Motion of Satellite under the Effect of Oblateness of Earth and Atmospheric Drag

Jaita Sharma∗1, B. S. Ratanpal†1, U. M. Pirzada‡2 and Vishant Shah§1

1Department of Applied Mathematics, Faculty of Technology & Engineering, The M. S. University of Baroda, Vadodara - 390 001, India
2School of Science and Engineering, Navrachana University, Vadodara - 391 410, India

Abstract

The equations governing motion of the satellite under the combined effects of oblate Earth and atmospheric drag have been studied, for a fixed initial position and three different initial velocities, till satellite collapses on Earth. In this study, we have considered exponential atmospheric density model and implemented R-K-Gill method. The minimum and maximum values of orbital elements and their variation over a time for different initial velocities have been reported.

Keywords- Motion of satellite, Oblateness of Earth, Atmospheric drag, R. K. Gill method

AMS Subject Classification- 0F05,70F10,70F15

1 Introduction

The study of motion of the satellite and its life span is the topic of interest of many researchers over the past few decades. When the orbit of the satellite is low Earth orbit (LEO), the perturbation due to oblateness of Earth and atmospheric drag plays very important role. Various analytic, semi-analytic and numerical techniques are adopted for solving perturbed equations of motion. Raj[1] extensively studied the motion of satellite under the oblateness of Earth and also by considering atmospheric drag. They solved the equations of motion by applying KS transformations [2]. King-Hele[3] solved the equations of the motion of a satellite analytically by considering oblateness of Earth. The motion of satellite in the terrestrial upper atmosphere was studied by Sehnal[4]. Knowles et. al.[5] analyzed the effect of geomagnetic storm’s driven by solar eruption on upper atmosphere of Earth and its effect on motion of satellite. The dynamics of satellite motion around the oblate Earth using rotating frame were developed by Yan and Kapila[6]. The Hamilton equations for the motion of satellite under the Earth’s oblateness and atmospheric drag were derived and solved using canonical transformation by Khalil[7]. Bezdvěk and Vokrouhlický[8] presented a semi-analytic theory for long-term dynamics of a low Earth orbit of artificial satellites, they considered both oblateness of Earth and atmospheric drag. Some statistical measures were used by them to compare the observations over the computer efficiency. The resonance in satellite motion under air drag was studied by Bhardwaj and Sethi[9]. Hassan et.
al. \[10\] tried to find a solution of equations governing motion of artificial satellites under the effect of an oblateness of Earth by using KS variables. The authors then applied Picard’s iterative method to find the solution. The algorithm is prescribed by the authors depends on initial guess solution. The differential equations governing relative motion of the satellite under the oblateness of Earth and atmospheric drag were derived and solved by Chen and Jing \[11\], the wide application of their work is in satellite attitude control and orbital maneuver for inter-planetary missions. The satellite rotational dynamics was studied and simulated by Lee et. al.\[13\], they used Lie group variational integrator approach. Reid and Misra\[12\] studied the effect of aerodynamic forces on the formation flight of satellite. The analytic solution in terms of Keplerian angular elements of satellite orbit under atmospheric drag was studied by Xu and Chen\[14\]. Al-Bermani et. al.\[15\] investigated the effect of atmospheric drag and zonal harmonic \(J_2\) for the near Earth orbit satellite namely Cosmos1484. The analytic solution of motion of satellite by considering combined effect of Earth’s gravity and air drag was found by Delhaise\[16\] using Lie transformations. Aghav and Gangal\[17\] designed and simplified the orbit determination algorithm for low Earth orbit navigation.

In this paper we have used R-K Gill method to solve the equations of motion of satellite under the influence of oblateness of Earth and atmospheric drag in the low Earth orbit. We have analyzed results for 1–day, 1–month, 6–months and till satellite collapses on Earth, by considering fixed initial position and three different initial velocities. The orbital elements have been computed for the above mentioned period. We have considered the initial velocities in such a way that satellite will remain in low Earth orbit.

The paper is organized as follows: section 2 describes the model. Solution of equations governing motion of the satellite under oblateness of Earth and atmospheric drag and calculation of orbital elements are reported in section 3. Section 4 contains discussion and concluding remarks.

## 2 The Model

The equation of motion of satellite without any additional perturbing force other than gravitational force between Earth and satellite is given by

\[
\ddot{\vec{r}} = -\frac{\mu}{r^3} \vec{r},
\]

where \(\mu = GM\), \(G\) is gravitational constant and \(M\) is mass of Earth. In the presence of perturbation, additional perturbing acceleration must be added on the right side of equation (2.1). Since we are considering perturbation due to oblateness of Earth and perturbation due to atmospheric drag, the equation of motion can be written as

\[
\ddot{\vec{r}} = -\frac{\mu}{r^3} \vec{r} + \vec{a}_O + \vec{a}_A,
\]

where \(\vec{a}_O\) is acceleration due to oblateness of Earth and \(\vec{a}_A\) is acceleration due to atmospheric drag. The second order equation (2.2) can be written as following set of two first order differential equations

\[
\begin{align*}
\dot{\vec{r}} &= \vec{v}, \\
\dot{\vec{v}} &= -\frac{\mu}{r^3} \vec{r} + \vec{a}_O + \vec{a}_A.
\end{align*}
\]
In the Cartesian co-ordinate system the system of equations (2.3) takes the form,

\[
\begin{align*}
\dot{x} &= v_x, \\
\dot{y} &= v_y, \\
\dot{z} &= v_z, \\
\dot{v}_x &= -\frac{\mu x}{r^3} + \vec{a}_O_x + \vec{a}_{A_x}, \\
\dot{v}_y &= -\frac{\mu y}{r^3} + \vec{a}_O_y + \vec{a}_{A_y}, \\
\dot{v}_z &= -\frac{\mu z}{r^3} + \vec{a}_O_z + \vec{a}_{A_z},
\end{align*}
\]

(2.4)

where \(\vec{a}_O_x, \vec{a}_O_y\) and \(\vec{a}_O_z\) are components of acceleration due to oblateness of Earth in the direction \(x, y\) and \(z\) axis respectively and \(\vec{a}_{A_x}, \vec{a}_{A_y}\) and \(\vec{a}_{A_z}\) are components of acceleration due to atmospheric drag in \(x, y\) and \(z\) axis respectively.

The Earth’s gravitational potential can be modeled in terms of zonal harmonics Battin [18]. In the expression the value of \(J_2\) zonal coefficient is 400 times higher than other \(J_n\) zonal coefficient, \(n \geq 3\). Hence we consider only \(J_2\) into account. If these higher order zonal coefficients are neglected and taking the gradient of scalar potential function then the components of acceleration due to oblateness of Earth in the direction of \(x, y\) and \(z\) direction respectively are,

\[
\begin{align*}
\vec{a}_O_x &= -\frac{3\mu R^2 J_2 x(x^2 + y^2 - 4z^2)}{2r^7}, \\
\vec{a}_O_y &= -\frac{3\mu R^2 J_2 y(x^2 + y^2 - 4z^2)}{2r^7}, \\
\vec{a}_O_z &= -\frac{3\mu R^2 J_2 z(3x^2 + 3y^2 - 2z^2)}{2r^7},
\end{align*}
\]

(2.5)

where \(R = 6378.1363\) km is radius of Earth, \(\mu = GM = 398600.436233\) km\(^3\)/sec\(^2\) and \(J_2 = 1082.63 \times 10^{-6}\).

The acceleration due to atmospheric density is given by

\[
\vec{a}_A = -\frac{1}{2}\rho \frac{C_D A}{m} |\vec{v}_r| \vec{v}_r,
\]

(2.6)

where \(\rho\) is atmospheric density, \(C_D\) is drag coefficient, \(A\) is cross sectional area of the satellite perpendicular to velocity vector, \(m\) is mass of satellite and \(\vec{v}_r\) is satellite velocity vector relative to an atmosphere.

We take the simple exponential atmospheric model for which atmospheric density given by,

\[
\rho = \rho_{pa} e^{\left[\frac{(r_{pa} - r)}{H}\right]},
\]

(2.7)

where \(\rho_{pa}\) is the density at initial perigee point, \(r_{pa}\) is the initial distance of satellite from Earth’s surface, \(r = |\vec{r}|\) and \(H\) is scale height. The ratio \(B^* = \frac{C_D A}{m}\) is called the Ballistic coefficient.

We assume that the atmosphere rotates at the same angular speed as Earth. With this assumption the relative velocity vector is given by Wiesel [19]

\[
\vec{v}_r = \vec{v} - \vec{\omega} \times \vec{r},
\]

(2.8)

where, \(\vec{\omega}\) is the inertial rotation vector of the Earth given by

\[
\vec{\omega} = \omega_e \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix},
\]

(2.9)
where, \( \omega_e = 7.292115486 \times 10^{-5} \text{ rad/sec} \). The cross product of the (2.8) and (2.9) gives three components of the relative velocity vector as

\[
\vec{v}_r = \begin{bmatrix}
v_x + \omega_e r_y \\
v_y - \omega_e r_x \\
v_z
\end{bmatrix}.
\] (2.10)

Substituting (2.7), (2.10) and \( B^* \) in (2.6), we get the components of acceleration due to atmospheric drag in the direction of \( x, y \) and \( z \) axis respectively as

\[
a_{A_x} = -\rho pa e \frac{r pa - r_H}{r^7} \sqrt{(v_x + \omega_e r_y)^2 + (v_y - \omega_e r_x)^2 + v_z^2 (v_x + \omega_e r_y) B^*},
\]

\[
a_{A_y} = -\rho pa e \frac{r pa - r_H}{r^7} \sqrt{(v_x + \omega_e r_y)^2 + (v_y - \omega_e r_x)^2 + v_z^2 (v_y - \omega_e r_x) B^*},
\] (2.11)

\[
a_{A_z} = -\rho pa e \frac{r pa - r_H}{r^7} \sqrt{(v_x + \omega_e r_y)^2 + (v_y - \omega_e r_x)^2 + v_z^2 v_z B^*}.
\]

Substituting (2.5) and (2.11) in (2.4), we get equations of motion of satellite under oblateness of Earth and atmospheric drag as

\[
\dot{x} = v_x,
\]

\[
\dot{y} = v_y,
\]

\[
\dot{z} = v_z,
\]

\[
\frac{v_x}{r^3} = -\frac{\mu x}{r^3} - 3\mu R^2 J_2 x(x^2 + y^2 - 4z^2) + \rho pa e \frac{r pa - r_H}{r^7} \sqrt{(v_x + \omega_e r_y)^2 + (v_y - \omega_e r_x)^2 + v_z^2 (v_x + \omega_e r_y) B^*},
\]

\[
\frac{v_y}{r^3} = -\frac{\mu y}{r^3} - 3\mu R^2 J_2 y(x^2 + y^2 - 4z^2) + \rho pa e \frac{r pa - r_H}{r^7} \sqrt{(v_x + \omega_e r_y)^2 + (v_y - \omega_e r_x)^2 + v_z^2 (v_y - \omega_e r_x) B^*},
\] (2.12)

\[
\frac{v_z}{r^3} = -\frac{\mu z}{r^3} - 3\mu R^2 J_2 z(3x^2 + 3y^2 - 2z^2) + \rho pa e \frac{r pa - r_H}{r^7} \sqrt{(v_x + \omega_e r_y)^2 + (v_y - \omega_e r_x)^2 + v_z^2 v_z B^*}.
\]
### Table 1: Density at Initial Perigee Point and Scale Height

| $r_{pa}$ (km) | $\rho_{pa}$ (kg/m$^3$) | $H$ (km) |
|--------------|----------------|---------|
| 0            | 1.225          | 7.249   |
| 25           | $3.899 \times 10^{-2}$ | 6.349   |
| 30           | $1.774 \times 10^{-2}$ | 6.682   |
| 40           | $3.972 \times 10^{-3}$ | 7.554   |
| 50           | $1.057 \times 10^{-3}$ | 8.382   |
| 60           | $3.206 \times 10^{-4}$ | 7.714   |
| 70           | $8.770 \times 10^{-5}$ | 6.549   |
| 80           | $1.905 \times 10^{-5}$ | 5.799   |
| 90           | $3.396 \times 10^{-6}$ | 5.382   |
| 100          | $5.297 \times 10^{-7}$ | 5.877   |
| 110          | $9.661 \times 10^{-8}$ | 7.263   |
| 120          | $2.438 \times 10^{-8}$ | 9.473   |
| 130          | $8.484 \times 10^{-9}$ | 12.636  |
| 140          | $3.845 \times 10^{-9}$ | 16.149  |
| 150          | $2.070 \times 10^{-9}$ | 22.523  |
| 180          | $5.464 \times 10^{-10}$ | 29.740  |
| 200          | $2.784 \times 10^{-10}$ | 37.105  |
| 250          | $7.248 \times 10^{-11}$ | 45.546  |
| 300          | $2.418 \times 10^{-11}$ | 53.628  |
| 350          | $9.518 \times 10^{-12}$ | 53.298  |
| 400          | $3.725 \times 10^{-12}$ | 58.515  |
| 450          | $1.585 \times 10^{-12}$ | 60.828  |
| 500          | $6.967 \times 10^{-13}$ | 63.822  |
| 600          | $1.454 \times 10^{-13}$ | 71.835  |
| 700          | $3.614 \times 10^{-14}$ | 88.667  |
| 800          | $1.170 \times 10^{-14}$ | 124.64  |
| 900          | $5.245 \times 10^{-15}$ | 181.05  |
| 1000         | $3.019 \times 10^{-15}$ | 268.00  |

For the exponential atmospheric model the scale height ($H$) and $\rho_{pa}$ can be computed from the table-1, Vallado(20).

### 3 Solution and Calculation of Orbital Elements

Using R-K Gill method, we have solved the differential equations (2.12) by fixing the initial position and varying the initial velocity. We have fixed the initial position $\vec{r}_0 = [0, -5888.9727, -3400]$ in kilometers, ballistic coefficient $B^* = 0.095$ m$^2$/kg and three different initial velocities are considered as (i) $\vec{v}_0 = [7.6, 0, 0]$, (ii) $\vec{v}_0 = [7.7, 0, 0]$ and (iii) $\vec{v}_0 = [7.8, 0, 0]$ in km/sec$^2$. For each of these three initial velocities which lead to low Earth orbit we have solved the system of differential equations (2.12) using R-K Gill method. We have analyzed each of three cases till satellite hits on the Earth. We have obtained the intermediate values of scale height ($H$) and $\rho_{pa}$ by applying interpolation on values described in Table-1. The minimum and maximum values of orbital elements for each of these three initial velocities over different time periods are shown in table-2, table-3 and table-4 respectively.
Table 2: \( \vec{r}_0 = [0, -5888.9727, -3400] \) km; \( \vec{v}_0 = [7.6, 0, 0] \) km/sec; \( B^* = 0.096 \ m^2/kg \); Satellite collapses after 3222.64583333333 days.

| Orbital Elements | 1 Day       | 30 Days      | 180 Days     | 3222 Days    |
|------------------|-------------|--------------|--------------|--------------|
| a                | 6.7010 x 10^7 | 6.7070 x 10^7 | 6.6990 x 10^7 | 6.7070 x 10^7 |
| \( \epsilon \)   | 0.0144      | 0.0161       | 0.0149       | 0.0165       |
| t                | 0.5236      | 0.5242       | 0.5236       | 0.5242       |
| \( \Omega \)     | 0           | 6.2829       | 0            | 6.2829       |
| \( \omega \)     | 0.4938      | 1.8496       | 7.1754 x 10^{-4} | 6.2829 |
| f                | 0           | 6.2820       | 5.7651 x 10^{-4} | 6.2828 |

Table 3: \( \vec{r}_0 = [0, -5888.9727, -3400] \) km; \( \vec{v}_0 = [7.7, 0, 0] \) km/sec; \( B^* = 0.096 \ m^2/kg \); Satellite collapses after 6523.69583333334 days.

| Orbital Elements | 1 Day       | 30 Days      | 180 Days     | 6523 Days    |
|------------------|-------------|--------------|--------------|--------------|
| a                | 6.8787 x 10^7 | 6.8837 x 10^7 | 6.8772 x 10^7 | 6.8837 x 10^7 |
| \( \epsilon \)   | 0.0101      | 0.0117       | 0.0097       | 0.0121       |
| t                | 0.5236      | 0.5242       | 0.5236       | 0.5242       |
| \( \Omega \)     | 0           | 6.2829       | 0            | 6.2829       |
| \( \omega \)     | 4.6148      | 4.9996       | 1.4771 x 10^{-4} | 6.2825 |
| f                | 0           | 6.2727       | 0            | 6.2831       |

Table 4: \( \vec{r}_0 = [0, -5888.9727, -3400] \) km; \( \vec{v}_0 = [7.8, 0, 0] \) km/sec; \( B^* = 0.096 \ m^2/kg \); Satellite collapses after 9150.11111111111 days.

| Orbital Elements | 1 Day       | 30 Days      | 180 Days     | 9150 Days    |
|------------------|-------------|--------------|--------------|--------------|
| a                | 7.0076 x 10^7 | 7.0126 x 10^7 | 7.0088 x 10^7 | 7.0128 x 10^7 |
| \( \epsilon \)   | 0.0366      | 0.0381       | 0.0382       | 0.0385       |
| t                | 0.5236      | 0.5242       | 0.5236       | 0.5242       |
| \( \Omega \)     | 0           | 6.2829       | 0            | 6.2831       |
| \( \omega \)     | 4.6014      | 4.9048       | 1.7723 x 10^{-4} | 6.2828 |
| f                | 0           | 6.2770       | 0            | 6.2831       |

The orbit of the satellite for satellite with initial position \( \vec{r}_0 = [0, -5888.9727, -3400] \) km, initial velocity \( \vec{v}_0 = [7.8, 0, 0] \) km/sec and ballistic coefficient \( B^* = 0.096 \ m^2/kg \); for 1 day, 3 days and 7 days are shown in figure 1 respectively from left to right.
Figure 1: Orbit of Satellite 1 day, 3 days and 7 days from left to right

The graphs of angle of inclination, argument of perigee, eccentricity, longitude of ascending node, semi-major axis and true anomaly with initial position $\vec{r}_0 = [0, -5888.9727, -3400]$ km, initial velocity $\vec{v}_0 = [7.8, 0, 0]$ km/sec and ballistic coefficient $B^* = 0.096$ $m^2/kg$; for 1 day, 3 days and 7 days are shown in figure 2, figure 3 and figure 4 respectively.
Figure 2: Angle of inclination, argument of perigee, eccentricity, longitude of ascending node, semimajor axis and true anomaly with initial position $\vec{r}_0 = [0, -5888.9727, -3400]$ km, initial velocity $\vec{v}_0 = [7.8, 0, 0]$ km/sec and $B^* = 0.096 m^2/Kg$; for 1 day
Figure 3: Angle of inclination, argument of perigee, eccentricity, longitude of ascending node, semimajor axis and true anomaly with initial position \( \vec{r}_0 = [0, -5888.9727, -3400] \) km, initial velocity \( \vec{v}_0 = [7.8, 0, 0] \) km/sec and ballistic coefficient \( B^* = 0.096 \text{ m}^2/\text{Kg} \); for 3 days
Figure 4: Angle of inclination, argument of perigee, eccentricity, longitude of ascending node, semi major axis and true anomaly with initial position $\vec{r}_0 = [0, -5888.9727, -3400]$ km, initial velocity $\vec{v}_0 = [7.8, 0, 0]$ km/sec and ballistic coefficient $B^* = 0.096 m^2/Kg$; for 7 days
4 Discussion and Concluding Remarks

The equations governing motion of the satellite under the oblateness of Earth and atmospheric drag have been solved using R-K Gill over a short as well as long time duration. In table-2 to table-4 minimum and maximum values of orbital elements over a different time period have been reported. From these tables it can be seen that satellite will sustain for longer time in low Earth orbit if initial velocity is \( \vec{v}_0 = [7.8, 0, 0] \text{ km/sec} \), initial position \( \vec{r}_0 = [0, -5888.9727, -3400] \text{ km} \) and ballistic coefficient \( B^* = 0.096 \text{ m}^2/\text{kg} \). From figure 1, it can be seen that even for a shorter period of time (1 day, 3 days and 7 days), oblateness of earth and atmospheric drag effects the orbit of the satellite. The salient features of solution of equation of motion under oblateness of Earth and Atmospheric drag are:

1. The choice of initial position and velocities are such that initially satellite's position is in \(YZ\) plane and velocities are applied in X-direction.

2. The variation of argument of perigee is almost linear and increasing.

3. The variation of longitude of ascending node is almost linear and decreasing.

4. The eccentricity increases and then decreases over a longer time.

5. The true anomaly varies between 0 to 6.2832.

4. There is significant decline in semi-major axis over a long time duration.

5. For initial position \( \vec{r}_0 = [0, -5888.9727, -3400] \text{ km} \), initial velocity \( \vec{v}_0 = [7.8, 0, 0] \text{ km/sec} \) and ballistic coefficient \( B^* = 0.096 \text{ m}^2/\text{kg} \); satellite collapses on Earth after 9150.111111111111 days, the height of satellite from surface of Earth on 9150\(^{th}\) day is approximately 43 km.

The particular care have been taken for step size of numerical integration in order to have stability for R-K Gill method. The analysis suggest that with mentioned initial position and initial velocities the maximum time the satellite can survive is 9150.111111111111 days under the oblateness of Earth and Atmospheric drag.
References

[1] J. X. Raj, Analytical and Numerical Predictions for Near Earth’s Satellite Orbits with KS Uniform Regular Canonical Equations, PhD Thesis, Vikram Sarabhai Space Centre, India (2007).

[2] E. L. Stiefel and G. Scheifele, Linear and Regular Celestial Mechanics, Springer-Verlag, Berlin, Heidelberg, New York (1971).

[3] D. G. King-Hele, The effect of Earth’s oblateness on the orbit of a near satellite, Proc. R. Soc. London A, Math. Phys. Sci. 247 (1958) 49-72.

[4] L. Sehnal, The Earth upper atmosphere and the motion of Artificial Satellites, Publications of the Department of Astronomy, Beograd, 10 (1980) 5-13.

[5] S. H. Knowles, J. E. Picon, S.E. Thonnard and A. C. Nicholas, The effect of Atmospheric drag on Satellite orbits during the Bastille day event, Solar Physics, 204 (2001) 387-397.

[6] Q. Yan and V. Kapila, Analysis and Control of Satellite orbits around oblate Earth using perturbation method, Proc. 40th IEEE Conf. Dec. Cont., Orlando, Florida (2001) 1517-1522.

[7] KH. I. Khalil, The drag exerted by an oblate rotating atmosphere on artificial satellite, Appl. Math. Mech., 23 (2002) 1016-1028.

[8] A. Bezdvek and D. Vokrouhlický, Semianalytic theory of motion for close-Earth spherical satellite including drag and gravitational perturbations, Planet. Spa. Sci., 52 (2004) 1233-1249.

[9] R. Bhardwaj and M. Sethi, Resonance in satellite’s motion under airdrag, Ame. J. App. Sci., 3 (2006) 2184-2189.

[10] I. A. Hassan, Z. M. Hayman and M. A. F. Basha, Pre-solution of perturbed motion of artificial satellite, Proc. First Middle East Africa IAU- Regional Meet.-1 (2008) 16-16.

[11] W. Chen and W. Jing, Dynamic equations of relative motion around an oblate earth with air drag, Journal of Aero-space Engineering, 25 (2012) 21-31.

[12] T. Reid and A. K. Misra, Formation flight of satellite in the presence of atmospheric drag, J. Aero. Engin. Sci. Appl., 3 (2011) 64-91.

[13] D. Lee, J. C. Springmann, S.C. Spangelo and J. W. Cutler, Satellite dynamics simulator development using Lie group variational integrator, Proc. AIAA Mod. Sim. Tech.2010, 1-20.

[14] G. Xu, X. Tianhe, W. Chen and T. Yeh, Analytical solution of satellite orbit disturbed by atmospheric drag, Mon. Not. R. Astron. Soc., 410 (2011) 654-662.

[15] M. J. F. Al-Bermani, Abed Al-Ameer H. Ali, A. M. Al-Hashmi, A. S. Baron, Effect of atmospheric drag and zonal harmonic on Cosmos1484 satellite orbit, J. Kufa - Phys., 4 (2012) 1-9.

[16] F. Delhaise, Analytical treatment of air drag and earth oblateness effect upon an artificial satellite, Cel. Mecha. Dyna. Astron., 52 (1991) 85-103.

[17] S. T. Aghav and S. A. Gangal, Simplified orbit determination algorithm for low earth orbit satellite using spaceborne GPS navigation sensor, Arti. Sat., 49 (2014) 81-99.
[18] R. H. Battin, An Introduction to Mathematics and Methods of Astrodynamics, AIAA Education Series, New York (1987).

[19] W. E. Wiesel, Modern Astrophysics, Aphelion Press (2003).

[20] D. A. Vallado, Fundamentals of Astrodynamics and Applications, Microcosm Press and Kluwer Academic Publisher (2004).

[21] M. Grewal, L. Weill and A. Andrews, Global Positioning System, Intertial, Navigation and Integration, A John Wiley and Sons Inc. Publication (2007).