Quantum superpositions and definite perceptions: envisaging new feasible experimental tests.

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Abstract

We call attention on the fact that recent unprecedented technological achievements, in particular in the field of quantum optics, seem to open the way to new experimental tests which might be relevant both for the foundational problems of quantum mechanics as well as for investigating the perceptual processes.

1 Introductory considerations.

The crucial problem of quantum mechanics can be summarized in a very elementary way: if one assumes that the linear evolution law of the theory governes all natural processes, then, while in actual measurement processes (and in all those measurement like processes we are obliged to admit ... are going on more or less all the time, more or less everywhere) linear superpositions of macroscopically different situations occur, we perceive only one among the many potential outcomes. In a way or another (more about this in what follows) we have to recognize that, at least at the level of our definite perceptions, the linear laws of quantum mechanics are violated. Here we will not discuss the extremely delicate and controversial problem of the perceptual process, but we will limit ourselves to call attention on the fact that recent unprecedented technical improvements, specifically in the field of quantum optics, seem to allow the preparation of linear superpositions of states such that the terms of the superposition are able to trigger definite and different perceptions of a conscious observer. In spite of the fact that everybody would feel confident in anticipating the main characteristics of the final exit of the experiments we are going to discuss we think that it is interesting to analyze them since they exhibit some completely new features and they could, in principle, allow to get some interesting indications about the perceptual process.

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2 The problem and some standard solutions.

Usually one circumvents the just mentioned problem of reconciling our definite perceptions with the waviness, the indeterminacy characterizing linear superpositions, by pointing out that in actual situations one can avoid to even mention the conscious observer since the different apparatus states whose superpositions are generated in measurement like process are macroscopically different (the most elementary case being the one of different positions of the macroscopic pointer of the apparatus). Even though the issue under discussion is rather controversial, everybody would agree that the above remark is absolutely pertinent, at least for all practical purposes. For clarity sake, let us elucidate this point by analyzing the puzzling situation we have just mentioned from the point of view of various positions which one can take about the theory and its interpretation.

The textbook solution. With the above expression we denote the position \( [2, 3] \) of the founding fathers of the theory, i.e. the one which is usually referred to as “the orthodox interpretation”. It plainly amounts to accept that there are two evolution laws governing natural processes, the first one characterizing microscopic systems which is linear and deterministic, the second one entering into play when different macroscopic effects are triggered by different microscopic situations which is phenomenologically described by the nonlinear and stochastic process of wave packet reduction.

Obviously, such a solution has to face problems of consistency and of lack of mathematical precision and is considered as unsatisfactory by most scientists involved in the debate about the conceptual implications of quantum mechanics, but this is not the point we want to make here. For our purposes it is simply useful to remark that if one takes such a position the problem of our definite perceptions in the above considered cases simply disappears: even before we “look” at the apparatus, due to the fact that wave packet reduction has taken place, the macroscopic system is already in a macroscopically precise situation which matches our definite perception about it.

The decoherence argument. According to this point of view \([4, 5]\) no wave packet reduction takes place, but the quantum nature of the “macroscopic pointer” (i.e. its being in a superposition) cannot play, in practice, any testable role since the interference effects to which it could (in principle) give rise are hidden or suppressed by various mechanisms, the most typical one being its unavoidable coupling with the environment and the ensuing decoherence. This fact can be put in a precise mathematical form by recalling an elementary theorem about composite systems:

Suppose we consider an entangled normalized state \( |\Psi(1, 2)\rangle \) of a composite system \( S = S_1 + S_2 \):

\[
|\Psi(1, 2)\rangle = \alpha |\phi^{(1)}\rangle \otimes |A^{(2)}\rangle + \beta |\chi^{(1)}\rangle \otimes |B^{(2)}\rangle,
\]

(1)
and let us evaluate, for such a state, the expectation value of an arbitrary projection operator $P_1$ of the Hilbert space of $S_1$. We have:

\[
\langle \Psi(1, 2) | P_1 | \Psi(1, 2) \rangle = |\alpha|^2 \langle \phi^{(1)} | P_1 | \phi^{(1)} \rangle + |\beta|^2 \langle \chi^{(1)} | P_1 | \chi^{(1)} \rangle + 2 \text{Re} \left\{ \alpha^* \beta \langle \phi^{(1)} | P_1 | \chi^{(1)} \rangle \langle A^{(2)} | B^{(2)} \rangle \right\}.
\]

(2)

This simple equation expresses the well known fact that if the states $|A^{(2)}\rangle$ and $|B^{(2)}\rangle$ of the system which is entangled with $S_1$ coincide ($\langle A^{(2)} | B^{(2)} \rangle = 1$), then the interference effects are fully exploitable, while if they are orthogonal ($\langle A^{(2)} | B^{(2)} \rangle = 0$), any conceivable test on the system $S_1$ gives exactly the same outcome as the one implied by the statistical mixture of pure states of $S_1$ in which a fraction $|\alpha|^2$ of the systems are in the state $|\phi^{(1)}\rangle$ and a fraction $|\beta|^2$ in the state $|\chi^{(1)}\rangle$, i.e. the situation characterized by the statistical operator:

\[
\rho = |\alpha|^2 |\phi^{(1)}\rangle \langle \phi^{(1)} | + |\beta|^2 |\chi^{(1)}\rangle \langle \chi^{(1)} |.
\]

(3)

The conclusions which are relevant for the problem we are investigating should then be obvious: the macroscopically different final apparatus states (which must be identified with the states $|\phi^{(1)}\rangle$ and $|\chi^{(1)}\rangle$ of the above example) become immediately entangled with the environment and the states of the environment associated to the different terms (i.e. the states corresponding to $|A^{(2)}\rangle$ and $|B^{(2)}\rangle$) are (practically) orthogonal. Since when an observer “looks” at the pointer he cannot “perceive” all environmental degrees of freedom, his perceptions too, can, in practice, be identified with those triggered by either the state $|\phi^{(1)}\rangle$, i.e. the one making legitimate the claim “the pointer points at 1” or by the state $|\chi^{(1)}\rangle$, associated to the fact that “the pointer points at 2”.

The Bohmian point of view. As is well known, the de Broglie-Bohm theory \cite{Bohm1, Bohm2, Bohm3} is a nonlocal and deterministic hidden variable theory which completely agrees with quantum mechanics and attributes to all particles of the universe a perfectly definite position at all times. For the case under consideration the theory claims that the pointer is definitely either in one or in the other of the two positions 1 and 2, while its wavefunction (which coincides with the quantum mechanical one) is different from zero both in regions 1 and 2.

Within this framework, the fundamental interplay of the position variables and the wavefunction describing the Hilbert space evolution of the physical system is the fundamental ingredient guaranteeing the agreement with quantum mechanics. For instance, a particle can be claimed to follow, in the two slits experiment, one and only one of the two “possible paths” but the very fact that its wavefunction is different from zero also on the path which it does not follow guarantees that, on repetition of the experiment, one gets the interference pattern on the screen.
However, it is one of the nicest and most relevant features of the theory that, while for a microsystem the full wave function plays the just mentioned fundamental role, in the case (like the one which interests us) of a macro-object which is in a precise region but whose wave function is different from zero also in another region, one can resort to the “effective wavefunction” to describe its evolution. The effective wavefunction is simply the one obtained by dropping the so called “empty wave”, i.e. its part referring to the region where there is no object. This is obviously an approximation, but it can be proved to hold to an extremely high degree of accuracy.

Once more, with reference to our example, we could claim that the conscious observer is confronted with an unambiguous and precisely definite macroscopic situation: the pointer is at a definite place and only the value of the wavefunction at that place matters.

The spontaneous localization point of view. Recently, a new way out from the difficulties connected with the measurement problem has been suggested. It is based on the consideration of nonlinear and stochastic modifications of the Schrödinger equation. Such modifications are due to terms describing the occurrence of spontaneous localization processes affecting all elementary constituents of any composite system. The characteristic trait of the approach derives from the fact that the dynamical modifications are such that they leave (practically) unaltered all predictions of standard quantum mechanics for microscopic systems but they induce an extremely rapid (within millionths of a second) suppression of the superpositions of macroscopically distinguishable states.

With reference to our problem, the various dynamical reduction models which have been discussed in the literature lead (even though in a consistent way and as a consequence of a universal dynamical equation) to the same conclusion as the standard theory supplemented by the assumption of wave packet reduction: the superpositions of macroscopically different states corresponding to different pointer positions are dynamically forbidden. Once more, the conscious observer faces a macroscopically definite situation: the quantum ambiguity connected with superpositions has already been disposed of by the universal dynamics which characterizes such models.

The conclusion one can draw from the previous analysis should be obvious: in a way or another, quite independently of the position one takes about the theory and its interpretation the standard wisdom about perceptions is restored: the perceptions of the conscious observer can be considered, for all practical purposes, as triggered by unambiguous macroscopic situations. It is therefore interesting to investigate whether one can devise situations in which one can be sure that the perceptual process is directly triggered by a genuine superposition of different states which, when considered by themselves, would give rise to different perceptions.

Before coming to discuss this point, a short clarifying digression is appropriate.

\footnote{We do not want to be misunderstood: we consider the macro-objectification problem as a quite relevant one deserving hard work by theoretical physicists. But this fact can be disregarded for our present purposes.}
3 A short digression.

To prepare the reader to the subsequent analysis it seems useful to recall a question which has been debated in the literature several years ago [13]. Suppose we send a spin 1/2 particle in a state which is the eigenstate corresponding to the eigenvalue +1 of the observable $\sigma_x$, through a Stern-Gerlach apparatus devised to measure $\sigma_z$. We all know that a subsequent experiment aimed to detect whether the particle is in the upper or in the lower of the two possible paths for it will give an unambiguous outcome: either the particle will activate the counter on the upper path or the one on the lower one. Now comes the question: does the reduction of the wave packet take place at the moment of the detection or has it already taken place during the process of traversing the inhomogeneous magnetic field? B. d’Espagnat has given the clear-cut answer: the reduction cannot take place when the particle goes through the region of the Stern-Gerlach apparatus for the simple reason that everybody would agree that if one were able to build an apparatus which undoes exactly what the first one did (i.e. it induces the time-reversed evolution), it would recombine the two wave packets and it would restore the spin eigenstate $|\sigma_x = 1\rangle$ for the particle. As a consequence, a measurement of $\sigma_x$ would give, with certainty, the outcome +1. On the contrary, if consideration were given to the statistical mixture corresponding to the assumption that the reduction has already taken place, the unfolding of the “reverse” process would assign probability 1/2 to the outcome $\sigma_x = 1$. The question admits therefore a clear cut experimental answer.

Obviously, one could object that this is a “truly gedanken experiment” and that the test is possible only “in a science fiction world” in which one can actually build an apparatus which undoes precisely what the first Stern-Gerlach apparatus did to the particle. However, we all know the way to circumvent this (appropriate) criticism by resorting to systems involving photons. Such a procedure brings down the above experiment from “the heaven of the conceptual possibilities” to “the real world of the actual feasibility”.

In fact, let us consider [14] a photon with plane polarization at 45 degrees impinging on a birifrangent crystal whose ordinary ray is characterized by the horizontal and the extraordinary ray by the vertical polarization. From the crystal two spatially well separated rays emerge, or, to be precise, after having traversed the crystal the photon is in the superposition of being in the ordinary ray and having a horizontal polarization and of being in the extraordinary ray and having a vertical polarization. To check that the superposition is still present is quite easy: one can use a mirror reflected crystal which recombines the two states and makes them to interfere. After traversing it the photon is polarized at 45 degrees, as one can easily check by putting a polarizer in front of a photon detector. So we have here an example of a situation in which it is possible (and easy) to test that before impinging on the second crystal the superposition of the “spatially separated states” is still there.

We are now ready to discuss the experiment we want to propose.
4 A challenge for experimental quantum optics.

The idea we are going to present in this Section is based on the simple remark that the human visual perceptual apparatus is characterized by an extreme sensitivity. As is well known, the threshold for visual perception is of about 7 photons. To our knowledge this is the only firmly established case in which a truly microscopic system can trigger directly a definite perception. So, let us start by considering the case in which a bunch, let us say of 10 photons coming from a region A, propagates towards the eye of a human observer. The bunch hits the retina of the observer and triggers the definite perception “a luminous spot at A”. Now we can consider an analogous situation in which the bunch comes from a region B, spatially separated and perceptively distinct from A. Again we can perform the experiment and the conscious observer will perceive “a luminous spot at B”.

Here comes the challenge for people working in quantum optics. Suppose we are able to prepare a state which is the superposition of the two previously considered states, i.e. the state:

\[ |\Psi(1,2)\rangle = \frac{1}{\sqrt{2}} (|10 \text{ photons from A}\rangle + |10 \text{ photons from B}\rangle). \] (4)

We also assume, and this is a crucial point of the proposal, that there is the practical possibility of testing that actually the superposition persists for the time we will be interested in. This means that we have a mean to recombine the two states and to put into evidence their interference effects, just in a way analogous to the one we have analyzed above to test the fact that the single photon is in a superposition after traversing the birifrangent crystal. We do not know whether the present technology already allows such a performance, but Profs. De Martini and Zeilinger have told me that the desired result would certainly be attainable within a short time, and that, for the time being, to actually prepare it and to perform the interference test is mainly a matter of money. Some relevant steps in this direction have been performed recently [18, 19].

Given these facts, we think that everybody will have clear the novelty of the situation we are envisaging with respect to the one analyzed in Section 2. Here we prepare a state such that we can test explicitly that the quantum coherence between its terms persists up to the moment in which the conscious observers “looks” at it. The interesting fact is that, even though the two superposed states refer to a microscopic system, they can trigger definite and different perceptions. Here one cannot invoke wave packet reduction, the decoherence induced by the entanglement with the environment, the possibility of making the approximations involved in the Bohmian analysis, or the decoherence induced by the spontaneous localizations before the observer looks at the photons. So we can proceed. We put our observer in place of the apparatus testing that the superposition is still there

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2A state like the one we are considering has been discussed for the first time by Albert and Vaidman [13] (see also [16]) to point out an alleged difficulty of the dynamical reduction program. We have already answered [17] to this criticism proving its unappropriateness. In spite of that, the analysis of these authors has obviously played a precise role for the present paper. The relevant fact, however, is that at the time of the work by Albert and Vaidman the possibility of preparing such a state was a pure speculative assumption, while what we are pointing out in this paper is that now to prepare such a state seems practically feasible.
and we investigate what are his perceptions. We think everybody will agree about the probable outcome of the experiment: the observer will have definite perceptions and will not end up in a confused state of mind. Moreover his perceptions will be distributed randomly between “I perceive a spot at A” and “I perceive a spot at B”.

But we also think that to perform the experiment is of some relevance and that no one would feel fully confident in answering to questions like: which will be the precise unfolding of the perception process in this peculiar and fundamentally new situation? To make clear our point of view let us describe in greater details a precise set of experiments (among the various ones) that can be devised.

We have an apparatus which can prepare three different kinds of photon bunches:

i) Ten photons emanating from A,

ii) Ten photons emanating from B,

iii) The above state (4).

In the first run of experiments we stimulate the visual apparatus of the observer by choosing randomly the set-ups leading to the preparations i) or ii). Obviously the observer will perceive a random sequences of “spots from A” and “spots from B”. In the second run we stimulate the visual apparatus of the observer by choosing always set-up iii). We expect that he will still perceive a random sequence of “spots from A” and “spots from B”. But now we raise the questions: are we sure that the specific details of the perception will be the same? For instance, are we sure that the reaction times and/or the possible perceptual failures will be the same in the two cases? These questions are legitimate and do not admit a naive answer (more about this in the next Section) since one must recall that, loosely speaking, in the second run the brain has to do the “extra job” (with respect to the one of the first run) of “reducing the superposition” of the initial nervous stimuli.

These arguments should have made clear why we consider of some interest to actually perform the experiment: it might be that we can get from it some unexpected information, which could be of some relevance. At any rate we will at least obtain the experimental proof that a system like our sensory apparatus, contrary to the complicated and macroscopic system used to test that the superposition is still there, is actually able to “reduce the statevector”.

One could then think of more ambitious programs. For instance, the late Prof. Borsellino has repeatedly called my attention on the fact that even quite small living organisms can be trained to react to a light stimulus. One could then investigate the reactions of such organisms to see whether some differences occur when one subjects them to one of the two above procedures. Many variants of the indicated process can be devised. Obviously it is rather hazardous to guess that one could really learn something from such experiments since many loopholes will in any case remain open, but at the same time it seems to us that they have some intrinsic interest.
5 Making our argument more plausible.

To fully appreciate our point of view and the reasons for which we believe that the proposed experiment might be relevant it is appropriate to discuss a possible naive objection against it. One could argue along the following lines. Consider the first run of experiments and the case in which the preparation procedure leads to situation i). The photons impinge on the observer’s retina and trigger the transmission of the nervous signal. Let us consider an oversimplified description of the unfolding of the process from the initial stimulus to an intermediate stage in which part of the brain is involved:

\[ |\text{stimulus i); brain state } R;...\rangle \rightarrow |\text{stimulus i); brain state } S_i;...\rangle. \quad (5)\]

Here R stays obviously for “ready to react to the stimulus” and we have left unspecified (for reasons which will become clear in a moment) the states of the other parts of the perceptual apparatus which will enter into play, in particular those referring to the final stage in which a definite perception emerge. Analogous considerations apply to the case in which the initial state is the state ii):

\[ |\text{stimulus ii); brain state } R;...\rangle \rightarrow |\text{stimulus ii); brain state } S_{ii};...\rangle. \quad (6)\]

One could then remark that since in each of the experiments of the second run the initial state is the superposition of the two above initial states, according to the linear nature of quantum evolution, one would have:

\[ \frac{1}{\sqrt{2}} [|\text{stimulus i); brain state } R;...\rangle + |\text{stimulus ii); brain state } R;...\rangle] \]
\[ \Rightarrow \frac{1}{\sqrt{2}} [|\text{stimulus i); brain state } S_i;...\rangle + |\text{stimulus ii); brain state } S_{ii};...\rangle]. \quad (7)\]

For such a state, however, the subsequent occurrence of wave packet reduction (due e.g. to the emergence of a definite conscious perception or to the uttering by the observer of a sentence stating which perception he has experienced, or to the coupling of his brain states with the environment), will lead precisely to one of the final states (5) or (6). Thus, the proposed experiment would be meaningless: at the end there is intrinsically no difference between the two types of runs we have proposed.

However, it has to be stressed that the above argument is entirely based on the assumption that the linear and deterministic evolution characterizes all physical processes and on the hypothesis, characteristic of standard quantum mechanics, that the reduction process is essentially an instantaneous process. Moreover it is also assumed that such a process simply selects one of the two terms of the superposition without in any way affecting it. This is a very restrictive and unrealistic assumption. Actually, the only request that is necessary to yield agreement with our definite experience is that the final state belongs to a precise eigenmanifold (among the two possible ones) of the observable “my perception about the location of the spot”. Now, the idea that the assumptions listed above are inappropriate is shared by an increasing number of scientists. We can
mention the lucid statement by Leggett \[20\]: one might imagine that there are corrections
to Schrödinger’s equation which are totally negligible at the level of one, two, or even one
hundred particles but play a major role when the number of particles involved becomes
macroscopic, and the repeated assertions of R. Penrose \[21\] as emblematic examples. The
very fact that the dynamical reduction program has raised so much interest and has been
so convincingly supported by J.S. Bell, shows that the scientific community is contemplat-
ing seriously the idea that one should give up the unrestricted validity of the linear nature
of the evolution. If this is the case, the physical processes which present themselves as
the most natural candidates to get some evidence of a violation of the standard dynam-
ics are the mesoscopic processes and those processes like our definite perceptions which
(even if we know very little about them) seem to require that reduction actually takes
place. It is therefore reasonable to entertain the idea that in the case under discussion a
non linear and stochastic dynamics, the one leading to wave packet reduction and to a
definite perception, governs the process we are analyzing. Accordingly, there is no reason
to exclude that in spite of the fact that the evolutions described by Eqs. \(5\) and \(6\) rule
the process when the initial states are i) or ii), respectively, the correct equation which
actually describes the unfolding of the process and which has to replace Eq. \(7\) in case
iii) will take the form:

\[
\frac{1}{\sqrt{2}} [\text{stimulus i); brain state R;...} + \text{stimulus ii); brain state R;...}]
\Rightarrow \frac{1}{\sqrt{2}} [\text{stimulus i); brain state (Si)*;...} + \text{stimulus ii); brain state (Sii)*;...}]. \tag{8}
\]

where the states (Si)* and (Sii)* are physically different (in some respect) from the cor-
responding states Si and Sii appearing in Eqs. \(5\) and \(6\). Alternatively, one could
contemplate the possibility (see the next section for an illuminating ex-
ample) that the non linear evolution might lead to a final state like the one in Eq. \(7\), in which, however,
a linear superposition with coefficients quite near but not exactly equal to \(1/\sqrt{2}\), appears.
Then, at the moment in which the subsequent evolution, e.g. the final act of perception
or any other physical mechanism, will induce the reduction, the actual state of the brain
would be different according to whether we are performing experiments of the first or
of the second type, respectively. In turn, these mesoscopic differences could give rise to
slight differences concerning the reaction mechanisms of the nervous system, such as, e.g.,
the average reaction times and/or possible perceptual failures.

That the present argument has to be taken seriously, can be shown by making refer-
ce to one of the theoretical models which give a mathematically precise (even though
phenomenological) account of the reduction process, i.e., the Spontaneous Localization
Models \[9, 10, 11, 12\]. Within such models the dynamical processes leading to the final
definite outcome and/or perception take place during appropriate time intervals which
are precisely defined by the physical context. Actually, just to answer the criticisms of
ref. \[15, 16\], we have analyzed \[17\] (even though only in a qualitative way) precisely
the visual perception mechanism. What we have proved is that one can summarize the
unfolding of the process in the following way. As soon as the superposition of the two
stimuli excite the retina, two nervous signals start and propagate along two different axons. The process goes on and the relative weights of the two terms of the superposition change due to the stochastic processes affecting the ions which cross the Ranvier nodes to transmit the signal. Only when, in the process of nervous transmission along the axons to the lateral geniculate body and to the higher visual cortex, the number of ions which occupy different positions in the two states of the superposition reaches an appropriate threshold, the suppression can be considered complete: only one of the two signals survives and triggers an unambiguous perception process. Has it has been shown in [17] the completion of the process requires times comparable to the perception time. This means that for an appreciable fraction of such a time the superposition of the two ”nervous signals” is still present, the two terms having comparable weights. In turn, each term will be associated to smeared electric effects involving few particles but characterized by a spreading of the wavefunctions associated to them. Then, if one takes into account the very structure of the theory, one realizes that the genuinely stochastic processes implying the “spontaneous localization” of precise particles of the system, are governed by the overall wavefunction and, as a consequence, they might exhibit different features when one consider either situation i) or situation ii) versus situation iii). But a localization of a particle can lead, as discussed in refs. [9, 10, 11, 12], to the excitation or dissociation of the ion to which it belongs. In brief, the actual unfolding of the process in case iii) is by no means the “linear combination” of the processes occurring in the cases i) and ii). Thus, when the reduction will actually come to an end and the definite perception will emerge, it is not unreasonable to admit that some precise and permanent record of the fact that the perceptual apparatus has been triggered by case iii) and not by either case i) or case ii), might remain. Whether such differences can have a systematic effect leading for instance to an increase of perceptual failures and errors or to a change in the reaction time is a fact that we cannot give for granted but which we cannot even exclude.

Obviously, due to the fact that the considered dynamical reduction models have only a phenomenological status and that it has not be possible to test them against quantum mechanics, due to our extremely vague knowledge of perceptual processes and so on, one cannot take at its face value the above argument and one cannot be precise about the differences we have to look for. But the general argument is surely correct: if some mechanism is present in nature which actually leads to reductions (and as such it must violate the linear nature of standard quantum mechanics) then this fact by itself implies that, after the reduction process has taken place and has led to a definite “outcome/perception”, the final state of all systems which have entered into play in the process will be, in general, different according whether the final perception has been triggered by a state which could lead to different perceptions or it has been triggered by a state which can only lead to the one which actually occurred. To clarify further this point it seems useful to discuss in detail a dynamical reduction toy model which allows to focus the specific features of dynamical reduction theories in the case in which the reduction process becomes competitive with the hamiltonian evolution.
6 A detailed study of a simple dynamical reduction model.

In this section we will consider a dynamical reduction model of the type of those introduced in refs. [9-12]. However, since we intend to mimic the unfolding of a dynamical process in which a microscopic state triggers a process which takes an appreciable time (let us say of the order of the perceptual times) to lead to the situation in which the nonlinear and stochastic processes inducing the reduction become effective, we cannot disregard (as one usually does when discussing the suppression of superpositions of macroscopically different states - like “\(|\text{pointer here}\rangle + |\text{pointer there}\rangle\)”), the Hamiltonian part of the evolution equation. This can be easily understood: as discussed in refs. [9-12], in the case of superpositions corresponding to different locations of a macroscopic object the effective spontaneous localizations occurs with an extremely high frequency \(\lambda \approx 10^7 \text{sec}^{-1}\).

Then, for a macroscopic system, the Hamiltonian evolution during two successive localizations cannot change appreciably the state of the system and can be disregarded. As a consequence the state

\[ |\Psi\rangle = a|\text{pointer here}\rangle + b|\text{pointer there}\rangle. \tag{9} \]

will be transformed, with probabilities exactly equal to \(|a|^2\) and \(|b|^2\) into one of the two terms of eq.(9).

The situation we are facing here is quite different: we are considering the unfolding of a physical process which at the beginning involves few particles, and consequently no appreciable contribution can derive from the nonlinear terms and the evolution is essentially the Hamiltonian one. Subsequently, the linear and nonlinear dynamical terms become competing up to the moment (after about \(10^{-2}\text{sec}\)) in which the reducing terms govern the game. To get an idea of how things could go for the case we are interested in we have to consider a model in which the time between two successive reduction processes is such that, during it, the Hamiltonian evolution can modify appreciably the state of the system and can be disregarded. As a consequence the state

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The most elementary model in which a competition between the Hamiltonian and the stochastic and nonlinear terms can be explicitly studied is the one of a system whose associated Hilbert space is two-dimensional. Obviously, such a toy model has nothing to do with the actual unfolding of a perceptual process. But our aim is simply to identify some precise formal features which characterize the evolution. Actually, for our limited purposes, it will not even be necessary to discuss the dynamical behaviour at the individual level (i.e. to follow the stochastic evolution of each individual physical system), the relevant information being deducible directly from the statistical operator.

We will therefore consider a dynamics of the GRW or of the CSL type (individual discontinuous or continuous reductions) and the evolution equation for the statistical operator that it implies. Let us be more precise about the details of the process. We have a system S in a two dimensional Hilbert space and we suppose that the linear part of its
evolution is governed by the Hamiltonian:

\[
H = \hbar \omega \begin{vmatrix} 0 & A \\ A^* & 0 \end{vmatrix}, |A| = 1, \tag{10}
\]

while the analogous of the spontaneous localizations are reductions on the one-dimensional manifolds identified by the projection operators:

\[
P_+ = \begin{vmatrix} 1 & 0 \\ 0 & 0 \end{vmatrix}, P_- = \begin{vmatrix} 0 & 0 \\ 0 & 1 \end{vmatrix}. \tag{11}
\]

The reduction processes are assumed to take place with a mean frequency \(\lambda\).

The dynamical evolution equation for the statistical operator is of the Quantum Dynamical Semigroup type and reads:

\[
\frac{d\rho}{dt} = \frac{-i}{\hbar} [H, \rho] + \lambda P_+ \rho P_+ + \lambda P_- \rho P_- - \lambda \rho. \tag{12}
\]

If we write

\[
\rho(t) = \begin{vmatrix} r(t) & \beta(t) \\ \beta^*(t) & 1 - r(t) \end{vmatrix}, \tag{13}
\]

the evolution equations are:

\[
\frac{dr(t)}{dt} = 2\omega \left[ \text{Im}(A)\text{Re}(\beta(t)) - \text{Re}(A)\text{Im}(\beta(t)) \right] \tag{14}
\]

\[
\frac{d\text{Re}(\beta(t))}{dt} = \omega \text{Im} \left[ A \left\{ 1 - 2r(t) \right\} \right] - \lambda \text{Re}(\beta(t))
\]

\[
\frac{d\text{Im}(\beta(t))}{dt} = -\omega \text{Re} \left[ A \left\{ 1 - 2r(t) \right\} \right] - \lambda \text{Im}(\beta(t)).
\]

A general theorem by Spohn \[22\] guarantees that, in the finite dimensional case, if a Quantum Dynamical Semigroup equation admits a steady solution, then the general solution converges to it for \(t \to +\infty\). In our case the steady solution is \(\rho(t) = \frac{1}{2} I\). Taking into account this fact one easily writes the general solution of our problem:

\[
r(t) = \begin{vmatrix} 2\omega \frac{\text{Re}(A)\text{Im}(\beta(0)) - \text{Im}(A)\text{Re}(\beta(0))}{\lambda^2 \Delta} + \frac{4\omega^2 \left[ 1 - 2r(0) \right]}{\lambda^2 \Delta \left[ 1 - \Delta \right]} \right\} e^{-\frac{\lambda}{2}(1+\Delta)t}

- \left\{ 2\omega \frac{\text{Re}(A)\text{Im}(\beta(0)) - \text{Im}(A)\text{Re}(\beta(0))}{\lambda^2 \Delta} + \frac{4\omega^2 \left[ 1 - 2r(0) \right]}{\lambda^2 \Delta \left[ 1 - \Delta \right]} \right\} e^{-\frac{\lambda}{2}(1-\Delta)t} + \frac{1}{2}
\]

\[
\text{Re}(\beta(t)) = a\text{Im}(A)e^{-\frac{\lambda}{2}(1+\Delta)t} + b\text{Im}(A)e^{-\frac{\lambda}{2}(1-\Delta)t}

+ [\text{Re}(A)\text{Re}(\beta(0)) + \text{Im}(A)\text{Im}(\beta(0))] \text{Re}(A)e^{-\lambda t}
\]

\[\text{Im}(\beta(t)) = [\text{Re}(A)\text{Re}(\beta(0)) + \text{Im}(A)\text{Im}(\beta(0))] \text{Im}(A)e^{-\lambda t}

- a\text{Re}(A)e^{-\frac{\lambda}{2}(1+\Delta)t} - b\text{Re}(A)e^{-\frac{\lambda}{2}(1-\Delta)t}. \tag{15}\]
In the above equation we have put:
\[ \Delta = \sqrt{1 - 16 \varepsilon^2}, \varepsilon = \frac{\omega}{\lambda}. \tag{16} \]

We have now to choose precise values for \( \omega \) and \( \lambda \). Let us suppose that \( \varepsilon \) is smaller than \( \frac{1}{4} \). From eqs. (15) we see that \( \rho(t) \to \frac{1}{2} I \) for \( t \to +\infty \), independently from its initial value \( \rho(0) \). This, in particular, means that for extremely long times the reducing dynamics will not work as one would like, i.e., it will not transform the pure state \( |\Psi\rangle = a \begin{pmatrix} 1 \\ 0 \end{pmatrix} + b \begin{pmatrix} 0 \\ 1 \end{pmatrix} \)
into either \( \begin{pmatrix} 1 \\ 0 \end{pmatrix} \) or \( \begin{pmatrix} 0 \\ 1 \end{pmatrix} \) with the desired probabilities \( |a|^2 \) and \( |b|^2 \), respectively. \[3\]

Suppose now that \( \varepsilon \) is quite small. Then, within the time interval \( \lambda^{-1} << t << (4\lambda \varepsilon^2)^{-1} \) we have:
\[ e^{-\lambda t} \simeq 0, e^{-4\lambda \varepsilon^2 t} \simeq 1, \tag{17} \]
and the statistical operator is practically diagonal with diagonal elements corresponding (almost exactly) to the probability of reduction on the desired manifolds. In fact, evaluating, for the considered time interval and at first order in the small parameter \( \varepsilon \), the explicit form of the element \( r(t) \) of \( \rho(t) \) for two different initial conditions, i.e.:
\[ \rho_{\text{Mixt}}(0) = \begin{pmatrix} |a|^2 & 0 \\ 0 & |b|^2 \end{pmatrix}, \]
\[ \rho_{\text{Pure}}(0) = \begin{pmatrix} |a|^2 & ab^* \\ a^*b & |b|^2 \end{pmatrix}, \tag{18} \]
corresponding, respectively, to the statistical mixture
\[ \rho_{\text{Mixt}}(0) = |a|^2 P_+ + |b|^2 P_-; \tag{19} \]
and to the pure state
\[ \rho_{\text{Pure}}(0) = |\Psi\rangle \langle \Psi|, |\Psi\rangle = a \begin{pmatrix} 1 \\ 0 \end{pmatrix} + b \begin{pmatrix} 0 \\ 1 \end{pmatrix}, \tag{20} \]
we have:
\[ r_{\text{Mixt}}(t) = |a|^2, r_{\text{Pure}}(t) = |a|^2 - 2\varepsilon \left[ \text{Re}(A)\text{Im}(ab^*) - \text{Im}(A)\text{Re}(ab^*) \right]. \tag{21} \]

In the considered approximation the off-diagonal elements of \( \rho(t) \) are equal to zero in the first case and of order \( \varepsilon \) in the second.

Eqs. (17) and (21) teach us a quite interesting lesson:

i. The reduction regime can last only for a certain (possibly extremely long) time interval,

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\[3\] Actually, in the realistic case of the GRW and CSL models the situation is even worse since \( \rho(t) \) does not have a precise limit for \( t \to +\infty \) (see, however, the comments following eq. (21)).
ii. The dynamical reduction process can lead, in such a time interval, to different situations according whether the initial condition corresponds to a statistical mixture of two states with weights $|a|^2$ and $|b|^2$ or to the superposition with coefficients $a$ and $b$ of the same states.

Obviously, the two above facts do not change in any way the relevant implications of the GRW and CSL theories. In fact, such theories lead to the objectification of the positions of macroscopic bodies, and, more generally, to a definite average mass density distribution (the average being evaluated on cells of the order of $10^{-15} \text{cm}^3$ around any chosen space point $r$). If one would establish a correspondence between the dynamics of these theories and the one of the toy model we have analyzed in this section, one should take into account that the average effective localization frequency for a macroscopic object within the GRW and CLSL models corresponds to the choice $\lambda \simeq 10^7 \text{sec}^{-1}$. Moreover, the hamiltonian cannot induce, during the time interval between two reductions, appreciable transitions between the different macrostates to which the reduction mechanism leads, so that one can assume $\omega \simeq 0$. Concluding, in the macroscopic case the dynamical reduction models lead to a definite macroscopic situation in extremely short times and such a situation persists (in absence of other dynamical processes) practically forever.

Let us now analyze the case in which the hamiltonian evolution and the reduction mechanism are competitive. Since we want to mimic a perception process we could tentatively make in the above toy model the following choice for the parameters characterizing it:

$$\lambda \simeq 10^2 \text{sec}^{-1}, \varepsilon \simeq 10^{-4}. \quad (22)$$

The first choice corresponds to the idea that the time which is necessary for a definite perception to emerge is of the order of one hundredth of a second, the second is done in order to guarantee that the statistical distribution of the reduced states is (almost) perfectly respected, i.e., that the asymptotic regime leading to the equal weight distribution of the two outcomes (independently from the triggering state), cannot become immediately effective. That this is the case follows from the fact that the above choice implies $4\lambda \varepsilon^2 \simeq 10^{-6} \text{sec}^{-1}$.

We can now discuss some particular cases. Suppose one chooses in eqs. (18), $a = b = \sqrt{\frac{1}{2}}$, and in eq. (10), $A = i$. Then eqs. (21) become:

$$r_{\text{Mixt}}(t) = \frac{1}{2}, r_{\text{Pure}}(t) = \frac{1}{2} - \varepsilon \simeq \frac{1}{2} - 10^{-4}. \quad (23)$$

If we take this result as giving some indications about the perception processes we have analyzed in section 5, we are led to conclude that when we trigger the perceptual apparatus with the exact 50%-50% mixture of the states $|\text{stimulus i}>$ and $|\text{stimulus ii}>$, the observer perceives precisely as many ”spots at A” as ”spots at B”. On the contrary, when triggered by the superposition $\sqrt{\frac{1}{2}} [ |\text{stimulus i}> + |\text{stimulus ii}>)$, he will tend to perceive a slightly larger number of ”spots at B” than ”spots at A”.

Two remarks are at order:

i: We do not intend to suggest that the toy model has to be taken as appropriately mimicking the perceptual process. Our analysis has to be considered as a rigorous proof
that when the reduction process takes some time to be completed and is competing with other processes, then, within the only known consistent theory describing such a process, some precise differences between the outcomes generated by an initial statistical mixture and a pure state (for which the squares of the moduli of its coefficients coincide with the weights of the statistical mixture) can, and, in general will, actually emerge.

ii. We stress that if one goes through the same calculation by changing the sign of the second term of the superposition we have chosen as the initial state (i.e. if consideration is given to the state \( \frac{1}{\sqrt{2}} [\text{stimulus } i) - [\text{stimulus } ii) ] \)), then the expression for \( r(t) \), in the considered approximation, is \( r(t) \simeq \frac{1}{2} + \varepsilon \). The change of sign in front of \( \varepsilon \) is extremely important. If the corrections to \( \frac{1}{2} \) would not cancel when the + or - signs are considered, one could use the difference in the probabilities of the two outcomes to set up a device allowing faster-than-light signalling. This cannot happen, and actually there are many proofs (the first one having been exhibited by J. Bell [23]) that the GRW theory (as well as all models involving dynamical reductions of the type of CSL) cannot lead to superluminal communication.

Concluding: we have analyzed in detail a toy model of dynamical reduction in which the hamiltonian dynamics and the reduction mechanism compete and we have proved that it allows to distinguish between an initial state corresponding to a linear superposition and a statistical mixture with the appropriate weights. Obviously, since we have dealt with a two dimensional Hilbert space, the difference could only consists in a slight discrepancy of the probabilities of the two outcomes. It is trivial (even though a little bit tedious) to show that when analogous dynamical conditions (the competing of the linear and nonlinear parts of the dynamics) occur in a system whose Hilbert space is higher-dimensional and in which the reductions occur on degenerate manifolds, the differences originating from different initial situations could also derive from the different composition of the final ensemble within the manifolds to which reductions lead. We have worked out detailed calculations for a four-dimensional case with reduction on two orthogonal two-dimensional subspaces: we do not reproduce our results here since they involve quite cumbersome formulae. Those who have followed our argument can easily grasp that things actually go as indicated above.

The analysis of this section makes, in our opinion, quite plausible the general argument developed in the previous sections.

7 Concluding remarks.

We think that the above analysis, even though it gives only some general hints about what might emerge in the experiments we have proposed, shows that it is worth to actually perform them. The proposal identifies new ways (which only recent technological developments have made possible) of investigating such fundamental processes like the violation of the linear nature of quantum mechanics and the formation of definite perceptions. As already mentioned, R. Penrose [21], as well as many other scientists, have repeatedly suggested that strict links should exists between the possible violation of the linear Schrödinger evolution and the perception mechanism. The present proposal points
out that there is a possibility of performing experiments which might throw some light precisely on the crucial problems we have mentioned.

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