Full-Waveform Inversion in Time Domain by Increased Frequency Iteratively

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Abstract. With seismic imaging techniques using numerical solutions of seismic acoustic wave equations, full-waveform inversion in the time domain can be a tool for estimating high-resolution models from complex geological conditions. However, full-waveform inversion in the time domain has several drawbacks, namely if you determine the initials of the model that is far from the global minimum (the true model) will give a stacked result at the local minimum and to overcome this problem is to use the method of increasing the frequency iteratively. With this method, at least it will keep away the results from a local minimum and close to a global minimum. The result of using the iterative frequency increment method on the full-waveform inversion time domain produces an image that resembles the original synthetic model and produces an updated velocity value that is not too different from the true velocity value of the synthetic model.

1. Introduction

Full-waveform Inversion, or we can say, FWI is the imaging technique of seismic waves that estimates high-resolution models from the subsurface by minimizing the misfit function \cite{1} between modeled and observed data and uses local optimization \cite{2} The FWI method is well to apply in complex geological conditions by simulating the propagation of seismic waves using numerical solutions of seismic wave equations. Minimizing the misfit function itself is optimization algorithm to get a global minimum of the misfit function and the non-linear relationship between seismic data and subsurface model parameters \cite{4}.

Research on the FWI method with synthetic data has long been carried out by several universities in the world since 2000. However, this method was first introduced in 1983 by Lailly, then continued by Tarantola in 1984, and in 1986, Mora introduced the method too \cite{5} In 2010, Magrave et al. reviewed developments and proposed the FWI is an iterative cycle that can involve forward modeling, impedance inversion, velocity model updates, and pre-stack migration \cite{3}.

2. Data and Methodology

First, we use two synthetic models with different levels of difficulty. Figure 1a shows model 1 with a depth of 474 m and a distance of 1194 m, the model resembles salt diapir. With 4 layer that have velocity in each layer 1000 m/s, 1200 m/s, 1400 m/s and 1500 m/s. In the last model Figure 1b is the Marmousi model with gridpoint intervals \(dx = 20\) m with the lowest velocity of 1500 m/s and the highest velocity of 4700 m/s. For forwarding modeling from Figure 2, we use the acoustic wave equation and use the...
finite difference time-space domain. In the gradient computation of the objective function from Figure 2, we use the initial velocity as the forward wavefield and use the true velocity as the back-propagation wavefield. In the process of calculating the back-propagation wavefield, there is a forward model that reversed in time.

![Figure 1](image1.png)

**Figure 1.** True velocity and initial velocity (a) synthetic model 1 and (b) Marmousi model

In the calculation of gradient optimization from Figure 2, we use conjugate direction instead of solve the large computation of the Hessian inverse matrix. The step-length calculation uses a back-trace method which aims to eliminate the increased misfit value due to cycle-skipping. The step-length algorithm can be calculated through the following stages.

First, misfit \( f_0 \) calculation,

\[
f_0 = s(x_0) - s(x_{obs})
\]  

(1)

calculate the initial step length \( a_0 \).

\[
a_0 = \frac{s(x_0)}{CD}
\]  

(2)

Use back-tracking to get updated step-length calculated,

\[
s(x_1) = s(x_{obs}) + a_0 \times CD \times k
\]  

(3)

![Figure 2](image2.png)

**Figure 2.** FWI research flow chart
Update model,

\[ f_1 = s(x_1) - s(x) , \]  
\[ f_0 > f_1 , \text{if not, it is repeated by increasing or decreasing} \] 
\[ s(x_2) = s(x_1) + a_0 \ast CD \ast k \]  
\[ (5) \]

Update model,

\[ f_2 = s(x_2) - s(x_{obs}) \]  
\[ f_0 > f_2 , \text{if not, it is repeated by increasing or decreasing} \] 
\[ \gamma = \frac{(f_0-f_2)+(f_1-f_0)}{(2f_0+2f_1-2f_2)} \]  
\[ (7) \]

\[ a_n = a_0 \gamma \]  
\[ (8) \]

Stop if misfit \( s(x)^{(i)} + \alpha_n G < \text{misfit} \) \( s(x)^{(i)} + \alpha_0 G \). After calculating the step-length (\( \alpha \)) and gradient values (\( CD \)), the model updated with the following equation:

\[ s(x)^{(i+1)} = s(x)^{(i)} + \alpha CD \]  
\[ (9) \]

Stop If the model update matches the true model, otherwise the model update will be repeated until the model update approaches the global minimum or the true model.

3. Result and Discussion

3.1. FWI results for synthetic model 1

There are two FWI processes carried out in the synthetic model 1. First, the synthetic model only uses one frequency (ricker wavelet with frequency 10 Hz) in 1-50 iterations and results can be seen in Figure 3. In 20-50 iterations, there are many multiple effects produced resulting in layer boundaries between layers 2 and 3 that do not match the true model. One of the reasons is the use of model initials are far from global minimums.

![Figure 3. Updated velocity full-waveform inversion synthetic model 1 only uses one frequency (10 Hz).](image)

The FWI process carried out in synthetic model 1 is using frequency 5 Hz in iterations 1-10 (frequency changes in every 10 iterations). It’s that mean in every 10 iterations used single dominant frequency (on
ricker wavelet) and so on until the last iteration is 40 (which uses a frequency of 10 Hz) for the results can be seen in Figure 4. In 1-5 iterations, there is a slower reduction in misfit reduction than the previous process which only uses a frequency of 10 Hz. However, in 10-40 iterations, the resulting model is better than the previous process. With a reduced multiple effect (due to the use of low frequencies) so that the layer boundaries between layers 2 and 3 match the true synthetic model.

For a comparison of the velocity profile generated from the two FWI processes, it can be seen in Figure 5 with the green dotted line as the initial velocity, the black dots are FWI and the blue line is the true model. The effect of wiggle produced in the FWI increased frequency process is iteratively less than the FWI process that only uses one frequency. This is also due to the use of low frequencies in iterations 1-10 which reduce the ringing effect on the shot gather resulting from the forward modeling process. With the minimal wiggle effect on the resulting waves resulting in the level of layer boundary resolution in the FWI increased frequency iteratively better than the FWI process only uses one frequency. So that the resulting misfit value in the FWI increased frequency process is iteratively less than the FWI process only uses one frequency which can be seen in Figure 6.

**Figure 4.** Updated velocity full-waveform inversion synthetic model 1 by increased frequency iteratively (5, 7, 9, 10 Hz)

**Figure 5.** Velocity Profile comparison synthetic model 1 (a) only uses one frequency (10 Hz) at distance 252 m, 900 m and (b) by increased frequency iteratively (5, 7, 9, 10 Hz) at distance 252 m, 900 m.
Figure 6. Misfit from (a) synthetic model 1 only uses one frequency (10 Hz) and (b) synthetic model 1 by increased frequency iteratively (5, 7, 9, 10 Hz).

3.2. FWI results for synthetic model Marmousi

After being tested in model 1 and resulting in iteratively better use of the FWI by increased frequency process. And then in testing the Marmousi models. Iterations 1-10 using a frequency of 2 Hz (on ricker wavelet) helps in reducing the resulting local minimum so that in 10 iterations the resulting model has begun to approach the global minimum even though the resulting resolution is still not good.

Figure 7. Updated velocity full-waveform inversion synthetic model Marmousi by increased frequency iteratively (2, 4, 6, 8, 10 Hz)

Figure 8. Velocity profile comparison synthetic model Marmousi (a) at distance 4480 m, (b) at distance 8000 m and (c) Misfit from synthetic model Marmousi
To improve the resolution of the result of iteration 10, the frequency is increased in every 10 iterations, so that at iteration 50 the results are very close to the global minimum which can be seen in Figure 7. In Figure 8b (which is the most complex layer in the Marmousi model) using the FWI process by increased frequency iteratively at least gives the velocity profile shape according to the true velocity profile form. The misfit results resulting from this process can be seen in Figure 8c.

4. Conclusion
Our case studies concluded that determining the initial model close to the global minimum on the full-waveform inversion time domain has an important role in simplifying the gradient optimization method to find the global minimum and not stop at the local minimum. Determination of the maximum frequency value on the full-waveform inversion time domain can affect the results of the final interpretation if using too high a frequency on shot gather will get multiple effects which results in the image not being resolved properly. Applying the method of increasing the frequency to the full-waveform inversion of the time domain gives the final velocity model close enough to the true velocity model even though the initial velocity model used is far from the true velocity model, using a frequency of 2 Hz in iterations 1-10 to reduce the resulting local minimum, then increasing the frequency in every ten iterations will increase the resolution.

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