Generating EPR beams in a cavity optomechanical system

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We propose a scheme to produce continuous variable entanglement between phase-quadrature amplitudes of two light modes in an optomechanical system. For proper driving power and detuning, the entanglement is insensitive with bath temperature and $Q$ of mechanical oscillator. Under realistic experimental conditions, we find that the entanglement could be very large even at room temperature.

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Entanglement is the key resource of the field of quantum information. Light is the perfect medium to distribute entanglement among distant parties. Entangled light with continuous variable (CV) entanglement between phase-quadrature amplitudes of two light modes is widely used in teleportation, entanglement swap, dense coding, etc. [1]. This type of entangled state is also called Einstein-Podolsky-Rosen (EPR) state. The EPR beams have been generated experimentally by a nondegenerate optical parameter amplifier [2], or Kerr nonlinearity in an optical fiber [3]. The later one is simpler and more reliable. The Kerr nonlinearity is used to generate two independent squeezed beams. With interference at a beam splitter, the EPR entanglement is obtained between output beams. However, Kerr nonlinearity in fiber is very weak, which limits entanglement between output beams.

It was found that strong Kerr nonlinearity appeared in an optomechanical system consisting of a cavity with a movable boundary [4, 5, 6]. Besides, the single-mode squeezing could be made insensitive with thermal noise [5, 6]. The later one is simpler and more reliable. The Kerr nonlinearity is used to generate two independent squeezed beams. With interference at a beam splitter, the EPR entanglement is obtained between output beams. However, Kerr nonlinearity in fiber is very weak, which limits entanglement between output beams.

We propose a scheme to produce continuous variable entanglement between phase-quadrature amplitudes of two light modes in an optomechanical system. For proper driving power and detuning, the entanglement is insensitive with bath temperature and $Q$ of mechanical oscillator. Within the experimentally available parameters [13, 14], we find the maximum two-mode squeezing could be higher than 16 dB under room temperature. The entanglement of formation (EOF) between two modes is larger than 5 [15]. Since the coupling efficiency between cavity and fiber could be larger than 99% in the WGM cavity system [16], we neglect the coupling induced noises in this paper.

FIG. 1: (Color online) Experimental setup. Cavity modes $a$ and $b$, which are driven by four lasers, couple to the mechanical mode $a_m$.

As shown in Fig. 1 we consider an optomechanical system consisting of a WGM cavity with a movable boundary. There are two cavity modes $a$ and $b$ with the same frequency but the opposite momentum. They are coupling with the same mechanical oscillation mode $a_m$ and driven by four lasers, two from the right-hand side with frequencies $\omega_L$ and $\omega_{L'}$, the other two from the left-hand side with frequencies $\omega_{L''}$ and $\omega_{L'''}$.

\[ a_{in} \]

\[ a_{out} \]

\[ b_{in} \]

\[ b_{out} \]

\[ H = H_0 + H_d + H_I, \]

\[ H_0 = \frac{1}{2} \omega_m q^2 + \frac{1}{2} m \omega_m L^2 q^2, \]

\[ H_d = g a \cdot \delta, \]

\[ H_I = \alpha \hat{a}^\dagger \hat{a} \hat{a}^\dagger \hat{a}^\dagger \hat{a}^\dagger + \beta \hat{b}^\dagger \hat{b} \hat{b}^\dagger \hat{b}^\dagger, \]
Here \(a, b\) and \(a_m\) are the annihilation operators for the optical and mechanical modes, \(\omega_p\) and \(\omega_m\) are their angular frequencies. \(\Omega_j\) with \(j = a, b\) is the driving amplitude and defined as \(\Omega_j = 2\sqrt{P_j/\hbar\omega_{Lj}}\), where \(P_j\) is the input laser power and \(\gamma = 1/\tau\) is the photon loss rate into the output modes. \(\nu\) is the coupling strength between cavity modes \(a\) and \(b\). For the WGM cavity system, it ranges from 100 MHz to 10 GHz [13, 20]. The dimensionless parameter \(\eta = (\omega_p/\omega_m)(x_m/R)\) is used to characterize optomechanical coupling, with \(x_m = \sqrt{\hbar/m\omega_m}\) the zero-point motion of the mechanical resonator [21], \(m\) its effective mass, and \(R\) a cavity radius. In typical WGM cavity systems we find \(\eta \sim 10^{-4}\).

We define the normal modes \(a_1 = (a + b)/\sqrt{2}\) and \(a_2 = (a - b)/\sqrt{2}\). We suppose the conditions that \(\Omega_a - \Omega_b = 0\) and \(\Omega_a^2 + \Omega_b^2 = 0\) are satisfied. The Hamiltonian can be written as

\[
H = \hbar(\omega_p + \nu)a_1^\dagger a_1 + \hbar(\omega_p - \nu)a_2^\dagger a_2 + \hbar\omega_m a_m^\dagger a_m
+ \hbar\Omega_2 \bar{a}_2 e^{-i\omega_t t} + \Omega_2 \bar{a}_2 e^{-i\omega_t t} + \text{H.c.}
\]

\[
+ \hbar\nu a_m(a_1^\dagger a_1 + a_2^\dagger a_2)(a_1^\dagger + a_2^\dagger + a_m + a_m),
\]

where \(\Omega_1 = \Omega_a + \Omega_b\) and \(\Omega_2 = \Omega_a^\prime - \Omega_b^\prime\). We define the detuning \(\Delta_1 = \omega_p - \omega_p - \nu\) and \(\Delta_2 = \omega_p - \omega_p + \nu\). As shown in Fig. 11 with beam splitters and Faraday rotator, we can get the output mode of \(a_1\) and \(a_2\). We assume both cavity and oscillator modes are weakly dissipating at rates \(\gamma\) and \(\gamma_m\), respectively, where \(\gamma_m \ll \omega_m\).

We can quantum Langevin equations [22]

\[
\dot{a}_j = i\Delta_j a_j - i\nu\omega_m a_j(a_m + a_m^\dagger) - \frac{\Omega_j}{2} a_j - \gamma a_j(t)
+ \sqrt{\gamma} a_j^\text{in}
\]

\[
\dot{a}_m = -i\nu\omega_m \sum_{j=1}^{2} a_j^\dagger a_j - (i\omega_m + \gamma_m a_m + \sqrt{\gamma_m} a_m^\text{in}(t)
\]

where thermal noise inputs are defined as correlation functions \(\langle a_{m1}^\dagger(t), a_{m1}^\dagger(t') \rangle = n_m \delta(t - t')\), \(\langle a_{m1}^\dagger(t), a_{m1}^\dagger(t') \rangle = (a_{m1}^\dagger(t), a_{m1}^\dagger(t')) = 0\), \(\langle a_{m2}^\dagger(t), a_{m2}^\dagger(t') \rangle = (a_{m2}^\dagger(t), a_{m2}^\dagger(t')) = (a_{m1}^\dagger(t), a_{m1}^\dagger(t')) = 0\), with \(n_m\) the thermal occupancy number of thermal bath for oscillator mode. We suppose cavity modes couple with vacuum bath.

To simplify Eqs. (3) and (4), we apply a shift to normal coordinate, \(a_j \rightarrow a_j + \alpha_j, a_m \rightarrow a_m + \beta\). \(\alpha_j\) and \(\beta\) are \(c\) numbers, which are chosen to cancel all \(c\) number terms in the transformed equations. We find they should fulfill the following requirements: \(\beta \simeq -\eta(\alpha_1^2 + \alpha_2^2)\), and \(i\Delta_j\alpha_j + 2i\eta^2 \omega_m\alpha_j(\alpha_1^2 + \alpha_2^2) - \frac{\Omega_j}{2} \beta - i\frac{\Omega_j}{2} = 0\). Because \(\gamma_m \ll \omega_m\), the imaginary part of \(\beta\) can be neglected. In the limit \(\Delta_j \gg 2\eta^2 \omega_m(\alpha_1^2 + \alpha_2^2)\), we find \(\alpha_j \simeq \Omega_j/\sqrt{\gamma^2 + 4\Delta_j^2}\). In the limit \(|\alpha| \gg |\langle p\rangle|\), the Langevin equations are linearized as

\[
\dot{a}_j = -i\nu\omega_m a_j(a_m + a_m^\dagger) + i\Delta_j - \gamma a_j + \sqrt{\gamma} a_j^\text{in}(t)
\]

\[
\dot{a}_m = -i\nu\omega_m \sum_{j=1}^{2} (a_{m1}^\dagger a_{p} + a_{p} a_{m1}^\dagger) - (i\omega_m + \gamma_m a_m)
\]

\[
+ \sqrt{\gamma_m a_m^\text{in}},
\]

where \(j = 1, 2\) and \(\Delta_j = \Delta_j + 2\eta^2 \omega_m(\alpha_1^2 + \alpha_2^2)\). We suppose \(\Delta_1 < 0\) and \(\Delta_2 > 0\). We define \(\delta = (\Delta_2 - \Delta_1)/2 - \omega_m\) and \(d = -(\Delta_1 + \Delta_2)/2\).

In the limit \(\omega_m \gg \delta, \delta, \gamma, \gamma_m\), the Langevin equations [5] and [6] can be simplified as

\[
\dot{a}_1 = -i\nu\omega_m a_1 a_m - \gamma a_1 + \sqrt{\gamma} a_1^\text{in},
\]

\[
\dot{a}_2 = -i\nu\omega_m a_2 a_m - \gamma a_2 + \sqrt{\gamma} a_2^\text{in},
\]

\[
\dot{a}_m = i\delta a_m - i\nu\omega_m (\alpha_1^\dagger a_1 + a_2^\dagger a_2) - \gamma_m a_m + \sqrt{\gamma_m} a_m^\text{in}.
\]

(7)

With proper detuning and input power, we can always tune the cavity mode amplitude \(\alpha_1 = \alpha_2 = \alpha\). Define the Fourier components of the intracavity field by \(a(t) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} e^{-i(\omega + t)\omega} a(\omega) d\omega\). In the limit \(\delta \gg \omega, \gamma_m\), we can adiabatically eliminate the \(a_m\) mode. We get \(a_m(\omega) \simeq \eta \bar{a}_m(\alpha_1^\dagger a_1 + a_2^\dagger a_2) - \sqrt{\gamma_m} a_m^\text{in}\). Then we have quantum Langevin equations for \(a_1(\omega)\) and \(a_1^\dagger(\omega)\)

\[
-i\omega a_1(\omega) = -i\gamma a_1(\omega) + ig a_2^\dagger(\omega) - \frac{\gamma}{2} a_1(\omega)
\]

\[
+ \sqrt{\gamma} a_1^\text{in}(\omega) + \sqrt{\gamma} a_1^\text{in} a_m(\omega),
\]

\[
i\omega a_2^\dagger(\omega) = i\gamma a_2^\dagger(\omega) + i\gamma a_1(\omega) - \frac{\gamma}{2} a_2^\dagger(\omega)
\]

\[
+ \sqrt{\gamma} a_2(\omega) - \sqrt{\gamma} a_m(\omega),
\]

where \(g = \eta^2(\alpha_1^2 + \alpha_2^2)/\delta, \bar{\gamma}_m = (\eta|\alpha_1\omega_m|/\delta)^2 \gamma_m\), and \(g' = g + d\). In Eq. (15), we neglect the phase of \(\alpha\) because it is not important.

Denote \(\bar{a}(\omega) = (a_1(\omega), a_2^\dagger(\omega), a_m(\omega))\), \(\bar{a}^\dagger(\omega) = (a_1^\dagger(\omega), a_2(\omega))^t\) and \(\bar{a}_{m}^\dagger(\omega) = (a_m(\omega), a_m^\dagger(\omega))\). We get the following matrix equation

\[
A\bar{a}(\omega) = \sqrt{\gamma} a_1^\text{in}(\omega) + \sqrt{\gamma} a_m^\text{in},
\]

where

\[
A = \begin{pmatrix}
-\nu + i\gamma' & ig & \nu - i\gamma' \\
-ig & -\nu + i\gamma' & -i\gamma' \\
\end{pmatrix}.
\]

Using boundary conditions \(a_1^\text{out}(\omega) = a_1^\text{in}(\omega) + \sqrt{\gamma} a_1(\omega)\) for \(j = 1, 2\), we can calculate output field as

\[
a_1^\text{out}(\omega) = G(\omega)a_1^\text{in}(\omega) - H(\omega)a_1^\dagger(\omega) + I(\omega)a_m(\omega),
\]

\[
a_2^\text{out}(\omega) = G(\omega)a_2^\dagger(\omega) - H(\omega)a_1^\text{in}(\omega) - I(\omega)a_m(\omega),
\]

(10)
where \( G(\omega) = (\omega^2 + \frac{\gamma_2}{4} + g^2 - g^2 - ig^2)/\Delta(\omega) \), \( H(\omega) = ig^2/\Delta(\omega) \), \( I(\omega) = (-\omega + \frac{\gamma_2}{4} - ig^2 + ig)\sqrt{\gamma_m}/\Delta(\omega) \), and \( \Delta(\omega) = (-\omega + \frac{\gamma_2}{4})^2 + g^2 - g^2 \).

Let us define the dimensionless position and momentum operators of fields \( X_j^{out}(\omega) = [a_j^{out}(\omega) + a_j^{out}(\omega)] \) and \( P_j^{out}(\omega) = [a_j^{out}(\omega) - a_j^{out}(\omega)]/i \), for \( j = 1, 2 \). We define the correlation matrix of the output field as \( V_{ij} = \langle (\xi_j, \xi_j, \xi_j)/\Delta(\omega) \rangle \), where \( \xi = (X_1^{out}, P_1^{out}, X_2^{out}, P_2^{out}) \). We calculate the correlation matrix with Eq. (10). Up to local unitary transformation, the standard form of it is

\[
V_S = \begin{pmatrix}
  n & 0 & k_x & 0 \\
  0 & n & 0 & -k_x \\
  k_x & 0 & n & 0 \\
  0 & -k_x & 0 & n
\end{pmatrix},
\]

where \( n = \{(\omega^2 + \frac{\gamma_2}{4} + g^2 - g^2)^2 + (g^2 + g^2)^2 + [(\omega + g^2 - g^2) + \frac{\gamma_2}{4}]\gamma_m(2\Delta \pm 1)/\Delta(\omega)^2 \}, k_x = \sqrt{V_{14} + V_{24}}, \)

where \( V_{14} = -2g\gamma(\omega^2 + \frac{\gamma_2}{4} + g^2 - g^2)/\Delta(\omega)^2 \), \( V_{24} = \{2g\gamma^2 + [(\omega + g^2 - g^2) + \frac{\gamma_2}{4}]\gamma_m(2\Delta \pm 1)/\Delta(\omega)^2 \}. \) This is the symmetric Gaussian state. The EOF for the symmetric Gaussian states is defined as

\[
EF = C_+(n - k_x) \log_2 [C_+(n - k_x)] - C-(n - k_x) \log_2 [C-(n - k_x)]
\]

where \( C\pm(x) = (x^{-1/2} \pm x^{1/2})^2/4 \). \( V \) describes an entanglement state if and only if \( n - k_x < 1 \). Based on the standard form of matrix \( V \), we also find that \( \langle \delta^2(X_1 + X_2) \rangle = \langle \delta^2(P_1 - P_2) \rangle = n - k_x \). We define the two-mode squeezing as \( S = -10 \log_{10}(n - k_x) \).

As shown in Fig. 2 the bigger the cavity mode amplitude \( \alpha \), the larger the output entanglement. Because \( \alpha_j \simeq \Omega_j / \sqrt{\gamma^2 + 4\Delta^2} \), the output entanglement is proportional to driving amplitude. But the peak of entanglement is splitted into two symmetric peaks when driving is very strong. The splitting distance is proportional to driving power. Increasing driving power can decrease the entanglement too. This is because adiabatic elimination condition \( \omega \ll \delta \) are not valid around peaks for very strong driving. So the driving power should be neither too big nor too small. For the specific \( \alpha \) and \( \delta \), we find there is an optimum \( d \) which makes entanglement maximum and the entanglement peaks appear near \( \omega = 0 \). The optimum \( d \) is \( d_0 = \sqrt{(\gamma^2 + 4\Delta^2)} \), corresponding to squeezing \( S_0 = -10 \log_{10}(d_0/\sqrt{\gamma^2}) \) and entanglement which is obtained from Eq. (12) with \( n - k_x = 4(d_0/\gamma) \). It is obvious that the higher the input power, the smaller the optimum \( d \). In the mean time, we find that decreasing the mechanical Q factor nearly does not change the entanglement spectrum if \( d \) is around its optimum value and the condition \( \omega_m/Q \ll \delta \) is fulfilled. Leaving other parameters unchanged, \( Q \) could be as low as 300. Considering the difficulty of increasing the mechanical oscillator \( Q \), the above finding makes our scheme more practical.

We also test the stability of our scheme. As shown
in Fig. 4 the optimum $d$ is around 0.07$\gamma$ if $\alpha = 1000$, $\delta/2\pi = 10$ MHz. To maintain such high entanglement, we need to precisely control the $d$ down to 0.02$\gamma \sim 2\pi \times 60$ kHz. $d$ is defined as $d = -(\Delta'_1 + \Delta'_2)/2 = -(\Delta_1 + \Delta_2)/2 - 4\pi^2 \omega_m |\alpha|^2$. The higher entanglement is needed, the more precise detuning and driving power is required at the same time. To maintain the entanglement as high as Fig. 4, the laser spectrum width should be less than 60 kHz and the driving power fluctuation should be less than 1%. The lower entanglement between two beams is needed to adopt $\omega$ at the same time. To maintain the entanglement as high as Fig. 7, $\omega_m/2\pi = 73.5$ MHz, $T = 300K$, $\gamma_m = \omega_m/30,000$, $\gamma/2\pi = 3.2$ MHz, and $|\alpha| = 1000$.

Before conclusion, we briefly discuss the approximations we used. Our scheme needs the steady states existing, which requires $\langle a^\dagger a \rangle \ll |\alpha|^2$. During numerical calculation, $\langle a^\dagger a \rangle$ is in the order of $10^3$, which is much less than $|\alpha|^2 \sim 10^6$. The other two approximations are rotating wave approximation $\omega_m \gg \delta, d, \gamma, \gamma_m$ and adiabatical elimination $\delta \gg \omega, \gamma_m$, which can be fulfilled independently. For $\alpha \sim 10^3$, the driving amplitude $\Omega$ is in the order of $10^{11}$ Hz, which is much lower than the distance between adjacent cavity modes $\Delta \omega = c/(Rn_0) \times 5 \times 10^{12}$ Hz, where $c$ is the light speed in a vacuum, $n_0$ the refractive index of silica. Therefore the approximation that one laser only drives one cavity mode is valid. Laser power is needed in the order of 10 mW, which is available in the laboratory.

In conclusion, we have proposed a scheme to generate EPR lights in an optomechanical system. Two sideband modes, which couple with the mechanical mode, are driven by lasers. After adiabatically eliminating the the mechanical mode, we find that the output sideband modes are highly entangled. The higher power of the driving laser, the larger entanglement of the output light. To maintain the entanglement, we need to precisely control the driving power and laser frequency at the same time. With proper parameters, the entanglement is insensitive to the thermal noise and mechanical $Q$ factor. We test the scheme by experimental available parameters. Though in this paper we focus on WGM cavity systems, our scheme can be realized in other optomechanical systems, as long as the mechanical mode frequency is much larger than the cavity decay rate.

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