Chapter 8
Power Take-Off Systems for WECs

Amélie Tétu

8.1 Introduction, Importance and Challenges

The power take-off (PTO) of a wave energy converter is defined as the mechanism with which the absorbed energy by the primary converter is transformed into useable electricity. The primary converter can for example be an enclosed chamber for an oscillating water column or a point absorber buoy. The PTO system is of great importance as it affects not only directly how efficiently the absorbed wave power is converted into electricity, but also contributes to the mass, the size and the structural dynamics of the wave energy converter.

By having this direct influence on the wave energy converter, the PTO system has a direct impact on the levelised cost of energy (LCoE) [1]. The PTO system has a direct effect on the efficiency of power conversion; hence, it has a direct impact on the annual energy production. The PTO system affects directly the capital cost of a device by accounting for typically between 20–30% of the total capital cost [2]. The reliability of the PTO system affects the availability (the energy production) and the operation and maintenance cost.\(^1\) The influence of the PTO on the LCoE is schematized in Fig. 8.1. A study made by the Partnership for Wave Energy in Denmark investigated the influence of the PTO system for four different wave energy converters [3]. The impact of the PTO efficiency and the reduction in cost of the PTO system on the LCoE were the PTO variables studied; Fig. 8.2 shows the results.

\(^1\)For more information on those economic variables, the reader is referred to Chap. 5 of this book.
For both an increase in efficiency and a reduction in cost of the PTO, a decrease of the LCoE is observed. Even though an increase in PTO efficiency has a bigger effect on the LCoE, both parameters have a significant impact on the LCoE showing the importance of the PTO system in a wave energy converter.

But the task of designing a cost-efficient PTO system is definitively not an easy one. The main challenge comes from the intrinsic properties of the energy resource. Ocean energy presents high variability. As shown in Fig. 8.3, the surface elevation varies irregularly in time and can induces high amplitude displacements, accelerations and forces on a body in a very short period of time. At other instants, the waves present low amplitude displacements, accelerations and forces. Those two extreme regimes present different dynamic load patterns and in both cases, the PTO system should be as efficient as possible.
As mentioned earlier, many different types of PTO systems exist, and the type of PTO chosen for a particular wave energy converter is often strongly correlated with
the type of converter. The different main paths for wave energy to electricity conversion are schematised in Fig. 8.4.

![Fig. 8.4 Different paths for wave energy to electricity conversion](image)

A systematic comparison of the different types of PTO is a difficult task to accomplish as limited data is available and one particular device can be bound to only two types of systems. The types of PTO systems can be categorized into five main categories and are described in the following sections.

### 8.2.2 Air Turbines

Air turbines as a mean for converting wave power into mechanical power are mostly used in oscillating water column (see Chap. 2). The idea is to drive a turbine with the oscillating air pressure in an enclosed chamber as a consequence of the oscillating water level, induced by the ocean waves (see Fig. 8.5). The main challenge comes from the bidirectional nature of the flow. Non-returning valves to rectify the air flow combined with a conventional turbine is one solution. However, this configuration is complicated, has high maintenance cost and for prototype size the valves become too large to be a viable option. Another solution is to use a self-rectifying air turbine that converts an alternating air flow into a unidirectional rotation.
Several types of self-rectifying turbines have been proposed in the last 40 years, and new ideas are still being pursued to find an efficient reliable PTO system for the OWC systems, the main ones being:

- Wells type turbines
- Impulse turbines
- Denniss-Auld turbines

Wells type turbine was the first self-rectifying turbine to be developed and is named after its inventor A.A. Wells. It consists of a symmetrical rotor composed of many aerofoil blades positioned around a hub with the normal of their chords planes aligned with the axis of rotation (see Fig. 8.6a). When the rotor is in movement, the rotational speed induces an apparent flow angle $\alpha$, which in turn creates a lift force perpendicular to the apparent flow direction and a drag force parallel to the apparent flow direction (see Fig. 8.6b). Those forces can be decomposed into axial ($F_x$) and tangential force ($F_\theta$). For some given value $\alpha$, the

![Fig. 8.5 Schematic of a wave energy converter where an air turbine is employed](image1)

![Fig. 8.6 Illustration of a self-rectifying Wells turbine (taken from [37])](image2)
direction of the tangential force is independent of the sign of \( \alpha \), and the rotor will rotate in a single direction regardless of the direction of the air flow.

The Wells turbine is the simplest of all the self-rectifying turbines and probably the most economical option for wave energy conversion. The Azores Pico Plant [4] and the LIMPET in Islay, U.K., [5] are both equipped with this type of turbine. One major drawback of the Wells turbines is that they are not self-starting: the rotor has to be initially accelerated by an external source of energy.

To overcome the drawbacks of the Wells turbine, the so called impulse turbine was developed. The idea is to redirect the air flow by using guide vanes in order to directly transfer the kinetic energy of the air flow into the tangential force component on the rotor blades, as depicted in Fig. 8.7. The guide vanes can either be fixed or pitched. The pitching mechanism can either be self-controlled by the air flow or controlled by another active mechanism, for example hydraulic actuator [6]. This extra feature increases the amount of moving parts of the turbine and therefore decreases the reliability and increases the operation and maintenance cost of the turbine. On the other hand, the pitching mechanism increases considerably the efficiency of the turbine, cf. Fig. 8.8.

![Fig. 8.7 Schematic of the cross-section at the aerofoils level of an impulse turbine (taken from [7])](image-url)
An impulse turbine with self-pitch-controlled guide vanes was tested at the NIOT plant in India and showed a threefold increase in total efficiency with respect to the previously installed Wells turbine [7].

The Denniss-Auld self-rectifying turbine is based on the Wells turbines configuration where the aerofoils blades can rotate around their neutral position in order to achieve optimal angle of incident flow (see Fig. 8.9). The rotation of the blades is controlled by measuring the pressure in the chamber. This type of turbine was installed in the MK1 OWC full-scale prototype deployed in New South Wales, Australia in 2005 [8].

**Fig. 8.8** Comparison of efficiency for different self-rectifying air turbines under irregular flow conditions (taken from [38])

Table 8.1 summarises the technological advantages and inconveniences for the self-rectifying air turbines mentioned above.

**Fig. 8.9** a Schematic of an aerofoil and b illustration of the aerofoil pitching sequence in oscillating flow for a Denniss-Auld turbine (taken from [39])
When the energy capture mechanism is based on the movement of a body in response to the interaction with the waves, as is the case for some point absorbers and attenuators, conventional rotary electrical machines are not directly compatible. Hydraulic converter is often the solution chosen to interface the wave energy converter with the electrical generator since they are well suited to absorb energy when dealing with large forces at low frequencies. In this particular case, the energy path is usually reversed with respect to traditional hydraulic system. The movement of the body is feeding energy into a hydraulic motor, which in turn translates the energy to an electrical generator.

A schematic of a hydraulic PTO system for wave energy conversion is depicted in Fig. 8.10. A point absorber connected to an hydraulic cylinder moves up and down with respect to an actuator, forcing fluid through controlled hydraulic manifolds to a hydraulic motor, which in turn drives the electric generator. Accumulators are also added to the system so as to smoothen the supply of high pressure fluid in the system by either providing or accumulating hydraulic energy when necessary. For wave energy conversion, radial piston motor is often favoured as it is well suited for high loads, low velocity applications.

### Table 8.1 Advantages and inconvenients for different self-rectifying turbine employed in wave energy conversion

| Turbine type       | Advantages                                      | Inconvenients                                           |
|--------------------|------------------------------------------------|--------------------------------------------------------|
| Wells turbine      | • Technologically simple                       | • Narrow flow range at which the turbine operates at useful efficiencies |
|                    |                                                 | • Poor starting characteristics                         |
|                    |                                                 | • High operational speed and consequent noise           |
|                    |                                                 | • High axial thrust                                     |
| Impulse turbines   | • Good starting characteristics                | • Large number of movable parts for the self-pitching configuration |
|                    | • Low operational speed                        |                                                        |
|                    | • Wide range of flow coefficients at which the turbine operates at useful efficiencies |                                                        |
| Denniss-Auld turbines | • Low operational speed                  | • Large number of movable parts                        |
|                    | • Wide range of flow coefficients at which the turbine operates at useful efficiencies |                                                        |
Many issues arise when choosing a hydraulic PTO system for wave energy conversion. Fluid containment of the hydraulic system has to be addressed with regards to performance and environmental impacts. The use of biodegradable transformer oil has been reported to address the environmental issue [9]. Efficiency of the whole system is also of importance. Due to the variability of the energy resource, hydraulic systems often include several hydraulic gas accumulators that can store the absorbed peak loads and smoothen the wave energy conversion from the motor. Digital displacement motors [10], based on radial piston motor, were developed in order to increase the part-load efficiency of hydraulic motor and facilitate their controllability [11]. Hydraulic systems are composed of many moving parts, and the seals of the piston will wear over time which can increase drastically the maintenance cost. This has to be kept in mind while designing a hydraulic PTO system. Another issue to address is the protection of the PTO system in the event of extreme conditions, where the hydraulic actuator exceeds its design travel and damage the system. One solution is to include mechanical limit to the stroke [12] or to use radial hydraulic piston [13].

### 8.2.4 Hydro Turbines

Hydro turbines are employed in overtopping devices or hydraulic pump systems using seawater as fluid. In overtopping type of devices, the water reaching over a ramp accumulates in a basin, and its potential energy is converted using low-head turbines and generators (see Fig. 8.11).
Hydro turbines have the benefit of being a mature technology that has been used for many decades for power generation. Kaplan turbines were used in the Wave Dragon device [14] and the Danish Wave Power system [15]. A Kaplan turbine is a reaction turbine that comprises a rotating element called a runner fully immersed in water, enclosed within a pressure casing (see Fig. 8.12). The turbine is equipped with adjustable (or fixed) guide vanes regulating the flow of water to the turbine runner. The blades of the runner are also adjustable from an almost flat profile for low flow conditions to a heavily pitched profile for high flow conditions.

Hydro turbines can operate at efficiency values of an excess of 90 % and require low maintenance. For wave energy conversion, the bottleneck resides in the energy extraction from the waves being able to deliver sufficient head and flow for the Kaplan turbine generator unit to be economical.
8.2.5 **Direct Mechanical Drive Systems**

A direct mechanical drive PTO system consists on translating the mechanical energy of an oscillating body subjected to waves into electricity by means of an extra mechanical system driving a rotary electrical generator. This type of PTO system is illustrated in Fig. 8.13. For example, the mechanical conversion system can comprise gear box, pulleys and cables. Flywheel can be integrated in a rotation based system so as to accumulate or release energy and thereby smooth out power variations [16].

![Fig. 8.13 Illustration of a direct mechanical drive PTO system](image)

One advantage of that type of PTO system is that only up to three energy conversions are necessary, resulting in high efficiency. On the other hand, the direct mechanical drive system undergoes uncountable load cycles, and reliability of this type of system still needs to be proven.

8.2.6 **Direct Electrical Drive Systems**

Direct electrical drive PTO systems refer to systems for which the mechanical energy captured by the primary converter is directly coupled to the moving part of a linear electrical generator [17, 18]. Development of permanent magnets and advances in the field of power electronics have rendered this solution attractive. Figure 8.14 illustrates a direct electrical drive PTO system. A translator on which alternating polarity magnets are mounted is coupled to a buoy. The ocean waves induce a heaving motion to this system with respect to a relatively stationary stator equipped with coils, inducing electrical current in the stator.
As the wave motion is directly converted to electricity in direct electrical drive PTO systems, rectification is necessary before conversion into a sinusoidal fixed voltage and frequency waveform for grid connection. This can be done either passively or actively [19]. Careful design of the mounting structure is also necessary in order to maintain fine air gaps between the translator and the stator.

8.2.7 Alternative PTO Systems

Other types of PTO system for wave energy conversion are investigated. One alternative makes use of dielectric elastomer [20, 21]. The principle is to coat with electrodes a membrane of dielectric elastomer. The mechanical energy from the waves deforms the membrane, reducing the capacitance and thereby increasing the electrical potentials of charges residing in the electrodes. Although promising simulation results have been shown, the technology is still far from mature.

8.3 Control Strategy of Power Take-Off System

8.3.1 Introduction

Ocean waves have a broad frequency band that changes with time and season, and present extreme events. On the other hand, wave energy converters are often designed with an oscillator having a narrow frequency range, i.e. their efficiency in absorbing wave energy peaks near their natural frequency ($\omega_0$) [22]. This is represented schematically in Fig. 8.15.
In order to render the wave energy converter more efficient by increasing the overlap between the (changing) ocean wave spectrum and the response of the converter, some tuning is necessary. The process of adapting the wave energy converter to behave as in resonance over a broad band of frequencies is referred to as control. The physical characteristics of the wave energy converter, like size, mass and shape, are often difficult to vary according to the incoming waves, but the behaviour of the converter can be adjusted by acting on the stiffness and/or the damping of the system. These variables are accessible through the PTO system of a wave energy converter. By controlling the behaviour of wave energy converters through their PTO system, one can increase the efficiency of the system and hence its cost-effectiveness. Furthermore, in the event of extreme conditions, the wave energy converter should automatically switch to safe operation mode in order to insure its survivability. This implies a controlled system where the forces exerted on the system are monitored regularly. However necessary, control of the PTO system of wave energy converters introduces complexity to the system, which in turn lowers the reliability of the system and increases maintenance cost. The influence of the control strategy on the structural fatigue also needs to be considered [23]. Careful design of the control strategy is imperative in order to ensure cost-effective converters.

8.3.2 Types of Control Strategy

Control can be achieved on different time scales. Some of the device properties can be adjusted according to the current wave conditions, or sea state, over a period of some minutes to hours (also referred to as slow tuning). Furthermore, to allow for the irregularity of the incoming waves, the device properties should also be adapted according to the incoming wave for achieving best response, and this is referred to as fast tuning or wave-to-wave tuning.

For an unconstrained point absorber in sinusoidal wave, two conditions need to be fulfilled in order to achieve optimum control, or in other words optimum energy absorption [24]:

---

2For a deeper understanding of the hydrodynamics of wave energy converters, the reader is referred to Chap. 6 of this book.
(1) The velocity of the oscillator is in phase with the dynamic pressure of the incoming wave.

(2) The amplitude of the motion of the oscillator at the resonance condition needs to be adjusted so that the amplitude of the incident wave is twice the amplitude of the radiated wave from the oscillator.

The first condition corresponds to adjusting the phase of the velocity with the phase of the incoming wave and is, therefore, often referred to as phase control. According to the second condition, the amplitude of the motion of the oscillator has to be adjusted by damping in order to achieve maximum energy conversion efficiency. If the damping is set too low, the oscillator will move too much with regards to the wave and little power will be extracted. In the same way, if the damping is too high, the amplitude of the motion will be limited, resulting in low power extraction. Hence, appropriate damping on the PTO system is fundamental.

There are many different control strategies, cf. [25]. Some of the main common ones are briefly detailed in the following.

### 8.3.2.1 Passive Loading Control

The damping coefficient is defined as the ratio of the force to velocity for linear motions, or torque to angular velocity for rotating motions. The damping coefficient is frequency dependent and can be either determined numerically or derived from experimental tank testing. This control strategy corresponds to adjusting the damping coefficient provided by the PTO system for a given sea state condition. For example, for rotating motion the PTO system will provide a given counter torque for a certain angular velocity of the shaft. The force-velocity (torque-angular velocity) relationship can be linear, as well as exponential or even having more advanced features (see Fig. 8.16). This technique can also be used to limit the range of movement of a device in order to avoid damaging the device in extreme wave conditions.

**Fig. 8.16** Various types of linear and non-linear passive loading
8.3.2.2 Latching Control

Latching control is a non-linear control strategy that consists of stalling the device when its velocity is zero and releasing it when the excitation force has a given phase that maximises energy absorption [26], as illustrated in Fig. 8.17. This type of control requires a PTO system that can react quickly to a given control command, like for example a hydraulic PTO system [25], and has been shown to increase significantly the absorbed energy of different devices in irregular wave conditions [27–30]. The main drawback of this strategy is that it requires the knowledge of the future wave profile in order to know when to fix and release the device, and accurate algorithms for wave prediction of wave algorithms are a challenge in itself. Latching can also lead to very large forces and it becomes less effective for two bodies system.

![Fig. 8.17 Illustration of the latching control where a heaving body is kept at a fixed vertical position for a certain time interval in order to achieve phase control](image)

8.3.2.3 Reactive Loading Control

One consequence of optimum control is that some energy is returned into the sea for a small fraction of the oscillation cycle [31]; for this reason optimum control is also known as reactive control. Reactive loading control can be used to widen the frequency band of the wave energy converter around the natural frequency [25].

Any wave energy converter has inertia, which consists of the intrinsic inertia of the converter plus the inertia of the adjacent water. A wave energy converter is also often associated with a stiffness term. When pushing a body down in the water and releasing it, the body will come back to its original position after some oscillations in the same way as a mass spring system in the presence of friction would behave. Inertia is the resistance to acceleration, and stiffness is the resistance to deflection. Intuitively, those two variables should be minimised. Reactive loading control strategy aims at maximising the energy absorption at all frequencies by dynamically adjusting the spring constant (stiffness), the inertia and the damping of the oscillator.
Even though this strategy can enhance wave energy absorption [32, 33] as illustrated in Fig. 8.18, it leads to reversible and very complex PTO mechanisms. Many different suboptimal control strategies have been proposed for wave energy conversion to simplify the problem [34–36].

![Fig. 8.18 Comparison of the power that can be absorbed from a given sinusoidal wave by a semi-submerged sphere heaving with optimum condition for three different control strategies (taken from [42])](image)

### 8.4 Conclusion

This chapter introduced what a PTO system of a wave energy converter is, described the different types of PTO systems and presented the concept of control, with the overall objective of showing how crucial this subsystem is. The efficiency of the PTO system directly affects the annual energy production of the machine, and the choice of components has a direct influence on the cost of the whole converter and the maintenance cost of the system. An efficient, maintenance-free and reliable PTO system is fundamental in order to reach the goal of cost-effectiveness for wave energy conversion.

### References

1. SI OCEAN: Ocean energy: cost of energy and cost reduction opportunities (2013)
2. Previsic, M, Bedard, R., Hagerman, G., Siddiqui, O.: System level design, Performance and costs for San Francisco California Pelamis offshore wave power plant, E2I EPRI Global—006A—SF report (2004)
3. Marquis, L.A.: PTO system. Presentation at a meeting of the Danish Partnership for wave energy (2014)
4. http://www.pico-owc.net/
5. Alcorn, R.G., Beattie, W.C.: Power quality assessment form the LIMPET wave-power station. In: Proceedings of the 11th International Offshore and Polar Engineering Conference (ISOPE '01), vol. 1, pp. 575–580 (2001)
6. Thielbaut, F., Sullivan, D.O., Kratch, P., Ceballos, S., Lopez, J., Boake, C, Bard, J., Brinquete, N., Varandas, J, Gato, L.M.C., Alcorn, R., Lewis, A.W.: Testing of a floating OWC device with movable guide vane impulse turbine power take-off. In: Proceedings of the 9th European Wave and Tidal Energy Conference, Southampton UK, paper no. 159 (2011)
7. Takao, M., Setoguchi, T.: Air turbines for wave energy conversion. Int. J. Rotating Mach. 2012 (2012). Article ID 717398
8. http://www.oceanlinx.com/projects/past-projects/mk1-2005
9. http://ecogeneration.com.au/news/pelamis_wave_power_powers_up_in_north_portugal/42829
10. Artemis intelligent power LTD. http://www.artemisip.com/
11. Ehsan, Md, Rampen, W.H.S., Salter, S.H.: Modeling of digital-displacement pump-motors and their application as hydraulic drives for nonuniform loads. J. Dyn. Syst. Meas. Contr. 122, 210–215 (2000)
12. Henderson, R.: Design, simulation, and testing of a novel hydraulic power take-off system for the Pelamis wave energy converter. Renew. Energy 31, 271–283 (2006)
13. Babarit, A., Guglielmi, M., Clément, A.H.: Declutching control of a wave energy converter. Ocean Eng. 36, 1015–1024 (2009)
14. Wave Dragon, http://www.wavedragon.net/
15. Nielsen, K., Remmer, M., Beattie, W.C. 1993. Elements of large wave power plants, 1st European Wind Energy Conference (EWEC)
16. Yoshida, T., Sanada, M., Morimoto, S., Inoue, Y: Study of flywheel energy storage system for power leveling of wave power generation system. In: Proceedings of the 15th International Conference on Electrical Machines and Systems, pp. 1–5 (2010)
17. Baker, N.J., Mueller, M.A.: Direct Drive Wave Energy Converters, pp. 1–7. Power Engineering, Revue des Energies Renouvelables (2001)
18. Mueller, M.A.: Electrical generators for direct drive wave energy converters. IEE Proc. Gener. Transm. Distrib. 149, 446–456 (2002)
19. Baker, N.J., Mueller, M.A., Brooking, P.R.M.: Electrical power conversion in direct drive wave energy converters. In: Proceedings of the European Wave Energy Conference, Cork, Ireland, pp. 197–204 (2003)
20. Moretti, G., Fontana, M., Vertechy, R.: Model-based design and optimization of a dielectric elastomer power take-off for oscillating wave surge energy converters. Meccanica 50, 2797–2813 (2015)
21. Vertechy, R., Rosati Papini, G.P., Fontana, M.: Reduced model and application of inflating circular diaphragm dielectric elastomer generators for wave energy harvesting. J. Vibr. Acoust. 137, 011004 (2015)
22. Falnes, J.: Ocean Waves and Oscillating Systems. Cambridge University Press (2002)
23. Ferri, F., Ambühl, S., Fischer, B., Kofof, J.P.: Balancing power output and structural fatigue of wave energy converters by means of control strategies. Energies 7, 2246–2273 (2014)
24. Budal, K., Falnes, J.: A resonant point absorber of ocean-wave power. Nature 256, 478–479 (1975)
25. Salter, S.H., Taylor, J.R.M., Caldwell, N.J.: Power conversion mechanisms for wave energy. Proc. Inst. Mech. Eng. Part M: J. Eng. Marit. Environ. 216, 1–27 (2002)
26. Budal, K., Falnes, J.: Interacting point absorbers with controlled motion. Power from Sea Waves, pp. 381–399. Academic Press, London (1980)
27. Korde, U.A.: Control system applications in wave energy conversion. In: Proceedings of the OCEANS 2000 MTS/IEEE Conference and Exhibition, Providence, Rhode Island, USA, vol. 3, pp. 1817–1824 (2000)
28. Babarit, A., Duclos, G., Clément, A.H.: Comparison of latching control strategies for a heaving wave energy in random sea. Appl. Ocean Res. 26, 227–238 (2004)
29. Babarit, A., Clément, A.H.: Optimal latching control of a wave energy device in regular and irregular waves. Appl. Ocean Res. 28, 77–91 (2006)
30. Falcão, A.F.O.: Phase control through load control of oscillating-body wave energy converters with hydraulic PTO system. Ocean Eng. 35, 358–366 (2008)
31. Budal, K., Falnes, J.: Optimum operation of improved wave-power converter. Mar. Sci. Commun. 3, 133–150 (1977)
32. Korde, U.A.: Efficient primary energy conversion in irregular waves. Ocean Eng. 26, 625–651 (1999)
33. Valério, D., Beirão, P., Sá de Costa, J.: Optimisation of wave energy extraction with the Archimedes wave swing. Ocean Eng. 34, 2330–2344 (2007)
34. Price, A.A.E.: New perspectives on wave energy converter control. Ph.D. thesis. University of Edinburgh (2009)
35. Hals, J.: Modelling and phase control of wave-energy converters. Ph.D. thesis. Norwegian University of Science and Technology (2010)
36. Hansen, R.H.: Design and control of the power take-off system for a wave energy converter with multiple absorbers. Ph.D. thesis. Aalborg University (2013)
37. Raghunathan, S.: The Wells air turbine for wave energy conversion. Prog. Aerosp. Sci. 31, 335–386 (1995)
38. Setoguchi, T., Santhakumar, S., Maeda, H., Takao, M., Kaneko, K.: A review of impulse turbines for wave energy conversion. Renew. Energy 23, 261–292 (2001)
39. Finnigan, T., Auld, D.: Model testing of a variable-pitch aerodynamic turbine. In: Proceedings of the 13th (2003) International Offshore and Polar Engineering Conference, Honolulu, Hawaii, USA, May 25–30 2003
40. Drew, B., Plummer, A.R., Sahinkaya, M.N.: A review of wave energy converter technology. Proc. Inst. Mech. Eng. Part A: J. Power Energy 223, 887–902 (2009)
41. IIT. 2016. Basic principles of turbomachines. http://nptel.ac.in/courses/112104117/chapter_7/ 7_11.html. Accessed 19 Jan 2016
42. Falnes, J., Hals, J.: Heaving buoys, point absorbers and arrays. Philos. Trans. R. Soc. A 370, 246–277 (2012)