Stable Torsion Torque Control Based on Friction-Backlash Analogy for Two-Inertia Systems with Backlash

Juan Padron, Yuki Yokokura, Toshimasa Miyazaki, Student Member, Yusuke Kawai, Member, Kiyoshi Ohishi, Fellow

Torsion torque control (TTC) has been studied in applications ranging from industrial robots to electric vehicles. However, its performance is severely compromised when there is joint backlash, causing limit cycles and inducing unstable behavior. This study proposes a stable TTC scheme for two-inertia systems with joint backlash based on an analogy between static friction and backlash phenomena. First, a transformation of the joint torsion dynamics into a first-order system is applied, and a proportional torsion torque controller is proposed. Then, an analogy between the transformed joint torsion dynamics with backlash and one-mass systems with static friction is established. Based on this analogy, a combination of a switched disturbance observer and an impact torque suppression controller is proposed, ensuring rapid control over backlash and stable reengagement with reduced gear collision impact. The effectiveness of the proposed method is verified through simulation and experimental results.

Keywords: Two-mass System, Torsion Torque Control, Disturbance Observer, Backlash, Static Friction

1. Introduction

Torsion torque control (TTC) has been gaining attention over the last years, especially in the automotive (1) and robotics fields (2). TTC allows to directly control the torsion torque input to the load-side in mechatronic systems, improving backdrivability in robotic applications (3), and improving driver’s comfort in automotive applications (4).

However, many of these applications suffer from backlash on the joint shaft, which affects the performance of TTC due to the hard nonlinearity and causes discomfort in human-machine applications. Even in the case of robots with harmonic drives, wear in the gear causes the torque transmission to deteriorate and produce backlash, making it an unavoidable issue that must be taken into account.

The backlash phenomenon has been studied since more than 70 years (5), and until now several approaches have been proposed, such as switched control (6), fractional-order disturbance observers (7), and model-predictive control (MPC) (8). MPC has demonstrated to reduce the impact caused by shaft reengagement, but has the disadvantage of being computationally intensive. More recent approaches involve final state control (FSC) (9), receding horizon sliding control (RHSC) (10) and compensation through smooth inverse backlash models (11). However, all of these methods require a load-side encoder, which cannot measure exactly torsion torque and it is difficult to implement in robots.

On the other hand, torsion torque sensors allow to exactly measure the torsion torque and have been applied for vibration control (12), stable contact of robot with environment (12)

© 2021 The Institute of Electrical Engineers of Japan.
based on scheduled torsion torque-derivative feedback is proposed for suppressing the impact torque caused by gear collision and for stabilizing the gear reengagement. Followed by this, a switched torsion disturbance observer (referred from now on as STDOb) is proposed to reduce the gear disengagement time during torque zero-crossings. The STDOb works now on as this, a switched torsion disturbance observer (referred from posed for suppressing the impact torque caused by gear collision and for stabilizing the gear reengagement. Followed by using a high-order DOB when gears are disengaged, and a lower-order DOB after gears reengage, achieving fast backlash overcoming while preventing instability typically seen in high-order DOB. The effectiveness of the proposed method is demonstrated by numerical simulations and experiments, together with two application examples on direct teaching and load torque control.

2. Conventional Torsion Torque Control

2.1 I-P Torsion Torque Controller The I-P TTC scheme shown in Fig. 1 is selected as conventional method. This system is composed by an I-P torsion torque controller, torsional damping feedback for preventing vibrations and friction compensation on the motor side, and it has demonstrated a good performance in human-robot applications such as direct teaching.

Design is done by the transfer function of Eq. (1), which is a third-order transfer function. $K_{m}, K_{g}$ and $J_{s}$ parameters are determined to achieve the desired pole allocation.

$$\frac{\tau_{s}}{\tau_{ref}} = \frac{n_{0}}{s^3 + d_{2}s^2 + d_{1}s + d_{0}}. \quad (1)$$

The coefficients of Eq. (1) are given as follows:

$$d_{2} = \frac{K_{s}J_{s}}{J_{m}R_{g}} \quad d_{1} = \frac{K_{s}J_{s}K_{m}}{J_{m}R_{g}} + \frac{K_{g}}{J_{s}R_{g}} + \frac{K_{s}}{J_{m}R_{g}}$$

$$d_{0} = \frac{K_{s}J_{s}}{J_{m}R_{g}} \quad n_{0} = \frac{K_{s}J_{s}}{J_{m}R_{g}} \quad (2)$$

The desired transfer function is designed as:

$$\frac{\tau_{s}}{\tau_{ref}} = \frac{g_{TTC}^{\tau_{s}}}{(s + g_{TTC})^2}, \quad (3)$$

where $g_{TTC}$ is the torsion torque control bandwidth.

Figure 1: Conventional I-P TTC block diagram

2.2 Backlash in Two-Inertia Systems

In order to test the performance of the conventional method when backlash is present, numerical simulations using the dead-zone backlash model are performed. Simulation results are shown in Fig. 4. When there is no backlash in the system, the torsion torque response follows the reference successfully. However, when backlash is considered, the torque tracking performance is severely deteriorated, showing repeated gear disengagement and gear collisions that may compromise comfort during human-machine interaction, and also damage the mechanical components.

For this paper, we consider a setting of a geared motor, gear coupling with backlash and a load, shown in Fig. 3, together with its equivalent block diagram. $\theta_{m}, \theta_{l}$ and $\theta_{ref}$ stand for the motor-side rotation angle, load-side rotation angle and q-axis current. $J_{m}, J_{l}, K_{s}, R_{g}, D_{m}$ and $D_{l}$ represent the motor-side inertia, load-side inertia, torque constant, gear reduction ratio, motor-side and load-side viscous friction constant, respectively. The experimental setting includes a torque sensor and a load-side encoder, but the load-side encoder is only used for observation. Motor and load-side velocities are used for backlash identification. The backlash width $2\beta$ was identified as 16 mrad.

$$\tau_{s} = \begin{cases} 
K_{s}(\theta_{s} - \beta), & \text{if } \theta_{s} \geq \beta \\
0, & \text{if } |\theta_{s}| < \beta \\
K_{s}(\theta_{s} + \beta), & \text{if } \theta_{s} \leq -\beta 
\end{cases} \quad (4)$$

In order to test the performance of the conventional method when backlash is present, numerical simulations using the dead-zone backlash model are performed. Simulation results are shown in Fig. 4. When there is no backlash in the system, the torsion torque response follows the reference successfully. However, when backlash is considered, the torque tracking performance is severely deteriorated, showing repeated gear disengagement and gear collisions that may compromise comfort during human-machine interaction, and also damage the mechanical components.
3. Analogy Between Static Friction and Backlash

3.1 Approximating Torsion Dynamics as a First-Order Plant

According to Fig. 3, backlash is modelled as a dead-zone element in the torsional joint between the motor-side and the load-side. However, by applying a high-bandwidth velocity control and a force-position integrated PI velocity controller, the motor-side velocity \( \omega_m \) is effectively controlled, and the torsion torque dynamics transform into a first-order plant, as shown in Fig. 5. This method has been previously proposed to establish a torque-velocity duality for the two-inertia system in contact with environment (15). For this paper, this approximation is applied to establish an analogy between a one-mass system with static friction and the transformed first order torsion torque dynamics with backlash.

Note that this transformation ignores the load-side dynamics, and rather considers \( \omega_l \) as a disturbance to be compensated (11).

3.2 Friction-Backlash Analogy: Similarities Between the Stick-Slip Phenomenon and the Gear Disengagement-Collision Problem

The transformed first order joint torsion torque dynamics are now compared with a one-mass plant with static friction, illustrated in Fig. 6. Note that the actual frictional behavior may be more complicated than a simple dead-zone element, and placement of the integrators for both plants is also different. However, looking at the input-output relationship in Fig. 7 for a one-mass plant with friction under velocity control, and a two-inertia plant with backlash under torsion torque control, we see that the resulting phenomena are similar. This is explained as follows:

- Gear disengagement (analogous to stick phenomenon): Gear disengagement occurs when motor-side and load-side gears are disengaged and the torque sticks to zero. This is analogous to the stick phenomenon caused by
static friction, where velocity sticks to zero until static friction is overcome.

- **Gear collision (analogous to slip phenomenon):** Gear collision occurs when the backlash gap is traversed and the gears collide as they reengage. Because of the accumulated torsion velocity, a large impact torque occurs. This is analogous to the slip phenomenon of static friction, where a sharp increase in velocity occurs because of the excessive accumulated torque as the mass begins to move.

Based on such analogy, we adapt the methodology for stick-slip compensation proposed in (13). This approach, which works by combining a variable-order DOb and high-

**4. Proposed Method for Gear Collision Impact Reduction: The Impact Torque Suppressor**

**4.1 Torsion Torque Control Based on Proportional Control and Torsion Disturbance Observer**

In order to apply the proposed methodology, a TTC scheme based on proportional control and a torsion disturbance observer (TDOb) is employed, which is shown in Fig. 8. The TDOb compensates for load-side velocity $\omega_l$ and other disturbances present in the torsional dynamics, such as variations in $K_s$ and backlash. The total estimated disturbance is given by the $\dot{\omega}_{dis}$ term. The TDOb resembles the environment observer proposed in (10), but applied to the joint torsion dynamics.

If the TDOb compensates for all disturbances and gears are engaged (system is in linear zone), then the transfer function from $\tau_s^{ref}$ to $\tau_s$ is given as:

$$\tau_s = \frac{K_s \tau_{TTC} R_g^{-1}}{s + K_s \tau_{TTC} R_g^{-1}} = g_{TTC} \frac{1}{s + g_{TTC}}$$  \hspace{1cm} (5)

The gain $K_{TTC}$ of the proportional controller is then designed from Eq. (5) as:

$$K_{TTC} = g_{TTC} R_g^{-1}$$  \hspace{1cm} (6)

where $g_{TTC}$ is the desired bandwidth. When $\tau_s^{ref}$ is a step reference, $\omega_l$ increases as a ramp, so the TDOb is designed with a first-order DOB structure.

**4.2 The ITS**

From the torsion torque dynamics with backlash described in Eq. (4), the torsional jerk just before and after gears collide and impact occurs is derived as follows:

$$\dot{\tau}_s = \begin{cases} 0, & \text{if } |\theta_l| < \beta \quad \text{(Before collision)} \\ K_s \omega_s, & \text{if } |\theta_l| > \beta \quad \text{(After collision)} \end{cases}$$  \hspace{1cm} (7)

It is clear from Eq. (4) that, in order to prevent excessive impact torque due to gear collision, the torsion velocity $\omega_s$...
must be taken to zero just before the gears reengage. This is the usual approach taken in automotive powertrain applications. However, such approach requires an exact backlash model and exact load position measurement, which cannot be done with a torque sensor.

Based on the previously explained static friction and backlash analogy, we treat the gear collision impact suppression problem of backlash in the same manner as the slip suppression problem of static friction, and apply the method described in \( m(i) \), which is based on applying gain-scheduled high-frequency damping. Adapting such method for gear collision impact suppression, we propose the impact torque suppressor (ITS) shown in Fig. 9, which is given as follows:

\[
\omega_{js} = \begin{cases} 
\frac{R_g \omega_{ref}}{K_j} \tau_s & \text{if } |\tau_s| - |\tau_s^{ref}| > K_m \\
0 & \text{if else}
\end{cases}
\]  \( (8) \)

where \( \omega_{js} \) is the output of the ITS. The switching conditions given in Eq. (8) are such that the ITS is activated only when gear collisions occurs. The first condition detects when overshoot due to collision impact occurs, while the second condition detects if excessive jerk occurs. \( K_m \) is the torque threshold, while \( |\tau_s^{ref} - \tau_s| \) is the jerk threshold, which is the maximum jerk that is produced in the linear case for a step input of amplitude \( \tau_s^{ref} \).

If \( \omega = \omega_{TTC} \cdot \omega_{js} \) (where \( \omega_{ref} \) is the reference from the TDOB and \( P \) controller), then the torsional jerk when the ITS is activated during gear collision is given as follows:

\[
\tau_s = \frac{R_g \omega_{ref}}{K_j} \tau_s - R_g \omega_{js}
\]

If the pseudodifferentiator bandwidth \( g_{js} \) is large enough, \( \tau_s \), \( \omega_{js} \rightarrow \tau_s \). Rearranging terms:

\[
\tau_s \left( \frac{1}{K_s} + \frac{1}{K_j} \right) = \tau_s \left( \frac{1}{K_{eq}} \right) = \omega_{ref} R_g^{-1} - \omega_l
\]  \( (9) \)

Finally:

\[
\tau_s = K_{eq} \omega_l^s
\]  \( (10) \)

where \( \omega_l^s = \omega_{ref} R_g^{-1} - \omega_l \) and \( K_{eq} = \frac{K_m}{K_j} \). We will call \( K_{eq} \) the equivalent torsion spring constant. By applying the proposed ITS, \( K_{eq} \) can be varied. In order to reduce the gear collision impact, the \( K_{eq} \) term is designed so that the equivalent spring constant \( K_{eq} \) is softer than the actual spring constant \( K_s \). That is, \( K_{eq} \) should be designed so that \( K_{eq} < K_s \) so that:

\[
K_{eq} = \frac{K_m K_s}{K_m + K_j} < K_s \rightarrow K_{eq} \omega_l^s < K_s \omega_l^s
\]  \( (11) \)

Thus, the gear collision torque is reduced. By decreasing \( K_{eq} \) the collision torque can be further reduced, but its minimum value is restricted by the current limit value of the actuator. From Eqs. (8), (11) and (12), we can see that the ITS works as an active nonlinear spring (see Fig. 10).

Considering the motor-side velocity control being of very high bandwidth, the closed-loop transfer function from \( \tau_s^{ref} \) to \( \tau_s \) when the ITS is activated is given as follows:

\[
\frac{\tau_s}{\tau_s^{ref}} = \frac{K_s R_g^{-1} K_{TTC} (s + g_{js}) s D(s)}{D(s)}
\]  \( (13) \)

where \( D(s) = (s + g_{js})(1 - Q(s))(s^2 + \frac{K_j}{K_m}) + \frac{K_s}{K_j} s^2 Q(s) \) and \( Q(s) \) represents the TDOB Q-filter. The Bode dia-

![Figure 9: Block diagram of ITS](image-url)

![Figure 10: Physical interpretation of proposed ITS as an active nonlinear spring](image-url)

![Figure 11: Bode diagram of closed-loop transfer function from \( \tau_s^{ref} \) to \( \tau_s \) when ITS is activated as \( K_{eq} \) is varied.](image-url)
Stable Torsion Torque Control Based on Friction-Backlash Analogy for Two-Inertia Systems with Backlash (Juan Padron et al.)

(a) Case 0 - I-P TTC
(b) Case 1 - P TTC and first order TDOb
(c) Case 2 - P TTC and first order TDOb with ITS (Proposed)

Figure 12: Simulation results using ITS

Figure 13: Simulation results using the proposed ITS (zoomed view).

gram as $K_{eq}$ is varied is shown in Fig. 11, where it can be seen that reducing $K_{eq}$ results in an equivalent reduction of the total spring constant over the frequency range of the ITS. The ITS frequency range depends on the pseudo-differentiator bandwidth $g_{jd}$, which should be selected as high as possible while avoiding excessive noise amplification (20).

4.3 Simulation Results Figs. 12 and 13 show the simulation results for the proposed and conventional methods (refer to Table 1 for simulation and experimental parameters). For case 0 (I-P controller) the torsion torque response is marginally unstable, as the gears reengage and collide continuously. For case 1 (P TTC + first-order TDOb), the response is stable but still has a large collision torque and bouncing on zero torque crossings.

On the other hand, the proposed ITS (case 2) manages to...
Figure 16: STDOb switching process (using $a_2^\max = 1$). The inverse backlash model is shown for reference.

Figure 17: Design of STDOb regarding resonant mode: (a) Bode diagram of $(1 - Q(s, \alpha_1, \alpha_2))$ and plant frequency response. (b) Closed loop transfer function $\frac{1}{T(s)}$. Dashed lines indicate response for intermediate values of $\alpha_1$ and $\alpha_2$

4.4 Experimental Results  
Figs. 14 and 15 show the experimental results using the ITS. The proposed ITS effectively reduces the gear collision torque while also preventing bouncing, yielding a stable response over zero-torque crossings and ensuring seamless reengagement. The collision torque using proposed ITS is reduced in more than 70% compared to the conventional cases 0 and 1.

5. Proposed Method for Gear Disengagement Time Reduction: The Switched Torsion Disturbance Observer (STDOb)

Reducing the gear disengagement time, which is the amount of time for which gears are disengaged and there is no torque transmission ($\tau_s = 0$), is extremely important in both powertrain/automotive and robotic applications. For automotive applications, a large disengagement time will delay the vehicle acceleration after pushing the accelerator pedal, causing discomfort in the driver. In robotic applications, a large disengagement time may induce instability when an outer loop, such as velocity or position control is implemented.

Based on the friction-backlash analogy, we treat the gear disengagement time reduction problem in the same manner as the stick phenomenon, and the method for stick compensation described in [19], where a variable order DOB is used and switched between the linear and nonlinear zones of friction. In this case, we propose a switched torsion disturbance observer (STDOb) for backlash that switches its order, using a high-order DOB on the nonlinear zone, where the gears are still disengaged, and a low-order DOB on the linear zone after gears have reengaged, without compromising stability due to the use of high-order DOB. For that purpose, we propose a switched Q-filter given as:

$$Q(s, \alpha_1, \alpha_2) = \begin{cases} \frac{g_0}{s + g_0} & \text{(Zero-order component)} \\ \frac{\alpha_1 g_1 s}{(s + g_0)(s + g_1)} + \frac{\alpha_2 g_2 s^2}{(s + g_0)(s + g_1)(s + g_2)} & \text{(High-order components)} \end{cases}$$

where $\alpha_1$ and $\alpha_2$ are scheduling gains that allow to switch the order of the STDOb between a zeroth ($N = 0$, $\alpha_1 = 0$, $\alpha_2 = 0$), first ($N = 1$, $\alpha_1 = 1$, $\alpha_2 = 0$) and second-order ($N = 2$, $\alpha_1 = 1$, $\alpha_2 = 1$) DOB (please refer to [20] for details on the switched Q-filter implementation). The STDOb poles are designed such that the poles corresponding to the higher order components $g_1$ and $g_2$ are smaller than the zero-order component $g_0$, in order to avoid excessive noise effects from the high order components.

The switching is based on whether $\tau_s = 0$ or not. The Q-filter of the STDOb is switched by the following switching law

$$Q(s, \alpha_1, \alpha_2) = \begin{cases} Q(s, 1, \alpha_1^\max) & \text{if } |\tau_s| \leq \epsilon \\ Q(s, 1, 0) & \text{if } |\tau_s| > \epsilon \end{cases}$$

where $\epsilon$ is the switching threshold for the STDOb. From Fig. 2, gear disengagement occurs when $\tau_s = 0$, so ideally $\epsilon$ should be chosen as zero. However, in order to prevent undesired switching due to sensor noise, $\epsilon$ is chosen to be as small as possible while considering the sensor noise. Specifically, $\epsilon$ should be at least higher than the standard deviation of sensor noise. $\alpha_1^\max$ can be selected between 0 and 1 to adjust the compensation strength. Note that $\alpha_1$ is set to 1, as a first-order TDOB is required to compensate for $\alpha_2$, so the STDOb switches between first and second-order. The switching process is illustrated in Fig. 16, and the complete block diagram is shown in Fig. 21.
5.1 STDOb design procedure regarding resonant mode  Given that the two-inertia plant has a resonant mode, it must be also considered in the design of the STDOb. From Fig. 21.A, the closed-loop transfer function from $\tau_s^{ref}$ to $\tau_s$ in linear region after backlash has been overcome ($|\theta_f| > \beta$) and the ITS is deactivated is given as:

$$\tau_s = \frac{K_T K_m s}{\left(1 - Q(s, \alpha_1, \alpha_2)\right) \left(s^2 + \frac{K_g}{J_l} R_g K_m + M(s)\right)}$$

(18)

$$M(s) = K_g R_g Q(s, \alpha_1, \alpha_2) s^2 + K_T K_m s.$$  (19)

In order to suppress the resonant mode, the STDOb bandwidth need to be wide enough. This is achieved by either increasing poles $[g_0, g_1, g_2]$ or by increasing the STDOb order by increasing $\alpha_1$ and $\alpha_2$. This is demonstrated in the Bode diagrams of Fig. 17.

5.2 STDOb Design Procedure for $\alpha_2^{max}$  When $|\theta_f| < \beta$ backlash occurs and $\tau_s$ becomes zero due to the gear disengagement. Thus, the block diagram of Fig. 21.A is simplified to the one in Fig. 21.B. In order to overcome backlash, $|\theta_f| \geq \beta$ must hold. So how quick backlash is overcome is determined by the torsion velocity $\omega_s$, which is derived as follows:

$$\omega_s(s) = \tau_s^{ref} K_{TTC} R_g \frac{1}{1 - Q(s, \alpha_1, \alpha_2)} - \omega_l.$$  (20)

As $\omega_l$ is not controllable during backlash, $(1 - Q(s, \alpha_1, \alpha_2))^{-1}$ determines how quick $\omega_s$ increases.

5.3 Simulation Results  Fig. 19 show the simulation results for the STDOb as $\alpha_2^{max}$ is varied between 0 and 1. All cases include an ITS on the inner loop.

As $\alpha_2^{max}$ is increased, the gear disengagement time is reduced, with the smallest disengagement time achieved for $\alpha_2^{max} = 1$. However, the gear collision torque is also increased, so there is a trade-off between disengagement time and collision torque reduction.

5.4 Experimental Results  Fig. 20 shows the exper-
Figure 21: Block diagram of proposed torsion torque control method: (A) Complete block diagram (B) Block diagram of proposed method during backlash ([θ] < β → τ_s = 0)

Table 1: Simulation and exp. parameters

| Parameter                              | Value |
|----------------------------------------|-------|
| Motor-side Inertia                     | J_m   |
| Load-side Inertia                      | J_l   |
| Gear ratio                             | R_g   |
| Torque Constant                        | K_t   |
| Torsion Spring Constant                | K_t   |
| Viscous friction constant              | D_m   |
| Load-side friction (seen from motor-side) | D_l   |
| Backlash width                         | τ_d   |
| STDOb poles                            | [q_0, q_1, q_2] |
| STDOb switching threshold              | ε     |
| Torsion torque bandwidth               | ω_d   |
| ITS pseudodifferentiator bandwidth     | ω_i   |
| ITS gain                               | K_i   |
| Equivalent spring constant             | K_e   |
| ITS torque threshold                   | K     |
| PI vel. cont. bandwidth                | ω_p   |
| FPIDO bandwidth                        | ω_F   |

Figure 22: Trade-off relationship between disengagement time and collision torque when combining ITS and STDOb

6. Discussion of Results

Table 2 shows a numerical comparison of the conventional and proposed methods based on gear disengagement time, gear collision torque and the presence of bouncing. The conventional method using I-P controller has the worst performance overall. The second conventional method using a P TTC with first-order TDOb slightly improves performance, but still shows severe bouncing.

On the other hand, by applying the proposed ITS for gear collision torque suppression, the maximum collision torque is reduced in more than 70% compared to conventional methods 1 and 2, while also suppressing the bouncing phenomenon and ensuring smooth gear reengagement.

Finally, by applying the proposed STDOb for gear disengagement time reduction, the disengagement time is reduced in almost 28% compared to the ITS with first-order TDOb case. While the collision torque almost doubled, it is still lower than that of the conventional methods. As shown in Fig. 19 and 22, there is a trade-off between gear collision torque and gear disengagement time when combining the STDOb and the ITS. By adjusting the magnitude of the STDOb switching gain α_s,max, the control designer can achieve the desired disengagement time and collision torque depending on the specifications of the control objective, such as mechanical characteristics and actuator capacity.

6.1 Application Example: Direct Teaching

Figs. 23 and 24 shows an application example of the proposed method on direct teaching, or zero-torque control (τ_s^ref = 0). For the conventional method using I-P control, the joint backlash induces severe limit cycles that affect performance and gives a jerky sensation to the operator. On the other hand, the proposed method using ITS and STDOb ensures a smooth response even in the presence of backlash.

6.2 Application Example 2: Load torque control

Fig. 25 shows another application example using load-side torque control as an outer loop with the proposed method. A zero-torque reference was employed, and the plant was driven by a load-side motor to evaluate its performance. Results are shown in Fig. 26, showing that the proposed method is stable even when driven by an outer loop.
Table 2: Performance comparison of tested methods in experiment

| Control Scheme | Disengagement Time [s] | Collision Torque [Nm] | Bouncing |
|----------------|-------------------------|-----------------------|----------|
| I-P TTC (Conventional) | 0.892 | 3.43 | YES |
| P TTC + first-order TDOb (Conv. 2) | 0.597 | 2.81 | YES |
| P TTC + first-order TDOb + ITS (Proposed w/ collision suppression) | 0.210 | 0.71 | NO |
| P TTC + ITS + STDOb (Proposed w/ collision suppression and disengagement time reduction) | 0.151 | 1.90 | NO |

Figure 23: Direct teaching (zero-torque control) results: (a) I-P torque control (conventional); (b) P control with ITS and STDOb (proposed)

Figure 24: Direct teaching (zero-torque control) results over long span for slow and fast motion: (a) I-P torque control (conventional); (b) P control with ITS and STDOb (proposed)

7. Conclusion

This paper proposes a torsion torque control strategy for two-inertia systems with backlash based on a friction-backlash analogy. It was demonstrated that by applying a high-bandwidth motor-side velocity controller, the joint torsion torque dynamics are transformed into a first-order plant. From this, an analogy between a one-mass plant with static friction and the first-order approximated joint dynamics with backlash was established, and the similarity between the stick-slip compensation problem and the backlash compensation problem was demonstrated. From this, a combina-

References

(1) S. Wakui, T. Enmei, H. Fujimoto, Y. Hori and K. Omata, “Gear Collision Reduction of In-Wheel-Motor by Joint Torque Control Using Load-Side High-Resolution Encoder,” IEEE International conference on Mechatronics, pp. 550-555, 2019.
(2) Y. Yokokura and K. Ohishi, “Fine Load-Side Acceleration Control Based on Torsion Torque Sensing of Two-Inertia System,” IEEE Transactions on Industrial Electronics, vol. 67, no. 1, pp. 768-777, 2020.
(3) Y. Kawai, Y. Yokokura, K. Ohishi and T. Miyazaki, “Smooth Human Interaction Control using Torsion Torque Controller and Motor-side Normalization Compensator Focusing on Back-forward Drivability,” IEEE Journal of Industry Applications, vol. 8, no. 2, pp. 322-333, 2019.
(4) F. Bottiglione, A. Sorniotti and L. Shead, “The effect of half-shaft torsion dynamics on the performance of a traction control system for electric vehicles,” Journal of Automobile Engineering, vol. 226, no. 9, pp. 1145-1159, April 2012.
(5) A. Tustin, “The Effects of Backlash and of Speed-Dependent Friction on the Stability of Closed-Cycle Control Systems,” J. Inst. Elec. Eng, vol. 94, no. 2A, pp. 143-151, 1947.
(6) M. Nordin and P.O. Gutman, “Non-linear Speed Control of Elastic Systems with Backlash,” Proc. 31st IEEE Conference on Decision and Control, 2000, vol.4, pp. 4060-4065.
(7) C. Ma and Y. Hori, “Backlash Vibration Suppression Control of Torsional System by Novel Fractional Order PIDk Controllers,” IEEE Journal of Industry Applications, vol. 124, no. 3, pp. 312-317, 2004.
(8) P. Rostalski, T. Besselmann, M. Barič, F. Van Belzen and M. Morari, “A hybrid approach to modelling, control and state estimation of mechanical systems with backlash”, International Journal of Control, vol. 80, no. 11, pp. 1729-1740, 2007.
(9) A. Yamaguchi, K. Ohishi, Y. Yokokura, T. Miyazaki and K. Sasazaki, "Backlash-based Shock Isolation Control for Jerk Reduction in Clutch Engagement," IEEE Journal of Industry Applications, vol. 8, no. 2, pp. 160-169, 2019.
(10) Y. Li, A. Hansen, J.K. Hedrick and J. Zhang, "A receding horizon sliding control approach for electric powertrain with backlash and flexible half-
Example of STDOb tuning depending on backlash width

As the backlash width $2\beta$ increases, the gear collision torque also increases, factor that may induce instability in the proposed method. In this section, an example of tuning the STDOb for reducing the gear collision impact torque is presented. This process is based on reducing the integral compensation of the STDOb during backlash (when $|r_1| \leq \epsilon$) based on Eqs. (21)-(23), and is depicted in the flowchart of app. Fig. 1. app. Fig. 2 shows that by adjusting both $\alpha_2$ and $\alpha_1$ during backlash, the gear collision can be further reduced if required.

| $\alpha_1$ | $\alpha_2$ | $\alpha_3$ | $\alpha_4$ |
|---------|---------|---------|---------|
| 1       | 0.75    | 0.25    | 0       |
| 1       | 0.5     | 0.25    | 0       |
| 1       | 0       | 0.25    | 0       |
| 1       | 0       | 0       | 0       |

app. Fig. 3: Simulation results of proposed method to load-side inertia variation. Load inertia $J_l$ is varied from 10 to 0.1 times the nominal value $J_{ln}$

Performance of proposed method considering load inertia variation

The proposed method performance to load-side inertia $J_l$ variation is evaluated in app. Fig. 3. The load-side inertia was varied from 10 to 0.1 times the nominal value $J_{ln}$. Although convergence time is slowed for lighter inertia values, it is confirmed that stability is preserved even for load-side inertia variation, so the proposed method is also suitable for use in industrial robots.

Juan Padron (Student Member) received the B.E degrees in electronics engineering from Universidad Simon Bolívar, Caracas, Venezuela, and the M.E degree in electrical engineering from Nagaoka University of Technology, Niigata, Japan, in 2016 and 2019, respectively. He is now a Ph.D. candidate at Nagaoka University of Technology. His research interests include motion control, haptics and compensation of nonlinear phenomena in mechatronics. He is a member of the Institute of Electrical Engineering of Japan (IEEJ) and the IEEE Industrial Electronics Society (IEEE IES).
Stable Torsion Torque Control Based on Friction-Backlash Analogy for Two-Inertia Systems with Backlash (Juan Padron et al.)

Yusuke Kawai (Member) received B.E. degree in electrical engineering and the Ph.D. degree in Science of Technology Innovation from Nagaoka University of Technology, Nagaoka, Japan, in 2015 and 2020. He was Postdoctoral Research Associate at Nagaoka University of Technology from 2020 to 2021. He has been an Assistant Professor at National Institute of Technology, Ichinoseki College since 2021. His research interests include motion control and Haptics. He is a member of the Institute of Electrical Engineering of Japan (IEEE) and the IEEE Industrial Electronics Society (IEEE IES).

Yuki Yokokura (Member) received the B.E. and M.E. degrees in electrical engineering from Nagaoka University of Technology, Niigata, Japan, in 2007 and 2009, respectively, and the Ph.D. degree in integrated design engineering from Keio University, Yokohama, Japan. He was a Visiting Fellow at Keio University, and a Postdoctoral Fellow at Nagaoka University of Technology in 2011. Dr. Yokokura was a JSPS (Japan Society for the Promotion of Science) Research Fellow (DC2 and PD) from 2010 to 2011. He was an Assistant Professor with Nagaoka University of Technology from 2012 to 2020, and he has been an Associate Professor since 2020. His research interests include motion control, motor drive, power electronics, and real-world haptics.

Kiyoshi Ohishi (Fellow) received the B.E., M.E., and Ph.D. degrees in electrical engineering from Keio University, Yokohama, Japan, in 1981, 1983, and 1986, respectively. From 1986 to 1993, Prof. Ohishi was an Associate Professor with Osaka Institute of Technology, Osaka, Japan. Since 1993, he has been with Nagaoka University of Technology, Niigata, Japan. He became a Professor in 2003. His research interests include motion control, mechatronics, robotics and power electronics. He received twice "IEEE Distinguished Paper Award" from IEEJ in 2002 and 2009, respectively. He is an IEEE Fellow member from 2015. He is a Senior AdCom Member of IEEE IES Society from 2016, and he was an AdCom Member (elected) of IEEE IES Society for 12 years from 2004. He received the Outstanding Paper Awards at IECON’85 and Best Paper Awards at IECON’02, IECON’04 from the IEEE Industrial Electronics Society. He is a General chair of IEEE IECON2015, AMC2010, AMC2016 and AMC2018.

Toshimasa Miyazaki (Member) received the B.E. and M.E., and Ph.D. degrees all in electrical engineering, from Nagaoka University of Technology, Niigata, Japan, in 1994, 1996, and 1999, respectively. From 1999 to 2009, he was an Associate Professor with Nagaoka National College of Technology, Niigata, Japan. Since 2010, he was an Associate Professor with Nagaoka University of Technology, Niigata, Japan. Since 2020, he has been a Professor with Nagaoka University of Technology, Niigata, Japan. His research interests include motion control and power electronics. He is a member of the IEEE Industrial Electronics Society (IEEE IES), the Institute of Electrical Engineers of Japan (IEEE), and the Society of Instrument and Control Engineers (SICE).