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Capture and acceleration of electrons by the ultrarelativistic wakefield

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Abstract. Modeling results of the external injection of electrons into wakefield excited by the ultrarelativistic ion driving bunch in plasma are presented. Capture and acceleration of the injected electron bunch in the wakefield are described by the system of relativistic dynamic equations. Initial parameters of the plasma and the ion bunch are similar to that in the experiment AWAKE which is currently carried out at CERN. Characteristics of the accelerated electron bunch (percentage of captured electrons and energy dispersion) are studied at small angle of injection relative to the direction of ion driver propagation and far enough from the driver where the wakefield structure is strongly nonlinear.

Electrons acceleration by a wakefield in plasma is a promising method of accelerating particles to ultrarelativistic energies, as plasma wakefields behind a proton driver experience much weaker limitations on the accelerating gradient [1]. Accelerated electrons can be obtained from the background plasma, serving as a medium for excitation of the wakefield [2], or as a result of crossing a sharp boundary of inhomogeneous plasma by laser pulse [3]. Nevertheless it is believed that the achievement of the maximum capture and acceleration of electrons can be provided by injection of external electrons from the outside [4].

To model wakefields in plasma, hydrodynamic equations of a cold electron liquid were used [5]. The high energy ultra-relativistic driving ion bunch (with proton energy of 423 GeV) considered below evolves slowly in rarefied plasma. Neglecting scattering and stopping of ions, it is possible to assume that speed and shape of the ion bunch does not change during investigated process of the wake field generation. In this case, assuming axial symmetry of the driver moving along $OZ$ axis, all quantities can be expressed through only two independent variables (that implies the self-similar solution with the phase velocity of wakefields $v_{ph}$ equals to the velocity of driving ion bunch $v_b$):

$$\xi = k_{p0}(z - v_b t), \quad \rho = k_{p0} r,$$

with

$$k_{p0} = \omega_{p0}/c, \quad \omega_{p0} = \sqrt{4\pi n_0 e^2/m_e},$$

where $\xi$ is dimensionless comoving coordinate, $r$ is the radial cylindrical coordinate, $k_{p0}$ is wave vector, $\omega_{p0}$ is the electron plasma frequency, $n_0$ is a background electron plasma density, $e, m_e$ are electron charge and mass, $c$ is the speed of light and $v_b$ is the constant driver velocity. Using
these comoving variables, the hydrodynamic relativistic equations for background electrons and the Maxwell equations for fields, the wakefield structure is described by a single function, the wakefield potential \[5, 6\]:

\[
\Psi(\xi, \rho) = \gamma - q_z \beta_b^{-1},
\]

\[
\gamma = \sqrt{1 + q^2}, \quad q_z = p_z/(mc),
\]

where \(\gamma\) is gamma-factor and \(q_z\) is longitudinal part of dimensionless momentum \(q = p/(mc)\) of background electrons.

Proton driver had the following energy and form

\[
\gamma_b = (1 - \beta_b^2)^{-\frac{1}{2}} = 450, \quad n_b = 0.7n_0 \exp \left[ -\frac{\xi^2 + \rho^2}{2(k_{p0}\sigma)^2} \right],
\]

where \(\beta_b^2 = v_b^2/c^2\), \(n_0 = 7 \times 10^{14}\) cm\(^{-3}\), i.e., \(k_{p0} = 49.7\) cm\(^{-1}\); \(k_{p0}\sigma = 1\) is dimensionless driver bunch radius. The \(\gamma_b\)-factor of proton bunch corresponds to energy of 423 GeV. The choice of these parameters was made using approximate values of parameters in a current experiment AWAKE in CERN \[7\]. With these parameters of the proton bunch and plasma the nonlinearity of the wake wave increases with distance behind the driver until the structure of the wave destroys. Figure 1 shows the dimensionless potential of the wakefield in the region of the 30-th period of the wave (counting from the proton bunch). This region was chosen for injection of external electrons as the most promising for capture of particles, because here the wave still retains its structure, but already has a rather nonlinear form. The potential \(\Psi(\xi, \rho)\) determines accelerating and focusing forces acting on relativistic electron moving along the wave axis in wakefields

\[
F_z = \frac{\partial \Psi}{\partial \xi}, \quad F_r = \frac{\partial \Psi}{\partial \rho}.
\]

The motion of \(i\)-th injected electron in wakefield was described by the system of dynamic equations \[8\]

\[
\frac{dq_{zi}}{d\zeta} = \frac{1}{\beta_{zi}} \frac{\partial \Psi}{\partial \xi}, \quad \frac{d\xi_i}{d\zeta} = 1 - \beta_b \beta^{-1}_{zi},
\]

\[
\frac{dq_{zi}}{d\zeta} = \frac{1}{\beta_{zi}} \frac{\partial \Psi}{\partial \rho} \cos \phi_i, \quad \frac{dx_i}{d\zeta} = q_{xi},
\]

\[
\frac{dq_{yi}}{d\zeta} = \frac{1}{\beta_{zi}} \frac{\partial \Psi}{\partial \rho} \sin \phi_i, \quad \frac{dy_i}{d\zeta} = q_{yi},
\]

with

\[
\beta_{zi} = q_{zi}/\gamma_{ei}, \quad \gamma_{ei} = \sqrt{1 + q_e^2},
\]

where \(\phi_i = \arctan(y_i/x_i)\), \(\gamma_{ei}\) is relativistic factor and \(q_{zi}\) is the longitudinal part of momentum of an injected electron. The system is written in three dimensional coordinates \((\xi, x, y)\), so it provides a correct dynamics of electrons that do not fall exactly on the axis of the wakefield, and, therefore, have a moment of rotation. It can describe the motion of electrons even in a non-axisymmetric field.

A homogeneous cylinder of external electrons with dimensionless radius 0.4 and length 3.1 (which corresponds to \(8.0 \times 10^{-3}\) cm and \(62.4 \times 10^{-3}\) cm respectively) consisting of 799 particles was directed to the axis of the wave from the periphery \((\rho \simeq 4)\) where the value of \(\Psi\) is negligible. Electrons of the injected cloud had parameters

\[
\gamma_e = 40, \quad \alpha = -1^\circ,
\]
Table 1. Parameters of captured electron bunch at the final iteration of modeling.

| Parameter                  | Value     |
|----------------------------|-----------|
| Electrons amount           | 193       |
| Average $\mu$              | -203.37   |
| Average $\rho$             | 0.25      |
| Dispersion $\mu$           | 0.07      |
| Average energy (GeV)       | 8.23      |
| Energy dispersion (GeV)    | 0.10      |

Figure 1. The structure of the wakefield potential $\Psi(\xi, \rho)$ near the injection region.

where $\gamma_e$ corresponds to 20.48 MeV and $\alpha$ is the angle between the vector of the initial momentum of an electron and the axis of the wave in $(\xi, x)$ plane. We injected electrons into region of positive $\partial\Psi/\partial\xi$ otherwise particles definitely would be not trapped by the wave.

Modeling was carried out over the distance $z \simeq 20.12$ m while dephasing length $L_{dp} = \gamma_e^2 \lambda_{pl}$ (where $\lambda_{pl}$ is the plasma wave length) in our conditions is about 291 m. With the parameters of injection described above, the fraction of captured and accelerated electrons is about 24%. Trajectories of two representative electrons (one trapped and another non-trapped) are shown in figure 2(a). Moving from the periphery, particles fall into the region of a strong focusing and accelerating field near the wave axis. The amplitude of electron oscillation, under the action of transverse forces, drops in the process of trapping, while its energy along longitudinal direction rapidly increases up to $\simeq 8.23$ GeV with dispersion of about 1% [figure 2(b)]. Parameters of the trapped and accelerated electron bunch are presented in table 1.

It should be noted that initial length of injected bunch is about of the half of plasma wave length, but in the process of trapping the trapped bunch is strongly compressing while non-
trapped particles are lost for acceleration. As a result, the length of trapped and accelerated bunch along the axis is relatively small that determines a small energy spread in the bunch [9].

The distribution of electrons in the transverse plane is $y$-symmetrical, as is expected (figure 3). In spite of the fact that the total spread in this plane is about $(-1; 1)$, the major part of electrons is concentrated near the axis, so the r.m.s. radius is about 0.21.

In conclusion, the processes of electron bunch capture, compression and acceleration in the nonlinear wakefield generated by the ultrarelativistic ion drive bunch were modeled and analyzed. It is shown that far enough from the driver (about 30-th period of the wake wave), where the wake field structure is strongly nonlinear, the electron bunch, externally injected at a small angle to the axis with relatively small injection energy, can be effectively captured and accelerated. The fraction of trapped electrons injected at the angle of 1° with energy of 20 MeV on the half of the wakefield period is 23%. Longitudinal compression of the electron bunch in process of capture provides a small energy spread of 1% in the bunch accelerated to 8 GeV. Optimization of the injection process to increase the fraction of trapped electrons and of their final energy together with minimization of the energy spread of accelerated electron bunch will be the subject of future investigation.
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