Acceleration of the cosmic expansion induced by symmetry breaking

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It is proved that in order to obtain a model of the accelerated cosmic expansion the thing one only need to do is to add a perturbation term to the Einstein-Hilbert Lagrangian. This term leads to some symmetry breaking terms in the fields equation, which makes the cosmic expansion accelerating. A vacuum de Sitter solution is obtained. A new explanation of the acceleration of the cosmic expansion is presented. In this model the changing of the expansion from decelerating to accelerating is an intrinsic property of the universe without need of an exotic dark energy. The acceleration of the cosmic expansion is induced by the symmetry breaking perturbation of the gravitational energy. The cosmological constant problem, the coincidence problem and the problem of phantom divide line crossing are naturally solved. The results of the model are roughly consistent with the observations.

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I. Introduction

There is, by now, convincing evidence for new physics associated with the gravitational interaction on cosmological distance scales [1]. It is fashionable to attribute these phenomena to esoteric sources of energy-momentum [2], but it is conceivable that gravity itself should be modified by adding terms to the action for General Relativity [3]. Very recently, there have been proposals for constructing generalizations of teleparallel gravity [4] which followed the spirit of modified gravity [3] as a generalization of general relativity. The interest in these theories was aroused by the claim that their dynamics differ from those of general relativity but their equations are still second order in derivatives and, therefore, they might be able to account for the accelerated expansion of the universe and remain free of pathologies. It has been shown, however, that these theories are not locally Lorentz invariant [5]. Gravity is usually considered to be irrelevant as far as the physics of elementary particles is concerned and, in particular, in the context of the spontaneous symmetry breaking mechanism. For many years only fewer works [6] have been done in this direction and no work suggests a relation between symmetry breaking and cosmological phenomena. The aim of this letter is to show that the acceleration of the cosmic expansion can be produced by a symmetry breaking. By adding a perturbation term to the Einstein-Hilbert Lagrangian a new cosmological model is obtained. In this model the field equation include some symmetry breaking terms which play the role of the so-called "dark energy" and make the cosmic expansion accelerate. A vacuum de Sitter solution is obtained. The cosmological constant problem, the coincidence problem and the problem of phantom divide line crossing are naturally solved. The coefficient of the perturbation term is found

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to be a new cosmological parameter describing the evolution of the universe.

II. Field equations

We start from the action

\[ S = \frac{1}{2\kappa^2} \int \sqrt{-g} \left( R + \alpha \Gamma^2 + \mathcal{L}_m \right) d\Omega, \] (1)

where \( \kappa^2 = 8\pi G_N \) with the bare gravitational constant \( G_N \),

\[ \Gamma = \left\{ \{ \sigma^{\mu}_{\nu} \} \{ \rho^{\nu}_{\mu} \} - \{ \mu^{\nu}_{\rho} \} \{ \sigma^{\mu}_{\rho} \} \right\} g^{\rho\sigma}, \] (2)

with the Christoffel symbol \( \{ \nu^{\mu}_{\sigma} \} \), and \( \alpha \) is a new parameter describing the evolution of the universe. We will see that \( \alpha \) is very small and then the term \( \alpha \Gamma^2 \) can be considered as a perturbation term. The variational principle yields the field equations for the metric \( g_{\mu\nu} \):

\[ (1 + 2\alpha \Gamma) \left( R_{\rho\sigma} - \frac{1}{2} R g_{\rho\sigma} \right) + \frac{\alpha}{2} g_{\rho\sigma} \Gamma^2 - 2\alpha \Gamma,_{\mu} \frac{\partial \Gamma}{\partial g^{\rho\sigma},_{\mu}} = \kappa^2 T_{\rho\sigma}. \] (3)

where \( R_{\rho\sigma} \) is the Ricci curvature tensor of \( \{ \sigma^{\mu}_{\nu} \} \), \( R = g^{\rho\sigma} R_{\rho\sigma} \), and

\[ T_{\rho\sigma} = -2 \frac{\delta \mathcal{L}_m}{\delta g^{\rho\sigma}} \]

is the energy-momentum of the matter fields. These equations can be re-arranged in the Einstein-like form

\[ R_{\mu\nu} - \frac{1}{2} R g_{\mu\nu} = -\frac{\alpha g_{\mu\nu} \Gamma^2}{2(1 + 2\alpha \Gamma)} + \frac{2\alpha \Gamma,_{\lambda}}{(1 + 2\alpha \Gamma)} \frac{\partial \Gamma}{\partial g^{\mu\nu},_{\lambda}} + \frac{\kappa^2}{(1 + 2\alpha \Gamma)} T_{\mu\nu}. \] (4)

The first and the second term on the right are not covariant and represent the "dark" energy-momentum which has the same origin with the energy-momentum of the gravitational field. We will see that it is this perturbational gravitational energy-momentum that induces the acceleration of the cosmic expansion.

III. Cosmological model

We now investigate the cosmological dynamics for the models of this theory. We consider a flat Friedmann-Lemaitre-Robertson-Walker spacetime with the metric

\[ g_{\mu\nu} = \text{diag} \left( -1, a(t)^2, a(t)^2, a(t)^2 \right), \] (5)

where \( a(t) \) is a scale factor. For the matter Lagrangian \( \mathcal{L}_m \) in Eq. (1) we take into account non-relativistic matter and radiation. The non-vanishing components of the Christoffel symbol are

\[ \{ 0^{0}_{0} \} = 0, \{ 0^{0}_{1} \} = \{ 0^{0}_{2} \} = 0, \{ 1^{0}_{1} \} = a \dot{a} \delta_{ij}, \]

\[ \{ 0^{0}_{i} \} = 0, \{ 1^{0}_{i} \} = \{ 2^{0}_{i} \} = \frac{\dot{a}}{a} \delta_{ij}, \{ 3^{0}_{i} \} = 0, i, j, k, ..., = 1, 2, 3. \] (6)
and

\[
\Gamma = \{\sigma_{\mu} \{\rho_{\nu} \} - \{\mu_{\nu} \} \{\sigma_{\rho} \}\} g^{\rho\sigma} = -6 \frac{\dot{a}}{a^2} = -6H^2, \tag{7}
\]

where \( H \equiv \dot{a}/a \) is the Hubble parameter and a dot represents a derivative with respect to cosmic time \( t \). The field equations (4) take the forms:

\[
3H^2 = \kappa^2 (\rho + \rho_{de}), \tag{8}
\]

and

\[
-2 \dot{H} - 3H^2 = \kappa^2 (p + p_{de}), \tag{9}
\]

or

\[
\frac{\ddot{a}}{a} = -\frac{\kappa^2}{6} ((\rho + \rho_{de}) + 3 (p + p_{de})), \tag{10}
\]

where

\[
\rho_{de} = \frac{18\alpha H^2 (6H^2 - 144\alpha H^4 + \kappa^2 p)}{\kappa^2 (1 - 30\alpha H^2)}, \tag{11}
\]

\[
p_{de} = \frac{6\alpha H^2 (6H^2 + 5\kappa^2 p)}{\kappa^2 (1 - 30\alpha H^2)}, \tag{12}
\]

are the density and the pressure of the "dark energy" \( \alpha \Gamma^2 \). The state equation of the "dark energy" is then

\[
w_{de} = \frac{p_{de}}{\rho_{de}} = \frac{1}{3} \frac{6H^2 + 5\kappa^2 p}{36H^2 - 144\alpha H^4 + \kappa^2 p}. \tag{13}
\]

The equations (8) and (9) yield

\[
\dot{H} = -\frac{\kappa^2}{2 (1 - 48\alpha H^2)} (\rho + p). \tag{14}
\]

In the vacuum we have a de Sitter solution

\[
\dot{H} = 0. \tag{15}
\]

For dust matter

\[
p = 0
\]

the equations (8), (9), (11), (12) and (13) become

\[
3H^2 = 108\alpha H^4 \frac{1 - 24\alpha H^2}{1 - 30\alpha H^2} + \kappa^2 \rho, \tag{16}
\]

\[
\dot{H} = -\frac{3H^2 (1 - 18\alpha H^2)}{2 (1 - 30\alpha H^2)}, \tag{17}
\]
\[ \rho_{de} = \frac{108 \alpha H^4}{\kappa^2 (1 - 30 \alpha H^2)}, \]  
(18)

\[ p_{de} = \frac{36 \alpha H^4}{\kappa^2 (1 - 30 \alpha H^2)}, \]  
(19)

and

\[ w_{de} = \frac{1}{3 (1 - 24 \alpha H^2)}. \]  
(20)

The evolution of \( w_{de} \) is illustrated in Fig. 1.

FIG. 1: The evolution of \( w_{de} \).

The equations (11), (12), and (13) indicate that the property of the ”dark energy” depends on the pressure \( p \) of the ordinary matter. In dust matter or vacuum, however, the property of the ”dark energy” depends only on the spacetime geometry as indicated by the equations (18), (19), and (20).

Letting

\[ w_{de} = -1 \]

and

\[ H_0 = 74 \text{km/sec/Mpc} \simeq 2.4 \times 10^{-18} \text{sec}^{-1} \]

one can compute

\[ \alpha = \frac{1}{18 (74)^2} = 1.0145 \times 10^{-5} \text{(km/sec/Mpc)}^{-2} = 9.6451 \times 10^{33} \text{sec}^2. \]  
(21)

Then letting

\[ w_{de} = -\frac{1}{3} \]

we obtain

\[ H = 90.631 \text{km/sec/Mpc}. \]  
(22)

Observations of type Ia supernovae at moderately large redshifts (\( z \sim 0.5 \) to 1) have led to the conclusion that the Hubble expansion of the universe is accelerating \( ^1 \). This is consistent also with microwave background measurements \( ^2 \). According to the result of \( ^3 \), \( H = 90.631 \text{km/sec/Mpc} \) corresponds to \( z \sim 0.88 \).
which is consistent with the observations.

Using the formula

\[ H = \frac{\dot{a}}{a} = -\frac{1}{1+z} \frac{\dot{z}}{z}, \]  

(23)

we have

\[ \dot{H} = \frac{dH}{dt} = \frac{dH}{dz} \frac{\dot{z}}{z} = -H (1+z) \frac{dH}{dz}. \]

Then the equation (17) leads to

\[ -H (1+z) \frac{dH}{dz} = -\frac{3H^2 (1 - 18\alpha H^2)}{2(1 - 30\alpha H^2)}, \]

and can be integrated

\[ z = \left( \frac{H}{H_0} \right)^{2/3} \left( \frac{18\alpha H^2 - 1}{18\alpha H_0^2 - 1} \right)^{2/9} - 1. \]

(24)

The function \( z = z(H) \) is illustrated in Fig. 2 which is roughly consistent with the observations.

The equation (8) indicates that during the evolution of the universe \( H^2 \) decreases owing to decreasing of the matter density \( \rho \). This makes \( w_{de} \) descend during the evolution of the universe. The evolution of the function \( w_{de} = w_{de} (\alpha H^2) \) given by (20) is illustrated in Fig. 1. According to (20) when \( H^2 = \frac{1}{12\alpha} \), \( w_{de} = -\frac{1}{3} \), \( \alpha \Gamma^2 \) changes from "visible" to dark as indicated by (10). If \( H^2 > \frac{1}{12\alpha} \), it decelerates the expansion, if \( H^2 < \frac{1}{12\alpha} \), it accelerates the expansion. When \( H^2 = \frac{1}{18\alpha} \), \( w_{de} \) crosses the phantom divide line \( -1 \). In other words, the expansion of the universe naturally includes a decelerating and an accelerating phase. \( \alpha \) given by (21) can be seen as a new constant describing the evolution of the universe.

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