Indirect adaptive control of nonlinearily parameterized nonlinear dissipative systems

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Abstract

In this note we address the problem of indirect adaptive (regulation or tracking) control of nonlinear, input affine dissipative systems. It is assumed that the supply rate, the storage and the internal dissipation functions may be expressed as nonlinearily parameterized regression equations where the mappings (depending on the unknown parameters) satisfy a monotonicity condition—this encompasses a large class of physical systems, including passive systems. We propose to estimate the system parameters using the “power-balance” equation, which is the differential version of the classical dissipation inequality, with a new estimator that ensures global, exponential, parameter convergence under the very weak assumption of interval excitation of the power-balance equation regressor. To design the indirect adaptive controller we make the standard assumption of existence of an asymptotically stabilizing controller that depends—possibly nonlinearly—on the unknown plant parameters, and apply a certainty-equivalent control law. The benefits of the proposed approach, with respect to other existing solutions, are illustrated with examples.

KEYWORDS

adaptive control, dissipative systems, nonlinear systems

1 | INTRODUCTION

The problem of adaptive control of nonlinear systems has attracted the attention of researchers for several years, see References 1–4 for a survey of the literature. The development of direct adaptive controllers, where we estimate directly the parameters of a full information stabilizing controllers, is stymied by the so-called matching condition Reference 1(section 3.3), which imposes severe constraints on the class of systems for which it is applicable. This obstacle is avoided for the case of systems with particular (triangular) structures, but this structural assumption is just mathematically motivated and is rarely verified in physical systems. Although it is possible, in some cases, to transform a general nonlinear system into a triangular one this requires the solution of a partial differential equations, which is difficult to find. For this reason, most of the recent attention has been centered on indirect adaptive controllers, where we estimate the parameters of the plant and then compute the parameters of the controller.

The implementation of indirect adaptive controllers is, in general, complex and very computationally demanding. This is mainly due to the fact that the parameterization that is used to obtain the linear regression equation (LRE) needed for their implementation is based on the state space model of the system dynamics, which involves complicated signal and parameter relations that are unrelated with the physical properties of the system. An additional difficulty is that, in order to obtain a linear relation in the LRE, it is often necessary to overparametrize the vector of unknown parameters.
This approach has very serious shortcomings, in particular the need of more stringent excitation conditions stemming from the fact that the parameter search takes place in a bigger dimensional space with nonunique minimizing solutions—see References 5 and 6 and the detailed discussion in Reference 7 (section 1). This situation has severely stymied the practical implementation of adaptive control techniques in many critical applications.

A route, pursued by some control researchers, to overcome this difficulty is to replace the complicated expression of the regressor using function approximation, for instance neural networks or fuzzy controllers. Unfortunately, as always with function approximation-based techniques, although they might lead to successful designs, there is no solid theoretical guarantee that the procedure will work. A second alternative is to use high-gain based schemes, like sliding modes or fractional power controllers that—as is well-known—suffer from their extreme sensitivity to the unavoidable presence of noise in the system, rendering them unfeasible in most practical applications.

For robotic applications it was suggested in Reference 10 (section 2.2) to use the parameterization of the power-balance equation to design an indirect adaptive controller. In an independent line of research, in Reference 11 (section 12.6.2) this parameterization was used for the identification of the robot parameters. The main advantage of this approach is that the resulting parameterization avoids the cumbersome terms related to the Coriolis and centrifugal forces matrix. This is a significant simplification that drastically reduces the complexity and computational demands. To the best of our knowledge, such an approach was never actually pursued, because the excitation requirements for the consistent estimation of the parameters is “very high”—see Reference 7 (remark 16). In the recent paper 12 a procedure to overcome, for the first time, this fundamental problem was proposed. Toward this end, a recent technique of generation of new LRE with “exciting” regressors developed in Reference 13 was used.

The main contributions of the paper, which include several generalizations of the results in References 7 and 12, may be summarized as follows.

C1 As indicated above most of the research on adaptive control has relied on the use of LREs, which are usually obtained overparameterizing the regression equations. In contrast with this approach we consider here the case where the uncertain parameters enter into the system dynamics in a nonlinear way and construct a nonlinearly parameterized regression equation (NLPRE). The interested reader is referred to References 7 and 14 for recent reviews of the literature dealing with NLPRE.

C2 In contrast to the results in References 7 and 12 where, to prove parameter convergence, it is necessary to assume some a priori nonverifiable conditions, in this paper we use the parameter estimator proposed in Reference 15—called G+D estimator—that ensures (global exponential) parameter convergence assuming only the extremely weak condition of interval excitation 16,17 of the original vector regressor. An additional advantage of the G+D estimator is that it can deal with a class of NLPREs—in particular, separable ones.

C3 We extend the technique, restricted in Reference 12 to Euler–Lagrange systems, to the much broader class of dissipative systems, 18,19 which contains as a particular case Euler–Lagrange and port-Hamiltonian systems.

C4 The use of the power balance equation, instead of the full dynamics of the system, to obtain the parameterization needed for an indirect adaptive control implementation, yields a significant numerical complexity reduction. The impact of such a simplification in the practical feasibility and the excitation requirements of the scheme can hardly be overestimated.

The remainder of the paper is organized as follows. In Section 2 we identify the class of systems for which our adaptive control result is applicable and present the problem formulation. In Section 3 we present the derivation of a NLPRE for the estimation of the unknown plant parameters proceeding from the dissipation inequality. We also recall in this section the standard parameterization that imposes the, rarely verified, assumption of linearity in the parameters of the system dynamics. Section 4 presents the proposed adaptive control scheme. Simulation results, which illustrate the performance of the proposed controller are presented in Section 5. The paper is wrapped-up with concluding remarks in Section 6.

Notation: $I_n$ is the $n \times n$ identity matrix. For $x \in \mathbb{R}^n$, we denote the square of the Euclidean norm as $|x|^2 := x^T x$. The action of an operator $F : \mathcal{L}_{\text{oc}} \rightarrow \mathcal{L}_{\text{oc}}$ on a signal $u(t)$ is denoted as $F[u]$. In particular, we define the derivative operator $\mathbf{p}[u] := \frac{du(t)}{dt}$. All mappings are assumed smooth and all dynamical systems are assumed to be forward complete. Given a function $f : \mathbb{R}^n \rightarrow \mathbb{R}$ we define the differential operator $\nabla f := \frac{df}{dx}^T$. To simplify the notation, whenever clear from the context, the arguments of the various functions are omitted.
2 | PROBLEM FORMULATION

2.1 | Plant description

Consider the input affine nonlinear system

\[
\begin{align*}
\dot{x} &= f(x, \theta) + g(x, \theta)u_p \\
y_p &= h(x, \theta) + j(x, \theta)u_p,
\end{align*}
\]

(1)

where \(x(t) \in \mathbb{R}^n\) is the system state, \(u_p(t) \in \mathbb{R}^{n_p}\) and \(y_p(t) \in \mathbb{R}^{n_p}\) are the port variables, \(\theta \in \mathbb{R}^q\) is a vector of unknown parameters and

\[
\begin{align*}
f &: \mathbb{R}^n \times \mathbb{R}^q \to \mathbb{R}^n, \\
g &: \mathbb{R}^n \times \mathbb{R}^q \to \mathbb{R}^{n \times n_p} \\
h &: \mathbb{R}^n \times \mathbb{R}^q \to \mathbb{R}^{n_p}, \\
j &: \mathbb{R}^n \times \mathbb{R}^q \to \mathbb{R}^{n_p \times n_p}.
\end{align*}
\]

To be able to treat systems with external sources, the vector \(u_p\) is assumed to be of the form \(u_p = \text{col}(u, E)\), where \(u(t) \in \mathbb{R}^m\) are the control signals and \(E(t) \in \mathbb{R}^{n_p - m}\) are external signals that represent uncontrollable sources (or loads).

2.2 | Assumptions

We make the following assumptions on the system.

A1 [Measurements] The system state \(x\) is measurable.

A2 [Dissipativity] The system (1) is dissipative with respect to the supply rate \(s : \mathbb{R}^{n_p} \times \mathbb{R}^{n_p} \times \mathbb{R}^q \to \mathbb{R}\), that is, there exist a storage function \(S : \mathbb{R}^n \times \mathbb{R}^q \to \mathbb{R}_+\) and an internal dissipation function \(d : \mathbb{R}^n \times \mathbb{R}^q \to \mathbb{R}_+\) such that

\[
\dot{S}(x, \theta) = -d(x, \theta) + s(u_p, y_p, \theta).
\]

(2)

A3 [Parameterization] The functions \(s, S\) and \(d\) admit the following separable NLPRE representation

\[
\begin{align*}
s(u_p, y_p, \theta) &= \phi_s^\top(u_p, y_p)G_s(\theta) + b_s(u_p, y_p) \\
S(x, \theta) &= \phi_S^\top(x)G_s(\theta) + b_S(x) \\
d(x, \theta) &= \phi_d^\top(x)G_d(\theta) + b_d(x),
\end{align*}
\]

(3)

where the functions

\[
\begin{align*}
\phi_s &: \mathbb{R}^{n_p} \times \mathbb{R}^{n_p} \to \mathbb{R}^{p_s}, \\
\phi_S &: \mathbb{R}^n \to \mathbb{R}^{p_s}, \\
\phi_d &: \mathbb{R}^n \to \mathbb{R}^{p_d} \\
b_s &: \mathbb{R}^{n_p} \times \mathbb{R}^{n_p} \to \mathbb{R}, \\
b_S &: \mathbb{R}^n \to \mathbb{R}, \\
b_d &: \mathbb{R}^n \to \mathbb{R}
\end{align*}
\]

and the mappings \(G_s : \mathbb{R}^q \to \mathbb{R}^{p_s}, G_S : \mathbb{R}^q \to \mathbb{R}^{p_s}\) and \(G_d : \mathbb{R}^q \to \mathbb{R}^{p_d}\) are known and

\[
p := p_s + p_s + p_d \geq q.
\]

A4 [Monotonicity] Define the mappings \(G : \mathbb{R}^q \to \mathbb{R}^p\)

\[
G(\theta) := \begin{bmatrix} G_s(\theta) \\ G_S(\theta) \\ G_d(\theta) \end{bmatrix}
\]

(4)
and \( \mathcal{W} : \mathbb{R}^q \to \mathbb{R}^q \)

\[
\mathcal{W}(\theta) := TG(\theta),
\]

where \( T \in \mathbb{R}^{q \times q} \) is chosen by the designer. Assume there is a positive definite matrix \( P \in \mathbb{R}^{q \times q} \) such that \( \mathcal{W}(\theta) \) is strongly \( P \)-monotonic\(^{20,21} \). That is,

\[
(a - b)^T P \left[ \mathcal{W}(a) - \mathcal{W}(b) \right] \geq \rho |a - b|^2 > 0, \quad \forall \ a, b \in \mathbb{R}^q,
\]

with \( a \neq b \) and for some \( \rho > 0 \).

**A5** [Stabilizability] Given a desired bounded trajectory for the state vector \( x_*(t) \in \mathbb{R}^n \), with bounded derivative. Define the state tracking error \( \hat{x} := x - x_* \). There exists a mapping \( \beta : \mathbb{R}^n \times \mathbb{R}^q \times \mathbb{R}_{\geq 0} \to \mathbb{R}^m \), such that the closed-loop system

\[
\dot{x} = f(x, \theta) + g(x, \theta) \left[ \beta(x, \theta, t) \right]
\]

has an error dynamics

\[
\dot{\hat{x}} = f_*(\hat{x}, \theta, E, t),
\]

whose origin is uniformly asymptotically stable (UAS).

### 2.3 Control objective

Design an estimator of the plant parameters \( \theta \)

\[
\dot{x}_\theta = f_\theta(x_\theta, \hat{x}, t)
\]

\[
\dot{\hat{\theta}} = h_\theta(x_\theta, \hat{x}, t),
\]

where \( f_\theta : \mathbb{R}^{n_\theta} \times \mathbb{R}^n \times \mathbb{R}_{\geq 0} \to \mathbb{R}^{n_\theta} \) and \( h_\theta : \mathbb{R}^{n_\theta} \times \mathbb{R}^n \times \mathbb{R}_{\geq 0} \to \mathbb{R}^q \) which ensures global, exponential convergence of the parameter errors \( \hat{\theta} := \hat{\theta} - \theta \) and such that the (certainty-equivalent) indirect adaptive control \( u = \beta(x, \hat{\theta}, t) \) ensures UAS of the zero equilibrium of the adaptively controlled error system

\[
\dot{\hat{x}} = f_*(\hat{x}, \theta, E, t) + g(\hat{x} + x_*, \theta) \left[ \beta(\hat{x} + x_*, \hat{\theta} + \theta, t) - \beta(\hat{x} + x_*, \theta, t) \right],
\]

Consequently,

\[
\lim \inf \hat{x}(t) = 0,
\]

with all signals bounded provided the initial errors \( \hat{x}(0) \) are sufficiently small.

### 2.4 Discussion

The following remarks are in order.

**P1** We have restricted ourselves to a local stabilization objective using static—versus dynamic—state-feedback controllers. As will become clear below, the extension to the case where the controller is dynamic follows verbatim.
However, to obtain global results in tracking it is necessary to strengthen the UAS Assumption A5 to global exponential stability. This additional assumption is not necessary in regulation tasks with static state-feedback controllers for which global asymptotic stabilization of the known parameter controller is sufficient for global stabilization of the adaptive one. See Reference 7(remark 12) for a discussion on this issue.

**P2** A convenient representation of the supply rate $s$ is the one corresponding to the so-called (QSR) dissipativity\textsuperscript{18} that is given as

$$s(u_p, y_p) = y_p^TQy_p + 2y_p^TSu_p + u_p^TRu_p,$$

where $Q$, $S$ and $R$ are $n_p \times n_p$ constant matrices, with $Q$ and $R$ symmetric. It is clear that the properties of passivity and finite $L_2$-gain are particular cases of (Q,S,R) dissipativity.\textsuperscript{18,19} In reference to the first property, and with some abuse of notation, we will refer to (2) as “power-balance” equation, which is the differential version of the classical dissipation inequality including the dissipation function $d$.

**P3** As is well-known\textsuperscript{1,6} the difference between direct and indirect adaptive controllers is that, in the former, it is assumed that there exists a parameter-dependent controller that achieves the control objective, while in the latter we additionally assume that the plant depends on some unknown parameters and that there exists a mapping between the plant and the controller parameters that allows us to compute the control signal. In direct adaptive control we estimate directly the parameters of the controller. On the other hand, in its indirect version the parameters of the plant are estimated and then the parameters of the controller are computed via the aforementioned mapping. To simplify the presentation in the problem formulation above we have obviated this latter step—embedding the mapping between the plant and controller parameters in the function $\beta$.

**P4** The Assumption A4 of strong $P$-monotonicity of $\mathcal{V}(\theta)$ is similar to the one used in Reference 7. The form adopted here was first proposed in Reference 15. As indicated in References 7,19,20,21 it is possible to verify the monotonicity assumption from the Jacobian of $\mathcal{V}(\theta)$ invoking Reference 7(lemma 1).

## 3 | DERIVATION OF THE NEW SYSTEMS PARAMETERIZATION

In this section we present the first contribution of the paper, namely the derivation of a NLPRE for the estimation of the unknown parameters $\theta$ proceeding from the “power-balance” equation (2)—to which we will refer in the sequel as power-balance equation parameterization (PBEP). As indicated in the Introduction this is a generalization of the procedure proposed for robot manipulators in References 11 and 10, see also Reference 12. We also recall in Section 3.2 that, imposing the assumption that the functions $f$ and $g$ of (1) admit a linear parameterization—usually doing some overparameterization—it is possible to apply standard filtering techniques to derive a LRE without the requirement of dissipativity. The advantages of the new PBEP, with respect to the latter one, are discussed in Section 3.3 below.

### 3.1 | Power-balance equation parameterization

**Proposition 1.** Consider the nonlinear system (1) satisfying Assumptions A1–A3. Fix an LTI, stable filter

$$F(\hat{p}) = \frac{\hat{\lambda}}{\hat{p} + \lambda},$$

with $\lambda > 0$. Define the signals

$$Y := F(\hat{p})[b_s - b_d] - \hat{p}F(\hat{p})[b_S]$$

$$\Omega := \begin{bmatrix} - F(\hat{p})[\phi_s] \\ \hat{p}F(\hat{p})[\phi_S] \\ F(\hat{p})[\phi_d] \end{bmatrix}.$$
The following NLPRE holds

\[ Y = \Omega^T G(\theta), \quad (12) \]

where \( G(\theta) \) is defined in (4).

**Proof.** We carry out the next operations

\[
\dot{S} = -d + s \quad \quad (\Leftrightarrow (2)) \\
pF(p)[S] = -F(p)[d] + F(p)[s] \quad \quad (\Leftrightarrow F(p)[\cdot]) \\
pF(p)[\phi_3 G_3(\theta) + b_3] = -F(p)[\phi_3 G_3(\theta) + b_3] + F(p)[\phi_3 G_{\phi}(\theta) + b_3] \quad \quad (\Leftrightarrow (3)) \\
F(p)[b_3] - F(p)[b_3] - pF(p)[b_3] = \left[ -F(p)[\phi_3^T] \quad pF(p)[\phi_3^T] \quad F(p)[\phi_3^T] \right] G(\theta) \quad \quad (\Leftrightarrow \theta = \text{const}, (4)) \\
Y = \Omega^T G(\theta) \quad \quad (\Leftrightarrow (11), (12)).
\]

\[ \blacksquare \]

### 3.2 Standard linear parametrization

To derive the standard parameterization we make the following assumption.

**A6 [Standard LPRE]** The vector field \( f \) and the elements of the matrix \( g \), that is, \( g_{ij} \) for \( i = 1, \ldots, n \), \( j = 1, \ldots, n_p \), admit the following parameterization

\[ f(x, \theta) = w_f(x) C(\theta) + b_f(x) \quad (13a) \]

\[ g_{ij}(x, \theta) = \phi_{3i}^T(x) C(\theta) + b_{ij}(x) \quad (13b) \]

with the mapping \( C : \mathbb{R}^q \to \mathbb{R}^{n_x} \), \( n_w \geq q \), and known functions

\[ w_f : \mathbb{R}^n \to \mathbb{R}^{nx}, \quad \phi_{3i} : \mathbb{R}^n \to \mathbb{R}^{n_w} \]

\[ b_f : \mathbb{R}^n \to \mathbb{R}, \quad b_{ij} : \mathbb{R}^n \to \mathbb{R}. \]

To streamline the presentation of the result we define the matrices

\[
w_g(x, u_p) := \begin{bmatrix} \sum_{j=1}^{n_p} \phi_{3i}^T(x) u_{pj} \\ \vdots \\ \sum_{j=1}^{n_p} \phi_{3i}^T(x) u_{pj} \end{bmatrix}, \quad B_g(x) := \begin{bmatrix} b_{i1}(x) & \cdots & b_{in_p}(x) \\ \vdots & \vdots & \vdots \\ b_{i1}(x) & \cdots & b_{in_p}(x) \end{bmatrix}, \quad (14)\]

and notice that \( w_g : \mathbb{R}^n \times \mathbb{R}^{n_p} \to \mathbb{R}^{nx}\) and \( B_g : \mathbb{R}^n \to \mathbb{R}^{nx} \).

**Proposition 2.** Consider the nonlinear system (1) verifying Assumption A6. Define the signals

\[
Y := pF(p)[x] - F(p)[b_f + B_g u_p] \\
\Omega^T := F(p)[w_f + w_g]
\]

where \( F(p) \) is defined in (10). Define the extended vector of parameters

\[ \Theta := C(\theta). \quad (16) \]
The following LRE holds

\[ Y = \Theta^\top \Theta \]  

(17)

**Proof.** The following set of operations is carried out

\[
\begin{align*}
\dot{x} &= (w_f + w_g) \Theta + b_f + B_g u_p \\
pF(p)[x] &= F(p)[(w_f + w_g) \Theta] + F(p)[b_f + B_g u_p] \\
pF(p)[x] - F(p)[b_f + B_g u_p] &= F(p)[w_f + w_g] \Theta \\
Y &= \Theta^\top \Theta
\end{align*}
\]

(\( \Leftrightarrow (1), (13), (16) \))

(\( \Leftrightarrow F(p)[\cdot] \))

(\( \Leftrightarrow \Theta = \text{const} \))

(\( \Leftrightarrow (15) \)).

\[ \blacksquare \]

### 3.3 Discussion

The following remarks are in order.

**P5** The main advantages of the PBEP with respect to the standard one include the following:

(i) The reduced complexity in the derivation of the corresponding parameterization (12) and (17) is evident comparing (11) with (15). The examples given in Section 5 will further illustrate this point.

(ii) Mathematical modeling of physical systems usually proceeds from a classification of its components into energy-storing and energy-dissipating, which appear explicitly in the storage and dissipation functions, respectively. These elements are interconnected among themselves and the external sources via the physical laws, for example, Kirchhoff’s or Newton’s—see Reference 22. In many cases, including mechanical, electrical, and electromechanical systems, the system parameters verify the “monotonicity” Assumption **A4**.

(iii) In contrast with the remark above, in the state space description (1) the physical parameters will enter the functions \( f \) and \( g \) multiplied among themselves rendering harder the verification of Assumption **A6**. Very often, for example, in robotics, it is possible to overparameterize the functions to comply with the linearity requirement. It is well-known that overparameterization has very severe shortcomings, see References 5 and 7 for a detailed discussion on this point.

**P6** A state-space realization of (11a) is given by

\[
\begin{align*}
\dot{X}_Y &= -\lambda(x_Y + b_S) + b_d - b_s \\
Y &= -\lambda(x_Y + b_S).
\end{align*}
\]

On the other hand, a state-space realization for (11b) is

\[
\begin{align*}
\dot{\Omega} &= \begin{bmatrix} -\lambda x_{\Omega_1} - \phi_s \\
-\lambda(x_{\Omega_1} - \phi_S) \\
-\lambda x_{\Omega_3} + \phi_d \end{bmatrix} \\
\Omega &= \lambda \text{col}(x_{\Omega_1}, \phi_S - x_{\Omega_2}, x_{\Omega_3}).
\end{align*}
\]

It is clear that the state space description of (15) is very similar to the one given above for (11), hence it is omitted for brevity.

**P7** As thoroughly discussed in References 19 and 22, many physical systems can be described by port-Hamiltonian models of the form

\[
\begin{align*}
\dot{x} &= [J(x) - \mathcal{R}] \nabla H(x) + g(x) u_p \\
y_p &= g^\top(x) \nabla H(x),
\end{align*}
\]
where \( J(x) = -J^T(x) \) and \( R = R^T \geq 0 \) is a constant matrix, representing the interconnection and dissipation structures and \( H : \mathbb{R}^n \to \mathbb{R}_+ \) is the energy function of the system. These systems are passive and they satisfy the power-balance equation (2) with

\[
s = u_p^T y_p, \quad S = H, \quad d = \nabla H^T R \nabla H.
\]

In many practical examples the energy function is of the form \( H(x) = \frac{1}{2}x^T Q x \), with \( Q > 0 \). In this case, Assumption A4 is satisfied introducing a reparameterization of the form \( Q^T R Q \) for the elements of the dissipation function \( d \).

P8 It is clear from the derivations above that \( F(p) \) can be replaced in both propositions by any strictly proper stable LTI filter. This degree of freedom can be exploited to attenuate the deleterious effect of measurement noise.

P9 In some applications part of the external sources, denoted \( E \in \mathbb{R}^{n_p-m} \) in the system description of Section 2, are unknown. If they enter in the supply rate in the form

\[
s(u_p, y_p) = s_k(u, y_p) + s_u^T(u, y_p) E,
\]

with known functions \( s_k \) and \( s_u \) and they are constant, it is possible to incorporate these uncertain parameters into the vector \( \theta \) and derive a new NLPRE that includes them.

P10 For the sake of clarity of presentation, in Assumption A3 we suppose that the supply rate \( s \) and the dissipation functions \( d \) admit independent parameterizations of the form (3a) and (3c). From the proof of Proposition 1 it is clear that we can replace this by the existence of a parameterization of the form

\[
s(u_p, y_p, \theta) + d(x, \theta) = \phi_{sd}^T(u_p, y_p, x) G_{sd}(\theta) + b_{sd}(u_p, y_p, x),
\]

with known functions \( \phi_{sd} \) and \( b_{sd} \). This variation will, in general, yield simpler expressions for the regressors.

4 | INDIRECT ADAPTIVE CONTROL

In this section we present the second main contribution of the paper, namely the construction of a UAS indirect adaptive controller for systems satisfying Assumptions A1–A5 using the PBEP (12) of Proposition 1. As indicated in point C2 of the Introduction the key step is the utilization of the G+D estimator of Reference 15(proposition 7) that we briefly recall in the lemma below—whose proof is given in the previous reference. Replacing this estimates in the controller of Assumption A5 yields the proposed adaptive controller.

4.1 | The G+D estimator of Reference 15(proposition 7)

As expected from an identification-based procedure some excitation assumptions will be required. However, as shown in Reference 15 it is possible to achieve global, exponential convergence of the parameter error imposing the following extremely weak interval excitation assumption of the regressor vector \( \Omega \) of the NLPRE (12).

A7 [Boundedness and Interval Excitation] The regressor vector \( \Omega \) of the NLPRE (12) is bounded* an interval exciting.

That is, there exists constants \( C_c > 0 \) and \( t_c > 0 \) such that

\[
\int_0^{t_c} \Omega(s) \Omega^T(s) ds \geq C_c I_q.
\]

Lemma 1. Consider the NLPRE (12) with \( \zeta(\theta) \) satisfying Assumption A4 and \( \Omega \) verifying Assumption A7. Define the G+D interlaced estimator

\[
\dot{\theta}_g = \gamma_g \Omega(Y - \Omega^T \dot{\theta}_g), \quad \theta_g(0) = \theta_{g0} \in \mathbb{R}^p, \quad \Phi = -\gamma_g \Omega \Omega^T \Phi, \quad \Phi(0) = I_p,
\]

with

\[
\Phi = -\gamma_g \Omega \Omega^T \Phi, \quad \Phi(0) = I_p,
\]
\[
\dot{\theta} = \gamma PT \Delta [\mathcal{Y} - \Delta \mathcal{G}(\hat{\theta})], \quad \hat{\theta}(0) = \theta_0 \in \mathbb{R}^q,
\]  
(18c)

with tuning gains \(\gamma_g > 0, \gamma > 0\), and we defined

\[
\Delta := \det(I_p - \Phi),
\]
(19a)

\[
\mathcal{Y} := \text{adj}(I_p - \Phi)[\hat{\theta}_g - \Phi \theta_0],
\]
(19b)

where \(\text{adj} \{ \cdot \}\) denotes the adjugate matrix. Then, for all \(\theta_0 \in \mathbb{R}^p\) and \(\theta_0 \in \mathbb{R}^q\), we have the exponential convergence

\[
\lim_{t \to \infty} \tilde{\theta}(t) = 0,
\]
(20)

with all signals bounded.

4.2 Main adaptive stabilization result

In the proposition below we present an indirect adaptive controller for the system (1) that ensures UAS of the closed-loop.

**Proposition 3.** Consider the non-linearly parameterized, nonlinear system (1) satisfying Assumptions A1–A5, with \(\Omega\), defined in (11b) of Proposition 1, satisfying Assumption A7. Let the adaptive control be given by

\[
u = \beta(x, \hat{\theta}, t),
\]

where \(\hat{\theta}\) is generated via the G+D parameter estimator of Lemma 1. Then, the zero equilibrium of the adaptive error system (8) is UAS. Consequently, (9) holds with all signals bounded provided the initial errors \(\tilde{x}(0)\) are sufficiently small.

**Proof.** First, notice that using (12) the error equation for the estimator is given by

\[
\dot{\hat{\theta}} = -\gamma \Delta^2 PT[\mathcal{G}(\hat{\theta}) - \mathcal{G}(\theta)] =: F_2(\hat{\theta}, t).
\]

In Reference 15 (proposition 7) the Lyapunov function candidate

\[
V(\hat{\theta}) := \frac{1}{2} \hat{\theta}^\top P^{-1} \hat{\theta},
\]

is used to show that, under Assumptions A4 and A7, its origin is **globally exponentially stable** .

Second, the state equation of the closed-loop system takes the form

\[
\dot{x} = f_s(x, \theta, t) + \chi(\tilde{x}, \hat{\theta}, t),
\]

where we defined the perturbation term

\[
\chi(\tilde{x}, \hat{\theta}, t) := g(\tilde{x} + x_s, \theta)
\]

\[
\begin{bmatrix}
\beta(\tilde{x} + x_s, \hat{\theta} + \theta, t) - \beta(\tilde{x} + x_s, \theta, t) \\
0
\end{bmatrix},
\]

which satisfies \(\chi(\tilde{x}, 0, t) = 0\). The overall dynamics of the closed-loop system clearly has a cascade form

\[
\dot{x} = F_1(\tilde{x}, \hat{\theta}, t)
\]

\[
\dot{\hat{\theta}} = F_2(\hat{\theta}, t),
\]
(21)

with \(F_1(0, 0, t) = 0\) and \(F_2(0, t) = 0\). Moreover, in Lemma 1 it is shown that all signals of the G+D estimator are bounded, consequently there exists a constant \(c > 0\) such that
\[
\sup_{t \geq 0} \sup_{|\delta| \leq c} \| \nabla_\delta F_2(\hat{\theta}, \delta) \| < \infty,
\]

where \( \| \cdot \| \) is the induced matrix norm. Assumption A5 ensures that the origin of the \( \bar{x} \) subsystem is an UAS equilibrium of the unperturbed system. Invoking Reference 23 (theorem 3.1) we conclude that the closed-loop system (21) has a UAS equilibrium at the origin.

5 | EXAMPLES

In this section the application of the indirect adaptive controller of Proposition 3 is illustrated with two different examples.

5.1 | A port-Hamiltonian system

Consider the LTI port-Hamiltonian system

\[
\dot{x} = \begin{bmatrix} 0 & -a \\ a & 0 \end{bmatrix} x + \begin{bmatrix} \theta \\ \theta^2 \end{bmatrix} u
\]

\[
y_p = \begin{bmatrix} \theta & \theta^2 \end{bmatrix} x,
\]

(22)

where \( a \) is known and the state is measurable, hence ensuring Assumption A1 of Proposition 3. The system is \( u \mapsto y_p \) is passive with storage function \( S(x) = \frac{1}{2} |x|^2 \) and admits the NLPRE (3a) with

\[
\phi_s = u, \quad G_s(\theta) = \begin{bmatrix} \theta \\ \theta^2 \end{bmatrix}.
\]

Hence Assumptions A2 and A3 are satisfied. Clearly, selecting \( T = [1 \quad 0] \) and \( P = 1 \) ensures the monotonicity Assumption A4 with \( \rho = 1 \).

Assume the control objective is to stabilize the zero equilibrium. The closed-loop polynomial for a linear state feedback of the form \( u = -\frac{1}{\theta} k^T x \) is given by

\[
s^2 + (k_1 + k_2 \theta)s + a(a - \theta k_1 + k_2),
\]

which is a Hurwitz polynomial for \( k = \text{col}(1, \theta) \). Therefore, the static state feedback

\[
\beta(x, \theta) = -\frac{1}{\theta} x_1 - x_2,
\]

ensures Assumption A5. Finally, since the regressor of the NLPRE (12) is given by

\[
\Omega = -F(p)[\beta(x, \hat{\theta})x],
\]

it is clear that the interval excitation Assumption A7 holds for all \( x(0) \neq 0 \). Since all assumptions of Proposition 3 are satisfied applying the adaptive controller

\[
u = \beta(x, \hat{\theta}),
\]

to the system (22), with \( \hat{\theta} \) generated with the G+D estimator of Lemma 1, ensures UAS of the closed-loop system.

In contrast with the situation above if we adopt the standard linear parameterization of Section 3.2 and apply a gradient estimator to the overparameterized LRE the resulting adaptive controller will fail. Indeed, for the system (22) the overparameterized LRE (17) is satisfied with
Therefore, the error dynamics for the gradient estimator have the form

\[ \dot{\Theta} = -\gamma \Omega^2 \Theta, \]
\[ x_\Omega = -\lambda x_\Omega + u, \]
\[ \Omega = \lambda x_\Omega, \]

where the solution of (23a) is

\[ \tilde{\Theta}(t) = \tilde{\Theta}(0)e^{-\gamma \int_0^t \Omega^2(\tau) d\tau}. \]

It follows that the gradient estimator of \( \Theta \) will ensure parameter convergence if and only if \( u(t) \not\in L^2 \). This condition will not be satisfied since, according to the control objective, it is desired that \( x \to 0 \) which implies that \( u \to 0 \) and, thus, \( x_\Omega \to 0 \).

### 5.2 An electrical circuit

Consider the electrical circuit depicted in Figure 1. The dynamics of this system is described by

\[
\begin{bmatrix}
\theta_1 & 0 \\
0 & \theta_1^a
\end{bmatrix} \dot{x} = \begin{bmatrix}
0 & 0 \\
0 & -\theta_2
\end{bmatrix} x + \begin{bmatrix}
-x_2 \\
x_1
\end{bmatrix} u + \begin{bmatrix}
E \\
0
\end{bmatrix}
\]
\[ y_p = x_1, \]

where the physical meaning of the state vector \( x \) and the parameters \( \theta_1 > 0 \) and \( E > 0 \) are given in the figure, with \( a \in \mathbb{R} \).

The system \( E \leftrightarrow x_1 \) is passive with storage function

\[ S(x) = \frac{1}{2}(\theta_1 x_1^2 + \theta_1^a x_2^2). \]

Moreover, it admits the NLPRE (3a) with

\[ b_s(x) = Ex_1, \quad \phi_s(x) = \frac{1}{2} \begin{bmatrix} x_1^2 \\ x_2^2 \end{bmatrix}, \quad \phi_d(x) = x_2^2, \quad \varphi_s(\theta) = \begin{bmatrix} \theta_1 \\ \theta_1^a \end{bmatrix}, \quad \varphi_d(\theta) = \theta_2. \]
Hence Assumptions A2 and A3 are satisfied. Clearly, selecting

\[ T = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}. \]

and \( P = \rho I_2 \) ensures the monotonicity Assumption A4 for any \( \rho > 0 \).

The set of assignable equilibria is given by

\[ \{ x \in \mathbb{R}^2 \mid E x_1 - \theta_2 x_2^2 = 0 \}. \]  \hspace{1cm} (26)

Assume the control objective is to regulate the voltage at \( x_2 = x_{2*} := \kappa \in \mathbb{R} \). It is possible to show that with the static state feedback

\[ u = \beta(x, \theta) := -k_p \left( \frac{\theta_2 \kappa^2}{E} x_2 - \kappa x_1 \right) + \frac{E}{\kappa} \]  \hspace{1cm} (27)

with free gain \( k_p > 0 \), the task is accomplished—that is, Assumption A5 is satisfied. To prove it, notice first that from the assignable equilibrium set (26), we get the value of \( x_1 \) at the equilibrium as \( x_{1*} := \frac{E x_2}{\kappa^2} \). Also, from the first equation of (24) with \( \dot{x} = 0 \), we obtain the value of \( u \) at the equilibrium, that is, \( u_* := \frac{E}{\kappa} \). Now, with \( \dot{(\cdot)} := (\cdot) - (\cdot)_* \), consider the Lyapunov function \( W = S(\bar{x}) \). Setting \( \dot{x} = 0 \) in (24), we get

\[ \begin{bmatrix} E \\ 0 \end{bmatrix} = - \begin{bmatrix} 0 & 0 \\ 0 & -\theta_2 \end{bmatrix} x - \begin{bmatrix} -x_{2*} \\ x_{1*} \end{bmatrix} u_* \]  \hspace{1cm} (28)

which substituted into (24) produces

\[ \begin{bmatrix} \theta_1 & 0 \\ 0 & \theta_2 \end{bmatrix} \dot{\bar{x}} = \begin{bmatrix} 0 & 0 \\ 0 & -\theta_2 \end{bmatrix} \bar{x} + \begin{bmatrix} -x_{2*} \\ x_{1*} \end{bmatrix} \bar{u} + \begin{bmatrix} -\bar{x}_2 \\ \bar{x}_1 \end{bmatrix} (\bar{u} + u_*). \]  \hspace{1cm} (29)

Thus, the time derivative of \( W \) is

\[ \dot{W} = -\theta_2 \bar{x}_2^2 + \left( \frac{\theta_2 \kappa^2}{E} \bar{x}_2 - \kappa \bar{x}_1 \right) \bar{u}. \]

From (27) and the definition of \( u_* \), it follows that \( \bar{u} = \beta - u_* \). Substituting the later into the last equation produces

\[ \dot{W} = -\theta_2 x_2^2 + k_p \left( \frac{\theta_2 \kappa^2}{E} \bar{x}_2 - \kappa \bar{x}_1 \right) \left( \frac{\theta_2 \kappa^2}{E} x_2 - \kappa x_1 \right) \]

\[ = -\theta_2 x_2^2 - k_p \left( \frac{\theta_2 \kappa^2}{E} \bar{x}_2 - \kappa \bar{x}_1 \right)^2 , \]

where, invoking LaSalle’s Invariance principle, we can conclude that \( x \to x_* \).

To implement the adaptive controller of Proposition 3 we compute from (11)

\[ Y = EF(p)[x_1], \quad \Omega = \begin{bmatrix} \frac{1}{2} p F(p) \left[ \begin{bmatrix} x_1^2 \\ x_2^2 \end{bmatrix} \right] \\ F(p)[x_1^2] \end{bmatrix}, \]

and define the mapping \( \mathcal{G}(\theta) = \text{col}(\theta_1, \theta_1^2, \theta_2) \).
FIGURE 2  Simulation results for power balance parametrization and G+D estimator

FIGURE 3  Simulation results for gradient estimator (30) and standard parametrization

On the other hand, the standard parameterization of Proposition 2 is computed with

\[
\begin{align*}
Y &= pF(p)[x] \\
\Omega^T &= F(p) \begin{bmatrix} -x_2 u + E & 0 & 0 \\ 0 & x_1 u & -x_2 \end{bmatrix},
\end{align*}
\]

with the overparameterized vector \( \Theta = \text{col} \left( \frac{1}{\theta_1}, \frac{1}{\theta_2}, \frac{1}{\theta_3} \right) \).

For the simulations we consider the adaptive controller \( u = \beta(x, \hat{\theta}) \), where the estimate \( \hat{\theta} \) is generated either by the G+D estimator of Lemma 1 or from a standard gradient estimator for the overparameterized LRE (17). That is,

\[
\dot{\hat{\Theta}} = \gamma \Omega(Y - \Omega^T \hat{\Theta}). \tag{30}
\]

The parameter values of the system are \( \theta_1 = 1, \theta_2 = 1.5, E = 15, \alpha = 2, k_p = 10, \) and \( \kappa = 15 \).

The gains of the G+D estimator and filter constant were taken as \( \gamma_g = 100, \gamma = 50, \) and \( \lambda = 10 \), respectively. The results of the simulation are shown in Figure 2. As seen from the figures the parameter estimates converge to their respective (Figure 2A) values and \( x_2(t) \rightarrow \kappa = 15 \) (Figure 2B), as desired.

In contrast with the situation above if we adopt the standard linear parameterization of Section 3.2 and apply the gradient estimator (30) to the overparameterized LRE the resulting adaptive controller will fail. This comes from the fact...
that the regulation task requires that $x \rightarrow x^*$. Therefore, from $27$, $u \rightarrow u^* = \frac{E}{k}$. Since $x$ and $u$ converge to a constant, then $\Omega \rightarrow \Omega^*$—that is, $\Omega$ converges to a constant as well. Thus, $\Omega$ is not PE.

The simulation results of this second scenario are shown in Figure 3 for estimator gain $\gamma = 30$ and the remaining parameters selected as before. In Figure 3A, the estimation performance is shown for $\Theta$. Zooming in the plot of the estimated parameters in this figure, it can be seen that $\hat{\Theta}(t) \rightarrow \text{col}(1.00, 0.17, 0.25)$, however, $\Theta = \text{col}(1.00, 1.00, 1.50)$. That is, the estimation is deficient since $\hat{\Theta}$. This lead to an erroneous estimate of $\theta_2$ and, as consequence of that, the regulation performance is poor. This is evident in Figure 3B where $x_2$ is not driven to its setpoint $\kappa = 15$.

6 | CONCLUSIONS

We have presented in the article a procedure to identify the parameters of nonlinear, non-linearly parameterized, dissipative systems of the form (1). The method is based on the power balance equation of the system (2), avoiding in this way the messy computations and stringent excitation requirements related with the standard parameterization of the systems vector field and input matrix given in (13). Invoking the G+D parameter estimator proposed in Reference 15, which ensures global exponential convergence of the parameter error under very weak regressor excitation assumptions, we proposed an indirect adaptive controller that guarantees UAS of the closed-loop system.

Adaptive control was one of the main research topics in control from the 70s to the mid-90s. The development and analysis of the problem’s many and varied solutions over all these years, unquestionably played an absolutely major role in guiding us to our present understanding of control theory in general. Although many critical issues remained open, a large part of the control community moved away from the field. Partly responsible for this unfortunate situation was the deviation of the problem formulation from a self-tuning procedure to the—more mathematically tractable but of little practical relevance—stabilization technique.

For at least two reasons it is reasonable to expect a renewed interest in adaptive control and identification theories in the near future. On one hand, to comply with the increasing performance requirements imposed to modern control systems it is necessary to develop efficient controller tuning procedures, that a well-formulated, adaptive control theory can provide. On the other hand, in recent years we have witnessed an explosion of references to the hyped-up artificial intelligence field, which is simply the application of a neural network-based structure to a massive collection of data, whose success in some particular applications has been widely publicized. Obviously, the interest for a scientific theoretical field of “procedures that work in some examples” is highly questionable. Because of the prevalence of non-linear parameterizations in neural networks, very little theoretical understanding is available on adaptive neural networks—a situation that was already denounced 25 years ago and is still prevalent. Development of solid theoretical foundations are essential to turn this tide, an endeavor where adaptive control should play a central role.

CONFLICT OF INTEREST

The authors have declared no conflict of interest.

DATA AVAILABILITY STATEMENT

Data sharing not applicable to this article as no datasets were generated or analyzed during the current study.

ENDNOTE

*Boundedness of $\Omega$ is a blanket assumption made to avoid technicalities in the proofs.

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