Classical Decay of the Non-SUSY-Preserving Configuration of Two D-Branes.

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Abstract

We have studied a problem of the tachyon mediated D-brane - D-brane annihilation from the underlying world-volume gauge field theory point of view. The initial state was chosen in the form of two D-branes crossing at a non-zero angle, which is non-supersymmetric configuration generically. This state was not a groundstate of the theory and the problem at hand was a model, where we had rather precise control over the behavior of the theory. Some applications of this model to the D-brane physics and conclusions about stability of several configurations were made. By taking a T-dual picture of this process on a $T^2$ torus, we derived known conclusion about the instability of a system consisting of D-0 and D-2 branes, and found its decay modes.
1 Introduction.

The Dirichlet $p$-branes in the type II superstring theories are defined as objects, on which the fundamental strings can end [1]. The low energy dynamics of D-branes comes from massless modes in the open-string sector of a worldsheet theory. In the case of a flat supergravity background, it is given by a gauge field theory dimensionally reduced from 10 dimensions down to $p+1$ [2, 3]. For slowly varying fields this theory can be approximated by a world-volume Dirac-Born-Infeld theory with a gauge group $U(n)$, and for small perturbations around flat D-brane surface it can be approximated further by a $U(n)$ super-Yang-Mills theory. We will use the latter approximation, although it is discovered lately that there are important phenomena missing in the Y-M approximation. For example, the process in which a fundamental string emanates from a D-brane (cf. [7, 8]).

The translation between the languages of the (super-)Yang-Mills theory on a world-volume and the perturbative string theory with D-branes included can be done in the following manner. If in the language of a 10-dimensional $U(n)$ Yang-Mills in some particular gauge, we have a solution for a vector field $\hat{A}_\mu$

$$\hat{A}_\mu = \frac{1}{2} \left( \begin{array}{c} A^{(1)}_\mu(x_0...x_p) \\ 0 \\ A^{(2)}_\mu(x_0...x_p) \\ ... \\ 0 \\ A^{(n)}_\mu(x_0...x_p) \end{array} \right),$$

then we say, that $\frac{1}{2\pi\alpha'}A^{(k)}_i(x_0...x_p)$ define transverse positions of $n$ D$p$-branes for $i = p + 1, ..., 9$, and $A^{(k)}_a(x_0...x_p)$ for $a = 0..p$ is just a world-volume gauge field, living in a D-brane ($k$). All the off-diagonal excitations around this configuration, like $A^{(i)}_{ij}$, correspond to the fundamental open strings stretched from the brane ($i$) to the brane ($j$).

As an example of using this dictionary, one can consider a D-string twice wound around the circle $S^1$. In the language of the gauge fields it corresponds to a twisted boundary condition on a circle, which permutes diagonal elements of a $U(2)$ matrix.

There are different configurations of D-branes preserving some number of supersymmetries. One D-brane, for example, breaks down one half of all the supersymmetries. Other known configurations can preserve other part of supersymmetries, like 1/4 or 3/8 ([9, 10]). Any D-brane configuration preserving at least some of supersymmetries is a true groundstate of the theory.

In this paper we describe in some details the behavior of a system consisting of two D-strings crossing at a non-zero angle (parameter $\theta$ is a tangent of the crossing angle) and separated in one more spatial direction by the distance $a$. This configuration breaks down all the supersymmetries so it is not a groundstate and can decay into something else. We found that this configuration indeed was unstable when $\theta - 2\pi\alpha' a^2 > 0$. The decay mode of this system can be effectively described in

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1 for reviews see [4, 5, 6]
the string theory language as a process which intercommutes the ends of D-strings. The bent string emerging as a result moves away from the crossing point. We found that the characteristic time of this decay is of order $\tau \approx l_s(\theta - a^2/l_s^2)^{-\frac{1}{2}}$. By taking this theory on a $T^2$ and by T-dualizing it, we find similar conditions for unstability of a D2-D0 system, when the angle between two D-strings translates into the complex structure of a $\tilde{T}^2$.

2 D-strings crossing.

The model we have studied was the configuration of two D-strings, which are separated along $x^2$ coordinate by distance $a$, and are rotated one relatively to another in the $(x^1, x^3)$ plane by a non-zero angle. Everywhere in the text we use the parameter, $\theta$ which is a tangent of a crossing angle between these two D-strings. The static force between two crossing D-strings was calculated using string technics in one loop order in $[11]$.

We would like to emphasize, that since the configuration considered is not BPS, the spectrum will have quantum corrections.

Conventions: In this paper we use dimensionless units, by taking $\frac{1}{2\pi\alpha'} = l_s^2 = 1$ and the signature is taken $(-,+,...+)$. Greek indices $\mu, \nu...$ are 10-dim indices, $a, b...$ are world-volume indices or SU(2) group indices, depending on a context and $i, j...$ are indices in the directions transverse to a D-brane.

Using the property that U(1) subgroup of U(2) corresponds to the center of mass motion and decouples, we will use SU(2) as the gauge group of the theory.

2.1 First approach. Off-diagonal excitations around two-strings-crossing configuration.

In this work we consider unstable classical solutions containing bosonic fields only. Our starting point was a 10-dim Yang-Mills SU(2) vector field $A_\mu$, with the bosonic part of the Lagrangian density given by

$$\mathcal{L} = -\frac{1}{2} \text{Tr} F_{\mu\nu} F^{\mu\nu}. \quad (2.2)$$

Let’s consider excitations around field configuration, corresponding to two D-strings crossing at some non-zero angle:

$$\hat{A}_\mu = \frac{1}{2} \begin{pmatrix} a_\mu & 0 \\ 0 & -a_\mu \end{pmatrix}, \quad (2.3)$$

where $a_\mu = (0, 0, a, \theta x, 0, 0)$, where plain $x$ will denote a spatial coordinate $x_1$ along the D-string. We have considered the case with D-strings only, so the value of $p = 1$ is taken.

In this chapter we will focus on modes corresponding to a string, stretched between branes “1”
and “2” only\footnote{It means we will not consider the motion of a brane itself for awhile.}, so we take perturbations in the form

$$\delta \hat{A}_\mu = \frac{1}{2} \begin{pmatrix} 0 & A^\mu \\ A^*_\mu & 0 \end{pmatrix}$$

(2.4)

It turns out that the tachyon excitation lives in this sector.

Performing dimension reduction, we get the 2-dimensional theory with “mixing” for fields $A_0, A_1, \Phi_2$ and $\Phi_3$, and $x$-dependent “mass” term of form $m^2 = a^2 + \theta^2 x^2$ for every other field $\Phi_k$ in this problem. The eigenstates and eigenvalues of the Lagrangian $x$–differential operator we got, correspond to the masses we would have had, if we had no explicit $x$–dependence in this operator.

### 2.2 Lagrangian and Hamiltonian formulation of the world-volume theory.

It turns out that the search for a static spectrum is easier to perform in the Hamiltonian formulation of that theory. We will not fix any gauge freedom present in the Hamiltonian, since we are considering classical excitations and any allowed gauge transformations can be undone later, if one needs it, in terms of the classical solutions.

The canonical momenta conjugated to the fields $A_{0,1}, \Phi_{2,9}$ are $\pi_{0,9}$ and the Hamiltonian for this system looks like

$$H = \int dx (2\pi^*_0 \pi_{1,9} + (A_0 \partial_1 \pi_1 + ia \pi_2 + i\theta x \pi_3) + c.c.)$$

$$+ \frac{1}{2} \begin{pmatrix} A_1^* & \Phi_2^* & \Phi_3^* \end{pmatrix} \begin{pmatrix} a^2 + x^2 \theta^2 & ia \partial_1 & ix \theta \partial_1 - i \theta \\ ia \partial_1 & -\partial_1^2 + x^2 \theta^2 & -ax \theta \\ ix \theta \partial_1 + 2 i \theta & -ax \theta & -\partial_1^2 + a^2 \end{pmatrix} \begin{pmatrix} A_1 \\ \Phi_2 \\ \Phi_3 \end{pmatrix} - \frac{1}{16} (A_{\mu} A^*_{\nu})^2$$

(2.5)

Let

$$\mathbb{H} = \begin{pmatrix} a^2 + x^2 \theta^2 & ia \partial_1 & ix \theta \partial_1 - i \theta \\ ia \partial_1 & -\partial_1^2 + x^2 \theta^2 & -ax \theta \\ ix \theta \partial_1 + 2 i \theta & -ax \theta & -\partial_1^2 + a^2 \end{pmatrix}$$

(2.6)

From the \textit{bona fide} quadratic ($x^2$) behavior of the “potential” (as well as from the point of view of a stretched string) we would expect the normal modes of a differential operator $\mathbb{H}$ to be the oscillatory ones. This ansatz indeed gives us the lowest normalizable static eigenstate as:

$$T \propto \begin{pmatrix} i \\ 0 \\ 1 \end{pmatrix} e^{-\frac{ax^2}{2}},$$

(2.7)

with $\mathbb{H} T = (a^2 - \theta) T$, and it becomes tachyonic if $a^2 < \theta$.

The second lowest eigenvalue of this operator is $(a^2 + \theta)$, it can not become tachyonic, so it can not lead to any instability.
2.3 \( T^4 \) member of Lagrangian. “Stable” tachyonic vev.

Since the eigenstates \((V_k)\) of \(H\) form a complete basis in the Hilbert space \(H^3\), we can write arbitrary \(A_1\) and \(\Phi_{2..9}\) as

\[
A_1(t, x) = \Sigma_k Z_{1,k}(t) V_k(x) = -i Z_0(t) e^{-\frac{a^2}{2}} + \Sigma_{k\neq0} Z_{1,k}(t) V_k(x) \tag{2.8}
\]

\[
\Phi_3(t, x) = \Sigma_k Z_{3,k}(t) V_k(x) = Z_0(t) e^{-\frac{a^2}{2}} + \Sigma_{k\neq0} Z_{3,k}(t) V_k(x) \tag{2.9}
\]

\[
\Phi_i(t, x) = \Sigma_{k\neq0} Z_{i,k}(t) V_k(x), \quad i = 2, 4..9 \tag{2.10}
\]

Of all the original fields, only \(A_1\) and \(\Phi_3\) contain a tachyon. Supposing that no other fields are excited, and we are interested in the terms in the Hamiltonian containing \(Z_0\) only, then we can substitute (2.10) into \(H\) and integrate over \(x\). We get the following “potential” for (static) \(Z_0\):

\[
H = \sqrt{\frac{\pi}{\theta}} \left( (a^2 - \theta) Z_0^* Z_0 + \frac{1}{2\sqrt{2}} (Z_0^* Z_0)^2 \right) + \ldots \tag{2.11}
\]

So when \(a^2 - \theta < 0\) we have the symmetry spontaneously broken, and even if we have \(Z_0(t = 0) = 0\), it evolves with time to a non-zero value:

\[
\langle Z_0^* Z_0 \rangle = - (a^2 - \theta) \sqrt{2} \tag{2.12}
\]

Assuming \(\langle Z_0 \rangle\) real we get the following stable static vevs\(^3\) for the initial fields

\[
\langle A_1 \rangle = -i \sqrt{- (a^2 - \theta) \sqrt{2} e^{-\frac{a^2}{2}}}, \tag{2.13}
\]

\[
\langle \Phi_3 \rangle = \sqrt{- (a^2 - \theta) \sqrt{2} e^{-\frac{a^2}{2}}}, \tag{2.14}
\]

and no other fields evolve to a non-zero static value due to this process.

2.4 Decay of the initial configuration.

The initial configuration corresponds to two D-strings crossed. Since at the initial moment of time the gauge field is zero, the Wilson’s line is trivial, so the D-strings in the original picture are distinguishable entities. The U(2) gauge field \(A_1\) becomes non-zero as the time goes on, so that the

\[
\langle A_1 \rangle = \frac{1}{2} \left( \begin{array}{cc} 0 & \langle A_1(x) \rangle \\ \langle A_1^*(x) \rangle & 0 \end{array} \right), \tag{2.15}
\]

gives a non-trivial Wilson’s line for the original (fixed in the given framework) diagonal \(\Phi_3\) configuration

\[
U_p(\infty, -\infty) = P \exp(i \int dx \langle A_1(x) \rangle) = \left( \begin{array}{cc} \cos(c) & -\sin(c) \\ \sin(c) & \cos(c) \end{array} \right), \tag{2.16}
\]

where \(c = \sqrt{\frac{\pi}{\theta\sqrt{2}}} \sqrt{- (a^2 - \theta)}\)

\(^3\)Under the term vev we assume just a classical solution for the equations of motion.
We immediately see, that such a tachyon mediated evolution mixes the ends of two original D-strings. They do not belong to different distinguishable D-strings anymore. A problem with that perturbative solution we found is that it does not exchange ends completely \((c \neq \pi/2)\), but just mixes them. This is just an artifact of taking a perturbative approach and including off-diagonal excitations only. The decay of the resulting system, consisting of two bent D-strings cannot be found in the framework of this chapter, since we did not consider any change for diagonal part of \(\hat{\Phi}\) matrix, which describes the positions of two D-branes. The bent D-strings will obviously tend to straighten themselves, so they effectively will move away from the crossing point. We will show in the next chapter, that the running away solution is indeed a stable classical solution for a full non-perturbative interacting theory.

3 Unstable and stable configurations.

Since we have established a condition when the tachyonic mode arises: \(a^2 - \theta < 0\), (or in dimensionful quantities \(a^2 - l_s^2\theta < 0\)) we can restrict ourselves to the simpler case of \(a = 0\) and we will not lose anything interesting. The physics of the process with a tachyonic mode and \(a \neq 0\) is the same as the one considered below. Something different can happen when we take a boundary value, \(a^2 - \theta = 0\), and we might have a classical perturbative flat direction, free for the system to move along. It may be unstable when properly considered, with all the higher order terms included.

3.1 Full interacting SU(2) theory. Unstable solution.

\[
\mathcal{L} = -\frac{1}{2} \text{Tr} F_{\mu\nu} F^{\mu\nu}. \tag{3.17}
\]

After dimensional reduction to \(1+1\) dimensions the Lagrangian becomes:

\[
\mathcal{L}_R = \frac{1}{2} (F_{01}^a)^2 + \frac{1}{2} (D_0 \Phi_1^a)^2 - \frac{1}{2} (D_1 \Phi_3^a)^2 - \frac{1}{4} (\epsilon^{abc} \Phi_i^a \Phi_j^b)^2 \tag{3.18}
\]

Let’s consider such classical field configurations only, which have non-zero fields \(A_1^2, \Phi_1^3, \Phi_3^3\), and all the other fields are vanishing. As we have established in the previous chapter, those chosen \(A_1^2, \Phi_3^3\), are sufficient to describe a tachyon, and the \(\Phi_3^3\) will describe the string’s position.

(N.B. that the field \(A_0\) provides us with one equation of motion, the 3.19(1) below, even vanishing by itself.)

To clarify notation we will use from now on the following symbols:

\[
A \equiv A_1^2, \quad \Phi_1 \equiv \Phi_3^3, \quad \Phi_3 \equiv \Phi_3^3
\]

\(^4\)Upper \(a\) index is a SU(2) group index, running 1..3
The corresponding equations of motion are

\begin{align*}
(1) & \quad -\dot{A}' + \Phi_3 \dot{\Phi}_1 - \Phi_3 \dot{\Phi}_1 = 0 \\
(2) & \quad -\ddot{A} + \Phi_3 \Phi_1' - \Phi_3' \Phi_1 = A(\Phi_1^2 + \Phi_3^2) = 0 \\
(3) & \quad -\ddot{\Phi}_1 + \Phi_1'' - A' \Phi_3 - 2A \Phi_3' - A^2 \Phi_1 = 0 \\
(4) & \quad -\ddot{\Phi}_3 + \Phi_3'' + A' \Phi_1 + 2A \Phi_1' - A^2 \Phi_3 = 0
\end{align*}

\hfill (3.19)

The static solution, corresponding to the initial configuration of two crossing 1-branes is given in our original gauge by:

\[ A = 0, \quad \Phi_1 = 0, \quad \Phi_3 = \theta x. \]

Let this gauge, where the initial configuration looks this way be called \textbf{Gauge-A}.

As we know from the previous chapter, this solution is unstable relative to the perturbation of the form \( \delta A = -\epsilon, \ \delta \Phi_1 = \epsilon \). Now, we will show this explicitly using different methods.

\section*{3.2 Gauge-A and Gauge-B.}

We will call \textbf{Gauge-B} any gauge, where \( \Phi_3 \) defines bent strings. Any transformation between \( A \rightarrow B \) is given by a transformation non-trivial at one of the spatial infinities, while any \( B \rightarrow B' \) or \( A \rightarrow A' \) is given by a gauge transformation trivial at the spatial infinities.

For our initial configuration, it’s obvious, that any \textbf{Gauge-A} has the trivial Wilson’s line \( U_p \), while any of \textbf{Gauge-B} has purely off-diagonal \( U_p \).

Local and time-independent SU(2) gauge transformation of form

\[ g(x) = \begin{pmatrix} f_1(x) & f_2(x) \\ -f_2(x) & f_1(x) \end{pmatrix} \]

where

\[ (f_1(x))^2 = \frac{1}{2}(1 + \text{th}(\alpha x)) \]

\[ (f_2(x))^2 = \frac{1}{2}(1 - \text{th}(\alpha x)) \]
with \( \alpha \) – is an arbitrary parameter, is an example of a gauge transformation, which takes us to the \textbf{Gauge-B}. In this particular gauge, the initial static configuration of fields plus perturbation of our kind (in the \textbf{Gauge-A}: \( A = -\epsilon, \Phi_1 = \epsilon, \Phi_3 = \theta x \)) looks like:

\[
A = \frac{\alpha}{\ch x} - \epsilon
\]

\[
\Phi_1 = -\frac{\theta x}{\ch x} + \epsilon \th x
\]

\[
\Phi_3 = \theta x \th x + \frac{\epsilon}{\ch x}
\]

The initial configuration in the \textbf{Gauge-B} has the Wilson’s line

\[
U_p(\infty, -\infty) = P \exp(i \int_{-\infty}^{+\infty} dx \hat{A}(x)) = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}, \tag{3.20}
\]

completely off-diagonal, as expected, so these ‘bent’ strings are really cross-connected (as it was obvious in the \textbf{Gauge-A}).

If we take \( \epsilon \) of form \( \epsilon(t, x) = e^{i\omega t} \epsilon(x) \), we’ll find that in the approximation linear in \( \epsilon \), the unstable solution with \( \omega^2 = -\theta < 0 \) exists and corresponds to \( \epsilon(x) \propto e^{-\theta x^2/2} \). So this is indeed the direction of decay for our original configuration. By taking \( \epsilon(t, x) = 0 \) and slightly “kicking” this initial field configuration, we will move along this downward direction until a new (non-static) equilibrium is found.

Evolution of the Wilson’s line in the \textbf{Gauge-B} looks like

\[
U_p(\infty, -\infty) = P \exp(i \int_{-\infty}^{+\infty} dx \hat{A}(x) - \hat{\epsilon}(x)) \approx \begin{pmatrix} \epsilon(t) & 1 - \epsilon(t)^2/2 \\ -(1 - \epsilon(t)^2/2) & \epsilon(t) \end{pmatrix}, \tag{3.21}
\]

so the classical motion downward in this direction is going to mix the ends of D-strings.

### 3.3 Elementary estimation of a decay rate for the scattering process.

We have considered the following initial configuration (in the \textbf{Gauge-A}):

\[
\Phi_3^2 = \theta x, \quad \Phi_2^2 = \text{const} - vt, \tag{3.22}
\]

similar to the configurations discussed in the section (2), for the D-strings crossing at some distance \( a \), except that now the D-strings are moving. This configuration satisfies equations of motion, so it is an (unstable) solution. To estimate this instability for different values of \( v \) - relative velocities of D-strings, we notice, that the characteristic time of D-string decay, as prompted by \( \exp(\sqrt{\theta t}) \) behavior of \( \epsilon(t) \) is \( \tau_d = \theta^{-1/2} \) while the characteristic time of interaction is \( \tau_i = \theta/v \). So if \( v \leq \theta^{3/4} \) these decay processes are important.

**NB:** The estimation can break down, since the decay rate \( \tau_D = (\theta - a^2)^{-3/4} \) was calculated in the assumption of static brane configuration.
At time $t > 0$

$0 < A < A(t=0)$

Asymptotically, at large time $t$

Figure 2: Evolution of the initial configuration in the Gauge-B.

### 3.4 Stable final configurations.

On the other hand, the final configuration (as $t \to \infty$), is supposed to have $A = 0$ in one of the Gauges-B, and the equations of motion for the fields $A$, $\Phi_1$, $\Phi_3$ imply the wave equations for fields $\Phi_1$ and $\Phi_3$

$$-\ddot{\Phi}_{1,3} + \Phi''_{1,3} = 0, \text{ with relation } \Phi_1 = \text{const } \Phi_3.$$  

So we can eliminate $\Phi_1$ from the final state completely by a global gauge transformation.

To show that this configuration is stable, we need to notice only, that for such an asymptotic wavelike behavior of $\Phi_3$, it becomes arbitrarily big as time $t \to \infty$: i.e. the D-strings become widely separated, and the separation increases with the time.

$$\min \| \Phi_3(t) \| \geq \theta t$$

The linearized equation of motion 3.19(2) for a small $\delta A \propto e^{i \omega t}$ perturbation around such a solution looks like:

$$\omega^2 \delta A = -\Phi_3 \delta \Phi_1' + \Phi_3' \delta \Phi_1 + (\Phi_3)^2 \delta A,$$  

and the eventually huge positive part $(\Phi_3)^2$ kills everything else. So there are no “downward” directions which can change $A$, but there is a flat direction which leaves $A$ intact and moves $\Phi_1$ in the $\Phi_3$ direction. As was mentioned above, this can be undone by some global gauge transformation, which does not change anything physically.

### 3.5 Open questions.

Since the behavior of $\Phi_3(t)$ was not found explicitly for the intermediate times, we cannot say for sure which of the final states the system chooses, i.e. what is the final profile of a wave-like solution.

**NB:** The Born-Infeld action was approximated by a Yang-Mills action to consider the stability of a static solution, and it was a good approximation – i.e. for a small deviation from a flat D-brane surface. The Born-Infeld action is more appropriate for the purpose of describing a running away
and bent D-string. As we have seen above, interaction between these runaway D-strings decreases as the separation increases, and at some point we can neglect the interaction with the other D-string completely, thus the Born-Infeld action becomes Nambu-Goto action, describing a lone runaway string.

For a 1+1-dimensional worldsheet in the classical case the question will be very simple, it is just a choice of a proper parametrization for a coordinate $\sigma$ along the string. We cannot choose $X_1 = \sigma$ as before, but taking a wavelike solution (in the light cone coordinates $x_+ = \frac{1}{2}(\sigma + \tau)$ and $x_- = \frac{1}{2}(\sigma - \tau)$) in a form of

$$
\begin{align*}
X_0 &= x_+ - x_- \\
X_1 &= f_1(x_+) + f_1(x_-) \\
X_3 &= f_3(x_+) + f_3(x_-),
\end{align*}
$$

with an extra-condition

$$
f_1'^2 + f_3'^2 = 1
$$

we find that it satisfies the Nambu-Goto equations of motion, and (3.25) provides us inexplicit expression for $\sigma(X_1(\tau = 0))$ giving in principle the shape $X_3(X_1, \tau)$ as expected.

### 4 Crossing strings on a torus and a dual torus.

#### 4.1 The toroidal compactification.

We put the D-strings crossing on a torus $T^2$, which compactifies dimensions $x^1$ and $x^3$ (cf. fig. 3). Since we have considered the U(2) group only, it describes our initial configuration for two D-strings with a winding numbers $1+1$ along the foundations of a torus, so they are parallel to the sides of a torus. Thus the method considered describes only tori with zero flux and such a modular parameter $\tau$ so $\text{Im} \ \tau/\text{Re} \ \tau = \theta$.

The decay approximation is valid when the dimensions of the torus are much larger than the tachyon field size, $l_s \theta^{-1/2}$. The final configuration in the case of decay corresponds to one D-string with a winding number $1_1 + 1_3 = 2$ around this $T^2$.

#### 4.2 T-dual picture.

If we consider a picture in a torus $\tilde{T}^2$, T-dual to original one along one direction (say $x^1$) only, then we have the initial configuration of a D-2 brane (from a D-string along the $x^3$ on a $T^2$) and a D-0

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5If we have one D-string wound more than once, say $n$ times, then the corresponding Yang-Mills theory, which is to describe this D-string, must have a gauge group U(N), with N not less than n. Otherwise we cannot embed any non-trivial Wilson’s line distinguishing between different turns. Alternatively, if we think about the theory in a small region far from the torus’ borders, n-wound theory is a gauge U(n) theory. So the case of U(2) describes two single-wound D-strings (or one double wound), and this is possible for non-parallel D-strings only in the case when sides of a torus are parallel to our D-strings.
brane (from a D-string wound around the $x^1$). From the previous discussion we are aware of the instability of this configuration, we can conclude that in the final state D-0 brane 'dissolves' in a D-2 brane wrapped around $\tilde{T}^2$ and, in principle, we can describe this process of dissolution. The final state can be best described as once wrapped D-2 brane on a $\tilde{T}^2$ with one unit of uniformly distributed flux (cf. [6] ch.4.4).

Thus the combination of D-0 and D-2 branes compactified on tori are unstable\footnote{this fact can be easily understood in terms of energy minimization (cf. [6] and ref. thereof)}. The measure of the instability for this picture is hidden in the background magnetic field $B$. By introducing a very small $B$ we can control the behavior like we did in the original case using $\theta = \text{Im} \tau / \text{Re} \tau$ (under T-duality $\theta \leftrightarrow \frac{V}{\tau^2} B$).

5 Final Remarks.

The virtue of the model considered was that we were able to study the decay process involving tachyonic excitation explicitly, and one can perform numerical computations in principle, to find an exact shape of the final state. We understand that our example of the Gauge-B was chosen arbitrarily, and perhaps it is not the best one. But using it we can predict an asymptotic behavior of the final state.

We did not consider here any contribution from fermionic fields. We have assumed superstring
theory as a starting point for our calculations, so that we did not have a bosonic tachyon apriori (with a mass of order $1/l_s$).

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