(\rho - \pi/2 - \pi/4)]$ for $\rho \gg 1$, and $H_{m}^{(2)}(\rho) = [H_{m}^{(1)}(\rho)]^{*}$.

#### A.2. Nonzero magnetic field

At the Dirac point ($\epsilon = \delta = 0$), the radial part of the wavefunction, being a general solution of equation (9), reads

$$
\phi_{m}(r) = \frac{f_{m}(\rho)}{t_{m}} \begin{pmatrix}
0 \\
0 \\
0 \\
0
\end{pmatrix}
$$

+[\alpha_{3} \begin{pmatrix}
0 \\
0 \\
0 \\
0
\end{pmatrix} + \alpha_{4} \begin{pmatrix}
0 \\
0 \\
0 \\
0
\end{pmatrix}],
$$

(58)

where $f_{m}(\rho) = \exp(-m \ln \rho - \rho^{2}/4)$, $\tilde{f}_{m}(\rho) = 1/f_{m}(\rho)$, $\alpha_{j}$ are arbitrary complex coefficients (taking different values depending whether the mode-matching analysis is carried out for the wave incoming from $r = 0$ given by $\phi_{m}^{\pm}(r)$ or $\phi_{m}(r)$; see appendix B), and the remaining symbols are the same as in equation (9).

At finite dopings ($\epsilon \neq 0$ or $\delta \neq 0$), the radial wavefunctions are given by

$$
\phi_{m}^{\pm}(r) = \begin{pmatrix}
a_{m}^{\pm}(\epsilon, \delta; \rho) \\
b_{m}^{\pm}(\epsilon, \delta; \rho) \\
c_{m}^{\pm}(\epsilon, \delta; \rho) \\
d_{m}^{\pm}(\epsilon, \delta; \rho)
\end{pmatrix}
$$

(l = 1, 2),

(59)

where the spinor components can be written as

$$
a_{m,1}^{\pm}(\epsilon, \delta; \rho) = 2(\pi^{1/2})^{2} \frac{t_{m}}{m+1} \left( \frac{\gamma_{\pm}}{2}, m+1, -\frac{\rho^{2}}{2} \right),
$$

(60)

$$
b_{m,1}^{\pm}(\epsilon, \delta; \rho) = -\left( \delta + \tilde{\epsilon} \right)^{2} \frac{t_{m}}{m+1} \left( \frac{\gamma_{\pm}}{2}, m+1, -\frac{\rho^{2}}{2} \right),
$$

(61)

$$
c_{m,1}^{\pm}(\epsilon, \delta; \rho) = \frac{\left( \delta + \tilde{\epsilon} \right)^{2}}{m+1} \left( \frac{\gamma_{\pm}}{2}, m+1, -\frac{\rho^{2}}{2} \right),
$$

(62)

and

$$
a_{m,2}^{\pm}(\epsilon, \delta; \rho) = 2(\pi^{1/2})^{2} \frac{t_{m}}{m+1} \left( \frac{\gamma_{\pm}}{2}, m+1, -\frac{\rho^{2}}{2} \right),
$$

(63)

$$
b_{m,2}^{\pm}(\epsilon, \delta; \rho) = -\left( \delta + \tilde{\epsilon} \right)^{2} \frac{t_{m}}{m+1} \left( \frac{\gamma_{\pm}}{2}, m+1, -\frac{\rho^{2}}{2} \right),
$$

(64)

$$
c_{m,2}^{\pm}(\epsilon, \delta; \rho) = \frac{\left( \delta + \tilde{\epsilon} \right)^{2}}{m+1} \left( \frac{\gamma_{\pm}}{2}, m+1, -\frac{\rho^{2}}{2} \right),
$$

(65)

and

$$
\phi_{m}^{\pm}(r) \simeq \sqrt{\frac{2}{\pi e r}} \exp \left[ i \left( \tilde{\epsilon} r - \frac{1}{2} \frac{\pi m - 1}{4} \right) \right] \begin{pmatrix}
\frac{1}{\mp 1} \\
\frac{1}{\pm 1}
\end{pmatrix}
$$

(A3)

and

$$
\phi_{m}^{\pm}(r) \simeq \sqrt{\frac{2}{\pi e r}} \exp \left[ -i \left( \tilde{\epsilon} r - \frac{1}{2} \frac{\pi m - 1}{4} \right) \right] \begin{pmatrix}
\frac{1}{\mp 1} \\
\frac{1}{\pm 1}
\end{pmatrix}
$$

(A4)

6 We use $H_{m}^{(1)}(\rho) \approx \sqrt{2/(\pi \rho)} \exp[i(\rho - \pi/2 - \pi/4)]$ for $\rho \gg 1$, and $H_{m}^{(2)}(\rho) = [H_{m}^{(1)}(\rho)]^{*}$.
We have further defined $\gamma_\pm = (\delta^2 + \varepsilon^2) \pm \sqrt{\varepsilon^2 (4\delta^2 + r^2) - \delta^2 r^2}$. $L_{n}^0(x)$ is the generalized Laguerre polynomial (see [54], Chapter 22), $U(a, b, x)$ denotes the confluent hypergeometric function [55], $\Gamma(z) = \int_0^\infty x^{z-1}e^{-dx}$ is the Euler gamma function, $\mathcal{F}(a; b; z) \equiv \Gamma(z) \Gamma(b) \Gamma(b/a)$ with $pF_q(a_1, \ldots, a_p; b_1, \ldots, b_q; z)$ denoting the generalized hypergeometric function [60], and the remaining symbols are the same as in equation (9) in the main text.

### Appendix B. Transmission eigenvalues

The charge conservation conditions for the interfaces between the disk area and the leads $(r = R_l$ and $r = R_o$; see also figure 1) can be written, in terms of the radial wavefunctions presented in appendix A, as

$$
\phi_{\pm}^o(R_l) + r_{\pm}^o \phi_{\text{rad}}(R_l) = \phi_0(R_l),
$$

$$
\phi_0(R_l) = r_{\pm}^o \phi_{\text{rad}}(R_o) + r_{\pm}^o \phi_{\text{rad}}(R_o),
$$

where we have represented wavefunctions in the leads following equations (12) and (13) in the main text. If the disk area is undoped and unbiased ($\varepsilon = \delta = 0$), the function $\phi_0(R_l)$ is given by equation (A5). Taking the limit of $|U_{\text{val}}| \to \infty$ for the leads (i.e. choosing the functions $\phi_{\text{rad}}^o(r)$ and $\phi_{\text{rad}}^o(r)$ as given by equations (A3) and (A4)) and solving the system of linear equations following from equation (B1), one gets the closed-form expression for $T^{\pm}_{\text{val}}$ transmission eigenvalues for a given angular momentum quantum number $m$ (see equation (17) in the main text).

For a more general case of finite dopings in the disk area ($\varepsilon \neq 0$ or $\delta \neq 0$), the limit $|U_{\text{val}}| \to \infty$ leads, combined with radial wavefunctions of the form $\phi_{\text{rad}}^o(r)$ and $\phi_{\text{rad}}^o(r)$ (see equations (A6)-(A8)) for the disk area, bring us to the system of linear equations

$$
\begin{pmatrix}
-1 & -1 & a_{01}^m & a_{02}^m & a_{m1}^m & a_{m2}^m & 0 & 0 \\
-1 & -1 & b_{01}^m & b_{02}^m & b_{m1}^m & b_{m2}^m & 0 & 0 \\
1 & 1 & c_{01}^m & c_{02}^m & c_{m1}^m & c_{m2}^m & 0 & 0 \\
i & -i & d_{01}^m & d_{02}^m & d_{m1}^m & d_{m2}^m & 0 & 0 \\
0 & 0 & a_{01}^m & a_{02}^m & a_{m1}^m & a_{m2}^m & -R & -R \\
0 & 0 & b_{01}^m & b_{02}^m & b_{m1}^m & b_{m2}^m & iR & iR \\
0 & 0 & c_{01}^m & c_{02}^m & c_{m1}^m & c_{m2}^m & R & -R \\
0 & 0 & d_{01}^m & d_{02}^m & d_{m1}^m & d_{m2}^m & -iR & iR \\
(\varepsilon, \delta; \rho_0) & (\varepsilon, \delta; \rho_0) & (\varepsilon, \delta; \rho_0) & (\varepsilon, \delta; \rho_0) & (\varepsilon, \delta; \rho_0) & (\varepsilon, \delta; \rho_0) & (\varepsilon, \delta; \rho_0) & (\varepsilon, \delta; \rho_0)
\end{pmatrix}
\begin{pmatrix}
\psi^{(i)}(R_l) \\
\psi^{(i)}(R_l) \\
\psi^{(i)}(R_l) \\
\psi^{(i)}(R_l) \\
\psi^{(i)}(R_l) \\
\psi^{(i)}(R_l) \\
\psi^{(i)}(R_l) \\
\psi^{(i)}(R_l)
\end{pmatrix}
= \begin{pmatrix}
1 \\
-i \\
0 \\
0 \\
0 \\
0 \\
0 \\
0
\end{pmatrix},
$$

with $\rho_0 = R_l/|l_B|, \rho_1 = R_o/|l_B|$ and $R = \sqrt{R_l/R_o}$. The elements of the reflection and transmission matrices $\mathbf{R}_{K,m}$ and $\mathbf{T}_{K,m}$ occurring in equation (B2) differ from the corresponding elements of $\mathbf{R}_{K,m}$ and $\mathbf{T}_{K,m}$ (see also equation (15) in the main text) only via phase factors, which are insignificant when calculating transmission eigenvalues. Solving equation (B2), one obtains the matrices $\mathbf{R}_{K,m}$ and $\mathbf{T}_{K,m}$ for the $K$ valley and the angular momentum quantum number $m$. The reflection and transmission matrices for the $K'$ valley can be obtained from an analogous procedure, starting from radial wavefunctions modified according to $(\phi_1, \phi_2, \phi_3, \phi_4)^T \rightarrow (\phi_1, -\phi_2, \phi_3, -\phi_4)^T$, with a substitution $\delta \to -\delta$.

### Appendix C. Mode matching for a disk in 2DEG

For Schrödinger electrons in the Corbino setup, current conservation at $r = R_l$ and $r = R_o$ leads to four independent matching conditions

$$
\frac{d\psi^{(i)}_l(R_l)}{dr} = \frac{d\psi^{(i)}_l(R_l)}{dr}, \quad \frac{d\psi^{(i)}_l(R_l)}{dr} = \frac{d\psi^{(i)}_l(R_l)}{dr},
$$

$$
\frac{d\psi^{(i)}_{l,i}(R_l)}{dr} = \frac{d\psi^{(i)}_{l,i}(R_l)}{dr}, \quad \frac{d\psi^{(i)}_{l,i}(R_l)}{dr} = \frac{d\psi^{(i)}_{l,i}(R_l)}{dr},
$$

determining the coefficients $r_i, t_i, C_i$ and $D_i$, defined via equations (30) and (31) in the main text. Let us further define the wavefunctions in the disk area

$$
\psi^{(d)}_{l,i}(r) = \frac{1}{r} W_{\Omega,l/2} \left( -\frac{r^2}{2l^2_B} \right) \text{ and } \psi^{(d)}_{l,i}(r),
$$

$$
= \frac{1}{r} W_{-\Omega,l/2} \left( \frac{r^2}{2l^2_B} \right),
$$

where $\Omega = (l - k^2l^2_B)/2, \ k = \sqrt{2mE/\hbar^2}$ and $W_{\pm\Omega}(x)$ is the Whittaker function [55]. (In turn, $\psi^{(i)}_{l,i}(r) = C_i\psi^{(d)}_{l,i}(r) + D_i\psi^{(d)}_{l,i}(r)$.) The transmission probability for the angular momentum quantum number $l$ can now be written as

$$
T_l = |t_l|^2 = \frac{1}{|M_l|^2} \left( \frac{4}{\pi l^2 B R_o} \right)^2,
$$

where

$$
M_l = H^{(1)}_{l+1}(K R_l) H^{(2)}_{l+1}(K R_o) \times \left[ \delta \psi^{(d)}_{l,1}(R_l) \delta \psi^{(d)}_{l,1}(R_l) - \delta \psi^{(d)}_{l,1}(R_l) \delta \psi^{(d)}_{l,1}(R_l) \right] + K^2 \left[ \delta \psi^{(d)}_{l,1}(K R_l) \right] \left[ \delta \psi^{(d)}_{l,1}(K R_l) \right] 
\times \left[ \psi^{(d)}_{l,1}(R_l) \psi^{(d)}_{l,1}(R_l) - \psi^{(d)}_{l,1}(R_l) \psi^{(d)}_{l,1}(R_l) \right] 
+ K H^{(1)}_{l+1}(K R_l) \delta \psi^{(d)}_{l,1}(K R_o) 
\times \left[ \psi^{(d)}_{l,1}(R_l) \psi^{(d)}_{l,1}(R_l) - \psi^{(d)}_{l,1}(R_l) \psi^{(d)}_{l,1}(R_l) \right] 
+ KH^{(1)}_{l+1}(K R_l) \delta \psi^{(d)}_{l,1}(K R_o) \times \left[ \psi^{(d)}_{l,1}(R_l) \psi^{(d)}_{l,1}(R_l) - \psi^{(d)}_{l,1}(R_l) \psi^{(d)}_{l,1}(R_l) \right]
$$
\[ \begin{align*}
\times \left[ \partial_{\nu_{i}}^{(d)} \left( R_{\nu} \right) \partial_{\nu_{j}}^{(d)} \left( R_{\nu} \right) - \partial_{\nu_{i}}^{(d)} \left( R_{\nu} \right) \partial_{\nu_{j}}^{(d)} \left( R_{\nu} \right) \right] \\
+ K \left[ \partial_{\nu_{i}}^{(d)} \left( K R_{\nu} \right) \partial_{\nu_{j}}^{(d)} \left( K R_{\nu} \right) + \partial_{\nu_{i}}^{(d)} \left( R_{\nu} \right) \partial_{\nu_{j}}^{(d)} \left( R_{\nu} \right) - \partial_{\nu_{i}}^{(d)} \left( R_{\nu} \right) \partial_{\nu_{j}}^{(d)} \left( R_{\nu} \right) \right],
\end{align*} \]
and the derivatives are given by
\[ \begin{align*}
\partial_{\nu_{i}}^{(d)} \left( \rho \right) &= H_{\nu_{i}}^{(d)} \left( \rho \right) - \frac{1}{\rho} H_{1-i}^{(d)} \left( \rho \right), \\
\partial_{\nu_{i}}^{(d)} \left( r \right) &= -\frac{1}{r} \left[ \left( 2 \mu \Omega_{r} + 1 + \frac{\lambda_{0} r^{2}}{2 \ell_{B}^{2}} \right) W_{\nu_{i} \Omega_{r}, 1/2} - \frac{\lambda_{0} r^{2}}{2 \ell_{B}^{2}} \right] + 2W_{1+\nu_{i} \Omega_{r}, 1/2} - \frac{\lambda_{0} r^{2}}{2 \ell_{B}^{2}},
\end{align*} \]
for \( \alpha = 1, 2, \) and \( \lambda_{0} = \mp(1)\).
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