Abstract

A model of natural inflation with an effectively trans-Planckian decay constant can be easily achieved by the “phase locking” mechanism while keeping field values in the effective field theory within the Planck scale. We give detailed description of “phase locked” inflation based on this mechanism. We also construct supersymmetric natural inflation based on this mechanism and show that the model is consistent with low scale supersymmetry. We also investigate couplings of the inflaton with the minimal supersymmetric standard model to achieve an appropriate reheating process. Interestingly, in a certain class of models, we find that the inflation scale is related to the mass of the right-handed neutrino in a consistent way with the seesaw mechanism.
1 Introduction

Cosmic inflation [1] (see also Ref. [2]) is a very successful paradigm of modern cosmology which not only solves various problems of the standard cosmology such as the horizon problem and the flatness problem, but also provides the origins of the large scale structure of the universe and the fluctuation of the cosmic microwave background (CMB) radiation [3]. Precise measurements of the CMB [4, 5] have shown that the so-called slow-roll inflation [6, 7] is remarkably successful.

Among various models of cosmic inflation, so-called natural inflation [8] has been considered to be one of the most attractive models, in which a pseudo-Nambu-Goldstone (NG) boson (pseudo-NGB) associated with spontaneously breaking of an approximate symmetry plays a role of the inflaton. The shape of the inflaton potential is well controlled by small explicit breaking of the spontaneously broken symmetry. After the announcement of a large tensor fraction in the CMB, \( r = O(0.1) \), by the BICEP2 collaboration [9], natural inflation has received renewed attention since it shows a perfect fit with the BICEP2 result [10].

Worrisome features of natural inflation are, however, the requirements of a decay constant larger than the Planck scale and the trans-Planckian variation of the inflaton field during inflation. Such trans-Planckian decay constant and field variation are not easy to be justified within the framework of the effective field theory valid below the Planck scale. There the Lagrangian is at the best given by a series expansion of fields with higher dimensional operators suppressed by the Planck scale. In this viewpoint of the effective field theory, the trans-Planckian decay constant and the field variation seem inevitably sensitive to the theory beyond the Planck scale.

In order to avoid troubles of Planck suppressed operators, so far, many attempts have been made where the trans-Planckian decay constant and field variation are realized in effective ways while keeping actual field values within the Planck scale. For example, alignment between potentials of natural inflation generated by multiple explicit breaking effects leads to the one with an effective decay constant larger than the Planck scale [11] (see also Ref. [12, 13] for recent discussions). In N-flation [14], many pseudo-NGBs are introduced and their collective dynamics lead to an effectively large decay constant. The monodromic behaviors of the pseudo-NGB also realize models with an effective decay constant larger than the Planck scale, which is originally proposed in models based on string theory [15, 16] and are gen-
eralized in field theoretic approaches [17, 18, 19, 20]. It is also pointed out 
that natural inflation potential generated by an anomaly in a large $N_c$ gauge 
dynamics has an effective decay constant much larger than the scale of the 
spontaneous symmetry breaking [21].

In Ref. [22], the authors proposed another class of models which achieve 
trans-Planckian field variations in an effective way in a field theoretic ap-
proach. There, the inflaton potential exhibits monodromic behavior due to 
the “phase locking” mechanism between two pseudo-NGBs, which can be 
immediately applied to natural inflation. In this paper, we refer to models 
in this class as “phase locked” inflation, and give detail discussion on the 
models.

We also propose a supersymmetric model of natural inflation based on 
the “phase locking” mechanism. In particular, we show that it is possible to 
construct a model consistent with low-scale supersymmetry in our approach. 
It should be emphasized that in the most models of supersymmetric natural 
inflation so far proposed, the explicit symmetry breaking term leading to the 
inflaton potential is proportional to the constant term in the superpotential, 
i.e. the gravitino mass. Therefore, in order to reproduce the measured prop-
erties of the CMB, the gravitino mass turns out to be very large and are 
not compatible with low scale supersymmetry in a multi-TeV range. In our 
model, on the other hand, the explicit symmetry breaking is separated from 
the $R$-symmetry breaking, and hence, the model does not require a large 
gravitino mass even for a high scale inflation.

The organization of this paper is as follows. In the next section, we 
give detailed descriptions of generic features of natural inflation based on the 
“phase locking” mechanism. We also discuss details of the inflation dynamics 
such as the initial condition of the inflaton. In section 3 we show a simple 
model of the supersymmetric natural inflation based on “phase locked” in-
fation which is compatible with low scale supersymmetry breaking. We also 
discuss the reheating process and some interesting features of the model. The 
final section is devoted to discussion and conclusions.

2 Phase Locked Inflation

In this section, we give a detailed description of the “phase locking” mecha-
nism proposed in Ref. [22], which can be applied to natural inflation so that 
the model has an effective decay constant larger than the Planck scale.
2.1 Brief review on natural inflation

Before going to details of the mechanism, let us briefly review natural inflation [8]. In natural inflation, the inflaton is identified with a pseudo-NGB associated with spontaneously breaking of an approximate symmetry. When the explicit breaking of the symmetry is dominated by a single breaking term, the inflaton potential is given by,

$$ V = \Lambda^4 (1 - \cos(a/f)) , $$

where $\Lambda$ denotes a parameter encoding the explicit breaking while $f$ is the decay constant of the pseudo-NGB $a$. From this potential, we immediately find that the slow-roll parameters,

$$ \epsilon = \frac{M_{Pl}^2}{2} \left( \frac{V'}{V} \right)^2 = \frac{M_{Pl}^2}{2f^2} \frac{\sin^2(a/f)}{(1 - \cos(a/f))^2} , $$

$$ \eta = \frac{M_{Pl}^2 V''}{V} = \frac{M_{Pl}^2}{f^2} \frac{\cos(a/f)}{(1 - \cos(a/f)))} , $$

satisfy the slow-roll conditions, $\epsilon \ll 1$ and $|\eta| \ll 1$, only for

$$ f \gg M_{Pl} \quad \text{and} \quad |a| \gg M_{Pl} . $$

Thus, if we simply identify the decay constant $f$ with the scale of spontaneous symmetry breaking by a vacuum expectation value (VEV), i.e.

$$ X \sim O(f) e^{ia/f} , $$

where $X$ denotes a field which is responsible for spontaneous symmetry breaking, the large decay constant requires a VEV much larger than the Planck scale. As cautioned in introduction, such a VEV larger than the Planck scale is difficult to be justified in the effective field theory below the Planck scale. We lose control over the inflaton potential from contributions of higher dimensional operators suppressed by the Planck scale.

2.2 Effectively large decay constant by phase locking

The decay constant $f$ is, however, not necessary to be of the order of the scale of spontaneous symmetry breaking. In fact, a decay constant much larger than the VEVs can be realized in a very simple manner, the “phase locking” mechanism proposed in Ref. [22].
To illustrate the mechanism, let us consider a model with an approximate $U(1)$ symmetry under which two complex scalar fields $\phi$ and $S$ have charges of $N$ and 1, respectively. The scalar potential consistent with the $U(1)$ symmetry is given by

$$V = V(\phi \phi^*, SS^*, \phi^* S^N).$$  

(5)

Now, let us assume that both $\phi$ and $S$ obtain VEVs in a similar size. In such a case, we have two candidates of the NGB, i.e. the phases of $\phi$ and $S$;

$$\phi|_{\text{NGB}} = |\langle \phi \rangle| e^{i \text{arg} \phi}, \quad S|_{\text{NGB}} = |\langle S \rangle| e^{i \text{arg} S}.$$  

(6)

Then, due to terms depending on $\phi^* S^N$, one of the NGB candidates obtains a large mass from\(^1\)

$$V \propto \phi^* S^N + \text{h.c.} = 2|\langle \phi \rangle| |\langle S \rangle|^N \cos(\text{arg} \phi - N \text{arg} S).$$  

(7)

As a result, the remaining NGB which will be identified with the inflaton corresponds to a combination,

$$a \propto N \text{ arg} \phi + \text{arg} S.$$  

(8)

Putting all together, the canonically normalized NGB, $a$, is given by,

$$\phi|_{\text{NGB}} = |\langle \phi \rangle| e^{i N a/f}, \quad S|_{\text{NGB}} = |\langle S \rangle| e^{i a/f}, \quad f \equiv \sqrt{2N^2 |\langle \phi \rangle|^2 + 2|\langle S \rangle|^2}.$$  

(9)

Remarkably, the decay constant $f$ can be much larger than the VEVs $\langle \phi \rangle \simeq \langle S \rangle$ for $N \gg 1$.

To generate the inflaton potential as in Eq. (1), we softly break the $U(1)$ symmetry by

$$V = M^3 S + \text{h.c.}.$$  

(10)

Here, one may consider that the explicit breaking parameter $M^3$ is a spurious field whose $U(1)$ charge is $-1$. The inflaton potential is dominated by the

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\(^1\)Here, we suppressed phases of the potential terms for simplicity.
contribution of this explicit breaking term. Substituting Eqs. (9) to (10), we obtain the desirable inflaton potential \(^2\)

\[
V(a) = 2|M^3 \langle S \rangle | (1 - \cos(a/f)) .
\]

(11)

Here, we have eliminated a constant phase and a sign inside the cosine by shifting \(a\), and added a constant term to the potential so that the cosmological constant vanishes at the vacuum. In this way, we obtain a model of natural inflation with a decay constant much larger than the scale of symmetry breaking.

It should be noted that the potential term Eq. (7) tightly interrelates the phases of \(\phi\) and \(S\) so that

\[
\arg S = \frac{1}{N} \arg \phi , \quad \text{i.e.} \quad S \propto \phi^{1/N} ,
\]

(12)
in the effective theory below the breaking scale. We refer to this phenomenon as “phase locking”. In view of the phase locking mechanism, the effectively large decay constant can be understood in the following way. Due to the phase locking, when \(\phi\) rotates \(2\pi\), \(S\) rotates only \(2\pi/N\). Since the inflaton potential is provided by the explicit breaking term on the phase rotation symmetry of \(S\) as in Eq. (10), the periodicity of the potential in terms of the phase of \(\phi\) is effectively enlarged to \(2\pi N\). Besides, for a large \(N\), the pseudo-NGB \(a\) is mainly composed of the phase of \(\phi\) due to its large charge \(N\) as long as \(\langle \phi \rangle \sim \langle S \rangle\). Therefore, the inflaton dynamics is approximately identified with the dynamics of the phase of \(\phi\), which can take effectively larger field values than the VEVs of \(\phi\) and \(S\). In Fig. 1, we show the inflaton trajectory on the \((\arg S, \arg \phi)\) plane for \(N = 5\) \(^3\).

The explicit breaking of the \(U(1)\) symmetry may also be provided by non-perturbative dynamics. For example, let us assume a QCD-like theory which exhibits spontaneous breaking of chiral symmetries at a dynamical

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\(^2\) For a very large \(N\), the scalar potentials which lock the phases of \(\phi\) and \(S\), i.e. the \(\phi^* S^N\)-dependent terms, are highly suppressed due to their high mass dimensions. In this case the potential in Eq. (7) is highly suppressed and hence the mode proportional to \(\arg \phi - N \arg S\) becomes lighter than \(a\) which ruins our discussion \(^{22}\). A possible solution to this problem is discussed at the end of this section.

\(^3\) For large \(N\), distances between the valleys of the potential is small and hence the tunneling process between valleys might disturb the inflation dynamics (see Fig. 3 in Appendix). We estimate the tunneling rate in Appendix and find that the tunneling process is irrelevant for sufficiently large \(\langle \phi \rangle\) and \(\langle S \rangle\).
Figure 1: (Left) A scalar potential as a function of the phases of $S$ and $\phi$ for $N = 5$. The inflaton trajectory corresponds to the valley is shown by the solid line. Arrowheads indicate the height of the inflaton potential along the trajectory, and the field values which maximize and minimize the potential in Eq. (11) are denoted by points. (Right) The shallow inflaton potential in Eq. (11) along the valley in the left panel as a function of $\arg[\phi]$.

scale $\Lambda_{\text{dyn}}$ far below the VEVs of $\phi$ and $S$. We couple $S$ to the gauge theory via a Yukawa coupling,

$$\mathcal{L}_{\text{int}} = y SQ\bar{Q},$$

where $Q$ and $\bar{Q}$ are fermion fields charged under the gauge symmetry and $y$ is a constant. Since the $U(1)$ symmetry is now anomalous, the scalar potential of the pseudo-NGB $a$ is generated.

We first consider the case where there are fermions which are charged under the gauge symmetry and are lighter than $\Lambda_{\text{dyn}}$. In this case, as is the case with the QCD-axion [24, 25, 26], the potential of $a$ is given by

$$V \simeq m_f \Lambda_{\text{dyn}}^3 (1 - \cos (\arg S)),$$

$$\simeq m_f \Lambda_{\text{dyn}}^3 (1 - \cos (a/f)).$$

(14)

where $m_f$ is the mass of the light fermions. Here, we have assumed that the light fermions have masses of the same order, for simplicity.

Next, let us consider the case where there is no light fermion which is charged under the gauge symmetry. In this case, it is difficult to calculate
the scalar potential of $a$; the simple cosine form such as the one in Eq. (14) is not guaranteed. For large $N_c$ dynamics, however, the potential is expanded as \[ V(a) = \Lambda_{\text{dyn}}^4 \frac{a^2}{f^2} \left( 1 + O \left( \frac{a^2}{f^2 N_c^2} \right) \right). \] (15)

For $a < fN_c$, the potential is approximated by a quadratic one. We can say that the effective decay constant is further enhanced by $N_c$,

$$f_{\text{eff}} \simeq N_c f \simeq N_c |\langle \phi \rangle|.$$ (16)

This enhancement is also helpful to have a successful natural inflation model from small field values \[21\].

Before closing this section, let us discuss an issue for a very large $N$. In this case, the phase locking potential in Eq. (7) is highly suppressed due to their high mass dimensions, and hence, the other inflaton candidate, $b \propto \arg \phi - N \arg S$, becomes lighter than $a$ for a given $M^2$ unless the VEVs of $S$ and $\phi$ are very close to $M_{\text{PL}}$. Unfortunately, however, $b$ cannot be a good candidate of the inflaton since the effective decay constant of $b$ is given by $f/N$, and hence, we cannot achieve an effectively large decay constant for $b$.

This problem can be evaded if we generalize the idea to a generic approximately $U(1)$ symmetric model and introduce complex scalar fields $\phi_i$ with $U(1)$ charges of $q_i$ ($i = 1, 2, \cdots, |q_i| \leq |q_{i+1}|, q_1 = 1$). We further assume that the $U(1)$ symmetry is spontaneously broken by the VEVs of all the complex scalar fields in similar sizes. With this generalization, the potential terms which corresponds to the phase locking potential in Eq. (7) are not necessarily suppressed since we have fields with variety of charges. Therefore, in this case, we expect that the pseudo-NGB $a$ associated with the spontaneous symmetry breaking resides in the complex scalar fields;

$$\phi_i \rightarrow |\langle \phi_i \rangle| \exp \left[ i q_i \frac{a}{f} \right],$$

$$f = \sqrt{2 \sum_i q_i^2 |\langle \phi_i \rangle|^2},$$ (17)

while other phase directions are strongly fixed by the phase locking potentials. The larger charge the field has, the more pseudo-NGB component the field has. Then, by explicitly breaking the $U(1)$ symmetry by a linear term of
the field with the smallest charge, the inflaton potential is that of natural inflation with the decay constant \( f \), which can be much larger than a VEV of each field.

### 2.3 Inflaton Dynamics and Initial Condition

Finally, let us discuss how inflation begins in our set up. Since we assume that the VEVs of \( \phi \) and \( S \) are below the Planck scale, it is natural to expect that the universe is in a symmetric phase at a very early epoch at around the Planck time. Then, it is generically expected that cosmic strings are formed when the universe experiences the \( U(1) \) phase transition.\(^4\) Then, as the energy of the universe further drops, the explicit breaking of the \( U(1) \) symmetry becomes important, and domain walls are formed in between cosmic strings. As a result, the universe is dominated by networks of strings and domain walls.\(^5\) This situation is quite similar to the evolution of the string-domain wall network in the case of the QCD-axion \(^6\) which is confirmed by numerical simulations.\(^6\)

In the string-domain wall network, we can find domain walls surrounded by cosmic strings around which the phases of \( \phi \) and \( S \) on the inflaton trajectory shown in Fig. 1 wrap. The energy density of this type of the domain walls is as large as \( \Lambda^4 \). A typical thickness of domain walls is as large as \( m_a^{-1} \), where \( m_a \) is the mass of the inflaton.\(^6\) On the other hand, the Hubble radius of a domain wall-dominated universe is as large as \( M_{\text{Pl}}/(m_a f) \). For \( f > M_{\text{Pl}} \), domains walls are thicker than the Hubble radius of the domain wall dominated universe. Thus, a region inside a domain wall can be regarded as an independent universe due to causality. Well inside domain walls, the field value of the inflaton is close to the “Maximum” in Fig. 1 where the slow-roll conditions are satisfied. That is, natural inflation begins in a region well inside the domain wall. This situation is the same as topological inflation.\(^6\)

\(^4\)Here, the cosmic strings corresponds to the so-called global string which is expected to be stable if it stretches across the Hubble volume.

\(^5\)We assume that the universe is open and enough long-lived; otherwise the universe collapses before entering this phase.

\(^6\)There also exist domain walls surround by cosmic strings around which the phases of \( \phi \) and \( S \) not on the inflaton trajectory wrap. The energy density of this type is as large as \(|\langle \phi \rangle||\langle S \rangle|^N |M_{\text{Pl}}^{N-3} |.\)
Table 1: Charge assignment of the inflaton sector.

## 3 Supersymmetric Natural Inflation

In this section, we propose a simple model of supersymmetric natural inflation compatible with low scale supersymmetry breaking, where the effectively trans-Planckian decay constant is realized by the phase locking mechanism.

### 3.1 Phase locking and NG multiplet

According to the recipe to construct models with the phase locking mechanism, we assume that the model possesses an approximate $U(1)$ symmetry where the explicit breaking leads to the inflaton potential. Concretely, we introduce four chiral multiplets $\phi, \bar{\phi}, S$ and $\bar{S}$ with $U(1)$ charges of $+N$, $-N$, $+1$ and $-1$, respectively. We also assume that the model possesses the $R$-symmetry under which the above chiral multiplets are neutral.

In order for them to obtain non-zero VEVs and to have their phases locked with each other, we further introduce three chiral multiplets, $Y_1$, $Y_2$ and $Y_3$ with the $R$ charges of 2 but with vanishing $U(1)$ charges. With these charge assignments, the generic superpotential is given by

$$W = \sum_{i=1,2,3} Y_i F_i (\phi \bar{\phi}, S \bar{S}, S^N \bar{\phi}, \bar{S}^N \phi),$$

where $F_i$ are generic holomorphic functions. With this superpotential, all directions except for the pseudo-NG multiplet are fixed, and the phases are locked.

For simplicity, we further introduce discrete symmetries $Z_2 \times Z_{2C}$ symmetry whose charge assignments are given in Table 1. The generic leading order superpotential is given by

$$W = Y_1 (\lambda_{1\phi} \phi \bar{\phi} - v_1^2) + Y_2 (\lambda_{2\phi} \phi \bar{\phi} + \lambda_{2S} S \bar{S} - v_2^2) + \lambda_3 Y_3 (S^N \bar{\phi} + \bar{S}^N \phi)/M_{Pl}^{N-1}. $$

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where $\lambda_1\phi$, $\lambda_2\phi$, $\lambda_2 S$ and $\lambda_3$ denote dimensionless coupling constants while $v_{1,2}$ are dimensional constants in a similar size with each other. We have eliminated the coupling of $Y_1$ with $S\bar{S}$ by taking appropriate linear combinations of $Y_1$ and $Y_2$. We take $\lambda_1\phi$, $v_{1}^2$, $\lambda_2 S$ and $v_{2}^2$ to be real and positive without loss of generality. As we will discuss later, we find the $Z_2 \times Z_{2C}$ symmetry is helpful to suppress the decay of the inflaton into gravitinos, though not mandatory. It should be emphasized that the other higher dimensional operators suppressed by $M_{\text{Pl}}$ are not very relevant for the following discussion since we assume that the VEVs of $\phi$’s and $S$’s are below the Planck scale.

Now, let us extract a multiplet of the pseudo-NGB associated with the $U(1)$ symmetry breaking. First, with the $F$ term conditions of $Y_1$ and $Y_2$, the chiral multiplets $\phi$, $\bar{\phi}$, $S$ and $\bar{S}$ are reduced to

$$
\phi \rightarrow v_\phi e^{\rho}, \quad \bar{\phi} \rightarrow v_\phi e^{-\rho}, \\
S \rightarrow v_S e^\sigma, \quad \bar{S} \rightarrow v_S e^{-\sigma},
$$

(20)

where $v_\phi = v_1/\sqrt{\lambda_1\phi}$ and $v_S = v_2/\sqrt{\lambda_2 S}$, and $\rho$ and $\sigma$ are the chiral multiplets. Then, the $F$ term condition of $Y_3$ further requires

$$
e^{N\sigma - \rho} + e^{-N\sigma + \rho} = 0 \iff \sigma = \frac{\rho}{N} + i\pi \frac{2n + 1}{2N}, \quad (n = 0, 1, \cdots, 2N - 1),
$$

(21)

which is nothing but the phase locking. In the following, we fix the phase locking direction to $\sigma = \rho/N + i\pi/(2N)$ by rotating $S$ and $\bar{S}$ by $e^{-i\pi n/N}$ and $e^{+i\pi n/N}$.\footnote{Note that the superpotential in Eq. (19) is invariant under this rotation by further rotating $Y_3$ by $-1$.}

According to the discussion in the previous section, we identify $\rho$ as the main component of the pseudo-NG multiplet for a large $N$ as long as $v_\phi \sim v_S$. The $F$-term condition of $Y_3$ generates the non-trivial potential mainly to $\sigma$\footnote{We may explicitly extract the massless chiral multiplet in the diagonalized base of $\rho$ and $\sigma$, although the following discussion is not significantly altered.} With this identification, we refer to $\rho$ as the inflaton multiplet whose imaginary part of the scalar component plays a role of the inflaton in natural inflation. We also refer to the real part of the inflaton multiplet as the s-inflaton.
3.2 Inflation dynamics

In order to give a potential to the inflaton via explicit $U(1)$ breaking, we introduce a chiral multiplet $X$ and spurious fields $\epsilon$ and $\bar{\epsilon}$ whose charge assignments are given in Table 1. We assume that the $Z_{2C}$ symmetry is not explicitly broken by these fields, and hence, we fix $\epsilon = \bar{\epsilon}$. By using these multiplet, we introduce explicit $U(1)$ breaking terms,

$$\Delta W = X(\epsilon S + \epsilon \bar{S}) = \epsilon X(S + \bar{S}) \ .$$

(22)

By substituting the NG modes appeared in Eqs. (21) and (22), we obtain the superpotential of the pseudo-NG multiplet,

$$\Delta W = \epsilon v S X \left( \exp \left[ \frac{\rho}{N} + i \frac{\pi}{2N} \right] + \exp \left[ -\frac{\rho}{N} - i \frac{\pi}{2N} \right] \right) ,$$

(23)

which has a similar structure with the superpotential used in the supergravity chaotic inflation [32]. From this superpotential, we again find that the defining region of the imaginary part of $\rho$, i.e. the inflaton, is effectively enhanced from $[0, 2\pi]$ to $[0, 2\pi N]$ due to the phase locking in Eq. (21). Then, by remembering that the kinetic term of $\rho$ is mainly originated not from $S$’s but from $\phi$’s, and hence, the canonically normalized inflaton multiplet $A$ is given by $A \sim \rho / v_{\phi}$, the effective decay constant appearing in Eq. (23) is $f \sim N v_{\phi}$. In this way, we can successfully supersymmetrize the phase locking mechanism which provides an effective decay constant larger than the VEVs in the linearly realized $U(1)$ symmetric models, i.e., $f = O(N \times v_{\phi, S})$.

Now, let us discuss behaviors of the scalar fields during inflation. As in the generic model of the chaotic inflation in supergravity [32], $X$ obtains a non-zero $F$-term, $F_X$, during inflation, which provides the inflaton potential. The scalar components of $X$ as well as the s-inflaton are, on the other hand, stabilized at their origins due to couplings to $XX^\dagger$ in the Kahler potential, assuming that those couplings provide them with positive squared masses of the order of the Hubble scale squared during inflation. After inflation, $F_X$ vanishes as the inflaton oscillation decays. In this way, our model realizes the natural inflation model in the same way as the non-SUSY models. It

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9 The resultant superpotential is analogous to the one given in Ref. [33] to construct natural inflation, which in our terminology is given by $W \propto X \sin[\rho/N]$. It should be emphasized that our model not only provides a simple ultra-violet completion to their model but also realizes the large decay constant via the phase locking mechanism at the same time.
should be emphasized that we have not required any $R$-symmetry breaking to realize the explicite $U(1)$ breaking effective potential in Eq. (23). Thus, the energy density of the universe during inflation which is proportional to the size of the explicite $U(1)$ braking is not related to $R$-symmetry breaking, i.e. the size of the gravitino mass. Therefore, in our model, the gravitino can be small even for high scale inflation, and hence, it is compatible with low scale supersymmetry breaking.

Finally, let us show the inflaton potential explicitly. Since we assume that the VEVs of $\phi$'s and $S$'s are below the Planck scale, the relevant Kahler potential terms of $\rho$ originate from,

\[ K = \phi\phi^\dagger + \bar{\phi}\bar{\phi}^\dagger + SS^\dagger + \bar{S}\bar{S}^\dagger \]

\[ = v_\phi^2(e^{(\rho+\bar{\rho})^\dagger} + e^{-(\rho+\bar{\rho})}) + v_S^2(e^{(\rho+\bar{\rho})}/N + e^{-(\rho+\bar{\rho})}/N) \]

\[ \approx v_\phi^2(e^{(\rho+\bar{\rho})^\dagger} + e^{-(\rho+\bar{\rho})}) \]

\[ = v_\phi^2 + v_\phi^2(\rho + \bar{\rho})^2 + \cdots \] (24)

where we have assumed $v_{\phi} \sim v_S$ and $N \gg 1$. Around the origin of the $s$-inflaton, the canonically normalized pseudo-NG multiplet $A$ is given by

\[ A = \sqrt{2}v_{\phi}\rho , \] (25)

as expected. Then, from Eqs. (23) and (25), the potential of the inflaton is given by

\[ V(a) = 2\epsilon^2 v_S^2 \left(1 - \cos \left(\frac{2a}{Nv_\phi} + \frac{\pi}{N}\right)\right) , \] (26)

where $a = \sqrt{2}\text{Im}A$. The effective decay constant is

\[ f = Nv_\phi/2, \] (27)

which is far larger than the VEV of $\phi$ if $N \gg 1$. In Fig. 2, we show the energy scale $\sqrt{\epsilon v_S}$ and the mass of the inflaton $m = \sqrt{2}\epsilon v_S/f$ for a given $f$. [8]

Here, we have used the observed magnitude of the curvature perturbation, $P_\zeta \simeq 2.2 \times 10^{-9}$ [5], and assumed that the number of e-foldings at the pivot scale of 0.002 Mpc$^{-1}$ is 55.
3.3 Reheating Processes

In this subsection, let us discuss the reheating process in our set up. For a while, we assume that the $Z_2 \times Z_2C$ symmetry introduced in the previous section is also respected in the interactions for the reheating process. Here, it should be noted that the inflaton possesses a $Z_2$ odd parity around the vacuum, i.e. at $F_X = 0$:

$$
\langle \rho \rangle = i\pi \left( \frac{N(2k + 1) - 1}{2} \right), \quad \langle \sigma \rangle = i\pi \left( k + \frac{1}{2} \right) \quad (k = 0, 1).
$$

That is, after $\phi$ and $S$ obtain non-vanishing VEVs, $Z_2 \times Z_2C$ is broken down to a $Z_2$ symmetry. This remaining $Z_2$ symmetry forbids the decay of the inflaton into gravitinos \cite{34,35,36}, and hence, the model with this symmetry is free from the gravitino problem induced by the inflaton decay.

Despite this advantage of the $Z_2$ symmetry, the symmetry at the same time forbids the decay of the inflaton into the radiation composed of the Standard Model particles unless they are also charged under the $Z_2$ symmetry\textsuperscript{10}. Fortunately, however, it is not difficult to make the Standard Model fields have non-trivial charges under the symmetry. For example, we may consider the charge assignment given in Table 2\textsuperscript{11} in which we have promoted the $Z_2 \times Z_2C$ symmetry to a $Z_4 \times Z_4C$ symmetry which has some

\textsuperscript{10}See also Ref. \cite{37} for related discussion.

\textsuperscript{11} With the charge assignment given in Table 2, the bilinear term of the Higgs, $H_uH_d$ have the same charges as $SS$ and $\phi\phi$. The super potential terms $W \supset Y_1H_uH_d, Y_2H_uH_d$ are allowed by the symmetry. When the squared mass terms of the Higgs doublets ($m^2_{H_u,d})$, are smaller than those of $S, \bar{S}, \phi$ and $\bar{\phi}$ ($m^2_{S,\phi}$), the vacuum with $\langle \langle H_u,d \rangle \rangle \ll \langle \langle S \rangle \rangle, \langle \langle S \rangle \rangle, \langle \langle \phi \rangle \rangle, \langle \langle \bar{\phi} \rangle \rangle$ is destabilized. In order to avoid such a problem we assume ei-
Table 2: Charge assignment of the inflaton sector and of the minimal supersymmetric standard model sector \((\bar{u}, Q, \bar{e}, d, L, H_u, H_d)\), and of the right-handed neutrinos \(N_R\).

|        | \(\phi\) | \(\phi\) | \(S\) | \(\bar{S}\) | \(Y_{1,2}\) | \(Y_3\) | \(X\) | \(\epsilon\) | \(\bar{\epsilon}\) |
|--------|---------|---------|-------|-------|--------|--------|-------|--------|--------|
| \(U(1)\) | \(+N\) | \(-N\) | \(+1\) | \(-1\) | 0      | 0      | 0     | \(+1\) | \(-1\) |
| \(U(1)_R\) | 0      | 0      | 0     | 0     | 2      | 2      | 2     | 0      | 0      |
| \(Z_4\) | \((-)^{N+1}\) | \((-)^{N+1}\) | \(-\) | \(-\) | \(+\) | \(-\) | \(+\) | \(+\) | \(+\) |
| \(Z_{4C}\) | \(\phi \leftrightarrow \bar{\phi}\) | \(S \leftrightarrow \bar{S}\) | \(\epsilon \leftrightarrow \bar{\epsilon}\) |
| \(\bar{u}, Q, \bar{e}\) | 0 | 0 | 0 | 0 | 0 |
| \(d, L\) | 1 | 1 | 0 | 0 | 1 |
| \(H_u\) | \(i\) | \(i\) | \(-\) | \(-\) | \(i\) |
| \(H_d\) | \(i\) | \(i\) | \(-\) | \(-\) | \(i\) |
| \(N_R\) | \(i\) | \(i\) | \(-\) | \(-\) | \(i\) |

With the above charge assignment, the dominant decay mode is provided by the Kahler potential term,

\[
K = \frac{y_1}{2M_{Pl}} X^\dagger N_R N_R + \text{h.c.,} \\
(29)
\]

which leads to the decay rate;

\[
\Gamma = \frac{y_1^2 m^3}{8\pi M_{Pl}} \\
(30)
\]

where \(y_1\) is a constant. The reheating temperature is

\[
T_{\text{RH}} \equiv \sqrt{\frac{90}{\pi^2 g_s}} \sqrt{\Gamma M_{Pl}} \\
= 1.6 \times 10^9 \text{ GeV} \times y_1 \left(\frac{m}{1.5 \times 10^{13} \text{ GeV}}\right)^{3/2} \left(\frac{g_s}{200}\right)^{-1/2}, \\
(31)
\]

\(\text{ther} m_{H_u,d}^2 > m_{S,\phi}^2\) or somewhat suppressed couplings between \(Y_{1,2}\) and \(H_u H_d\). The latter possibility can be realized, for example, by the so-called SUSY zero mechanism.

\(^{12}\)Here, we have assigned vanishing \(R\)-charges to the Higgs doublets so that the model is consistent with the Pure Gravity Mediation model \(^{38}\) where the so-called \(\mu\)-term is generated from \(R\)-symmetry breaking \(^{39, 40}\).
where $g_\ast$ is the effective degree of freedom of radiation. For $y_1 = O(1)$, the reheating temperature is high enough for successful thermal leptogenesis [41, 42]. Since the inflaton directly decays into the right-handed neutrino, non-thermal leptogenesis [43] is also possible.

As an interesting feature of the promoted discrete symmetry, $Z_4 \times Z_{4C}$, the masses of the right-handed neutrinos are interrelated to the explicit $U(1)$ breaking parameter $\epsilon$. That is, due to $Z_4 \times Z_{4C}$, the masses of the right-handed neutrinos are generated from the following coupling to the inflaton sector,

$$W = \frac{y_2}{2M_{Pl}} (\bar{\epsilon}S - \epsilon S) N_R N_R,$$

where $y_2$ is a constant. As a result, the right-handed neutrino masses are given by,

$$M_R = 2y_2 \frac{\epsilon v_S}{M_{Pl}} = 1.4 \times 10^{14} \text{ GeV} \times y_2 \frac{\epsilon v_S}{1.7 \times 10^{32} \text{ GeV}^2},$$

which are appropriate for the seesaw mechanism [23].

Let us also comment on the reheating process when the model does not respect the $Z_2 \times Z_{2C}$ symmetry. In this case, the inflaton decays into other particles through the supergravity effect without introducing any particular couplings between the inflaton and the Standard Model fields [44, 45]. On top of those spontaneous decays, the inflaton may decay, for example, via the superpotential term,

$$W = y_3 X H_u H_d$$

where $y_3$ is a constant, which tends to result in a very high reheating temperature [45]. It should be noted that without the $Z_2 \times Z_{2C}$ symmetry, the inflaton in general decays into gravitinos, which leads to overproduction of the lightest supersymmetric particle and/or spoils the success of the Big-Bang Nucleosynthesis. To evade this inflaton induced gravitino problem, we need to assume either the discrete symmetries as we did in this paper or to assume the fields in the SUSY breaking sector much heavier than the inflaton [47].

\footnote{If $y_3$ is larger than $m/M_{Pl}$, inflation ends not by the fast roll of the inflaton but by the “waterfall” of the higgs multiplets [46]. In this case, the predictions on the spectral index and the tensor fraction are different from that of the standard natural inflation.}
4 Discussion and Conclusions

In this paper, we gave detailed descriptions of “phase locked” inflation where the effectively trans-Planckian decay constant for natural inflation is achieved by the “phase locking” mechanism [22]. As we have discussed, the effectively trans-Planckian decay constant originates from the charge assignments of fields which break the $U(1)$ symmetry. This mechanism should be contrasted with other attempts to realize the effectively trans-Planckian decay constant field variation by, for example, alignment between several potentials of natural inflation or by using the collective behaviour of multi-inflatons of natural inflation. In our model, on the other hand, the effectively trans-Planckian decay constant can be achieved by only two fields without having alignments.

We also construct a model of supersymmetric natural inflation based on the “phase locking” mechanism. The advantageous feature of our model is that the explicit symmetry breaking of the $U(1)$ symmetry is separated from the $R$-symmetry breaking, and hence, the model does not lead to a large gravitino mass even for a high inflation energy scale. Therefore, our model is compatible with low scale supersymmetry.

We have also discussed how the reheating process proceeds in this model. In particular, we found that the decay of the inflaton can be well controlled by discrete symmetries which forbids the decay modes into gravitinos. We also found that the masses of the right-handed neutrinos appropriate for the seesaw mechanism can be related to the inflation scale in a certain class of models with discrete symmetries.

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A Tunneling between valleys

In this section, we estimate the tunneling rate between the valleys of the inflaton potential, whose trajectory is shown in Fig. 3 by the red arrow. For a large $N$, distance between valleys is short, and hence the tunneling process might disturb the inflation dynamics. To be concrete, we consider the two field model presented in section 2. The potential along the tunneling trajectory is sketched in the right panel of Fig. 3 with the parameters

$$\Delta b = \frac{2\pi}{N^2 f} \sim \frac{2\pi}{N} |\langle \phi \rangle|,$$

$$\epsilon \sim \frac{2\pi}{N} \Lambda^4, \quad \Lambda^4 \sim 10^{-8} M_{pl}^4,$$

$$U \sim \frac{1}{M_{pl}^{N-3}} |\langle \phi \rangle| |\langle S \rangle| N,$$

$$\mu^2 \sim \frac{1}{M_{pl}^{N-3}} |\langle \phi \rangle| |\langle S \rangle| N^2 \frac{N^2}{|\langle \phi \rangle|^2}. \quad (35)$$

Here, $b \sim \text{arg}S/N \times f$ denotes the canonically normalized field along the trajectory.
Note that $U > \epsilon$ is required so that $b$ does not move classically, which gives lower bound on $\langle \phi \rangle$ and $\langle S \rangle$. Under this assumption, the size of the core of the bounce solution of the tunneling process in the thin wall approximation \cite{48}, $U^{1/2}\Delta b/\epsilon$, is larger than the thickness of the skin of the bounce solution, $\mu^{-1}$. Thus the thin wall approximation is applicable. The tunneling rate per unit volume per unit time is given by \cite{48}

\begin{align*}
\Gamma & \sim \mu^4 \exp(-S_0), \\
S_0 & \sim \frac{27}{2\epsilon^3 \pi^2} U^2 \Delta b^4 = \frac{27\pi^6}{N} \frac{|\langle \phi \rangle|^6 |\langle S \rangle|^2}{\Lambda^{12} M_{Pl}^{2N-6}}.
\end{align*}

(36)

Since $\Lambda \sim 10^{-2} M_{Pl} \ll M_{Pl}$, $S_0$ is far larger than one for sufficiently large but sub-Planckian $\langle \phi \rangle$ and $\langle S \rangle$. The tunneling process shown in Fig. 3 can be suppressed. We note that the lower bound on $\langle \phi \rangle$ and $\langle S \rangle$ from the conditions $U/\epsilon$ and $S_0 \gg 1$ is relaxed by the generalization discussed at the end of section 2.

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