Estimation of the interaction between groundwater and surface water based on the flow routing using an improved nonlinear Muskingum-Cunge method

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Highlights:

1. Update the nonlinear Muskingum-Cunge flood routing method with lateral inflow and applies it to estimate GW-SW interaction.
2. Parameter sensitivity is analyzed qualitatively and quantitatively to reflect the influence of different parameters on the simulation results.
3. The applicability of improved methods is verified through the flood cases documented in literature and one case measured from Zhongtian River, China.
Abstract

The interaction of groundwater (GW) and surface water (SW) not only sustains the runoff in dry seasons, but also plays an important role in regulating aquatic ecosystems. Hydrological engineers proposed the idea of modeling flood routing using the Muskingum-Cunge method, by ignoring GW-SW interaction during flooding which may however contribute to the outflow. This study proposes an improved nonlinear Muskingum-Cunge flood routing model considering lateral inflow, which is denoted as NMCL1 and NMCL2 that can simulate the flood routing and calculate GW-SW exchange. In addition, both the linear and nonlinear lateral inflow (with the channel inflow) are discussed, and both the stable lateral inflow due to GW-SW exchange and the transient/conventional lateral inflow changing with the river inflow are considered for the first time. Sensitivity analysis has shown that different parameters have different effects on the simulation results. Three different flood cases documented in literature with one measured from Zhongtian River, China, were selected to compare the classical and the updated Muskingum-Cunge methods. Two different floods of the River Wye are selected to verify the accuracy of the calibrated model. Comparison has shown that, for several cases, the proposed method is capable of obtaining the optimal simulation results. For the case of Zhongtian River, the proposed method can estimate the GW-SW interaction and lateral inflow reliably. The proposed method inherits the ability of Maskingum-Cunge in flood routing. Moreover, the new Muskingum-Cunge method can quantify GW-SW exchange, and the estimation has reliably owned to the nonlinearity and sign flexibility of the calculated exchange process.
1. Introduction

Groundwater (GW) and surface water (SW) are two components of a hydrologic system, which interact at various spatiotemporal scales and regulate ecological and biogeochemical processes (Alley et al., 2002; Boano et al., 2014; Cardenas, 2008; Krause et al., 2011; Shuai et al., 2017; Winter et al., 1998). It is of critical importance to quantify the GW-SW interaction for maintaining the river ecosystem and managing water resources (Ford, 2005; Sophocleous, 2002; Welch et al., 2013). The quantification is however nontrivial because the GW-SW interaction interface is difficult to directly locate and the interaction itself varies with scales, thus has highly motivated the carrying out of this study.

This study tries to evaluate GW-SW exchange in lumped flow routing, a classical procedure in hydrology to determine the flow hydrograph. Several hydrologic models were derived by integrating spatially the hydraulic equations, which can be viewed as the finite-element approach that integrates the hydraulic differential equations. This is the case for the well-known Muskingum-Cunge (MC) model (Cunge, 1969), which, although based on a linear MC, allows the parameters $K$ and $x$ determined according to the channel and flow characteristics assuming the parameters constant during the passage of flood wave over the considered study reach, and therefore mimics the behavior of a hydraulic parabolic model. Note that the original storage equation in the MC model is linear with time. This model was successfully used to solve various flood routing problems (Chow et al., 1988; Gill, 1978; Linsley et al., 1982; O'Donnell, 1985; Wilson, 1974). Though the flood wave movement process in a river or channel is a nonlinear process, the traditional MC method simulates the process as linear models. To be realistic with the nonlinear process of flood movement in rivers, models like the Muskingum method are modified to account for the nonlinearity of the flow movement process. (Beven, 2001; Karahan
et al., 2013; Niazkar and Afzali, 2015; Yoon and Padmanabhan, 1993). Therefore, nonlinear versions of the storage equation were proposed. There are two ways of accounting nonlinearity in the flood routing process. Gill (1978) firstly introduced the nonlinear storage equation using the exponent of the Muskingum storage equation as the third parameter. The other way is the quasi-linear modelling approach in which the parameters of the model remain constant over a routing interval time, implying adoption of linear process modelling over the routing time interval, but varying from one time interval to the next time interval proposed by Ponce and Chaganti (1994), Todini (2007), Price (2009), and Perumal and Price (2013). These models account for nonlinearity of flood wave movement in a manner by varying the parameters in a physically based manner.

Accuracy of the nonlinear model was then significantly improved by optimizing parameters (Karahan et al., 2013; Niazkar and Afzali, 2015; Ouyang et al., 2015) or adjusting the nonlinear storage equation in addition to parameter optimization (Easa, 2013). The latter used nonlinear equations containing parameters with discrete values in sub periods during a flood event, and most importantly, it neglected the lateral flow within the routing reach. To overcome this apparent limitation, state-of-the-art MC models with multi-parameters and lateral flow were developed by several researchers. For example, Karahan (2014) proposed a new nonlinear MC flood routing model by considering the influence of lateral flow, and used a cuckoo search algorithm to estimate the parameters. Ayvaz and Gurarslan (2017) considered lateral flow and partitioned the inflow hydrograph into different sub-regions with sub-region dependent parameters. Kang et al. (2017) included lateral inflow in the MC method and divided the inflow range into several levels for parameter calculation. Barbetta et al. (2017) extended the variable-parameter Muskingum stage hydrograph routing method (Perumal et al., 2010) by enabling the
streamflow assessment with significant lateral inflow along the river reach.

The MC method considering the constant lateral inflow, however, may not be reliable to estimate the exchange volume between GW and SW, due to two main reasons. Firstly, the lateral inflow was assumed to be linear with the channel inflow (Ayvaz and Gurarslan, 2017; Karahan et al., 2015), which is no longer valid for the GW-SW exchange. This assumption is only feasible when considering the slope flow between sections, because the slope flow is dominantly driven by precipitation, which may have a linear relationship with the upstream inflow of the river channel. The exchange process of GW and SW can differ significantly from the slope flow. The GW-SW interaction is not only affected by surface flow, but also related to subsurface water dynamics represented by the fluctuation of groundwater head. Affected by unsteady precipitation and complex aquifer heterogeneity, the GW level dynamics tend to exhibit nonlinear and heterogeneous characteristics. Secondly, there is no constant term for slope flow assumption (Rodriguez-Iturbe and Valdés, 1979), meaning that when the upstream flow of the river channel is zero, the lateral inflow must be zero, which is inconsistent with the exchange between GW and SW. The exchange between GW and SW is primarily controlled by the water level difference between them; therefore, the interaction is not zero in most cases even when the river inflow is zero. Hence, it is necessary to improve the model when estimating the GW-SW interaction using the nonlinear Muskingum method with lateral inflow.

To address the issues mentioned above, this study updates the nonlinear Muskingum-Cunge flood routing method with lateral inflow and then applies it to estimate GW-SW interaction. The estimated GW-SW exchange in the improved model in this study includes the sum of vertical and lateral inflow. Section 2 briefly introduces the traditional MC methods and explains improvement in incorporating the lateral inflow. Section 3 performs the parameter sensitivity
analysis and selects different groups of literature data to estimate the model parameters, the channel routing, and the exchange volume of GW and SW. Section 4 applies the new model for a field study conducted by the authors, and evaluates the lateral inflow and exchange volume estimated by the traditional MC method and the proposed one, as well as the thermal method. Section 5 discusses the application and deficiency of the proposed methods. Lastly, Section 6 reports the main conclusions.

2. Methodology development

2.1 MC Models

(1) MC models without the lateral flow (MC1, MC2, and NMC1)

The conventional MC method is based on the channel storage equation and water balance equation (Geem, 2006; Gill, 1978; Kim et al., 2001; Mohan, 1997; Tung, 1985; Yoon and Padmanabhan, 1993). The earliest linear Muskingum channel storage equation assumes that the storage \( S \) of the reach is linearly related to the flow \( Q \):

\[
S = K[\chi I + (1 - \chi)Q] \tag{1}
\]

where \( \chi \) represents the relative proportion of the influence of inflow and outflow on the channel storage. \( K \) and \( \chi \) are determined according to the channel and flood characteristics, and are assumed to be constant. The water balance equation and the storage equation are decomposed in the time section, and the flow routing equation can be obtained (named MC1):

\[
Q_t = C_0I_t + C_1I_{t-1} + C_2Q_{t-1} \tag{2}
\]

where \( C_0 \), \( C_1 \) and \( C_2 \) are functions of the Muskingum method with respect to parameters \( K \) and \( \chi \), the sum of which is equal to 1. While the time step \( (\Delta t) \) is set, the three coefficients can be obtained using the recommended equations by O'Donnell (1985).
Cunge (1969) proved that the Muskingum method is the second-order approximate solution of diffusion wave. He connected the Muskingum method with diffusion wave in hydraulics and proposed the Muskingum-Cunge (MC) method. While this form is the same as the earliest Muskingum method, it has clear hydraulic significance. For a certain reach, the method first gives a single wide reference flow, estimates the wave velocity and hydraulic diffusivity in the diffusion wave equation, and then, as a constant, uses the difference scheme to solve the motion wave equation. In the MC version, $K$ and $x$ are calculated using the following formulas derived by Cunge (1969) (named MC2):

$$K = \frac{\Delta x}{c}$$  \hspace{1cm} (3)

$$x = \frac{1}{2} \left(1 - \frac{I}{c \cdot S \cdot \Delta x} \right)$$  \hspace{1cm} (4)

in which $\Delta x$ represents the reach length (i.e., the space interval), $c$ is the flood wave celerity, $I$ is the unit width discharge, and $S$ is the channel bed slope.

Both the Muskingum method (MC1) and the Muskingum-Cunge method (MC2) assume the linear relationship between flow and storage, according to the graphical method (Moghaddam et al., 2016). This assumption is not valid for some reaches of the river, motivating the nonlinear Muskingum model (Mohan, 1997), such as the nonlinear Maskingum method (named NMC1) proposed by Gill (1978):

$$S_t = K[xI + (1-x)Q]$$  \hspace{1cm} (5)

$$Q_t = \frac{1}{1-x} \left( \frac{S_t}{K} \right)^m - x \cdot I_t$$  \hspace{1cm} (6)

where the new parameter $m$ is the exponent of the power-law relationship between the
accumulated storage and the weighted flow. When \( m = 1 \), equation (5) reduces to the specific linear relation. \( K \) in the nonlinear models is the storage parameter for the river reach assumed to be equal to the travel time of flood peaks that move through the reach. Also, \( x \) is the dimensionless weight which indicates a weight between inflow and outflow on storage depends on the channel type. The graphical method used for historical inflow and outflow hydrographs is inappropriate for the nonlinear Muskingum models. Hence, the determination of \( K \) and \( x \) is difficult, requiring advanced methods (Moghaddam et al., 2016).

(2) MC models with lateral flow (MCL1)

All the models mentioned above ignore the lateral flow existing in the river reach in the actual flood events. O'Donnell (1985) assumed that the lateral flow entering the river reach was directly proportional to the inflow with a proportionality factor \( \beta \), and then proposed the first MC model with the lateral flow:

\[
\frac{dS_t}{dt} = (1 + \beta)I_t - Q_t \tag{7}
\]

\[
S_t = K[(1 + \beta)xI_t + (1-x)Q_t] \tag{8}
\]

\[
Q_{lat} = \beta I \tag{9}
\]

Integrating the nonlinear continuity and storage equations, which takes lateral flow into consideration, one obtains:

\[
S_t = K[(1 + \beta)xI_t + (1-x)Q_t]^m \tag{10}
\]

\[
Q_t = \frac{1}{1-x} \left( \frac{S_t}{K} \right)^{\frac{1}{m}} - \frac{x(1+\beta)}{1-x}I_t \tag{11}
\]

Tung (1985) and Geem (2006) suggested using the inflow at the previous time step (\( I_{t-1} \)) rather than at the current time (\( I_t \)) when calculating the outflow (\( Q_t \)). Geem (2014) discussed
this issue and conformed that under the condition of optimal parameters when calculating $Q_t$, $I_{t-1}$ is better than $I_t$ (named MCL1):

$$Q_t = \frac{1}{1-x} \left( \frac{S_t}{K} \right)^{\frac{1}{m}} - \frac{x(1+\beta)}{1-x} I_{t-1}$$  \hfill (12)

### 2.2 Improve MC method to calculate GW-SW interaction (NMCL1 and NMCL2)

The MC methods reviewed above assume that the lateral inflow is linearly proportional to the inflow. The zero-value inflow leads to zero lateral inflow, which cannot explain the potential lateral inflow due to GW-SW interaction. To solve this issue, it is hereby first assumed that a stable GW-SW interaction process exists before the flood event, and add a constant representing the stable exchange volume in the storage (named NMCL1):

$$S_t = K[(1+\beta)xI_t + (1-x)Q_t + e]^m$$  \hfill (13)

where $e$ represents the stable lateral inflow due to GW-SW exchange. We can then estimate the sum of GW-SW interaction and lateral inflow using:

$$Q_{lat} = I_t\beta + e$$  \hfill (14)

Another option is to consider the nonlinear relationship between the lateral inflow and the channel inflow, leading to the following storage (named NMCL2):

$$S_t = K[xI_t + (1-x)Q_t + e + I_t^\beta]^m$$  \hfill (15)

where $\beta$ no longer represents the ratio coefficient of lateral inflow, but represents the power exponent. The sum of the vertical inflow due to GW-SW interaction and the transient lateral inflow can be calculated as:

$$Q_{lat} = I_t^\beta + e$$  \hfill (16)

Both the linear and nonlinear relations between the inflow and the lateral inflow are
considered in this study and will be checked against real-world data in the next section.

When using the MC methods mentioned above to simulate flood routing, the primary objective is to minimize the discrepancy between the simulated and measured outflow. For this purpose, the sum of the squared errors (SSQ), the Nash efficiency coefficient (NSE), and the root mean square error (RMSE) are used as the objective functions.

The calculated $S_{t+1}$ and $Q_{t+1}$ may have negative or complex values for the generated parameters of $K$, $x$, and $m$ (Karahan et al., 2013). To solve this problem, the following penalty functions are used:

$$
S_{t+1}^* = \begin{cases} 
S_{t+1} & \text{if } S_{t+1} \geq 0 \\
\lambda_1 |S_{t+1}| & \text{if } S_{t+1} < 0 \\
\lambda_1 |RS_{t+1}| & \text{if } S_{t+1} \text{ is complex}
\end{cases}
$$ (17)

$$
Q_{t+1}^* = \begin{cases} 
Q_{t+1} & \text{if } Q_{t+1} \geq 0 \\
\lambda_2 |Q_{t+1}| & \text{if } Q_{t+1} < 0 \\
\lambda_2 |RQ_{t+1}| & \text{if } Q_{t+1} \text{ is complex}
\end{cases}
$$ (18)

where $\lambda_1$ and $\lambda_2$ are the penalty constants, and $R()$ is the real part of the complex number $S_{t+1}$ and $Q_{t+1}$. The equations state that if $S_{t+1}$ or $Q_{t+1}$ is less than zero, $S_{t+1}^*$ or $Q_{t+1}^*$ is then used in the routing procedure. Note that the two constants $\lambda_1$ and $\lambda_2$ can be case dependent. Preliminary tests are needed before starting the actual search process to determine $\lambda_1$ and $\lambda_2$.

The particle swarm optimization (PSO) approach, which is an intelligent cluster optimization algorithm (Eberhart and Kennedy, 1995), is applied for parameter optimization.

3. Model Validation

3.1 Sensitivity analysis of MC parameters

Parameter sensitivity analysis is performed by adjusting the five parameters (listed below).
We choose the improved NMCL1 and NMCL2 methods, which consider the nonlinear lateral inflow (i.e., the portion changing nonlinearly with the river inflow) and the stable lateral inflow due to GW-SW exchange.

The observed flow at the River Wyre in UK (O’Donnell, 1985) is selected here for sensitivity analysis. This example involves inflow with multi peaks (Figure 1). The significant amount of lateral flow resulted in the observed outflow far greater than the inflow. The optimal solution vectors are obtained by the PSO algorithm: $x = 0.2127$, $K = 24.6451$, $m = 1.0261$, $\beta = 2.3489$, $e = 3.1677$ for NMCL1 (13), and $x = -0.8860$, $K = 10.7691$, $m = 1.0216$, $\beta = 1.2426$, $e = 7.3229$ for NMCL2 (15). The corresponding SSQ is 98 and 125.1, the NSE is 0.9966 and 0.9957, and the RMSE is 1.75 and 1.977, for NMCL1 and NMCL2, respectively. Figure 1 shows the observed and fitted flow using NMCL1 and NMCL2.

Figure 1 shows that there the best-fit solutions differ slightly from the observed flow at the rising limb, and the solutions fit better the recession limb. The results of NMCL2 are slightly larger than those of NMCL1, and so are the estimation results of the GW-SW exchange. The sum of the GW-SW exchange calculated by these two methods are larger than the river inflow, and all the values are positive, implying that the surface water gains more water from groundwater than it loses, and/or the lateral inflow of this flood is large. The separation of GW-SW interaction and lateral inflow for River Wyre is not discussed here due to the lack of necessary information or methodologies (note that baseflow separation cannot distinguish the lateral and vertical inflows).

In the sensitivity analysis, the bounds for the five parameters $x$, $k$, $m$, $e$, and $\beta$ in NMCL1, NMCL2 and the sensitivity of each parameter are shown in Table 1. The comprehensive sensitivity is calculated according to the equations proposed by Kabala (2001) and Doherty (2004). The selection of value range is based on the optimal parameter size simulated by PSO.
algorithm. In the traditional Muskingum method, the value range of parameter $x$ is generally between 0-0.5. After preliminary simulation, it is found that the value range does not get better simulation results. Besides, in order to highlight the influence of parameter $x$ change on simulation results, this study refers to (Ayvaz and Gurarslan, 2017), and sets the parameter range as -0.9-0.9. In these bounds, most parameters take ten different values with a uniform interval, except for parameter $x$ which takes five values. In the process of simulation, it is found that the change of parameter $x$ has a great influence on the result; selecting 5 different values can accurately show its sensitivity. The results are shown in Figure 2.

Figure 2(a)–(e) shows the simulated flood process by adjusting $x$, $K$, $m$, $e$ and $\beta$. Figure 2(a) shows that the model solution is more sensitive to parameter $x$ than the others, considering the largest fluctuation of model results by adjusting $x$. The influence of variable $x$ on the model result is similar between the two methods (NMCL1 and NMCL2). A larger $x$ causes a larger fluctuation and more irregularity in the modeled outflow. According to Table 1, it can also be seen that the sensitivity of parameter $x$ is only next to that of parameter $m$, which is ten times larger than that of parameter $K$. Here $x$ is the weighting factor allocating the contribution of the wedge to the total storage. A larger $x$ (>0) means more storage for the wedge, so that more river inflow (from the channel) can directly transfer to outflow, resulting in a stronger fluctuation of the outflow, especially during the advance of a flood event.

The change of $K$ also has big influence on the model results (Figure 2(b)). A larger $K$ generates a smoother outflow curve with a lower flood peak. In the rising stage, the smaller the $K$ is, the larger the calculated outflow, while the opposite is true for the recession stage. The simulated curves using different $K$ converge at the beginning stage of the flood decline. According to Table 1, the sensitivity of parameter $K$ is small, only greater than parameter $e$. The
sensitivity calculated by NMCL1 method is one fifth of that calculated by NMCL2 method. It shows that the smaller the parameter $K$ is, the greater the sensitivity. $K$ is the storage constant expressing the ratio between storage and discharge, which may also be viewed as the lag or travel time through the reach (Karahan, 2012). When $K$ is smaller, the flood rise and fall difference of the reach is larger, and the water storage capacity of the channel is also reduced. When $K$ is equal to 1, the channel water storage is very small and the flood fluctuation increases. At this time, the inflow of the river is constant, and the outflow needs to be increased to balance the water storage of the river, which leads to the amplification of the outflow and the increase of the fluctuation.

$m$ is an index in Eq. (13) and (15) and affects flood simulation results (Figure 2(c)). According to Table 1, $m$ shows the biggest impact on the outflow. In order to further show the influence of parameter $m$ on the simulation results, $m=0.7$ is added in the picture. Which shown when $m$ continuous decreases, the simulated flow process also exhibits apparent fluctuations. $m$ is used as power exponents to improve the relation between accumulated storage and weighted flow. When $m$ is smaller, the storage decreases, the reciprocal of $m$ becomes larger. Under certain inflow conditions, when storage decreases, the outflow increases, when $m$ continues to decrease, the fluctuation of outflow hydrograph will increase significantly.

The sensitivity of parameter $e$ to the simulated outflow, as shown in Figures 2(d) and 2(f) and Table 1, is the smallest. A larger $e$ leads to a greater outflow, and the simulated flood curves exhibit the similar pattern. In the range of $e$ set up for sensitivity analysis, the maximum value of the simulated outflow changes from 97 m$^3$/s to 107 m$^3$/s in both methods, and the estimated exchange volume and the sum of the transient lateral flow and the vertical inflow change from 80 m$^3$/s to 90 m$^3$/s (in NMCL1) and from 82 m$^3$/s to 92 m$^3$/s (in NMCL2), respectively. According
to the sensitivity calculation, the change of parameter $e$ is equal to the change of exchange volume, which means that the change of stable lateral inflow causes the same change of exchange volume i.e. $\Delta EX = \Delta e$. The change of this parameter is the similar to that of the simulated outflow, i.e. $\Delta Q_{out} = \frac{\Delta e}{1 - x}$.

The parameter $\beta$ shown in Figure 2(e) and 2(g) and Table 1 is more sensitive to the outflow than $e$. This sensitivity is stronger for the NMCL2 method than the NMCL1 method. This discrepancy may be due to the different role of $\beta$ in the two methods: $\beta$ is an index in the NMCL2 (15), while it is a multiplier in the NMCL1 (13). The maximum value of the simulated outflow changes from 71 m$^3$/s to 129 m$^3$/s in NMCL1 and from 69 m$^3$/s to 169 m$^3$/s in NMCL2. The estimated GW-SW exchange change from 49 m$^3$/s to 117 m$^3$/s (in NMCL1), 46 m$^3$/s to 167 m$^3$/s (in NMCL2), respectively. A smaller $\beta$ leads to a smoother flood curve with smaller outflow. The simulated change is higher during the middle stage of the flood curve segment then the rising and declining limbs (Figure 2(e) and 2(g)).

In general, $m$ is the most sensitive parameter and has the greatest impact on the simulated outflow and routing shape. When the parameter is small enough, it also has a large impact on the simulation results. The influence of the change of parameter $x$ on the result is second only to that of parameter $m$. The parameter $e$ is the least sensitive one and the influence on the simulation results is equal. The change of parameter $\beta$ also has big influence on the simulation results, but can maintain the basic flood routing shape. If the adjustment range for the parameters is expanded, the results obtained above need to be re-evaluated.

3.2 Validation via published data

The paper selects three types of flood cases from literature denoted as Wilson (1974),
Features of these floods include a smooth single peak, unsmooth multiple peaks, and a large amount of lateral inflow contribution, representing three common flood features. Model parameters are fitted by the PSO algorithm. Using the optimal parameters, the results simulated by different MC methods are compared. Table 2 lists the error analysis of flood routing simulation results using different methods for the three cases.

3.2.1 Flow data from Wilson (1974)

The first example (Wilson, 1974) was a smooth single-peak hydrograph with low lateral flow contribution. Wilson’s data is one of the most extensively used benchmark applications in evaluating the performance of the linear and nonlinear Muskingum flood routing methods (Ayvaz and Gurarslan, 2017). We use the six MC methods mentioned above to simulate the flood routing in Wilson's data.

As listed in Table 2, the simulation results of the conventional linear MC1 and MC2 methods are poor in matching the data (with the highest SSQ), followed by the non-linear NMC1 method. The result of the NMCL2 method is better (with a lower SSQ, a larger NSE, and a smaller RMSE). The best-fit models are MCL1 and NMCL1 according to the values of SSQ, NSE, and RMSE. Other researchers also simulated this case. For example, O'Donnell (1985) used a linear, three-parameter model taking the contribution from the lateral flow into consideration, whose SSQ was 815.68. Karahan et al. (2013) used a nonlinear, three-parameter model with a SSQ of 36.76. The SSQ calculated by the NMCL1 method is 36.3% lower than the SSQ value obtained by Karahan et al. (2013), showing a better fit for the method developed by this study.
The observed maximum outflow is 85 m$^3$/s, and the best-fit result is 84.43 m$^3$/s using MCL1 and 84.45 m$^3$/s using NMCL1. The simulated occurrence time of the flood peak is also consistent with the observed one. Using the PSO algorithm by running 10,000 times to estimate the optimal parameters in MCL1 and NMCL1, we obtain the best-fit parameters: $x = 0.2467$, $K = 0.3346$, $m = 1.9479$, $\beta = -0.0233$ (MCL1), and $x = 0.2580$, $K = 0.2513$, $m = 2.0053$, $e = 1.1989$, $\beta = -0.0450$ (NMCL1). The best-fit parameters in the two methods are similar.

Figure 3 shows that all the six methods can capture the overall shape of the observed flood routing, although with subtle differences. For example, for the MC1 and MC2 methods, the estimated outflow peak time is an one hour earlier than the observed one, and the simulated outflow declines slower than the data in the recession stage. For the MCL1 and NMCL1 methods, the simulated outflow profiles are steeper than those simulated by the MC1 and MC2 methods and hence can match better the observed flood routing.

Figure 3 also shows that, during this flood event, the GW and SW exchange is very small. The values estimated by MCL1 and NMCL2 are similar and negative (i.e., recharge to groundwater). The value estimated by NMCL1 method is positive at the beginning, and then becomes negative as the inflow/outflow increases. When the flow starts to decrease, it gradually decreases to 0 and then rises, reaching positive eventually. With the increase of rainfall, the river stage is rapidly increased, but the groundwater increases slowly. These combination of river stage and groundwater resulted in the vertical recharge to groundwater from surface water. In the later stage of rainfall, the groundwater level gradually increases and the river stage then decreases, the GW-SW interaction changes to discharge to river from groundwater. At the last phase of rain event, the GW-SW interaction gradually returns to the initial state before rainfall.
3.2.2 Flow data from O’Donnell (1985)

The flood data observed at the River Wyre, UK (O’Donnell, 1985) has considerable lateral inflow. We use the six MC methods mentioned above to simulate the flood routing.

Table 2 confirms that the conventional methods, including MC1, MC2, and NMC1 which ignore the lateral inflow, fail to simulate the flood routing, while the MCL1 and NMCL2 methods work much better. The solution of NMCL1 matches the data the best with a relatively low SSQ (=98), while the SSQ in O’Donnell (1985) was 468.84 and the solution in O’Donnell (1985) did not match the flood. The simulated maximum outflow using the NMCL1 method is 100.14 m³/s, close to the observed one (=98.89 m³/s). The best-fit parameters after running the PSO algorithm for 10,000 times are: \( x = 0.1704, \ K = 24.8942, \ m = 1.0307, \) and \( \beta = 2.5414 \) for MCL1; \( x = -0.8860, \ K = 10.7691, \ m = 1.0216, \) and \( e = 7.3229, \) \( \beta = 1.2426 \) for NMCL2; and \( x = 0.2127, \ K = 24.6451, \ m = 1.0261, \) and \( \beta = 2.3489, \) \( e = 3.1677 \) for NMCL1. In these three methods, the optimal value for parameter \( m \) is similar, while the other parameters have different values in different models.

Figure 4 shows the first three conventional MC methods (MC1, MC2, and NMC1) fail to simulate the flood routing process. In this case, the inflow is much smaller than the outflow, which indicates that there are a lot of lateral inflow, so the influence of lateral inflow should be considered when simulating the outflow. Formula (1)-(6) show that the first three methods assume that the storage capacity of the river is defined by the difference between inflow and outflow, and the lateral inflow supplying the reach is not considered. For River Wyre, the contribution of lateral inflow to river water storage can no longer be neglected. The simulation result of MCL1, NMCL1 and NMCL2 are similar, all of them can fit well with the observed outflow hydrograph.
The simulated GW-SW exchange are larger than the river inflow, and all the values are positive, implying that the surface water gains more water from groundwater than it loses, and/or the lateral inflow of this flood is large. The maximum GW-SW exchange was 85.89 m³/s (according to MCL1), 86.45 m³/s (NMCL2), and 82.8 m³/s (NMCL1), respectively. The exchange routing procedure estimated by the three methods are also similar.

3.2.3 Flow data from Viessman and Lewis (2003)

Viessman and Lewis (2003) observed a flow hydrograph with double peaks. The resultant error objective functions of the six MC methods fitting the observed flood routing are listed in Table 2. The SSQ is large due to the large flood magnitude. The two conventional MC methods have the biggest errors (SSQ=124310 and 110620 respectively). The best method is NMCL1, based on the objective functions which are 71708 for the SSQ, 0.9835 for the NSE, and 54.661 for the RMSE. The SSQ is lower than that reported in literature, such as 76758 in Easa (2014), 73379 in Haddad et al. (2015), and 74812 in Moghaddam et al. (2016), implying that the NMCL1 method is slightly better than the methods used in the literature.

The two observed flood peaks are 1509.3 m³/s and 1248.8 m³/s, respectively. The two simulated flood peaks using the NMCL1 method are 1450.2 m³/s and 1228.6 m³/s, respectively, which are slightly smaller than the observed ones.

The optimal parameters obtained by the PSO algorithm are: \( x = 0.0938, \ K = 0.287, \ m = 1.4956, \) and \( \beta = -0.0032 \) for MCL1; \( x = 0.0922, \ K = 0.2696, \ m = 1.5038, \) and \( \beta = -0.3272 \) for NMCL2 and \( x = 0.1057, \ K = 0.1990, \ m = 1.5408, \) and \( \beta = -0.02488, \) and \( e = 20.6572 \) for NMCL1. Hence, the same parameter in these three different methods is close to each other.

Figure 5 shows that the flood hydrographs simulated by the six methods are generally
consistent with the shape of the observed outflow. In the transition period between the two flood peaks, the error between the simulation results and the observation is large. In the Muskingum method, it is considered that there is a direct corresponding relationship between channel storage and the linear combination of inflow and outflow, which represents a single flood in actual channel. This case is an obvious double peak flood, which means that multiple floods are superimposed, and the hydrograph presents two dependent moving waves. So, in the simulation routing of this case, the error of simulation results between two flood peaks is large. This dataset implies low contribution to outflow from the lateral inflow and the exchange of GW-SW. The modeled lateral inflow and vertical inflow due to GW-SW exchange are negligible compared to the modeled and measured inflow and outflow.

3.3 Validation using two different floods in a same river

In order to further verify the calibrated model, two different floods in the same river are selected for simulation. The two floods data observed at the River Wye, UK (O'Donnell, 1985). We use the three improved MC methods mentioned above to simulate the flood routing. Two different flood processes of Wye River are recorded in O'Donnell (1985). The duration of one flood is 198 hours and the peak value of outflow is 969 m$^3$/s, and the counterparts of the other flood are 108 hours and 323 m$^3$/s, respectively. First, we select the longer duration flood for optimal parameter calibration, and substitute the calibrated parameters into another flood for simulation. Flood 1 represents the flood duration of 198 hours and flood 2 represents the flood duration of 108 hours.

The optimal parameters are estimated according to the data of flood 1. The best-fit parameters after running the PSO algorithm for 10,000 times are: $x = 0.304$, $K = 0.147$, $m = 1.727$, and $\beta = 0.049$ for MCL1; $x = 0.314$, $K = 0.094$, $m = 1.787$, $e = 8.150$, and $\beta = 0.401$ for NMCL2;
and \( x = 0.316, \ K = 0.098, \ m = 1.784, \) and \( \beta = 0.014, \ e = 13.110 \) for NMCL1. In these three methods, the optimal value for same parameters \( x, \ K, \ m \) are similar. This shows that the PSO algorithm accurately captures the characteristics of the flood, and the calculated parameters are in line with the basic situation of the flood.

By analyzing the error of the simulation results, the MCL1, NMCL1 and NMCL2 methods were confirmed successfully simulate the flood process. The simulation results of the three methods are very close, the simulation result of flood 1 is better than that of flood 2. The simulation results of flood 1 from three models are very close and all the NSEs are about 0.98. While the simulation results of flood 2 are different, and the MCL method is the best (NSE = 0.93).

Figure 6 shows that all the three methods can capture the overall shape of the observed flood routing. The three methods in flood 1 are the closest, the simulated hydrographs almost overlap. While, the flood peak time simulated by the three methods is 6 hours ahead of actual time, and the simulated peak discharge is less than the observed value. In the initial and recession stage of flood, the simulation hydrograph of the methods fit well with the observation. The simulation results of flood 2 hydrograph are not as good as that of flood 1. In the initial stage of flood, the simulated value is larger than the observed value, but the simulated results capture the time and size of the flood peak, and the recession process is close to the observed one.

Figure 6 also shows that, during this flood event, the sum of the lateral inflow and the vertical inflow due to the interaction between GW and SW is small. The exchange volume estimated by the three methods are all positive, which indicates that the river was recharged by groundwater in flood events. The values estimated by NMCL1 and NMCL2 are similar. The two methods estimate more exchange volume and simulate smoother hydrograph. MCL1 method estimates less exchange volume and the simulated hydrograph fluctuates greater. The maximum
GW-SW exchange in flood 1 was 55.97 m$^3$/s (according to MCL1), 25.03 m$^3$/s (NMCL2), and 28.82 m$^3$/s (NMCL1), respectively. The maximum GW-SW exchange in flood 2 using the three methods are similar and around 18 m$^3$/s.

The results mentioned above confirmed that the new method proposed in this paper can be applied to different flood simulation of the same river. Although there are differences between the two flood simulation results, the simulation results are reliable (NSE > 0.9).

4. Applications

4.1 Field investigation of GW-SW interaction at Zhongtian River, China

Zhongtian River is a second-order stream in Jiangsu Province, southeast China. The basin contains mainly low hills, with the elevation between 17.8 m and 531.5 m. The climate of the study site is subtropical monsoon, and the temperature and humidity significantly differ between seasons. The annual average temperature is 17.5°C, and the annual rainfall is 1,149.7 mm. The rainfall in summer reaches 50% of the yearly rainfall, and the winter rainfall accounts for only 12.7% of the annual total (according to data collected from 1971 to 2010). The headwaters of this reach are in the Tianmu Lake Water Preserve, and the selected river reach is approximately 30 m long and 10 m wide. Two Levelogger Edge data loggers (3001, Solinst) were used to monitor the river stages at upstream and downstream, respectively. Because thereal time measured flow was not obtained, the river stage observed continuously is utilized in the MC routing based on the assumption of linear relationship between river stage and discharge.

Four cross sections were set up from upstream to downstream in the streambed to measure hydrologic and thermal properties, and the four cross sections are composed of 11 thermal
monitoring points (see Figure 1 of Lu et al. 2020). The hydrodynamic and thermal conditions in
the river, groundwater, and hyporheic zone were continuously recorded and analyzed for ten days
from 11/23/2014 to 12/02/2014. Three rainfall events of 89 mm, 9 mm, and 9 mm were observed
on Nov. 23-25, Nov. 27, and Nov. 29-30, respectively. The stream stage had a 1~2 days delay
after the rainfall event. Although the total rainfall had a small magnitude and short duration, the
steeply sloping catchment caused a sizable increase in the stream stage.

1D analytical solutions for heat transport can be used to estimate the hyporheic flow when
the flow has a vertical component. Three popular methods (with analytical solutions), including
the amplitude ratio (Ar) approach, the phase shift (Δϕ ) approach, and the combination of Ar
and Δϕ , have been widely used to calculate vertical flow components for the case of
downwelling flow. The amplitude-based method (Ar) in Hatch is usually more stable and robust
than the phase-shift based version and the combination method (Irvine et al., 2015). We tried the
three popular methods and found that these three methods generated deviating results, which we
attribute to the sensitivity of the Ar method to non-ideal input data (see Hatch et al. (2006)).
Therefore, we chose to use the Ar method because of its robustness and consistent results.

Water flux at each of the thermal monitoring point is calculated by VFLUX, and the fluxes
for cross section by the weighted point flux results are shown in Figure 7. The rate of water flux
change at S1 and S4 is the largest, and the flux across S1 changes from small upward flow to
larger downward flow, from -2×10^{-6} m/s to 5×10^{-6} m/s (the negative and positive values represent
upward and downward flow, respectively), while the flux at S4 changes from larger upward flux
to smaller upward flux, from -7×10^{-6} m/s to 1×10^{-6} m/s. The temporal change of the flow
direction along the four downstream sections starts first at S1 and finishes at S4. Hence, the
upstream section responds to rainfall rapidly with the largest rate of change and the earliest
downward flux, while the downstream sections respond to rainfall slowly. In the initial stage, the surface water gains water from groundwater. Due to rainfall, the amount of water gained from groundwater decreases gradually. The flux direction at S1 in the upstream changes rapidly from upward flow to downward flow; meanwhile, the surface water loses water and recharge groundwater, and the amount of supply increases gradually. In the later observation period, the surface water in the three downstream sections returns water to subsurface and supplies groundwater. Based on the simulated water flux at S1, the downward flux is larger than the upward flux, indicating that the loss of surface water is larger than the supply from groundwater.

4.2 Estimation of interaction between GW and SW

The measured water level data are used to calibrate model parameters in the PSO optimal algorithm, and the flood routing and exchange volume are simulated by different MC methods.

Figure 8 shows that the simulated outflow using the MC1 method is the worst, almost identical to inflow; while the NMCL2 method provides the best fit, with the resultant SSQ of 0.014, the NSE of 0.99, and the RMSE as small as 0.008. In this case, the error of NMCL1 is larger than NMCL2, with the resultant SSQ of 0.224, the NSE of 0.98, and the RMSE of 0.031.

Regarding the exchange volume estimated by the models, the results of MCL1 and NMCL1 are similar at the starting point whose value is ~0 m³/s and the results are negative in the whole simulation period. Among them, the downward flow simulated by MCL1 is larger than NMCL1. It shows that in MCL1 and NMCL1 method, during the whole simulation period, the flow is downward due to GW-SW exchange and with little conventional horizontal lateral inflow. The results simulated by NMCL2 are positive at the beginning. The flow direction changes from downward (represented by positive flow) to upward (negative values) around the rising point of the hydrograph. The GW-SW exchange calculated by NMCL2 methods is converted from
positive to negative close to the starting point of the flow rise. It shows that the rising of streamflow caused by rainfall results in the increase of downward recharge to groundwater. The solutions of NMCL2 methods remain negative during almost the whole observation period. Except for the beginning at the observation, the overall simulated outflow is decreasing slowly.

According to the calculation formula, in MCL1(10), the stable groundwater recharge caused by the exchange of GW-SW is not considered, i.e. parameter e. And the GW-SW exchange has a single linear relationship with the inflow, therefore, the direction of the simulated exchange volume is completely determined by the parameter β. At this time, the optimal parameter β is a negative number, which results in the simulated exchange volume is downward flow in the whole period, and there is no direction change phenomenon caused by rainfall. In the NMCL1(13), the optimal β and e parameters are small, and the parameter β is negative causing the calculated exchange volume to be negative. When the parameter e is very small, the method NMCL1 is similar to MCL1, and the estimated direction and size of exchange volume have a single linear relationship with parameter β. In the NMCL2(15), because parameter β is exponential, the positive and negative of the simulated exchange volume cannot be completely determined by this parameter, it is inflow, parameter β and e that is jointly determined. Therefore, this method can not only correctly simulate the exchange volume, but also reflect the dynamic process of direction change caused by the increase inflow of rainfall.

Figure 9 is a boxplot showing the GW-SW exchange calculated by MCL1, NMCL2, and NMCL1. The MCL1and NMCL1 method provides all negative, while the results of MCL1 are bigger than NMCL1. The simulation results of MCL1 method are much larger than that of NMCL1, which is directly related to the optimal parameter β of the two methods. Among the
three methods, the result of the NMCL1 method has the narrowest distribution ranging from 0 m³/s to -0.25 m³/s, while the result of the MCL1 method has the broadest distribution ranging from 0 m³/s to -1.25 m³/s. The average results of the MCL1, NMCL1 and NMCL2 methods are -0.6 m³/s, -0.15 m³/s and -0.05 m³/s respectively.

According to Figures 8 and 9, only the GW-SW exchange estimated by the NMCL2 methods is slightly greater than 0 at the initial stage, indicating that the quantitative difference of these methods in the interaction of surface water and groundwater is small, and the amount of surface water gained from groundwater and the lateral inflow is slightly greater than that due to loss. With the rise of the river water level, the exchange volume gradient changes, and the simulation results become negative, indicating that the surface water mainly loses water. The solutions of the MCL1 and NMCL1 method are negative, showing that the surface water recharge groundwater account for the majority of the GW-SW exchange volume.

It can be seen from Figure 7 that the vertical flux direction of the four sections has changed during the observation, which proves that the GW-SW exchange does exist, and the amount of water supplied by groundwater is smaller than the loss of surface water. Besides, the GW-SW exchange volume is small, which is similar to the result of the NMCL2 method whose interquartile range is ~0.13 m³/s, closest to the thermal based result. In addition, the transition time of the GW-SW exchange volume gradient is closest to the rising point of the water level. This feature reflects the observed fact that the river water level rises due to rainfall, which simultaneously enhances the loss of surface water and strengthens the exchange between GW and SW. The trend of the simulation results of the two methods is consistent. Compared with the thermal based method, the flood routing method proposed by this study is more accurate and easier to operate, which can simulate the overall change of GW-SW exchange volumes. When
the field investigation is limited, more reliable dynamic data of flood routing can be obtained by numerical simulation. In addition, before the field survey, the numerical simulation method can also be used to predict the flow routing, so as to make the field work more feasible.

5. Discussion

5.1 Model comparison: Capture GW-SW interaction

For flood routing: Two new models (named NMCL1, NMCL2) were proposed by this study to calculate flood routing by considering lateral inflow. Both the linear and nonlinear relationship between the lateral inflow and the river inflow are considered, in addition to the stable part of lateral flow, which can cover various situations of the lateral inflow. The testing results showed that our model is superior to the conventional MC models in fitting the hydrograph in various flood events with different lateral inflow. Particularly, the NMCL1 method, which generates the least error and assumes the linear relationship between the lateral inflow and the river inflow, can obtain the best simulation results.

For lateral inflow and the GW-SW exchange: Another contribution of this study is to estimate the GW-SW exchange when using the MC method to simulate the flood routing procedure. Previous studies ignored the GW-SW exchange, although considering the lateral inflow. Both literature review and the field application, as well as the calculated hyporheic flow using 1D analytical solutions for heat transport, showed that the NMCL1 and NMCL2 methods proposed by this study can reliably estimate the exchange of GW and SW, and the exchange volume calculated is in accordance with the calculated hyporheic flow. The method proposed by this study can be combined with groundwater flood early warning (Adams et al., 2010; Gotkowitz et al., 2014; Hughes et al., 2011; Linn et al., 2003) to prevent flood hazards. In addition, it can also be applied to explore the properties of hyporheic zone during flood.
5.2 Model parameters

There had been many discussions in literature on the optimal parameter estimation method (Kang et al., 2017; Karahan et al., 2015), but there is a lack of discussion in the influence of parameter change on model results. Parameter sensitivity analysis conducted in this study revealed several conclusions: 1) parameter $m$ has the greatest influence on the simulation results, when $m$ is small, the fluctuation of outflow hydrograph will increase significantly; 2) the second sensitive parameters are $x$ and $\beta$ in NMCL2 method, which have big influence on the simulation results; 3) parameter $K$ is less sensitive, but when it is small enough, it will also have a huge impact on simulation results; and 4) the parameter $e$ and $\beta$ in NMCL1 method is the least sensitive one, the change of parameter $e$ will lead to the linear correlation between the change of outflow and exchange volume. It can be concluded that exponential parameters have the greatest influence on simulation results and the proportion of inflow and outflow in the storage also has great influence on the simulation results. Besides, the storage parameter $K$ cannot take too small value. The value of parameter $m$ should be close to 1.

Although six methods are compared in this study, some parameters in different methods have the same meaning. For example, for the flow data documented in Wilson (1974) and Viessman and Lewis (2003) where the lateral inflow is small, the parameters optimized by the PSO algorithm are similar for different methods. However, in River Wyre (O'Donnell, 1985) with a large lateral inflow, the best-fit parameters are quite different in different methods. Although NMCL2 can simulate better results, its optimal parameters cannot be explained by the definition of parameters in traditional MC method. In order to better meet the observed value, it is necessary to change the range of parameters. The traditional MC method only considers the relationship between river storage and inflow/outflow, but does not consider the influence of
lateral inflow on river discharge. Therefore, the definition and selection range of parameters in traditional MC method are mainly applied to single flood without lateral inflow. If the traditional MC methods are used to simulate the flooding processes with significant lateral inflow and multiple flood, the results will produce large errors. In order to get better simulation results, some parameters will exceed the range defined by MC theory. Therefore, the optimal range of parameters simulated in this paper is not limited to the traditional parameters, and some parameter anomalies obtained can just show the discrepancy between the flood process and the traditional MC assumption. These outliers cannot be simply eliminated. How to ensure the accuracy while explaining the physical meaning of parameters is the future direction of parameter estimation and model improvement.

Besides, due to the limited data, only a few examples were selected in this study. In the parameter sensitivity analysis, only one case with obvious lateral inflow was analyzed, while three cases were considered in methodology comparison. Further discussion is needed by considering different flood events.

6. Conclusions

In this paper, new MC methods used in the modeling of flood events with lateral flow contribution are proposed. The nonlinear relationship between lateral inflow and river inflow is considered, as well as the constant term of lateral flow due to GW-SW interaction. Besides, while simulating the flood outflow routing, the sum of the vertical inflow due to GW-SW exchange and the conventional lateral inflow is also estimated. Three kinds of representative cases Wilson (1974), Wyre (1985), and Viessman (2003) are selected from the literature, and the simulation results of the proposed methods are compared with the traditional MC methods.
According to the simulation results, the improved model is better than the traditional method in fitting the flood hydrograph. Among them, the NMCL1 method has the smallest error and can get the best simulation results.

Moreover, two different floods in the River Wye are selected to verify the calibrated model. According to the data of one flood, the optimal parameters are calculated as the parameter conditions of the other flood simulation, and three improved methods are used to simulate the flood hydrograph. The outflow hydrographs of two floods were successfully simulated.

The case with large lateral inflow (Wyre) was selected for parameter sensitivity analysis. According to the sensitivity analysis, the influence of different parameters on the results is further understood. High sensitivity parameters and low sensitivity parameters were distinguished. Exponential parameters have the greatest influence on simulation results and the proportion of inflow and outflow in the storage also has great influence on the simulation results. For high sensitive parameters, parameter estimation is needed before simulation, while for low sensitive parameters, empirical values can be selected according to the range of parameter analysis results. In order to get better simulation results, the optimal range of parameters simulated in this paper is larger than the theoretical range. These parameter anomalies obtained can just show the discrepancy between the flood process and the traditional MC assumption.

How to ensure the accuracy while explaining the physical meaning of parameters is the future direction of parameter estimation and model improvement.

Through different classic cases and a real field, the accuracy and reliability of the new MC methods are verified. The proposed methods put forward a direction of improvement of MC methods considering lateral flow. Moreover, the new numerical simulation method to calculate the sum of the vertical inflow due to GW-SW exchange and the conventional lateral inflow is
important of studying river ecosystem and the sustainable management of water resources.
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References

Adams B, Bloomfield JP, Gallagher AJ, Jackson CR, Rutter HK Williams AT (2010) An early warning system for groundwater flooding in the Chalk. Q J Eng Geol Hydroge 43:185-193

Alley WM, Healy RW, LaBaugh JW, Reilly TE (2002) Hydrology - Flow and storage in groundwater systems.
Ayvaz MT Gurarslan G (2017) A new partitioning approach for nonlinear Muskingum flood routing models with lateral flow contribution. J Hydrol 553:142-159

Barbetta S, Moramarco T, Perumal M (2017) A Muskingum-based methodology for river discharge estimation and rating curve development under significant lateral inflow conditions. J Hydrol 554:216-232

Beven KJ (2001) Rainfall-runoff modelling: The primer. Wiley, New York

Boano F, Harvey JW, Marion A, Packman AI, Revelli R, Ridolfi L, Worman A. (2014) Hyporheic flow and transport processes: Mechanisms, models, and biogeochemical implications. Rev Geophys 52:603-679

Cardenas MB (2008) Surface water-groundwater interface geomorphology leads to scaling of residence times. Geophys Res Lett 35:L08402

Chow VT, Maidment DR, Mays LW (1988) Applied Hydrology. McGraw-Hill Book Company, New York

Cunge JA (1969) On the subject of a flood propagation computation method (Muskingum Method). J Hydraul Res 7:205-230

Doherty J (2004) PEST model-independent parameter estimation users manual. Watermark Numerical Computing, Brisbane, Australia

Easa SM (2013) Improved nonlinear Muskingum model with variable exponent parameter. J Hydrol Eng 18:1790-1794

Easa SM (2014) Closure to “Improved nonlinear Muskingum model with variable exponent parameter” by Said M. Easa. J Hydrol Eng 19:07014008

Eberhart R, Kennedy J (1995) A new optimizer using particle swarm theory. In: Proceedings of the Sixth International Symposium on Micro Machine and Human Science. Nagoya, Japan 39-43

Ford R (2005) Water/surface-water interactions on contaminant transport with application to an arsenic contaminated site, Environmental Research Brief. United States Environmental Protection Agency, Report number: EPA/600/S-05/002

Geem ZW (2006) Parameter estimation for the nonlinear Muskingum model using the BFGS technique. J Irrig Drain Eng 132:474-478

Geem ZW (2014) Issues in optimal parameter estimation for the nonlinear Muskingum flood routing model. Eng Optimiz 46:328-339
Gill MA (1978) Flood routing by the Muskingum method. J Hydrol 36:353-363
Gotkowitz MB, Attig JW, McDermott T (2014) Groundwater flood of a river terrace in southwest Wisconsin, USA. Hydrogeol J 22:1421-1432
Haddad OB, Hamedi F, Fallah-Mehdipour E, Orouji H, Mariño MA (2015) Application of a hybrid optimization method in Muskingum parameter estimation. J Irrig Drain Eng 141:04015026
Hatch CE, Fisher AT, Revenaugh JS, Constantz J, Ruehl C (2006) Quantifying surface water-groundwater interactions using time series analysis of streambed thermal records: Method development. Water Resour Res 42:W10410
Hughes AG, Vounaki T, Peach DW, Ireson AM, Jackson CR, Butler AP, Bloomfield JP, Finch J, Wheater HS (2011) Flood risk from groundwater: Examples from a Chalk catchment in southern England. J Flood Risk Manag 4:143-155
Irvine DJ, Lautz LK, Briggs MA, Gordon RP, McKenzie JM (2015) Experimental evaluation of the applicability of phase, amplitude, and combined methods to determine water flux and thermal diffusivity from temperature time series using VFLUX 2. J Hydrol 531:728-737
Kabala ZJ (2001) Sensitivity analysis of a pumping test on a well with wellbore storage and skin. Adv Water Resour 24:483-504
Kang L, Zhou LW, Zhang S (2017) Parameter estimation of two improved nonlinear Muskingum models considering the lateral flow using a hybrid algorithm. Water Resour Manag 31:4449-4467
Karahan H (2012) Predicting Muskingum flood routing parameters using spreadsheets. Comput Appl Eng Educ 20:280-286
Karahan H (2014) Discussion of “Improved nonlinear Muskingum model with variable exponent parameter” by Said M. Easa. J Hydrol Eng 19:07014007
Karahan H, Gurarslan G, Geem ZW (2013) Parameter estimation of the nonlinear Muskingum flood-routing model using a hybrid harmony search algorithm. J Hydrol Eng 18:352-360
Karahan H, Gurarslan G, Geem ZW (2015) A new nonlinear Muskingum flood routing model incorporating lateral flow. Eng Optimiz 47:737-749
Kim JH, Geem ZW, Kim ES, (2001) Parameter estimation of the nonlinear Muskingum model using harmony search. J Am Water Resour As 37:1131-1138
Krause S, Hannah DM, Fleckenstein JH, Heppell CM, Kaeser D, Pickup R, Pinay G, Robertson AL, Wood PJ (2011) Inter-disciplinary perspectives on processes in the hyporheic zone. Ecohydrology 4:481-499

Linn F, Masie M, Rana, A (2003) The impacts on groundwater development on shallow aquifers in the lower Okavango Delta, northwestern Botswana. Environ Geol 44:112-118

Linsley R, Kohler MA, Paulhus J (2003) Hydrology for engineers. McGraw-Hill, New York

Lu C, Ji K, Zhang Y, Fleckenstein JH, Zheng C, Salsky K (2020) Event-Driven Hyporheic Exchange during Single and Seasonal Rainfall in a Gaining Stream. Water Resour Manag 34:4617-4631

Moghaddam A, Behmanesh J, Farsijani A (2016) Parameters estimation for the new four-parameter nonlinear Muskingum model using the particle swarm optimization. Water Resour Manag 30:2143-2160

Mohan S (1997) Parameter estimation of nonlinear muskingum models using genetic algorithm. J Hydraul Eng-Asce 123:137-142

Niazkar M, Afzali SH (2015) Assessment of modified honey bee mating optimization for parameter estimation of nonlinear Muskingum models. J Hydrol Eng 23: 04014055

O'Donnell T (1985) A direct three-parameter Muskingum procedure incorporating lateral inflow. Hydrol Sci J 30:479-496

Ouyang AJ, Liu LB, Sheng, Z, Wu F (2015) A class of parameter estimation methods for nonlinear Muskingum model using hybrid invasive weed optimization algorithm. Math Probl Eng 2015:573894

Perumal M, Moramarco T, Sahoo B, Barbeta S (2010) On the practical applicability of the VPMS routing method for rating curve development at ungauged river sites. Water Resour Res 46:W03522

Perumal M, Price RK (2013) A fully mass conservative variable parameter McCarthy-Muskingum method: Theory and verification. J Hydrol 502:89-102

Ponce VM, Chaganti PV (1994) Variable parameter Muskingum-Cunge method revisited. J Hydrol 162:433-439

Price RK (2009) Volume conservative non-linear flood routing. J.Hydraul/Eng. ASCE 135:838-845

Rodriguez-Iturbe I, Valdés JB (1979) The geomorphologic structure of hydrologic response. Water Resour Res 15:1409-1420

Shuai P, Cardenas MB, Knappett PSK, Bennett PC, Neilson BT (2017) Denitrification in the banks of fluctuating rivers: The effects of river stage amplitude, sediment hydraulic conductivity and dispersivity, and ambient groundwater flow. Water Resour Res 53:7951-7967
Sophocleous M (2002) Interactions between groundwater and surface water: The state of the science. Hydrogeol J 10:52-67

Todini E (2007) A mass-conservative and water storage consistent variable parameter Muskigum-Cunge approach. Hydrol. Earth Syst Sci 11:1645-1659

Tung YK (1985) River flood routing by nonlinear Muskingum method. J Hydraul Eng-ASCE 111:1447-1460

Viessman W, Lewis GL (2003) Introduction to hydrology. Prentice Hall India (P) Limited, New Jersey

Welch C, Cook PG, Harrington GA, Robinson NI (2013) Propagation of solutes and pressure into aquifers following river stage rise. Water Resour Res 49:5246-5259

Wilson EM (1974) Engineering hydrology. Wiley, New York

Winter T, Harvey J, Franke O, Alley W (1998) Ground water and surface water a single resource. U.S. Geological Survey, 1139

Yoon J, Padmanabhan G (1993) Parameter Estimation of Linear and Nonlinear Muskingum Models. J Water Res Plan Man 119:600-610
Table 1. The simulation ranges and sensitivities of different parameters of the two methods. The contents in brackets represent units, “*” represents dimensionless unit, “\" represents inapplicability for the sensitivity.

| Method | NMCL1 | | | NMCL2 | | |
|--------|-------| | |-------| | |
| Parameter | $x$ | $K$ | $m$ | $e$ | $\beta$ | $x$ | $K$ | $m$ | $e$ | $\beta$ |
| Unit | * | h | * | m$^3$/s | * | * | h | * | m$^3$/s | * |
| Range | 0.9~0.9 | 15~ | 0.92~ | 0~ | 1.35~ | -0.9~ | 0.92~ | 2.3~ | 1.04~ |
| | 35 | 1.12 | 10 | 3.35 | 0.9 | 1~21 | 1.12 | 12.3 | 1.44 |
| Sensitivity for Outflow | 63962 | 1453 | 150316 | 376 | 2081 | 106468 | 7329 | 149125 | 175 | 33077 |
| Sensitivity for Interaction | \ | \ | \ | 227 | 6673 | \ | \ | \ | | 254 | 56432 |
Table 2. Three error objective functions using six MC methods in three cases of flooding. The contents in brackets represent units, ‘*’ represents dimensionless unit.

| Datasets         | Objective function | MC1  | MC2  | NMC1 | MCL1  | NMCL1 | NMCL2 |
|------------------|--------------------|------|------|------|-------|-------|-------|
| Wilson [1974]    | SSQ(m⁶/s²)         | 605.63 | 448.45 | 178.98 | 19.59  | 17.55  | 24.11  |
|                  | NSE(*)             | 0.9504 | 0.9633 | 0.9854 | 0.9984 | 0.9986 | 0.9980 |
|                  | RMSE(m³/s)         | 5.247 | 4.515 | 2.852 | 0.944  | 0.893  | 1.047  |
| O'Donnell [1985] | SSQ(m⁶/s²)         | 50686.00 | 13618.00 | 29043.00 | 122.09 | 98.01  | 125.13 |
|                  | NSE(*)             | -0.7425 | 0.5318 | 0.0015 | 0.9958 | 0.9966 | 0.9957 |
|                  | RMSE(m³/s)         | 39.799 | 20.629 | 30.126 | 1.953  | 1.750  | 1.977  |
| Viessman and Lewis [2003] | SSQ(m⁶/s²) | 124310.00 | 110620.00 | 82227.00 | 73811.00 | 71708.00 | 73774.00 |
|                  | NSE(*)             | 0.9714 | 0.9745 | 0.9811 | 0.9830 | 0.9835 | 0.9830 |
|                  | RMSE(m³/s)         | 71.969 | 67.891 | 58.533 | 55.457 | 54.661 | 55.443 |
Figure Caption

Figure 1. Flood routing simulation at the River Wyre using the NMCL1 and NMCL2 models. $Q_{in}$ and $Q_{out}$ are the measured inflow and outflow, respectively. $Q_{NMCL1}$ and $Q_{NMCL2}$ are the best-fit outflow using the NMCL1 model and the NMCL2 model, respectively, and $Q_{EX}$ represents the sum of the transient/conventional lateral inflow and the stable inflow due to GW-SW interaction.

Figure 2. Results of the parameter sensitivity analysis. $Q_{out}$ is the measured outflow. $Q_{NMCL1}$ and $Q_{NMCL2}$ are the best-fit outflow with the optimized parameters, and $Q_{EX}$ represents the sum of the transient/conventional lateral inflow and the stable inflow/vertical inflow due to GW-SW exchange. (a)–(e) represent the influence of parameters $x$, $K$, $m$, $e$ and $\beta$ on the modeled outflow, respectively. (f)–(g) represent the influence of parameters $e$ and $\beta$ on the estimated exchange volume between GW and SW, respectively. The number in the upper left corner of each figure “1” and “2” represent the method NMCL1 and NMCL2, respectively.

Figure 3. The measured versus the modeled results for the flow data from Wilson (1974). $Q_{in}$ and $Q_{out}$ are the measured inflow and outflow, respectively. $Q_{MC1}$ is the estimated outflow using MCL1, $Q_{MC2}$ is the estimated outflow using MC2, $Q_{NMC1}$ is the estimated outflow using NMC1, $Q_{MCL1}$ is the estimated outflow using MCL1, $Q_{NMCL1}$ is the estimated outflow using NMCL1, $Q_{NMCL2}$ is the estimated outflow using NMCL2. $Q_{EX}$ represents the sum of the transient/conventional lateral inflow and the vertical inflow due to the interaction between GW and SW.

Figure 4. The measured versus the simulated flow for River Wyre from O’Donnell (1985). $Q_{in}$
and $Q_{\text{out}}$ are the measured inflow and outflow, respectively. $Q_{\text{MC1}}$ is the estimated outflow using MCL1, $Q_{\text{MC2}}$ is the estimated outflow using MC2, $Q_{\text{NMC1}}$ is the estimated outflow using NMC1, $Q_{\text{MCL1}}$ is the estimated outflow using MCL1, $Q_{\text{NMC1}}$ is the estimated outflow using NMCL1, $Q_{\text{NMCL2}}$ is the estimated outflow using NMCL2. $Q_{\text{EX}}$ represents the sum of the transient/conventional lateral inflow and the vertical inflow due to the interaction between GW and SW.

Figure 5. The measured versus the modeled results for the flow data from Viessman and Lewis (2003). $Q_{\text{in}}$ and $Q_{\text{out}}$ are the measured inflow and outflow, respectively. $Q_{\text{MC1}}$ is the estimated outflow using MCL1, $Q_{\text{MC2}}$ is the estimated outflow using MC2, $Q_{\text{NMC1}}$ is the estimated outflow using NMC1, $Q_{\text{MCL1}}$ is the estimated outflow using MCL1, $Q_{\text{NMC1}}$ is the estimated outflow using NMCL1, $Q_{\text{NMCL2}}$ is the estimated outflow using NMCL2. $Q_{\text{EX}}$ represents the sum of the transient/conventional lateral inflow and the vertical inflow due to the interaction between GW and SW.

Figure 6. The measured versus the modeled results for the two different flow data of the River Wye from O’Donnell (1985). $Q_{\text{in}}$ and $Q_{\text{out}}$ are the measured inflow and outflow, respectively. $Q_{\text{MCL1}}$ is the estimated outflow using MCL1, $Q_{\text{NMC1}}$ is the estimated outflow using NMCL1, $Q_{\text{NMCL2}}$ is the estimated outflow using NMCL2. $Q_{\text{EX}}$ represents the sum of the transient/conventional lateral inflow and the vertical inflow due to the interaction between GW and SW.
Figure 7. Flux across the four cross sections S1, S2, S3 and S4. The average flux of each section is estimated by VFLUX using the measured temperature at these four sections. The negative flux means upward flow, and the positive flux means downward flow.

Figure 8. The measured versus the simulated flow for Zhongtian River. $Q_{in}$ and $Q_{out}$ are the measured inflow and outflow, respectively. $Q_{MC1}$ is the estimated outflow using MCL1, $Q_{MC2}$ is the estimated outflow using MC2, $Q_{NMC1}$ is the estimated outflow using NMC1, $Q_{NMCL1}$ is the estimated outflow using MCL1, $Q_{NMCL2}$ is the estimated outflow using NMCL2. $Q_{EX}$ represents the sum of the lateral inflow and the flow due to GW-SW interaction.

Figure 9. Boxplot of the GW-SW exchange volume. The GW-SW exchange volume and the sum of lateral and vertical inflow estimated by MCL1, NMCL1, and NMCL2 are analyzed by the box plot. The green box represents the data in the middle of the upper and lower quartiles. The middle line in the green box represents the median, and the small box in the middle represents the average.
Figure 1
Figure 2
Figure 3
Figure 4
Figure 7

Average Flux (×10^{-6} m/s)

Time (h)

downward flow
upward flow

S1
S2
S3
S4
Figure 8
Figure 9

![Box plot showing flow (m³/s) for MCL1, NMCL1, and NMCL2. The plot includes the 25% to 75% range, median line, mean, and outliers.]