Form factors of $\eta_c$ in light front quark model

Chao-Qiang Geng$^{1,3,4a}$ and Chong-Chung Lih$^{2,3,4b}$

$^1$College of Mathematics & Physics,
Chongqing University of Posts & Telecommunications, Chongqing, 400065, China

$^2$Department of Optometry, Shu-Zen College of Medicine and Management, Kaohsiung Hsien, Taiwan 452

$^3$Physics Division, National Center for Theoretical Sciences, Hsinchu, Taiwan 300

$^4$Department of Physics, National Tsing Hua University, Hsinchu, Taiwan 300

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Abstract

We study the form factors of the $\eta_c$ meson in the light-front quark model. We explicitly show that the transition form factor of $\eta_c \to \gamma^* \gamma$ as a function of the momentum transfer is consistent with the experimental data by the BaBar collaboration, while the decay constant of $\eta_c$ is found to be $f_{\eta_c} = 230.5_{-61.0}^{+52.2}$ and $303.6_{-116.4}^{+115.2}$ MeV for $\eta_c \sim c\bar{c}$ by using two $\eta_c \to \gamma\gamma$ decay widths of $5.3 \pm 0.5$ and $7.2 \pm 2.1$ keV, given by Particle Data Group and Lattice QCD calculation, respectively.
I. INTRODUCTION

A neutral meson decaying into two photons is the simplest exclusive process since there is only one form factor involved. Experimental searches have been concentrated on the transition form factors of $P \rightarrow \gamma^* \gamma$ ($P = \pi^0, \eta$ and $\eta'$), $F_{P \gamma}(Q^2)$, in terms of the momentum transfer $Q^2$. In particular, the anomalous result for the pion transition form factor of $F_{\pi \gamma}(Q^2)$ at the large $Q^2$ has been reported by the BaBar collaboration [1] in comparison with the theoretical expectation [2] as well as the recent data by the Belle collaboration [3]. In addition, the transition form factors of $\eta, \eta' \rightarrow \gamma^* \gamma$, i.e. $F_{\eta \gamma, \eta' \gamma}(Q^2)$, have also been measured by BaBar [4, 5] in the regions up to 40 and 35 GeV$^2$, respectively. These form factors as functions of $Q^2$ allow us to not only extract information on the meson wave functions, but also check the pQCD predictions. Recently, we have studied the transition form factors of $F_{P \gamma}(Q^2)$ at the large $Q^2$ in the light-front quark model (LFQM) [6, 7] and shown that our results can fit with all the data, including those at the large $Q^2$ regions.

There is another interesting form factor, which is related to the $\eta_c \rightarrow \gamma^* \gamma$ transition. The measurements on this form factor have been done by both L3 [8] and BaBar [9] Collaborations based on the process of $e^+ e^- \rightarrow e^+ e^- \gamma^* \gamma^* \rightarrow e^+ e^- \eta_c$ in the range of $Q^2$ from 2 to 50 GeV$^2$. Since $\eta_c$ is composed of two massive charm quarks, it is important to know the behavior of the $\eta_c \rightarrow \gamma^* \gamma$ transition form factor at a high $Q^2$ momentum transfer to examine the validity of the pQCD calculations in this heavy meson as well as compare with those of the light pseudoscalars. The transition form factor of $F_{\eta_c \gamma}(Q^2)$ has been investigated by various QCD models [10]. In this paper, we study this form factor for $Q^2$ up to 50 GeV$^2$ in the LFQM. We will also simultaneously evaluate the $\eta_c$ meson decay constant $f_{\eta_c}$. This form factor is important for us to understand the pseudoscalar charmonium meson $\eta_c$. Moreover, the precise knowledge of $f_{\eta_c}$ can help us to examine other related processes, such as $B \rightarrow \eta_c K$.

By analogy with the $\pi^0 \gamma$ and $\eta^{(')} \gamma$ cases, we use the light front approach based on the simple meson wave function structure of $Q\bar{Q}$ ($Q = u, d, s, c$) pairs, constrained by the experimental measured decay width of $\eta_c \rightarrow \gamma\gamma$ as well as the mass of $\eta_c$.

This paper is organized as follows. In Sec. II, we present the transition form factors for $Q\bar{Q} \rightarrow \gamma^* \gamma$. In Sec. III, we show our numerical results on the transition form factor of $\eta_c \rightarrow \gamma^* \gamma$ and the decay constant of $\eta_c$. We give our conclusions in Sec. IV.
II. THE FORM FACTORS

To extract the transition form factor $F_{\eta c \gamma}$, we first write the decay amplitude of $\eta_c \rightarrow \gamma^* \gamma^*$ as

$$A(\eta_c \rightarrow \gamma^*(q_1, \epsilon_1) \gamma^*(q_2, \epsilon_2)) = i e^2 F_{\eta c \gamma}(q_1^2, q_2^2) \varepsilon_{\mu\nu\rho\sigma} \epsilon_1^\mu \epsilon_2^\nu q_1^\rho q_2^\sigma,$$

where $F_{\eta c \gamma}(q_1^2, q_2^2)$ is a symmetric function under the interchange of $q_1^2$ and $q_2^2$. The light front approach [12, 13] provides a framework for the relativistic quark model in which a consistent and relativistic treatment of quark spins and the center-of-mass motion can be carried out. In this framework, we consider the meson wave function as a combination of the $Q\bar{Q}$ Fock states. In the quark-flavor mixing scheme, the state of $\eta_c$ can be parameterized as

$$|\eta_c\rangle = -\theta_c \sin(\phi - \theta_y) |\psi_q\rangle - \theta_c \cos(\phi - \theta_y) |\psi_s\rangle + |\psi_c\rangle,$$

where $\theta_c$, $\phi$, and $\theta_y$ are the mixing angles and $|\psi_q\rangle = \frac{1}{\sqrt{2}} |u\bar{u} + d\bar{d}\rangle$, $|\psi_s\rangle = |s\bar{s}\rangle$ and $|\psi_c\rangle = |c\bar{c}\rangle$ in the quark flavor basis. Consequently, $F_{\eta c}(q_1^2, q_2^2)$ can be found by summing up the relevant $Q\bar{Q}$ Fock states. From the quark-$Q\bar{Q}$ meson loops shown in Fig. 1, we get

$$A(Q\bar{Q} \rightarrow \gamma^*(q_1) \gamma^*(q_2)) = e_Q e_{Q'} N_c \int \frac{d^4 p_3}{(2\pi)^4} \Lambda_{Q\bar{Q}} \left\{ \frac{\text{Tr} \left[ \gamma_5 i (\not{p}_3 + m_Q) \gamma_5 i (\not{p}_2 + m_Q) \right]}{p_3^2 - m_Q^2 + i\epsilon} \gamma_5 \frac{\gamma_5 i (\not{p}_1 + m_Q)}{p_1^2 - m_Q^2 + i\epsilon} \right\} + (\epsilon_1 \leftrightarrow \epsilon_2, q_1 \leftrightarrow q_2),$$

$$\times (p_1(3) \leftrightarrow p_3(1), m_Q \leftrightarrow m_{Q'}),$$

where $N_c$ is the number of colors, $\Lambda_{Q\bar{Q}}$ is a vertex function related to the momentum distribution amplitude of the $Q\bar{Q}$ Fock state, and $m_Q = m_{Q'}$ is the Q-quark mass. In the LFQM,
the amplitude can be solved in principle by the light-front QCD bound state equation [12]. Here, however, we chose phenomenological Gaussian type of the amplitude [6, 7, 13]. Similar to the procedures in Refs. [6, 7], after integrating over \( p_3^- \) and calculating the trace in Eq. (3), we obtain the form factor \( F_{Q\bar{Q}}(q_1^2, q_2^2) \) in Eq. (11) to be:

\[
F_{Q\bar{Q}}(q_1^2, q_2^2) = -\frac{16}{9} \frac{N_c}{3} \int \frac{dx d^2 k_\perp}{(2\pi)^3} \Phi(x, k_\perp^2) \frac{1}{1-x} \frac{m_Q + (1-x)m_Qk_\perp^2\Theta}{x(1-x)q_2^2 - m_Q^2 - k_\perp^2} + (q_2 \leftrightarrow q_1),
\]

with

\[
\Phi(x, k_\perp^2) = \sqrt{\frac{x(1-x)}{2M_0^2}} \phi_{Q\bar{Q}}(x, k_\perp), \quad \Theta = \frac{1}{\Phi(x, k_\perp^2)} \frac{d\Phi(x, k_\perp^2)}{dk_\perp^2},
\]

where

\[
M_0^2 = \frac{m_Q^2 + k_\perp^2}{x} + \frac{m_Q^2 + k_\perp^2}{1-x},
\]

\[
\phi_{Q\bar{Q}}(x, k_\perp) = N \sqrt{\frac{dk_\perp}{dx}} \exp\left(-\frac{\bar{k}^2}{2\omega_{Q\bar{Q}}^2}\right),
\]

with \( N = 4(\pi/\omega_{Q\bar{Q}}^2)^{3/4} \), \( \bar{k} = (k_\perp, k_z) \), \( k_z = (x+1/2)M_0 \), and \( \omega_{Q\bar{Q}} \) being the parameter related to the physical size of a pseudoscalar state of \( Q\bar{Q} \) in the wave function. If \( q_1^2 \equiv Q^2 \) and \( q_2^2 = 0 \), i.e. one of the photons is on its mass shell, from Eq. (11) we derive

\[
F_{Q\bar{Q}}(Q^2) \equiv F_{Q\bar{Q}}(Q^2, 0) = -\frac{16}{9} \frac{N_c}{3} \int \frac{dx d^2 k_\perp}{(2\pi)^3} \Phi(x, k_\perp^2) \frac{1}{1-x} \left\{ \frac{m_Q + (1-z)m_Qk_\perp^2\Theta}{x(1-x)Q^2 - m_Q^2 - k_\perp^2} - \frac{m_Q}{m_Q^2 + k_\perp^2} \right\}. \tag{7}
\]

Consequently, the transition form factor of \( F_{\eta_c\gamma} \) can be found by

\[
F_{\eta_c\gamma}(Q^2) = -\theta_c \sin(\phi - \theta_y) F_{q\bar{q}}(Q^2) - \theta_c \cos(\phi - \theta_y) F_{s\bar{s}}(Q^2) + F_{c\bar{c}}(Q^2), \tag{8}
\]

with \( F_{Q\bar{Q}}(Q^2) \) \((Q = q, s, c)\) given in Eq. (7). The decay width of \( \eta_c \to \gamma\gamma \) is related to the form factor of \( F_{\eta_c\gamma}(Q^2 = 0) \) by

\[
\Gamma_{\eta_c \to \gamma\gamma} = \frac{(4\pi\alpha)^2}{64\pi} m_{\eta_c}^3 |F_{\eta_c\gamma}(0)|^2. \tag{9}
\]

Based on the experimental data of \( \Gamma_{\eta_c \to \gamma\gamma} \), from Eqs. (7)-(9) we can extract the free parameter in the meson wave function.

The decay constant of the \( \eta_c \) meson is defined by the matrix element

\[
\langle 0 | q\gamma_\mu\gamma_5 q | \eta_c(P) \rangle = i f_{\eta_c} P_\mu. \tag{10}
\]
Combining the above with Eq. (2), we obtain the decay constant of $\eta_c$ to be

$$f_{\eta_c} = -\theta_c \sin(\phi - \theta_y) f_{q\bar{q}} - \theta_c \cos(\phi - \theta_y) f_{s\bar{s}} + f_{c\bar{c}},$$  \hspace{1cm} (11)$$

where the explicit expressions of $f_{Q\bar{Q}}$ ($Q = q, s, c$) are given by [7, 15]

$$f_{Q\bar{Q}} = 4 \sqrt{N_c} \sqrt{2} \int \frac{dx d^2k_\perp}{2(2\pi)^3} \phi_{Q\bar{Q}}(x, k_\perp) \frac{m_Q}{\sqrt{m_Q^2 + k_\perp^2}}.$$  \hspace{1cm} (12)$$

III. NUMERICAL RESULT

To numerically calculate the transition form factor of $F_{\eta_c\gamma}$ in Eq. (8), we need to specify the parameters in the meson wave functions, in particular the meson scale parameters of $\omega_{Q\bar{Q}}$ in Eq. (6). From Eq. (9), $F_{\eta_c\gamma}(0)$ can be fixed by $\Gamma_{\eta_c \rightarrow 2\gamma}$, which can be used to determine the value of $\omega_{c\bar{c}}$. Note that the scale parameters of $\omega_{q\bar{q}, s\bar{s}}$ can be found in Refs. [6, 7]. In our calculations, we use $m_{\eta_c} = 2981.0 \pm 1.1$ MeV \[16\] and two inputs for the decay width of $\eta_c \rightarrow \gamma\gamma$:

$$5.3 \pm 0.5 \text{ keV (I)} \quad \text{and} \quad 7.2 \pm 2.1 \text{ keV (II)},$$  \hspace{1cm} (13)$$
given by Particle Data Group (PDG) \[16\] and Lattice QCD prediction \[18\], which lead to

$$|F_{\eta_c\gamma}(0)| = 0.069 \pm 0.003 \text{ GeV}^{-1} \text{ (I)} \quad \text{and} \quad 0.081 \pm 0.011 \text{ GeV}^{-1} \text{ (II)},$$  \hspace{1cm} (14)$$
respectively. The mixing angles have been studied in Ref. [14]. To illustrate the effects of the mixings, we take

(a) $(\theta_c, \theta_y, \phi) = (-1.0^\circ, -21.2^\circ, 39.3^\circ),$  
(b) $(\theta_c, \theta_y, \phi) = (-0.9^\circ, -21.2^\circ, 42.0^\circ),$  
(c) $(\theta_c, \theta_y, \phi) = (0^\circ, -, -).$  \hspace{1cm} (15)$$

We note that the case (a) corresponds the center values used in Ref. [14], while the case (c) represents that $\eta_c$ is a pure $c\bar{c}$ state. We also note that there is no other free parameter to adjust the light front wave function of $c\bar{c}$. Now, we can fit the parameter of $\omega_{c\bar{c}}$ from Eqs. (7) and (8) with $q^2 = 0$ and a given charm quark mass. In the $\overline{MS}$ scheme, one has that $\bar{m}_c(\bar{m}_c) = 1.29^{+0.05}_{-0.11}$ GeV \[16\]. However, at the higher order in QCD, the center value is enhanced, while the range of the pole charm quark mass is even broader \[17\]. In our numerical calculation, $m_c$ is a free input parameter.
FIG. 2. $|F_{\eta c\gamma}(Q^2)/F_{\eta c\gamma}(0)|$ as a function of $Q^2$ in the LFQM, where the bands for LFQM I and II correspond to the calculations based on $\Gamma_{\eta c\to \gamma\gamma}$ given by PDG [16] and Lattice QCD calculations [18], respectively, while the stars represent the experimental data by the BaBar collaboration [9].

In Fig. 2, we show the numerical results for $F_{\eta c\gamma}(q^2)/F_{\eta c\gamma}(0)$ in the LFQM for the first set of the mixing angles in Eq. (15), where I and II represent the two inputs of $\Gamma_{\eta c\to \gamma\gamma}$ in Eq. (13), respectively. From the figure, we see that both predictions in the LFQM I and II are consistent with the BaBar experimental data even though they are about 10% smaller than the data points in the range $(7.5 \sim 20)$ GeV. We remark that our results do not change much for the other sets of the mixing angles in Eq. (15). As an illustration, we can also fit the result of the LFQM I by a double pole form

$$\frac{F_{\eta c\gamma}(Q^2)}{F_{\eta c\gamma}(0)} = \frac{1}{1 + (Q/\alpha)^2 - (Q/\beta)^4}$$

where $\alpha = 2.2$ and $\beta = 4.7$ in GeV and we have ignored the errors for the input parameters.

Apart from the transition form factor, from the fitted values of the meson scale parameters of $\omega_{Q\bar{Q}}$ and Eq. (12), we can simultaneously determine the range of the $\eta_c$ decay constant $f_{\eta_c}$. Our results on $f_{\eta_c}$ with different sets of the mixing angles in Eq. (15) and the two inputs of (I) and (II) on $\Gamma_{\eta c\to \gamma\gamma}$ in Eq. (13) as well as the experimental data from the CLEO collaboration [19] and the Lattice QCD prediction [18] are summarized in Table. I. From the table, we see that our results of $f_{\eta_c}$ from the LFQM I are smaller than those from the
The $\eta_c$ decay constant $f_{\eta_c}$ in the LFQM with three sets of the mixing angles in Eq. (15) and two input parameters in Eq. (13), given by PDG [16] and Lattice QCD [18], respectively.

|               | $\Gamma_{\eta_c \to \gamma \gamma}$ (keV) | $f_{\eta_c}$ (MeV) |
|---------------|------------------------------------------|-------------------|
| LFQM I        | 5.3 ± 0.5 [16]                           | (a) 194.0$^{+33.3}_{-47.0}$, (b) 196.9$^{+33.7}_{-44.0}$, (c) 230.5$^{+52.2}_{-61.0}$ |
| LFQM II       | 7.2 ± 2.1 [18]                           | (a) 243.6$^{+127.5}_{-84.4}$, (b) 249.2$^{+143.2}_{-84.4}$, (c) 303.6$^{+115.2}_{-116.4}$ |
| Lattice QCQ   | 7.2 ± 2.1 [18]                           | 394.7 ± 2.4       |
| CLEO [19]     | -                                        | 335 ± 52 ± 47 ± 12 ± 25 |

LFQM II due to the use of a smaller decay width of $\eta_c \to \gamma \gamma$. We note that the center values of our results on $f_{\eta_c}$ in Table I correspond to the use of $m_c = 1.29$ GeV. It is clear that in order to match the experimental result in the CLEO collaboration [19] and the Lattice QCD value [18], a smaller mixing case with a larger $m_c$ is favored. We emphasize that our predictions for $f_{\eta_c}$ are sensitive to the charm quark mass $m_c$. In Fig. 3, we show the charm quark mass dependence for the $\eta_c$ decay constant in the non-mixing case (c) in Eq. (15).

From the figure, we observe that to fit the CLEO data [19] or the Lattice QCD value [18], a large value of $m_c$ is needed.

![Graph of $\eta_c$ decay constant ($f_{\eta_c}$) as a function of $m_c$ in the LFQM without the mixing, i.e. $\theta_c = 0$.](image-url)
IV. CONCLUSIONS

We have studied the transition form factor of $\eta_c \to \gamma^*\gamma$ and the decay constant of the $\eta_c$ meson in the LFQM. In particular, we have illustrated the transition form factor of $\eta_c \to \gamma^*\gamma$ as a function of the momentum transfer $Q^2$. We have shown that although our results are consistent with the experimental data by the BaBar collaboration, they are about 10% smaller than the data points for $Q^2$ in the range of 7.5~20 GeV. We have also evaluated the decay constant of $\eta_c$. We have shown that it is sensitive to the mixing angles as well as the mass of the charm quark. Explicitly, for $\eta_c \sim c\bar{c}$, i.e. a pure $c\bar{c}$ state ($\theta_c = 0$), we have found that $f_{\eta_c} = 230.5^{+52.2}_{-61.0}$ and 303.6$^{+115.2}_{-116.4}$ MeV in the LFQM I and II based on the two sets of input parameters, $\Gamma_{\eta_c \to \gamma\gamma} = 5.3 \pm 0.5$ and $7.2 \pm 2.1$ keV, given by Particle Data Group and Lattice QCD calculation, respectively. Both results are within the error of $335 \pm 52 \pm 47 \pm 12 \pm 25$ MeV measured by the CLEO collaboration, but they are somewhat smaller than 394.7 $\pm$ 2.4 MeV predicted by the Lattice QCD, in which $\Gamma_{\eta_c \to \gamma\gamma} = 7.2 \pm 2.1$ keV is used like the LFQM II. However, the Lattice QCD result can easily be accounted when a larger value of the charm quark mass is used. Future precision measurements on the decay width of $\eta_c \to \gamma\gamma$ are clearly needed in order to determine the $\eta_c$ decay constant in the LFQM.

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