I review lattice studies of the hadrons: $D_s(2317)$, $X(3872)$, and $Y(4260)$.

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1. Introduction

I review lattice QCD results relevant to the recently discovered hadrons: $X(3782)$, $Y(4260)$, and $D_s(2317)$, because these seem, to me at least, to be the most interesting states from the perspective of solving and understanding QCD.

The physical picture behind lattice QCD calculations is that an interpolating operator creates a hadron in the QCD vacuum, and after a specific time interval the hadron is destroyed. The choice of interpolating operator is particularly important for hadrons that are thought to be a hybrid meson or a molecule of hadrons.

For example, for a $1^{--}$ state in the charmonium system, possible interpolating operators are

\begin{align}
O_1 &= \bar{c}\gamma_i c \\
O_2 &= \epsilon_{ijk}\bar{c}\gamma_5 F_{jk} c \\
M_2 &= (\bar{c}\gamma_i c)(\bar{q}q)
\end{align}

where $F_{jk}$ is the QCD field strength tensor, and $c$ and $q$ are creation operators for the charm and light quark respectively. Operator $O_2$ is a hybrid meson operator because it contains excited glue.

A critical issue for molecular interpolating operators, such as $M_2$ in equation 3, is whether the state is a genuine bound state or two mesons weakly interacting. One technique that is widely used, was developed by the Kentucky group, is to study the volume dependence of certain amplitudes in the calculation. For non-interacting scattering states the amplitude is proportional to the inverse of the volume, but for resonances the amplitude is independent of the volume.

An important issue is how close the parameters of unquenched lattice QCD calculations are to the QCD in the real world. The MILC collaboration have pion masses as low as 240 MeV, a dynamical range of lattice spacings between 0.06 and 0.15 fm, and 2+1 flavours of sea quarks. As reviewed by Schierholz at this conference, due to algorithm breakthroughs, other collaborations are now doing lattice calculations with comparable parameters. For example the ETM collaboration have accurate results at two lattice spacings with pion masses just under 300 MeV with 2 flavours of sea quarks. Unfortunately, the published results on the new heavy hadrons use older data sets, that are either quenched or unquenched with pion masses above 500 MeV.

The effect of the sea quarks could be important for hadrons close to threshold, such as the $D_s(2317)$ and $X(3872)$. In an unquenched lattice calculation, a sea quark loop in a meson has the quark content of $q_1\bar{q}_2q_3\bar{q}_4$. This dynamics is important for the mixing of tetraquark and quark-antiquark states. Also this diagram contains the dynamics of two meson decay that are sometimes included.
in quark models via coupled channel techniques \(^1\)\(^6\).

2. Lattice results for \(D_s(2317)\).

This state was discovered by BaBar and confirmed by CLEO, and BELLE \(^1\)\(^2\). The quantum numbers are thought to be \(J^P = 0^+\). The quark model predictions for the \(J^P = 0^+\) strange-charm meson were above the \(DK\) threshold. The experimental signal for the \(D_s(2317)\) was below the \(DK\) threshold with a small width. The closeness of the mass of the \(D_s(2317)\) to the mass of the \(DK\) threshold has caused some people to speculate that the experimental \(D_s(2317)\) is a hadron molecule.

After the discovery of the \(D_s(2317)\), Bali \(^7\) estimated the mass of the lightest strange-charm \(0^+\) meson to be 2.57(11) GeV from lattice QCD and suggested this provided some evidence for non-\(\tau\bar{s}\) interpretation of the \(D_s(2317)\). UKQCD, obtained the mass 2404(57) MeV for \(0^+\) using charm-strange interpolating operators, and claimed consistency between lattice and mass of \(D_s(2317)\) \(^8\). Lin et al. \(^9\) recently reported the mass of the \(D_s(0^+)\) to be 2379(40) MeV, from a quenched lattice QCD calculation at a single lattice spacing.

Another way of determining whether a state is a molecule or not is to compute the leptonic decay constant of the state \(^10\)\(^11\).

\[ \langle 0 | \bar{s}s | D_{0^+} \rangle = M_{D_{0^+}} f_{D_{0^+}} \]

There are some normalisation issues for leptonic decay constant that are discussed in \(^12\). A molecular state would have a small coupling to the \(\bar{s}s\) operator at the origin. From partially unquenched QCD UKQCD obtained \(^12\) \(f_{D_{0^+}} = 340(110)\) MeV. This is large on the scale relative to the pion decay constant and so is inconsistent with the \(D_{0^+}\) being molecular. The size of the leptonic decay constant can not discriminate between a localised 4 quark state and quark anti-quark state \(^12\). Other uses of \(f_{D_{0^+}}\) in phenomenology are discussed in \(^12\).

After the discovery of the \(D_s(2317)\), UKQCD studied the spectrum of the \(B_s\) states using unquenched lattice QCD \(^13\). Results for the masses of four \(L=1\) strange-bottom mesons were presented, as well as arguments for all four states to have narrow widths. For example, UKQCD \(^13\) obtained \(M(B_{s0}) - M(B_s) = 386 \pm 31\) MeV that is under the \(B_s\) \(K\) threshold. One of the results from \(^13\): \(M(B_{s2}) - M(B_s) = 534 \pm 52\) MeV, can be compared against the preliminary result from D0 \(^14\) of 469 \(\pm 1.4 \pm 1.5\) MeV.

UKQCD used unquenched lattice QCD to compute the decay width of 160 MeV for the lightest P-wave \(B\) meson to decay to the S-wave \(B\) meson and a pion \(^11\). Also an effective hadronic coupling for the decay of the lightest P-wave \(B_s \rightarrow BK\) was found to be of similar size to the coupling for the decay \(K(1412)\) to \(K\pi\). Since, the \(K(1412)\) is not thought to be molecular, this is additional evidence that the lightest P-wave \(B_s\) meson is not molecular.

3. Lattice results for \(X(3872)\).

The \(X(3872)\) was first discovered by Belle \(^1\)\(^2\). The mass is 3872.0 \(\pm 0.6 \pm 0.5\) MeV and the width is less than 2.3 MeV \(^1\)\(^2\). The \(X(3872)\) is thought to have \(J^{PC} = 1^{++}\) quantum numbers. The assignment \(J^{PC} = 2^{-+}\) for the \(X(3872)\) has not been ruled out \(^1\)\(^2\). The mass of \(X(3872)\) is very close to the \(D^0\overline{D}^{*0}\) threshold and this motivated the suggestion that the \(X(3872)\) is a molecule (other possibilities are reviewed in \(^1\)).

Quenched lattice calculations \(^15\) of the charmonium spectrum find the first excited state with \(J^{PC} = 1^{++}\) above 4.00(8) GeV (statistical errors only) using quark and anti-quark interpolating operators.

Chiu and Hsieh \(^16\) have used quenched lattice QCD to study the \(1^{++}\) state in
charmonium using molecular and diquark-antidiquark operators. They see a state at $3890 \pm 30$ MeV \cite{Chiu16} that has the expected volume dependence for a resonance. One concern about the results of Chiu and Hsieh \cite{Chiu16} is that their quark masses are large ($m_c a = 0.8$) in lattice units. This could mean that the systematic errors due to the non-zero lattice spacing are potentially large. Using the same lattice setup Chiu et al. \cite{Chiu17} computed the $f_D$ and $f_{D_s}$ decay constants and obtained good agreement with the recent experimental measurements by CLEO-c, so this is a crosscheck on their systematic errors.

4. Lattice results on $Y(4260)$.

The $Y(4260)$ (with $J^{PC} = 1^{--}$) was first seen by BaBar and has been confirmed by CLEO \cite{CLEO1,CLEO2}. Although there are many suggestions for the quark and glue content of the $Y(4260)$, perhaps the most popular one is that the state is a non-exotic hybrid meson (this still needs confirming of course) \cite{CLEO1,CLEO2}.

There have been a lot of lattice calculations that studied the exotic charmonium meson with $J^{PC} = 1^{-+}$. Although $J^{PC} = 1^{--}$ for the $Y(4260)$ hadron, the mass of the $1^{-+}$ state gives some indication of the hybrid excitation energy. In figure 4, I plot the mass difference between the $1^{-+}$ states and the S-wave states for both heavy and light quarks using data from \cite{UKQCD,CP-PACS}, as well as the experimentally determined masses of the $Y(3940)$ and $Y(4260)$ states. This shows that the mass of the $Y(4260)$ state is close to the mass of the $1^{-+}$ masses, but the mass of the $Y(3940)$ is too low \cite{Chiu16}.

In the heavy quark limit the decay width of the exotic $1^{-+}$ meson has been computed via lattice QCD \cite{Luo-Liu-JPB}. The excited potential de-excites to the standard heavy quark potential (Coulomb + linear) via the emission of light quark-antiquark pair. This transition was used to estimate the widths for $1^{-+} \rightarrow \chi_b S$ as $\sim 80$ MeV and the width for $1^{-+} \rightarrow \chi_b \eta$ to be less than 1 MeV, where $S$ is a scalar $0^{++}$ meson. The message from the lattice gauge theory calculation in \cite{Luo-Liu-JPB} is that the decay width of hybrid mesons to states that include light flavour singlet mesons could be sizeable.

Luo and Liu \cite{Luo-Liu} studied the non-exotic hybrid mesons in charmonium using a quenched lattice QCD calculation. The masses of the ground and excited states that coupled to the $1^{--}$ operator in equation 1 were 3.094(18) GeV (close to $J/\psi$) and 3.682(81) GeV (close to $\psi(2S)$). The masses of the ground and excited states that coupled to the $1^{--}$ hybrid meson operator in equation 2 were 3.099(62) GeV (close to $J/\psi$) and 4.379(149) GeV (close to $Y(4260)$). My main criticism of the work is that they use multi-exponentials fit models to fit single channel correlators. They used reasonable techniques, but that this type of fitting is still hard to do.

Chiu and Hsieh \cite{Chiu22} used hybrid and molecular operators (equations 1, 2, and 3) and additional operators to study the $1^{--}$ hadron in charmonium using a quenched lattice QCD calculation at a single lattice spacing. For the first excited state of the hybrid $1^{--}$ operator they obtain the mass 4501(178)(215) MeV in reasonable agreement with the calculation of Luo and Liu \cite{Luo-Liu}. 

| Bottom | UKQCD | CP-PACS | Juge et al. |
|--------|-------|---------|------------|
| Charm  | MILC  | MILC    | CP-PACS    |
| Light  |       |         |            |

$M(1^{+-}) - M_S$ mass splitting

\begin{tabular}{|c|c|c|c|c|}
\hline
Bottom & UKQCD & CP-PACS & Juge et al. \\
\hline
Char    & MILC  & MILC    & CP-PACS    \\
\hline
Light   &       &         &            \\
\hline
Y(3940) &       &         &            \\
Y(4260) &       &         &            \\
\hline
\end{tabular}
(whose result is reduced by taking the continuum limit). Using a molecular operator of the form \((q\bar{c}c\bar{q})\), Chiu and Hsieh \(^{22}\) obtained the mass 4238±31 MeV and they used the Kentucky volume method \(^{3}\) to show that the state was a resonance. Hence Chiu and Hsieh \(^{22}\) favour a molecular interpretation of the \(Y(4260)\).

Burch et al. \(^{23,24}\) have studied the explicitly mixing between the hybrid operator and \(\bar{q}q\) operator for the \(1^{-+}\) state using NRQCD in a quenched lattice QCD calculation. The \(\sigma_{M/Qa}\) term in the NRQCD Lagrangian is the one that mixes the hybrid and \(\bar{q}q\) operators to the order that they work. For the ground state \(Y\) they obtain the mixture

\[
|Y\rangle = 0.99826(6) |\bar{Q}Q\rangle - 0.059(1) |\bar{Q}Qg\rangle
\]

so the hybrid \( |\bar{Q}Qg\rangle\) contribution to the ground heavy-heavy \(1^{-+}\) state is small. A similar calculation for the first excited \(1^{-+}\) hadron would be useful to help understand the \(Y(4260)\), unfortunately the NRQCD expansion is not very reliable for charmonium.

5. Conclusions

My first "no-brainer" conclusion is that the above lattice calculations need to be repeated with modern unquenched lattice QCD data sets. It is particularly important to study the effect of sea quarks on the tetraquark/molecular interpolating operators. In quenched QCD, it seems that tetraquark/molecular and \(\bar{q}q\) operators couple to distinct states, however sea quarks will in principle cause these two type of operators to mix. The molecular versus quark-antiquark picture is also an issue for light hadrons such as the \(a_0(980)\). Two recent lattice calculations \(^{10,25}\) disagree on the quark content of the \(a_0(980)\).

In QCD there are also glueball interpolating operators. The potential effect of glueball dynamics on vector and pseudoscalar states in charmonium is discussed in \(^{26}\).

References

1. E.S. Swanson, Phys. Rept. 429 (2006) 243, hep-ph/0601110,
2. S. Godfrey, (2006), hep-ph/0605152,
3. N. Mathur et al., Phys. Rev. D70 (2004) 074508, hep-ph/0406196,
4. MILC, C. Bernard et al., (2006), hep-lat/0609053,
5. ETM, K. Jansen and C. Urbach, (2006), hep-lat/0610015,
6. C. Michael, PoS LAT2005 (2006) 008, hep-lat/0509023,
7. G.S. Bali, Phys. Rev. D68 (2003) 071501, hep-ph/0305209,
8. UKQCD, A. Dougall et al., Phys. Lett. B569 (2003) 41, hep-lat/0307001,
9. H.W. Lin et al., (2006), hep-lat/0607035,
10. UKQCD, C. McNeile and C. Michael, Phys. Rev. D74 (2006) 014508, hep-lat/0604009,
11. UKQCD, C. McNeile, C. Michael and G. Thompson, Phys. Rev. D70 (2004) 054501, hep-lat/0404010,
12. UKQCD, G. Heroioza, C. McNeile and C. Michael, Phys. Rev. D74 (2006) 014510, hep-lat/0604001,
13. UKQCD, A.M. Green et al., Phys. Rev. D69 (2004) 094505, hep-lat/0312007,
14. P. Castagnini, (2006), hep-ex/0605051,
15. P. Chen, Phys. Rev. D64 (2001) 034509, hep-lat/0006019,
16. TWQCD, T.W. Chiu and T.H. Hsieh, (2006), hep-ph/0603207,
17. T.W. Chiu et al., Phys. Lett. B624 (2005) 31, hep-ph/0506266,
18. K.J. Juge, J. Kuti and C.J. Morningstar, Phys. Rev. Lett. 82 (1999) 4400, hep-ph/9902336,
19. C. Michael, (2003), hep-lat/0302001,
20. UKQCD, C. McNeile, C. Michael and P. Pennanen, Phys. Rev. D65 (2002) 094505, hep-lat/0201006,
21. X.Q. Luo and Y. Liu, Phys. Rev. D74 (2006) 034502, hep-lat/0512044,
22. TWQCD, T.W. Chiu and T.H. Hsieh, Phys. Rev. D73 (2006) 094510, hep-lat/0512029,
23. T. Burch, K. Orginos and D. Toussaint, Phys. Rev. D64 (2001) 074505, hep-lat/0103025,
24. MILC, T. Burch and D. Toussaint, Phys. Rev. D68 (2003) 094504, hep-lat/0305008,
25. N. Mathur et al., (2006), hep-ph/0607110,
26. UKQCD, C. McNeile and C. Michael, Phys. Rev. D70 (2004) 034506, hep-lat/0402012,