Feasible scheme for measuring experimentally the speed of the response of quantum states to the change of the boundary condition

Guang Ping He
School of Physics & Engineering and Advanced Research Center,
Sun Yat-sen University, Guangzhou 510275, China

When the boundary condition of a quantum system changes, how fast will it affect the state of the system? Here we show that if the response takes place immediately, then it can allow superluminal signal transfer. Else if the response propagates in space with a finite speed, then it could give a simple explanation why our world shows classicality on the macroscopic scale. Furthermore, determining the exact value of this speed can either clarify the doubts on static experiments for testing Bell’s inequality, or support the pilot-wave interpretation of quantum mechanics. We propose an experimental scheme for measuring this speed, which can be implemented with state-of-art technology, e.g., single-electron biprism interference.

PACS numbers: 03.65.Ta, 42.50.Xa, 03.65.-w, 03.65.Yz

I. INTRODUCTION

Quantum mechanics achieved great success in the past century and was proven correct in almost any physical process on any scale. But due to some anti-intuitional features of quantum mechanics, people keep wondering why our world has to be quantum. Some even doubted the completeness of quantum mechanics. Many quantum interpretation theories thus arose, e.g., the Copenhagen interpretation, statistical interpretation, pilot-wave interpretation, and many worlds interpretation, etc. But many details of these theories generally involve some quantities which are not the observables of quantum mechanics, therefore hardly any existing experiment can prove or disprove these interpretations.

Nevertheless, here we will propose an experimental scheme that can provide some clues to the details of how quantum mechanics works. Thus it may serve as a starting point for picking the correct quantum interpretation. We consider the following problem. It is well-known that once the Hamiltonian and boundary condition of a quantum system are provided, the state of the system and its time evolution are completely determined by Schrödinger equation (or Klein-Gordon/Dirac equations in the relativistic case). But how fast is the state determined by these elements? More specifically, when the boundary conditions change, how fast will the wavefunction in another location of space be affected?

It is important to note that what we consider here is different from the existing results obtained from systems with time-dependent Hamiltonian and/or fast changing boundary conditions (e.g., Refs. 1 2), where it was assumed that at any given time $t$, the wavefunction $\psi(t)$ in the whole space satisfies the Schrödinger equation of the same $t$. In literature, this assumption was widely adopted in quantum mechanics, despite that it was not clearly stated as an assumption most of the time. But it could violate the theory of relativity even if we replace Schrödinger equation with Klein-Gordon or Dirac equation. In this paper, instead of adopting this assumption, we are interested in whether the Hamiltonian/boundary condition at a given time will affect the wavefunction at all the locations of the space instantaneously, and if the answer is no, then how fast the effect will occur. It is worth noting that the problem cannot be solved only within the framework of basic quantum mechanical formalism (e.g., Schrodinger/Klein-Gordon/Dirac equations). It has to rely on a certain interpretation of the quantum theory to supply details on how the quantum system “gets information” on the boundary condition.

We will show below that it is possible to measure experimentally the speed how fast the response to the change of boundary condition will take place. We will also show theoretically that if the speed is infinite, then it can allow superluminal signal transfer and thus conflicts with the theory of relativity. Else if the speed is finite, then it may provide a clue to the long-time puzzling open problem why our world looks classical on the macroscopic scale despite that all microscopic processes are quantum. Furthermore, knowing the exact value of this speed can help us to understand how a quantum system “knows” the status of the boundary condition, which can help to judge whether the pilot-wave theory or other quantum interpretations seems more appropriate, and prove or disprove the doubts on static experiments for testing Bell’s inequality. Therefore, implementing our scheme can provide results which will develop our understanding on quantum mechanics, and bring us closer to the answer of John Wheeler’s big question “why the quantum”.

II. IMPORTANCE OF THE SPEED

Before going to the details of our experimental scheme, let us first consider the Gedanken experiment illustrated in Fig. 1, to see why it is important to determine the
speed \( v \) with which a quantum system responds to the change of the boundary condition. A particle source produces single mode quantum particles with speed \( v_0 \). The quantum particles can be either photons, electrons, or neutrons, etc., and \( v_0 \) can be either equal to or smaller than the speed of light \( c \). The output power of the source is carefully controlled so that it produces only one particle at a time. Similar to the “which-way” experiment \[4\], let the particles pass a double-slit wall (in fact we use pinholes instead of slits) and then reach the screen. There is a barrier behind pinhole A that can choose to either open or close the pinhole. The status of pinholes A and B thus serves as the boundary condition for the quantum state in the space. Let pinhole B be opened all the time. According to quantum mechanics, when pinhole A is opened, a double-slit interference pattern should be observed on the screen. On the other hand, if pinhole A is closed, the interference pattern will disappear, while only the single-slit diffraction caused by pinhole B will be observed. Now consider the following question: suppose that pinhole A was initially opened, and is closed at time \( t_1 \), then how fast will the pattern observed on the screen show a response to this change of the status of pinhole A?

On one hand, if the response takes place immediately, then it seems to allow superluminal signal transfer and conflict with the theory of relativity. This is because an observer at a certain location \( S \) of the screen can deduce whether a distant pinhole A is closed or not by the pattern he observed. For simplicity, let \( S \) be a point corresponding to a dark fringe of the interference pattern when both pinholes A and B are opened. Suppose that the particle source was initially shut down but turned on right after \( t_1 \). The observer then waits for a finite time interval \( \Delta t \) \((\Delta t > (b_1 + b_2)/v_0)\) so that a sufficient number of particles can reach the screen from the source. Now if the observer detected a certain amount of particle flux on point \( S \) between the time \( t_1 + (b_1 + b_2)/v_0 \) and \( t_1 + \Delta t \), he can deduce that pinhole A was closed. Else, if he found that point \( S \) is still dark after \( t_1 + \Delta t \), then he concludes that pinhole A was not closed at \( t_1 \). Therefore, if the distance \( a_2 \) is sufficiently larger than \((b_1 + b_2)c/v_0\) so that \( a_2 > c\Delta t \), a superluminal signal is transferred from pinhole A to point \( S \). Although in practice the interference will be too weak to detect if \( a_1 \) and \( a_2 \) are very large, in principle it still makes a difference in the observed pattern. More importantly, unlike the spooky action at a distance realized with Einstein-Podolsky-Rosen entangled pairs \[3\] where the observers cannot predict the outcome before performing the measurement, in our case the difference in the pattern can indeed deliver a preassigned signal. Therefore this result seems impossible according to the theory of relativity.

On the other hand, suppose that it takes a finite time before it becomes possible to observe the response to the boundary condition corresponding to the change of the status of pinhole A. That is, the response propagates in space with a finite speed \( v \). Then it is puzzling what is traveling through space delivering the information of the boundary condition. At the first glance, it seems to be a natural interpretation that these traveling objects are the particles themselves, who reach the boundary so that they “know” how many pinholes are opened. But previous experiments showed that even if the particle source produces only one single particle at a time, the interference pattern will still present if both pinholes are opened. Then it may seem weird to assume that a single particle reaches both pinholes simultaneously by itself. But no matter the particle reaches a pinhole in its entirety or by parts, as long as this interpretation is correct, it seems natural that we should find \( v = v_0 \) (within the precision allowed by the uncertainty principle). This is what the mainstream interpretation theories of quantum mechanics predict. But besides this picture, there are also other interpretations on how quantum systems “know” the boundary condition. For instance, in literature there were doubts on whether static experiments for testing Bell’s inequality can provide a convincing conclusion, because “the settings of the instruments are made sufficiently in advance to allow them to reach some mutual rapport by exchange of signals with velocity less than or equal to that of light” \[6,7\]. If this is also the case in our experiment, then it is possible that the particle source, the slits, and the screen somehow “know” the status of each other with or without the existence of the particle. Thus \( v \) will not have to be equal to \( v_0 \). According to the de Broglie-Bohm pilot-wave interpretation of quantum mechanics \[8,9,10\], the traveling objects carrying the information of the boundary condition can be viewed as the pilot waves set up in space by the Hamiltonian and boundary condition, and the particle travels by following a certain pilot wave. In this picture, both \( v = v_0 \) and \( v \neq v_0 \) are allowed in theory \[8\]. Specifically, if \( v = c \), it may indicate that the traveling objects are
the virtual photons being exchanged between the particle (or the source) and the boundary so that they “know” the existence of each other, as described in quantum field theory. Besides these interpretations, there can even be other mechanism that we may currently be unaware of. Thus it is natural to assume in general that \( v_0 \leq v \leq c \).

Obviously, if the value of \( v \) can be measured, it can help us understand the mechanism better. Furthermore, as long as \( v \) is finite, we can have a simple interpretation on the classicality of our macroscopic world. Consider the following case. Suppose that the source was initially turned off, then turned on and off intermittently after \( t_1 \). At each interval, it is turned on only for a short period of time so that it produces only one particle at the most. Let \( \varepsilon \) denote the time between each interval it is turned on. \( \varepsilon \) should be sufficiently long to guarantee that the particle produced in the previous interval already reached the screen and completely interacted with and was absorbed by the screen, before the source is turned on in the next interval. Now if pinhole A was initially closed, and the observer at point \( S \) does not know whether pinhole A is opened or closed at \( t_1 \), what pattern will he find on the screen?

Since we assumed that the response to the change of the status of pinhole A propagates in space with a finite speed \( v \), if pinhole A is opened at \( t_1 \), point \( S \) will not be affected immediately. Therefore, if pinhole A is sufficiently far away from the source and the screen, the interference pattern should not be observed at point \( S \) for a period of time, because pinhole A is initially closed. But will the interference pattern be observed later? If yes, then what makes the forthcoming particle interfere? Note that it is assumed that the source produces only one particle at a time, and the previous particle was already absorbed by the screen long before the next particle is produced. Thus anything (for conciseness in the description, we call it as pilot wave hereafter, no matter what it really is) generated by the previous particle (if any) should no longer exist when the next particle comes out. So if we assume that the pilot wave delivering the status of the boundary condition is generated by the particle itself, then there is nothing left from the previous particle to guide the next one. The next particle has to generate its own pilot wave to sense the boundary. Therefore, similar to the previous particles, the forthcoming particles will not form an interference pattern at point \( S \) either, as long as pinhole A is so far away from the source and the screen, that each particle already reached and was absorbed by the screen from the source before its own pilot wave can reach the screen from pinhole A. Consequently, interference can never be observed in this Gedanken experiment. If this is indeed the case, then it gives a clue on why our world looks classical on the macroscopic scale even though every single particle is ruled by quantum mechanics and has wave-particle duality – simply because in the real world most quantum particles are closely surrounded by and interacting with other particles on the microscopic scale, and they act as pinhole B and the screen to each other, so that other boundary condition at a distance (which does not have to be really far away on the macroscopic sense) is relatively too far to have these quantum particles fully display their corresponding wave-like behaviors such as interference. Of course, to fully explain the classicality of our macroscopic world, we need to further study whether this mechanism also plays a crucial role in every other complicated physical process. Thus it is still too early to make a deterministic conclusion. Nevertheless, we would like to pinpoint out that this interpretation has the advantage that it does not require any new physical postulation. As long as \( v \) is finite, the above mechanism is valid, while \( v \) has to be finite as long as superluminal signal transfer is impossible. That is, this interpretation is based merely on the validity of Special Relativity. Note that other existing interpretations on the classicality of the macroscopic world generally involve new postulations which may not have been proven. Therefore our interpretation looks promising and worth further investigation.

On the contrary, if the interference pattern can indeed be observed in this Gedanken experiment, then it seems to suggest that the pilot wave is generated by the boundary instead of the particle itself. If so, then its speed could be independent of the type of the particle. This picture is also interesting since the corresponding mechanism will be worth studying, and it may even be related to the interpretation of space-time structure and gravity.

Either way, we can see that the corresponding physical picture is interesting. Intuitively, it seems very possible that the result would be \( v = v_0 \). However, as mentioned in the Introduction, this speed cannot be calculated from basic quantum mechanical formulas (e.g., Schrödinger equation) without involving any quantum interpretation theory. Therefore, nothing should be taken for granted unless it is proven by experiments. Here we are not going to reach a conclusion theoretically. Instead, we will propose a feasible scheme to measure the speed \( v \).

III. THE EXPERIMENTAL SCHEME

The experimental apparatus for measuring this speed is illustrated in Fig. 2. It is similar to the above Gedanken experiment, but none of the pinholes needs to be far away so that it is practical to be implemented. There are two wheels rotating clockwise (when viewing from the particle source) along the same axis with the same speed \( \omega \). The shape of the front wheel is a sector of angle \( \alpha \). It is located right behind pinhole A so it will cover pinhole A from time to time as it rotates. The shape of the rear wheel is a sector of angle \( \beta \). It is located right in front of the screen so it will cover a certain area of the screen as it rotates. Now suppose that the speed \( \omega \) is very high, and the output power of the particle source is carefully controlled so that it produces no more than one single particle at a time. In one round of the rotation of the wheels, at time \( t_1 \) the left edge \( \text{OE}_1 \) of the front
wheel meets pinhole A so that A is going to be covered by the front wheel (see Fig. 3a). At a later time \( t_2 \), the right edge \( OE_2 \) of the front wheel starts to leave pinhole A so that A will be opened hereafter (see Fig. 3b). Let \( OE_0 \) denote the line on the screen corresponding to the position of the left edge of the rear wheel at \( t_1 \), and \( OE_3 \) denote the line on the screen corresponding to the position of the right edge of the rear wheel at \( t_2 \). Then \( OE_0E_3 \) forms a sector with angle \( \gamma = \beta - \alpha \) (supposing that the diameter of the pinhole is negligible).

Now consider the pattern we will observe within the sector area \( OE_0E_3 \) of the screen after a sufficiently long period of time during which the wheels completed many rounds of rotation. Note that in each round, the area \( OE_0E_3 \) of the screen is completed covered during the period from \( t_1 \) to \( t_2 \). On the other hand, pinhole A is kept opened before \( t_1 \) and after \( t_2 \). That is, the part of the screen within the area \( OE_0E_3 \) is exposed only when both pinholes A and B are opened. Therefore, if the change of the boundary condition (i.e., the status of pinhole A in our case) takes effect instantaneously (i.e., the response to the change propagates in space with an infinite speed), only the interference pattern will be observed within the area \( OE_0E_3 \). Meanwhile, a mix pattern will be observed on the screen outside the area \( OE_0E_3 \), which is not only the interference pattern obtained when both pinholes A and B are opened, but also overlaps with the single-slit diffraction pattern when pinhole A is covered by the front wheel.

On the contrary, if the response has a finite speed \( v \), then the mix pattern will also be observed in some parts of the area \( OE_0E_3 \). This is because pinhole A is covered during \( t_1 \) to \( t_2 \). And though it is opened at \( t_2 \), the screen will not be affected until \( t_2 + \Delta T \), where \( \Delta T = \frac{a_2}{v} \). Therefore, if any particle reaches the screen during \( \Delta T \), it will contribute to form the single-slit diffraction pattern as if pinhole A was not opened yet. Since the rear wheel is rotating with the speed \( \omega \), during \( \Delta T \) its right edge sweeps through an angle

\[
\delta = \omega \Delta T = \frac{\omega a_2}{v},
\]

thus leaving a narrow sector of angle \( \delta \) at the left of \( OE_3 \) uncovered. Consequently, the mix pattern should be presented in this sector area. Similarly, though pinhole A starts to close at \( t_1 \), it will not take effect on the screen until \( t_1 + \Delta T \). Therefore, the interference pattern instead of the mix pattern will be observed in a narrow sector of angle \( \delta \) to the left of \( OE_0 \). As a whole, there is still a sector area of the screen in which only the pure interference pattern will be observed. The sector has the same angle as that of the sector \( OE_0E_3 \), but its position is like rotating the sector \( OE_0E_3 \) clockwise along the axis of the wheels by angle \( \delta \), while the pattern observed inside the sector (the position of the fringes) does not rotate since the relative position of the pinholes and the screen stays unvaried.

To observe the changed angle, in the experiment we can initially rotate the wheels at an extremely slow speed \( \omega_s \), while putting a photographic plate on the screen and exposing for a long period of time (just a little longer than that in an ordinary double-slit experiment without the wheels), so that there can be a sufficient amount of particles reaching the screen to form a visible pattern for reference. This eliminates the need for single-particle detectors. Then we rotate the wheels at a very fast speed \( \omega_f = \omega_s + \Delta \omega \), while putting another photographic plate on the screen and exposing for the same period of time. By comparing the current pattern with the initial one, the angle \( \Delta \delta \equiv \delta_f - \delta_s \) can be measured, where \( \delta_f = \frac{\omega_f a_2}{v} \) and \( \delta_s = \frac{\omega_s a_2}{v} \). Thus the speed of the response to the change of the boundary condition can be obtained as

\[
v = \frac{\Delta \omega a_2}{\Delta \delta}.
\]

\[\text{IV. FEASIBILITY AND DISCUSSIONS}\]

In practice, since we would like to determine whether the speed \( v \) equals to the speed \( v_0 \) of the particles in the experiment, it is recommended to use particles with a non-vanishing mass, so that \( v_0 < c \). Therefore single-electron interference \[11, 12, 13, 14\] and single cold-atom interference \[15, 16\] can both be used. Take for example, consider the apparatus of the single-electron interference experiment in Ref. [11], whose specification was provided in Ref. [12]. It used a convergent electrostatic biprism to take the place of the double-slit. The distance (denoted as \( b \) in Ref. [12]) between the biprism and the screen (before magnified by the projector lenses) is \( a_2 = 6cm \). When the wire potential of the biprism is 24V, an interference pattern with a fringe spacing of 1000\( \AA \) can be obtained (see Fig. 5(e) of Ref. [12]). To implement our experimental scheme, all we need is to add the wheels shown in Fig. 2 to their apparatus. The wheels should
be made of the same sort of non-magnetic insulating materials used in the biprism so they will not charge up nor disturb the magnetic field of the lenses. Let the radius of both the front and rear wheels be \( R = 10 \text{ cm} \). Since it is sufficient for us to observe the angle \( \delta \) at only one of the edges of the sector \( OE_0E_3 \), e.g., \( OE_0 \), the sector \( OE_0E_3 \) can be much larger than the size of the entire visible area of the interference pattern. Therefore the angles \( \alpha \) and \( \beta \) need not to be too small nor precise so that they can be prepared easily. The hardest part of the experiment may be that the wheels need to be placed correctly, so that the tip \( E_0 \) of the rear wheel falls within the visible area of the interference pattern on the screen when the front wheel starts to cover one half of the biprism. This is also how the angle \( \theta \) between the left edges of two wheels (as shown in Fig. 3a) is determined. Once this is done, we rotate the wheels at a low speed, e.g., \( \omega_s \sim 10 \text{ round per second} \), and turn on the whole system to get a pattern for reference. As long as the wheels are correctly placed, half of the pattern we observe now should be identical to the interference pattern without the wheels (i.e., Fig. 5(e) of Ref. [12]), while the other half should be blurred by the single-slit diffraction pattern (which should look like a mix of Fig. 5(a) and Fig. 5(e) of Ref. [12]). This can also be used as an approach to verify whether the wheels are placed correctly. After that, we rotate the wheels at a high speed \( \omega_f = 500 \text{ round per second} \). Then as long as the speed we want to measure (i.e., \( v \) in Eq. (2)) is finite, the position of the dividing boundary between the interference region and the mixed region in the pattern currently observed should be different from that of the pattern previously observed at low rotation speed \( \omega_s \) of the wheels. And the difference is most significant on the location of the screen corresponding to the far end of the rear wheel (the tip \( E_0 \) when the front wheel starts to cover one half of the biprism. Rigorously, according to Eq. (2), the change of the position of the dividing boundary in the pattern around this location is

\[
x \equiv R \Delta \delta = R \Delta \omega_2/v.
\]

The speed of the electron in the experiment in Refs. [11, 12] is \( v_0 = 1.5 \times 10^8 \text{ m/sec} \). Therefore if the speed \( v \) equals to \( v_0 \), then we get \( x = R \Delta \omega_2/v_0 = 1232 \text{ A} \). Even if \( v \) equals to the speed \( c \) of light, which is the maximum allowed by the theory of relativity, there is still \( x = R \Delta \omega_2/c = 616 \text{ A} \). Both values are comparable to the fringe spacing (1000 A) thus are detectable.

On the other hand, if we merely want to determine whether the speed \( v \) is finite or not without caring the relationship between \( v \) and \( v_0 \), then single-photon double-slit interference experiments will be more convenient. In this case, the distance \( a_2 \) between the double-slit and the screen in free space can be \( \sim 10^2 \text{ times larger than that of the single-electron interference experiment, and can be made even larger with optical fibers. Therefore we can observe a significantly larger } x \text{ and measure it with higher precision. The visible area of the interference pattern is also larger, thus the radius of the wheels can be increased too, so that the speed } \omega_f \text{ can be lower. The disadvantage of these experiments with photons is that } v_0 \text{ is exactly the speed of light. Then if the experimental result shows that } v \text{ also equals to the speed of light, it can hardly provide any information on whether the status of the boundary condition is learned by the photons themselves or by something else being exchanged between the experimental instruments.}

No matter which type of experiments is used, as long as the result shows that \( v \) is indeed finite, then it seems to support our above interpretation why our macroscopic world shows classicality though any physical process on the microscopic scale is quantum. Moreover, if experiments with different types of particles all prove that \( v = v_0 \), then we can conclude that the particles learn the boundary condition by themselves. Thus the doubts [6, 7] on static experiments for testing Bell’s inequality can be clarified. Also, in double-slit interference type of phenomena, logically this result can even be understood as an evidence showing that a single particle indeed passes the two slits simultaneously. Or if we find \( v \neq v_0 \), then it will suggest that there are pilot waves or other intriguing mechanism which deliver the information on the status of the boundary, that travel separately from the particles.

We thank Prof. Hua-Zhong Li and Prof. Sofia Wechsler for valuable discussions. The work was supported in part by the NSF of China under grant Nos. 10975198 and 10605041, the NSF of Guangdong province under grant
No.9151027501000043, and the Foundation of Zhongshan University Advanced Research Center.

[1] M. Moshinsky, *Phys. Rev.* **81**, 347 (1951).
[2] A. del Campo, J. G. Muga, and M. Kleber, *Phys. Rev. A* **77**, 013608 (2008).
[3] R. Clifton, J. Bub, and H. Halvorson, *Found. Phys.* **33**, 1561 (2003).
[4] S. S. Afshar, *Proc. SPIE* **5866**, 229 (2005).
[5] D. Salart, A. Baas, C. Branciard, N. Gisin, and H. Zbinden, *Nature* **454**, 861 (2008).
[6] J. S. Bell, *Physics* **1**, 195 (1964).
[7] A. Aspect, J. Dalibard, and G. Roger, *Phys. Rev. Lett.* **49**, 1804 (1982).
[8] D. Bohm, *Phys. Rev.* **85**, 166 (1952).
[9] D. Bohm, *Phys. Rev.* **85**, 180 (1952).
[10] J. S. Bell, *Found. Phys.* **12**, 989 (1982).
[11] P. G. Merli, G. F. Missiroli, and G. Pozzi, *Am. J. Phys.* **44**, 306 (1976).
[12] O. Donati, G. P. Missiroli, and G. Pozzi, *Am. J. Phys.* **41**, 639 (1973).
[13] A. Tonomura, J. Endo, T. Matsuda, T. Kawasaki, and H. Ezawa, *Am. J. Phys.* **57**, 117 (1989).
[14] J. -Y. Chesnel, A. Hajaji, R. O. Barrachina, and F. Frémont, *Phys. Rev. Lett.* **98**, 100403 (2007).
[15] D. S. Milne-Brownlie, M. Foster, J. F. Gao, B. Lohmann, and D. H. Madison, *Phys. Rev. Lett.* **96**, 233201 (2006).
[16] H. T. Schmidt, et al., *Phys. Rev. Lett.* **101**, 083201 (2008).