Choice of Offers by means of Multi-criteria Decision Systems, using Fuzzy Logic. Case Study

A M Prada¹, F S Blaga¹ and M Ursu²

¹Universitatea din Oradea, Facultatea de Inginerie Manageriala si Tehnologica, str.Universității nr.1, Oradea 410087, Romania
²Universitatea din Oradea, Facultatea de Inginerie Electrica si Tehnologia Informatiei, str.Universității nr.1, Oradea 410087, Romania

E-mail: andrei.prada.97@gmail.com

Abstract. This paper shows in detail the steps for the creation of an application in Matlab, using multi-criteria decision systems based on fuzzy logic. The aim of the application is to find the optimal offer for a client, based on the scores attained by simulation. In order to make the optimal offer in terms of quality and price for thermoplastic material parts, the main elements that are taken into account for the final price are cost of raw material, its quality and the price of the mould. The price of the raw material is mainly influenced by the annual necessary volume and by the project duration. The dimensions, the tolerances of dimensions, shapes and positions, the surface roughness, the coordinates of the critical points and the warranty of the mould have all a great influence on the price of the mould. This paper has two main chapters, of which the first one presents the selection procedure of an offer, and the definition of the multi-criteria decision systems that are used to evaluate the final offer, based on the accuracy of the choice for raw material and mould. The second chapter presents a case study which demonstrates the possibility to make such an application. The input quantities of the decision system must be carefully defined in order to attain results as coherent as possible.

1. Introduction

When there are multiple possible alternatives, the choice of an offer is a very complex process which depends on many criteria. Papers [1] and [2] are the basic theoretical support for the conceiving of the application. Paper [3] states “Multi-Attribute Decision Making is the most well-known branch of decision making. It is a branch of a general class of Operations Research (or OR) models which deal with decision problems under the presence of a number of decision criteria. This super class of models is very often called multi-criteria decision making (or MCDM). In cases in which the number of attributes is large (e.g., more than a few dozens), attributes may be arranged in a hierarchical manner. That is, some attributes may be major attributes. Each major attribute may be associated with several sub-attributes. Similarly, each sub-attribute may be associated with several sub-sub-attributes and so on. Although some MADM methods may explicitly consider a hierarchical structure in the attributes of a problem, most of them assume a single level of attributes (e.g., no hierarchical structure)”. Paper [4] states “there have been a variety of decision support systems, with definitional differences across disciplines and countries. These include a system focus, a model focus, an application focus, and decision aids (the latter a European term). Each of these views is described with examples. The focus of the paper is on model-focused decision support systems using multicriteria models, and decision aids, of which differences between multi-attribute utility theory, analytic hierarchy process, French and Belgian outranking methods, and other approaches are discussed. The underlying
preference function assumptions of each approach are compared. The many very rich applications throughout the world are reviewed”.

Paper [5] describes the fuzzy logic concept. It belongs to the much larger field of the fuzzy mathematics, a non-binary kind of logic where the truth values of the variables can be anywhere between 0 and 1 inclusively. The fuzzy logic proposes the concept of partial truth, which can vary between 100% true or 100% false. This approach contrasts with the Boolean logic, where the truth values can be only 0 for “false” and 1 for “true”.

The fuzzy logic is based on the fact that, in everyday life, humans make decisions by means of imprecise non-numerical information. Thus, the fuzzy sets are mathematical means to represent the imprecision of this information, so that these models will be able to recognize, represent, manage, use and interpret vague and uncertain information.

The fuzzy logic can be applied in many fields, of which the most well-known are the control theory and the artificial intelligence.

Paper [6] shows a multi-attribute decision system based on fuzzy sets which selects a project of many more available ones, for a construction company.

2. The Procedure for the Selection of an Offer

2.1. General Considerations

Starting from the next premises, the elaboration of an application in Matlab with Fuzzy Logic toolbox will begin.

The most important elements that are taken into account for the evaluation of an offer are the price of the material, the prices for material tests (when necessary, the compatibility of the material with the standards of the client is checked), the price of the mould and its dimensions.

The process begins from the design of the part, of which the following information is selected in order to make the offer:

- dimensions, tolerances for dimensions, position and shape, roughness, coordinates of critical points etc. – the mould specifications will be formulated using this information, which will influence the price and dimensions of the mould;
- specifications about the material and about its validation tests, several standards imposed or agreed by the client – the costs of the raw material investment for the fabrication of the part will be assessed.

An application will be made for this purpose, based on three decision structures which are useful in order to evaluate the costs of the offer. The output quantities of the decision systems “Choice of Material” (“Alegere material”) and “Choice of Mould” (“Alegere matrfă”) are the input quantities for the third decision structure, of which the output quantity will be a score that allows the evaluation of the offer as objectively as possible. All these are shown in Figure 1. Next, the elaboration steps of the three decision structures will be shown.

![Figure 1. Basic diagram.](image-url)
2.2. Definition of the decision systems based on fuzzy sets

2.2.1. Decision system for the choice of material

Figure 2 shows the decision system for the choice of the material, based on fuzzy sets. In order to implement the multi-criteria decision system for the evaluation of the material type the following steps must be taken:

A. Definition of the criteria (quantities for the decision process) which will be used for the evaluation of the material type:
   1. Price of material (“Pret material”);
   2. Compatibility to standard (“Compatibilitate cu standard”);
   3. Price of material tests (“Pret teste de material”).

B. Definition of the variation ranges for the input quantities.

The range for Price of material is shown by equation (1)

\[ D_{\text{Price of material}} = [0; 10] \]  

(1)

The range for Compatibility to standard is shown by equation (2)

\[ D_{\text{Compatibility to standard}} = [0; 100] \]  

(2)

The range for Price of material tests is shown by equation (3)

\[ D_{\text{Price of material tests}} = [2600; 2800] \]  

(3)

C. Definition of the linguistic variables associated to each input quantity

The linguistic variable price_of_material is associated to the input quantity Price of material (Figure 2.a).

The linguistic variable compatibility_to_standard is associated to the input quantity Compatibility to standard (Figure 2.b).

The linguistic variable price_of_material_tests is associated to the input quantity Price of material tests (Figure 2.c).

D. Definition of the linguistic terms associated to each input quantity

The linguistic terms associated to the input linguistic variable price_of_material are shown by relation (4):

\[ TL_{\text{price_of_material}} = \{ \text{very_low, low, average, high, very_high} \} \]  

(4)

The linguistic terms associated to the input linguistic variable compatibility_to_standard are shown by relation (5):

\[ TL_{\text{compatibility_to_standard}} = \{ \text{very_low, low, average, high, very_high} \} \]  

(5)

The linguistic terms associated to the input linguistic variable price_of_material_tests are shown by relation (6):

\[ TL_{\text{price_of_material_tests}} = \{ \text{very_low, low, average, high, very_high} \} \]  

(6)

E. Definition of the ranges for the output quantity

The range for Choice of material is shown by relation (7)

\[ D_{\text{Choice of material}} = [0, 100] \]  

(7)

F. Definition of the linguistic variables associated to the output quantity

The linguistic variable Choice_of_material is associated to the output quantity Choice of material (Figure 3).

G. Definition of linguistic terms associated to the output quantity

The linguistic terms associated to the output linguistic variable Choice_of_material are shown in relation (8):

\[ TL_{\text{Choice_of_material}} = \{ \text{very_unsuitable, unsuitable, satisfactory, suitable, very_suitable} \} \]  

(8)
Figure 2. Input quantities of the decision system Choice of material: a) price_of_material; b) compatibility_to_standard; c) price_of_material_tests

Figure 3. Score of Choice_of_material.
Decision system for the choice of material.

The dependance between the input quantities and the output quantity was defined by means of 125 inference rules:

1. If (price_of_material is very_low) and (compatibility_to_standard is very_low) and (price_of_material_tests is very_low) then (Choice_of_material is satisfactory)
2. If (price_of_material is very_low) and (compatibility_to_standard is very_low) and (price_of_material_tests is low) then (Choice_of_material is satisfactory)
3. If (price_of_material is very_low) and (compatibility_to_standard is very_low) and (price_of_material_tests is average) then (Choice_of_material is unsuitable)

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54. If (price_of_material is average) and (compatibility_to_standard is very_low) and (price_of_material_tests is high) then (Choice_of_material is very_unsuitable)
55. If (price_of_material is average) and (compatibility_to_standard is very_low) and (price_of_material_tests very_high) then (Choice_of_material is very_unsuitable)
56. If (price_of_material is average) and (compatibility_to_standard is low) and (price_of_material_tests is very_low) then (Choice_of_material is satisfactory)

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123. If (price_of_material is very_high) and (compatibility_to_standard is very_high) and (price_of_material_tests is average) then (Choice_of_material is satisfactory)
124. If (price_of_material is very_high) and (compatibility_to_standard is very_high) and (price_of_material_tests is high) then (Choice_of_material is unsuitable)
125. If (price_of_material is very_high) and (compatibility_to_standard is very_high) and (price_of_material_tests very_high) then (Choice_of_material is very_unsuitable)

Figure 5 shows the surface that was generated by the 125 rules of the decision system Choice of material.
2.2.2. Decision system for the choice of mould

Figure 6 shows the decision system for the selection of the mould, based on fuzzy sets. In order to implement the multi-criteria decision system for the evaluation of the mould the following steps must be taken:

A. Definition of the criteria (input quantities of the decision process) which will be used to evaluate the mould:
   1. Dimensions of mould;
   2. Price of mould.

B. Definition of the ranges for the input quantities

The range of Dimensions of mould is shown in relation (9):

\[ D_{\text{Dimensions of mould}} = [1100, 1300] \] (9)

The range of Price of mould is shown in relation (10):

\[ D_{\text{Price of mould}} = [50000, 70000] \] (10)

C. Definition of the linguistic variables associated to each input quantity

The linguistic variable dimensions_of_mould is associated to the input quantity Dimensions of mould (Figure 6.a).

The linguistic variable price_of_mould is associated to the input quantity Price of mould (Figure 6.b).

D. Definition of the linguistic terms associated to each input quantity

The linguistic terms associated to the input linguistic variable dimensions_of_mould are shown in relation (11):

\[ TL_{\text{dimensions_of_mould}} = \{\text{very_low, low, average, high, very_high}\} \] (11)

The linguistic terms associated to the input linguistic variable price_of_mould are shown in relation (12):

\[ TL_{\text{price_of_mould}} = \{\text{very_low, low, average, high, very_high}\} \] (12)

E. Definition of the ranges for the output quantity

The range for Choice of mould is shown in relation (13):

\[ D_{\text{Choice of mould}} = [0, 100] \] (13)

F. Definition of the linguistic variables associated to the output quantity

The linguistic variable Choice_of_mould is associated to the output quantity Choice of mould (Figure 7).

G. Definition of the linguistic terms associated to the output quantity

The linguistic terms associated to the output linguistic variable Choice_of_mould are shown in relation (17):

\[ TL_{\text{Choice_of_material}} = \{\text{very_unsuitable, unsuitable, satisfactory, suitable, very_suitable}\} \] (14)
Figure 6. Input quantities for the decision system Choice of mould:
   a) dimensions_of_mould; b) price_of_mould

Figure 7. Score Choice_of_mould

Figure 8. Decision system for the choice of the mould.

The dependance between the input quantities and the output quantity was defined by means of 25 inference rules:

1. If (dimensions_of_mould is very_low) and (price_of_mould is very_low) then (Choice_of_mould is very_suitable)
2. If (dimensions_of_mould is very_low) and (price_of_mould is low) then (Choice_of_mould is suitable)
3. If (dimensions_of_mould is very_low) and (price_of_mould is average) then (Choice_of_mould is satisfactory)

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11. If (dimensions_of_mould is average) and (price_of_mould is very_low) then (Choice_of_mould is suitable)
12. If (dimensions_of_mould is average) and (price_of_mould is low) then (Choice_of_mould is suitable)
13. If (dimensions_of_mould is average) and (price_of_mould is average) then (Choice_of_mould is satisfactory)

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23. If (dimensions_of_mould is very_high) and (price_of_mould is average) then (Choice_of_mould is unsuitable)
24. If (dimensions_of_mould is very_high) and (price_of_mould is high) then (Choice_of_mould is very_unsuitable)

25. If (dimensions_of_mould is very_high) and (price_of_mould is very_high) then (Choice_of_mould is very_unsuitable)

Figure 9 shows the surface generated by the 25 rules of the decision system Choice of mould.

2.2.3. Decision system for the choice of offer

Figure 12 shows the decision system for the evaluation of the offer, based on fuzzy sets. In order to implement the multi-criteria decision system for the evaluation of the offer the following steps must be taken:

A. Definition of the criteria (input quantities of the decision process) which will be used to evaluate the offer:
   1. Choice of material
   2. Choice of mould

B. Definition of the ranges for the input quantities

The range for Choice of material is shown in relation (15):

\[ D_{choice\ of\ material} = [0,100] \]  \( (15) \)

The range for Choice of mould is shown in relation (16):

\[ D_{choice\ of\ mould} = [0,100] \]  \( (16) \)

C. Definition of the linguistic variables associated to each input quantity

The linguistic variable Choice_of_material is associated to the input quantity Choice of material (Figure 10.a).

The linguistic variable Choice_of_mould is associated to the input quantity Choice of mould (Figure 10.b).

D. Definition of the linguistic terms associated to each input quantity

The linguistic terms associated to the input linguistic variable Choice_of_material are shown in relation (17):

\[ TL_{Choice\ of\ material} = \{very\ unsuitable, unsuitable, satisfactory, suitable, very\ suitable\} \]  \( (17) \)

The linguistic terms associated to the input linguistic variable Choice_of_mould are shown in relation (14).
E. Definition of the ranges for the output quantity
The range for Offer is shown in relation (18)

\[ D_{\text{offer}} = [0, 100] \] (18)

F. Definition of the linguistic variables associated to the output quantity
The linguistic variable Offer is associated to the output quantity Offer (Figure 11).

G. Definition of the linguistic terms associated to the output quantity
The linguistic terms associated to the output linguistic variable Offer are shown in relation (19):

\[ TL_{\text{offer}} = \{ \text{very_unsuitable}, \text{unsuitable}, \text{satisfactory}, \text{suitable}, \text{very_suitable} \} \] (19)

**Figure 10.** Input quantities for the decision system Offer: a) Choice_of_material; b) Choice_of_mould

**Figure 11.** Score of Offer.
Figure 12. Decision system for the evaluation of the offer.

The dependance between the input quantities and the output quantity was defined by means of 25 inference rules:
1. If (Choice_of_material is very_unsuitable) and (Choice_of_mould is very_unsuitable) then (Offer is very_unsuitable)
2. If (Choice_of_material is very_unsuitable) and (Choice_of_mould is unsuitable) then (Offer is unsuitable)
3. If (Choice_of_material is very_unsuitable) and (Choice_of_mould is satisfactory) then (Offer is unsuitable)

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11. If (Choice_of_material is satisfactory) and (Choice_of_mould is very_unsuitable) then (Offer is unsuitable)
12. If (Choice_of_material is satisfactory) and (Choice_of_mould is unsuitable) then (Offer is satisfactory)
13. If (Choice_of_material is satisfactory) and (Choice_of_mould is satisfactory) then (Offer is satisfactory)

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23. If (Choice_of_material is very_suitable) and (Choice_of_mould is satisfactory) then (Offer is suitable)
24. If (Choice_of_material is very_suitable) and (Choice_of_mould is suitable) then (Offer is very_suitable)
25. If (Choice_of_material is very_suitable) and (Choice_of_mould is very_suitable) then (Offer is very_suitable)

Figure 13 shows the surface generated by the 25 rules of the decision system Offer.

Figure 13. Surface generated by the decision system Offer.
3. Case Study
The part from which the application conceiving started is **Bezel LH FN**, an ornamental component of the trunk assembly for the client BOS GmbH, final client Volkswagen. Figure 14 shows the three-dimensional CAD model of this part.

![Figure 14. Part Bezel LH FN](image)

The initial premises about the evaluation of the raw material proposed for offering are presented next. The price and the compatibility with the standard imposed by the client will also be considered. Normally, only the materials 100% compatible to the standard are accepted, but, in crisis or force-majeure situations, alternative materials are also accepted, if they have similar properties to the standard and, of course, if they are agreed by the final customer, in our case Volkswagen.

According to Table 1, it is preferable to choose the first version of material because it is 100% compatible to the standard, although it is more expensive than the other two options. The table shows the prices of the materials and their compatibility-to-standard coefficient.

| Name                                | Price (EUR/kg) | Compatibility to standard (%) |
|-------------------------------------|----------------|-------------------------------|
| Luran S KR 2863C Q463 UV soul       | 6,04           | 100                           |
| PC/ASA Gebablend A soul black 4PK   | 3,40           | 60                            |
| BADALAC ASA/PC 120 H UV soul        | 3,95           | 60                            |

As for the testing of the materials, only the testing price will be considered, and the testing laboratory with the lowest prices will be selected. The testing prices are shown in Table 2.

| Test                      | Laboratory | Price (EUR) |
|---------------------------|------------|-------------|
| Flammability              | SGS        | 444,00      |
|                           | ASO        | 419,80      |
|                           | IMAT-UVE   | 478,00      |
| Light exposure test       | SGS        | 1560,00     |
|                           | ASO        | 1602,50     |
|                           | IMAT-UVE   | 1657,50     |
| Colorimetric evaluation   | SGS        | 138,00      |
|                           | ASO        | 134,80      |
|                           | IMAT-UVE   | 140,00      |
| Scratch resistance        | SGS        | 197,00      |
|                           | ASO        | 230,00      |
|                           | IMAT-UVE   | 205,00      |
| Mar resistance            | SGS        | 311,00      |
|                           | ASO        | 309,80      |
|                           | IMAT-UVE   | 305,00      |

Total SGS = 2,650,00 EUR; Total ASO = 2,696,90; Total IMAT-UVE = 2,785,50 EUR;

Table 3 shows the ranges of the input quantities.
Table 3. Ranges of the input quantities.

| Input quantity          | Range          |
|------------------------|----------------|
| Price of material      | [0;10]         |
| Compatibility to standard | [0;100]      |
| Price for material testing | [2.600;2.800] |

The scores of Table 4 were attained by means of simulation.

Table 4. Scores attained by simulation.

| Material                          | Testing laboratory | Price of material (EUR) | Compatibility to standard [%] | Price of material testing (EUR) | Score | Interpretation       |
|----------------------------------|--------------------|-------------------------|-------------------------------|--------------------------------|-------|----------------------|
| Luran S KR 2863C Q463 UV soul    | SGS                | 6,04                    | 100                           | 2.650,00                        | 76,5  | Suitable choice      |
| Luran S KR 2863C Q463 UV soul    | ASO                | 6,04                    | 100                           | 2.696,90                        | 63    | Suitable choice      |
| Luran S KR 2863C Q463 UV soul PC/ASA | IMAT-UVE                | 6,04                    | 100                           | 2.785,50                        | 47,3  | Satisfactory choice  |
| Gebablend A soul black 4PK PC/ASA | SGS                | 3,40                    | 60                            | 2.650,00                        | 67,5  | Satisfactory choice  |
| Gebablend A soul black 4PK PC/ASA | ASO                | 3,40                    | 60                            | 2.696,90                        | 58,1  | Satisfactory choice  |
| Gebablend A soul black 4PK PC/ASA | IMAT-UVE                | 3,40                    | 60                            | 2.785,50                        | 43    | Satisfactory choice  |
| BADALAC ASA/PC 120 H UV soul     | SGS                | 3,95                    | 60                            | 2.650,00                        | 62,1  | Suitable choice      |
| BADALAC ASA/PC 120 H UV soul     | ASO                | 3,95                    | 60                            | 2.696,90                        | 58,3  | Satisfactory choice  |
| BADALAC ASA/PC 120 H UV soul     | IMAT-UVE                | 3,95                    | 60                            | 2.785,50                        | 43,3  | Satisfactory choice  |

Figure 15 shows the best score attained by simulation.

Figure 15. Choice of material (“Choice of material”) – the best score attained by simulation.
As for the evaluation of the mould, the price will be considered and the average or under average prices will be selected, but also the dimensions of the mould will be taken into account. In our case, it is preferable that the dimensions (length and width) of the mould do not exceed 1200 mm, so that it can be mounted on the table of the injection machine with 400 tons closing force. Otherwise, it will be necessary to mount the mould on a bigger injection machine, which will increase the price of the part because of the standard hourly rate of each machine. Considering the number of cavities of the mould is specified by the Reinert company according to the dimensions, geometry, mass of the part and annual quantity requested by the client, and the fact that the average value of the injection cycle is only estimated in the offering phase, these two aspects will be neglected. Thus, in our case it is preferable to take the offer based on the price specified by ZMT company (60.400 EUR), and the application was made using this consideration. Table 5 shows the specifications for the moulds.

| Mould maker | Dimensions of mould [mm] | Number of cavities | Injection cycle duration (s) | Price (EUR) |
|-------------|--------------------------|--------------------|------------------------------|-------------|
| DZK         | 1150*350*630             | 1                  | 50                           | 67.500,00   |
| ZMT         | 1100*500*680             | 1                  | 50                           | 60.400,00   |
| KK          | 1200*700*770             | 1                  | 55                           | 63.500,00   |
| RYD         | 1300*500*670             | 1                  | 50                           | 52.980,00   |

Table 6 shows the ranges of the input quantities.

| Input quantity | Range         |
|----------------|---------------|
| Price of mould | [50.000; 70.000] |
| Dimensions of mould | [1.100; 1.300]   |

The scores shown in Table 7 were attained by means of simulation.

| Mould maker | Dimensions of mould [mm] | Price of mould [EUR] | Score | Interpretation |
|-------------|--------------------------|----------------------|-------|----------------|
| DZK         | 1.150                    | 67.500               | 22.1  | Unsuitable choice |
| ZMT         | 1.100                    | 60.400               | 49.1  | Satisfactory choice |
| KK          | 1.200                    | 63.500               | 32.6  | Unsuitable choice |
| RYD         | 1.300                    | 52.980               | 48.3  | Satisfactory choice |

Figure 16 shows the best score attained by simulation.
Table 8 shows the ranges of the input quantities.

| Input quantity       | Range  |
|----------------------|--------|
| Choice of material   | [0;100]|
| Choice of mould      | [0;100]|

The score shown in Table 9 was attained by simulation.

| Score Choice of material | Score Choice of mould | Score Offer | Interpretation         |
|--------------------------|-----------------------|-------------|------------------------|
| 76,5                     | 49,1                  | 72          | Suitable choice        |

Figure 17 shows the score attained by simulation.

4. Conclusions
Considering the case study presented in chapter 3, it can be concluded that the designed application gave consistent results.

According to the attained score, the choice of the offer proved itself to be a suitable one. The relevance of the results depends greatly on the accurate definition of the ranges for the input quantities and on the right number of the proposed linguistic variables.

Concerning the chosen raw material, the quality of material (represented by the standard compatibility coefficient) proved itself to be more important than the price, which was almost twice the price of the other two alternative materials.

About the choice of the mould, its dimensions were decisive at the expense of the price. Considering that this is a project with low annual requirement, the choice of a too big mould would have increased the final cost of the part because it would have been necessary to mount the mould on a bigger and stronger injection machine.

The choice of the offer was evaluated as suitable according to the two attained scores: suitable choice of material and satisfactory choice of the mould.

The future research will follow the increase of the decision quality using fuzzy sets by refining the inference rules. Also, a user interface will be designed.
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