Chiral symmetry-breaking schemes and dynamical generation of masses and field mixing

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Abstract. In this paper we review dynamical generation of field mixing after chiral symmetry breaking. We also study the explicit form of discrete transformations of flavor states in a two-flavor scalar model with field mixing. We find that CPT symmetry is spontaneously broken on flavor vacuum because of its dynamically generated condensate structure.

1. Introduction

The relation of flavor oscillations with CPT and Lorentz and transformations has been analyzed in modern theoretical physics literature [1, 2, 3, 4]. Deformations of the Lorentz dispersion relations, as the ones encountered in the treatment of neutrino oscillations in quantum field theory (QFT) [5, 6], were explicitly studied in Ref. [3], by using the formalism developed in Ref. [7]. Analogous results can be inferred from more fundamental and exotic scenarios: In string theory, for example, the D-foam model [8] predicts deformations of the Lorentz dispersion relation of the order of some power of $E/M_P$, where $E$ is the energy of a particle and $M_P$ is the Planck mass. In this context, the scattering between open strings and D0-branes is modeled by an effective action which describes the dynamical generation of mixing via flavor vacuum condensates [9, 10]. This result was also studied from a non-perturbative algebraic point of view in the case of two [11] and three flavors [12].

String theory also inspired the Standard Model extension (SME) by Colladay and Kostelecky [13]. In that context, CPT and Lorentz violating terms were added to the Standard Model Lagrangian. Neutrino oscillations were there explicitly studied [14] and modified dispersion relations connected with an underlying Planck scale physics were found. Following these developments, many authors dedicated efforts to both theoretical and phenomenological implications of SME or similar models (see e.g. [15, 16, 17]). It can be also shown [18] that bounds on the parameters of SME can be fixed in connection with generalized uncertainty principle [19].

In this paper we briefly review the dynamical mechanism of flavor vacuum condensate and field mixing generation presented in Ref. [11]. As a consequence, Lorentz and CPT symmetries
are spontaneously broken. We here limit our study to the case of discrete symmetries in a simple effective model of two flavor scalar fields with mixing \[20, 21\]. We explicitly show that this model presents a spontaneous symmetry breaking (SSB) of \(T\) and \(CPT\): while these are exact symmetries of the considered Lagrangian, the flavor vacuum is not left invariant by their action. To this end, the transformation properties of flavor creation and annihilation operators are explicitly derived, in the case of parity, charge conjugation and time reversal.

The paper is organized as it follows: In Section 2 we review the mechanism of dynamical mixing generation described in Ref. \[11\]. In Section 3 quantization of flavor (scalar) fields is reviewed \[20, 21\]. Discrete symmetries in the flavor representation are studied in Section 4. Finally, in Section 5, conclusions and perspectives are presented.

2. Dynamical field mixing generation
Following Ref.\[11\], let us consider a Lagrangian density \(L\) that is invariant under the global chiral-flavor group \(G = SU(2)_L \times SU(2)_R \times U(1)_V\). Let the fermion field be a flavor doublet \(\psi = \begin{bmatrix} \tilde{\psi}_1 \\ \tilde{\psi}_2 \end{bmatrix}\).

Under a generic chiral-group transformation \(g\), the field \(\psi\) transforms to \(\psi'\), where \[22\]

\[
\psi' = g \psi = \exp \left[ i \left( \phi + \omega \cdot \sigma + \frac{\sigma_5}{2} \gamma_5 \right) \right] \psi .
\]

Here \(\sigma_j, j = 1, 2, 3\) are the Pauli matrices and \(\phi, \omega, \omega_5\) are real-valued transformation parameters of \(G\). Noether’s theorem implies the conserved vector and axial currents

\[
J^\mu = \overline{\psi} \gamma^\mu \psi , \quad J^\mu = \overline{\psi} \gamma^\mu \frac{\sigma}{2} \psi , \quad J_5^\mu = \overline{\psi} \gamma^\mu \gamma_5 \frac{\sigma}{2} \psi ,
\]

and the ensuing conserved charges

\[
Q = \int d^3x \, \psi^\dagger \psi , \quad Q = \int d^3x \, \psi^\dagger \frac{\sigma}{2} \psi , \quad Q_5 = \int d^3x \, \psi^\dagger \frac{\gamma_5}{2} \psi .
\]

From these we recover the Lie algebra of the chiral-flavor group \(G\), i.e.

\[
\begin{align*}
[Q_1, Q_j] &= i \varepsilon_{ijk} Q_k , & [Q_1, Q_{5,j}] &= i \varepsilon_{ijk} Q_{5,k} , & [Q_{5,i}, Q_{5,j}] &= i \varepsilon_{ijk} Q_k , \\
[Q, Q_{5,j}] &= [Q, Q_j] = 0 .
\end{align*}
\]

Here \(i, j, k = 1, 2, 3\) and \(\varepsilon_{ijk}\) is the Levi-Civita pseudo-tensor.

To proceed, let us recall \[22\] that SSB is characterized by the existence of some local operator(s) \(\phi(x)\) so that, on the vacuum \(|\Omega\rangle\), it is

\[
\langle [N_i, \phi(0)] \rangle = \langle \phi_i(0) \rangle \equiv v_i \neq 0 ,
\]

where \(\langle \cdots \rangle \equiv \langle \Omega| \cdots |\Omega\rangle\). Here \(v_i\) are the order parameters and \(N_i\) represent group generators from the quotient space \(G/H\), with \(H\) being the stability group. In our case \(N_i\) will be taken as \(Q\) and \(Q_5\) according to the SSB scheme under consideration.

By analogy with quark condensation in QCD \[22\], we will limit our considerations to order parameters that are condensates of fermion-antifermion pairs. To this end we introduce the following composite operators

\[
\Phi_k = \overline{\psi} \sigma_k \psi , \quad \Phi_k^5 = \overline{\psi} \sigma_k \gamma_5 \psi , \quad \sigma_0 \equiv \mathbb{I} ,
\]

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with \( k = 0, 1, 2, 3 \). For simplicity we now assume \( \langle \Phi^5 \rangle = 0 \).

Let us now consider three specific SSB schemes \( G \rightarrow H \):

i) SSB sequence corresponding to a single mass generation is \([22, 23]\)

\[
SU(2)_L \times SU(2)_R \times U(1)_V \rightarrow U(2)_V .
\]

The broken-phase symmetry (which corresponds to dynamically generated mass matrix \( M = m_0 I \)) is characterized by the order parameter

\[
\langle \Phi_0 \rangle = v_0 \neq 0, \quad \langle \Phi_k \rangle = 0, \quad k = 1, 2, 3.
\]

One can easily check that this is invariant under the residual symmetry group \( H = U(2)_V \) but not under the full chiral group \( G \).

ii) As a second case we consider the SSB pattern

\[
SU(2)_L \times SU(2)_R \times U(1)_V \rightarrow U(1)_V \times U(1)_V^3,
\]

which is responsible for the dynamical generation of different masses \( m_1, m_2 \). In this case the order parameters take the form

\[
\langle \Phi_0 \rangle = v_0 \neq 0, \quad \langle \Phi_3 \rangle = v_3 \neq 0.
\]

iii) Finally, we consider the SSB scheme

\[
SU(2)_L \times SU(2)_R \times U(1)_V \rightarrow U(1)_V \times U(1)_V^3 \rightarrow U(1)_V,
\]

which is responsible for the dynamical generation of field mixing.

Let us introduce

\[
\Phi_{k,m} = \bar{\psi} \sigma_k \psi , \quad k = 1, 2, 3 ,
\]

where \( m \) indicates that \( \psi \) is now a doublet of fields \( \psi = [\psi_1 \psi_2]^T \) in the mass basis. The SSB condition now reads

\[
\langle \Phi_{1,m} \rangle \equiv v_{1,m} \neq 0.
\]

Hence we find [11] that a necessary condition for a dynamical generation of field mixing within chiral symmetric systems, is the presence of exotic pairs in the vacuum, made up by fermions and antifermions with different masses \(^1\):

\[
\langle \bar{\psi}_i(x) \psi_j(x) \rangle \neq 0, \quad i \neq j.
\]

In other words, field mixing requires mixing at the level of the vacuum condensate structure. This conclusion is consistent with analogous results obtained in the context of QFT treatment of neutrino oscillations, in which case a flavor vacuum has the structure of a non-trivial condensate \([5, 6]\). Moreover, this is an agreement with Ref. [9], where, as previously mentioned, this structure is recovered via dynamical symmetry breaking in a specific effective model generated by string-brane scattering. We remark that the above result is basically model independent (the only assumption made was the global chiral symmetry), and has a non-perturbative nature.

\(^1\) Generally also diagonal condensate may be present.
In the mean-field approximation the vacuum condensate responsible for (14) formally resembles the aforementioned flavor vacuum

$$|0\rangle_{e,\mu} = \prod_k \prod_r \left[ (1 - \sin^2 \theta V_k^2) - \eta^2 \sin \theta \cos \theta V_k (\alpha^{r^\dagger}_{k,1} \beta^r_{k,2} - \alpha^{r^\dagger}_{k,2} \beta^r_{k,1}) + \eta^2 \sin^2 \theta V_k U_k (\alpha^{r^\dagger}_{k,1} \beta^r_{k,2} - \alpha^{r^\dagger}_{k,2} \beta^r_{k,1}) + \sin^2 \theta V_k U_k (\alpha^{r^\dagger}_{k,1} \beta^r_{k,2} - \alpha^{r^\dagger}_{k,2} \beta^r_{k,1}) \right] |0\rangle_{1,2},$$

where

$$U_k = A_k \left( 1 + \frac{|k|^2}{(\omega_{k,1} + m_1)(\omega_{k,2} + m_2)} \right), \quad V_k = A_k \left( \frac{|k|}{\omega_{k,1} + m_1} - \frac{|k|}{\omega_{k,2} + m_2} \right),$$

with $A_k = \sqrt{\frac{(\omega_{k,1} + m_1)(\omega_{k,2} + m_2)}{4\omega_{k,1}\omega_{k,2}}}$. In this case [11]:

$$v_{1,m} = 2 \sin 2\theta \int d^3k \left( \frac{m_2}{\omega_{k,2}} - \frac{m_1}{\omega_{k,1}} \right).$$

Let us remark that such considerations are very general. Flavor vacuum was originally introduced in the context of neutrino oscillation physics [5] but the same structure can be equally recovered by studying the bosonic case [21]. Therefore, just for technical simplicity, we here study the properties of flavor vacuum in this last case. However, it is understood that the origin of such condensate and the consequent CPT SSB have to be retraced in a generation mechanism similar to the one described in this Section.

### 3. Scalar field mixing

Let us consider the effective Lagrange density

$$\mathcal{L}(x) = \partial^\mu \varphi^\dagger_j(x) \partial_\mu \varphi_j(x) - \varphi^\dagger_j(x) M^2 \varphi_j(x),$$

where

$$\varphi_j(x) = \begin{bmatrix} \varphi_A(x) \\ \varphi_B(x) \end{bmatrix}, \quad M^2 = \begin{bmatrix} m_A^2 & m_{AB}^2 \\ m_{AB}^2 & m_B^2 \end{bmatrix},$$

which describes the dynamics of two coupled (mixed) scalar fields that we will call flavor fields, in analogy with the terminology used in quark and neutrino physics. The Lagrange density (20) can be diagonalized thanks to the following transformation:

$$\varphi_j(x) = U \varphi_m(x), \quad U = \begin{bmatrix} \cos \theta & \sin \theta \\ -\sin \theta & \cos \theta \end{bmatrix},$$

where $\tan 2\theta = 2 m_{AB}^2 / (m_B^2 - m_A^2)$. Therefore, $\mathcal{L}$ becomes

$$\mathcal{L}(x) = \partial^\mu \varphi^\dagger_m(x) \partial_\mu \varphi_m(x) - \varphi^\dagger_m(x) M^2_d \varphi_m(x),$$

where

$$\varphi_m(x) = \begin{bmatrix} \varphi_1(x) \\ \varphi_2(x) \end{bmatrix}, \quad M^2_d = \begin{bmatrix} m_1^2 & 0 \\ 0 & m_2^2 \end{bmatrix}.$$ (23)

The Lagrange density (22) describes two free scalar particles with masses $m_1$ and $m_2$. They can be thus expanded as:

$$\varphi_j(x) = \int \frac{d^3k}{2\omega_{k,j}(2\pi)^3} \left[ a_{k,j} e^{-i\omega_{k,j}t} + b_{k,j}^* e^{i\omega_{k,j}t} \right] e^{i\mathbf{k} \cdot \mathbf{x}}, \quad j = 1, 2.$$
where the covariant annihilation and creation operators satisfy the following commutation relations:

\[ [a_{k,i}, a_{p,j}^\dagger] = [b_{k,i}, b_{p,j}^\dagger] = 2\omega_{k,i} (2\pi)^3 \delta(k - p) \delta_{ij} , \]

and annihilate the mass vacuum:

\[ a_{k,j}|0\rangle_{1,2} = b_{k,j}|0\rangle_{1,2} = 0 , \]

i.e. the ground state of the system. We now expand flavor fields in a similar way:

\[ \varphi_\sigma(x) = \int \frac{d^3k}{2\omega_{k,\sigma}(2\pi)^3} \left[ a_{k,\sigma}(t) e^{-i\omega_{k,\sigma}t} + b_{k,\sigma}^\dagger(t) e^{i\omega_{k,\sigma}t} \right] e^{ikx}, \quad \sigma = A, B \]

with \( \omega_{k,\sigma} = \sqrt{|k|^2 + \mu_{\sigma}^2} \) and \( \mu_{\sigma} \) are mass parameters which have to be specified. From mixing transformation (21), it follows\(^2\):

\[ a_{k,A}(t) = \int \frac{d^3x}{i} e^{i(\omega_{k,\sigma}(t) - k \cdot x)} i \partial_0 (\cos \theta_{\varphi_1}(x) + \sin \theta_{\varphi_2}(x)) , \]

and similar expressions work for the other operators. Explicitly:

\[
\begin{pmatrix}
    a_{k,A} \\
    b_{-k,A} \\
    a_{k,B} \\
    b_{-k,B}
\end{pmatrix}
= 
\begin{pmatrix}
    c_\theta \rho_{11}^k & c_\theta \lambda_1^k & s_\theta \rho_{22}^k & s_\theta \lambda_2^k \\
    c_\theta \lambda_{1A}^k & c_\theta \lambda_{1B}^k & s_\theta \lambda_{2A}^k & s_\theta \lambda_{2B}^k \\
    -s_\theta \rho_{2B}^k & -s_\theta \lambda_{2B}^k & c_\theta \rho_{1B}^k & c_\theta \lambda_{1B}^k \\
    -s_\theta \lambda_{2B}^k & -s_\theta \lambda_{2B}^k & c_\theta \rho_{1B}^k & c_\theta \lambda_{1B}^k
\end{pmatrix}
\begin{pmatrix}
    a_{k,1} \\
    b_{-k,1} \\
    a_{k,2} \\
    b_{-k,2}
\end{pmatrix},
\]

where \( c_\theta \equiv \cos \theta, s_\theta \equiv \sin \theta, \)

\[ \rho_{\sigma j}^k = |\rho_{\sigma j}^k| e^{i(\omega_{k,\sigma} - \omega_{k,j})t}, \quad \lambda_{\sigma j}^k = |\lambda_{\sigma j}^k| e^{i(\omega_{k,\sigma} + \omega_{k,j})t}, \quad (\sigma, j) = (A, 1), (B, 2), \]

where

\[ |\rho_{\sigma j}^k| = \frac{1}{2} \left( \frac{\omega_{k,\sigma}}{\omega_{k,j}} + 1 \right), \quad |\lambda_{\sigma j}^k| = \frac{1}{2} \left( \frac{\omega_{k,\sigma}}{\omega_{k,j}} - 1 \right). \]

Note that this is a canonical transformation:

\[ [a_{k,\sigma}(t), a_{p,\rho}^\dagger(t)] = [b_{k,\sigma}(t), b_{p,\rho}^\dagger(t)] = 2\omega_{k,\sigma} (2\pi)^3 \delta(k - p) \delta_{\sigma\rho}. \]

Let us notice that we have not specified the mass parameters \( \mu_{\sigma} \). The situation here is similar the one encountered in QFT in curved spacetime [24] where one has an infinite set of creation and annihilation operators related by a Bogolyubov transformation. In our case, the Bogolyubov transformation is of the type:

\[
\begin{pmatrix}
    \tilde{a}_k \\
    \tilde{b}_k^\dagger
\end{pmatrix}
= 
\begin{pmatrix}
    \rho_k \lambda_k \\
    \lambda_k \rho_k
\end{pmatrix}
\begin{pmatrix}
    a_k \\
    b_{-k}
\end{pmatrix},
\]

where

\[ |\rho_k| = \frac{1}{2} \left( \frac{\tilde{\omega}_k}{\omega_k} + 1 \right), \quad |\lambda_k| = \frac{1}{2} \left( \frac{\tilde{\omega}_k}{\omega_k} - 1 \right). \]

Here the time dependence of creation and annihilation operators indicates that flavor fields are interacting fields. Actually, this interacting model can be solved exactly, without perturbation expansion.
These are related to the usual Bogoliubov coefficients [22] as:
\[ \rho_B^k = \rho^k \sqrt{\frac{\omega_k}{\omega_k}}, \quad \lambda_B^k = \lambda^k \sqrt{\frac{\omega_k}{\omega_k}}, \] (34)
which satisfy the usual relation \(|\rho_B^k|^2 - |\lambda_B^k|^2 = 1\). In Ref. [25], it was shown that different choices of \(\mu_\sigma\) affect the form of the Casimir force between two plates. Typical choices studied in literature are [26] \(\mu_A = m_1, \mu_B = m_2\) and \(\mu_A = m_A, \mu_B = m_B\).

Therefore, one can define the flavor vacuum as the state which is annihilated by flavor annihilation operators at a fixed time \(t\):
\[ a_{k,\sigma}(t) |0(t)\rangle_{A,B} = b_{k,\sigma}(t) |0(t)\rangle_{A,B} = 0. \] (35)
This is characterized by a boson-condensate structure, similar to the one encountered in Eq. (16). In the case \(\mu_A = m_1, \mu_B = m_2\) one gets [21]:
\[ A,B \langle 0(t)|a_{k,j}^\dagger a_{k,j} |0(t)\rangle_{A,B} = A,B \langle 0(t)|b_{k,j}^\dagger b_{-k,j} |0(t)\rangle_{A,B} = \sin^2 \theta |V_k|^2 2 \omega_{k,j} (2\pi)^3, \] (36)
where \(|V_k| = |\lambda_k|_{\mu_A=m_1,\mu_B=m_2}\), analogous to the function encountered in Eq. (17). This structure is responsible for the CPT symmetry breaking discussed below.

A flavor state is defined as an excitation of the flavor vacuum:
\[ |a_{k,\sigma}(t)\rangle \equiv a_{k,\sigma}^\dagger(t) |0(t)\rangle_{A,B}. \] (37)
These are eigenstates of the flavor charge
\[
Q_\sigma(t) = i \int d^3x \, \varphi_\sigma^\dagger(x) \, \partial_0 \, \varphi_\sigma(x) \\
= \int \frac{d^3k}{2 \omega_{k,\sigma} (2\pi)^3} \left( a_{k,\sigma}^\dagger(t) a_{k,\sigma}(t) - b_{k,\sigma}^\dagger(t) b_{k,\sigma}(t) \right),
\] (38)
at fixed time \(t\)
\[ Q_\sigma(t) |a_{k,\sigma}(t)\rangle = |a_{k,\sigma}(t)\rangle, \quad \sigma = A, B. \] (39)

4. Discrete symmetries
In this Section we study the behavior of flavor annihilation and creation operators under parity, charge conjugation and time reversal. We will see that CPT symmetry is spontaneously broken on the flavor vacuum.

4.1. Parity
The parity transformation of the flavor scalar fields is given by:
\[ P \varphi_\sigma(x) P^{-1} = \eta_{\sigma,p} \varphi_\sigma(\tilde{x}), \] (40)
where \(P\) is the unitary parity operator and \(\tilde{x} = (t, -x)\). As usual, \(|\eta_{\sigma,p}|^2 = 1\). By using the explicit expansion (26), we find:
\[
P \varphi_\sigma(x) P^{-1} = \int \frac{d^3k}{2 \omega_{k,\sigma} (2\pi)^3} \left[ P a_{k,\sigma}(t) P^{-1} e^{-i\omega_{k,\sigma} t} + P b_{-k,\sigma}(t) P^{-1} e^{i\omega_{k,\sigma} t} \right] e^{ik \cdot x} \\
= \eta_{\sigma,p} \int \frac{d^3k}{2 \omega_{k,\sigma} (2\pi)^3} \left[ a_{k,\sigma}(t) e^{-i\omega_{k,\sigma} t} + b_{-k,\sigma}^\dagger(t) e^{i\omega_{k,\sigma} t} \right] e^{-i k \cdot \tilde{x}}.
\] (41)
Transformations of creation and annihilation operators follow:
\[
P a_{k,\sigma}(t) P^{-1} = \eta_{\sigma,\nu} a_{-k,\sigma}(t) \quad P b_{k,\sigma}(t) P^{-1} = \eta_{\sigma,\nu}^* b_{-k,\sigma}(t),
\]
\[
P a_{k,\sigma}^\dagger(t) P^{-1} = \eta_{\sigma,\nu}^* a_{-k,\sigma}(t) \quad P b_{k,\sigma}^\dagger(t) P^{-1} = \eta_{\sigma,\nu} b_{-k,\sigma}^\dagger(t).
\]
The flavor vacuum is invariant under parity transformation:
\[
P |0(t)\rangle_{A,B} = |0(t)\rangle_{A,B}.
\]
In fact, the explicit form of \(P\) is the same as in the free case [27]:
\[
P = \exp \left\{ \frac{i}{2} \int \frac{d^3k}{2\omega_{k,\sigma}(2\pi)^3} \left[ \left( a_{-k,\sigma}^\dagger(t) a_{k,\sigma}(t) + b_{-k,\sigma}^\dagger(t) b_{k,\sigma}(t) \right) - \eta_{\sigma,\nu} \left( a_{k,\sigma}^\dagger(t) a_{k,\sigma}(t) + b_{k,\sigma}^\dagger(t) b_{k,\sigma}(t) \right) \right] \right\},
\]
from which Eq.(44) follows. A flavor state (37) transforms as
\[
P |a_{k,\sigma}(t)\rangle = |a_{-k,\sigma}(t)\rangle,
\]
and the flavor charge (38) remains invariant, i.e.
\[
[P, Q_{\sigma}(t)] = 0.
\]

4.2. Charge conjugation
The charge conjugation transformation of the flavor scalar fields is given by:
\[
C \varphi_\sigma(x) C^{-1} = \eta_{\sigma,c} \varphi_\sigma^\dagger(x),
\]
where \(C\) is the unitary charge conjugation operator. As usual, \(|\eta_{\sigma,c}|^2 = 1\). Once more, by using the explicit expansion (26), we find:
\[
C \varphi_\sigma(x) C^{-1} = \int \frac{d^3k}{2\omega_{k,\sigma}(2\pi)^3} \left[ C a_{k,\sigma}(t) C^{-1} e^{-ikx} + C b_{k,\sigma}^\dagger(t) C^{-1} e^{ikx} \right]
\]
\[
= \eta_{\sigma,c} \int \frac{d^3k}{2\omega_{k,\sigma}(2\pi)^3} \left[ a_{k,\sigma}(t) e^{ikx} + b_{k,\sigma}^\dagger(t) e^{-ikx} \right].
\]
Transformations of creation and annihilation operators follow:
\[
C a_{k,\sigma}(t) C^{-1} = \eta_{\sigma,c} b_{k,\sigma}(t) \quad C b_{k,\sigma}(t) C^{-1} = \eta_{\sigma,c}^* a_{k,\sigma}(t),
\]
\[
C a_{k,\sigma}^\dagger(t) C^{-1} = \eta_{\sigma,c}^* b_{k,\sigma}^\dagger(t) \quad C b_{k,\sigma}^\dagger(t) C^{-1} = \eta_{\sigma,c} a_{k,\sigma}^\dagger(t).
\]
The flavor vacuum is invariant under charge conjugation:
\[
C |0(t)\rangle_{A,B} = |0(t)\rangle_{A,B}.
\]
In fact, the explicit form of \(C\) is
\[
C = \exp \left\{ \frac{i}{2} \int \frac{d^3k}{2\omega_{k,\sigma}(2\pi)^3} \left[ \left( b_{k,\sigma}^\dagger(t) a_{k,\sigma}(t) + a_{k,\sigma}^\dagger(t) b_{k,\sigma}(t) \right) - \eta_{\sigma,c} \left( a_{k,\sigma}^\dagger(t) a_{k,\sigma}(t) + b_{k,\sigma}^\dagger(t) b_{k,\sigma}(t) \right) \right] \right\},
\]
from which Eq.(52) follows. A flavor state (37) transforms as

$$C |a_{k, \sigma}(t)\rangle = |b_{k, \sigma}(t)\rangle,$$

(54)

while flavor charge (38) reverses its sign

$$C Q(t) C^{-1} = \int \frac{d^3k}{2\omega_{k, \sigma}(2\pi)^3} \left( b_{k, \sigma}^\dagger(t) b_{k, \sigma}(t) - a_{k, \sigma}^\dagger(t) a_{k, \sigma}(t) \right) = -Q(t),$$

(55)

as expected.

4.3. Time reversal

The time reversal transformation of the flavor scalar fields is given by:

$$T \phi_\sigma(x) T^{-1} = \eta_{\sigma, T} \phi_\sigma(-\hat{x}),$$

(56)

where T is the antiunitary time reversal operator. As usual, $|\eta_{\sigma, T}|^2 = 1$. By using the explicit expansion (26), we find:

$$T \phi_\sigma(x) T^{-1} = \int \frac{d^3k}{2\omega_{k, \sigma}(2\pi)^3} \left[ T a_{k, \sigma}(t) T^{-1} e^{i\omega_{k, \sigma} t} + T b_{k, \sigma}^\dagger(t) T^{-1} e^{-i\omega_{k, \sigma} t} \right] e^{-ik \cdot x}$$

$$= \eta_{\sigma, T} \int \frac{d^3k}{2\omega_{k, \sigma}(2\pi)^3} \left[ a_{k, \sigma}(-t) e^{i\omega_{k, \sigma} t} + b_{k, \sigma}^\dagger(-t) e^{-i\omega_{k, \sigma} t} \right] e^{ik \cdot x}. $$

(57)

Transformations of creation and annihilation operators follow:

$$T a_{k, \sigma}(t) T^{-1} = \eta_{\sigma, T} a_{-k, \sigma}(-t), \quad T b_{k, \sigma}(t) T^{-1} = \eta_{\sigma, T}^* b_{-k, \sigma}(-t),$$

(58)

$$T a_{k, \sigma}^\dagger(t) T^{-1} = \eta_{\sigma, T} a_{-k, \sigma}^\dagger(-t), \quad T b_{k, \sigma}^\dagger(t) T^{-1} = \eta_{\sigma, T} b_{-k, \sigma}^\dagger(-t).$$

(59)

By using canonical commutation relations (31) one gets:

$$a_{-k, \sigma}(-t) = \sum_{\rho = A, B} \frac{1}{2\omega_{k, \rho}(2\pi)^3} \left[ \left[ a_{k, \sigma}(t), a_{-k, \rho}^\dagger(t) \right] a_{k, \rho}(t) - \left[ a_{-k, \sigma}(t), b_{k, \rho}(t) \right] b_{k, \rho}^\dagger(t) \right],$$

(60)

and similar relations for the other operators. If one now look at flavor vacuum transformation properties

$$|0(-t)\rangle_{A, B}^T = T |0(t)\rangle_{A, B},$$

(61)

it is evident that time-reversal symmetry is spontaneously broken. This could also be deduced by looking at the transformation of the flavor charge (38):

$$T Q(t) T^{-1} = \int \frac{d^3k}{2\omega_{k, \sigma}(2\pi)^3} \left( a_{k, \sigma}^\dagger(-t) a_{k, \sigma}(-t) - b_{k, \sigma}^\dagger(-t) b_{k, \sigma}(-t) \right) = Q_{\sigma}(-t),$$

(62)

i.e. $[Q_{\sigma}(t), T] \neq 0$ in a non-trivial way.
4.4. CP and CPT symmetry
From the previous considerations it is evident that CP is an exact symmetry in the flavor representation:
\[ CP |0(t)\rangle_{A,B} = |0(t)\rangle_{A,B}. \] (63)
However, from Eq.(61), it follows that CPT symmetry is spontaneously broken on the flavor vacuum:
\[ |0(-t)\rangle_{A,B}^\Theta = \Theta |0(t)\rangle_{A,B}, \] (64)
where \( \Theta \equiv CPT \). This is a consequence of the transformation law of creation and annihilation operators:
\[ \Theta a_{k,\sigma}(t) \Theta^{-1} = \eta_{\sigma} b_{k,\sigma}(-t) \]
\[ \Theta a_{k,\sigma}^\dagger(t) \Theta^{-1} = \eta_{\sigma} b_{k,\sigma}^\dagger(-t) \]
\[ \Theta b_{k,\sigma}(t) \Theta^{-1} = \eta_{\sigma}^* a_{k,\sigma}(-t), \]
\[ \Theta b_{k,\sigma}^\dagger(t) \Theta^{-1} = \eta_{\sigma}^* a_{k,\sigma}^\dagger(-t). \] \[ \Theta Q_\sigma(t) \Theta^{-1} = \int \frac{d^3k}{2\omega_{k,\sigma}(2\pi)^3} \left( b_{k,\sigma}^\dagger(-t) b_{k,\sigma}(-t) - a_{k,\sigma}^\dagger(-t) a_{k,\sigma}(-t) \right) \]
\[ = -Q_\sigma(-t). \] (67)

5. Conclusions
In this paper, we have briefly reviewed the mechanism of dynamical generation of field mixing via flavor vacuum condensate, as introduced in Ref. [11]. Then, we have studied the properties of flavor creation and annihilation operators under discrete symmetry transformations, in an effective model describing a flavor scalar doublet with mixing. We have proved that these transformations are symmetries of the Lagrangian, but they do not annihilate the flavor vacuum. Therefore spontaneous breaking of CPT symmetry occurs. This SSB can be driven by a mechanism analogous to the one described in Section 2.

A similar symmetry breaking occurs in literature, e.g. in Refs. [13, 14], where explicit Lorentz and CPT violating terms are added to the Standard Model Lagrangian. A link between our approach and these previous studies should be determined. In this connection, also Lorentz/Poincaré symmetry breaking should be explicitly analyzed [28].

Recently [29], an extension of the Standard Model were proposed, by noticing that flavor mixing naturally implies Poincaré symmetry breaking. Then it is evident that a careful study of the symmetry properties of the flavor vacuum may represent an important indication of new physics. In this direction, the analysis of vacuum structure in connection with dynamical mixing generation [9, 10, 11, 12] reveals its important rôle.

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\[ This \ is \ not \ true \ for \ the \ three \ flavor \ case, \ where \ CP \ symmetry \ can \ be \ explicitly \ broken \ because \ of \ a \ complex \ phase \ in \ the \ mass \ matrix. \]
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