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POWER THRESHOLD FOR NEUTRAL BEAM CURRENT DRIVE

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ABSTRACT. For fully non-inductive current drive in tokamaks using neutral beams, there is a power and density threshold condition, setting a minimum value for \( P_{3/2}/n_3^2 \). If this condition is not met, a stationary state cannot occur, and a tokamak discharge will collapse. This is a consequence of the coupling between current and electron temperature, or between current drive efficiency and energy confinement time.

This note is intended to point out a potentially important consideration in the development of tokamaks based on fully non-inductive current drive, particularly for designs for which non-inductive current buildup is contemplated. The analysis has been motivated by the observation of slowly collapsing discharges in the DIII-D tokamak during fully non-inductive neutral beam current drive (NBCD) experiments. Under some conditions in these discharges, after establishment of a state in which both the thermal energy and the toroidal current are provided by the neutral beams, both the current and the thermal energy decay with a characteristic time of a few seconds (see Fig. 2). This appears to be a direct consequence of the coupling which results from the current dependence of the energy confinement time and the dependence of the current drive efficiency on temperature. Because the threshold falls in the parameter range of interest for present experiments and future designs, it is important to take this phenomenon into account.

This threshold behaviour can be understood qualitatively by noting that the energy confinement time improves with increasing current under a wide variety of tokamak conditions. In addition, the current drive efficiency improves with increasing electron temperature, but this improvement saturates when the electron temperature is sufficiently high, because of increased direct coupling between the neutral beam and the thermal ions. On the time-scale of the current evolution, the electron temperature and the plasma current are directly related through the confinement time. If the neutral beam power is too low, then at any given temperature the current drive efficiency is too low to provide enough current to sustain the assumed temperature. Alternatively, at any current, the energy confinement time is too short to provide sufficient temperature to sustain that current. Thus, both the current and the temperature collapse on the inductive time-scale.

The important features of this process can be extracted from a zero-dimensional model. For simplicity, we assume that the density \( n = \langle n_e(r) \rangle \) is independent of time. We also take equal electron and ion temperatures, \( \langle T_e \rangle = \langle T_i \rangle = T \). The thermal energy content of the plasma is determined by

\[
3n \frac{dT}{dt} = -3nT \frac{T}{T_E} + P \tag{1}
\]

where \( T_E \) is the global energy confinement time, \( V \) is the volume, and \( P \) is the deposited power. All units are SI, except that the temperature is measured in eV. The power \( P \) is assumed to go to both heating and current drive. The evolution of the plasma current is given by

\[
\frac{dT}{dt} = -I + P \frac{\eta_{CD}}{nR} \tag{2}
\]

where \( \tau_{LR} \) is the magnetic time constant, \( R \) is the major radius and \( \eta_{CD} \) is the current drive figure of merit (\( \approx n_{CD}/P_{CD} \)).

The magnetic time constant is the ratio of the inductance to the resistance of the plasma, or approximately

\[
\tau_{LR} = k_{LR} T^{3/2} \tag{3}
\]

where

\[
k_{LR} \approx \frac{\mu_0 \kappa a^2 (\ln \frac{8R}{a} - 2 + \frac{1}{4})}{2(\eta_0 T_0^{3/2})} \tag{4}
\]

Here, \( \kappa \) is the plasma elongation, \( a \) is the minor radius, \( \xi \) is the internal inductance and \( \eta_0 \) is the resistivity at temperature \( T_0 \). Typically, in large machines the inductance is of the order of a few microhenries, the resistance is a few times \( 10^{-4} \) Ohm, and \( \tau_{LR} \) falls in the range of tens to hundreds of seconds.

For the energy confinement time, there is a wide variety of possible choices. For this analysis, we use Goldston's scaling [1] because it provides an adequate fit to large tokamak data, contains the important
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dependences on current and power, and has a simple form that is useful for analytic manipulation:

\[ \tau_E = k_E \frac{I}{P^{1/2}} \quad (5) \]

where

\[ k_E = 3.3 \times 10^{-6} C_E \frac{\kappa^{1/2} A_i^{1/2} R^{1.75}}{a^{0.57}} \quad (6) \]

A, is the plasma ion atomic mass and \( C_E \) is a multiplier to account for possible confinement enhancement (the NBCD experiments on DIII-D show \( C_E \sim 1 \)). We note that the qualitative conclusions presented here regarding threshold behaviour do not depend on the particular choice of confinement scaling rule. Any scaling relation for which \( \tau_E \) improves with \( I \) and worsens with \( P \) will lead to similar behaviour.

Typically, \( \tau_E \) is a fraction of a second, much smaller than \( \tau_{UR} \), and so the temperature is always in quasi-equilibrium on the time-scale of the current evolution:

\[ \frac{3nT}{\tau_E} = \frac{P}{eV} \quad (7) \]

If, as with the Goldston scaling, \( \tau_E \) is not an explicit function of temperature, \( T \) can be solved for

\[ T = \frac{1}{3} \frac{P\tau_E}{n} \quad (8) \]

Neutral beams are the source for non-inductive current drive in the cases discussed here. However, it should be noted that a power threshold will also occur for any other current drive scheme that shows similar dependence on electron temperature. The second term in Eq. (2) is the beam driven current, \( I_{CD} \). This is obtained by averaging the local (one-dimensional) current over the plasma profiles. The local current is

\[ J_{CD} = J_{circ} \left[ 1 - \frac{Z_b}{Z_{eff}} (1 - G) \right] \quad (9) \]

The first term is the circulating fast ion current and the second term is the electron response, including finite aspect ratio effects in \( G(Z_{eff}, \epsilon) \). Note that the electron correction depends on beam charge \( (Z_b) \), average ion charge \( (Z_{CD}) \), and inverse aspect ratio \( (\epsilon) \), and can be treated as a simple multiplier of the fast ion current: \( J_{CD} = J_{circ} F \). There is a weak dependence on temperature because the trapping factor \( G \) depends on the collisionality regime, but this is ignored. The factor \( G \) has been calculated by Start and Cordey [2] for the collisionless regime, and their numerical results are well approximated by

\[ G = (1.55 + 0.85/Z_{CD}) \epsilon - (0.20 + 1.55/Z_{CD}) \epsilon. \]

The fast ion current is

\[ J_{circ} = e Z_b S_b \tau_{\epsilon} v_{\epsilon 0} \xi_0 I(x, y) \quad (10) \]

where \( S_b = P/eV_{\epsilon 0} \) is the source strength \( (E_{\epsilon 0} \) is the injected beam energy), \( \tau_{\epsilon} = [(4\pi e)^2 3m m_{\epsilon}^2]/ [16 \sqrt{\pi} e^4 n L^2 \ln \Lambda] \) is the slowing-down time, \( v_{\epsilon 0} \) is the injected beam velocity and \( \xi_0 \) is the cosine of the angle between the injected beam and the toroidal direction. The distribution of fast ions is accounted for in the integral

\[ I(x, y) = \frac{1}{\pi} \left( \frac{1 + \xi^2}{\xi^2 + 1} \right)^{1/2} \int_0^1 \left( \frac{1 + \xi^2}{1 + \xi^2 + 1} \right)^{v+1} du \]

Mikkelsen and Singer [3] give an approximate fit to this function: \( I(x, y) = x^2/(4 + 3y) + (1.39 + 0.617 \epsilon) x^2 + x^3 \). The arguments are \( x = v_{\epsilon 0}/v_{\epsilon} \), where \( v_{\epsilon} = v_{\epsilon 0} [3 \sqrt{\pi}/4 (m_{b}/m_e)(Z_{eff}/Z_{CD})]^{1/3} \) is the velocity at which equal momentum goes to electrons and ions, and \( y = Z_{\epsilon}/3 \). The mass-weighted charge factor is

\[ F = \frac{Z_2}{(Z_b \sum n_i Z_i^2/m_i)} \]

Defining \( T_c \) as the temperature at which \( v_{\epsilon} = v_{\epsilon 0} \) gives \( x = (T_c/T)^{1/2} \), with \( T_c = 6.76 \times 10^{-2} A_i^{1/3} \times (Z_2/Z_{CD})^{2/3} E_{\epsilon 0} \). Putting all of this together and averaging over the plasma cross-section gives

\[ I_{CD} = k_{CD} \frac{PT^{3/2}}{n} \frac{(T_c/T)^{3/2}}{c_1 + c_2 (T_c/T) + (T_c/T)^{3/2}} \quad (11) \]

with

\[ k_{CD} = 1.39 \times 10^{18} A_i^{1/2} \frac{Z_b}{Z_{CD} \ln \Lambda} \frac{1}{R E_{\epsilon 0}^{1/2}} \quad (12) \]

where \( c_1 = 4 + 3y \) and \( c_2 = 1.39 + 0.617 \epsilon \).

Substitution of \( I_{CD} \) in Eq. (2) gives
\[ k_{L/R} T^{3/2} \frac{dI}{dt} = -I \left[ 1 - k_{CD} \frac{P T^{3/2}}{I n} \frac{1}{c_1 (T/T_c)^{3/2} + c_2 (T/T_c)^{1/2} + 1} \right] \]

(13)

The possibility of a threshold is clear from this expression. If the relationship between \( T \) and \( I \) (determined by the energy confinement time scaling) is such that there is a maximum in the second term in brackets, then a stationary state can only exist if the value of that maximum exceeds 1.

We define normalized current and temperature variables, \( u = I/I_* \) and \( v = T/T_* \):

\[ I_* = k_{CD} \frac{T_c^{3/2}}{c_1} \frac{P}{n} \]

(14)

\[ T_* = \frac{k_B}{3 \text{ eV}} \frac{k_{CD} T_c^{3/2}}{c_1} \frac{P^{3/2}}{n^2} \]

Equation (8) gives the temperature in terms of the plasma current simply as

\[ v = u \]

(15)

and the time dependence is given by

\[ \frac{du}{dt} = - \frac{1}{u^{1/2}} + \frac{1}{u^{3/2} + \alpha u^{1/2} + \beta} \]

(16)

where \( \hat{t} = t/k_{L/R} T_*^{3/2} \), \( \alpha = (c_2/c_1) (T_c/T_*) \), and \( \beta = (1/c_1) (T_c/T_*)^{3/2} \).

First, we consider possible stationary solutions to Eq. (16). The right-hand side of this equation is sketched in Fig. 1. Note first that, at low values of \( P^{3/2}/n^2 (T_c/T_* < 1) \), there are no stationary solutions; \((dI/dt)\) is always negative, and the plasma collapses to zero current and temperature for any initial values. At higher values of \( P^{3/2}/n^2 \) there are two stationary solutions; only the one at higher values of \( I \) and \( T \) is stable. If the initial current is below the lower stationary point, the plasma will collapse even if sufficient power is applied. Thus, there is a threshold condition on \( P^{3/2}/n^2 \) for the existence of a stationary state; also, a minimum initial current is required for reaching this state via non-inductive current buildup.

The existence of a minimum current for any power level has important implications for the design of tokamaks that start up with no inductive drive.

The steady state solution of Eq. (16) is given by the roots of

\[ 0 = u^{3/2} + (\alpha - 1) u^{1/2} + \beta \]

(17)

These roots can be found analytically. If the quantity \( v = (\alpha - 1)^{1/2} + \beta^{3/4} \) is positive, there is one real root which is non-physical \((u^{1/2} < 0)\). If \( v \) is negative, there are two positive real roots. The threshold condition is \( v \leq 0 \), giving \( T_c/T_* \geq C_2/c_1 \) or

\[ \frac{P^{3/2}}{n^2} \geq \frac{C_2}{(k_B/3 \text{ eV}) k_{CD} T_c^{1/2}} \]

(18)

where \( C_2 = c_1 + 3(c_1/4) = 1.39 + 0.61(Z_b/3)^{0.7} + 3(1+Z_b/4)^{1/3} \). In terms of the beam, plasma and machine parameters, this becomes

\[ \frac{P^{3/2}}{n^2} \geq 7.96 \times 10^{-31} \frac{Z_b}{A_1^{1/3} \xi_0} \left( \frac{Z_{eff}}{Z_d} \right)^{1/3} \times \frac{C_2 \ln \Lambda}{C_B A_1^{1/2} F} \kappa^{1/2} R^{0.25} a^{2.37} \]

(19)

Note that this condition depends most strongly on the size of the plasma and is independent of beam energy.

FIG. 1. Schematic of \( du/dt \) versus \( u \) (Eq. (16)) for low, threshold and high values of \( P^{3/2}/n^2 \). Arrowheads indicate the direction in which the system state moves.
Well above the threshold, the stationary solution approaches $u = 1$, or

$$I = I_* = 2.44 \times 10^{16} \frac{E_{b0} \epsilon_0}{Z_b}$$

$$\times \frac{Z_2}{Z_{eff} (4 + Z_2)} \frac{F}{\ln \Lambda} \frac{P}{n R}$$

$$T = T_* = 5.08 \times 10^{34} \frac{E_{b0} \epsilon_0}{Z_b}$$

$$\times \frac{Z_2}{Z_{eff} (4 + Z_2)} \frac{F}{\ln \Lambda} \frac{P^{3/2}}{n^2 R V}$$

Figure 2 shows the time behaviour of two NBCD discharges in DIII-D that are near the power threshold. In these discharges, the target plasma is prepared inductively, but the inductive drive is removed at the time when the beams are turned on. The plasma current is shown in Fig. 2(a) and the time behaviour of $P^{3/2}/n^2$ in Fig. 2(b). Discharge 57456 is slightly above the threshold, and the current remains approximately constant from the end of the initial transient at 1250 ms to the decrease of injected neutral beam power at 2050 ms [4]. The value of $P^{3/2}/n^2$ for discharge 61539 is approximately two-thirds of the value for discharge 57456; the current decays throughout the beam pulse — slowly at first and then more rapidly as the temperature falls. The temperature shows a similar time dependence.

The interpretation of the data for these discharges is complicated by the fact that the density is also time dependent (as are other plasma parameters). The density is affected by the beam current, which changes the particle source, and by the power and current which change the particle confinement time. For example, the increase in $P^{3/2}/n^2$ for discharge 61539 at about 2600 ms results from a decrease of the density associated with a movement of the plasma. Since $Z_{eff}$ increased by a factor of 1.4 at the same time, the threshold condition was not exceeded.

In Fig. 3, we summarize the results for a number of fully non-inductive NBCD discharges. The measured value of $P^{3/2}/n^2$ is plotted versus the calculated threshold value (Eq. (19)). For these plasmas, the coefficient $C_E$ varied between 0.6 and 1.2, and was determined by fitting the measured confinement data to Eq. (5).

This phenomenon may be important because the threshold conditions for power, density and current are in the range of both present NBCD experiments and
FIG. 4. Steady state current versus power, with density as a parameter, for (a) DIII-D and (b) ITER conditions. The dashed lines indicate the unstable branch. Parameters for DIII-D:

\[ a = 0.63, R = 1.65, \kappa = 1.7, Z_{eq} = 2.0, \ln \Lambda = 18, C_e = 2.0, A_t = 2, A_s = 2, \xi_0 = 0.71, E_{so} = 75 \text{ keV}. \]

Parameters for ITER:

\[ a = 2.2, R = 5.8, \kappa = 1.8, Z_{eq} = 1.5, \ln \Lambda = 19, C_e = 2.0, A_t = 2.5, A_s = 2, \xi_0 = 0.87, E_{so} = 1 \text{ MeV}. \]

future large tokamak designs. Figure 4 shows the stable and unstable stationary conditions of the plasma current versus the applied power for parameters characteristic of DIII-D and ITER. The corresponding temperature curves have qualitatively similar behaviour, except that the asymptotic dependence for high power is \( T \propto P^{3/2} \), whereas the current follows \( I \propto P \) (see Eqs (20) and (21)). For example, for ITER at \( n = 6 \times 10^{19} \text{ m}^{-3} \), the minimum power is about 120 MW, and the current and temperature at the threshold are 13.8 MA and 11.9 keV, respectively (this temperature does not depend on density). As shown in Fig. 4(b), the current rises very rapidly with power because the asymptotic condition \( (I \propto P) \) is not approached until the power is in the range of 500–1000 MW. Figure 4(a) also shows a strong dependence of current on power near the threshold.

For ITER, this analysis applies in the startup and heating phases. Once the alpha particle power becomes comparable to the neutral beam power, the heating and current drive are no longer simply related. In the threshold expression (Eqs (18) and (19)), the power is replaced by

\[ P \rightarrow P_{CD}(P_{CD} + P_a)^{1/2} \quad (22) \]

where \( P_{CD} \) is the neutral beam current drive power and \( P_a \) is the heating due to fusion generated alpha particles. For example, if \( Q = P_{\text{fusion}}/P_{\text{CD}} = 5 \), the threshold current drive power is reduced by \( 1/\sqrt{2} \).

An important element not included in this calculation is the bootstrap current. The bootstrap current is dependent on the details of the density and temperature profiles, as well as on the collisionality of the electrons. Thus, it is awkward to include it in a zero-dimensional formulation such as this. If the bootstrap current is included, a term proportional to \( T/\nu \) with a factor accounting for profiles and collisionality should be added to the right-hand side of Eq. (2). The bootstrap effect will also appear as a positive term, proportional to \( u^{-3/2} \) on the right-hand side of Eq. (16), with a correction for collisionality which changes the dependence of this term to \( u^{1/2} \) at small \( u \). The behaviour of this term is qualitatively similar to that of the current drive term, and inclusion of the bootstrap current will not remove the threshold but will only modify the threshold conditions.

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EMPIRICAL TRANSPORT COEFFICIENTS COMPARED WITH QUASI-LINEAR FLUCTUATION INDUCED TRANSPORT

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ABSTRACT. Empirical scaling relations for the electron heat diffusivity $\chi_e$, the diffusion coefficient $D$ and the inward drift velocity $v_n$ in the Ohmic, L- and H-regimes of ASDEX are checked against anomalous transport predicted for electrostatic or magnetic fluctuations. It is stressed that $\chi_e$ and $D$ exhibit the same dependence on the fluctuation spectrum, which explains the coupling of these diffusivities observed in all confinement regimes. The ratios $\chi_e/D \approx 3.3$ and $v_n/D = 0.5 T_e^2 \delta T_e/3 \delta T_e$ are shown to be general results of quasi-linear fluctuation induced transport and to agree with the empirical coefficients in the Ohmic, L- and H-regimes. Also discussed are the consequences of $\chi_e \propto D$ for the interpretation of the saturated and improved Ohmic confinement, and of $v_n \propto D$ for the density profile shape.

The quasi-linear effects of electric and magnetic field fluctuations (connected with plasma turbulence) on particle and heat fluxes, bootstrap current, core pinch flux and electrical conductivity have been studied previously [1]. On the basis of the drift kinetic equation, these transport quantities were calculated for given fluctuation spectra of $\Phi$ and $A$, where $\Phi$ is the fluctuating electrostatic potential and $A$ is the fluctuating parallel component of the vector potential. One advantage of this treatment is that the conclusions obtained are not restricted to the special fluctuation spectrum of a single driving instability. In addition, the uncertainties in the self-consistent determination of saturated spectra are avoided. The problems due to the presence of many instabilities and to unidentified non-linear saturation mechanisms are well known. Quasi-linear theory can help to understand the scalings of transport coefficients and the relations among various coefficients. It must be noted, however, that quasi-linear theory neglects non-linear terms which are important at saturation so that, with some applications, incorrect conclusions will result.

Strong turbulence models predict a linear dependence of the transport coefficients on the fluctuation level. Application of these coefficients in self-consistent simulations of the plasma profiles can lead to profiles near marginal stability, where a weak turbulence description is more appropriate. This is why quasi-linear and weak turbulence models, which predict a quadratic variation of the diffusivities with the fluctuation level, can be relevant.

Neglecting the equilibrium radial electric field term and the $\omega_{pe}/m$ term in the electron particle flux $\Gamma_e$ and the electron heat flux $q_e$ of Ref. [1] yields

$$\Gamma_e = -\frac{\sqrt{\pi}}{4} \rho_{pe} \frac{v_{te}}{\eta_0 n_q} \sum_{m, n} \left( \frac{e \Phi_{mn}}{\sqrt{2} T_e} \right)^2 \frac{m_0}{m - n q}$$

$$q_e = -\frac{\sqrt{\pi}}{4} \rho_{pe} \frac{v_{te}}{R q} n_0 T_e \sum_{m, n} \left( \frac{e \Phi_{mn}}{\sqrt{2} T_e} \right)^2 \frac{m_0}{m - n q}$$

The corresponding fluxes due to magnetic fluctuations are obtained by replacing $\Phi$ by an expression proportional to $A$ [1]. Here, $\rho_{pe}$ is the electron poloidal gyroradius, $v_{te}$ is the electron thermal speed, $R$ is...