MOCCA Code for Star Cluster Simulations - II. Comparison with N-body Simulations

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ABSTRACT
We describe a major upgrade of a Monte Carlo code which has previously been used for many studies of dense star clusters. We outline the steps needed in order to calibrate the results of the new Monte Carlo code against N-body simulations for large N systems, up to N = 200000. The new version of the Monte Carlo code (called MOCCA), in addition to the old version, incorporates direct FewBody integrator for three- and four-body interactions, and new treatment of the escape process based on Fokushige & Heggie (2000). Now stars which fulfil the escape criterion are not removed immediately, but can stay in the system for a certain time which depends on the excess of the energy of a star above the critical energy. They are called potential escapers. FewBody integrator allows to follow all interaction channels, which are important for the rate of creation of various types of objects observed in star clusters, and assures that the energy generation by binaries is treated in a meaner similar to the N-body model.

There are at most three parameters which have to be adjusted against N-body simulations for large N. Two (or one, depends on the chosen approach) connected with the escape process and one responsible for determination of the interaction probabilities. The adopted free parameters are independent on N. They allow MOCCA code to reproduce N-body results, in a reasonably precision, not only for the rate of cluster evolution and the cluster mass distribution, but also for the detailed distributions of mass and binding energy of binaries. Additionally, the code can follow the rate of formation of blue stragglers and black hole - black hole binaries. The code computes interactions between binaries and single stars up to a maximum separation rpmax, and it is found that the MOCCA code needs rather large value of rpmax to get agreement with the N-body simulations.

The MOCCA code is at present the most advanced code for simulations of real star clusters. It can follow the cluster evolution in details comparable to N-body code, but orders of magnitude faster.

Key words: stellar dynamics – methods: numerical – globular clusters: evolution

1 INTRODUCTION
This is the second paper in a new series of papers in which we attempt to describe the development of the MOCCA (MOnte Carlo Cluster simulAtor) code and its application to the simulations of star cluster evolution. The first in the series [Hypki & Giersz 2011] described in detail recent developments of the previous version of the Monte Carlo code (Giersz, Heggie, & Hurley 2008, and references therein) and the first results of simulations concerning blue stragglers (BSS) in an evolving star cluster environment. In this paper, we further develop the code and perform a very detailed comparison with N-body simulations of large N stellar systems up to N = 2 × 105.

The MOCCA code [Hypki & Giersz 2011] is at present the most advanced numerical code for stellar dynamical simulations, capable to follow evolution of real star clusters in detail comparable to N-body simulations, but orders of magnitude faster (several hours for N = 2 × 106). The dynamical ingredients of the Monte Carlo code are essentially the same as those described in Giersz 2006 and Giersz, Heggie, & Hurley 2008, whose code embodies several features introduced by Stodol´ skiewicz 1986, whose code was in turn based on that originally devised by Henon 1971. Two main features distinguish the MOCCA code from the previous version of the Monte Carlo code: (i) it now incorporates dynamical interactions between binary and sin-
Lagrangian points and between pairs of binaries based on Fregeau et al. (2004); (ii) it replaces the treatment of the escape process in the static tidal field based on Baumgardt (2001) by one in accordance the theory proposed by Fokushige & Heggie (2000). The escape process is not instantaneous any more, an object needs time to find its way around the Lagrangian points $L_1$ and $L_2$ to escape. The MOCCA code incorporates most of the processes which are important during stellar system evolution e.g.: the relaxation process, the main engine of dynamical cluster evolution; stellar evolution according to Hurley, Pols, & Tout (2000) for the evolution of single stars, supplemented by the methods of Hurley, Tout, & Pols (2002) for internal evolution of binary stars (BSE code) and also simple approach for colliding stars according to McScatter interface to BSE by Heggie, Portegies Zwart, & Hurley (2006), the escape process in the static tidal field of the parent galaxy; direct few body integrator to follow interactions between binaries and single stars and other binaries; mass segregated initial cluster configuration according to Baumgardt, De Marchi, & Kroupa (2008) and Subr, Kroupa & Baumgardt (2008).

There are several factors which motivate this work. Star clusters are the focus of many intensive observational campaigns (e.g. Bedin et al. 2001; Bedin, Piotto et al. 2003; Grindlay et al. 2001; Piotto et al. 2002; Kafri et al. 2003; Kafka et al. 2004; Richer et al. 2004; Anderson et al. 2006; Milone et al. 2011) and references therein), which are now turning to the examination of the parameters of their populations of different kinds of binaries, BSS and other "peculiar" objects. Dynamical models are needed for the design and interpretation of observational programmes: how is the period distribution and the spatial distribution of binaries affected by dynamical evolution? What is the influence of environment and dynamical evolution on the formation of "peculiar" objects? Understanding the abundance, spatial distribution and channels of formation of BSS can only be attempted by a technique which follows simultaneously both BSS dynamics and internal evolution. While N-body technique may ultimately be the method of choice for such studies, systems of the size of most globular clusters are likely to remain beyond reach for some years, simply because of the number of stars and the size of the binary population. After all, it is only recently that the "hardest" open clusters M67 (Hurley et al. 2005) and "easiest", loosely bound and distant globular cluster Palomar 14 (Zoonoozi et al. 2011) have been modelled at the necessary level of sophistication. To efficiently compute detailed models of large star clusters and to investigate the influence of initial parameters on a cluster's global and local observational properties we need a technique which is much faster than the N-body code and at the same time can give the same level of information about every object in the cluster as the N-body code does. The MOCCA code is such a technique.

One of the drawbacks of the non direct techniques (also the Monte Carlo one) compared to the N-body model is a necessity of use of free parameters which try to describe the complexity of physical processes naturally covered in the direct code. The most important free parameters (from the point of view of MOCCA) are connected with the relaxation process ($\gamma$ coefficient in the Coulomb logarithm), escape process in the static tidal field and dynamical interactions between different objects (where the parameter, $r_{pmax}$, is the maximum pericentre distance between interacting objects for which few-body interactions are calculated explicitly). The usual method to determine the free parameters is a comparison with the results of N-body simulations. For the previous version of the Monte Carlo code the comparison was done only for small $N$ systems (up to $N = 24000$). The code was successfully used to simulate evolution of real star clusters: M67 (Giersz, Heggie, & Hurley 2008), M4 (Heggie & Giersz 2008a), NGC6397 (Giersz & Heggie 2009; Heggie & Giersz 2009) and 47Tuc (Giersz & Heggie 2011). Despite those successes there were some doubts connected with the $N$ scaling of the escape process, which implementation was based on Baumgardt (2001). To fully trust the MOCCA code it has to be tested for larger $N$, and not only for the global parameters like evolution of the total cluster mass or Lagrangian radii, but also against the properties and spatial distributions of binaries and BSS. That kind of comparison will show how far we can trust results of Monte Carlo simulations and which processes cannot be properly described in the framework of the MOCCA code.

This paper begins in Sec.2 with a summary of the features which have been added to the Monte Carlo scheme during the construction of the new version of the code, the MOCCA code. We also show there how we calibrate the free parameters of the MOCCA code with results of $N$-body simulations. Next (Sec. 3) we describe the similarities and differences between MOCCA and N-body simulation and discuss the possible reasons for that. The final section summarises our conclusions, and discusses some limitations and future developments of the MOCCA code.

2 TECHNIQUE

The MOCCA code (Hypki & Giersz 2011) is an updated version of the Monte Carlo code developed in Giersz (1998, 2001, 2008); Giersz, Heggie, & Hurley (2008). In addition to the description of the relaxation process which is responsible for the dynamical system evolution it includes synthetic stellar evolution of single and binary stars using prescriptions described by Hurley, Pols, & Tout (2000) and Hurley, Tout, & Pols (2002) and direct integration procedures for small $N$ subsystems based on the FewBody code Fregeau et al. (2004). One of the more important updates is a better description of the escape process according to Fokushige & Heggie (2000). Now the escape of an object from the system is not instantaneous anymore, but time delayed. The theory of Fokushige & Heggie (2000) incorporates a number of parameters which they had to determine empirically, and which depend on the system under consideration; here we shall determine these parameters by comparing the results with those of N-body simulations. In the N-body code formation of binaries and subsequent their interactions follow naturally from the movement of stars in the system under the mutual gravitational force. In the MOCCA code first we need to check if the interaction is due be computing its probability and then if it is so we execute the direct integration procedure for small $N$ (3- or 4-body) subsystem to find out the outcome of the interaction. The interaction probability depends, among others, on the maximum value of the pericentre distance, $r_{pmax}$. The larger this distance the larger the number of interactions, which are weaker on average. Choosing a proper value of $r_{pmax}$ is crucial for a balance between code efficiency and its accuracy, e.g. the number of BSS observed in the system strongly depends on $r_{pmax}$.

2.1 Delayed escape

As it was pointed out in Fokushige & Heggie (2000) and Baumgardt (2001) the process of escape from a cluster in a steady tidal field is extremely complicated. Some stars which fulfil the energy criterion for escape (binding energy of the star greater than...
the critical energy $E_{\text{crit}} = -1.5(GM/r_t)$, where $G$ is the gravitational constant, $M$ is the total mass and $r_t$ is the tidal radius, see Spitzer (1987) can still be trapped inside the potential well. Some of those stars can be scattered back to lower energy before they escape from the system. These two factors cause the cluster lifetime to scale nonlinearly with relaxation time for tidally limited clusters (Baumgardt 2001), in contrast with what would be expected from the standard theory. The efficiency of this effect decreases as the number of stars increases. To account for the process described above in the previous version of the Monte Carlo code an additional free parameter $\alpha$ was introduced (Giersz, Heggie, & Hurley 2008). The critical energy for escaping stars was approximated by: 

$$E_{\text{crit}} = -x_{\text{tid}}(GM/r_t),$$

where $x_{\text{tid}} = 1.5 - \alpha(\ln(\gamma)N/N)^{1/4}$, $\alpha$ was approximated by 2.5 and $\gamma$ is the coefficient in the Coulomb logarithm equal to 0.11 for equal mass case and 0.02 for unequal mass case (see Giersz, Heggie, & Hurley 2008, and reference therein). This prescription was also tested by Chatterjee et al. (2010) in their Monte Carlo simulations of star clusters. They are three drawbacks of this approach: (i) the effective tidal radius for Monte Carlo simulations is $r_{\text{eff}}/r_t = r_t/x_{\text{tid}}$ and it is smaller than $r_t$. Therefore, for Monte Carlo simulations, a system was slightly too concentrated (as measured by the ratio between the tidal radius and the half-mass radius) compared to N-body simulations; (ii) the escape process is instantaneous. A star which energy is greater than the critical energy is promptly removed from the system; (iii) the coefficient $x_{\text{tid}}$ is an explicit function of $N$. Its $N$ dependence was calibrated only for low $N$ system Giersz, Heggie, & Hurley (2008), so one can have some doubts about the rate of system evolution for large $N$.

To overcome the drawbacks which are described above, we decided to apply the theory described in Fokushige & Heggie (2000). According to this theory the time-scale for escape is given by:

$$t_e = \frac{2y_{\text{tid}}\sqrt{6}(GM)^{3/2}\omega^{1/2}}{\pi(E - E_{\text{crit}})^{3}},$$

(1)

where $\omega$ is the angular velocity of a cluster around a parent galaxy and $E$ is the energy of a star. $y_{\text{tid}}$ is a coefficient, which slightly depends on the system structure, and can be approximated by 0.38.

The probability of escape in a time-step $\Delta t$ of a star with energy greater than $E_{\text{crit}}$ is given by:

$$P_e(\Delta t) = 1 - \exp(-\Delta t/t_e),$$

(2)

According to Figure 9 in Fokushige & Heggie (2000) the equation (2) matches the simulation results very poorly not only for the escape time scale but also for the overall shape of escape probability distribution. Indeed strictly it is known only to give an upper limit to the rate of escape, and they found empirically that the true rate of escape is smaller by about a factor of 10. So we can treat $y_{\text{tid}}$ in equation (1) as a free parameter and adjust it by comparison with N-body simulations.

To better represent the empirical shape of probability distribution, which can not be properly represent, even with appropriate choice of $y_{\text{tid}}$, we decided to use approximation to equation (2) suggested by Fokushige & Heggie (2000) for the probability distribution of escape times. For the purposes of the MOCCA code, this approach is strictly formal and does not have any physical meaning. The probability of escape in a time-step $\Delta t$ is given by:

$$P_f(\Delta t) = 1 - (1 + a\Delta t)^{-b},$$

(3)

where $a$ and $b$ are coefficients which slightly depend on the structure of the system and $\tilde{t} = \omega\Delta t/\tilde{E}^2$, where $\tilde{E} = (E - E_{\text{crit}})/E_{\text{crit}}$. In the MOCCA code they are a free parameters which are fitted against results of N-body simulations. As a first guess, values equal to 3.0 and 0.8 can be adopted.

So, to model in MOCCA escape process according to Fokushige & Heggie (2000) we have to adjust against N-body simulations the free parameters: $y_{\text{tid}}$ or $a$ and $b$. The big advantage of this approach is that the probability of escape does not explicitly depend on the number of stars in the system and that the escape process introduced into the MOCCA code generally follows the one in N-body systems.

### 2.2 Probability of interactions

In the hyperbolic 2-body approximation, the total cross section for interaction between a binary and a star, or another binary, with a pericentre distance less than $r_{\text{max}}$ is given by:

$$\sigma = \pi p^2 = \pi r_{\text{max}}^2 \left(1 + \frac{2GM_{123}}{r_{\text{max}}V^2}\right),$$

(4)

\[\text{Table 1. Initial conditions for N-body simulations}\]

| Cluster          | N=24000 (M67) | N=100000 (NGC6397) | N=200000 |
|------------------|--------------|-------------------|---------|
| $N_r$            | 12000        | 95000             | 195000  |
| $N_b$            | 12000        | 5000              | 5000    |
| Binary fraction  | 0.5          | 0.05              | 0.025   |
| M(0)             | $1.869 \times 10^4 M_\odot$ | $5.177 \times 10^4 M_\odot$ | $1.001 \times 10^5 M_\odot$ |
| Initial model    | Plummer      | Plummer           | Plummer |
| Initial tidal radius | 32.2pc       | 52.4pc            | 35.8pc  |
| IMF of stars     | Kroupa\textsuperscript{a} | Kroupa\textsuperscript{a} | Kroupa\textsuperscript{a} |
| IMF of binaries  | Kroupa\textsuperscript{b} | Kroupa\textsuperscript{b} | Kroupa\textsuperscript{b} |
| Mass ratio       | Uniform      | Uniform           | Uniform |
| Binary eccentricities | thermal\textsuperscript{c} | thermal\textsuperscript{c} | thermal\textsuperscript{c} |
| Binary semi-major axes | Uniform\textsuperscript{d} | Uniform\textsuperscript{d} | Uniform\textsuperscript{d} |
| SN kick distribution | Gaussian\textsuperscript{e} | Gaussian\textsuperscript{e} | flat\textsuperscript{f} |
| Metallicity      | 0.02         | 0.001             | 0.001   |

\textsuperscript{a}Kroupa, Tout & Gilmore (1999) with mass range between 0.1 and 500 M_\odot

\textsuperscript{b}Kroupa, Gilmore & Tout (1991) eq.(1) with mass range between 0.2 and 100 M_\odot

\textsuperscript{c}thermal distribution modified according to Hurley et al. (2005) eq.(1)

\textsuperscript{d}uniformly distributed in the logarithm in the range 2(R_1 + R_2) to 50 AU

\textsuperscript{e}Gaussian distribution with $\sigma = 190$ km/s

\textsuperscript{f}uniform distribution with kick velocities between 0 to 100 km/s
where \( p \) is the impact parameter, \( V \) is the relative velocity between binary and star or other binary, \( m_{123} \) is the mass of interacting objects. The second term in the brackets in equation (4) describes the so-called gravitational focusing term. The larger the \( r_{\text{pmaz}} \), the larger the probability of interaction. In general, the energy outcome from the interactions should not depend on \( r_{\text{pmaz}} \) for \( r_{\text{pmaz}} \gg A \), where \( A \) is a binary semi-major axis \((\text{Heggie} 1975)\). For large enough \( r_{\text{pmaz}} \) there is a balance between positive and negative binary binding energy changes. So the tail of \( r_{\text{pmaz}} \) describes the so-called gravitational focusing term. The larger the energy outcome from the interactions should not depend on energy generation. Of course, a larger \( r_{\text{pmaz}} \) means a larger impact parameter and larger number of interactions with modest energy changes are missing. 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are (but see comments about statistical fluctuations below).

The best values were chosen “by b and N-body means N-body results. The evolution of the system total mass as a function of time

\[ E = \begin{cases} \frac{v^2}{2} + \phi + \frac{1}{2} \omega^2 (z^2 - 3x^2) & \text{for N-body} \\ \frac{v^2}{2} + \phi & \text{for MOCCA} \end{cases} \]

where \( \phi \) is the potential \( v \) is the velocity dispersion, and \( x \) and \( z \) are coordinates with origin at the cluster centre. The last term for N-body depends on the centrifugal and tidal forces. In the case of the MOCCA code this term is not present, so the star energy is not exactly comparable in the both methods. The MOCCA system has spherically symmetrical shape instead of “oblate”. This could lead to some differences in the time to escape between MOCCA and N-body code, which we attempt by overcame by choice of \( y_{tid} \) or \( a \) and \( b \).

The escape criterion used in the old version of the Monte Carlo code (Giersz, Heggie, & Hurley 2008) gives reasonable agreement with N-body results (despite it was calibrated only for low \( N \)), although the rate of evolution is systematically to fast. This assures that the models of real star clusters computed with the old Monte Carlo code are relevant. Nevertheless, the MOCCA code with the new description of the escape process (based on Fokushige & Heggie 2000) gives more consistent results with N-body, not only with respect to the evolution of the global parameters, but also with respect to the detailed properties of binary distributions. What is also important, it is \( N \) independent and can be safely used for any \( N \).

In the rest of the paper we use \( N(0) \) in the escape procedure instead of \( N(t) \) for scaling time from Monte Carlo to N-body units. For all other processes the scaling given in equation (6) was used.

To determine the free parameters described above (\( y_{tid} \) or \( a \) and \( b \)) we have to run several simulations with a different number of stars and different values for the parameters, and then compared results with N-body simulations. The best values were chosen "by eye", we did not attempt to assess how accurate and how unique they are (but see comments about statistical fluctuations below).

The results are presented in Figs 2 and 3. As one can see, the dependence on \( y_{tid} \) is stronger than on \( a \) and \( b \). The "best" values are \( y_{tid} = 4.0 \) and \( a = 3.0, b = 0.7 \). The values for \( a \) and \( b \) are close to the values given in Fokushige & Heggie (2000) in Table 1 there, \( y_{tid} \) is about ten times larger than given in Fokushige & Heggie (2000) (eq. (9) there), but taking into account that the time scale given in equation (2) is too short (by about 1 dex) the value 4.0 is very close to the empirical value which Fokushige & Heggie (2000) found. Only for the simulations with \( N = 24000 \) the evolution rate is slightly too slow for MOCCA, for time larger than about 2 Gyr, and it seems that \( y_{tid} = 3.0 \) is a better choice than \( y_{tid} = 4.0 \). For other \( N \) (100000 and 200000) MOCCA simulations follow the N-body results very well and choice of \( y_{tid} = 4.0 \) gives better agreement with N-body.

To assess the influence of statistical fluctuations on the obtained results, three simulations with exactly the same initial conditions but with different sequences of random numbers were run. The results are given in Fig 3 for \( N = 24000 \). The fluctuations for this model are largest, but clearly smaller than the difference connected
with different values of the free parameters. For larger $N$ the fluctuations are practically negligible. For other global quantities the fluctuations are similar like for the total mass. They are small and practically negligible. For other global quantities the fluctuations are similar like for the total mass. They are small and practically negligible.

Having determined $a$, $b$ or $\gamma_{\text{tid}}$ we now turn to find out the best value for $r_{p_{\text{max}}}$. As it was argued above the value of $r_{p_{\text{max}}}$ will have big impact on the number and distribution of binaries and BSS in the system. In the MOCCA code BSS is defined exactly as it was done in Hurley’s N-body simulations. A main sequence star is identified as BSS when its mass is greater than $M_{\text{to}}$, where $M_{\text{to}}$ is the turn off mass. As can be seen from Figs 6 and 7 the requirements set by the N-body simulation for the number of BSS and binaries are rather contradictory from the point of view of MOCCA results.

To get the best agreement for the evolution of the total number of binaries, MOCCA needs as large as possible $r_{p_{\text{max}}}$. $r_{p_{\text{max}}}$ = 27$A$ seems to be a good choice. On the other hand to match N-body results for the number of BSS MOCCA prefers modest values of $r_{p_{\text{max}}}$ - equal to about $A$. That conclusion is also true for $= 24000$ and $N = 200000$. It seems that a reasonable compromise between the evolution of the total number of binaries and BBS is given for the $r_{p_{\text{max}}}$ suggested by the theoretical considerations given above in Sec 2.2 namely for three-body interactions $r_{p_{\text{max}}} = 27A$ and for four-body interactions $r_{p_{\text{max}}} = A$. One can thing that this contradictory requirements set by the number of BSS and binaries can be explained by the different definitions of binary in N-body and MOCCA simulations. In the N-body results presented here, binary is identified with a regularised binary (rather hard), so called KS binary. In MOCCA we follow all binaries, even very soft ones. So, generally in MOCCA simulations we should expect larger number of binaries than in N-body. To quantify this we checked the number of non KS binaries in N-body simulations. This number was rather small (at most about 30) and could explain observed differences only partially.
It is worth to note that, as one can expect, for the MOCCA code the evolution of the Lagrangian radii and the total mass do not depend on the value of \( r_{\text{pm}} \), the total energy generation in three- and four-body interactions does not depend on \( r_{\text{pm}} \) provided that \( r_{\text{pm}} \) is not too small or too large. The situation is different when the FewBody integrations are switch off in MOCCA and only cross sections are used for energy generation in binary interactions (MOCCA-NoFB). Then there is a very strong dependence on \( r_{\text{pm}} \) for the Lagrangian radii evolution. Larger \( r_{\text{pm}} \) means larger probability for interactions. Each interaction generates on average the same amount of energy (according to the adopted cross section), so for larger \( r_{\text{pm}} \) more energy is generated by binaries, and the system expands faster than in the case of MOCCA simulations (with FewBody integrator). It seems that the best values of \( r_{\text{pm}} \) for MOCCA-NoFB are 0.54 and 0.25, for three- and four-body interactions, respectively.

In the remainder of the paper the best values of the free parameters given in this section \( y_{\text{tid}} = 4.0, a = 3.0, b = 0.7, r_{\text{pm}} = 2.4 \) and \( A \) for three- and four-body interactions, respectively will be used for comparison of different system and binary properties obtained in MOCCA and N-body simulations.

### 3.2 Half mass time and potential escapers

Having chosen the free parameters \( a, b \) and \( y_{\text{tid}} \), which determine the rate of escape from the tidally limited cluster we can now check the results of simulations for a tidally limited and single mass system presented in Baumgardt (2001). Namely: the \( N \) dependence of the half-time, time when system contains half of its initial mass, and the evolution of the number of potential escapers for different \( N \). According to the model presented by Baumgardt (2001) the half-time should scale as the relaxation time to the power 3/4 instead of the linear scaling with the relaxation time predicted by the standard theory. On Figure 8 is shown how the half mass time as a function of \( N \) for MOCCA simulations from \( N = 4K \) to \( N = 256K \). It also shows different scaling laws fitted to the simulation data. As we can see for \( N \) up to 16\( K \) (the range for models presented in Baumgardt (2001)) the scaling proportional to \( t_{rh}^{3/4} \) is acceptable within error bars (3\( \sigma \) - \( \sigma \) is estimated from 10, 7, 5 and 3 simulations with the same initial conditions but with different random sequences for \( N = 4k, N = 16k, N = 64k \) and \( N = 256k \), respectively). For larger \( N \) the scaling is less steep and the whole range of \( N \) can be reasonably well fitted with scaling \( t_{rh}^{0.61} \) or \( N^{0.54} \). The reason for the discrepancies between results predicted by the theory scaling and those obtained from the simulations may be attributed to the facts that the star energy in MOCCA does not contain terms attributed to the non inertial reference frame and tidal force. The scaling \( N^{0.54} \) is close to the results obtained by Lamers et al. (2005) and Gieles & Baumgardt (2008).

Figure 8 shows the evolution of the potential escaper fraction with time for different initial number of stars. The initial model setup is responsible for the 15 per cent potential escapers at the beginning. The cluster starts with primordial escapers, because the escape energy, \( E_{\text{crit}} \), is lower than the edge potential of the initial King model. The different behaviour of the number of the potential escaper for different \( N \) at the beginning is in agreement with the results presented by Baumgardt (2001). The initial increase of the potential escapers probably indicates the phase when cluster evolves towards equilibrium after removal in a short time substantial amount of mass (Baumgardt 2001). This increase is largest for small \( N \) systems, for largest \( N \) is barely visible. The number of potential escapers decrease with time until the core collapse, when it starts to rise again. The comparison of Figure 9 for \( N = 4k \) and \( N = 16k \) with Figure 11 in Baumgardt (2001) suggests that the core collapse is delayed for MOCCA. The detailed inspection with Figure 8 in Baumgardt (2001) shows that the delay is rather modest not larger than \( 5 - 10 \) per cent. The reasons for this can be connected with the fact that the \( r_{\text{pm}} = A \) chosen for simulations was to small and the probability for binary formation in the three body interactions can be slightly too small in MOCCA. Indeed, larger \( r_{\text{pm}} \) = 27\( A \) and larger formation probability both brings MOCCA results to much better agreement with N-body, but still core collapse in MOCCA is slightly delayed. This suggests that there are other factors which could be responsible for this disagreement, e.g. the \( \gamma \) coefficient in the Coulomb logarithm, or the rate (in N-body and MOCCA) in which small \( N \) clusters...
can regain equilibrium after the initial substantial mass loss. The observed disagreement does not influence the results for the half mass time and the evolution of the potential number of escapers before the collapse time. We can conclude that the MOCCA code reproduce reasonably well results presented by Baumgardt (2001) for small N-body simulations.

### 3.3 Results of comparison

The comparison between MOCCA and N-body results will proceed in three steps. First, the evolution of the global parameters (total mass, Lagrangian radii, core radius) will be compared. Second, the evolution of the binary properties (number, energy, mass and number distributions) will be checked. Third, properties of the peculiar objects like BSS and black holes (BH) will be compared. Most of the figures presented below, will additionally display the result of MOCCA simulations with a switched off FewBody integrator, and a switched on interaction cross sections (MOCCA-NoFB). This will help the reader assess how well the simplified MOCCA-NoFB (very similar to the old version of the Monte Carlo code, which was previously used to successfully simulate evolution of real star clusters) can follow N-body results and for which cluster properties it is enough to use the much faster and simplified code.

#### 3.3.1 Global parameters

The comparison between N-body and MOCCA results was partially discussed already in Sec.3.1 for the total mass evolution. It was shown that the agreement between both techniques is very good, only for \( N = 24000 \) it was barely acceptable (see Fig.8). The evolution of the core radius (defined according to Casertano & Hut (1985)) for \( N = 200000 \) is shown in the Fig.10. The agreement between MOCCA and N-body is very good. As one can expect, MOCCA-NoFB gives a slightly too large core radius. This, as it was explained in Sec.3.1, is connected with the overestimation of the binary energy generation in the cross section regime for a large enough \( r_{\text{max}} \). The large fluctuations in the core radius visible in the figure for N-body and MOCCA simulations are connected with the movement of a massive BH or BH-BH binary in the system. The mass of the BH objects is about \( 30 - 50 M_\odot \). Its movement in the system is connected with kicks acquired in interactions. If the massive object is present in the core the core radius is smaller, than when it is in the halo. The mass of the massive object is comparable to the core mass. When all the most massive objects are removed from the system, because of strong interactions with other massive binaries or stars, the evolution of the core radius is again "smooth". The evolution of the core radius is generally similar for the other models (but without a large fluctuations) with exception for \( N = 100000 \) when N-body results are systematically slightly below the MOCCA (see Fig.23).

The evolution of the half mass radius for \( N = 100000 \) model is shown on the Fig.11. The evolution of all models from the very beginning is very similar, although the N-body is slightly below both MOCCA models. The differences start to build up around the core collapse time (bump), which seems to be around 17 Gyr for N-body. For MOCCA models the core collapse time is around 20 Gyr. The bump in the half mass radius is also visible for MOCCA, but less pronounced. For \( N = 24000 \) and \( N = 200000 \) the half mass radius for MOCCA models is systematically slightly below the N-body models. The differences start to build up from the beginning, and are biggest around the time when the stellar evolution stops to be the dominant process of cluster expansion (time when the indirect heating connected with the stellar mass loss becomes smaller than the heating connected with the binary energy generation). Then the evolution of the half mass radius starts to converge for both N-body and MOCCA models. The same behaviour can be observed for other Lagrangian radii (1% and 10%). The reason for such a behaviour is unclear and can be attributed to different mutual strengths of the physical processes which operate during the different phases of cluster evolution. Faster mass segregation in N-body than in MOCCA can generate more extended cluster. Stellar evolution responsible for the loss of stellar mass can substantially blow up the cluster, particularly at the initial phases of evolution. Both N-body and MOCCA models delay the same stellar evolution prescription (Hurley, Pols, & Tout 2000; Hurley, Tout, & Pols 2002), so we cannot expect that the amount of mass loss is different in both models. However, the mass segregation acting together with the stellar mass loss can substantially amplify the expansion effect. If the most massive stars lose their envelopes when they are already mass segregated the effect on cluster expansion is largest. Finally, the larger binary energy generation in N-body simulations...
Figure 12. Evolution of the average mass inside 50% Lagrangian radius for \( N = 200000 \). Red line - MOCCA, green line - MOCCA-NoFB and blue line - N-body.

Figure 13. Evolution of the total binding energy of binaries for \( N = 200000 \). Red line - MOCCA, green line - MOCCA-NoFB and blue line - N-body.

(see Fig. 13, increase of the total binary binding energy) can be responsible for faster Lagrangian radii expansion. The evolution of the average mass inside 50% Lagrangian radius and evolution of the total binary binding energy for \( N = 200000 \) are shown on Figs. 12 and 13. These figures are representative for all models and Lagrangian radii. Indeed, it seems that the mass segregation is slightly stronger in N-body than in MOCCA. This suggests that the larger mass segregation in N-body model can be responsible, at least partially, for the slightly discrepant evolution of the Lagrangian radii. The evolution of the total binary binding energy, is from the very beginning rather similar, until later time (about 4 Gyr), when more bound binaries are formed in N-body. The final formation of very hard binary is visible in MOCCA and N-body (see discussion in Sec. 3.3.2), but not for MOCCA-NoFB. When binary is removed from the system, because of interactions, all models again are similar. It is worth to note that the results of simulations for MOCCA and MOCCA-NoFB are very similar until the late phases of evolution.

3.3.2 Binary properties

The evolution of the number of binaries was already presented in Sec. 3.1 during discussion about the determination of the free model parameters. We know that the evolution of the total number of binaries is slightly too slow for MOCCA. The difference starts to build up at the time when stellar evolution becomes less and less important (see Fig. 6). The average binary mass and the binding energy distributions will be discussed for model \( N = 200000 \). The results for this model are representative for other models and additionally shows build up of the average binary mass and binding energy for massive binaries. This build up is possible only for the model \( N = 200000 \) for which the distribution of supernova (SN) kicks is uniform between 0 and 100 km/s and lets larger, than in other models (Maxwellian distribution with \( \sigma = 190 \text{ km/s} \) for SN kicks), fraction of BHs to be bound to the system.

The evolution of the average binary mass for different regions of the system is shown in Figs. 14, 15, and 16. The agreement between N-body and MOCCA results is very good in all regions. This is despite the fact, that in N-body there is a smaller number of binaries than in MOCCA and Lagrangian radii are slightly different. It seems that the average binary mass and its distribution does not depend on the number of binaries, which is probably a result of exactly the same binary initial conditions for both models and similar mass spectrum of removed or destroyed binaries in the models.

For the region inside the core, the build up of the mass of binaries is clearly visible. The big fluctuations are connected with the movement of the massive binaries, which because of hard interactions with other stars are kicked out from the core and then because of mass segregation sink again to the centre. Finally, they are kicked out of the system and the average mass of binaries starts to become less chaotic and its changes are rather small. The drop and then increase of the average mass around 14.5 Gyr is connected with the core collapse. The high central density makes binary-binary interactions very effective and substantial number of relatively wide and massive binaries are destroyed causing drop of the binary average mass. Looking on the average masses in the different cluster regions the mass segregation is clearly visible. In the centre the mass is about twice as large as in the halo. For the \( N = 200000 \) model the difference between MOCCA and MOCCA-NoFB is very small, except in the core when the increase of the average mass is less pronounced for MOCCA-NoFB. The sharp increase of the average mass, close to the cluster dissolution time, is connected with the fact that only the most massive binaries are able to stay in the system, less massive ones are successively removed.

For other N models the behaviour of the average binary mass is similar to the one described above, except that in the core there is no large fluctuation (there are no very massive binaries there) and for late phases of cluster evolution small discrepancies between N-body and MOCCA start to show up.

The evolution of the average binary binding energy for different regions of the system is shown in Figs. 17, 18, and 19. The best agreement between N-body and MOCCA results is for the core region. The build up of the binary binding energy connected with the formation of massive BH-BH binary is clearly visible. It has to be stressed that the increase of the binary binding energy is not connected with the core collapse and core bounce. It is purely connected with the formation of a very massive BH-BH binary and then in the increase of its binding energy in interactions with stars and other binaries. The binary is finally removed from the system and then the average binary binding energy suddenly drops. For other cluster regions the agreement between N-body and MOCCA.
is less satisfactory. The average binary binding energy for N-body is systematically larger than for MOCCA. The differences start to build up just after the time when the stellar evolution stops to dominate the cluster global evolution. It seems that in N-body simulations harder binaries can be produced in the core before they are kicked out to the outer parts of the system. Maybe this is connected with the fact that in MOCCA code only binaries are allowed, the higher hierarchies are not allowed. Triples and quadruples are artificially disrupted to binaries and single stars. It is well known that in N-body simulations substantial numbers of triples and quadruples are formed (e.g. Mikkola 1984; McMillan, Hut & Makino 1991; Heggie & Hut 2003; Hurley et al. 2005, and reference therein). They can interact with other objects in the system and produce, on average slightly harder binaries. Differently than for the average binary mass the average binary binding energy for MOCCA-NoFB is clearly smaller than for MOCCA and N-body. In MOCCA-NoFB simulations Heggie’s cross section was used (see Sec.2.2). Therefore, on average the binary binding energy is changing by 40% in every interaction. It seems that such large energy changes are too big to produce a substantial population of binaries with energies smaller than the energy needed for binary escape from the system and larger than the minimum binding energy needed to generate binary escape in next interactions. Smaller energy changes will allow binaries to populate such a binding energy region. For the MOCCA simulations with $r_{\text{pmax}} = 2A$ for binary-single interactions, the average change of binding energy is about 18%. It seems, from the point of view of under abundance of binaries with large enough energies in the outer parts of the system, that this number is still too large. Indeed, larger $r_{\text{pmax}}$ gives much better agreement with the N-body results (the larger $r_{\text{pmax}}$ the smaller average change of binary binding energy), but is still not perfect. We should keep in mind that forcing too large $r_{\text{pmax}}$ to get smaller binding energy changes in interactions we create a much larger number of BSS comparable to N-body simulations (see Sec.2.2 and Fig.7). In this paper we decided not to use extremely large values of $r_{\text{pmax}}$, putting more emphasis on the number of BSS than on the distributions of the average binary binding energy. The dependence of the number of BSS on $r_{\text{pmax}}$ is much stronger than for the bi-
3.3.3 Black holes and blue stragglers

N-body simulations used for the comparison with MOCCA provided additional information about global system parameters and detailed data about binaries and about BHs and BSS. Some data on binary binding energy. But if one is more interested in binary binding energy distribution, much larger \( r_{\text{pmax}} \), say 9\( a \) or even higher, should be used. It is worth to stress that MOCCA-NoFB cannot reproduce the binary binding energy distribution observed in N-body and MOCCA.

For other N-body models the behaviour of the average binary binding energy is similar to the one described above, except that in the core there is no large buildup of very hard binaries. For \( N = 24000 \) the discrepancies between N-body and MOCCA are largest. This is probably connected with the fact that this model contains as much as 50\% binaries (10, 20 times more than other models), and small differences in binary energy generation may produce larger discrepancies in properties of binary distributions.

BSS was already used in Sec.3.1 to put some constrains on \( r_{\text{pmax}} \). In this section we will try to assess how well MOCCA code can reproduce the N-body results in respect to behaviour of "peculiar" objects, which are rare and are formed in interaction channels, which are not very probable.

In Fig.20 evolution of BSS for different \( r_{\text{pmax}} \) for \( N = 24000 \) is shown. The dependence of the number of BSS on \( r_{\text{pmax}} \) is different than in \( N = 100000 \) and \( N = 200000 \). There is no sharp increase of the maximum number of BSS for \( r_{\text{pmax}} > 3A - A \) (see Fig.11). For \( N = 24000 \) there is a very weak, if any, dependence of the number of BSS on \( r_{\text{pmax}} \). The fluctuations of the number of BSS connected with the stochasticity of the system and MOCCA model are comparable in the size with differences related to \( r_{\text{pmax}} \) - about 5 BSS. The differences observed on Figs.7 and 20 are rather surprising and point in the direction of strong dependence on the binary fraction. It was 50\% in \( N = 24000 \), at least ten times more than in the other models. This result contradicts the naive thinking that a large number of binaries should amplify the effect of large \( r_{\text{pmax}} \). Maybe, a large binary fraction makes binary-binary interactions more probable and more binaries are destroyed or binary properties are changed in such a way that formation of BSS is less probable, somehow balancing the larger BSS formation for larger \( r_{\text{pmax}} \). Generally, MOCCA is able to reproduce the evolution of the number of BSS given in N-body simulations provided a reasonable values of \( r_{\text{pmax}} \) is chosen. MOCCA-NoFB gives much smaller number of BSS than other models. That is not surprising, because important for BSS formation channels are missing in the cross section approach.

There is a long debate about BH kick velocities in SN explosions. We tested three possible variants in the paper (guided by choices done in N-body simulations): First, Maxwellian distribution with \( \sigma = 190 \) km/s (Phinney 1992), and Maxwellian distribution with \( \sigma = 190 \) km/s but with the maximum kick velocity set to 50 km/s (reduction of the kick velocity to maximum 50 km/s is the procedure used in Jarrod Hurley’s N-body simulation) - \( k_{\text{fallb}} = 0 \) and \( k_{\text{fallb}} = 2 \) in the figures, respectively. Second, uniform distribution with kick velocities between 0 and 100 km/s, and uniform distribution with kick velocities between 0 and 50 km/s - \( k_{\text{fallb}} = 0 \) and \( k_{\text{fallb}} = 2 \) in the figures, respectively (only for \( N = 200000 \) model). Third, Maxwellian distribution with \( \sigma = 190 \) km/s but the kick velocity is finally modified by...
the amount of mass which falls back on BH during SN explosion \citep{Belczynski2002} - $k_{f\text{alb}} = 1$ in the figures. The amount of mass fall back depends on the core mass of a star, just before the supernova explosion. For the core mass greater than $7.6 M_\odot$ the kick velocity is 0 km/s.

The discussion of the influence of the SN kick velocity definition on the system evolution and on properties of BH and BH-BH binaries will be presented for the $N = 100000$ model. For other models the different kick velocities do not have strong influence on the system parameters. For $N = 240000$ the escape velocity is very small, so even for $k_{f\text{alb}} = 1$ case only a few BH will stay in the system after SN kicks and they will be quickly removed by interactions from the system. For $N = 200000$ the SN kick velocities for $k_{f\text{alb}} = 0$ and $k_{f\text{alb}} = 2$ cases were already relatively small and a substantial fraction of BHs were left in the system. Therefore in the case $k_{f\text{alb}} = 1$ the BHs fraction did not change substantially and the system evolution was not influenced strongly. For $N = 100000$ the situation is different. Only in the case $k_{f\text{alb}} = 1$ substantial number of BHs left in the system strongly influencing its evolution.

On Figs.\ref{fig:21} and\ref{fig:22} the number of BHs and BH-BH binaries are shown for $N = 100000$ and for different description of the SN kick velocities. The striking feature of these figures is the fact that there is no BHs and BH-BH binaries left in the system after SN kicks for case $kic = 0$ and even for $kick = 2$. Kick velocities are too large and all BHs escape immediately from the system. Only in the case $k_{f\text{alb}} = 1$ kick velocities are small enough to keep a substantial number of BHs in the system. The most massive BHs have masses larger than $25 M_\odot$, so they very quickly form in three-body interactions massive binaries with mass about $50 M_\odot$. The probability of binary formation in the three-body interactions is a very strong function of masses \citep{Heggie1975}. As can be seen from Fig.\ref{fig:21} the number of BH binaries present at the same time in the system can be as large as 5. They are the most massive objects in the system, but this does not mean that all those binaries are in the same time in the core. They strongly interact with other stars/BHs or binaries and acquire kicks which move them outside the core. When they become hard enough they are one by one removed from the system by a final strong interaction. In the course of interactions each massive BH-BH binary can remove several stars, therein a few BHs.

There are acting like a vacuum cleaner quickly reducing the number of single BHs in the system, which is clearly visible in Figs.\ref{fig:21} and\ref{fig:22} The probability of binary-single interaction depends on the mass of interacting objects. The larger the mass the larger the probability and the larger number of hard interactions connected with substantial energy generation. The hardest and most massive binaries can produce much more energy in interactions than other binaries. Therefore, one can expect that such a large amount of energy is able to change the system structure showing up distinct features not visible in models with $k_{f\text{alb}} = 0$ and $k_{f\text{alb}} = 2$.

On Figs.\ref{fig:22} and\ref{fig:23} are shown evolution of the core radius for $N = 100000$. Red line - $k_{f\text{alb}} = 0$, green line - $k_{f\text{alb}} = 1$ and blue line - $k_{f\text{alb}} = 2$. The cluster lives shorter. Therefore, not only BH with
masses of several hundred $M_\odot$ (IMBH) can influence the system structure, which can be detected by observations, but also presence in the system of several massive stellar mass BHs can leave observational imprint on the cluster structure. That is a very interesting possibility worth to further check. Similar behaviour was reported in Hurley (2007) for stochastic core radius evolution powered by single rather massive BH and in Merritt et al. (2004), Mackey et al. (2008) for strong core expansion of massive star cluster powered by large number of stellar mass BHs (in this case the core radius evolution is rather smooth because of a large number of BHs which forms bound subsystem in the central part of the system).

The behaviour of the half mass and the core radii described above for $N = 100000$ is also visible for $N = 200000$. The fact that in this case, for different $k_{fallb}$ the number of BHs and BH-BH binaries are very similar, is interesting. This suggests that not the number of BH-BH binaries is crucial for the discussed above behaviour but the mass of BH-BH binary. For $k_{fallb} = 1$ case more massive BHs can be formed than in other $k_{fallb}$ because of the mass fallback during SN explosion. Therefore, the most massive binary which can form in this case is substantially more massive than in other cases. Such a binary can generate more energy in interactions and remove more stars and binaries from the system than a less massive binary. It is possible that in the case when two or more massive BH-BH binaries are formed in the system, they will interact between themselves and quickly harden. There is a non zero probability that before they escape, they will merge in collision interactions with “ordinary” stars/binaries and form more massive BH, which very quickly will form again binary. The sequence can be repeated a few times forming a seed IMBH in the system. That is an interesting new road for the possible creation of IMBH in globular clusters. This possible scenario strongly rely on the number of non KS binaries is rather small and can not be entirely responsible for the observed differences. For binary binding energy distribution evidences are rather indirect and the observed dependence on $r_{p_{max}}$ could be also, at least partially, attributed to unidentified yet physical processes or some systematical errors in MOCCA code. Therefore, we decided to opt for the compromise value of $r_{p_{max}}$ than use larger values suggested by the total number of binaries and their distributions. We estimate that the error made in the total number of binaries by use of this compromise value is of order 5 per cent of the number of binaries.

4 CONCLUSIONS

In this paper we have presented an advanced Monte Carlo code (MOCCA) for the evolution of rich star clusters, including most aspects of dynamical interactions involving binary and single stars, internal evolution of single and binary stars and complicated process of escape in the static tidal field. The direct integration of a few body problem was introduced on the base of FewBody code developed by Fregeau et al. (2004). The stellar and binary internal evolution was done according to BSE code (Hurley, Pols, & Tout 2000). The description of the escape process was based on the theory presented by Fokushige & Heggie (2000). MOCCA code is able to follow all channels of interaction up to binary-binary hierarchy including merging of stars, and the escape is not immediate any more, stars need time to find their way to escape from the $L_1$ and $L_2$ Lagrangian points. The probability of escape and probability for interaction are characterized by some free parameters which were adjusted by comparison of MOCCA and N-body simulation results for large $N$ systems, up to $N = 200000$.

It was shown that the free parameters of the MOCCA code can be successfully calibrated against N-body simulations and that the free parameters do not depend on $N$. MOCCA code is not only able to follow evolution of the cluster total mass, Lagrangian radii and the core radius, but also is able to reproduce in an reasonably accuracy distributions of binary parameters and number of BH-BH binaries and BSS. It also reproduces well the results obtained by Baumgardt (2001) for single mass tidally limited systems for the half-mass time and evolution of potential escapers. The code is able to cope with very diverse systems, from single mass, isolated system without primordial binaries to multi mass, tidally limited system with large fraction of binaries. It was shown that simplified and
faster version of the MOCCA code (without direct FewBody integrator - MOCCA-NoFB) is a method of choice for projects which aim is to investigate the evolution of star clusters from the point of view their global properties. For other purposes, particularly when properties of “peculiar” objects and their distributions are in area of interests one should use slower MOCCA code. It is worth to note that MOCCA and MOCCA-NoFB simulations presented in this paper needs only about three and two hours, respectively, to be completed on a standard Opetron 2.4Ghz CPU.

Despite these successes the MOCA code has still some known shortcomings, which we summarise here.

(i) Higher-order multiples: It is widely argued that primordial triples and higher multiples should be incorporated into simulations along with primordial binaries. In any case, hierarchical triples form abundantly in binary-binary interactions (Mikkola 1984). Such higher-order multiples are ignored in the present Monte-Carlo code, they are counted and artificially disrupted (Hypki & Giersz 2011). In our models the absence of these rotational effects can be compensated by a modest alteration of the initial conditions.

(ii) Rotation: the Monte Carlo code is based on spherical symmetry, and would require rather fundamental and very difficult reconstruction in order to cope with cluster rotation. Rotation accelerates the rates of core collapse and mass segregation (e.g. Fiestas, Spurzem & Kim 2006, Kim et al. 2008, and references therein). In our models the absence of these rotational effects can be compensated by a modest alteration of the initial conditions.

(iii) Static tide: the effect of tidal shocks have been extensively studied and it would be possible to add the effects as another process altering the energies and angular momenta of the stars in the simulations. The addition of tidal shocks will be more important when modelling Galactic globular clusters than open clusters, which usually are confined inside the Galactic disk.

Despite these limitations, some of which are difficult to cure, the MOCA code presented in this paper shows its potential power in simulations of real star clusters, from open clusters to rich globular clusters. Monte Carlo models are feasible in a reasonable time (a few days) for globular clusters, which are, still for some time, too large for direct N-body simulations. The MOCA code is able to provide data as detailed as N-body code can do. Only those two codes can provides such comprehensive information. Even when N-body simulations eventually become possible, Monte Carlo models will remain as a quicker way of exploring the parameter space for the large scale N-body simulations.

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