From qubits and actions to the Pauli–Schrödinger equation

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Abstract
Here I show that a classical or quantum bit state plus one simple operation, an action, are sufficient ingredients to derive a quantum dynamical equation that rules the sequential changes of the state. Then, by assuming that a freely moving massive particle is the qubit carrier, it is found that both, the particle position in physical space and the qubit state, change in time according to the Pauli–Schrödinger equation. So, this approach suggests the following conjecture: because it carries one qubit of information the particle motion has its description enslaved by the very existence of the internal degree of freedom. It is compelled to be described no more classically but by a wavefunction. I also briefly discuss the Dirac equation in terms of qubits.

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1. Introduction
Quantum mechanics (QM) can be instructed either by adopting the schemes proposed by its inventors (Born, Heisenberg, Schrödinger and Jordan) or, more rigorously, following Dirac, within the Hilbert space framework, or even using Feynman’s path integral approach. As it is believed to be pedagogically more appealing, almost all textbooks prefer to begin with the nonrelativistic approach, discussing the wave–particle dualism, wavefunctions, the Schrödinger equation (SE), Hilbert space, noncommutative operators, etc. However, looking at the emblematic dynamical equations of Schrödinger and Dirac, one notes that the SE is less fundamental than Dirac’s relativistic equation, because this one contains inherently the internal degree of freedom spin, while it is absent in the former. In between there is the Pauli–Schrödinger equation (PSE), which was derived by Pauli when applying the low-energy approximation in the Dirac equation. Although being nonrelativistic, the PSE is more complete than the SE because the spin is inherently present, while in the SE the spin must be added as an extra degree of freedom. See [1] for a detailed discussion.

At the beginning of the 1980s the possibility of quantum computation was foreseen by authors like Benioff, Feynman and Deutsch [2–4], and their work influenced a mini-revolution that began in the 1990s, which threw a new perspective on QM, mainly in its interpretation and potentialities to explain new phenomena; in the last 15 years, we have witnessed huge theoretical developments along with ingenious experiments involving a few or single atoms or molecules, electrons and photons. So the understanding of quantum physics has widened, shaping the new arena called quantum information theory (QIT) that borrowed many concepts of classical information theory. In this context there seems to exist a recognition [5] that QM is a special kind of information theory immersed in Hilbert space, and characterized by a reversible logic [6–8].

In this connection, by using elementary concepts of communication theory, such as bits and gates, but represented in the framework of the Hilbert space, I will show here that a quantum evolution equation for one classical bit (Cbit, as defined in [9]) or quantum bit (qubit) of information can be derived. Then, by asking what could be the carrier of one qubit (or spin 1/2), the natural choice is a particle of mass $m$ characterized by its kinetic energy, and this information is introduced in the qubit dynamical equation. This procedure is sufficient to derive the PSE that rules the time evolution of both, the qubit and the particle, its carrier. So the particle is compelled to display wave properties, i.e. to be represented by a wavefunction; it ceases to be a particle in the Newtonian sense to become a kind of wave–particle hybrid and the particle dynamics become enslaved by the qubit dynamics. The qubit/spin evolution acquires ascendancy over the particle motion, being at the root of the observed quantum properties.
of matter. Last but not least, the Dirac equation and its solution are briefly discussed in terms of qubits.

2. Cbits and actions

In classical information theory the numbers in \( \mathbb{Z}_2 = \{0, 1\} \) are associated with bits, as in a relay or in a memory storage device. One can go one step further and associate a particular representation with the numbers 0 and 1: a column matrix representing one classical bit of information, the Cbit state, \( 1 \rightarrow (\alpha) \) and \( 0 \rightarrow (\beta) \), like the states ‘up’ and ‘down’ for the spin 1/2. These states can be written in the more familiar form of Dirac’s kets and bras, the kets \( |x\rangle \), \( |\bar{x}\rangle \) (\( x = 1 \), \( \bar{x} = 0 \)) for \( x, \bar{x} \in \mathbb{Z}_2 \), and the bras are transposed, so \( \mathcal{H}_2^\ast = \langle 1|, 0| \rangle \) is the dual space of \( \mathcal{H}_2 = \{0, 1\} \). The qubit \( \langle x| \) is a generalization of the Cbit, with \( a \) and \( \beta \) being complex numbers. The simplest operators to be used are the identity \( I \), \( I|x\rangle = |x\rangle \), and the NOT \( X \) that inverts the Cbit state, \( X|x\rangle = |\bar{x}\rangle \). Each set of numbers \( \alpha, \beta \) commutes with all others.

Assuming the parameters \( \alpha \) and \( \beta \) in \( U(\alpha, \beta) \) to be real, with \( \alpha^2 + \beta^2 = 1 \) and set \( \alpha^2 = \beta^2 = 1 \), so \( \alpha, \beta \in \mathbb{R}_2 \) is the set of all real numbers on a circle of radius 1. So acting on a Cbit one gets a qubit, \( U(\alpha, \beta)|x\rangle = \alpha|x\rangle + \beta|\bar{x}\rangle \). Two consecutive operations give \( U(\alpha_2, \beta_2)U(\alpha_1, \beta_1) = U(\alpha_3, \beta_3) \), and as \( \alpha_2^2 + \beta_2^2 = \alpha_3^2 + \beta_3^2 = 1 \), it follows that \( \alpha_3 \alpha_2^* + \beta_3 \beta_2^* = 1 \), \( \alpha_3 \beta_2^* + \beta_3 \alpha_2^* = 0 \), and \( (\alpha_1, \beta_1) \notin \mathbb{R}_2 \) and \( U(\alpha_1, \beta_1) \) is not an element of the group, unless one of the four coefficients is zero, therefore any probabilistic interpretation for \( \alpha^2 \) and \( \beta^2 \) fails. Moreover, the inverse action is \( U^{-1}(\alpha, \beta) = \bar{\alpha}I + \bar{\beta}X \), where the new parameters \( \bar{\alpha} = \alpha/(\alpha^2 - \beta^2) \) and \( \bar{\beta} = -\beta/(\alpha^2 - \beta^2) \) are not in \( \mathbb{R}_2 \). Due to the reality of \( \alpha \) and \( \beta \), \( U(\alpha, \beta) \) is a self-adjoint operator \( U^\dagger(\alpha, \beta) = U(\alpha, \beta) \), however, it is not unitary since \( U^\dagger(\alpha, \beta) \neq U^{-1}(\alpha, \beta) \); although the norm \( \|U(\alpha, \beta)|x\rangle\| = 1 \) is parameter independent, this is not true for the inverse \( \|U^{-1}(\alpha, \beta)|x\rangle\| = \|\alpha^2 - \beta^2 \|^{-1} \). Thus, if we want to construct an evolution operator \( U(\alpha, \beta) = \prod_{j=1}^n (\alpha_jI + \beta_jX) \), with \( \alpha_j^2 + \beta_j^2 = 1 \), that is also reversible, we are in trouble. Since the inverse of \( U(\alpha, \beta) \) is \( U(\bar{\alpha}, \bar{\beta}) \), for a sequence of \( n \) inverse actions we have \( U_n^{-1}(\bar{\alpha}, \bar{\beta}) = \prod_{j=1}^n U_j(\bar{\alpha}_j, \bar{\beta}_j) \), however as \( \alpha_j^2 + \beta_j^2 = (\alpha_j^2 - \beta_j^2)^{-1} \), therefore there is no normalization.

2.1. Coefficients on a circle of unit radius

I now assume the parameters \( \alpha \) and \( \beta \) in \( U(\alpha, \beta) \) to be real, with \( \alpha^2 + \beta^2 = 1 \) and set \( \alpha^2 = \beta^2 = 1 \), so \( \alpha \) and \( \beta \) are real numbers. The simplest operators to be used are the identity \( I \), \( I|x\rangle = |x\rangle \), and the NOT \( X \) that inverts the Cbit state, \( X|x\rangle = |\bar{x}\rangle \). Each set of numbers \( \alpha, \beta \) commutes with all others. A sequence of actions \( \alpha, \beta \) carries the evolution \( |x\rangle \rightarrow |\bar{x}\rangle \). Due to the reality of \( \alpha \) and \( \beta \), \( U(\alpha, \beta) \) is a self-adjoint operator \( U^\dagger(\alpha, \beta) = U(\alpha, \beta) \), however, it is not unitary since \( U^\dagger(\alpha, \beta) \neq U^{-1}(\alpha, \beta) \); although the norm \( \|U(\alpha, \beta)|x\rangle\| = 1 \) is parameter independent, this is not true for the inverse \( \|U^{-1}(\alpha, \beta)|x\rangle\| = \|\alpha^2 - \beta^2 \|^{-1} \). Thus, if we want to construct an evolution operator \( U(\alpha, \beta) = \prod_{j=1}^n (\alpha_jI + \beta_jX) \), with \( \alpha_j^2 + \beta_j^2 = 1 \), that is also reversible, we are in trouble. Since the inverse of \( U(\alpha, \beta) \) is \( U(\bar{\alpha}, \bar{\beta}) \), for a sequence of \( n \) inverse actions we have \( U_n^{-1}(\bar{\alpha}, \bar{\beta}) = \prod_{j=1}^n U_j(\bar{\alpha}_j, \bar{\beta}_j) \), however as \( \alpha_j^2 + \beta_j^2 = (\alpha_j^2 - \beta_j^2)^{-1} \), therefore there is no normalization.

2.2. Reversibility and complex coefficients

In order to establish the reversibility of a sequence of actions, the domain of \( \alpha \) and \( \beta \) must be extended to the field of complex numbers because the conditions \( \alpha^2 + \beta^2 = 1 \) and \( \alpha^2 - \beta^2 = 1 \) implies \( |\beta|^2 + \beta^2 = 0 \) must be satisfied. This happens for \( \alpha \) real and \( \beta \) pure imaginary, \( \beta = -i\beta \). Since one is left with one free parameter only, a natural parameterization is \( \alpha = \cos \xi \) and \( \beta = -i\sin \xi \) (\( \xi \) real), thus the action \( U(\alpha, \beta) \equiv U(\xi) = \cos \xi I - i\sin \xi X \) is a universal unitary operator mapping a Cbit or a qubit into a qubit, \( U(\xi)|x\rangle = \cos \xi |x\rangle - i\sin \xi |\bar{x}\rangle \). So, the complex nature of \( U(\xi) \) is due to the reversibility property. A sequence of actions

\[
U_n(\xi) = \prod_{j=1}^n (\cos \xi_j I - i\sin \xi_j X). 
\]
Thus \( U(\phi) \) becomes \( U(n\vec{\xi}) = \exp[-in\vec{\xi} \cdot \vec{X}] \), which stands for a sequence of actions, or an evolution. The previously undetermined intervals between actions become equally spaced, see figure 1(b), characterizing the uniformization of their distribution. In order to turn the distribution dense, I shall look for a differential equation for \( U(n\vec{\xi}) \) by taking first the difference between two consecutive values of \( n \) and then dividing by \( \vec{\xi} \).

\[
U \left( (n + 1)\vec{\xi} \right) - U \left( n\vec{\xi} \right) = \left( e^{-i\xi \cdot \vec{X}} - 1 \right) \exp[-in\vec{\xi} \cdot \vec{X}].
\]

The limit to a continuous parameter is obtained for \( n \gg 1 \) and \( \vec{\xi} \ll 1 \), keeping, however, the product \( n\vec{\xi} \) finite. A linear differential equation results, \( i\hbar \partial U(\tau)/\partial \tau = XU(\tau) \), and \( U(\tau) = e^{i\tau \cdot \vec{X}} \), where \( \tau \) is the continuous ordering parameter of the actions, or a local time in arbitrary units, that should be set according to the clock to be used. Writing \( |x_r\rangle = U(\tau)|x_0\rangle \) the evolution equation \( i\hbar \partial |x_r\rangle /\partial \tau = \vec{X}\langle x_r| \) says how a qubit evolves as a function of \( \vec{X} \), which is the generator of changes.

Defining the generator \( G = \mu I + v \vec{X} \), \( \mu \) and \( v \) being two real parameters, the evolution equation is generalized,

\[
\imath \hbar \frac{d}{d\tau} |\psi_\tau\rangle = G |\psi_\tau\rangle,
\]

with \( U(\tau) = e^{-i\tau \mu I} e^{-i\tau v \vec{X}} \) for the evolution operator. Different from the factor \( e^{-i\tau \mu I} \) that does really affect the evolution of a qubit, the phase factor \( e^{-i\tau v \vec{X}} \) is apparently not significant because, besides a global phase factor, it does not entail any change when acting on a Cbit or a qubit. The eigenvalues and eigensates of \( G \) are, respectively,

\[ G_{\pm 1} = (\mu \pm v), \quad |G_{\pm 1}\rangle = (|1\rangle \pm |0\rangle)/\sqrt{2}. \]

A general solution to equation (5) is \( |\psi_\tau\rangle = \sum_{\sigma=\pm 1} e^{i\sigma \epsilon \tau /\hbar} |\sigma\rangle \) where \( G_{\sigma} = \mu + \sigma v \). Now conjecturing about the qubit carrier, it is assumed to be a massive particle \( |0\rangle \) and the parameter \( \mu \) is chosen to represent its energy; thus the change \( X \rightarrow G \) was important because it allowed the introduction of that particle property. \( G \) can be identified as a Hamiltonian, and for an arbitrary initial condition the mean value is \( \langle \psi_\tau|G|\psi_\tau\rangle = \mu + v(|c_1|^2 - |c_-|^2) \); while \( \mu \) is the particle kinetic energy, the second term is the qubit energy that exists only when it is coupled to some field \( \psi \neq 0 \).

3. The qubit carrier and the PSE

The spatial localization of the carrier must be introduced into equation (5), thus for a qubit state \( |\psi_0\rangle = a_0|x_0\rangle + b_0|\bar{x}_0\rangle \), the parameters \( a_0, b_0 \) should depend on the position \( q \), thus \( |\psi_0(q)\rangle = a_0(q)|x_0\rangle + b_0(q)|\bar{x}_0\rangle \) becomes the state of the whole system, with normalization condition \( \int dq \ |a_0(q)|^2 + \int dq \ |b_0(q)|^2 = 1 \). The qubit state becomes correlated to the particle position that influences its probability outcomes \( |a_0(q)|^2 \) and \( |b_0(q)|^2 \). Coordinate dependence should also be present in the generator, so \( G(q) = \mu(q)I + v \vec{X} \) and the parameter \( v \) is assumed \( q \)-independent because interaction between both degrees of freedom is not considered. The evolved state is \( U(\tau)|\psi_0(q)\rangle = |\psi(q, \tau)\rangle = a_\tau(q)|x_\tau\rangle + b_\tau(q)|\bar{x}_\tau\rangle \), with amplitudes

\[
a_\tau(q) = e^{-i\tau \mu(q)} (a_0(q) \cos \nu \tau - i b_0(q) \sin \nu \tau),
\]

\[
b_\tau(q) = e^{-i\tau \mu(q)} (-ia_0(q) \sin \nu \tau + b_0(q) \cos \nu \tau),
\]

with \( a_0(q) \) and \( b_0(q) \) to be determined from the motional dynamical equation. So, the qubit was merged with the spatial motion of its carrier within a single equation, meaning that the evolution—the qubit sequence of actions as well as the change in the spatial configuration of the carrier—is measured by a single clock. To determine the parameters \( a_0(q) \) and \( b_0(q) \) they should obey some differential equation, then \( \mu(q) \) must also depend on \( \sigma/\hbar \) or/and its powers. However, instead of trying to guess the functional form \( \mu(q, \sigma/\hbar) \), it is better to take advantage of the available information from Hamiltonian mechanics, so I define \( \mu \) as the kinetic energy of a nonrelativistic particle, \( \mu \equiv T(p) = p^2/2m \), where \( p \) is the linear momentum in some reference frame. Equation (5) becomes \( \hbar \partial |\psi(\tau)\rangle /\partial \tau = [T(p)I + \epsilon_0 \vec{v} \cdot \vec{X}] |\psi(p, \tau)\rangle \). Since \( T(p) \) has dimension of energy, the second term in brackets should also have the same dimensionality. So the constants \( \epsilon_0 \) and \( \vec{v} \) have both dimension of energy. One can also choose some unit to measure the dimensionless time \( \tau, \tau = t/\hbar \), so the dynamical equation becomes

\[
\hbar \partial |\psi(p, t)\rangle /\partial \tau = [T(p)I + \epsilon_0 \vec{v} \cdot \vec{X}] |\psi(p, t)\rangle,
\]

where \( \hbar = \kappa_0 \), \( \epsilon_0 = \epsilon_0^\prime \). Note that the constant \( \hbar \) has dimension of energy \( \times \) time and \( \epsilon_0 \) has dimension of energy. An arbitrary initial condition assumes that the particle momentum and the qubit state are correlated and the probability amplitude

\[
|\tilde{\psi}_0(p, 0)\rangle = |\tilde{\psi}_0(p)\rangle = a_0(p)|x_0\rangle + b_0(p)|\bar{x}_0\rangle
\]

depends on the particle momentum and it contains all the available information. In momentum space the evolution operator is \( U(\tau) = \exp[-i(T(p)I + \epsilon_0 \vec{v} \cdot \vec{X})/\hbar] \) and the solution to equation (8) is

\[
|\tilde{\psi}(p, t)\rangle = e^{-i\epsilon_0 T(p)/\hbar} \left[ a_\tau(p)|x_\tau\rangle + b_\tau(p)|\bar{x}_\tau\rangle \right]
\]

with \( a_\tau(p) = \cos(\epsilon_0 \nu/\hbar) a_0(p) - i \sin(\epsilon_0 \nu/\hbar) b_0(p) \) and \( b_\tau(p) = \cos(\epsilon_0 \nu/\hbar) b_0(p) + i \sin(\epsilon_0 \nu/\hbar) a_0(p) \), and the
particle mean energy is \( \langle \hat{\psi}(p, \tau) | \hat{H}(p) | \hat{\psi}(p, \tau) \rangle = T(p) + \sum_{\alpha} \text{Re}(\alpha_p \langle \hat{\rho}_\alpha(p) \rangle) \). Since coordinate and momentum are conjugated variables the statevector in coordinate representation is
\[
|\psi(q, t)\rangle = \psi_{0\alpha}(q, t) |x_0\rangle + \psi_{1\alpha}(q, t) |\bar{x}_0\rangle
\]
and \( \psi_{0\alpha}(q, t), \psi_{1\alpha}(q, t) \) are the amplitudes for the qubit being in states \( |x_0\rangle, |\bar{x}_0\rangle \), they can be written as Fourier transforms
\[
\psi_{\alpha}(q, t) = \int \frac{dp}{2\pi} \omega_{pq}/|\hbar| e^{-i\omega T(p)/\hbar} \left( \frac{a(p, t)}{b(p, t)} \right)^{\alpha}.
\]
(10)
The constant \( \hbar_1 \) is introduced to set the correct dimensionality, and it has the same units as \( \hbar_0 \), nonetheless nothing can be said about being the same constant, unless confirmed by experiment. In equation (10)
\[
\left( \frac{\dot{a}(p, t)}{\dot{b}(p, t)} \right) = \int dq e^{-i\omega pq/\hbar} \left( \frac{a(q', t)}{b(q', t)} \right)^{\alpha}
\]
(11)
with \( a(q', t) = \cos(\epsilon_0 t/\hbar_0) a_0(q') - i\sin(\epsilon_0 t/\hbar_0) b_0(q') \) and \( b(q', t) = \cos(\epsilon_0 t/\hbar_0) b_0(q') + i\sin(\epsilon_0 t/\hbar_0) a_0(q') \). So even without a direct interaction between the qubit and its carrier, the probability for measuring the qubit in state \( |x_0\rangle \) or \( |\bar{x}_0\rangle \), becomes affected by its position.

Using equations (10) and (11) and manipulating equation (8) it is not hard to verify that one can substitute the c-number \( p \) by the derivative \(-i\hbar_1 \partial / \partial \hbar q \), and we can rewrite that equation as
\[
\hbar_0 \frac{\partial}{\partial t} |\psi(q, t)\rangle = \left[ \frac{1}{2m} \left( \begin{array}{c}
-q_i \\
+\epsilon_0 v X
\end{array} \right) - i \hbar_1 \frac{\partial}{\partial q} \right] |\psi(q, t)\rangle.
\]
(12)
The terms in brackets stand for the particle and qubit Hamiltonian in coordinate and matrix representation, so the parameter \( \mu \) becomes determined. In the presence of an energy conserving potential \( V(q) \) the PSE takes its familiar form, with Hamiltonian \( H = H_0 + i/2 \rightleftharpoons X = [\{-i\hbar_1/2(2m)\}^{3/2}/\partial^2/\partial^2 q \} + V(q)\]\). The particle described by equation (12) has now blurred classical properties (it loses the sharp trajectory in phase space), its best representation is given by a wavefunction and the appearance of quantum properties are due to the qubit it is carrying. Any further generalization is trivial and immediate: (i) from 1D to 3D in spacial coordinates, \( \partial / \partial q \rightarrow \nabla \) and (ii) since any \( 2 \times 2 \) matrix can be expanded in the basis formed by the unit matrix \( I \) and Pauli matrices \( (\sigma_x, \sigma_y, \text{and } \sigma_z) \), then \( \nabla \rightarrow \nabla - \vec{\sigma} \).

4. Dirac equation: two qubits of information

A few words about Dirac equation \( i\hbar \frac{\partial}{\partial t} |\Psi_D(t)\rangle = \hat{H}_D |\Psi_D(t)\rangle \), its Hamiltonian is \( \hat{H}_D = i\hbar \frac{\partial}{\partial t} |\psi_D(t)\rangle \), its four-dimensional matrices \( \delta \), \( \beta \) satisfy the relations \( \alpha \gamma \alpha = \alpha \gamma \beta = 0 \) and \( \beta^2 = 0 \). These matrices can be expressed as tensor products of two-dimensional matrices, each one acting on its own qubit, \( \alpha = X_1 \otimes (X_2 \otimes X_2), Y_1 = X_1 \otimes Y_1 \), \( Z_1 = X_1 \otimes Z_1 \). c \( \alpha \hat{p} = \hat{X}_1 \otimes (c \hat{p} \cdot \hat{\sigma}_2) \) and \( \beta = Z_1 \otimes \hat{X}_1 \). Thus Dirac’s Hamiltonian can be written in terms of tensor products acting on independent D-2 Hilbert subspaces \( \hat{H}_D = Z_1 \otimes (mc^2 I_2) + X_1 \otimes (c \hat{p} \cdot \hat{\sigma}_2) \). Squaring \( \hat{H}_D \) one gets the relativistic energy \( \hat{E}_D^2 = \hat{E}_p^2 I_1 \otimes I_2 \), where \( \hat{E}_p^2 = mc^2 + c^2 \hat{p}^2 \). The time-dependent equation reduces into direct products of \( 2 \times 2 \) matrices
\[
\left[ I_1 \otimes (i\hbar \frac{\partial}{\partial t} I_2) - Z_1 \otimes (mc^2 I_2) - X_1 \otimes (c \hat{p} \cdot \hat{\sigma}_2) \right]
\times |\Psi_D(t)\rangle = 0,
\]
which is invariant under Lorentz transformation, and the solutions are
\[
|\psi^k(\hat{p}, t)\rangle = N_k e^{-i\lambda t E_p}
\]
\[
\times \left[ |1\rangle \langle 0| + |0\rangle \langle 1| \right] \left[ c \hat{p} \cdot \hat{\sigma}_2 \right] \frac{1}{mc^2 + \lambda E_p} |\psi(\hat{p})\rangle_2.
\]
(13)
with \( \lambda = \pm 1 \) standing for positive and negative energy solutions and \( N_k \) is a normalization constant. The qubit 2 in equation (13) represents the particle state whereas the Cbit 1 is apparently ancillary, it works as a selector: the projector \( |1\rangle \langle 1|_1 \) selects the nonrelativistic component \( |\psi(\hat{p})\rangle_2 \) while \( |0\rangle \langle 0|_1 \) projects the relativistic complement. Also interesting is that all the \( \gamma_\mu \) matrices have the structure of the direct product of two-qubit operators \( \gamma^0 = Z_1 \otimes I_2, \gamma^1 = iX_1 \otimes X_2 = Y_1 \otimes Y_2, \gamma^3 = iY_1 \otimes Z_2 \).

5. Concluding remarks

As long as the qubit is not probed (for a spin, there is no external magnetic field probing it), \( \nu = 0 \), equation (12) reduces to two uncoupled SEs
\[
i\hbar \frac{\partial}{\partial t} \left( \begin{array}{c}
|\psi_{0\alpha}(q, t)\rangle \\
|\psi_{1\alpha}(q, t)\rangle
\end{array} \right) = H_0 \left( \begin{array}{c}
|\psi_{0\alpha}(q, t)\rangle \\
|\psi_{1\alpha}(q, t)\rangle
\end{array} \right),
\]
(14)
for two wavefunctions \( |\psi_{0\alpha}(q, t)\rangle, |\psi_{1\alpha}(q, t)\rangle \) in one qubit/spin state that is not activated. The two equations are redundant thus the relevant information resides in one of them only, going to the usual spinless SE. So why do quantum properties of the particle still persist even when the correlation between a qubit/spin and its carrier is broken? It is worth stressing that by setting \( \nu = 0 \) the particle motion is not ruled (back) by classical physics (Hamilton equations) but by the usual SE, although classical physics was crucial to arrive at (14). The answer is: even not being activated the qubit/spin is still carried by the particle, the internal degree of freedom and the particle make one single object, although not entangled. One is left with an equation (SE) that does not keep any clue about the presence of a qubit/spin; nonetheless, it is still there although not manifestly evident. That is why the SE is used without any mention of spin if not needed; otherwise, this internal degree of freedom must be added in order to explain the observed phenomena. In conclusion, because it is carrying one qubit of information, the particle shifts its behavior from the classical picture, it acquires new properties such as the particle-wave duality and a probabilistic behavior where the uncertainty relations represent one facet.
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