Gravity solutions duals to non-relativistic field theories

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Abstract. In this note we discuss solutions of type IIB supergravity with an isometry group of non-relativistic scaling symmetry and in a special case, the well-known Schrödinger symmetry. In particular we discuss supersymmetric backgrounds with Schrödinger isometry and renormalization group flow geometries with Schrödinger symmetry at the fixed points. We also discuss scale-invariant gravity backgrounds with a general dynamical exponent. This is a summary based on the following works: N. Bobev and A. Kundu, JHEP 0907:098,2009 [arXiv:0904.2873 [hep-th]]; N. Bobev, A. Kundu and K. Pilch, JHEP 0907:107,2009 [arXiv:0905.0673 [hep-th]].

1. Introduction

In recent years gauge-gravity duality has emerged both as a remarkable concept and a novel tool. For a recent and application oriented review of this duality we refer the reader to [1]. The techniques of gauge-gravity duality have been extensively used in understanding aspects of strong coupling dynamics in relativistic systems ranging from quark-gluon plasma to several condensed matter systems [2]. There are several strongly coupled systems which are inherently non-relativistic. Over the past couple of years there has been active research towards constructing holographic duals of non-relativistic field theories.

The primary focus has been to consider non-relativistic field theories which exhibit scale invariance. This scale invariance is characterized by an anisotropic scaling between the spatial and the temporal directions and this anisotropy is parametrized by a real number known as the dynamical exponent. For a special value of this dynamical exponent the non-relativistic scaling symmetry can be extended to include a special conformal generator. The corresponding symmetry group of this extended algebra is known as the Schrödinger group. Physical systems such as free fermions or the cold atoms at unitarity have been observed to possess this symmetry [3]. More details on non-relativistic conformal field theories can be found in e.g. [4].

One of the guiding principles behind any holographic construction is to match the global symmetry group of the gauge theory with the isometry group of the proposed dual gravitational background. This has been the basic philosophy behind the pioneering work in [5, 6] where a five-dimensional Schrödinger symmetric background was proposed and the subsequent work in [7, 8, 9] where this background was embedded in ten dimensional supergravity. In this note, we will discuss several other gravitational backgrounds obtained as solutions of type IIB supergravity which possess such non-relativistic scaling symmetry (and for a special value of the
dynamical exponent the Schrödinger symmetry) as the isometry group. These backgrounds are generally obtained by deforming well-known AdS-backgrounds in a specific way.

It is an interesting question to consider the possible values of dynamical exponents realizable within a ten-dimensional type IIB supergravity set-up. In general, given an AdS$_5 \times X_5$ solution it is possible to find a deformed background which has a non-relativistic scaling symmetry characterized by the corresponding dynamical exponent. In this case the dynamical exponents are related to the spectrum of scalar or vector harmonics on the internal manifold $X_5$ [10, 11].

Note however that it is unclear whether the dual field theories of such supergravity backgrounds share any common feature with real systems (such as the cold atoms at unitarity). Therefore it is not clear what lessons we might learn about strongly coupled non-relativistic field theories using these gravity backgrounds. Hence we stand on the following ground: the supergravity backgrounds we discuss are interesting enough to consider in their own rights and should further be treated as toy models perhaps capable of providing insights into strong coupling dynamics.

The article is organized as follows: in Section 2 we discuss some details of the Schrödinger group and the five-dimensional background which possess this isometry. We move on to Section 3 to describe the embedding of this low-dimensional background into ten dimensional supergravity via a solution generating technique. Guided by these results we discuss new gravity solutions in Section 4. This includes supersymmetric backgrounds with Schrödinger symmetry, RG-flow backgrounds with Schrödinger symmetry at the fixed points and gravity backgrounds having scaling symmetry with a general dynamical exponent. Finally we conclude with open questions and future directions in Section 5.

2. The Schrödinger group and the gravity background

We begin with a brief review of the so called Schrödinger group, denoted by Schr($d$). It is the symmetry group of the Schrödinger equation in free space and is the non-relativistic analogue of the relativistic conformal group. The generators of the Schrödinger group are: temporal translation, spatial translations, rotations, Galilean boosts, dilatation and a special conformal transformation.

The corresponding Schrödinger algebra in $d$-spatial dimensions can be obtained by the so called light cone reduction of the relativistic conformal algebra in $(d+1)$-spatial dimensions. We unpack briefly the operational meaning of the light cone reduction as follows[5]. Let us start with the relativistic conformal group in $(d+1)$-spatial dimensions, denoted by SO($d+2,2$). First we identify the light cone momentum of the SO($d+2,2$) group with the Galilean mass operator. Now we choose all operators in the parent SO($d+2,2$) group that commute with this light cone momentum. This set of operators close under the Schrödinger algebra. Therefore by construction, Schr($d$) $\subset$ SO($d+2,2$).

In this algebra the dilatation and the Galilean mass operator (or the number operator) are simultaneously diagonalizable. So any representation of the Schrödinger algebra is labeled by two numbers. Note that for physical systems the eigenvalues of the number operator should be discrete.

It is simple to understand why this construction is inherently non-relativistic. The Schrödinger algebra thus constructed consists of a dilatation operator under which space and time scale differently: $t \rightarrow \lambda^2 t, \vec{x} \rightarrow \lambda \vec{x}$ where $\lambda$ is a real number, breaking the Lorentz invariance.

Following our ideology discussed in the introduction, we may ask ourselves whether it is possible to construct a gravitational background with the Schrödinger isometry group. It turns out that such a background can be uniquely realized by deforming the standard AdS metric (we set the radius to unity) in the Poincaré patch in the following manner

$$ds^2 = -\frac{1}{z^4}du^2 + \frac{1}{z^2}(-2dudv + d\vec{x}^2 + dz^2), \quad (1)$$
where the term proportional to \(du^2\) is the deformation, \(u\) and \(v\) are the light-cone coordinates, \(\vec{x}\) is the spatial 2-directions and \(z\) is the radial coordinate (in other words the renormalization group scale for the dual field theory). The infrared (IR) of the geometry is obtained by setting \(z \to \infty\) and the ultraviolet (UV) is located at \(z \to 0\). Note that the background in (1) degenerates to a one-dimensional line segment in the strict UV limit and therefore does not have a meaningful UV boundary. In practice this problem can be circumvented by introducing a small but non-vanishing cut-off \(z_{\text{cut-off}}\). However, this remains an obstacle in the efforts for a systematic analysis of holographic renormalization in this spacetime. On the other hand, the IR limit of this geometry is well-defined: it simply is an AdS-space in the light cone coordinates.

The coordinates \(u\) and \(\vec{x}\) are respectively identified with the time and the spatial directions of the dual field theory. The \(v\)-coordinate is an additional coordinate present purely in the bulk geometry. Recall that the representations of the Schrödinger group are labeled by two conserved numbers: the conformal dimension and the particle number. Therefore it would be natural to compactify the \(v\)-direction and identify the corresponding discrete mass spectrum with the discrete particle number. Note that this is a special requirement imposed by hand and \textit{a priori} there is nothing that prevents the \(v\)-direction to be an extended one.

The spacetime in (1) has very interesting causal structure. It is conformal to a pp-wave spacetime known to be non-distinguishing [12, 13]. This means that although there are no closed causal curves in the above background, there are distinct points in the spacetime with identical past and future sets. This is the causal structure of a Galilean field theory.

3. Embedding the gravity background in supergravity

We will now briefly discuss the embedding of such backgrounds in supergravity. Shortly after the spacetime in (1) was proposed in [5, 6], an algorithmic approach of constructing similar solutions from known asymptotically AdS solutions of type IIB supergravity was laid out in [7, 8, 9]. This algorithm makes use of a solution generating technique known as the null Melvin twist [14, 15] or the TsT transformation [8]. The basic idea behind the null Melvin twist is simple: it generates a new solution of supergravity equations of motion upon applying a well-defined set of algebraic symmetry operations (involving boosts, T-duality and shift-symmetry) on a given supergravity background. In order to apply these symmetry operations we need to pick one compact and one non-compact \(U(1)\)-isometry in the given ten dimensional background.

Let us start with a Freund-Rubin type solution of type IIB supergravity of the form \(\text{AdS}_5 \times X_5\)

\[
\begin{align*}
\text{AdS}_5 & \quad \text{ds}^2 + \text{ds}^2_{X_5} \ , \\
F_5 & \quad = (1 + \ast) \text{vol}_{\text{AdS}_5} ,
\end{align*}
\]

(2)

where \(X_5\) is an Einstein manifold. We further assume that \(X_5\) has at least one \(U(1)\)-isometry and \(K\) denotes the one-form dual to the corresponding Killing vector. Now we apply the null Melvin twist involving \(K\) and one spatial direction of the AdS-part of the background. This operation yields the following Schrödinger symmetric background [11]

\[
\begin{align*}
\text{Schr}_5 & \quad \text{ds}^2 + \text{ds}^2_{X_5} \ , \\
F_5 & \quad = (1 + \ast) \text{vol}_{\text{Schr}_5} ,
\end{align*}
\]

(3)

where

\[
\begin{align*}
\text{ds}^2_{\text{Schr}_5} & \quad = -\frac{\Omega}{z^4} du^2 + \frac{1}{z^2} \left( -2dudv + d\vec{x}^2 + dz^2 \right) , \quad \text{with} \quad \Omega = ||K||^2 , \\
B_{(2)} & \quad = \frac{1}{z^2} K \wedge du .
\end{align*}
\]

(4)

(5)

Note the presence of the function \(\Omega(X_5)\) in front of the deformation term in the metric. The general form of the metric generated using the null Melvin twist technique is therefore of the warped product form \(\text{Schr}_5 \times_w X_5\).
In [7, 8, 9], $X_5$ was chosen to be $S^5$ and the Hopf fiber was chosen to be the compact $U(1)$ along which the twist was performed. The Killing vector associated with the Hopf fiber has a constant norm, and therefore in this case we get $\Omega = 1$ upon proper normalization. The five dimensional piece of this background identically matches with the one in (1). Upon Kaluza-Klein reduction on the compact $S^5$ we obtain the background given in (1) along with a massive vector field (coming from dimensionally reducing the $B_{(2)}$-field on $S^5$). Therefore we obtain an Einstein-Proca effective action in five dimensions which contains the background in (1) as a solution.

Following the discussions in [9] we will now briefly comment on the meaning of the null Melvin twist process in the dual gauge theory. Let us recall that the gauge theory dual to $AdS_5 \times S^5$ is the $\mathcal{N} = 4$ super Yang-Mills (SYM) and the Hopf fiber direction in $S^5$ corresponds to an $U(1)_R$ symmetry of the gauge theory. Without the Melvin deformation, compactifying the light like direction $v$ amounts to the Discrete Light Cone Quantization (DLCQ) of the original $\mathcal{N} = 4$ SYM. The Melvin deformation adds a twist to the momentum generator along the light like direction by the momentum generator along the $U(1)$-direction used in the Melvin twist process. Thus in this case, we end up with the $R$-symmetry deformed DLCQ version of the $\mathcal{N} = 4$ SYM, which possess the Schrödinger symmetry\(^1\).

Note that the original $\mathcal{N} = 4$ SYM is a maximally supersymmetric theory. However, as shown in [7, 8] this non-relativistic deformation coming from the twist by the $R$-symmetry direction breaks it completely. Therefore the resulting background in [7, 8, 9] is not supersymmetric. A question naturally arises: can we construct Schrödinger symmetric gravity backgrounds possessing some amount of supersymmetry. We will address this question in the next section.

4. New gravity solutions

4.1. Twisting with a general $U(1)$ and supersymmetry

Let us take the specific example when $X_5 = S^5$. Choosing the Hopf fiber as the twisting direction is merely a convenience. Note that the $S^5$ has an $SO(6)$ isometry group and we can apply a more general three-parameter twist on the Cartan $U(1)^3$ subgroup [17]. However, in this general construction $\Omega$ is indeed a function of the $S^5$-coordinates and therefore it is not \textit{a priori} known how to obtain an effective five-dimensional description.

It is possible that the function $\Omega$ vanishes on some locus in $X_5$, therefore locally the deformation term may disappear. However, such phenomenon have been analyzed carefully in [18, 19] and it has been concluded that as long as $\Omega$ is non-zero on an open set in $X_5$, the asymptotic and the causal structure of the background are consistent with the dual non-relativistic field theory. This is indeed the case in all examples we consider.

Since we have a more general twist at our disposal, we can revisit the question related to supersymmetry asked at the end of the previous section. Namely we can ask whether the backgrounds in (4) preserve any supersymmetry for some choice of the one-form $K$. One approach to address this issue is to analyze which Killing spinors of the undeformed $AdS_5 \times X_5$ background survive the non-relativistic deformation.

Interestingly, we can address this issue without referring to any particular choice of the internal manifold $X_5$. The only constraint we impose is that $X_5$ is a Sasaki-Einstein manifold with at least one $U(1)$-isometry (implying the existence of a non-trivial $K$). Recall that $X_5$ needs to be Einstein in order to satisfy the supergravity equations of motion. The condition that $X_5$ is also a Sasaki manifold comes from requiring that the undeformed $AdS_5 \times X_5$ background allows for at least one Killing spinor.

Under these assumptions, by analyzing the supersymmetry variation equations it is possible to obtain a necessary and sufficient condition under which a Killing spinor in the undeformed

\(^1\) Alternatively, the Melvin twist can be thought of as modifying the ordinary products of fields to the so called star product. See \textit{e.g.} [16] for more details on such constructions.
AdS$_5 \times X_5$ background will also be a Killing spinor of the Schr$_5 \times u_\epsilon X_5$ background and vice versa. This condition consists of one algebraic and one differential constraint of the following form [11]:

$$\left( 1 + \Gamma^{14} \right) \epsilon = 0 \quad \text{and} \quad \mathcal{L}_K \epsilon = 0 \ ,$$

where $\epsilon$ is the Killing spinor, $\Gamma$ denotes the purely real ten-dimensional Dirac gamma matrices, the indices 1, 4 refers to the flat coordinates corresponding to the $u$ and $v$-directions respectively and $\mathcal{L}_K$ denotes the Lie derivative along the isometry direction associated with the Killing vector $K$. Observe that none of the superconformal Killing spinors of the undeformed AdS-background survives the $\Gamma^{14}$ projection condition. Note however, that this does not impose any condition on the additional Killing spinors (with opposite $\Gamma^{14}$-chirality) which may arise only at the Schrödinger background [20].

Let us now go back to the example when $X_5 = S^5$. The Lie derivative acting on the Killing spinors of the undeformed background along the Hopf fiber direction gives the non-zero R-charge of the corresponding Killing spinor. This is in direct violation of the differential constraint imposed in (6) and thus breaks all supersymmetry. This is what was observed in [7, 8]. For the sphere, choosing an appropriate U(1) preserves two real supercharges in general, which can be enhanced to four real supercharges for special choices of the U(1) [11].

Before concluding this section, let us offer some more comments based on other examples of Sasaki-Einstein manifolds. The next simplest example is the $T^{1,1}$ which has an SU(2)$\times$SU(2) × U(1)$_R$ symmetry [21]. There are also two infinite families of five-dimensional Sasaki-Einstein manifolds with explicitly known metrics. These are the $Y^{p,q}$ family [22] with SU(2)$\times$U(1)$\times$U(1)$_R$ symmetry and the $L^{p,q,r}$ family [23, 24] with U(1)$^2 \times$U(1)$_R$ symmetry. Note that for a general Sasaki-Einstein manifold there exists a Killing vector with constant norm associated with the U(1)$_R$-symmetry. This is known as the Reeb vector. Choosing the Reeb vector in the twist will violate the differential constraint in (6) and break supersymmetry completely. However for all Sasaki-Einstein manifolds with explicitly known metrics, we can preserve two real supercharges by choosing an appropriate U(1) other than the R-symmetry. In these cases, the undeformed dual CFT is a quiver gauge theory. Therefore the non-relativistic deformation of such theories is to be understood as the deformed DLCQ of the corresponding quiver gauge theory.

4.2. RG flow geometry with Schrödinger symmetry

Let us now switch gears to discuss RG flow gravity backgrounds dual to non-relativistic field theories. Our approach here is solely based on the solution generating technique discussed earlier. We start with the well-known supergravity solutions dual to relevant deformations of $\mathcal{N} = 4$ SYM and apply the null Melvin twist on them and generate the non-relativistic analogue of the flow.

First let us review the relevant deformation of $\mathcal{N} = 4$ SYM in some details. This theory has three adjoint chiral superfields, which we denote by $\Phi_i$, $I = 1, 2, 3$. If we give mass to one of these superfields by adding a mass perturbation of the form $m_1 \Phi^2_1$ to the superpotential, then the theory flows to an IR $\mathcal{N} = 1$ fixed point with SU(2)$\times$U(1)$_R$ symmetry [25]. The dual gravity background of this flow was constructed in [26] and is known as the Pilch-Warner background.

More generally we can also start with a $\mathbb{Z}_2$ quiver gauge theory with an SU($N$)$\times$SU($N$) gauge group, two hypermultiplets and a pair of adjoint chiral superfields, denoted by $\{\Phi_1, \Phi_2\}$. This defines a CFT with $\mathcal{N} = 2$ supersymmetry. Now we can add the following mass perturbation

$$\Delta W \sim m_1 \Phi^2_1 + m_2 \Phi^2_2 \ .$$

Under this mass perturbation, this theory flows to a family of $\mathcal{N} = 1$ IR fixed points parametrized by the ratio of the two mass parameters. When $m_1 = m_2$, we get the Pilch-Warner fixed
point and for \( m_1 = -m_2 \) we get the Klebanov-Witten fixed point [27]. The gravity solution interpolating between these two fixed points have been constructed in [28].

Since these gravity solutions are known\(^2\), we can use the machinery of the null Melvin twist to generate the non-relativistic analogue of these flow and interpolating backgrounds. Interestingly for all these backgrounds the internal manifold allows for one \( U(1) \)-isometry direction to be used in the Melvin twist procedure. This exercise has been carried out in detail in [17] and it has been shown explicitly that this fixed point structure persists in the Melvin twisted backgrounds which are now dual to the Schrödinger symmetric fixed points. The non-relativistic field theories are again understood as the deformed DLCQ of the corresponding relativistic field theory.

There are some interesting features of these backgrounds. For example, the Schrödinger symmetric Pilch-Warner fixed point is a perfectly regular warped product space with rotation-like terms present in the metric [17]. Furthermore, this background has internal NS and RR-fluxes. To the best of our knowledge, this is the only known Schrödinger symmetric background with all the above-mentioned features.

### 4.3. Gravity solutions with general dynamical exponents

Note that so far we have looked for gravity solutions having a scaling symmetry of the form: 
\[
t \to \lambda^2 t, \quad \vec{x} \to \lambda \vec{x},
\]
which is the scaling symmetry of the Schrödinger group. However we can attempt for a more general scaling symmetry of the form: 
\[
t \to \lambda^n t, \quad \vec{x} \to \lambda \vec{x},
\]
where \( n \) is some real number known as the dynamical exponent. It is known that for \( n \neq 2 \) the symmetry algebra does not have a special conformal generator and therefore the corresponding symmetry group is just the Galilean group with a dilatation operator. We call this the Galilean scaling algebra\(^3\).

Unlike the Schrödinger algebra, the representations of Galilean scaling algebra are labeled by a single number: a dimension.

Based on the known results in the Schrödinger (\( i.e. \ n = 2 \)) case in (4) we generalize our ansatz as follows:
\[
ds^2 = -\frac{\Omega}{z^{2n}} du^2 + \frac{1}{z^2} (-2dudv + d\vec{x}^2 + dz^2) + ds^2_{X_5},
\]
\[
B_{(2)} = \frac{1}{z^n} \mathcal{A} \wedge du, \quad F_{(5)} = (1 + \ast) \text{vol}_{X_5},
\]
where \( X_5 \) is still an Einstein manifold, \( \Omega \) is an unknown function on \( X_5 \) and \( \mathcal{A} \) is an unknown one-form both of which we need to fix from the equations of motion.

The equations of motion separate in two parts: the Maxwell equation and the Einstein equation. The Maxwell equation gives a massive Proca equation along with a transversality condition for the one-form \( \mathcal{A} \). Taken together, this results in a Laplace equation for \( \mathcal{A} \). On the other hand the Einstein equation gives an inhomogeneous Laplace equation for the scalar function \( \Omega \), where the inhomogeneous part consists of the energy-momentum contribution coming from the one-form \( \mathcal{A} \).

In order to solve the Maxwell equation, we need to know the spectrum of transverse vector harmonics on \( X_5 \). Upon choosing an appropriate vector harmonic on \( X_5 \) the Maxwell equation reduces to an algebraic relation between the dynamical exponent \( n \) and the eigenvalue of the Laplacian on the corresponding vector harmonic. This algebraic relation determines the allowed values for the dynamical exponents. Then the inhomogeneous Laplace equation for \( \Omega \) can in principle be solved by expanding the source term into scalar harmonics on \( X_5 \). Note that as a special case we can set \( \mathcal{A} = 0 \) and reproduce the results of [10].

\(^2\) These backgrounds are not always known analytically, but for the twist procedure we do not need an exact analytic form of the background.

\(^3\) We thank M. Rangamani for suggesting this name.
For example, when $X_5 = S^5$ the allowed dynamical exponents are all positive integers greater than one. In this case, we have constructed some explicit solutions in [11]. For a general internal manifold the allowed dynamical exponents can be irrational real numbers as is the case for $T^{1,1}$ [11].

5. Conclusion
We have discussed several interesting gravity backgrounds with a general scaling symmetry. This scaling symmetry is characterized by a real number called the dynamical exponent and we have found a plethora of such dynamical exponents allowed within solutions of the supergravity equations of motion.

For the special case of $n = 2$, i.e. when the symmetry group is enhanced to the full Schrödinger group, we have found backgrounds with some amount of supersymmetry. It is therefore natural to ask whether the backgrounds with general dynamical exponents preserve any supersymmetry. This appears to be a difficult problem in complete generality, however it would be very interesting to analyze this for specific examples. This would be particularly interesting since the supersymmetric extension of the Galilean scaling algebra has not been studied extensively in the current literature.

Our backgrounds with general dynamical exponents are warped product spaces and therefore it is not known how to obtain a low-dimensional consistent truncation of these backgrounds. However, for special choices of the internal manifold $X_5$ and for special values of the dynamical exponent the scalar function $\Omega$ may become a pure number; thereby facilitating a straightforward Kaluza-Klein compactification. It would be very interesting to explore this issue further.

Interestingly, although there are no curvature singularities in our backgrounds with general dynamical exponent, the Poincaré patch is geodesically incomplete for $n \geq 2$ [29]. For the Schrödinger case (i.e. when $n = 2$), it is possible to extend this patch to a global one which is everywhere regular [29]. However, the analogue of such a global patch is not known for backgrounds with other dynamical exponents. It will be an interesting issue to consider in future.

It is always interesting to find black hole solutions. The only known black hole solutions within this context exist in the asymptotically Schrödinger spacetime. Such solutions are obtained by using the null Melvin twist on the known AdS-Schwarzschild solution. However, for a general dynamical exponent there is no such solution generating technique and one needs to solve the equations of motion in its full glory. It would therefore be very interesting to find black hole solutions with general dynamical exponent within ten-dimensional supergravity or perhaps a lower dimensional effective model. By virtue of the gauge-gravity duality such a solution will automatically provide an arena for studying aspects of a scale-invariant strongly coupled non-relativistic field theory at finite temperature.

In this note we have restricted ourselves within type IIB supergravity. One can also look for similar solutions within type IIA and 11-dimensional supergravity. Some such analysis can be found in e.g. [30, 31].

Let us conclude by noting that we are able to find several interesting examples of gravity solutions candidates for a dual of scale-invariant non-relativistic field theory. These field theories are obtained by the so called dipole deformation (in Schrödinger symmetric cases) or deformation by higher dimensional Lorentz violating operators (for a general dynamical exponent) of the conventional gauge theories arising from massless string modes living on the worldvolume of the D-branes. To be a truly compelling proposal, one needs to identify and extract the features of strong coupling dynamics within this framework which might be relevant for real physical systems. These are exciting prospects for future research.
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