Learning with Heterogeneous Side Information Fusion for Recommender Systems

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Collaborative filtering (CF) based methods have become the most popular technique for recommender systems (RSs). In recent years, various types of side information such as social connections among users and metadata of items have been introduced into CF and shown to be effective for improving recommendation performance. Moreover, side information can alleviate data sparsity and cold start problems facing conventional CF based methods. However, previous works process different types of information separately, thus losing information that might exist across different types of side information. In this work, we study the application of Heterogeneous Information Network (HIN), which offers flexible representation of different types of side information, to enhance CF based recommendation methods. Since HIN could be a complex graph representing multiple types of relations between entity types, we need to tackle two challenging issues facing HIN-based RSs: How to capture the complex semantics that determines the similarities between users and items in a HIN, and how to fuse the heterogeneous side information to support recommendation. To address these issues, we apply metagraph to HIN-based RSs and solve the information fusion problem with a “matrix factorization (MF) + factorization machine (FM)” framework. For the MF part, we obtain the user-item similarity matrix from each metagraph and then apply low-rank matrix approximation to obtain latent features for both users and items. For the FM part, we apply FM with Group lasso (FMG) on the features obtained from the MF part to train the recommending model and at the same time identify the usefulness of the metagraphs. Experimental results on two large real-world datasets, i.e., Amazon and Yelp, show that our proposed approach is better than FM and other state-of-the-art HIN-based recommendation methods.

Additional Key Words and Phrases: Recommender Systems, Collaborative filtering, Heterogeneous Information Networks, Matrix Factorization, Factorization Machine

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1 INTRODUCTION

In the big data era, people are overwhelmed by the huge amount of information on the Internet, making recommender systems (RSs) an indispensable tool for getting interesting information. Collaborative filtering (CF) has been the most popular recommendation method in the last decade [Herlocker et al. 1999; Koren 2008], which tries to predict a user’s preferences based on users who are similar to him/her. In recent years, researchers try to incorporate auxiliary information, or side information, to enhance CF. For example, social connections among users [Ma et al. 2011; Xiao et al. 2019; Zhao et al. 2017a], reviews of items [Ling et al. 2014; McAuley and Leskovec 2013], metadata attached to commodities [Wang et al. 2018], or locations of users and items [Ye et al. 2011; Zheng et al. 2012], have been shown to be effective for improving recommendation performance.

However, a major limitation of most existing methods is that various types of side information are processed independently [Haghighat et al. 2016], leading to information loss across different types of side information. This limitation becomes more and more severe, because modern websites record rich side information about their users and contents [Pan 2016] and it would be a huge loss to their business if the side information cannot be fully utilized to improve performance. For example, on Yelp (https://www.yelp.com/), a website recommendation business to users, users can follow other users to form a social network, businesses have categories and locations, and users can write reviews on businesses. If each type of side information is processed in isolation, information that exists across different types of side information will be neglected. Therefore, a unifying framework is needed to fuse all side information for producing effective recommendations.

Heterogeneous information networks (HINs) [Shi et al. 2017; Sun et al. 2011] have been proposed as a general data representation tool for different types of information, such as scholar network [Sun et al. 2011] and knowledge graph [Wang et al. 2015a]. Thus, it can also be used to model rich side information for RSs [Shi et al. 2015; Yu et al. 2014]. Figure 1 shows an example HIN on Yelp, and Figure 2 shows a network schema defined over the entity types User, Review, Aspect, Business, etc. Based on the network schema, we can design metapaths [Shi et al. 2017; Sun et al. 2011], which are sequences of node types, to compute the similarities between users and businesses for generating recommendations. For example, we can define complicated metapaths such as User → Review → Aspect → Review → User → Business, to measure similarities between user and business based on similar reviews written by users about the same aspect. In summary, we can unify rich side information with HIN and design metapaths to compute user-item similarities induced from different semantics for making effective recommendation.

There are two major issues facing existing HIN-based RSs. The first issue is that metapaths are not enough for representing the rich semantics for HIN-based RSs. We refer to it as semantic limitation. Figure 1 shows a concrete example, where the metapath User → Review → Aspect → Review → User → Business is used to capture users’ similarity since both users write reviews and mention the same aspect (seafood) in the review texts. However, if we want to capture the similarity induced by the two users’ reviews mentioning the same aspect (such as seafood) and ratings on the same business (such as Royal House), then metapath is not able to capture this semantic. Thus, we need a better way to capture such complicated semantics. Recently, Huang et al. [Huang et al. 2016] and Fang et al. [Fang et al. 2016] proposed to use metagraph (or meta-structure) for computing similarity between homogeneous types of entities (e.g., using Person to search Person) over HINs, which is more powerful than metapath in capturing complex semantics. However, they did not
explore metagraphs for entities of heterogeneous types, which are essential for RSs. In this paper, we extend metagraph to capture similarities of complex semantics between users and items (businesses) in recommendation scenarios.

The second issue is about similarity fusion, i.e., how to fuse the similarities produced by different semantics between users and items for HIN-based RSs. Our goal to achieve accurate predictions of the users’ ratings on items can be formulated as a matrix completion problem on the user-item rating matrix. There are two principled ways to do so. One way to predict missing ratings for HIN is to compute a similarity between users and items based on each metapath, and then learn a weighing mechanism to explicitly combine the similarities from different metapaths to approximate the user-item rating matrix [Shi et al. 2015]. This approach and does not utilize latent features derivable from a metapath. Thus, the similarity matrix could be too sparse to contribute to the final ensemble. The other way is to first factorize each user-item similarity matrix to obtain user and item latent features, and then use all latent features to recover the user-item rating matrix [Yu et al. 2014]. This method solves the sparsity problem associated with each similarity matrix. However, it does not fully utilize the latent features because it ignores the interactions among the latent features from different metapaths and captures only linear interactions among the latent features. Therefore, existing HIN-based recommendation methods [Shi et al. 2015; Yu et al. 2014] suffer from information loss in various ways.

Fig. 1. An example of HIN built for Royal House on Yelp.

Fig. 2. The Network Schema for the HIN in Figure 1. A: aspect extracted from reviews; R: reviews; U: users; B: business; Cat: category of business; Ci: city.
To address the above challenges, we propose a new systematic way to fuse rich side information in HIN for recommendation. First, instead of using metapaths for recommendation [Shi et al. 2015; Yu et al. 2014], we introduce the concept of metagraph to the recommendation problem, which allows us to incorporate more complex semantics into HIN-based RSs. Second, instead of computing the recovered matrices directly from the metagraphs, we utilize the latent features from all metagraphs. Based on matrix factorization (MF) [Koren 2008; Mnih and Salakhutdinov 2007] and factorization machine (FM) [Rendle 2012], we propose a “MF + FM” framework for our metagraph based RS in HIN. We first compute the user-item similarity matrix from each metagraph and then factorize the similarity matrix to obtain a set of user and item vectors representing the latent features of users and items. Finally, after obtaining sets of user and item latent features from the metagraphs, we use FM to assemble them to predict the missing ratings that users give to the items. This method enables us to capture nonlinear interactions among all of the latent features, which has been demonstrated to be effective in MF-based RS [Rendle 2012]. To further improve the performance of the “MF+FM” framework, we propose to use group lasso [Jacob et al. 2009] with FM (denote as FMG) to learn the parameters for selecting metagraphs that contribute most to recommendation effectiveness. As a result, we can automatically determine for different applications which metagraphs are the most effective, and for each group of user and item features from a metagraph, how the features should be weighed. Experimental results on two large real-world datasets, Amazon and Yelp, show that our framework significantly outperforms recommendation methods that are solely based on MF, FM, or meta-path in HIN.

Preliminary results of this manuscript have been reported in [Zhao et al. 2017b] published in KDD, where MF is designed to extract latent features from metagraphs, and then FMG is proposed for metagraphs fusion and selection. In this full version, we propose and highlight the systematic framework “MF + FM” (see Figure 3), which provides a complete solution for fusing various side information for RS in a HIN. For the best of our knowledge, upon the publishing of the KDD version [Zhao et al. 2017b], this is the first study on comprehensive exploring of HIN-based RSs. In this manuscript, we conduct extensive research, respectively, on recommendation strategy designing (metagraph construction), feature extraction, fusion and selection, which constitutes a novel and effective paradigm not only for RS, but also for other HIN-based applications. Hence, the contributions of this work lie in broader domains. To be specific, besides proving hand-crafted metagraphs in the experiments, we give practical guidelines about how to design metagraphs for RSs in Section 4.1. Then for feature extraction, we propose a more effective method based on nuclear norm regularization (NNR) to extract metagraph based latent features in Section 4.3.2. Furthermore, for feature selection, a novel nonconvex variant of group lasso regularization [Candès et al. 2008] is designed to improve metagraph selection performance in Section 5.2.2. It leads to a difficult and challenging optimization problem by integrating all above components together, thus in Section 5.3.2 we design another effective and efficient solver based on stochastic variance reduced gradient (SVRG) [Xiao and Zhang 2014] compared the proposed nmAPG solver in KDD version [Zhao et al. 2017b]. Finally, additional experiments are performed, respectively, to support the increased components in other RS scenarios and other HIN-based prediction problems in Section 6.9. Finally, we give practical suggestions to apply our framework to other RS scenarios and other HIN-based prediction problems in Section 7. Our code is available at https://github.com/HKUST-KnowComp/FMG.

Notation. We denote vectors and matrices by lowercase and uppercase boldface letters, respectively. In this paper, a vector always denote row vector. For a vector $x$, $\|x\|_2 = (\sum_{i=1}^n |x_i|^2)^{1/2}$ is its $\ell_2$-norm. For a matrix $X$, its nuclear norm is $\|X\|_* = \sum_i \sigma_i(X)$, where $\sigma_i(X)$’s are the singular values of $X$; $\|X\|_F = (\sum_{i,j} X_{ij}^2)^{1/2}$ is its Frobenius norm and $\|X\|_1 = \sum_{i,j} |X_{ij}|$ is its $\ell_1$-norm. For two matrices $X$
and \( Y, \langle X, Y \rangle = \sum_{i,j} X_{ij} Y_{ij} \) and \([X \odot Y]_{ij} = X_{ij} Y_{ij}\) denotes the element-wise multiplication. For a smooth function \( f, \nabla f(x) \) is its gradient at \( x \).

2 RELATED WORK

In this section, we review existing works related to HIN, RS with side information, and FM.

2.1 Heterogeneous Information Networks (HINs)

HINs have been proposed as a general representation for many real-world graphs or networks [Joshua 2012; Kong et al. 2013b; Shi et al. 2017; Sun and Han 2013; Sun et al. 2011].

A metapath is a sequence of entity types defined by the HIN network schema. Based on metapath, several similarity measures, such as PathCount [Sun et al. 2011], PathSim [Sun et al. 2011], and PCRW [Lao and Cohen 2010] have been proposed, and research has shown that they are useful for entity search and as similarity measure in many real-world networks. After the development of metapath, many data mining tasks have been enabled or enhanced, including recommendation [Shi et al. 2015; Yu et al. 2013, 2014], similarity search [Shi et al. 2014; Sun et al. 2011], clustering [Shi et al. 2013; Wang et al. 2015a], classification [Jiang et al. 2017; Kong et al. 2013a; Wang et al. 2017, 2015b], link prediction [Sun et al. 2012; Zhang et al. 2014], malware detection [Fan et al. 2018a; Hou et al. 2017], and opioid user detection [Fan et al. 2018b].

Recently, metagraph (or meta-structure) has been proposed for capturing complicated semantics in HIN that metapath cannot handle [Fang et al. 2019, 2016; Huang et al. 2016]. However, in existing research, metagraph is limited to entity similarity problems where entities have the same type. In this paper, we extend metagraph to the recommendation problem, where we need to compute the similarity between heterogeneous types of entities, i.e., users and items.

2.2 Recommendation with Heterogeneous Side Information

Modern recommender systems are able to capture rich side information such as social connections among users and metadata and reviews associated with items. Previous works have explored different methods to incorporate heterogeneous side information to enhance CF based recommender systems. For example, [Ma et al. 2011] and [Zhao et al. 2017a], respectively, incorporate social relations into low-rank and local low-rank matrix factorization to improve the recommendation performance, and heterogeneous item relations are explored for recommendation in [Kang et al. 2018]. In [Ling et al. 2014; McAuley and Leskovec 2013], review texts are analyzed together with ratings in the rating prediction task. [Ye et al. 2011] proposed a probabilistic model to incorporate users’ preferences, social network and geographical information to enhance point-of-interests recommendation. In [Wang et al. 2019], knowledge graph is used to enhance the item representation in textual content recommendation. In [Xiao et al. 2019], social connections and textual features are processed together in a deep learning framework for content recommendation. [Zheng et al. 2012] proposed to integrate users’ location data with historical data to improve the performance of point-of-interest recommendation. [Wang et al. 2018] proposed a graph embedding based methods to incorporate side informations of commodities to improve the recommendation performance of e-commerce systems. These previous approaches have demonstrated the importance and effectiveness of heterogeneous information in improving recommendation accuracy. However, most of these approaches deal with different heterogeneous information separately, hence losing important information that might exist across the information sources.

HIN-based recommendation has been proposed to avoid the disparate treatment of different types of information. Based on metapath, several approaches have attempted to tackle the recommendation task based on HIN. In [Yu et al. 2013], metapath based similarities are used as regularization terms in matrix factorization. In [Yu et al. 2014], multiple metapaths are used to
learn user and item latent features, which are then used to recover similarity matrices combined by a weighted mechanism. In [Shi et al. 2015], users’ ratings to items are used to build a weighted HIN, based on which metapath based methods are used to measure the similarities of users for recommendation. The combination of different metapaths are explicit, using the similarities instead of latent features. In [Shi et al. 2018], metapath based embedding is utilized for HIN-based RS, and the authors further utilize deep learning in HIN for recommendation [Han et al. 2018; Hu et al. 2018]. However, existing HIN-based methods are all relying on metapath, thus failing to capture complex semantics underlying the similarities between users and items. And these approaches do not make full use of the metapath based features, whereas our framework based on “MF + FM” aims to accomplish. Besides, these methods cannot do the metagraph selection either. In one word, we propose a comprehensive and powerful pipeline for HIN-based RSs by more effectively fusing heterogeneous side information. Moreover, it can be easily adapted for other HIN-based problems.

2.3 Factorization Machine (FM)  
FM [Rendle 2012] is a popular and powerful recommendation framework, which can model non-linear interactions among features, e.g., the rating information, categories of items, texts, time. Many approaches and systems have been developed based on FM [Hong et al. 2013; Rendle and Schmidt-Thieme 2010]. Different from previous approaches which only consider explicit features, we generate latent features by low-rank approximation on similarity matrices generated from different metagraphs. Moreover, our framework can do feature selection in groups (corresponding to metagraphs) automatically. In [Yan et al. 2014], coupled group lasso is proposed to select by one row or column from second-order weight matrix in FM, while in this framework, our framework can select by block from second-order weight matrix in FM. Moreover, in this paper, we are the first to adopt nonconvex regularization for feature selection in FM.

3 “MF + FM” FRAMEWORK  
The proposed framework is illustrated in Figure 3. The input to the MF part is a HIN, e.g., the one in Figure 1. To solve the semantic limitation issue, we design metagraphs instead of metapaths to capture complex semantics that exists between users and items in a HIN, e.g., those in Figure 4 and 5. Let there be L metagraphs. The MF part, introduced in Section 4, computes from the L metagraphs L user-item similarity matrices, denoted by $R^1, R^2, \ldots, R^L$. Since these similarity matrices tend to be very sparse, we apply low-rank matrix approximation on similarity matrices generated from two low-dimension matrices, representing the latent features of users and items. The output of the MF part is the L groups of latent features for users and items. Since existing methods only compute metapath based similarities, we design a new algorithm to compute the user-item similarities from metagraphs.

The objective of the MF part is to utilize the latent features to learn a recommendation model that is more effective than previous HIN-based RSs. This addresses the similarity fusion issue. FMG (see Section 5) has two advantages over previous methods: 1) FM can capture non-linear interactions among features [Rendle 2012], which is more effective than linear ensemble model adopted in previous HIN-based RS [Yu et al. 2014], 2) by introducing group lasso regularization, we can automatically select the useful features and in turn the useful metagraphs for a recommendation application, avoiding laborious feature and metagraph engineering when a new HIN is encountered. Specifically, for a user-item pair, user $u_i$ and item $b_j$, we first concatenate the latent features $u^1_i, u^2_i, \ldots, u^L_i$ and $b^1_j, b^2_j, \ldots, b^L_j$ from all of the metagraphs to create a feature vector, using rating $R_{ij}$ as label. We then train our FMG model with group lasso regularization method to select the useful features in the groups, where each group corresponds to one metagraph. The selected features are in grey in Figure 3. Finally, to efficiently train FMG, we propose two algorithms, one is
Fig. 3. The proposed "MF + FM" framework. In the MF part, latent features are extracted from user-item similarity matrices derived from metagraphs on a HIN (e.g., Figure 1). In the FM part, latent features are concatenated and then fed into FMG to predict missing ratings. In the bottom, the features in grey are selected by group lasso regularizers.

Based on the proximal gradient algorithm [Parikh and Boyd 2014] and the other on the stochastic variance reduced gradient algorithm [Xiao and Zhang 2014] (see Section 5.3).

**Remark 3.1.** The main contribution of this paper is to solve the information fusion problem in HIN by the proposed “MF + FM” framework. More importantly, the designed pipeline and methods can be applied to RSs as well as other HIN-based problems, e.g., intent recommendation [Fan et al. 2019], fraud detection [Hu et al. 2019], malware detection in software systems [Fan et al. 2018a; Hou et al. 2017], opioid user detection [Fan et al. 2018b], or medical diagnosis [Hosseini et al. 2018]. Through this paper, we also give practical suggestions about how to apply our framework to existing RS or other HIN-based problems, thus we believe the proposed framework has practical values in broader application domains.

4 METAGRAPH CONSTRUCTION AND FEATURE EXTRACTION

In this section, we elaborate on the MF part for metagraph based feature extraction. In Section 4.1, we introduce the method for constructing metagraphs in HIN. Then, we show how to compute the
user-item similarity matrices in Section 4.2. Finally, in Section 4.3, we obtain latent features from the user-item matrices using MF-based approaches. The main novelty of our approach is the design of the MF part, which extracts and combines latent features from each metagraph before they are fed to the FM part. Further, as existing methods are only for computing similarity matrices based on metapaths, we show how similarity can be computer for metagraphs.

4.1 Construction of Metagraphs

We first give the definitions of HIN, Network Schema for HIN, and Metagraph [Fang et al. 2019, 2016; Huang et al. 2016; Sun et al. 2011]. Then, we introduce how to compute metagraph based similarities between users and items in a HIN.

**Definition 1 (Heterogeneous Information Network).** A heterogeneous information network (HIN) is a graph $G = (V, E)$ with an entity type mapping $\phi: V \rightarrow A$ and a relation type mapping $\psi: E \rightarrow R$, where $V$ denotes the entity set, $E$ denotes the link set, $A$ denotes the entity type set, and $R$ denotes the relation type set, and the number of entity types $|A| > 1$ or the number of relation types $|R| > 1$.

**Definition 2 (Network Schema).** Given a HIN $G = (V, E)$ with the entity type mapping $\phi: V \rightarrow A$ and the relation type mapping $\psi: E \rightarrow R$, the network schema for network $G$, denoted by $T_G = (A, R)$, is a graph, in which nodes are entity types from $A$ and edges are relation types from $R$.

In Figures 1 and 2, we show, respectively, an example of HIN and its network schema from the Yelp dataset. We can see that we have different types of nodes, e.g., User, Review, Restaurant, and different types of relations, e.g., Write, CheckIn. The network schema defines the relations between node types, e.g., User $\xrightarrow{\text{CheckIn}}$ Restaurant, Restaurant $\xrightarrow{\text{LocateIn}}$ City. Thus, we can see that HIN is a flexible way for representing various information in an unified manner. The definition of metagraph is given below.

**Definition 3 (Metagraph).** A metagraph $M$ is a directed acyclic graph (DAG) with a single source node $n_s$ (i.e., with in-degree 0) and a single sink (target) node $n_t$ (i.e., with out-degree 0), defined on a HIN $G = (V, E)$. Formally, $M = (V_M, E_M, A_M, R_M, n_s, n_t)$, where $V_M \subseteq V$ and $E_M \subseteq E$ are constrained by $A_M \subseteq A$ and $R_M \subseteq R$, respectively.

As introduced above [Fang et al. 2019, 2016; Huang et al. 2016], compared to metapath, metagraph can capture more complex semantics underlying the similarities between users and items. In fact, metapath is a special case of metagraph. Thus, in this paper, we introduce the concept of metagraph for HIN-based RS. In Figures 4 and 5, we show the metagraphs on Yelp and Amazon datasets, respectively, used in our experiments. In these figures, $R^{-1}$ represents the reverse relation of $R$.

For example, for $M_3$ in Figure 4, $B \xrightarrow{\text{CheckIn}^{-1}} U$ means $U$ checks in a business $B$. From Figure 4 and 5, we can see that each metagraph has only one source ($U$) and one target ($B$) node, representing a user and an item in the recommendation scenario.

Since there could be many metagraphs in a HIN and they are not equally effective, we give three guidelines for the selection of metagraphs: 1) All metagraphs designed are from the network schema. 2) Domain knowledge is helpful in the selection of good metagraphs because some metagraphs correspond to traditional recommendation strategies that have been proven to be effective [Shi et al. 2015; Yu et al. 2014]. For example, $M_2$ and $M_3$ in Figure 4, respectively, represent social recommendation and the well-known user-based CF. In practice, an understanding of existing recommendation strategies and application semantics is essential for the design of good metagraphs; 3) It is better to construct shorter metagraphs. In [Sun et al. 2011], the authors have shown that
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Fig. 4. Metagraphs used for the Yelp dataset (Ca: Category; Ci: City; St: State; Sr: Star, the average number of stars a business obtained).

Fig. 5. Metagraphs used for the Amazon dataset (Ca: Category; Br: Brand of the item).

Longer metapaths decrease the performance because they tend to have more noises. This result is applicable to metagraphs as well.
4.2 Computation of Similarity Matrices

We use $M_3$ and $M_0$ in Figure 4 to illustrate the computation of metagraph based similarities. In previous work, commuting matrices [Sun et al. 2011] have been employed to compute the count-based similarity matrix of a metapath. Suppose we have a metapath $\mathcal{P} = (A_1, A_2, \ldots, A_l)$, where $A_i$’s are node types in $\mathcal{A}$ and denote the adjacency matrix between type $A_i$ and type $A_j$ by $W_{A_i,A_j}$.

Then the commuting matrix for $\mathcal{P}$ is defined by the multiplication of a sequence of adjacency matrices:

$$C_P = W_{A_1,A_2}W_{A_2,A_3} \cdots W_{A_{l-1},A_l},$$

where $C_P(i,j)$, the entry in the $i$-th row and $j$-th column, represents the number of path instances between object $x_i \in A_1$ and object $x_j \in A_l$ under $\mathcal{P}$. For example, for $M_3$ in Figure 4, $C_{M_3} = W_{UB}W^\top_{UB}W_{UB}$, where $W_{UB}$ is the adjacency matrix between type $U$ and type $B$, and $C_{M_3}(i,j)$ represents the number of instances of $M_3$ between user $u_i$ and item $b_j$. In this paper, for a metagraph $M$, the similarity between a source object and a target object is defined as the number of instances of $M$ connecting the source and target objects. In the remainder of this paper, we adopt the term similarity matrix instead of commuting matrix for clarity.

From the above introduction, we can see that metapath based similarity matrix is easy to compute. However, for metagraphs, the problem is more complicated. For example, consider $M_0$ in Figure 4, there are two ways to pass through the metagraph, which are $(U, R, A, R, U, B)$ and $(U, R, B, R, U, B)$. Note that $R$ represents the entity type $\textit{Review}$ in HIN. In the path $(U, R, A, R, U, B)$, $(R, A, R)$ means if two reviews mention the same $A$ ($\textit{Aspect}$), then they have some similarity. Similarly, in $(U, R, B, R, U, B)$, $(R, B, R)$ means if two reviews rate the same $B$ ($\textit{Business}$), they have some similarity too. We should decide how similarity should be defined when there are multiple ways to pass through the metagraph from the source node to the target node. We can require a flow to pass through either path or both paths in order to be considered in similarity computation. The former strategy simply splits a metagraph into multiple metapaths, thus suffering from information loss. Therefore, we adopt the latter, but it requires one more matrix operation in addition to simple multiplication, i.e., element-wise product. Algorithm 1 depicts the algorithm for computing count-based similarity based on $M_0$ in Figure 4. After obtaining $C_{S_0}$, we can get the whole similarity matrix $C_{M_0}$ by multiplying the sequence of matrices along $C_{M_0}$. In practice, not limited to $M_0$ in Figure 4, the metagraph defined in this paper can be computed by two operations (Hadamard product and multiplication) on the corresponding matrices.

**Algorithm 1** Computing similarity matrix based on $M_0$.

1. Compute $C_{P_1} : C_{P_1} = W_{RB}W^\top_{RB}$.
2. Compute $C_{P_2} : C_{P_2} = W_{RA}W^\top_{RA}$.
3. Compute $C_{S_r} : C_{S_r} = C_{P_1} \odot C_{P_2}$.
4. Compute $C_{M_0} : C_{M_0} = W_{UR}C_{S_r}W^\top_{UR}W_{UB}$.

By computing the similarities between all users and items for the $l$-th metagraph $M$, we can obtain a user-item similarity matrix $R^l \in \mathbb{R}^{m \times n}$, where $R^l_{ij}$ represents the similarity between user $u_i$ and item $b_j$ along the metagraph, and $m$ and $n$ are the number of users and items, respectively. Note that $R^l_{ij} = C_{M_l}(i,j)$ \footnote{To maintain consistency with the remaining sections, we change the notation $C$ into $R$.} if $C_{M_l}(i,j) > 0$ and 0 otherwise. By designing $L$ metagraphs, we can get $L$ different user-item similarity matrices, denoted by $R^1, \ldots, R^L$.
4.3 Latent Feature Generation

In this part, we elaborate on how to generate latent features for users and items from the $L$ user-item similarity matrices. Since the similarity matrices are usually very sparse, using the matrices directly as features will lead to the high-dimensional learning problem, resulting in overfitting. Motivated by the success of low-rank matrix completion for RSs [Candès and Recht 2009; Koren 2008; Mnih and Salakhutdinov 2007], we propose to generate latent features using matrix completion methods.

Specifically, the nonzero elements in a similarity matrix are treated as observations and the others are taken as missing values. Then we find a low-rank approximation to this matrix. Matrix factorization (MF) [Koren 2008; Mnih and Salakhutdinov 2007] and nuclear norm regularization (NNR) [Candès and Recht 2009] are two popular approaches for matrix completion. Generally, MF leads to nonconvex optimization problems, while NNR leads to convex optimization problems. NNR is easier to optimize and has better theoretical guarantee on the recovery performance than MF. Empirically, NNR usually has better performance and the recovered rank is often much higher than that of MF [Yao and Kwok 2015]. In this paper, we generate metagraph based latent features with both methods and conduct experiments to compare their performance (shown in Section 6.6). The technical details of these two methods are introduced in the remaining part of this section.

4.3.1 Matrix Factorization. Consider a user-item similarity matrix $R \in \mathbb{R}^{m \times n}$, let the observed positions be indicated by 1’s in $\Omega \in \{0, 1\}^{m \times n}$, i.e., $[P_{\Omega}(X)]_{ij} = X_{ij}$ if $\Omega_{ij} = 1$ and 0 otherwise. $R$ is factorized as a product of $U \in \mathbb{R}^{m \times F}$ and $V \in \mathbb{R}^{n \times F}$ by solving the following optimization problem:

$$\min_{U, B} \frac{1}{2} \|P_{\Omega}(UB^T - R)\|_F^2 + \frac{\mu}{2} \left(\|U\|_F^2 + \|B\|_F^2\right),$$

(1)

where $F \ll \min(m, n)$ is the desired rank of $R$, and $\mu$ is the hyper-parameter controlling regularization.

We adopt the gradient descent based approach for optimizing (1), which is popular in RSs [Koren 2008; Mnih and Salakhutdinov 2007]. After optimization, we take $U$ and $B$ as the latent features of users and items, respectively.

4.3.2 Nuclear Norm Regularization. Although MF can be simple, (1) is not a convex optimization problem, so there is no rigorous guarantee on the recovery performance. This motivates our adoption of nuclear norm, which is defined as the sum of all singular values of a matrix. It is also the tightest convex envelope to the rank function. This leads to the following nuclear norm regularization (NNR) problem:

$$\min_{X} \frac{1}{2} \|P_{\Omega}(X - R)\|_F^2 + \mu \|X\|_*,$$

(2)

where $X$ is the low-rank matrix to be recovered, and $\mu$ is the hyper-parameter controlling regularization. Nice theoretical guarantee has been developed for (2), which shows that $X$ can be exactly recovered given sufficient observations [Candès and Recht 2009]. These advantages make NNR popular for low-rank matrix approximation [Candès and Recht 2009]. Thus we adopt (2) to generate latent features, using the state-of-the-art AIS-Impute algorithm [Yao and Kwok 2015] in optimizing (2). It has fast $O(1/T^2)$ convergence rate, where $T$ is the number of iterations, with low per-iteration time complexity. In the iterations, a Singular Value Decomposition (SVD) of $X = P\Sigma Q^T$ is maintained ($\Sigma$ only contains the nonzero singular values). When the algorithm stops, we take $U = P\Sigma^{\frac{1}{2}}$ and $B = Q\Sigma^{\frac{1}{2}}$ as user and item latent features, respectively.
4.4 Complexity Analysis

We analyze the time complexity of the MF part, which includes similarity matrix computation and latent feature generation. For similarity matrix computation, the core part is matrix multiplication. Since the adjacency matrices tend to be very sparse, they can be implemented very efficiently as sparse matrices. Moreover, for MF and NNR, according to [Yao and Kwok 2015; Yao et al. 2018], the computation costs in each iteration are $O(||\Omega||_1 \cdot F + (m+n)F)$ and $O(||\Omega||_1 \cdot F + (m+n)F^2)$, respectively, where $||\Omega||_1$ is the number of nonzero elements in the similarity matrix, $m$ and $n$ are the dimensions of the similarity matrix, and $F$ is the rank used in the factorization of the similarity matrix.

5 METAGRAPH BASED FEATURES FUSION AND SELECTION

In this section, we describe the FM part for fusing multiple groups of metagraph based latent features. Existing HIN-based RS methods [Shi et al. 2015; Yu et al. 2014] only use linear combination of different metapath based features and thus ignore the interactions among features. To resolve this limitation, we apply FM to capture the interactions among metagraph based latent features and non-linear interactions among features (i.e., second-order interactions) when fusing various side information in HIN. In Section 5.1, we show how FM performs prediction utilizing the metagraph based latent features. Then we introduce two regularization terms in Section 5.2, which can achieve automatic metagraph selection. In Section 5.3, we depict the objective function and propose two optimization methods for it.

5.1 Combining Latent Features with FM

In this section, we introduce our FM-based algorithm for fusing different groups of latent features. As described in Section 4.3, we obtain $L$ groups of latent features of users and items, denoted by $U^1, B^1, \ldots, U^L, B^L$, from $L$ metagraph based user-item similarity matrices. For a sample $x^n$ in the observed ratings, i.e., a pair of user and item, denoted by $u_i$ and $b_j$, respectively, we concatenate all of the corresponding user and item features from the $L$ metagraphs:

$$
x^n = [u^1_i, \ldots, u^L_i, b^1_j, \ldots, b^L_j] \in \mathbb{R}^d,
$$

where $d = 2 \sum_{l=1}^L F_l$, and $F_l$ is the rank of the factorization of the similarity matrix for the $l$-th metagraph obtained with (1) or (2). $u^l_i$ and $b^l_j$, respectively, represent user and item latent features generated from the $l$-th metagraph, and $x^n$ is a $d$-dimension vector representing the feature vector of the $n$-th sample after concatenation.

Given all of the features in (3), the predicted rating for the sample $x^n$ based on FM [Rendle 2012] is computed as follows:

$$
y^n(w, V) = b + \sum_{i=1}^d w_i x^n_i + \sum_{i=1}^d \sum_{j=1}^d \langle v_i, v_j \rangle x^n_i x^n_j,
$$

where $b$ is the global bias, and $w \in \mathbb{R}^d$ represents the first-order weights of the features. $V = [v_i] \in \mathbb{R}^{d \times K}$ represents the second-order weights for modeling the interactions among the features, and $v_i$ is the $i$-th row of the matrix $V$, which describes the $i$-th variable with $K$ factors. $x^n_i$ is the $i$-th feature in $x^n$. The parameters can be learned by minimizing the mean square loss:

$$
\ell(w, V) = \frac{1}{N} \sum_{n=1}^N (y^n - y^n(w, V))^2,
$$

where $y^n$ is an observed rating for the $n$-th sample, and $N$ is the number of all observed ratings.
5.2 Metagraph Selection with Group Lasso

We need to tackle two problems when FM is applied to metagraph based latent features. The first problem is that noise may arise when there are too many metagraphs, thus impairing the predicting capability of FM. This is because not all metagraphs are useful for recommendation because the semantics captured in a metagraph may have little effect on recommendation behavior in the real world. The second problem is computational cost. All of the features are generated by MF, which means that the design matrix (i.e., features fed to FM) is dense. It increases the computational cost for learning the parameters of the model and that of online recommendation. To alleviate these two problems, we propose two novel regularization terms to automatically select useful metagraphs during training process. They can be categorized into convex and nonconvex regularizations, and either of them enables our model to automatically select useful metagraphs during the training process.

5.2.1 Convex Regularization. The convex regularizer is the $\ell_{2,1}$-norm regularization, i.e., group lasso regularization [Jacob et al. 2009], which is a feature selection method on a group of variables. Given the pre-defined non-overlapping $G$ groups $\{I_1, \ldots, I_G\}$ on the parameter $p$, the regularization is defined as follows.

$$\phi(p) = \sum_{g=1}^{G} \eta_g \|p_{I_g}\|_2,$$

(6)

where $\|\cdot\|_2$ is the $\ell_2$-norm, and $\eta_g$ is a hyper-parameter. In our model, the groups correspond to the metagraph based features. For example, $U_l$ and $B_l$ are the user and item latent features generated by the $l$-th metagraph. For a pair of user $i$ and item $j$, the latent features are $u_{l,i}$ and $b_{l,j}$. There are two corresponding groups of variables in $w$ and $V$ according to (4). Thus, with $L$ metagraphs, $w$ and $V$ each has $2L$ groups of variables.

For the first-order parameters $w$ in (4), which is a vector, group lasso is applied to the subset of variables in $w$. Then we have:

$$\hat{\phi}(w) = \sum_{l=1}^{2L} \hat{\eta}_l \|w^l\|_2,$$

(7)

where $w^l \in \mathbb{R}^{F_l}$, which models the weights for a group of user or item features from one metagraph, and $\hat{\eta}_l$ is a hyper-parameter. For the second-order parameters $V$ in (4), we have the regularizer as follows:

$$\hat{\phi}(V) = \sum_{l=1}^{2L} \hat{\eta}_l \|V^l\|_F,$$

(8)

where $V^l \in \mathbb{R}^{F_l \times K}$, the $l$-th block of $V$ corresponds to the $l$-th metagraph based features in a sample, and $\|\cdot\|_F$ is the Frobenius norm.

5.2.2 Nonconvex Regularization. While convex regularizers usually make optimization easy, they often lead to biased estimation. For example, in sparse coding, the solution obtained by the $\ell_1$-regularizer is often not as sparse and accurate compared to capped-$\ell_1$ penalty [Zhang 2010]. Besides, in low-rank matrix learning, the estimated rank obtained with the nuclear norm regularizer is often very high [Yao et al. 2018]. To alleviate these problems, a number of nonconvex regularizers, which are variants of the convex $\ell_1$-norm, have been recently proposed [Yao and Kwok 2016; Yao et al. 2018]. Empirically, these nonconvex regularizers usually outperform the convex ones.
by the above observations, we propose to use nonconvex variant of (7) and (8) as follows:

$$\hat{\psi}(w) = \sum_{l=1}^{2L} \hat{\eta}_l \kappa \left( \left\| w^l \right\|_2 \right), \quad \hat{\psi}(V) = \sum_{l=1}^{2L} \hat{\eta}_l \kappa \left( \left\| V^l \right\|_F \right),$$

(9)

where $\kappa$ is a nonconvex penalty function. We choose $\kappa(|\alpha|) = \log (1 + |\alpha|)$ as the log-sum-penalty (LSP) [Candès et al. 2008], as it has been shown to give the best empirical performance on learning sparse vectors [Yao and Kwok 2016] and low-rank matrices [Yao et al. 2018].

5.2.3 Comparison with existing methods. Yu et al. studied recommendation techniques based on HINs [Yu et al. 2014] and applied matrix factorization to generate latent features from metapaths. Ratings are predicted using a weighted ensemble of the dot products of user and item latent features from every single metapath: $\hat{r}(u_i, b_j) = \sum_{l=1}^{L} \theta_l \cdot u_i^l \cdot b_j^l$, where $\hat{r}(u_i, b_j)$ is the predicted rating for user $u_i$ and item $b_j$ and $u_i^l$ and $b_j^l$ are the latent features for $u_i$ and item $b_j$ from the $l$-th metapath, respectively. $L$ is the number of metapaths used, and $\theta_l$ is the weight for the $l$-th metapath latent features. However, the predicting method is not adequate, as it fails to capture the interactions between features across different metapaths, and between features within the same metapath, resulting in a decrease of the prediction performance for all of the features. In addition, previous works on FM [Hong et al. 2013; Rendle 2012; Yan et al. 2014] only focus on the selection of one row or column of the second-order weight matrix, while $\hat{\phi}$ in our method selects a block of rows or columns (defined by metagraphs). Moreover, we are the first to adopt nonconvex regularization, i.e., $\hat{\psi}$, for weights selection in FM.

5.3 Model Optimization
Combining (5) and (9), we define our FM with Group lasso (FMG) model with the following objective function:

$$h(w, V) = \frac{1}{N} \sum_{n=1}^{N} (y^n - \hat{y}^n(w, V))^2 + \lambda \hat{\psi}(w) + \tilde{\lambda} \hat{\psi}(V).$$

(10)

Note that when $\kappa(\alpha) = |\alpha|$ in (9), we get back (7) and (8). Thus, we directly use the nonconvex regularization in (10).

We can see that $h$ is nonsmooth due to the use of $\hat{\phi}^w$ and $\hat{\phi}^V$, and nonconvex due to the nonconvexity of loss $\ell$ on $w$ and $V$. To alleviate the difficulty on optimization, inspired by [Yao and Kwok 2016], we propose to reformulate (10) as follows:

$$\tilde{h}(w, V) = \ell(w, V) + \kappa_0 \hat{\phi}(w) + \kappa_0 \tilde{\phi}(V),$$

(11)

where $\ell(w, V) = \ell(w, V) + g(w, V)$, $\kappa_0 = \lim_{\beta \to 0} \kappa'(|\beta|)$ and

$$g(w, V) = \tilde{\lambda} \left[ \hat{\psi}(w) - \kappa_0 \hat{\phi}(w) \right] + \tilde{\lambda} \left[ \hat{\psi}(V) - \kappa_0 \tilde{\phi}(V) \right].$$

Note that $\tilde{h}$ is equivalent to $h$ based on Proposition 2.1 in [Yao and Kwok 2016]. A very important property for the augmented loss $\tilde{\ell}$ is that it is still smooth. As a result, while we are still optimizing a nonconvex regularized problem, we only need to deal with convex regularizers.

In Section 5.3.1, we show how the reformulated problem can be solved by the state-of-the-art proximal gradient algorithm [Li and Lin 2015]; moreover, such transformation enables us to design a more efficient optimization algorithm with convergence guarantee based on variance reduced methods [Xiao and Zhang 2014]. Finally, the time complexity of the proposed algorithms is analyzed in Section 5.3.3.
Remark 5.1. nmAPG was previously used in our paper [Zhao et al. 2017b]. Here, we show that it can still be applied to the new model (11). Besides, we further propose to use SVRG and show in Section 6.8 that it is much more efficient than nmAPG.

5.3.1 Using nmAPG Algorithm. To tackle the nonconvex nonsmooth objective function (11), we propose to adopt the PG algorithm [Parikh and Boyd 2014] and, specifically, the state-of-the-art non-monotonic accelerated proximal gradient (nmAPG) algorithm [Li and Lin 2015]. It targets at optimization problems of the form:

\[
\min_x F(x) \equiv f(x) + g(x),
\]

where \(f\) is a smooth (possibly nonconvex) loss function and \(g\) is a regularizer (can be nonsmooth and nonconvex). To guarantee the convergence of nmAPG, we also need \(\lim_{\|x\|_{L^2} \to \infty} F(x) = \infty\), \(\inf_x F(x) > -\infty\), and there exists at least one solution to the proximal step, i.e., \(\text{prox}_{\gamma g}(z) = \arg \min_x \frac{1}{2} \|x - z\|_2^2 + \gamma g(x)\), where \(\gamma \geq 0\) is a scalar [Li and Lin 2015].

The motivation of nmAPG is two fold. First, nonsmoothness comes from the proposed regularizers, which can be efficiently handled if the corresponding proximal steps have cheap closed-form solution. Second, the acceleration technique is useful for significantly speeding up first order optimization algorithms [Li and Lin 2015; Yao and Kwok 2016; Yao et al. 2017], and nmAPG is the state-of-the-art algorithm which can deal with general nonconvex problems with sound convergence guarantee. The whole procedure is given in Algorithm 2. Note that while both \(\hat{\phi}\) and \(\bar{\phi}\) are nonsmooth in (11), they are imposed on \(w\) and \(V\) separately. Thus, for any \(\alpha, \beta \geq 0\), we can also compute proximal operators independently for these two regularizers following [Parikh and Boyd 2014]:

\[
\text{prox}_{\alpha \hat{\phi} + \beta \bar{\phi}}(w, V) = \left( \text{prox}_{\alpha \hat{\phi}}(w), \text{prox}_{\beta \bar{\phi}}(V) \right).
\]

These are performed in steps 5 and 10 in Algorithm 2. The closed-form solution of the proximal operators can be obtained easily from Lemma 1 below. Thus, each proximal operator can be solved in one pass of all groups.

Lemma 1 ([Parikh and Boyd 2014]). The closed-form solution of \(p^* = \text{prox}_{\lambda \phi}(z)\) (\(\phi\) is defined in (6)) is given by \(p^*_{I_g} = \max \left(1 - \frac{\eta_g}{\|z_{I_g}\|_2}, 0\right) z_{I_g}\) for all \(g = 1, \ldots, G\).

It is easy to verify that the above assumptions are satisfied by our objective \(h\) here. Thus, Algorithm 2 is guaranteed to produce a critical point for (11).

5.3.2 Using SVRG Algorithm. While nmAPG can be an efficient algorithm for (11), it is still a batch-gradient based method, which may not be efficient when the sample size is large. In this case, the stochastic gradient descent (SGD) [Bertsekas 1999] algorithm is preferred as it can incrementally update the learning parameters. However, the gradient in SGD is very noisy. To ensure the convergence of SGD, a decreasing step size must be used, making the speed possibly even slower than batch-gradient methods.

Recently, the stochastic variance reduction gradient (SVRG) [Xiao and Zhang 2014] algorithm has been developed. It avoids diminishing step size by introducing variance reduced techniques into gradient updates. As a result, it combines the best of both worlds, i.e., incremental update of the learning parameters while keeping non-diminishing step size, to achieve significantly faster converging speed than SGD. Besides, it is also extended for the problem in (12) with nonconvex objectives [Allen-Zhu and Hazan 2016; Reddi et al. 2016]. This allows the loss function to be smooth
Algorithm 2 nmAPG algorithm for (11).

1: Initiate $w_0, V_0$ as Gaussian random matrices;
2: $w_1 = w_1 = w_0, V_1 = V_0, c_1 = h(w_1, V_1); g_1 = 1, \delta = 10^{-3}, a_0 = 0, a_1 = 1, \text{step-size } \alpha;$
3: for $t = 1, 2, \ldots, T$ do
4: \hspace{1em} $y_t = w_t + \frac{a_t}{a_t'} (w_t - w_t) + \frac{a_t-1}{a_t} (w_t - w_t - 1);$ 
5: \hspace{1em} $y_t = V_t + \frac{a_t}{a_t'} (V_t - V_t) + \frac{a_t-1}{a_t} (V_t - V_t);$ 
6: \hspace{1em} $w_{t+1} = \text{prox}_{\alpha_k \lambda \delta} (w_t - \alpha \nabla \ell(w_t, V_t));$
7: \hspace{1em} $V_t+1 = \text{prox}_{\alpha_k \lambda \delta} (V_t - \alpha \nabla \ell(w_t, V_t));$
8: \hspace{1em} \text{if } h(w_{t+1}, V_{t+1}) \leq c_{t} - \delta \Delta_{t}; \text{ then}$
9: \hspace{2em} $w_{t+1} = w_{t+1}, V_{t+1} = V_{t+1};$
10: \hspace{1em} end if
11: \hspace{1em} $w_{t+1} = \text{prox}_{\alpha_k \lambda \delta} (w_{t} - \alpha \nabla \ell(w_{t}, V_{t}));$
12: \hspace{1em} $V_{t+1} = \text{prox}_{\alpha_k \lambda \delta} (V_{t} - \alpha \nabla \ell(w_{t}, V_{t}));$
13: \hspace{1em} \text{if } h(w_{t+1}, V_{t+1}) < h(w_{t+1}, V_{t+1}) \text{ then}$
14: \hspace{2em} $w_{t+1} = w_{t+1}, V_{t+1} = V_{t+1};$
15: \hspace{1em} end if
16: \hspace{1em} end if
17: \hspace{1em} $a_{t+1} = \frac{1}{2} (\sqrt{4 a_t^2 + 1} + 1);$ 
18: \hspace{1em} $q_{t+1} = \eta q_t + 1, c_{t+1} = \frac{1}{q_{t+1}} (q_{t+1} c_t + h(w_{t+1}, V_{t+1}));$
19: \hspace{1em} end for
20: return $w_{T+1}, V_{T+1}.$

(possibly nonconvex) but the regularizer still needs to be convex. Thus, instead of working on the original problem (10), we work on the transformed problem in (11).

To use SVRG, we first define the augmented loss for the $n$-th sample as $\tilde{\ell}_n(w, V) = (y^n - y^n(w, V))^2 + \frac{1}{N} g(w, V).$ The whole procedure is depicted in Algorithm 3. A full gradient is computed in step 4, a mini-batch $B$ of size $m_b$ is constructed in step 6, and the variance reduced gradient is computed in step 7. Finally, the proximal steps can be separately executed based on (13) in step 8.

As mentioned above, the nonconvex variant of SVRG [Allen-Zhu and Hazan 2016; Reddi et al. 2016] cannot be directly applied to (10). Instead, we apply it to the transformed problem (11), where the regularizer becomes convex and the augmented loss is still smooth. Thus, Algorithm 3 is guaranteed to generate a critical point of (11).

5.3.3 Complexity Analysis. For nmAPG in Algorithm 2, the main computation cost is incurred in performing the proximal steps (step 5 and 10) which cost $O(NKd);$ then the evaluation of function value (step 7 and 11) costs $O(NKd)$ time. Thus, the per-iteration time complexity for Algorithm 2 is $O(NKd).$ For SVRG in Algorithm 3, the computation of the full gradient takes $O(NKd)$ in step 5; then $O(m_bBKd)$ time is needed for steps 6-10 to perform mini-batch updates. Thus, one iteration in Algorithm 2 takes $O((N + m_b B)Kd)$ time. Usually, $m_b B$ shares the same order as $N$ [Allen-Zhu and Hazan 2016; Reddi et al. 2016; Xiao and Zhang 2014]. Thus, we set $m_b B = N$ in our experiments. As a result, SVRG needs more time to perform one iteration than nmAPG. However, due to stochastic updates, SVRG empirically converges much faster as shown in Section 6.8.
which are denoted as CIKM-Yelp and CIKM-Douban, respectively. Note that four datasets are used

Algorithm 3 SVRG for (11).

1: Initiate $w_0, V_0$ as Gaussian random matrices, mini-batch size $m_b$;
2: $w_1^B = w_0, V_1^B = V_0$ and step-size $\alpha$;
3: for $t = 1, 2, \ldots, T$ do
4: $w_{t+1}^0 = w_t^B, V_{t+1}^B = V_t^B$;
5: $g_{t+1}^{\bar{w}} = \nabla w_t^B f(w_t, V_t), g_{t+1}^{V_t} = \nabla V_t^B f(w_t, V_t)$;
6: for $b = 0, 1, \ldots, B - 1$ do
7: Uniformly randomly sample a mini-batch $B$ of size $m_b$;
8: $m_b^b = \frac{1}{m_b} \sum_{i_b \in B} (\nabla w_t^b f(w_t, V_t) - \nabla V_t^b (w_t, V_t)) + \hat{g}_{t+1}^B$;
9: $w_{t+1}^{b+1} = \text{prox}_{\alpha \lambda \phi} \left( w_{t+1}^b - \alpha m_b^b \right)$;
10: $V_{t+1}^{b+1} = \text{prox}_{\alpha \lambda \phi} \left( V_{t+1}^b - \alpha m_b^b \right)$;
11: $w_{t+1} = \frac{1}{B} \sum_{b=1}^B w_{t+1}^b, V_{t+1} = \frac{1}{B} \sum_{b=1}^B V_{t+1}^b$;
12: end for
13: return $w_{T+1}, V_{T+1}$.

6 EXPERIMENTS

In this section, we conduct extensive experiments to demonstrate the effectiveness of our proposed framework. We first introduce the datasets, evaluation metrics and experimental settings in Section 6.1. In Section 6.2, we show the recommendation performance of our proposed framework compared to several state-of-the-art recommendation methods, including MF-based and HIN-based methods. We analyze the influence of the parameter $\lambda$, which controls the weight of convex regularization term, in Section 6.3, and the influence of $\lambda$ in the nonconvex regularization term in Section 6.4. To further understand the impact of metagraphs on performance, we discuss the performance of each single metagraph in Section 6.5. In Section 6.6, we compare the performance between NNR and MF in extracting the features. In Section 6.7, we show the influence of $K$ of FMG. Finally, the two proposed two optimization algorithms described in Section 5.3 are compared in Section 6.8, and their scalability is studied in Section 6.9.

6.1 Setup

To demonstrate the effectiveness of HIN for recommendation, we mainly conduct experiments using two datasets with rich side information. The first dataset is Yelp, which is provided for the Yelp challenge.² Yelp is a website where a user can rate local businesses or post photos and review about them. The ratings fall in the range of 1 to 5, where higher ratings mean users like the businesses while lower rates mean users dislike business. Based on the collected information, the website can recommend businesses according to the users’ preferences. The second dataset is Amazon Electronics,³ which is provided in [He and McAuley 2016]. As we know, Amazon highly relies on RSs to present interesting items to its users. We extract subsets of entities from Yelp and Amazon to build the HIN, which includes diverse types and relations. The subsets of the two datasets both include around 200,000 ratings in the user-item rating matrices. Thus, we identify them as Yelp-200K and Amazon-200K, respectively.

²http://jmcauley.ucsd.edu/data/amazon/
³http://jmcauley.ucsd.edu/data/yelp/

2020,
Vol. 1, No. 1, Article . Publication date: September 2019.
to compare the recommendation performance of different methods, as shown in Section 6.2. To evaluate other aspects of our model, we only conduct experiments on the first two datasets, i.e., the Yelp-200K and Amazon-200K datasets. The statistics of our datasets are shown in Table 1. For the detailed information of CIKM-Yelp and CIKM-Douban, we refer the readers to [Shi et al. 2015].

Table 1. Statistics of the Yelp-200K and Amazon-200K datasets.

| Relations(A-B)       | Number of A | Number of B | Number of (A-B) | Avg Degrees of A/B |
|----------------------|-------------|-------------|------------------|---------------------|
| Amazon-200K          |             |             |                  |                     |
| User-Review          | 59,297      | 183,807     | 183,807          | 3.1/1               |
| Business-Category    | 20,216      | 682         | 87,587           | 4.3/128.4           |
| Business-Brand       | 95,33       | 2,015       | 9,533            | 1/4.7               |
| Review-Business      | 183,807     | 20,216      | 183,807          | 1/9.1               |
| Review-Aspect        | 183,807     | 10          | 796,392          | 4.3/79,639.2        |
| Yelp-200K            |             |             |                  |                     |
| User-Business        | 36,105      | 22,496      | 191,506          | 5.3/8.5             |
| User-Review          | 36,105      | 191,506     | 191,506          | 5.3/1               |
| User-User            | 17,065      | 17,065      | 140,344          | 8.2/8.2             |
| Business-Category    | 22,496      | 869         | 67,940           | 3/78.2              |
| Business-Star        | 22,496      | 9           | 22,496           | 1/2,499.6           |
| Business-State       | 22,496      | 18          | 22496            | 1/1,249.8           |
| Business-City        | 22,496      | 215         | 22,496           | 1/104.6             |
| Review-Business      | 191,506     | 22,496      | 191,506          | 1/8.5               |
| Review-Aspect        | 191,506     | 10          | 955,041          | 5/95,504.1          |

Table 2. The density of rating matrices in the four datasets (Density = \#Ratings / \#Users x \#Items).

|               | Amazon-200K | Yelp-200K | CIKM-Yelp | CIKM-Douban |
|---------------|-------------|-----------|-----------|-------------|
| Density       | 0.015%      | 0.024%    | 0.086%    | 0.630%      |

To evaluate the recommendation performance, we adopt the root-mean-square-error (RMSE) as our metric, which is the most popular for rating prediction in the literature [Koren 2008; Ma et al. 2011; Mnih and Salakhutdinov 2007]. It is defined as RMSE = \(\sqrt{\frac{\sum_{y^n \in R_{test}} (y^n - \hat{y}^n)^2}{|R_{test}|}}\), where \(R_{test}\) is the set of all the test samples, \(\hat{y}^n\) is the predicted rating for the \(n\)-th sample, \(y^n\) is the observed rating of the \(n\)-th sample in the test set. A smaller RMSE value means better performance.

We compare the following baseline models to our approaches.

- **RegSVD** [Paterek 2007]: The basic matrix factorization model with \(L_2\) regularization, which uses only the user-item rating matrix. We use the implementation in [Guo et al. 2015].
- **FMR** [Rendle 2012]: The factorization machine with only the user-item rating matrix. We adopt the method in Section 4.1.1 of [Rendle 2012] to model the rating prediction task. We use the code provided by the authors.4
- **HeteRec** [Yu et al. 2014]: It is based on metapath based similarity between users and items. A weighted ensemble model is learned from the latent features of users and items generated by applying matrix factorization to the similarity matrices of different metapaths. We implemented it based on [Yu et al. 2014].
- **SemRec** [Shi et al. 2015]: It is a metapath based recommendation technique on weighted HIN, which is built by connecting users and items with the same ratings. Different models are learned from different metapaths, and a weight ensemble method is used to predict the users’ ratings. We use the code provided by the authors.5

4http://www.libfm.org/
5https://github.com/zzqsmall/SemRec

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• **FMG:** The proposed framework (Figure 3) with convex group lasso regularizer in (7) and (8) used with factorization machine. The model is proposed in our previous work [Zhao et al. 2017b].

• **FMG(LSP):** Same as FMG, except nonconvex group lasso regularizer in (9) is used.

Note that it is reported in [Shi et al. 2015] that SemRec outperforms the method in [Yu et al. 2013], which uses metapath based similarities as regularization terms in matrix factorization. Thus, we do not include [Yu et al. 2013] in the comparison. All experiments run in a server (OS: CentOS release 6.9, CPU: Intel i7-3.4GHz, RAM: 32GB).

On Amazon-200K and Yelp-200K datasets, we use the metagraphs in Figures 5 and 4 for HeteRec, SemRec, FMG, and FMG(LSP), while on CIKM-Yelp and CIKM-Douban, we use the metapaths provided in [Shi et al. 2015] for these four methods. To get the aspects (e.g., A in Figures 4 and 5) from review texts, we use a topic model software Gensim [Řehůřek and S. 2010] to extract topics from the review texts and use the extracted topics as aspects. The number of topics is set to 10 empirically.

In Section 6.2, we use the four datasets in Table 2 to compare the recommendation performance of our models and the baselines. For the experimental settings, we randomly split the whole dataset into 80% for training, 10% for validation and the remaining 10% for testing. The process is repeated five times and the average RMSE of the five rounds is reported. Besides, for the parameters of our models, we set $\bar{\lambda} = \lambda = \lambda$ in Eq. (10) for simplicity, and $\lambda$ is set to obtain the optimal value on different validation datasets. As in [Zhao et al. 2017b], $F$ and $K$ are set to 10 for its good performance and computational efficiency. From Sections 6.3 to Section 6.8, to explore the influences of different settings of the proposed framework, we create two smaller datasets, Amazon-50K and Yelp-50K, where only 50,000 ratings are randomly sampled from Amazon-200K and Yelp-200K. Finally, in Section 6.9, we conduct experiments for the FM part with the two optimization algorithms presented in Sec 5.3 to demonstrate the scalability of the proposed framework. Datasets of different scales are created from Amazon-200K and Yelp-200K, and the parameters $\lambda, F, K$ are set to 0.1, 10, 10, respectively.

### 6.2 Recommendation Effectiveness

The RMSEs of all of the methods evaluated are shown in Table 3. The relative decrease of RMSEs achieved by FMG compared to the baselines is shown in Table 4. For CIKM-Yelp and CIKM-Douban, we directly report the performance of SemRec from [Shi et al. 2015] since the same amount of training data is used in our experiment. Besides, the results of SemRec on Amazon-200K are not reported, as the programs crashed due to large demand of memory.

| Method   | Amazon-200K | Yelp-200K | CIKM-Yelp | CIKM-Douban |
|----------|-------------|-----------|-----------|-------------|
| RegSVD   | 2.9656±0.0008 | 2.5141±0.0006 | 1.5323±0.0011 | 0.7673±0.0010 |
| FMR      | 1.3462±0.0007 | 1.7637±0.0004 | 1.4342±0.0009 | 0.7524±0.0011 |
| HeteRec  | 2.5368±0.0009 | 2.3475±0.0005 | 1.4891±0.0005 | 0.7671±0.0008 |
| SemRec   | —           | 1.4603±0.0003 | 1.1559(*)  | 0.7216(*)  |
| FMG      | **1.1953±0.0008** | **1.2583±0.0003** | **1.1167±0.0011** | **0.7023±0.0011** |
| FMG(LSP) | 1.1980±0.0010 | 1.2593±0.0005 | 1.1255±0.0012 | **0.7035±0.0013** |

Firstly, we can see that our FMG model, including the convex and nonconvex ones, consistently outperforms all baselines on the four datasets. This demonstrates the effectiveness of the proposed framework shown in Figure 3. Note that the performance of FMG and FMG(LSP) are very close, but
FMG(LSP) needs fewer features to achieve such performance, which supports our motivation to use nonconvex regularization for selecting features. In the following two sections, we will compare in detail the two regularizers.

Secondly, from Table 3 and 4, we can see that comparing to RegSVD and FMR, which only use the rating matrix, SemRec and FMG, which use side information from metagraphs, are significantly better. In particular, the sparser the rating matrix, the more obvious is the benefit produced by the additional information. For example, on Amazon-200K, FMG outperforms RegSVD by 60%, while for CIKM-Douban, the percentage of RMSE decrease is 8.5%. Note that the performance of HeteRec is worse than FMR, despite the fact that we have tried our best to tune the model. This aligns with our discussion in Section 5 that a weighting ensemble of dot products of latent features may cause information loss among the metagraphs and fail to reduce noise caused by having too many metagraphs. These demonstrate the effectiveness of the proposed FMG for fusing various side information for recommendation.

When comparing the results of FMG and SemRec, we find that the performance gap between them are not that large, which means that SemRec is still a good method for rating prediction, especially when comparing to the other three baselines. The good performance of SemRec may be attributed to the reason that it incorporates rating values into HIN to create a weighted HIN, which can better capture the metagraph or metapath based similarities between users and items.

### 6.3 The Impact of Convex Regularizer

In this part, we study the impact of group lasso regularizer for FMG. Specifically, we show the trend of RMSE by varying $\lambda$ (with $\lambda = \tilde{\lambda} = \lambda$ in (10)), which controls the weights of group lasso. The RMSE of Amazon-50K and Yelp-50K are shown in Figure 6(a) and (b), respectively. We can see that with $\lambda$ increasing, RMSE decreases first and then increases, demonstrating that $\lambda$ values that are too large or too small are not good for the performance of rating prediction. Specifically, on Amazon-50K, the best performance is achieved when $\lambda = 0.06$, and on Yelp-50K, the best is when $\lambda = 0.05$. Next, we give further analysis of these two parameters in terms of sparsity and the metagraphs selected by group lasso.

#### 6.3.1 Sparsity of $\mathbf{w}, \mathbf{V}$

We study the sparsity of the learned parameters, i.e., the ratio of zeros in $\mathbf{w}, \mathbf{V}$, after learning. We define NNZ (number of non zeros) as $\frac{nnz_{\mathbf{w}} + nnz_{\mathbf{V}}}{n_{\mathbf{w}} + n_{\mathbf{V}}}$, where $nnz$ is the total number of nonzero elements in $\mathbf{w}$ and $\mathbf{V}$, and $n_{\mathbf{w}}$ and $n_{\mathbf{V}}$ are the number of entries in $\mathbf{w}$ and $\mathbf{V}$, respectively. The smaller NNZ, the fewer the nonzero elements in $\mathbf{w}$ and $\mathbf{V}$, and the fewer the metagraph based features left after training. The trend of NNZ with different $\lambda$’s is shown in Figure 7. We can see that with $\lambda$ increasing, NNZ becomes smaller, which aligns with the effect of group lasso. Note that the trend is non-monotonic due to the nonconvexity of the objective.

#### 6.3.2 The Selected Metagraphs

In this part, we analyze the selected features in FMG. From Figure 6(a) and (b), we can see that RMSE and sparsity are good when $\lambda = 0.06$ on Amazon-50K and $\lambda = 0.05$ on Yelp-50K. Thus, we want to show the selected metagraphs and their user and item
features in these configurations. Recall that in Eq. (4), we introduce $w$ and $V$, respectively, to capture the first-order weights for the features and second-order weights for interactions of the features. Thus, after training, the nonzero values in $w$ and $V$ represent the selected features, i.e., the selected metagraphs. We list in Table 5 the selected metagraphs corresponding to nonzero values in $w$ and $V$ from the perspective of both users and items.

Table 5. The selected metagraphs by FMG and FMG(LSP) on Amazon-50K and Yelp-50K datasets. We show the selected latent features from the perspective of users and items and from both first-order and second-order parameters.

|       | User-Part | Item-Part |
|-------|-----------|-----------|
|       | first-order | second-order | first-order | second-order |
| Amazon-50K | FMG | $M_1$-$M_3$, $M_5$ | $M_1$-$M_6$ | $M_2$, $M_3$, $M_5$, $M_6$ | $M_2$, $M_5$, $M_6$ |
|        | FMG(LSP)  | $M_1$, $M_5$ | -           | $M_2$, $M_5$ | -           |
| Yelp-50K | FMG | $M_1$-$M_4$, $M_5$, $M_8$ | $M_1$-$M_3$, $M_5$, $M_8$ | $M_1$-$M_5$, $M_8$, $M_9$ | $M_3$, $M_8$ |
|        | FMG(LSP)  | $M_1$, $M_5$, $M_4$, $M_8$ | $M_2$, $M_5$, $M_8$ | $M_1$-$M_5$, $M_8$ | $M_8$ |

From Table 5, we can observe that the metagraphs with style like $U \rightarrow * \leftarrow U \rightarrow B$ are better than those like $U \rightarrow B \rightarrow * \leftarrow B$. We use $U \rightarrow * \leftarrow U \rightarrow B$ to represent metagraphs like $M_2$, $M_5$, $M_6$, $M_9$ in Figure 4 (Yelp) and $M_2$, $M_5$, $M_6$ in Figure 5 (Amazon), and $U \rightarrow B \rightarrow * \leftarrow B$ to represent metagraphs like $M_4$, $M_5$, $M_6$, $M_7$ in Figure 4 and $M_3$, $M_4$ in Figure 5. On Yelp-50K, we can see that metagraphs like $M_2$, $M_3$, $M_8$, $M_9$ tend to be removed while $M_4$ - $M_7$ are removed. This means that on Yelp, recommendations by friends or similar users are better than those by similar items. Similar observations can be made on Amazon-50K, i.e., $M_3$, $M_4$ tend to be removed. Furthermore, on both datasets, complex structures like $M_9$ in Figure 4 and $M_6$ in Figure 5 are found to be important for item latent features. This demonstrates the importance of the semantics captured by metagraphs, which are ignored in previous metapath based RSs [Shi et al. 2015; Yu et al. 2013, 2014].

### 6.4 Impact of Nonconvex Regularizer

In this part, we study the performance of the nonconvex regularizer. We conduct experiments on Amazon-50K and Yelp-50K datasets to compare the results of the convex and nonconvex regularizers.

The results are reported in the same manner as in Section 6.3. The RMSEs of the nonconvex regularizer on Amazon-50K and Yelp-50K are shown in Figures 6(a) and (b), respectively. We observe
Fig. 7. The trend of NNZ by varying $\lambda$ on the Amazon-50K and Yelp-50K datasets. On Amazon-50K, FMG performs best when $\lambda = 0.06$, and FMG(LSP) is the best when $\lambda = 0.5$. On Yelp-50K, FMG performs best when $\lambda = 0.05$, and FMG(LSP) best when $\lambda = 0.1$.

that the trend of the nonconvex regularizer is similar to that of the convex regularizer. Specifically, on Amazon, the best performance is achieved when $\lambda = 0.5$, and on Yelp, the best is when $\lambda = 0.1$.

As in Section 6.3.1, we also use NNZ to show the performance of FMG(LSP) in Figure 7. We can see that with $\lambda$ increasing, NNZ becomes smaller. Note that the trend is also non-monotonic due to the nonconvexity of the objective. Besides, NNZ of the parameters of FMG(LSP) is much smaller than that of FMG when the best performance on both Amazon-50K and Yelp-50K is achieved. This is due to the effect of nonconvexity of LSP, which can induce larger sparsity of the parameters with a smaller loss of performance gain.

Next, we analyze the selected features by FMG(LSP). As in FMG, we show the selected metagraphs when the best performance is achieved in Figure 6, i.e., $\lambda = 0.5$ on Amazon-50K and $\lambda = 0.1$ on Yelp-50K. The results of Amazon-50K and Yelp-50K are also shown in Table 5, and the observation is very similar to that of FMG, i.e., metagraphs with style like $U \rightarrow \ast \leftarrow U \rightarrow B$ are better than those like $U \rightarrow B \rightarrow \ast \leftarrow B$. On Yelp-50K, metagraphs like $M_2, M_3, M_8$ tend to be selected while $M_4 - M_7$ are removed, while on Amazon-50K $M_3, M_4$ tend to be removed.

Besides sparsity trends and selected metagraphs, we emphasize an interesting discovery here. From Figure 7, we can see that on both Amazon-50K and Yelp-50K the NNZ of FMG(LSP) is smaller than that of FMG when they both obtain the best performance. For example, on Amazon-50K, FMG performs best with $\lambda = 0.06$ and NNZ = 0.52, while FMG(LSP) performs best with $\lambda = 0.5$ and NNZ = 0.25. Similar cases exist on Yelp-50K. In other words, to obtain the best performance, nonconvex regularizers can induce larger sparsity, which means they can select useful features more effectively, i.e., they can achieve comparable performance with fewer selected metagraphs.

### 6.5 Performance with Single Metagraph

In this part, we compare the performance of different metagraphs separately on Amazon-50K and Yelp-50K. In the training process, we use only one metagraph for user and item features and then predict with FMG and evaluate the results obtained by the corresponding metagraph. Specifically, we run experiments to compare RMSE of each metagraph in Figures 4 and 5. The RMSE of each metagraph is shown in Figure 8. Note that we show for comparison the RMSE when all metagraphs are used, which is denoted by $M_{all}$.
From Figure 8, we can see that on both Amazon-50K and Yelp-50K, the performance is the best when all metagraph based user and item features are used, which demonstrates the usefulness of the semantics captured by the designed metagraphs in Figures 4 and 5. Besides, we can see that on Yelp-50K, the performance of $M_4 - M_7$ is the worst, and on Amazon-50K, the performance of $M_3 - M_4$ is also among the worst three. Note that they are both metagraphs with style like $U \rightarrow B \rightarrow \ast \leftarrow B$. Thus, it aligns with the observation in the above two sections that metagraphs with style like $U \rightarrow \ast \leftarrow U \rightarrow B$ are better than those like $U \rightarrow B \rightarrow \ast \leftarrow B$. These similar observations described in these three sections can be regarded as domain knowledge, which indicates that we should design more metagraphs with style $U \rightarrow \ast \leftarrow U \rightarrow B$.

Finally, for $M_9$ on Yelp-50K and $M_9$ on Amazon-50K, we can see that their performance are among the best three, which demonstrates the usefulness of the complex semantics captured in $M_9$ on Yelp-50K and $M_9$ on Amazon-50K.

### 6.6 Feature Extraction Methods

In this part, we compare the performance of different feature extraction methods in MF part, i.e., NNR and MF described in Section 4.2. Note that, the parameter $F$ of MF and $\mu$ of NNR will lead to different number of latent features for different similarity matrices. Figure 9 shows the performance with different $d$, i.e., total length of the input features. We can see that latent features from NNR have slightly better performance than MF, while the feature dimension resulting from NNR is much larger. These observations support our motivation to use these two methods in Section 4.3, which is that NNR usually has better performance while the recovered rank is often much higher than that of MF. Thus, we can conclude that if we want the best performance, NNR is better for extracting features, while MF is more suitable for trade-off between performance and efficiency.

### 6.7 Rank of Second-Order Weights Matrix

In this part, we show the performance trend by varying $K$, which is the rank of the second-order weights $V$ in the FMG model (see Section 5). For the sake of efficiency, we conduct extensive experiments on Amazon-50K and Yelp-50K and employ the MF-based latent features. We set $K$ to values in the range of $[2, 3, 5, 10, 20, 30, 40, 50, 100]$, and the results are shown in Figure 10. We can
see that the performance becomes better with larger $K$ values on both datasets and reaches a stable performance after $K = 10$. Thus, we fix $K = 10$ for all other experiments.

![Fig. 9. The performance of latent features obtained from MF and NNR.](image1)

![Fig. 10. The trend of RMSE of FMG w.r.t. $K$.](image2)

### 6.8 Optimization Algorithm

In this part, we compare the SVRG and nmAPG algorithms proposed in Section 5.3. Besides, we also use SGD as a baseline since it is the most popular algorithm for models based on factorization machine [Hong et al. 2013; Rendle 2012]. Again, we use the Amazon-50K and Yelp-50K datasets. As suggested in [Xiao and Zhang 2014], we compare the efficiency of various algorithms based on RMSE w.r.t. the number of gradient computations divided by $N$.

The results are shown in Figure 11. We can observe that SGD is the slowest among all three algorithms and SVRG is the fastest. Although SGD can be faster than nmAPG at the beginning, the diminishing step size used to guarantee convergence of stochastic algorithms finally drags SGD down to become the slowest. SVRG is also a stochastic gradient method, but it avoids the problem of diminishing step size using variance reduced technique, which results in even faster speed than nmAPG. Finally, as both SVRG and nmAPG are guaranteed to produce a critical point of (10), they have the same empirical prediction performance. Therefore, in practice, the suggestion is to use SVRG as the solver because of the faster speed and empirically good performance.

![Fig. 11. The trend of RMSE of SVRG, nmAPG, and SGD w.r.t. the number of gradient computations divided by $N$.](image3)
6.9 Scalability

In this part, we study the scalability of our framework. We extract a series of datasets of different scales from Amazon-200K and Yelp-200K according to the number of observations in the user-item rating matrix. The specific values are [12.5K, 25K, 50K, 100K, 200K].

The time cost on Amazon and Yelp are shown in Figure 12. For simplicity, we only show the results of FMG with SVRG and nmAPG algorithms. From Figure 12, the training time is almost linear to the number of observed ratings, which aligns with the analysis in Section 5.3.3 and demonstrates that our framework can be applied to large-scale datasets.

7 CONCLUSION AND FUTURE WORKS

In this paper, we present a heterogeneous information network (HIN) based recommendation framework and introduce a principled way of fusing various side information in HIN. By using metagraphs derived from the HIN schema, we can capture similarities of rich semantics between users and items. From each metagraph, we obtain a user-item matrix, to which we apply matrix factorization and nuclear norm regularization to obtain the user and item latent features in an unsupervised way. After that, we use a group lasso regularized factorization machine to fuse different groups of latent features extracted from different metagraphs to predict the links between users and items, i.e., to recommend items to users. To solve the nonconvex nonsmooth optimization...
problem, we propose two algorithms, one is based on the proximal gradient algorithm and the other on stochastic variance reduced gradient algorithm. Experimental results demonstrate the effectiveness of our framework.

In addition to the technical solutions developed in our framework, we give suggestions on how our proposed framework can be used effectively. For example, for the construction of metagraphs, we suggest to apply domain knowledge in well-known recommendation strategies (e.g., user-based CF) to design good metagraphs and avoid metagraphs that are too large. For feature extraction methods, we note that if efficiency is important, MF is preferred to NNR. For metagraph selection methods, nonconvex regularizers can select useful metagraphs more effectively. For optimization methods, SVRG is always preferred for its empirically superior performance and fast convergence speed. With these suggestions, our framework can be quickly applied to not only HIN-based RSs for other scenarios, but also other HIN-based problems, e.g., intent recommendation [Fan et al. 2019], fraud detection [Hu et al. 2019], malware detection in software systems [Fan et al. 2018a; Hou et al. 2017], opioid user detection [Fan et al. 2018b], or medical diagnosis [Hosseini et al. 2018].

In the future, we plan to explore automatic methods [Quanming et al. 2018] to generate metagraphs instead of hand-crafting them as done in this paper. Thus, our framework can be quickly applied to new domains. Further, our framework is a two-stage process, i.e., the MF part and FM part, where we only utilize label information (ratings) to train FM while not to generate latent features from multiple metagraphs. This may also lead to information loss. Therefore, we plan to explore whether better latent features can be obtained if ratings are exploited in MF to generate latent features. To achieve this, a joint modeling of the two parts or an end-to-end deep learning model may be considered.

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