Charge Asymmetry and Photon Energy Spectrum in the Decay $B_s \rightarrow l^+l^−γ$

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Abstract

We consider the structure-dependent amplitude of the decay $B_s \rightarrow l^+l^−γ$ ($l = e, \mu$) in a model based on the effective Hamiltonian for $b\bar{s} \rightarrow l^+l^−$ containing the Wilson coefficients $C_7, C_9$ and $C_{10}$. The form factors characterising the matrix elements $\langle γ|\bar{s}\gamma_\mu(1 \mp \gamma_5)b|B_s\rangle$ and $\langle γ|\bar{s}\sigma_\mu\nu(1 \mp \gamma_5)b|B_s\rangle$ are taken to have the universal form $f_V \approx f_A \approx f_T \approx f_{B_s} M_{B_s} R_s/(3E_γ)$ suggested by recent work in QCD, where $R_s$ is a parameter related to the light cone wave function of the $B_s$ meson. Simple expressions are obtained for the charge asymmetry $A(x_γ)$ and the photon energy spectrum $dΓ/dx_γ(x_γ = 2E_γ/M_{B_s})$. The decay rates are calculated in terms of the decay rate of $B_s \rightarrow γγ$. The branching ratios are estimated to be $Br(B_s \rightarrow e^+e^−γ) = 2.0 \times 10^{-8}$ and $Br(B_s \rightarrow μ^+μ^−γ) = 1.2 \times 10^{-8}$, somewhat higher than earlier estimates.

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1 Introduction

The rare decay $B_s \rightarrow l^+l^-\gamma$ is of interest as a probe of the effective Hamiltonian for the transition $b\bar{s} \rightarrow l^+l^-$, and as a testing ground for form factors describing the matrix elements $\langle \gamma | \bar{s}\gamma_{\mu}(1 \mp \gamma_5)b|\bar{B}_s\rangle$ and $\langle \gamma |\bar{s}\sigma_{\mu\nu}(1 \mp \gamma_5)b|\bar{B}_s\rangle$ [1, 2]. The branching ratio for $B_s \rightarrow l^+l^-\gamma$ can be sizeable in comparison to the non-radiative process $B_s \rightarrow l^+l^-$, since the chiral suppression of the latter is absent in the radiative transition. We will be concerned mainly with the structure-dependent part of the matrix element, since the correction due to bremsstrahlung from the external leptons is small and can be removed by eliminating the end-point region $s_{l^+l^-} \approx M_{B_s}^2$. (For related studies of radiative B decays, we refer to the papers in Ref. [3].)

Our objective is to calculate the decay spectrum of $B_s \rightarrow l^+l^-\gamma$ using form factors suggested by recent work in QCD [4]. These form factors have the virtue of possessing a universal behaviour $1/E_\gamma$ for large $E_\gamma$, as well as a universal normalization. These features can be tested in measurements of $B^+ \rightarrow \mu^+\nu\gamma$ and $B_s \rightarrow \gamma\gamma$. We derive simple formulae for the photon energy spectrum $d\Gamma/dx_\gamma$, $x_\gamma = 2E_\gamma/m_{B_s}$, and the charge asymmetry $A(x_\gamma)$, defined as the difference in the probability of events with $E^+ > E^-$ and $E^+ < E^-$, $E^\pm$ being the $l^\pm$ energies. This asymmetry is large over most of the $x_\gamma$ domain. Predictions are obtained for the branching ratios $Br(B_s \rightarrow e^+e^-\gamma)$ and $Br(B_s \rightarrow \mu^+\mu^-\gamma)$ which are somewhat higher than those estimated in previous literature [1, 2].

2 Matrix Element and Differential Decay Rate

The effective Hamiltonian for the interaction $b\bar{s} \rightarrow l^+l^-$ has the standard form [5]

$$H_{\text{eff}} = \frac{\alpha G_F V_{tb}V_{ts}^*}{\sqrt{2\pi}} \left\{ C_9^\text{eff} (\bar{s}\gamma_{\mu}P_Lb)\bar{l}\gamma_{\mu}l + C_{10}^\text{eff} (\bar{s}\gamma_{\mu}P_Lb)\bar{l}\gamma_{\mu}\gamma_5l - 2\frac{C_7}{q^2} \bar{s}\sigma_{\mu\nu}q^\nu (m_bP_R + m_sP_L)\bar{b}\gamma_{\mu}l \right\}$$ (1)

where $P_{L,R} = (1 \mp \gamma_5)/2$ and $q$ is the sum of the $l^+$ and $l^-$ momenta. For the purpose of this paper, we will neglect the small $q^2$-dependent terms in $C_{9,10}^\text{eff}$, arising from one-loop contributions of four-quark operators, as well as long-distance effects associated with $c\bar{c}$ resonances. The Wilson coefficients in Eq. (1) will be taken to have the constant values

$$C_7 = -0.315 , \quad C_9 = 4.334 , \quad C_{10} = -4.624 \quad (2)$$

To obtain the amplitude for $B_s \rightarrow l^+l^-\gamma$, one requires the matrix elements $\langle \gamma |\bar{s}\gamma_{\mu}(1 \mp \gamma_5)b|\bar{B}_s\rangle$ and $\langle \gamma |\bar{s}\sigma_{\mu\nu}(1 \mp \gamma_5)b|\bar{B}_s\rangle$. We parametrise these in the same
way as in Ref. [1, 2]

\[ \langle \gamma(k) | s\gamma_\mu b | \bar{B}_s(k + q) \rangle = e \epsilon_{\mu'\nu'\rho'\sigma'} \epsilon^{*\nu\rho\sigma} k^{*\mu} f_V(q^2)/M_{B_s} \]

\[ \langle \gamma(k) | s\gamma_\mu \gamma_5 b | \bar{B}_s(k + q) \rangle = -i e \epsilon_{\mu'\nu'\rho'\sigma'} \epsilon^{*\nu\rho\sigma} k^{*\mu} f_A(q^2)/M_{B_s} \]

\[ \langle \gamma(k) | s\sigma_\mu q'' b | \bar{B}_s(k + q) \rangle = -e \epsilon_{\mu'\nu'\rho'\sigma'} \epsilon^{*\nu\rho\sigma} k^{*\mu} f_T(q^2) \]

\[ \langle \gamma(k) | s\sigma_\mu \gamma_5 q'' b | \bar{B}_s(k + q) \rangle = -i e \epsilon_{\mu'\nu'\rho'\sigma'} \epsilon^{*\nu\rho\sigma} k^{*\mu} f'_T(q^2) \]

The form factors $f_V, f_A, f_T$ and $f'_T$ are dimensionless, and related to those of Aliev et al [1] by

\[ f_V = g/M_{B_s}, f_A = f/M_{B_s}, f_T = -g_1/M_{B_s}^2, f'_T = -f_1/M_{B_s}^2 \]

The matrix element for $\bar{B}_s \rightarrow l^+ l^- \gamma$ can then be written as (neglecting terms of order $m_s/m_b$)

\[ \mathcal{M}(|\bar{B}_s \rightarrow l^+ l^- \gamma) = \frac{\alpha G_F}{2\sqrt{2}\pi} e V_{tb} V_{ts}^* \frac{1}{M_{B_s}} \]

\[ \left[ \epsilon_{\mu'\nu'\rho'\sigma'} \epsilon^{*\nu\rho\sigma} k^{*\mu} \left( A_1 \bar{\gamma}^\mu l + A_2 \bar{\gamma}^{\mu'\gamma_5} l \right) \]

\[ + i \left( \epsilon^{*\mu}(k \cdot q) - (\epsilon^{*\mu} q) k_\mu \right) \left( B_1 \bar{\gamma}^\mu l + B_2 \bar{\gamma}^{\mu'\gamma_5} l \right) \right] \]

where

\[ A_1 = C_9 f_V + 2 C_7 \frac{M_{B_s}^2}{q^2} f_T \]

\[ A_2 = C_{10} f_V \]

\[ B_1 = C_9 f_A + 2 C_7 \frac{M_{B_s}^2}{q^2} f'_T \]

\[ B_2 = C_{10} f_A \]

(In the coefficient of $C_7$, we have approximated $m_b M_{B_s}$ by $M_{B_s}^2$). The Dalitz plot density in the energy variables $E_{\pm}$ is

\[ \frac{d\Gamma}{dE_+ dE_-} = \frac{1}{256\pi^3 M_{B_s}} \sum_{\text{spin}} |\mathcal{M}|^2 \]
\[ \sum_{\text{spin}} |\mathcal{M}|^2 = \left| \frac{\alpha G_F V_{tb} V_{ts}^*}{\sqrt{2\pi}} \right|^2 \frac{1}{M_{B_s}^2} \]

\[ \left\{ (|A_1|^2 + |B_1|^2) \left[ q^2 \{(p_+ \cdot k)^2 + (p_- \cdot k)^2\} + 2m_t^2(q \cdot k)^2 \right] + (|A_2|^2 + |B_2|^2) \left[ q^2 \{(p_+ \cdot k)^2 + (p_- \cdot k)^2\} - 2m_t^2(q \cdot k)^2 \right] + 2\text{Re}(B_1^*A_2 + A_1^*B_2)q^2 \left[ (p_+ \cdot k)^2 - (p_- \cdot k)^2 \right] \right\} \tag{7} \]

It is convenient to introduce dimensionless variables

\[ x_\gamma = 2E_\gamma/M_{B_s} \, , \, x_\pm = 2E_\pm/M_{B_s} \, , \, \Delta = x_+ - x_- \, , \, r = m_t^2/M_{B_s}^2 \tag{8} \]

in terms of which \( q^2 = M_{B_s}^2(1 - x_\gamma) \). Taking \( x_\gamma \) and \( \Delta \) as the two coordinates of the Dalitz plot, phase space is defined by

\[ |\Delta| \leq vx_\gamma \, , \, v = \sqrt{1 - 4m_t^2/q^2} = \sqrt{1 - 4r/(1 - x_\gamma)} \, , \, 0 \leq x_\gamma \leq 1 - 4r \tag{9} \]

In terms of \( x_\gamma \) and \( \Delta \), the differential decay width takes the form

\[ \frac{d\Gamma}{dx_\gamma d\Delta} = \mathcal{N} \left\{ (|A_1|^2 + |B_1|^2) \left[ \frac{(1 - x_\gamma)(x_\gamma^2 + \Delta^2)}{8} + \frac{1}{2}rx_\gamma \right] + (|A_2|^2 + |B_2|^2) \left[ \frac{(1 - x_\gamma)(x_\gamma^2 + \Delta^2)}{8} - \frac{1}{2}rx_\gamma \right] + 2\text{Re}(B_1^*A_2 + A_1^*B_2)(1 - x_\gamma)\frac{1}{4}x_\gamma \Delta \right\} \tag{10} \]

where \( \mathcal{N} = \frac{\alpha^2 G_F^2/(256\pi^4)}{|V_{tb}V_{ts}^*|^2 M_{B_s}^5} \). The last term is linear in \( \Delta \) and produces an asymmetry between the \( l^+ \) and \( l^- \) energy spectra.

We will derive from Eq. (10) two distributions of interest:

(i) The charge asymmetry \( A(x_\gamma) \) defined as

\[ A(x_\gamma) = \frac{\left( \int_{0}^{vx_\gamma} \frac{dr}{dx_\gamma d\Delta} \int_{-vx_\gamma}^{0} \frac{dr}{dx_\gamma d\Delta} \right) d\Delta}{\int_{-vx_\gamma}^{vx_\gamma} \frac{dr}{dx_\gamma d\Delta} d\Delta} \tag{11} \]

\[ = \frac{3}{4}v(1 - x_\gamma) \left\{ (|A_1|^2 + |B_1|^2)(1 - x_\gamma + 2r) + (|A_2|^2 + |B_2|^2)(1 - x_\gamma - 4r) \right\} \]
The photon energy spectrum

\[
\frac{d\Gamma}{dx_\gamma} = \frac{\alpha^3 G_F^2 |V_{tb}|^2 |V_{ts}|^2 M_{B_s}^5 v x_\gamma^3}{768\pi^4} \left[ (|A_1|^2 + |B_1|^2)(1 - x_\gamma + 2r) + (|A_2|^2 + |B_2|^2)(1 - x_\gamma - 4r) \right]
\]

(12)

To proceed further, we must introduce a model for the form factors which appear in the functions \(A_{1,2}\) and \(B_{1,2}\) defined in Eq. (8).

3 Model for Form Factors

First of all, we note that the form factors \(f_T\) and \(f'_T\) defined in Eq. (3) are necessarily equal, by virtue of the identity

\[
\sigma_{\mu\nu} = \frac{i}{2} \epsilon_{\mu\nu\alpha\beta} \sigma^{\alpha\beta} \gamma_5
\]

(13)

This was pointed out by Korchemsky et al [4]. We therefore have to deal with three independent form factors \(f_V, f_A\) and \(f_T\). These have been computed in Ref. [4] using perturbative QCD methods combined with heavy quark effective theory. For the vector and axial vector form factors of the radiative decay \(B^+ \to l^+ \nu \gamma\), and their tensor counterpart, defined as in Eq. (3), these authors obtain the remarkable result

\[
f_V(E_\gamma) = f_A(E_\gamma) = f_T(E_\gamma) = \frac{f_{B_s} M_{B_s}}{2 E_\gamma} \left( Q_u R - \frac{Q_b}{m_b} \right) + \mathcal{O}\left( \frac{\Lambda^2_{QCD}}{E_\gamma^2} \right)
\]

(14)

where \(R\) is a parameter related to the light-cone wave-function of the \(B\) meson, with an order of magnitude \(R^{-1} \sim \bar{\Lambda} = M_B - m_b\), where the binding energy \(\bar{\Lambda}\) is estimated to be between 0.3 and 0.4 GeV. Applying the same reasoning to the form factors for \(B_s \to l^+ l^- \gamma\), we conclude that

\[
f_V(E_\gamma) = f_A(E_\gamma) = f_T(E_\gamma) = \frac{f_{B_s} M_{B_s}}{2 E_\gamma} \left( -Q_s R_s + \frac{Q_b}{m_b} \right) + \mathcal{O}\left( \frac{\Lambda^2_{QCD}}{E_\gamma^2} \right)
\]

(15)

In what follows, we will neglect the term \(Q_b/m_b\), and approximate the form factors by

\[
f_{V,A,T}(E_\gamma) \approx \frac{f_{B_s} M_{B_s}}{2 E_\gamma} \frac{1}{3 \bar{\Lambda}_s} \frac{1}{3 \bar{\Lambda}_s x_\gamma}
\]

(16)

where \(\bar{\Lambda}_s = M_{B_s} - m_b\) will be taken to have the nominal value 0.5 GeV. Several of our results will depend only on the universal form \(f_{V,A,T}(E_\gamma) \sim 1/E_\gamma\), independent
of the normalization. As pointed out in [4], a check of the behaviour \( f_{V,A} \sim 1/E_\gamma \) in the case of \( B^+ \to \mu^+\nu\gamma \) is afforded by the photon energy spectrum, which is predicted to be

\[
\frac{d\Gamma}{dx_\gamma} \sim \left[ f_V^2(E_\gamma) + f_A^2(E_\gamma) \right] x_\gamma^3 (1 - x_\gamma)
\]

\[
\sim x_\gamma (1 - x_\gamma)
\]

(17)

In the case of the reaction \( B_s \to l^+l^-\gamma \), the normalization of the tensor form factor \( f_T(E_\gamma) \) at \( E_\gamma = M_{B_s}/2 \) (i.e. \( x_\gamma = 1 \)) can be checked by appeal to the decay rate of \( B_s \to \gamma\gamma \). To see this connection, we note that the matrix element of \( B_s \to \gamma(l,k) + \gamma(l',k') \) can be obtained from that of \( B_s \to l^+l^-\gamma \) by putting \( C_9 = C_{10} = 0 \), and replacing the factor \( (e f_T C_7/q^2)(\bar{l}\gamma\mu l) \) by \( f_T(x_\gamma = 1) \epsilon_\mu' \). This yields the matrix element

\[
\mathcal{M}(\bar{B}_s \to \gamma(\epsilon,k)\gamma(\epsilon',k')) = -i \frac{G_F e^2}{\sqrt{2} \pi^2} (V_{tb} V'_{ts}^*) \cdot \left[ A^+ F_{\mu\nu} F'^{\mu\nu} + i A^- F_{\mu\nu} \tilde{F}^{\mu\nu} \right]
\]

with

\[
A^+ = -A^- = \frac{1}{4} M_{B_s} f_T(x_\gamma = 1) C_\gamma.
\]

(18)

The result for \( A^\pm \) coincides with that obtained in Refs. [7, 8, 9] when \( f_T(x_\gamma = 1) = -\frac{Q_{fB}}{\Lambda_s} = \frac{1}{3} \frac{f_{B_s}}{\Lambda_s} \). (In Ref. [3, 4], the role of the parameter \( \Lambda_s \) is played by the constituent quark mass \( m_s \).) Thus the decay width of \( B_s \to \gamma\gamma \),

\[
\Gamma(B_s \to \gamma\gamma) = \frac{M_{B_s}^3}{16\pi} \left| \frac{G_F e^2}{\sqrt{2} \pi^2} V_{tb} V'_{ts}^* \right|^2 (|A_+|^2 + |A_-|^2)
\]

(19)

serves as a test of the normalization factor \( f_T(x_\gamma = 1) \).

We remark, parenthetically, that the calculation of \( B_s \to \gamma\gamma \), based on an effective interaction for \( b \to s\gamma\gamma \), produces the amplitudes \( A^+ \) and \( A^- \) given in Eq. (18) in the limit of retaining only the ‘reducible’ diagrams related to the transition \( b \to s\gamma \). Inclusion of ‘irreducible’ contributions like \( bs \to c\bar{c} \to \gamma\gamma \) introduces a correction term in \( A_- \) causing the ratio \( |A_+/A_-| \) to deviate from unity. Estimates in Ref. [4, 5] yield values for this ratio between 0.75 and 0.9. The branching ratio \( Br(B_s \to \gamma\gamma) \) is estimated at \( 5 \times 10^{-6} \), with an uncertainty of about 50%.

Having specified our model for the form factors \( f_V(x_\gamma), f_A(x_\gamma) \) and \( f_T(x_\gamma) \), we proceed to present results for the spectrum and branching ratio of \( B_s \to l^+l^-\gamma \). We use \( M_{B_s} = 5.3 \text{ GeV} \), \( f_{B_s} = 200 \text{ MeV} \) and, where necessary, \( \Lambda_s = 0.5 \text{ GeV} \) in the normalization of the form factors in Eq. (16).
4 Results

4.1 Charge Asymmetry

With the assumption of universal form factors \( f_V = f_A = f_T \sim \frac{1}{x_\gamma} \), the asymmetry \( A(x_\gamma) \) in Eq. (11) assumes the simple form

\[
A(x_\gamma) = \frac{3}{4} v^2 C_{10} \left[ C_9 + 2 C_7 \left( \frac{1}{1-x_\gamma} \right) \right] (1-x_\gamma) \left( 1 - x_\gamma + 2 r \right) + C_{10}^2 (1-x_\gamma - 4 r) \tag{20}
\]

This is plotted in Fig. 1, and is clearly large and negative over most of the \( x_\gamma \) domain, changing sign at \( x_\gamma = 1 + \frac{2 C_7}{C_9} \). (A negative asymmetry corresponds to \( l^- \) being more energetic, on average, than \( l^+ \) in the decay \( \bar{B}_s(= b\bar{s}) \to l^+ l^- \gamma \).) The average charge asymmetry is

\[
\langle A \rangle = \frac{3}{4} \int_0^{1-4r} dx_\gamma v^2 x_\gamma (1-x_\gamma) 2 C_{10} \left( C_9 + 2 C_7 \left( \frac{1}{1-x_\gamma} \right) \right) \left( 1 - x_\gamma + 2 r \right) + C_{10}^2 (1-x_\gamma - 4 r) \tag{21}
\]

and has the numerical value \( \langle A \rangle_e = -0.28 \), \( \langle A \rangle_\mu = -0.47 \) for the modes \( l = e, \mu \), the difference arising essentially from the end-point region \( x_\gamma \approx 1 - 4 r \).

4.2 Photon Energy Spectrum

With the form factors of Eq. (16), the photon energy spectrum simplifies to

\[
\frac{d\Gamma}{dx_\gamma} = \frac{1}{3} N v x_\gamma \left\{ (1-x_\gamma + 2 r) (C_9 + 2 C_7 \left( \frac{1}{1-x_\gamma} \right) )^2 + (1-x_\gamma - 4 r) C_{10}^2 \right\} \tag{22}
\]

where the constant factor \( N \) is defined after Eq. (10). It is expedient to write this distribution in terms of the decay rate of \( \bar{B}_s \to \gamma \gamma \). We then obtain the prediction

\[
\frac{d\Gamma(\bar{B}_s \to l^+ l^- \gamma)}{\Gamma(\bar{B}_s \to \gamma \gamma)} = \left\{ \frac{2 \alpha}{3 \pi} \frac{x_\gamma^3}{(1-x_\gamma)^2} v (1-x_\gamma + 2 r) \right\} \tag{23}
\]

\[
\cdot \left( \frac{1}{x_\gamma} \right)^2 \left\{ \eta_9 (1-x_\gamma) + 1 \right\}^2 + \left\{ \eta_{10} (1-x_\gamma) \right\}^2 \frac{1-x_\gamma - 4 r}{1-x_\gamma + 2 r}
\]

The first factor (in curly brackets \( \{ \} \)) is the QED result expected if the decay \( \bar{B}_s \to l^+ l^- \gamma \) is interpreted as a Dalitz pair reaction \( \bar{B}_s \to \gamma \gamma^* \to \gamma l^+ l^- \), without form factors. The factor \( (1/x_\gamma)^2 \) results from the universal behaviour \( f_{V,A,T} \sim 1/x_\gamma \) given in Eq. (10), while the last factor is the electroweak effect associated with the coefficients \( \eta_9 = C_9/(2 C_7) \) and \( \eta_{10} = C_{10}/(2 C_7) \). This distribution is plotted in Figs. 2 and 3, where the QED result is shown for comparison.
4.3 Rates and Branching Ratios

From the photon spectrum given in Eq. (23), we derive the ‘conversion ratios’

\[ R_l = \frac{\int_{0}^{1} - 4r \, \frac{dt}{dx} (B_s \to l^+ l^- \gamma)}{\Gamma(B_s \to \gamma \gamma)} \]  \hspace{1cm} (24)

The numerical values are \( R_e = 4.0\% \) and \( R_\mu = 2.3\% \). These are to be contrasted with the QED result given by

\[ R_l(QED) = 2\alpha \frac{3}{3\pi} \left[ (1 - 18r^2 + 8r^3) \ln \frac{1 + \sqrt{1 - 4r}}{1 - \sqrt{1 - 4r}} + \sqrt{1 - 4r} \left(-\frac{7}{2} + 13r + 4r^2\right) \right] \]  \hspace{1cm} (25)

which yields \( R_e(QED) = 2.3\% \), \( R_\mu(QED) = 0.67\% \). The absolute branching ratios of \( \bar{B}_s \to l^+ l^- \gamma \), obtained by taking \( Br(B_s \to \gamma \gamma) = 5 \times 10^{-7} \) [4, 5] are \( Br(\bar{B}_s \to e^+ e^- \gamma) = 2.0 \times 10^{-8} \), \( Br(\bar{B}_s \to \mu^+ \mu^- \gamma) = 1.2 \times 10^{-8} \). Our results for the average charge asymmetry \( \langle A \rangle_l \), the conversion ratios \( R_l \) and the branching ratios are summarized in Table 1.

5 Comments

(i) The branching ratios calculated by us are somewhat higher than those obtained in previous work [1, 2], which used a different parametrization of the form factors \( f_V, f_A, f_T, f'_T \) based on QCD sum rules [1] and light-front models [3]. In particular these parametrizations do not satisfy the relation \( f_T = f'_T \) which, as noted in [4], follows from the identity \( \sigma_{\mu\nu} = \frac{i}{2} \epsilon_{\mu\nu\alpha\beta} \sigma^{\alpha\beta} \gamma_5 \).

(ii) Our predictions for the charge asymmetry \( \langle A \rangle \) and the conversion ratio \( \Gamma(\bar{B}_s \to l^+ l^- \gamma)/\Gamma(\bar{B}_s \to \gamma \gamma) \) are independent of the parameter \( \Lambda_s \) which appears in the form factor in Eq. (16). The branching ratios in Table 1 assume \( Br(\bar{B}_s \to \gamma \gamma) = 5 \times 10^{-7} \), and can be rescaled when data on this channel are available.

(iii) A full analysis of the decay \( \bar{B}_s \to l^+ l^- \gamma \) requires inclusion of the bremsstrahlung amplitude corresponding to photon emission from the leptons in \( B_s \to l^+ l^- \). This contribution is proportional to \( f_B m_l \) and affects the photon energy spectrum in the small \( x_\gamma \) region. We have calculated the corrected spectrum for \( B_s \to l^+ l^- \), following the procedure in [14], and the result is shown in Fig. 4 for the case \( l = \mu \). As anticipated, the correction is limited to small \( x_\gamma \), and can be removed by a cut at small photon energies.
(iv) The QCD form factors in Eq. (16) are valid up to corrections of order \((\Lambda_{\text{QCD}}/E_\gamma)^2\). In the small \(x_\gamma\) region, arguments based on heavy hadron chiral perturbation theory suggest form factors dominated by the \(B^*\) pole with the appropriate quantum numbers, for example,

\[
f_V(x_\gamma) \sim \frac{1}{M_{B_s}^2 (1 - x_\gamma) - M_{B_s}^2}
\]

Defining \(M_{B^*_s} - M_{B_s} = \Delta M\), this form factor has the behaviour \(f_V(x_\gamma) \sim \frac{1}{x_\gamma + \delta}\), with \(\delta \approx 2\Delta M/M_{B_s} \approx 0.02\). We have investigated the effect of replacing the QCD form factor of Eq. (16) by a different universal form \(f_{V,A,T}(x_\gamma) = f_{B_s}/(3\bar{\Lambda}_s(x_\gamma + \delta))\), and found only minor changes in the numbers given in Table 1. In general, one must expect some distortion in the spectrum at low \(x_\gamma\), compared to that shown in Figs. 1-4.

(v) We will examine separately the predictions for \(A(x_\gamma)\) and \(d\Gamma/dx_\gamma\) in the reaction \(B_s \to \tau^+\tau^-\gamma\), in which the bremsstrahlung part of the matrix element plays a significant role [11]. We will consider also refinements due to the \(q^2\)-dependent term in \(C_9^{\text{eff}}\), and the effects of \(c\bar{c}\) resonances.

In view of their clear signature, non-negligible branching ratios and interesting dynamics, the decays \(B_s \to l^+l^-\gamma\) could form an attractive domain of study at future hadron colliders.

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Decay | Average Charge Asymmetry \(\langle A \rangle\) | Conversion Ratio \(\frac{\Gamma(B_s \rightarrow l^+l^-\gamma)}{\Gamma(B_s \rightarrow \gamma\gamma)}\) | Branching Ratio \(\frac{\Gamma(B_s \rightarrow l^+l^-\gamma)}{\Gamma(B_s \rightarrow \text{all})}\)  
--- | --- | --- | ---  
\(\bar{B}_s \rightarrow e^+e^-\gamma\) | -0.28 | 4.0% | 2.0 \times 10^{-8}  
\(\bar{B}_s \rightarrow \mu^+\mu^-\gamma\) | -0.47 | 2.3% | 1.2 \times 10^{-8}  

Table 1: Average charge asymmetry, Conversion ratio and Branching ratio for the decays \(\bar{B}_s \rightarrow e^+e^-\gamma\) and \(\bar{B}_s \rightarrow \mu^+\mu^-\gamma\). (Last column assumes \(Br(\bar{B}_s \rightarrow \gamma\gamma) = 5 \times 10^{-7}\))

![Asymmetry versus \(x_\gamma\)](image_url)

Figure 1: Asymmetry versus \(x_\gamma\)
Figure 2: Photon Energy Distribution for $\bar{B}_s \rightarrow e^+e^-\gamma$, normalized to $\bar{B}_s \rightarrow \gamma\gamma$. (Dashed line is the QED result.)

Figure 3: Photon Energy Distribution for $\bar{B}_s \rightarrow \mu^+\mu^-\gamma$, normalized to $\bar{B}_s \rightarrow \gamma\gamma$. (Dashed line is the QED result.)
Figure 4: Photon Energy Spectrum in $\bar{B}_s \rightarrow \mu^+ \mu^- \gamma$, with bremsstrahlung (solid line) and without bremsstrahlung (dashed line)