Multiplayer Repeated Game of Environmental Protection between Government and Enterprises

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Abstract. An important source of environmental pollution in our country is industrial enterprises, which have caused great damage to the ecological environment. The government continues to pay more attention to environmental protection, while enterprises aim to maximize their own interests, to the detriment of the overall social interests. We propose a probability game with economic incentive mechanism between government and polluting enterprises. In the game, each enterprise decides whether to invest in environmental protection. Considering the benefits of selfish enterprises, local governments, as managers, adopt a zero determinant (ZD) strategy for enterprises. This strategy has played an important role in minimizing social costs. It has introduced the entire society's environmental protection investment into the game. We simulate the game model to evaluate the performance of the ZD strategy numerically.

Keywords. Zero determinant; environmental protection game; social cost.

1. Introduction
People have realized that extensive economic development has paid huge resources and environmental costs, and resource allocation and economic development methods must be transformed. The use of collaboration between the government and enterprises is conducive to environmental protection and pollution control, as well as reducing social costs. However, the government's environmental protection investment and the social cost of each enterprise's environmental protection investment, as well as the factors affecting the cost have not been fully studied. In this paper, we use game theory to find ways to minimize government and corporate environmental protection investment and social costs.

The openness of environmental pollution corporate behavior provides companies with opportunities to demonstrate selfish behavior. For example, companies can choose to "free ride" [1], which will harm the overall social interests. Therefore, a challenging issue is to encourage cooperation between enterprises to promote high efficiency in pollution control [2]; at the same time, the environmental externalities of industrial enterprises’ pollution discharge make the Nash equilibrium pollution discharge exceed the Pareto optimal discharge, resulting in "tragedy of the commons" [3]. It can be seen that environmental governance is not only a technical problem, but also a realistic dilemma under the conflict of different interest demands and behavior orientations of complex
stakeholders [4]. Therefore, classical game theory is widely used to reveal the interests of multiple stakeholders and has achieved fruitful results.

At present, various game models have been studied in many fields, such as prisoner’s dilemma game, public goods game, snowdrift game and Stackelberg game [5-7] and other game models are applied to optimization control [8], frequency sharing [9], Evolutionary Cooperation [10]. Assuming that all participants expect their own costs to be the lowest, regardless of the costs of other participants, study their Nash equilibrium conditions [11-14]. But in most cases, the Nash equilibrium state is not able to reach the lowest social cost. In fact, every participant wants to benefit from the strategies of other participants at the lowest cost. Through PoA [15, 16] the Nash equilibrium conditions are further studied. The PoA is the ratio between the social cost in the Nash equilibrium state and the global optimal social cost. However, it is difficult to achieve the best social cost in a non-cooperative game. Authors in Ref. [17] proposed a mixed equilibrium, which calculates PoA by defining the average approximation cost as the average social optimal value.

In this paper, we consider the zero determinant (ZD) strategy [6] proposed by Press and Dyson to reveal the relationship between game strategy and expected cost. Participants with ZD policies can control the expected costs of other participants regardless of their policies. In this paper, we propose a repeated game model with economic incentives for government and enterprises to invest in environmental protection. In the aspect of environmental protection, we use zero determinant strategy to enterprises as the administrator. The government has considered the environmental protection cost of the whole society. The environmental protection exceeds the private interests of each enterprise. Then, the government can adjust the cost by implementing the zero determinant strategy, regardless of the strategy chosen by other enterprises.

In this article, we use the zero determinant (ZD) strategy to reveal the relationship between game strategy and cost. Participants with ZD strategy can control the opponent's expected cost regardless of the opponent's strategy. We propose a repeated game model with economic incentives for government and corporate environmental protection investment. The government uses the zero-determinant strategy to enterprises as an environmental protection administrator. The government has considered the environmental protection costs of the entire society, surpassing the private interests of individual enterprises. And the government can adjust costs by implementing a zero-determinant strategy, regardless of the strategy chosen by other companies.

The rest of this paper is arranged as follows. The second part summarizes the game model of pollution control between government and enterprises. The corresponding analysis of the zero determinant strategy is introduced in the third part. In the fourth section, the proposed strategy is verified numerically. Finally, the fourth section summarizes the full text.

2. Game Model of Pollution Control
This paper studies the social cost of environmental pollution control, which consists of local government (G) and a number of industrial enterprises (E) investing in environmental protection funds. They play games with each other in terms of investment cost. Local government G is responsible for economic incentives and social costs. This paper explores the implementation of zero determinant (ZD) strategy by local government G to reduce social costs to the greatest extent. The ZD strategy is actually a conditional probability strategy, which was first studied in the two-person repeated prisoner’s dilemma game. No matter which strategy the opponent adopts, the party using the ZD strategy can set the opponent's expected cost, or control it to a certain ratio. The trade-off between investment in environmental protection funds and pollution control costs (e.g. purchase of pollution control equipment, investment in labor costs) is very different for each enterprise. We assume that each enterprise has the same cost investment. Industrial enterprise E pursues maximum private interests, while local government G considers that the overall environmental and social costs exceed its personal interests. In this study, we assume that pollution control costs are more than the investment in environmental protection in advance.
In repeated game of environmental protection, enterprise decides whether to invest in environmental protection. Suppose each enterprise's environmental protection investment cost $w$, and pollution cost $h$ (we set it as the punishment of local government for polluting enterprises). If an enterprise invests in pollution control equipment and manpower to protect the environment, the enterprise is regarded as a partner ($C$); Otherwise, the enterprise is regarded as a traitor ($D$) and is at risk of being punished for environmental pollution by the government. However, because of the strength of government inspection (the strength of government inspection can be expressed by the cost of government investment in environmental protection), not every traitor can be punished, we assume that the probability of punishment for traitors is $\tau$ ($0 \leq \tau \leq 1$).

We have designed an environmental protection game to minimize the cost of social environmental pollution input, with an economic incentive mechanism. The government $G$ considers the overall social interest, not the individual interest. Considering that there are $N$ participants repeating the game of environmental protection, in each round of game, each enterprise decides whether to invest in environmental protection. The results of each game round $r$ are indicated by $S(r) = \{s_1, s_2, ..., s_t, ..., s_T\}$, where $s_t \in \{G, C, D\}$ are the strategies of the government and enterprises $i$. And $L = S(r - 1)$ was the result of the last round $r - 1$. All the results of each round of games are collectively recorded as a set $\Omega$. For example, in a game of three players, $\Omega = \{CCC, CCD, CDC, CDD, DCC, DCD, DDC, DDD\}$ . So $L_j \in \Omega (j = 1, 2, 3, ...8)$ . The cost of the government or enterprise $i$ with results $L$ in the previous round is expressed by the following formula:

$$U_i^L = \begin{cases} W_i & (s_i = C) \\ \tau H_i & (s_i = D) \end{cases} \quad (1)$$

Assuming there are three participants (one government, two companies) in the game, figure 1 shows the cost of each participant under the results of their cooperation and betrayal respectively. The reputation mechanism will affect repeated games, so in order to encourage companies to cooperate, suppose that when all companies choose to cooperate (pollution control), each partner will have economic incentives $F_i (F < W_i)$. In the repeated game, if the result of the previous round of government and enterprise selection is $L_j$, the probability of the current round of government G choosing cooperation is $p_{Gj}^{L_j}$, and the probability of enterprise 1 and enterprise 2 choosing cooperation is $p_{1j}^{L_j}$ and $p_{2j}^{L_j}$ respectively, then the process is a Markov process. The probabilistic strategy of local government G and industrial enterprise E determines the transition matrix of Markov process, as shown in figure 2.

| Result $L_j$ | CCC | CCD | CDC | CDD | DCC | DCD | DDD |
|--------------|-----|-----|-----|-----|-----|-----|-----|
| Cost of Government $U_{1j}$ | $W_i - F_i$ | $W_i - F_i$ | $W_i - \tau H_i$ | $W_i - \tau H_i$ | $W_i - \tau H_i$ | $W_i - \tau H_i$ | $W_i - \tau H_i$ |
| Cost of Enterprise 1 $U_{1j}$ | $W_i - F_i$ | $W_i - \tau H_i$ | $W_i - \tau H_i$ | $W_i - \tau H_i$ | $W_i - \tau H_i$ | $W_i - \tau H_i$ | $W_i - \tau H_i$ |
| Cost of Enterprise 2 $U_{2j}$ | $W_i - \tau H_i$ | $W_i - \tau H_i$ | $W_i - \tau H_i$ | $W_i - \tau H_i$ | $W_i - \tau H_i$ | $W_i - \tau H_i$ | $W_i - \tau H_i$ |

**Figure 1.** All possible results of three participants’ environmental protection investment game (one government, two companies) and corresponding costs for each participant.

| $p_{G1}^{L_j}p_{1j}^{L_j}$ | $p_{G2}^{L_j}p_{2j}^{L_j}$ | $p_{G1}^{L_j}p_{1j}^{L_j}(1-p_{1j}^{L_j})$ | $p_{G2}^{L_j}p_{2j}^{L_j}(1-p_{2j}^{L_j})$ | $p_{G1}^{L_j}(1-p_{1j}^{L_j})p_{1j}^{L_j}$ | $p_{G2}^{L_j}(1-p_{2j}^{L_j})p_{2j}^{L_j}$ | $(1-p_{G1}^{L_j})p_{G1}^{L_j}(1-p_{1j}^{L_j})p_{1j}^{L_j}$ | $(1-p_{G2}^{L_j})p_{G2}^{L_j}(1-p_{2j}^{L_j})p_{2j}^{L_j}$ | $(1-p_{G1}^{L_j})(1-p_{1j}^{L_j})(1-p_{2j}^{L_j})$ | $(1-p_{G2}^{L_j})(1-p_{2j}^{L_j})(1-p_{1j}^{L_j})$ |
|----------------|----------------|----------------|----------------|----------------|----------------|----------------|----------------|----------------|----------------|

**Figure 2.** An 8×8 3-player game Markov transition matrix.
The cost vector of the government or enterprise \( i \) is \( U_i = [U_i^{L_1},...,U_i^{L_j},...,U_i^{L_n}]^T \), where \( U_i^{L_j}(j \in \{1,2,...,|I|\}) \) is the cost corresponding to the \( j \) (the cost vector of the government is \( U_G \)). Figure 1 shows cost examples corresponding to all possible outcomes of the environmental protection game among 3 participants (1 government and 2 enterprises). The set of elements in one row in figure 1 represents the cost vector of an enterprise. The government is responsible for reducing costs. Its strategy is represented by a vector \( P_G = [p_G^{L_1},...,p_G^{L_j},p_G^{L_{j+1}},...,p_G^{L_n}]^T \), which is the conditional cooperation possibility related to each possible result of the previous round of games. The government’s goal is to optimize the expected cost of each enterprise (social cost), expressed by \( S_C \). Every enterprises should have more motivation to cooperate in repeated games, in order to pursue their long-term interests.

We considered a repeated game of government and enterprise investment in environmental protection. Enterprises that choose not to invest in environmental protection may be punished by the government, while each enterprise pays a cost for investing in environmental protection. As we all know, enterprises are self-interested, so there is a dislocation between the interests of enterprises and public society, which leads to a dilemma of environmental protection. Every participant who pursues his own interests will bring sub-optimal social costs. In order to achieve social optimality, the impact of personal interests on social costs should be minimized. As a manager, the government can unilaterally handle the social cost by implementing ZD strategy, no matter what strategy other enterprises adopt.

In \( N-1 \) enterprises, the number of defectors is set to be \( m \). The cooperator will not be punished, and the punishment probability of traitor \( i \) is \( \tau \). We consider a one-step memory strategy, which is more advantageous than a strategy with long memory. This round of action depends only on the results of the previous round. If local government \( G \) chose to cooperate in the last round, and the number of industrial enterprises \( E \) that chose to cooperate is \( n \) (\( n \leq N-1 \)), the pollution cost of each defector is \( \tau H \), and the probability that the government will take cooperation in this round is expressed as \( p_G^{C,n} \). Similarly, if local government \( G \) chose to betray in the last round, the number of industrial enterprises \( E \) that chose to cooperate is \( n \) (\( n \leq N-1 \)), the pollution cost of each traitor is \( \tau H \), the probability of adopting cooperative government in this round is expressed as \( p_G^{B,n} \).

3. Game Model and Zero Determinant Strategy

When all enterprises and governments play the game, local government \( G \) takes an appropriate probability strategy vector, expressed as \( p_G = [p_G^{C,N-1},...,p_G^{C,n},...,p_G^{C,n+1},...,p_G^{C,0},p_G^{B,n},...,p_G^{B,0}] \). This process is a Markov process (\( v^TM = v^T \)), \( M \) is expressed as the transition matrix, which depends on the probability strategy of the local government \( G \) and industrial enterprise \( E \), and \( v \) is the corresponding steady state vector. Define a matrix \( M' = M - I \), \( I \) is identity matrix. Performing a series of column transformations on column \( i \) of the matrix, the determinant value will not change, and all elements of column \( i \) will only be related to the strategy vector of the local government \( G \). The column \( i \) can be expressed as a vector \( \bar{p}_G = [-1+p_G^{C,n-1},...,1+p_G^{C,n},...,1+p_G^{C,0},p_G^{B,n},...,p_G^{B,0}] \). Taking three participants as an example, as shown in figure 3, the dot product of the steady state vector \( v \) of Markov matrix and any vector \( f \) is equal to a determinant, wherein the fourth column can only be controlled by the government and is expressed as a vector \( \bar{p}_G \). Replacing the last column of the matrix \( M' \) with \( U_G \), which is the cost vector of local government \( G \), the \( v \cdot U_G = det(p_1,...,p_G,...,p_N,U_G) \) is the expected cost of local government \( G \). We get the linear combination of the expected cost of local government \( G \) and all enterprises, according to the linear relationship between the cost vector and the expected cost, as follows:
\[
\sum_{i=1}^{N-1} \alpha_i E_i + \omega E_G + \xi = \frac{\det\left( p_1, \ldots, p_G, \ldots, p_N, \sum_{i=1}^{N-1} \alpha_i s_i + \omega s_G + \xi u \right)}{\det (p_1, \ldots, p_G, \ldots, p_N, u)}
\]  

(2)

where, \( \alpha_i (i = 1, 2, N - 1) \), \( \omega \) and \( \xi \) are constants, and \( u \) is a vector with all values of 1. If the local government G chooses a strategy, according to the equation (3):

\[
\bar{p}_G = \sum_{i=1}^{N-1} \alpha_i s_i + \omega s_G + \xi u
\]

(3)

\[
v^T \cdot f = \begin{vmatrix}
-1 + p_1 \alpha_1 \omega & -1 + p_2 \alpha_1 \omega & -1 + p_3 \alpha_1 \omega & -1 + p_4 \alpha_1 \omega & -1 + p_5 \alpha_1 \omega & f_1 \\
-1 + p_1 \alpha_2 \omega & -1 + p_2 \alpha_2 \omega & -1 + p_3 \alpha_2 \omega & -1 + p_4 \alpha_2 \omega & -1 + p_5 \alpha_2 \omega & f_2 \\
-1 + p_1 \alpha_3 \omega & -1 + p_2 \alpha_3 \omega & -1 + p_3 \alpha_3 \omega & -1 + p_4 \alpha_3 \omega & -1 + p_5 \alpha_3 \omega & f_3 \\
-1 + p_1 \alpha_4 \omega & -1 + p_2 \alpha_4 \omega & -1 + p_3 \alpha_4 \omega & -1 + p_4 \alpha_4 \omega & -1 + p_5 \alpha_4 \omega & f_4 \\
-1 + p_1 \alpha_5 \omega & -1 + p_2 \alpha_5 \omega & -1 + p_3 \alpha_5 \omega & -1 + p_4 \alpha_5 \omega & -1 + p_5 \alpha_5 \omega & f_5 \\
-1 + p_1 \alpha_6 \omega & -1 + p_2 \alpha_6 \omega & -1 + p_3 \alpha_6 \omega & -1 + p_4 \alpha_6 \omega & -1 + p_5 \alpha_6 \omega & f_6 \\
-1 + p_1 \alpha_7 \omega & -1 + p_2 \alpha_7 \omega & -1 + p_3 \alpha_7 \omega & -1 + p_4 \alpha_7 \omega & -1 + p_5 \alpha_7 \omega & f_7 \\
-1 + p_1 \alpha_8 \omega & -1 + p_2 \alpha_8 \omega & -1 + p_3 \alpha_8 \omega & -1 + p_4 \alpha_8 \omega & -1 + p_5 \alpha_8 \omega & f_8 \\
\end{vmatrix}
\]

\( v^T \cdot f = \det\left( p_1, \ldots, p_G, \ldots, p_N, \bar{p} \right) = 0 \). There is a linear relationship between the cost of local government G and industrial enterprise E, namely \( \sum_{i=1}^{N-1} \alpha_i E_i + \omega E_G + \xi = 0 \). Considering that each industrial enterprise E plays the same role, we make the coefficient \( \alpha_i = \mu \). We set \( \omega = 0 \), equation (3) can be simplified as

\[
\bar{p}_G^G = \sum_{i=1}^{N-1} \mu s_i + \xi u
\]

(4)

Among them, \( s_i \) is the cost vector of all enterprises. The equation of the industrial enterprise total expected cost: \( \sum_{i=1}^{N-1} \mu E_i + \xi = 0 \), and \( \bar{p}_G = \sum_{i=1}^{N-1} \mu s_i + \xi u \), then get the society total expected cost

\[
\sum_{i=1}^{N-1} E_i = -\frac{\xi}{\mu}
\]

(5)

Where \( E_i \) is the cost of enterprise \( i \) (\( i = 1, 2, \ldots, N - 1 \)), \( \xi \) and \( \mu \) are constants.

To link environmental pollution with social and economic losses, the industrial enterprise total expected cost is defined as the social cost (Sc).

In a game with \( N \) game participants (a government and N-1 enterprises), \( F \) is economic incentive and \( \tau \) is penalty ratio, and the costs of environmental protection investment and punishment are \( W \) and \( H \) respectively. According to formula (4), the following formula can be obtained:

\[
\begin{align*}
p_{C,n}^G &= 1 + \mu \left( (n+1)W + (N-n)\tau H \right) + \xi \\
p_{P,n}^G &= \mu \left( nW + (N-n)\tau H \right) + \xi \\
p_{C,N-1}^G &= 1 + \mu N(W - F) + \xi
\end{align*}
\]

(6)

Among them \( n \leq N - 1 \), which is the number of enterprises cooperating. According to the above formula, we have:
\[
\begin{align*}
    p_{c,n-2}^G &= 1 + \mu((N-1)W + \tau H) + \xi \\
    p_{d,n-1}^G &= \mu((N-1)W + \tau H) + \xi \\
    p_{d,0}^G &= \mu NH \tau + \xi
\end{align*}
\]

We can use \( p_{c,n-2}^G \) and \( p_{d,0}^G \) to express two constants:

\[
\begin{align*}
    \mu &= \frac{p_{d,0}^G - p_{c,n-2}^G + 1}{(H \tau - W)(N-1)} \\
    \xi &= \frac{(p_{c,n-2}^G - 1)HN\tau - (H \tau + W(N-1)p_{d,0}^G)}{(H \tau - W)(N-1)}
\end{align*}
\]

According to equations (5) and (8), the social cost can be written as

\[
S_c = \frac{(p_{c,n-2}^G - 1)HN\tau - (H \tau + W(N-1)p_{d,0}^G)}{p_{c,n-2}^G - p_{d,0}^G - 1}
\]

The parameters \( W, H, N, \tau \) are constants for a given game. Based on equation (8), express \( p_{c,n-2}^G \) and \( p_{d,0}^G \) as a function of \( n \). The function depends on the coefficients \( \mu, W \) and \( H \). Pollution control cost \( W \) is smaller than punishment cost \( H \), with probability constraint \( p_{c,n}^G, p_{d,n}^G \in [0,1] \).

By solving inequality (7), in the feasible region, the minimum social cost is \( S_c = \tau H(N+1) + W(N-1) \) if \( p_{d,0}^G \) and \( p_{c,n-2}^G \) is satisfied \( p_{d,0}^G HN\tau - (p_{c,n-2}^G - 1)(H \tau + W(N-1)) = 0 \). So when \( \frac{W}{H} < \tau < 1 \), can find the minimum social cost \( \tau H(N+1) + W(N-1) \).

Through the above analysis, no matter which strategy industrial enterprise \( E \) chooses, local government \( G \) can set a certain value to the social cost. Under certain conditions, the minimum social cost can be obtained.

4. Numerical Simulation
We first consider the situation of three participants (1 local government and 2 industrial enterprises) repeating the environmental protection game, assuming the following costs: \( W = 0.4, H = 0.6, \) penalty ratio \( \tau = 0.3 \). For convenience of description, environmental protection investment and penalty are set as relative costs, so as to evaluate the effect of the government's implementation of ZD strategy. The strategy vector of local government is \( p_G \) (corresponding to the state sequence \([CC, CCD, CDC, CDD, DCC, DDC, DDC, DDD]\)), and the strategies of two enterprises are randomly selected. We take \([1,0,1,0,0,1,0] \) and \([1,0,1,0,1,0] \) as examples. Figure 4 shows that when the local government \( G \) implements a given ZD strategy, the social total cost converges to a certain value regardless of the industrial enterprise strategy.
Figure 4. In the repeated environmental protection game of $N = 3$ and $\tau = 0.3$, through the ZD strategy of local government G, the expected cost of the two enterprises finally tends to a fixed value.

5. Conclusion
Enterprises invest in pollution to protect the social environment. However, due to the self-interest of enterprises, the social cost is usually not optimal. The recommended zero determinant strategy got by the government is an efficient method, which can minimize effectively the social cost of pollution control. This article shows that the government can control and set the social cost to a certain value, and has nothing to do with the strategies of other enterprises.

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