Strongly Coupled Dark Energy Cosmologies: preserving $\Lambda$CDM success and easing low scale problems
I – Linear theory revisited

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ABSTRACT

In this first paper we discuss the linear theory and the background evolution of strongly coupled (SCDEW) models. In these models, today energy density is dominated by Warm Dark Matter which, like baryons, is uncoupled. Dark Energy is a scalar field $\Phi$, whose coupling to an ancillary low–density Cold Dark Matter (CDM) component is an essential model feature. Such coupling, in fact, allows the formation of cosmic structures, in spite of very low WDM particle masses ($\sim 100$ eV). SCDEW models yield Cosmic Microwave Background and linear Large Scale features substantially undistinguishable from $\Lambda$CDM, but thanks to the very low WDM masses they strongly alleviates $\Lambda$CDM issues on small scales, as confirmed via numerical simulations in the II associated paper. Moreover SCDEW cosmologies significantly ease the coincidence and fine tuning problems of $\Lambda$CDM, and, by using a field theory approach, we also outline possible links with inflationary models. We also discuss a possible fading of the coupling at low redshifts which prevents non linearities on the CDM component. The coupling intensity and the WDM particle mass, being extra parameters in respect to $\Lambda$CDM, are found to be substantially constrained a priori so that, if SCDEW is the underlying cosmology, we expect most data to fit also $\Lambda$CDM predictions.

Key words: cosmology: dark matter galaxies: evolution - formation methods:linear programs

1 INTRODUCTION

When SNIa Hubble diagrams revitalized $\Lambda$CDM, one could hardly guess how successful this cosmology would be in meeting cosmological data. In spite of its coincidence and fine tuning problems, therefore, $\Lambda$CDM is surely the benchmark for any attempt to improve our understanding of cosmology.

In this series of works we present a detailed analysis of a new class of cosmologies that preserves all successes of $\Lambda$CDM models on large and intermediate scales. In turn, they improve the agreement with the observed dark matter distribution on small scales (e.g. Oh et al. 2011; Walker & Peñarrubia 2011), at the same time easing the coincidence and fine tuning problems of $\Lambda$CDM, while possibly launching a bridge between the reheating stages in the late inflationary regime and today’s Dark Energy (DE) nature.

These cosmological models are based on the presence of three dark components: i) a very–warm dark matter component (WDM), uncoupled (like the baryons), and constituting the observable dark matter at the present time; ii) a scalar field $\Phi$ which acts as Dark Energy; iii) a peculiar Cold Dark Matter component (CDM), whose coupling to $\Phi$ played an essential role in cosmic history, but whose density at $z = 0$ naturally became almost negligible. We dubbed these models SCDEW (Strongly Coupled DE + WDM).

The basic features of these models were discussed in two previous papers (Bonometto, Sassi, & La Vacca 2012; Bonometto & Mainini 2014; BSLV12 and BM14 respectively, hereafter). BSLV12 dealt with background components: they showed that a purely kinetic scalar field $\Phi$ strongly coupled to a CDM component would both exhibit densities $\propto a^{-4}$ ($a$ : scale factor), during the radiative eras, expanding along a tracker solution, with (primeval) density parameters $\Omega(0.1 \%)$. The derelativisation of a further WDM component eventually turns these components into quintessential DE and a tiny CDM component. The radiative eras of these models are therefore characterized by three low density components, in top of the usual $\gamma$’s and $\nu$’s:
WDM, coupled CDM and scalar field, accounting for constant fractions of the overall density, and sharing similar densities. BM14 then deals with fluctuation evolution, showing that CMB anisotropy and polarization spectra, in these models, are hardly distinguishable from the ΛCDM benchmark, while coupled CDM fluctuations continue to grow also between their entry in the horizon and matter–radiation equality, being so able, in spite of the low CDM density, to revitalize WDM fluctuations on scales suffering and early free streaming, even when the WDM particle mass $m_w$ is quite small.

The $\Phi$–CDM coupling is therefore an essential feature in the early model evolution. After WDM has derelativized, instead, the $\Phi$–CDM coupling is unessential, while leading to technical difficulties, as $\delta_c$ (CDM fluctuation amplitudes) tend to approach unity, developing then some early non-linearities, typically involving $\ll 1\%$ of the total mass. However, switching off the $\Phi$–CDM coupling after the break of “conformal invariance”, yields more comfortably tractable models.

The plan of the paper is as follows. In the next Section we shall formulate the Lagrangian theory for strongly coupled Dark Energy cosmologies. It confirms the prediction on the coupled $\Phi$–CDM component densities, while outlining possible links with inflationary theories. In Section 3 we discuss the evolution of background parameters, focusing on the exit from the primeval stationary regime. We also widen the range of models in respect to BSLV12 and BM14 papers, by allowing for $\Phi$–CDM conundrums, free us from its coincidence paradox and fine tunings.

2 COUPLED DARK ENERGY IN THE EARLY UNIVERSE

There are direct evidences that Dark Matter (DM) is a physical cosmic component, mostly clustering with observable baryons. Doubts have been cast, on the contrary, on the true nature of Dark Energy (DE); here we assume it to be a self–interacting scalar field $\Phi$, keeping essentially unclustered. In this scheme, both dark components interact with baryons and radiations just gravitationally, so that the energy pseudo–conservation equation

$$T^{(\rho)}_{\mu\nu} + T^{(\Phi)}_{\mu\nu} = 0 \quad (1)$$

holds (here $T^{(\rho,b,d)}_{\mu\nu}$ are the stress–energy tensors of DM and DE, their traces reading $T^{(\rho)}_{\mu\nu}$). The most common option is that the sum vanishes thanks to separate vanishings of both terms. The alternative option that

$$T^{(\Phi)}_{\mu\nu} = +CT^{(c)}\Phi_{,\mu}, \quad T^{(c)}_{\mu\nu} = -CT^{(c)}\Phi_{,\nu}, \quad (2)$$

however, is also widely considered in the literature (e.g. Ellis et al. 1998, Wetterich 1995, Amendola 1999, Amendola & Tocchinile Valentini 2002, Macciò et al. 2004, Baldi et al. 2010), together with other possible options for energy transfer between DM and DE (e.g. Lopez Honorez et al. 2010). The coupling

$$C = b/m_p = (16\pi/3)^{1/2}\beta/m_p, \quad (3)$$

sets the intensity of the energy flow from DM to DE. The option of scale dependent $C$ is also discussed in the literature, namely in connection with specific models (e.g. Macciò & Bonometto 2004).

Let the background metric then read

$$ds^2 = a^2(\tau)(d\tau^2 - d\lambda^2), \quad (4)$$

($\tau$ : conformal time, $d\lambda$ : the spatial element); eqs. (2) then yields

$$\ddot{\Phi} + 2\dot{\Phi} = -a^2V' + Ca^2\rho_c, \quad \dot{\rho}_c + 3\dot{\Phi} = -C\rho_c\Phi, \quad (5)$$

$\rho_c$ being the DM density, while $V(\Phi)$ is a self–interaction potential, as is required within quintessential DE models.

The rational is to allow for an energy flow from DM (supposed to be cold: CDM) to DE, so keeping a significant DE density also at high $\tau$ and easing one of the coincidence problems of ΛCDM. In this way, the DE field $\Phi$ could keep a steady fraction (some percents) of CDM density during the whole cosmic expansion, although being essentially kinetic at redshift $z > z_c$, where the DE state parameter shifts from $\sim -1$ to $\sim +1$. This feature is generic, independently of the shape of $V(\Phi)$.

2.1 An early $\Phi$–CDM coupling

Within a field theory context, a possible assumption is that CDM is a non–relativistic Dirac spinor field $\psi$, interacting with $\Phi$ through a Yukawa–like lagrangian

$$\mathcal{L}_m = -\mu f(\Phi/m)\bar{\psi}\psi; \quad (6)$$

here 2 mass scales, $m = m_p/b$ and $\mu = g\mu_p$ are introduced for dimensional reasons, $m_p$ being the Planck mass. In particular, $b$ coincides with the coupling parameter in eq. (5) (see below).

By assuming a kinetic part of the scalar field lagrangian $\mathcal{L}_k \sim \partial_{\mu}\Phi\partial^{\mu}\Phi$, its equation of motion reads

$$\ddot{\Phi} + 2\dot{\Phi} = -a^2V' + a^2\rho_c f' / f, \quad (7)$$

once we work out that

$$\rho_c = -\mathcal{L}_w = \mu f(\Phi/m_p)\bar{\psi}\psi, \quad (8)$$

in the absence of a significant kinetic term for the spinor quanta. Notice that the number density operator for the spinor field $n \propto \psi\bar{\psi}$, so that

$$\rho_c \propto f(\Phi/m)a^{-3} \quad (9)$$

according to the findings of Das et al. 2006.

Eq. (7) is consistent with the former eq. (5) if $f'/f = b/m_p$, so that, if the function inserted in the Lagrangian (6) has the form

$$f = \exp(-b\Phi/m_p) \quad (10)$$

we re–obtain the coupled–DE theories of eqs. (3). Also $\rho_c$, according to eq. (9), then exhibits a sort of exponential scaling, unless we assume that the argument of the exponential

$$b\Phi/m_p = \ln(\tau/\tau_c), \quad (11)$$

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\( \tau \), being a (unconstrained) reference time; then \( f = \tau / \tau \) and
\[
\dot{\Phi} = \frac{m_\Phi}{67}. \tag{12}
\]

Let us then suppose to be in a radiative era, \( \Phi \) bearing essentially kinetic energy; then the \( \text{r.h.s.} \) of eq. (12) reads \( \alpha^2 p b / m_\Phi \), while the whole equation means that \( \rho_c = (m_\Phi/m_\tau)^2 \) (thence \( \rho_c \propto \alpha^{-4} \)) or, equivalently,
\[
\left( \frac{\dot{a}}{a} \right)^2 = \frac{8 \pi}{3} \frac{\rho_c}{m_\Phi^2} a^2 (2\beta^2 \rho_c). \tag{13}
\]

By comparing this equation with the Friedmann eq., we deduce a constant early density parameter for CDM
\[
\Omega_c \equiv \frac{\rho_c}{\rho} = \frac{1}{2 \beta^2}. \tag{14}
\]

In turn, \( \Phi \) being purely kinetic, owing to eq. (12), its energy density
\[
\rho_\Phi \equiv \frac{\dot{\Phi}^2}{2 \alpha^2} = \left( \frac{m_\Phi}{b} \right)^2 \frac{1}{2 \alpha^2 \tau^2} \rho_c - \frac{\rho_c}{2}. \tag{15}
\]

coinciding with its pressure. The state parameter of the future quintessential field is then \( w = +1 \), while its early density parameter
\[
\Omega_\Phi = \frac{\Omega_c}{2} = \frac{1}{4 \beta^2}. \tag{16}
\]

In connection with the scale dependence found for \( \rho_c \), let us outline that, if we read \( \mathcal{L}_m \) as a mass term of the Dirac spinor, its mass scales \( \propto \alpha^{-1} \) (as though being redshifted). As a matter of fact, both \( \Phi \) and CDM energy densities scale as \( \alpha^{-4} \). In the absence of coupling they would scale as \( \alpha^{-6} \) and \( \alpha^{-3} \), respectively. The flow of energy from CDM to \( \Phi \) is tuned to yield a faster (slower) CDM \( \Phi \) field dilution.

The key issue, however, shown in BSLV12, is that these conditions are an attractor: even significantly perturbing “initial conditions”, the cosmic evolution rapidly settles on the behavior here described. Accordingly, such kind of evolution could last since a very early epoch, even since the late inflationary stages.

We might tentatively assume that cosmic reheating was due to the very \( \Phi \) field decaying into \( \psi \) field quanta, a process stopping as soon as the attractor solution is attained. Interactions of \( \psi \) with other fields would then allow the \( \Phi \) field energy to reheat other cosmic components, including both \( \gamma \)’s & \( \nu \)’s, as well as any other component then belonging to the thermal soup.

The ensuing self-similar expansion, that can be defined conformally invariant (in the sense discussed, e.g., by Parker 1969,1971), can only be broken if a component scaling differently from \( \alpha^{-4} \) achieves a significant density. Let us finally notice that, if the attractor solution is followed since inflation, owing to the logarithmic time dependence of \( \Phi \), its today’s value is just \( \sim 60 \) times its value at the end of the reheating stages.

### 3 EXIT FROM THE STATIONARY REGIME

In any reasonable cosmological model, the non–relativistic baryon density would eventually break the conformally invariant expansion at a redshift \( z_b \gtrsim 500 \). Adding just a baryon component to radiative components (coupled and uncoupled) is however insufficient to meet observations. Among viable possibilities, BM14 outlined the option of including a Warm DM (WDM) component of thermal origin, with a temperature parameter \( T_w \). Its derativatization occurs when \( T_w \sim m_w \) (mass of WDM quanta) at a redshift \( z_w \). At \( z \gg z_w \), being \( P_w \simeq \rho_w / 3 \), it is \( \rho_w \propto T_w^4 \) and WDM is one of the components of the thermal soup; then, at \( z \ll z_w \), being \( P_w \ll \rho_w \) and therefore \( \rho_w \propto T_w^3 \), WDM shall overcome the radiative component density so that the early stationary regime reaches an end.

This also sets an end to the attractor solution of the coupled components. During their successive evolution, as in any model allowing for CDM–DE coupling with small or large \( \beta \), the state parameter of the field component must turn from \( +1 \) to \( \sim -1 \) about a suitable redshift \( z \); this is needed to allow a fair amount of today’s DE and, in any approach based on a \( \Phi \) self interaction potential \( V(\Phi) \), the \( z \) value is obtained by tuning some parameter(s) inside \( V(\Phi) \) expression itself. SCDEW models behave similarly. It is just convenient to make use of the first order field equation
\[
\dot{\Phi}_1 + \frac{\dot{a}}{a} \Phi_1 = \frac{1 + w}{2} C \rho_c a^2, \tag{17}
\]

instead of eq. (5); here \( \Phi_1 \equiv \Phi \) and \( 2 \dot{\Phi}_1 = 1 + 3w - d \log (1 + w) / \log a \). This equation, shown by BSLV12, requires \( w(a) \) to be assigned, instead of a potential \( V(\Phi) \), so allowing us to select \( z \) directly, without arguing about unstable \( V(\Phi) \) expressions. More explicitly, let us assume that
\[
w = 1 \frac{1 - A}{1 + A} \text{ with } A = \left( \frac{a}{a_{\pm}} \right)^\epsilon \tag{18}
\]

with \( a_{\pm} = (1 \pm z_w)^{-1} \). In BM14 we also discussed the (minimal) dependence of results on the parameter \( \epsilon = 2.9 \) here), whose arbitrariness replaces the choice of \( V(\Phi) \) expression.

An option, not mentioned by BSLV12 and BM14, is that \( \beta \) gradually fades, after the rise of \( \rho_w \) broke conformal invariance. No feature of this class of cosmologies really depends on a late coupling and we shall see that, if \( \beta \) fading occurs late enough, just minor quantitative changes arise.

Let us then consider the option that \( \beta \sim \exp(-a/a_{\beta}) \) with \( a_{\beta} = D \times a_w \). If we expect \( \beta \) fading to be a sort of consequence of the loss of conformal invariance, if should be \( D \gg 1 \). Accordingly, different options will be labelled by the value of the delay (del.)
\[
d = \log_{10} D \tag{19}
\]

which is an extra parameter we introduce here. In Figure 4 we show the scale dependence of cosmic components in 4 different cases, passing from the option of ever lasting coupling, down to the case of delay \( d = 1 \). In this Figure and anywhere in paper I, we select the following parameter values:
\[
\Omega_{\phi}, \Omega_{h} = 0.7 \quad 0.045 \quad 0.685 \quad 2.726 \quad 3.04 \quad 0.968
\]

Here \( \Omega_{\phi}, \Omega_{h} = 0.7, 0.045 \), then \( T_{CMB}, N_{\nu}, n_s \) are the present DE and baryon density parameters, the present Hubble parameter in units of 100 (km/s)/Mpc, the CMB temperature, the number of (almost) massless \( \nu \)’s, the primaval scalar fluctuation index, respectively. In the list, \( \Omega_{\phi} \), \( \Omega_{h} \), and \( \Omega_{\nu} \) are not included. Neglecting \( \gamma \)’s \( \nu \)’s, their sum is \( 1 - \Omega_{\phi} - \Omega_{h} \), while \( \Omega_{\nu} \) is fixed by the selection of the coupling constant \( \beta \) and \( \Omega_{h} \) covers the residual gap. Notice that, at \( z = 0 \), \( \Omega_{\nu} \ll 1/2 \beta^2 \) being, typically, \( \sim 10^{-3} \Omega_{\phi} \) (see Figure 1).

The option \( d = 4 \), that will be selected in Paper II, is characterized by \( a_{\beta} \sim a_{\pm} \); as shown in Figure 4 for \( d = 4 \) density evolution is just marginally affected by the fading of coupling.

An earlier decoupling, instead, bears a number of consequences. Among them, a rise of today’s CDM density parameter \( \Omega_{\nu} \) because the energy leaking from CDM to DE has a stop. If \( d \gtrsim 2 \), the CDM rise is limited and, in this range of \( d \) values,
Figure 1. Scale dependence of the densities of cosmic components after the break of conformal invariance. All models in this plot have $\beta = 10$, $m_w = 90\text{ eV}$. Delay (del) values from 1 to $\infty$ (everlasting coupling) are considered. The thick curves labelled “cou” concern an everlasting coupling. The behavior of densities does not suffer major changes, in respect to this option, unless $d < 2$.

![Figure 1](image)

the present CDM contribution to the cosmic budget is $\sim 1/100$ of baryonic one.

A related effect is the formation of a deep in $\rho_\phi a^4$ evolution. As a matter of fact, until $\Phi$ is purely kinetic, its density keeps $\rho_\phi = m_\phi^2/(2h a^2 z^2)$, thanks to the energy inflow from CDM. In the radiative era, when $\tau \propto a$, $\rho_\phi a^4$ is then constant. The transition to matter domination should strengthen $\rho_\phi$ decline; however, when this occurs, coupled CDM density starts to increase, so providing a stronger $\Phi$ feeding. If the feeding stops before the kinetic–potential transition, occurring at $z_{\text{ph}}$, a progressive dilution of $\rho_\phi$ is unavoidable. However, this produces a significant deep only if $d < 2$. Owing to these reasons, in the rest of this Paper I we shall never consider options $d < 2$. We shall also verify that models with any $d > 2$ exhibit just minor differences.

Before concluding this discussion on background evolution let us outline that the relation between expansion factor $a$ (or redshift) and ordinary time $t$ are almost identical in SCDEW and $\Lambda$CDM models, they never exceed $0.01\%$ for any values of coupling $\beta$ and delay $d$.

4 LINEAR FLUCTUATION EVOLUTION

In BM14 fluctuation evolution is studied in detail. Out of horizon initial conditions are determined and the system of differential equations, holding after the entry into the horizon, is numerically solved. Here, let us only remind some peculiar aspects, that this problem does not share with similar cosmological models. In particular, field fluctuations are conveniently described by a variable $\varphi$, related to the quintessential field $\phi$ as follows:

$$\phi = \Phi + \frac{b}{m_p} \varphi,$$

(20)

$\Phi$ being the background field. At the first perturbative order $\varphi$ fulfills the equation

$$\dddot{\varphi} + 2 \frac{a}{\dot{a}} \ddot{\varphi} + \frac{\dot{\Phi}}{a} \dot{\varphi} + k^2 \varphi + a^2 V''(\Phi) \varphi = 2 \beta^2 \Omega_c \left( \frac{a}{\dot{a}} \right)^2 \delta_c.$$

(21)

Derivatives are taken in respect to the conformal time, as the metric reads

$$ds^2 = a^2(\tau) \left[ d\tau^2 - (\delta_{ij} + h_{ij}) dx_i dx_j \right].$$

(22)

Moreover, $\delta_c$ are the fluctuations in the coupled CDM component and $h$, the trace of the $3$–tensor $h_{ij}$, describes gravity. Finally, $k$ yields the mass scale $M = (4\pi/3) \rho (2\pi/k)^3$ of the fluctuation.

Most terms in eq. (21) have therefore a transparent meaning, apart of the term $V''(\Phi)$, where we apparently refer to a $\phi$ self–interaction potential. Aiming to skip any reference to such quantity, BM14 show that one can use the relation

$$2V'' = \frac{A}{1 + A} \left\{ \frac{\dot{\varphi}}{a} + \epsilon \right\} + 6 + \beta \frac{\dot{\Phi}^2}{\Phi} \right\}$$

(23)

with $\epsilon = -6$. Here, $A$ and $\epsilon$ are the quantities defining the $w(a)$ behavior in eq. (11). All variables in this expression are known at each step of a linear evolutionary algorithm.

It is then significant that, within this formulation, also in the fluctuation equations only $\Phi$ and $\dot{\Phi}$ appear, while $\Phi$ values never matter. BM14 made use of these relations to modify the public algorithm CMFAST and here we shall report results obtained with this code.

4.1 Early growth of coupled CDM fluctuations

Figure 2 and 3 then show a few examples of linear evolution, for fluctuations in the different cosmic components, from before their entry in the horizon, down to $z = 0$. The background cosmological parameters are the same of Figure 1. The mass of the warm component is anywhere 90 eV, its range being fixed by the arguments discussed below, in the next subsection. The $k$ values selected for Figure 2, where the delay parameter is $d = 2$, correspond to the mass scales of $1.8 \times 10^{14} h^{-1} M_\odot$ (or $8 h^{-1} \text{Mpc}$, top panel), $1.4 \times 10^{12} h^{-1} M_\odot$ (central panel), and $8.7 \times 10^{10} h^{-1} M_\odot$ (lower panel). Figure 3 is for 2 intermediate scales and $d = 3$. For sake of comparison, the evolution of baryon and CDM fluctuations in a $\Lambda$CDM model with the same cosmological parameters is also overplotted.

Before discussing the most evident feature in all plots, i.e., the rapid growth of coupled CDM fluctuations, let us outline a result made evident by the top Figure 2, the low–$z$ growth of fluctuations, namely in (scales close to) the linear regime, is the same in $\Lambda$CDM and SCDEW cosmologies. This is visible in spite of the (slightly) different final amplitude, and is also approximately true for greater non–linear $k$ values. Numerical outputs confirm that discrepancies between $\Lambda$CDM and SCDEW growth factors, for $k \lesssim 0.5$, are below the expected precision of the algorithms used. Aside of that, it is worth outlining that SCDEW models are also indistinguishable from $\Lambda$CDM, as far as CMB spectra are concerned. Discrepancies between SCDEW and $\Lambda$CDM, for $\beta \geq 5$ are again below the expected precision of the algorithms used.

As far as growth of coupled CDM fluctuations is concerned, the key feature is that it is not discontinued at the time when the horizon attains the fluctuation size. In $\Lambda$CDM, uncoupled CDM fluctuation grow much more slowly, namely until radiation exceeds CDM density, as radiation fluctuations (still coupled to baryons) have stopped their growth, being in the sonic regime. This extra growth is expected, in spite of the small density $\rho_c \simeq \rho_\phi/2\beta^2$ ($\rho_\phi :
Figure 2. Evolution of density fluctuations in the model indicated in the frame for $3k$ values; from top to bottom they correspond to masses $\sim 1.8 \times 10^{14} - 1.35 \times 10^{12} - 8.7 \times 10^{10} M_\odot$, respectively. The first $k$ value then corresponds to a comoving length scale of $8h^{-1}$ Mpc, at the boundaries between linear and non-linear behaviors; the second and third $k$ values lay at the top and bottom limits of the galactic mass scales. Dotted curves show the evolution of CDM and baryons, for the same scales, in a $\Lambda$CDM model.

Figure 3. As previous Figure, for $d = 3$, and 2 intermediate scales.

4.2 WDM fluctuation regeneration

Large amplitude CDM waves cause a restart of fluctuations in WDM, as soon as it becomes non-relativistic. As is also visible in Figures 2 and 3, WDM derelativizes at $z_{w,dec} \sim 10^9$. Derelativization is linked to the value taken by the $T_w/T_\gamma$ ratio, between WDM and photon temperatures. If WDM particles decoupled from the thermal soup when its density was $(\pi^2/30)g_{w,dec}T^4$, today $(T_w/T_\gamma)^3 \approx (2/g_{w,dec})(1 + 3N_\nu/22) (N_\nu : \text{number of (almost) massless } \nu \text{'s in the cosmic background}; g_{w,dec}: \text{number of relativistic degrees of freedom at the time of WDM decoupling})$, while

$$
\Omega_{w,0} h^2 \approx \frac{45c^2}{4\pi^4} \left( \frac{T_w}{T_\gamma} \right)^3 \frac{m_w}{10^4 T_\gamma^4}.
$$

(24)
For $N_v = 3$ we have then
\[
\Omega_{\text{w}} h^2 \simeq 0.115 \frac{m_w/\text{eV}}{g_{\text{w,dec}}} \tag{25}
\]
so that $T_w/T_\gamma \sim 0.32$ if $\Omega_{\text{w,0}} h^2 \simeq 0.12$ and $m_w \simeq 90 \text{ eV}$. Accordingly, $T_w \simeq m_w$ at a redshift $z \simeq 10^5-10^6$, in agreement with what is shown in the plots, these effects being all included in modified CMBFAST. In particular, our algorithm takes into account the contributions of coupled CDM and $\Phi$ field to the thermal soup.

At the time of WDM derelativization, CDM fluctuations $\delta_c$ exceed then any other fluctuations, by a scale dependent factor $F_\delta \sim 10^2-10^3$ for galactic scales. There is no boost to the CDM–WDM gravity due to coupling, but the CDM density excess $\delta\rho_c = \rho_c \delta_c \simeq (\rho_\ell/2)^2 \delta_c$ is boosted by the factor $F_\delta$, compensating the division by $2\beta^2 \sim 10^2$ due to the small CDM density. As the factor $F_\delta$ depends on the time elapsed since the entry in the horizon, the restart of WDM fluctuations should be more and more effective towards greater $k$ values. In turn, WDM particle escape velocity from smaller size fluctuations is smaller, so that the effect is only partially visible in the final spectra.

On the contrary, this very effect is unchallenged in baryons, whose mean velocities are negligible. Also the (later) restart of their fluctuations is mostly due to CDM. In the absence of residual particle velocities, baryon fluctuations growth even exceeds WDM fluctuations, both at high $z$ and at $z = 0$, namely at high $k$.

Since $\sim z_{\text{w,der}}$, WDM density starts to dilute just $\sim a^{-3}$, and primeval conformal invariance is broken. Also baryon density growth violates such invariance, of course, but baryon density overcomes radiative components at a redshift $< z_{\text{w,der}}$. In principle, it can make sense that a break of conformal invariance and $\beta$ fading are related events: when WDM has turned non relativistic, the $\beta$ coupling appears like a residual pleonasm. Leaving apart detailed options to model a relation between these effects, we just quantify it with the parameter defined in eq. (19). For instance, for $d = 2$ $(3, 4)$, $\beta$ fades when $z \sim 5000$ $(500, 50)$. Values of $d \gg 6$ return an ever lasting $\beta$-coupling.

### 4.3 WDM particle mass selection

WDM fluctuation regeneration in SCDEW models contrasts with undisputed free streaming effects in standard $\Lambda$WDM models, causing a large–$k$ cutoff to the spectral function
\[
\Delta^2(k) = \frac{1}{2\pi^2} k^3 P(k) , \tag{26}
\]
($P(k) = (\bar{\delta}(k))^2$ : transferred spectrum), because of the erasing of any fluctuation entering the horizon before WDM has derelativized. With $m_w \simeq 90 \text{ eV}$, the decline of $\Delta^2$ starts at $k \simeq 0.5 \text{ hMpc}^{-1}$. $\Delta^2$ being already damped by a factor $\sim 10^2$ at $k \simeq 1 \text{ hMpc}^{-1}$. Accordingly, viable AWDM models currently refer to WDM masses $m_w \simeq 2-3 \text{ keV}$, allowing fluctuations to survive up to $k \simeq 20-30 \text{ hMpc}^{-1}$.

Unfortunately, the need of such a large $m_w$ partially vanifies the reasons to choose warm instead of cold DM, trying to exploit residual motions of low–mass particles, e.g., to reduce the number of expected MW satellites or to prevent them settling on a NFW (Navarro, Frank & White 1997) profile. Recent simulations confirm a relation between $m_w$ and the size of a core. According to Macciò et al. (2012), in the dwarf galaxy mass range, the size of the core
\[
R_{\text{core}} \sim \left( \frac{1.0}{h^{-1}\text{kpc}} \right) \left( \frac{100 \text{ eV}}{m_w} \right)^{1.8} , \tag{27}
\]

![](Figure4.png)

**Figure 4.** Primeval density parameter of WDM and coupled CDM–$\Phi$ components in the radiation dominated era, for $m_w = 90 \text{ eV}$. The upper (lower) dashed line yields CDM ($\Phi$) density. The total density of the coupled components is the “parallel” solid curve. The slightly rising solid line is WDM density parameter.

If $m_w \sim 2 \text{ keV}$, e.g., it is $r_{\text{core}} \sim 5 h^{-1}\text{pc}$ while a fair core size, fitting observations, requires $m_w \sim 80-110 \text{ eV}$, a mass scales yielding fair cores but no galaxies. On the contrary, the linear theory of strongly coupled–DE models exhibits a full restart of fluctuations.

Before concluding that a solution to low–scale $\Lambda$CDM problems is found, however, we shall first consider in detail the full shape of linear spectra and use them to perform ad–hoc numerical simulations. The former aim is fulfilled here below, the latter one is the target of Paper II (Macciò et al. 2015).

Meanwhile, let us outline that, although selecting $m_w \sim 90 \text{ eV}$ is dictated, first of all, by the opportunity of exploiting this intrinsic feature of SCDEW cosmologies, there are a few other elements in favor of this choice. The very Figure 1 shows that the contributions of WDM, CDM and $\Phi$ to the primeval thermal soup are all within half order of magnitude. Even though this is due to the choice of $\beta$ and $m_w$, it would be reckless to conclude that it fixes their ranges, but it would also be hard to believe all that to be fully casual. Figure 4 then shows a strictly related point:

The primeval density parameters $\Omega_i$, plotted there are for CDM, $\Phi$ and WDM. The WDM density parameter is not strictly $\beta$ independent as, for very low $\beta$ values, its level is eaten by a non–negligible $3/4\beta^2$ contribute. Should however $m_w$ be greater by $\Delta m$, WDM derelativize earlier by $\Delta a \simeq \Delta m/T_0$ and the nearly flat WDM line should be lower. In order to recover the nearly–coincidence between primeval $\Omega_i$, fairly greater $\beta$ values should then be selected, as the dependence on $\beta$ is quadratic. The opposite would occur for lower $m_w$ values, with the risk to approach the limits of the physical range for $\beta$’s.

This plot should be however taken together with a third argument, which can be made only after giving details on spectra (see Figure 3 below): equal normalizations for CMB angular spectra and $\sigma_8$, in $\Lambda$CDM and SCDEW models, should be required; this fixes a narrow $\beta$ range, once $m_w = 90 \text{ eV}$ is selected. In turn, Figure 4 returns $m_w = 90 \text{ eV}$ for that $\beta$ range. Altogether, the two constraints are close to a system of 2 algebraic equations with 2 unknown, although yielding softer constraints. $\beta \simeq 10$ and
4.4 CDM non–linearities

Recovering (WDM or) baryon fluctuations thanks to coupled CDM action, on those scales where they had been erased, requires its fluctuations to be 100–1000 times wider than uncoupled CDM fluctuations on those scales where they had been erased, requiring its fluctuations to be 100–1000 times wider than uncoupled CDM fluctuations. An estimate of the effect can be thought to be just as light. A quantitative estimate of such boost is not immediately feasible, although one must however recall that even if coherence is partially lost, that part of \( \delta_b \) will enter a unified growth regime. Accordingly, the effect of marginal non-linearities can be thought to be just a slight boost of the CDM+baryon fluctuation amplitudes. A quantitative estimate of such boost is not immediately feasible, although one must however recall that \( \Omega_c \ll \Omega_b \), so that the very uncertainty shall not be too large.

When considering the spectral function in the next Section, which are a product of linear algorithms, one must therefore discriminate among different possible meanings of apparently high \( \Delta^2(k) \) for the CDM component at high \( k \).

There is however a clear conclusion for the above discussion: SCDEW models predict the formation of early non–linear CDM structures up to a mass \( \sim 10^7-10^8 h^{-1} M_\odot \). With a present CDM motion, so that we can expect a substantial loss of coherence between the distribution of CDM and other cosmic components, with top spectral contribution eventually affecting wavelengths smaller than the original fluctuations \( \delta_c \). Accordingly, no excessive effects are expected on scales never overcoming a mild non–linearity.

At an advanced stage, evolved CDM non–linearities shall then be non–linear structures embedded in an almost unperturbed continuum, taking also into account that, altogether, the CDM density parameter has reduced to \( \Omega_c \ll \Omega_b \), so that the perturbing bodies shall be relatively rare.

Let us also outline that, also after \( \beta \) fading, the linear algorithm will continue to treat CDM and baryons separately. As a consequence, the CDM fluctuation \( \delta_c \) can still overcome unity while the baryon fluctuation \( \delta_b \) is still \( \ll 1 \). However, in these conditions, CDM and baryon obey the same equations of motion and the physical variable is

\[
\delta_{cb} = \frac{\Omega_c \delta_c + \Omega_b \delta_b}{\Omega_c + \Omega_b}.
\]

Unless this is non–linear, physical non–linearities do not exist. Also if, slightly before \( \beta \) fading, \( \delta_c \) approached unity, but without badly loosing coherence, the physical post–fading variable will be \( \delta_{cb} \); however, even if coherence is partially lost, that part of \( \delta_c \) still coherent with \( \delta_b \) will enter a unified growth regime. Accordingly, the effect of marginal non-linearities can be thought to be just a slight boost of the CDM+baryon fluctuation amplitudes. A quantitative estimate of such boost is not immediately feasible, although one must however recall that \( \Omega_c \ll \Omega_b \), so that the very uncertainty shall not be too large.

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\[ m_{\nu} = 90 \text{ eV} \] are at the middle of the narrow allowed bands. If N–body simulations will confirm that this selection favours observational profiles, etc., we may say that this follows from the optimal parameter choice within the model.
density \( \sim 10^8 h^{-1} M_\odot/(h^{-1} \text{Mpc})^3 \), we expect their average comoving distance to be \( \sim 1 h^{-1} \text{Mpc} \), a value comparable with the average intergalactic distance. Smaller residual CDM structures could also exist, if not embedded in greater structures in a hierarchical formation process. No further speculation is however possible, to better define the specific features these “objects” could exhibit, apart of the rather obvious but probably semplistic statement that they might occupy the nucleus of existing galaxies.

5 FLUCTUATION SPECTRA

The modified version of CMFBEAST enables us to evaluate the transfer functions and the spectral functions \( \Delta^2(k, z) \) (see eq. (29)). Figures 8 show them at \( z = 0 \), for specific models, with separate curves for different components. Results are compared with \( \Lambda \)CDM.

For \( \log k \lesssim -1 \) the spectra of all cosmic components almost overlap. At greater \( k \) values, baryons and WDM exhibit a gap in respect to \( \Lambda \)CDM. In the absence of strongly coupled CDM, such hint to decline would turn into a fast decrease at slightly higher \( k \)’s, a characteristic of standard \( \Lambda \)WDM cosmologies. Here, on the contrary, thanks to the action of coupled CDM, we see a fast recovery. Baryon and WDM spectra therefore re-approach the \( \Lambda \)CDM spectrum and, in the case of \( d = 4 \), overcome it. In turn, the (coupled) CDM spectra are constantly above baryon and WDM; for \( d = 2 \), however, the CDM spectrum does first decline, as baryons and WDM do, then gradually rises above them; on the contrary, for \( d = 4 \), the CDM spectrum keeps constantly above \( \Lambda \)CDM and there is just a hint of slower increase where baryons and WDM begin their gap.

The most significant point, however, is the comparison with \( \Lambda \)CDM. Figures 5 show a greater spectral discrepancy for lower \( d \) values. If we however consider \( \beta \neq 10 \) values (not shown in the Figures), we have a similar evolution of CDM fluctuations, for the \( \beta \) compensation discussed in Section 4.1, but equally large CDM fluctuations are less efficient to recreate WDM and baryon fluctuations, because of the smaller CDM mass attracting them. Accordingly, the discrepancy with \( \Lambda \)CDM is even greater, in spite of CDM having spectra similar to the ones shown here. As a matter of fact, with \( \beta = 10 \), the discrepancy of baryon and WDM spectra with \( \Lambda \)CDM is not so great, never exceeding 1 o.o.m.

There is another point, already visible in the spectral functions in Figure 5 but conveniently stressed in Figure 6 at high \( k \) the baryon spectrum starts to exceed the WDM spectrum. It is not a negligible effect. At \( k \approx 30 h^{-1} \text{Mpc} \), the baryon spectrum exceeds average by \( \sim 60 \% \), while the WDM spectrum exhibits a deficiency, still in respect to average, by \( \sim 25 \% \). In Figure 6, curves referring to models with \( d = 2, 3, 4 \) are shown to overlap. The Figure is done for \( m_w = 90 \text{eV} \); as expected, a significant increase of \( m_w \) reduces the gap.

It may be significant to see the shift between SCDEW and \( \Lambda \)CDM, when looking at the linear m.s. fluctuation

\[
\sigma_r^2 = \int_0^\infty dk \Delta^2(k) W^2(k R) \tag{29}
\]

(mass variance). There are different forms for the window function \( W(k R) \); in this work all computations were coherently performed by using an exponential window. The \( \sigma_r \) behavior is shown in Figure 7 for \( d = 4 \) where, within the models considered here, it is greatest. As recalled in the frame, this is a linear computation, and the slight discrepancies appear on scales \( R < 1 h^{-1} \text{Mpc} \) (\( \sim 3.5 \times 10^{11} h^{-1} M_\odot \)), surely non-linear today. The expected overall effect, after the extensive non-linear \( k \)-mode mixing, is expected to be slightly greater.

The expression (29) can be also used to study the \( \beta \) dependence of \( \sigma_R \) (mass variance on the scale of \( 8 h^{-1} \text{Mpc} \)) on \( \beta \) and \( m_w \). In Figure 8 we keep \( m_w \) fixed and study the \( \sigma_R \) dependence on \( \beta \) for models with \( d = 4 \). The main finding shown in this plot is that, in order to approach the \( \sigma_R \) value of \( \Lambda \)CDM, we need \( \beta \approx 10 \).

Let us then outline the difference between total spectra of models differing for the \( d \) values. As shown in Figure 9 discrepances increase with \( k \). For \( \log k \approx 1.5 (M \approx 2.8 \times 10^9 h^{-1} M_\odot) \)
the discrepancy is $\sim 2$, when we compare $d = 4$ and $d = 2$; it becomes $< 20\%$ when we compare $d=3$ and 4. The above feature however exhibits just a mild $\beta$ dependence (not shown in the Figure): for lower (higher) $\beta$ values, the discrepancy is slightly smaller (greater).

These spectra were accurately considered to choose the SCDEW model most suitable to study non-linear effects. Coupled CDM non-linearities are safely milder with $d = 2$. (They would be even milder for $d<2$ but, as shown in Figure 1, the $d$ dependence of the model becomes stronger and, in particular, a significant gap in $\rho_\Phi$ behavior becomes unavoidable.) As discussed in previous Section, even in the $d = 2$ case we expect some impact of forming CDM non-linearities, probably slightly rising the spectrum, more and more as $k$ increases. Owing then to the limited difference between spectra, shown in Figure 2, we decided to make use of the $d = 4$ spectrum, assuming that, probably, it best represents the $d = 2$ case, once the upward shifts due to forming CDM non-linearities are included.

6 DISCUSSION

A number of experiments are in progress, aiming to inspect the nature of dark cosmic components, e.g. BOSS, HETDEX, DES, LSqST, and Euclid. Even assuming a linear expression $w = w_0 + w_1(1 - a)$ for DE state parameter, however, errors on $w_0$ and $w_1$ can hardly be pushed below 10% and same 10%, respectively (e.g. Joachimi & Bridle 2010). Better hopes exist to detect violations of the standard relation between scale factor $a$ and growth factor $G$ time dependences.

They could arise from violation of General Relativity if, e.g., the gravitational action $R$ (Riemann scalar) is added a suitable function $f(R)$. In this case the dynamics of any component, baryons and DM, would be directly modified at large distances. A large deal of work has deepened this option. N-body simulations have been also recently performed (Puchwein et al. 2013).

An alternative option is that only DM dynamics is modified, as occurs in “ordinary” coupled DE theories. Also this option has been widely debated, N-body simulations of these models have been performed since several years (Macciò et al. 2004; Caldera–Cabral et al. 2006; Baldi et al. 2010; Li & Barrow 2011; Baldi 2012), finding that baryon distribution would also change, over very large scales, as an indirect consequence. Since a few years these models have been shown to be consistent with data if $\beta \lesssim 0.1$, limits being widened if coupling is considered together with a non-vanishing neutrino mass, even finding a $\sim 2-\sigma$ constraint on the coupling $\beta$ about $\sim 0.09$ (La Vacca et al. 2009; Kristiansen et al. 2010; Pettorino et al. 2012). More recently, the coupling option was also shown to ease the tension between Planck and Hubble telescope $H_0$ estimates, allowing Xia (2013) to turn limits into a $> 3-\sigma$ detection, yielding $\beta = 0.078 \pm 0.022$ and, consistently, $H_0 = 74.8 \pm 2.8$. Also within this option, significantly affecting the dynamics of the whole DE, future data are expected to outline apparent $a(t)$–$G(t)$ discrepancies.

Strongly Coupled cosmologies (SCDEW models), on the contrary, predict no change in $a(t)$ or $G(t)$ in respect to $\Lambda$CDM and therefore, a fortiori, the $a(t)$–$G(t)$ relation should be found do be consistent with $\Lambda$CDM predictions. The very equation of state of DE remains somehow arbitrary, although state parameters $w < 1$ would require suitable extensions of the approach described in this work. All above experiments are therefore expected to yield results seemingly consistent with $\Lambda$CDM cosmologies.

This point has been accurately verified in this work. In comparison with BSLV12 and BM14, two further points were outlined here: (i) By providing a Lagrangian approach to the coupling between $\Phi$ and $\psi$ fields (DE and CDM, respectively) and stressing the logarithmic growth of the former one, we outlined the possibility that $\Phi$ is both inflaton and DE. (ii) The risk of CDM fluctuations
reaching an early non–linear dynamics was also outlined and circumscribed.

We shall further discuss the (i) point in another work. As far as the (ii) point is concerned, it is strictly linked to the minimal assumption of a constant interaction constant $\beta$. We tentatively suggested to overcome such option by admitting $\beta$ to fade, once it accomplished its aim to allow us an (almost) conformally invariant cosmic expansion through cosmic ages. Possible mechanisms relating conformal invariance break to $\beta$ fading, however, were not discussed. Other options, like CDM particle annihilation, etc., were also not debated.

Altogether, the picture considered remains fully viable and leads to a phenomenological picture quite close to $\Lambda$CDM. In its theoretical framework, however, most of the unpleasant fine tunings and coincidences of $\Lambda$CDM are significantly eased.

SCDEW and $\Lambda$CDM cosmologies are however distinguishable through a number of observables: First of all, SCDEW suggests a DM particle with mass $\sim 100$ eV. If this kind of warm–hot DM replaces the CDM of $\Lambda$CDM models, in the absence of strong coupling there would be almost no structure in the Universe. Even the option of mixed cold and hot–warm DM is far from fitting data.

This however occurs over mass scales $\lesssim 10^7 h^{-1} M_\odot$ (let us however recall that, altogether, coupled CDM density is $\sim 1\%$ of the very baryon density, today).

This risks to weaken the predictivity of linear codes. When growing non linear, CDM fluctuation gravitation becomes stronger than linear codes compute, speeding up the growth of other components. This effect is however weakened by $\rho_c$, becoming even smaller than $1/2 \beta^2$, as the Universe turns from radiation to matter dominated, and by the fact that other component fluctuations have recovered such a significant amplitude to yield an -- at least -- comparable push. Accordingly, we suggested the extra gravitational push of CDM to bear effects equivalent to shifting the parameter $d$ from 2 to 4. Although based on qualitative arguments, this option is not unlikely to approach the real effect. Accordingly, $N$–body simulations in the associated Paper II are based on the $d = 4$ option.

All that implies a clear prediction: that early CDM primeval structures have formed. Although being extremely rare because of the low overall CDM density ($\sim 1/100$ of baryon density), they could be the main existing structures at very high redshifts ($z \gg 10$) and might have an effect at later times on the formation of the first stars and/or cosmic reionization, we plan to address these issues in future work.

In our opinion, however, the strongest argument in favour of SCDEW models is the apparent easing of fine tunings. The only point which could be made against them is the need of a twofold DM, an option however often considered on phenomenological bases. DM in SCDEW cosmologies, however, is just the warm one; CDM is a sort of handyman component, fist allowing DE and inflaton to bear effects equivalent to shifting the parameter $d$ from 2 to 4. As far as the (ii) point is concerned, it is strictly linked to the minimal assumption of a constant interaction constant $\beta$. We tentatively suggested to overcome such option by admitting $\beta$ to fade, once it accomplished its aim to allow us an (almost) conformally invariant cosmic expansion through cosmic ages. Possible mechanisms relating conformal invariance break to $\beta$ fading, however, were not discussed. Other options, like CDM particle annihilation, etc., were also not debated.

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