Nonlocal lattice fermion models on the 2d torus

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Abstract

Abelian fermion models described by the SLAC action are considered on a finite 2d lattice. It is shown that modification of these models by introducing additional Pauli – Villars regularization supresses nonlocal effects and provides agreement with the continuum results in vectorial U(1) models. In the case of chiral fermions the phase of the determinant differs from the continuum one.

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1 Introduction

In a recent paper [1] we considered lattice fermion models on the 2d lattice with Wilson action improved by Pauli – Villars (PV) regularization. It was shown that in spite of the chiral symmetry breaking for finite lattice spacing $a$, its effects are suppressed by PV regularization and the model provides a good agreement with the continuum results both in the perturbative and nonperturbative region. Nevertheless lack of chiral symmetry for finite $a$ is not quite harmless as to get a good agreement for large external fields one needs big lattices.

So it would be highly desirable to have a formulation which preserves chiral symmetry for a finite lattice spacing as well.

”No-go” theorem [2] forbids any local formulation preserving chiral symmetry, therefore we consider a nonlocal SLAC model proposed originally in paper [3]. It is known that the original formulation of the model requires introduction of nonlocal counterterms [4, 5], which makes it unacceptable for practical calculations. However the model can be improved in the same way as it has been done for Wilson action by introducing additional gauge invariant PV regularization which suppresses the contribution of momenta close to the edge of the Brillouin zone. It was shown [6] that for anomaly free models on an infinite lattice in the framework of perturbation theory all nonlocal effects can be suppressed in this way and one gets a manifestly chiral invariant formulation of anomaly free models.

The propose of this paper is to check these results both perturbatively and nonperturbatively for the models on the 2d finite lattice. We found that in the case of vectorial models this approach works leading to a reasonable agreement with continuum results. However for chiral fermions, even when perturbative anomaly is absent, we observed a discrepancy in the value of the phase of continuum and lattice determinants. The origin of this discrepancy is discussed.
2 Vectorial lattice model

In this section we consider the vectorial model described by the action \[ I_{VS} = \sum_{x,y,\mu} \bar{\psi}(x) \gamma_\mu D_\mu(x - y) \exp[i \sum_{z=1}^{N/2} A_\mu(z)] \psi(y). \] (1)

Here \(-N/2 + 1 \leq x_\mu \leq N/2, \mu = 0, 1\). The lattice spacing is chosen to be equal to 1. We restrict ourselves by the case of a constant external fields

\[ A_\mu = \frac{2\pi}{N} h_\mu, \quad h_\mu = \text{const}. \]

The Fermi field \(\psi\) satisfies antiperiodic boundary conditions. \(D_\mu(x)\) is the lattice (SLAC) derivative:

\[ D_\mu(x) = \frac{1}{N^2} \sum_{p=-\frac{N}{2}+1}^{\frac{N}{2}} iP_\mu(p) \exp\left(2\pi i \left(p - \frac{1}{2}\right)x,\right) \]

where

\[ P_\mu(p) = \frac{2\pi}{N} \left(p_\mu - \frac{1}{2}\right), \quad \frac{-N/2 + 1}{N} \leq p_\mu - mN \leq \frac{N/2}{N}, \quad m = 0, \pm 1, \ldots. \]

The action (1) is gauge invariant but it is not local.

A straightforward calculation gives for the vectorial fermion determinant normalized to 1 at \(h = 0\), the following expression:

\[ D_{VS} = \prod_{p=-\frac{N}{2}+1}^{\frac{N}{2}} \frac{B^2(p, h)}{B^2(p, 0)}, \] (2)

where \(B^2(p, h) = \sum_{\mu=0}^{N/2} B^2_\mu(p, h)\), \(B_\mu\) is the Fourier transform of the covariant derivative

\[ B_\mu(p, h) = \frac{\pi}{N} \sum_{z=-N/2+1}^{N/2} (-1)^{z+1} \frac{\sin\frac{2\pi}{N}(p_\mu - h_\mu - \frac{1}{2})z}{\sin\frac{\pi z}{N}}. \] (3)

The dependence of \(B_\mu/2\pi\) on \((p_\mu - h_\mu - 1/2)/N\) is presented at Fig.1. One sees that on a finite lattice, the saw-tooth form of the SLAC derivative is
smoothen and contrary to the infinite lattice case may be approximated by a continuous curve. This fact will be important for further discussion.

The lattice determinant (2) has to be compared with the known result for the continuum theory [7, 8, 9]

\[ D_{VC} = e^{-2\pi h_1^2} \prod_{n=1}^{\infty} |F[n, h]F[n, -h]|^2, \tag{4} \]

where

\[ F[n, h] = \frac{1 + e^{-2\pi(n-1/2)+2\pi i(h_0+ih_1)}}{1 + e^{-2\pi(n-1/2)}}. \]

The determinants (2), (4) satisfy the following symmetry properties:

\[ D[h_0, h_1] = D[h_1, h_0] = D[-h_0, h_1] = D[h_0 + n_0, h_1 + n_1], \]

where \( n_0, n_1 = 0, \pm 1, \pm 2, \ldots \). Therefore it is sufficient to consider the fields \( h_\mu \) only in the range \( 0 \leq h_0 \leq h_1 \leq 1/2 \).
Using the eq. (2) one can easily write the expression for the diagrams with two and more external $h$-lines. It looks as follows:

$$\frac{\partial^2}{\partial h_\mu \partial h_\nu} \ln D_{VS} = \sum_{p=-N/2+1}^{N/2} \left\{ 2\delta_{\mu\nu} \cdot \frac{B_{\mu}^2(p, h) + B_\mu(p, h)B''_\mu(p, h)}{B^2(p, h)} - \frac{4B_\mu(p, h)B'_\mu(p, h)B_\nu(p, h)B'_\nu(p, h)}{[B^2(p, h)]^2} \right\}, \quad (5)$$

where $'$ means a derivative with respect to $h$. We wish to study the behaviour of different diagrams at $N \to \infty$. Computer simulations of the second and fourth order diagrams as functions of $N$ were performed using eqs (3) and (4). The results are presented at Fig.2. One sees that the polarization operator $\Pi_{VS}(0)$ and the diagram with four external lines $\Gamma_{0000}(0)$ diverge like $N$. These calculations show that the SLAC action does not provide the correct continuum limit. A mass renormalization which one
could expect on the basis of power counting in the continuum theory does not save the situation as the diagrams with more than two external lines diverge in the limit $N \to \infty$.

We shall try to improve the model by introducing additional PV regularization according to [6]. The regularized action looks as follows:

$$I_{VR} = I_{VS} + I_{PV},$$

$$I_{PV} = \sum_r \left\{ \sum_{x, y, \mu} \overline{\phi}_r(x) \gamma_\mu \mathcal{D}_\mu(x - y) \exp[i \sum_{z = x, \mu} A_\mu(z)] \phi_r(y) + \sum_x M_r \overline{\phi}_r(x) \phi_r(x) \right\}.$$  

Here $\phi_r$ are Bose and Fermi PV fields having the same spinorial and internal structure as $\psi$.

The estimates of asymptotic behaviour of different diagrams, analogous to the ones presented above, show that if one uses one PV field the diagrams have asymptotics $\sim M^2 N^2$. Therefore to suppress the contribution of momenta close to the edge of the Brillouin zone which are responsible for nonlocal effects, one has to choose $M \ll \frac{1}{N}$. Such small values of PV field masses are not acceptable as they are comparable with masses of physical particles. Moreover, as we shall see from numerical analysis, the model with one PV field is very sensitive to the particular choice of $M$ and therefore the results are not stable.

To get reliable results one needs to introduce at least three PV fields. In this case the diagrams have asymptotic behaviour $M^4 N^2$, and choosing $\frac{1}{N} \ll M \ll \frac{1}{\sqrt{N}}$ one can suppress the contribution of momenta close to the edge of the Brillouin zone.

Below we present the results of numerical calculations for the cases of 1 and 3 PV fields.

The regularized determinant has a form

$$D_{VR} = D_{VS}[h]D_{PV}[h],$$

where $D_{VS}[h]$ is given by eq. (2) and $D_{PV}$ is defined as follows:

$$D_{PV} = \prod_r \prod_{p = -N/2 + 1}^{N/2} \left( \frac{B^2(p, h) + M_r^2}{B^2(p, 0) + M_r^2} \right)^{c_r}.$$
Here $c_r = 1$ or $-1$ corresponds to the case of Fermi or Bose PV field.

**Fig.3.** Vectorial determinants $D_V$ as functions of $M$ at $h_0 = h_1 = 0.2$:

1. $D_{VC}$ on the torus;
2. $N = 32$, 3 PV fields;
3. 1 PV field;
4. $N = 160$;

The results of calculations are presented at Fig.3. One sees that in the case of one PV fields the agreement is achieved for $N=160$ only at one particular value of $M_0 = 0.026 - 0.033$. Even small variation in the value of $M$ leads to a large discrepancy between lattice and continuum results. In the case of 3 PV fields the lattice results agree with the continuum in the rather big interval of values of PV fields masses. For $N=160$ the agreement is observed for $M_1 = 0.04 \leq M \leq M_2 = 0.2$. These values are practically independent of $h$, except for the case $h_\mu \to 0.5$. Comparing the data at $N=32$ and $N=160$ one notes that if $N$ grows the mass values decrease and the minimal value of $M_1$ at 3 PV fields behaves like $N^{-3/4}$.

Let us discuss the behaviour of the lattice determinant in the case of 3 PV fields in some more details. Due to the fact that the covariant
derivative $B_\mu(p, h)$ sharply goes to zero at the border of the Brillouin zone, it is instructive to separate the products in eqs (2), (7) into three parts:

$$D_{VR} = D_{in}D_bD_a.$$ (7)

Here $D_{in}$ is a product over $p$ excluding external points, $D_b$ is the product over $p$ belonging to the edges of the Brillouin zone and $D_a$ is the product of $p$ corresponding to the vertices of the zone.

Let us start with $D_{in}$. According to the behaviour of $B_\mu(p, h)$ (see Fig.1), in this region difference between regularized diagrams and the continuous ones is of order $1/MN$. Near the border of this region $B_\mu(p, h)$ and its derivatives are of order 1. It allows to expand the corresponding terms in regularized diagrams over $M^2$. In the case of 3 PV fields the first nonvanishing term is $\sim M^4$ leading to the asymptotic behaviour $\sim M^4N^2$. Summarizing these estimates we get

$$D_{VR} = D_{VC} \left(1 + O \left(1/MN \right) + O(M^4N^2) \right), \quad N \to \infty,$$ (8)

when $\frac{1}{N} \ll M \ll \frac{1}{\sqrt{N}}$.

For the further estimates of values $D_a$ and $D_b$, using numerical data presented at Fig.1, we approximate the covariant derivative $B_\mu(p, h)$ by the following two lines (ignoring oscillations):

$$B_\mu(p, h) \approx \frac{2\pi}{N}(p_\mu - h_\mu - 1/2), \quad p_\mu = -N/2 + 2, \ldots, N/2 - 1;$$

$$B_\mu(p + N/2, h) \approx -2\pi(p_\mu - h_\mu - 1/2), \quad p_\mu = 0, 1.$$ (9)

The value of $D_a$ at $|h_\mu| \leq 1/2$ has the form following from the eqs (6) and (9):

$$D_a = D_{a0}[h_0, h_1]D_{a0}[h_0, -h_1]D_{a0}[h_1, h_0]D_{a0}[h_1, -h_0]/D_{a0}^4[0, 0].$$

Here

$$D_{a0}[h_0, h_1] = \frac{[(\frac{1}{2} - p)^2 + 2(M/2\pi)^2](\frac{1}{2} - p)^2}{[(\frac{1}{2} - p)^2 + (M/2\pi)^2]^2},$$
where \((\frac{1}{2} - p)^2 = \sum_{\mu} (\frac{1}{2} - p_\mu)^2\). One sees that at all values of \(h\) in the interval \(|h_\mu| \leq 1/2\) except the trivial case \(|h_0| = |h_1| = 1/2\) when \(D_{VR} = D_{VC} = 0\)

\[ D_a \to 1 \quad \text{at} \quad M(N) \to 0. \quad (10) \]

Now we consider the value \(D_b\). Its expression at \(|h_\mu| \leq 1/2\) looks as follows:

\[ D_b = D_{b0}[h_0, h_1] D_{b0}[h_0, -h_1] D_{b0}[h_1, h_0] D_{b0}[h_1, -h_0]/D_{b0}^4[0, 0]. \]

Here

\[ D_{b0}[h_0, h_1] = \frac{N/2-1}{\prod_{p_0 = -N/2+2} [G[p_0, h] + 2(M/2\pi)^2][G[p_0, h]'} \]

where \(G[p_0, h] = \frac{1}{\pi^2} (p_0 - h_0 - \frac{1}{2})^2 + (\frac{1}{2} - h_1)^2\).

Let us extend in the last equation from the interval \(-N/2 + 2 \leq p_0 \leq N/2 - 1\) to \(-\infty < p_0 < \infty\). In additional domain \(|p_0 - \frac{1}{2}| \geq \frac{N-1}{2}\) the value \(D_{b0}\) behaves like

\[ \sum_{|p_0 - \frac{1}{2}| \geq \frac{N-1}{2}} \ln \left(1 + 2(M/2\pi)^2\right) \approx \left(\frac{MN}{\pi}\right)^2 \sum_{p_0 = N} \frac{1}{p_0^2} \lesssim M^2 N. \]

Using this estimation one can transform \(D_{b0}\) to the form

\[ D_{b0}[h_0, h_1] = \frac{H[h, M\sqrt{2}]H[h, 0]}{H^2[h, M]} \left(1 + O(M^2 N)\right), \]

where \(H[h, M] = \text{ch} \, 2\pi N \sqrt{\left(\frac{1}{2} - h_1\right)^2 + \left(\frac{M}{2\pi}\right)^2 - \cos 2\pi \left(h_0 + \frac{1}{2}\right)}\).

According to this formula, we get for the value \(D_b\) at \(|\frac{1}{2} - h_\mu| \gg M\) the following expression:

\[ D_b = \exp\left\{-\frac{M^4 N}{32\pi^3} \left[ (1/2 - h_1)^{-3} + (1/2 + h_1)^{-3} + 
\quad + (1/2 - h_0)^{-3} + (1/2 + h_0)^{-3} - 32 \right]\right\}. \quad (11) \]

It follows from eqs (7), (8), (10), (11) that for \(\frac{1}{N} \ll M \ll \frac{1}{\sqrt{N}}\) our regularized lattice model agrees with the continuum toron model for the fields \(h_\mu\) in the interval \(|\frac{1}{2} \pm h_\mu| \gtrsim (M^4 N)^{1/3}\).
Fig. 4. Vectorial determinants $D_V$ as functions of $h_1$ at $h_0 = 0.2$:

1. $D_{VC}$ on the torus;
2. $D_{VR}$ with 3 PV fields:
   2. $D_{VR}$ with $N=32$: $M = 0.15$;
   3. $D_{VR}$ with $N=160$: $M = 0.04$;

In the region $(M^4N)^{1/3} \gg \frac{1}{2} \pm h_\mu \gg M$, the lattice determinant decreases sharply when $|h_\mu| \to 1/2$ and the agreement with the continuum model is lost. This effect is due to vanishing of the covariant derivative $B_\mu(p,h)$ at the border of the Brillouin zone, and as our calculations show it comes from the contribution of the edges of the Brillouin zone.

If we denote the value of $h$ at which $D_{VR}$ and $D_{VC}$ start to differ by more than 5% by $h^*$, i.e when $D_{VR}/D_{VC} = \xi = 0.95$, it follows from eq. (12) that

$$|h^*_\mu| = \frac{1}{2} - \left(\frac{M^4N}{32\pi^3\ln\xi^{-1}}\right)^{1/3}.$$  

One sees that in the limit $N \to \infty$ the regularized model agrees with the continuum toron model in the whole interval of $h$ values.

These analytic results are in a good agreement with numerical calculations of $D_{VR}$ and $D_{VC}$ according to eqs (6), (4). At Fig. 4 the values of $D_{VR}$
and $D_{VC}$ calculated for different $h$ and $M$ are presented. One sees that for $N=160$ 3 PV fields provide a good agreement with the continuum for $|h| < 0.4$.

3 Phase of the lattice chiral determinant

In this section we consider the possibility to use the SLAC action for regularizing chiral U(1) model. Obviously such a regularization could be successful only for anomaly free models, as SLAC action is chiral invariant and therefore cannot reproduce anomaly. However our discussion in the previous section shows that even in the case when perturbative anomalies are compensated as in 11112 model, this regularization most probably fails. Indeed the possibility to suppress the contribution of momenta close to the border of the Brillouin zone in anomaly free models by introducing gauge invariant vectorial interaction of PV fields is related to the fact that usually we deal with a finite number of divergent diagrams, whose sum is anomaly free models is parity conserving. For example in the 2d U(1) model with 4 positive chirality fermions with charge 1 and one negative chirality fermion with charge 2 the sum of anomalous second order diagrams is purely vector like. For nonzero external momenta all higher order diagrams are convergent and the contribution to them of momenta close to $\pi/a$ is negligible. This is the reason why PV regularization was successfully applied to the SLAC action on the infinite lattice, see [6]. However in the toron model with SLAC action, as was shown in the previous section, there are ”divergent” diagrams with more than two external lines. That means the contribution of boundary momenta $p_\mu \sim \frac{\pi}{a}$ is important not only for two-point diagram, but for other diagrams as well. The sum of the diagrams with more than 2 external lines is not vector-like and therefore cannot be suppressed by vectorial PV interaction. Using PV regularization one can easily obtain the agreement with the continuum case for the modulus of determinant. However vectorial PV interaction does not influence the phase of the determinant. Therefore the SLAC action can provide a correct phase only if for some reasons the contribution of boundary momenta cancel by itself. Moreover it
is easy to show that nonzero contribution to phase comes from the diagrams with more than two external lines and hence anomaly cancelation does not help.

So it is sufficient to check the phase of a model with one positive chirality fermion described by the standard SLAC action (I), where one has to consider \( \psi \) as the Weyl fermion

\[
\psi = \frac{1 + \gamma^3}{2}\psi.
\]

No PV fields are needed.

A straightforward calculation gives the result

\[
D_+S = \prod_{p=-N/2+1}^{N/2} \frac{B_0(p,h) + iB_1(p,h)}{B_0(p,0) + iB_1(p,0)},
\]

(12)

where \( B_\mu(p,h) \) is the covariant derivative defined by the eq. (3) (see also Fig.1).

The corresponding expression for the determinant in continuum theory on the torus looks as follows [7, 8, 9]:

\[
D_+C = e^{i\pi h_1(n_0 + i n_1)} \prod_{n=1}^{\infty} F[n, h]F[n, -h],
\]

(13)

where \( F[n, h] \) is defined by eq (4).

The determinants (12), (13) satisfy the following symmetry properties:

\[
D_+[h_0, h_1] = D^*_+[h_1, h_0] = D^*_+[-h_0, h_1].
\]

Due to chiral invariance of the SLAC action the lattice determinant (13) is periodic in \( h \):

\[
D_+S[h_0, h_1] = D_+S[h_0 + n_0, h_1 + n_1], \quad n_0, n_1 = 0, \pm 1, \pm 2, \ldots
\]

The continuum theory is anomalous and the corresponding determinant satisfies the condition

\[
D_+C[h_0 + n_0, h_1 + n_1] = e^{i\pi(n_0 h_1 - n_1 h_0)}D_+C[h_0, h_1].
\]
It follows that one can hope at most on the agreement only for the fields $h_\mu$ in the interval $|h_\mu| \leq 0.5$. It is sufficient to consider $0 \leq h_0 \leq h_1 \leq 0.5$.

Let us firstly make analytic estimates of $\text{Arg} D_{+S}$. In the previous section we showed that due to sharp decreasing of $B_\mu(p,h)$ near the border of the Brillouin zone this region gives a considerable contribution which may spoil the agreement with the continuum case. For that reason we shall study the behaviour of $\text{Arg} D_{+S}$ in the interior and boundary part of the zone separately. Using the notations introduced in the previous section we present $\text{Arg} D_{+S}$ as a sum

$$\text{Arg} D_{+S} = \text{Arg} D_{+\text{in}} + \text{Arg} D_{+b} + \text{Arg} D_{+a} \mod 2\pi. \quad (14)$$

To estimate the separate terms we adopt a linear approximation (9) of the covariant derivative $B_\mu(p,h)$. Eq. (12) leads to the following expression for different terms:

$$\text{Arg} D_{+\text{in}} = \sum_{p=-N/2+2}^{N/2-1} \frac{\text{arctg} \frac{h_0 (p_1 - \frac{1}{2}) - h_1 (p_0 - \frac{1}{2})}{\sum_{\mu} (p_\mu - h_\mu - \frac{1}{2}) (p_\mu - h_\mu - \frac{1}{2})}}{\mod 2\pi}, \quad (15)$$

$$\text{Arg} D_{+a} = \text{arctg} \frac{16 h_0 h_1 (h_0^2 - h_1^2)}{4 (h_0^2 - h_1^2)^2 - 16 h_0^2 h_1^2 + 1},$$

$$\text{Arg} D_{+b} = \text{Arg} D_{+b0}[h_0, h_1] + \text{Arg} D_{+b0}[-h_0, -h_1] +$$

$$+ \text{Arg} D_{+b0}[-h_1, h_0] + \text{Arg} D_{+b0}[h_1, -h_0] \mod 2\pi,$$

$$\text{Arg} D_{+b0}[h_0, h_1] = \sum_{p_0=1}^{N/2-1} \frac{\text{arctg} \frac{2 h_0 N (h_1 - \frac{1}{2})}{(p_0 - \frac{1}{2})^2 + N^2 (h_1 - \frac{1}{2})^2 - h_0^2}}{\mod 2\pi}. \quad (16)$$

Due to the symmetry properties of the determinant $D_{+S}$ only the diagrams with more than 2 external lines contribute to $\text{Arg} D_{+\text{in}}$. These lattice diagrams differ from the corresponding continuum diagrams by the terms of order $1/N$. Therefore for $|h_\mu| < 0.5$ we have

$$\text{Arg} D_{+\text{in}} \to \text{Arg} D_{+C}, \quad N \to \infty. \quad (16)$$

To estimate $\text{Arg} D_{+b0}[h_0, h_1]$ we expand the $\text{arctg} x$ in the Taylor series. Keeping only the first term we replace the sum by the integral and substitute
Fig. 5. Arguments of positive chiral determinants $\text{Arg } D_+$ as functions of $h_1$ at $h_0 = 0.4$:

1 – $\text{Arg } D_{+C}$ on the torus; 2, 3 – $\text{Arg } D_{+S}$ on the lattice:

2 – computed by eq. (12) at $N = 32$ and $160$,

3 – estimated by eqs (14) – (17) at $N \rightarrow \infty$

this expression into the formula for $\text{Arg } D_{+b_0}$. In this way we get

$$\text{Arg } D_{+b} = 2h_1 \arctg \frac{2h_0}{1 - 2h_0^2} - 2h_0 \arctg \frac{2h_1}{1 - 2h_1^2}, \quad N \rightarrow \infty.$$  \hspace{1cm} (17)

The calculations show that for all $h$ in the interval under consideration

$$| \text{Arg } D_{+b} | \ll | \text{Arg } D_{+\text{in}} |, \quad N \rightarrow \infty.$$

Finally, calculating $\text{Arg } D_{+a}$ with the help of eq. (16) we find

$$\text{Arg } D_{+a} = \text{Arg } D_{+\text{in}}, \quad N \rightarrow \infty.$$

Our estimates show that the contribution of the edges of the Brillouin zone to the $\text{Arg } D_{+S}$ is small, but the contribution of the vertices is approximately equal to the contribution of the interior and their sum is two times
bigger than the continuum value:

\[ \text{Arg } D_{+S} = 2 \text{Arg } D_{+C}. \]

So we have here some kind of doubling phenomenon which is somewhat reminiscent to the phenomenon observed by Bodwin and Kovacs \[10\] in Rabin’s formulation of Schwinger model \[11\].

Analytic estimates given above are in a good agreement with the results of numerical calculations presented at Fig.5. For all values of \( h \) \( \text{Arg } D_{+S} \) is two times bigger than the corresponding continuum value.

4 Discussion

In this paper we analyzed the U(1) model on a finite 2d lattice described by the gauge invariant nonlocal SLAC action. It was shown that in the case of a constant gauge field (toron model) SLAC action generates infinite series of divergent (in the limit \( N \to \infty \)) diagrams with more than two external lines and the value of the lattice fermion determinant does not agree with the known exact result for the continuum toron model. Modification of the SLAC action by introducing additional PV regularization cures this decease in the case of vectorial interaction. A minimal number of PV fields necessary for such regularization is equal to three.

The peculiar feature of the SLAC action on a finite lattice is large contribution of momenta corresponding to the vertices of the Brillouin zone. In the case of the vectorial interaction contribution of these momenta results in sharp decreasing of the lattice determinant in the narrow region near \( |h_\mu| = 1/2 \). However in the limit \( N \to \infty \) the width of this region tends to zero and the model agrees with the continuum result for all \( |h_\mu| < 1/2 \).

In the case of axial interaction the existence of divergent diagrams with more than two external lines does not allow to cancel the contribution of momenta \( |p| \sim \frac{\pi}{a} \) by introducing PV regularization. As in the vectorial case the boundary momenta give a large anomalous contribution. As a result the phase of the lattice chiral determinant is two times bigger than the corresponding continuum phase.
Our results show that SLAC action supplemented by additional PV regularization solves successfully the problem of fermion spectrum doubling in the case of vectorial interaction, but fails to describe correctly chiral models interacting with a constant gauge field.

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References

[1] A.A. Slavnov, N.V. Zverev, hep-lat 9708022.
[2] H.B. Nielsen, M. Ninomiya, Nucl. Phys. B105 (1981) 219.
[3] S. Drell, M. Weinstein, S. Yankielovitz, Phys. Rev. D14 (1976) 487, 1627.
[4] L. Karsten, J. Smit, Nucl. Phys. B144 (1978) 536.
[5] L. Karsten, J. Smit, Phys. Lett. B85 (1979) 100.
[6] A.A. Slavnov, Nucl. Phys. (Proc. Suppl.) B42 (1995) 166.
[7] L. Alvarez-Gaume, G. Moore, C. Vafa, Comm. Math. Phys. 6 (1986) 1.
[8] R. Narayanan, H. Neuberger, Phys. Lett. B348 (1995) 549.
[9] C.D. Fosco, S. Randjbar-Daemi, Phys. Lett. B354 (1995) 383.
[10] G. Bodwin, E. Kovacs, Phys. Rev. D35 (1987) 3198.
[11] J. Rabin, Phys. Rev. D24 (1981) 3218.