Variation of the Amati relation with cosmological redshift: a selection effect or an evolution effect?

Li-Xin Li*
Max-Planck-Institut für Astrophysik, 85741 Garching, Germany

Accepted 2007 April 23. Received 2007 April 21; in original form 2007 April 12

ABSTRACT
Because of the limit in the number of gamma-ray bursts (GRBs) with available redshifts and spectra, all current investigations on the correlation among GRB variables use burst samples with redshifts that span a very large range. Evolution and selection effects have thus been ignored, which might have an important influence on the results. In this Letter, we divide the 48 long-duration GRBs studied by Amati into four groups with redshift from low to high, each group containing 12 GRBs. Then we fit each group with the Amati relation \( \log E_{\text{iso}} = a + b \log E_{\text{peak}} \), and check whether the parameters \( a \) and \( b \) evolve with the GRB redshift. We find that \( a \) and \( b \) vary with the mean redshift of the GRBs in each group systematically and significantly. Monte Carlo simulations show that there is only a \( \sim 4 \) per cent chance that the variation is caused by the selection effect arising from the fluence limit. Hence our results may indicate that GRBs evolve strongly with cosmological redshift.

Key words: cosmology: theory – gamma-rays: bursts – gamma-rays: observations.

1 INTRODUCTION
A remarkable achievement in the observation of gamma-ray bursts (GRBs) has been the identification of several very good correlations among the GRB observables [see Schaefer (2007) for a review]. Based on several of these correlations, some people have eagerly proposed the use of GRBs as standard candles to probe the cosmological Hubble diagram to very high redshift (Schaefer 2003; Dai, Liang & Xu 2004; Ghirlanda et al. 2004a; Lamb et al. 2005; Firmani et al. 2006; Schaefer 2007, and references therein). Enlightening comments and criticism on GRBs as standard candles can be found in Bloom, Frail & Kulkarni (2003) and Friedman & Bloom (2005).

All of the GRB correlations have been obtained by fitting a hybrid GRB sample without discriminating in terms of the redshift. Indeed, the redshift in the sample usually spans a very large range: from \( z \sim 0.1 \) up to \( z \sim 6 \). This is of course caused by the fact that we do not have a large enough number of GRBs with measured redshifts limited in a small range. Then, inevitably, the effects of the GRB evolution with redshift, and the selection effects, have been ignored. This raises an important question about whether the relations that people have found reflect the true physics of GRBs or whether they are just superficial. [See Band & Preece (2005) for a nice discussion on the selection effect and the correlation between the GRB peak spectral energy and the isotropic-equivalent/jet collimated energy.]

For objects distributed from \( z \sim 0.1 \) to \( \sim 6 \), it is hard to believe that they do not evolve. There is cumulative evidence suggesting that long-duration GRBs prefer to occur in low-metallicity galaxies (Fynbo et al. 2003; Hjorth et al. 2003; Le Floc’h et al. 2003; Sollerman et al. 2005; Fruchter et al. 2006; Stanek et al. 2006). With a sample of five nearby GRBs, Stanek et al. (2006) have found that the isotropic energy of GRBs is anticorrelated with the metallicity in the host galaxy (see, however, Wolf & Podsiadlowski 2007). It is well known that metallicities evolve strongly with the cosmological redshift (Kewley & Kobulnicky 2005; Savaglio et al. 2005). Hence the evolution of GRBs with redshift is naturally expected (see e.g. Langer & Norman 2006).

In this Letter, we use the Amati relation as an example to test the cosmic evolution of GRBs. The Amati relation is a correlation between the isotropic-equivalent energy of long-duration GRBs and the peak energy of their integrated spectra in the GRB frame (Amati et al. 2002):

\[ \log E_{\text{iso}} = a + b \log E_{\text{peak}}. \]  (1)

The isotropic-equivalent energy \( E_{\text{iso}} \) is defined in the 1–10 000 keV band in the GRB frame.

With a sample of 41 long GRBs with firmly determined redshifts and peak spectral energy, Amati (2006) has obtained \( a = -3.35 \) and \( b = 1.75 \) with the least-squares method (\( E_{\text{peak}} \) in keV and \( E_{\text{iso}} \) in \( 10^{52} \) erg); and \( a = -4.04 \) and \( b = 2.04 \) with the maximum likelihood method with an intrinsic dispersion in the relation (1) being included. Long GRBs detected by Swift and having measured redshifts and \( E_{\text{peak}} \) are found to be consistent with the Amati relation (Amati 2007).

The difference in the values of the parameters obtained with the two methods can be explained as follows. The maximum likelihood method directly probes the intrinsic relation between the two

*E-mail: lxli@mpa-garching.mpg.de

© 2007 The Author. Journal compilation © 2007 RAS

Downloaded from https://academic.oup.com/mnrasl/article-abstract/379/1/L55/1044284 on 29 July 2018
variables, \( x = \log E_{\text{iso}} \) and \( y = \log E_{\text{peak}} \) (D’Agostini 2005). However, roughly speaking, the least-squares method estimates the average value of \( x \) at a given \( y \), \( \langle x \rangle = a' + b'y \). Teerikorpi (1984) has shown that, when \( x \) has a Gaussian distribution with a dispersion \( \sigma_x \), and the relation \( x = a + by \) has an intrinsic dispersion \( \sigma_y \) in \( y \), \( b' \) is related to \( b \) by

\[
b' = b \left( 1 + \frac{b^2 \sigma_y^2}{\sigma_x^2} \right)^{-1}.
\]

Amati (2006) has found that \( \sigma_x \approx 0.9 \), \( b \approx 2.04 \) and \( \sigma_y \approx 0.15 \). Then by equation (2) we have \( b' \approx 1.83 \), which is close to the value of 1.75 obtained by the least-squares method.

To test whether the Amati relation varies with the cosmological redshift, in this Letter we separate a sample of 48 long GRBs [consisting of the 41 long GRBs from Amati (2006) and seven additional Swift long GRBs from Amati (2007)] into four groups by GRB redshift. That is, we sort the GRBs by their redshifts, and divide them into four groups with redshifts distributed from low values to high values. Each group contains 12 GRBs (for details see Section 2). We then fit each group by equation (1) and calculate the mean redshift, and check whether the values of \( a \) and \( b \) evolve with redshift.

As we will see, the values of \( a \) and \( b \) strongly vary with redshift. The variation is not likely to arise from the selection effect and hence may indicate that GRBs evolve strongly with the cosmological redshift.

Throughout this Letter, we follow Amati (2006) to adopt a cosmology with \( \Omega_m = 0.3 \), \( \Omega_{\Lambda} = 0.7 \) and \( H_0 = 70 \text{ km s}^{-1} \text{ Mpc}^{-1} \).

### 2 VARIATION OF THE AMATI RELATION WITH COSMOLOGICAL REDSHIFT

To test whether the Amati relation (1) evolves with redshift, we separate a sample of 48 long GRBs into four groups according to the redshift of the GRBs, then fit each group with equation (1).

The sample contains 41 long GRBs from table 1 of Amati (2006) and seven additional Swift long GRBs from Amati (2007). The additional seven Swift long GRBs are 060115, 060124, 060206, 060418, 060707, 060927 and 061007. Since a GRB sample with \( z \leq 0.1 \) is very incomplete, we select only GRBs with \( z > 0.1 \) and hence GRB 060218 (\( z = 0.0331 \)) is not included. GRB 060614 is also excluded from our sample because of the very large uncertainty in its \( E_{\text{peak}} \) (Amati et al. 2007).

A least-squares fit to the 48 GRBs as a single sample with equation (1) leads to \( a = -3.42 \), \( b = 1.78 \), with \( \chi^2_r = 5.9 \). \( \chi^2_r \) is the reduced \( \chi^2 \), i.e., the \( \chi^2 \) of the fit divided by the number of degrees of freedom. A maximum likelihood fit, which includes an intrinsic dispersion \( \sigma_x \) in \( \log E_{\text{peak}} \) in the relation (1), leads to \( a = -4.08 \), \( b = 2.04 \) and \( \sigma^2 = 0.14 \). These results are consistent with that obtained with 41 GRBs by Amati (2006).

The redshift of the 48 GRBs spans a range of 0.17–5.6. GRB 030329 has the minimum redshift (\( z = 0.17 \)). GRB 060927 has the maximum redshift (\( z = 5.6 \)). The mean redshift is \( \langle z \rangle = 1.685 \). We separate the 48 GRBs into four groups with redshifts from low to high, each group containing 12 GRBs:

- (A) \( 0.1 < z < 0.84 \), \( \langle z \rangle = 0.56 \);
- (B) \( 0.84 \leq z < 1.3 \), \( \langle z \rangle = 1.02 \);
- (C) \( 1.3 \leq z < 2.3 \), \( \langle z \rangle = 1.76 \);
- (D) \( 2.3 \leq z \leq 5.6 \), \( \langle z \rangle = 3.40 \).

The least-squares fit to each group of GRBs by equation (1), taking into account the errors in both \( E_{\text{peak}} \) and \( E_{\text{iso}} \), is shown in Fig. 1. Immediately one can see that, except for Group B, the \( \chi^2_r \) for each group is smaller than that obtained by fitting the whole sample of GRBs. This fact indicates that treating the GRBs at different redshifts as a single sample may increase the data dispersion (see Fig. 3 below).

We find that the values of \( a \) and \( b \) vary with the mean redshift of the GRBs monotonically. In Fig. 2 we plot \( a \) and \( b \) against \( \langle z \rangle \). Clearly, \( a \) and \( b \) are correlated/anticorrelated with \( \langle z \rangle \). The Pearson linear correlation coefficient between \( a \) and \( \langle z \rangle \) is \( r(a, \langle z \rangle) = 0.975 \), corresponding to a probability \( P = 0.025 \) for a zero correlation. The correlation coefficient between \( b \) and \( \langle z \rangle \) is \( r(b, \langle z \rangle) = -0.960 \), corresponding to a probability \( P = 0.040 \) for a zero correlation.

A least-squares linear fit to \( a \langle z \rangle \) (the solid line in the upper panel of Fig. 2) leads to

\[ a = -4.58(\pm0.36) + 0.43(\pm0.17)z, \]

with \( \chi^2_r = 0.13 \). A least-squares linear fit to \( b \langle z \rangle \) (the solid line in the lower panel of Fig. 2) leads to

\[ b = 2.32(\pm0.15) - 0.207(\pm0.066)z, \]

with \( \chi^2_r = 0.31 \).

The results indicate that \( a \) and \( b \) strongly evolve with the cosmological redshift.

In Fig. 3 we plot the deviation of fit (\( x \); see Bevington & Robinson 1992; Li & Paczyński 2006) against the mean redshift of GRBs. There is not a clear trend for \( x \) to vary with \( \langle z \rangle \). However, it appears that the deviation of fit of each group is smaller than that of the whole sample.

Generalized, the variations of \( a \) and \( b \) are not independent – see e.g. Li & Paczyński (2006).
Variation of the Amati relation with redshift

L57

Figure 2. The fitted values of $a$ and $b$ against the mean redshift of GRBs. Each data point with error bars represents a group of GRBs (A, B, C and D from left to right). The solid lines are least-squares linear fits to $a-\langle z \rangle$ and $b-\langle z \rangle$.

Figure 3. The deviation of fit. Each circle corresponds to a group of GRBs (A, B, C and D, from left to right). The star represents the result obtained by fitting the whole sample (48 GRBs), which is $s=0.16$.

3 IS THE VARIATION CAUSED BY THE SELECTION EFFECT?

To check whether the variation of $a$ and $b$ with cosmological redshift is caused by the selection effect, we use Monte Carlo simulations to generate a sample of GRBs according to a pre-assumed Amati relation (1) and with a limit in the observed GRB fluence. Then we divide the sample into four groups by the GRB redshift and fit each group by equation (1), just as we did in Section 2.

The lower limit in the bolometric fluence, $F_{\text{bol, lim}}$, leads to a lower limit in the isotropic-equivalent energy of a detectable burst at redshift $z$:

$$E_{\text{iso, lim}} = 4\pi D_{\text{com}}^2 (1+z) F_{\text{bol, lim}},$$

(5)

where $D_{\text{com}}$ is the comoving distance to the burst.

In Fig. 4 (upper panel) we plot the isotropic energy of the 48 GRBs in the sample of Amati (2006, 2007) against their redshifts. The isotropic energy is clearly correlated with the redshift, with a Pearson linear correlation coefficient $r=0.437$ and a probability $P=0.0019$ for a zero correlation. The dashed line in the figure is the limit given by equation (5) with $F_{\text{bol, lim}}=1.2 \times 10^{-6} \text{ erg cm}^{-2}$, which reasonably well represents the selection effect.

The distribution of the redshifts of the GRBs in the sample is plotted in the lower panel of Fig. 4. It can be fitted by a log-normal distribution, with a mean $\mu=0.151$ and a dispersion $\sigma=0.332$ in $\log z$. The $\chi^2$ of the fit is 0.25. Then the frequency distribution in $\log z$ is

$$f_1(\log z) = \frac{1}{\sqrt{2\pi}\sigma} \exp \left[ -\frac{(\log z - \mu)^2}{2\sigma^2} \right],$$

(6)

the integration of which over $\log z$ (from $-\infty$ to $\infty$) is unity.

The distribution of the isotropic-equivalent energy is also described by a log-normal distribution, with a mean $= 1.09$ and a dispersion $= 0.85$ in $\log E_{\text{iso}}$ (in units of $10^{52}$ erg).

Define $x = \log E_{\text{iso}}$ and $y = \log E_{\text{peak}}$, where $E_{\text{iso}}$ is in units of $10^{52}$ erg and $E_{\text{peak}}$ is in keV. Assume that the Amati relation is valid and independent of the redshift, and for a given $x$ we have $y = mx + p$ with an intrinsic dispersion $\sigma^2$ in $y$. Then, for a given $x$, the Gaussian
distribution of $y$ is given by

$$f_2(y) = \frac{1}{\sqrt{2\pi} \sigma^2} \exp \left[ -\frac{(y - mx - p)^2}{2\sigma^2} \right]. \quad (7)$$

By our maximum likelihood fit results in Section 2, we take $m = 0.49$, $p = 2.00$ and $\sigma^2 = 0.14$.

The Monte Carlo simulation is done as follows. First, we generate $N$ redshifts with the distribution in equation (6). Then, at each redshift, we generate an isotropic-equivalent energy with a log-normal distribution (mean of log $E_{iso} = 1.09$, dispersion = 0.85) and satisfying $E_{iso} > E_{lim}$, Finally, for any pair of ($z, E_{iso}$), we generate a peak energy $E_{peak}$ according to the distribution in equation (7). Then we have a sample of $N$ GRBs, where each GRB has a redshift, a peak spectral energy and an isotropic-equivalent energy. These GRBs satisfy the distributions described above, and the select condition defined by equation (5) (with $F_{bol,lim} = 1.2 \times 10^{-4}$ erg cm$^{-2}$).

With the above approach, we generate $N = 4000$ GRBs. We divide them into four groups by redshift, and each group contains 1000 GRBs. Then we fit each group of GRBs by equation (1) and obtain the values of $a$ and $b$, and check the evolution of $a$ and $b$ with the mean redshift ($\langle z \rangle$). We repeat the process 10 000 times, each time with a different sample of 4000 GRBs. We find that $a$ and $b$ indeed vary with $\langle z \rangle$:

$$a = -3.57 + 0.105z, \quad b = 1.84 - 0.0347z. \quad (8)$$

This variation is caused by the selection effect, i.e. the limit in equation (5). If we turn off the limit, we find that $a$ and $b$ do not evolve with $\langle z \rangle$. However, comparing equation (8) to equations (3) and (4), we find that the selection effect is not likely to be the cause for the evolution in equations (3) and (4), since the $a$ and $b$ in equation (8) evolve too slowly with $z$. Even if we increase the value of $F_{bol,lim}$ to $10^{-3}$ erg cm$^{-2}$, we get $da/dz = 0.22$ and $db/dz = -0.065$, the values of which are still too small to explain the evolution in equations (3) and (4).

Of course, equation (8) only describes the average evolution of $a$ and $b$ for the 10 000 runs. For each run, the evolution may deviate from equation (8). To obtain the chance probability for the evolution in equations (3) and (4) to arise from the selection effect, we have used the Monte Carlo simulation described above to generate $N = 48$ GRBs and have repeated the process 500 times. For each 48 GRBs obtained in each run, we separate them into four groups and calculate $da/dz$ and $db/dz$ just as we did to the 10 000 runs of 4000 GRBs. The results are shown in Fig. 5.

Based on our simulations (500 runs of 48 GRBs), we find that the probability of getting a pair of $(da/dz, db/dz)$ with $da/dz > 0.43$ and $db/dz < -0.207$ is 0.04. Hence we have only a $\sim 4$ per cent chance that the variation presented in Section 2 is caused by the selection effect.

4 CONCLUSIONS

If GRBs do not evolve with redshift and selection effects are not important, we would expect that the Amati relation does not change with redshift. Hence, from the variation of the Amati relation with redshift, we may get some clues on the cosmic evolution of GRBs.

By dividing the 48 GRBs in Amati (2006, 2007) into four groups by their redshifts and fitting each group separately, we have found that the isotropic-equivalent energy and the peak spectral energy of GRBs remain correlated in each group, even with a smaller dispersion than that for the whole sample. However, the parameters $a$ and $b$ in the Amati relation (1) evolve strongly with redshift (equations 3 and 4).

Although the selection effect arising from the limit in the GRB fluence may cause a similar variation of $a$ and $b$ (equation 8), generally the variation is too slow to explain what we have found for the observed GRBs (equations 3 and 4). With Monte Carlo simulations we have shown that there is only a $\sim 4$ per cent chance that the observed variation is caused by the selection effect. Hence the variation of the Amati relation with redshift that we have discovered may reflect the cosmic evolution of GRBs and indicates that GRBs are not standard candles.

Our results are limited by the small number of GRBs in the sample: we have 48 GRBs in total, and only 12 GRBs in each group. To get a more reliable conclusion, the number of GRBs with well-determined redshifts and spectra needs to be significantly expanded. Since the launch of Swift, the fraction of GRBs with measured redshifts has increased rapidly. However, unfortunately, because of the narrow energy range of the Burst Alert Telescope (BAT) on Swift, the fraction of bursts that have accurately determined peak spectral energy and isotropic-equivalent energy has not increased proportionally. The Gamma-ray Large Area Space Telescope (GLAST) scheduled for launch in late 2007 holds more promise for this purpose (Omodei 2006).

We must also stress that our treatment of the selection effect has been greatly simplified. The GRBs in the sample were detected and measured by different instruments, hence the selection effect is much more complicated. A more careful consideration of the various selection biases is required to determine whether the observed evolution of the Amati relation reflects the cosmic evolution of GRBs.

No matter what the conclusion will be (whether the variation of parameters is caused by the GRB evolution effect or by the selection effect), our results suggest that it is very risky to use GRBs with...
redshifts spanning a large range as a single sample to draw conclusions on the physics by statistically analysing the correlations among observables. Although we have only tested the Amati relation, it would be surprising if any of the other relations (e.g. the Ghirlanda relation: Ghirlanda, Ghisellini & Lazzati 2004b) does not change with redshift.

ACKNOWLEDGMENTS

The author thanks the referee Dr P. O’Brien for a very helpful report. This Letter was based on a presentation by the author at the debate on ‘Through GRBs to $\Omega$ and $\Lambda’ during the conference ‘070228: The Next Decade of GRB Afterglows’ held in Amsterdam, 2007 March 19–23. The author acknowledges all the attendees at the debate for exciting and inspiring discussions.

REFERENCES

Amati L., 2006, MNRAS, 372, 233
Amati L., 2007, Il Nuovo Cim, C, in press (astro-ph/0611189)
Amati L. et al., 2002, A&A, 390, 81
Amati L., Della Valle M., Frontera F., Guidorzi C., Montanari E., Pian E., 2007, A&A, 463, 913
Band D., Preece R. D., 2005, ApJ, 627, 319
Bevington P. R., Robinson D. K., 1992, Data Reduction and Error Analysis for the Physical Sciences. McGraw–Hill, New York
Bloom J. S., Frail D. A., Kulkarni S. R., 2003, ApJ, 594, 674
D’Agostini G., 2005, preprint (physics/0511182)
Dai Z. G., Liang E. W., Xu D., 2004, ApJ, 612, L101
Firmani C., Avila-Reese V., Ghisellini G., Ghirlanda G., 2006, MNRAS, 372, L28
Friedman A. S., Bloom J. S., 2005, ApJ, 627, 1
Fruchter A. S. et al., 2006, Nat, 441, 463
Fynbo J. P. U. et al., 2003, A&A, 406, L63
Ghirlanda G., Ghisellini G., Lazzati D., Firmani C., 2004a, ApJ, 613, L13
Ghirlanda G., Ghisellini G., Lazzati D., 2004b, ApJ, 616, 331
Hjorth J. et al., 2003, ApJ, 597, 699
Kewley L., Kobulnicky H. A., 2005, in de Grijs R., Gonzalez Delgado R. M., eds, Starbursts: From 30 Doradus to Lyman Break Galaxies. Springer–Verlag, Berlin, p. 307
Lamb D. Q. et al., 2005, preprint (astro-ph/0507362)
Langer N., Norman C. A., 2006, ApJ, 638, L63
Le Floc’h E. et al., 2003, A&A, 400, 499
Li L.-X., Paczyński B., 2006, MNRAS, 366, 219
Omodei N., 2006, in Holt S. S., Gehrels N., Nousek J. A., eds, Gamma-Ray Bursts in the Swift Era. Am. Inst. Phys., NY, p. 642
Savaglio S. et al., 2005, ApJ, 635, 260
Schaefer B. E., 2003, ApJ, 583, L67
Schaefer B. E., 2007, ApJ, 660, 16
Sollerman J., Östlin G., Fynbo J. P. U., Hjorth J., Fruchter A., Pedersen K., 2005, New. Astron., 11, 103
Stanek K. Z. et al., 2006, Acta Astron., 56, 333
Teerikorpi P., 1984, A&A, 141, 407
Wolf C., Podsiadlowski P., 2007, MNRAS, 375, 1049

This paper has been typeset from a TeX/LaTeX file prepared by the author.