Non-Markovian effects on charging of quantum batteries

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(Dated: October 18, 2019)

In general, a quantum battery unavoidably interacts with its surroundings. Here, we study memory effects on energy and ergotropy of a quantum battery in the framework of open system dynamics, where the battery and charger are individually allowed to access a bosonic environment. Our investigation shows that the battery can be fully charged and all the energy stored in the battery can be extracted in the presence of non-Markovian dynamics, in addition, its energy can be preserved for long times compared with Markovian dynamics. Our results indicate that memory effects can play a significant role in improving the performance of quantum batteries.

PACS numbers: 03.65.Yz, 42.50.Lc, 03.65.Ud, 05.30.Rt

I. INTRODUCTION

Quantum systems are open systems where they inevitably interact with their surroundings. Open system dynamics can be characterized by either Markovian or non-Markovian approach [1, 2]. Quantum batteries (QBs) are quantum systems that are capable to store energy, making it useable for later goals. The role of quantum effects on the problem of energy storage has been extensively studied in last years [3–11].

In fact, well known examples of classical batteries are electrochemical devices converting chemical energy to electrical one. They can be disposable or rechargeable using electricity and so on. Therefore, batteries are very comfortable for their multiple use, and their presence in everyday life has transformed them to essential elements. On the other hand, studies over the past few decades show that many modern-day technologies are based on the principles of quantum mechanics [12–15], are much more efficient than their classical ones, so, the battery also needs to be miniaturized. In recent years, the study of QBs have attracted much more attention (There is worldwide interest in exploiting QBs) as small quantum systems used for temporary energy storage and transferring it from production centers to consumption centers [3].

As mentioned above, the research on QBs and the investigation of the stability of stored energy in the battery are very important; in addition, how to charge the battery in order to use QBs in different areas of technology should be strongly considered.

In most research on QBs, they are considered as closed systems that do not interact with their environments. Whenever technologies are used in the microscopic domain, the openness of the QBs must be regarded in any reasonable physical description. Recently, in a new approach, the QB charging process has been viewed from an open system approach [9]. In the light of this approach and given that the interaction of real QBs with the outside world is unavoidable, this is worthwhile investigating the effects of quantum memory in the process of charging QBs and the work can be extracted from the battery alone.

In this paper, we study the role of dissipation on the energy, ergotropy and discharge time of the battery. For this purpose, we consider a battery, B, which is connected to a charger, A, [8, 10], each of which interacts separately with its environment. The battery and the charger are both composed of a two-level or qubit. We accurately obtain the one excitation time evolution of the total system and then examine the efficiency of our proposed mechanism on how to charge the battery and amount of work can be extracted from it for both Markovian and non-Markovian dynamics. Specifically, we study the different ways in which the dissipation environment and coupling with the charger contribute to the process. Here, we find, in the underdamped regime, the non-Markovian dynamics plays a significant role in increasing the stored energy and the extractable work from the battery. Further, we especially illustrate the maximum stored energy can be preserved for long times in the presence of memory effects unlike Markovian dynamics. Moreover, we demonstrate the charging and discharging process are highly dependent on physical parameters such as the coupling constant between the battery and the charger and the decay rates of the environments.

It is important to say that since our proposed model is physically very close to reality and the preparation of its laboratory settings is not so complicated, the results obtained here can be very influential for future researches.

The paper is organized as follows. In Sec. II a review of the QBs is provided. We represent our model and how we can obtain the time evolution of the system is explained in Sec. III. We focus on the results for Markovian and non-Markovian dynamics in Sec. IV. In order to investigate the stability of stored energy in the battery, we consider discharge time for two types dynamics in Sec. V. The paper concludes in Sec. VI.

II. QUANTUM BATTERIES

In this section, we briefly introduce QBs. A QB is a physical quantum system that stores energy. Each quantum system with distinct energy eigenstates can therefore be considered as a quantum battery for sustainable practical purposes. The energy can be stored at energy levels and coherence. Such energy storage always depend on the time-independent reference
Hamiltonian $H$ where Hilbert space of the battery is assumed to be finite. The useful energy can be extracted from a $QB$ in state $\rho$ is equal to the difference between the system’s internal energy $E_{\rho} = \text{tr}(\rho H)$ of state $\rho$ and the energy of the energetically lowest, accessible state $\sigma_p$, which is passive by definition. In other words, the stored energy is defined as
\[ \mathcal{W}_\rho = \text{tr}(\rho H) - \text{tr}(\sigma_p H). \] (1)

Therefore, it equals to the ergotropy introduced in [16, 17]. Passive states are defined as those states from which no work can be extracted in a cyclic unitary process $H$ and a given state $\rho$, where are written in their respective eigenbasis (increasing for $H$, decreasing for $\rho$)
\[ H = \sum_j \varepsilon_j |\varepsilon_j\rangle\langle\varepsilon_j|, \quad \varepsilon_{j+1} \geq \varepsilon_j \quad \forall j, \]
\[ \rho = \sum_j r_j |\varepsilon_j\rangle\langle r_j|, \quad r_{j+1} \leq r_j \quad \forall j. \] (2)

The state $\rho$ is passive with respect to $H$ if and only if 1. $\rho$ and $H$ are diagonal in the same basis
\[ [\rho, H] = 0, \] (3)
2. $\rho$ contains no population inversion, i.e.,
\[ \varepsilon_i < \varepsilon_j \Rightarrow r_i \geq r_j. \] (4)

We have denoted passive states by $\sigma_p$, thus it takes the following form
\[ \sigma_p = \sum_j r_j |\varepsilon_j\rangle\langle \varepsilon_j|. \] (5)

Note that no work can be extracted from a passive state, hence systems with passive quantum states are regarded as empty battery. Any state that is not passive will be called active [4]. Charging a $QB$ is to change its state from an initial state $\rho$ to a more energetic state and, conversely, using the battery will take it to a lower energy state. Here, charging (discharging) processes are not necessarily unitary and the quantum system may interact with its surrounding. Also, the ratio between the extractable work and the energy value of the battery can be written as
\[ R_\rho = \frac{\mathcal{W}_\rho}{E_\rho}, \] (6)
in addition, its instantaneous charging power is defined by
\[ p(t) = \frac{d\mathcal{W}_\rho}{dt}. \] (7)

In fact, how fast can a battery charged (or discharged), it depends on its power.

In the next section, we introduce the open quantum battery where the $QB$ access to a dissipative environment.

III. MODEL

Here, we consider a model consisting of four components: two two-level systems (two qubits) interacting with each other at given time interval, such that one is considered as a quantum battery $\mathcal{B}$ and another as charger $\mathcal{A}$, and each qubit is locally contacted with an amplitude damping reservoir $E$ (see Fig.1). So, the total Hamiltonian is given by
\[ H = H_0 + H_{int}, \] (8)
where
\[ H_0 = \frac{\omega_A}{2} (\sigma_A^+ I + I) + \frac{\omega_B}{2} (\sigma_B^+ I + I) + \sum_k (\omega_k^A \sigma_k^A \sigma_k^A \sigma_k^A + \sum_k \omega_k^B \sigma_k^B \sigma_k^B \sigma_k^B), \] (9)
in which two first terms indicate the free Hamiltonians of the charger and the $QB$ as well as two last terms show the free Hamiltonians of independent environments coupled to the charger and the battery, respectively. The transition frequency of the battery is indicated by $\omega_B$ and it is equal to $\omega_A = \omega_0 + \delta$ for the charger, this difference is due to an external and controlled magnetic field applied to the charger. Let $\sigma_k^j (j = A, B)$ denotes the Pauli operator in $z$ direction of the systems $A$ and $B$, respectively. Also, $\omega_k^j (j = A, B)$ is the frequency of the $k$th mode of the environments $A$ and $B$, respectively, $a_k$ and $b_k$ are the creation (annihilation) operators corresponding to the $k$th mode of the environments.

The interaction Hamiltonian is defined as
\[ H_{int} = \kappa (\sigma_+^A \sigma_+^B + \sigma_-^A \sigma_-^B) + \sum_k (\sigma_+^A \sigma_k^B + \sigma_-^A \sigma_k^B) + \sum_k (\sigma_+^B \sigma_k^A + \sigma_-^B \sigma_k^A) \] (10)
where the first term describes the interaction between the two qubits with the coupling constant $\kappa$, and two last summations show the coupling between the charger and the battery to their environments, that $g_k^j (j = A, B)$ is the coupling constant between the $j$th qubit and the $k$th mode of its corresponding environment. Here, $\sigma_\pm = \frac{1}{2} (\sigma_+ \pm \sigma_-)$ are the raising and lowering Pauli operators of the $j$th qubit.
In this work, we exactly obtain the one excitation time evolution of the total system. For this purpose, we work in the interaction picture defined by
\[ H_{int}(t) = e^{iH_0 t} H_{int} e^{-iH_0 t}, \]
in which the Hamiltonian takes the following form:
\[ H_{int}(t) = H^I_{AB} + H^I_t, \]
where
\[ H^I_{AB} = \kappa(\sigma^+_A \sigma^+_B e^{i\delta t} + \sigma^-_A \sigma^-_B e^{-i\delta t}), \]
and
\[ H^I_t = \sum_k (\sigma^+_A g^A_k a_k e^{(\omega_A - \omega^A_k) t} + \sigma^-_A g^+_A a^+_k e^{-i(\omega_A - \omega^A_k) t}) + \sum_k (\sigma^+_B g^B_k b_k e^{(\omega_B - \omega^B_k) t} + \sigma^-_B g^+_B b^+_k e^{-i(\omega_B - \omega^B_k) t}). \]

In the following, it is simple to investigate the number operator
\[ N = \sum_{j=A,B} (\sigma^+_j \sigma^-_j + \sum_k a^+_k a_k + \sum_k b^+_k b_k), \]
is a constant of motion i.e.; \([H, N] = 0\). At this point, let us consider the exact solution of the time-dependent Schrodinger equation, confining ourselves to the one-excitation subspace. First, assume the two qubits are in a linear combination of states with one excitation at \(t = 0\) and environments in vacuum state, which is determined by \(|0^B_k\rangle\). Thus, the initial state is
\[ |\psi(0)\rangle = |\mu(0)\rangle |e\rangle |g\rangle + \nu(0) |g\rangle |e\rangle \otimes |0^B_k\rangle, \]
where \(|\mu(0)\rangle^2 + \nu(0) |\nu(0)\rangle^2 = 1\), also, \(|g\rangle\) and \(|e\rangle\) are the ground and excited state of qubits, respectively. Hence, its time evolution will be
\[ |\psi(t)\rangle = |\mu(t)\rangle |e\rangle |g\rangle + \nu(t) |g\rangle |e\rangle \otimes (\sum_k \eta^A_k(t) |1^A_k\rangle |0^B_k\rangle + \sum_k \eta^B_k(t) |0^A_k\rangle |1^B_k\rangle), \]
where \(|1^A_k\rangle\) and \(|1^B_k\rangle\) characterize a state of the corresponding environment with one excitation in the \(k\)th mode. After some calculations, one can find the following differential equations for the probability coefficients \(\mu(t), \nu(t)\) and \(\eta^A_k(t), \eta^B_k(t)\):
\[ \dot{\mu}(t) = -i(\kappa e^{i\delta t} \mu(t) + \sum_k \eta^A_k(t) g^+_A a_k e^{(\omega_A - \omega^A_k) t}), \]
\[ \dot{\nu}(t) = -i(\kappa e^{i\delta t} \mu(t) + \sum_k \eta^B_k(t) g^+_B b_k e^{(\omega_B - \omega^B_k) t}), \]
\[ \eta^A_k(t) = -i \mu(t) g^+_A e^{-i(\omega_A - \omega^A_k) t}, \]
\[ \eta^B_k(t) = -i \nu(t) g^+_B e^{-i(\omega_B - \omega^B_k) t}. \]

Using two last differential equations in Eq. (18) and initial conditions \(\eta^A_k(0) = \eta^B_k(0) = 0\), the probability coefficients \(\eta^A_k(t), \eta^B_k(t)\) can be written as
\[ \eta^A_k(t) = -i g^A_k \int_0^t \mu(t') e^{-(\omega_A - \omega^A_k) t'} dt', \]
\[ \eta^B_k(t) = -i g^B_k \int_0^t \nu(t') e^{-(\omega_B - \omega^B_k) t'} dt'. \]
By applying above equations, the differential equations for \(\mu(t)\) and \(\nu(t)\) can be obtained as
\[ \dot{\mu}(t) = -i \kappa e^{i\delta t} \mu(t) - \int_0^t k_A(t - t') \mu(t') dt', \]
\[ \dot{\nu}(t) = -i \kappa e^{i\delta t} \nu(t) - \int_0^t k_B(t - t') \nu(t') dt', \]
where the kernels \(k_A(t - t')\) and \(k_B(t - t')\) are given by
\[ k_A(t - t') = \sum_k |g_k^A|^2 e^{i(\omega_A - \omega^A_k)(t - t')}, \]
\[ k_B(t - t') = \sum_k |g_k^B|^2 e^{i(\omega_B - \omega^B_k)(t - t')}. \]

For simplicity, we assume \(k_A(t - t') = k_B(t - t') = k(t - t')\). In continuum limit, the correlation function \(k(t - t')\) can be expressed as
\[ k(t - t') = \int_0^\infty d\omega J(\omega) e^{i(\omega - \omega^0)(t - t')}, \]
where \(J(\omega)\) is the spectral density of the environments. For the model considered in our study, it is taken as
\[ J(\omega) = \frac{\gamma \lambda^2}{2\pi[(\omega - \omega - \Delta)^2 + \lambda^2]}, \]
where \(\gamma\) is coupling constant which is connected to the relaxation time of the qubit \(\tau_R \approx \gamma^{-1}\) and \(\lambda\) is the width of the Lorentzian spectrum that is linked to the environment correlation time \(\tau_G \approx \lambda^{-1}\) and \(\Delta\) is the detuning of \(\omega\) and the central frequency of the environment. Therefore, one can obtain the correlation function \(k(t - t') = \frac{\gamma}{\lambda} e^{-(\lambda + \Delta)(t - t')}\).
At this step, let us define
\[ z(t) = \mu(t) + \nu(t), \]
\[ w(t) = \mu(t) - \nu(t). \]
Then, by regarding Eq. (21), one can write the following equations
\[ \dot{z}(t) = -i \kappa e^{i\delta t} z(t) - \int_0^t k(t - t') z(t') dt', \]
\[ \dot{w}(t) = -i \kappa e^{i\delta t} w(t) - \int_0^t k(t - t') w(t') dt'. \]
Applying the Laplace transform, the above equations becomes
\[ s \tilde{z}(s) - z(0) = -i \kappa \tilde{z}(s - i\delta) \tilde{k}(s) \tilde{z}(s), \]
\[ s \tilde{w}(s) - w(0) = i \kappa \tilde{w}(s - i\delta) \tilde{k}(s) \tilde{w}(s), \]
in which \(\tilde{z}(s), \tilde{w}(s)\), and \(\tilde{k}(s)\) are the Laplace transforms of \(z(t), w(t)\), and \(k(t - t')\), respectively. Finally, after some calculations, we can find (see Appendix A)
\[ \tilde{\mu}(s) = \frac{s - i\delta + \tilde{k}(s - i\delta) \mu(0) - i \kappa \nu(0)}{\left[s + \tilde{k}(s)(s - i\delta + \tilde{k}(s - i\delta)) + \lambda^2\right]}, \]
\[ \tilde{\nu}(s) = \frac{s - i\delta + \tilde{k}(s - i\delta) \nu(0) - i \kappa \mu(0)}{\left[s + \tilde{k}(s)(s - i\delta + \tilde{k}(s - i\delta)) + \lambda^2\right]} \]
Now, using inverseLaplace transform \(\tilde{\mu}(s)\) and \(\tilde{\nu}(s)\) in Eqs. (28) and (29), it is easy to obtain \(\mu(t)\) and \(\nu(t)\).
IV. RESULTS

According to Eq. (17), two-qubit system state $\rho_{AB}$ at a generic time instance $t$ can be written as

$$ \rho_{AB}(t) = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & |\mu(t)|^2 & \mu(t)\eta^*(t) & 0 \\ 0 & \mu(t)\eta^*(t) & |\nu(t)|^2 & 0 \\ 0 & 0 & 0 & \sum_k (|\eta_k(t)|^2 + |\eta_k^B(t)|^2) \end{pmatrix}. $$

(30)

It is straightforward to calculate internal energy of the battery

$$ E_B(t) = \omega_0|\nu(t)|^2, $$

and the ergotropy in Eq. (1) can be evaluated as

$$ W_B(t) = \frac{\omega_0}{2}[2|\nu(t)|^2 - 1 + 2|\nu(t)|^2 - 1]. $$

(32)

It is clear from Eqs. (31) and (32) that the energy $E_B(t)$ and ergotropy $W_B(t)$ of battery, for initial total state Eq. (16), depend on only the probability amplitude $\nu(t)$.

At this point, we assume at $t = 0$ the battery is empty, i.e., its state is $|g\rangle$ and the charger is in excited state, $|e\rangle$, so the initial state in Eq. (16) is $|\psi(0)\rangle = |e\rangle|g\rangle \otimes |0^*\rangle|0^B\rangle$. We analyze our results in two Markovian and non-Markovian dynamics. In order to distinguish the strong coupling regime from the weak coupling regime, we can define the dimensionless positive parameter $R \equiv \frac{\omega}{\gamma}$. It has been demonstrated that for $R \ll 1$ the dynamics is divisible (Markovian), while for $R \gg 1$ it becomes non-divisible (non-Markovian). Moreover, the ratio between $\gamma$ and $\kappa$ plays an important role in the energy transfer. Hence, we display two distinct regimes: an underdamped regime occurring for $\kappa \gg \gamma$, and an overdamped regime for $\kappa \ll \gamma$.

A. Markovian dynamics

Dynamical behaviors of the local energy $E_B(t)$ (dashed line) and the ergotropy $W_B(t)$ (solid line) of the battery are plotted in Fig. 2(a) and (b) as a function of $\omega_0t$ for the overdamped regime and the underdamped regime, respectively, where we have used Markovian dynamics ($R = 0.01$) for environments.

In Fig. 2, we have $\kappa = 0.1\omega_0$ for the red lines and $\kappa = 0.4\omega_0$ for the blue lines where the other parameters are $\Delta = 0$ and $\delta = 0.2\omega_0$. Also, we have considered $\gamma = \omega_0$ and $\lambda = 100\omega_0$ in Fig. 2(a), $\gamma = 0.05\omega_0$ and $\lambda = 5\omega_0$ in Fig. 2(b).

Looking at Fig. 2, we clearly see that whereas we cannot extract work from the battery in case of the overdamped regime, ergotropy is not zero for a finite time interval in the underdamped regime. In other words, in the underdamped regime and Markovian evolution, by increasing the coupling constant between the charger and the battery, we can extract work from the battery.

As one can see, the amount of the battery energy in Fig. 2(b) is larger than in Fig. 2(a), since the coupling between the battery and charger is stronger than the coupling between the battery and its Markovian environment, and for this reason we observe the damping fluctuations of the energy. These results indicate that in the Markovian dynamics, significant work cannot be extracted from the battery.

B. Non-Markovian dynamics

This subsection deals with the dynamics of energy and ergotropy of the battery in which non-Markovian reservoirs ($R = 10$), are considered. We present plots of time evolution of $E_B(t)$ (dashed line) and $W_B(t)$ (solid line) of the battery (both in units of $\omega_0$) as a function of $\omega_0t$ for local Markovian dynamics ($R = 0.01$). (a) we have overdamped regime where $\kappa = 0.1\omega_0$ for the red dashed line and $\kappa = 0.4\omega_0$ for the blue dashed line and the other parameters are $\Delta = 0$, $\gamma = \omega_0$, $\lambda = 100\omega_0$ and $\delta = 0.2\omega_0$; (b) Underdamped regime, we choose $\gamma = 0.05\omega_0$, $\lambda = 5\omega_0$ and the other parameters are the same as in (a).

![Figure 2](image-url)  

**Fig. 2**: (Color online). The energy $E_B(t)$ (dashed line) and ergotropy $W_B(t)$ (solid line) of the battery (both in units of $\omega_0$) as a function of $\omega_0t$ for local Markovian dynamics ($R = 0.01$). (a) we have overdamped regime while $\kappa = 0.1\omega_0$ and $\kappa = 0.4\omega_0$ for overdamped regime, respectively, where we have used Markovian dynamics ($R = 0.01$) for environments.

In Fig. 3(a), we have the overdamped regime while Fig. 3(b) shows the underdamped regime. In the figure, the blue lines display the case where $\kappa = 0.4\omega_0$ and the red lines indicate the case of $\kappa = 0.1\omega_0$. In addition, let us take $\gamma = \omega_0$, $\lambda = 0.1\omega_0$ in Fig. 3(a), $\gamma = 0.05\omega_0$ and $\lambda = 5\omega_0$ in Fig. 3(b), also we consider $\Delta = 0$ and $\delta = 0.2\omega_0$ as Fig. 2.

Fig. 3(a) illustrates that the work can be extracted form the battery in overdamped regime compared with the Fig. 2(a), and also the amount of stored battery energy is larger. In the underdamped regime, Fig. 3(b), the amount of the stored energy and the extractable work are increased compared with Fig. 3(a) and Fig. 2(b). Also as can be seen, there are periodic behaviors that this is useful for operational and practical tasks.

In following, the ratio between $E_B(t)$ and $W_B(t)$, i.e., $R_B(t)$ of the battery, for underdamped regime ($\kappa = 0.9\omega_0$), are plotted in Fig. 4; in which we have local Markovian dynamics ($R = 0.01$) in Fig. 4(a) and local non-Markovian dynamics ($R = 10$) in Fig. 4(b); other parameters are the same as in Fig. 2. As can be seen, the battery is fully charged in the non-Markovian dynamics and all the stored energy can be extracted form it, in addition, this behavior is periodic unlike Markovian dynamics where the ratio becomes zero after some oscillation.
FIG. 3: (Color online). The energy $E_B(t)$ (dashed line) and ergotropy $W_B(t)$ (solid line) of the battery (both in units of $\omega_0$) as a function of $\omega_0 t$ for local Non-Markovian dynamics ($R = 10$). (a) Overdamped regime we have $\gamma = \omega_0$ and $\lambda = 0.1\omega_0$; (b) Underdamped regime, we have $\gamma = 0.05\omega_0$, $\lambda = 0.005\omega_0$. The other parameters are the same as in Fig. 2.

C. discharge time

Here, we suppose that the charger is removed after the battery is fully charged and we investigate the time the battery loses its energy due to the interaction with its environment. For this aim, we assume the system is under the conditions as Fig. 4(b) and the battery is fully charged at time $\tau$, i.e., $\mu(\tau) = 1$, then according to Eq. 17, the total state at time $\tau$ will be

$$|\psi(\tau)\rangle = |e\rangle |g\rangle \otimes |0_B^A\rangle |0_B^B\rangle.$$  \hspace{1cm} (33)

Supposing the charger is instantly removed and the battery interacts only with its environment.

For simplicity, we consider $\tau = 0$ in Eq. 33, and we obtain the time which the energy of the battery is wasted because of the dissipation. We display the energy dynamics of the battery in the absence of the charger in Fig. 5. Dashed red line presents Markovian dynamics $R = 0.01$ and Dotted blue and dotted-dashed black lines show non-Markovian dynamics for $R = 10$ and $R = 100$, respectively; in addition, we have considered $\Delta = 0$. Also, the green solid line indicates non-Markovian dynamics for $R = 10$ and $\Delta = 10\omega_0$. Figure 5 demonstrates that the discharge time in the presence of non-Markovian environment is larger than Markovian environment. In addition, it illustrates the discharge time becomes much larger by increasing the value of the non-Markovianity. Moreover, as can be seen in inset, we see the energy of the battery will be preserved for long times in the non-Markovian dynamics by using $\Delta = 10\omega_0$.

As abstract, a battery can be fully charged and its energy can be preserved in the presence of non-Markovian environment.

FIG. 4: The $R_B(t)$ of the battery as a function of $\omega_0 t$ for underdamped regime $\kappa = 0.9\omega_0$. (a) local Markovian dynamics, (b) local non-Markovian dynamics.

FIG. 5: The energy $E_B(t)$ of the battery in the absence of the charger as a function of $\omega_0 t$. Dotted blue and dotted-dashed black lines show non-Markovian dynamics for $R = 10$ and $R = 100$, respectively, and dashed red line indicates Markovian dynamics $R = 0.01$. The green solid line shows non-Markovian dynamics for $R = 10$ and $\Delta = 10\omega_0$. Inset is plotted for long times.

V. CONCLUSION

This paper provided a model for quantum batteries, inspired by the inevitable interaction of quantum systems with its environment. In the model considered, the quantum battery interacts with the charger, while each of them is in contact with a dissipative environment. We demonstrated that the energy of battery cannot be usefully extracted in the Markovian dynamics, whereas the memory effects lead the battery to be fully charged in non-Markovian dynamics and also all
its energy can be extracted. In addition, we illustrated that the discharge time of the battery in non-Markovian dynamics is longer than the Markovian dynamics, and the stored energy can be preserved for long times in the presence of non-Markovian dynamics. These results suggest that memory effects can play an important role in improving the performance of quantum batteries in the framework of open system approach.

ACKNOWLEDGMENTS

This work has been supported by the University of Kurdistan. F. T. Tabesh and S. Salimi thank Vice Chancellorship of Research and Technology, University of Kurdistan.

Appendix A: Derivation of Eqs. (28), (29) in Sec. III

To this end, we begin with first equation of (27) in the main text

\[ s \tilde{z}(s) - z(0) = -i \kappa \tilde{z}(s - i \delta) - \tilde{k}(s) \tilde{z}(s), \]  

(A1)

and replace \( s \) with \( s + i \delta \), then we have

\[ \tilde{z}(s + i \delta)[s + i \delta + \tilde{k}(s + i \delta)] = z(0) - i \kappa \tilde{z}(s), \]  

(A2)

now, exchange \( \delta \) by \(- \delta\) in Eq.(A2), the above equation takes the following form

\[ \tilde{z}(s - i \delta) = \frac{z(0) - i \kappa \tilde{z}(s)}{[s - i \delta + k(s - i \delta)]}, \]  

(A3)

Substituting Eq.(A3) into (A1), we obtain

\[ \tilde{z}(s) = z(0)(\frac{1}{s + k(s) + \frac{i\kappa}{Q(s)}} - \frac{i\kappa}{Q(s)[s + k(s)] + \kappa^2}), \]  

(A4)

where \( Q(s) = s - i\delta + \tilde{k}(s - i\delta) \). Similarly, starting from the second equation in (27), one can get the following equation for \( \tilde{w}(s) \)

\[ \tilde{w}(s) = w(0)(\frac{1}{s + k(s) + \frac{i\kappa}{Q(s)}} + \frac{i\kappa}{Q(s)[s + k(s)] + \kappa^2}). \]  

(A5)

On the other hand, by following definitions

\[ \tilde{\mu}(s) = \frac{1}{2} \tilde{\tilde{z}}(s) + \tilde{\tilde{w}}(s) \],

\[ \tilde{\nu}(s) = \frac{1}{2} \tilde{\tilde{z}}(s) - \tilde{\tilde{w}}(s) \],

(A6)

and some straightforward calculations, they can be written as

\[ \tilde{\mu}(s) = \frac{[s - i\delta + \tilde{k}(s - i\delta)]\mu(0) - i\kappa \nu(0)}{[s + k(s)](s - i\delta + k(s - i\delta)] + \kappa^2} \]  

(A7)

\[ \tilde{\nu}(s) = \frac{[s - i\delta + \tilde{k}(s - i\delta)]\nu(0) - i\kappa \mu(0)}{[s + k(s)](s - i\delta + k(s - i\delta)] + \kappa^2} \]  

(A8)

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