ON THE FORM FACTORS OF THE $D_s^+ \rightarrow \phi \, \mu^+ \, \nu_\mu$ DECAY

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Abstract

We apply the infinite mass effective theory, when a heavy quark mass tends to infinity, and Chiral perturbation theory at the quark level, based on the extended Nambu – Jona – Lasinio model with linear realization of chiral $U(3) \times U(3)$ symmetry, to calculate the form factors of the $D_s^+ \rightarrow \phi \, \mu^+ \, \nu_\mu$ decay up to the first order in current $s$ – quark mass. The theoretical results are compared with experimental data and found to be in good agreement.

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Introduction

In our recent publications [1-3] we considered the form factors of the semileptonic $D \to \bar{K}^*(\bar{K}) e^+ \nu_e$ decays both in the chiral limit [1,2] and at first order in current $s$-quark mass expansion [3]. For the description of $D$-mesons we applied the infinite mass effective theory (IMET) [4,5], when the $c$-quark mass $M_c$ tends to infinity, i.e. $M_c \to \infty$. In the IMET, we describe the long - distance physics within Chiral perturbation theory at the quark level (CHPT) [6], based on the extended Nambu-Jona-Lasinio (ENJL) model with linear realization of chiral $U(3) \times U(3)$ symmetry [7]. The IMET supplemented by (CHPT)$_q$ has been successfully applied to the description of the fine structure of the mass spectra of non-strange $D (D^*)$ and strange $D_s^+ (D_s^{*+})$ and leptonic constants, caused by first order corrections in current-quark mass expansion [8]. The computation of the probabilities of strong and electromagnetic $D^*$ decays performed within IMET and (CHPT)$_q$ has given good results compared with experimental data [9].

In this paper we apply IMET and (CHPT)$_q$ to the calculation of the form factors of the $D_s^+ \to \phi \mu^+ \nu_\mu$ decay, keeping corrections up to first order in current $s$-quark mass.

1. The form factors of the $D_s^+ \to \phi \mu^+ \nu_\mu$ decay

The amplitude of the $D_s^+ \to \phi \mu^+ \nu_\mu$ decay is determined as follows

$$M(D_s^+ \to \phi \mu^+ \nu_\mu) = -\frac{G_F}{\sqrt{2}} V_{cs}^* <\phi(Q)|\bar{s}(0)\gamma_\alpha(1 - \gamma^5)c(0)|D_s^+(p) > \ell^\alpha (1)$$

where $G_F = 1.166 \times 10^{-5}$ GeV$^{-2}$ is the Fermi weak coupling constant, $|V_{cs}| = 0.975$ is the CKM mixing matrix element, $s(0)$ and $c(0)$ are the $s$ and $c$ current quark fields with $N$ colour degrees of freedom each, and $\ell^\alpha = \bar{u}(k_{\mu^+})\gamma^\alpha(1 - \gamma^5)v(k_{\mu^+})$ is the weak leptonic current.

The hadronic matrix element

$$M_\alpha(D_s^+ \to \phi) = <\phi(Q)|\bar{s}(0)\gamma_\alpha(1 - \gamma^5)c(0)|D_s^+(p) >$$

can be parametrized in terms of four form factors [3].
\[ M_\alpha \left( D_s^+ \to \phi \right) = i a_1 (q^2) \epsilon_\alpha^* (Q) - i a_2 (q^2) (\epsilon^* (Q) \cdot p) (p + Q) \alpha \\
- i a_3 (q^2) (\epsilon^* (Q) \cdot p) (p - Q) \alpha \\
- 2 b(q^2) \epsilon_{\alpha \beta \mu \nu} \epsilon^{* \beta} (Q) p^\mu Q^\nu, \]  

(3)

where \( q^2 \) is the square invariant mass of the lepton pair such that \( m_\mu^2 \leq q^2 \leq (M_{D_s^+} - M_\phi)^2 \) where \( M_{D_s^+} \), \( M_\phi \) and \( m_\mu \) are the masses of the \( D_s^+ \), \( \phi \) and \( \mu^+ \) mesons respectively and \( \epsilon^* \) is the polarisation tensor of the outgoing \( \phi \) meson.

We shall seek the form factors \( a_i(q^2) \) \( (i = 1, 2, 3) \) and \( b(q^2) \) in the form of an expansion in powers of the current \( s \)-quark mass upto first order terms

\[ a_i(q^2) = a_i^{(0)}(q^2) + a_i^{(1)}(q^2), \]
\[ b(q^2) = b^{(0)}(q^2) + b^{(1)}(q^2) \]

(4)

The form factors \( a_i^{(0)}(q^2) \) \( (i = 1, 2, 3) \) and \( b^{(0)}(q^2) \) are determined in the chiral limit \( (\text{ch.l.}) \). They are calculated in the same way as the corresponding form factors for the process \( D \to \bar{K}^* e^+ \nu_e \) \( [1,3] \), that is

\[ a_1^{(0)}(q^2) = \sqrt{\frac{3}{8}} M_* \]
\[ a_2^{(0)}(q^2) = \sqrt{\frac{3}{8}} \frac{M_*}{M_D^2} \left[ \frac{q^2}{M_D^2 - q^2} \right. \\
+ \left. \frac{M_D^2 - q^2}{M_*^2} \left( 1 - \frac{2 m M_D}{M_D^2 - q^2} \right) \ln \left( 1 + \frac{M_*^2}{M_D^2 - q^2} \right) \right], \]
\[ a_3^{(0)}(q^2) = -\sqrt{\frac{3}{8}} \frac{M_*}{M_D^2} \left[ \frac{2 M_D^2 - q^2}{M_D^2 - q^2} \right. \\
- \left. \frac{M_D^2 - q^2}{M_*^2} \left( 1 - \frac{2 m M_D}{M_D^2 - q^2} \right) \ln \left( 1 + \frac{M_*^2}{M_D^2 - q^2} \right) \right], \]
\[ b^{(0)}(q^2) = \sqrt{\frac{3}{8}} \frac{1}{M_*} \ln \left( 1 + \frac{M_*^2}{M_D^2 - q^2} \right). \]

(5)

Here we have denoted \( M_* = \sqrt{M_D \bar{v}'} / 2 \) where \( M_D = 1.86 \text{ GeV} \) is the mass of the charmed pseudoscalar meson at the chiral limit and \( \bar{v}' = 4 \Lambda = \)
2.66 GeV. The parameter $\Lambda$ appears as the cut-off in the Euclidian 3-dimensional momentum space evaluation of constituent quark loop diagrams. This cut-off $\Lambda$ is connected with the scale of spontaneous breaking of chiral symmetry (SBCS) in $(\text{CHPT})_q$ via the relationship $\Lambda = \Lambda_\chi / \sqrt{2} = 0.67$ GeV at $\Lambda_\chi = 0.94$ GeV [6].

The form factors $a^{(1)}_i(q^2)$ ($i = 1, 2, 3$) and $b^{(1)}(q^2)$ are determined by the matrix element [3]

$$M^{(1)}_\alpha (D^+_s \to \phi) = -i m_0 s_i \times \int d^4 x < h (Q)| T(\bar{s}(x)s(x)\bar{s}(0)\gamma_\alpha (1 - \gamma^5) c(0))|D^+_s(p) >_{\text{ch.}f}. \quad (6)$$

In accordance with the procedure expounded in [1-3,8,9] we can reduce the matrix element (6) to the expression

$$M^{(1)}_\alpha (D^+_s \to \phi) = g_D m_0 s_i \int d^4 x \int_{-\infty}^{\infty} d z_0 \theta (-z_0) \times \times < \phi(Q)| T(\bar{s}(x)s(x)\bar{s}(0)\gamma_\alpha \left(\frac{1 + \gamma^0}{2}\right) \gamma^5 s(z_0,0))|0 >_{\text{ch.}f}. \quad (7)$$

obtained at leading order in the large $N$ and $M_c$ expansion. The coupling constant $g_D$ has been calculated in [9]

$$g_D = \frac{2\sqrt{2} \pi}{\sqrt{N}} \left(\frac{M_D^2}{M_c v'}\right)^{1/2} \quad (8)$$

The r.h.s. of (7) involves only the light quark fields. Therefore for the evaluation of (7) we can apply $(\text{CHPT})_q$ [1-3,6,8,9]. Since the leading order of the r.h.s. of (7) in current quark mass expansion is fixed by the factor $m_0 s_i$, so the matrix element $< \phi(Q)| T(\ldots)|0 >$ has to be calculated in the chiral limit.

In order to evaluate the matrix element (7) let us compare this with the matrix element $M^{(1)}_\alpha (D \to \bar{K}^*)$ describing the $D \to \bar{K}^*$ transition at the first order in current $s$-quark mass expansion [3]

$$M^{(1)}_\alpha (D \to \bar{K}^*) = g_D m_0 s_i \int d^4 x \int_{-\infty}^{\infty} d z_0 \theta (-z_0) \times \times$$
where \( q = u \) or \( d \) for \( D^0 \) or \( D^+ \), respectively.

By applying the formulas of quark conversion (Ivanov [6]) one can show that, between matrix elements \( M^{(1)}_\alpha (D^+_s \to \phi) \) and \( M^{(1)}_\mu (D \to \bar{K}^*) \), there is the relationship

\[
M^{(1)}_\alpha (D^+_s \to \phi) = 2 M^{(1)}_\alpha (D \to \bar{K}^*) .
\] (10)

Readers can verify this relationship by noting that the \( \phi \) meson possesses the quark structure \( (\bar{s}s) \).

By virtue of the relationship (10) the form factors \( a^{(1)}_i (q^2) \) \((i = 1, 2, 3)\) and \( b^{(1)} (q^2) \) read [3]

\[
a^{(1)}_1 (q^2) = \sqrt{3} \frac{m_0 s}{M_s} \frac{\bar{v}}{4 m} M_D \ell n \left( \frac{\bar{v}}{4 m} \right),
\]

\[
a^{(1)}_2 (q^2) = -a^{(1)}_3 (q^2) = b^{(1)} (q^2),
\]

\[
b^{(1)} (q^2) = \sqrt{3} \frac{m_0 s}{M_s} \frac{\bar{v}}{4 m} \frac{M_D}{M_B^2 - q^2} \left[ 1 - \ell n \left( 1 + \frac{M_D^2}{M_B^2 - q^2} \right) \right].
\] (11)

Now we can get the numerical values of the form factors at \( q^2 = 0 \)

\[
a_1 (0) = a^{(0)}_1 (0) + a^{(1)}_1 (0) = 0.96 + 0.28 = 1.24 \text{ (GeV)},
\]

\[
a_2 (0) = a^{(0)}_2 (0) + a^{(1)}_2 (0) = 0.14 + 0.06 = 0.20 \text{ (GeV)}^{-1},
\]

\[
a_3 (0) = a^{(0)}_3 (0) + a^{(1)}_3 (0) = -0.42 - 0.06 = -0.48 \text{ (GeV)}^{-1},
\]

\[
b (0) = b^{(0)} (0) + b^{(1)} (0) = 0.21 + 0.06 = 0.27 \text{ (GeV)}^{-1}.
\] (12)

One finds that the first order corrections in current \( s \)-quark mass expansion are between 14 and 43\%. The form factors \( a_i (q^2) \) \((i = 1, 2, 3)\) and \( b (q^2) \) are connected with the standard form factors \( A_i (q^2) = (i = 1, 2, 3)\) and \( V (q^2) \) via the relations [1]
\[ A_1(0) = \frac{1}{M_{D_s} + M_\phi} a_1(0) = 0.43 \]
\[ A_2(0) = (M_{D_s} + M_\phi) a_2(0) = 0.60 \]
\[ A_3(0) = (M_{D_s} + M_\phi) a_3(0) = -1.44 \]
\[ V(0) = (M_{D_s} + M_\phi) b(0) = 0.81 \]

where \( M_{D_s} = 1.97 \text{ GeV} \) and \( M_\phi = 1.02 \text{ GeV} \) [10].

The theoretical values are in good agreement with the experimental data [11,12]

\[
(R_v)_\text{th} = \frac{V(0)}{A_1(0)} = 1.9, \quad (R_v)_\text{exp} = \begin{cases} 1.8 \pm 0.9 \pm 0.1 \text{ [11]} \\ 1.4 \pm 0.5 \pm 0.3 \text{ [12]} \end{cases}
\]
\[
(R_2)_\text{th} = \frac{A_2(0)}{A_1(0)} = 1.4, \quad (R_2)_\text{exp} = \begin{cases} 1.1 \pm 0.6 \pm 0.1 \text{ [11]} \\ 0.9 \pm 0.6 \pm 0.3 \text{ [12]} \end{cases}
\]

**Conclusion**

By applying IMET supplemented by \((\text{CHPT})_q\) we have evaluated the form factors of the \( D^+_s \to \phi \mu^+ \nu_\mu \) decay up to first order in current s-quark mass. The theoretical predictions compare reasonably well with experimental data. The proposed approach allows us to set up a relationship (10) between chiral corrections to the form factors of the decays \( D^+_s \to \phi \mu^+ \nu_\mu \) and \( D \to \bar{K}^* e^+ \nu_e \). Unfortunately, the possibility of the experimental investigation of the relationship (10) goes beyond the available accuracy of present day experimental abilities.

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