Rayleigh–Bénard convection in rotating nanofluids layer with feedback control subjected to magnetic field

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Abstract. Magnetic field on Rayleigh–Bénard convective instability in rotating feedback–controlled nanofluids layer heated from below has been examined for the boundaries of free–free, rigid–free and rigid–rigid. Model applied to nanofluids associated with the Brownic movement and thermophoresis mechanism. A normal–mode linear stability assessment has been performed, the eigenvalue solution has been extracted by using single term Galerkin technique and computed by employing Maple software. It is found that the influences of magnetic field, feedback control, rotation are to slow down the thermal instability.

1. Introduction
Heat transfer enhancement in thermoscience and thermal engineering progress where the utilization of fluid additives is often involved. The solid nanoscaled either non–metal or metal nanoparticles are suspended in the base fluids in order to alter the properties of energy transportation, flow and thermal transfer characteristics of the fluids [1]. Ghasemi et al. [2] and Hamada et al. [3] used nanofluids based in water comprising various nanoparticles types such as silver Ag, Copper Cu and alumina Al₂O₃ for numerical calculation in nanofluids with magnetic field effect. Later, Yadav et al. [4] investigated the thermal instability of the combined effects of rotational magnetic field for alumina Al₂O₃–H₂O nanofluid. Later, Yadav et al. [5, 6] analyzed the effects of rotation and magnetic field independently for nanofluids of alumina–water Al₂O₃–H₂O and copper–water Cu–H₂O respectively.

Scientists and scientists began researching the impacts of rotation and magnetic field several centuries earlier. Chandrasekhar [7] indicates that Rayleigh’s critical value could be enhanced through the use of magnetic field and rotation effects, thus stabilized the system. Yadav et al. [8], Chand [9], Gupta et al. [10] and Yadav et al. [11] have demonstrated and studied the stabilizing impacts of magnetic field and rotation on nanofluids layer thermal instability. At the same time, Chand and Rana [12] and Yadav et al. [13] have introduced other relevant impacts on the thermal instability of the nanofluid layer in the presence of the magnetic field.

The utilizing of feedback control is used for convective thermal stabilization has been pioneered a few years ago by many scientists and researchers. It was shown by Tang and Bau
that the substitution feedback control impact can considerably raising critical Rayleigh number on convective instability. Bau [16] noted that the same control strategies used to stabilize the Marangoni-Benard convection can be employed to control the Rayleigh-Benard convection. Further, control strategy developed by Bau [16] has been applied by Arifin et al. [17], Hashim and Siri [18], Kechil and Hashim [19], Bachok et al. [20], Siri et al. [21] and Khalid et al. [22] to study the effect of feedback control with other effects.

In this study, Rayleigh–Bénard convection in rotating Al$_2$O$_3$–H$_2$O and Cu–H$_2$O nanofluids with feedback control and magnetic field for lower and upper free, lower rigid upper free as well as lower and upper rigid is investigated. Analysis of linear stabilization depending on normal mode was used and the eigenvalue was acquired using the technique of Galerkin. Numerical computations of the Taylor number $Ta$, magnetic Chandrasekhar number $H$ and feedback control $K$ parameters are computed by Maple software and presented graphically.

2. Mathematical Formulation

The infinite parallel plane of the nanofluids layer heated from below confined to the effects of rotation, magnetic field and feedback control between $z = 0$ and $z = d$ is studied. The temperature and volumetric fraction of nanoparticles at the lower and upper boundaries are referred to as $T_0^*$ and $\phi_0^*$ at $z = 0$; and $T_u^*$ and $\phi_u^*$ at $z = d$. The estimation of Oberbeck–Boussinesq flow that has been introduced depends on the following:

$$\nabla^* \cdot u^* = 0,$$

$$\rho_f \left[ \frac{\partial u^*}{\partial t^*} + (u^* \cdot \nabla^*)u^* \right] = -\nabla^*p^* + \mu \nabla^2 u^* - 2\rho_f \Omega^* \times u^* + g \{ \phi^* \rho_p + (1 - \phi^*) \rho_f [1 - \alpha_T(T^* - T_u^*)] \} + \frac{\mu_e}{4\pi} (H^* \cdot \nabla^*)H^*, \quad (2)$$

$$\frac{\partial T^*}{\partial t^*} + (u^* \cdot \nabla^*)T^* = \kappa \nabla^2 T^* + (\rho c)_p D_B \nabla^* \phi^* \cdot \nabla^* T^* + \frac{\partial \phi^*}{\partial t^*} + (u^* \cdot \nabla^*)\phi^* = D_B \nabla^2 \phi^* + \frac{D_T}{T_u^*} \nabla^2 T^* \quad \text{(3)}$$

where $u^*$ is the velocity, $\rho_f$ is the density of the fluid, $t^*$ is the time, $p^*$ is the pressure, $\mu$ is the viscosity, $\Omega^* = (0, 0, \Omega)$ is the angular velocity, $g$ is the gravity, $\phi^*$ is the volumetric fraction of nanoparticles, $\rho_p$ is the particles density, $\alpha_T$ is the thermal volumetric coefficient, $T^*$ is the temperature, $H^*$ is the magnetic permeability, $\mu_e$ is the magnetic permeability of the fluids, $H$ is the magnetic field, $c$ is the specific heat, $\kappa$ is the nanofluids thermal conductivity, $D_B$ is the Brownian motion and $D_T$ is the thermophoretic diffusion coefficient.

The modified Maxwells equation are, Chandrasekhar [7] :

$$\left[ \frac{\partial}{\partial t^*} + (u^* \cdot \nabla^*) \right] H^* = (H^* \cdot \nabla^*)u^* + \eta \nabla^2 \cdot H^*, \quad (5)$$

$$\nabla^* \cdot H^* = 0, \quad \text{(6)}$$

where $\eta = \frac{1}{4\pi \mu_e h}$ and $h$ are fluid resistivity and electrical conductivity.
Equations (1)–(6) are nondimensionalized using the following definitions

\[ (x^*, y^*, z^*) = L(x, y, z), \quad p^* = \frac{\mu \alpha_f p}{L^2}, \quad \phi^* = \frac{\phi^* - \phi^*_l}{\phi^*_u - \phi^*_l}, \quad (H_x^*, H_y^*, H_z^*) = \frac{(H_x^s, H_y^s, H_z^s)}{H_0^*}, \]

\[ \psi^*_z = \alpha_f \frac{\psi^*_z}{L}, \quad t^* = \frac{L^2 t}{\alpha_f}, \quad (u^*, v^*, w^*) = \alpha_f \left(\frac{u}{L}, v, w\right), \quad T = \frac{T^* - T^*_a}{\Delta T^*}, \quad x_z = L \frac{x_z^*}{H_0^*} \]

where \( \alpha_f = \frac{k}{(p c)_f} \) is defined as the thermal diffusiveness and \( \psi^*_z \) is the \( z \)-component rotational vorticity.

The following non-dimensional variables are obtained after nondimensionalization of Equations (1)–(6):

\[ \nabla \cdot \mathbf{u} = 0, \quad \frac{1}{Pr} \left[ \frac{\partial \mathbf{u}}{\partial t} + \mathbf{u} \cdot \nabla \mathbf{u} \right] = -\nabla p + \nabla^2 \mathbf{u} - Rm \mathbf{e}_z + Ra T \mathbf{e}_z - Rn \phi \mathbf{e}_z - \sqrt{T} \mathbf{a} \times \mathbf{e}_z + H \frac{Pr}{Pm} (H \cdot \nabla) \mathbf{H}, \]

\[ \frac{\partial T}{\partial t} + \mathbf{u} \cdot \nabla T = \nabla^2 T + \frac{N_B}{L_n} \nabla \phi \cdot \nabla T + \frac{N_A N_B}{L_n} \nabla \mathbf{T} \cdot \nabla T, \]

\[ \frac{\partial \phi}{\partial t} + \mathbf{u} \cdot \nabla \phi = \frac{1}{L_n} \nabla^2 \phi + \frac{N_A}{L_n} \nabla^2 T, \]

\[ \frac{\partial \mathbf{H}}{\partial t} + \mathbf{u} \cdot \nabla \mathbf{H} = (H \cdot \nabla) \mathbf{u} + \frac{Pr}{Pm} \nabla^2 \mathbf{H}, \]

\[ \nabla \cdot \mathbf{H} = 0, \]

here \( Pm = \frac{\mu}{\rho_f \eta^*} \) is the magnetic Prandtl number and \( H = \frac{\mu \rho^2 L^2}{4 \pi \rho_f \nu^*} \) is the magnetic Chandrasekhar number, where the kinematic viscosity of nanofluids is \( \nu = \frac{\mu}{\rho_f} \).

The proposed representation of normal mode technique is

\[ (u', T', \phi', \psi_z') = [W(z), \Theta(z), \Phi(z), \Psi(z)] e^{i[(a_x x + a_y y) + st]}, \]

where \( W(z), \Theta(z), \Phi(z) \) and \( \Psi(z) \) are vertical velocity disturbance, temperature, volumetric fraction of nanoparticles and vorticity due to the rotation amplitudes; the wavenumbers in the directions of \( x \) and \( y \) are \( a_x \) and \( a_y \); \( s = i \omega \), where \( i = \sqrt{-1} \) and \( \omega \) is real and dimensionless frequency.

\[ (D^2 - a^2)^2 W - HD^2 W - \sqrt{T} a D \Psi - a^2 Ra \Theta + a^2 Rn \Phi = 0, \]

\[ W + \left[ D^2 - a^2 + \frac{N_B}{L_n} D - 2 \frac{N_A N_B}{L_n} D \right] \Theta - \frac{N_B}{L_n} D \Phi = 0, \]

\[ W - \frac{N_A}{L_n} (D^2 - a^2) \Theta - \frac{1}{L_n} (D^2 - a^2) \Phi = 0, \]

\[ (D^2 - a^2)^2 \Psi + \sqrt{T} a (D^2 - a^2) DW - HD^2 \Psi = 0, \]

where \( D = \frac{d}{dz} \) and \( a = \sqrt{a_x^2 + a_y^2} \) is the wavenumber.
Following Bau’s proportional feedback control [16], the continually circulated actuators and sensors are sorted in such a manner that an actuator is placed directly below it for each sensor. It is possible to determine a control $q(t)$ using the proportional–integral–differential (PID) controller of the form

$$q(t) = r + K \left[ e(t) \right], e(t) = \hat{m}(t) + m(t),$$

(19)

where $r$ is a control calibration, $e(t)$ is a measurement error, $\hat{m}(t)$, from the required reference value, $m(t)$, $K$ is a scalar controller gain where $K = K_P + K_D \frac{d}{dt} + K_L \int_0^t dt$, $K_P$ is a proportional gain, $K_D$ is a differential gain, and $K_L$ is an integral gain. On the basis of (19), the actuator modifies the temperature of the heated surface of one sensor plane and proportional feedback control using a proportional relationship between the lower, $z = 0$ and the upper, $z = 1$ thermal boundaries for disturbance field,

$$T'(x, y, 0, t) = -KT'(x, y, 1, t),$$

(20)

where $T'$ is the fluid temperature variance from its conductive value. Under the appropriate boundary conditions, Equations (15)–(18) are solved by considering the proportional controller, $K$ positioned at the lower boundary of nanofluids layer. Therefore, we will have:

**Lower free and upper free boundaries**

$$W = D^2W = \Theta(0) + K\Theta(1) = \Phi = \Psi = D\Psi = 0 \text{ at } z = 0,$$

$$W = D^2W = D\Theta = \Phi = D\Psi = 0 \text{ at } z = 1.$$  

(21)

**Lower rigid and upper free boundaries**

$$W = DW = \Theta(0) + K\Theta(1) = \Phi = \Psi = D\Psi = 0 \text{ at } z = 0,$$

$$W = D^2W = D\Theta = \Phi = D\Psi = 0 \text{ at } z = 1.$$  

(22)

**Lower rigid and upper rigid boundaries**

$$W = DW = \Theta(0) + K\Theta(1) = \Phi = \Psi = D\Psi = 0 \text{ at } z = 0,$$

$$W = DW = D\Theta = \Phi = D\Psi = 0 \text{ at } z = 1.$$  

(23)

The technique of Galerkin is employed to discover an approximate solution to the system. In a series of basic functions, the variables are as shown below:

$$W = \sum_{i=1}^N A_iW_i, \quad \Theta = \sum_{i=1}^N B_i\Theta_i, \quad \Phi = \sum_{i=1}^N C_i\Phi_i, \quad \Psi = \sum_{i=1}^N D_i\Psi_i,$$

(24)

where constants are $A_i, B_i, C_i$ and $D_i$ and base functions are $W_i, \Theta_i, \Phi_i$ and $\Psi_i$ where $i = 1, 2, 3, ..$ will be selected for the trial function that meets the lower and upper boundaries of free–free, rigid–free and rigid–rigid, respectively.

$$W_i = \sin(z\pi), \quad \Theta_i = z(2 - z), \quad \Phi_i = \sin(z\pi), \quad \Psi_i = z(3z - 2z^2),$$

(25)

$$W_i = z^2(1 - z)(3 - 2z), \quad \Theta_i = z(2 - z), \quad \Phi_i = z(1 - z), \quad \Psi_i = z(3z - 2z^2),$$

(26)

$$W_i = z^2(1 - z)^2, \quad \Theta_i = z(2 - z), \quad \Phi_i = z(1 - z), \quad \Psi_i = z(3z - 2z^2).$$

(27)

Substitute (24) in Equations (15)–(18) and create the left–hand expressions of those equations (residuals) orthogonal to the trial functions, thus achieving a system of $4N$ linear algebraic equations in the unknown $4N$. The vanishing of the coefficients determinant generates the system’s eigenvalue equation. One can consider $Ra$ as the eigenvalue solution.
3. Results and Discussion

Analysis of linear stabilization is conducted to investigate the control on Rayleigh–Bénard convection in rotating nanofluids layer subjected to the magnetic field. The nanofluid model includes two significant impacts of Brownian movement and thermophoresis. By using the lower–upper boundary conditions; free–free, rigid–free and rigid–rigid, the sensitiveness of the critical Rayleigh number $Ra_c$ to changes in the physical difference parameters of Taylor number $Ta$, feedback control $K$ and magnetic Chandrasekhar number $H$ are studied. Comparative analysis of the thermal instability for alumina–water $\text{Al}_2\text{O}_3$–$\text{H}_2\text{O}$ and copper–water $\text{Cu}$–$\text{H}_2\text{O}$ nanofluids is done. Following Yadav et al. [5], the aforementioned values of the parameters for alumina–water $\text{Al}_2\text{O}_3$–$\text{H}_2\text{O}$ nanofluids in the representative values of dimensionless parameters are $Le = 5000$, $N_A = 5$, $N_B = 0.00775$ and $Rn = 0.122$. Meanwhile, for copper–water $\text{Cu}$–$\text{H}_2\text{O}$ nanofluids in the representative values of dimensionless parameters are $Le = 5000$, $N_A = 0.5$, $N_B = 0.0085$ and $Rn = 0.325$. The stationary convection curves of Rayleigh number $Ra$ against wavenumber $\alpha$ for various values for parameter are shown in Figures 1–3. The global minimum marginal curve known as critical Rayleigh number $Ra_c$ is presented in Figures 4–6. We employed alumina–water $\text{Al}_2\text{O}_3$–$\text{H}_2\text{O}$ and copper–water $\text{Cu}$–$\text{H}_2\text{O}$ nanofluids suspensions for numerical computation.

For the boundary conditions considered by using single–term Galerkin method, the resulting eigenvalue problem for different values of $Ta$, $K$, $H$, $Rn$, $Ln$, $N_B$ and $N_A$ are obtained. Calculations are performed first under the limiting case of nanofluids to validate the solution procedure, i.e. for regular fluids when $Rn = 0$, $N_A = 0$ and $N_B = 0$. The critical Rayleigh number $Ra_c$ for different values of magnetic Chandrasekhar number $H$ in the absence of Taylor number $Ta$ and feedback control $K$ are obtained and compared to the results in Chandrasekhar [7] and Yadav et al. [4] shown in Tables 1 and 2 for lower and upper boundaries of rigid–rigid and rigid–free. We notice from the tables that the agreement is very excellent and that the technique used is thus accurate.

**Table 1.** Comparisons of $Ra_c$ for different values of $H$ with Chandrasekhar [7] and Yadav et al. [4] for regular fluids in the absence of rotation in lower and upper rigid boundaries

| $H$  | Chandrasekhar [7] | Yadav et al. [4] | Present Study |
|------|-------------------|------------------|--------------|
| 0    | 1707.80           | 1707.83          | 1707.76      |
| 10   | 1945.90           | 1945.74          | 1945.74      |
| 50   | 2802.10           | 2801.96          | 2802.00      |
| 100  | 3757.40           | 3757.23          | 3757.23      |
| 200  | 5488.60           | 5489.18          | 5488.54      |
| 500  | 10110.00          | 10119.76         | 10109.28     |
| 1000 | 17103.00          | 17160.41         | 17109.06     |

**Table 2.** Comparisons of $Ra_c$ for different values of $H$ with Chandrasekhar [7] and Yadav et al. [4] for regular fluids in the absence of rotation in lower rigid upper free boundaries

| $H$  | Chandrasekhar [7] | Yadav et al. [4] | Present Study |
|------|-------------------|------------------|--------------|
| 0    | 1100.75           | 1100.77          | 1100.76      |
| 25   | 1699.40           | 1699.36          | 1699.32      |
| 50   | 2217.60           | 2217.28          | 2217.45      |
| 250  | 5613.30           | 5613.01          | 5613.25      |
| 500  | 9304.50           | 9306.52          | 9305.07      |
| 1000 | 16119.00          | 16137.56         | 16120.56     |
| 1500 | 22592.00          | 22660.07         | 22600.07     |
Figure 1. Variation of $K$ on $Ra$ against wavenumber $a$, for nanofluids of Al$_2$O$_3$–H$_2$O and Cu–H$_2$O

Figure 1 analyzed the effect of feedback control, $K = 30$ and 35, for the plot of $Ra$ versus wavenumber, $a$ in various types of boundary conditions of alumina–water Al$_2$O$_3$–H$_2$O and copper–water Cu–H$_2$O nanofluids. Clearly, the increasing values of the feedback control, $K$ significantly postponed the onset of convection which is the result is equivalent to Bau [16]. The sensors identify the nanofluid from its conductive state physically and then the actuators are guided to eliminate any disruptions. Therefore, the same control strategy of feedback control implemented by the previous researchers; [14]–[16], [20] and [21], in fluids layer can be effectively used to stabilize the nanofluids layer heated from below. By comparing these two types of nanofluids considered in this problem, feedback control $K$ plotted for alumina–water Al$_2$O$_3$–H$_2$O nanofluids showing a much better performance than copper–water Cu–H$_2$O nanofluids, thus reveals that alumina–water Al$_2$O$_3$–H$_2$O nanofluids stabilize more than copper–water Cu–H$_2$O nanofluids.

Figure 2 analyzed the effect of increasing the values of Taylor number, $Ta = 1000$ and 2500, for the plot of $Ra$ versus wavenumber, $a$ in various types of boundary conditions for nanofluids of alumina-water Al$_2$O$_3$–H$_2$O and copper-water Cu–H$_2$O. It has been found, the effect of elevating the values of $Ta$ inhibit the onset of convection [5] and [7]. This is due to the rotation mechanism that induces vorticity into the nanofluids layer, causes the nanofluids to move in the horizontal planes with higher velocity and velocity at the vertical planes is reduces thus slow down the process of heat transfer [9].

Figure 3 represents the variation values of $Ra$ with wavenumber, $a$ for various values of magnetic Chandrasekhar number, $H = 100$ and 150 in various boundary conditions for nanofluids of alumina-water Al$_2$O$_3$–H$_2$O and copper-water Cu–H$_2$O. The impact of increasing the values of $H$ has a strong magnetic field stabilizing impact and postpone the Rayleigh–Bénard convection [4]. This is due to the variations of the magnetic field implemented contributes to Lorentz force variability, where the Lorentz force generates greater resistance on the horizontal field than that of the vertical field thus delay the convection. The increasing values of magnetic Chandrasekhar number, $H$ for alumina–water Al$_2$O$_3$–H$_2$O nanofluid have higher values of critical Rayleigh number, $Ra_c$ than copper–water Cu–H$_2$O nanofluid. From the findings, it is noted that alumina–water Al$_2$O$_3$–H$_2$O nanofluid exhibit higher stability than copper–water Cu–H$_2$O
Figure 2. Variation of $Ta$ on $Ra$ against wavenumber $a$, for nanofluids of Al$_2$O$_3$–H$_2$O and Cu–H$_2$O.

Figure 3. Variation of $H$ on $Ra$ against wavenumber $a$, for nanofluids of Al$_2$O$_3$–H$_2$O and Cu–H$_2$O.

nanofluid and the most stable boundaries are rigid–rigid compared to free–free and rigid–free.

The effect of various feedback control values, $K = 15$ and 35 for critical Rayleigh number, $Ra_c$ depending on Taylor number, $Ta$ has been illustrated in Figure 4 in various boundary conditions for nanofluids of alumina–water Al$_2$O$_3$–H$_2$O and copper–water Cu–H$_2$O. As expected, the increasing values of $K$ and $Ta$ stabilizes the system and alumina–water Al$_2$O$_3$–H$_2$O nanofluid is more stable than copper–water Cu–H$_2$O nanofluid.
Figure 4. Variation of $K$ on $Ra_c$ against $Ta$, for nanofluids of $\text{Al}_2\text{O}_3$–$\text{H}_2\text{O}$ and $\text{Cu}$–$\text{H}_2\text{O}$

Figure 5. Variation of $H$ on $Ra_c$ against $K$, for nanofluids of $\text{Al}_2\text{O}_3$–$\text{H}_2\text{O}$ and $\text{Cu}$–$\text{H}_2\text{O}$

Figure 6. Variation of $Ta$ on $Ra_c$ against $H$, for nanofluids of $\text{Al}_2\text{O}_3$–$\text{H}_2\text{O}$ and $\text{Cu}$–$\text{H}_2\text{O}$
Illustration in Figure 5 demonstrates the impact of the selected values of magnetic Chandrasekhar number $H = 100$ and 150 for critical Rayleigh number $Ra_c$ depending on feedback control $K$ for lower and upper boundaries of free–free, rigid–free and rigid–rigid. As discussed earlier, the impact of rising the values of magnetic Chandrasekhar number $H$ and feedback control $K$ stabilizes the system. At the same time, alumina–water $\text{Al}_2\text{O}_3$–$\text{H}_2\text{O}$ nanofluids shows an excellence performance than copper–water $\text{Cu}$–$\text{H}_2\text{O}$ nanofluids.

The variety of the critical number of Rayleigh, $Ra_c$ for the magnetic Chandrasekhar number, $H$ depending on various Taylor number, $Ta = 500$ and 1500 values are plotted in Figure 6. This plots indicate that as $Ta$ and $H$ increases, the $Ra_c$ shows a positive increment and stabilized the system in various boundary conditions for nanofluids of alumina–water $\text{Al}_2\text{O}_3$–$\text{H}_2\text{O}$ and copper–water $\text{Cu}$–$\text{H}_2\text{O}$.

4. Conclusion
The control problem of Rayleigh–Bénard convective instability in magnetic rotating nanofluids is heated from below for lower and upper boundaries free–free, rigid–free and rigid–rigid. The numerical computations are represented considering two types of nanofluids of water–based fluids and nanoparticles of alumina $\text{Al}_2\text{O}_3$ and copper $\text{Cu}$ suspended in it. The implemented model for nanofluids incorporates the combination effects of Brownie movement and thermophoresis. A normal–mode linear stability assessment is used, the eigenvalue solution acquired is analyzed using the method of Galerkin numerically and calculated using Maple software.

The obtained results from the investigation can be concluded as the following:

(i) In the system, alumina–water $\text{Al}_2\text{O}_3$–$\text{H}_2\text{O}$ and copper–water $\text{Cu}$–$\text{H}_2\text{O}$ nanofluids thermal instability are regarded. It is found that alumina nanoparticles $\text{Al}_2\text{O}_3$ when deposited in liquids improve the system’s stability faster than copper nanoparticles $\text{Cu}$ when feedback control, rotation and magnetic field are present. Therefore, alumina–water $\text{Al}_2\text{O}_3$–$\text{H}_2\text{O}$ nanofluid is the most stable nanofluids compared to copper–water $\text{Cu}$–$\text{H}_2\text{O}$ nanofluid.

(ii) The increasing values of magnetic Chandrasekhar number, $H$, feedback control, $K$ and Taylor number, $Ta$ significantly stabilized the system.

(iii) Finally, three different varieties of lower and upper boundaries, the most stable system is when both bounding surfaces are rigid and the most unstable system is when both bounding surfaces are free.

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