Recent Developments in the Theory of Heavy Quarks

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Abstract

Recent developments in the theory of heavy quarks are reviewed. In the area of heavy quark fragmentation, there has been progress in the study of both perturbative and nonperturbative processes, including the identification of new observable nonperturbative fragmentation parameters. There has also been considerable theoretical activity in the study of inclusive rare and semileptonic decays of bottom hadrons.

UCSD/PTH 93-31, JHU-TIPAC-930027

October 1993

* On leave from The Johns Hopkins University, Baltimore, Maryland.

To appear in the Proceedings of the Advanced Study Conference on Heavy Flavours, Pavia, Italy, September 3-7, 1993.
1. Introduction

The properties of hadrons containing a heavy quark $Q$ simplify considerably in the limit $m_Q \to \infty$. If the hadron contains only heavy quarks, this is because of asymptotic freedom. If the hadron contains light quarks, this is because for $m_Q \gg \Lambda_{QCD}$ the light degrees of freedom become insensitive to the mass $m_Q$; as far as they are concerned, the heavy quark acts simply as a non-recoiling source of color.\(^1\) In the latter case, hyperfine effects associated with the heavy quark chromomagnetic moment also decouple, and a new “heavy quark spin-flavor symmetry” emerges. There is an extensive literature in which these simplifications have been used to predict hadronic spectra and decay rates, among other things. In this talk I will describe more recent work, in which they have been applied to heavy quark fragmentation and inclusive decays.

2. Heavy Quark Fragmentation

There has been much recent progress in describing the fragmentation to hadrons of heavy quarks which are produced at high energies in collisions. In the case of fragmentation to a hadron containing a single heavy quark, the symmetries of the heavy quark effective theory have been applied to identify new nonperturbative fragmentation parameters which describe the anisotropies of the light degrees of freedom. For fragmentation to hadrons containing two heavy quarks, such as to “onium” states, perturbative QCD has been used to compute fragmentation functions and probabilities in an expansion in $\alpha_s(m_Q)$ and $m_Q/E$, where $E$ is the energy of the quarkonium in the rest frame of the event. We will discuss each of these systems in turn.

2.1. Heavy-Light Systems

We begin by applying the heavy quark symmetries to fragmentation processes in which no new heavy quarks are produced.\(^2\) In the limit $m_Q \to \infty$ such a process factorizes into short-distance and long-distance pieces. The heavy quark is first produced via some high energy interaction, such as the decay of a virtual photon, which may typically be calculated in perturbation theory. This initial stage occurs on a time short compared to the time scale $1/\Lambda_{QCD}$ of the nonperturbative strong interactions. Later, over a longer, hadronic time scale, a fragmentation process occurs which eventually produces a physical hadron containing the heavy quark. Since this slower process only involves the redistribution of energies small compared to $m_Q$, the velocity of the heavy quark remains unchanged once it
has been produced perturbatively, and its mass and spin decouple from the nonperturbative dynamics.

Let us now consider this sequence of events in somewhat more detail. We imagine that we begin with a heavy quark which has been ejected at relativistic speed from a hard reaction. The axis linking the rest frame of the heavy quark to the center of mass frame of the hard process is a preferred direction, which we call the axis of fragmentation. The interactions which couple to the heavy quark spin, such as the chromomagnetic moment operator, are suppressed by at least one power of the small parameter $\Lambda_{\text{QCD}}/m_Q$, and hence the rate of heavy quark spin flip is very slow on the hadronic time scale $1/\Lambda_{\text{QCD}}$.

We might imagine the early stages of fragmentation to involve the production of excited hadrons containing the heavy quark, which then rapidly decay to lighter excited states. So long as these decay times are shorter than the typical spin flip times $m_Q/\Lambda_{\text{QCD}}^2$, the heavy quark spin will be unaffected by this evolution.

However, eventually the light degrees of freedom will reach a state whose lifetime is longer than the time required to flip the heavy quark spin. In the heavy quark limit, if the light degrees of freedom have angular momentum $j > 0$, the physical hadrons are a nearly degenerate doublet with spin $J = j \pm \frac{1}{2}$. The trademark of a sufficiently long-lived state is that the members of this doublet are well separated resonances; in that case, the doublet lives long enough for the two eigenstates of total angular momentum to become incoherent with each other. This, in turn, will happen only if the heavy quark and the light degrees of freedom have enough time to exchange their spin orientations before the doublet decays.

It is at this point that things become interesting, since the dynamics now allows the different helicity states of the light degrees of freedom (and of the heavy quark) to mix with each other. This effect, as we shall see below, is observable in the subsequent decays of the long-lived states, in such a way so as to give us information about the fragmentation process itself. We now introduce a new set of fragmentation parameters appropriate to HQET. When light degrees of freedom with angular momentum $j$ are formed by fragmentation, they can be produced in one of $2j + 1$ helicity states along the fragmentation axis. Parity invariance requires that the probability of forming a given helicity state cannot depend on the sign of the helicity $j^3$. However, the fragmentation need not be entirely isotropic; states with different magnitudes $|j^3|$ can arise with different probabilities. For the examples we will consider here, we can characterize the situation as follows: for a system of light degrees
of freedom of spin $j$, let $w_j$ be the probability that fragmentation leads to a state with the maximum value of $j^3$. Then the parameter $w_j$ may take values between 0 and 1.

Of the systems available for study, the most interesting is the $j = \frac{3}{2}$ excited meson doublet in the charm system. These states have been identified as the spin-1 $D_1(2420)$ and the spin-2 $D_2^*(2460)$, and their decays to the ground state mesons $D$ and $D^*$ via pion emission have been observed.\(^3\) The production of these resonances via fragmentation is described by the parameter $w_{3/2}$, which is the probability of producing the light degrees of freedom in a state of helicity $\pm \frac{3}{2}$ along the fragmentation axis, rather than $\pm \frac{1}{2}$. Since the $D_1$ and $D_2^*$ peaks are well separated, these two components of the doublet become incoherent before they decay, and it becomes appropriate to describe them as a mixed state containing $D_1$ or $D_2^*$ with fixed probabilities. If we make the further assumption that the initial charm quark is polarized left-handed (it is straightforward to extend this to a general or mixed polarization), then the populations of the helicity states of the excited mesons are easy to calculate. For example, the helicity states of the $D_2^*$ are populated with the following relative probabilities, where the helicity runs from $-2$ to $+2$ across the table:

$$p(D_2^*, h) = \begin{pmatrix} \frac{1}{2} w_{3/2} & \frac{3}{8} (1 - w_{3/2}) & \frac{1}{4} (1 - w_{3/2}) & \frac{1}{8} w_{3/2} & 0 \end{pmatrix}. \quad (2.1)$$

Given these probabilities, we may now compute the angular distribution for the observed strong decay $D_2^* \to D + \pi$. We define $\theta$ to be the angle between the momentum of the pion and the fragmentation axis, in the rest frame of the $D_2^*$. Normalizing to the full width $\Gamma$, we find

$$\frac{1}{\Gamma} \frac{d\Gamma}{d \cos \theta} (D_2^* \to D\pi) = \frac{1}{4} [1 + 3 \cos^2 \theta - 6 w_{3/2} (\cos^2 \theta - \frac{1}{3})]. \quad (2.2)$$

This distribution is independent of the initial polarization state of the charm quark (this is not necessarily the case for the decay of the $D_1$; see Ref. 2.). Note that for the isotropic production of the light degrees of freedom, $w_{3/2} = \frac{1}{2}$, the pion is also emitted isotropically. However, an ARGUS analysis of this decay\(^4\) found no significant population of the helicity states $\pm 2$ of the $D_2^*$, implying that $w_{3/2}$ is small. Our own fit yields

$$w_{3/2} < 0.24, \quad 90\% \, \text{conf.} \quad (2.3)$$

This is an intriguing result, for which we have as yet no satisfactory physical interpretation.

A similar quantity may soon be measured for the $j = 1$ heavy charmed baryon doublet, the spin-$\frac{1}{2}$ $\Sigma_c$ and the spin-$\frac{3}{2}$ $\Sigma_c^*$. The $\Sigma_c$ has been observed with a mass of 2453 MeV;
while the $\Sigma_c^*$ has not yet been found, its mass is known to be greater than 2530 MeV. In any case, the peaks for the two states are known to be well separated.\(^3\) Hence an analysis of the fragmentation parameter $w_1$ relevant to this system may be performed by observing the strong decay $\Sigma_c^* \to \Lambda_c + \pi$. Defining $\theta$ as before, we find the angular distribution of the pion to be

$$\frac{1}{\Gamma} \frac{d\Gamma}{d\cos\theta} (\Sigma_c^* \to \Lambda_c \pi) = \frac{1}{4} \left[ 1 + 3 \cos^2 \theta - \frac{9}{2} w_1 (\cos^2 \theta - \frac{1}{3}) \right].$$  \hspace{1cm} (2.4)

In this case, isotropic production of the light degrees of freedom corresponds to $w_1 = \frac{2}{3}$.

This quantity, when it is obtained, will be useful for another reason. The ground state $\Lambda_Q$ baryon has light degrees of freedom with total angular momentum $j = 0$; the spin of this baryon is carried entirely by the heavy quark. Hence if a sample of $\Lambda_Q$ baryons arises from an initial sample of polarized heavy quarks, they should themselves be strongly polarized.\(^5\) (We have in mind charm and bottom quarks produced at the $Z_0$ peak, which have polarizations $P_c = 0.67$ and $P_b = 0.94$, respectively.) The $\Lambda_Q$ would, in fact, carry all of the polarization of the heavy quark if it were produced only directly, but there is a potentially significant pollution from $\Lambda_Q$'s produced via the cascade decays $(\Sigma_Q, \Sigma_Q^*) \to \Lambda_Q + \pi$. If (as is the case in the charm system) the excited doublet $(\Sigma_Q, \Sigma_Q^*)$ lives long enough that spin exchange interactions between the heavy quark and the light degrees of freedom have time to take place, the initial polarization of the heavy quark will be diluted. The actual polarization of the $\Lambda_Q$ baryons may be computed in terms of the anisotropy parameter $w_1$ and a parameter $A$ which describes the probability of producing a $j = 1$ (and isospin one) diquark relative to one with $j = 0$ (and isospin zero), summed over the nine possible helicity and isospin states. If $P$ is the initial polarization of the heavy quark, we find the polarization of the sample of $\Lambda_Q$'s to be

$$P_\Lambda = \frac{1 + (1 + 4w_1)A/9}{1 + A} P.$$  \hspace{1cm} (2.5)

The parameter $A$ is related to one which appears in the Lund Monte Carlo\(^6\) and may be estimated to be 0.45, with large errors. If, motivated by the results for charmed mesons, we take $w_1 = 0$, we find a polarization $P_\Lambda = 0.72P$. Note that the polarization retention $P_\Lambda$ grows with $w_1$. Finally, the brief treatment given here depends on the assumption that the $\Sigma_Q$ and $\Sigma_Q^*$ resonances are well separated. While this is known to be true in the case of charm, it may not necessarily be true for bottom. For a more general analysis in which this latter possibility is taken into account, see Ref. 2.
2.2. Heavy-Heavy Systems

We now turn to the much rarer possibility that the fragmentation process generates an additional pair of heavy quarks, rather than just light degrees of freedom. Braaten et al. have shown that the production of heavy “onium” states at high energies is dominated by such fragmentation processes, which may be calculated perturbatively. A typical process would be the production of charmonium from the fragmentation of a high energy charm quark produced in $Z_0$ decay, $Z_0 \to \bar{c}c \to \psi + \bar{c}c + X$. Braaten et al. have shown that such a process factorizes to leading order in $m_c/E$, where $E$ is the energy of the $\psi$:

$$d\Gamma(Z^0 \to \psi(E) + X) = \sum_i \int_0^1 dz \, d\hat{\Gamma}(Z_0 \to i(E/z) + X, \mu) \, D_{i\to\psi}(z, \mu). \quad (2.6)$$

Here $d\hat{\Gamma}$ is the cross-section for the $Z_0$ to produce the parton $i$ with energy $E/z$, and $D_{i\to\psi}(z, \mu)$ is a process-independent fragmentation function for the parton $i$ to fragment to a $\psi$ with momentum fraction $z$. The scale $\mu$ is a factorization scale which is introduced to separate the short- and long-distance physics; note that the energy of the $\psi$ does not appear in $D_{i\to\psi}(z, \mu)$, which may be evaluated at a low-energy scale $\mu \approx m_c$. The renormalization group may then be used to resum the leading logarithms between $m_c$ and the high-energy scale $E$.

The fragmentation functions may be evaluated straightforwardly and perturbatively in $\alpha_s(m_\psi)$, but the results are somewhat messy. Of more interest are the fragmentation probabilities, obtained by integrating over $z$. In many cases it can be shown that the probabilities $P = \int_0^1 dz \, D(z, \mu)$ are in fact independent of $\mu$, at least to leading order. Some typical probabilities which Braaten et al. report are

$$P_{c\to\psi} \approx 1.2 \times 10^{-4}, \quad P_{c\to\eta_c} \approx 1.2 \times 10^{-4}, \quad P_{c\to\eta} \approx 1.2 \times 10^{-4},$$

$$P_{b\to B_c} \approx 3.8 \times 10^{-4}, \quad P_{b\to B_{c}^*} \approx 5.4 \times 10^{-4}. \quad (2.7)$$

These results may be extended in a number of ways. First, one may compute not only the probability of a charm quark to fragment to a $\psi$, but the fraction of the time that the $\psi$ is transversely rather than longitudinally aligned. We define $\zeta$ to be the ratio of the probability of producing a transversely aligned $\psi$ to the total production probability. Then a straightforward perturbative calculation gives $\zeta = 0.69$, corresponding to a small excess of transversely aligned $\psi$'s. (This fraction is independent of the heavy quark mass,

* I am grateful to T. C. Yuan for providing me with updated versions of these numbers.
and is hence the same for Υ’s.) In leading logarithmic approximation, the corresponding ratio for gluon fragmentation to ψ’s has the same value and is also μ-independent. Hence, at a hadron collider, where ψ’s are produced by both quark and gluon fragmentation, the fraction of transversely aligned ψ’s is also ζ = 0.69.

This fact has an immediate application to the study of direct ψ production at hadron colliders, where it is important to distinguish these ψ’s from those produced via the weak decays of b quarks. The alignment of the ψ may be observed in the angular distribution of its leptonic decay, ψ → ℓ⁺ℓ⁻, which is parameterized as

\[ \frac{dΓ}{d\cos θ} \propto 1 + α \cos^2 θ . \] (2.8)

Here θ is the angle between either of the leptons and the alignment axis in the ψ rest frame. For ζ = 0.69, we find α = 0.053, a small (5%) asymmetry. By contrast, for the weak decay b → ψsℓ⁺ℓ⁻ we find α ≈ −0.46. Hence the lepton angular distribution may provide a useful tool for separating these two sources of ψ’s in a collider environment.

Finally, we have applied these perturbative methods to estimating the probability of fragmentation to baryons which contain two heavy quarks, such as bbq, ccq and bcq states. There are a host of possible “doubly heavy” spin-\( \frac{1}{2} \) and spin-\( \frac{3}{2} \) baryons; we found the largest production probabilities to be

\[ P(c → Σ_{cc}, Σ^*_{cc}) \sim 2 \times 10^{-5} ; \]
\[ P(b → Σ_{bc}, Σ^*_{bc}) \sim 5 \times 10^{-5} ; \] (2.9)
\[ P(b → Λ_{bc}) \sim 4 \times 10^{-5} . \]

While there is considerable uncertainty in these estimates (they should be trusted at best within a factor of two), this is enough to tantalize us with the hope that these states may one day be observed at hadron colliders such as an upgraded Tevatron.

3. Inclusive Heavy Hadron Decays

There has been much recent progress in the analysis of inclusive decays of heavy hadrons. More precisely, these are decays to final states in which some, but not all, of the quantum numbers of the decay products are known. For example, we might specify the identity and momenta of certain weakly interacting particles, such as leptons or photons, or we might restrict the flavor of the final hadrons. The decays are inclusive in that we
sum over all final states which can be produced by the long-distance, nonperturbative
strong interactions, subject to certain constraints which are determined by short-distance,
perturbative physics.

We are typically interested in studying quark-level transitions induced by short-
distance weak interactions, such as $b \rightarrow c \ell \bar{\nu}$, $b \rightarrow s \gamma$, or $b \rightarrow s e^+ e^-$. In a lagrangian renor-
malized far below the weak scale, these interactions appear as local non-renormalizable
operators with coefficients which contain all dependence on the interesting short-distance
physics. For example, the low-energy lagrangian contains terms

$$
\frac{G_F}{\sqrt{2}} V_{cb} \bar{c} \gamma_\mu (1 - \gamma^5) b \bar{\ell} \gamma^\mu (1 - \gamma^5) \nu
$$

and

$$
\frac{4G_F}{\sqrt{2}} V_{tb} V_{ts}^* \frac{e}{16\pi^2} m_b c_7(\mu) \bar{s} \sigma_{\mu\nu} b L F^{\mu\nu},
$$

and we would like to extract the parameters $V_{cb}$ and $c_7(m_b)$ from experiment. In order
to do this, however, it is necessary to understand how these quark-level operators induce
transitions in the physical hadrons which are actually observed.

The difficulty is that exclusive decays, such as $B \rightarrow K^* \gamma$ in the case of an underlying
$b \rightarrow s \gamma$ transition, are typically not accessible theoretically. This is because they require
the calculation of nonperturbative hadronic matrix elements such as $\langle K^* | \bar{s} \sigma_{\mu\nu} b_L | B \rangle$, for
which there are no rigorous techniques except in certain situations of enhanced symmetry.
It is simpler, in a sense, to consider inclusive decays, in which all possible final states are
summed over. One may constrain the kinematics by fixing the momenta of the nonhadronic
quanta in the final state, and one may assume in the sum that the flavor of the final hadron
may be tagged experimentally. In that case, what one needs instead of $\langle K^* | \bar{s} \sigma_{\mu\nu} b_L | B \rangle$
is the sum over all strange hadrons of the probabilities of producing them,

$$
W = \sum_{X_s} \langle B | \bar{b}_L \sigma_{\alpha\beta} s R | X_s \rangle \langle X_s | \bar{s} R \sigma_{\mu\nu} b_L | B \rangle.
$$

There is an analogous expression, of course, for inclusive semileptonic weak $b$ decays, and
so forth.

If the energy which is released in the decay is large, then the inclusive rate may be
modeled simply by the decay of a free $b$ quark. The heuristic argument for this is simply
that the quark-level transition takes place over a time scale which is much shorter than
the time it takes for the quarks in the final state to materialize as physical hadrons. Hence
these two parts of the hadronic decay process do not interfere with each other. Once the
short-distance interaction has taken place, the probability is unity that the quarks which
have been produced will hadronize somehow; we do not need to know the (incalculable)
probabilities of hadronization to individual final states if we sum over all of them. This free
quark decay model (FQDM) has been the basis, until recently, of all theoretical studies of
inclusive heavy quark decays.

It is intuitively clear that the FQDM will be a good approximation when the energy
release in the decay is much larger than $\Lambda_{\text{QCD}}$, and when the final quark is far from its
mass shell after the short-distance interactions have taken place. It has been shown by
Chay et al.\textsuperscript{10} that the FQDM may in fact be justified more rigorously within QCD, via
an operator product expansion. To do this, one notes that the optical theorem may be
used to rewrite an inclusive sum such as (3.3) as the imaginary part of a forward scattering amplitude,

$$ W = 2 \text{Im} T = 2 \text{Im} \langle B | T \{ \bar{b}_L \sigma^{\alpha\beta} s_R, \bar{s}_R \sigma^{\mu\nu} b_L \} | B \rangle. \quad (3.4) $$

There is now no reference in the expression to any explicit strange hadronic state $X_s$. Let
us denote by $P^\mu$ the momentum of the virtual strange quark in the time-ordered product.
Since the scale of $P^\mu$ is set by the available energy $m_b$, over almost all of the Dalitz plot we
have $P^2 \gg m_s^2$ and the strange quark is far from its mass shell. (The point in the Dalitz
plot is fixed by the kinematics of the rest of the event, in this case the energy of the photon.
For semileptonic decays, it would be set by the lepton kinematic invariants.) If we stay
away from those regions where $P^2 \approx m_s^2$, it is appropriate to expand the nonlocal hadronic
object $T$ in an operator product expansion, as a series of local operators suppressed by the
large off-shell momentum.

The insight of Chay et al. was to notice that it is more convenient to expand directly
in inverse powers of the large mass $m_b$. This is almost the same thing as expanding in the
off-shellness of the strange quark, since $P^\mu$ scales with $m_b$; it only differs near those
extreme regions of the Dalitz plot where the expansion breaks down. But this technique is
considerably more powerful, because it allow us to expand in operators of the heavy quark
effective theory\textsuperscript{1}) (HQET). The expansion of $T$ then has the structure

$$ T \to \frac{1}{m_b} \left[ O_0 + \frac{1}{2m_b} O_1 + \frac{1}{4m_b^2} O_2 + \cdots \right], \quad (3.5) $$

where the operator $O_n$ is of dimension $3 + n$ and in HQET takes the form

$$ O_n = c_n \bar{h} \Gamma D_{\mu_1} \cdots D_{\mu_n} h. \quad (3.6) $$
The effective fields $h(x)$ which have replaced the quark fields $b(x)$ are obtained by the usual HQET rescaling.\textsuperscript{1)}

The advantage of the HQET formulation is that the heavy quark symmetries may be used to compute the necessary matrix elements, up to the desired order in $1/m_b$. At leading order in the expansion of $T$, the matrix element which is needed is of the form

$$\langle B | \bar{h} \Gamma h | B \rangle.$$  

This may be computed exactly in HQET; it is proportional to the Isgur-Wise function evaluated at the zero-recoil point, where it is absolutely normalized. Chay \textit{et al.} pointed out that the result obtained by truncating $T$ at this leading operator reproduced the free quark decay model prediction for the inclusive rate. Hence the FQDM was shown to be the first term in a controlled expansion in $m_b$, with all corrections formally of order $\Lambda_{QCD}/m_b$. This was a considerable improvement from its earlier status as an \textit{ad hoc}, if intuitively reasonable, model.

In fact, Chay \textit{et al.} showed that the situation is considerably better than it appears at first glance, because all corrections of relative order $\Lambda_{QCD}/m_b$ vanish if the decay rate is written in terms of the bottom \textit{quark}, rather than \textit{meson}, mass. (The distinction is significant, since the decay rate is proportional to $m_b^5$.) The result follows simply from the fact that all matrix elements of HQET operators with one derivative vanish at the point of zero momentum transfer.\textsuperscript{11)} This is a surprising and intriguing result, since there are certainly $1/m_b$ corrections to the individual exclusive modes; however in the inclusive sum they conspire to cancel.

The recent progress in the field has arisen due to the realization that the leading non-vanishing corrections, of relative order $1/m_b^2$, may be computed simply in terms of two nonperturbative parameters. All the necessary matrix elements of operators $O_2$ may be expressed in terms of the matrix elements of two dimension-five operators\textsuperscript{12)}:

$$\langle B | \bar{h} (iD)^2 h | B \rangle = 2m_B \lambda_1,$$

$$\langle B | \bar{h} (-\frac{i}{2}\sigma^{\mu\nu})G_{\mu\nu} h | B \rangle = 6M_B \lambda_2.$$  

The parameters $\lambda_1$ and $\lambda_2$ appear also in the expansion of the physical hadron masses in terms of the bare quark mass,

$$M_B = m_b + \bar{\Lambda} - \frac{1}{2m_b}(\lambda_1 + 3\lambda_2),$$

$$M_B^* = m_b + \bar{\Lambda} - \frac{1}{2m_b}(\lambda_1 - \lambda_2).$$  

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Here $\bar{A}$ is the energy of the light degrees of freedom.\textsuperscript{11,13} The physical splitting between the vector and pseudoscalar states may be used to extract $\lambda_2$, which represents the additional energy of the $b$ quark in the hadron due to its chromomagnetic interactions with the light degrees of freedom. We find $\lambda_2 = 0.12\text{ GeV}^2$. Unfortunately, $\lambda_1$, which represents the $b$ quark’s residual kinetic energy in the bound state, is not easily related to any such observable. It is probably reasonable to allow it to vary within the range $-1\text{ GeV}^2 < \lambda_1 < 1\text{ GeV}^2$.

We can now obtain the leading nonperturbative corrections to the inclusive width $\Gamma$, as well as to other inclusive observables. The expansion will generally take the form

$$\Gamma \propto m_b^5 \left[ 1 + \frac{1}{4m_b^2} f(\lambda_1, \lambda_2) + \cdots \right].$$

Here the ellipses denote corrections arising from yet higher order terms in the operator product expansion, plus QCD radiative corrections which we have not included. We stress that while such radiative effects, such as real and virtual gluon emission, can be as large as the terms which we consider here, they may simply be added in as necessary.

We close by tabulating some recent results which have been obtained with these methods. The most extensive work has concerned inclusive semileptonic $b \to c$ transitions.\textsuperscript{14} One may obtain the leading corrections to the full inclusive width, as well as corrections to the lepton energy spectrum. Since Prof. Bigi has discussed his own results at this conference, I have little to add. The corrections to the lepton spectrum are typically a few percent, except near the endpoint of maximum energy. There the effect grows dramatically, but these divergences are a signal of the breakdown of the operator product expansion and are not to be trusted. The computed spectrum must be smeared over lepton energies of a few hundreds of MeV for the results to be sensible. Unfortunately it is not possible to probe the very endpoint of the lepton energy spectrum in detail using these techniques. Hence the study of the charmless semileptonic transition $b \to u\ell\bar{\nu}$ is not practical.

These techniques have also been applied to the rare $b \to s$ transitions $B \to X_s\gamma$ and $B \to X_s\ell^+\ell^-$. For the two-body final state, we find corrections to the inclusive width,\textsuperscript{15,16}

$$\Gamma \propto m_b^5 \left[ 1 + \frac{1}{2m_b^2} (\lambda_1 - 9\lambda_2) \right],$$

and to the average photon energy,\textsuperscript{15}

$$\langle E_\gamma \rangle = \frac{m_b}{2} \left[ 1 - \frac{1}{2m_b^2} (\lambda_1 + 3\lambda_2) \right].$$
Note that it is the quark mass $m_b$ which appears in these expressions, not the meson mass. The corrections are typically a few percent, which tells us that the FQDM is in fact a good approximation when applied to these rare $b$ decays. Finally, we observe that the form of the correction to $\langle E_\gamma \rangle$ is just what we would expect, since, according to eq. (3.9), $-(\lambda_1 + 3\lambda_2)/2m_b$ is precisely the shift in the energy of the free $b$ quark when it is bound in the $B$ meson.

For the $X_s\ell^+\ell^-$ final state, the form of the result is too unwieldy to reproduce here, and we refer the reader to Ref. 15 for details. We have calculated the leading corrections to the lepton invariant mass spectrum, $d\Gamma/d\hat{s}$, where $\hat{s} = (P_{\ell^+} + P_{\ell^-})^2/m_b^2$. They are somewhat larger than for $b \to s\gamma$, typically on the order of ten percent. However, one must be careful to stay away from the $\psi$ resonance region in $\hat{s}$, where additional four-quark operators can contribute and the operator product expansion is not to be trusted.

**Acknowledgements**

It is a pleasure to thank the organizers of the conference for their warm hospitality and flawless organization. This work was supported by the Department of Energy under contract DE-FG03-90ER40546 and by the Texas National Laboratory Research Commission under grant RGFY93-206.

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