Inductance calculation of fractional slot concentrated windings

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Abstract. Inductance calculation of the generator with permanent magnets and a collector-type rotor is shown in the paper. Calculation is carried out taking into account the core features. The accuracy of the results is verified with the experiments.

1. Introduction
Synchronous machines with excitation of permanent magnets become more and more popular due to better energy characteristics compared with asynchronous machines having, by the way, significantly lower mass and overall dimensions. Moreover, the developers and manufacturers use fractional slot concentrated winding i.e., winding with a number of slots per pole and phase is less than one in order to increase the reliability and manufacturability of such machines. The growing popularity of such windings is due to the ability to realize a high number of poles while reducing the technological limitations on the width of stator slot and simplifying the manufacturing technology because of the lack of crossing the frontal parts.

However, the use of fractional slot concentrated winding sets a number of tasks for developers, such as accounting the features of magnetomotive force generated by windings. Particularly, the presence of a wide spectrum of high harmonics and subharmonics in the MMF curve complicates the task of determining the inductances significations, without knowledge of which it is impossible to predict the characteristics of the machine while designing it.

2. Calculation of inductances in synchronous machines with permanent magnets.
The magnetic field in the air gap contains the whole line of high harmonics and subharmonics, causing an increased differential leakage inductance in synchronous machines with permanent magnets and fractional slot concentrated winding. The value of such inductance can reach 50 or more percent of the inductance armature reaction, which is a function not only of the air gap value and configuration, but also of the design of the rotor, geometry and magnetic properties of permanent magnets.

For the design of the rotor with radially magnetized magnets, armature reaction inductance along the \( d \) and \( q \) axes is practically the same (\( X_{ad} \approx X_{aq} \)) and determined by the magnetic resistance of the air gap and magnetic resistance of the magnet (for high-coercivity magnets \( \mu_{ad} \approx \mu_0 \));

For SMPM with tangentially magnetized magnets, magnetic resistance along the \( d \) and \( q \) axes is determined by the air gap and the magnetic resistances of the magnets. Along the \( q \) axle magnetic resistance is determined by the value of the air gap and the design. Significant difference between the
magnetic resistances along \(d\) and \(q\) axes causes different character of the field distribution, and, consequently different values of field harmonics inductance differ from the main harmonics.

The ratios for calculation differential leakage inductance for the salient pole synchronous machine with classical winding were obtained in [4]. We obtain analogous ratios for synchronous machines with permanent magnets and fractional slot concentrated winding and a collector rotor.

The basis for inductance calculation is given by the well-known expression:

\[
X = \omega L = \omega \frac{\psi}{i}
\]

where \(\omega\) - angular frequency.

The linkage is obtained from the distribution of air-gap induction:

\[
B(\alpha) = \lambda(\alpha) \sum_{\nu=0}^{\nu} F_{\nu}(\alpha) \cdot
\]

Here \(F_{\nu}(\alpha)\) - amplitude of \(\nu\)- harmonics of the magnetomotive force;

\(\lambda(\alpha)\) - equivalent conductivity of the air gap.

The law of the magnetomotive force distribution is given by the nature of the currents distribution along the bore of the machine.

The model of the machine with a smooth stator and a salient pole rotor is considered here instead of the real stator and the rotor slot structure in SMPM. Then the conductivity of the air gap [5] can be represented as:

\[
\lambda(\alpha) = \lambda_0 + \sum_{k=1}^{k} \lambda_k \cos[k(Z_2 \theta - n\alpha)]
\]

Here \(n = Z_2 = 2p\), \(\theta = \omega t - \theta_0\) - the angle between the axis of a stator winding phase and axis \(d\) of the rotor pole, \(\alpha\) - angular coordinate, measured from the axis of the stator winding phase.

We confine to the constant component of the first harmonics, while the angle between the axes and actual coordinate are expressed in electric degrees in calculation of equivalent conductivity of the air gap without a sufficient reduction in accuracy. Then the ratio for conductivity will take the form:

\[
\lambda(\alpha) = \lambda_0 + \lambda_k \cos2(\omega t - \alpha - \theta_0).
\]

The magnetizing force of a three-phase winding with a symmetrical phase load can be presented by the following series:

\[
F = \sum_{\nu} F_{\nu}(\omega t + \nu \alpha)
\]

where \(F_{\nu}\) - the harmonic amplitude of the order \(\nu\) [6] for windings with the number of slots per pole and phase is less than one \(q < 1\), while \(q = \frac{c}{d}\):

\[
F_{\nu} = \frac{m\sqrt{2} W_1 k_{\nu} \omega_1 t}{\pi \nu} I
\]

where \(I\) - effective value of the current in phase, \(m\) - number of phases, \(W_1\) - number of turns in phase \((W_1 = c \cdot W_k\) - for \(d\) - is even, \(W_1 = 2c \cdot W_k\) - for \(d\) - odd), \(t\) - time taken from the moment when the current is maximum in phase \(A\). The sign «» in formula (1) refers to the straight-running harmonics, «+» - to reversible harmonics.

Formulas for calculation the winding coefficients for \(\nu\)- harmonics [6]:

\[
k_{\nu\nu} = k_{\nu
y} \cdot k_{p\nu},
\]
\[ k_{yv} = \sin v \frac{\pi}{m_c}, \]
\[ k_{pv} = \frac{\sin \left( \frac{c - 2v}{m_i} \frac{\pi}{2} \right)}{c \cos \frac{v\pi}{m_c}}, \]

where \( m_z \) – number of phase zones (editorial sections) of which fractional slot concentrated windings consist. Moreover, the number of phase zones is a multiple of the number of phases \( m \), while \( m_z = 2m \) for \( d \) – is odd and \( m_z = m \) for \( d \) – is even.

Then the flux density in the air gap is:
\[ B(\alpha) = \left( \sum_v F_v \cos(\omega t + \omega v\alpha) \right) \left[ \lambda_0 + \lambda_i \cos 2(\omega t - \theta_0) \right] = \]
\[ = \lambda_0 \cdot \sum_v F_v \cos(\omega t + \omega v\alpha) + \lambda_i \sum_v F_v \cos(\omega t - \theta_0) \cdot \cos(\omega t + \omega v\alpha) = \]
\[ = \sqrt{2} B_0 \left[ \lambda_0 \cdot \sum_v k_{v} \cos(\omega t + \omega v\alpha) + \lambda_i \sum_v k_{v} \cos 2(\omega t - \theta_0) \cdot \cos(\omega t + \omega v\alpha) \right] = \]
\[ = \sqrt{2} B_0 \left[ \lambda_0 \sum_v \frac{k_{v}}{v} \cos(\omega t + \omega v\alpha) + \lambda_i \sum_v \frac{k_{v}}{v} \cos(3\omega t + \alpha(2 + \nu) - 2\theta_0) + \frac{1}{2} \sum_v \frac{k_{v}}{v} \cos(\omega t - \alpha(2 + \nu) - 2\theta_0) \right] \]

where
\[ B_0 = \frac{mW_1}{\pi}. \]

The second and the third terms in the sum (2) contain field harmonics of the order \( 2 \pm \nu \) and \( 2 \pm \nu \), which varying with angular frequencies \( \omega t \) and \( 3\omega t \). In the form of an inductive resistance of differential scattering \( \omega t \), the fields of the fundamental frequency are taken into account, so the second term in the sum (2) must be discarded.

We calculate the flux linkage of phase A as the sum of the flux linkages of each of the field harmonics that vary with the fundamental frequency:
\[ \psi_a = \sum \psi_v = \sum W_v k_{a,v} \Phi_v \]

where
\[ \Phi_v = \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} B_v(\alpha) l_\delta d\alpha = \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} B_v(\alpha) l_\delta \frac{D}{2} d\alpha = \]
\[ = \frac{2C_1 k_{a,v}}{\nu^2} \cos(\omega t) - \frac{2C_2 k_{a,v}}{(2 \pm \nu)\nu} \cos(\omega t - 2\theta_0) \]

Here are coefficients \( C_1 = \sqrt{2} l_\delta B_0 D l_\delta, C_2 = \sqrt{2} l_\delta B_0 D l_\delta \frac{\lambda_i}{2}, \)

\( D \) - diameter of the stator boring, \( l_\delta \) - its active length.
Then:
\[
\psi_a = \sum \psi_v = \sqrt{2} I W B D l \left[ \lambda_0 \sum_{v} \frac{k_{o,v}^2}{v^2} - \frac{\lambda_1}{2} \sum_{v} \frac{k_{o,v}^2}{(2 + v)v} \cos (\omega t - 2\theta_0) \right].
\]

We consider the moment of time \( \omega t = 0 \). At \( \theta_0 = 0 \) the current is purely longitudinal:
\[
I_{dm} = \sqrt{2} I
\]
and the ratio \( \psi_a / I_{dm} \) give the value of the inductance \( L_{ad} \) along the longitudinal axis, which includes both the inductance of the anchor reaction along the \( d \) axis, and the inductance of the differential scattering along this axis. At \( \theta_0 = \pi / 2 \), the current will be purely quadrature:
\[
I_{qm} = \sqrt{2} I
\]
and the ratio \( \psi_a / I_{qm} \) will give an inductance value along the quadrature axis including both the inductance of the armature reaction along the \( q \) axis and the differential scattering inductance along this axis. Multiplying these expressions by \( \omega \), we obtain the corresponding inductive resistances:
\[
x_{ad} = x_0 \left[ \lambda_0 \sum_{v} \frac{k_{o,v}^2}{v^2} - \frac{\lambda_1}{2} \sum_{v} \frac{k_{o,v}^2}{(2 + v)v} \right] = x_0 \left[ \lambda_0 \xi_0 - \frac{\lambda_1}{2} \xi_1 \right]
\]
\[
x_{aq} = x_0 \left[ \lambda_0 \sum_{v} \frac{k_{o,v}^2}{v^2} + \frac{\lambda_1}{2} \sum_{v} \frac{k_{o,v}^2}{(2 + v)v} \right] = x_0 \left[ \lambda_0 \xi_0 + \frac{\lambda_1}{2} \xi_1 \right]
\]
where
\[
x_0 = \frac{\omega m W^2 D l_\delta}{\pi}
\]

The values of the coefficients \( \xi_0 = \sum \frac{k_{o,v}^2}{v^2} \) and \( \xi_1 = \sum \frac{k_{o,v}^2}{(2 + v)v} \) for the most possible fractions of \( q \) three-phase windings are given in Table 1, in calculating the coefficients the harmonics are taken into account to \( V = 30 \), further calculation of the harmonics does not affect the accuracy of the calculation of the coefficients.

| №  | \( q \)  | \( \xi_0 \)  | \( \xi_1 \) |
|----|--------|-------------|-------------|
| 1. | 1/2    | 1.0778      | 1.0269      |
| 2. | 2/5    | 0.0645      | 0.0616      |
| 3. | 3/8    | 0.1143      | 0.158       |
| 4. | 4/11   | 0.0141      | 0.0141      |
| 5. | 5/14   | 0.0397      | 0.0445      |
| 6. | 6/17   | 6.23 \cdot 10^{-3} | 6.22 \cdot 10^{-3} |
| 7. | 7/20   | 0.0183      | 0.0195      |
| 8. | 8/23   | 3.491 \cdot 10^{-3} | 3.488 \cdot 10^{-3} |

It is necessary to determine the conductivities of the air gap to calculate the inductances of the armature reaction along the \( d \) and \( q \) axis. The maximum value of the conductivity \( (\lambda_{aq}) \) will be when the axis of the phase coincides with the \( d \) axis with \( c – \) even, and with the \( q \) axis at \( c – \) odd (fig. 1). The
minimal value of the conductivity ($\lambda_{ad}$), therefore, will be when the axis of the phase coincides with the $q$ axis when $c$ is even and with the $d$ axis at $c – odd$ (fig. 2). Then the values of the coefficients $\lambda_0$ and $\lambda_1$ in ratios (3) and (4) are found in the following way:

$$
\lambda_0 = \frac{\lambda_{ad} + \lambda_{aq}}{2},
$$

$$
\lambda_1 = \lambda_{aq} - \lambda_{ad}.
$$

![Figure 1. Position of the rotor with the maximum value of inductance in a stator winding: a) $c – even$; b) $c – odd$.](image1)

![Figure 2. Position of the rotor with the minimum value of inductance in a stator winding: a) $c – even$; b) $c – odd$.](image2)

The armature reaction conductivity $\lambda_{ad}$ is determined by the magnetic resistance $R_{ad}$ to the passage of the magnetic flux $\Phi_a$ produced by the magnetic force of the armature reaction $F_a$, the value of which is found from the circuit for replacing the magnetic circuit for the corresponding design of the machine. Accordingly, the specific conductivities of the armature reaction along the both axes:

$$
\lambda_{ad} = \frac{k_d}{\pi l_\delta} \frac{1}{R_{ad}}.
$$
\[ \lambda_{aq} = \frac{k_q}{\tau l_a R_{aq}} \]

where \( \tau = \frac{\pi(D - 2\delta)}{2p} \) - pole division, \( k_d \) and \( k_q \) form coefficients along the \( d \) and \( q \) axes.

3. Results

Experimental studies were carried out on the real samples of electric machines with magnetoelectric excitation and fractional slot concentrated winding to evaluate the correctness of the proposed method of the armature reaction inductance calculation. The comparison of the total inductances was done because it is impossible to make experiments with the separate armature reaction inductances. The values of the calculation of the total inductances by the traditional method and with the method described in this paper are given.

The results of calculation of the inductances for synchronous electric machines with the power of 0.37 and 0.75 kW with \( q = \frac{2}{5} \) and the power of 2.2 kW with \( q = \frac{3}{8} \), and their values determined by the experiments are shown in Table 2.

| Motor | 0.37 kW | 0.75 kW | 2.2 kW |
|-------|---------|---------|--------|
| Inductance | \( x_{d}, \text{Ohm} \) | \( x_{q}, \text{Ohm} \) | \( x_{d}, \text{Ohm} \) | \( x_{q}, \text{Ohm} \) | \( x_{d}, \text{Ohm} \) | \( x_{q}, \text{Ohm} \) |
| Experiment | 4.744 | 9.708 | 6.377 | 11.97 | 0.993 | 2.086 |
| Analytical calculation (traditional method) | 4.353 | 6.148 | 5.33 | 7.92 | 0.789 | 1.15 |
| Analytical calculation (refined calculation) | (-14.2\%) | (2.5\%) | (-3\%) | (-3.7\%) | (2.6\%) | (-8.8\%) |

4. Conclusion

The proposed method of the calculation of synchronous inductances with differential leakage gives significantly smaller error than the traditional method of the calculation as shown by the analysis of the results presented in Table 3, and can be applied to the calculation of the electric machines of this type.

References

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