Effective field theory contribution to charge symmetry breaking $NN$ scattering

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Abstract

The effect of isospin breaking pion $s$-wave rescattering is included in elastic $NN$ scattering at low energies using effective field theory. Although this mechanism gave a large contribution to charge symmetry breaking in $np \rightarrow d\pi^0$, the effect is rather small in $pp$ vs. $nn$ scattering parameters and in the $^3$H-$^3$He binding energy difference. This smallness is caused by large cancellation of the up-down quark mass difference contribution and electromagnetic effects to the $np$ mass difference.

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Charge symmetry is the most accurate special case of general flavour symmetry. It is trivially broken by the electromagnetic interaction, notably the Coulomb force in comparisons of the $pp$ and $nn$ systems and by the magnetic interaction in the $np$ system. Other well known sources are the $np$ mass difference and $\eta\pi$- as well as $\rho\omega$-meson mixing. These in turn may be related to the up- and down-quark mass difference - the microscopic flavour symmetry breaking in QCD. One might consider remarkable the fact that, although the relative quark mass difference is large ($\geq 10\%$), the symmetry breaking at the observable hadron level is two orders of magnitude smaller.

Charge symmetry breaking (CSB) has been studied for the mirror system $pp$ vs. $nn$ for many decades [1], while its appearance in the $np$ system was first seen only a decade ago [2] as the difference $\Delta A = A_n - A_p$ elastic analyzing powers and is presently being searched for also in pionic inelasticity in the reaction $np \rightarrow d\pi^0$ [3]. The CSB observables have been seen in calculations to be sensitive to different combinations of sources. For example, in $np$ scattering above 300 MeV the $np$ mass difference in OPE dominates, while at $\approx 200$ MeV $\rho\omega$ meson mixing and the magnetic interaction become about equally important [4]. Of traditional CSB mechanisms in pion production $\eta\pi$ mixing is important and was seen to dominate at threshold [5], while at higher energies the $np$ mass difference becomes more important [6]. The CSB effects in the $np$ system change the isospin of the two baryons (class IV in the terminology of Ref. [7]), while in $pp$ and $nn$ the isospin is conserved (class III). In class III the main contribution is expected to be the $\rho\omega$ meson mixing [1, 8].

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Figure 1: CSB mechanisms arising from the up-down quark mass difference in pion rescattering: in $np \rightarrow d\pi^0$ (a), in $NN$ elastic scattering with a nucleonic (b) and $\Delta$ intermediate state (c).

Two-meson exchange in CSB has been studied earlier extensively by Coon and collaborators [9, 10] and in charge dependence in e.g. Refs. [9, 11, 12, 13].

Recently a new mechanism related to the $ud$ quark mass difference in QCD based effective field theory was suggested for the CSB forward-backward asymmetry of the cross section in $np \rightarrow d\pi^0$ [14]. It consists of CSB $s$-wave rescattering of the pion from the second nucleon. This rescattering (depicted in Fig. 1a) appears in effective field theory through the second term of the isospin symmetry violating Lagrangian [13, 16]

$$L = \frac{\delta m_N}{2} \left( N^\dagger \tau_0 N - \frac{2}{DF^2_\pi} N^\dagger \phi_0 \phi \cdot \tau N \right),$$

where the nucleon isospin is represented by the Pauli matrices $\tau$, $F_\pi = 186$ MeV is the pion decay constant and $\delta m_N$ is the up and down quark mass difference effect in the nuclear masses. The denominator is in principle $D = 1 + \phi^2/F^2_\pi$, but $D = 1$ is used here. The isospin violation here originates from the rather significant quark mass difference $m_d - m_u \approx$ a few MeV. In addition to the bare quark mass difference one should include an electromagnetic contribution $\delta m_N$ to the nucleon mass difference changing the effective CSB strength parameter [14]. We shall come to this correction later.

The above mechanism was seen to be a major contributor to the asymmetry in CSB pion production. However, it is clear that returning the emitted pion back to the first nucleon it can also contribute to elastic scattering as shown in Fig. 1b and 1c. The question is only whether its contribution really is isospin violating and of what type. The aim of this paper is to investigate this interaction and its effect to the difference of the $^1S_0$ scattering lengths (experimentally estimated to be $\Delta a = a_{pp} - a_{nn} = 1.5 \pm 0.5$ fm). Furthermore, a simple estimate of this effect to the $^3$H-$^3$He binding energy difference is made.

It is straightforward algebra to see that, with the conventions of Fig. 1 and neglecting the baryon kinetic energies, the diagram 1b yields in the momentum space a CSB interaction of the form

$$V_N(q) = \frac{\delta m_N}{2} \frac{f_\pi^2}{\mu^2} \int \frac{d^3k}{(2\pi)^3} \frac{(k^2 - q^2/4)(\tau_{10} + \tau_{20})}{[\mu^2 + (k + q/2)^2][\mu^2 + (k - q/2)^2]},$$

where $f^2/4\pi = 0.076$ is the pion-nucleon coupling constant, $\mu$ the pion mass and $\tau_{i0}$ refers to the $z$ component of the isospin operator of the $i$th nucleon. With the intermediate $\Delta$ (Fig.
\[ V_\Delta(q) = \frac{4 \delta m_N}{9 F_\pi^2} \frac{f^{*2}}{\mu^2} \int \frac{d^3k}{(2\pi)^3} \frac{(k^2 - q^2/4)(\tau_{10} + \tau_{20})(\omega_+ + \omega_- + \Delta)}{\omega_+ + \omega_- + \Delta} \]  

where now the \( \pi N \Delta \) coupling constant is \( f^{*2}/4\pi = 0.35 \) from the width of the \( \Delta(1232) \) and \( \Delta \) is the mass difference between the \( \Delta(1232) \) isobar and the nucleon (the real part of the \( \Delta \) pole is used). Also a shorthand notation has been introduced for the pion energy with \( \omega_\pm = \mu^2 + (k \pm q/2)^2 \). In addition, monopole form factors \( (\Lambda^2 - \mu^2)/(\Lambda^2 + q^2) \) are inserted for the pion emission and absorption vertices. Clearly the above potentials belong to class III in the classification of [7], which violates charge symmetry between \( pp \) and \( nn \) but not in the \( np \) system. For positive \( \delta m_N \) they tend to make the \( nn \) interaction more attractive.

An interesting point in these CSB contributions is that the coefficient multiplying the integrals could be numerically large as compared with the coefficients in Refs. [10, 17] for CSB arising from the \( np \) mass difference. However, the dimension is different (depending also on the integral). One may ask whether the contribution could be even unrealistically large to exclude this mechanism from CSB. An explicit calculation is necessary to answer this question.

One may note that there is large uncertainty in the exact value of \( \delta m_N \) with estimates ranging mostly between 2 and 3 MeV depending on electromagnetic corrections to the \( np \) mass difference. For the moment the value \( \delta m_N = 2.4 \) MeV has been used in these results, which represented the total CSB strength for the reaction \( np \rightarrow d\pi^0 \) in Ref. [14] (including also the electromagnetic contribution to the \( np \) mass difference). The contribution to \( \Delta a \) scales linearly with \( \delta m_N \). We shall return to the effect of the electromagnetic corrections later.

The above integrals are numerically easy to perform and, in the same way as in Ref. [17], the resulting potential is then transformed into the coordinate space where the final calculations are done. Simple fits of the integrals with a form

\[ V(q) = AB \frac{B^2}{B^2 + q^2} \]  

for the first (\( k^2 \) dependent) parts and

\[ V(q) = AB \frac{B^2}{B^2 + q^2} \frac{C^2}{C^2 + q^2} \]  

lead to a tolerable agreement (although not as perfect as in Ref. [17]) with the exact results for \( V_N \) and \( V_\Delta \) (Fig. 2). In the coordinate representation these turn to Yukawa functions or their derivatives, shown in Fig. 3. These are very large potentials, indeed, for charge asymmetry, an order of magnitude larger than in Ref. [17] for class IV, but this may be in line with chiral power counting arguments, which stipulate that class III should be stronger than class IV [13, 15]. In these figures the coefficients of the \( (\tau_{10} + \tau_{20}) \) operators are shown, so the total difference of the \( pp \) vs. \( nn \) interaction will get still another factor of 4.

\[ 1 \text{ One might note that there is also a contribution with a structure } i(\tau_1 \pm \tau_2)(\sigma_1 \pm \sigma_2) \cdot k \times q. \]  

With the above static approximation for the baryons this vanishes in the integration over \( k \). However, if the baryon kinetic energies are taken into account, there is also an odd term in the angular dependence of the denominators allowing a nonzero class IV part as found in Ref. [17]. At low energies this correction, however, should be significantly smaller than the potentials (2-3).
The charge symmetric interaction between the nucleons is taken to be the phenomenological Reid soft core potential \([18]\). This is then also supplemented with explicit excitation of \(N\Delta\) intermediate states by the coupled channels method \([17]\). No other CSB effects are included in the present calculation except Eqs. (2-3) (Figs. 1b,c).

The results for the effective range parameter differences \(\Delta a = a_{pp} - a_{nn}\) and \(\Delta r_0 = r_{0,pp} - r_{0,nn}\) in the low energy expansion \(p \cot \delta_0 \approx -1/a + \frac{1}{2}r_0 p^2\) are presented in Table I. It can be seen that this mechanism with the above strength and the monopole form factor mass \(\Lambda = 1\) GeV gives a considerable contribution to \(\Delta a\), about one third of its experimental value and of the same sign (i.e. the \(nn\) interaction is the more attractive of the two). The fraction is even larger, if one considers that perhaps 0.4 fm in \(\Delta a\) may be attributed simply to different kinetic energies arising from the \(np\) mass difference \([19]\). One half of the calculated effect here comes from \(V_N\) and the other half from the \(\Delta\) contribution \(V_\Delta\).

The column labeled \(\Delta E\) is the contribution to the \(^3\!H-^3\!He\) binding energy difference using the simple prescription

\[
\Delta E_{GS} = (40\, \Delta a + 1600\, \Delta r_0)\, \text{keV/fm}
\]

obtained by Gibson and Stephenson for separable potentials \([20]\). This is likely an overestimate but gives an idea of the order of magnitude of the effect \([10]\). Here the relevant empirical result is \(\Delta E_{\text{expt}} \approx 76\pm 24\) keV after removing the "trivial" Coulomb repulsion and the effect of the \(np\) mass difference in the kinetic energy.

For model dependence one can vary the form factors. With softer form factors one normally expects smaller results. On line 3 the form factor has been taken to be of the dipole form with the same cut-off mass (or as well monopole vertices and a dipole formfactor in \(\pi N\) scattering). The result is about 25% smaller as might be expected.
A more interesting and more fundamental comparison is to a phase-equivalent coupled channels calculation with explicit $N\Delta$ intermediate states included in the charge symmetric scattering. Details of the $NN \leftrightarrow N\Delta$ transition potential including both $\pi$ and $\rho$ exchanges are given in Ref. \cite{17}. The diagonal $^1S_0$ Reid soft core potential must be adjusted by a repulsion of $381 e^{-3\mu r}/(\mu r)$ MeV to refit the phase shift from the coupled channels with the original at $E_{lab} = 2$ MeV. By unitarity, the $NN$ wave function should be depleted at short distances and consequently the mechanisms 1b,c somewhat suppressed. This is, indeed, the case as seen on lines 4–5 of Table I, but the decrease is not very large. Since the $N\Delta$ excitation must be an essential part of isospin one $NN$ scattering, this may be considered as the most realistic estimate.

| Model                          | $\Delta a$ (fm) | $\Delta r_0$ (fm) | $\Delta E$(keV) |
|-------------------------------|-----------------|-------------------|-----------------|
| Reid SC, $NN$, only Fig. 1b   | 0.28            | 0.006             | 20              |
| Reid SC, $NN + N\Delta$, Figs. 1b,c | 0.55          | 0.012             | 41              |
| Reid SC, Figs. 1b,c, dipole ff | 0.40            | 0.009             | 30              |
| Coupled channels               | 0.50            | 0.010             | 36              |
| Coupled channels, dipole ff    | 0.37            | 0.007             | 27              |
| Experiment \cite{1}            | 1.5 ± 0.5       | 0.10 ± 0.12       | 76 ± 24 \cite{10}|

Table 1: CSB effective range parameters and $^3$H-$^3$He binding energy differences for various models described in the text.
In principle the presence of the \( \Delta \) makes new diagrams possible, e.g. those with one or both pion-baryon vertices being \( \pi \Delta \Delta \) or pion rescattering off the \( \Delta \). The knowledge of these is much inferior to \( \pi NN \) or \( \pi N \Delta \). These mechanisms are also of higher order and are not discussed in this work. However, one could note that also the above unitarity depletion effect is of higher order in this sense, so that conservatively one can only say that the effect of coupling to the \( N \Delta \) intermediate states is only of the order of 10% in the CSB observables.

The above obtained results appear to indicate that CSB pion rescattering could be potentially an important effect also in elastic \( NN \) scattering as it was in \( np \to d\pi^0 \). However, we have not yet considered another isospin violating term in the effective Lagrangian \[15, 16\]

\[
\mathcal{L} = \frac{\delta m_N}{2} \left( N^\dagger \tau_0 N + \frac{2}{DF_{2\pi}} N^\dagger (\phi_0 \phi \cdot \tau - \phi \cdot \phi \tau_0) N \right),
\]

of electromagnetic origin. Here \( \delta m_N \) is the electromagnetic contribution to the \( np \) mass difference, typically estimated to be of the order of -1 or -2 MeV. In Ref. \[14\] this gave a similar contribution as Eq. 1 and the strength parameter changed there \( \delta m_N \to \delta m_N - \delta m_N/2 \). With \( \delta m_N \) negative this increased the effect. However, in the present case of \( NN \) scattering the effect turns out to be the change \( \delta m_N \to \delta m_N + 2\delta m_N \). Now the two mass difference terms tend to cancel each other and the above results should be scaled accordingly by a factor \( (\delta m_N + 2\delta m_N)/(2.4 \text{ MeV}) \) shown as a function of \( \delta m_N \) in Fig. 4. It can be seen that for the most reasonable range of \( |\delta m_N| \) between 0.5 and 1.5 MeV \[14\] the strength of the CSB potentials decreases into a fraction of the original. For example, using \( \delta m_N = -0.76 \pm 0.30 \) MeV from the Cottingham formula \[21\] yielding the strength 2.4 MeV for Ref. \[14\], the final results here would be only a quarter of the results in Table I. This means that if the present situation of understanding the \( pp \) vs. \( nn \) difference (in particular \( \Delta a \)) is considered satisfactory, this understanding is not significantly disturbed by the present mechanism even if it is large in pion production in \( np \to d\pi^0 \).

In summary, a new QCD-based \( \pi N \) rescattering contribution has been incorporated in CSB elastic \( NN \) scattering using effective field theory. This is potentially a strong effect as was seen for CSB in pion production. However, contrary to Ref. \[14\] in this class III interaction the quark mass difference and electromagnetic mass contributions tend to cancel, so that the effect in e.g. \( \Delta a \) actually becomes rather small. Thus even the large CSB contribution found in \( np \to d\pi^0 \) can be accommodated without compromising the understanding of \( pp \) vs. \( nn \) differences.

References

[1] G. A. Miller, B. M. K. Nefkens and I. Slaus, Phys. Rep. 194, 1 (1990); G. A. Miller and W. T. van Oers, in Symmetries and Fundamental Interactions in Nuclei, eds. W. C. Haxton and E. M. Henley, World Scientific (Singapore 1995).

[2] R. Abegg et al., Phys. Rev. Lett. 56, 2571 (1986), Phys. Rev. D 39, 2464 (1989); L. D. Knutson et al., Phys. Rev. Lett. 66, 1410 (1991); S. E. Vigdor et al., Phys. Rev. C 46, 410 (1992); J. Zhao et al., Phys. Rev. C 57, 2126 (1998).

[3] TRIUMF experiment E704, spokespersons A. Opper and E. Korkmaz.
Figure 4: The scaling factor needed for consistency with the $np$ mass difference and its electromagnetic part.

[4] A. G. Williams, A. W. Thomas and G. A. Miller, Phys. Rev. C 36, 1956 (1987); M. J. Iqbal and J. A. Niskanen, Phys. Rev. C 38, 2259 (1988).

[5] J. A. Niskanen, Few-Body Systems 26, 241 (1999).

[6] J. A. Niskanen, M. Sebestyen, and A. W. Thomas, Phys. Rev. C 38, 838 (1988).

[7] E. M. Henley and G. A. Miller, in Mesons and Nuclei, Vol. I, eds. M. Rho and D. H. Wilkinson, North Holland (Amsterdam 1979).

[8] S. A. Coon and R. C. Barrett, Phys. Rev. C 36, 2189 (1987).

[9] S. A. Coon and M. D. Scadron, Phys. Rev. C 26, 2402 (1982).

[10] S. A. Coon and J. A. Niskanen, Phys. Rev. 53, 1154 (1996).

[11] T. E. O. Ericson and G. A. Miller, Phys. Lett. 132B, 32 (1983); Phys. Rev. C 36, 2707 (1987).

[12] C. Y. Cheung and R. Machleidt, Phys. Rev. C 34, 1181 (1986).

[13] J. L. Friar and U. van Kolck, Phys. Rev. C 60, 034006 (1999).

[14] U. van Kolck, J. A. Niskanen and G. A. Miller, Phys. Lett. B 493, 65 (2000), nucl-th/0006042.

[15] U. van Kolck, Ph.D. dissertation, U. of Texas (1993); Few-Body Syst. Suppl. 9, 444 (1995).

[16] S. Weinberg, in Chiral Dynamics: Theory and Experiment, eds. A. M. Bernstein and B. R. Holstein, Springer (Berlin 1995), hep-ph/9412326.

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[17] J. A. Niskanen, Phys. Rev. C 45, 2648 (1992).

[18] R. V. Reid, Ann. Phys. (N.Y.) 50, 411 (1968).

[19] E. M. Henley, in Isospin in Nuclear Physics, ed. D. H. Wilkinson (North Holland, Amsterdam).

[20] B. F. Gibson and G. J. Stephenson, Jr., Phys. Rev. C 8, 1222 (1973).

[21] J. Gasser and H. Leutwyler, Phys. Rep. 87, 77 (1982).