Nonleptonic weak decays of charmed mesons

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ABSTRACT

A previous analysis of two-body Cabibbo allowed nonleptonic decays of $D^0$ mesons and of Cabibbo allowed and first-forbidden decays of $D^+$ and $D_s^+$ has been adjourned using more recent experimental data and extended to the Cabibbo forbidden decays of $D^0$. Annihilation and W-exchange contributions as well as final state interaction effects (assumed to be dominated by nearby resonances) have been included and are in fact crucial to obtain a reasonable agreement with the experimental data, which show large flavour SU(3) violations. New fitting parameters are necessary to describe rescattering effects for Cabibbo forbidden $D^0$ decays, given the lack of experimental informations on isoscalar resonances. We keep their number to a minimum - three - using phenomenologically based considerations. We also discuss CP violating asymmetries.

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1. Introduction

A theoretical description of exclusive nonleptonic decays of charmed hadrons based on general principles is not yet possible. Even if the short distance effects due to hard gluon exchange can be resummed and an effective hamiltonian has been constructed (recently, at next-to-leading order [1]), the evaluation of its matrix elements requires nonperturbative techniques. Waiting for future progress in lattice QCD calculations one has to rely on approximate methods and/or models.

The largely different lifetimes of charmed hadrons make it clear that the infinitely heavy quark limit is quite far from the actual situation. Therefore, the expansion in inverse powers of the heavy quark mass characteristic of heavy quark effective theory [2] is presumably not a useful tool in this case. Moreover, the methods of HQET are not obviously extended to cope with exclusive hadronic decays. On the other hand, the simple factorized ansatz for the matrix elements is known not to describe properly Cabibbo allowed $D^0$ decays. The color-suppression of some contributions seems in fact to be stronger than the factor 1/3 expected from QCD [3] and the data exhibit large phase differences between amplitudes with definite isospin. We are thus forced, still using the factorization approximation as a starting point of the matrix element evaluation, to include important corrections due to rescattering effects in the final states. This we do assuming the dominance of nearby resonances and taking from experiment - when possible - their masses and widths. We also include W-exchange and annihilation contributions that turn out to be larger than generally believed. The presence of nearby resonances may well have the effect of increasing these terms relative to their naive PCAC estimates.

In two previous papers the Cabibbo allowed [4] two-body decays of charmed mesons were described in the framework discussed above and the model was applied to the analysis of Cabibbo first-forbidden decays of the charged mesons $D^+$ and $D_s^+$ and to their CP violating asymmetries.

The considerable success of that analysis prompts us to extend it to the Cabibbo forbidden two-body decays of $D^0$. The recent experimental determination of the branching ratio $B.R.(D^+ \rightarrow \pi^+\pi^0) = 0.25\pm0.07\%$ [5] - that agrees with our prediction [6] - allows
to perform an amplitude analysis on the complex of $D \to \pi\pi$ decays, that shows a large ($\simeq 90^\circ$) phase difference between $I = 0$ and $I = 2$ amplitudes. Moreover, a comparison of the $I = 2$ amplitude with the $I = \frac{3}{2}$ from $D^+ \to \pi^0\bar{K}$ shows a considerable violation of flavour SU(3) in the direction of larger $\pi\pi$ amplitudes; on the other hand, it is known since a long time and recently confirmed [4] that the ratio of $D^0$ decay branching fractions to $K^+K^-$ and to $\pi^+\pi^-$ is much larger than the SU(3) prediction (i.e. 1), showing an opposite pattern of SU(3) breaking in exotic and nonexotic channels. Another striking signal of the importance of SU(3) violations is given by the value, quite similar to other Cabibbo forbidden decays, of the BR($D^0 \to K^0\bar{K}^0$) that should be vanishing in the symmetric limit.

Our model describes satisfactorily the experimental situation. For what concerns SU(3) breaking in exotic channels it is the combination of several small effects that yields a large result. These effects would not be enough to explain the large $K^+K^-$ to $\pi^+\pi^-$ ratio and to produce a nonvanishing decay rate to $\bar{K}^0K^0$: in the nonexotic channels the rescattering effects are essential.

In order to introduce these rescattering effects we need to know masses, widths and couplings of yet unobserved spinless, isoscalar resonances with positive and negative parity and masses around 1.9 GeV. One expects for each parity two resonances of this type, a SU(3) singlet and a member of an octet, that generally mix among themselves. Such a large number of new parameters to fit eight new data (or limits) for branching ratios is obviously unappealing, unless some arguments can be given to reduce it. In the following we will show that reasonable phenomenological assumptions may reduce the number of new parameters to three.

We have to determine these by a fit to the data. Before doing that we repeated the fit to all Cabibbo allowed and to charged meson first-forbidden decay branching ratios, which in the meantime have got lower error bars and in some cases have also changed. The model is therefore passing a more demanding test.

2. Decay amplitudes in the factorized approximation

The effective weak hamiltonian for Cabibbo allowed nonleptonic decays of charmed particles is given by ($U_{ij}$ are elements of the CKM matrix)

$$H_{\Delta f}^{\Delta S} = \frac{G_F}{\sqrt{2}} U_{ud} U_{cs}^{*} \left[ C_2 \bar{s}^\alpha \gamma_{\mu} (1 - \gamma_5) c_\alpha \bar{u}^\beta \gamma^\mu (1 - \gamma_5) d_\beta + 
C_1 \bar{u}^\alpha \gamma_{\mu} (1 - \gamma_5) c_\alpha \bar{s}^\beta \gamma^\mu (1 - \gamma_5) d_\beta \right] + \text{h.c.},$$

(2.1)
while for $\Delta C = -\Delta S$ processes the hamiltonian is obtained from the same equation with the substitution $s \leftrightarrow d$. The effective weak hamiltonian for Cabibbo first-forbidden nonleptonic decays is

$$H^{\Delta C = \pm 1, \Delta S = 0}_{\text{eff}} = \frac{G_F}{\sqrt{2}} U_{ud} U_{cd}^* \left[ C_1 Q_1^d + C_2 Q_2^d \right] +$$

$$+ \frac{G_F}{\sqrt{2}} U_{us} U_{cs}^* \left[ C_1 Q_1^s + C_2 Q_2^s \right] -$$

$$- \frac{G_F}{\sqrt{2}} U_{ub} U_{cb}^* \sum_{i=3}^{6} C_i Q_i + \text{h.c.}, \quad (2.2)$$

In eq. (2.2) the operators are [8]:

$$Q_1^d = \bar{u}^\alpha \gamma_\mu (1 - \gamma_5) d_\beta \bar{d}^\beta \gamma^\mu (1 - \gamma_5) c_\alpha,$$

$$Q_2^d = \bar{u}^\alpha \gamma_\mu (1 - \gamma_5) d_\alpha \bar{d}^\beta \gamma^\mu (1 - \gamma_5) c_\beta,$$

$$Q_3 = \bar{u}^\alpha \gamma_\mu (1 - \gamma_5) c_\alpha \sum_q \bar{q}^\beta \gamma^\mu (1 - \gamma_5) q_\beta,$$

$$Q_4 = \bar{u}^\alpha \gamma_\mu (1 - \gamma_5) c_\beta \sum_q \bar{q}^\beta \gamma^\mu (1 - \gamma_5) q_\alpha,$$

$$Q_5 = \bar{u}^\alpha \gamma_\mu (1 - \gamma_5) c_\alpha \sum_q \bar{q}^\beta \gamma^\mu (1 + \gamma_5) q_\beta,$$

$$Q_6 = \bar{u}^\alpha \gamma_\mu (1 - \gamma_5) c_\beta \sum_q \bar{q}^\beta \gamma^\mu (1 + \gamma_5) q_\alpha. \quad (2.3)$$

The operator $Q_1^s$ ($Q_2^d$) in eq. (2.2) is obtained from $Q_1^d$ ($Q_2^d$) with the substitution ($d \rightarrow s$). In eqs. (2.1) and (2.3) $\alpha$ and $\beta$ are colour indices (that we will omit in the following formulae) and in the “penguin” operators $q (\bar{q})$ is to be summed over all active flavours ($u, d, s$).

If we neglect mixing with the third generation ($U_{ub} = 0$ and $U_{us} U_{cs}^* = -U_{ud} U_{cd}^* = \sin \theta_C \cos \theta_C$) then the three effective hamiltonians

$$\frac{H^{\Delta C = \Delta S}_{\text{eff}}}{\cos^2 \theta_C}, \quad \frac{-H^{\Delta S = 0}_{\text{eff}}}{\sqrt{2} \sin \theta_C \cos \theta_C}, \quad \frac{H^{\Delta C = -\Delta S}_{\text{eff}}}{\sin^2 \theta_C} \quad (2.4)$$

form a $U$-spin triplet. Therefore, in the limit of exact flavour SU(3) symmetry a number of relations between decay amplitudes should hold. We shall discuss some of them in Section 4 and we will see that they are often violated rather strongly.

We have evaluated the coefficients $C_i$ at the scale 1.5 GeV using the two-loop anomalous dimension matrices recently calculated by A. Buras and collaborators [1], assuming
\[ \Lambda_4^{\overline{MS}} = 300 \text{ MeV.} \] This value, which corresponds to the best agreement between the experimental data and the theoretical results on the exclusive decay channels of D mesons, is compatible with the experimental determination from LEP measurements \[3\]. The coefficients at next-to-leading order are renormalization scheme dependent: we assume in the following the values obtained using the ”scheme independent prescription” of Buras et al., namely \[ C_1 = -0.628, C_2 = 1.347, C_3 = 0.027, C_4 = -0.057, C_5 = 0.015, C_6 = -0.070. \]

In the factorized approximation the matrix elements of \( H_{\text{eff}} \) are written in terms of matrix elements of currents, \( (V^q;q')^\mu = \bar{q}' \gamma^\mu q \) and \( (A^q;q')^\mu = \bar{q}' \gamma^\mu \gamma_5 q \).

We recall the definitions of the decay constants for pseudoscalar (\( \pi, K \ldots \)) and vector (\( \rho, K^\star \ldots \)) mesons

\[
<V_i(p,\lambda)|V^\mu|0> = M_i \, f_{V_i} \, \epsilon^\mu(\lambda),
\]

and the usual definitions \[4\] for the matrix elements of the currents:

\[
<V_i|V^\mu|P_j> = (p_i^\mu + p_j^\mu - \frac{M_j^2}{q^2} q^\mu) \, f_{+}(q^2) + \frac{M_j^2}{q^2} q^\mu \, f_0(q^2),
\]

\[
<V_i|A^\mu|P_j> = i \, (M_j + M_i) \, A_1(q^2) \left( \epsilon^\mu \cdot \frac{q^2}{q^2} - \frac{q^\mu}{q^2} \right) - i \, A_2(q^2) \left( \epsilon^\mu \cdot \frac{q_j^2}{M_j + M_i} - \frac{M_j^2}{q^2} q^\mu \right) + i \, 2 \, M_i \, A_0(q^2) \left( \epsilon^\mu \cdot \frac{q_j^2}{q^2} - q^\mu \right),
\]

\[
<V_i|V^\mu|P_j> = 2 \, V(q^2) \left( \epsilon^\mu \cdot \frac{\epsilon^\sigma \nu_{\rho \sigma}}{M_j + M_i} p_j^\rho p_i^\sigma \right).
\]

To avoid the presence of a spurious singularity at \( q^2 = 0 \), one has to require that

\[
f_{+}(0) = f_0(0) \equiv v_{qq}, \tag{2.7}
\]

\[
A_0(0) = \frac{M_i + M_j}{2 \, M_i} A_1(0) + \frac{M_i - M_j}{2 \, M_i} A_2(0) \equiv a_{qq}. \tag{2.8}
\]

The semileptonic decay rate for \( D^0 \rightarrow K^- e^+ \nu \) \[10\] indicates a value \( v_{cs} \simeq 0.79 \); assuming SU(3) symmetry for the weak charges, we will set

\[
v_{cs} = v_{cd} = v_{cu} = 0.79 \tag{2.9}
\]
Using the definition (2.8) and the data on the semileptonic decay $D^0 \to K^{*-} e^+ \nu$ we obtain $a_{cs} \simeq 0.54 \pm 0.13$ (E653 Collaboration) or $a_{cs} \simeq 0.71 \pm 0.16$ (E691 Collaboration). Different lattice QCD calculations give similar results: in average $a_{cs} \simeq 0.74 \pm 0.15$. The limited statistics for the data on $D^0 \to \rho^- e^+ \nu$ decay does not allow an analysis of the different form factors, but within large errors the measured branching fraction is larger than theoretical predictions based on quark models. Lattice results and results obtained in the framework of QCD sum rules by using (2.8) on SU(3) breaking are inconclusive because of large errors on the $A_2$ form factor.

Since the data on $D$ meson decays show large SU(3) breaking effects and since the axial charges are not protected by the so-called Ademollo-Gatto theorem, in our fit we allowed $a_{cs}$ and $a_{cd} = a_{cu}$ to vary between .5 and 1 independently. The values chosen by the fit are $a_{cs} = 0.59$, consistent with experimental data, and $a_{cd} = 1$ (in fact, an even better fit would be obtained allowing larger values for $a_{cd}$). We note that these values do not agree with the direct QCD sum rule calculation of $A_0(q^2)$ performed in [14], where the authors conclude that SU(3) breaking effects should be small ($a_{cs}/a_{cu} = 1.10 \pm 0.05$) and the $q^2$ dependence of the form factors compatible with a polar dependence dominated by the $0^-$ pole.

The $q^2$ dependence of the form factors is assumed to be dominated by the nearest singularity. This entails for the $c$-quark decay terms the usual simple-pole form factors

$$f_+(q^2) = \frac{v_{cu(s,d)}}{1 - q^2/M_{D^*_+(1^-)}^2},$$
$$f_0(q^2) = \frac{v_{cu(s,d)}}{1 - q^2/M_{D^*_+(0^+)}^2}, \quad (2.10)$$

and analogous expressions, with the mass of the lightest particle with appropriate quantum numbers, for the other form factors.

In the W-exchange and annihilation terms, however, the large and time-like $q^2$ values needed, together with the suggested existence of resonances with masses near to the $D$-meson mass, make a prediction based on the lightest mass singularity unjustified. These terms depend on the matrix elements of current divergences between the vacuum and two-meson states. We write them, with the help of the equations of motion, in the
way indicated in the following examples:

\[
< K^- \pi^+ | \partial^\mu (V_s^d)_{\mu} | 0 > = i (m_s - m_d) < K^- \pi^+ | \bar{s}d | 0 > \equiv \\
= i (m_s - m_d) \frac{M_s^2}{f_D} W_{PP}, \\
< K^- \rho^+ | \partial^\mu (A_s^d)_{\mu} | 0 > = i (m_s + m_d) < K^- \rho^+ | \bar{s}\gamma_5 d | 0 > \equiv \\
= - (m_s + m_d) \frac{2 M_\rho}{f_D} \epsilon^* \cdot p_K W_{PV}.
\]  

(2.11)

We assume SU(3) symmetry for the matrix elements of scalar and pseudoscalar densities, and express all of them in terms of \(W_{PP}, W_{PV}\). In our approach the \(W_i\)'s are free parameters of the fit. Their magnitude turns out to be considerably larger than what one would obtain assuming form factors dominated by the pole of the lightest scalar or pseudoscalar meson, i.e. \(K^*_0(1430)\) or \(K(497)\).

We note that to obtain the amplitudes for Cabibbo first-forbidden decays one has to evaluate matrix elements like \(< \eta | \bar{d} \gamma^\mu \gamma_5 d | 0 >\) or, for penguin operators, \(< \eta | \bar{d} \gamma_5 d | 0 >\). To get the correct result it is necessary to take into account the anomaly of the singlet axial current; we have followed the method discussed in [15]. In this scheme the \(\eta - \eta'\) mixing angle, \(\theta_{\eta\eta'}\), results to be equal to \(-10^\circ\). Remarkably, this value which is consistent with the Gell-Mann Okubo mass formula, is also perfectly compatible with the experimental value of \(\Gamma(\eta \to \gamma\gamma)\) obtained by two-photon production experiments [16]. Therefore the \(\eta - \eta'\) mixing angle is not a parameter of the fit, as it was in our previous analyses.

If the final \(K\) meson in Cabibbo allowed decays is neutral, it has been observed as a short-lived neutral \(K, K_S\). There is therefore an interference between Cabibbo allowed \(D \to \bar{K}^0 + X\) and doubly suppressed \(D \to K^0 + X\) decay amplitudes. We have fitted the experimental data \(\Gamma_{\text{exp}}[6]\) using the definition

\[
\Gamma(D \to K_S + X) \equiv \frac{1}{2} \Gamma_{\text{exp}}(D \to \bar{K}^0 + X),
\]

(2.12)

and included FSI modifications also for the doubly Cabibbo suppressed part of the amplitude. The correction due to this effect is not negligible and it helps in obtaining a better fit to the experimental data.

We write now a few examples of amplitudes for Cabibbo allowed and first forbidden
decays in the factorized approximation:

\[ A_w(D^0 \rightarrow K^-\pi^+) = \]

\[ -\frac{G_F}{\sqrt{2}} U_{ud} U_{cs}^* \left[ (C_2 + \xi C_1) < K^-|(V_c^c)\mu|D^0 > < \pi^+|(A_d^d)\mu|0 > + \right. \]

\[ + (C_1 + \xi C_2) < K^-\pi^+|(V_s^d)\mu|0 > < 0|(A_u^s)^\mu|D^0 > \right] = (2.13) \]

\[ -\frac{G_F}{\sqrt{2}} U_{ud} U_{cs}^* \left[ (C_2 + \xi C_1) f_\pi < K^-|\partial^\mu(V_c^c)\mu|D^0 > + \right. \]

\[ + (C_1 + \xi C_2) f_D < K^-\pi^+|\partial^\mu(V_s^d)\mu|0 > \].

In (2.13) the two terms correspond to c-quark decay and W-exchange, respectively.

\[ A_w(D^+ \rightarrow K_S\rho^+) = \]

\[ -\frac{G_F}{2} U_{ud} U_{cs}^* \left[ (C_2 + \xi C_1) < K_0^0|(V_c^c)\mu|D^+ > < \rho^+|(V_u^d)\mu|0 > + \right. \]

\[ + (C_1 + \xi C_2) < K_0^0|(A_d^d)\mu|0 > < \rho^+|(A_u^s)^\mu|D^+ > \right] + \]

\[ + \frac{G_F}{2} U_{us} U_{cd}^* \left[ (C_2 + \xi C_1) < 0|(A_d^c)\mu|D^+ > < K_0^0\rho^+|(A_u^s)^\mu|0 > + \right. \]

\[ + (C_1 + \xi C_2) < 0|(A_d^d)\mu|0 > < \rho^+|(A_u^s)^\mu|D^+ > \right] = (2.14) \]

\[ -\frac{G_F}{2} U_{ud} U_{cs}^* \left[ (C_2 + \xi C_1) f_\rho M_\rho e^{\mu^\rho} < K_0^0|(V_s^c)\mu|D^+ > + \right. \]

\[ + (C_1 + \xi C_2) f_K < \rho^+|\partial\mu(A_u^s)^\mu|D^+ > \right] + \]

\[ + \frac{G_F}{2} U_{us} U_{cd}^* \left[ (C_2 + \xi C_1) f_D < K_0^0\rho^+|\partial\mu(A_u^s)^\mu|0 > + \right. \]

\[ + (C_1 + \xi C_2) f_K < \rho^+|\partial\mu(A_u^s)^\mu|D^+ > \].

In (2.14) we have given an example of an amplitude with \( K^0 \) and \( K^0 \) interfering contributions, that also contains an annihilation term.

\[ A_w(D^+ \rightarrow \pi^0\pi^+) = \]

\[ -\frac{G_F}{\sqrt{2}} U_{ud} U_{cd}^* \left[ (C_2 + \xi C_1) < \pi^0|(V_d^c)\mu|D^+ > < \pi^+|(A_d^d)\mu|0 > + \right. \]

\[ + (C_1 + \xi C_2) < \pi^+|(V_u^c)\mu|D^+ > < \pi^0|(A_d^d)\mu|0 > \right] = (2.15) \]

\[ -\frac{G_F}{\sqrt{2}} U_{ud} U_{cd}^* \left[ (C_2 + \xi C_1) f_\pi < \pi^0|\partial\mu(V_c^c)\mu|D^+ > + \right. \]

\[ - (C_1 + \xi C_2) \frac{f_\pi}{\sqrt{2}} < \pi^+|\partial\mu(V_u^c)\mu|D^+ > \right] = \]

\[ + \frac{G_F}{2} U_{ud} U_{cd}^* \left( C_1 + C_2 \right) \left( 1 + \xi \right) f_\pi < \pi^+|\partial\mu(V_u^c)\mu|D^+ > . \]
\[ A_w(D^0 \to \pi^- \pi^+) = \]
\[ = -\frac{G_F}{\sqrt{2}} U_{ud} \bar{U}_{cd}^* \left( C_2 + \xi C_1 \right) |\bar{\pi^+}(V^e_d)_\mu|D^0 > < \pi^+|A^d_u^\mu|0 > + \]
\[ + \frac{G_F}{\sqrt{2}} U_{ub} \bar{U}_{cb}^* \left[ (C_4 + \xi C_3) |\bar{\pi^+}(V^e_d)_\mu|D^0 > < \pi^+|A^d_u^\mu|0 > + \right. \]
\[ - 2 (C_6 + \xi C_5) |\bar{\pi}^- \pi^+|\bar{u}u|0 > < 0|\bar{u}\gamma_5 c|D^0 > + \]
\[ + 2 (C_6 + \xi C_5) |\bar{\pi}^- |d\bar{c}|D^0 > < \pi^+|\bar{u}\gamma_5 d|0 > \]
\[ = -\frac{G_F}{\sqrt{2}} U_{ud} \bar{U}_{cd}^* \left( C_2 + \xi C_1 \right) f_\pi < \pi^- |\bar{\pi}(V^e_d)_\mu|D^0 > + \]
\[ + \frac{G_F}{\sqrt{2}} U_{ub} \bar{U}_{cb}^* \left[ (C_4 + \xi C_3) f_\pi < \pi^- |\bar{\pi}(V^e_d)_\mu|D^0 > + \right. \]
\[ + 2 i (C_6 + \xi C_5) < \pi^- \pi^+|\bar{u}u|0 > \frac{f_D M_D^2}{m_u + m_c} + \]
\[ - 2 i (C_6 + \xi C_5) < \pi^- |d\bar{c}|D^0 > \frac{f_\pi M_\pi^2}{m_u + m_d} \right]. \]  \tag{2.16}

In (2.15) and (2.16) we give examples of Cabibbo forbidden decay amplitudes, to the second of which penguin operators contribute.

The parameter \( \xi \) appearing in the above equations should be equal to \( \frac{1}{3} \) in QCD. Since however other colour suppressed, nonfactorizable contributions have been neglected we will consider it as a free parameter to be fitted to the experimental data, following [3]. The result of the fit favours a value \( \xi \approx 0.3 \).

3. Final state interaction effects

We make the assumption that FSI are dominated by resonant contributions, and we neglect the phase-shifts in exotic channels. In the mass region of pseudoscalar charmed particles there is evidence, albeit not very strong [3], for a \( J^P = 0^- K(1830) \) (with \( \Gamma = 250 \text{ MeV} \) and an observed decay to \( K\phi [17] \)) and a \( J^P = 0^- \pi(1770) \) with \( \Gamma = 310 \text{ MeV} [18] \). The coupling of an octet of \( 0^- \tilde{P} \) resonance to \( 0^- + 1^- (PV) \) channels is determined from charge conjugation and SU(3) symmetry to be:

\[ h f_{\mu} (\partial_{\mu} \bar{P}_a) V^\mu_b P_c. \]  \tag{3.1}

Equation (3.1) implies:

\[ \frac{\Gamma(\tilde{K} \to PV)}{\Gamma(\tilde{\pi} \to PV)} = 0.8 \approx \frac{\Gamma_{\text{exp}}(\tilde{K})}{\Gamma_{\text{exp}}(\tilde{\pi})}, \]  \tag{3.2}

consistent with the assumption that the \( \tilde{P} \) resonances decay predominantly into the lowest lying \( P \) and \( V \) mesons, which we shall make for simplicity.
For Cabibbo allowed and doubly forbidden $D^0$ decays and for Cabibbo first and doubly forbidden $D^+$ decays, the FSI effect modifies the amplitudes in the following way:

$$\mathcal{A}(D \to V_h P_k) = \mathcal{A}_w(D \to V_h P_k) + c_{hk}[\exp(i\delta_8) - 1] \sum_{h'k'} c_{h'k'} \mathcal{A}_w(D \to V_{h'} P_{k'}) , \quad (3.3)$$

In (3.3) $c_{hk}$ are the normalized ($\sum c_{hk}^2 = 1$) couplings $\tilde{P}PV$ and

$$\sin\delta_8 \exp(i\delta_8) = \frac{\Gamma(\tilde{P})}{2(M_{\tilde{P}} - M_D) - i\Gamma(\tilde{P})} , \quad (3.4)$$

where $\tilde{P}$ is the resonance appropriate to the decay channel considered ($\tilde{\pi}$ or $\tilde{K}$). Equation (3.4) determines $\delta_8$ up to a 180° ambiguity. The choice can be made according to the number of resonances and bound states [19] that are present in the channel at lower energies, each one of them increasing the phase-shift by $\pi$. In this case the $\tilde{P}$ resonance may be assumed as the third resonance in the channel, with $\delta_8$ close to $\frac{1}{2}\pi$ (in our recent preprint a different choice was made for the $PV$ channels).

For Cabibbo forbidden $D^0 \rightarrow PV$ decays one expects $\tilde{\eta}^0$ and $\tilde{\eta}'$ resonances to take also part to rescattering. Due to charge conjugation invariance, the singlet components have vanishing coupling and the combined effect of the two expected isoscalar resonances may be described by a phase attached to the isosinglet octet part of the decay amplitudes. This phase is the only added parameter for these channels to be fitted. The fitted value is $\sim 243^\circ$, corresponding to two resonances with masses one below and the other above the $D^0$ mass.

Coming now to FSI effects for parity violating $D \rightarrow PP$ decays, we note that some evidence exists for a $J^P = 0^+$ resonance $K^*_0$ (with mass $1945\pm10\pm20$ MeV, width $201\pm34\pm79$ MeV and $52\pm14\%$ branching ratio in $K\pi$ [20]). No $a_0$ isovector resonance has been observed up to now in the interesting mass region. In [4] we assumed its existence and we estimated its mass from the equispacing formula

$$M^2_{a_0} = M^2_{K^*_0} - M^2_K + M^2_\pi . \quad (3.5)$$

In the fit we allowed the mass and the width of $K^*_0$ to vary within the experimental bounds. The best fit values are 1928 MeV and 300 MeV, respectively. From Eq. (3.5) we get $M_{a_0} = 1869$ MeV.

The SU(3) and $C$ invariant coupling of a scalar $S$ resonance to two pseudoscalar mesons is:

$$\sqrt{\frac{3}{2}} g_{888} d_{abc} P_a P_b S_c + g_{818} (P_a P_0 + P_0 P_a) S_a +$$

$$+ g_{881} P_a P_0 S_0 + g_{111} P_0 P_0 S_0 . \quad (3.6)$$
In (3.6) $a, b, c = 1 \ldots 8$ are SU(3) indices, $P_0$ and $S_0$ are SU(3) singlets.

Assuming that the $S$ resonances decay dominantly to a pair of mesons belonging to the lowest mass nonet, one obtains from (3.6) the branching ratio for $(K^*_0 \to K\pi)$ as a function of the ratio of coupling constants $r = g_{818}/g_{888}$. The further assumption of nonet symmetry would imply $r = 1$. The experimental data allow two possible values for $r$, one positive (and consistent with 1) and another negative and close to $-1$. The best fit value of $r$ is $-0.84$, corresponding to a branching ratio for the decay of the $K^*_0$ resonance in $K\pi$ of about 64%.

The description of rescattering effects for Cabibbo forbidden $D^0$ decays is complicated by the presence of yet unobserved $f_0$ and $f'_0$ isoscalar resonances, which should be singlet-octet mixtures. Denoting by $|f_0\rangle$ the lower mass state, we define

$$
|f_0\rangle = \sin \phi \, |f_8\rangle + \cos \phi \, |f_1\rangle \tag{3.7}
$$

$$
|f'_0\rangle = -\cos \phi \, |f_8\rangle + \sin \phi \, |f_1\rangle.
$$

The results will also depend on the parameters $a = g_{881}/g_{888}$ and $c = g_{111}/g_{888}$, see (3.6).

To reduce the number of new parameters, we assume that these scalar resonances behave similarly to the tensor mesons $f_2$ (1270) and $f'_2$ (1525): the $f'_2$ is very weakly coupled to $\pi\pi$, and the $f_2$ has in turn a small coupling to $KK$. In order to forbid the $f'_0 \to \pi\pi$ decay, we required $a$ and the mixing angle $\phi$ to be related by

$$
a = \frac{1}{\sqrt{2} \tan \phi}.
$$

The value $\tan \phi = \sqrt{2}$ would then imply a vanishing branching ratio for the decay $f_0 \to KK$. The best fit value is $\tan \phi = 1.14$, not very far from 1.41.

The fit has therefore two new free parameters, $\Delta^2 = M_{f'_0}^2 - M_{f_0}^2$ and $\phi$. For any pair $(\Delta^2, \phi)$ there are two possible values for $c$, that are solutions of a quadratic condition coming from the requirement of unitarity of the rescattering transformation.

4. Comparison with experimental data on Branching Ratios

Starting from the weak amplitudes $A_w$ defined in Section 2 and modifying them with FSI effects as explained in the previous Section, we evaluated the rates for all Cabibbo allowed two-body decays and for Cabibbo forbidden $D^+ \to D^0K^+$ and $D^+_s \to D^0K^+$ decays as functions of the parameters of the fit. These are $\xi$, the parameters $W_{PP}$ and $W_{PV}$ of the annihilation contributions, the axial charges $a_{cu} = a_{cd}$ and $a_{cs}$, the mass and width of the scalar $K^*_0$ resonance. We use the values $(m_u, m_d, m_s, m_c) = (4.5, 7.4, 150, 1500 \text{ MeV})$ for
the quark masses. The decay constants are \((f_\pi, f_K, f_\rho = f_{K^*}, f_\omega, f_\phi) = (133, 160, 216, 156, 233 \text{ MeV})\). The pole masses in the form factors \(f_{K^*} = f_{K^*}\) corresponding to yet undetected charmed particles have been taken to be \(M_{D^*(0^+)} = 2470 \text{ MeV}\) and \(M_{D^{*+}(0^+)} = 2600 \text{ MeV}\). Other parameters relevant for decay to final states containing \(\eta(\eta')\) have been fixed following \[13\]. The results are presented in Tables 1, 2, 3.

The best-fit results are reported in column three of the Tables. The total \(\chi^2\) is 80.6. The 25 data points for Cabibbo allowed decays contribute 61.8 and the 12 data points for Cabibbo first forbidden decays 18.8. The best fit parameters are \(\xi = -0.027, r = -0.84, W_{PP} = -0.29, W_{PV} = +0.29, a_{cu} = a_{cd} = 1.0, a_{cs} = 0.59, M_{K_0^*} = 1928 \text{ MeV}\) and \(\Gamma_{K_0^*} = 300 \text{ MeV}\). A separate fit to Cabibbo allowed data alone gives quite similar values for the parameters. In the Tables we have also reported the theoretical predictions for the decays to final states containing \(K_L\), in order to show the importance of interference effects with doubly Cabibbo-suppressed amplitudes.

We note that the relatively large SU(3) violation present in the data for exotic \(D^+\) decays \[3\]

\[
R_+ = \left| \frac{U_{cs}}{U_{cd}} \right|^2 \frac{\Gamma(D^+ \to \pi^+\pi^0)}{\Gamma(D^+ \to KS\pi^+)} \approx 3.55, 
\]

(instead of \(R_+ = 1\)) is well reproduced by the fitted data, as it also happened in \[3\]. This point has been discussed in detail in \[21\] and more recently in \[22\]. The reason is that several SU(3) breaking effects, each one rather small, contribute coherently to enhance \(R_+\).

We have also evaluated, not including them in the fit, two recently measured doubly Cabibbo forbidden branching ratios. The experimental data are \[23, 24\]

\[
R_0^{K\pi} = \frac{B.R.(D^0 \to \pi^-K^+)}{B.R.(D^0 \to \pi^+K^-)} = (0.77 \pm 0.25 \pm 0.25) \times 10^{-2} \tag{4.2}
\]

\[
R_+^{K'} = \frac{B.R.(D^+ \to \phi K^+)}{B.R.(D^+ \to \phi\pi^+)} = (5.8^{+3.2}_{-2.6} \pm 0.7) \times 10^{-2}, \tag{4.3}
\]

and the theoretical predictions using the best-fit parameters are \(R_0^{K\pi} = 0.89 \times 10^{-2}\) and \(R_+^{K'} = 0.76 \times 10^{-2}\).

In the SU(3) limit the U-spin properties of the hamiltonian should give \(R_0^{K\pi} = \tan^4 \theta_C = 0.26 \times 10^{-2}\), also discussed in \[22\]. Our model predicts \(R_0^{K\pi}\) to be 3.4 times larger than this value. This is due mainly to the W-exchange contributions, that should vanish in the symmetric limit and have opposite signs in the two amplitudes, and also to rescattering effects.
On the other hand the theoretical value for $R'_\pm$ is much smaller than the experimental datum, that however differs from zero by only 2.5 standard deviations. We note that in a factorized model the decay $D^+ \to \phi K^+$ may only proceed through annihilation and rescattering. It is therefore difficult to reproduce the present very large value for $R'_\pm$.

We consider now $D^0$ decay processes, and in particular Cabibbo first-forbidden decays. The $D^0$ meson is a $U$-spin singlet and it should only decay to $U$-spin triplet states, if flavour SU(3) is a good symmetry. Therefore in that limit several relations among decay amplitudes hold. For the parity violating $D^0 \to PP$ decays they are:

$$A(D^0 \to K^0 K^0) = 0,$$  \hspace{1cm} (4.4)

$$A(D^0 \to K^+ K^-) = -A(D^0 \to \pi^+ \pi^-) = \frac{A(D^0 \to K^+ \pi^-)}{\tan \theta_C} = -\tan \theta_C A(D^0 \to K^- \pi^+)$$ \hspace{1cm} (4.5)

(the last equality corresponds to the symmetry prediction discussed after (4.2)),

$$\frac{1}{\sqrt{3}} A(D^0 \to K_S \pi^0) = \cos \theta_{\eta \eta'} A(D^0 \to K_S \eta) + \sin \theta_{\eta \eta'} A(D^0 \to K_S \eta'),$$ \hspace{1cm} (4.6)

$$-\frac{1}{\sqrt{3}} A(D^0 \to \pi^0 \pi^0) = \cos \theta_{\eta \eta'} A(D^0 \to \pi^0 \eta) + \sin \theta_{\eta \eta'} A(D^0 \to \pi^0 \eta'),$$ \hspace{1cm} (4.7)

$$-A(D^0 \to \pi^0 \pi^0) = (1 - \tan^2 \theta_{\eta \eta'}) A(D^0 \to \eta \eta) + 2 \tan \theta_{\eta \eta'} A(D^0 \to \eta \eta') + \frac{1}{\sqrt{3}} \cos \theta_{\eta \eta'} A(D^0 \to \pi^0 \eta) - \cos \theta_{\eta \eta'} A(D^0 \to \pi^0 \eta'),$$ \hspace{1cm} (4.8)

$$A(D^0 \to \eta \eta') = \tan \theta_{\eta \eta'} A(D^0 \to \eta \eta) + \sqrt{3} (\sin \theta_{\eta \eta'} A(D^0 \to \pi^0 \eta) - \cos \theta_{\eta \eta'} A(D^0 \to \pi^0 \eta')).$$ \hspace{1cm} (4.9)

Comparing the above formulae to the experimental data [6], [7], [23], we note that (4.4) is definitely not true, the moduli of the amplitudes in (4.5) are in the ratios (1:0.59:1.15:0.67) instead of being equal, relation (4.6) is compatible with the data, but only with large phases, and finally no data exist for (4.7), (4.8), (4.9).

In the factorization scheme the amplitude $A(D^0 \to K^0 \bar{K}^0)$ vanishes, and we may only obtain a nonzero rate in that channel through rescattering from the other decay channels. Analogously, the small SU(3) breaking effects ($f_\pi \neq f_K$, $M_D^2 - M_\pi^2 > M_B^2 - M_K^2, \ldots$) are not enough to reproduce the large ratio of $K^+ K^-$ to $\pi^+ \pi^-$ rates. In Table 4 we show the results of our best fit to the Cabibbo first-forbidden decay rates. As explained in Section 3, to the parameters previously determined we added two more free parameters, $\Delta$ and $\phi$, and chose the value of another one, $c$, between the two solutions of a quadratic consistency condition. The fit we obtain is good ($\chi^2 = 1.7$ with 4 data points). The best fit results for
the parameters are \( \Delta = \sqrt{M_{f_0}^2 - M_{f_0}'^2} = 1205 \text{ MeV}, \phi = 48.7^\circ \) and \( c \simeq -2.69 \). The corresponding masses and widths of the scalar and isoscalar resonances are \( M_{f_0} = 1778 \text{ MeV}, \) \( \Gamma_{f_0} = 361 \text{ MeV}, \) \( M_{f_0}' = 2148 \text{ MeV} \) and \( \Gamma_{f_0}' = 389 \text{ MeV}. \)

For parity conserving decays, the relations obtained from flavour symmetry are less restrictive in view of the few existing data. For instance, the relation corresponding to (4.4), namely

\[
\mathcal{A}(D^0 \to K^0 \bar{K}^0) + \mathcal{A}(D^0 \to K^0 K^{*0}) = 0, \quad (4.10)
\]

is valid in the factorization approximation and rescattering does not spoil its validity in our model. Only upper limits exist experimentally. Similarly, other SU(3) predictions are

\[
\begin{align*}
\mathcal{A}(D^0 \to & K^+ K^{-}) = - \mathcal{A}(D^0 \to \pi^+ \rho^-) \\
\mathcal{A}(D^0 \to & K^- K^{*+}) = - \mathcal{A}(D^0 \to \pi^- \rho^+) .
\end{align*}
\quad (4.11)
\]

The W-exchange terms strongly violate these relations and our predictions are therefore at variance with them. Data for \( \pi \rho \) final states are unfortunately still missing.

The experimental data and the fitted values for parity conserving Cabibbo first-forbidden \( D^0 \to PV \) decays are also reported in Table 4. The only parameter added to those determined in the previous fits is the phase-shift of the isosinglet octet part of the decay amplitudes. This parameter turns out to be \( 243^\circ \) and the quality of the fit is reasonably good \((\chi^2 = 7.7\) for 4 data points\).  

5. CP violation

It is well known that CP violating effects show up in a decay process only if the decay amplitude is the sum of two different parts, whose phases are made of a weak (CKM) and a strong (final state interaction) contribution. The weak contributions to the phases change sign when going to the CP-conjugate process, while the strong ones do not. Let us denote a generic decay amplitude of this type by

\[
\mathcal{A} = A e^{i\delta_1} + B e^{i\delta_2} \quad (5.1)
\]

and the corresponding CP conjugate amplitude by

\[
\bar{\mathcal{A}} = A^* e^{i\delta_1} + B^* e^{i\delta_2} \quad (5.2)
\]

The CP violating asymmetry in the decay rates will be therefore

\[
a_{CP} \equiv \frac{|\mathcal{A}|^2 - |\bar{\mathcal{A}}|^2}{|\mathcal{A}|^2 + |\bar{\mathcal{A}}|^2} = \frac{2 \Im(AB^*) \sin(\delta_2 - \delta_1)}{|A|^2 + |B|^2 + 2 \Re(AB^*) \cos(\delta_2 - \delta_1)} . \quad (5.3)
\]
Both factors in the numerator of eq. (5.3) should be nonvanishing to have a nonzero effect. Moreover, to have a sizeable asymmetry the moduli of the two amplitudes $A$ and $B$ should not differ too much.

In Cabibbo first-forbidden $D$ decays, the penguin terms in the effective hamiltonian (2.2) provide the different phases of the weak amplitudes $A$ and $B$. Having obtained a reasonable description of the decay processes, including a model for their strong phases, we may envisage the more ambitious goal to derive CP violating asymmetries using our model for the phase-shifts. The asymmetries resulting are around $10^{-3}$, somewhat larger than previously expected. We stress however that the actual numbers may vary appreciably for parameter variations that still give reasonable fits to the decay rates.

For $D^{+}$ (and $D_s^{+}$) decays the total charge allows to directly measure the rates to be combined in the asymmetry (5.3). In the neutral $D$ decays, however, the need of tagging the decaying particle to tell its charm and the possibility of $D - ar{D}$ mixing make (5.3), as it stands, not directly measurable. Let us consider the particular case of an $e^+e^-$ factory at the $\psi''$, assume to tag $D^0$ by semileptonic decays and define

$$a_{CP}^{\text{eff.}} = \frac{|A(f_i, e^-)|^2 - |A(f_i, e^+)|^2}{|A(f_i, e^-)|^2 + |A(f_i, e^+)|^2}. \tag{5.4}$$

Limiting our considerations to the simplest case, $D$ decays to a CP self-conjugate final state ($f_i = \pm f_i$), and considering the time-integrated asymmetry, one obtains

$$a_{CP}^{\text{eff.}} = \frac{\left(1 - \frac{|p|^2}{|q|^2}\right) \left(1 + \frac{|\bar{A}|^2}{|A|^2}\right) + \frac{1 - y^2}{1 + x^2} \left(1 + \frac{|p|^2}{|q|^2}\right) \left(1 - \frac{|\bar{A}|^2}{|A|^2}\right)}{\left(1 + \frac{|p|^2}{|q|^2}\right) \left(1 + \frac{|\bar{A}|^2}{|A|^2}\right) + \frac{1 - y^2}{1 + x^2} \left(1 - \frac{|p|^2}{|q|^2}\right) \left(1 - \frac{|\bar{A}|^2}{|A|^2}\right)}. \tag{5.5}$$

In (5.5) $A = \mathcal{A}(D^0 \rightarrow f_i)$ is the decay amplitude, the mass matrix eigenstates are defined as

$$|D_{S,L}\rangle \propto |D^0\rangle \pm \frac{q}{p} |\bar{D}^0\rangle$$

and the mixing parameters are

$$x = \frac{2 (M_S - M_L)}{\Gamma_S + \Gamma_L}, \quad y = \frac{\Gamma_S - \Gamma_L}{\Gamma_S + \Gamma_L}.$$

The mixing for charmed mesons is experimentally known to be small ($|x| < 0.083$, $y < 0.085$) \cite{3} and the theoretical calculations of the short-distance contributions give very small predictions. A reliable calculation of long-distance terms is problematic, even more so for CP violation in the mass matrix and the ratio $p/q$. We did not consider at all the
time-dependence in the asymmetries, since these depend on the phase of $p/q$. We note anyhow that the smallness of $x$ and $y$ prevents the development in time of appreciable asymmetries even if the phases would allow this. We expect moreover that the modulus $|p/q|$ will differ from one by a small amount, $O(10^{-3})$. Therefore we can expand (5.5) in the small quantities

$$a_{CP} = \frac{1}{2} \left( 1 - \frac{|A|^2}{|A|^2} \right) \quad \text{and} \quad \Re\epsilon = \frac{1}{4} \left( |p| - 1 \right),$$

with the result

$$a_{CP}^{\text{eff.}} \simeq a_{CP} (1 - \delta) - 2 \delta \Re\epsilon + \ldots$$

(5.6)

In (5.6) we have used the definition $\delta = 1 - (1 - y^2) / (1 + x^2)$. Experimentally $\delta < 0.014$. Therefore, if $a_{CP}$ is not much smaller than $\Re\epsilon$, the measured asymmetry will be $a_{CP}^{\text{eff.}} \simeq a_{CP}$.

We report in Table 5 the values of $a_{CP}$ that we obtain in our model for several decay processes. We chose to give only the results that correspond to a good fit for the branching ratios, even if the predictions for some other decay channels are also large. For parity conserving $D^0$ decays the final states are not CP eigenstates, and the corresponding formula for $a_{CP}^{\text{eff.}}$ is more complicated; however, we note that for amplitudes of not too different absolute values, as it happens in Cabibbo first-forbidden decays, and given the smallness of the mixing parameters, the result is again $a_{CP}^{\text{eff.}} \simeq a_{CP}$. If the final state contains a neutral kaon, normally identified by its decay to $\pi^+\pi^-$, one has to disentangle the CP violating effects in $D$ and $K$ decays. How to do this for the $D^+$ decays has been discussed in [3]. We note that these channels are not very promising candidates to look for CP violating effects in $D$ decays.

We evaluated the central value for $a_{CP}$ choosing for the Maiani – Wolfenstein parameters $(\rho, \eta)$ the values (0.2, 0.3), following a recent analysis of CKM parameters [25], and $U_{cb} = 0.040$. We varied $(\rho, \eta)$ in the one-sigma region obtained in [25] for $f_B = 200 \pm 40$ MeV. The error given in Table 5 reflects only this uncertainty, which is already quite large.

6. Conclusion

We have presented a generally successful description of the complex of two-body nonleptonic decays of charmed mesons. We fitted 45 experimental branching ratios with eleven free parameters, that assume values close to the expected ones at the minimum of the $\chi^2 = 90$. 

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We note that the large SU(3) breaking effects shown by the data are well reproduced in our results. Rescattering (FSI) effects are particularly important in this respect: their parameters are derived from experimental data on nearby resonances. The masses of these being unequal, flavour SU(3) breaking is induced through the difference in the phase shifts for each isospin channel.

W-exchange/annihilation contributions are substantial in many cases. The danger of getting too big decay rates for $D_s$ Cabibbo-favoured decays has been avoided in our model imposing chiral symmetry requirements.

Moreover, the rather large final state phase shifts and “penguin” operator contributions lead us to envisage CP violating asymmetries larger than $10^{-3}$ in some decay channels.
| $f_i$       | $\text{B.R.}_{\text{exp}}(D^0 \to f_i)$ | $\text{B.R.}_{\text{th}}(D^0 \to f_i)$ |
|------------|---------------------------------------|--------------------------------------|
| $K^-\pi^+$ | 4.01 ± 0.14                           | 4.03                                 |
| $K_S\pi^0$ | 1.02 ± 0.13                           | 0.78                                 |
| $K_L\pi^0$ | −                                     | 0.57                                 |
| $K_S\eta$  | 0.34 ± 0.06                           | 0.46                                 |
| $K_L\eta$  | −                                     | 0.34                                 |
| $K_S\eta'$ | 0.83 ± 0.15                           | 0.84                                 |
| $K_L\eta'$ | −                                     | 0.67                                 |
| $\bar{K}^{*0}\pi^0$ | 3.0 ± 0.4 | 3.49                                 |
| $K_S\rho^0$ | 0.55 ± 0.09                           | 0.49                                 |
| $K_L\rho^0$ | −                                     | 0.39                                 |
| $K^{*-}\pi^+$ | 4.9 ± 0.6    | 4.69                                 |
| $K^-\rho^+$ | 10.4 ± 1.3                             | 11.19                                |
| $\bar{K}^{*0}\eta$ | 1.9 ± 0.5   | 0.51                                 |
| $\bar{K}^{*0}\eta'$ | < 0.11 | 0.005                                |
| $K_S\omega$ | 1.0 ± 0.2                           | 1.12                                 |
| $K_L\omega$ | −                                     | 1.04                                 |
| $K_S\phi$  | 0.415 ± 0.060                          | 0.42                                 |
| $K_L\phi$  | −                                     | 0.48                                 |

**TABLE 1**

Decay branching ratios in percent for Cabibbo allowed two-body $D^0$ nonleptonic decays. In the first column the experimental data are reported (upper bounds are 90% c.l.) [6], in the second column the theoretical values obtained in the best fit. See text for further explanations.
| $f_i$       | B.R._exp($D^+ \to f_i$) | B.R._th($D^+ \to f_i$) |
|------------|-------------------------|------------------------|
| $K_S\pi^+$ | $1.37 \pm 0.15$         | 1.08                   |
| $K_L\pi^+$ | --                      | 1.43                   |
| $K^0\pi^+$ | $2.2 \pm 0.4$           | 0.64                   |
| $K_S\rho^+$| $3.30 \pm 1.25$         | 5.28                   |
| $K_L\rho^+$| --                      | 6.49                   |
| $\pi^+\pi^0$ | $0.25 \pm 0.07$      | 0.17                   |
| $\pi^+\eta$ | $0.75 \pm 0.25$       | 0.36                   |
| $\pi^+\eta'$ | $< 0.9$               | 0.79                   |
| $K^0K^+$   | $0.78 \pm 0.17$        | 0.86                   |
| $\rho^0\pi^+$ | $< 0.14$       | 0.17                   |
| $\rho^+\pi^0$ | --                    | 0.37                   |
| $\rho^+\eta$ | $< 1.2$             | 0.0002                 |
| $\rho^+\eta'$ | $< 1.5$            | 0.13                   |
| $\omega\pi^+$ | $< 0.7$          | 0.035                  |
| $\phi\pi^+$ | $0.67 \pm 0.08$     | 0.59                   |
| $K^0K^{*+}$ | --                    | 1.70                   |
| $\overline{K}^0K^+$ | $0.51 \pm 0.10$ | 0.25                   |

**TABLE 2**

Decay branching ratios in percent for Cabibbo allowed and first-forbidden two-body $D^+$ nonleptonic decays.

In the first column the experimental data are reported (upper bounds are 90% c.l.)\cite{3}, in the second column the theoretical rates obtained in the best fit.

See text for further explanations.
| $f_i$              | $\text{B.R}_{\text{exp}}(D^+_s \to f_i)$ | $\text{B.R}_{\text{th}}(D^+_s \to f_i)$ |
|-------------------|---------------------------------|---------------------------------|
| $K_SK^+$          | 1.75 ± 0.35                     | 2.53                            |
| $K_LK^+$          | –                               | 2.26                            |
| $\pi^+\eta$      | 1.90 ± 0.40                     | 1.33                            |
| $\pi^+\eta'$     | 4.7 ± 1.4                       | 5.89                            |
| $\rho^+\eta$     | 10.0 ± 2.2                      | 9.49                            |
| $\rho^+\eta'$    | 12.0 ± 3.0                      | 2.61                            |
| $K^*^0K^+$        | 3.3 ± 0.5                       | 3.86                            |
| $K_SK^{*+}$       | 2.1 ± 0.5                       | 1.44                            |
| $K_LK^{*+}$       | –                               | 1.93                            |
| $\phi\pi^+$      | 3.5 ± 0.4                       | 2.89                            |
| $\omega\pi^+$    | < 1.7                           | 0.0                             |
| $\rho^0\pi^+$    | < 0.28                          | 0.080                           |
| $\rho^+\pi^0$    | –                               | 0.080                           |
| $K^+\pi^0$       | –                               | 0.16                            |
| $K^+\eta$        | –                               | 0.27                            |
| $K^+\eta'$       | –                               | 0.52                            |
| $K^0\pi^+$       | < 0.7                           | 0.43                            |
| $K^{*+}\pi^0$    | –                               | 0.029                           |
| $K^+\rho^0$      | –                               | 0.24                            |
| $K^{*+}\eta$     | –                               | 0.024                           |
| $K^{*+}\eta'$    | –                               | 0.024                           |
| $K^+\omega$      | –                               | 0.072                           |
| $K^+\phi$        | < 0.25                          | 0.015                           |
| $K^{*0}\pi^+$    | –                               | 0.33                            |
| $K^0\rho^+$      | –                               | 1.95                            |

**TABLE 3**

Same as Table 2 for $D^+_s$ nonleptonic decays.
| $f_i$     | $\text{B.R.}_{\text{exp}}(D^0 \rightarrow f_i)$ | $\text{B.R.}_{\text{th}}(D^0 \rightarrow f_i)$ |
|-----------|---------------------------------------------|---------------------------------------------|
| $\pi^0\eta$       | --                                      | 0.058                                      |
| $\pi^0\eta'$      | --                                      | 0.17                                       |
| $\eta\eta$        | --                                      | 0.10                                       |
| $\eta\eta'$       | --                                      | 0.22                                       |
| $\pi^0\pi^0$      | 0.088 ± 0.023                           | 0.116                                      |
| $\pi^+\pi^-$      | 0.159 ± 0.012                           | 0.159                                      |
| $K^+K^-$          | 0.454 ± 0.029                           | 0.456                                      |
| $K^0\overline{K}^0$ | 0.11 ± 0.04 | 0.093                                      |
| $\omega\pi^0$     | --                                      | 0.008                                      |
| $\rho^0\eta$      | --                                      | 0.024                                      |
| $\rho^0\eta'$     | --                                      | 0.010                                      |
| $\omega\eta$      | --                                      | 0.19                                       |
| $\omega\eta'$     | --                                      | 0.0001                                     |
| $\phi\pi^0$       | --                                      | 0.11                                       |
| $\phi\eta$        | --                                      | 0.057                                      |
| $K^{*0}\overline{K}^0$ | < 0.08 | 0.099                                      |
| $\overline{K}^{*0}K^0$ | < 0.15 | 0.099                                      |
| $K^{*+}K^-$      | 0.34 ± 0.08                             | 0.45                                       |
| $K^{*-}K^+$      | 0.18 ± 0.10                             | 0.28                                       |
| $\rho^+\pi^-$    | --                                      | 0.82                                       |
| $\rho^-\pi^+$    | --                                      | 0.65                                       |
| $\rho^0\pi^0$    | --                                      | 0.17                                       |

**TABLE 4**

Decay branching ratios in percent for Cabibbo first-forbidden two-body $D^0$ nonleptonic decays. See the caption of Table 2 and text for details.
| decay channel           | $10^3 \times a_{CP}$ | decay channel           | $10^3 \times a_{CP}$ |
|-------------------------|-----------------------|-------------------------|-----------------------|
| $D^+ \rightarrow \rho^0\pi^+$ | $-1.17 \pm 0.68$      | $D^+ \rightarrow \overline{K}^0 K^+$ | $-0.51 \pm 0.30$      |
| $D^+ \rightarrow \rho^+\pi^0$  | $+1.28 \pm 0.74$      | $D^0 \rightarrow \pi^0\eta$ | $+1.43 \pm 0.83$      |
| $D^0 \rightarrow K^{*0}\overline{K}^0$ | $+0.67 \pm 0.39$      | $D^0 \rightarrow \pi^0\eta'$ | $-0.98 \pm 0.57$      |
| $D^0 \rightarrow \overline{K}^{*0}\overline{K}^0$ | $+0.67 \pm 0.39$      | $D^0 \rightarrow \eta\eta'$ | $+0.50 \pm 0.29$      |
| $D^0 \rightarrow K^{*+}K^-$   | $-0.038 \pm 0.022$    | $D^0 \rightarrow \eta\eta'$ | $+0.28 \pm 0.16$      |
| $D^0 \rightarrow K^{*-}K^+$   | $-0.16 \pm 0.09$      | $D^0 \rightarrow \pi^0\pi^0$ | $-0.54 \pm 0.31$      |
| $D^0 \rightarrow \rho^+\pi^-$  | $-0.37 \pm 0.22$      | $D^0 \rightarrow \pi^+\pi^-$ | $+0.02 \pm 0.01$      |
| $D^0 \rightarrow \rho^-\pi^+$  | $+0.36 \pm 0.21$      | $D^0 \rightarrow K^+K^-$ | $+0.13 \pm 0.8$       |
| $D^0 \rightarrow \rho^-\pi^-$  | $+0.36 \pm 0.21$      | $D^0 \rightarrow \eta\eta'$ | $+0.50 \pm 0.29$      |
| $D^0 \rightarrow \rho^-\pi^+$  | $+0.36 \pm 0.21$      | $D^0 \rightarrow K^0\overline{K}^0$ | $-0.28 \pm 0.16$      |

**TABLE 5**

CP-violating decay asymmetries for some $D^+$ and $D^0$ Cabibbo forbidden decays. See text for explanation.
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