A STUDY ON DISTINCT FUZZY COLOURINGS AND DOMINATION PARAMETERS IN DOUBLE LAYERED FUZZY GRAPH

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Abstract: In Double Layered Fuzzy Graph, it is discussed that, the Fuzzy Vertex Colouring, Fuzzy Edge Colouring, Fuzzy Total Colouring and determined bounds for Fuzzy Chromatic Number, Fuzzy Edge Chromatic Number and Fuzzy Total Chromatic number. Also established some of the Domination parameters such as Fuzzy Dominator Chromatic Number, Inverse Domination, Connected Inverse Domination.

Keywords: Fuzzy Vertex Colouring, Fuzzy Edge Colouring, Fuzzy Total Colouring, Fuzzy Dominator Chromatic Number, Inverse Domination, Connected Inverse Domination and Double Layered Fuzzy Graph.

1. Introduction:
Here we have taken a graph to be a simple FG, n-order & m-size. Zadeh.L.A[9] was insinuated Fuzzy relation concept in the year 1965. Rosenfeld was interpose the theoretical conception like cycle and connectedness and also intercalate the fuzzy graph. Fuzzy graphs colouring was interpolate by Mu’noz et al in [6]. Fuzzy Chromatic number as a Fuzzy number same the $\alpha$-cut’s. The FVC same by Onagh and Eslahchi [3] Developing Fuzzy Vertex Colouring in 2006. Term of Family of Fuzzy sets extended to Fuzzy Total Colouring in by Lavanya. S and Sattanathan. R[4]. Dominator colouring and dominator chromatic number was introduced by Raluca Gera et al. Total colouring was insert by Bezsad afrnd Vizing between 1964 and 1968. A.Somasundram and S.Somasundram insert the impression of domination in FG. S.C.Sigarkanti, it focused on the Inverse Domination in graphs. In dispute the FG and DLFG and to argue the conceit are FVC, FEC, FTC, FDCN, FID and CID also established Results on FG and DLFVC, DLFEC, DLFTC, DLFDCN, DLFID and DLCID dissucessed and illustrated with examples.

2. Definitions

2.1. Definition
Let $G_{DL} = (\sigma_{DL}, \mu_{DL})$ be a DLFG. A $\subseteq D$ of $V$ is called DLFDS of DLF(G) if $\forall \ v \in V - D, \ u \in D \ni (u, v)$ is strong arc.

2.2. Definition
A DLF(G) is a PDLFC which that each vertex to DLF(G) Dominate every vertex of atleast some colour class.

3. FCN of FG and DLFG

3.1. Theorem
If $G = (\sigma, \mu)$ is a simple FG of Fuzzy Cycle of order n, then
\[ \chi_F(G) = \begin{cases} 2, & \text{if } n = 2k, \\ 3, & \text{if } 2k + 1. \end{cases} \]

**Proof:**

This theorem can be proved in 2 cases,

1. **Case**
   
   When \( n \) - even, vertex set \( V_1 \) consisting of even numbers, it is coloured by \( C_1 \) and \( C_2 \) preferentially without strike the concept of Fuzzy Proper Colouring and hence it is desired 2 colours. Thus the Fuzzy Chromatic number of \( F(G) \) is \( \chi_F(G) = 2 \) if \( n = 2k \), \( \forall k = 1, 2, 3, \ldots \).

2. **Case**
   
   When \( n \) - odd, vertex set \( V_2 \) has odd numbers, it is also coloured by two distinct colours \( C_1 \) and \( C_2 \) possibility and end vertex of \( V_2 \) must be coloured by colour \( C_3 \). Hence it required 3 colours, therefore \( \chi_F(G) = 3 \), if \( n \) = odd.

Thus \( \chi_F(G) = \begin{cases} 2, & \text{if } n = \text{even} \\ 3, & \text{if } n = \text{odd}. \end{cases} \)

3.2. **Example**

For \( n = 10 \) and \( n = 11 \). Then

\[ \chi_F(G) = \begin{cases} 2, & \text{if } n = 10 \\ 3, & \text{if } n = 11. \end{cases} \]

**Case 1:**

For \( n = 10 \) for \( C_{10} \) is \( \chi_F(G) = 2 \).
**Figure 3.2.1** $C_{10}$

**Figure 3.2.2** $\chi_F(C_{10}) = 2$

**Table 1**: $\chi_F(C_{10})$ for FG:

| Vertices $v_i$ (1 to 10) | C1 (blue) | C2 (red) | $V_i \Gamma = \sigma$ | $\gamma_i \Lambda \gamma_j = 0$ |
|-------------------------|-----------|----------|---------------------|-------------------------------|
| 1                       | (0.1)     | 0        | (0.1)               | 0                             |
| 2                       | 0         | (0.2)    | (0.2)               | 0                             |
| 3                       | 0         | (0.3)    | (0.3)               | 0                             |
| 4                       | (0.4)     | 0        | (0.4)               | 0                             |
| 5                       | (0.5)     | 0        | (0.5)               | 0                             |
| 6                       | 0         | (0.6)    | (0.6)               | 0                             |
| 7                       | 0         | (0.7)    | (0.7)               | 0                             |
| 8                       | (0.8)     | 0        | (0.8)               | 0                             |
| 9                       | (0.1)     | 0        | (0.1)               | 0                             |
| 10                      | 0         | (0.8)    | (0.8)               | 0                             |

$\chi_F(C_{10}) = 2$. 
Case 2:
For $n = 11$ and the FCN of $C_{11}$ is $\chi_F(G) = 3$.

![Figure 3.2.3 Fuzzy cycle $C_{11}$](image1)

![Figure 3.2.4 $\chi_F(C_{11}) = 3$](image2)

Table 2: $\chi_F(C_{11}) = 3$ for FG:

| Vertices $v_i$ | C1 (Green) $v_i$ | C2 (Orange) $v_i$ | C3 (Black) $v_i$ | $V_2^\Gamma = \sigma$ | $\gamma_1 A \gamma_j = 0$ |
|---------------|-----------------|-----------------|-----------------|-------------------|------------------|
| 1             | (0.1)           | 0               | 0               | (0.1)             | 0                |
| 2             | 0               | (0.2)           | 0               | (0.2)             | 0                |
| 3             | 0               | (0.3)           | 0               | (0.3)             | 0                |
| 4             | (0.4)           | 0               | 0               | (0.4)             | 0                |
| 5             | (0.5)           | 0               | 0               | (0.5)             | 0                |
| 6             | 0               | (0.6)           | 0               | (0.6)             | 0                |
| 7             | 0               | (0.7)           | 0               | (0.7)             | 0                |
| 8             | (0.8)           | 0               | 0               | (0.8)             | 0                |
| 9             | (0.9)           | 0               | 0               | (0.9)             | 0                |
| 10            | 0               | (0.7)           | 0               | (0.7)             | 0                |
| 11            | 0               | 0               | (0.8)           | (0.8)             | 0                |
Therefore $\chi_{F}(C_{11}) = 3$.

3.3. Theorem

If $G_{DL} = (\sigma_{DL}, \mu_{DL})$ is a DLFG of Fuzzy Cycle of order $n$, then

$$\chi_{DLF}(G) = \begin{cases} 2, & \text{if } n = 2k, \\
3, & \text{if } n = 2k + 1. \end{cases}$$

Proof:

This theorem can be proved in 2 cases,

1. Case
When $n$ - even, vertex set $V_1$ consisting of even numbers, it is coloured by $C_1$ and $C_2$ preferentially without strike the concept of Fuzzy Proper Colouring and hence it is desired 2 colours. Thus the Double Layered Fuzzy Chromatic number of $DLF(G)$ is $\chi_{DLF}(G) = 2$ if $n = 2k, (\forall k=1,2,3,...)$.

2. Case
When $n$ - odd, vertex set $V_2$ has odd numbers, it is also coloured by two distinct colours $C_1$ and $C_2$ possibility and end vertex of $V_2$ must be coloured by colour $C_3$. Hence it required 3 colours, therefore $\chi_{DLF}(G) = 3$, if $n$ - odd.

Thus $\chi_{DLF}(G) = \begin{cases} 2, & \text{if } n = \text{even} \\
3, & \text{if } n = \text{odd}. \end{cases}$

3.4. Example

For $n = 8$ and $n = 7$. Then

$$\chi_{DLF}(G) = \begin{cases} 2, & \text{if } n = 8 \\
3, & \text{if } n = 7. \end{cases}$$

Case 1:

For $n = 8$ $\chi_{DLF}(C_8) = 2$. 
Table 1: $\chi_{\text{DLF}}(C_8) = 2$ for DLF(G):

| Vertices $v_i (i \text{ to } 16)$ | C1 (Orange) | C2 (Blue) | $V_i \Gamma_{DL} = \sigma_{DL}$ | $\gamma_i \Lambda \gamma_j = 0$ |
|----------------------------------|-------------|-----------|-----------------|-----------------|
| 1                                | (0.1)       | 0         | (0.1)           | 0               |
| 2                                | 0           | (0.2)     | (0.2)           | 0               |
Therefore $\chi_{DLF}(C_8) = 2$

Case 2:
For $n = 7$ and the $\chi_{DLF}(C_7) = 3$.

Figure 3.4.4 Fuzzy cycle $C_7$

Figure 3.4.5 DLFG of $C_7$
Figure 3.4.6 $\chi_{\text{DLF}}(C_7) = 3$

Table 2: $\chi_{\text{DLF}}(C_7) = 3$ for DLF(G):

| Vertices $v_i$ (1 to 13) | $C_1$ (Orange) | $C_2$ (Blue) | $C_3$ (Red) | $V_2 \Gamma_{\text{DLF}} = \sigma_{\text{DL}} \gamma_i, \Lambda \gamma_j = 0$ |
|--------------------------|----------------|--------------|-------------|--------------------------------------------------|
| 1                        | (0.1)          | 0            | 0           | 0                                                 |
| 2                        | 0              | (0.2)        | 0           | (0.2)                                             |
| 3                        | 0              | (0.3)        | 0           | (0.3)                                             |
| 4                        | (0.4)          | 0            | 0           | (0.4)                                             |
| 5                        | (0.4)          | 0            | 0           | (0.4)                                             |
| 6                        | 0              | 0            | (0.6)       | (0.6)                                             |
| 7                        | 0              | (0.5)        | 0           | (0.5)                                             |
| 8                        | 0              | (0.2)        | 0           | (0.2)                                             |
| 9                        | (0.1)          | 0            | 0           | (0.1)                                             |
| 10                       | 0              | (0.3)        | 0           | (0.3)                                             |
| 11                       | 0              | 0            | (0.2)       | (0.2)                                             |
| 12                       | (0.5)          | 0            | 0           | (0.5)                                             |
| 13                       | 0              | (0.2)        | 0           | (0.2)                                             |
Hence \( \chi_{DLF}(C_7) = 3 \).

4. Fuzzy Index Number of FG and DLFG

4.1. Theorem
If \( G_{DL} = (\sigma_{DL}, \mu_{DL}) \) is a FG of a Fuzzy Cycle of size \( m \), Then

\[
\chi^E_F(G) = \begin{cases} 
  2, & \text{if } m = 2k, \\
  3, & \text{otherwise}. \quad (\forall k = 1, 2, 3, \ldots).
\end{cases}
\]

Proof:
This theorem can be proved in 2 cases,

1. Case
When \( m \) - even,
Edge set \( E_1 \) consisting of even numbers, it is coloured by \( C_1 \) and \( C_2 \) preferentially without strike the concept of Fuzzy Proper Colouring and hence it is desired 2 colours. Thus the Fuzzy Chromatic number of \( F(G) \) is \( \chi^E_F(G) = 2 \) if \( m = 2k, (\forall k = 1, 2, 3, \ldots) \).

2. Case
When \( m \) - odd,
Edge set \( E_2 \) has odd numbers, it is also coloured by two distinct colours \( C_1 \) and \( C_2 \) possibility and end edge of \( E_2 \) must be coloured by colour \( C_3 \). Hence it required 3 colours, therefore

\( \chi^E_F(G) = 3, \) if \( m \) = odd.

Thus \( \chi^E_F(G) = \begin{cases} 
  2, & \text{if } m = \text{even} \\
  3, & \text{if } m = \text{odd}.
\end{cases} \)

4.2. Example
For \( m = 14 \) and \( m = 15 \).

Then

\[
\chi^E_F(G) = \begin{cases} 
  2, & \text{if } m = 14 \\
  3, & \text{if } m = 15.
\end{cases}
\]

Case 1:
For size \( m = 14 \) and the \( \chi^E_F(C_{14}) = 2 \).
Table 1: $\chi^E_F(C_{14}) = 2$ for FG:

| Edges $e_i$ (1 to 14) | C1 (violet) | C2 (Orange) | $\max_{i,j}(\gamma_i(uv) = \mu(uv))$ | $\gamma_i \wedge \gamma_j = 0$ |
|----------------------|-------------|-------------|-----------------------------------|-----------------------------|
| 1                    | (0.2)       | 0           | (0.2)                             | 0                           |
| 2                    | 0           | (0.1)       | (0.1)                             | 0                           |
| 3                    | 0           | (0.3)       | (0.3)                             | 0                           |
| 4                    | (0.2)       | 0           | (0.2)                             | 0                           |
|   |   |   |   |   |
|---|---|---|---|---|
| 5 | (0.3) | 0 | (0.3) | 0 |
| 6 | 0 | (0.4) | (0.4) | 0 |
| 7 | 0 | (0.2) | (0.2) | 0 |
| 8 | (0.4) | 0 | (0.4) | 0 |
| 9 | (0.6) | 0 | (0.6) | 0 |
| 10 | 0 | (0.6) | (0.6) | 0 |
| 11 | 0 | (0.6) | (0.6) | 0 |
| 12 | (0.6) | 0 | (0.6) | 0 |
| 13 | (0.6) | 0 | (0.6) | 0 |
| 14 | 0 | (0.6) | (0.6) | 0 |

Therefore $\chi^E_{F}(C_{14}) = 2$.

**Case 2:**

For $m = 15$ and the $\chi^E_{F}(C_{15}) = 3$. 
Figure 4.2.3 Fuzzy cycle $C_{15}$

Figure 4.2.4 $\chi^{E}_{F}(C_{15}) = 3$

Table 2: $\chi^{E}_{F}(C_{15}) = 3$ for FG:

| Edges | C1 (Blue) | C2 (Orange) | C3 (Red) | $\max \gamma(\text{uv}) = \mu(\text{uv})$ | $\gamma_i \wedge \gamma_j = 0$ |
|-------|-----------|-------------|----------|--------------------------------|--------------------------------|
| $v_i = 1$ (1 to 15) | (0.2) | 0 | 0 | (0.2) | 0 |
| 2 | 0 | (0.1) | 0 | (0.1) | 0 |
| 3 | 0 | (0.3) | 0 | (0.3) | 0 |
| 4 | (0.2) | 0 | 0 | (0.2) | 0 |
Hence $\chi^{E}_{DLF}(C_{15}) = 3$.

4.3. Theorem

If $G_{DL} = (\sigma_{DL}, \mu_{DL})$ is a DLFG of Fuzzy Cycle of size $m$, then $\chi^{E}_{DLF}(G) = 3$.

Proof:

Let $E = \{1, 2, \ldots, 2+k\}$ be a set of edges on DLF($G$). Let $\Gamma_{DL} = \{\gamma_1, \ldots, \gamma_{2+k}\}$ be family of Double Layered Fuzzy set of $E$ in $G_{DL} = (\sigma_{DL}, \mu_{DL})$. Let us colour the set of edges of $E$ by colour $C_1$ and $C_2$ as a choices and the end edge of $E$ must be coloured by $C_3$ without affecting the concept of Double Layered Fuzzy Proper Edge Colouring it required 3 colours. Hence $\chi^{E}_{DLF}(G) = 3$, if $m = 2+k$, $\forall m > 2$ ($k = 1, 2, 3, \ldots$).

4.4. Example

For $m = 6$, then $\chi^{E}_{DLF}(C_6) = 3$. 
Figure 4.4.1 Fuzzy cycle $C_6$

Figure 4.4.2 DLF($C_6$)

Figure 4.4.3 $\chi_{DLF}^E(C_6) = 3$
Table 1: $\chi^{E}_{DLF}(C_6) = 3$ for DLF(G):

| B Edges e_i=i(1 to 18) | Colour 1 (Green) | Colour 2 (Blue) | Colour 3 (Orange) | max $\gamma_i(uv)$ $= \mu_{DLF}(uv)$ | $\gamma_i \neq \gamma_j = 0$ |
|-------------------------|------------------|-----------------|-------------------|-------------------------------|------------------|
| 1                       | (0.1)            | 0               | 0                 | (0.1)                        | 0                |
| 2                       | 0                | (0.1)           | 0                 | (0.1)                        | 0                |
| 3                       | 0                | (0.3)           | 0                 | (0.2)                        | 0                |
| 4                       | (0.3)            | 0               | 0                 | (0.3)                        | 0                |
| 5                       | 0                | 0               | (0.3)             |                               | 0                |
| 6                       | 0                | (0.5)           | 0                 | (0.5)                        | 0                |
| 7                       | 0                | (0.1)           | 0                 | (0.1)                        | 0                |
| 8                       | (0.3)            | 0               | 0                 | (0.3)                        | 0                |
| 9                       | 0                | (0.4)           | 0                 | (0.4)                        | 0                |
| 10                      | (0.5)            | 0               | 0                 | (0.5)                        | 0                |
| 11                      | 0                | (0.2)           | 0                 | (0.2)                        | 0                |
| 12                      | (0.1)            | 0               | 0                 | (0.1)                        | 0                |
| 13                      | 0                | 0               | (0.1)             | (0.1)                        | 0                |
| 14                      | 0                | 0               | (0.2)             | (0.2)                        | 0                |
| 15                      | 0                | 0               | (0.3)             | (0.3)                        | 0                |
| 16                      | 0                | 0               | (0.4)             | (0.4)                        | 0                |
| 17                      | 0                | 0               | (0.4)             | (0.4)                        | 0                |
| 18                      | 0                | 0               | (0.4)             | (0.4)                        | 0                |

$\chi^{E}_{DLF}(C_6) = 3$

5. Fuzzy Total Chromatic Number on FG and DLFG

5.1. Theorem
If $G = (\sigma, \mu)$ is a FG of Fuzzy Cycle of order m, Then

$$\chi^{T}_{F}(G) = \begin{cases} 3, & \text{if } n = 3k, \\ 4, & \text{otherwise, } (\forall \ k = 1,2,3,\ldots). \end{cases}$$

Proof:

It is proved in two cases:

Case 1:
Let us colour the set $V_i = V_1 \cup E_1$ by using the colours C_1 and C_2 alternatively without affecting the concept of Fuzzy Total Colouring and the either edge or vertex must be coloured by C_3 and hence it required 3 colours, therefore $\chi^{T}_{F}(G) = 3$, if $n = 3k$, ($\forall \ k = 1,2,3,\ldots$).
Case 2:
Let us colour the set $V_2' = V_2 \cup E_2$ by using the colours $C_1$, $C_2$ and $C_3$ alternatively without affecting the concept of Fuzzy Total Colouring and the final edge or vertex must be coloured by $C_4$. Hence it required 4 colours, therefore $\chi_{FT}(G) = 4$, if $n = 3k + 1$ ($\forall k = 1, 2, 3, ..$).

5.2. Example
For $n = 12$ and $n = 13$. Then

$$\chi_{FT}(G) = \begin{cases} 
3, & \text{if } n = 12, \\
4, & \text{if } n = 13.
\end{cases}$$

Case 1:
For $n = 12$ and $\chi_{FT}(C_{12}) = 3$.

![Figure 5.2.1 Fuzzy cycle C_{12}](image1)

![Figure 5.2.2 \(\chi_{FT}(C_{12}) = 3\)](image2)

Table 1: $\chi_{FT}(C_{12}) = 3$ for FG:

| Vertices and Edges $V \cup E$ | C1 (Green) | C2 (Black) | C3 (Blue) | max $\gamma_i(v)$ = $\sigma(v)$ | $\gamma_i \land \gamma_j = 0$ |
|-------------------------------|------------|------------|-----------|-------------------------------|--------------------------|
| $V_i = \{1 \text{ to } 12\}$   | (0.1)      | 0          | 0         | (0.1)                         | 0                        |
| $e_i = \{1 \text{ to } 12\}$   |            |            |           |                               |                          |
|    | 0   | 0   | (0.2) | (0.2) | 0   |
|----|-----|-----|-------|-------|-----|
| 2  | 0   | 0   | (0.3) | 0     | (0.3)| 0   |
| 3  | 0   | (0.4)| 0     | (0.4) | 0   |
| 4  | 0   | 0   | (0.5) | (0.5) | 0   |
| 5  | (0.7)| 0   | 0     | (0.7) | 0   |
| 6  | (0.6)| 0   | 0     | (0.6) | 0   |
| 7  | 0   | 0   | (0.6) | (0.6) | 0   |
| 8  | 0   | (0.7)| 0     | (0.7) | 0   |
| 9  | 0   | (0.9)| 0     | (0.9) | 0   |
| 10 | 0   | 0   | (0.8) | (0.8) | 0   |
| 11 | (0.7)| 0   | 0     | (0.7) | 0   |
| 12 | 0   | (0.2)| 0     | (0.2) | 0   |
| 1  | 0   | 0   | (0.1) | (0.1) | 0   |
| 2  | 0   | (0.3)| 0     | (0.3) | 0   |
| 3  | (0.2)| 0   | 0     | (0.2) | 0   |
| 4  | 0   | 0   | (0.3) | (0.3) | 0   |
| 5  | 0   | (0.4)| 0     | (0.4) | 0   |
| 6  | 0   | (0.2)| 0     | (0.2) | 0   |
| 7  | 0   | (0.4)| 0     | (0.4) | 0   |
| 8  | 0   | (0.6)| 0     | (0.6) | 0   |
| 9  | (0.6)| 0   | 0     | (0.6) | 0   |
| 10 | (0.6)| 0   | 0     | (0.6) | 0   |
| 11 | 0   | 0   | (0.4) | (0.4) | 0   |
| 12 | 0   | (0.6)| 0     | (0.6) | 0   |

Therefore $\chi^f(C_{12}) = 3$.

*Case 2:*

For $n = 13$ The Fuzzy Total Chromatic number of fuzzy cycle $C_{13}$ is $\chi^f(G) = 4$. 
Table 1: $\chi^r_F(C_{12}) = 3$ for FG:

| Vertices and Edges $V \cup E$ | C1 (Blue) | C2 (Black) | C3 (Red) | C4 (Green) | $\max_i \gamma_i(v)$ | $\gamma_i \land \gamma_j = 0$ |
|-------------------------------|-----------|------------|----------|------------|---------------------|------------------|
| $v_i:i(1 \text{to} 13)$      | $e_i:i(1 \text{to} 13)$ | (0.1)       | 0        | 0          | 0                   | (0.1) 0          |
| 1                             |           | 2          | 0        | (0.2)      | 0                   | (0.2) 0          |
| 2                             |           | 3          | 0        | (0.3)      | 0                   | (0.3) 0          |
| 3                             |           | 4          | 0        | 0          | (0.4)               | (0.4) 0          |
| 4                             |           | 5          | 0        | (0.5)      | 0                   | (0.5) 0          |
| 5                             |           | 6          | (0.7)    | 0          | 0                   | (0.7) 0          |
| 6                             |           | 7          | 0        | 0          | (0.6)               | (0.6) 0          |
| 7                             |           | 8          | 0        | (0.6)      | 0                   | (0.6) 0          |
| 8                             |           | 9          | (0.7)    | 0          | 0                   | (0.7) 0          |
| 9                             |           | 10         | 0        | (0.9)      | 0                   | (0.9) 0          |
| 10                            |           | 11         | 0        | 0          | (0.8)               | (0.8) 0          |
| 11                            |           | 12         | (0.7)    | 0          | 0                   | (0.7) 0          |
| 12                            |           | 13         | (0.7)    | 0          | 0                   | (0.7) 0          |
| 13                            |           | 1          | 0        | (0.2)      | 0                   | (0.2) 0          |
| 1                             |           | 2          | 0        | (0.1)      | 0                   | (0.1) 0          |
| 2                             |           | 3          | (0.3)    | 0          | 0                   | (0.3) 0          |
| 3                             |           | 4          | 0        | 0          | (0.2)               | (0.2) 0          |
| 4                             |           | 5          | 0        | (0.3)      | 0                   | (0.3) 0          |
| 5                             |           | 6          | (0.4)    | 0          | 0                   | (0.4) 0          |
| 6                             |           | 7          | 0        | (0.2)      | 0                   | (0.2) 0          |
| 7                             |           | 8          | (0.4)    | 0          | 0                   | (0.4) 0          |
| 8                             |           | 9          | (0.6)    | 0          | 0                   | (0.6) 0          |
| 9                             |           | 10         | 0        | (0.6)      | 0                   | (0.6) 0          |
| 10                            |           | 11         | 0        | (0.4)      | 0                   | (0.4) 0          |
| 11                            |           | 12         | (0.5)    | 0          | 0                   | (0.5) 0          |
Thus, $\chi_T(C_{12}) = 3$.

5.3. Theorem

If $G_{DL} = (\sigma_{DL}, \mu_{DL})$ is a DLFG of Fuzzy Cycle of order $n$, Then

$$\chi_{DLF}^T(G) = \begin{cases} 4, & \text{if } n = 3 \\ 5, & \text{otherwise, } (\forall \; k = 1, 2, 3, \ldots). \end{cases}$$

Proof:

Let $DLF(G)$ has $2n$ vertices. The vertex set and edge sets are divided $V_1$ and $V_2$ and also $E_1$ and $E_2$. Let $V_1 = \{v_1, v_2, v_3\}$ and $V_2 = \{v_1, v_2, v_3, \ldots, v_{3+k}\}$ be a subset of vertex set on Double Layered Fuzzy Graph of $G_{DL} = (\sigma_{DL}, \mu_{DL})$.

Case 1:

colour the vertices and also edges $V_1 = V_1 \cup E_1$ by using the colours $C_1$ and $C_2$ preferentially without strike the concept of Double Layered Fuzzy Total Colouring and the final edge or vertex must be coloured by $C_3$ and $C_4$ hence it is desired 4 colours, therefore $\chi_{DLF}^T(G) = 4$, if $n = 3$ $(\forall \; k = 1, 2, 3, \ldots)$.

Case 2:

Similarly colour the vertices and edges $V_2 = V_2 \cup E_2$ by using the colours $C_1$ and $C_2$ possibility without rather than the concept of Double Layered Fuzzy Total Colouring and the final edge or vertex must be coloured by $C_3$ and $C_4$ and using the colour $C_5$. Hence it is required 5 colours, therefore $\chi_{DLF}^T(G) = 5$, if $n = 3 + k$, $(\forall \; k = 1, 2, 3, \ldots)$.

5.4. Example

For $n = 3$ and $n = 3 + k$, Then

$$\chi_{DLF}^T(G) = \begin{cases} 4, & \text{if } n = 3 \\ 5, & \text{if } n = 3 + k. \end{cases}$$

Case 1:

For $n = 3$ and the $\chi_{DLF}^T(C_3) = 4$.

Figure 5.4.1 Fuzzy cycle $C_3$

Figure 5.4.2 DLFC$C_3$
Figure 5.4.3 $\chi_{DLF}^T(C_3) = 4$

Table 1: $\chi_{DLF}^T(C_3) = 4$ for DLF(G):

| Vertices and Edges | C1 (Violet) | C2 (Red) | C3 (Green) | C4 (Orange) | max $\gamma_i(v) = \sigma_{DL}(v)$ | $\gamma_i \land \gamma_j = 0$ |
|-------------------|-------------|----------|------------|-------------|-------------------------------|-----------------------------|
| $v_i = i(1 \text{ to } 6)$ | (0.2) 0 0 0 (0.2) 0 | 0 (0.6) 0 0 (0.6) 0 | 0 0 (0.5) 0 (0.5) 0 | 0 0 (0.2) 0 (0.2) 0 | |
| $e_j = j(1 \text{ to } 9)$ | 0 (0.2) 0 0 (0.2) 0 | 0 0 (0.2) 0 (0.2) 0 | 0 0 (0.2) 0 (0.2) 0 | 0 0 (0.2) 0 (0.2) 0 | |
| 1                | (0.2) 0 0 0 (0.2) 0 | 0 (0.6) 0 0 (0.6) 0 | 0 0 (0.5) 0 (0.5) 0 | 0 0 (0.2) 0 (0.2) 0 | |
| 2                | 0 (0.6) 0 0 (0.6) 0 | 0 0 (0.5) 0 (0.5) 0 | 0 0 (0.2) 0 (0.2) 0 | 0 0 (0.2) 0 (0.2) 0 | |
| 3                | 0 0 (0.5) 0 (0.5) 0 | 0 0 (0.2) 0 (0.2) 0 | 0 0 (0.2) 0 (0.2) 0 | 0 0 (0.2) 0 (0.2) 0 | |
| 4                | 0 0 (0.2) 0 (0.2) 0 | 0 0 (0.2) 0 (0.2) 0 | 0 0 (0.2) 0 (0.2) 0 | 0 0 (0.2) 0 (0.2) 0 | |
| 5                | 0 0 (0.2) 0 (0.2) 0 | 0 0 (0.2) 0 (0.2) 0 | 0 0 (0.2) 0 (0.2) 0 | 0 0 (0.2) 0 (0.2) 0 | |
| 6                | 0 (0.2) 0 0 (0.2) 0 | 0 0 (0.2) 0 (0.2) 0 | 0 0 (0.2) 0 (0.2) 0 | 0 0 (0.2) 0 (0.2) 0 | |
| 7                | 0 0 (0.2) 0 (0.2) 0 | 0 0 (0.2) 0 (0.2) 0 | 0 0 (0.2) 0 (0.2) 0 | 0 0 (0.2) 0 (0.2) 0 | |
| 8                | 0 0 (0.2) 0 (0.2) 0 | 0 0 (0.2) 0 (0.2) 0 | 0 0 (0.2) 0 (0.2) 0 | 0 0 (0.2) 0 (0.2) 0 | |
| 9                | 0 0 (0.2) 0 (0.2) 0 | 0 0 (0.2) 0 (0.2) 0 | 0 0 (0.2) 0 (0.2) 0 | 0 0 (0.2) 0 (0.2) 0 | |

Therefore $\chi_{DLF}^T(C_3) = 4$. 
Case 2:
For \( n = 3 + k \) and \( \chi_{DLF}^{T}(C_n) = 5 \).
It is true that $\chi_{DLF}(G) = 5$.

### 6. Fuzzy Dominator Chromatic Number of FG and DLFG

#### 6.1. Theorem
If $G = (\sigma, \mu)$ is a FG of “Fuzzy Cycle” of order $m$, then $\chi_{F^d}(G) = 3$.

#### 6.2. Example
For $n = 15$, then $\chi_{F^d}(C_{15}) = 3$.

![Figure 6.2.1 Fuzzy cycle $C_{15}$](image1)

![Figure 6.2.2 $\chi_{F^d}(C_{15}) = 3$](image2)

**Table 1:** $\chi_{F^d}(C_{15}) = 3$ for FG:

| Vertices $v_i$ (1 to 15) | C1 (Blue) | C2 (Orange) | C3 (Red) | $V \Gamma = \sigma$ | $\gamma_i \land \gamma_j = 0$ |
|-------------------------|-----------|-------------|----------|---------------------|-----------------------------|
| 1                       | 0         | 0           | (0.1)    | (0.1)               | 0                           |
| 2                       | (0.2)     | 0           | 0        | (0.2)               | 0                           |
| 3                       | 0         | (0.3)       | 0        | (0.3)               | 0                           |
| 4                       | 0         | (0.4)       | 0        | (0.4)               | 0                           |
| 5                       | (0.5)     | 0           | 0        | (0.5)               | 0                           |
| 6                       | 0         | 0           | (0.7)    | (0.7)               | 0                           |
|   |   |   | (0.6) | (0.6) |   |
|---|---|---|-------|-------|---|
| 7 | 0 | 0 | (0.6) | (0.6) | 0 |
| 8 | (0.6) | 0 | 0 | (0.6) | 0 |
| 9 | 0 | (0.7) | 0 | (0.7) | 0 |
| 10 | 0 | (0.9) | 0 | (0.9) | 0 |
| 11 | (0.8) | 0 | 0 | (0.8) | 0 |
| 12 | 0 | 0 | 0 | (0.9) | 0 |
| 13 | 0 | 0 | (0.7) | (0.7) | 0 |
| 14 | (0.7) | 0 | (0.7) | (0.7) | 0 |
| 15 | 0 | (0.8) | 0 | (0.8) | 0 |

Therefore $\chi_{F}^{d}(C_{15}) = 3$.

6.3. Theorem
If $G_{DL} = (\sigma_{DL}, \mu_{DL})$ is a DLFG of “Fuzzy Cycle” of order $n$, Then $\chi_{DLF}^{d}(G) = 3$.

6.4. Example
For $n = 8$, then $\chi_{DLF}^{d}(C_{8}) = 3$.

![Figure 6.4.1 Fuzzy cycle $C_{8}$](image1)

![Figure 6.4.2 DLF($C_{8}$)](image2)
Figure 6.4.3 $\chi_{DLF}^d (C_8) = 3$

Table 1: $\chi_{DLF}^d (C_8) = 3$ for FG:

| Vertices $v_i$ (i=1 to 16) | C1 (Blue) | C2 (Orange) | C3 (Red) | $V_{DL} = \sigma_{DL}$ | $\gamma_i \land \gamma_j = 0$ |
|-----------------------------|-----------|-------------|-----------|-------------------------|-------------------------------|
| 1                           | 0         | 0           | (0.1)     | (0.1)                   | 0                             |
| 2                           | (0.2)     | 0           | 0         | (0.2)                   | 0                             |
| 3                           | (0.3)     | 0           | 0         | (0.3)                   | 0                             |
| 4                           | 0         | (0.4)       | 0         | (0.4)                   | 0                             |
| 5                           | 0         | (0.4)       | 0         | (0.4)                   | 0                             |
| 6                           | (0.4)     | 0           | 0         | (0.4)                   | 0                             |
| 7                           | (0.5)     | 0           | 0         | (0.5)                   | 0                             |
| 8                           | 0         | 0           | (0.6)     | (0.6)                   | 0                             |
| 9                           | (0.2)     | 0           | 0         | (0.2)                   | 0                             |
| 10                          | 0         | (0.1)       | 0         | (0.1)                   | 0                             |
| 11                          | 0         | 0           | (0.3)     | (0.3)                   | 0                             |
| 12                          | 0         | (0.2)       | 0         | (0.2)                   | 0                             |
| 13                          | (0.3)     | 0           | 0         | (0.3)                   | 0                             |
| 14                          | 0         | (0.4)       | 0         | (0.4)                   | 0                             |
| 15                          | 0         | 0           | (0.2)     | (0.2)                   | 0                             |
| 16                          | 0         | (0.4)       | 0         | (0.4)                   | 0                             |

Thus $\chi_{DLF}^d (C_8) = 3$. 

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7. Fuzzy Inverse Domination of FG and DLFG

7.1. Theorem
Suppose $G = (\sigma, \mu)$ is a FG of “Fuzzy Cycle” of order $m$, Then

$$
\gamma_{\text{FD}}^{-1}(G) = \begin{cases} 
    k + 1, & \text{if } n = 3k \\
    2k, & \text{if } n = 3k + 1 \\
    2k + 1, & \text{if } n = 3k + 2.
\end{cases}
$$

7.2. Example
Case 1: $\gamma_{\text{FD}}^{-1}(G) = k + 1$, if $n = 3k$.

Figure 7.2.1 Fuzzy cycle $C_3$, $n = 3$.

Thus $\gamma_{\text{FD}}^{-1}(C_3) = 2$.

Case 2: $\gamma_{\text{FD}}^{-1}(G) = 2k$, $\forall q = 1 + k$, if $n = 3k + 1$.

Figure 7.2.2 Fuzzy cycle $C_{10}$
Thus $\gamma^{-1}_F(C_{10}) = 6$.

Case 3: $\gamma^{-1}_F(G) = 2k + 1, (\forall \ p = 1 + k)$

![Figure 7.2.3 Fuzzy cycle $C_{14}$](image)

Thus $\gamma^{-1}_F(C_{14}) = 9$.

7.3. Theorem

If $G_{DL} = (\sigma_{DL}, \mu_{DL})$ is a DLF of “Fuzzy Cycle” of order $n$, Then

$$\gamma^{-1}_{{DLF}}(G) = \begin{cases} 
  n + q, & \text{if } n = 4k - 1 \\
  n + p, & \text{if } n = 4k \\
  n + q, & \text{if } n = 4k + 1 \\
  n + p, & \text{if } n = 4k + 2, (\forall, \ k = 1, 2, 3, \ldots)
\end{cases}$$

7.4. Example

Case 1: $\gamma^{-1}_{{DLF}}(G) = n + q$, if $n = 4k - 1$. 
Thus $\gamma^{-1}_{DLF}(C_7) = 10$.

Case 2: $\gamma^{-1}_{DLF}(G) = n + p$, if $n = 4k$.

Thus $\gamma^{-1}_{DLF}(C_8) = 12$.

Case 3: $\gamma^{-1}_{DLF}(G) = n + q$, if $n = 4k + 1$. 
Thus \( \overline{\gamma}^{-1}_{DLF}(C_9) = 12. \)

**Case 4:** \( \gamma^{-1}_{DLF}(G) = n + p, \) if \( n = 4k + 2. \)

Thus \( \gamma^{-1}_{DLF}(C_6) = 8. \)
8. Connected Inverse Domination of FG and DLFG

8.1. Theorem
If $G_{DL} = (\sigma_{DL}, \mu_{DL})$ is a DLFG of “Fuzzy Cycle” of order $n$,

$$\gamma^{-1}_{DLCF}(G) = \begin{cases} 
  n + p , & \text{if } n = 4k - 1 \\
  n + q , & \text{if } n = 4k \\
  n + p , & \text{if } n = 4k + 1 \\
  n + q , & \text{if } n = 4k + 2 , \forall , k = 1,2,3,... 
\end{cases}$$

Note:
Connected Inverse Domination does not exist FG where as it appeared in DLFG.

8.2. Example
Case 1: $\gamma^{-1}_{DLCF}(G) = n + p , \text{ if } n = 4k - 1$

![Figure 8.2.1 Fuzzy cycle C₃](image1)

There for $\gamma^{-1}_{DLCF}(C₃) = 4$.

Case 2: $\gamma^{-1}_{DLCF}(G) = n + q , \text{ if } n = 4k$

![Figure 8.2.2 DLCF(C₃)](image2)
Case 3: $\gamma_{DLCF}^d(G) = n + p$, if $n = 4k + 1$.
Figure 8.2.6 DLCF(C₃)

Therefor \( \gamma^{-1}_{DLCF} (C₃) = 6 \).

Case 4: \( \gamma^{-1}_{DLCF} (G) = n + q \), if \( n = 4k + 2 \), \( \forall \), \( k = 1, 2, 3, \ldots \)

Figure 8.2.7 Fuzzy cycle C₆, Figure 8.2.8 DLF(C₆)
9. Conclusion:
In this paper, it is determined several Chromatic Numbers of FG and DLFG and also discussed some of the Domination parameters such as Fuzzy Inverse Domination and Connected Inverse Domination.

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