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Decrease in Accuracy of a Rotational SINS Caused by its Rotary Table's Errors

Regular Paper

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Abstract We call a strapdown inertial navigation system (SINS) that uses the rotation auto-compensation technique (which is a common method to reduce the effect of the bias errors of inertial components) a ‘rotational SINS’. In a rotational SINS, the rotary table is an important component, rotating the inertial sensor assembly back and forth in azimuth to accomplish error modulation. As a consequence of the manufacturing process, errors may exist in rotary tables which decrease the navigation accuracy of rotational SINSs. In this study, the errors of rotary tables are considered in terms of installation error, wobble error and angular error, and the models of these errors are established for the rotational SINS. Next, the propagation characteristics of these errors in the rotational SINSs are analysed and their effects on navigation results are discussed. Finally, the theoretical conclusions are tested by numerical simulation. This paper supplies a good reference for the development of low-cost rotational SINSs, which usually have low accuracy rotary tables and which may be used in robots, intelligent vehicles and unmanned aerial vehicles (UAVs).

Keywords Strapdown Inertial Navigation System (SINS), Rotation Technique, Rotary Table Error, Robot Navigation

1. Introduction

Navigation systems supply the necessary information for the control systems of robots, vehicles and UAVs [1]. SINS is one of the most commonly-used navigation systems, since it can output comprehensive navigation information, such as attitude, velocity and position [2]. Besides, SINS is an autonomous navigation system, which means it is not influenced by external environmental interference, including natural factors and human factors [3].

The inertial measurement unit (IMU), consisting of gyros and accelerometers, is the main component of a SINS [4]. There are several types of gyros and accelerometers, such as liquid-floated gyros, optical gyros, micro-electromechanical system (MEMS) gyros, flexure accelerometers, MEMS accelerometers, and so on. In robots’ and small UAVs’ navigation systems, MEMS gyros and MEMS accelerometers are most often used [5], mainly due to size and cost factors. However, MEMS gyros usually have low accuracy [6], which leads to large navigation errors.

The rotation auto-compensation technique can efficiently reduce the bias errors of inertial components so as to improve navigation accuracy [7]. The technique has been
widely used in SINSs with optical gyros, which are usually equipped for ships and can supply accurate navigation information for several days, such as AN/WSN-7B [8] and MK49 [9]. Nowadays, the rotational SINSs with MEMS gyros has not been produced yet, but the rotation auto-compensation technique has been proposed for use in MEMS IMUs and has significantly improved the accuracy of inertial components [10]. In addition, rotational north-finding systems with MEMS gyros have been studied and good experimental results have been seen [11-12]. So, it can be deduced that the rotation auto-compensation technique can greatly improve the navigation accuracy of MEMS SINS and that rotational MEMS SINSs show significant potential for application. In this study, some pre-work was conducted for the development of rotational MEMS SINSs.

In a rotational SINS, the IMU is mounted on a rotary table [13]. The rotary table rotates the IMU back and forth to modulate the bias errors of the gyros and accelerometers. In rotational SINSs with optical gyros, the rotary tables are highly accurate but are large and costly. If these rotary tables are used in rotational MEMS SINSs, the MEMS-advantages of size and cost will be greatly diminished. Therefore, a kind of small-sized, low-cost rotary tables should be used in rotational MEMS SINSs. However, this kind of rotary tables have low accuracy.

The rotary table is an important component of the rotational SINS, and its errors affect navigation accuracy. Especially for low-accuracy rotary tables, a large decrease in navigation accuracy may be brought about by the rotary tables’ errors. Currently, most of the researches on the rotational SINSs are about modulation theory [14-15], rotation scheme [16] and initial alignment [17], but the research on the effect of the rotary table’s errors has not been found.

In this study, the errors of the rotary table in a rotational SINS are classified and modelled while its error propagation characteristics are analysed. Detailed navigation error formulae are derived which supply a good reference for the selection of rotary tables for low-cost MEMS rotational SINSs. Firstly, in Section 2, the basic principle behind the rotation auto-compensation technique can be simply explained as follows: the errors of the three gyros are denoted by $\epsilon_x$, $\epsilon_y$, and $\epsilon_z$; the rotation rate of the rotary table is denoted by $\omega_t$; and then the projection of $\epsilon'$ on the n-frame can be expressed by:

$$\epsilon'' = C_n^b C_b^r \epsilon'$$

$$= C_n^b \begin{bmatrix} \cos \omega_t t & -\sin \omega_t t & 0 \\ \sin \omega_t t & \cos \omega_t t & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \epsilon_x \\ \epsilon_y \\ \epsilon_z \end{bmatrix}$$

$$= C_n^b \begin{bmatrix} \epsilon_x \cos \omega_t t - \epsilon_y \sin \omega_t t \\ \epsilon_x \sin \omega_t t + \epsilon_y \cos \omega_t t \\ \epsilon_z \end{bmatrix}$$

In this paper, the coordinate systems are defined as follows: the inertial coordinate system is the reference coordinate for the inertial components, denoted by $i$; the navigation coordinate system is chosen as the local east-north-up (ENU) coordinate, denoted by $n$; the body coordinate system is vehicle-carried and denoted by $b$; the rotary table coordinate system is fixed with the rotary table and denoted by $r$. Take the gyros’ errors for example, the principle behind the rotation auto-compensation technique can be simply explained as follows: the errors of the three gyros are denoted by $\epsilon'$; the rotation rate of the rotary table is denoted by $\omega_t$; and then the projection of $\epsilon'$ on the n-frame can be expressed by:

$$\epsilon'' = C_n^b C_b^r \epsilon'$$

$$= C_n^b \begin{bmatrix} \cos \omega_t t & -\sin \omega_t t & 0 \\ \sin \omega_t t & \cos \omega_t t & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \epsilon_x \\ \epsilon_y \\ \epsilon_z \end{bmatrix}$$

$$= C_n^b \begin{bmatrix} \epsilon_x \cos \omega_t t - \epsilon_y \sin \omega_t t \\ \epsilon_x \sin \omega_t t + \epsilon_y \cos \omega_t t \\ \epsilon_z \end{bmatrix}$$
where \( C^n_r \) is the transition matrix from the b-frame to the n-frame and \( C^b_r \) is the transition matrix from the r-frame to the b-frame. During one rotation period, if the vehicle's attitude stays the same, then \( C^n_r \) is a constant matrix and the errors of the two horizontal gyros are modulated to sine functions. The bias errors are constant values, so, according to Eq. (1), their accumulations are zero after one period and will not affect the navigation accuracy. Similarly for the gyros, the two horizontal accelerometers are also modulated.

3. Error modelling of the rotary table

For the rotary table in a rotational SINS, there are three main types of errors: installation error, wobble error and angular error. In this section, the physical causes of these errors are analysed and then models are established.

3.1 Modelling of the installation error

The vertical axis of the rotary table should be parallel to the vertical axis of the body, which means that the z-axis of the r-frame should coincide with the z-axis of the b-frame. However, there exists an error angle due to the installation technology, which is called the 'installation error'. Assuming that the r-frame is the coordinate system coinciding with the initial r-frame, then the installation error can be expressed through the transition matrix from the b-frame to the r-frame, which can be written as:

\[
C^r_b = \begin{bmatrix}
\cos \alpha & \sin \alpha & 0 \\
-\sin \alpha & \cos \alpha & 0 \\
0 & 0 & 1
\end{bmatrix} \tag{2}
\]

The installation error can be expressed through the transition matrix from the b-frame to the r-frame, which consists of two error-angle parameters, denoted by \( \alpha \), \( \beta \):

\[
C^b_r = \begin{bmatrix}
\cos \alpha & 0 & -\sin \alpha \\
0 & 1 & 0 \\
\sin \alpha & 0 & \cos \beta \end{bmatrix} \begin{bmatrix}
1 & 0 & 0 \\
0 & \cos \beta & \sin \beta \\
0 & -\sin \beta & \cos \beta
\end{bmatrix} \tag{3}
\]

\[
C^b_r = \begin{bmatrix}
\cos \alpha & \sin \alpha & -\cos \beta \\
0 & \cos \beta & \sin \beta \\
-\sin \alpha & -\cos \beta & \cos \alpha
\end{bmatrix}
\]

Assuming \( \alpha \), \( \beta \) are both small quantities, then Eq. (3) can be approximated as:

\[
C^b_r \approx \begin{bmatrix}
1 & 0 & -\alpha \\
0 & 1 & \beta \\
\alpha & -\beta & 1
\end{bmatrix} \tag{4}
\]

Considering the installation error of the rotary table, the gyros' outputs can be expressed by:

\[
\omega_{ir}' = C^r_b C^n_r \omega_{ir}^b = \begin{bmatrix}
\cos \alpha & \sin \alpha \omega_t & -\alpha \cos \omega_t + \beta \sin \omega_t \\
-\sin \alpha & \cos \omega_t & \alpha \sin \omega_t + \beta \cos \omega_t \\
0 & 0 & 1
\end{bmatrix} \omega_{ir}^b \tag{5}
\]

The accelerometers' outputs can be expressed by:

\[
f_{ir}' = C^r_b f_{ir}^b = \begin{bmatrix}
\cos \alpha \omega_t & \sin \alpha \omega_t & -\alpha \cos \omega_t + \beta \sin \omega_t \\
-\sin \alpha \omega_t & \cos \omega_t & \alpha \sin \omega_t + \beta \cos \omega_t \\
0 & 0 & 1
\end{bmatrix} f_{ir}^b \tag{6}
\]

Usually, before using a rotational SINS, some calibration operations will be performed to compensate for the errors in the system. Eq. (5) and Eq. (6) supply the reference for designing the calibration experiment for the installation error of the rotary table.

3.2 Modelling of the wobble error

The vertical axis of the table board \( r_z \) should coincide with the rotation axis of the rotary table \( r_{0z} \), actually there exists an error angle between the two axes, denoted by \( \theta \). Assuming that the rotation rate of the rotary table is \( \omega_z \), then the table board rotates at a speed of \( \omega_z \) around its vertical axis. Beyond that, the misalignment between the two axes makes \( r_z \) rotate around \( r_{0z} \) at the speed of \( \omega_z \), which can be seen in Figure 3. Assuming \( P \) is a point at the edge of the table board and does not rotate, from Figure 3 it is clear that \( P \) jumps up and down when the rotary table rotates, so the table board wobbles and the error is called the 'wobble error'. When \( r_z \) rotates 180 degrees to \( r_z' \), the P point jumps to \( P' \) and has the maximum amplitude \( |PP'| \), which is usually used to measure the wobble error. Assuming that \( \theta \) is a small
angle, the diameter of the table board is denoted by \( d \) and then \( \theta \) can be expressed by:

\[ \theta = \frac{d}{l p p'} \]  

(7)

The wobble error changes the rotation rate between the r-frame and the b-frame, which can be decomposed into two types of motion [19]: the r-frame does the coning motion relative to the b-frame caused by the wobble error, whereby the coning rate is \( \omega_c \) and the half-cone angle is \( \theta \); the r-frame rotates at the speed of \( \omega_c \) around \( \mathbf{r}_z \), caused by the rotation of the rotary table. Using the quaternion \( \mathbf{Q}_r(t) \) to express the coning motion, it can be written as:

\[
\mathbf{Q}_r(t) = \begin{bmatrix}
\cos(\theta/2)
\\
\sin(\theta/2) \cos(\omega_c t + \eta)
\\
\sin(\theta/2) \sin(\omega_c t + \eta)
\\
0
\end{bmatrix}
\]  

(8)

where \( \eta \) is a coning parameter describing the relation between the r-frame and the b-frame at the initial time. Using the quaternion \( \mathbf{Q}_b(t) \) to express the rotation, it can be written as:

\[
\mathbf{Q}_b(t) = \begin{bmatrix}
\cos(\omega_c t/2)
\\
-\sin(\omega_c t/2) \sin(\omega_c t + \eta)
\\
\sin(\omega_c t/2) \sin(\omega_c t + \eta)
\\
\sin(\omega_c t/2) \cos(\theta)
\end{bmatrix}
\]  

(9)

Then, the r-frame to the b-frame quaternion is:

\[
\mathbf{Q}(t) = \mathbf{Q}_r(t) \mathbf{Q}_b(t)
\]

(10)

According to the quaternion differential equation [20]:

\[
\dot{\mathbf{Q}}(t) = \frac{1}{2} \mathbf{Q}(t) \otimes \mathbf{\omega}_e
\]  

(11)

where \( \otimes \) is the quaternion multiplication symbol, \( \dot{\mathbf{Q}}(t) \) is the differentiation of \( \mathbf{Q}(t) \) and \( \mathbf{\omega}_e \) is the angular rate of the r-frame relative to the b-frame projected on the r-frame. It can be got as:

\[
\mathbf{\omega}_e = 2 \mathbf{Q}^* (t) \mathbf{Q}(t)
\]

(12)

where \( \mathbf{Q}^* \) is the dual quaternion of \( \mathbf{Q}(t) \). If the wobble error is zero, \( \mathbf{\omega}_e = [0 \ 0 \ \omega_c] \), which is equal to the rotation rate.

According to the relation between the quaternion and the transition matrix, \( \mathbf{C}^b_r \) can be written as:

\[
\mathbf{C}^b_r = \begin{bmatrix}
B_{11} & B_{12} & B_{13} \\
B_{21} & B_{22} & B_{23} \\
B_{31} & B_{32} & B_{33}
\end{bmatrix}
\]  

(13)

where:

\[
\begin{align*}
B_{11} &= \cos(3\omega_c t + 2\eta) - \cos(\omega_c t)\cos(2\eta) \\
B_{12} &= \sin(3\omega_c t + 2\eta) - \sin(\omega_c t)\cos(2\eta) \\
B_{13} &= \sin(2\omega_c t + \eta) + \theta \\
B_{21} &= \sin(3\omega_c t + 2\eta) + \sin(\omega_c t)\cos(2\eta) \\
B_{22} &= \cos(3\omega_c t + 2\eta) + \cos(\omega_c t)\cos(2\eta) \\
B_{23} &= -\cos(2\omega_c t + \eta) + \theta \\
B_{31} &= -\sin(\omega_c t + \eta)\sin\theta \\
B_{32} &= \cos(\omega_c t + \eta)\sin\theta \\
B_{33} &= \cos\theta
\end{align*}
\]

(14)

Considering the wobble error of the rotary table, the gyro's outputs can be expressed by:

\[
\mathbf{\omega}_e^g = \mathbf{C}^b_r \mathbf{\omega}_e^r
\]

(15)

The accelerometers' outputs can be expressed by:

\[
\mathbf{f}_e^a = \mathbf{C}^b_r \mathbf{f}_e^a
\]

(16)
Eq. (15) and Eq. (16) supply the reference for designing the calibration experiment for the wobble error of the rotary table.

3.3 Modelling of the angular error

From Figure 2, it can be seen that after solving the attitude of the IMU, the rotation angle of the rotary table is needed to get the attitude of the vehicle. However, there exists an error in the rotation angle, which is called the ‘angular error’ and mainly comes from two aspects: one concerns the errors from the angular measuring device of the rotary table; the other is caused by the desynchronizing between the rotary table and the SINS, which means that the angle value transmitted to the SINS does not match the values of the inertial components. Usually, the maximum angular measuring error is given by the introduction of the rotary table and the device of the rotary table; the other is caused by the installation error. Then, the angular error of the rotary table is expressed by a constant value; accordingly, the angular error of the rotary table is derived as:

$$\delta \phi = C_{0}^{b} \bar{b}_{r}^{b}$$

(17)

Considering the angular error of the rotary table, the gyros’ outputs can be expressed by:

$$\omega_{ir}^{f} = C_{b}^{b'} C_{r}^{b'} C_{b}^{f} \omega_{ir}^{b}$$

(18)

The accelerometers’ outputs can be expressed by:

$$f_{ir}^{f} = C_{b}^{b'} C_{r}^{b'} C_{b}^{f} f_{ir}^{b}$$

(19)

Eq. (18) and Eq. (19) supply the reference for designing the calibration experiment for the angular error of the rotary table.

4. The effect of the rotary table’s errors on navigation accuracy

In this section, the effects of the rotary table’s errors on position accuracy and attitude accuracy will be analysed and formulae will be derived.

4.1 The effect of the installation error

The installation error changes the relative attitude between the IMU and the vehicle and has no effect on the IMU position-solving. From Figure 2, it can be seen that the rotational SINSs’s position is equal to the IMU’s position, so the installation error does not affect position accuracy.

From Figure 2, it can be seen that during the attitude solution process, the transition matrix from the b-frame to the r-frame is needed to get the vehicle attitude from the IMU attitude. But, according to the analysis of Section 3.1, the transition matrix is affected by the installation error, and so the attitude accuracy is also affected by the installation error.

The attitude of the vehicle is denoted as follows: the pitch angle is $\tau$, the roll angle is $\gamma$, and the heading angle is $\psi$. Then, the transition matrix from the b-frame to the n-frame can be expressed by [21]:

$$C_{b}^{n} = \begin{bmatrix}
C_{11} & C_{12} & C_{13} \\
C_{21} & C_{22} & C_{23} \\
C_{31} & C_{32} & C_{33}
\end{bmatrix}$$

(20)

where:

$$C_{11} = \cos \gamma \cos \psi + \sin \gamma \sin \psi \sin \tau$$

$$C_{12} = \sin \gamma \cos \psi$$

$$C_{13} = -\cos \gamma \sin \psi$$

$$C_{21} = -\cos \gamma \sin \psi \cos \tau$$

$$C_{23} = \cos \gamma \cos \psi$$

$$C_{31} = -\sin \gamma$$

$$C_{32} = \cos \gamma$$

$$C_{33} = \cos \gamma$$

Due to the installation error, the calculated $\hat{C}_{b}^{n}$ can be derived as:

$$\hat{C}_{b}^{n} = C_{b}^{a} C_{r}^{f}$$

$$= C_{b}^{a} C_{b}^{b'} C_{r}^{b'} C_{b}^{f}$$

(21)

$$= C_{b}^{a} C_{b}^{b'} C_{r}^{b'} C_{b}^{f}$$

(22)
where the \( r' \)-frame is the theoretical \( r \)-frame with no installation error. Let \( \Delta \tau \), \( \Delta \gamma \) and \( \Delta \psi \) stand for the pitch angle error, the roll angle error and the heading angle error respectively. Then, according to the relation between \( C^b_n \) and the attitude, we have:

\[
\sin(\tau + \Delta \tau) = C_{32} + \beta C_{33} \\
\tan(\gamma + \Delta \gamma) = (-C_{31} - \alpha C_{33}) / (\alpha C_{31} - \beta C_{32} + C_{33}) \tag{23}
\]

\[
\tan(\psi + \Delta \psi) = (C_{12} + \beta C_{13}) / (C_{22} + \beta C_{23})
\]

Assuming that \( \Delta \tau \), \( \Delta \gamma \), \( \Delta \psi \), \( \alpha \) and \( \beta \) are all small quantities, then the attitude errors can be deduced from Eq. (23), such that:

\[
\Delta \tau = \beta \cos \gamma \\
\Delta \gamma = \alpha + \beta \sin \gamma \tan \tau \tag{24}
\]

\[
\Delta \psi = \beta \sin \gamma / \cos \tau
\]

It can be seen from Eq. (24) that the attitude errors are related to the vehicle's attitude. When \( \tau - \gamma - \psi = 0 \), it can be calculated that \( \Delta \tau = \beta, \Delta \gamma = \alpha, \Delta \psi = 0 \), which is equal to the installation error. However, when the pitch angle approaches 90 degrees, Eq. (24) is no longer applicable. This is because the attitude of SINS degenerates to two degrees of freedom and it is the attitude which needs to be redefined rather than Eq. (20). The same problem occurs in the following analysis (and the explanation will not be repeated).

4.2 The effect of the wobble error

As with the installation error, a new velocity/position error is not introduced by the wobble error. However, it can be seen from Eq. (13) that the transition matrix from the \( r \)-frame to the \( b \)-frame is affected by the wobble error, so the attitude accuracy is affected. Similar to the derivation in Section 4.1, the calculated \( \tilde{C}_b^n \) can be derived as:

\[
\tilde{C}_b^n = C^n_b C'^n_b = C^n_b C'^n_b \\
= \begin{bmatrix}
C_{11} & C_{12} & C_{13} \\
C_{21} & C_{22} & C_{23} \\
C_{31} & C_{32} & C_{33}
\end{bmatrix}
\begin{bmatrix}
B_{11} & B_{12} & B_{13} \\
B_{21} & B_{22} & B_{23} \\
B_{31} & B_{32} & B_{33}
\end{bmatrix}
\begin{bmatrix}
\cos \omega t & \sin \omega t & 0 \\
-sin \omega t & \cos \omega t & 0 \\
0 & 0 & 1
\end{bmatrix}
\]

(25)

where:

\[
D_{11} = C_{1,1} \cos^2 \theta + C_{1,1} \sin \omega t + \eta \sin \theta \\
D_{12} = C_{1,1} \cos^2 \theta + C_{1,1} \sin \omega t + \eta \sin \theta \\
D_{13} = C_{1,1} \sin \omega t + \eta \sin \theta - C_{1,2} \cos \omega t + \eta \sin \theta + C_{1,3} \cos \theta \\
D_{21} = C_{2,1} \cos^2 \theta + C_{2,1} \sin \omega t + \eta \sin \theta \\
D_{22} = C_{2,1} \cos^2 \theta + C_{2,1} \sin \omega t + \eta \sin \theta \\
D_{23} = C_{2,1} \sin \omega t + \eta \sin \theta - C_{2,2} \cos \omega t + \eta \sin \theta + C_{2,3} \cos \theta \\
D_{31} = C_{3,1} \cos^2 \theta + C_{3,1} \sin \omega t + \eta \sin \theta \\
D_{32} = C_{3,1} \cos^2 \theta + C_{3,1} \sin \omega t + \eta \sin \theta \\
D_{33} = C_{3,1} \sin \omega t + \eta \sin \theta - C_{3,2} \cos \omega t + \eta \sin \theta + C_{3,3} \cos \theta
\]

(26)

According to the relation between \( C^b_n \) and the attitude, the following equations can be derived:

\[
\sin(\tau + \Delta \tau) = C_{3,1} \cos^2 \theta + C_{3,1} \sin \omega t + \eta \sin \theta \\
\tan(\gamma + \Delta \gamma) = -C_{3,1} \cos^2 \theta + C_{3,1} \sin \omega t + \eta \sin \theta - C_{3,2} \cos \omega t + \eta \sin \theta + C_{3,3} \cos \theta \\
\tan(\psi + \Delta \psi) = C_{3,2} \cos^2 \theta + C_{3,2} \sin \omega t + \eta \sin \theta \\
\tan(\psi + \Delta \psi) = C_{3,3} \cos^2 \theta + C_{3,3} \sin \omega t + \eta \sin \theta \\
\tan(\psi + \Delta \psi) = C_{3,3} \cos^2 \theta + C_{3,3} \sin \omega t + \eta \sin \theta \\
\tan(\psi + \Delta \psi) = C_{3,3} \cos^2 \theta + C_{3,3} \sin \omega t + \eta \sin \theta + C_{3,3} \cos \theta
\]

(27)

Assuming that \( \Delta \tau \), \( \Delta \gamma \), \( \Delta \psi \) and \( \eta \) are all small quantities, then the attitude errors can be deduced from Eq. (27), such that:

\[
\Delta \tau = \theta \cos \cos \omega t + \eta \\
\Delta \gamma = \theta \sin \sin \omega t + \eta \tan \cos \omega t + \eta \\
\Delta \psi = \theta \cos \omega t + \eta \sin \gamma / \cos \tau
\]

(28)

It can be seen from Eq. (28) that the attitude errors brought about by the wobble error are related to the attitude angle, the half-cone angle \( \eta \) and the rotation angle \( \omega t \).

4.3 The effect of the angular error

It can be seen from Figure 2 that the rotation angle has nothing to do with the position solution, so its error does not affect the position accuracy. However, the rotation angle is needed in the attitude calculation process, so its error affects the attitude accuracy. Similarly with the derivations in Section 4.1 and 4.2, the calculated \( \tilde{C}_b^n \) can be derived as:

\[
\tilde{C}_b^n = C^n_b C'^n_b = C^n_b C'^n_b \\
= \begin{bmatrix}
D_{11} & D_{12} & D_{13} \\
D_{21} & D_{22} & D_{23} \\
D_{31} & D_{32} & D_{33}
\end{bmatrix}
\begin{bmatrix}
\cos \omega t & \sin \omega t & 0 \\
-sin \omega t & \cos \omega t & 0 \\
0 & 0 & 1
\end{bmatrix}
\]

(25)
According to the relation between $\mathbf{C}^b_8$ and the attitude, the following equations can be derived:

\[
\sin(\tau + \Delta \tau) = -C_{31} \sin \delta \phi + C_{32} \cos \delta \phi
\]
\[
\tan(\gamma + \Delta \gamma) = \frac{-(C_{31} \cos \delta \phi + C_{32} \sin \delta \phi)}{C_{33}}
\]
\[
\tan(\psi + \Delta \psi) = \frac{-C_{11} \sin \delta \phi + C_{12} \cos \delta \phi}{-C_{21} \sin \delta \phi + C_{22} \cos \delta \phi}
\]

Assuming that $\Delta \tau$, $\Delta \gamma$, $\Delta \psi$ and $\delta \phi$ are all small quantities, then the attitude errors can be deduced from Eq. (30):

\[
\Delta \tau = \delta \phi \sin \gamma
\]
\[
\Delta \gamma = -\delta \phi \cos \gamma \tan \tau
\]
\[
\Delta \psi = -\delta \phi \cos \gamma / \cos \tau
\]

It can be seen from Eq. (31) that the attitude errors are related to the attitude angle and the angular error. When $\tau - \gamma = 0$, it can be calculated that $\Delta \tau = \Delta \gamma = 0$, $\Delta \psi = -\delta \phi$, which is equal to the error angle.

5. Simulation results and discussion

From the above analysis, it can be concluded that the rotary table's errors have no effect on position accuracy and that only the installation error, the wobble error and the angular error affect attitude accuracy. In this section, the effects of the rotary table's errors to attitude will be simulated and the simulation results will be compared with the theoretical ones. In order to verify the derived formulas, the simulation conditions are set as follows:

1) The rotary table rotates the IMU back and forth in azimuth through four positions (0 deg., 90 deg., 180 deg., 270 deg.). The dwell time for each position is 290 seconds and the rotation rate is 10 deg./s, so a complete rotation period is 20 minutes.

2) In order to directly reflect the effect of the rotary table’s errors, the other errors (such as the inertial components' errors and the initial alignment error) are not considered in the simulation.

3) From Eqs. (24), (28) and (31), it can be seen that the attitude errors are related to the vehicle's attitude. So, in the simulation, the vehicle's attitude varies as shown in Figure 4: the roll angle increases to 30 degrees at a speed of 6 deg./s at 500 seconds, the pitch angle increases to 30 degrees at a speed of 6 deg./s at 1100 seconds, the pitch angle decreases to 0 degrees at a speed of -6 deg./s at 1700 seconds, and the roll angle decreases to 0 degrees at a speed of -6 deg./s at 2300 seconds. The total simulation time is 3600 seconds and the velocity of the vehicle is maintained at zero throughout the whole simulation.

4) Both the rotational SINS with a perfect rotary table and the one with an imperfect rotary table are simulated. The attitude error response to the rotary table's errors is acquired by comparing the attitude results of the two systems. Next, the predicted errors calculated by the error model derived in the paper are compared with the simulated ones so as to verify the correctness of the theory.

According to the analysis in Section 3.1, the installation error can be expressed by two parameters $\alpha$, $\beta$. In the simulation, the two error angles are both set at 5 minutes of angle, which corresponds to low installation accuracy. The attitude errors of the rotational SINS are shown in Figure 5 and the specific results of some special time points are referred to in Table 1, Table 2 and Table 3. It is clear that the calculated values agree well with the simulated values and that the attitude errors reach 0.1 degrees under the simulation conditions.
The attitude errors brought about by the installation error

| Simulation Time (Seconds) | Roll Angle Error (deg.) |
|--------------------------|-------------------------|
| 0 (Initial time)         | 0.083                   |
| 1105 (Pitch angle 30 deg.) | 0.107                  |
| 1705 (Pitch angle 0 deg.) | 0.083                   |

Table 1. The roll error brought about by the installation error

| Simulation Time (Seconds) | Pitch Angle Error (deg.) |
|--------------------------|--------------------------|
| 0 (Initial time)         | 0.083                   |
| 505 (Roll angle 30 deg.) | 0.072                   |
| 2305 (Roll angle 0 deg.) | 0.083                   |

Table 2. The pitch error brought about by the installation error

The diameter of the table board is assumed to be 100 mm and the maximum vibration amplitude is set as 1 mm, which represents low machining accuracy. Next, from Section 3.2, it can be seen that the half-cone angle is $\theta = 1/100 \text{rad} = 0.01 \text{rad}$. Without loss of generality, the initial installation angle $\eta$ is set as 30 degrees. The attitude errors are shown in Figure 6. It is clear that the calculated values agree well with the simulated ones, and the attitude errors almost reach 1 degree under the simulation condition, which greatly decreases the attitude accuracy. There are several spikes in the curves of Figure 6 corresponding to the rotation process which can be seen in detail in Figure 7. This is because the attitude errors are related to the rotation angle, as shown in Eq. (22). The specific results of some special time points are referred to in Table 4, Table 5 and Table 6.

The angular error is set as 1 minute of angle, which corresponds to a low-accuracy rotary table. The attitude errors are shown as in Figure 8. The specific results of some special time points are referred to in Table 7, Table 8 and Table 9. It is clear that the calculated values agree well with the simulated values and the attitude errors are less than 0.02 degrees under the simulation conditions.

Table 3. The heading error brought about by the installation error

| Simulation Time (Seconds) | Heading Angle Error (deg.) |
|--------------------------|-----------------------------|
| 0 (Initial time)         | 0                           |
| 505 (Roll angle 30 deg.) | 0.042                       |
| 1105 (Pitch angle 30 deg.) | 0.046               |
| 1705 (Pitch angle 0 deg.) | 0.042                       |
| 2305 (Roll angle 0 deg.) | 0                           |

Table 4. The roll error brought about by the wobble error

| Simulation Time (Seconds) | Roll Angle Error (deg.) |
|--------------------------|-------------------------|
| 0 (Initial time)         | 0.286                   |
| 309 (Turntable 90 deg.)  | -0.286                  |
| 609 (Turntable 180 deg.) | 0.286                   |
| 909 (Turntable 270 deg.) | -0.286                  |
| 1105 (Pitch angle 30 deg.) | -0.430               |
| 1209 (Turntable 0 deg.)  | 0.430                   |
| 1705 (Pitch angle 0 deg.) | 0.286                   |

Table 5. The pitch error brought about by the wobble error

| Simulation Time (Seconds) | Heading Angle Error (deg.) |
|--------------------------|-----------------------------|
| 0 (Initial time)         | 0                           |
| 309 (Turntable 90 deg.)  | 0.496                       |
| 505 (Roll angle 30 deg.) | 0.430                       |
| 609 (Turntable 180 deg.) | 0.496                       |
| 909 (Turntable 270 deg.) | 0.496                       |
| 1105 (Pitch angle 30 deg.) | -0.248               |
| 1209 (Turntable 0 deg.)  | 0.248                       |
| 1509 (Turntable 90 deg.) | 0.496                       |

Table 6. The heading error brought about by the wobble error
From the above simulation, it is clear that the derived formulas can well describe the attitude errors brought about by the rotary table’s errors. Under the simulation conditions, the wobble error has the greatest impact on attitude accuracy.

6. Conclusions

The decrease in the accuracy of a rotational SINS caused by its rotary table’s errors is analysed in this study. The rotary table’s errors are classified according to three types: installation error, wobble error and angular error. The effects of these errors on navigation accuracy are analysed and simulations are carried out. From the theoretical analysis and the simulation results, the following conclusions can be drawn:

1) The rotary table’s errors do not affect the position accuracy of the rotational SINS. However, the installation error, wobble error and angular error have an influence on attitude accuracy.
2) The attitude errors brought about by the installation error, wobble error and angular error are related to the vehicle’s attitude. Moreover, the attitude errors brought about by the wobble error are related to the rotation angle.

The derived propagation formulae of the rotary table’s errors supply a good reference for the analysis of the navigation accuracy of a rotational SINS. Furthermore, the theoretical results could serve as the basis for the design of a rotational SINS’s rotary table and are especially useful for low-cost MEMS rotational SINSs, which could become widely used in robots, intelligent vehicles and UAVs in the future.

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