Area Spectral Efficiency and Coverage for Mixed Duplexing Networks with Directional Transmissions

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Abstract—In this paper, we consider a system of small cells assuming full duplex (FD) capable base stations (BSs) and half duplex (HD) user equipment (UEs). We investigate a mixed duplexing cellular system composed of FD and HD cells, when BSs are using directional transmissions. A stochastic geometry based model of the proposed system is used to derive the coverage and area spectral efficiency (ASE) of both BSs and UEs. The effect of FD cells on the performance of the mixed system is presented under different degree of directionality at the BSs. We show that enabling directional transmissions at the BSs yields significant ASE and coverage gain in both downlink and uplink directions. With directional transmissions, the ASE increases rapidly with the number of FD cells while the drop in the coverage rate due to FD operations reduces significantly.

Index Terms—Area spectral efficiency, beamforming, full duplex, stochastic geometry, outage.

I. INTRODUCTION

Beam-centric design is expected to be one of the key concepts for achieving higher throughput and spectral efficiency in the Fifth Generation (5G) of cellular networks [1], [2]. In this concept, directional transmissions are employed to increase the received signal quality at the intended users. As the signal energy is concentrated in a narrow region with directional transmissions, this helps to increase signal-to-noise ratio (SNR) for a given link distance while reducing the interference among the users.

Another key technology considered for 5G wireless systems is full duplex (FD) communication with simultaneous transmission and reception on the same carrier. 5G wireless systems are being developed by the 3GPP New Radio (NR) standardization activity to meet performance requirements for IMT-2020. 3GPP has decided that NR will support paired and unpaired spectrum using frequency (FDD) and time (TDD) division duplexing operations, and will strive to maximize commonality between the technical solutions [3], allowing a flexible duplexing. The ability to assign transmission resources simultaneously to different transmission directions will allow efficient utilization of the available spectrum, enable future FD solutions. Albeit its potential benefits, it has been reported that simultaneous downlink and uplink transmissions increase the interference floor in a network, introduce a trade-off between area spectral efficiency (ASE) and coverage [4]–[6]. In this study, our goal is to provide further insights on this trade-off due to FD communications when directional transmissions are employed at the base stations (BSs) by considering the beam-centric design philosophy of the 5G networks.

In literature, FD operation in wireless networks have been investigated considering different scenarios [7]–[10]. While Tong et al. [7] analyze the throughput of a wireless network with FD radios using stochastic geometry in an ad-hoc setting, Lee et al. [8] derive the throughput of a mixed network considering only downlink and/or FD BSs. Alves et al. [9] show the impact of residual self-interference on the spectral efficiency for a dense network along with FD operation. In [10], the authors propose a scheme which allows a partial overlapping between uplink and downlink bands to maximize the gains with FD operations in each cell. Nevertheless the papers [8]–[10] mentioned above assume that the user equipment (UEs) to have FD capabilities, which is not practical given existing FD circuit designs [11]. In addition, these studies investigate the ASE without assessing the outage probability in the network. In our earlier work [6], we analyze the performance of mixed duplexing cellular systems, i.e., mixed system, composed of FD and half duplex (HD) BSs with omni-directional antennas. We show that the fraction of FD cells can be used as a design parameter to target different ASE vs. coverage trade-offs; in particular, by increasing the amount of FD cells in the mixed system, the overall ASE increases at the cost of a drop in terms of coverage, and vice-versa. Psomas et al. [12] quantify the impact of directionality in FD cellular networks, where all the BSs are in FD mode. The case of UEs with FD capability is compared against the case of UEs with only HD capability, where the latter case is shown to have more potential from both performance as well as practical implementation perspectives. Among the existing papers addressing FD for wireless networks in cellular systems, to the best of our knowledge, there is no comprehensive study that addresses the ASE vs. coverage trade-off in mixed systems for the uplink and the downlink, with directional transmissions at the BSs, available.

In this paper, we consider a mixed system where the BSs are using directional transmissions. A stochastic geometry-based model is utilized to investigate the impact of directional transmissions on the performance of a mixed system. We derive signal-to-interference-plus-noise ratio (SINR) for both uplink and downlink by taking the impact of all intra and inter-cell interference. We also provide a model to calculate residual self-interference under FD operation with directional transmission. In particular, we analyze the ASE vs. coverage trade-off of the mixed system as a function of the proportion of FD cells under different degree of directionality at the BSs. Among our main findings, we show that the trade-
off between ASE and coverage due to FD communications decreases when directional transmissions are employed at the BSs, i.e., increasing the number of FD cells increases the ASE significantly with a small loss in the coverage of the network.

The remainder of this paper is organized as follows. In Section II, we describe the system model. We show our formulation for computing the SINR and ASE for both downlink and uplink directions in Section III and Section IV respectively. In Section V we present and discuss the results while the conclusions are drawn in Section VI.

II. SYSTEM MODEL

In this section, we describe the mathematical models for BS deployment, directional transmission, residual self-interference for FD communications with directional transmissions, wireless channel, and power control.

A. Deployment and Duplexing

We consider a network where BSs are distributed according to a homogeneous and isotropic spatial Poisson point process (SPPP) $\Phi_B$ with density $\lambda_B$. We assume that the BSs are capable of both HD and FD modes, while the UEs are limited to only HD operation. The probabilities of a BS to be in FD mode, downlink HD mode, and uplink HD mode are denoted by $\rho_F$, $\rho_D$, and $\rho_U$, respectively, where $\rho_F + \rho_D + \rho_U = 1$. Based on Thinning theorem [13], the locations of the BSs in FD, downlink HD, and uplink HD modes can be modeled as independent SPPPs denoted by $\Phi^{F}_B$, $\Phi^{D}_B$, and $\Phi^{U}_B$, where the corresponding densities of the SPPPs are $\rho_F\lambda_B$, $\rho_D\lambda_B$, and $\rho_U\lambda_B$, respectively.

Each UE is assumed to be served by the nearest BS, which leads to a Voronoi tessellation where the generators of the tessellation are the BS locations, and the distribution of UE location is uniform in a Voronoi cell. We further assume that each BS in FD mode serves one uplink UE and one downlink UE on the same resources while each BS in HD mode communicates with one UE in the either downlink or uplink direction, i.e., UEs are always in HD mode. We denote the set of downlink and uplink UEs served by the FD BSs on the same resource as $\Phi^{F,D}_U$ and $\Phi^{F,U}_U$, respectively. Similarly, the set of downlink and uplink UEs served by the HD BSs on the same resource are expressed as $\Phi^{H,D}_U$ and $\Phi^{H,U}_U$, respectively. It is worth noting that $\Phi^{F,D}_U$, $\Phi^{F,U}_U$, $\Phi^{H,D}_U$, and $\Phi^{H,U}_U$ are not SPPPs since the UEs are associated with the nearest BS. Nevertheless, we model these subsets as SPPP for the sake of tractable analysis, which has also been considered in [6], [14], [15] as this approximation yields well-aligned SINR distributions. We can obtain the densities of $\Phi^{F,D}_U$, $\Phi^{F,U}_U$, $\Phi^{H,D}_U$, and $\Phi^{H,U}_U$ as $\lambda_{U,F,D} = \rho_F\lambda_B$, $\lambda_{U,F,U} = \rho_U\lambda_B$, $\lambda_{U,H,D} = \rho_D\lambda_B$, and $\lambda_{U,H,U} = \rho_U\lambda_B$, respectively, by exploiting Thinning theorem [13]. In addition, we assume that the set $\Phi^{U}_U$ of all UEs, which is the union $\Phi^{F,D}_U \cup \Phi^{F,U}_U \cup \Phi^{H,D}_U \cup \Phi^{H,U}_U$; $\Phi^U$ is an SPPP and its density is the sum of each subset’s density, which is $(\rho_F + 1)\lambda_B$ [15], $\Phi^{F,D}_U$, $\Phi^{F,U}_U$, $\Phi^{H,D}_U$, and $\Phi^{H,U}_U$ are also assumed to be independent of one another and independent of $\Phi^F_B$, $\Phi^D_B$, and $\Phi^U_B$ to maintain model tractability, which are also considered in the previous work [6], [9], [10].

B. Directional Transmission

We consider directional antennas at the BSs and omnidirectional antennas at the UEs. For FD communications, we assume that each BS employs two directional antennas; one for the uplink and one for the downlink, and each of them is steerable to an independent direction. For the sake of tractability, we consider a directional antenna model which characterizes the main lobe and the side lobes of the antenna pattern with single variables as

$$G(\theta) = \begin{cases} G_B & \text{if } \theta \leq |\theta_B/2|, \\ G_S & \text{if } \theta \geq |\theta_B/2|, \end{cases}$$

where $\theta$ is the radiation angle in local coordinate systems of the antenna, $G_B$ is the antenna gain for the main lobe of the antenna radiation pattern which spans $\theta_B$ degrees and $G_S$ is the antenna gain for the side lobe of the antenna radiation pattern which spans the rest of the angles, i.e., $\theta_S = 2\pi - \theta_B$ degrees as illustrated in Fig.11. It is worth noting that various antenna models are proposed for system-level investigations in the literature (e.g., [17]–[19] and the references therein). It is possible to integrate these mathematical models in our derivations in the following sections or approximate them by introducing multiple gain levels to (1). Since our goal in this study is to focus on the insights for the coverage and ASE in a mixed FD and HD network with a directional antenna setting at the BSs via a tractable approach, we provide our results by considering the model given in (1).

C. Residual Self-Interference in Full Duplex Mode

We consider a self-interference between transmit and receive branches in FD mode which takes directional antenna model described in Section II-B as illustrated in Fig.11. Since we assume that the directional antennas at the receive and transmit chains are steerable independently, i.e., one direction for uplink UE and another direction for downlink UE in FD BS, the amount of the self-interference is a function of the antenna orientations. For example, if uplink UE and downlink UE are on the opposite sides relative to the BS location, the amount of the interference can be amplified or attenuated significantly due to the the antenna gains depending on the orientations of the transmit and receive antennas. We model the residual self-interference $I_{SI}$ as

$$I_{SI} = \begin{cases} 0 & \text{if } C^1_{SI} = 0, \\ A_P g^2_{B} \Phi^F_B & \text{if } C^1_{SI} \geq 0, \quad |\phi_{tx}| \leq \beta_B/2, \quad |\pi - \phi_{rx}| \leq \theta_B/2, \\ A_P g^2_{B} G_S & \text{if } C^2_{SI} = 0, \\ \Phi^F_B & \text{if } C^2_{SI} \geq 0, \quad |\phi_{tx}| \geq \beta_B/2, \quad |\pi - \phi_{rx}| \leq \theta_B/2, \quad |\phi_{tx}| \leq \theta_B/2, \\ \text{or} & \nonumber \\ A_P g^2_{B} G_S & \text{if } C^3_{SI} = 0, \quad |\phi_{tx}| \geq \beta_B/2, \quad |\pi - \phi_{rx}| \geq \theta_B/2, \quad |\phi_{tx}| \geq \theta_B/2, \\ A_P g^2_{B} G^2 & \text{if } C^4_{SI} = 0, \quad |\phi_{tx}| \geq \theta_B/2, \quad |\pi - \phi_{rx}| \geq \theta_B/2, \quad |\phi_{tx}| \geq \theta_B/2, \quad |\phi_{tx}| \geq \theta_B/2. \end{cases}$$

where $\phi_{tx}$ and $\phi_{rx}$ are the transmit and receive antenna boresights in local coordinates of the BS, respectively, and $A_P$ is the effective self-interference cancellation gain which is a function of path loss, analog, and digital cancellation mechanisms at the BS.
D. Wireless Channel and Power Allocation

We consider different wireless channel models between the devices considering the device type, which is also recommended by 3GPP for BS-to-BS, BS-to-UE, and UE-to-UE links [20]. Without loss of generality, we assume that the path loss model between BS and UE, UEs, and BSs are

\[ \text{PL}_1(d) = K_1 d^{-\alpha_1}, \quad \text{PL}_2(d) = K_2 d^{-\alpha_2}, \quad \text{and} \quad \text{PL}_3(d) = K_3 d^{-\alpha_3}, \]

respectively, where \(\alpha_1, \alpha_2,\) and \(\alpha_3\) are the path loss exponents, and \(K_1, K_2,\) and \(K_3\) are the signal attenuations at distance \(d = 1.\) We consider that the transmitted signal is exposed to Rayleigh fading, which leads to exponentially distributed received signal power \(\sim \exp(\mu)\) with mean \(\mu^{-1}.

In the downlink, the BS transmission powers are assumed to be identical and denoted as \(P_B.\) For the uplink, we consider fractional power control where each UE, which is at distance \(R\) from its serving BS transmits with power \(P_u K_1^{-1} R^{-\alpha_1},\) where \(\epsilon \in [0, 1]\) is the power control factor [14].

III. SINR DISTRIBUTIONS

In this section, we provide the analytic expressions of SINR complementary cumulative distribution functions (CCDFs) and ASE in downlink and uplink. Since our system model considers a mixed network which includes BSs in FD mode and BSs in HD in both uplink and downlink directions at the same time, we first evaluate SINR CCDFs in FD cell and HD cell separately. Subsequently, we derive ASE and coverage rate considering the complete network.

Regardless of the duplexing mode, all of the active downlink BSs and all of the active uplink UEs in other cells interfere the intended link in both downlink and uplink. In a HD cell, both uplink UEs in the neighboring FD cells and their corresponding BSs interfere the intended received signal; that is either in downlink or in uplink as illustrated in Fig.1c and Fig.1d, respectively. On the other hand, in the downlink of an FD cell, the uplink UE interferes the downlink signal for the UE located in the same FD cell as shown in Fig.1b. In the uplink of an FD cell, the BS interferes itself based on the model given in (2), as shown in Fig.1e.

A. Downlink SINR Distribution in an FD Cell

The downlink SINR at a UE of interest in an FD cell can be expressed as

\[ \gamma_{\text{FD, UE}} = \frac{P_{\text{RX, UE}}}{N_0 + I_D + I_U}, \]

where \(N_0\) is the noise power at the UE, and \(P_{\text{RX, UE}}\) is the received signal power from the serving BS, given by

\[ P_{\text{RX, UE}} = P_B G_{\text{B}} g_{b} K_1 r^{-\alpha_1}, \]

where \(r\) is the distance between the UE and its serving BS. The serving BS is indicated by \(b,\) and \(g_{b}\) denotes the Rayleigh fading affecting the signal from the BS \(b.\) \(I_D\) and \(I_U\) are the total interference received at the UE from all the downlink transmissions and from all the uplink transmissions, respectively.

The total interference from all the downlink transmissions including all FD cells \((\Phi_{\text{FD}})\) and all HD downlink cells \((\Phi_{\text{HD}})\) can be defined as

\[ I_D = \sum_{b \in (\Phi_{\text{FD}}) \cup \Phi_{\text{HD}}} G_{b}^{\text{DD}} P_{b} g_{b} K_1 R^{-\alpha_1}, \]

where \(R\) is the distance between the UE of interest and the interfering BS \(b,\) and \(g_{b}\) denotes the Rayleigh fading for this link, and \(G_{b}^{\text{DD}}\) is the effective antenna gain expressed as

\[ G_{b}^{\text{DD}} = \begin{cases} G_{b} & \text{if } C_{1}^{\text{DD}} \text{ holds,} \\ G_{b} & \text{if } C_{2}^{\text{DD}} \text{ holds,} \end{cases} \]

where \(C_{1}^{\text{DD}}\) and \(C_{2}^{\text{DD}}\) are the conditions that the transmit beam of the interfering BS \(b\) is or is not pointed towards the UE of interest, which occurs with the probability of \(p_{1}^{\text{DD}} = \theta_B/2\pi\) and the probability of \(p_{2}^{\text{DD}} = \theta_S/2\pi,\) respectively.

Considering that UEs employ omni-directional antennas, the sum of interference from all the uplink transmissions, i.e., \(I_U,\) can be expressed as

\[ I_U = P_U \sum_{u \in (\Phi_{\text{U}}) \cup (\Phi_{\text{HD}})} K_1^{-1} Z_u^{\alpha_1} h_u K_2 D_u^{-\alpha_2}, \]

where \(Z_u\) is the distance between the uplink UE \(u\) and its serving BS, \(D_u\) is the distance between the uplink UE \(u\) and the UE of interest, and \(P_u K_1^{-1} Z_u^{\alpha_1} h_u\) is the transmit power of the uplink UE \(u.\) The symbol \(h_u\) denotes the Rayleigh fading for the channel between the \(u\)th uplink UE and the UE of interest.

By using (3) and (4), we can express the SINR CCDF for a given link distance \(R\) as

\[ P[\gamma_{\text{FD, UE}} > y | r = R] = P \left[ \frac{P_B G_{\text{B}} g_{b} K_1 R^{-\alpha_1}}{N_0 + I_D + I_U} > y \right] \]

\[ = P \left[ g_{b} > yP_B^{-1} G_{\text{B}}^{-1} K_1^{-1} R^{\alpha_1}(N_0 + I_D + I_U) \right] \]
(8) \quad e^{-\mu y P_B^{-1} G_B^{-1} K_1^{-1} R^{n+1} N_0} \quad L_{I_D+I_U}(\mu y P_B^{-1} G_B^{-1} K_1^{-1} R^{n+1})

where (a) follows from the fact that \( g_{b_0} \sim \exp(\mu) \). The Laplace transform of the total interference can be written as

\[
L_{I_D+I_U}(s) = \mathbb{E}_{\Phi_B^{D}, \Phi_B^{H} \cup \Phi_U^{D}, \Phi_U^{H}, g_b, h_b, Z_u} e^{-s(I_D+I_U)}
\]

where \( s = \mu y P_B^{-1} G_B^{-1} K_1^{-1} R^{n+1} \) and \( \Phi_B^{DD}, \Phi_B^{CD}, \Phi_U^{D}, \Phi_U^{H} \) are assumed to be independent SPPs, we rewrite (9) as

\[
L_{I_D+I_U}(s) = \frac{\mathbb{E}_{\Phi_B^{D}, \Phi_B^{H}, g_b, c_{dd}} e^{-sI_D}}{L_x(s)} \times \frac{\mathbb{E}_{\Phi_U^{D}, \Phi_U^{H}, Z_u} e^{-sI_U}}{L_y(s)}
\]

By taking the expectation over \( c_{dd} \), the first term in (10) can be written as

\[
L_x(s) = \sum_{i=1}^{2} P_B^{DD} e^{-s \sum_{k \in \{1,2\}} \mu_i P_B g_b K_1^{-1}}
\]

where \( \mu_i^1 = G_B^D \) when \( C_i^{DD} \) holds. By applying the Probability Generating Functional (PGFL) (13) of the SPPP to (11), it can be further written as

\[
L_x(s) = \sum_{i=1}^{2} P_B^{DD} e^{-2 \pi \lambda_B (\rho_B + \rho_U)} f_{\alpha_B}^\infty \left( \frac{\pi K_2 P_B k_B}{\pi K_1 P_B v^{n+1}} \right) e^{s \frac{\pi K_2 P_B k_B}{\pi K_1 P_B v^{n+1}}}dv
\]

By following the similar steps, the second term in (10), i.e., \( L_y(s) \), can be written as

\[
L_y(s) = e^{-2 \pi (\rho_B + \rho_U) \lambda_B} f_{\alpha_B}^\infty \left( \frac{\pi K_2 P_B k_B}{\pi K_1 P_B v^{n+1}} \right) e^{s \frac{\pi K_2 P_B k_B}{\pi K_1 P_B v^{n+1}}}dv
\]

It is worth noting that the lower extreme of integration in (12) is \( R \) as the distance between the closest interfering BS and the UE of interest is greater than \( R \). However, the closest interfering BS is also in its own cell, the lower extreme of integration in (13) becomes 0. Assuming that there is no power control, i.e., \( \epsilon = 0 \), (17) can be rewritten as

\[
L_y(s) = e^{-2 \pi \lambda_B (\rho_B + \rho_U)} f_{\alpha_B}^\infty \left( \frac{\pi K_2 P_B k_B}{\pi K_1 P_B v^{n+1}} \right) e^{s \frac{\pi K_2 P_B k_B}{\pi K_1 P_B v^{n+1}}}dv
\]

Finally, we obtain the CCDF of the downlink SINR in a FD cell for a mixed system,

\[
P[\gamma_{FD,UE} > y] = \int_0^\infty P[\gamma_{FD,UE} > y|r]f_r(R)dR
\]

\[
= \int_0^\infty e^{-s N_0} L_x(s) L_y(s) f_r(R)dR,
\]

where \( s = \mu y P_B^{-1} G_B^{-1} K_1^{-1} R^{n+1} \) and \( f_r(R) \) is given by

\[
f_r(R) = e^{-\pi R^2} 2\pi R,
\]

as the UE of interest is associated with the nearest BS and the BS deployment follows SPPP (13), (21).

**B. Uplink SINR Distribution in an FD Cell**

The uplink SINR for the BS of interest in a FD cell of the mixed system is given by

\[
\gamma_{PD,BS} = \frac{P_{RX,BS}}{N_1 + I_D + I_U + I_{SI}}
\]

where \( N_1 \) is the noise power at the BS, \( I_D \) and \( I_U \) are the total interference received at the BS from all other downlink transmissions and from all the uplink transmissions, respectively, \( I_{SI} \) represents the residual self-interference due to being in FD mode, which is discussed in Section II-C and \( P_{RX,BS} \) is the received signal power from the uplink UE. \( P_{RX,BS} \) can be expressed as

\[
P_{RX,BS} = P_U G_B h_{u_0}^I K_1^{(1-c)} r^{\alpha_1} (r-1)
\]

where \( r \) is the link distance between the BS of interest and its uplink UE, and \( h_{u_0}^I \) denotes the Rayleigh fading for this link. \( I_D \) and \( I_U \) can be expressed as

\[
\begin{align*}
I_D &= \sum_{b \in \{\Phi_B^D \cup \Phi_B^H\}} G_b^D P_B g_b K_3 L_b^{-\alpha_2} \\
I_U &= \sum_{u \in \{\Phi_U^D \cup \Phi_U^H\}: X_u > Z_u} G_u^U P_U h_u^I K_2^{-1} r^{\alpha_1} X_u^{-\alpha_2}
\end{align*}
\]

respectively, where \( L_b \) and \( X_u \) are the distance between the interfering BS \( b \) and the BS of interest and the distance between the interfering uplink UE \( u \) and the BS of interest, respectively; \( Z_u \) is the distance between the interfering uplink UE \( u \) and its serving BS. It is worth noting that we introduce the condition \( \{X_u > Z_u\} \) in (19) for all \( u \in \{\Phi_U^D \cup \Phi_U^H\} \) as it guarantees that the distance \( Z_u \) of the interfering UE \( u \) to its serving BS is shorter than the distance from \( u \) to the victim BS, which is also taken into account in (15). In (18), \( G_u^D \) is the effective antenna gain by

\[
G_u^D = \begin{cases}
G_u^B & \text{if } C_u^D \text{ holds,} \\
G_B G_S & \text{if } C_u^2 \text{ holds,} \\
G_S & \text{if } C_u^3 \text{ holds,}
\end{cases}
\]

where \( C_u^D, C_u^2, \) and \( C_u^3 \) are the condition that the receiving beam of the BS of interest and the transmit beam of the interfering BS \( b \) are aligned to each other, the condition that either receiving beam of the BS of interest or the transmit beam of the interfering BS \( b \) is aligned to each other, and the condition that neither receiving beam of the BS of interest nor the transmit beam of the interfering BS \( b \) are aligned to each other, which occur with the probabilities of \( p_u^D = \theta_B^2/4\pi^2, p_u^2 = 2\theta_B^2\theta_S/4\pi^2, \) and \( p_u^3 = \theta_B^2/4\pi^2, \) respectively. Similarly, in (19), the effective antenna gain in the uplink, i.e., \( G_u^U \), can be defined as

\[
G_u^U = \begin{cases}
G_u^B & \text{if } C_u^U \text{ holds,} \\
G_S & \text{if } C_u^2 \text{ holds,} \\
G_B & \text{if } C_u^3 \text{ holds,}
\end{cases}
\]

where \( C_u^U \) and \( C_u^2 \) are the condition that the receiving beam of the BS of interest is or is not oriented towards the interfering...
where the Laplace transform of 

\[ \phi(s) = \sum_{i} \Phi_{U}^{(s)} \times \sum_{i} \gamma_{i} \times e^{-is\gamma_{i}} \]  

(23)

where \( s = \mu y P_{U}^{-1} G_{B}^{-1} K_{1}^{(e-1)} R_{\alpha}(1-\epsilon) \) and \( c_{dU} \in \mathbb{C}^{D_{U}} = \{ C_{D_{1}}, C_{D_{2}}, C_{D_{3}} \} \) and \( c_{uU} \in \mathbb{C}^{U} = \{ C_{1}, C_{2}, C_{3} \} \).

The first term in (23) can be written as

\[ H_{B}(s) = \sum_{i=1}^{3} P_{i}^{(s)} e^{-s\gamma_{i}} \]

(24)

where \( P_{i}^{(s)} = G_{D_{i}}^{U} \) when \( C_{i}^{D_{i}} \) holds true. The lower extreme of integration in (24) is zero as the closer interferer BS (either FD or HD) can be at any distance greater than 0.

The second term in (23) can be written as

\[ H_{U}(s) = \sum_{i=1}^{2} P_{i}^{(s)} U \times \]

\[ \mathbb{E}_{\Phi_{U}^{(s)}, \gamma_{U}^{(s)}, Z_{U}} \left[ e^{-s \sum_{u \in \Phi_{U}^{(s)}} y_{u} \gamma_{u} P_{u} K_{u}^{\alpha-1}} \right] \]

(25)

where \( P_{i}^{(s)} = G_{D_{i}}^{U} \) when \( C_{i}^{U} \) holds true. The lower extreme of integration in (25) is zero also but the constraint \( \{ Z_{U} < y \} \) makes sure that only UEs from the other cells are included in the interference term. The integration in (25) can be further simplified using integration by parts as given in [6]. When there is no power control (\( \epsilon = 0 \), it can be written as:

\[ H_{U}(s) = \sum_{i=1}^{2} P_{i}^{(s)} U \times e^{-2\pi(\rho_{F} + \rho_{U}) \lambda_{B} f_{R}^{\infty} (1 - \pi z_{U})} \]

(28)

Similar to (15), the expression for CCDF of SINR in HD cell is given by:

\[ P_{[\gamma_{HD,UE}] > y} = \int_{0}^{\infty} e^{-sN_{0}} L_{x}(s) \cdot L_{y}(s) f_{R}(R) dR, \]

(29)

where \( s = \mu y P_{B}^{-1} G_{B}^{-1} K_{1}^{(e-1)} R_{\alpha}(1-\epsilon) \) and \( f_{R}(R) \) is given by

\[ f_{R}(R) = e^{-\pi \rho_{H} R^{2}} \frac{2\pi \nu \lambda_{B} R}{2} \]

(30)

\[ \gamma_{HD,BS} = \frac{P_{RX,BS}}{N_{1} + I_{D} + I_{U}}. \]
The CCDF of $\gamma_{HD,BS}$ is given by,
\[
P[\gamma_{HD,BS} > y] = \int_0^\infty P[\gamma_{HD,BS} > y|r = R] f'_r(R) dR, \tag{31}
\]
where
\[
P[\gamma_{HD,BS} > y|r = R] = e^{-sN_i} H_x(s) H_y(s), \tag{32}
\]
with $s = \mu y P_U^{-1} G_B^{-1} K_i^{(e-1)} R^{(1-e)}$, where, $H_x(s)$, and $H_y(s)$ are given in (24) and (25), respectively.

IV. AVERAGE RATE

The average rate per hertz can be computed as \[4, 14\]
\[
E[C] = \int_0^\infty P[\log_2(1 + \gamma) > u] du. \tag{33}
\]

By using (8), the average downlink rate in an FD cell is given by
\[
E[C_{FD,UE}] = \int_0^\infty P[\log_2(1 + \gamma_{FD,UE}) > u] du
= \int_0^\infty \int_0^\infty e^{-\mu(2^u - 1)} P_U^{-1} G_B^{-1} K_i^{(e-1)} R^{(1-e)} N_0 \times
L_{D+U1}(\mu(2^u - 1) P_U^{-1} G_B^{-1} K_i^{(e-1)} R^{(1-e)}) f_r(R) dR du. \tag{34}
\]

By using (22), the average uplink rate in a FD cell is given by
\[
E[C_{FD,BS}] = \int_0^\infty P[\log_2(1 + \gamma_{FD,BS}) > u] du
= \int_0^\infty \int_0^\infty e^{-\mu(2^u - 1)} P_U^{-1} G_B^{-1} K_i^{(e-1)} R^{(1-e)} N_0 \times
L_{D+U1}(\mu(2^u - 1) P_U^{-1} G_B^{-1} K_i^{(e-1)} R^{(1-e)}) f'_c(R) dR du. \tag{35}
\]

Similarly, the average downlink and uplink rates in a HD cell, i.e., $E[C_{HD,UE}]$, $E[C_{HD,BS}]$, respectively, can be derived. Combining the rates of FD and HD cells, the average downlink and uplink rates of the complete network are given by
\[
E[C_{D}] = \rho_F E[C_{FD,UE}] + \rho_D E[C_{HD,UE}], \tag{35}
\]
and
\[
E[C_{U}] = \rho_F E[C_{FD,BS}] + \rho_U E[C_{HD,BS}], \tag{36}
\]
respectively. Hence, the downlink and uplink ASEs of the mixed network can be obtained from (35) and (36), respectively, as $ASE_D = \lambda_B E[C_{D}]$ and $ASE_U = \lambda_B E[C_{U}]$.

V. NUMERICAL RESULTS

We evaluate the ASE and coverage of a mixed HD and FD network in the case of directional transmission at the BSs. We also provide results with traditional TDD HD (THD) systems, in which, all the BSs are involved only in HD operations, i.e., a BSs schedules either uplink or downlink transmission. We simulate different THD systems while varying the proportion of cells in downlink and uplink transmissions. The network parameters are tabulated in Table I. In the case of directional transmissions at the BSs, we consider two different antenna patterns listed as: 1) $\theta_B = 35^\circ$ with $G_B = 15$ dBi, $G_S = 0$ dBi, 2) $\theta_B = 90^\circ$ with $G_B = 7$ dBi, $G_S = 0$ dBi [17].

| Parameter                | Value          |
|--------------------------|----------------|
| Bandwidth                | 10 MHz         |
| BS Density [nodes/m²]    | $10^{-3}$      |
| Thermal Noise Density    | $-174$ dBm/Hz  |
| Outage SINR Threshold    | 8 dB           |
| Noise Figure             | 9 dB (UE), 8 dB (BS) |
| $K_1, K_2, K_3$          | 8.8 x 10⁻⁴    |
| $\alpha_1, \alpha_2, \alpha_3$ | 3.67         |
| $A$                      | 1.20 dB        |
| $c$                      | 0              |

Figs. 2 and 3 show the ASE vs. coverage trade-off in downlink and uplink for different antenna settings, respectively, when $P_B = 24$ dBm and $P_U = 23$ dBm. It includes the performance of different mixed and THD systems. In the case of THD systems, $\rho_D = 1$, and $\rho_U = 1$ represent the scenarios of all the BSs scheduled in downlink and uplink transmissions, respectively. The overall coverage rates of the mixed system in downlink and uplink are computed as $(\rho_F \Theta_{DF,DL} + \rho_D \Theta_{HD,DL})/(\rho_F + \rho_D)$ and $(\rho_F \Theta_{DF,UL} + \rho_D \Theta_{HD,UL})/(\rho_F + \rho_D)$, respectively, where $\Theta_{FD,DL}$, $\Theta_{HD,DL}$, and $\Theta_{HD,UL}$ are the coverage of FD downlink, FD uplink, HD downlink, and HD uplink in the mixed system, respectively. The coverage is defined as the fraction of UEs in a non-outage region, where an outage happens if the received SINR is below the outage SINR threshold. The trade-off in the mixed system is presented for a given percentage of FD BSs, i.e., $\rho_F$. The remaining BSs in the mixed system are equally divided into HD downlink and HD uplink modes, i.e., $\rho_D = \rho_U = (1 - \rho_F)/2$. In all antenna configurations, by increasing the number of BSs in FD mode, both downlink and uplink ASE increase at the cost of lower coverage. This trade-off occurs as the number of transmissions and the aggregated interference in each direction increase for higher $\rho_F$.

For both mixed and the THD systems, beamforming at the BSs provides gain both in terms of ASE and coverage. For example, in the mixed system with $\rho_F = 0.4$, with increasing the beamforming gain, i.e., changing $\theta_B$ from 90° to 35°, the ASE increases by 77% and 79% in the downlink and the uplink while the downlink and uplink coverage rates are increased by 9% and 19%, respectively. Applying beamforming at the BSs provides beamforming gain to both the downlink and the uplink transmissions while reducing the interference from the other nodes. In the downlink, the interference from the neighboring BS, and in the uplink the interference from both the UE and BS of the neighboring cell, and the self-interference decrease. Beamforming provides gain to all the mixed and the THD systems. Moreover, with higher beamforming gain, as we increase the number of FD cells, the rate of increment in ASE is much higher than the rate of decrement in the coverage.

In the case of THD systems, increasing the number of transmissions in the downlink direction ($\rho_D \to 1$) reduces both ASE and coverage in both downlink and uplink directions. It is because a downlink transmission generates interference from a BS which is generally stronger than the interference from a UE transmitting in the uplink direction. This also
Fig. 2: Trade-off between ASE and coverage in downlink with directional transmissions.

holds for the mixed system, where for lower values of $\rho_F$, most of the cells are in HD mode, including both uplink and downlink transmissions. Therefore, both downlink and uplink performances in the mixed system are superior to the THD systems consisting of higher number of downlink transmissions.

VI. CONCLUSION

In this paper, we investigate a mixed duplexing cellular system composed of FD and HD cells with directional transmission at the BSs. We consider a stochastic geometry-based model to derive the SINR complementary CDF and the ASE for the downlink and uplink directions. We study the impact of FD cells on the ASE vs. coverage trade-off of mixed systems for different beamforming configurations at the BSs. We show that the beamforming at the BSs increases the performance of both mixed as well as the traditional HD systems significantly in both uplink and downlink directions. With higher beamforming gain, as we increase the number of FD cells, the gain in ASE increases rapidly with a small loss in the coverage of the network. Further extensions to our study could include opportunistic scheduling, more comprehensive antenna patterns including residual interference models for FD communications, beamforming at the UE side, and operation in millimeter wave frequencies.

REFERENCES

[1] “NGMN 5G white paper,” March 2015. [Online]. Available: www.ngmn.org
[2] P. Marsch et al., “5G radio access network architecture: design guidelines and key considerations,” IEEE Comm. Mag., vol. 54, no. 11, pp. 24–32, 2016.
[3] 3GPP, “Study on new radio access technology: Physical layer aspects,” TR 38.802, v.14.0.0, Apr. 2016.
[4] S. Goyal, P. Liu, and S. S. Panwar, “User selection and power allocation in full-duplex multicell networks,” IEEE Tran. on Vehicular Technology, vol. 66, no. 3, pp. 2408–2422, 2017.
[5] S. Goyal et al., “Full duplex cellular systems: Will doubling interference prevent doubling capacity?” IEEE Comm. Mag., vol. 53, no. 5, pp. 121–127, May 2015.
[6] S. Goyal, C. Galiotto, N. Marchetti, and S. S. Panwar, “Throughput and coverage for a mixed full and half duplex small cell network,” CoRR, vol. abs/1602.09115, 2016.
[7] Z. Tong and M. Haenggi, “Throughput analysis for wireless networks with full-duplex radios,” in 2015 IEEE WCNC. Mar. 2015, pp. 717–722.
[8] J. Lee and T. Quek, “Hybrid full-half-duplex system analysis in heterogeneous wireless networks,” IEEE Tran. Wireless Commun., vol. 14, no. 5, pp. 2883–2895, May 2015.
[9] H. Alves et al., “On the average spectral efficiency of interference-limited full-duplex networks,” in IEEE CROWNCOM, 2014, pp. 550–554.
[10] A. AlAmmouri et al., “In-band full-duplex communications for cellular networks with partial uplink/downlink overlap,” in 2015 IEEE GLOBECOM, Dec 2015, pp. 1–7.
[11] A. Sabharwal et al., “In-band full-duplex wireless: Challenges and opportunities,” IEEE J. Sel. Areas Commun., vol. 32, no. 9, pp. 1637–1652, Sept 2014.
[12] C. Psomas et al., “Impact of directionality on interference mitigation in full-duplex cellular networks,” IEEE Tran. on Wireless Commun., vol. 16, no. 1, pp. 487–502, Jan 2017.
[13] M. Haenggi, Stochastic Geometry for Wireless Networks. Cambridge University Press, 2013.
[14] T. D. Novlan, H. S. Dhillon, and J. G. Andrews, “Analytical modeling of uplink cellular networks,” IEEE Tran. on Wireless Commun., vol. 12, no. 6, pp. 2669–2679, June 2013.
[15] B. Yu, S. Mukherjee, H. Ishii, and L. Yang, “Dynamic TDD support in the LTE-B enhanced local area architecture,” in IEEE GLOBECOM Wkshps, Dec. 2012, pp. 585–591.
[16] F. Baccelli and B. Błaszczyszyn, Stochastic Geometry and Wireless Networks, Volume 1, Theory, NOW Publishers, 2009.
[17] 3GPP, “Technical specification group radio access network; spatial channel model for multiple input multiple output (MIMO) simulations (Release 10),” TR 25.996, v.10.0.0, Jun. 2011.
[18] ———, “Study on 3d channel model for LTE,” TR 36.873, v.12.0.0, Jun. 2014. [Online]. Available: www.3gpp.org
[19] F. F. Gunnarsson et al., “Downscaled base station antennas - a simulation model proposal and impact on HSPA and LTE performance,” in IEEE VTC, Sep. 2008.
[20] 3GPP, “Further enhancements to LTE time division duplex (TDD) for downlink-uplink (DL-UL) interference management and traffic adaptation,” TR 36.828, v.11.0.0, Jun. 2012.
[21] J. G. Andrews, F. Baccelli, and R. K. Ganti, “A tractable approach to coverage and rate in cellular networks,” IEEE Tran. on Commun., vol. 59, no. 11, pp. 3122–3134, Nov 2011.
[22] 3GPP. “Further Advancements for E-UTRA Physical Layer Aspects (Release 9),” Mar. 2010, 3GPP TR 36.814 V9.0.0 (2010-03).