Cosmology with non-minimally coupled Yang–Mills field

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We discuss cosmological model with homogeneous and isotropic Yang-Mills field non-minimally coupled to gravity through an effective mass term. In this model conformal symmetry is violated which leads to possibility of inflationary expansion. Parameters of non-minimal coupling have relatively “natural” values in the regime of sufficiently long acceleration stage.

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I. INTRODUCTION

Vector fields were suggested in cosmology as alternative to scalar fields as inflation and dark energy drivers [1–4]. Apart from being physically appealing as well-understood and common component of all existing theories, they may be useful, in particular, in generating appropriate curvature perturbations [5–7]. Especially attractive seem to be non-Abelian models, which admit magnetic type field configurations compatible with isotropy and homogeneity of space-time [8–11] and may therefore be used in the standard Friedmann-Robertson-Walker setting without averaging needed in the case of vector singlet to avoid anisotropy. If their dynamics is ruled by the standard Yang-Mills (YM) conformally invariant action, the corresponding equation of state is that of the hot universe [8], but violation of conformal symmetry may lead to different equations of state. In particular, the Born-Infeld lagrangian generates an equation of state which interpolates between the hot regime at low densities and a zero acceleration regime at high density [12–15]. Stronger violation of conformal symmetry in the purely YM sector may produce accelerating expansion of inflationary dark energy type [16] exhibiting typically a finite acceleration period. Various realizations of this scenario were suggested [17–23] with different motivation. Cosmological perturbations in the YM cosmology
were studied in \[25–27\]. It is also worth noting that if one treats inflation as associated with the Higgs sector of some gauge theory, the excitation of the Yang-Mills component becomes inevitable and modifies inflationary regime in a non-trivial way \[28–30\].

Here we would like to investigate cosmological dynamics of Yang-Mills filed non-minimally coupled to gravity via curvature-dependent mass term. Earlier proposals of non-minimally coupled Yang-Mills in cosmology include \[31–34\], but our approach here is different. To introduce non-minimal coupling one usually probe various combination the Riemann tensor with the field strength and the vector potential. In the latter case the gauge invariance of the theory is destroyed at classical level, but one consider this just as the simplified for cosmological purposes effect of spontaneous symmetry breaking mechanism of the gauge theory giving mass to $W$-boson.

II. THE MODEL

We introduce the non-minimal coupling of the YM potential $A^a_\mu$ to the Ricci tensor generically depending on two constants together with the ordinary mass term. This form is motivated by absence of higher derivatives in the resulting equations, so the theory seems to be ghost-free. With this in mind we choose as the lagrangian the sum of the Einstein-Hilbert term, the standard YM term and the non-minimal term $L = L_{EH} + L_{YM} + L_{NM}$, namely:

$$L = -\frac{M_{Pl}^2}{2}R - \frac{1}{4} F_\mu^a F^{a\mu} + \left(\nu R / 4 - m^2 / 2\right) A_\mu^a A^{a\mu} + (\lambda - \nu) S_{\mu\nu} A^{a\mu} A^{a\nu}.$$  \hspace{1cm} (1)

Here $M_{Pl} = 1/\sqrt{8\pi G}$ is the modified Plank mass, and $\nu, \lambda$ are new dimensionless parameters and $m^2$ is the mass term which looks natural within such a model. The Ricci coupling is split into the scalar curvature coupling and coupling to Schouten tensor, defined in $D$ dimensions as

$$S_{\mu\nu} = \frac{1}{D-2} \left( R_{\mu\nu} - \frac{R}{2(D-1)} g_{\mu\nu} \right),$$ \hspace{1cm} (2)

its advantage will be clear later.

It is convenient to choose the length scale $l = (1/e M_{Pl})$ and replace the mass by the dimensionless parameter $\mu = (m/e M_{Pl})^2$. The interval in dimensionless coordinates reads

$$ds^2 = l^2 \left\{ -N^2 dt^2 + a^2 \left[ d\chi^2 + \Sigma_k^2(\chi)(d\theta^2 + \sin^2 \theta d\varphi^2) \right] \right\},$$ \hspace{1cm} (3)

where $\Sigma_k(\chi) = \{\sin \chi, \chi, \sinh \chi\}$ for the closed, flat and open universe, labeled by $k = 1, 0, -1$, correspondingly.
The scalar curvature and the Schouten tensor contain the second order derivatives

\[ R = 6 \left[ \frac{\dot{a}}{aN} / N + 2 \dot{a}^2 / a^2 N^2 + k / a^2 \right]; \quad (4) \]

\[ S_0 = \frac{\dot{a}}{aN} / N + \dot{a}^2 / 2 a^2 N^2 - k / 2 a^2, \quad S_i = a^2 / 2 a^2 N^2 + k / 2 a^2. \quad (5) \]

In General Relativity it is common to integrate by parts separating the total derivative

\[ \frac{1}{2} R \sqrt{-g} = 3 \left[ a^3 (\dot{a} / aN) + 2 a \dot{a}^2 / N + k aN \right] = 3 \left[ k aN - a \dot{a}^2 / N \right] + \text{div}. \quad (6) \]

The ansatz for the YM potential preserving the isotropy and homogeneity was constructed in \[8\] for all \( k \). In the temporal gauge \( A_0^a = 0 \) we have

\[ A_i^a A^{aj} = \delta_i^j \frac{(k - f)^2}{a^2}, \quad -\frac{1}{4} F_{\mu \nu}^a F_{\alpha \beta}^{\mu \nu} = \frac{3 f^2}{2 a^2 N^2} - \frac{3 (k - f^2)^2}{2 a^4}. \quad (7) \]

The same as for the curvature term, one can shift the second derivative in the scalar curvature coupling term to the gauge field:

\[ \frac{1}{4} R A_\mu^a A^{\mu} \sqrt{-g} = 3 \dot{f} (k - f) \dot{a} / N + \frac{3}{2} (k - f)^2 \left[ kN / a + \dot{a}^2 / aN \right] + \text{div}. \quad (8) \]

Note that the term \( 3 \dot{f} (k - f) \dot{a} / N \) looks quite similar to the topological term

\[ \varepsilon^{\mu \nu \alpha \beta} F_{\mu \nu}^a F_{\alpha \beta}^a \sqrt{-g} = 3 \dot{f} (k - f^2). \]

It has the form of the interaction with an “axion” \( \dot{a} / N \) — which is a common trick to make the topological term to contribute into dynamics.

The coupling of the YM field amplitude to the Schouten tensor does not contain the second derivative terms:

\[ S_{\mu \nu} A^{\mu} A^{\nu} \sqrt{-g} = \frac{3}{2} (k - f)^2 \left[ kN / a + \dot{a}^2 / aN \right]. \quad (9) \]

Collecting all the terms together and omitting the factor 3, one obtains the following effective one-dimensional Lagrangian:

\[ L_{\text{eff}} = k a N - \frac{a \dot{a}^2}{N} + \frac{a \dot{f}^2}{2 N} - \frac{(k - f^2)^2 N}{2 a} + L_{NM}, \quad \text{where} \]

\[ L_{NM} = -\frac{\mu}{2} (k - f)^2 a N + \nu \dot{f} (k - f) \dot{a} / N + \frac{\lambda}{2} (k - f)^2 \left[ \frac{kN}{a} + \frac{\dot{a}^2}{aN} \right]. \quad (10) \]

So there arise three different dynamical terms in \( L_{NM} \) due to non-minimal coupling, which can be switched off by making zero the corresponding coupling constants.
III. SLOW-ROLL

Our goal is to describe the slow-roll inflation within the above model, so we will make some preparations to make future analysis a bit easier. First, we omit the contribution of the curvature term during the phase of the fast expansion, though it contains a very interesting mode of the cosmological sphaleron in the case $k = 1$ due to the YM self-interaction potential \[35, 36\], which will be discussed elsewhere. So, in the what follows, $k = 0$.

Next, for the YM mode the conformal field amplitude $\psi = f/a$ is quite natural, while the metric variable can be chosen in the exponential form $a = \exp \alpha$. This allows us to simplify the contribution of the metric into the kinetic term, so the full Lagrangian now takes the form:

$$L = \frac{e^{3\alpha}}{2N} \left[ -A(\psi) \dot{\alpha}^2 + 2B(\psi) \dot{\alpha} \dot{\psi} + \dot{\psi}^2 \right] - Ne^{3\alpha} V(\psi),$$  \hspace{1cm} (11)

The above dynamical system has the constraint, obtained by the variation with respect to $N$, which in the form of the Friedman equation reads:

$$\frac{\dot{\alpha}^2}{2} A(\psi) = \dot{\alpha} B(\psi) + \frac{\dot{\psi}^2}{2} + N^2 V(\psi).$$  \hspace{1cm} (12)

The variation with respect to $\alpha$ and $\psi$ gives the equations of motion:

$$[N^{-1} e^{3\alpha} (B \dot{\psi} - A \dot{\alpha})]' = \frac{3e^{3\alpha}}{2N} \left[ -A \dot{\alpha}^2 + 2B \dot{\alpha} \dot{\psi} + \dot{\psi}^2 - 2N^2 V \right],$$  \hspace{1cm} (13)

$$[N^{-1} e^{3\alpha} (\dot{\psi} + B \dot{\alpha})]' = \frac{e^{3\alpha}}{2N} \left[ -A \dot{\psi} \dot{\alpha} + 2B \dot{\psi} \dot{\psi} - 2N^2 V \psi \right].$$  \hspace{1cm} (14)

Here $A_\psi, B_\psi, V_\psi$ denote the partial derivative with respect to $\psi$. Instead of fixing the time gauge we may proceed with an invariant description, choosing the $\alpha$ as an independent variable. Introducing the Hubble parameter, $H \equiv da/Ndt$, we may rewrite the system as

$$H [H e^{3\alpha} (B \psi' - A)]' = -\frac{3}{2} H^2 e^{3\alpha} \left[ A - 2B \psi' - \psi'^2 + 2H^{-2}V \right],$$  \hspace{1cm} (15)

$$H \left[ H e^{3\alpha} (\psi' + B) \right]' = -\frac{1}{2} H^2 e^{3\alpha} \left[ A_\psi - 2B_\psi \psi' + 2H^{-2}V_\psi \right],$$  \hspace{1cm} (16)

where prime denotes the derivative with respect to $\alpha$. Using the constraint, the potential term can be expressed as

$$2H^{-2}V = A - 2B \psi' - \psi'^2,$$  \hspace{1cm} (17)

and used in the above equations.
Now let us introduce the slow-roll parameters which are usually used to detect the inflationary stage in the dynamics of the system:

\[ \epsilon = - \frac{\dot{H}}{H^2 N} = - \frac{H'}{H}, \quad \delta = - \frac{\dot{\psi}}{H \psi N} = - \frac{\psi'}{\psi}. \]  

(18)

Of course they are also independent on the choice of gauge. We may rewrite the system of equations (15–16), replacing \( \psi', \psi'', H' \) by \( \delta, \delta', \epsilon \) and using the constraint to avoid appearance of the \( H^{-2} \) term:

\[
\begin{align*}
(\epsilon - 3)(B\psi\delta + A) + (B\psi \psi + B)\psi\delta^2 - B\psi\delta' &= -3(A + 2B\psi\delta - \psi^2\delta^2), \\
(\epsilon - 3)(\psi\delta - B) + \psi(\delta^2 - \delta') - B\psi\delta &= \\
-\frac{1}{2}(A\psi + 2B\psi\delta) - \frac{1}{2}[A + 2B\psi\delta - \psi^2\delta^2](\ln V)_\psi.
\end{align*}
\]

(19)

Finally collect terms with different powers of \( \epsilon, \delta \):

\[
\begin{align*}
A\epsilon + 3B\psi\delta &= B\psi\delta' - (B\psi \psi + B - 3\psi)\psi\delta^2 - B\psi\delta\epsilon, \\
3B + \frac{1}{2}[A\psi + A(\ln V)_\psi] + [B(\ln V)_\psi - 3] \psi\delta - B\epsilon &= \psi^2\delta' + \left[\frac{\psi}{2}(\ln V)_\psi - 1\right]\psi\delta^2 - \psi\delta\epsilon.
\end{align*}
\]

(20)

Since the constraint was already incorporated in these two equations, they provide the independent conditions on the slow-roll parameters. The first equation gives:

\[ \epsilon = -(3B\psi/A)\delta + O(\delta^2). \]

(23)

Then the second equation implies the relation on the initial state of the system:

\[ \delta + \frac{A[A(\ln V)_\psi + A\psi + 6B]}{2\psi[A(\ln V)_\psi - 3A + 3B^2]} = O(\delta^2, \delta'). \]

(24)

If the initial conditions \( \{\psi_i, \psi'_i\} \) ensure both \( \delta_i = -\psi'_i/\psi_i \ll 1 \) and the l.h.s. of the relation (24) to vanish, then one has \( \delta' \sim O(\delta^2), \epsilon \sim \delta \) which signals a slow-roll regime. Now assume for simplicity that neither the quantity \( 3B\psi/A \) itself, nor its derivative with respect to \( \psi \) is singular in the corresponding region of the phase space, to ensure that \( \epsilon' \sim O(\delta^2) \) as well. Also one has to ensure that the value \( H^2 \) from the constraint (17) is positive for the chosen initial conditions: this is the additional restriction, which was not taken into account before.

To estimate the number of \( e \)-folds gained by the scale factor during the slow-roll inflation, one can just treat the expression (24) as the function of \( \psi : \delta(\psi) \). Then by definition \( d\alpha = -[\psi\delta(\psi)]^{-1}d\psi \) and

\[ N_{\text{e-folds}} = \alpha_e - \alpha_i = - \int_{\psi_i}^{\psi_e} \frac{d\psi}{\psi\delta(\psi)}. \]

(25)
where the exit from the slow-roll may occur when $|\delta(\psi_e)| = 1$. The full expression is rather complicated, yet one can find that in the trans-Planckian region, $|\psi_i| \gg 1$, $\psi^2 \gg |\mu|,$

$$\delta(\psi) \approx -\frac{[2\nu - \psi(\lambda + 1)][5\nu - 3\psi(\lambda + \nu)]}{(3\nu^2 + 4\lambda\nu - \lambda - 2\nu + 2)\psi^2}. \tag{26}$$

In addition, the constraint (17) will provide $H^2 > 0$ if $A + 2B\psi\delta - \psi^2\delta^2 > 0$.

For the convenient choice of the parameters, say, $\lambda = -1$ or $\lambda = -\nu$ one can easily get the answer. Indeed, one has now $\delta \sim 1/\psi$ and $N_{e-\text{folds}} \sim \psi_i$. Of course, the value $N_{e-\text{folds}}$ is calculated up to the factor of unity, since the exit condition, $\delta_e \sim 1$, has just the same level of accuracy. And in general case the analytic analysis is quite complicated. So we now better proceed with a brief numerical calculations.

### IV. NUMERIC ANALYSIS AND OUTLOOK

It is convenient to solve numerically the system (13–14) in the gauge $N = 1$. Mention that the system of equations $M_{ij}(q)\ddot{q}^j = \Phi_i(q, \dot{q})$ has a singularity when $\Delta = \det(M)$ vanishes. In our case the corresponding determinant is $\Delta \sim B^2(\psi) + A(\psi)$. But the slow-roll initial state automatically ensures that the regime is non-singular (the dynamics can not be ‘slow’ in the vicinity of singularity).

For simplicity we restrict numerical experiments here by the two-dimensional parameter domain $(\nu, \lambda)$ of non-minimal couplings to gravity setting the mass parameter $\mu$ zero. The corresponding behavior of $\delta(\psi)$ is shown on the Fig. 1. It appears that the sub-domains of the slow-roll initial states are nearly one-dimensional in a wide range of $\psi_i$. This is true in the transplanckian region but changes substantially when the field amplitude goes below the Planck scale.

There is no practical sense in investigating such a complicated domain of the initial states. So we then focus on the transplanckian regime. The numerical solutions confirm that the line $\lambda = -\nu$ generates the stable slow-roll inflation in the area $\nu > 1$. The number of $e$-folds does not actually depend on the value of $\nu$, and is proportional to $\psi_i$, as can be seen on the Fig. 2. But it is very sensitive to the condition $\lambda = -\nu$. Even the small deviations up to percent may increase or decrease $N_{e-\text{folds}}$ up to several times. Finally, on the exit of the inflationary stage when $\psi_e \sim 1$, the solutions shown on the Fig. 2 meet the square-root singularity due to the vanishing system determinant $\Delta$. In fact, the singularity can be avoided for other choices of initial data, but the numerical experiments performed here have shown that these usually do not provide the good inflation.
It would be interesting to look whether there is any physics beyond the condition \( \lambda = -\nu \). In this case the non-minimal coupling term in four dimensions will be

\[
L_{NM} = -\nu[R_{\mu\nu} - (5/12)g_{\mu\nu}]A^\mu A^\nu.
\]

The geometrical tensor appearing here is close to, but not exactly coinciding with the Einstein tensor. In such a model the stable inflationary regime exists starting in the transplanckian region when the YM field amplitude rolls down to the Planck scale, while the number of e-folds is proportional to the initial value of the field amplitude, \( N_{e-\text{folds}} \approx 0.6\psi_i \).

Another special property of the proposed non-minimally coupled YM model is that typically there is no smooth exit from the inflationary stage into the radiation dominated universe. The solutions end with a square-root singularity, where the derivatives \( \dot{\alpha} \) and \( \dot{\psi} \) diverge. In this area the system demonstrates chaotic behavior due to the essentially non-linear coupling of the YM field to the metric which then evolves from one singularity to another. The solutions corresponding to radiation dominated universe can be obtained when the initial energy of the YM field is small, so that non-linear terms of the non-minimal coupling can be neglected. Probably, the singular region between the inflationary stage and the hot universe may correspond to a phase transition in the universe, when other matter fields are included.

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FIG. 1: Some sets in the parameter space (filled with grey color) which satisfy the energy constraint and provide the slow-roll dynamics: $|\delta(\psi, \nu, \lambda)| < 0.1$ and $H^2(\psi, \nu, \lambda) > 0$.

FIG. 2: A set of 16 solutions for $\psi_i = 50, 100, 150, 200$ and $\nu = -\lambda = 1, 5, 10, 15$. Solid lines for the metric exponent, $\alpha(t)$, dot lines for the YM field amplitude, $\psi(t)$. Their linear similarity verifies that the trans-Planckian region and the parameter line $\nu = -\lambda > 1$ is a stable domain for the slow-roll inflationary scenario.