Communications with decode-and-forward relays in mesh networks

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Abstract

We consider mesh networks composed of groups of relaying nodes which operate in decode-and-forward mode, where each node from a group relays information to all the nodes in the next group. We study these networks in two setups, one where the nodes have complete channel state information from the nodes that transmit to them, and another when they only have the statistics of the channel. We derive recursive expressions for the probabilities of errors of the nodes and present several implementations of detectors used in these networks. We compare the mesh networks with multihop networks, the latter being formed by a set of parallel sections of multiple relaying nodes. We demonstrate with numerous simulations that there are significant improvements in performance of mesh over multihop networks in various scenarios.

1 Introduction

An old concept in radio communications, known as relaying, has been in the center of interest of various studies in communications [Ephremides, 2002, Goldsmith and Wicker, 2002, Ribeiro et al., 2008]. To a good extent, this interest has been driven by commercial applications and by the ability of wireless networks to exploit relaying so that they have reduced energy consumption and thereby increased lifetime. In general, relaying is used to provide for improved error performance and capacity [Liu et al., 2009].

In networks with relaying, nodes cooperate in moving information from source to destination [Liu et al., 2009]. In the study of these networks, the important notion of cooperative diversity was introduced in [A. Sendonaris et al., 1998], and then extended in [A. Sendonaris et al., 2003a] and [J. N. Laneman et al., 2004]. A class of wireless networks that are formed by a set of parallel sections of multiple relaying nodes was thoroughly studied in the literature [Lin et al., 2006, T. Wang et al., 2007]. They are known as multi-branch multi-hop networks, and they can reach a full diversity order [T. Wang et al., 2007]. However, unlike mesh networks, these networks do not allow for full connectivity among nodes.

Optimum detection in a network demands knowledge of all its channels. There is some work on this subject in the literature, and especially on particular network topologies. For example, in a recent paper, the problem of detection by a set of sensors and the communication of their information were
considered together [B. Chen et al., 2004]. Various fusion rules were proposed that correspond to different scenarios in terms of what is known for deriving the rules. The study was performed for multi-branch multi-hop topologies, but not for general mesh networks.

An analysis of the maximum likelihood (ML) detector in cooperative multibranched multi-hop networks was presented in [D. Chen and J. N. Laneman, 2006]. A closed form expression was obtained for the bit error rate of a network with one relay. The difficulties with ML detection in DF networks have led to several suboptimal alternatives. In [A. Sendonaris et al., 2003b], a variant of the maximal-ratio combiner (MRC), the $\lambda$-MRC, was introduced to combine signals from several branches. In order to explicitly obtain a full diversity order, the cooperation MRC (CMRC) was proposed in [T. Wang et al., 2007]. For this detector, the weight of the channel of relay-destination was chosen to maximize the equivalent SNR for the channel. In that paper the multibranch multi-hop topology was considered. It is interesting to mention that a multi-node cooperative network can be viewed as a virtual MIMO system [Liu et al., 2009, A. K. Sadek et al., 2007]. In successive phases the relays combine the received signals from previous relays and the source using the MRC criterion. The system is not optimal and it can be adapted to a mesh network.

Most of the above solutions assume perfect knowledge of the channel state information (CSI). However, having such knowledge can be expensive. For instance, in a sensor network the battery consumption can be high if the nodes have to inform about their CSI in a variable environment. In other circumstances it could be impossible to have complete knowledge of the CSI. For these cases it is important to develop schemes that operate with less information about the channel: instead of full information one has available only the channel statistics. There is some previous work in the literature on detection using channel statistics. In [A. Bravo et al., 2006], the average SNR is used as a weight for combining signals in a multi-branch multi-hop relay network. In [Lin et al., 2006], optimum ML fusion rules are derived for a joint sensor-communication problem with a multi-branch multi-hop topology.

Data traffic in a sensor network is usually small. We take this statement as true, although it is not always the case, and we assume that most of the available channels can be used for cooperation and we do not put a limit to this number. This is a usual assumption in the literature, see [T. Wang et al., 2007]. A consequence of this assumption is that the spectral efficiency is reduced. In the literature, there are examples of simple networks where the error probability and the spectral efficiency are improved at the same time [A. Sendonaris et al., 2003b]. However, it is not straightforward to extend these results to arbitrary networks. In this paper, we look for the objective of minimizing the transmitted power, or, what is related to it, minimizing the error probability, with no restrictions in the number of available channels.

The cooperative detection problems mentioned above were posed for multibranched and/or multi-hop networks. For them, the optimal ML solution was abandoned in favor of tractable solutions. In this paper we present and analyze the optimal maximum a posteriori (MAP) detector for cooperative mesh networks, and we provide explicit analytical solution for a general topology. We consider optimal detection in two cases related to the channels in the network. One is when the CSI is available and the other when the channel statistics are only known. The first case was studied in part by the authors in [A. Bravo and P. M. Djurić, 2009]. The relaying in the network is DF with uncoded and symbol-by-symbol demodulators. In summary, the two main contributions in our paper are the following: (a) we derive optimal MAP detectors for the nodes in mesh networks.
and (b) we propose several implementations of the detectors for both, the case of known CSI and scenarios of known channel statistics. In the paper we do not consider the problems of routing and protocols that may arise, for example, in ad hoc mesh networks. How they relate to the proposed schemes here will be addressed elsewhere.

The paper is organized as follows. In Section 2 we formulate the problem. In Section 3 we provide the general MAP solution that holds for known CSI as well as for known channel statistics. In Sections 4 and 5 we present detectors in the first and higher groups of nodes, respectively, and provide the specific solutions for the above two scenarios. Simulation results that demonstrate the performance of the mesh networks and how they compare with the multibranch and multi-hop networks are shown in Section 6. In the last Section 7 we have some concluding remarks about our findings.

2 Problem statement

We observe a mesh network where its nodes are grouped into relay groups and where the notation \( R_i^{(k)} \) signifies the \( i \)th node of the \( k \)th group. We denote the source by \( R_1^{(0)} \) and the destination by \( R_1^{(K+1)} \), the superscript \( K+1 \) implying that a message is relayed \( K \) times on its way from the source to the destination. For example, \( R_4^{(3)} \) refers to the fourth node from the third group of nodes. We use the symbol \( n^{(k)} \) to denote the total number of relay nodes in group \( k \). In Fig. 1 we show a drawing of a general mesh network of our interest.

![Figure 1: A general mesh network with \( K \) groups of relaying nodes.](image)

In this paper, for comparison purposes, we also work with multi-branch multihop networks. Multi-branch multihop networks can be obtained from mesh networks when we remove connections between nodes so that each node in a group is connected to only one node from the previous group, as in Fig. 2. Clearly, a wireless network formed by a set of interconnected nodes can operate as a mesh network or as a multi-branch multihop network. In mesh networks, a node receives and
processes information from more than one node from a previous relay group, whereas in a multihop network, it does only from one node.

Figure 2: A general multi-branch multi-hop network with $K$ groups of relaying nodes.

We consider binary modulations only (the extension to other modulations is straightforward), and we assume phase coherent reception. The received signal by node $R^{(k)}_j$ and transmitted by node $R^{(k-1)}_i$, where $k \geq 1$, is denoted by

$$y^{(k)}_{ij} = h^{(k)}_{ij} x^{(k-1)}_i + w^{(k)}_{ij},$$

where $x^{(k-1)}_i \in \{-1, 1\}$ is the transmitted symbol, $h^{(k)}_{ij}$ is the channel fading between $R^{(k-1)}_i$ and $R^{(k)}_j$, and $w^{(k)}_{ij}$ is the channel noise. In this formulation, $h^{(k)}_{ij} > 0$ is the real-valued fading envelope of the channel. We denote the transmitted symbol by the source with $x$ and not $x^{(0)}_1$ so that we avoid superfluous notation. The noise is modeled as a zero mean Gaussian random variable with variance $\sigma^2$, i.e., $w^{(k)}_{ij} \sim \mathcal{N}(0, \sigma^2)$, and it is considered identically distributed in all the channels.

We assume a simple protocol where every node has access to an orthogonal channel. This can be implemented with time division, frequency division or hybrid multiplexing. This assumption could be considered unrealistic, but with a little traffic offered in a network, as is frequently the case in sensor networks, most of the available channels can be used for cooperation with the node source of information and the time/frequency channels can be reused by distant nodes. We also point out that in a real system the number of available channels is not necessarily small. For example, the Zigbee standard for sensor networks [ZigBee Alliance, 2009] envisages a hybrid access for its next release in order to increase the number of channels and, thus, reduce message collisions.

We distinguish two scenarios:

1. all the channels in the network are completely known (CSI is available to all the nodes), and
2. only the statistics of the channels are known, where it is assumed that the channels follow the
Rayleigh distribution, i.e., for the fading of a channel $h$ we write
\[ h \sim \frac{2h}{\sigma_h^2} e^{-\frac{h^2}{\sigma_h^2}}, \quad h \geq 0. \tag{2} \]

The signal-to-noise ratio (SNR) in the former case is defined by
\[ \gamma = \frac{h^2}{\sigma^2}, \tag{3} \]
and in the latter, by
\[ \gamma = \frac{\sigma_h^2}{\sigma^2}. \tag{4} \]

Obviously, we can have a combination of the two scenarios, but we will not consider it here.

Given the available information, the objective is to determine the optimal decisions at each node and study the performance of the systems under the two scenarios.

## 3 General solution

In this section we derive the decision rule of the nodes under the assumptions from the previous section. We denote the probability of correct decision of node $i$ in the $k$th group by $P_i^{(k)} = P(x_i^{(k)} = x|x)$ and the probability of error by $P_i^{(k-1)} = P(x_i^{(k-1)} = -x|x)$. In the sequel, we assume $P(x = 1) = P(x = -1)$.

Clearly, the nodes $R_j^{(1)}$, $j = 1, 2, \cdots, n^{(1)}$ make their decisions according to
\[ L_j^{(1)} = \frac{P(x = 1|y_{ij}^{(1)})}{P(x = -1|y_{ij}^{(1)})} = \frac{f(y_{ij}^{(1)}|x = 1)}{f(y_{ij}^{(1)}|x = -1)}, \tag{5} \]
where $j = 1, 2, \cdots, n^{(1)}$, and $f(y_{ij}^{(1)}|x = s)$ is the likelihood of $x = s$, with $s$ being 1 or -1.

The following groups of nodes will receive in general more than one signal and a decision is made by using all of them. Let the signals received by node $R_j^{(k)}$ be denoted by the vector $y_j^{(k)} = [y_{ij}^{(k)}, y_{2j}^{(k)}, \cdots, y_{n^{(k-1)}j}^{(k)}]^\top$, and let the decisions of the nodes in the previous group, be given by $x^{(k-1)} = [x_1^{(k-1)}, x_2^{(k-1)}, \cdots, x_{n^{(k-1)}}^{(k-1)}]^\top$. Then we have
\[ L_j^{(k)} = \frac{P(x = 1|y_j^{(k)})}{P(x = -1|y_j^{(k)})} = \frac{f(y_j^{(k)}|x = 1)}{f(y_j^{(k)}|x = -1)} \]
\[ = \frac{\sum_{x^{(k-1)}} f(y_j^{(k)}|x^{(k-1)}) P(x^{(k-1)}|x = 1)}{\sum_{x^{(k-1)}} f(y_j^{(k)}|x^{(k-1)}) P(x^{(k-1)}|x = -1)} \]
\[ = \frac{\sum_{x^{(k-1)}} P(x^{(k-1)}|x = 1) \prod_{i=1}^{n^{(k-1)}} f(y_{ij}^{(k)}|x_i^{(k-1)})}{\sum_{x^{(k-1)}} P(x^{(k-1)}|x = -1) \prod_{i=1}^{n^{(k-1)}} f(y_{ij}^{(k)}|x_i^{(k-1)})}. \tag{6} \]
The decision rule at node $R_j^{(k)}$ is

$$x_j^{(k)} = \text{sgn} \left( \log \left( L_j^{(k)} \right) \right).$$

For $k > 2$ and general mesh networks, the decision rule (7) requires that a node must know the states or statistics of the channels and the joint probability mass function (pmf) of correct decision of its relaying nodes, $P(x^{(k-1)}|x = 1)$ and $P(x^{(k-1)}|x = -1)$. The former requirement in most studies is assumed satisfied. The second requirement is much more restrictive, and in the sequel we propose ways of dealing with it.

We note that in (6), we have likelihoods of the form $f(y_j^{(k)}|x)$. For them we have the following claim:

Claim 1: For the likelihood $f(y_j^{(k)}|x)$, $k = 1, 2$ in mesh networks, we can write

$$f \left( y_j^{(k)} | x \right) = \prod_{i=1}^{n^{(k)}} f \left( y_{ij}^{(k)} | x \right).$$

Proof: The proof follows directly from the conditional independence of the channels and the decision independence of the nodes in the first group.

From Claim 1, for the likelihood ratio in nodes of group 1 and 2, $L_{ij}^{(k)}$, $k = 1, 2; j = 1, 2, \ldots, n^{(k)}$, we can write

$$L_j^{(k)} = \prod_{i=1}^{n^{(k-1)}} \frac{f(y_{ij}^{(k)}|x = 1)}{f(y_{ij}^{(k)}|x = -1)}$$

$$= \prod_{i=1}^{n^{(k-1)}} \frac{f(y_{ij}^{(k)}|x_i^{(k-1)} = 1)P_i^{(k-1)} + f(y_{ij}^{(k)}|x_i^{(k-1)} = -1)\overline{P}_i^{(k-1)}}{f(y_{ij}^{(k)}|x_i^{(k-1)} = -1)P_i^{(k-1)} + f(y_{ij}^{(k)}|x_i^{(k-1)} = 1)\overline{P}_i^{(k-1)}},$$

where

$$P_i^{(k-1)} = P(x_i^{(k-1)} = 1|x = 1) = P(x_i^{(k-1)} = -1|x = -1)$$

is the probability of correct decision, and

$$\overline{P}_i^{(k-1)} = 1 - P_i^{(k-1)}$$

is the probability of error, with $P^{(0)} = 1$.

The likelihood ratio in (9), in general, does not have the same simple form for $k > 2$ because of the dependence among the decisions of the relaying nodes. However, if we approximate

$$P \left( x^{(k-1)} | x \right) \approx \prod_{i=1}^{n^{(k-1)}} P \left( x_i^{(k-1)} | x \right),$$

we will obtain suboptimal detectors that use (9), as is shown in Section 5. We note that with the assumption (12), we only need to know the marginal probabilities of correct decisions of the relaying nodes.
A symmetry property, similar to (10) and (11), can be stated for $P(x^{(k-1)}|x = 1)$ and $P(x^{(k-1)}|x = -1)$. We make the following claim:

Claim 2: If the channel likelihoods satisfy $f(y_{ij}^{(k)}|x_i^{(k-1)}) = f(-y_{ij}^{(k)}|x_i^{(k-1)})$ and $f(-y_{ij}^{(k)}|x_i^{(k-1)}) = f(y_{ij}^{(k)}|x_i^{(k-1)})$ for all $k$, we have

$$P\left(x^{(k)}|x = 1\right) = P\left(-x^{(k)}|x = -1\right).$$  (13)

Proof: See Appendix 1.

We note that if the conditions for Claim 2 are satisfied, for optimal processing of the received signals all the information about the joint pmf of correct decisions needed by a node in the $k$th group is in $P(x^{(k-1)}|x = 1)$. Then, for the likelihood ratio (6) we can formally write

$$L_j^{(k)} = \frac{\sum_{x^{(k-1)}} P(x^{(k-1)}|x = 1) \prod_{i=1}^{n^{(k-1)}} f(y_{ij}^{(k)}|x_i^{(k-1)})}{\sum_{x^{(k-1)}} P(-x^{(k-1)}|x = 1) \prod_{i=1}^{n^{(k-1)}} f(y_{ij}^{(k)}|x_i^{(k-1)})},$$  (14)

where all the probabilities are conditioned on $x = 1$.

4 Detectors in the first group of nodes

In Section 2, we described two scenarios, one where the necessary CSI is available to the nodes, and another where only statistics of the channels are known. Here we describe the detectors of the nodes in the first group and we present their performances.

4.1 Completely known channels

In the case of known channels and based on the assumptions from Section 2, we can express the likelihood as follows:

$$f\left(y_{1j}^{(1)}|h_{1j}^{(1)}, x\right) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{(y_{1j}^{(1)} - h_{1j}^{(1)} x)^2}{2\sigma^2}}.$$  (15)

The decision rule of node $R_j^{(1)}$ is based on

$$L_j^{(1)} = \frac{p(y_{1j}^{(1)}|x = 1)}{p(y_{1j}^{(1)}|x = -1)},$$  (16)

which simplifies to

$$x_j^{(1)} = \text{sgn} \left(y_{1j}^{(1)}\right).$$  (17)

For the nodes in this group, we can easily find the probability of error, and it is given by

$$P_j^{(1)} = Q\left(\sqrt{\gamma_{1j}^{(1)}}\right),$$  (18)
with \( Q(z) = 1 - \Phi(z) \), where \( \Phi(z) = \int_{-\infty}^{z} \frac{1}{\sqrt{2\pi}} e^{-\frac{t^2}{2}} dt \), and \( \gamma_{1j}^{(1)} \) is the SNR of the channel that links the transmitter and the \( j \)th node in group one (see (3)).

### 4.2 Channels with known statistics

When the channels are not known, and instead the nodes only have available the channel statistics, we proceed as follows [Niu et al., 2006]. We write for the likelihoods

\[
f \left( y_{1j}^{(1)} \mid x \right) = \int_{0}^{\infty} f \left( y_{1j}^{(1)} \mid h_{1j}^{(1)}, x \right) f(h_{1j}^{(1)}) dh_{1j}^{(1)},
\]

(19)

where \( f(h_{1j}^{(k)}) \) is given by (2).

For simplicity, we rewrite the last integral without subscripts and superscripts, and we get

\[
f(y \mid x) = \int_{0}^{\infty} \frac{1}{\sqrt{2\pi} \sigma^2} \exp \left( -\frac{(y-hx)^2}{2\sigma^2} \right) \frac{2h}{\sigma^h} \exp \left( -\frac{h^2}{\sigma^h} \right) dh.
\]

It is easy to show that \( f(y \mid x) \) can further be expressed as

\[
f(y \mid x) = \sqrt{\frac{2}{\pi \sigma^2 \sigma_h}} \exp \left( -\frac{y^2}{2\sigma^2} \right) \int_{0}^{\infty} h \exp \left( -\mu h^2 + 2\nu h \right) dh
\]

(20)

where

\[
\mu = \frac{x^2}{2\sigma^2} + \frac{1}{\sigma_h^2}, \quad \nu = \frac{yx}{2\sigma^2}.
\]

(21)

We can show that the integral in the above equation can be analytically solved. We get

\[
\int_{0}^{\infty} h \exp \left( -\mu h^2 + 2\nu h \right) dh = \frac{1}{2\mu} + \frac{\nu}{\mu} \sqrt{\frac{\pi}{\mu}} \exp \left( \frac{\nu^2}{\mu} \right) \Phi \left( \nu \sqrt{\frac{2}{\mu}} \right).
\]

(22)

After substituting the solution (22) in (20) and returning all the superscripts and subscripts, we obtain

\[
f(y_{1j}^{(1)} \mid x) = \sqrt{\frac{2 \sigma^2 a^2}{\pi \sigma_h^2}} \exp \left( -\frac{y_{1j}^{(1)^2}}{2\sigma^2} \right) \left( 1 + \sqrt{2\pi} a x y_{1j}^{(1)} \Phi \left( a x y_{1j}^{(1)} \right) \exp \left( \frac{a^2 y_{1j}^{(1)^2}}{2} \right) \right),
\]

(23)

where

\[
a = \frac{\sigma_h}{\sigma \sqrt{2\sigma^2 + \sigma_h^2}}.
\]

In Fig. 3 we plotted the probability density functions (pdfs) of \( y \) given \( x = \pm 1 \) for SNR = 3dB for the two cases of interest, when the channels are completely known and when their statistics are only known. As expected, the latter case leads to deterioration in performance because then the
pdfs are shifted towards $y = 0$ and are flatter in comparison to the pdfs that correspond to known channels.

For the probability of error at node $R_j^{(1)}$, we have

$$
\overline{P}_j^{(1)} = P \left( L_{1j}^{(1)} < 1 | x = 1 \right) = P \left( y < 0 | x = 1 \right),
$$

(24)

where

$$
y_{1j}^{(1)} \sim f(y_{1j}^{(1)} | x = 1),
$$

i.e.,

$$
\overline{P}_j^{(1)} = \int_{-\infty}^{0} \sqrt{\frac{2}{\pi}} \frac{\sigma^2}{\sigma^2_h} \exp \left( -\frac{y^2}{2\sigma^2} \right) \left( 1 + \sqrt{2\pi}ay\Phi(ay) \exp \left( \frac{a^2y^2}{2} \right) \right) dy. \quad (25)
$$

Note that $\overline{P}_j^{(1)}$ is the probability of error of a coherent binary signaling over a Rayleigh fading channel [Proakis 2001]. The above integral can be solved and expressed in a closed form by using
integration by parts, and the result is given by

\[ T_j^{(1)} = \frac{1}{2} \left(1 - \sqrt{\frac{2}{\gamma} - 1}\right). \]  

(26)

5 Detectors in the groups of nodes beyond the first group

The detectors for the nodes in groups \( k > 1 \) are challenging to implement because of the need to know the joint probabilities of correct decisions of the relaying nodes (see (7)). It is interesting to point out that there is one exception to the need for pmfs, and it is the case of mesh networks with two nodes per group, known CSI, and assumptions as in Claim 2. Here, we describe four types of detectors for the nodes \( R_i^{(k)}, k > 1, i = 1, 2, \ldots, n^{(k)} \). We also briefly describe the detectors in multihop networks.

5.1 Estimation of the joint pmf based on Monte Carlo sampling (MCS)

The joint pmf of interest is \( P(x^{(k)}|x) \), and we can show that it can be expressed as

\[ P(x^{(k)}|x) = P\left(\operatorname{sgn}\left(\log L_1^{(k)}\right), \operatorname{sgn}\left(\log L_2^{(k)}\right), \ldots, \operatorname{sgn}\left(\log L_n^{(k)}\right) \right), \]  

(27)

where the random variables \( L_i^{(k)} \) are not independent.

From (6), we see that the \( L_j^{(k)} \) s are expressed via the decisions \( x^{(k-1)} \), which allows us to propose a computational method for obtaining the joint pmf. The procedure is the following:

1. Each node \( R_j^{(k)} \) generates samples \( y_{ij}^{(k)(m)} \), where \( i = 1, 2, \ldots, n^{(k-1)}, m = 1, 2, \ldots, M \) from the mixture distribution given by

\[ f(y_{1j}, y_{2j}, \ldots, y_{ij}, \ldots, y_{n^{(k-1)}j}) = \sum_{x^{(k-1)}} P(x^{(k-1)}|x) \prod_{i=1}^{n^{(k-1)}} f\left(y_{ij}^{(k)} | x_i^{(k-1)}\right). \]  

(28)

2. The node computes the signs of the obtained \( \log L_j^{(k)} \), \( i = 1, 2, \ldots, n^{(k)} \) at the drawn \( y_{ij}^{(k)(m)} \), \( j = 1, 2, \ldots, n^{(k)} \) and stores them as a vector in a counter.

3. The process is repeated \( M \) times.

4. From the obtained outcomes, the node obtains the estimates of the joint probabilities.

The generation of samples in the case of a mixture Gaussian is easy. When the channel statistics are known only, the sampling of \( y_{ij} \) must come from a distribution given by (23). To that end, one could apply rejection sampling.

Clearly, at the end, all the nodes in the \( k \)-th group will have different estimates of the joint pmf. The next group may combine all these estimates, for example, by taking the average of the estimates.
This approach becomes tedious when the number of nodes in a group gets large. For example, when \( n^{(k)} = 10 \), one has to work with a joint pmf of a 10-dimensional vector, which, in general, requires the computation of 1023 estimates. However, there are two ways to decrease the computational load of this approach and they are (1) to have a designated node in a group compute the joint pmf and (2) under an assumption of symmetry, instead of computing the values of \( 2^{n^{(k)} - 1} \) elements, a node computes only \( n^{(k)} + 1 \) elements.

### 5.2 Estimation of the joint pmf based on pilot signals (PS)

Suppose that node \( R_j^{(k)} \), \( k > 2 \) receives messages from \( n^{(k-1)} \) relay nodes, \( y_{ij}^{(k)} \). In the rest of the subsection, we suppress the subscript \( j \) and the superscripts \( (k) \). For example, for the received signals \( y_{ij}^{(k)} \), we write \( y_i \).

Define the variable \( z_i = 1 \) if \( y_i > 0 \) and \( z_i = 0 \) if \( y_i < 0 \). Let also

\[
\kappa(z_1, z_2, \cdots, z_n) = \sum_{i=1}^{n} z_i 2^{i-1} = \sum_{i=1}^{n} \kappa_i 2^{i-1},
\]

where \( \kappa_i \in \{0, 1\} \) identifies the \( i \)th binary digit of \( \kappa \) represented in binary notation. Thus, for example, \( \kappa = 0 \) corresponds to \( \{y_1 < 0, y_2 < 0, \cdots, y_n < 0\} \) and \( \kappa = 1 \) to \( \{y_1 > 0, y_2 < 0, \cdots, y_n < 0\} \). In other words, the joint pmf of \( z_1, z_2, \cdots, z_n \) can succinctly be represented by the pmf of \( \kappa \).

Similarly, we represent the joint pmf of the transmitted symbols of the relaying nodes. Let \( \zeta \) and \( \zeta_i \) be defined by

\[
\zeta(x_1, x_2, \cdots, x_n) = \sum_{i=1}^{n} (x_i + 1) 2^{i-2} = \sum_{i=1}^{n} \zeta_i 2^{i-1}, \quad \zeta_i \in \{0, 1\},
\]

and the joint pmf of \( x_i \) is equivalent to the pmf of \( \zeta \).

We want to estimate \( p(\zeta) \) from pilot data (where the source, say, repeatedly transmits \( x = 1 \)). Then the node first estimates the pmf of \( \kappa \) and, based on the model for the channels, estimates the pmf of \( \zeta \). We can write for \( \kappa = 0, 1, \cdots, 2^n - 1 \)

\[
p(\kappa) = \sum_{l=0}^{2^n-1} p(\kappa|\zeta = l)p(\zeta = l),
\]

where the probabilities \( p(\kappa|\zeta = l) \) are known, i.e., we can show that

\[
p(\kappa|\zeta = l) = \prod_{i=1}^{n} p_{c,i}^{\delta_i(\kappa, \zeta)} p_{e,i}^{1-\delta_i(\kappa, \zeta)},
\]

where

\[
\delta_i(\kappa, \zeta) = \begin{cases} 
1, & \kappa_i = \zeta_i \\
0, & \text{otherwise}
\end{cases}
\]
where \( \kappa_i = \zeta_i \) means
\[
\zeta_i = \frac{x_i + 1}{2},
\]
and \( p_{c,i} \) is the probability of correct decision based on the transmitted signal from the \( i \)th node, and \( p_{e,i} \) is the corresponding probability of error, that is \( p_{e,i} = 1 - p_{c,i} \). This probability of error is obtained from \([26]\).

We can represent the probabilities of \( \kappa \) by
\[
p_{\kappa} = \begin{bmatrix} p(\kappa = 0) & p(\kappa = 1) & \cdots & p(\kappa = 2^n - 1) \end{bmatrix}^T
\]
and the probabilities of \( \zeta \) by
\[
p_{\zeta} = \begin{bmatrix} p(\zeta = 0) & p(\zeta = 1) & \cdots & p(\zeta = 2^n - 1) \end{bmatrix}^T.
\]

We can also construct a \( 2^n \times 2^n \) matrix, \( P \) with probabilities as the ones defined by \([32]\). Then we can write
\[
p_{\kappa} = P p_{\zeta}.
\]  \hspace{1cm} (34)

We note that the matrix \( P \) is a doubly stochastic matrix, that is, its rows and columns sum up to one.

We reiterate that we estimate the vector \( p_{\kappa} \) from the pilot data, and we denote it by \( \hat{p}_{\kappa} \). Then the estimate of \( p_{\zeta} \) is readily obtained from
\[
\hat{p}_{\zeta} = P^{-1} \hat{p}_{\kappa}.
\]  \hspace{1cm} (35)

This estimate, however, may produce negative probabilities (although the sum of all the elements of \( \hat{p}_{\zeta} \) equals one), which is due to errors in estimating \( p_{\kappa} \). Thus, we change the set of equations \([34]\) to the following minimization problem with linear constraints:
\[
\hat{p}_{\kappa} = P(p_{\zeta} + \epsilon),
\]  \hspace{1cm} (36)
where \( \epsilon \) has minimum norm and adds to 0. In Appendix 2, we show that the solution vector \( \hat{p}_{\zeta} \) to the system \([36]\) has elements given by
\[
[\hat{p}_{\zeta}]_i = (b_i - \xi)^+,
\]  \hspace{1cm} (37)
where \((x)^+\) is the positive part of \( x \), \( i = 1, 2, \cdots, 2^n \), \( b = P^{-1} \hat{p}_{\kappa} \), and \( \xi \) is a parameter related to the Lagrange multipliers. It is worth pointing out that \([37]\) is in fact an upside-down waterfilling scheme.

In summary, according to this scheme the source node transmits symbols \( x \) known to the rest of the network. The nodes in groups \( k > 1 \) can estimate from the received signals from their relaying groups the joint pmfs \( P(x^{(k-1)}|x) \) as just shown. When the estimation is completed, the nodes have the necessary information for processing of the signals with unknown transmitted symbols. Obviously, a drawback of the scheme is that some transmitted symbols are used for estimating joint pmfs and some power is used for estimating the pmfs.
5.3 Estimation of the joint pmf based on a predefined joint pmf (PJP)

The biggest values of the joint pmf $P(x_1^{(k-1)}, x_2^{(k-1)}, \ldots, x_n^{(k-1)}|x = 1)$ are for combinations when most of the arguments $x_i^{(k-1)} = 1$. This suggests that we approximate the joint pmf by

$$P(x_1^{(k-1)}, x_2^{(k-1)}, \ldots, x_n^{(k-1)}|x = 1) = \begin{cases} p_c, & \left| \left\{ i : x_i^{(k-1)} = 1, i \in \{1, 2, \ldots, n^{(k-1)}\} \right\} \right| \geq N_f, \\ 0, & \text{otherwise} \end{cases}$$

where $p_c$ is probability obtained from the number of combinations of the elements of $x^{(k-1)}$ that satisfy the condition in (38), $|\cdot|$ is the cardinality of the set, and $N_f \leq n^{(k-1)}$. In other words, the probability of correct decision is constant and different from zero whenever the number of arguments $x_i^{(k-1)} = 1$ is big enough. This detector is efficient because only a few terms in the numerator of the likelihood ratio (6) are different from 0. Claim 2 is used for selecting the significant terms of the denominator of (6). This simple detector does not require the use of pilot signals.

5.4 Estimation of the joint pmf based on the assumption of independent decisions (ID)

When for $k > 2$ we approximate the joint pmf of $x^{(k-1)}$ by (12), for decision making we can use the likelihood ratio (9). If we take the logarithm on both sides of (9), and we define

$$l_{ij}^{(k)} = \log L_{ij}^{(k)} = \log \left( \tilde{L}_{ij}^{(k)} P_i^{(k-1)} + \tilde{P}_i^{(k-1)} \right) - \log \left( P_i^{(k-1)} + \tilde{L}_{ij}^{(k)} \tilde{P}_i^{(k-1)} \right),$$

where

$$l_{ij}^{(k)} = \log f(y_{ij}^{(k)}|x = 1) - \log f(y_{ij}^{(k)}|x = -1),$$

and

$$\tilde{L}_{ij}^{(k)} = \frac{f(y_{ij}^{(k)}|x_i^{(k-1)} = 1)}{f(y_{ij}^{(k)}|x_i^{(k-1)} = -1)}$$

representing the likelihood ratio for the transmitted symbol of $R_i^{(k-1)}$, we can write for the overall loglikelihood ratio

$$l_j^{(k)} = \sum_{i=1}^{n^{(k-1)}} l_{ij}^{(k)}.$$  \hspace{1cm} (42)

The decision rule then simplifies to

$$x_j^{(k)} = \text{sgn} \left( l_j^{(k)} \right).$$  \hspace{1cm} (43)
For the probability of error at $R_j^{(k)}$, we can write

$$\mathcal{P}_j^{(k)} = P \left( \sum_{i=1}^{n^{(k-1)}} l_{ij}^{(k)} < 0 \mid x = 1 \right) = P \left( \sum_{i=1}^{n^{(k-1)}} \log \left( \hat{L}_{ij}^{(k)} P_i^{(k-1)} + \mathcal{P}_i^{(k-1)} \right) - \log \left( P_i^{(k-1)} + \hat{L}_{ij}^{(k)} \mathcal{P}_i^{(k-1)} \right) < 0 \mid x = 1 \right).$$

(44)

We note that under these assumptions the new random variables $l_{ij}^{(k)}$ are independent. Clearly, if the $y_{ij}^{(k)}$s are identically distributed, so are the loglikelihood ratios $l_{ij}^{(k)}$.

The implementation of this detector requires information about the probability of correct decision of the nodes in the previous group $P_i^{(k-1)}$, $i = 1, \ldots, n^{(k-1)}$. These probabilities can readily be computed as described in subsections A and B. The computations in this case are much simpler because the problem is broken into $n^{(k-1)}$ independent problems, where in each problem we estimate the probability of detection (error) of only one relaying node at a time.

Another possibility is that the nodes compute their own probabilities of errors from (44) and transmit them to the nodes of the next group. The advantage of this approach is that there is no need for transmitting pilot signals. We note that in the transmission of the probability, some form of quantization must be adopted. In Section 6 we study the influence of quantization via simulations.

We now briefly describe the implementation of the scheme in the cases of completely known channels and known channel statistics, respectively.

5.4.1 Completely known channels

For the conditional distributions of the observations, we can write

$$f(y_{ij}^{(k)} \mid x = 1) = P_i^{(k-1)} \mathcal{N}(h_{ij}^{(k)}, \sigma^2) + \mathcal{P}_i^{(k-1)} \mathcal{N}(-h_{ij}^{(k)}, \sigma^2)$$

and for the likelihood ratios,

$$\hat{L}_{ij}^{(k)} = \exp \left( \frac{2 y_{ij}^{(k)} h_{ij}^{(k)}}{\sigma^2} \right),$$

(46)

whereas for the loglikelihood terms we have

$$h_{ij}^{(k)} = \log \left( \exp \left( \frac{2 y_{ij}^{(k)} h_{ij}^{(k)}}{\sigma^2} \right) P_i^{(k-1)} + \mathcal{P}_i^{(k-1)} \right) - \log \left( P_i^{(k-1)} + \exp \left( \frac{2 y_{ij}^{(k)} h_{ij}^{(k)}}{\sigma^2} \right) \mathcal{P}_i^{(k-1)} \right),$$

(47)
We can also find the distribution of $l_{ij}^{(k)}$ by change of variables. We obtain

$$f(l_{ij}^{(k)} | x = 1) = \frac{1}{2\sqrt{2\pi} h_{ij}^{(k)}} \left( \frac{e_{ij}^{(k)} (2P_{i}^{(k-1)} - 1)}{P_{i}^{(k-1)} e_{ij}^{(k)} (2P_{i}^{(k-1)} - 1) - P_{i}^{(k-1)} e_{ij}^{(k)} - P_{i}^{(k-1)} e_{ij}^{(k)}} \right) \times \left( P_{i}^{(k-1)} e_{ij}^{(k)} - \frac{1}{2} \left( \frac{\sigma e_{ij}^{(k)}}{h_{ij}^{(k)}} \log \frac{P_{i}^{(k-1)} e_{ij}^{(k)} (2P_{i}^{(k-1)} - 1)}{P_{i}^{(k-1)} e_{ij}^{(k)} (2P_{i}^{(k-1)} - 1) - P_{i}^{(k-1)} e_{ij}^{(k)} - P_{i}^{(k-1)} e_{ij}^{(k)}} + \frac{h_{ij}^{(k)}}{\sigma} \right)^2 \right). \quad (48)$$

The probabilities of error of the nodes in group $k \geq 2$ are obtained by the recursive equation (44). A closed form analytical solution of the recursive equation in the general case, however, cannot be obtained.

5.4.2 Channels with known statistics

The probability of error at the nodes in group $k \geq 2$ is the same, that is, it is given by (44), where now

$$\tilde{L}_{ij}^{(k)} = \frac{1 + \sqrt{2\pi a y_{ij}^{(k)} \Phi(ay_{ij}^{(k)})} \exp \left( \frac{a^2 y_{ij}^{(k)}^2}{2} \right)}{1 - \sqrt{2\pi a y_{ij}^{(k)} \Phi(-ay_{ij}^{(k)})} \exp \left( \frac{a^2 y_{ij}^{(k)}^2}{2} \right)} \quad (49)$$

is obtained by using (41) and (23). The probabilities of error of the nodes in groups $k \geq 2$ are again obtained numerically by the recursive equation (44).

5.5 Detectors for multihop networks

In multihop networks, the variables $y_{ij}^{(k)}, i = 1, 2, \ldots, n^{(k-1)}$, $k \geq 1$, are conditionally independent. Therefore, for detection at the relay nodes one uses the likelihood ratio in (9). The last node in the network, $R_{1}^{(K+1)}$, collects the information of all the nodes from the previous group and uses the decision rule (43), where the sum loglikelihood $l_{ij}^{(k)}$ is calculated from (42). For this node, the obtained results under the independence assumption apply.

6 Simulations

We conducted many experiments where we compared the performances of the mesh and multihop networks shown in Figs. 1 and 2, respectively. In the experiments, the channels were Rayleigh distributed and, for better understanding of the results and easier comparisons, the SNRs were the
Figure 4: Probabilities of error for mesh and multihop networks as functions of number hops for the case of known statistics.

same for all the nodes. Most of the simulations were for the scenario where the channel statistics are known. This case presents two main advantages in real networks: (a) the information about the statistics is provided just once for each node and (b), the probabilities of correct detection are evaluated also once. By contrast, when detecting with complete CSI at the nodes, the channel information must be updated if the CSI changes, and the probability of correct detection has to be evaluated accordingly.

First, we considered a mesh network with a decision rule based on complete knowledge of the probabilities of correct decision of the nodes of the previous groups and based on (7). The results of this setup were a benchmark for comparison. The joint pmfs of correct/incorrect decisions of the groups for \( k > 1 \) were computed using the MCS method.

In Fig. 4 we show in the curve Mesh MCS the probabilities of error of mesh networks with different number of hops when all the joint pmfs were known. In the simulations, the number of nodes per group was 10 and the SNR = 3 dB. The error probability decreased as the number of hops increased until a point where it remained constant with the number of hops. The spatial redundancy introduced by the mesh network lowered the error probability until an error floor was reached.
In Fig. 4, we also present the performance of the mesh network using the PS method. As can be seen, the obtained results are close to the ones of mesh networks that have complete information about the pmfs.

To these curves, four other curves are also displayed. One of them shows the performance of the PJP detector. With the curve Mesh ID Q=4 bits, we show the performance of the detector based on the ID detector, and where the probabilities of correct decision are quantized with four bits. Note that a node uses the probabilities of correct decision of previous nodes and quantizing these variables reduces the amount of information to be distributed. Finally, with the Multihop ID Q=4 bits curve we plotted the performance of the multihop network, and with the MRC curve, we displayed the performance of the MRC detector [Proakis 2001]. For the multihop network, the probabilities of correct decision of the previous node were again quantized with four bits. The MRC detector was implemented by combining the signals arriving to a node using the MRC criterion.

The worst performance was clearly achieved by the multihop network. Also, its performance deteriorated steadily as the number of hops increased. The performance of the network whose nodes had to estimate the joint decision pmfs of the nodes in the previous group performed almost as well as the network whose nodes knew the joint pmfs. Surprisingly well performed the mesh network that used the independence assumption and where the nodes quantized their probabilities with four bits. Next in performance of the mesh networks came the one that employed the simple detector and finally, the worst was the network with MRC detectors.

The analytical study of the quantization effect on the ID detectors is difficult. Instead, we performed experiments where the number of bits for representing the quantized probabilities was either one bit (the quantized probabilities were zero and one) or four bits (for 16 probability values, that is, the quantized probabilities were 0, 0.0666, ···, 1). We also obtained results when the probabilities were not quantized. The results are shown in Fig. 5. There we see the probability of error as a function of the number of hops. The nodes assumed independence of the decisions of the nodes of the previous group. The SNR was 3 dB. In all the mesh networks with 10 nodes per group, the BER decreased with the number of hops. The results show that four bits were enough for achieving almost as good performance as the one when there was no quantization. Interestingly, using only one bit for quantization also yielded good results. Note that representing the probability of correct decision with one bit is a form of selection combining: the nodes with low probability withdraw themselves from combining in the following group of nodes.

In the case of five nodes per group (curve Mesh ID 5 nods/grp Q=4 bits) the decrease in probability of error probability was slower. The performance of this network was worse than that with 10 nodes per group by an order of magnitude. In Fig. 5 we also plotted the performance of two multihop networks. One of them was the same network whose performance is shown in Fig. 4 and the other corresponds to the multihop network with CMRC detectors (Multihop CMRC 10 nods/grp) [T. Wang et al., 2007]. Note that the CMRC detector uses complete information about the channel. Nevertheless, it has a larger error probability than the multihop ID detector (curve Multihop ID 10 nods/grp Q=4 bits), where only channel statistics are used. In both cases the error probability increases very fast with the number of hops.

Finally, in Fig. 5 we present curves of probability of error versus SNR for a mesh network with MCS detectors, a mesh network with ID detectors and four bits for quantization, and a multihop network. The number of nodes per group was 10 and the curves were obtained after 9 hops.
From the experiment we can draw the following conclusions: a) the assumption of independence is adequate and gives results that are very close to the case of dependence between the probabilities of correct decision of the nodes in a given group and b) the mesh networks offer great advantages in performance over multihop networks.

7 Conclusions

In this paper we studied the performance of mesh networks whose nodes operate in the decode-and-forward mode. In particular, we investigated the probability of error in these networks as a function of SNR, number of nodes per group, and number of hops. We studied two cases, where in the first case the nodes knew the CSI and in the second, the nodes worked only with the statistics of the channels. We compared the mesh networks with multihop networks and showed the gain in performance of the former with respect to the latter. In the presented work we dealt with
Figure 6: Probabilities of error for mesh and multihop networks as functions of SNR for MAP detectors.

binary modulations, but the generalization of the proposed detectors to deal with \( M \)-ary, \( M > 2 \), modulations is not difficult.

Appendix 1

Proof of Claim 2

We prove the claim by induction. Suppose that the channel likelihoods satisfy

\[
\begin{align*}
    f(y_{ij}^{(k)}|x_i^{(k-1)}) &= f(-y_{ij}^{(k)} - x_i^{(k-1)}) \\
    f(y_{ij}^{(k)}|x_i^{(k-1)}) &= f(-y_{ij}^{(k)} - x_i^{(k-1)}) \\
    f(y_{ij}^{(k)}|x_i^{(k-1)}) &= f(y_{ij}^{(k)} - x_i^{(k-1)})
\end{align*}
\]

and that the claim is true for \( k - 1 \),
i.e. $P(x^{(k-1)}|x = 1) = P(-x^{(k-1)}|x = -1)$ Then we have the likelihood ratio[6]

$$L_j^{(k)}(y_j^{(k)}) = \frac{\sum_{x^{(k-1)}} \prod_{i=1}^{n^{(k-1)}} f(y_{ij}^{(k)}|x_i^{(k-1)})P(x^{(k-1)}|x = 1)}{\sum_{x^{(k-1)}} \prod_{i=1}^{n^{(k-1)}} f(y_{ij}^{(k)}|x_i^{(k-1)})P(x^{(k-1)}|x = -1)}$$

$$= \frac{\sum_{x^{(k-1)}} \prod_{i=1}^{n^{(k-1)}} f(-y_{ij}^{(k)}|x_i^{(k-1)})P(-x^{(k-1)}|x = -1)}{\sum_{x^{(k-1)}} \prod_{i=1}^{n^{(k-1)}} f(-y_{ij}^{(k)}|x_i^{(k-1)})P(-x^{(k-1)}|x = 1)},$$

(50)

where we have used the inductive hypothesis and the symmetry of the likelihoods. We can reorder the sums by replacing $x$ with $-x$ to get

$$L_j^{(k)}(y_j^{(k)}) = \frac{\sum_{x^{(k-1)}} \prod_{i=1}^{n^{(k-1)}} f(-y_{ij}^{(k)}|x_i^{(k-1)})P(x^{(k-1)}|x = -1)}{\sum_{x^{(k-1)}} \prod_{i=1}^{n^{(k-1)}} f(-y_{ij}^{(k)}|x_i^{(k-1)})P(x^{(k-1)}|x = 1)}$$

$$= \frac{1}{L_j^{(k)}(-y_j^{(k)})}.$$  

(51)

From[51], we have the equality

$$P(L_1^{(k)}(y_1^{(k)}) \geq 1, \ldots, L_j^{(k)}(y_j^{(k)}) \geq 1, \ldots| x = 1) =$$

$$P(L_1^{(k)}(-y_1^{(k)}) \leq 1, \ldots, L_j^{(k)}(-y_j^{(k)}) \leq 1, \ldots| x = 1).$$  

(52)

Next we show that

$$f(-y_j^{(k)}|x = 1) = \sum_{x^{(k-1)}} \prod_{i=1}^{n^{(k-1)}} f(-y_{ij}^{(k)}|x_i^{(k-1)})P(x^{(k-1)}|x = 1)$$

$$= \sum_{x^{(k-1)}} \prod_{i=1}^{n^{(k-1)}} f(y_{ij}^{(k)}|x_i^{(k-1)})P(-x^{(k-1)}|x = -1)$$

$$= f(y_j^{(k)}|x = -1)$$  

(53)

and therefore

$$P(L_1^{(k)}(y_1^{(k)}) \geq 1, \ldots, L_j^{(k)}(y_j^{(k)}) \geq 1, \ldots| x = 1) =$$

$$P(L_1^{(k)}(y_1^{(k)}) \leq 1, \ldots, L_j^{(k)}(y_j^{(k)}) \leq 1, \ldots| x = -1)$$  

(54)

proving the claim for the $k$th step.

The proof of the claim for $k = 1$ is immediate. This completes the proof of Claim 2.

Appendix 2

The system[36] can be expressed as the following optimization problem:
minimize \( \sum \epsilon_i^2 \)
subject to \( p_i^{(k-1)} + \epsilon_i = b_i, \ i = 1, \ldots, n^{(k-1)} \)
\(-p_i^{(k-1)} \leq 0, \ i = 1, \ldots, n^{(k-1)} \)
\( \sum \epsilon_i = 0. \) \( (55) \)

The last two conditions make sure that the vector \( p^{(k-1)} \) is a probability vector because, as can easily be proved, \( b \) is a vector whose elements add to one. The problem (55) is convex and for it the Karush-Kuhn-Tucker (KKT) condition is necessary and sufficient for the optimal solution \cite{Luo2006}. More specifically, the KKT condition is

\[
\nabla \sum \epsilon_i^2 + \sum_{i=1}^{n^{(k-1)}} \lambda_i \nabla (-p_i^{(k-1)} - 1) + \sum_{i=1}^{n^{(k-1)}} \nu_i \nabla (p_i^{(k-1)} + \epsilon_i - b_i) + \nu_e \nabla \sum \epsilon_i = 0 \\
-p_i^{(k-1)} \leq 0 \quad i = 1, \ldots, n^{(k-1)} \\
p_i^{(k-1)} + \epsilon_i = b_i \quad i = 1, \ldots, n^{(k-1)} \\
\sum \epsilon_i = 0 \\
\lambda_i (-p_i^{(k-1)} - 1) = 0, \ \lambda_i \geq 0, \ i = 1, \ldots, n^{(k-1)}.
\]

(56)

After applying the partial derivatives, for the KKT condition we obtain

\[
\epsilon_i + \frac{1}{2} (\nu_i + \nu_e) = 0; \quad i = 1, \ldots, n^{(k-1)} \\
\lambda_i = \nu_i; \quad i = 1, \ldots, n^{(k-1)} \\
\lambda_i \geq 0, \lambda_i = 0 \text{ if } p_i^{(k-1)} \geq 0, \quad i = 1, \ldots, n^{(k-1)} \\
p_i^{(k-1)} \geq 0, \quad i = 1, \ldots, n^{(k-1)} \\
\sum \epsilon_i = 0 \\
p_i^{(k-1)} + \epsilon_i = b_i \quad i = 1, \ldots, n^{(k-1)}. \quad (57)
\]

We define the variable

\[
\xi = \sum \frac{\lambda_i}{M}, \quad (58)
\]

where \( M \) is the number of variables \( p_i^{(k-1)} \) equal to zero. With it and the KKT condition (57), the solution to the system (55) can be expressed in a compact way as given by (37).

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