Role of a periodic varying deceleration parameter in Particle creation with higher dimensional FLRW Universe

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Abstract

The present search focus on the mechanism of gravitationally influenced particle creation (PC) in higher dimensional Friedmann-Lemaitre-Robertson-Walker (FLRW) cosmological models with cosmological constant (CC). The solution of the corresponding field equations is obtained by assuming a periodically varying deceleration parameter (PVDP) i.e. \( q = m \cos kt - 1 \) [Shen and Zhao, Chin. Phys. Lett., 31 (2014) 010401] which gives a scale factor \( a(t) = a_0 \left[ \tan \left( \frac{kt}{2} \right) \right] ^\frac{1}{m} \), where \( a_0 \) is the scale factor at the current epoch. Here \( k \) displays the PVDP periodicity and can be regarded as a parameter of cosmic frequency, \( m \) is an enhancement element that increases the PVDP peak. Here, we investigated periodic variation behavior of few quantities such as the deceleration parameter \( q \), the energy density \( \rho \), PC rate \( \psi \), the entropy \( S \), the CC \( \Lambda \), Newton's gravitational constant \( G \) and discuss their physical significance. We have also explored the density parameter, proper distance, angular distance, luminosity distance, apparent magnitude, age of the universe, and the look-back time with redshift \( z \) and have observed the role of particle formation in-universe evolution in early and late times. The periodic nature of various physical parameters is also discussed which are supporting the recent observations.

Keywords: FRW metric, Particle creation, Periodic varying deceleration parameter, Observational Parameters.

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1 Introduction

The phenomenon of universe expansion rate becomes an attention point for all cosmologists, astrophysicists, and astronomers. This fact has been confirmed by observations (Supernovae Ia, CMB, BAO, etc). In 1998 and following years, some surprising results have been obtained by several groups of astronomers [1–7] to estimate the universe expansion rate. These groups have estimated the separations and predicted the accelerated expansion of the cosmos, which is probably going to continue forever in view of SN Ia observations. It is predicted by these observations that something is responsible for this expansion. As a solution, the cosmological constant befits the context in this regard. Also, on large scales, the universe is having flat geometry as predicted by the cosmic microwave background (CMB) [8–13]. Since there is not sufficient matter in the universe, so to deliver this flatness, the same 'dark energy' may be a candidate. In addition, the impact of DE appears to fluctuate, with the expansion of the universe decreasing and increasing over the period of time [14–17]. CC \( \Lambda \) is the simplest candidate for Dark Energy (DE) yet it should be, to a great degree, modified to fulfill the present estimation of the dark energy.

In present time a dynamical cosmological term \( \Lambda(t) \) has attracted the attention of researchers as it resolve the cosmological constant problem. There is substantial observational evidence to detect Einstein’s cosmological constant, or a part of the universe’s material content that changes gradually over time to behave like \( \Lambda \). The birth of the universe was caused by an excited vacuum fluctuation that caused the Super-fresh to adopt an inflationary expansion. The release of the vacuum’s stored energy results in subsequent heating.
The cosmological term, which is the measurement of empty space energy, generates a repulsive force against the gravitational attraction between galaxies. A repulsive force against the gravitational attraction between the galaxies is created by the cosmological term, which is the measurement of empty space energy. If the cosmological parameter occurs, since mass and energy are identical, the energy it describes counts as mass. If the cosmological term is huge enough, it may result in inflation by its energy plus the matter in the universe.

In the relativistic cosmological models the study of particle creation has drawn by many authors [18–40]. Prigogine [41,42] gave the first theoretical approach of particle creation. Schrodinger [43] has discussed the possibility of particle creation production as a result of space time curvature. Generally in curved space time a unique vacuum state does not exist. So there is ambiguity in the physical perception and the description of particles becomes even more complicated [44,45]. There are two general approaches to understand the physical concept of particle production: (i) the technique of adiabatic vacuum state [18], and (ii) the technique of instantaneous Hamiltonian diagonalization [46]. Singh et al. [47] discussed statefinder diagnostic in particle creation. Recently, Dixit et al. [47] have searched particle creation in FLRW higher dimensional universe with gravitational and cosmological constants.

Recently a lot of cosmologists and astrophysicists are there addressed the FRW models with particle creation problem [48–50]. Zimdahl, Yuan Qiang, and their colleague [51,52] studied the models of particle creation with SN 1a data and showed the result is consistent and the universe is in an accelerating phase. At the moment, [53] is exploring a different type of matter formation. Many researchers [54] have recently paid great attention to the cosmology of the production of ‘adiabatic’ particles that are gravitationally induced and explain the present accelerated expansion. The particle outputs of non-minimally coupled scalar fields of light will lead to an early accelerated universe [55] due to the change in space-time geometry.

The new advances in super-string theory and super gravitational theory have inspired physicists’ interest in exploring the evolution of the universe in higher-dimensional space-times. Kaluza [57] and Klein [58] suggested an eminent five-dimensional theory in which gravity and electromagnetism are combined with additional dimensions. Many cosmologist [59,60] have worked on higher dimensional theory. A marvelous review of the higher dimensional unified theory which has an excellent discussion about the cosmological and astrophysical implications of extra dimensions been presented by Overduin and Wesson [61]. For describing the nature of gravitational constant and cosmological constant, Harko and Mak [62] has been proposed a different type of theory which is related to matter creation.

The purpose and objective of our paper are to study the output of particles and the generation of entropy in the higher-dimensional FLRW model with periodically varying deceleration parameter. Here, in this model we discuss particle creation and entropy generation which has a Big Rip singularity [63,70]. Moreover, we investigate some cosmological quantities such as the energy density ($\rho$), the PC rate ($\psi$), the entropy ($S$), the deceleration parameter ($q$), etc which are dependent on time. These quantities demonstrate the behavior of periodic variation with singularity-I. For an oscillating cosmic model, a time-dependent PVDP has been discussed in [71]. Cosmological oscillating models have been also discussed in literature [72,74].

The paper has the following structure. The derivation of the field equations in FLRW is presented in Section 2. We find the solution of field equations for FLRW space times in Section 3. We outline estimates of some other physical and kinematic parameters using PVDPP in section 4, this section has two subsections: the model with particle creation and the model without particle creation. Interpretation of the derived results has been given in section 5. Kinematic tests are discussed in Section 6. Finally, conclusion are summarized in section 7.

## 2 Explicit Field Equations in FLRW

Consider the Friedman-Lemaître-Robertson-Walker (FLRW) metric for a (d+2)-dimensional homogeneous, isotropic, and flat model

$$ds^2 = dt^2 - a^2(t) [dr^2 + r^2 dx_d^2]$$

(1)

where $a$ is time dependent scale factor, and $dx_d^2$ define as

$$dx_d^2 = d\theta_1^2 + \sin^2 \theta_1 d\theta_2^2 + \ldots .. \sin^2 \theta_{d-1} d\theta_d^2$$

(2)
Considering that the universe is full of a perfect fluid whose energy-momentum tensor is defined as

$$T_{ij} = \rho \left(1 + \frac{p}{\rho}\right) u_i u_j - p \delta_{ij},$$  \hfill (3)

Here $\rho$ & $p$ stand for the energy density and pressure of the fluid respectively, $u_i$ is the $(d + 2)$ velocity vector which satisfy $u_i u^i = 1$. Einstein field equations (EFEs) with time-varying $\Lambda$ are given by

$$R_{ij} - \frac{R}{2}\delta_{ij} = -\Lambda g_{ij} - 8\pi G T_{ij},$$  \hfill (4)

where $R_{ij}$, $g_{ij}$, and $R$ are the Ricci tensor, the metric tensor and the Ricci scalar, respectively. And $G$ & $\Lambda$ indicates time dependent gravitational constant and cosmological constant. By Eqs. (1) and (3), the EFEs (4) reduce to

$$\frac{d^2 + d}{2} H^2 = 8\pi G \rho + \Lambda$$ \hfill (5)

and

$$d(H + H^2) + \frac{(d^2 - d)}{2} H^2 = -8\pi G p + \Lambda,$$ \hfill (6)

where $H = \frac{\dot{a}}{a}$ and dot shows a derivative with respect to cosmic time ($t$). On differentiating Eq. (5), we find

$$(d^2 + d)H \dot{H} = 8\pi (\dot{G}(t)\rho + G\dot{\rho}(t)) + \dot{\Lambda}(t)$$ \hfill (7)

Multiplying by $(-1)$ in Eq. (5) and adding with Eq (6), we find

$$d \dot{H} = -8\pi G (\rho + p)$$ \hfill (8)

On solving Eqs. (7) and (8), continuity equation is obtained as

$$\dot{\rho} + \rho(d + 1) \left(1 + \frac{p}{\rho}\right) H = -\left(\rho \frac{\dot{G}}{G} + \frac{\dot{\Lambda}}{8\pi G}\right)$$ \hfill (9)

With the help of Eqs. (5) and (9), we obtain

$$\rho(t) + p = -\frac{d \dot{H}}{8\pi G},$$ \hfill (10)

$$\Lambda(t) = \frac{(d^2 + d)}{2} H^2 - 8\pi G \rho,$$ \hfill (11)

3 Particle Development Thermodynamics

We assume that the early universe particle substance consists of a non-interacting relativistic fluid having a particle number density $n$ and equation of state (EoS) as follows:

$$\omega = \frac{p}{\rho},$$ \hfill (12)

and

$$n = n_0 \left(\frac{\rho}{\rho_0}\right)^{\frac{1}{1+\omega}},$$ \hfill (13)

where $\omega$ is EoS parameter which lies in the interval $-1 < \omega \leq 1$. However, the Supernova SN 1a, and Cosmic background radiation (CMB) data [11] show that for the accelerating universe, equation of state parameter is lying in the range $-1.3 < \omega \leq -0.79$. $n_0 \geq 0$ and $\rho_0 \geq 0$ are the current values of the particle number and energy density The particle number density gives the equilibrium equation

$$\dot{n} + nH(1 + d) = n\psi(t).$$ \hfill (14)

Here $\psi(t)$ stands for time-dependent PC rate. On the side, $\psi(t) > 0$ show a particle source, $\psi(t) < 0$ indicate particle disappearance and $\psi(t) = 0$ gives no particle production.
Using Eqs. (12), (9) & (13), we obtain

\[ \frac{1}{\rho} \frac{d\rho}{dt} + (d + 1)(\omega + 1)H = -\left( \frac{\dot{G}}{G} + \frac{\dot{\Lambda}}{8\pi G \rho} \right), \]  

(15)

\[ \frac{\dot{n}}{n} = \frac{1}{(1 + \omega)\rho} \frac{d\rho}{dt}. \]  

(16)

To find particle creation function we use Eqs. (14) − (16):

\[ \psi(t) = -\left( \frac{1}{1 + \omega} \right) \left( \frac{\dot{G}(t)}{G(t)} + \frac{\dot{\Lambda}(t)}{8\pi G(t)} \right). \]  

(17)

The entropy \( S \) generated during PC at temperature \( T \) follows this relation

\[ p \frac{dV}{dt} + \frac{d(\rho V)}{dt} = T \frac{dS}{dt}, \]  

(18)

where \( V = a^{(d+1)} \) is a spatial volume. In cosmological fluid, entropy is define as

\[ S = \frac{\rho(1 + \omega)a^{(d+1)}}{T}. \]  

(19)

From Eq. (18) and Eq. (9) we find,

\[ \frac{dS}{dt} = \frac{(1 + \omega)\rho a^{(d+1)}}{T} \psi(t). \]  

(20)

The entropy as a function of \( \psi(t) \) (PC rate) is obtained from the Eq. (20) as

\[ S(t) = S_0 e^{\int \psi(t) dt}, \]  

(21)

where \( S_0 \) is an integrating constant. Here, we have summarized the formation of PC \( \psi(t) \) and entropy \( S(t) \) which is time-dependent and based on the previous studies. In these references [41, 42, 68, 75], we can found discussion in detail.

4 Role of PVDP in Solutions

We consider a time-dependent periodic varying deceleration parameter (PVDP) [71] to solve field equations as

\[ q = m \cos(kt) - 1, \]  

(22)

where \( k \) & \( m \) figures as +ve constants. Here the periodicity of the PVDP is defined by \( k \) which can be viewed as a parameter of cosmic frequency. The peak of the PVDP is increased by the enhancement factor \( m \). In this model, the universe begins with a period of deceleration and expands in a cyclic history into a phase of super-exponential growth. Here by the definitions \( q = -1 - \frac{H}{H^2} \) & \( \dot{a} = aH \) we obtain \( \dot{H} = -mH^2 \cos kt \). After integration of Eq. (22), we obtain

\[ H = \frac{k}{k_1 + m \sin(kt)} \]  

(23)

where \( k_1 \) is an integrating constant. We may consider \( k_1 = 0 \) without losing generality. Hence the Hubble function becomes

\[ H = \frac{k}{m \sin(kt)} \]  

(24)

By integration of Eq. (24), we find the \( a(t) \) (scale factor) such as,

\[ a(t) = a_0 \left[ \tan \left( \frac{kt}{2} \right) \right]^{\frac{1}{m}}. \]  

(25)
where \( a_0 \) is the scale factor at the present epoch.

By using the constraints \( H_0 = 69.2 \) and \( q_0 = -0.52 \) from the recent observational Hubble data (OHD) and joint light curves (JLA) data \(^{76,77}\) in Eq. (22), we find a relation between \( k \) & \( m \) i.e. \( k = H_0 \cos^{-1} \left( \frac{m+1}{m} \right) \). From this relation we obtain the value of \( m \) for different value of \( k \).

Now, in these subsections, we obtain the cosmological solutions of with particle creation & without particle creation.

### 4.1 With Particle Creation

Using equations \(^{10}-^{12} \& \,^{24}\), we find time dependent energy density \( \rho \) & time dependent cosmological constant \( \Lambda \), respectively as

\[
\rho = \frac{dk^2 \cos (kt)}{8m (\sin (kt))^2 (\omega + 1) \pi G},
\]

\[
\Lambda = \frac{d(d + 1)k^2}{2m^2 (\sin (kt))^2} - \frac{dk^2 \cos (kt)}{m (\sin (kt))^2 (\omega + 1)}.
\]

It is obviously that, to be valid for both solutions \( \omega \neq -1 \) is required.

Equations (17), (26) and (27) gives particle creation rate as

\[
\psi = \frac{(d + 1)k}{m \sin (kt)} + \left( \frac{-2k (\cos (kt) - \sin (kt)) \cos (kt)}{(\omega + 1)} \right) (\omega + 1).
\]

Finally, we obtained the entropy production during the particle creation from Eqs. (21) and (28) as

\[
S(t) = S_0 \exp \left[ \frac{4 \ln (\csc (kt) - (d + 1)(\omega + 1) \cot (kt)) d}{(\omega + 1) m} + \frac{\ln (\cos (kt) - 2 \ln (\sin (kt)))}{(\omega + 1)} \right].
\]

### 4.2 Without Particle Creation

We get the normal particle conservation law of standard cosmology in the absence of particle creation, which implies \( \psi(t) = 0 \). The following equations are used in continuity Eq. (9) for this conservation law:

\[
\dot{\rho} + \left[ \rho (1 + d) + p(1 + d) \right] \frac{\dot{a}}{a} = 0,
\]

i.e.

\[
\frac{\dot{\rho}}{\rho} = -(d + 1)(1 + \omega) \frac{\dot{a}}{a}.
\]

\[
\dot{\Lambda}(t) + 8\pi \rho \dot{G}(t) = 0.
\]

After the integration of Eq. (31), we obtain

\[
\rho = \rho_0 a^{-(1 + \omega)(d + 1)}.
\]

Here \( \rho_0 \) is a positive constant. From Eq. (25) and (33), we find,

\[
\rho = \rho_0 \left( a_1 \tan \left( \frac{kt}{2} \right) \right)^{\frac{(1+\omega)(d+1)}{m}}.
\]

After solving Eqs. (10), (32) and (34), we get the gravitational constant \( G(t) \) and cosmological constant \( \Lambda(t) \):

\[
G = -\frac{dk^2 \cos (kt) (\tan \left( \frac{kt}{2} \right))}{4m \pi \sin^3 (kt) (1 + \omega) \rho_0 \left( a_0 \tan \left( \frac{kt}{2} \right) \right)^{\frac{(1+\omega)(d+1)}{m}}} \left( 1 + \left( \tan \left( \frac{kt}{2} \right) \right)^2 \right)^2.
\]

\[
\Lambda = \frac{d(d + 1)k^2}{2(m \sin (kt))^2} - \frac{2dk^2 \cos (kt) \tan \left( \frac{kt}{2} \right)}{m (1 + \omega) \sin (kt) \left( 1 + \left( \tan \left( \frac{kt}{2} \right) \right)^2 \right)^2}.
\]
5 Results and Discussions

Figure 1(a) and 1(b) corresponding to the Eq. (26) & Eq. (34), portrays the behavior of periodic variation of the energy density ($\rho$) with and without particle creation versus cosmic time (t) for the dimension 5 and three distinct values of $m$ and $k$. It is observed from the figure 1(a) that energy density ($\rho$) with particle creation has Big Rip singularities at the cosmic time $t = \frac{n\pi}{k}$, where $n$ is an integers ($n = 0, 1, 2, 3, 4....$). Since the PVDP is depending on the choice of the values of $k$ and $m$ and we consider only three distinct values so there exists the cosmic singularities corresponding to the different time period as $t = 0, 12.56, 25.12......$ for $k = 0.25$, $t = 0.6, 2.56, 12.56......$ for $k = 0.5$ and $t = 0.3, 1.14, 6.28......$ for $k = 1$. The interesting aspect is that it begins with a large value at stating time in a given cosmic period and decreases to a minimum $\rho$, and then increases again with the growth of time. The minimum energy density occurs at the time $t = \frac{(n+1)\pi}{2k}$.

From the figure 1(b), we observe that energy density without particle creation has also periodic singularities at the cosmic time $t = \frac{(2n-1)\pi}{k}$, where $n$ is an integers ($n = 1, 2, 3, 4....$). In this figure various cosmic singularities are exists for different values of $k$, i.e. $t = 12.56, 37.68, 87.92......$ for $k = 0.25$, $t = 6.28, 18.84, 43.96......$ for $k = 0.5$ and $t = 3.14, 9.42, 21.98......$ for $k = 1$. In beginning of the evolution of the universe, it was infinitely large, indicating the Big-bang scenario. Then it dropped first rapidly, and slowly. It approaches to a smallest value of $\rho$ and as time progresses, it increases again. Since $t$ tends to $t_s$, energy density diverge strongly for all values of $k$ and $m$ i.e. $\rho$ tends to infinite so it has Type-I singularity.

In this model we analyze the cosmological term $\Lambda$ which will determine the universe’s nature. This is plotted in Fig. 2(a) and 2(b) corresponding to the Eq. (27) & Eq. (36) respectively for with and without particle creation.

Here, in Figure 2(a), the cosmological term $\Lambda$ exhibits a periodic variation with singularity at cosmic time $t = \frac{n\pi}{k}$ ($n = 0, 1, 2, 3....$) for all three values of $m$ and $k$ corresponding to FLRW metric. And in 2(b) figure cosmological term $\Lambda$ has singularity at cosmic time $t = \frac{(2n-1)\pi}{k}$. We see that within a given cycle cosmological constant $\Lambda$ starts from large positive values and approach to the smallest positive value of $\Lambda$ and again increases with the growth of time i.e. this is a big rip singularity. Thus, the nature of $\Lambda$ in present models is supported by observational evidences.
Figure 2: (a) Cosmological constant with particle creation vs. cosmic time $t$, (b) Cosmological constant without particle creation vs. cosmic time $t$

Figures 3(a) and 3(b) demonstrate evolutionary behavior of pressure $p$ with respect to time $t$ for all values of $k$ and $m$ corresponding of FLRW metric for dimension 5 ($d=5$). From the figure 3(a) it is ascertained that the pressure have a periodic variation with singularity at cosmic time $t = \frac{n\pi}{k}$ ($n = 0, 1, 2, 3, ...$). Here, we find a repeated cyclic pattern, where at the beginning pressure decreases from large positive value to large negative values and it keeps flowing on.

And in figure 3(b) nature of pressure is also periodic and contain singularity at cosmic time $t = \frac{(2n-1)\pi}{k}$, where $n$ is an integers ($n = 1, 2, 3, 4, ...$) i.e. $t = \frac{\pi}{k}, \frac{3\pi}{k}, \frac{5\pi}{k}, ...$. It is negative throughout the evolution. From the figure we observe that pressure is decreasing function of time $t$ & it begins from an large negative value and reaches zero. We find this is repeated cycle pattern. We see that pressure $p$ is high negative at an early stage however it decreases as time will increase.
Figure 3: (a) Pressure for particle creation versus cosmic time $t$, (b) Pressure for no particle creation versus cosmic time $t$.

Figure 4(a) shows periodic variation behavior of particle creation $\psi$ with cosmic time $t$ for all three values of $m$ and $k$. From the figure it is observed that particle creation ($\psi$) has 'Big Rip' singularities at the cosmic time $t = \frac{n\pi}{k}$, i.e. $t = 0, \frac{\pi}{k}, \frac{2\pi}{k}, \frac{3\pi}{k}, \ldots$, where $n$ is an integers ($n = 0, 1, 2, 3, 4, \ldots$). We can see from the figure that the nature of time dependent particle creation $\psi$ is consistent with the standard observational data.

In the case of no particle creation, Figure 4(b) demonstrate the periodic variation of gravitational constant with singularities at the cosmic time $t = \frac{(2n-1)\pi}{k}$. From the figure, we can see that the gravitational constant is only an increasing function for all values of $m$ and $k$ as expected.
Figure 4: (a) Particle creation versus cosmic time $t$, (b) Gravitational constant versus cosmic time $t$

Figure 5 demonstrates entropy creation with cosmic time $t$ for different values of $k$ and $m$. It has periodic singularity at cosmic time $t = \frac{n\pi}{k}$ for all values of $k$ and $m$, here $n$ is an integers ($n = 1, 2, 3, 4, \ldots$) i.e. $t = \frac{\pi}{k}, \frac{2\pi}{k}, \frac{3\pi}{k}, \ldots$.

6 Kinematic Test

Now, as suggested in the preceding section, we derive some kinematic relationships of the model.
6.1 The Density Parameter

The density parameters for matter $\Omega_{\text{matt}}(t)$ and vacuum $\Omega_\Lambda(t)$ are

$$\Omega_{\text{matt}}(t) = \frac{\rho}{\rho_c}$$

i.e.

$$\Omega_{\text{matt}}(t) = \frac{2m \cos(kt)}{(\omega + 1)(d + 1)}$$

$$\Omega_\Lambda(t) = \frac{\rho_\Lambda}{\rho_c} = 1 - \frac{2m \cos(kt)}{(\omega + 1)(d + 1)}.$$  

where $\rho_c = \frac{d(d+1)H^2}{16\pi G}$.

From equations (38) and (39), total density parameter $\Omega_T$ is

$$\Omega_T = \Omega_{\text{matt}} + \Omega_\Lambda = 1.$$  

The inflationary scenario favors this solution which is equivalent to the normal Einstein gravity. The preliminary outcomes, as per the high redshift of Supernovae and CNB, indicate that the universe may accelerate, with a dominant contribution to its energy density being received in the form of a cosmological constant. Wide range of values of $\Omega_{\text{matt},0}$ and $\Omega_{\Lambda0}$ (the present cosmic matter and vacuum energy density parameters) has been offered by the recent measurements. SNe Ia observation together with the total energy density constraints from CMB [78] and combined gravitating lens and stellar dynamical analysis [79] contribute to $\Omega_{\text{matt},0} \sim 0.3$ and $\Omega_{\Lambda0} \sim 0.7$.

6.2 Proper Distance Redshift

$d(z) = a0r(z)$ is the acceptable distance between the source and observer, where $r(z)$ is the radial distance of the target at light emission in terms of redshift, such as

$$r(z) = \int_t^{t_0} \frac{dt}{a(t)},$$

$$r(z) = \int_0^z \frac{dz}{a_0 H(z)},$$

Hence

$$d(z) = \int_0^z \frac{2mdz}{ka_0(1+z)(1+\frac{1}{(1+z)^{2m}})}.$$  

Figure 6: Proper Distance vs Redshift z

For some selected values of $m$ and $k$ the proper distance as a function of redshift shown in figure 6. We observed that particle creation gives rise to proper distance.
6.3 Angular distance Redshift

The another important Kinematic test is angular diameter distance redshift which is denoted by $d_A$. This is the ratio of physical transverse size of an object to its angular size (in radians). It is given by in term of $z$ such as

$$d_A = \frac{d(z)}{(1 + z)}.$$  \hspace{1cm} (44)

Using the Eq. (43), angular distance redshift is

$$d_A = \frac{1}{(1 + z)} \int_0^z \frac{2mdz}{ka_0(1 + z)(1 + \frac{1}{(1+z)^m})}.$$ \hspace{1cm} (45)

In Figure 7, we draw the variation of angular distance vs redshift $z$ for three values of $m$ and $k$. This shows that particle creation enhance the angular distance. The angular diameter distance initially increases with increasing $z$ and gradually starts to decrease for all values of $k$ and $m$ ($k = 0.25$, $m = 0.4800$ and $k = 0.5$, $m = 0.48001$ $k = 1$, $m = 0.48005$).

![Angular distance vs Redshift](a)

Figure 7: Angular distance vs Redshift $z$

6.4 Luminosity Distance Redshift

This is pointed out by the observations of Ia Supernova \[5, 6\] that universe expansion is in the accelerating phase. In this Redshift plays an important role. luminosity distance versus time $t$ \[80, 81\] is an important observational tool to study the evolution of the universe. The expansion of the universe causes the light which is emitted by the stellar object to get redshifted. The concept of distance states the explanation of the expanding universe which is linked with the observations. It has been defined in various ways in the literature. Specifically, luminosity distance is the distance defined by the luminosity of a stellar object and has an important role in astronomy. One can also compute the rate of expansion of the universe through the observational measurements of luminosity distance. Here luminosity distance $D_L$ in terms of redshift has been derived.

Expression for the Luminosity distance determining flux of the source with red shift $z$ is

$$D_L = a_0 c(1 + z) \int_0^{t_o} \frac{dt}{a(t)},$$ \hspace{1cm} (46)

where $c$ expresses speed of light and $a_0$ shows the present value of the scale factor.

$$= c(1 + z) \int_0^z \frac{dz}{H(z)}.$$ \hspace{1cm} (47)

Since $H(z) = \frac{k(1+z)}{2m} \left( 1 + \frac{1}{(1+z)^m} \right)$, so Eq. (49) become,

$$D_L = c(1 + z) \int_0^z \frac{2mdz}{k(1 + z)(1 + \frac{1}{(1+z)^m})}.$$ \hspace{1cm} (48)
6.5 Apparent Magnitude

The distance modulus $\mu$ which is difference of apparent magnitude and absolute magnitude and related to the luminosity distance define as

$$\mu = m - M = 25 + 5 \log_{10} \left( \frac{D_L}{Mpc} \right),$$

where $m$ denotes apparent magnitude and $M$ denotes absolute magnitude respectively.

For finding the small red shift, we use the following equation of $D_L$ given as

$$D_L = \left( \frac{cz}{H_0} \right),$$

There are a lot of supernova of low red shift whose apparent magnitudes are known. we have find absolute magnitude $M$ of Type Ia supernova(SNIa) \cite{82,83} by using $z = 0.026$ and $m = 16.08$ in Eq. (51) as follows:

$$M = 5 \log_{10} \left( \frac{H_0}{0.026c} \right) - 8.92,$$

From this we obtain apparent magnitudes $m$ as

$$m = 16.08 + 5 \log_{10} \left( \frac{D_LH_0}{0.026c} \right),$$

$$m = 16.08 + 5 \log_{10} \left( \frac{H_0(1 + z)}{0.026} \int_0^z \frac{dz}{H(z)} \right),$$

$$m = 16.08 + 5 \log_{10} \left( \frac{H_0(1 + z)}{0.026} \int_0^z \frac{2mdz}{ka_0(1 + z)(1 + \frac{1}{(1+z)^m})} \right),$$
Figure 9 depicts apparent magnitude versus redshift $z$ for the observational values of $(m, k)$.

### 6.6 Age of the universe versus redshift $z$

The age of the universe is defined as

$$ t_0 = \int_{0}^{t_0} dt = \int_{0}^{\infty} \frac{dz}{H(z)(1+z)}, \quad (55) $$

Since $H(z) = \frac{k(1+z)}{2m} \left(1 + \frac{1}{(1+z)^m}\right)$, so Eq. (55) becomes

$$ t_0 = \int_{0}^{\infty} \frac{2mdz}{k(1+z)^2(1+\frac{1}{(1+z)^m})}, \quad (56) $$

In Fig. 10, the curves are drawn for $H_0$ $t$ with respect to redshift $z$. It shows that $H_0$ $t$ is in increasing order with increasing $z$ for all values of $k$ and $m$ ($k = 0.25, m = 0.4800$ & $k = 0.5, m = 0.48001$ and $k = 1, m = 0.48005$). Consequently, this gives the universe age $t_0$ as 10.34 Gyrs and 12.36 Gyrs. The age of our universe, according to WMAP info, is approximately 13.73 Gyrs. So, the closest available theoretical value of $t_0$ is Gyrs 12.36. However the drawn curves are best suited and in full agreement with observed results for $H_0 = 69.2$. 

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(a) Figure 9: Apparent magnitude vs Redshift $z$

(a) Figure 10: Age of the universe versus Redshift $z$
6.7 Look-back Time Redshift

The look-back time is the difference between the age of universe at the present time $z = 0$ and the age of the universe when a specific right ray was produced at redshift $z$ \[84\]. For a given redshift $z$, scale factor $a$ is related to the $a_0$ by the relation $a = a_0(1 + z)^{-1}$, since $1 \leq (1 + z) < \infty$ is the inverse of the scale factor $0 < a \leq 1$, for both $z$ and $(1 + z)$ are potentially misleading and nonlinear functions of fundamental quantities.

In terms of the scale factor $a$ and $H = d\ln(a)/dt$, the look-back time is

$$t_L = \int_0^{t_0} dt = \int_1^a \left( \frac{dt}{d\ln(a)} \right) d\ln(a)$$

(57)

$$t_L = \int_1^a \frac{da}{aH(a)}$$

(58)

The look back time can be written in terms of redshift ($z$):

$$t_L = \int_0^t dt = \int_0^z \left( \frac{dt}{d\ln(a)} \right) dz$$

(59)

Using the Equation $a = a_0(1 + z)^{-1}$ to determine $dt/dz$ as:

$$\frac{dz}{dt} = \frac{1}{a} \left( \frac{\dot{a}}{a} \right) = -(1 + z)H$$

(60)

$$t_L = \int_0^z \frac{dz}{(1 + z)H(z)}$$

(61)

$$t_L = \int_0^z \frac{2mdz}{ka_0(1 + z)^2 \left( 1 + \frac{1}{(1 + z)^m} \right)}$$

(62)

Figure 11: Look-back time versus redshift $z$.

We have observed in Fig 11 that the look-back time is increases with redshift ($z$) for all values of $k$ and $m$.

7 Concluding remarks

In the present work, we consider a periodically varying deceleration parameter to reconstruct the cosmic history. We also analyzed the mechanism of particle creation admitting variations of the cosmological constant $\Lambda$ and the gravitational constant $G$ for higher-dimensional FLRW space-times.
Apparently, the universe’s dynamic properties have been periodically produced by the PVDP. Within the specified cosmic duration and defined by the cosmic frequency parameter of the model, the energy density $\rho$ & pressure $p$ vary cyclically. The magnitude of these physical parameters turns infinitely large at some finite time. This behavior contributes to the singularity of type I as categorized by Nojiri et al. [85]. There seems to be a Big Rip singularity at a certain finite period during the cosmic replication of the process since $a \to \infty$, $\rho \to \infty$ and $|p| \to \infty$.

We notice that for the particle creation, behavior of the parameters repeat after the time period $t = \frac{n\pi}{k}$ i.e. the big Rip occurs periodically after a time gap. Similar behavior of the parameters is also for without particle creation for the time period $t = \frac{(2n-1)\pi}{k}$.

Here we observed that particle creation $\psi$ and entropy $S$ has periodic variation with singularity at the cosmic time $t = \frac{n\pi}{k}$ i.e. these have Big-Rip singularity. We also note that gravitational constant $G$ has cyclic nature with singularity at the cosmic time $t = \frac{(2n-1)\pi}{k}$. From the figures we analyze that cosmological constant $\Lambda$ has singularity for both cases particle creation and without particle creation. Then, we investigated the periodic variation of several cosmological quantities such as energy density $\rho$, pressure $p$, particle creation rate $\psi$, entropy $S$, etc. which have Big Rip singularities.

We have also explored some more parameters through some kinematics tests like Proper distance, Angular distance, Luminosity distance ($D_L$), Apparent Magnitude ($m$), Age of the Universe, and Look back time ($t_L$) with respect to redshift ($z$). These results are found to be compatible with the current observations.

Hence, our constructed model and their solutions have good agreement with the observational data and physically acceptable. Therefore, for more understanding of the characteristics of the particle creation in our universe’s evolution within the framework of FLRW metric, the solution demonstrated in this paper may be helpful.

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