On superpotentials for nonlinear sigma-models with eight supercharges

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Abstract

Using projective superspace techniques, we consider 4D \( \mathcal{N} = 2 \) and 5D \( \mathcal{N} = 1 \) gauged supersymmetric nonlinear sigma-models for which the hyper-Kähler target space is (an open domain of the zero section of) the cotangent bundle of a real-analytic Kähler manifold. As in the 4D \( \mathcal{N} = 1 \) case, one may gauge those holomorphic isometries of the base Kähler manifold (more precisely, their lifting to the cotangent bundle) which are generated by globally defined Killing potentials. In the U(1) case, by freezing the background vector (tropical) multiplet to a constant value of its gauge-invariant superfield strength, we demonstrate the generation of a chiral superpotential, upon elimination of the auxiliary superfields and dualisation of the complex linear multiplets into chiral ones. Our analysis uncovers a \( \mathcal{N} = 2 \) superspace origin for the results recently obtained in hep-th/0601165.

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Recently, Bagger and Xiong \cite{1} have presented a $\mathcal{N} = 1$ superspace formulation for four- and five-dimensional supersymmetric nonlinear sigma-models with eight supercharges. In particular, they have proved the existence of a superpotential whenever the hyper-Kähler target manifold possesses a tri-holomorphic isometry. Although the corresponding potential was previously obtained in components\footnote{For nonlinear sigma-models with eight supersymmetries, it follows from the famous 2D $\mathcal{N} = 4$ analysis of \cite{3} that the presence of a potential triggers the appearance of a central charge in the supersymmetry algebra, with the central charge (the potential) being proportional to (the square of) a tri-holomorphic Killing vector field. This is why such sigma-models are often called massive. The structure of potentials in 2D $\mathcal{N} = 1, 2$ supersymmetric sigma-models was studied in \cite{3}, and these results were also re-cast in superfield form in \cite{4}.} \cite{2}, the superspace treatment is clearly quite enlightening in several respects.

The aim of the present note is to uncover a manifestly supersymmetric origin\footnote{Only four supersymmetries are kept manifest in the $\mathcal{N} = 1$ superspace formulation of \cite{4}.} for the results in \cite{1}. There are two general approaches to keep manifest eight supersymmetries: harmonic superspace (see \cite{5} for a review) and projective superspace (see \cite{6, 7} and references therein). They are related and complementary to each other \cite{8}. Projective superspace used here is ideally suited if one is interested in re-casting the $\mathcal{N} = 2$ results in terms of $\mathcal{N} = 1$ superfields. Specifically, this approach allows one to keep $\mathcal{N} = 2$ supersymmetry under control without leaving $\mathcal{N} = 1$ superspace (an example of superspace holography).

Our starting point will be the 4D $\mathcal{N} = 2$ and 5D $\mathcal{N} = 1$ supersymmetric nonlinear sigma-models studied in \cite{8, 9, 10}. Although embracing only a subclass in the general family of supersymmetric actions for self-interacting polar multiplets \cite{6}, these models are especially interesting in the sense that they naturally extend the general 4D $\mathcal{N} = 1$ supersymmetric nonlinear sigma-model \cite{11}

$$
\int d^4 \theta K(\Phi^I, \bar{\Phi}^{\bar{I}}), \quad I, \bar{J} = 1, \ldots, n, \quad (1)
$$

with $K(\Phi, \bar{\Phi})$ the Kähler potential of a Kähler manifold $\mathcal{M}$ of complex dimension $n$. The extension consists of the two steps:

(i) replace the chiral $\Phi$ and antichiral $\bar{\Phi}$ dynamical variables with so-called arctic $\Upsilon(w)$ and antarctic $\bar{\Upsilon}(w)$ projective multiplets \cite{5, 7}

$$
\Upsilon(w) = \sum_{n=0}^{\infty} \Upsilon_n w^n, \quad \bar{\Upsilon}(w) = \sum_{n=0}^{\infty} (-1)^n \bar{\Upsilon}_n \frac{1}{w^n}, \quad w \in \mathbb{C} \setminus \{0\} ; \quad (2)
$$

for nonlinear sigma-models with eight supersymmetries, it follows from the famous 2D $\mathcal{N} = 4$ analysis of \cite{3} that the presence of a potential triggers the appearance of a central charge in the supersymmetry algebra, with the central charge (the potential) being proportional to (the square of) a tri-holomorphic Killing vector field. This is why such sigma-models are often called massive. The structure of potentials in 2D $\mathcal{N} = 1, 2$ supersymmetric sigma-models was studied in \cite{3}, and these results were also re-cast in superfield form in \cite{4}.
(ii) replace the Lagrangian (1) with

\[ L = \oint \frac{dw}{2\pi iw} \int d^4 \theta K(\Upsilon, \bar{\Upsilon}) \right|, \]  

with the contour around the origin. The \( \Upsilon(w) \) and \( \bar{\Upsilon}(w) \) are 4D \( \mathcal{N} = 2 \) or 5D \( \mathcal{N} = 1 \) superfields obeying the constraints

\[ \nabla_{\dot{\alpha}}(w) \Upsilon(w) = \nabla_{\dot{\alpha}}(w) \bar{\Upsilon}(w) = 0, \]  

where

\[ \nabla_{\dot{\alpha}}(w) = \begin{pmatrix} \nabla_{\alpha}(w) \\ \bar{\nabla}_{\dot{\alpha}}(w) \end{pmatrix}, \quad \nabla_{\alpha}(w) \equiv w D^1_{\alpha} - D^2_{\alpha}, \quad \bar{\nabla}_{\dot{\alpha}}(w) \equiv \bar{D}^1_{\dot{\alpha}} + w \bar{D}^2_{\dot{\alpha}}, \]  

and \( D^i_{\alpha} = (D^i_{\alpha}, \bar{D}^i_{\dot{\alpha}}) \) are the 4D \( \mathcal{N} = 2 \) or 5D \( \mathcal{N} = 1 \) spinor covariant derivatives (see [10] for our 5D conventions). The constraints imply that the dependence of the component superfields \( \Upsilon_n \) and \( \bar{\Upsilon}_n \) on \( \theta^a \) and \( \bar{\theta}^{\dot{a}} \) is uniquely determined in terms of their dependence on \( \theta^1 \) and \( \bar{\theta}^{\dot{1}} \). In other words, the projective superfields depend effectively on half the Grassmann variables which can be chosen to be the spinor coordinates of 4D \( \mathcal{N} = 1 \) superspace

\[ \theta^a = \theta^a_1, \quad \bar{\theta}^{\dot{a}} = \bar{\theta}^{\dot{a}}_2. \]  

As a result, one can deal with reduced superfields \( U|, D_{\alpha}^1 |, \bar{D}_{\dot{\alpha}}^1 |, \ldots \) and 4D \( \mathcal{N} = 1 \) spinor covariant derivatives \( D_{\alpha} \) and \( \bar{D}_{\dot{\alpha}} \) defined via the bar-projection:

\[ U| = U(x, \theta^a_1, \bar{\theta}^{\dot{a}}_2)_{\theta_2 = \bar{\theta}^{\dot{2}} = 0}, \quad D_{\alpha} = D^1_{\alpha} \bigg|_{\theta_2 = \bar{\theta}^{\dot{2}} = 0}, \quad \bar{D}_{\dot{\alpha}} = \bar{D}^1_{\dot{\alpha}} \bigg|_{\theta_2 = \bar{\theta}^{\dot{2}} = 0}. \]  

For the leading components \( \Upsilon_0| = \Phi \) and \( \Upsilon_1| = \Gamma \), the constraints (4) give

\[ \begin{align*}
\bar{D}^1_{\dot{\alpha}} \Phi &= 0, \\
\bar{D}^2_{\dot{\alpha}} \Gamma &= 0, \\
D &= 4 ; \\
\bar{D}^1_{\dot{\alpha}} \Phi &= 0, \\
\bar{D}^2_{\dot{\alpha}} \Gamma &= \partial_5 \Phi, \\
D &= 5.
\end{align*} \]  

The action functional generated by the Lagrangian \( \mathcal{L} \), eq. (3), can be shown to follow from a manifestly supersymmetric action [7, 10].

The supersymmetric sigma-model (3) respects all the geometric features of its 4D \( \mathcal{N} = 1 \) predecessor (1). The Kähler invariance of (1)

\[ K(\Phi, \bar{\Phi}) \rightarrow K(\Phi, \bar{\Phi}) + \left( \Lambda(\Phi) + \bar{\Lambda}(\bar{\Phi}) \right) \]  

(9)
turns into

\[ K(Υ, \bar{Υ}) \rightarrow K(Υ, \bar{Υ}) + \left( \Lambda(Υ) + \bar{Λ}(\bar{Υ}) \right) \]  

(10)

for the model (3). A holomorphic reparametrization \( \Phi^I \mapsto f^I(Φ) \) of the Kähler manifold has the following counterparts

\[ Υ^I(w) \mapsto f^I(Υ(w)) \]  

(11)

in the 4D \( \mathcal{N} = 2 \) and 5D \( \mathcal{N} = 1 \) cases, respectively. Therefore, the physical superfields

\[ \left. Υ^I(w) \right|_{w=0} = Φ^I, \quad \left. \frac{dΥ^I(w)}{dw} \right|_{w=0} = Γ^I, \]  

(12)

should be regarded, respectively, as a coordinate of the Kähler manifold and a tangent vector at point Φ of the same manifold. That is why the variables \( (Φ^I, Γ^I) \) parametrize the tangent bundle \( T\mathcal{M} \) of the Kähler manifold \( \mathcal{M} \).

The supersymmetric sigma-model (3) can be gauged in complete analogy with the famous gauging procedure for its 4D \( \mathcal{N} = 1 \) predecessor (1), the latter developed in [12, 13, 14, 15, 16, 17, 18]. We should first recall the relevant geometric prerequisites [12]. Let \( X \) and \( \bar{X} \) be the holomorphic and antiholomorphic parts of a Killing vector,

\[ X = X^I(Φ) \frac{∂}{∂Φ^I}, \quad \bar{X} = \bar{X}^I(\bar{Φ}) \frac{∂}{∂\bar{Φ}^I}, \]  

(13)

Their important properties are [12]

\[ [X, K] = X^I K_I = i D(Φ, \bar{Φ}) + η(Φ), \]

\[ [\bar{X}, K] = \bar{X}^\bar{I} \bar{K}_{\bar{I}} = -i D(Φ, \bar{Φ}) + \bar{η}(\bar{Φ}), \]  

(14)

where \( D = \bar{D} \) is the so-called Killing potential, and \( η \) is a holomorphic function. From these relations one deduces

\[ X^I K_{IJ} = X^I g_{IJ} = i D_J, \quad \bar{X}^\bar{I} \bar{K}_{\bar{IJ}} = \bar{X}^\bar{I} g_{\bar{IJ}} = -i D_{\bar{J}}. \]  

(15)

The gauging of the supersymmetric sigma-model (3) gives

\[ L_{\text{gauged}}(V) = \oint \frac{dw}{2\pi i} \int d^4θ \left\{ K(Υ, \bar{Υ}) + e^{iV L_\bar{X}} - \frac{1}{iV L_\bar{X}} D(Υ, \bar{Υ}) V \right\} \]  

(16)

with \( V \) the gauge potential, and \( L_\bar{X} \) the Lie derivative along the vector \( \bar{X}(\bar{Υ}) \). The gauge potential is described by the so-called tropical multiplet [6, 7, 10]

\[ V(w) = \sum_{n=-∞}^{+∞} V_n w^n, \quad \nabla_\vec{a}(w)V(w) = 0, \quad V_n = (-1)^n V_{-n}, \]  

(17)
possessing the gauge freedom

$$\delta V(w) = i\left(\Lambda(w) - \Lambda(w)\right), \tag{18}$$

with the gauge parameter $\Lambda(w)$ an arctic superfield. Although our discussion is restricted to the U(1) case, a nonabelian generalisation is obvious.

In 4D $\mathcal{N} = 2$ superspace, as a manifestation of the Higgs effect, charged hypermultiplets can be made massive by freezing a U(1) vector multiplet to a constant value $\mu$ of its chiral gauge-invariant superfield strength $W$ [19, 20] (the same procedure also works in the 5D $\mathcal{N} = 1$ case, where the superfield strength $W$ is real). The gauge freedom (18) can be used to bring a vector multiplet of constant superfield strength to the form\(^3\)

$$V_0(w) = -\frac{1}{w} \left\{ \bar{\mu} (\theta_1^2) - \mu (\bar{\theta}_2^2) \right\} - 2 \left\{ \bar{\mu} \theta_1 \theta_2 + \mu \bar{\theta}_1 \bar{\theta}_2 \right\} + w \left\{ \mu (\bar{\theta}_1^2) - \bar{\mu} (\theta_2^2) \right\}. \tag{19}$$

Here the parameter $\mu$ is arbitrary complex if $D = 4$. It turns out to be real, $\mu = \bar{\mu}$, in the case $D = 5$ [10]. It is useful to define the arctic and antarctic components of $V_0$:

$$V_0(w)\big| = V_+(w) + V_-(w), \quad V_+ = w \mu \bar{\theta}_2, \quad V_- = -\frac{1}{w} \bar{\mu} \theta_2. \tag{20}$$

Important for our consideration are the following obvious properties

$$V^2_+ = V^2_- = 0. \tag{21}$$

They imply that the second term in (16) contains only two contributions

$$\frac{e^{iV_0L_\bar{X}} - 1}{iV_0L_\bar{X}} DV_0 = V_0 D(\Upsilon, \bar{\Upsilon}) + \frac{i}{2} V_0^2 [\bar{X}(\bar{\Upsilon}), D(\Upsilon, \bar{\Upsilon})], \tag{22}$$

with the bar-projection assumed.

With the aid of (14), the first term in (22) can be represented as (the bar-projection is assumed)

$$V_0 D(\Upsilon, \bar{\Upsilon}) = -iV_+ [X, K] + iV_- [\bar{X}, K] + iV_+ \eta(\Upsilon) - iV_- \bar{\eta}(\bar{\Upsilon}). \tag{23}$$

\(^3\)From the point of view of supersymmetry without central charge, eq. [19] defines the Wess-Zumino gauge and, to preserve it, any supersymmetry transformation should be accompanied by an induced gauge transformation. This generates a central charged supersymmetry transformation which makes the expression for $V_0(w)$ super Poincaré-invariant, see the second reference in [10] for more details. See also [4] for a similar mechanism in two dimensions.
Here the expression in the second line does not contribute to the Lagrangian (16),
\[ \oint \frac{dw}{2\pi i w} V(\eta(Y)) = 0 \]  \tag{24}

With the aid of (15), the second term in (22) can be represented as
\[ \frac{i}{2} V^2 [\bar{X}(\bar{Y}), D(Y, \bar{Y})] = V_{+} V_{-} [X, [\bar{X}, K]] . \]  \tag{25}

The relations obtained allow us to rewrite the Lagrangian, \( \mathcal{L}_{\text{gauged}}(V_0) \), as follows:
\[ \mathcal{L}' = \mathcal{L}_{\text{gauged}}(V_0) = \oint \frac{dw}{2\pi i w} \int d^4 \theta K(Y, \bar{Y}) \bigg| , \quad Y^I = Y^I - i V_{+} X^I(Y) . \]  \tag{26}

Among the component superfields of the modified arctic multiplet
\[ \mathcal{Y}(w) \bigg| = \sum_{n=0}^{\infty} \mathcal{Y}_n w^n , \]  \tag{27}

the leading component does not change at all, \( \mathcal{Y}_0 = \Phi \), while the next-to-leading one, \( \mathcal{Y}_1 = \Gamma \), obeys the modified linear constraint
\[ -\frac{1}{4} \bar{D}^2 \Gamma^I = -i \mu X^I(\Phi) , \quad D = 4 ; \]
\[ -\frac{1}{4} D^2 \Gamma^I = -i \mu X^I(\Phi) + \partial_5 \Phi^I , \quad D = 5 . \]  \tag{28}

The expressions appearing in both sides of these relations transform as holomorphic target-space vectors. It is worth pointing out that such generalised constraints for 4D \( \mathcal{N} = 1 \) complex linear superfields were designed many years ago [21].

The auxiliary superfields \( \mathcal{Y}_2, \mathcal{Y}_3, \ldots \), and their conjugates, can be eliminated with the aid of the corresponding algebraic equations of motion
\[ \oint dw w^{n-1} \frac{\partial K(Y, \bar{Y})}{\partial Y^I} = 0 , \quad n \geq 2 . \]  \tag{29}

Their elimination can be carried out using the ansatz[^4]
\[ \mathcal{Y}^I_n = \sum_{p=0}^{\infty} G^I_{j_1 \ldots j_{n+p}} \bar{L}_1 \ldots \bar{L}_p (\Phi, \bar{\Phi}) \Gamma^j_1 \ldots \Gamma^{j_{n+p}}_1 \bar{\Gamma}^l_1 \ldots \bar{\Gamma}^{l_p}_1 , \quad n \geq 2 . \]  \tag{30}

[^4]: It is explained in [9] how to eliminate the auxiliary superfields in the case of symmetric Kähler spaces, and the example of \( \mathcal{M} = \mathbb{C}P^n \) is explicitly elaborated.

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Upon elimination of the auxiliary superfields, the Lagrangian (26) takes the form

\[ \mathcal{L}_{tb}(\Phi, \bar{\Phi}, \Gamma, \bar{\Gamma}) = \int d^4\theta \left\{ K(\Phi, \bar{\Phi}) - g_{IJ}(\Phi, \bar{\Phi}) \Gamma^I \bar{\Gamma}^J \right. 

\left. + \sum_{p=2}^{\infty} \mathcal{R}_{I_1 \ldots I_p \bar{J}_1 \ldots \bar{J}_p}(\Phi, \bar{\Phi}) \Gamma^{I_1} \ldots \Gamma^{I_p} \bar{\Gamma}^{\bar{J}_1} \ldots \bar{\Gamma}^{\bar{J}_p} \right\}, \tag{31} \]

where the tensors \( \mathcal{R}_{I_1 \ldots I_p \bar{J}_1 \ldots \bar{J}_p} \) are functions of the Riemann curvature \( R_{IJKL}(\Phi, \bar{\Phi}) \) and its covariant derivatives. Each term in the action contains equal powers of \( \Gamma \) and \( \bar{\Gamma} \), since the original model (26) is invariant under rigid U(1) transformations

\[ \Upsilon(w) \rightarrow \Upsilon(e^{i\alpha}w) \iff \Upsilon_n(z) \rightarrow e^{in\alpha}\Upsilon_n(z). \tag{32} \]

For the theory with Lagrangian \( \mathcal{L}_{tb}(\Phi, \bar{\Phi}, \Gamma, \bar{\Gamma}) \), we can develop a dual formulation involving only chiral superfields and their conjugates as the dynamical variables. In the five-dimensional case, for concreteness, consider the first-order action

\[ S_{tb} - \int d^5x \left\{ \int d^2\theta \Psi_I \left( \partial_5 \Phi^I_\ell - i \mu X^I_\ell(\Phi) + \frac{1}{4} \bar{D}^2 \Gamma^I_\ell \right) + \text{c.c.} \right\} 

\[ = S_{tb} + \int d^5x \left\{ \int d^2\theta \Psi_I \Gamma^I_\ell + \int d^2\theta \overline{\Psi}_I \left( i \mu X^I(\Phi) - \partial_5 \Phi^I_\ell \right) + \text{c.c.} \right\}, \]

where \( S_{tb} = \int d^5x \mathcal{L}_{tb}(\Phi, \bar{\Phi}, \Gamma, \bar{\Gamma}) \), and the tangent vector \( \Gamma^I_\ell \) is now complex unconstrained, while the one-form \( \Psi_I \) is chiral, \( \bar{D}_a \Psi_I = 0 \). Upon elimination of \( \Gamma \) and \( \bar{\Gamma} \), with the aid of their equations of motion, the action turns into \( S_{cb}[\Phi, \bar{\Phi}, \Psi, \bar{\Psi}] \). Its target space is the cotangent bundle \( T^*M \) of the Kähler manifold \( M \).

Let us consider the superpotential term in four dimensions

\[ i \mu \int d^2\theta \Psi_I X^I(\Phi) = e^{i\sigma} \int d^2\theta \Psi_I \tilde{X}^I(\Phi), \quad \tilde{X}^I(\Phi) = |\mu| X^I(\Phi). \tag{33} \]

In terms of the redefined holomorphic Killing vector \( \tilde{X}^I(\Phi) \), the superpotential is defined uniquely up to a phase factor, \( e^{i\sigma} = i \mu/|\mu| \), and this agrees with [1].

Massive supersymmetric sigma-models on the cotangent bundles of complex projective spaces \( \mathbb{C}P^n \) and Grassmannians \( G(k, n) \) possess interesting topological solutions, see [23, 24] and references therein. It would be interesting to extend this study to more general target spaces \( T^*M \). Our work provides a constructive approach to generate such sigma-models.

Note added: The author has been informed that a similar construction for susy nonlinear sigma-models in 6D is presently under investigation [25].
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