Measurement of Single Muons at Forward Rapidity in $p + p$ Collisions at $\sqrt{s} = 200$ GeV
and Implications for Charm Production

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Muon production at forward rapidity ($1.5 \leq |\eta| \leq 1.8$) has been measured by the PHENIX experiment over the transverse momentum range $1 \leq p_T \leq 3$ GeV/c in $\sqrt{s} = 200$ GeV $p+p$ collisions at the Relativistic Heavy Ion Collider. After statistically subtracting contributions from light hadron decays an excess remains which is attributed to the semileptonic decays of hadrons carrying heavy flavor, i.e., charm quarks or, at high $p_T$, bottom quarks. The resulting muon spectrum from heavy flavor decays is compared to PYTHIA and a next-to-leading order perturbative QCD calculation. PYTHIA is used to determine the charm quark spectrum that would produce the observed muon excess. The corresponding differential cross section for charm quark production at forward rapidity is determined to be $d\sigma_{cc}/dy|_{y=1.6} = 0.243 \pm 0.013$ (stat.) $\pm 0.105$ (data syst.) $\pm 0.049$ (PYTHIA syst.) mb.

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I. INTRODUCTION

Measurements of heavy quark production in proton-proton ($p+p$) interactions at collider energies serve as important tests for perturbative Quantum Chromodynamics (pQCD). Bottom production at the Tevatron collider ($\sqrt{s} = 1.8$ and 1.96 TeV/c) is reasonably well described by a recent Fixed Order Next-to-Leading Logarithm (FONLL) calculation \[2\]. Charm production at FNAL, which has only been measured at relatively high $p_T (>5$ GeV/c), is $\approx 50\%$ higher than the FONLL prediction \[3\]. However, theoretical and experimental uncertainties are large, such that significant disagreement between theory and data cannot be claimed.

Measurements at Brookhaven National Laboratory’s Relativistic Heavy Ion Collider (RHIC), by both the PHENIX and STAR experiments, have provided a wealth of information on mid-rapidity open charm production in collisions at $\sqrt{s_{NN}} = 130$ GeV ($p+p$) and $\sqrt{s_{NN}} = 200$ GeV ($p+p$, $d+Au$, and $Au+Au$) down to $p_T \approx 0.5$ GeV/c. Semileptonic decay of produced charm quarks is the primary source of high $p_T$ leptons after contributions from known (light hadron) sources are subtracted. Both PHENIX \[7, 8, 9, 10, 11, 12, 13, 14, 15, 16\] and STAR \[17, 18\] have made statistical measurements of charm production via single-electron spectra. STAR has also made a direct measurement of charm production through reconstruction of hadronic decay modes of D mesons \[17\]. In $p+p$ collisions at $\sqrt{s_{NN}} = 200$ GeV PHENIX finds $\frac{d\sigma_{cc}/dy}{dy} = 0.123 \pm 0.012(\text{stat.}) \pm 0.045(\text{syst.})$ mb \[12\]. STAR finds a somewhat higher central value, $\frac{d\sigma_{cc}/dy}{dy} = 0.30 \pm 0.04(\text{stat.}) \pm 0.09(\text{syst.})$ mb \[17\], but the two measurements are consistent within the stated errors. Both measurements are noticeably (2-4×) higher than PYTHIA (a leading order pQCD event generator) \[19\] see experimental references for specific parameter sets\[1\] and FONLL \[20\]. Again, quantitative disagreement cannot be established with current experimental and theoretical errors. However, we note that there is some debate about whether charm quarks are heavy enough to be reliably treated by pQCD \[21\].

Such measurements also serve as an important baseline for charm production in proton-nucleus or deuteron-nucleus ($p+A$ or $d+A$), and nucleus-nucleus ($A+B$) collisions \[22, 23, 24, 25\]. In the absence of any nuclear effects, charm production (since it is a point-like process) is expected to scale with the number of binary nucleon-nucleon collisions ($N_{coll}$), which depends on the impact parameter of the nuclear collision and can be obtained from a Glauber calculation \[26\]. The degree of scaling for any given centrality bin is quantified by the nuclear modification factor:

$$R_{AB} = \frac{1}{N_{coll}} \times \frac{dN^{AB}/dy}{dN^{pp}/dy}. \quad (1)$$

Deviations from this scaling ($R_{AB} \neq 1$) in $p+A$ or $d+A$ collisions quantify cold nuclear matter effects (such as initial state energy loss \[24, 25\], $\Delta$), and shadowing \[33, 34, 35, 36, 37\]. Any such deviation must be understood so that in $A+B$ collisions contributions to $R_{AB} \neq 1$ from hot nuclear matter effects (such as medium energy loss \[38\] and references therein\[29\]) and cold nuclear matter effects can be disentangled. In $d+Au$ collisions both PHENIX and STAR find little or no effect of cold nuclear matter on charm production ($R_{dAu} \approx 1$ over the measured lepton $p_T$ range \[14, 17\]). This contrasts with measurements of open charm in $Au+Au$ collisions: although the total charm production appears to scale with $N_{coll}$ \[8\], there is a strong suppression of lepton spectra for $p_T > 2$ GeV/c that increases with centrality \[11, 12, 15\]. Furthermore the elliptical flow of nonphotonic single electrons, as measured by PHENIX in $Au+Au$ collisions \[14, 15, 16\], implies that the charm quarks interact strongly with the created medium.

Finally, since the initial formation of open and closed charm are both sensitive to initial gluon densities \[33, 40\], open charm production serves as an appropriate normalization for $J/\psi$ production. The production of $J/\psi$ mesons is expected to be sensitive to the production of a quark gluon plasma (QGP), should one be formed in $A+B$ collisions \[41, 42, 43, 44, 45, 46\]. In order to understand $J/\psi$ production differences in $A+B$ collisions compared to $p+p$ and $p+A$ collisions it is important to take into account any differences in the charm quark production in each of the different systems.

Until now, open charm measurements at RHIC have been limited to mid-rapidity. Measurements at forward rapidity are interesting for a variety of reasons. First is the need to constrain theoretical calculations over a wide kinematic range. The importance of this is demonstrated by the D0 measurement of bottom production at large rapidity ($\sqrt{s} = 1.8$ TeV, $p_T > 5$ GeV/c, 2.4 < $y_B$ < 3.2), as deduced from the production of high $p_T$ muons \[1\]. Significant theoretical improvements resulted from the effort to reduce what was, initially, a discrepancy between theory and experiment that increased with increasing rapidity \[3\]. Second, significant cold nuclear effects have been seen in RHIC collisions at forward rapidity. PHENIX \[17\], BRAHMS \[48, 49\], and STAR \[50\] have all measured light hadron production in $d+Au$ collisions at forward rapidity and have found significant deviations from $R_{dAu} = 1$. It will be interesting to see whether charm production follows a similar pattern. Finally, open charm production at forward rapidity needs to be understood to fully interpret PHENIX $J/\psi$ measurements at forward rapidity \[24, 25, 51, 52, 53\].

In this paper we report on the measurement of muon production at forward rapidity (1.5 ≤ $|y| ≤ 1.8$), in the range 1 < $p_T < 3$ GeV/c, in $\sqrt{s} = 200$ GeV $p+p$ collisions by the PHENIX experiment. The upper limit of the $p_T$ range is determined by available statistics. The vertex-independent muon yield is statistically extracted by calculating and subtracting contributions from
light mesons (\(\pi\)'s and \(K\)'s) which decay into a muon, and hadrons which penetrate through the muon arm absorber material. In the absence of new physics, and in the \(p_T\) range measured in this analysis, such muons come dominantly from the decay of hadrons containing a charm quark (with small contributions from decays of hadrons containing a bottom quark and decays of light-vector mesons). PYTHIA is used to determine the charm quark spectrum that would produce the observed vertex-independent muon signal, and extract the differential cross section for charm production at forward rapidity.

In Section III we describe the methodology used to extract the vertex-independent muon yield; and details from abundant light hadrons, which are subtracted to obtain the vertex-independent muon signal. This section includes details on the run, event and track selection criteria; values obtained for contributions to the muon yield from abundant light hadrons, which are subtracted to obtain the vertex-independent muon yield; and details on the systematic error analysis. In Section IV we extract the differential cross section for charm production at \(y = 1.6\), integrated over \(p_T\). Finally, in Section V we compare to other measurements, draw conclusions, and discuss the prospects for such measurements with improved data sets currently under analysis.

II. THE PHENIX EXPERIMENT

The PHENIX experiment \(^{54}\), shown in Figure II is a large multipurpose set of detectors optimized for measuring relatively rare electromagnetic probes (photons, muons, and electrons) of the spin structure of the proton and of the hot dense matter created in ultrarelativistic heavy ion collisions. The data acquisition system and multilevel triggers are designed to handle the very different challenges presented by \(p + p\) collisions (relatively small events at very high rates) and \(Au + Au\) collisions (very large events at relatively low rates) with little or no deadtime \(^{53, 56}\). Event characterization devices, such as the Beam-Beam Counters \(^{57}\) used in this analysis, provide information on the vertex position, start time, and centrality of the collision. The two muon arms cover 1.2 < \(|\eta|\) < 2.4 in pseudorapidity and \(\delta\phi = 2\pi\) in azimuth. The two central arms, which each cover \(|\eta| < 0.35\) and \(\delta\phi = \pi/2\), are not used in this analysis.

The Beam-Beam Counters (BBCs) \(^{57}\) each consist of 64 quartz radiators instrumented with mesh dynode PMTs and arranged in a cylinder coaxial with the beam. The BBCs are placed on either side of the collision vertex and cover 3.0 < \(|\eta|\) < 3.9. Each channel has a dynamic range extending to 30 MIPs. The BBCs measure the arrival times of particles on both sides of the collision vertex, \(t_S\) and \(t_N\). From the average of these times we determine the event start time. From their difference we obtain the position of the vertex along the beam direction, \(z_{vtx}\). The BBCs also provide the minimum bias interaction trigger, which requires that there be at least one hit in each BBC and that \(|z_{vtx}| < 38\) cm.

The muon arms \(^{53}\) are coaxial with the beam on opposite sides of the collision vertex. By convention the arm on the South (North) end of the interaction region is assigned negative (positive) \(z\) coordinates and rapidity. For the 2001/2 run period, in which the data for this paper were collected, only the South muon arm was operational. Each muon arm is comprised of a Muon Tracker (MuTR) and a Muon Identifier (MuID). The MuTR makes an accurate measurement of particle momenta. The MuID allows coarse resolution track reconstruction through a significant amount of steel absorber. Together the muon arm detectors provide significant pion rejection (> 250 : 1, increasing with decreasing momentum) through a momentum/penetration-depth match.

Before ever reaching the MuTR detectors a particle must pass through the pre-MuTR absorber: 20 cm of copper (the nosecone) plus 60 cm of iron (part of the MuTR magnet). The nominal nuclear interaction lengths of iron and copper are \(\lambda_F^{Fe} = 16.7\) cm and \(\lambda_F^{Cu} = 15.3\) cm (although this varies with particle species and energy, see Section III.F). Therefore the pre-MuTR absorber presents a total thickness of \(4.9\lambda_F/\cos\theta\), where \(\theta\) is the polar angle of a particle’s trajectory. This absorber greatly reduces the MuTR occupancy and provides the first level of pion rejection.

Each MuTR arm consists of three stations of cathode strip chambers installed in an eight-sided conical magnet \(^{52}\). The radial magnetic field \((\int B \cdot dl = 0.72\) T·m at 15 degrees, \(B(\theta) \approx B(15^\circ) \tan(\theta)/\tan(15^\circ)\)) bends particles in the azimuthal direction. Each station occupies a plane perpendicular to the beam axis and consists of multiple ionization gaps (3 gaps for the two stations closest to the collision vertex, 2 gaps for the last station) which have their charge imaged on two cathode strip planes, one hit in each BBC and that \(|z_{vtx}| < 38\) cm.

The MuID arm consists of five steel absorber plates\(^ {53}\) which decay into a muon, and hadrons which penetrate through the muon arm absorber material. In the absence of new physics, and in the \(p_T\) range measured in this analysis, such muons come dominantly from the decay of hadrons containing a charm quark (with small contributions from decays of hadrons containing a bottom quark and decays of light-vector mesons). PYTHIA is used to determine the charm quark spectrum that would produce the observed vertex-independent muon signal, and extract the differential cross section for charm production at forward rapidity.

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three-sided ground electrode and an anode wire, mounted inside a PVC gas enclosure. A readout channel ("two-pack") is formed by wire-ORing the 16 anode wires of two tubes which are mounted in planes perpendicular to the beam axis and staggered by half of a cell width (0.5 cm). This provides redundancy, eliminates geometric inefficiency due to the cell walls, and reduces the maximum drift time for charge collection. Digital readout of the two-pack signals provides a coarse one-dimensional hit position ($\sigma = 9 \text{ cm}/\sqrt{12} = 2.6 \text{ cm}$). The tubes in each gap are mounted in six individual panels, each of which contains two layers of two-packs (horizontally and vertically oriented), thus providing two-dimensional information.

The first MuID absorber plate (thickness = 20 cm - South; 30 cm - North) also serves as the return yoke of the MuTR magnet. Successive plates (identical for the two arms) are 10, 20 and 20 cm thick, thus totaling $4.8\lambda_f/\cos\theta$ (5.4$\lambda_f/\cos\theta$) for the South (North) arm. Due to ionization energy loss a particle must have a momentum at the vertex which exceeds $2.31 \cos\theta \text{ GeV}/c$ ($2.45 \cos\theta \text{ GeV}/c$) to penetrate to the most downstream MuID gap of the South (North) arm.

Steel plates surrounding the beam pipe guard against backgrounds caused by low-angle beam-beam collision products which scrape the beam pipe near the MuID z-location (7-9 m) or shine off the RHIC DX magnets immediately downstream of each MuID arm. Steel blocks in the RHIC tunnels guard against penetrating radiation generated by the incoming beams scraping against beamline components (primarily the final focusing quadrupoles).

The MuID also contributes information to the first-level trigger decision. For the 2001/2 run, during which the data for this analysis were collected, a relatively coarse trigger was implemented using LeCroy 2372 Memory Lookup Units (MLUs). Each gap was divided into quadrants with a horizontal and vertical split going through the beam axis. Signals from tubes in an individual gap/orientation (layer) and quadrant were logically combined. Only gaps 0, 2, 3 and 4 were used in the trigger due to the 16-bit input limitation of the MLUs. The penetration depth required for the trigger to fire was programmable. The MuID-1Deep trigger fired if more than 6 out of 8 layers in a particular quadrant were hit (indicating the possibility that the event contained a particle penetrating to MuID gap 4). The MuID-1Shallow trigger fired if 3 of the 4 most shallow layers (horizontal and vertical layers in gaps 0 and 2) were hit for a particular quadrant.

III. METHOD FOR EXTRACTION OF MUONS FROM CHARM DECAY

Inclusive muon candidates, $N_I$, are those particles which are successfully reconstructed to the last MuID gap (gap 4). These consist of four components: 1) “free-decay muons”, $N_D$, which result from the decay of light hadrons ($\pi$ and $K$ mesons) before reaching the pre-MuTR absorber, 2) “punchthrough hadrons”, $N_P$, which penetrate the entire detector and are thus misidentified as muons.
3) “background tracks”, \(N_B\), which in \(p + p\) collisions are dominated by hadrons which decay into a muon after reaching the MuTR, and 4) “vertex-independent muons”, \(N_{\mu}\), which are primarily due to the decay of heavy flavor mesons.

Figure 2 shows a schematic depiction of the relative yield per event of these different contributions as a function of flightpath into the muon arms, as described below.

The number of hadrons is large and essentially independent of flightpath until the first absorber layer is reached. In each absorber layer these hadrons undergo strong interactions with a probability \(1 - \exp(-L/\lambda)\), where \(L\) is the length of absorber material traversed, and \(\lambda\) is the species and \(p_T\)-dependent nuclear interaction length determined in Section III F. Most of these interacting hadrons are effectively eliminated as possible muon candidates. However, a small fraction of hadrons penetrate the entire absorber thickness. These punchthrough hadrons are indistinguishable from muons.

The decay lengths for \(\pi^0\)’s (\(c\tau = 780\) cm) and \(K^0\)’s (\(c\tau = 371\) cm) are long compared to the flightpath from the vertex to the absorber. Therefore, the fraction of decay muons from these sources is relatively small, but increases linearly with the flightpath until the first absorber layer is reached. A hadron which decays prior to the pre-MuTR absorber into a muon that is inside the detector acceptance is indistinguishable from a muon originating at the vertex. After the first absorber layer the number of free-decay muons remains constant by definition.

Hadrons which decay in the MuTR volume are a relatively small contribution since most are absorbed prior to reaching the MuTR, the Lorentz-dilated decay lengths are long compared to the length of the MuTR volume (South \(\approx 280\) cm, North \(\approx 420\) cm), and a particle which decays in the MuTR is less likely to be reconstructed. Such tracks are partially accounted for in the calculation of punchthrough hadrons (see Section III F) and the remaining fraction falls under the category of background tracks (Section III G). This small contribution is not shown.

Without a high-resolution vertex detector muons from various sources (the decay of open heavy flavor hadrons, the decay of quarkonia, the decay of light vector mesons, and Drell-Yan production) originate indistinguishably close to the collision vertex. Thus their yield is independent of the flightpath and independent of the vertex position. Since inclusive muon candidates, by definition, penetrate to MuID gap 4, we measure the combined yield at \(z \approx 870\) cm.

Figure 2 shows a sample distribution of the inclusive muon candidate yield as a function of collision vertex (\(z_{\text{vtx}}\)), and its decomposition into the four different contributions. The yield of free-decay muons is seen to have a linear dependence that is set to 0 at \(z_{\text{vtx}} = z_{\text{abs}} - \lambda_D\). Here \(z_{\text{abs}} = -40\) cm is the upstream face of the pre-MuTR absorber (indicated by the thick solid line), and \(\lambda_D\) is the effective absorption length, beyond which there are no free-decay muons. \(\lambda_D\) was found to be nearly iden-
of punchthrough hadrons and vertex-independent muons also have no \( z_{vtx} \) dependence. Note that the ratio of different contributions to the inclusive muon candidate spectrum is \( p_T \) dependent.

In order to extract the cross section for charm production we first need to determine the yield of vertex-independent muons, \( N_\mu(p_T) \), the amount beyond that due to light hadrons and fake backgrounds. As described in Section III A we select good runs, events, and tracks, and restrict our acceptance to regions where the detector performance was optimal, and the acceptance vs. \( z_{vtx} \) was nearly constant. Next, as described in Sections III B and III C we obtain the yield of inclusive muon candidates vs. \( p_T \) and \( z_{vtx} \), corrected for acceptance and efficiency: \( N_I(p_T, z_{vtx}) \). In Section III D we describe a data-driven hadron generator. This generator is used in Section III E in which we describe how the vertex dependence of the inclusive muon candidate yield allows us to obtain the yield of muons from light-meson decay before the pre-MuTR absorber, similarly corrected and binned: \( N_D(p_T, z_{vtx}) \). This generator is also used in Section III F in which we describe how we use hadrons which stop in MuID gap 3 (the penultimate gap), together with simulations of hadron penetration in the MuID absorber, to obtain the yield of punchthrough hadrons in MuID gap 4: \( N_P(p_T, z_{vtx}) \). The yield of fake tracks, \( N_{FB}(p_T, z_{vtx}) \), determined from simulations described in Section III G is found to be small.

The yield of vertex-independent muons is determined by subtracting the contributions from light hadrons and fake backgrounds and averaging over \( z_{vtx} \) bins:

\[
N_\mu(p_T) = \frac{1}{N_{\mu, vtx}} \sum_{j=1}^{N_{\mu, vtx}} N_I(p_T, z_{vtx}^j) - N_D(p_T, z_{vtx}^j) - N_P(p_T, z_{vtx}^j) - N_{FB}(p_T, z_{vtx}^j),
\tag{2}
\]

where \( d^2/2\pi p_T dp_T d\eta \) is implicit in all terms of the equation.

We convert this into a cross section via

\[
\frac{d^2\sigma_\mu(p_T)}{2\pi p_T dp_T d\eta} = \frac{\sigma_{pp}^{BB}}{\epsilon_{BBC}^{c,\rightarrow\mu}} \frac{d^2N_\mu(p_T)}{2\pi p_T dp_T d\eta}.
\tag{3}
\]

Here \( \sigma_{pp}^{BB} \) is the cross section of the BBC trigger for \( p + p \) interactions and \( \epsilon_{BBC}^{c,\rightarrow\mu} \) is the efficiency of the BBC trigger for events in which a charm quark is created and decays into a muon. Substituting \( p \rightarrow p \) introduces negligible error due to the small mass of the muon, the only component left after the subtraction. As described in Section III H systematic errors are determined for each component and combined into a term that applies to the overall normalization and a term that applies to the \( p_T \) dependence of the spectrum.

We use PYTHIA to derive the \( p_T \)-dependent differential cross section for the production of charm quarks responsible for the vertex-independent muon yield. This procedure is very similar to that in references [5, 6, 7, 8, 9, 10, 11, 12, 13], and is described in detail, along with the associated systematic error analysis, in Section IV

\section{A. Data Reduction}

\subsection{1. Data Sets and Triggering}

Runs were selected for this analysis based on stable detector operation using the same criteria as an earlier analysis of \( J/\psi \) production [51]. Further runs were eliminated due to the presence of large beam-related back-
grounds entering the back of the detector.

We select only those events in the vertex range \(-20 < z_{vtx} < 30\) cm. This minimizes the \(z_{vtx}\) dependence of the detector acceptance and allows us to treat the amount of absorber material as a simple function of polar angle, ignoring complications in the pre-MuTR absorber near the beampipe.

The decision to collect an event was made by the Local Level-1 Trigger \([55]\) within 4 \(\mu s\) of the collision. Input to the trigger decision was given by the BBC (collision with a valid event vertex) and the MuID (reconstructed penetrating track). Each trigger could be independently scaled down, so that it occupied a predetermined fraction of the available bandwidth, by selecting every \(N_i\)th instance, where \(N_i\) is the scaledown factor for the \(i\)th trigger. Three different data sets were selected from the good runs for different aspects of the data analysis:

- **BBC**: To extract \(N_D\) we need to measure the \(z_{vtx}\) dependence of \(N_I\). For this we need the unbiased collision vertex distribution, which we obtain from a set of events collected with the BBC trigger: \(N_{\text{BBCN}} > 1, \&\& N_{\text{BBCS}} > 1, \&\& |z_{vtx}| < 38\) cm, where \(N_{\text{BBCN}}\) and \(N_{\text{BBCS}}\) are the number of hits in the North and South BBC respectively. \(\sigma_{BB\text{C}}\) was found to be \(21.8 \pm 2.1\) nb using a van der Meer scan \([60]\). There were \(1.72 \times 10^7\) BBC triggered events passing our vertex selection criteria in this data set, corresponding to a sampled luminosity of \(\int L dt = 0.79\) nb\(^{-1}\).

- **MuID-1Deep &\& BBC (MID)**: In order to extract \(N_I, N_D\) and \(N_B\) we used events selected with the MID trigger, which enriched the sample of events with tracks penetrating to MuID gap 4. For the MID and BBC data sets we used identical run selection criteria. The total number of sampled BBC triggers for this data set was \(5.77 \times 10^5\), corresponding to a sampled luminosity of \(\int L dt = 26.5\) nb\(^{-1}\).

- **MuID-1Shallow &\& BBC (MIS)**: In order to extract \(N_P\) we need a data set which provides an unbiased measurement of the number of particles which penetrate to MuID gap 3. Since the MID trigger required tracks to penetrate to MuID gap 4 it could not be used. Instead we used the MIS trigger, which only used information from MuID gaps 0-2. We used a subset of runs for which the scaledown factor for this trigger was only 10, corresponding to a sampled luminosity of \(\int L dt = 1.72\) nb\(^{-1}\).

2. **Track Selection**

The Muon arm reconstruction algorithm starts by finding “roads” (collections of hits in the MuID which form straight, two-dimensional lines) and then combining them with hits in the MuTR to form “tracks”. We apply strict cuts on both road and track parameters in order to reduce backgrounds, see Table I.

The resulting purity of the selected tracks is demonstrated in Figure 4. This figure shows \(p \delta \theta\), the angular deviation through the pre-MuTR absorber, scaled by the particle’s momentum, \(p \delta \theta\). For muons (and hadrons not undergoing a strong interaction in the pre-MuTR absorber) one expects the distribution of this quantity to be well described by the standard multiple scattering formula. The different panels show \(p \delta \theta\) for different \(p_T\) bins with fits (normalization only) to the expected distribution.
TABLE I: Road and track cuts. Here $D_p$ is the penetration depth, defined to be the most downstream MuID gap with at least one hit (from the horizontal or vertical layer) associated with the track; $z$ is the coordinate along the beam; $x$ and $y$ are transverse to each other and to the beam axis; the vertex cut refers to the transverse position of the MuID road projected to the $xy$ plane at $z = 0$; and the slope cut refers to the direction cosine of the road in each transverse direction.

| Road cuts | Track cuts |
|-----------|------------|
| Vertex cut, $\sqrt{x^2 + y^2} < 100$ cm @ $z = 0$ | Track fit quality, $\chi^2$/dof $\leq 10$ |
| Slope cut, $\sqrt{\left(\frac{dy}{dz}\right)^2 + \left(\frac{dx}{dz}\right)^2} > 0.25$ | $\#$ Associated MuTR hits, $N_{MuTR} > 12$ |
| $\geq 1$ associated hit in MuID gap 4 | (out of a possible 16) |

FIG. 5: The angular deflection, $\delta\theta$ is the angular difference between the reconstructed particle trajectory at the collision vertex and at MuTR station 1. The momentum used to scale $\delta\theta$ is the average of the momentum reconstructed inside the MuTR magnet ($p_{vtx}$) and the momentum extrapolated to the vertex ($p_{vtx}$).

\[
\left(\sqrt{2}\right)(13.6 \text{ MeV/c})/(4\sqrt{K}) \text{ rad} = 133 \text{ rad-MeV/c}.
\]

The integral beyond $3\sigma_{\delta\theta}$ is $\approx 5\%$ and is largely due to hadrons which have a strong interaction in the pre-MuTR absorber and are still reconstructed as a muon candidate. Such tracks are accounted for in the calculation of the punchthrough hadron yield, as described below.

3. Acceptance Restriction

We further restricted the acceptance of muon candidates for this analysis in two ways:

1. We required tracks to pass through $\theta/\phi$ regions in which the Monte Carlo detector response strictly agreed with the measured response. This was established by agreement between the number of data hits and Monte Carlo hits assigned to tracks in each $\theta/\phi$ region of the detector.

2. We required tracks to lie within a pseudorapidity range, $1.5 < |\eta| < 1.8$, a region over which the acceptance depends only weakly on the collision $z_{vtx}$ location.

B. Acceptance and Efficiency

We factorized the acceptance and efficiency for tracks penetrating to a particular MuID gap, $i$, into four components:

1. $\varepsilon_{\text{acc}}^i$: the acceptance of a perfectly working detector with the acceptance restrictions described above. This quantity ($\approx 50\%$) is normalized to $2\pi \delta\eta$ ($\delta\eta = 0.3$) and accounts for non-sensitive structural members in between the cathode strip chambers and chamber regions removed from consideration for the entirety of this analysis.

2. $\varepsilon_{\text{rec}}^i$: the efficiency of reconstructing a track that fell within the accepted region. This quantity is somewhat low (64%) due to detector problems in this first physics run that have been subsequently resolved.

3. $\varepsilon_{\text{user}}^i$: the efficiency for reconstructed tracks to pass the cuts listed in Table I.

4. $\varepsilon_{\text{trig}}^i$: the efficiency of the MuID trigger to fire in events with selected tracks.

$\varepsilon_{\text{acc}}^i$, $\varepsilon_{\text{rec}}^i$, and $\varepsilon_{\text{user}}^i$ were evaluated with a GEANT simulation using single muons thrown with a realistic $p_T$ spectrum into the muon arms. The applied detector response incorporated measured detector performance. Reductions in efficiency due to occupancy are negligible in $p + p$ collisions. Run-to-run variations were ignored since we selected runs in which the detector performance was similar and stable. Efficiency values for tracks penetrating to MuID gap 4 were parameterized in terms of $z_{vtx}$ and $p_T$ and are listed in Table III. There are minor differences in these parameterizations for particles with different charge sign.

We also determined the efficiencies for tracks which only penetrate to MuID gap 3, since these are needed to obtain the yield of punchthrough hadrons. These were found to scale from the efficiencies for tracks penetrating to MuID gap 4: $\varepsilon_{\text{acc}}^3 \varepsilon_{\text{rec}}^3 \varepsilon_{\text{user}}^3 = \varepsilon_{\text{scale}}^3 \varepsilon_{\text{acc}}^4 \varepsilon_{\text{rec}}^4 \varepsilon_{\text{user}}^4$, where $\varepsilon_{\text{scale}}^3 = 0.66$, $\varepsilon_{\text{scale}}^3$ is less than one because the MuID and the road reconstruction algorithm are optimized for deeply penetrating particles. Particles which do not penetrate to the last gap have poorer resolution matching to MuTR tracks (due to reduced lever arm and smaller number of hits) and are also more susceptible to MuID inefficiencies.
TABLE II: Trigger, acceptance, track reconstruction and track selection efficiencies. Systematic errors for these quantities are given in Tables V and VII.

| Quantity | Value |
|----------|-------|
| $\varepsilon^{4+}_{\text{acc}}$ | $0.51 \times (1 - 11 \exp(-5.9p_T)) \times (1 + 0.0015z_{\text{vtx}})$ |
| $\varepsilon^{-}_{\text{acc}}$ | $0.50 \times (1 - 531 \exp(-7.5p_T)) \times (1 + 0.0013z_{\text{vtx}})$ |
| $\varepsilon^{4+}_{\text{rec}}$ | 0.64 |
| $\varepsilon^{-}_{\text{rec}}$ | 0.64 |
| $\varepsilon^{4+}_{\text{user}}$ | $0.74 \times (1 - 0.0019z_{\text{vtx}})$ |
| $\varepsilon^{-}_{\text{user}}$ | $0.74 \times (1 - 0.0009z_{\text{vtx}})$ |
| $\varepsilon^{4}_{\text{scale}}$ | 0.66 |
| $\varepsilon^{3}_{\text{trig}}$ | 0.86 |
| $\varepsilon^{3}_{\text{BBC}}$ | 0.97 |
| $\varepsilon^{4}_{\text{trig}}$ | 0.75 |

Trigger efficiencies, $\varepsilon^{4}_{\text{trig}}$ and $\varepsilon^{4}_{\text{trig}}$, are also listed in Table III. These were evaluated using the BBC data set, which did not require the MuID trigger to fire.

$$\varepsilon^{4}_{\text{trig}} = \frac{N_{\text{M1D}} \times S_{\text{M1D}}}{(N_{\text{BBC}} \times S_{\text{BBC}}),}$$

where $N_{\text{M1D}}$ is the number of selected tracks penetrating to MuID gap 4 for events in which the M1D trigger fired, $S_{\text{M1D}}$ is the scaledown factor applied to the M1D trigger, and similarly for the BBC trigger. $\varepsilon^{3}_{\text{trig}}$ was also evaluated according to Equation 4 but with $N_{\text{M1D}} \rightarrow N_{\text{M1S}}$ and $\varepsilon^{3}_{\text{trig}} \rightarrow \varepsilon^{3}_{\text{BBC}}$.

Since both the M1D and M1S triggers required the BBC trigger in coincidence with the MuID trigger we must also account for the BBC trigger efficiency for events in which a reconstructed muon is created via charm quark decay: $\varepsilon^{3}_{\text{BBC}}$. The BBC efficiency was evaluated for events in which a J/$\psi$ was produced in the muon arm acceptance using PYTHIA+GEANT simulations 50. The BBC efficiency was also evaluated for events in which $\pi^0$s were produced in the central arm acceptance 50 using data triggered without a BBC requirement. The BBC efficiency under both conditions was found to have a similar value that we therefore adopt: $\varepsilon^{3}_{\text{BBC}} = 0.75$.

Systematic errors for all acceptance and efficiency corrections are discussed in Section IIII and listed in Tables V and VII.

C. Inclusive Muon Candidates

We first form two sets of collision vertex ($z_{\text{vtx}}$) histograms with 10cm bins: one histogram for all interactions selected with the BBC trigger, and a series of histograms for interactions selected with the M1D trigger and having a good muon candidate within a $p_T$ bin (1 < $p_T$ < 3 GeV/c, 200 MeV/c bins). The muon candidate histograms are formed separately for each charge sign. Entries into each histogram are scaled by the appropriate trigger scaledown factor. The muon candidate histograms are divided by the minimum bias histogram to give $N_I(p_T, z_{\text{vtx}})$, as shown in Figure 4. Systematic errors shown in this figure are discussed in Section IIII and listed in Table V.

D. Hadron Generator

In order to determine the contributions to the inclusive muon yield from free-decay muons (Section IIII), and punchthrough hadrons (Section IIII) we make use of a data-driven hadron generator. The input for this generator is obtained from PHENIX measurements in $\sqrt{s} = 200$ GeV $p + p$ collisions at $y = 0$ 63. 64 using the following procedure:

1. $\pi^+$ and $\pi^-$ spectra at $y = 0$ (0 < $p_T$ < 3.5 GeV/c) are fit to a power law. We assume factorization in $y$ and $p_T$ and scale the spectra fit at $y = 0$ according to:

$$N_{\pi^+} = N_{\pi^0}(p_T)\exp((-1.65^2/2\sigma_y^2)), \quad \text{with} \quad \sigma_y = 2.5. \quad \text{This factorization is observed both in PYTHIA and in BRAHMS} \quad 49 \quad \text{data measured at} \quad y = 1 \quad \text{and} \quad y = 2.2.$$

2. We use a similar procedure to obtain the charged kaon yield at $y = 1.65$, but we need to first extrapolate the yield at $y = 0$ beyond the current measurement limit ($p_T < 2$ GeV/c).

We start by forming the isospin averaged $K/\pi$ ratio vs. $p_T$ at $y = 0$. For $p_T < 2$ GeV/c we use charged particles, $(K^+ + K^-)/(\pi^+ + \pi^-)$ 55. We use neutral particles, $K^0/\pi^0$, for $2 < p_T < 6.5$ GeV/c 66. We then fit this combined ratio to the form $f(p_T) = A(1 - B \exp(-Cp_T))$. Next, we normalize this function separately to the $K^+/\pi^+$ and $K^-/\pi^-$ ratios for $p_T < 2$ GeV/c. Finally, we multiply by the corresponding charged pion spectrum to obtain $N_{K^+}^y(p_T)$, our parameterization of the mid-rapidity charged kaon $p_T$ spectra extending out to 3.5 GeV/c.

As with pions, we need to extrapolate this parameterization of the yield at $y = 0$ to obtain the yield at $y = 1.65$. One possibility is to assume boost invariance of the $K/\pi$ ratio. However, PYTHIA gives a slightly narrower rapidity distribution for kaons than for pions, resulting in a kaon yield at $y = 1.65$ that is only 85% of that predicted with the boost invariance assumption. We split the difference between these two assumptions:

$$N_{K^+}^y = 92.5\% N_{K^+}^y(p_T)\exp((-1.65^2/2\sigma_y^2)),$$

where, again, $\sigma_y = 2.5$.

3. The $p$ and $\bar{p}$ spectra are assumed to have the same shape as the pion spectra with normalization factors set to the measured values at $y = 0$, $p_T = 3$ GeV/c (0.4 for $p/\pi^+$, 0.24 for $\bar{p}/\pi^-$) 65.
The exact form used for the $p, \bar{p}$ spectra is unimportant. They obviously do not contribute to the yield of decay muons and their contribution to the yield of punchthrough hadrons is greatly suppressed due to their relatively short nuclear interaction length. Systematic errors associated with this hadron generator are discussed in Section III and listed in Table VI.

### E. Free-Decay Muons

In Figure 6 one can clearly see the linear dependence in the yield of inclusive muon candidates vs. $z_{vtx}$ at low transverse momentum ($p_T < 2 \text{ GeV}/c$). This dependence is due to muons from the decay of abundant light hadrons ($\pi$’s and $K$’s) prior to the first absorber material at $z_{abs} = -40 \text{ cm.}$ We fit these histograms with the function $a + bz_{vtx}$. After multiplying by $dz/dl_p = \cos(<\theta>) = 0.947$ the slope, $b$, and its fit error give, respectively, the yield per unit length of decay flightpath of muons from hadron decay, $dN_D(p_T)/dl_p$, and the statistical error on this quantity. Results are shown in Figure 7. Systematic errors shown in this figure are discussed in Section III and listed in Table VII.

This procedure does not provide a quantitative measure of the decay muon spectrum above $p_T \sim 2 \text{ GeV}/c$, even though a substantial fraction of the inclusive muons are decay muons up to significantly higher $p_T$. This is due to the fact that at high $p_T$ the decay slopes decrease (Lorentz time dilation) as do the statistics, both of which make it more difficult to quantify the decay component directly. In order to extend our estimate of decay muons to higher $p_T$ we use our hadron generator, described in Section III. We simulate the decay of hadrons into the muon arms and obtain predicted $p_T$ spectra (per unit length) of muons from hadron decay separately for each charge sign. We then normalize these predicted spectra to the measured spectra. The normalized predicted spectra are shown as the dashed lines in Figure 7. The predicted spectral shape agrees with the data where we have a statistically significant measurement. The absolute normalization of the prediction is within 7% of the measured value, easily consistent within errors.

We obtain $N_D(p_T, z_{vtx})$ from the product of $dN_D(p_T)/dl_p$ and the average value of the decay flightpath, $l_{fp} = \lambda_D + [z_{vtx} - z_{abs}]/\cos(\theta)$, for each $z_{vtx}$ bin.

### F. Punchthrough Hadrons

A hadron penetrating to MuID gap 4 is impossible to distinguish from a muon. However, we can cleanly
identify hadrons in shallower gaps and then extrapolate their yield to obtain the yield of punchthrough hadrons in MuID gap 4.

Figure 8 shows the longitudinal momentum ($p_z$, the momentum projected onto the beam axis) distribution of particles that stop in MuID gap 3. The sharp peak at $p_z \approx 2.2 \text{GeV}/c$ corresponds to charged particles which stopped because they ranged out in the absorber plate between gaps 3 and 4 (this includes both muons and also hadrons which only suffered ionization energy loss.) The width of this peak is due to the 20 cm (11.4 $X_0$) absorber thickness between MuID gaps 3 and 4, and energy-loss fluctuations in all the preceding absorber layers. Particles at momenta beyond the peak ($p_z > 3 \text{GeV}/c$) form a relatively pure sample of hadrons, with only a small contamination due to inefficiencies in MuID gap 4. 

In order to extrapolate this measured spectrum for hadrons stopping in MuID gap 3 to the spectrum of punchthrough hadrons which penetrate to MuID gap 4 we start by assuming exponential absorption of hadrons entering the muon arm absorber material. With this assumption we obtain an expression for the “gap 3 inclusive yield”, those hadrons that reach at least MuID gap 3:

$$N_{i}^{3}(p_{T}, \theta) = N_{\text{etx}}^{i}(p_{T}, \theta) \exp(-L_{3}(\theta)/\lambda_{2}(p_{T})), \quad (5)$$

where $i$ indicates the contributing hadron species ($\pi^{\pm}, K^{\pm}, p, \bar{p}$), $N_{\text{etx}}^{i}(p_{T}, \theta)$ is the yield at the vertex of the $i$th species, $L_{3}(\theta)$ is the amount of absorber material traversed to reach MuID gap 3, and $\lambda_{2}(p_{T}, \theta)$ is the $p_{T}$-dependent nuclear interaction length of the $i$th species. We can write a similar expression for the punchthrough hadron yield:

$$N_{i}^{p}(p_{T}, \theta) = N_{\text{etx}}^{i}(p_{T}, \theta) \exp(-L_{4}(\theta)/\lambda_{1}(p_{T})), \quad (6)$$

where $L_{4}(\theta)$ is the amount of absorber material traversed to reach MuID gap 4.

By taking the difference between these two equations we obtain an expression for the gap 3 exclusive yield:

$$N_{3}^{\text{stop}}(p_{T}, \theta) = N_{3}^{3}(p_{T}, \theta) - N_{3}^{p}(p_{T}, \theta) = N_{\text{etx}}^{i}(p_{T}, \theta) \exp(-L_{3}(\theta)/\lambda_{2}(p_{T})) \times (1 - \exp((L_{3}(\theta) - L_{4}(\theta))/\lambda_{1}(p_{T}))), \quad (7)$$

In our measurement we cannot identify the species comprising the gap 3 exclusive yield, but we do know their charge sign. As a result, Equation (7) can be rewritten as two equations with six unknowns for each $p_{T}$ bin:

$$N_{3}^{+}\text{stop}(p_{T}, \theta) = N_{3}^{+}(p_{T}, \theta) - N_{3}^{-}(p_{T}, \theta)$$

$$= \sum_{i=\pi^{+}, K^{+}, p, \bar{p}} N_{\text{etx}}^{i}(p_{T}, \theta) \exp(-L_{3}(\theta)/\lambda_{2}(p_{T})) \times (1 - \exp((L_{3}(\theta) - L_{4}(\theta))/\lambda_{1}(p_{T}))), \quad (8)$$

$$N_{3}^{-}\text{stop}(p_{T}, \theta) = N_{3}^{-}(p_{T}, \theta) - N_{3}^{-}(p_{T}, \theta)$$

$$= \sum_{i=\pi^{-}, K^{-}, \bar{p}} N_{\text{etx}}^{i}(p_{T}, \theta) \exp(-L_{3}(\theta)/\lambda_{2}(p_{T})) \times (1 - \exp((L_{3}(\theta) - L_{4}(\theta))/\lambda_{1}(p_{T}))), \quad (9)$$

Based on measured cross sections for various species $67$, we chose to reduce the number of unknowns with the following assumption:

$$\lambda_{K^{+}} = \lambda_{long},$$

$$\lambda_{p} = \lambda_{\pi^{+}} = \lambda_{\pi^{-}} = \lambda_{K^{-}} = \lambda_{short},$$

$$\lambda_{\bar{p}} = 0.$$  

We further assume that $\lambda_{short}$ and $\lambda_{long}$ have the form $a + b(p_{T}[\text{GeV}/c] - 1)$.
FIG. 7: Yield per unit length of (left) positively and (right) negatively charged free-decay muons. Points are the measured values determined by linear fits to the inclusive muon candidate yield (Figure 8). Error bars indicate statistical errors for those fits. p_T bins with a negative (non-physical) slope in those fits are shown with a line at the 90% C.L. (statistical errors only) and an arrow pointing down. See Section III D for details. Dashed lines are the predictions for each sign from a data-driven hadron generator normalized to the measured points. The χ^2/DOF for these fits are quoted. See Section III D for details. The width of the lines corresponds to σ_o (see Table V), the error on the ratio of free-decay muons to inclusive muon candidates.

Shaded bands on each point show the systematic errors that affect the p_T shape of the inclusive muon candidate spectrum. These last two systematic errors (Table V) need to be included in the total error budget for the yield of free-decay muons, σ_o/N (Table V), but are displayed separately since they are common to all components of the inclusive muon candidate yield, see Equations 12 and 13. Systematic errors are discussed in Section III D.

Results of these fits are shown in Figure 8.

With these values for λ_i(p_T) and the hadron generator input spectra, we could directly apply Equation 8 to obtain the final punchthrough spectra. However, we made one further correction, described below, after finding that our assumption of exponential absorption does not hold when applied to GEANT simulations of the punchthrough process.

Using our GEANT-based PHENIX simulation program, we generated data sets with both the FLUKA [68] and GHEISHA [69] hadronic interaction packages. Input spectra for these data sets were given by our decay hadron generator. We selected all particles which did not decay before the pre-MuTR absorber. “Truth” values for the punchthrough and gap 3 exclusive yields were obtained by splitting those particles based on the absence (gap 3 exclusive) or presence (punchthrough) of associated charged particles with η > 100 MeV in MuID gap 4. We varied η from 50 – 300 MeV and saw no significant change in the results.

Using the known input spectra, known values for L_{5,4}(θ), and truth values for gap 3 exclusive yield, we applied Equation 7 to the Monte Carlo data sets to obtain λ_i(p_T). Due to statistical limitations we integrated our results over θ and into two p_T bins: 1 < p_T < 2 GeV/c and p_T > 2 GeV/c. Values extracted for λ(p_T) for the different hadronic interaction packages are listed in Table III. These values are consistent with those found for our measured data, listed above.

Inserting these values for λ(p_T) into Equation 6 we obtained a prediction for N_{p_T}(p_T, θ) for the different hadronic interaction packages are listed in Table IV. One can see that these ratios deviate significantly from unity and that the two hadronic input spectra differ.
TABLE III: Nuclear interaction lengths, $\lambda_i(p_T)$, for different particle species and $p_T$ bins (in GeV/c) for FLUKA and GHEISHA. Statistical errors on these values are $\approx$ factors:

| Species | $1 < p_T < 2$ | $p_T > 2$ | $1 < p_T < 2$ | $p_T > 2$ |
|---------|---------------|-----------|---------------|-----------|
| $\pi^+$ | 19.6          | 24.5      | 16.0          | 21.1      |
| $\pi^-$ | 19.4          | 24.8      | 15.0          | 19.3      |
| $K^+$   | 24.4          | 29.6      | 24.9          | 30.8      |
| $K^-$   | 20.5          | 24.2      | 17.2          | 21.2      |

interaction packages disagree on the direction of the deviation: the exponential absorption model tends to overpredict the punchthrough yield for FLUKA ($R_{N_p(p_T)}^{FLUKA} < 1$) and underpredict it for GHEISHA ($R_{N_p(p_T)}^{GHEISHA} > 1$).

Relatively little data exists in the relevant momentum range that would allow us to conclude which, if either, of the hadronic interaction package is correct. Measurements by RD10 and RD45 [70] of the penetration depth of identified hadrons found that GHEISHA did well for protons and FLUKA did not. But neither did well for pions and no data exists for kaons. Furthermore, the results are sensitive to the definition of a penetrating particle: For RD10/45 an incoming particle with any associated charged particles in the $120 \times 120$ cm$^2$ detector area at a particular depth was defined to have penetrated to that depth. In our measurement we reconstruct particle trajectories and MuID hits are not associated with a road unless they are within a narrow search window surrounding the projected trajectory. Thus we are relatively insensitive to the leakage of a showering hadron.

As a result of these uncertainties on the applicability of our exponential absorption model we incorporate a species and $p_T$-dependent correction factor to Equation 5:

$$N_{i'}(p_T, \theta) = C^i(p_T) N_{i,ext}(p_T, \theta) \exp(-L_4(\theta)/\lambda^i(p_T)).$$

(10)

The correction factors for pions and kaons are obtained from the average of the values of $R_{N_p(p_T)}$ for the two packages, $\langle R \rangle_i(p_T) = \langle R_{N_p(p_T)}^{FLUKA} + R_{N_p(p_T)}^{GHEISHA} \rangle / 2$, which are listed in Table IV. The values of $\langle R \rangle_i(p_T)$ for a given species are not the same for the different $p_T$ bins, so we assume the values are valid at the average $p_T$ of each bin ($p_T = 1.25$ GeV/c and $2.31$ GeV/c respectively) and use a linear extrapolation in $p_T$ to obtain the final correction factors:

$$C^i(p_T) = \langle R \rangle_i(1 < p_T < 2 \text{GeV/c}) + (\langle R \rangle_i(p_T > 2 \text{GeV/c}) - \langle R \rangle_i(1 < p_T < 2 \text{GeV/c})) \frac{p_T \text{[GeV/c]} - 1.25}{2.31 - 1.25}$$

(11)

We assume that $p$'s and $\bar{p}$'s have the same correction factors as the corresponding sign pions. Since $p$'s and
TABLE IV: Ratios, $R^{PLUKA}_{N_{i,T,p}^p}$, of truth values for the punchthrough hadron yield to those predicted assuming exponential hadron absorption for different particle species and $p_T$ bins (in GeV/c), for FLUKA and GHEISHA. Average values of the ratios for the two different hadronic interaction packages, $\langle R \rangle_{i,T,p}$, are smoothed across the $p_T$ bin at 2 GeV/c to obtain correction factors for the exponential absorption model. Statistical errors on these quantities are $\approx 10\%$. The maximum fractional difference in the ratios for the two different packages (32\%) is incorporated into the systematic error estimate, as shown in Table VII.

| Species | $1 < p_T < 2$ | $p_T > 2$ | Description |
|---------|---------------|-----------|-------------|
| $\pi^+$ | 0.76          | 0.86      | $R^{PLUKA}_{N_{i,T,p}^p}$ |
| $\pi^-$ | 0.91          | 0.75      |             |
| $K^+$   | 0.91          | 1.00      |             |
| $K^-$   | 1.17          | 1.06      |             |
| $\bar{\pi}^+$ | 1.48 | 1.04 | $R^{GHEISHA}_{N_{i,T,p}^p}$ |
| $\bar{\pi}^-$ | 1.47 | 1.09 |             |
| $K^+$   | 1.31          | 1.07      |             |
| $K^-$   | 2.21          | 1.69      |             |
| $\bar{\pi}^+$ | 1.12 | 0.95 | $\langle R \rangle_{i,T,p}$ |
| $\bar{\pi}^-$ | 1.19 | 0.92 |             |
| $K^+$   | 1.11          | 1.04      |             |
| $K^-$   | 1.67          | 1.38      |             |
| $\pi^+$ | 32\%          | 10\%      | $\delta R_{N_{i,T,p}^p}/C_{i,T,p}$ |
| $\pi^-$ | 24\%          | 18\%      |             |
| $K^+$   | 18\%          | 3\%       |             |
| $K^-$   | 32\%          | 22\%      |             |

G. Background Tracks

The main source of tracks which are not accounted for in the yield of punchthrough hadrons and free-decay muons, and which are not due to vertex-independent muons, are light hadrons which penetrate the pre-MuTR absorber, decay into a muon, and are still reconstructed as a valid track.

A simulation of single pions thrown into the muon arm acceptance shows that the number of hadrons which decay after the pre-MuTR absorber and penetrate to MuID gap 4 is only 5-10\% (increasing with increasing $p_T$) of the $z_{vtx}$-averaged number of free-decay muons, $N_D(p_T, z_{vtx} = 0)$. This ratio will be suppressed by the...
fact that tracks which decay are less likely to be reconstructed successfully. It is further suppressed by our punchthrough calculation procedure: the number of such tracks which stop in MuID gap 3 is roughly half the number that penetrate to gap 4; these will be counted in our calculation of the punchthrough hadron yield.

We express our estimate for the yield of background tracks not otherwise accounted for as \( N_B(p_T) = 5\% \times N_D(p_T, z_{vtx} = 0) \). The systematic uncertainty assigned to this quantity, \( \pm 5\% \times N_D(p_T, z_{vtx} = 0) \), covers the extreme possibilities that the \( N_B/N_D \) is unsuppressed or fully suppressed by reconstruction and punchthrough procedures, as described above.

This estimate was verified in a simulation of \( \pi^- \)'s and \( K^- \)'s which were thrown into the muon arm acceptance and fully reconstructed. The reconstructed track information, together with the Monte Carlo truth information, allows us to eliminate uncertainties due to mis-reconstruction of the track \( p_T \) and due to determination of whether a track which penetrated to the last gap did so in a reconstructible fashion. Systematic errors on this estimate are discussed in Section III I.

H. Vertex-Independent Muons

Figure 14 shows the yield of inclusive muon candidates, \( N_I(p_T, z_{vtx}) \), with contributions from individual components (free-decay muons, punchthrough hadrons, and background tracks) shown as well as their sum. The vertex-independent muons can be seen as the clear excess above the calculated background sources. The systematic error bands shown on the component sums are discussed in Section III I and listed in Tables VII and VIII.

We obtain the yield of vertex-independent muons by applying Equation 2 in each \( p_T \) bin, subtracting the hadronic contributions from the inclusive muon candidate yield, and averaging over \( z_{vtx} \) bins. This yield is shown, before averaging over \( z_{vtx} \) to demonstrate the expected vertex independence, in Figure 12.

We make one final correction for momentum scale. The observed mass of the \( J/\psi \), reconstructed with the same code and in the same data set, is higher than the nominal value by \( \approx 100 \text{ MeV}(3\%)^{51} \). However, in a higher statistics data set the momentum scale accuracy is verified to within 1\% by our observation of the accepted value for the mass of the \( J/\psi \) \[52\]. Also, the peak observed in the falling spectrum, artificially hardens the measured spectrum. For \( 1 < p_T < 3 \text{ GeV}/c \) this hardening increases the normalization of the yield by \( \approx 3.7\% \). However, this is accounted for in our efficiency determination since we use a realistic \( p_T \) spectrum as input. Therefore we apply no explicit correction for this effect.

The final values for the vertex-independent muon cross section, obtained from Equation 2, are shown in Figure 13. Points in this figure have been placed at the average \( p_T \) value of the bin contents to account for bin shifting in the steeply falling distributions. Systematic errors shown in this figure are discussed in Section III I and listed in Tables VII and VIII.

I. Systematic Errors

Many sources of systematic error on the yield of vertex-independent muons, \( N_\mu \), are common to the different components of the inclusive muon candidate yield. In order to account for this we rewrite Equation 2 (making the \( p_T \) and \( z_{vtx} \) dependencies implicit) as:

\[
N_\mu = N_I \times (N_{IP}/N_I) = N_I \times ((N_I - N_D - N_P - N_B)/N_I) = N_I \times (1 - R_D - R_P - R_B),
\]

where \( R_j = N_j/N_I \) is the fraction of the inclusive muon candidate yield attributed to the \( j \)th component. We can now write the systematic error on \( N_\mu \) as:

\[
\sigma_{N_\mu} = \sqrt{\left(\sigma_{N_I}/N_I\right)^2 N_\mu^2 + \left(\sigma_{N_D}/N_I\right)^2 N_\mu^2 + \left(\sigma_{N_P}/N_I\right)^2 N_\mu^2 + \left(\sigma_{N_B}/N_I\right)^2 N_\mu^2},
\]

(13)

\( \sigma_{N_{IP}} \), as determined below, is displayed in Figures 11 and 12. Note that the errors for positives are significantly larger than for negatives. This is due to the much larger relative contribution to positive inclusive muon candidates from punchthrough hadrons, which is due to the relatively small size of the \( K^+ \) nuclear interaction cross section.

Error sources contributing to \( \sigma_{N_I} \) are quantified in Table V. Error sources contributing to \( \sigma_{N_D} \) and \( \sigma_{R_D} \) are quantified in Table VII. Error sources contributing to \( \sigma_{N_P} \) and \( \sigma_{R_P} \) are quantified in Table VIII. Note that in these tables we separately list errors that affect the overall normalization \( (\sigma/N_{true}) \) and the shape of the \( p_T \) spectrum \( (\sigma/N_{true}) \). The error on \( R_B \) is taken to be 100\% of its estimated value: \( \sigma_{R_B} = N_B/N_I = 0.05 \times N_D(z_{vtx} = 0)/N_I \).

Values for \( \sigma_{N_I}/N_I \) are displayed in Figure 6. Values for \( \sigma_{R_D} \) and \( \sigma_{R_P} \) are displayed in Figures 10 and 7, respectively. We insert \( \sigma_{N_I}/N_I \), \( \sigma_{R_D} \), \( \sigma_{R_P} \), and \( \sigma_{R_B} \) into Equation 13 as part of the final systematic error on \( N_\mu \).

To get the vertex-independent muon cross section, as defined in Equation 8 and displayed in Figures 13 and 14, we need to add in quadrature the errors on \( N_\mu, \sigma_{IP}, \sigma_{IP}, \sigma_{IP}, \sigma_{IP} \cdot \sigma_{IP} \). The error on \( N_\mu \) is obtained from the components above according to Equation 13. As mentioned
FIG. 11: Points show the yield of (left) positively and (right) negatively charged inclusive muon candidates vs. \( z_{vtx} \) for different \( p_T \) bins with statistical errors. Dotted, solid and dashed lines show contributions from decay muons, punchthrough hadrons and background tracks, respectively. Shaded bands show the systematic error around the sum of these components, as listed in Tables V-VII and discussed in Section III.

\[
\eta \frac{dN}{dp_T} \propto \frac{1}{\sqrt{\pi} \sigma^2} \exp\left(-\frac{p_T^2}{\sigma^2}\right)
\]

FIG. 12: Points show the yield of (left) positively and (right) negatively charged vertex-independent muons vs. \( z_{vtx} \) for different \( p_T \) bins with statistical errors. The dashed lines show the yield for each \( p_T \) bin, averaged over \( z_{vtx} \). The shaded bands around 0 show the systematic error on the sum of the contributions to the inclusive muon candidate yield from light-hadronic sources, as listed in Tables V-VII and discussed in Section III.
FIG. 13: $p_T$ spectrum of vertex-independent muons. Error bars indicate statistical errors. One point with unphysical (less than zero) extracted yield is shown as an arrow pointing down from the 90% C.L.U.L. Shaded bands indicate systematic errors, as listed in Tables V-VII and discussed in Section III.

above, $\sigma_{BB\mu}^{pp}$ was determined by a van der Meer scan to be 21.8 mb, with an error of 9.6% [60]. We assign an error of 5% to $\varepsilon_{C\bar{C}\mu}$, which was found to be 0.75 through two different methods [51, 60].

IV. CHARM CROSS SECTION

The charm production cross section obtained from the yield of vertex-independent muons (or from the yield of non-photonic electrons, or $D$ mesons) is necessarily model dependent since we do not measure the charm quarks directly. We use PYTHIA to convert our measurement of the vertex-independent muon yield into an estimate of the differential charm production cross section at forward rapidity, $d\sigma_{c\bar{c}}/dy|_{y=1.6}$, in a procedure very similar to that used in PHENIX measurements of charm production at $y = 0$ [5, 8, 9, 10, 11, 12, 13]. We use PYTHIA version 6.205 with parameters tuned to reproduce charm production data at SPS and FNAL [71] and single electron data at the ISR [72, 73, 74]. Tuned parameters are listed in Table VIII. The meaning of each parameter is more thoroughly defined in the PYTHIA manual [75].

Vertex-independent muon sources, predicted by a PYTHIA simulation using the same parameters (except that MSEL is set to 2 to generate unbiased collisions), are listed in Table IX. These sources include decays of hadrons containing a heavy quark, and light vector mesons with a decay length too short to be measured with the existing experimental apparatus ($\rho, \omega, \phi$). Their $p_T$ spectra are shown in Figure 14. Contributions from quarkonium decays, Drell-Yan and $\tau$ lepton decays are negligible. This shows that vertex-independent muon production in our acceptance is dominated by muons from decay of charm hadrons, although for $p_T > 2.5$ GeV/c the contribution from decays of hadrons containing a bottom quark is starting to become important.

FIG. 14: PYTHIA calculation showing the major contributions to the vertex-independent muon $p_T$ spectrum. Solid, dashed and dotted lines show the yield from charm, bottom and short-decay length light vector mesons ($\rho, \omega, \phi$) respectively.

This simulation also gives the distribution of charm quarks ($p_T$ vs. $y$) that produce a muon in our acceptance, as shown in Figure 15. This demonstrates that the vertex-independent muons we measure sample charm quarks down to $p_T \approx 1$ GeV/c, over a narrow rapidity slice centered at $y = 1.6$.

FIG. 15: PYTHIA results for the $p_T$ vs. $y$ distribution (linear $z$-scale) of charm quarks that produce a muon in the PHENIX acceptance.

Figure 16 shows a comparison of the measured vertex-independent negative muon spectrum (from Figure 13) to the prediction of this default PYTHIA simulation and to a FONLL calculation [20, 77]. One can see that the measured values significantly exceed both predictions.
TABLE V: Sources of systematic error on the calculation of $N_I$, the yield of inclusive muon candidates.

| Error Source          | $\sigma / N_{\text{norm}}$ | $\sigma / N_{\text{PT}}$ | Comment                                                                 |
|-----------------------|----------------------------|---------------------------|-------------------------------------------------------------------------|
| Momentum Scale        | 6.0%                       | $(p_T^{\text{GeV/c}} - 1) \times 1.3$% | Taken to be 100% of the correction.                                     |
| $\varepsilon_{\text{acc}}$ | 10%                       | $(p_T^{\text{GeV/c}} - 1) \times 1.5$% | Taken to be 20% of the correction.                                      |
| $\varepsilon_{\text{rec}}$ | 9.0%                       | 0                         | Taken to be 25% of the correction.                                      |
| $\varepsilon_{\text{user}}$ | 5.0%                       | $(p_T^{\text{GeV/c}} - 1) \times 5.0$% | Error on $\sigma_{\text{norm}}$ taken to be 20% of the correction.     |
| $\varepsilon_{\text{trig}}$ | 4.7%                       | 0                         | Taken to be the difference in $\varepsilon_{\text{trig}}$ obtained with different procedures. |
| $\sigma_{N_I}/N_I$    | 16.3%                      | $(p_T^{\text{GeV/c}} - 1) \times 5.4$% | Add all rows in quadrature.                                             |

The spectrum also appears to be somewhat harder than the PYTHIA spectrum with the parameters listed in Table VIII.

We scale the charm (only) contribution to the PYTHIA vertex-independent muon $p_T$ spectrum such that the total spectrum (including the small contributions from open bottom and vector mesons) matches the central values of the measured vertex-independent negative muon spectrum. Only statistical errors are used in the fit. Note, larger systematic errors for the positive muon spectrum preclude a significant measurement for that charge sign. We multiply the scale factor from the fit (2.27) by the PYTHIA value for the charm production cross section, $d\sigma_{c\bar{c}}/dy_{y=1.6}^{\text{PYTHIA}} (0.107 \text{ mb})$, to obtain $d\sigma_{c\bar{c}}/dy_{y=1.6}^{\text{PHENIX}} = 0.243 \pm 0.013\text{(stat.) mb}$. We distinguish between two different sources of systematic uncertainty on the extraction of the charm cross section: 1) uncertainty in the PYTHIA calculation, and 2) uncertainty in the data, which is largely independent of PYTHIA.

We determined the uncertainty in the data ($\pm 43$%) by refitting PYTHIA to the data at the minimum and maximum of the 1$\sigma$ systematic error band.

We determined the uncertainty in the PYTHIA calculation with a systematic study in which we varied simulation parameters, extracted the new simulated vertex-independent negative muon spectrum, normalized to the measured spectrum, and extracted $d\sigma_{c\bar{c}}/dy_{y=1.6}^{\text{PHENIX}}$ for the modified parameter sets. We varied PDF libraries, the hard scattering scale, the charm quark mass, the
TABLE VIII: Tuned PYTHIA parameters (default settings for this analysis) for determination of charm production cross section central value.

| Parameter | Value | Meaning |
|-----------|-------|---------|
| MSEL      | 4     | Heavy quark production every event (gluon fusion + q/\bar{q} annihilation). |
| MSTP(32)  | 4     | Hard scattering scale, \( Q^2 = \hat{s} \). |
| MSTP(33)  | 1     | Use K-factor. |
| MSTP(52)  | 2     | Use PDF libraries. |
| MSTP(51)  | 4046  | Select CTEQ5L PDF libraries [26]. |
| MSTP(91)  | 1     | Use Gaussian distribution for intrinsic \( k_T \). |
| PMAS(4.1)| 1.25  | \( m_c \) (GeV/c). |
| \( D^+/D^0 \) | 0.32  | Default charm chemistry ratio. |

The PYTHIA charm cross section varies substantially \( \Delta(\sigma_{c\bar{c}}/dy)_{y=1.6}^{PYTHIA} \approx 4 \) for the chosen parameter sets. However, the extracted experimental charm cross section is relatively stable \( \Delta(\sigma_{c\bar{c}}/dy)_{y=1.6}^{PHENIX} < 0.36 \). This is due to the fact that the parameter set changes have relatively minor effects on the shape of the predicted vertex-independent muon \( p_T \) spectrum, and we obtain the experimental charm cross section by normalizing the PYTHIA charm cross section by the ratio of the measured and predicted muon \( p_T \) spectra.

One way to visualize this is to plot (see Figure 17) the vertex-independent muon yield in our acceptance per event in which a \( c\bar{c} \) pair is created for the different PYTHIA parameter sets. Due to our procedure, parameter sets which give similar vertex-independent muon yields per \( c\bar{c} \) event in the low \( p_T \) region (which dominates the fit) will necessarily give similar values for \( d\sigma_{c\bar{c}}/dy|_{y=1.6}^{PHENIX} \), whatever the PYTHIA charm cross section is.

The largest variation in the predicted muon yield at \( p_T = 1 \) GeV/c per \( c\bar{c} \) event is seen for simulations in which the intrinsic \( k_T \) is varied from its default value \( < k_T >= 1.5 \) GeV/c to the value expected from arguments based on Fermi momentum (case 4c, \( < k_T >= 0.3 \) GeV/c), or to a value which best reproduces the measured spectrum at higher \( p_T \) (case 4d, \( < k_T >= 3.0 \) GeV/c). These parameter sets also result in the largest variation in \( d\sigma_{c\bar{c}}/dy|_{y=1.6}^{PHENIX} \), as shown in Table X. We use the cross section values obtained in this pair of simulations to define the systematic uncertainty in our measurement due to the uncertainty in our PYTHIA calculation. This gives us our final answer: \( d\sigma_{c\bar{c}}/dy|_{y=1.6} = 0.243 \pm 0.013 \) (stat.) \( \pm 0.105 \) (data syst.) mb.

Figure 18 shows the PHENIX charm rapidity spectrum. The result of this analysis (mirrored about \( y = 0 \))

\[ p+p \rightarrow \mu + X \ (y = -1.6, \sqrt{s} = 200 \text{ GeV}) \]
since this is a symmetric collision system) is plotted along with the result for $d\sigma_{c\bar{c}}/dy|_{y=0}$ [13]. In order to compare with the data at $y = 0$ the systematic uncertainty on the data from this analysis is shown as the quadrature sum of the two sources of systematic uncertainty described above (data and PYTHIA). Theoretical curves from PYTHIA (case 1 and case 5a), FONLL [20, 77], and an NLO calculation from Vogt [83] are also displayed.

In the top panel of the figure PYTHIA with the default parameter set (Case 1) is fit to the two PHENIX points with statistical and systematic errors added in quadrature. Other theory curves are normalized so that they are equal at $y = 0$ in order to allow shape comparisons. As shown in Table X, different PYTHIA parameter sets differ in the predicted ratio $d\sigma_{c\bar{c}}/dy|_{y=1.6}^{PYTHIA}/d\sigma_{c\bar{c}}/dy|_{y=0}^{PYTHIA}$ by > 30%. Unfortunately, current systematic error bars preclude any conclusions about the charm production rapidity shape.

In the bottom panel of the figure the theory curves are unnormalized to allow an absolute comparison. The quoted theoretical uncertainty bands for the FONLL and NLO calculations are also shown. We note that, although our data are above the FONLL prediction, the error bars touch. This is in contrast to the situation for the vertex-independent muon cross section, shown in Figure 16, where the data are significantly above the prediction. The larger disagreement in the vertex-independent muon cross section is presumably due to different treatment of the fragmentation process in PYTHIA and FONLL [20, 77, 83].

V. CONCLUSION AND OUTLOOK

We have measured muon production at forward rapidity ($1.5 \leq |y| \leq 1.8$), in the range $1 < p_T < 3$ GeV/c, in $\sqrt{s} = 200$ GeV $p + p$ collisions at RHIC. We determined and subtracted the contribution from light hadron sources ($\pi, K, p$) to obtain the vertex-independent muon yield which, for the $p_T$ range measured in this analysis, and in the absence of new physics, arises dominantly from the decay of $D$ mesons.

We normalized the PYTHIA muon spectrum resulting from the production of charm quarks to obtain the differential cross section for charm production at forward rapidity: $d\sigma_{c\bar{c}}/dy|_{y=1.6} = 0.243 \pm 0.013(\text{stat.}) \pm 0.105(\text{data syst.})^{+0.045}_{-0.087}(\text{PYTHIA syst.})$ mb. This is compatible with PHENIX charm measurement at $y = 0$, although even further above predictions from PYTHIA and FONLL. Large systematic uncertainties in the current measurement preclude statements about the rapidity dependence of the charm cross section.
TABLE X: Results for PYTHIA simulations with different parameter sets used to explore the systematic error on the charm cross section due to model uncertainties. The top of the table details the different parameter sets tested. Unless otherwise noted, parameters are the same as those listed in Table VIII. The bottom of the table gives the results for different simulations: The 1st column identifies the simulation; the 2nd column gives the total charm production cross section given the chosen PYTHIA parameter set; the 3rd column gives the differential charm production cross section at $y = 1.6$; the 4th column gives the normalization factor needed to fit the PHENIX data; the 5th column gives the differential charm production cross section at $y = 1.6$ for PHENIX data (the product of the 3rd and 4th columns); the 6th column gives the fractional difference between the results for each simulation compared to the simulation with the default PYTHIA parameter set; the last column gives the ratio $d\sigma_{c\bar{c}}/dy|_{y=1.6}/d\sigma_{c\bar{c}}/dy|_{y=0}$. 

| Case | PYTHIA Settings |
|------|-----------------|
| 1    | Default settings, see Table VIII |
| 2a   | MSTP(51) = 4032, CTEQ4L PDF libraries [78]. |
| 2b   | MSTP(51) = 5005, GRV94LO PDF libraries [79]. |
| 2c   | MSTP(51) = 5012, GRV98LO PDF libraries [80]. |
| 2d   | MSTP(51) = 3072, MRST (c-g) PDF libraries [81]. |
| 3a   | MSTP(32) = 1, $Q^2 = (s^2 + t^2 + u^2)/(s^2 + t^2 + u^2)$. |
| 3b   | MSTP(32) = 2, $Q^2 = p_T^2 + (m_3^2 + m_4^2)/2$. |
| 3c   | MSTP(32) = 3, $Q^2 = \min(-t, -u)$. |
| 3d   | MSTP(32) = 5, $Q^2 = -t$. |
| 4a   | PMAS(4,1) = $m_c = 1.15$ GeV/c. |
| 4b   | PMAS(4,1) = $m_c = 1.35$ GeV/c. |
| 4c   | PARP(91) = $< k_T > = 0.3$ GeV/c. |
| 4d   | PARP(91) = $< k_T > = 3.0$ GeV/c. |
| 4e   | MSTP(68) = 2, Maximum virtuality scale and matrix element matching scheme. PARP(67) = 4, Multiplicative factor applied to hard scattering scale. |
| 5a   | PARP(31) = $K$-factor = 1, MSEL = 1, Hard scattering enabled. |
| 5b   | PARP(31) = $K$-factor = 1, MSEL = 1, Hard scattering enabled, All other parameters untuned. |
| 6    | $D^+/D^0 = 0.45$ [82]. |
| 7    | Open bottom and vector mesons scale with charm. |

| Case | PYTHIA Settings | $\sigma_{c\bar{c}}^{\text{PYTHIA}}$ (mb) | $d\sigma_{c\bar{c}}/dy|_{y=1.6}^{\text{PYTHIA}}$ (mb) | Normalization to Data | $d\sigma_{c\bar{c}}/dy|_{y=1.6}^{\text{PHENIX}}$ (mb) | $\Delta d\sigma_{c\bar{c}}/dy|_{y=1.6}^{\text{PHENIX}}$ (%) | $d\sigma_{c\bar{c}}/dy|_{y=1.6}^{\text{PYTHIA}}/d\sigma_{c\bar{c}}/dy|_{y=0}^{\text{PYTHIA}}$ |
|------|-----------------|-------------------------------|----------------|----------------|-------------------|-------------------|-----------------------------|
| 1    | 0.658 0.107     | 2.27                          | 0.243          | –              | 0.67              |
| 2a   | 0.691 0.111     | 2.10                          | 0.232          | -4.5           | 0.69              |
| 2b   | 0.698 0.112     | 2.09                          | 0.233          | -3.9           | 0.71              |
| 2c   | 0.669 0.109     | 2.18                          | 0.238          | -1.7           | 0.73              |
| 2d   | 0.551 0.088     | 2.67                          | 0.236          | -2.9           | 0.71              |
| 3a   | 1.520 0.243     | 1.12                          | 0.271          | 11.8           | 0.84              |
| 3b   | 0.863 0.139     | 1.63                          | 0.226          | -6.7           | 0.71              |
| 3c   | 1.501 0.242     | 1.11                          | 0.267          | 10.2           | 0.84              |
| 3d   | 1.104 0.178     | 1.45                          | 0.258          | 6.4            | 0.78              |
| 4a   | 0.905 0.145     | 1.73                          | 0.252          | 3.7            | 0.67              |
| 4b   | 0.487 0.078     | 2.91                          | 0.226          | -6.7           | 0.64              |
| 4c   | 0.658 0.104     | 2.81                          | 0.292          | 20.4           | 0.66              |
| 4d   | 0.658 0.104     | 1.50                          | 0.156          | -35.8          | 0.63              |
| 4e   | 0.658 0.106     | 2.09                          | 0.220          | -9.2           | 0.63              |
| 5a   | 0.435 0.068     | 3.91                          | 0.266          | 9.4            | 0.80              |
| 5b   | 0.385 0.058     | 4.67                          | 0.271          | 11.7           | 0.79              |
| 6    | 0.658 0.107     | 2.38                          | 0.255          | 5.0            | 0.67              |
| 7    | 0.658 0.107     | 2.20                          | 0.236          | -2.9           | 0.67              |
The systematic uncertainty in the data is dominated by uncertainty on the determination of the fractional contribution of decay muons. This will be improved with higher statistics data sets (already collected) which will allow better measurements of the $z_{\text{vtx}}$ dependence of particle production. Final results for identified particle $p_T$ distributions in $p + p$ collisions by BRAHMS will also be invaluable for improving the input to our hadron generator. The systematic uncertainty in PYTHIA is dominated by differences observed when the intrinsic $< k_T >$ is varied. In order to reduce this uncertainty we need to reduce the allowed parameter space by improving the measurement of the high $p_T$ portion of the vertex-independent muon spectrum, where the error is dominated by the uncertainty in the yield of punchthrough hadrons. Data sets (already collected) with higher statistics, and with hadrons stopping in MuID gap 2, will allow a completely data-driven approach to the calculation of the punchthrough yield. This will eliminate the reliance on hadronic interaction simulation packages, differences in which are the largest source of systematic error at high $p_T$. Analogous measurements are also being carried out for $d + Au$, $Cu + Cu$, and $Au + Au$ collisions at $\sqrt{s_{NN}} = 200$ GeV. These will allow determination of the magnitude of nuclear modification effects on charm production at forward rapidity.

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