Summertime thermal regime of water downstream of the Krasnoyarsk hydroelectric power plant

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Abstract. Summertime hydrothermal regime of the Yenisei River downstream of the Krasnoyarsk hydroelectric power plant is modeled based on a deterministic approach. To that end, the Fourier equation is used and the following physical processes contributing to the heat exchange between water and the surroundings are taken into consideration: absorption of direct and scattered solar radiation by water, absorption of downwelling thermal infrared radiation from the atmosphere by water surface, thermal infrared radiation back from the water surface, convection of heat and heat loss due to evaporation of water. A clearskies river thermal regime under no wind is studied in a 124-km stream reach below the power plant and the obtained results are compared against temperatures recorded at gauging stations.

1. Introduction

The Yenisei is the largest river in Russia in terms of runoff (624.41 km3/year). Water temperature and the streamflow velocity of the river have changed after commissioning a series of hydroelectric power plants, referred to as the Yeniseisky HPP Cascade (Sayano-Shushenskaya HPP, Mainskaya HPP, and Krasnoyarsk HPP). Changes in the temperature pattern entail changes in hydrological conditions of the river (e.g. absence of an ice crust) and biochemical processes (the growth of river flora and fauna). This inevitably has affected the environmental situation in the nearby areas. Krasnoyarsk HPP (second largest after Sayano-Shushenskaya HPP) is the major anthropogenic factor influencing the Yenisei River and the area near the city of Krasnoyarsk. In this paper we study summer thermal regime of the Yenisei River downstream of the Krasnoyarsk HPP.

2. Mathematical modeling

The hydrothermal river regime can be described by a second-order Fourier equation [1-3]:

$$\frac{\partial T_w(x,t)}{\partial t} = -V(x) \frac{\partial T_w}{\partial x} + \lambda \frac{\partial^2 T_w}{\partial x^2} + \frac{W(t) B(x)}{\rho c S(x)}$$

(1)

Here c is the specific thermal capacity of water, $T_w$ - the cross-sectional average water temperature, $t$ (hour) - the time, $x$ (km) - the distance from of the dam, $\lambda$ – the heat conductivity coefficient, $B$(m) – the stream width, $V$ (km/h) – the average streamflow velocity found as the ratio between the water discharge through the dam body $Q$(m³/sec) and the stream cross-section $S$(m²):
\[ V = \frac{Q}{S} \]  \hspace{1cm} (2)

\( W(t) \) is the total surface heat flux equal to:

\[ W(t) = W_s + W_a - W_w + W_c - W_e, \]  \hspace{1cm} (3)

where \( W_s \) is solar radiation absorbed by water; \( W_a \) is atmospheric TIR absorbed by water; \( W_w \) is TIR from water surface to the atmosphere; \( W_c \) is convective heat transfer, and \( W_e \) is the loss of heat due to evaporation.

In the system of coordinates moving at a velocity \( V(x) \) Equation (3) is rewritten as

\[ \frac{dT_w(t)}{dt} = \frac{W(t)B(x(t))}{pcS(x(t))}, \]  \hspace{1cm} (4)

\( S/B \approx d \) is the mean stream channel depth.

Solution of (4) is found from the expression

\[ T_w(t) = \frac{1}{pcd} \int_{t_0}^{t} W(t) dt + T_w(0, t_0). \]  \hspace{1cm} (5)

Here \( T_w(0, t_0) \) is the outflow temperature of water leaving the dam.

3. Physical modeling

3.1. Solar radiation

Solar radiation reaching the Earth’s atmosphere consists of direct solar radiation attenuated while going through the atmosphere and the direct solar radiation scattered in the atmosphere.

The power of extraterrestrial solar radiation incident on the Earth’s atmosphere equals [4]

\[ S = S_0E \cos \theta, \]  \hspace{1cm} (6)

where \( S_0 = 1367 \) W/m\(^2\) is the solar constant, \( E \) - the eccentricity correction factor, \( \theta \) - the zenith angle

\[ \cos \theta = \sin \varphi \cdot \sin \delta + \cos \varphi \cdot \cos \delta \cdot \cos \Omega, \]  \hspace{1cm} (7)

where is the geographic latitude of the location and \( \delta \) is solar declination depending on the time of the year. So, on July 3, 2016 solar declination was \( \delta = 23.06^\circ \) and \( E = 0.966 \) [5]. For the Krasnoyarsk HPP we have \( \varphi = 55.94^\circ \).

\[ \Omega = \omega t, \]  \hspace{1cm} (8)

where \( \Omega \) is the hour angle, \( t \) – the time, \( \omega \) – the angular velocity of the Earth’s rotation around its axis.

From expressions (6) and (7) it follows that

\[ S = S_0E(A + B \cos \omega t), \]  \hspace{1cm} (9)

where \( A = \sin \varphi \cdot \sin \delta, B = \cos \varphi \cdot \cos \delta \).
The sunrise and sunset time equation is determined as
\[
\cos \omega t_0 = -\tan \varphi \cdot \tan \delta = -\frac{A}{B}.
\]  

(10)

Equation (10) yields two values for the hour angle: the negative root $-\omega t_0$, refers to the sunrise and the positive one, $\omega t_0$, to the sunset. Solving this equation for our conditions yields $t_v = 4:08$ the time of sunrise and $t_z = 21:36$ the time of sunset. Hence the day length is 17:28 hours and the true noon ($t_n$) is at 12:52 local time.

Since $t$ is the time from noon, we further have
\[
\cos \omega t = \cos \left( \frac{\pi}{12} (t - t_n) \right).
\]  

(11)

Temporal behavior of extraterrestrial solar power is shown in figure 1 (Curve 1). So, solar power at true noon is $S = 1100 \, W/m^2$.

Only part of solar radiation reaches the Earth's surface, the rest is scattered and absorbed by molecules of various gases, water droplets, ice crystals and aerosol impurities present in the atmosphere. Attenuation of a light beam propagating in absorbing medium obeys the Bouguer-Lambert-Beer law. According to this law the radiation power reaching the water surface is
\[
S' = S \cdot \exp \left( -\frac{\tau_0}{\cos \theta} \right)
\]  

(12)

where $\tau_0$ is the optical thickness of atmosphere for $\cos \theta = 1 (\theta = 0$, which corresponds to the sun in zenith). For $\tau_0 \ll 1$ expression (12) can be expanded in Taylor series
\[
S' = S_0 E \cos \theta - S_0 E \tau_0.
\]  

(13)

Temporal behavior of the solar power passed through atmosphere is shown in figure 1 (Curve 2). So, at true noon it is $S' = 900 \, W/m^2$.

Upon reaching a plane water surface, solar radiation is partially reflected. The reflection coefficient $R$ is calculated by the Fresnel's formula
\[ R = \frac{1}{2} \left[ \frac{\sin^2(\varphi - i)}{\sin^2(\varphi + i) + \tan^2(\varphi - i)} \right], \]  

(14)

where \( \varphi = (90 - \theta) \) is the angle of incidence and the angle of refraction \( i \) is found from the relation

\[ \sin i = \frac{\sin \varphi}{n}, \]  

(15)

where \( n=1.33 \) is the refractive index of clear water.

Dependence of the reflection coefficient on the angle of incidence is shown in figure 2.

![Figure 2. Reflection coefficient versus angle of incidence.](image)

The part of solar energy that has successfully penetrated through the atmosphere and has not been reflected by water surface (1-R) is absorbed by water at a depth of about one meter thereby increasing its temperature [6]. Thus the solar power absorbed by water is

\[ W_s = (1 - R) S', \]  

(16)

Temporal behavior of the absorbed solar power is shown in figure 1 (Curve 3). At true noon \( W_s = 811 \text{ W/m}^2 \).

3.2. Thermal radiation

3.2.1. Emission from water. The Stefan-Boltzmann law states that the heat energy emitted by an absolutely black body is proportional to the fourth power of its temperature

\[ A = \sigma T^4, \]  

(17)

where \( T \) is temperature in degrees Kelvin and \( \sigma \) is the Stefan-Boltzmann constant. According to Wien's displacement law, the black-body radiation curve peaks at a wavelength \( \lambda_{\text{max}} \), inversely proportional to the absolute temperature \( T \).

\[ \lambda \approx \frac{0.29}{T}. \]  

(18)

For \( T = 283^\circ K \), \( \lambda \approx 10 \mu m \). This wavelength is absorbed at a depth of 100 \( \mu \)m and since it is much less than the total water depth the energy emitted by water, according to the Stefan-Boltzmann law, is
\[ W_w \approx \varepsilon_w \sigma (273 + T_w)^4, \]  
\[ \text{(19)} \]

where \( \varepsilon_w = 0.995 \) [7] is the emission coefficient of water. Expanding in Taylor’s series yields
\[ W_w \approx A + BT_w, \]  
\[ \text{(20)} \]

Where \( A=313.36, \quad B=4.59. \)

Energy emitted by water on July 3, 2016 at night temperature \( T_2 = 7^\circ C \) and day temperature \( T_2 = 20^\circ C \) was \( W_{w1} = 346 \, W/m^2 \) and \( W_{w2} = 413 \, W/m^2 \), respectively.

3.2.2. Atmospheric emission. Water surface absorbs atmospheric thermal radiation and gets warmer. Energy emitted by the Earth’s atmosphere obeys the same Stefan-Boltzmann law.
\[ W_a \approx \varepsilon_a \sigma (273 + T_a)^4. \]  
\[ \text{(21)} \]

On July 3, 2016 the atmospheric temperature was \( T_{a1} = 15 \) day and \( T_{a2} = 25 \) at night.

The atmospheric emission coefficient \( \varepsilon_a \) was found as [8]:
\[ \varepsilon_a = 1 - 0.4 \cdot \exp \left( -\frac{100e_a}{T_a + 273} \right), \]  
\[ \text{(22)} \]

where \( e_a \) — the pressure of water vapor:
\[ e_a = \frac{H}{100} e_s, \]  
\[ \text{(23)} \]

\( e_s \) — the pressure of saturated vapor:
\[ e_s = 6.1 \cdot \exp \left( \frac{17.27 \cdot T_a}{273 + T_a} \right). \]  
\[ \text{(24)} \]

\( H \) is the atmospheric humidity.

Table 1 summarizes our results on atmospheric emission calculations.

| \( T_{a2},^\circ C \) | \( H, \% \) | \( e_s, \text{mBar} \) | \( e_a, \text{mBar} \) | \( \varepsilon_a \) | \( W_a, \text{W/m}^2 \) |
|------------------------|-----------|----------------|----------------|--------|----------------|
| Night                  | 15        | 85             | 14.99          | 12.74  | 0.995          | 388.21          |
| Day                    | 25        | 45             | 25.97          | 11.68  | 0.992          | 443.60          |

3.3. Heat exchange between water surface and atmosphere

3.3.1. Evaporation. The energy spent on water evaporation, \( W_e \), is estimated as [6]:
\[ W_e = \rho L f(w)(e_s - e_a), \]  
\[ \text{(25)} \]

where \( L=2.26 \times 10^6 \, J/kg \) is the latent heat of evaporation, \( f(w) \) is the wind function determined as
\[ f(w) = a + bw. \]  
\[ \text{(26)} \]

\( w \) is the wind velocity, \( a \approx 3 \times 10^{-9}, \quad b \approx \left( \frac{1}{a} \right) \times 10^{-9}. \)
When the pressure of water vapor exceeds that of saturated water vapor, water evaporates and the water temperature drops. In a reverse situation we deal with condensation of water vapor and the water temperature increases.

| Table 2. Energy spent on water evaporation. |
|-------------------------------------------|
| $w = 0 \, m/sec$ | $w = 1.6 \, m/sec$ |
| Night | 15.25 | 16.62 |
| Day | 96.88 | 105.57 |

3.3.2. Convection. Convective heat exchange between water surface and atmosphere is estimated as [6]:

$$W_c = 0.61 \rho L_f(w) \cdot (T_w - T_a).$$  

(27)

If $T_w < T_a$ water temperature will grow due to convection and it will go down if on the contrary.

| Table 3. Convective heat exchange. |
|-----------------------------------|
| $w=0 \, m/sec$ | $w=1.6 \, m/sec$ |
| Night | 33.09 | 36.06 |
| Day | 20.67 | 22.54 |

4. Results

We consider a 124-km reach of the Yenisei River downstream the dam of the Krasnoyarsk HPP. The reach is divided by 4 cross-section lines at (0.5, 40, 77, 124 km) with gauging stations at the first, second and forth section lines to measure water temperature. The first station is located next to the dam ($x = 0.5$ km) and measures water temperature leaving the dam. The other two are located at 40 km and 124 km downstream. Streamflow velocity is assumed constant from section to section and is found from Eq. (1) at $Q = 2900 \, m^3/sec$. The $S$ is equal to the cross-sectional area of the downstream lowest reach section. Flow time between section lines is found as the section-to-section distance divided by the flow velocity. Temperature measurements at the gauging stations are taken at time, $t_g$ (at 08:00 and 20:00 hour). Water leaves the dam at time $t_0 = t_g - t_i$ where $t_i$ is the length of time within which water from the dam reaches a cross-section line. Water temperature was computed using Eq. (5) which now has the form:

$$T_w(t) = \frac{1}{\rho c d} \sum_{i=2}^{4} W_i \Delta t_i + T_w(0, t_0), \Delta t_1 = t_i - t_{i-1}.$$  

(28)

where $i$ is the reach section number, $\Delta t_i$ – the flow time between the $i-1$ and $i$ section lines, $W_i \Delta t_i$ – the energy received by water along each reach section. Power $W_i$ depends on day or night time, water and atmospheric temperature, water vapor pressure, and humidity. Morphometric and hydrophysical characteristics of the river reach sections are summarized in Table 4.

| Table 4. Morphometric and hydrophysical characteristics. |
|---------------------------------------------------------|
| Cross-section line number | 1 | 2 | 3 | 4 |
| Dam-to-section line distance, [km] | 5 | 40 | 77 | 124 |
| Width B, [m] | 520 | 830 | 580 | 450 |
| Cross-sectional area S, [m^2] | 1834 | 2254 | 2452 | 2513 |
| Stream flow velocity, [km/hour] | 5.7 | 4.6 | 4.3 | 3.7 |
Flow time between adjacent cross-section lines, \( \Delta t_i \), [hours]

\[
\begin{array}{|c|c|c|c|}
\hline
\text{Relation B/S, } [m^{-1}] & 7.6 & 9.5 & 11.9 \\
\hline
\text{Calculated} & 0.283 & 0.368 & 0.236 & 0.179 \\
\text{Measured} & 7.2 & 8.0 & 10 \\
\hline
\end{array}
\]

| Tw July 3 at 08.00, °C | Calculated | Measured |
|------------------------|------------|----------|
|                        | 7.8        | 7.2      |
|                        | 8.7        | 9.0      |
|                        | 10.1       | 10.3     |

| Tw July 3 at 20.00, °C | Calculated | Measured |
|------------------------|------------|----------|
|                        | 9.0        | 7.2      |
|                        | 9.9        | 9.0      |
|                        | 10.3       | 10.6     |

5. Conclusion

We have proposed a simple model for simulating summertime hydrothermal regime of a river based on calculation of water temperature in a co-ordinate system moving with water. The physically based estimation of water heat budget takes into account absorption of solar radiation by water surface, emission and absorption of atmospheric TIR by water, convective heating of water as well as heat loss due to evaporative processes. The temporal fluctuation pattern of direct and scattered solar radiation depends on the zenith angle and atmospheric absorption. The dominant water heating factor is solar radiation during daytime and atmospheric TIR at night. Water temperatures 124 km downstream of the Krasnoyarsk HPP on the Yenisei River computed using the proposed model with consideration of morphometric characteristics are close to the recorded temperatures observed at the gauging stations, which proves that the deployed physical-mathematical model provides an adequate description of the actual hydrothermal processes.

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