Anomaly puzzle in $N = 1$ supersymmetric electrodynamics as artifact of dimensional reduction.

K.V. Stepanyantz *

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Moscow State University, physical faculty,
department of theoretical physics.
117234, Moscow, Russia

Abstract

Calculations of the two-loop $\beta$-function for $N = 1$ supersymmetric electrodynamics are compared for regularizations by higher derivatives and by the dimensional reduction. The renormalized effective action are found to be the same for both regularizations. However, unlike the dimensional reduction, the higher derivative regularization does not lead to anomaly puzzle, because it allows to perform correct calculation of diagrams with insertions of counterterms. In particular, using this method a contribution of diagrams with insertions of counterterms is calculated exactly to all orders. This contribution appears to be 0 if the theory is regularized by the dimensional reduction. We argue, that this result follows from mathematical inconsistency of the dimensional reduction and is responsible for the anomaly puzzle.

1 Introduction.

It is well known [1, 2, 3, 4], that in supersymmetric theories the axial and the trace of the energy-momentum tensor anomalies are components of a chiral scalar supermultiplet. Adler-Bardeen theorem [5, 6] asserts that there are no radiative corrections to the axial anomaly beyond the one-loop approximation, while the trace anomaly is proportional to the $\beta$-function [7] to all orders. Therefore this seems to imply, that the $\beta$-function in supersymmetric theories should be exhausted by the first loop [8]. It really takes place in models with $N = 2$ supersymmetry [9]. However explicit perturbative

*E-mail:stepan@theor.phys.msu.su
calculations find higher order corrections to the $\beta$-functions of $N = 1$ supersymmetric theories, regularized by dimensional reduction [10, 11, 12]. This contradiction is usually called "anomaly puzzle".

Many papers are devoted to attempts of solving the anomaly puzzle in supersymmetric theories. For example, in [13] the anomaly puzzle is argued to be a consequence of the difference between usual and Wilsonian effective actions. In particular, the authors noted, that the nontrivial contribution to the $\beta$-function come from the so-called Konishi anomaly [14, 15]. Investigation of this contribution in [13] and investigation of instanton contributions in [16] lead to construction of the so called exact Novikov, Shifman, Vainshtein and Zakharov (NSVZ) $\beta$-function. For $N = 1$ supersymmetric electrodynamics considered in this paper the NSVZ $\beta$-function has the following form:

$$\beta(\alpha) = \frac{\alpha^2}{\pi} \left( 1 - \gamma(\alpha) \right)$$  \hspace{1cm} (1)

where $\gamma(\alpha)$ is the anomalous dimension of the matter superfield. Explicit perturative calculations using regularization by the dimensional reduction (DRED) verify the NSVZ $\beta$-function up to two-loop order. Nevertheless, three loop results obtained in [17, 18, 19] do not agree with the NSVZ $\beta$-function. However [20] this disagreement can be eliminated by a special choice of renormalization scheme, a possibility of such choice being highly nontrivial [21]. In principle it is possible to relate DRED scheme and NSVZ scheme order by order [22] in the perturbation theory.

A very simple and beautiful solution of anomaly puzzle, different from the solution of [13], was presented in [23]. The main idea of this paper is that the higher order corrections in NSVZ $\beta$-function are due to anomalous Jacobian under the rescaling of the fields done in passing from holomorphic to canonical normalization. In the case of supersymmetric electrodynamics holomorphic normalization means, that the renormalized action is written as

$$S_{\text{ren}} = \frac{1}{4e^2} Z_3(\Lambda/\mu) \text{Re} \int d^4x d^2\theta W_a C^{ab} W_b +$$
$$+ Z(\Lambda/\mu) \frac{1}{4} \int d^4x d^4\theta \left( \phi^* e^{2V} \phi + \tilde{\phi}^* e^{-2V} \tilde{\phi} \right),$$ \hspace{1cm} (2)

while in the canonical normalization

$$S_{\text{ren}} = \frac{1}{4e^2} Z_3(\Lambda/\mu) \text{Re} \int d^4x d^2\theta W_a C^{ab} W_b +$$
$$+ \frac{1}{4} \int d^4x d^4\theta \left( \phi^* e^{2V} \phi + \tilde{\phi}^* e^{-2V} \tilde{\phi} \right),$$ \hspace{1cm} (3)

In the former case the $\beta$-function is supposed to be exhausted at the one loop, while in the latter case it coincides with the NSVZ result. In principle this solution is different from the one, given by Shifman and Vainshtein. Moreover, it contradicts to the results of explicit two-loop calculations, made by dimensional reduction. The authors of [23]
supposed, that in the holomorphic normalization the $\beta$-function is exhausted at the one loop if one uses higher covariant derivative regularization [24, 25], supplemented by the Pauli-Villars. It is known [26], that this regularization always yields the same result for one-loop logarithmic divergences as the dimensional regularization (reduction). Explicit two-loop calculations for theories, regularized by higher derivatives (HD), were made first in [27, 28] for $N = 1$ supersymmetric electrodynamics and gave zero two-loop contribution to the $\beta$-function. This result implies absence of anomaly puzzle in view of the solution proposed in [23]. However it is not quite clear why different regularizations give different results for the scheme independent two-loop $\beta$-function. In principle, in [27] we noted, that using of the higher derivative regularization leads to a nontrivial contribution of diagrams with insertions of one-loop counterterms, which is absent for dimensional reduction. However the detailed analysis of this result was not yet made.

In this paper calculation of diagrams with insertions of counterterms is analyzed for the DRED and HD regularization. DRED technique proposed by Siegel [29] consists of continuing in the number of space-time dimensions from 4 to $n$, where $n$ is less than 4, but keeping the numbers of components of all other tensors fixed. It is important to note, that such regularization is mathematically inconsistent [30]. As a consequence, a straightforward application of DRED to the calculation of axial anomaly gives incorrect zero result, because DRED does not break chiral symmetry. It is necessary to stress an essential difference between the DRED and the dimensional regularization (DREG), which is mathematically consistent and allows to calculate anomalies [31]. In principle, it is possible to calculate axial anomaly even within the DRED technique. However, for this purpose it is necessary to impose some mathematically inconsistent conditions, for example $\text{tr}(AB) \neq \text{tr}(BA)$ [32]. Another possibility is an attempt to go to $n > 4$ in the DRED scheme [33], that also leads to some contradictions.

In the present paper we argue, that the above mentioned contradictions of DRED lead to the incorrect result for the sum of diagrams with insertions of counterterms and, therefore, for the $\beta$-function. These arguments are confirmed by comparison between calculations of the two-loop $\beta$-function for the $N = 1$ supersymmetric electrodynamics by DRED and HD regularization.

The paper is organized as follows:

In section 2 we consider $N = 1$ supersymmetric electrodynamics and different ways of its regularization. In particular, in this section we remind main contradictions of DRED, pointed out by Siegel. Calculation of the two-loop contribution to the $\beta$-function and relationship between this contribution and Konishi anomaly are analyzed in section 3 using different regularizations. In section 4 the sum of diagrams with insertions of counterterms on the matter lines regularized by HD is calculated exactly to all orders. Section 5 contains some concluding remarks. Finally, appendix contains a derivation of the exact result for a sum of diagrams with insertions of counterterms.
2 Supersymmetric electrodynamics and its regularization.

2.1 $N = 1$ supersymmetric electrodynamics.

$N = 1$ supersymmetric electrodynamics in the superspace is described by the following action:

$$S_0 = \frac{1}{4e^2} \text{Re} \int d^4x d^2\theta W_a C^{ab} W_b + \frac{1}{4} \int d^4x d^4\theta \left( \phi^* e^{2V} \phi + \tilde{\phi}^* e^{-2V} \tilde{\phi} \right).$$

(4)

Here $\phi$ and $\tilde{\phi}$ are chiral superfields

$$\phi(y, \theta) = \varphi(y) + \bar{\theta}(1 + \gamma_5)\psi(y) + \frac{1}{2}\bar{\theta}(1 + \gamma_5)f(y);$$

$$\tilde{\phi}(y, \theta) = \tilde{\varphi}(y) + \bar{\theta}(1 + \gamma_5)\tilde{\psi}(y) + \frac{1}{2}\bar{\theta}(1 + \gamma_5)f(y),$$

(5)

where $y^\mu = x^\mu + i\bar{\theta}\gamma^\mu\gamma_5\theta/2$. Two Majorana spinors $\psi$ and $\tilde{\psi}$ form one Dirac spinor

$$\Psi = \frac{1}{\sqrt{2}} \left( (1 + \gamma_5)\psi + (1 - \gamma_5)\tilde{\psi} \right).$$

(6)

$V$ in (4) is a real superfield

$$V(x, \theta) = C(x) + i\sqrt{2}\gamma_5\xi(x) + \frac{1}{2}(\bar{\theta}\theta)K(x) + \frac{i}{2}(\bar{\theta}\gamma_5\theta)H(x) + \frac{1}{2}(\bar{\theta}\gamma^\mu\gamma_5\theta)A_\mu(x) +$$

$$+ \sqrt{2}(\bar{\theta}\theta)\theta \left( i\gamma_5 \chi(x) + \frac{1}{2}\gamma^\mu\gamma_5\partial_\mu \xi(x) \right) + \frac{1}{4}(\bar{\theta}\theta)^2 \left( D(x) - \frac{1}{2}\partial^2 C(x) \right),$$

(7)

where, in particular, $A_\mu$ is an Abelian gauge field. The superfield $W_a$ in the Abelian case is defined by

$$W_a = \frac{1}{16} D(1 - \gamma_5) D \left[ (1 + \gamma_5) D_a V \right],$$

(8)

where $D$ is the supersymmetric covariant derivative

$$D = \frac{\partial}{\partial \theta} - i\gamma^\mu \theta \partial_\mu.$$

(9)

2.2 Higher derivative regularization.

In order to regularize model (4) by HD its action should be modified as follows:
\[ S_0 \to S = S_0 + S_\Lambda = \]
\[ = \frac{1}{4e^2} \text{Re} \int d^4x d^2 \theta W_a C^{ab} \left( 1 + \frac{\partial^{2n}}{\Lambda^{2n}} \right) W_b + \]
\[ + \frac{1}{4} \int d^4x d^4 \theta \left( \phi^* e^{2V} \phi + \bar{\phi}^* e^{-2V} \bar{\phi} \right). \] (10)

Note, that in the Abelian case the superfield \( W^a \) is gauge invariant, so that the higher derivative term contains usual derivatives.

Quantization of (10) can be made using standard technique described in [34] and is not considered here. It is necessary to mention only that the gauge invariance was fixed by adding of

\[ S_{gf} = -\frac{1}{64e^2} \int d^4x d^4 \theta \left( V D^2 \bar{D}^2 \left( 1 + \frac{\partial^{2n}}{\Lambda^{2n}} \right) V + V \bar{D}^2 D^2 \left( 1 + \frac{\partial^{2n}}{\Lambda^{2n}} \right) V \right), \] (11)

where

\[ D^2 \equiv \frac{1}{2} \bar{D}(1 + \gamma_5)D; \quad \bar{D}^2 \equiv \frac{1}{2} \bar{D}(1 - \gamma_5)D. \] (12)

After adding of such terms the free part of the action for the superfield \( V \) is written in the most simple form

\[ S_{\text{gauge}} + S_{gf} = \frac{1}{4e^2} \int d^4x d^4 \theta V \partial^2 \left( 1 + \frac{\partial^{2n}}{\Lambda^{2n}} \right) V. \] (13)

In the Abelian case diagrams containing ghost loops are certainly absent.

The superficial degree of divergence for the model (10) is equal to (see e.f. [27])

\[ \omega_\Lambda = 2 - 2n(L - 1) - E_\phi(n + 1), \] (14)

where \( L \) is a number of loops and \( E_\phi \) is a number of external \( \phi \)-lines. According to (14) divergences remain in one-loop diagrams even for \( n \geq 2 \). In order to regularize these divergences it is necessary to insert in the generating functional Pauli-Villars determinants [6]:

\[ Z = \int DV \; D\phi \; D\bar{\phi} \prod_i \left( \text{det} PV(V, M_i) \right)^{c_i} \exp \left\{ i \left[ \frac{1}{4e^2} \int d^4x d^4 \theta V \partial^2 \left( 1 + \frac{\partial^{2n}}{\Lambda^{2n}} \right) V - \frac{1}{4e^2} \left( Z_3(\Lambda/\mu) - 1 \right) \int d^4x d^4 \theta V \Pi_{1/2} \partial^2 \left( 1 + \frac{\partial^{2n}}{\Lambda^{2n}} \right) V + \right. \right. \]
\[ + \frac{1}{4} Z(\Lambda/\mu) \int d^4x d^4 \theta \left( \phi^* e^{2V} \phi + \bar{\phi}^* e^{-2V} \bar{\phi} \right) + \]
\[ + \int d^4x d^4 \theta JV + \int d^4x d^2 \theta \left( j_\phi + \tilde{j}_\phi \right) + \int d^4x d^2 \bar{\theta} \left( j^*_\phi + \tilde{j}^*_\phi \right) \right] \right\}. \] (15)
where

\[
\left( \det PV(V,M) \right)^{-1} = \int D\Phi D\bar{\Phi} \exp \left\{ i \left[ Z(\Lambda/\mu) \frac{1}{4} \int d^4x d^4\theta \left( \Phi^* e^{2V} \Phi + \bar{\Phi}^* e^{-2V} \bar{\Phi} \right) \right. \right. 
\]
\[
\left. \left. + \frac{1}{2} \int d^4x d^2\theta M \bar{\Phi} \Phi + \frac{1}{2} \int d^4x d^2\bar{\theta} M \bar{\Phi}^* \Phi^* \right] \right\},
\]

(16)

and the coefficients \( c_i \) satisfy equations

\[
\sum_i c_i = 1; \quad \sum_i c_i M_i^2 = 0.
\]

(17)

Below we will assume, that \( M_i = a_i \Lambda \), where \( a_i \) are some constants. Insertion of Pauli-Villars determinants allows to cancel remaining divergences in all one-loop diagrams, including diagrams with insertions of counterterms.

In our notations the generating functional \( W \) is defined by

\[
W = -i \ln Z
\]

(18)

and an effective action is obtained by making a Legendre transformation:

\[
\Gamma = W - \int d^4x d\theta J V - \int d^4x d^2\theta \left( j \phi + \bar{j} \bar{\phi} \right) - \int d^4x d^2\bar{\theta} \left( j^* \phi^* + \bar{j}^* \bar{\phi}^* \right),
\]

(19)

where \( J, j \) and \( \bar{j} \) is to be eliminated in terms of \( V, \phi \) and \( \bar{\phi} \), through solving equations

\[
V = \frac{\delta W}{\delta J}; \quad \phi = \frac{\delta W}{\delta j}; \quad \bar{\phi} = \frac{\delta W}{\delta \bar{j}}.
\]

(20)

Due to the supersymmetric gauge invariance

\[
V \rightarrow V - \frac{1}{2} (A + A^+); \quad \phi \rightarrow e^A \phi; \quad \bar{\phi} \rightarrow e^{-A} \bar{\phi},
\]

(21)

where \( A \) is an arbitrary chiral scalar superfield, the renormalized action can be written as

\[
S_{\text{ren}} = \frac{1}{4e^2} Z_3(\Lambda/\mu) \text{Re} \int d^4x d^2\theta W_a C^{ab} \left( 1 + \frac{\partial^{2n}}{\Lambda^{2n}} \right) W_b + 
\]
\[
+ Z(\Lambda/\mu) \frac{1}{4} \int d^4x d^4\theta \left( \phi^* e^{2V} \phi + \bar{\phi}^* e^{-2V} \bar{\phi} \right).
\]

(22)

Here \( e = e(\Lambda/\mu) \) is a renormalized coupling constant, while the bare coupling constant \( e_0 \) is given by

\[
\frac{1}{e_0} = \frac{1}{e(\Lambda/\mu)^2} Z_3(\Lambda/\mu)
\]

(23)
and does not depend on $\mu$.

Having obtained $S_{\text{ren}}$, it is possible to find the $\beta$-function and the anomalous dimension, which in our notations are defined by

$$\beta = \frac{d}{d\ln \mu} \left( \frac{e^2}{4\pi} \right); \quad \gamma = \frac{d\ln Z}{d\ln \mu}.$$  \hspace{1cm} (24)

### 2.3 Dimensional reduction and dimensional regularization.

Although HD regularization can be easily applied to calculations of quantum corrections in the supersymmetric electrodynamics [27, 28, 36], use of this regularization encounters considerable technical difficulties in non-Abelian gauge theories due to complicated structure of vertexes. That is why the HD regularization was not applied to calculations so often as the DREG [31] or DRED [29].

In the DREG method calculations of quantum corrections are formally performed in the space-time with dimension $n \neq 4$. However this method is not well-suited for supersymmetric theories, because DREG does not preserve invariance of the action with respect to the supersymmetry transformations. The reason is that a necessary condition for supersymmetry is equality of Bose and Fermi degrees of freedom, which can take place only for integer $n$.

A modification of DREG so as to render it compatible with supersymmetry was made by Siegel [29]. According to Siegel’s method the $n = 4$ Lagrangian is dimensionally reduced to $n < 4$ dimensions. Then a vector $A_\mu$ is split into an $n$ component vector $\tilde{A}_\mu$ and $\epsilon = 4 - n$ "$\epsilon$-scalars" $\hat{A}_\mu$, but the total number of bosons remains $n$ independent. It is important, that the dimension $n$ should be less than 4, so that

$$\delta^\nu_\mu \tilde{\delta}_\nu = n,$$  \hspace{1cm} (25)

where $\delta^\nu_\mu$ is 4-dimensional Kronecker symbol and $\tilde{\delta}_\nu$ is $n$-dimensional Kronecker symbol.

However as pointed out in [30] there remain ambiguities with dimensional reduction associated with treatment of the Levi-Civita symbol. For example, the product

$$\varepsilon^{\alpha\beta\gamma\delta} \varepsilon_{\alpha\beta\gamma\delta} \varepsilon^{\mu\nu\rho\tau} \varepsilon_{\mu\nu\rho\tau}$$  \hspace{1cm} (26)

depends on a way of calculation and can be equal either to 0 or to

$$(n - 4)(n - 3)^2(n - 2)^2(n - 1)^2n,$$  \hspace{1cm} (27)

Therefore the dimension reduction is mathematically consistent only for integer $n \leq 4$. Moreover, if $n < 4$ the $\gamma_5$-matrix should be chosen so that

$$\{\gamma_5, \tilde{\gamma}_\mu\} = 0; \quad \gamma_5^2 = 1.$$  \hspace{1cm} (28)

As a consequence it is possible obtain (see [21] for details), that
(n - 4) \text{tr} \left( \gamma_5 \tilde{\gamma}_\nu \tilde{\gamma}_\alpha \tilde{\gamma}_\beta \right) = 0. \quad (29)

Due to (28) both gauge and chiral symmetries are unbroken. In DREG this problem can be solved by use of $\gamma_5$ with the following properties:

\[
\{ \gamma_5, \gamma_\mu \} = 0, \quad \mu = 0, \ldots, 3; \quad [\gamma_5, \gamma_\mu] = 0, \quad \mu > 3,
\]

which allow to derive the axial anomaly unambiguously [31]. Nevertheless (see review [21] and references therein) DRED is usually believed to be a satisfactory regularization for supersymmetric theories. From the practical point of view in order to perform calculations by DRED it is necessary to use 4-dimensional algebra of $\gamma$-matrices and calculate the remaining integrals in the dimension $n \neq 4$ [34].

3 Calculation of the two-loop $\beta$-function.

3.1 Two-loop Feynman diagrams.

Feynman diagrams giving nontrivial contributions to the two-loop $\beta$-function for $N = 1$ supersymmetric electrodynamics are presented at Figure 1. These diagrams can be naturally divided into three parts:

1. one-loop diagrams (1) and (2);
2. two-loop diagrams (without subtraction diagrams) (3) – (8);
3. subtraction diagrams (9) – (12), containing insertions of one-loop counterterms.

3.2 Higher derivatives regularization.

Divergent part of the two-loop effective action for $N = 1$ supersymmetric electrodynamics was obtained in [27, 36] \footnote{In [36] contributions of diagrams (1) – (8) was found correctly, but the contribution of diagrams (9) – (12) was omitted.}. The result is

\[
\Delta \Gamma^{(2)}_{\text{V}} = \text{Re} \int d^2 \theta \, \frac{d^4 p}{(2\pi)^4} W_a(p) C^{ab} W_b(-p) \left( f_1 + f_2 + f_{2PV} + f_3 \right), \quad (31)
\]

where

\[
f_1 = -\frac{i}{2} \left( \int \frac{d^4 k}{(2\pi)^4} \frac{1}{k^2(k+p)^2} - \sum_i c_i \int \frac{d^4 k}{(2\pi)^4} \frac{1}{(k^2 - M_i^2)((k+p)^2 - M_i^2)} \right) \quad (32)
\]

is a one-loop result,
\[ f_2 = -e^2 \int \frac{d^4k}{(2\pi)^4} \frac{d^4q}{(2\pi)^4} \frac{(k + p + q)^2 + q^2 - k^2 - p^2}{k^2(1 + (-1)^n k^{2n}/\Lambda^{2n})(k + q)^2(k + p + q)^2q^2(q + p)^2} \]  

(33)

is a sum of diagrams (3) – (8) without contributions of Pauli-Villars fields,

\[ f_{2\nu} = e^2 \sum_i c_i \int \frac{d^4k}{(2\pi)^4} \frac{d^4q}{(2\pi)^4} \frac{1}{k^2(1 + (-1)^n k^{2n}/\Lambda^{2n})} \times \]
\[ \left( \frac{(k + p + q)^2 + q^2 - k^2 - p^2}{((k + q)^2 - M_i^2)((k + p + q)^2 - M_i^2)(q^2 - M_i^2)((q + p)^2 - M_i^2)} + \right) \]
\[ + \frac{4M_i^2}{((k + q)^2 - M_i^2)((q^2 - M_i^2)((q + p)^2 - M_i^2))} \]  

(34)

is a contribution of diagrams (3) – (8) with internal loop of Pauli-Villars fields and

\[ f_3 = -\frac{ie^2}{2\pi^2} \ln \frac{\Lambda}{\mu} \sum_i c_i \int \frac{d^4k}{(2\pi)^4} \frac{M_i^2}{(k^2 - M_i^2)^2((k + p)^2 - M_i^2)} \]  

(35)

is a sum of diagrams (9) – (12), which have insertions of the one-loop counterterms.

According to the calculations performed in [27, 36] contribution (34) is finite and does not affect divergent part of the effective action. It is important, because some diagrams with Pauli-Villars loop in principle can contain divergencies. However, Pauli-Villars regularization always implies existence of divergent graphs, but these divergences should be cancelled in the sum of all diagrams, that actually takes place in the considered case.

The other contributions in (31) are given by

\[ f_1 = \frac{1}{16\pi^2} \ln \frac{\Lambda}{p} + O(1); \]
\[ f_2 = \frac{1}{16\pi^2} \frac{\alpha}{\pi} \ln \frac{\Lambda}{p} + O(1); \]
\[ f_3 = -\frac{1}{16\pi^2} \frac{\alpha}{\pi} \ln \frac{\Lambda}{\mu} + o(1). \]

(36)

The corresponding two-loop contribution to the effective action is \( (\Gamma_R = S_{ren} + \Delta \Gamma) \)

\[ \Delta \Gamma^{(2)}_V = \frac{1}{16\pi^2} \text{Re} \int d^2\theta \frac{d^4p}{(2\pi)^4} W_a(p)C^{ab}W_b(-p) \left( \ln \frac{\Lambda}{p} + \frac{\alpha}{\pi} \ln \frac{\mu}{p} + O(1) \right), \]  

(37)

so that it is not necessary to add any new counterterms for cancellation of the two-loop divergencies.
\[
\Delta S = -\frac{1}{16\pi^2} \text{Re} \int d^2 \theta \frac{d^4 p}{(2\pi)^4} W_a(p) C^{ab} W_b(-p) \left( \ln \frac{\Lambda}{\mu} + \text{finite terms} + O(\alpha^2) \right) + \\
+ \text{terms with matter superfields},
\]

so that

\[
\frac{1}{\alpha_0} = \frac{1}{\alpha \left( \Lambda / \mu \right)} - \frac{1}{\pi} \ln \frac{\Lambda}{\mu} + O(\alpha^2).
\]

According to equation (24) this expression corresponds to the following two-loop \( \beta \)-function:

\[
\beta_{HD} = \frac{\alpha^2}{\pi} + O(\alpha^4).
\]

3.3 Dimensional reduction.

A result corresponding to (31), obtained by DRED, is given by

\[
\Delta \Gamma^{(2)}_V = \text{Re} \int d^2 \theta \frac{d^4 p}{(2\pi)^4} W_a(p) C^{ab} W_b(-p) \left( \tilde{f}_1 + \tilde{f}_2 + \tilde{f}_3 \right),
\]

where

\[
\tilde{f}_1 = -\frac{i}{2} \int \frac{d^n k}{(2\pi)^4} \frac{1}{k^2 (k + p)^2},
\]

\[
\tilde{f}_2 = -\epsilon^2 \int \frac{d^n k}{(2\pi)^4} \frac{d^n q}{(2\pi)^4} \frac{(k + p + q)^2 + q^2 - k^2 - p^2}{k^2 (k + q)^2 (k + p + q)^2 q^2 (q + p)^2},
\]

\[
\tilde{f}_3 = 0.
\]

Using notations of [33]

\[
I \equiv (p^2)^{2-n/2} \int \frac{d^n k}{k^2 (k + p)^2} = \pi^{n/2} \Gamma \left( 2 - n/2 \right) B \left( n/2 - 1, n/2 - 1 \right);
\]

\[
J \equiv (p^2)^{4-n} \int \frac{d^n k}{(k^2)^{3-n/2} (k + p)^2} = \pi^{n/2} \frac{\Gamma \left( 4 - n \right)}{\Gamma \left( 3 - n/2 \right)} B \left( n - 3, n/2 - 1 \right);
\]

\[
Z \equiv (p^2)^{5-n} \int \frac{d^n k \; d^n q}{q^2 k^2 (k + p)^2 (q + p)^2 (k - q)^2} = \frac{1}{n - 4} \left( (6n - 20)IJ - (2n - 6)I^2 \right)
\]

(45)
the two-loop contribution to the effective action, calculated by DRED, can be written as

$$\Delta \Gamma^{(2)}_V = \text{Re} \int d^2 \theta \frac{d^4 p}{(2\pi)^4} W_a(p) C^{ab} W_b(-p) \times$$

$$\times \left( \frac{1}{2(2\pi)^4} (p/\mu_0)^{n-4} I + \frac{e^2}{(2\pi)^8} (p/\mu_0)^{2n-8} (2IJ - I^2 - p^2 Z) \right),$$

(46)

where a constant $\mu_0$ is present because the dimension of the coupling constant depends on the space-time dimension $n$ [35].

Having calculated the integrals we obtain, that

$$\Delta \Gamma^{(2)}_V = \frac{1}{16\pi^2} \text{Re} \int d^2 \theta \frac{d^4 p}{(2\pi)^4} W_a(p) C^{ab} W_b(-p) \times$$

$$\times \left( -\frac{1}{4-n} + \ln \frac{\mu_0}{p} + \frac{\alpha}{\pi} \left( -\frac{1}{2(4-n)} + \ln \frac{\mu}{\mu_0} \right) + \text{finite terms} + O(1) \right).$$

(47)

First two terms here correspond to the one-loop integral $\tilde{f}_1$, and the third term corresponds to the two-loop integral $\tilde{f}_2$.

In order to cancel divergences in $\Delta \Gamma^{(2)}_V$, it is necessary to add counterterms

$$\Delta S = \frac{1}{16\pi^2} \text{Re} \int d^2 \theta \frac{d^4 p}{(2\pi)^4} W_a(p) C^{ab} W_b(-p) \times$$

$$\times \left( -\frac{1}{4-n} + \ln \frac{\mu}{\mu_0} + \frac{\alpha}{\pi} \left( -\frac{1}{2(4-n)} + \ln \frac{\mu}{\mu_0} \right) + \text{finite terms} + O(\alpha^2) \right).$$

(48)

Therefore the DRED $\beta$-function defined by (24) is equal to

$$\beta_{DRED} = \frac{\alpha^2}{\pi} + \frac{\alpha^3}{\pi^2} + O(\alpha^4),$$

(49)

while the renormalized effective action coincides with the one, obtained by HD regularization. Unlike (40) expression (49) agrees with the NSVZ $\beta$-function (1).

### 3.4 Comparison between HD and DRED regularizations.

Comparing the calculations, described above, we see that the difference of the $\beta$-functions comes from the different results for the sum of diagrams with insertions of one-loop counterterms. In order to understand why these results are different it is necessary to note, that the sum of diagrams with insertions of counterterms in the considered approximation is equal to Konishi anomaly [14, 15]. The existence of Konishi anomaly can be explained by following arguments:
Let us consider the following expression:

\[
\text{Im} \left[ \bar{D}^2 \left( \phi^* e^{2V} \phi + \tilde{\phi}^* e^{-2V} \tilde{\phi} \right) \right].
\]  
(50)

Using equation (5) and (7), it is easy to see, that in components it will contain (among other terms)

\[-\bar{\theta} \theta \partial_\mu \left( \bar{\Psi} \gamma^\mu \gamma_5 \Psi \right),\]

where the Dirac spinor \(\Psi\) is defined by (6). It is well known [37], that the conservation of the axial current is broken by quantum corrections and in particular

\[
\langle \bar{\theta} \theta \partial_\mu \left( \bar{\Psi} \gamma^\mu \gamma_5 \Psi \right) \rangle = -\bar{\theta} \theta \frac{1}{8\pi^2} F_{\mu\nu} \tilde{F}^{\mu\nu}.
\]  
(52)

Hence due to the supersymmetry \(^2\)

\[
\text{Im} \left\langle \bar{D}^2 \left( \phi^* e^{2V} \phi + \tilde{\phi}^* e^{-2V} \tilde{\phi} \right) \right\rangle = \frac{1}{2\pi^2} \text{Im} \left( W_a C^{ab} W_b \right).
\]

(53)

Performing supersymmetry transformations it is easy to see, that if imaginary part of a chiral superfield is equal to 0, then this superfield is a real constant. Therefore, from (53) we obtain, that

\[
\left\langle \bar{D}^2 \left( \phi^* e^{2V} \phi + \tilde{\phi}^* e^{-2V} \tilde{\phi} \right) \right\rangle = \frac{1}{2\pi^2} W_a C^{ab} W_b + \text{const}.
\]

(54)

Applying

\[-\frac{1}{2} \int d^4x D^2 = \int d^4x d^2\theta\]

(55)

to (54) and taking a real part of the result, we obtain, that

\[
\left\langle \frac{1}{4} \int d^4x d^4\theta \left( \phi^* e^{2V} \phi + \tilde{\phi}^* e^{-2V} \tilde{\phi} \right) \right\rangle = -\frac{1}{16\pi^2} \text{Re} \int d^4x d^2\theta W_a C^{ab} W_b.
\]

(56)

It is important to note, that if the axial anomaly is found to be equal to 0 due to some reason and the supersymmetry is unbroken, instead of (56) we will automatically obtain

\[
\left\langle \frac{1}{4} \int d^4x d^4\theta \left( \phi^* e^{2V} \phi + \tilde{\phi}^* e^{-2V} \tilde{\phi} \right) \right\rangle = 0.
\]

(57)

This takes place if the calculations are made by DRED. Really, if we calculate supergraphs, then it is impossible to impose requirements similar to \(\text{tr}(AB) \neq \text{tr}(BA)\). Therefore, it is necessary to use original variant of DRED, proposed in [29] with \(n < 4\)

\(^2\)Note, that our arguments can not be considered as a derivation of Konishi anomaly, because (51) does not contain all terms of (50), proportional to \(\bar{\theta} \theta\). A strict derivation of the Konishi anomaly can be found in [14, 15]. Our goal is only to remind relation between axial anomaly and Konishi anomaly.
and $\gamma_5$, satisfying (28). It means, that the chiral symmetry is not broken in the regularized theory and anomaly is equal to 0 due to the mathematical inconsistency of DRED. As a consequence Konishi anomaly is also equal to 0, that, in turn, leads to zero result for the sum of diagrams with insertions of counterterms. Taking into account that in the two-loop approximation these diagrams should cancel the other contributions, DRED leads to a nontrivial two-loop correction and to the anomaly puzzle.

4 Exact result for diagrams with insertions of counterterms.

In the previous section we found, that the difference between HD and DRED $\beta$-functions originated from different results for the sum of diagrams with insertions of counterterms. In DRED this sum is equal to zero, because this regularization does not allow to calculate Konishi anomaly, which can be found correctly by HD regularization. In order to confirm such arguments in this section a sum of diagrams with insertion of counterterms is calculated exactly to all orders using HD regularization.

The result obtained in appendix can be written in the following form:

$$\exp(i\Gamma) = \exp \left\{ -i \ln \frac{1}{16\pi^2} \text{Re} \int d^4x \, d^2\theta \, W_a C^{ab} W_b + \text{finite terms} \right\} \times$$

$$\times \int D\phi \, D\Phi \prod_i \left( \det P^\prime V(V, M_i) \right)^{\chi_i} \exp \left\{ i \left[ \frac{1}{16e^2} \int d^4x \, d^4\theta \, \partial^2 \left( 1 + \frac{\partial^2n}{\Lambda^2n} \right) V - \frac{1}{4e^2} \left( Z_3(\Lambda/\mu) - 1 \right) \right] + \frac{1}{4} \int d^4x \, d^4\theta \left( \phi^* e^{2V} \phi + \tilde{\phi}^* e^{-2V} \tilde{\phi} \right) + \int d^4x \, d^4\theta \left( J \phi + j \tilde{\phi} \right) \right\},$$

(58)

where $J$, $j$ and $\tilde{j}$ should be eliminated in terms of $V$, $\phi$ and $\tilde{\phi}$ and

$$\left( \det P^\prime V(V, M) \right)^{-1} \equiv \int D\Phi \, D\Phi^\dagger \exp \left\{ i \left[ \frac{1}{4} \int d^4x \, d^4\theta \left( \Phi^* e^{2V} \Phi + \tilde{\Phi}^* e^{-2V} \tilde{\Phi} \right) + \frac{1}{2} \int d^4x \, d^2\theta \, M \Phi \Phi + \frac{1}{2} \int d^4x \, d^2\bar{\theta} \, M \Phi^* \Phi^* \right\}.$$  

(59)

(The difference between this definition and (16) is the absence of $Z$ in definition of $\det^\prime$)

Equation (58) can be formally written in the more simple form:
\[
\langle \exp \left( i(Z - 1) \frac{1}{4} \int d^4x d^4\theta \left( \phi^* e^{2V} \phi + \tilde{\phi}^* e^{-2V} \tilde{\phi} \right) \right) \rangle = \\
= \exp \left( -i \ln Z \frac{1}{16\pi^2} \text{Re} \int d^4x d^2\theta W_a C^{ab} W_b + \text{finite terms} \right), \quad (60)
\]

Therefore, although expression (58) is rather complicated, its essence is quite simple: rescaling of \( \phi \) and \( \tilde{\phi} \) produces a factor

\[
\exp \left\{ -i \ln Z \frac{1}{16\pi^2} \text{Re} \int d^4x d^2\theta W_a C^{ab} W_b + \text{finite terms} \right\}. \quad (61)
\]

From (58) we conclude, that the sum of diagrams with insertions of counterterms gives the following contribution to the \( \beta \)-function:

\[
\Delta \beta = \frac{\alpha^2}{\pi} \frac{d\ln Z}{d\ln \mu} = \frac{\alpha^2}{\pi} \gamma(\alpha). \quad (62)
\]

Therefore, if the \( \beta \)-function obtained by HD regularization is defined by the one-loop approximation, the sum of all diagrams without insertions of counterterms on the matter lines will give NSVZ \( \beta \)-function (1). Such result can be obtain after rescaling of matter superfields \( \phi \to Z^{-1/2} \phi \), which convert renormalized action (22) to

\[
S_{ren} = \frac{1}{4e^2} Z_3(\Lambda/\mu) \text{Re} \int d^4x d^2\theta W_a C^{ab} \left( 1 + \frac{\partial^{2n}}{\Lambda^{2n}} \right) W_b + \\
+ \frac{1}{4} \int d^4x d^4\theta \left( \phi^* e^{2V} \phi + \tilde{\phi}^* e^{-2V} \tilde{\phi} \right). \quad (63)
\]

Then the diagrams with insertions of counterterms will be evidently absent. Such possibility was investigated in [23] by other methods.

Another possibility to obtain (1) is using of DRED. In this case the sum of diagrams with insertions of counterterms is calculated incorrectly and is equal to 0. As a consequence we obtain anomaly puzzle, which appears as an artifact of mathematical inconsistency of DRED. However, the renormalized effective action is the same for both regularizations and the only difference between these regularizations is a value of the \( \beta \)-function.

Note, that if HD regularization leads to the one-loop \( \beta \)-function [27], the result obtained in DRED will give the NSVZ \( \beta \)-function after redefinition of the coupling constant, that is in agreement with the results of [20, 22].

Finally it is necessary to mention, that equations similar to (60) were also obtained in [13] and [23]. However anomalous contribution in (58) was not identified with the sum of diagrams with insertions of counterterms. Nevertheless, the derivation of this equation presented in [13] is in a certain degree similar to the derivation, given in this paper.
In this paper calculation of the two-loop $\beta$-function for $N = 1$ supersymmetric electrodynamics was analyzed for regularizations by DRED and HD. Now let us summarize the results.

1. Konishi anomaly really gives a nontrivial contribution to the $\beta$-function, as it was pointed out in [13]. This anomaly can be identified with the sum of Feynman diagrams with insertions of counterterms. Nevertheless, contribution of these diagrams does not change form of the renormalized effective action.

2. Existence of rescaling anomaly, investigated in [23], can be easily explained by the diagram technique: Diagrams with insertions of counterterms (on lines of matter superfields) are present only if the renormalized action contains

$$\frac{1}{4}Z \int d^4x \, d^4\theta \left( \phi^* e^{2V} \phi + \bar{\phi}^* e^{-2V} \bar{\phi} \right),$$

that corresponds to the holomorphic normalization of matter superfields. After rescaling $\phi \to Z^{-1/2} \phi$, such diagrams disappear. Therefore, due to the Konishi anomaly these transformations will be anomalous.

3. If the $\beta$-function is calculated by a mathematically consistent regularization, which does not break supersymmetry, then the anomaly puzzle is absent (at least at the two-loop level). This means, that the $\beta$-function is completely defined by the one-loop approximation. An example of such regularization is regularization by HD, considered in this paper. It is important, that HD regularization allows to obtain an exact expression for the sum of subtraction diagrams, which agrees with the results found in [13] and [23] from other arguments. Possibly the NSVZ $\beta$-function can be obtained if one uses HD regularization and subtractions at some scale $\mu$, as it was proposed in [27]. However, this can be checked only at the three loops, because starting from the three-loop approximation coefficients of the $\beta$-function depend on the renormalization scheme. This work has been already finished and the corresponding paper is in preparation.

4. Ambiguities of DRED lead to the incorrect result for the axial anomaly and, as a consequence, for the Konishi anomaly. This, in turn, produces incorrect $\beta$-function and the anomaly puzzle. At the present moment I do not know how to impose requirements like $\text{tr}(AB) \neq \text{tr}(BA)$, allowing to find anomalies by DRED, within the explicitly supersymmetric technique of calculations.

It is necessary to note, that so far we considered only the Abelian case. For the supersymmetric Yang-Mills theory using of higher covariant derivative regularization [38] leads to very involved calculations, because in this case Feynman rules become much more complicated. In this case using of usual derivatives can considerably simplify the calculations. However, such regularization breaks the gauge invariance. Nevertheless, even in case of noninvariant regularization it is possible to obtain gauge invariant renormalized effective action by a special choice of subtraction scheme [39, 40]. For Abelian
supersymmetric theories such scheme was proposed in [41]. Construction of invariant
renormalization procedure for supersymmetric non-Abelian models is in progress.

Also I would like to mention, that NSVZ $\beta$-function was obtained not only in DRED,
but also in the differential renormalization (DiffR) [42]. The calculations were made
in [43] for $N = 1$ supersymmetric Yang-Mills theory. At present I can not trace the
origin of the multiloop corrections in this case and believe, that it would be interesting
to compare the calculation of this paper with the corresponding results, obtained in
DiffR.

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Appendix.

A Exact sum of diagrams with insertions of counterterms.

In order to prove (58) it is necessary to consider 1PI diagrams with insertions of
counterterms on lines of the matter superfield. A sum of diagrams, containing internal
$V$-lines, is finite due to the HD regularization. Hence, nontrivial contributions can
be given only by diagrams with a single loop of matter superfields without internal
$V$-lines. A sum of such contributions can be calculated exactly. Really, it is easy to
see, that the effective line

$$\ldots = \ldots + \ldots + \ldots + \ldots,$$

where crosses denote contributions from counterterms, corresponds to the following
propagators:
\[
\frac{1}{Z} \left( \begin{array}{cccc}
0 & -\frac{D^2 D^2}{16(\partial^2 + m^2/Z^2)} & \frac{m D^2}{4Z(\partial^2 + m^2/Z^2)} & 0 \\
\frac{D^2 D^2}{16(\partial^2 + m^2/Z^2)} & 0 & 0 & \frac{m D^2}{4Z(\partial^2 + m^2/Z^2)} \\
\frac{m D^2}{4Z(\partial^2 + m^2/Z^2)} & 0 & 0 & \frac{D^2 D^2}{16(\partial^2 + m^2/Z^2)} \\
0 & \frac{m D^2}{4Z(\partial^2 + m^2/Z^2)} & \frac{D^2 D^2}{16(\partial^2 + m^2/Z^2)} & 0
\end{array} \right)
\]

(66)

\((m = 0 \text{ for } \phi \text{ and } \tilde{\phi} \text{ lines and } m = M \text{ for lines of Pauli-Villars fields.})\) The first string of this matrix corresponds to the propagators \(\phi - \phi, \phi - \phi^*, \phi - \tilde{\phi}, \phi - \tilde{\phi}^*,\) the second string corresponds to the propagators \(\phi^* - \phi, \phi^* - \phi^*, \phi^* - \tilde{\phi}, \phi^* - \tilde{\phi}^*\) e t.c.

These expressions should be substituted to the diagrams

\begin{align*}
\begin{array}{c}
\includegraphics[width=0.3\textwidth]{diagram.jpg} \\
\text{where circles denotes the effective vertexes}
\end{array}
\end{align*}

\[(67)\]

\begin{align*}
\begin{array}{c}
\includegraphics[width=0.3\textwidth]{diagram2.jpg}
\end{array}
\end{align*}

\[(68)\]

and

\begin{align*}
\begin{array}{c}
\includegraphics[width=0.3\textwidth]{diagram3.jpg}
\end{array}
\end{align*}

\[(69)\]

which are proportional to

\[
Z \left( \begin{array}{cccc}
0 & 1 & 0 & 0 \\
1 & 0 & 0 & 0 \\
0 & 0 & 0 & -1 \\
0 & 0 & -1 & 0
\end{array} \right) \quad \text{and} \quad Z \left( \begin{array}{cccc}
0 & 1 & 0 & 0 \\
1 & 0 & 0 & 0 \\
0 & 0 & 0 & 1 \\
0 & 0 & 1 & 0
\end{array} \right)
\]

(70)

respectively. Then it is quite evident, that the expression for diagrams (67) will differ from the corresponding expression for usual one-loop diagrams.
by the substitution $M \to M/Z$ and is equal to

$$-rac{i}{2} \Re \int d^2 \theta \frac{d^4 p}{(2\pi)^2} W_a(p, \theta) C^{ab} W_b(-p, \theta) \times$$

$$\times \int \frac{d^4 k}{(2\pi)^2} \left( \frac{1}{k^2(k+p)^2} - \sum_i c_i \frac{1}{(k^2 - M_i^2/Z^2)((k+p)^2 - M_i^2/Z^2)} \right).$$

(72)

Subtracting the result for the one-loop diagrams from this expression we obtain the following result for the diagrams with insertions of counterterms:

$$-rac{i}{2} \Re \int d^2 \theta \frac{d^4 p}{(2\pi)^2} W_a(p, \theta) C^{ab} W_b(-p, \theta) \times$$

$$\times \sum_i c_i \int \frac{d^4 k}{(2\pi)^2} \left( \frac{1}{k^2 - M_i^2((k+p)^2 - M_i^2/Z^2)} - \frac{1}{k^2 - M_i^2/Z^2((k+p)^2 - M_i^2/Z^2)} \right).$$

(73)

Performing Wick rotation and taking into account that

$$\frac{1}{2} \int \frac{d^4 k}{(2\pi)^4} \frac{1}{(k^2 + M^2/Z^2)((k+p)^2 + M^2/Z^2)} -$$

$$-\frac{1}{2} \int \frac{d^4 k}{(2\pi)^4} \frac{1}{(k^2 + M^2)((k+p)^2 + M^2)} =$$

$$= -\frac{1}{16\pi^2} \left( \ln \frac{M}{pZ} + \sqrt{1 + \frac{4M^2}{p^2Z^2}} \arctanh \sqrt{\frac{p^2}{4M^2/Z^2 + p^2}} - \ln \frac{M}{p} - \sqrt{1 + \frac{4M^2}{p^2}} \arctanh \sqrt{\frac{p^2}{4M^2 + p^2}} \right) =$$

$$= \frac{1}{16\pi^2} \ln Z + \text{finite terms},$$

(74)

the sum of the considered 1PI diagrams with insertions of counterterms appears to be

$$-\ln Z \frac{1}{16\pi^2} \Re \int d^4 x d^2 \theta W_a C^{ab} W_b + \text{finite terms}.$$ 

(75)

It is easy to see, that in terms of the generating functional this equation can be written as (58).
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Figure 1: Feynman diagrams giving nontrivial contributions to the two-loop $\beta$-function.