New constraint on the minimal SUSY GUT model from proton decay

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We present results of reanalysis on proton decay in the minimal SU(5) SUSY GUT model. Unlike previous analyses, we take into account a Higgsino dressing diagram of dimension 5 operator with right-handed matter fields (RRRR operator). It is shown that this diagram gives a significant contribution for $p \rightarrow K^+ \tau$. Constraints on the colored Higgs mass $M_C$ and the sfermion mass $m_{\tilde{f}}$ from Super-Kamiokande limit become considerably stronger than those in the previous analyses: $M_C > 6.5 \times 10^{16}$ GeV for $m_{\tilde{f}} < 1$ TeV. The minimal model with $m_{\tilde{f}} \lesssim 2$ TeV is excluded if we require the validity of this model up to the Planck scale.

1 Introduction

Supersymmetric grand unified theory (SUSY GUT) is strongly suggested by gauge coupling unification around $M_X \sim 2 \times 10^{16}$ GeV. In this theory, the hierarchy between the weak scale and the GUT scale $M_X$ is protected against radiative corrections by supersymmetry. Also, this theory makes successful prediction for the charge quantization. Proton decay is one of the direct consequences of grand unification. The main decay mode $p \rightarrow K^+ \tau$ in the minimal SU(5) SUSY GUT model has been searched for with underground experiments, and the previous results have already imposed severe constraints on this model. Recently new results of the proton decay search at Super-Kamiokande have been reported. The bound on the partial lifetime of the $K^+ \tau$ mode is $\tau(p \rightarrow K^+ \tau) > 5.5 \times 10^{32}$ years (90% C.L.), where three neutrinos are not distinguished.

There are many analyses on the nucleon decay in the minimal SU(5) SUSY GUT model. In the previous analyses, it was considered that the contribution from the dimension 5 operator with left-handed matter fields ($LLLL$ operator) was dominant for $p \rightarrow K^+ \tau$. In particular a Higgsino dressing diagram of the $RRRR$ operator has been estimated to be small or neglected in these analyses. It has been concluded that the main decay mode is $p \rightarrow K^+ \tau$, and the decay rate of this mode can be suppressed sufficiently by adjusting relative phases between Yukawa couplings at colored Higgs interactions.

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$a$ Talk given by T. Nihei in International Symposium on Supersymmetry, Supergravity, and Superstring (SSS99), Seoul, Korea, June 23-27, 1999, based on the published work [1].

$b$ After we finished our analysis, the latest limit of the Super-Kamiokande $\tau(p \rightarrow K^+ \tau) > 6.7 \times 10^{32}$ years (90% C.L.) was reported. An adaptation to the updated experimental limit is straightforward (See Eq. (5)).
In this talk, we present results of our analysis on the proton decay including the $RRRR$ operator in the minimal SU(5) SUSY GUT model. We calculate all the dressing diagrams (exchanging the charginos, the neutralinos and the gluino) of the $LLLL$ and $RRRR$ operators, taking account of various mixing effects among the SUSY particles, such as flavor mixing of quarks and squarks, left-right mixing of squarks and sleptons, and gaugino-Higgsino mixing of charginos and neutralinos. For this purpose we diagonalize mass matrices numerically to obtain the mixing factors at ‘ino’ vertices and the dimension 5 couplings. Comparing our calculation with the Super-Kamiokande limit, we derive constraints on the colored Higgs mass and the typical mass scale of squarks and sleptons. We find that these constraints are much stronger than those derived from the analysis neglecting the $RRRR$ effect.

2 Dimension 5 operators in the minimal SU(5) SUSY GUT model

Nucleon decay in the minimal SU(5) SUSY GUT model is dominantly caused by dimension 5 operators, which are generated by the exchange of the colored Higgs multiplet. The dimension 5 operators relevant to the nucleon decay are described by the following superpotential:

$$W_5 = -\frac{1}{M_C} \left\{ \frac{1}{2} C^{ijkl}_{5L} Q_k Q_i L_j + C^{ijkl}_{5R} U^c_k U^c_i D^c_j E^c_l \right\}.$$  \hfill (1)

Here $Q$, $U^c$ and $E^c$ are chiral superfields which contain a left-handed quark doublet, a charge conjugation of a right-handed up-type quark, and a charge conjugation of a right-handed charged lepton, respectively, and are embedded in the 10 representation of SU(5). The chiral superfields $L$ and $D^c$ contain a left-handed lepton doublet and a charge conjugation of a right-handed down-type quark, respectively, and are embedded in the $\overline{5}$ representation. A mass of the colored Higgs superfields is denoted by $M_C$. The indices $i, j, k, l = 1, 2, 3$ are generation labels. The first term in Eq. (1) represents the $LLLL$ operator which contains only left-handed SU(2) doublets. The second term in Eq. (1) represents the $RRRR$ operator which contains only right-handed SU(2) doublets. The coefficients $C_{5L}$ and $C_{5R}$ in Eq. (1) at the GUT scale are determined by Yukawa coupling matrices as

$$C^{ijkl}_{5L} = (Y_D)_{ij} (V^T P Y_U V)_{kl},$$
$$C^{ijkl}_{5R} = (P^* V^* Y_D)_{ij} (Y_U V)_{kl},$$  \hfill (2)

See also Ref. [26].
where $Y_U$ and $Y_D$ are diagonalized Yukawa coupling matrices for $10 \cdot 10 \cdot 5_H$ and $10 \cdot 5 \cdot 5_H$ interactions, respectively. The unitary matrix $V$ is the Cabibbo-Kobayashi-Maskawa (CKM) matrix at the GUT scale. The matrix $P = \text{diag}(P_1, P_2, P_3)$ is a diagonal unimodular phase matrix with $|P_i| = 1$ and $\det P = 1$. We parametrize $P$ as

$$P_i / P_3 = e^{i\phi_{13}}, \quad P_2 / P_3 = e^{i\phi_{23}}.$$  

The parameters $\phi_{13}$ and $\phi_{23}$ are relative phases between the Yukawa couplings at the colored Higgs interactions, and cannot be removed by field redefinitions \[16\]. The expressions for $C_{5L}$ and $C_{5R}$ in Eq. (4) are written in the flavor basis where the Yukawa coupling matrix for the $10 \cdot 5 \cdot 5_H$ interaction is diagonalized at the GUT scale. Numerical values of $Y_U$, $Y_D$ and $V$ at the GUT scale are calculated from the quark masses and the CKM matrix at the weak scale using renormalization group equations (RGEs).

In this model, soft SUSY breaking parameters at the Planck scale are described by $m_0$, $M_{3/2}$ and $A_X$ which denote universal scalar mass, universal gaugino mass, and universal coefficient of the trilinear scalar couplings, respectively. Low energy values of the soft breaking parameters are determined by solving the one-loop RGEs \[17\]. The electroweak symmetry is broken radiatively due to the effect of a large Yukawa coupling of the top quark, and we require that the correct vacuum expectation values of the Higgs fields at the weak scale are reproduced. Thus we have all the values of the parameters at the weak scale. The masses and the mixings are obtained by diagonalizing the mass matrices numerically. We evaluate hadronic matrix elements using the chiral Lagrangian method \[18\]. The parameters $\alpha_p$ and $\beta_p$ defined by $\langle 0 | \epsilon^{abc} (d_R^a u_R^b | u_L^c | p) \rangle = \alpha_p N_L$ and $\langle 0 | \epsilon^{abc} (d_L^a u_L^b | u_L^c | p) \rangle = \beta_p N_L$ ($N_L$ is a wave function of the left-handed proton) are evaluated as $0.003 \text{GeV}^3 \leq \beta_p \leq 0.03 \text{GeV}^3$ and $\alpha_p = -\beta_p$ by various methods \[19\]. We use the smallest value $\beta_p = -\alpha_p = 0.003 \text{GeV}^3$ in our analysis to obtain conservative bounds. For the details of the methods of our analysis, see Ref. \[1\].

3  **RRRR contribution to the proton decay**

The dimension 5 operators consist of two fermions and two bosons. Eliminating the two scalar bosons by gaugino or Higgsino exchange (dressing), we obtain the four-fermion interactions which cause the nucleon decay \[4, 11\]. In the one-loop calculations of the dressing diagrams, we include all the dressing diagrams exchanging the charginos, the neutralinos and the gluino of the $LLLH$ and $RRRR$ dimension 5 operators. In addition to the contributions from the dimension 5 operators, we include the contributions from dimension 6 operators.
mediated by the heavy gauge boson and the colored Higgs boson. Though the effects of the dimension 6 operators ($\sim 1/M_X^2$) are negligibly small for $p \to K^+\tau$, these could be important for other decay modes such as $p \to \pi^0 e^+$. The major contribution of the $LLLL$ operator comes from an ordinary diagram with wino dressing. The major contribution of the $RRRR$ operator arises from a Higgsino dressing diagram depicted in Fig. 1. The circle in this figure represents the complex conjugation of $C_{ijkl}^{5R}$ in Eq. (2) with $i = j = 1$ and $k = l = 3$. This diagram contains the Yukawa couplings of the top quark and the tau lepton. The importance of this diagram has been pointed out in Ref. [15] in the context of a SUSY SO(10) GUT model. The contribution of this diagram has been estimated to be negligible or simply ignored in previous analyses in the minimal SU(5) SUSY GUT [4, 10–14]. In particular, the authors of Ref. [11] calculated the diagram in Fig. 1. However the effect was estimated to be small, because the authors assumed a relatively light top quark ($m_t \sim 50$ GeV). We use an experimental value of the top quark mass, and show that this diagram indeed gives a significant contribution in the case of the minimal SU(5) SUSY GUT model also.

Before we present the results of our numerical calculations, we give a rough estimation for the decay amplitudes for a qualitative understanding of the results. In the actual calculations, however, we make full numerical analyses including contributions from all the dressing diagrams as well as those from dimension 6 operators. We also take account of various effects such as mixings between the SUSY particles.

Aside from the soft breaking parameter dependence arising from the loop calculations, relative magnitudes between various contributions can be roughly understood by the form of the dimension 5 operator in Eq. (4). Counting the CKM suppression factors and the Yukawa coupling factors, it is easily shown that the $RRRR$ contribution to the four-fermion operators $(u_R s_R)(d_L \nu_{\tau L})$ and
(u_{RSR})(d_L
u_{\tau L}) is dominated by a single Higgsino dressing diagram exchanging \tilde{t}_R (the right-handed scalar top quark) and \tilde{\tau}_R (the right-handed scalar tau lepton). For \(K^+\mu_\mu\) and \(K^+\tau\tau\), the \(RRRR\) contribution is negligible, since it is impossible to get a large Yukawa coupling of the third generation without small CKM suppression factors in this case. The main \(LLLL\) contributions to \((u_Ld_L)(s_L\nu_{\mu L})\) and \((u_Ls_L)(d_L\nu_{\tau L})\) consist of two classes of wino dressing diagrams; they are \(\tilde{c}_L\) exchange diagrams and \(\tilde{t}_L\) exchange diagrams \(^{[1]}\). Neglecting other subleading effects, we can write the amplitudes (the coefficients of the four-fermion operators) for \(p \rightarrow K^+\nu_c\) as,

\[
\text{Amp.}(p \rightarrow K^+\nu_c) \sim [P_2A_c(\tilde{c}_L) + P_3A_c(\tilde{t}_L)]_{LLLL},
\]

\[
\text{Amp.}(p \rightarrow K^+\nu_{\mu L}) \sim [P_2A_{\mu}(\tilde{c}_L) + P_3A_{\mu}(\tilde{t}_L)]_{LLLL},
\]

\[
\text{Amp.}(p \rightarrow K^+\nu_{\tau L}) \sim [P_2A_{\tau}(\tilde{c}_L) + P_3A_{\tau}(\tilde{t}_L)]_{LLLL} + [P_1A_{\tau}(\tilde{R}_L)]_{RRRR},
\]

where the subscript \(LLLL\) \((RRRR)\) represents the contribution from the \(LLLL\) \((RRRR)\) operator. The \(LLLL\) contributions for \(A_{\tau}\) can be written in a rough approximation as \(A_{\tau}(\tilde{c}_L) \sim g^2Y_LV^*_{ub}V^*_{cd}V_{ce}m_\tilde{\tau}/(GM_C^2)\) and \(A_{\tau}(\tilde{t}_L) \sim g^2Y_YV^*_{tb}V^*_{cd}V_{ce}m_\tilde{\tau}/(GM_C^2)\), where \(g\) is the weak \(SU(2)\) gauge coupling, and \(m_\tilde{\tau}\) is a mass of the wino \(\tilde{\tau}\). A typical mass scale of the squarks and the sleptons is denoted by \(m_f\). For \(A_{\mu}\) and \(A_{c}\), we just replace \(V^*_{ub}\) in the expressions for \(A_{\tau}\) by \(Y_LV^*_{ub}\) and \(Y_YV^*_{ub}\), respectively. The \(RRRR\) contribution is also evaluated as \(A_{\tau}(\tilde{R}_L) \sim Y_L^2Y_YV^*_{tb}V^*_{cd}V_{ce}m_\mu/(GM_C^2)\), where \(\mu\) is the supersymmetric Higgsino mass. The magnitude of \(\mu\) is determined from the radiative electroweak symmetry breaking condition, and satisfies \(|\mu| > |m_\tilde{\mu}|\) in the present scenario. Relative magnitudes between these contributions are evaluated as follows. The magnitude of the \(\tilde{c}_L\) contribution is comparable with that of the \(\tilde{t}_L\) contribution for each generation mode: \(|A_{\tau}(\tilde{c}_L)| \sim |A_{\tau}(\tilde{t}_L)|\). Therefore, cancellations between the \(LLLL\) contributions \(P_2A_{\tau}(\tilde{c}_L)\) and \(P_3A_{\tau}(\tilde{t}_L)\) can occur simultaneously for three modes \(p \rightarrow K^+\nu_{iL}\) \(\ (i = e, \mu, \tau)\) by adjusting the relative phase \(\phi_{23}\) between \(P_2\) and \(P_3\) \(^{[1]}\). The magnitudes of the \(LLLL\) contributions satisfy \(|P_2A_{\tau}(\tilde{c}_L) + P_3A_{\tau}(\tilde{t}_L)| > |P_2A_{\tau}(\tilde{c}_L) + P_3A_{\tau}(\tilde{t}_L)| > |P_2A_{\tau}(\tilde{c}_L) + P_3A_{\tau}(\tilde{t}_L)| \) independent of \(\phi_{23}\). On the other hand, the magnitude of \(A_{\tau}(\tilde{R}_L)\) is larger than those of \(A_{\tau}(\tilde{c}_L)\) and \(A_{\tau}(\tilde{t}_L)\), and the phase dependence of \(P_2A_{\tau}(\tilde{R}_L)\) is different from those of \(P_2A_{\tau}(\tilde{c}_L)\) and \(P_3A_{\tau}(\tilde{t}_L)\). Note that \(A_{\tau}(\tilde{c}_L)\) and \(A_{\tau}(\tilde{t}_L)\) are proportional to \(\sim 1/(\sin\beta\cos\beta) = \tan\beta + 1/\tan\beta\), while \(A_{\tau}(\tilde{R}_L)\) is proportional to \(\sim (\tan\beta + 1/\tan\beta)^2\), where tan \(\beta\) is the ratio of the vacuum expectation values of the two Higgs fields. Hence the \(RRRR\) contribution is more enhanced than the \(LLLL\) contributions for a large \(\tan\beta\) \(^{[17]}\).
Figure 2: Decay rates $\Gamma(p \to K^+ \nu_i)$ ($i = e, \mu$ and $\tau$) as functions of the phase $\phi_{23}$ for $\tan \beta = 2.5$. The other phase $\phi_{13}$ is fixed at $210^\circ$. The CKM phase is taken as $\delta_{13} = 90^\circ$. We fix the soft SUSY breaking parameters as $m_0 = 1$ TeV, $M_{\tilde{g}} = 125$ GeV and $A_X = 0$. The sign of the supersymmetric Higgsino mass $\mu$ is taken to be positive. The colored Higgs mass $M_C$ and the heavy gauge boson mass $M_V$ are assumed as $M_C = M_V = 2 \times 10^{16}$ GeV. The horizontal lower line corresponds to the Super-Kamiokande limit $\tau(p \to K^+ \nu) > 5.5 \times 10^{32}$ years, and the horizontal upper line corresponds to the Kamiokande limit $\tau(p \to K^+ \nu) > 1.0 \times 10^{32}$ years.

4 Numerical results

Now we present the results of our numerical calculations [1]. For the CKM matrix we fix the parameters as $V_{us} = 0.2196$, $V_{cb} = 0.0395$, $|V_{ub}/V_{cb}| = 0.08$ and $\delta_{13} = 90^\circ$ in the whole analysis, where $\delta_{13}$ is a complex phase in the CKM matrix in the convention of Ref. [20]. The top quark mass is taken to be 175 GeV [21]. The colored Higgs mass $M_C$ and the heavy gauge boson mass $M_V$ are assumed as $M_C = M_V = 2 \times 10^{16}$ GeV. We require constraint on $b \to s\gamma$ branching ratio from CLEO [22] and bounds on SUSY particle masses obtained from direct searches at LEP [23], LEP II [24] and Tevatron [25].

Let us focus on the main decay mode $p \to K^+ \nu$. We first discuss the effects of the phases $\phi_{13}$ and $\phi_{23}$ parametrizing the matrix $P$ in Eq. (1). In Fig. 3 we present the dependence of the decay rates $\Gamma(p \to K^+ \nu_i)$ on the phase $\phi_{23}$. As
an illustration we fix the other phase $\phi_{13}$ at 210°, and later we consider
the whole parameter space of $\phi_{13}$ and $\phi_{23}$. The soft SUSY breaking parameters
are also fixed as $m_0 = 1$ TeV, $M_{3X} = 125$ GeV and $A_X = 0$ here. The sign
of the Higgsino mass $\mu$ is taken to be positive. With these parameters, all the
masses of the scalar fermions other than the lighter $\tilde{t}$ are around 1 TeV, and
the mass of the lighter $\tilde{t}$ is about 400 GeV. The lighter chargino is wino-like
with a mass about 100 GeV. This figure shows that there is no region for
the total decay rate $\Gamma(p \to K^+\tau)$ to be strongly suppressed, thus the whole
region of $\phi_{23}$ in Fig. 4 is excluded by the Super-Kamiokande limit. The phase
dependence of $\Gamma(p \to K^+\tau)$ is quite different from those of $\Gamma(p \to K^+\mu)$
and $\Gamma(p \to K^+\tau)$. Though $\Gamma(p \to K^+\mu)$ and $\Gamma(p \to K^+\tau)$ are highly
suppressed around $\phi_{23} \sim 160°$, $\Gamma(p \to K^+\tau)$ is not so in this region. There
exists also the region $\phi_{23} \sim 300°$ where $\Gamma(p \to K^+\tau)$ is reduced. In this
region, however, $\Gamma(p \to K^+\mu)$ and $\Gamma(p \to K^+\tau)$ are not suppressed in turn.
Note also that the $K^+\tau$ mode can give the largest contribution.

This behavior can be understood as follows. For $\tau_\mu$ and $\tau_\tau$, the effect of the $RRRR$ operator is negligible, and the cancellation between the $LLLL$
contributions directly leads to the suppression of the decay rates. This can-
cellation indeed occurs around $\phi_{23} \sim 160°$ for both $\tau_\mu$ and $\tau_\tau$ simultaneously
in Fig. 4. For $\tau_\tau$, the situation is quite different. The similar cancellation
between $P_3A_\tau(\tilde{c}_L)$ and $P_3A_\tau(\tilde{t}_L)$ takes place around $\phi_{23} \sim 160°$ for $\tau_\tau$ also.
However, the $RRRR$ operator gives a significant contribution for $\tau_\tau$. Therefore,
$\Gamma(p \to K^+\tau)$ is not suppressed by the cancellation between the $LLLL$
contributions in the presence of the large $RRRR$ operator effect. Notice that it is
possible to reduce $\Gamma(p \to K^+\tau)$ by another cancellation between the $LLLL$
contributions and the $RRRR$ contribution. This reduction of $\Gamma(p \to K^+\tau)$
indeed appears around $\phi_{23} \sim 300°$ in Fig. 4. The decay rate $\Gamma(p \to K^+\mu)$
is rather large in this region. The reason is that $P_2A_\tau(\tilde{c}_L)$ and $P_3A_\tau(\tilde{t}_L)$
are constructive in this region in order to cooperate with each other to cancel the
large $RRRR$ contribution $P_1A_\tau(t_R)$, hence $P_2A_\mu(\tilde{c}_L)$ and $P_3A_\mu(\tilde{t}_L)$ are also
constructive in this region. Thus we cannot reduce both $\Gamma(p \to K^+\tau)$ and
$\Gamma(p \to K^+\mu)$ simultaneously. Consequently, there is no region for the total
decay rate $\Gamma(p \to K^+\mu)$ to be strongly suppressed. In the previous analysis [8]
the region $\phi_{23} \sim 160°$ has been considered to be allowed by the Kamiokande
limit $\tau(p \to K^+\tau) > 1.0 \times 10^{32}$ years (90% C.L.) [8]. However the inclusion
of the Higgsino dressing of the $RRRR$ operator excludes this region. We also
examined the whole region of $\phi_{13}$ and $\phi_{23}$ with the same values for the other
parameters as in Fig. 4. We found that we cannot reduce both $\Gamma(p \to K^+\tau)$
and $\Gamma(p \to K^+\mu)$ simultaneously, even if we adjust the two phases $\phi_{13}$ and
$\phi_{23}$ anywhere.
Next we consider the case where we vary the parameters we have fixed so far. The relevant parameters are the colored Higgs mass $M_C$, the soft SUSY breaking parameters and $\tan \beta$. The partial lifetime $\tau(p \rightarrow K^+\bar{\nu})$ is proportional to $M_C^2$ in a very good approximation, since this mode is dominated by the dimension 5 operators. Using this fact and the calculated value of $\tau(p \rightarrow K^+\bar{\nu})$ for the fixed $M_C = 2 \times 10^{16}$ GeV, we can obtain the lower bound on $M_C$ from the experimental lower limit on $\tau(p \rightarrow K^+\bar{\nu})$. In Fig. 3, we present the lower bound on $M_C$ obtained from the Super-Kamiokande limit as a function of the left-handed scalar up-quark mass $m_{\tilde{u}_L}$. Masses of the squarks other than the lighter $\tilde{t}$ are almost degenerate with $m_{\tilde{u}_L}$. The soft breaking parameters $m_0$, $M_{gX}$ and $A_X$ are scanned within the range of $0 < m_0 < 3$ TeV, $0 < M_{gX} < 1$ TeV and $-5 < A_X < 5$, and $\tan \beta$ is fixed at 2.5. Both signs of $\mu$ are considered. The whole parameter region of the two phases $\phi_{13}$ and $\phi_{23}$ is examined. The solid curve represents the bound derived from the Super-Kamiokande limit $\tau(p \rightarrow K^+\bar{\nu}) > 5.5 \times 10^{32}$ years, and the dashed curve represents the corresponding result without the $RRRR$ effect. The left-hand side of the vertical dotted line is excluded by other experimental constraints. The dash-dotted curve represents the bound derived from the Kamiokande limit on the neutron partial lifetime $\tau(n \rightarrow K^0\bar{\nu}) > 0.86 \times 10^{32}$ years.
0 < M_{gX} < 1\,\text{TeV} \text{ and } -5 < A_X < 5, \text{ and } \tan\beta \text{ is fixed at 2.5. Both signs of } \mu \text{ are considered. The whole parameter region of the two phases } \phi_{13} \text{ and } \phi_{23} \text{ is examined. The solid curve in this figure represents the result with all the } LLLL \text{ and } RRRR \text{ contributions. It is shown that the lower bound on } M_C \text{ decreases like } \sim 1/m_{\tilde{u}_L} \text{ as } m_{\tilde{u}_L} \text{ increases. This indicates that the } RRRR \text{ effect is indeed relevant, since the decay amplitude from the } RRRR \text{ operator is roughly proportional to } \mu/(M_C m_{\tilde{u}}^2) \sim 1/(M_C m_{\tilde{f}}), \text{ where we use the fact that the magnitude of } \mu \text{ is determined from the radiative electroweak symmetry breaking condition and scales as } \mu \sim m_{\tilde{f}}. \text{ The dashed curve in Fig. 3 represents the result in the case where we ignore the } RRRR \text{ effect. In this case the lower bound on } M_C \text{ behaves as } \sim 1/m_{\tilde{u}_L}^2, \text{ since the } LLLL \text{ contribution is proportional to } m_{\tilde{W}}/(M_C m_{\tilde{f}}^2).

The solid curve in Fig. 3 indicates that the colored Higgs mass \( M_C \) must be larger than \( 6.5 \times 10^{16} \, \text{GeV} \) for \( \tan\beta = 2.5 \) when the typical sfermion mass is less than 1 TeV. On the other hand, there is a theoretical upper bound of the colored Higgs mass \( M_C \lesssim 4 \times 10^{16} \, \text{GeV} \) from an analysis of RGEs when we require the validity of the minimal SU(5) SUSY GUT model up to the Planck scale. Then it follows from Fig. 3 that the minimal SU(5) SUSY GUT model with the sfermion masses less than 2 TeV is excluded for \( \tan\beta = 2.5 \). The \( RRRR \) effect plays an essential role here, since the lower bound on \( m_{\tilde{f}} \) would be 600 GeV if the \( RRRR \) effect were ignored. We also find that the Kamiokande limit on the neutron partial lifetime \( \tau(n \to K^0\tau) > 0.86 \times 10^{32} \, \text{years} \) (90\% C.L.) already gives a comparable bound with that derived here from the Super-Kamiokande limit on \( \tau(p \to K^+\tau) \), as shown by the dash-dotted curve in Fig. 3.

Fig. 4 shows the \( \tan\beta \) dependence of the lower bound on the colored Higgs mass \( M_C \) obtained from the Super-Kamiokande limit. Here we vary \( m_0, M_{gX} \text{ and } A_X \) as in Fig. 3. The phases \( \phi_{13} \text{ and } \phi_{23} \) are fixed as \( \phi_{13} = 210^\circ \text{ and } \phi_{23} = 150^\circ \). The result does not change much even if we take other values of \( \phi_{13} \text{ and } \phi_{23} \). The region below the solid curve is excluded if \( m_{\tilde{u}_L} \) is less than 1 TeV. The lower bound reduces to the dashed curve if we allow \( m_{\tilde{u}_L} \) up to 3 TeV. It is shown that the lower bound on \( M_C \) behaves as \( \sim \tan^2\beta \) in a large \( \tan\beta \) region, as expected from the fact that the amplitude of \( p \to K^+\tau \) from the \( RRRR \) operator is roughly proportional to \( \tan^2\beta/M_C \). On the other hand the \( LLLL \) contribution is proportional to \( \sim \tan\beta/M_C \), as shown by the dotted curve in Fig. 4. Thus the \( RRRR \) operator is dominant for large \( \tan\beta \). Note that the lower bound on \( M_C \) has the minimum at \( \tan\beta \approx 2.5 \). Thus also it has been pointed out that there exists an upper bound on \( M_C \) given by \( M_C \lesssim 2.5 \times 10^{16} \, \text{GeV} \) (90\% C.L.) if we require the gauge coupling unification in the minimal contents of GUT superfields.
we can conclude that for other value of $\tan \beta$ the constraints on $M_C$ and $m_f$ become severer than those shown in Fig. 3. In particular the lower bound on $m_f$ becomes larger than $\sim 2$ TeV for $\tan \beta \neq 2.5$.

The constraints obtained from the figures can be expressed as follows:

\[
\left( \frac{M_C}{6.5 \times 10^{16} \text{ GeV}} \right) \lesssim \left( \frac{\tau_{\exp}(p \rightarrow K^+\nu)}{5.5 \times 10^{32} \text{ years}} \right)^{\frac{1}{2}} \left( \frac{\beta_p}{0.003 \text{ GeV}^2} \right) \left( \frac{1 \text{ TeV}}{m_f} \right) \left( \frac{\tan \beta}{10} \right)^2
\]

for $\tan \beta \approx 2.5$, and

\[
\left( \frac{M_C}{5.0 \times 10^{17} \text{ GeV}} \right) \lesssim \left( \frac{\tau_{\exp}(p \rightarrow K^+\nu)}{5.5 \times 10^{32} \text{ years}} \right)^{\frac{1}{2}} \left( \frac{\beta_p}{0.003 \text{ GeV}^2} \right) \left( \frac{1 \text{ TeV}}{m_f} \right) \left( \frac{\tan \beta}{10} \right)^2
\]

for $\tan \beta \gtrsim 5$. 

(5)
where $\tau_{\exp}(p \to K^+\bar{\nu})$ is an experimental lower limit for the partial lifetime of the decay mode $p \to K^+\bar{\nu}$.

5 Conclusions

We have reanalyzed the proton decay including the $RRRR$ dimension 5 operator in the minimal SU(5) SUSY GUT model. We have shown that the Higgsino dressing diagram of the $RRRR$ operator gives a dominant contribution for $p \to K^+\bar{\nu}_\tau$, and the decay rate of this mode can be comparable with that of $p \to K^+\bar{\nu}_{\mu}$. We have found that we cannot reduce both the decay rate of $p \to K^+\bar{\nu}_\tau$ and that of $p \to K^+\bar{\nu}_{\mu}$ simultaneously by adjusting the relative phases $\phi_{13}$ and $\phi_{23}$ between the Yukawa couplings at the colored Higgs interactions. We have obtained the bounds on the colored Higgs mass $M_C$ and the typical sfermion mass $m_{\tilde{f}}$ from the new limit on $\tau(p \to K^+\bar{\nu})$ given by the Super-Kamiokande: $M_C > 6.5 \times 10^{16}$ GeV for $m_{\tilde{f}} < 1$ TeV. The minimal SU(5) SUSY GUT model with $m_{\tilde{f}} \lesssim 2$ TeV is excluded if we require the validity of this model up to the Planck scale.

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