Small Active and Sterile Neutrino Masses from the TeV Scale

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Abstract

A new mechanism for understanding small neutrino masses using only simple new physics at the TeV scale is proposed. As an application, it is shown how it can naturally lead to the mass hierarchy of the so called bimaximal mixing in the case of three active neutrinos, or the (3+1) scenarios for sterile neutrinos, using only the SU(2)_L quantum numbers of the particles.

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I. INTRODUCTION

The most commonly used way to understand the smallness of neutrino mass \textsuperscript{[1]} compared to that of the charged fermions is the seesaw mechanism \textsuperscript{[2]}, according to which one has,

\[ M_\nu \simeq -M_D^T M_R^{-1} M_D. \] (1)

For choices of the Dirac mass matrix \(M_D\) dictated by simple unified models, one expects the typical scale of the seesaw masses to be around 10^{10} \text{ GeV} or higher for neutrino masses required to fit atmospheric and solar data \textsuperscript{[3,4]}. In this class of models, neutrino masses and mixings are dictated by physics at superheavy scales. An important question to ask (and has often been discussed in literature) therefore is whether the neutrino masses can arise from new physics nearby the weak scale, e.g., in the TeV range so that one may hope to probe them in the collider or other indirect experiments.

There exist several TeV scale scenarios for neutrino masses in the literature: (i) radiative scenarios that use extensions of the standard model by extra charged Higgs bosons \textsuperscript{[5]} (ii)
scenarios using bulk neutrinos [6–8] or bulk scalars [9,10] and (iii) a more recent one [11] that uses the seesaw mechanism but with naturally suppressed Dirac masses $M_D$.

In this letter, we propose a new mechanism that uses higher dimensional operators [12] that lead to a naturally suppressed neutrino masses using only simple new physics at the TeV scale. The higher dimensional operator could arise in a number of ways from TeV scale physics depending on the nature of extra symmetries and the particle content of the theory. We will illustrate our mechanism using the effective low energy theory at the weak scale.

For simplicity, we will assume the low energy theory to be the standard model plus an extra Higgs doublet and a Higgs singlet field. We will assume also the theory to have softly broken global symmetries such as $L = L_e + L_\mu + L_\tau$ and other linear combination of these quantum numbers depending on the kind of neutrino mixings that one wants to obtain. The general principle we will adopt is that global symmetries will only be broken by soft terms in the potential and any other interaction with dimension $d \geq 4$ will respect the symmetry.

As an application of our mechanism, we apply it to understand the lightness of the sterile neutrino, that is required to provide a simultaneous fit to solar, atmospheric and the LSND data [13]. We find that in simple models that implement our strategy, the sterile neutrino, even though is a standard model gauge singlet, may get a small mass comparable to that of familiar neutrinos due to the higher dimensional operators, leading naturally, for instance, to a (3+1) scenario which has recently been discussed [14,15]. We must however mention that the (3+1) scheme is generically constrained such that it may work only for certain domain of values of the mass difference $\Delta m^2_{LSND}$ [14,15], and there are also recent papers [16] noting that it may be least favoured as the explanation of all neutrino anomalies, though it has not been ruled out, and so, it remains of some interest.

II. THE NEW MECHANISM

Before presenting our proposal, let us review the higher dimensional mechanism suggested by Weinberg [12]. According to this mechanism, one writes down the nonrenormalizable operator involving the standard model fields that can lead to neutrino masses,

$$\mathcal{L} = \frac{f_{ab}}{\Lambda} \left( L^T_{ia} C^{-1} L_{jb} H_k H_l \epsilon_{i k} \epsilon_{j l} \right),$$

where $i, j, k, l$ are $SU(2)_L$ indices and $\Lambda$ is the scale of new physics. This operator breaks explicitly the lepton number. When $H$ develops a vacuum expectation value (vev), neutrinos acquire masses,

$$m_\nu = f \frac{\langle H \rangle^2}{\Lambda}.$$  

From Eq. (3), it is clear that this formula is similar to the seesaw formula in Eq. (1) with Dirac mass replaced by the Higgs vev that determines the weak scale and to get neutrino masses in eV range or less implies the scale of new physics $\Lambda \geq 10^{13}$ GeV or so. We should mention that a recent study [17] has classified several other non renormalizable operators of dimension smaller than 11 that may give rise to neutrino masses. However those equivalent to the ones we will use here were not considered there.
Let us now add to the SM representation content a singlet scalar field $\chi$ which carries lepton number $L = -1$ and a second scalar doublet $H'$ with $L = 0$. We then impose the discrete $Z_2$ symmetry on the model under which $H'$ and $\chi$ are odd while all other fields are even. Then the higher dimensional operator given above is forbidden by lepton number conservation. The lowest dimensional operator that is invariant under $L$ is given by:

$$L = \frac{f}{M^3} (LH\chi)^2 + \frac{f'}{M^3} (LH'\chi)^2,$$

where $M$ is now the new scale of physics. Once $H, H'$ and $\chi$ develop vev's we obtain the following expression to neutrino masses:

$$m_\nu = \frac{\langle \chi \rangle^2}{M^3} \left( f\langle H \rangle^2 + f'\langle H' \rangle^2 \right).$$

If the vev of $\langle \chi \rangle$ is suppressed, then $M$ can be in the TeV range. We now show that this happens quite naturally in our model.

To study the vevs of the Higgs fields, we write down the most general Higgs potential consistent with the symmetries we have imposed on the model, i.e., the gauge symmetry and global $U(1)_L$. However to generate a natural small vacuum to $\chi$ we consider in the potential terms that break explicitly the lepton number,

$$V(H, H', \chi) = \mu_H^2 H^\dagger H + \frac{\lambda_1}{2} (H^\dagger H)^2 + \mu_{H'}^2 H'^\dagger H' + \frac{\lambda_2}{2} (H'^\dagger H')^2 + \lambda_3 H^\dagger HH'^\dagger H'$$

$$+ \frac{\mu_\chi^2}{2} \chi^\dagger \chi + \frac{\lambda_4}{2} (\chi^\dagger \chi)^2 + \lambda_4 H^\dagger H \chi^\dagger \chi + \lambda_4' H'^\dagger H' \chi^\dagger \chi - MH^\dagger H' \chi + h.c. \quad (6)$$

First note that the $M$ term in the potential breaks the global $L$-symmetry softly. For $\mu_{H,H'}^2 < 0$, the potential leads to both $H$ and $H'$ fields having nonzero vev's that break the electroweak symmetry. Minimization of the potential gives for the parameterization,

$$\begin{pmatrix} \langle H \rangle \\ \langle H' \rangle \end{pmatrix} = v \begin{pmatrix} \cos \beta \\ \sin \beta \end{pmatrix}, \quad (7)$$

the mixing angle:

$$\tan^2 \beta = \frac{\lambda_2 \mu_H^2 - \lambda_3 \mu_{H'}^2}{\lambda_1 \mu_{H'}^2 - \lambda_3 \mu_H^2}. \quad (8)$$

If one assumes the $d_R$ and $e_R$ to be odd under the $Z_2$ symmetry, then one can choose $\langle H' \rangle \ll \langle H \rangle$ as in the case of large $\tan \beta$ supersymmetric models. For $\mu_\chi^2 > 0$ and $\lambda_4 \geq 0$, the $M$ term in the potential then induces a vev for the $\chi$ field given by:

$$\langle \chi \rangle = \frac{M \langle H \rangle \langle H' \rangle}{\mu_\chi^2 + \lambda_4^2 \langle \chi \rangle^2 + v^2 \cos^2 \beta (\lambda_4^2 + \lambda_4' \tan^2 \beta)}.$$

If we assume $v_d \simeq \frac{\mu_\chi}{10}$ and $M_\chi \sim M \sim 20$ TeV, we get $\langle \chi \rangle \simeq 0.3$ GeV, without any fine tuning of parameters. Note that this leads to breaking of lepton number. There is however no zero mass particle since the $M$ term in the potential breaks lepton number explicitly. In
fact the masses of the real fields in $\chi$ become of order of the the scale $M$. It is worth noticing
that this is nothing but a singlet realization of the so called Type II see-saw mechanism [18].
Indeed in all our analysis above one can use a electroweak triplet carrying the same lepton
number instead of the singlet $\chi$, with the richness on the phenomenology of such a field. For
what we want to discuss hereafter, whether $\chi$ is a singlet or a triplet will not be relevant.

Using Eq. (5) and $f \sim 0.1$, we get $m_\nu \simeq 0.04$ eV. It is interesting that the muon neutrino
mass required to explain the atmospheric neutrino deficit emerges without any fine tuning
of parameters. This is the new mechanism which we want to exploit in the rest of the paper
to understand neutrino mass patterns. This mechanism has also the potential to be useful in
understanding small neutrino masses in low string scale theories, where the highest available
scale is in the multi TeV range. Moreover, it may even generate a naturally small sterile
 neutrino mass. We should notice that the present mechanism can also be seen as a Type II
see-saw realization of the dimension nine operator $L^2H^6$. Notice that such class of operators
was explicitly excluded in the general analysis of Ref. [17].

Let us conclude this section describing an underlying theory that could lead to the higher
dimensional operators as part of the low energy effective theory. One possibility is to have at
the 10 TeV scale three new singlet fields of mass of order 10 TeV: $N_{1,2,3}$; such that $N_{1,2}$ have
$L = \pm 1$ respectively whereas $N_3$ has $L = 0$ and is odd under $Z_2$. The invariant potential
involving these fields is then given by:

$$
\mathcal{L} = LH N_2 + M_1 N_1 N_2 + N_3 \chi N_1 + M_3 N_3 N_3 + h.c.
$$

(10)

$M_{1,3}$ are assumed to be of order 10 TeV. This theory leads in the low energy limit to the
effective theory described in (4) (see Fig. 1). We now move into considering how the desired
pattern for the neutrino masses can be obtained.

### III. BIMAXIMAL MIXING PATTERN

In this section, we apply the mechanism of the previous section to derive the bimaximal
neutrino mixing pattern. For this purpose, we recall that in the previous section, we used
the softly broken global symmetry $L$. Suppose, in addition, we have a softly broken global
$L' = L_e - L_\mu - L_\tau$. Then, the allowed higher dimensional terms in the Eq. (5) would have
generation labels in them and would lead to a mass matrix of the form:

$$
\mathcal{M}_\nu^{(0)} = m \begin{pmatrix}
0 & c_\theta & s_\theta \\
-c_\theta & 0 & 0 \\
s_\theta & 0 & 0
\end{pmatrix},
$$

(11)

where $c_\theta$ ($s_\theta$) represents the function $\cos \theta$ ($\sin \theta$). It is well known that this leads to the
bimaximal neutrino mixing pattern (see for instance Refs. [19–23]):

$$
U_{BM} = \begin{pmatrix}
\frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} & 0 \\
\frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & 0 \\
\frac{s_\theta}{\sqrt{2}} & \frac{s_\theta}{\sqrt{2}} & c_\theta
\end{pmatrix}.
$$

(12)

As it is evident from (11), at this stage, there is no mass splitting between $\nu_1$ and $\nu_2$
that can lead to oscillations. There is however mass splitting between $\nu_{1,2}$ and $\nu_3$ with
$\Delta m^2 = m^2$, which give rise to oscillations with the amplitude $\sin^2 2\theta$. As we saw before, for the parameters chosen in the previous section, this is precisely in the range required to solve the atmospheric neutrino anomaly, since then $\Delta m^2_{\text{atm}} = m^2 \approx 1.6 \times 10^{-3} \text{ eV}^2$. The interesting point to reemphasize in this connection is that the neutrino mass spectrum is an inverted one.

In order to generate the splitting between the $\nu_1$ and $\nu_2$, we need to introduce soft breaking of $L' = L_e - L_\mu - L_\tau$ symmetry so that an entry such as $m_{ee}$ or $m_{\mu\mu}$ can be generated. For solar neutrino anomaly solution via the large angle MSW mechanism, one would require $m_{ee}$ to be of order $10^{-4} \text{ eV}$ and for resolution via the vacuum oscillation solution, one would require $m_{ee} \sim 10^{-9} \text{ eV}$. For this purpose it is convenient to introduce a field $\delta$ with $L = 0$ but $L' = -1$. The vev of this field can also be induced by the mechanism developed in the last section, but this time we choose the parameters such that $\langle \delta \rangle / M \sim 10^{-1}$. This allows us to introduce a higher dimensional operator of the type:

$$\frac{(L_e H)^2 \chi^2 \delta^2}{M^5}. \quad (13)$$

Once all vevs are substituted, one gets $m_{ee} \sim 4 \times 10^{-4} \text{ eV}$. This favors the large angle MSW solution to the solar neutrino problem. In fact, the solar mass parameter is then given as $\Delta m^2_{\odot} \approx 2m^2(\langle \delta \rangle / M)^2 \approx 3 \cdot 10^{-5} \text{ eV}^2$. A smaller vacuum, $\langle \delta \rangle \sim 1 \text{ GeV}$, however, will produce $m_{ee} \sim 3 \times 10^{-9} \text{ eV}$ for vacuum solution.

**IV. THE (3+1) SCHEME**

The above neutrino mass scheme does not explain the LSND results. In order to include it into our model, the simplest possibility in our framework appears to be the so called (3+1) scheme, that may account for the explanation of all neutrino anomalies [14–16]. For this purpose, we include a left-handed singlet neutrino in the model, $\nu_s$, that carries its own lepton number $L_s$, and two odd scalar singlets, besides $\chi$, which we call $\delta_1$ and $\delta_2$. In order to get the final mass pattern, extra softly broken global symmetries are needed. Following our principle, we take those symmetries to be linear combinations of the four lepton numbers: $L_{e,\mu,\tau,s}$. Let us consider, for instance, three of such global symmetries: $U(1)_L^1$; $U(1)_L^2$ and $U(1)_L^3$, where $L_{1,2,3}$ represents our specific choice for the linear combinations of lepton numbers. We have taken such symmetries among the total active lepton number, $L = L_e + L_\mu + L_\tau$; the sterile lepton number, $L_s$; the total lepton number, $L_T = L + L_s$; and the combinations: $L' = L_e - L_\mu - L_\tau$ and $L'' = L_s - L$. We always pick up $L_2 = L'$ to maintain as much as possible our previous results. The quantum numbers associated to the chosen symmetries and carried by the scalar fields are depicted in Table I, where three different models are considered. All these models have the same output, they provide the following higher dimensional mass terms (at the lower order) that conserve such symmetries:

i) Active flavour mass terms:

$$f_{ij} L_i H L_j H \frac{\chi^2}{M^3}, \quad (14)$$

where the Yukawa couplings $f_{ij}$; for $i = e\mu\tau$, are only constrained by the $L'$ symmetry. These terms give rise to the same mass matrix as in Eq. (11), and same order of parameters, that now we assume.
ii) Active-sterile couplings:

\[
\frac{g_e L_e H' \nu_s \chi^2 \delta_1}{M^3} + (g_\mu L_\mu + g_\tau L_\tau) \frac{H' \nu_s \chi^2 \delta_2}{M^3}.
\]

(15)

In this case the Yukawa couplings are all non zero, and if \(\langle \delta_{1,2} \rangle\) are both the same order then the mass couplings, \(\mu_i \sim \langle H' \rangle \langle \chi \rangle^2 / M^2\), are comparable among each other. For simplicity one may take the light hierarchy \(g_e : g_\mu : g_\tau \approx 1 : c_\theta : s_\theta\). This choice is suggested by the fact that they are governed by \(L'\) as well as the active mass terms. Some amount of tuning may be needed in our example, but small deviations of our choice will not affect our conclusions. However, this particular choice will allow us to make more explicit calculations later on.

iii) Finally, the sterile mass term:

\[
h \nu_s \nu_s \chi^2 \delta_1 \delta_2 M^3.
\]

(16)

That gives rise to the sterile mass, \(m_s \sim \langle \chi \rangle^2 \langle \delta \rangle / M^3\). Now, if we take \(\langle \chi \rangle \sim 0.3\) GeV, as before with \(M \sim 20\) TeV and \(\langle \delta \rangle \sim 400\) GeV, we get for \(h \sim 1\), the sterile mass \(m_s \sim 2\) eV, which is adequate to explain the mass difference required by the LSND data.

After introducing expectation values we get, in the basis \((\nu_\alpha, \nu_s)\), the mass matrix,

\[
\mathcal{M} = \begin{pmatrix}
\mathcal{M}^{(0)} & \tilde{\epsilon} m_s \\
\tilde{\epsilon}^\dagger m_s & m_s
\end{pmatrix},
\]

(17)

where \(\epsilon_i = \mu_i / m_s \sim (g_i / h) \langle H' \rangle / \langle \delta \rangle\). Thus, by taking \(g \sim \mathcal{O}(1)\), this gives us the hierarchy on masses, \(m \ll \mu_i < m_s\) that will reproduce the features of the \((3+1)\) scheme.

Let us now analyze how the required parameters appear from Eq. (17). First thing to notice is that at the lower order in \(\epsilon\), the unitary mixing matrix has the form:

\[
U = \begin{pmatrix}
U_{BM} & \tilde{\epsilon} \\
\tilde{\epsilon}^\dagger U_{BM} & 1
\end{pmatrix}.
\]

(18)

Therefore, as an immediate conclusion we have that \(U_{e3} = 0\), as desired by CHOOZ. Also, LSND arises with \(\sin^2 \theta_{LSND} = 4 |\epsilon_e \epsilon_\mu|^2\), that constrains the active to sterile couplings to values well in our preferred range of parameters (\(\mathcal{O}(10^{-1})\)).

Next, while the sterile mass remains almost unperturbed, all other mass terms in the active block get a see-saw type correction \([15]\),

\[
\mathcal{M}^{(0)} \rightarrow \mathcal{M}^{(0)} - \tilde{\epsilon} \tilde{\epsilon}^\dagger m_s.
\]

(19)

The effect of this correction is to break the degeneracy of \(\nu_1\) and \(\nu_2\) induced by \(L'\), since in general it introduces small diagonal terms (among others). Thus, a nice feature of this model is that we do not need to invoke any other mechanism to generate those corrections. They come naturally due to the breaking of the other symmetries.

Let us now consider as an specific example the above suggested hierarchy of the \(g\) Yukawa couplings. We then take \(\tilde{\epsilon}^\dagger \sim (|\tilde{\epsilon}| / \sqrt{2}) (1, c_\theta, s_\theta)\). After rotating the active sector by \(U_{BM}\), \(\mathcal{M}^{(0)}\) becomes \(\text{Diag}(m, -m, 0)\), where \(m = \sqrt{\Delta m^2_{atm}} \approx 0.04\) eV, as before. The same rotation projects \(\tilde{\epsilon}\) along the direction \((1, 0, 0)\). Hence, \(\nu_3\) remain exactly massless, and
the 1-4 sector remains mixed with a mixing angle $\tan \beta \approx |\vec{\epsilon}|$. Therefore, after a complete diagonalization, one gets the exact mixing matrix:

$$U_{\text{mix}} = \begin{pmatrix}
\frac{c_\beta}{\sqrt{2}} & \frac{1}{\sqrt{2}} & 0 & \frac{s_\beta}{\sqrt{2}} \\
\frac{c_\beta s_\theta}{\sqrt{2}} & \frac{c_\theta}{\sqrt{2}} & \frac{s_\beta}{\sqrt{2}} & \frac{c_\beta s_\theta}{\sqrt{2}} \\
\frac{s_\beta}{\sqrt{2}} & \frac{c_\theta}{\sqrt{2}} & -c_\beta & \frac{s_\beta}{\sqrt{2}} \\
-s_\beta & 0 & 0 & c_\beta
\end{pmatrix}.$$  \hspace{1cm} (20)

The mass spectrum we then get is,

$$m_1 = m - |\vec{\epsilon}|^2 m_s ,$$

$$m_2 = -m ,$$

$$m_3 = 0 ,$$

$$m_4 = m_s (1 + |\vec{\epsilon}|^2) .$$  \hspace{1cm} (21)

Let us see how the solar neutrino mass arise. Looking at the spectrum (21), we require $\Delta m^2_{21} = m_2^2 - m_1^2 = \Delta m^2_{\odot}$, that renders,

$$|\vec{\epsilon}|^2 \approx \frac{2m}{m_s} - \frac{\Delta m^2_{\odot}}{2m m_s} .$$  \hspace{1cm} (22)

This may imply a quite large cancellation on the parameters of the theory, and is a manifestation of the constrained solution we are discussing. The required fine tuning would, however, be not too large. For large mixing angle solutions one needs that $|\vec{\epsilon}|^2$ be adjusted at the level of 10% at most, such that the difference $2m - |\vec{\epsilon}|^2$ be of the order of $10^{-3}$.

By taking $m_s \sim 1 - 2$ eV as the LSND scale (just as it came from our previous general analysis), for the large mixing MSW solution of solar neutrino problem, one gets: $|\vec{\epsilon}|^2 \approx 0.04 - 0.08$. Therefore, our prediction for the LSND oscillation probability is

$$\sin^2 2\theta_{\text{LSND}} = \cos^2 |\vec{\epsilon}|^4 \approx (0.8 - 3.2) \times 10^{-3},$$  \hspace{1cm} (23)

where the extreme right hand side has been evaluated assuming maximal mixing in the active sector. We should comment that, as already observed in the literature \[14,15\], this also pushes Bugey and CDHS to the limit. With our set of parameters we get: $\sin^2 2\theta_{\text{Bugey}} \sim 0.04 - 0.08$ and $\sin^2 2\theta_{\text{CDHS}} \sim 0.08 - 0.16$. On the other hand, this could also be interpreted as a chance to have accessible signals of $\nu_{e,\mu}$ disappearance in future experiments. Given our (inverted) hierarchy, other mixing parameters are given as

$$\sin^2 2\theta_{\text{atm}} = \sin^2 2\theta \cdot \left(1 - \frac{1}{2} |\vec{\epsilon}|^2\right) \approx 0.96 - 0.98 ,$$

$$\sin^2 2\theta_{\odot} = (1 - |\vec{\epsilon}|^2) \approx 0.92 - 0.96 ,$$  \hspace{1cm} (24)

which means that solar and atmospheric are both explained by large mixing angle solutions in our model.
V. CONCLUDING REMARKS

We have presented a new mechanism that may generate small neutrino masses at tree level which does not need the existence of large scales for new physics but rather make use of higher dimensional operators generated at the TeV scale. In the realization of our mechanism the smallness of the neutrino masses are due to the large mass suppression on dimension seven operators which are involved in the generation of masses and due to the presence of a relatively small vacuum of a scalar particle which (softly) breaks down lepton number. Such a vacuum arises from a singlet scalar realization of the so called Type II see-saw mechanism. As a direct application we have explored the generation of textures that may explain the neutrino anomalies. Given the flavour symmetry $L' : L_e - L_\mu - L_\tau$ we generate the bimaximal neutrino mixing pattern, which we further extended by adding extra symmetries and a sterile neutrino which also obtains a eV mass, giving rise to the (3+1) scenario that may account for including LSND in the scheme. Our mechanism may be useful in theories where the fundamental (string) scale is in the TeV range, since it relays only on brane physics for the generation of neutrino masses.

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TABLES

| Field | Model I  | Model II | Model III |
|-------|----------|----------|-----------|
|       | $L_T$    | $L'$     | $L''$     | $L_T$    | $L'$     | $L_s$    | $L$       | $L'$     | $L_s$    |
| $\chi$ | -1       | 0        | 1         | -1       | 0        | 0        | -1        | 0        | 0        |
| $\delta_1$ | 0       | -1       | -2        | 0        | -1       | -1        | 1         | -1       | -1       |
| $\delta_2$ | 0       | 1        | -2        | 0        | 1        | -1        | 1         | -1       | 1        |

TABLE I. Assignments of charges for the scalar fields in three different models with the flavour symmetries as shown, which give rise to the (3+1) scenario. Here $L_T = L_s + L$; $L = L_e + L_\mu + L_\tau$; $L' = L_e - L_\mu - L_\tau$; and $L'' = L_s - L$. Standard Higgs doublets are in all cases chargeless under this symmetries.
FIGURE 1. Tree-level realization of the effective operator in Eq. (4).