Hints of Modified Gravity in Cosmos and in the Lab?

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(Dated: April 23, 2019)

General Relativity (GR) is consistent with a wide range of experiments/observations from millimeter scales up to galactic scales and beyond. However, there are reasons to believe that GR may need to be modified because it includes singularities (it is an incomplete theory) and also it requires fine-tuning to explain the accelerating expansion of the universe through the cosmological constant. Therefore, it is important to check various experiments and observations beyond the above range of scales for possible hints of deviations from the predictions of GR. If such hints are found it is important to understand which classes of modified gravity theories are consistent with them. The goal of this review is to summarize recent progress on these issues. On sub millimeter scales we show an analysis of the data of the Washington experiment [1] searching for modifications of Newton’s Law on sub-millimeter scales and demonstrate that a spatially oscillating signal is hidden in this dataset. We demonstrate that even though this signal cannot be explained in the context of standard modified theories (viable scalar tensor and \( f(R) \) theories), it is a rather generic prediction of nonlocal gravity theories. On cosmological scales we review recent analyses of Redshift Space Distortion (RSD) data which measure the growth rate of cosmological perturbations at various redshifts and show that these data are in some tension with the \( \Lambda \)CDM parameter values indicated by Planck/2015 CMB data at about 3σ level. This tension can be reduced by allowing for an evolution of the effective Newton constant that determines the growth rate of cosmological perturbations. We conclude that even though this tension between the data and the predictions of GR could be due to systematic/statistical uncertainties of the data, it could also constitute early hints pointing towards a new gravitational theory.

I. INTRODUCTION

General Relativity (GR) has been tested in a wide range of scales starting from sub-mm scales out to supercluster \( O(100 \text{Mpc}) \) scales. Even though no statistically significant evidence has been found so far indicating deviations from GR, there are theoretical arguments and experimental/observational hints that indicate that GR may need to be modified on both the smallest and the largest probed scales.

From the theoretical point of view, it is clear that GR has to face the following challenges:

- It predicts the existence of unphysical singularities which indicate that it is a physically incomplete theory.
- It is nonrenormalizable and inconsistent with Quantum Field Theory (QFT) at high energies due to the prediction of black hole formation when small scales are probed via scattering experiments.
- It can not explain the observed accelerating expansion of the universe unless extreme fine tuning is assumed.

From the experimental/observational point of view GR has been well tested on solar system scales where the PPN parameters measuring deviations from GR have been shown to reduce to the values predicted by GR at an accuracy level of about \( 10^{-5} \) [2]. On larger and smaller scales however the constraints on deviations from GR are not as strong. In fact there have been claims for hints of deviations from GR predictions even on solar system scales (e.g. Pioneer anomaly [3]).

On galactic scales, the deviation of star velocities from the velocities expected in the presence of visible matter in the context of GR indicates that the Einstein equation \( G_{\mu \nu} = T_{\mu \nu}^{\text{lum}} \) (where \( T_{\mu \nu}^{\text{lum}} \) is the energy momentum tensor of luminous matter) is violated. The usual approach is restoring consistency between the two sides of the Einstein equation has been to modify the right side of the Einstein equation and write it in the form \( G_{\mu \nu} = T_{\mu \nu}^{\text{dm}} + T_{\mu \nu}^{\text{lum}} \) where \( T_{\mu \nu}^{\text{dm}} \) is the energy momentum tensor of matter that interacts only gravitationally (dark matter [4]). An alternative approach is to modify the left side of the Einstein equation as \( G_{\mu \nu} + G_{\mu \nu}^{\text{TeVeS}} = T_{\mu \nu}^{\text{lum}} \) leading to a modified version of GR: Tensor Vector Scalar theory (TeVeS) [5]. Clearly a combination of the two above solutions is also possible leading to \( G_{\mu \nu} + G_{\mu \nu}^{\text{TeVeS}} = T_{\mu \nu}^{\text{lum}} + T_{\mu \nu}^{\text{dm}} \).

The frontiers of current gravitational research lie on the two extreme scales that gravitational experiments/observations can currently probe: sub-mm scales where a wide range of experiments [6] search for new types of forces and cosmological scales of a few \( \text{Mpc} \) or larger where observations of the growth rate of cosmological perturbations through Redshift Space Distortions [7–12] or Weak Lensing [13–16] can probe the gravitational laws and the consistency of GR with data. Current re-
search on these frontier scales is the focus of the present review.

Small scale gravity experiments [17–37] probe sub-mm scales searching for new forces on these scales. The forces between test masses are measured at various distances and compared with the expected forces on the basis of known physics. Deviations from the null result corresponding to Newtonian gravitational interaction are fit to specific parametrizations that are well motivated based on theoretical arguments. The most commonly used parametrization for fitting the above deviations of gravitational experiment data is the Yukawa parametrization, where the effective gravitational interaction potential is expressed as

$$V_{\text{eff}} = -G \frac{M}{r} (1 + \alpha e^{-mr}) \quad (1.1)$$

corresponding to a spatially varying effective Newton’s constant of the form

$$G_{\text{eff}}(r) = G(1 + \alpha e^{-mr}) \quad (1.2)$$

Eq. (1.2) depends on the parameters $\alpha$ and $m$, which denote the amplitude and the range of Yukawa force. Fig. 1 shows current constraints from small scale gravity experiments. For $\alpha \approx 1$, the range of this Yukawa exponential is constrained to be less than about 0.1 mm [6].

For example, consider a generic form of $f(R)$ theories with an $R^2$ correction of the form [44]

$$f(R) = R + \frac{1}{6m^2} R^2 \quad (1.3)$$

The generalized Einstein-Hilbert action is of the form

$$S_R = \frac{1}{16\pi G} \int d^4x \sqrt{-g} f(R) + S_{\text{matter}} \quad (1.4)$$

Varying action (1.4) with respect to the metric leads to the dynamical equations

$$f'(R) R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} f(R) = 8\pi G T_{\mu\nu} + \nabla_\mu \nabla_\nu f'(R) - g_{\mu\nu} \square f'(R) \quad (1.5)$$

Assuming that $f(R)$ has the form of Eq. (1.3), in the weak field limit $g_{\mu\nu} = h_{\mu\nu} + \eta_{\mu\nu}$, it is straightforward to show that a solution for the metric perturbation $h_{00}$ in the presence of a point mass $M$ takes the form [49]

$$h_{00} = \frac{2GM}{r} \left(1 + \frac{1}{3} e^{-mr}\right) \quad (1.6)$$

which compared to the usual Newtonian case has the correction factor $\frac{1}{3} e^{-mr}$. A comparison with the Yukawa ansatz (1.2) implies that $\alpha = \frac{1}{3}$. A similar form of modified Newtonian force is obtained for massive Brans-Dicke (BD) theories [49] and for Kaluza-Klein theories [48, 50]. In those cases the phenomenological parameters $\alpha, m$ depend on the fundamental parameters of the theories (e.g. mass of scalar field, Brans-Dicke parameter $\omega$, size and number of extra dimensions).

The above Yukawa parametrization is well motivated theoretically and is currently the standard parametrization used to fit experimental residuals of the Newtonian force. However, alternative parametrizations may also be theoretically motivated in the context of other theoretical models and they may in fact provide better fits to experimental residuals with respect to the Newtonian force on sub-mm scales. For example some brane theories favor a power law residual parametrization [51–54]. A purely phenomenological approach could also consider arbitrary parametrizations (e.g. spatially oscillating parametrizations) of residual forces designed so that they provide the best fit to residual force data.

Stability of the theories that lead to a Yukawa type of modified Newton’s law usually implies that $m^2 > 0$ [49]. The case $m^2 < 0$ is usually associated with instabilities [55, 56] of the underlying theories and also with an oscillating behaviour of the additional term modifying the Newtonian gravitational force. Despite the fact that in these cases we may have no Newtonian limit, such a spatially oscillating term can escape detection if its spatial wavelength is smaller than a fraction of a mm [49]. This case will be discussed in Section III along with an example of a healthy theory (nonlocal gravity [57–59]) that predicts such spatial oscillations without the presence of ghosts/instabilities [60–62].

FIG. 1. A review of current constraints based on the Yukawa parametrization (1.1) for deviation from Newton’s law. From Ref. [6].

The Yukawa parametrization is the most commonly used parametrization for testing for deviations from Newton’s law on sub-mm scales. It is generic and well motivated theoretically as it is a natural prediction in the context of a wide range of modified gravity theories including Brans-Dicke [38–40], scalar-tensor [41–44] and $f(R)$ theories [45–47]. It is also a natural prediction of theories involving compactified extra dimensions such as Kaluza-Klein theories [48].
On the other frontier of testing GR, cosmological scales, the properties of the gravitational theory can be probed by measuring the growth rate of cosmological perturbations through the measurement of peculiar velocities of galaxies (obtained using Redshift Space Distortion (RSD) data [8, 63]) and through weak gravitational lensing [13, 64, 65]. In the presence of perturbations, the perturbed metric in the Newtonian gauge takes the form

\[ ds^2 = -(1 + 2\phi)dt^2 + a^2(1 - 2\psi)dx^2 \]  

(1.7)

where \( \phi \) and \( \psi \) are potentials with \( \phi \) corresponding to the Newtonian potential. These two potentials in general obey modified Poisson equations of the following form

\[ \nabla^2 \phi = 4\pi G_{\text{eff}} a^2 \rho \delta_m, \]  

(1.8)

\[ \nabla^2(\phi + \psi) = 8\pi G_L a^2 \rho \delta_m, \]  

(1.9)

where \( \delta_m \) in the linear matter overdensity, \( \rho \) is the mean matter density and \( a \) is the cosmic scale factor. The potential \( \phi \) can be probed using growth of density perturbations observations through RSD data [8, 63] and \( \phi + \psi \) is usually probed using weak lensing data [13, 64, 65]. In Eq. (1.8) and Eq. (1.9) we also have the parameters \( G_{\text{eff}} \) and \( G_L \) which in GR are equal and constant

\[ G_{\text{eff}} = G_L = G_N \]  

(1.10)

while in modified gravity theories they can be spacetime dependent. Therefore a basic question arises. “How can the actual data constrain possible scale or redshift dependence of these parameters?” Here we focus on the \( G_{\text{eff}} \) that is associated with the Newtonian potential \( \phi \) and can be constrained using RSD data measuring the growth of density perturbations.

Early hints of modifications of GR are most likely to come from experiments/observations at the frontier scales: sub-mm and cosmological scales. Important questions that need to be addressed in this context are the following:

- Is GR consistent with currently available data on each scale?
- Even if it is consistent what is the optimum parametrization of the effective Newton’s constant \( G_{\text{eff}} \) in providing the best quality of fit to the data?
- If there is such parametrization providing a better fit to the data, then what are the theoretical models that support it?

These questions will be the focus of the present brief review.

The structure of this review is the following: In Section II, we focus on cosmological scales and review the phenomenological predictions of modified gravity theories on the observable growth rate of matter density perturbations which can be used as a probe of gravitational physics on cosmological scales. We also focus on Redshift Space Distortions (RSD) as a probe of the growth rate of matter density perturbations and use an extended compilation of RSD data to identify the tension level between the \( \Lambda \)CDM parameter values favoured by Planck 2015 [66] and the corresponding parameter values favoured by the RSD growth data. The effect of an evolving with redshift \( z \) effective Newton’s constant \( G_{\text{eff}}(z) \) on the level of this tension is reviewed and the qualitative features of the best fit form of \( G_{\text{eff}}(z) \) are identified. The consistency of these qualitative features with specific modified gravity theories is also discussed. In Section III we focus on sub-mm scales and identify the quality of fit of a novel oscillating residual force parametrizations on the data of the Washington small scale gravity experiment. The consistency of this parametrization with specific modified gravity models (\( f(R) \) theories and nonlocal gravity) is also discussed. Finally, in Section IV we conclude, summarize and discuss interesting extensions of the reviewed research.

II. HINTS OF MODIFIED GRAVITY ON COSMOCAL SCALES

II.1. RSD Data: Analysis and Phenomenological Implications

A particularly useful probe of the growth rate of density perturbations is weak lensing [13, 64, 65]. Recent \( \Lambda \)CDM parameter constraints emerging from a tomographic weak gravitational lensing analyses indicates a 2-3 \( \sigma \) tension in the \( \sigma_8 - \Omega_m \) parameter space between the parameter values favoured by Planck 2015 [66] (which can be seen in Table I) and specific weak lensing survey data [13, 67].

| Table I. The Planck15/\( \Lambda \)CDM parameters as reported in Ref. [66] |
|-----------------------------------|------------------|
| Parameter                        | Planck15/\( \Lambda \)CDM Values [66] |
| \( \Omega_b h^2 \)               | 0.02225 ± 0.00016 |
| \( \Omega_c h^2 \)               | 0.1198 ± 0.0015   |
| \( n_s \)                        | 0.9645 ± 0.0049   |
| \( H_0 \)                        | 67.27 ± 0.66      |
| \( \Omega_m \)                   | 0.3156 ± 0.0091   |
| \( w \)                          | -1               |
| \( \sigma_8 \)                   | 0.831 ± 0.013     |

This tension is demonstrated in Fig. 2. On the left panel we show the \( 1 - 2\sigma \), \( \sigma_8 - \Omega_m \) best fit parameter contours obtained by the Kilo Degree Survey (KiDS) [13] superposed with the corresponding Planck 2015 [66] contours. On the right panel we show the \( 1 - 2\sigma \), \( S_8 - \Omega_m \) (\( S_8 \equiv \sigma_8 (\Omega_m^{0.5})^{0.5} \)) best fit parameter contours obtained by the Dark Energy Survey (DES) [67] superposed with the corresponding Planck 2015 [66] contours. In both cases a \( 2-3\sigma \) tension between the Planck15/\( \Lambda \)CDM best fit and the weak lensing best fit parameter values is ev-
ident. This tension may be either due to systematics of the weak lensing or Planck15/ΛCDM data or could be an early hint of new gravitational physics since the weak lensing data are much more sensitive to the growth of cosmological perturbations (gravitational physics) than the CMB data which only probe this growth rate through the ISW effect on very large scales (low l).

As is clearly seen from Fig. 2 weak lensing data appear to favour a lower value for Ω_m compared to the value of Ω_m favoured by Planck15/ΛCDM. The requirement of lower Ω_m made by the weak lensing data may also be viewed as a requirement of weaker gravity than implied by GR (Planck15/ΛCDM) at low redshifts. An interesting question therefore emerges: “Is the same tension for weaker gravity at low redshifts and tension with Planck15/ΛCDM also favoured by other probes of the growth rate of density perturbation like the RSD data?”

The RSD surveys probe the growth of matter density perturbations by detecting the distortion of the power spectrum of perturbations which are induced by peculiar velocities. This distortion probes the peculiar velocities of galaxies on large scales which in turn can be used to obtain the growth rate of perturbations \( f(a) = \frac{\delta\rho}{\langle\rho\rangle} \), where \( a \) is the scale factor and \( \delta(a) \equiv \delta\rho/\rho \) is the linear matter overdensity growth factor. Combined with density rms fluctuations within spheres of radius \( R = 8h^{-1}Mpc \) which may be written as \( \sigma_s(a) = \sigma_s(\frac{a}{1+z})(1) \), the observable product \( f\sigma_s(a) \) measured by RSD surveys at various redshifts \( z \) (or values of the scale factor \( a \)) may be expressed in terms of the present value of \( \sigma_s(1) = \sigma_s \) and the derivative of \( \delta(a) \) with respect to the scale factor \( a \) as

\[
  f\sigma_s(a) \equiv f(a) \cdot \sigma(a) = \frac{\sigma_s}{\delta(1)} \cdot a \cdot \delta'(a) \tag{2.1}
\]

This combination, i.e. Eq.\( (2.1) \), at various redshifts is published by various surveys as a probe of the growth of matter density perturbations.

Given the background expansion rate \( H(z) \) which can be parametrized as \( wCDM \)

\[
  E(a)^2 = \frac{H(a)^2}{H_0^2} = \Omega_m a^{-3} + (1 - \Omega_m) a^{-3(1+w)} \tag{2.2}
\]

the theoretically predicted functional form of \( \delta(a) \) and therefore of \( f\sigma_s(a) \) can be obtained on sub-Hubble scales by solving the dynamical growth equation [7]

\[
  \delta''(a) + \left( \frac{3}{a} + \frac{H'(a)}{H(a)} \right) \delta'(a) - \frac{3}{2} \frac{\Omega_m G_{eff}(a,k)/G_N}{a^5 H(a)^2/H_0^2} \delta(a) = 0 \tag{2.3}
\]

or in redshift space

\[
  \delta'' + \frac{(H^2)'}{2 H^2} \frac{1}{1+z} \delta' = \frac{3}{2} \frac{(1+z) H_0^2 G_{eff}(z,k)}{H^2 G_N \Omega_m \delta} \tag{2.4}
\]

In Eqs. \( (2.3), \ (2.4) \) possible deviations from GR are expressed by allowing for a scale and redshift-dependent effective Newton’s constant \( G_{eff} = G_{eff}(a,k) \). It should be stressed that an observed value of \( G_{eff} \) that is not constant and/or differs from the Newton’s constant value \( G_N \) on solar system scales does not necessarily mean that GR is violated. It could also mean that dark energy clusters on sub-Hubble scales and/or that there is a coupling between dark matter and dark energy. Both of these effects would lead to a modification of Eq. \( (2.3) \) from its standard form with \( G_{eff} = G_N \).

In the context of standard GR (\( G_{eff} = G_N \)) and assuming a \( wCDM \) background \( (2.2) \) it is straightforward to solve Eq. \( (2.3) \) numerically with initial conditions deep in the matter era \( \delta(a) \sim a \) and obtain the solution \( \delta(a,w,\Omega_m) \) and then use \( (2.1) \) to obtain the theoretically predicted form of \( f\sigma_s(z,a,\sigma_s, w,\Omega_m) \) in the context of GR. A fit of this theoretical prediction to the observed RSD datapoints \( f\sigma_s(zi) \) can lead to constraints on the parameters \( \sigma_s, w, \Omega_m \). The comparison of these constraints with the corresponding Planck15/ΛCDM constraints can
be a measure of the consistency of the RSD data with Planck15/ΛCDM in the context of GR.

A fit along the above lines has been implemented in Refs. [7, 12] where \( \chi^2 \) was constructed by defining the vector

\[
V^i(z_i, \Omega_m, \sigma_8, g_n) = f \sigma_{8i} - f \sigma_S(z_i, \sigma_8, w, \Omega_m)
\]  

where \( f \sigma_{8i} \) are the RSD datapoints and \( f \sigma_S(z_i, \sigma_8, w, \Omega_m) \) is the theoretical prediction at the same redshift \( z_i \). The best fit \( \sigma_8, w, \Omega_m \) parameter values were obtained [7, 12] by minimizing

\[
\chi^2(\sigma_8, w, \Omega_m) = V^i C^{-1}_{ij} V^j
\]

where \( C_{ij} \) is the covariance matrix assumed to be diagonal except of the WiggleZ survey 3 \( \times \) 3 subset [68]. Thus, the covariance matrix may be written as

\[
C^{\text{growth, total}}_{ij} = \begin{pmatrix}
\sigma_1^2 & 0 & 0 & \cdots \\
0 & C_{WiggleZ} & 0 & \cdots \\
0 & 0 & \cdots & \sigma_N^2
\end{pmatrix}
\]  

(2.7)

where [68]

\[
C_{WiggleZ}^{\text{WiggleZ}} = 10^{-3} \begin{pmatrix}
6.400 & 2.570 & 0.000 \\
2.570 & 3.969 & 2.540 \\
0.000 & 2.540 & 5.184
\end{pmatrix}
\]  

(2.8)

The rest of the non-diagonal terms are assumed to be 0, implying no correlation among the corresponding datapoints. This assumption is an approximation which as discussed below using Monte Carlo simulations has a relatively small effect on the derived best fit parameter values [12].

A wide range of \( f \sigma_8 \) datasets have been used to constrain cosmological model parameters. Three of the largest such compilations have been constructed in Refs. [7, 12]. In Ref. [7] a compilation of 34 \( f \sigma_8 \) datapoints was constructed including datapoints published until 2016. In an attempt to minimize correlations among datapoints a second compilation consisting of 18 \( f \sigma_8 \) datapoints was constructed which included those datapoints that appeared to have minimal levels of correlation (originating from different redshift surveys and different patches in the sky). This more robust compilation is shown in Table VI in Appendix A. The third more recent compilation [12] is the largest \( f \sigma_8 \) dataset published to date consisting of 63 distinct datapoints (Table V in Appendix A).

Despite the possible correlations among the datapoints of this compilation, it contains interesting useful information which has been extracted in the detailed analysis of Ref. [12]. The growth rate of cosmological perturbations is obtained from the RSD data by comparing the observed power spectrum of large scale structures in redshift space \( P_{\text{obs}}(k_{\text{ref, } \perp}, k_{\text{ref, } \parallel}, z) \) with the expected isotropic (due to the cosmological principle) true underlying spectrum \( P_{\text{matter}}(k, z) \sim \delta \rho / \rho(k, z)^2 \) where \( k_{\text{ref, } \perp} \) is the Fourier scale wavevector component parallel to the line of sight and \( k_{\text{ref, } \parallel} \) is the corresponding wavevector perpendicular to the line of sight in the context of a given reference (fiducial) cosmology used to convert the measured angles and redshifts to distances. The true statistically isotropic power spectrum depends only on the magnitude of the true Fourier scale wavevector. The observed spectrum of perturbations is distorted for two reasons:

- **Incorrect Fiducial Cosmology:** The redshift surveys measure galaxy redshifts and angles of galaxies. In order to construct the correlation function and thus the power spectrum, these angles and redshifts need to be converted to comoving coordinates. This conversion requires the assumption of a particular form of \( H_{\text{ref}}(z) \) (a reference or fiducial cosmology) which is not necessarily identical with the true cosmology \( H(z) \). The use of an incorrect fiducial cosmology \( H_{\text{ref}}(z) \) would lead to an incorrect distorted nonisotropic power spectrum \( P_{\text{obs}}(k_{\text{ref, } \perp}, k_{\text{ref, } \parallel}, z) \) which may be shown [69, 70] to be connected with the galaxy power spectrum \( P_g(k_{\text{ref, } \perp}, k_{\text{ref, } \parallel}, z) \) obtained with the correct cosmology \( H(z) \) with the relation [69]

\[
P_{\text{obs}}(k_{\text{ref, } \perp}, k_{\text{ref, } \parallel}, z) = \frac{d_A(z)^2 H(z)}{d_A(z)^2 H_{\text{ref}}(z)} P_g(k_{\text{ref, } \perp}, k_{\text{ref, } \parallel}, z)
\]

(2.9)

where \( d_A(z) \) is the angular diameter distance. This geometric distortion of the correlation function and the power spectrum due to the use of the incorrect fiducial cosmology is known as the Alcock-Paczynski (AP) effect.

Even if the correct cosmology was used for the conversion of angles-redshifts to distances, the power spectrum \( P_g \) is still nonisotropic. The reason for this remaining distortion are the peculiar velocities of galaxies which encapsulate the information for the gravitational growth of perturbation. Thus, the second effect that distorts the observed power spectrum is the peculiar velocity effect.

- **Peculiar Velocities:** Peculiar velocities add an extra component to the cosmological redshifts thus perturbing the real positions of galaxies \( x_r \) along the line of sight to a new position \( x_p \) of the form [71, 72]

\[
x_p = x_r + (1 + z) \frac{\ddot{x} \cdot \vec{v}}{H(z)}
\]

(2.10)

This distortion of galaxy positions due to their peculiar velocities leads to an additional distortion of the observed power spectrum of the form...
where \( b(z) \) is the bias factor (the ratio of the galaxy overdensities over the underlying matter overdensities) and \( \beta(z) \equiv \frac{\langle z(z) \rangle}{k(z)} \) is the linear redshift space distortion parameter. The wavenumbers \( k_{\text{ref} \perp} \) and \( k_{\text{ref} \parallel} \) obtained using the fiducial cosmology are connected to the wavenumbers \( k\parallel \) and \( k\perp \) in the true cosmology as \( k_{\text{ref} \parallel} = \frac{H(z) k_{\parallel}}{H(z)} \) and \( k_{\text{ref} \perp} = \frac{d_A(z)}{d_{A\text{fid}}(z)} k_{\perp} \). Using Eq. (2.11), the measured distorted power spectrum \( P_g(k_{\text{ref} \perp}, k_{\text{ref} \parallel}, z) \) and the isotropy of the true power spectrum, the parameter \( \beta(z) \) can be inferred and from it the bias free product \( f\sigma_8 \) can be derived.

Each one of the datapoints of Tables V and VI is constructed under the assumption of a particular fiducial cosmology. Thus an Alcock-Paczyński correction factor needs to be imposed to each one of the datapoints converting them to the values corresponding to the true cosmology \( H(z) \). If an \( f\sigma_8 \) measurement has been obtained assuming a fiducial ΛCDM cosmology \( H(z) \), the corresponding \( f\sigma_8 \) obtained with the true cosmology \( H(z) \) is approximated as [8]

\[
f\sigma_8(z) \simeq \frac{H(z)d_A(z)}{H'(z)d'_{A\text{fid}}(z)} f\sigma'_8(z) = q(z, \Omega_m, \Omega_m') f\sigma_8(z)
\]

(2.12)

This equation should be taken as a rough order of magnitude estimate of the AP effect as it appears in somewhat different forms in the literature [69, 73, 74].

This correction is small (at most it can be about 2–3% at redshifts \( z \approx 1 \) for reasonable values of \( \Omega_m \)) [12]. The magnitude of this factor is demonstrated in Fig. 3 for typical values of fiducial and true cosmologies.

With the predictions of specific models obtained by solving Eq. (2.4) with matter domination initial conditions and using Eq. (2.1) for specific cosmological models: Planck15/ΛCDM with GR \( (G_{\text{eff}} = G_N) \), the best fit ΛCDM model to the \( f\sigma_8 \) data (with a reduced value of \( \Omega_m \)) and a modified gravity model where the background expansion is given by the Planck15/ΛCDM parameters while \( G_{\text{eff}} \) is allowed to vary with redshift with a specific

![Plot of the correction factor q as a function of the redshift z](image)

**FIG. 3.** Plot of the correction factor \( q \) as a function of the redshift \( z \) (from Ref. [12]).

The fiducial model corrected datapoints of Tables V and VI are shown in Figs. 4 and 5, respectively along with the predictions of specific models obtained by solving Eq. (2.4) with matter domination initial conditions and using Eq. (2.1) for specific cosmological models: Planck15/ΛCDM with GR \( (G_{\text{eff}} = G_N) \), the best fit ΛCDM model to the \( f\sigma_8 \) data (with a reduced value of \( \Omega_m \)) and a modified gravity model where the background expansion is given by the Planck15/ΛCDM parameters while \( G_{\text{eff}} \) is allowed to vary with redshift with a specific

![Plot of \( f\sigma_8 \) as a function of the redshift z for the full growth rate data set of Table V in Appendix A.](image)

**FIG. 4.** Plot of \( f\sigma_8 \) as a function of the redshift \( z \) for the full growth rate data set of Table V in Appendix A. The green dashed line describes the best fits of WMAP7/ΛCDM while the red one the best fits of Planck15/ΛCDM. The blue dashed line describes the best fit ΛCDM \( (\Omega_m = 0.28 \pm 0.02) \) indicated by Table V while the black one corresponds to an evolving \( G_{\text{eff}}(z) \) parametrization with a Planck15/ΛCDM background. The 20 earlier published data from the compilation are denoted as red points whereas the 20 latest published points are denoted as orange points (from Ref. [12]).

![Plot of \( f\sigma_8 \) as a function of the redshift z for the 18 growth rate dataset of Ref. [7].](image)

**FIG. 5.** Plot of \( f\sigma_8 \) as a function of the redshift \( z \) for the 18 growth rate dataset of Ref. [7]. The green dashed line describes the best fit of ΛCDM \( (\Omega_m = 0.21) \), the red one the best fit of Planck15/ΛCDM. The blue dot-dashed one corresponds to an evolving \( G_{\text{eff}}(z) \) parametrization with a Planck15/ΛCDM background.
FIG. 6. The $1\sigma - 3\sigma$ contours level in the parametric space $(w, \sigma_8, \Omega_m)$ using the collection of the 18 points presented in Ref. [7]. The blue contours describe the best fit of the data, the light green contours correspond to Planck15/ΛCDM while the light blue are constructed from the Planck data assuming a wCDM background.

FIG. 7. The $1\sigma - 4\sigma$ confidence contours in the parametric space $(\Omega_m - \sigma_8)$ using the full dataset of $f\sigma_8$ (Table V) from Ref. [12]. The blue contours correspond to the best fit obtained using the full compilation of $f\sigma_8$ data from Table V (left panel), the 20 early data (middle panel) and the 20 late data (right panel). The light green contours describe the contours for the Planck15/ΛCDM model.

parametrization described below so that the best fit to the $f\sigma_8$ data is obtained. In both Figs. 4 and 5 it is clear that the Planck15/ΛCDM prediction (red dashed line) is somewhat higher than the majority of the $f\sigma_8$ datapoints indicating that the growth rate is too large in this model. As shown in Figs. 4 and 5, this growth rate at low $z$ can be reduced (thus improving the fit to the data) by either decreasing $\Omega_m$ while maintaining GR and ΛCDM (green line) or by allowing for a $G_{\text{eff}}$ that evolves with redshift so that it is reduced at low $z$ (blue line).

The tension between a Planck15/ΛCDM background (GR) and the growth data of Fig. 5 is shown more clearly in Fig. 6 where we show the likelihood contours in two dimensional subspaces of the parameter space $\sigma_8, w, \Omega_m$. In each plot, the third parameter has a fixed value indicated by Planck15/ΛCDM.

The blue $1\sigma - 2\sigma$ parameter contours are obtained using the growth data of Fig. 5 while the red dot corresponds to the Planck15/ΛCDM. Clearly, there is a $2-3\sigma$ tension between the growth data contours and the best fit Planck15/ΛCDM parameter values. The Planck15 best fit wCDM parameter contours are also shown indicating that if the equation of state parameter $w$ is allowed to vary, the tension level between the growth data parameter $1\sigma - 2\sigma$ contours (blue contours) and the Planck15 [66] contours is significantly reduced.

An interesting question to address is the following: “How does the level of tension between the growth data and the Planck15/ΛCDM best fit parameter values evolve with time of publication?” or “Are early growth data at the same tension level with Planck15/ΛCDM as more recently published data?” This question has been addressed in Ref. [12] using the data of Table V and reviewed in what follows.

The evolution of the tension level is demonstrated in Fig. 7 where we show the $(\Omega_m - \sigma_8)$ ΛCDM ($w = -1$)...
The evolving tension between Planck15/ΛCDM parameter values and $f\sigma_8$ data may also be described by defining the residuals

$$\delta f\sigma_8(z_i) \equiv \frac{f\sigma_8(z_i)^{\text{data}} - f\sigma_8(z_i)^{\text{Planck15/ΛCDM}}}{\sigma_8}$$  \hspace{1cm} (2.13)$$

Using these residuals, the 20 point moving average residual may be defined as

$$\overline{f\sigma_8}_j \equiv \sum_{i=j-20}^{j} \frac{\delta f\sigma_8(z_i)}{20}$$  \hspace{1cm} (2.14)$$

These residual datapoints versus time of publication along with the corresponding 20 point moving average from Eq. (2.14), are shown in Fig. 8 (from Ref. [12]). There is a clear trend for reduced tension with Planck15/ΛCDM in more recently published data.

More recent $f\sigma_8$ datapoints tend to probe higher redshifts and thus they also tend to have higher errors. This is demonstrated in Fig. 9 where we show the 20 point moving average of the datapoint errorbars and redshifts versus time of publication. Both of them show an increasing trend especially for more recent data. At higher redshifts the universe is matter dominated and GR is approximately restored in most models and thus there is degeneracy in the predictions of different models. Thus, more recent datapoints that tend to probe higher redshifts have less constraining power on cosmological models.

In fact, the increase of the average redshift is a possible explanation for the reduced tension of the recent data.
with Planck15/ΛCDM due to the degeneracy that exists between models at high z. This degeneracy can also be observed in Fig. 4, where for high z the four curves coincide.

As discussed above, the tension between $f \sigma_8$ data and Planck15/ΛCDM may be reduced by either reducing $\Omega_m$ or by extending GR and allowing for an evolving $G_{\text{eff}}$. Such an evolving $G_{\text{eff}}(z)$ may be described by a parametrization of the form

$$G_{\text{eff}}(a, g_a, n) = \frac{1 + g_a(1-a)^n - g_a(1-a)^{2n}}{G_N}$$

$$= 1 + g_a \left(\frac{z}{1+z}\right)^n - g_a \left(\frac{z}{1+z}\right)^{2n} \quad (2.15)$$

where $g_a$ and $n$ are parameters to be fit. This parametrization for $n = 2$ has been used for the construction of the Figs. 6 and 7.

The parametrization (2.15) is well motivated and consistent with solar system experiments. The solar system constraints entail for the first derivative that [75]

$$\lim_{z \to 0} G'_{\text{eff}}(z) \simeq 0 \Rightarrow \left| \frac{1}{G_N} \frac{dG_{\text{eff}}(z)}{dz} \right|_{z=0} < 10^{-3} h^{-1} \quad (2.16)$$

This constraint implies that unless a Chameleon type mechanism [76] is present we must have $n \geq 2$ in the parametrization (2.15). The solar system experiments also leave the second derivative unconstrained since [75]

$$\left| \frac{1}{G_N} \frac{d^2G_{\text{eff}}(z)}{dz^2} \right|_{z=0} < 10^5 h^{-2} \quad (2.17)$$

Finally, at high redshifts, the Big Bang Nucleosynthesis provides the following additional constraint at the 1σ level [77]

$$|G_{\text{eff}}/G_N - 1| \leq 0.2 \quad (2.18)$$

which is also consistent with the parametrization (2.15).

In the context of this parametrization, which was presented in Refs. [7, 12], two extra parameters have been inserted ($g_a$ and $n$) describing the possible deviation from GR. Setting $\Omega_m = \Omega_{m, \text{Planck15}}$ and $\sigma_8 = \sigma_{8, \text{Planck15}}$, i.e. eliminating the tension with respect to $\Omega_m$ and $\sigma_8$, we see that the value of $\chi^2$ gets reduced significantly for $g_a \neq 0$ (using the data of Table VI in the Appendix A and setting $n = 2$ it was found that $g_a = -1.16 \pm 0.341$).

This corresponds to the blue curve of Fig. 5. The best fit values of $g_a$ for various values of $n$ are also shown in Table III and the corresponding forms of $G_{\text{eff}}(z)$ are shown in Fig. 10.

Notice that a significant reduction of the gravitational constant is required at low $z$ to fit the $f \sigma_8$ data with $\Omega_m = \Omega_{m, \text{Planck15}}$. Such a large reduction is inconsistent with other cosmological observations (e.g. CMB large scale power spectrum where the ISW effect dominates [7] or the distance moduli of SNIa when their dependence on $G_{\text{eff}}(z)$ is taken into account [78]) and therefore it is unlikely that the tension implies only the existence of evolving $G_{\text{eff}}(z)$. It is more likely that the tension is also (or only) due to other factors like a reduced value of $\Omega_m$ or systematic/statistical errors of the $f \sigma_8$ data.

The evolution of the tension level with time of publication of the $f \sigma_8$ data may also be seen by deriving the best fit value of the parameter $g_a$ assuming $n = 2$ and a Planck15/ΛCDM background while using 20 datapoint subsamples from Table V starting from the earliest to the latest subsample (from left to right in Fig. 11).

### Table III. The best fit values of $g_a$ along with the 1σ errors bars for various values of $n$.

| $n$  | $g_a$         |
|------|--------------|
| 0.343| $-1.200 \pm 1.025$ |
| 1    | $-0.944 \pm 0.253$ |
| 2    | $-1.156 \pm 0.341$ |
| 3    | $-1.534 \pm 0.453$ |
| 4    | $-2.006 \pm 0.538$ |
| 5    | $-2.542 \pm 0.689$ |
| 6    | $-3.110 \pm 0.771$ |

![Figure 10](image1.png)

**FIG. 10.** Plot of $G_{\text{eff}}/G_N$ as a function of $a$ considering the values of $n$ and $g_a$ from Table III (from Ref. [7])

![Figure 11](image2.png)

**FIG. 11.** The 1σ range of the parameter $g_a$ from the compilation of Table V. The red square point denotes the best fit of $g_a$ obtained from the compilation of Table V along with its error bar (from Ref. [12])

Clearly the absolute value of $g_a$ required to eliminate the tension with Planck15/ΛCDM decreases significantly for...
more recent data indicating also the reduced level of the tension for more recent data. The best fit value of $g_a$ for the full dataset of Table V ($g_a = -0.91 \pm 0.17$) is also indicated in Fig. 11 (red point).

II.2. Consistency of Reduced $G_{\text{eff}}(z)$ with Modified Gravity Theories

The best fit form of $G_{\text{eff}}(z)$ which appears to indicate reduced strength of gravity at low $z$ may lead to constraints on the fundamental parameters of modified theories of gravity. In fact, it may be shown that the simplest modified gravity theories including $f(R)$ and scalar-tensor theories tend to be inconsistent with a decreasing $G_{\text{eff}}(z)$ especially in a $\Lambda$CDM and in a phantom cosmological background [78].

In scalar-tensor gravity the action has the following form [41, 79]

$$S = \int d^4x \sqrt{-g} \left[ \frac{1}{2} F(\phi) R - \frac{1}{2} Z(\phi) g^{\mu\nu} \partial_\mu \phi \partial_\nu \phi - U(\phi) \right] + S_m,$$

(2.19)

where we have set $8\pi G = 1$. It is clear from Eq. (2.19) that the action depends on the scalar field $\phi$. Throughout this subsection we also consider $Z(\phi) = 1$. The line element for a flat Friedmann-Robertson-Walker (FRW) metric is

$$ds^2 = -dt^2 + a^2(t) \left[ dr^2 + r^2 (d\theta^2 + \sin^2 \theta \, d\phi^2) \right].$$  

(2.20)

By varying the action (2.19) with respect to the inverse metric, considering that the scalar field is homogenous and that the background is that of a perfect fluid, the dynamical equations of motion are of the following form

$$3FH^2 = \rho + \frac{1}{2} \dot{\phi}^2 - 3H \dot{\phi} + U \quad (2.21)$$

$$-2F \ddot{H} = (\rho + p) + \dot{\phi}^2 + \ddot{F} - H \dot{F} \quad (2.22)$$

Usually it is convenient to express Eqs. (2.21) and (2.22) in terms of the redshift $z$. We define the squared rescaled Hubble parameter as

$$q(z) \equiv E^2(z) = \frac{H^2(z)}{H_0^2} \quad (2.23)$$

After an additional rescaling of the potential ($U \to U \cdot H_0^2$) the equation of motion for $F(z)$ is [7, 41, 79]

$$F''(z) + \left[ \frac{q'(z)}{2q(z)} - \frac{2}{1+z} \right] F'(z) - \frac{1}{(1+z) q(z)} q'(z) F(z) + 3 \frac{1+z}{q(z)} \Omega_m = -\phi'(z)^2 \quad (2.24)$$

where the prime denotes from now on differentiation with respect to $z$.

In scalar-tensor theories the effective Newton’s constant may be expressed as [80]

$$G_{\text{eff}}(z)/G_N = \frac{1}{F(z)} \left[ \frac{F'(z)}{F(z)} + 2F_{\phi\phi}^2 \right] \quad (2.25)$$

where $G_N$ is the usual Newton’s constant in GR.

Using the best fit form of $G_{\text{eff}}(z)$ on the left hand side of Eq. (2.25), we may obtain the corresponding form of $F(z)$ and then use Eq. (2.24) with a $q(z)$ corresponding to Planck15/$\Lambda$CDM to find the corresponding form of $\phi'(z)^2$. Therefore the question that we want to address is: “Can the weakening effect of gravity indicated by the growth data be due to an underlying scalar-tensor theory?”

If this effect is due to an underlying scalar-tensor theory, then the new reconstructed scalar field must obey $\phi'(z)^2 > 0$ so that the theory is self consistent. However a decreasing $G_{\text{eff}}$ with redshift at low $z$ is inconsistent with $\phi'(z)^2 > 0$ as it is demonstrated numerically in the following Fig. 12.

From Fig. 12 it is clear that at low $z$, $\phi'(z)^2$ is negative. As a result this behaviour can not be supported by a self consistent scalar tensor theory.

![Plot of $\phi'(z)^2$ with redshift $z$ for various values of $n$](image)

The best fit values of $g_a$ shown in Table III were assumed with a Planck15/$\Lambda$CDM background.

This numerical result may also be demonstrated analytically. For a $w$CDM background we have

$$q(z) = \Omega_m (1+z)^3 + (1-\Omega_m) (1+z)^{3(1+w)} \quad (2.26)$$

For low $z$, we can expand the dynamical Newton’s con-
stant $G_{\text{eff}}(z)$, which up to the second order is of the form

$$G_{\text{eff}}(z) = G_{\text{eff}}(0) + G'_{\text{eff}}(0)z + \frac{z^2}{2}G''_{\text{eff}}(0)$$  \hspace{1cm} (2.27)

Applying the solar system constraints for the first derivative of $G_{\text{eff}}(z)$, i.e. Eq. (2.16), Eq. (2.27) is rewritten as

$$G_{\text{eff}}(z) = G_{\text{eff}}(0) + \frac{z^2}{2}G''_{\text{eff}}(0)$$  \hspace{1cm} (2.28)

It is straightforward to show that the constraint of Eq. (2.16) implies that $F'(0) \approx 0$. Therefore, setting $G_N = F(0) = 1$ and differentiating $G_{\text{eff}}(z)$ with respect to $z$ we obtain

$$G''_{\text{eff}}(0) = F''(0) \left(-1 + \frac{F'(0)}{\phi'(0)^2}\right)$$  \hspace{1cm} (2.29)

Furthermore, using Eq. (2.26) in Eq. (2.29) and setting $z = 0$, Eq. (2.30) is derived

$$3 - 3w(-1 + \Omega_{m}) - 3\Omega_{m} - \phi'(0)^2 - F''(0) = 0$$  \hspace{1cm} (2.30)

Substituting it to Eq. (2.29), the second derivative of $G_{\text{eff}}(z)$ takes the following form [78]

$$G''_{\text{eff}}(0) = 9(1+w)(-1+\Omega_{m})+\frac{9(1+w)^2(-1+\Omega_{m})^2}{\phi'(0)^2} + 2\phi'(0)^{2}$$  \hspace{1cm} (2.31)

Fixing a ΛCDM background, i.e. setting $w = -1$, Eq. (2.27) takes the form

$$G_{\text{eff}}(z) \approx G_{\text{eff}}(0) + \frac{1}{2}G''_{\text{eff}}(0)z^2 = G_{\text{eff}}(0) + \phi'^2(0)z^2$$  \hspace{1cm} (2.32)

which is always an increasing function of $z$ if we assume that the kinetic term of $\phi'(z)$ is always positive, an assumption which is crucial if we want to have a self-consistent theory. This is demonstrated in Fig. 13.

![FIG. 13. The second derivative of $G_{\text{eff}}$ in the parametric space ($\phi'(0) - w$), setting $\Omega_{m} = 0.3$. With blue we denote the parameter values in which $G''_{\text{eff}}(0) < 0$, while the brown regions describes $G''_{\text{eff}}(0) > 0$ which is achieved only for $w > -1$.](image)

From the above analysis the following result is extracted: If a ΛCDM background is assumed, any $G_{\text{eff}}(z)$ initially decreasing with $z$ leads to a reconstructed scalar-tensor negative kinetic term for some range of low $z$ [78].

Since the magnitude of the best fit parameter $g_a$ is relatively large, it is important to test its consistency with other observational probes and in particular with the low $l$ angular power spectrum of the CMB which is affected by the ISW effects and therefore can probe the strength of gravity. Using MGCAMB [81] the predicted CMB angular power spectrum may be derived assuming a Planck15/ΛCDM background cosmology and a $G_{\text{eff}}(z)$ parametrized by the ansatz (2.15). Such an analysis [7] indicates that for $l \gtrsim 80$ the CMB angular power spectrum remains practically unaffected by the evolving form of $G_{\text{eff}}(z)$ and thus the Planck15/ΛCDM best fit parameter value for $\Omega_{m}$ remains also practically unaffected. On the other hand, the low $l$ CMB spectrum is affected significantly due to the ISW effect. This is demonstrated in Fig. 14 [7] where the measured low $l$ values of the $C_l$ components are superposed with the theoretical prediction obtained for an evolving $G_{\text{eff}}(z)$ for various values of the parameters $g_a$ and $n$.

![FIG. 14. Top panel: The theoretically predicted CMB power spectra for the best fit parameter values $g_a$ for various values of $n$ are not consistent with the observed values of $C_l$s. Therefore, the tension between the Planck15/ΛCDM best fit parameter values and those indicated by the $f\sigma_8$ data can not be attributed solely to an evolving $G_{\text{eff}}(z)$. Bottom panel: The theoretically predicted low $l$ CMB power spectra for $n = 2$ and various values of $g_a$. Only values $|g_a| \lesssim 0.5$ are consistent with the observed CMB power spectrum (from Ref. [7]).](image)
Clearly the low l CMB power spectra impose strong constraints on the allowed values of $g_a$ and only the range $|g_a| \lesssim 0.5$ appears to be consistent with the observed values of low $l C'_{l}$s.

III. HINTS OF MODIFIED GRAVITY ON SUB MILLIMETRE SCALES

III.1. Review of the Washington Experiment

As discussed in the Introduction, the small scale frontier of the gravitational physics research is on sub-mm scales. This scale, however, is also connected with macro-physics and with dark energy. In fact, the dark energy scale may be written as

$$\lambda_{\text{de}} \equiv \sqrt{\hbar c/\rho_{\text{de}}} \approx 0.085\text{mm} \quad (3.1)$$

where it is assumed that $\Omega_{m} = 0.3$ and $H_0 = 70\text{kmsec}^{-1}\text{Mpc}^{-1}$. Hence, if the accelerating expansion is connected with modified gravity, it is natural to expect signatures of modified theories of gravity on scales $\lambda \approx 0.1\text{mm}$.

In the last decade a large number of experiments [1, 6, 24, 82] have imposed constraints on parametrizations which are extensions of Newton’s gravitational potential.

One of the most sensitive such experiments which have imposed the best constraints so far on the Yukawa parametrization discussed in the Introduction is the Washington experiment [23] which consists of three similar setups (Experiments I, II, III). It is based on a torsion-balance set-up shown in Fig. 15.

![FIG. 15. The Washington Experiment set-up (from Ref. [23])](image)

It consists of a fiber pendulum, 82 cm long, attached to a thin plate ring (yellow in Fig. 15) placed above a rotating plate with holes (blue in Fig. 15). The blue ring was an attractor which, like the pendulum ring, contained ten equally spaced holes with diameters about 9.5mm. The test-bodies used to measure the gravitational interaction were the holes exerting a torque of the form

$$N(\phi) = -\frac{\partial V(\phi)}{\partial \phi} \quad (3.2)$$

where $V(\phi)$ is the potential energy of the attractor ring-pendulum system when the holes of the ring twisted and formed an angle $\phi$ with respect to the pendulum.

The torque residuals that were measured in this experiment were fit assuming two different forms of a gravitational potential: A Yukawa parametrization of the form

$$V_{\text{eff}} = -GM/r(1 + \alpha e^{-mr}) \quad (3.3)$$

and a power law parametrization of the form [25]

$$V_{\text{eff}} = -GM/r \left(1 + \beta_k \left[\frac{1\text{mm}}{r}\right]^{k-1}\right) \quad (3.4)$$

This power law ansatz emerges naturally from some brane world models [51–54]. The torque residuals from the Newtonian torques are shown Fig. 16 along with the predicted residuals in the context of the above generalized gravitational potentials for specific parameter values.

![FIG. 16. The residuals of the datapoints used in the analysis of Ref. [23] (datapoints of Experiment I). The solid curve corresponds to residuals of the Yukawa parametrization of Eq. (3.3) for $a = 1$, $m^{-1} = \lambda = 250\mu\text{m}$, whereas the dot-dashed describes the power law of Eq. (3.4), where $k = 5$ and $\beta_k = 0.005$ (from Ref. [23]).](image)

III.2. Yukawa and Oscillating Newtonian Potential in $f(R)$ Theories

The simplest form of $f(R)$ theories is $f(R) = R + \frac{1}{6m^2}R^2 + ...$. In the weak field limit for $m^2 > 0$ the theory is self consistent and stable [45, 83] leading to Yukawa type correction to the gravitational potential. For $m^2 <$
0 the Yukawa type gravitational potential gets modified and the exponentially suppressed correction transforms to an oscillating correction of the form

\[ V_{\text{eff}} = -G \frac{M}{r} (1 + \alpha \cos(m \theta)) \]  

(3.5)

where \( \theta \) is a parameter.

In order to demonstrate the validity of the modified Newtonian potentials (3.3) and (3.5) in the context of \( f(R) \) theories, consider the generalized Einstein-Hilbert action \([49, 84, 85]\)

\[ S_R = \frac{1}{16\pi G} \int d^4 x \sqrt{-g} f(R) + S_{\text{matter}} \]  

(3.6)

where \( R \) is the Ricci scalar. It is easy to show that this action can be rewritten in the equivalent form \([86, 87]\)

\[ S_{\text{BD}} = \frac{1}{16\pi G} \int d^4 x \sqrt{-g} [f(\phi) + f(\phi)(R - \phi)] + S_{\text{matter}} \]  

(3.7)

By varying action (3.7) with respect to the scalar field \( \phi \) and assuming \( f_\phi \neq 0 \) we obtain

\[ \phi = R \]  

(3.8)

Setting \( f(R) = R + \frac{1}{6m^2} R^2 \), and defining \( \Phi \equiv 1 + \frac{1}{3m^2} \), then it is straightforward to rewrite Eq. (3.7) as

\[ S_{\text{BD}} = \frac{1}{16\pi G} \int d^4 x \sqrt{-g} \left[ \Phi R - \frac{3}{2} m^2 (\Phi - 1)^2 \right] + S_{\text{matter}} \]  

(3.9)

which is the action of a massive BD scalar field with \( \omega = 0 \).

Furthermore, varying Eq. (3.9) with respect to the inverse metric and the scalar field \( \Phi \), we obtain the dynamical equations

\[ \Phi \left( R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R \right) = 8\pi G T_{\mu\nu} + \nabla_\mu \partial_\nu \Phi - g_{\mu\nu} \Box \Phi - g_{\mu\nu} \frac{3}{2} m^2 (\Phi - 1)^2 \]  

(3.10)

\[ \Box \Phi = \frac{8\pi G}{3} T + m^2 ((\Phi - 1)^2 + (\Phi - 1) \Phi) \]  

(3.11)

respectively. Considering the weak gravitational field for a point mass of the form

\[ T_{\mu\nu} = \text{diag}(M\delta(\vec{r}), 0, 0, 0) \]  

(3.12)

the quantities \( \Phi \) and \( g_{\mu\nu} \) can be expanded as

\[ \Phi = 1 + \varphi \]  

(3.13)

\[ g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu} \]  

(3.14)

Substituting Eqs. (3.13) and (3.14) in the dynamical equations and keeping terms up to linear order we obtain the perturbative dynamical equations around the vacuum solution as

\[ (\Box - m^2) \varphi = -\frac{8\pi G}{3} M \delta(\vec{r}) \]  

(3.15)

\[ -\frac{1}{2} \left[ \Box \left( h_{\mu\nu} - \eta_{\mu\nu} \frac{h}{2} \right) \right] = 8\pi G T_{\mu\nu} + \partial_\mu \partial_\nu \varphi - \eta_{\mu\nu} \Box \varphi \]  

(3.16)

where \( h = h_{\mu\nu} \). For static configurations, these equations convert to

\[ \nabla^2 \varphi - m^2 \varphi = -\frac{8\pi G}{3} M \delta(\vec{r}) \]  

(3.17)

\[ \nabla^2 h_{00} - \nabla^2 \varphi = -8\pi G M \delta(\vec{r}) \]  

(3.18)

\[ \nabla^2 h_{ij} - \delta_{ij} \nabla^2 \varphi = -8\pi G M \delta(\vec{r}) \delta_{ij} \]  

(3.19)

which lead to the following weak field solution for \( \varphi \) and \( h_{\mu\nu} \):

\[ \varphi = \frac{2GM}{3r} e^{-mr} \]  

(3.20)

\[ h_{00} = \frac{2GM}{r} \left( 1 + \frac{1}{3} e^{-mr} \right) \]  

(3.21)

\[ h_{ij} = \frac{2GM}{r} \delta_{ij} \left( 1 - \frac{1}{3} e^{-mr} \right) \]  

(3.22)

Thus, the Yukawa generalization for the gravitational potential of a point mass is obtained under the assumption \( m^2 > 0 \) while for \( m^2 < 0 \) an oscillating solution is obtained

\[ \varphi = \frac{2GM}{r} \left( 1 + \frac{1}{3} Cos(|m|r + \theta) \right) \]  

(3.23)

where \( \theta \) is a parameter that describes an arbitrary phase. This solution leads to an oscillating gravitational potential of the form

\[ V_{\text{eff}} = -\frac{h_{00}}{2} = -\frac{GM}{r} \left( 1 + \frac{1}{3} Cos(|m|r + \theta) \right) \]  

(3.24)

In order to study the stability of these solutions, we allow for a time-dependent perturbation \( \delta \varphi \), \( \varphi = \varphi_0(t) + \)
\( \delta \varphi(r, t) \), where \( \varphi_0 \) is the unperturbed part of the solution. The perturbed part \( \delta \varphi(r, t) \) satisfies the following equation

\[
-\ddot{\delta \varphi} + \nabla^2 \delta \varphi - m^2 \delta \varphi = 0 \tag{3.25}
\]

which for positive \( m^2 \) is the usual Klein-Gordon equation and leads to a wavelike stable solution (even if we consider higher order terms in the initial Lagrangian [88]). However if a negative \( m^2 \) is considered, Eq. (3.17) leads to instabilities and exponentially increasing perturbations [49, 58, 59].

Thus, \( f(R) \) theories are unable to predict oscillatory behavior of the Newtonian potential without the presence of tachyonic instabilities unless higher-order terms in the action or a nontrivial background energy momentum tensor are considered. As discussed in the following, however, such oscillatory behavior is more natural in the context of nonlocal gravity theories.

### III.3. Fit of Oscillating Parametrization on the Washington Experiment Data

In the context of the Washington experiment [23] the data were reported as differences between the measured torques for a Yukawa type potential and the expected torques from a Newtonian potential. These differences (residuals) have been reported in three different experiments denoted as Experiment I, II and III respectively in Ref. [25]. Each experiment involved variations of the attractor and detector thickness in such a way that the systematic errors were minimized.

Therefore a total of \( N = 87 \) residuals points [49] were shown [1, 24, 49, 82, 89] along with the residual curves. These \( 87 \) residual points could be either statistical fluctuations around a Newtonian gravitational potential or could emerge from generalized gravitational potentials, e.g. Eq. (3.3), deviating from the Newtonian potential.

In Ref. [49] the \( 87 \) residual datapoints \( \delta \tau = \tau - \tau_N \) from the three experiments were fit to the following parametrizations:

\[
\begin{align*}
\delta \tau_1(\alpha', m', r) &= \alpha' \\
\delta \tau_2(\alpha', m', r) &= \alpha'e^{-m'r} \\
\delta \tau_3(\alpha', m', r) &= \alpha' \cos(m'r + \frac{3\pi}{4})
\end{align*} \tag{3.26-3.28}
\]

i.e. an offset Newtonian, a Yukawa and an oscillating ansatz where \( \alpha' \) and \( m' \) are the parameters that were fitted. The parameter \( \theta' \) was fixed in \( \theta' = \frac{2\pi}{3} \) since it provided the best fit compared to other selected phases. The primes were used in order to avoid confusion with the fundamental parameters of Eq. (3.3). It is important to note that the connection between the dotted and undotted parameters \( \alpha, m \) and \( \theta \) is not obvious unless specific details of the apparatus of the experiment are known. This is discussed in detail in Ref. [49].

The parametrizations (3.26)-(3.28) were used in Ref. [49] to minimize \( \chi^2(\alpha', m') \) which was defined the usual way as

\[
\chi^2(\alpha', m') = \sum_{j=1}^{N} \frac{(\delta \tau(j) - \delta \tau_i(\alpha', m', r_j))^2}{\sigma_j^2} \tag{3.29}
\]

where \( j \) referred to the \( j^{th} \) residual of the experiment, \( i \) to the selected parametrization (\( i \) runs from 1 to 3) and \( N = 87 \). In the following Table IV the best fit values of \( \chi^2 \) for each parametrization are shown.

| Parametrization | \( \chi^2 \) |
|-----------------|-------------|
| \( \delta \tau = \alpha'e^{-m'r} \) | 85.5 |
| \( \delta \tau = \alpha' \cos(m'r + \frac{3\pi}{4}) \) | 70.7 |
| \( \delta \tau = \alpha' \) | 85.4 |

Table IV indicates that the value of \( \chi^2 \) for the oscillating parametrization, i.e. Eq. (3.28), is significantly smaller (\( \chi^2 \approx -15 \)) compared with the other two parametrizations. For the oscillating parametrization, the best fit value of the spatial frequency \( m \) was obtained as \( m \approx 65 mm^{-1} \) corresponding to a wavelength \( \lambda = \frac{2\pi}{m} \approx 0.1 mm \). The corresponding \( 1\sigma \) and \( 2\sigma \) contours in the parametric space \( (\alpha', m') \) are shown in Fig. 17 for the oscillating parametrization and in Fig. 18 for the Yukawa parametrization which does not provide a better fit than the offset Newtonian potential.

![FIG. 17. The 1\sigma and 2\sigma contours in the parametric space (\( \alpha', m' \)) for the oscillating parametrization (3.28). The quality of fit is significantly improved compared to the Newtonian ansatz. (from Ref. [49])](image)
(0.004 fN·m, 65 mm⁻¹) of the oscillating parametrization is more than 3σ for a two-parameter parametrization. However, the existence of multiple minima in the parameter space (α', m') with similar depths reduces the statistical significance of this signal. The existence of such additional minima is demonstrated in Fig. 20 where we show χ²(m') for which each m' minimization with respect to α' has been performed. For example the minima corresponding to m' ≃ 195 mm⁻¹ and m' ≃ 202 mm⁻¹ have comparable depths with the main minimum at m' ≃ 65 mm⁻¹ but they are effectively higher harmonics of this deepest minimum.

In order to estimate the significance of the deepest χ² minimum at m ≃ 65 mm⁻¹ Monte Carlo simulations were performed [49] (100 realizations) of the 87 data from the Washington experiment assuming a Gaussian distribution of the residuals around a Newtonian potential (δτ = 0) and standard deviation equal to the errorbars of the residuals. Using each Monte-Carlo realization χ²(α', m', θ) was minimized for m' in the range 0–100 mm⁻¹ using the oscillating parametrization (3.28). The depth δχ² ≡ χ²(α' = 0) − χ²_{min}, corresponding to each simulated Monte-Carlo dataset was then compared to the corresponding depth δχ² ≃ 15 of the real data. About 10% of the simulated Newtonian data had a larger depth δχ² than the real data (Fig. 21). Thus, the probability that the oscillating signal in the Washington experiment data is a statistical fluctuation is about 10%.

Therefore, there is an oscillation signal in the data whose origin could be either statistical, systematic or physical. In the later case, it is important to identify physical theories that are consistent with such an oscillating signal since as discussed above such a signal is not consistent with the simplest modified gravity theories as it is associated with instabilities. As shown in the next section however a class of theories involving infinite derivatives in the Lagrangian (nonlocal theories of gravity) naturally predict the existence of such oscillations on sub-mm scales.
III.4. Oscillating Newtonian Potential from Non-Local Gravity Theories

The Lagrangian of non-local gravity theories may be written as [90]

\[ L_{IDG} = \frac{1}{8\pi G} \sqrt{-g} \left[ R + \alpha \left( R F_1(\Box) R + R^\mu\nu F_2(\Box) R_{\mu\nu} + R^\mu\nu\rho\sigma F_3(\Box) R_{\mu\nu\rho\sigma} \right) \right] \] (3.30)

where

\[ F_i(\Box) = \sum_{n=0}^{\infty} f_{i,n} \left( \frac{\Box}{M^2} \right)^n \] (3.31)

Such a Lagrangian involving infinite higher derivative terms may offer the solution to some basic problems of GR, such as the behaviour of GR at small scales (GR predicts singularities at small scales). A related issue is the existence of unrenormalisable UV divergences in GR [91]. These divergences can be alleviated if an Einstein-Hilbert action with higher derivative terms is considered [92]. These higher terms, however, are related with instabilities at the quantum level since the gravitational propagator imposed from these theories has a spin 2 component, which leads to an unhealthy classical vacuum theory (unstable). These extra problems can be resolved if we take infinite number of higher derivatives in the action, i.e. making the theory nonlocal, which modifies appropriately the gravitational propagator [90]. These infinite derivatives are usually condensed in an exponential term for the avoidance of introducing new poles. [60, 61, 93, 94]

Thus nonlocal gravity theories provide the following advantages:

- They soften the UV divergences present at the quantum level along with the singularities of Big Bang and Black Holes. [90, 95].
- They modify the Newtonian potential at the scale of nonlocality \( m \), removing the divergences of the Newtonian potential at \( r = 0 \), while in many cases they predict the existence decaying spatial oscillations of the gravitational potential on scales smaller than the nonlocality scale \( m \). [57–59, 96]
- They are consistent with cosmological observations as they can provide a mechanism producing the observed accelerating expansion of the universe. [97–100]

In the nonlocal theories the modified Newtonian potential is [57]

\[ V_{eff}(r) = -\frac{GM}{r} f(r, m) \] (3.32)

where

\[ f(r, m) = \frac{1}{\pi} \int_{-\infty}^{+\infty} dk \frac{\sin(k r) e^{-\tau(k, m)}}{k} \] (3.33)

Setting \( \tau \) as

\[ \tau = \frac{k^{2n}}{m^{2n}} \] (3.34)

which is a typical form for \( \tau \) and \( n = 1 \), then Eq.(3.33) is rewritten as

\[ f(r) = Er f(m \frac{r}{2}) \] (3.35)

which for \( \bar{r} \equiv m \ r << 1 \) takes a linear form and for \( \bar{r} \gg 1 \) it approaches unity. \( f(r) \) is shown in Fig. 22, for two different values of \( n \)

![Fig. 22. The plot of \( f(r) \) for \( n = 1 \) and \( n = 20 \) denoted with dashed green and red line respectively along with the fit of (3.36),(3.37) (from Ref. [49])](image-url)

For large \( n \), \( f(r) \) can be approximated very well by the following functions [49]

\[ f(r) = \alpha_1 \bar{r} \quad 0 < \bar{r} < 1 \] (3.36)

\[ f(r) = 1 + \alpha_2 \frac{\cos(\bar{r} + \theta)}{\bar{r}} \quad 1 < \bar{r} \] (3.37)

where \( \alpha_1 = 0.544, \alpha_2 = 0.572, \theta = 0.885\pi \).

Such models are interesting since not only they are free from UV divergences and singularities but the have
a well-defined Newtonian limit. Therefore, this type of oscillating behavior may have been the origin of the oscillating signal in the data of the Washington experiment discussed in the previous sub-section.

IV. CONCLUSIONS

In the present brief review, we discussed experimental and observational data on the smallest and the largest scales where gravity can be directly probed with current technology. We demonstrated that current data indicate the presence of hints of modified gravity in both the cosmological and the sub-millimeter scales. Concerning the cosmological scales, we showed that the best fit Planck15/ΛCDM σ8 − Ωm parameter values are more than 3σ away from the corresponding best fit parameter values obtained using the latest RSD growth rate data fσ8, assuming a Planck15/ΛCDM background cosmology [7]. This tension has also been observed from various weak gravitational lensing analyses [13, 67].

The tension can be reduced either by reducing the value of Ωm in the context of a ΛCDM cosmology or by allowing for an evolving Newton’s constant G eff(z) leading to weaker gravity at z < 1. In particular we showed that a Planck15/ΛCDM H(z) cosmological background with a well motivated form of G eff(z) = 1 + g a (z + Ωm) n − g a (z + Ωm) 2n , which is a decreasing function of z (for ga < 0), can be significantly more consistent with the full dataset of fσ8 from Ref. [12]. This type of evolution cannot be reproduced in scalar-tensor theories with a ΛCDM background, since it leads to negative kinetic term of the scalar field φ(z). One possible way to reproduce a decreasing G eff(z) in scalar-tensor theories would be to assume for a wCDM expansion background with w > −1. For example we demonstrated that for a wCDM background, G eff(z) can be a decreasing function of z in the context of scalar-tensor theories.

Finally on sub-mm scales higher derivative gravity models generically predict sub-mm spatial oscillations of the gravitational potential. Hints for such oscillations have been demonstrated to exist in the Washington torsion-balance experiment.

Thus we have demonstrated the existence of hints for deviations from GR on both the largest scales where a G eff(z < 1) < G N is favored and on the smallest probed scales (sub-mm) where an oscillating G eff(r) is favored. It is therefore important to clarify if these hints are due to systematic or statistical effects or they constitute early manifestation for new physics. This clarification may be achieved by considering new larger datasets focusing on the redshifts/scales where these hints appear (z ≃ 0.3 and r ≃ 80μm).

ACKNOWLEDGEMENTS

This research is co-financed by Greece and the European Union (European Social Fund-ESF) through the Operational Programme “Human Resources Development, Education and Lifelong Learning” in the context of the project “Strengthening Human Resources Research Potential via Doctorate Research” (MIS-5000432), implemented by the State Scholarships Foundation (IKY)

Appendix A: Data Used in the Analysis

| Index | Dataset | fσ8(z) | Refs. | Year | Fiducial Cosmology |
|-------|---------|--------|-------|------|---------------------|
| 1     | SDSS-LRG | 0.35   | 0.440 ± 0.050 | [101] | 30 October 2006 (Ωm, ΩK, σ8) = (0.25, 0, 0.756) [102] |
| 2     | VVDS    | 0.77   | 0.490 ± 0.18  | [101] | 6 October 2009 (Ωm, ΩK, σ8) = (0.25, 0, 0.78) |
| 3     | 2dFGRS  | 0.17   | 0.510 ± 0.060 | [101] | 6 October 2009 (Ωm, ΩK) = (0.3, 0, 0.9) |
| 4     | 2MRS    | 0.02   | 0.314 ± 0.048 | [103], [104] | 13 November 2010 (Ωm, ΩK, σ8) = (0.266, 0, 0.65) |
| 5     | SDSS-LRG-200 | 0.25 | 0.3512 ± 0.0583 | [106] | 20 October 2011 (Ωm, ΩK, σ8) = (0.3, 0, 0.814) |
| 6     | SDSS-LRG-200 | 0.37 | 0.4602 ± 0.0378 | [106] | 9 December 2011 (Ωm, ΩK, σ8) = (0.276, 0, 0.8) |
| 7     | SDSS-LRG-60 | 0.25 | 0.3665 ± 0.0601 | [106] | 9 December 2011 (Ωm, ΩK, σ8) = (0.276, 0, 0.8) |
| 8     | SDSS-LRG-60 | 0.37 | 0.4031 ± 0.0586 | [106] | 9 December 2011 (Ωm, ΩK, σ8) = (0.276, 0, 0.8) |
| 9     | WiggleZ | 0.44   | 0.413 ± 0.080  | [68] | 12 June 2012 (Ωm, ΩK, σ8) = (0.27, 0, 0.71) |
| 10    | WiggleZ | 0.60   | 0.390 ± 0.063  | [68] | 12 June 2012 (Ωm, ΩK, σ8) = (0.27, 0, 0.71) |
| 11    | WiggleZ | 0.73   | 0.437 ± 0.072  | [68] | 12 June 2012 (Ωm, ΩK, σ8) = (0.27, 0, 0.71) |
| 12    | 6dFGS   | 0.067  | 0.423 ± 0.055  | [107] | 4 July 2012 (Ωm, ΩK, σ8) = (0.27, 0.76) |
| 13    | SDSS-BOSS | 0.30 | 0.407 ± 0.055  | [108] | 11 August 2012 (Ωm, ΩK, σ8) = (0.25, 0, 0.804) |
| 14    | SDSS-BOSS | 0.40 | 0.419 ± 0.041  | [108] | 11 August 2012 (Ωm, ΩK, σ8) = (0.25, 0, 0.804) |
| 15    | SDSS-BOSS | 0.50 | 0.427 ± 0.043  | [108] | 11 August 2012 (Ωm, ΩK, σ8) = (0.25, 0, 0.804) |
| 16    | SDSS-BOSS | 0.60 | 0.433 ± 0.067  | [108] | 11 August 2012 (Ωm, ΩK, σ8) = (0.25, 0, 0.804) |
| 17    | Vipers  | 0.80   | 0.470 ± 0.080  | [109] | 9 July 2013 (Ωm, ΩK, σ8) = (0.25, 0.82) |
| 18    | SDSS-DR7-LRG | 0.35 | 0.429 ± 0.089  | [110] | 8 August 2013 (Ωm, ΩK, σ8) = (0.25, 0.809) [111] |
| 19    | GAMA    | 0.18   | 0.360 ± 0.090  | [112] | 22 September 2013 (Ωm, ΩK, σ8) = (0.27, 0, 0.8) |
| 20    | GAMA    | 0.38   | 0.440 ± 0.060  | [112] | 22 September 2013 (Ωm, ΩK, σ8) = (0.27, 0, 0.8) |
| 21    | BOSS-LOWZ | 0.32 | 0.384 ± 0.095  | [113] | 17 December 2013 (Ωm, ΩK, σ8) = (0.274, 0, 0.8) |
| Index | Dataset | \(z\) | \(f\sigma_8(z)\) | Refs. | Year | Fiducial Cosmology |
|-------|---------|-------|----------------|-------|------|-------------------|
| 1     | 6dFGS+SnIa | 0.02  | 0.428 ± 0.0463 | [124] | 2016 | (\(\Omega_m, h, \sigma_8\)) = (0.3, 0.683, 0.8) |
| 2     | SnIa+LRAS | 0.02  | 0.398 ± 0.065  | [105], [104] | 2011 | (\(\Omega_m, \Omega_K\)) = (0.3, 0) |
| 3     | 2MASS    | 0.02  | 0.314 ± 0.048  | [103], [104] | 2010 | (\(\Omega_m, \Omega_K\)) = (0.266, 0) |
| 4     | SDSS-veloc | 0.10  | 0.370 ± 0.130  | [116] | 2015 | (\(\Omega_m, \Omega_K\)) = (0.3, 0) |
| 5     | SDSS-MGS  | 0.15  | 0.490 ± 0.145  | [115] | 2014 | (\(\Omega_m, h, \sigma_8\)) = (0.31, 0.67, 0.83) |
| 6     | 2dFGRS   | 0.17  | 0.510 ± 0.060  | [101] | 2009 | (\(\Omega_m, \Omega_K\)) = (0.3, 0) |
| 7     | GAMA     | 0.18  | 0.360 ± 0.090  | [112] | 2013 | (\(\Omega_m, \Omega_K\)) = (0.27, 0) |
| 8     | GAMA     | 0.38  | 0.440 ± 0.060  | [112] | 2013 | (\(\Omega_m, h, \sigma_8\)) = (0.30715, 0.6777, 0.8288) |
| 9     | SDSS-LRG-200 | 0.25  | 0.3512 ± 0.0583 | [106] | 2011 | (\(\Omega_m, \Omega_K\)) = (0.25, 0) |
| 10    | SDSS-LRG-200 | 0.37  | 0.4692 ± 0.0378 | [106] | 2011 | (\(\Omega_m, h, \sigma_8\)) = (0.30715, 0.6777, 0.8288) |
| 11    | BOSS-LOWZ | 0.32  | 0.384 ± 0.095  | [113] | 2013 | (\(\Omega_m, \Omega_K\)) = (0.274, 0) |
| 12    | SDSS-CMASH | 0.50  | 0.488 ± 0.060  | [120] | 2013 | (\(\Omega_m, h, \sigma_8\)) = (0.30715, 0.6777, 0.8288) |
| 13    | WigleZ   | 0.44  | 0.413 ± 0.080  | [68]  | 2012 | (\(\Omega_m, h\)) = (0.27, 0.71) |
| 14    | WigleZ   | 0.60  | 0.390 ± 0.063  | [68]  | 2012 | (\(\Omega_m, h\)) = (0.27, 0.71) |

TABLE VI: A compilation of robust and independent \(f\sigma_8(z)\) measurements from different surveys. In the columns we show in ascending order with respect to redshift, the name and year of the survey that made the measurement, the redshift and value of \(f\sigma_8(z)\) and the corresponding reference and fiducial cosmology.
|   |   |   |   |   |   |
|---|---|---|---|---|---|
| 15 | WiggleZ | 0.73 | 0.437 ± 0.072 | 68 | 2012 |
| 16 | Vipers PDR-2 | 0.60 | 0.550 ± 0.120 | 126 | 2016 |
| 17 | Vipers PDR-2 | 0.86 | 0.400 ± 0.110 | 126 | 2016 |
| 18 | FastSound | 1.40 | 0.482 ± 0.116 | 118 | 2015 |

(\(\Omega_m, \Omega_b = (0.3, 0.045)\))

(\(\Omega_m, \Omega_K = (0.270, 0)\))
