More loosely bound hadron molecules at CDF?

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In a recent paper we have proposed a method to estimate the prompt production cross section of \(X(3872)\) at the Tevatron making the assumption that this particle is a loosely bound molecule of a \(D\) and a \(D^*\), with a binding energy as small as \(0.25 \pm 0.40\) MeV. The motivation for this study is that, after the Belle discovery, CDF and D0 confirmed the \(X(3872)\) in proton-antiproton collisions \(p\bar{p}\) and it seems at odds with common intuition that such a loosely bound molecule could be produced promptly (i.e. not from \(B\) decay) in a high energy hadron collision environment. This was also one of the initial motivations to consider the possibility that the \(X(3872)\) could be, instead of a molecule, a 'point-like' hadron resulting from the binding of a diquark and an antidiquark \(D^0\bar{D}^0\) \(D^0\), following the interpretation proposed by Jaffe and Wilczek \(\bar{D}^0\bar{D}^0\) and pentaquark baryons (antidiquark-antidiquark-quark).

To start let us summarize the content of \[1\]. Let us suppose that \(X(3872)\) is an \(S\)-wave bound state of two \(D\) mesons, namely a \(1/\sqrt{2}(D^0\bar{D}^0 + \bar{D}^0D^0)\) molecule (we will use the shorthand notation \(D^0\bar{D}^0\)) \(^1\). The molecule production cross section will be proportional to the number of \(D^0\bar{D}^0\) pairs in the event. Thus the \(X(3872)\) prompt production cross section at the Tevatron could be written as:

\[
\sigma(p\bar{p} \rightarrow X(3872)) \sim \left| \int d^3k \langle X|D\bar{D}^*(k)\rangle\langle D\bar{D}^*(k)|p\bar{p}\rangle^2 \right| \leq \int_R d^3k |\psi(k)|^2 d^3k |\langle D\bar{D}^*(k)|p\bar{p}\rangle|^2 \leq \int_R d^3k |\langle D\bar{D}^*(k)|p\bar{p}\rangle|^2 \sim \sigma(p\bar{p} \rightarrow X(3872))^{\text{max}} \tag{1}
\]

where \(k\) is the relative 3-momentum between the \(D(p_1), D^*(p_2)\) mesons. \(\psi(k) = \langle X|D\bar{D}^*(k)\rangle\) is some normalized bound state wave function characterizing the \(X(3872)\). \(R\) is the integration region where \(\psi(k)\) is significantly different from zero. The matrix element \(\langle D\bar{D}^*(k)|p\bar{p}\rangle\) can be computed using standard matrix-element/hadronization Monte Carlo programs (MC) like Herwig \[9\] and Pythia \[12\] \(^2\). We require our MC tools to generate \(2 \rightarrow 2\) QCD events with some loose partonic generation cuts as detailed in \[1\] \(^3\).

As for the determination of the region \(R\) in \[1\] we estimate it having in mind a naive gaussian ansatz for the bound state wave function. It is straightforward to estimate the momentum spread of the gaussian by

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\(^1\) Such a molecule has the correct \(1^{++}\) quantum numbers of the \(X(3872)\).
\(^2\) Open charm meson pairs generated with hadronization Monte Carlo are ordered as a function of their relative center-of-mass 3-momenta. If more than one \(D^0\bar{D}^0\) pair is found in the event, we select the pair having the smaller relative 3-momentum \(k\). As a first step we select those pairs which pass the kinematical cuts used in the data analysis made by the CDF collaboration.
\(^3\) Configurations with one gluon recoiling from a \(c\bar{c}\) pair, are those configuration expected to produce two collinear charm quarks and in turn collinear open charm mesons. The parton shower algorithms in Herwig and Pythia treat properly these configurations at low \(p_\perp\) whereas they are expected to be less important at higher \(p_\perp\).
assuming a strong interaction Yukawa potential between the two $D$ mesons. Given that the binding energy $\mathcal{E}_0$ is $\mathcal{E}_0 \sim M_X - M_D - M_{D^*} = -0.25 \pm 0.40$ MeV we find that $r_0 \sim 8$ fm ($8.6 \pm 1.1$ fm) and applying the (minimal) uncertainty principle relation, we get the gaussian momentum spread $\Delta p \sim 12$ MeV.

Given the very small binding energy we can estimate $k$ to be as large as $k \simeq \sqrt{2\mu(-0.25+0.40)} \simeq 17$ MeV, $\mu$ being the reduced mass of $D^0D^{*0}$ system, or of the order of the center of mass momentum $k = \sqrt{\lambda(m_X^2, m_{D^*}^2, m_{D}^2)/2m_X} \simeq 27$ MeV. These considerations imply that we can restrict the integration region to a ball $\mathcal{R}$ of radius $4 \sim [0, 35]$ MeV.

Keeping $k$ inside $\mathcal{R}$ we estimate a $\sigma(pp \to X(3872))$ which is about 30 times smaller than the most conservative estimate ($\sigma \sim 3.2$ nb) of the minimal prompt production cross section measured at CDF [1]. In the analysis proposed in [11] the experimental cross section is estimated from data to be $\sigma \sim 30 \div 70$ nb reinforcing the negative result obtained with our theoretical calculation that would rather be 300 times smaller than the measured one. This fact would undoubtedly put in serious trouble the molecular interpretation of $X(3872)$.

In this note we intend to start from the main result discussed in [11] where it is argued that the effect of final state interactions in the $D^0D^{*0}$ system is such that two corrections should be made to our previous calculation: i) the ball $\mathcal{R}$ should be enlarged to include momenta up to $\Lambda \sim 300$ MeV; ii) a correction factor to the cross section we compute (see (1)) should be considered so that the actual cross section $\sigma^*$ including the full effect of final state interaction is

$$\sigma^*(pp \to X(3872)) = \sigma(k < \Lambda) \times \frac{6\pi\sqrt{2\mu|\mathcal{E}_0|}}{\Lambda}$$

Assuming that this is the correct way of discussing the $X(3872)$ production we observe that, besides reconciling the experimental result with the theoretical computation for the $X$, this mechanism should enhance the occurrence of an hypothetical new molecule, the $D^*D^{*0}$, which otherwise would be suppressed as one could infer by looking at data on $D_s$ production at Tevatron [14] (as shown in Fig. 1, $D_s$ is on average $\sim 5$ times less probable than $D^0$). The same data are used to tune our Monte Carlo (Herwig in this calculation) with respect to $D_s$ production as shown in Fig. 1.

![FIG. 1: Differential charm cross section measured in fully hadronic charm decays using 5.8 pb$^{-1}$ at CDF [14]. The error bars represent the total uncertainty of the measurement. The ratio in the production of $D^0$ and $D_s$ is 4.4 whereas the ratios in the production of $D^*$ and $D_s$ is about 2.2. The dot-dashed lines are the result of the Monte Carlo simulation done with Herwig rescaling the normalizations of the distributions by a factor $K = 1.5$. This value is in very good agreement with the $K$ factor found in [1] ($K = 1.8$) using data on $d\sigma/d\Delta\phi$, $\Delta\phi$ being the angle between $D^0$ and $D^{*-}$ mesons produced at CDF within some definite cuts in rapidity and transverse momentum.](image)

The $X_s(1^{++})$ molecule should exist as a partner with strange light quarks of the $X(3872)$. One could expect it to be a more compact molecule with respect to the $X(3872)$ as $\eta$ particle exchange forces would be at work. This enlarges the spread $\Delta p$ in the relative momentum and naturally makes the ball $\mathcal{R}$ larger. We will postulate a binding energy for $X_s$ as small as that of the $X(3872)$ and its mass is expected to be $M_{X_s} = 4080$ MeV. $X_s$ could decay into $J/\psi \pi\pi$ with a narrow width because of its mass and flavor content (it cannot decay into $K^+K^-J/\psi$ (via $\phi$) because of phase space; it cannot either decay to $J/\psi f_0(980)$ because of quantum numbers), or in $D_sD_{s}\gamma$. Using the Herwig hadronization algorithm to compute $\sigma(k < \Lambda)$ in (2) we obtain

$$\sigma^*(pp \to X_s(4080)) = 1 \div 3 \text{ nb}$$

4 Which corresponds to a $k_0$ of the Gaussian at $\sim 27$ MeV and a spread of $+12$ MeV.
where the value of 3 nb is found pushing the Λ value up to 600 MeV (following some considerations on the possible values of the Λ cutoff made in [11]). We obtain definitely similar results using Pythia [12].

Such numbers should put the $X_s(4080)$ molecule in the conditions to be observed at CDF. We would find rather surprising that no such state is found assuming that the mechanism (2) is correct thus we encourage searches of this resonance.

On the other hand we cast some doubts on the possibility that final state interactions can indeed play such a pivotal role as described in [11]. First of all we remind that Watson formulae [13] used in [11] are valid for $S$-wave scattering, whereas a relative three-momentum $k$ of 300 MeV indicates that higher partial waves should be taken into account.

Most importantly, we have verified in our MC simulations that as the relative momentum $k$ in the center of mass of the molecule is taken to be up to 300 MeV, then other hadrons (on average more than two) have a relative momentum $k < 100$ MeV with the $D$ or the $D^*$ constituting the molecule (see Fig. 2). On the other hand the Migdal-Watson theorem for final state interactions requires that only two particles in the final state participate to the strong interactions causing them to rescatter. In other words the extra hadrons involved in the process do necessarily interfere in an unknown way with the mesons assumed to rescatter into an $X(3872)$. This is particularly true as one further enlarges the dimensions of the momentum ball $R$ as required in [11].

Tetraquarks with a $[cs][\bar{c}\bar{s}]$ might also occur, and one expects the lightest of this family to be a scalar at about 3930 MeV, as estimated in [15]. Computing the prompt production cross section is an harder task though. This would require some specific model for the fragmentation of partons into diquarks allowing to extract from data a ratio of the production rate of $[cs]$ and $[cq]$ diquarks. In turn this would allow, for example, to estimate the prompt production cross section of the $X_s$ under the hypothesis that the $X(3872)$ produced at CDF is a tetraquark. A simple model of parton to diquark fragmentation could be drawn along the lines discussed in [16] where the case of light diquarks was treated. Yet we prefer to postpone such estimate as soon as the first data on exotic hadron production will be available from LHCb and ALICE.

In this note we show that starting from the results discussed in [11] we should expect an enhancement in the prompt production cross section of an hypothetical new $X_s(4080)$ molecular loosely bound resonance constituted by a $D_sD^*_s$ pair. We estimate such cross section to be between 1 and 3 nb at the Tevatron. On the other hand we cast some doubts on the applicability of the Watson theorem for final state interactions in the calculation at hand. We show that in the hadronization shower the number of hadrons in a momentum volume $R(k)$ tends to grow with $k$ whereas the final state interactions formulae used in [11] (see [13]) should involve only two hadrons at a time.

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