Pressure control strategy for high-pressure tubing

Guanwei Lin¹,³, Linhan Wang² and Kangrui Chen¹

¹Department of Electronic Engineering, College of Information Science and Technology, Jinan University, Guangzhou, Guangdong 510632, China
²Department of Mathematics, College of Information Science and Technology, Jinan University, Guangzhou, Guangdong 510632, China
³Email: lin_guanwei@163.com

Abstract. The entry and ejection of fuel into and out of high-pressure oil tube are the basis of many fuel engines. However, the intermittent working process of fuel entering and exiting will be leading to pressure changes in high-pressure oil tube. Furthermore, it will affect the efficiency of an engine. Therefore, it can be seen important to control high-pressure oil tube effectively. Also, it will be increasing the efficiency of an engine, while having a greatly impact on the development of engine manufacturing and energy industry. This paper is to focus on solving pressure control under high-pressure oil tube. The problems are stable pressure at the inlet of high-pressure oil tube and high-pressure oil tube with no pressure reducing valve and only one nozzle. Regarding the first one, we used the least square method to fit an elastic modulus and pressure data. Then, a differential equation model of pressure and time were set inside high-pressure oil tube. Meanwhile, we also worked on a delayed injection strategy and traversal search algorithm to have results. So, the optimal delayed injection time is while the one-way valve opening time is 3.06 ms. The maximum offset pressure is about 0.3 MPa. The second case which is based on the first case. In order to achieve pressure control, linking the pressure at the inlet of high-pressure oil tube to a cam and using the principle of mass conservation. So, it is stated that the pressure can be stabilized as much as possible when the angular velocity is 0.0275 rad/ms. Finally, we verified the feasibility of the model through simulation analysis.

1. Introduction

The pressure change of the high-pressure oil tube deeply affects the efficiency of an engine. Many scholars studied high-pressure oil tube in different ways. For instance, Liu Xuelong et al. [1] established a simulation model and discussed main parameters which affect oil injection. They also optimized the characteristics of high-pressure oil tube to improve performance of high-pressure common rail system. Furthermore, Lv Xiaochen and his Colleagues [2] examined structure of the high-pressure oil tube, and discussed how the structure affects performance parameters of the fuel system, such as the oil supply pressure of a single pump, oil injection pressure and circulation oil injection quantity.

Although many scholars analysed the pressure of the high-pressure oil tube, most of them focus on the parameter research of the high-pressure oil tube. There are few people examine the pressure change of the oil tube caused by the intermittent process of fuel entering and exiting. The paper is to proposal a set of feasible pressure control scheme for high-pressure oil tube by studying the whole process of fuel entering and exiting and using the least square method, traversal search algorithm, differential equation, mass conservation and other methods, which has contributed to the development of this field. The
detailed parameters and working process of the high-pressure oil tube and relevant data are all from CUMCM-2019 Contest [3].

2. Pressure control of high-pressure oil tube with constant pressure at inlet

In order to keep the pressure of the high-pressure tube as stable as possible, it is required that the oil inlet and oil injection of the high-pressure tubing work together regularly. Firstly, there are four relationships can be used to obtain the relationship between internal pressure and internal oil density after a period of time t. They are the relationship between injection rate and time, and the flow in and out of the high-pressure tubing, the relationship between pressure and density and the relationship between elastic modulus and pressure. So, we developed the relationship between pressure and time \( \frac{dP}{dt} = f(P, t) \) by combining the density change, the flow rate of fuel oil inlet tube and the flow rate of fuel oil outlet tube [3]. However, it is found that the stabilizing effect was not ideal before optimization. Therefore, we adopt another strategy to find out the time-delay oil supply point through traversal. And, the inside oil pressure can be met and kept within the required pressure. Please see the Figure 1 as the schematic diagram of high-pressure tube [3].

![Figure 1. High pressure-tubing diagram.](image)

2.1. Establishment of mathematical model

According to the relevant data of elastic modulus and pressure [3], we can fit out

\[
E = 0.029P^2 + 3.077P + 1572
\]

The fitting curve is shown as Figure 2:

![Figure 2. The fitting curve of P and E.](image)

The relationship between pressure change and density change [3][4][5][6],

\[
dP = \frac{E}{\rho} \, d\rho
\]

(2)

Variation in density in high pressure tubing,

\[
d\rho = \frac{(\rho_H \rho_{in|\rho_{100}} - \rho_{out}) dt}{V_{cu}}
\]

(3)

where \( \rho_H \) is the density at the higher pressure, and the \( V_{cu} \) is the tubing volume [3].
The flow rate of oil into the tubing [3][5],

$$Q_{in} = CA \left( \frac{2AP}{\rho H} \right)^{\frac{1}{2}}, \Delta P = 160 - P$$

The flow rate of the oil jet tubing [3],

$$Q_{out} = \begin{cases} 
100t, & 0 < t < 0.2 \text{ ms} \\
20, & 0.2 \text{ ms} \leq t < 2.2 \text{ ms} \\
-100t + 240, & 2.2 \text{ ms} \leq t < 2.4 \text{ ms}
\end{cases}$$

Simultaneous equations (1), (2), (3), (4), (5), we can get

$$\frac{dP}{dt} = \frac{E_{2}\rho(Q_{in} - Q_{out})}{\rho v_{cu}} = f(P, t)$$

2.2. Model resolution

According to length and diameter of the inner cavity of the high-pressure tubing [3], its volume can be calculated as $V_{cu} = 39269.91 \text{ mm}^3$. Solve equation (6) and use MATLAB to draw the curve of P changing with t, as shown in Figure 3.

In one cycle of high-pressure oil injection, within a certain 100 ms, it is necessary to close inlet one-way valve at a certain time after the high-pressure pipe injection in order to stabilize the pressure as at 100 MPa inside the tubing. And, it is the time which the high-pressure oil returns to 100 MPa after the injection. As shown in Figure 3, if high-pressure tubing is opened simultaneously for oil intake and oil injection, the curve obtained will deviate a lot from 100 MPa in a certain section, and the curve with 100 MPa as the secant line goes up and down asymmetrically. Therefore, it is not difficult to analyse the reasons behind. During the period of 0.2 – 2.2 ms, the high-pressure tubing has a relatively large injection rate and small oil inlet rate. It also makes the pressure drop significantly and causes a largely deviation of 100 MPa in the pressure.

In order to achieve the upper and lower symmetry of the curve, we started feeding the oil for a period of time $t_d$ and spray instead the oil intake and oil injection at the same time. In other words, it means we took the delayed injection measures to achieve the purpose of stable pressure around 100 MPa.

In order to obtain the delay time, we need to find the target condition, that is, the maximum oil pressure offset reaches the minimum under a delay injection time. The set is obtained:

$$L = \{ \text{Max } \{P_d \text{ and } 100\} | (d = 1, 2, 3, ...) \}$$

In the equation (7), $P_d$ is a pressure change in the tube under the d delay injection time, and the optimal delay injection time is the delay $t_d$ which corresponds to the minimum value of the L set. We use the traversal search algorithm to solve the migration time $t_d$. The initial value of $t_d$ is 0 ms, the step size is 0.01 ms, and the final value is 90 ms (to ensure that the oil intake time is less than 100 ms). Finally, it is found that the optimal delay is 0.23 ms. After the adjustment, the curve of pressure P in the high-pressure tube with time t is shown in Figure 4.

![Figure 3. Before the adjustment.](image1)

![Figure 4. After the adjustment.](image2)
As shown in Figure 4, when the tube pressure returns to 100 MPa again, the one-way valve is closed and the pressure is stable. At this point, the opening time of the one-way valve is about 2.78 ms. The maximum pressure deviation is 0.41 MPa without applying the delayed fuel injection strategy. After that, the deviation is changed to 0.3 MPa and the stabilizing effect is increased by 27%.

3. Pressure control of high-pressure oil tube with no pressure reducing valve of only one nozzle

From the data provided in the attachment [3], we concluded that the cam edge curve and the needle valve motion curve. The plunger moves upward while the fuel in the plunger chamber is compressed. When the pressure in the plunger chamber is greater than the pressure in the high-pressure tubing, the one-way valve connected with the plunger chamber and the high-pressure tubing is opened. So, the fuel enters the high-pressure tubing. In a very short time, the oil pump and tube pressure on both sides of the valve is equal and stable at about 100 MPa. Therefore, the effective spray area of nozzle and the lift of the needle valve can be calculated from the movement curve of the needle valve. Also, the injection flow was concluded. Furthermore, we analysed the relationship between the cam motion curve and the oil supply rate. The ideal cam angular velocity can be obtained to meet the pressure stability in the tube at about 100 MPa. The actual high-pressure tube is shown in Figure 5 and Figure 6 [3].

![Figure 5. Schematic diagram of actual working process of high-pressure tubing.](image1)

![Figure 6. Schematic diagram of oil injector nozzle after magnification.](image2)

3.1. Establishment of mathematical model

According to the cam edge curve data [3], the difference between the longest polar diameter and the shortest polar diameter of the cam is

$$\Delta k = 7.2390 - 2.4130 = 4.8260 \text{ (mm)}.$$  

Therefore, with the change of the polar diameter, the maximum change of the volume of the plunger cavity is

$$\Delta V = \pi r_{pc}^2 \Delta k = 94.7583 \text{ (mm}^3).$$

where $r_{pc}$ is the radius of plunger cavity, and $r_{pc} = 2.5 \text{ mm}[3]$.  

From equation (2), when the pressure is 0.5 MPa, the density is $\rho_{0.5} = 0.8045 \text{ mg/mm}^3$. Therefore, when the plunger is at bottom dead center, the total mass of oil in the plunger cavity is

$$m_{\text{total}} = (\Delta V + 20) \times \rho_{0.5} = 92.3230 \text{ (mg)}.$$  

When the plunger moves to the top dead center, it is easy to know that the pressure in the plunger cavity and the pressure in the high-pressure oil tubing reach equilibrium. At this time, the mass of oil remaining in the plunger cavity is

$$m_{\text{residual}} = 20 \times \rho_{100} = 17 \text{ (mg)},$$  

where $\rho_{100} = 0.85 \text{ mg/mm}^3$. Therefore, in one movement period of the cam, the oil supply quantity of the plunger cavity is

$$\Delta m = m_{\text{total}} - m_{\text{residual}} = 75.3230 \text{ (mg)}.$$  

Now we analyze the oil supply process of the oil pump. When the cam is located at bottom dead center, the oil pump is filled with oil, and the polar angle is 3.14 rad, which is the starting polar angle. We assume that there is a critical point before the plunger rises to the top dead center. Before the plunger
reaches the critical point, the oil pressure in the oil pump cannot be greater than 100 MPa of the oil tubing, that is, the valve cannot be opened. When the critical point is reached, the valve opens, the oil pressure in the oil pump \( P_c = 100 \text{ MPa} \), and the density \( \rho_c = 0.85 \text{ mg/mm}^3 \). Therefore, the volume of oil at the critical point is

\[
V_c = \frac{m_{\text{total}}}{\rho_c} = 108.6135 \text{ mm}^3.
\]

And then, we have the polar diameter at the critical point is

\[
k_c = 2.7259 \text{ mm}.
\]

Now we analyse the movement process of needle valve. According to the geometric relationship, we have the radius of the circle obtained by cutting the cone with the plane at the bottom of the needle valve [3]

\[
r_t = [h_0 + h(t)] \times \tan 9', \tag{8}
\]

where \( h_0 \) is the height from cone apex when needle valve lift is 0, and \( h(t) \) is the height of the needle valve from the apex of the cone after a period of movement. Therefore, the difference in area is

\[
\Delta S = \pi r_t^2 - \pi r_{nv}^2, \tag{9}
\]

where \( r_{nv} \) is the radius of needle valve[3].

According to the data of the needle valve movement curve and the cam edge curve in the attachment[3][7], we can fit the data to obtain \( h(t) \) and \( k(\theta) \)

\[
h(t) = \begin{cases} h_1(t), & 0 < t \leq 0.45 \\ 2, & 0.45 < t \leq 2 \\ h_2(t), & 2 < t \leq 2.46 \\ 0, & 2.46 < t \leq 100 \end{cases}, \tag{10}
\]

\[
k(\theta) = 0.002397\theta^6 - 0.04517\theta^5 + 0.2593\theta^4 - 0.2872\theta^3 - 0.9499\theta^2 - 0.09409 + 7.247. \tag{11}
\]

If the initial position is at bottom dead center, then \( \theta = \omega t + 3.14 \) [3], so the change curve of cam polar diameter with respect to time can be expressed as \( g(t) \). \( Q_{in} \) is the amount of oil entering the oil tube per unit time, and because the oil pump inner bottom area is fixed, we have

\[
Q_{in} = g(t)\pi r_{p_c}^2 \tag{12}
\]

We know that \( Q_{out} = CA \left( \frac{2(P_{\text{tube}}-P_0)}{\rho_{100}} \right)^{\frac{1}{2}} \) [3], where \( P_0 \) is the atmospheric pressure, \( P_{\text{tube}} \) is the pressure in the oil tube, and \( \rho_{100} \) is the fuel density when the air pressure is 100 MPa. As we can be seen from Figure 6, the value of \( A \) is the smaller of \( S_n \) and \( \Delta S \), where \( S_n \) is the area of the nozzle. From the critical condition \( S_n = \Delta S \), we can have two critical time points,

\[
t_1 = 0.33 \text{ ms}, t_2 = 2.12 \text{ ms}.
\]

According to the size relationship between \( S_n \) and \( \Delta S \) at different times, \( A \) has different values, and we can have different \( Q_{out} \).

\[
Q_{out} = \begin{cases} \frac{C\Delta S \left( \frac{2(P_{\text{tube}}-P_0)}{\rho_{100}} \right)^{\frac{1}{2}}}{2}, & 0 < t \leq t_1 \\ \frac{CS_n \left( \frac{2(P_{\text{tube}}-P_0)}{\rho_{100}} \right)^{\frac{1}{2}}}{2}, & t_1 < t \leq t_2 \\ \frac{C\Delta S \left( \frac{2(P_{\text{tube}}-P_0)}{\rho_{100}} \right)^{\frac{1}{2}}}{2}, & t_2 < t \leq t_s \\ 0, & t_s < t \leq T \end{cases} \tag{13}
\]

And \( t_s = 2.45 \text{ ms}, T = 100 \text{ ms} \) [3].

3.2. Model resolution

As the oil pressure in the tube should be as stable as possible, the input oil and the output oil need to meet the conservation of mass, that is

\[
m_{\text{out}} = m_{\text{in}} \tag{14}
\]
6

When the plunger moves upward, the pressure in the oil pump and the oil pipe on both sides of the valve is equal by increasing the pressure in the oil pump, opening the valve opens and entering the oil pipe. So, the volume of input oil and output oil can be regarded as equal. We assume that $V_{in}$ represents the oil supply volume for one revolution of the cam, while $V_{out}$ shows the oil volume for one cycle of oil injection. In order to make the total fuel supply amount equal to the total fuel injection amount, it is required that

$$aV_{in} = bV_{out}$$  (15)

where $a$ and $b$ are coefficients. At the same time, it is also required that the total time of oil supply is equal to the total time of oil injection, so it can be obtained that

$$aT_{in} = bT_{out}$$  (16)

where $T_{in}$ is the time for one revolution of the cam, and $T_{out}$ is the cycle of the oil injection nozzle [3].

The angular velocity of cam rotation is

$$\omega = \frac{2\pi}{T_{in}}$$  (17)

Simultaneous equations (8), (9), (10), (13), (15), (16), (17), and we set $P_{tube} = 100 \text{ MPa}$, we can have

$$V_{out} = 38.7912 \text{ mm}^3$$
$$\omega = 0.02750 \text{ rad/ms}$$

Therefore, when the angular velocity $\omega$ of the cam is equal to 0.02750 rad/ms, the pressure in the high-pressure oil pipe is as stable as possible at about 100 MPa.

4. Model verification

In this section, we simulated and analysed each part by compiling MATLAB [8] program. We also drew that the pressure variation curve with time in the oil tube to check whether our model can effectively control the pressure variation in the high-pressure oil tube.

4.1. Verification of results in section 2

According to the existing data [3], we established the best delayed injection time model and obtained that the opening time of the one-way valve is 2.78 ms. Moreover, the stabilizing effect with delayed injection is 27% higher than before. Figure 3 and 4 are the pressure variation curves in the high-pressure oil tube while we were writing MATLAB programs to simulate. As can be seen from the figure, the model has good pressure stabilization effect. And, the maximum pressure deviation is only 0.3 MPa. Therefore, the model is feasible.

4.2. Verification of results in section 3

We concluded that the pressure in the high-pressure oil tube can be kept at about 100 MPa when the cam has an angular velocity of 0.0275 rad/ms. We write programs to simulate the working process of the oil tube. The Figure 7 is the pressure curve of the high-pressure oil tube obtained by simulating for 2 s.

![Figure 7. Pressure curve after simulating for 2 s.](image)
We can obtain from the figure that the pressure of the high-pressure oil tube fluctuates up and down at 100 MPa, but the fluctuation range is not large, and the maximum pressure offset is about 3 MPa. Therefore, the model is feasible.

5. Conclusions
To be concluded, this paper mainly analyzed the pressure control of high-pressure oil tube and provided feasible schemes for pressure control of high-pressure oil tube under two different conditions. As shown in the Table 1.

| Conditions                                      | Measures                                      |
|------------------------------------------------|-----------------------------------------------|
| High-pressure oil tube with constant pressure at inlet | Oil injection is delayed by **0.23 ms** and the opening time of one-way valve is **3.06 ms** |
| High-pressure oil tube with no pressure reducing valve of only one nozzle | The angular velocity of cam is **0.0275 rad/ms** |

We developed a feasible solution for the pressure control of high-pressure oil tubing. In the future, it could contribute wisdom to engine manufacturing industry and energy-saving industry.

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