Minimum $L^0$-Norm Two-Dimensional Phase Unwrapping Algorithm Based on the Derivative Variance Correlation Map

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Abstract. A new minimum $L^0$-norm two-dimensional phase unwrapping algorithm, based on the derivative variance correlation map, is proposed. In the algorithm, the novel derivative variance correlation map can truly reflect wrapped phase quality, ensuring a more reliable unwrapped result. After the definition of the derivative variance correlation map and the principle of the proposed algorithm are present, the performance of the new algorithm has been tested by use of wrapped phase data from an interferometric synthetic aperture radar experiment. Experimental result has verified that the proposed algorithm can cope well with the intractable noisy wrapped data.

1. Introduction

Two-dimensional phase unwrapping arises in several branches of applied physics and engineering, such as optical interferometry, magnetic resonance imaging, and synthetic aperture radar interferometry [1]. The wrapped phase data are measured or calculated from several coherent multidimensional signals and are the wrapped form of the true phase data. The wrapped phase data are distributed in the interval $[-\pi, \pi]$ and must somehow be unwrapped in order to provide an estimate of the true physical quantity of interest.

In the past two decades, a number of phase unwrapping algorithms have been proposed [2-3]. These algorithms can be grouped into three classes [4]: path-following algorithms, region algorithms, and global algorithms. Each of them is used to handle some sorts of problem successfully and each has its own drawbacks. Among different two-dimensional phase unwrapping approaches, the minimum $L^0$-norm method is gaining attention because the unwrapped phase distribution can be obtained by efficient mathematical techniques [5]. The method imposes a constraint on the solution such that the phase differences of the wrapped phase values agree with those of the unwrapped phase in the minimum $L^0$-norm sense.

In the minimum $L^0$-norm algorithms, the process of generating weighting coefficient is needed because weighting coefficient is used to determine whether the phase inconsistencies are masked correctly. Generally, the algorithm uses a quality map, which is an array of values that indicates the pixels’ reliability of the wrapped phase map, to define the weighting coefficient. However, the intrinsic drawbacks of existing quality maps constrain the application of these algorithms. For example, correlation map [1] is only available for SAR(synthetic aperture radar) data, and fringe modulation [6] is insensitive to surface structure changes that do not cast a shadow. Phase derivative variance map [7], which is generally believed as the most reliable measure of phase quality, cannot be responsive to
highly varying regions that have been corrupted by undersampling when these regions may have good phase quality and therefore high quality values. These quality maps often fail to handle wrapped phase data that contain error sources, such as phase discontinuities, noise and undersampling. In order to deal with those intractable wrapped phase data, a new minimum $L^0$-norm two-dimensional phase unwrapping algorithm based on derivative variance correlation map is proposed. In the algorithm, a novel quality map we have proposed, derivative variance correlation map [8], can truly reflect wrapped phase quality, ensuring a more reliable unwrapped result.

2. Derivative variance correlation map
In our new minimum $L^0$-norm phase unwrapping algorithm, the weighting coefficient is derived from derivative variance correlation map (DVCM). The DVCM is defined by equation

$$q_{m,n} = \frac{\sum_{i=m-l/2}^{m+l/2} \sum_{j=n-l/2}^{n+l/2} (\Delta x_{i,j} - \Delta x_{m,n})^2 + \sum_{i=m-l/2}^{m+l/2} \sum_{j=n-l/2}^{n+l/2} (\Delta y_{i,j} - \Delta y_{m,n})^2}{l \times l} \cdot (1 - \frac{\sum_{i=m-l/2}^{m+l/2} \sum_{j=n-l/2}^{n+l/2} \cos \phi_{i,j})^2 + \sum_{i=m-l/2}^{m+l/2} \sum_{j=n-l/2}^{n+l/2} \sin \phi_{i,j})^2}{l \times l},$$

(1)

where the quality of the pixel $(m, n)$ is calculated from its $1 \times 1$ neighborhoods. The terms $\Delta x_{i,j}$ and $\Delta y_{i,j}$ can be computed respectively by the formulas

$$\Delta x_{i,j} = W(\phi_{i+1,j} - \phi_{i,j})$$

(2)

$$\Delta y_{i,j} = W(\phi_{i,j+1} - \phi_{i,j})$$

(3)

$\Delta x_{i,j}$ and $\Delta y_{i,j}$ are the mean values of the partial derivatives of the phase in $1 \times 1$ window. In equation (2) and equation (3), $W$ is the wrapping operator

$$W(\phi_{i,j}) = \phi_{i,j} + 2\pi k_{i,j},$$

(4)

and $k_{i,j}$, an unknown integer, is chosen in such a way that $W(\phi_{i,j}) \in (-\pi, \pi]$. The DVCM is sensitive to rapid changing wrapped phase data and able to recognize the phase data with error sources, such as phase discontinuities, noise and undersampling, by assigning them with low-quality values. Therefore, the quality map can overcome the phase derivative variance map’s disadvantage and can be expected to generate a more reliable weighting coefficient. The DVCM of the wrapped phase map can be obtained by using the wrapped phase data and equation (1).

3. New Minimum $L^0$-norm algorithm based on DVCM
If $\phi_{m,n}$ is the wrapped phase data for pixel $(m, n)$ of an $N \times N$ image, the unwrapped phase $\phi_{m,n}$ in the minimum $L^0$-norm solution can be obtained as the solution of the following equation:

$$(\phi_{i+1,j} - \phi_{i,j}) \xi(i, j) + (\phi_{i,j+1} - \phi_{i,j}) \eta(i, j)$$

$$- (\phi_{i,j} - \phi_{i-1,j}) U(i-1, j) + (\phi_{i,j} - \phi_{i,j-1}) V(i, j-1) = c(i, j)$$

(5)

where

$$U(i, j) = \frac{\xi U_q(i, j)}{(\phi_{i+1,j} - \phi_{i,j} - \Delta x_{i,j})^2 + \xi U_q(i, j)} \quad i = 0, \cdots, M - 2, \quad j = 0, \cdots, N - 1$$

$$U(i, j) = 0 \quad \text{otherwise}$$

\[ U(i, j) = \frac{\xi U_q(i, j)}{(\phi_{i+1,j} - \phi_{i,j} - \Delta x_{i,j})^2 + \xi U_q(i, j)} \quad i = 0, \cdots, M - 2, \quad j = 0, \cdots, N - 1 \]

$$U(i, j) = 0 \quad \text{otherwise}$$
The is an adjustable parameter, which provides a compromise between how closely the requirement solution gradients matches the measured gradients and the rate of convergence of the phase unwrapping algorithm. The weighting coefficients, \( U_q(i,j) \) and \( V_q(i,j) \), are defined by

\[
U_q(i,j) = \min\left(q_{i+1,j}^2, q_{i,j}^2\right)
\]

\[
V_q(i,j) = \min\left(q_{i,j+1}^2, q_{i,j}^2\right)
\]

where \( q \) is the corresponding quality value of the DVCM. Because the weights \( U(i,j) \) and \( V(i,j) \) in equation (5) are the function of the updated solution, the equation (5) must be solved by means of the Preconditioning Conjugate Gradient (PCG) method in a doubly iterative structure. The implementation process of the new algorithm is summarized as follows:

1. For the iteration counter \( k = 0 \), values of the original solution array \( \phi_0 \) are chosen.
2. Compute the residual wrapped phase data \( R(i,j) = W\{\phi(i,j) - \phi(i,j)\} \).
3. Determine whether the residual wrapped phase contains residues by means of the residues detecting method. If no residues exist, it goes to step 4. Otherwise, it goes to step 5.
4. Unwrap \( R(i,j) \) with the simplest linear scanning path-following algorithm and the unwrapped result is \( W^{-1}\{R(i,j)\} \). The final solution becomes \( \phi(i,j) = \phi(i,j) + W^{-1}\{R(i,j)\} \). The total process is stopped.
5. Calculate weights \( U_k(i,j) \) and \( V_k(i,j) \) from equation (6) and (7) and then compute the \( c_k(i,j) \) from equation (8). Solve equation (5) with the Preconditioning Conjugate Gradient (PCG) method.
6. The iteration counter \( k = k+1 \) is update.
7. If \( k \geq k_{\text{max}} \) the calculation is stopped. Otherwise, it goes to step 2.

In the next section, the performance of the new algorithm will be tested by use of an experimental interferometric synthetic aperture radar (IFSAR) wrapped data. The unwrapped result by use of the new algorithm also will be compared with that by use of the Goldstein’s branch-cut algorithm\[9\], which is considered as an excellent algorithm that should always be tried first\[1\].

4. Experimental result

The experimental wrapped phase map, which is shown in figure 1(a), was obtained from an IFSAR experiment\[1\]. The wrapped phase map is 512×512 pixels in size. The images depicting wrapped phase in the range\( [-\pi, \pi] \) are scaled between black and white for display. Derivative variance correlation maps and unwrapped phase images are also scaled between black and white to cover the full dynamic range.

The wrapped phase fringes in figure 1(a) are indicative of terrain height and analogous to a contour map. As it can be seen, there are patches of noise where the phase fringes are broken up and the terrain height cannot be determined. An effective phase unwrapping algorithm must be able to unwrap the valid phase data around and between the noisy patches.

The figure 1(b) shows the DVCM of figure 1(a). The figure 1(c) shows the result of applying the new algorithm on figure 1(a). In this case, when the maximum number of iteration \( k_{\text{max}} = 4 \) and the adjustable parameter \( \varepsilon = 0.0001 \) are chosen, the satisfied result can be obtained. The corresponding three-dimensional profile of figure 1(c) is shown in figure 1(e). The unwrapped result of Goldstein’s branch-cut algorithm is shown in figure 1(d) and its corresponding three-dimensional plot is depicted.
in figure 1(f). As it can be seen, the new algorithm unwraps the IFSAR data correctly, producing consistent unwrapped result without being corrupted by the noisy patches. On the contrary, although Goldstein’s branch-cut algorithm correctly unwraps most of the wrapped phase data, there is a large region of error in the lower-right portion of the image. Apparently, the proposed algorithm is more effective to cope well with the intractable noisy wrapped data, obtaining an unwrapped result that is consistent and correct.

Figure 1. Unwrapping of a wrapped phase image obtained from an IFSAR experiment: (a) wrapped phase image; (b) derivative variance correlation map of (a); (c) unwrapped result by use of the proposed algorithm; (d) unwrapped result by use of Goldstein’s branch-cut algorithm; (e) three-dimensional rendering of (c); (f) three-dimensional rendering of (d).

5. Conclusion
In summary, a new minimum L₀-norm two-dimensional phase unwrapping algorithm based on derivative variance correlation map has been proposed and demonstrated. The derivative variance correlation map is directly derived from the wrapped phase map and used to define the weighting
coefficient. Because the quality map can overcome the drawbacks of the phase derivative variance map, which is generally believed as the most reliable measure of phase quality, it can truly reflect phase quality of the wrapped phase data, ensuring the minimum $L^0$-norm phase unwrapping algorithm retrieves a more reliable unwrapped result. The proposed algorithm is more effective to cope well with the intractable noisy wrapped data than the Goldstein’s branch-cut algorithm.

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