Stability Analysis and T-S Fuzzy Dynamic Positioning Controller Design for Autonomous Surface Vehicles Based on Sampled-Data Control

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ABSTRACT This article investigates the stability analysis and the fuzzy dynamic positioning (DP) controller design for an autonomous surface vehicle (ASV) based on sampled-data. Firstly, the Takagi-Sugeno (T-S) fuzzy model for the ASV with dynamic positioning system (DPS) is established. Then the criteria for asymptotically stability analysis and controller synthesis are provided by means of linear matrix inequalities. And less conservative results can be obtained by introducing convex reciprocal inequalities. Finally, simulation result is shown that the fuzzy sampled-data controller is effective to guarantee that the states of the ASV are stable and have good DP performance under the external disturbance.

INDEX TERMS Autonomous surface vehicles, dynamic positioning system, T-S fuzzy model, sampled-data control.

I. INTRODUCTION
Autonomous surface vehicle (ASV) is a flexible surface robot carrier platform with independent navigation. Generally speaking, ASV has a rigid structure, and its shape is very small such as wave piercing boats, sliding boats, hydrofoil boats, etc. Due to its independent navigation ability, flexibility, and high efficiency in accomplishing various complex tasks, ASV has very important application value in civil, military and coastal construction, such as marine resource exploration, coastal defense, hydrographic research, etc., especially in some fields like high risk and beyond the physiological limit of human body, the ASV has irreplaceable value. Recently, ASV has become a hot research area, and a large number of literature has been reported for ASV, which includes heading control [1], [2]; roll stabilization [4]; mooring control [3]; fault tolerated [5]; and path following [6]; tracking control [7]; neural network control [8] and so on. For instance, [8] deal with the issue about neural network control for the ASV in leader–follower formation with unknown local and leader dynamics. In [9], the formation control issue for underactuated ASV with ocean disturbance and uncertainties is discussed by using the control method of dynamic surface. In [10], the method about dynamic output feedback controller for an ASV based on network model is discussed, which considers the packet dropouts between sampler and control station. In [11], based on the finite time control (FHC) framework, the trajectory tracking control for ASV with unknown variables is studied.

Fuzzy model is an effective method for realizing the modeling and control of nonlinear systems, such as T-S fuzzy model, and sector nonlinear and local approximation is usually used in the fuzzy model [12], [13]. So far, many scholars have focused on the fuzzy ASV system [14], [15]. In [16], based on the fuzzy theory, a fuzzy learning controller is proposed for ASV, where the proposed controller has obvious advantages than the traditional controller in reducing energy and external interference; in [17], combined with the adaptive control and modern theory, a fuzzy adaptive controller is proposed, which can overcome the model error and suppress the external interference at the same time. In [18], based on ontology knowledge, a fuzzy support agent is proposed to provide control model for ASV system. In [19], a T-S fuzzy DP controller is designed for an ASV in network.
environments which considered the network-induced delay. In [20], by constructing a fuzzy control law, the problem about distributed maneuvering for multiple ASVS with state constraint and unknown kinetics is discussed. In [7], by combining with Nussbaum method and the adaptive fuzzy backstepping technology, an adaptive fuzzy trajectory tracking control scheme for ASV is proposed.

Dynamic positioning ship (DPS) is a marine vessel with a computer-control system, which used the dynamic positioning (DP) system to maintain their horizontal position and heading, and it has been used for different kinds of vessels like offshore support vessel, cruise vessel, shuttle tankers and so on. (see [21]). Compared with the conventional vessel which use anchor moored to keep position, the DPS has higher positioning accuracy and flexibility. Thus, the control problem for DPS has attracted considerable attention recently. In [22], the robust controller for the vessels with DPS is designed based on mixed sensitivity and uncertain model. The issue about optimal $H_{\infty}$ control for a DPS is focused on by [23] with T-S fuzzy models. In [24], the issue about input saturation and robust nonlinear control for the DPS under extern unknown disturbance is discussed. In [25], the problem of thrust distribution for DPS with cycloidal propeller is discussed, and the sequential quadratic programming algorithm is introduced for the thrust distribution. In [26], a new adaptive constraint control algorithm is introduced for DPS with input amplitude and rate saturation. By adding mass terms, a new robust controller for DP ships has been designed in [27].

Recently, sampled-data system has become an important topic, because modern control systems widely used the digital computers to control continuous-time systems (see [28]–[30]). Until now, the system has attracted many attention of scholars, and considerable results have been reported for that system (see [31]–[39]). The ASV with DPS is also a sampled-data system, where a variety of sensors is used by digital computers to obtain several kinds of sensor information, such as higher accuracy position, heading, velocity and so on. However, few literatures have been reported for nonlinear sampled-data ASV with DPS based on T-S fuzzy model. Since the sampled-data ASV with DPS is a complex nonlinear system. So, how to propose a T-S fuzzy model and control approach to improve the DP performance is a challenging work. Besides, how to improve the Lyapunov–Krasovskii functional (LKF) by adding novel item or introducing novel approach such as reciprocally convex combination approach to get less conservative results, which are the motivations of the paper.

In this brief, the issue about sampled-data control for fuzzy ASV with DPS is investigated. The T-S fuzzy model for the system is established firstly. Then the lower bound and upper bound of the network-induced delays between ASV and control center is both taken into consideration. In terms of LMI approach, Lyapunov theorems is involved for the stability analysis. And less conservative result can be obtained by introducing convex reciprocal inequalities. Then, the designed method of sampled-data controller is introduced to provide good DP performance under the external disturbances. Finally, a simulation of an ASV is conducted to illustrate the effectiveness of the proposed methods.

This note is organized as follow. The problem formulation is introduced in section 2. Section 3 introduces the stability analysis and controller design for the system. A simulation result is given in section 4 to validate the effectiveness of the proposed algorithm.

II. PROBLEM FORMULATION

The motion equations of the ASV, which is equipped with thruster and dynamic positioning system, are considered as follow.

$$M\ddot{v}(t) + Du(t) + G\eta(t) = u(t) + w(t)$$  \hspace{1cm} (1)

where

$$M = \begin{bmatrix} m_{11} & 0 & 0 \\ 0 & m_{22} & m_{23} \\ 0 & m_{32} & m_{33} \end{bmatrix}, \quad D = \begin{bmatrix} n_{11} & 0 & 0 \\ 0 & n_{22} & n_{23} \\ 0 & n_{32} & n_{33} \end{bmatrix},$$

$$G = \begin{bmatrix} g_{11} & 0 & 0 \\ 0 & g_{22} & 0 \\ 0 & 0 & g_{33} \end{bmatrix},$$

where $\eta(t) = [x_a(t) \ y_a(t) \ \psi(t)]^T$ represents the positions $x_p(t), \ y_p(t)$ and yaw angle $\psi(t)$ which is based on the earth-fixed frame. $v(t) = [p(t) \ q(t) \ r(t)]^T$ represents the body-fixed velocity, where $p(t), q(t), r(t)$ represents surge velocity, sway velocity and the yaw velocity, respectively.

$u(t)$ represents the control input; $w(t)$ represents the exogenous disturbances like wave, wind, ocean currents which belongs to $L_2[0, \infty)$; $M, D$ represent the inertia and linear damping matrix respectively, $G$ represent mooring forces, and

$$\dot{\eta}(t) = J(\psi)v(t)$$  \hspace{1cm} (2)

where $J(\psi)$ represents the transformation matrix between the two coordinates mentioned above, which is presented.
in Fig. 1, and

\[
J (\psi(t)) = \begin{bmatrix}
\cos (\psi(t)) & -\sin (\psi(t)) & 0 \\
\sin (\psi(t)) & \cos (\psi(t)) & 0 \\
0 & 0 & 1
\end{bmatrix}
\]

Define

\[
x(t) = \begin{bmatrix}
\eta(t) \\
\nu(t)
\end{bmatrix}^T = \begin{bmatrix}
x_p(t) \\
y_p(t) \\
\psi(t) \\
p(t) \\
q(t) \\
r(t)
\end{bmatrix}^T
\]

Let

\[
-M^{-1}D = \begin{bmatrix}
a_{11} & 0 & 0 \\
0 & a_{22} & a_{23} \\
0 & a_{32} & a_{33}
\end{bmatrix}
\]

\[
M^{-1} = \begin{bmatrix}
d_{11} & 0 & 0 \\
0 & d_{22} & d_{23} \\
0 & d_{32} & d_{33}
\end{bmatrix}
\]

\[
-M^{-1}G = \begin{bmatrix}
b_{11} & 0 & 0 \\
0 & b_{22} & b_{23} \\
0 & b_{32} & b_{33}
\end{bmatrix}
\]

Substituting (2) and (3) into (1) yields

\[
\dot{x}(t) = Ax(t) + Bu(t) + B_ww(t),
\]

where

\[
A = \begin{bmatrix}
0 & 0 & 0 & \cos (\psi(t)) & -\sin (\psi(t)) & 0 \\
0 & 0 & 0 & \sin (\psi(t)) & \cos (\psi(t)) & 0 \\
b_{11} & 0 & 0 & 0 & 0 & 1 \\
0 & b_{22} & b_{23} & 0 & a_{22} & a_{23} \\
0 & b_{32} & b_{33} & 0 & a_{32} & a_{33}
\end{bmatrix}
\]

\[
B = B_w = \begin{bmatrix}
0 & 0 & 0 \\
0 & 0 & 0 \\
0 & 0 & 0 \\
0 & 0 & 0 \\
0 & d_{32} & d_{33}
\end{bmatrix}
\]

Then the T-S fuzzy model for system (4) can be described by following rules.

Mode Rule 1: IF \( z_i(t) \) is \( f_{i1}, \cdots, f_{i8}(t) \) is \( f_{in} \), THEN

\[
\dot{x}(t) = A_{i1}x(t) + B_1u(t) + B_ww(t), \quad i = 1, 2, \ldots, n
\]

\[
y(t) = C_{i1}x(t),
\]

where \( f_{ij} \) denote the fuzzy set, \( n \) denotes the number of rule, \( z_1(t), z_2(t), \ldots, z_n(t) \) are premise variables, \( y(t) \) is controlled output, and \( C_i \) are known matrices.

Assumed that yaw angle \( \psi(t) \) is varying between \( -\pi/2 \) and \( \pi/2 \), so the exact T-S fuzzy model is obtained by the following three rules.

Model Rule 1:
IF \( \psi(t) \) is about \( 0 \), THEN

\[
\dot{x}(t) = A_1x(t) + B_1u(t) + B_ww(t),
\]

\[
y(t) = C_1x(t).
\]

Model Rule 2:
IF \( \psi(t) \) is about \( \pi/2 \), THEN

\[
\dot{x}(t) = A_2x(t) + B_2u(t) + B_ww(t),
\]

\[
y(t) = C_2x(t).
\]

Model Rule 3:
IF \( \psi(t) \) is about \( -\pi/2 \), THEN

\[
\dot{x}(t) = A_3x(t) + B_3u(t) + B_ww(t),
\]

\[
y(t) = C_3x(t).
\]

where

\[
A_1 = \begin{bmatrix}
0 & 0 & 0 & 1 & -\alpha & 0 \\
0 & 0 & 0 & \alpha & 1 & 0 \\
0 & 0 & 0 & 0 & 0 & 1 \\
b_{11} & 0 & 0 & a_{11} & 0 & 0 \\
0 & b_{22} & b_{23} & 0 & a_{22} & a_{23} \\
0 & b_{32} & b_{33} & 0 & a_{32} & a_{33}
\end{bmatrix}
\]

\[
A_2 = \begin{bmatrix}
0 & 0 & 0 & \beta & -1 & 0 \\
0 & 0 & 0 & 1 & \beta & 0 \\
0 & 0 & 0 & 0 & 0 & 1 \\
b_{11} & 0 & 0 & a_{11} & 0 & 0 \\
0 & b_{22} & b_{23} & 0 & a_{22} & a_{23} \\
0 & b_{32} & b_{33} & 0 & a_{32} & a_{33}
\end{bmatrix}
\]

\[
A_3 = \begin{bmatrix}
0 & 0 & 0 & \beta & 1 & 0 \\
0 & 0 & 0 & -1 & \beta & 0 \\
0 & 0 & 0 & 0 & 0 & 1 \\
b_{11} & 0 & 0 & a_{11} & 0 & 0 \\
0 & b_{22} & b_{23} & 0 & a_{22} & a_{23} \\
0 & b_{32} & b_{33} & 0 & a_{32} & a_{33}
\end{bmatrix}
\]

\[
B_l = B_{wi} = \begin{bmatrix}
0 & 0 & 0 \\
0 & 0 & 0 \\
0 & 0 & 0 \\
0 & 0 & 0 \\
0 & d_{32} & d_{33}
\end{bmatrix}
\]

The overall fuzzy model inferred from (6)-(8) is represented as:

\[
\dot{x}(t) = \sum_{i=1}^{3} \mu_i(z(t)) [A_ix(t) + B_lu(t) + B_ww(t)],
\]

\[
y(t) = \sum_{i=1}^{3} \mu_i(z(t))C_ix(t),
\]

where

\[
\mu_i(z(t)) = \frac{\omega_i(z(t))}{\sum_{i=1}^{3} \omega_i(z(t))} \geq 0, \quad i = 1, 2, 3,
\]

\[
\omega_i(z(t)) = \prod_{j=1}^{n} f_{ij}(z_j(t)),
\]

\[
\sum_{i=1}^{3} \mu_i(z(t)) = 1,
\]

\[
z(t) = [z_1(t), z_2(t), \ldots, z_n(t)].
\]
Controller Rule 3:

If $\psi(t)$ is about 0, THEN

$$u(t) = K_1 x(t_k + \tau_k). \quad (11)$$

Controller Rule 2:

If $\psi(t)$ is about $\frac{\pi}{2}$, THEN

$$u(t) = K_2 x(t_k + \tau_k). \quad (12)$$

Controller Rule 3:

If $\psi(t)$ is about $-\frac{\pi}{2}$, THEN

$$u(t) = K_3 x(t_k + \tau_k). \quad (13)$$

where $\tau_k$ represents the sampler-to-controller network-induced delay. $K_1, K_2, K_3$ are state-feedback gain matrices, then the overall fuzzy controllers are described as

$$u(t) = \sum_{j=1}^{3} \mu_j(z(t))K_j x(t_k + \tau_k), \quad \tau_k \leq t < t_{k+1}, \quad k = 0, 1, 2, \ldots \quad (14)$$

Substituting (14) into Eq. (9), we obtain

$$\dot{x}(t) = \sum_{i=1}^{3} \sum_{j=1}^{3} \mu_i(z(t))\mu_j(z(t)) [A_i x(t)]$$

$$+ B_i K_j x(t_k + \tau_k) + B_{wi} w(t)] \quad (15)$$

Define

$$\tau(t) = t - t_k - \tau_k. \quad (16)$$

where the time-varying delay $\tau(t)$ is piecewise-linear satisfying $\tau(t) \in [\tau_m, \tau_M]$. Thus, the sampled-data system in (15) is converted to a system with time-varying delay as follow:

$$\dot{x}(t) = \sum_{i=1}^{3} \sum_{j=1}^{3} \mu_i(z(t))\mu_j(z(t)) [A_i x(t)]$$

$$+ B_i K_j x(t_k + \tau(t)) + B_{wi} w(t)], \quad t > 0 \quad (17)$$

The paper's object is finding sampled-data controllers $K_1, K_2, K_3$ satisfying:

1) The system (17) with $w(t) = 0$ is asymptotically stable;

2) To reject the varying environment disturbances such as wave, current and wind, it is required that $||y(t)||_2 < \gamma ||w(t)||_2$ for all non-zero $w(t) \in L_2[0, \infty)$ at zero initial condition, where $\gamma > 0$.

**Remark 1:** Compared with the references which use continuous-time control method for ASV, the sampled-data control and network-induced delays are both included in the controller (14), which has more significance. If we ignore the sampled-data control and network-induced delays, the controller is similar with the one in [40]. So, the controller in [40] is a special case of (14).

**Remark 2:** It is noted that the lower and upper bounds of the network-induced delays are both considered in the paper, and it has more significance than [41], where the lower bound of the time-varying delay is assumed to be zero. Actually, the delay is often varying in a range, so the lower bound is not restricted to be zero and it will lead to conservatism.

To get the main results, the following lemma is shown.

**Lemma 1 (42):** For given matrix $M \in \mathbb{R}^{n \times n} > 0$, scalars $r_1$ and $r_2$, and a differentiable function $\omega \in \mathbb{R}^n$, then the following inequality holds:

$$\left( \int_{r_1}^{r_2} \omega^T(s)ds \right) M \left( \int_{r_1}^{r_2} \omega(s)ds \right) \leq (r_2 - r_1) \int_{r_1}^{r_2} \omega^T(s)M \omega(s)ds \quad (18)$$
III. MAIN RESULTS

In this section, we firstly exhibit the sufficient stability criteria of network-based ASV with DPS by establishing Lyapunov-Krasovskii functional. Then the sampled-data controller is designed to analyse the stability criteria.

**Theorem 1:** For constant delay \( \tau_m \geq 0 \), the system (17) is asymptotically stable with \( H_\infty \) norm bound \( \gamma \), if there exist positive matrices \( P, S, Q_1, Q_2, R_1, R_2, R_3 \) such that

\[
\begin{bmatrix}
R_3 & S \\
* & R_3
\end{bmatrix} > 0,
\]

\[
\Psi_{ij} = \begin{bmatrix}
\Gamma_{ij}^{P_i} P_i B_i K_j & R_1 & R_2 & P_i B_{wi}^T A_i^T \\
* & \Gamma_{22} & \Gamma_{23} & \Gamma_{24} & 0 & K_i^T B_i^T \\
* & * & \Gamma_{33} & T & 0 & 0 \\
* & * & * & \Gamma_{44} & 0 & 0 \\
* & * & * & * & -\gamma^2 I & B_{wi}^T \\
* & * & * & * & * & \Gamma_{66}
\end{bmatrix}
\]

\[
< 0
\]

where

\[
\Gamma_{ij}^{P_i} = P_i A_i + A_i^T P_i + Q_1 + Q_2 - R_1 - R_2 + C_i^T C_i
\]

\[
\Gamma_{22} = -2R_3 + S + S^T
\]

\[
\Gamma_{23} = R_3 - S
\]

\[
\Gamma_{24} = R_3 - S
\]

\[
\Gamma_{33} = -Q_1 - R_1 - R_3
\]

\[
\Gamma_{44} = -Q_2 - R_2 - R_3
\]

\[
\Gamma_{66} = -R^{-1}
\]

\[
R = \tau_m^2 R_1 + \tau_m^2 R_2 + (\tau_m - \tau_m^2) R_3
\]

Proof. The Lyapunov-krasovskii function is chosen as

\[
V(t) = \sum_{i=1}^{4} V_i(t), \quad t \in [t_k, t_{k+1})
\]

\[
V_1(t) = x(t)^T P x(t)
\]

\[
V_2(t) = \int_{t-\tau_m}^{t} x(s)^T Q_1 x(s) ds + \int_{t-\tau_m}^{t} x(s)^T Q_2 x(s) ds
\]

\[
V_3(t) = \tau_m \int_{1+\theta}^{1+\theta} \dot{x}(s) R_1 \dot{x}(s) ds \theta
\]

\[
+ \tau_m \int_{1+\theta}^{1+\theta} \dot{x}(s) R_2 \dot{x}(s) ds \theta
\]

\[
V_4(t) = (\tau_m - \tau_m) \int_{t-\tau_m}^{t} \dot{x}(s) R_3 \dot{x}(s) ds \theta
\]

Calculating the derivative of \( V(t) \), it can be obtained that:

\[
\dot{V}_1(t) = 2x(t)^T P \dot{x}(t)
\]

\[
\dot{V}_2(t) = x(t)^T Q_1 x(t) - x(t-\tau_m)^T Q_1 x(t-\tau_m) + x(t)^T Q_2 x(t) - x(t-\tau_m)^T Q_2 x(t-\tau_m)
\]

\[
\dot{V}_3(t) = \tau_m^2 \dot{x}(t)^T R_1 \dot{x}(t) + \tau_m^2 \dot{x}(t)^T R_2 \dot{x}(t)
\]

\[
- \tau_m \int_{t-\tau_m}^{t} \dot{x}(s) R_1 \dot{x}(s) ds
\]

\[
- \tau_m \int_{t-\tau_m}^{t} \dot{x}(s) R_2 \dot{x}(s) ds
\]

\[
\dot{V}_4(t) = (\tau_m - \tau_m) \int_{t-\tau_m}^{t} \dot{x}(s) R_3 \dot{x}(s) ds
\]

Substituting (23) into (22) that

\[
\dot{V}(t) \leq \sum_{i=1}^{3} \sum_{j=1}^{3} \mu_i(z(t)) \mu_j(z(t)) \xi_i^T(t) (\xi_j)
\]

\[
+ [A_i \quad B_i K_j - 0 \quad 0]^T (\tau_m^2 R_1 + \tau_m^2 R_2)
\]

\[
+ (\tau_m - \tau_m^2) R_3 \begin{bmatrix}
A_i \quad B_i K_j - 0 \quad 0
\end{bmatrix} \xi(t)
\]

\[
(24)
\]
where

\[
\Xi_{ij} = \begin{bmatrix}
\Gamma_{11} & P^T B_i K_j & R_1 & R_2 \\
\ast & \Gamma_{22} & \Xi_{23} & \Xi_{24} \\
\ast & \ast & \Gamma_{33} & S \\
\ast & \ast & \ast & \ast & \Gamma_{44}
\end{bmatrix}
\]

\[
\hat{\Xi}_{ij} = P^T A_i + A_i^T P + Q_1 + Q_2 - R_1 - R_2
\]

\[
\zeta(t) = \begin{bmatrix} x^T(t) & x^T(t - \tau(t)) & x^T(t - \tau_m) & x^T(t - \tau_M) \end{bmatrix}^T
\]

By Schur complement, (20) guarantee:

\[
\Xi_{ij} + [A_i B_i K_j \ 0 \ 0]^T \begin{pmatrix} \tau_m R_1 + \tau_M^2 R_2 \\
+ (\tau_M - \tau_m)^2 R_3 \end{pmatrix}[A_i B_i K_j \ 0 \ 0] < 0 \tag{25}
\]

Therefore, there exists a small enough scalar \(\sigma > 0\) to guarantee \(\hat{V}(t) < -\sigma ||x(t)||^2\) when \(x(t) \neq 0\). Hence, the system (10) with \(w(t) = 0\) is asymptotically stable.

Next, we will prove that the system (17) has robust stability and satisfies the \(H_\infty\) performance \(\gamma\) under external disturbances. Then, under zero initial conditions, for all nonzero \(w(t) \in L_2[0, \infty)\), the \(H_\infty\) performance is considered:

\[
J_{cw} = \int_0^\infty \left[ y^T(s)y(s) - \gamma^2 w^T(s)w(s) \right] ds, \quad \gamma > 0 \tag{26}
\]

then

\[
y^T(t)y(t) - \gamma^2 w^T(t)w(t) + \hat{V}(t) \\
\leq \sum_{i=1}^3 \sum_{j=1}^3 \mu_i(z(t))\mu_j(z(t))\zeta^T(t) \Theta_{ij} \\
+ [A_i B_i K_j \ 0 \ 0 \ B_{wi}]^T \begin{pmatrix} \tau_m R_1 + \tau_M^2 R_2 \\
+ (\tau_M - \tau_m)^2 R_3 \end{pmatrix} \\
\times [A_i B_i K_j \ 0 \ 0 \ B_{wi}] \zeta(t) \tag{27}
\]

where

\[
\zeta(t) = \begin{bmatrix} x^T(t) & x^T(t - \tau(t)) & x^T(t - \tau_m) & x^T(t - \tau_M) \end{bmatrix}^T
\]

\[
\Theta_{ij} = \begin{bmatrix}
\Gamma_{11} & P^T B_i K_j & R_1 & R_2 & PB_{wi} \\
\ast & \Gamma_{22} & \Xi_{23} & \Xi_{24} & 0 \\
\ast & \ast & \Gamma_{33} & S & 0 \\
\ast & \ast & \ast & \ast & \ast & -\gamma^2 I
\end{bmatrix}
\]

By Schur complement, (20) guarantee

\[
\Theta_{ij} + [A_i B_i K_j \ 0 \ 0 \ B_{wi}]^T \\
\times \begin{pmatrix} \tau_m R_1 + \tau_M^2 R_2 + (\tau_M - \tau_m)^2 R_3 \end{pmatrix} \\
\times [A_i B_i K_j \ 0 \ 0 \ B_{wi}] < 0 \tag{28}
\]

From (28), we have:

\[
V(\infty) + V(0) + \int_0^\infty \left( ||y(t)||^2_2 - \gamma^2 ||w(t)||^2_2 \right) dt \leq 0 \tag{29}
\]

Then, under zero initial conditions, we can obtain \(V(0) = 0\) and \(V(\infty) \geq 0\). According to (29), for all nonzero \(w(t) \in L_2[0, \infty), ||y(t)||_2 \leq \gamma ||w(t)||_2\) is obtained, which means that \(J_{cw} \leq 0\). The proof is completed.

Remark 3 Compared with [41], if \(\tau_m = 0, Q_1 = Q_2 = 0, R_2 = R_3 = 0\), then LKF is similar with the one in [41]. Therefore, the LKF in [41] is a special case of (9). So, the result in Theorem 1 has less conservatism than [41].

Remark 4 Compared with [43] and [44], which use Jensen inequality directly to estimate the integral term \(- (\tau_M - \tau_m) \int_{t-\tau_M}^{t-\tau_m} \hat{y}^T(s)R_3 \hat{y}(s)ds\), and we use a substep to make the results less conservative which called reciprocally convex combination inequalities. Therefore, the conservativeness of stability criterion is greatly reduced.

Furthermore, the sampled-data controller (14) will be proposed to stabilize the system (17) based on the following Theorems.
Theorem 2: For scales $\tau_M \geq \tau_m > 0$, $\gamma > 0$, if there exist positive matrices $\bar{P}, \bar{S}, \bar{Q}_1, \bar{Q}_2, \bar{R}_1, \bar{R}_2, \bar{R}_3$, such that:

$$
\begin{bmatrix}
\bar{R}_3 & \bar{S} \\
\ast & \bar{R}_3
\end{bmatrix} > 0,
$$

$$
\bar{\Psi}_{ij} = 
\begin{bmatrix}
\bar{\Gamma}_{11} & B_i \bar{\tilde{K}}_j & \bar{\tilde{R}}_1 & \bar{\tilde{R}}_2 & B_{wi} & \bar{P}A_i^T & \bar{P}C_i^T \\
* & \bar{\Gamma}_{22} & \bar{\Gamma}_{23} & \bar{\Gamma}_{24} & 0 & \bar{K}_j B_i^T & 0 \\
* & * & \bar{\Gamma}_{33} & \bar{\tilde{S}} & 0 & 0 & 0 \\
* & * & * & \bar{\Gamma}_{44} & 0 & 0 & 0 \\
* & * & * & * & -\gamma^2 I & \bar{P}B_{wi}^T & 0 \\
* & * & * & * & * & \bar{\Gamma}_{66} & 0 \\
* & * & * & * & * & * & I
\end{bmatrix} < 0
$$

Then, the system (17) is asymptotically stable with $H_\infty$ performance $\gamma$. Moreover, the gain of the controller $K_j$ is given that

$$
K_j = \bar{\tilde{K}}_j \bar{P}^{-1}
$$

Proof: By noticing that $-\bar{P} \bar{R}_i^{-1} \bar{P} \leq \bar{R} - 2\bar{P}$, let $\xi = \text{diag} \{P^{-T}, P^{-T}, P^{-T}, P^{-T}, I, I, I\}$. Defining $\bar{P} = P^{-1}$, $\bar{K}_j = K_j P^{-1}$, $\bar{R} = P^{-T} R P^{-1}$, $\bar{S} = P^{-T} S P^{-1}$, $\bar{Q}_1 = P^{-T} Q_1 P^{-1}$, $\bar{Q}_2 = P^{-T} Q_2 P^{-1}$, $\bar{R}_3 = P^{-T} R_3 P^{-1}$, $i = 1, 2, 3$. Pre- and post-multiplying (20) with $\xi$ and $\xi^T$, then according to Schur complement, we can see that (31) is satisfied. This completed the proof.

IV. NUMERICAL EXAMPLES

In the section, one example for an ASV is used to verify the effectiveness of the given method. Considered the $M, D$ and
$G$ in model (1) as follow ([1]):

$$
M = \begin{bmatrix}
1.0852 & 0 & 0 \\
0 & 2.0575 & -0.4087 \\
0 & -0.4087 & 0.2153 \\
\end{bmatrix},
$$

$$
D = \begin{bmatrix}
0.0865 & 0 & 0 \\
0 & 0.0762 & 0.1510 \\
0 & 0.0151 & 0.0031 \\
\end{bmatrix},
$$

$$
G = \begin{bmatrix}
0.0389 & 0 & 0 \\
0 & 0.0266 & 0 \\
0 & 0 & 0 \\
\end{bmatrix}.
$$

Let $\alpha = \sin 2^\circ$ and $\beta = \cos 88^\circ$, we have obtained by the equation as shown at the bottom of the page.

Let

$$
C_i = \begin{bmatrix}
1 & 0 & 0 & 1 & 0 & 0 \\
0 & 1 & 0 & 0 & 1 & 0 \\
0 & 0 & 1 & 0 & 0 & 1 \\
\end{bmatrix}, \quad i = 1, 2, 3.
$$

Firstly, we compare the results in Theorem 2 with the references which use the traditional T-S fuzzy sampled-data control method. We choose the same parameters, and then the maximum upper bound of sampling period $\tau_M$ obtained by different methods are listed in Table 1, which shows that the results in the paper are less conservative than the references mention above. Besides, the lower and upper bounds of the time-varying delay are both considered in the paper, which has more significance than [41].

Then, the ship’s initial state is $x_i(t) = [1 1 0 1 0 0]^T$. Assuming the sampling interval $\tau_m = 0$, $\tau_M = 0.25$, and the network-induced delays $\tau_k$ is assumed to be varied stochastically in $(\tau_m, \tau_M)$, then the $H_\infty$ performance is obtained that $\gamma_{\min} = 5.621$, which improves the corresponding ones in [41] above 135.94%. Besides, the results in Table 1 are shown in Figs.4-6. Fig. 4-5 shows the responses of the ASV state of velocities of ASV. Fig.6 shows response of velocities of ASV.

It is easy to know that the ASV can be stabilized by the designed controller without external disturbances. Considered the external environment disturbance as follow

$$
w(t) = 0.1 \sin(t)
$$

| Method  | [45] | [46] | [47] | Theorem2 |
|--------|------|------|------|----------|
| $\tau_M$ | 0.25 | 0.264 | 0.453 | 0.532 |

Sampled-data vessel with DPS in [41]. We choose the same sampling period obtained from Theorem 2 are larger than those in [44]–[46], which shows that the results in the paper is more conservative than the references mention above.

Then, we can obtain the controller gain matrix, is obtained by the equation as shown at the top of the next page. The responses of the ASV state are shown in Figs.4-6. Fig. 4-5 shows the responses of $x$, $y$ directions and yaw angle $\psi$ of ASV. Fig.6 shows response of velocities of ASV.

$$
A_1 = \begin{bmatrix}
0 & 0 & 0 & 1.0000 & -0.0349 & 0 \\
0 & 0 & 0 & 0.0349 & 1.0000 & 0 \\
0 & 0 & 0 & 0 & 0 & 1.0000 \\
-0.0358 & 0 & 0 & -0.0797 & 0 & 0 \\
0 & -0.0208 & 0 & 0 & -0.0818 & -0.1224 \\
0 & -0.0394 & 0 & 0 & -0.2254 & -0.2468 \\
\end{bmatrix},
$$

$$
A_2 = \begin{bmatrix}
0 & 0 & 0 & 0.0349 & -1.0000 & 0 \\
0 & 0 & 0 & 1.0000 & 0.0349 & 0 \\
0 & 0 & 0 & 0 & 0 & 1.0000 \\
-0.0358 & 0 & 0 & -0.0797 & 0 & 0 \\
0 & -0.0208 & 0 & 0 & -0.0818 & -0.1224 \\
0 & -0.0394 & 0 & 0 & -0.2254 & -0.2468 \\
\end{bmatrix},
$$

$$
A_3 = \begin{bmatrix}
0 & 0 & 0 & 0.0349 & 1.0000 & 0 \\
0 & 0 & 0 & -1.0000 & 0.0349 & 0 \\
0 & 0 & 0 & 0 & 0 & 1.0000 \\
-0.0358 & 0 & 0 & -0.0797 & 0 & 0 \\
0 & -0.0208 & 0 & 0 & -0.0818 & -0.1224 \\
0 & -0.0394 & 0 & 0 & -0.2254 & -0.2468 \\
\end{bmatrix},
$$

$$
B_i = B_{wi} = \begin{bmatrix}
0 & 0 & 0 \\
0 & 0 & 0 \\
0 & 0 & 0 \\
0.9215 & 0 & 0 \\
0 & 0.7802 & 1.4811 \\
0 & 1.4811 & 7.4562 \\
\end{bmatrix}, \quad i = 1, 2, 3.
$$
Then the responses of the ASV state are shown in Fig. 7-9, which indicates that the $\eta(t)$ and velocities $\upsilon(t)$ are stable in a short time. That is, the designed sampled-data controllers can stabilize the ASV and have good DP performance even if the external disturbance exists.

V. CONCLUSION
This article discusses the fuzzy sampled-data control problem for ASV with DPS. T-S fuzzy model for the ASV has been established, and both the lower and upper bounds of network-induced delays have been considered in the model. By integrating the convex reciprocal inequalities and Lyapunov theorems, adequate stability criterion is derived and less conservative result is obtained. The simulation result shows that the proposed fuzzy sampled-data controller is effective in providing good DP performance for the ASV. In the future, we will study new inequalities such as Wirtinger-based inequalities to improve the results of this article.

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