ANALYSIS ON COURSE OUTCOMES OF
COMPUTATIONAL AND NUMERICAL ANALYSIS
SUBJECT

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Abstract

Computational and Numerical Analysis is one of the core topics for Computational Mathematics in Engineering Mathematics. Students required to learn different methods of analysis as well as MATLAB programming to solve a given problem. The objective of this paper is to analyze the final exam questions of Computational and Numerical Analysis subject. There are four course outcomes for the Computational and Numerical Analysis subject. Five questions were set for final and each question carries 20 marks. The Bloom Taxonomy for the questions are from comprehension, application, analysis and synthesis level. A total of 115 students from Chemical Engineering and Mechanical Engineering departments took the final examination. To analyze this subject, the results of the final examination of students from Chemical and Mechanical Engineering departments are tabulated in EXCEL and transformed into WINSTEPS. The Computational and Numerical Analysis questions can be categorized into four groups. They are difficult, mediocre, easy and very easy. The ability of the Chemical and Mechanical Engineering students cannot be divided into any group. A misfit item is identified from Point-Measure Correlation, Outfit MNSQ and Outfit z-Standard. Since one item is out of the three measures, therefore there is one misfit question for the Computational and Numerical Analysis final examination. The person-item distribution map showed the questions which
belong to difficult, mediocre, easy and very easy group. Generally Course Outcome 1 was difficult for the students. This question is from the analysis level from Bloom Taxonomy. Course Outcome 2 was average and Course Outcome 3 was easy for this batch of students. The Rasch model able to classify the difficulty level of questions versus the Course Outcomes of Computational and Numerical Analysis subject.

**Keywords:** Analysis, Computational and Numerical Analysis, Difficult topics, Final Exam, Rasch model

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I. **Introduction**

Students at Faculty of Engineering and the Built Environment, SEGi University undergo 4 Engineering Mathematics modules, namely Engineering Mathematics I, Engineering Mathematics II, Engineering Statistics and Computational and Numerical Analysis. There are 4 engineering departments at the Faculty of Engineering and the Built Environment at SEGi University Malaysia. They are Department of Electronic and Electrical Engineering, Department of Mechanical Engineering, Department of Chemical Engineering and Department of Civil Engineering. Students in first and second year are compulsory to take all the Engineering Mathematics module. These modules will be given a strong foundation of Mathematics for the students to further the engineering subjects.

Passing exams and going to the next semester will be the main objective of the undergraduate students. But as an educator, the questions given to the students need to be appropriate for them in order to test their knowledge and understanding. Rasch model is able to classify the ability of the students and classify the difficulty of the exam questions.

Evaluation and measurement in teaching and learning is important to ensure continuous quality improvement in education. However, the average grade of the cumulative grade (CGPA) based on the raw min score is not very accurate from the aspect of lineage and it has no validity in the measurement. The Rasch model has been used to measure the students’ performance [VIII], [VI], [V], [II], [IV] and it is found that if the students’ achievement is higher than the mean of questions that means that the students able to answer the exam questions within the scope of the subject [VII]. If the students’ achievement is lower than the mean of questions then immediate solution of the problem should be addressed in order to improve the understanding [IX].

II. **Methodology**

The final examination for Computational and Numerical Analysis was conducted in session 2018 / 2019. The students undergo a teaching of 14 weeks in a semester. Students will have a mid-term test and an assignment to be submitted as the coursework component. The weightage for the written test is 20%, MATLAB test is 10% and the weightage for the assignment is 10% whereas the weightage for final examination is 60%. The total percentage of the test, assignment and final examination will determines the grade of a student for the Computational and Numerical Analysis subject.
A total of 5 subjective questions were designed and validated by a lecturer. The duration of the final examination is 3 hours and the total marks are 100. A total of 115 students from the Bachelor of Chemical Engineering and Bachelor of Mechanical Engineering departments sat for the final exam of Computational and Numerical Analysis. Table 1 shows the students’ profile.

Table 1: Students’ profile

| Department     | Number of students |
|----------------|--------------------|
| Chemical       | 61                 |
| Mechanical     | 54                 |
| Total          | 115                |

As a first step, grades were compiled in the EXCEL *prn format. The grades were transferred using Bond and Fox [IX] known AS WINSTEPS. It is a Rasch model analysis software used to obtain logit values. In the Rasch model, the probability of success can be estimated for the maximum likelihood of an event as

\[ P(\theta) = \frac{e^{\beta - \delta_i}}{1 + e^{\beta - \delta_i}} \]  \hspace{1cm} (1)

where \( e \) refers to the base of the natural algorithm or Euler’s number of 2.7183, \( \beta \) represents student’s ability while \( \delta_i \) is an item or task difficulty.

Table 2 list the Course Outcome for the Computational and Numerical Analysis subject.

Table 2: Course Outcome for Computational and Numerical Analysis subject

| Course Outcome | Description                                                                 |
|----------------|------------------------------------------------------------------------------|
| 1              | Solve numerical models using numerical methods in linear system of equations and nonlinear equation. |
| 2              | Apply numerical methods in differentiation and integration.                  |
| 3              | Solve numerical problems in ordinary differential equation.                  |
| 4              | Perform the numerical analysis by using MATLAB.                             |

Table 3 shows the Programme Outcome for Computational and Numerical Analysis subject.
Table 3: Programme Outcome for Computational and Numerical Analysis subject

| Programme Outcome | Description                           |
|-------------------|---------------------------------------|
| 1                 | Engineering knowledge                 |
| 2                 | Problem analysis                      |
| 3                 | Design / development of solutions     |
| 4                 | Investigation                         |
| 5                 | Modern tool usage                     |
| 6                 | The engineer and society              |
| 7                 | Environment and sustainability         |
| 8                 | Ethics                                |
| 9                 | Communication                         |
| 10                | Individual and team work              |
| 11                | Lifelong learning                     |
| 12                | Project management and finance        |

Table 4 illustrates the distribution of final exam questions together with the marks.

Table 4: Final exam questions

| Question | Description | Marks |
|----------|-------------|-------|
| 1        | The following function, \( f(x) = \frac{5 - 2 \cos 3x}{4x} \) is used to obtain data as shown in the table below. Derive Lagrange interpolating polynomials in radians to approximate \( f(1.0367) \) for the following data. Hence, calculate the percentage of relative error. (Calculate in 5 decimal places) | 20 |
| 2(a)     | The most common approaches for numerical integration are Newton-Cotes formulas. Three of the most widely used Newton-Cotes formulas are the Trapezoidal rule, Simpson’s 1/3 rule and Simpson’s 3/8 rule. Evaluate the following integral | |
\[
\int_{x_0}^{x_2} \frac{9x^2}{2 + 3x^4} \, dx
\]

(i) Analytically.  

(ii) Using Trapezoidal rule for \( h=0.25 \). (Calculate in 5 decimal places) 

(iii) Calculate the relative error percentage.

(b) Estimate the second derivative of \( f(x)=1.29\sin x - 0.84\cos x \) in radians with a step size of \( h=0.2 \) at \( x=2.4741 \) by using

(i) the forward difference approximation.  

(ii) The high-accuracy differentiation. 

(iii) From (i) and (ii), determine which approximation is more accurate. Explain why?

3

Given the following non-linear equation
\[
f(x)=3.2x^2 - \cos x - 0.95
\]

By using the Bisection method, estimate the root in radians using initial guesses of \( x_a = 0.25 \) and \( x_b = 1.38 \) and accurate to within \( \epsilon = 0.005 \).

4

Use Gauss Elimination method with partial pivoting to solve the following system of linear equations. (Calculate in 3 decimal places)

\[
\begin{align*}
0.189x_1 - 0.598x_2 + 2.379x_3 &= -4.224 \\
-2.711x_1 + 5.392x_2 - 3.364x_3 &= 1.571 \\
1.924x_1 - 0.394x_2 + 4.178x_3 &= 5.122
\end{align*}
\]

5

Solve the following initial value problem

\[
x_2y + 4 \frac{dy}{dx} - x^3 \frac{dy}{dx} = 2x^3 \text{ and } y(0.25) = -1.78
\]

by using Heun's method with \( h=0.2 \) and \( 0.25 \leq x \leq 1.05 \).

The details entry of each final question is given in Table 5.
Table 5: Entry number for final questions

| Question | Course Outcome | Programme Outcome | Bloom Level | Taxonomy | Description |
|----------|----------------|-------------------|-------------|----------|-------------|
| 1        | 1              | 2                 | 4           | Analysis |             |
| 2(a)(i)  | 2              | 1                 | 2           | Comprehension |             |
| 2(a)(ii) | 2              | 1                 | 2           | Comprehension |             |
| 2(a)(iii)| 2              | 1                 | 3           | Application |             |
| 2(b)(i)  | 2              | 1                 | 3           | Application |             |
| 2(b)(ii) | 2              | 1                 | 3           | Application |             |
| 2(b)(iii)| 2              | 1                 | 5           | Synthesis |             |
| 3        | 1              | 2                 | 3           | Application |             |
| 4        | 1              | 2                 | 3           | Application |             |
| 5        | 3              | 2                 | 4           | Analysis |             |

III. Results and Discussion

The Research analysis for this paper is divided into three parts, where the first part is called the “Person Measure”, the second part is the “Item Measure”, next is the “Item Statistics” and the last is “Person-Item Distribution Map”. These outputs are extracted from Bond & Fox Steps [X].

Fig. 1 shows the summary statistics for individuals. Individuals represent the student who took Computational and Numerical Analysis subject. The person summary reveals a strong reliability of Cronbach Alpha = 0.70 and person reliability = 0.41. The result of separation was 0.84 indicating that the students cannot be divided into any groups.

SUMMARY OF 115 MEASURED (EXTREME AND NON-EXTREME) Person

| TOTAL | MODEL | INFIT | OUTFIT |
|-------|-------|-------|--------|
| SCORE | COUNT | MEASURE | S.E. | MNSQ | ZSTD | MNSQ | ZSTD |
|-------|-------|---------|------|------|------|------|------|
| MEAN  | 37.2  | .60     | .38  |
| SEM   | .7    | .06     | .03  |
| P.SD  | 7.7   | .68     | .34  |
| S.SD  | 7.7   | .69     | .34  |
| MAX.  | 50.0  | 2.47    | 1.56 |
| MIN.  | 16.0  | -.78    | .21  |
| REAL RMSE | .52 TRUE SD | .44 SEPARATION | .84 Person RELIABILITY | .41 |
| MODEL RMSE | .51 TRUE SD | .45 SEPARATION | .87 Person RELIABILITY | .43 |
| S.E. of Person MEAN = .06 |

Person RAW SCORE-TO-MEASURE CORRELATION = .86
CRONBACH ALPHA (KR-20) Person RAW SCORE "TEST" RELIABILITY = .70 SEM = 4.20

Fig. 1: Summary statistics for individual
Fig. 2 shows the summary statistics for the 10 items involved in this study. “Items” represents the questions tested on the final test. The item summary summarizes very high reliability of 0.94 and item separation = 4.02. The value of the item separation indicates that there are 3 groups classifiable from the questions as “difficult”, “mediocre”, “easy” and “very easy”. The value for the mean is 0.

SUMMARY OF 10 MEASURED (NON-EXTREME) Item

|          |          |          |          |          |          |          |
|----------|----------|----------|----------|----------|----------|----------|
|          | TOTAL    | COUNT    | MEASURE  | S.E.     | MNSQ     | ZSTD     |
|----------|----------|----------|----------|----------|----------|----------|
| MEAN     | 427.4    | 109.5    | .00      | .08      | 1.04     | .19      |
| SEM      | 32.5     | 5.5      | .12      | .01      | .06      | .44      |
| P.SD     | 97.4     | 16.5     | .37      | .02      | .19      | 1.33     |
| S.SD     | 102.7    | 17.4     | .39      | .02      | .20      | 1.40     |
| MAX.     | 543.0    | 115.0    | .53      | .13      | 1.40     | 2.37     |
| MIN.     | 176.0    | 60.0     | -.75     | .07      | .78      | 1.78     |

| REAL RMSE| .09 | TRUE SD | .36 | SEPARATION | 4.02 | Item | RELIABILITY | .94 |
| MODEL RMSE| .09 | TRUE SD | .36 | SEPARATION | 4.26 | Item | RELIABILITY | .95 |

S.E. OF Item MEAN = .12

Fig. 2: Summary statistics for items

Fig. 3 shows the item statistics for the exam questions. Item statistics enable to check if there is any misfit item or unfit questions. If there is any misfit question, the question should be deleted or rephrase. To identify whether an item fits the instrument, a three step calculation is examined. They are Point-Measure Correlation (PMC) value, Outfit Mean Square (MNSQ) and Outfit z-Standard (ZSTD). An item is classified as non-fit if it fails to meet all three criteria. The range for PMC is $0.4 < x < 0.8$ [I]. From Fig. 3, item 1, item 5, item 2b(ii) and item 4 are out of the PMC range. The range for Outfit MNSQ is $0.5 < MNSQ < 1.5$ [III]. Item 1 is out of Outfit MNSQ range. The range for Outfit z-Standard is $-2 < z < 2$ [X]. Item 1 is out of the range of Outfit z-Standard. Since item 1 is out of range of PMC, Outfit MNSQ and Outfit z-Standard therefore item 1 is considered misfit.
| ENTRY | TOTAL | TOTAL | MODEL | INFIT | OUTFIT | PTMEASUR-AL | EXACT MATCH | Item |
|-------|-------|-------|-------|-------|--------|-------------|-------------|------|
| NUMBER | SCORE | COUNT | MEASURE | S.E. | MNSQ | ZSTD | MNSQ | ZSTD | CORR. | EXP. | OBS% | EXP% | Item |
| 1 | 176 | 60 | .53 | .09 | 1.40 | 2.37 | 1.73 | 2.74 | A | .28 | .59 | 24.6 | 22.8 | 1 |
| 2 | 461 | 115 | -.04 | .08 | 1.13 | .97 | 1.25 | 1.08 | B | .35 | .47 | 35.5 | 34.9 | 5 |
| 3 | 407 | 115 | .24 | .07 | 1.23 | 1.89 | 1.14 | .81 | C | .53 | .55 | 18.7 | 22.9 | 2a(i) |
| 4 | 543 | 115 | -.75 | .13 | 1.17 | .65 | .57 | -1.02 | D | .35 | .28 | 79.4 | 76.2 | 2b(ii) |
| 5 | 397 | 115 | .29 | .07 | 1.06 | .59 | .95 | -.26 | E | .58 | .56 | 15.0 | 20.8 | 2a(ii) |
| 6 | 491 | 115 | -.23 | .10 | 1.01 | .09 | .92 | -.12 | F | .33 | .36 | 53.3 | 55.4 | 4 |
| 7 | 463 | 115 | -.36 | .07 | .90 | -.83 | .82 | 1.07 | C | .61 | .57 | 25.2 | 22.3 | 2b(i) |
| 8 | 436 | 115 | .10 | .07 | .80 | -1.76 | .64 | -1.96 | b | .61 | .51 | 31.8 | 30.8 | 3 |
| 9 | 499 | 115 | -.05 | .08 | .78 | -1.78 | .69 | -1.38 | a | .53 | .47 | 45.8 | 36.9 | 2b(iii) |
| MEAN | 427.4 | 109.5 | .00 | .08 | 1.04 | .2 | .95 | -.2 | | 37.3 | 37.1 |
| P.SD | 97.4 | 16.5 | .37 | .02 | .19 | 1.3 | .33 | 1.3 | | 18.3 | 17.3 |

**Fig. 3: Item statistics**

Person problem-solving skills and item difficulty were mapped side by side on the same vertical line with the logit unit. Fig. 4 refers to the Person-Item Distribution Map (PIDM). The discussion aims at the performance of the items spread on the logit scale. The scale for the items is made up samples ranging from -0.75 to 0.53 where the most difficult item and the most able exam takers were laid out on top of the scale.

On the left side, each student was represented by their department number for example CH05 meaning a student from Chemical Engineering department and ME16 representing a student who took Computational and Numerical Analysis final from Mechanical Engineering department. The right hand side illustrates the test item which was represented by numbers.

PIDM shows that the final exam questions can be divided into four categories, namely difficult, mediocre, easy and very easy. Question 1 is also identified as misfit item. This question need to be rephrase so that it becomes smaller parts and have more questions. Thus the 20 marks given to question 1 is an analysis level in Bloom Taxonomy. Students lose marks in this questions as they skip many steps. They had difficulty in showing the steps. Answers from calculator were not shown in the scripts. Thus this question become difficult for this exam.

Question 2a(i), question 2a(ii), question 2b(i) and question 3 are in the mediocre group. In question 2a(i) some students forget to integrate. This question is comprehension level only.

Question 2a(ii) is also comprehension level. Students need to use numerical method which is Trapezoidal rule to integrate the function.

Question 2b(i) is an application level in the Bloom Taxonomy. This question required students to use the forward difference approximation method to solve. This method is similar to the limit method from differentiation.

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Question 3 is using bisection method to estimate the root of the non-linear equation. This is application level question. The initial guesses are given for this question.

Question 2a(iii), question 2b(iii) and question 5 belong to the easy group. Question 2a(iii) is application level question. This question is easy as the students need to calculate the relative error percentage from the question 2a(i) and question 2a(ii). Once students can solve question 2a(i) and question 2a(ii), they just need to use those values to calculate the relative error percentage.

Question 2b(iii) is synthesis level question. Students need to identify which method between the forward difference approximation and the high-accuracy differentiation is more accurate. This question becomes easy for students since they manage to solve the question 2a(i) the forward difference approximation and 2a(ii) the high-accuracy differentiation.

Question 5 is analysis level in Bloom Taxonomy. Students need to solve the initial value problem using Heun’s method. Since students aware of Heun’s method which is a straight forward method, this question falls under easy category.

The questions that categorize in very easy group are question 2b(ii) and question 4. Question 2b(ii) required students to estimate the second derivative of a function using the high-accuracy differentiation. This question belongs to application level of Bloom Taxonomy.

Question 4 is also application level question. This question is very easy for students as they need to use Gauss elimination method with partial pivoting to solve the system of linear equations. Since students have prior knowledge of matrix, this question seems to be very easy for them.

![Person-Item Distribution Map](image)

Fig. 4: Person-Item Distribution Map
Table 6 summarizes the results of the person-item distribution map.

Table 6: Results of the person-item distribution map

| Question | Course Outcome | Level of Bloom’s Taxonomy | Category |
|----------|----------------|---------------------------|----------|
| 1        | 1              | Analysis                   | Difficult |
| 2(a)(i)  | 2              | Comprehension              | Mediocre |
| 2(a)(ii) | 2              | Comprehension              | Mediocre |
| 2(a)(iii)| 2              | Application                | Easy     |
| 2(b)(i)  | 2              | Application                | Mediocre |
| 2(b)(ii) | 2              | Application                | Very Easy|
| 2(b)(iii)| 2              | Synthesis                  | Easy     |
| 3        | 1              | Application                | Mediocre |
| 4        | 1              | Application                | Very Easy|
| 5        | 3              | Analysis                   | Easy     |

IV. Conclusion

The present study brings an important conclusion where the final test questions (items) for Computational and Numerical Analysis course was well designed and constructed. The summary statistics for individual cannot categorize the students into any group as they have the same ability. The summary statistics for item shows that the questions can be categorize into four groups. The person-item distribution map grouped the exam question into difficult, mediocre, easy and very easy category. The item statistics found one misfit question. The item needs to rephrase to reduce the difficulty level.

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