Setting Interim Deadlines to Persuade

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Abstract

A principal funds a multistage project and retains the right to cut the funding if it stagnates at some point. An agent wants to convince the principal to fund the project as long as possible, and can design the flow of information about the progress of the project in order to persuade the principal. If the project is sufficiently promising ex ante, then the agent commits to providing only the good news that the project is accomplished. If the project is not promising enough ex ante, the agent persuades the principal to start the funding by committing to provide not only good news but also the bad news that a project milestone has not been reached by an interim deadline. I demonstrate that the outlined structure of optimal information disclosure holds irrespective of the agent’s profit share, benefit from the flow of funding, and the common discount rate.

Keywords: dynamic information design, informational incentives, interim deadline, multistage project.

JEL Classification Numbers: D82, D83, G24, 031.
1 Introduction

The development of any innovation requires investment of both time and capital, while the outcome of this investment is inherently stochastic. Usually, the investor, being the principal, retains the option to stop funding the innovative project if at some point it proves unsuccessful. It is widely documented that the agent running the project tends to prefer the principal to postpone the stopping of the funding to enjoy either the extra funds or an additional chance to turn her research idea into a success story. In such an agent-principal relationship, the agent’s technological expertise and the quality of her innovative idea often allow her to manipulate the principal by designing how and when the outcomes of the research and development process are announced.

Recently, venture capital firms have started to pour billions into startups focused on the development of quantum computers, which are known for their technological complexity and difficulty of construction. The economic viability of quantum computing is questioned by a number of experts; however, the startups promise the investors a completed product in the foreseeable future. For instance, a quantum startup PsiQuantum announced to potential investors that it hopes to develop a commercially-viable quantum computer within five years and managed to raise more than $200 million in 2019.

This paper studies the implications of the agent’s control of information during the progress of a research and development project when the agent and the principal disagree about the timing of when to abandon the research idea. I ask: What is the degree of transparency to which an agent should commit before starting to work on an innovative project? In particular, which terms for self-reporting on the progress of the project should a startup propose while discussing the term sheet with a venture capitalist? As I show, depending on the properties of the project, the startup would strategically choose both the timing for the disclosure of updates on the progress of the project and the type of news it discloses - either good or bad.

I study a game between a startup and an investor. The startup controls the information on the progress of the project and has the power to propose the terms

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1 Agency conflict in which the agent prefers the principal to postpone abandoning the project that the agent is working on is studied in Admati and Pfleiderer (1994); Gompers (1995); Bergemann and Hege (1998, 2005); Cornelli and Yosha (2003).

2 “The Quantum Computing Bubble.” Financial Times, August 25, 2022.

3 “Bristol Professor’s Secretive Quantum Computing Start-Up Raises £179m.” The Telegraph, November 16, 2019.
for self-reporting on it to the venture capitalist.\textsuperscript{4} The startup has an intertemporal commitment power and commits to a dynamic information policy, which can be interpreted as designing the terms of the contract specifying how the information on the progress of the project is disclosed over time as the project unfolds. In return, the investor continuously provides funds and chooses when to stop funding the project.

The project has two stages and evolves stochastically over time toward completion, conditional on continuous investment in it. The completion of each of the stages of project occurs according to a Poisson process. The completion of the first stage serves as a milestone, such as the development of a prototype, while completion of the second stage achieves the project. The investor gets a lump-sum project completion profit if and only if he stops investing after the project is completed and before an exogenous project completion deadline, and the startup prefers the principal to postpone stopping the funding.\textsuperscript{5}

As the investor receives the reward only after a prolonged period of investment, he initially invests without being able to see if the investment is worthwhile. Hence, it is individually rational for the investor to start investing only if he is sufficiently optimistic regarding the future of the project. An important feature of the setting that I consider is that the information is symmetric at the outset: not only the investor, but also the startup is unable to find out if the project will bring profit to the investor or not - this can be inferred only as time goes on and some evidence is accumulated. The only tool that the startup has for persuading the investor to start investing is the promise of future reports on the progress of the project.

Clearly, the good news about the completion of the project is valuable to the investor as it helps him to stop investing in a timely manner. Further, as evidence regarding the project accumulates over time, failure to pass the milestone in a reasonable time makes the project unlikely to be accomplished in time - and the investor prefers to stop investing after observing such bad news. When designing the information policy, the startup chooses optimally between the provision of these two types of evidence in order to postpone the investor’s stopping decision for as long as possible.

I show that the startup’s choice of information policy depends on the ex ante attractiveness of the project for the investor. The attractiveness is captured by the flow cost-benefit ratio of the project. Thus, a project is relatively more attractive ex ante to the investor when its flow investment cost is lower, its project completion

\textsuperscript{4}I discuss the reasoning behind this assumption in Section 3.2.

\textsuperscript{5}I discuss the reasons for the presence of the project completion deadline in Section 3.1.
profit is higher, or the Poisson rate, at which completion of one stage of the project occurs, is higher.

When the project is sufficiently attractive ex ante to the investor, promises to provide information only on the completion of the project serve as a sufficiently strong incentive device to motivate the investor to start the funding at the outset. Further, the future news on the completion of the project does not harm the total expected surplus generated by the interaction of the startup and investor, while the future news on the project being poor decreases the surplus that the startup can potentially extract from the investor. Accordingly, the startup commits to providing only the good news, but not the bad news on the project in the future: it discloses the completion of the project and postpones the disclosure in order to ensure the extraction of as much surplus as possible from the investor. In the context of quantum computing, the startup optimally chooses and announces to the venture capitalist the date by which it plans to have a fully developed quantum computer. When the date comes, the startup reports completion if the quantum computer has been completed; if not, the startup reports the completion as soon as it occurs.

The situation changes when the project does not look promising to the investor ex ante. In that case, if the startup commits to disclosing only the completion of the project, the investor will not have the sufficient motivation to start investing in it. Thus, the startup extends the information policy to encompass not only the good news but also the bad. As in the case of the promising project, the startup discloses the project’s completion and does so without any postponement, thereby fully exploiting its preferred incentive tool. In addition, the startup sets a date at which the bad news is released if the milestone of the project has not yet been reached - this date is the interim reporting deadline.

Setting the interim deadline, the startup chooses a deterministic date, which it optimally postpones. As the startup prefers the investor to postpone stopping the funding, it prefers the interim deadline to be at a later expected date. Further, the completion of the stages of project according to a Poisson process makes both the startup and the investor risk-averse with respect to the date of the interim deadline. Thus, the startup prefers to set the interim deadline at a deterministic date and to postpone it as late in time as possible in order to extract all the surplus from the investor. In the context of quantum computing, the startup optimally chooses and announces a fixed date by which it plans to have a prototype of the quantum computer. When the date comes, reporting having the prototype at hand convinces the investor to continue the funding, and reporting not having the prototype leads to termination of the project.
Finally, I demonstrate that the outlined structure of the optimal information disclosure holds for a broad class of preferences of the startup and the investor. I allow for profit-sharing between the startup and the investor, varying degrees of the startup’s benefit from the flow of funding, and exponential discounting, and show that the startup prefers not to set any interim deadlines whenever the project is sufficiently promising to the investor. The future disclosure of the completion of the project promises investor profit in exchange for a prolonged investment, while the disclosure of the stagnation of the project at the interim deadline promises investor only saved costs, as further investment stops. Thus, when the project is attractive, the startup can make the funding and the beneficial experimentation relatively longer by setting no interim deadlines.

2 Related literature

My paper is related to the literature on dynamic information design. The closest paper in this strand of literature is by Ely and Szydlowski (2020). Similarly to my paper, they study the optimal persuasion of a receiver facing a lump-sum payoff to incur costly effort for a longer time. In my model, as in theirs, the sender is concerned to satisfy the beginning-of-the-game individual rationality constraint of the receiver when choosing the information policy. Further, the “leading on” information policy in Ely and Szydlowski (2020) has a similar flavor to the “postponed disclosure of completion” information policy in my paper: promises of news on completion of the project serve as an incentive device sufficient to satisfy the receiver’s individual rationality constraint.

However, there are several substantial differences between Ely and Szydlowski (2020) and my paper. While in their model the state of the world is static and drawn at the beginning of the game, in my model it evolves endogenously over time, given the receiver’s investment. As a result, the initial disclosure used in the “moving goalposts” policy in Ely and Szydlowski (2020) cannot provide additional incentives for the receiver in my model. The sender in my model uses another incentive device to incentivize the receiver to opt in at the initial period: she commits to an interim deadline at which she discloses that the first stage of the project is not completed.

Another closely related paper is by Orlov et al. (2020). The main similarity to my paper lies in the sender’s incentive to postpone the receiver’s irreversible stopping decision. The sender in their paper prefers to backload the information provision, which is in line with the properties of the optimal information policy in my paper. However, there are a number of substantial differences between
our papers. In Orlov et al. (2020), the sender does not have the intertemporal commitment power. Further, the receiver obtains a payoff at each moment of time, and thus the sender does not need to persuade the receiver to opt in at the beginning of the game.

Ely (2017); Renault et al. (2017); Ball (2019) also analyze dynamic information design models. However, their papers focus on persuading a receiver who chooses an action and receives a payoff at each moment of time, whereas in my paper the receiver takes an irreversible action and receives a lump-sum project completion payoff. Henry and Ottaviani (2019) consider a dynamic Bayesian persuasion model in which, similarly to my model, the receiver needs to take an irreversible decision. However, the incentives of the sender and receiver differ from my model: the receiver wants to match the static state of the world and the sender is concerned with both the receiver’s action choice and with the timing of that choice. Basak and Zhou (2020) study dynamic information design in a regime change game. The optimal information policy in their model resembles the interim deadline policy in my model: at a fixed date, the principal sends the recommendation to attack if the regime is substantially weak by that time.

My paper is also related to the literature on the dynamic provision of incentives for experimentation (Bergemann and Hege, 1998, 2005; Curello and Sinander, 2020; Madsen, 2022). The closest papers in this strand of literature are by Green and Taylor (2016) and Wolf (2017). Similarly to my model, both papers consider design of a contract regarding a two-stage project, in which the completion of stages arrives at a Poisson rate. In Green and Taylor (2016), there is no project completion deadline and the quality of the project is known to be good, while in Wolf (2017) the quality of the project is uncertain. In contrast to my paper, both papers focus on a canonical moral-hazard problem and give the power to design the terms of the contract to the investor (principal) rather than the startup (agent). In particular, the contract in Green and Taylor (2016) specifies the terms for the agent’s reporting on the completion of the first stage of the project. Similarly to my model, the optimal reporting takes the form of a deterministic interim deadline: at a principal-chosen date, the agent truthfully reports if she has already completed the first stage, which determines the further funding of the project.

6In a broad sense, my paper also relates to the small strand of theoretical literature on dynamic startup-investor and startup-worker relations under information asymmetry (Kaya, 2020; Ekmekci et al., 2020). However, while these papers focus on the signaling of the type of startup, I study the provision of information by the startup on the progress of the project.
3 The model

3.1 Setup

I consider a game between an agent (she, sender) and a principal (he, receiver). Time is continuous and there is a publicly observable deadline $T$, $t \in [0, T]$. For each $t$, the principal chooses sequentially to invest in the project ($a_t = 1$) or not ($a_t = 0$). The flow cost of the investment is constant and given by $c$. The choice of $a_t = 0$ at some $t$ is irreversible and induces the end of the game.

The assumption that the project needs to be completed in finite time is natural in many economic settings. The main interpretation for $T$ is an expected change in market conditions that renders the project unprofitable. In the context of a research and development project, $T$ could stand for the date at which the competitor’s innovative product is expected to enter the market, or the date at which the competitor is expected to get a patent on the competing innovation.

The state of the world at time $t$ is captured by the number of stages of the project completed by $t$, $x_t$, and the project has two stages, $x_t \in \{0, 1, 2\}$. The state process begins at the state $x_0 = 0$ and, conditional on the continuation of the investment by the principal, it increases at a Poisson rate $\lambda > 0$. Information on the number of stages completed is controlled by the agent. Thus, when making investment decisions, the principal relies on the information provided by the agent.

The project brings the profit $v$ if and only if the second stage of the project has been completed by the time of stopping, and a payoff of 0, otherwise. I assume that all of the profit goes to the principal. This assumption simplifies the exposition without affecting the main results of the paper. I relax this assumption and consider the profit-sharing between the agent and the principal in Section 6.

There is a conflict of interest between the agent and the principal as the agent benefits from using the funds provided by the principal for running the project, possibly diverting them for her benefit. Thus, the agent faces the flow payoff of $c$ and wants the principal to postpone his irreversible decision to stop as long as possible.

I study the agent’s choice of information provision to the principal. The agent chooses an information policy to maximize her expected long-run payoff. I assume that the agent has the power to announce and commit to a policy. An information

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7The results for the setting without a deadline are easily obtained by considering $T \to \infty$. They are presented in Appendix E.

8The absence of the principal’s commitment to an investment policy and the irreversibility of the stopping decision capture the venture capitalist’s option to abandon the project, e.g., in the case of its negative net present value.
**Policy** \( \sigma \) is a rule that for each date \( t \) and for each past history \( h(t) \) specifies a probability distribution on the set of messages \( M \). The history includes all past and current realizations of the process and all past message draws and principal’s action choices.

When choosing an information policy, the agent faces a rich strategy space. First, she can choose if the information on the completion of the first, or second, stage of the project will be disclosed by the policy. Second, she can choose how to disclose the completion of a stage of the project: for instance, to do so immediately or to postpone the disclosure.

The timing of the game is as follows. First, at \( t = 0 \), the agent publicly commits to an information policy \( \sigma \). Second, at each \( t \) the principal observes the message generated by the information policy and makes her investment decision. The game ends when the principal chooses to stop investing or at \( T \), if he keeps investing until \( T \). I assume that whenever indifferent about investing or not, the principal chooses to invest, and whenever indifferent about disclosing information or not, the agent chooses not to disclose.

Throughout the paper, I use the following intuitive notational convention: for any two dates at which the principal stops investing, \( S \) and \( \tau \),

\[
S \land \tau := \min \{S, \tau\},
\]
\[
S \lor \tau := \max \{S, \tau\}.
\]

### 3.2 Discussion of assumptions

The main interpretation of the considered dynamic information design problem is the contracting between the agent (startup) and the principal (investor) on the terms of reporting on the completion of stages of the project that are not publicly observed. The terms could take the form of a proposed formal reporting schedule or a schedule of meetings with the investor. Non-observability of the stage completions stems from the fact that, while the technology is being developed in the lab, the principal either does not have sufficient expertise to assess the progress or the full access to the lab.

I assume that the principal does not have the power to propose the terms for reporting to the agent and, e.g., make her fully disclose the progress achieved in the lab. The most natural interpretation of such an asymmetry in the bargaining power is the asymmetry in the market for private equity: there are sufficiently many investors willing to invest in a particular technology or sufficiently few startups working on the technology.\(^9\) For instance, investors’ interest in quantum

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\(^9\)In the alternative interpretation of the model, contracting concerns internal corporate re-
computing has grown markedly in recent years, while there are reports of a shortage of human capital in this industry.\textsuperscript{10,11} Another example is the communication software industry, which has recently experienced increased investment activity.\textsuperscript{12}

As the agent enjoys the power of full control over the information on the progress of the project, she is completely free to offer what is disclosed and when. In particular, the contract between the agent and the principal can specify that the completion of the second stage of the project is disclosed with a delay rather than immediately. The agent who has an advantage in expertise over the principal can rationalize such a condition by saying that before the success is reported to the principal, it is worth re-checking the data, which takes time.

Even though the principal can not dictate to the agent which information and how she should disclose, the principal can potentially hire an external monitor who would visit the lab and prepare an additional report on the progress of the project. In that case, the contract signed between the agent and the principal will account for both free information that the agent promised to provide and additional costly information which the principal obtains with the help of a monitor. In the baseline version of the model, I assume that the principal can not use the help of a monitor. This can be rationalized by the shortage of experts in the field, which makes hiring a monitor prohibitively costly. Alternative interpretation is that the agent restricts the principal’s access to additional information on the progress of the project by stating that a potential information leak would put the technology being developed at risk.\textsuperscript{13}

The information policy relies upon the agent’s commitment power, which holds not only within each date but also between the dates. The agent’s commitment within each date follows from prohibitively high legal costs of cooking up the evidence. The agent’s intertemporal commitment stems from the rigidity of terms and form of reporting fixed in the contract that the agent and the principal sign.

\textsuperscript{10}“The Quantum Computing Bubble.” \textit{Financial Times}, August 25, 2022.

\textsuperscript{11}“Quantum Computing Funding Remains Strong, but Talent Gap Raises Concern”, a report by McKinsey Digital, https://www.mckinsey.com/business-functions/mckinsey-digital/our-insights/quantum-computing-funding-remains-strong-but-talent-gap-raises-concern/.

\textsuperscript{12}“This Is Insanity: Start-Ups End Year in a Deal Frenzy.” \textit{Best Daily Times}, December 07, 2020.

\textsuperscript{13}In particular, this rationale was used to restrict the investors’ access to information on the progress of the project in the case of Theranos, see “What Red Flags? Elizabeth Holmes Trial Exposes Investors’ Carelessness.” \textit{The New York Times}, November 04, 2021.
at the outset of the game.

4 No-information and full-information benchmarks

4.1 No-information benchmark

First, I consider the simple case when the information policy is given by \( \sigma^{NI} \): the same message \( m \) is sent for all histories \( h(t) \) and all dates \( t \). Thus, the agent provides no information regarding the progress of the project. As I demonstrate, in this case the principal starts investing in the project if and only if it is sufficiently promising for the principal from the ex ante perspective and invests until a deterministic interior date.

As no news arrives, the principal bases his decision about when to stop investing on his unconditional belief regarding the completion of the second stage of the project. I denote the unconditional belief that \( n \) stages of the project were completed by \( t \), by \( p_n(t) \), i.e., \( p_n(t) := \Pr(x_t = n) \). The state of the world is fully determined by \( p(t) \) given by

\[
\begin{align*}
p_0(t) &= e^{-\lambda t}, \\
p_1(t) &= \lambda t e^{-\lambda t}, \\
p_2(t) &= 1 - e^{-\lambda t} - \lambda t e^{-\lambda t}.
\end{align*}
\]

The principal’s sequential choice of \( a_t \in \{0, 1\} \) can be restated equivalently as the choice of deterministic stopping time \( S^{NI} \in [0, T] \) chosen at \( t = 0 \).\(^{14}\) Given the principal’s continuous investment, the probability of completion of the second stage of the project, \( p_2(t) \), increases monotonously over time, making obtaining the payoff \( v \) more likely. However, postponing the stopping is costly.

To decide on \( S^{NI} \), the principal trades off the flow benefits and flow costs of postponing the stopping decision, while keeping the individual rationality constraint in mind. The flow cost of postponing the stopping for \( \Delta t \) is given by \( c \cdot \Delta t \) and the flow benefit is given by \( v \cdot p_1(t) \lambda \Delta t \).\(^{15}\) Thus, the necessary condition for the principal’s stopping at some interior moment of time \( 0 < S < T \) is given by

\[
v \cdot p_1(S) \lambda = c. \tag{1}
\]

\(^{14}\)Note that the dynamic belief system that he faces is deterministic in a sense of being fully specified from \( t = 0 \) perspective.

\(^{15}\)To observe this, note that the probability of the completing both the first and second stages within a very short time \( \Delta t \) is negligibly small; thus, during some \( \Delta t \), the principal receives the project completion payoff \( v \) iff the first stage has already been completed.
Let 
\[ \kappa := \frac{c}{v \lambda}, \]

the ratio of the flow cost of investment \( c \) to the gross project payoff \( v \) normalized using \( \lambda \), the rate at which a project stage is completed in expectation. The intuitive interpretation of \( \kappa \) is the flow cost-benefit ratio of the project. \( \kappa \) is an inverse measure of how ex ante promising the project is for the principal. (1) is equivalently given by \(^{16}\)

\[ \frac{p_1 (S)}{\text{flow benefit of waiting}} = \frac{\kappa}{\text{flow cost of waiting}} \tag{2} \]

and presented graphically in Figure 1. As the state process transitions monotonously from 0 to 1 and then to 2, \( p_1 (t) \) first increases and after some time starts to decrease. Thus, the principal has two candidate interior stopping times satisfying (2), \( \bar{S} \) and \( \bar{S}^{NI} \). The principal prefers to postpone stopping to \( \bar{S}^{NI} \), as during \( (\bar{S}, \bar{S}^{NI}) \) the flow benefits are higher than the flow costs.

![Figure 1](image-url)

**Figure 1:** Principal’s choice under no information:
- **left plot:** postponing stopping increases the chance of getting a project payoff \( v \);
- **right plot:** principal trades off costs and benefits and optimally stops at \( \bar{S}^{NI} \).

The forward-looking principal can guarantee himself a payoff of 0 if he does not start investing at \( t = 0 \). Thus, he will choose to start investing at \( t = 0 \) only if his flow gains accumulated up to \( T \wedge \bar{S}^{NI} \) are larger than his flow losses, and his expected payoff is given by

\[ V^{NI} := \max \left\{ 0, \int_0^{T \wedge \bar{S}^{NI}} (v \cdot p_1 (s) \lambda - c) \, ds \right\}. \tag{3} \]

\(^{16}\)Here I WLOG express the flow benefits and flow costs of investing for the principal in different units of measurement.
Geometrically, the integral in (3) represents the difference between the shaded areas in Figure 2 that correspond to the accumulated gains and losses. The principal starts investing at \( t = 0 \) if, given \( T \) and \( \lambda \), the normalized cost-benefit ratio \( \kappa \) is low enough, so that the shaded area of the accumulated gains is at least as large as that of the accumulated losses. I denote such a cutoff value of \( \kappa \) by \( \kappa^{NI}(T, \lambda) \) and summarize the principal’s choice without information in Lemma 1.

\[
\bar{S}^{NI} = \begin{cases} 
\bar{S}^{NI}, & \text{if } \frac{1}{\lambda} \leq T \text{ and } \kappa \geq e^{-\lambda T} \lambda T \\
T, & \text{otherwise}
\end{cases}
\]  

(4)

In both plots the expected accumulated gains are higher than the losses, so the principal starts to invest at \( t = 0 \).

**Lemma 1.** Assume no information regarding the progress of the project arrives over time. Denote the time at which the principal stops investing by \( S^{NI} \). If \( \kappa > \kappa^{NI}(T, \lambda) \), then the principal does not start investing in the project, i.e., \( S^{NI} = 0 \). If \( \kappa \leq \kappa^{NI}(T, \lambda) \), then the principal’s choice of stopping time is given by

\[
S^{NI} = \begin{cases} 
\bar{S}^{NI}, & \text{if } \frac{1}{\lambda} \leq T \text{ and } \kappa \geq e^{-\lambda T} \lambda T \\
T, & \text{otherwise}
\end{cases}
\]  

the closed-form expressions for \( \bar{S}^{NI} \) and \( \kappa^{NI}(T, \lambda) \) are presented in the proof in Appendix C.
4.2 Full-information benchmark

Here, I consider the case in which the information policy is given by $\sigma^{FI}$: $M = \{m_0, m_1, m_2\}$ and the message $m_n$ is sent for all $t$ such that $x_t = n, n \in \{0, 1, 2\}$. Thus, the principal fully observes the progress of the project at each $t$. I characterize the cutoff level of the cost-benefit ratio below which the principal is willing to start investing. Further, I show that the principal chooses to stop when no stages of the project are completed and the project completion deadline $T$ is sufficiently close.

At each $t$, the principal uses the information on the number of stages completed to decide either to stop investing or to postpone the stopping. The news on completion of the second stage of the project makes the principal stop immediately, as this way he immediately receives the project payoff $v$ and stops incurring the costs of further investment. If only the first stage of the project is completed, the principal faces the following trade-off. The instantaneous probability that the second stage will be completed during $\Delta t$ is given by $\lambda \Delta t$, which is constant over the time. Thus, the expected benefit of postponing the stopping for $\Delta t$ is given by $v \cdot \lambda \Delta t$. Meanwhile, the expected cost of postponing the stopping is given by $c \cdot \Delta t$. As a result, if $\kappa \leq 1$, then the principal who knows that the first stage of the project has already been completed invests until either the completion of the second stage or until the project deadline $T$ is reached.

Consider now the case in which the principal knows that the first stage has not yet been completed. The principal’s trade-off with respect to the stopping decision is now more involved. Postponing the stopping for $\Delta t$ leads to the completion of the first stage of the project with the instantaneous probability $\lambda \Delta t$. Completion of the first stage of the project at some $t$ implies that the principal receives the continuation value of the game, conditional on having the first stage completed. I denote the continuation value of the principal at time $t$ under full information and conditional on the completion of first stage of the project by $V_{t|1}^{FI}$. This is given by

$$V_{t|1}^{FI} = \left(v - \frac{c}{\lambda}\right)\left(1 - e^{-\lambda(T-t)}\right).$$

The principal’s expected benefit from postponing the stopping for $\Delta t$ is given by $V_{t|1}^{FI} \cdot \lambda \Delta t$ and the cost of postponing the stopping is, as before, given by $c \cdot \Delta t$. The continuation value, $V_{t|1}^{FI}$, shrinks over time and approaches 0 as the project deadline $T$ approaches. This is because the shorter the time left before the project

\footnote{This benchmark corresponds to equilibrium in the setting, where the principal has the full power to propose the terms of self-reporting to the agent.}

\footnote{See the derivation in the proof of Lemma 2 in the Appendix.}
deadline, the less likely it is that the second stage of the project will be completed before $T$. If at some $t$, and given that no stages are completed yet, the expected net benefit of investing reaches 0, it is optimal for the principal to stop at that $t$.\footnote{If at $t$ the expected benefit of investing becomes lower than the cost, then, after $t$, the net expected benefit remains negative. Thus, it is optimal for the principal to stop investing precisely at $t$.} I denote this date by $S_0^P$ and plot it in Figure 3.

Figure 3: The principal optimally sets an interim deadline $t = S_0^P$ under full information: given that the first stage of the project has not been completed by $S_0^P$, it is optimal to stop investing at $S_0^P$.

As the principal has an incentive to stop at $S_0^P$ only if he knows that the first stage or the milestone of the project has not been reached, the economic interpretation of $S_0^P$ is that it is the interim deadline that the principal sets for the project. If the milestone has not been reached by the interim deadline, then it is sufficiently unlikely that the project will be completed before the project deadline $T$. Thus, it is optimal for the principal to “give up” on the project and stop investing at $t = S_0^P$. If the milestone is reached by the interim deadline, then the principal has an incentive not to stop investing until either the second stage is completed or $T$ is hit.

Finally, given the plan to stop either at the interim deadline, or at the completion of the second stage of the project, it is individually rational to start investing only if the principal’s expected payoff from the $t = 0$ perspective is non-negative. I denote the upper bound for the normalized cost-benefit ratio such that the principal starts investing at $t = 0$ by $\kappa^{FI}(T, \lambda)$. Intuitively, $\kappa^{FI}(T, \lambda) > \kappa^{NI}(T, \lambda)$: whenever the principal is willing to start investing under no information, he is also willing to start under the full information. I summarize the principal’s choice
under full information in Lemma 2.

**Lemma 2.** Assume that the progress of the project is fully observable at each moment in time. If \( \kappa > \kappa^{FI}(T, \lambda) \), where \( \kappa^{FI}(T, \lambda) > \kappa^{NI}(T, \lambda) \), then the principal does not start investing in the project. If \( \kappa \leq \kappa^{FI}(T, \lambda) \), the principal invests either until the random date at which the second stage of the project is completed, \( t = \tau_2 \), or until the interim deadline, \( t = S^P_0 \), at which he stops if the first stage has not yet been completed. Formally, the time at which the principal stops investing is a random variable \( \tau \) given by:

\[
\tau = \begin{cases} 
\tau_2 \wedge T, & \text{if } x_{S^P_0} \neq 0 \\
S^P_0, & \text{otherwise}
\end{cases}
\]

where \( S^P_0 = T + \frac{1}{\lambda} \log \left( \frac{1-2\kappa}{1-\kappa} \right) \) and the expression for \( \kappa^{FI}(T, \lambda) \) is presented in the proof in Appendix C.

Assume now that the agent chooses which information to provide to the principal. As for \( \kappa > \kappa^{FI}(T, \lambda) \) the principal is not willing to start investing even under full information, there is no way in which the agent can strategically conceal the information to her benefit. In Section 5, I assume \( \kappa \leq \kappa^{FI}(T, \lambda) \) and analyze how the agent can strategically provide information on the progress of the project and extract the principal’s surplus.

## 5 Agent’s choice of information policy

In this Section, I present how the agent’s choice of information policy changes with the ex ante attractiveness of the project, which is captured by the cost-benefit ratio \( \kappa \). In Section 5.1, I start with Proposition 1 which summarizes the comparative statics result. In Sections 5.2-5.3, I proceed with the detailed discussion of the economic mechanisms that determine the outlined structure of the optimal information policy. Throughout Section 5, I maintain the following technical assumption:

**Assumption 1.** \( e^{\lambda T} > \lambda T (\lambda T + 1) + 1 \).

For the sake of a clearer exposition, this assumption rules out the case in which \( T \) is so low that whenever the principal is willing to start investing in the no-information benchmark, he invests until \( T \). Relaxing this assumption does not change the the comparative statics result in Proposition 1 qualitatively.\(^{20}\)

\(^{20}\)I discuss the implications of relaxing this assumption in the proof of Proposition 1.
5.1 The structure of optimal information disclosure

Proposition 1. There exist cost-benefit ratio cutoffs $κ^{ND}(T, λ), κ^{ND}(T, λ) < κ^{NI}(T, λ)$, and $κ(T, λ), κ^{NI}(T, λ) < κ(T, λ) < κ^{FI}(T, λ)$, such that, depending on the cost-benefit ratio of the project, the optimal information policy has the following form:

1. when $κ ≤ κ^{ND}(T, λ)$, the agent provides no information and the principal invests until $T$;
2. when $κ^{ND}(T, λ) < κ ≤ κ(T, λ)$, the agent discloses only the completion of the second stage of the project and does that with the postponement;
3. when $κ(T, λ) < κ < κ^{FI}(T, λ)$, the agent immediately discloses the completion of the second stage of the project whenever it occurs and specifies a deterministic interim deadline, at which it discloses if the first stage is already completed;
4. when $κ ≥ κ^{FI}(T, λ)$, the agent provides no information as the principal’s long-run payoff is non-positive even under full information.

Figure 4 illustrates Proposition 1 and presents the partition of the cost-benefit ratio space based on the corresponding forms of the optimal information policy.

The structure of optimal disclosure presented in Proposition 1 follows the simple and intuitive pattern. The lower is the value of cost-benefit ratio, the higher is ex ante attractiveness of the project to the principal. First, for $κ ≤ κ^{ND}(T, λ)$, the project is so attractive that the principal is willing to keep investing until the project deadline $T$ even in the no-information benchmark. Thus, there is no need to disclose any information. For the higher values of $κ$, there emerges a room for strategic disclosure, and the higher is the value of $κ$ (i.e., the lower is the ex ante attractiveness of the project), the more information the agent has to disclose to incentivize the principal. For $κ ≥ κ^{FI}(T, λ)$, the project gets so
unattractive that the principal can not strictly benefit from investing even in the full-information benchmark. In this extreme case, the agent chooses not to disclose any information.

The most important part of the result in Proposition 1 demonstrates which additional pieces of information the agent chooses to disclose and when she chooses to discloses them as $\kappa$ gets higher and higher. When $\kappa \in (\kappa^{ND}(T, \lambda), \tilde{\kappa}(T, \lambda)]$, the agent discloses only the completion of the second stage of the project and does not promise any information on the completion of the first stage of the project. Further, as $\kappa$ increases from $\kappa^{ND}(T, \lambda)$ to $\tilde{\kappa}(T, \lambda)$, the agent adjusts the timing of the disclosure: she postpones the disclosure of the second stage completion less and less and discloses immediately for $\tilde{\kappa}(T, \lambda)$. For $\kappa \in (\tilde{\kappa}(T, \lambda), \kappa^{FI}(T, \lambda))$, the agent not only discloses the completion of the second stage of the project immediately, but also provides information on the completion of the first stage at the interim deadline that she optimally chooses.

In the subsequent Sections, I provide details on the mechanisms that shape the aforementioned comparative statics results. I omit the trivial case of non-disclosure under $\kappa \leq \kappa^{ND}(T, \lambda)$ and start the discussion from the optimal information policy under $\kappa \in (\kappa^{ND}(T, \lambda), \tilde{\kappa}(T, \lambda)]$.

5.2 Postponed disclosure of project completion

In this Section, I restrict attention to $\kappa \in (\kappa^{ND}(T, \lambda), \tilde{\kappa}(T, \lambda)]$ and explain why the optimal information policy has the particular form presented in the Proposition 1: the agent discloses only the completion of the project and does this with the postponement.

5.2.1 Agent’s problem

To characterize the agent’s choice of information policy, I consider an equivalent problem, in which the agent directly chooses the stochastic history-contingent length of investment subject to the principal’s individual rationality constraints that ensure optimality of such action process for the principal. An investment schedule is a random variable $\tau: \Omega \to [0, T]$ defined on the probability space $(\Omega, \mathcal{F}, \mathbb{P})$ and adapted to the filtration $F = \left(\mathcal{F}_t\right)_{t \geq 0}$ generated by the stochastic process $x_t$. As I demonstrate in Section 5.2.2, restricting attention to random variables adapted to the natural filtration of $x_t$ is without loss of generality for the agent’s equilibrium expected payoff when $\kappa \in (\kappa^{ND}(T, \lambda), \tilde{\kappa}(T, \lambda))$.\footnote{In other words, there is no need for external randomization devices to optimally incentivize the principal when $\kappa \in (\kappa^{ND}(T, \lambda), \tilde{\kappa}(T, \lambda)]$.}
Informally, an investment schedule $\tau$ is a random variable with support $[0, T]$ specified by a rule that suggests when to stop investing depending on the history of previous realizations of the number of completed stages $x_t$.\footnote{The stopping rules from the no-information and full-information benchmarks are given in Lemmas 1 and 2, respectively. Further examples of such rules include “stop 1 minute after the second stage of the project is completed” and “stop at $t = S$ if only the first stage of the project is completed by $t = S$.”} The agent chooses this rule at $t = 0$. $P(x_\tau = 2)$ captures the belief about two stages of the project completed by $\tau$, the random time of stopping in the future, and $E[\tau]$ captures the expected length of investment from $t = 0$ perspective.

Given an investment schedule $\tau$, the long-run payoff of the agent and the principal are given, respectively, by

$$W(\tau) := E[\tau] c,$$
$$V(\tau) := P(x_\tau = 2) v - E[\tau] c.$$ 

As an investment schedule $\tau$ is an action recommendation rule, the action recommendations generated by this rule have to be obedient for the principal. In other words, at each date and for each possible history the principal’s actions suggested by $\tau$ have to be optimal for the principal. An object useful for characterizing if an investment schedule $\tau$ generates obedient action recommendations is given by the principal’s continuation value at some interim date $t$ promised by the investment schedule $\tau$. This continuation value depends on the beliefs of the principal.

As the principal does not commit to a policy at $t = 0$, he rationally updates his belief given an investment schedule $\tau$ and assesses the costs and benefits of either further following the investment schedule $\tau$ provided by the agent or deviating from it. The absence of stopping by some date $t$ and, thus, the fact that the game continues at $t$ serves as a source of inference for the principal. First, he forms a belief regarding the number of completed stages of the project by $t$, conditional on the game still continuing at $t$, $P(x_t = n|t < \tau)$. Second, he forms a belief regarding the number of completed stages of the project at the random date of stopping in the future, $\tau$, $P(x_\tau = n|t < \tau)$.

Given the absence of stopping by $t$, the principal’s expected payoff promised by the schedule is given by $P(x_\tau = 2|t < \tau) v - E[\tau - t|t < \tau] c$. The principal’s expected payoff from stopping at $t$ is given by $P(x_t = 2|t < \tau) v$. The principal’s continuation value at $t$ given the investment schedule $\tau$ is the difference between these two expected payoffs, I denote it by $V_t(\tau)$:
\[ V_t(\tau) := [P(x_\tau = 2|t < \tau) - P(x_t = 2|t < \tau)]v - E[\tau - t|t < \tau]c. \] (6)

This way of formulating the continuation value is intuitive: if the continuation value \( V_t(\tau) \) gets negative then it is not valuable to continue investing for the principal, and he is better-off stopping immediately rather than following the schedule. The following Lemma shows the necessary and sufficient conditions for an investment schedule \( \tau \) to generate obedient action recommendations for the principal.

**Lemma 3.** An investment schedule \( \tau \) is the principal’s best response to at least one information policy \( \sigma \) if and only if
\[ V_t(\tau) \geq 0, \forall t \geq 0 \text{ and } V_t^{NI} < 0, \] (7)
where \( V_t^{NI} \) is the principal’s optimal continuation value in the absence of any additional information from the agent starting from the date \( t \).

\( V_t(\tau) \geq 0, \forall t \geq 0 \) ensures that the principal does not want to stop before the date of stopping suggested by the investment schedule is reached, and \( V_t^{NI} < 0 \) ensures that the principal does not want to continue conditional on reaching the date of stopping suggested by the investment schedule. Conditions from Lemma 3 constitute the system of constraints for the agent’s problem.

As the agent chooses an investment schedule \( \tau \) to maximize her long-run payoff, the constraint \( V_t^{NI} < 0 \) is inactive at optimum.\(^{23}\) Thus, without loss of generality, I omit this constraint from the agent’s problem, and the problem that the agent solves at \( t = 0 \) is given by
\[
\max_{\tau \in T} \{c \cdot E[\tau]\} \quad \text{s.t. } V_t(\tau) \geq 0, \forall t \geq 0, \] (8)
where \( T \) is the set of stopping times with respect to the natural filtration of \( x_t \). As the principal’s choice to postpone the stopping of funding is costly, it is natural to interpret the system of constraints in (8) as the system of principal’s individual rationality constraints.

The agent’s problem is complex, and thus I solve it in steps. First, I characterize the investment schedule, which solves the relaxed version of (8) with the principal’s individual rationality constraints only for some initial periods. Second, I demonstrate that there exists an investment schedule solving the relaxed agent’s problem and satisfying the full system of the principal’s individual rationality constraints (7). This investment schedule pins down optimal information policy.

\(^{23}\)Otherwise, the agent can prolong the expected funding by choosing a different \( \tau \).
5.2.2 Solution to the relaxed agent’s problem

In this Section, I consider the relaxed agent’s problem and discuss its solution. This sheds light on the technical intuition behind the key properties of the optimal information policy. The agent’s relaxed problem for the parametric case of $\kappa \in (\kappa_{ND} (T, \lambda), \kappa_{NI} (T, \lambda))$ is given by (8) with the principal’s individual rationality constraint only for $t \in [0, S_{NI}]$. The agent’s relaxed problem for the parametric case of $\kappa \in (\kappa_{NI} (T, \lambda), \tilde{\kappa} (T, \lambda))$ is given by (8) with the principal’s individual rationality constraint only for $t = 0$.

Consider the agent’s long-run payoff given an investment schedule, $W (\tau)$. This can be restated equivalently as follows:

$$W (\tau) = [W (\tau) + V (\tau)] - V (\tau)$$

$$= P (x_\tau = 2) v - [P (x_\tau = 2) v - E [\tau] c].$$

(9)

The solution to the agent’s relaxed problem for both considered parametric cases follows a simple idea: the optimal investment schedule ensures that the total surplus is maximal and that the principal’s surplus is minimal. Consider a schedule $\tau$ such that the stopping occurs after the completion of the second stage of the project, unless the project deadline $T$ was hit, i.e., the schedule satisfies the condition $\tau \geq \tau_2 \land T$. Such a schedule leads to

$$P (x_\tau = 2) = P (x_T = 2).$$

(10)

Given a schedule $\tau$ satisfying (10), if $\tau$ is individually rational for the principal at date $t = 0$ then the total surplus generated achieves its upper bound and is given by $P (x_T = 2) v$, which depends on the exogenously given project deadline $T$ and the profit $v$. However, the stopping only after the second stage completion is not individually rational for the principal at $t = 0$ when the cost of funding is sufficiently high, the profit is sufficiently low, or the expected time until a project stage completion is sufficiently high.

Lemma 4 elaborates on the cost-benefit ratio cutoff value $\tilde{\kappa} (T, \lambda)$: it distinguishes the case in which stopping only after the second stage completion is individually rational at $t = 0$ from the case in which it is not. Based on this partition, when $\kappa \in (\kappa_{ND}, \tilde{\kappa} (T, \lambda)]$, I call the project ex ante promising for the principal.

**Lemma 4.** For each $(T, \lambda)$ there exists $\tilde{\kappa} (T, \lambda)$, $\kappa_{NI} (T, \lambda) < \tilde{\kappa} (T, \lambda) < \kappa_{FI} (T, \lambda)$, such that if $\kappa \leq \tilde{\kappa} (T, \lambda)$ ($\kappa > \tilde{\kappa} (T, \lambda)$) then an investment schedule $\tau$ in which stopping after $\tau_2 \land T$ happens with probability one is individually rational at $t = 0$ (not individually rational at $t = 0$) for the principal.
For \( \kappa \in (\kappa^{ND}(T, \lambda), \tilde{\kappa}(T, \lambda)) \), the schedule \( \tau \geq \tau_2 \wedge T \) is individually rational for the principal at \( t = 0 \), and it maximizes the total surplus. In addition to choosing \( \tau \geq \tau_2 \wedge T \), it is optimal for the agent to choose the investment schedule with a higher expected date of stopping the funding to extract all the principal’s surplus subject to his individual rationality constraints. For \( \kappa \in (\kappa^{NI}(T, \lambda), \tilde{\kappa}(T, \lambda)) \), the agent chooses such \( \tau \) that the principal’s individual rationality constraint at \( t = 0 \) is binding. As a result, \( V(\tau) = V^{NI} \), i.e., the principal gets his no-information benchmark payoff given by 0.

For \( \kappa \in (\kappa^{ND}(T, \lambda), \kappa^{NI}(T, \lambda)) \), as in the no-information benchmark the principal invests until \( \bar{S}^{NI} \) with certainty, the agent chooses the investment schedule as to postpone the start of information provision at least until \( \bar{S}^{NI} \). Further, the agent chooses \( \tau \) with a higher expected date of stopping so that the principal’s individual rationality constraint at \( t = \bar{S}^{NI} \) is binding. The absence of stopping until at least \( \bar{S}^{NI} \) and the fact that individual rationality constraint binds at \( t = \bar{S}^{NI} \) taken together imply that \( V(\tau) = V^{NI} \), i.e., from \( t = 0 \) perspective, the principal gets her no-information benchmark payoff, which is non-negative and given by (3).

The next Lemma summarizes the necessary conditions for an investment schedule to solve the agent’s relaxed problem when the project is promising. These conditions are shared both by the relaxed problem formulated for the case of \( \kappa \in (\kappa^{ND}(T, \lambda), \kappa^{NI}(T, \lambda)) \) and the relaxed problem formulated for the case of \( \kappa \in (\kappa^{NI}(T, \lambda), \tilde{\kappa}(T, \lambda)) \). The conditions that are both necessary and sufficient for an investment schedule to solve the agent’s relaxed problem are presented in the Proof of Lemma 5.

**Lemma 5.** Assume \( \kappa \in (\kappa^{ND}, \tilde{\kappa}(T, \lambda)) \). If an investment schedule \( \tau \) solves agent’s relaxed problem, then

1. with probability one, stopping occurs after \( \tau_2 \wedge T \);  
2. \( V(\tau) = V^{NI} \), where \( V^{NI} \) is the principal’s expected payoff in the no-information benchmark, given by (3).

### 5.2.3 Optimal information policy

In this Section, I show that there exists an information policy that both solves the agent’s relaxed problem and satisfies the full system of the individual rationality constraints. Given this, as Lemma 5 describes the solution to the relaxed problem, it also sheds light on the properties of the optimal information policy for the case of a promising project. These properties have a clear-cut economic
interpretation as an investment schedule $\tau$ can be easily interpreted in terms of action recommendations for the principal.

An investment schedule $\tau$ can be without loss of generality implemented using a direct recommendation mechanism - a simple policy which has $M = \{0, 1\}$, where $m = 1$ received at date $t$ is a recommendation to continue investing at $t$ for the principal and $m = 0$ received at date $t$ is a recommendation to stop investing at $t$. Keeping this in mind, it is clear from Lemma 5 that the optimal information policy has to satisfy the following conditions. First, whenever the agent recommends the principal to stop, the second stage of the project is already completed. Second, the recommendation to stop is postponed so that the principal’s individual rationality constraint is binding, which manifests in $V(\tau) = V^{NI}$. The first condition presents the key feature of the optimal information policy for the case of promising project: the agent discloses the completion of the second stage of the project, but stays silent regarding the completion of the first stage of the project. The intuition behind the agent’s choice is simple: a recommendation to stop when no stages of the project are completed and the project deadline $T$ is close does indeed incentivize the principal; however, it also reduces the total surplus generated that can be extracted via the agent’s control of information. Meanwhile, the recommendation to stop when the two stages of the project are completed incentivizes the principal without reducing the total surplus generated. When $\kappa \leq \tilde{\kappa}(T, \lambda)$, a partially informative policy that discloses only the completion of the second stage provides sufficient incentives to the principal, and thus the agent uses it.

I proceed with obtaining an investment schedule that not only satisfies the conditions in Lemma 5 and solves the relaxed problem, but also satisfies the full system of the principal’s individual rationality constraints in Lemma 3. Ensuring both is non-trivial. For instance, consider a mechanism that implements an investment schedule solving the agent’s relaxed problem and assume it recommends to continue for $t \in [0, S^*)$, then at $S^*$ recommends stopping if the second stage is already completed, but recommends to continue at all the subsequent dates $t \in (S^*, T]$. A no stopping recommendation drawn at $S^*$ reveals that the state is either 0 or 1. Clearly, after sufficient time passes after $S^*$, the principal would attach a high probability to the second stage already being completed and would

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24 The connection between an investment schedule $\tau$ and a direct recommendation mechanism implementing the schedule $\tau$ is simple: whenever, based on the evolution of the state process, $\tau$ suggests stopping the funding, the direct recommendation mechanism sends the message $m = 0$.

25 The “leading on” information policy in Ely and Szydlowski (2020) is similar: the only information that the policy provides is that the task is already completed and, thus, it is time to stop investing.
potentially be tempted to deviate from the recommendation to continue.\footnote{In other words, $V_t(\tau)$ drifts down over time and can get negative at some date.} However, a \textit{direct recommendation mechanism that implements an optimal investment schedule exists}. I present it in Proposition 2.

**Proposition 2.** Assume $\kappa \in (\kappa^{ND}(T, \lambda), \tilde{\kappa}(T, \lambda)]$. The optimal mechanism does not provide a recommendation to stop during $t \in [0, S^*)$. At $t = S^*$, if the second stage of the project is already completed, then the mechanism recommends the principal to stop. If the second stage of the project is not yet completed, then the mechanism recommends the principal to stop at the moment of its completion $t = \tau_2$. Formally,

$$
\tau = S^* \lor (\tau_2 \land T),
$$

where $S^*$ is chosen such that $V(\tau) = V^{NI}$, i.e., the respective constraint in the system of principal’s individual rationality constraints is binding.

The recommendation mechanism starting from $S^*$ generates recommendations to stop if the second stage is completed. As the recommendation to stop comes immediately at the completion of the second stage for all $t > S^*$, hearing no recommendation to stop reveals that the state is either 0 or 1. Further, as time goes on, the principal attaches a higher and higher probability to the state being 1, which ensures obedience to the recommendation to continue at each date. Further, the start of information provision $S^*$ is sufficiently postponed to ensure that the principal’s individual rationality constraint is binding either at $t = \bar{S}^{NI}$ or at $t = 0$.

The choice of $S^*$ is driven by extraction of the principal’s surplus and depends on $\kappa$ in an intuitive way. First, consider the case $\kappa \in (\kappa^{ND}, \kappa^{NI}(T, \lambda)]$, the principal is willing to start investing and invests until $t = \bar{S}^{NI}$ in the no-information benchmark. The agent’s optimal choice is to set $S^* > \bar{S}^{NI}$. Given such an information policy, the principal does not stop at $\bar{S}^{NI}$, the date of stopping in the no-information benchmark, and with probability one continues to invest during $t \in [\bar{S}^{NI}, S^*)$ even though the mechanism provides absolutely no information for all $t < S^*$. This is driven by the fact that the expected benefit from stopping at some future date $t \in [S^*, T]$ and obtaining the project payoff $v$ with certainty compensates the flow losses of investing during $t \in [\bar{S}^{NI}, S^*)$.\footnote{Similarly to the “leading on” information policy in Ely and Szylkowski (2020), the promises of future disclosure of the completion of the project are used as a “carrot” to make the receiver continue investing beyond the point at which he stops in the no-information benchmark.} Further, the agent sufficiently postpones $S^*$ to ensure that she extracts the principal’s surplus and the principal gets precisely $V^{NI} \geq 0$. 

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In the case $\kappa \in (\kappa^{NI}(T,\lambda), \bar{\kappa}(T,\lambda)]$, the principal is not willing to start in the no-information benchmark as his expected payoff from investing is negative. Thus, the agent chooses $S^*$ to guarantee that the principal gets his reservation value $V^{NI} = 0$ and is thus willing to start investing at $t = 0$. The value of $S^*$ is relatively lower as compared to the previous case: as the project is less attractive, to provide the principal sufficient incentives, the agent needs to start the information provision regarding the completion of the project earlier.

Finally, there exist many information policies that both solve the relaxed agent’s problem and satisfy the full system of constraints (7). This constitutes an important advantage for the agent: she can choose an optimal policy that is easier to implement from the real-world perspective, depending on the particular environment. In the optimal mechanism from Proposition 2, the recommendation to stop at some date $t$ depends only on the current state of the world $x_t$. In an alternative delayed disclosure mechanism, the recommendation to stop arrives with a pre-specified delay after the second stage was completed. Thus, the recommendation depends only on the past history and not on the current state of the world. In an optimal delayed disclosure mechanism, the delay becomes smaller as more time passes. I characterize such a mechanism in Appendix D.28

Recall that, as Lemma 5 suggests, the key idea of the optimal information policy is that the agent postpones the disclosure of the completion of the project to extract more surplus, which makes the principal’s individual rationality constraint bind. The higher the cost-benefit ratio of the project $\kappa$ becomes, the higher additional value the agent’s information policy needs to provide to the principal to ensure that his active individual rationality constraint is satisfied. The implication of this for the optimal information policy is presented in Lemma 6.

**Lemma 6.** Assume $\kappa \in (\kappa^{ND}(T,\lambda), \bar{\kappa}(T,\lambda)]$. Given the recommendation mechanism implementing an optimal investment schedule $\tau$, for a fixed Poisson rate $\lambda$, the expected length of investment $E[\tau]$ decreases in the cost-benefit ratio $\kappa$.

The intuition is that the higher the cost-benefit ratio of the project becomes, the sooner after the second stage of the project is completed the agent recommends the principal to stop. For the cost-benefit ratio as high as $\bar{\kappa}(T,\lambda)$, the agent provides the recommendation to stop immediately at the date of completion of the second stage of the project. Further, for $\kappa > \bar{\kappa}(T,\lambda)$, the optimal information

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28The rich set of optimal direct recommendation mechanisms in my model encompasses both mechanisms in which the information provision depends only on the current state, similarly to the optimal mechanism in Ely and Szydlowski (2020), and the mechanisms that use delay, similarly to the delayed beep from Ely (2017).
policy satisfying the conditions in Lemma 5 ceases to be individually rational for the principal. As I show in the next Section, for $\kappa > \kappa(T, \lambda)$, in addition to immediate disclosure of the project completion, the agent provides the information regarding the completion of the first stage of the project.

5.3 Immediate disclosure of completion and an interim deadline

When $\kappa > \kappa(T, \lambda)$, the project is not promising for the principal and any investment schedule in which stopping occurs after $\tau_2 \wedge T$ with probability one violates the principal’s individual rationality constraint. In other words, from the ex ante perspective the future reports disclosing only the completion of the project do not motivate the principal to start investing. Thus, an investment schedule that provides an individually rational expected payoff to the principal should assign a positive probability not only to stopping after the completion of the project, but also to stopping in either state 0, when no stages of the project are completed, or state 1, when only the first stage of the project is completed. I present the necessary conditions for an investment schedule to be optimal when the project is not promising in Lemma 7.

**Lemma 7.** Assume $\kappa \in (\kappa(T, \lambda), \kappa_{FI}(T, \lambda))$. If an investment schedule $\tau$ solves agent’s problem, then it satisfies the conditions

1. conditional on no completed stages of the project, stopping of the funding happens with a positive probability;
2. conditional on one completed stage of the project, stopping of the funding happens with probability zero;
3. conditional on two completed stages of the project, stopping of the funding happens immediately (at $t = \tau_2$) and with probability one.

Stopping when only the first stage of the project is already completed is clearly inefficient. In state 1, the principal prefers to continue investing until the completion of the second stage and this principal’s incentive to wait is aligned with the agent’s incentive to postpone the stopping. Further, stopping in state 1 does not help overcome the problem of the violated individual rationality constraint under $\kappa > \kappa(T, \lambda)$. Meanwhile, assigning a positive probability to stopping when no stages are completed helps to overcome the problem of violated individual rationality constraint, as the principal benefits from stopping at some date $t$ when the
first stage of the project is not yet completed and the project deadline $T$ is sufficiently close. Further, the agent clearly prefers the stopping of funding after the completion of the second stage rather than in state 0 as the former does not harm the total surplus generated. Thus, a schedule that is optimal assigns probability 1 to immediate stopping when the second stage is completed.

Lemma 7 implies that in an investment schedule, optimal for the agent, stopping after the completion of the second stage of the project happens immediately and stopping also happens given that no stages of the project are completed - i.e., at the interim deadline chosen by the agent, which I denote by $S_0^A$. Thus, Lemma 7 drastically simplifies the strategy space in the agent’s design problem: it is only left to characterize the optimal interim deadline. At the outset of the game, the agent designs a device that privately randomizes over the interim deadlines $S_0^A$. That is, the agent publicly chooses a distribution $F_{S_0^A}$, then an interim deadline is drawn according to it and privately observed by the agent. Next, the information starts to flow. The action recommendation to stop the funding satisfies the following investment schedule

$$
\tau = \begin{cases}
S_0^A, & \text{if } x_{S_0^A} = 0 \\
\tau_2 \wedge T, & \text{otherwise},
\end{cases}
$$

(11)

where the principal knows only the distribution $F_{S_0^A}$, but not the draw of $S_0^A$.

Given that the completion of the second stage of the project is disclosed immediately, stopping at the interim deadline in state 0 leads to a loss of expected further investment flow for the agent, and a potential savings from abandoning a “stagnating” project for the principal. The agent’s payoff can be without loss of generality restated as the expected loss in future investment due to stopping at the interim deadline $S_0^A$ in state 0 (rather than at $\tau_2 \wedge T$). Given this, the agent’s problem can be expressed as

$$
\min_{F_{S_0^A}} \mathbb{E}_{F_{S_0^A}} \left[ \mathbb{P} \left( x_{S_0^A} = 0 \right) \mathbb{E} \left[ \tau_2 \wedge T - S_0^A | x_{S_0^A} = 0 \right] \right],
$$

(12)

subject to the system of the principal’s individual rationality constraints, which also have a natural interpretation as the expectation of principal’s savings on the future investment, which discontinues at $S_0^A$ in state 0, minus the loss in the project completion profit due to stopping the funding at $S_0^A$ in state 0.\(^{29}\)

Inspecting the agent’s expected loss in future investment in (12) reveals that if the agent postpones the interim deadline $S_0^A$, then two effects arise. First, the

\(^{29}\)The principal’s long-run payoff is given in (44).
probability that stopping at the interim deadline will happen decreases. Second, the expected loss in total surplus due to stopping at the interim deadline rather than at $\tau_2 \land T$ decreases. Thus, the agent’s expected loss in future investment is decreasing in the date of interim deadline and the agent prefers an interim deadline with a later expected date.

Agent’s choice of the interim deadline distribution $F_{S_0^A}$ is affected by the two factors. First, as the expected loss in future investment in (12) is decreasing and convex in the date of the interim deadline, and thus the agent is risk-averse with respect to random interim deadlines. Thus, given some random interim deadline, the agent directly benefits from inducing a mean-preserving contraction. Second, the agent benefits from inducing a mean-preserving contraction indirectly. Inspecting the principal’s long-run payoff for some fixed $S_0^A$ reveals that the principal is also risk-averse with respect to random interim deadlines. Thus, inducing a mean-preserving contraction makes the principal better-off and relaxes the individual rationality constraint at $t = 0$, hence, allowing the agent to postpone the expected interim deadline further. As a result the optimal for the agent interim deadline takes the form of a deterministic date. In other words, it is optimal for the agent to publicly announce the interim deadline $S_0^A$ at the outset, so that the principal knows it.

The agent has an incentive to postpone the interim deadline and uses her control of the information environment to postpone the deadline as much as possible so that the principal’s individual rationality constraint at $t = 0$ binds. Figure 5 demonstrates the principal’s long-run payoff as a function of the interim deadline, which I denote by $S_0$. It is maximized at the principal-preferred interim deadline $S_0^P$, which was characterized in Lemma 2. The agent-preferred interim deadline $S_0^A$ yields the principal’s expected payoff of 0.

![Figure 5: Principal’s long-run payoff, $V$, as a function of an interim reporting deadline chosen by the agent, $S_0$.](image)
The next Proposition summarizes the optimal investment schedule, which can be without loss of generality implemented using a direct recommendation mechanism:

**Proposition 3.** Assume \( \kappa \in (\tilde{\kappa}(T, \lambda), \kappa^{FI}(T, \lambda)) \). The optimal information policy is given by a direct recommendation mechanism that generates

(a) the recommendation to stop at the moment of completion of the second stage of the project, \( t = \tau_2 \), and

(b) a conditional recommendation to stop at the publicly announced interim deadline \( t = S^A_0 \). At \( S^A_0 \), stopping is recommended with certainty if the first stage of the project has not yet been completed.

Formally,

\[
\tau = \begin{cases} 
S^A_0, & \text{if } x_{S^A_0} = 0 \\
\tau_2 \land T, & \text{otherwise},
\end{cases}
\]

where \( S^A_0 \) is chosen so that the principal’s individual rationality constraint at \( t = 0 \) is binding, i.e., \( V(\tau) = 0 \).

A stopping recommendation at any date other than the interim deadline \( t = S^A_0 \) fully reveals that project is accomplished. Further, observing a recommendation to stop at the interim deadline, the principal learns that the milestone of the project has not yet been reached and becomes sufficiently pessimistic that the project will be completed by \( T \).

A notable feature of the optimal information policy when the project is ex ante unattractive is its uniqueness. The only optimal instrument through which the agent fine tunes the incentive provision to the principal is the choice of interim deadline, and there is a unique optimal way to set the deadline to make the principal’s individual rationality constraint bind.

I proceed by considering the comparative statics of the interim deadline. Both the agent-preferred and the principal-preferred interim deadline, \( S^A_0 \) and \( S^P_0 \), respectively, increase in the exogenous deadline \( T \). This is because less time pressure relaxes the principal’s individual rationality constraint and allows the agent to postpone the deadline further in order to extract the principal’s surplus.

As the cost-benefit ratio increases up to \( \kappa^{FI} \), the agent-preferred deadline converges to the principal-preferred deadline. An increase in the cost-benefit ratio of the project makes the principal’s individual rationality constraint tighter.\(^{30}\) As a

---

\(^{30}\)This is because the increase in \( \kappa \) makes the principal’s instantaneous benefit from waiting decrease, and the normalized instantaneous cost of waiting becomes higher.
result, for a higher $\kappa$, in the absence of completion of the first stage, the principal is willing to wait for a shorter time before stopping. Thus, both the interim deadline preferred by the principal $S_0^P$ and the interim deadline chosen by the agent $S_0^A$ are lower for a higher $\kappa$. Further, for a higher $\kappa$ the agent has to choose an information policy relatively closer to the full-information benchmark to ensure that the individual rationality constraint at $t = 0$ is satisfied. Hence, the agent-chosen interim deadline $S_0^A$ approaches $S_0^P$, the interim deadline preferred by the principal. The comparative statics of $S_0^P$ and $S_0^A$ with respect to the cost-benefit ratio of the project $\kappa$ are presented in Figure 6.

![Figure 6: Interim deadline chosen by the agent $S_0^A$ (dashed) and preferred by the principal $S_0^P$ (thick), as functions of the cost-benefit ratio of the project $\kappa$.](image)

6 General preferences

In this Section, I allow for profit-sharing between the agent and the principal, varying degree of the agent’s benefit from the flow of funds, and exponential discounting, and demonstrate that the optimal information policy still has the same properties as in the baseline model.

First, I assume that the agent and the principal share the project completion profit $v$: the principal gets $\alpha \cdot v$, while the agent gets $(1 - \alpha) \cdot v$, $\alpha \in (0, 1]$. Thus, now the agent benefits not only from the flow of funds provided by the principal for running the project but also from the share in the profit. The assumption that the agent gets a share in the project completion profit is natural in many situations. In particular, the research documents that the entrepreneurs in innovative startups are up to some extent driven by giving vent to their entrepreneurial mindset and
bringing their innovative ideas to life (Gundolf et al., 2017). In such a context, a positive profit share of the agent captures that the agent is motivated by the success of the project.

Second, I assume that given a flow cost of $c$ incurred by the principal, the agent obtains a flow benefit $\beta c, \beta \geq 0$. $\beta$ can be interpreted as the agent’s marginal benefit from using the funds provided by the principal for funding the project. Alternatively, for $\beta \in [0,1]$ the loss of $1 - \beta$ of the amount of the transfer at each date can be interpreted as the transaction costs. Finally, setting $\beta = 0$ for some $\alpha < 1$ allows for abstracting from the agent’s motives for diverting the funds and considering the agent motivated only by the success of the project.

Third, I allow for exponential discounting at a rate $r > 0$. Thus, the present value of a profit obtained at a date $t$ is given by $ve^{-rt}$ and the present value of a stream of funding up to date $t$ is given by $\frac{1}{r} (1 - e^{-rt}) c$. The following Proposition demonstrates that given the more general preference specification, the structure of the optimal disclosure, present in the baseline model, preserves.

Proposition 4.

(a) When the cost-benefit ratio of the project is low, $\kappa \leq \tilde{\kappa}(T, \lambda, r, \alpha)$, the optimal investment schedule $\tau$ satisfies $\tau \geq \tau_2 \wedge T$, i.e., the agent recommends the principal to stop only after the completion of the second stage of the project.

(b) When $\kappa > \tilde{\kappa}(T, \lambda, r, \alpha)$, the optimal investment schedule $\tau$ assigns positive probability both to the stopping in state 2 and state 0, i.e., the agent not only discloses the completion of the second stage of the project, but also specifies an interim deadline for the completion of the first stage.

Similarly to the baseline model, allowing the principal to stop after the project completion brings profit to the principal and thus leads to a relatively higher total surplus, which the agent can extract. Meanwhile, allowing the principal to stop at the interim deadline does not increase total surplus and serves solely as an expected payoff transfer from the agent to the principal. To see that, note that stopping when the first stage of the project is still incomplete allows the principal to save on the further costs of funding the project when over time the project proves to be “unsuccessful”. This can not be beneficial for the agent as she does not internalize the costs of running the project. Further, stopping at the interim deadline is strictly detrimental for the agent as she strictly prefers the principal to postpone the stopping of funding when no stages of the project are completed.\(^{31}\)

\(^{31}\)The probability of project success and stock of obtained funds are non-decreasing in the
When the project is sufficiently ex-ante attractive, the agent can motivate the principal to start funding the project without promising to stop the stagnant project at the interim deadline, and this is strictly beneficial for the agent. Thus, when the project is promising, the agent sets no interim deadlines, which in expectation gives her more funds and more experimentation for free.

7 Conclusion

A transparent flow of information is crucial for the successful management of any innovative project. However, the researcher, who controls the information on the progress of the project, often tends to have different motives than the investor. This leads to the question of how a researcher chooses the transparency of the flow of information about the progress of a project in order to manipulate the investor’s funding decisions. I address this question in a dynamic information design model in which the agent commits to providing information to the principal with an incentive to postpone the principal’s irreversible stopping of the funding.

I contribute to the dynamic information design literature by studying the problem of the dynamic provision of information regarding the progress of a multistage project, which evolves endogenously over time and needs to be completed before a deadline. I show that the agent’s choice of which pieces of information to provide and when depends on the project being either ex ante attractive for the principal or not. In the case of a promising project, the agent provides only the good news that the project is completed and postpones the reports. In the case of an unattractive project, to motivate the principal to start funding the project the agent not only reports the completion of the project, but also helps the principal to find out when the project stagnates. To achieve this, the agent announces an interim deadline for the project – a certain date at which she recommends the principal to cut the funding of the project if the milestone of the project has not been reached.

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Appendix

A Notational conventions

The state process is given by \( x_t, \forall t \in \mathbb{R}_+ \), defined on the probability space \((\Omega, \mathcal{F}, P), t \in \mathbb{R}_+\). Its natural filtration is denoted by \( F = (\mathcal{F}_t)_{t \geq 0} \). Throughout Appendices B and C, the following notational conventions are used:

1. I denote the random time at which the \( n \)th stage of the project is completed by \( \tau_n \). Formally, \( \tau_n \in \mathbb{R}_+ \) is a continuously distributed random variable that represents the first hitting time of \( x_t = n \).

2. Consider some stopping time \( \tau \). Whenever \( "\tau" \) stands as a term in an inequality, it stands for a realization of the stopping time \( \tau \) and it should be read as “for each \( \omega \in \Omega \) and corresponding \( \tau(\omega) \”).

   Example 1. \( "\tau_2 \land T \geq \tau" \) should be read as “\( \tau_2(\omega) \land T \geq \tau(\omega) \), for all \( \omega \in \Omega \)”.

   Example 2. “for all \( t \in [S, \tau)" \) should be read as “for all \( t \in [S, \tau(\omega)) \), for all \( \omega \in \Omega \”.

3. The continuation value of the agent at time \( t \), given \( \tau \), and conditional on \( t < \tau \):

\[
W_t(\tau) := E[\tau - t| t < \tau] c.
\]

4. The total (continuation) surplus at time \( t \), given \( \tau \), and conditional on \( t < \tau \):

\[
SV_t(\tau) := W_t(\tau) + V_t(\tau) .
\]

5. Shorthand for posterior beliefs:

\[
q_n(t) := P(x_t = n| t < \tau), \\
r_n(t) := P(x_\tau = n| t < \tau).
\]

B The principal’s continuation value

Here I present the alternative formulation of the principal’s continuation value (6). This helps me to study some of its properties for further use in Appendix C. The continuation value of the principal at time \( t \) and given the investment
schedule \( \tau \) is given by (6). Postponing the stopping for \( \Delta t \) brings a benefit in the form of project completion payoff \( v \) iff the second stage of the project is completed within \( \Delta t \). As \( x_t \) follows the Poisson process, the probability of two increments in a very short time \( \Delta t \) is negligibly small. Thus, during some \( \Delta t \), the principal gets the project completion payoff \( v \) iff the second stage of the project is completed within \( \Delta t \). Thus, postponing the stopping for \( \Delta t \) brings the principal \( v \) with probability \( \lambda q_1(t) \Delta t \). The second stage is not completed within \( \Delta t \) with the complementary probability of \( 1 - \lambda q_1(t) \Delta t \). The principal’s continuation value is thus given by

\[
V_t(\tau) = (v \lambda q_1(t) - c) \Delta t + (1 - \lambda q_1(t) \Delta t) V_{t+\Delta t}(\tau) = v \lambda (q_1(t) - \kappa) \Delta t + (1 - \lambda q_1(t) \Delta t) V_{t+\Delta t}(\tau).
\]

Differentiating both sides w.r.t. \( \Delta t \) and considering \( \lim_{\Delta t \to 0} \) yields

\[
0 = v \lambda (q_1(t) - \kappa) - \lambda q_1(t) V_t(\tau) + \dot{V}_t(\tau),
\]

which, after rearranging becomes

\[
\dot{V}_t(\tau) = \lambda q_1(t) V_t(\tau) + v \lambda (\kappa - q_1(t)). \tag{13}
\]

### C Proofs

**Proof of Lemma 1.** The beliefs regarding the number of stages of the project completed by time \( t \), \( x_t \), evolve according to the Poisson process. The principal’s unconditional beliefs are given by \( p_0(0) = 1 \) and for any \( t \) such that the game still continues,

\[
\begin{align*}
\dot{p}_0(t) &= -\lambda p_0(t), \\
\dot{p}_1(t) &= \lambda (p_0(t) - p_1(t)), \\
\dot{p}_2(t) &= \lambda p_1(t),
\end{align*}
\]

where \( p_0(t) = e^{-\lambda t} \) and \( p_1(t) = \lambda t e^{-\lambda t} \), \( p_2(t) = 1 - p_0(t) - p_1(t) \). The principal’s problem is given by

\[
\max_{S \in [0,T]} \{ v \cdot p_2(S) - c \cdot S \}. \tag{15}
\]

I start with analyzing the choice of \( S \) for the interior solution case, \( S \in (0,T) \). F.O.C. for (15) is given by

\[
v \cdot \dot{p}_2(S) = c, \tag{16}
\]

or, equivalently, \( p_1(S) = \kappa \). There are two values satisfying (16): \( \bar{S} \) and \( \bar{S}^{NI} \), \( \bar{S} < \bar{S}^{NI} \). At each \( t \in (\bar{S}, \bar{S}^{NI}) \) the principal receives a net positive payoff flow.
Thus, stopping at $\bar{S}$ is not optimal and the only candidate for optimal stopping is $\bar{S}^{NI}$. Further, one can obtain the closed form expression for the interior stopping time $\bar{S}^{NI}$ from (16):

$$\bar{S}^{NI} = -\frac{1}{\lambda} W_{-1}(-\kappa),$$

(17)

where $W_{-1}(x)$ denotes the negative branch of the Lambert $W$ function. $\bar{S}^{NI}$ is well-defined for any $\kappa < e^{-1}$.

Thus, the solution to (15) could potentially be $0, \bar{S}^{NI}$, or $T$. I proceed with a useful lemma.

**Lemma 8.** The following is true regarding the principal’s continuation value in the no-information benchmark, $V^{NI}_t$: if $V^{NI}_t \geq 0$, for some $t \in [0, \bar{S}^{NI} \wedge T]$, then $V^{NI}(s) \geq 0$, for all $s \in [t, \bar{S}^{NI} \wedge T]$.

**Proof.** The principal’s continuation value in the no-information benchmark is given by

$$V^{NI}_t = \left[p_2(T \wedge \bar{S}^{NI}) - p_2(t)\right] v - \left(T \wedge \bar{S}^{NI} - t\right) c.\tag{18}$$

Further,

$$V^{NI}_t = v\lambda \left(\kappa - e^{-\lambda t}\right) = v\lambda (\kappa - p_1(t)).$$

$p_1(t) \leq \kappa$ for all $t \in [0, \bar{S}]$ and $p_1(t) \geq \kappa$ for all $t \in [\bar{S}, \bar{S}^{NI} \wedge T]$. Thus, $V^{NI}_t$ increases for $t \in [0, \bar{S}]$, decreases for $t \in [\bar{S}, T \wedge \bar{S}^{NI}]$, and $V^{NI}_t(T \wedge \bar{S}^{NI}) = 0$, which establishes the result. \(\Box\)

Lemma 8 implies that if $V^{NI}_0 \geq 0$ and the principal chooses to opt in at $t = 0$, then $V^{NI}_t \geq 0$, $t \in [0, \bar{S}^{NI} \wedge T]$, i.e., he invests until $t = T \wedge \bar{S}^{NI}$. This implies that the solution to (15) is either $T \wedge \bar{S}^{NI}$ or 0.

Finally, at $t = 0$ the principal chooses to start investing or not. The condition for the principal to start investing at $t = 0$ is given by

$$V^{NI}_t \geq 0.$$  

(19)

To specify the set of parameters for which (19) is satisfied, I obtain the cutoff value of $\kappa$ given $T$ and $\lambda$. Such a parameterization is intuitive: $\kappa$ above the cutoff level corresponds to a project with sufficiently high normalized cost-benefit ratio and implies that the principal does not opt in. I denote this cutoff by $\kappa^{NI}(T, \lambda)$. This solves (19) holding with equality. Two cases are possible.

$^{32}\bar{S}$ is a local minimum of the objective.
Case 1: $T \leq \bar{S}^{NI} \iff T \leq -\frac{1}{\lambda} \mathcal{W}_{-1}(-\kappa)$. This inequality is satisfied when either $\frac{1}{\lambda} > T$ or $\kappa \leq e^{-\lambda T} \lambda T$.

Given $T \leq \bar{S}^{NI}$, (19) holding with equality becomes

$$p_2(T) v - Tc = 0.$$  

Solving it for $\kappa$ yields

$$\kappa = e^{-\lambda T} \left( \frac{e^{\lambda T} - 1}{\lambda} \right).$$

Case 2: $T > \bar{S}^{NI}$. This inequality is satisfied when $\frac{1}{\lambda} \leq T$ and $\kappa > e^{-\lambda T} \lambda T$.

Given $T > \bar{S}^{NI}$, (19) holding with equality becomes

$$vp_2(\bar{S}^{NI}) - c\bar{S}^{NI} = 0 \iff v \left( 1 - p_0(\bar{S}^{NI}) - p_1(\bar{S}^{NI}) \right) = c\bar{S}^{NI},$$

where (recall that $\hat{p}_2(\bar{S}^{NI}) = \frac{\xi}{v}$)

$$p_0(\bar{S}^{NI}) = \frac{1}{\lambda^2 S^{NI}} p_2(\bar{S}^{NI}) = \frac{c}{\lambda^2 S^{NI} v} = \frac{\kappa}{\lambda S^{NI}}$$

and

$$p_1(\bar{S}^{NI}) = \frac{1}{\lambda} \hat{p}_2(\bar{S}^{NI}) = \frac{c}{\lambda v} = \kappa.$$

Consequently,

$$vp_2(\bar{S}^{NI}) - c\bar{S}^{NI} = v - v \cdot \kappa \left( 1 + \lambda \bar{S}^{NI} + \frac{1}{\lambda S^{NI}} \right).$$

Let $y := \lambda \bar{S}^{NI}$. Note that, by definition, $y > 1$. Then $\kappa = ye^{-y}$, and so

$$\left( vp_2(\bar{S}^{NI}) - c\bar{S}^{NI} \right) / v = 1 - e^{-y} \left( 1 + y + y^2 \right).$$

It follows that $V^{NI}(0)$ is nonnegative whenever $\lambda S^{NI} \geq y_0 \doteq 1.79328$, which is equivalent to

$$\kappa \leq \kappa_0 \doteq 0.298426.$$

Finally, putting the two cases together yields

$$\kappa^{NI}(T, \lambda) = \begin{cases} 
\kappa_0 \doteq 0.298426, & \text{if } \frac{1}{\lambda} \leq T \text{ and } \kappa \geq e^{-\lambda T} \lambda T \\
e^{-\lambda T} \left( \frac{e^{\lambda T} - 1}{\lambda} \right), & \text{otherwise.}
\end{cases}$$  

(20)

Proof of Lemma 2. The principal chooses $a_t \in \{0, 1\}$ sequentially given the observed realizations of $x_t \in \{0, 1, 2\}$. Whenever the principal observes $t = \tau_2$, he immediately chooses $a_t = 0$ and gets $v$.

Consider the case $x_t = 1, t < T$, i.e., the first stage of the project has already been completed. As $x_t$ follows a Poisson process, in expectation it would take
\[ \lambda \] units of time for the second stage to be completed and its completion brings the principal the value of \( v \). Thus, the necessary and sufficient condition for the principal to invest at \( t \) when \( x_t = 1, t < T \) is given by

\[ v - c \cdot \frac{1}{\lambda} \geq 0 \iff \kappa \leq 1 \]

Assume that \( \kappa \leq 1 \) holds and \( x_t = 1 \); thus, the principal chooses to invest at \( t \). In that case, the principal invests until \( \tau_2 \lor T \). To see this, recall that the only news that the principal can receive given \( x_t = 1, t < T \) is the completion of the second stage of the project, \( \tau_2 \), which leads to immediate stopping. At each \( t < \tau_2 \lor T \) such that \( x_t = 1 \), choosing \( a_t = 0 \) yields an instantaneous expected payoff of 0, while choosing \( a_t = 1 \) yields an instantaneous expected payoff of \( \lambda v \Delta t - c \Delta t \).

Thus, \( \kappa \leq 1 \) suffices for the principal to invest until \( \tau_2 \lor T \).

Consider now the case of \( x_t = 0, t < T \), i.e., no stages of the project have yet been completed. Postponing the stopping for \( \Delta t \) brings the instantaneous expected payoff of \( V_{t_1}^{FI} \lambda \Delta t - c \Delta t \), where \( V_{t_1}^{FI} \) is the principal’s continuation value at time \( t \) under full information, conditional on the completion of the first stage of the project. I proceed with obtaining the expression for \( V_{t_1}^{FI} \). By definition, the principal gets \( v \) whenever the second stage is completed not later than \( T \). The principal invests until \( \tau_2 \lor T \), and knows that at \( t \) the first stage of the project is already completed; thus, \( V_{t_1}^{FI} \) is given by

\[ V_{t_1}^{FI} = v \mathbb{P}(\tau_2 \leq T| x_t = 1) - c \mathbb{E}[\tau_2 \lor T - t| x_t = 1] \cdot x_t = 1 \text{ corresponds to the time between two consecutive Poisson arrivals, and thus has exponential distribution. First, consider } \mathbb{P}(\tau_2 \leq T| x_t = 1):

\[ \mathbb{P}(\tau_2 \leq T| x_t = 1) = 1 - e^{-\lambda(T-t)} \]

Next, consider \( \mathbb{E}[\tau_2 \lor T - t| x_t = 1] \):

\[ \mathbb{E}[\tau_2 \lor T - t| x_t = 1] = \mathbb{P}(\tau_2 \leq T| x_t = 1) \int_t^T z \cdot \frac{\lambda e^{-\lambda(z-t)}}{\mathbb{P}(\tau_2 \leq T| x_t = 1)} dz + \mathbb{P}(\tau_2 > T| x_t = 1) T - t \]

\[ = \frac{1}{\lambda} \left( 1 - e^{-\lambda(T-t)} \right) + t - e^{-\lambda(T-t)}T + \mathbb{P}(\tau_2 > T| x_t = 1) T - t \]

\[ = \frac{1}{\lambda} \left( 1 - e^{-\lambda(T-t)} \right) \] (21)

Thus,

\[ V_{t_1}^{FI} = v \left( 1 - e^{-\lambda(T-t)} \right) - c \frac{1}{\lambda} \left( 1 - e^{-\lambda(T-t)} \right) \]

\[ = \left( v - \frac{c}{\lambda} \right) \left( 1 - e^{-\lambda(T-t)} \right) . \] (22)
From (22) one observes that $V_{t1}^{FI}$ decreases in $t$. If the net instantaneous benefit given by $V_{t1}^{FI} \lambda \Delta t - c \Delta t$ gets as low as 0 at some $t$, then the principal chooses to stop investing at this $t$. I denote the time at which the net instantaneous benefit reaches 0 by $S_0^P$. $S_0^P$ can be obtained from $(\lambda V_{1}^{FI} (S_0^P) - c) \Delta t = 0$. Thus,

$$S_0^P = T + \frac{1}{\lambda} \log \left( \frac{1 - 2\kappa}{1 - \kappa} \right).$$

(23)

The principal is willing to start investing iff at $t = 0$ the expected payoff from investing at $t = 0$ covers the costs of investing, i.e. $(\lambda V_{1}^{FI} (0) - c) \Delta t \geq 0$. From (23), this corresponds to $S_0^P \geq 0$. I denote the upper bound on the cost-benefit ratio $\kappa$ such that the principal chooses to start investing in $t = 0$ under full information by $\kappa^{FI} (T, \lambda)$, I solve $S_0^P = 0$ for $\kappa$ and obtain

$$\kappa^{FI} (T, \lambda) = \frac{1 - e^{-\lambda T}}{2 - e^{-\lambda T}}.$$  

(24)

In summary, under full information, if $\kappa \leq \kappa^{FI} (T, \lambda)$, then the principal starts investing at $t = 0$. Further, he stops at $S_0^P$ if the first stage of the project has not been completed by that time. Otherwise, he proceeds to invest until $\tau_2 \wedge T$.

\[
\square
\]

**Proof of Proposition 1.** I provide the proof for each of the four parametric cases below.

1. The case of $\kappa \leq \kappa^{ND} (T, \lambda)$.

$\kappa^{ND} (T, \lambda)$ is defined as follows: for any $\kappa \leq \kappa^{ND} (T, \lambda)$, the principal invests until $T$ in the no-information benchmark. From Lemma 1, if the principal is willing to start investing, i.e., $\kappa \leq \kappa^{NI} (T, \lambda)$, then

$$S^{NI} = \overline{S}^{NI} \wedge T.$$  

For the sake of instruction, below I consider relaxing the Assumption 1 and demonstrate how the relation between $\kappa^{ND} (T, \lambda)$ and $\kappa^{NI} (T, \lambda)$ changes between Case a (relaxed Assumption 1) and Case b (Assumption 1 holds).

Case a. $e^{\lambda T} \leq \lambda T (\lambda T + 1) + 1$. In this case, whenever the principal is willing to start investing in the no-information benchmark, she invests until $T$, i.e., $\kappa^{ND} (T, \lambda) = \kappa^{NI} (T, \lambda)$, where $\kappa^{NI} (T, \lambda)$ is given by (20). To see that, first, consider the extreme sub-case in which $T < \frac{1}{\lambda}$. As $-\lambda \overline{S}^{NI}$ must belong to $-1$ axis of Lambert $W$ function, it has a lower bound corresponding to $\frac{1}{\lambda}$. Thus, $T < \overline{S}^{NI}$ for any $\kappa (T, \lambda)$. Second, consider $\lambda T \in [1, \lambda T]$, where $\lambda T$ solves $e^{\lambda T} = \lambda T (\lambda T + 1) + 1$. In this case, from (17), if $\kappa (T, \lambda) \leq e^{-\lambda T} (\lambda T) \geq e^{-\lambda T} \lambda T$, respectively), then $T \leq \overline{S}^{NI} (T \geq \overline{S}^{NI}$, respectively). However, $\kappa^{NI} (T, \lambda) \leq e^{-\lambda T} \lambda T$. Thus, $\kappa^{ND} (T, \lambda) = \kappa^{NI} (T, \lambda)$.  

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Case b. $e^{\lambda T} > \lambda T (\lambda T + 1) + 1$. As before, it holds that if $\kappa(T, \lambda) \leq e^{-\lambda T} \lambda T$ ($\kappa(T, \lambda) \geq e^{-\lambda T} \lambda T$), then $T \leq S^{NI}$ ($T \geq S^{NI}$, respectively). Denote 

$$
\kappa^{ND}(T, \lambda) := e^{-\lambda T} \lambda T.
$$

As $\kappa^{NI}(T, \lambda) > \kappa^{ND}(T, \lambda)$, two cases emerge. If $0 < \kappa \leq \kappa^{ND}(T, \lambda)$, then $T \leq S^{NI}$, and from $\kappa \leq \kappa^{NI}(T, \lambda)$, it holds that $S^{NI} = T$ and as the agent does not strictly benefit from disclosing any information, she chooses non-disclosure. If $\kappa > \kappa^{ND}(T, \lambda)$, then $T > S^{NI}$ and the agent can potentially benefit from information disclosure.

2. The case of $\kappa^{ND}(T, \lambda) < \kappa \leq \kappa(T, \lambda)$.

The result is established in Proposition 2.

3. The case of $\kappa(T, \lambda) < \kappa < \kappa^{FI}(T, \lambda)$.

The result is established in Proposition 3.

4. The case of $\kappa \geq \kappa^{FI}(T, \lambda)$.

The principal’s long-run payoff in the full-information benchmark non-positive. Thus, the agent can not strictly benefit from information disclosure and chooses non-disclosure.

\[\square\]

Proof of Lemma 3. Necessity. Assume $V_t(\tau) < 0$ for some $t$. In that case, it is optimal for the principal to deviate to stopping at $t < \tau$. Thus, there is no information policy $\sigma$, for which this $\tau$ is the principal’s best reply. Assume $V^{NI}_\tau \geq 0$. Thus, the principal deviates to stopping at $t > \tau$, and there is no $\sigma$, for which this $\tau$ is the best reply.

Sufficiency. Assume (7) holds. $V_t(\tau) \geq 0$ for all $t < \tau$ implies that the principal prefers to continue rather than to stop the funding for all $t < \tau$. Thus, it can not be that case that the principal stops before $\tau$. Further, $V^{NI}_\tau < 0$ implies that, conditional on reaching the date of stopping $\tau$, it is better for the principal to stop immediately rather than to stop at $t > \tau$.

Consider a direct recommendation mechanism $\sigma$ with $M = \{0, 1\}$ such that whenever, based on the evolution of the state process, the considered investment schedule $\tau$ suggests stopping the funding, the direct recommendation mechanism sends the message $m = 0$ to the principal. As it is optimal for the principal to stop at $\tau$, $\tau$ is the principal’s best reply to $\sigma$.

\[\square\]

Proof of Lemma 4. Consider the recommendation mechanism immediately disclosing the completion of the second stage of the project; it is given by $\tau = \tau_2 \wedge T$. There exists such $\tilde{\kappa}(T, \lambda)$ that solves the principal’s binding $t = 0$ individual rationality constraint when $\tau = \tau_2 \wedge T$:
\[ V(\tau_2) = 0, \quad (25) \]

where

\[ V(\tau_2) = p_2(T) v - E[\tau_2 \wedge T] c \]

\[ = v \left(1 - e^{-\lambda T} - \lambda T e^{-\lambda T}\right) - c \frac{1}{\lambda} \left(2 - 2 e^{-\lambda T} - \lambda T e^{-\lambda T}\right). \quad (26) \]

The solution to equation (25) is given by

\[ \tilde{k}(T, \lambda) = \frac{1 - e^{\lambda T} + \lambda T}{2 - 2 e^{\lambda T} + \lambda T}. \quad (27) \]

Further, \( \kappa > \tilde{k}(T, \lambda) \Rightarrow V(\tau_2) < 0 \) and \( \kappa \leq \tilde{k}(T, \lambda) \Rightarrow V(\tau_2) \geq 0 \).

**Proof of Lemma 5.** Consider the case of \( \kappa \in (\kappa^{ND}(T, \lambda), \kappa^{NI}(T, \lambda)] \). The agent’s relaxed problem for this case has the individual rationality constraints only for \( t \in [0, \bar{S}^{NI}] \), and it is given by

\[ \max_{\tau \in \mathcal{T}} \left\{ c \cdot E[\tau] \right\} \]

s.t. \( V_t(\tau) \geq 0, \forall t \in \left[0, \bar{S}^{NI}\right] \),

where \( V_t(\tau) \) is given by (6) and \( \mathcal{T} \) is the set of stopping times with respect to the natural filtration of \( x_t \).

Consider the candidate investment schedule \( \tau \) such that \( \tau \geq \bar{S}^{NI} \lor (\tau_2 \wedge T) \) and \( V(\tau) = V^{NI} \), where \( V^{NI} \) is given by (3). I start with arguing that the candidate \( \tau \) satisfies the system of individual rationality constraints. From Lemma 1, given candidate \( \tau \), the principal invests until \( \bar{S}^{NI} \) with certainty and the constraints in (28) are satisfied for all \( t \in [0, \bar{S}^{NI}] \). Further, \( \tau \) implies that \( V_{\bar{S}^{NI}}(\tau) = 0 \), i.e., the individual rationality constraint at \( t = \bar{S}^{NI} \) is binding.

I proceed with arguing that the candidate \( \tau \) maximizes the agent’s objective function in (28). The agent’s objective can be WLOG written out as:

\[ W(\tau) = \underbrace{P(x_\tau = 2)}_{\text{total surplus}} v - \underbrace{V(\tau)}_{\text{principal’s surplus}}. \quad (29) \]

By Lemma 4, an investment schedule \( \tau \) that assigns probability one to \( \tau \geq \tau_2 \wedge T \) satisfies the individual rationality constraint at \( t = 0 \) in (28). Note that, given \( \tau \geq \tau_2 \wedge T \), the total surplus in (29) is given by \( P(x_T = 2) v \), i.e., total surplus achieves its upper bound determined by the exogenously given project deadline \( T \). The principal’s surplus in (29) is given by \( V(\tau) = V^{NI} \), i.e., principal’s surplus achieves its lower bound specified by (3). This can be seen from the principal’s decision problem, in which he best replies to an information policy \( \sigma \). As \( \sigma \) allows
the principal to condition his actions on the information regarding the evolution of the state process, the principal’s equilibrium payoff can not be lower than \(V^{NI}\), his equilibrium payoff when he is restricted to choosing actions without conditioning them on the information about the state process. Thus, \(\tau\) solves the relaxed problem (28).

Consider the case of \(\kappa \in (\kappa^{NI}(T, \lambda), \tilde{\kappa}(T, \lambda))\). The agent’s relaxed problem for this case has the individual rationality constraint only for the initial period, and it is given by

\[
\max_{\tau \in T} \{ c \cdot E[\tau]\}
\]

s.t. \(V(\tau) \geq 0\),

where \(V(\tau) = P(x_\tau = 2)v - E[\tau]c\).

Consider candidate investment schedule \(\tau\) such that \(\tau \geq \tau_2 \wedge T\) and \(V(\tau) = V^{NI}\). For such \(\tau\), agent’s expected payoff (29) is given by \(P(x_T = 2)v - V^{NI}\). As discussed for the parametric case \(\kappa \in (\kappa^{ND}(T, \lambda), \kappa^{NI}(T, \lambda))\], the first term is at its upper bound. To see that the second term is at its lower bound, note that, from Lemma 1, \(V^{NI} = 0\), and thus the individual rationality constraint in (30) is binding. Hence, \(\tau\) solves the relaxed problem (30).

**Proof of Proposition 2.** The proof covers the case \(\kappa \in (\kappa^{ND}(T, \lambda), \kappa^{NI}(T, \lambda))\] and the case \(\kappa \in (\kappa^{NI}(T, \lambda), \tilde{\kappa}(T, \lambda))\] separately.

1. **The case of \(\kappa \in (\kappa^{ND}(T, \lambda), \kappa^{NI}(T, \lambda))\).**

I start with proving the existence of \(S^*\) such that \(V(\tau) = V^{NI}\). Assume that \(S^* > \tilde{S}^{NI}\). For all \(t \in [\tilde{S}^{NI}, S^*)\), stopping never occurs, at \(t = S^*\) it occurs if \(x_{S^*} = 2\), and for all \(t \in (S^*, \tau)\) it occurs at \(t = \tau_2 \wedge T\). For \(t \in [S^*, \tau)\), the absence of stopping induces posteriors \(q_n(t)\). Further, for \(t \in [S^*, \tau)\) the principal discounts future benefits from postponing stopping using the probability of the state being 2. Hence, the continuation value at \(t = \tilde{S}^{NI}\) can be written as

\[
V_{\tilde{S}^{NI}}(\tau) = v\lambda \left( \int_{\tilde{S}^{NI}}^{S^*} p_1(z) dz + \int_{S^*}^{T} (q_1(z) - \kappa)(1 - P(x_z = 2)) dz \right). \tag{31}
\]

The principal’s long-run payoff is given by

\[
V(\tau) = \int_{0}^{\tilde{S}^{NI}} (v \cdot p_1(s) \lambda - c) ds + V_{\tilde{S}^{NI}}(\tau),
\]

where \(\int_{0}^{\tilde{S}^{NI}} (v \cdot p_1(s) \lambda - c) ds = V^{NI}\). Thus, to ensure that \(S^*\) makes the individual rationality constraint bind at \(t = \tilde{S}^{NI}\), i.e., \(V(\tau) = V^{NI}\), it is necessary and sufficient that \(V_{S^{NI}}(\tau) = 0\). Using (31), this equation can be written as

\[
\int_{\tilde{S}^{NI}}^{S^*} \kappa - p_1(z) dz = \int_{S^*}^{T} (q_1(z) - \kappa)(1 - P(x_z = 2)) dz.
\]
Let \( g(S) := \int_{S^{-}}^{S^+} \kappa - p_1(z) \, dz \) and \( k(S) := \int_{S}^{S^+} (q_1(z) - \kappa) (1 - P(x_z = 2)) \, dz \), \( S \in [\bar{S}^{NI}, \tau) \). \( q_1(t) \geq \kappa \), for all \( t \in [S^*, T) \). Thus, \( g(S^{NI}) = 0, k(S^{NI}) > 0 \).

Further, \( p_1(t) < \kappa \), for all \( t \in (S^{NI}, T] \). Hence, \( g(T) > 0, k(T) = 0 \). Finally, \( p_1(t) \leq \kappa \), for all \( t \in [\bar{S}^{NI}, T] \) implies that \( g'(S) \geq 0 \), for all \( S \in [\bar{S}^{NI}, T] \), and \( q_1(t) \geq \kappa \), for all \( t \in [S^*, T] \) implies that \( k'(S) \leq 0 \), for all \( S \in [S^*, T] \). Thus, by the intermediate value theorem, there exists \( S^* \) solving \( V_{S^{NI}}(\tau) = 0 \). Thus, there exists \( S^* > \bar{S}^{NI} \) such that principal’s individual rationality constraint is binding at \( t = \bar{S}^{NI} \).

I proceed with proving that the investment schedule \( \tau \) satisfies the conditions in Lemma 3 and thus it is obedient.

First, consider \( t \leq \bar{S}^{NI} \). The principal’s continuation value for all \( t \in [0, \bar{S}^{NI}] \) can be written as

\[
V_t(\tau) = \int_{\bar{S}^{NI}}^{S^{NI}} v\lambda(p_1(s) - \kappa) \, ds + V_{S^{NI}}(\tau).
\]

Given the binding individual rationality constraint, it becomes

\[
V_t(\tau) = \int_{\bar{S}^{NI}}^{S^{NI}} v\lambda(p_1(s) - \kappa) \, ds, \quad \text{for all } t \in [0, \bar{S}^{NI}].
\]

Finally, note that \( V_t(\tau) \) above is equivalent to \( V_{t}^{NI} \) given by (18). Lemma 1 implies that given \( \kappa \in (\kappa^{ND}(T, \lambda), \kappa^{NI}(T, \lambda)] \), \( V^{NI}(0) = V(\tau) \geq 0 \). Further, Lemma 8 implies that \( V(\tau) \geq 0 \Rightarrow V_t(\tau) \geq 0, \forall t \in [0, \bar{S}^{NI}] \).

Second, consider \( t \in [\bar{S}^{NI}, S^*) \). Given \( \kappa \in (\kappa^{ND}(T, \lambda), \kappa^{NI}(T, \lambda)] \), \( p_1(t) \leq \kappa, \forall t \in [\bar{S}^{NI}, S^*) \). Thus, \( V_{S^* NI} = 0, \forall t \in [\bar{S}^{NI}, S^*) \). The principal’s continuation value is given by

\[
V_t(\tau) = \int_{\bar{S}^{NI}}^{S^*} v\lambda(p_1(s) - \kappa) \, ds + V_{S^*}(\tau) \quad (32)
\]

As \( p_1(t) \leq \kappa, \forall t \in [\bar{S}^{NI}, S^*) \), \( \int_{\bar{S}^{NI}}^{S^*} v\lambda(p_1(s) - \kappa) \, ds \leq 0 \) and it is increasing in \( t \). As \( V_{S^{NI}}(\tau) = 0 \), where \( V_{S^{NI}}(\tau) \) is given by (31), it follows that \( V_t(\tau) \geq 0, \forall t \in [S^{NI}, S^*) \).

Third, consider \( t \in [S^*, \tau) \). The absence of stopping at \( t \geq S^* \) reveals that \( x_t \neq 2 \). Thus, \( q_1(t) = \frac{p_1(t)}{p_0(t) + p_1(t)} = \frac{\lambda}{1 + \lambda}, \forall t \in [S^*, \tau] \), and, thus, \( q_1(t) > 0 \). Further, \( q_1(S^*) > \kappa \). The continuation value \( \forall t \in [S^*, \tau] \) is given by

\[
V_t(\tau) = E\left[ \int_{0}^{\tau} v\lambda(q_1(z) - \kappa) \, dz \mid t < \tau \right].
\]

Thus, \( V_t(\tau) \geq 0, \forall t \in [S^*, \tau] \).

2. The case of \( \kappa^{NI}(T, \lambda) < \kappa \leq \tilde{\kappa}(T, \lambda) \).
I start with proving the existence of $S^*$ such that $V(\tau) = 0$. For all $t \in [0, S^*)$, stopping never occurs, at $t = S^*$ it occurs if $x_{S^*} = 2$, and for all $t \in (S^*, T]$ it occurs at $t = \tau_2 \wedge T$. The principal’s long-run payoff can be written as

$$V(\tau) = v\lambda \left( \int_0^{S^*} p_1(z) - \kappa dz + \int_{S^*}^T (q_1(z) - \kappa) (1 - P(x_z = 2)) dz \right).$$

(33)

To ensure that $S^*$ makes the individual rationality constraint bind at $t = 0$, it is necessary and sufficient that $V(\tau) = 0$. The next step of the proof consist of inspecting (33) to establish that there exists $S^*$ ensuring that $V(\tau) = 0$. It follows the respective part from the proof for the parametric case $\kappa^{ND}(T, \lambda) < \kappa \leq \kappa^{NI}(T, \lambda)$, imposing $\bar{S}^{NI} = 0$ in it everywhere; thus, I omit it for the sake of space.

I proceed with proving that the investment schedule $\tau$ satisfies the conditions in Lemma 3 and thus it is obedient. The principal’s continuation value is given by (32). As $\kappa \in (\kappa^{NI}(T, \lambda), \bar{\kappa}(T, \lambda)]$, it follows from Lemma 1 that $V^NI_t = 0, \forall t \in [0, S^*)$. First, assume $S^* \leq \bar{S}^{NI}$. From the proof of Lemma 1, it follows that $p_1(t) \leq \kappa, \forall t \in [0, \bar{S}]$, and $p_1(t) \geq \kappa, \forall t \in [\bar{S}, \bar{S}^{NI}]$. Thus,

$$\int_t^{\bar{S}^{NI}} v\lambda (p_1(s) - \kappa) ds \geq \int_0^{\bar{S}^{NI}} v\lambda (p_1(s) - \kappa) ds, \forall t \in [0, \bar{S}^{NI}].$$

(34)

As $V_t(\tau)$ is given by (32), $V(\tau) = 0$ and (34) imply that $V_t(\tau) \geq 0, \forall t \in [0, S^*)$.

Second, assume $S^* \geq \bar{S}^{NI}$. As $V(\tau) = 0$ and $\int_0^{\bar{S}^{NI}} v\lambda (p_1(s) - \kappa) ds < 0$, it must be that $V(\bar{S}^{NI}) > 0$. Further, $\int_0^{\bar{S}^{NI}} v\lambda (p_1(s) - \kappa) ds$ increases in $t$ for $t \in [\bar{S}^{NI}, S^*)$. Thus, $V_t(\tau) \geq 0, \forall t \in [0, S^*)$.

Finally, the proof that $V_t(\tau) \geq 0, \forall t \in [S^*, \tau)$ follows the the respective part of the proof for the parametric case $\kappa \in (\kappa^{ND}(T, \lambda), \kappa^{NI}(T, \lambda)]$; thus, I omit it for the sake of space.

Proof of Lemma 6. I provide the proof for the parametric cases $\kappa^{ND}(T, \lambda) < \kappa \leq \kappa^{NI}(T, \lambda)$ and $\kappa^{NI}(T, \lambda) < \kappa \leq \bar{\kappa}(T, \lambda)$ separately.

1. The case of $\kappa^{ND}(T, \lambda) < \kappa \leq \kappa^{NI}(T, \lambda)$.

Under any obedient optimal policy, the principal’s individual rationality constraint is binding, thus, $V(\tau) = V^{NI}$, or equivalently $p_2(T) v - E[\tau] c = p_2(\bar{S}^{NI}) v - \bar{S}^{NI} c$. Thus,

$$E[\tau] = \frac{1}{\lambda\kappa} \left( p_2(T) - p_2(\bar{S}^{NI}) \right) + \bar{S}^{NI}.$$

Differentiating both sides with respect to $\kappa$ yields

$$\frac{\partial E[\tau]}{\partial \kappa} = \frac{e^{-T\lambda} (1 + T\lambda) - e^{-S^{NI}\lambda} - \kappa}{\kappa^2 \lambda}.$$
The equation
\[ e^{-T\lambda} (1 + T\lambda) - e^{-S\lambda} - \kappa = 0 \]
can be equivalently rewritten as
\[ e^{-T\lambda} - e^{-S\lambda} = \kappa - e^{-T\lambda} T\lambda. \]

It has a unique solution corresponding to \( \kappa = \kappa^{ND} (T, \lambda) := e^{-T\lambda} T\lambda. \) As \( \kappa > \kappa^{ND} (T, \lambda) \), it holds that \( \partial E [\tau] / \partial \kappa < 0. \)

2. The case of \( \kappa^{NI} (T, \lambda) < \kappa \leq \bar{\kappa} (T, \lambda) \).

The principal’s long-run payoff under any obedient optimal policy is given by
\[ E [\tau] c = p^2 (T) v. \]
Rewriting it equivalently, \( E [\tau] = \frac{1}{\lambda} p^2 (T) \Rightarrow \partial E [\tau] / \partial \kappa < 0. \)

Proof of Lemma 7. Lemma 4 implies that if a schedule \( \tau \) assigns zero probability to stopping in states 0 and 1 then \( V (\tau) < 0 \) and the individual rationality constraint is violated. Thus, the necessary condition for a schedule \( \tau \) to be individually rational under \( \kappa \in (\bar{\kappa} (T, \lambda), \kappa^{FI} (T, \lambda)) \) is that it assigns a positive probability to stopping not only in state 2, but also to stopping in either state 0 or state 1. Consider a schedule \( \tau \) that assigns a positive probability to stopping in state 1. Consider an alternative schedule \( \tau' \) which is induced by reallocating the probability mass of stopping in state 1 to stopping at \( \tau_2 \wedge T \). Lemma 2 suggests that in state 1 the principal strictly benefits from postponing the stopping until the second stage of the project is completed. Thus, \( V (\tau') > V (\tau) \). Further, under \( \tau' \) the principal invests strictly longer, in expectation. Thus, \( W (\tau') > W (\tau) \). Thus, for a schedule to be optimal it should not assign a positive probability to stopping in state 1.

Next, consider a schedule \( \tau \) which assigns a positive probability to stopping in states 0 and 2. Assume that the stopping in state 0 happens at date \( S \), which can be either deterministic or stochastic: if \( x_S = 0 \) then \( \tau = S \), otherwise, \( \tau \geq \tau_2 \wedge T \) and there exists \( \omega \in \Omega \) such that \( \tau (\omega) > \tau_2 (\omega) \), i.e., with a positive probability, stopping in state 2 happens strictly after the date of transition to state 2. Assume that \( V (\tau) = 0 \). Consider the following investment schedule \( \tilde{\tau} \): if \( x_S = 0 \) then \( \tilde{\tau} = \tilde{S}, E [\tilde{S}] > E [S] \), otherwise, \( \tilde{\tau} = \tau_2 \wedge T \), and \( V (\tilde{\tau}) = 0 \). Further, from (9), the agent’s objective is given by
\[ W (\tilde{\tau}) - W (\tau) = (SV (\tilde{\tau}) - V (\tilde{\tau})) - (SV (\tau) - V (\tau)) = SV (\tilde{\tau}) - SV (\tau). \]

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The change from \( \tau \geq \tau_2 \land T \) to \( \tau = \tau_2 \land T \) induces no loss in total surplus as the measure of \( \omega \in \Omega \) satisfying the event \( \{ \tau_2 \leq T \} \) is equal for both schedules. Further, the change from conditional stopping at \( S \) to conditional stopping at \( \tilde{S} \) induces an increase in total surplus as \( P(x_{\tilde{S}} = 0) < P(x_S = 0) \) and thus, in the latter case, conditional stopping happens less frequently. Hence, \( SV(\tilde{\tau}) \geq SV(\tau) \). Thus, for a schedule that assigns positive probability to stopping in states 0 and 2 to be optimal, it is necessary that stopping in state 2 happens at \( \tau_2 \) with probability 1.

\[ \square \]

**Proof of Proposition 3.** Given Lemma 7, the space of candidate optimal investment schedules under \( \kappa \in (\tilde{\kappa}(T, \lambda), \kappa_{FI}(T, \lambda)] \) simplifies to schedules such that stopping in state 2 happens at \( \tau_2 \), and also stopping in state 0 happens with positive probability. Thus, to characterize the optimal schedule under \( \kappa \in (\tilde{\kappa}(T, \lambda), \kappa_{FI}(T, \lambda)] \), I need to characterize the assignment of the probability mass of stopping in state 0 that is optimal for the agent given the principal’s individual rationality constraints. To do this, I consider the agent’s optimal design of a device that randomizes over the dates of stopping in state 0.

At \( t = 0 \), the agent chooses a distribution \( F_{\rho} \) on \([0, T] \), observable to both the agent and the principal. \( \rho \) stands for the random date at which the stopping occurs if the state is 0 by that date. \( \rho \) is drawn at \( t = 0 \) according to \( F_{\rho} \), which is independent from the state process \( x_t \), and the draw privately observed by the agent.

To formulate the agent’s design problem, I start with characterizing the welfare implications of stopping in state 0 for the agent and principal. A few useful objects are \( SV_{t|0}(\tau_2) \) and \( V_{t|0}(\tau_2) \). \( SV_{t|0}(\tau_2) \) is the time \( t \) continuation total surplus given that \( x_t = 0 \) at \( t \) and completion of the second stage of the project is immediately disclosed whenever it happens, \( \tau = \tau_2 \land T \):

\[
SV_{t|0}(\tau_2) = v P(\tau_2 \leq T | x_t = 0) = v \left[ 1 - e^{-\lambda(T-t)} - \lambda(T-t) e^{-\lambda(T-t)} \right]. \tag{35}
\]

\( V_{t|0}(\tau_2) \) is the principal’s time \( t \) continuation value given that \( x_t = 0 \) and completion of the second stage of the project is immediately disclosed, \( \tau = \tau_2 \land T \):

\[
V_{t|0}(\tau_2) = v P(\tau_2 \leq T | x_t = 0) - c E[\tau_2 \land T - t | x_t = 0],
\]

where \( v P(\tau_2 \leq T | x_t = 0) \) is given by (35) and
\[ \mathbb{E} [\tau_2 \wedge T - t | x_t = 0] \]
\[ = P (\tau_2 \leq T | x_t = 0) \int_0^T z \cdot \frac{\lambda^2 (z - t) e^{-\lambda (z - t)}}{P (\tau_2 \leq T | x_t = 0)} \, dz + P (\tau_2 > T | x_t = 0) (T - t) \quad (36) \]
\[ = \frac{2}{\lambda} - \frac{2}{\lambda} e^{-\lambda (T - t)} - e^{-\lambda (T - t)} (T - t). \]

I proceed with a useful lemma.

**Lemma 9.** Given an investment schedule

\[ \tau = \begin{cases} 
\rho, & \text{if } x_\rho = 0 \\
\tau_2 \wedge T, & \text{otherwise,} 
\end{cases} \quad (37) \]

where \( \rho \) has a publicly observable distribution \( F_\rho \) on \([0, T]\), \( \rho \) is independent of the state process \( x_t \) and is drawn at \( t = 0 \), and the draw is unobservable to the players, the total surplus at date \( t \) can be written as

\[ SV_t (\tau) = SV_t (\tau_2) - \mathbb{E}_{F_\rho} \left[ P (x_\rho = 0 | t < \tau) SV_{\rho|0} (\tau_2) \right], \]

and the principal’s expected payoff at date \( t \) can be written as

\[ V_t (\tau) = V_t (\tau_2) - \mathbb{E}_{F_\rho} \left[ P (x_\rho = 0 | t < \tau) V_{\rho|0} (\tau_2) \right], \]

for all \( t \geq 0 \).

**Proof.** By construction, \( SV_t (\tau) \) corresponds to the expected value of the project completion payoff under stopping policy \( \tau \) conditional on stopping not having happened by \( t \), i.e., \( t < \tau \). Given (37), the principal gets \( v \) either if the second stage is completed before \( \rho \) or if the first stage is completed before \( \rho \) and the second stage is completed before \( T \). Note that when \( t < \rho, t < \tau \) implies that the state is either 0 or 1, and, when \( t \geq \rho, t < \tau \) implies that the state is 1. Thus,

\[ SV_t (\tau) = v \mathbb{E}_{F_\rho} \left[ P \left( \{ x_\rho = 1 \} \cap \{ \tau_2 \leq T \} | t < \tau \right) + P (x_\rho = 2 | t < \tau) \right]. \]

Further, for each realization of \( \rho \),

\[ P \left( \{ x_\rho = 1 \} \cap \{ \tau_2 \leq T \} | t < \tau \right) = P (x_\rho = 1 | t < \tau) P (\tau_2 \leq T | x_\rho = 1). \]

Thus,
\[ SV_t(\tau) = v \mathbb{E}_{\rho}\left[ P\left( x_\rho = 1|t < \tau \right) P\left( \tau_2 \leq T|x_\rho = 1 \right) + P\left( x_\rho = 2|t < \tau \right) \right]. \]  

(38)

\( SV_{\rho|0}(\tau_2) \) corresponds to the expected value of the project completion payoff when \( x_\rho = 0 \). In that case, \( v \) is obtained when the completion of the second stage happens not later than \( T \). Thus, \( SV_{\rho|0}(\tau_2) = \mathbb{E}_{\rho}\left[ v P\left( \tau_2 \leq T|x_\rho = 0 \right) \right] \). Therefore,

\[
SV_t(\tau_2) - \mathbb{E}_{\rho}\left[ P\left( x_\rho = 0|t < \tau \right) SV_{\rho|0}(\tau_2) \right] \\
= P\left( x_T = 2|t < \tau \right) v - \mathbb{E}_{\rho}\left[ P\left( x_\rho = 0|t < \tau \right) v P\left( \tau_2 \leq T|x_\rho = 0 \right) \right] \tag{39} \\
= v \mathbb{E}_{\rho}\left[ P\left( x_T = 2|t < \tau \right) - P\left( x_\rho = 0|t < \tau \right) P\left( \tau_2 \leq T|x_\rho = 0 \right) \right].
\]

Thus, given (38) and (39), to complete the proof of the first result of the Lemma 9, it suffices to show that,

\[
P\left( x_T = 2|t < \tau \right) - P\left( x_\rho = 0|t < \tau \right) P\left( \tau_2 \leq T|x_\rho = 0 \right) \\
= P\left( x_\rho = 2|t < \tau \right) + P\left( x_\rho = 1|t < \tau \right) P\left( \tau_2 \leq T|x_\rho = 1 \right)
\]

Using the full probability formula,

\[
P\left( x_T = 2|t < \tau \right) = \\
P\left( x_\rho = 0|t < \tau \right) P\left( \tau_2 \leq T|x_\rho = 0 \right) \\
+ P\left( x_\rho = 1|t < \tau \right) P\left( \tau_2 \leq T|x_\rho = 1 \right) \\
+ P\left( x_\rho = 2|t < \tau \right) P\left( \tau_2 \leq T|x_\rho = 2 \right).
\]

Hence,

\[ SV_t(\tau) = SV_t(\tau_2) - \mathbb{E}_{\rho}\left[ P\left( x_\rho = 0|t < \tau \right) SV_{\rho|0}(\tau_2) \right], \text{ for all } t \geq 0. \tag{40} \]

I proceed with proving the second result of Lemma 9. First, applying (40) to \( V_t(\tau) \) yields

\[
V_t(\tau) = SV_t(\tau) - \mathbb{E}_{\rho}\left[ c \mathbb{E}[\tau|t < \tau] \right] \\
= SV_t(\tau_2) - \mathbb{E}_{\rho}\left[ P\left( x_\rho = 0|t < \tau \right) SV_{\rho|0}(\tau_2) - c \mathbb{E}[\tau|t < \tau] \right]. \tag{41}
\]
Further, for each realization of $\rho$:

$$
E[\tau | t < \tau] = P(x_\rho = 0 | t < \tau) E[\tau | x_\rho = 0] + P(x_\rho = 1 | t < \tau) E[\tau | x_\rho = 1] + P(x_\rho = 2 | t < \tau) E[\tau | x_\rho = 2]
$$

$$= P(x_\rho = 0 | t < \tau) \rho + P(x_\rho = 1 | t < \tau) E[\tau_2 \land T | x_\rho = 1] + P(x_\rho = 2 | t < \tau) E[\tau_2 \land T | x_\rho = 2]$$

$$= P(x_\rho = 0 | t < \tau) \rho + P(x_\rho = 1 | t < \tau) E[\tau_2 \land T | t < \tau] - P(x_\rho = 0 | t < \tau) E[\tau_2 \land T | x_\rho = 0]$$

$$= E[\tau_2 \land T | t < \tau] - P(x_\rho = 0 | t < \tau) \left( E[\tau_2 \land T | x_\rho = 0] - \rho \right)$$

where the second equality uses the full probability formula.

Plugging (42) into (41) yields

$$SV_t(\tau_2) - E[F_\rho \left(c E[\tau_2 \land T | t < \tau] \right) - E[F_\rho \left(P(x_\rho = 0 | t < \tau) \left( SV_{\rho|0}(\tau_2) - c E[\tau_2 \land T - \rho | x_\rho = 0] \right) \right)$$

$$= V_t(\tau_2) - E[F_\rho \left(P(x_\rho = 0 | t < \tau) V_{\rho|0}(\tau_2) \right), \forall t \geq 0.$$

I proceed to formulating the agent’s problem. The agent’s objective can be represented as

$$c E[\tau] = SV(\tau) - V(\tau).$$

Using Lemma 9,

$$SV(\tau) - V(\tau) = SV(\tau_2) - V(\tau_2) - E[F_\rho \left[P(x_\rho = 0 | t < \tau) \left( SV_{\rho|0}(\tau_2) - c E[\tau_2 \land T - \rho | x_\rho = 0] \right) \right]$$

$$= SV(\tau_2) - V(\tau_2) - c E[F_\rho [P(x_\rho = 0 | t < \tau) E[\tau_2 \land T - \rho | x_\rho = 0]]]$$

The individual rationality constraint for the principal can be expressed as

$$V_t(\tau_2) \geq 0, \forall t \geq 0 \iff V_t(\tau_2) \geq E[F_\rho \left[P(x_\rho = 0 | t < \tau) V_{\rho|0}(\tau_2) \right], \forall t \geq 0.$$

Finally, (43) yields the objective and (44) yields the individual rationality constraint for the agent’s problem

$$\min_{F_\rho} \left\{ E[F_\rho [P(x_\rho = 0 | t < \tau) E[\tau_2 \land T - \rho | x_\rho = 0]]] \right\}$$

$$s.t. E[F_\rho \left[P(x_\rho = 0 | t < \tau) \left( c E[\tau_2 \land T - \rho | x_\rho = 0] - SV_{\rho|0}(\tau_2) \right) \right] \geq -V_t(\tau_2), \forall t \geq 0.$$
I proceed in two steps: first, I formulate and solve the relaxed version of (45); second, I demonstrate that the solution to the relaxed problem satisfies the full system of constraints in (45). The relaxed problem has the principal’s individual rationality constraint only for $t = 0$:

$$\begin{align*}
\min_{F_\rho} \{ E_{F_\rho} [P (x_\rho = 0) E [\tau_2 \wedge T - \rho | x_\rho = 0]] \} \\
\text{s.t. } E_{F_\rho} [P (x_\rho = 0) \left( c E [\tau_2 \wedge T - \rho | x_\rho = 0] - SV_{\rho \mid 0} (\tau_2) \right)] \geq -V (\tau_2).
\end{align*}$$ (46)

The Lagrangian function for the problem is

$$\mathcal{L} = E_{F_\rho} [P (x_\rho = 0) E [\tau_2 \wedge T - \rho | x_\rho = 0]] - \mu \left( E_{F_\rho} \left[ P (x_\rho = 0) \left( c E [\tau_2 \wedge T - \rho | x_\rho = 0] - SV_{\rho \mid 0} (\tau_2) \right) \right] \right) + V (\tau_2),$$

where $P (x_\rho = 0) = e^{-\lambda^2}$, $E [\tau_2 \wedge T - \rho | x_\rho = 0]$ is given by (36), $SV_{\rho \mid 0} (\tau)$ is given by (35).

I obtain the F.O.C., which needs to hold for each value of $\rho$ that has a positive probability in $F_\rho$:

$$e^{-\lambda^T} \left( c \left( 2e^{-\lambda (T - \rho)} - 1 \right) (\mu - 1) - \mu \lambda v \left( e^{-\lambda (T - \rho)} - 1 \right) \right) = 0.$$ (47)

The derivative of the left-hand side of (47) w.r.t. $\rho$ is given by $e^{-\lambda^2} \rho \left( 2c + \mu (\lambda v - 2c) \right)$. As $\kappa^F (T, \lambda) < \frac{1}{2}$, the derivative is positive. Thus, there exists at most one $\rho$ that satisfies the FOC (47). Thus, the optimal $F_\rho$ is degenerate. I denote it with $S^A_0$; the interim deadline.

I proceed with characterizing the optimal $S^A_0$:

$$\begin{align*}
\min_{S \in [0, T]} \{ P (x_S = 0) E [\tau_2 \wedge T - S | x_S = 0] \} \\
\text{s.t. } P (x_S = 0) \left( c E [\tau_2 \wedge T - S | x_S = 0] - SV_{S \mid 0} (\tau_2) \right) \geq -V (\tau_2).
\end{align*}$$ (48)

The system of F.O.C. is given by

$$\begin{align*}
e^{-\lambda^T} c \left( 2e^{-\lambda (T - S)} - 1 \right) (\mu - 1) & \geq 0 \text{ if } S = 0 \\
e^{-\lambda^T} \mu \lambda v \left( e^{-\lambda (T - S)} - 1 \right) & = 0 \text{ if } S \in (0, T) \\
e^{-\lambda^T} \left( \frac{e^{-\lambda (T - S)}}{\lambda} - \lambda (T - S) \right) & \leq 0 \text{ if } S = T \\
\frac{e^{-\lambda^T}}{\lambda} \left( 2 \left( e^{-\lambda (T - S)} - 1 \right) - \lambda (T - S) \right) & = 0 \text{ if } \mu > 0.
\end{align*}$$

Assume $\mu = 0$. In this case, the first F.O.C. w.r.t $S$ yields $-ce^{-\lambda^T} \left( 2e^{-\lambda (T - S)} - 1 \right)$. The expression is negative for all $S \in (0, T)$. Thus, $\mu > 0$, and optimal $S$ solves the
binding constraint. Thus, I proceed with inspecting the corresponding equation given by

\[
\frac{c}{\lambda} e^{-\lambda T} \left( 2 \left( e^{-\lambda (T - S)} - 1 \right) - \lambda (T - S) \right)
- \nu e^{-\lambda T} \left( \left( e^{-\lambda (T - S)} - 1 \right) - \lambda (T - S) \right) = - V(\tau_2),
\]

where \( V(\tau_2) \) is given by (26).

The solution to (49) is given by

\[
S = \frac{1}{\lambda} \left[ \gamma + W(-\gamma e^{-\gamma}) \right],
\]

where \( \gamma = e^{\lambda_T \frac{1-2\kappa}{1-\kappa}} \) and \( W(.) \) denotes the Lambert \( W \) function.

Denote the 0 and \(-1\) branches of the Lambert \( W \) function by \( W_0(.) \) and \( W_{-1}(.) \). \( \kappa \in \left( 0, \frac{1}{2} \right) \), thus, \( \gamma > 0 \). (50) depends on \( \gamma \) and for each \( \gamma \neq 1 \) corresponds to two points as the Lambert \( W \) function has two branches. The values of (50) as a function of \( \gamma \) are presented in Figure 7. They are given by

\[
S = \begin{cases} 
\frac{1}{\lambda} \left[ \gamma + W_1(-\gamma e^{-\gamma}) \right], & \text{if } \gamma < 1 \\
0, & \text{if } \gamma = 1 \\
\frac{1}{\lambda} \left[ \gamma + W_0(-\gamma e^{-\gamma}) \right], & \text{if } \gamma > 1 
\end{cases}
\]

Figure 7: Roots of equation (49) as a function of the parameter \( \gamma \):
- root corresponding to branch 0 of the Lambert \( W \) function - thick;
- root corresponding to branch \(-1\) of the Lambert \( W \) function - dashed.

\( \gamma \) is decreasing in \( \kappa \), and \( \gamma_{|\kappa = \kappa^{FI}} = 1 \). As \( \kappa \leq \kappa^{FI} \), which corresponds to \( \gamma \geq 1 \), the solution to (49) is given by

\[
S_A = 0, \quad S_B = \frac{1}{\lambda} \left[ \gamma + W_0(-\gamma e^{-\gamma}) \right].
\]
As the objective of (48) is decreasing in $S$ and $S_B > S_A$, the solution to (48) is given by
\[ S_0^A = \frac{1}{\lambda} \left[ \gamma + W_0 \left( -\gamma e^{-\gamma} \right) \right], \quad \gamma = e^{\lambda T} \frac{1 - 2\kappa}{1 - \kappa}. \] (51)

Finally, I can describe the solution to (46): $\tau$ is the stopping time such that stopping occurs either at the moment of completion of the second stage of the project or at $S_0^A$, conditional on the absence of the completion of the first stage of the project, i.e.
\[ \tau = \begin{cases} S_0^A, & \text{if } x_{S_0^A} = 0 \\ \tau_2 \land T, & \text{otherwise}, \end{cases} \] (52)
where $S_0^A$ is given by (51).

I proceed with the second part of the proof: I demonstrate that (52) satisfies the full system of constraints in (45), and thus solves (45). To do this, I need to demonstrate that $V_t(\tau) \geq 0$, for all $t \in [0, \tau)$. If the recommendation mechanism $\tau$ is given by (52), then, for $t < S_0^A$ the absence of stopping at some $t$ reveals that $x_t \neq 2$. Thus,
\[ q_1(t) = \frac{p_1(t)}{p_1(t) + p_0(t)} = \frac{\lambda t}{1 + \lambda t}, \quad \forall t < S_0^A. \]
Hence, $q_1(t) > 0$, for all $t < S_0^A$. Further, for $t \geq S_0^A$, the absence of stopping reveals that $x_t = 1$. Thus, $q_1(t) = 1$, for all $t \geq S_0^A$.

Writing out $V_t(\tau)$ based on (13) yields
\[ \dot{V}_t(\tau) = \lambda q_1(t) V_t(\tau) + v\lambda (\kappa - q_1(t)). \] (53)

$q_1(0) = 0$ and $q_1(t) > 0$, for all $t < S_0^A$. I define $\tilde{t}$ as the solution of $\frac{\lambda t^2}{1 + \lambda t} = \kappa$. $q_1(t) < \kappa$, for all $t \in \left[0, \tilde{t} \land S_0^A\right]$. I argue that $V(\tau) \geq 0 \Rightarrow V_t(\tau) \geq 0$, for all $t \in \left(0, \tilde{t} \land S_0^A\right)$. Assume that this is not true, then $\exists \tilde{t}$ such that $\tilde{t} := \inf \{ t \in \left(0, \tilde{t} \land S_0^A\right) : V_t(\tau) < 0 \}$. As $V_t(\tau)$ is continuous in $t$, it follows that $V_{\tilde{t}}(\tau) = 0$, and by the mean value theorem there must be $\tilde{t} \in \left(0, \tilde{t}\right)$ such that $V_{\tilde{t}}(\tau) \leq 0$. But this is in contradiction with the fact that $V_{\tilde{t}}(\tau) \geq 0$ and (53).

Consider now $t \in \left[\tilde{t} \land S_0^A, \tau\right)$. The continuation value can be written as
\[ V_t(\tau) = \mathbb{E} \left[ \int_t^\tau v\lambda (q_1(z) - \kappa) \, dz \mid t < \tau \right]. \] (54)
As $\kappa < \frac{1}{2}$ and $q_1(t) = 1$, for all $t \in \left[S_0^A, \tau\right)$, it holds that $q_1(t) \geq \kappa, \forall t \in \left[\tilde{t} \land S_0^A, \tau\right)$. Thus, it can be seen from (54) that $V_t(\tau) \geq 0, \forall t \in \left[\tilde{t} \land S_0^A, \tau\right)$.

\[ \square \]
Proof of Proposition 4. I assume it is not the case that $\alpha = 1$ and $\beta = 0$ as, otherwise, agent is indifferent and discloses no information. I start with proving existence of $\tilde{\kappa}$ and then proceed to proving that when the project is promising, an investment schedule, in which stopping never occurs in state 0, is optimal. Proving existence of $\tilde{\kappa}$ follows the steps of the proof of Lemma 4. The principal’s expected payoff is given by

$$V(\tau) = \alpha P(x, \tau = 2) v E[e^{-r\tau} | \tau_2 \leq \tau] - E\left[\int_0^{\tau_2} e^{-rs} ds\right] c.$$ 

$\tilde{\kappa}$ solves $V(\tau_2) = 0$, or, equivalently

$$\alpha P(x, \tau_2 \wedge T = 2) v E[e^{-r\tau_2 \wedge T} | \tau_2 \leq T] = E\left[\int_0^{\tau_2 \wedge T} e^{-rs} ds\right] c, \quad (55)$$

where $P(x, \tau_2 \wedge T = 2) = p_2(T)$. Solving (55) for $\kappa$ yields

$$\tilde{\kappa}(T, \lambda, r, \alpha) = \frac{1}{\lambda \alpha} \frac{P(x, \tau_2 \wedge T = 2) E[e^{-r\tau_2 \wedge T} | \tau_2 \leq T]}{E\left[\int_0^{\tau_2 \wedge T} e^{-rs} ds\right]}.$$ 

Finally, $V(\tau)$ decreases in $\kappa$. Thus, if $\kappa < \tilde{\kappa}(T, \lambda, r, \alpha)$, then an investment schedule $\tau = \tau_2 \wedge T$ satisfies the principal’s individual rationality constraint.

Consider now the agent’s expected payoff $W(\tau)$ given by

$$W(\tau) = (1 - \alpha) P(x, \tau = 2) v E[e^{-r\tau} | \tau_2 \leq \tau] + E\left[\int_0^{\tau} e^{-rs} ds\right] \beta c.$$ 

Consider the case $\kappa \leq \tilde{\kappa}(T, \lambda, r, \alpha)$. Consider an investment schedule $\tau$ given by (37), i.e., such that stopping happens either immediately at the moment of the second stage completion, or in state 0 at a possibly random interim deadline. Further, consider an alternative investment schedule $\hat{\tau} = \tau_2 \wedge T$. Given the two investment schedules, $P(x, \hat{\tau} = 2) > P(x, \tau = 2)$. Further, $E[e^{-rs} | \tau_2 \leq \hat{\tau}] = E[e^{-rs} | \tau_2 \leq \tau]$ and $E\left[\int_0^{\tau} e^{-rs} ds\right] > E\left[\int_0^{\hat{\tau}} e^{-rs} ds\right]$. As $W(\hat{\tau}) > W(\tau)$ and $\kappa < \tilde{\kappa}(T, \lambda, r, \alpha)$, the agent prefers to implement an investment schedule $\hat{\tau}$ rather than $\tau$.

Consider now the case $\kappa > \tilde{\kappa}(T, \lambda, r, \alpha)$. The application of the arguments from the proof of Lemma 7 establishes the result.

\[\square\]

D Disclosure of project completion with a deterministic delay

**Proposition 5.** Assume $\kappa \in (0, \kappa_{NI}(T, \lambda)]$ and $T > \bar{S}_{NI}$. The optimal mechanism provides no information until $t = S_{NI}$. At each $t \geq S_{NI}$, it generates a
recommendation to stop iff the second stage of the project was completed at date $\pi(t)$ in the past, where

$$\pi(t) = -\frac{1}{\lambda} \left( 1 + \frac{1}{\lambda} W_{-1}(\frac{1}{\kappa} e^{-\lambda \lambda t}) \right),$$

where $W_{-1}(\cdot)$ denotes the $-1$ branch of Lambert $W$ function.

The mechanism from Proposition 5 does not recommend stopping until the second stage of the project is completed, and thus maximizes the total surplus. The mechanism makes the principal’s individual rationality constraint bind, $V_{SNI}(\tau) = 0$. The absence of a stopping recommendation after $t = \tilde{S}^{NI}$ induces posterior beliefs $q_1(t) = \kappa, \forall t \geq \tilde{S}^{NI}$. Note that the principal’s expected instantaneous payoff within $\Delta t$ is given by

$$v \cdot q_1(t) \lambda \Delta t - c \cdot \Delta t = v \lambda \Delta t (q_1(t) - \kappa).$$

No information is provided until $\tilde{S}^{NI}$ and after $\tilde{S}^{NI}$ the mechanism keeps the principal’s expected instantaneous payoff precisely at 0, $\forall t \geq \tilde{S}^{NI}$. As a result, the principal’s continuation value is kept at 0 for all $t \in [\tilde{S}^{NI}, \tau)$.

The delay is given by $t - \pi(t)$. At the beginning of the disclosure, $t = \tilde{S}^{NI}$, the delay is $\tilde{S}^{NI}$. To keep the belief regarding state $1$ constant, the delay decreases for all $t \in (\tilde{S}^{NI}, \tau)$.

**Proof of Proposition 5.** Posterior beliefs at date $\pi$ induced by the disclosure of the absence of second stage completion are given by

$$q_0(\pi) = \frac{p_0(\pi)}{p_0(\pi) + p_1(\pi)},$$

$$q_1(\pi) = \frac{p_1(\pi)}{p_0(\pi) + p_1(\pi)}.$$

As no other evidence is provided during $(\pi, t]$, the beliefs evolve according to

$$q_0(s) = \frac{e^{-\lambda s}}{1 + \lambda \pi},$$

$$q_1(s) = \frac{e^{-\lambda \lambda \lambda (s + \pi)} (s + \pi)}{1 + \lambda \pi},$$

where $s \geq \pi$.

The belief regarding state 1 at current date $t$ is given by

$$q_1(t) = \frac{e^{-\lambda (t - \pi) \lambda t}}{1 + \lambda \pi}. \quad (56)$$
The dynamic of the state is the same as in the no-information benchmark until $t = \bar{S}_{NI}$. Therefore,

$$q_0 (\bar{S}_{NI}) = p_0 (\bar{S}_{NI}) = \frac{\kappa}{\lambda S_{NI}} \quad \text{and} \quad q_1 (\bar{S}_{NI}) = p_1 (\bar{S}_{NI}) = \kappa.$$  

The dynamics for $t \geq \bar{S}_{NI}$ then is $q_1 (t) = \kappa$, $\dot{q}_1 (t) = 0$. Solving from (56),

$$\pi = \frac{1}{\lambda} \left( 1 + \frac{1}{\lambda} W_{-1} (-\frac{1}{\kappa} e^{-1-\lambda t}) \right).$$

The recommendation mechanism $\tau$ is obedient. $\tau \geq \tau_2 \land T$ implies that the recommendation to stop comes only if the second stage of the project has already been completed, and thus immediate stopping is clearly optimal for the principal. The recommendation not to stop is also obedient. $V_t (\tau) \geq 0, \forall t \in [0, \bar{S}_{NI})$ is formally demonstrated in the proof of obedience for Proposition 2. I proceed by showing that $V_t (\tau) = 0, \forall t \in [\bar{S}_{NI}, \tau)$. Writing out $V_t (\tau)$ in the recursive form yields

$$V_t (\tau) = (v \lambda q_1 (t) - c) \Delta t + (1 - \lambda q_1 (t) \Delta t) V_{t+\Delta t} (\tau)$$

$$= v \lambda (q_1 (t) - \kappa) \Delta t + (1 - \lambda q_1 (t) \Delta t) V_{t+\Delta t} (\tau).$$

As $q_1 (t) = \kappa$, $\forall t \in [\bar{S}_{NI}, \tau)$, it becomes

$$V_t (\tau) = (1 - \lambda q_1 (t) \Delta t) V_{t+\Delta t} (\tau), \forall t \in [\bar{S}_{NI}, \tau).$$

Differentiating both sides w.r.t. $\Delta t$ yields

$$0 = -\lambda q_1 (t) V_{t+\Delta t} (\tau) + \dot{V}_{t+\Delta t} (\tau).$$

This differential equation together with the boundary condition $V_T (\tau) = 0$ has a unique solution $V_t (\tau) = 0$ for all $t \in \left[ \bar{S}_{NI}, T \right]$. 

\[ \square \]

E The case of no project completion deadline

Importantly, the presence of a hard project deadline $T$ serves as one of the necessary and sufficient conditions for the agent to commit to an interim reporting deadline. Without a hard deadline $T$, the principal’s incentives under full information are different. Recall from Lemma 2 the principal’s incentive to continue investing decreases in the length of absence of the first stage completion. In the case $T \to \infty$, the continuation value $V_t^{FI}$ is constant and given by $v (1 - \kappa)$. As a result, the principal’s incentive to continue investing given the absence of stage completion does not change over time. Thus, if the principal opts in, he never chooses to stop investing before the completion of the second stage occurs. As a
result, setting an interim deadline stops serving as an agent’s tool to incentivize the principal’s investment. The agent’s information policy in the case of no project deadline is given in Lemma 10.

**Lemma 10.** Assume that $T \to \infty$. In that case, if $\kappa < \frac{1}{2}$, then the agent uses the information policy presented in Proposition 1, Case 2.

**Proof of Lemma 10.** Under full information and the absence of an exogenous deadline, the principal assigns value $v_x$ to each state $x \in \{0, 1, 2\}$. Clearly, $v_2 = v$ as the principal stops immediately and gets $v$. In state 1, at each $t$ the principal gets $v \Delta t$ with probability $\lambda \Delta t$ and pays $c \Delta t$. As a result, the principal’s continuation value is constant. Assume that $\kappa < 1$, as otherwise $c \geq \lambda v$ and the principal chooses not to invest in state 1. As the principal’s continuation value in state 1 does not change over time,

$$0 = \lambda \cdot (v_2 - v_1) - c,$$

and so

$$v_1 = v - \frac{c}{\lambda} = v(1 - \kappa).$$

Thus, the principal wants to invest in state 0 if $c \leq \lambda v_1$, i.e., $\kappa \leq \frac{1}{2}$.

Finally, as the information regarding $\tau_1$ is not decision-relevant for the principal, for $\kappa < \frac{1}{2}$, the agent chooses the information policy that discloses only the completion of the second stage of the project and optimally postpones the disclosure to make the principal’s individual rationality constraint bind.

□