Applicability of an effective conductivity approach in modeling thoracic impedance changes

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Abstract. This paper describes numerical simulations of the influence of conductivity changes inside a volume conductor on impedance changes measured on its surface. A simple model based on the finite element method has been developed to estimate an applicability of the effective conductivity theory in human chest modeling. The model consisted of a cylinder with two concentric spheres inside. Simulations were performed for two cases: first the geometry was changing and material properties were constant within each subdomain, next, the geometry was constant but conductivity values were changing for each phase of cardiac cycle. In the considered range of geometry changes dependence between impedance changes calculated for two models was linear. Performed simulations showed that effective conductivity approach can be utilized when studying dynamic processes involved by volume changes of internal organs.

1. Background
Impedance measurement of heart activity has a great value when trying to evaluate its mechanical function. However, since the mechanism of bioimpedance signal formation is very complex the problem of signal sources identification and separation is still present. The application of an anatomically detailed finite element chest model allows for spatial sensitivity analysis. Having the spatial sensitivity distribution one can assess the impact of components from spatially separable sources. When trying to analyze impedance changes during cardiac cycle the dynamic model has to be generated. The first method to obtain a dynamic model is to generate several static geometric models with finite element mesh, each of them corresponding to the different cardiac cycle phase. Because of the complexity of the human chest it is time consuming and complicated method. The purpose of the article is to examine whether changes in the geometry of individual organs may be replaced by effective conductivity changes inside organs volume.

2. Methods
2.1. Effective conductivity theory
The basic model for a dielectric mixture consists of isotropic dielectric spheres with permittivity \( \varepsilon_i \) embedded in an isotropic dielectric environment with permittivity \( \varepsilon_e \). If permittivity values of that two components and the fraction \( f \) of the total volume that is occupied by the inclusion phase are known, the average permittivity can be calculated using Maxwell Garnett formula [1].

Since the relation between permittivity and conductivity states that [2] \( \varepsilon = \sigma/(j\omega\varepsilon_0) \), the Maxwell Garnett formula can be converted as follows:
\[
\sigma_{\text{eff}} = \sigma_e + 3f\sigma_e \frac{\sigma_i - \sigma_e}{\sigma_i + 2\sigma_e - f(\sigma_i - \sigma_e)} \tag{1}
\]

where: \(\sigma_{\text{eff}}\) – effective conductivity, \(\sigma_e\) – conductivity of environment, \(\sigma_i\) – conductivity of inclusion.

The equation (1) describes the case of homogenous inclusion. Considered model, however, contains inhomogeneous structure – a layered sphere (figure 1).

![Figure 1. Homogenous medium with shell-sphere inclusion.](image)

To obtain effective conductivity of that structure, the effective conductivity \(\sigma_{\text{eff}}'\) of shell-sphere combination is calculated from equation (1) assuming that \(f=(r_2/r_1)^3\), then the result is used to calculate effective conductivity \(\sigma_{\text{eff}}\) of the inclusion-medium system:

\[
\sigma_{\text{eff}} = \sigma_e + 3f\sigma_e \frac{\sigma_{\text{eff}}' - \sigma_e}{\sigma_{\text{eff}}' + 2\sigma_e - f(\sigma_{\text{eff}}' - \sigma_e)} \tag{2}
\]

where \(f\) is the fraction of the total volume that is occupied by the inclusion phase with radius \(r_1\).

Although the theory in most cases is applied to mixtures and heterogeneous materials with many small inclusions, it is reasonable to apply it also in the considered model:

a) the mixing formula satisfies the limiting processes for vanishing inclusion phase

\[f \to 0 \quad \Rightarrow \quad \sigma_{\text{eff}} \to \sigma_e\] \tag{3}

and vanishing host medium

\[f \to 1 \quad \Rightarrow \quad \sigma_{\text{eff}} \to \sigma_i\] \tag{4}

therefore there are no restrictions on the fraction \(f\) of the total volume occupied by the inclusion,

b) the theory was constructed by finding the solution of the Laplace equation, considered model also applies quasi-static approach,

c) the ratio between the size of inhomogeneities and the wavelength of the electromagnetic field of 20 kHz frequency for considered human body tissues is less than \(1.15 \times 10^{-3}\) [3], so it is much smaller than unity that satisfies the homogenization condition [1].

2.2. The model

The simplified finite element model of a human chest was built (figure 2). It consisted of a cylinder of 0.4 m height and 0.12 m radius with two concentric spheres inside (left ventricle). Simulations were performed for two different sets of model's parameters. First, the geometry was changing and material properties were constant within each subdomain. Next, the geometry was constant but conductivity values were changing for each phase of cardiac cycle.
2.2.1. Changing geometry

Analyses were performed for five different phases of the cardiac cycle. The first phase corresponds to systole and the last corresponds to diastole. The volume $V_b$ of the inner sphere of radius $r_2$ changes from $4.46 \cdot 10^{-5}$ m$^3$ to $1.13 \cdot 10^{-4}$ m$^3$ that corresponds to changes of a blood volume inside a ventricle during heart cycle [4]. The radius $r_1$ of the second sphere changes so as to ensure constant volume of the difference between outer and inner sphere (table 1). That "shell" corresponds to left ventricle wall with constant volume of heart muscle $V_m$ and variable thickness that changes from $10^{-2}$ m in systole to $6.62 \cdot 10^{-3}$ m in diastole.

![Figure 2. The model of a volume conductor: a) 3D view, b) configuration of electrodes.](image)

Table 1. Effective conductivity values for different phases of cardiac cycle.

| phase of the heart cycle | $V_b$ [m$^3$] | $r_2$ [m] | $r_1$ [m] | $\sigma_{eff}'$ [S/m] | $\sigma_{eff}$ [S/m] |
|--------------------------|--------------|-----------|-----------|----------------------|---------------------|
| systole                  | $4.46 \cdot 10^{-5}$ | 0.022     | 0.0320    | 0.2907               | 0.2050              |
| .                        | $5.79 \cdot 10^{-5}$ | 0.024     | 0.0330    | 0.3054               | 0.2270              |
| .                        | $7.36 \cdot 10^{-5}$ | 0.026     | 0.0341    | 0.3203               | 0.2560              |
| .                        | $9.20 \cdot 10^{-5}$ | 0.028     | 0.0353    | 0.3350               | 0.2951              |
| diastole                 | $1.13 \cdot 10^{-4}$ | 0.030     | 0.0366    | 0.3492               | 0.3492              |

The model consists of three subdomains with different material properties. Conductivity of the $V_b$ region corresponds to blood conductivity $\sigma_2 = 0.5$ S/m. The second subdomain, "shell" volume $V_m$, has heart muscle conductivity $\sigma_1 = 0.22$ S/m [3]. $\sigma_c = 0.1$ S/m was assigned to the remaining volume of the cylinder.

2.2.2. Changing effective conductivity

To evaluate whether, for the purpose of thorax impedance modeling, changes of the blood volume inside a ventricle can be replaced with changes of the effective conductivity inside the considered region, Maxwell Garnett formula was applied. First, the effective conductivity $\sigma_{eff}'$ of the blood-muscle structure was calculated (equation 1), then the result was used to calculate effective conductivity $\sigma_{eff}$ of the ventricle-thorax system (equation 2). Obtained results are presented in table 1.
2.3. Simulation
Changes of impedance value between two electrodes on the cylinder surface (four-electrode technique, configuration shown in figure 2b) were calculated for each “cardiac cycle phase,” first, for changing volumes and constant conductivities within each subdomain, next, for constant spheres volume but variable effective conductivity values inside the modeled left ventricle.

3. Results
In the considered range of geometry changes dependence between impedance changes calculated for two models – with changing geometry (ΔZ_{geom}) and that with changing effective conductivity (ΔZ_{eff}) – was linear, ΔZ_{eff} = 0.98 ΔZ_{geom}. The relation between impedance changes and blood volume changes was linear for two considered models (figure 3). This proves that changes of geometry parameters can be replaced with effective conductivity changes while maintaining constant volume of considered organs. Relationship between ΔZ_{eff} and ΔZ_{geom} was analyzed for different values of blood σ_2 (0.43; 0.73) and lungs σ_e (0.04; 0.22) conductivities (figure 4) [5]. Obtained correlation coefficient between ΔZ_{eff} and ΔZ_{geom} ranged from 0.993 to 0.993 that indicated strong correlation between that two results.

4. Conclusions
Performed simulations indicate that effective conductivity approach can be utilized in cases of relatively big models when studying dynamic processes involved by volume changes of internal organs.

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Corrigendum: Applicability of an effective conductivity approach in modeling thoracic impedance changes

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