Stochastic Damage Constitutive Relationship of Steel-Reinforced Concrete Bond-Slip

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1. Introduction

The steel-reinforced concrete structures have been widely used in practical engineering due to their excellent durability, economical advantage, and antiseismic performance. The experimental research shows that there is a bond-slip problem between profile steel and concrete in steel-reinforced concrete structures, and the bond-slip problem has a significant adverse effect on the working performance of the steel-reinforced concrete structure [1–12]. At present, domestic and foreign scholars have made many achievements in the design and calculation theory of steel-reinforced concrete, but there are still many problems that have not yet reached a consensus. In particular, the bond-slip problem between profile steel and concrete is usually simplified or ignored in the calculation and analysis of steel-reinforced concrete structures. When studying the damage of steel-reinforced concrete structures, international scholars pay more attention to the study of damage effects on concrete performance, while the bonding damage of the interface between profile steel and concrete is neglected.

Based on the bond-slip properties of steel-reinforced concrete interface, the study of the bonding damage mechanism of steel-reinforced concrete interface will provide an important theoretical basis for the study of the damage performance of steel-reinforced concrete structures. However, concrete is a multicomponent, multiphase, and heterogeneous composite material; its internal constituent structure includes cement stone, aggregates of different shapes and sizes, and various capillaries (void structures) formed during the concrete preparation process [13], so the mechanical properties of concrete have obvious discreteness and randomness. Based on the study of bond-slip properties of steel-reinforced concrete interface, the randomness of concrete material properties is considered, a stochastic damage model based on the mesomechanical model of steel-reinforced concrete interface is established to extend the deterministic constitutive model to the probabilistic version. The structural relationship objectively reflects the non-determinism of the mechanical properties of steel-reinforced concrete materials under different stress stages and provides
a reference for the establishment of damage constitutive relations of steel-reinforced concrete under complex stress conditions.

2. Damage Model and Damage Index Definition

2.1. Analysis on Bonding Stress. The bonding force between profile steel and concrete is the key to ensure that the profile steel works with concrete. The bonding force between profile steel and concrete is mainly consisted of three parts: chemical bonding force, frictional resistance, and mechanical interaction. Researchers have shown that the chemical bonding force exists only in the original forming state of members. Once the bond-slip on the joint surface between profile steel and concrete occurs, the cement crystal will be sheared and crushed, and the chemical bonding force will be lost and converted into frictional resistance and mechanical interaction [10]. At the same time, the chemical bonding force will be generated within the diffusion length of the chemical bonding force that has not yet slipped, so that the frictional resistance, the mechanical interaction, and the chemical bonding force of the nonslip section can constitute a new force to jointly bear the external load and achieve a new balance (Figure 1).

2.2. Mesomechanical Model. According to the characteristics of the above-mentioned steel-reinforced concrete interface bonding, the spring-friction block model of J. Eeibl is introduced to simulate the force of the interface [14] (Figures 2 and 3). In Figure 2, the steel-reinforced concrete bonding surface of the steel-reinforced concrete pull-out test can be simulated by numerous spring-friction block elements as shown in Figure 3. Assume that the profile steel and concrete outside the bonding surface are rigid; consider only the force on the bonding surface, not the deformation of the profile steel and concrete, and then the external force of each spring-friction block element on the bonding surface is unified.

2.3. Analysis of Microscopic Element Mechanism. As shown in Figure 4, the spring and the friction block are connected in parallel, and the slip controller limits the slip starting position of the friction block, so that the friction block does not slip before the spring is broken. \( P_i \) is the external force borne by the spring in the spring-friction block element, \( P_f \) is the external force borne by the friction block in the spring-friction block element, and \( P_i = P_f (\Delta < \Delta_0) \) or \( P_i = P (\Delta \geq \Delta_0) \). \( \Delta_s \) is the tensile displacement of the spring, \( \Delta_f \) is the sliding displacement of the friction block, and \( \Delta_f = S (S < S_0) \) or \( \Delta_f = S - S_0 (S \geq S_0) \). As shown in Figure 5, the left graph shows the load-displacement curve of the spring and the friction block, and the right graph shows the overall load-displacement curve of the spring-friction block element. \( P_0 f \) is the sliding friction resistance of the friction block, \( \Delta_m \) is the ultimate sliding displacement of the friction block, and \( \Delta_0 \) is the ultimate tensile displacement of the spring. Only the spring is stressed when the slip of the bonding surface is less than the ultimate displacement \( \Delta_0 \) of the spring, and the load-displacement curve is an oblique straight line (from point 0 to point A); meanwhile the spring is broken when the slip is greater than or equal to the ultimate displacement \( \Delta_0 \) of the spring. It may be assumed that the maximum static friction force of the friction block is less than or equal to the ultimate bearing capacity of the spring, and then the friction block begins to slide, generating sliding friction, and its load-displacement curve is a horizontal straight line (from point A to point B).

When the slippage of the bonding surface \( S \) is less than the ultimate displacement \( \Delta_0 \) of the spring, the load on the steel-reinforced concrete interface is borne by the chemical bonding force, there is no frictional resistance or mechanical interaction, and the frictional resistance does not slip relatively; when the slip is equal to the ultimate displacement \( \Delta_0 \), the chemical bonding force disappears, the spring breaks, the load on the steel-reinforced concrete interface begins to be borne by the mechanical interaction and the friction resistance, and the friction blocks begin to slip relatively to each other and produce sliding friction; when the amount of slip continues to increase, due to the fact that the sliding friction remains unchanged and the spring-friction block elements cannot resist the continuous increase of external load, the adjacent spring-friction block elements will participate in the force in turn; when the slippage reaches the ultimate displacement \( \Delta_m \) of the friction block, the friction block falls off, the frictional resistance and the mechanical interaction are lost, and the spring-friction block element is completely broken; when all spring-friction block elements on the bonding surface are broken, the bonding surface is completely destroyed.

2.4. Definition of Damage Index. Assume that the interface damage of steel-reinforced concrete follows the continuum damage mechanics, and the interface damage variable is defined as the ratio of the fractured area to the total area; that is,

\[
D = \frac{A_w(S)}{A},
\]

where \( A_v \) is the damage area, that is, the area of the fractured interface; \( A \) is the original area, that is, the total bonded area of profile steel and concrete.

For the spring element, the damage area defined on the bonding surface is

\[
A_w(S) = \sum_{i=1}^{Q} H(S - S_0 - \Delta_0_i) dA_i,
\]

where \( A_i \) is the cross-sectional area of the \( i \)-th spring element; \( Q \) is the total number of spring elements on the original bonding surface; \( \Delta_0_i \) is the ultimate tensile displacement of the \( i \)-th spring, that is, \( \Delta_0 \) of the \( i \)-th spring. \( H(s) \) is Heaviside equation; that is,

\[
H(s - \Delta_i) = \begin{cases} 
0, & s \leq \Delta_0_i, \\
1, & s > \Delta_0_i.
\end{cases}
\]

Substituting equation (2) into equation (1), the damage variable \( D(S) \) of the spring element can be obtained as
**Figure 1:** Sketch of the bonding stress distribution.

**Figure 2:** Steel-reinforced concrete pull-out test.

**Figure 3:** Spring-friction block equivalent model.

**Figure 4:** Spring-friction block microelement.
According to the stochastic damage theory, the spring stiffness on the bonding surface of profile steel and concrete and the friction coefficient of the friction block should be stochastic, but doing this will result in very complex subsequent calculations. To simplify the calculation, it may be assumed that the spring stiffness and friction coefficient of friction block in all spring-friction block elements on the bonding surface are constant, and the ultimate deformation of the spring and friction block is considered as a stochastic variable; we also assume that $\Delta(z)$ and $\Delta(y)$ are stochastic variables that follow the same rules of distribution, so the spring damage index $D(S)$ and the friction block stochastic index $D_f(S)$ are also stochastic variables that follow the same rules of distribution.

### 3. Stochastic Damage Constitutive Relation

#### 3.1. Mesomechanical Model Failure Mode

The damage process of the bonding interface mesomechanical model is divided into four stages: no damage stage, elastic damage stage, plastic damage stage, and complete damage stage (Figure 6). In the no damage stage, the external force is less than or equal to the ultimate bearing capacity of the spring, the external force is entirely borne by the spring, the friction block has no relative slip, and work $W_p(S)$ done by the external force is all converted by the spring into elastic internal energy $W_e(S)$. As the load continues to increase, due to $\Delta(y)$ being a stochastic variable, the spring begins to rupture at the smaller point where $\Delta(y)$ is smaller, and the mesomechanical model enters the elastic damage stage. At this time, the external force work corresponding to the broken part of the spring is transformed into the spring breaking energy $W_d(S)$. After the spring is broken, the corresponding external force is borne by the friction block, and the model enters the plastic damage stage. At this stage, the friction block produces relative slip. When the external force is constant, the bond-slip continues to increase. Due to the fact that both $\Delta(z)$ and $\Delta(y)$ are stochastic variables, the friction block begins to fall off at the smaller part of $\Delta(z)$. The work of external forces is partly converted by friction into internal energy and partly into the fracture energy of the friction block. With the further increase of the slippage, the damaged area of the spring and friction block continues to increase, until all springs and friction blocks on the entire bonding surface are destroyed, and the model enters the stage of complete damage. It can be seen that the external force is converted into the elastic potential energy which is stored and the fracture potential energy is consumed.

#### 3.2. Stochastic Damage Constitutive Relation

Based on the above analysis of the failure mode of the bond-slip mesomechanical model of the steel-reinforced concrete interface, according to the law of conservation of energy, the energy balance equation can be obtained as

$$
\begin{cases}
W_p(S) = W_e(S), & S < S_0, \\
W_p(S) = W_e(S) - W_d(S), & S \geq S_0.
\end{cases}
$$

In equation (6),
Substituting equation (7) and equation (8) into equation (6), the following can be obtained:

\[
\begin{align*}
W_d (S) &= \frac{1}{A} \int_{S_0}^{S} \lim_{Q \to \infty} \left( \sum_{i=1}^{Q} E x H (S - \Delta_i) A_i dx + \int_{S_0}^{S} P_f^0 \frac{1}{A} Q \to \infty \sum_{i=1}^{Q} H (S - \Delta_i) A_i dx \right) \lim_{Q \to \infty} \frac{1}{A} Q \to \infty \sum_{i=1}^{Q} H (S - \Delta_i) A_i dx, \\
&= \int_{S_0}^{S} E x D (x) dx + \int_{S_0}^{S} P_f^0 D (x) D_f (x) dx
\end{align*}
\]

Substituting equation (7) and equation (8) into equation (6), the following can be obtained:

\[
\begin{align*}
\int_{S_0}^{S} P (x) dx &= \frac{1}{2} E S^2 \quad S < S_0, \\
\int_{S_0}^{S} P (x) dx &= \frac{1}{2} E S^2 + \int_{S_0}^{S} P_f^0 D (x) dx - \int_{S_0}^{S} E x D (x) dx - \int_{S_0}^{S} P_f^0 D (x) D_f (x) dx \quad S \geq S_0.
\end{align*}
\]

The derivative of \( S \) in equation (9) is
Studies have shown that the microscopic defects and congenital damage of concrete follow the lognormal distribution [12]. Therefore, it is assumed that the damage index and stiffness of the profile steel and concrete bonding surface are subject to lognormal distribution; that is, $\log D \sim (\mu_d, \sigma^2_d)$ and $\log E \sim (\mu_e, \sigma^2_e)$.

Then the average form of equation (10) can be written as

$$\mu_s(S) = \mu_eS, \quad S < S_0,$$

$$\mu_s(S) = \mu_eS[1 - \mu_D(S)] + \mu_E, S_0 \leq S \leq \sigma_D(S), \quad S \geq S_0.$$  \hspace{1cm} (11)

Similarly, its variance form can be written as

$$\text{var}P(S) = S^2 \sigma_e^2, \quad S < S_0,$$

$$\text{var}P(S) = S^2 \left( \sigma_e^2 + \sigma_D^2, \sigma_D^2 \right) + \mu_E \text{var} \left( 2 \sigma_D^2 + \mu_D^2 + \sigma_D^2 \sigma_{D_j}^2, \right) \mu_D \text{var} \left( 2 \sigma_D^2 + \mu_D, \right), \quad S \geq S_0.$$  \hspace{1cm} (12)

4. Experimental Research

To verify the accuracy of the damage model established in this paper, the pull-out test data of medium-sized steel-reinforced concrete in the literature [3] are compared with the numerical calculation results of the model.

4.1. Design and Production of Test Pieces. The experimental design of this paper is mainly based on the axial pull-out test (as shown in Figure 7). All profile steels used in the test pieces are made of two I-channel steel and two 6 mm thick steel plates (Figure 8) to embed the resistance strain gauges in the flanges and webs (longitudinal) (measuring the longitudinal bonding stress of the flange and the web, as well as the distribution along the anchor length). Production of the combined I-beam and arrangement of embedded measuring points is as follows: ① On the web of one of the two channel steel plates and in the middle of one of the two bonded steel plates, a 3 mm × 15 mm longitudinal through length groove is precisely by a milling machine, and a circular hole with a diameter of 6 mm is drilled longitudinally along the middle of the other ungrooved channel steel plates and the steel plates according to certain spacing requirements (from dense to thin) (for embedding slip sensors). ② Use a momentary strong adhesive (T-1 502 glue) to longitudinally paste the resistance strain gauge from dense to sparse according to certain spacing requirements (depending on the embedded length of the steel used for the test piece, the groove steel web, and the upper and lower flange steel plates, each patch is 8–12 pieces); after the insulation piece is insulated and moisture-proof, it is taken out from two ends of the groove through the connecting wire, and then the groove is filled, compacted, and smoothed with epoxy resin, the groove is made with acetone, and the steel surface outside the groove is cleaned with acetone. ③ After the epoxy resin is substantially solidified, the ungrooved channel steel and the corresponding grooved buried resistance strained channel steel are bonded into the I-beam by epoxy resin glue, and the two channel steels are tightly coupled and uniformly pressed for a certain time. To avoid the epoxy resin glue in the construction to block the reserved hole on the ungrooved steel web, the hole on the bonding side of the channel is protected with a tape in advance, and the clay is filled in the hole. ④ After the two channels of steel are firmly bonded, the outer surface of the flange is cleaned with acetone. Then, two corresponding lengths of steel plate, respectively, adhere to the outside of the two flanges of the combined channel steel by epoxy glue and put pressure on it evenly for a certain time to make it tightly bound. ⑤ After the outer steel plate of the flange is firmly bonded, the combined I-beam surface is thoroughly cleaned with acetone, and a slip sensor is installed on its flange and webs (Figure 8).

Geometry and section size of the I-beam are as follows: depth of section $h_s = 112$ mm; flange width $b_f = 96$ mm; flange thickness $t_f = 14.5$ mm; web thickness $t_w = 10.6$ mm; web height $h_w = 83$ mm; sectional area $A_s = 3602 \text{ mm}^2$; flange and web surface area $A_f = 835 \text{ mm}^2$ and $A_w = 2767 \text{ mm}^2$; perimeter $C_s = 572$ mm.

To understand the bond-slip mechanism, the main influencing factors, the distribution of bonding stress, and slip along the anchor length, four factors of the tensile test need to be considered comprehensively: concrete strength grade ($f_c$), concrete protective layer thickness ($c$), steel anchor length ($l_o$), and transverse hoop ratio ($\rho_{sv}$). According to the orthogonal test design principle, each factor considers 4 levels of design (regardless of the orthogonal conditions of interaction), and a total of 16 test pieces were designed.

The concrete of the test piece is based on the design strength grade, refers to the relevant provisions of the “Concrete and Reinforced Concrete Construction Manual,” and combines them with the relevant research results of the Building Engineering Materials Testing and Testing Center.
of Xidian University. The main materials for making concrete specimens are Qinling cement, Weihe sand, gravel (the diameter is 5~8 mm), and tap water.

The concrete is manually mixed and mechanically vibrated and poured evenly in batches at one time (make 4 thumps every day), and damaging the slip sensors and strain measuring points arranged on the embedded steel flanges and webs of the test piece is avoided as much as possible, and the vibrating place is strictly controlled during vibrating. At the same time as the specimen is poured, each batch of test pieces is made 3 cube strength test blocks (150×150×150 mm³) in the same batch of concrete. These test blocks and the test pieces were maintained under the same conditions for 28 days. According to the standard test method, unidirectional axial compression tests are carried out on the reserved concrete test block, and its relevant mechanical performance indexes are obtained. To protect the embedded profile steel with a slip sensor, the test piece adopts horizontal pouring concrete (as is shown in Figures 9 and 10).

The parameters for pulling out test pieces are listed in Table 1. The test pieces of interior steel are as follows: Φ16 (HRB335) for the main rib, Φ6 (HPB235) and Φ8 (HPB235) for the stirrup, and Q235 for both the steel plate and the channel steel. According to the metal tensile test method (GB228-87), the mechanical properties of the steel are tested.

4.2. Test Loading Scheme. The test is carried out in the Seismic Structure Laboratory of Xi’an University of Architecture and Technology. The loading devices used were a
100-ton pseudohydrostatic servo actuator and a 5000 kN long column tester, where the former is used for shaft pull-out and push-pull repeated loading tests and the latter for shaft push and short column tests.

The lower end of the test piece is free, and the upper end is the load end. The full section of the upper concrete is fixed by the steel plate (thickness of 40 mm), the beam, and the gantry device. The pull-out force is applied to the embedded steel and is transmitted from the steel to the concrete through the bonding between the steel and the concrete. The bonding stress between the steel and concrete and its relative slip occur at the loading end (upper end) of the test piece and gradually develop along the anchor length with the pull-out force increasing. This loading mode is similar to the stress state of the tension zone of the steel-reinforced concrete beam column.

The load used in the test is vertical static load, and the loading mode is monotonic axial load and repeated axial load. To avoid eccentricity during the test, physical alignment is carried out in a professional manner: the electronic dial gauges are placed symmetrically on both sides of the middle part of the specimen; by preloading and adjusting the loading device until the reading of the 2,000 measuring instruments is close, the basic alignment is considered.

Axial pull-out loading program is as follows: loading 2 tons per stage before the loading end of the test piece starts to slip and loading 1 ton per stage after the slip occurs. When the sliding reaches about 5 mm, the load tends to be stable, adopting the displacement of 2 mm or 4 mm which is controlled to be loaded until the steel is pulled out by about 200 mm. The proposed load and displacement are controlled by the hydraulic servo control system in a timely manner, monitored, and recorded online by a computer. This loading method can fully develop the slip between profile steel and concrete, so that the bonding splitting failure shape of the steel-reinforced concrete member is more apparent.

Axial launch and short column loading procedure is as follows: Load at 2 tons or 2.5 tons per stage, stabilize the data for about 2 minutes after reaching intended load, and then record the data and carry out the next level of loading until the ultimate load is reached. The intended load is directly controlled by the dial of the testing machine, monitored, and recorded by the computer at the right time. The test includes the bond load on the specimen, the relative slip strains of reinforcement and concrete at the loaded and free ends, the profile steel and its distribution along the anchor length, the distribution of relative slip of the reinforcement and concrete at the joint surface, and the crack width.

Measurement of end bond-slip is as follows: Electronic dial indicators (or dial indicators) are installed at the 4 corners of the loading end and the free end of the specimen to directly measure the relative slip value at the 4 corners of the end section which can be directly measured and the average value of the 4 tables can be used as the end bond-slip value. The data measured by the electronic dial indicator (or

![Figure 9: The embedded profile steel with strain and slip detection points has been set.](image1)

![Figure 10: Forming and pouring test pieces.](image2)
dial gauge) are collected and monitored by the computer at the right time.

Measurement of internal bond-slip is as follows: The relative slip (distribution) of the profiled flange and web on the concrete joint is measured directly by the slip sensor prepositioned inside and outside and web surfaces of the profiled flange and the measured data are collected and monitored by the computer at the right time. The test loading and testing device are shown in Figure 11.

4.3. Test Results and Analysis

4.3.1. Failure Mode and Process of the Test Piece. The final failure mode of a part of the test piece in the axial pull-out test of steel-reinforced concrete is shown in Figure 12.

(1) Bonding Splitting Failure. This kind of damage occurs in specimens with low reinforcement ratio, thin thickness of profile steel protective layer, and long embedded part of profile steel. In the test, most of the axes pulled out of the test piece suffered such damage. Destruction characteristics are as follows: the test piece has a short duration from initial cracking to failure. When the fracture occurs, the crack penetrates the entire specimen along the longitudinal direction of the profile steel. At the initial stage of loading, there are basically no cracks in the test piece, and there is no relative slip between the profile steel and concrete. When the load reaches the ultimate bond load of 20% to 40%, the chemical bond force in the bond force at the loading end of the test piece is basically lost. There is a slight slip between profile steel and concrete (about 0.05mm), but there is still no crack on the surface of the concrete; as the load increases, the slip at the loading end gradually increases; when the load reaches about the ultimate bond load (i.e., the test piece reaches the ultimate bond strength), the crack passes through the upper and lower parts of the test piece and extends to the full length. The hooping begins to bend and

| Test piece number | Concrete strength $f_{cu}$ (N/mm²) | Protective cover thickness $C_a$ (mm) | Sectional dimension (mm²) | Steel buried depth $l_a$ (mm) | Hoop form | Stirrup reinforcement ratio $\rho_{sv}$ (%) | The ratio of longitudinal reinforcement $\rho_l$ (%) | Steel ratio $\rho_a$ (%) |
|-------------------|----------------------------------|-------------------------------------|--------------------------|-----------------------------|----------|------------------------------------------|------------------------------------------|---------------------|
| A-1 (1)           | C60                              | 40                                  | $180 \times 200$         | 340                         | $\Phi 6@180$ | 0.18                                      | 3.35                                      | 10.01               |
| A-2 (2)           | C60                              | 60                                  | $220 \times 240$         | 540                         | $\Phi 8@100$ | 0.26                                      | 2.29                                      | 6.82                |
| A-3 (3)           | C60                              | 80                                  | $260 \times 280$         | 740                         | $\Phi 8@115$ | 0.34                                      | 1.66                                      | 4.95                |
| A-4 (4)           | C60                              | 100                                 | $300 \times 320$         | 940                         | $\Phi 8@135$ | 0.42                                      | 1.26                                      | 3.75                |
| A-5 (5)           | C50                              | 40                                  | $180 \times 200$         | 540                         | $\Phi 6@105$ | 0.26                                      | 2.29                                      | 6.82                |
| A-6 (6)           | C50                              | 60                                  | $220 \times 240$         | 340                         | $\Phi 6@135$ | 0.34                                      | 1.66                                      | 4.95                |
| A-7 (7)           | C50                              | 80                                  | $260 \times 280$         | 940                         | $\Phi 6@125$ | 0.26                                      | 1.26                                      | 3.75                |
| A-8 (8)           | C50                              | 100                                 | $300 \times 320$         | 740                         | $\Phi 6@105$ | 0.18                                      | 3.35                                      | 10.01               |
| A-9 (9)           | C40                              | 40                                  | $180 \times 200$         | 740                         | $\Phi 6@125$ | 0.26                                      | 3.35                                      | 10.01               |
| A-10 (10)         | C40                              | 60                                  | $220 \times 240$         | 940                         | $\Phi 6@145$ | 0.18                                      | 2.29                                      | 6.82                |
| A-11 (11)         | C40                              | 80                                  | $260 \times 280$         | 340                         | $\Phi 8@95$  | 0.42                                      | 1.66                                      | 4.95                |
| A-12 (12)         | C40                              | 100                                 | $300 \times 320$         | 540                         | $\Phi 8@100$ | 0.34                                      | 1.26                                      | 3.75                |
| A-13 (13)         | C30                              | 40                                  | $180 \times 200$         | 940                         | $\Phi 6@95$  | 0.34                                      | 3.35                                      | 10.01               |
| A-14 (14)         | C30                              | 60                                  | $220 \times 240$         | 740                         | $\Phi 8@110$ | 0.42                                      | 2.29                                      | 6.82                |
| A-15 (15)         | C30                              | 80                                  | $260 \times 280$         | 340                         | $\Phi 6@125$ | 0.18                                      | 1.66                                      | 4.95                |
| A-16 (16)         | C30                              | 100                                 | $300 \times 320$         | 540                         | $\Phi 8@130$ | 0.26                                      | 1.26                                      | 3.75                |

Note: The thickness of the protective layer of the test piece related to the longitudinal ribs is 25 mm; the configuration of the longitudinal ribs of the test piece is 6Φ16 (HRB335); the number in the parentheses of the test piece is the number corresponding to the picture (photo) at the back.
**Figure 11:** Test loading and testing device.

**Figure 12:** Pulling out the final crack and damage state of the test piece.
the profile steel is slowly pulled out, but is not bent yet. Longitudinal shear occurs between profile steel and concrete. However, due to the different design parameters of the test pieces, the crack width and failure process are clearly different, and the thickness of the protected layer and the anchor length have a significant impact. When the anchoring length is larger (>720 mm) and the thickness of the protective layer is relatively smaller (≤60 mm), the crack is smaller, and the damage process is relatively slow. When the anchoring length is smaller (<720 mm) and the thickness of the protective layer is larger (>60 mm), the crack is larger, and the damage process is relatively sudden. When the loading end slip is larger, the influence of the hoop ratio on the damage begins to be obvious: the number and width of the cracks of the test piece with a large hoop ratio (≥0.30%) hardly increase; and the crack width of the specimen with a lower hoop ratio (<0.30%) continues to increase until they are connected.

The bonding splitting failure belongs to brittle failure and should be avoided in the design of steel-reinforced concrete structural components.

(2) Bond Anchorage (i.e., Steel Pull-Out) Failure. This damage occurs in the test piece with a relatively large hoop ratio, shorter profile steel buried, and a large thickness of the profile steel protective layer. In the test, the damage occurred in specimens A-11 and A-15 pulled out of the axis. Destruction characteristics are as follows: when the load reaches about 80% of the ultimate bond load, the microcracks perpendicular to the flange begin to appear on the inner side of the steel wing margin protection layer loaded at the end of the test piece but then continue to load until the profile steel is pulled out. The shear cracks on the inside of the protective layer around the profile steel are mainly extended to the full length of the profile steel within a small thickness range and expanded slightly to the outside of the protective layer. The hooping and the profile steel are usually in an elastic working state. After the test, it was found that the surface of the extracted profile steel was accompanied by a layer of crushed concrete powder and was mixed with fine particles.

The bond anchorage failure is also a brittle failure and should be avoided in the design of steel-reinforced concrete structural components.

(3) Profile Steel (under Tension) Yield Failure. This damage occurs in the test piece with a large hoop ratio, thick steel protective layer, and long profile steel buried. In the test, the A-4 specimen pulled out of the axis caused such damage destruction characteristics: before the load reaches the tensile yield load (i.e., the tensile profile steel yield reaches the tensile yield strength), the test piece has almost no cracks, and there is no obvious relative slip between the profile steel and concrete; when the load is close to the tensile yield load (i.e., the profile steel is close to the tensile yield strength), the crack occurs locally at the loading end of the test piece, and the crack width and the relative slip amount of the profile steel are small. After that, if the loading continues, the profile steel (end) is usually subjected to tensile yield and undergoes strengthening during the stage until it is pulled out.

The profile steel (under tension) yield failure is ductile failure; that is, the bond bearing capacity (or bond strength) between profile steel and concrete is greater than the tensile ultimate bearing capacity (or tensile strength) of profile steel. In the design of steel-reinforced concrete structural components, it (under tension) should be advocated.

4.3.2. The Comparison of Test Results and Theoretical Calculation Results. There are many factors involved in the bond-slip of steel-reinforced concrete, which are concealed and difficult to measure. Therefore, in previous research and engineering practice on bond-slip of steel-reinforced concrete, it is often used to simplify the problem by using the value of the applied load at the loading end of the test piece and the relative slip between the profile steel and concrete measured. Then the average bonding stress along the anchorage length of the profile steel is derived, and the relationship between the average bonding stress and the relative slip of the profile steel and concrete at the loading end of the test piece is fitted. In this way, the data are obtained from the test about the load value of the loading end of various test pieces and the relative slip between profile steel and concrete is statistically analyzed, and the simplified calculation formulas of average bonding stress and average bond strength which are convenient for engineering practice are proposed. The relationship between the average bonding stress along the (full) anchorage length and the effective anchorage length of the profile steel and the relative slip between the loaded-end steel and the concrete of the specimen are given, respectively. This research will lay a theoretical foundation for the study of the conversion rules of the steel-reinforced concrete bond-slip along the anchoring length of the profile steel.

In the numerical calculation, it is assumed that the spring damage index $D(e)$ and the friction block damage index $D_f(e)$ follow the same evolution law [6]:

$$D(S) = 1 - \exp \left[ -\frac{1}{2} \left( \frac{S - S_0}{aS_0} \right)^2 \right] \geq S \geq S_0,$$  (13)

$$D_f(S) = 1 - \exp \left[ -\frac{1}{2} \left( \frac{S - S_0}{bS_0} \right)^2 \right] \geq S \geq S_0,$$  (14)

where $a$ and $b$ are parameters related to the component size.

Equations (13) and (14) are substituted into equations (11) and (12). The P-S mean value curve and single variance fluctuation curve of the steel-reinforced concrete can be calculated. The corresponding test curves of A-13 and A-14 are shown in Figure 13.

Figure 13 shows the mean P-S curve calculated by the spring-friction block model theory and the fluctuation range of the P-S curve with one-fold variance as the changing amplitude. It can be seen that the variation law of the P-S test curve of each test piece is consistent with the change rule of the P-S curve of mean value being calculated theoretically. Among them, the P-S test curves of test pieces A-10 and
A-14 all fall within the range of one-fold variation of the mean value, and most of the test curves of test piece A-13 also fall within the range of one-fold variation of the mean value. It is shown that the theoretical model of spring-friction block stochastic damage established in this paper can simulate the P-S transformation law of the interface between profile steel and concrete in the mean sense, and it can accurately predict the error range brought by the constitutive relationship analysis results due to the discreteness of concrete material properties and the randomness of defects.

5. Summary of This Chapter

This paper presents a model of the bond-slip stochastic failure of the interface. Through theoretical analysis and experimental verification, the following conclusions are drawn:

(1) Based on the research on the bonding effect of steel-reinforced concrete, the mechanism of bond-slip failure at the interface of steel-reinforced concrete was analyzed.

(2) According to the distribution of chemical bonding force, frictional resistance, and mechanical bite force at the interface between profile steel and concrete and the transformation law between them, a mesoscopic model based on spring-friction block is established.

(3) Considering the discreteness of concrete performance and the randomness of defects and applying the stochastic damage theory, according to the energy transformation and conservation law of damage evolution process, the stochastic failure model of steel-reinforced concrete surface is established.

(4) The comparison between the numerical results of the model and the tensile test results of reinforced concrete shows that the theoretical model of stochastic destruction of spring-friction blocks can well average the simulate value of the P-S conversion rule at the interface of the profile steel and concrete, and the model can accurately predict the error range brought about by the constitutive relationship analysis results due to the dispersion of the properties of the concrete material performance and the randomness of the defects. This model will provide a theoretical basis for the simulation analysis of the refined damage of steel-reinforced concrete structures [15].

Data Availability

All the data, models, and code generated or used during the study appear in the submitted article.

Conflicts of Interest

The authors declare that they have no conflicts of interest.

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