Radiation-induced large-scale structure during the reionization epoch: the autocorrelation function

Rupert A. C. Croft* and Gabriel Altay
Department of Physics, Carnegie Mellon University, Pittsburgh, PA 15213, USA

Accepted 2008 May 22. Received 2008 May 19; in original form 2007 September 12

ABSTRACT
The structures produced during the epoch of reionization by the action of radiation on neutral hydrogen are in principle different from those that arise through gravitational growth of initially small perturbations. We explore the difference between the two mechanisms using high-resolution cosmological radiative transfer. Our computations use a Monte Carlo code which ray-traces directly through smoothed particle hydrodynamics (SPH) kernels without a grid, preserving the high spatial resolution of the underlying hydrodynamic simulation. Because the properties of the first sources of radiation are uncertain, we simulate a range of models with different source properties and recombination physics. We examine the morphology of the neutral hydrogen distribution and the reionization history in these models. We find that at fixed mean neutral fraction, structures are visually most affected by the existence of a lower limit in source luminosity, then by galaxy mass-to-light ratio, and are minimally affected by changes in the recombination rate and amplitude of mass fluctuations. We concentrate on the autocorrelation function of the neutral hydrogen, $\xi_{HI}(r)$, as a basic quantitative measure of radiation-induced structure. All the models we test exhibit a characteristic behaviour, with $\xi_{HI}$ becoming initially linearly antibiased with respect to the matter correlation function, reaching a minimum bias factor $b \sim 0.5$ when the universe is $\sim 10$–20 per cent ionized. After this $\xi_{HI}$ increases rapidly in amplitude, overtaking the matter correlation function. It keeps a power-law shape, but flattens considerably, reaching an asymptotic logarithmic slope of $\gamma_{HI} \simeq -0.5$. The growth rate of $H_1$ fluctuations is exponentially more rapid than gravitational growth over a brief interval of redshift $\Delta z \sim 2$–3.

Key words: cosmology: observations – large-scale structure of Universe.

1 INTRODUCTION
In the standard cosmological model, the large-scale structure in the density field grows from small initial perturbations through the mechanism of gravitational instability. Statistical measures of this structure can be used to both verify the growth mechanism (see e.g. Bernardeau et al. 2002 and references therein) and quantify the initial perturbations. A different kind of growth of structure is expected when we consider the neutral hydrogen density field during the epoch of reionization (see e.g. the review by Loeb & Barkana 2001). In this case, bubbles of ionized material first around bright sources and grow as the ionization fronts overlap until the universe is fully ionized. Statistical measures applied to this ‘radiation-induced structure’ (hereafter RIS) can be used in a similar way to the gravitational instability picture above, but this time to verify the process of reionization and the nature of the sources of radiation. The effect of RIS is likely to be qualitatively different from that of gravity, and as a result the statistical signatures will be different. Our aim in this paper is to explore the differences, using ray-traced simulations of reionization. We aim to both find out how reionization is different from gravity in the way it forms structure and how to use these differences to categorize reionization scenarios. The statistical properties of RIS have been explored in many other works, e.g. using the power spectrum of $H_1$ fluctuations by Furlanetto et al. (2004a,b), Zaldarriaga, Furlanetto & Hernquist (2004), Morales & Hewitt (2004), and using Minkowski functionals by Gleser et al. (2006). In the present paper we will focus on the autocorrelation function.

Theoretical studies of reionization include both analytic work, e.g. Miralda-Escude, Haehnelt & Rees (2000), Wyithe & Loeb (2003), Cen (2003), Liu et al. (2004), Furlanetto et al. (2004), and numerical simulations, e.g. Razoumov & Scott (1999), Abel, Norman & Madau (1999), Gnedin (2000), Ciardi et al. (2001), Sokasian, Abel & Hernquist (2001), Razoumov et al. (2002), Sokasian et al. (2004).

Recently, $N$-body simulations with radiative transfer (RT) post-processing have been performed by Iliev et al. (2006) and Zahn.

*E-mail: rcroft@cmu.edu

© 2008 The Authors. Journal compilation © 2008 RAS
Downloaded from https://academic.oup.com/mnras/article-abstract/388/4/1501/980912 by guest on 30 July 2018
et al. (2007) with box sizes as large as 100 h⁻¹ Mpc and able to resolve haloes down to masses of ~2 × 10⁹ M⊙. Kohler, Gnedin & Hamilton (2007) have performed simulations with an extremely large box size (up to 1280 h⁻¹ Mpc) in which hydrodynamics and RT are coupled self-consistently. These simulations rely on higher resolution simulations to calibrate the subgrid (<10 h⁻¹ Mpc) physics. Other recent advances in simulating reionization include the work of Trac & Cen (2007), a hybrid N-body dark matter and RT approach, a code which was used to study the growth of bubbles during reionization by Shin, Trac & Cen (2007).

Analytic work has been carried out using perturbation theory to predict inhomogeneities in the density of neutral hydrogen and photons (Zhang, Hui & Haiman 2007), as well as Press–Schechter based analyses (e.g. Furlanetto et al. 2004). Most relevant to the work here, the autocorrelation function of 21-cm emission has been examined in analytic models by Wyithe & Morales (2007) and Barkana (2007).

In the present paper we do not aim to simulate particular observational probes of this epoch, such as the Doppler scattering of cosmic microwave background (CMB) photons on relativistic electrons, or the 21-cm emission from neutral hydrogen. Instead, we concentrate on the differences between 〈ξ(ξ) for the density field and the density field evolved by RIS. In principle, this will be directly observable in the future, through the various observational probes (e.g. Carilli et al. 2004; Peterson, Pen & Wu 2005; Valdés et al. 2006).

Although we leave detailed exploration of this to future work, it is important to stress that the information regarding the neutral hydrogen distribution in the simulation will need to be combined with further assumptions and modelling in order to derive observational predictions. For the 21-cm brightness temperature fluctuations their relation to the neutral hydrogen density fluctuations is given by (e.g. Zaldarriaga et al. 2004)

\[ \delta T(\nu) \simeq 26x_h (1 + \delta_h) \left( \frac{T_s - T_{\text{CMB}}}{T_s} \right) \left( \frac{\Omega_h h^2}{0.022} \right) \times \left[ \frac{0.15}{\Omega_m h^2} \left( \frac{1 + z}{10} \right) \right]^{1/2} \text{mK,} \]

where \( \delta T(\nu) \) is the temperature difference relative to the CMB at frequency \( \nu \), \( x_h \) is the hydrogen neutral fraction, \( (1 + \delta_h) \) is the gas density in units of the cosmic mean, \( T_s \) is the spin temperature and \( T_{\text{CMB}} \) is the CMB temperature. A usual simplifying assumption made in this case is (e.g. Ciardi & Madau 2003) that \( T_s \gg T_{\text{CMB}} \), implying that \( \delta T \propto (1 + \delta_h) x_H \). Redshift distortions also need to be taken into account in order to put the simulations into the observed space. Other probes of the H I distribution include coherent structure in the polarization of the CMB (which can be predicted by propagating linearly polarized plane CMB waves through the simulation, e.g. Doré et al. 2007).

The format of this paper is as follows. In Section 2 we describe the simulations, including the N-body outputs and the RT code. We also describe our different models for the sources of ionizing radiation and different physical conditions we simulate. In Section 3 we show how global properties such as the mean ionized fraction evolve as a function of redshift in the different runs, and then in Section 4 examine the morphology of the neutral and ionized hydrogen density field. In Section 5 we measure the autocorrelation function in our different models and explore how it can differentiate between them. We summarize our results and discuss them in Section 6.

2 SIMULATIONS

We work in the context of the standard cosmological constant dominated universe, with parameters \( \Omega_m = 0.7, \Omega_b = 0.3, \Omega_{\Lambda} = 0.04 \) and a Hubble constant \( H_0 = 70 \text{ km s}^{-1} \text{ Mpc}^{-1} \). The initial linear power spectrum is cluster-normalized with a linearly extrapolated amplitude of \( \sigma_8 = 0.9 \) at \( z = 0 \). The RT is carried out as post-processing on N-body simulation outputs.

2.1 N-body outputs

We run our N-body simulations with the cosmological hydrodynamic code GADGET (Springel, Yoshida & White 2001). Our fiducial simulation is run in a 40 h⁻¹ Mpc cubical volume. Iliev et al. (2006) have shown in tests using subvolumes of a larger (100 h⁻¹ Mpc) simulation that 30 h⁻¹ Mpc is the smallest box size length for which the scatter in reionization histories in different volumes is reasonably small. We use 256³ dark matter and 256³ gas particles. The mass resolution is therefore 6.05 × 10⁷ M⊙ for gas particles and 3.93 × 10⁵ M⊙ for dark matter. We also run higher resolution models in tests, as detailed below. We do not include radiative cooling or star formation when computing the gas dynamics so that our simulations are similar to those carried out by Sokasian et al. (2001). We run the RT as post-processing, so that there is no coupling between the hydrodynamics and radiation. In this respect, the gas serves to trace the dark matter distribution closely (there is little difference at these high redshifts). We choose our sources of radiation to be associated with dark matter haloes (see below).

In addition to the fiducial run, we run another model with identical box size and particle number but with different random initial phases in order to roughly indicate the effect of simulation cosmic variance. We also run a model with a different amplitude of mass fluctuations (\( \sigma_8 = 0.7 \)). For resolution tests, we run a simulation with 128³ gas and dark matter particles in a box of size 20 h⁻¹ Mpc (the same mass resolution as the fiducial run) and a simulation with 256³ gas and dark matter particles in a box of size 20 h⁻¹ Mpc (eight times better mass resolution than the fiducial run). All models were started at \( z = 50 \) and run until \( z = 5.2 \). We output snapshots of the density field every 25 Myr, so that there are approximately 40 snapshot files per run.

2.2 Radiative transfer

After choosing models for the sources of radiation (see below) we carry out ray-tracing simulations of RT to study the evolution of the neutral hydrogen density. The code we use to do this carries out Monte Carlo RT to follow photon packets through the distribution of matter. The code is based on that used by Croft (2004) to study the fluctuating radiation background field at lower redshift but incorporates time-dependent RT in a Monte Carlo manner similar to the CRASH code of Maselli, Ferrara & Ciardi (2003) [we actually only treat hydrogen here, so we are in fact closer to the earlier work of Ciardi et al. (2001)]. The code in the present paper traces directly through the smoothed particle hydrodynamics (SPH) particle kernels (see also Kessell-Deynet & Burkert 2000; Susa 2006; Yoshida et al. 2007 and Dale et al. 2007 for non-grid-based RT and Semelin, Combes & Baek 2007 and Oxley & Wolfe for SPH codes that trace through a Barnes–Hut tree) and so requires no regreidding of the density field between outputs. The spatial resolution of the RT is therefore in principle higher in dense regions than would be possible with a uniform grid. This approach is described in detail in Altay, Croft & Pelupessy (2008), where test problems are carried.
out. We also outline some of the relevant features of the code briefly below.

We note that, as with other codes which use a Monte Carlo approach, the resolution of the code is limited by the number of photon packets used. With a small number, shot noise will degrade the angular coverage of radiation around each source and the benefit of not re-assigning the particles to a grid will not be apparent. In order for the ionized density structure to be limited by the resolution of the SPH particles themselves, we need a large number of packets. We make sure that this condition is achieved for the correlation function through a convergence test, changing the total number of packets (see below and Section 5.1).

In the present paper we model only the hydrogen component of the Universe (assumed to comprise 0.76 of the baryonic mass). We also do not explicitly follow the temperature evolution of the gas, beyond taking temperatures of particles to be $10^5$ K when they are ionized and $T_{\text{CMB}}$ when they are not. We follow collisional ionization, photoionization and recombinations using the rates given in Cen (1992). We randomly sample source photons from a power-law distribution, $F_\gamma \propto \nu^\alpha$, of photon energies (more details on the sources are given in Section 2.3, below). In the present paper, photon packets are monochromatic and are emitted isotropically from sources, again using a random number generator to pick directions.

Every time a ray is traced through a particle, the number of recombination photons which have been produced in that particle since the last time it was visited are added to a stack. When the stack size reaches one packet, a recombination photon packet is emitted from the particle where this occurs. The frequency of the recombination radiation is given by the Milne relation (Osterbrock 1989).

We have one numerical code parameter, $c_1$, which sets the size of the photon packet, and therefore the time resolution of the code. This parameter $c_1$ is the number of fully neutral simulation gas particles which could be ionized by one photon packet if the energy in that packet was split up into 13.6-eV photons. For example, if a packet consists of $N_\gamma$ 13.6-eV photons and a gas particle contains $N_{\text{H}_1}$ neutral hydrogen atoms then $c_1 = N_\gamma / N_{\text{H}_1}$. By trying runs with different values of $c_1$ we change how well the code can resolve the recombination time-scale by governing the average interval between rays visiting particles and updating their ionization states. Packets of recombination radiation as well as source radiation are governed by $c_1$ so that shot noise arising from the discreteness of recombination modelling can be controlled. We choose a suitable value of $c_1$ by carrying out convergence tests (see e.g. Section 5.1). In practice we find that $c_1 = 0.33$ is adequate for modelling clustering of H I and is the value we use in our fiducial simulations. This results in 0.5–1.0 × $10^5$ photon packets being used in each of the runs.

Aalti et al. (2008) is mainly concerned with presenting a closely related code, the publicly available code SPHRAY, which has many additional features to the one used in this paper, including the ability to model temperature evolution and helium species.

2.3 Sources/runs

Eventually, observations of the RIS at the epoch of reionization will be useful as probes of the sources of radiation as well as cosmology. The statistical measures of this structure are likely to be correlated with the properties of the sources, their luminosities, lifetimes and clustering. One of the goals of this paper is to study the RIS produced by various extremely simple models for the ionizing source population, in order to see how they can be differentiated (principally through the autocorrelation function, which we focus on) and which features appear to be generic to the models considered.

The range of possible sources for reionization is extremely wide, including decaying dark matter (e.g. Mapelli, Ferrara & Pierpaoli 2006), primordial black holes (Peng & Fang 2002; Ricotti, Ostriker & Mack 2008), high-redshift miniquasars (e.g. Madau et al. 2004), Population III stars (e.g. Sokasian et al. 2004), Population II stars (e.g. Sokasian et al. 2003), some more or less likely than others. Rather than attempting to simulate particular models in detail, we restrict ourselves to simply parametrized models which relate the ionizing radiation intensity directly to the dark matter distribution. This is on the understanding that in most reasonable models of reionization there would be some relationship between the two (either through galaxies and stars associated with dark matter overdensities, or directly through dark matter clumps decaying to ionizing photons). In particular, we associate sources of radiation to dark matter haloes.

This approach has been used also by Melléma et al. (2006), who use a constant mass-to-light ratio to assign ionizing radiation to each halo, as well as by Zahn et al. (2007), who populate each halo with a single ionizing source whose luminosity is proportional to host halo mass. McQuinn et al. (2007) have also simulated 17 different variations of this type of model with various ionizing photon efficiencies, prescriptions for feedback and minihaloes. McQuinn et al. (2006) focus on the morphology of H I regions.

All the runs we use for the main studies in this paper have a simulation box length of $40 h^{-1}$ Mpc and particle number of $2 \times 256^3$, although as explained in Section 2.1, for resolution studies we have some runs with different mass resolutions and box sizes. We find haloes using a standard friends-of-friends routine, with a linking length of 0.2 times the mean interparticle separation. The minimum halo mass we use as a source in our fiducial run is $1.6 \times 10^9 h^{-1}$ M⊙, containing only eight (gas and dark matter) particles. This is approaching the scale of minihaloes expected to host numerous weak ionizing sources and provide small-scale clumpiness to the intergalactic medium (IGM), but as with the calculations of Zahn et al. (2007) (who do have a smaller particle mass) it is still approximately an order of magnitude too large. Although we only resolve such haloes with a small number of particles, we have checked using simulations with eight times better mass resolution (see Section 5.1) that this does not affect our calculation of the autocorrelation function of neutral hydrogen, the statistic we focus on. In general, the limited mass resolution of simulations will affect results both through the absence of sinks (minihaloes) and sources hosted by small haloes. Recent simulations (e.g. Santos et al. 2007) are beginning to address this directly through increases in simulation particle number. We return to this point in Sections 5.1 and 6.2.

We use the same density field for 10 of the 12 main runs, carrying out the RT as post-processing using different source prescriptions. The other two are a low-fluctuation amplitude model ($\sigma_8 = 0.7$) run with the same random phases and another model with the same amplitude as the fiducial case but with different phases. The 12 runs are differentiated by their different halo mass-to-light ratios, treatment of the relationship between halo mass and ionizing radiation, the recombination rate and spectrum of radiation. An overview is given in Table 1, and they are described in detail below. We label the different simulations by short descriptive names rather than numbers or letters in order to avoid the necessity of the reader referring back to a table when examining the results.
The total integrated luminosity to $z = 6$ is therefore less than that in the fiducial run.

(vii) No recomb, 2 × recomb. In these runs, the recombination rate of ionized hydrogen was either set to zero or doubled, and the number of recombination photon packets adjusted accordingly. Otherwise, the runs are the same as the fiducial run. These models can be thought of as parametrizing the effects of changing the clumping factor of unresolved gas.

(viii) Other random phases. As a rough indicator of the effects of cosmic variance, this run has the same parameters as the fiducial run, but uses a different underlying density field realization.

We evolve all the simulations to redshift $z = 5.5$, irrespective of whether they have achieved full reionization by then.

### 3 Global Evolution

Because we have many different simulation runs with different treatments of recombination and models for the sources, we need to decide at which redshifts to compare our measures of clustering. The different runs reach 50 per cent mean ionized fraction by mass, $x_m$, at redshifts ranging from $z = 9$ to 6, so that comparing them at the same redshift will mean comparing different stages of reionization. We will therefore concentrate in this paper on snapshots taken at

---

**Table 1.** Radiative transfer simulations discussed in this paper. Further details are given in Section 2.3.

| Simulation | (Luminosity integrated to $z = 6$)/fiducial | Spectrum $\alpha$ $(F_\nu \propto \nu^\alpha)$ | Recombination rate | Box length ($h^{-1}$ Mpc) | Particle number | Comments |
|------------|--------------------------------------------|---------------------------------------------|--------------------|----------------------------|----------------|----------|
| Fiducial   | 1.0                                        | −4.0                                       | 1.0                | 40.0                       | 2 × 256³       |          |
| L/2        | 0.5                                        | −4.0                                       | 1.0                | 40.0                       | 2 × 256³       |          |
| L/4        | 0.25                                       | −4.0                                       | 1.0                | 40.0                       | 2 × 256³       |          |
| L/8        | 0.125                                      | −4.0                                       | 1.0                | 40.0                       | 2 × 256³       |          |
| L indep M  |                                            | −4.0                                       | 1.0                | 40.0                       | 2 × 256³       | Luminosity independent of halo mass |
| $M > 10^{10} h^{-1} M_\odot$ | 1.0                                        | −4.0                                       | 1.0                | 40.0                       | 2 × 256³       | Lower limit on halo mass for sources |
| $M < 10^{10} h^{-1} M_\odot$ | 1.0                                        | −4.0                                       | 1.0                | 40.0                       | 2 × 256³       | Upper limit on halo mass for sources |
| $\sigma_\nu = 0.7$ | 0.87                                      | −4.0                                       | 1.0                | 40.0                       | 2 × 256³       | Low amplitude of mass fluctuations |
| No recomb. | 1.0                                        | −4.0                                       | 1.0                | 40.0                       | 2 × 256³       |          |
| 2 × recomb | 1.0                                        | −4.0                                       | 2.0                | 40.0                       | 2 × 256³       |          |
| $\nu^{-2}$ spectrum | 1.0                                      | −2.0                                       | 1.0                | 40.0                       | 2 × 256³       |          |
| Hires      | 1.0                                        | −4.0                                       | 1.0                | 20.0                       | 2 × 256³       | Same parameters as fiducial, different ICs |
| Small      | 1.0                                        | −4.0                                       | 1.0                | 20.0                       | 2 × 128³       |          |
| Different $c_1$ | 1.0                                    | −4.0                                       | 1.0                | 40.0                       | 2 × 256³       | Three runs: photon packet $c_1 = 3.0, 1.0, 0.1$ |

(i) The fiducial run. Here the instantaneous luminosity of sources is proportional to the dark matter halo mass, with $L = L_\odot \times M_{\text{halo}} / h^{-1} M_\odot$. In our fiducial run, we take $L_\odot = 2.7 \times 10^{31} \text{ erg s}^{-1} (h^{-1} M_\odot)^{-1}$, and a source spectrum with $F_\nu \propto \nu^{-2}$, appropriate for Population II stellar sources (e.g. Sokasian et al. 2003). This simple model is also used by Zahn et al. (2007), who assume a number of photons proportional to halo mass [with a conversion factor of $3.1 \times 10^{41}$ photons s$^{-1} (h^{-1} M_\odot)^{-1}$]. This corresponds to our fiducial model having a luminosity per halo mass four times larger than that of Zahn et al. Using the computation of Zahn et al., the source output of our fiducial model can be considered to be very roughly equivalent to one with Population II stars forming with an efficiency of $f_\text{esc} = 0.1$ from a Salpeter initial mass function, with a stellar lifetime of $\Delta t = 5 \times 10^5 \text{ yr}$ and an escape fraction of $f_{\text{esc}} = 0.04$. The recombination rate in the fiducial simulation is taken to be that computed directly from the gas density, and diffuse recombination photons are treated.

(ii) Runs L/2, L/4 and L/8. These are the same as the fiducial run in every respect except that the ionizing luminosity has been reduced by an overall factor of 2, 4 or 8. These models can be considered to be equivalent to e.g. reducing the efficiency of star formation, and/or the ionizing escape fraction with respect to the fiducial run. The L/4 model therefore corresponds to the model simulated by Zahn et al. (2007).

(iii) L indep M. We use the same halo list as the fiducial run, but instead of a luminosity proportional to halo mass, we assign the same luminosity to all haloes. The total integrated luminosity to $z = 6$ is set to be equal to that in the fiducial run.

(iv) $M > 10^{10} h^{-1} M_\odot, M < 10^{10} h^{-1} M_\odot$. For these two runs, the same halo source list is used as in the fiducial run, but with either an upper or lower cut-off applied in the mass of a halo which can host a source. As in the previous model above, the total integrated luminosity to $z = 6$ is set to be equal to the fiducial run. These runs can be considered to roughly model the effects of feedback which might cause disruption of galaxy sources in small haloes.

(v) $\sigma_\nu = 0.7$. This run is the same as the fiducial run, except using a simulation which has a significantly lower amplitude of mass fluctuations, leading to later halo formation times. The source luminosity was proportional to the halo mass in the same fashion as for the fiducial run, also with $L_\odot = 2.7 \times 10^{31} \text{ erg s}^{-1} (h^{-1} M_\odot)^{-1}$.
the same mean ionized fractions in the different runs, rather than at the same redshifts.

In Fig. 1 we show how the mean mass-weighted neutral fraction (linear scale) and mean mass-weighted ionized fraction (log scale) vary with redshift for the different runs. As expected, the run without recombinations ionizes first, reaching a 1 per cent ionized fraction by redshift $z = 13$ and being 1 per cent neutral at $z = 7.8$. As reionization proceeds, the relationship between mean ionized fraction $x_m$ and redshift is approximately exponential, $x_m = e^{-m(z-1)}$ (here $z_i$ is the redshift when the model is fully ionized) for this and the other models. This is roughly true for all models, except for the model where only large galaxies (halo masses $M > 10^{10} M_\odot$) are sources, and the model with $\sigma_8 = 0.7$, both of which have a substantially more rapid change in ionization with redshift, $x_m = e^{-m(z-\zeta_i)}$. This can be explained by the fact that galaxies massive enough to be sources only form relatively late in these two models.

The models where the luminosity per dark matter halo atom was varied, fiducial, $L/2, L/4, L/8$ reach the $x_m = 0.5$ point later by $\Delta z = 1$ for each halving of the luminosity. The slope of log $x_m$ versus $z$ is very similar for $x_m < 0.1$ for these models but then changes as reionization proceeds. We note that the $L/4$ model reaches $x_m = 0.5$ at $z \sim 7$, similar to the model of Zahn et al. (2007), which has a similar spectrum and source luminosity (see Section 2.3 above).

More physical insight can be gained by looking at two other quantities as a function of redshift, the number of recombinations per atom and the photon mean free path (MFP), which are plotted in Fig. 2. To compute the former, we divide the cumulative number of recombination photons by the total initial number of hydrogen atoms in the simulation volume. We plot a symbol on the curves at the point when the ionized fraction by mass reaches $x_m = 0.5$ and another at $x_m = 0.99$ (not all models reach this stage). Most of the models have below 0.5 recombination photons per atom by the time they have fully ionized, indicating that recombinations do not play the major role in the process of reionization. The curves for the different models tend to flatten off slightly for the last half of the reionization process. One striking feature of these curves is the importance of the amplitude of mass fluctuations, $\sigma_8$. As we would expect, the low-amplitude model ($\sigma_8 = 0.7$), having less clumping has a lower recombination rate. When compared at the point when $x_m = 0.5$, this model has experienced 3.2 times fewer recombinations than the fiducial model. This is greater than the ratio of $\sigma_8^2$ for the two models (1.65), due to non-linearity of clustering and the greater cosmic time in the low-amplitude model to get to this point.

The photon MFP as a function of redshift is also shown in Fig. 2. The mean path-length between absorptions has been calculated by computing for each photon packet (including sources and recombination rays) the distance that it travels from the point of emission to the location along the ray when it has been reduced in energy by a factor $1/e$. For each point plotted in Fig. 2 we have computed the average of the MFPs for the most recently computed 5 x $10^4$ photon packets. This is a relatively small number and as a result, sawtooth features in the curves can be seen, which correspond to the times at which the new density field snapshots were inputted (every 25 Myr). We have tested with more widely spaced outputs (50 Myr) that this does not affect the underlying curves. Another artefact of the measurement is the behaviour at large MFP. When averaging the path-length, we only include photon packets that have not wrapped more than once round the box. As a result, when the volume is close to fully ionized, the average is taken over a biased sample,
those photons which have run into an absorber in less than a box length. The MFP we plot therefore falls. Our reason for wrapping the photon packets only once round the box is that at late times, as the RISs become comparable to the size of the volume, the simulation will become inaccurate in any case. Rather than increasing the computational time of the simulation by wrapping more than once, which may or may not increase its accuracy, we prefer to restrict our use of numerical results (e.g. \( \xi \)) to regimes for which tests of resolution and box size show that they are reliable.

The overall behaviour of the MFP is as we would expect, with a rapid rise for most models at early times, with an MFP corresponding roughly to the size of the \( \text{H} \text{n} \) regions around sources. For example at redshift \( z = 14 \), when the ionized fraction is \( \sim 10^{-3} \), the MFP in the fiducial model is \( \sim 10 h^{-1} \text{kpc} \). In the \( \nu^{-2} \) spectrum case, the low cross-section of neutral hydrogen for the hard photons means that the MFP starts off nearly an order of magnitude larger, \( \sim 300 h^{-1} \text{kpc} \). The hardest photons will travel much farther than this mean, and there is a low level of ionization (ionized fraction \( x_m \gtrsim 10^{-5} \)) present throughout the whole volume, even at these very early redshifts.

In order to understand how reionization proceeds, it is instructive to look at the relation between neutral fraction and density. This is plotted for the fiducial run in Fig. 3 at four different stages in the evolution of the model. In each panel, we also show a histogram of particle density values and neutral fractions as well as a scatter plot of one versus the other. If we go from panel to panel, we can see that the high-density regions are reionized first (the ‘inside-out’ scenario also found in Iliev et al. 2006). For example, when \( x_m = 0.1 \), the cloud of partially ionized particles to the right-hand side of the panel is centred around \( \rho /(\langle \rho \rangle) \sim 10 \), whereas at \( x_m = 0.7 \) it is centred around \( \rho /(\langle \rho \rangle) \sim 3 \). Within this cloud of points, the higher rate of recombinations in high-density regions means that within the partially ionized volume there is a trend for higher neutral fractions at high densities. This trend is additional to the opposite trend for high-density regions to reach this partially ionized state first. As the universe comes close to being fully ionized, \( x_m = 0.97 \), larger fraction of particles close to the mean density are still fully neutral and more particles with the highest densities have neutral fractions \(<10^{-6} \). As a quantitative measure of this, we have computed the median density of particles with a neutral fraction greater than 0.1. We find that it decreases as ionization proceeds, being median \( \rho_{\text{HI} > 0.1} = 2.58, 2.48, 1.80 \) and 0.93 for \( x_m = 0.1, 0.3, 0.7 \) and 0.97, respectively.

In the next section, we examine the morphology of the neutral and ionized regions in order to investigate this in more detail. For now, we can compare the scatter plots of neutral fraction and density in different models. We do this at \( x_m = 0.5 \) in Fig. 4. The absence of a cloud of partially ionized points in the ‘no recomb’ simulation is expected, as once particles are ionized, they drop off the bottom of the plot. Also as expected, the \( \nu^{-2} \) spectrum simulation and the ‘2 x recomb’ run have a greater density of particles in this region than the fiducial model. With the ‘L/4’ model, if we look closely at the histogram on the y-axis, we can see that the spread of neutral fractions is distributed somewhat differently to the fiducial case. There are more intermediate values, and less close to neutral and

**Figure 3.** Scatter plot of gas density (in units of the mean) against hydrogen neutral fraction for particles in the fiducial simulation run (see Section 2.3). We show results for four different output times, characterized by the mean mass-weighted ionized fraction \( x_m \) which appears in the panel labels. In each panel, we also show as shaded areas histograms of the number of particles in bins of hydrogen neutral fraction and also histograms binned by density, \( \rho /(\langle \rho \rangle) \). The height of each histogram bin is on a log scale.

**Figure 4.** Scatter plot of gas density (in units of the mean) against hydrogen neutral fraction for particles in six different simulation runs (taken from the 12 in Section 2.3). We show results for all simulations at the time when the mean mass-weighted ionized fraction \( x_m = 0.5 \). In each panel, we also show as shaded areas histograms of the number of particles in bins of hydrogen neutral fraction and also histograms binned by density, \( \rho /(\langle \rho \rangle) \). The height of each histogram bin is on a log scale.
in the highly ionized tail. We shall see later that this difference will manifest itself in the morphology of the structures also. In the ‘$\sigma_8 = 0.7$’ model, the situation is more similar to the fiducial case.

4 MORPHOLOGY

Just as looking at structure in galaxy redshift surveys (e.g. Schectman et al. 1996) revealed filaments, voids and clusters, the RIS is expected to give rise to a complex morphology. In the case of structure in the density field, reproducing the visual characteristics of observational data was one of the drivers in searching for the correct theory of structure formation, and cosmological N-body simulations are expected to give rise to a ‘cosmic web’ with the same appearance (e.g. Bond, Lofman & Pogosyan 1996). Deciding how to view the morphology of RIS is complicated by the question of whether to plot the neutral fraction, neutral density, ionized density or ionized fraction, each of which can lead to a potentially different impression. Also, unlike pure density fluctuations, which evolve relatively slowly, the exact time the RIS is plotted as reionization proceeds can yield very different results. Again, we will choose to compare different models at the same value of $x_m$.

In Fig. 5, we show a thin ($1 \, h^{-1} \, \text{Mpc}$) slice through the fiducial simulation volume, at the time when $x_m = 0.5$. We use a two-dimensional colour scale to show both the density and the neutral fraction of hydrogen, as well as overplotting the positions of the individual sources of radiation, associated with dark matter haloes. It is apparent from the plot that low-density regions where there are only a few isolated sources have not yet created noticeable bubbles, but that the highly clustered regions, associated with filaments and protoclusters, have appreciable Stromgren spheres around them. Iliev et al. (2006) have measured the size of the bubbles in their simulations by fitting spheres into the ionized regions, finding a median bubble radius of $\sim 5 \, h^{-1} \, \text{Mpc}$ (see their fig. 13) at this late stage of reionization. Many other studies, e.g. Shin et al. (2007) and Zahn et al. (2007), have been carried out on the size of bubbles in simulations. Visually, our simulation appears to be broadly consistent with the sizes found by Iliev et al., and we leave to future work detailed statistical characterization of the size and shape of voids in the neutral hydrogen.

For now, we will comment on the obvious differences apparent between the morphology of RIS in the simulation and that which can be seen in the underlying density field. The RIS has a much higher contrast level, with the neutral fraction in H II regions being $\sim 10^5$ times less than in the rest of the volume. The edges of H II regions are consequently much sharper than those of voids in the matter distribution, lending themselves to easier detection by void-finding techniques (e.g. Colberg et al. 2005).

The matter density field over the redshift range relevant to reionization evolves little compared to the neutral density field. Reionization (say the change from $x_m = 0.001$ to 0.999) in these models takes place over an change in scalefactor $a$ of $\sim 2$, and as $\Omega_m \sim 1$ to good approximation at these redshifts, linear growth of matter fluctuations occurs by the same factor. Because of this, most of the change in the morphology and structure of neutral density field occurs in the RIS. Plotting a slice through the neutral density field at different epochs allows us to see this well. In Fig. 6, we show this for three different models (Fiducial, $M > 10^{10}$, $M < 10^{10}$), at times when $x_m$ varied from 0.1 to 0.9 in steps of 0.2.

Figure 5. A thin ($1 \, h^{-1} \, \text{Mpc}$) slice through the fiducial simulation volume ($40 \, h^{-1} \, \text{Mpc}$ wide), at redshift $z = 8.5$, when the mean mass-weighted H neutral fraction $x_m = 0.5$. We show both the densities in units of the mean and the H neutral fraction using a two-dimensional colour scale. The positions of individual sources of ionizing radiation are also shown as (red) points.
McQuinn et al. (2007) have shown that the large-scale morphology of ionized hydrogen bubbles depends most strongly on $x_m$ and the properties of the ionizing sources, and is relatively less affected by the specific subgrid model used to determine small-scale source suppression and clumping factors. In Fig. 6, we can see that models in the same column (same value of $x_m$) are indeed fairly similar. However, because we use the mass-weighted ionization fraction $x_m$ rather than the volume-weighted fraction $x_v$, our conclusions about the similarity of the morphologies is somewhat different than those of McQuinn et al. (2007). For example, the panel with $x_m = 0.5$ (middle panel) for the fiducial model (top row) seems to be most similar to the $x_m = 0.3$ (second panel) for the $M > 10^{10} M_\odot$ model (middle row). This is because in the $M > 10^{10} M_\odot$ model, the ionized material is all concentrated in large bubbles, whereas in the fiducial model there are many smaller H II regions around the fainter sources which are hard to see but which nevertheless account for much of the mass in ionized material.

A good way to see that this is the case is to refer to Fig. 7, which shows the same slices through the same models, but this time plotting the ionized density rather than neutral density. The H II regions around the fainter sources are clearly seen in the top and bottom rows. The early H II regions appear sharper and perhaps more spherical when seen directly in terms of their ionized density, rather than as shadows in the neutral hydrogen plot (Fig. 6). This is understandable because of the fact that the edges of the bubbles and the totality of bubbles that are smaller than the slice thickness (1 $h^{-1}$ Mpc) in size will be somewhat obscured in Fig. 6 by neutral hydrogen that lies in front of or behind the bubble but is still in the slice.

In Figs 7 and 6 we see little difference between the $M < 10^{10} h^{-1} M_\odot$ model and the fiducial model, indicating that the absence of the most massive haloes does not greatly affect the morphology, as long as we compare at the same value of $x_m$. If we look at the panels at the farthest right-hand side of these plots, the end stages of reionization ($x_m = 0.9$), we can see that the models with small haloes allowed do have H I remnants with more ragged edges and more small-scale structure apparent in them than the $M > 10^{10} h^{-1} M_\odot$ model. In future work, it would be interesting to investigate the mass function of the disconnected H I remnants present at these times as they may have constraining power (as well as being likely detectable in 21-cm emission). The $x_m$ value at which the ionized regions percolate seems likely to depend on the source model also, as e.g. the $x_m = 0.5$ panel for the $M > 10^{10} h^{-1} M_\odot$ model consists less of disconnected H II regions than the other two panels. This makes sense, as the ionized material is more clustered, being closer to the more massive haloes.

This $x_m = 0.5$ epoch is therefore a good one at which to compare the morphology of the other models also. In Fig. 8, we show the same thin slice through the neutral density field for the 12 different models. The panel most different from the others is not surprisingly that for the ‘other random phases’ model, for which the slice intersects a fairly spherical bubble (at the bottom of the plot) and a large region which has little sign of reionization (in the middle). Comparing this panel to the fiducial model, it might seem as though the latter is closer to the percolation stage, although they both have the same $x_m = 0.5$. In the later stages of reionization, when the ionized structures are a substantial fraction of the box size, we must naturally be careful in our interpretation of the morphology, due
Figure 7. Same as Fig. 6 except that we show the ionized density: plotted are slices (1 h⁻¹ Mpc thick) through the ionized H density field in three simulation runs as a function of mean mass-weighted ionized fraction, $x_m$. The top row shows the fiducial run, the middle row the $M > 10^{10} h^{-1} M_\odot$ run and the bottom row the $M < 10^{10} h^{-1} M_\odot$ run (see Section 2.3 for full descriptions). In each panel we use a log scale, with light shades representing high ionized H density. From left- to right-hand side, we show results for $x_m = 0.1, 0.3, 0.5, 0.7$ and 0.9.

5 AUTOCORRELATION FUNCTION

Quantitative comparisons of the structure in the various models can be carried out by looking at the autocorrelation function, $\xi$. We will focus on $\xi(r)$ measured for the neutral density distribution, although we will briefly examine $\xi(r)$ for the ionized density field. We note that the Fourier transform of $\xi(r)$, the power spectrum $P(k)$ of fluctuations, has been studied in reionization models by many authors. We compute $\xi(r)$ for the gas density directly from the particle positions in the simulation, and for $\xi_{\text{HI}}(r)$ we weight each particle by its neutral fraction:

$$\xi_{\text{HI}}(r) = \frac{\sum N_p (x_{\text{HI}})_j (x_{\text{HI}})_k}{N_{p,e} (x_{\text{HI}})^2} - 1,$$

where $N_p$ is the number of pairs of particles in a bin centred at $r$, $N_{p,e}$ is the expected number for a random distribution, $(x_{\text{HI}})_j$ and $(x_{\text{HI}})_k$ are the neutral fractions of particles in a pair and $\langle x_{\text{HI}} \rangle$ is the mean neutral fraction by mass. As with our examination of morphology of the neutral density, we will compute $\xi(r)$ for different simulation outputs chosen by their mass-weighted ionized fraction, $x_m$.

5.1 Resolution and box size tests

In order to test the range of validity of our $\xi(r)$ results, we carry out several resolution and box size tests. Because the bubble-like structures which overlap during the end stages of reionization occupy a large volume, one would expect that the simulation box size may have a strong effect on our results. We therefore compute $\xi(r)$ for the neutral gas and the total gas density for two simulations with different box sizes (we vary the box side length by a factor of 2) but the same mass and spatial resolution (this is kept the same as our fiducial model). The results are show in Fig. 9, where the
Figure 8. Slices (1 h⁻¹ Mpc thick) through the neutral H density field in the 12 simulation runs described in Section 2.3. We show all results at output times that correspond to a mean mass-weighted ionized fraction, \( x_m = 0.5 \). The density is shown on a log scale (light colours for higher neutral density). Regions which are completely black generally have a neutral fraction \(< 10^{-6}\).

three panels show \( \xi(r)_\rho \) and \( \xi(r)_{HI} \) at different stages of reionization (parametrized by \( x_m \)) in the fiducial model.

We can see that the \( \xi(r)_{\rho} \) curves are extremely similar over the range \( 0.05 < r < 4 h^{-1} \) Mpc for the three values of \( x_m \). In particular, a power-law fit to the curves over this range gives virtually identical parameters. This is a very good thing, because it shows that no nonlinear gravitational mode coupling has taken place with large-scale density modes of the order of the box size. This is one advantage of working at these high redshifts (\( z \sim 8 \)) for \( x_m = 0.8 \) in this case), where we can see that even a 20 h⁻¹ Mpc box is large enough to study gravitational clustering over this range of length-scales. The correlation function of the neutral gas density has a broadly similar behaviour. The position of the break in the power-law form of \( \xi(r) \) is due to the finite size of the simulation volume and so we will restrict our analysis of \( \xi(r) \) to smaller scales.

Interestingly, the panel with results closest to the end of reionization (\( x_m \)) does not show any greater disagreement for large \( r \) for \( \xi(r)_{HI} \) than for \( \xi(r)_{\rho} \). This means that at least over this limited range of scales, our \( \xi_r \) measurements will also be reliable for the neutral density. The large-scale cut-off does however appear on different scales for the HI than for \( \rho \). For example, the cut-off is on smaller scales for the \( x_m = 0.2 \) panel, and larger for the \( x_m = 0.8 \) panel.

---

\( \xi(r) \) and \( \xi(r)_{H^I} \) at different stages of reionization (parametrized by \( x_m \)) in the fiducial model.
This is to be expected, as the structure in the H I and ρ result from different physical processes and so will respond to the finite box size in different ways.

On the smaller scales (r < 0.1h⁻¹ Mpc), there appears to be a rapid drop-off in ξ, for the neutral density for both box sizes. We shall see below when we consider mass and spatial resolution that our results will not be useful below these scales in any case.

For our next test, we keep the box size fixed at 20h⁻¹ Mpc but vary the particle number (and hence mass resolution) by a factor of 8. We also vary the spatial (force) resolution by a factor of 2. The coarser mass/spatial resolution is the one we use in our fiducial case (only with a 40h⁻¹ Mpc box). The dark matter haloes which we use to place our sources of radiation have the same lower mass cut-off (1.6 × 10⁶ h⁻¹ M⊙) in both runs, which corresponds to eight times fewer particles in the low-resolution case. The results for ξ(r)ρ and ξ(r)H I are shown in Fig. 10, again for different values of x₀. The ξ(r)ρ curves show good agreement on large scales (the two simulations were run with the same initial phases for the density field). On scales r < 0.15h⁻¹ Mpc, the two curves diverge, indicating the effects of mass and spatial resolution on the gravitational evolution of the gas density field. We note that on scales comparable to this the clustering in the gas density will be influenced by cooling and star formation, which we do not include here in any case. The ξ(r)H I correlation functions also agree well down to r < 0.15h⁻¹ Mpc, when they diverge even more sharply. We note that as with the box size test, the worst disagreement on large scales occurs rather unexpectedly for the early stages of reionization (x₀ = 0.2).

The final simulation parameter which we vary is c₁, the maximum number of simulation particles that can be ionized by a single photon packet. This parameter (see Section 2.2 for more details) is inversely proportional to the total number of photon packets used to carry out the Monte Carlo RT. For larger values of c₁, the radiation field will not be as smooth, and there will be more shot noise in the neutral density field. Our fiducial value of c₁ = 0.3, and in Fig. 11 we show what it happens when this is varied from c₁ = 3 to 0.1, with all other simulation parameters the same as our fiducial model. The ξ(r)ρ curves are almost identical, as might be expected, with the small differences due to the fact that reionization occurs at slightly different times in the different c₁ runs. The curves for ξ(r)H I are also very similar, and there is no apparent systematic effect even when c₁ is made 10 times larger than our fiducial value. At least as far as the correlation function is concerned, we have therefore converged to stable results with our fiducial number of photon packets.

The tests in this section have therefore revealed that our results for ξ(r) should be reliable over at least the scales 0.15 < r < 4h⁻¹ Mpc. We will concentrate on this range in our analysis, e.g. looking at the power-law nature of ξ(r). For looking at larger scales, approaching the scale of bubbles at the time of percolation, larger simulation volumes should be run in the future. We note that because ξ(r) for a 20h⁻¹ Mpc volume converged with a larger box on scales below r < 4h⁻¹ Mpc it is probably safe to assume that we can draw information from our fiducial volume (40h⁻¹ Mpc box) on scales up to r ~ 8h⁻¹ Mpc.

5.2 The evolution of ξ(r)

We show ξ(r) for the fiducial model in Fig. 12, for values of the ionized fractions x₀ ranging from 0.1 to 0.99, which corresponds to a redshift range of z = 10.2 to 7.6. Before reionization starts, when x₀ = 0, ξ(r) and ξH I(r) are identical, by definition. However, once x₀ has reached 0.1, one can see that ξH I(r) is somewhat lower than...
explore qualitatively the behaviour of $\xi_\rho(r)$ as reionization proceeds. We note that this behaviour (a decrease and then an increase in the amplitude of clustering) has been seen in analytic calculations of the 21-cm brightness correlations (e.g. Wyithe & Morales 2007).

As $x_m$ increases, $\xi_\rho(r)$ exhibits the usual linear growth, with the amplitude increasing as expected under gravitational instability, and the shape remaining constant. As can be seen from the solid lines in Fig. 12, this gravitational amplification of structure is very small (a factor of $\sim 1.5^2$) over the time of reionization. $\xi_{H_1}(r)$ on the other hand shows dramatic growth, by a factor of 100 or more over the same interval, as we would predict e.g. by examining the morphology of structure in the $H_1$ density field in Fig. 6. The initial stage of reionization affects primarily the high-density regions around sources of radiation. Removing their contribution from the clustering of $H_1$ leads to an antibias on all scales in $\xi_{H_1}(r)$ with respect to $\xi_\rho(r)$. After this, the effect of removing highly clustered neutral regions competes with the amplifying effect of RIS, with the latter winning after $x_m \sim 0.35$, raising $\xi_{H_1}(r)$ above $\xi_\rho(r)$ at this point. As the RIS grows in scale, the shape of $\xi_{H_1}(r)$ begins to change, with the slope of the power-law region becoming flatter, reaching $\gamma_{H_1} \sim 0.5$ when $x_m = 0.99$.

The relationship between $\xi_{H_1}(r)$ and $\xi_\rho(r)$ can be examined by plotting the bias as a function of scale, defined by

$$b_{H_1}(r) = \left[ \frac{\xi_{H_1}(r)}{\xi_\rho(r)} \right]^{1/2}.$$  

(4)

This quantity is plotted in Fig. 13 for the fiducial model, where it can be seen that $b_{H_1}(r)$ is approximately constant with scale for $r \lesssim 5 h^{-1} \text{Mpc}$ for $x_m < 0.35$, with a value less than 1 for low values of $x_m$. After this, $b_{H_1}(r)$ takes on a positive power-law slope. We can fit $\xi_{H_1}(r)$ and $\xi_\rho(r)$ with power laws (equation 3), with slopes $\gamma_{H_1}$ and $\gamma_\rho$, and correlation lengths $r_{0,H_1}$ and $r_{0,\rho}$. If we do this, then we also expect the slope of a power-law fit to $b_{H_1}$,

$$b_{H_1}(r) = (r/r_{0,H_1})^{-\gamma_{H_1}},$$  

(5)

to give $\gamma_{H_1} = (1/2)(\gamma_{H_1} - \gamma_\rho)$. In this case, $r_{0,H_1}$ will be the separation for which the correlation function of $\rho$ and $H_1$ have equal amplitude.
a dependence on the properties of the sources of radiation or the physics of reionization. We again compute $\xi_{HI}(r)$ at output times corresponding to different specific values of $x_m$.

The results for our 12 models are shown in Fig. 15, along with power-law fits (described more fully in Section 5.4 below) to the region $4.0 > r > 0.2\ h^{-1}\ Mpc$. It is apparent that all models display behaviour broadly similar to the fiducial case, with a form reasonably approximate to a power law over $\sim 1.5$ decades in scale. As indicated by our tests above, the break on scales $r \sim 7\ h^{-1}\ Mpc$ is largely caused by the finite box volume and is expected to be similar in all cases. All models have a certain level of antibias between $\xi_{HI}(r)$ and $\xi_{\rho}(r)$ at first, with the most extreme antibias being reached in the ’L/8’ model. The ’$M > 10^{10}\ h^{-1}\ M_{\odot}$’ model has only a small level of antibias, indicating that the large bubbles formed rapidly by the very luminous sources quickly modulate the $HI$ field.

The correlation function becomes shallower and its amplitude increases dramatically for all models towards the end of reionization. When $x_m = 0.99$, the models all have a very similar amplitude and a shallow power-law slope $\gamma_{HI} \sim 0.5$. It should be noted that model ’L/8’ and ’L indep. M’ do not have outputs for the very end stages of reionization. The latter has very similar behaviour to the fiducial case, and the former is rather different, as we shall see when we consider the evolution of the power-law fit parameters.

Overall one can see from the panels in Fig. 15 that the RIS leads in all models to a strong increase in $\xi_{HI}(r)$ over the course of reionization. Unlike linear growth of fluctuations under GI, the slope of the correlations changes, reaching a similar shallow value in all models. No obvious features with any particular physical scale are created in $\xi_{HI}(r)$ in any of our variations of the reionization scenario. Despite the relatively wide variety of models employed, the differences in $\xi_{HI}(r)$ are subtle, and will be explored in the $r_{0.1h} - \gamma_{HI}$ space below.

### 5.4 Power-law fits

The galaxy–galaxy correlation function has only small deviations from power-law behaviour over $\sim 3$ decades in length-scale (e.g. Peebles 1980 and Zehavi et al. 2002), despite the undoubted complexities of galaxy formation. The dark matter correlation function in simulations also exhibits this behaviour (e.g. Jenkins et al. 1998; Kravtsov et al. 2004). It is therefore not unreasonable to expect that this type of scale invariance might also be created by RIS. We have seen from Fig. 15 that this is indeed the case. In this subsection we examine the power-law behaviour of $\xi_{HI}(r)$ more quantitatively.

We fit power laws to the region $4.0 > r > 0.2\ h^{-1}\ Mpc$, based on the resolution and box size tests of Section 5.1. The power-law fits were carried out assuming Poisson errors on the $\xi_{HI}(r)$ points, so that the error is $\sigma = [1 + \xi_{HI}(r)]/\sqrt{N_p}$, where $N_p$ is the number of pairs of particles in a bin. We have tried changing $N_p$ to include only pairs of particles above a threshold neutral fraction (e.g. $x_{HI} = 0.5$), but this has a negligible effect on the fit parameters. Also, changing the fitting region to $1.0 > r > 0.2\ h^{-1}\ Mpc$ does not change any of our conclusions below.

In the top two panels of Fig. 16, we show the dependence of the power-law fit parameters $r_{0.1h}$ and $\gamma_{HI}$ on $x_m$ for our different models. Looking at $r_{0.1h}$ first, we can see that there is a strong dependence on $x_m$. The models all follow a trend roughly equivalent (within a factor of 2 in $r_{0.1h}$ for all but one model) to $\log_{10} r_{0.1h} \sim 0.06/(1.0 - x_m)^2$, with the $M > 10^{10} h^{-1} M_{\odot}$ model being the most extreme outlier. The fact that the lines for nearly all the different models are clustered together so tightly is rather surprising, and

### 5.3 $\xi(r)$ for the different models of reionization

We have seen that $\xi_{HI}(r)$ in the fiducial model has a similar shape to that which arises under gravitational instability. We now investigate $\xi_{HI}(r)$ in the other reionization models in order to see if there is

![Figure 14. Clustering of ionized hydrogen: we show the autocorrelation function, $\xi(r)$, for ionized H density field (dashed lines) and the gas density field (solid lines) in the fiducial simulation (see Section 2.3). We show results for 11 different output times, for which we have labelled the ionized H curves with the mean mass-weighted ionized fraction at that time, $x_m$.](https://example.com/fig14.png)
Figure 15. The autocorrelation function $\xi$ for all 12 models described in Section 2.3. The dashed lines are $\xi(r)$ for the neutral H\textsc{i} and solid lines for the gas density. The dotted lines show power-law fits to the H\textsc{i} $\xi(r)$ for the region $4.0 > r > 0.2 h^{-1}$ Mpc. For all panels except those for the ‘L/8’ and ‘L indep M’ models, we show results for the same values of the mass-weighted mean ionized fraction as in Fig. 12, i.e. $x_m = 0.1–0.9$ in steps of 0.1 and then $x_m = 0.97$ and 0.99. For the ‘L indep M’ models, the $x_m = 0.99$ lines are not shown.

appears to be an indication that $x_m$ plays a dominant role setting $r_{0,\text{H}_\text{i}}$. At least in this fashion, the neutral fraction governs the structure in the neutral hydrogen density field. The visual morphology of different models with the same $x_m$ values (e.g. Fig. 8) did appear to be noticeably different, rather more than one would expect give the tight locus of $r_{0,\text{H}_\text{i}}$--$x_m$ curves. Given the visual impression of the H\textsc{i} slices, however, it is understandable that the model with perhaps the greatest difference to the others (the $M > 10^{10} h^{-1} M_\odot$ model) is also discrepant in terms of $r_{0,\text{H}_\text{i}}$. For any value of $x_m$, this model has a larger $r_{0,\text{H}_\text{i}}$ than the others.

The $r_{0,\text{H}_\text{i}}$ differences between models are small, but measurable, as are the purely gravitational instability based differences (note that the models reach different values of $x_m$ at different redshifts, so that the underlying $\xi_\rho$ curves will be different). We note (discussed further in Section 6.2) that the value of $x_m$ will not be available a priori from observational data. Therefore to measure $r_{0,\text{H}_\text{i}}$ differences and discriminate between models it will be necessary to measure $r_{0,\text{H}_\text{i}}$ when given values of the slope $\gamma_{\text{H}_\text{i}}$ are reached, for example. As we discuss further below, the spread between $r_{0,\text{H}_\text{i}}$ for different models we have simulated is of the order of 50 per cent of the mean of the $r_{0,\text{H}_\text{i}}$ values when $\gamma_{\text{H}_\text{i}} > 0.8$.

We do find a wider variation in the values of the slope of $\xi_{\text{H}_\text{i}}$, for fixed values of $x_m$. This can be seen from the middle panel of Fig. 16, where e.g. $\gamma_{\text{H}_\text{i}}$ varies from 1.6 to 0.6 when $x_m = 0.5$. The fiducial model has $\gamma_{\text{H}_\text{i}} = 0.72$ when $x_m = 0.5$. If we look horizontally to find out at what $x_m$ value the other models have the same slope, we find a range from $1 - x_m = 0.34$ (for the L/4 model) to $1 - x_m = 0.60$ (for the $M > 10^{10} h^{-1} M_\odot$ model). All curves (except for the L/8 model, which does not get more than half-ionized) do follow the same pattern, with the slope getting asymptotically flatter as reionization proceeds, all ending up with $\gamma_{\text{H}_\text{i}} \sim 0.5$. The rapidity with which this asymptotic value is reached does vary with the

$\copyright$ 2008 The Authors. Journal compilation $\copyright$ 2008 RAS, MNRAS 388, 1501–1520
Radiation-induced large-scale structure

In the region to the left-hand side of the plot, clustering of H\(_{\text{I}}\) in all models is dominated by the clustering in the underlying density field. The RIS then takes over, and again all models follow a rather similar locus of \(\xi_{\text{H}\text{I}}\) parameters. We have seen above that the variation between the \(r_{0,\text{H}\text{I}}=\gamma_{\text{H}\text{I}}\) lines is mostly due to the variation of \(\gamma_{\text{H}\text{I}}\) with \(x_{\text{m}}\) in the different models.

Overall, the change in slope for the different models can be qualitatively explained by the different length-scales of the RIS features that occur in each, even at the same stages of reionization \((x_{\text{m}}\) values). There is a competition between small- and large-scale features that sets the slope of the correlation function. The behaviour of \(r_{0,\text{H}\text{I}}\) is more puzzling, and we shall return to these questions in Section 6.2.

5.5 The growth factor of perturbations

We have seen in previous sections that once reionization is visually progressing, the amplitude of fluctuations in the H\(_{\text{I}}\) field grows extremely rapidly. It is interesting to compare this quantitatively to the growth expected of perturbations under gravitational instability. In the latter case, the amplitude of linear density perturbations will grow at the same rate independent of scale, so that \(\xi_{\rho}(r)\) will retain the same shape, but increase in amplitude by an overall factor \([g(z)]^2\) which can be computed from first-order perturbation theory (e.g. Peebles 1980). At the high redshifts relevant here \((z \gtrsim 7)\), the LCDM universe behaves in a manner very close to an Einstein–de Sitter model so that the linear growth factor \(g(z) \propto 1/(1+z)\). From looking at the solid lines in Fig. 15, we have seen that \(r_{0,\rho}\) (the separation at which the matter correlation function is equal to unity) is between 0.1 and 0.5 \(h^{-1}\) Mpc for the models over the range of redshifts when reionization occurs, so that linear theory should be accurate over most of the range of scales indicated by our resolution and box size tests (Section 5.1). We can also see from Fig. 15 that \(\xi_{\rho}(r)\) does keep the same shape, and increases in amplitude slowly, as expected. This can also be inferred from Fig. 17, where

![Figure 16. Parameters for power-law fits to the autocorrelation function of the H\(_{\text{I}}\) density in all 12 models described in Section 2.3. Top panel: correlation length \(r_{0,\text{H}\text{I}}\) as a function of mass-weighted ionized H density \(x_{\text{m}}\). Middle panel: slope \(\gamma_{\text{H}\text{I}}\) as a function of \(x_{\text{m}}\). Bottom panel: \(r_{0,\text{H}\text{I}}\) versus \(\gamma_{\text{H}\text{I}}\).](image)

![Figure 17. The growth factor of perturbations in the fiducial model. We show results for the gas density field as solid lines and the H\(_{\text{I}}\) density field as dashed lines, with a different line for three different comoving scales. In each case we normalize by the amplitude of fluctuations at redshift \(z = 10.5\). The dotted line is a curve with \(g \propto e^{a(z)}\).](image)
we plot the square root of the ratio of the correlation function $\xi\rho(r)$ at redshift $z$ to that at redshift $z = 10.5$, as a function of $z$. This is proportional to the growth factor of perturbations between redshifts. There are three lines on the plot, corresponding to $r = 0.2$, 1 and $7 \ h^{-1}\text{Mpc}$. The lines are close together, indicating that the shape of $\xi\rho(r)$ does not change dramatically.

In Fig. 17, we plot the same quantity for $\xi_{HI}(r)$, for the same $r$ values, for the fiducial model of reionization. In this case, we see that the growth of perturbations is much more rapid, due to the RIS, than the linear density growth. We draw a smooth curve corresponding to $g(z) \propto e^{\alpha z^\beta}$ alongside the simulation growth factor for $r = 1 \ h^{-1}\text{Mpc}$ in order to show how extreme the growth in fluctuations as a function of redshift is during the epoch of reionization. The growth is faster than exponential over the short interval between $z \sim 10$ and 8, it is roughly an exponential function of a power law.

We also see that the different scales exhibit different growth rates, with the large-scale fluctuations changing most rapidly. On $7 \ h^{-1}\text{Mpc}$ scales, the fluctuations are first suppressed, with the square root of $\xi_{HI}(r)$ decreasing by a factor of $\sim 5$, before rapidly increasing after $z = 9.5$. This stronger behaviour relative to the smaller scales results in the flattening of the correlation function. The roughly parallel nature of the curves after $z \sim 8.5$ indicates that at the late stages of reionization the flatter power-law form of $\xi_{HI}(r)$ has been reached, and the amplitude grows similarly on all scales.

If we now consider the growth of perturbations in the different models, a better way to compare them is to look at the growth as a function of $x_m$. We show how the amplitude of $\text{HI}$ fluctuations (proportional to the square root of the correlation function) changes with mean ionized fraction. In order to compare this relative growth between different models, we choose a normalization for the fluctuations, picking $x_m = 0$. In this respect what we show is different from the usual growth factor which quantifies how the amplitude of fluctuations varies with redshift. In Fig. 18 we show this for the $r = 1 \ h^{-1}\text{Mpc}$ scale. As the models become more ionized, from left- to right-hand side, the amplitude of fluctuations first dips and then rises steeply. The model which dips the least on these scales is the $M > 10^{10} \ M_{\odot}$ model, as we expect from looking at Fig. 15. The main growth phase of fluctuations starts between $x_m = 0.1$ and 0.4, depending on the model, with an exponential relationship between $g$ and $x_m$.

The curves appear to converge towards the end of reionization, so that all models exhibit roughly the same amount of growth, within $\pm 50$ per cent from the start of reionization until $x_m = 0.97$. The models which started off with less growth in $\xi_{HI}$ therefore have steeper dependence on $x_m$. We find that $g \propto e^{3.5x_m}$ approximately holds over the range $x_m = 0.2–0.8$ for the fiducial model. The steepest curve has $g \propto e^{2x_m}$ (for the $L > 10^{10} \ h^{-1}\text{Mpc}$ model).

If we look at the growth factor from the point of view of the parameters varied in each model, we can see that decreasing the luminosity (from the fiducial model through $L/2$, $L/4$ and $L/8$) monotonically changes how abrupt the growth of $\text{HI}$ fluctuations is during the bulk of the reionization process. The model with least luminous sources ($L/8$) exhibits fastest growth with respect to $x_m$, although of course with respect to $z$, this is not necessarily the case. We have examined the growth factor versus $z$ for the different models (not plotted) and find that the growth of fluctuations for all models has (at least for the $1 \ h^{-1}\text{Mpc}$ scale) a form roughly consistent with the $g(z) \propto e^{\alpha z^\beta}$ curve drawn in Fig. 17. To zeroth order, the growth of $\text{HI}$ fluctuations as a function of $z$ does not seem to be dependent on the source physics.

### 5.6 Babul and White model fit

We have seen that a simple power-law fit to the $\text{HI}$ correlation function works well on scales for which the simulation has sufficient box size and spatial resolution. As reionization proceeds, the power law becomes shallower, moving away from the slope of the underlying matter correlation function. While this paper’s focus is on numerical modelling and phenomenology of $\text{HI}$ clustering rather than analytic modelling, may nevertheless be instructive to consider a different fitting function, for $\xi_{HI}(r)$, derived from very simple model. Many analytic models of bubble growth during reionization, of varying complexity (e.g. using the excursion set formalism, Furlanetto et al. 2004, and perturbation theory, Zhang et al. 2007) have been proposed, and most recently, tests of the model of Furlanetto, Zhan & Hernquist have been carried out by Zahn et al. (2007) using RT simulations. Here we consider the model of Babul & White (1991) originally proposed by those authors as a simple description of the way galaxy clustering could be modulated by ‘spheres of avoidance’ around quasars.

The model makes the simplifying assumption that the sources are Poisson distributed and that they give rise to spheres of avoidance of a fixed physical size. The two-point correlation function of material distribution uniformly in the interbubble regions is then (Babul & White 1991)

$$1 + \xi_{\text{interbubble}}(r) = \exp \left\{ \frac{f_s}{2} \left[ \left( \frac{r}{2R_b} \right)^3 - 3 \left( \frac{r}{2R_b} \right) + 2 \right] \right\}, \quad r \leq 2R_b,$$

$$= 1, \quad r > 2R_b.$$  

Here $f_s = 4\pi n_b R_b^2/3$ is the nominal filling factor of spherical bubbles of radius $r_b$ and mean number density $n_b$. If we furthermore assume that the distribution of sources is independent of the density distribution, modulation of the $\text{HI}$ by bubbles leads to an $\text{HI}$ correlation function given by

$$1 + \xi_{HI}(r) = \left[ 1 + \xi_{\text{interbubble}} \right] \left[ 1 + \xi_{\rho} \right].$$  

---

© 2008 The Authors. Journal compilation © 2008 RAS, MNRAS 388, 1501–1520

Downloaded from https://academic.oup.com/mnras/article-abstract/388/4/1501/980912 on 30 July 2018
values did vary monotonically with $x_m$ with $R_b = 3.2 h^{-1}$ Mpc for $x_m = 0.4$, $R_b = 7.4 h^{-1}$ Mpc for $x_m = 0.5$ and $R_b = 8.1 h^{-1}$ Mpc for $x_m = 0.7$. Above $x_m = 0.8$, $R_b$ is saturated at a value of $9.0 h^{-1}$ Mpc, possibly indicating the limitations of finite box size.

The other parameter, the filling factor $f_b$ can also be examined, and could also be a useful diagnostic. In this case the value steadily increases, with $f_b = 0.06, 0.12, 0.25, 0.46, 0.81, 1.4, 2.7, 3.9$ for $x_m = 0.4, 0.5, 0.6, 0.7, 0.8, 0.9, 0.97, 0.99$. We note from the definition of $f_b$ above that it is a nominal filling factor, and as a result $f_b$ will become greater than unity as bubbles overlap in the late stages of reionization.

The difference between $\xi_{\text{interbubble}}$ for the different models (all with $x_m = 0.5$) is shown in Fig. 20. We can see that the peak amplitude of the curves varies by a factor of $\sim 6$ between models, but that they all display the same shallow curve behaviour with a cut-off. As we would expect, the model with the largest obvious bubbles in Fig. 8, $M > 10^{10} h^{-1}$ M$_\odot$ has the largest amplitude and largest cut-off scale. A few of the models (e.g. L/4 and L/8) have a negative $\xi_{\text{interbubble}}$ and are not plotted. For clarity, we do not show the model fits, but we note that the $M > 10^{10}$ has the largest value of $f_b$, 0.27, and the L/2 model has the smallest value, 0.06. These values for the bubble filling factor appear by eye to be at least indicative of the trends seen in Fig. 8.

6 Summary and Discussion

6.1 Summary

In this paper we have explored the RISs produced during the epoch of reionization and compared them to the results of gravitational instability. Our conclusions can be summarized as follows.

(1) For most of the models we have tested, we find that the mean ionized fraction of hydrogen increases exponentially, $x_m = e^{-(z-i)}$ (where $z$ is the redshift of full ionization) as reionization proceeds. For models adjusted to have emitted the same number of source

---

Figure 19. A fit of the functional form taken from Babul & White (1991) to the autocorrelation function of interbubble material (see equation 7). We show results for the fiducial simulation, for eight output times with different values of the mean mass-weighted ionized H density, $x_m$. The fits were carried out to points with $r > 0.3 h^{-1}$ Mpc.

Figure 20. The autocorrelation function of interbubble material (see equation 7). We show results for the 12 simulations described in Section 2.3, for the output times when the mean mass-weighted ionized H density, $x_m = 0.5$. The difference between $\xi_{\text{interbubble}}$ for the different models (all with $x_m = 0.5$) is shown in Fig. 20. We can see that the peak amplitude of the curves varies by a factor of $\sim 6$ between models, but that they all display the same shallow curve behaviour with a cut-off. As we would expect, the model with the largest obvious bubbles in Fig. 8, $M > 10^{10} h^{-1}$ M$_\odot$ has the largest amplitude and largest cut-off scale. A few of the models (e.g. L/4 and L/8) have a negative $\xi_{\text{interbubble}}$ and are not plotted. For clarity, we do not show the model fits, but we note that the $M > 10^{10}$ has the largest value of $f_b$, 0.27, and the L/2 model has the smallest value, 0.06. These values for the bubble filling factor appear by eye to be at least indicative of the trends seen in Fig. 8.
ionizing photons by \( z = 6 \), there is still quite a wide spread in the redshift of reionization, with models reaching \( x_{\text{m}} = 0.5 \) from \( z = 9.0 \) to 6.4 (the last one to reionize is the low \( \sigma_8 \) model).

(2) At a fixed \( x_{\text{m}} \), the morphology of the RIS is most strongly affected by the lower cut-off in source luminosity, which changes the size of bubbles. The mass-to-light ratio of sources also has a substantial effect, but the recombination rate and the amplitude of mass fluctuations, \( \sigma \), only minimally change the appearance of the \( \text{H} \) i density field.

(3) The \( \text{H} \) i correlation function, \( \ell_{\text{H} \, \text{I}} \), exhibits a generic behaviour for all models tested. \( \ell_{\text{H} \, \text{I}} \) initially becomes linearly anti-correlated with respect to the matter \( \xi_d \) as the high-density \( \text{H} \) i around sources is ionized. The linear bias factor reaches a minimum of \( b \sim 0.5 \) when \( x_{\text{m}} \sim 0.1–0.2 \). The amplitude of \( \ell_{\text{H} \, \text{I}} \) then increases rapidly, and \( \ell_{\text{H} \, \text{I}} \) keeps a scale-invariant shape, but the power-law index flattens to an asymptotic value of \( \gamma_{\text{H} \, \text{I}} \approx -0.5 \).

(4) We find that \( r_{\text{H} \, \text{I}} \), the correlation length of \( \ell_{\text{H} \, \text{I}} \), has essentially the same functional relationship with \( x_{\text{m}} \) in all but one of the models we tested. How the power-law index \( \gamma_{\text{H} \, \text{I}} \) varies with \( x_{\text{m}} \) on the other hand depends much more widely on the different source and physics prescriptions adopted.

(5) The growth factor of \( \text{H} \) i perturbations is seen to change much more rapidly than that of gravitationally evolving matter perturbations over a redshift range \( \Delta z \approx 2–3 \) during which the bulk of reionization occurs. We find perturbations on a scale of \( 1 \, \text{h}^{-1} \text{Mpc} \) to be evolving \( \propto \exp(\Delta z) \) compared to \( \propto \alpha(t) \) for gravitational growth. This is valid for all models tested, so that the source physics does not appear to affect the relation.

(6) During the late stages of reionization, the shape evolution of \( \ell_{\text{H} \, \text{I}} \) can be approximately reproduced by a simple model due to Babul & White (1991) in which ionizing sources are uncorrelated with the density field and produce spherical bubbles. Fitting the parameters of this model to \( \ell_{\text{H} \, \text{I}} \) therefore forms a method for inferring simple morphological characteristics from measurements of \( \ell_{\text{H} \, \text{I}} \).

### 6.2 Discussion

In this paper, we have largely avoided discussing directly observational probes of the RIS and the reionization epoch. This said, many of our findings can be related closely to the possible results of a survey of 21-cm brightness, as a function of angular position and wavelength. We have concentrated mainly on the correlation function of the \( \text{H} \) i, and so in the most likely scenario in which there have been enough early X-ray sources to heat up the primordial gas just before reionization occurs, its temperature is higher than the CMB and we are in the emission regime. In this case, the 21-cm brightness temperature is independent of the spin temperature. The brightness temperature \( T \propto \Delta_{\text{HI}}(1 + \delta) \), where \( \delta \) is the overdensity of hydrogen and \( \Delta_{\text{HI}} \) is the neutral fraction (see e.g. Madau 1997; Di Matteo, Ciardi & Miniati 2004; Barkana 2007).

Looking at the autocorrelation function of the \( \text{H} \) i, the power-law behaviour we see is striking. For structure growing through gravitational instability, the initial fluctuations are scale invariant over a large range and they evolve in a scale-free manner (Peebles 1974; Davis & Peebles 1977). In the case of \( \text{RIS} \), the onset of significant clustering is extremely rapid, and it might be expected that this would lead to features in the correlation function related to e.g. the Stromgren radius of sources dominant at that time. On the other hand, cosmic variance would lead to a substantial scatter in the scalelength of features from place to place. Our fiducial simulation volume, at 40 comoving \( h^{-1} \text{Mpc} \) side length, also limits our ability to capture the late stages of reionization. Wyithe & Loeb (2004) have predicted a comoving radius of bubbles \( \sim 40 \, \text{h}^{-1} \text{Mpc} \) at the end of the overlap stage. We have seen in Section 5.6 that the simple Babul and White model fit does capture the shape of \( \xi(r) \) reasonably well even though there is a distinct bubble scale in the model. Scale invariance therefore appears relatively easy to achieve and detection of departures from it probably requires a wide range of scales to be available.

Furlanetto, McQuinn & Hernquist (2006) have used the analytic model of Furlanetto et al. (2004) to describe how the \( \text{H} \) i bubble size is related to both the bias of galaxies and the underlying matter power spectrum. The success of such analytic models in comparisons to simulations (e.g. Zahn et al. 2007) has opened up the way for their use in analysing future 21-cm observations. Our related work, attempting to directly simulate a relatively wide range of models, has found that the autocorrelation function of 21-cm emission can be used to infer the broad signatures of RIS compared to gravitational structure (e.g. non-monotonic growth, flatter asymptotic slope \( \gamma_{\text{H} \, \text{I}} \approx -0.5 \)) which will help in the analysis of the first observations.

Among the other analytic models which have been developed, the perturbation theory approach of Zhang et al. (2007) is different from many in that it does not make a step function bubble approximation to the \( \text{H} \) i distribution. As our simulation approach includes recombinations and the contribution of residual \( \text{H} \) i in the ionized regions, future comparisons will be beneficial. For example, Zhang et al. (2007) compute the rapid rise in the bias of \( \text{H} \) i clustering as a function of redshift, finding some qualitatively similar results to ours, although they find a scale-independent bias on large scales.

One important point to note when looking at the comparisons between different reionization models (e.g. Fig. 8 or Fig. 15) is that the results have been plotted for the same mean mass-weighted neutral fraction \( \langle x_{\text{m}} \rangle \) for all models. This quantity is not the one which could be simply extracted from the 21-cm observations, for example, and so such model comparisons could not be easily carried out in practice. In principle, it should be possible eventually to use the measured evolution with redshift of the \( \text{H} \) i correlation function itself (e.g. as in Fig. 17) in order to discriminate between models. This is similar to the situation with galaxy clustering, for example, where the mean mass in galaxies (or the mean halo mass) is not easy to measure, but can be inferred by a combination of measured clustering and a theoretical model for the clustering of haloes of different masses.

One aspect which we have not covered is the effect of redshift distortions on the RIS. We expect the autocorrelation function to be affected by the usual line-of-sight amplification (Kaiser 1987) on large scales and small-scale suppression from the velocity dispersion (e.g. Peebles 1980) on small scales. The latter effect in particular is likely to be strongly modified by the fact that most of the \( \text{H} \) i around bright sources and hence in dense regions is ionized early on. These effects will likely be important in the use of 21-cm emission maps to carry out tests of cosmic geometry (see e.g. Nusser 2005). It would be simple enough in future work to look at the simulations in redshift space, such as would be seen with the observational data.

In carrying out our numerical experiments, we have simulated a range of models, which we expect to have many of the features likely in most scenarios for the reionization of the Universe. It was not possible to be completely general, however, and it is certainly possible to imagine other interesting models and tests of the physics that could be included. For example, McQuinn et al. (2007) ran as
one of their many models one in which all cells in the simulation were set to the mean density. This had the effect of changing the amount of structure in the ionization fronts, and of course drastically reducing the recombination rate.

With our post-processing RT carried out on the hydrogen distribution only, by keeping track of the ionization state but not predicting the temperature we necessarily simplified much of the physics involved in reionization. As our intention was to capture the broad differences between RIS and gravitational structure, we do not regard this as important. However, in future work extending our simulation approach so that it is directly relevant to upcoming observational data, we plan to capture more detail. For example, the code SPHray (Altay et al. 2008), which represents an extension of the present method, includes different ionized species and explicit temperature evolution. Other works, such as Ciardi et al. (2006) and McQuinn et al. (2007), specifically include the effect of minihaloes, sources below the resolution scale of the simulation. The latter group finds that they have a strong effect on the growth of large bubbles in the late stages of reionization. In the present paper, we have seen from our resolution studies that the autocorrelation function at least is not sensitive to increases in mass resolution and small-scale structure, at least to the accuracy and range of models that we have considered. In principle, the high spatial resolution of our gridless ray-tracing approach could allow the effects of Lyman-limit systems to be modelled, which become dominant in limiting the photon MFP when the universe is mostly ionized (e.g. Miralda-Escude et al. 2000).

Future work on realistic models should also include radiative cooling in the formation of sources and modelling of specific sources and spectra. Quasars and miniquasars have been investigated by including the formation and growth of black holes together with a model for feedback directly into cosmological SPH simulations (Pelupessy, Di Matteo & Ciardi 2007; Di Matteo et al. 2008; Sijacki et al. 2007). Using such models as sources in RT calculations would enable us to investigate how the RIS caused by harder sources is different from softer sources, in ways which are not constrained by the simple association of source and halo mass as has been carried out here.

ACKNOWLEDGMENT

This work was supported by the NASA Astrophysics Theory Programme, contract NNG 06-GH88G and NSF grant AST-0507665.

REFERENCES

Abel T., Norman M. L., Madau P., 1999, ApJ, 523, 66
Altay G., Croft R. A. C., Pelupessy F. I., 2008, MNRAS, 386, 1931
Babul A., White S. D. M., 1991, MNRAS, 253, 31v
Barkana R., 2007, MNRAS, 376, 1784
Bernardeau F., Colombi S., Gaztañaga E., Scoccimarro R., 2002, Phys. Rep., 367, 1
Bond J. R., Loeb M., Pogosyan D., 1996, Nat, 380, 603
Carilli C. L., Furlanetto S., Briggs F., Jarvis M., Rawlings S., Falcke H., 2004, New Astron. Rev., 48, 1029
Cen R., 1992, ApJS, 78, 341
Cen R., 2003, ApJ, 591, 12
Ciardi B., Madau P., 2003, MNRAS, 359, 1
Ciardi B., Ferrara A., Marri S., Raimondo G., 2001, MNRAS, 324, 381
Ciardi B., Scannapieco E., Stoehr F., Ferrara A., Iliev I. T., Shapiro P. R., 2006, MNRAS, 366, 689
Colberg J. M., Sheth R. K., Diaferio A., Gao L., Yoshida N., 2005, MNRAS, 360, 216
Croft R. A. C., 2004, ApJ, 610, 642
Dale J., Ercolano B., Clarke C., 2007, MNRAS, 382, 1759
Davis M., Peebles P. J. E., 1977, ApJS, 34, 425
Di Matteo T., Ciardi B., Miniati F., 2004, MNRAS, 355, 1053
Di Matteo T., Colberg J. M., Springel V., Hernquist L., Sijacki D., 2008, ApJ, 676, 33
Doré O., Holder G., Alvarez M., Iliev I. T., Mellema G., Pen U.-L., Shapiro P. R., 2007, Phys. Rev. D, 76, 3002
Furlanetto S. R., Zaldarriaga M., Hernquist L., 2004a, ApJ, 613, 16
Furlanetto S. R., Zaldarriaga M., Hernquist L., 2004b, ApJ, 613, 1
Furlanetto S. R., McQuinn M., Hernquist L., 2006, MNRAS, 365, 115
Gleser L., Nusser A., Ciardi B., Desjacques V., 2006, MNRAS, 370, 1329
Gnedin N. Y., 2000, ApJ, 535, 530
Iliev I. T., Mellema G., Pen U.-L., Merz H., Shapiro P. R., Alvarez M. A., 2006, MNRAS, 369, 1625
Jenkins A. et al., 1998, ApJ, 499, 20
Kaiser N., 1984, ApJ, 284, L9
Kaiser N., 1987, MNRAS, 227, 1
Kessel-Deynet O., Burkert A., 2000, MNRAS, 315, 713
Kohler K., Gnedin N. Y., Hamilton A. J. S., 2007, ApJ, 657, 15
Kravtsov A. V., Berlind A. E., Weinberg R. H., Klypin A. A., Gottlöber S., Allgood B., Primack J. R., 2004, ApJ, 609, 35
Liu J. R., Fang L. Z., Feng L. B., Bi H. G., 2004, ApJ, 605, 591
Loeb A., Barkana R., 2001, ARA&A, 39, 19
McQuinn M., Lidz A., Zahn O., Dutta S., Hernquist L., Zaldarriaga M., 2007, MNRAS, 377, 1043
Madau P., Rees M. J., Volonteri M., Haardt F., Oh S. P., 2004, ApJ, 604, 484
Mapelli M., Ferrara A., Pierpaoli E., 2006, MNRAS, 369, 1719
Masielli A., Ferrara A., Ciardi B., 2003, MNRAS, 345, 379
Mellema G., Iliev I. T., Alvarez M. B., Shapiro P. R., 2006, New Astron., 11, 374
Miralda-Escude J., Haehnelt M., Rees M. J., 2000, ApJ, 530, 1
Moraes M. F., Hewitt J., 2004, ApJ, 615, 7
Nusser A., 2005, MNRAS, 364, 743
Osterbrock D. E., 1989, Astrophysics of Gaseous Nebulae and Active Galactic Nuclei. University Science Books, Mill Valley, CA
Oxley S., Woolfson M. M., 2003, MNRAS, 343, 900
Peebles P. J. E., 1974, ApJ, 189, L51
Peebles P. J. E., 1980, The Large-scale Structure of the Universe. Research supported by the National Science Foundation.
Princeton Univ. Press, Princeton, NJ, p. 435
Pelupessy F. I., Di Matteo T., Ciardi B., 2007, ApJ, 665, 107
Peterson J. B., Pen U., Wu X., 2005, in Kassim N., Perez M., Junor M., Henning P., eds, ASP Conf. Ser. Vol. 345, From Clark Lake to the Long Wavelength Array: Bill Erickson’s Radio Science. Astron. Soc. Pac., San Francisco, p. 441
Ping H., Fang L.-Z., 2002, ApJ, 568, L1
Razoumov A. O., Scott D., 1999, MNRAS, 309, 287
Razoumov A. O., Norman M. L., Abel T., Scott D., 2002, ApJ, 572, 695
Ricotti M., Ostriker J. P., Schecter P. L., 1996, ApJ, 470, 172
Santos M. G., Amblard A., Pritchard J., Trac H., Cen R., Cooray A., 2007, preprint (arXiv:0708.2424)
Semelin B., Combes F., Baek S., 2007, A&A, 474, 365
Secthman S. A., Landy S. D., Oemler A., Tucker D. L., Lin H., Kirkshner R. P., Schecter P. L., 1996, ApJ, 470, 172
Shin M.-S., Trac H., Cen R., 2007, ApJ, preprint (arXiv:0708.2425)
Sijacki D., Springel V., Di Matteo T., Hernquist L., 2007, MNRAS, 380, 877
Sokasian A., Abel T., Hernquist L., 2001, New Astron., 6, 359
Sokasian A., Abel T., Hernquist L., Springel V., 2003, MNRAS, 344, 607
Sokasian A., Yoshida N., Abel T., Hernquist L., Springel V., 2004, MNRAS, 350, 47
Springel V., Yoshida N., White S. D. M., 2001, New Astron., 6, 79
Susa H., 2006, PASJ, 58, 445
Telfer R. C., Zheng W., Kriss G. A., Davidsen A. F., 2002, ApJ, 565, 773
Trac H., Cen R., 2007, ApJ, 671, 1

© 2008 The Authors. Journal compilation © 2008 RAS, MNRAS 388, 1501–1520

Downloaded by guest on 30 July 2018
Tumlinson J., Shull M., Venkatesan A., 2003, ApJ, 584, 608
Wyithe J. S. B., Loeb A., 2003, ApJ, 586, 693
Wyithe J. S. B., Loeb A., 2004, Nat, 432, 194
Wyithe J. S. B., Morales M. F., 2007, MNRAS, 379, 1647
Yoshida N., Oh S. P., Kitayama T., Hernquist L., 2007, ApJ, 663, 687
Zahn O., Lidz A., McQuinn M., Dutta S., Hernquist L., Zaldarriaga M., Furlanetto S. R., 2007, ApJ, 654, 12
Zaldarriaga M., Furlanetto S. R., Hernquist L., 2004, ApJ, 608, 622
Zehavi I. et al., 2002, ApJ, 571, 172
Zehavi I. et al., 2005, ApJ, 630, 1
Zhang J., Hui L., Haiman Z., 2007, MNRAS, 375, 324

This paper has been typeset from a \TeX/\LaTeX file prepared by the author.