Transient and Free vibration analysis of a thermo-electric-elastic by using Stochastic hybrid numerical method

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Abstract. This paper presents the transient and Free vibration analysis of an infinite thermo-electric-elastic solid cylinder is analysed stochastically by using hybrid numerical method (combined Galerkin finite element and Newmark finite difference method). The governing equations for the thermo-electric-elastic solid cylinder is made up of 6mm class, involving the mechanical, electrical and thermal fields and studied by free and forced vibration as boundary conditions. The dynamic finite element equations are obtained by assumed shape functions. After assembling the Mass, Stiffness and Damping and matrices, the global transient equations are established in time field. The resulting equilibrium equations are solved by using the finite difference technique with suitable time instants. By using material constants displacement, velocity and acceleration of vibrations are obtained with various time values and the nondimensional frequencies are also obtained by different values of nondimensional wave number. Numerical work is carried out by the materials Barium titanate and Cadmium selenide. The outcomes are tabulated and represented graphically.

1. Introduction

The thermo-electric-elastic materials has many applications in various fields of science and technology, namely, atomic physics, industrial engineering, thermal power plants, sub marine structures, aerospace, chemical pipe and metallurgy. The dynamic behaviour and wave propagation in structures have been used to safety evaluation of stress field in engineering structures.

High frequency vibration of Piezoelectric crystal plates have been studied by R.D. Mindlin [1]. In [2] V.K. Nelson and S. Karthikeyan dealt with Vibration of pyro electric sandwich plate. Flexural vibration of Pyro composite solid cylinder*, have been analyzed by S. Karthikeyan and V.K. Nelson [3]. Axisymmetric vibration of Pyro composite solid cylinder is studied by S. Karthikeyan and V.K. Nelson [4]. In [5] H.S. Paul, D.P. Raju and T.R. Balakrishnan dealt with Free vibrations of Piezo electric layer of Hexagonal (6mm) class. Finite Element Modelling of a Layered, Multiphase Magneto electro elastic cylinder subjected to an axisymmetric temperature Distribution is studied by N. Ganesan, A.Kumaravel and Rajusethuraman [6]. In [7] George.R. Buchanan has been analyzed by Galerkin finite element derivation for vibration of a thermo piezo electric structure. Seyed Mahmoud Hosseini, Farzad Shahabian Transient analysis of thermo elastic waves in thick hollow cylinders using a stochastic hybrid numerical method, considering Gaussian mechanical properties are studied by Seyed Mahmoud Hosseini, Farzad
2. Constitutive Equation

The governing equations of linear thermo-electric-elasticity are, [7]

\[
\begin{align*}
T_{ij} + F_i &= \rho u_i, \quad T_{ij} = T_{jk}, \text{for } k \neq j, \quad D_{ij} = 0, \quad q_{k,i} + T_{ij} = 0, \\
T_{ij} &= C_{ijkl} S_{kl} - \epsilon_{ijkl} E_{k} - \beta_{ij} T, \quad S_{ij} = \frac{1}{2}(u_{k,j} + u_{j,k}), \\
D_{ij} &= \epsilon_{ijkl} S_{kl} + \epsilon_{ijkl} E_{k} - P_i T, \quad E_k = -\phi_k, \\
\eta &= \beta_{ij} S_{ij} + P_i E_k + a T, \quad a = \rho C_v T_0^{-1}, \quad q_l = -k_{ij} T_j
\end{align*}
\]

(1)

Where \( T_{ij} \) and \( S_{ij} \) are the stress and strain tensors, the remaining terms \( u_k, D_k \) and \( E_k \) are mechanical displacement, the electric displacement, the electric field respectively. \( T \) is the temperature change from a reference temperature \( T_0 \), \( q_{ij} \) the heat flux, \( \phi \) the electric potential, \( \eta \) the entropy, \( \rho \) the mass density and \( C_{ijkl}, \epsilon_{ijkl}, \beta_{ij}, \epsilon_{ijkl}, P_i \) are the elastic, piezoelectric, stress coefficient, dielectric, pyroelectric material constants respectively and \( C_v \) the specific heat. The usual notation is displayed, and the comma followed by a lower case denotes partial differentiation of that coefficient with respect to independent variable. The coefficient of material constants, strain displacement, electric field and heat flux are in Appendix.

The assumed solution for an infinite solid cylinder along with \((l,m,0)\) direction with cylindrical polar coordinates \((r,\theta,z,t)\) are as, by [7]

\[
\begin{align*}
u_1(r,\theta,z,t) &= U(r) \exp i(kl z + km \theta - \omega t) \\
u_2(r,\theta,z,t) &= V(r) \exp i(kl z + km \theta - \omega t) \\
u_3(r,\theta,z,t) &= \left(\frac{i}{h}\right) W(r) \exp i(kl z + km \theta - \omega t) \\
\psi(r,\theta,z,t) &= i \left(\frac{C_{44}}{e} \right) \left(\frac{\Psi(r)}{h}\right) \exp i(kl z + km \theta - \omega t) \\
\Theta(r,\theta,z,t) &= \left(\frac{C_{44}}{ \beta} \right) \left(\frac{\Theta(r)}{h}\right) \exp i(kl z + km \theta - \omega t)
\end{align*}
\]

\[
(2)
\]

Where \( k \) is the wave number and \( \omega \) is frequency. \( l = \cos \theta, m = \sin \theta, l^2 + m^2 = 1 \) and \( i = \sqrt{-1} \). Introduce the nondimensional wave number \( \varepsilon = kh \) (\( h \) is the thickness of the cylinder).

3. Boundary condition

In traction free boundary conditions, the body force and the densities are absence. By using these, the governing equations can be written as,
\[
T_{ij, j} = \rho \ddot{u}_k \\
D_{k,k} = 0 \\
q_{k,k} + T_0 \dot{\phi}_k = 0
\] 
\rightarrow \quad (3)

The equation (1) can be rewritten as,
\[
C_{ijr} u_{r,st} - e_{ikl} \phi_{s,t} - \beta_{jk} T_j = \rho \ddot{u}_k ,
\]
\[
e_{ikl} u_{k,ji} + e_{ikl} \phi_{s,tk} - P_{ij} T = 0 ,
\]
\[
\beta_{ij} u_{ij} - P_{ik} \dot{\phi}_k + aT - \frac{k_{ij}}{T_0} T_{ij} = 0 ,
\]
\rightarrow \quad (4)

4. Finite Element Technique

The assumed shape functions are required to construct the finite element formulation, that the corresponding mechanical, electrical and thermal fields are as follows,
\[
u_i = [N]^e_{
u} \{u^e\}, \quad \psi = [N]^e_{\psi} \{\psi^e\} , \quad \Theta = [N]^e_{\Theta} \{\Theta^e\}
\]

where \([N]^e_{
u}, [N]^e_{\psi}\) and \([N]^e_{\Theta}\) are unknown nodal points and \([N]^e_{
u}, [N]^e_{\psi}\) and \([N]^e_{\Theta}\) are corresponding shape functions and \(e\) is the element level degrees of freedom. In a wave propagation of an infinite solid cylinder, along the thickness direction, Power series are most commonly used for shape functions.

The solution of an infinite solid cylinder can be assumed as,
\[
u_i (r, \theta, z, t) = z^{-1} \exp i(kl z + km \theta - pt) \{U\}
\]
\[
u_2 (r, \theta, z, t) = z^{-1} \exp i(kl z + km \theta - pt) \{V\}
\]
\[
u_3 (r, \theta, z, t) \left( \frac{i}{h} \right) \frac{z^{-1}}{h} \exp i(kl z + km \theta - pt) \{W\}
\]
\[
\psi (r, \theta, z, t) = \left( \frac{C_{44}}{\beta_{33}} \right) \frac{i}{h} \frac{z^{-1}}{h} \exp i(kl z + km \theta - pt) \{\psi\}
\]
\[
\Theta (r, \theta, z, t) = \left( \frac{C_{44}}{\beta_{33}} \right) \frac{1}{h} \frac{z^{-1}}{h} \exp i(kl z + km \theta - pt) \{\Theta\}
\]
\rightarrow \quad (5)

for \(j = 1 \text{ to } 4\)

Substitute the free boundary condition, shape function, equation (5) in (4) and to make integrate over the suitable corresponding volume. Obtain a completely coupled system of the following Global Stiffness \([K]\), Damping \([C]\) and Mass \([M]\) matrices are,
The equation (6) contains elasticity in Cartesian coordinates and are obtained by using the assumed
shape functions solutions of equation (5). The four noded element is considered and an operator matrix
on the equation (1) to form \([B_u],[B_\phi]\) and \([B_\theta]\) as,

\[
\begin{bmatrix}
M^{\epsilon_{uu}} & 0 & 0 \\
0 & 0 & 0 \\
0 & 0 & 0 \\
\end{bmatrix}
\begin{bmatrix}
\ddot{u}^\epsilon \\
\ddot{u}^\epsilon \\
\ddot{u}^\epsilon \\
\end{bmatrix}
+\begin{bmatrix}
0 & 0 & 0 \\
0 & 0 & 0 \\
C^{\epsilon_{\theta\theta}} - C^{\epsilon_{\theta\phi}} & C^{\epsilon_{\phi\phi}} \\
\end{bmatrix}
\begin{bmatrix}
\ddot{\phi}^\epsilon \\
\ddot{\phi}^\epsilon \\
\ddot{\phi}^\epsilon \\
\end{bmatrix}
+\begin{bmatrix}
K^{\epsilon_{uu}} & K^{\epsilon_{u\phi}} & -K^{\epsilon_{u\phi}} \\
K^{\epsilon_{u\phi}} & -K^{\epsilon_{\phi\phi}} & K^{\epsilon_{\phi\phi}} \\
0 & 0 & K^{\epsilon_{\phi\phi}} \\
\end{bmatrix}
\begin{bmatrix}
\ddot{u}^\epsilon \\
\ddot{u}^\epsilon \\
\ddot{\phi}^\epsilon \\
\end{bmatrix}
= \begin{bmatrix}
0 \\
0 \\
0 \\
\end{bmatrix}
\]  \rightarrow (6)

\[
(K^{\epsilon_{uu}}) = \int[B_u]^T[C][B_u]dV, \quad (K^{\epsilon_{u\phi}}) = \int[B_u]^T[e][B_\phi]dV \\
(K^{\epsilon_{\phi\phi}}) = \int[B_\phi]^T[k][B_\phi]dV \\
\]

Where \( [C^{\epsilon_{\theta\theta}}] = \int[N_\theta]^T[\beta]^T[N_\theta]dV, \quad [C^{\epsilon_{\phi\phi}}] = \int[N_\phi]^T[p]^T[N_\phi]dV, \quad [M^{\epsilon}] = \int[N]^T[\rho][N]dV \)

dV = 2\pi dr d\theta dz

The equation (6) contains elasticity in Cartesian coordinates and are obtained by using the assumed
shape functions solutions of equation (5). The four noded element is considered and an operator matrix
on the equation (1) to form \([B_u],[B_\phi]\) and \([B_\theta]\) as,
Where the matrices $[B_u]$, $[B_w]$ and $[B_{\phi}]$ are assumed shape function matrices. Every element in each matrix contained smaller sub-matrices that are multiplied by various type of in-plane functions. The nondimensional frequencies and the corresponding shape functions are obtained by solving the above Eigen value problem without external forces.

Assemble all element equations, the required frequency equation is,

$$- \rho^2 [M][u] - i\omega [C][u] + [K][u] = 0 \quad \rightarrow (7)$$

where $[M] = \begin{bmatrix} M_{uu} & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$, $[C] = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ C_{\theta\theta} & -C_{\phi\theta} & C_{\phi\phi} \end{bmatrix}$ and $[K] = \begin{bmatrix} K_{uu} & K_{u\phi} & -K_{u\theta} \\ K_{u\phi} & K_{\phi\phi} & K_{\phi\theta} \\ -K_{u\theta} & K_{\phi\theta} & K_{\theta\theta} \end{bmatrix}$

by using the method of concatenation, the above equation was solved and resulting Eigen values are denoted the frequencies. Numerical illustrations are obtained by using the material constants values of Barium titanate and Cadmium selenide.

5. Newmark’s finite difference formulation

In on forced vibration boundary conditions, the electric displacement is vanished. By using these conditions, the equations of motion can be written in the form,

$$T_{ij,j} + F_i = \rho \ddot{u}_k$$
$$D_{k,k} = 0,$$
$$q_{k,k} + T_{ij,j} = 0 \quad \rightarrow (8)$$

Substitute equation (5) and corresponding shape function values are in (8) and to make integration over the corresponding volume. Equation (6) can be change over to the following Stiffness $[K]$, Damping $[C]$, Mass $[M]$ and force $[F]$ matrices,

$$\begin{bmatrix} M_{uu} & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} \dddot{u} \\ \dddot{\phi} \end{bmatrix} + \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ C_{\theta\theta} & -C_{\phi\theta} & C_{\phi\phi} \end{bmatrix} \begin{bmatrix} \dddot{\phi} \\ \dddot{\theta} \end{bmatrix} = \begin{bmatrix} K_{uu} & K_{u\phi} & -K_{u\theta} \\ K_{u\phi} & K_{\phi\phi} & K_{\phi\theta} \\ -K_{u\theta} & K_{\phi\theta} & K_{\theta\theta} \end{bmatrix} \begin{bmatrix} \dddot{u} \\ \dddot{\phi} \\ \dddot{\theta} \end{bmatrix} + \begin{bmatrix} f_u \\ f_\phi \end{bmatrix} \quad \rightarrow (9)$$

(i.e) $[M][\dddot{U}] + [C][\dddot{U}] + [K][U] = [F] \quad \rightarrow (10)$

After getting the global Stiffness, Damping, Mass and force matrices of each element, consider the following shape functions for each element

$$\Phi = [N_1 \quad N_2], \quad U = \begin{bmatrix} u_i \\ u_{i+1} \end{bmatrix}, \quad \hat{u} = \Phi U$$

$$\Phi = [N_1 \quad N_2], \quad T = \begin{bmatrix} T_i \\ T_{i+1} \end{bmatrix}, \quad \hat{T} = \Phi T$$

The following dynamic finite element governing equation can be obtained for each element by using linear shape function are
\[ [M], [\Phi] + [C], [\Phi] + [K], [\Phi] = [F],_{l} \rightarrow (12) \]

Where \( \Phi^{T} = [\tilde{u}_{i}, \tilde{T}_{i}, \tilde{u}_{i+1}, \tilde{T}_{i+1}]^{T} \)

\( \varepsilon^{i} \) is \( \varepsilon^{i} \) th element and \( \alpha^{i} \) stand for \( \alpha^{i} \) th node. After assembling of all matrices of each of the elements, the coupled thermos elastic problem in the form of stochastic frame work can be rewritten as,

\[ [M], [\Phi] + [C], [\Phi] + [K], [\Phi] = [F] \rightarrow (12) \]

The above equation is in transient nature. By applying the Newmark’s integration method, and to approximate the time derivatives and to solve the equations of forced vibration. About this method, the functions and its corresponding derivatives are approximated by the following assumptions.

Initially to calculate the value of displacements \( \{U_{t+\Delta}\} \), the constitute equation can be evaluated at time \( t_{r+\Delta} \) as,

\[ [M], [U_{1}] + [C], [U_{1}] + [K], [U_{1}] = [F],_{l} \rightarrow (13) \]

Where \( \Delta t \) = time step, \( t_{r} = step number \). Rearranging the above equation,

\[ \{U_{r},\} = [M]^{-1}([F] - [C] \{U_{r}\} - [K] \{U_{r}\}) \]

The displacement matrix \( \{U_{1}\} \), velocity matrix \( \{U_{2}\} \) and acceleration matrix \( \{U_{3}\} \) can be found by using following equations,

\[ \{U_{1}\} = \frac{[K]_{mg}}{([F]_{eff})}, \rightarrow (14) \]

Where \( \{K\}_{mg} = [K] + a_{0}*[M] + a_{1}*[C] \)

\[ ([F]_{eff}) = [F] + [M]*([a_{i} * \{U_{i}\} + a_{i} * \{U_{i-1}\} + a_{i} * \{U_{i+1}\}] + [C]*([a_{i} * \{U_{i}\} + a_{i} * \{U_{i-1}\} + a_{i} * \{U_{i+1}\}] + [K]*([a_{i} * \{U_{i}\} + a_{i} * \{U_{i-1}\} + a_{i} * \{U_{i+1}\}] \rightarrow (15) \]

\[ \{U_{2}\} = a_{0} * \{(U_{1})_{i+1}\} - \{(U_{1})_{i}\} - a_{1} * \{(U_{1})_{i}\} - a_{2} * \{(U_{1})_{i}\}, \rightarrow (16) \]

Once the value of displacements \( \{(U)_{t+\Delta}\} \) at time \( t_{r+\Delta} \) are obtained then solving equation (14) the velocities \( \{U_{1}\} \) at \( t_{r+\Delta} \) are obtained respectively.

In equations (14), (15) and (16), \( a_{0}, a_{1}, a_{2}, a_{3}, a_{4}, a_{5}, a_{6} and a_{7} \) are Newmark parameters and the values are given in Appendix. In Newmark difference method the best convergence can be reached by choosing \( \alpha = 0.25 and \delta = 0.5 \) in the above parameters.

6. Analysis and Results

The values of displacement \( \{(U)_{t+\Delta}\} \), velocity \( \{(U)_{t+\Delta}\} \) and acceleration \( \{(U)_{t+\Delta}\} \) for the materials BaTiO3/Cdse are obtained. The terms for non-dimensional are necessary to solve the problem and assumed as follows. The thermo elastic elastic terms are assumed by Ref [2, 3&4] and the additional non-dimensional terms were derived based upon the constitute equations. The non-dimensional terms for the infinite solid cylinder analysis are as follows,

\[ C_{ij} = \frac{c_{ij}}{c_{44}}, \quad K = \frac{e}{h}, \quad \beta_{ij} = \frac{\beta_{ij}}{\beta_{3}}, \quad p_{ij} = \frac{p_{ij}c_{44}}{e_{33}^{2} \beta_{3}}, \quad K_{ij} = \left(\frac{\rho c_{44}^{2}}{\beta_{2} \beta_{3} hT_{0}}\right)K_{ij} \]

The displacement \( \{(U)_{t+\Delta}\} \), velocity \( \{(U)_{t+\Delta}\} \) and acceleration \( \{(U)_{t+\Delta}\} \) are obtained by using MATLAB software. The natural frequencies are obtained by various values of dimensionless wave number. The displacement, velocity and acceleration values are obtained and represented graphically, with different time variants.
7. Conclusion
This transient and free vibration analysis of an infinite thermo-electric-elastic solid cylinder is solved by using hybrid numerical method. The cylinder is of 6mm class with \( \ell = \cos 30^\circ \) and \( m = \sin 30^\circ \). Thickness of the cylinder is taken by \( h = 0.05\text{mm} \). The constitute equations for thermo-electric-elastic materials are derived by using Finite element formulations with free/forced vibrations. The coupled mass, stiffness and damping matrices, the global transient equations are executed in time domains. The Newmark finite difference method with suitable time step is used and the equilibrium equations are solved. The values of displacement, velocity, acceleration and the natural frequencies are obtained. The numerical illustrations have been carried out for Barium titanate and Cadmium selenide. Solutions are tabulated and illustrate graphically. The nondimensional wave number versus the displacement, velocity and acceleration values are calculated with suitable time variants. Finally, it is observed that in the case of Free vibration, the natural frequencies of thermo-electric-elastic infinite solid cylinder made by the material BaTiO3 is significantly greater than that of CdSe material. The damping effect analyzed through the imaginary part of complex frequencies. In the case of Transient, different values of nondimensional wave number, if the time domain increased when the values of the displacement, velocity and acceleration of BaTiO3 material is significantly greater than that of CdSe material.

8. Numerical illustration and Figures
In Transient State

| \( \epsilon \)  | \( t = 0.1 \) | Displacement | \( t = 0.2 \) | Displacement | \( t = 0.3 \) | Displacement |
|---|---|---|---|---|---|---|
|   | BaTiO3 | CdSe | BaTiO3 | CdSe | BaTiO3 | CdSe |
| 0.2 | 0.6782+0.3025i | 0.48356-0.2442i | 0.71287+0.3206i | 0.51236+0.24928i | 0.74768-0.36797i | 0.62638-0.1071i |
| 0.4 | 0.7995+0.1515i | 0.61796+0.21587i | 0.83912+0.3174i | 0.66762+0.12178i | 0.84570-0.51221i | 0.7851-0.1239i |
| 0.45 | 0.8389+0.1413i | 0.7689+0.3378i | 0.8839-0.24622i | 0.77137+0.44247i | 0.932837-0.4341i | 0.8405-0.4203i |
| 0.55 | 0.9047+0.1984i | 0.8124-0.2728i | 0.92097+0.3661i | 0.842394+0.4061i | 0.96658-0.81854i | 0.8785+0.3886i |
| 0.6 | 0.9333-0.1062i | 0.8303-0.3314i | 0.9673-0.37853i | 0.877906+0.4194i | 1.31344-0.82872i | 0.92637+0.3559i |
| 0.7 | 1.0959-0.7192i | 0.8801+0.2296i | 1.5648-0.63875i | 0.94878+0.3831i | 1.7431+0.54803i | 0.9751-0.5926i |
| 0.8 | 1.5679+0.5596i | 0.9654+0.1825i | 1.61582-0.9407i | 0.98437+0.53359i | 1.9021+0.639i | 1.3588+0.7481i |
| 1.0 | 1.7072+0.4167i | 0.9876+0.2378i | 1.94376+0.8049i | 1.19945+0.8141i | 2.1281+0.9774i | 1.4835-0.6118i |
| 2.0 | 2.1462-1.1246i | 1.1892+0.8043i | 2.3236+0.9778i | 1.66858+0.73455i | 2.77695+0.73054i | 1.72795+0.9808i |
| 2.7 | 2.4625-0.9029i | 1.5262-0.8436i | 2.4812-0.85454i | 1.84618+0.43583i | 2.87467-0.6152i | 2.4706-0.1519i |
| 3.5 | 2.5332+1.2196i | 2.3759+1.2273i | 2.6679+0.99892i | 2.40436+1.23997i | 3.127-1.83242i | 2.8783+0.5788i |
| 4.0 | 2.8771+1.3103i | 2.5789-0.9884i | 3.19369+1.5716i | 2.92477-1.33406i | 3.4416+1.43755i | 3.1515+0.9992i |

| \( \epsilon \)  | \( t = 0.1 \) | Velocity | \( t = 0.2 \) | Velocity | \( t = 0.3 \) | Velocity |
|---|---|---|---|---|---|---|
|   | BaTiO3 | CdSe | BaTiO3 | CdSe | BaTiO3 | CdSe |
| 0.2 | 0.5023-0.3273i | 0.3166+0.1333i | 0.5651+0.2861i | 0.3674-0.275i | 0.5978+0.2792i | 0.4920+0.2782i |
| 0.4 | 0.5447-0.3273i | 0.3983-0.3688i | 0.5894-0.4546i | 0.4796-0.3377i | 0.6302-0.4817i | 0.5322-0.3858i |
| 0.45 | 0.5918+0.2499i | 0.4713+0.3334i | 0.6006+0.4747i | 0.5396-0.3367i | 0.6895+0.4792i | 0.6744-0.1015i |
| 0.55 | 0.6647+0.3307i | 0.5706+0.3355i | 0.7293+0.3861i | 0.6595-0.4377i | 0.7521-0.4818i | 0.7572-0.2965i |
| 0.6 | 0.6978+0.3307i | 0.6361-0.3688i | 0.7862+0.2862i | 0.7195-0.3977i | 0.8041-0.4602i | 0.7923-0.3406i |
In Free vibration

| ϵ  | Acceleration | Frequency |
|----|--------------|-----------|
|    | Batio3       | Cdsε      | Batio3    | Cdsε      | Batio3    | Cdsε      |
| 0.2 | 0.2652+0.1862i | 0.2398-0.1376i | 0.2904-0.2178i | 0.25915+0.15754i | 0.3323-0.3466i | 0.26331+0.2666i |
| 0.4 | 0.2884+0.2011i | 0.2662+0.1666i | 0.3033+0.2861i | 0.28622+0.2667i | 0.3712-0.3362i | 0.32662+0.2667i |
| 0.6 | 0.3261-0.2545i | 0.2913+0.1635i | 0.3929-0.2313i | 0.3539-0.2376i | 0.4101+0.3603i | 0.3962-0.3376i |
| 0.8 | 0.3335+0.2830i | 0.3188+0.1884i | 0.4104-0.3488i | 0.3741+0.2666i | 0.44380-0.48502i | 0.4174+0.4668i |
| 1.0 | 0.4351+0.2745i | 0.3719-0.2377i | 0.4795+0.2861i | 0.3949-0.3377i | 0.4862+0.4861i | 0.4719-0.4377i |
| 2.0 | 0.4647+0.2891i | 0.3810+0.2991i | 0.5648-0.3387i | 0.4839-0.3377i | 0.5928+0.3862i | 0.5222+0.4466i |
| 4.0 | 0.5653-0.2548i | 0.4550+0.3333i | 0.5767-0.4953i | 0.55415+0.2719i | 0.60666+0.4962i | 0.56036+0.31352i |
| 8.0 | 0.6521+0.4264i | 0.4987+0.4526i | 0.6698-0.4948i | 0.5772-0.4610i | 0.71901+0.4068i | 0.6581+0.3642i |
| 16.0| 0.7165+0.3978i | 0.6332+0.3666i | 0.7469-0.6546i | 0.6982-0.5376i | 0.7651+0.6861i | 0.7532+0.50622i |
| 32.0| 0.7852+0.3458i | 0.7413+0.1381i | 0.8511-0.9743i | 0.7866+0.4549i | 0.86486+0.99237i | 0.8547+0.3668i |
| 64.0| 0.8572+0.2867i | 0.7511+0.2142i | 0.8789+0.9859i | 0.8547+0.3812i | 0.8832-0.9999i | 0.8645+0.1725i |
| 128.0| 0.9295+0.4171i | 0.8797-0.4376i | 0.9531+0.9861i | 0.8851+0.4917i | 0.9938-0.9952i | 0.9126+0.26466i |
Figure 1. Nondimensional wave number Vs Displacement with transient state

Figure 2. Nondimensional wave number Vs Velocity with transient state
**Figure 3.** Nondimensional wave number Vs Acceleration with transient state

**Figure 4.** Nondimensional wave number Vs natural frequency with free vibration

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Appendix A The strain displacement and The electric potential $\phi$ and heat flux $q_{i,j}$

$$S_1 \frac{\partial u}{\partial r}, \quad S_2 = \frac{1}{r} \left( \frac{\partial v}{\partial \theta} + u \right), \quad S_3 = \frac{\partial w}{\partial z},$$

$$S_4 = \frac{\partial v}{\partial r} + \frac{1}{r} \frac{\partial w}{\partial \theta} + u, \quad E_1 = \frac{\partial \phi}{\partial r}, \quad E_2 = \frac{1}{r} \frac{\partial \phi}{\partial \theta}, \quad E_3 = \frac{1}{r^2} \frac{\partial \phi}{\partial z}$$

and

$$q_i = -k_{11} \frac{\partial T}{\partial r}, \quad q_2 = -k_{22} \frac{\partial T}{\partial \theta}, \quad q_3 = -k_{33} \frac{\partial T}{\partial z}$$

Appendix B The material constants for crystal class 6mm and Newmark Parameters:

$$\epsilon_0 = \begin{bmatrix} c_{11} & c_{12} & c_{13} & 0 & 0 & 0 \\ c_{21} & c_{22} & c_{23} & 0 & 0 & 0 \\ c_{31} & c_{32} & c_{33} & 0 & 0 & 0 \\ 0 & 0 & 0 & c_{44} & 0 & 0 \\ 0 & 0 & 0 & 0 & c_{55} & 0 \\ 0 & 0 & 0 & 0 & 0 & c_{66} \end{bmatrix}, \quad \epsilon_\alpha = \begin{bmatrix} e_{11} & 0 & 0 \\ 0 & e_{11} & 0 \\ 0 & 0 & e_{11} \end{bmatrix}$$

$$\beta_0 = \begin{bmatrix} \beta_{11} \\ \beta_{22} \\ \beta_{33} \end{bmatrix}, \quad \beta_\alpha = \begin{bmatrix} \beta_{11} \alpha^\alpha (\Delta t) \beta_{13} \\ \beta_{22} \alpha^\alpha (\Delta t) \beta_{23} \\ \beta_{33} \alpha^\alpha (\Delta t) \beta_{33} \end{bmatrix}, \quad \beta_+ = \begin{bmatrix} \beta_{11} \alpha^\alpha (\Delta t) \beta_{13} \\ \beta_{22} \alpha^\alpha (\Delta t) \beta_{23} \\ \beta_{33} \alpha^\alpha (\Delta t) \beta_{33} \end{bmatrix}$$

$$a_0 = \frac{1}{\alpha^\alpha (\Delta t)}, \quad a_1 = \frac{\delta}{(\alpha^\alpha (\Delta t))}, \quad a_2 = \frac{1}{(2^\alpha \alpha - 1)}, \quad a_3 = \frac{\Delta t}{(\alpha^\alpha - 1)}, \quad a_4 = \frac{\Delta t (\alpha^\alpha - 1)}{(\alpha^\alpha)}, \quad a_5 = \alpha^\alpha (\Delta t)$$

$$p_0 = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}, \quad p_\alpha = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}, \quad p_+ = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$a_0 = \frac{\Delta t}{(\alpha^\alpha - 1)}, \quad a_1 = \frac{\Delta t (\alpha^\alpha - 1)}{(\alpha^\alpha)}, \quad a_2 = \alpha^\alpha (\Delta t)$$