Edge magnetoplasmon excitations in a Quantum Dot in high magnetic fields

Subhasis Sinha
The Institute of Mathematical Sciences, Madras 600 113, India.
(March 24, 2022)

Abstract

We investigate the collective magnetoplasmon excitations in a quantum dot containing finite number of electrons in the high magnetic field limit. We consider the electrons in the lowest Landau level and neglect mixing between the higher Landau levels. The dispersion relation of these edge modes are estimated following the energy weighted sum-rule approach. In this finite size system the edge magnetoplasmon modes have different multipolarities (or angular momentum $l$). Their dependence on the magnetic field and on the system size is investigated. With increasing magnetic field, energy of these collective modes decreases and in the bulk limit they become gapless. We also consider the breathing mode of a dot in the presence of a strong magnetic field, and the energy of this mode approaches the cyclotron frequency $\hbar \omega_c$.

PACS numbers: 73.20.Dx, 72.15.Rn
Recent developments in nanofabrication technology have made it possible to manufacture electronic systems with reduced dimensionality. Quantum dot is one such nanosystem, which is very interesting both experimentally and theoretically. A quantum dot is a two-dimensional electron system where the electrons are confined within a finite area by applying a gate voltage \[1\]. It is an example of a finite size quantum system where the number of electrons can be varied from a few electrons to a few thousand electrons. This nanosystem shows many interesting properties in the presence of a magnetic field. One of the interesting properties in the presence of strong magnetic field is the chiral edge magnetoplasmon excitations. The edge modes in the quantum Hall bar have been studied theoretically using one dimensional field theory and numerical diagonalisation of the few particle system \[2,3\]. The edge modes of a two-dimensional electron system in the presence of positive ion background have been studied by several authors \[4,5\]. Giovanazzi et al \[6\] have calculated the dispersion relation of the edge modes of the electron gas confined in a jellium disk in the long wave-length limit. Recently collective excitations in the finite size nanostructures like quantum dot have become very interesting, because many device parameters can be controlled by adjusting the gate voltages. The number of electrons in the system can also be varied. In particular, the dipole excitation in a quantum dot with parabolic confinement, and in the presence of a strong magnetic field has been discussed in ref. \[7\]. Evidence has been found for a strong collective dipole mode in the presence of a magnetic field in far infrared spectroscopy experiment \[8\]. Recently the collective charge and spin excitation in a dot containing about 200 electrons has been observed experimentally \[9\]. Non parabolic confinement has been observed in this experimental device, but the exact nature of the confinement is not well understood. The edge magneto-plasmon modes of a quantum dot with finite number of electrons have been studied by using magneto-hydrodynamics by several authors \[10–13\].

The collective excitations in the finite fermion systems like nuclei and metal clusters have been extensively studied within random phase approximation (RPA) and using sum-rule approach in the last years by several authors \[14,15\]. This method has also been applied to analyze the multipole excitations in the two dimensional quantum dot and antidot systems \[16,17\]. Most of the theoretical works have been done numerically or by using classical magneto-hydrodynamics. The main aim of this work is to estimate the collective modes of a dot in the presence of a strong magnetic field microscopically, and also to obtain analytical results for the dispersion relations of these modes.

In this paper we study the low-lying multipole excitations and the breathing mode of a quantum dot in the presence of a strong magnetic field by using RPA sum-rule approach. We consider the electrons in the lowest Landau level, and derive a simple semiclassical energy functional for the electrons. By minimising the energy functional we obtain the ground state density profile of the electrons. Then we generalise the sum-rules in the presence of a magnetic field. Different moments of the RPA strength distribution function are calculated by using the ground state density profile. Finally we obtain the analytic expressions for the dispersion relation of the low-lying magnetoplasmon modes with different multi-polarities in a quantum dot. With increasing magnetic field, the energy of these modes decreases and in the bulk limit these modes become gap-less. Neglecting the $1/N$ corrections, the dispersion relation takes a very general form, where only one parameter contains the information about the two-body interaction and the shape of the density profile. Also, the $1/N$ corrections
of these modes are calculated. In the strong magnetic field limit the dispersion relation of these modes agrees with the dispersion relation obtained from hydrodynamics [10]. We also calculate the energy of breathing mode excitation, which is much larger than the energy of low-lying edge modes. We obtained the most general form of the dispersion relation of the edge excitations in strong magnetic field limit. We also calculated the finite size $N$ dependent corrections of the energy of these modes. Analyzing the dispersion relation in a strong magnetic field, we have shown the stability of the self-consistent semicircular density profile, which shows that this semi circular density is a better ansatz for ground state density than commonly used flat density profile.

The hamiltonian of the two-dimensional electron system in the presence of a magnetic field in the perpendicular direction is given by,

$$H = \sum_i \frac{1}{2m^*}[\hat{p}_{xi} + \frac{1}{2}m^*\omega_c y_i)^2 + (\hat{p}_{yi} - \frac{1}{2}m^*\omega_c x_i)^2] + \sum_i \frac{1}{2}m^*\omega_0^2 r_i^2 + \sum_{i<j} V(|\vec{r}_i - \vec{r}_j|),$$

(1)

where $m^*$ is the effective mass, $\omega_0$ is the frequency of the external parabolic confinement, and $\omega_c$ is the cyclotron frequency $\frac{eB}{m^*c}$. Neglecting the two-body interaction, the single particle Hamiltonian can be solved exactly. The single-particle energy levels are,

$$E_{n,m} = (2n + |m| + 1)\Omega - \frac{m}{2}\omega_c,$$

(2)

where $\Omega = \sqrt{\omega_0^2 + \frac{\omega_c^2}{4}}$, and $m$ is the angular momentum quantum number, and $n$ denotes Landau level index. In the presence of a strong magnetic field all electrons occupy the lowest Landau level(LLL), energy levels become almost degenerate and they form a band. In the lowest Landau level the single particle wave functions are,

$$\psi_m(\vec{r}) = \frac{1}{\sqrt{\pi l_0^2}} \frac{1}{\sqrt{m!}} (\frac{z}{l_0})^m e^{-r^2/2l_0^2},$$

(3)

where, $l_0 = \sqrt{\frac{\hbar}{m^*\Omega}}$, and $z = x + iy$ is the complex coordinate. In the high magnetic field limit the effect of higher Landau levels can be neglected and the many body wave function can be constructed out of these lowest Landau level basis. For electrons at the filling factor $\nu = 1$ the Laughlin wave function can be obtained by constructing the Van-der Monde determinant of the above single particle states,

$$\psi(\vec{r}_1, \vec{r}_2, ..., \vec{r}_n) = \Pi_{i<j}(z_i - z_j)e^{-\sum_i z_i \bar{z}_i/2l_0^2}.$$

(4)

In general we can write the ground state many body wave function constructed out of the LLL as,

$$\psi(\vec{r}_1, \vec{r}_2, ..., \vec{r}_i, ..) = f(z_1, z_2, .. z_i, ..)e^{-\sum_i z_i \bar{z}_i/2l_0^2}.$$

(5)

We can calculate the energy functional in the lowest Landau level, by calculating the expectation value of the Hamiltonian with respect to the ground state wave function given above.
After doing some algebra and using some properties of the Bargman space \[18\], we arrive at the following energy functional,

\[
E[\psi] = \frac{1}{2} \hbar \omega_c \int |\psi|^2 d^2 r + m^* \Omega (\Omega - \omega_c/2) \int r^2 |\psi|^2 d^2 r \\
+ \frac{1}{2} \int d^2 r_1 \int d^2 r_2 V(|\vec{r}_1 - \vec{r}_2|) |\rho(r_1, r_2)|^2 d^2 r_2, \tag{6}
\]

where,

\[
|\rho(R_1, R_2)|^2 = <\vec{r}_1, \vec{r}_2, ... \vec{r}_i, ... | \sum_{ij} \delta(\vec{R}_1 - \vec{r}_i) \delta(\vec{R}_2 - \vec{r}_j) |\vec{r}_1, \vec{r}_2, ... \vec{r}_i, ... >. \tag{7}
\]

Within Hartree approximation we can write down a simple local density functional for lowest Landau level,

\[
E[\rho] = \int d^2 r \left[ \frac{1}{2} \hbar \omega_c + m^* \Omega (\Omega - \omega_c/2) r^2 \right] |\rho(r)|^2 \\
+ \frac{1}{2} \int d^2 r_1 \int d^2 r_2 \rho(r_1) V(|r_1 - r_2|) \rho(r_2). \tag{8}
\]

In the case of a flat droplet, the Hartree energy typically goes as \(\sim N^{3/2}\), and the exchange correlation energy goes as \(\sim N\), where \(N\) is the number of particles in the droplet \[19\]. Hence in the large \(N\) limit, the exchange term may be neglected and we obtain the above simple looking energy functional. We minimize the free energy \(E - \mu N\) with respect to \(\rho\), keeping the total number of particles \(N\) fixed, and obtain the following integral equation for the density,

\[
m^* \Omega (\Omega - \omega_c/2) r^2 + \int d^2 r' V(|\vec{r} - r'|) |\rho(r')| = \mu - \frac{1}{2} \hbar \omega_c. \tag{9}
\]

For coulomb interaction, \(V(r) = e^2/r\) the solution of the above integral equation is \[10,20\],

\[
\rho(r) = n_0 \sqrt{1 - \left( \frac{r}{R} \right)^2}, \tag{10}
\]

where, \(n_0 = \frac{3N}{2\pi R^2}\), and \(R^3 = \frac{3e^2 \pi N}{8m^* \Omega (\Omega - \omega_c)}\). From this simple energy functional we obtain the nonperturbative density of the electrons.

We now consider the edge magnetoplasmon excitations in the dot. Magnetoplasmon excitation are the multipole excitations of a quantum dot in the presence of a magnetic field. We denote the multipolarity of the operator by the number \(k\). The excitation with multipolarity \(k\) is generated by the excitation operator \(F = z^k\), where \(z\) is the complex coordinate \(x + iy\). Excitation energies of the above modes can be estimated by sum-rule method. Many useful quantities may be calculated from the strength function, that is defined as

\[
S_\pm(E) = \sum_n |<n|F_\pm|0>|^2 \delta(E - E_n), \tag{11}
\]

where, \(E_n\), and \(|n\rangle\) are the excitation energy and excited state respectively, and \(F_+ = F\), and \(F_- = F^\dagger\). Various energy-weighted sum rules can be derived through the moments of the strength function, and are written below,
$$m_k^{\pm} = \frac{1}{2} \int E^k(S_+(E) \pm S_-(E))dE$$
$$= \frac{1}{2} \langle 0|F^\dagger(\hat{H} - E_0)^kF|0 \rangle \pm \langle 0|F(\hat{H} - E_0)^kF^\dagger|0 \rangle). \quad (12)$$

Some useful moments can be written in terms of the commutators of the excitation operator $F$ with the many body Hamiltonian $H$, and they are given below,

$$m_0^- = \frac{1}{2} \langle 0|[F^\dagger, F]|0 \rangle, \quad (13)$$
$$m_1^+ = \frac{1}{2} \langle 0|[F^\dagger, [H, F]]|0 \rangle, \quad (14)$$
$$m_2^- = \frac{1}{2} \langle 0|[J^\dagger, J]|0 \rangle, \quad (15)$$
$$m_3^+ = \frac{1}{2} \langle 0|[J^\dagger, [H, J]]|0 \rangle, \quad (16)$$

where $J = [H, F]$. In the presence of magnetic field +$k$ and -$k$ collective modes split, and the corresponding strength distributions are sharply peaked at these collective frequencies. Near the collective excitation energy, we can approximate the strength distribution by delta function, $S^\pm = \sigma^\pm \delta(E - E_{c^\pm})$. Since for the multipole modes $m_q^0 = 0$, we obtain $\sigma_+ = \sigma_-$. Within this approximation the low lying multipole modes can be estimated by using the above moments of the strength distribution function and can be written as,

$$E_{c^\pm} = \sqrt{\frac{m_3^+}{m_1^+} - \frac{3}{4} \left(\frac{m_2^-}{m_1^+}\right)^2} \pm \frac{m_2^-}{2}, \quad (17)$$

where, $\pm$ sign correspond to $k$ and -$k$ multipolarities. Similar expression for the excitation frequencies of the edge multipole modes can be derived by using the variational principle. Given the $N$ electron ground state $|0 \rangle$, it is possible to find the collective excitation energy and the collective state $|c \rangle$, if one able to find an operator $O^\dagger$, which satisfies the following equation of motion,

$$[\hat{H}, O^\dagger] = \hbar \omega_{coll} O^\dagger. \quad (18)$$

The state $O^\dagger|0 \rangle$ has the excitation energy $\hbar \omega_{coll}$. This excitation energy is obtained from,

$$\hbar \omega_{coll} = \frac{\langle 0|[O, [\hat{H}, O^\dagger]]|0 \rangle}{\langle 0|[O, O^\dagger]|0 \rangle}. \quad (19)$$

For $k = 1$ the dipole excitation operator is,

$$O^\dagger = \frac{1}{2} \sum_i (z_i - \frac{i}{m^* \Omega} \hat{p}_{i+}), \quad (20)$$

where $p_+ = p_x + ip_y$. Similarly, we may take the variational ansatz for the higher multipole excitations in the following form,

$$O^\dagger = F + aJ \quad ; \quad J = [H, F], \quad (21)$$
where $a$ is the variational parameter, $F = \sum_i z_i^k$, and $J = \left( \frac{\hbar k}{m^*} \right) \sum_i z_i^{(k-1)} \hat{D}_+ + \frac{im^* \omega_c}{2} z_i$. The collective excitation energy in terms of the the energy weighted sum rules is,

$$\hbar \omega_{\text{coll}} = \frac{m_1^+ + 2am_2^- + a^2 m_3^+}{2am_1^+ + a^2 m_2^-}. \quad (22)$$

After minimising the above expression with respect to the variational parameter $a$, the following expression for the collective excitation frequency is obtained,

$$\hbar \omega_{\text{coll}} = \sqrt{m_3^+ - \frac{3}{4} \left( \frac{m_2}{m_1^+} \right)^2 + \frac{m_2^-}{2}}. \quad (23)$$

This agrees with the previous expression in eqn.(17), obtained by the approximate form the strength distribution.

Now we explicitly evaluate the the sum-rules by calculating the commutators. After doing some algebra the sum-rules can be written in the following form. The first energy weighted moment $m_1^+$ is given by,

$$m_1^+(k) = \frac{\hbar^2 k^2}{m^*} < r^{2(k-1)} >, \quad (24)$$

where $< ... >$ denotes the average weighted with the ground state density. The contribution for the third moment $m_3^+$ coming from the kinetic energy of the hamiltonian $m_3^+(T)$ can be written as,

$$m_3^+(T) = \left( \frac{\hbar^2 k^2}{m^*} \right) \frac{1}{m^*} \int d^2 r r^{2(k-2)} |\nabla \psi|^2 \rho(r) - 2 \int d^2 r r^{2(k-3)} \rho(r) \frac{\partial V_H}{\partial r} \rho(r), \quad (25)$$

Similarly the contribution of the external potential term in $m_3^+$ is,

$$m_3^+(V) = \frac{\hbar^2 k^2}{m^*} km^* \omega_0^2 < r^{2(k-1)} >. \quad (26)$$

The most important contribution comes from the electron-electron interaction term, and is given by,

$$m_3^+(ee) = \frac{\hbar^4 k^2}{2m^*} \left[ \int d^2 r \nabla^2 V_H(r) r^{2(k-1)} \rho(r) + 2(k-1) \int d^2 r \frac{\partial V_H}{\partial r} r^{2(k-3)} \rho(r) \right] \int d^2 r' \rho'(r') \frac{1}{|r-r'|} e^{-i\theta} |r'| \rho(r'), \quad (27)$$

where $V_H(r)$ is the Hartree potential. In the presence of a magnetic field there is nonvanishing second moment $m_2^-$,

$$m_2^- = \frac{\hbar^2 k^2}{m^*} \left[ -\hbar m^* \omega_c k < r^{2(k-1)} > + 2\hbar(k-1) < r^{2(k-2)} \hat{l}_z > \right], \quad (28)$$
where $\hat{l}_z$ is the angular momentum operator with eigenvalue $m$. We now consider the limit in which the magnetic field is large, so that all electrons are in the lowest Landau level. Neglecting the mixing of higher Landau levels, if we consider the wave function in the following form,

$$\psi(z_i) = f(z_1, z_2, ..., z_i, ...)e^{-\sum z_i z_i/2\hbar^2},$$  \quad (29)

then the above sum-rules can be simplified and written in terms of moments of radius $r$. In the lowest Landau level the sum-rules $m^+_3$ and $m^-_2$ can be written as,

$$m^+_3(T) + m^+_3(V) = \frac{\hbar^2 k^2}{m^*}\frac{\hbar^2 m^* \Omega^2(3((k-1) - \frac{k}{2} \frac{\omega_c}{\Omega})^2 + 1)}{r^{2(k-1)}} >$$

$$-3\hbar^3 m^* \Omega (k-1)^2 (2(k-2) - k(\frac{\omega_c}{\Omega})) < r^{2(k-2)} >$$

$$+4\hbar^2 (k-1)^2 (k-2)^2 < r^{2(k-3)} >,$$  \quad (30)

$$m^-_2 = \frac{\hbar^2 k^2}{m^*}[-hm^* \omega_c k < r^{2(k-1)} >$$

$$+2\hbar (k-1)(m^* \Omega < r^{2(k-1)} > - \hbar (k-1) < r^{2(k-2)} >].$$  \quad (31)

We can write down the expressions for the ratios of moments in the following form,

$$\frac{m^-_2}{m^+_1} = 2(k-1)(\Omega - \frac{\omega_c}{2}) - \omega_c + \frac{\alpha_1}{R^2},$$  \quad (32)

$$\frac{m^+_3(T) + m^+_3(V)}{m^+_1} = \hbar^2 \Omega^2(1 + 3(k-1 - \frac{k}{2} \frac{\omega_c}{\Omega})^2) \frac{\alpha_2}{R^2} + \frac{\beta_2}{R^4}.$$  \quad (33)

For large number of electrons, if we neglect the terms $O(1/R^2)$ and the higher order terms, then the excitation energies can be written in the most general form, given below,

$$E_{coll}(k) = (k-1)\hbar(\Omega - \frac{\omega_c}{2}) + \sqrt{\hbar^2 \Omega^2 + \Delta(e)} - \frac{\omega_c}{2}.$$  \quad (34)

The coefficients $\alpha_1, \alpha_2, \beta_2$ depend on the density of the electrons, and the function $\Delta(e)$ depends on the nature of the two body interaction and also on the shape of the density profile. Now we can estimate the low-lying multipole excitations by using the semi-circular density profile. The parameters in the above form of the collective frequency can be evaluated by using the semicircular density profile $\rho = n_0 \sqrt{1 - (r/R)^2}$,

$$\Delta(e) = 2\hbar^2 \Omega (\Omega - \frac{\omega_c}{2}) \frac{\Gamma(k+\frac{3}{2})}{\Gamma(k)\Gamma(3/2)} - k,$$  \quad (35)

$$\alpha_1 = \hbar \Omega (k-1)(2k+1),$$  \quad (36)

$$\alpha_2 = -\frac{3}{2} \hbar^2 \Omega^2 (k-1)(2k+1)(2(k-2) - k(\frac{\omega_c}{\Omega})),$$  \quad (37)

$$\beta_2 = \hbar^2 \Omega^2 (k-1)(k-2)(2k+1)(2k-1).$$  \quad (38)

Since the semi-circular density profile is the exact solution of the Hartree equation, the dispersion relation of the edge modes obtained by using this density is a non-perturbative
result. Further the asymptotic results do not contain the coupling constant $e^2/l_0$, only the finite size corrections depend on the coupling constant. This density profile goes to zero at the turning point $R$ and the diffusive tail is absent in this case. But the diffusive tail in the exact density distribution can only contribute $1/N$ corrections to the spectrum of low-lying excitations [15]. From the energy levels of the non-interacting electrons, we can estimate the magnetic field above which all $N$ particles go into the lowest Landau level. This gives the condition $2\hbar\Omega > (N-1)(\Omega - \omega_c/2)$ (or $\omega_0/\omega_c < 1/\sqrt{N}$). If $k << N$, which is true for the low-lying modes, then in the strong field limit we can expand the expression for collective modes in terms of $\Delta(e)/\hbar^2\Omega^2$, and obtain the following result in strong magnetic field limit

$$E_c(k) = \omega_0 \frac{\omega_0}{\omega_c} \frac{\Gamma(k + 1/2)}{\Gamma(k)\Gamma(3/2)} + O((\frac{\omega_0}{\omega_c})^4).$$

(39)

This result now be compared with the dispersion relation obtained by, Shikin et al [11] using classical hydrodynamics,

$$\omega_k^\ominus = \sqrt{k\Omega_{kk}^2 + \omega_c^2/4 - \omega_c/2},$$

(40)

$$k\Omega_{kk}^2 = \omega_0^2 \frac{\Gamma(k + 1/2)}{\Gamma(k)\Gamma(3/2)},$$

(41)

where $\hbar\omega_k^\ominus$ is the same as $E_c(k)$ in our notation. If we expand this expression in terms of $\omega_0/\omega_c$, the leading term agrees with the dispersion relation obtained from sum-rule approach.

For comparison, we may also estimate the edge excitations in a quantum dot by using the density profile of the non-interacting electrons at the filling factor $\nu = 1$. In the large $N$ limit, the density profile of the system can be approximated by, $\rho_0 \theta(R - r)$, where, $\rho_0 = \frac{1}{\pi l_0^2}$. This density profile is very sharp near the edge, and therefore singularities arise in evaluating $\Delta$, but after doing the integrals and then taking the limit($r \to R$) at the edge, the singularities cancel out and we obtain finite value of the parameter $\Delta$. The parameter $\Delta$ in this case can be derived as,

$$\Delta(e) = 2\hbar\Omega \frac{e^2}{l_0^2} \sqrt{\frac{N}{\hbar \omega_0}} \left[1 - \sum_{m=1}^{k} \frac{1}{2m - 1}\right].$$

(42)

This result agrees with the value obtained by Giovanazzi et al [4]. In this case the excitation energies of various multipole modes vanishes at different values of magnetic field. The approximate value of the magnetic field, where excitation energy of the $k$ th mode becomes negative is,

$$\left(\frac{\omega_0}{\omega_c}\right)^{3/2} < \frac{e^2}{\sqrt{2N_l} \hbar \omega_0} \left[\sum_{m=1}^{k} \frac{1}{2m - 1}\right] - 1,$$

(43)

where $l_0 = \sqrt{\frac{\hbar}{m^* \omega_0}}$. Energy of the higher multipole modes vanish at lower magnetic field. For example the mode with multipolarity $k = 10$ becomes gapless at a magnetic field $\sim 9.65T$, in a dot with $N = 40$ and $\hbar \omega_0 = 5.4meV$. Softening of these edge modes indicate the edge reconstruction of the dot and the formation of new ground state. This phenomena indicates the instability of the flat density. The critical magnetic field where the instability sets in is obtained from [19-21].
\[
\left( \frac{\omega_0}{\omega_c} \right)^{3/2} \approx \left[ \frac{0.5139 e^2}{l_0 \hbar \omega_0 \sqrt{N}} \right].
\] (44)

Although \( \Delta \) is negative for both the cases, the excitation energy for low-lying modes for the semi-circular density profile are positive and asymptotically go to zero, which indicates the stability of the ground state. For comparison, we have shown in Fig.1 \( \Delta \) for different multipolarities, evaluated using two different ground state densities. In Fig.2, the variation of few low-lying modes with magnetic field is shown, using the dispersion law given in eqn.(34) and eqn.(35). These modes vanish as \( \sim 1/B \) with the increasing magnetic field. These low-lying modes are important for the thermodynamics of the system at very low temperatures.

Finally we consider the breathing mode of the dot within the same formalism. The breathing mode is excited by the excitation operator, \( F = r^2 \). The operator \( J \) is,

\[
J = [H, F] = \left( -i \frac{\hbar}{m^*} \right) \left[ \hat{p}_x x + x \hat{p}_y + y \hat{p}_y \right].
\] (45)

The first moment \( m_1^+ \) is given by,

\[
m_1^+ = \frac{2 \hbar^2}{m^*} < r^2 >.
\] (46)

The third moment can be written in the following way,

\[
m_3^+ = \frac{1}{2} \frac{\partial^2}{\partial \eta^2} < \eta | \hat{H} | \eta > |_{\eta=0},
\] (47)

where \( | \eta > = e^{\eta J} | 0 > \). Now the moment \( m_3^+ \) can be evaluated by using the scaling properties of the wave function [22],

\[
e^{\eta J} | \psi > = e^{-2 \eta \hbar^2/m^*} | \psi (x e^{-2 \eta \hbar^2/m^*}, y e^{-2 \eta \hbar^2/m^*}) > .
\] (48)

From the above scaling property of the wave function, we obtain,

\[
m_3^+ = 8 \left( \frac{\hbar^2}{m^*} \right)^2 [ < \hat{T} > + \frac{1}{2} m^* \Omega^2 < r^2 > ] + 2 \left( \frac{\hbar^2}{m^*} \right)^2 E_{int},
\] (49)

\[
= 8 \left( \frac{\hbar^2}{m^*} \right)^2 [ m^* \Omega^2 < r^2 > ] + 2 \left( \frac{\hbar^2}{m^*} \right)^2 E_{int},
\] (50)

where \( \hat{T} \) is the kinetic energy operator and \( E_{int} \) is the interaction energy of the system. In deriving the second step, we used the properties of the lowest-Landau level wave functions. The general expression of the excitation energy of the breathing mode is derived as,

\[
E_b = \sqrt{4 \hbar^2 \Omega^2 + \frac{\hbar^2}{m^*} \frac{E_{int}}{< r^2 >}}.
\] (51)

Using the semi-circular density for the ground state of the electrons, we obtain the non-perturbative result for the breathing mode,
\[ E_b = 2\hbar \Omega \sqrt{1 + \frac{1}{4}(1 - \frac{\omega_c}{2\Omega})}. \] (52)

In the strong field limit this mode approaches the bulk collective mode \( \hbar \omega_c \).

To summarise, we have considered the quantum dot in the lowest Landau level. We have derived a simple local density functional for the electrons in the LLL within the Hartree approximation. For the two-body coulomb interaction a semicircular density profile of the electrons has been derived. We have estimated the low-lying edge multipole modes within the sum-rule approach. Since the semi-circular density is exact within Hartree approximation, we obtain the non-perturbative results for the dispersion realtions of the collective modes by using this density profile. The energy of these collective modes decreases with increasing magnetic field. In the strong field limit the most general expression for the dispersion relation of the edge modes has been derived and is given by,

\[ E(k) = \sqrt{\Omega^2 + \Delta(e) - \frac{k^2}{2} + (k - 1)\omega_0 \frac{\omega_c}{\omega_0}}. \]

The parameter \( \Delta(e) \) depends on the exact nature of two-body interaction and on the number of electrons in the system. We evaluate the parameter \( \Delta(e) \) by using exact density, as well as by using the density profile of the non interacting electrons at fillin factor \( \nu = 1 \). In both cases \( \Delta(e) \) is negative. In the case of flat density the energy of the low-lying edge modes become zero at some magnetic field and the softening of the edge modes indicate the instability of the ground state. But for the semi-circular ground state density the low-lying modes are positive and asymptotically go to zero, which shows the stability of the self-consistent density profile. We have also calculated the breathing mode of a quantum dot, and in the strong magnetic field the energy of the breathing mode approaches the cyclotron frequency \( \omega_c \). The main results of this paper are, the derivation of the most general dispersion relation of the edge modes in a quantum dot in a strong magnetic field. The \( 1/N \) corrections of the energy of these modes are also obtained. Stability of the self consistent density profile has been shown by analysing the dispersion relation of the low-lying edge modes. This result shows that the semicircular density is a better ansatz for the ground state density than commonly used flat density profile, in the strong magnetic field limit. These edge modes are important for the edge excitation and edge reconstruction of quantum dot. They also determine the low-temperature thermodynamic properties of the quantum dot.

I would like to thank M. V. N Murthy for his helpful comments. I would also like to thank Tapash Chakraborty and R. Shankar for critical reading of the manuscript.
REFERENCES

[1] T. Chakraborty, Quantum Dot, A Survey of the Properties of Artificial Atoms, (Elsevier, North-Holland, 1999)
[2] X. G. Wen , Phys. Rev. B 44, 5708 (1991).
[3] M. Stone , Phys. Rev. B 42, 212 (1990)
[4] S. Giovanazzi, L. Pitaevskii and S. Stringari, Phys. Rev. Lett. 72, 3230 (1994).
[5] E. Lipparini, N. Barberan, M. Barranco, M. Pi, and L. Serra et al, Phys. Rev. B, 56, 12375 (1997); A. Emperador, M. Barranco, E. Lipparini, M. Pi, and L. Serra cond-mat/9902357.
[6] P. A. Maksym and T. Chakraborty, Phys. Rev. Lett. 65, 108 (1990).
[7] Ch. Sikorski and U. Merkt, Phys. Rev. Lett. 62, 2164 (1989).
[8] T. Demel, D. Heitmann, P. Grambow, and K. Ploog, Phys. Rev. Lett. 64, 788 (1990).
[9] C. Schuller, K. Keller, G. Biese, E. Ulrichs, L. Rolf, C. Steinebach, and D. Heitmann, Phys. Rev. Lett. 80, 2673 (1998).
[10] V. Shikin, S. Nazin, D. Heitmann, and T. Demel, Phys. Rev. B, 43, 11903(1991).
[11] D. B. Mast, A. J. Dahm, and A. L. Fetter, Phys. Rev. Lett. 54, 1706 (1985).
[12] B. Partoeus, A. Matulis and F. M. Peeters, cond-mat 9712066
[13] Z. L. Ye and E. Zaremba , Phys. Rev. B 50, 17217 (1994).
[14] E. Lipparini and S. Stringari, phys. Rep. 175, 103 (1989).
[15] M. Brack, Rev. Mod. Phys, 65, 677 (1993).
W. de Heer, Rev. Mod. Phys, 65, 611 (1993).
[16] S. Sinha, Preprint
[17] L. Serra, M. Barranco, A. Emperador, M. Pi, and E. Lipparini, e-print cond-mat 9806104
[18] S. M. Girvin and T. Jach , Phys. Rev. B 29, 5617 (1984).
[19] A. H. MacDonald, S. R. Eric Yang and M. D. Johnson , Aust. J. Phys 46, 345 (1993).
[20] E. Lieb, J. P. Solovej and J. Yngvason , Phys. Rev. B 51, 10646 (1995).
[21] M. Ferconi and G. Vignale, Phys. Rev. B 56, 12108 (1997).
[22] O. Bohigas, A. M. Lane, and J. Martorell, Phys. Rep. 51, 267 (1979).
FIGURES

FIG. 1. The parameter $\Delta(e)$ in units of $\hbar^2 \omega_0^2$, for different multipolarities $k$, for a dot with $N = 50$, $\frac{e^2}{\hbar^2 \omega_0} = 0.8$ and $\omega_c/\omega_0 = 10$. The open triangles represent values calculated with semi-circular density and the solid triangles denote the same parameter for flat density profile.

FIG. 2. The variation of four lowest multipole modes with magnetic field, in a quantum dot with semi-circular density profile. Different values of $k$ denote different multipolarities.
Fig. 2 (sinha)