Strangeness spin, magnetic moment and strangeness
configurations of the proton

C. S. An,¹ D. O. Riska,²† and B. S. Zou³,¹‡

¹ Institute of High Energy Physics, CAS,
P.O.Box 918, Beijing 100049, China
² Helsinki Institute of Physics and Department of Physical Sciences,
POB 64, 00014 University of Helsinki, Finland
³ CCAST (World Lab.), P.O.Box 8730, Beijing 100080, China

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Abstract

The implications of the empirical signatures for the positivity of the strangeness magnetic moment \(\mu_s\) and the negativity of the strangeness contribution to the proton spin \(\Delta_s\) on the possible \(uuds\bar{s}\) configurations of five quarks in the proton are analyzed. The empirical signs for the values for these two observables can only be obtained in configurations where the \(uuds\) subsystem is orbitally excited and the \(\bar{s}\) antiquark is in the ground state. The configurations in which the \(\bar{s}\) is orbitally excited, which include the conventional \(K^+\Lambda^0\) configuration, with exception of that in which the the \(uuds\) component has spin 2, yield negative values for \(\mu_s\). Here the strangeness spin \(\Delta_s\), the strangeness magnetic moment \(\mu_s\) and axial coupling constant \(G_A^s\) are calculated for all possible configurations of the \(uuds\bar{s}\) component of the proton. In the configuration with \([4]_{FS}[22]_{F}[22]_{S}\) flavor-spin symmetry, which is likely to have the lowest energy, \(\mu_s\) is positive and \(\Delta_s \simeq G_A^s \simeq -1/3\mu_s\).
I. INTRODUCTION

Four recent experiments on parity violation in electron-proton scattering suggest that the strangeness magnetic moment of the proton $\mu_s$ is positive \cite{1,2,3,4}. In contrast most theoretical calculations have led to negative values for this observable \cite{5}, with but few exceptions \cite{6,7,8,9,10,11}. The implications of the empirical result for the configuration of the constituents of the proton is considered by a calculation of $\mu_s$ for all positive parity configurations of the $uudss\bar{s}$ system with one constituent in the first orbitally excited state, which may be contained in the proton, have recently been studied in Ref.\cite{12}. The formalism summarized in that reference is elaborated here in more detail along with a few minor corrections. In addition it is now applied to the strangeness contribution to the proton spin $\Delta_s$ and the corresponding strangeness axial form factor $G_A^s$.

In the case of the strangeness contribution to the proton spin $\Delta_s$, the empirical indications are that it is very small \cite{13} or small and negative ($-0.10 \pm 0.06$) \cite{14,15}. Extrapolation of the empirical values for the strangeness axial form factor $G_A^s$ to low $Q^2$ indicates a non-zero negative value for that quantity \cite{16}.

Here it is noted that $\mu_s$ is positive and that both $\Delta_s$ and $G_A^s$ are negative and smaller than $\mu_s$ in magnitude in the $uudss\bar{s}$ configuration, which is likely to have the lowest energy, where the $\bar{s}$ quark is in the ground state and the $uuds$ system is in the $P$–state. If in contrast the strange antiquark is in the $P$–state and the 4 quarks are in their ground state the $\mu_s$ has the opposite – and empirically contraindicated sign. The latter configurations include that of a fluctuation of the proton into a kaon and a strange hyperon, which is well known to lead to a negative value for the strangeness magnetic moment \cite{17,18,19,20}.

In this manuscript a detailed description of the wave functions of the $uudss\bar{s}$ configurations that may contained in the proton. The manuscript falls into 6 sections. Section 2 contains the definitions of the strangeness observables and flavor wave functions for the $uudss\bar{s}$ components of the proton along with a discussion of their expected hyperfine splittings. The configurations, in which the $uuds$ system is in the ground state, are considered in section 3 and the corresponding configurations, in which the $uuds$ system is in the $P$–state are considered in section 4. The implications of the empirical values for $\mu_s$ and $\Delta_s$ for the probability of the $ss\bar{s}$ in the proton are considered in section 4. A summarizing discussion is given in section 5.
II. THE STRANGENESS OBSERVABLES AND FLAVOR WAVE FUNCTIONS FOR $uuds\bar{s}$ COMPONENTS

In the non-relativistic quark model, the strangeness magnetic moment $\mu_s$ and the strangeness contribution to the proton spin $\Delta_s$ are defined as the expectation value of the following two operators in the proton state:

$$\vec{\mu}_s = e \sum_i \hat{S}_i \left( \hat{l}_i + \hat{\sigma}_i \right),$$
$$\vec{\sigma}_s = \hat{\sigma}_s + \hat{\sigma}_{\bar{s}}, \quad (1)$$

where $\hat{S}_i$ is the strangeness counting operator with eigenvalue $+1$ for $s$ and $-1$ for $\bar{s}$ quark and $m_s$ is the constituent mass of the strange quark. The strangeness axial form factor is in turn defined as the matrix element of the strangeness axial current operator

$$\vec{A}_s = \vec{\gamma}_s i \gamma_5 + \vec{\gamma}_{\bar{s}} i \gamma_5. \quad (2)$$

The matrix element of this operator in the proton is denoted as $G_A^s$. In the non-relativistic limit and at $Q^2 = 0$ one has $G_A^s = \Delta_s$.

A key ingredient for the calculation of the matrix elements of these operators are the flavor wave functions for the $uuds\bar{s}$ components in the proton. These are usually constructed by coupling $uuds$ flavor wave functions with the $\bar{s}$ flavor wave function. There are four possible flavor symmetry patterns for the $uuds$ system: $[4]_F$, $[31]_F$, $[22]_F$ and $[211]_F$ in the Weyl tableaux of the $SU(3)$ group [21, 22]. Combination of these with the anti-quark with flavor symmetry $[1]^\ast_F$ leads to the following pentaquark multiplet representations of $SU(3)$:

$$[4]_F \otimes [1]^\ast_F = 10 \oplus 35, \quad (3)$$
$$[31]_F \otimes [1]^\ast_F = 8 \oplus 10 \oplus 27, \quad (4)$$
$$[22]_F \otimes [1]^\ast_F = 8 \oplus \bar{10}, \quad (5)$$
$$[211]_F \otimes [1]^\ast_F = 1 \oplus 8. \quad (6)$$

Here the numbers in boldface on the right-hand-side of the equations indicate the dimensions of the pentaquark representations. As an example the tentative $\theta^+$-pentaquark belongs to the baryon anti-decuplet $\bar{10}$ representation. Since the proton belongs to the baryon octet representation, the possible flavor symmetry patterns for the $uuds$ in the proton are limited
to \([22]_F, [211]_F\) and \([31]_F\). The corresponding flavor wave functions can be obtained as in refs. \([21, 22]\). For convenience we list the relevant ones here explicitly.

For the mixed flavor symmetry representations \([22]_F\) and \([211]_F\), there are 2 and 3 independent flavor wave functions, respectively:

\[
|22\rangle_F = \frac{1}{\sqrt{24}} \left[ 2|uuds\rangle + 2|uusd\rangle + 2|dsuu\rangle + 2|sduu\rangle - |duus\rangle - |udus\rangle \\
-|sudu\rangle - |usdu\rangle - |suud\rangle - |dusu\rangle - |usud\rangle - |udsu\rangle \right],
\]

(7)

\[
|22\rangle_F' = \frac{1}{\sqrt{8}} \left[ |udus\rangle + |sudu\rangle + |dusu\rangle + |usdu\rangle - |duus\rangle - |usdu\rangle \\
-|suud\rangle - |udsu\rangle \right],
\]

(8)

\[
|211\rangle_F = \frac{1}{\sqrt{16}} \left[ 2|uuds\rangle - 2|uusd\rangle - |duus\rangle - |usdu\rangle - |sudu\rangle - |usdu\rangle \\
+|suud\rangle + |dusu\rangle + |usud\rangle + |udsu\rangle \right],
\]

(9)

\[
|211\rangle_F' = \frac{1}{\sqrt{48}} \left[ 3|udus\rangle - 3|duus\rangle + 3|suud\rangle - 3|usud\rangle + 2|dsuu\rangle - 2|sduu\rangle \\
-|sudu\rangle + |usdu\rangle + |dusu\rangle - |usud\rangle \right],
\]

(10)

\[
|211\rangle_F'' = \frac{1}{\sqrt{6}} \left[ |sudu\rangle + |usdu\rangle + |dusu\rangle - |usud\rangle - |dusu\rangle - |sduu\rangle \right].
\]

(11)

In the case of the mixed flavor symmetry \([31]_F\) for the \(uuds\) system there is a need to separate the isospin 1/2 and 3/2 states, both of which are listed in Ref. \([21]\). The reason for the presence of two separate classes of wave functions with the flavor \([31]_F\) is the following.

In the case of flavor \(SU(3)\) model, the \(\Delta^+\) resonance is composed \(uud\) valence quarks with the flavor symmetry \([3]_F\). In the consideration of the \(uuds\bar{s}\) component of this baryon, and its \(uuds\) subsystem, the latter may also have the mixed flavor symmetry \([31]_F\). This flavor state of the \(uuds\) subsystem is therefore a combination of a \(\Delta^+\) and a proton component. There are 6 independent flavor wave functions for \(uuds\) of the flavor symmetry \([31]_F\). Among them, by using the weight diagram method \([22]\), the three wave functions labeled as \(\psi^0\) in Ref. \([21]\) are found to correspond to isospin 1/2. These are explicitly:

\[
|31\rangle_F = \frac{1}{\sqrt{18}} \left[ 2|uusd\rangle + 2|suud\rangle + 2|usdd\rangle - |sudu\rangle - |usdu\rangle - |dsuu\rangle \\
-|udsu\rangle - |dusu\rangle - |sduu\rangle \right],
\]

(12)

\[
|31\rangle_F' = \frac{1}{12} \left[ 6|uuds\rangle - 3|duus\rangle - 3|udsu\rangle - 4|dsuu\rangle - 4|sduu\rangle + 5|sudu\rangle \\
+5|usdu\rangle + 2|uusd\rangle - |suud\rangle - |dusu\rangle - |usud\rangle - |udsu\rangle \right],
\]

(13)

\[
|31\rangle_F'' = \frac{1}{\sqrt{48}} \left[ -3|duus\rangle + 3|udsu\rangle - 3|dusu\rangle + 3|usdu\rangle - 2|dsuu\rangle + 2|sduu\rangle \\
-|sudu\rangle + |usdu\rangle - |suud\rangle + |usud\rangle \right].
\]

(14)
The color symmetry of all the \textit{uuds} configurations is limited to be \([211]_C\) in order to combine with the \(\bar{s}\) antiquark to form a color singlet (cf.\([14]\)). The Pauli principle requires that corresponding orbital-flavor-spin states have the mixed symmetry \([31]_{XFS}\) so as to combine with the mixed color symmetry state \([211]_C\) to form the required completely antisymmetric 4-quark state \([1111]\). Since the intrinsic parity is positive for a quark and negative for an anti-quark, the \textit{uuds}\(\bar{s}\) component in a proton, which has positive parity, requires that either the \(\bar{s}\) is in the P-state with the \textit{uuds} system in the ground state with spatial symmetry \([4]_X\) or that one of the quarks is in the P-state so that the \textit{uuds} system has mixed spatial symmetry \([31]_X\).

There are several configurations of the \textit{uuds}\(\bar{s}\) system that have positive parity, isospin \(1/2\), spin \(1/2\) and one unit of orbital angular momentum. The spin-dependent hyperfine interaction between the quarks splits these states, so that the configurations with the lowest energy may be expected to be those with highest probability for admixture in the proton.

In most models for the hyperfine interaction between quarks in the baryon is spin dependent. In the common colormagnetic hyperfine interaction model, the spin dependence is such that the spin singlet state has lower energy than the spin triplet state \([23]\). This is also the case for the instanton induced interaction model \([24]\). Finally, the schematic flavor and spin dependent interaction model:

\[
H' = -C \sum_{i<j} \vec{\lambda}_i \cdot \vec{\lambda}_j \vec{\sigma}_i \cdot \vec{\sigma}_j ,
\]

which gives the qualitative description with correct ordering of the states in low lying part of the baryon spectrum in all flavor sectors \([26, 27]\), implies that antisymmetric flavor and spin configurations have the lowest energy. In this interaction \(C\) is a constant that represents an average of the spin-spin part of the interaction expected to be mediated by pseudoscalar meson exchange \([25]\). Phenomenologically \(C \sim 20 - 30\) MeV.

In next two sections, the strangeness spin \(\Delta_s\) and magnetic moment \(\mu_s\) are calculated for all possible configurations of the \textit{uuds}\(\bar{s}\) system that have the quantum numbers of the proton.
III. THE CONFIGURATIONS WITH THE \textit{uuds} SYSTEM IN ITS GROUND STATE

In the configurations where the \textit{uuds} quarks are in their ground state, the spatial state has complete symmetry \([4]_X\), and accordingly their flavor-spin state has to have the mixed symmetry \([31]_{FS}\). All the different flavor and spin state symmetry configurations that can combine to the required \([31]_{FS}\) mixed symmetry combination have been listed in Table II [28]. In the table the matrix elements of the quadratic Casimir operator of \(SU(6)\), \(C^2_2\), multiplied by the constant \(C\) in the flavor-spin interaction (15) are also listed so as to give an indication of the energy splitting between these configurations. The requirement of positive parity requires that for these configurations the strange antiquark in the \(uuds\) system has to be in the \(P\) state.

TABLE I: Flavor and spin configurations of the \(uuds\) quark states in the ground state [28] and the corresponding operator matrix elements.

| \(uuds\) ground state | \(\bar{s}_s\) | \(\Delta_s(P_{ss})\) | \(\mu_s (\frac{m_s}{m_s} P_{ss})\) | \(-C C^2_2\) |
|------------------------|-----------|-----------------|-----------------|-----------------|
| \([31]_{FS}[211]_F[22]_S\) | -         | \(-1/3\)         | \(-1/3\)         | \(-16C\)         |
| \([31]_{FS}[211]_F[31]_S\) | \(13/36\) | \(85/216\)       | \(-95/216\)      | \(-13\frac{1}{3}C\) |
| \([31]_{FS}[22]_F[31]_S\)  | \(1/2\)   | \(5/12\)         | \(-5/12\)        | \(-9\frac{1}{3}C\) |
| \([31]_{FS}[31]_F[22]_S\)  | -         | \(-1/3\)         | \(-1/3\)         | \(-8C\)          |
| \([31]_{FS}[31]_F[31]_S\)  | \(65/108\) | \(281/648\)      | \(-259/648\)     | \(-5\frac{1}{3}C\) |
| \([31]_{FS}[31]_F[4]_S\)   | -         | \(1/6\)          | \(7/6\)          | \(0\)            |

The wave functions of the proton state with spin \(+1/2\) that have any one of these symmetries may be written in the general form:

\[
|p, \frac{1}{2}\rangle = A_{ss} \sum_{abc} \sum_{s,m,m',s'} C_{1s,im}^{[4]} C_{1m',s'}^{[14]} C_{[31]_a[211]_a}^{[31]_a} C_{[F][S]_c}[F]_b[S]_c[211]_C,\bar{Y}_{im'}\bar{X}_{s,s'}\varphi(\{\vec{r}_i\}). \tag{16}
\]

Here \([F]_b, [S]_c\) and \([211]_C,\bar{a}\) represent the flavor, spin and color state wave functions, denoted by their Weyl tableaux respectively. The sums over \(a, b\) and \(c\) run over the configurations of the \([31][F][S]\) representations of \(S_4\) permutation group for which the corresponding Clebsch-Gordan coefficients \(C^{a}_{b,c}\) do not vanish. The value of the first of these CG coefficients are
$C^{[14]}_{[31]a[21]a} = \pm \frac{1}{\sqrt{3}} \, [21]$. Finally $\bar{Y}_{im\nu}$ and $\bar{\chi}_{s'}$ denote the orbital and spin states of the anti-strange quark respectively. In Eq. (16) $A_{ss}$ is the amplitude of the $s\bar{s}$ component in the proton and $\varphi(\{\vec{r}_i\})$ is a symmetric function of the coordinates of the $uud\bar{s}$.

The wave functions of the $uuds$ subsystem may be organized in groups according to their spin states. Consider first the states which have the spin symmetry $[22]_S$. To these belong the configuration $[211]_F[22]_S$, which is expected to have the lowest energy among all the configurations with the mixed flavor-spin symmetry $[31]_{FS}$. As the total spin of the $uuds$ system with this symmetry is $S = 0$, and the angular momentum space is isotropic, it gives no contribution to $\mu_s$ and $\sigma_s$. In these configurations only the $\bar{s}$ quark contributes to $\mu_s$ and $\Delta_s$ (and $G_A^s$):

\[ \mu_s = -\frac{1}{3} \frac{e}{2m_s} P_{s\bar{s}}, \quad (17) \]

\[ \Delta_s = -\frac{1}{3} P_{s\bar{s}}. \quad (18) \]

Here $P_{s\bar{s}}$ is the probability of this configuration. In units of the nuclear magnetons $\mu_s$ takes the value:

\[ \mu_s = -\frac{1}{3} \frac{m_p}{m_s} P_{s\bar{s}}. \quad (19) \]

In this configuration the numerical value of $\mu_s$ is negative and $\sim 2$ times $\Delta_s$, as $m_p \approx 2m_s$.

The spin of the $uuds$ system in the states which have the spin symmetry $[31]_S$ is $S = 1$. If we neglect the interaction of quarks, then the wave function of the proton in the angular momentum space may be written in the general form:

\[ |p, \frac{1}{2} \rangle = \frac{A_{s\bar{s}}}{\sqrt{2}} \left( C^{\frac{1}{2}, \frac{1}{2}}_{11,10} \left[ C^{11}_{11,10} |11\rangle_{A_S}|10\rangle_{B_X} + C^{11}_{10,11} |10\rangle_{A_S}|11\rangle_{B_X} \right] |\frac{1}{2} \rangle_{BS} \right. \]

\[ + C^{\frac{1}{2}, \frac{1}{2}}_{10,11} \left[ C^{10}_{11,1-1} |11\rangle_{A_S}|1-1\rangle_{B_X} + C^{10}_{1-1,11} |1-1\rangle_{A_S}|11\rangle_{B_X} \right] |\frac{1}{2} \rangle_{BS} \right) + \]

\[ C^{\frac{1}{2}, \frac{1}{2}}_{00,1-1} \left[ C^{00}_{11,1-1} |11\rangle_{A_S}|1-1\rangle_{B_X} + C^{00}_{1-1,11} |1-1\rangle_{A_S}|11\rangle_{B_X} \right] |\frac{1}{2} \rangle_{BS}. \quad (20) \]

Here for the sake of abbreviation the subindices $A_S$ and $B_X$ represent the spin state of the $uuds$ system and the orbital state of the $\bar{s}$ quark respectively. Explicit evaluation of the matrix elements with these wave functions for $\mu_s$ and $\Delta_s$ leads to the results:

\[ \mu_s = -\frac{1}{2} \frac{m_p}{m_s} \left( 1 - \frac{1}{3} \bar{\sigma}_s \right) P_{s\bar{s}}. \quad (21) \]

\[ \Delta_s = \frac{1}{3} \left( 1 + \frac{1}{2} \bar{\sigma}_s \right) P_{s\bar{s}}. \quad (22) \]
Here the $\bar{\sigma}_s$ is the expectation value of the z-component of the spin of the $s$ quark in the configuration where $s_z = 1$. (Note that Eq. (21) here corrects Eq.(4) in [12]). The numerical value of $\bar{\sigma}_s$ depends on the detailed configuration as shown below. The results in all cases satisfy the inequality $\bar{\sigma}_s < 1$ as shown in Table II. This result implies that $\mu_s$ in all these configuration is negative and that $\Delta_s$ is positive, in contradiction with experiment.

The final possibility when the spatial state of the $uuds$ system is that the spin state of the $uuds$ system is completely symmetric: $[4]_S$. In this case it has spin $S = 2$ and as a consequence the total angular momentum of the $\bar{s}$ quark has to be $j = \frac{3}{2}$ in order to combine with the $uuds$ system to form the proton state with spin $+1/2$. The wave function then may be expressed in a way analogous to Eq.(20). The relevant matrix elements for these configurations are found to be:

$$\mu_s = \frac{7m_p}{6m_s} P_{ss}, \quad \Delta_s = \frac{1}{6} P_{s \bar{s}}.$$

(23)

(24)

This configuration thus yields positive values for both $\mu_s$ and $\Delta_s$ as well as $G_s^A$. This configuration is very unlikely to have a large probability of the proton, as it is expected to have an energy of 100 - 150 MeV above all the other configurations with the mixed flavor-spin symmetry $[31]_F S$.

The (tedious) calculation of the average spin value $\bar{\sigma}_s$ in Eq.(22) may illustrated by the following explicit calculation for the case of the configuration $[31]_F S [22]_F [31]_S$. The results for this and the other configurations are listed in Table II.

There are three combinations of the mixed symmetry states $[22]_F$ and $[31]_S$ that combine to the mixed symmetry state $[31]_F S$. The explicit expressions for these are the following [21]:

$$|[31]_{FS_1}\rangle = \frac{1}{\sqrt{2}} \left( |[22]_{F_1}\rangle |[31]_S\rangle + |[22]_{F_2}\rangle |[31]_S\rangle \right),$$

(25)

$$|[31]_{FS_2}\rangle = \frac{1}{2} \left( \sqrt{2} |[22]_{F_1}\rangle |[31]_S\rangle + |[22]_{F_2}\rangle |[31]_S\rangle - |[22]_{F_2}\rangle |[31]_S\rangle - |[22]_{F_1}\rangle |[31]_S\rangle \right),$$

(26)

$$|[31]_{FS_3}\rangle = \frac{1}{2} \left( \sqrt{2} |[22]_{F_2}\rangle |[31]_S\rangle - |[22]_{F_2}\rangle |[31]_S\rangle - |[22]_{F_1}\rangle |[31]_S\rangle - |[22]_{F_1}\rangle |[31]_S\rangle \right).$$

(27)

where the $[22]_F$ flavor functions are given by Eqs.(7,8) and the three spin wave functions are

$$|[31]_S\rangle = \frac{1}{\sqrt{12}} [\uparrow \uparrow \uparrow \downarrow - |\uparrow \uparrow \downarrow \rangle - |\uparrow \downarrow \uparrow \rangle - |\downarrow \uparrow \uparrow \rangle],$$

(28)
\[ |[31]s_2 > = \frac{1}{\sqrt{6}} [2| \uparrow \uparrow \uparrow \rangle - | \uparrow \downarrow \uparrow \rangle - | \downarrow \uparrow \uparrow \rangle] ; \]  
\[ |[31]s_3 > = \frac{1}{\sqrt{2}} [ | \uparrow \downarrow \uparrow \uparrow \rangle - | \downarrow \uparrow \uparrow \uparrow \rangle]. \]  

From Eqs. (25-27) one obtains

\[ \bar{\sigma}_s = \frac{1}{3} (\bar{\sigma}_{FS_1} + \bar{\sigma}_{FS_2} + \bar{\sigma}_{FS_3}) . \]  

It then follows from Eqs. (28-30) that

\[ \bar{\sigma}_s = \frac{1}{2} , \]  

and at last, from Eqs. (21,22),

\[ \mu_s = - \frac{5}{12} \frac{m_p}{m_s} P_{s\bar{s}} , \]  

\[ \Delta_s = \frac{5}{12} P_{s\bar{s}} . \]  

The matrix elements needed for the values of \( \Delta_s \) and \( \mu_s \) in all other configurations may be calculated by the same method.

**IV. THE CONFIGURATIONS WITH THE \( \bar{s} \) IN ITS GROUND STATE**

In the configurations, where the \( \bar{s} \) antiquark is in the ground state, the lowest possible orbital configuration allowed by the requirement of positive parity for the \( uuds \) state is that with orbital angular momentum of \( L = 1 \). The corresponding spatial state has the mixed symmetry labeled by the Weyl tableau \([31]_X\). The possible symmetries of the flavor-spin state are then either complete symmetry, \([4]_{FS}\), or the mixed symmetries \([31]_{FS}, [22]_{FS}\) and \([211]_{FS}\). All the flavor and spin symmetry configurations that can combine to these configurations, and which can be a component of the proton, are listed in Table II. The wave functions of the proton with spin +1/2 may for all of these symmetry configurations be expressed in the general form:

\[ |p, +\frac{1}{2} \rangle = \sum_{abcde} \sum_{M,s_s} \sum_{M_s,s_s'} C_{JM,s_s}^{[4]} C_{IM,s_s}^{[14]} C_{[31]a[211]b}^{[31]a} C_{[31]a[FS]c}^{[31]a} C_{[FS]d[S]e}^{[FS]d[S]e} \]  

\[ [31]_X, m(b)[F]_{d[S]s_s(e)}[211]_c(a) \chi_{s_s'}(\{r_i\}). \]  

Here \( J \) is the total angular momentum of the \( uuds \) system \( M \) the corresponding \( z \)-component. The orbital angular momentum of these \( uuds \) states is \( L = 1 \), with the \( z \)-component \( m \).
These configurations may also be organized in groups according to their spin symmetry. The method described explicitly in the previous section may then be applied to evaluate the matrix elements that are required for $\mu_s$ and $\Delta_s$.

Of these configurations, the configuration $[4]_{FS}[22]_{F}[22]_S$ is expected to have the lowest energy. For this and all the configurations that have the mixed spin symmetry $[22]_S$ the desired matrix elements for $\mu_s$ and $\Delta_s$ are:

$$\mu_s = \frac{m_p}{3m_s}(1 + 2\bar{l}_s)P_{ss}, \quad (36)$$

### Table II: Flavor and spin configurations of the $uuds$ quark states in the first orbitally excited state $[28]$ with total angular momentum $J = 1$ and the corresponding operator matrix elements.

| $uuds$ p-state | $\bar{l}_s$ | $\bar{\sigma}_s$ | $\Delta_s(P_{ss})$ | $\mu_s(m_p/m_s \ P_{ss})$ | $-CC_2^{(6)}$ |
|----------------|-----------|----------------|-------------------|------------------------|-------------|
| $[4]_{FS}[22]_{F}[22]_S$ | 1/4 | - | -1/3 | 1/2 | -28C |
| $[4]_{FS}[31]_{F}[31]_S$ | 1/4 | 7/9 | -2/27 | 73/108 | -21\(\frac{2}{3}\)C |
| $[31]_{FS}[211]_{F}[22]_S$ | 13/48 | - | -1/3 | 37/72 | -16C |
| $[31]_{FS}[211]_{F}[31]_S$ | 119/432 | 13/36 | -23/108 | 707/1296 | -13\(\frac{2}{3}\)C |
| $[31]_{FS}[22]_{F}[31]_S$ | 1/4 | 1/2 | -1/6 | 7/12 | -7\(\frac{2}{3}\)C |
| $[31]_{FS}[31]_{F}[22]_S$ | 13/48 | - | -1/3 | 37/72 | -8C |
| $[22]_{FS}[211]_{F}[31]_S$ | 1/4 | 5/12 | -7/36 | 5/9 | -5\(\frac{1}{3}\) |
| $[31]_{FS}[31]_{F}[31]_S$ | 295/1296 | 65/108 | -43/324 | 2371/3888 | -5\(\frac{2}{3}\) |
| $[22]_{FS}[22]_{F}[22]_S$ | 1/4 | - | -1/3 | 1/2 | -4C |
| $[211]_{FS}[211]_{F}[22]_S$ | 11/48 | - | -1/3 | 35/72 | 0 |
| $[31]_{FS}[31]_{F}[4]_S$ | 43/216 | - | 1/6 | 497/648 | 0 |
| $[211]_{FS}[211]_{F}[31]_S$ | 119/432 | 65/108 | -43/324 | 811/1296 | 2\(\frac{1}{3}\)C |
| $[22]_{FS}[31]_{F}[31]_S$ | 13/54 | 5/12 | -7/36 | 251/324 | 2\(\frac{2}{3}\)C |
| $[22]_{FS}[22]_{F}[4]_S$ | 1/4 | - | 1/6 | 3/4 | 4C |
| $[211]_{FS}[22]_{F}[31]_S$ | 1/4 | 1/2 | -1/6 | 7/12 | 6\(\frac{2}{3}\) |
| $[211]_{FS}[211]_{F}[4]_S$ | 23/72 | - | 1/6 | 157/216 | 8C |
| $[211]_{FS}[31]_{F}[22]_S$ | 11/48 | - | -1/3 | 35/72 | 8C |
| $[211]_{FS}[31]_{F}[31]_S$ | 31/144 | 13/36 | -23/108 | 227/432 | 10\(\frac{2}{3}\)C |
\[
\Delta_s = -\frac{1}{3} P_{s\bar{s}}. \tag{37}
\]

Here the \( \bar{l}_s \) is the average value of the z-component of the orbital angular momentum of the \( s \) quark in the \( uuds \) system for \( m = 1 \). The numerical value for \( \bar{l}_s \) depends on the detailed configuration and may be calculated by the same method as that described above for the calculation of \( \bar{\sigma}_s \) in the previous section. The results of \( \bar{l}_s \) for various \([31]_X\) configurations with \( uuds \) total angular momentum \( J = 1 \) are listed in Table II along with the corresponding values of \( \bar{\sigma}_s \). The values for \( \bar{l}_s \) are in every case smaller than \( 1/2 \): \( \bar{l}_s < 1/2 \). Note that the values of \( \bar{l}_s \) and \( \bar{\sigma}_s \) given in Ref.\([12]\) for the mixed \([31]_F\) flavor symmetry are not correct due to the neglect of the isospin decomposition and are corrected in Table II.

In the case of the lowest energy configuration \([4]_{FS}[22]_F[22]_S\) \( \bar{l}_s = 1/4 \) and consequently

\[
\mu_s = \frac{m_p}{2m_s} P_{s\bar{s}}. \tag{38}
\]

The value of \( \mu_s \) in this configuration is consequently positive, and since \( m_p/m_s \simeq 2 \), it is approximately equal to the probability of that configuration in the proton. Moreover for this configuration \( \mu_s \approx -3\Delta_s \), a relation that agrees with the present experimental values within their uncertainty range.

Because of its low energy, this is the most likely \( s\bar{s} \) component in the proton. The \([4]_{FS}[22]_F[22]_S\) \( uuds \) wave function has the simple form:

\[
|[4]_{FS}\rangle = \frac{1}{\sqrt{2}} \{[22]_{F_1}[22]_{S_1} + [22]_{F_2}[22]_{S_2} \}. \tag{39}
\]

The explicit form for the two flavor components \([22]_{F_1}\) and \([22]_{F_2}\) are given in Eqs.\([4,8]\). The two corresponding spin functions are readily constructed by the substitutions \( u \leftrightarrow \uparrow \) and \( d, s \leftrightarrow \downarrow \) with an additional \( 1/\sqrt{2} \) in the normalization factor. To complete the wave function for this configuration one needs the explicit antisymmetric color-space wave function:

\[
|[1111]_{CX}[211]_C[31]_X;m\rangle = \sum_{a=1}^{3} C^{[1]}_{[a]} [211]_a [31]_a [1111]_a C^{[3]}_{X}. \tag{40}
\]

As the operators that are considered here do not depend on color, the explicit color wave functions with the mixed symmetry \([211]_C\) are not required. The explicit spatial wave functions may be constructed by reference to Eqs.\([28,30]\), with the substitution of ground state wave functions for the 3 constituents denoted \( \uparrow \) and a \( P- \)state wave function, multiplied by the spherical harmonic \( Y_{1m} \), for the constituent denoted \( \downarrow \).
For the states which have spin symmetry $[31]_S$ the total angular momentum of the $uuds$ system $J$ may take the values 0 and 1. For $J = 0$ the results are:

\[ \mu_s = -\frac{m_p}{m_s} P_{s\bar{s}}, \]  
\[ \Delta_s = P_{s\bar{s}}. \]  

These are the only configurations with the $uuds$ system in an orbitally excited state, which lead to negative values for the strangeness magnetic moment. As the lowest one of these configurations lies $\sim 140 - 200$ MeV above the configuration with $[4]_{FS}[22]_F[22]_S$ symmetry, it is unlikely to have a large probability as a component of the proton.

In the case when $J = 1$ the corresponding results are

\[ \mu_s = \frac{m_p}{3m_s} (1 + \bar{l}_s + \bar{\sigma}_s) P_{s\bar{s}}, \]  
\[ \Delta_s = -\frac{1}{3}(1 - \bar{\sigma}_s) P_{s\bar{s}}. \]  

Finally for the states which have spin symmetry $[4]_S$ the results are:

\[ \mu_s = \frac{1}{3} \frac{m_p}{m_s} \left( \frac{5}{2} - \bar{l}_s \right) P_{s\bar{s}}, \]  
\[ \Delta_s = \frac{1}{6} P_{s\bar{s}}. \]  

As the strangeness contribution to the proton spin is positive for these configurations in apparent conflict with the experimental situation, as these are expected to require a relatively high energy of excitation, they are not likely to have a large probability in the proton.

V. THE PROBABILITY OF THE $uuds\bar{s}$ COMPONENT

Above the possibility for transition matrix elements between the $uuds\bar{s}$ and the $uud$ components of the proton was not considered, as only diagonal matrix elements in configurations with $s\bar{s}$ components were calculated. The calculation of such transition matrix elements demands an explicit model for the spatial wave functions for determination of the overlap. These transition matrix elements modify the proportionality coefficients between the strangeness magnetic moment and the probability of the $s\bar{s}$ components in the proton. Referring to the analogous situation in the case of decay of the $\Delta(1232)$ considered in ref. [31], these contributions are expected to allow a smaller values for $P_{s\bar{s}}$ than eg. Eq. [38] with the
present empirical values for $\mu_s$. As an example in the case of the $[31]_X[4]_FS[22]_F[22]_S$ configuration, the effect of the transition matrix elements would lead to modification of the expression (48) for $\mu_s$ by an additional factor:

$$\mu_s = \frac{m_p}{2m_s}(1 + \delta)P_{s\bar{s}}. \quad (47)$$

Here the relative modification caused by transition matrix elements is contained in the term $\delta$. Because the $uuuds$ configuration is a $P$--state, this term would have the general form:

$$\delta \sim C_0m_s\sqrt{\langle r^2 \rangle} \sqrt{\frac{P_{uuud}}{P_{s\bar{s}}}}, \quad (48)$$

where $P_{uuud} + P_{s\bar{s}}$. Here $C_0$ is a factor that takes into account the overlap between the wave function for the $uuud\gamma$ state and the $uuuds\bar{s}$ component of the proton, and $\langle r^2 \rangle$ is the mean square radius of the $uuuds\bar{s}$ component. The mass factor $m_s$ arises from the $s\bar{s} \to \gamma$ vertex. Provided that these wave functions have the same phase, it follows from (47) that the empirical value for $\mu_s$ sets an upper limit for $P_{s\bar{s}}$. As an example the harmonic oscillator model employed in ref. [31], in which $\sqrt{\langle r^2 \rangle} = 1/\omega$, the value for the coefficient $C_0$ is $C_0 = 3/8 \approx 0.37$. Since $2m_s \sim m_p$ and in the oscillator model $m_s\sqrt{\langle r^2 \rangle} = m_s/\omega \sim 1...1.5$ the following approximate relation emerges for $\mu_s$ in the $[31]_X[4]_FS[22]_F[22]_S$ configuration:

$$\mu_s \sim P_{s\bar{s}} + (0.37...0.55)\sqrt{P_{s\bar{s}} - P_{s\bar{s}}^2}. \quad (49)$$

If this equation is solved for the $s\bar{s}$ configuration probability $P_{s\bar{s}}$ with the mean result for $\mu_s$ given by the SAMPLE experiment ($\mu_s = 0.37 \pm 0.2 \pm 0.26 \pm 0.07$) [1] as input one obtains $P_{s\bar{s}} \approx 0.17...0.22$. By (47) this probability for the $s\bar{s}$ configuration gives $\Delta_s = -0.06...-0.07$, which values falls well within the range of values for this quantity ($-0.10 \pm 0.06$) that is determined from the recently enlarged set of inclusive and semi-inclusive polarized deep inelastic scattering data [15]. Note that in the case of $\Delta_s$ or $G^s_A$, the contribution from transition matrix elements relative to the diagonal matrix elements is much smaller than in the case of $\mu_s$.

The inclusion of the $[31]_X[4]_FS[22]_F[22]_S$ type of pentaquark components not only reproduces well the strangeness magnetic moment and spin of the proton, but also is consistent with the observed excess of $\bar{d}$ over $\bar{u}$ [32] and the quark spin contribution [15] in the proton. In this configuration, only $udud\bar{d}$ and $udus\bar{s}$ pentaquark components are allowed for the
proton but no $uudd\bar{u}$ component. The quark wave function for the proton may then be expressed as:

$$|p> = A_{3q}|uud> + A_{dd}|[ud][ud]\bar{d}> + A_{ss}|[ud][us]\bar{s}> ,$$

with the normalization condition $|A_{3q}|^2 + |A_{dd}|^2 + |A_{ss}|^2 = 1$. To reproduce the observed light flavor sea quark asymmetry in the proton, $\bar{d} - \bar{u} = 0.12$ [32], one then needs $P_{dd} \equiv |A_{dd}|^2 = 12\%$, while to reproduce the strangeness spin of the proton, $\Delta_s = -0.10 \pm 0.06$ [15], one needs $P_{ss} = (12 - 48)\%$. With more than 24\% of this kind of pentaquark component in the proton, this can reproduce the observed $u$-quark spin contribution in the proton $\Delta_u = 0.85 \pm 0.17$ [15] as well, since the pentaquark components give no contribution to the $\Delta_u$ and $\Delta_u = \frac{1}{3}|A_{3q}|^2$. Instead they give a contribution: $\Delta L_q = \frac{1}{3}(P_{dd} + P_{ss})$ of the proton spin through the orbital angular momentum of quarks. In Ref. [15], the spin contributions from both $\bar{d}$ and $\bar{s}$ antiquarks were found to be negative, while that from $\bar{u}$ is very uncertain even as to its sign and could be far less polarized than $\bar{d}$ and $\bar{s}$. The pentaquark configuration considered here gives such quark spin contributions, with $-\frac{1}{3}P_{dd}$ from $\bar{d}$ and $-\frac{1}{3}P_{ss}$ from $\bar{s}$ and zero from $\bar{u}$ antiquarks. The total $d$-quark spin contribution is predicted to be $\Delta_d = -\frac{1}{3}(1 - P_{ss})$ which is only slightly smaller than the lower end of the observed value $\Delta_d = -(0.33 \sim 0.56)$. In Ref. [15], it was also found that the available data do not require the gluon to be polarized.

VI. DISCUSSION

The complete analysis given above of all the positive parity configurations of the form $uudss\bar{s}$ with spin $1/2$ and an at most one unit of orbital angular momentum reveals that the present empirical signs for $\mu_s$ and $\Delta_s$ imply that the $uuds$ subsystem has to be orbitally excited and that the $s$ antiquark is in its ground state. The configurations of the $uudss\bar{s}$ system, which can agree with the empirical indications that $\mu_s$ is positive, and that $\Delta_s$ is negative and smaller in magnitude than $\mu_s$, are the configurations with $[31]_X[22]_S$ and $[31]_X[31]_S$ that have $J = 1$. These configurations do not include the conventional $K\Lambda^0$ and $K\Sigma^0$ configurations, which yield negative signs for $\mu_s$ [17, 18, 19, 20].

The configuration $[31]_X[4]_F[22]_F[22]_S$ stands out by the fact that its energy is some 140-200 MeV lower than any other in the flavor-spin interaction model of Eq. (15). Moreover its energy is more than 240 MeV lower than the lowest energy configuration with the $s$ in the $P-$state, which would correspond to the $K\Lambda^0$ loop fluctuations. The lower excitation energy
should increase the probability of this configuration as a component in the proton. Two recent diquark cluster models \cite{29,30} proposed for the tentative $\theta^+$-pentaquark correspond to similar $uuds$ configurations as $[31]X[4]_{FS}[22]_F[22]_S$ for the $uuds\bar{s}$ component in the proton with the common feature that the $\bar{s}$ quark in its ground state, and hence similar values for the strangeness spin and magnetic moment for the proton \cite{12}.

Another interesting point worth noting is that a negative value for the strange electric form factor $G_E^s$ is hinted at by the data \cite{4}. This indicates that the strange quarks have a larger rms radius than the anti-strange quarks in the proton. The $uuds\bar{s}$ configurations that give the empirical signs for the strangeness spin and magnetic moment do have that feature, as the strange antiquark is in the ground the $uuds$ component is in the orbitally excited state.

The possible smaller than expected role of the long range $K\Lambda^0$ and $K\Sigma^0$ fluctuations, which lead to negative values for $\mu_s$, has recently been analyzed in Ref.\cite{33}. The conclusion drawn in that work is that the main contribution to $\mu_s$ from these “chiral loops” arise from their high momentum components, the determination of which a priori falls outside of the convergence range of the effective chiral field theory approach. As a consequence the loop expansion as calculated by effective field theory methods may not give realistic results. Recent numerical QCD lattice results that have been obtained in the quenched approximation have given both positive \cite{10} and negative results for $\mu_s$ \cite{34} and therefore have not settled the issue of the sign of $\mu_s$.

The diquark cluster configuration or similar $[31]X[4]_{FS}[22]_F[22]_S$ $uuds$ configurations for the $uuds\bar{s}$ system all lead to positive values for the strangeness magnetic moment, negative values for the strangeness spin or axial form factor and negative strangeness electric form factor of the proton at low values of momentum transfer. Moreover they also give a natural explanation for the mass ordering of the $N^*(1440)1/2^+$, $N^*(1535)1/2^-$ and $\Lambda^*(1405)1/2^-$ resonances by admixture of large pentaquark components $[28]$, which is otherwise very puzzling in the conventional $3q$ constituent quark model. In the diquark cluster pentaquark configuration $[29,35]$, the $N^*(1440)1/2^+$ is composed of the $[ud][ud]\bar{d}$ with two $[ud]$ scalar diquarks in relative P-wave, the $N^*(1535)1/2^-$ is composed of the $[ud][us]\bar{s}$ with diquarks in the ground state, and $\Lambda^*(1405)1/2^-$ is composed of the $[ud][us]\bar{u}$ and $[ud][ds]\bar{d}$. The large admixture of $[ud][us]\bar{s}$ in the $N^*(1535)1/2^-$ resonance makes it heavier than the $N^*(1440)1/2^+$ and the $\Lambda^*(1405)1/2^-$ along with a large coupling to the channels involving strangeness,
such as $N\eta$ and $K\Lambda$.

All facts and discussions given above suggest that there are significant $qqq\bar{q}$ components in baryons and they may be mainly in colored quark cluster configurations rather than in “meson cloud” configurations.

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[1] D. T. Spayde et al., Phys. Lett. **B583**, 79 (2004)
[2] K. Aniol et al., Phys. Rev. **C69**, 065501 (2004)
[3] F. Maas et al., Phys. Rev. Lett. **94**, 152001 (2005)
[4] D. S. Armstrong et al., Phys. Rev. Lett. **95**, 092001 (2005)
[5] D. H. Beck and R. D. McKeown, Ann. Rev. Nucl. Part. Sci. **51**, 189 (2001)
[6] H. Weigel et al., Phys. Lett. **B353**, 20 (1995)
[7] P. Geiger and N. Isgur, Phys. Rev. **D55**, 299 (1997)
[8] S.-T. Hong, B.-Y. Park and D. P. Min, Phys. Lett. **B414**, 229 (1997)
[9] V. Lyubovitskij et al., Phys. Rev. C66, 055204 (2002)
[10] R. Lewis, W. Wilcox and R. M. Woloshyn., Phys. Rev. **D67**, 013003 (2003)
[11] A. Silva, H. C. Kim and K. Goeke, Eur. Phys. **A22**, 481 (2004)
[12] B. S. Zou and D. O. Riska, Phys. Rev. Lett **95**, 072001 (2005)
[13] A. Airapetian et al. (HERMES collaboration), Phys. Rev. **D71**, 012003 (2005)
[14] B. W. Filippone and X.-D. Ji, Adv. Nucl. Phys. **26**, 1 (2001)
[15] D. de Florian, G. A. Navarro and R. Sassot, Phys. Rev. **D71**, 094018 (2005).
[16] S. F. Pate, Phys. Rev. Lett. **92**, 082002 (2004)
[17] M. Musolf and M. Burkhardt, Z. Phys. **C61**, 433 (1984)
[18] H. Forkel, F. S. Navarra and M. Nielsen, Phys. Rev. C61, 055206 (2000)
[19] L. Hannelius and D. O. Riska, Phys. Rev. C62, 045204 (2000)
[20] X.S.Chen et al., Phys. Rev. C70, 015201 (2004)
[21] J. Q. Chen, Group Representation Theory for Physicists, World Scientific, Singapore (1989)
[22] Zhong-Qi Ma, Group Theory in Physics (in Chinese) Science Press, Beijing, (1998).
[23] S. Capstick and N. Isgur, Phys. Rev. D34, 2809 (1986)
[24] U. Loring, B. Metsch and H. Petry, Eur. Phys. J. A10, 395 (2001), Eur. Phys. J. A10, 447 (2001)
[25] D. O. Riska and G. E. Brown, Nucl. Phys. A653, 251 (1999)
[26] L. Ya. Glozman and D. O. Riska, Phys. Rep. 268, 263 (1996)
[27] L. Ya. Glozman and D. O. Riska, Nucl. Phys. A603, 326 (1996); Nucl. Phys. A620, 510 (1997)
[28] C. Helminen and D. O. Riska, Nucl. Phys. A699, 624 (2002)
[29] R. Jaffe and F. Wilczek, Phys. Rev. Lett. 91, 232003 (2003)
[30] E. Shuryak and I. Zahed, Phys. Lett. B589, 21 (2004)
[31] Q. B. Li and D. O. Riska, nucl-th/0507008
[32] G.T.Garvey, J.C.Peng, Prog. Part. Nucl. Phys. 47, 203 (2001), and references therein.
[33] J. Donoghue et al. hep-ph/0412035
[34] D. Leinweber et al., Phys. Rev. Lett. 94, 212001 (2005)
[35] A.Zhang et al., hep-ph/0403210, High Energy Phys. Nucl. Phys. 29, 250 (2005).
[36] B. C. Liu and B. S. Zou, nucl-th/0503069

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