Testing and validation of a \( \dot{B} \) algorithm for cubesat satellites

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Abstract. For most satellite missions, it is essential to decrease the satellite angular velocity. The \( \dot{B} \) algorithm is a common algorithm to stabilize the spacecraft by using magnetorquers. Controlling the satellite using the magnetorquers is part of the attitude control subsystem detumbling mode. Due to oscillating disturbances in the space environment, the required initial conditions needs analysis. As a consequence, the satellite stays in \( B \) detumbling mode for the entire operation. In the detumbling mode, the spacecraft oscillates around its spatial axes. The purpose of this paper is to extend the \( B \) algorithm with a disturbances compensation module and to achieve reduction of satellite’s angular velocity. The developed algorithm is found to be able to reduce satellite’s angular velocity up to 10-11 degrees.

1. Introduction

The success of a space mission depends on sophisticated attitude control subsystem (ACS). As the ACS can only be activated if the spinning about the satellite's own axes remains within certain boundaries, uncontrolled oscillation should be minimized, even if a precisely determined orientation is not required for the particular mission. Due to the increasing importance of small and very small satellites such as CubeSat satellites, which are mainly operating in a low earth orbit (LEO) under a relatively strong earth magnetic field, the relevance of magnetic attitude control systems has also grown [1, 2]. This is pushed by magnetorquers' low prices, their space saving assembly, and the independence of propellant. Therefore, they are often the only actuators in CubeSat missions. They are integrated inside the solar panels to obtain the necessary electrical power [3]. The \( B \) law, a purely magnetic control scheme, was originally proposed in [4]. Since then, this control law has been adopted as the primary detumbling solution in small satellites such as Pico Satellite Solar Cell Testbed-2 (PSSCT 2) [5] and T-SAT1 [6]. While the aforementioned studies present satisfactory detumbling performances, the initial conditions are difficult to estimate in real applications. The tuning factor, for instance, is generally chosen based on trial and error experiments. However, the initial factor chosen may not reject the oscillating disturbances effectively, and the control law may fail in case the disturbances change. Therefore, the satellite is not detumbled effectively.

The aim of this work is to present a novel yet convenient \( \dot{B} \) algorithm to subside the effect of oscillating disturbances in the space environment. The study presents a solution to control the current by a pulse width modulation (PWM) signal and proposes a gain factor function to obtain the optimum torque to reject the disturbance. The method has been simulated and tested further on a CubeSat to validate the feasibility of the proposed scheme. The paper is organized as follows: the conventional and the novel \( \dot{B} \) algorithms are introduced followed by the description of the mission and the strategy.
The validation procedure using the CubeSat and the obtained results are presented later followed by the simulation results. The conclusion describes the findings of the study.

2. Β algorithm

The open-loop control principle for the magnetic ACS is often used in space missions to first detumble the spacecraft after its deployment and second, to force the spinning into certain boundaries defined by the main ACS as initial conditions. Since this approach comprises an open-loop control system without feedback, the correction angle accuracy of the Β controller is limited to less than 15 degrees.

Equation 1 shows the Β control law, which gives the dipole momentum \( M \) (A⋅m\(^2\)) as a function of a gain factor \( K \) and the time derivative of the magnetic field vector. However, determination of \( \dot{B} \) is highly challenging, as the satellite might rotate with an uncertain angular frequency \( \omega \) (rad/s). If that frequency is known, \( \dot{B} \) (T) is given by Equation 2, were \( B_\omega \) denotes the magnetic field, e.g., measured with a magnetometer. However, if the angular frequency is not known, for instance for strongly spinning satellites, the time derivation of the magnetic field can only be approximated linearly with Equation 3, where \( B_1 \) and \( B_0 \) denote two different measurements taken within a time \( \Delta t \) (s).

\[
\begin{align*}
M &= -K \times \dot{B} \\
\dot{B} &= B_\omega \times \omega \\
B &= \frac{B_1 - B_0}{\Delta t}
\end{align*}
\]

Figure 1 shows a cyclic sequence of the Β algorithm. First, the magnetic field is measured within a control cycle of the cycle time \( T_{cycle} \). Second, its derivation is estimated subsequently. Then, the dipole momentum according to the Β control law is generated by the magnetorquers within the time \( T_{actuator} \). At the end of the cycle, all actuator coils have to be deactivated to avoid potential influences on the following cycle.

![Figure 1. Β controller blocks](image)

Because of both, inaccuracies in the magnetic field measurements and oscillating disturbances in the space environment, the satellite might not be stabilized properly, and its oscillation escalates. In fact, this could prevent the satellite from reaching the initial conditions for its main ACS. As a result, the satellite will try to detumble for the complete operating time, remaining in a constant oscillation.
3. Mission and strategy

There are several models for the forecast of different space disturbances. For a CubeSat mission, it can be assumed that there is a combination of disturbances with the total amount shown in Table 1 [7].

| Constraints                  | Description                                          |
|------------------------------|------------------------------------------------------|
| Orbit                        | Circular at 500 km with inclination of 0°             |
| Size                         | 0.1 x 0.1 x 0.1 m³                                   |
| Mass                         | 1 kg                                                 |
| Max. dipole momentum         | 0.1 A·m²                                             |
| Rel. inaccuracy PWM          | 0.00015%                                            |
| Rel. inaccuracy magnetometer | 5.888%                                               |
| External disturbance torque  | \[5 + \sin(\Omega_0 t)] \cdot 10^{-7} \text{ Nm}\] |

The parameter \(\Omega_0\) denotes the Earth’s orbital frequency.

The gain factor \(K\) of the \(\hat{B}\) controller needs to be tuned to compensate for the disturbance torque. Simulations based on the modified \(\hat{B}\) algorithm show increasing damping behaviors. However, it is necessary to control the current driving the magnetorquers smoothly instead of only switching between maximum current and off state. Furthermore, a correlation between the gain factor and the angular position is needed as well.

4. Validation

4.1. Current control

The current, which is driven through the magnetorquers in order to generate a dipole momentum, is controlled by a pulse width modulation (PWM) signal. To use the novel \(\hat{B}\) algorithm, this signal must be adjusted continuously between 0 % and 100 %. It is furthermore necessary to validate the ability of the PWM signal generator to follow a given trajectory signal.

Several specific PWM duty cycles were set up and their response on the magnetic field of the earth has been measured with the satellite’s magnetometers to determine a linear relation between the PWM duty cycle, \(DC_{PWM}\), and the measured magnetic field \(B_{meas}\) presented in Equation 4.

\[
B_{meas} = 0.1189 \cdot DC_{PWM} \cdot \mu \text{T}
\]  

(4)

Therefore, the duty cycle can be estimated within inaccuracy boundaries:

\[
DC_{PWM} = B_{meas} / 0.1189 \cdot \mu \text{T}
\]  

(5)

With this correlation, a trajectory can be specified and the response of the satellite can be plotted. Figure 2 shows the ability of the magnetorquers to follow the specified path, although a high accuracy cannot be achieved. The variations are caused by various reasons, e.g. high measurement inaccuracy of the magnetometers (see Table 1) and influences of certain magnetic disturbances on the testing environment.
4.2. Optimal gain factor $K$
Assuming that the satellite is under influence of a disturbance torque, e.g. as in Table 1, and that the derivation of the magnetic field caused by the satellite’s spinning remains at a low level less than $0.01 \mu{T}$, the periodically changing controlled output torque $M_{\text{output}}$ can be optimized to compensate for the oscillating disturbances.

$$M_{\text{output}} = \sin(\Phi) \cdot 10^{-7} \text{ N.m}$$

The controlled output torque is a function of the body position of the cubesat, and $\Phi$ denotes variation of the cubesat position along its nadir axis. Figure 3 illustrates the oscillating disturbances, the controlled output torque and the aggregate amount for different orbit positions. Basically, it also shows the effective torque induced by the magnetorquers to detumble the cubesat itself.

Assuming Equation 6 as the preferred torque output function and $B$ as fixed, the gain factor function can be set by using Equation 7.

$$K(\Phi) \approx [5.42914695 \cdot \Phi - 206.318465] \cdot 10^{-7} \text{ N.m/} \mu{T}$$

The result is plotted in Figure 4, including a linear approximation.

5. Testing

5.1. Simulation
According to Table 1, a CubeSat on a circular earth oriented orbit with inclination of $i=0$ was assumed to simulate the satellite dynamics in form of the Euler angles by using MATLAB/Simulink. It is an open loop command during rocket injection/b-detumbling mode. Hereby, the pitch axis is decoupled from roll and yaw axis. As the roll and yaw axis behave similarly, it suffices to observe only one of them.
Figure 3. PWM signal setting and response

Figure 4. PWM signal setting and response
Figure 5 shows the roll angle response within 40 orbit cycles. It can be seen that, due to the oscillating disturbances, the satellite is performing an undamped oscillating movement within 90± deg. The satellite is stabilized around 0 degree; however, the accuracy of the angle control is not satisfying.

![Figure 5](image)

**Figure 5.** Satellite's roll angle simulation result with traditional \( \hat{B} \) controller

Figure 6 shows simulation result of the roll attitudes up to \( 10^{11} \) degree. Even though the response exhibits more noise compared to Figure 5, the amplitude of the oscillation remains within an acceptable range during 40 orbit cycles (comment: confusing oscillation stays within 40 degree). The result is satisfactory; however, the observed oscillation is due to the inaccurately measured magnetic field and the resulting derivations in the gain factor itself. That is indeed the simulated performance. We may have variation/degradation during in-orbit flights.

![Figure 6](image)

**Figure 6.** Satellite's roll angle simulation result with the new \( \hat{B} \) controller algorithm

### 6. Conclusion

The gain function \( K(\phi) \) proposed for the novel \( \hat{B} \) control algorithm can effectively compensate for the typical space disturbances during a LEO CubeSat mission. This linear gain function only depends on the actual angular position in space in a circular orbit with 0º inclination. In addition, simulations showed that the oscillation of the satellite angles can be reduced within 40 orbit cycles compared to the existing algorithms. However, the satellite's controlled axis could not be fully stabilized to zero errors. Therefore, based on the specific mission objectives and constraints, suitable limitations of the acceptable attitude accuracies must be defined. Further work will not only focus on implementation of the \( \hat{B} \) algorithm in a real space environment, but also on improved magnetic field measurements, in order to decrease inaccuracies for the gain function optimization.
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