The Mixed Liu Estimator in Stochastic Restricted Linear Measurement Error Model

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1. Introduction

When we use the linear regression model to deal with the problem, some regression explanatory cannot be observed, and the values of the regression explanatory often have measurement. If we directly use these values to set the model, the estimator of regression coefficient may not be a consistent estimator. In order to overcome this problem, the statisticians and econometricians have proposed the linear measurements model. Fuller [2] and Cheng and Van Ness [3] have discussed the model.

It is well known that, in standard linear regression model when there exists collinearity, the ordinary least squares estimator is no longer a good estimator. Some statisticians are discussed how to deal with collinearity. One method is to consider the biased estimator, such as Stein [4]; Hoerl and Kennard [5]; Liu [6]; Yang and Chang [7]; and Wu and Yang [8] et al. Another method is to consider the linear restriction and stochastic linear restrictions [9], such as Li and Yang et al. [10].

In the linear measurement error model, the collinearity problem may also lead the estimator unstable. In order to deal with this problem, Saleh and Shalabh [11] considered the ridge estimator and Ghapani and Babdi [1] considered the Liu estimator. When the linear restrictions or stochastic linear restrictions are satisfied in linear measurement error model, Li et al. [12] proposed some new estimators and discussed the properties of these estimators under Pitman’s closeness criterion. Saleh and Shalabh [11] discussed the preliminary test ridge estimator in the linear measurement error model with linear restrictions, Li and Yang [13] consider the weighted mixed estimator. Ghapani et al. [14] have discussed the weighted ridge estimator in linear measurement error model with stochastic linear restrictions. There are many researches on Liu estimator for different models done by various researchers. To mention a few, Arashi et al. [15], Kibria [16], Alheety and Kibria [17], Ghapani [18], and, very recently, Li et al. [19] are among them.

In this paper, we use a different method to propose a new mixed Liu estimator by construct a Lagrange function. Furthermore, we discuss the properties of the new estimator.

The rest of the paper is organized as follows. In Section 2, we propose the mixed Liu in linear measurement error model with stochastic linear restrictions, and the properties of the new estimator are studied in Section 3. A simulation study has been conducted to support the theoretical results in Section 4, and some conclusion remarks are given in Section 5.
2.1. The Liu Estimator. Let us study the following linear measurement error model:

\[
\begin{align*}
y &= Z\beta + \epsilon, \\
X &= Z + \Delta,
\end{align*}
\]

where \(y = (y_1, y_2, \ldots, y_n)'\) denotes an \(n \times 1\) vector of response variables, \(\hat{\beta}\) shows a \(p \times 1\) vector of unknown parameters, and \(Z\) denotes an \(n \times p\) matrix of unobservable values of explanatory variables which can be observed through the matrix \(X\) with the measurement error \(\Delta' = (\delta_1, \delta_2, \ldots, \delta_n)\), where \(\delta_i, i = 1, 2, \ldots, n\) are \(p \times 1\) uncorrelated random vectors with \(E(\delta_i) = 0, \text{Var}(\delta_i) = \Sigma\). We assume that the common variance \(\Sigma\) of measurement errors associated with the explanatory variables is known. And, we also suppose that \(\epsilon\) is an \(n \times 1\) vector of unobservable random errors with \(E(\epsilon) = 0, \text{Var}(\epsilon) = \sigma^2I\). We let \(\epsilon\) and \(\Delta\) be mutually independent, and we assume the \(i\)th rows of matrices \(Z\) and \(X\) with \(z_i'\) and \(x_i'\), respectively.

Fuller [2] has introduced the consistent estimator of \(\beta\), which is presented as follows:

\[
\hat{\beta} = (X'X - n\sum)\beta = (X'X - n\sum)^{-1}X'y,
\]

and this estimator is obtained by solving the following function:

\[
S_1(\beta) = \arg\min_\beta \left\{ (y - X\beta)'(y - X\beta) - n\beta'\delta - \sum \beta \right\}.
\]

In order to deal with the collinearity problem, Ghapani and Babdi [1] introduced a Liu estimator (LE), and this estimator can be obtained as follows: based on (3), consider the following objective function:

\[
S_2(\beta) = \arg\min_\beta \left\{ (y - X\beta)'(y - X\beta) - n\beta'\delta - \sum \beta \\
+ (d\hat{\beta} - \beta)'(d\hat{\beta} - \beta) \right\}.
\]

Dealing with (4), we can obtain

\[
\hat{\beta}(d) = (X'X - n\sum +I)^{-1}(X'y + d\hat{\beta}), \quad 0 < d < 1.
\]

In Section 2.2, we will present the new estimator.

2.2. The Mixed Liu Estimator. In this article, we suppose that the stochastic linear restrictions on the parametric component are of the following form:

\[
h = H\beta + e, e \sim N(0, \sigma^2W),
\]

where \(h\) is a \(q \times 1\) observable random vector, \(H\) is a \(q \times p\) known matrix with rank \((H) = q\) for \(q < p\), and \(e\) shows a \(q \times 1\) error vector with \(E(e) = 0\) and \(\text{Var}(e) = \sigma^2W\), and we also assume that \(W\) is a known positive definite matrix. Furthermore, we also suppose that \(e\) is stochastically independent of \(\epsilon\) and \(\Delta\).

Based on models (1) and (6), using the mixed method, we can minimize the following equation:

\[
S_3(\beta) = \arg\min_\beta \left\{ (y - X\beta)'(y - X\beta) - n\beta'\delta - \sum \beta \\
+ (h - H\beta)'W^{-1}(h - H\beta) \right\},
\]

with respect to \(\beta\). By (7), we obtain the mixed estimator (ME):

\[
\hat{\beta}_{\text{ME}} = (X'X - n\sum +H'W^{-1}H)^{-1}(X'y + H'W^{-1}h).
\]

Ghapani and Babdi [1] proposed a mixed Liu estimator, which is defined as follows:

\[
\hat{\beta}_{\text{MLE}} = (X'X - n\sum +H'W^{-1}H)^{-1} \times \left((X'X - n\sum +I)X'y + H'W^{-1}h\right).
\]

Now, we will propose a new mixed Liu estimator. Consider the following function:

\[
S_4(\beta) = \arg\min_\beta \left\{ (y - X\beta)'(y - X\beta) - n\beta'\delta - \sum \beta \\
+ (h - H\beta)'W^{-1}(h - H\beta) + (d\hat{\beta}_{\text{ME}} - \beta)'(d\hat{\beta}_{\text{ME}} - \beta) \right\}.
\]

Dealing with (9), we can obtain

\[
\hat{\beta}_{\text{NMLE}}(d) = (X'X - n\sum +H'W^{-1}H + I)^{-1} \times (X'y + H'W^{-1}h + d\hat{\beta}_{\text{ME}}).
\]

where \(0 < d < 1\) denotes the biasing parameter and \(\hat{\beta}_{\text{ME}}\) denotes the mixed estimator, and we call this estimator as a new mixed Liu estimator.

The new estimator can also be written as follows:

\[
\hat{\beta}_{\text{NMLE}}(d) = (X'X - n\sum +H'W^{-1}H + I)^{-1} \times (X'y + H'W^{-1}h + d\hat{\beta}_{\text{ME}}).
\]

By the definition of the new estimator, we can see that the new estimator is a general estimator which contains \(\hat{\beta}_{\text{ME}}, \hat{\beta}_{\text{MLE}}, \hat{\beta}_{\text{NMLE}}(d),\) and \(\hat{\beta}_{\text{MLE}}\) as special cases.

If \(d = 1\), \(\hat{\beta}_{\text{NMLE}}(d) = \hat{\beta}_{\text{ME}}\).

If \(H = 0\), \(\hat{\beta}_{\text{NMLE}}(d) = \hat{\beta}(d)\).

If \(d = 1\) and \(H = 0\), \(\hat{\beta}_{\text{NMLE}}(d) = \hat{\beta}\).

In Section 3, we will study the asymptotic properties of these estimators.

3. The Properties of These Estimators

In this section, we will give the comparison of the new estimator with some estimators. Firstly, we give the properties of these estimators.

3.1. Large Sample Properties of These Estimators. Though the exact distribution and small sample properties of these estimators are difficult to obtain, in this paper, we use the large sample asymptotic approximation theory to study the asymptotic distribution of the estimators. We assume that the parameter \(\beta\) is identifiable and we also assume that as \(n\) tends to infinity, the limits of \(n^{-1}(Z'Z + H'W^{-1}H), n^{-1}(Z'Z + H'W^{-1}H + dI),\) and \(n^{-1}(Z'Z + H'W^{-1}H + I)\) exist and \(E\) denotes the global expectation taken at the true value \(\beta\).
Theorem 1. \( \hat{\beta}_{\text{NMLE}}(d) \) is asymptotically normally distributed. The asymptotically mean and variance of \( \hat{\beta}_{\text{NMLE}}(d) \) are, respectively, given as \( \text{E}[\hat{\beta}_{\text{NMLE}}(d)] = M_1^{-1}M_d\beta \) and \( \text{AVar}[\hat{\beta}_{\text{NMLE}}(d)] = M_1^{-1}M_dM_0^{-1}(B + \sigma^2(Z'Z + H'W^{-1}H))M_0^{-1}M_dM_1^{-1} \), where \( M_d = n^{-1}(Z'Z + H'W^{-1}H + dl) \) and \( B = (n\sigma^2 + \beta'Z'\beta)\Sigma \).

Proof. By Fung et al. [20], \( E(X'X) = Z'Z + n\Sigma \), we have

\[
X'X = Z'Z + n\sum + O_p(n^{1/2}).
\]

Therefore, we may write

\[
n^{-1}(X'X + H'W^{-1}H + I) = n^{-1}(Z'Z + H'W^{-1}H + I) + \sum + O_p(n^{-1/2}), \quad (14)
\]

\[
n^{-1}(X'X + H'W^{-1}H + dl) = n^{-1}(Z'Z + H'W^{-1}H + dl) + \sum + O_p(n^{-1/2}), \quad (15)
\]

\[
n^{-1}(X'X + H'W^{-1}H) = n^{-1}(Z'Z + H'W^{-1}H) + \sum + O_p(n^{-1/2}). \quad (16)
\]

Thus, by (14)–(16), we can obtain that

\[
\sqrt{n} \hat{\beta}_{\text{NMLE}}(d) = \left\{ n^{-1}(Z'Z + H'W^{-1}H + I) + O_p(n^{-1/2}) \right\}^{-1}
\times \left\{ n^{-1}(Z'Z + H'W^{-1}H + dl) + O_p(n^{-1/2}) \right\}
\times \left\{ n^{-1}(Z'Z + H'W^{-1}H) + O_p(n^{-1/2}) \right\}^{-1} n^{-1/2}(X'y + H'W^{-1}h)
\]

\[
= \left[ I + O_p(n^{-1/2}) \right]^{-1} \left\{ n^{-1}(Z'Z + H'W^{-1}H + I) + O_p(n^{-1/2}) \right\}^{-1}
\times \left\{ n^{-1}(Z'Z + H'W^{-1}H + dl) + O_p(n^{-1/2}) \right\}
\times \left\{ n^{-1}(Z'Z + H'W^{-1}H) + O_p(n^{-1/2}) \right\}^{-1} n^{-1/2}(X'y + H'W^{-1}h).
\]

Then, we get

\[
\text{AVar}[\hat{\beta}_{\text{NMLE}}(d)] = M_1^{-1}M_dM_0^{-1}(B + \sigma^2(Z'Z + H'W^{-1}H))M_0^{-1}M_dM_1^{-1}.
\]

Corollary 1. \( \hat{\beta}_{\text{MLE}} \) has asymptotically normal distribution with \( E[\hat{\beta}_{\text{MLE}}] = \beta \) and \( \text{AVar}[\hat{\beta}_{\text{MLE}}] = M_0^{-1}(B + \sigma^2(Z'Z + H'W^{-1}H))M_0^{-1}. \)

Corollary 2. \( \hat{\beta}(d) \) has asymptotically normal distribution with \( E[\hat{\beta}(d)] = G_d\beta \) and \( \text{AVar}[\hat{\beta}(d)] = G_d(Z'Z)^{-1}(B + \sigma^2Z'Z)(Z'Z)^{-1}G_d, \) where \( G_d = (Z'Z + I)^{-1}(Z'Z + dl). \)

By [1], we know that \( E[\hat{\beta}_{\text{MLE}}(d)] = M_0^{-1}(G_dZ'Z + H'W^{-1}H)\beta \) and \( \text{AVar}[\hat{\beta}_{\text{MLE}}(d)] = M_0^{-1}[G_dBG_d + \sigma^2(G_dZ'ZG_d + H'W^{-1}H)]M_0^{-1}. \)
3.2. Comparisons among Biased Estimators. In this subsection, we will present the comparison of the new estimator to the $\hat{\beta}_{\text{ME}}$, $\hat{\beta}(d)$, and $\hat{\beta}_{\text{MLE}}(d)$ under the mean-squared error matrix. Firstly, we present the mean-squared error matrix of an estimator $\hat{\theta}$ of $\theta$ is defined as

$$\text{MSEM}(\hat{\theta}) = E((\hat{\theta} - \theta)'(\hat{\theta} - \theta)) = \text{Var}(\hat{\theta}) + \text{Bias}(\hat{\theta})\text{Bias}(\hat{\theta})' ,$$

where $\text{Bias}(\hat{\theta}) = E(\hat{\theta} - \theta)$ denotes the bias vector. In order to present the main results, we give some lemmas.

$$\begin{align*}
\text{AMSEM}[\hat{\beta}_{\text{MLE}}(d)] &= M_0^{-1}M_0^{-1}\left[ B + \sigma^2(Z'Z + H'W^{-1}H)\right]M_0^{-1}M_0^{-1} + b_1b_1', \\
\text{AMSEM}[\hat{\beta}_{\text{ME}}] &= M_0^{-1}\left[ B + \sigma^2(Z'Z + H'W^{-1}H)\right]M_0^{-1} , \\
\text{AMSEM}[\hat{\beta}(d)] &= G_d(Z'Z)^{-1}\left[ B + \sigma^2(Z'Z + H'W^{-1}H)\right]G_d + b_2b_2' , \\
\text{AMSEM}[\hat{\beta}_{\text{MLE}}(d)] &= M_0^{-1}\left[ G_dBG_d + \sigma^2(G_dZ'ZG_d + H'W^{-1}H)\right]M_0^{-1} + b_3b_3' ,
\end{align*}$$

where $b_1 = M_1^{-1}M_0\beta$, $b_2 = (G_d - I)\beta$, and $b_3 = (M_1^{-1}(G_dZ'Z + H'W^{-1}H) - I)\beta$.

In order to compare the $\hat{\beta}_{\text{MLE}}(d)$ to $\hat{\beta}_{\text{ME}}(d)$, we consider the asymptotic AMSEM differences:

$$\begin{align*}
\text{V}_1 &= \text{AMSEM}[\hat{\beta}_{\text{ME}}] - \text{AMSEM}[\hat{\beta}_{\text{MLE}}(d)] \\
&= D_1 - b_1b_1' , \\
\text{V}_2 &= \text{AMSEM}[\hat{\beta}_{\text{MLE}}] - \text{AMSEM}[\hat{\beta}_{\text{MLE}}(d)] \\
&= D_2 + b_2b_2' - b_1b_1' , \\
\text{V}_3 &= \text{AMSEM}[\hat{\beta}_{\text{MLE}}(d)] - \text{AMSEM}[\hat{\beta}_{\text{MLE}}(d)] \\
&= D_3 + b_3b_3' - b_1b_1' ,
\end{align*}$$

where

$$\begin{align*}
D_1 &= M_0^{-1}\left[ B + \sigma^2(Z'Z + H'W^{-1}H)\right]M_0^{-1} - M_1^{-1}M_0M_0^{-1}\left[ B + \sigma^2(Z'Z + H'W^{-1}H)\right]M_0^{-1}M_0^{-1} , \\
D_2 &= G_d(Z'Z)^{-1}\left( B + \sigma^2Z'Z\right)(Z'Z)^{-1}G_d - M_1^{-1}M_0M_0^{-1}\left[ B + \sigma^2(Z'Z + H'W^{-1}H)\right]M_0^{-1}M_0^{-1} , \\
D_3 &= M_0^{-1}\left[ G_dBG_d + \sigma^2(G_dZ'ZG_d + H'W^{-1}H)\right]M_0^{-1} - M_1^{-1}M_0M_0^{-1}\left[ B + \sigma^2(Z'Z + H'W^{-1}H)\right]M_0^{-1}M_0^{-1} .
\end{align*}$$

\textbf{Lemma 1} (see [21]). 0 Suppose that $M$ be a positive matrix, namely, $M > 0$ and $a$ be some vector, then $M - ad' \geq 0$ if and only if $d M^{-1}a \leq 1$.

\textbf{Lemma 2} (see [9]). Let $n \times n$ matrices $M > 0$, $N \geq 0$, then $M > N$ if and only if $\lambda_{\text{max}}(NM^{-1}) < 1$.

Now, we give the comparison of the estimator $\hat{\beta}_{\text{MLE}}(d)$ to the $\hat{\beta}_{\text{ME}}$ in the MSEM sense.

\textbf{Theorem 2}. The $\hat{\beta}_{\text{MLE}}(d)$ is better than the estimator $\hat{\beta}_{\text{ME}}$ in the MSEM sense, if and only if $b_1'(D_2 + b_2b_2')^{-1}b_1 \leq 1$.

\textbf{Proof}. Now we prove

$$D_1 = M_0^{-1}\left[ B + \sigma^2(Z'Z + H'W^{-1}H)\right]M_0^{-1} - M_1^{-1}M_0M_0^{-1}\left[ B + \sigma^2(Z'Z + H'W^{-1}H)\right]M_0^{-1}M_0^{-1} > 0 .$$

Let $H = M_0^{-1}\left( B + \sigma^2(Z'Z + H'W^{-1}H)\right)M_0^{-1}$, we have $H > 0$, then we can write $D_1$ as follows:

$$D_1 = H - M_1^{-1}M_0HM_0M_0^{-1} = (1 - d)M_1^{-1}M_0(M_0^{-1}H + (1 - d)M_0^{-1}H)^{-1}M_0^{-1}M_0^{-1} .$$

Since $0 < d < 1$, $M_0^{-1} > 0$, and $H > 0$, we have $D_1 > 0$. By Lemma 1, we have $\hat{\beta}_{\text{MLE}}(d)$ is better than the estimator $\hat{\beta}_{\text{ME}}$ in the MSEM sense, if and only if $b_1'(D_2 + b_2b_2')^{-1}b_1 \leq 1$. 

\textbf{Theorem 3}. When

$$\lambda_{\text{max}}\left[ M_1^{-1}M_0M_0^{-1}\left[ B + \sigma^2(Z'Z + H'W^{-1}H)\right]M_0^{-1}M_0^{-1}\left[ (G_d(Z'Z)^{-1}(B + \sigma^2Z'Z)(Z'Z)^{-1}G_d)^{-1}\right]\right] \leq 1 ,$$

the $\hat{\beta}_{\text{MLE}}(d)$ is better than the estimator $\hat{\beta}(d)$ in the MSEM sense, if and only if $b_1'(D_2 + b_2b_2')^{-1}b_1 \leq 1$.

\textbf{Proof}. Since

$$G_d(Z'Z)^{-1}(B + \sigma^2Z'Z)(Z'Z)^{-1}G_d > 0 ,$$

$M_1^{-1}M_0M_0^{-1}\left[ B + \sigma^2(Z'Z + H'W^{-1}H)\right]M_0^{-1}M_0^{-1} > 0 ,$$

(29)
then by Lemma 2, when

$$
\lambda_{\max}\left[ M_1^{-1}M_dM_0^{-1}\left( B + \sigma^2(Z'Z + H'W^{-1}H) \right) M_0^{-1}M_dM_1^{-1} \cdot \left( G_d(Z'Z)^{-1}(B + \sigma^2Z'Z)(Z'Z)^{-1}G_d \right)^{-1} \right] \leq 1,
$$

we have $D_3 > 0$, then by Lemma 1, we get that the new estimator is superior to the $\hat{\beta}(d)$ in the MSEM sense, if and only if $b_1'(D_3 + b_1b_1')^{-1}b_1 \leq 1$. \hfill \Box

Theorem 4. When

$$
\lambda_{\max}\left[ M_1^{-1}M_dM_0^{-1}\left( B + \sigma^2(Z'Z + H'W^{-1}H) \right) M_0^{-1}M_dM_1^{-1} \cdot \left( M_0^{-1}[G_dBG_d + \sigma^2G_dZ'ZG_d + H'W^{-1}H]M_0^{-1} \right)^{-1} \right] \leq 1.
$$

then by Lemma 2, when

we have $D_3 > 0$, then by Lemma 1, we get that the new estimator is superior to the $\hat{\beta}_{\text{MLE}}(d)$ in the MSEM sense, if and only if $b_1'(D_3 + b_1b_1')^{-1}b_1 \leq 1$. \hfill \Box

4. Monte Carlo Simulation Experiments

In this section, we will conduct a Monte Carlo simulation experiment is designed to show the performance of these estimators. Following McDonald and Galarneau [22], we may get the explanatory variables by using

$$
z_{ij} = (1 - \rho^2)^{1/2}w_{ij} + \rho w_{i(p+1)},
$$

where $w_{ij}$ are got by the standard normal distribution and $\rho$ is chosen so that the correlation between any two variables is $\rho^2$. Three different values of the correlation are used, namely, 0.9, 0.95, and 0.99. The real values of the parameter vector $\beta$ are chosen as the eigenvector of the matrix $Z'Z$ corresponding to the largest eigenvalue. Moreover, we have considered the explanatory variable as $\rho = 4$. We also assume that $\Sigma = \text{diag}(0.01, \ldots, 0.01)$ and $\sigma = 1, 5$, and 10. The sample size is taken to be 50, 100, and 150.

The stochastic linear restrictions of $H$ are generated by norm distributions and $e \sim N(0, \sigma^2I)$. Note that in this paper we did not introduced any estimators of the shrinkage parameter $d$; therefore, we only consider some values of $d$ such that $0 < d < 1$. We generated 5000 data sets containing the explanatory variables and the dependent variable. The simulated mean-squared error (MSE) is used to compare the estimators such that it can be computed as follows:

$$
\tilde{\beta}_r = \frac{\sum_{r=1}^{5000}(\hat{\beta}_r - \beta)(\hat{\beta}_r - \beta)}{5000}
$$

where $\hat{\beta}_r$ is any estimator considered in this paper in the $r^{th}$ repetition. All computations are performed using the R Program.

We have summarized the results of the simulation in Tables 1–5. We can conclude the following from the tables.

(1) The new estimator is always superior to the ME and LE.

(2) The new estimator is superior to the MLE in most cases. When the $\rho^2 = 0.99$, that is the multicollinearity is serve, the new estimator is superior to the MLE.

(3) When the $n$ is small, the new estimator performs well.
### Table 1: MSE values of the estimator for different values of $d$ and $\rho$ when $\sigma = 1$ and $n = 50$.  

| $\rho$ | 0.1 | 0.2 | 0.3 | 0.4 | 0.5 | 0.6 | 0.7 | 0.8 | 0.9 |
|-------|-----|-----|-----|-----|-----|-----|-----|-----|-----|
| OME   | 0.3466 | 0.3466 | 0.3466 | 0.3466 | 0.3466 | 0.3466 | 0.3466 | 0.3466 | 0.3466 |
| LE    | 0.3630 | 0.3740 | 0.3851 | 0.3964 | 0.4080 | 0.4196 | 0.4315 | 0.4436 | 0.4558 |
| MLE   | 0.2849 | 0.2914 | 0.2979 | 0.3046 | 0.3113 | 0.3182 | 0.3251 | 0.3322 | 0.3394 |
| NMLE  | 0.2839 | 0.2905 | 0.2972 | 0.3040 | 0.3109 | 0.3179 | 0.3249 | 0.3321 | 0.3393 |

### Table 2: MSE values of the estimator for different values of $d$ and $\rho$ when $\sigma = 1$ and $n = 100$.  

| $\rho$ | 0.1 | 0.2 | 0.3 | 0.4 | 0.5 | 0.6 | 0.7 | 0.8 | 0.9 |
|-------|-----|-----|-----|-----|-----|-----|-----|-----|-----|
| OME   | 0.1493 | 0.1493 | 0.1493 | 0.1493 | 0.1493 | 0.1493 | 0.1493 | 0.1493 | 0.1493 |
| LE    | 0.1551 | 0.1567 | 0.1583 | 0.1600 | 0.1616 | 0.1633 | 0.1650 | 0.1667 | 0.1684 |
| MLE   | 0.1375 | 0.1388 | 0.1401 | 0.1414 | 0.1427 | 0.1440 | 0.1453 | 0.1466 | 0.1479 |
| NMLE  | 0.1375 | 0.1388 | 0.1401 | 0.1414 | 0.1427 | 0.1440 | 0.1453 | 0.1466 | 0.1479 |

### Table 3: MSE values of the estimator for different values of $d$ and $\rho$ when $\sigma = 1$ and $n = 150$.  

| $\rho$ | 0.1 | 0.2 | 0.3 | 0.4 | 0.5 | 0.6 | 0.7 | 0.8 | 0.9 |
|-------|-----|-----|-----|-----|-----|-----|-----|-----|-----|
| OME   | 0.1009 | 0.1009 | 0.1009 | 0.1009 | 0.1009 | 0.1009 | 0.1009 | 0.1009 | 0.1009 |
| LE    | 0.1040 | 0.1048 | 0.1055 | 0.1063 | 0.1070 | 0.1077 | 0.1085 | 0.1093 | 0.1100 |
| MLE   | 0.0953 | 0.0959 | 0.0965 | 0.0971 | 0.0977 | 0.0984 | 0.0990 | 0.0996 | 0.1002 |
| NMLE  | 0.0953 | 0.0959 | 0.0965 | 0.0971 | 0.0977 | 0.0984 | 0.0990 | 0.0996 | 0.1002 |

### Table 4: MSE values of the estimator for different values of $d$ and $\rho$ when $\sigma = 1$ and $n = 200$.  

| $\rho$ | 0.1 | 0.2 | 0.3 | 0.4 | 0.5 | 0.6 | 0.7 | 0.8 | 0.9 |
|-------|-----|-----|-----|-----|-----|-----|-----|-----|-----|
| OME   | 0.0928 | 0.0932 | 0.0932 | 0.0932 | 0.0932 | 0.0932 | 0.0932 | 0.0932 | 0.0932 |
| LE    | 0.0967 | 1.0154 | 1.0867 | 1.1606 | 1.2370 | 1.3159 | 1.3975 | 1.4815 | 1.5682 |
| MLE   | 0.6152 | 0.6461 | 0.6781 | 0.7112 | 0.7454 | 0.7807 | 0.8171 | 0.8545 | 0.8931 |
| NMLE  | 0.6132 | 0.6451 | 0.6778 | 0.7115 | 0.7461 | 0.7816 | 0.8180 | 0.8554 | 0.8936 |
5. Conclusions

In this paper, we use a new method to propose a new mixed estimator in the linear measurement error model and we also discuss the properties of the new estimator. A Monte Carlo simulation experiment is designed to evaluate the performances of the estimators in terms of the simulated mean-squared error criterion. Simulation results indicated that the new estimator performed better than the rest of the estimators when the multicollinearity problem exists in the data. Therefore, the new estimator can be an alternative to the existing estimators especially in the presence of highly correlated data.

Data Availability

The data used to support the findings of this study are available from the corresponding author upon request.

Conflicts of Interest

The author declares that they have no conflicts of interest.

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Table 4: MSE values of the estimator for different values of $d$ and $\rho$ when $\sigma = 5$ and $n = 50$.

| $\rho$ | 0.1 | 0.2 | 0.3 | 0.4 | 0.5 | 0.6 | 0.7 | 0.8 | 0.9 |
|-------|-----|-----|-----|-----|-----|-----|-----|-----|-----|
| OME   | 7.3456 | 7.3456 | 7.3456 | 7.3456 | 7.3456 | 7.3456 | 7.3456 | 7.3456 | 7.3456 |
| LE    | 7.6629 | 7.8677 | 8.0756 | 8.2868 | 8.5011 | 8.7187 | 8.9395 | 9.1634 | 9.3906 |
| MLE   | 6.1826 | 6.3046 | 6.4285 | 6.5541 | 6.6815 | 6.8108 | 6.9418 | 7.0746 | 7.2092 |
| NMLE  | 6.1587 | 6.2846 | 6.4119 | 6.5407 | 6.6711 | 6.8029 | 6.9363 | 7.0713 | 7.2077 |

Table 5: MSE values of the estimator for different values of $d$ and $\rho$ when $\sigma = 10$ and $n = 50$.

| $\rho$ | 0.1 | 0.2 | 0.3 | 0.4 | 0.5 | 0.6 | 0.7 | 0.8 | 0.9 |
|-------|-----|-----|-----|-----|-----|-----|-----|-----|-----|
| OME   | 58.8476 | 58.8476 | 58.8476 | 58.8476 | 58.8476 | 58.8476 | 58.8476 | 58.8476 | 58.8476 |
| LE    | 38.5358 | 48.9573 | 60.7964 | 74.0531 | 88.7275 | 104.8194 | 122.3290 | 141.2563 | 161.6011 |
| MLE   | 24.7887 | 27.6342 | 30.2419 | 33.4217 | 36.9038 | 40.6882 | 44.7747 | 49.1635 | 53.8544 |
| NMLE  | 21.0234 | 24.2775 | 27.6287 | 31.2570 | 35.1625 | 39.3452 | 43.8051 | 48.5421 | 53.5563 |
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