MULTIFRAGMENTATION AND THE RENORMALIZATION GROUP

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Abstract

The possible usefulness of the renormalization group (RG) method in Nuclear Physics is pointed out in this talk in the context of the nuclear multifragmentation. The presentation is rather superficial and sketchy, to indicate the main lines only. But I believe that the idea is warrant of further, more careful studies.

1 The Renormalization Group

Motto: "We should rather eliminate a degree of freedom if it is not playing an important role in a given observable. The complications left behind, such as the effective interactions, form factors, etc. are easier to cope with than the complications related to a truly dynamical variable." A successful implementation of this idea can be found in quantum chemistry where the d,f and g electrons are taken into account by means of the form factors rather than keeping them as true dynamical degrees of freedom. In an analogous manner, the application of the RG method for the multifragmentation processes is based on the effective treatment of the nuclear bound state structure.

The RG method has two independent bonuses. It (i) helps to identify those few microscopic coupling constants which characterize a given dynamics \[1\], and (ii) provides a non-perturbative tool for solving quantum field theories.

Elementary interactions: Suppose that the elementary ("microscopic") description of the dynamics is given by the hamiltonian \(H_k[\psi_N, \Phi_M, \chi(A,Z)]\), where the fields \(\psi_N\) and \(\Phi_M\) stand for few low lying nucleons and mesons, respectively. The nuclei belong to the field \(\chi(A,Z)\). This hamiltonian is supposed to describe the dynamics up to \(k = \Lambda \approx \text{few GeV/c single particle momentum}\)

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and it contains interaction terms like $\bar{\psi}_N \psi_N \Phi_M$, $\Phi_M^4$, $\chi^{(A,Z)}_{(A_1+Z_1+Z_2)} \chi(A_1,Z_1) \chi(A_2,Z_2)$. The local field are convoluted with form factors to form the vertices.

**RG flow:** Such a model is not particularly interesting because the large number of the parameters spoils practicality and the predictive power. The remedy of the problem is the blocking, or the RG step. It consists of the lowering of the cutoff, $k \rightarrow k' = k - \Delta k$ by eliminating particles within the momentum shell $k - \Delta k \leq p \leq k$ and by constructing the effective dynamics governed by $H_{k-\Delta k}$. The coupling constants appearing in the new hamiltonian take new values, $g_n \rightarrow g'_n = B_n(g; k, k')$, their modification represent the dynamics of the modes eliminated. The result is the cutoff dependence of the coupling constants $g_n(k)$, the RG flow.

What is the use of the RG flow? Suppose that we need the expectation value $\langle A(p_{\text{obs}}) \rangle$ of an observable at the momentum scale $p_{\text{obs}}$. The contributions of particles with momentum $p > p_{\text{obs}}$ are suppressed in the expectation value. The basic principle of the RG method is to use the effective dynamics with cutoff slightly above $p_{\text{obs}}$ to obtain the expectation value. As a result, the coupling constants $g_n(p)$ express the effective strength of the interactions at a given scale. In order to see the true scale dependence the dimension of the coupling constant is expressed by the cutoff, $k$, and one works with dimensionless coupling constants.

**Universality:** The blocking step allows us to eliminate a large number of coupling constants from the hamiltonian. This is achieved by linearizing the blocking transformation $B(g; k, k')$ around a fixed point, $g^* = B(g^*; k, k')$. Let us denote the eigenvalues of the linearized blocking, $\partial g_n B_m(g^*; k, k')$ by $\lambda_n(k/k')$. Two successive blocking steps give the relation $\lambda_n(x) \lambda_n(y) = \lambda_n(xy)$. The only continuous solution of this equation is $\lambda_n(x) = x^{\nu_n}$. The parameter $\nu_n$ introduced in this manner is called the critical exponent. The coupling constants with $\nu < 0$ or $\nu > 0$ are called irrelevant, or relevant, respectively. This classification corresponds to the scaling regime, the range of the scale $k$ where the linearization is reliable. The use of this property obtained in the linearized level is that it asserts that the RG flow becomes independent of the initial value of the irrelevant coupling constants after leaving the vicinity of a fixed point.

There are two other important developments to mention:

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2It is important to bear in mind that the irrelevance is *not* the claim that the coupling constant is weak. It concerns instead the difference between the running coupling constants and the fixed point.
• The kinetic energy is the dominant piece of the Hamiltonian at high energy and the models consisting of free, massless particles represent perturbative fixed points.

• The relevant (irrelevant) coupling constants are the renormalizable (non-renormalizable) ones.

The result is the universality, the claim that the initial value of the non-renormalizable coupling constants at the shortest length scale can safely be set zero and it is enough to adjust the renormalizable coupling constants only.

The weak point of applying the RG method in Nuclear Physics lies here. One may say that one of the fundamental difficulties of Nuclear Physics is the closeness of its characteristic scales\(^3\): one cannot talk about hadronic degrees of freedom below 1 fm and the strong interaction is already negligible beyond few dozens of fm. The UV and IR "neighbors" of Nuclear Physics, QCD and QED are better placed because they have longer scaling regimes which allow the sufficiently strong suppression of the non-universal interactions and thereby eliminate the need of the non-renormalizable coupling constants. It remains to be seen if the critical exponents of the non-renormalizable coupling constants are sufficiently negative in Nuclear Physics to eliminate enough coupling constants at \(\Lambda\).

**Wegner-Houghton equations:** In order to follow the RG flow of a large number of coupling constants one needs a specially powerful version of the RG method. This is based on infinitesimal RG steps where \(\Delta k/k \ll 1\). \(\Delta k/k\) acts as a new small parameter and the RG equation obtained in the one-loop level becomes exact as \(\Delta k/k \to 0\) \(^2\).

**Time evolution:** A characteristic feature of the multifragmentation processes is that the conservation laws play an important role in bringing the system out of equilibrium. One encounters similar situations in the experimental studies of glassy materials \(^3\). The potential energy of glasses is an extremely complicated function of the microscopic parameters and possesses a large number of minima. In the quenching experiment one starts at a temperature where the kinetic energy is well above the peaks of the potential energy. After thermalization the temperature is suddenly lowered what makes the system trapped at the closest potential minimum. The time evolution and the relaxation follow different laws before and after quenching. This is reminiscent the break-up step in the multifragmentation because the ergodicity

\(^3\)This results from the strongness of the coupling constants, c.f. the separation of the scales by \(\alpha\) in atomic physics.
is abruptly broken in both cases. The method of the dynamical renormalization group (DRG) was developed to deal with such problems and to classify the possible time evolutions [4]. This method would, in addition, provide a systematic description of the statistical aspects of the multifragmentation.

Soft modes: The detectors pick up long time and long distance observables. This makes one believe that the soft modes, the light mesons are particularly important for the multifragmentation processes. One might envision the nuclei and the nucleons emerging from the multifragmentation process as being surrounded by the classical meson fields which are responsible for their interactions. The resulting picture is similar to the cloudy bag model scenario.

The RG method tailored for the multifragmentation problem may consists of a TDHF scheme where the actual cutoff is time dependent and follows the average kinetic energy of the particles.

2 Mixed Phase

Another application of the RG strategy which might be interesting in the study of the nuclear multifragmentation is the description of the spinodal phase separation. Let us consider the partition function

\[
Z_\Lambda(\Phi) = \int D[\phi] e^{-\frac{1}{\bar{\hbar}} S_k[\phi]} \delta \left( L^{-d} \int d^d x \phi(x) - \Phi \right)
\]

where the Landau-Ginsburg free-energy, \( S[\phi] \), corresponds to the phase with spontaneously broken symmetry,

\[
S_k[\phi] = \int d^d x \left[ \frac{1}{2} (\partial_\mu \phi(x))^2 + V_k(\phi(x)) \right].
\]

The dimensionless parameter \( \bar{\hbar} \) is introduced to organize the loop-expansion and \( L^d \) is the volume. The mean-field solution predicts metastability for \( \Phi_{sp}(0) \leq |\Phi| \leq \Phi_0(0) \), where

\[
k^2 + \partial_\phi^2 V_k(\Phi_{sp}(k)) = 0, \quad k^2 \Phi_0(k) + \partial_\phi V_k(\Phi_0(k)) = 0.
\]

Consider the fluctuations with momentum \( p < k_{cr} \) and \( |\Phi| \leq \Phi_{sp}(p) \). Here \( k_{cr} \) denotes the onset of the instability, i.e. \( \Phi_0(k) > 0 \) for \( k < k_{cr} \). The amplitude of these modes increases exponentially in time because the inverse propagator \( G^{-1}(p^2) = p^2 + \partial_\phi^2 V_p(\Phi) \) is negative.

The RG proceeds by the successive elimination of the modes, and the construction of the effective action

\[
e^{-\frac{1}{\bar{\hbar}} S_k - \Delta_k[\phi_{IR}]} = \int D[\phi_{UV}] e^{-\frac{1}{\bar{\hbar}} S_k[\phi_{IR} + \phi_{UV}]},
\]
where the Fourier transforms $\tilde{\phi}_{IR}(p)$ and $\tilde{\phi}_{UV}(p)$ are non-vanishing for $p < k - \Delta k$ and $k - \Delta k < p < k$, respectively. The action $S_k[\phi]$ has a local maximum at $\phi_{UV}(k) = 0$ in the spinodal unstable region. Since $S_k[\phi]$ is bounded from below the spinodal instability is characterized by the appearance of a nontrivial saddle point, $\tilde{\phi}_k^{sp}(x)$, in the blocking (4). The saddle point is the solution of the projection of the equation of motion of $S_k$ into the momentum space shell $k - \Delta k < p < k$. The blocking relation is
\[ e^{-\frac{1}{\hbar}S_{k-\Delta k}[\phi_{IR}]} = \sum_{\alpha} e^{-\frac{1}{\hbar}S_k[\phi_{IR} + \phi_k^{sp}] - \mu(X_{k,\alpha})} \int \frac{dX_{k,\alpha}}{\sqrt{\text{det}' \frac{\partial^2 S_k[\phi_{IR} + \phi_k^{sp}] + \phi_k^{sp}}{\partial \phi_{UV} \partial \phi_{UV}}}} (1 + O(\hbar)), \] (5)
where the summation is over the different saddle points. $\text{det}'$ denotes the determinant in the subspace of the modes with momentum $k - \Delta k < p < k$ which is orthogonal to the saddle point and $\mu(X_{k,\alpha})$ stands for the integral measure of the zero modes $X_{k,\alpha}$. We find a tree-level renormalization because the saddle point depends on the infrared background field, $\tilde{\phi}_k^{sp} = \tilde{\phi}_k^{sp}[\phi_{IR}]$. In the case of a single saddle point (5) reads as
\[ S_{k-\Delta k}[\phi_{IR}] = S_k[\phi_{IR} + \phi_k^{sp}[\phi_{IR}]] + O(\hbar) \] (6)
The numerical computation of the effective action was reported in ref. [5] by taking into account the plane wave saddle points. The conclusion can be generalized by assuming the continuity of the coupling constants in the cutoff in the unstable region with the following result [6]:

- The mixed phase extends over the metastable region of the mean-field solution, i.e. the saddle point is nontrivial not only for $|\Phi| \leq \Phi_{sp}(k)$ but whenever $|\Phi| \leq \Phi_0(k)$.

- The amplitude of the saddle point $\phi_k^{sp}(x)$ is such that the background field plus the saddle point sweeps through the whole unstable region, $\max_x |\Phi + \phi_k^{sp}(x)| = \Phi_0(k)$.

- The effective action is degenerate in the mixed phase, i.e. it is left unchanged by the fluctuations $\max_x |\Phi + \phi_{UV}(x)| \leq \Phi_0(k)$. The gradient expansion ansatz
\[ S_k = \int d^d x \left[ \frac{1}{2} Z_k(\phi(x))(\partial_\mu \phi(x))^2 + V_k(\phi(x)) \right] \] (7)
yields a simple quadratic effective action
\[ S_k^{\text{mixed}} = \frac{1}{2} Z_k^{\text{stable}}(\Phi_0(k)) \int d^d x \left[ (\partial_\mu \phi(x))^2 - k^2 \phi^2(x) \right] + O \left( \frac{\Delta k}{k} \right) \] (8)
in the mixed phase where \(Z^{\text{stable}}\) is read off from the solution of the RG equation in the stable region. Notice that this result is exact, the only correction being \(O(\Delta k/k)\) which can be arbitrarily small.

Notice that the last point implies the Maxwell construction. In fact, \(V_{k=0}(\Phi)\) is the free energy density which is flat for \(-\Phi_0(0) \leq \Phi \leq \Phi_0(0)\) according to (8). The tree-level renormalization of the potential \(V(\phi) = -0.05\phi^2 + 0.2\phi^4/4!\) is depicted in Fig. 1.

It is worthwhile mentioning that the correlation functions can easily be obtained in the mean-field approximation. The real time evolution can, as well, be incorporated since it is actually a saddle point.

![Fig. 1. \(V_k(\Phi)\) for different values of \(k\) showing the evolution towards the Maxwell construction at \(k = 0\).](image)

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