Supernatural A-term Inflation

CHIA-MIN LIN\(^1\) AND KINGMAN CHEUNG\(^2\)

Department of Physics, National Tsing Hua University, Hsinchu, Taiwan 300

Physics Division, National Center for Theoretical Sciences, Hsinchu 300, Taiwan

\(^3\)Division of Quantum Phases & Devices, School of Physics, Konkuk university, Seoul 143-701, Korea

May 7, 2009

Abstract

Following \([10]\), we explore the parameter space of the case when the supersymmetry (SUSY) breaking scale is lower, for example, in gauge mediated SUSY breaking model. During inflation, the form of the potential is \(V_0\) plus MSSM\(^1\) (or A-term\(^2\)) inflation. We show that the model works for a wide range of the potential \(V_0\) with the soft SUSY breaking mass \(m \sim O(1)\) TeV. The implication to MSSM (or A-term) inflation is that the flat directions which is lifted by the non-renormalizable terms described by the superpotential \(W = \lambda_p \phi^{p-1}/M_P^{p-3}\) with \(p = 4\) and \(p = 5\) are also suitable to be an inflaton field for \(\lambda_p = O(1)\) provided there is an additional false vacuum term \(V_0\) with appropriate magnitude. The flat directions corresponds to \(p = 6\) also works for \(0 \lesssim V_0/M_P^4 \lesssim 10^{-40}\).

\(^1\)cmlin@phys.nthu.edu.tw, \(^2\)cheung@phys.nthu.edu.tw

1 Introduction

Inflation \([1, 2, 3]\) (for review, \([4, 5, 6]\)) is an vacuum-dominated epoch in the early Universe when the scale factor grew exponentially. This scenario is used to set the initial condition for the hot big bang model and to provide the primordial density perturbation as the seed of structure formation. In the framework of slow-roll inflation, the slow-roll parameters are defined by

\[
\epsilon \equiv \frac{M_P^2}{2} \left(\frac{V'}{V}\right)^2, \tag{1}
\]

\[
\eta \equiv M_P^2 \frac{V''}{V}, \tag{2}
\]

where \(M_P = 2.4 \times 10^{18}\) GeV is the reduced Planck mass. The spectral index can be expressed in terms of the slow-roll parameters as

\[
n_s = 1 + 2\eta - 6\epsilon. \tag{3}
\]
The latest WMAP 5-year result prefers the spectral index around \( n_s = 0.96 \) \( [7] \). The spectrum is given by

\[
P_R = \frac{1}{12\pi^2 M_P^2 V^2}.
\]

With the slow-roll approximation the value of the inflaton field \( \phi \), in order to achieve \( N \) e-folds inflation, is

\[
N = M_P^{-2} \int_{\phi_{end}}^{\phi(N)} \frac{V}{V'} d\phi.
\]

From observation \( [7] \) \( P_R^{1/2} \simeq 5 \times 10^{-5} \) at \( N \approx 60 \) (we call this CMB (Cosmic Microwave Background) normalization).

The scalar potential for the inflaton field \( \phi \) of supernatural inflation \( [8, 9] \) (during inflation) is

\[
V = V_0 + \frac{1}{2} m^2 \phi^2
\]

where \( V_0 \equiv M_S^4 \) is the SUSY breaking scale with \( M_S \approx 10^{11} \) GeV and the second term is the soft mass term with \( m \sim O(1) \) TeV in order to address the hierarchy problem of the Standard Model (SM). Supernatural inflation is very attractive but predicts the spectral index \( n_s > 1 \). In \( [10] \), a model is proposed with the potential includes the A-term and non-renormalizable terms

\[
V = V_0 + \frac{1}{2} m^2 \phi^2 - A \frac{\lambda_p \phi^p}{p M_P^{p-3}} + \lambda_p^2 \phi^{2(p-1)} M_P^{2(p-3)}.
\]

Where \( 4 \leq p \leq 9 \) for the Minimal Supersymmetric Standard Model (MSSM) \( [11] \). When \( M_S \) is the gravity mediated SUSY breaking scale it is shown that the spectral index can naturally be \( n_s = 0.96 \), because the potential is naturally of a hilltop form (so called hilltop inflation \( [12, 13] \)).

The soft mass \( m \) is related to \( M_S \) via

\[
m \sim \sqrt{3} C M_S^2 / M_P
\]

\[
= C m_{\frac{3}{2}},
\]

where \( [14] \)

\[
C \sim 1 \quad \text{gravity-mediated}
\]

\[
C \gg 1 \quad \text{gauge/gaugino-mediated}
\]

\[
C \sim 10^{-3} \quad \text{anomaly-mediated}.
\]

As we can see from Eq. \( [9] \), for gravity-mediated SUSY breaking, \( M_S \sim 10^{11} \) GeV. For gauge mediated SUSY breaking, \( M_S \) can be as low as \( O(1) \) TeV. From Eq. \( [6] \), it is clear that the original model of supernatural inflation cannot work for lower \( M_S \). However, for the model Eq. \( [7] \), it certainly can. The reason is even if we have \( V_0 = 0 \) the potential still can accommodate for a successful inflation model if inflation occurs near a saddle point where \( V' = V'' = 0 \), and it is called MSSM inflation or A-term inflation in the literature \( [15, 16, 17, 18] \). In this paper, we will explore the parameter space for a successful inflation model from Eq. \( [7] \) near a saddle point with a wide range of \( V_0 \) from \( M_S = 1 \) TeV to \( M_S = 10^{11} \) GeV.

The paper is organized as follows. We present the calculation and result in Section 2. Our conclusions are summarized in Section 3.
2 Calculation

As in the case of MSSM inflation, the potential (Eq. (7)) has a saddle point where \( V' = V'' = 0 \) if \( m \) and \( A \) are related via

\[
m^2 = \frac{A^2}{8(p - 1)}
\]

(11)

The saddle point is at \( \phi = \phi_0 \) where

\[
\phi_0 = \left( \frac{m M_p^{p-3}}{\lambda_p \sqrt{2p - 2}} \right)^{\frac{1}{p-2}}.
\]

(12)

The potential at the saddle point is

\[
V(\phi_0) \sim m^2 \phi_0^2 + V_0
\]

(13)

Near the saddle point, the potential can be described as

\[
V = V(\phi_0) + \frac{1}{6} V'''(\phi_0) (\phi - \phi_0)^3
\]

(14)

where

\[
V'''(\phi_0) \sim \frac{m^2}{\phi_0}
\]

(15)

From Eq. (5), the number of e-folds is

\[
N = \frac{2}{M_P^2} \left( \frac{V_0 \phi_0}{m^2} + \phi_0^3 \right) \left( \frac{1}{\phi - \phi_0} \right)
\]

(16)

The slow roll parameters \( \epsilon \) and \( \eta \) are

\[
\epsilon = \frac{2(V_0 \phi_0 + \phi_0^3)^2}{N^4 M_P^6}
\]

(17)

\[
\eta = \frac{m^2 (\phi - \phi_0)}{V_0 + m^2 \phi_0^2} = -\frac{2}{N}
\]

(18)

Inflation ends when \( |\eta| \sim 1 \). The spectral index is (for \( N = 50 \))

\[
n_s \simeq 1 + 2\eta = 1 - \frac{4}{N} = 0.92
\]

(19)

which is the same as A-term inflation. The spectrum is given by

\[
P_R = \frac{V_0 + m^2 \phi_0^2}{12\pi^2} \frac{N^4 M_P^2}{4(V_0 \phi_0 + \phi_0^3)^2}
\]

(20)

After impose CMB normalization (\( P_R = (5 \times 10^{-5})^2 \)) for \( m = O(1) \) TeV, we obtain the relation between \( V_0 \) and \( \phi_0 \). We show the result on Fig. (1).

As we can see from Fig. (1), for \( 10^{-60} \lesssim V_0/M_P^4 \lesssim 10^{-40} \), the predictions is just the same as MSSM (or A-term) inflation. Actually the lower bound can go down to zero, and recovers MSSM (or A-term) inflation. For \( 10^{-40} \lesssim V_0/M_P^4 \lesssim 10^{-30} \), the existence of \( V_0 \) allows also \( p = 4 \) and \( p = 5 \) to fit CMB normalization with \( \lambda_p = 1 \). For \( p = 4 \), from Eq. (12), we have \( \phi_0 \simeq 10^{-8} M_P \), corresponds to \( V_0/M^4_P \sim 10^{-35} \). Similarly, \( p = 5 \) corresponds to \( V_0/M^4_P \sim 10^{-40} \).
3 Conclusion

In this paper, we generalize the model of [10] by considering smaller $V_0$. Our model is similar to hybrid inflation because the additional $V_0$ which eventually should gracefully exist by a waterfall field. However, it is different from the usual hybrid inflation because inflation occurs near a saddle point. The result is for $\lambda_p \sim 1$ and $m \sim O(1)$ TeV, the $p = 6$ flat direction can works with $0 \lesssim V_0 / M_p^4 \lesssim 10^{-40}$. The $p = 4$ and $p = 5$ flat directions can also play the role of an inflaton field with appropriate $V_0$.

Acknowledgement

This work was supported in part by the NSC under grant No. NSC 96-2628-M-007-002-MY3, by the NCTS, and by the Boost Program of NTHU. This work is also supported in parts by the WCU program through the KOSEF funded by the MEST (R31-2008-000-10057-0). We are grateful to David Lyth and Kazunori Kohri for discussions.
References

[1] A. A. Starobinsky, Phys. Lett. B 91, 99 (1980).
[2] K. Sato, Mon. Not. Roy. Astron. Soc. 195, 467 (1981).
[3] A. H. Guth, Phys. Rev. D 23, 347 (1981).
[4] D. H. Lyth and A. Riotto, Phys. Rept. 314, 1 (1999) arXiv:hep-ph/9807278.
[5] D. H. Lyth, Lect. Notes Phys. 738, 81 (2008) arXiv:hep-th/0702128.
[6] A. Linde, Lect. Notes Phys. 738, 1 (2008) arXiv:0705.0164 [hep-th]]
[7] E. Komatsu et al. [WMAP Collaboration], arXiv:0803.0547 [astro-ph].
[8] L. Randall, M. Soljacic and A. H. Guth, Nucl. Phys. B 472, 377 (1996) arXiv:hep-ph/9512439.
[9] L. Randall, M. Soljacic and A. H. Guth, arXiv:hep-ph/9601296.
[10] C. M. Lin and K. Cheung, arXiv:0901.3280 [hep-ph].
[11] T. Gherghetta, C. F. Kolda and S. P. Martin, Nucl. Phys. B 468, 37 (1996) arXiv:hep-ph/9510370.
[12] L. Boubekeur and D. H. Lyth, JCAP 0507, 010 (2005) arXiv:hep-ph/0502047.
[13] K. Kohri, C. M. Lin and D. H. Lyth, JCAP 0712, 004 (2007) arXiv:0707.3826 [hep-ph]]
[14] K. Dimopoulos and D. H. Lyth, Phys. Rev. D 69, 123509 (2004) arXiv:hep-ph/0209180.
[15] R. Allahverdi, K. Enqvist, J. Garcia-Bellido and A. Mazumdar, Phys. Rev. Lett. 97, 191304 (2006) arXiv:hep-ph/0605035.
[16] D. H. Lyth, JCAP 0704, 006 (2007) arXiv:hep-ph/0605283.
[17] J. C. Bueno Sanchez, K. Dimopoulos and D. H. Lyth, JCAP 0701, 015 (2007) arXiv:hep-ph/0608299.
[18] R. Allahverdi, Mod. Phys. Lett. A 23, 2799 (2008) arXiv:0812.3628 [hep-ph].