Calculations of $\bar{p}$-nuclear states within the Paris $\bar{N}N$ potential model

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Abstract. We performed calculations of $\bar{p}$-nuclear quasi-bound states using an optical potential derived from the 2009 version of the Paris $\bar{N}N$ potential model. Self-consistent treatment of strong energy dependence of the considered $S$- and $P$-wave $\bar{p}N$ scattering amplitudes is essential for proper evaluation of $\bar{p}$ binding energies and widths. The $P$-wave potential has very small effect on the calculated $\bar{p}$ binding energies, however, it reduces the corresponding widths in large measure. The Paris $S$-wave potential supplemented by a phenomenological $P$-wave term yields the $\bar{p}$ binding energies and widths very close to those obtained within the RMF model consistent with $\bar{p}$-atom data.

1. Introduction

Analyses of $\bar{p}$-atom data [1, 2, 3] and $\bar{p}$ scattering off nuclei at low energies [4, 5] revealed that the antiproton interaction with the nuclear medium near threshold and at low densities is dominated by $\bar{p}$ annihilation. The experimental data could be well fitted by a $\bar{p}$-nuclear optical potential, the imaginary part of which considerably exceeds the attractive real part. But deep in the nuclear interior, the phase space for $\bar{p}$ annihilation is significantly reduced. This could result in relatively long lifetime of $\bar{p}$ in the nuclear medium [6].

The knowledge of $\bar{p}$-nucleus interactions at various densities and kinematical conditions will be employed and further improved in forthcoming experiments with $\bar{p}$-beams at FAIR [7]. Simulations of the considered processes in a wide range of $\bar{p}$-beam momenta are being performed within the Giessen Boltzmann-Uehling-Uhlenbeck (GiBUU) transport model [8] in which the $\bar{p}$ optical potential in nuclear matter serves as input. The study of the $\bar{p}$ interaction with the nuclear medium is thus topical.

In this work, we report on our very recent calculations of $\bar{p}$ quasi-bound states in selected nuclei across the periodic table [9], using the $\bar{p}N$ scattering amplitudes stemming from the 2009 version of the Paris potential [10]. This potential was confronted by Friedman et al. [11] with $\bar{p}$-atom data and measured low-energy $\bar{p}$ elastic scattering and annihilation cross sections. Their analysis disclosed the necessity to include the $P$-wave part of the $\bar{p}N$ potential. The Paris $S$-wave potential supplemented by a phenomenological $P$-wave term was found to fit the data on low-density, near-threshold $\bar{p}$-nucleus interaction. We were thus tempted to explore the effect of the $P$-wave interaction on the calculated binding energies and widths of $\bar{p}$-nuclear states.

In Section 2, we briefly introduce the model applied in our calculations. Few representative results of our study are presented and discussed in Section 3.
2. Model

The binding energies $B_\bar{p}$ and widths $\Gamma_\bar{p}$ of $\bar{p}$-nuclear quasi-bound states are determined by solving self-consistently the Dirac equation

$$[-i\vec{\alpha} \cdot \vec{\nabla} + \beta m_\bar{p} + V_{\text{opt}}(r)]\psi_\bar{p} = \epsilon_\bar{p}\psi_\bar{p},$$

where $m_\bar{p}$ is the mass of the antiproton and $\epsilon_\bar{p} = -B_\bar{p} - i\Gamma_\bar{p}/2$, ($B_\bar{p} > 0$). The complex optical potential $V_{\text{opt}}(r)$ is of the form

$$2E_\bar{p}V_{\text{opt}}(r) = q(r) + 3\vec{\alpha} \cdot \alpha(r)\vec{\nabla}.$$  

(2)

The $S$-wave part $q(r)$ is constructed in a ‘t$\rho$’ form as

$$q(r) = -4\pi \frac{\sqrt{s}}{m_N} \left[F_0 \frac{1}{2} \rho_p(r) + F_1 \left(\frac{1}{2} \rho_p(r) + \rho_n(r)\right)\right],$$

(3)

where $E_\bar{p} = m_\bar{p} - B_\bar{p}$ and $\rho_p(r)$ [$\rho_n(r)$] is the proton (neutron) density calculated within the Relativistic Mean Field (RMF) model NL-SH [12]. The isospin 0 and 1 in-medium amplitudes $F_0$ and $F_1$ are constructed from the free-space $\bar{p}N$ amplitudes using the multiple scattering approach of Wass et al. [13] (WRW) which accounts for Pauli correlations in the nuclear medium.

$$F_1 = \frac{f_{\bar{p}n}^S(\delta\sqrt{s})}{1 + \frac{1}{4}\xi_k \frac{2}{m_N} f_{\bar{p}n}^S(\delta\sqrt{s})\rho(r)}, \quad F_0 = \frac{[2f_{pp}^S(\delta\sqrt{s}) - f_{\bar{p}n}^S(\delta\sqrt{s})]}{1 + \frac{1}{4}\xi_k \frac{2}{m_N} [2f_{pp}^S(\delta\sqrt{s}) - f_{\bar{p}n}^S(\delta\sqrt{s})]\rho(r)}.$$  

(4)

Here, $f_{\bar{p}n}^S$ ($f_{pp}^S$) denotes the free-space $\bar{p}n$ ($\bar{p}\bar{p}$) $S$-wave scattering amplitude as a function of $\delta\sqrt{s} = \sqrt{s} - E_{\text{th}}$ with $E_{\text{th}} = m_N + m_\bar{p}$; $\rho(r) = \rho_p(r) + \rho_n(r)$ is the nuclear density distribution and $\xi_k$ is the Pauli correlation factor (for details see [9]).

The $P$-wave part is written as

$$\alpha(r) = 4\pi \frac{m_N}{\sqrt{s}} \left(f_{pp}^P(\delta\sqrt{s})\rho_p(r) + f_{\bar{p}n}^P(\delta\sqrt{s})\rho_n(r)\right),$$

(5)

where $f_{pp}^P(\delta\sqrt{s})$ and $f_{\bar{p}n}^P(\delta\sqrt{s})$ denote the $P$-wave $\bar{p}\bar{p}$ and $\bar{p}n$ free-space scattering amplitudes, respectively. We do not consider medium modifications of the $P$-wave amplitudes because the $P$-wave interaction contributes mainly near the nuclear surface (due to its gradient form) where the nuclear densities are relatively low. Friedman et al. [11] revealed that the potential constructed from the Paris $S$- and $P$-wave amplitudes fails to fit the $\bar{p}$-atom data. On the contrary, the potential based on the Paris $S$-wave and phenomenological $P$-wave amplitude $f_{\bar{p}N}^P = 2.9 + i1.8$ fm$^3$ fits the data well [11]. We performed calculations using both the Paris and phenomenological $P$-wave interactions.

In Fig. 1, we demonstrate strong energy dependence of the Paris $\bar{p}N$ scattering amplitudes used in our calculations. In the upper panel, the in-medium $\bar{p}\bar{p}$ (left) and $\bar{p}n$ (right) $S$-wave amplitudes evaluated at the saturation density $\rho_0 = 0.17$ fm$^{-3}$ are plotted as a function of the energy shift $\delta\sqrt{s} = E - E_{\text{th}} \leq 0$. While the in-medium $\bar{p}\bar{p}$ amplitude is attractive in the entire energy range, the real part of the in-medium $\bar{p}n$ amplitude is attractive for $\delta\sqrt{s} \leq -70$ MeV with slightly repulsive dip near threshold. The lower panel of Fig. 1 shows the Paris free-space $P$-wave amplitudes below threshold.

The strong energy dependence of the $\bar{p}N$ amplitudes shown in Fig. 1 calls for proper self-consistent evaluation of the energy shift $\delta\sqrt{s}$. It is expressed using the Mandelstam variable

$$s = (E_N + E_\bar{p})^2 - (\vec{p}_N + \vec{p}_\bar{p})^2,$$

(6)
Figure 1. Energy dependence of the Paris 09 \( \bar{p}p \) and \( \bar{p}n \) two-body cm scattering amplitudes used in the present calculations. Upper panel: in-medium (Pauli blocked) \( S \)-wave amplitudes for \( \rho_0 = 0.17 \text{ fm}^{-3} \); lower panel: free-space \( P \)-wave amplitudes.

where \( E_N = m_N - B_{Nav} \) with \( B_{Nav} = 8.5 \text{ MeV} \) being the average binding energy per nucleon. When the interaction of \( \bar{p} \) with a nucleon takes place in a nucleus, the momentum dependent term in Eq. (6) does not vanish and gives rise to an additional downward energy shift [14, 15, 16]. The shift \( \delta \sqrt{s} \) can then be approximated as

\[
\delta \sqrt{s} = E_{th} \left( 1 - \frac{2(B_{\bar{p}} + B_{Nav})}{E_{th}} + \frac{(B_{\bar{p}} + B_{Nav})^2}{E_{th}^2} - \frac{T_{\bar{p}}}{E_{th}} - \frac{T_{Nav}}{E_{th}} \right)^{1/2} - E_{th},
\]

where the kinetic energy per nucleon \( T_{Nav} \) and the \( \bar{p} \) kinetic energy \( T_{\bar{p}} \) were calculated as the corresponding expectation values of the kinetic energy operator [9].

3. Results
We employed the above introduced in-medium \( S \)-wave and free-space \( P \)-wave scattering amplitudes derived from the Paris \( NN \) potential, as well as the phenomenological \( P \)-wave potential fitted by Friedman and Gal to \( \bar{p} \) atom data [11] and constructed the \( S + P \)-wave \( \bar{p} \)-nucleus optical potential [Eq. (2)]. The potential was then used in the self-consistent calculations of \( \bar{p} \)-nuclear quasi-bound states in selected nuclei across the periodic table.
We performed static, as well as dynamical calculations. While in the static calculations, the nuclear core remains unaffected by the presence of the antiproton, in the dynamical calculations, the antiproton polarizes the nuclear core, causing changes in the nuclear density distribution. It is to be noted that the response of the nuclear core to the antiproton is not instant and the characteristic time could be longer than the lifetime of the antiproton inside a nucleus [17, 18]. As a result, the antiproton probably annihilates before the nuclear core is fully polarized. Present static and dynamical calculations should be considered as two limiting scenarios.

In Fig. 2, we present $1s\bar{p}$ binding energies (left) and widths (right) as a function of mass number $A$, calculated statically (upper panel) and dynamically (lower panel) with the Paris $S$-wave (circles), Paris $S + P$-wave (pluses), and Paris $S$-wave + phen. $P$-wave (crosses) potentials. The binding energies calculated dynamically, $B_{\text{dyn}}^\pi \approx 200$ MeV, are somewhat larger than those obtained in static calculations, $B_{\text{stat}}^\pi \sim 150 - 200$ MeV; the polarization effects decrease with the mass number $A$. In dynamical and static calculations alike, both the Paris and phenomenological $P$-wave interaction terms do not affect much the $\bar{p}$ binding energies which are comparable with the binding energies evaluated using merely the $S$-wave potential.

Figure 2. $1s\bar{p}$ binding energies (left) and widths (right) in various nuclei, calculated statically (upper panel) and dynamically (lower panel) using the $S$-wave Paris potential (blue empty circles), including the Paris $P$-wave potential (red pluses), and the phenomenological $P$-wave potential (black crosses). Results for the phenomenological RMF potential from Ref. [16] (phen $V_{\text{opt}}$, green empty squares) are shown for comparison.
On the other hand, the $\bar{p}$ widths calculated dynamically are noticeably larger than the widths calculated statically. It is caused by the increase of the central nuclear density, which exceeds the decrease of the $pN$ amplitudes due to the larger energy shift with respect to threshold ($\delta \sqrt{s} \sim -255$ MeV in the dynamical case vs. $\delta \sqrt{s} \sim -200$ MeV in the static case).

The $\bar{p}$ widths exhibit considerably larger dispersion for the different potentials. The Paris $S$-wave potential yields sizable widths in all nuclei, $\Gamma_{\bar{p}} \geq 250$ MeV. When the $P$-wave interaction term is included the $\bar{p}$ widths are reduced significantly. The effect is even more pronounced for the Paris $P$-wave interaction, when the widths $\Gamma_{\bar{p}} \leq 150$ MeV, exhibiting strong $A$-dependence. On the contrary, the widths calculated with the phenomenological $P$-wave term [11], as well as only with the $S$-wave potential vary much less with $A$.

It is to be noted that the form of the $S + P$-wave potential is a result of delicate interplay between the $S$- and $P$-wave parts which are linked together, as well as the balance between the real and imaginary parts of the $P$-wave amplitudes since they control the range of the potential. It was demonstrated by Friedman and Gal [11] that the Paris $S$-wave potential supplemented by the contribution of the Paris $P$-wave amplitudes fails to achieve reasonable fit to $\bar{p}$-atom data. The real and imaginary parts of the Paris $P$-wave had to be scaled by different factors in order to obtain satisfactory description of the data. We may infer that the real and imaginary parts of the Paris $P$-wave amplitudes are not well balanced in the energy region relevant to $\bar{p}$-nuclear states calculations.

Finally, Fig. 2 shows that the Paris $S$-wave + phen. $P$-wave potential yields very similar $\bar{p}$ binding energies and widths as the phenomenological RMF approach [16]. The widths are in the range $\sim 200 - 230$ MeV, exhibiting strong $A$-dependence. The agreement between the phenomenological $\bar{p}$-optical potential [16], parameters of which were tuned to fit the $\bar{p}$-atom data, and the Paris $S$-wave potential supplemented by the phenomenological $P$-wave term is quite impressive.

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