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Theoretical interpretation of the nuclear structure of $^{88}$Se within the ACM and the QPM models.

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Abstract. The four-parameter algebraic collective model (ACM) Hamiltonian is used to describe the nuclear structure of $^{88}$Se. It is shown that the ACM is capable of providing a reasonable description of the excitation energies and relative positions of the ground-state band and $\gamma$ band. The most probable interpretation of the nuclear structure of $^{88}$Se is that of a transitional nucleus. The Quasiparticle-plus-Phonon Model (QPM) was also applied to describe the nuclear motion in $^{88}$Se. Preliminary calculations show that the collectivity of second excited state $2^+_2$ is weak and that this state contains a strong two-quasiparticle component.

1. Introduction
Recently, the nuclear structure of the very neutron-rich nucleus $^{88}$Se was studied in Ref. [1]. The $\gamma$ decay scheme in $^{88}$Se has been constructed using data from an neutron-induced fission of $^{235}$U. The measurement was performed with the EXILL array of Ge detectors at the PF1B cold-neutron beam facility of the Institut Laue-Langevin (ILL) [2], Grenoble. The level scheme of $^{88}$Se was established using the $\gamma-\gamma-\gamma$ coincidence technique. A low $2^+_1$ energy hints at the onset of quadrupole deformation and the identification of possible members of a $2^+_2$ band provides evidence for $\gamma$ vibrations.

2. ACM model
The algebraic collective model (ACM), introduced as a computationally tractable version of the Bohr model (BM) [3], which describes collective quadrupole vibrational modes in deformed nuclei. Detailed information of the ACM can be found in Ref. [4]. This model is characterized by a well defined algebraic subgroup chain and it allows collective-model calculations to be made using a simple routine procedure.

A great advantage of the ACM is that fully converged calculations can be performed easily for a range of Hamiltonians [5]. This enables one to determine the extent to which a particular BM Hamiltonian can realistically describe experimental data. Such calculations for various BM Hamiltonians prepare the way for more general, but still solvable, algebraic collective
models that include intrinsic degrees of freedom, similarly to the unified model of Bohr and Mottelson [6, 7, 8, 9, 10].

A general purpose ACM Hamiltonian is given, for example, in the form

$$\hat{H}(M, \alpha, \kappa, \chi) = \frac{-5^2}{2M} + \frac{1}{2} M[(1 - 2\alpha)\beta^2 + \alpha\beta^4]$$

$$- \chi \beta \cos 3\gamma + \kappa \cos^2 3\gamma$$

(1)

$$\nabla^2 = \frac{1}{\beta^2} \frac{\partial}{\partial \beta} \beta^4 \frac{\partial}{\partial \beta} + \frac{1}{\beta^2} \Lambda^2$$

(2)

where

$$\nabla^2$$

is the Laplacian on the 5-dimensional collective model space [4].

The above Hamiltonian, expressed in terms of the quadrupole deformation parameters \(\beta\) and \(\gamma\), serves as a useful starting point for a description of a wide range of nuclear collective spectra. Moreover, it has been shown that this kind of Hamiltonian can be exploited in the study of a phase transition between axial and triaxial deformations [11] or in the study of the second-order phase transition of a model nucleus, from a spherical to a deformed phase, with \(\alpha\) as a control parameter [8]. For \(\alpha = 0\) the potential is that of a spherical harmonic oscillator, \(\frac{1}{2}M\beta^2\), while for \(\alpha > 0.5\) it has a minimum for a non-zero value of \(\beta\), which increases with \(\alpha\) increases. Moreover, as the effective mass parameter \(M\) of the Hamiltonian (1) increases, the kinetic energy decreases and the result is a decrease of the vibrational \(\beta\) fluctuations of the model about its equilibrium deformation. Thus, the value of the parameter \(\alpha\) controls the \(\beta\) deformation of the model and the parameter \(M\) controls its rigidity. By adjusting the parameters \(\alpha\) and \(M\) a model with any equilibrium value of the \(\beta\) deformation and any degree of rigidity may be constructed. In Ref. [5] it has been shown that parameter values in the range \(0 < \alpha < 2.0\) and \(10 < M < 100\) are sufficient to describe the \(\beta\) deformations and rigidities of the observed nuclear collective states.

The last fourth term of the ACM Hamiltonian (1), proportional to \(\cos^2 3\gamma\), allows a triaxial minimum to appear in the potential energy surface (PES). A delicate competition between all the first three terms in the potential and the second and the third one in particular will determine whether the potential energy minimum will remain axially symmetric (the term proportional to \(\cos 3\gamma\) dominates) or will be driven to a triaxial minimum by the last term.

3. ACM calculations for transitional nuclei

The detailed spectroscopy of the nuclei in the isotopic chain of Se, Ge, Sr and Kr clearly reveals the existence of \(\gamma\)-soft collective structure. Within the ACM one thus may be tempted to test the validity of the Wilets-Jean (WJ) limit that is characterised by a completely \(\beta\)-rigid and \(\gamma\)-independent potential which leads to characteristic degeneracies of some angular momentum states (e.g. 3\(^+\), 4\(^+\)). After having applied the WJ limit to \(^{88}\)Se we have found the ground state energies were correctly described but the \(\gamma\)-bandhead was about 200 keV higher than the experimental one and its excited doublet of states (3\(^+\), 4\(^+\)) was almost 1 MeV higher than the state (3\(^+\), 4\(^+\)) observed at 1798.6 keV. We have also tested the triaxial rigid rotor ACM which also provides a very good description of the ground-state band energies and the two known \(\gamma\)-band energies are very well described, see Fig. 1b. In this interpretation, the state (3\(^+\), 4\(^+\)) observed at 1798.6 keV would be described as a 3\(^+_1\) state and the calculated 4\(^+_2\) state would be located at a very high energy. This is, however, not expected for \(\gamma\)-soft nuclei. Even though the rigid triaxial rotor scenario cannot be excluded based on the available experimental data it is most probably not very realistic for a nucleus with only 4 neutrons beyond \(N = 50\).

As a weak quadrupole deformation of \(\beta = 0.22(2)\) was derived for \(^{88}\)Se [1], we have tested a scenario where this nucleus is assumed to be close to the transitional point between the spherical and deformed phase described by \(\alpha = 0.5\). Firstly, a parameter value \(\alpha\) should be determined
in order to provide an optimum description of the observed spectrum. Fig. 2 shows that using the parameters $M = 70$, $\chi = \kappa = 0$ and in the interval $1 < \alpha < 4$ the calculated energies remain close to those of the WJ limit. When the parameter value $\alpha$ tends to 0.5, the energies of the calculated states $2^+_2$, $4^+_1$, and $3^+_1$, $4^+_2$ tend to the experimental values 1242.5 and 1798.6 keV. Thus, in order to model a transitional nucleus in the framework of the ACM model, we have chosen the following nuclear potential parameters $M = 70$, $\alpha = 0.55$, $\chi = -0.8$ and $\kappa = 0$. The non-zero value of the parameter $\chi$ is needed to introduce a small axial deformation in this studied nucleus. The minimum of the corresponding potential is not localized around a fixed value of $\beta$. The potential is very $\beta$-soft as seen in Fig. 3.

The ACM spectrum obtained with the above parameters is shown in Fig. 1c where it is compared to its experimental counterpart and with shell model (SM) calculations [1]. It is seen in both models that the calculated splitting of the levels $3^+_1$ and $4^+_2$ is almost the same. The position of the $2^+_2$ state is reasonably well described.

It should be noted that for very $\beta$-soft potentials large centrifugal stretching effects cannot be avoided. These manifest themselves by ground-state band energies that fall systematically below their experimental counterparts (see Fig. 1c).

The transitional interpretation doesn’t favour very strong transitions that would be described within the WJ limit as $\Delta \nu = 2$, where $\nu$ is the seniority quantum number. This conclusion has been confirmed by SM calculations. In particular, the reduced transition probability $B(E2)$ of transitions $4^+_2(\nu = 3) \rightarrow 2^+_1(\nu = 1)$ or $2^+_2(\nu = 2) \rightarrow g.s(\nu = 0)$ with $\Delta \nu = 2$ is not strong with respect to the transitions with $\Delta \nu = 1$.

Some of the transition rates, e.g. $B(E2; 2^+_2 \rightarrow 2^+_1)$ are much larger than the SM ones but it should be stressed that the values of these transitions change in the vicinity of the transitional
Figure 2. Energy doublets $2^+_2, 4^+_1$ and $3^+_1, 4^+_2$ as a function of parameter $\alpha$. The triangles represent the experimental energy values of excited states $2^+_2$ and $(3_1, 4_2)^+$ of $^{88}\text{Se}$.

In the current interpretation, the ACM and SM indicate that a spin of $5^+_1$ or $6^+_1$ can be attributed to the state at 2708.0 keV, experimentally identified in this work. Angular correlations with sufficient statistics could confirm with certainty this hypothesis on this spin assignment.

4. QPM model

A detailed description of the QPM can be found, for example, in Refs [13, 14]. In this work, the even-even nucleus $^{88}\text{Se}$ has been studied, this is why only the vibrating even-even core has been taken into account in the model.

The intrinsic Hamiltonian can be written as

$$\hat{H}_{\text{int}} = \hat{H}_{\text{av}} + \hat{H}_{\text{pair}} + \hat{H}_{\text{l, res}}$$

where $\hat{H}_{\text{av}}$ is an axially symmetric quadrupole and hexadecapole-deformed average mean-field, which can be approximated by the Nilsson single-particle Hamiltonian, $\hat{H}_{\text{pair}}$ is a monopole pairing interaction, and $\hat{H}_{\text{l, res}}$ is a long-range residual multipole-multipole interaction.

$$\hat{H}_{\text{pair}} = -G_n \sum_{ij \in n} a_i^\dagger a_j^\dagger a_j a_i - G_p \sum_{ij \in p} a_i^\dagger a_i^\dagger a_j a_j$$

and

$$\hat{H}_{\text{l, res}} = -1/2 \sum_{\tau = p, n} \sum_{\lambda \mu} \kappa_{\lambda \mu} M_{\lambda \mu}^{\tau}(\vec{r}) M_{\lambda \mu}^{\tau}(\vec{r})$$

where $a_i^\dagger$ and $a_i$ are particle creation and annihilation operators of the single-particle state $i$; $G_n$ and $G_p$ are the neutron and proton pairing strength constants; $M_{\lambda \mu}^{\tau}$ is the multipole-multipole operator of the multipolarity $\lambda$, and its projection $\mu$, and $\kappa_{\lambda \mu}$ stands for the corresponding long-range residual interaction strength constant.

In practical calculations with deformed nuclei the strength constants should explicitly depend on $\mu$ (the projection of $\lambda$ on the symmetry axis) and should be fixed by the requirement to obtain good agreement between the energies of the lowest RPA solutions and the corresponding experimental phonon energies in even-even nuclei [15, 16].
The Nilsson-potential parameters used in our calculations are those recommended in Soloviev’s monograph [15]. Initial values of the quadrupole- and hexadecapole-deformation parameters and of the ground-state pairing gaps are taken from the finite-range liquid-drop model plus shell-correction method calculations [17]. Experimental energies or the systematics of the quadrupole and octupole one-phonon states in the even-even cores are used to deduce the strengths of the multipole-multipole interactions $\kappa_{\lambda\mu}$, where no distinction is made between interaction strengths for protons and neutrons.

5. The microscopic structure of a low-lying state $2^+_2$ in $^{88}$Se obtained with the QPM

In this section, we will study the nature of the state $2^+_2$ at 1242.5 keV in $^{88}$Se. This nucleus consists of 34 protons and 54 neutrons or 4 neutrons above the magic shell $N = 50$. The aim of this theoretical study is to show which Nilsson orbits contribute to the collective motion in $^{88}$Se. In order to perform QPM calculations, the shape parameters $\varepsilon_2 = 0.166$ and $\varepsilon_4 = -0.02$ of the ground state were taken from Ref. [17]. To see the stability of the results with respect to changes of the quadrupole deformation parameter $\varepsilon_2$ we have performed several calculations with $\varepsilon_2$ ranging from 0.106 to 0.256, see Fig. 4. In this figure, the probability to find a given configuration in the excited state $2^+_2$ is represented by a value $X^2 - Y^2$, where $X$ and $Y$ represent forward and backward amplitudes defining the quasiparticle-RPA phonon operator $\hat{Q}^\dagger_{\lambda\mu i}$

$$\hat{Q}^\dagger_{\lambda\mu i} = \sum_{i,\nu,\nu'} X_i^{\nu,\nu'} \hat{a}_{\nu'}^\dagger \hat{a}_\nu^\dagger - \sum_{i,\nu,\nu'} Y_i^{\nu,\nu'} \hat{a}_{\nu}^\dagger \hat{a}_{\nu'}.$$

We observed that the two two-quasiparticle components $\pi 3/2[312] \otimes \pi 1/2[310]$ and $\nu 3/2[422] \otimes \nu 1/2[431]$ are essentially independent of the deformation parameter $\varepsilon_2$ within the chosen range. On the other hand, the two-quasiparticle configuration $\nu 3/2[422] \otimes \nu 1/2[420]$ is sensitive
to variations of the quadrupole deformation $\varepsilon_2$, as shown in Fig. 4. The configuration $\nu^3/2[422] \otimes \nu^1/2[420]$ is always dominant except for the case of a low deformation $\varepsilon_2 \lesssim 0.136$.

For $\varepsilon_2 = 0.22(2)$, empirically derived in Ref. [1], the QPM predicts two dominant configurations $\nu^3/2[422] \otimes \nu^1/2[420]$ and $\pi^3/2[312] \otimes \pi^1/2[310]$ with amplitudes of 60% and 15% respectively. Thus, the excited state of the nucleus $^{88}$Se is mainly composed of two-quasiparticle excitations $\nu^3/2[422] \otimes \nu^1/2[420]$ and $\pi^3/2[312] \otimes \pi^1/2[310]$. The Nilsson orbitals $\nu^3/2[422]$ and $\nu^1/2[420]$ originate from the spherical orbits $\nu d_{5/2}, g_{7/2}$ respectively.

6. Conclusions
The ACM favours a shape transitional interpretation of the nuclear structure of $^{88}$Se. Within this interpretation $^{88}$Se is described as a nucleus close to the transitional point between a spherical and an axially deformed shape.

The QPM has been used to study the microscopic structure of the excited state $2^+_2$. We have found that the degree of collectivity of this state is low because it has a dominant two-quasiparticle structure with main configurations $\nu^3/2[422] \otimes \nu^1/2[420]$ and $\pi^3/2[312] \otimes \pi^1/2[310]$. It should be noted that the QPM calculations have been done with the standard Nilsson parameters recommended in the Soloviev’s monograph [15] may not be perfectly adapted for the $A = 80−90$ mass region. A migration of the neutron $g_{7/2}$ orbit to higher energies might be expected when the proton $g_{9/2}$ orbit is empty. This is in agreement with the shell model calculations [1] where the occupation of the neutron orbit $g_{7/2}$ is low. On the other hand there are indications [18] that even with the migration of the neutron $g_{7/2}$ orbit toward higher energies the $1/2[420]$ Nilsson orbit will play a role for the deformations in question. Thus, a comparison of the SM and QPM calculations raises important questions that will be investigated in more detail in future work.

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