Simulation of the Free Electron Fraction for Cosmological Parameters and Possible Constraints on the Neutrinos

M L Abdelali and N Mebarki
Laboratoire de Physique Mathematique et Subatomique
Mentouri University, Constantine, Algeria

E-mail: mabdelali1@gmail.com

Abstract. A new simulation code to constrain the cosmological parameters and computes the free electron fraction evolution is presented. The code algorithm, inputs-outputs and possible preliminary results are also discussed.

1. Introduction
The evolution of the universe is controlled by the cosmological parameters which are principally constrained by observations of supernovae type Ia and cosmic microwave background (CMB) anisotropies. Codes, like CMBFAST, CMB-easy or CAMB, are developed to compute these parameters from the CMB anisotropies observations based on several experiments like WMAP, PLANCK or COBE [1]-[3]. These codes compute the cosmological fluid perturbations of the Einstein Boltzmann equations. To affine the cosmological parameters, the primary and secondary anisotropies have to be estimated including new theories like inflation. The code which we have developed uses new approach to constrain the cosmological parameters. This method is based on a theoretical estimation after the computation of a unique Boltzmann equation. The paper is organized as follows: In section 2, we present the code for the Simulation of free electron fraction for Cosmological Parameters Estimation (SCOPE) as well as its inputs and outputs. In section 3, the physical settings of the computed equation are discussed. In section 3, we show the possible constrains on neutrinos and finally, in the last section, we draw some of the preliminary results and conclusion.

2. SCOPE code, its inputs and outputs
The new code denoted by SCOPE (or Simulation of free electron fraction for Cosmological Parameters Estimation) [4]-[6] is a C++ code based on a Monte Carlo algorithm. It has as an objective to constraint the cosmological parameters through a numerical computation of the free electron fraction. Its evolution is governed essentially by the Boltzmann equation. The main steps of the algorithm are:

1- Seed the principal cosmological parameters with randomly generated numbers;
2- Make a numerical integration of the evolution equation of the free electron fraction;
3- Check at all steps of integration the mathematical condition of the free electron fraction;
4- Do not take into account the combination if the condition is not valid;
5- Continue the integration until the acceptance of the combination;
6- Return to the first step until the needed number of cosmological parameters combinations.

1 To whom any correspondence should be addressed.
Throughout the computation, the code, checks two principal conditions: the first is that the sum of the cosmological parameters of the fluid species (baryon, dark matter, dark energy and relativistic particles) represents a flat universe and the second is that the free electron fraction, at all steps of integration, remains coherent (it can not be negative or exceed one).

The code is decomposed into three principal parts. In the first part, it generates the random numbers and makes combinations that verify the first condition. In the second part, it reads the list of these combinations, then computes the free electron fraction and finally accepts the appropriate ones. The third part consists of a Maple program to analyze and plot the results. It is very important to mention that one combination in SCOPE is composed of three random numbers representing the cosmological parameters of baryons, dark matter and dark energy where the one for the relativistic particles is given as an input. The inputs to the main part (which is the second one) are of two types. The first inputs are introduced through the interactive black screen when the code is launched. They represent:

- Number of simulated combinations.
- Present day CMB temperature to compute photon contribution to the cosmological parameter of relativistic particles.
- Present day neutrino parameter that contributes to the cosmological parameter of relativistic particles.
- CMB temperature at the beginning of the recombination to compute the scale factor at this era.
- CMB temperature at the end of the recombination or the start of re-ionization to compute the scale factor for the end of the integration.
- Helium fraction that is one of the evolution equation parameters.
- Number of iterations in the numerical integration.

The Second input is the data file containing the list of simulated combinations. The outputs of the main part are three lists of combinations for accepted, refused by exceeding one and refused by being negative. The set of accepted combinations will constrain the cosmological parameters.

3. Physical settings

SCOPE computes the Boltzmann evolution of the free electron fraction in a cosmological process of the Hydrogen recombination. The evolution equation can be applied after the end of Helium recombination and beginning of the Hydrogen recombination to the re-ionization era where the first stars are formed and their ultraviolet radiation can ionize the early universe gas. The CMB temperature of the recombination (resp. re-ionization) is believed to be 0.25eV [7]. (resp.0.0023eV). These two temperatures are strongly related to the cosmological parameters. When these two SCOPE inputs change, the accepted intervals of the cosmological parameters in the outputs will change too. Now, when the universe is cooled up at $T \sim 1$ eV, the neutrinos decoupled from the primordial plasma. Protons, electrons and photons remained tightly coupled by two main types of scattering processes namely: Compton and Coulomb scatterings $e + \gamma \rightarrow e + \gamma$ and $e + p \rightarrow H + \gamma$ respectively.

The Boltzmann equation for a process of interaction of two particles $1 + 2 \rightarrow 3 + 4$ is given by:

$$a^{-3} \frac{\partial (n_1 a^3)}{\partial t} = n_{12}^0 \sigma v \left( \frac{n_1 n_4}{n_2 n_3} - \frac{n_1 n_2}{n_1 n_3} \right)$$  \hspace{1cm} (1)

where $a$ is the universe expansion scale factor and $(\sigma v)$proportional to the interaction rate, represents the total thermally averaged annihilation cross section for the process. For the recombination process, the interaction involve $(e, p)$ resulting in $(H, \gamma)$ and in this case the Boltzmann evolution equation of the free electron fraction [8]$x_e = n_e / n_b$ ($n_b = n_e + n_H$ is the baryonic number density) reads:
$$\frac{dx_e}{dt} = C_r\left[(1 - x_e)\beta - x_e^2 n_b \alpha \right]$$  \hspace{1cm} (2)

where $\beta$ is the collision photo-ionization rate and is given by:

$$\beta \approx \alpha \left(\frac{m_e T}{2\pi} \right)^{3/2} e^{-\frac{B_1}{T}}$$  \hspace{1cm} (3)

$\alpha$ is the recombination rate to the excited states of hydrogen which has the following expression:

$$\alpha = \frac{64 \pi}{(2\pi)^{1/2}} \frac{e^4}{m_e} \left(\frac{B_1}{T} \right)^{1/2} \varphi_2$$  \hspace{1cm} (4)

where the reduction factor $C_r$ is just the ratio of the net decay rate to the sum of the decay and ionization rates from the $n=2$ level (first excited state) or the probability that an atom in the first excited state reaches the ground state before being photo-ionized. Here the factor $\varphi_2$ is:

$$\varphi_2(t < 6000 K) \approx 0.448 ln \left(\frac{B_1}{T} \right)$$  \hspace{1cm} (5)

here $B_1$ is the hydrogen ionization energy and $n_i$ is the number density for the species "i" ($n_i^0$ is the equilibrium number density where the chemical potential vanishes). It is worth to mention that the free electron fraction evolution equation derived from the Boltzmann equation is physically independent of models, the various gauges of metric perturbation and the inflation parameters.

Notice that the cosmological parameters will contribute in two parts: the first one in the baryon number density $n_b$ related to the baryon cosmological parameter and the second in the conversion equation as a function of the scale factor that is:

$$\frac{dx_e}{dt} = aH \frac{dx_e}{da}$$  \hspace{1cm} (6)

where the Hubble parameter $H$ is given by the Friedman equation in flat space:

$$H = \sqrt{\Omega_A h^2 + (\Omega_{dm} h^2 + \Omega_b h^2)a^{-3} + \Omega_r h^2 a^{-4}}$$  \hspace{1cm} (7)

The cosmological energy densities parameters $\Omega_A$, $\Omega_{dm}$, $\Omega_b$ and $\Omega_r$ are for the present time dark energy (represented by the cosmological constant), dark matter, baryonic and relativistic matter respectively. Here the parameter $h = H_0$ and $a_0 = 1$. The two different contributions to the evolution equation of the baryon part of the matter impose that SCOPE need to generate the baryon and dark matter contribution separately rather than the cosmological parameter for all matter. Moreover, the SCOPE simulation code gives a theoretical estimation with little and defined inputs. The code results are independent of the measure errors and secondary contaminations that affect the CMB estimations of the cosmological parameters.

## 4. Possible constraints on neutrino

The possibility that neutrino to be massive or relativistic make it to contribute to different parts in eqs.(6) and (7). The relativistic neutrino contributes to the relativistic particles cosmological parameter related to that of photons as[9]:

$$\Omega_r = \Omega_\nu + \Omega_\nu$$  \hspace{1cm} (8)
\[ \Omega_\nu = \frac{7}{8} N_{\text{eff,neutrino}} \left( \frac{4}{11} \right)^{4/3} \Omega_\gamma \]  

\[ \Omega_\nu h^2 = \frac{\Sigma m_\nu}{94 \text{eV}} \]  

\( N_{\text{eff,neutrino}} \) is the effective number of neutrinos. The massive neutrino contributes to the dark matter cosmological parameter as hot dark matter. Now, considering the neutrinos as relativistic and as massive in two time experiment allow measuring up the changes in the different accepted ranges of the different cosmological parameters. The two time experiment allows measuring up the cosmological parameters evolution for both relativistic and massive neutrino. The relativistic neutrino cosmological parameter will vanish for massive neutrino. The contribution of these neutrinos will be considered as part of the total dark matter. The difference in the dark matter contribution in two computations that have the two different neutrino contribution type will be considered as an allowed massive neutrino contribution [9]:

By requiring that the neutrinos do not close the universe, this means that \( \Omega_\nu < 1 \), one gets an upper bound \( \Sigma m_\nu < 15 \text{eV} \) (using \( h = 0.7 \)). Measurements of tritium \( \beta \) decay find that \( \Sigma m_\nu < 6 \text{eV} \), observations of the cosmic microwave background, galaxy clustering and type Ia supernovae leads to a stronger bound \( \Sigma m_\nu < 1 \text{eV} \) and upper limit 0.6 eV from the structure formation [10]. The SCOPE code allows the calculation of the cosmological parameters evolution of both the relativistic and massive neutrinos at the same time. Of course the neutrino cosmological parameter will vanish for massive neutrinos. The contribution of these neutrinos will be as a part of the dark matter component. The difference in the dark matter contribution in the two computations with different neutrino contribution type will be considered as an allowed massive neutrino contribution (see eq.10)

5. Preliminary results and conclusion:
Using various CMB temperatures at the beginning and the end of recombination SCOPE code computation yields to the following result and behavior of the cosmological parameters:

1- The dark matter cosmological parameter expectation value was greater for the massive neutrinos case.
2- The allowed massive neutrino contribution is greater for the beginning of the earlier recombination
3- The computed neutrino mass challenges the lower limit 0.6 eV from neutrino experiments and an upper limit 0.6 eV from the structure formation.

Figs.(1)-(4) display the scatters plots of the energy densities parameters variation of the baryonic and dark matter, dark energy and \( \Sigma m_\nu \) respectively as a function of the recombination temperature using SCOPE simulation code.

The future improvements and developments that can be ported to the code come in two principal ways:

1- First, new constrains can be added to the actual hierarchy:
   a- Make the accepted ranges of the free electron fraction more restraint like imposing a small fraction at the end of iteration and at the beginning of the re-ionization era.
   b- Compute the optical depth that is related to this free electron fraction and make new acceptance ranges.
   c- Make SCOPE part of more complicated code for CMB estimations of the cosmological parameters.

2- Second, the computation is constrained by new free electron fraction evolution equation. The Boltzmann evolution equation is founded on both the Friedmann-Robertson-Walker (FRW) cosmological model for the evolution of flat universe and standard Quantum-Electro-Dynamics (QED) theory for the interaction of the recombination process. The inclusion of new components that affect
the interactions or corrections to one of these standard theories can result in different evolution pattern and different constrains on the cosmological parameters.

Figure 1. Simulated baryonic energy density parameter as a function of the recombination temperature

Figure 2. Simulated dark matter energy density parameter as a function of the recombination temperature
Figure 3. Simulated dark matter energy density parameter as a function of the recombination temperature.

Figure 4. Simulated sum of the neutrinos masses as a function of the recombination temperature.
Acknowledgement
This work is supported by the Algerian Ministry of education and research and DGRSDT.

References
[1] Seljak U. and Zaldarriaga M 1996 *APJ* 469 437
[2] Zaldarriaga M, Seljak U and Bertschinger E 1998 *APJ* 494 491
[3] Zaldarriaga, M and Seljak U 2000 *APJ* 129 431
[4] Abdelali M L and Mebarki N 2013 “A new non cosmological redshift effect”, 9th international Conference in Subatomic Physics and applications (CIPSA) Mentouri University, Constantine, Algeria, Sept. 30th-Oct. 02nd
[5] Abdelali M L and Mebarki N 2013 “A new Primordial magnetic field evolution and signatures on the CMB spectrum”, 9th international Conference in Subatomic Physics and applications (CIPSA) Mentouri University, Constantine, Algeria, Sept. 30th-Oct. 02nd
[6] Mebarki N 2014 “A New Non Cosmological redshift, Implications on the UHECR propagation and distances recalibration” 15th JEM-EUSO International Meeting June 9-13, Palermo, Italy
[7] Hu W and Dodelson S 2002 *Ann. Rev. Astron. Astrophys.* 40 171
[8] Peebles P J E 1968 *Astrophysical Journal* 153 1
[9] Hannestad S 2006 “Neutrinos in Cosmology” Progress in Particle and Nuclear Physics, International Workshop in Astroparticle and Nuclear Physics, Ettore Majorana Center for Scientific Culture309.
[10] Lesgourgue J and Pastor S 2006 *Phys. Rep.* 429 307