Constraints on the Neutrino Parameters from the ‘Rise-up’ in the Boron Neutrino Spectrum at Low Energies

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Abstract

The rise-up in boron neutrino spectrum at low energies has been studied within the framework of ‘pure LMA’ scenario. Indirect bounds on the spectral ‘upturn’ have been obtained from the available solar neutrino data. These bounds have been used to demonstrate the efficacy of the precision measurements of the ‘upturn’ for further constraining the neutrino parameter space allowed by SNO salt phase data. The sterile neutrino flux has been constrained in the light of the recent 766.3 Ty KamLAND spectral data.

Neutrino Physics is passing through a phase of spectacular development. Vast amount of solar and atmospheric neutrino data has been accumulated and the neutrino deficits have been established to be the consequence of non-standard neutrino physics. The most recent steps in this direction are the pioneering results from SNO and KamLAND experiments. The SNO experiment provided a model independent proof of solar neutrino oscillations and the terrestrial disappearance of reactor $\overline{\nu}_e$ in the KamLAND experiment has provided a further confirmation of the neutrino oscillation solution of the solar neutrino problem (SNP). This gives us confidence in the oscillation solution of the atmospheric neutrino anomaly.

The neutral current measurements at SNO [1] have, conclusively, established the oscillations of solar neutrinos. After the evidence of terrestrial antineutrino disappearance in a beam of electron antineutrinos reported by KamLAND [2], all other [3] explanations of the solar neutrino deficit can, at best, be just subdominant effects. After these pioneering experiments, there is no scope for doubting the physical reality of neutrino mass and the consequent oscillations. KamLAND is the first experiment to explore the neutrino parameter space relevant to SNP with a beam of terrestrial neutrinos and has, convincingly, demonstrated the existence of neutrino oscillations confined to the large mixing angle (LMA) region. The total

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event rate as well as the spectrum distortion at KamLAND are in good agreement with the LMA expectations. Recently, updated analyses of all the available solar and reactor neutrino data including KamLAND and SNO salt phase data have been presented [4]. However, even after the confirmation of the LMA MSW mechanism as a dominant solution of SNP, the oscillation parameters are not precisely known. A precise determination of these parameters will be of great importance for theory as well as phenomenology of neutrino oscillations in particular and particle physics in general.

The solar neutrino experiments have, already, entered a phase of precision measurements for oscillation parameters. On the other hand, the LMA solution is facing a deeper scrutiny. In fact, the completeness of the LMA solution is being questioned [5] and the scope for some possible subdominant transitions is being explored [6, 7] vigorously. Does the LMA solution satisfactorily explain all the solar neutrino data? Are there any observations indicating new physics beyond LMA? These are some of the relevant questions being posed. It is also the high time to put the LMA predictions to closer experimental scrutiny. There are, at least, two generic predictions of LMA [6] which point towards life beyond LMA. One of these is the prediction of a high argon production rate, \( Q_{Ar} \approx 3SNU \), for the Homestake experiment which is about 2\( \sigma \) above the observed rate. Another generic prediction of the LMA scenario is the ‘spectral upturn’ at low energies. Within the LMA parameter space, the survival probability should increase with decrease in energy and for the best fit point, the upturn could be as large as 10-15\% between 8MeV and 5MeV [6]. However, neither the SuperKamiokande (SK) nor SNO have reported any statistically significant ‘rise-up’ in the observed neutrino survival probability. Both these predictions of LMA can only be tested in the forthcoming phase of high precision measurements in the solar neutrino experiments and are crucial for confirmation of the LMA solution.

The distortions in the neutrino spectrum are an important factor in resolving the solar neutrino problem. These distortions arise due to the energy dependence of the survival probability as a result of which neutrinos with different energies survive in different proportions leading to distortions in the observed spectrum. Experimentally, the boron neutrinos are the most accessible source for the study of the distortions in the observed spectrum since the SK and SNO detect the boron neutrinos in the small energy bins over a wide energy range. Since, the LMA has emerged as a solution of the SNP, the spectrum distortions within the LMA scenario are of paramount importance for the final confirmation of the LMA as a solution of the SNP and, also, for possible physics beyond LMA.

In the present work, we focus on the ‘rise-up’ in the neutrino spectrum at low energies and demonstrate how a precision measurement of the ‘upturn’ can be used to further constrain the neutrino parameter space allowed by the SNO salt phase data. In the absence of concrete experimental results on the ‘rise-up’, we obtain indirect bounds on the ‘rise-up’ in the boron neutrino spectrum by comparing the boron neutrino survival probability obtained from the experiments with the asymptotic value of the corresponding LMA survival probability.

The apparent lack of the ‘rise-up’ in the observed boron neutrino spectrum at low energies has been sought to be explained by introducing subdominant transitions into sterile neutrinos [6] and/or antineutrinos [7]. In the present work, it has been shown that the ‘rise-up’ in the boron neutrino spectrum can be reduced significantly by choosing a suitable point within
the LMA parameter space itself. We examine the status of this ‘rise-up’ within the pure LMA scenario in the following manner. From the SNO salt phase data and the value of the neutrino mixing angle ‘\( \theta \)’ obtained from the global analyses, indirect bounds on the ‘rise-up’ are obtained. The constraints on the ‘rise-up’ and the boron neutrino survival probability are combined to further constrain the neutrino parameter space allowed by the SNO.

The LMA survival probability \[8\], to a very good approximation, can be written as

\[
P = \frac{1}{2} + \frac{1}{2} \cos 2\theta \cos 2\theta_m, \quad (1)
\]

where the mixing angle in matter is given by

\[
\cos 2\theta_m = \frac{\cos 2\theta - \beta}{\sqrt{(\cos 2\theta - \beta)^2 + \sin^2 2\theta}} \quad (2)
\]

and the ratio of matter to vacuum effects ‘\( \beta \)’ is given by

\[
\beta = \frac{2\sqrt{2} G_F N_e E}{\Delta m^2}. \quad (3)
\]

\( E \) is the energy of the neutrino and \( N_e \) is the electron number density at the point of maximal boron neutrino production i.e.e at. \( x = r/R_S = 0.05 \) where \( R_S \) is the solar radius so that

\[
G_F N_e = 0.4714 \times 10^{-11} \text{eV} \quad (4)
\]

at this point \[9\]. The energy dependence of the LMA survival probability \( P \) given by eqn. (1) is shown in Fig.1 (dashed line) along with its asymptotic value \( \sin^2 \theta \) (dotted line). The survival probability averaged over the production region of the boron neutrinos \[9\] has been plotted as a solid line. It can be easily seen that the analytical expression (1) is in fairly good agreement with the exact numerical result. The value of \( P \) is slightly increased by averaging over the production region. Moreover, the earth regeneration effects will, also, increase the survival probability only by a small amount. It can be seen that the percentage increase in the survival probability from the earth regeneration effects equals the day-night asymmetry. The expected day-night asymmetry at SNO is about 3% \[10\]. Thus, eqn. (1) is a fairly good approximation to survival probability.

Equation (1) can be written as

\[
P = \sin^2 \theta + R, \quad (5)
\]

where

\[
R = \cos 2\theta \cos^2 \theta_m \quad (6)
\]

is the rise-up in the survival probability. Obviously, \( R \) is always positive and increases with decrease in energy. The survival probability \( P \) is an increasing function of both \( \Delta m^2 \) and \( \theta \) in the allowed LMA region in contrast to \( R \) which is an increasing function of \( \Delta m^2 \) and a decreasing function of \( \theta \), within this region. The ‘rise-up’ \( R \) becomes zero for maximal mixing. Since, maximal mixing is rejected at 5.4 standard deviations, the ‘rise-up’ cannot be zero. Hence, a non-zero ‘rise-up’ is an inescapable consequence of the LMA scenario.
Global analysis of the SNO salt phase data along with other solar and reactor neutrino data yields \[ \Delta m^2 = 7.1^{+1.2}_{-0.6} \times 10^{-5} \text{eV}^2; \] \[ \theta = 32.5^{+2.4}_{-2.3} \text{deg}. \] (7) (8)

For these LMA parameters, we have

\[ \sin^2 \theta = 0.289^{+0.038}_{-0.036}, \] \[ P = 0.362^{+0.036}_{-0.031}, \] \[ R = 0.074^{+0.044}_{-0.025}. \] (9) (10) (11)

It is clear that R is about three standard deviations above zero and is large enough to be measured experimentally.

The value of the survival probability for the boron neutrinos can be calculated from the SNO CC and NC rates using the relation

\[ P = \frac{\phi_{CC}^{SNO}}{\phi_{NC}^{SNO}} \] (12)

where we have assumed transitions into active flavors only. Transitions into sterile neutrinos can be important and will be studied elsewhere [12]. Even though, neither SK nor SNO has reported any statistically significant ‘rise-up’, one can infer the ‘rise-up’ at 6.4MeV from SNO CC/NC ratio. Since, \( \sin^2 \theta \) is constrained by equation (9), we can constrain R using eqns. (5) and (9). In this manner, we can obtain an indirect upper bound on R. However,
the value of $\theta$ obtained from the global analyses is not model independent as a result of which the value of $R$ obtained in this manner will be model dependent and will be valid only within the LMA scenario.

The pure $D_2O$ data from SNO \[\text{I}\] gives

\[
\phi_{CC}^{SNO} = 1.76^{+0.108}_{-0.103} \times 10^6 \text{cm}^{-2} \text{s}^{-1},
\]

\[
\phi_{NC}^{SNO} = 6.42^{+1.66}_{-1.67} \times 10^6 \text{cm}^{-2} \text{s}^{-1},
\]

where the statistical and the systematic errors have been combined in quadratures. From equation (12), we have

\[
P = 0.274^{+0.073}_{-0.073}.
\]

Using the LMA value of $\theta$ and equation (5), one can obtain

\[
R = -0.015^{+0.082}_{-0.081},
\]

which is not, significantly, different from zero. However, one can obtain an upper bound on $R$ from equation (16) viz.

\[
R \leq 0.120
\]

at 90\%C.L. It may be worthwhile to mention that the NC rate given in equation (14) has been obtained without any assumptions regarding the energy dependence of the survival probability. If one assumes an undistorted boron neutrino spectrum and, hence, an energy independent survival probability, SNO pure $D_2O$ data gives

\[
\phi_{NC}^{SNO} = 5.09^{+0.637}_{-0.608} \times 10^6 \text{cm}^{-2} \text{s}^{-1}.
\]

Using this value instead of the value quoted in equation (14) would give

\[
P = 0.346^{+0.048}_{-0.046}
\]

and

\[
R = 0.057^{+0.061}_{-0.058}
\]

in agreement with the LMA values given in eqns. (10) and (11). However, the LMA survival probability being energy dependent, the use of the value quoted in equation (18) for deriving constraints on neutrino parameters will not be internally consistent \[\text{I}\].

The most recent SNO salt phase data \[\text{II}\]

\[
\frac{\phi_{CC}^{SNO}}{\phi_{NC}^{SNO}} = 0.306^{+0.035}_{-0.035}
\]

can, also, be used to obtain the new bounds on $P$ and $R$ viz.

\[
P = 0.306^{+0.035}_{-0.035},
\]

\[
R = 0.017^{+0.052}_{-0.050}.
\]
This value of ‘rise-up’ will be used henceforth. The value of P given in equation (22) is smaller than the mean LMA value by an amount

\[ 0.057^{+0.068}_{-0.056} \]  

(24)

which is one standard deviation above zero. We shall explore the allowed LMA region to reduce the difference between the LMA values of P, R and their experimental values given by eqns. (22) and (23) respectively which imply the following upper bounds on P and R:

\[ P \leq 0.363, \]  

(25)

\[ R \leq 0.102, \]  

(26)

at 90%C.L. As noted earlier, the ‘rise-up’ R becomes smaller for smaller values of \( \Delta m^2 \) and larger values of \( \theta \). However, a larger value of \( \theta \) leads to an increase in the value of P. In fact, the experimental value of P is already greater than the mean LMA value and cannot be increased further. Hence, we consider the constraints (25) and (26) on P and R simultaneously. This can be achieved by plotting the constant P and constant R curves in the allowed parameter space. The curves corresponding to 90% C.L. upper bounds on P and R have been plotted in Fig.2 within the LMA parameter space allowed by the SNO. The overlap region below P and R curves is the region of parameter space allowed by the bounds on ‘rise-up’ and survival probability obtained above. The resulting upper bounds on \( \Delta m^2 \) and \( \theta \) are

\[ \Delta m^2 \leq 7.9 \times 10^{-5}eV^2, \]  

(27)

\[ \theta \leq 33.7 \text{ deg}, \]  

(28)

at 90% C.L. Thus, the ‘rise-up’ in the boron neutrino spectrum can be used to further restrict the neutrino parameter space. In fact, the bound on the ‘rise-up’ derived from SNO salt-phase data selects lower values of \( \Delta m^2 \) consistent with the conclusions reached by Aliani et al [4] who incorporated the SNO spectrum data in the global analysis. Therefore, the ‘pure LMA’ scenario will get rejected at more than 90% C.L. if the future precision measurements favor \( \Delta m^2 > 7.9 \times 10^{-5}eV^2 \). The value of \( \Delta m^2 \) larger than \( 7.9 \times 10^{-5}eV^2 \) will be clear signature of physics beyond LMA being manifest in the oscillations of solar boron neutrinos. The inclusion of the earth regeneration effect as well as the averaging over the production region will only decreases the value of \( \Delta m^2 \) and the upper bound mentioned above will, still, remain valid.

The curves P=0.342 and R=0.070 corresponding to 1.02\( \sigma \)C.L. are also shown in Fig. 2 below which there is no overlap. These two curves intersect at

\[ \Delta m^2 = 6.5 \times 10^{-5}eV^2, \]  

(29)

\[ \theta = 31.4 \text{ deg}. \]  

(30)

For these values of \( \Delta m^2 \) and \( \theta \), the difference between the LMA values of P, R and their experimental values (22) and (23) is the least (about one standard deviation). This can be regarded as the best fit point in the SNO allowed parameter space. The values of \( \Delta m^2 \) and
Figure 2: The constant $P$ and the constant $R$ curves plotted within the neutrino parameter space allowed by SNO salt phase data.

$\theta$ obtained from the global analyses of all the solar neutrino data \[10\] are very close to the values obtained here.

While this work was in progress, KamLAND reported 766.3 Ty spectrum data \[14\] which has been combined with the solar neutrino data by several authors \[4, 10, 14\]. The two main implications of the new KamLAND data are the increase in the value of $\Delta m^2$ to $8.3^{+0.40}_{-0.37} \times 10^{-5}$ eV$^2$ and a decrease in the best-fit value of $\theta$ to $31.3^{+1.9}_{-1.3}$ deg \[10\]. The best fit value of $\Delta m^2$ obtained in \[10\] is larger than the upper bound derived here (eqn. (27)) hinting towards possible new physics beyond LMA.

The inclusion of the earth regeneration effects will increase the values of $P$ and $R$ by only about 3% which is too small as compared to the rise-up (which is about 28%) [see Fig. 3]. Moreover, the LMA values of $P$ and $R$ are, already, larger than their experimental values. The earth regeneration effect will, therefore, further increase their values enhancing the mismatch between the theory and experiment. This would make the upper bound on $\Delta m^2$ even more restrictive.

For these values of $\Delta m^2$ and $\theta$, $P$ and $R$ will, now, become

\[
P = 0.376^{+0.017}_{-0.014}, \tag{31}
\]
\[
R = 0.106^{+0.023}_{-0.025}, \tag{32}
\]

in place of eqns. (10) and (11). The difference of $P$ from its experimental value (eqn. (22)) is given by

\[
0.076^{+0.039}_{-0.038} \tag{33}
\]

which is $2\sigma$ above zero. Hence, there is considerable difference between the experimental and theoretical values of $P$ in the LMA scenario and we are constrained to go beyond
the pure LMA scenario. A natural candidate for these transitions would be the spin flavor precession (SFP) driven transitions into antineutrinos. Since, there are very stringent bounds on the solar antineutrino flux [15], the transitions into antineutrinos cannot account for this difference. Hence, we attribute the whole of this difference to the transitions into sterile neutrinos and obtain [16]

\[ P(\nu_e \to \nu_e) = \frac{x \sin^2 \alpha}{1 - x \cos^2 \alpha}, \]
\[ P(\nu_e \to \nu_\mu) = (1 - P(\nu_e \to \nu_e)) \sin^2 \alpha, \]
\[ P(\nu_e \to \nu_S) = (1 - P(\nu_e \to \nu_e)) \cos^2 \alpha, \]

where

\[ x = \frac{\phi_{CC}}{\phi_{NC}}, \]

and

\[ P(\nu_e \to \nu_\mu) = 1 - P_{LMA}. \]

Here, \( \alpha \) is the sterile mixing angle. From these equations, we obtain

\[ P(\nu_e \to \nu_e) = \frac{x (1 - P_{LMA})}{1 - x}, \]

and

\[ \sin^2 \alpha = 1 - \frac{P_{LMA} - x}{1 - 2x + xP_{LMA}}. \]

Using equation (21) for \( x \) and equation (31) for \( P_{LMA} \), we obtain

\[ \sin^2 \alpha = 0.861^{+0.091}_{-0.077}, \]
and

\[ P(\nu_e \rightarrow \nu_e) = 0.275^{+0.055}_{-0.049}. \quad (42) \]

The pure sterile solution \((\sin^2 \alpha = 0)\) is disfavored at 11.2 standard deviations. From equation (36), we obtain

\[ P(\nu_e \rightarrow \nu_s) = 0.101^{+0.066}_{-0.069}. \quad (43) \]

at 3σC.L. which implies

\[ P(\nu_e \rightarrow \nu_s) \leq 0.299. \quad (44) \]

The sterile flux is non-zero at about 1.5 standard deviations. A more elaborate analysis is needed to constrain the sterile component using the approach adopted here and will be presented elsewhere [12].

In conclusion, the ‘rise-up’ in the boron neutrino spectrum at low energies has been studied within the framework of the LMA scenario. Indirect bounds on the rise-up have been obtained from the available solar neutrino data. These bounds have been used to demonstrate as to how a precision measurement of the rise-up can be used to further constrain the neutrino parameter space allowed by the SNO salt phase data. It is found that the pure LMA solution is sufficient to explain the SNO salt phase data for \(\Delta m^2 \leq 7.9 \times 10^{-5}eV^2\) and \(\theta \leq 33.7\) deg since larger values of \(\Delta m^2\) will violate the upper bound given in equation (26). However, the most recent global analyses [4, 10, 14] of the solar neutrino and the recent KamLAND data favor a value of \(\Delta m^2\) which violates this upper bound. Consequently, pure LMA solution seems to be disfavored and other subdominant transitions seem unavoidable. The theoretical and experimental values of the boron neutrino survival probability in the pure LMA scenario for the most recent LMA parameters differ by two standard deviations. This discrepancy is too large to be explained by the subdominant SFP transitions into antineutrinos. In the present work, this discrepancy has been attributed to the subdominant transitions into the sterile neutrinos. It is concluded that the sterile neutrino flux in this scenario could be as large as 0.299 times the boron neutrino flux at 3σ.

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