The Transverse Electron Scattering Response Function of $^3\text{He}$

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Abstract

The transverse response function $R_T(q, \omega)$ for $^3$He is calculated using the configuration space BonnA nucleon-nucleon potential, the Tucson-Melbourne three-body force, and the Coulomb potential. Final states are completely taken into account via the Lorentz integral transform technique. Non-relativistic one-body currents plus two-body $\pi$- and $\rho$-meson exchange currents as well as the Siegert operator are included. The response $R_T$ is calculated for $q=174, 250, 400, \text{ and } 500 \text{ MeV/c}$ and in the threshold region at $q=174, 324, \text{ and } 487 \text{ MeV/c}$. Strong MEC effects are found in low- and high-energy tails, but due to MEC there are also moderate enhancements of the quasi-elastic peak (6%-10%). The calculation is performed both directly and via transformation of electric multipoles to a form that involves the charge operator. The contribution of the latter operator is suppressed in and below the quasielastic peak while at higher energies the charge operator represents almost the whole MEC contribution at the lowest $q$ value. The effect of the Coulomb force in the final state interaction is investigated for the threshold region at $q=174 \text{ MeV/c}$. Its neglect enhances $R_T$ by more than 10% in the range up to 2 MeV above threshold. In comparison to experimental data one finds relatively good agreement at $q=250$ and 400 MeV/c, while at $q=500$ MeV/c, presumably due to relativistic effects, the theoretical quasi-elastic peak position is shifted to somewhat higher energies. The strong MEC contributions in the threshold region are nicely confirmed by data at $q=324$ and 487 MeV/c.

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I. INTRODUCTION

Electromagnetic interactions in the trinucleon systems play an important role in testing NN and 3N forces as well as nucleonic current operators. Among the many reaction observables available are the response functions which determine the inclusive electron scattering cross-section

\[ \frac{d^2\sigma}{d\Omega \, d\omega} = \sigma_{Mott} \left[ \frac{Q^4}{q^4} R_L(q, \omega) + \left( \frac{Q^2}{2q^2} + \tan^2 \left( \frac{\theta}{2} \right) \right) R_T(q, \omega) \right] \]  

where \( \omega \) is the electron energy loss, \( q \) is the magnitude of the electron momentum transfer, \( \theta \) is the electron scattering angle and \( Q \equiv \{q, \omega\}, Q^2 = q^2 - \omega^2 \). \( R_L(q, \omega) \) and \( R_T(q, \omega) \) are called the longitudinal and transverse response functions respectively. In order to calculate either of these response functions one needs to be able to take into account all final states (usually in the continuum) which are connected to the ground state via the current or charge operators. This can be accomplished in several ways. For example recent calculations of these response functions by Golak et al. [1] and Deltuva et al. [2] base their calculations on Faddeev techniques while we employ the Lorentz integral transform (LIT) [3, 4] method. A recent review article on the LIT approach is given in [5].

The longitudinal response is driven by the nuclear charge density operator and has recently been calculated covering large parts of the non-relativistic regime in Refs. [1, 2, 4]. It is notable that although some of these groups use considerably different calculational techniques they obtain similar results for \( R_L(q, \omega) \). All these non-relativistic calculations of \( R_L(q, \omega) \) show quite good agreement with experiment for modest momentum transfers i.e. \( q < 400 \text{ MeV/c} \). However for larger \( q \) the position of the quasi-elastic peak is sensitive to relativistic corrections in the kinetic energy. In [6] the non-relativistic calculation was extended up to \( q=700 \text{ MeV/c} \) by choosing a proper reference frame, where relativistic effects on the kinetic energy are minimized. A subsequent transformation of the theoretical results to the laboratory frame led in fact to a much better agreement with data. Concerning the realistic NN interaction model there appears to be a relative insensitivity to which model is used. As far as 3N forces are concerned there is no unique picture. For \(^3\text{He}\) their inclusion improves the agreement with data, whereas for \(^3\text{H}\) one observes the opposite effect [4].

In \( R_T(q, \omega) \) it is the nuclear transverse current density which drives the response. This current density can be expressed as the sum of various components: the normal one-body currents with their relativistic corrections, two-body currents arising from meson exchange
NN forces (MEC) and isobar excitations, three-body currents arising from NNN forces. In [1] the AV18 NN potential [7] with the UrbanaIX NNN potential [8] were used and one-body currents as well as \( \pi \)- and \( \rho \)-MEC were taken into account to calculate \( R_T(q, \omega) \) at \( q=200 \), 300, 400, and 500 MeV/c and the low-energy \( R_T \) at various \( q \). In [2] the CD-Bonn potential and its coupled channel extension CD-Bonn+\( \Delta \) [9] was taken. The one-body current, \( \pi \)- and \( \rho \)-MEC and \( \Delta \)-currents were considered. In addition to computing \( R_T \) at \( q=300 \) and 500 MeV/c near threshold responses at various \( q \) were also shown.

Here we present the first fully realistic computation of \( R_T \) with the LIT method (in [10] the LIT method was applied to the \( R_T \) of \(^4\text{He}\) but with approximations for the MEC and using a semirealistic NN potential only). We use the configuration space BonnA potential [11] (hereinafter referred to as BonnRA) together with the TM’ [12] NNN potential to calculate the response at \( q=174, 250, 400, 500 \) MeV/c and at \( q=324 \) and 487 MeV/c in the near threshold region. Our reason for choosing the BonnRA potential is that the MECs are uniquely defined in the case of a boson exchange based potential. We would like to emphasize that the LIT method allows us to include consistently the Coulomb interaction in initial and final states, which is not done in [1, 2]. For the electromagnetic current operator we include the non-relativistic one-body operators plus, as in [1, 2], the \( \pi \) and \( \rho \) two-body MEC currents. As is well known the MEC are intimately connected to details of the Hamiltonian through the requirement of charge conservation. With boson-exchange potentials like the BonnRA and partially for CD-Bonn (contains two effective \( \sigma \)-mesons with partial-wave dependent parameters) the form of the MEC are determined by explicit knowledge of the boson-nucleon coupling. For phenomenological potentials such as the AV18 which was used in [1] one can construct a consistent \( \pi \)- and \( \rho \)-MEC [13, 16, 17] by interpreting the isovector part of the given potential model as due to an effective \( \pi \) and \( \rho \) exchange.

Our calculation is performed in two ways depending on how we treat the electric multipole operators. One method, which we refer to as the direct method, simply uses the current operators \textit{per se} in the electric multipoles. In the second method the electric multipole operators are transformed via use of the continuity equation into a form which includes the charge operator. We refer to this latter form of the electric multipole operator as the Siegert form. If both the continuity equation were fulfilled exactly and dynamic equations were solved exactly then these two ways would lead to the same results. Since, as in our
case, a realistic nuclear force includes components additional to one-boson exchange potentials, such as momentum-dependent NN forces and 3N forces, the continuity equation is only approximately fulfilled when one employs only the dominant, well established MECs. Therefore performing calculations in the two ways allows us on one hand to find out to what extent the $\pi$ and $\rho$ exchange currents we use are compatible with the realistic nuclear force employed. On the other hand, via use of the charge operator it permits us to take into account a part of the additional MEC thus checking their possible relevance. In [1, 2] such an investigation has not been carried out (in [1] the Siegert operator is only used for reactions with real photons).

II. NUCLEAR FORCES AND THE CURRENT OPERATOR

The transverse response $R_T$ which depends on the transverse nuclear current density operator $J_T$ is given by

$$R_T(q, \omega) = \sum_{M_0} \sum df \langle \Psi_0 | J_T^\dagger(q, \omega) | \Psi_f \rangle \cdot \langle \Psi_f | J_T(q, \omega) | \Psi_0 \rangle \cdot \delta(E_f - E_0 + q^2/(2M_T) - \omega).$$

(2)

Here $M_T$ is the mass of the target nucleus, $\Psi_0$ and $\Psi_f$ denote the ground and final states, respectively, while $E_0$ and $E_f$ are their eigenenergies,

$$(h - E_0)\Psi_0 = 0, \quad (h - E_f)\Psi_f = 0,$$

(3)

where $h$ is the intrinsic nuclear non-relativistic Hamiltonian. States of our system are represented by products of normalized center of mass plane waves $\varphi(P_{0,f})$ and internal substates $\Psi_{0,f}$ entering (2). Correspondingly, the current operator $J$ in (2) is related to the primary current operator $\bar{J}$ as follows,

$$J \delta(P_f - P_0 - q) = \langle \varphi(P_f) | \bar{J} | \varphi(P_0) \rangle,$$

(4)

where the matrix element is defined in the center of mass subspace. The cross section we need corresponds to the laboratory reference frame and we set in (1) $P_0 = 0$. The quantity $J_T$ is that component of $J$ which is orthogonal to $q$. The second summation (integration) in (2) goes over all final states belonging to the same energy $E_f$, and $M_0$ is the projection of the ground state angular momentum $J_0$.

The Hamiltonian $h$ includes the kinetic energy terms, the 2N and 3N force terms, and the proton Coulomb interaction term. As in [4] the ground state $\Psi_0$ is calculated via an
expansion in basis functions which are correlated sums of products of hyperradial functions, hyperspherical harmonics and spin-isospin functions. In the present work the 2N + 3N interactions are taken as the Coulomb+ BonnRA+TM' (Λ=2.835 fm−1) as in [4]. The TM' cut-off parameter Λ properly fixes the 3H binding energy to 8.47 MeV.

We perform a non-relativistic calculation. The current \( \mathbf{J} \) includes one-body and two-body operators. The one-body current operator as obtained from (4) is

\[
\mathbf{j}^{(1)} = \sum_{k=1}^{A} [ \mathbf{j}(k)_{\text{spin}} + \mathbf{j}(k)_{p} + \mathbf{j}(k)_{q} ]
\]

where \( A \) is the number of nucleons in the target nucleus and

\[
\mathbf{j}(k)_{\text{spin}} = e^{i \mathbf{q} \cdot \mathbf{r}'_{k}} \frac{i (\sigma_{k} \times \mathbf{q})}{2M} G_{M}(k), \\
\mathbf{j}(k)_{p} = e^{i \mathbf{q} \cdot \mathbf{r}'_{k}} \frac{\mathbf{p}_{k}}{M} G_{E}(k), \\
\mathbf{j}(k)_{q} = e^{i \mathbf{q} \cdot \mathbf{r}'_{k}} \frac{\mathbf{q}}{2M} G_{E}(k).
\]

Here \( \mathbf{r}'_{k} = \mathbf{r}_{k} - \mathbf{R}_{cm} \), \( \mathbf{p}'_{k} = \mathbf{p}_{k} - \mathbf{P}_{cm}/A \), and \( \sigma_{k} \) are the relative coordinate, momentum, and spin operator of the \( k \)-th particle and \( M \) denotes the nucleon mass, while \( \mathbf{R}_{cm} \) and \( \mathbf{P}_{cm} \) are the center of mass coordinate and momentum variables of the \( A \)-body system. The component \( \mathbf{j}_{q} \) does not contribute to \( \mathbf{J}_{T} \). However, separate multipoles as defined below depend on this component.

In the above expressions we use the notation

\[
G_{E,M}(k) = G_{E,M}^{p}(Q^{2}) \frac{1 + \tau_{zk}}{2} + G_{E,M}^{n}(Q^{2}) \frac{1 - \tau_{zk}}{2}
\]

where \( G_{E,M}^{p,n} \) are the Sachs form factors and \( \tau_{zk} \) denotes the third component of the isospin operator of the \( k \)-th nucleon. With our procedure the computational labour is reduced when the number of \( \omega \)-dependent form factors is reduced [5]. To this end we use the approximation

\[
G_{E}(Q^{2}) \approx G_{E}(Q^{2})_{av} \gamma(Q^{2}_{av})
\]

where \( \gamma(Q^{2}_{av}) = G_{E}(Q^{2}_{av})/G_{E}(Q^{2}_{av})_{av} \), \( Q^{2}_{av} = q^{2} - \omega_{av}^{2} \) and \( \omega_{av} = q^{2}/(2M) \). Similarly for the one-body spin current we use

\[
G_{M}^{p}(Q^{2}) \approx \tilde{\mu}_{p}(Q^{2}_{av}) G_{E}(Q^{2}) \\
G_{M}^{n}(Q^{2}) \approx \tilde{\mu}_{n}(Q^{2}_{av}) G_{E}(Q^{2})
\]

where

\[
\tilde{\mu}_{p}(Q^{2}_{av}) = \frac{G_{M}^{p}(Q^{2}_{av})}{G_{E}(Q^{2}_{av})}, \\
\tilde{\mu}_{n}(Q^{2}_{av}) = \frac{G_{M}^{n}(Q^{2}_{av})}{G_{E}(Q^{2}_{av})}
\]
For the usual dipole magnetic form factors, as used in this work, the above relations are fulfilled exactly and we have checked that the approximation provides a very good accuracy for \( G_E^m \). In a future extension of our work to a high-\( q \) region, \( q > 500 \text{ MeV/c} \), we will use more sophisticated nucleon form factor fits. In these cases the above relations are only approximately fulfilled although we have checked that they still lead to excellent accuracy.

The neutron electric form factor we use here is taken from [13] as used in [14]. With (6-8) the one-body current is replaced by

\[
J^{(1)}(q) = \sum_{k=1}^{A} \frac{e^{i\mathbf{q} \cdot \mathbf{r}_k}}{M} \left\{ \left( \mathbf{p}_k' + \frac{\mathbf{q}}{2} \right) \left[ \frac{1 + \tau z_k}{2} + \gamma(Q_{av}^2) \frac{1 - \tau z_k}{2} \right] + \frac{i(\sigma_k \times \mathbf{q})}{2} \left[ \bar{\mu}_p(Q_{av}^2) \frac{1 + \tau z_k}{2} + \bar{\mu}_n(Q_{av}^2) \frac{1 - \tau z_k}{2} \right] \right\}. \tag{9}
\]

The dominant contributions to the two-body current \( J^{(2)} \) arise from the \( \pi \)- and \( \rho \)-meson exchange currents. These currents are usually expressed in terms of "Seagull" and "true exchange" pieces. Thus we write here

\[
J^{(2)} = J_{\pi}^{SG} + J_{\pi}^{ex} + J_{\rho}^{SG} + J_{\rho}^{ex}. \tag{10}
\]

We list in Appendix A the coordinate space representations of these currents with the corresponding values of coupling constants etc. Momentum space forms of these meson exchange currents are related to these coordinate space forms, apart from the multiplicative isovector electric form factor \( G_E^v(Q^2) = (G_E^p(Q^2) - G_E^n(Q^2))/2 \), via

\[
j^b_a(q)e^{i\mathbf{q} \cdot \mathbf{R}_{cm}} = \int d^3x \ e^{i\mathbf{q} \cdot \mathbf{x}} \ j^b_a(x), \tag{11}\]

where the super/sub-scripts above are those corresponding to the right hand side of (10).

Finally we use the current operator \( J \) in the form

\[
J = G_E^p(Q^2)J^{(1)} + 2G_E^v(Q^2)J^{(2)}. \tag{12}\]

III. MULTIPOLe EXPANSION OF THE TRANSVERSE RESPONSE

The dynamic calculations are performed in separate subspaces belonging to fixed angular momentum \( J \) and its projection \( M \) (see also [5]). One can account for \( M \)-dependencies analytically via performing a multipole expansion of \( R_T \). To this end we use a decomposition
into multipoles of the transverse current. This decomposition shall also allow us employing
an alternative expression for the transition operator, see below. The transverse current is
represented as
\[ J_T = 4\pi \sum_{\lambda=\text{el, mag}} \sum_{jm} i^{j-\epsilon} T_{jm}^{\lambda}(q) Y_{jm}^{(\lambda)\ast}(\hat{q}). \] (13)

Here \( \hat{q} = q^{\ast} q \) and \( Y_{jm}^{(\lambda)} \) are electric and magnetic vector spherical harmonics \(^{18}\) and \( \epsilon = 0 \)
when \( \lambda = \text{el} \) or \( \epsilon = 1 \) when \( \lambda = \text{mag} \). This then allows the transverse response to be written
as
\[ R_T(q, \omega) = \frac{4\pi}{2J_0 + 1} \sum_{\lambda=\text{el, mag}} \sum_{J} (2J + 1) (R_T)^{J\lambda}_j \] (14)

where
\[ (R_T)^{J\lambda}_j = \sum d\Omega \langle q^{J\lambda}_{JM} | \Psi_f(J, M) \rangle \langle \Psi_f(J, M) | q^{J\lambda}_{JM} \rangle \delta(E_f - E_0 - \omega), \] (15)

\( J \) and \( M \) are the final state angular momentum and its projection, and \( |q^{J\lambda}_{JM}\rangle \) is given by
\[ |q^{J\lambda}_{JM}\rangle = [T_j^j \otimes |\Psi_0(J_0)\rangle]_{JM}. \] (16)

In Eq. (15) \( M \) is arbitrary.

In terms of the more standard multipoles and vector spherical harmonics \( Y_{jm}^l \) we can write
\[ T_{jljm}^{\text{el}} = \left( \frac{j+1}{2j+1} \right)^{1/2} T_{j-1jm}^{\text{el}} + \left( \frac{j}{2j+1} \right)^{1/2} T_{jm}^{\text{el}+1}, \] (17)
\[ T_{jm}^{\text{mag}} = T_{jm}^{j} \] (18)

where
\[ T_{jm}^{l} = \frac{1}{4\pi i^{j-\epsilon}} \int d\Omega q \left( Y_{jm}^l(\hat{q}) \cdot \mathbf{J}(q, \omega) \right). \] (19)

Since charge has to be conserved it is well known that the above expression for \( T_{jljm}^{\text{el}} \) can be
rewritten as
\[ T_{jljm}^{\text{el}} = \left( \frac{j+1}{j} \right)^{1/2} \frac{\omega}{q} \rho_{jm} + \left( \frac{2j+1}{j} \right)^{1/2} T_{jm}^{\text{el}+1} \] (20)

where \( \rho_{jm} \) is a charge multipole of the charge density operator \( \rho \) defined by
\[ \rho_{jm}(q) = \frac{1}{4\pi i^{j}} \int d\Omega q Y_{jm}(\hat{q}) \rho(q). \] (21)

We shall refer to the form of \( T_{jljm}^{\text{el}} \) in (17) as the direct form and to that in (20) as the Siegert
form. The first term of (20) will be called Siegert operator, while the second term is the
residual term. Appendix B gives the multipole operators \( T_{jljm}^l \) for the one-body currents
while Appendix C lists them for the \( \pi \) and \( \rho \) exchange currents.
IV. CALCULATION OF THE RESPONSE

The techniques we use in calculating the response have been largely set out in [4]. Here we add some extra detail which arises in the case of the transverse response. The Lorentz transform of the partial response \((R_T)^{j\lambda}_{J}\) of Eq. (15) is given by

\[
\Phi^{j\lambda,\alpha}_{J}(q,\sigma_R,\sigma_I) = \sum_n (R_T)^{j\lambda,\alpha}_{J}(q,\omega_n) + \int d\omega \frac{(R_T)^{j\lambda,\alpha}_{J}(q,\omega)}{(\omega - \sigma_R)^2 + \sigma_I^2}.
\]

(22)

The sum in (22) corresponds to transitions to discrete levels with excitation energy \(\omega_n\). In our \(A=3\) case there exists only one discrete contribution corresponding to M1 elastic scattering. In (22) the response is supplied with an additional superscript \(\alpha\). It specifies separate contributions to the response \((R_T)^{j\lambda}_{J}\) of Eq. (15), e.g. a given \(\alpha\) determines the isospin of the final state. In addition it specifies contributions that correspond to components of the multipole operators with different nucleon form factor dependencies.

It was pointed out above that one-body and two-body currents have different \(\omega\)-dependence through their different form factors. Therefore we need to calculate the responses with the individual parts of the current i.e. \(J^{(1)}(J^{(1)},J^{(2)},\text{ and } J^{(2)}\). The corresponding partial response functions will carry the superscripts \(\alpha = \{11, 12, 22\}\) so that the response \((R_T)^{j\lambda}_{J}\) would be expressed as

\[
(R_T(q,\omega))^{j\lambda}_{J} = (G_E^p(Q^2))^{2}(R_T(q,\omega))^{j\lambda,11}_{J} + 4G_E^p(Q^2)G_E^v(Q^2)(R_T(q,\omega))^{j\lambda,12}_{J} + 4(G_E^v(Q^2))^{2}(R_T(q,\omega))^{j\lambda,22}_{J}.
\]

(23)

Additional \(\omega\)-dependence of the electric multipole operators arises when they are used in the Siegert form, i.e. in the form of Eq. (20). Due to the additional \(\omega\)-dependence of the first term in (20) we calculate separately the response originating from this term \(\sim \omega^2\), the response originating from the second term in (20) and the cross-term response \(\sim \omega\). For the same reason as in Eq. (23) each of these responses is in turn broken into the one-body piece, the two-body piece and the cross piece which are calculated separately. The superscript \(\alpha\) in (22) enumerates all these various cases, so that the response \((R_T)^{j\lambda,\alpha}_{J}\) multiplied by products of nucleon form factors times \(\omega_n\), \(n = 0, 1, 2\).

As described in [3] the transforms are determined dynamically. In the present case the
transforms $\Phi^{j,\alpha}_{j}\lambda,\alpha$ are obtained from
\[
\Phi^{j,\alpha}_{j}(q, \sigma_{R}, \sigma_{I}) = \langle \tilde{\psi}^{j,\alpha}_{JM} | \psi^{j,\alpha}_{JM} \rangle, \quad |\tilde{\psi}^{j,\alpha}_{JM}\rangle = [h - \sigma_{R} + i\sigma_{I}]^{-1}|q^{j,\alpha}_{JM}\rangle.
\] (24)

The calculation (24) is $M$-independent and is performed in separate subspaces belonging to given isospin and parity. Parities are determined by the multipole order $j$ and the choice of $\lambda=el/mag$. For a given $\lambda$, parity, and $J$ only one value of $j$ is possible in our case.

To pass to responses one needs to invert the transforms. This may be done either separately for each transform $\Phi^{j,\alpha}_{j}$ using Eq. (22) or for their sums at the same $\alpha$. One may define the responses $R^{\alpha}_{T} = \sum_{\lambda=el,mag} R^{\lambda,\alpha}_{T}$, where (c.f. (14))
\[
R^{\lambda,\alpha}_{T}(q, \omega) = \frac{4\pi}{2J_{0} + 1} \sum_{J,j}(2J + 1)(R^{\lambda,\alpha}_{T})^{j,\alpha}_{j}(q, \omega).
\] (25)

One also defines the corresponding transforms
\[
\Phi^{\lambda,\alpha}(q, \sigma_{R}, \sigma_{I}) = \frac{4\pi}{2J_{0} + 1} \sum_{J,j}(2J + 1)\Phi^{j,\alpha}_{j}(q, \sigma_{R}, \sigma_{I}).
\] (26)

They are related to the responses (25) in the same way as in (22). All the various $\Phi^{\lambda,\alpha}$ are inverted separately to get $R^{\lambda,\alpha}_{T}$. Our inversion method and more information concerning the inversion can be found in [5, 19, 20]. We take $\sigma_{I} = 20$ MeV and distinguish between the two isospin cases $T=1/2$ and $3/2$, since the corresponding responses have different thresholds and thus the inversion can be carried out more precisely. After having inverted both cases we sum up the two results. For the magnetic part of the response we invert the M1 transition to the final state with $J^{\pi} = \frac{1}{2}^{+}$ separately, since, as mentioned above, it contains an elastic contribution. This elastic contribution can easily be determined by choosing a very small value for $\sigma_{I}$ and thereafter its effect on the transform can be subtracted leading to a LIT of a purely inelastic response.

As mentioned earlier charge conservation leads to the equality of the Siegert and direct forms of the transverse electric multipole operator. This provides an important check on our procedures especially with respect to the implementation of the MECs. The BonnRA potential contains more than just $\pi$- and $\rho$-meson exchange but we expect that taking account of MECs from only these two exchanged particles should lead to the dominant MEC contribution in our kinematical range, while additional MEC effects are partially taken care of by the Siegert operator. A good test for the implementation of the MEC is provided by
using a simple $\pi + \rho$ OBEP with their corresponding MECs. In this case charge conservation should be exact and the transverse response should be independent of whether one uses the Siegert or the direct form of $T_{jm}^{el}$. We have made such tests at $q=10$, 300, and 500 MeV/c and found very good agreement between the results of the two calculations \[21\].

V. RESULTS AND DISCUSSION

We have selected the momentum transfers $q = 174$, 250, 400, and 500 MeV/c for a calculation of $R_T(q,\omega)$ in a large $\omega$-range. In addition we consider the low-$\omega$ part of $R_T$ at $q = 174$, 324, and 487 MeV/c for which cases we take a maximal value of $J = 7/2$. For the other $q$-values a different choice for $J_{\text{max}}$ is made: 11/2 ($q=250$ MeV/c), 15/2 ($q=400$ MeV/c), and 19/2 ($q=500$ MeV/c). We have checked that with these settings very good convergences of the multipole expansions of $R_T$ are obtained in the requested energy ranges.

In the discussion we compare results calculated with the various current operators of section II (both with direct and Siegert forms) representing the following contributions: (a) one-body, (b) one-body and implicit MEC via Siegert operator, (c) one-body, $\pi$- and $\rho$- MEC, and (d) one-body, $\pi$- and $\rho$-MEC plus additional MEC via Siegert operator. If exact charge conservation was satisfied then the results of the direct calculation (c) would agree with those of the Siegert form (d).

In Figs. 1 and 2 we show the various current contributions to $R_T$. It is readily seen that there are rather strong MEC effects: 15-30 MeV above threshold MEC enhance $R_T$ by more than 30% for the two higher $q$-values (very close to threshold even by up to 200%, see Fig. 4); they increase the quasi-elastic peak height by 10% ($q=174$, 250 MeV/c), 7% ($q=400$ MeV/c), and 6% ($q=500$ MeV/c); for lower $q$ they also lead to large effects in the high-energy tail (e.g. at pion threshold: increases of 180% ($q=174$ MeV/c), 95% ($q=250$ MeV/c), 22% ($q=400$ MeV/c), and 5% ($q=500$ MeV/c)). In general, relative contributions of MEC are determined mainly by distances $|\omega - \omega_{\text{peak}}|$. This is natural since the peaks correspond to maximum contributions of one-body operators.

It is also seen that Siegert contributions remain quite small in and below the quasi-elastic peak. On the other hand they become more important with increasing energy (e.g., at pion threshold and $q=174$ (250) MeV/c, enhancements are of 130% (55%) of the one-body contribution). In addition to the fact that in general MEC contributions are rather
small in the peak as compared to one-body contributions, Siegert contributions are strongly suppressed in and below the peak by the factor $\omega/q$ in (20). The approximate transition operator we discuss takes account of MEC only via the Siegert operator, i.e. the charge operator from (20). As it is seen from Fig. 2, in the tail region this approximation provides the response rather close to the true one at the lowest $q$ value $q = 174$ MeV/c. This agrees with the well-known fact that in moderate energy photodisintegration processes ($\omega = q$) MEC contributions are largely included by the Siegert operator.

It is interesting to note that there exist additional Siegert MEC contributions beyond the $\pi$- and $\rho$-MEC entering the direct calculation. This is due to the fact that $\pi$ and $\rho$ exchanges constitute only the dominating part of a consistent exchange current with the BonnRA potential. Other two-body currents are induced by momentum and spin-orbit dependent potential terms. In addition also three-body currents, originating from the TM-3NF, could lead to Siegert contributions. Effects of the Siegert operator beyond $\pi$- and $\rho$-MEC were also found in the proton-deuteron radiative capture with the BonnCD+$\Delta$ potential [22]. It is seen from Fig. 1 and Fig. 2 that such effects are small for energies far away from the photon point. Indeed, there the complete calculation via direct inclusion of MEC operators and the complete alternative calculation that involves the Siegert operator have led to results close to each other. However, closer to the photon point the additional Siegert contributions can lead to corrections of the order of 10%.

In Fig. 3 we show our $R_T$ results in comparison to experimental data. For $q = 250$ and 400 MeV/c one finds good agreement. However, data are not precise enough to allow a definite conclusion about the MEC contribution. As opposed to the lower $q$ cases we find at $q = 500$ MeV/c a difference between the theoretical and experimental peak positions. The shift amounts to about 5-10 MeV. Relativistic effects, in particular those arising from corrections to the kinetic energy, might be responsible for this difference. In fact in [6] it was shown for the longitudinal response function $R_L(q, \omega)$ that such effects lead at $q = 500$ MeV/c to a shift of the peak position by 6 MeV.

In Fig. 4 we depict various $R_T$ theoretical and experimental low-energy results at $q = 0.882$, 1.64, and 2.47 fm$^{-1}$ corresponding to about 174, 324, and 487 MeV/c, respectively. We do not show the contribution of the Siegert operator, since, as shown in Fig. 1, its effect is very small at low energies. One sees that the MEC contribution can be very important, e.g. at $q = 487$ MeV/c one finds an increase of about 200% close to threshold. Contrary to the cases
shown in Fig. 3 one can make a definite conclusion about the MEC contribution. It is evident that they lead to a considerably improved agreement between theory and experiment. For the two higher \( q \)-values theoretical and experimental results agree very well, whereas for \( q = 174 \) MeV/c the theoretical result underestimates experimental data somewhat below 10 MeV. A better theoretical description of the \( q = 174 \) MeV/c data is found in [1], where the AV18 NN potential and the UrbanaIX 3N-force is used as nuclear interaction. However, the Coulomb force was not included in the final state interaction. The effect of such a neglect is illustrated in Fig. 4 for the case in discussion. Within 2 MeV above threshold it leads to an increase of more than 10\%, while at 5 MeV above threshold the effect still amounts to 4\%. In this way the theoretical results are shifted closer to the experimental data, but the effect is too small to reach a good agreement at low energies.

We summarize our results as follows. We have calculated the transverse form factor \( R_T(q, \omega) \) considering besides one- and two-body currents also the so-called Siegert operator. As nuclear interaction we have taken the BonnRA NN potential and the Tucson-Melbourne TM’ 3N-force. Since we are particularly interested in the MEC effects and the role of the Siegert operator we have chosen the BonnRA potential, for which the important \( \pi \)- and \( \rho \)-exchange currents are directly determined by the potential model. It is true that also for more phenomenological NN potentials, e.g. AV18, a consistent \( \pi \)- and \( \rho \)-MEC can be constructed [15, 16, 17], but to this end one has to interpret the isovector part of the phenomenological potential as an effective \( \pi \)- and \( \rho \)-exchange.

We find that MEC provide very strong contributions both at lower energies and in the high-energy tail while giving a moderate increase to the height of the quasi-elastic peak. Siegert contributions are unimportant in and below the quasi-elastic peak. They become considerably more sizeable at higher energies, but additional MEC contributions have also to be taken into account and thus a calculation, where in addition to the one-body current, MEC currents are taken into account via only the Siegert operator is not sufficient. On the other hand also a calculation with only one-body currents plus \( \pi \)- and \( \rho \)-MEC may not be sufficient at higher energies, since, as we have shown, effects due to additional two- and three-body currents can become important. To include at least a part of these additional exchange effects it is better to work also in this case with the Siegert operator. The appropriate place to study the structure of MEC is the energy region below the quasielastic peak. Indeed, the contributions of MEC are large in this region while they are rather small in the peak,
and beyond the peak they are partly represented by the Siegert operator, i.e. the charge operator.

In comparison to experiment relatively good agreement is obtained at $q=250$ and 400 MeV/c, while at $q=500$ MeV/c, presumably because of relativistic effects, the position of the theoretical quasi-elastic peak is located somewhat above the experimental one. Close to threshold one finds very strong MEC contributions. They are necessary in order to achieve a good description of the experimental data at $q=324$ and 487 MeV/c. Also at $q=174$ MeV/c they lead to an improved agreement with experiment, but in the range from threshold to 5 MeV above the theoretical result underestimates data somewhat.

In future we plan to investigate the momentum range $500 \text{ MeV/c} \leq q \leq 1 \text{ GeV/c}$ considering relativistic corrections for the one-body current operator and performing the calculation in a reference frame where relativistic effects in the kinetic energy are minimized [6]. We also plan to study isobar current contributions including $\Delta(1232)$ degrees of freedom.

VI. ACKNOWLEDGMENT

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APPENDIX A: CONFIGURATION SPACE $\pi$ AND $\rho$ MECS

For convenience we list below the well known $\pi$ and $\rho$ configuration space exchange currents.

$$j^\pi_{SG}(x) = \frac{f^2_0}{m^2_{\pi}} \sum_{i<j} (\tau_i \times \tau_j) \times \left[ (\sigma_i \cdot \nabla_i) \delta(x - r_j) - (\sigma_j \cdot \nabla_j) \delta(x - r_i) \right] + \sum_{k=1}^{3} h_k^\pi \gamma_k(\mu^\pi_k, |r_i - r_j|),$$  \hspace{1cm} (A1)

$$j^\rho_{ex}(x) = \frac{1}{4\pi} \frac{f^2_0}{m^2_{\pi}} \sum_{i<j} (\tau_i \times \tau_j) \times \left( \sigma_i \cdot \nabla_i \right) \left( \sigma_j \cdot \nabla_j \right) \times \left[ (\sigma_i \times \nabla_i) \times \delta(x - r_j) - (\sigma_j \times \nabla_j) \times \delta(x - r_i) \right] + \sum_{k=1}^{3} h_k^\rho \gamma_k(\mu^\rho_k, |r_i - r_j|),$$  \hspace{1cm} (A2)

$$j^\rho_{SG}(x) = \frac{1}{4\pi} \left( \frac{g_\rho}{2M} \right)^2 \left( 1 + \frac{f_\rho}{g_\rho} \right)^2 \sum_{i<j} (\tau_i \times \tau_j) \times \left( \sigma_i \times \nabla_i \right) \cdot \left( \sigma_j \times \nabla_j \right) \times \left[ (\sigma_i \times \nabla_i) \times \delta(x - r_j) - (\sigma_j \times \nabla_j) \times \delta(x - r_i) \right] + \sum_{k=1}^{3} h_k^\rho \gamma_k(\mu^\rho_k, |r_i - r_j|).$$  \hspace{1cm} (A3)

$$j^\rho_{ex}(x) = \frac{1}{(4\pi)^2} \left( \frac{g_\rho}{2M} \right)^2 \left( 1 + \frac{f_\rho}{g_\rho} \right)^2 \sum_{i<j} (\tau_i \times \tau_j) \times \left( \sigma_i \times \nabla_i \right) \cdot \left( \sigma_j \times \nabla_j \right) \times \left[ (\sigma_i \times \nabla_i) \times \delta(x - r_j) - (\sigma_j \times \nabla_j) \times \delta(x - r_i) \right] + \sum_{k=1}^{3} h_k^\rho \gamma_k(\mu^\rho_k, |r_i - r_j|).$$  \hspace{1cm} (A4)

Here $Y(m, r) = e^{-mr}/r$. We list the coupling constants, the masses $\mu^\alpha_k$, and the regularization constants $h^\alpha_k$, where $\alpha = \pi$ or $\rho$, taken from [11]:

$$f^2_0 = \frac{1}{4\pi} f^2_{\pi NN} = 0.0805, \quad \frac{g^2_\rho}{4\pi} = 1.2, \quad \frac{f_\rho}{g_\rho} = 6.1,$$

$$\mu_1^\alpha = m_\alpha, \quad \mu_2^\alpha = \Lambda_\alpha + 10 \text{ MeV}, \quad \mu_3^\alpha = \Lambda_\alpha - 10 \text{ MeV},$$

$$\Lambda_\pi = 1.3 \text{ GeV}, \quad \Lambda_\rho = 1.2 \text{ GeV},$$

$$h_1^\alpha = 1, \quad h_2^\alpha = \frac{(\mu_3^\alpha)^2 - (\mu_1^\alpha)^2}{(\mu_3^\alpha)^2 - (\mu_2^\alpha)^2}, \quad h_3^\alpha = \frac{(\mu_2^\alpha)^2 - (\mu_1^\alpha)^2}{(\mu_3^\alpha)^2 - (\mu_2^\alpha)^2}.$$  \hspace{1cm} (A5, A6, A7)
APPENDIX B: $T_{jm}^l$ MULTIPOLES OF ONE-BODY CURRENTS

In the following the non-relativistic expressions of the electric and magnetic multipoles for the one-body currents are written. Each of them is decomposed in a convection and a spin current. For the magnetic multipoles one has

$$T_{jm}^l = \sum_i [T_{jm}^{l,\text{spin}}(i) + T_{jm}^{l,\text{conv}}(i)]$$

with:

$$T_{jm}^{l,\text{spin}}(i) = \frac{1}{M} \frac{q}{2} \left(\frac{\mu_p + \mu_n}{2} + \frac{\mu_p - \mu_n}{2} \tau_{zi}\right) \left\{ \sqrt{\frac{j}{2j+1}} \frac{j_{j+1}(qr'_i)}{j_{j+1}} [Y_{j+1}(\mathbf{r}'_i) \otimes \sigma_i]_{jm} + \sqrt{\frac{j+1}{2j+1}} \frac{j_{j-1}(qr'_i)}{j_{j-1}} [Y_{j-1}(\mathbf{r}'_i) \otimes \sigma_i]_{jm} \right\},$$

$$T_{jm}^{l,\text{conv}}(i) = \frac{1}{M} \left(\frac{1+\gamma}{2} + \frac{1-\gamma}{2} \tau_{zi}\right) \frac{j_j(qr'_i)}{j_j} [Y_j(\mathbf{r}'_i) \otimes \partial_i]_{jm} .$$

The quantity \(\partial'_\mu\) is defined by the relationship \(-i\partial'_\mu = p'_\mu\). If the last Jacobi vector is defined as \(\mathbf{\xi}_{A-1} = \sqrt{(A-1)/A} [\mathbf{r}_A - (A-1)^{-1} \sum_{i=1}^{A-1} \mathbf{r}_i]\) then

$$\partial'^{(A)}\mu = \left[\frac{A-1}{A}\right]^{1/2} \frac{\partial}{\partial \xi_{A-1,\mu}} .$$

Similarly we write the one-body multipoles contributing to $T_{jm}^{el}$ as

$$T_{jm}^{l} = \sum_i [T_{jm}^{l,\text{spin}}(i) + T_{jm}^{l,\text{conv}}(i)]$$

where $l = j \pm 1$. One obtains

$$T_{jm}^{j\pm1,\text{spin}}(i) = -\frac{1}{M} \frac{q}{2} \left(\frac{\mu_p + \mu_n}{2} + \frac{\mu_p - \mu_n}{2} \tau_{zi}\right) \frac{j_{j \pm 1}(qr'_i)}{j_{j \pm 1}} [Y_{j \pm 1}(\mathbf{r}'_i) \otimes \sigma_i]_{jm}$$

and

$$T_{jm}^{j\pm1,\text{conv}}(i) = \pm \frac{1}{M} \left(\frac{1+\gamma}{2} + \frac{1-\gamma}{2} \tau_{zi}\right) \left\{ j_{j \pm 1}(qr'_i) [Y_{j \pm 1}(\mathbf{r}'_i) \otimes \partial_i]_{jm} + \sqrt{\frac{j+1}{2j+1}} \frac{j_{j \pm 1}(qr'_i)}{j_{j \pm 1}} [Y_{j \pm 1}(\mathbf{r}'_i) \otimes \partial_i]_{jm} \right\} .$$

The term proportional to $j_j(qr'_i)$ in (B6) above cancels when one forms the electric multipole (17).
APPENDIX C: $T^{L}_{JM}$ MULTipoles of $\pi$ AND $\rho$ MECS

Here the $T^{L}_{JM}$ multipoles are given for the "12" pair. The total result should be multiplied by 3 to account for three pairs of identical particles in the trimucleons.

1. $\pi$-Seagull

$$T^{L}_{JM} = \frac{\sqrt{4\pi}}{2} \frac{f_{0}}{m_{\pi}} \sum_{\ell \rho \sigma' \ell' \rho' \sigma' \ell'' \rho''} i^{\sigma+\sigma'+1} (-1)^{\sigma+\ell+\ell'} [1 + (-1)^{\ell+\rho}] \hat{r} \hat{L} \hat{\rho} \hat{\sigma}' \hat{L}'$$

$$\left( \begin{array}{c} \ell \ 1 \\ 0 \ 0 \ 0 \end{array} \right) \left( \begin{array}{c} \sigma' \ \ell \ L \\ 0 \ 0 \ 0 \end{array} \right) \left( \begin{array}{c} L \ \ell \ \sigma' \\ \sigma \ \ell \ L \\ \sigma \ \ell \ L \\ \sigma \ \ell \ L \\ \sigma \ \ell \ L \\ \sigma \ \ell \ L \end{array} \right) \left( \begin{array}{c} 1 \ \ell \ L' \\ \ell \ \rho \ \sigma' \ J \ L' \\ \ell \ \rho \ \sigma' \ J \ L' \end{array} \right)$$

$$j_{\sigma'}(qz) j_{\ell} \left( \begin{array}{c} qr \ 2 \end{array} \right) (H^{\pi}(r))' \left[ [Y_{\sigma}(\hat{z}) \otimes Y_{\sigma}(\hat{r})]^{L} \otimes \Sigma_{12}^{j} \right]_{M} (\tau_{1} \times \tau_{2})_{z} . \quad (C1)$$

Here $r = r_{2} - r_{1}$, $z = -[(r_{1} + r_{2})/2 - R_{cm}]$, $\hat{r} = r/r$, $\hat{z} = z/z$, $(H^{\pi}(r))' = dH^{\pi}(r)/dr$, and $H^{\pi}(r) = \sum_{k=1}^{3} h_{k}^{\pi} Y(\mu_{k}, r)$, where the constants are given by Eqs. (A5), (A7). We denote the spin-coupling $[\sigma_{1} \otimes \sigma_{2}]_{\rho, m}$ by $\Sigma_{12}^{j}$.

2. $\pi$-Exchange Current

The multipole for the true-$\pi$ exchange is the sum

$$T^{L}_{JM} = T^{L,X1}_{JM} + T^{L,X2}_{JM} + T^{L,X3}_{JM}$$

where

$$T^{L,X1}_{JM} = -\frac{\sqrt{4\pi}}{2} \frac{4}{2} \frac{f_{0}}{m_{\pi}} \sum_{\ell \rho \sigma' \ell' \rho' \sigma' \ell'' \rho''} i^{\sigma+\sigma'+1} (-1)^{\sigma} \hat{r} \hat{L} (\hat{L}')^{2} \hat{\rho} \hat{\sigma}' \hat{L}'$$

$$\left( \begin{array}{c} 1 \ 1 \ \rho \\ 0 \ 0 \ 0 \end{array} \right) \left( \begin{array}{c} \sigma' \ \ell \ L \\ 0 \ 0 \ 0 \end{array} \right) \left( \begin{array}{c} 1 \ \ell \ L' \\ 0 \ 0 \ 0 \end{array} \right) \left( \begin{array}{c} L' \ \rho \ \sigma \\ L \ \sigma \ \ell' \ J \ L' \end{array} \right) \left( \begin{array}{c} 1 \ \ell \ L' \\ \ell' \ \rho \ \sigma' \ J \ L' \end{array} \right)$$

$$j_{\sigma'}(qz) \Phi_{\sigma',\ell}(q, r) \left[ [Y_{\sigma}(\hat{z}) \otimes Y_{\sigma}(\hat{r})]^{L} \otimes \Sigma_{12}^{j} \right]_{M} (\tau_{1} \times \tau_{2})_{z} . \quad (C2)$$
\[ T^{L,X}_\alpha = \frac{-\sqrt{4\pi} 4\sqrt{3} q}{i^{J-\epsilon}} \left( \frac{f_0}{m_\pi} \right)^2 \sum_{\ell \ell' \sigma' \sigma \ell' L' J} \sum_{\ell' L J} \iota^{\sigma + \sigma' + 1} (-1)^{\ell + (\hat{L})^2} \hat{L} \hat{\sigma} \hat{\sigma'} \hat{L'} \hat{J'} \hat{j}^2 \]

\[
\begin{pmatrix}
1 & \ell & J' \\
0 & 0 & 0
\end{pmatrix}
\begin{pmatrix}
1 & J' & \sigma \\
0 & 0 & 0
\end{pmatrix}
\begin{pmatrix}
L & 1 & L' \\
0 & 0 & 0
\end{pmatrix}
\begin{pmatrix}
L' & \ell & \sigma' \\
0 & 0 & 0
\end{pmatrix}
\begin{pmatrix}
\sigma & \ell & f \\
L & f & \sigma
\end{pmatrix}
\begin{pmatrix}
1 & \ell & J' \\
1 & 1 & 1
\end{pmatrix}
\begin{pmatrix}
L' & f & L \\
L & 1 & J
\end{pmatrix}
\]

\[ j_{\sigma'}(qz) \Phi_{\sigma,\ell}^{(2)}(q, r) \left[ [Y_{\sigma'}(\hat{z}) \otimes Y_{\sigma}(\hat{r})]^L \otimes \Sigma_{12}^{[1]} \right]_M (\tau_1 \times \tau_2)_z . \quad (C4) \]

Here \( \Phi_{\sigma,\ell}^{(n)}(q, r) = \sum_{k=1}^{3} h_{\pi}^{k} \phi_{\sigma,\ell}^{(n)}(q, r, \mu_{\pi}^{k}) \), where the functions \( \phi_{\sigma,\ell}^{(n)}(q, r, m) \) are defined in \[27\]. The multipoles (\[C1\]–\[C4\]) are real.

3. \( \rho \)-Exchange Currents

The multipoles of the \( \rho \)-exchange currents can be obtained from the above \( \pi \)-exchange currents by means of the following replacements: \( \mu_{\pi}^{k} \rightarrow \mu_{\rho}^{k}, h_{\pi}^{k} \rightarrow h_{\rho}^{k} \),

\[ \left( \frac{f_0}{m_\pi} \right)^2 \rightarrow \frac{1}{4\pi} \left( \frac{g_\rho}{2M} \right)^2 \left( 1 + \frac{f_\rho}{2g_\rho} \right)^2, \quad (C5) \]

and by inserting in each equation above the factor

\[ -6(\rho)^1 \begin{pmatrix} 1 & 1 & 1 \\ 1 & \rho & 1 \end{pmatrix}, \quad (C6) \]

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CAPTIONS TO FIGURES

FIG. 1: Effects of the various contributions on $R_T$ in the quasi-elastic region at $q=174$ (upper left), 250 (upper right), 400 (lower left), and 500 MeV/c (lower right): one-body (dotted), one-body + implicit MEC via Siegert operator (dashed), one-body + $\pi$-MEC + $\rho$-MEC (dashed dotted), one-body + $\pi$-MEC + $\rho$-MEC + additional MEC via Siegert operator (solid).

FIG. 2: As Fig. 1 but for the high-energy region.

FIG. 3: Comparison of theoretical and experimental $R_T$ at $q=250$ (upper panel), 400 (middle panel), and 500 MeV/c (lower panel). Theoretical $R_T$ with contributions: one-body (dotted) and one-body + $\pi$-MEC + $\rho$-MEC + additional MEC via Siegert operator (solid). Experimental data from [23] (triangles), [24] (circles), and [25] (squares).
FIG. 4: Comparison of theoretical and experimental low-energy $R_T$ at $q=0.882$ (upper panel), 1.64 (middle panel), and 2.47 fm$^{-1}$ (lower panel). Notation of curves as in Fig.3, but additional curve in upper panel for total result in case that Coulomb force is neglected in the final state interaction (dash-dotted). Experimental data from [26].
The plots show the function $R_T$ for different values of $q$, which is measured in MeV/c. The $x$-axis represents the energy $\omega$ in MeV, while the $y$-axis represents $R_T$ in $10^{-3}$ MeV. Four different plots are shown:

- **$q=174$ MeV/c**
- **$q=250$ MeV/c**
- **$q=400$ MeV/c**
- **$q=500$ MeV/c**

Each plot contains curves of different styles, indicating different theoretical models or fits. The curves trend downward as $\omega$ increases, reflecting a decrease in $R_T$ with increasing energy. The specific details of the curves are not specified in the image.
