Chiral Lagrangian
from gauge invariant, nonlocal, dynamical quark model

Hua Yang\textsuperscript{a,b}, Qing Wang\textsuperscript{a,c}, Qin Lu\textsuperscript{a}

\textsuperscript{a}Department of Physics, Tsinghua University, Beijing 100084, China
\textsuperscript{b}Institute of Electronic Technology, Information Engineering University, Zhengzhou 450004
\textsuperscript{c}Institute of Theoretical Physics, Academia Sinica, Beijing 100080, China

Abstract

Parameters of Gasser Leutwyler chiral Lagrangian are proved saturated by
dynamical quark self energy $\Sigma(k^2)$ in a gauge invariant, nonlocal, dynamical
quark model.

Much of low-energy QCD can be encoded into a series parameters appearing in a chiral
Lagrangian, expanded to some finite order of low energy expansion. Attempts have been
made to understand these parameters: it is shown that low lying vector mesons will saturate
the parameters [1]. To go beyond phenomenological level, the anomaly contribution was
taken as the main source of the parameters [2], we call this type investigation the anomaly
approach, it leads result

\begin{equation}
8L_1 = 4L_2 = -2L_3 = 24L_7 = -8L_8 = L_9 = -2L_{10} = \frac{N_c}{48\pi^2}
\end{equation}

which are close to experiment result except $L_7$ and $L_8$ which have wrong signs. The defi-
ciency of this calculation lies in its independence of interaction: if we switch off the strong
interaction and discuss a system of free quark field with external sources, the anomaly calcu-
lation can still be performed without any change. Then it seems that (1) is not due to strong
interaction among quarks and gluons, but rather an artificial result. Another type research,
we call it dynamical approach, mainly consider the dynamical effect [3], in which the main
source of the parameters is from dynamical quark self energy $\Sigma(k^2)$. This approach has
advantage of maintaining chiral symmetry and momentum dependence of dynamical quark

*Mailing address
mass, in the mean time avoiding introduce in the theory the hard constituent quark mass to cause wrong bad ultraviolet behavior of the theory. But it does not explain why it can offer the better numerical result (without wrong sign problem for $L_7$ and $L_8$) than anomaly approach. In fact, anomaly contribution and dynamical quark self energy contribution are two independent sources, if the anomaly contribution play role, according to (1), it will be dominant at all parameters and then there is no room left for $\Sigma(k^2)$ to play role to match the experiment data, except for $L_7$ and $L_8$. If the anomaly contribution donot play role, it must be cancelled in some sense and after the cancellation, we need to show the remaining dynamical effect (which may or may not be dominant by dynamical quark self energy) can still recover or improve the result (4). It is purpose of this work to judge these two possibilities. We will show the second choice is correct, the cancellation do happen in dynamical approach and remanent contribution from dynamical quark self energy $\Sigma(k^2)$ can provide values for parameters of chiral Lagrangian consistent with experiment data.

Consider QCD in presence of external scalar, pseudoscalar, vector and axial vector sources,

$$J(x) = \not{\psi}(x) + \not{\phi}(x)\gamma_5 - s(x) + ip(x)\gamma_5,$$

The generating functional in Minkovski space is

$$Z[J] = \int \mathcal{D}\psi \mathcal{D}\bar{\psi} \mathcal{D}U \mathcal{D}\bar{U} e^{i\int d^4x \left[ \mathcal{L}(\psi, \bar{\psi}, \Psi, \bar{\Psi}, A_\mu) + \bar{\psi}J\psi \right]}$$  \hspace{1cm} (2)

where $\psi, \Psi, A_\mu$ are light, heavy and gluon fields respectively. $\mathcal{L}(\psi, \bar{\psi}, \Psi, \bar{\Psi}, A_\mu)$ is Lagrangian of QCD. The chiral Lagrangian relate this generating functional by

$$Z[J] = \int \mathcal{D}U \ e^{iS_{GL}[U, J]}$$  \hspace{1cm} (3)

$U$ is pseudo goldstone boson (PGB) field, $S_{GL}[U, J]$ is Gasser and Leutwyler (GL) chiral Lagrangian [4],

$$S_{GL}[U, J] = S_{\text{normal}}[U, J] + S_{\text{anomaly}}[U, J],$$  \hspace{1cm} (4)

$$S_{\text{normal}}[U, J] = \int d^4x \left\{ \frac{1}{4} F_0^2 \text{tr} [\nabla^\mu U] [\nabla_\mu U + U \chi + U^\dagger \chi] + L_1 [\text{tr} (\nabla^\mu U^\dagger \nabla_\mu U)]^2 
+ L_2 [\text{tr} (\nabla^\mu U^\dagger \nabla_\mu U) [\nabla^\nu U^\dagger \nabla_\nu U] + L_3 [\text{tr} (\nabla^\mu U^\dagger \nabla_\mu U)^2] 
+ L_4 [\text{tr} (\nabla^\mu U^\dagger \nabla_\mu U) [\nabla^\nu U + U^\dagger \chi + L_5 [\text{tr} (\nabla^\mu U^\dagger \nabla_\mu U (\chi^\dagger U + U^\dagger \chi)] 
+ L_6 [\text{tr} (\chi^\dagger U + U^\dagger \chi)]^2 + L_7 [\text{tr} (\chi^\dagger U - U^\dagger \chi)]^2 + L_8 [\text{tr} (\chi^\dagger U \chi^\dagger U + U^\dagger \chi U^\dagger U)] 
- iL_9 [\text{tr} (\chi^\dagger U + U^\dagger \chi)]^2 + L_7 [\text{tr} (\chi^\dagger U - U^\dagger \chi)]^2 + L_8 [\text{tr} (\chi^\dagger U \chi^\dagger U + U^\dagger \chi U^\dagger U)] 
- iL_9 [\text{tr} (\chi^\dagger U + U^\dagger \chi)]^2 + L_7 [\text{tr} (\chi^\dagger U - U^\dagger \chi)]^2 + L_8 [\text{tr} (\chi^\dagger U \chi^\dagger U + U^\dagger \chi U^\dagger U)] 
- iL_9 [\text{tr} (\chi^\dagger U + U^\dagger \chi)]^2 + L_7 [\text{tr} (\chi^\dagger U - U^\dagger \chi)]^2 + L_8 [\text{tr} (\chi^\dagger U \chi^\dagger U + U^\dagger \chi U^\dagger U)] 
- iL_9 [\text{tr} (\chi^\dagger U + U^\dagger \chi)]^2 + L_7 [\text{tr} (\chi^\dagger U - U^\dagger \chi)]^2 + L_8 [\text{tr} (\chi^\dagger U \chi^\dagger U + U^\dagger \chi U^\dagger U)] 
- iL_9 [\text{tr} (\chi^\dagger U + U^\dagger \chi)]^2 + L_7 [\text{tr} (\chi^\dagger U - U^\dagger \chi)]^2 + L_8 [\text{tr} (\chi^\dagger U \chi^\dagger U + U^\dagger \chi U^\dagger U)] \right\} + O(p^6) \text{ terms}$$  \hspace{1cm} (5)

$$S_{\text{anomaly}}[U, J] = S_{\text{WZW}}[U, J] + O(p^6) \text{ terms},$$  \hspace{1cm} (6)
where
\[ \chi(x) = 2B_0[s(x) + ip(x)] \]  \hspace{1cm} (7)
and \( S_{\text{WZW}}[U, J] \) is Wess-Zumino-Witten action given in Ref. [5]. Up to order of \( p^4 \), \( S_{\text{anomaly}}[U, J] \) is completely known, but \( S_{\text{normal}}[U, J] \) left fourteen parameters \( F_0, B_0, L_1, \ldots, L_{10}, H_1, H_2 \) need to be calculated. To reveal the source of these parameters, we improve the conventional dynamical approach by building up a gauge invariant, nonlocal, dynamical (GND) quark model. The action in GND model is assumed to be \( S_{\text{eff}}[\psi, \overline{\psi}, U, J] \), it relate to our generating functional by
\[ Z[J] = \int \mathcal{D}U \mathcal{D}\psi \mathcal{D}\overline{\psi} \ e^{iS_{\text{eff}}[\psi, \overline{\psi}, U, J]} . \]  \hspace{1cm} (8)
The r.h.s. of above equation can be seen as a result of integrating out heavy quark and gluon fields and integrate in the PGB field \( U \) in (3). If we further integrate out light quark field in above generating functional, we obtain GL result (3). So \( S_{\text{eff}}[\psi, \overline{\psi}, U, J] \) can be seen as an intermediate stage action to relate fundamental QCD with phenomenological chiral Lagrangian.

Compare (3) and (8), we find GL chiral Lagrangian relate to GND model by
\[ e^{iS_{\text{GL}}[U, J]} = \int \mathcal{D}\psi \mathcal{D}\overline{\psi} \ e^{iS_{\text{eff}}[\psi, \overline{\psi}, U, J]} , \]  \hspace{1cm} (9)
\( S_{\text{eff}}[\psi, \overline{\psi}, U, J] \) is required to be invariant under following local \( U_L(3) \otimes U_R(3) \) chiral transformations:
\[ \begin{align*}
\psi(x) &\rightarrow \psi'(x) = [V_R(x)P_R + V_L(x)P_L]\psi(x) \\
J(x) &\rightarrow J'(x) = [V_R(x)P_L + V_L(x)P_R][J(x) + i\partial] [V_R^\dagger(x)P_R + V_L^\dagger(x)P_L] \\
U(x) &\rightarrow U'(x) = V_R(x)U(x)V_L^\dagger(x) .
\end{align*} \]  \hspace{1cm} (10)
Notice that \( U \) field has standard decomposition \( U(x) = \Omega(x)\Omega(x) \) and \( \Omega(x) \) field, under transformation (10), transform as \( \Omega(x) \rightarrow \Omega'(x) = h^\dagger(x)\Omega(x)V_L^\dagger(x) = V_R(x)\Omega(x)h(x) \) with \( h(x) \) depend on \( V_R, V_L \) and \( \Omega \), represent an induced hidden local \( U(3) \) symmetry to keep transformed \( \Omega \) be a representative element at coset class.

To implement local chiral symmetry explicitly, we take a special local chiral transformation \( V_R(x) = \Omega^\dagger(x), V_L(x) = \Omega(x) \), the corresponding hidden symmetry transformation is \( h(x) = 1 \),
\[ \begin{align*}
\psi_\Omega(x) &\equiv [\Omega^\dagger(x)P_R + \Omega(x)P_L]\psi(x) \\
J_\Omega(x) &\equiv [\Omega(x)P_R + \Omega^\dagger(x)P_L] [J(x) + i\partial] [\Omega(x)P_R + \Omega^\dagger(x)P_L] \\
&\equiv -s_\Omega(x) + i\rho_\Omega(x)\gamma_5 + \phi_\Omega(x) + \phi^\dagger_\Omega(x)\gamma_5 \\
U_\Omega(x) &\equiv 1 .
\end{align*} \]  \hspace{1cm} (11)
On rotated basis, we can rewrite (9) as

\[ e^{iS_{GL}[U,J]} = \frac{\int D\psi D\bar{\psi} e^{iS_{\text{det}}[\psi_\Omega, \bar{\psi}_\Omega, 1, J_\Omega]} \int D\psi D\bar{\psi} e^{i\int d^4x \overline{\psi}(x)[i\partial_x + J(x)]\psi(x)}}{\int D\psi D\bar{\psi} e^{i\int d^4x \overline{\psi}(x)[i\partial_x + J(x)]\psi(x)}} \]

\[ = N' \frac{\int D\psi_\Omega D\bar{\psi}_\Omega e^{iS_{\text{det}}[\psi_\Omega, \bar{\psi}_\Omega, 1, J_\Omega]} \int D\psi_\Omega D\bar{\psi}_\Omega e^{i\int d^4x \overline{\psi}_\Omega(x)[i\partial_x + J_\Omega(x)]\psi_\Omega(x)}}{\int D\psi_\Omega D\bar{\psi}_\Omega e^{i\int d^4x \overline{\psi}_\Omega(x)[i\partial_x + J_\Omega(x)]\psi_\Omega(x)}} \] (13)

where \( N' \equiv \int D\psi D\bar{\psi} e^{i\int d^4x \overline{\psi}(x)[i\partial_x + J(x)]\psi(x)} = \text{Det}[i\partial_x + J(x)] \). In the last equality, we have taken chiral rotation (11) for functional integration measure both in numerator and denominator. The possible anomalies caused by this rotation are cancelled between numerator and denominator. Since we are only interested in \( U \) dependence of the theory, pure source terms \( N' \) is irrelevant and therefore can be treated as a normalization factor.

Result (13) tells us that \( S_{\text{eff}} \) should have following structure

\[ S_{\text{eff}}[\psi_\Omega, \bar{\psi}_\Omega, 1, J_\Omega] = \int d^4x \overline{\psi}_\Omega(x)[i\partial_x + J_\Omega(x)]\psi_\Omega(x) + S_{\text{int}}[\psi_\Omega, \bar{\psi}_\Omega, 1, J_\Omega] \] (14)

where \( S_{\text{int}} \) is interaction part caused by color gauge interaction. If we switch off color gauge interaction which means we are dealing with free fermion fields, there will be no effective Lagrangian (\( S_{\text{GL}} = 0 \)). \( S_{\text{int}} \) should include those fermion self interaction terms caused by integrate out gluon and heavy quark fields in underlying QCD and integrate in local goldstone boson fields \( U \). Among these, the most important effect related to chiral symmetry at low energy region is spontaneous chiral symmetry breaking (SCSB) which require quark has a nontrivial momentum dependent self energy \( \Sigma(k^2) \), its effects can be introduced into the theory by adding in \( S_{\text{int}} \) a self energy term

\[ -\int d^4x \overline{\psi}_\Omega(x)\Sigma(\partial_x^2)\psi_\Omega(x) . \] (15)

Just this term itself is not enough, since it is not invariant under local chiral symmetry transformations. To make it invariant, in conventional dynamical approach [3], a non-integratable face factor is introduced into theory which cause very complex formulae and authors in [3] even donot put their analytical result in their papers. We donot use non-integratable face factor, instead we note that local chiral symmetry transformation on rotated variable is

\[ \psi_\Omega(x) \rightarrow \psi'_\Omega(x) = h^\dagger(x)\psi_\Omega(x) \]
\[ J_\Omega(x) \rightarrow J'_\Omega(x) = h^\dagger(x)[J_\Omega(x) + i\partial_x]h(x) . \] (16)

Original local chiral symmetry now is realized as a hidden local symmetry. Once the theory is constructed to be invariant under this hidden symmetry, it is invariant under original local chiral symmetry. Since interaction part in \( S_{\text{eff}} \) should be invariant on local chiral
symmetry, we need at least to generalize self energy term (13) to be invariant on hidden local symmetry (16). To achieve this, we change the ordinary derivative $\partial_{x}$ to hidden symmetry covariant derivative $\nabla_{x}^{\mu} = \partial_{x}^{\mu} - iv_{\Omega}^{\mu}(x)$ (the overline on $\nabla_{x}^{\mu}$ is to denote the difference with covariant derivative appeared in (3)). (16) tells us $v_{\Omega}(x)$ transform as $v_{\Omega}^{\mu}(x) \rightarrow v_{\Omega}^{\mu}(x) = h_{\Omega}(x)v_{\Omega}^{\mu}(x)h(x) + ih_{\Omega}(x)\partial_{x}^{\mu}(h(x))$ which lead $\nabla_{x}^{\mu} \rightarrow \nabla_{x}^{\mu} = h_{\Omega}(x)\nabla_{x}^{\mu}h(x)$. The modified chiral invariant interaction action now is

$$S_{\text{int}}[\psi_{\Omega}, \overline{\psi}_{\Omega}, 1, J_{\Omega}] = - \int d^{4}x \, \overline{\psi}_{\Omega}(x)\Sigma(\nabla_{x}^{2})\psi_{\Omega}(x).$$

(17)

This action is not the complete part of interaction, but it is the minimal part of interaction which respect local chiral symmetry with dynamical quark and SCSB. If we take the idea of dynamical perturbation originally from Pagel-Stokar \[6\] and developed in Ref. \[7\], in which at the leading order of the expansion, all perturbative effects are ignored and only nonperturbative effect considered in the theory is that from quark self energy $\Sigma(k^{2})$. (17) in this sense can be seen as a result of leading order expansion from dynamical perturbation.

In GND model, quark fields dependence is bilinear and can be exactly integrated out, the result GL Lagrangian from (13) is

$$S_{\text{GL}}[U, J] \approx S_{\text{GND}}[U, J] \equiv -i\text{Tr} \, \ln[i\partial + J_{\Omega} - \Sigma(\nabla^{2})] + i\text{Tr} \, \ln[i\partial + J_{\Omega}].$$

(18)

Use the Schwinger proper time formulation developed in \[8\], we can compute the $\Sigma(\nabla^{2})$ dependent determinant in (18). The result is

$$-i\text{Tr} \, \ln[i\partial + J_{\Omega} - \Sigma(\nabla^{2})]$$

$$= \int d^{4}x \text{tr}_{f} \left[ B_{0}F_{0}^{2}S_{\Omega} + C_{1}a_{\Omega}^{2} + C_{2}[d_{a}a_{\Omega}^{\mu}]^{2} + C_{3}(d_{a}a_{\Omega}^{\mu} - d_{a}a_{\Omega}^{0})(d_{a}a_{\Omega,\nu} - d_{a}a_{\Omega,\nu})
+ C_{4}[a_{a}^{2}]^{2} + C_{5}a_{a}^{\mu}a_{0}^{\nu}a_{\Omega,\mu}a_{\Omega,\nu} + C_{6}a_{a}^{2} + C_{7}p_{\Omega}^{2} + C_{8}s_{a}a_{\Omega}^{2} + C_{9}V_{\Omega,\mu\nu}^{\mu\nu}V_{a,\mu\nu} + C_{10}V_{\Omega,\mu\nu}^{\mu\nu}a_{\Omega,\mu}a_{\Omega,\nu}
+ C_{11}p_{\Omega}d_{a}a_{\Omega}^{\mu}ight] + O(p^{6}) + \text{imaginary terms}$$

(19)

where $\text{tr}_{f}$ is trace for flavor indices. Covariant derivative for function $f$ and $V_{\Omega}^{\mu\nu}$ are defined as

$$d^{\mu}f \equiv \partial_{x}^{\mu}f - iv_{\Omega}^{\mu}f + ifv_{\Omega}^{\mu}$$

$$V_{\Omega}^{\mu\nu} = \partial_{x}^{\mu}v_{\Omega}^{\nu} - \partial_{x}^{\nu}v_{\Omega}^{\mu} - iv_{\Omega}^{\mu}v_{\Omega}^{\nu} + iv_{\Omega}^{\nu}v_{\Omega}^{\mu}.$$  

(20)

The $\Sigma$ dependence for coefficients appeared in (19) are

$$F_{0}^{2}B_{0} = 4 \int d\bar{k} \Sigma_{k}X_{k}$$

(21)

$$C_{1} = 2 \int d\bar{k} \left[ (-2\Sigma_{k}^{2} - k^{2}\Sigma_{k}^{2})X_{k}^{2} + (2\Sigma_{k}^{2} + k^{2}\Sigma_{k}^{2})\frac{X_{k}}{L^{2}} \right]$$

(22)
\[C_2 = -2 \int dk \left[ -2A_k X_k^3 + 2A_k \frac{X_k^2}{\Lambda^2} - A_k \frac{X_k}{\Lambda^4} + \frac{k^2}{2} \Sigma_k'^2 \frac{X_k}{\Lambda^2} - \frac{k^2}{2} \Sigma_k'^2 \Sigma_k^2 X_k \right]
\]
\[C_3 = - \int dk \left[ -2B_k X_k^3 + 2B_k \frac{X_k^2}{\Lambda^2} - B_k \frac{X_k}{\Lambda^4} + \frac{k^2}{2} \Sigma_k'^2 \frac{X_k}{\Lambda^2} - \frac{k^2}{2} \Sigma_k'^2 \Sigma_k^2 X_k \right]
\]
\[C_4 = 2 \int dk \left[\left(\frac{4\Sigma_k^4}{3} - \frac{2k^2 \Sigma_k^2}{3} + \frac{k^4}{18}\right)(6X_k^4 - \frac{6X_k^3}{\Lambda^2} + \frac{3X_k^2}{\Lambda^4} - \frac{X_k}{\Lambda^6}) + (-4\Sigma_k^2 + \frac{k^2}{2})(-2X_k^3
\]
\[+ \frac{2X_k^2}{\Lambda^2} - \frac{X_k}{\Lambda^4} - \frac{X_k}{\Lambda^2} + X_k^2 \right]
\]
\[C_5 = \int dk \left[\left(-\frac{4\Sigma_k^4}{3} + \frac{2k^2 \Sigma_k^2}{3} + \frac{k^4}{18}\right)(6X_k^4 - \frac{6X_k^3}{\Lambda^2} + \frac{3X_k^2}{\Lambda^4} - \frac{X_k}{\Lambda^6}) + 4\Sigma_k^2(-2X_k^3 + \frac{2X_k^2}{\Lambda^2}
\]
\[- \frac{X_k}{\Lambda^4} + \frac{X_k}{\Lambda^2} - X_k^2 \right]
\]
\[C_6 = 2 \int dk \left[\left(3\Sigma_k^2 + 2k^2 \Sigma_k \Sigma_k'\right)X_k^2 + \left[2\Sigma_k^2 - k^2(1 + 2\Sigma_k \Sigma_k')\right] \frac{X_k}{\Lambda^2} \right]
\]
\[C_7 = 2 \int dk \left[\left(\Sigma_k^2 + 2k^2 \Sigma_k \Sigma_k'\right)X_k^2 + k^2(1 + 2\Sigma_k \Sigma_k') \frac{X_k}{\Lambda^2} \right]
\]
\[C_8 = 4 \int dk \left[\left(-4\Sigma_k^3 + k^2 \Sigma_k\right)X_k^3 + (4\Sigma_k^3 - k^2 \Sigma_k) \frac{X_k^2}{\Lambda^2} - \left(2\Sigma_k^3 - \frac{1}{2} k^2 \Sigma_k\right) \frac{X_k}{\Lambda^4} + 3\Sigma_k \frac{X_k}{\Lambda^2} - 3\Sigma_k X_k \right]
\]
\[C_9 = - \int dk \left[\left(\frac{1}{3} k^2 \Sigma_k \Sigma_k'' + \frac{1}{3} \Sigma_k' \Sigma_k''\right)X_k + \left(C_k - D_k\right) \frac{X_k}{\Lambda^2} - \left(C_k - D_k\right) \frac{X_k}{\Lambda^2} - 2E_k X_k^3
\]
\[+ 2E_k \frac{X_k^2}{\Lambda^2} - E_k \frac{X_k}{\Lambda^4} \right]
\]
\[iC_{10} = 4 \int dk \left[ -2F_k X_k^3 + 2F_k \frac{X_k^2}{\Lambda^2} - F_k \frac{X_k}{\Lambda^4} + \frac{k^2}{2} \Sigma_k'^2 \frac{X_k}{\Lambda^2} - \frac{k^2}{2} \Sigma_k'^2 X_k^2 \right]
\]
\[C_{11} = -4 \int dk \left[ -\left(\Sigma_k + \frac{1}{2} k^2 \Sigma_k''\right) \frac{X_k}{\Lambda^2} + \left(\Sigma_k + \frac{1}{2} k^2 \Sigma_k''\right) X_k^2 \right]
\]
where
\[
\int dk \equiv iN \int \frac{dk^4}{(2\pi)^4} e^{\frac{k^2 - \Sigma^2(-k^2)}{\Lambda^2}} \quad \Sigma_k \equiv \Sigma(-k^2) \quad X_k \equiv \frac{1}{k^2 - \Sigma^2(-k^2)}
\]
\[A_k = \frac{2}{3} k^2 \Sigma_k \Sigma_k' (1 + 2\Sigma_k \Sigma_k') + \frac{1}{3} \Sigma_k'^2 (1 + 2\Sigma_k \Sigma_k') - \frac{1}{3} k^2 \Sigma_k'^2 (\Sigma_k'^2 + \Sigma_k \Sigma_k'') + \frac{1}{6} k^4 (\Sigma_k'^2 + \Sigma_k \Sigma_k'')
\]
\[B_k = \frac{2}{3} k^2 \Sigma_k (1 + 2\Sigma_k \Sigma_k') + \frac{1}{3} \Sigma_k'^2 (1 + 2\Sigma_k \Sigma_k') - \frac{1}{3} k^2 \Sigma_k'^2 (\Sigma_k'^2 + \Sigma_k \Sigma_k'') + \frac{1}{18} k^4 (\Sigma_k'^2 + \Sigma_k \Sigma_k'')
\]
\[+ \frac{1}{6} k^2 (1 + 2\Sigma_k \Sigma_k')
\]
\[C_k = \frac{1}{3} - \frac{1}{3} \Sigma_k \Sigma_k' - \frac{1}{2} k^2 \Sigma_k'^2
\]
\[D_k = \frac{1}{2} k^2 \Sigma_k'^2 + \frac{1}{6} k^2 \Sigma_k \Sigma_k' (1 + 2\Sigma_k \Sigma_k') + \frac{2}{9} k^4 \Sigma_k' \Sigma_k'' (1 + 2\Sigma_k \Sigma_k') + \frac{2}{9} k^4 \Sigma_k'^2 (\Sigma_k'^2 + \Sigma_k \Sigma_k'')
\]
\[+ \frac{1}{3} k^2 \Sigma_k \Sigma_k' (\Sigma_k'^2 + \Sigma_k \Sigma_k'')
\]
\[E_k = -\frac{1}{6} k^2 \Sigma_k \Sigma_k' (1 + 2\Sigma_k \Sigma_k')^2 - \frac{1}{9} k^4 \Sigma_k'^2 (1 + 2\Sigma_k \Sigma_k')^2
\]
\[ F_k = -\frac{4}{3}k^2\Sigma_k\Sigma'_k + \frac{4}{3}k^2(\Sigma_k\Sigma'_k)^2 - \frac{2}{3}\Sigma_k^2 + \frac{2}{3}\Sigma_k^3\Sigma'_k - \frac{1}{3}k^2\Sigma_k^2(\Sigma'_k + \Sigma_k\Sigma'_k) + \frac{1}{9}k^4(\Sigma'_k + \Sigma_k\Sigma'_k) + \frac{1}{3}k^2(1 + 2\Sigma_k\Sigma'_k) - \frac{1}{2}k^2. \]

The result for \( C_\equiv F_0^2 \) in (22) is just the well known Pagel-Stokar formula [3], if we take momentum cutoff \( \Lambda \) be infinity. The part of \( \Sigma(k) \) independent quark determinant in (19) is just the result of anomaly approach by taking limit of \( \Sigma \) approach, the effective action is \( i \equiv C \), it relate to anomaly result mentioned in the beginning of this paper. In fact, in anomaly approach, the effective action is \( i \equiv C \), which is just the result of anomaly approach by taking limit of \( \Sigma \equiv \text{const} \to 0 \). (Note due to possible infrared divergence, limit of \( \Sigma \to 0 \) must be taken after the momentum integration). The nonzero coefficients \( C_i \) for the case of infinite momentum cutoff \( \Lambda \) is

\[
\begin{align*}
C_2 &\to \frac{N_c}{24\pi^2}, \quad C_3 \to \frac{N_c}{24\pi^2}(\ln \frac{\Sigma^2}{\Lambda^2} + \gamma + 1), \quad C_4 \to \frac{N_c}{24\pi^2}(\ln \frac{\Sigma^2}{\Lambda^2} + \gamma + 4), \\
C_5 &\to -\frac{2N_c}{24\pi^2}(\ln \frac{\Sigma^2}{\Lambda^2} + \gamma + 2), \quad C_6 \to \frac{N_c}{8\pi^2}\Lambda^2, \quad C_7 \to \frac{N_c}{8\pi^2}\Lambda^2, \\
C_9 &\to -\frac{N_c}{48\pi^2}(\ln \frac{\Sigma^2}{\Lambda^2} + \gamma), \quad iC_{10} \to \frac{N_c}{12\pi^2}(\ln \frac{\Sigma^2}{\Lambda^2} + \gamma + 2).
\end{align*}
\]

The pure imaginary terms in (19) are completely known at phenomenological level and its calculation in terms of \( \Sigma \) is already performed in Ref. [4] and proved exactly recover the Witten’s result [5], we donot explicitly write down their detail structures.

With help of (12), (18) and (19) lead relation,

\[
\begin{align*}
L_1 &= \frac{1}{2}L_2 = \frac{C_5}{32} - \frac{C_9}{16} + \frac{iC_{10}}{32}, \quad L_3 = \frac{C_4 - 2C_5 + 6C_9 - 3iC_{10}}{16}, \\
L_4 &= 0, \quad L_5 = \frac{C_8}{16B_0}, \quad L_6 = 0, \quad L_7 = \frac{C_2}{48} - \frac{C_{11}}{48B_0}, \\
L_8 &= \frac{C_2}{16} + \frac{C_6}{16B_0^2} - \frac{C_7}{16B_0^2} + \frac{C_{11}}{16B_0}, \quad L_9 = \frac{4C_9 + iC_{10}}{8}, \\
L_{10} &= \frac{-C_3 + C_9}{2}, \quad H_1 = \frac{C_3 + C_9}{4}, \quad H_2 = \frac{C_2}{8} + \frac{C_6}{8B_0^2} + \frac{C_7}{8B_0^2} - \frac{C_{11}}{8B_0}. \quad (24)
\end{align*}
\]

where \( \overline{C}_i \equiv C_i - \lim_{\Sigma \to 0} C_i \quad i = 1, 2, \ldots, 11 \). Term \( -\lim_{\Sigma \to 0} C_i \) is of special interest, since it relate to anomaly result mentioned in the beginning of this paper. In fact, in anomaly approach, the effective action is \( i \equiv C \), which is just the result of (3) with (24) by taking \( C_i \) values at \( \Sigma = 0 \) and revert all signs. One can easily check this reproduce result (3) in which all ultraviolet divergence are cancelled each other for \( L_i \) parameters. The interpretation of this result is that from (18), the contribution of anomaly play no role in the final result, it is completely cancelled by dynamical quark self energy dependent part, only the remainder after cancellation play role in the final parameters \( L_i \).

In (24), parameter \( B_0 \) needs special treatment. Since with help of (21), we find \( F_0^2B_0 \) is generally divergent. To renormalize this condensate, we note that \( F_0 \) and \( m_\Lambda(\overline{\psi}\psi)_\Lambda \) (\( m_\Lambda \) is
bare current quark mass) is renormalization invariant or more general, the \( \chi \) field defined in (7) is renormalization invariant, i.e. \( \chi(x) = 2B_0[s(x) + ip(x)] = 2B_r[s_r(x) + ip_r(x)] \) with \( B_r, s_r, p_r \) are renormalized \( B_0, s, p \). Correspondingly, renormalized quark condensate \( \langle \bar{\psi}\psi \rangle_r \) is \( \langle \bar{\psi}\psi \rangle_r = -N_fF_0^2B_r \). So replacing the scalar and pseudoscalar sources and \( B_0 \) with renormalized ones donot change value of \( \chi \) field. With renormalized sources, we can replace \( B_0 \) in (24) with \( B_r \). In this paper, we donot directly calculate and use \( B_0 \), instead we calculate and use \( B_r \). The renormalization point is chosen to be at scale of 1GeV.

Now, once the quark self energy \( \Sigma(k^2) \) was input into the formulae, we can get all parameters in Gasser-Leutwyler chiral Lagrangian. In conventional dynamical approach [3], the ignorance of \( \Sigma(k^2) \) is parametrized by following ansatz

\[
\Sigma(k^2) = \frac{(A + 1)m^3}{k^2 + Am^2}
\]

which satisfy \( \Sigma(m^2) = m \) and shares qualitative similarities with solutions of improved ladder SD equation. It is finite, positive, monotonically decreasing functions with \( 1/k^2 \) behavior at large \( q^2 \) and \( \Sigma'(0) < 0 \). The constituent quark mass \( m \) is determined for each choice of \( A \) for \( F_0 = 93MeV \) from (24). For \( A = 1, 2, 3, 4 \), we obtain \( m = 379, 350, 331, 317MeV \) respectively. With ansatz (25), the result parameters are listed in TABLE.I. We see that the wrong sign problem for \( L_7 \) and \( L_8 \) in anomaly calculation is corrected now and result parameters are roughly consistent with experiment data.

To further trace the relation of GND model with underlying theory QCD. Note that the parameters in the chiral Lagrangian are recently expressed in terms of QCD Green’s functions [10] and for quark two point Green’s function \( \Phi^{\rho \rho}_{ij}(x,y) \), at large \( N_c \) limit, [10] gives equation,

\[
[i\bar{\psi} + i\Phi^{\rho \rho}_{ij}^{-1} + J_\Omega + \tilde{\Xi}]^{\sigma \rho}(x,y) + \sum_{n=1}^{\infty} \int d^4x_1 \cdots d^4x_n d^4x'_1 \cdots d^4x'_n (\frac{-i}{n!}) (N_c g^2)^n \\
\times G^{\sigma_1 \cdots \sigma_n}_{\rho_1 \cdots \rho_n}(x, y, x_1, x'_1, \cdots, x_n, x'_n) \Phi^{\rho_1 \rho_1}_{ij}(x_1, x'_1) \cdots \Phi^{\rho_n \rho_n}_{ij}(x_n, x'_n) = 0.
\]

(26)

Now consider the coincidence limit of two point quark Green’s function in QCD in presence of external sources,

\[
\frac{1}{N_c} \langle 0| T\bar{\psi}^{an}_\alpha(x)\psi^{\kappa}_\alpha(x')|0 \rangle_{QCD} = -i \frac{\delta \ln Z[J]}{N_c \delta J^{(an)(\kappa)}(x)}
\]

\[
= \frac{1}{N_c} \int DU \frac{\delta S_{GL}[U,J]}{\delta J^{(an)(\kappa)}(x)} e^{iS_{GL}[U,J]} = \frac{1}{N_c} \int DU \frac{\Phi^{(an)(\kappa)}[U,J](x, x)}{e^{iS_{GL}[U,J]}}
\]

where we have used (3) and \( \Phi^{(an)(\kappa)}[U,J](x, x) \) is
\[ \Phi^{(\alpha \beta \xi)}(U, J)(x, x) \equiv \frac{1}{N_c} \frac{\delta S_{\text{GL}}[U, J]}{\delta J^{(\alpha \beta \xi)}(x)} \approx \frac{1}{N_c} \frac{\delta S_{\text{GND}}[U, J]}{\delta J^{(\alpha \beta \xi)}(x)}. \] (28)

Use (18), the rotated \( \Phi \) become

\[ \Phi^{(\alpha \beta \xi)}(U, J)(x, x) = \text{Tr} \left[ \frac{-i}{i \hat{\mathcal{D}} + J_{\Omega} - \Sigma(\nabla^2)} \delta (J_{\Omega} - \Sigma(\nabla^2)) \right] \] (29)

which imply

\[ \Phi_{\Omega}(x, y) = -i \int d^4z \left[ i \hat{\mathcal{D}} + J_{\Omega} - \Sigma(\nabla^2) \right]^{-1}(x, z) \delta(z - y) + \Delta(z, y) \] (30)

where \( \Delta(z, y) \) relate to \( \delta \Sigma(\nabla^2)/\delta J \). Compare to (26), we find, present choice of \( S_{\text{int}} \) is equivalent to take following approximation

\[ \left[ (1 + \Delta)^{-1}[\Sigma(\nabla^2) - \Delta(i \hat{\mathcal{D}} + J_{\Omega})] \right]^{\sigma\rho}(x, y) \approx \tilde{\Sigma}^{\sigma\rho}(x, y) + \sum_{n=1}^{\infty} \int d^4x_1 \cdots d^4x_n d^4x_1' \cdots d^4x_n' \frac{(-i)^{n+1}(N_c g^2)^n}{n!} \times G^{\sigma_1 \cdots \sigma_n}_{\rho_1 \cdots \rho_n}(x, y, x_1, x_1', \cdots, x_n, x_n'). \] (31)

If we further drop correlation functions \( G^{\sigma_1 \cdots \sigma_n}_{\rho_1 \cdots \rho_n}(x, y, x_1, x_1', \cdots, x_n, x_n') \) with \( n > 1 \) and ignore the external sources in above equation (\( \Delta \) therefore must be ignored), as mentioned in Ref. [10], (31) then is just Schwinger-Dyson equation for quark propagator. We only consider the term of quark self energy with argument of \( \nabla^2 \) which is the minimal generalization from pure \( \Sigma(\partial^2) \) term to source dependent terms satisfying local chiral symmetry. Include in source terms, just self energy term in l.h.s. of (31) is not enough to match all contributions of r.h.s. of equation, but if underlying QCD can provide correct predictions for parameters in chiral Lagrangian, the fact of \( \Sigma(k^2) \) dominance in the parameters of chiral Lagrangian imply that the terms we dropped at this work should not play so important role. we will leave these terms in addition to self energy term to balance equation (31) in future investigations.

In conclusion, all 12 parameters in \( p^4 \) order \( SU(3) \) chiral Lagrangian are explicitly expressed in terms of functions of quark self energy \( \Sigma(k^2) \). We have shown that the original result given from anomaly approach is completely cancelled. Take suitable quark self energy ansatz to perform numerical calculation, we find after cancellation of anomaly contribution, the dynamical quark self energy do can provide parameter values roughly consistent with experimental data.

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TABLE I. Values, multiplied by $10^3$, for the parameters of the order $p^4$ chiral Lagrangian calculated in GND model with quark self energy determined by ansatz (25). anomaly: anomaly approach result; expt: experimental values.

|     | $L_1$ | $L_2$ | $L_3$ | $L_4$ | $L_5$ | $L_6$ | $L_7$ | $L_8$ | $L_9$ | $L_{10}$ |
|-----|-------|-------|-------|-------|-------|-------|-------|-------|-------|---------|
| A=1 | 0.927 | 1.85  | -6.55 | 0     | 1.79  | 0     | -0.570| 1.56  | 3.79  | -5.18   |
| A=2 | 0.771 | 1.54  | -5.53 | 0     | 1.63  | 0     | -0.501| 1.39  | 2.60  | -3.91   |
| A=3 | 0.708 | 1.42  | -5.09 | 0     | 1.51  | 0     | -0.449| 1.26  | 2.13  | -3.38   |
| A=4 | 0.674 | 1.35  | -4.85 | 0     | 1.41  | 0     | -0.413| 1.17  | 1.90  | -3.08   |
| anomaly | 0.792 | 1.58  | -3.17 | 0     | 0     | 0     | 0.263 | -0.792| 6.33  | -3.17   |
| expt | 0.9±0.3 | 1.7±0.7 | -4.4±2.5 | 0±0.5 | 2.2±0.5 | 0±0.3 | -0.4±0.15 | 1.1±0.3 | 7.4±0.7 | -6.0±0.7 |