I. INTRODUCTION

Most parameter fitting studies assume that the spectrum of primordial cosmological perturbations is a fixed power of the co-moving wave-number $k$ over the range of scales being probed by observations. At most, a parameter describing a running of the spectrum is introduced. However, it is possible that the primordial spectrum has particular features which are poorly described assuming a slow running superimposed over a spectrum with fixed slope. In fact, it was found \[1\] that making an ansatz for the spectrum which includes special features may improve the fit to the data.

Recently, it has been realized that the “matter bounce” provides a possible alternative to cosmological inflation for producing a scale-invariant spectrum of primordial adiabatic fluctuations on large scales, is a break in the power spectrum at a characteristic scale, below which the spectral index changes from $n=1$ to $n=3$. We study the constraints which current cosmological data place on the location of such a break, and more generally on the position of the break and the slope at length scales smaller than the break. The observational data we use include the WMAP five-year data set (WMAP5), other CMB data from BOOMERanG, CBI, VSA, and ACBAR, large-scale structure data from the Sloan Digital Sky Survey (SDSS, their luminous red galaxies sample), Type Ia Supernovae data (the “Union” compilation), and the Sloan Digital Sky Survey Lyman-α forest power spectrum (Lyα) data. We employ the Markov Chain Monte Carlo method to constrain the features in the primordial power spectrum which are motivated by the matter bounce model. We give an upper limit on the length scale where the break in the spectrum occurs.

In a contracting universe, the energy density of radiation increases faster than that of matter. Hence, if the initial state contained both matter and radiation, it is inevitable that a change in the equation of state from matter to radiation will occur, very much like a transfer from radiation to matter occurs in the expanding branch. As we show in the following section, scales which exit the Hubble radius in the radiation phase acquire an $n=3$ spectrum. If the cosmological bounce is close to time-symmetric, we should expect the break in the slope of the spectrum of primordial perturbations to occur on a cosmologically interesting scale, close to the scale which re-enters the Hubble radius during the expanding phase at the time $t_{eq}$ of equal matter and radiation.

In this Note we study the constraints which can be put from current observations on the transition scale where the spectral index changes from $n=3$ to $n=1$. Comparisons of a scale dependent primordial power spectrum with observational data can also been found in Ref.\[10\].

The outline of this Note is as follows: in the following Section we explain why in a matter bounce model a break in the spectrum from $n=1$ on large scales to $n=3$ on small scales will occur. In Section 3 we will discuss our method and the data used. Our numerical results are presented in Section 4, and we end with a discussion.

II. BREAK IN THE SPECTRUM IN THE MATTER BOUNCE SCENARIO

Let us review the computation of the spectrum of cosmological perturbations on super-Hubble scales starting from vacuum initial fluctuations on sub-Hubble scales. We work in longitudinal gauge (see e.g. \[11\] for a detailed review of the theory of cosmological fluctuations and \[12\] for a shorter overview) in which the metric is
given by
\[ ds^2 = a^2(\eta)\left[ (1 + 2\Phi)d\eta^2 - (1 - 2\Phi)dx^2 \right], \]
where \( \eta \) is conformal time, the \( x \) denote co-moving spatial coordinates and \( \Phi(x, \eta) \) is the generalized gravitational potential which carries the information about the fluctuations. As long as we are far from the bounce time, a good quantity to characterize the magnitude of the inhomogeneities is \( \zeta \), the curvature fluctuation in co-moving coordinates which is expressed in terms of \( \Phi \) via
\[ \zeta = \frac{2}{3}(\mathcal{H}\Phi' + \Phi)\frac{1}{1+w} + \Phi, \]
with \( \mathcal{H} \) denoting the Hubble expansion rate in conformal time. If our universe has emerged from a time-symmetric bounce, we would expect all scales which enter the Hubble radius during a radiation-dominated contraction around the bounce point, we would expect that the scale-invariant power spectrum on super-Hubble scales

\[ P_\zeta(k, \eta) \sim k^3|\zeta_k(\eta)|^2 \sim k^3|v_k(\eta)|^2 \sim k^3|v_k(\eta_H(k))|^2 \eta_H(k)^2 \sim \text{const}, \]
where the first step is the definition of the power spectrum, the second step uses \([3]\), the third the temporal scaling \([3]\) of the dominant mode, and in the final step we have inserted the vacuum initial conditions \([4]\), the Hubble crossing relation \( \eta_H(k) \sim k^{-1} \) and the scaling of the scale factor as a function of conformal time (see also \([6]\) where the calculation was done following the evolution of \( \Phi \) from initial Hubble radius crossing in the contracting phase until the post-bounce expanding phase).

In the radiation phase (see \([13]\) for an earlier analysis of the evolution of fluctuations in a contracting radiation phase) we have \( p = 1/2 \) and hence \( \nu = -1/2 \) which leads to the two values for \( \alpha \) which are \( \alpha = 1 \) and \( \alpha = 0 \). Hence
\[ \nu_k(\eta) = c_1\eta + c_2, \]
where \( c_1 \) and \( c_2 \) are constants. Thus, the dominant mode of \( \nu \) is constant on super-Hubble scales (the corresponding mode of \( \zeta \) is growing). A calculation analogous to \([10]\) yields
\[ P_\zeta(k, \eta) \sim k^3|\zeta_k(\eta)|^2 \sim k^3|v_k(\eta)|^2 \sim k^3|v_k(\eta_H(k))|^2 \sim k^2, \]
where we have used the constancy of \( \nu \) on super-Hubble scales and the vacuum initial conditions. This is an \( n_s = 3 \) spectrum.

To summarize this section, we have shown that inhomogeneities originating as quantum vacuum perturbations on sub-Hubble scales and crossing the Hubble radius in a matter-dominated contracting phase obtain a scale-invariant \( n_s = 1 \) spectrum, whereas those exiting the Hubble radius during a radiation-dominated contracting phase get an \( n_s = 3 \) spectrum.

If our universe has emerged from a time-symmetric bounce, we would expect all scales which enter the Hubble radius after the time \( t_{eq} \) of equal matter and radiation to have a scale-invariant spectrum, whereas those re-entering earlier would have an \( n_s = 3 \) spectrum. If the bounce is asymmetric because of the production of radiation around the bounce point, we would expect that the transition scale is somewhat smaller than the scale which re-enters the Hubble radius at \( t_{eq} \). In the following, we will study what constraints which current data can give on the scale at which the break in the index of the power spectrum occurs.

In the matter phase we have \( p = 2/3 \) and hence \( \nu = -3/2 \) which leads to the two values for \( \alpha \) which are \( \alpha = 2 \) and \( \alpha = -1 \). The second mode is growing in the contracting phase. Making use of vacuum initial conditions \([5]\) at Hubble radius crossing, we obtain a scale-invariant power spectrum on super-Hubble scales
III. METHOD AND DATA

In our study, we perform a global analysis using the publicly available MCMC package CosmoMC\(^2\). Our theory input is as follows: We assume purely adiabatic initial conditions. Our theory parameter space vector is:

\[
P \equiv \left( \omega_b, \omega_c, \Theta_s, \tau, n_s, A_s, k_s, n_s2 \right),
\]

(13)

where \(\omega_b \equiv \Omega_b h^2\) and \(\omega_c \equiv \Omega_c h^2\), in which \(\Omega_b\) and \(\Omega_c\) are the physical baryon and cold dark matter densities relative to the critical density, \(\Theta_s\) is the ratio (multiplied by 100) of the sound horizon to the angular diameter distance at decoupling, \(\tau\) is the optical depth to re-ionization. The remaining parameters are related to the primordial scalar power spectrum \(P_\chi(k)\) which is parameterized as [13]:

\[
\ln P_\chi(k) = \begin{cases} 
\ln A_s(k_{s0}) + (n_s - 1) \ln \left( \frac{k}{k_{s0}} \right), & k \leq k_s, \\
\ln A_s'(k_{s0}) + (n_{s2} - 1) \ln \left( \frac{k}{k_{s0}} \right), & k > k_s,
\end{cases}
\]

(14)

where \(A_s(k_{s0})\) and \(A_s'(k_{s0})\) are defined as the amplitudes of initial power spectrum before and after the step. \(n_s\) and \(n_{s2}\) measure the spectral index before and after the step in the power spectrum, a step which is located at the co-moving wavenumber \(k_s\), and we match the power spectrum at \(k_s\) during the numerical calculation in order to guarantee the continuity of the power spectrum. For the pivot scale we set \(k_{s0} = 0.05\ Mpc^{-1}\). We assume that the universe is spatially flat and work in the context of the LCDM framework with equation of state \(w = -1\) for the dark energy component.

In our calculations, we have taken the total likelihood to be the products of the separate likelihoods of CMB, SNIa, LSS and Lya. Alternatively defining \(\chi^2 = -2 \log L\), we get

\[
\chi^2_{total} = \chi^2_{CMB} + \chi^2_{SN1a} + \chi^2_{LSS} + \chi^2_{Lya}.
\]

(15)

Using a Markov chain Monte Carlo routine the package then determines the best fit theory parameter values and the corresponding statistical error bars. The data sets we have used are the following: In terms of CMB data, we have included in our parameter fitting the WMAP5 temperature and polarization power spectra with the routine for computing the likelihood supplied by the WMAP team\(^3\). We also included some small-scale CMB measurements, such as BOOMERanG [16], CBI [17], VSA [18] and the newly released ACBAR data [19]. Besides the CMB information, we also combine information about the matter power spectrum from the “Luminous Red Galaxies” sample from the Sloan Digital Sky Survey [20].

| TABLE I. Constraints on the parameters \(n_s\) and \(k_s\) from CMB, LSS, SNIa with/without Lyman \(\alpha\) data. We have considered two cases: either fixing \(n_s2 = 3\) or keeping \(n_s2\) free. The results are obtained by marginalizing over the other theory parameters. |
|------------------|------------------|
| CMB+LSS+SN      | CMB+LSS+SN+Lyα   |
| \(n_s\) fix 3    | \(n_s\) free     |
| 0.957 ± 0.0112   | 0.957 ± 0.0115   |
| 0.960 ± 0.0114   | 0.959 ± 0.0115   |
| \(k_s\) > 0.289(2\(\sigma\)) | \(k_s\) > 0.195(2\(\sigma\)) | \(k_s\) > 1.02(2\(\sigma\)) | \(k_s\) > 1.05(2\(\sigma\)) |

\(\dagger\) Available at: [http://cosmologist.info/cosmomc/](http://cosmologist.info/cosmomc/)

\(\dagger\) Available at the LAMBDA website: [http://lambda.gsfc.nasa.gov/](http://lambda.gsfc.nasa.gov/)

2 and supernova SNIa data. In the calculation of the likelihood from SNIa we have marginalized over the “nuisance parameter” [21]. The supernova data we use is the “Union” compilation (307 sample) [22]. We also consider small scale information obtained from the Lyman-\(\alpha\) forest power spectrum from the Sloan Digital Sky Survey (SDSS) [23], however we also keep its unclear systematics in mind [24, 25]. Furthermore, we make use of the Hubble Space Telescope (HST) measurement of the Hubble parameter \(H_0 \equiv 100\ km\ s^{-1}\ Mpc^{-1}\) by using a Gaussian likelihood function centered around \(h = 0.72\) with standard deviation \(\sigma = 0.08\) [20].

IV. NUMERICAL RESULTS

In this section we show the results from the global fitting to the observational data. We focus on \(k_s\), the scale at which the break in the power spectrum occurs, and \(n_s, n_s2\), the power index parameters, and we marginalize over the other cosmological parameters.

The numerical results are listed in Tables I. We give the results obtained in two different calculations: in one we keep free the parameters \(n_s\) and \(k_s\) and fix \(n_s2 = 3\), in the other we keep \(n_s, k_s\) and \(n_s2\) all to be free parameters.

In Fig. 1 we plot the one dimensional probability distribution of \(k_s\) obtained by fitting with observational data. In this calculation, we have fixed the second power index to be \(n_s2 = 3\) and let the first index \(n_s\) as well as \(k_s\) be free. The black solid line is obtained if we only use the CMB, LSS and SN Ia data, the red dashed line results by including in addition the Lyman \(\alpha\) data. The probability distribution is obtained by marginalizing over the other cosmological parameters.

We find that with current data we can obtain a limit on \(k_s\). Since the data (in particular the Lyman \(\alpha\) data) tends to indicate that there is slightly less structure on small scales than a scale-invariant primordial power spectrum would predict, and the matter bounce model predicts more power on small scales than is obtained from a scale-invariant spectrum, we get a lower bound on the \(k\)-value of the break point. From CMB, LSS and SN data alone, we get \(k_s > 0.289 \ Mpc^{-1}\) at 2\(\sigma\) C.L.. Taking into account the Lyman \(\alpha\) data, the limit on \(k_s\) can be pushed to a much smaller length scale: the 2\(\sigma\) limits can be pushed to \(k_s > 1.02 \ Mpc^{-1}\).
In Fig. 2 we plot the results obtained by keeping both \( n_{s2} \) and \( k_s \) free. We find there are only weak constraints on \( n_{s2} \). The data mildly prefers a red rather than a blue tilt (such as the “matter bounce” would predict), and prefers the position of the break in the power spectrum to be at the value \( k_s = 19 \, h \, Mpc^{-1} \). The interpretation of this result is that a primordial spectrum which is almost scale-invariant, characterized by a fixed power index \( n_s \), fits the large-scale structure data well, but that there is less structure on scales below the break point.

![FIG. 1: Constraints on \( k_s \) from the observations, assuming a spatially flat universe. The vertical axis is the probability, the horizontal axis co-moving scale. The black solid line shows the probability distribution for the position \( k_{st} \) of the break in the spectrum obtained using the CMB + LSS + SN Ia data. The red dashed line is obtained by additionally taking into account the Lyα data.](image1)

![FIG. 2: Constraints on \( k_s \) and the second power index \( n_{s2} \) from CMB + LSS + SN Ia + Lyα data.](image2)

V. SUMMARY

There are theoretical models such as the “matter bounce” which predict a transition in the power spectrum from being approximately scale-invariant on length scales larger than some distinguished scale to being approximately scaling as \( n_s = 3 \) in shorter length scales. In this note, we have studied the constraints on the value of \( k_s \), the co-moving momentum at which the break occurs. The current cosmological data put a 2\( \sigma \) upper limit which is \( k_s \sim o(1) \, h Mpc^{-1} \). Future data sets will be able to set much tighter constraints on \( k_s \).

Acknowledgements

We acknowledge the use of the Legacy Archive for Microwave Background Data Analysis (LAMBDA). Support for LAMBDA is provided by the NASA Office of Space Science. We have performed our numerical analysis on the Shanghai Supercomputer Center (SSC). We thank Yi-Fu Cai for helpful discussions. This work is supported in part by the National Natural Science Foundation of China under Grant Nos. 90303004, 10533010 and 10675136 and by the Chinese Academy of Science under Grant No. KJCX3-SYW-N2. RB wishes to thank the Institute of High Energy Physics for hospitality and financial support, and also the KITPC for hospitality and support during the program “Connecting Fundamental Physics with Cosmological Observations”. RB is also supported by an NSERC Discovery Grant and by funds from the CRC program.

[1] A. Shafieloo and T. Souradeep, “Estimation of Primordial Spectrum with post-WMAP 3 year data,” Phys. Rev. D 78, 023511 (2008) [arXiv:0709.1944 [astro-ph]]; T. Souradeep and A. Shafieloo, “Early Universe With CMB Anisotropy,” Prog. Theor. Phys. Suppl. 172, 156 (2008);
R. Sinha and T. Souradeep, “Post-WMAP assessment of infrared cutoff in the primordial spectrum from inflation,” Phys. Rev. D 74, 043518 (2006) [arXiv:astro-ph/0511808];
A. Shafieloo and T. Souradeep, “Primordial power spectrum from WMAP,” Phys. Rev. D 70, 043523 (2004) [arXiv:astro-ph/0312174]; P. Hunt and S. Sarkar, “Multiple inflation and the WMAP ‘glitches’ II. Data analysis and cosmological parameter extraction,” Phys. Rev. D 76, 123504 (2007) [arXiv:0706.2443 [astro-ph]]; R. K. Jain, P. Chingangbam, J. O. Gong, L. Sriramkumar and T. Souradeep, “Double inflation and the low CMB multipoles,” JCAP 0901, 009 (2009) [arXiv:0809.3915]
A. Lewis and S. Bridle, “Cosmological parameters from R. Brandenberger and X. Zhang, “The Transfer of Adiabatic Fluctuations through a Nonsingular Cosmological Bounce,” arXiv:0707.4677 [hep-th].

D. Wands, “Duality invariance of cosmological perturbation spectra,” Phys. Rev. D 60, 023507 (1999) arXiv:gr-qc/9809062.

F. Finelli and R. Brandenberger, “On the generation of a scale-invariant spectrum of adiabatic fluctuations in cosmological models with a contracting phase,” Phys. Rev. D 65, 103522 (2002) arXiv:astro-ph/0404441.

B. Feng, J. Q. Xia and J. Yokoyama, “Scale dependence of the primordial spectrum from combining the three-year WMAP, Galaxy Clustering, Supernovae, and Lyman-alpha forests,” JCAP 0705, 020 (2007) arXiv:0704.1181 [hep-th].

B. Feng, J. Q. Xia and J. Yokoyama, “Scale dependence of the primordial spectrum from combining the three-year WMAP, Galaxy Clustering, Supernovae, and Lyman-alpha forests,” JCAP 0705, 020 (2007) arXiv:astro-ph/0608365.

V. F. Mukhanov, H. A. Feldman and R. H. Brandenberger, “Theory of cosmological perturbations. Part 1. Classical perturbations. Part 2. Quantum theory of perturbations. Part 3. Extensions,” Phys. Rept. 215, 203 (1992).

R. H. Brandenberger, “Lectures on the theory of cosmological perturbations,” Lect. Notes Phys. 646, 127 (2004) arXiv:hep-th/0306071.

R. Brandenberger and X. Zhang, “The Trans-Planckian Problem for Inflationary Cosmology Revisited,” arXiv:0903.2065 [hep-th].

A. Lewis and S. Bridle, “Cosmological parameters from CMB and other data: a Monte-Carlo approach,” Phys. Rev. D 66, 103511 (2002) arXiv:astro-ph/0205436.

A. Kosowsky and M. S. Turner, “CBR anisotropy and the running of the scalar spectral index,” Phys. Rev. D 52, 1739 (1995) arXiv:astro-ph/9504071; J. E. Lidsey, A. R. Liddle, E. W. Kolb, E. J. Copeland, T. Barreiro and M. Abney, “Reconstructing the inflaton potential: An overview,” Rev. Mod. Phys. 69, 373 (1997) arXiv:astro-ph/9508078; S. Hannestad, S. H. Hansen, F. L. Villante and A. J. S. Hamilton, “Constraints on inflation from CMB and Lyman-alpha forest,” Astrophys. J. 17, 375 (2002) arXiv:astro-ph/0103047; S. L. Bridle, A. M. Lewis, J. Weller and G. Efstathiou, “Reconstructing the primordial power spectrum,” Mon. Not. Roy. Astron. Soc. 342, L72 (2003) arXiv:astro-ph/0302306; B. Feng, X. Gong and X. Wang, “Assessing the Effects of the Uncertainty in Reheating Energy Scale on Primordial Spectrum and CMB,” Mod. Phys. Lett. A 19, 2377 (2004) arXiv:astro-ph/0301111.

C. J. MacTavish et al., “Cosmological parameters from the 2003 flight of BOOMERANG,” Astrophys. J. 647, 799 (2006) arXiv:astro-ph/0507503.

A. C. S. Readhead et al., “Extended Mosaic Observations with the Cosmic Background Imager,” Astrophys. J. 609, 498 (2004) arXiv:astro-ph/0402359.

C. Dickinson et al., “High sensitivity measurements of the CMB power spectrum with the extended Very Small Array,” Mon. Not. Roy. Astron. Soc. 353, 732 (2004) arXiv:astro-ph/0402498.

C. L. Reichardt et al., “High resolution CMB power spectrum from the complete ACBAR data set,” arXiv:0801.1491 [astro-ph].

M. Tegmark et al., “Cosmological Constraints from the SDSS Luminous Red Galaxies,” Phys. Rev. D 74 (2006) 123507, arXiv:astro-ph/0608632.

E. Di Pietro and J. F. Claeskens, “ Quintessence models faced with future supernovae data,” Mon. Not. Roy. Astron. Soc. 341, 1299 (2003) arXiv:astro-ph/0207532.

M. Kowalski et al., “Improved Cosmological Constraints from New, Old and Combined Supernova Datasets,” Astrophys. J. 686, 749 (2008) arXiv:0804.4142 [astro-ph].

P. McDonald et al. [SDSS Collaboration], “The Lyman-alpha Forest Power Spectrum from the Sloan Digital Sky Survey,” Astrophys. J. Suppl. 163, 80 (2006) arXiv:astro-ph/0405013.

P. McDonald et al. [SDSS Collaboration], “The Linear Theory Power Spectrum from the Lyman-alpha Forest in the Sloan Digital Sky Survey,” Astrophys. J. 635, 761 (2005) arXiv:astro-ph/0407377.

E. Komatsu et al. [WMAP Collaboration], “Five-Year Wilkinson Microwave Anisotropy Probe (WMAP) Observations: Cosmological Interpretation,” Astrophys. J. Suppl. 180, 330 (2009) arXiv:0803.0547 [astro-ph].

G. L. Fogli et al., “Observables sensitive to absolute neutrino masses (Addendum),” Phys. Rev. D 78, 033010 (2008) arXiv:0805.2517 [hep-ph].

W. L. Freedman et al. [HST Collaboration], “Final Results from the Hubble Space Telescope Key Project to Measure the Hubble Constant,” Astrophys. J. 553, 47 (2001) arXiv:astro-ph/0012376.