CARDASSIAN EXPANSION: CONSTRAINTS FROM COMPACT RADIO SOURCE ANGULAR SIZE VERSUS REDSHIFT DATA

ZONG-HONG ZHU and MASA-KATSU FUJIMOTO
National Astronomical Observatory, 2-21-1, Osawa, Mitaka, Tokyo 181-8588, Japan; zong-hong.zhu@nao.ac.jp, fujimoto.masa-katsu@nao.ac.jp

Received 2002 July 25; accepted 2002 August 13

ABSTRACT

The “Cardassian Expansion Scenario” was recently proposed by Freese & Lewis as an alternative to a cosmological constant in explaining the current accelerating universe. In this paper we investigate observational constraints on this scenario from recent measurements of the angular size of high-z compact radio sources compiled by Gurvits and coworkers. We show that the allowed intervals for n and z_eq, the two parameters of the Cardassian model, are heavily dependent on the value of the mean projected linear size l. However, the best fit to the current angular size data prefers the conventional flat cold dark matter model to this Cardassian expansion proposal, though the latter is cosmologically credible and compatible with the Θ-z diagram for some values of l.

Subject headings: cosmological parameters — cosmology: theory — distance scale

1. INTRODUCTION

The standard big bang cosmological model is based on four cornerstones: the Hubble expansion, the cosmic microwave background radiation, primordial big bang nucleosynthesis, and structure formation. Recent observations of the Hubble relation of distant Type Ia supernovae have provided strong evidence for the acceleration of the universe (Perlmutter et al. 1998, 1999; Riess et al. 1998). While current measurements of the cosmic microwave background anisotropies favor a spatially flat universe with cold dark matter (CDM; de Bernardis et al. 2000; Lange et al. 2001), both the deuterium abundance measured in four high-redshift hydrogen clouds seen in absorption against distant quasars (Burles & Tytler 1998a, 1998b; combined with baryon fraction in galaxy clusters from X-ray data; see White et al. 1993) and the large-scale structure in the distribution of galaxies (Bahcall 2000; Peacock et al. 2001) have made a strong case for a low-density universe (for a recent summary, see Turner 2002). All these observations can be concordantly explained by the hypothesis that there exists, in addition to CDM, a dark energy component with negative pressure in our universe (Turner 1998). The existence of this component has also been independently confirmed by other observations such as age estimates of old high-redshift galaxies (Dunlop et al. 1996; Krauss 1997; Alcaniz & Lima 1999) and gravitational lensing (Kochanek 1996; Chiba & Yoshii 1999; Futamase & Hamana 1999; Jain et al. 2001; Dev et al. 2001; Ohyama et al. 2002).

During the past several years, a huge number of candidates for the dark energy component have been proposed, such as a cosmological constant (Weinberg 1989; Carroll et al. 1992; Krauss & Turner 1995; Ostriker & Steinhardt 1995), a frustrated network of topological defects (such as cosmic strings or domain walls; Vilenkin 1984; Davis 1987; Kamionkowski & Toumbas 1996) and an evolving scalar field (referred to by some as quintessence; Ratra & Peebles 1988; Frieman et al. 1995; Coble et al. 1995; Caldwell et al. 1998). Despite great effort to pin down the amount and the nature of the dark energy, a convincing mechanism with a solid basis in particle physics that explains the accelerating universe is still far off. Very recently, Freese & Lewis (2002) proposed a “Cardassian Expansion Scenario” in which the universe is flat, matter dominated, and accelerating but contains no vacuum contribution. The main point of this scenario is to modify the standard Friedman-Robertson-Walker equation as follows:

$$H^2 = A\rho + B\rho^n,$$

where $H \equiv \dot{R}/R$ is the Hubble parameter (as a function of time), $R$ is the scale factor of the universe, and the energy density $\rho$ contains only ordinary matter and radiation (Freese & Lewis 2002). The second term, which may arise as a consequence of brane world cosmologies, drives the acceleration of the universe at a late epoch when it is dominant. The authors claimed that this Cardassian model survives observational tests such as the cosmic microwave background radiation, the age of the universe, the cluster baryon fraction, and structure formation, and they are now studying possible observational tests (Freese & Lewis 2002). In this paper, we give the first observational constraint on this scenario from recent measurements of the angular size of high-z compact radio sources made by Gurvits, Kellerman, & Frey (1999). We show that, although this Cardassian expansion proposal is cosmologically credible and compatible with the Θ-z diagram for some values of the mean projected linear size l, it is disfavored by the best fit to the current angular size data when compared with the conventional flat ΛCDM model. Our result is very similar to the one of Avelino & Martins (2002), who used Type Ia supernovae data to show another particular solution of a brane world scenario being disfavored. There is a common point among these analyses: both models predict a universe with unreasonably low matter density.

2. ANGULAR SIZE DATA ANALYSIS

We begin by evaluating the angular diameter distance as a function of redshift z as well as the parameters of the model. Following the notation of Peebles (1993), we define the redshift dependence of $H$ as $H(z) = H_0E(z)$. For the Ansatz of equation (1) and a flat universe with only matter
(baryonic and CDM), Freese & Lewis (2002) get
\[
E^2(z; n, z_{eq}) = \left[1 + (1 + z_{eq})^{3(1-n)}\right]^{-1} (1 + z)^3 \\
+ \left\{1 - \left[1 + (1 + z_{eq})^{3(1-n)}\right]^{-1}\right\}(1 + z)^{3m},
\]
where \(n\) and \(z_{eq}\) are the two parameters of the Cardassian model. Note that \(z_{eq}\) is the redshift at which the two terms of equation (1) are equal. The coefficients of the Ansatz of equation (1) can be written as \(A = 8\pi G/3\) and \(B = H_0^2(1 + z_{eq})^{3(1-n)}\rho_0\left[1 + (1 + z_{eq})^{3(1-n)}\right]^{-1}\), where \(H_0 = 100 h\) km s\(^{-1}\) Mpc\(^{-1}\) is the Hubble constant and \(\rho_0\) is the matter density of the universe at the present time. It is straightforward to show that the angular diameter distance is given by
\[
d_A(z; n, z_{eq}) = \frac{c}{H_0} \frac{1}{1 + z} \int_0^{z} \frac{dz'}{E(z'; n, z_{eq})}.
\]

In order to give an observational constraint on the Cardassian model parameters \(n\) and \(z_{eq}\), we analyze the angular size data for millisecond radio sources recently compiled by Gurvits et al. (1999). This data set is 145 sources binned into 12 redshift bins, with about the same number of sources per bin (Fig. 1). The lowest and highest redshift bins are centered at redshifts \(z = 0.52\) and \(z = 3.6\), respectively. We determine the model parameters \(n\) and \(z_{eq}\) through a \(\chi^2\) minimization method. The range of \(n\) spans the interval [0, 1] in steps of 0.01, while the range of \(z_{eq}\) spans the interval [0, 2] in steps of 0.02.

\[
\chi^2(l; n, z_{eq}) = \sum_i \left(\frac{\theta(z_i; l; n, z_{eq}) - \theta_{oi}}{\sigma_i^2}\right)^2,
\]
where \(\theta(z_i; l; n, z_{eq}) = l/d_A\) is the angle subtended by an object of proper length \(l\) transverse to the line of sight and \(\theta_{oi}\) is the observed values of the angular size with errors \(\sigma_i\) of the \(i\)th bin in the sample. The summation is over all the observational data points.

In the Cardassian model for a flat universe containing only the matter, the matter density in units of critical density, \(\rho_c = 3H_0^2/8\pi G\), is \(\Omega_m = F(n, z_{eq})\equiv [1 + (1 + z_{eq})^{3(1-n)}]^{-1}\). Instead of specifying \(\Omega_m\), we consider both \(n\) and \(z_{eq}\) as independent parameters, while \(\Omega_m\) (or \(F\)) is treated as the output of the fitting result. As was pointed out by Gurvits et al. (1999), Lima & Alcaniz (2002), and Alcaniz (2002) when one uses the angular size data to constrain the cosmological parameters, the results will be strongly dependent on the characteristic length \(l\). Therefore, instead of assuming a specific value for \(l\), we have worked on the interval \(l = 15-30\) h\(^{-1}\) pc. In Figure 2, we show contours of constant likelihood (95.4% and 68.3%) in the plane \((n, z_{eq})\) for several values of \(l\), i.e., \(l = (20, 21, 24, 28)\) h\(^{-1}\) pc. Table 1 summarizes our best fits for different values of \(l\); e.g., at \(l = 20\) h\(^{-1}\) pc, the best fit occurs for \(n = 0.17\) and \(z_{eq} = 0.48\) (hence \(\Omega_m = 0.27\)). In order to make the analysis independent of the choice of the characteristic length \(l\), we also minimize equation (4) for \(l, n\), and \(z_{eq}\), which gives \(l = 22.6\) h\(^{-1}\) pc, \(n = 0.0\), and \(z_{eq} = 0.62\) (hence \(\Omega_m = 0.19\)) as the best fit with \(\chi^2 = 4.52\) and 9 degrees of freedom. Therefore, it seems that the best fit to the current angular size data prefers the conventional flat \(\Lambda\)CDM model to this Cardassian expansion proposal, though the latter is cosmologically credible and compatible with the \(\Theta-z\) diagram for some values of \(l\).

3. CONCLUSIONS AND DISCUSSION

Although the evidence for an accelerating universe is increasing from various astronomical observations, understanding the mechanism based on particle physics is still one of the most important challenges in modern cosmology. The Cardassian Expansion Scenario proposed by Freese & Lewis (2002) is one intriguing possibility, because it assumes the universe is flat, matter dominated, and accelerating, but contains no vacuum contribution. We have used the updated angular size data to give the first observational constraint for this scenario. As is shown, although this Cardassian model is credible and compatible with the \(\Theta-z\) diagram for some specific values of \(l\), it is disfavored by the present angular size data because of its prediction of a universe with unreasonably low matter density.

Ignorance of the characteristic length \(l\) is one of the major uncertainties in the present analysis. One method to overcome it is to do the analysis over a large enough range of \(l\) to include almost all of the possibilities and then to calculate the probability distribution for the model parameters by integrating over \(l\) (Chen & Ratra 2002). However, this will loosen the cosmological constraints, making a larger range of model parameters plausible. In this sense, the Cardassian proposal will be more compatible with the present angular size data if we take into account the uncertainty of \(l\).

Another uncertainty comes from the possibility that the source linear size is dependent on the source luminosity and redshift, i.e., the sources are not “true” standard rods (Gurvits et al. 1999; Vishwakarma 2001). Parameterizing the effects of this dependence as \(l \rightarrow lL^\gamma(1 + z)^\beta\), Gurvits et al. (1999) and Vishwakarma (2001) have shown that the analysis with and without \(\beta\) and \(\gamma\) are basically consistent. It
is reasonable, for the data compiled by Gurvits et al. has been minimized for this dependence by discarding low values of luminosities and extreme values of spectral indices (Gurvits et al. 1999; Vishwakarma 2001).

We would like to thank L. I. Gurvits for sending us his compilation of the angular size data and helpful explanation of his data, J. S. Alcaniz, J. A. S. Lima, A. G. Riess, and Z.-Q. Shen for their help, and P. Beyersdorf for polishing up the English. Z.-H. Zhu is also grateful to all TAMA300 members and the staff of NAOJ for their hospitality and help during his stay. Finally, our thanks go to the anonymous referee for valuable comments and useful suggestions. This work was supported by a Grant-in-Aid for Scientific Research on Priority Areas (14047219) from the Ministry of Education, Culture, Sports, Science, and Technology.

### TABLE 1

| $l$ ($h^{-1}$ pc) | $n$   | $z_{eq}$ | $\Omega_{m}(F)$ | $\chi^2$ |
|-------------------|-------|----------|-----------------|---------|
| 15.0.................. | 0.88  | 1.38     | 0.42            | 5.06    |
| 16.0.................. | 0.76  | 1.78     | 0.32            | 4.90    |
| 17.0.................. | 0.67  | 2.00     | 0.25            | 4.78    |
| 18.0.................. | 0.51  | 0.94     | 0.27            | 4.69    |
| 19.0.................. | 0.33  | 0.58     | 0.29            | 4.63    |
| 20.0.................. | 0.17  | 0.48     | 0.27            | 4.58    |
| 21.0.................. | 0.03  | 0.44     | 0.26            | 4.54    |
| 24.0.................. | 0.00  | 0.82     | 0.14            | 4.56    |
| 28.0.................. | 0.00  | 1.52     | 0.06            | 5.21    |
| 30.0.................. | 0.00  | 2.00     | 0.04            | 5.92    |
| Best fit: 22.6........ | 0.00  | 0.62     | 0.19            | 4.52    |
REFERENCES

Alcaniz, J. S. 2002, Phys. Rev. D, 65, 123514
Alcaniz, J. S., & Lima, J. A. S. 1999, ApJ, 521, L87
Avelino, P. P., & Martins, C. J. A. P. 2002, ApJ, 565, 661
Bahcall, N. A. 2000, Phys. Rep., 333, 253
Burles, S., & Tytler, D. 1998a, ApJ, 499, 699
———. 1998b, ApJ, 507, 732
Caldwell, R., et al. 1998, Phys. Rev. Lett., 80, 1582
Carroll, S., et al. 1992, ARA&A, 30, 499
Chen, G., & Ratra, B. 2002, ApJ, 582, in press (astro-ph/0207051)
Chiba, M., & Yoshii, Y. 1999, ApJ, 510, 42
Coble, K., et al. 1995, Phys. Rev. D, 55, 1851
Davis, R. L. 1987, Phys. Rev. D, 35, 3705
de Bernardis, P., et al. 2000, Nature, 404, 955
Dev, A., et al. 2001, preprint (astro-ph/0104076)
Dunlop, J. S., et al. 1996, Nature, 381, 581
Freese, K., & Lewis, M. 2002, Phys. Lett. B, 540, 1
Frieman, J., et al. 1995, Phys. Rev. Lett., 75, 2077
Futamase, T., & Hamana, T. 1999, Prog. Theor. Phys., 102, 1037
Gurvits, L. I., Kellerman, K. I., & Frey, S. 1999, A&A, 342, 378
Jain, D., Dev, A., Punchapakesan, N., Mahajan, S. & Bhatia, V. B. 2001,
preprint (astro-ph/0105551)
Kamionkowski, M., & Toumbas, N. 1996, Phys. Rev. Lett., 77, 587

Kochanek, C. S. 1996, ApJ, 466, 638
Krauss, L. M. 1997, ApJ, 480, 466
Krauss, L. M. & Turner, M. S. 1995, Gen. Relativ. Gravitation, 27, 1137
Lange, A. E., et al. 2001, Phys. Rev. D, 63, 042001
Lima, J. A. S., & Alcaniz, J. S. 2002, ApJ, 566, 15
Ohyama, Y., et al. 2002, AJ, 123, 2903
Ostriker, J. P., & Steinhardt, P. J. 1995, Nature, 377, 600
Peacock, J. A., et al. 2001, Nature, 410, 169
Peebles, P. J. E. 1993, Principles of Physical Cosmology (Princeton:
Princeton Univ. Press)
Perlmutter, S., et al. 1998, Nature, 391, 51
———. 1999, ApJ, 517, 565
Ratra, B., & Peebles, P. J. E. 1988, Phys. Rev. D, 37, 3406
Riess, A. G., et al. 1998, AJ, 116, 1009
Turner, M. S. 1998, in ASP Conf. Ser. 165, The Galactic Halo, ed. B. K.
Gibson, T. S. Axelrod, & M. E. Putnam (San Francisco: ASP), 431
———. 2002, ApJ, 576, L101
Vilenkin, A. 1984, Phys. Rev. Lett., 53, 1016
Vishwakarma, R. G. 2001, Classical Quantum Gravity, 18, 1159
Weinberg, S. 1989, Rev. Mod. Phys., 61, 1
White, S. D. W., et al. 1993, Nature, 366, 429