Cluster structure in light neutron-rich nuclei

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Abstract. We investigated molecular states and cluster gas-like states in light nuclei based on microscopic calculations with antisymmetrized molecular dynamics. In particular, we focused on excited states of \(^{13}\text{B}\) and \(^{8}\text{He}\). It is suggested that deformed bands, \(K^\pi = 3/2^-\) and \(K^\pi = 1/2^+\), with molecular orbital structures appear in the excited states of \(^{13}\text{B}\). In the results of \(^{8}\text{He}\), we proposed possible existence of the \(0^+_2\) state with dineutron gas-like structure of \(^4\text{He}+2n+2n\), which is associated with the 3α-cluster structure of the \(^{12}\text{C}(0^+_2)\) state.

1. Introduction
Cluster physics started a long time ago as possible existence of α particles in nuclei was considered in 1930’s. In 1960’s and 1970’s, microscopic studies with cluster models has been developed and applied to nuclei covering the sd-shell region as well as the p-shell region. From 1990’s, remarkable progress of cluster physics has been made in the development of experimental and theoretical studies on unstable nuclei while involving the advance of innovative theoretical models. It has been revealed that cluster aspect often appears in light unstable nuclei as well as stable nuclei. Moreover, in order to understand various exotic phenomena discovered in unstable nuclei, we need to extend the concept of cluster to a further general meaning of “spatial correlation of multi nucleons”.

The conventional cluster structure is based on weak coupling picture. Clusters are formed by tightly bound nucleons, and inter-cluster motion is important degree of freedom. Let us turn to structure of unstable nuclei. Neutron halo structure, where valence neutron motion around the cluster core is important, is regarded a kind of cluster structure. In case of two neutron halo nuclei, di-neutron correlation is one of hot subjects concerning cluster aspect. Another interesting structure is molecular orbital structures, which has been discovered in Be isotopes, and also has been suggested in heavier nuclei like Ne isotopes[1]. Recently, further novel cluster structure has been suggested in excited state of stable nuclei as well as unstable nuclei. For example, 3α gas state of the second \(0^+\) state in \(^{12}\text{C}\) has been recently proposed by Tohsaki et al.[2], and attracts great interests concerning with the BEC(Bose Einstein condensation) phenomena in nuclear system. In heavier-mass nuclei like sd-shell nuclei, coexistence of cluster and mean-field natures becomes more essential. For example, it is suggested that cluster effect gives contribution in largely deformed states such as superdeformation in \(^{32}\text{S}\) and \(^{40}\text{Ca}\). Moreover, multi-nucleon correlation may promote such exotic structure as decoupling of proton and neutron deformation in unstable nuclei as is discussed in \(^{16}\text{C}\). These facts indicate that “cluster”(in the extended meaning) is one of the essential features in nuclear many-body system. Then, it is a key issue to make systematic study focusing on cluster phenomena.
In this paper, we focused on molecular states and cluster gas-like states in light nuclei. Based on theoretical calculations with the antisymmetrized molecular dynamics, we discussed molecular states in $^{13}\text{B}$, and dineutron structure in $^{8}\text{He}$.

This paper is organized as follows. In the next section, we briefly explain the formulation of theoretical framework. The molecular orbital states are reported in section 3, and cluster gas-like states are discussed in section 4. Finally, we give a summary in section 5.

2. Formulation

The detailed formulation of the AMD method for nuclear structure study is described in [3, 4, 5]. An AMD wave function is given by a Slater determinant of Gaussian wave packets;

$$\Phi_{\text{AMD}}(Z) = \frac{1}{\sqrt{A!}} A\{\varphi_1, \varphi_2, \ldots, \varphi_A\},$$

where the $i$th single-particle wave function is written by a product of spatial($\phi$), intrinsic spin($\chi$) and isospin($\tau$) wave functions as,

$$\varphi_i = \phi_{X_i} \chi_i \tau_i,$$

$$\phi_{X_i}(r_j) \propto \exp\{-\nu (r_j - \frac{X_i}{\sqrt{\nu}})^2\},$$

$$\chi_i = \left(\frac{1}{2} + \xi_i\right)\chi_1 + \left(\frac{1}{2} - \xi_i\right)\chi_1.$$  

$\phi_{X_i}$ and $\chi_i$ are spatial and spin functions, and $\tau_i$ is isospin function which is fixed to be up(proton) or down(neutron). Accordingly, an AMD wave function is expressed by a set of variational parameters, $Z \equiv \{X_1, X_2, \ldots, X_A, \xi_1, \xi_2, \ldots, \xi_A\}$. In calculations of B, we performed energy variation after spin parity projection(VAP) within the AMD model space, as was done in the previous studies[5, 6], while we applied the AMD+GCM to He isotopes. In the AMD+GCM calculations, we adopted the total oscillator quanta and deformation as the constraints in order to obtain basis wave functions.

3. Molecular orbital structure

Molecular orbital states are favoured in low-lying states of neutron-rich Be isotopes[1]. In $^{12}\text{Be}$, the intruder state with the dominant $2h\omega$ configuration drops down to the ground state because of the developed $2\alpha + 4n$ molecular-orbital structure. This phenomena of the intruder configuration in the $^{12}\text{Be}$ ground state is known to be breaking of neutron magicity. On the other hand, a neighbouring nucleus, $^{13}\text{B}$, has the ground state with normal $0h\omega$ configuration. Instead, it is expected that molecular orbital states may appear in the excited states of $^{13}\text{B}$.

$^{13}\text{B}$ is a nucleus with neutron magic number $N = 8$, and its ground state is the $3/2^-$ state with the neutron $p$-shell closure. Above the $3/2^-$ ground state of $^{13}\text{B}$, many states are experimentally known to exist in the excitation energy $E_x \geq 3.5$ MeV region. Unfortunately, spins and parities are unknown for most of the excited states. Recently, the state at 4.83 MeV has been assigned to be a $1/2^+$ state by $^{4}\text{He}(^{12}\text{Be},^{13}\text{B}\gamma)X$ experiments[7]. Because of its strong production via proton-transfer to the $^{12}\text{Be}(0^+)$ state, this excited state is suggested to be a proton intruder state.

The calculated energy levels of the negative- and positive-parity states of $^{13}\text{B}$ are shown in Fig. 1. The effective interaction is same as that adopted in the previous $^{12}\text{Be}$ calculations[6]. we obtained many excited states with various $J^\pi$ in the region $E_x \geq 4$ MeV above the ground $3/2^-$ state. Now, we focus on the deformed bands obtained in the calculations. We obtained three largely deformed bands, $K^\pi = 3/2^-$, $K^\pi = 1/2^+$ and $K^\pi = 1/2^-$ (solid lines). These bands are composed of intruder states or well-developed cluster states.

In this paper, we focused on molecular states and cluster gas-like states in light nuclei. Based on theoretical calculations with the antisymmetrized molecular dynamics, we discussed molecular states in $^{13}\text{B}$, and dineutron structure in $^{8}\text{He}$.
In the \( K^\pi = 3/2^- \) band, two valence neutrons occupy an approximately positive-parity orbital, and the other two neutrons and a proton are in orbitals with dominant negative-parity component. Since the negative- and positive-parity orbitals of the valence nucleons are associated with the \( p \)-orbits and \( sd \)-orbits, respectively, we can roughly describe the states in the \( K^\pi = 3/2^- \) band as the neutron \( 2\hbar \omega \) excited configurations. What is interesting is that the positive-parity orbital of the last two neutrons is largely deformed and has nodes along the \( 2\alpha \) direction (longitudinal axis). This orbital well corresponds to the so-called \( \sigma \) orbital in the molecular orbital picture [1, 8, 9, 10, 11, 12]. It is surprising that the predicted \( K^\pi = 3/2^- \) band has a similar neutron structure to that of the \(^{12}\text{Be} \) ground state. Thus, we conclude that the \( K^\pi = 3/2^- \) is the band of the intruder neutron \( 2\hbar \omega \) states, and can be interpreted as \(^{12}\text{Be}(0^+_1)+p\), where the \(^{12}\text{Be}(0^+_1)\) has the intruder configuration and the additional proton strongly couples to the deformed core.

The band-head state \( 1/2^+_1 \) of the \( K^\pi = 1/2^+ \) is the proton intruder state with a large deformation. We tentatively assign this state to the experimental \( 1/2^+(4.83 \text{ MeV}) \) state. In this state, \( 1\hbar \omega \) excitation occurs in the proton shell. Namely, the last proton occupy a \( \sigma \)-like orbital, which is quite similar to that of the highest neutron orbital in the \( K^\pi = 3/2^- \) band and also that in the \(^{12}\text{Be}(0^+_1)\). Although this proton intruder structure seems to contradict the naive expectation that neutron excitations are favoured in neutron-rich nuclei, however, it can be naturally understood in the molecular orbital picture with the lowering mechanism of the \( \sigma \) orbital due to the developed two-center structure. The band-head \( 1/2^+ \) state is predicted at \( E_x = 8 \text{ MeV} \) in the present calculation. The observed state at 4.83 MeV, which has been recently assigned to be a \( 1/2^+ \) state by \(^4\text{He}(^{12}\text{Be},^{13}\text{B}\gamma)X \) experiments [7], is a candidate of this state. Because of strong production via the proton-transfer to the \(^{12}\text{Be}(0^+_1)\) state and analysis of angular dependence, Ota et al. suggested that this \( 1/2^+ \) state may have large component of the proton intruder configuration. It is consistent with the present result though the theoretical excitation energy is slightly higher than the experimental value.

As for other excited states (disconnected filled circles in Fig. 1), intrinsic deformation is small or as large as normal deformation at most. These are considered to be the \( 0\hbar \omega \) or neutron \( 1\hbar \omega \) configurations.

![Figure 1](image-url). Excitation energies of the negative- and positive-parity states of \(^{13}\text{B} \) calculated by VAP. The basis wave functions obtained by VAP are superposed. Filled circles are the energies of the \( J^\pi_n \) states, for which the VAP calculations were done. Open circles are the energies of the \( J^\pi_n \) states, which were obtained by diagonalization of Hamiltonian by superposing wave functions, but the VAP calculations were not performed for the corresponding \( J^\pi_n \) states.
4. Cluster gas-like states

Recently, Tohsaki et al. proposed a new type of cluster structure in the second 0\(^+\) state of \(^{12}\)C, where 3 \(\alpha\) clusters are weakly interacting[2]. This state is called “alpha condensation” in association with Bose-Einstein Condensation(BEC) which was suggested in dilute nuclear matter by Röpke et al.[13]. One of our aim is to search for such cluster gas-like states with a dilute density in other nuclei. In analogy to the alpha condensation, dineutron condensation in neutron matter is a recent key issue in physics of unstable nuclei. In real nuclear systems, one should focus on dineutron correlation in finite nuclei such as halo nuclei and extremely neutron-rich nuclei, or that in neutron skin at a surface region of neutron-rich nuclei, as is discussed in two-neutron halo nuclei like \(^6\)He and \(^{11}\)Li. From a Let us consider \(^8\)He from a point of view of the dineutron condensation, because this has a extreme proton-neutron ratio 1:3, and two pairs of neutrons might be possible because it has four valence neutrons around the \(^3\)He core. In analogy to \(^{12}\)C, the ground state of \(^8\)He may have a feature of the neutron \(p_{1/2}\) closure or the SU(3)-limit p-shell configuration. Instead of the ground state, one can speculate the dineutron gas-like state with developed \(^4\)He+2n+2n structure in excited states.

We calculated the ground and excited states of He isotopes with the method of AMD+GCM. In the present work, we used effective nuclear interactions consisting of the central force(Volkov[14] or MV1[15]), the spin-orbit force and Coulomb force. We used the spin-orbit force of the G3RS force[16] with a strength of \(u_{ls} = 2000\) MeV. We carefully tuned the interaction parameters in the central force by taking care of energies of subsystems. \(^6\)He, \(^8\)He and \(^{10}\)He were calculated with AMD+GCM by using 2 sets(v58 and m56) of the interaction parameters. The adopted interaction parameters are described in table 1. In order to demonstrate characteristics of the effective interactions, we show the relative energies of subsystems and the nucleon-nucleon scattering lengths calculated with these interactions. We estimate the energies of the \(^4\)He, \(^4\)He-n, and \(^4\)He-\(^4\)He state with the \((0s)^4\) \(\alpha\) clusters for simplicity. The adopted v58 and m56 interactions systematically reproduce the energies of \(^4\)He, \(^6\)He and \(^8\)He, though they overestimate the \(^{10}\)He energy. The v58 interaction well describes the energies of subsystems except for the fault of the too strong neutron-neutron interaction, while the m56 interaction reasonably reproduces the global features of the subsystem energies.

**Table 1.** The theoretical values of scattering length \(a_s(a_l)\) for singlet(triplet) even channel, one- and two-neutron separation energies and 2\(\alpha\) threshold energy of \(^8\)Be calculated by using the v58 and the m56 interactions. Volkov No.2 with \(w = 0.42, b = h = 0, m = 0.58\) and MV1-case(3) with \(w = 0.44, b = h = 0.15, m = 0.56\) are adopted as the central force of the v58 and m56 interactions, respectively.

|                | exp.  | v58  | m56  |
|----------------|-------|------|------|
| \(a_s\) (fm)   | 5.42 (p-n) | 9.7  | 4.2  |
| \(a_l\) (fm)   | -16.5 (n-n) | 9.7  | >100 |
| \(S_n\) (\(^3\)He) (MeV) | -0.9 | -0.7 | -0.4 |
| \(2E(\(^4\)He)−E(\(^4\)He−\(^4\)He)\) (MeV) | -0.1 | 0.6  | -0.6 |
| \(S_{2n}\) (\(^6\)He) (MeV) | 1.0  | 1.3  | 1.1  |
| \(S_{2n}\) (\(^8\)He) (MeV) | 2.1  | 3.0  | 2.0  |

The calculated energy levels of \(^8\)He are illustrated in Fig. 2. In both of the m56 and v58 results, the \(2^+_1\) state is the lowest excited state, and the \(0^+_2\) state appears in the same energy region as the \(1^+_1, 2^+_1\) and \(3^+_1\) states. Compared with the experimental data, the \(2^+_1\) excitation energy are reproduced well in the m56 calculation. Although the excitation energies depends on the interaction, however, it is important that the relative level structure among the excited states is not sensitive to the adopted interaction. The \(0^+_2\) state is theoretically suggested to
appear at about 5 MeV higher energy than $2^+_1$ state. What is striking is that the $0^+_2$ state has a remarkably large neutron radius ($r_{\text{m.s.t.}}=3.1$ fm in case m56) compared with the ground state($r_{\text{m.s.t.}}=2.6$ fm in case m56) because of the developed $^4\text{He}+2n+2n$ structure. In the obtained wave function for the $0^+_2$ state, which is given by a superposition of the basis AMD wave functions, the amplitude is found to be widely distributed into the basis wave functions with various spatial configuration of $^4\text{He}+2n+2n$. This indicates a gas-like feature that the dineutrons are rather freely moving around the $^4\text{He}$ core. Therefore, we consider that the $0^+_2$ state is the candidate of the cluster gas-like state with two dineutrons around the $^4\text{He}$ core.

![Figure 2](image_url)  
**Figure 2.** Energy levels of $^8\text{He}$. The calculated results are those with the m56 and v58 interactions.

In order to investigate the features of dineutron cluster structure in the $0^+$ states of $^8\text{He}$, we extracted the $2n$-cluster motion from the obtained $^8\text{He}(0^+)$ wave functions by assuming a dineutron written by the $(0s)^2$ wave function. We assume a simple core $(^4\text{He}+2n)_{0^+}$ which is equivalent to the SU(3)-limit $^6\text{He}(0^+)$, and form the $^6\text{He}^{SU(3)}(0^+)-2n$ cluster wave function with the $L=0$ relative motion between the core $^6\text{He}^{SU(3)}(0^+)$ and the $2n$ cluster. We calculated the reduced width amplitudes $r_y(r)$ for the $2n$-cluster motion by taking the overlap of the $^6\text{He}^{SU(3)}(0^+)-2n$ cluster wave functions with the $^8\text{He}$ wave functions. In the analysis of the reduced width amplitudes, which correspond to the $2n$-cluster motion in $^8\text{He}$, the ground state is found to be dominated by the $p_{3/2}$ sub-shell closed component with a mixing of SU(3)-limit $^4\text{He}+2n+2n$ structure. On the other hand, the $^8\text{He}(0^+_2)$ state has the large amplitude of the dineutron cluster in the long distance region around $r=4-6$ fm. Considering that the other $2n$ cluster exists inside the $^6\text{He}^{SU(3)}(0^+)$ core, it is regarded that the $^8\text{He}(0^+_2)$ has the component of the developed $^4\text{He}+2n+2n$ clustering, where two dineutrons are moving in the $L=0$ orbit. Because of the analogy to $^{12}\text{C}$, the $^8\text{He}(0^+_2)$ is considered to contain the dineutron gas-like structure.

5. Summary

We theoretically investigated molecular states and cluster gas-like states in light nuclei based on calculations with antisymmetrized molecular dynamics. The theoretical results of $^{13}\text{B}$ suggest that the intruder states with molecular orbital structure construct the rotational bands, $K=3/2^-$ and $K=1/2^+$, starting from 5 MeV and 8 MeV, respectively. The ground and excited states of He isotopes were also investigated. It was found that the ground state of $^8\text{He}$ is dominated by the $p_{3/2}$ sub-shell closed component with the mixing of SU(3)-limit $^4\text{He}+2n+2n$ structure. The present results suggest that the $0^+_2$ state may appear above the $2^+_1$ state. By analyzing dineutron structure, it was found that this state has a significant component of the
developed $^4\text{He}+^2n+^2n$ structure where two dineutrons are moving around the $^4\text{He}$ core in $S$ wave to a dilute density. Because of the analogy of the $^2n$-cluster motion in the $^8\text{He}(0^+_2)$ state with the $\alpha$-cluster motion in the $^{12}\text{C}(0^+_2)$ state, we consider that the predicted $0^+_2$ state is the candidate of the dineutron gas-like state. In future study, widths of the excited states should be carefully investigated by taking into account the continuum coupling in order to confirm the stability of the resonances against particle decays.

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