BIT ERROR AND BLOCK ERROR RATE TRAINING FOR ML-ASSISTED COMMUNICATION

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ABSTRACT

Even though machine learning (ML) techniques are being widely used in communications, the question of how to train communication systems has received surprisingly little attention. In this paper, we show that the commonly used binary cross-entropy (BCE) loss is a sensible choice in uncoded systems, e.g., for training ML-assisted data detectors, but may not be optimal in coded systems. We propose new loss functions targeted at minimizing the block error rate and SNR deweighting, a novel method that trains communication systems for optimal performance over a range of signal-to-noise ratios. The utility of the proposed loss functions as well as of SNR deweighting is shown through simulations in NVIDIA Sionna.

1. INTRODUCTION

Machine learning (ML) has revolutionized a large number of fields, including communications. The availability of software frameworks, such as TensorFlow [2] and, recently, NVIDIA Sionna [3], has made implementation and training of ML-assisted communication systems convenient. Existing results in ML-assisted communication systems range from the atomistic improvement of data detectors (e.g., using deep unfolding) [4–7] to model-free learning of end-to-end communication systems [8–10]. Quite surprisingly, only little attention has been devoted to the question of how ML-assisted communication systems should be trained. In particular, the choice of the cost function is seldom discussed (see, e.g., the recent overview papers [11, 12]) and—given the similarity between communication and classification—one usually resorts to an empirical cross-entropy (CE) loss [13–18]. The question of training a communication system for good performance over a range of signal-to-noise ratios (SNRs) is another issue that has not been seriously investigated. Systems are usually trained on samples from only one SNR [4, 9], or on samples uniformly draw from the targeted SNR range [5, 15, 17], apparently without questioning how this may affect performance for different SNRs.

In this paper, we investigate how ML-assisted communication systems should be trained. We first consider the case where the intended goal is to minimize the uncoded bit error rate (BER) and discuss why the empirical binary cross-entropy (BCE) loss is indeed a sensible choice in uncoded systems, e.g., for data detectors in isolation. However, in most practical communication applications, the relevant figure of merit is the (coded) block error rate (BLER), as opposed to the BER, since block errors cause undesirable retransmissions [19, Sec. 9.2], whereas (coded) bit errors themselves are irrelevant.\textsuperscript{1} We underscore that minimizing the (coded) BER is not equivalent to minimizing the BLER. This observation calls into question the common practice of training coded systems with loss functions that penalize individual bit errors (such as the empirical BCE), and thus optimize for the (irrelevant) coded BER instead of the BLER. In response, we propose a range of novel loss functions that aim at minimizing the BLER by penalizing bit errors jointly. We also show that training on samples that are uniformly drawn from a target SNR range will focus primarily on the low-SNR region while neglecting high-SNR performance. As a remedy, we propose a new technique called SNR deweighting. We evaluate the impact of the different loss functions as well as of SNR deweighting through simulations in NVIDIA Sionna [3]. All proofs, as well as additional analysis, are included in the extended arXiv version [1].

2. TRAINING FOR BIT ERROR RATE

ML-assisted communication systems are typically trained with a focus on minimizing the (uncoded) BER [5, 17], under a tacit assumption that the learned system could then be used in combination with a forward error correction (FEC) scheme to ensure reliable communication.\textsuperscript{2} Due to the similarity between detection and classification, the strategy typically consists of (approximately) minimizing the empirical BCE\textsuperscript{3} on a training set \( D = \{(b^{(n)}, y^{(n)})\}_{n=1}^{N} \), where \( b = (b_1, \ldots, b_K) \) is the vector of bits of interest (even in uncoded systems, one is interested in multiple bits, e.g., when using higher-order constellations, multiple OFDM subcarriers, or multi-user transmission), \( y \in \mathcal{Y} \) is the channel output, and \( n \) is the sample index. In fact, this strategy appears to be so obvious that it is often not motivated—let alone questioned—at all.

\textsuperscript{1}For this reason, physical layer (PHY) quality-of-service is assessed only in terms of BLER (not BER) in 3GPP LTE and other standards. Reference [20] notes that the relation between BER and BLER can be inconsistent.

\textsuperscript{2}The discussion also applies to systems that already include FEC, but we argue in Secs. 1 and 3 that minimizing the coded BER is a category mistake.

\textsuperscript{3}When we speak of the BCE between vectors, we mean the sum of binary CEs between the individual components as defined in (3), and not the categorical CE between the bit-vector and its estimate (as used, e.g., in [8–10]).
2.1. Minimizing the BCE Learns the Posterior Marginals

An “ML style” justification is to note that the expected BCE between the bit vector \( b \) and its estimate \( \hat{f}(y) = (f_1, \ldots, f_K) \) can be written as \( \sum_k H(b_k | y) + \mathbb{E}_y [D(p_{b_k | y} \| f_k)] \), where \( H(\cdot | \cdot) \) and \( D(\cdot \| \cdot) \) are the conditional and relative entropy. The expected BCE is thus minimized when the estimates \( f_k(y) \) equal the true posterior marginals \( p_{b_k | y} \).\footnote{This assumes that the transmitter is not trainable, so that \( H(b | y) \) is a constant. See [21] for a discussion that includes trainable transmitters.} Once the posterior is learned, simple thresholding (at \( \frac{1}{2} \)) results in BER-optimal data detection. The expected BCE is not available, but resorting to an empirical proxy through stochastic gradient descent is so common by now that it is often not even mentioned anymore.

We now argue explicitly—using the framework of empirical risk minimization (ERM)—that minimizing the empirical (as opposed to the expected) BCE can learn the true posterior marginals. We do not claim that this result is “novel,” but an explicit derivation seems unavailable in the literature. In the ERM framework, one learns a function

\[
\hat{f} = \arg \min_{f \in \mathcal{F}} L(f, \mathcal{D}),
\]

where \( \mathcal{F} \subseteq \{ f : \mathcal{Y} \to [0,1]^K \} \) is the set of admissible functions \( f = (f_1, \ldots, f_K) \) and

\[
L(f, \mathcal{D}) = \sum_{n=1}^N l_{\text{BCE}}(b^{(n)}, f(y^{(n)})),
\]

is the empirical risk, which here is induced by the BCE loss

\[
l_{\text{BCE}}(b, f) = -\sum_{k=1}^K b_k \log(f_k) + (1-b_k) \log(1-f_k). \tag{3}
\]

In principle, the empirical risk would be minimal if

\[
f(y^{(n)}) = b^{(n)}, \quad n = 1, \ldots, N. \tag{4}
\]

The optimal \( f \) would therefore make hard decisions on the training data set that—with hindsight—are always right. However, there are \textit{a priori} no restrictions on how such a function \( f \) responds to an input \( y \) that is not contained in \( \mathcal{D} \): We are at the danger of overfitting. ERM with a BCE loss may therefore be a reasonable strategy primarily in one of the following two settings: Either \( \mathcal{F} \) is “inflexible” or the range \( \mathcal{Y} \ni y \) is “small” compared to \( \mathcal{D} \). In either case, (4) cannot be satisfied and overfitting is prevented.\footnote{It has been argued that learned systems may also generalize to new inputs \textit{even when} they achieve perfect accuracy on the training dataset [22, 23]. An investigation of such settings is, however, beyond the scope of this paper.} The first case is more relevant in practice but more difficult to analyze. We therefore focus on the second case, which we formalize through the following assumption:

**Assumption 1.** We assume that \( \mathcal{D} \) is large and representative of the underlying posterior marginals \( p_{b_k | y} \) in the sense that, for some \( 0 < \varepsilon < 1 \) and for all \( k \) and all \( (b, y) \in \{0,1 \}^2 \times \mathcal{Y} \),

\[
|p_{b_k | y}(b = 1 | y) - \frac{1}{|\mathcal{N}(y)|} \sum_{n \in \mathcal{N}(y)} f_k^{(n)}| \leq \varepsilon, \tag{5}
\]

where \( \mathcal{N}(y) = \{ n \in \{1, \ldots, N \} : y^{(n)} = y \} \).

**Proposition 1.** Under Ass. 1, ERM with \( \mathcal{F} = \{ f : \mathcal{Y} \to [0,1]^K \} \) and BCE loss learns the posterior marginals up to precision \( \varepsilon \),

\[
|p_{b_k | y}(b = 1 | y) - \hat{f}_k(y)| \leq \varepsilon, \quad \forall y \in \mathcal{Y}, \quad k = 1, \ldots, K. \tag{6}
\]

The proof of this proposition (as well as of all following propositions) is included in the arXiv version [1, Sec. 7.1].

It should be interesting to translate this result to the case where \( \mathcal{Y} \) is uncountable but \( \mathcal{F} \) is “inflexible,” or even to the interpolating case described in [22]. We also note that, while the BCE is the most natural and probably most widely used loss in this context, it is by no means the only option. In fact, an analogous version of Prop. 1 holds for the mean square error (MSE) loss \( l_{\text{MSE}} : \{0,1 \}^K \times [0,1]^K \to \| b - f \|_2^2 / K \).

**Proposition 2.** Under Ass. 1, ERM with \( \mathcal{F} = \{ f : \mathcal{Y} \to [0,1]^K \} \) and MSE loss learns the posterior marginals up to precision \( \varepsilon \),

\[
|p_{b_k | y}(b = 1 | y) - \hat{f}_k(y)| \leq \varepsilon, \quad \forall y \in \mathcal{Y}, \quad k = 1, \ldots, K. \tag{7}
\]

2.2. Posterior vs. Posterior Marginals

We now draw attention to a subtle but conceptually important point: The loss in (3) considers the sum of empirical BCEs between the individual components of \( b \) and \( f \), and we have shown that this loss can be used to learn the posterior marginals \( p_{b_k | y}, k = 1, \ldots, K \). But this is not equivalent to learning the joint posterior \( p_{b | y} \), since we do not learn the conditional dependencies between the different bits \( b_k \). As a consequence of the summation of the component BCEs, \( f \) approximates the posterior as a product of independent distributions. For an information-theoretic perspective, see also [1, Sec. 7.2].

3. TRAINING FOR BLOCK ERROR RATE

3.1. The Difference Between BER and BLER Optimality

Learning to minimize the BLER in (block-)coded systems is not tantamount with learning to minimize the BER in those systems. To see this, consider a (block-)coded system in which the bits \( b = (b_1, \ldots, b_K) \) are encoded into codewords \( c = \text{enc}(b) \in \mathcal{C} \) for reliable data transmission. (In contrast to Sec. 2, we now look at multiple bits from the same data stream.) Optimal (coded) BER is obtained when we decode on the basis of the posterior probabilities \( p(b_k | y) \), which—as we have seen—can be learned, e.g., with a BCE loss function:

\[
\hat{b}_k = \arg \max_{b_k \in \{0,1 \}} p_{b_k | y}(b_k | y), \quad k = 1, \ldots, K. \tag{8}
\]

Perhaps surprisingly, this need not coincide with BLER-optimal decoding, which is achieved by the decoding rule

\[
\hat{b} = \text{dec}(\arg \max_{c \in \mathcal{C}} p_c | y)(c | y), \tag{9}
\]

where \( \text{dec} = \text{enc}^{-1} \) is the inverse mapping of the encoder. The reason is as follows: Even though the data bits \( b \) may be independent \textit{a priori}, their conditional distribution given the channel output, \( p_{b | y}(b | y) \), is in general no longer so, \( p_{b | y}(b | y) \neq \prod_{k=1}^K p_{b_k | y}(b_k | y) \). We have the following result:
Proposition 3. Bit error rate (BER) optimal decoding in (block-) coded communication systems need not coincide with block error rate (BLER) optimal decoding.

Since the BCE and MSE loss learn the posterior marginals instead of the joint posterior, they are inherently aimed at solving the BER-optimal decoding problem (8), but not the BLER-optimal problem (9) which is relevant in practice.

3.2. Loss Functions for Block Error Rate Optimization

We now propose several loss functions that aim at minimizing the usage of well-known smooth approximations to the Max loss. Among these are the SmoothMax, the LogSumExp, and the 2-norm loss.

\[ l_{\text{SM}}(\mathbf{b}, \ell; \alpha) = \sum_{k=1}^{K} x_k \exp(\alpha x_k) / \left( \sum_{k=1}^{K} \exp(\alpha x_k) \right), \quad \text{LogSumExp} \] (12)

with \( x_k = l_{\text{BCE}}(b_k, p(\ell_k)) \), where \( p(\cdot) \) maps from logits to probabilities. However, the max loss has the undesirable property that, for any given \((\ell, \mathbf{b})\), only one of the partial derivatives with respect to \( \ell_k \) is nonzero. We therefore also propose the usage of well-known smooth approximations to the Max loss. Among these are the SmoothMax loss with parameter \( \alpha \) (which we set to \( \frac{1}{2} \) in our experiments)

\[ l_{\text{SM}}(\mathbf{b}, \ell; \alpha) = \sum_{k=1}^{K} x_k \exp(\alpha x_k) / \left( \sum_{k=1}^{K} \exp(\alpha x_k) \right), \quad \text{LogSumExp} \] (12)

and the \( p \)-norm loss for \( p \geq 1 \) (with regularizer \( \gamma = 10^{-8} > 0 \))

\[ l_p(\mathbf{b}, \ell; \gamma) = (x_1^p + \cdots + x_K^p + \gamma)^{\frac{1}{p}}. \quad \text{(14)} \]

Remark. A popular loss for learning end-to-end communication systems is the categorical CE (CCE) between the transmitted and guessed message [8–10]. By identifying messages with the blocks of a block code, the CCE can be seen as a loss that optimizes the BLER. CCE-based learning, however, seems to be feasible only for very short blocks of \( K \approx 8 \) bits.

Fig. 1. The average loss of different BER (left) and BLER (right) losses is just as SNR-dependent as BER and BLER.

4. SNR DEWEIGHTED TRAINING

ML-assisted communication systems often learn a single set of parameters while operating over a range of SNRs. To perform well over an entire range, training data should be sampled from the targeted SNR range. However, the aggregate loss of the training set will then be dominated by low-SNR data samples. Consequently, training will focus on low-SNR performance, because a small relative improvement at low SNR will affect the cost much more than a large relative improvement at high SNR. Fig. 1 showcases the issue by visualizing the average loss when using an LDPC code with a classical BP decoder over an AWGN channel for the different loss functions as a function of SNR (normalized such that the average loss at 0 dB is 1). Evidently, the loss depends strongly on the SNR. In fact, the average losses closely mirror the bit/block error rates.\(^6\)

To compensate for this effect, we propose SNR deweighted training: Training consists of multiple epochs with \( M \) batches per epoch and \( N \) Monte-Carlo (MC) samples per batch. We partition every batch \( \{1, \ldots, N\} \) into \( J \ll N \) sets \( N(j), j = 1, \ldots, J \), each of which we associate with an SNR value that is selected from a uniform (in dB) grid which covers the desired range of operation. The loss of the \( n \)-th batch is defined as

\[ l_n = \frac{1}{N} \sum_{j=1}^{J} \sum_{n \in N(j)} w^{(j)}(t^{(n)}), \quad m = 1, \ldots, M, \quad \text{(15)} \]

where \( t^{(n)} = l((b_m^{(n)}, \ell_m^{(n)})) \) is the loss of the \( n \)-th MC sample in the \( m \)-th batch. The weights \( w^{(j)} \) are initialized to 1 and updated after every epoch: To balance the loss over the SNR range, we accumulate the loss over all samples with the same SNR, \( l^{(j)} = \sum_{n \in N(j)} t^{(n)} \). The weights for the next epoch are set to the inverse cumulative losses, plus a constant \( \delta > 0 \) that bounds the weight for stability: \( w^{(j)} = (l^{(j)} + \delta)^{-1} \).

To avoid global loss scaling, we normalize the weights by dividing by the weight at the grid center, \( w^{(j/2)} \), i.e., \( w^{(j)} = w^{(j)} / w^{(j/2)} \), before continuing training.

Alternatively, one might also perform SNR deweighting by using the loss of a fixed baseline (e.g., a classical communication system) to deweight the training samples, instead of using the adaptive reweighting strategy described here.

\(^6\)Note that the BER curve is shaped more like the BER losses (BCE/MSE), whereas the BLER curve is shaped more like the BLER losses. This supports the insight that BCE or MSE do not optimally target the BLER.
5. SIMULATION RESULTS

We evaluate the utility of the different losses and of SNR deweighting through simulations in NVIDIA Sienna v0.12.1 [3]. We consider a novel deep unfolded interleaved detection and decoding (DUIDD) receiver [24] for a 5G MIMO-OFDM wireless system with 4 single-antenna UEs and one 16-antenna base station. We use a short rate-matched (80, 60) 5G LDPC code based on a (520, 100) code with lifted base graph (BG) 2. The coded bit stream is mapped to QPSK symbols, which are transmitted over a 3GPP UMa line-of-sight wireless channel. The channel is estimated using pilots and a least-square estimator with linear interpolation across frequency and time.

In the first experiment, we consider the difference between BER and BLER performance with different losses for training (Fig. 2), as well as with an untrained “classical” receiver [24]. We learn a single parameter set by training over a \([-10, 10]\) dB interval (without SNR deweighting). We start by pre-training the receiver for 2500 batches of \(N = 200\) MC samples with the BCE (or MSE) loss. We then fine-tune the receiver by training with the respective loss functions for another 2500 batches. Because we do not use SNR deweighting, the low-SNR region dominates training. The results show that the BER losses (BCE and MSE, solid) have the best BER-performance in the dominant low-SNR region (1), but that the BLER losses (dashed) have superior BLER-performance at low-SNR (2). Somewhat surprisingly, we observe that in the high-SNR regime—which is neglected during training, since we do not use SNR deweighting—the BER losses outperform the BLER losses in terms of BER (2) as well as BLER (3). The improvement in BLER-performance of the best BLER loss compared to the best BER loss is 0.62 dB at a BLER of 1% (4).

In a second experiment, we select the product loss to consider the impact of different SNR training methods (Fig. 3). In the left figure, we compare naïve training over a large SNR range of \([-10, 10]\) dB (\(R_{[-10,10]}\)) with SNR deweighted training over that same range (\(DW_{[-10,10]}\)), as well as with training at a single SNR point at \(-5\) dB (\(P_{-5}\)). The results show that naïve training over the range, as well as training only at a single low-SNR point achieves good relative performance at low SNR (5) but comparably bad performance at high SNR (7). In fact, training only at a low-SNR point leads to a complete breakdown at very high SNRs (in this experiment). Finally, SNR deweighted training achieves well-balanced performance even when training over such a large SNR range. SNR deweighted training outperforms naïve training over the range by 0.54 dB at a BLER of 1% (7).

In the right figure of Fig. 3, we perform the same experiment, but training only over a smaller range of SNRs \([-4.6, 10]\) dB in the BLER waterfall region (or a single point therein). The results show significant convergence between the different training methods in this case. Training on a single SNR point (\(P_{5dB}\)) achieves slightly better performance than its competitors at high SNR (9), but performs worse at low SNR (8). SNR deweighted training still enjoys a (tiny) advantage over naïve training over the SNR range at high SNR (9), while naïve training over the SNR range enjoys an (even tinier) advantage at the low-SNR end of the waterfall (8). These results highlight that SNR plays an important role in training: naïve training over a range focuses excessively on low SNRs. Training at a single SNR sometimes works well, but sometimes leads to bad surprises. In contrast, SNR deweighted training seems to be robust and provide uniformly good performance.

The gains that are afforded in these experiments by BLER specific losses and by SNR deweighting (0.62 dB and 0.54 dB, respectively) may be modest. However, we emphasize that these gains are not caused by a more elaborate receiver or more training data, but simply by using a more appropriate loss function. As such, they are effectively available for free.

6. CONCLUSIONS

We have turned the spotlight on the impact that different loss functions and SNRs have on the training of ML-assisted communication systems. Seemingly obvious losses, such as empirical BCE, turn out to be suboptimal for minimizing the BLER and are outperformed by BLER-specific losses. We have also shown that naïve training over a range of SNRs will focus excessively on the low-SNR (high-loss) region and neglect high-SNR performance. To compensate for this effect, we have proposed SNR deweighting. The findings of this paper are not meant as final answers to the question of how to train communication systems, but rather as a starting point to some of the relevant issues and considerations.

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7. A second experiment on a simple trainable LDPC decoder for a single-input single-output (SISO) complex AWGN channel is included in the supplementary material of the extended arXiv version [1].

8. For pre-training, we applied BCE loss and trained on the same SNR range or point as in the latter refinement step, respectively, but without deweighting.
7. REFERENCES

[1] R. Wiesmayr, G. Marti, C. Dick, H. Song, and C. Studer, “Bit Error and Block Error Rate Training for ML-Assisted Communication,” arXiv:2210.14103, Mar. 2023.

[2] M. Abadi et al., “TensorFlow: Large-scale machine learning on heterogeneous systems,” 2015, [Online]. Available: https://www.tensorflow.org/.

[3] J. Hoydis, S. Cammerer, F. Ait Aoudia, A. Vem, N. Binder, G. Marcus, and A. Keller, “Sionna: An open-source library for next-generation physical layer research,” arXiv:2203.11854, Mar. 2022.

[4] T. J. O’Shea, T. Erpek, and T. C. Clancy, “Deep learning based MIMO communications,” arXiv:1707.07980, Jul. 2017.

[5] N. Samuel, T. Diskin, and A. Wiesel, “Learning to detect,” IEEE Trans. Signal Process., vol. 67, no. 10, pp. 2554–2564, May 2019.

[6] M. Khani, M. Alizadeh, J. Hoydis, and P. Fleming, “Adaptive neural signal detection for massive MIMO,” IEEE Trans. Wireless Commun., vol. 19, no. 8, pp. 5635–5648, Aug. 2020.

[7] A. Balatsoukas-Stimming and C. Studer, “Deep unfolding for communications systems: A survey and some new directions,” in IEEE Int. Workshop Signal Process. Sys. (SiPS), Oct. 2019.

[8] S. Dörner, S. Cammerer, J. Hoydis, and S. ten Brink, “Deep learning based communication over the air,” IEEE J. Sel. Topics Signal Process., vol. 12, no. 1, pp. 132–143, Feb. 2018.

[9] F. A. Aoudia and J. Hoydis, “Model-free training of end-to-end communication systems,” IEEE J. Sel. Topics Signal Process., vol. 37, no. 11, pp. 2503–2516, Nov. 2019.

[10] J. Song, C. Häger, J. Schröder, T. J. O’Shea, E. Agrell, and H. Wymeersch, “Benchmarking and interpreting end-to-end learning of MIMO and multi-user communication,” IEEE Trans. Wireless Commun., vol. 21, no. 9, pp. 7287–7298, Sep. 2022.

[11] A. Ly and Y.-D. Yao, “A review of deep learning in 5G research: Channel coding, massive MIMO, multiple access, resource allocation, and network security,” IEEE Open J. Commun. Soc., vol. 2, pp. 396–408, Feb. 2021.

[12] M. A. Albreem, A. H. Alhabbash, S. Shahabuddin, and M. Juntti, “Deep learning for massive MIMO uplink detectors,” IEEE Commun. Surveys Tuts., vol. 24, no. 1, pp. 741–766, Dec. 2021.

[13] T. Gruber, S. Cammerer, J. Hoydis, and S. ten Brink, “On deep learning-based channel decoding,” in 2017 51st Ann. Conf. Inf. Sciences Syst. (CISS), Mar. 2017, pp. 1–6.

[14] W. Xu, Z. Wu, Y.-L. Ueng, X. You, and C. Zhang, “Improved polar decoder based on deep learning,” in IEEE Int. workshop Signal Process. Sys. (SiPS), Oct. 2017, pp. 1–6.

[15] E. Nachmani, E. Marciano, L. Lugosch, W. J. Gross, D. Burshtein, and Y. Be’ery, “Deep learning methods for improved decoding of linear codes,” IEEE J. Sel. Topics Signal Process., vol. 12, no. 1, pp. 119–131, Feb. 2018.

[16] S. Cammerer, F. A. Aoudia, S. Dorner, M. Stark, J. Hoydis, and S. ten Brink, “Trainable communication systems: Concepts and prototype,” IEEE Trans. Commun., vol. 68, no. 9, pp. 5489–5503, Sep. 2020.

[17] M. Honkala, D. Korpi, and J. M. Huttunen, “DeepRx: Fully convolutional deep learning receiver,” IEEE Trans. Wireless Commun., vol. 20, no. 6, pp. 3925–3940, Jun. 2021.

[18] H. Song, X. You, C. Zhang, and C. Studer, “Soft-output joint channel estimation and data detection using deep unfolding,” in Proc. IEEE Int. Theory Workshop, Oct. 2021, pp. 1–5.

[19] E. Dahlman, S. Parkvall, and J. Skold, 5G NR : The Next Generation Wireless Access Technology, Academic Press, Sep. 2020.

[20] V. Lipovac, “Practical consistency between bit-error and block-error performance metrics up to application layer,” Wireless Personal Communications, vol. 93, no. 3, pp. 779–793, Dec. 2014.

[21] M. Stark, F. A. Aoudia, and J. Hoydis, “Joint learning of geometric and probabilistic constellation shaping,” in Proc. IEEE Global Commun. Conf. (GLOBECOM), Dec. 2019, pp. 1–6.

[22] A. J. Wyner, M. Olson, J. Bleich, and D. Mease, “Explaining the success of adaboost and random forests as interpolating classifiers,” J. Machine Learning Research (JMLR), vol. 18, no. 1, pp. 1558–1590, May 2017.

[23] M. Belkin, D. J. Hsu, and P. Mitra, “Overfitting or perfect fitting? Risk bounds for classification and regression rules that interpolate,” Proc. Advances Neural Inf. Process. Syst. (NeurIPS), vol. 31, Dec. 2018.

[24] R. Wiesmayr, C. Dick, J. Hoydis, and C. Studer, “DUIDD: Deep-unfolded interleaved detection and decoding for MIMO wireless systems,” in Asilomar Conf. Signals, Syst., Comput., Oct. 2022.