SO(3) monopoles, vortices and confinement in SU(2) gauge theory

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We report on further progress in our programme of understanding confinement in 3d and 4d SU(2) gauge theory in terms of Z(2) monopoles. A sufficient condition for confinement was previously translated into Z(2) monopole correlation inequalities in a related SO(3) gauge theory. We shall discuss the physical picture underlying this scenario and present some Monte Carlo evidence concerning the monopole correlation inequalities.

It is an old idea that confinement in Z(N) gauge theories can be understood in terms of Z(N) vortices (also called fluxons) linking with the Wilson loops (see e.g. [1]). In three dimensions a Z(N) vortex is a closed stack of plaquettes (forming a loop) with a nontrivial Z(N) element on them. These are excitations very much analogous to Peierls contours in the 2d Ising model.

This vortex condensation mechanism was generalised to SU(N) gauge theories by Mack and Petkova [2]. Here the magnetic vortices also belong to Z(N), the centre of SU(N) but there is a major difference as compared to the Abelian case. While the Abelian magnetic fluxon necessarily has a thickness of one lattice spacing, in SU(N) lattice gauge theory, due to the continuous nature of the group, it can be spread out over several lattice spacings. In fact, it is exactly the spreading of the vortices that makes it possible for them to survive at weak coupling. It was emphasized by several authors that the sufficiently fast (exponential) spreading of the magnetic flux is an essential feature of any confining SU(N) gauge theory [2]. The general picture emerging from these investigations was that it is probably the thick, spread out vortices interlocking with the Wilson loop that are responsible for confinement at weak coupling. Unfortunately it is extremely hard to trace the dynamics of these fluxons at different length scales and although they provide a nice intuitive picture, not much progress was made in obtaining a quantitative understanding of the mechanism.

For the past few years there has been a programme to achieve this goal and rigorously demonstrate that confinement persists down to arbitrarily small couplings in SU(2) gauge theory [4]. In the following we shall summarise the main ideas underlying this programme. Here we can present only a rough intuitive picture and for more technical details the reader is referred to [4] and especially [8]. The latter is the most detailed presentation so far, although in the context of the analogous 2d SU(2) principal chiral model.

In the original formulation of Mack and Petkova, thick vortices were detected by introducing a vortex container, special boundary conditions along a thick torus linking with the Wilson loop, to trap the vortex inside the torus. Instead of this we essentially use the whole lattice as a vortex container. The Wilson loop confinement criterion can be translated into an exponentially fast spreading of a vortex winding around the cubic lattice, as a function of the finite lattice size [5]. In order to detect thick vortices winding around the lattice we use an equivalent representation of the SU(2) gauge theory in terms of a dual Ising model (Z(2) gauge theory in the 4d case) interacting with an SU(2)/Z(2) gauge theory. Since the action of the SU(2)/Z(2) system is insensitive to the Z(2) centre, a thin Z(2) vortex (i.e. a stack of plaquettes) winding around the lattice does not cost an energy proportional to its length, however due to the periodic boundary conditions it signals the presence of a thick vortex winding around the lattice. In this way in order to trace thick vortices we have to detect only thin

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1Throughout this paper we shall consider gauge theories in three dimensions but our arguments can be easily adopted to the physically more relevant 4d case.
ones.

Contrary to $\mathbb{Z}(N)$ gauge theories, $\text{SU}(N)$ gauge theories admit $\mathbb{Z}(N)$ monopoles, elementary lattice cubes that have a net $\mathbb{Z}(N)$ flux flowing out of them. (In the $\mathbb{Z}(N)$ case these are absent due to the Bianchi identity.) These monopoles are gauge invariant objects, not like the $\text{U}(1)$ monopoles that appear in this model only after a partial gauge fixing. For a Monte Carlo study involving both types of monopoles, see [6].

The presence of $\mathbb{Z}(2)$ monopoles means that in our $\text{SU}(2)$ model $\mathbb{Z}(2)$ vortices can either be closed or bounded by a pair of monopoles (a monopole loop in 4d). A vortex going all the way around the lattice is globally quite similar to the one which is bounded by a nearby pair of monopoles; the difference between them is only a small local excitation. In the $\mathbb{Z}(2) \times \text{SU}(2)/\mathbb{Z}(2)$ formulation the monopoles of the $\text{SU}(2)/\mathbb{Z}(2)$ gauge theory turn out to couple to the dual Ising model and provide an external magnetic field for the Ising spins. If they can produce a nonzero effective magnetic field for the dual Ising model than the Ising spins are ordered which in turn means disorder i.e. confinement in the original gauge theory. The presence of a nonzero effective external field for the dual Ising model can be rigorously established provided that two types of monopole correlation inequalities are satisfied.

At this point we want to emphasize that the presence of $\mathbb{Z}(2)$ monopoles is not necessary for confinement. It is of course not small monopoles but large spread out vortices that are really responsible for confinement. However, as we have already seen, some of the large vortices naturally occur “tagged” with a pair of monopoles, i.e. they are not closed but have a small gap instead. It is exactly these tagged vortices that couple to the dual $\mathbb{Z}(2)$ system and make it possible to establish confinement in terms of the $\mathbb{Z}(2)$ variables. In other words, if we were to constrain out $\mathbb{Z}(2)$ monopoles, confinement would not be lost but we would not be able to see it in terms of the $\mathbb{Z}(2)$ variables.

Now returning to the monopole inequalities, the first of these is a factorisation inequality pertaining to monopole pairs (loops in 4d), this was discussed to some length in [6]. The second inequality that we would need to prove is that the probability of having a nearby monopole pair with their flux winding around the lattice, is greater than some finite non-zero number. In other words, the free energy of the above configuration is a bounded function of the lattice size. A simple semiclassical estimate supports this assumption in 3 and 4 dimensions (but not above 4). In the remainder of the present paper we briefly present the results of a Monte Carlo measurement of this quantity as a function of the lattice size.

We could do the measurement only in the 3d case since in 4d due to the long lifetime of the large vortices we could not obtain enough statistics. We measured the probability of the configuration with two fixed adjacent monopoles, their flux winding around the lattice; normalised by the probability of having no monopoles at all and no vortex going around the lattice in the given direction. The measurement was done by generating a series of configurations using a local heat bath algorithm with the Villain form of the $\text{SU}(2)/\mathbb{Z}(2)$ action

$$S[U, \sigma] = \beta \sum_p \sigma_p \text{Tr} U_p,$$

(1)

where the $\sigma_p$’s are $\mathbb{Z}(2)$ valued plaquette variables that are summed over in the partition function to ensure the desired $\mathbb{Z}(2)$ invariance. We had to ensure that the $\beta$ we chose was already in the weak coupling region since we were interested in the weak coupling behaviour of the quantity measured. Unfortunately in 3d the crossover between weak and strong coupling is not so sharp as it is in 4d and in 3d the specific heat peak at the crossover is missing. Therefore we used the exponential falling of the monopole density as a criterion for weak coupling. We chose $\beta = 11.5$, as can be seen in Figure [6] this is already well in the exponential regime of the monopole density.

Since (at a given $\beta$) on larger lattices there are typically more monopoles, the probability of the configurations with exactly one an zero monopole pairs decreased very rapidly with increasing lattice size. This meant that the quantity we measured was given as a ratio of two numbers, both becoming very small on larger lattices, however their ratio was expected to be fairly sta-
Figure 1. The $\mathbb{Z}(2)$ monopole density as a function of $\beta$ on a $8^3$ lattice.

Figure 2. The probability of two adjacent monopoles with their connecting fluxon going around the lattice divided by the probability of having no monopoles at all and no fluxon going around the lattice in the given direction.

This made the signal less accurate on larger lattices and eventually prevented us from going beyond a lattice size of $10^3$. Even at this point we typically needed several thousand configurations to get a signal at all. Our results are summarised in Figure 2 which shows the measured probability as a function of the lattice size. As we expect, the probability of the two-monopole configuration with the long fluxon does not decrease with the lattice size. This is consistent with the semiclassical estimate and makes it quite improbable that it can go to zero as $L \to \infty$.

If confinement is present we expect an exponential spreading of the flux, i.e. its free energy to approach zero exponentially on sufficiently large lattices. In our framework we would need to prove only the substantially weaker condition that the flux free energy does not diverge with the lattice size. It is remarkable that even this weaker property is absolutely nontrivial to prove rigorously and we had to resort to Monte Carlo.

Finally we would like to point out that each step in the programme outlined here has been given precise formulation and to complete it we would need to prove the correlation inequalities discussed above.

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