Numerical restoration of surface vortices in Nb films measured by a scanning SQUID microscope

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Abstract. In the present work, we investigated a vortex profile appeared on a pure Nb film (500 nm in thickness, 10 mm x 10 mm) by using a scanning SQUID microscope. We found that the local magnetic distribution thus observed is broadened compared to a true vortex profile in the superconducting film. We therefore applied the numerical method to improve a spatial resolution of the scanning SQUID microscope. The method is based on the inverse Biot-Savart law and the Fourier transformation to recover a real-space image. We found that the numerical analyses give a smaller vortex than the raw vortex profile observed by the scanning microscope.

1. Introduction

While the penetration depth \( \lambda \) is one of the fundamental material parameters in a bulk type-II superconductor, a penetration depth \( \lambda \) is prolonged to an effective penetration depth \( \Lambda_{\text{eff}} \) in a film of small thickness \( d \) [1]. In our preceding studies, we investigated a vortex distribution in a concave decagon (star) and a Pacman superconducting plate by scanning SQUID microscopy for amorphous Mo80Ge20 films [2] [3] [4], the penetration depth in Mo80Ge20 film is rather extended compared to a sample size. This gives us an opportunity of investigating a de facto mesoscopic superconducting property even by using a micrometer-sized superconducting system.

In the present work, we measured vortex profiles on a pure Nb film (10 mm x 10 mm) by using a scanning SQUID microscope. The thickness of the Nb film is chosen to be 500 nm so as to minimize the enlarging effect of the penetration depth in a very thin film. We apply the numerical method to improve the spatial resolution of the scanning SQUID microscope in terms of the inverse Biot-Savart law [5]. We found the penetration depth of Nb film by correcting the effect of broadening of the vortex profile when the observation point is deviated from the surface. The penetration depth can be determined by the measurement of lower critical magnetic field \( H_{c1} \) while it would be difficult to determine it definitely due to the demagnetization effect. We consider that our method
alternatively offers a direct method to determine the penetration depth in the superconducting film.

2. Vortex in a thin superconductor

In the mixed state of type-II superconductors, the magnetic flux penetrates into the superconductor in the form of flux quanta. By combining Maxwell’s equation and the second London equation, one obtains an expression for spatial dependence of the local magnetic field $\mathbf{h}$ as

$$\nabla^2 \mathbf{h} - \frac{\mathbf{h}}{\lambda^2} = -\frac{\Phi_0}{\lambda^2} \delta(\mathbf{r} - \mathbf{r}_0) \mathbf{e}_z$$

where a center of the vortex is located at $r_0$. The solution for this equation is given by the zero-th order Bessel function:

$$h(r) = \frac{\Phi_0}{2\pi\lambda^2} K_0\left(\frac{r}{\lambda}\right),$$

where $K_0$ is a Hankel function. At a large distance from the vortex origin ($r \to \infty$), $h(r)$ decays exponentially as

$$h(r) \approx \frac{\Phi_0}{2\pi\lambda^2} \left(\frac{\pi}{2r}\right) e^{-\frac{\pi}{2\lambda r}},$$

where $\Phi_0 (=2.07 \times 10^{-15} \text{ Tm}^2)$ is the flux quantum. On the other hand, the distribution of the magnetic flux line tends to spread over quickly as an observation point departs from the sample surface.

In Figure 1, we show the flux line distribution both inside and outside of the thin film as originally elucidated by Carneiro and Brandt [6]. The vortex profile inside and outside of a superconducting film is very dependent on the film thickness. Vortex profile inside of the thinner film is remarkably different from that of the bulk superconductor. This is the origin of the prolonged penetration depth in a thinner film. This is the reason why we choose an enough thickness of 500 nm for the Mo$_{80}$Ge$_{20}$ films used in the present studies. We find that the magnetic field penetrates into the superconducting film so as to give a radial distribution from the vortex center. It is difficult to measure a true local magnetic field by using a SQUID microscope because an observation point is somewhat deviated from the sample surface. Therefore, it is extremely difficult to

![Figure 1](image-url)
observe a true vortex profile because the magnetic flux lines become broad very quickly when the observation point departs from the sample surface. In this work, we attempt to recover a true magnetic field distribution by using the inverse Biot-Savart law, which has been demonstrated to work properly to refine the vortex distribution in our relevant publication [5].

3. The inverse Biot-Savart Law

We use the algorithm of using the inverse Biot-Savart law algorithm developed by our group [5]. The magnetic flux measured by a SQUID coil is given by

$$\Phi(\vec{R}) = \frac{\mu_0}{2 \cos \theta} \int \frac{d^3k}{(2\pi)^2} (ik_x \sin \theta + k_\perp \cos \theta) \times \vec{m}_\perp(k_\perp) e^{-i k_\perp R_z} \mathcal{F}(k_\perp)$$

where $\vec{R} = (R_x, R_y, R_z)$, $\theta$ is a declined angle of the SQUID sensor from the sample surface, and $\vec{k}$ is the wave number vector. We do not denote some of notations due to the limited page s allotted to this paper, and the reader should refer to Ref. [5] to identify the definition of some terminologies. The integral kernel is given by the following formula as

$$\mathcal{F}(k_\perp) = \int_{-\infty}^{\infty} dx \int_{-\infty}^{\infty} dy f \left( \frac{x}{\cos \theta}, y \right) e^{ik_\perp x \tan \theta + ik_\perp y}$$

The Fourier transform $m_\perp(k)$ of the magnetic moment $m_\perp(r)$ is expressed by

$$\vec{m}_\perp(k_\perp) = \frac{2 \cos \theta \Phi(k_\perp)}{i k_x \sin \theta + k_\perp \cos \theta} \mathcal{F}(k_\perp)^{-1}$$

where $\Phi(k_\perp)$ is the Fourier transform of the measured flux image. The real space image can be obtained by the inverse Fourier transformation of $\vec{m}_\perp(k_\perp)$. We obtain the current distribution on the surface with the aid of the Maxwell equation.

We consider that this algorithm [5] outlined here is quite useful to recover a true local field around a vortex. This is also useful to estimate the penetration depth $\lambda$ by processing the broadening effect of the local magnetic flux measured by the scanning SQUID microscope at the observation point.

4. Measurements of Nb film by SQUID microscope

We measured a 500-nm thick Nb film (10 mm ×10 mm in size) with a scanning SQUID microscope (SQM-2000, Seiko Instruments, Inc.) with a pickup coil (10 $\mu$m in diameter). The reason we used the thick Nb film is to avoid a prolonged penetration depth, which appears in a very thin film as a Pearl vortex [1].

In Figure 2(a), we show an original image of a single vortex observed by the scanning SQUID microscope at 4K. If we observe the several vortices in a single image, it would be very difficult to discuss the quantization condition and the profile determined by the penetration depth. Therefore, we
searched for an image where a single vortex can be seen in the scanning area. After the several attempts of the in-field-cooled measurements, we were successful in obtaining such an image of an isolated vortex as shown in Figure 2(a).

We converted Figure 2(a) into a contour presentation to see a three-dimensional XYZ distribution. Then, we rotated the contour so as to be able to see it in the X-Z plane from the front side. In Figure 2(c), we show a front-side view of the single vortex obtained in such a manner. The vortex has a half width at half maximum (HWHM) of 5.4 µm. We may regard this value of 5.4 µm as an effective penetration depth if we consult a form of Eq. (3) having an exponential decay factor. Note that the local magnetic field distribution at observation points of Figure 2(a) is conducted not on the sample-surface plane but on a scanning plane somewhat lifted from the surface. This means that the observed vortex image is influenced by the broadening effect compared to a virtual true vortex image in the Nb film.

Hayashi et al. [5] successfully developed an algorithm to recover virtual true local magnetic fields on the sample surface. We applied the inverse Biot-Savart law to the original image of Figure 2(a), and obtained a restored image as shown in Figure 2(b), where a lift-off distance is taken as 1.6 µm. When we compare the restored image of Figure 2(b) with the original image of Figure 2(a), the profile of vortex becomes clearer. We take into account the deviation of the magnetic field distribution from the surface of Nb film in recovering an image on the sample-surface plane.

On the other hand, in Figure 2(c), we show an original image of vortex projected on the x-z plane while a restored image is projected on the x-z plane. In Figure 2(d), we assume that a distance from pickup coil to the surface of sample is 1.4 µm. We estimate a half width at half maximum (HWHM) as a measure of the penetration depth. The HWHM of the original image 5.4 µm, while the HWHM of the

**Figure 2.** The vortex image in Nb film. (a) Original image of vortex by SQUID microscope. (b) Restored image of vortex by SQUID microscope. (c) Original image of vortex on the x-z plane. (d) Restored image of vortex on the x-z plane.
restored image is 4.0 μm. We note that the width of the peak decreased by 25%. We conclude that our numerical method is useful for extracting a true image coming from a difference of the sample surface and the observation point.

In Figure 3, we show the FWHM as a function of distance between pickup coil and sample surface. We found that HWHM increases rather monotonically as a function of a lift-off distance. This is consistent with the idea of local field broadening as indicated in Figure 1.

5. Flux quantization of vortex

In investigating the penetration depth in Nb film, we first confirmed the validity of our original data by the SQUID microscope by evaluating the quantization condition of the magnetic flux in the mixed state for a single quantum of flux as [7]

$$\Phi_0 = \frac{hc}{2e} = 2.0678 \times 10^{-15} \text{Tm}^2 .$$  \hspace{1cm} (7)

We investigated the $\Phi_0$ value of the image observed by SSM satisfies the condition of the flux quantization with the following method.

In Figure 4, the red-colored square, which appeared at the surrounding area of the vortex, gives the magnetic flux density area of the flux quantization while the four areas indicated in green are used to evaluate an averaged background field. We can get a value of the flux quantization for the obtained image with the follow equation

$$\Phi = \int(B-<B>)dS$$  \hspace{1cm} (8)

where $B$ means the magnetic flux density of the measured flux quantization and $<B>$ means the average of the background magnetic flux density. The calculated value of $B$ was found to be $3.135 \times 10^{-5}$ T and an averaged magnetic flux density $<B>$ was found to be $2.166 \times 10^{-5}$ T. We obtained the square area of $2.100 \times 10^{-10}$ m², and found the $\Phi_0$ value of the image observed by SSM as $2.035 \times 10^{-15}$ T·m². This satisfied the condition of the flux quantization well. We found that the quantization condition is in good agreement with the actual flux evaluated by our SQUID microscope for a single isolated vortex appeared in the Nb film (see Figure 4).
6. Conclusion

We restore the penetration depth from the vortex profile when the observation point is deviated from the surface in the thick Nb film. We applied the Hayashi method [5] of the inverse Biot-Savart law analysis to recover a vortex image on the surface of the film. According to this numerical processing, we recovered a compressed vortex image in size, and confirmed that this process is useful to recover the deviation from a true local magnetic field. We investigated the $\Phi_0$ value for the image obtained by SSM, and confirmed that it satisfies the flux quantization well. Further systematic studies are needed to elucidate remaining subjects on the SQUID microscopy.

7. Acknowledgement

This work is partially supported by a Grant-in-Aid for Young Scientists (B) (No.26800192) and a Grant-in-Aid for Scientific Research (A) (No.16H02450) from JSPS.

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