Parallel electric field amplification by phase-mixing of Alfvén waves

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ABSTRACT

Context. Previous numerical studies have identified “phase mixing” of low-frequency Alfvén waves as a mean of parallel electric field amplification and acceleration of electrons in a collisionless plasma.

Aims. Theoretical explanations are given of how this produces an amplification of the parallel electric field, and as a consequence, also leads to enhanced collisionless damping of the wave by energy transfer to the electrons.

Methods. Our results are based on the properties of the Alfvén waves in a warm plasma which are obtained from drift-kinetic theory, in particular, the rate of their electron Landau damping.

Results. Phase mixing in a collisionless low-β plasma proceeds in a manner very similar to the visco-resistive case, except for the fact that electron Landau damping is the primary energy dissipation channel. The time and length scales involved are evaluated. We also focus on the evolution of the parallel electric field and calculate its maximum value in the course of its amplification.

Key words. Magnetohydrodynamics (MHD)-waves-Sun:Corona

1. Introduction

At finite wave-numbers in the direction perpendicular to the ambient magnetic field, Alfvén waves produce a compression of the plasma which results in the creation of a parallel electric field via the thermo-electric effect, i.e. due to electron pressure fluctuations along the magnetic field lines. This electric field, whose magnitude increases with \( k_z \), leads to wave-particle interactions, and hence, to collisionless damping of the wave.

Importance of this parallel electric field was pointed out already some time ago by (Hasegawa & Chen 1976). Indeed, they argue that “resonant absorption” (Hasegawa & Chen 1974) is a manifestation of mode conversion from the MHD Alfvén wave (AW) to the kinetic Alfvén wave (KAW) and that the physical mechanism of the heating depends on the collisionless absorption of the KAW. Although the original motivation was heating electrons in laboratory fusion plasmas, this electric field was also proposed as a mechanism which can accelerate electrons in space plasmas (Hasegawa 1976; Hasegawa & Mima 1978, Hasegawa 1985; Goertz & Boswell 1979) and for understanding solar coronal heating (Irons 1978).

(Hevaerts & Priest 1983) also introduced the idea of “phase-mixing” to improve the efficiency of AW dissipation. Their theory is based on visco-resistive magnetohydrodynamics (MHD). Since then, MHD phase mixing has attracted a significant amount of attention in the context of heating open magnetic structures in the solar corona (Parker 1991; Nakariakov et al. 1997; Bothe et al. 2004; De Moortel et al. 2006; Hood et al. 2005). Popular excitation mechanisms for coronal AWs in open magnetic structures are photospheric motions and chromospheric reconnection events, respectively for the low-frequency and high-frequency range of the spectrum.

Phase mixing can be understood as the refraction of the wave while it propagates along a magnetic field with transverse variation in the Alfvén velocity, i.e. the progressive increase of its \( k_z \). This is a special occurrence of conservative energy cascade (Bian & Tsiklauri 2008), a phenomenon generally attributed to non-linear interactions. Therefore, it is not surprising that phase mixing produces amplification of the parallel electric field that accompanies the AW in a collisionless plasma, although this cannot be understood within the framework of ideal MHD theory which assumes \( E_\parallel = 0 \).

Previous numerical studies of phase mixing in a collisionless plasma have identified its implication in the generation of a parallel electric field and acceleration of electrons (Tsiklauri et al. 2005, Tsiklauri & Haruki 2008), see also (Genot et al. 1999, 2004) in the magnetospheric context. As stated above, the same features were established already some time ago, by Hasegawa and Chen, for resonant absorption. Here, we provide a detailed discussion of the role played by phase mixing in both parallel electric field amplification and enhanced electron Landau damping of AWs in a collisionless plasma.

The calculations are based on the drift-kinetic theory presented in Section II, which is valid in the limit of low-frequency fluctuations with \( \omega \ll \omega_{ci}, \omega_{ce} \) being the ion cyclotron frequency. Phase mixing and enhanced electron Landau damping of AWs in a collisionless low-β plasma are considered in Section III. Parallel electric field amplification is analyzed in Section IV. Conclusions and discussions are provided in Section V.

2. Kinetic properties of the Alfvén wave in a warm collisionless plasma

Our starting point is the linearized drift-kinetic equation for the electrons:

\[
\partial_t f_i + v_i \nabla_i f_i - \frac{e}{m_e} E_\parallel \partial_{\parallel} f_i = 0
\]

The latter is supplemented by Maxwell’s equations. Faraday’s law is

\[
E_\parallel = -\nabla_\parallel \phi - \frac{1}{\varepsilon} \frac{\partial A_\parallel}{\partial t}.
\]
\( \phi \) is the electric potential, \( A_i \) the parallel component of the vector potential. The parallel component of Ampere’s law reads
\[
\nabla^2 A_i = \frac{4\pi e}{c} \int v_i f_i dv_i.
\]
(3)

The above system is closed by the quasi-neutrality condition, which in the limit \( k_i \rho_i \ll 1 \), reads
\[
n_0\rho_i^2 \nabla^2 \frac{e\phi}{T_0} = \int f_i dv_i.
\]
(4)

\( \rho_i \) is the thermal ion Larmor radius at the temperature \( T_0 \) and \( n_0 \) is the background density. We set the Boltzmann constant to unity which means that the temperature has the unit of energy. While this so-called gyrokinetic Poisson equation [Eq.(4)] includes the effect associated with the perpendicular ion polarization drift, the electron response along the perturbed field lines is described by the drift-kinetic equation [Eq.(1)].

We assume a small deviation \( f_1 \) from an equilibrium Maxwellian distribution \( f_0 \):
\[
f_0(v_i) = \frac{n_0}{\sqrt{\pi c e}} e^{-v_i^2/2c^2}.
\]
(5)

The electron density perturbation is defined as \( n_e = \int f_1 dv_i \) and the parallel current perturbation as \( J_i = -e \int v_i f_1 dv_i = -e n_0 u_{i|e} \), \( u_{i|e} \) being the electron parallel velocity, and the electron pressure perturbation is defined as \( P_e = m_e \int v_i^2 f_1 dv_i \). Hence, Ampere’s law and Poisson law can be written respectively as \( \nabla^2 A_i = -e/(4\pi c) J_i \) and \( \rho_i^2 \nabla^2 \frac{e\phi}{T_0} = n_e/m_e \). On one hand, taking the zeroth order moment of the electron kinetic equation provides the electron continuity equation:
\[
\frac{\partial n_e}{\partial t} + n_0 k_i u_{i|e} = 0
\]
(6)

On the other hand, the first moment provides the parallel electron momentum equation:
\[
n_0 m_e \frac{\partial u_{i|e}}{\partial t} = -i k_i P_e - n_0 e E_i.
\]
(7)

It is usual to refer to the last equation as the Ohm’s law and \( P_e = m_e T_0 \) for an isothermal plasma. Therefore, there are two possible sources of parallel electric field associated with the electron dynamics: inertia and pressure (or density) variations along the field lines. The continuity equation combined with Poisson law, yields a vorticity equation:
\[
\frac{\partial}{\partial t} \frac{e\phi}{T_0} \nabla^2 \frac{e\phi}{T_0} + \frac{c}{4\pi e n_0} i k_i \nabla^2 A_i = 0
\]
(8)

Neglecting first the effects of electron inertia and electron pressure gradient in Ohm’s law yields the MHD Ohm’s law \( E_i = 0 \), i.e.
\[
\frac{1}{c} \frac{\partial A_i}{\partial t} = -i k_i \phi.
\]
(9)

Introducing the stream and flux function for the velocity \( u_{\cdot|e} = z \times \nabla \phi, \) and the magnetic field \( B_{\cdot|e} / \sqrt{4\pi n_0 m_i} = z \times \nabla \psi, \) defined as \( \phi = (c/B_0) \) and \( \psi = -A_i / \sqrt{4\pi n_0 m_i}, \) gives
\[
\frac{\partial}{\partial t} \nabla^2 \phi = v A_i \nabla^2 \psi,
\]
(10)
\[
\frac{\partial \psi}{\partial t} = v A_i k_i \phi,
\]
(11)

with \( v_A = B_0 / \sqrt{4\pi n_0 m_i} \) being the Alfvén velocity. These two equations are the standard linearized reduced-MHD equations describing shear-Alfvén waves with frequency:
\[
\omega = \pm v_A k_i.
\]
(12)

In the case where the parallel electric field is produced by density fluctuation in Ohm’s law, we have \( E_i = -i k_i T_0 n_e/m_0 \). Using the Poisson equation, \( E_i = -i k_i \rho_i \nabla^2 \phi, \) which also reveals the vortical nature of the parallel electric field. The parameter \( \rho_i = c_i/\omega_\alpha = \sqrt{T_0/T_0 k_i} \) is the ion gyroradius at the electron temperature. By including this parallel electric field in Ohm’s law, an extension of the previous reduced-MHD system now takes the form
\[
\frac{\partial}{\partial t} \nabla^2 \phi = v A_i k_i \nabla \psi,
\]
(13)
\[
\frac{\partial \psi}{\partial t} = v A_i k_i (\psi - \rho_i^2 \nabla^2 \phi),
\]
(14)

which describes the dynamics of kinetic Alfvén waves with frequency
\[
\omega = \pm v_A k_i \sqrt{1 + \rho_i^2 k_i^2}.
\]
(15)

It is worth noticing that equations (13)-(14) can also be obtained directly from two-fluid MHD theory by retaining the Hall and electron pressure effects in Ohm’s law [Bian & Tsiklauri 2009]. Using the above results, it is easily seen that for kinetic Alfvén waves, the magnitude of the parallel electric field is related to \( B_{\perp} \) by
\[
E_i = \frac{v_A}{c} k_i \frac{k_i^2 \rho_i^2}{1 + k_i^2 \rho_i^2} B_{\perp}.
\]
(16)

The above fluid derivation of the Alfvén wave frequency gives the same result as its kinetic counterpart, however the latter, which is presented below, is more complete in the sense that it also provides the imaginary part associated with Landau damping. The electron kinetic equation can be solved for the perturbed distribution function \( f_1 \), i.e.,
\[
f_1 = \frac{e}{m_e} E_i \frac{2n_0}{\sqrt{\pi k_i^2 \rho_i^2}} \frac{v_i/v_{te}}{\sqrt{1 + \omega^2 k_i^2 v_{te}^2}} e^{-v_i^2/2c^2}.
\]
(17)

Some notations are introduced: \( x = v_i/v_{te}, \) \( \alpha = \omega/k_i v_{te} \) and
\[
Z_\alpha(\alpha) = \frac{1}{\sqrt{\pi}} \int \frac{x^\alpha}{x - \alpha} e^{-x^2} dx,
\]
(18)

with \( Z_\alpha(\alpha) \) being the standard plasma dispersion function. We also summarize some properties of the functions \( Z_\alpha: Z_1 = 1 + \alpha Z_2, Z_2 = \alpha Z_1. \) Moreover, in the limit \( \alpha \ll 1 \)
\[
Z_\alpha(\alpha) \sim -2\alpha + i \sqrt{\alpha^2 + 1 - \alpha^2},
\]
(19)

Using the above properties, it follows that the density and current perturbations are related to the parallel electric field through:
\[
\int f_1 dv_i = \frac{2ie_n}{m_e k_i v_{te}} [1 + \alpha Z_\alpha(\alpha)] E_i,
\]
(20)

for the density, and
\[
\int f_1 v_i dv_i = \frac{2ie_n \alpha}{m_e k_i v_{te}} [1 + \alpha Z_\alpha(\alpha)] E_i,
\]
(21)
Hence, the relation between parallel current and parallel electric field is
\[ J \parallel = \frac{-i\omega}{4\pi k_e^2}\frac{E}{i\omega_c} \left[1 + aZ_0(\alpha)\right]E_\parallel. \]  

(22)

It is convenient to define a collisionless plasma conductivity \( \sigma \) as
\[ J \parallel = \sigma E_\parallel. \]  

(23)

Its imaginary part results in the dispersion of the Alfven wave and the its real part yields the collisionless dissipation. In the limit \( \alpha \equiv \omega/k_{\|}v_{te} \ll 1 \), the real part is
\[ \sigma_\epsilon = e^2m_e^{1/2}n_0\omega^2/4k_e^2. \]  

(24)

This also gives the energy per united time transferred to the electrons through the relation:
\[ Q = Re(J\epsilon) \]  

(25)

i.e.
\[ Q = \frac{\sqrt{\pi}\omega^2}{k_e^2/\omega_c} U_{E_i} \]  

with \( A_{De} \) being the electron Debye length and \( U_{E_i} = |E^2/\omega_c|/8\pi \) being the energy density of the parallel component of the electric field. It is in fact a standard result that the asymptotic \( \omega t \gg 1 \) averaged power transferred to electrons, \( Q = \int |E|E_i > d|\parallel| \) due to the presence of an harmonic electric field fluctuation
\[ E_\parallel = \cos(kz - \omega t) \]  

is
\[ Q = -e^2E_\parallel^2/2m_e[k_{\|}v_{te}] |\parallel|/\omega_c. \]  

(27)

This can easily be verified from Eq.(1) and for a Maxwellian distribution it is equivalent to Eq.(26). Using the relation between \( E_\parallel \) and \( B_\perp \), \( Q \) can finally be expressed in term of the magnetic energy, \( U_B = B_\perp^2/8\pi \).
\[ Q = \frac{\sqrt{\pi}\omega^2}{k_e^2/\omega_c} \left[1 + k_{\|}^2/k_{\perp}^2\right] U_B. \]  

(28)

The coefficient of proportionality between \( Q \) and \( U_B \), which has the dimension of the inverse of a time, is nothing else than the Landau damping rate.

The Landau damping rate is now directly obtained, without any reference to its physical meaning, from the complex dispersion relation. The kinetic dispersion relation is obtained from:
\[ \nabla_k^2 A_\| = i\omega\left(k_\perp^2 A_{De} c^2[1 + aZ_0(\alpha)]E_\parallel + \rho_e^2k_\perp^2 \nabla_\perp^2 \phi = i/k_{\|}[1 + aZ_0(\alpha)]E_\parallel \right) \]  

and
\[ E_\parallel = -ik_\perp\phi + ioA_\perp/k_{\|}. \]  

(33)

This is the general complex dispersion relation for the dispersive Alfven wave. In the limit \( \alpha \ll 1 \), it reads
\[ \omega^2 = k_\perp^2v_{te}^2\left[1 + k_\perp^2/\rho_e^2(1 - i\sqrt{\alpha})\right]. \]  

(30)

Its real part corresponds to the frequency of the kinetic Alfven wave which was also derived from fluid theory above. Its imaginary part, which corresponds to the Landau damping rate [see also Eq.(28)] and reads
\[ \gamma(k) = \frac{\sqrt{\pi}}{2}v_{te}k_\perp^2/4k_e^2. \]  

(31)

Most calculations above were finalized in the limit \( \alpha \ll 1 \), in the opposite limit of \( \alpha \gg 1 \) one obtains the frequency and damping rate of the inertial Alfven wave, which has its parallel electric field balanced by the electron inertia in Ohm’s law. For frequency \( \omega \sim k_\perp v_A, \alpha \sim v_A/v_{te} \), hence the kinetic Alfven wave regime corresponds to \( v_A/v_{te} \ll 1 \) and the inertial Alfven wave regime to \( v_A/v_{te} \gg 1 \). In the following we continue to focus on the warm plasma regime corresponding to \( 1 \gg \beta_e \gg m_e/m_i \).

3. Phase Mixing

Phase mixing of a shear Alfven wave packet can be considered in the framework of an eikonal description:
\[ \frac{dx}{dt} = \nabla_k\omega, \]  

(32)
\[ \frac{dk}{dt} = -\nabla_s\omega, \]  

(33)

with \( \omega = \pm k_{\||}v_A \). These are the characteristics of the wave-kinetic equation
\[ \frac{de_\perp}{dt} + \nabla_\perp\omega \nabla_\perp e_\perp - \nabla_s \nabla_\perp e_\perp = -\gamma(k)e_\perp. \]  

(34)

In the latter equation \( e_\perp \) are the amplitudes of the wave-packets corresponding to \( \omega = \pm k_{\||}v_A \) and \( \gamma(k) \) is a wave-number dependent damping rate. Following the trajectory of a wave-packet in phase space \((x,k)\), its amplitude evolves according to:
\[ \frac{de_\perp}{dt} = -\gamma(k)e_\perp. \]  

(35)

The latter equation is integrated to give
\[ e_\perp(t) = e_\perp(0)\exp(-\int \gamma(k)dt). \]  

(36)

The principle of phase-mixing is simple: for any damping rate \( \gamma \) which is an increasing function of \( k \), any mechanism producing an increase in \( k \) as a function of time results also in a smaller damping time scale. This is precisely the situation when the Alfven wave packet propagates along field lines with a transverse variation of the Alfven speed: the wave packet is sheared. In this case, say \( v_A(x) = v_A', x \) being the unit vector in the parallel direction and \( x \) the transverse coordinate, then
\[ \frac{dk}{dt} = k_{\||}v_A', \]  

(37)

with by definition \( v_A' = v_A/L_{\perp} \). \( L_{\perp} \) being the characteristic length of the transverse inhomogeneity and \( k_{\||} = k_{\||}(t = 0) \). This means that \( k_{\||} \) increases linearly with time due to differential advection of the wave packets along the field lines, i.e.
\[ k_{\||}(t) = k_{\||}(0)t, \]  

(38)

where we have taken \( k_{\||}(t = 0) = 0 \) without loss of generality.

For a resistive MHD Ohms’ law, \( E_\parallel = \eta J_{\||} \) the following results are well known. The damping rate is \( \gamma(k) = \eta(k_\perp^2 + k_\perp^2)/4\pi = \alpha_\perp^2(k_\perp^2 + k_\perp^2) \), this is the fourier transform of the operator responsible for magnetic diffusion in the induction equation. Hence
\[ e_\perp(t) = e_\perp(0)\exp[-Dm_e^2 \int (1 + v_A'^2c^2)dt]. \]  

(39)
which in the limit \( t \gg v_{\perp A}^{-1} \) yields
\[
e_s(z) \sim e_s(0) \exp\left(-\frac{D_m v_A^2 k_{\perp}^2}{3} t^2 \right) \tag{40}
\]
Since, \( z = v_A t \), we also have
\[
e_s(t) \sim e_s(0) \exp\left(-\frac{D_m v_A^2 \omega^2}{3v_{pe}^2} t^2 \right), \tag{41}
\]
for an Alfven wave excited at \( z = 0 \) with frequency \( \omega \). In a collisionless plasma, when the dissipation is provided by electron Landau damping, with damping rate \( \gamma = \sqrt{\pi v_A^2 k_{\perp}^2 \rho_s^2} / 2 \nu_{le} \), the equivalent expressions are:
\[
e_s(t) = e_s(0) \exp\left(-\frac{\sqrt{\pi} v_A^2 k_{\perp}^2}{6v_{le}} \rho_s^2 \omega^2 \right), \tag{42}
\]
and also
\[
e_s(z) = e_s(0) \exp\left(-\frac{\sqrt{\pi} v_A^2 k_{\perp}^2}{6v_{le}} \rho_s^2 \omega^2 z^2 \right), \tag{43}
\]
for an Alfven wave excited at \( z = 0 \) with frequency \( \omega \). Hence, the phase mixing time scale is
\[
\tau_{pm} \sim \frac{v_A^{1/3} L_A^{2/3}}{\rho_s^{1/3} k_{||}} \tag{44}
\]
and the phase mixing length scale is
\[
l_{pm} \sim \frac{v_A^{2/3} v_{le}^{1/3} L_A^{1/3}}{\rho_s^{1/3} \omega}. \tag{45}
\]
Notice that the scaling of the phase mixing length scale with the frequency \( \omega \) in the spatial problem is different from that of resistive MHD phase mixing since the collisionless conductivity associated with electron Landau damping depends on \( \omega \), contrary to Spitzer conductivity. However, the dependence with time or distance of the decay law, like \( \exp(-\alpha_1 t^2) \) or \( \exp(-\alpha_2 z^2) \), are similar to resistive MHD phase mixing. The physical reason is obviously the common scaling of the damping rate \( \gamma(k) \) with \( k_{\perp} \) in the collisionless and collisional case.

The following comments are due. The enhanced electron Landau damping associated with phase mixing was first considered by Voitenko & Goossens (2000a). They derived a relation identical to Eq.45 [see equations (30) and (11) in Voitenko & Goossens (2000a)]. Moreover, results of the Particles-In-Cell (PIC) simulations carried by Tsiklauri & Haruki (2008) have produced \( l_{pm} \propto \omega^{-\zeta} \) with \( \zeta \approx 1.0 \), for the dependence of the phase mixing length scale \( l_{pm} \) with frequency \( \omega \). They also report that the parallel electric field associated with the Alfven wave is primarily balanced by the electron pressure gradient in their simulations. They attribute the scaling of \( l_{pm} \) with \( \omega \) to the effect of an "anomalous resistivity" due to "scattering of particles by magnetic fields" which "plays an effective role of collisions". Here, we emphasize that their PIC simulation results can be accurately interpreted as the "normal" effect of electron Landau damping of the KAW since it gives \( l_{pm} \propto \omega^{-\zeta} \) with \( \zeta = 1 \). We now elaborate on the parallel electric field amplification which is observed in the simulations.

### 4. Parallel electric field generation

For an Alfven wave created by a source through perturbation of the background magnetic field, a parallel electric field is produced, provided \( k_{\perp} \) is finite, which is given by Eq.(16):
\[
E_{||} = \frac{v_A}{c} k_{||} \frac{k_{\perp} \rho_s^2}{\sqrt{1 + k_{\perp}^2 \rho_s^2}} B_{||} \tag{46}
\]
It is this parallel electric field which is responsible for the Landau damping of the wave (see above). For a given \( k_{\perp} \) and \( \delta B_{||} \), this electric field is amplified provided that the \( k_{\perp} \) associated with the wave field is also amplified, \( E_{||} \) being a monotonic increasing function of \( k_{\perp} \). However, \( E_{||}(k_{\perp}) \) also reaches a plateau for \( k_{\perp} \rho_s \sim 1 \), which is the boundary between the MHD and the dispersive regime. Indeed,
\[
E_{||} = \frac{v_A}{c} k_{||} \frac{k_{\perp} \rho_s^2}{\sqrt{1 + k_{\perp}^2 \rho_s^2}} B_{||} \tag{47}
\]
for \( k_{\perp} \rho_s \ll 1 \) and \( E_{||} \) reaches its maximum, of the order of
\[
E_{||} = \frac{v_A}{c} k_{||} \rho_s B_{||} \tag{48}
\]
when \( k_{\perp} \rho_s \sim 1 \) or larger. Therefore, significant amplification of this parallel electric field can only occur in the range of wave-numbers where the wave is non-dispersive, i.e. it behaves as a shear-Alfven wave with frequency \( \omega \approx \pm k_{\perp} v_A \) and, hence, it can be subject to standard phase mixing.

From the results of the previous section we obtain the dependence with time of the parallel electric field strength during the phase mixing process:
\[
\tilde{E}_{||}(t) = v_A^2 k_{||} L_{A}^2 t \exp\left(-\frac{\sqrt{\pi} v_A^2 k_{||}^2}{6v_{le}} \rho_s^2 \omega^2 \right), \tag{49}
\]
where a normalized electric field \( \tilde{E}_{||} = E_{||} / (B_{||}(0) v_A/c) \) has been defined. The variation with time \( \tilde{E}(t) \) has the form \( \beta t \exp(-\alpha t^2) \), with a growth phase followed by a decay phase typical of the alternating field aligned current during phase mixing. Since \( z = v_A t \), then
\[
\tilde{E}_{||}(z) = v_A^2 k_{||} L_{A}^2 \rho_s^2 \exp\left(-\frac{\sqrt{\pi} v_A^2 k_{||}^2}{6v_{le}} \rho_s^2 \omega^2 z^2 \right), \tag{50}
\]
which has the form \( \beta t \exp(-\alpha t^2) \), for an Alfven wave excited at \( z = 0 \) with frequency \( \omega \). The above defined phase mixing time/length scales are precisely the scales associated with the amplification of the parallel electric field, i.e. the time/length scales for the parallel electric field to reach its maximum value given by:
\[
\tilde{E} \sim \frac{\omega v_{le}^{1/3} \rho_s^{4/3}}{v_A^{1/3} L_A^{1/3}} \tag{51}
\]
with \( \omega \approx k_{||} v_A \)

### 5. Conclusions

Previous PIC (Particles-In-Cell) simulations of "collisionless phase mixing" of Alfven waves (Tsiklauri et al. 2005a,b, Tsiklauri & Haruki 2008) have identified its relation to the generation of a parallel electric field and acceleration of electrons. Importance of this parallel electric field was first pointed out by Hasegawa and Chen in the context of resonant absorption, who
also showed that the dominant energy dissipation of the Alfven wave, in a collisionless low-$\beta$ plasma, involves energy transfer to the electrons (Hasegawa & Chen 1976). The role of electron Landau damping in “collisionless phase mixing” was also considered by (Voitenko & Goossens 2000a,b).

Focusing on the “kinetic” regime of the dispersive Alfven wave, when $v_A/v_{te} \ll 1$, we provided a detailed discussion of the role played by phase mixing in both parallel electric field amplification and enhanced electron Landau damping of the wave.

Qualitatively, the physics of collisionless phase mixing can be summarized as follow. A parallel electric field accompanies the propagation of Alfven waves with finite $k_\perp$ because they compress the plasma. The magnitude of this electric field is an increasing function of $k_\perp$ that saturates in the dispersive range when $k_\perp \rho_s \sim 1$ or larger. Therefore, any mechanism that produces an increase in $k_\perp$ also leads to the amplification of the parallel electric field associated with the Alfven wave. Phase mixing is such a mechanism, independently of the energy dissipation channel. Phase mixing is a special occurrence of energy-conserving cascade (Bian & Tsiklauri 2008). Such a cascade, predominantly involving perpendicular wave-numbers, is generally attributed to non-linear interactions, i.e. to turbulence. Existence of this parallel electric field and the dependence of its magnitude with $k_\perp$ yield a Landau damping rate which scales like $k_\perp^2$, just as visco-resistive damping. This can be demonstrated very simply in the framework of drift-kinetic theory. Therefore, in a collisionless plasma, phase mixing leads to enhanced electron Landau damping of the Alfven wave in a manner which is very similar to the well-studied case of enhanced visco-resistive damping. As a consequence, once the wave has Landau damped in a collisionless low-$\beta$ plasma, its energy has been transferred to the electrons, and the time and length scales involved have been evaluated for small amplitude perturbations. Moreover, we studied the evolution of the magnitude of the parallel electric field in the course of its amplification and calculated its maximum value.

We argued that the scaling of the phase mixing length scale with frequency, $l_{pm} \propto \omega^{-\zeta}$ and $\zeta \approx 1$, reported by (Tsiklauri & Haruki 2008) has a simple interpretation in term of electron Landau damping. PIC simulations of collisionless phase mixing are valuable tools because they can provide direct information on the modification of the electron distribution function involved in the acceleration process, see (Tsiklauri et al. 2005a,b), a feature which the present kind of analysis is not capable of. For this, a theoretical framework is needed, e.g. quasi-linear theory. This is the subject of ongoing work.

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