Four-Family $\mathcal{N} = 1$ Supersymmetric Pati-Salam Models from Intersecting D6-Branes

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Abstract: We investigate the construction of four-family $\mathcal{N} = 1$ supersymmetric Pati-Salam models from Type IIA $T^6/Z_2 \times Z_2$ orientifold with intersecting D6-branes. Utilizing the deterministic algorithm introduced in Ref. [1], we obtain 274 types of models with three rectangular tori and distinct gauge coupling relations at string scale, while 6 types of models with two rectangular tori and one titled torus. In both cases, there exists a class of models with gauge coupling unification at string scale. In particular, for the models with two rectangular tori, one titled torus and gauge coupling unification, the gaugino condensations are allowed, and thus supersymmetry breaking and moduli stabilization are possible for further phenomenological study.
1 Introduction

One of the main motivations of string phenomenology is to find a unifying $\mathcal{N} = 1$ supersymmetric quantum field theory, a competent framework among various extensions of the four-dimensional Standard Model (SM). D-branes play an important role in constructing interesting models at the phenomenological level, especially in Type I, Type IIA and Type IIB string theories. Chiral fermions appear at

- worldvolume singularities of D-branes [2–8]; and
- intersecting loci of D-branes in the internal space [9].

Intersecting $D6$-branes on Type IIA orientifolds have been used to construct three-family non-supersymmetric models and grand unified models [10–30]. Even though these models satisfy the Ramond-Ramond (RR) tadpole cancellation conditions, there are Neveu-Schwarz-Neveu-Schwarz (NS-NS) tadpoles remaining due to their non-supersymmetric nature. Moreover, the string scale is close to the Planck scale since the intersecting $D6$-branes are not transversal in the internal space. As a result, there are large Planck scale corrections
at the loop level, leading to the gauge hierarchy problem. As a remedy, a large number of supersymmetric standard-like models and grand unified models [31–49] have been constructed, with the gauge hierarchy problem solved. We refer to [50] for a comprehensive review for such kind of non-supersymmetric and supersymmetric models.

Among these supersymmetric models, Pati-Salam models has been a prominent road to the Standard Model, without adding any extra U(1) symmetry around the electroweak level. In Refs. [40], Cvetič, Liu and one of us (TL) showed how to systematically construct \( \mathcal{N} = 1 \) supersymmetric Pati-Salam models from intersecting \( D6 \)-branes on Type IIA \( \mathbb{T}^6/\mathbb{Z}_2 \times \mathbb{Z}_2 \)-orientifold. After \( D \)-brane splitting and supersymmetric preserving Higgs mechanism applied, the Pati-Salam gauge symmetry \( SU(4)_C \times SU(2)_L \times SU(2)_R \) eventually breaks down to the SM gauge symmetry. Due to the supersymmetry breaking and moduli stabilization triggered by their two confining groups in the hidden sectors, these models do have realistic and phenomenological consequences, as shown in Refs. [43, 51, 52]. Until very recent, intriguingly all possible 202752 three-family \( \mathcal{N} = 1 \) supersymmetric Pati-Salam models on \( \mathbb{T}^6/\mathbb{Z}_2 \times \mathbb{Z}_2 \) have been found and classified with 33 type of independent models according to gauge coupling relations [1].

Most of these above achievements are based on three-family model buildings. In fact, the study of the SM with four-families is also worthy of attention due to flavor democracy hypothesis. In [53], a democratic mass matrix model was introduced to mainly fix the mass gap problem between three families of fermions of the SM, as well as the hierarchy problem of the Yukawa couplings. Allowing three families of fermions, one gets typical SM predictions, such as a low mass of the top quark and the inequality between three neutrino masses [54]. If one allows four families of fermions in the democratic mass matrix model, three families of precisely massless neutrinos and a massive neutrino can be realized without the assumption on a larger hierarchy of the Yukawa couplings. Moreover, via a slight breaking of democracy, the three massless neutrinos obtain small masses. Then the flavor problem of the SM, can be solved naturally by putting the flavor democracy hypothesis due to the important role democratic mass matrix model plays.

As a matter of fact, the SM does not make a theoretical prediction on the number of families. The only restriction for the number of SM families comes from the requirement made by the Quantum Chromodynamics (QCD). The asymptotic freedom of QCD provide an upper bound 8 for the number of families as discussed in [55]. And if we allow four families of fermions for SM, many open issues of SM can be solved in a natural way. For example, introducing a fourth massive generation to SM can alter the cross section and decay channels of the Higgs particles [56]. When the Yukawa couplings of the fourth generation particles are large enough, these particles are natural candidates for electroweak symmetry breaking [57]. Motivated by constructing realistic four-family SM, in this paper, we concentrate on building four-family \( \mathcal{N} = 1 \) supersymmetric models with gauge symmetry \( SU(4)_C \times SU(2)_L \times SU(2)_R \) on \( \mathbb{T}^6/\mathbb{Z}_2 \times \mathbb{Z}_2 \) orientifold with intersecting \( D6 \)-branes as an extension to [1, 40]. We obtain four-family Pati-Salam models with gauge coupling unification at string scale or near string scale. In particular, there are models without any filler brane required as well.

The paper is organized as follows. In Section 2 we will briefly review the basics of
model building from intersecting $D6$-branes on $T^6/\mathbb{Z}_2 \times \mathbb{Z}_2$ orientifold. Constraints on $D6$-brane configuration, such as RR tadpole cancellation condition and supersymmetric condition are also reviewed. In Section 3, we present the symmetry breaking mechanism for $U(4)_C \times U(2)_L \times U(2)_R$, i.e., $D6$-brane splitting and the Higgs mechanism. Four-family of chiral fermion condition and various symmetry relations, such as T-dualities are also discussed. Section 4 is devoted to the phenomenological features of the four-family models. For each class of model, we list its chiral spectrum for the open string sector. In Section 5, we draw conclusions and briefly discuss the limitations of our work. The four-family Pati-Salam models are presented in the Appendix.

2 Basics of $T^6/(\mathbb{Z}_2 \times \mathbb{Z}_2)$-Orientifolds Model Construction

To construct realistic and four-family supersymmetric models, we recall the basics of model construction from Type IIA string theory compactified on $T^6/(\mathbb{Z}_2 \times \mathbb{Z}_2)$-orientifold, with $D6$-branes intersecting at general tilted angles, under similar settings as in [31] and [33].

We begin with the orientifold $T^6/(\mathbb{Z}_2 \times \mathbb{Z}_2)$, where $D6$-branes can be naturally viewed as general 3-cycles. Using the canonical isomorphism $T^6 \simeq \mathbb{T}^2 \times \mathbb{T}^2 \times \mathbb{T}^2$, we can easily write down the coordinate chart of the orientifold. Let $z_i, i = 1, 2, 3$ be the complex coordinates of the $i$th torus, respectively. Let $\theta$ and $\omega$ be the two generators of the abelian group $\mathbb{Z}_2 \times \mathbb{Z}_2$. In coordinate $(z_1, z_2, z_3)$, we define orientifold actions $\theta$ and $\omega$ of $\mathbb{Z}_2 \times \mathbb{Z}_2$ on $T^6$ by

$$\theta(z_1, z_2, z_3) := (-z_1, -z_2, z_3),$$

$$\omega(z_1, z_2, z_3) := (z_1, -z_2, -z_3).$$

In addition, we define actions $\Omega$ and $R$ of $\mathbb{Z}_2 \times \mathbb{Z}_2$ on $T^6$, by $\Omega$ the parity-reversion on the world-sheet, and $R$ the complex conjugate of $T^6$ as a complex manifold. By the very definition of an orientifold, where the world-sheet parity and complex conjugate make no difference, there are exactly four components of the $T^6/(\mathbb{Z}_2 \times \mathbb{Z}_2)$-orientifold, namely the image of $T^6$ under the action of $\Omega R$, $\Omega R \theta$, $\Omega R \omega$ and $\Omega R \theta \omega$. These components are objects bearing RR charges, and thus $D6$-branes are introduced to cancel their RR charges.

Generally, $Dp$-branes are $(p + 1)$-dimensional objects in the spacetime, where strings start from and land on. In our case, viewed from the internal space $T^6/\mathbb{Z}_2 \times \mathbb{Z}_2$, the $D6$-branes are 3-dimensional objects. It is sufficient to identify $D6$-branes and 3-cycles for physical considerations. The powerful Eilenberg-Zilber theorem tells us that

$$H_3(T^6; \mathbb{Z}) \simeq H_3(T^2 \times T^2 \times T^2; \mathbb{Z}) \simeq H_1(T^2; \mathbb{Z}) \times H_1(T^2; \mathbb{Z}) \times H_1(T^2; \mathbb{Z}) \simeq \mathbb{Z}^2 \times \mathbb{Z}^2 \times \mathbb{Z}^2.$$

In the following discussion, we denote $[a_i], [b_i], i = 1, 2, 3$ as generators of $H_3(T^2; \mathbb{Z})$ for the $i$th torus respectively. Under the orientifold action $\mathbb{Z}_2 \times \mathbb{Z}_2$ and taking world-sheet into consideration, we find that there are only two patterns for the lattice $H_1(T^2; \mathbb{Z}) \simeq \mathbb{Z}^2$: the rectangular one and the tilted one [11, 33, 35, 45]. In the basis $[a_i], [b_i], i = 1, 2, 3$, the former is generated by $[a_i], [b_i]$ over $\mathbb{Z}$, while the latter is generated by $[a_i], [b_i]$ over $\mathbb{Z}$ with
Let $\tilde{a}_i := \frac{1}{2}[a_i + b_i]$. Under the basis $[a_1], [b_1], [a_2], [b_2], [a_3], [b_3]$, we can represent a 3-cycle $[\Pi_a]$ of the orientifold $T^6/(\mathbb{Z}_2 \times \mathbb{Z}_2)$ by the coordinate $(n_{a_1}^1, 2^{-\beta_1}l_{a_1}^1) \times (n_{a_2}^2, 2^{-\beta_2}l_{a_2}^2) \times (n_{a_3}^3, 2^{-\beta_3}l_{a_3}^3)$, where $n_{a_i}^i, l_{a_i}^i$ are all integers, $\beta_i$ takes 0 when the $i$th torus is rectangular and takes value 1 for the tilted case. Under the action of $\Omega R$, a $D6$-brane $[\Pi_a]$ becomes $[\Pi_a] = (n_{a_1}^1, -l_{a_1}^1) \times (n_{a_2}^2, -l_{a_2}^2) \times (n_{a_3}^3, -l_{a_3}^3)$, with $[\Pi_a]$ short-handed notation for the image of $[\Pi_a]$. To sum up, a $D6$-brane $[\Pi_a]$ and its orientifold image $[\Pi_{\omega}]$ take the form

$$
\begin{align}
[\Pi_a] &= (n_{a_1}^1, 2^{-\beta_1}l_{a_1}^1) \times (n_{a_2}^2, 2^{-\beta_2}l_{a_2}^2) \times (n_{a_3}^3, 2^{-\beta_3}l_{a_3}^3)
[\Pi_{\omega}] &= (n_{a_1}^1, -2^{-\beta_1}l_{a_1}^1) \times (n_{a_2}^2, -2^{-\beta_2}l_{a_2}^2) \times (n_{a_3}^3, -2^{-\beta_3}l_{a_3}^3).
\end{align}
$$

Denote $[\Pi_{\Omega R}] = 2^3(1,0) \times (1,0) \times (1,0)$, then we have

$$
\begin{align}
[\Pi_{\Omega R}] &= 2^3(1,0) \times (1,0) \times (1,0),
[\Pi_{\Omega R,\omega}] &= -2^{-3-\beta_2-\beta_3}(1,0) \times (0,1) \times (0,1),
[\Pi_{\Omega R,\theta}] &= -2^{-3-\beta_1-\beta_3}(1,0) \times (0,1) \times (0,1),
[\Pi_{\Omega R,\theta\omega}] &= -2^{-3-\beta_1-\beta_2}(1,0) \times (0,1) \times (0,1),
\end{align}
$$

where $[\Pi_{\Omega R,\omega}], [\Pi_{\Omega R,\theta}], [\Pi_{\Omega R,\theta\omega}]$ are the images of $[\Pi_{\omega}]$ under the action of $\omega, \theta$ and $\theta \omega$ respectively. The coefficient $2^3$ comes from identifying 8 $O6$-branes $(\pm 1, 0) \times (\pm 1, 0)$, under the quotient of $\Omega$ and $R$ actions. The intersection number of $D6$-branes can be easily computed as

$$
\begin{align}
I_{ab} &= [\Pi_a][\Pi_b] = 2^{-k}(n_{a_1}^1 l_{b_1}^1 - n_{b_1}^1 l_{a_1}^1)(n_{a_2}^2 l_{b_2}^2 - n_{b_2}^2 l_{a_2}^2)(n_{a_3}^3 l_{b_3}^3 - n_{b_3}^3 l_{a_3}^3),
I_{ab'} &= [\Pi_a][\Pi_{b'}] = -2^{-k}(n_{a_1}^1 l_{b_1}^1 + n_{b_1}^1 l_{a_1}^1)(n_{a_2}^2 l_{b_2}^2 + n_{b_2}^2 l_{a_2}^2)(n_{a_3}^3 l_{b_3}^3 + n_{b_3}^3 l_{a_3}^3),
I_{aa'} &= [\Pi_a][\Pi_{a'}] = -2^{-3-k}(n_{a_1}^1 n_{a_2}^2 n_{a_3}^3 l_{a_1}^1 l_{a_2}^2 l_{a_3}^3),
\end{align}
$$

where $k = \beta_1 + \beta_2 + \beta_3$. If we denote $[\Pi_{O6}] = [\Pi_{\Omega R}] + [\Pi_{\Omega R,\omega}] + [\Pi_{\Omega R,\theta}] + [\Pi_{\Omega R,\theta\omega}]$, we have

$$
\begin{align}
I_{aO6} &= [\Pi_a][\Pi_{O6}] = 2^{-3-k}(-l_{a_1}^1 l_{a_2}^1 l_{a_3}^1 + l_{a_1}^1 n_{a_2}^2 n_{a_3}^3 + l_{a_1}^1 n_{a_2}^1 n_{a_3}^3 + l_{a_1}^1 n_{a_2}^1 n_{a_3}^1).
\end{align}
$$

The spectrum of intersecting $D6$-branes is given in Table 1.

| Sectors | Representations |
|---------|----------------|
| $aa$    | $U(N_a/2)$ vector multiplets |
| $ab + ba$ | 3 adjoint chiral multiplets |
| $ab' + b'a$ | $I_{ab}$ ($\square_a, \square_b$) fermions |
| $aa' + a'a$ | $I_{aa'}$ ($\square_a, \square_a$) fermions |

For three-family $\mathcal{N} = 1$ supersymmetric Pati-Salam model building in Type IIA orientifolds on $T^6/(\mathbb{Z}_2 \times \mathbb{Z}_2)$ with intersecting $D6$-branes in which the $SU(4)_C \times SU(2)_L \times SU(2)_R$
gauge symmetries arise from $U(n)$ branes. To get four families of fermions, we require

$$I_{ab} + I_{ab'} = 4,$$
$$I_{ac} = -4, \quad I_{ac'} = 0.$$  \hfill (2.6)

The conditions $I_{ab} + I_{ab'} = 4$ and $I_{ac} = -4$ ensure that there are four families of SM fermions, and the condition $I_{ac'} = 0$ means that stack $a$ of D6-branes are parallel to the orientifold image of stack $c$ of D6-branes. Thus there should be open strings stretching between those two stack of D6-branes. The light scalar from the NS scalar will obtain mass $Z_{ac'}/4\pi\alpha'$, with $Z_{ac'}$ the minimal squared length of the stretching string. Similarly, the light fermions from the R sector acquire the same masses \[13, 14, 39\]. These light scalars and fermions form the Higgs fields needed to break the Pati-Salam gauge symmetry to the SM gauge symmetry.

In addition, there are two main constraints for D6-brane configurations and O6-plane configurations, namely the **RR Tadpole Cancellation Condition** and **Supersymmetry Condition**. As the sources of RR charges, D6-branes and O6-planes should satisfy the Gauss’ law, for the flux of RR fields through the compact space $\mathbb{T}^6/(\mathbb{Z}_2 \times \mathbb{Z}_2)$ without boundary should be conserved. This form of Gauss’ law is the so-called RR tadpole cancellation condition. To satisfy this condition, stacks of $N_a, a = 1, 2, 3$ D6-branes are often needed to be introduced as the so-called filler brane, with $a$ running through three families of gauge groups. Then the RR tadpole cancellation condition reads

$$3 \sum_{a=1}^3 N_a[\Pi_a] + 3 \sum_{a=1}^3 N_a[\Pi_a'] - 4[\Pi_{O6}] = 0,$$  \hfill (2.7)

and the coefficient 4 before $[\Pi_{O6}]$ comes from the $-4$ RR charges in the D6-brane charge unit. Other than O6-planes in Equation (2.7), we can also introduce D6-branes between the four O6-planes to cancel out the total RR charges, and rewrite Equation (2.7) free of O6-planes. To do this, we note

$$A_a = -n_a^1 n_a^2 n_a^3, \quad B_a = n_a^1 l_a^1 l_a^2 l_a^3, \quad C_a = l_a^1 n_a^2 n_a^3, \quad D_a = l_a^1 l_a^2 n_a^3;$$
$$\tilde{A}_a = -l_a^1 l_a^2 n_a^3, \quad \tilde{B}_a = l_a^1 n_a^2 l_a^3, \quad \tilde{C}_a = l_a^1 n_a^2 n_a^3, \quad \tilde{D}_a = n_a^1 n_a^2 l_a^3.$$  \hfill (2.8)

Therefore the tadpole cancellation conditions 2.7 can be expressed in the form

$$-2^k N^{(1)} + 3 \sum_{a=1}^3 N_a A_a = -16,$$
$$-2^k N^{(2)} + 3 \sum_{a=1}^3 N_a B_a = -16,$$
$$-2^k N^{(3)} + 3 \sum_{a=1}^3 N_a C_a = -16,$$
$$-2^k N^{(4)} + 3 \sum_{a=1}^3 N_a D_a = -16,$$  \hfill (2.9)

- 5 -
where $2N^{(i)}, i = 1, 2, 3, 4$ are the number of filler branes between the four $O6$-branes, as shown in Table 2.

Furthermore, the four-dimensional $\mathcal{N} = 1$ supersymmetric conditions require 1/4 supercharges conserved under (i) orientation reversion of $D6$-branes; and (ii) group action of $\mathbb{Z}_2 \times \mathbb{Z}_2$. As shown in [9], the four-dimensional $\mathcal{N} = 1$ supersymmetry is preserved under the orientation reversion if and only if rotations of $D6$-branes with respect to $O6$-planes are elements of SU(3), while their total rotation angles equal to 0. When the four-dimensional $\mathcal{N} = 1$ supersymmetry is preserved under orientation reversion, it will be preserved under the $\mathbb{Z}_2 \times \mathbb{Z}_2$-action manifestly. The supersymmetric condition can be written as [33]

$$x_A \tilde{A}_a + x_B \tilde{B}_a + x_C \tilde{C}_a + x_D \tilde{D}_a = 0,$$

$$A_a/x_A + B_a/x_B + C_a/x_C + D_a/x_D < 0, \quad a = 1, 2, 3, 4$$

(2.10)

with $x_A = \lambda, x_B = \lambda^2 \beta_2 + \beta_1/\chi_2 \chi_3, x_D = \lambda^2 \chi_1 \chi_2$. The positive parameter $\lambda$ is introduced to put Equation (2.10) on an equal scaling, and $\chi_i = R_i^2/R_i^1, i = 1, 2, 3$ is the complex structure modulus parameter for the $i$th torus respectively.

### 3 Gauge Symmetry Breaking via Brane Splittings

To obtain SM or standard-like models via the mechanism of intersecting $D6$-branes, there should be at least two extra $U(1)$ gauge symmetries for either supersymmetric models or non-supersymmetric models, as a result of the constraints on the quantum number of the right handed electron [14, 31–33]. Among these two $U(1)$ gauge symmetries, one is lepton number symmetry $U(1)_L$ and another $U(1)_{I3R}$ is an analogy for right-hand weak isospin. We have the hypercharge $Q_Y$ expressed in the form

$$Q_Y = Q_{I3R} + \frac{Q_B - Q_L}{2}.$$  

(3.1)

The baryonic charge $Q_B$ arises from $U(1)_B$, via the decomposition $U(3)_C \simeq SU(3)_C \times U(1)_B$. On the other hand, since the $U(1)_{I3R}$ gauge field should be massless, the gauge group $U(1)_{I3R}$ must come from the non-abelian component of $U(2)_R$ or USp symmetry, otherwise the $U(1)_{I3R}$ will acquire mass from the $B \wedge F$ couplings. To get an anomaly free $U(1)_{B-L}$, the $U(1)_L$ symmetry should come from some non-abelian group for similar reasons. In previous studies of supersymmetric model building [31, 32], $U(1)_{I3R}$ comes from USp groups. These models indeed have two extra anomaly-free $U(1)$ symmetries, and have at least 8 Higgs doublets. One could in principal break their symmetry groups down to the SM.
symmetry, but cannot do this without violating the D-flatness and F-flatness, thus the supersymmetry.

In this paper, as introduced in [52], we study a generalized version of the four-family MSSM models. In these models the b or c stacks are with twice the numbers of D6-branes. Then the gauge symmetries of these generalized four-family models can be broken to the standard four-family gauge symmetries $SU(4)_C \times SU(2)_L \times SU(2)_R$ via the Higgs mechanism. Taking gauge symmetries $U(4) \times U(4)_L \times U(2)_R$ as example, we consider a U(4) gauge theory with a scalar field in the adjoint representation. By choosing appropriate rotations commuting with the generators of the Lie algebra of SU(4), one can break U(4) to U(2) $\times$ U(2), and finally to U(2). We choose the rotation for U(4) scalar field as

$$\Phi_1 = \begin{pmatrix} V_1 & 0 & 0 & 0 \\ 0 & V_1 & 0 & 0 \\ 0 & 0 & -V_1 & 0 \\ 0 & 0 & 0 & -V_1 \end{pmatrix}. \quad (3.2)$$

We find that the U(4) gauge symmetry breaks spontaneously to U(2) $\times$ U(2), as the matrix in (3.2) lies in the center of the Lie algebra of U(4). What left to us is to break down U(2) $\times$ U(2) to U(2). For each U(2) component in U(2) $\times$ U(2), the generators of its Lie algebra are the standard Pauli matrices. If we choose the rotation matrix for U(2) $\times$ U(2) as

$$\Phi_2 = \begin{pmatrix} 0 & 0 & V_2 & 0 \\ 0 & 0 & 0 & V_2 \\ V_2 & 0 & 0 & 0 \\ 0 & V_2 & 0 & 0 \end{pmatrix}, \quad (3.3)$$

we find that U(2) $\times$ U(2) breaks down to U(2). We also note that a mass of

$$m^2 = 8g^2 V_2^2 \quad (3.4)$$

is acquired through the above process. For models with gauge symmetry U(4) $\times$ U(2)$_L \times$ U(4)$_R$ and U(4) $\times$ U(4)$_L \times$ U(4)$_R$, one can similarly breaks the symmetry down to U(4) $\times$ U(2)$_L \times$ U(2)$_R$ following the above procedure. The anomalies of the overall U(1) symmetries are canceled by the generalized Green-Schwarz mechanism [13, 14, 31], while their fields get massive from the linear $B \wedge F$ couplings.

The gauge symmetry SU(4)$_C \times$ SU(2)$_L \times$ SU(2)$_R$ can be further broken down to SM gauge symmetry by D6-brane splitting and Higgs mechanism. Firstly, one can split stack a of $N_a = 8$ D6-branes into stack $a_1$ of $N_{a_1} = 6$ D6-branes and stack $a_2, N_{a_2} = 2$ D6-branes. Then the U(4)$_C$ symmetry breaks down to U(3) $\times$ U(1). We denote the numbers of symmetric and anti-symmetric representations for SU(4)$_C$, SU(2)$_L$ and SU(2)$_R$ by $n^a$, $n^a_i$, $n^a_i^h$, $n^a_i^b$, $n^a_{i'}$, and $n^a_{i'}^h$. After splitting, the symmetric and anti-symmetric representations of SU(4)$_C$ descend to symmetric representations of SU(3)$_C$ and U(1)$_{B-L}$, and antisymmetric representations of SU(3)$_C$. Note that there are $I_{a_1,a_2}$ new fields, arising from the intersection of $a_1$ stack and $a_2$ stack of D6-branes. The anomaly-free gauge symmetry is SU(3)$_C \times$ U(1)$_{B-L}$, a subgroup of SU(4)$_C$. 


Similarly, the stack $c$ of $N_c = 4$ D6-branes can be broken into stack $c_1$ of $N_{a_1} = 2$ D6-branes and stack $c_2$ $N_{a_2} = 2$ D6-branes. Then the $U(2)_R$ symmetry breaks down to $U(1)_{I3R}$. The symmetric representations of $SU(2)_R$ descend to the symmetric representations of $U(1)_{I3R}$ only. Also, there are $I_{c_1c_2}$ new fields, arising from the intersection of $c_1$ stack and $c_2$ stack of D6-branes. The anomaly-free gauge symmetry is $U(1)_{I3R}$; a subgroup of $SU(2)_R$. After splitting, the gauge symmetry of our model breaks down to $SU(3)_C \times SU(2)_L \times U(1)_{B-L} \times U(1)_{I3R}$.

To get just the SM gauge symmetry, we assume the minimal squared distance $Z_{a_2c_1}^2$ between $a_2$ stack and the orientifold image of $c_1$ stack of D6-branes to be very small. Then there are $I_{a_2c_1}$ chiral multiplets of light fermions, arising from the open string stretching between $a_2$ stack of D6-branes and the orientifold image of $c_1$ stack of D6-branes. These particles break down $SU(3)_C \times SU(2)_L \times U(1)_{B-L} \times U(1)_{I3R}$ to the SM gauge symmetry, playing the same role as the right-handed neutrinos and their complex conjugates. Meanwhile, they preserve the D-flatness and F-flatness, thus the supersymmetry. In conclusion, the whole symmetry-breaking chain is

\[
SU(4)_C \times SU(2)_L \times SU(2)_R \xrightarrow{a \rightarrow a_1 + a_2} SU(3)_C \times SU(2)_L \times SU(2)_R \times U(1)_{B-L}
\]

\[
\xrightarrow{c \rightarrow c_1 + c_2} SU(3)_C \times SU(2)_L \times U(1)_{I3R} \times U(1)_{B-L} \xrightarrow{\text{Higgs Mechanism}} SU(3)_C \times SU(2)_L \times U(1)_{Y}.
\]

The process of dynamical supersymmetry breaking has been studied in [36] for D6-brane models from Type IIA orientifolds. The kinetic function for a stack $a$ of D6-branes is of the form[43]

\[
f_a = \frac{1}{4\kappa_a}(n_a^1 n_a^2 n_a^3 s - n_a^1 \beta_a^3 u_1^1 - \frac{l_a^1 n_a^2 l_a^3 u_2^2}{2\beta_2 + \beta_3} - \frac{l_a^1 l_a^2 n_a^3 u_3^3}{2\beta_1 + \beta_2}),
\]

where $\kappa_a$ is a constant with respect to the gauge groups, for instance $\kappa_2 = 1$ for $SU(N_2)$. We use moduli parameters $s$ and $u^i, i = 1, 2, 3$ in supergravity basis, which are related to four dimensional dilation parameter $\phi_4$ and complex structure moduli parameters $U^i, i = 1, 2, 3$ as following

\[
\text{Re}(s) = \frac{e^{-\phi_4}}{2\pi} \frac{\text{Im}(U^1) \text{Im}(U^2) \text{Im}(U^3)}{|U^1 U^2 U^3|},
\]

\[
\text{Re}(u^1) = \frac{e^{-\phi_4}}{2\pi} \sqrt{\frac{\text{Im}(U^1)}{\text{Im}(U^2) \text{Im}(U^3)}} \frac{U^2 U^3}{U^1},
\]

\[
\text{Re}(u^2) = \frac{e^{-\phi_4}}{2\pi} \sqrt{\frac{\text{Im}(U^2)}{\text{Im}(U^1) \text{Im}(U^3)}} \frac{U^1 U^3}{U^2},
\]

\[
\text{Re}(u^3) = \frac{e^{-\phi_4}}{2\pi} \sqrt{\frac{\text{Im}(U^3)}{\text{Im}(U^1) \text{Im}(U^2)}} \frac{U^1 U^2}{U^3}.
\]

In our present models, the $U^i, i = 1, 2, 3$ can be computed as in [45]

\[
U^1 = i \chi_1, \quad U^2 = i \chi_2, \quad U^3 = \frac{2 \chi_2^2 + 4 i \chi_3}{4 + \chi_3^2}.
\]
Moreover, the Kähler potential takes the form of

\[ K = -\ln(S + S') - \sum_{i=1}^{3} \ln(U_i + U'_i). \] (3.9)

Note that the three moduli parameter \( \chi_1, \chi_2, \chi_3 \) are not independent, as they can be expressed in terms of \( x_A, x_B, x_C, x_D \) and the latter parameters are related by the supersymmetric condition (2.10). Actually, one can determine \( \chi_1, \chi_2, \chi_3 \) up to an overall coefficient, namely an action of dilation on these parameters. So one has to stabilize this dilation to determine all the moduli parameters. Previous studies [58–60] employ mechanisms like gaugino condensation to stabilize this overall coefficient, dictating that there should be at least two USp groups in the hidden sectors. Moreover, the one-loop beta functions [40]

\[ \beta_i^g = -3(N^{(i)}_2 + 1) + 2|I_{ai}| + |I_{bi}| + |I_{ci}| + 3(N^{(i)}_2 - 1) \]

(3.10)

for each USp\((N^{(i)})\) arising from \(2N^{(i)}\) filler branes are required to be negative. However, in this paper, to include other potential mechanisms, we do not restrict ourselves only to models with at least two USp groups in the hidden sectors. The gauge coupling constant related to stack \( a \) of D6-branes is

\[ g_a^{-2} = |\text{Re}(f_a)|, \]

(3.11)

and the coupling constant of stack \( b \) and stack \( c \) of D6-branes are determined in the same way. The kinetic function for U(1)\(_Y\) is a linear combination of those for SU(4)\(_C\) and SU(2)\(_R\), as shown in [11, 43]

\[ f_Y = \frac{3}{5}(\frac{2}{3}f_a + f_c). \]

(3.12)

The coupling constant \( g_Y \) is determined by

\[ g_Y^{-2} = |\text{Re}(f_Y)|. \]

(3.13)

At tree-level, the gauge couplings have the relation

\[ g_a^2 = \alpha g_b^2 = \beta \frac{5}{3} g_Y^2 = \gamma (\pi e^{-\phi_1}), \]

(3.14)

where \( \alpha, \beta, \) and \( \gamma \) are ratios between the strong coupling and the weak coupling, and hypercharge coupling, respectively.

### 3.1 T-Duality and its Variations

In string theory, if two models are related by T-duality, these models are considered equivalent. By applying T-duality, one can tremendously simplify the process of searching inequivalent models. Before the discussion of T-duality, we first point out two obvious symmetries that relate equivalent models.

(i) Two models are equivalent, if they are related by a permutation of three \( \mathbb{T}^2 \); and
two $D6$-models are equivalent if their wrapping numbers on any two $\mathbb{T}^2$ are in opposite signs, while are the same on the third $\mathbb{T}^2$.

The above two symmetries are known as the $D6$-brane Sign Equivalent Principle (DSEP). Then, follow the convention of [40], we introduce Type I and Type II T-dualities. Type I duality transformation acts on arbitrary two $\mathbb{T}^2$, say the $j$th and $k$th $\mathbb{T}^2$. The wrapping numbers on these tori transform as follows

$$(n^i_a, l^i_a) \mapsto (-l^i_a, n^i_a), \quad (n^j_a, l^j_a) \mapsto (l^j_a, -n^j_a), \quad (n^k_a, l^k_a) \mapsto (l^k_a, n^k_a),$$

when Type I T-duality applies. Recall the definitions in Equation (2.8), Type I T-duality only makes an exchange between $(A_a, \tilde{A}^a), (B_a, \tilde{B}^a), (C_a, \tilde{C}^a), (D_a, \tilde{D}^a)$. Moreover, Type I duality transformation is often combined with the trivial two $\mathbb{T}^2$ exchange, and we call the combination an extended Type I T-duality.

As for Type II T-duality, it acts on all three different $\mathbb{T}^2$. For instance, if we pick the $i$th, $j$th and $k$th $\mathbb{T}^2$, the wrapping numbers on these tori transform as

$$(n^i_a, l^i_a) \mapsto (-n^i_a, l^i_a), \quad (n^j_a, l^j_a) \mapsto (l^j_a, n^j_a), \quad (n^k_a, l^k_a) \mapsto (l^k_a, n^k_a).$$

In [40], Type II T-duality often combines with the interchange between $b$ and $c$ stacks of $D6$-branes

$$b \leftrightarrow c$$

associated to SU(2)\(_L\) and SU(2)\(_R\) gauge groups.

If we composite Type I T-duality and DSEP, we will get a variation of Type II T-duality. Under this symmetry transformation, the wrapping numbers on all three $\mathbb{T}^2$ change as

$$(n^1_a, l^1_a) \mapsto (n^1_a, -l^1_a), \quad (n^2_a, l^2_a) \mapsto (n^2_a, -l^2_a), \quad (n^3_a, l^3_a) \mapsto (n^3_a, -l^3_a),$$

$$b \leftrightarrow c.$$

It’s easy to see that under variation of Type II T-duality, only the signs of $\tilde{A}_a, \tilde{B}_a, \tilde{C}_a, \tilde{D}_a$ in Equation (2.8) change.

However, it is worth mentioning that, the variation (3.18) of Type II T-duality is not an equivalence in our construction of four-family supersymmetric models, if the model is not invariant under SU(2)\(_L\) and SU(2)\(_R\) interchange. This observation makes sense at least phenomenologically. For a four-family supersymmetric model, one can obtain a new model by exchanging the $b$ and $c$ stacks of $D6$-branes associated to the SU(2)\(_L\) and SU(2)\(_R\) groups, as the quantum numbers for SU(2)\(_L\) and SU(2)\(_R\) in the particle spectrum will exchange, so will the gauge couplings for these two groups at the string level. We will present with examples of this inequivalence in the next section.
3.2 Supersymmetric 4-family Models

Employing the deterministic algorithm in [1], we do not restrict the number of USp groups, and consider two cases, one without any titled torus, and the other with the third torus to be tilted, without loss of generality. Note that four-family models are completely different from the three-family models, thus the argument in [40] to exclude which torus is tilted or not cannot be directly applied to our case, due to the even number of generations.

3.2.1 Models without Tilted Torus

We obtain six classes of 274 supersymmetric four-family models with the deterministic algorithm introduced in [1] with representative models presented in Appendix A. The classification is based on the gauge groups, T-equivalences and phenomenological considerations such as gauge coupling relations.

Model 14 is a class of its own, as it is the representative model without tilted torus achieving exact gauge coupling unification at the string scale. The Higgs-like particles in this model arise from the intersection at b and c′ stacks of D6-branes, while the Higgs doublets arise from the massless open string states in a N = 2 subsector and form vector-like pairs.

The second class of models has no USp group, which means the tadpole cancellation conditions are satisfied without any filler brane as a rare case. These models are represented by Model 15 and Model 16, which are independent models of T-equivalence. These models are first four-family examples having no confining USp groups to achieve approximate gauge coupling unification at the string scale.

The third class of models includes Model 17 and Model 18 with negative β function and positive β function respectively.

The fourth class of models includes Model 19-21 with two USp groups. These models are independent under T-duality.

As discussed in [59] there are at least two confining USp groups needed, with negative β function, and thus allow for gaugino condensations, these models would need alternative mechanisms to break the supersymmetry. Furthermore, we can observe from the spectrum tables that Model 15, 17 and 20 do not have the proper Higgs doublets with quantum number (1, 2, 2) under $U(4)_C \times U(2)_L \times U(2)_R$ gauge symmetry, from neither the b and c or c′ stacks of brane intersection nor the massless open string states in a N = 2 subsector. Thus, we do not have the SM fermion Yukawa couplings at renormalizable level which are invariant under the global $U(1)_C \times U(1)_L \times U(1)_R$ symmetry in these models.

The fifth class of models no less than three USp groups, and includes Models 19-25. Among these models, Model 24 and Model 25 are related by T-dualities with b and c stacks of branes swapping. To see this, we show how Model 24 and Model 25 are related. To begin with, a stacks of D6-branes in Model 24 and Model 25 are related by the DSEP:

$$(-1,0) \times (-1,1) \times (1,1) \xrightarrow{\text{DSEP (i)}} (-1,0) \times (1,1) \times (-1,1)$$

$$\xrightarrow{\text{DSEP (i)}} (1,0) \times (1,1) \times (1,-1).$$

And the b stack of D6-branes in Model 24 are related to the c stack of D6-branes in Model 25, by Type I T-duality (3.15), the DSEP, and the interchange (3.17) of b and c.
stacks:

\[(0, 1) \times (-1, 1) \times (-3, 1) \xrightarrow{\text{DSEP (i)}} (0, 1) \times (-3, 1) \times (-1, 1) \]

\[\xrightarrow{\text{Type I T-duality}} (0, 1) \times (-1, -3) \times (1, 1) \]

\[\xrightarrow{\text{DSEP (ii) and } b \leftrightarrow c} (0, 1) \times (1, 3) \times (-1, -1). \]

While the \(c\) stack of \(D6\)-branes in Model 24 and the \(b\) stack of \(D6\)-branes in Model 25 are related by Type II T-duality, DSEP and the \(b \leftrightarrow c\) exchange:

\[(1, 2) \times (-1, 0) \times (-1, 1) \xrightarrow{\text{Type II T-duality}} (-1, 2) \times (0, -1) \times (1, -1) \]

\[\xrightarrow{\text{DSEP (i)}} (-1, 2) \times (1, -1) \times (0, -1) \]

\[\xrightarrow{\text{DSEP (ii) and } b \leftrightarrow c} (-1, 2) \times (-1, 1) \times (0, 1). \]

Even though these two models are related by the generalized T-duality as above, they are not phenomenologically equivalent. Model 24 achieves \(U(4)\) and \(U(2)_R\) unification, while Model 25 has \(U(4)\) and \(U(2)_L\) unification due to \(b\) and \(c\) stacks swapping. The Higgs particles of Model 24 come from intersection of \(b\) and \(c'\) stacks of \(D6\)-branes. Since all the beta functions of models within this class are negative, we may break the supersymmetry and stabilize the moduli via gaugino condensations.

The sixth class of models are with large wrapping numbers 5, 6, 7, 8, 9, 10, 11, 13, 15, 17, represented by Models 26-35 which did not appear in the former search. Three-family models with large wrapping number 5 have been found in [51], but it’s the first time to find four-family models with wrapping numbers at this scale.

### 3.2.2 Models with One Tilted Torus

Employing the deterministic algorithm, we obtain in total 6 types of gauge coupling relations with represented models presented in Appendix B.

Model 36 is a class of its own, as it is the only type of models with one tilted torus achieving exact gauge coupling unification at the string scale. The Higgs particles in this model arise from the massless open string states in a \(N = 2\) subsector and form vector-like pairs because the \(b\) stack branes for these models are parallel to \(c\) stack brane images on the third two-tori. Since all the beta functions are negative in this model, we can break the supersymmetry and stabilize the moduli via gaugino condensations.

The second class of models includes Model 37 and Model 38, and has no USp group. These models are related by type II T-duality. More specifically, we will below show how they are related by T-dualities explicitly. It’s easy to find that the \(a\) stacks of both models are related by DSEP and Type II T-duality

\[(1, -1) \times (-1, 0) \times (-1, -1) \xrightarrow{\text{DSEP}} (-1, 0) \times (1, -1) \times (-1, -1) \]

\[\xrightarrow{\text{Type II T-duality}} (1, 0) \times (-1, 1) \times (-1, -1). \]
Note that the $b$ stack of wrapping numbers of Model 38 are obtained by applying the variation of Type II T-duality and DSEP on the $b$ stack of wrapping numbers of Model 37:

$$(0, 1) \times (-1, -2) \times (1, 1) \xrightarrow{\text{variation of Type II T-duality}} (-1, 2) \times (0, -1) \times (1, 1) \xrightarrow{\text{DSEP}} (-1, 2) \times (0, -1) \times (1, -1).$$

The $c$ stacks of Model 37 and Model 38 are related by applying DSEP twice:

$$(1, 0) \times (1, -2) \times (1, 1) \xrightarrow{\text{DSEP}} (1, -2) \times (0, 1) \times (1, 1) \xrightarrow{\text{DSEP}} (1, -2) \times (-1, 0) \times (-1, 1).$$

Models 39-42 with one USp group in the hidden sector are related by T-dualities in a similar way. The Higgs particles in Model 37 again come from the massless open string states in a $N = 2$ subsector and form vector-like pairs because the $b$ stack branes for these models are parallel to $c$ stack brane on the third two-tori. Since there are no USp groups in the hidden sectors, gaugino condensations do not work in this case. One needs to stabilize the modulus and break the supersymmetry via different mechanism. For Model 39, there are four Higgs doublets arising from $N = 2$ subsectors due to the parallel of $b$ stack branes and $c$ stack brane on the third two-tori. In addition, there are models 43 and 44 with distinct gauge coupling relations at string scale.

4 Phenomenological Analysis

4.1 Models without Tilted Torus

We begin with Model 14. The gauge group of Model 14 is $U(4) \times U(2)_L \times U(2)_R \times \text{USp}(2) \times \text{USp}(4)$. We tabulate the full spectrum of chiral particles of Model 14 in Table 3. Interestingly, both Model 15 and Model 16 do not have any USp group, and then they do not have any exotic particles charged under USp groups as well. We tabulate the full spectrum of chiral particles of Model 15 below as a representative for this class.

Model 17 and Model 18 are constructed with one USp group. We tabulate the full spectrum of chiral particles of Model 17 in Table 5 as a representative of this class of models.

Models 19-21 are built with two USp groups. The gauge groups for Model 19, Model 20 and Model 21 are $U(4) \times U(2)_L \times U(2)_R \times \text{USp}(2) \times \text{USp}(4)$, $U(4) \times U(2)_L \times U(2)_R \times \text{USp}(2) \times \text{USp}(4)$, $U(4) \times U(2)_L \times U(2)_R \times \text{USp}(2)^2 \times \text{USp}(4)$, respectively. We tabulate the full spectrum of chiral particles in Model 20 in Table 6.

Models 22-25 are built with at least three USp groups. Their gauge groups are $U(4) \times U(2)_L \times U(2)_R \times \text{USp}(2)^2 \times \text{USp}(4)$, $U(4) \times U(2)_L \times U(2)_R \times \text{USp}(2)^2 \times \text{USp}(4)$, $U(4) \times U(2)_L \times U(2)_R \times \text{USp}(2)^3 \times \text{USp}(4)$, and $U(4) \times U(2)_L \times U(2)_R \times \text{USp}(2)^3 \times \text{USp}(4)$, respectively. Note that Model 24 and Model 25 are related by T-duality, but are not phenomenologically equivalent. This can be easily seen from the fact that Model 24 has $U(4)$ and $U(2)_R$ gauge
Table 3. Spectrum of chiral particles of Model 14

| Model 14 | SU(4)_C × SU(2)_L × SU(2)_R | Q_{4C} | Q_{2L} | Q_{2R} | Q_{em} | B − L | Field |
|----------|-----------------------------|--------|--------|--------|--------|--------|-------|
| ab       | 8 × (4, 2, 1, 1)            | 1      | −1     | 0      | −1/2   | 1/2, 1/2 | Q_L, L_L |
| ac       | 4 × (4, 2, 1, 1)            | −1     | 0      | 1      | −1/2   | 1/2, 1/2 | Q_L, L_L |
| bc       | 8 × (1, 2, 1, 1)            | 0      | −1     | −1     | 0      | 0, 1    | H' |
| b1       | 4 × (1, 1, 2, 1)            | 0      | −1     | 0      | ±1/2   | 0       | 0     |
| a4       | 2 × (4, 1, 1, 1)            | 1      | 0      | 0      | ±1/2   | 1/2, 1/2 | Q_R, L_R |
| e4       | 2 × (1, 1, 1, 1)            | 0      | 0      | −1     | ±1/2   | 0       | 0     |
| 1        | 1 × (1, 1, 1, 1)            | 0      | 0      | 0      | ±1     | 0       | 0     |
| b'       | 2 × (1, 2, 1, 1)            | 0      | −1     | 1      | 0, 1   | 0       | H'_s, H'_d |
| b''      | 2 × (1, 2, 1, 1)            | 0      | 1      | −1     | 0, 1   | 0       | 0     |

Table 4. Spectrum of chiral particles of Model 15

| Model 15 | SU(4)_C × SU(2)_L × SU(2)_R | Q_{4C} | Q_{2L} | Q_{2R} | Q_{em} | B − L | Field |
|----------|-----------------------------|--------|--------|--------|--------|--------|-------|
| ab       | 8 × (4, 2, 1)               | 1      | −1     | 0      | −1/2   | 1/2    | Q_L, L_L |
| ac       | 4 × (4, 2, 1)               | −1     | 0      | 1      | −1/2   | 1/2    | Q_L, L_L |
| bc       | 16 × (1, 1, 1)              | 0      | −1     | 1      | 0      | 0, 1   | H' |
| b1       | 5 × (1, 3, 1)               | 0      | 2      | 0      | ±1     | 0      | 0     |
| a4       | 5 × (1, 1, 1)               | 0      | 2      | 0      | 0      | 0      | 0     |
| e4       | 7 × (1, 1, 1)               | 0      | 0      | −2     | 0      | ±1     | 0     |
| 1        | 9 × (1, 1, 1)               | 0      | 0      | 2      | 0      | ±1     | 0     |
| b'       | 8 × (1, 2, 2)               | 0      | −1     | −1     | 0      | 0, 1   | H' |
| b''      | 8 × (1, 2, 2)               | 0      | 1      | 1      | 0      | 0      | 0     |

Table 5. Spectrum of chiral particles of Model 17

| Model 17 | SU(4)_C × SU(2)_L × SU(2)_R × USp(2) | Q_{4C} | Q_{2L} | Q_{2R} | Q_{em} | B − L | Field |
|----------|-----------------------------|--------|--------|--------|--------|--------|-------|
| ab       | 6 × (4, 2, 1, 1)            | 1      | −1     | 0      | −1/2   | 1/2    | Q_L, L_L |
| ac       | 2 × (4, 2, 1, 1)            | −1     | 1      | 0      | −1/2   | 1/2    | Q_L, L_L |
| bc       | 8 × (1, 1, 1, 1)            | 0      | −1     | 1      | 0, 1   | 0      | H' |
| e2       | 4 × (1, 1, 1)               | 0      | 0      | 1      | ±1/2   | 0      | 0     |
| b1       | 3 × (1, 1, 1, 1)            | 0      | 2      | 0      | 0      | 0      | 0     |
| a4       | 3 × (1, 1, 1, 1)            | 0      | 2      | 0      | 0      | 0      | 0     |
| e4       | 7 × (1, 1, 1, 1)            | 0      | 0      | −2     | 0      | ±1     | 0     |
| 1        | 9 × (1, 1, 1, 1)            | 0      | 0      | 2      | 0      | 0      | 0     |
| b'       | 6 × (1, 2, 2)               | 0      | −1     | −1     | 0      | ±1     | 0     |
| b''      | 6 × (1, 2, 2)               | 0      | 1      | 1      | 0      | 0     | 0     |
coupling unification at the string scale, while Model 25 has U(4) and U(2)_L gauge coupling unification. We represent the full spectrum of chiral particles in Model 24 in Table 7.

The exotic particles charged by USp groups may form bound states and composite particles at some intermediate energy scale, as the strong coupling dynamics of the USp groups requires. The composite particles are consistent with anomaly cancellation conditions, as in the QCD case. These composite particles thus are charged only under the SM gauge symmetry [34]. There are essentially two kinds of neutral bound states. The first one comes from decomposing the rank 2 anti-symmetric representation of the USp groups into two fundamental representations, and then taking the pseudo inner product of the fundamental representations. The second one comes from the rank 2N anti-symmetric representation of USp(2N) for N ≥ 2. The first bound state is similar to a meson that is the inner product of

| Model 20 | SU(4)_C × SU(2)_L × SU(2)_R × USp(4)^2 | Q_{4C} | Q_{2L} | Q_{2R} | Q_{em} | B − L | Field |
|----------|----------------------------------------|--------|--------|--------|--------|--------|-------|
| ab       | 6 × (1, 2, 1, 1, 1)                     | 1      | −1     | 0      | −1     | −1     | Q_L, L_L |
| ab'      | 2 × (1, 1, 2, 1, 1)                     | −1     | −1     | 0      | −1     | −1     | Q_L, L_L |
| ac       | 4 × (1, 1, 2, 1, 1)                     | −1     | 0      | 1      | 1      | −1     | Q_R, L_R |
| bc       | 4 × (1, 1, 2, 1, 1)                     | 0      | −1     | 1      | 0      | ±1     | 0     |
| c2       | 2 × (1, 1, 2, 1, 1)                     | 0      | 0      | −1     | −1     | 0      | 0     |
| a4       | 3 × (1, 1, 2, 1, 1)                     | 0      | 1      | 0      | 1      | 0      | 0     |
| b4       | 4 × (1, 1, 2, 1, 1)                     | 0      | 0      | 1      | 0      | ±1     | 0     |
| c4       | 1 × (10, 1, 1, 1, 1)                    | 0      | 0      | 1      | 2      | 2      | H'    |
| q         | 1 × (1, 1, 1, 1)                        | 0      | 0      | ±1     | 0      | 0      | 0     |
| q         | 1 × (1, 1, 1, 1)                        | 0      | 0      | ±1     | 0      | 0      | 0     |
| q         | 5 × (1, 1, 1, 1)                        | 0      | 0      | 2      | ±1     | 0      | 0     |
| q         | 27 × (1, 1, 1, 1)                       | 0      | 0      | 2      | 0      | 0      | 0     |
| bc'      | 7 × (1, 2, 1, 1)                        | 0      | 1      | 1      | 0      | ±1     | 0     |
| b         | 27 × (1, 1, 1, 1)                       | 0      | 1      | 1      | 0      | ±1     | 0     |

| Model 24 | SU(4)_C × SU(2)_L × SU(2)_R × USp(2)^2 × USp(4) | Q_{4C} | Q_{2L} | Q_{2R} | Q_{em} | B − L | Field |
|----------|-----------------------------------------------|--------|--------|--------|--------|--------|-------|
| ab'      | 2 × (1, 2, 1, 1, 1)                           | 1      | −1     | 0      | −1     | −1     | Q_L, L_L |
| ac       | 4 × (1, 1, 2, 1, 1, 1)                        | −1     | 0      | 1      | 1      | −1     | Q_R, L_R |
| bc       | 2 × (1, 1, 2, 1, 1, 1)                        | 0      | 1      | −1     | 0      | ±1     | 0     |
| b        | 1 × (1, 1, 2, 1, 1, 1)                        | 0      | −1     | −1     | 0      | ±1     | 0     |
| a3       | 1 × (1, 1, 1, 1, 2, 1)                        | −1     | 0      | 0      | −1     | −1     | H'    |
| a4       | 2 × (1, 1, 1, 2, 1, 1)                        | 1      | 0      | 0      | −1     | −1     | H'    |
| b4       | 3 × (1, 1, 2, 1, 1, 1)                        | 1      | 0      | 0      | −1     | −1     | H'    |
| c2       | 2 × (1, 1, 2, 1, 2, 1)                        | 0      | 1      | 0      | −1     | −1     | H'    |
| c4       | 1 × (1, 1, 2, 1, 1, 1)                        | 0      | 0      | 1      | −1     | −1     | H'    |
| q         | 1 × (1, 1, 1, 1, 1)                           | 0      | 0      | 2      | ±1     | 0      | 0     |
| q         | 1 × (1, 1, 1, 1, 1)                           | 0      | 0      | 2      | ±1     | 0      | 0     |

Table 6. Spectrum of chiral particles of Model 20

Table 7. Spectrum of chiral particles of Model 24
a fundamental representation and an anti-fundamental representation of SU(3)C in QCD. The second bound state is a USp(2N) singlet, which is an analog to a baryon being a rank 3 anti-symmetric representation of SU(3)C. Models 14, 20 and 25 contain the second kind of bound states. Now we take Models 14, 20 and 25 as examples to show explicitly how bound states are formed.

The composite particle spectrum for Model 14 is listed in Table 8. The confining group is USp(4), with two charged intersection. The mixing of intersection a4 and c4 results in the chiral supermultiplets (4, 1, 2, 1, 1). Moreover, the confined particle spectrum for Model 20 in Table 9. The confining group is USp(4), and has three charged intersection. Besides self-confinement, it is also viable to form mixed-confinement between sections within the same confining group. The chiral supermultiplets (4, 2, 1, 1, 1), (1, 2, 2, 1, 1) and (4, 1, 2, 1, 1) are yielded by the mixed-confinement between intersections a4, b4 and c4.

For Model 25, the composite particle spectrum is given in Table 10. There are two confining groups USp(4) and USp(2), each with two charged intersections. The mixed-confinement between intersections b2, c2 yields the chiral supermultiplet (1, 2, 2, 1, 1, 1), while the mixed-confinement between intersections a4, c4 yields the chiral supermultiplet (4, 1, 2, 1, 1, 1). Note that when there is only one charged intersection, mixed-confinement will not be formed, thus only the tensor representations from self-confinement are left. Checking from the composite particle spectra, one finds that no new anomaly is introduced to the remaining gauge symmetry. Thus our models are free of anomalies. The above analysis for composite particles applies for all our models except Model 15 and Model 16 without any confining group.

| Table 8. The composite particle spectrum for Model 14 |
|-----------------------------------------------|
| Model 14 | Confining Force | Intersection | SU(4)C × SU(2)L × SU(2)R × USp(2) × USp(4) | Confined Particle Spectrum |
|-----------------------------------------------|
| | USp(4) | a4 | 2 × (4, 1, 1, 1, 1) | 3 × (6, 1, 1, 1, 1), 3 × (10, 1, 1, 1, 1), 4 × (4, 1, 2, 1, 1) |
| | USp(4) | b4 | 2 × (1, 1, 2, 1, 1) | 3 × (1, 1, 1, 1, 1), 3 × (1, 1, 3, 1, 1) |
| | USp(2) | c4 | 4 × (1, 2, 1, 1, 1) | 10 × (1, 1, 1, 1, 1), 10 × (1, 1, 3, 1, 1) |

| Table 9. The composite particle spectrum for Model 20 |
|-----------------------------------------------|
| Model 20 | Confining Force | Intersection | SU(4)C × SU(2)L × SU(2)R × USp(4)² | Confined Particle Spectrum |
|-----------------------------------------------|
| | USp(4) | a4 | 2 × (4, 1, 1, 1, 1) | 3 × (6, 1, 1, 1, 1), 3 × (10, 1, 1, 1, 1), 6 × (4, 2, 1, 1, 1) |
| | | b4 | 3 × (1, 2, 1, 1, 1) | 6 × (1, 1, 1, 1, 1), 12 × (1, 2, 2, 1, 1) |
| | | c4 | 4 × (1, 1, 2, 1, 1) | 10 × (1, 1, 1, 1, 1), 10 × (1, 1, 3, 1, 1), 8 × (4, 1, 2, 1, 1) |
| | USp(4) | c2 | 1 × (1, 1, 2, 4, 1) | 1 × (1, 1, 3, 1, 1), 1 × (1, 1, 1, 1, 1) |
4.2 Models with One Tilted Torus

In this section, we show basic phenomenological properties of models with one tilted torus. Model 36 represents the models with exact gauge coupling unification at the string level so far. The gauge symmetries therein are $U(4) \times U(2)_L \times U(2)_R \times USp(2)^2$ with two confining groups in the hidden sector. The full spectrum of this model is shown in Table 3.

| Model 36 | SU(4)$_C \times SU(2)_L \times SU(2)_R \times USp(2)^2 \times USp(2)$ | $Q_{AC}$ | $Q_{2L}$ | $Q_{2R}$ | $Q_{em}$ | $B - L$ | Field |
|----------|------------------------------------------------|--------|--------|--------|--------|--------|------|
| $ab'$    | $4 \times (1,2,1,1,1)$                         | $-1$   | $-1$   | $-1$   | $-\frac{1}{2}$ | $-\frac{1}{2}$ | $0$   | $H_{u}, H_{d}$ |
| $ac'$    | $4 \times (1,1,1,1,1)$                         | $0$    | $0$    | $0$    | $0$    | $0$    | $0$   | $H_{u}, H_{d}$ |
| $b$      | $4 \times (1,2,1,1,1)$                         | $-2$   | $0$    | $0$    | $0$    | $0$    | $0$   | $H_{u}, H_{d}$ |
| $c$      | $4 \times (1,1,1,1,1)$                         | $-2$   | $0$    | $0$    | $0$    | $0$    | $0$   | $H_{u}, H_{d}$ |
| $bc$     | $8 \times (1,1,1,1,1)$                         | $-2$   | $0$    | $0$    | $0$    | $0$    | $0$   | $H_{u}, H_{d}$ |

Model 37 and Model 38 have no USp group. The gauge group for the two models is $U(4)_C \times U(2)_L \times U(2)_R$. Since there is no USp group, the gaugino condensation mechanism will not work. Thus, one need to find other mechanism for supersymmetry breaking. Also, there are no exotic particles in these two models, as exotic particles are charged under USp groups. We show the full chiral spectrum in the open string sectors for Model 37 in Table 12.

| Model 37 | SU(4)$_C \times SU(4)_L \times SU(4)_R$ | $Q_{AC}$ | $Q_{2L}$ | $Q_{2R}$ | $Q_{em}$ | $B - L$ | Field |
|----------|----------------------------------------|--------|--------|--------|--------|--------|------|
| $ab'$    | $4 \times (1,1,1,1,1)$                  | $-1$   | $-1$   | $-1$   | $-\frac{1}{2}$ | $-\frac{1}{2}$ | $0$   | $H_{u}, H_{d}$ |
| $ac'$    | $4 \times (1,1,1,1,1)$                  | $0$    | $0$    | $0$    | $0$    | $0$    | $0$   | $H_{u}, H_{d}$ |
| $b$      | $4 \times (1,1,1,1,1)$                  | $0$    | $0$    | $0$    | $0$    | $0$    | $0$   | $H_{u}, H_{d}$ |
| $c$      | $4 \times (1,1,1,1,1)$                  | $0$    | $0$    | $0$    | $0$    | $0$    | $0$   | $H_{u}, H_{d}$ |
| $bc$     | $4 \times (1,1,1,1,1)$                  | $0$    | $0$    | $0$    | $0$    | $0$    | $0$   | $H_{u}, H_{d}$ |
Models 39, 40, 41, 42 have only one USp(2) group in the hidden sector. Their gauge symmetry is all $U(4) \times U(4)_L \times U(2)_R \times USp(2)$. We note that Model 39 and Model 40 are T-dual to each other, as well as Model 41 and Model 42. But they are clearly not equivalent models at phenomenological level due to b and c stacks of brane swapping. In Model 39, we have SU(3)$_C$ and U(1)$_Y$ gauge coupling unification at the string level, while in Model 40 this gauge unification get swapped to SU(3)$_C$ and SU(2)$_L$ gauge coupling unification at the string level. Similarly, this b and c stacks of brane swapping appear between Model 41 and Model 42 as well. The whole spectrum of chiral particles of Model 39 in Table 13.

| Model 39 | SU(4)$_C$ × SU(4)$_L$ × SU(2)$_R$ × USp(2) | $Q_{ac}$ | $Q_{4L}$ | $Q_{2R}$ | $Q_{em}$ | $B - L$ | Field |
|---------|---------------------------------|--------|--------|--------|--------|---------|-------|
| ab      | 4 × (4, 4, 1, 1)                 | 1      | -1     | 0      | $\frac{1}{3}$, $\frac{2}{3}$, -1, 0 | $\frac{1}{3}$, $\frac{2}{3}$, -1, 0 | $Q_L$, $L_L$ |
| ac      | 4 × (2, 1, 1, 2)                 | -1     | 0      | 1      | $\frac{1}{2}$, $\frac{2}{2}$, -1, 0 | $\frac{1}{2}$, $\frac{2}{2}$, -1, 0 | $Q_R$, $L_R$ |
| bc$'$   | 4 × (4, 2, 1, 1)                 | 0      | 1      | 1      | 0      | 0      | $H'$   |
| b'i     | 4 × (1, 2, 1, 3)                 | 0      | 0      | 2      | 0      | $\pm \frac{1}{2}$ | 0      |
| b       | 2 × (1, 10, 1, 1)                | 0      | 2      | 0      | $\frac{1}{3}$, $\frac{2}{3}$, -1 | $\frac{1}{3}$, $\frac{2}{3}$, -1 | 0      |
| a       | 2 × (1, 1, 2, 1)                 | -2     | 0      | -1     | -$\frac{1}{3}$, $\frac{2}{3}$ | -$\frac{1}{3}$, $\frac{2}{3}$ | 0      |
| a       | 3 × (1, 1, 3, 1)                 | 0      | 0      | 2      | 0      | $\pm 1$ | 0      |
| a       | 3 × (1, 1, 1, 1)                 | 0      | 0      | 2      | 0      | 0      | 0      |
| bc      | 6 × (1, 2, 2, 1)                 | 0      | 1      | -1     | 0      | $\pm 1$ | 0      |
| bc      | 6 × (1, 2, 2, 1)                 | 0      | -1     | 1      | 0      | $H_L$, $H_d$ |

For the models with one tilted torus, the models represented by Model 36 are the only class with gauge coupling unification and carry two confining USp groups. Thus gaugino condensation can trigger supersymmetry breaking and moduli stabilization [36].

5 Discussions and Conclusions

Utilizing the deterministic algorithm, we obtain various classes of four-family supersymmetric models from intersecting $D6$-branes on $T^6/Z_2 \times Z_2$ orientifold, with and without tilted torus. In total, there are 274 physical independent four-family supersymmetric models without tilted torus, and 6 physical independent four-family supersymmetric models with the third torus to be the tilted one, without loss of generality.

For models without tilted torus, Model 14 represents the model with gauge coupling unification at the string scale. Models 15 and 16 are the rare models without any USp group in the hidden sectors, with tadpole cancellation conditions satisfied. Models 22-26 are with at least two confining USp groups. Thus gaugino condensation can be triggered to break the supersymmetry and stabilize the moduli. Moreover, there are Models 27-35 with wrapping numbers absolutely larger than 5 which was not reached for three family supersymmetric Pati-Salam models as discussed in [1].

For models with one tilted torus, Models 37 and 38 satisfy the tadpole cancellation conditions without any filler branes. Models such as Models 39 and 40 are related with b and c stacks of branes swapping. Model 36 represents the models with exact gauge coupling unification at the string scale, with two confining USp groups allowing gaugino condensation as well. This class of models would be ideal for further phenomenology model buildings (such...
as in [61]) as gaugino condensation can be triggered to break the supersymmetry and stabilize the moduli, while gauge coupling unified at string scale.

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A Four-Family Standard Models from Intersecting D6-Branes without Tilted Tori

In the appendix, we list all representative four-family models obtained from our random scanning method. In the first columns for each table, $a, b, c$ represent three stacks of D6-branes, respectively. Also in the first columns, 1, 2, 3, 4 is a short-handed notation for the filler branes along the $ΩR, ΩRω, ΩRθω$ and $ΩRθ$ 06-planes, respectively. The second columns for each table list the numbers of D6-branes in every stack, respectively. In the third columns of each table, we record wrapping numbers of each D6-brane configuration, and designate the third $T^2$ to be tilted.

The rest columns of each table record intersection numbers between stacks. For instance, in the $b$ column of Table 37, from top to bottom, the numbers represent intersection numbers $I_{ab}, I_{bb}, I_{cb}$, etc. As usual, $b'$ and $c'$ are the orientifold $ΩR$ image of $b$ and $c$ stacks of D6-branes. We also list the relation between $x_A, x_B, x_C, x_D$, which are determined by the supersymmetry condition Equation (2.10), as well as the relation between the moduli parameter $χ_1, χ_2, χ_3$. The one loop $β$ functions $β^p_0$ are also listed. To have a clearer sight of gauge couplings, we list them up in the caption of each table, which makes it easier to check whether they are unified.

Table 14. D6-brane configurations and intersection numbers of Model 14, and its gauge coupling relation is $g_a^2 = g_b^2 = g_c^2 = (\frac{7}{3}g_Y^2) = \frac{4}{3} \sqrt{2} \pi e^{α_4}$.

| stack | $N$ | $t^i \times (n^i, t^j) \times (n^i, t^j)$ | $\blacksquare$ | $\blacksquare$ | $\blacksquare$ | $b'$ | $c'$ | $c'$ |
|-------|-----|---------------------------------|-------------|-------------|-------------|-----|-----|-----|
| a     | 8   | $(-1,0) \times (-1,1) \times (1,2)$ | 1           | 1           | 0           | 0   | 0   | 2   |
| b     | 4   | $(0,1) \times (-1,2) \times (-1,2)$ | 3           | -3          | -4          | 0   | -8  | -4  |
| c     | 4   | $(1,1) \times (-1,0) \times (-1,2)$ | 1           | -1          | -           | -4  | 0   | -2  |
| 1     | 2   | $(1,0) \times (1,0) \times (1,0)$  | $x_A = \frac{1}{2}x_B = \frac{1}{2}x_C = \frac{1}{2}x_D$ |
| 4     | 4   | $(0,1) \times (1,0) \times (1,0)$  | $β^p_1 = -2, β^p_3 = 0$ |
|       |     |                                 | $χ_1 = \frac{1}{√5}, χ_2 = \frac{1}{√5}, χ_3 = \frac{1}{√5}$ |

Table 15. D6-brane configurations and intersection numbers of Model 15, and its gauge coupling relation is $g_a^2 = \frac{7}{9}g_b^2 = \frac{7}{9}g_c^2 = \frac{L^3}{2πx}(\frac{7}{9}g_Y^2) = \frac{4}{3} \sqrt{2} \pi e^{α_4}$.

| stack | $N$ | $t^i \times (n^i, t^j) \times (n^i, t^j)$ | $\blacksquare$ | $\blacksquare$ | $\blacksquare$ | $b'$ | $c'$ | $c'$ |
|-------|-----|---------------------------------|-------------|-------------|-------------|-----|-----|-----|
| a     | 8   | $(-1,0) \times (-1,1) \times (1,1)$ | 0           | 0           | 0           | 0   | 0   | 0   |
| b     | 4   | $(-1,2) \times (0,1) \times (-1,3)$ | 5           | -5          | -           | -4  | -16 | 0   |
| c     | 4   | $(1,2) \times (-2,1) \times (-1,1)$ | -7          | -9          | -           | -    |
|       |     |                                 | $x_A = \frac{1}{2}x_B = \frac{1}{2}x_C = \frac{1}{2}x_D$ |
|       |     |                                 | $χ_1 = \frac{1}{√3}, χ_2 = \frac{1}{√3}, χ_3 = \frac{1}{√3}$ |
Table 16. D6-brane configurations and intersection numbers of Model 16, and its gauge coupling relation is $g_5^2 = \frac{10}{g_6^2} = \frac{10}{g_7^2} = \frac{50}{g_8^2} = \frac{2}{\sqrt{34}} \sqrt{34} \pi e^{\phi^2}$.

| stack | N | $(n_1^i, l_1^i) \times (n_2^j, l_2^j) \times (n_3^k, l_3^k)$ | $a$ | $b$ | $c$ | $c'$ |
|-------|---|-------------------------------------------------|-----|-----|-----|-----|
| a     | 8 | $(-1, 0) \times (1, 1) \times (-1, 1)$           | 0   | 0   | 8   | -4  |
| b     | 4 | $(-1, 2) \times (0, 1) \times (-1, 2)$           | 4   | -4  | -   | -6  |
| c     | 4 | $(-1, 2) \times (-1, 2) \times (-1, 1)$          | 8   | 24  |    | -   |

$x_A = x_B = \frac{x_C}{\sqrt{34}} = \frac{x_D}{\sqrt{34}}$

$\chi_1 = \frac{\sqrt{34}}{34}, \chi_2 = \frac{1}{\sqrt{34}}, \chi_3 = \frac{1}{\sqrt{34}}$

Table 17. D6-brane configurations and intersection numbers of Model 17, and its gauge coupling relation is $g_5^2 = \frac{2}{g_6^2} = \frac{4}{g_7^2} = \frac{10}{g_8^2} = \frac{2}{\sqrt{34}} \sqrt{34} \pi e^{\phi^2}$.

| stack | N | $(n_1^i, l_1^i) \times (n_2^j, l_2^j) \times (n_3^k, l_3^k)$ | $a$ | $b$ | $c$ | $c'$ |
|-------|---|-------------------------------------------------|-----|-----|-----|-----|
| a     | 8 | $(-1, 0) \times (-1, 1) \times (1, 1)$           | 0   | 0   | 6   | -2  |
| b     | 4 | $(-1, 2) \times (0, 1) \times (-1, 2)$           | 3   | -3  | -   | -8  |
| c     | 4 | $(1, 2) \times (-2, 1) \times (-1, 1)$           | -7  | -9  | -   | -4  |
| 2     | 2 | $(1, 0) \times (0, 1) \times (0, 1)$             |     |     |     |     |

$x_A = x_B = \frac{x_C}{\sqrt{34}} = \frac{x_D}{\sqrt{34}}$

$\beta_3^2 = -2$

$\chi_1 = \frac{1}{\sqrt{34}}, \chi_2 = \frac{1}{\sqrt{34}}, \chi_3 = 2 \sqrt{34}$

Table 18. D6-brane configurations and intersection numbers of Model 18, and its gauge coupling relation is $g_5^2 = g_6^2 = 3 g_7^2 = \frac{4}{g_8^2} = \frac{10}{\sqrt{34}} \sqrt{34} \pi e^{\phi^2}$.

| stack | N | $(n_1^i, l_1^i) \times (n_2^j, l_2^j) \times (n_3^k, l_3^k)$ | $a$ | $b$ | $c$ | $c'$ |
|-------|---|-------------------------------------------------|-----|-----|-----|-----|
| a     | 8 | $(-1, 0) \times (-1, 2) \times (1, 1)$           | 1   | -1  | 4   | 0   |
| b     | 8 | $(-1, 1) \times (0, 1) \times (-1, 1)$           | 0   | 0   | -   | 0   |
| c     | 8 | $(1, 1) \times (-1, 1) \times (-1, 1)$           | -4  | -12 | -   | -2  |
| 3     | 4 | $(0, 1) \times (1, 0) \times (0, 1)$             |     |     |     |     |

$x_A = x_B = x_C = \frac{x_D}{\sqrt{34}}$

$\beta_3^2 = 2$

$\chi_1 = \sqrt{2}, \chi_2 = \frac{1}{\sqrt{2}}, \chi_3 = \sqrt{2}$

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Table 19. D6-brane configurations and intersection numbers of Model 19, and its gauge coupling relation is $g_\alpha^2 = 2g_\beta^2 = g_\gamma^2 = \frac{\sqrt{3}g_\varphi^2}{\pi} = \frac{8\sqrt{3}}{3} \times 10^{1/4} \pi e^{\phi^4}$.

| stack | $N$ | $(n^4, l^4) \times (n^3, l^3) \times (n^2, l^2)$ | $b$ | $b'$ | $c$ | $c'$ | $1$ | $4$ |
|-------|----|---------------------------------|-----|-----|-----|-----|-----|-----|
| $a$   | 8  | $(-1, 1) \times (-1, 1) \times (1, 2)$ | -1  | 1   | 0   | -4  | 0   | 0   |
| $b$   | 8  | $(0, 1) \times (-1, 1) \times (-1, 1)$ | 0   | 0   | -4  | 2   | -6  | -2  |
| $c$   | 4  | $(1, 1) \times (-1, 0) \times (-1, 2)$ | -1  | -1  | -4  | 0   | 0   | -2  |
| 1     | 2  | $(1, 0) \times (0, 1) \times (1, 0)$ | $x_A = x_B = x_C = 2x_D$ |
| 4     | 4  | $(0, 1) \times (0, 1) \times (1, 0)$ | $\beta_1^4 = -4, \beta_1^5 = 0$ |

Table 20. D6-brane configurations and intersection numbers of Model 20, and its gauge coupling relation is $g_\alpha^2 = \frac{4}{11}g_\beta^2 = \frac{2}{7}g_\gamma^2 = \frac{1}{2}(\frac{4}{3}g_\varphi^2) = \frac{1}{11} \sqrt{3} \times 10^{1/4} \pi e^{\phi^4}$.

| stack | $N$ | $(n^4, l^4) \times (n^3, l^3) \times (n^2, l^2)$ | $b$ | $b'$ | $c$ | $c'$ | $2$ | $4$ |
|-------|----|---------------------------------|-----|-----|-----|-----|-----|-----|
| $a$   | 8  | $(-1, 0) \times (1, 1) \times (-1, 2)$ | 1   | 1   | 6   | -2  | -4  | 0   |
| $b$   | 4  | $(-3, 2) \times (1, 2) \times (0, 1)$ | 1   | -1  | -4  | 0   | 0   | 3   |
| $c$   | 4  | $(-2, 1) \times (-1, 2) \times (-1, 2)$ | 5   | 27  | 0   | 0   | 0   | 3   |
| 2     | 4  | $(1, 0) \times (0, 1) \times (0, 1)$ | $x_A = \frac{3}{2}x_B = \frac{3}{2}x_C = \frac{3}{2}x_D$ |
| 4     | 4  | $(0, 1) \times (0, 1) \times (1, 0)$ | $\beta_1^4 = -5, \beta_1^5 = 5$ |

Table 21. D6-brane configurations and intersection numbers of Model 21, and its gauge coupling relation is $g_\alpha^2 = g_\beta^2 = 4g_\gamma^2 = \frac{20}{11}(\frac{5}{3}g_\varphi^2) = \frac{32}{9} \sqrt{2} \times 10^{1/4} \pi e^{\phi^4}$.

| stack | $N$ | $(n^4, l^4) \times (n^3, l^3) \times (n^2, l^2)$ | $b$ | $b'$ | $c$ | $c'$ | $2$ | $4$ |
|-------|----|---------------------------------|-----|-----|-----|-----|-----|-----|
| $a$   | 8  | $(-1, 0) \times (1, 1) \times (-1, 2)$ | 1   | 1   | 4   | 0   | -4  | 0   |
| $b$   | 8  | $(-1, 1) \times (-1, 1) \times (0, 1)$ | 0   | 0   | -4  | 0   | 0   | 2   |
| $c$   | 4  | $(-2, 1) \times (1, 2) \times (-1, 2)$ | 5   | 27  | 0   | 0   | 0   | 4   |
| 2     | 12 | $(1, 0) \times (0, 1) \times (0, 1)$ | $x_A = \frac{3}{2}x_B = \frac{3}{2}x_C = \frac{3}{2}x_D$ |
| 4     | 4  | $(0, 1) \times (0, 1) \times (1, 0)$ | $\beta_1^4 = -5, \beta_1^5 = 4$ |

\( x_1 = 2\sqrt{2}, x_2 = \frac{1}{\sqrt{2}}, x_3 = \frac{1}{\sqrt{2}} \)}
Table 22. D6-brane configurations and intersection numbers of Model 22, and its gauge coupling relation is $g_a^2 = g_b^2 = 2 g_c^2 = \frac{10}{7} (\frac{5}{3} g_Y^2) = 2 \sqrt{\frac{3}{4}} \pi e^4$.

| stack | U(4) $\times$ U(2)$_L$ $\times$ U(2)$_R$ $\times$ USp(2)$^2$ $\times$ USp(4) |
|-------|---------------------------------------------------------------|
|       | $n^1 \times (n^2, I^2) \times (n^3, I^3)$ | $\beta$ | $b$ | $b'$ | $c'$ | $c$ | $2$ | $3$ | $4$ |
| a     | (1, 0) $\times$ (1, 1) $\times$ (1, 1) | 0 | 0 | 4 | 0 | -4 | 0 | 0 | -1 | 1 |
| b     | (0, 1) $\times$ (1, 1) $\times$ (1, 1) | 2 | -2 | 0 | 0 | -12 | 1 | 0 | 0 | 0 |
| c     | (1, 0) $\times$ (1, 1) $\times$ (1, 1) | 0 | 0 | 0 | 0 | -2 | 0 | 2 | 0 |

Table 23. D6-brane configurations and intersection numbers of Model 23, and its gauge coupling relation is $g_a^2 = g_b^2 = \frac{5}{7} g_c^2 = \frac{25}{16} (\frac{5}{3} g_Y^2) = \frac{2}{3} 11^{3/4} \pi e^4$.

| stack | U(4) $\times$ U(2)$_L$ $\times$ U(2)$_R$ $\times$ USp(2)$^2$ $\times$ USp(8) |
|-------|---------------------------------------------------------------|
|       | $n^1 \times I^1 \times (n^2, I^2) \times (n^3, I^3)$ | $\beta$ | $b$ | $b'$ | $c'$ | $c$ | $1$ | $2$ | $3$ | $4$ |
| a     | (1, 0) $\times$ (0, 1) $\times$ (0, 1) | 0 | 0 | 4 | 0 | -4 | 0 | 0 | 1 | 1 |
| b     | (1, 0) $\times$ (0, 1) $\times$ (0, 1) | 1 | -1 | - | 0 | -4 | 6 | -2 | 0 | 1 |
| c     | (1, 0) $\times$ (0, 1) $\times$ (0, 1) | 0 | 12 | - | - | - | 3 | 1 | -1 | 1 |

Table 24. D6-brane configurations and intersection numbers of Model 24, and its gauge coupling relation is $g_a^2 = \frac{3}{2} g_b^2 = g_c^2 = (\frac{7}{4} g_Y^2) = 2 3^{3/4} \pi e^3$.

| stack | U(4) $\times$ U(2)$_L$ $\times$ U(2)$_R$ $\times$ USp(2)$^2$ $\times$ USp(4) |
|-------|---------------------------------------------------------------|
|       | $n^1 \times I^1 \times (n^2, I^2) \times (n^3, I^3)$ | $\beta$ | $b$ | $b'$ | $c'$ | $c$ | $1$ | $2$ | $3$ | $4$ |
| a     | (1, 0) $\times$ (1, 1) $\times$ (1, 1) | 0 | 0 | 0 | 4 | -4 | 0 | 0 | 0 | -1 | 1 |
| b     | (0, 1) $\times$ (1, 1) $\times$ (1, 1) | -2 | -2 | - | 2 | -4 | -1 | 3 | 0 | 0 |
| c     | (1, 0) $\times$ (1, 1) $\times$ (1, 1) | -1 | 0 | - | 0 | 2 | 0 | -1 | 1 | 1 | 1 |

| a     | (1, 0) $\times$ (1, 1) $\times$ (1, 1) | $x_A = \frac{3}{2} x_B = \frac{3}{4} x_C = \frac{3}{4} x_D$ |
| b     | (0, 1) $\times$ (1, 1) $\times$ (1, 1) | $\beta^a = -5, \beta^b = -1, \beta^c = -4, \beta^d = -3$ |
| c     | (0, 1) $\times$ (1, 1) $\times$ (1, 1) | $x_1 = \frac{3}{4}, x_2 = \frac{1}{V_8}, x_3 = \frac{3}{2}$ |
Table 25. D6-brane configurations and intersection numbers of Model 25, and its gauge coupling relation is $g_a^2 = g_b^2 = \frac{3}{14}g_c^2 = \frac{5}{4}(\frac{g_1}{2}g_2^2) = 2 \frac{3^3}{4\pi}e^{\phi^4}$.

| model 25 | $U(4) \times U(2)_L \times U(2)_R \times USp(2)^3 \times USp(4)$ | \(N\) | \((n^1, l^1) \times (n^2, l^2) \times (n^3, l^3)\) | \(n\) | \(b\) | \(b'\) | \(c\) | \(c'\) | 1 | 2 | 3 | 4 |
|-----------|------------------------------------------------|-------|------------------------------------------------|-------|-------|-------|-------|-------|---|---|---|---|
| a         | 8                                             | (1, 0)\times(1, 1)\times(1, -1) | 0     | 0     | 0     | 0     | 0     | 0     | 0     | 1 | -1|
| b         | 4                                             | (−1, 2)\times(−1, 1)\times(0, 1) | 1     | -1    | -1    | -1    | -1    | 4     | 2     | 2 | 0 | 0 | 0 |
| c         | 4                                             | (0, 1)\times(1, 3)\times(−1, −1) | -2    | 2     | -     | -     | -     | -3    | 1     | 0 | 0 | 0 | 0 |

Table 26. D6-brane configurations and intersection numbers of Model 26, and its gauge coupling relation is $g_a^2 = \frac{11}{16}g_b^2 = \frac{11}{8}g_c^2 = \frac{5}{3}g_1^2 = \frac{5}{6}g_2^2 = \frac{8}{5\sqrt{2}}\frac{3^3}{4\pi}e^{\phi^4}$.

| model 26 | $U(4) \times U(2)_L \times U(2)_R \times USp(2)^3$ | \(N\) | \((n^1, l^1) \times (n^2, l^2) \times (n^3, l^3)\) | \(n\) | \(b\) | \(b'\) | \(c\) | \(c'\) | 1 | 2 | 3 | 4 |
|-----------|------------------------------------------------|-------|------------------------------------------------|-------|-------|-------|-------|-------|---|---|---|---|
| a         | 8                                             | (1, −1)\times(1, 0)\times(1, 1) | 0     | 0     | -3    | 7     | -4    | 0     | -1    | 0 | 1 | 0 | 1 |
| b         | 4                                             | (−5, 2)\times(1, 1)\times(−1, 0) | 3     | 0     | -     | -     | -     | 2     | 0     | 2 | 5 | 0 | 0 |
| c         | 4                                             | (−3, 1)\times(−1, 1)\times(1, 1) | 0     | 12    | -     | -     | -     | 1     | 3     | 3 | 3 | 3 | 3 |

Table 27. D6-brane configurations and intersection numbers of Model 27, and its gauge coupling relation is $g_a^2 = \frac{6}{7}g_b^2 = \frac{7}{9}g_c^2 = \frac{5}{3}g_1^2 = \frac{5}{9\sqrt{2}}\frac{3^3}{4\pi}e^{\phi^4}$.

| model 27 | $U(4) \times U(2)_L \times U(2)_R$ | \(N\) | \((n^1, l^1) \times (n^2, l^2) \times (n^3, l^3)\) | \(n\) | \(b\) | \(b'\) | \(c\) | \(c'\) | 1 | 2 | 3 | 4 |
|-----------|-----------------------------------|-------|------------------------------------------------|-------|-------|-------|-------|-------|---|---|---|---|
| a         | 8                                 | (−2, −1)\times(1, 1)\times(1, 1) | 0     | -     | 12    | -     | -     | -     | -     | - | - | 0 | 0 |
| b         | 4                                 | (−1, 1)\times(6, 2)\times(−1, 0) | 4     | -4    | -     | -     | -     | -     | -     | - | - | - | - |
| c         | 4                                 | (1, 1)\times(−1, 0)\times(−2, 2) | 0     | 0     | -     | -     | -     | -     | -     | - | - | - | - |

\[x_A = \frac{x_B}{7} x_C = \frac{13}{7} x_D\]

\[\beta_1^2 = -1, \beta_2^2 = 2, \beta_3^2 = -1\]

\[\chi_1 = \sqrt{3}, \chi_2 = 3\sqrt{3}, \chi_3 = 2\sqrt{3}\]
Table 28. D6-brane configurations and intersection numbers of Model 28, and its gauge coupling relation is $g_\alpha^2 = \frac{47}{24} g_\beta^2 = \frac{14}{5} g_\gamma^2 = \frac{39}{8} g_\delta^2 = \frac{1}{4} \sqrt{\frac{7}{2}} 23^{3/4} \pi e^{\phi^4}$.

| model 28 | $U(4) \times U(2)_L \times U(2)_R \times USp(2) \times USp(4)$ |
| --- | --- |
| stack | $N$ | $(n^1, l^1) \times (n^2, l^2) \times (n^3, l^3)$ | $b$ | $b'$ | $b''$ | $c$ | $c'$ | $c''$ | 1 | 2 |
| a | 8 | $(-1, -1) \times (1, 0) \times (2, 1)$ | 1 | -1 | 5 | 9 | -4 | 0 | 0 | -2 |
| b | 4 | $(-7, 2) \times (1, 1) \times (-1, 0)$ | -5 | 5 | - | - | 9 | 11 | 0 | -2 |
| c | 4 | $(-2, 1) \times (-2, 1) \times (-2, 1)$ | 5 | 27 | - | - | - | - | 1 | 4 |
| 1 | 2 | $(1, 0) \times (1, 0) \times (1, 0)$ | $x_A = \frac{\pi}{4} x_B = 40 \chi_C = \frac{\pi}{4} x_D$ | $\beta_1^2 = -5, \beta_2^2 = 4$ |
| 2 | 4 | $(1, 0) \times (0, 1) \times (0, 1)$ | $\chi_1 = \sqrt{23}, \chi_2 = \frac{3 \sqrt{23}}{4}, \chi_3 = 4 \sqrt{23}$ |

Table 29. D6-brane configurations and intersection numbers of Model 29, and its gauge coupling relation is $g_\alpha^2 = \frac{5}{24} g_\beta^2 = \frac{19}{8} g_\gamma^2 = \frac{95}{62} \frac{5}{3} g_\delta^2 = \frac{2}{45} \frac{86}{3} 23^{3/4} \pi e^{\phi^4}$.

| model 29 | $U(4) \times U(2)_L \times U(2)_R \times USp(2)^2$ |
| --- | --- |
| stack | $N$ | $(n^1, l^1) \times (n^2, l^2) \times (n^3, l^3)$ | $b$ | $b'$ | $b''$ | $c$ | $c'$ | $c''$ | 1 | 2 | 4 |
| a | 8 | $(-2, -1) \times (1, 0) \times (2, 1)$ | 0 | 0 | 6 | 10 | -4 | 0 | -2 | 2 |
| b | 4 | $(-8, 1) \times (1, 1) \times (-1, 0)$ | -7 | 7 | - | - | 15 | 11 | -1 | 0 |
| c | 4 | $(-3, 1) \times (-2, 1) \times (-2, 1)$ | 9 | 39 | - | - | - | - | 4 | 6 |
| 2 | 2 | $(1, 0) \times (0, 1) \times (0, 1)$ | $x_A = \frac{\pi}{4} x_B = 80 \chi_C = \frac{\pi}{4} x_D$ | $\beta_1^2 = 3, \beta_2^2 = 4$ |
| 4 | 2 | $(0, 1) \times (0, 1) \times (1, 0)$ | $\chi_1 = \sqrt{80}, \chi_2 = \frac{3 \sqrt{80}}{4}, \chi_3 = 2 \sqrt{80}$ |

Table 30. D6-brane configurations and intersection numbers of Model 30, and its gauge coupling relation is $g_\alpha^2 = \frac{700}{34} g_\beta^2 = 90 g_\gamma^2 = \frac{15}{7} \frac{5}{3} g_\delta^2 = \frac{46}{34} \frac{87}{3} 23^{3/4} \pi e^{\phi^4}$.

| model 30 | $U(4) \times U(2)_L \times U(2)_R \times USp(2)^2$ |
| --- | --- |
| stack | $N$ | $(n^1, l^1) \times (n^2, l^2) \times (n^3, l^3)$ | $b$ | $b'$ | $b''$ | $c$ | $c'$ | $c''$ | 1 | 3 |
| a | 8 | $(-1, -1) \times (-1, 0) \times (1, 1)$ | 0 | 0 | 5 | 0 | -4 | 0 | 0 | 0 |
| b | 4 | $(-2, 1) \times (-1, 1) \times (-4, 1)$ | 3 | 29 | - | - | 33 | -35 | -1 | 8 |
| c | 4 | $(1, 0) \times (-9, -2) \times (-1, 1)$ | 7 | -7 | - | - | - | - | 0 | 2 |
| 1 | 2 | $(1, 0) \times (1, 0) \times (1, 0)$ | $x_A = 42 x_B = \frac{\pi}{4} x_C = 42 x_D$ | $\beta_1^2 = -5, \beta_2^2 = 4$ |
| 3 | 2 | $(0, 1) \times (1, 0) \times (0, 1)$ | $\chi_1 = 2 \sqrt{7}, \chi_2 = 3 \sqrt{21}, \chi_3 = 4 \sqrt{7}$ |
Table 31. D6-brane configurations and intersection numbers of Model 31, and its gauge coupling relation is 
\( g_6^2 = \frac{1329}{44} g_5^2 = 5 g_5^2 = \frac{41}{7} (g_5^2)^2 = \frac{5}{3} \sqrt{185/3} \pi e^{\phi} \).

| model 31 | \( U(4) \times U(2)_L \times U(2)_R \) |
|----------|----------------------------------|
| stack    | \( (n^1, l^1) \times (n^2, l^2) \times (n^3, l^3) \) | \( r \) | \( n \) | \( b \) | \( b' \) | \( c \) | \( c' \) |
| \( a \)  | 8 | \((-1, 1) \times (-1, 0) \times (1, 1)\) | 0 | 0 | -5 | 9 | -4 | 0 | 0 |
| \( b \)  | 4 | \((-2, 1) \times (-2, 1) \times (-4, 1)\) | 13 | 51 | - | - | 45 | -35 | -1 |
| \( c \)  | 4 | \((1, 0) \times (-11, -2) \times (-1, 1)\) | 9 | -9 | - | - | - | 0 | 0 |

\( x_A = x_B = \frac{1}{3} x_C = \frac{1}{12} x_D \)
\( x_1 = \frac{1}{\sqrt{44}}, x_2 = \sqrt{\frac{1}{44}}, x_3 = 2 \sqrt{44} \)

Table 32. D6-brane configurations and intersection numbers of Model 32, and its gauge coupling relation is 
\( g_6^2 = \frac{1248}{41} g_5^2 = 11 g_5^2 = \frac{41}{19} (g_5^2)^2 = \frac{64}{121} \sqrt{77/3} \pi e^{\phi} \).

| model 32 | \( U(4) \times U(2)_L \times U(2)_R \times USp(14) \) |
|----------|----------------------------------|
| stack    | \( (n^1, l^1) \times (n^2, l^2) \times (n^3, l^3) \) | \( r \) | \( n \) | \( b \) | \( b' \) | \( c \) | \( c' \) | \( t \) |
| \( a \)  | 8 | \((-1, 1) \times (-1, 0) \times (1, 1)\) | 0 | 0 | -5 | 9 | -4 | 0 | 0 |
| \( b \)  | 4 | \((-2, 1) \times (-2, 1) \times (-4, 1)\) | 13 | 51 | - | - | 45 | -35 | -1 |
| \( c \)  | 4 | \((1, 0) \times (-11, -2) \times (-1, 1)\) | 9 | -9 | - | - | - | 0 | 0 |
| \( \tau \) | 14 | \((1, 0) \times (1, 0) \times (1, 0)\) | \( x_A = 56 x_B = \frac{112}{11} x_C = 56 x_D \) | \( \beta^\theta = -5 \) |
|          |                        | \( x_1 = 4 \sqrt{\frac{1}{44}}, x_2 = 2 \sqrt{\frac{1}{44}}, x_3 = 8 \sqrt{\frac{1}{44}} \) |

Table 33. D6-brane configurations and intersection numbers of Model 33, and its gauge coupling relation is 
\( g_6^2 = \frac{1680}{43} g_5^2 = 13 g_5^2 = \frac{43}{23} (g_5^2)^2 = \frac{284}{231} \sqrt{123/3} \pi e^{\phi} \).

| model 33 | \( U(4) \times U(2)_L \times U(2)_R \times USp(10) \) |
|----------|----------------------------------|
| stack    | \( (n^1, l^1) \times (n^2, l^2) \times (n^3, l^3) \) | \( r \) | \( n \) | \( b \) | \( b' \) | \( c \) | \( c' \) | \( t \) |
| \( a \)  | 8 | \((-1, 1) \times (-1, 0) \times (1, 1)\) | 0 | 0 | -5 | 9 | -4 | 0 | 0 |
| \( b \)  | 4 | \((-2, 1) \times (-2, 1) \times (-4, 1)\) | 13 | 51 | - | - | 45 | -35 | -1 |
| \( c \)  | 4 | \((1, 0) \times (-11, -2) \times (-1, 1)\) | 11 | -11 | - | - | - | 0 | 0 |
| \( \tau \) | 10 | \((1, 0) \times (1, 0) \times (1, 0)\) | \( x_A = 64 x_B = \frac{128}{11} x_C = 64 x_D \) | \( \beta^\theta = -5 \) |
|          |                        | \( x_1 = 8 \sqrt{\frac{1}{43}}, x_2 = 4 \sqrt{\frac{1}{23}}, x_3 = 16 \sqrt{\frac{1}{43}} \) |
Table 34. D6-brane configurations and intersection numbers of Model 34, and its gauge coupling relation is $g_a^2 = \frac{2176}{53}g_b^2 = 15g_c^2 = \frac{25}{17}(\frac{5}{3}g_Y^2) = \frac{192}{53}\sqrt{35}4/\pi e^\phi$.

| model 34 | $U(4) \times U(2)_L \times U(2)_R \times USp(6)$ |
|----------|-----------------------------------------------|
| stack    | $N$ | $(n^1,l^1) \times (n^2,l^2) \times (n^3,l^3)$ | $\Gamma_n$ | $\Gamma_{n'}$ | $b$ | $b'$ | $c$ | $c'$ | $1$ |
| $a$      | 8   | $(-1,1) \times (-1,0) \times (1,1)$          | 0          | 0            | -5 | 9  | -4 | 0  | 0   |
| $b$      | 4   | $(-2,1) \times (-2,1) \times (-1,1)$         | 13         | 51           | -  | -  | 57 | -55| -1  |
| $c$      | 4   | $(1,0) \times (-15,-2) \times (-1,1)$        | 13         | -13          | -  | -  | -  | -  | 0   |
| 1        | 6   | $(1,0) \times (1,0) \times (1,0)$            | $x_A = 72x_B = \frac{32}{27}x_C = 72x_D$ |               | $\beta_7^T = -5$ |               | $\chi_1 = 4\sqrt{\frac{5}{17}}, \chi_2 = 6\sqrt{17}, \chi_3 = 8\sqrt{\frac{2}{17}}$ |

Table 35. D6-brane configurations and intersection numbers of Model 35, and its gauge coupling relation is $g_a^2 = \frac{2736}{59}g_b^2 = 17g_c^2 = \frac{85}{47}(\frac{5}{3}g_Y^2) = \frac{64}{17}\sqrt{170}4/\pi e^\phi$.

| model 35 | $U(4) \times U(2)_L \times U(2)_R \times USp(2)$ |
|----------|-----------------------------------------------|
| stack    | $N$ | $(n^1,l^1) \times (n^2,l^2) \times (n^3,l^3)$ | $\Gamma_n$ | $\Gamma_{n'}$ | $b$ | $b'$ | $c$ | $c'$ | $1$ |
| $a$      | 8   | $(-1,1) \times (-1,0) \times (1,1)$          | 0          | 0            | -5 | 9  | -4 | 0  | 0   |
| $b$      | 4   | $(-2,1) \times (-2,1) \times (-1,1)$         | 13         | 51           | -  | -  | 63 | -65| -1  |
| $c$      | 4   | $(1,0) \times (-17,-2) \times (-1,1)$        | 15         | -15          | -  | -  | -  | -  | 0   |
| 1        | 2   | $(1,0) \times (1,0) \times (1,0)$            | $x_A = 80x_B = \frac{480}{17}x_C = 80x_D$ |               | $\beta_7^T = -5$ |               | $\chi_1 = 4\sqrt{\frac{10}{17}}, \chi_2 = 2\sqrt{170}, \chi_3 = 8\sqrt{\frac{6}{17}}$ |
B Four-Family Standard Models from Intersecting D6-Branes with One Tilted Torus

Table 36. D6-brane configurations and intersection numbers of Model 36, and its gauge coupling relation is $g_a^3 = g_b^3 = g_c^3 = (\frac{2}{3}g_Y^2) = 2\sqrt{2}\pi e^\alpha$.

| model 36 | $U(4) \times U(2)_L \times U(2)_R \times USp(2)^2$ |
|----------|-----------------------------------------------|
| stack    | $N$ $(n^1,l^1) \times (n^2,l^2) \times (n^3,l^3)$ | $n$ $b$ $b'$ $c$ $c'$ $2$ $4$ |
| $a$      | 8 $(1,1) \times (0,-1) \times (1,1)$ | 0 0 0 4 0 0 0 |
| $b$      | 4 $(1,0) \times (-4,1) \times (-1,-1)$ | -3 3 - - 0 0 4 |
| $c$      | 4 $(0,-1) \times (4,1) \times (1,1)$ | 3 -3 - - - - 4 |
| 2        | 2 $(1,0) \times (0,1) \times (0,-2)$ | $x_A = 4x_B = x_C = 4x_D$ |
| 4        | 2 $(0,1) \times (0,1) \times (-2,0)$ | $\beta_2^b = -2, \beta_3^b = -2$ |

Table 37. D6-brane configurations and intersection numbers of Model 37, and its gauge coupling relation is $g_a^3 = 2g_b^3 = 2g_c^3 = \frac{10}{1}(\frac{2}{3}g_Y^2) = 4\pi e^\alpha$.

| model 37 | $U(4) \times U(4)_L \times U(4)_R$ |
|----------|-----------------------------------------------|
| stack    | $N$ $(n^1,l^1) \times (n^2,l^2) \times (n^3,l^3)$ | $n$ $b$ $b'$ $c$ $c'$ |
| $a$      | 8 $(1,-1) \times (-1,0) \times (-1,-1)$ | 0 0 0 4 0 0 0 |
| $b$      | 8 $(0,1) \times (-1,-2) \times (1,1)$ | -2 2 - - 0 0 0 |
| $c$      | 8 $(1,0) \times (1,-2) \times (1,1)$ | 2 -2 - - - - - |
| 2        | $x_A = \frac{1}{2}x_B = x_C = \frac{1}{2}x_D$ |
| 4        | $x_A = \frac{1}{2}x_B = x_C = \frac{1}{2}x_D$ |
| $\chi_1 = 1, \chi_2 = \frac{1}{2}, \chi_3 = 2$ |

Table 38. D6-brane configurations and intersection numbers of Model 38, and its gauge coupling relation is $g_a^3 = 2g_b^3 = 2g_c^3 = \frac{10}{3}(\frac{2}{3}g_Y^2) = 4\pi e^\alpha$.

| model 38 | $U(4) \times U(4)_L \times U(4)_R$ |
|----------|-----------------------------------------------|
| stack    | $N$ $(n^1,l^1) \times (n^2,l^2) \times (n^3,l^3)$ | $n$ $b$ $b'$ $c$ $c'$ |
| $a$      | 8 $(1,0) \times (-1,1) \times (-1,-1)$ | 0 0 0 4 0 0 0 |
| $b$      | 8 $(1,-2) \times (0,-1) \times (-1,1)$ | 2 -2 - - 0 0 0 |
| $c$      | 8 $(1,-2) \times (-1,0) \times (-1,-1)$ | 2 -2 - - - - - |
| 2        | $x_A = x_B = \frac{1}{2}x_C = \frac{1}{2}x_D$ |
| 4        | $x_A = \frac{1}{2}x_B = x_C = \frac{1}{2}x_D$ |
| $\chi_1 = \frac{1}{2}, \chi_2 = 1, \chi_3 = 2$ |
Table 39. D6-brane configurations and intersection numbers of Model 39, and its gauge coupling relation is \( g_a^2 = 2g_b^2 = g_c^2 = \left(\frac{2}{3}\right)^2 = \frac{8}{3}\sqrt{2}\pi e^4 \).

| model 39 | \( U(4) \times U(4)_L \times U(2)_R \times USp(2) \) |
|---|---|
| stack | \( N \) | \((n^a, l^a) \times (n^b, l^b) \times (n^c, l^c)\) | \( b \) | \( b' \) | \( c \) | \( c' \) |
|\( a \) | 8 | \((1, -1) \times (1, 0) \times (1, 1)\) | 0 | 0 | 4 | 0 | -4 | 0 | 0 |
|\( b \) | 8 | \((0, 1) \times (-1, 2) \times (-1, 1)\) | 2 | -2 | - | - | 0 | 4 | 4 |
|\( c \) | 4 | \((-1, 0) \times (-1, -4) \times (1, -1)\) | -3 | 3 | - | - | - | - | 0 |
| 1 | 2 | \((1, 0) \times (1, 0) \times (-2, 0)\) | \( x_A = \frac{1}{2}x_B = \frac{2}{3}x_C = \frac{3}{4}x_D \) |
| \( \beta_2^a \) | \(-2\) |
| \( \chi_1 = \sqrt{2}, \chi_2 = \frac{1}{\sqrt{2}}, \chi_3 = 2\sqrt{2} \) |

Table 40. D6-brane configurations and intersection numbers of Model 40, and its gauge coupling relation is \( g_a^2 = g_b^2 = 2g_c^2 = \left(\frac{10}{3}\right)^2 = \frac{8}{3}\sqrt{2}\pi e^4 \).

| model 40 | \( U(4) \times U(2)_L \times U(4)_R \times USp(2) \) |
|---|---|
| stack | \( N \) | \((n^a, l^a) \times (n^b, l^b) \times (n^c, l^c)\) | \( b \) | \( b' \) | \( c \) | \( c' \) |
|\( a \) | 8 | \((-1, -1) \times (0, -1) \times (-1, -1)\) | 0 | 0 | 4 | 0 | -4 | 0 | 0 |
|\( b \) | 4 | \((-1, 0) \times (4, 1) \times (-1, 1)\) | -3 | -3 | - | - | 0 | -4 | 0 |
|\( c \) | 8 | \((0, -1) \times (2, -1) \times (-1, -1)\) | -2 | 2 | - | - | - | - | 4 |
| 2 | 2 | \((1, 0) \times (0, 1) \times (0, -2)\) | \( x_A = 2x_B = x_C = 4x_D \) |
| \( \beta_2^a \) | \(-2\) |
| \( \chi_1 = \sqrt{2}, \chi_2 = 2\sqrt{2}, \chi_3 = \sqrt{2} \) |

Table 41. D6-brane configurations and intersection numbers of Model 41, and its gauge coupling relation is \( g_a^2 = 2g_b^2 = g_c^2 = \left(\frac{2}{3}\right)^2 = \frac{8}{3}\sqrt{2}\pi e^4 \).

| model 41 | \( U(4) \times U(4)_L \times U(2)_R \times USp(2) \) |
|---|---|
| stack | \( N \) | \((n^a, l^a) \times (n^b, l^b) \times (n^c, l^c)\) | \( b \) | \( b' \) | \( c \) | \( c' \) |
|\( a \) | 8 | \((-1, 0) \times (1, 1) \times (-1, -1)\) | 0 | 0 | 4 | 0 | -4 | 0 | 0 |
|\( b \) | 8 | \((1, 2) \times (1, 0) \times (1, -1)\) | -2 | 2 | - | - | 0 | -4 | 4 |
|\( c \) | 4 | \((-1, 4) \times (0, -1) \times (1, -1)\) | 3 | -3 | - | - | - | - | 0 |
| 2 | 2 | \((1, 0) \times (0, 1) \times (0, -2)\) | \( x_A = \frac{1}{2}x_B = \frac{2}{3}x_C = \frac{3}{4}x_D \) |
| \( \beta_2^a \) | \(-2\) |
| \( \chi_1 = \frac{1}{\sqrt{2}}, \chi_2 = \frac{1}{\sqrt{2}}, \chi_3 = \sqrt{2} \) |
Table 42. D6-brane configurations and intersection numbers of Model 42, and its gauge coupling relation is $g_\alpha^2 = g_\beta^2 = g_\gamma^2 = 2g_\delta^2 = \frac{10}{9}(\frac{5}{2}g_\delta^2) = \frac{8}{3}\sqrt{2}\pi e^\phi$.

| model 42 | $U(4) \times U(2)_L \times U(4)_R \times USp(2)$ |
|----------|-----------------------------------------------|
| stack    | $N$ | $(n^a, l^a) \times (n^b, l^b) \times (n^c, l^c)$ | $a$ | $b$ | $c$ | $c'$ | 1 |
| a        | 8   | $(0, -4) \times (1, -1) \times (1, -1)$ | 0   | 0   | 3   | -4  | 0  | 0 |
| b        | 3   | $(0, -4) \times (1, -1) \times (1, -1)$ | -2  | 2   | -2  | 4   | 0  | 0 |
| c        | 8   | $(0, -4) \times (1, -1) \times (1, -1)$ | -2  | 2   | -2  | 4   | 0  | 0 |

Table 43. D6-brane configurations and intersection numbers of Model 43, and its gauge coupling relation is $g_\alpha^2 = g_\beta^2 = g_\gamma^2 = 2g_\delta^2 = \frac{16}{9}(\frac{2}{3}g_\delta^2) = \frac{16}{3}\sqrt{2}\pi e^\phi$.

| model 43 | $U(4) \times U(2)_L \times U(2)_R \times USp(2)$ |
|----------|-----------------------------------------------|
| stack    | $N$ | $(n^a, l^a) \times (n^b, l^b) \times (n^c, l^c)$ | $a$ | $b$ | $c$ | $c'$ | 1 |
| a        | 8   | $(0, 0) \times (1, -1) \times (1, -1)$ | 0   | 0   | 1   | 3   | -4  | 0  | 0 |
| b        | 3   | $(0, 0) \times (1, -1) \times (1, -1)$ | -3  | 2   | -2  | 4   | 0  | 0 |
| c        | 8   | $(0, 0) \times (1, -1) \times (1, -1)$ | -3  | 2   | -2  | 4   | 0  | 0 |

Table 44. D6-brane configurations and intersection numbers of Model 44, and its gauge coupling relation is $g_\alpha^2 = g_\beta^2 = g_\gamma^2 = 2g_\delta^2 = \frac{10}{9}(\frac{5}{2}g_\delta^2) = \frac{16}{3}\sqrt{2}\pi e^\phi$.

| model 44 | $U(4) \times U(2)_L \times U(2)_R \times USp(2)$ |
|----------|-----------------------------------------------|
| stack    | $N$ | $(n^a, l^a) \times (n^b, l^b) \times (n^c, l^c)$ | $a$ | $b$ | $c$ | $c'$ | 1 |
| a        | 8   | $(0, -4) \times (1, -1) \times (1, -1)$ | 0   | 0   | 1   | 3   | -4  | 0  | 0 |
| b        | 3   | $(0, -4) \times (1, -1) \times (1, -1)$ | 2   | -2  | -   | 6   | -6 | 0  | 0 |
| c        | 8   | $(0, -4) \times (1, -1) \times (1, -1)$ | 2   | -2  | -   | 6   | -6 | 0  | 0 |