Precision and uncertainties in mass scale predictions in SUSY $SO(10)$ with $SU(2)_L \times SU(2)_R \times U(1)_{B-L} \times SU(3)_C$ intermediate breaking

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Abstract. In a class of SUSY $SO(10)$ with $SU(2)_L \times SU(2)_R \times U(1)_{B-L} \times SU(3)_C$ $(g_{2L} \neq g_{2R})$ intermediate gauge symmetry, we observe that the prediction on the unification mass ($M_U$) is unaffected by Planck-scale-induced gravitational and intermediate-scale-threshold effects, although the intermediate scale ($M_I$) itself is subject to such corrections. In particular, without invoking the presence of additional lighter scalar degrees of freedom but including plausible and reasonable threshold effects, we find that interesting solutions for neutrino physics corresponding to $M_I \simeq 10^{10} - 10^{12}$ GeV and $M_U \simeq (5 - 6) \times 10^{17}$ GeV are permitted in the minimal models. Possibilities of low-mass right-handed gauge bosons corresponding to $M_I \simeq 1 - 10$ TeV consistent with the CERN-LEP data are pointed out in a number of models when threshold effects are included using effective mass parameters.

1 Introduction

Supersymmetric grand unified theories (GUTs) have been the subject of considerable attention over the past two decades [1–4]. While nonSUSY $SU(5)$ fails to unify the gauge couplings of the standard model, $SU(2)_L \times U(1)_Y \times SU(3)_C(\equiv G_{213})$, the SUSY $SU(5)$ and single step breaking of almost all SUSY GUTs exhibit remarkable unification of gauge and Yukawa couplings at $M_U \simeq 10^{16}$ GeV consistent with the recent CERN-LEP measurements. Compared to other GUTs, $SO(10)$ has several attractive features. The fermions contained in the spinorial representation $16 \subset SO(10)$ have just one extra member per generation which is the right-handed neutrino needed to generate light Majorana neutrino masses over a wide range of values through see-saw mechanism [5]. It explains why there is parity violation at low energies starting from parity conservation at the GUT scale [6,7]. It is the minimal left-right symmetric GUT with natural quark lepton unification and having $SU(2)_L \times SU(2)_R \times SU(4)_C$ [7] as its maximal subgroup. It is has the potentiality to guarantee $R$-parity conservation in the Lagrangian.

With $M_U \simeq M_N \simeq 10^{16}$ GeV, where $M_N = \text{degenerate right-handed Majorana neutrino mass}$, the grand desert model through see-saw mechanism predicts much smaller values of light left-handed Majorana neutrino masses than those needed for understanding neutrinos as hot dark matter (HDM) candidate along with experimental indications on atmospheric neutrino oscillations and neutrinoless double $\beta$ decay [8,9], unless substantially lower values of $M_N$ are obtained by a judicious dialing of the Yukawa coupling of the right-handed Majorana neutrino, or via nonrenormalizable operators. However, in such cases, one of the most attractive features like $b - \tau$ Yukawa unification for smaller values of $\tan \beta$ has to be sacrificed [10,11]. On the other hand, SUSY $SO(10)$ with an intermediate gauge symmetry like $SU(2)_L \times SU(2)_R \times U(1)_{B-L} \times SU(3)_C(\equiv G_{2131})$ [12–16] or $SU(2)_L \times SU(2)_R \times SU(4)_C(\equiv G_{2214})$ [17–20], while providing a more natural value for $M_N$, substantially lower than the GUT scale, has the potentialities to account for the $b - \tau$ Yukawa unification at the intermediate scale $M_I \simeq 10^9 - 10^{11}$ GeV. In this context it has been demonstrated that desirable values of $G_{2213}$-breaking scale with $M_I \simeq 10^9 - 10^{11}$ are possible provided that a number of scalar components of full $SO(10)$ Higgs representations are light with masses near the intermediate scale [13,14].

Gravitational corrections due to higher dimensional operators [21,23] and threshold effects due to superheavy particles have been shown to influence the GUT predictions significantly [17,23–25]. Since neither the superheavy masses contributing to threshold effects near the GUT scale, nor the coefficients of the higher dimensional operators contributing to gravitational corrections are determined by the grand unified theories, these corrections add to the uncertainties and inaccuracies of the model predictions. In order to remove such limitations of the GUTs, it is important to search for gauge symmetries and possible representations for which some of the uncertainties could be absent. For example, it has been demon-
strated through theorems that in all GUTs with $SU(2)_{L} \times SU(2)_{R} \times SU(4)_{C} \times U(1)_{em}$, the GUT-threshold and all gravitational corrections on $\sin^{2} \theta_{W}(M_{Z})$ and intermediate scale are absent [18,19]. The presence of $G_{224P}$ intermediate gauge symmetry has been found to be essential for these cancellations.

The purpose of this paper is two fold. For the first time, we demonstrate certain precise results in a class of GUTs with $G_{2213}(g_{2L} \neq g_{2R})$ intermediate gauge symmetry with $D$-parity broken at the GUT scale [26]. In particular we find that in $SO(10)$ the dominant effect due to the 5-dim. operator is absent on $M_{L}$ leading to the absence of such gravitational corrections on the proton lifetime for $p \rightarrow e^{+} \pi^{0}$. The threshold effects caused due to the spreading of masses around the intermediate scale are also found to be absent on $M_{L}$. Secondly, while exploring uncertainties in the intermediate scale predictions in SUSY $SO(10)$, we show, for the first time, that the $G_{2213}$ intermediate symmetry is allowed to survive down to $M_{I} \approx 10^{10} - 10^{13}$ GeV by threshold and gravitational corrections. We have investigated the impact of threshold effect in SUSY $SO(10)$ models with one pair of $126 \oplus 126$ and one or two pairs of $16 \oplus 16$ and find that even $M_{I} \approx 10$ TeV is allowed consistent with the CERN-LEP measurements [27] provided that the effective mass parameters at the intermediate or GUT-thresholds are few times heavier or lighter than the corresponding scales.

The paper is organized in the following manner. In Sec. 2 we discuss the analytic formulas for mass scales including threshold and gravitational corrections. In Sec. 3 we derive vanishing corrections due to 5-dim. operator on the GUT-scale and estimate gravitational corrections on the intermediate scale. In Sec. 4 we discuss the threshold effects and their impact on $M_{U}$ and $M_{I}$. The results are summarized with conclusions in Sec. 5.

## 2 Analytic Formulas for Mass Scales

We consider the following symmetry breaking pattern and derive the analytic formulas for the unification mass $M_{U}$ and the intermediate scale $M_{I}$ including one-loop, two-loop, gravitational and threshold corrections.

$$SO(10) \times SUSY \xrightarrow{210 \atop M_{U}} G_{2213} \times SUSY$$

$$\xrightarrow{S_{1} \atop M_{I}} G_{213} \times SUSY \xrightarrow{10 \atop M_{Z}} U(1)_{em} \times SU(3)_{C}$$

where the multiplet $S$ is a component of the $SO(10)$ representations $16 \oplus 16$ or $126 \oplus 126$ as the case may be. The renormalization group equations in the presence of the two gauge symmetries $G_{213}$ and $G_{2213}$ below the GUT scale can be written as

$$\frac{1}{\alpha_{i}(M_{Z})} = \frac{1}{\alpha_{i}(M_{I})} + \frac{a_{i}}{2\pi} \ln \frac{M_{I}}{M_{Z}} + \theta_{i} - \Delta_{i},$$

$$i = 1Y, 2L, 3C; \quad (1)$$

$$\frac{1}{\alpha_{i}(M_{U})} = \frac{1}{\alpha_{i}(M_{I})} + \frac{a_{i}'}{2\pi} \ln \frac{M_{I}}{M_{U}} + \theta_{i}' - \Delta_{i}' - \Delta_{i}^{NRO},$$

$$i = 2L, 2R, BL, 3C. \quad (2)$$

where the second term in the R.H.S. of eqs.(1) and (2) represents one-loop contributions and the third term of both the equations are the two-loop terms [28],

$$\theta_{i} = \frac{1}{4\pi} \sum_{j} B_{ij} \ln \frac{\alpha_{j}(M_{I})}{\alpha_{j}(M_{Z})},$$

$$\theta_{i}' = \frac{1}{4\pi} \sum_{j} B_{ij}' \ln \frac{\alpha_{j}(M_{I})}{\alpha_{j}(M_{U})}, \quad (3)$$

$$B_{ij} = \frac{b_{ij}}{a_{j}}, \quad B_{ij}' = \frac{b_{ij}'}{a_{j}}. \quad (4)$$

While the functions $\Delta_{i}$ include threshold effects at $M_{Z}$ and $M_{I}$,

$$\Delta_{i} = \Delta_{i}^{(Z)} + \Delta_{i}^{(I)}$$

$\Delta_{i}'$ include threshold effects at $M_{U}$. The expressions for $\Delta_{i}$ and $\Delta_{i}'$ are given in Sec. 5. The term $\Delta_{i}^{(NRO)}$ in eq. (2) contains higher-dimensional-operator effects which modify the boundary condition at $\mu = M_{U}$ as,

$$\alpha_{2L}(M_{U})(1 + \epsilon_{2L}) = \alpha_{2R}(M_{U})(1 + \epsilon_{2R})$$

$$\alpha_{BL}(M_{U})(1 + \epsilon_{BL}) = \alpha_{3C}(M_{U})(1 + \epsilon_{3C}) = \alpha_{G} \quad (5)$$

leading to

$$\Delta_{i}^{(NRO)} = - \frac{\epsilon_{i}}{\alpha_{G}}, \quad i = 2L, 2R, BL, 3C \quad (6)$$

where $\alpha_{G}$ is the GUT-fine-structure constant. Considering the boundary condition (5) along with eqs.(1), (2) and (6) we obtain the following analytic formulas for mass scales.

$$\ln \frac{M_{I}}{M_{Z}} = \frac{1}{(AB' - AB)(AL_{S} - A'L_{\theta}) + (A'J_{2} - AK_{2})}$$

$$- \frac{2\pi}{\alpha_{G}} (Ae'' - A'e') + (A'J_{A} - AK_{A})], \quad (7)$$

$$\ln \frac{M_{U}}{M_{Z}} = \frac{1}{(AB' - AB)(B'L_{S} - BL_{\theta}) + (BK_{2} - B'J_{2})}$$

$$- \frac{2\pi}{\alpha_{G}} (B'e' - B'e'') + (BK_{A} - B'J_{A})], \quad (8)$$

where

$$L_{S} = \frac{2\pi}{\alpha(M_{Z})} \left(1 - \frac{8}{3} \frac{\alpha(M_{Z})}{\alpha_{G}(M_{Z})}\right),$$

$$L_{\theta} = \frac{2\pi}{\alpha(M_{Z})} \left(1 - \frac{8}{3} \sin^{2} \theta_{W}(M_{Z})\right),$$

$$A = a_{2R}' + \frac{2}{3} a_{BL}' - \frac{5}{3} a_{2L}',$$

$$B = \frac{5}{3}(a_{Y} - a_{2L}) - (a_{2R}' + \frac{2}{3} a_{BL}' - \frac{5}{3} a_{2L}'),$$

$$A' = \left(a_{2R}' + \frac{2}{3} a_{BL}' + a_{2L}' - \frac{8}{3} a_{3C}'\right),$$
\[ B' = \frac{5}{3} a_Y + a_{2L} - \frac{8}{3} \alpha_{3C} \]

\[ J_2 = 2\pi \left[ \theta_{2R}' + \frac{2}{3} \theta_{BL}' + \frac{8}{3} \theta_2 - \frac{8}{3} \theta_3 \right], \]

\[ K_2 = 2\pi \left[ \frac{2}{3} \theta_{BL}' + \frac{8}{3} \theta_2 - \frac{8}{3} \theta_3 \right], \]

\[ \epsilon' = \epsilon_{2R} + \frac{2}{3} \epsilon_{BL} - \frac{8}{3} \epsilon_{3C}, \]

\[ J_\Delta = -2\pi \left[ \Delta_{2R} + \frac{2}{3} \Delta_{BL} - \frac{2}{3} \Delta_2 + \frac{5}{3} (\Delta_Y - \Delta_2) \right], \]

\[ K_\Delta = -2\pi \left[ \frac{2}{3} \Delta_{BL} + \Delta_2 - \frac{8}{3} \Delta_3 \right] + \frac{5}{3} \Delta_Y + \Delta_2 - \frac{8}{3} \Delta_3 \right]. \]

(9)

In the R.H.S. of eqs. (7) and (8) the first, second, third and fourth terms are one-loop, two-loop, gravitational and threshold contributions, respectively. The one-loop and the two-loop beta-function coefficients below the intermediate scale \((M_I)\) are given by [22, 28],

\[
\begin{pmatrix}
\alpha_Y \\
\alpha_{2L} \\
\alpha_{3C}
\end{pmatrix} = \begin{pmatrix}
\frac{33}{1} \\
\frac{1}{-3}
\end{pmatrix},
\]

\[
b_{ij} = \begin{pmatrix}
\frac{199}{9} & \frac{27}{5} & \frac{88}{9} \\
\frac{25}{9} & 24 & 14
\end{pmatrix}, \quad i, j = 1, 2, 3. \quad (10)
\]

Above the intermediate scale, the one-loop and two-loop beta-function coefficients are

\[
\begin{pmatrix}
\alpha_{2L}' \\
\alpha_{2L}'' \\
\alpha_{BL}' \\
\alpha_{3C}'
\end{pmatrix} = \begin{pmatrix}
n_{10} \\
6 + \frac{3}{2} n_{16} + \frac{9}{2} n_{126} \\
-3
\end{pmatrix},
\]

\[
b_{ij}' = \begin{pmatrix}
18 + 7 n_{10} & 3 n_{10} \\
3 n_{10} & 18 + 7 n_{10} + 7 n_{16} + 48 n_{126} \\
9 & 9 + \frac{3}{2} n_{16} + 72 n_{126}
\end{pmatrix}, \quad i, j = 2, 2R, BL, 3C. \quad (11)
\]

Including one- and two-loop corrections, we consider a variety of models taking the lighter multiplet to be

\[ S = pn_{126} + qn_{16} \]

where \(p\) and \(q\) are integers. Here \(n_{126} = 1\) or \(n_{16} = 1\) imply that the components \(\Delta_I(1, 3, -1, 1, 1) + \bar{\Delta}_I(1, 3, 1, 1, 1) \subset 126 + 126\) or \(\chi_I(1, 2, 1/2, 1) + \bar{\chi}_I(1, 2, -1/2, 1) \subset 16 + \bar{16}\) of \(SO(10)\) have masses close to \(M_I\). Here a minimal model is defined as the one with \(n_{126} = 1\) or \(n_{16} = 1\) where only one set of \(126 + 126\) or \(16 + \bar{16}\) is used for \(G_{2213}\) breaking. In addition the GUT scale symmetry breaking is carried out by only one representation like \(210\) or \(45\) which are needed for decoupling the parity and \(SU(2)_R\)-breakings.

There are nonminimal models in the literature as in refs. [13, 14] and in ref [12], the latter having \(n_{16} = 3\). It may be noted that the spontaneous breaking of \(SU(2)_R \times U(1)_{B-L}\) gauge symmetry by \(126\) guarantees automatic conservation of \(R\)-parity whereas the use of \(16\) instead of \(126\) leads to \(R\)-parity violation. In the latter case it is necessary to impose additional discrete symmetries to maintain the stability of the proton. We use the following input parameters for our analysis [27]

\[ \alpha^{-1}(M_Z) = 128.9 \pm 0.09, \quad \alpha_{3C} = 0.119 \pm 0.004, \]

\[ \sin^2 \theta(M_Z) = 0.23152 \pm 0.00032, \quad M_Z = 91.187 \text{ GeV}. \quad (12) \]

Our solutions including only one-loop and two-loop contributions in different models are shown in Table 1. For example, if \(n_{16} = 1\) and \(n_{126} = 0\) the two-loop values are \(M_I = 10^{16.9} \text{ GeV}\) and \(M_U = 10^{16.1} \text{ GeV}\). It is clear from Table 1 that upto two-loop level the models do not allow \(M_I = 10^{15} \text{ GeV}\) and in some cases \(M_I\) is even greater than \(M_U\) which are forbidden. Also it is to be noted that \(M_U\) for all models attains a constant value of \(10^{16.5} \text{ GeV}\). This phenomenon with occurrence of \(M_I \simeq M_U\) has led to invoke the existence of lighter scalar degrees of freedom in order to bring down the value of the intermediate scale with \(M_I \ll M_U\) [13, 14].

3 Gravitational Corrections on the Mass Scales

The mechanism of decoupling of parity and \(SU(2)_R\)-breakings is implemented in \(SO(10)\) by using the Higgs representation \(210\) or \(45\) for the symmetry breaking at the GUT scale. Out of these two, the representation \(45\) does not contribute to the gravitational corrections through the 5-dim. operator since \(T \gamma (F_{\mu\nu} \Phi_{45} F^{\mu\nu})\) vanishes identically. Thus confining to the minimal model and using \(210\) for the \(SO(10)\) symmetry breaking at the GUT scale, we demonstrate in this section how the prediction on the unification mass has vanishing correction due to the 5-dim. operator. We also show how the gravitational effect lowers the intermediate scale by at most two orders of magnitude from the SUSY GUT-scale.

3.1 Vanishing Gravitational Corrections on the Unification Scale

The super-Higgs representation \(210\) contains the singlet \(\xi(1, 1, 1)\) under \(SU(2)_L \times SU(2)_R \times SU(4)_C\) which has been
noted to be odd under $D$-symmetry that acts like the left-right discrete symmetry ($\equiv$ Parity) [26]. But the neutral component in $\chi(1,1,15)$ of $210$ is even under the same $D$-symmetry. $SO(10)$ can be broken to $G_{2213}$ without left-right discrete symmetry by assigning vacuum expectation value $(\xi(1,1,1)) = (\chi(1,1,15)) \approx M_I$. In this case it has been shown in ref. [25] that the nonrenormalizable Lagrangian containing the 5-dim. operator

$$\frac{-\eta}{2M_p} Tr (F_{\mu\nu} F^{\mu\nu})$$

(13)
yields via eqs.(5) and (9),

$$\epsilon_{2R} = -\epsilon_{2L} = -\epsilon_{3C} = \frac{1}{2} \epsilon_{BL} = \epsilon,$$

$$\epsilon = \frac{\eta}{16 M_p} \left[ \frac{3}{2} \frac{\alpha_{G}}{2\pi \alpha_{G}} \right]^{\frac{1}{2}},$$

$$\epsilon'' = \epsilon_{2L} + \epsilon_{2R} + \frac{2}{3} \epsilon_{BL} - \frac{8}{3} \epsilon_{3C} = 4\epsilon,$$

$$\epsilon' = \epsilon_{2R} + \frac{2}{3} \epsilon_{BL} + \frac{5}{3} \epsilon_{2L} = 4\epsilon,$$

(14)

where $\alpha_{G} = \frac{1}{27}$. It is important to note that $\epsilon' = \epsilon''$ identically which has strong bearing on the prediction of the GUT scale. From eqs.(7) and (8) we have the gravitational corrections due to the 5-dim. operator,

$$\ln \frac{M_I}{M_Z} \mid_{NRO} = \frac{2\pi(\epsilon' - A\epsilon'')}{\alpha_{G}(A'B' - A'B)},$$

(15)

$$\ln \frac{M_U}{M_Z} \mid_{NRO} = \frac{2\pi(B\epsilon'' - B'\epsilon')}{\alpha_{G}(A'B' - A'B)},$$

(16)

Now we demonstrate vanishing gravitational corrections to the unification mass in the following manner. In all models with decoupled parity and $SU(2)_R$ breaking where there are no additional $SU(2)_L$ or $SU(3)_C$-multiplets below GUT scale, except the SM-Higgs doublets near $M_Z$ and $\Delta_R \oplus \Delta_R$ or $\chi_R \oplus \chi_R$ near $M_I$ as the case may be,

$$a_{2L} = a'_{2L},$$

$$a_{3C} = a'_{3C}.$$

(17)

Using eq. (17) in eq.(9), we obtain

$$B = B' = \frac{5}{3} a_Y - \frac{2}{3} a_{BL} - a_{2R'},$$

$$A = a''_{2R} + \frac{2}{3} a_{BL} - \frac{5}{3} a_{2L'},$$

$$A' = a'_{2R} + \frac{2}{3} a_{BL} + a_{2L} - \frac{8}{3} a_{3C}.$$

(18)

Eqs. (15) and (16) then yield with the help of (14) and (18),

$$\ln \frac{M_I}{M_Z} = \frac{2\pi(A'\epsilon' - A\epsilon'')}{\alpha_{G} B(A - A')} = \frac{8\pi \epsilon}{B_{\alpha_{G}}},$$

$$\ln \frac{M_U}{M_Z} = \frac{2\pi(\epsilon'' - \epsilon')}{\alpha_{G}(A - A')} = 0.$$

(19)

The result given in (19) are valid both in SUSY and non-SUSY GUTs like $SO(10), SO(18)$, and $E_6$ etc as long as $\epsilon' = \epsilon''$ as in (14). This suggests an important aspect of the model that in $SO(10)$ with $D$-parity broken at the GUT scale, the GUT scale and the proton lifetime are unaffected due to gravitational corrections through the 5-dim. operator.

### 3.2 Gravitational Correction on the Intermediate Scale

We have examined the effect of gravitational correction on the intermediate scale originating from the 5-dim. operator given in eq. (13) in different models characterized by $(n_{16}, n_{126}) = (0,1), (1,0), (1,1), (2,0), (2,1), (1,2), (0,3)$ and (3,0). The one-loop coefficient $A$, $A'$, $B$, $B'$ and numerical results obtained in different cases are shown in Table 1. By varying $\eta$ parameter within the range $-10$ to $+10$, we obtain intermediate scale $M_I$ between $10^{14}$ to $10^{16}$ GeV. The maximal effects on $M_I$ is found to occur in the minimal model with $210$, $126$ $\oplus$ $\overline{126}$ and $10$ representations where $R$-parity is automatically conserved and we obtain the lowest possible value to be $M_I \approx 10^{14}$ GeV.

### 4 Threshold Effects

So far we have noted that the impact of gravitational corrections on intermediate scale $M_I$ could bring it down to $10^{14}$ GeV whereas the unification scale remains unaffected. The possibility of $M_I \approx 10^{10} - 10^{12}$ GeV with $b - \tau$ Yukawa unification at $M_I$ has been addressed in ref [13, 14] but with a number of additional Higgs scalars having masses near $M_I$, even though they do not contribute to the spontaneous symmetry breaking. But we demonstrate here that when threshold effects are taken into account, the scale $M_I$ fits into the desired range of values even if gravitational corrections are ignored and there are no additional scalar degrees of freedom (and superpartners) near the intermediate scale. We also note vanishing corrections on the unification mass ($M_U$) due to intermediate-scale-threshold effects.

From analytic formulas, the threshold-corrections for mass scales are

$$\Delta \ln \frac{M_I}{M_Z} = \frac{(A'J_\Delta - A K_\Delta)}{(A'B' - A'B)},$$

$$\Delta \ln \frac{M_U}{M_Z} = \frac{(B K_\Delta - B' J_\Delta)}{(A'B' - A'B)}.$$

(20)

$$\Delta \ln \frac{M_U}{M_Z} = \frac{(B K_\Delta - B' J_\Delta)}{(A'B' - A'B)}.$$

(21)

We assume the extended survival hypothesis to operate with the consequence that all scalar components of an $SO(10)$ representation which do not contribute to spontaneous symmetry breaking are superheavy. Only lighter degrees of freedom are those $G_{2213}$-components in $126 \oplus \overline{126}$ or $16 \oplus \overline{16}$ which contribute to spontaneous symmetry breaking at $M_I$. Similarly the lightest scalar components with masses near $M_Z$ are up and down type doublets originating from $10 \subset SO(10)$. The coloured triplets in $10$ have masses near the GUT scale. We compute threshold effects and...
on $M_I$ and $M_U$ using two different methods which have been adopted in the current literature: (4.1) Effective mass parameters and effective SUSY threshold have been introduced by Carena, Pokorski and Wagner [29] which have been also exploited in studying threshold effects in minimal SUSY GUTs [22]. Similarly SUSY SU(5) GUT-threshold effects have also been investigated by Langacker and Polonsky [22] by introducing another set of effective mass parameters near the GUT scale. For the present analysis we utilize the same set of effective mass parameters at the SUSY scale as in ref. [22] but use two new sets of effective mass parameters at $M_I$ and $M_U$. Although the effective mass parameters corresponding to SUSY threshold has been determined approximately using experimental measurements or well known estimations of the actual masses, such determinations for the effective mass parameters at higher thresholds has not been carried out due to lack of experimental data or adequate estimations on superheavy masses. In view of this we adopt the procedure similar to that outlined in [22] and assume these effective mass parameters to be few times heavier or lighter than the corresponding mass scales. (4.2) Without introducing effective mass parameters, threshold effects have been also computed conventionally by assigning specific and plausible values of masses to the superheavy scalar components in nonSUSY GUTs as well as SUSY theories [30–34]. This method will be adopted below in a separate analysis. Following a result due to Shifman, masses used for estimation of threshold effects have been assumed to be bare masses as the wave function renormalization has been shown to get cancelled by two-loop effects [35]. In both these cases we find interesting solutions even when the masses are assigned their expected values and taken to be few times heavier or lighter than the corresponding scales.

4.1 Threshold Effects with Effective Mass Parameters

Including threshold corrections, we have investigated three models corresponding to $(n_{16}, n_{126}) = (1, 0), (2, 0), (0, 1)$ with $45 \oplus 54$, for SO(10) breaking and other three models corresponding to $(n_{16}, n_{126}) = (1, 0), (2, 0), (0, 1)$ with $210$. The superheavy components in these models having masses near $M_I$ and $M_U$ are shown in Tables 2 and 3, respectively. It is clear that threshold effects on $M_U$ and $M_I$ can be estimated once $M'_I(i = 1Y, 2L, 3C)$ and $M''_I(i = 2L, 2R, BL, 3C)$ as defined in eqs. (22)-(24) below are known. In any model the superheavy masses near any particular symmetry breaking scale can be parametrized in terms of the corresponding effective mass parameters [22,29]. In the present model there are three such relations corresponding to the three symmetry breaking scales i.e., $\mu = M_{SUSY} = M_Z, \mu = M_I$ and $\mu = M_U$,

$$\Delta_i^Z = \sum_\alpha \frac{b'_\alpha}{2\pi} \ln \frac{M'_\alpha}{M_Z} = \frac{b'_i}{2\pi} \ln \frac{M'_I}{M_Z},$$

$$i = 1Y, 2L, 3C; \mu = M_Z; \quad (22)$$

$$\Delta_i^U = \sum_\alpha \frac{b''_\alpha}{2\pi} \ln \frac{M''_\alpha}{M_U} = \frac{b''_i}{2\pi} \ln \frac{M''_U}{M_U},$$

$$i = 2L, 2R, BL, 3C; \mu = M_U; \quad (23)$$

where $\alpha$ refers to the actual $G_{213}$ submultiplet near $\mu = M_Z, M_I$ or $G_{2213}$ submultiplet near $\mu = M_U$ and $M_A, M'_A$ or $M''_A$ refer to the actual component masses. The coefficients $b'_i = \sum b'_\alpha^{(\alpha)}$ and $b''_i = \sum b''_\alpha^{(\alpha)}$ have been defined in (22)-(24) following refs. [22,29]. The numbers $b'_i$ refer to the one-loop coefficients of the multiplet $\alpha$ under the gauge subgroup $U(1)_Y, SU(2)_L, SU(2)_R, SU(3)_C, U(1)_{B-L}$ etc. Using eqs.(9) and (20)-(24) we have obtained contributions to threshold effects on the two mass scales, $\Delta \ln \frac{M'_I}{M_Z}$ and $\Delta \ln \frac{M''_U}{M_U}$, as shown in Table 4 in terms of effective mass parameters $M'_I(i = 1Y, 2L, 3C)$ and $M''_I(i = 2L, 2R, BL, 3C)$. The numerical entries in Table 4 denote threshold effects at $M_Z$ estimated using the effective mass parameters of ref. [22].

Using one-loop coefficients from Tables 2-3 and effective mass parameters denoted as primes at the intermediate scale and as double primes at the GUT scale, the analytic expressions for threshold corrections are presented in Table 4 where different models have been also defined. A remarkable feature is that $\Delta \ln \frac{M'_I}{M_Z}$ has vanishing corrections due to intermediate-scale threshold effects as the corresponding expressions contain no term involving any of the parameters like $M'_L, M'_R, M'_B$ or $M'_C$. Further, the effective mass parameters $M'_I$ and $M''_I$ have vanishing contributions to the threshold effects on the unification mass. Another notable feature is that corrections due to $M'_L$ and $M'_C$ are absent in $\Delta \ln \frac{M''_U}{M_U}$. There is only a small correction due to $M'_Y$.

The relation (22) has been utilized in ref [22] to compute only one set of values of $M_I, M_Z$ in MSSM from the model predictions on $M_{SUSY}$. But, since such predictions are also model dependent, several other assumed values of effective mass parameters have been utilized for computation. At present no experimental or theoretical information is available on the actual values of superheavy masses around $M_I$ and $M_U$, although theoretically it is natural to assume these masses to spread around the corresponding scales by a factor bounded by $\frac{1}{10}$ to 10. In the present case, in the absence of actual values of component masses in the model, we make quite plausible and reasonable assumptions on $M'_I$ and $M''_I$ for computation. In our analysis the effective mass parameters $M'_I$ or $M''_I$ are taken to vary between $\frac{1}{10}$ to 5 times the relevant scale of symmetry breaking i.e., $M_I$ or $M_U$. Numerical solutions to different allowed values of mass scales corresponding to different choices of effective mass parameters are presented in Table 5.

Certain important features of these solutions are noteworthy. The minimal model-I with one set of $210, 16 \oplus 10$, and $10$ permits intermediate scale in the interesting range of $10^9 - 10^{13}$ GeV for reasonable choices of effective mass.
parameters having string scale unification $M_U \simeq (5 - 6) \times 10^{17}$ GeV. Also the $SO(10)$ model with $\{210, 126 \oplus \overline{126}\}$ and $10$ allows $M_I \simeq 5.3 \times 10^{11}$ GeV with high unification mass close to the string scale $M_{string} \simeq 5.6 \times 10^{17}$ GeV. We also note that the intermediate scale solution with $M_I \simeq 10^{11} - 10^{13}$ GeV is maintained irrespective of the fact whether a $210$ or a $45 \oplus 54$ or even a $45 \oplus 210$ are used for the GUT-scale symmetry breaking.

Although solutions with intermediate scale $M_I \simeq 10^{11} - 10^{13}$ GeV are also possible due to threshold effects with superheavy masses as shown in Sec. 4.2, a special and notable feature with effective mass parameters is the possibility of low-mass right-handed gauge bosons corresponding to $M_I \simeq 1 - 10$ TeV in all the six models, minimal or nonminimal. These solutions are also indicated in Table 5. Such low-mass right-handed gauge bosons might be testified through experimentally detectable $V + A$ currents in future [6].

4.2 Threshold Effects with Superheavy Masses

In this subsection, instead of using effective mass parameters, we estimate threshold effects with reasonable choices on values of the masses of superheavy components of Higgs scalars and their superpartners in three models corresponding to $(n_{16}, n_{126}) = (1, 0), (2, 0), (0, 1)$ with $45 \oplus 54$, and three other models corresponding to $(n_{16}, n_{126}) = (1, 0), (2, 0), (0, 1)$ with $210$

The expression for threshold effect in terms of actual superheavy component masses $M'_\alpha$ which have values near $M_I$ is given by eq. (23),

$$\Delta^I = \sum_\alpha \frac{b'^\alpha}{2 \pi} \ln \frac{M'_\alpha}{M_I}, \quad i = 1Y, 2L, 3C. \quad (25)$$

The superheavy components $\alpha$ contained in different models which have masses near $M_I$ are shown in Table 2. Similarly, the threshold effect at $\mu = M_U$ is given by eq. (24),

$$\Delta^U = \sum_\alpha \frac{b'^\alpha}{2 \pi} \ln \frac{M''_\alpha}{M_U}, \quad i = 2L, 2R, BL, 3C. \quad (26)$$

The superheavy components $\alpha$ contained in different models which have masses near $M_U$ are given in Table 3. While computing threshold effects, we have assumed all the multiplets belonging to an $SO(10)$ representation $'H'$ to have the same degenerate mass $M_H$. For example, all the superheavy components in $45$ given in Table 3 near $\mu = M_U$ have been subjected to the following degeneracy condition

$$M''(3, 1, 0, 1) = M''(1, 3, 0, 1) = M''(1, 1, 0, 8) = M_{45}.$$

Similarly for $\overline{210}, 54, 16 \oplus \overline{16}, 126 \oplus \overline{126}$ and $10$

$$M''(2, 2, \pm \frac{1}{3}, 6) = M''(2, 2, \pm \frac{1}{3}, 3) = \ldots = M_{210},$$
$$M''(3, 3, 0, 1) = M''(1, 1, \pm \frac{2}{3}, 6) = \ldots = M_{54},$$
$$M''(2, 1, \pm \frac{1}{3}, 3) = \ldots = M_{16}, \ldots$$

We have obtained contributions to threshold effect in terms of actual masses $M''_{ij}$ as shown in Table 3.

All the heavy masses near $\mu = M_I$ are assumed to have the same mass $M'$. We have obtained contributions to threshold effects on the two mass scales, $\Delta \ln \frac{M_I}{M'}$ and $\Delta \ln \frac{M'}{M_U}$ as shown in Table 4, in terms of superheavy masses. The last term i.e., the numerical entries denote threshold contributions at $\mu = M_Z$. From Table 5, it can be seen that the threshold contributions due to the superheavy masses $M_{16}$ or $M_{126}$ to $\Delta \ln \frac{M_U}{M_Z}$ are absent for all the models and the threshold contribution due to the representation $54$ cancels out from models IV, V and VI. These cancellations are understood due to a theorem by Mohapatra [36]. As before the masses are allowed to vary between $\frac{1}{3}$ and $5$ times the scale of relevant symmetry breaking i.e. $M_I$ or $M_U$. Numerical solutions to different allowed values of mass scales corresponding to different choices of superheavy masses are presented in Table 6. In Table 7, for models I, IV (II, V) we have intermediate scale $M_I \simeq 10^{14}(10^{15})$ GeV with unification scale $M'_U \simeq 10^{16}$GeV for reasonable choice of superheavy masses. In case of models III, VI the intermediate scale can be as low as $10^{10}$GeV with unification scale $3.3 \times 10^{15}$ GeV.

5 Summary and Conclusion

While investigating the possibility of $G_{2213}$ intermediate gauge symmetry in SUSY $SO(10)$, we have considered a number of models. At the two-loop level, we have noted that except for the nonminimal models with $3(16 \oplus \overline{16})$, $210, 10$ [10] and those of ref. [13,14,16] the RGEs in the minimal models do not permit $G_{2213}$ intermediate breaking scale $M_I$ to be substantially lower than $M_U$ when both gravitational and threshold effects are ignored. Including gravitational corrections in the minimal model, we observe that the prediction on unification mass remains unaffected by such Planck-scale-induced gravitational effects whereas the intermediate scale can be lowered by atmost two orders of magnitude through such corrections.

Including threshold corrections, we have considered various minimal and nonminimal models and obtained $M_I \simeq 10^{10} - 10^{13}$ GeV with high unification scale using plausible values of effective mass parameters but without using additional number of Higgs scalars at $M_I$ (models I, III, IV and VI with $n_{126} = 1$ or $n_{16} = 1$). Our choices of effective mass parameters are similar to those used in earlier investigations [22,29] and choices of superheavy masses near thresholds are similar to those used in earlier analyses [30-34]. An important feature of analytic result is that the GUT scale is unaffected by the spreading of masses near the intermediate scale although intermediate scale is itself changed significantly by the superheavy masses near the GUT scale. Thus the proton lifetime predictions in the model for $p \rightarrow e + \pi^0$ mode are unchanged by gravitational or intermediate scale threshold corrections. Another important aspect of this analysis is that even if the
permits the implementation of conventional see-saw mechanism for No. SP/S2/K-30/98 of Department of Science and Technology, The works of M.K.P and C.R.D. are supported by the project lowed naturally and such models where certain light degrees of freedom are allowed naturally and $R$-parity conservation is guaranteed.

| $SO(10)$ representation | $G_{213}$ multiplet | $b_{1L}^*, b_{1Y}^*, b_{2G}^*$ |
|--------------------------|---------------------|--------------------------|
| 16                       | (1, 0, 1)           | (0, 0, 0)                |
| 16$^*$                   | (1, −1, 1)          | (0, $\frac{2}{3}$, 0)   |
| 16$^*$                   | (1, 0, 1)           | (0, 0, 0)                |
| 16$^*$                   | (1, −1, 1)          | (0, $\frac{2}{3}$, 0)   |
| $^{16}[10]$              | (1, 1, 1)[(1, −1, 1)] | (0, $\frac{2}{3}$, 0)   |
| $^{126}$                 | (1, 2, 1)           | (0, $\frac{12}{5}$, 0)  |
| $^{126}$                 | (1, 0, 1)           | (0, 0, 0)                |
| $^{126}$                 | (1, −1, 1)          | (0, $\frac{2}{3}$, 0)   |
| $^{126}$                 | (1, −2, 1)          | (0, $\frac{12}{5}$, 0)  |

Table 2. The heavy Higgs content of the $SO(10)$ model with $G_{213}$ intermediate symmetry. The $G_{213}$ submultiplets acquire masses close to $M_t$ when $G_{213}$ is broken.

spreading of masses near the two thresholds are only few times heavier or lighter than the corresponding scales, the models result in $M_t \simeq 10^{11} - 10^{13}$ GeV either with effective mass parameters or with superheavy masses. We further observe that low-mass right-handed gauge bosons in the range $1 - 10$ TeV are permitted in the model only when threshold effects are computed with effective mass parameters. All relevant superheavy masses contributing to threshold effects have been assumed to be bare masses as their wave function renormalisation has been shown to be cancelled out by two-loop effects [35].

It may be noted that while the use of $^{126} \oplus ^{126}$ permits the implementation of conventional see-saw mechanism for neutrino masses with $R$-parity conservation, it is possible to use a generalized mechanism [37,38] with the choice $^{16} \oplus ^{10}$ such that one can get a see-saw like formula for light neutrino masses. In the latter case $R$-parity is violated and one needs to impose additional discrete symmetries to maintain the stability of the proton. We thus conclude that, $G_{213}(g_{2L} \neq g_{2R})$ with minimal choice of Higgs scalars is allowed as an intermediate gauge symmetry in SUSY $SO(10)$ model. The right-handed Majorana neutrinos associated with the intermediate scales obtained in this analysis are compatible with observed indications for light Majorana neutrino masses through see-saw mechanism [5,8,9].

Recently interesting investigations have been made in SUSY left-right gauge models while embedding $G_{213}$ in SUSY $SO(10)$ as an intermediate gauge group in the presence of higher dimensional operators [39]. It would be more interesting to study the impact of threshold effects in such models where certain light degrees of freedom are allowed naturally and $R$-parity conservation is guaranteed.

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Table 1. Mass scales and coefficients for different SUSY SO(10) models including one-loop and two-loop contributions. Also shown are the values of $M_I$ including the 5-dim. operator effect while $M_U$ remains unaffected.

| $n_{16}$ | $n_{126}$ | $A$ | $B$ | $A'$ | $B'$ | One-loop $M_I$ (GeV) | One-loop $M_U$ (GeV) | Two-loop $M_I$ (GeV) | Two-loop $M_U$ (GeV) | 5-dim. operator $\eta$ | $M_I$ (GeV) |
|----------|----------|-----|-----|------|------|---------------------|---------------------|---------------------|---------------------|---------------------|-------------|
| 1        | 0        | 6   | 4   | 16   | 4    | $10^{15.96}$        | $10^{16.5}$     | $10^{16.9}$        | $10^{16.11}$        | 8                   | $10^{15.9}$   |
| 0        | 1        | 6   | -4  | 24   | -4   | $10^{17.00}$        | $10^{16.5}$     | $10^{15.7}$        | $10^{16.11}$        | -8                  | $10^{14.3}$   |
| 2        | 0        | 4   | 2   | 18   | 2    | $10^{15.44}$        | $10^{16.5}$     | $10^{15.7}$        | $10^{16.12}$        | 8                   | $10^{15.8}$   |
| 0        | 2        | 4   | 2   | 14   | -14  | $10^{16.68}$        | $10^{16.5}$     | $10^{15.9}$        | $10^{16.11}$        | -8                  | $10^{15.6}$   |
| 1        | 1        | 4   | 2   | -6   | 26   | $10^{16.83}$        | $10^{16.5}$     | $10^{15.5}$        | $10^{16.09}$        | -8                  | $10^{14.9}$   |
| 2        | 1        | 4   | 2   | -8   | 28   | $10^{16.74}$        | $10^{16.5}$     | $10^{15.69}$       | $10^{16.12}$        | -8                  | $10^{15.2}$   |
| 1        | 2        | 4   | -16 | 36   | -16  | $10^{16.61}$        | $10^{16.5}$     | $10^{15.9}$        | $10^{16.12}$        | -8                  | $10^{15.6}$   |
| 0        | 3        | 4   | -24 | 44   | -24  | $10^{16.57}$        | $10^{16.5}$     | $10^{15.9}$        | $10^{15.90}$        | -8                  | $10^{15.8}$   |
| 3        | 0        | 4   | 2   | 0    | 20   |         | $10^{16.5}$     |         |         |         |         |

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Table 3. Same as Table II, but here the $G_{2213}$ submultiplets acquire masses close to $M_U$ when $SO(10)$ is broken.

| $SO(10)$ representation | $G_{2213}$ submultiplet | $b''_{2L}, b''_{2R}, b''_{3L}, b''_{SO}$ |
|--------------------------|--------------------------|--------------------------------------|
| 210                      | (2, 2, ±$\frac{1}{3}$, 6) | (6, 6, 4, 10)                        |
|                          | (2, 2, ±$\frac{1}{3}$, 3) | (3, 3, 2, 2)                         |
|                          | (2, 2, ±1, 1)              | (1, 1, 6, 0)                         |
|                          | (3, 1, ±$\frac{1}{3}$, 3) | (6, 0, 6, $\frac{2}{3}$)            |
|                          | (1, 3, ±$\frac{1}{3}$, 3) | (0, 6, 6, $\frac{4}{3}$)            |
|                          | (3, 1, 0, 8)              | (16, 0, 0, 9)                        |
|                          | (1, 3, 0, 8)              | (0, 16, 0, 9)                        |
|                          | (3, 1, 0, 1)              | (2, 0, 0, 0)                         |
|                          | (1, 3, 0, 1)              | (0, 2, 0, 0)                         |
|                          | (1, 1, 0, 8)              | (0, 0, 0, 3)                         |
| 45                       | (3, 1, 0, 1)              | (2, 0, 0, 0)                         |
|                          | (1, 3, 0, 1)              | (0, 2, 0, 0)                         |
|                          | (1, 1, 0, 8)              | (0, 0, 0, 3)                         |
| 54                       | (3, 3, 0, 1)              | (6, 6, 0, 0)                         |
|                          | (1, 1, ±$\frac{1}{3}$, 6) | (0, 0, 4, $\frac{2}{3}$)            |
|                          | (1, 1, 0, 8)              | (0, 0, 0, 3)                         |
|                          | (2, 2, ±$\frac{1}{3}$, 3) | (3, 3, 2, 2)                         |
| 16 [16]                  | (2, 1, $\frac{1}{2}$)[(2, 1, $\frac{1}{2}$, 1)] | ($\frac{1}{2}$, 0, $\frac{3}{2}$, 0) |
|                          | (2, 1, $\frac{1}{2}$)[(2, 1, $\frac{1}{2}$, $\frac{3}{2}$)] | ($\frac{1}{2}$, 0, $\frac{1}{2}$, 1) |
|                          | (1, 2, $\frac{1}{2}$)[(2, 1, $\frac{1}{2}$, $\frac{3}{2}$)] | (0, $\frac{1}{2}$, $\frac{1}{2}$, 1) |
| 126 [126]                | (1, 3, $\frac{1}{3}$)[(1, 3, $\frac{1}{3}$, 6)] | (0, 12, 3, $\frac{15}{2}$)          |
|                          | (1, 3, $\frac{1}{3}$)[(1, 3, $\frac{1}{3}$, 3)] | (0, 6, $\frac{3}{2}$, $\frac{3}{2}$) |
|                          | (3, 1, $\frac{1}{3}$)[(3, 1, $\frac{1}{3}$, 6)] | (12, 0, 3, $\frac{15}{2}$)          |
|                          | (3, 1, $\frac{1}{3}$)[(3, 1, $\frac{1}{3}$, $\frac{3}{2}$)] | (6, $\frac{3}{2}$, $\frac{3}{2}$) |
|                          | (3, 1, $\frac{1}{3}$)[(3, 1, $\frac{1}{3}$, 3)] | (2, 0, $\frac{1}{2}$, 0)            |
|                          | (2, 2, $\frac{1}{3}$)[(2, 2, $\frac{1}{3}$, $\frac{3}{2}$)] | (3, 3, 8, 2)                         |
|                          | (2, 2, $\frac{1}{3}$)[(2, 2, $\frac{1}{3}$, 3)] | (3, 3, 8, 2)                         |
|                          | (2, 2, 0, 8)[(2, 2, 0, 8)] | (8, 8, 0, 12)                        |
|                          | (2, 2, 0, 8)[(2, 2, 0, 1)] | (1, 1, 0, 0)                         |
|                          | (1, 1, $\frac{1}{2}$)[(1, 1, $\frac{1}{2}$, $\frac{3}{2}$)] | (0, 0, $\frac{1}{2}$, $\frac{1}{2}$) |
|                          | (1, 1, $\frac{1}{2}$)[(1, 1, $\frac{1}{2}$, $\frac{3}{2}$)] | (0, 0, $\frac{1}{2}$, $\frac{1}{2}$) |
| 10                       | (1, 1, ±$\frac{1}{3}$, 3) | (0, 0, $\frac{1}{2}$, $\frac{1}{2}$) |

submultiplets acquire masses close to $2213$ when $U_{SO}(10)$ is broken.
Table 4. Threshold effects on mass scales in different SUSY \( SO(10) \) models using effective mass parameters.

| MODEL | Representation content | \( \Delta \ln \frac{M_i}{M_Z} \) | \( \Delta \ln \frac{M_U}{M_Z} \) |
|-------|-------------------------|-------------------------------|-------------------------------|
| I     | \( 210, 16 \oplus T \_1, 10 \) | \( \frac{53}{2} \ln \frac{M_U'}{M_U} + \frac{135}{2} \ln \frac{M_C}{M_U} + \frac{27}{2} \ln \frac{M_C}{M_U} - 14 \ln \frac{M_U' M_C}{M_U} \) | -1.33 |
|       |                         | \( -\frac{81}{2} \ln \frac{M_U'}{M_U} + \frac{56}{3} \ln \frac{M_U'}{M_U} \) |                              |
| II    | \( 210, 2(16 \oplus 16), 10 \) | \( 28 \ln \frac{M_U'}{M_U} + 18 \ln \frac{M_U'}{M_U} - \frac{204}{3} \ln \frac{M_U'}{M_U} + 55 \ln \frac{M_U'}{M_U} + \frac{3}{2} \ln \frac{M_U'}{M_U} - 2.78 \) | +0.122 |
| III   | \( 210, 126 \oplus T \_2, 10 \) | \( -29 \ln \frac{M_U'}{M_U} - \frac{55}{3} \ln \frac{M_U'}{M_U} + 150 \ln \frac{M_U'}{M_U} - \frac{305}{3} \ln \frac{M_U'}{M_U} - \frac{9}{4} \ln \frac{M_U'}{M_U} - 1.56 \) | +0.105 |
| IV    | \( 45, 54, 16 \oplus T \_1, 10 \) | \( \frac{3}{2} \ln \frac{M_U'}{M_U} + \frac{7}{2} \ln \frac{M_U'}{M_U} + \frac{3}{2} \ln \frac{M_U'}{M_U} - 2 \ln \frac{M_U'}{M_U} + 1.17 \) | -1.33 |
|       |                         | \( \frac{9}{2} \ln \frac{M_U'}{M_U} + \frac{3}{2} \ln \frac{M_U'}{M_U} \) |                              |
| V     | \( 45, 54, 2(16 \oplus T \_2), 10 \) | \( 4 \ln \frac{M_U'}{M_U} + 2 \ln \frac{M_U'}{M_U} - \frac{35}{2} \ln \frac{M_U'}{M_U} + 11 \ln \frac{M_U'}{M_U} + \frac{3}{2} \ln \frac{M_U'}{M_U} - 2.78 \) | +0.122 |
|       |                         | \( \frac{35}{2} \ln \frac{M_U'}{M_U} + 11 \ln \frac{M_U'}{M_U} \) |                              |
| VI    | \( 45, 54, 126 \oplus T \_3, 10 \) | \( -17 \ln \frac{M_U'}{M_U} - \frac{41}{3} \ln \frac{M_U'}{M_U} + 18 \ln \frac{M_U'}{M_U} - \frac{35}{2} \ln \frac{M_U'}{M_U} + 0.105 \) | +0.105 |
|       |                         | \( +90 \ln \frac{M_U'}{M_U} - \frac{385}{3} \ln \frac{M_U'}{M_U} \) |                              |
|       |                         | \( -\frac{9}{4} \ln \frac{M_U'}{M_U} - 1.56 \) |                              |
Table 5. Predictions on mass scales $M_I$ and $M_U$ including threshold effect with effective mass parameters.

| Model | Two-loop (GeV) | $M_{1V}$ | $M_{3L}$ | $M_{2R}$ | $M_{1L}$ | $M_{2C}$ | $M_I$(GeV) | $M_U$(GeV) |
|-------|----------------|----------|----------|----------|----------|----------|------------|------------|
| I     | $M_I = 10^{16.9}$ | -        | $4M_U$   | $3M_U$   | $3M_U$   | $2.95M_U$| $1.36 \times 10^{11}$ | $5.2 \times 10^{17}$ |
|       | $M_U = 10^{16.11}$ | -        | $5M_U$   | $3.5M_U$ | $3.5M_U$ | $M_U$   | $2 \times 10^{11}$  | $1.5 \times 10^{17}$ |
|       |                | -        | $3M_U$   | $M_U$   | $M_U$   | $2.2M_U$| $1.6 \times 10^9$   | $6.5 \times 10^{17}$ |
| II    | $M_I = 10^{17.6}$ | $M_I$   | $3M_U$   | $4M_U$   | $2M_U$   | $2.5M_U$| $1.0 \times 10^{11}$ | $1.3 \times 10^{17}$ |
|       | $M_U = 10^{16.12}$ | $M_I$   | $3M_U$   | $3.3M_U$ | $3.3M_U$ | $2.27M_U$| $2.78 \times 10^{11}$ | $5.58 \times 10^{17}$ |
|       |                | $\frac{1}{3}M_I$ | $4M_U$   | $5M_U$   | $2M_U$   | $3M_U$   | $1.3 \times 10^9$   | $5.55 \times 10^{17}$ |
| III   | $M_I = 10^{15.2}$ | $M_I$   | $M_U$   | $M_U$   | $2M_U$   | $M_U$   | $1.11 \times 10^9$   | $1.46 \times 10^{16}$ |
|       | $M_U = 10^{16.11}$ | $3.5M_I$ | $2.27M_U$ | $3.5M_U$ | $3.5M_U$ | $2M_U$   | $5.3 \times 10^{11}$ | $5.5 \times 10^{17}$ |
|       |                | $M_I$   | $M_U$   | $M_U$   | $5M_U$   | $M_U$   | $1.2 \times 10^9$   | $1.46 \times 10^{16}$ |
| IV    | $M_I = 10^{16.9}$ | -        | $2M_U$   | $M_U$   | $M_U$   | $M_U$   | $1.84 \times 10^{12}$ | $3.34 \times 10^{17}$ |
|       | $M_U = 10^{16.11}$ | -        | $2M_U$   | $M_U$   | $M_U$   | $\frac{1}{3}M_U$ | $1.23 \times 10^{11}$ | $2.53 \times 10^{18}$ |
|       |                | -        | $3M_U$   | $\frac{1}{3}M_U$ | $\frac{4}{3}M_U$ | $\frac{4}{3}M_U$ | $1.2 \times 10^9$   | $6.6 \times 10^{19}$ |
| V     | $M_I = 10^{17.6}$ | $M_I$   | $M_U$   | $\frac{1}{3}M_U$ | $5M_U$   | $\frac{4}{3}M_U$ | $1.77 \times 10^{12}$ | $9.94 \times 10^{17}$ |
|       | $M_U = 10^{16.12}$ | $M_I$   | $\frac{1}{3}M_U$ | $M_U$   | $\frac{4}{3}M_U$ | $\frac{4}{3}M_U$ | $2.84 \times 10^{11}$ | $1.48 \times 10^{16}$ |
|       |                | $\frac{1}{3}M_I$ | $2M_U$   | $M_U$   | $M_U$   | $M_U$   | $6.3 \times 10^9$   | $6.7 \times 10^{17}$ |
| VI    | $M_I = 10^{15.2}$ | $5M_I$  | $\frac{1}{3}M_U$ | $M_U$   | $\frac{4}{3}M_U$ | $\frac{4}{3}M_U$ | $1.0 \times 10^{14}$ | $6.75 \times 10^{17}$ |
|       | $M_U = 10^{16.11}$ | $M_I$   | $\frac{1}{3}M_U$ | $3M_U$   | $M_U$   | $M_U$   | $4 \times 10^{11}$  | $1.46 \times 10^{16}$ |
|       |                | $5M_I$  | $\frac{1}{3}M_U$ | $5M_U$   | $M_U$   | $1.1M_U$ | $3.7 \times 10^9$   | $1.87 \times 10^{15}$ |

Table 6. Threshold effects on mass scales in different SUSY $SO(10)$ models using superheavy masses.

| MODEL | Representation content | $\Delta \ln \frac{M_I}{M_{12}}$ | $\Delta \ln \frac{M_U}{M_{12}}$ |
|-------|------------------------|-------------------------------|-------------------------------|
| I     | $210, 16 \oplus 16, 10$ | $-\frac{1}{2} \ln \frac{M_{10}}{M_{12}} + \frac{1}{2} \ln \frac{M_{12}}{M_{12}}$ | $\frac{1}{2} \ln \frac{M_{10}}{M_{12}} - \frac{1}{2} \ln \frac{M_{10}}{M_{12}}$ + 0.117 |
|       |                         | -1.33                         | +0.117                        |
| II    | $210, 2(16 \oplus 16), 10$ | $\frac{1}{4} \ln \frac{M_{10}}{M_{12}} - 2 \ln \frac{M_{10}}{M_{12}}$ | $\frac{1}{4} \ln \frac{M_{10}}{M_{12}} - \frac{1}{4} \ln \frac{M_{10}}{M_{12}}$ + 0.122 |
|       |                         | + $\frac{1}{3} \ln \frac{M_{12}}{M_{12}} + \ln \frac{M_{12}}{M_{12}}$ | +0.122                        |
|       |                         | -2.78                         |                               |
| III   | $210, 126 \oplus 126, 10$ | $-\frac{1}{2} \ln \frac{M_{10}}{M_{12}} + \frac{5}{2} \ln \frac{M_{126}}{M_{12}}$ | $-\frac{1}{4} \ln \frac{M_{10}}{M_{12}} - \frac{1}{2} \ln \frac{M_{10}}{M_{12}}$ + 0.105 |
|       |                         | + $\frac{1}{2} \ln \frac{M_{12}}{M_{12}} - 2 \ln \frac{M_{12}}{M_{12}}$ | +0.105                        |
| IV    | $45, 54, 16 \oplus 16, 10$ | $-\frac{1}{2} \ln \frac{M_{10}}{M_{12}} + \frac{1}{2} \ln \frac{M_{12}}{M_{12}}$ | $\frac{1}{2} \ln \frac{M_{10}}{M_{12}} - \frac{1}{2} \ln \frac{M_{10}}{M_{12}}$ + 0.117 |
|       |                         | -1.33                         | +0.117                        |
| V     | $45, 54, 2(16 \oplus 16), 10$ | $\frac{1}{4} \ln \frac{M_{10}}{M_{12}} - 2 \ln \frac{M_{10}}{M_{12}}$ | $\frac{1}{4} \ln \frac{M_{10}}{M_{12}} - \frac{1}{2} \ln \frac{M_{10}}{M_{12}}$ + 0.122 |
|       |                         | + $\frac{1}{3} \ln \frac{M_{12}}{M_{12}} + \ln \frac{M_{12}}{M_{12}}$ | +0.122                        |
|       |                         | -2.78                         |                               |
| VI    | $45, 54, 126 \oplus 126, 10$ | $-\frac{1}{2} \ln \frac{M_{10}}{M_{12}} - \frac{3}{2} \ln \frac{M_{126}}{M_{12}}$ | $-\frac{1}{4} \ln \frac{M_{10}}{M_{12}} - \frac{1}{2} \ln \frac{M_{10}}{M_{12}}$ + 0.105 |
|       |                         | + $\frac{1}{2} \ln \frac{M_{12}}{M_{12}} - 2 \ln \frac{M_{12}}{M_{12}}$ | +0.105                        |
|       |                         | -1.56                         |                               |
Table 7. Predictions on mass scales $M_I$ and $M_U$ including threshold effect with superheavy masses.

| Model | Two-loop(GeV) | $M_I$ | $M_{210}$ or $M_{45}$ | $M_{16}$ | $M_{126}$ | $M_{10}$ | $M_I$(GeV) | $M_U$(GeV) |
|-------|---------------|-------|-----------------------|---------|-----------|---------|------------|------------|
| I,IV  | $M_I = 10^{16.9}$ | -     | $\frac{1}{5}M_I$    | $\frac{1}{5}M_I$ | -         | $\frac{1}{5}M_I$ | $2.1 \times 10^{16}$ | $3.2 \times 10^{16}$ |
|       | $M_U = 10^{16.11}$ | -     | $\frac{1}{5}M_U$    | $5M_U$  | -         | $\frac{1}{5}M_U$ | $9.7 \times 10^{14}$ | $3.2 \times 10^{16}$ |
| II,V  | $M_I = 10^{17.6}$ | $\frac{1}{5}M_I$ | $\frac{1}{5}M_I$  | $\frac{1}{5}M_I$ | -         | $\frac{1}{5}M_I$ | $1.1 \times 10^{16}$ | $3.3 \times 10^{16}$ |
|       | $M_U = 10^{16.12}$ | $\frac{1}{5}M_I$ | $\frac{1}{5}M_U$  | $5M_U$  | -         | $\frac{1}{5}M_U$ | $1.7 \times 10^{13}$ | $3.3 \times 10^{16}$ |
| III,VI| $M_I = 10^{15.2}$ | $\frac{1}{5}M_I$ | $5M_U$    | -         | $\frac{1}{5}M_U$ | $5M_U$  | $2.3 \times 10^{10}$ | $6.3 \times 10^{15}$ |
|       | $M_U = 10^{16.11}$ | $\frac{1}{5}M_I$ | $5M_U$    | -         | $\frac{1}{5}M_U$ | $M_U$   | $1.1 \times 10^{11}$ | $9.5 \times 10^{15}$ |