Flavor-dependent $CP$ violation and electroweak baryogenesis in supersymmetric theories

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Abstract

We analyze electroweak baryogenesis in supersymmetric theories with flavor-dependent $CP$-violating phases. We generalize the standard approach to include the flavor dependence of the $CP$-violating sources and obtain an analytical approximate expression for the baryon asymmetry of the universe induced by these sources. It is shown that in the framework where the $\mu$-term is real and the chargino sources vanish, large flavor mixing might lead to a substantial baryon asymmetry through the squark contributions, once the condition to have a strong first-order phase transition induced by light right-handed up squarks is relaxed. We derive model independent bounds on the relevant up-squark left-right mass insertions. We show that in supersymmetric models with non-minimal flavor structure these bounds can be reached and the required baryon asymmetry can be generated, while satisfying the constraints coming from the electric dipole moments.

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I. INTRODUCTION

During the last few years, the data collected from the acoustic peaks in the cosmic microwave background radiation [1] has allowed to obtain a more precise measurement of the baryon asymmetry of the universe (BAU). This is expected to further improve in the near future with the MAP experiment [2] and the PLANCK satellite [3]. At the present time, the measurement of the baryon-to-entropy ratio is

\[ 0.7 \times 10^{-10} \lesssim \frac{n_B}{s} \lesssim 1.0 \times 10^{-10}, \]

where \( s = 2\pi^2 g_s T^3 / 45 \) is the entropy density and \( g_s \) is the effective number of relativistic degrees of freedom.

It is well known that in order to obtain an asymmetry starting from a symmetric state with a vanishing baryon number, three requirements must be satisfied: baryon number violation, \( C \) and \( CP \) violation, and departure from thermal and kinetic equilibrium [4]. In the standard model (SM) of electroweak interactions, the main source of \( CP \) violation comes from the phase \( \delta_{CKM} \) in the Cabibbo-Kobayashi-Maskawa (CKM) quark mixing matrix. Although this phase is able to account for the experimentally observed \( CP \) violation in the neutral \( K \)-mesons and, as recently observed, in the \( B_d \) system, it has been shown that it is not possible to generate sufficient BAU through \( \delta_{CKM} \) [5]. Furthermore, the strength of the phase transition is too weak in the SM and the universe is approximately in equilibrium [6] (for reviews on electroweak baryogenesis, see for instance Refs. [7, 8]).

In the context of supersymmetric (SUSY) extensions of the SM, it has been pointed out that the above problems can be in principle overcome [9]. Moreover, in the presence of light stops the electroweak phase transition can be strong enough for baryogenesis to take place [10–16]. Moreover, SUSY models contain new \( CP \)-violating sources beyond \( \delta_{CKM} \), namely the Higgs bilinear term, \( \mu \), and the soft breaking terms (gaugino and squark soft masses, bilinear and trilinear couplings). These can be classified as flavor-blind or flavor-dependent. The first category includes the phases of the \( \mu \)- and \( B \)-parameters, of the gaugino masses and the overall phase of the trilinear couplings \( A_{ij} \). Two of these phases can be eliminated by \( U(1)_R \) and \( U(1)_{PQ} \) transformations. The second category contains the phases of \( A_{ij} \) (after the overall phase is factored out), as well as the ones appearing in the off-diagonal elements of the soft squark masses. The low energy implications of these flavor-dependent phases on \( K \) and \( B \) meson \( CP \)-violating observables, as well as in rare decays,
have been extensively studied in the literature [17–19]. The question that naturally arises is whether the new SUSY phases could significantly enhance the $CP$-violating sources, so that supersymmetric electroweak baryogenesis could account for the observed baryon asymmetry of the universe.

A considerable amount of work has been done concerning the implications of the new flavor-independent $CP$-violating phases on generating an acceptable value of the baryon asymmetry [20–25]. It has been shown that in the minimal supersymmetric extension of the SM (MSSM), if the relative phase $\phi_\mu$ between the gaugino soft mass and the $\mu$ term is not too small, $\phi_\mu \gtrsim 0.04$, a considerable BAU can be generated through the scattering of the charginos with the bubble wall. However, the non-observation of the electric dipole moment (EDM) of the electron, neutron and mercury atom [26, 27] imposes severe constraints on the flavor-diagonal phases [28, 29], forcing them to be small. Since in this limit the theory does not acquire any new symmetry, one has to deal with a naturalness problem, and this is precisely the so-called SUSY $CP$ problem.

In particular, the EDM’s bound the phase $\phi_\mu$ to be $\lesssim 10^{-2}$, if the SUSY particle masses are not too heavy ($\lesssim 1$ TeV). It has recently been claimed that new contributions to the EDM of the electron could eventually rule out the electroweak baryogenesis scenario based on flavor-independent $CP$-violating phases [30]. A possible way to generate enough BAU while evading these constraints is to work in the heavy squark limit ($m_Q \sim 3$ TeV) [31].

However, if we assume that SUSY $CP$ violation has a flavor character such as in the SM [32], one is led to a scenario where all flavor conserving parameters as the $\mu$-term and gaugino masses are real. In this framework, the dominant sources of $CP$ violation that are relevant to electroweak baryogenesis are in general associated with the lightest of the right-handed up-squarks [33]. In the usual MSSM scenario with complex $\mu$ and gaugino masses, this contribution would be always subdominant, and could even be neglected when compared to that of the charginos and neutralinos [21, 22].

In this paper we study the effects of the flavor-dependent $CP$-violating phases on the mechanism of electroweak baryogenesis. We show that in generic SUSY models with non-universal soft SUSY breaking terms, and in particular with non-universal $A$-terms, the squark contributions to the BAU are far from being negligible. Although in this framework the $\mu$-term is real, flavor mixing might lead to a potentially large baryon asymmetry. As an example, we analyze the baryon asymmetry in supersymmetric models with Hermitian
flavor structures, a type of model that also provides an elegant solution for the EDM’s suppression [32, 34, 35].

Obtaining sufficiently accurate transport equations for particles propagating in the presence of a CP-violating bubble wall at the electroweak phase transition is crucial for the computation of the baryon asymmetry generated in the context of electroweak baryogenesis. In spite of the general consensus on the existence and nature of the CP-violating sources responsible for the baryon production, there is still a controversy in the literature in what concerns the strength and form of these sources as well as the transport equations to be used in the calculations [20–25]. For instance, the results obtained in the MSSM framework for the chargino and squark sources by making use of the continuity equations and the relaxation time approximation [21, 22] are different from those based on WKB-methods [23]. Furthermore, it has been claimed in Ref. [23, 24] that the dominant contribution to the chargino source, typically found within the MSSM and which is of the form \( \epsilon_{ij} v_i \partial_\mu v_j \) (\( v_{1,2} \) are the expectation values of the two Higgs doublets) [21, 22], is absent, thus leading to a suppressed baryon asymmetry [23, 24]. However, in a recent work [25] it is argued that such a suppression is in fact an artifact of the approximation used by the authors of Refs. [23, 24] in order to compute the CP-violating currents. In view of the above discussion, we adopt in the present work the approach of Refs. [20–22, 25] in order to derive the CP-violating currents and sources.

The paper is organized as follows. By using the closed time path formalism, we build in Section II the quantum Boltzmann equations to obtain the CP-violating sources relevant to baryogenesis and discuss their general flavor dependence. In Section III we compute the baryon asymmetry induced by these CP-violating sources. An analytical approximate expression for the asymmetry is also presented. In Section IV we obtain model independent bounds for the up-squark left-right mass insertions by requiring the baryon-to-entropy ratio to lie in the experimental range. We also consider some specific SUSY models with minimal and non-minimal flavor structure and discuss whether or not they can account for the required BAU. Our numerical results are presented in Section V. Finally, we present our concluding remarks in Section VI.
II. \textit{CP-Violating Sources for the Baryon Asymmetry}

Before computing the \textit{CP}-violating sources for the baryon asymmetry, we shall briefly recall some characteristics of non-equilibrium quantum field theory [36, 37]. During a first-order phase transition, the thermodynamical system is far from equilibrium. In order to keep an explicit time dependence, one should use the real-time finite temperature quantum field theory. The most used and powerful formalism to describe such a system is the so-called closed time path formalism (CTP), which is a generalization of the time contour integration to a closed time path. More precisely, the time integration is deformed to run from $-\infty$ to $+\infty$ and back to $-\infty$. The main effect of this closed time path is to double the field variables so that for each field we have four different real-time propagators on the contour.

In the case of a boson field $\phi$, we can write the corresponding Green functions in terms of the $2 \times 2$ matrices [38]

$$
\tilde{G}_\phi = \begin{pmatrix} G^t_\phi & G^<_\phi \\ G^>_\phi & -G^t_\phi \end{pmatrix},
$$

where

$$
G^>_\phi(x,y) = -i \langle \phi(x)\phi^\dagger(y) \rangle, \quad G^<_\phi(x,y) = -i \langle \phi^\dagger(y)\phi(x) \rangle,
$$

$$
G^t_\phi(x,y) = \theta(x,y) G^>_\phi(x,y) + \theta(y,x) G^<_\phi(x,y),
$$

$$
G^t_\phi(x,y) = \theta(y,x) G^>_\phi(x,y) + \theta(x,y) G^<_\phi(x,y).
$$

In what follows the subscript $\phi$ will be omitted to simplify our notation.

As mentioned in the introduction, an accurate computation of the \textit{CP}-violating sources responsible for electroweak baryogenesis is crucial to obtain a reliable estimate of the baryon asymmetry. Here we adopt the method developed by Kadanoff and Baym [36] to derive the quantum Boltzmann equations for a generic bosonic particle asymmetry. We shall compute the sources using the formalism based on CTP and the Dyson equations as described in Ref. [21].

In general it is possible to write the Dyson equation for any propagator $\tilde{G}$ as

$$
\tilde{G}(x,y) = \tilde{G}^0(x,y) + \int d^4x_1 \int d^4x_2 \tilde{G}^0(x,x_1) \tilde{\Sigma}(x_1,x_2) \tilde{G}(x_2,y) = \tilde{G}^0(x,y) + \int d^4x_1 \int d^4x_2 \tilde{G}(x,x_1) \tilde{\Sigma}(x_1,x_2) \tilde{G}^0(x_2,y),
$$

5
where $\tilde{G}^0$ are the non-interacting Green functions and $\tilde{\Sigma}$ are the self-energy functions, which can be also expressed in terms of the $2 \times 2$ matrices

$$
\tilde{\Sigma} = \begin{pmatrix} \Sigma^t & \Sigma^< \\ \Sigma^> & -\Sigma^\dagger \end{pmatrix},
$$

(6)
satisfying the relations

$$
\Sigma^t(x, y) = \theta(x, y) \Sigma^>(x, y) + \theta(y, x) \Sigma^<(x, y),
\Sigma^\bar{t}(x, y) = \theta(y, x) \Sigma^>(x, y) + \theta(x, y) \Sigma^<(x, y).
$$

(7)

Iterating Eq. (5) one finds

$$
\tilde{G}(x, y) = \tilde{G}^0(x, y) + \int d^4x_1 \int d^4x_2 \tilde{G}^0(x, x_1) \tilde{\Sigma}(x_1, x_2) \tilde{G}^0(x_2, y)
+ \int d^4x_1 \int d^4x_2 \int d^4x_3 \int d^4x_4 \tilde{G}^0(x, x_1) \tilde{\Sigma}(x_1, x_2) \tilde{G}^0(x_2, x_3) \tilde{\Sigma}(x_3, x_4) \tilde{G}^0(x_4, y)
+ \ldots .
$$

(8)

In order to compute the sources for the squark diffusion equations, we shall write the quantum Boltzmann equations (QBE) for a generic bosonic particle asymmetry,

$$
\frac{\partial n_\phi}{\partial T} + \nabla \cdot J_\phi = S ,
$$

(9)

where $n_\phi$ is the number density of particles minus antiparticles and $S$ is the associated $CP$-violating source. More specifically, we shall derive the QBE for the current

$$
\langle J_\phi^\mu(x) \rangle \equiv i \langle \phi^\dagger(x) \overleftarrow{\partial^\mu} \phi(x) \rangle \equiv (n_\phi(x), J_\phi(x)) ,
$$

(10)
or in terms of the Green functions,

$$
\langle J_\phi^\mu(x) \rangle = - \lim_{x \to y} (\partial^\mu_x - \partial^\mu_y) G^\leq(x, y) .
$$

(11)

It proves convenient to work in the coordinate system of the center of mass, which is defined by

$$
X = (T, \bar{R}) = \frac{1}{2} (x + y) , \quad \bar{x} = (t, \bar{r}) = x - y .
$$

Hence

$$
G^\leq(x, y) = G^\leq(t, \bar{r}, T, \bar{R}) = -i \langle \phi^\dagger(T - \frac{t}{2}, \bar{R} - \frac{\bar{r}}{2}) \phi(T + \frac{t}{2}, \bar{R} + \frac{\bar{r}}{2}) \rangle .
$$
The next step is to find a solution for $G^<$ when the system is not in equilibrium. This can be achieved by applying the operator $(\nabla^2 + m^2)$ on both sides of the equivalent representations of the Schwinger-Dyson equation (5):

$$ (\nabla^2_x + m^2) \tilde{G}(x, y) = \delta^4(x, y) \mathbb{1} + \int d^4x_1 \tilde{\Sigma}(x, x_1) \tilde{G}(x_1, y), \quad (12) $$

$$ \tilde{G}(x, y) (\nabla^2_y + m^2) = \delta^4(x, y) \mathbb{1} + \int d^4x_1 \tilde{G}(x, x_1) \tilde{\Sigma}(x_1, y), \quad (13) $$

where $\mathbb{1}$ is the identity matrix. Since $(\Sigma G)^< = \Sigma^t G^< - \Sigma^< G^t$ and $(G \Sigma)^< = G^t \Sigma^< - G^< \Sigma^t$, the equations for the $G^<$ component read as

$$ (\nabla^2_x + m^2) \tilde{G}^<(x, y) = \int d^4x_1 [\Sigma^t(x, x_1) G^<(x_1, y) - \Sigma^<(x, x_1) G^t(x_1, y)] , \quad (14) $$

$$ \tilde{G}^<(x, y) (\nabla^2_y + m^2) = \int d^4x_1 [G^t(x, x_1) \Sigma^<(x_1, y) - G^<(x, x_1) \Sigma^t(x_1, y)] . \quad (15) $$

The variation of the current is thus given by

$$ \frac{\partial J^\mu_\phi(X)}{\partial X^\mu} = -\partial^X \left\{ \lim_{x \to y} (\partial^\mu_x - \partial^\mu_y) G^<(x, y) \right\} $$

$$ = - \int d^4x_1 [\Sigma^t(X, x_1) G^<(x_1, X) - \Sigma^<(X, x_1) G^t(x_1, X)] $$

$$ - G^t(X, x_1) \Sigma^<(x_1, X) + G^<(X, x_1) \Sigma^t(x_1, X) \right\} . \quad (16) $$

Using Eqs. (4) and (7), we can compute the $CP$-violating source for a generic squark current:

$$ S = \frac{\partial n_\phi(X)}{\partial T} + \nabla \cdot J_\phi(X) = - \int d^4r' \int_{-\infty}^T dt' \left\{ \Sigma^>(X, x') G^<(x', X) - \Sigma^<(X, x') G^>(x', X) $$

$$ - G^>(X, x') \Sigma^<(x', X) + G^<(X, x') \Sigma^>(x', X) \right\} . \quad (17) $$

Inserting the interactions of the $\phi$ field with the background, the self-energy functions $\Sigma^>(x, y)$ can be written as follows

$$ \Sigma^>(x, y) = g(x) \delta^4(x - y) + g(x) G_{0>}^0(x, y) g(y) + \int d^4z \left[ g(x) \tilde{G}_{0>}^0(x, z) g(z) \tilde{G}_{0>}^0(z, y) g(y) \right]^> $$

$$ + \int d^4w \int d^4z \left[ g(x) \tilde{G}_{0>}^0(x, w) g(w) \tilde{G}_{0>}^0(w, z) g(z) \tilde{G}_{0>}^0(z, y) g(y) \right]^> + \ldots , \quad (18) $$

where the scalar function $g(x)$ describes the interaction of the field $\phi$ at position $x$ with the background fields.
Taking into account the chiral and flavor structures of the squarks, this generic squark source can be written as

\[
S_{ij}^{AB} = - \int d^3r' \int_0^T dt' \left\{ [g(X)]_{ij}^{AB} \, \delta^4 (X - x') \, \delta_{ij}^{AB} \, [G^< (x', X)]_j^B - [g(X)]_{ij}^{AB} \, \delta^4 (X - x') \, \delta_{ij}^{AB} \, [G^> (x', X)]_j^B \right. \\
- [G^> (X, x')]_i^A \, [g(x')]_{ij}^{AB} \, \delta^4 (X - x') \, \delta_{ij}^{AB} + [G^< (X, x')]_i^A \, [g(x')]_{ij}^{AB} \, \delta^4 (X - x') \, \delta_{ij}^{AB} \\
+ [G^< (X, x')]_i^A \, [g(x')]_{ik}^{AC} \, [G^< (x', X)]_k^C \, [g(X)]_{kj}^{CB} \, [G^< (x', X)]_j^B \\
- [G^> (X, x')]_i^A \, [g(x')]_{ik}^{AC} \, [G^> (x', X)]_k^C \, [g(X)]_{kj}^{CB} \\
+ [G^< (X, x')]_i^A \, [g(x')]_{ik}^{AC} \, [G^> (x', X)]_k^C \, [g(X)]_{kj}^{CB} + \mathcal{O}([g(x)]^3) \right\} , \tag{19}
\]

where \((i, j, k)\) are flavor indices and \((A, B, C)\) refer to \(L, R\) chiralities. In the above expression, we have kept only terms up to second order in \(g(x)\). Note that this formula is valid for squarks of any flavor and/or chirality. Thus, we have

\[
\delta^X_\mu J^\mu_\phi^A = \delta_{AB} \, \delta_{ij} \, S_{ij}^{AB} = S_{ii}^{AA} , \tag{20}
\]

with \(\phi^L_\mu \equiv \tilde{U}_L_i\) and \(\phi^R_\mu \equiv \tilde{U}_R_i\), the left and right-handed up-squark fields, respectively. As expected, the contributions of the terms proportional to \(\delta^4 (X - x')\) vanish\(^1\). The structure of the source can be symbolically depicted as in Fig. 1, where the \(\otimes\) denotes interactions with the background fields, parametrized by the scalar function \(g(x)\).

Before proceeding, let us consider the Higgs-squark interaction terms, arising from both SUSY conserving and SUSY breaking terms, that can be parametrized in the following way (flavor indices are omitted):

\[
\mathcal{L}_{\tilde{q}} = \tilde{U}_L^* [g(X)]^{LL} \tilde{U}_L + \tilde{U}_R^* [g(X)]^{RR} \tilde{U}_R + (\tilde{U}_L^* [g(X)]^{LR} \tilde{U}_R + \text{H.c}) . \tag{21}
\]

Note that in addition to the interactions with the background, the functions \([g(X)]^{AB}\) contain flavor and chirality mixing terms that originate from the soft breaking SUSY Lagrangian and become relevant when one computes higher order terms like those appearing in Fig. 1(b).

\(^1\) Throughout we are using the relations: \(G^< (x) = G^> (-x)\), \([G^< (x)]^* = -G^< (-x)\), \([G^> (x)]^* = -G^> (-x)\), which imply \(G^< (x) = [G^> (x)]^*\).
Within a generic supersymmetric extension of the SM, the functions $g(X)$ can be parametrized as follows:

$$
[g(X)]_{ij}^{LL} = \left( M_Q^2 + v_2^2(X) h_u^* h_u^T + D_L^2(X) \right)_{ij},
$$

$$
[g(X)]_{ij}^{RR} = \left( M_{\tilde{U}}^2 + v_2^2(X) h_u^T h_u^* + D_R^2(X) \right)_{ij},
$$

$$
[g(X)]_{ij}^{LR} = (-v_1(X) \mu h_u^* + v_2(X) (Y_u^A)^*)_{ij},
$$

$$
[g(X)]^{RL} = [g(X)]^{LR\dagger}.
$$

(22)

In the above equations, $h_u$ denotes the Yukawa coupling matrix for the up quarks and $v_{1,2}(x)$ are the $x$-dependent vacuum expectation values of the MSSM Higgs fields $H_{1,2}$, defined as $v_1 = v \cos \beta / \sqrt{2}$ and $v_2 = v \sin \beta / \sqrt{2}$, with $\tan \beta = v_2/v_1$. $M_Q$ and $M_{\tilde{U}}$ are the squark soft breaking mass matrices, $\mu$ is the Higgs bilinear term and $Y_u^A$ the soft trilinear terms, which are decomposed as

$$
(Y_u^A)_{ij} \equiv (h_u)_{ij} A_u^{ij},
$$

(23)

with no summation over $i, j$.

The $D$-term contributions are given by

$$
D_L^2(X) = \left( \frac{1}{2} - \frac{2}{3} \sin^2 \theta_W \right) \frac{g^2 + g'^2}{2} \left( v_1^2(X) - v_2^2(X) \right),
$$

$$
D_R^2(X) = \left( \frac{2}{3} \sin^2 \theta_W \right) \frac{g^2 + g'^2}{2} \left( v_1^2(X) - v_2^2(X) \right),
$$

(24)

g, g'$ are the $SU(2)$ and $U(1)$ weak couplings, respectively. These terms can be neglected
at the electroweak phase transition since they are suppressed by the weak couplings when compared to the terms proportional to the top Yukawa coupling.

Since we are interested in performing a basis independent calculation, we should express the current as an invariant quantity. Let us define the invariant current as

\[ \partial_\mu^X J_{\phi A}^\mu = \sum_{i=1}^{n_f} S_{i}^{AA} = \text{Tr} S^{AA}, \]  

(25)

where \( n_f \) is the number of up-squark flavors.

Under a generic rotation that transforms the left- and right-handed up squarks as \( \bar{U}_L \rightarrow W_L \bar{U}_L, \bar{U}_R \rightarrow Z_R \bar{U}_R \), we have

\[ [G(x, y)^{>\prec}]^L \rightarrow W_L [G(x, y)^{>\prec}]^L W_L^\dagger \equiv [\bar{g}(x)]^L, \]
\[ [G(x, y)^{>\prec}]^R \rightarrow Z_R [G(x, y)^{>\prec}]^R Z_R^\dagger \equiv [\bar{g}(x)]^R, \]  

(26)

Thus the invariant source which is given by

\[ \text{Tr} S^{AA} = -\int d^3r' \int_{-\infty}^{T} dt' \sum_{i,k=1}^{n_f} \left\{ [g(X)]_{ik}^{AC} [G^{>\prec}(X, x')]_{k}^C [g(x')]_{ki}^{CA} [G^{<\prec}(x', X)]_{i}^A \right. \]
\[ - [g(X)]_{ik}^{AC} [G^{<\prec}(X, x')]_{k}^C [g(x')]_{ki}^{CA} [G^{>\prec}(x', X)]_{i}^A + (X \leftrightarrow x') + \ldots \}, \]  

(28)

can be rewritten in any weak basis. The above equation can be recast in the form

\[ \text{Tr} S^{AA} = -2i \int d^3r' \int_{-\infty}^{T} dt' \sum_{i,k=1}^{n_f} \left\{ \text{Im} \left( [\bar{G}^{<\prec}(X, x')]_{k}^{C} [\bar{G}^{<\prec}(x', X)]_{i}^{A} \right) \right. \]
\[ \times \left. \left( [\bar{g}(X)]_{ik}^{AC} [g(x')]_{ki}^{CA} - [\bar{g}(x')]_{ik}^{AC} [g(X)]_{ki}^{CA} \right) + \ldots \right\}. \]  

(29)

Finally we obtain

\[ \text{Tr} S^{RR} = 4 \int d^3r' \int_{-\infty}^{T} dt' (v_1(X) v_2(x') - v_2(X) v_1(x')) \sum_{i,k=1}^{n_f} \left\{ \text{Im} \left( [\bar{G}^{<\prec}(X, x')]_{k}^{L} [\bar{G}^{<\prec}(x', X)]_{i}^{R} \right) \right. \]
\[ \times \left. \text{Im} \left( \mu \left( Z_R (Y_u^A)_{ik} W_L^\dagger \right \ W_L h_u^* Z_R^\dagger \right) \right\} . \]  

(30)
A convenient basis for the current associated with the right-handed up-squark $\tilde{U}_{Ri}$ is that where the Green functions are diagonal. For simplicity, we shall assume that the left-handed squark mass matrix $\tilde{M}_{LL}$ is already diagonal. Therefore one has $W_L = 1$. Moreover, we will also assume that the left-handed squarks are heavy ($m_Q \gg m_t$) and nearly degenerate, so that $\tilde{M}_{LL} \simeq \text{diag}(m_Q, m_Q, m_Q)$ and $\tilde{G}(x)_k \equiv \tilde{G}(x)^L$ for any flavor. The right-handed squark mass matrix $\tilde{M}_{RR}$ is diagonalized by a unitary matrix $Z_R$ so that

$$Z_R \tilde{M}_{RR}^2 Z_R^\dagger = d_{RR}^2,$$

where $d_{RR} = \text{diag}(m_{R1}, m_{R2}, m_{R3})$. In this case,

$$\text{Tr } S^{RR} = 4 \int d^3 r' \int d t' (v_1(X) v_2(x') - v_2(X) v_1(x')) \sum_{i=1}^{n_f} \{ \text{Im} \left( \left[ G^<(X, x') \right]^L \left[ G^<(x', X) \right]^R \right) \} \times \text{Im} \left( \mu \left[ Z_R \left( Y_u^A \right)^T h_u^* Z_R^\dagger \right]_{ii} \right) \}.$$

In order to obtain an analytical expression, it is useful to perform an expansion in the bubble wall velocity $v_w$. Such an expansion is well justified in the case of the MSSM, since bubbles are typically formed with thick walls ($L_w \sim (10 - 100)/T$) and propagate with extremely nonrelativistic velocities ($v_w \sim 0.1 - 0.01$) [39, 40]. Using the derivative expansion

$$v_i(x) = \sum_{n=0}^{\infty} \frac{1}{n!} \left( \frac{\partial^n}{\partial X^\mu} \right)^n v_i(X) \left( x^\mu - X^\mu \right)^n,$$

it is easy to check that the first nonzero contribution to Eq. (32) is given by the $n = 1$ term in the expansion (33). The associated contribution to the source is proportional to the function

$$v_1(X) \partial_X^\mu v_2(X) - v_2(X) \partial_X^\mu v_1(X) \equiv \frac{1}{2} v^2(X) \partial_X^\mu \beta(X),$$

which in turn implies that the source (32) is linear in the wall velocity $v_w$. Neglecting terms of higher order in $v_w$, which amounts to using the thermal equilibrium Green functions, one can obtain an explicit expression for the dominant contribution:

$$\text{Tr } S^{RR} = v^2(X) \partial_X \beta(X) \left( \sum_{i=1}^{n_f} I_{RR} \text{Im} \left( \mu \left[ Z_R \left( Y_u^A \right)^T h_u^* Z_R^\dagger \right]_{ii} \right) \right),$$
with $I_{RR}^i$ given by [21]

$$
I_{RR}^i = \int_0^\infty dk \frac{k^2}{4\pi^2\omega_Q \omega_R} \left[ \left( 1 + 2 \text{ Re} \left( n_Q^i \right) \right) I(\omega_Q, \Gamma_Q, \omega_R, \Gamma_R) + \left( 1 + 2 \text{ Re} \left( n_R^i \right) \right) I(\omega_R, \Gamma_R, \omega_Q, \Gamma_Q) \right] \text{Im} \left( \left[ \omega_R^i, \Gamma_R^i, \omega_Q^i, \Gamma_Q^i \right] \right) \right). 
$$

The squark equilibrium distribution functions are

$$
n_{R,Q}^i = \frac{1}{\exp \left( \frac{\omega_{R,Q}^i}{T} + i\Gamma_{R,Q}^i / T \right) - 1}, \quad (37)
$$

where the finite widths $\Gamma_{R,Q}^i$ account for the interactions with the plasma,

$$
\omega_Q^2 = k^2 + m_Q^2 + \bar{m}_Q^2(\phi, T), \quad \omega_R^2 = k^2 + m_R^2 + \bar{m}_R^2(\phi, T), \quad (38)
$$

and $\bar{m}_{Q,R}^2(\phi, T)$ are the field-dependent contributions to the squark masses, which include the temperature dependent self-energies $\Pi_{Q,Ri}(T)$ [41]. The functions $I$ and $G$ can be written as follows:

$$
I(a, b, c, d) = I_+(a, b, c, d) + I_-(a, b, c, d), \quad G(a, b, c, d) = G_+(a, b, c, d) + G_-(a, b, c, d), \quad (39)
$$

$$
I_\pm(a, b, c, d) = \frac{1}{2} \frac{1}{(a \pm c)^2 + (b + d)^2} \sin \left( 2 \arctan \frac{a \pm c}{b + d} \right), \quad G_\pm(a, b, c, d) = \frac{1}{2} \frac{1}{(a \pm c)^2 + (b + d)^2} \cos \left( 2 \arctan \frac{a \pm c}{b + d} \right). \quad (40)
$$

Before proceeding to the explicit computation of the baryon asymmetry, it is worth emphasizing that from Eqs. (35) and (36), the dominant contribution to $\text{Tr} \mathcal{S}^{RR}_R$ will be associated with the lightest of the right-handed squarks, typically $\tilde{t}_R$. In this framework, usually known as the light stop scenario, to satisfy the out-of-equilibrium conditions the mass of the right-handed stop should be small enough so that at the electroweak phase transition temperature $T_{ew}$ one has $m_{\tilde{t}_R}(T_{ew}) \sim m_t(T_{ew}) = h_t v_2(T_{ew})$. This is precisely the case we shall consider from now on. In this context, the right-handed soft breaking masses are chosen so that $m_{R1}^2 \simeq m_{R2}^2 \simeq m_Q^2$, $m_{R3}^2 \equiv m_R^2 \ll m_Q^2$. In particular, the parameter $m_R^2$ could be negative provided that no charge or color breaking minima appear [10].

In this case,

$$
\text{Tr} \mathcal{S}^{RR} \simeq v^2(X) \partial_X \beta(X) I_{RR}^i \text{Im} \left( \mu \left[ Z_R (Y_u^A)^T h_u^* Z_R^i \right]_{33} \right). \quad (41)
$$
In the limit where the trilinear terms are flavor conserving, one can easily recognize the well-known expressions obtained in Refs. [21, 22].

III. ELECTROWEAK BARYOGENESIS

In the present scenario, the baryon asymmetry generated in the broken phase is determined by the density of left-handed quarks\(^2\), \(n_L\), created in front of the bubble wall in the symmetric phase [20–23]. Such densities induce weak sphalerons to produce a nonvanishing baryon number [42–44]. If the system is near thermal equilibrium and the particles are weakly interacting, the particle densities \(n_i\) are given by

\[
n_i = \frac{k_i \mu_i T^2}{6},
\]

where \(\mu_i\) are the local chemical potentials and \(k_i\) are statistical factors equal to 2 (1) for bosons (fermions) and exponentially suppressed for particles with masses \(m_i \gg T\).

Assuming the supergauge interactions to be in thermal equilibrium and neglecting all Yukawa couplings except those corresponding to the top quark, it is possible to express \(n_L\) in terms of the densities of the chiral supermultiplet \(Q_i \equiv (q, \tilde{q})\), \(n_{Q_i} = n_{q_i} + n_{\tilde{q}_i}\), and of the right-handed top quark and squark, \(T = t_R + \tilde{t}_R\). Indeed, assuming that all the quarks have nearly the same diffusion constant, one obtains due to the strong sphaleron processes, \(n_{Q_1} = n_{Q_2} = 2(n_Q + n_T)\), implying the relation

\[
n_L = n_{Q_1} + n_{Q_2} = 5n_Q + 4n_T.
\]

We can write down a set of coupled diffusion equations for the relevant densities \(n_Q\), \(n_T\), \(n_H = n_{H_1} + n_{H_2}\), and \(n_h = n_{H_2} - n_{H_1}\). Ignoring the curvature of the bubble wall, all the quantities become functions of \(z \equiv r + v_w t\), the coordinate normal to the bubble wall surface. In the rest frame of the bubble wall we obtain

\[
D_q n_Q'' - v_w n_Q' - \Gamma_y N_1 - 6\Gamma_{ss} N_2 - \Gamma_m N_3 + \gamma_Q = 0,
\]

\[
D_q n_T'' - v_w n_T' + \Gamma_y N_1 + 3\Gamma_{ss} N_2 + \Gamma_m N_3 - \gamma_Q = 0,
\]

\[
D_h n_H'' - v_w n_H' + \Gamma_y N_1 - \Gamma_h \frac{n_H}{k_H} = 0,
\]

\[
D_h n_h'' - v_w n_h' + \rho \Gamma_y N_4 - (\Gamma_h + 4\Gamma_\mu) \frac{n_h}{k_H} = 0,
\]

\(^2\) In principle, the baryon asymmetry is determined by the density of all left-handed fermions, including leptons. However, in the usual electroweak baryogenesis scenario, there is essentially no lepton asymmetry.
where, according to Eq. (41), the CP-violating squark current is given by
\[ \gamma_Q \simeq v_w v^2(z) \beta'(z) I_{RR}^I \text{Im} \left( \mu \left[ Z_R (Y_u^A)^T h_u^* Z_R^T \right]_{33} \right) . \]  
Moreover,
\[ N_1 = \frac{n_Q}{k_Q} - \frac{n_T}{k_T} - \frac{n_H + \rho n_h}{k_H} , \quad N_2 = 2 \frac{n_Q}{k_Q} - \frac{n_T}{k_T} + 9 \frac{n_Q + n_T}{k_B} , \]
\[ N_3 = \frac{n_Q}{k_Q} - \frac{n_T}{k_T} , \quad N_4 = \frac{n_Q}{k_Q} + \frac{n_T}{k_T} - \frac{n_H + n_h/\rho}{k_H} , \]
with the parameter \( \rho \) varying in the range from 0 to 1. The coefficients \( k_B = 3 , k_Q = 6 , k_T = 9 , k_H = 12 \) are statistical factors and the quantities
\[ \Gamma_y \simeq \frac{27\zeta(3)^2}{2\pi^4} h_t^2 \sigma_T \simeq 2.4 \times 10^{-2} T , \]
\[ \Gamma_{ss} = 16 \alpha_s^4 T \simeq 3.3 \times 10^{-3} T , \]
\[ \Gamma_m = \frac{v^2(T)}{21 T} h_t^2 \sin^2 \beta \simeq 4.8 \times 10^{-2} T , \]
\[ \Gamma_h = \frac{v^2(T)}{140 T} \simeq 7.1 \times 10^{-3} T , \]
\[ \Gamma_\mu \simeq 0.1 T , \]
are reaction rates: \( \Gamma_y \) corresponds to the SUSY trilinear scalar interaction involving the third generation squarks and the Higgs \( H_1 \) plus all SUSY and soft breaking trilinear interactions arising from the superpotential term \( h_t H_2 QT \), \( \Gamma_{ss} \) is the strong sphaleron rate, \( \Gamma_h \) and \( \Gamma_m \) arise from the Higgs and axial top number violating processes, while \( \Gamma_\mu \) corresponds to the Higgs bilinear term. The numerical estimates of these rates are obtained assuming the top quark Yukawa coupling \( h_t = 1 \), the strong coupling constant \( \alpha_s = 0.12 \), \( v(T) \simeq T \) and \( \tan \beta = 10 \). Finally, the diffusion coefficients are given by
\[ D_q \simeq \frac{6}{T} , \quad D_h \simeq \frac{110}{T} . \]

Let us assume that \( \Gamma_y \) and \( \Gamma_{ss} \) are fast enough so that \( \Gamma_y D_h , \Gamma_{ss} D_q , \Gamma_y D_q \gg v_w^2 \). In this case \( N_1 = \mathcal{O}(\Gamma_y^{-1}) , N_2 = \mathcal{O}(\Gamma_{ss}^{-1}) \) and
\[ \frac{n_Q}{k_Q} - \frac{n_T}{k_T} - \frac{n_H + \rho n_h}{k_H} \simeq 0 , \quad 2 \frac{n_Q}{k_Q} - \frac{n_T}{k_T} + 9 \frac{n_Q + n_T}{k_B} \simeq 0 . \]

This implies
\[ n_Q = \frac{k_Q(9k_T - k_B)}{k_H(k_B + 9k_Q + 9k_T)} (n_H + \rho n_h) , \]
\[ n_T = -\frac{k_T(9k_T + 2k_B)}{k_H(k_B + 9k_Q + 9k_T)} (n_H + \rho n_h) . \]
\[ N_3 \simeq \frac{n_H + \rho n_h}{k_H}, \quad N_4 \simeq N_1 - \frac{1 - \rho^2}{\rho} \frac{n_h}{k_H}. \]  

(54)

Thus, Eq. (46) can be rewritten as:

\[ D_h n''_h - v_w n'_h + \rho \Gamma y N_1 - \left[ (1 - \rho^2) \Gamma y + \Gamma_h + 4 \Gamma \mu \right] \frac{n_h}{k_H} = 0. \]  

(55)

First we notice that for typical values of the scattering rate due to the top quark Yukawa coupling [20], the interaction rate \( (1 - \rho^2) \Gamma y \) is fast enough in a wide range of values of \( \rho \) (except those values very close to 1), leading to the solution

\[ n_h \simeq 0. \]  

(56)

Indeed, assuming \( v_w \simeq 0.1 \), the condition \( (1 - \rho^2) \Gamma y D_h \gtrsim v_w^2 \) together with Eqs. (49)-(50) imply that \( \rho \lesssim 0.998 \).

Next, from Eqs. (52), (53) and (42) we find:

\[ n_H \simeq \frac{k_H (k_B + 9k_Q + 9k_T)}{k_Q (9k_T - k_B)} n_Q, \]  

(57)

\[ n_T \simeq -\frac{k_T 9k_Q + 2k_B}{k_Q 9k_T - k_B} n_Q, \]  

(58)

\[ n_L \simeq \frac{9k_Q k_T - 5k_Q k_B - 8k_T k_B}{k_Q (9k_T - k_B)} n_Q. \]  

(59)

Taking a linear combination of Eqs. (43)-(45), which is independent of the fast rates \( \Gamma_y \) and \( \Gamma_{ss} \) we obtain an effective diffusion equation for \( n_Q \):

\[ D n''_Q - v_w n'_Q - \Gamma n_Q + \gamma = 0, \]  

(60)

where

\[ D = \frac{1}{\Delta} \left[ D_q (9k_Q k_T + k_Q k_B + 4k_T k_B) + D_h k_H (k_B + 9k_Q + 9k_T) \right], \]  

(61)

\[ \Gamma = \frac{k_B + 9k_Q + 9k_T}{\Delta} (\Gamma_m + \Gamma_h) \theta(z), \]  

(62)

\[ \gamma = \frac{k_Q (9k_T - k_B)}{\Delta} \gamma_Q \theta(z), \]  

(63)

\[ \Delta = 9k_Q k_T + k_Q k_B + 4k_T k_B + k_H (k_B + 9k_Q + 9k_T), \]  

(64)

and \( \theta(z) \) is the step function, which accounts for the fact that the rates \( \Gamma_m \) and \( \Gamma_h \) are active only in the broken phase.
To estimate the baryon asymmetry we need to solve Eq. (60) in the symmetric phase 
\((z \leq 0)\). Imposing the boundary conditions \(n_Q(\pm \infty) = 0\) and the continuity of \(n_Q\) and \(n'_Q\) at \(z = 0\), a simple analytical approximation can be given:

\[ n_Q(z) = A_Q e^{v_w z/D}, \quad (65) \]

where

\[ A_Q = \frac{1}{\lambda_+ D} \int_0^\infty dz' e^{-\lambda_+ z' \gamma(z')}, \quad \lambda_\pm = \frac{v_w \pm \sqrt{v_w^2 + 4 \Gamma D}}{2 D}. \quad (66) \]

Assuming that the weak sphalerons are inactive in the broken phase (i.e. inside the bubble), \(n_B\) will be constant in this phase. To find the value of this constant, one has to solve the diffusion equation for the baryon asymmetry in the symmetric phase:

\[ v_w n'_B = -\theta(-z) [n_F \Gamma_{ws} n_L(z) + R n_B(z)], \quad (67) \]

where \(n_F = 3\) is the number of families, \(R = \frac{5}{4} n_F \Gamma_{ws}\) is the relaxation coefficient [45] and \(\Gamma_{ws} \simeq 120 \alpha^5 \alpha T\) is the weak sphaleron rate. The above equation is easily integrated and we find

\[ n_B = -\frac{n_F \Gamma_{ws}}{v_w} \int_{-\infty}^0 dz e^{z R/v_w} n_L(z). \quad (68) \]

Using Eqs. (59) and (65), one gets

\[ n_B = -n_F \Gamma_{ws} \frac{9 k_Q k_B - 8 k_B \alpha T - 5 k_B k_Q}{k_Q(9k_T - 9k_B)} \left( \frac{DA_Q}{DR + v_w^2} \right). \quad (69) \]

To obtain a reliable approximation for the squark current, it is necessary to specify the Higgs profiles as functions of \(z\), i.e. the functions \(v(z)\) and \(\beta(z)\) which appear in Eq. (47). A simple expression is given by the kink approximation [22]

\[ v(z) = \frac{1}{2} v(T) \left[ 1 - \tanh \left( \alpha - \frac{2 \alpha}{L_w} z \right) \right], \quad (70) \]

\[ \beta(z) = \beta - \frac{1}{2} \Delta \beta \left[ 1 + \tanh \left( \alpha - \frac{2 \alpha}{L_w} z \right) \right], \quad (71) \]

where \(L_w/(2\alpha)\) parametrizes the thickness of the bubble wall, \(\alpha \simeq 3/2\) and \(\Delta \beta\) is the variation of the angle \(\beta(z)\) along the bubble wall, which lies in the range \(10^{-2} \gtrsim \Delta \beta \gtrsim 4 \times 10^{-3}\) for \(100 \text{ GeV} \leq m_A \leq 200 \text{ GeV}\) [39]. At first order in \(v_w\), all the dependence in \(z\) of the source \(\gamma_Q\) is given by the combination \(v^2(z)\beta'(z)\). Substituting the profiles (70) and
(71) into $A_Q$ (cf. Eq. (65)), it is possible to integrate explicitly the $z$ dependence. Finally we have

$$A_Q = \frac{v^2(T)\Delta \beta}{48D\lambda_+} \frac{k_Q(9k_T - k_B)}{\Delta} v_w \text{Im} \left( \mu \left[ Z_R^A(Y_A)^T h_u^* Z_R^T \right]_{33} \right) I_{RR}^T f_{\lambda_+},$$

(72)

with $I_{RR}^T$ given by Eqs. (36)-(40),

$$f_{\lambda_+} = \frac{2a(a-4)(a-2)}{a-6} e^{6a} \left( \frac{2F_1(1,3-a/2;4-a/2;e^{-2a}) + a(a-4)(a-2) \pi e^{-a} \text{cosec}(a\pi/2)}{-2(1+e^{2a})^{-3} e^{-4a} \left[ (a^2+8)e^{4a} + 2a(a-1)e^{2a} + a(a-2) \right]} \right),$$

(73)

$a = \lambda_+ L_w/(2\alpha)$ and $2F_1(a,b;c;z)$ is the hypergeometric function.

In Section V we will present our numerical results for the baryon asymmetry using the approximate solution given by Eqs. (69), (72) and (73).

IV. BARYON ASYMMETRY IN SUPERSYMMETRIC MODELS

In this section we study the flavor-dependent contribution to the squark current, which enters in Eq. (41) through the imaginary part

$$\text{Tr} \ S_{RR} \propto \text{Im} \left( \mu \left[ Z_R^A(Y_A)^T h_u^* Z_R^T \right]_{33} \right).$$

(74)

Our goal is to maximize the above quantity and, simultaneously, satisfy the experimental constraints on the off-diagonal $A$ terms as well as on the CKM mixing matrix.

To establish a direct connection between $S_{RR}$ and low energy supersymmetry phenomenology, let us express $S_{RR}$ in terms of the up left-right mass insertions, $\delta_{LR}^u$:

$$(\delta_{LR}^u)^{ij} \equiv \frac{v_2 \left( U^L (Y_A)^* U^R \right)_{ij} - v_1 \mu (d_u)_{ij}}{\langle m_{\tilde{q}}^2 \rangle},$$

(75)

normalized to $\langle m_{\tilde{q}}^2 \rangle$, the mean value of the squark masses; $U^{L,R}$ are the unitary matrices that diagonalize $h_u$,

$$U^L h_u^* U^R = d_u = \frac{1}{v_2} \text{diag} (m_u, m_c, m_t).$$

(76)

For the sake of simplicity and to maximize $\text{Tr} \ S_{RR}$, let us assume that $Z_R = \mathbb{1}$. We notice that the unitarity of $Z_R$ implies that any deviation from the identity (or its permutations) will introduce additional suppression factors in the CP-violating source. From Eqs. (74) and (75) we obtain

$$\text{Tr} \ S_{RR} \propto \hat{h}_u^2 \frac{\langle m_{\tilde{q}}^2 \rangle}{m_t} \text{Im} \left[ U_{L3}^R U_{L3}^R (\delta_{LR}^u)^{33} \right].$$

(77)
FIG. 2: Baryon asymmetry as a function of $\text{Im} (\delta_{LR}^u)_{3i}$ for different values of $m_A$. We assume $\mu = 500$ GeV, $v_w = 0.04$ and $\langle m_{\tilde{q}}^2 \rangle \simeq m_Q^2 = 1$ TeV$^2$. The other parameters are indicated in Table I. The dashed lines correspond to the lower and upper bounds of the observed baryon asymmetry given in Eq. (1).

Assuming for instance that the dominant contribution comes from large mixing between the $t$- and either the $u$- or $c$-quark, i.e. taking $U_{i3}^R = U_{33}^R = 1/\sqrt{2}$ ($i = 1, 2$), we have

$$\text{Tr} \ S_{RR} \sim \frac{1}{2} \mu h_i^2 \frac{\langle m_{\tilde{q}}^2 \rangle}{m_t} \text{Im} [ (\delta_{LR}^u)_{3i}^* ] .$$

In Fig. 2 we plot the baryon asymmetry as a function of $\text{Im} (\delta_{LR}^u)_{3i}$ for different values of $m_A$. It is seen that even for light pseudo-scalar masses $m_A$, large values of $\text{Im} (\delta_{LR}^u)_{3i}$ are required in order to reproduce the observed BAU. In Fig. 3 the baryon asymmetry is given as a function of $\text{Im} (\delta_{LR}^u)_{3i}$ for different values of $\mu$. We notice that it is possible to get a sufficient BAU if

$$\text{Im} [ (\delta_{LR}^u)_{3i}^* ] \simeq 0.14 , \text{ for } \mu \simeq 1 \text{ TeV} .$$

It is worth noticing that these bounds on the imaginary part of the mass insertions $(\delta_{LR}^u)_{3i}$, $i = 1, 2$ are compatible with the bounds obtained from the chargino contributions to $B_d - \bar{B}_d$ mixing and the $CP$ asymmetry of $B_d \to J/\psi K_S$ [46]. In addition to fulfilling...
FIG. 3: Baryon asymmetry as a function of $\text{Im} (\delta_{LR}^u)_{3i}$ for different values of $\mu$. We assume $m_A = 150$ GeV, $v_w = 0.04$ and $\langle m_q^2 \rangle \simeq m_Q^2 = 1$ TeV$^2$. The other parameters are taken as in Table I. The dashed lines correspond to the lower and upper bounds of the observed baryon asymmetry given in Eq. (1).

Regarding the latter, the current experimental bounds are [26]

$$d_n < 6.3 \times 10^{-26} \text{ e cm}, \quad d_{Hg} < 2.1 \times 10^{-28} \text{ e cm}.$$  \hspace{1cm} (80)

These values can be translated into bounds for the imaginary parts of the up and down $(\delta_{LR})_{11,22}$ [29]. In particular,

$$\text{Im} (\delta_{11}^u)_{LR} < 10^{-6} \quad \text{and} \quad \text{Im} (\delta_{11}^u)_{LR} < 10^{-7} - 10^{-8},$$  \hspace{1cm} (81)

from the neutron and mercury atom EDM’s, respectively.

Besides the underlying EDM problem that is associated with the imaginary parts of the diagonal elements of $\delta_{LR}^u$, one should also bear in mind the extensive array of constraints
on the non-diagonal entries. These stem from FCNC bounds, limits on rare decays and the observed amount of CP-violation in the $K$ and $B_d$ meson systems [49].

We can also express Eq. (74) in a different manner using the definition (23). Assuming $\mu$ real and keeping only terms proportional to $h_t$, one has

$$\text{Tr } S_{RR} \propto \mu h_t^2 |U^L_{3k}|^2 \text{Im} \left[ (Z_R)_{3j} A^u_{kj} U^R_{3j} U^R_{3l} (Z_R^*)_{3l} \right],$$

(82)

where a summation over the indices $j, k, l = 1, 2, 3$ is understood. In this case, the imaginary part of $\text{Tr } S_{RR}$ is given by the simple form:

$$\text{Tr } S_{RR} \propto \mu h_t^2 |U^L_{3k}|^2 |U^R_{33}|^2 \text{Im} (A^u_{k3}),$$

(83)

where $Z_R = 1$ has been assumed. The important point is that $S_{RR}$ is proportional to the imaginary part of $A^u_{k3}$, which are quantities weakly constrained by experimental data in contrast to other terms proportional to $A_{ij}$ ($i, j = 1, 2$) [47].

In the case of hierarchical Yukawa couplings, the matrix $U^R$ can be assumed to be very close to the identity and the maximum values for $U^L$ are bounded by the CKM matrix. In such scenario, we have

$$\text{Tr } S_{RR} \propto \mu h_t^2 |U^L_{3k}|^2 \text{Im} (A^u_{k3}) \lesssim \mu h_t^2 |V_{3k}^{CKM}|^2 \text{Im} (A^u_{k3}).$$

(84)

It is easy to see that the contribution proportional to $A^u_{13}$ and $A^u_{23}$ is very suppressed by $|V_{td}|^2$ and $|V_{ts}|^2$ and thus, the dominant contribution comes from the imaginary part of $A^u_{33}$ which is always constrained in GUT-inspired SUSY models by the EDM bounds. In particular, in the minimal supergravity inspired model where the $A$-terms are universal and the overall phase is constrained to be $\lesssim 10^{-1}$, this contribution becomes quite negligible and one cannot obtain a viable BAU through the squark current.

Therefore, the flavor-dependent phases will play an important role in baryogenesis only if significant mixing between the top quark and up and/or charm quark is present. This mixing can be obtained in supersymmetric models with non-universal soft breaking terms and, for instance, with nearly democratic Yukawa couplings. Of course, this means that processes like $t \to u + \gamma$, $t \to c + \gamma$ may have much larger branching ratios than those predicted in the SM.

To illustrate our results, let us now consider some simple textures for the quark and squark mass matrices at the electroweak scale. In case A we present the generic case of a
SUSY texture where the CKM matrix is dominated by the down quark mixing. Case B corresponds to SUSY models with Hermitian Yukawa couplings and trilinear terms at GUT scale. Of course, the RGE running down to the electroweak scale will induce some deviations from hermiticity, but their contributions will not be relevant to our BAU analysis. In case C, the textures are chosen such that the BAU produced at the electroweak scale is maximized and the phenomenological constraints are still satisfied.

Case A

We first consider the following simple texture for the trilinear matrix $A_u$:

$$A_u = A_t \begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & a e^{i \pi/2} \end{pmatrix},$$

(85)

with $a = \mathcal{O}(1)$ a real parameter. We also assume that the up quark Yukawa coupling matrix $h_u$ is diagonal, i.e. $h_u = d_u$, $U_L = 1$, $U_R = 1$. In this case one obtains

$$\text{Tr } S_{RR} \propto \mu h^2 t \Im (A^u_{33}) = a A_t \mu h^2 t .$$

(86)

Case B

Let us now assume that the up- and down-quark Yukawa couplings as well as the trilinear ones are Hermitian matrices. In this case the up-quark Yukawa coupling matrix is diagonalized by a unitary matrix $U$ such that $h_u = U^T d_u U^\dagger$. Since the trilinear matrix $A_u$ is Hermitian too, then $\Im A_{33} = 0$. The squark source $S_{RR}$ can be expressed as

$$\text{Tr } S_{RR} \propto \mu h^2 t |U_{33}|^2 \{ |U_{31}|^2 \Im (A^u_{13}) + |U_{32}|^2 \Im (A^u_{23}) \} .$$

(87)

It is easy to see that in order to maximize the imaginary part of $S_{RR}$, a large mixing between the third and the first (or the second) families of up quarks is needed. For instance, assuming maximal mixing between the $c$ and $t$ quarks, i.e.

$$U = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1/\sqrt{2} & -1/\sqrt{2} \\ 0 & 1/\sqrt{2} & 1/\sqrt{2} \end{pmatrix},$$

(88)
and the following Hermitian texture for the trilinear matrix $A_u$,

$$A_u = A_t \begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & b e^{i\pi/2} \\ 1 & b e^{-i\pi/2} & 1 \end{pmatrix}, \quad (89)$$

with $b$ a real parameter of order $O(1)$, one obtains

$$\text{Tr } S_{RR} \propto \frac{1}{4} \mu h_i^2 \text{Im } (A_{23}^u) = \frac{b}{4} A_t \mu h_i^2 \quad . \quad (90)$$

The corresponding texture for the down-quark Yukawa matrix $h_d$ is fixed by the structure of the CKM matrix, $V_{CKM} = U D^\dagger$, where $D$ is the mixing matrix that diagonalizes the down quark mass matrix, $h_d = D^T d_d D^*$, with $d_d = \frac{1}{v_1} \text{diag } (m_d, m_s, m_b)$. Thus, we have

$$h_d = U^T V_{CKM}^* d_d V_{CKM}^T U \quad . \quad (91)$$

**Case C**

In this case we take

$$A_u = A_t \begin{pmatrix} 1 & 1 & c e^{i\pi/2} \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix}, \quad U_L = \begin{pmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{pmatrix}, \quad U_R = \mathbb{I} \quad , \quad (92)$$

where $c = O(1)$. The squark source $S_{RR}$ is then given by

$$\text{Tr } S_{RR} \propto \mu h_i^2 \text{Im } (A_{13}^u) = c A_t \mu h_i^2 \quad . \quad (93)$$

Let us remark that in the previous examples most of the entries in the trilinear matrix $A_u$ were set for simplicity equal to one. It is clear that in general those elements can be nondegenerate and even complex.

In the next section we shall present some numerical examples for the textures considered above and compute the BAU generated by the corresponding $CP$-violating squark current.

**V. NUMERICAL RESULTS**

In order to produce the required BAU, it is needed not only to violate the $CP$ and $C$ symmetries, but the universe has also to be out of equilibrium during the stage of baryogenesis, to avoid any washout by the electroweak sphalerons. This can happen if the electroweak
phase transition is of first-order. To freeze the action of the sphalerons, their rate in the broken phase has to be smaller than the Hubble rate, i.e.

$$\Gamma_{ew}(T_c) < H(T_c)$$

where $$T_c$$ is the critical temperature of the phase transition. Using the sphaleron rate in the broken phase [50], this equation is translated into a condition on the vacuum expectation value of the Higgs fields at $$T_c$$:

$$\frac{v(T_c)}{T_c} \gtrsim 1.$$  \hfill (94)

In the standard model of electroweak interactions, this would imply a Higgs mass lighter than 43 GeV, which is already experimentally ruled out [27]. A way to avoid this constraint is to add scalar fields (with Higgs field-dependent masses) such that their contribution to the finite temperature potential will increase the strength of the phase transition. Naturally, the MSSM is an appealing alternative to the SM. In particular, it has been shown that the presence of a light right-handed top squark with small mixing considerably enhances the strength of the phase transition [10–12]. However, in this case a few constraints need to be satisfied. Within the MSSM, all these requirements impose restrictions to the allowed parameter space [16]: (i) a heavy pseudoscalar mass $$m_A$$ and a large tan $$\beta$$ regime, $$m_A \gtrsim 150$$ GeV, $$\tan \beta \gtrsim 5$$; (ii) heavy left-handed stops with a relatively small mixing, $$m_Q \gtrsim 1$$ TeV, $$0.25 \lesssim (A_t - \mu \cot \beta)/m_Q \lesssim 0.4$$; (iii) a light right-handed stop, 105 GeV $$\lesssim m_{\tilde{t}_R} \lesssim 165$$ GeV.

In the context of models with non-universal $$A$$ terms, we can expect to have more freedom and that some of these conditions may be relaxed. In fact, if we want to keep alive the light stop scenario, the constraints imposed by the electroweak phase transition (cf. Eq. (94)) are typically the same than in the one squark generation case, except that now we should require

$$|\langle \delta^n_{LR} \rangle_{ij} | \lesssim 0.4 \times \frac{m_im_Q}{\langle m^2_{\tilde{q}} \rangle} \approx 0.07,$$  \hfill (95)

if one assumes that $$\langle m^2_{\tilde{q}} \rangle = m^2_Q = 1$$ TeV$^2$.

From the last equation and the bound given in Eq. (79) (see also Figs. 2 and 3), we conclude that it is difficult to produce enough BAU using the flavor-dependent phases of the trilinear soft breaking terms unless $$\mu \gtrsim 1$$ TeV.

In Fig. 4 we present the baryon-to-entropy ratio $$n_B/s$$ as a function of the $$\mu$$ parameter for different up-quark Yukawa coupling and trilinear term textures (cases A, B and C considered.
in Section IV). The coefficients $a$, $b$, $c$ that appear in the textures (85), (89) and (92), are chosen so that the $CP$-violating squark source is maximized and the constraints coming from the electroweak phase transition are satisfied. We also require the lightest Higgs mass to be consistent with the present experimental lower bound, $m_{H} \gtrsim 109$ GeV [27]. In particular, to maximize the $CP$-violating source and, simultaneously, satisfy the bounds on the lightest right-handed squark mass, $m_{\tilde{t}_{R}} \gtrsim 105$ GeV, we take $a = 1$, $b = 1$ and $c = 1.8$. From the figure it is seen that textures A and B cannot generate enough BAU. This is related to the fact that the coefficients $a$ and $b$ cannot be increased without violating the bounds on $m_{\tilde{t}_{R}}$. It is also clear that the produced BAU in the Hermitian case (B) is further suppressed by a factor 1/4 due to the Yukawa quark mixings. On the other hand, texture C can produce the observed baryon asymmetry provided that the $\mu$ parameter is large enough. We also notice
FIG. 4: The baryon-to-entropy ratio $n_B/s$ as a function of the $\mu$ parameter for different up-quark Yukawa coupling and trilinear matrix textures (cases A, B and C considered in Section IV). We have chosen $a = 1$, $b = 1$ and $c = 1.8$ for the coefficients in the textures (85), (89) and (92), respectively. We assume $m_A = 150$ GeV and $v_w = 0.04$. The rest of the parameters are chosen according to Table I. The dashed lines correspond to the lower and upper bounds of the observed baryon asymmetry.

that in this case the coefficient $c$ is less constrained by $m_{\tilde{t}_R}$.

The baryon asymmetry as a function of the wall velocity $v_w$ is plotted in Fig. 5 for different values of $\mu$ and assuming the textures of case C, which succeeds in producing enough BAU. As we can see from the figure, to get a BAU compatible with the observational limits on $n_B/s$ (cf. Eq. (1)) large values of $\mu$ ($\mu \gtrsim 700$ GeV) are required.

We should emphasize however that the electroweak baryogenesis is a strongly out-of-equilibrium process and all our computation is based on several approximations. Unfortunately, it is very difficult to estimate the errors done during the calculation because typically they come from different sources. Nevertheless, we can conclude that the scenario with a light stop and $CP$ violation coming from flavor-dependent phases is in general disfavored. Of course, the constraints coming from the electroweak phase transition can be in principle relaxed in extensions of the MSSM with new scalar fields, for example by adding singlet
FIG. 5: The baryon-to-entropy ratio $n_B/s$ as a function of the wall velocity $v_w$ for different values of $\mu$ and $m_A = 150$ GeV. The other parameters are chosen as in Table I. We assume the textures given in case C of Section IV. The dashed lines correspond to the lower and upper bounds of the observed baryon asymmetry given in Eq. (1).

fields (NMSSM) [51].

In the present model, where the $CP$-violating sources arise from the flavor-dependent phases of the SUSY soft breaking trilinear terms, the strongest constraint comes from the lightest up squark mass. Indeed, two conditions that will push the lightest up-squark mass to smaller values must be fulfilled. First, $m_R \lesssim T_{ew}$, otherwise the contribution of the lightest right-handed squark to the BAU will be exponentially suppressed by a Boltzmann factor. Secondly, as we have seen from Fig. 3, $\text{Im} \left( \delta_{LR}^{\mu} \right)_{3i} \gtrsim 0.14$, which typically means that some of the $A_{ij}$’s have to be of the order or larger than $m_Q$. In other words, the $6 \times 6$ up-squark mass matrix will have large mixings.

In this paper, we have restricted ourselves to the range of parameters satisfying the upper bound (95) such that all the experimental and theoretical constraints on the squarks and the lightest Higgs are satisfied in the light stop scenario. An important question that remains is whether it is possible to find GUT patterns for the trilinear terms, SUSY soft-breaking squark masses and quark Yukawa couplings, such as to obtain large values of $\delta_{LR}^{\mu}$ and a strong
first-order phase transition, while still satisfying the experimental limits on the squark and Higgs masses, as well as the EDM’s constraints.

VI. CONCLUSIONS

Recent EDM bounds impose severe constraints on the usual scenario of supersymmetric electroweak baryogenesis based on flavor-conserving $CP$-violating phases. A natural solution to this problem is to work in the framework where all the flavor-conserving parameters, such as the $\mu$-term and gaugino masses are real. In this case, the dominant contribution to the baryon asymmetry is associated to the flavor-dependent $CP$-violating squark sources.

In this work we have studied in detail the impact of non-universal $A$ terms on the scenario of electroweak baryogenesis. By generalizing the standard approach, we have obtained the expression for the $CP$-violating squark sources with explicit flavor dependence. We have shown that if we impose on these terms the condition to have a strong first-order phase transition induced by a light right-handed squark, the baryon asymmetry of the universe produced at the electroweak scale is typically too small, thus disfavoring this scenario.

On the other hand, if we assume that the problem of the strength of the first-order electroweak phase transition is solved through another mechanism as it can happen in extensions of the MSSM with additional Higgs scalars, it is possible to have textures for the up- and down-quark Yukawa coupling matrices in order to maximize the $CP$-violating source. This however implies a large mixing between the top quark and one of the light up-quarks ($u$ or $c$) as well as large deviations from universality for the $A$ terms (typically, $\left(\delta_{LR}^u\right)_{13,23} \gtrsim 0.14$). Such large $\delta_{LR}^u$ could have important implications on flavor-changing top decays.

Supersymmetric electroweak baryogenesis is an attractive mechanism to explain the observed baryon asymmetry of the universe. Not only the physics involved in this process is directly related to low-energy observables, but it can also be testable in accelerator experiments in the near future. In particular, searches for a light Higgs boson and a light stop at LHC and Tevatron will constitute a test of the viability of this scenario.

There are still a few questions to be answered and controversial issues to be clarified. For instance, the precise details of the electroweak phase transition are still unknown and there is at present a debate regarding the structure of the $CP$-violating currents that are relevant to baryogenesis. Nevertheless, a considerable progress has been done during the past few
years, and the effort directed to resolve these problems could give us definite answers in the near future.

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