From the Foucault pendulum to the galactical gyroscope and LHC

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Abstract

The Lagrange theory of particle motion in the noninertial systems is applied to the Foucault pendulum, isosceles triangle pendulum and the general triangle pendulum swinging on the rotating Earth. As an analogue, planet orbiting in the rotating galaxy is considered as the giant galactical gyroscope. The Lorentz equation and the Bargmann-Michel-Telegdi equations are generalized for the rotation system. The knowledge of these equations is inevitable for the construction of LHC where each orbital proton “feels” the Coriolis force caused by the rotation of the Earth.

Key words. Foucault pendulum, triangle pendulum, gyroscope, rotating galaxy, Lorentz equation, Bargmann-Michel-Telegdi equation.

1 Introduction

In order to reveal the specific characteristics of the mechanical systems in the rotating framework, it is necessary to derive the differential equations describing the mechanical systems in the noninertial systems. We follow the text of Landau et al. (Landau et al. 1965).

Let be the Lagrange function of a point particle in the inertial system as follows:

\[ L_0 = \frac{m v_0^2}{2} - U \]  \hspace{1cm} (1)

with the following equation of motion

\[ m \frac{dv_0}{dt} = - \frac{\partial U}{\partial r} , \]  \hspace{1cm} (2)
where the quantities with index 0 corresponds to the inertial system.

The Lagrange equations in the noninertial system is of the same form as that in the inertial one, or,

\[ \frac{d}{dt} \frac{\partial L}{\partial \dot{\mathbf{v}}} = \frac{\partial L}{\partial \mathbf{r}}. \]  

(3)

However, the Lagrange function in the noninertial system is not the same as in eq. (1) because it is transformed.

Let us first consider the system \( K' \) moving relatively to the system \( K \) with the velocity \( \mathbf{V}(t) \). If we denote the velocity of a particle with regard to system \( K' \) as \( \mathbf{v}' \), then evidently

\[ \mathbf{v}_0 = \mathbf{v}' + \mathbf{V}(t). \]  

(4)

After insertion of eq. (4) into eq. (1), we get

\[ L'_0 = \frac{m\mathbf{v}'^2}{2} + m\mathbf{v}'\mathbf{V} + \frac{m}{2}\mathbf{V}^2 - U. \]  

(5)

The function \( \mathbf{V}^2 \) is the function of time only and it can be expressed as the total derivation of time of some new function. It means that the term with the total derivation in the Lagrange function can be removed from the Lagrangian. We also have:

\[ m\mathbf{v}'\mathbf{V}(t) = m\mathbf{V} \frac{d\mathbf{r}'}{dt} = \frac{d}{dt}(mr'\mathbf{V}(t)) - mr' \frac{d\mathbf{V}}{dt}. \]  

(6)

After inserting the last formula into the Lagrange function and after removing the total time derivation we get

\[ L' = \frac{m\mathbf{v}'^2}{2} - m\mathbf{W}(t)\mathbf{r}' - U, \]  

(7)

where \( \mathbf{W} = \frac{d\mathbf{V}}{dt} \) is the acceleration the system \( K' \).

The Lagrange equations following from the Lagrangian (7) are as follows:

\[ m \frac{d\mathbf{v}'}{dt} = -\frac{\partial U}{\partial \mathbf{r}'} - m\mathbf{W}(t). \]  

(8)

We see that after acceleration of the system \( K' \) the new force \( m\mathbf{W}(t) \) appears. This force is fictitious one because it is not generated by the internal properties of some body.

In case that the system \( K' \) rotates with the angle velocity \( \Omega \) with regard to the system \( K \), the radius vectors \( \mathbf{r} \) and \( \mathbf{r}' \) are identical and (Landau et al., 1965)

\[ \mathbf{v}' = \mathbf{v} + \Omega \times \mathbf{r}. \]  

(9)

The Lagrange function for this situation is (Landau et al., 1965)

\[ L = \frac{mv^2}{2} - m\mathbf{W}(t)\mathbf{r} - U + m\mathbf{v} \cdot (\Omega \times \mathbf{r}) + \frac{m}{2}(\Omega \times \mathbf{r})^2. \]  

(10)

The corresponding Lagrange equations for the last Lagrange function are as follows (Landau et al., 1965):

\[ m \frac{dv}{dt} = -\frac{\partial U}{\partial \mathbf{r}} - m\mathbf{W} + m\mathbf{r} \times \dot{\Omega} + 2m\mathbf{v} \times \Omega + m\Omega \times (\mathbf{r} \times \Omega). \]  

(11)
We observe in eq. (11) three so called inertial forces. The force \(mr \times \dot{\Omega}\) is connected with the nonuniform rotation of the system \(K'\) and the forces \(2mv \times \Omega\) and \(m\Omega \times r \times \Omega\) correspond to the uniform rotation. The force \(2mv \times \Omega\) is so called the Coriolis force and it depends on the velocity of a particle. The force \(m\Omega \times r \times \Omega\) is called the centrifugal force. It is perpendicular to the rotation axes and the magnitude of it is \(m\bar{r}\omega^2\), where \(\bar{r}\) is the distance of the particle from the rotation axis.

Equation (11) can be applied to many special cases. We apply it first to the case of the mathematical pendulum swinging in the gravitational field of the rotating Earth. In other words, to the so called Foucault pendulum.

## 2 Foucault pendulum

Foucault pendulum was studied by Léon Foucault (1819 - 1868) as the big mathematical pendulum with big mass \(m\) swinging in the gravitational field of the Earth. He used a 67 m long pendulum in the Panthéon in Paris and showed the astonished public that the direction of its swing changed over time rotating slowly. The experiment proved that the earth rotates. If the earth would not rotate, the swing would always continue in the same direction.\(^1\)

If we consider the motion in the system only with uniform rotation, then we write equation (11) in the form:

\[
\frac{d\mathbf{v}}{dt} = -\frac{\partial U}{\partial \mathbf{r}} + 2m\mathbf{v} \times \Omega + m\Omega \times \mathbf{r} \times \Omega. \tag{12}
\]

In case of the big pendulum, the vertical motion can be neglected and at the same time the term with \(\Omega^2\). The motion of this pendulum is performed in the horizontal plane \(xy\). The corresponding equations are as follows (Landau et al., 1965):

\[
\ddot{x} + \omega^2 x = 2\Omega_z \dot{y}, \quad \ddot{y} + \omega^2 y = -2\Omega_z \dot{x}, \tag{13}
\]

where \(\omega\) is the frequency of the mathematical pendulum without rotation of the Earth, or \(\omega = 2\pi/T\) and (Landau et al., 1965): \(T \approx 2\pi \sqrt{l/g}\), where \(T\) is the period of the pendulum oscillations, \(l\) is the length of the pendulum and \(g\) is the Earth acceleration.

After multiplication of the second equation of (13) by the imaginary number \(i\) and summation with the first equation, we get:

\[
\ddot{\xi} + 2i\Omega_z \dot{\xi} + \omega^2 \xi = 0 \tag{14}
\]

for the complex quantity \(\xi = x + iy\). For the small angle rotation frequency \(\Omega_z\) of the Earth with regard to the oscillation frequency \(\omega\), \(\Omega_z \ll \omega\), we easily find the solution in the form:

\[
\xi = e^{-i\Omega_z t}(A_1 e^{i\omega t} + A_2 e^{-i\omega t}), \tag{15}
\]

or,

\[
x + iy = e^{-i\Omega_z t}(x_0 + iy_0), \tag{16}
\]

\(^1\)Author performed the experiment with the Foucault pendulum inside of the rotunda of the Flower garden in Kroměříž (Moravia, Czech Republic)
where functions $x_0(t), y_0(t)$ are the parametric expression of the motion of the pendulum without the Earth rotation. If the complex number is expressed in the trigonometric form of (16), the $\Omega_z$ is the rotation of the complex number $x_0 + iy_0$. The physical meaning of eq. (16) is, that the plane of the Foucault pendulum rotates with the frequency $\Omega_z$ with regard to the Earth.

Galileo Galilei (1564 - 1642) - Italian scientist and philosopher - studied the mathematical pendulum before Foucault. While in a Pisa cathedral, he noticed that a chandelier was swinging with the same period as timed by his pulse, regardless of his amplitude. It is probable, that Galileo noticed the rotation of the swinging plane of the pendulum. However, he had not used this fact as the proof of the Earth rotation when he was confronted with the Inquisition tribunal. Nevertheless, his last words were “E pur si muove”.

3 The triangle pendulum

The triangle pendulum is the analogue of the Foucault pendulum with the difference that the pendulum is a rigid system composed from a two rods forming the triangle ABC. In the isosceles triangle it is $AC = CB = l = \text{const}$. The legs $AC = CB$ are supposed to be prepared from the nonmetal and nonmagnetic material, with no interaction with the magnetic field of the Earth. Point $C$ is a vertex at which the pendulum is hanged. The vertex is realized by the very small ball. Points $A$ and $B$ are not connected by the rod. The angle $ACB = \alpha$. The initial deflection angle of $CB$ from the vertical is $\varphi_0 + \alpha$, where $\varphi_0$ is the initial deflection angle from vertical.

To be pedagogical clear, let us give first the known theory of the simple mathematical pendulum (Amelkin, 1987).

The energetical equation of the pendulum is of the form ($\varphi$ is the deflection angle from vertical):

$$\frac{mv^2}{2} - mgl \cos \varphi = -mgl \cos \varphi_0,$$

from which follows, in the polar coordinates with $v = l \dot{\varphi}$

$$\dot{\varphi} + \frac{g}{l} \sin \varphi = 0.$$

We have for the very small angle $\varphi$ that $x \approx l \varphi$ and it means that from the last equation follows the equation for the harmonic oscillator

$$\ddot{x} + \frac{g}{l} x = 0.$$

The rigorous derivation of the period of pendulum follows from eq. (17). With $v = ds/dt = l d\varphi/dt$, we get

$$\frac{l}{2} \left( \frac{d\varphi}{dt} \right)^2 = g(\cos \varphi - \cos \varphi_0).$$

Then,

$$dt = \sqrt{\frac{T}{2g}} \frac{d\varphi}{\sqrt{\cos \varphi - \cos \varphi_0}}.$$

For the period $T$ of the pendulum, we have from the last formula:
\[ T = \frac{\sqrt{l}}{4} \int_{\varphi_0}^{\varphi_0} \frac{d\varphi}{\sqrt{\cos \varphi - \cos \varphi_0}}. \]  \hspace{1cm} (22)

Using relations \( \cos \varphi = 1 - 2\sin^2 \varphi/2 \), \( \cos \varphi_0 = 1 - 2\sin^2 \varphi_0/2 \), and substitution \( \sin \varphi/2 = k \sin \chi \), with \( k = \sin \varphi_0/2 \), we get

\[ d\varphi = \frac{2\sqrt{k^2 - \sin^2 \chi/2}}{\sqrt{1 - k^2 \sin^2 \chi}} d\chi \]  \hspace{1cm} (23)

and finally

\[ T = \frac{\sqrt{l}}{4} \int_{0}^{\pi/2} \frac{d\chi}{\sqrt{1 - k^2 \sin^2 \chi}}, \]  \hspace{1cm} (24)

where the integral in the last formula is so called the elliptic integral, which cannot be evaluated explicitly but only in the form of series.

Now, let us go back to the isosceles triangle pendulum. We write in the polar coordinates instead of the equation (17):

\[ \left( \frac{1}{2} m \ell^2 \dot{\varphi}^2 - mg \cos(\varphi - \alpha) \right) + \left( \frac{1}{2} m \ell^2 \dot{\varphi}^2 - mg \cos \varphi \right) = \text{const.} \]  \hspace{1cm} (25)

Then, after differentiation with regard to time, we get from the last equation the following one:

\[ 2\ddot{\varphi} + \frac{g}{l} (\sin(\varphi - \alpha) + \sin \varphi) = 0. \]  \hspace{1cm} (26)

It is easy to see that for \( \alpha = 0 \) the equation of motion is \( \ddot{\varphi} + \left(\frac{g}{l}\right) \sin \varphi = 0 \), which is the expected result because the triangle pendulum in this case is the mathematical pendulum.

The equilibrium state of the isosceles triangle pendulum is the state with \( \varphi = \alpha/2 \). The small swings are then performed in the interval

\[ \frac{\alpha}{2} - \varepsilon \leq \varphi \leq \frac{\alpha}{2} + \varepsilon. \]  \hspace{1cm} (27)

If we put

\[ \varphi = \chi + \alpha/2 \]  \hspace{1cm} (28)

the equation of motion for the variable \( \chi \) is

\[ \ddot{\chi} + \omega^2 \sin \chi = 0 \]  \hspace{1cm} (29)

with

\[ \omega = \sqrt{\frac{g}{l} \cos(\alpha/2)}. \]  \hspace{1cm} (30)

For very small angle \( \chi \) the equation (29) is the equation of the harmonic oscillator and in the situation of the rotation of the Earth it is possible to apply the same mathematical procedure as in case of the Foucault pendulum. The result is the same as we have described it (Landau et al., 1965). In other words, the triangle pendulum behaves as the Foucault
pendulum and it can be used as the table pendolino experiment for the demonstration of the Earth rotation.

The triangle pendulum with equal sides can be generalized to the situation with \( AC = l_1, BC = l_2 \) with masses \( m_1, m_2 \). Then, it is easy to show by the same procedure, that the original equation of motion of such generalized triangle pendulum is as follows:

\[
\ddot{\varphi} + \omega_1^2 \sin(\varphi - \alpha) + \omega_2^2 \sin(\varphi) = 0,
\]

where

\[
\omega_1^2 = \frac{m_1 gl_1}{m_1^2 l_1^2 + m_2^2 l_2^2}; \quad \omega_2^2 = \frac{m_2 gl_2}{m_1^2 l_1^2 + m_2^2 l_2^2}
\]

For \( l_1 = l_2 = l \) and \( m_1 = m_2 = m \), we get the isosceles pendulum and for \( \alpha = 0 \), we get the original simple mathematical pendulum.

The mathematical and physical analysis of the general triangle pendulum shows us that this pendulum has the same behavior as the Foucault pendulum. Or, in other words we can denote it as the triangle Foucault pendulum.

Let us still remark that while the magnetic needle of the compass rotates with the Earth (forced by the magnetic field of the Earth), the plane of motion of the Foucault pendulum and the triangle pendulum does not rotate with the Earth.

4 The galactical gyroscope

The gyroscope is usually defined as a device for measuring or maintaining orientation based on the principle of conservation of angular momentum. The essence of the device is the spinning wheel. We will show that the planet orbiting in the rotating galaxy is the galactical gyroscope because the orientation of the orbit is conserved reminding the classical gyroscope.

The force acting on the planet with mass \( m \) is according to Newton law

\[
F = -G \frac{mM}{r^2},
\]

where \( M \) is the mass of Sun, \( r \) being the distance from \( m \) to the Sun.

The corresponding equations of motion in the coordinate system \( x \) and \( y \) are as follows

\[
m\ddot{x} = -G \frac{mM}{r^2} \cos \varphi; \quad m\ddot{y} = -G \frac{mM}{r^2} \sin \varphi,
\]

or, with \( \sin \varphi = y/r, \cos \varphi = x/r \),

\[
\ddot{x} = -k \frac{x}{r^3}; \quad \ddot{y} = -k \frac{y}{r^3}, \quad k = GM, \quad r = \sqrt{x^2 + y^2}
\]

Using \( x = r \cos \varphi, y = r \sin \varphi \), we get instead of equations (35):

\[
(r - r^2 \dot{\varphi}^2) \cos \varphi - (2r \dot{\varphi} + r \ddot{\varphi}) \sin \varphi = -k \frac{\cos \varphi}{r^2}
\]

\[
(r - r^2 \dot{\varphi}^2) \sin \varphi + (2r \dot{\varphi} + r \ddot{\varphi}) \cos \varphi = -k \frac{\sin \varphi}{r^2}.
\]
In case that the motion of the planet is performed in the rotation system of a galaxy the equations (36), (37) are written in the form ($\Omega_z = \Omega$)

\[
(\ddot{r} - r\dot{\varphi}^2) \cos \varphi - (2\dot{r}\dot{\varphi} + r\ddot{\varphi}) \sin \varphi = -\frac{k \cos \varphi}{r^2} + 2\Omega \dot{y}
\]

(38)

\[
(\ddot{r} - r\dot{\varphi}^2) \sin \varphi + (2\dot{r}\dot{\varphi} + r\ddot{\varphi}) \cos \varphi = -\frac{k \sin \varphi}{r^2} - 2\Omega \dot{x},
\]

(39)

or,

\[
(\ddot{r} - r\dot{\varphi}^2) \cos \varphi - (2\dot{r}\dot{\varphi} + r\ddot{\varphi}) \sin \varphi = -\frac{k \varphi}{r^2} + 2\Omega (\dot{r} \sin \varphi + r \cos \varphi \dot{\varphi})
\]

(40)

\[
(\ddot{r} - r\dot{\varphi}^2) \sin \varphi + (2\dot{r}\dot{\varphi} + r\ddot{\varphi}) \cos \varphi = -\frac{k \sin \varphi}{r^2} - 2\Omega (\dot{r} \cos \varphi - r \sin \varphi \dot{\varphi}).
\]

(41)

After multiplication of eq. (40) by $\sin \varphi$ and eq. (41) by $\cos \varphi$ and after their subtraction we get

\[
2\dot{r}\ddot{\varphi} + r\dddot{\varphi} = -2\Omega \dot{r},
\]

(42)

or,

\[
\frac{d}{dt}(r^2 \dot{\varphi}) = -\Omega \frac{d}{dt}(r^2),
\]

(43)

or,

\[
\dot{\varphi} = -\Omega.
\]

(44)

It means that the angle velocity of the ellipse of a planet inside the rotating galaxy is $\Omega$ which is the angle velocity of the galaxy. Let us only remark that here we consider the well defined galaxy as the galaxy of elliptical form and not of the chaotic form. We do not consider here the “galaxy rotation problem” - the discrepancy between the observed rotation speeds of matter in the disk portion of spiral galaxies and the predictions of Newton dynamics considering the luminous mass - which is for instance discussed in http://en.wikipedia.org/wiki/Galaxy_spiral_problem.

5 Discussion

We have presented the Lagrange theory of the noninertial classical systems and we applied the theory to the so called Foucault pendulum, the isosceles triangle pendulum with two equal masses and to the triangle pendulum with the nonequal legs and masses. We have shown that Every pendulum is suitable for the demonstration of the rotation of the earth.

For the demonstration of the galaxy rotation, we have analyzed the elliptical motion of our planet and we have shown that the orbital motion of our planet can be used as gigantic gyroscope for the proof of the rotation of our galaxy in the universe. The orbit of our planet with regard to the rest of the universe has the stable stationary position while the galaxy rotates. The orbital planetary stability can be used as the method of the investigation of the rotation of all galaxies in the rest of the universe. To our knowledge this method was not still used in the galaxy astrophysics.

It is possible to consider also the rotation of the Universe. If we define Universe as the material bodies immersed into vacuum, then the rotation of the Universe is physically
meaningful and the orbit of our planet is of the constant position with regard to the vacuum as the rest system. The idea that the vacuum is the rest system is physically meaningful because only vacuum is the origin of the inertial properties of every massive body. In other words, the inertial mass $m$ in the Newton-Euler equation $F = ma$ is the result of the interaction of the massive body with vacuum and in no case it is the result of the Mach principle where the inertial mass is generated by the mass of rest of the Universe. At present time, everybody knows that Mach principle is absolutely invalid for all time of the existence of Universe.

Now, the question arises what is the description of the rotation in the general theory of relativity. If we use the the Minkowski element

$$ds^2 = c^2 dt'^2 - dx'^2 - dy'^2 - dz'^2$$

and the nonrelativistic transformation to the rotation system (Landau et al., 1988)

$$x' = x \cos \Omega t - y \sin \Omega t, \quad y' = x \sin \Omega t + y \cos \Omega t, \quad z = z'$$

then we get:

$$ds^2 = [c^2 - \Omega^2(x^2 + y^2)] dt'^2 - dx'^2 - dy'^2 - dz'^2 + 2\Omega y dx dt - 2\Omega x dy dt,$$

which is not relativistically invariant.

If we use the Minkowski element in the cylindrical coordinates

$$ds^2 = c^2 dt'^2 - dr'^2 - r'^2 d\varphi'^2 - dz'^2$$

and the transformation to the rotating system $r' = r, z' = z, \varphi' = \Omega t$ (Landau et al., 1988), we get the noninvariant element

$$ds^2 = [c^2 - \Omega^2 r^2] dt'^2 - 2\Omega r^2 d\varphi dt - dr'^2 - r'^2 d\varphi'^2 - dz'^2.$$  

So, we see that the rotational system can be used only for $r < c/\Omega$. For $r > c/\Omega$, the component $g_{00}$ is negative, which is in the contradiction with the principles of relativity.

However, according to special theory of relativity only Lorentz transformation can be inserted into the equation (45). Or,

$$dx' = \gamma(dx - vdt), \quad dt' = \gamma(t - vx/c^2), \quad \gamma = \frac{1}{\sqrt{1 - v^2/c^2}}.$$  

The radial coordinate of the rotational system is not contracted. Only the tangential coordinate $dx$. So, in the cylindrical coordinates, it is necessary to write $dx = r d\varphi, dx' = r d\varphi', v = \Omega r$. Using these ingredients we write:

$$d\varphi' = \gamma(d\varphi - \Omega dt), \quad dt' = \gamma(dt - (\Omega r^2/c^2)d\varphi).$$

After insertion of equations (51) into equation (48), we get for the interval $ds$ the original relation (48):

$$ds^2 = c^2 dt'^2 - dr'^2 - r'^2 d\varphi'^2 - dz'^2.$$  

We think that transformation (51) is correct because it it based on the Lorentz transformation, which has here the physical meaning of the relation between tangential elements $dx = r d\varphi, dx' = r d\varphi', v = \Omega r$ and the infinitesimal time relation.
The correctness of the transformation between inertial and rotation system is necessary because it enables to describe the motion of the particle and spin in the LHC by the general relativistic methods. The basic idea is the generalization so called Lorentz equation for the charged particle in the electromagnetic field $F^{\mu\nu}$ (Landau et all., 1988):

$$mc\frac{dv^\mu}{ds} = \frac{e}{c} F^{\mu\nu} v_\nu.$$  \hspace{1cm} (53)

In other words, the normal derivative is replaced by the covariant one and we get the general relativistic equation for the motion of a charged particle in the electromagnetic field and gravity (Landau et all., 1988):

$$mc \left( \frac{dv^\mu}{ds} + \Gamma^\mu_{\alpha\beta} v^\alpha v^\beta \right) = \frac{e}{c} F^{\mu\nu} v_\nu,$$  \hspace{1cm} (54)

where

$$\Gamma^\mu_{\alpha\beta} = \frac{1}{2} g^\mu\lambda \left( \frac{\partial g_{\lambda\alpha}}{\partial x^\beta} + \frac{\partial g_{\lambda\beta}}{\partial x^\alpha} - \frac{\partial g_{\alpha\beta}}{\partial x^\lambda} \right)$$  \hspace{1cm} (55)

are the Christoffel symbols derived in the Riemann geometry theory (Landau et all., 1988).

In case that we consider motion in the rotating system, then it is necessary to insert the metrical tensor $g_{\mu\nu}$, following from the Minkowski element for the rotation system. The construction of LHC with proton must be in harmony with equation (54) because orbital protons “feels” the Coriolis force from the rotation of the Earth.

The analogical situation occurs for the motion of the spin. While the original Bargmann-Michel-Telegdi equation for the spin motion is as follows (Berestetzki et all., 1988)

$$\frac{da^\mu}{ds} = 2\mu F^{\mu\nu} a_\nu - 2\mu' \mu v^{\mu} F^{\alpha\beta} v_\alpha a_\beta,$$  \hspace{1cm} (56)

where $\mu' = \mu - e/2m$ and $a_\mu$ is the axial vector, which follows also from the classical limit of the Dirac equation as $\bar{\psi}i\gamma_5 \gamma^\mu \psi$ (Rafanelli et al., 1964; Pardy, 1973), the general relativistic generalization of the Bargmann-Michel-Telegdi equation can be obtained by the analogical procedure which was performed with the Lorentz equation. Or,

$$\left( \frac{da^\mu}{ds} + \Gamma^\mu_{\alpha\beta} v^\alpha a^\beta \right) = 2\mu F^{\mu\nu} a_\nu - 2\mu' \mu v^{\mu} F^{\alpha\beta} v_\alpha a_\beta,$$  \hspace{1cm} (57)

where in case of the rotating system the metrical tensor $g_{\mu\nu}$ must be replaced by the metrical tensor of the rotating system. Then, the last equation will describe the motion of the spin in the rotating system.

The motion of the polarized proton in LHC will be described by the last equation because our Earth rotates. During the derivation we wrote $\Gamma^\mu_{\alpha\beta} v^\alpha a^\beta$ and not $\Gamma^\mu_{\alpha\beta} v^{\alpha} v^\beta$, because every term must be the axial vector. In other words, the last equation for the motion of the spin in the rotating system was not strictly derived but created with regard to the philosophy that physics is based on the creativity and logic (Pardy, 2005).

On the other hand, the equation (57) must evidently follow from the Dirac equation in the rotating system, by the same WKB methods which were used by Rafanelli, Schiller and Pardy (Rafanelli and Schiller, 1964; Pardy, 1973). The derived BMT equation in the metric of the rotation of the Earth are fundamental for the proper work of LHC because every orbital proton of LHC “feels” the rotation of the Earth and every orbital proton
spin “feels” the Earth rotation too. So, LHC needs equations (54) and (57) and vice versa.

The theory discussed in our article can be also applied to the pendulum where the fibre is elastic. The corresponding motion is then described by the wave equation with the initial and boundary conditions.

It is evident that there are many physical problems, classical and quantum mechanical considered in the rotation system. Some problems were solved and some problems will be solved in the future. Let us define some of these problems.

Mössbauer effect in the rotating system, Schrödinger equation for a particle in the rotating system, Schrödinger equation for the pendulum in the inertial system and in the rotating system, Schrödinger equation of H-atom in the rotating system, Schrödinger equation of harmonic oscillator in the rotating system, the Čerenkov effect in the rotating dielectric medium, the relic radiation in the rotating galaxy, The N-dimensional blackbody radiation in the rotating system, conductivity and superconductivity in the rotating system, laser pulse in the rotating system, Berry phase, Sagnac effect, and so on. All these problems can be formulated classically, or in the framework of the general theory of relativity with the Γ-connections corresponding to the geometry of the rotating system. We hope that the named problems are interesting and their solution will be integral part of the theoretical physics.

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