Sensitivity analysis of the effective centrifugal pump parameters using the EFAST method

Hamed Safikhani*

Department of Mechanical Engineering, Faculty of Engineering, Arak University, Arak 38156-88349, Iran

Abstract

In the present study, the effective parameters of centrifugal pumps are investigated using the EFAST Sensitivity Analysis (SA) method. The SA is performed using GMDH type artificial neural networks (ANN) which are based on validated numerical data of flow field in centrifugal pumps. There are four design variables namely: leading edge angle of blades on hub section ($\beta_{1_{Hub}}$), leading edge angle of blades on shroud section ($\beta_{1_{Shroud}}$), trailing edge angle of blades ($\beta_2$), and the stagger angle of blades on mid span ($\gamma_{mid}$) and there are two objective functions namely: efficiency ($\eta$) and the required NPSH of impeller. The results show that among design variables, $\beta_2$ has the highest effect on variations of $\eta$ (46%) and NPSH (45%). Except $\beta_2$, $\beta_{1_{Hub}}$ and $\gamma_{mid}$ has the highest effect on NPSH (33%) and $\eta$ (28%) respectively. The effects of all of the design variables on objective functions are shown in the results.

Keywords: Sensitivity analysis, EFAST method, centrifugal pumps, required NPSH.

* Corresponding author, Email: h-safikhani@araku.ac.ir, Tel. +98 86 32625726, Fax: +98 8632625001.
1. Introduction

Centrifugal pumps are the group of turbo machines which are used industrially in large scales. In recent years several researchers have investigated the different aspects of such pumps. Demeulenaere et al. [1] investigated an optimization process on centrifugal pumps using Fine/Design 3D environment of Numeca software and genetic algorithms. They tried to increase efficiency and head and decrease the $NPSH_r$ at two different flow rates and finally showed that the new blade geometry should have more curvature in the camber line definition. Nariman-zadeh et al. [2] presented a multi-objective optimization process on centrifugal pumps and suggested four optimal point that designer can select each of them. They tried to increase the hydraulic efficiency and head and decrease the input power. They did not use CFD in their simulation and just used the analytical equations for hydraulic efficiency, head and the input power. Safikhani et al. [3] investigated a multi-objective optimization process on centrifugal pumps. Combining CFD, GMDH type neural networks and NSGA II algorithm, they have presented Pareto front for centrifugal pumps.

Korakianitis et al. [4] developed specific speed versus specific diameter graphs suitable for the design and optimization of these smaller centrifugal pumps concentrating in dimensions suitable for ventricular assist devices (VADs) and mechanical circulatory support (MCS) devices. A combination of experimental and numerical techniques was used to measure and analyze the performance of 100 optimized pumps designed for this application. The data was presented in the traditional Cordier diagram of nondimensional specific speed versus specific diameter. Using these data, nine efficient designs were selected to be manufactured and tested in different operating conditions of flow, pressure, and rotational speed. The nondimensional results presented in this article enable preliminary design of centrifugal pumps for VADs and MCS.
devices. Wang et al. [5] proposed a method to optimize the design of a typical multistage centrifugal pump based on energy loss model and Computational Fluid Dynamics (ELM/CFD). Wang et al. [6] improved the efficiency of a centrifugal pump using optimization of a vanned diffuser. The steady simulations were carried out by solving the three-dimensional Reynolds-averaged Navier–Stokes equations with a shear stress transport turbulence model. Finally the efficiency of the optimal pump increased by 8.65% compared with the original scheme. The velocity distributions in the diffuser inlet and volute were improved and became more uniform. The total pressure in the diffuser and volute of the optimal pump was higher than that of the original pump. Zhao et al. [7] described the shape optimization of a low specific speed centrifugal pump at the design point. Some other researchers have also done some researches on optimizing of different engineering elements [8-13].

In centrifugal pumps there are a lot of geometrical parameters and using a sensitivity analysis the effective parameters should be defined. Sensitivity analysis refers to the study of “how uncertainty in model output (numerical and non-numerical) can be classified into different sources of uncertainty in model input factors” [14]. Saltelli et al. [15] have classified the sensitivity analysis methods into two groups: local and general. The local sensitivity analysis methods analyze the response of model output(s) by changing one of the parameters and maintaining the other parameters at central values; while the general sensitivity analysis methods investigate the general response of model output(s) (averaged over the variation of all the parameters) by searching a finite (or infinite) region. Although the local sensitivity analysis method is simple to use, it just analyzes one point at a moment; so nowadays, the general sensitivity analysis methods are preferred to the local ones.
As was mentioned, sensitivity analysis can specify the sensitive and insensitive parameters of a model. In this regard, Korayem et al. [16] investigated the use of different contact models in the AFM-based manipulation of biological cells in bio-environments. They employed the Sobol method to analyze the sensitivity of the modeling parameters of four contact mechanics models (PT, Hertz, DMT and JKR). Hertz model is very sensitive to the Young’s modulus, and the sensitivity of the adhesion energy in this model is zero (Hertz model disregards the effect of adhesion energy). Contrary to Hertz model, the other three models are highly sensitive to the adhesion energy as well as the elasticity modulus. All the models show little sensitivity to the parameters of particle radius and Poisson’s ratio.

Based on our information, no sensitivity analysis research has been carried out so far on centrifugal pumps. Therefore, sensitivity analysis is investigated in the present study using the EFAST method.

2. Defining the design variables

To parameterize the camber line curve, the simple Bezier method is used. Schematically definition of simple Bezier method is shown in Fig. 1. The design variables in this method are leading edge angle of blades on hub section \(\beta_{1\text{Hub}}\), leading edge angle of blades on shroud section \(\beta_{1\text{shroud}}\), trailing edge angle of blades \(\beta_2\), and the stagger angle of blades on mid span \(\gamma_{\text{mid}}\). In the present paper three sections are defined in the blades, first on hub, second one on shroud and the third one on the middle plane of hub and shroud, as shown in Fig. 2. It is supposed that \(\beta_2\) is the same at the three defined sections of blade. This problem is mathematically given by:

\[
\beta_{2\text{Hub}} = \beta_{2\text{shroud}} = \beta_{2\text{MidSpan}} = \text{DesignVariable}
\]

Moreover \(\beta_1\) at mid span is equal to the average of \(\beta_1\) at hub and shroud sections:
$\beta_{\text{MidSpan}} = \frac{\beta_{\text{Hub}} + \beta_{\text{Shroud}}}{2}$  \hspace{1cm} (2)

So there are four independent design variables namely: $\beta_{\text{Hub}}$, $\beta_{\text{Shroud}}$, $\beta_2$ and $\gamma_{\text{mid span}}$. In fact $\gamma_{\text{mid span}}$ is the average $\gamma$ of three sections. Design variables and their range of variations are shown in Table 1. The sensitivity analysis in the present paper is performed using the GMDH type Artificial Neural Network (ANN) models and CFD data which were presented in [3].

3. CFD and GMDH type ANN models

The sensitivity analysis (SA) presented in this paper is performed using GMDH type artificial neural networks (ANN) which are based on validated numerical data of flow field in centrifugal pumps. The details of numerical modeling and GMDH polynomials are presented in [3]. Some operating conditions are shown in Table 2 and moreover a sample of grid generation and pressure contour in numerical simulations are shown in Figs. 3 and 4 respectively.

4. Sensitivity analysis methods

An area of general sensitivity analysis methods that has attracted more attention is the variance-based methods. In these methods, the sensitivity index is computed as the share of each parameter in the overall output variance of the model. The general sensitivity analysis methods are implemented in four steps: (1) defining the inputs and the type of distribution of each input, (2) generating the samples for the input values, (3) computing the model’s output for each set of input samples and (4) determining the effect of each input factor on the output [17]. In this section, the variance-based sensitivity analysis methods have been reviewed. The variance-based general sensitivity analysis approaches can be used to obtain the first-order effect and the second-order effect (which include the interaction between other parameters) [18].
The Sobol method [19] is a model-independent general sensitivity analysis method which is based on variance analysis. This method can be used for nonlinear and non-uniform functions and models. For the model defined by function $Y=f(X)$, where $Y$ is the model output and $x(x_1,x_2,\ldots,x_n)$ is the vector of input parameters, Sobol suggested to decompose the function $f$ into summands of increasing dimensionality, where the integral of each term over its own input variables is zero. Sobol showed that, when all the inputs are perpendicular to one another, this resolution is unique and the output variance of the model ($V$) is the set of variances of each resolved term [19]:

$$V(Y) = \sum_{i=1}^{n} V_i + \sum_{i<j}^{n} V_{ij} + \ldots + V_{1\ldots n}$$

(3)

In relation (3), $V_i$ denotes the first-order effect for each input factor $x_i (V_i = V[E(Y|x_i)])$, and $V_{ij}$ ($V_{ij} = V[E[Y|x_i,x_j]]-V_i-V_j$) to $V_{1\ldots n}$ indicate the interactions between $n$ factors. Therefore, the shares allocated to parameters, and the interactions of parameters can be determined from the total output variance. The sensitivity index is obtained as the ratio of each order’s variance to the total variance ($S_i = V_i / V$ denotes the first-order sensitivity index, $S_{ij} = V_{ij} / V$ represents the second-order sensitivity index, and so on). The total sensitivity index (i.e., the overall effect of each parameter) is obtained as the summand of all the orders of sensitivity index for that parameter [19]:

$$S_{Ti} = S_i + \sum_{i \neq j} S_{ij} + \cdots$$

(4)

The EFAST method was presented by Cukier et al. [20] and was later improved by Saltelli et al. [21]. Like the Sobol method, this approach is also based on variance and it is independent of any assumption of linearity and uniformity between inputs and output(s). Contrary to the Sobol method, which uses multidimensional integrals to obtain the total variance and the partial
variances, this method converts the multidimensional integrals to one-dimensional ones by defining a transfer function and simplifies the procedure for the calculation of sensitivity indexes.

The EFAST method searches the n-dimensional space of the input factors (Unit Hypercube $\kappa^*$) by using a Search Curve defined by a set of parametric equations [21]:

$$x_i = \frac{1}{2} + \frac{1}{\pi} \arcsin\left(\sin(\omega_i s + \phi_i)\right)$$

(5)

where $\omega_i$ ($i = 1, 2, \cdots, n$) is the frequency related to factor $x_i$, $s$ is a variable that changes from $-\pi$ to $+\pi$, and $\phi_i$ specifies the starting point of the curve. The output variance of the model is approximated by means of Fourier analysis:

$$V(Y) = \frac{1}{2\pi} \int_{-\pi}^{\pi} f^2(s)ds - \left[\frac{1}{2\pi} \int_{-\pi}^{\pi} f(s)ds\right]^2 \approx$$

(6)

$$\sum_{j=-\infty}^{\infty} (A_j^2 + B_j^2) - (A_0^2 + B_0^2) \approx 2\sum_{j=1}^{n} (A_j^2 + B_j^2)$$

In the above relation, $f(s) = f(G_1(\sin(\omega_1 s)), G_2(\sin(\omega_2 s)), \ldots, G_n(\sin(\omega_n s)))$, $G(s)$ are the transfer functions, and $A_j$ and $B_j$ are the Fourier coefficients ($A_j = \frac{1}{2\pi} \int_{-\pi}^{\pi} f(s) \cos(js)ds, B_j = \frac{1}{2\pi} \int_{-\pi}^{\pi} f(s) \sin(js)ds$). By calculating the Fourier coefficients for the basic frequency ($\omega_i$) and its higher harmonics ($p\omega_i$), the partial first-order input variance ($V_i$) can be obtained.

$$V_i = \sum_{p\in \mathbb{Z}^+} (A_{p\omega_i}^2 + B_{p\omega_i}^2) = 2\sum_{p=1}^{n} (A_{p\omega_i}^2 + B_{p\omega_i}^2)$$

(7)

Also, like the Sobol method, the ratio of the first-order partial variance to total variance is used to compute the main sensitivity index. The total sensitivity index is obtained from relation (8) [22]:

...
\[ ST_i = 1 - \frac{V_{i}}{V} \] (8)

Variance \( V_{i} \) is obtained by changing all the parameters except parameter \( x_i \).

The Sobol method employs the Monte Carlo integral to obtain each partial variance; and in comparison with the EFAST method, it doesn’t use a transfer function; that’s why, it has a low computational efficiency. Algorithm of sensitivity analysis is shown in Fig. 5.

5. Results of sensitivity analysis

The results of sensitivity analysis for efficiency (\( \eta \)) and the Net Positive Suction Head (NPSH) in centrifugal pumps have been presented in this section. Employing the EFAST method, the sensitivity of four parameters: leading edge angle of blades on hub section (\( \beta_{1\,Hub} \)), leading edge angle of blades on shroud section (\( \beta_{1\,Shroud} \)), trailing edge angle of blades (\( \beta_2 \)) and the stagger angle of blades on mid span (\( \gamma_{mid} \)) have been explored for \( \eta \) and NPSH. Table 1 shows the intervals of changes of the investigated parameters.

Fig. 6 (a) shows the changes of the \( \eta \) with \( \beta_{1\,Hub} \) and indicates that with the increase of this parameter, the \( \eta \) diminishes with a sharp slope. As is observed in this figure, at low values of \( \beta_{1\,Hub} \), sensitivity is smaller and with the increase of \( \beta_{1\,Hub} \), the slope of the diagram becomes greater. So, by considering the results that indicate the effect of this parameter on the \( \eta \) the proper values for this parameter can be selected. As is shown in Fig. 6 (b), with the increase in the \( \beta_2 \), \( \eta \) also diminishes with a very sharp slope. So, the first most sensitive parameter is \( \beta_2 \).

The other investigated parameter is \( \beta_{1\,Shroud} \); and considering a near zero slope for the diagram showing the changes of \( \eta \) versus \( \beta_{1\,Shroud} \) (Fig. 6 (c)), this parameter is not considered to be a sensitive parameter either for the \( \eta \), and choosing different values for this parameter from its
range of changes doesn’t lead to a tangible change in the $\eta$ values. As Fig. 6 (d) demonstrates, the diagram showing the changes of the $\eta$ versus $\gamma_{mid}$ selected, and indicates that with the increase of this parameter, first, the $\eta$ decreases and then increases.

The changes of the NPSH with $\beta_{1\,Hub}$ have been shown in Fig. 7 (a). With the increase of $\beta_{1\,Hub}$, NPSH diminishes with a very sharp slope. As is observed in this figure, at low values of $\beta_{1\,Hub}$, sensitivity is smaller and with the increase of $\beta_{1\,Hub}$, the slope of the diagram becomes greater. So, by considering the results that indicate the effect of this parameter on the NPSH, the proper values for this parameter can be selected. Another sensitive parameter among the parameters is the $\beta_2$. According to Fig. 7 (b), with the increase of this parameter, NPSH also increases with a sharp slope.

The other investigated parameter is the $\beta_{1\,Shroud}$; and considering a near zero slope for the diagram showing the changes of the NPSH versus $\beta_{1\,Shroud}$ (Fig. 7(c)), this parameter is not considered to be a sensitive parameter either for the NPSH, and choosing different values for this parameter from its range of changes doesn’t lead to a tangible change in the NPSH values. Another sensitive parameter among the input parameters is $\gamma_{mid}$. According to Fig. 7 (d), with the increase of this parameter, first, the NPSH also increases and then decreases.

Fig.8 indicates more accurate analysis of the results obtained by the EFAST sensitivity analysis method. According to Fig.8, as expected, $\beta_2$ (with a sensitivity index of 46%), $\gamma_{mid}$ (with a sensitivity index of 28%) and $\beta_{1\,Hub}$ (with a sensitivity index of 21%) are of most significant sensitivity among four parameters in $\eta$. Also, According to Fig.8, $\beta_2$ (with 45% sensitivity) is the most important parameter and the parameters of $\beta_{1\,Hub}$ (with 33% sensitivity) and $\gamma_{mid}$ (with 22% sensitivity) respectively are the other effective parameters in NPSH.

6. Conclusion
The effective parameters of centrifugal pumps were investigated using the EFAST Sensitivity Analysis method. The SA was performed using GMDH type ANN which were based on validated numerical data of flow field in centrifugal pumps. There were four design variables namely: \( \beta_{1\ Hub} \), \( \beta_{1\ Shroud} \), \( \beta_2 \) and the stagger angle of blades on mid span \( \gamma_{\text{mid}} \) and there were two objective functions namely: \( \eta \) and the required NPSH of impeller. The results show that among design variables, \( \beta_2 \) has the highest effect on variations of \( \eta \) (46%) and NPSH (45%). Except \( \beta_2 \), \( \beta_{1\ Hub} \) and \( \gamma_{\text{mid}} \) has the highest effect on NPSH (33%) and \( \eta \) (28%) respectively. The effects of all of the design variables on objective functions were shown in the results (Fig. 8).

References

[1] Demeulenaere, A., Purwanto, A., Ligout, A., Hirsch, C., Dijkers, R. and Visser, F. “Design and Optimization of an Industrial Pump: Application of Genetic Algorithm and Neural Network”, *Proceedings of insert conference abbreviation, ASME fluid engineering summer conference*, , Houston, Texas, (2005).

[2] Nariman-zadeh, N., Amanifard, N., Hajiloo, A., Ghalandari, P. and Hoseinpoor, B. “Multi-Objective Pareto Optimization of Centrifugal Pumps using Genetic Algorithms”, *Proceedings of 11th WSEAS international conference on computers*, crete island, greece, (2007).

[3] Safikhani, H., Khalkhali, A. and Farajpoor, M. “Pareto based multi-objective optimization of centrifugal pumps using CFD, neural networks and genetic algorithms”, *Engineering Applications of Computational Fluid Mechanics*, 5, pp. 37-48 (2011).
[4] Korakianitis, T., Rezaienia, M., Gordon, P., Rahideh, A., Rothman, T. and Mozafari, S., “Optimization of Centrifugal Pump Characteristic Dimensions for Mechanical Circulatory Support Devices”, *ASAIO Journal*, **62** (5), pp. 545–551 (2016).

[5] Wang, C., Shi, W., Wang, X., Jiang, X., Yang, Y., Li, W. and Zhou, L., “Optimal design of multistage centrifugal pump based on the combined energy loss model and computational fluid dynamics”, *Applied Energy*, **187**, pp. 10-26 (2017).

[6] Wang, W., Shouqi, Y. and Ji, P., “Optimization of the diffuser in a centrifugal pump by combining response surface method with multi-island genetic algorithm”, *Proceedings of the Institution of Mechanical Engineers, Part E: Journal of Process Mechanical Engineering*, **231** (2), pp. 13-24 (2017).

[7] Zhao, A., Lai, Z., Wu, P., Cao, L. and Wu, D., “Multi-objective optimization of a low specific speed centrifugal pump using an evolutionary algorithm”, *Engineering Optimization*, **48** (7), pp. 1251-1274 (2016).

[8] Yun, X., Lei, T. and Shuliang, C., “Multiparameter and multiobjective optimization design of centrifugal pump based on orthogonal method”, *Proceedings of the Institution of Mechanical Engineers, Part C: Journal of Mechanical Engineering Science*, **231** (14), pp. 19-28 (2017).

[9] Safikhani, H., “Modeling and multi-objective Pareto optimization of new cyclone separators using CFD, ANNs and NSGA II algorithm”, *Advanced Powder Technology*, **27** (5), pp. 2277-2284 (2016).
[10] Safikhani, H. and Dolatabadi, H., “Multi-objective optimization of cooling of a stack of vertical minichannels and conventional channels subjected to natural convection”, *Applied Thermal Engineering, 96*, pp. 144-150 (2016).

[11] Damavandi, MD., Forouzanmehr, M. and Safikhani, H., “Modeling and Pareto based multi-objective optimization of wavy fin-and-elliptical tube heat exchangers using CFD and NSGA-II algorithm”, *Applied Thermal Engineering, 111*, pp. 325-339 (2017).

[12] Sadafi, MH., Hosseini, R., Safikhani, H., Bagheri, A., and Mahmoodabadi, MJ. “Multi-objective optimization of solar thermal energy storage using hybrid of particle swarm optimization, multiple crossover and mutation operator”. *International Journal of Engineering, 24* (3), pp. 366-76 (2011).

[13] Safikhani, H. and Eiamsa-ard, S. “Pareto based multi-objective optimization of turbulent heat transfer flow in helically corrugated tubes”, *Applied Thermal Engineering, 95*, pp. 275-280 (2016).

[14] Saltelli, A. and Sobol, M., “About the use of rank transformation in sensitivity analysis of model output”, *Reliability Engineering & System Safety, 50*, pp. 225-239 (1995).

[15] Saltelli, A., Chan, K. and Scott, E., “Sensitivity analysis Wiley series in probability and statistics”, *Wiley, New York*, (2000).

[16] Korayem, M., Rastegar, Z. and Taheri, M., “Sensitivity analysis of nano-contact mechanics models in manipulation of biological cell”, *Nanoscience and Nanotechnology, 2*, pp. 49-56 (2012).

[17] Tong, C., “Self-validated variance-based methods for sensitivity analysis of model outputs”, *Reliability Engineering & System Safety, 95*, pp. 301-309 (2010).
[18] Nossent, J., Elsen, P. and Bauwens, W., “Sobol’s sensitivity analysis of a complex environmental model”, *Environmental Modelling & Software*, 26, pp. 1515-1525 (2011).

[19] Sobol, I.M., “Sensitivity estimates for nonlinear mathematical models”, *Mathematical Modeling and Computational Experiments*, 14, pp. 407-414 (1993).

[20] Cukier, R., Levine, H. and Shuler, K., “Nonlinear sensitivity analysis of multiparameter model systems”, *Journal of computational physics*, 26, pp. 1-42 (1978).

[21] Saltelli, A., Tarantola, S. and Chan, K.-S., “A quantitative model-independent method for global sensitivity analysis of model output”, *Technometrics*, 41, pp. 39-56 (1999).

[22] Homma, T. and Saltelli, A., “Importance measures in global sensitivity analysis of nonlinear models”, *Reliability Engineering & System Safety*, 52, pp. 1-17 (1996).
Fig. 1: Blade camber line parameterization using simple Bezier method.
Fig. 2: Defining three sections on centrifugal pumps blade.
Fig. 3: a sample of CFD structured grid generation for centrifugal pumps.
Fig. 4: a sample of pressure contour in CFD simulations of centrifugal pumps.
Fig. 5: Algorithm of sensitivity analysis.
Fig. 6: The changes of $\eta$ with: (a) $\beta_{1\text{Hub}}$, (b) $\beta_2$, (c) $\beta_{1\text{Shroud}}$ and (d) $\gamma_{\text{mid}}$ in SA analysis.
Fig. 7: The changes of NPSH with: (a) $\beta_{1_{\text{Hub}}}$, (b) $\beta_2$, (c) $\beta_{1_{\text{Shroud}}}$ and (d) $\gamma_{\text{mid}}$ in SA analysis.
Fig. 8: Percent sensitivity of input parameter changes in the $\eta$ and NPSH.
Table 1: Design variables and their range of variations

| Design Variable | From (deg) | To (deg) |
|-----------------|------------|----------|
| $\beta_{1\text{Hub}}$ | 0 | 30 |
| $\beta_{1\text{Shroud}}$ | 60 | 89 |
| $\beta_2$ | 40 | 60 |
| $\gamma_{\text{mid span}}$ | 30 | 70 |
Table 2: The operating conditions in the simulations

| Parameter                  | Value          |
|----------------------------|----------------|
| Number of blades           | 7              |
| Rotational velocity (rpm)  | 2900           |
| Mass Flow (kg/s)           | 24.7 (BEP)     |
| Outlet static pressure (atm) | 3.2           |
**Brief technical biography of Authors:**

**Hamed Safikhani** is an assistant professor of mechanical engineering at the Arak University, I. R. Iran. He received his Ph.D. from the Amirkabir University of Technology in 2014. He is one of the members of “The Promised SORAYYA Technologist” science-based industries in I. R. Iran. He has co-authored more than 40 high quality journal and conference publications. His research interests include air pollution numerical modeling, HVAC systems, energy, two-phase and single-phase convective heat transfer in macro-micro and nanoscales.