Universal technique of elimination of power parametric excitation in general mechanical systems

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Abstract. A universal technique to transform general mechanical systems in rotative ones with a constant equivalent inertial characteristic is proposed. The technique is based on attaching of a kinematic chain with a program change of inertial characteristic to a reduction link. The inertial characteristic in addition to the main one forms the reduction link with a constant equivalent moment of inertia. Such modification of a mechanical drive eliminates parametric power drive excitation, reduces the level of dynamic load in links and drive joints, that results in reliability and system resource improvement.

1. Introduction
General issues of dynamics in mechanical systems are well studied in fundamental literature [1-9].

It is known that a dynamic simulation of mechanical systems motion is based on using a concept of a reduction link to which inertial characteristics of the system and external forces causing motion are reduced.

The variable equivalent inertia moment of the reduction link is a source of parametric excitation of inertial origin resulting in the increase and decrease of inertial characteristic of the reduction link in the system with a cyclic motion of links twice for one revolution of the reduction link. Thus, twice for the revolution it brings to the increase and decrease of motion velocity in the reduction link that generates the power moment of inertia forces in addition to the main flow, loading links and joints of the drive in general mechanical system.

2. Problem statement
Let us set and solve the problem of complete elimination of inertial parametric excitation in a general mechanical system by transformation of a general system with a variable value of inertial characteristic into a rotative system with a constant value of inertia equivalent moment.

The transformation of general mechanical system into the rotative one can be fulfilled by means of additional, embedded kinematic chain with a program variable reduced to a reduction link by the inertial characteristic, which combined with the equivalent moment of inertia in the main kinematic chain forms a constant equivalent moment of inertia in the modified system.

3. Theory
Lagrange equation for a reduction link of one-degree-of-freedom nondamper system with angular generalized coordinate $\phi$ and rigid links is
\[ J_{eq} \frac{d\omega}{dt} + \frac{1}{2} \frac{dJ_{eq}}{d\varphi} \omega^2 = Mg^{eq} - Mc^{eq}, \]

or

\[ J_{eq} \ddot{\varphi} + \frac{1}{2} \frac{dJ_{eq}}{d\varphi} \varphi^2 = Mg^{eq} - Mc^{eq}. \] (1)

where \( J_{eq} \) is a variable equivalent inertia moment of a reduction link defined from the equivalent of a kinetic energy of the reduction link and a kinetic energy of movable links of the system. \( \varphi \) is a generalized coordinate of the system; \( \omega = \dot{\varphi} \) is an angular velocity of the reduction link; \( t \) is time; \( Mg^{eq} - Mc^{eq} = M^{eq}_{exc} \) is a difference of equivalent power characteristics of external forces (excess moment of external forces).

Differential equation (1) is a non-linear second-order equation with variable coefficients before derivatives and, generally, has no analytical solution.

In the rotative system, the second component of the equation (1) becomes zero, and the equation (1) is simplified to the following:

\[ J_{eq} \frac{d\omega}{dt} = Mg^{eq} - Mc^{eq}, \] (2)

from which you can see that velocity vibrations of reduction link motion are entirely defined only by a power function of mismatch in external forces reduced to a reduction link – the differences of moments in equivalent driving forces and useful resistance forces. The equation (2) for most combinations of power characteristics and their linearization has the analytical solution.

Thus, if external equivalent forces depend on the motion velocity of a reduction link, i.e. when

\[ J_{eq} \frac{d\omega}{dt} = Mg^{eq}(\omega) - Mc^{eq}(\omega), \]

we can obtain an integral expression, having divided variables

\[ \int_{t_0}^{t_1} dt = J_{eq} \int_{\omega_0}^{\omega_1} \frac{d\omega}{Mg^{eq}(\omega) - Mc^{eq}(\omega)} \]

and, if difference can be presented as integrable function, that always can be achieved in linearization of components, the solution gives a series of motion time values in a function of velocity, i.e.

\[ t_1 = t_0 + J_{eq} \int_{\omega_0}^{\omega_1} \frac{d\omega}{Mg^{eq}(\omega) - Mc^{eq}(\omega)}, \] (3)

this series of values is easily transformed into inverse function \( \omega = \omega(t) \).

Similarly, the solution takes place for power characteristics depending on time:

\[ J_{eq} \frac{d\omega}{dt} = Mg^{eq}(t) - Mc^{eq}(t) \]

and

\[ \int_{\omega_0}^{\omega_1} d\omega = \frac{1}{J_{eq}} \left[ \int_{t_0}^{t_1} [Mg^{eq}(t) - Mc^{eq}(t)] dt \right] \]

or

\[ \omega_1 = \omega_0 + \frac{1}{J_{eq}} \left[ \int_{t_0}^{t_1} [Mg^{eq}(t) - Mc^{eq}(t)] dt \right]. \] (4)

In case of constant power characteristics values or their dependence on generalized coordinate \( \varphi \), selecting \( \Delta \varphi \), we can use integral expression:
\[ \frac{1}{2} J_{eq}(\Delta \omega)^2 = \int_{\phi_0}^{\phi} [(Mg(\phi) - Mc(\phi))] d\phi, \]

hence

\[ \Delta \omega = \sqrt{\frac{1}{J_{eq}} \int_{\phi_0}^{\phi} [(Mg(\phi) - Mc(\phi))] d\phi} \]

and

\[ \omega_1 = \omega_0 + \Delta \omega. \]  \hspace{1cm} (5)

If the machine has a combination of power characteristics dependent on various arguments, then the solution is possible only with the usage of numerical methods and linearization of power characteristics on intervals of integral computation.

Generalized coordinate \( \phi \) is divided into small intervals taken as an integration step. On each interval of function for equivalent power moments, \( Mg_{eq} \) and \( Mc_{eq} \) are assumed as constants, their difference can be written, for example, as

\[ Mg_{eq}(\omega) - Mc_{eq}(\phi) = M(\phi, \omega), \]

where the initial value \( M(\phi, \omega) \) is determined by initial values \( Mg_{eq}(\omega_0) \) and \( Mc_{eq}(\phi_0) \) with the subsequent step-by-step calculation of change in motion velocity of the reduction link and, as a result, the subsequent determination of level of dynamic loading in links and joints of a mechanical system in a machine drive.

4. Technical realization of the hypotheses

The embedded kinematic chain with a program changeable moment of inertia can be produced on the basis of a fixed flat cam mechanism which curvilinear profile provides a variable distance from a rotation axis of a reduction link for additional movable masses, thus changing their inertial characteristic in a transfer rotary motion. This inertial characteristic combined with the equivalent moment of inertia of the main mechanism potentially provides constancy of the equivalent moment of inertia in the modified system.

Let \( J(\phi) \) be a current variable value of equivalent moment of inertia dependent on generalized coordinate \( \phi \) in the main general mechanism, \( J(\phi)_{ad} \) be the equivalent inertia moment of additional movable masses of the embedded kinematic chain, then the condition is to be fulfilled

\[ J(\phi) + J(\phi)_{ad} = J_{const}. \]  \hspace{1cm} (6)

In each of two components in the equation (6) constant parts determined by rotative properties of the system are removed. This system comprises a motor spindle, a reducer, couplings, a crankshaft of the main mechanism, an arm 3 of rollers 4, minimum inertia moment \( Jr \) of rollers 4 in the embedded chain relative to axis 2 (fig. 1). The value \( Jr \) can be defined as \( Jr = 2m r_{min}^2 \), where \( m \) is a mass of one roller; \( r_{min} \) is a minimum size of a radial arrangement of mass center in a roller relative to axis 2.

The profile of cam 1 will determine radius \( \rho(\phi) \), obtained from calculation

\[ \rho(\phi) = r_{min} + r_r + \sqrt{\frac{J(\phi)_{ad}}{2m}}, \]  \hspace{1cm} (7)

where \( r_r \) is the radius of roller surface 4 sliding on a cam profile.

5. Results and discussion

The hypothesis of possible transformation of a general mechanical system in a rotative one to achieve a constant value of the equivalent inertial characteristic resulting in the elimination of parametric excitation in the system according to mathematical simulation of such transformation is technically
achievable. This is proved by the inventor's certificate [10] and engineering calculation of parameters for embedded kinematic chain.

![Figure 1. An embedded kinematic chain](image)

1 – a magnetic material cam; 2 – a common rotation axis of a reduction link; 3 – an arm for placement of rollers in embedded kinematic chain; 4 – rollers of the embedded kinematic chain.

The embedded kinematic chain (fig. 1) contains the arm 3, which is rigidly fixed with the reduction link, that has the rotation axis 2 on the arm 3 in special grooves and in diametrically opposite directions there are mutually balanced pairwise rollers 4, everyone with weight "m". The rollers can move radially in arm grooves, these relative movements occur due to running of rollers on the internal curvilinear surface of the cam 1 made of magnetic material. A certain arrangement of rollers on a curvilinear surface of a cam can be provided in another way, for example, by means of resilient elements creating necessary force-closure of an upper couple.

The device works as follows. When the reduction link rotates around the axis 2, the arm 3 rotates too, carrying rollers 4 that run on the cam 1 which profile is executed according to the equation (7). Then the change of equivalent inertia moment of mechanical drive in a general system is compensated by the alteration $J(\phi)ad$ due to the displacement of rollers 4 to the periphery of arm 3 or to its center thus, $J(\phi) + J(\phi)ad = J_{const}$, and derivative $\frac{dJ_{const}}{d\phi} = 0$.

Transformation of the general drive in the rotative one is technically realizable, it eliminates parametric power inertial excitation of the drive, the level of dynamic loading of links and joints in the drive of general mechanical system thereby decreases that leads to reliability of all elements of the drive and system resource improvement.

6. Conclusions

1) Transformation technique of general mechanical system in a rotative one by introducing an additional kinematic chain with a software-control of equivalent inertial characteristic to the system has been shown to be universal and technically implementable.

2) The constancy of inertial characteristic and equality to zero of its derivative by the generalized coordinate of modified drive means the elimination of parametric inertial power excitation in a power drive of any machines that reduces the level of dynamic loading in links and joints of the drive.

3) Calculation of program change of the inertial characteristic in the embedded kinematic chain is based on the knowledge of a variable part of equivalent moment of inertia in the main mechanism that
is defined by a known engineering calculation for energy criterion of mass reduction to a reduction link.

References

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