Analysis of heavy rainfall and barrier-jet evolution during Mei-Yu season using multiple Doppler radar retrievals: a case study on 11 June 2012

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ABSTRACT

An extremely heavy precipitation event that occurred over northern Taiwan during the Mei-Yu season is analysed using radar-network observations. Radiosonde observations demonstrate the existence of a barrier jet during this event, while radar data illustrate the gradual transition from trailing stratiform precipitation to parallel stratiform precipitation as the system approached and made landfall over Taiwan. In addition, line convection parallel to the orography was initiated at the southern edge of the main precipitation before merging to form a special Y-shaped echo near the coast, and this feature repeated twice within two hours. Through multiple analyses of three different Doppler radars every 30 minutes for a period of four hours, 3-D wind fields are retrieved over the ocean and the complex terrain of Taiwan using a variational algorithm, while the pressure and temperature structure are derived from the retrieved wind fields. The migration and intensity of the barrier jets at convective scales is revealed by a vorticity budget analysis. It is found that, taken together, the stagnated Mei-Yu front, the location and the strength of the barrier jet and cold pool, as well as orographic blockage over northern Taiwan explain the formation of this quasi-stationary and extremely heavy rainfall case. A schematic model of extreme heavy rainfall over complex terrain is presented.

Keywords: barrier jet, heavy rainfall, radar retrieval analysis

1. Introduction

The Mei-Yu front is a major high-impact weather system that brings heavy rainfall to Taiwan and causes significant damage. On average, the Mei-Yu front season starts in mid-May and ends in mid-June over southern China and Taiwan. During the Mei-Yu season, under the unstable environment of the East Asia Monsoon trough which extends from south-eastern China to southern Japan, mesoscale convective systems often evolve along the Mei-Yu frontal zone and move toward Taiwan. In many cases, the Mei-Yu front moves very slowly and becomes a quasi-stationary system for a few days. Due to the complex terrain of Taiwan, strong convection can form over relatively fine horizontal scales. Predicting the evolution and precipitation of these systems remains an outstanding challenge.

Many studies have focused on the heavy rainfall events that occur in northern Taiwan. From observations (radiosonde, surface observation and aircraft) made during the Taiwan Area Mesoscale Experiments (TAMEX) conducted in 1987, several studies found that a low-level jet (~1-km height), referred to as a barrier jet, strongly influences heavy rainfall over northern Taiwan (Chen and Li, 1995a, 1995b; Li et al., 1997; Li and Chen, 1998). Li et al. (1997) investigated a heavy rainfall event (with a barrier jet) over northern Taiwan using dual-Doppler data, with a particular emphasis on 3-D winds of the main precipitation system over the ocean and near the coast before landfall. Their results reveal the interactions between the frontal system, barrier jet and land-breeze near the northwest coast and help explain the persistence of several long-lived convective systems. Using the same case study as Li et al. (1997), Jou and Deng (1998) retrieved pressure and temperature fields in different convective cells; by presenting vertical structures of the cells in the same rainband, they demonstrated deep and shallow convection could occur in the same rainfall event. Yeh and Chen (2003) performed a theoretical study by initializing a regional model (7-km resolution) with a single upstream sounding for TAMEX Intensive
Observation Period (IOP) #3, confirming the existence of a barrier jet. Lin et al. (2011) and Chen et al. (2013) further examine the interaction between southwesterly flow and orography. They found that a horizontal resolution of around 1 km is required to resolve the dynamics (local circulations) and thermodynamics (pressure and temperature) when orographic interactions are included. All these studies demonstrate that the existence of the barrier jet is important for the occurrence of heavy rainfall over northern Taiwan. However, relatively few studies focus on the evolution of the barrier jet and its impact on strong convective cells.

In this study, a real case on 11 June 2012 which brought extremely heavy rainfall to northern Taiwan and caused flooding and serious damage is selected. Numerical weather prediction with high resolution (5-km) from the Central Weather Bureau (CWB) of Taiwan cannot capture the precipitation system stagnated over northern Taiwan; consequently, the extremely heavy rainfall event was poorly predicted. Recently, Chen et al. (2018) examined this case by using observations and simulations. Their study demonstrates how a favourable storm environment (barrier jet and high moisture content) accompanied with a frontal system causes long-lived convection cells over the northwestern Taiwan coast (before 1800 UTC 11 June) and within the Taipei basin (1800–2000 UTC, 11 June). With numerical simulation, they further proved that the orography blockage is an important factor for the extreme precipitation to occur. Wang et al. (2016) also studied this case with high-resolution model simulations (1.5 km in the horizontal). Their study focuses on dynamic and thermodynamic analyses of this event when the result of quantitative precipitation forecast (QPF) between 1800 and 2400 UTC (after 24-h lead-time of the forecast) of 11 June 2012 is similar to surface precipitation. However, their study also pointed out that the heavy rainfall occurred before 1800 UTC over the northwestern Taiwan coast remains challenging despite the application of a cloud-resolving model.

Therefore, the goal of this study is to further examine and document this case based on high-resolution observations (space and time) from the radar network. With observations from three different Doppler radar stations, the algorithms of Liou and Chang (2009) and Liou et al. (2012) are used to retrieve 3-D wind, pressure and temperature fields over the ocean and the complex terrain. The spatial and temporal evolution of the barrier jet and the thermodynamic fields are investigated in detail. The paper is structured as follows: the data resources and methodology of the retrieval algorithms are presented in Section 2. In Section 3, an overview of the extreme precipitation event is described. Section 4 discusses the results of retrievals in three different stages of the precipitation system. A conceptual model of these extreme heavy
rainfall events is also presented. Summary and future work are given in Section 5.

2. Data source and methodology

2.1. Data source

The observations utilized in this study are shown in Fig. 1. At the synoptic scale, weather maps at the surface, 850 hPa, and 200 hPa are from CWB. Radiosonde observations taken in Makung and Banqiao are used from 11 June to 12 June 2012. Satellite images of Multifunctional Transport Satellite (MTSAT) infrared imagery (IR) are used to track the Mesoscale Convective System (MCS). For ground observations, 10 surface stations which record pressure, temperature and winds around Taiwan are utilized. Finally, Automatic Rainfall and Meteorological Telemetry System (ARMTS, Kerns et al. 2010), which records hourly rainfall accumulations, are utilized.

Weather radar networks are useful to detect extreme weather system with high temporal and spatial resolution. Three weather radars have extensive coverage around northern of Taiwan as shown in Fig. 1. In this study, the precipitation event was monitored by three radars in the north of Taiwan: S-band RCWF (121.77°E, 25.07°N, 766m), C-band RCTP (121.21°E, 25.07°N, 10m) and the National Central University C-band dual-Polarimetric Doppler radar (hereafter NCU C-Pol, 121.18°E, 24.97°N, 196m). RCWF and RCTP are Doppler radars operated by CWB on Wu-Fen-Shan Mountain and Civil Aeronautics Administration (CAA) at Taiwan Taoyuan International Airport, respectively. Information of scanning strategies on the three radars can be found in Table 1. Regarding the retrieval algorithm, two consecutive volume scans (~10 min) from each radar are collected from 1200 to 1600 UTC, and the details can be found in Table 2.

A data quality control procedure is applied to all radar observations: for Doppler radars (RCWF and RCTP), high reflectivity and low radial wind speed are used to filter out the terrain and ground clutters. In this study, thresholds of reflectivity higher than 45 dBZ and radial wind speed lower than 2 m s\(^{-1}\) are used. For NCU C-POL radar, additional information available such as the cross-correlation coefficient (\(\rho_{HV}\)) is used to filter out undesired observations such as ground clutter, which in this case is rejected if \(\rho_{HV} < 0.85\). Observations have a quality control issue regarding velocity folding. Thus, an unfolding procedure is applied to radial wind in the spherical coordinate system. All radar observations are interpolated onto the 3-D Cartesian coordinates to CAPPI (Contain Altitude Plan Position Indicator) by horizontal resolution 1-km and vertical resolution 0.5-km for the retrieval algorithm.

2.2. Wind retrieval algorithm

In this study, a variational-based algorithm (Liou and Chang, 2009; Liou et al. 2012) is used to retrieve the 3-D wind fields with multiple Doppler radars. The system is named Wind Synthesis System using Doppler Measurements (WISSDOM, Liou et al., 2016). In Liou and Chang (2009), the uncertainties of the retrievals have been tested and validated with idealized experiments, and results show that the errors of horizontal winds are very small; larger uncertainty remains in vertical. In addition, the accuracy of the retrieved winds in real cases have been verified with an independent radar. This algorithm

### Table 1. Information of the three radar sites.

| Radar   | Wave length (cm) | Max range (km) | Elevation (°) | Variable | Nyquist (m s\(^{-1}\)) |
|---------|------------------|----------------|---------------|----------|------------------------|
| RCWF    | 10               | 300            | 0.5, 1.5, 2.4, 3.4, 4.3, 6.0, 9.9, 14.6, 19.5 | ZH, VR, SW | 26.6                   |
| RCTP    | 5                | 100            | 0.3, 1.0, 1.8, 2.5, 3.8, 5.0, 6.0, 7.0, 9.0, 11.0, 14.0, 17.0, 20.0 | ZH, VR, SW | 15.9                   |
| NCU-CPOL| 5                | 250            | 0.5, 1.4, 2.4, 3.4, 4.3, 6.0, 9.9, 14.6, 19.5 | ZH, VR, DR, KD, PH, RH | 31.9                   |

### Table 2. Observation time of radar data for the analysis time of retrieval algorithm.

| Time | RCWF | RCTP | NCU-CPOL |
|------|------|------|----------|
| 1    | 1200 | 1157 | 1150, 1205 | 1155, 1203 |
| 2    | 1230 | 1225, 1230 | 1220, 1235 | 1229, 1237 |
| 3    | 1300 | 1258, 1304 | 1255, 1305 | 1254, 1302 |
| 4    | 1330 | 1326, 1332 | 1325, 1335 | 1327, 1336 |
| 5    | 1400 | 1354, 1400 | 1355, 1405 | 1353, 1401 |
| 6    | 1430 | 1428, 1433 | 1425, 1435 | 1426, 1434 |
| 7    | 1500 | 1456, 1501 | 1455, 1505 | 1451, 1500 |
| 8    | 1530 | 1529, 1535 | 1525, 1535 | 1525, 1533 |
| 9    | 1600 | 1558, 1604 | 1555, 1605 | 1559, 1607 |
has been applied to diagnose different real case studies (Liou et al., 2013; Lee et al., 2014; Liou et al., 2016). Brief descriptions and major features of WISSDOM are given as follows, and the equations of the WISSDOM are demonstrated in the appendix. The details can be found in Liou and Chang (2009) and Liou et al. (2012).

The cost function of WISSDOM includes basic constraints for wind retrieval: a geometric relation to connect radial component and 3-D winds, an anelastic continuity equation, and the Laplacian smoothing term in space. In addition, extra constraints are applied in the algorithm: (1) a background term to cover the data-void region; (2) a vertical vorticity equation in the cost function to guarantee the balance of the vorticity budget. As mentioned in Liou and Chang (2009), this constraint can both improve the accuracy of the wind and thermodynamic retrievals. With these formulas as weak constraints, WISSDOM is able to retrieve the wind field along the radar baseline. By implementing the Immersed Boundary Method (IBM, Tseng and Ferziger, 2003), Liou et al. (2012) improved the WISSDOM so the retrieval algorithm can take into account the terrain effect.

2.3. Thermodynamic retrieval scheme
After obtaining 3-D wind components, the perturbations of pressure and temperature are retrieved based on Gal-Chen (1978). The algorithm uses the momentum equations to obtain thermodynamic fields. Furthermore, the contributions of vapour, cloud and rainwater are included to estimate the buoyancy force. In the retrieval scheme, a normalized pressure ($\pi$) is obtained. It is defined as

Fig. 2. Weather maps at 1200 UTC 11 June 2012: (a) surface; (b) 850-hPa; (c) 200-hPa; (d) observed surface pressure at different stations over Taiwan. The values in each station are: station number (first row), surface pressure (second row, unit: hPa), and surface temperature (third row, unit: Celsius degree). Green lines are the surface pressure contour of 999 and 1000 hPa.
\[ \pi = C_p \left( \frac{P^0}{P_0} \right)^{\frac{R}{C_p}}, \]  

where the function of \( p \) is the pressure, \( p_0 = 100 \text{kPa} \), \( R \) (unit: J kg\(^{-1}\) K\(^{-1}\)) is the gas constant, and \( C_p \) (unit: J kg\(^{-1}\) K\(^{-1}\)) is the specific heat capacity at a constant pressure. The perturbation of the virtual cloud temperature can be derived once the 3-D of pressure perturbation, \( \pi' \), is retrieved. It is defined as

\[ \theta'_c = \theta' + \left( 0.61q'_w - q_c \right) \theta_0. \]  

Where \( \theta_0 \) stands for potential temperature, \( q'_w \) (unit: g kg\(^{-1}\)) is the perturbation of the water vapour mixing ratio from its basic state and \( q_c \) (unit: g kg\(^{-1}\)) is the cloud water mixing ratio.

It should be noted that because the Neumann boundary conditions are used to solve the Poisson equation in the algorithm, the deviations from horizontal average (\( \pi - \langle \pi' \rangle \)) are obtained as a unique solution. To determine the perturbations (\( \pi' \)) of pressure in the entire domain, a single independent value of the pressure at each level is needed to estimate the average of pressure perturbation \( \langle \pi' \rangle \). This independent value could be obtained from in situ observation or numerical model output. In this study, the vertical information is used by Banqiao sounding profile.

### 2.4. Analysis configuration

The size of the analysis domain is shown in Fig. 1; it has dimensions of 250 km (west to east) by 260 km (north to south) with 1 km resolution in the horizontal direction, and 14.5 km with 0.5 km resolution in the vertical direction. The National Centers for Environmental Prediction (NCEP) \((1^\circ \times 1^\circ)\) Final operational global analysis...

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**Fig. 3.** Radiosonde profile of (a) Makung station (119.56°E, 23.56°N, 46734) and (b) Banqiao station (121.44°E, 24.99°N, 46692) at 1200 UTC 11 June 2012.

**Fig. 4.** Infrared imagery of brightness temperature (unit: °C) from Multifunctional Transport Satellite (MTSAT) on 11 June 2012 at 1200 UTC. The pink colour indicates the location of the MCS.
(FNL) for 1200 UTC 11 June was provided to WISSDOM as the initial guess and background field. In order to represent terrain effects near the surface, observations of horizontal wind from surface stations (or from automated rain gauges) are extrapolated to the lowest model level (0.5 km height) following the procedure of Tai et al. (2011). Since environment flow is mainly used to cover data-void region and the difference of horizontal wind speeds and directions from the global analyses are relatively small (not shown), the same background information is used for the entire retrieval period (4 h), that is the environmental flow is assumed to be relatively steady. To obtain perturbations of the thermodynamic fields, a radiosonde profile at the Banqiao station (see Fig. 1 for the location) is used. Since the retrievals are derived directly from WISSDOM, orographic effects are represented implicitly even though the current retrieval algorithm cannot fully resolve the thermodynamic field over the complex terrain. Therefore, the region which has less complex terrain and is covered by radar observations is selected as the retrieved area, and the domain of the thermodynamic retrievals is smaller than the domain of the 3-D wind.

3. Overview of the extreme rainfall event on 11 June 2012

The synoptic-scale weather favoured the development of convection over the Taiwan area. The surface weather map at 1200 UTC 11 June 2012 (Fig. 2a) shows that the Mei-Yu front is located to the north of Taiwan. At 850 hPa (Fig. 2b), a trough over southern China induced a confluent flow over Taiwan (i.e. a strong southwesterly flow appeared in the south while the northwesterly flow persisted in the north). A divergent flow occurred at 200 hPa (Fig. 2c). The distribution of surface pressure around Taiwan (Fig. 2d) clearly shows a pressure ridge on the windward side along the southwestern and western coasts of Taiwan. Figure 3 shows sounding profiles at 1200 UTC. The profile at Makung (Fig. 3a) presents the notable feature of the environment, a deep southwesterly flow extending from the surface to mid-levels. The profiles of temperature and dew point temperature indicate that the relative humidity is high, which means abundant water vapour is distributed around the west of the Taiwan region. Compared to the Makung station, the profile at Banqiao (Fig. 3b) illustrates intensified southwesterly wind below 850 hPa. Figures 2d and 3 imply that a barrier jet existed before the severe weather system occurred over northern Taiwan. The value of the Convective Available Potential Energy (CAPE) is less than 1000 J kg$^{-1}$ at the Banqiao station, indicating a weak instability environment near Taiwan.

The infrared satellite images (Fig. 4) reveal that a MCS formed in northwest Taiwan and then moved towards and stayed over northern Taiwan as it matured. The MCS lasted for 12 h and dissipated at 2200 UTC (not shown). The rainfall accumulation from 1400 UTC 11 June to 0000 UTC 12 June exceeded 400 mm in northern Taiwan (Fig. 5a). In Fig. 5b, the auto-station at Yangmei (near the maximum accumulation of rainfall; see the cross in Fig. 5a) shows that the most intense hourly rainfall reached 123 mm hr$^{-1}$ between 1500 and 1600 UTC. At the Banqiao weather station (Fig. 5c), the temperature dropped and sea level pressure increased between 1200 and 1600 UTC when the precipitation system moved and landed over northern Taiwan. In
Fig. 6. The maximum reflectivity of RCWF radar on 11 June 2012 at: (a) 1200 UTC; (b) 1230 UTC; (c) 1300 UTC; (d) 1330 UTC; (e) 1400 UTC; (f) 1430 UTC; (g) 1500 UTC; (h) 1530 UTC; (i) 1600 UTC. C1, C2, C3 and C4 indicate the main convection of the precipitation system (reflectivity ≥ 40 dBZ) in different stages. S1, S2 and S3 are the line convection that occurred in different stages. (j) The contours of 40 dBZ shows repeated Y-shaped reflectivity at 1430 UTC and 1530 UTC.
addition, the wind speed near the surface also became stronger (not shown) before landing.

The evolution of the precipitation system is illustrated by radar observations. Figure 6 shows the maximum reflectivity of the RCWF radar at 30-minute intervals from 1200 UTC to 1600 UTC. At 1200 and 1230 UTC, a main convection centre C1 (defined as reflectivity $>40$ dBZ and length $>150$ km) approached northern Taiwan. During this time interval, a line convection S1 (length of approximately 50 km), oriented in an east-west direction, occurred along the southwest part of the main precipitation system and continued to develop (Fig. 6a and b). When the system moved toward Taiwan, the main convection C1 weakened at 1300 and 1330 UTC according to the reflectivity (Fig. 6c and d). C1 continued to move southeast towards northern Taiwan and eventually merged with S1. The convection centre C2 was formed at 1400 UTC (Fig. 6e), strengthening the main convection. Based on the leading edge of the strong reflectivity ($>40$ dBZ), we determined that the precipitation system landed over northern Taiwan at 1400 UTC. After landing, a line convection S2 (Fig. 6f) was triggered again at the south-western part of the main precipitation system parallel to the orography. S2 merged with C2 at 1500 UTC, yielding C3 (Fig. 6g). A new line convection, S3, resembling S2 (Fig. 6h) formed at 1530 UTC; subsequently it merged with the main convection, yielding C4 (Fig. 6i).

Parker and Johnson (2000) categorized linear convection of the MCS into three different types based on the environmental flow, speed of translation, and the

**Fig. 7.** Hovmöller time series (every 6-min) of radar reflectivity (contours are 20, 30, 40, 45dBZ): (a) Averaged within 10-km in the longitudinal (121.1°E), and the latitudinal range between 24.7°N and 26°N from 1100 to 1900 UTC on 11 June 2012; (b) Zoomed detail in between 1400 and 1600 UTC and averaged within 10-km from south-west (latitudinal: 23.0°N, longitudinal: 119.3°E) to north-east (latitudinal: 27.2°N, longitudinal: 123.2°E).
Table 3. Features on the precipitation system in three stages.

| Stage | State                        | Time (UTC) | Description                                |
|-------|------------------------------|------------|--------------------------------------------|
| 1     | Fast speed (15 km hr⁻¹)      | 1200 to 1230 | The precipitation demonstrates a squall line structure |
| 2     | Landing (transition)         | 1300 to 1400 | New cell merges into main convection region |
| 3     | Quasi-stationary            | 1430 to 1600 | Extremely heavy rainfall continues at the same region |

Fig. 8. Retrieved result at 1200 UTC 11 June 2012: (a) vertical velocity (colour shaded, unit: m s⁻¹) at 5 km and convergence area (green contour, interval is 0.5 × 10⁻³ s⁻¹) at 1 km; (b) horizontal wind speed (unit: m s⁻¹) and wind vector at 1 km height (blue colour shows wind retrieved with radar observations, and grey colour indicates retrieved wind beyond radar observations); (c) vertical cross-section (see Fig. 8a) of radar reflectivity (colour shaded, unit: dBZ) and the horizontal wind speed (contour lines); (d) cross-section of vertical wind (colour shaded, unit: m s⁻¹) and wind vector relative to the system motion.

The reflectivity pattern of the precipitation system: trailing stratiform (TS), leading stratiform (LS) and parallel stratiform (PS). According to their categorization, the precipitation system changed from TS (1200-1230 UTC; Fig. 6a–b) to PS (1430-1600 UTC; Fig. 6f–i). By plotting contours of reflectivity values greater than 40 dBZ, the
The main convection almost remains stationary; therefore, the system is defined as quasi-stationary for 2 h after landing and the intense convection is located around the same area in northern Taiwan (Fig. 6e–i). This feature corresponds to 1500–1600 UTC, when maximum hourly rainfall occurred (Fig. 5b). After 1600 UTC, the system gradually decayed and moved away from Taiwan.

According to the system translation speed (estimated by the strong line echo, $Z > 40$ dBZ, of the radar observation) and the pattern of the rainband, we divide the lifecycle of the precipitation system into three different stages (see Table 3). The first 30-min (1200–1230 UTC) is stage one. During this time, the system moved rapidly (15 km hr$^{-1}$) from the southern China to the northern Taiwan. Between 1300 UTC and 1400 UTC, the system underwent a transitional period: the line convection $S_1$ merged with $C_1$ (see Fig. 6), and the united convection $C_2$ landed in northern Taiwan. This is defined as the second stage. During the third stage (after 1430 UTC), the line convection $S_2$ and $S_3$ with Y-shaped echo formed and merged with the main convection, and the displacement of the system became quasi-stationary.

It should be noted that these synoptic-scale features are commonly seen during the Mei-Yu season in Taiwan. However, how to form a quasi-stationary precipitation system with repeated Y-shaped echoes is not fully known. In the next section, the structure of the precipitation system and its evolution in time are investigated using a retrieval technique.

4. Retrievals for the precipitation system

Dynamical and thermodynamical fields are retrieved from 1200 UTC to 1600 UTC every 30 minutes, and the discussion is divided into three stages as described in the previous section.

4.1. The first stage: pre-frontal convection over the ocean (TS type, 1200–1230 UTC)

The retrievals for the first stage reveal a storm structure over the ocean. At 1200 UTC, the precipitation system ($C_1$) and line convection ($S_1$) are both over the ocean. At 1 km height, strong convergence and upward motions in front of the system (Fig. 8a) are retrieved by WISSDOM. In addition, a strong southwesterly wind (isotach $> 15$ m s$^{-1}$) along the west coast (Fig. 8b) is shown. The strong horizontal wind at this level (1 km) matches the intensified southwesterly flow (barrier jet) observed at the Banqiao station (Fig. 3b). The retrieved wind also illustrates the size and the location of the barrier jet along the northwestern coast of Taiwan.
Figure 9 examines the profile of wind speed at two locations: one is away from the coast over the ocean (BG), and the other is along the coast inside the barrier jet (BJ). The profile at BG shows the flow has maximum wind around 5-km height. On the other hand, the local maximum of the winds is shown at BJ to be around 1–1.5 km height. Figure 9 also shows that the retrieved wind is able to capture the barrier jet in northern Taiwan. The cross-section (see Fig. 8a, line 1-1’) of isotach (overlaid reflectivity) in Fig. 8c and vertical velocity (overlaid wind vector relative to the system motion) in Fig. 8d display a typical squall line structure (Houze et al. 1989); the strong convection extends to 10 km with reflectivity larger than 30 dBZ, and the echo top is over 12 km. The maximum upward motion is established at 5–6 km height. The convection zone is located at the leading edge of the precipitation system, and the stratiform region trails the precipitation system. The wind vector shows typical squall line features: one is rear-to-front flow with downdrafts at low level, and the other is front-to-rear flow followed by updrafts at mid-levels in the leading convective area. The maximum vertical velocity during the first stage could be up to 5 m s\(^{-1}\). The CAPE at Banqiao station indicates weak instability (< 1000 J kg\(^{-1}\)) of the environment. However, the intensity of the updraft motion could be underestimated by the retrieval algorithm as mentioned in Liou et al. (2012) and Lee et al. (2014).

The retrieved pressure (Fig. 10a) at low levels shows that a local low is distributed over the precipitation system. Figure 10b shows the retrieved temperature perturbations; the cold region is generally distributed around the leading edge of the precipitation system, and the dashed line indicates the location of cold outflow boundary at 1-km height.
precipitation system C$_1$, and the warm region is south of the retrieved domain near the island of Taiwan. When the horizontal wind vector (relative to southwesterly flow of Makung station) at 1 km height overlaps the temperature perturbation, the line convection S$_1$ is triggered in the warm area by the outflow of the cold pool (dashed line in Fig. 10b). The vertical cross-section of pressure in Fig. 10c illustrates a low pressure centred at ~2.5 km height. In Fig. 10d, a cold pool area due to evaporative cooling of precipitation exists below 2 km height, and the warm core settles in the mid-level (~7 km). The cross-section of temperature also shows a tilted structure of upward motion where the cold pool is behind it. The retrieval diagnostics demonstrate that both dynamical and thermodynamical mechanisms support the development of the convection.

Before the Mei-Yu front system landed in Taiwan and interacted with the orography, the dynamic and thermodynamic retrievals show a structure of pre-frontal convection over the ocean. Compared to observations of strong low-level wind at the Banqiao station and the conceptual model of a squall line, the retrieved fields confirm that reliable meteorological state variables can be obtained from observations with three radars.

4.2. The second stage: encountering barrier jet over Northern Taiwan (transition, 1300–1400 UTC)

During the second stage, the retrievals show transition features when the precipitation system landed over Taiwan. Before the line convection S$_1$ merges with the
Fig. 12. Retrieved thermodynamic perturbations from 1300 to 1400 UTC of 2-km height: (a), (c) and (e) are horizontal pressure fields (unit of pressure: hPa); (b), (d) and (f) are temperature fields (unit of temperature: K). Horizontal wind vector in (f) is relative to southwesterly flow of Makung station and the dashed line indicates the location of cold outflow boundary at 1-km height.
main convection C₁ (Fig. 6d and e), the low-level convergence (at 1-km height) and vertical velocity (at 5-km height) at 1330 UTC reveal that the convection C₁ was weakening (not shown). After merging, the intensity of the convergence field and updraft motion at 1400 UTC (Fig. 11a) is maintained similarly to the first stage.

On the other hand, Fig. 11b shows the region of the barrier jet (isotach > 15 m s⁻¹) was wider and stronger than the result at 1200 UTC (Fig. 8b). In the transition period, the cross-section (see Fig. 11a, line 2-2') of the reflectivity (Fig. 11c) shows that a new convection (~121°E) developed in the south-west of the main precipitation system. The isotach in Fig. 11c illustrates that an intense wind (southwesterly) exists between 2 and 5 km on the southwest side of the strong reflectivity. The retrieved vertical motion in Fig. 11d shows a tilted structure (southwest to northeast), and the updraft is much stronger than the downdraft.

The pressure perturbations at 2-km height from 1300 to 1400 UTC are shown in Fig. 12a, c, and e. A meso-low centre (L₁) is distributed over the retrieved domain from 1300 to 1400 UTC. The low of L₁ is caused and followed by the precipitation system. In the meantime, a relative weak low, L₂, is located inland of northern Taiwan. The intensity of the low pressure (L₂) is weaker before convection S₁ united with C₁ (Fig. 12a and c), and it is strengthened (Fig. 12e) after the two systems merged over northern Taiwan. The pressure gradient force due to

Fig. 13. Same as Fig. 8 but at 1530 UTC 11 June 2012. The location of cross-section of (c) and (d) is in Fig. 13a.
L2 over land would accelerate area I and decelerate area II of the southwesterly wind (Fig. 12e). The forcing in area I is superimposed over the original barrier jet and this could explain the enhancement and extension of the barrier jet at 1400 UTC (Fig. 11b).

The temperature fields (Fig. 12b, d and f) at 2-km height illustrate that the warm region stays in the southwest of the retrieval domain. From the sounding data for Makung and Banqiao stations, the wind shear below 700 hPa indicates warm advection, as in Li et al. (1997). Therefore, it is expected that the enhanced barrier jet would advect more humid and warm air from the south of the ocean and that the positive temperature perturbation would be larger at 1400 UTC (Fig. 12f). The convection S2 with Y-shaped echo was triggered in this warmer area because the outflow of the cold pool near the surface encountered the barrier jet (dashed line in Fig. 12f). Moreover, the time series of the temperature field (Fig. 12b, d, f) shows the cold region evolved from the northwest to the northeast of the domain when the precipitation system approached and landed over northern Taiwan.

4.3. The third stage: repeated local convection with evolved barrier jet (PS type, 1430–1600 UTC)

The retrievals present the features of quasi-stationary precipitation system in northern Taiwan during this stage. Figure 13a shows that a strong convergence zone and upward motion both occurred inland. In addition, the
maximum of the barrier jet established itself inland and moved parallel to the orography (Fig. 13b). Figure 13c shows a well-developed storm in the cross-section (see Fig. 13a, line 3-3'). Compared to the second stage, the isotachs illustrate even stronger horizontal winds extending from low to mid-levels southwest of the main convection (the maximum reflectivity is > 45 dBZ). The vertical velocity in Fig. 13d presents a tilted upward motion from low to high levels. The local maximum of the updraft at the low level corresponds to the strong convergence in Fig. 13a and the downward motion indicates the location of the torrential heavy rainfall. In addition, the maximum upward motion at this stage is more than 6 m s\(^{-1}\), which is larger than that at the first and second stages.

The cross-section of pressure and temperature perturbation parallel to the orography of northern Taiwan is shown in Fig. 14. When the precipitation system stagnated over northern Taiwan, a strong low remained inland at 1530 UTC (Fig. 14a). As in the analysis of Fig. 12e, the horizontal pressure gradient force explains the enhancement of the barrier jet in Fig. 13b. In addition, downward motion associated with a heavy rainfall area (Fig. 13d) caused an extended cold pool region (Fig. 14b and d) that almost covered the entire inland area of northern Taiwan. The line convection S\(_3\) was triggered in the same way as S\(_2\) over the warm region (Fig. 14b). The cross-section of the pressure and temperature fields (Fig. 14c and d) reveal a tilted (from southwest tilted to northeast) low pressure coupled with a warm area. The southwesterly inflow is parallel to the orientation of the tilted structure (from southwest to northeast) of the precipitation, and the flow could bring warmer and more

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**Fig. 15.** The horizontal wind direction relative to the system motion and the total horizontal wind speed at 8-km height: (a) at 1200 UTC, the system motion is 15 km h\(^{-1}\) towards south-east; (b) at 1530 UTC, the system motion is \(~0\) km h\(^{-1}\) (quasi-stationary).

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**Fig. 16.** Retrieved wind profiles (averaged over precipitation area) from 1200 UTC to 1600 UTC. The speed (shaded and contour, unit: m s\(^{-1}\)) and direction (vector) of the horizontal wind are relative to the system motion as indicated in Table 3. Red dashed line divides the precipitation types: PS, transition and PS.
humid air from the ocean. The features of the inflow and the slanting structure of the precipitation could have made the convection last longer.

Compared to the observed temperature decrease of $\sim 5 ^\circ C$ at Banqiao station (Fig. 5c), the temperature perturbations at 2 km height decreased around 2 to 3 degrees from 1300 UTC to 1500 UTC around the Banqiao area. This validates the retrieved temperature is reasonable quantitatively.

4.4. Discussion of the evolution of the barrier jet and precipitation system

The 4-h evolution of the precipitation system from 1200 to 1600 UTC are further analysed and summarized here. As mentioned in Section 3, the precipitation pattern transitioned from TS to PS type. By calculating the horizontal wind direction relative to the system motion at the first stage (1200 UTC, Fig. 15a) and the third stage (1500

![Fig. 17.](image-url)
UTC, Fig. 15b) at 8 km height, the results show that the precipitation system has much stronger southwesterly winds at high levels after the system landed and stayed over northern Taiwan. The enhanced horizontal wind (southwesterly) caused the stratiform precipitation region at the downstream wind location. The orientation of temperature perturbations (from north-south to northeast-southwest) at low level between 1300 and 1400 UTC (Fig. 12b, d, and f) also corresponds to the transition of the precipitation type from TS to PS.

To investigate the evolution of the vertical wind shear as the precipitation system transitioned from ocean to land, average wind profiles over the precipitation area (defined by reflectivity > 0 dBZ) are plotted. Figure 16 shows the mean of wind speed and horizontal wind direction relative to the system motion (see Table 3) from 1200 UTC to 1600 UTC. The horizontal wind speed below 1.5 km height decreases, but it increases above 4.5 km height with time. The enhanced horizontal wind above mid-levels corresponds to the stronger updraft revealed at the third stage (not shown). In general, the time series of wind profiles present larger vertical wind shear at the third stage (PS-type) than at the first stage (TS-type). The change of the wind shear in vertical can be also found by radiosonde between 1200 UTC of 11 June and 0000 UTC of 12 June at Banqiao station.

The mechanism of the evolved barrier jet is further investigated with the vertical vorticity signature and vertical vorticity budget (tendency, tilting and stretching). Since the horizontal wind are retrieved with at least two radars at each time, and the errors of the retrieved horizontal wind are very small (Liou and Chang 2009, see their Table 1), it is able to analyse the vertical vorticity quantitatively. The horizontal pattern of 1 km vertical vorticity \( \zeta = \frac{\partial v}{\partial x} - \frac{\partial u}{\partial y} \) at 1400 UTC (Fig. 17a) shows that positive (negative) \( \zeta \) distributed left (right) wing of the barrier jet (blue vector J1 in Fig. 17a). When computing the different terms of the vorticity budget, a positive (negative) vorticity tendency occurs at the right (left) wing of the barrier jet, implying displacement of the barrier jet. In addition, further examination confirms that stretching and tilting dominates the positive vorticity tendency at 1400 UTC (Fig. 17b). This feature matches the intense convection and the location of upward motion retrieved by WISSDOM in Fig. 11a. The evolution of the vorticity at the right wing of the barrier jet \( (J_1) \) in Fig. 17a can be estimated as follows: the vertical vorticity is about \(-8 \times 10^{-4} \text{ s}^{-1}\) and the vorticity tendency in vertical

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**Fig. 18.** Schematic diagrams to demonstrate the mechanism of the extremely heavy rainfall event. (a) Pre-frontal convection forms a TS-type precipitation over the ocean, and the line convection is triggered due the cold outflow encountering a warm and humid southwesterly flow; (b) the strengthened cold pool and enhanced barrier jet repeatedly triggered the Y-shaped echo line convection, then merge with the main convection to form a PS-type precipitation over northern Taiwan. The location of the Mei-Yu front and the warm/humid southwesterly flow (red arrow) illustrate the environmental condition of the synoptic scale over Taiwan. The location of the cold pool and the orography in northern Taiwan blocks the displacement of the main convection.
direction is up to 0.5×10^{-6} \text{ s}^{-2}. Assuming the vorticity tendency is constant, the vertical vorticity at the right wing of $J_1$ would increase to 2.7×10^{-3} \text{ s}^{-1} over 1.5-hr. The increment (2.7×10^{-3}) due to the tendency exceeds the original negative vorticity; hence, the vorticity at the right wing of the barrier jet ($J_1$) would become positive. The vorticity tendency thus moves the barrier jet inland (east) at 1530 UTC as shown in Fig. 17c. The schematic diagram in Fig. 17d illustrates the migration and strengthening of the barrier jet. The vertical wind shear shows a horizontal vertical vortex. When the strong convection landed, the barrier jet enhanced by the intense pressure gradient force (from southwest to northeast, Fig. 12e) and moved inland by the positive vertical vorticity tendency from tilting and stretching via convective updrafts (blue vector in Fig. 17d).

Based on this study, the schematic diagram in Fig. 18 illustrates the mechanism behind the formation of TS and PS precipitation over northern Taiwan in this case:

1. As shown in Fig. 18a, the Mei-Yu front is accompanied by a MCS (prefrontal precipitation) located over the ocean to the north of Taiwan. At the same time, Taiwan is surrounded by southwesterly flow and the barrier jet already exists along the coast due to orographic effect over Taiwan. The cold outflow of the main precipitation system encounters a warm and humid prevailing wind (southwesterly flow), and then triggers a line convection over the ocean.

2. When the main precipitation moves southeast and approaches Taiwan, the line convection is pushed by the southwesterly flow and merges with the main precipitation system. In addition, strong convection occurs near the location of the barrier jet, and it strengthens the pressure gradient force (PGF) at low levels. The PGF is co-located with the barrier jet in the same direction; therefore the barrier jet is enhanced after the system lands over Taiwan. The upward motion of the strong convection tilts and stretches the horizontal vortex, as shown in Fig. 17d. This mechanism further causes the barrier jet to move east towards the mountain area.

3. Since the Mei-yu front moves slowly after strong convection occurs inland, a PS-type precipitation is formed (Fig. 18b). The strong convection was blocked by orography and thus maintained in the north of Taiwan. In the meantime, the stratiform area produces a wide cold region over north of Taiwan, and the cold outflow encounters the barrier jet and triggers a new line convection. Because the outflow is partially blocked by the orography, the line convection is oriented from southwest to northeast. The main precipitation and new convection then display a Y-shaped echo structure.

4. The line convection is pushed towards and united with the main precipitation system by the enhanced barrier jet. This feature of the Y-shaped convection can repeat several times when the barrier jet and cold outflow coexist in the right locations. Overall, the locations of the cold pool, barrier jet, orography and the quasi-stationary Mei-Yu front over northern Taiwan form a mechanism to maintain the intensity of the convection and render the precipitation system quasi-stationary. Extremely heavy rainfall can thus occur for an extended period of time in the same area. Once the front moved further south, the outflow due to precipitation no longer coupled well with the barrier jet and this mechanism could not be maintained.

5. Summary
An extremely heavy rainfall occurring on 11 June 2012 during the Mei-Yu season was investigated in this study. By using multiple Doppler radar observations, wind, pressure and temperature fields were retrieved in time and space in order to examine the interaction of dynamics, thermodynamics, and terrain effect on the mesoscale and convective scale. The precipitating system is divided into three phases, and the evolution of the barrier jets at the convective scale is revealed for the first time by analyzing the migration and intensity of the barrier jet within the first 4 hours when the heavy rainfall took place. At the first stage (TS-type), typical mesoscale convection occurred south of the front, and the retrievals presented a squall-line structure when the system remained over the ocean in the north of Taiwan. The horizontal wind at low levels demonstrated that a barrier jet had already appeared along the west coast at this stage. A line convection initialized by southwesterly flow developed and merged with the main precipitation system. The system landed over northern Taiwan and went through a transition period at the second stage: strong convection with upward motion inland favoured the eastward displacement of barrier jet. In the meantime, thermodynamic retrievals illustrated a strengthened meso-low located inland of northern Taiwan, and the pressure gradient force enhanced the intensity of the barrier jet. At the third stage (PS-type), the location of the cold pool and the orography over northern Taiwan act as obstacles. When the cold outflow encounters the barrier jet, Y-shaped echo convection appeared at the cold pool outflow convergence zone and then united to the main convection. These features form a favorable condition to maintain a strong convection around the same location.
and cause extremely heavy rainfalls in a short period of time.

The results of this study can provide useful information to examine and validate the model simulation and data assimilation at the convective scale. Future work is to assimilate radar observations to investigate if the QPF can be further improved. Moreover, the dual-polarization parameters observed by NCU-CPOL can be studied in order to understand the microphysical process in the near future.

Note

1. The Central Mountain Range, which is Taiwan’s dominant topographical feature, has a peak near 4 km and an average height around 2 km.

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Disclosure statement

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### Appendix A

#### Formulas of WISSDOM

There are five (weak) constraints in the cost function of WISSDOM for obtaining the optimal 3-D winds, and they are:

1. The geometric relation to connect the retrieved Cartesian wind components and the radial velocity directly measured by the individual radar;

\[
J_1 = 2 \sum_{l=1}^{2} \sum_{x,y,z=1}^{N} g_{l,i} (T_{1,l,t})^2, \tag{A1}
\]

\[
T_{1,l,t} = (V_{l,t})_{x} - \frac{(x-P_{l})}{r_{l}} u_{l} - \frac{(y-P_{l})}{r_{l}} v_{l} - \frac{(z-P_{l})}{r_{l}} w_{l} + W_{T,l}, \quad \text{and}
\]

\[
r_{l} = \sqrt{(x-P_{l})^2 + (y-P_{l})^2 + (z-P_{l})^2},
\]

where the subscripts \(t\) (from 1 to 2) indicates the number of time levels, and \(i\) (from 1 to \(N\)) is for the number of radar sites. \(V_{l,t}\) is the radial winds observed by \(l\)th radar at time \(t\), \((u_{l}, v_{l}, w_{l})\) mean the 3-D wind at the location \((x,y,z)\) at time \(t\), and \(W_{T,l}\) represents the terminal velocity, which can be estimated using radar reflectivity data. \((P_{l}, P_{l}^{n}, P_{l}^{p})\) denote the coordinates of the \(l\)th radar, and \(r_{l}\) is the distance for each grid point to the \(l\)th radar.

2. The difference between the retrieved wind field (\(V_{l}\)) and the background winds (\(V_{B,l}\));

\[
J_2 = \sum_{l=1}^{2} \sum_{x,y,z} g_{2,i} (V_{l}-V_{B,l})^2, \tag{A2}
\]

This constraint provides auxiliary information in the data void region in the retrieved domain.

3. The anelastic continuity equation, expressed as:

\[
J_3 = \sum_{l=1}^{2} \sum_{x,y,z} \left[ \frac{\partial (\rho_u u_l)}{\partial x} + \frac{\partial (\rho_v v_l)}{\partial y} + \frac{\partial (\rho_w w_l)}{\partial z} \right]^2, \tag{A3}
\]

where \(\rho_{0}\) is the air density that varies with height only.

4. The simplified vertical vorticity equation which neglects the mixing and baroclinic terms, and it is formulated as:

\[
J_4 = \sum_{x,y,z} \left[ \frac{\xi}{r_{l}} + \frac{\partial}{\partial x} \left( \frac{\partial \tilde{u}_l}{\partial y} + \frac{\partial \tilde{v}_l}{\partial z} \right) + f \frac{\partial \tilde{v}_l}{\partial x} \right]^2, \tag{A4}
\]

where \(\xi = \frac{\partial^{2} \tilde{v}_l}{\partial x \partial z} - \frac{\partial^{2} \tilde{v}_l}{\partial y \partial z}\) is the vertical vorticity and \(f\) is the Coriolis parameter. The overbar represents a temporal average over the two time levels.

5. The Laplace smoothing filter, defined as:

\[
J_5 = \sum_{l=1}^{2} \sum_{x,y,z} g_{5,i} \left[ \nabla^2 (u_{l} + v_{l} + w_{l}) \right]^2, \tag{A5}
\]

where \(\nabla^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}\), and the purpose of this penalty term is to minimize the discrepancy between observation and data-void region.

The weighting coefficients in equations (A1 – A5) are given the same values as Liou et al. 2009.