The tunneling radiation of Kehagias-Sfetsos black hole under generalized uncertainty principle

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Abstract

We further the investigation on the Parikh-Kraus-Wilcezek tunneling radiation of Kehagias-Sfetsos black hole under the generalized uncertainty principle. We obtain the entropy difference involving the influence from the inequality. The two terms as generalizations of the Heisenberg’s uncertainty promote or retard the emission of this kind of black holes respectively.

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I. Introduction

Several years ago, P. Horava proposed a UV completion of gravity as a power-counting renormalizable theory of gravity with anisotropic scaling of space and time like \( x^i \rightarrow bx^i \) and \( t \rightarrow bt \) respectively and the spatial index \( i = 1, 2, \cdots, D \). Here \( D \) is dimensionality and \( z \) is called Lifshitz index [1, 2]. Within the frame of Horava-Lifshitz (HL) gravity mentioned above, a kind of black hole solution was obtained, called Kehagias-Sfetsos (KS) solution [3]. It is interesting that the KS black hole has two capture cross sections from inner and outer horizon due to the effect from HL scheme, but this kind of massive source becomes Schwarzschild black hole at infinity [3]. The various contributions have been paid to the KS black holes. According to the particle geodesics around the KS black hole, it was found that the particles will be either scattered or trapped near the horizon and interior space and the coupling constant belonging to the black hole governed by HL gravity will weaken the gravitation while modify the trajectories of the particle considerably [4]. In the Kraus-Parikh-Wilczek issue as Hawking radiation based on the semi-classical tunneling [5-11], the emission rate subject to the logarithmic entropy of KS black hole as an asymptotically flat solution of the HL gravity was calculated [12]. This work can help us to further understand the generalized gravity. The local thermodynamics of KS black hole was studied and the phase transition and stability of this kind of black hole were exhibited [13].

During the developments of quantum gravity and black holes, some new proposals such as the generalization of the Heisenberg’s uncertainty principle generated. The quantum gravity needs a minimal length of the order of the Planck length [14-20]. Further the generalized uncertainty principle (GUP) as the generalization of Heisenberg’s scheme was initiated and certainly modifies the quantum mechanics [21]. The GUP because of the existence of the fundamental scale of length attracted more attention [15, 22-27]. The generalization of the Heisenberg uncertainty relation in one-dimensional space can be used to cure the divergence from states density near the black hole horizon and relate the entropy of black hole to a minimal length as quantum gravity scale [28, 29]. The influence from the GUP on the Beckenstein-Hawking black hole entropy in the high-dimensional spacetime was discussed while the black hole radiation was investigated with the help of the tunneling formalism [28]. The Hawking tunneling radiation from black holes under the GUP corrections was also studied in the world that has more than four dimensions [29]. The tunneling radiation of a black hole involving a \( f(R) \) global monopole under GUP was considered [30]. Recently a parameter belonging to the GUP is estimated in virtue of the leading quantum corrections to the Newtonian potential [31] and the gravitational wave event GW150914 [32] respectively. The influence from the GUP seems not to be ignored.

It is necessary to promote the research on the tunneling radiation of the KS black holes based on the GUP. The terms associated with the Newtonian constant need to be added in the Heisenberg’s inequality once the gravity is taken into account [15, 22-27]. Research on the Hawking emission due to the KS black holes should not rule out the inequality including the gravitational corrections. As mentioned above, the tunneling radiation relating to logarithmic entropy of KS black hole
has been calculated with Kraus-Parikh-Wilczek technique [12]. Now we plan to reexamine the entropy and tunneling radiation of the same kind of black holes with the same technique while the Heisenberg’s uncertainty is generalized. We wonder how the GUP modifies the entropy difference and the tunneling probability of the KS black holes. We will list our results in the end.

II. The entropy difference of a radiating KS black hole under generalized uncertainty principle

Now we start to focus on the entropy of KS black hole involving the modified uncertainty principle. We introduce a metric of static and spherically symmetric black hole in the deformed HL gravity with \( \lambda = 1 \) [3],

\[
ds^2 = f(r)dt^2 - \frac{dr^2}{f(r)} - r^2(d\theta^2 + \sin^2 \theta d\phi^2) \tag{1}
\]

where

\[
f(r) = 1 + \gamma r^2 - \sqrt{\gamma^2 r^4 + 4\gamma Mr} \tag{2}
\]

and \( \gamma = \frac{16\mu^2}{\kappa^2} \) with constant parameters \( \mu \) and \( \kappa \). As a constant, \( M \) is positive. In the limit like \( r \to \infty \), the function \( f(r) \) subject to the KS black hole and shown in Eq.(2) approaches the corresponding component of Schwarzschild metric [3, 4, 13]. The component of KS black hole like function \( f(r) \) leads the outer and inner horizons as follow [3, 12, 13],

\[
r_{\pm} = M(1 \pm \sqrt{1 - \frac{1}{2\gamma M^2}}) \tag{3}
\]

Here we choose the outer ones \( r_{+} \) as event horizon \( r_{H} \). From Eq.(2) and (3), we obtain the Hawking temperature [12, 13],

\[
T_H = \frac{1}{8\pi} \frac{2\gamma r_H^2 - 1}{r_H(\gamma r_H^2 + 1)} \tag{4}
\]

We proceed our discussions in the context of GUP. The Heisenberg’s uncertainty principle can be generalized within the microphysics regime as [23, 27, 28, 33-43],

\[
\Delta x \Delta p \geq \frac{\hbar}{2}(1 - \frac{\alpha \Delta p}{\hbar} + \frac{(\beta p)^2}{\hbar^2} \Delta p^2) \tag{5}
\]

leading,

\[
y_- \leq x \leq y_+ \tag{6}
\]

with the choice,
\[ y_\pm = \left( \frac{l_p}{\hbar} \Delta p \right)_\pm \]
\[ = \frac{1}{2\beta^2}(\alpha + \frac{2\Delta x}{l_p}) \pm \frac{1}{2\beta^2}(\alpha + \frac{2\Delta x}{l_p}) \sqrt{1 - \left( \frac{2\beta}{\alpha + 2\Delta x} \right)^2} \] (7)

where \( \alpha \) and \( \beta \) are dimensionless positive parameters. The Planck length is denoted as \( l_p = \sqrt{\frac{\hbar G}{c^3}} \) with velocity of light in the vacuum \( c \). The terms with the Newtonian constant \( G \) provide the inequality with the gravitational effects. Following the procedure of Ref. [28, 29, 44], we choose,

\[ \Delta p' = \frac{\hbar}{l_p} y_- \]
\[ = \frac{\hbar}{2(\beta l_p)^2}(\alpha l_p + 2\Delta x)[1 - \sqrt{1 - \left( \frac{2\beta l_p}{\alpha l_p + 2\Delta x} \right)^2}] \] (8)

Approximately the uncertainty in the momentum from Eq.(8) is [44],

\[ \Delta p' \approx \frac{\hbar}{\alpha l_p + 2\Delta x} \] (9)

The combination of the GUP and the approximation (9) helps us to estimate the distance interval containing the gravitational corrections [44],

\[ \Delta x' \approx \Delta x[1 + \frac{(\beta l_p)^2}{2\Delta x(\alpha l_p + 2\Delta x)}] \] (10)

If the \( \beta \)-term in the GUP disappears, no gravitational effect will act on the distance interval no matter whether the other term with \( \alpha \) exists or not. Like Ref. [28, 29, 44], we select the corrected interval \( \Delta x' \) shown in Eq.(10) with the original size of black hole as the lower bound on the region like \( \Delta x = 2r_H \) to obtain the Hawking temperature of KS black hole under the GUP as follow,

\[ T_H' = \frac{1}{2\pi} \frac{\gamma \Delta x'^2 - 2}{\gamma \Delta x'^2 + 4} \]
\[ \approx T_H[1 + \frac{2\gamma^2 r_H^4 - 5\gamma r_H^2 - 1}{4r_H(2\gamma r_H^2 - 1)(\gamma r_H^2 + 1)(4r_H + \alpha l_p)\beta^2 l_p}]^{-1} \] (11)

Although the modified Hawking temperature has something to do with the parameters both \( \alpha \) and \( \beta \) from the gravitation, the gravitational effect will disappear if the variable \( \beta \) vanishes. The Bekenstein-Hawking entropy may be derived from the Hawking temperature by means of the following thermodynamics relation [5-7, 11],

\[ T_H = \frac{dE}{dS} \approx \frac{dM}{dS} \] (12)

where \( E \) is the total energy. In the case of GUP, the Hawking temperature \( T_H \) can be replaced as \( T_H' \). According to Eq.(12) and GUP, we perform the burden derivation to obtain the corrected entropy difference of the radiating KS black hole,
\[
\Delta S = \Delta S(\gamma, \alpha, \beta) = \pi(r^2_H - r^\prime_H) + \frac{3\pi C}{8}(\beta l_p)^2 \ln \frac{2\gamma r^2_H - 1}{2\gamma r^\prime_H - 1} \\
+ \frac{3\pi C}{16\sqrt{2}\gamma}(\alpha l_p)(\beta l_p)^2 \left( \ln \frac{\sqrt{2}\gamma r_H - 1}{\sqrt{2}\gamma r^\prime_H + 1} - \ln \frac{\sqrt{2}\gamma r^\prime_H - 1}{\sqrt{2}\gamma r_H + 1} \right) \\
+ \pi C\left[ \frac{1 - 6C}{64}\gamma (\alpha l_p)^2 + \frac{1 + 6C}{8} - \frac{2\gamma (\alpha l_p)^2}{(\beta l_p)^2} \ln \frac{4r^\prime_H + \alpha l_p}{4r_H + \alpha l_p} \right] \\
+ \frac{2\pi}{\gamma} \left[ 1 + \frac{1}{(\alpha l_p)^2} - \frac{\gamma}{8} C (\beta l_p)^2 \right] \ln \frac{r^\prime_H}{r_H} \\
+ \frac{\pi C}{2\gamma} \left( \frac{\gamma \alpha l_p}{8} - \frac{1}{\alpha l_p} \frac{1}{r_H} - \frac{1}{r^\prime_H} \right) (\beta l_p)^2 \\
\]  

(13)

where,

\[
C = \frac{8}{\gamma (\alpha l_p)^2 - 8} \\
\]

(14)

and \( r^\prime_H = M'(1 + \sqrt{1 - \frac{1}{2\gamma M^2} \beta}) \) which is similar to Eq.(3) while \( M' = M - \hbar \omega \). Here \( \omega \) is a shell of energy moving along the geodesics in the spacetime described by metric (1) [11]. In the similar process that we only set \( \beta = 0 \), the entropy change above return to be the results of Ref. [12],

\[
\Delta S_0 = -\pi(r^2_H - r^\prime_H) - \frac{2\pi}{\gamma} \ln \frac{r_H}{r^\prime_H} < 0 \\
\]  

(15)

In the case that \( \beta > 0 \), the parameter \( \alpha \) can amend the change in addition to the \( \beta \) terms in the Expression (13). The Hawking radiation as semiclassical quantum tunneling for KS black hole has been investigated with the help of Kraus-Parikh-Wilczek methodology while governed by the Heisenberg’s uncertainty principle of quantum mechanics [12]. The HL gravity that has a kind of solution named as KS black hole could be a candidate of quantum gravity which needs a minimal length of the order of the Planck length and initiates the GUP [14-21]. We reinvestigate the emission rate \( \Gamma = e^{\Delta S} [10, 11] \) and entropy of KS black hole under the GUP. In order to explore the influence from GUP on the quantum tunneling rate of the radiating KS black hole, we discuss the ratio \( \frac{\Delta S}{\Delta S_0} \) versus the variables \( \alpha \) and \( \beta \). The two parts with \( \alpha \) and \( \beta \) respectively making up the corrections contribute the opposite effects to the emission spectrum. The figure 1 declares that the bigger magnitude of \( \alpha \) will decrease the absolute value of the entropy change, which stimulates the emission rate of the KS black holes. It is found that the stronger influence from \( \beta \) part will increase the absolute value of \( \Delta S \) much more greatly in figure 2. The nature of entropy difference of this kind of black hole is negative, so greater absolute value of \( \Delta S \) retards the emission rate. The figures both demonstrate that the stronger HL coupling \( \gamma \) makes the whole curve of ratio \( \frac{\Delta S}{\Delta S_0} \) up, or equivalently make the radiation slower.

III. Discussion
In this work we study the entropy difference of radiating KS black hole under the generalized uncertainty principle. The GUP has two kinds of corrections. The negative part shown as $\alpha$-term makes the entropy change large, which promotes the radiation of KS black hole to keep it stable. The positive one like $\beta$-term leads absolute value of entropy difference $\Delta S$ whose nature is negative to be much larger if this term is greater, which means that the positive part added in the GUP will damp the emission of KS black holes. The stronger HL coupling $\gamma$ from the deformed gravity makes the ration $\frac{\Delta S}{\Delta S_0}$ greater, leading the black hole radiation slower.

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Figure 1: The solid, dashed, dot curves of the dependence of the ratio $\frac{\Delta S}{\Delta S_0}$ on $\alpha$ for $\gamma = 1, 1.1, 1.2$ respectively with $\beta = 5$ and $l_p = 1$ for simplicity.
Figure 2: The solid, dashed, dot curves of the dependence of the ratio $\frac{\Delta S}{\Delta S_0}$ on $\alpha$ for $\gamma = 1, 1.1, 1.2$ respectively with $\beta = 5$ and $l_p = 1$ for simplicity.