MULTI-PERIOD HAZARDOUS WASTE COLLECTION PLANNING WITH CONSIDERATION OF RISK STABILITY

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Abstract. Hazardous wastes are likely to cause danger to humans and the environment. In this paper, a new mathematical optimization model is developed for the multi-period hazardous waste collection planning problem. The hazardous wastes generated by each source are time-varying in weight and allow incomplete and delayed collection. The aim of the model is to help decision makers determine the weight of hazardous wastes to collect from each source and the transportation routes of vehicles in each period. In the developed model, three objectives are considered simultaneously: (1) minimisation of total cost over all periods, which includes start-up fee of vehicles, transportation cost of hazardous wastes, and penalty fee for the delayed collection; (2) minimisation of total transportation risk posing to the surrounding of routes over all periods; and (3) even distribution of transportation risk among all periods, also called risk stability. The developed multi-objective model is transformed into a single-objective one based on the weighted sums method, which is finally equated to a mixed 0-1 linear programming by introducing a set of auxiliary variables and constraints. Numerical experiments are computed with CPLEX software to find the optimal solutions. The computational results and parameters analysis demonstrate the applicability and validity of the developed model. It is found that the consideration of the risk stability can reduce the total transportation risk, the uneven distribution of the transportation risk among all periods, and the maximum number of vehicles used, though increasing the total cost to some extent.

1. Introduction. A waste is characterized as hazardous if it possesses one of the four characteristics: ignitability, corrosivity, reactivity or toxicity. Hazardous wastes can be generated as a result of a variety of production and manufacturing processes, such as wood preservation, inorganic pigment manufacturing, organic/inorganic chemicals manufacturing, pesticides manufacturing, explosives manufacturing, petroleum refining, iron and steel production, aluminum production, lead processing, veterinary pharmaceuticals manufacturing, ink formulation, coking, electroplating and other metal finishing operations, dioxin bearing, and production of certain chlorinated aliphatic hydrocarbons [1]. Hazardous wastes may pose risks to both the public and the environment, especially when poorly handled [2]. With the development of industrialization, hazardous waste management problem becomes more
serious, and the importance of efficient collection and recycling is raised. In order to protect the public and the environment, many countries have established laws and regulations that encourage and regulate the collection and recycling of hazardous wastes.

For the collection and recycling of hazardous wastes, the majority of relevant studies have been focused on routing decisions for transportation of hazardous wastes. Transportation of hazardous wastes calls for the determination of a set of routes for a fleet of vehicles to visit a set of sources of hazardous wastes generation. The routing decision may simultaneously affect several interested goals. For example, the selection of the routes with least risk may lead to an unacceptable increase in cost; conversely, reducing the cost may increase the risk. The transportation of hazardous wastes requires a balance between the cost and the risk. Consequently, a considerable amount of multi-objective optimization models have been developed in hazardous waste/material transportation-related studies, in order to take into account various goals simultaneously. As early as 1981, Shobrys [3] minimized a weighted combination of risk and cost for the selection of storage locations and transportation routes for hazardous substances. List and Mirchandani [4] introduced an integrated multi-objective model for routing and storing hazardous wastes. Three objectives are considered jointly: minimize cost, minimize risk, and equitably distribute risk among affected geographic areas. Risk equity is measured as the maximum risk per unit population for all the areas. Li et al. [5] studied the determination of optimal routes for dangerous goods transportation under conflicting objectives, including cost, safety, public and environmental exposure. Kang et al. [6] applied the concept of value at risk (VaR) to the routing problem of multiple trips and multiple types of hazardous materials, aiming at minimizing the global VaR value while satisfying equity constraints. Recently, models begin to engage more complex and realistic situations during hazardous waste/material transportation. For the bicriterion routing and scheduling problem arising in hazardous materials distribution planning, Androutsopoulos and Zografos [7] assumed the cost and risk attributes of each arc of the underlying transportation network are time-dependent, and sought to determine the non-dominated time-dependent paths for servicing a given and fixed sequence of customers within specified time windows. Fan et al. [8] formulated a bi-objective programming model that minimizes risk and cost for urban hazardous materials transportation, with consideration of road closure due to traffic controls, weather conditions, curfew and construction activities. Assadipour et al. [9] proposed a bi-objective optimization framework that considers both cost and risk for the rail-truck intermodal transportation of hazardous materials with terminal congestion.

All the models mentioned above are applicable only to single-period hazardous wastes transportation problems, in which the weight of hazardous wastes generated by each source is assumed to be a constant during the whole planning horizon ignoring the dynamic nature of the practical situations. In fact, the generation of hazardous wastes at each source varies with time, so it is advisable to have a multi-period model that takes into account such variations. Only limited researches have studied multi-period hazardous waste/material transportation problems. In order to enhance road network resilience with hazardous materials transportation in multiple periods, Chion [10] presented a mathematical optimization model to find period-dependent traffic responsive signal control. Tunaloğlu et al. [11] was concerned with the joint multiperiod problem of locating ultrafiltration facilities
and routing decisions for the collection and treatment of Olive Oil Mill Wastewater (OMWW), a kind of hazardous waste. In their problem, collections might be made on different periods for different olive mills. However, they did not consider the penalty fee due to the delayed collection, but at the end of each period the uncollected OMWW at each olive mill would result in contamination and storage costs. Rabbani et al. [12] extended a multi-objective mathematical model in the context of multi-period industrial hazardous waste management, in which the minimization of three objectives are related to the total cost, the transportation risk, and the site risk. In their setting, the total wastes of each treatment facility can be processed in the current period or be carried to the next period as inventory. Though the two above studies both considered the delayed collection/processing of hazardous wastes, the hazardous wastes at each source can only be completely collected/processed or completely uncollected/unprocessed in each period, and do not allow incomplete collection/processing, which means that a part of hazardous wastes are collected/processed in the current period, and the others are carried to the next period. To the best of our knowledge, there have been no studies considering the incomplete and delayed collection in the multi-period hazardous waste/material transportation problems.

In this paper, we consider that the hazardous wastes at each source in the current period allow an incomplete collection, and the uncollected hazardous wastes can be delayed to collect in the next period with incurring a penalty fee for the delayed collection. A new multi-objective mathematical optimization model is developed for the multi-period hazardous waste collection planning problem. Different with the previous hazardous waste/material transportation models, the decisions of our model in each period involve not only the transportation routes of vehicles but also the weight of hazardous wastes to collect from each source. The multi-period hazardous waste collection planning problem needs joint decision-making and optimization over all periods. Making decisions for each period separately, without taking into consideration the multi-period context, may result in a consequence that the transportation network is overloaded with hazardous wastes traffic, and the transportation risk increases considerably in a certain period. This inhomogeneity in the temporal distribution of transportation risk is detrimental to the long-term stable operation of the transportation network. In order to overcome this drawback and try to balance the distribution of transportation risk among all periods as much as possible, the developed multi-objective optimization model introduces a new objective of risk stability, in addition to minimisation of total cost and minimisation of total transportation risk. In the previous studies, the objective of risk equity is usually considered in the single-period hazardous waste/material management problems, and aims to evenly distribute the transportation risk among transportation areas/routes [4, 13, 14], which can be viewed as maximizing the risk homogeneity in the spatial distribution, while the objective of risk stability is suitable for the multi-period hazardous waste/material management problems, and aims to evenly distribute the transportation risk among all periods, which can be viewed as maximizing the risk homogeneity in the temporal distribution.

The contributions of this paper can be summarized as follows: (1) The incomplete and delayed collection of hazardous wastes is first considered in the multi-period hazardous waste collection planning problem, and a new multi-objective mathematical optimization model is developed, which can simultaneously determine the weight of hazardous wastes to collect from each source and the transportation routes
of vehicles in each period; (2) A objective of risk stability is first proposed, aiming to evenly distribute the transportation risk among all periods; (3) The developed multi-objective nonlinear programming model is equivalently transformed into a single-objective mixed 0-1 linear programming model based on the weighted sums method and by introducing a set of auxiliary variables and constraints; (4) Numerical experiments and parameters analysis are conducted to demonstrate the applicability and validity of the developed model.

The remainder of this paper is organized as follows. Section 2 presents the formal description and details of the mathematical model. Section 3 transforms and linearizes the developed model. Section 4 conducts numerical experiments and reports the computational results. Conclusions and suggestions for future research directions are provided in Section 5.

2. Problem description and formulation. Consider that there are one recycling center and multiple factories of hazardous wastes generation in a certain area. Let \( \{0\} \cup N \) denote the node set, in which node 0 corresponds to the recycling center and \( N \) represents the set of factories. The planning horizon consists of \(|T|\) periods, and in each period the weight of hazardous wastes generated by each factory is \( q_{ti}, t \in T, i \in N \). The recycling center is equipped with a homogeneous fleet \(|V|\) of vehicles with limited capacity \( Q \). In each period, every vehicle starts from the recycling center, collects hazardous wastes from each factory, and finally returns back to the recycling center. A transportation cost \( c_{ij} \) and a transportation risk \( r_{ij} \) will be incurred when transporting a unit weight of hazardous wastes from node \( i \) to node \( j \). Both the transportation cost and the transportation risk satisfy the triangle inequality, that is \( c_{ij} \leq c_{ik} + c_{kj}, r_{ij} \leq r_{ik} + r_{kj}, \forall i, j, k \in \{0\} \cup N \). In each period, the hazardous wastes at each factory allow an incomplete collection, and the uncollected hazardous wastes can be delayed to collect in the next period. At the end of each period, however, a unit weight of uncollected hazardous wastes would result in a penalty fee \( W \) due to the additional contamination and storage at factories.

The multi-period hazardous waste collection planning problem seeks to simultaneously determine the weight of hazardous wastes to collect from each factory and the transportation routes of vehicles in each period. It is worth noting that in each period, the problem relaxes the requirement of traditional vehicle routing problems that each node must be visited exactly once. In a certain period, a factory may not be visited when none of hazardous wastes is planned to collect from it. Three objectives need to be considered: first, to minimize the total cost over all periods, including the start-up fee of vehicles, the transportation cost of hazardous wastes, and the penalty fee for the delayed collection; second, to minimize the total transportation risk posed by hazardous wastes to the surrounding of routes over all periods; third, to evenly distribute the transportation risk among all periods, namely, to maximize the risk stability.

The three objectives are conflicting with each other. Minimizing the total cost may increase the total transportation risk, and vise verse; Minimizing the total transportation risk may aggravate the uneven distribution of the transportation risk among all periods, and vise verse. In the next subsection, a multi-objective mathematical optimization model is developed to consider the three objectives simultaneously based on some assumptions given as follows:

- The hazardous wastes generated by factories are the same type and can be transported together;
• The number of vehicles is sufficient enough to satisfy the needs of various collection planning;
• Every factory has enough storage capacity for hazardous wastes, and none of the factories has hazardous wastes stock at the beginning and end of the planning horizon;
• In each period, the weight of hazardous wastes generated by any a factory does not exceed the vehicle capacity;
• In each period, every factory can be visited by at most one vehicle;
• In each period, every vehicle can be used at most once.

2.1. Mathematical model. In this subsection, we develop a multi-objective mathematical optimization model for the problem. Notations which will be used in the model are first introduced as follows:

Sets
\{0\} \cup N: The set of all nodes, where \{0\} represents the recycling center, and \(N\) represents the set of factories;
\(T\): The set of periods;
\(V\): The set of vehicles;

Parameters
\(q_{ti}\): The weight of hazardous wastes generated by factory \(i\) in period \(t\), \(i \in N, t \in T\);
\(c_{ij}\): The transportation cost per unit weight from node \(i\) to node \(j\), \(i, j \in \{0\} \cup N\);
\(r_{ij}\): The transportation risk per unit weight of hazardous wastes from node \(i\) to node \(j\), \(i, j \in \{0\} \cup N\);
\(\omega\): The weight of each vehicle without load;
\(Q\): The capacity of each vehicle;
\(S\): The start-up fee of each vehicle;
\(W\): The penalty fee per unit weight of uncollected hazardous wastes at the end of each period;

Decision variables
\(x_{tvij}\): A binary variable; if node \(i\) is the precursor of node \(j\) on the route of vehicle \(v\) in period \(t\), it is equal to 1, otherwise 0, \(t \in T, v \in V, i, j \in \{0\} \cup N\);
\(s_{tvi}\): The weight of hazardous wastes to collect from factory \(i\) by vehicle \(v\) in period \(t\), \(t \in T, v \in V, i \in N\);

Intermediate variables
\(u_{ti}\): The weight of uncollected hazardous wastes at factory \(i\) at the end of period \(t\), \(t \in T, i \in N\);
\(p_{tvi}\): The loading weight of vehicle \(v\) leaving node \(i\) in period \(t\), \(t \in T, v \in V, i \in \{0\} \cup N\).

2.1.1. Objectives.

\[
\min \sum_{t \in T} \sum_{v \in V} \left[ S \sum_{j \in N} x_{tv0j} + \sum_{i \in \{0\} \cup N} \sum_{j \in \{0\} \cup N} c_{ij} (\omega + p_{tvi}) x_{tvij} \right] + \sum_{t \in T} \sum_{i \in N} Wu_{ti} \quad (1)
\]

The objective (1) is to minimize the total cost over all periods. In each period, the cost includes the start-up fee of vehicles, the transportation cost of hazardous wastes between nodes, and the penalty fee for the delayed collection. For each vehicle, the transportation cost of hazardous wastes from one node to other is quantified as a function of the transportation cost per unit weight, the weight of the vehicle without load.
load, and the loading weight of hazardous wastes.

\[
\min \sum_{i \in T} \sum_{v \in V} \sum_{i \in \{0\} \cup N} \sum_{j \in \{0\} \cup N} r_{ij}p_{tvi}x_{t\nu ij}
\]  

The objective (2) is to minimize the total transportation risk posed by hazardous wastes to the surrounding of routes over all periods. In each period, the transportation risk of each vehicle from one node to other is quantified as a function of the transportation risk per unit weight of hazardous wastes and the loading weight of hazardous wastes.

**Remark 1.** Many risk measures have been defined in previous hazardous waste/material management studies, such as societal risk (the product of the probability of a hazardous wastes accident times the consequences of that accident) [15] and population exposure (the number of people exposed to hazardous wastes) [9]. Without loss of generality, this paper does not specify a certain risk measure for the transportation risk, which makes the developed model more applicable.

**Remark 2.** In each period, the loading weight of each vehicle varies during the transportation process. Generally, the greater the loading weight of hazardous wastes, the greater the transportation cost/risk. Therefore, in the objectives (1) and (2), the transportation cost and transportation risk of each vehicle from one node to other are both loading weight-dependent, and quantified as a linear function of the loading weight of hazardous wastes without loss of generality. The relationship between the transportation cost/risk and the loading weight of hazardous wastes can also be modelled as a quadratic or nonlinear function. Many previous studies assume the transportation cost/risk from one node to other is a fixed value, regardless of the variation of vehicle loading weight [16, 17].

\[
\min \max_{t \in T} \sum_{v \in V} \sum_{i \in \{0\} \cup N} \sum_{j \in \{0\} \cup N} r_{ij}p_{tvi}x_{t\nu ij}
\]

The objective (3) is to maximize the risk stability. In this paper, the risk stability is enforced by minimizing the maximal transportation risk among all periods, which aims to improve the condition of those worst-off. Through adjusting the weight of hazardous wastes to collect and the transportation routes of vehicles in each period, the maximal transportation risk among all periods can be decreased. This “min-max” manner is similar to that in [18, 19].

2.1.2. **Constraints.**

\[
\sum_{j \in \{0\} \cup N} x_{t\nu ij} = \sum_{j \in \{0\} \cup N} x_{t\nu ji}, \quad t \in T, v \in V, i \in \{0\} \cup N
\]

Constraint (4) is the flow balance constraint of nodes in each period. Under this constraint, if a vehicle leaves the recycling center, it must return back to the recycling center; if a vehicle enters a factory, it must also leave the factory.

\[
\sum_{j \in N} x_{t\nu 0j} \leq 1, \quad t \in T, v \in V
\]

Constraint (5) ensures that each vehicle can be used at most once in each period.
Constraint (6) ensures that each factory can be visited at most once in each period.

\[ 0 \leq s_{tvj} \leq M \sum_{i \in \{0\} \cup N} x_{tvij}, \quad t \in T, v \in V, j \in N \] (7)

Constraint (7) ensures that in each period, the hazardous wastes at a factory can be collected by a vehicle only when the vehicle visits the factory, where \( M \) is a very large positive number.

\[ u_{ti} = u_{t-1,i} + q_{ti} - \sum_{v \in V} s_{tvv}, \quad t \in T, i \in N \] (8)

\[ u_{0i} = u_{iT,i} = 0, u_{ti} \geq 0, \quad t \in T, i \in N \] (9)

Constraint (8) is the weight balance constraint of hazardous wastes in each period. For a factory, the weight of uncollected hazardous wastes at the end of each period is equal to the weight of uncollected hazardous wastes at the end of former period plus the weight of hazardous wastes generated in this period minus the weight of hazardous wastes collected in this period. Constraint (9) ensures that the weight of hazardous wastes at each factory at the beginning and end of the planning horizon is zero.

\[ \sum_{i \in \{0\} \cup N} p_{tvv} x_{tvij} + s_{tvj} = p_{tvj}, \quad t \in T, v \in V, j \in N \] (10)

\[ p_{tv0} = 0, \quad t \in T, v \in V \] (11)

\[ 0 \leq p_{tvv} \leq Q, \quad t \in T, v \in V, i \in N \] (12)

Constraint (10) is the loading weight balance constraint of vehicles in each period. For a vehicle, the loading weight leaving a factory is equal to the loading weight arriving the factory and the weight of hazardous wastes collected from the factory. If vehicle \( v \) does not visit factory \( j \) in period \( t \), that is, \( \sum_{i \in \{0\} \cup N} x_{tvij} = 0 \), constraint (7) forces \( s_{tvj} = 0 \), then it can obtain \( p_{tvj} = 0 \) based on constraint (10). Constraint (11) ensures that the loading weight of each vehicle is zero when leaving the recycling center in each period. Constraint (12) is the vehicle capacity constraint, which ensures that the loading weight of each vehicle leaving any a factory in each period is less than the vehicle capacity. Meanwhile, constraints (10)-(12) can also ensure that there is no sub-tour among nodes in the route of each vehicle.

\[ x_{tvij} \in \{0, 1\}, x_{tvii} = 0, \quad t \in T, v \in V, i, j \in \{0\} \cup N \] (13)

Constraint (13) states the binary variables and self-loop elimination for each node.

3. Model transformation and linearization. In this section, we transform the developed multi-objective nonlinear model (1)-(13) into a single-objective nonlinear model, which is finally equated to a mixed 0-1 linear programming model by introducing a set of auxiliary variables and constraints.
3.1. Transformation to single-objective. The developed multi-objective optimization model (1)-(13) considers three objectives simultaneously, including minimizing the total transport cost, minimizing the total transportation risk, and maximizing the risk stability. In fact, the optimization of objective (3) is equivalent to solving the following programming:

\[
\min \bar{r}
\quad \text{s.t.} \quad \sum_{v \in V} \sum_{i \in \{0\} \cup N} \sum_{j \in \{0\} \cup N} r_{ij} p_{tv_i} x_{t v_i j} \leq \bar{r}, \; t \in T
\]

For multi-objective models, the weighted sums method can transform multiple objectives into an aggregated objective by multiplying each objective by a weight coefficient and summing up all weighted objectives [8, 20]. This paper uses the weighted sums method to transform the developed multi-objective model into a single-objective model, as follows

\[
\min \lambda_1 \sum_{t \in T} \sum_{v \in V} \left[ S \sum_{j \in N} x_{tv_0 j} + \sum_{i \in \{0\} \cup N} \sum_{j \in \{0\} \cup N} c_{ij} (\omega + p_{tv_i}) x_{tv_{ij}} \right] + \\
\lambda_1 \sum_{t \in T} \sum_{i \in N} W_{u_{ti}} + \lambda_2 \sum_{t \in T} \sum_{v \in V} \sum_{i \in \{0\} \cup N} \sum_{j \in \{0\} \cup N} r_{ij} p_{tv_i} x_{tv_{ij}} + \lambda_3 \bar{r}
\]

\[
\text{s.t.} \quad \text{Constraints (4) \textendash} (14),
\]

in which \( \lambda_i \geq 0, i = 1, 2, 3 \) are the weight coefficients of the three objectives respectively. In practical problems, the decision-maker can assign different weight coefficients to the three objectives, which reflects the preference of the decision-maker, and the model can then be used to analyze different scenarios involving various combinations of the three objectives.

3.2. Model linearization. The model (15) is initially a nonlinear programming model. Nonlinearity is manifested in the objective function, constraint (10) and constraint (14) containing the multiplication terms of two decision variables, \( p_{tv_i} x_{tv_{ij}} \). We introduce the auxiliary variable \( p_{tv_{ij}} \), which denotes the loading weight of vehicle \( v \) immediately after visiting node \( i \) and travelling to node \( j \) in period \( t \), \( t \in T, v \in V, i, j \in \{0\} \cup N \). Then we can have

\[
\sum_{j \in \{0\} \cup N} p_{tv_{ij}} = p_{tv_i}, \; t \in T, v \in V, i \in \{0\} \cup N.
\]

When vehicle \( v \) does not travel from node \( i \) to node \( j \) in period \( t \), i.e. \( x_{tv_{ij}} = 0 \), there should be \( p_{tv_{ij}} = 0 \); when vehicle \( v \) travels from node \( i \) to node \( j \) in period \( t \), i.e. \( x_{tv_{ij}} = 1 \), there should be \( p_{tv_{ij}} = p_{tv_i} \). Therefore, the following two equations holds:

\[
0 \leq p_{tv_{ij}} \leq M' x_{tv_{ij}}, \; t \in T, v \in V, i, j \in \{0\} \cup N, \quad (17)
\]

\[
0 \leq p_{tv_i} - p_{tv_{ij}} \leq M'(1 - x_{tv_{ij}}), \; t \in T, v \in V, i, j \in \{0\} \cup N, \quad (18)
\]

in which \( M' \) is a very large positive number.

Based on equations (16)-(18), the following equation can be obtained

\[
p_{tv_i} x_{tv_{ij}} = p_{tv_{ij}} x_{tv_{ij}} = p_{tv_{ij}}, \; t \in T, v \in V, i, j \in \{0\} \cup N.
\]

Equations (16) and (18) guarantee that the first equality holds, and equation (17) guarantees that the second equality holds.
Therefore, the model (15) is equivalent to the following linear programming

\[
\min \quad \lambda_1 \sum_{t \in T} \sum_{v \in V} \left[ S \sum_{j \in N} x_{tv0j} + \sum_{i \in \{0\} \cup N} \sum_{j \in \{0\} \cup N} c_{ij} \left( \omega x_{tvij} + p_{tvij} \right) \right] + \\
\lambda_2 \sum_{t \in T} \sum_{v \in V} \sum_{i \in \{0\} \cup N} \sum_{j \in \{0\} \cup N} r_{ij} p_{tvij} + \lambda_3 \bar{r}
\]

s. t. Constraints (4) – (9), (11) – (13), and (16) – (18),

\[
\sum_{i \in \{0\} \cup N} \sum_{v \in V} \sum_{j \in \{0\} \cup N} r_{ij} p_{tvij} \leq \bar{r}, \quad t \in T
\]

The model (20) eliminates the nonlinear terms of the model (15) and reduces to a mixed 0-1 linear programming model. The model (20) has \(|T|\*|V|*(|N|+1)^2\) binary decision variables, \(|T|\*|V|*(|N|+1)^2 + 2*|T|\*|V|*|N|+|T|*|N|+|T|*|V|+|N|+1\) nonnegative decision variables, and \(2*|T|\*|V|*(|N|+1)^2 + 6*|T|\*|V|*|N|+5*|T|\*|V|+2*|T|\*|N|+2*|N|+|T|\) constraints. The model (20) can be solved directly by mathematical optimization software such as CPLEX and LINGO, and the weight of hazardous wastes to collect from every factory and the transportation routes of vehicles in each period can be obtained.

4. Numerical experiments. In this section, we present an example to demonstrate the applicability and validity of the developed model. Following the description of the example, the computational results are reported, and then two experiments are conducted to investigate the influence of parameters on the results.

4.1. The description of the example. There is one recycling center and seven factories in a certain area. The recycling center and factories are labeled by nodes 0-7, in which node 0 is the recycling center, and nodes 1-7 are the factories, namely \(|N| = 7\). The planning horizon consists of three periods, namely \(|T| = 3\). The weight of hazardous wastes generated by each factory in each period is given in Table 1. Since there are a total of 7 factories, a maximum of 7 vehicles are needed in each period, so we set \(|V| = 7\). The transportation risk of hazardous wastes is measured by the number of population exposed along the transportation routes of hazardous wastes. Without loss of generality, the transportation cost and the number of people exposed of transporting a unit weight of hazardous wastes from one node to other are assumed to be symmetric, as given in Table 2. Other parameters are set as follows: the weight of each vehicle without load \(\omega = 5\) Tons; the vehicle capacity \(Q = 60\) Tons; the start-up fee of each vehicle \(S = 500\) $; and the penalty fee per unit weight of uncollected hazardous wastes \(W = 50\) $/Ton.

4.2. Computational results. The weight coefficients of the three objectives, \(\lambda_1 = 1, \lambda_2 = 2, \lambda_3 = 5\), are assigned based on the preference of the decision-maker. With the given network and parameters, the problem is solved using CPLEX version 12.8, on a computer with Intel Xeon CPU E5-1607 3.0 GHz and 32 G RAM. The model has 1344 binary decision variables, 1681 nonnegative decision variables, and 3727 constraints. The solution process takes 47 minutes and 46 seconds. The results are shown in Table 3. For comparison, the results of the scenario without consideration of the risk stability, i.e. \(\lambda_1 = 1, \lambda_2 = 2, \lambda_3 = 0\), are also provided in Table 3.
Table 1. The weight of hazardous wastes generated by each factory in each period (Tons)

| Period | Factory | 1   | 2   | 3   | 4   | 5   | 6   | 7   |
|--------|---------|-----|-----|-----|-----|-----|-----|-----|
| 1      |         | 28  | 19  | 21  | 24  | 32  | 20  | 22  |
| 2      |         | 19  | 27  | 23  | 16  | 25  | 11  | 14  |
| 3      |         | 12  | 20  | 15  | 10  | 14  | 13  | 17  |

Table 2. The transportation cost ($/Ton) and the number of population exposed (Pop/Ton) of transporting a unit weight of hazardous wastes from one node to another

| Node | 0      | 1      | 2      | 3      | 4      | 5      | 6      | 7      |
|------|--------|--------|--------|--------|--------|--------|--------|--------|
| 0    | 0/0    | 8/7    | 10/10  | 11/3   | 6/11   | 11/13  | 8/4    | 10/14  |
| 1    | 0/0    | 17/9   | 17/6   | 7/8    | 14/11  | 3/11   | 11/17  |        |
| 2    | 0/0    | 2/7    | 16/2   | 9/4    | 16/12  | 12/10  |        |        |
| 3    | 0/0    | 17/8   | 7/11   | 15/6   | 10/13  |        |        |        |
| 4    | 0/0    | 17/3   | 9/13   | 15/12  |        |        |        |        |
| 5    | 0/0    | 11/16  | 4/12   |        |        |        |        |        |
| 6    | 0/0    | 8/13   |        |        |        |        |        |        |
| 7    | 0/0    |        |        |        |        |        |        |        |

Table 3 shows that in the scenario without consideration of the risk stability, no penalty fee is incurred, which means that the hazardous wastes are completely collected in each period. In the scenario with consideration of the risk stability, the hazardous wastes are incompletely collected in the periods 1 and 2, and a total of 2665.7 $ penalty fee is incurred due to the delayed collection of hazardous wastes. Compared to the scenario without consideration of the risk stability, the total cost is increased by 14%, and the total transportation risk is decreased by 8.5% in the scenario with consideration of the risk stability. Although the decreased proportion of the total transportation risk is less than the increased proportion of the total cost, the difference between the maximal transportation risk over all periods and the minimal transportation risk over all periods is drastically reduced from 716 Pop to 16.7 Pop. Taken together, the consideration of the risk stability may increase the total cost to some extent, but the total transportation risk and the inequitable distribution of the transportation risk among all periods are both reduced. In addition, it can be observed that a maximum of 6 vehicles over all periods are used in the scenario without consideration of the risk stability, while the scenario with consideration of the risk stability needs 5 vehicles at most over all periods. In the developing countries such as India and China, the available budget for hazardous waste collection is usually restricted. It is more significant that the use of less vehicles over all periods can reduce the initial investment at the beginning of the planning horizon, especially when the collection vehicle is expensive.

Through solving the problem, the weight of hazardous wastes to collect from each factory and the transportation routes of vehicles in each period are obtained. Only the results of period 1 are reported due to the limited length of the paper, as shown in Figure 1. In period 1, the weight of hazardous wastes storing at each factory are 28, 19, 21, 24, 32, 20 and 22 respectively. A total of 5 transportation routes are
Table 3. The computational results

| Parameter          | Period 1 | Period 2 | Period 3 |
|--------------------|----------|----------|----------|
| No. of vehicles    | 6        | 4        | 4        |
| Transportation cost ($) | 6373    | 4704     | 5345.7   |
| Penalty fee ($)    | 0        | 0        | 1415.7   |
| Transportation risk (Pop) | 2442    | 2279     | 1955.7   |
| Total cost ($)     | 16355    | 18645.2  | 18445.2  |
| Total transportation risk (Pop) | 6447    | 5898.4   | 5898.4   |

\[ \lambda_1 = 1, \lambda_2 = 2, \lambda_3 = 0 \]

\[ \lambda_1 = 1, \lambda_2 = 2, \lambda_3 = 5 \]
formed: \(0 \rightarrow 1 \rightarrow 0, 0 \rightarrow 2 \rightarrow 3 \rightarrow 0, 0 \rightarrow 5 \rightarrow 0, 0 \rightarrow 6 \rightarrow 0, \) and \(0 \rightarrow 7 \rightarrow 0.\) The hazardous wastes at factories 1, 2, 5, 6, and 7 are completely collected. Only 16.7 Tons hazardous wastes at factory 3 are collected, and the remaining 4.3 Tons are delayed to collect. Factory 4 is not visited, which means the whole 24 Tons hazardous wastes are delayed to collect. From Table 3, it can be observed that the delayed collection of the hazardous wastes at factories 3 and 4 significantly reduces the transportation cost and transportation risk in period 1, but also incurs high penalty fee.

**Figure 1.** The results of period 1

4.3. **The analysis of parameters.** Based on the above problem, this subsection conducts two experiments to investigate the influence of the weight coefficient of the objective of risk stability, \(\lambda_3,\) and the penalty fee per unit weight of uncollected hazardous wastes, \(W,\) on the results. When the value of one parameter varies, the values of other parameters remain unchanged as set in above two subsections.

The purpose of the first experiment is to investigate the influence of the weight coefficient of the objective of risk stability, \(\lambda_3.\) From the perspective of the whole planning horizon, with varying the value of \(\lambda_3\) from 0 to 6, the total cost shows an increasing trend, while the total transportation risk shows a decreasing trend, as shown in Figure 2(a). For the objective of the risk stability, Figure 2(b) shows that varying the value of \(\lambda_3\) from 0 to 6 will gradually decrease the maximal transportation risk among all periods, but also gradually increase the minimal transportation risk among all periods, so that the difference between them (the longitudinal span of the grey shaded area) dwindles constantly. This illustrates that the “min-max” manner works effectively for improving the risk stability. Moreover, improving the risk stability is mainly achieved by adjusting the weight of hazardous wastes to collect in each period, which will inevitably lead to an increase in the total weight of hazardous wastes that are delayed to collect over all periods. As shown in Figure 2(b), the total weight of hazardous wastes that are delayed to collect over all periods is rising with increasing the value of \(\lambda_3.\)

The purpose of the second experiment is to investigate the influence of the penalty fee per unit weight of uncollected hazardous wastes, \(W.\) Intuitively, increasing the value of \(W\) will definitely lead to a decrease in the total weight of hazardous wastes that are delayed to collect over all periods. By varying the value of \(W\) from 30 to 70, Figure 3(b) confirms this intuition. In particular, when \(W\) is greater than 50, the total weight of hazardous wastes that are delayed to collect over all periods decreases rapidly. The variation of the value of \(W\) results in a consequence that the total cost is increasing first, decreasing afterwards, and increasing finally. By contrast, the variation of the transportation risk appears more regular. It can be
observed from Figure 3(a) and Figure 3(b) that with varying the value of $W$ from 30 to 70, the total transportation risk rises slowly, the maximal transportation risk among all periods is monotone increasing, and the minimal transportation risk among all periods is monotone decreasing. As a result, the difference between them (the longitudinal span of the grey shaded area) is widening constantly. This once again verifies the important role of adjusting the weight of hazardous wastes to collect in each period in improving the risk stability.

5. Conclusions and directions for future research. With the emphasis on public safety and environmental protection, the multi-period hazardous waste management problem has been a hot research topic. Although there have been some related studies, there is still a lack of multi-period joint decision-making and optimization from the perspective of the whole planning horizon. This paper considers that the hazardous wastes at each source allow incomplete collection and delayed
collection with incurring penalty fee. To the best of our knowledge, there have been no studies considering the incomplete and delayed collection in the multi-period hazardous wastes collection planning problem. The optimal collection planning, including the determination of the weight of hazardous wastes to collect from each source and vehicle scheduling in each period, needs to be made by global optimization.

A new multi-objective mathematical optimization model is developed for the multi-period hazardous waste collection problem, which includes three conflicting objectives related to cost, transportation risk, and risk stability. The cost includes the start-up fee of vehicles, the transportation cost of hazardous wastes between nodes, and the penalty fee for the delayed collection; the transportation risk refers to the potential risk posed by hazardous wastes to the surroundings of transportation routes; and the risk stability aims to evenly distribute the transportation risk among all periods, which is enforced by minimizing the maximal transportation risk among
all periods. With the help of the developed model, the weight of hazardous wastes to collect from each source and the transportation routes of vehicles in each period can be simultaneously determined. In order to solve the developed model, the three objectives are aggregated into a single objective based on the weighted sums method, and a set of auxiliary variables and constraints are introduced to eliminate the nonlinear terms in the model, which helps us equivalently transform the model into a mixed 0-1 linear programming.

In the numerical experiments, an example is presented, the results of which demonstrate the applicability and validity of the developed model. Compared to the scenario without consideration of the risk stability, the scenario with consideration of the risk stability can reduce the total transportation risk, the uneven distribution of the transportation risk among all periods, and the maximum number of vehicles used, though increasing the total cost to some extent. Furthermore, two experiments are conducted to investigate the influence of $\lambda_3$ and $W$ on the results. With varying the value of $\lambda_3$ from 0 to 6, the difference between the minimal transportation risk among all periods and the maximal transportation risk among all periods dwindles constantly, which illustrates that the “min-max” manner works effectively for improving the risk stability. Both two experiments show the important role of adjusting the weight of hazardous wastes to collect in each period in improving the risk stability.

There are some potential directions for future research. In this paper, the “min-max” manner is adopted to optimize the risk stability. Adopting other manners, such as minimizing variance, may be a direction for future research. The developed multi-objective model is transformed into a single-objective model based on the weighted sums method. Though the weighted sums method is popular and widely used, determining the weight coefficients may be a challenge. Solving the multi-objective model directly to find Pareto-optimal solutions could be a direction for future research. The computational effort is reasonable given the fact that this problem is a strategic decision making problem and it will be solved infrequently. Another future research is to develop an efficient heuristic to solve larger problems in a shorter time. Finally, it is valuable to apply the developed model into real-world cases including more considerations, such as the effects of weather and road conditions.

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