Observation and superselection in quantum mechanics

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Abstract

We attempt to clarify the main conceptual issues in approaches to ‘objectification’ or ‘measurement’ in quantum mechanics which are based on superselection rules. Such approaches venture to derive the emergence of classical ‘reality’ relative to a class of observers; those believing that the classical world exists intrinsically and absolutely are advised against reading this paper.

The prototype approach (Hepp) where superselection sectors are assumed in the state space of the apparatus is shown to be untenable. Instead, one should couple system and apparatus to an environment, and postulate superselection rules for the latter. These are motivated by the locality of any observer or other (actual or virtual) monitoring system. In this way ‘environmental’ solutions to the measurement problem (Zeh, Zurek) become consistent and acceptable, too. Points of contact with the modal interpretation are briefly discussed.

We propose a minimal value attribution to observables in theories with superselection rules, in which only central observables have properties. In particular, the eigenvector-eigenvalue link is dropped. This is mainly motivated by Ockham’s razor.

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1 Introduction

The original title of this paper was “To observe is to not observe”, but it was pointed out to the author that this represented a contradiction. Our first aim is to discuss certain lines of criticism that have been, or could be, leveled against resolutions of the measurement problem in quantum mechanics which essentially rely on the (algebraic) theory of superselection rules. Secondly, we will indicate how this theory may be combined with more recent ideas on decoherence and apparatus-environment coupling in order to counter the more pertinent critique. Thus we will arrive at the following point of view: the essence of a ‘measurement’, ‘fact’, or ‘event’ in quantum mechanics lies in the non-observation, or irrelevance, of a certain part of the system in question. The latter may well be the universe as a whole; one is not forced to make a ‘Heisenberg cut’ between system and observer, and in our analysis the observer (or IGUS = Information Gathering and Utilizing System in modern parlance (Gell-Mann and Hartle, 1990, 1993); whenever we speak of an observer in what follows, the reader may add ‘or IGUS’) relative to which the notion of non-observation or irrelevance is defined may be regarded as part of the system, and may be described by quantum mechanics if necessary. Without such irrelevance of some part of the system the notion of a fact (etc.) is meaningless in quantum mechanics, or, put differently, there can only be events once a specific algebra of ‘observables’ has been singled out. Any event that ‘happens’ only comes into existence relative to such a choice of ‘observables’, and on the assumption of the ignorance interpretation of mixed states. A world without parts declared or forced to be irrelevant is a world without facts - such a world may be preferable to ours. As we shall see, in practice facts owe their existence to the locality of the observer (a point of view the author learned from H.D. Zeh).

The measurement problem (cf. Busch et al. (1991), van Fraassen (1991) for an extensive discussion) is a special case of the enigma of classical behaviour within quantum mechanics. The precise formulation of the problem depends on the formalism one uses, and on the interpretative rules connecting the formalism to the
world. The fine kettle of fish is most evident if in the usual (von Neumann) liturgy one assumes a one-to-one correspondence between the physical properties of a system (in the sense of value attributions to observables) and its states. For in that case the formalism predicts the existence of states which seem to never occur. Such states are superpositions of eigenstates of operators which are ‘classical’ in the sense that the corresponding observables are empirically found to always possess sharp values.

The aim of this Introduction is to specify where the theory of superselection rules (on our reading) stands within the debate on the foundations of quantum mechanics, and to introduce the essential points of this theory with its physical interpretation. In section 2 we study a model, and we identify its main diseases in section 3. The best cure is investigated in section 4 in the form of the introduction of the environment into the problem. Section 5 then discusses points of contact with the modal interpretation of quantum mechanics. The final section contains some self-criticism, as well as a summary of our approach in the form of a question- and answer session.

In order to explain what we accept as chivalrous criticism of superselection approaches to the measurement problem, and which type of argument we reject as mock critique, we recall that in the context of quantum mechanics there exist two radically different views on the nature of ‘classical’ reality (for a good discussion cf. D’Espagnat (1990), Tsirelson (1994)). The difference between the two has been expressed in Khalfin and Tsirelson (1992, p. 904) by saying that “the ‘optimists’ investigate the emergence of classical reality relative to a class of observers, whereas the ‘pessimists’ acknowledge only absolute (independent) classical reality”. To elaborate on this point, we distinguish between two further positions, which have both been advocated as ‘realism’.

The first position, which we call A-realism, maintains that there exists a real world independently of the observer, and that one can make objective, observer-independent statements about it. This creed is meant to be contrasted with idealism, solipsism, and the like. It is very broad, and further subdivisions arise once an A-
realist specifies how (s)he relates the alleged real world to our observations, and what the aim of science should be. Thus almost opposite extremes such as naive realism and constructive empiricism (which is presented as an anti-realist persuasion in van Fraassen (1980)) both fall under A-realism.

This position has been attacked by some of the pioneers of quantum mechanics, and has been claimed to be inconsistent with it; the existence of the modal interpretation of quantum mechanics (van Fraassen, 1991, Kochen, 1985, Healey, 1989, Dieks, 1994a,b) shows that this latter claim is false. We see nothing objectionable to A-realism, and will adopt it in this paper.

The second position, B-realism, is in fact a specialization of A-realism, but in a good many papers on the foundations of quantum mechanics it is confused with (usually unqualified) ‘realism’ itself. It claims that this postulated real and independently existing world coincides with, or at least incorporates the classical world of ‘events’ and ‘facts’ that we observe around us. One may compare this with the pre-Copernican world view: the earth appears to be at rest, the sun revolving around it, and since it looks so to us, it must be real and true, independently of us. Some Anglo-Saxon more sympathetic to B-realism might instead describe it as the application of G.E. Moore’s common-sense realism to the interpretation of quantum mechanics. However, once it is realized that we should try to understand why we see things as we do, rather than explaining why these things (supposedly) ‘are’ the way we see them, one cannot help feeling awkward with B-realism.

Whether or not an author advocating superselection- or decoherence-type solutions to the measurement problem is an A-realist, (s)he will definitely reject B-realism at least when analyzing such solutions, for it is the whole point of these approaches to show that under certain conditions the classical world emerges relative to, say, local observables. Hence (s)he is an example of an optimist in the sense of the above quote. From the point of view of a B-realist, such solutions are at best valid ‘For All Practical Purposes (FAPP)’, and thus a large body of criticism on superselection- or decoherence-approaches can be summarized simply by saying that these approaches do not conform to B-realism. We believe that this type of critique
is based on a hallucination, for it is blind to the fact that the notion of classical reality itself is only valid FAPP (and not quantum mechanics, or resolutions of the measurement problem based on it).

In further motivating the point of view opposite to B-realism, we start by introducing some terminology. Quantum mechanics may be fruitfully thought of as having a kernel as well as a user interface. The kernel relates to observer-independent aspects of the real world. It primarily consists of the mathematical formalism, involving either operators, states, and transformations like time-evolution, or path integrals and an action functional, or some other mathematical machinery. Thus at the level of the kernel one may speak of eigenvectors, eigenvalues, and of mathematical expressions that are usually interpreted as expectation values of observables or transition probabilities. This particular physical interpretation is very delicate; it evidently does not belong to the mathematical formalism, but in our opinion it is not even part of the kernel at all (see below). In particular, it would be a mistake to assume the so-called ‘eigenvector-eigenvalue link’ (stating that an observable possesses a value if the state vector of the system is an eigenstate of the corresponding operator) as part of the kernel, and neither do the Born probabilities make physical sense at this stage.

Secondarily, an A-realist will want to relate the mathematical formalism to the observer-independent world through interpretative rules which are part of the kernel. Such a relation is not indispensable in order to confront the theory with observations, for one could introduce the physical interpretation of the mathematical formalism at the level of the observer. Indeed, the latter procedure is followed in the Copenhagen interpretation of quantum mechanics, and also in our exegesis of the theory of superselection rules given later on. But one would clearly feel more comfortable if the empirical content of the theory would follow from a direct physical interpretation in the kernel, amended by an objective description of the observer. In our opinion, this ideal situation has not (yet) been achieved in quantum mechanics (the modal interpretation being an attempt in that direction, cf. section 5 below).

The user interface specifies the connection between the real world and the ob-
server, regarded as part of the world. Hence (s)he/it is subject to the same laws (such as quantum mechanics) that govern the world. This interface is an objective account of those ingredients of the world which emerge only relative to the specification of an observer (or class of observers). Tautologically, observations in general are one example of such an ingredient. More daringly, we maintain that the entire classical world is another case in point.

The interpretation of mathematical expectation values in terms of measurements (particularly the eigenvector-eigenvalue rule), cross-sections, as well as more recent notions such as the probabilities of histories (Omnès, 1992, Gell-Mann and Hartle, 1990, 1993), properly belong to the user interface. Evidently, this interface cannot be created without a specification of the user. This is an elementary though crucial point: if we wish to explain from quantum mechanics why the world appears to us as it does, i.e., largely classically, we should expect this explanation to come from the user interface, and therefore be contingent on what or who the user is. (The question why the world is largely classical seems to us to be as little motivated as the question why the present King of France is bald.)

A somewhat related point was made by Zurek (1993, pp. 287-8): “Thus, the only sensible subject of considerations aimed at the interpretation of quantum theory - that is, at establishing correspondence between the quantum formalism and the events perceived by us - is the relation between the universal state vector and the states of memory (records) of somewhat special systems - such as observers - which are, of necessity, perceiving the Universe from within. It is the inability to appreciate the consequences of this rather simple but fundamental observation that has led to such desperate measures as the search for an alternative to quantum physics.”

Moreover, the necessity of identifying the user interface of a theory as the source of concepts naively thought to belong to the kernel even applies to classical physics, for instance in the problem of the emergence of time in the context of generally covariant field theories (cf. Barbour, 1994).

The theory of superselection rules is an important tool in attempts to make these additional points.

\footnote{We do not agree that such a search is a desperate move; the point is that it is misguided when motivated by the measurement problem.}
ideas precise. In its modern algebraic version, this theory was created by Haag and Kastler (1964), and the first application to the measurement problem was performed by Hepp (1972), who acknowledges that the main ideas are due to Fierz and Jost; also cf. Bohm (1951), Gottfried (1966), Jauch (1968) for pioneering insights in this direction; later work is e.g. Wan (1980), Beltrametti and Cassinelli (1981). A recent review, emphasizing technical points and explaining relevant aspects of the mathematical apparatus of operator algebras, is Landsman (1991) (to which the present paper is complementary). In what follows, we assume that the reader is somewhat familiar with the basic ideas of this approach, but we will keep technicalities to a minimum. Even if this familiarity is marginal or rusty, it will be easy to understand the key ideas on the basis of the example discussed in the next section. The interpretation of the formalism we offer is quite different from the one found in the literature (where one is usually satisfied with straightforward operationalistic ideas). A different motivation from the one given here to justify superselection rules is in Breuer et al. (1994).

In fact, the main idea is very simple. In conventional quantum mechanics any self-adjoint operator on a given Hilbert space $\mathcal{H}$ is deemed an observable, and therefore any unit vector in $\mathcal{H}$ corresponds to a pure state. However, a realistic observer will not actually monitor all conceivable correlations in the universe. Hence this setting may be modified by leaving out a certain set of operators from the ‘algebra of beables’ $\mathfrak{B}(\mathcal{H})$ (i.e., the algebra of all bounded operators on $\mathcal{H}$), to arrive at a remaining ‘algebra of observables’ $\mathfrak{A}$ (which we assume to be a von Neumann algebra for mathematical convenience). In our interpretation, the truncation of the original set $\mathfrak{B}(\mathcal{H})$ of beables to a (much) smaller set of observables $\mathfrak{A}$ is made by the ‘user’, who normally has little choice in doing so. This truncation therefore belongs to the user interface of quantum mechanics. We then regard a unit vector $\Psi$ in $\mathcal{H}$ as a state on $\mathfrak{A}$. Thus $\Psi$ determines a rule $\psi$ telling each operator $A$ in $\mathfrak{A}$ what

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2This means that $\mathfrak{A}$ consists of bounded operators, contains the unit operator, and is closed in the weak operator topology. Hence it is closed in the uniform operator topology as well, so that each von Neumann algebra is a $C^*$-algebra if it is equipped with the latter topology.

3This is a linear functional on $\mathfrak{A}$ which is positive (i.e., $\psi(A^*A) \geq 0$) and normalized ($\psi(\mathbb{1}) = 1$, where $\mathbb{1}$ is the unit operator in $\mathfrak{A}$).
its expectation value $\psi(A)$ is, namely $(A\Psi, \Psi)$. But now such a state may well be mixed on $\mathcal{A}$!

Therefore, we have to investigate the possible decompositions of $\Psi$ (or $\psi$). We make the simplifying assumption that the commutant $\mathcal{A}'$ of $\mathcal{A}$ on $\mathcal{H}$ is abelian (that is, commutative). This assumption amounts to the statement that $\mathcal{A}$ is represented without multiplicities, i.e., redundant repetitions of information. We say that $\Psi_1, \Psi_2 \in \mathcal{H}$ lie in different superselection sectors (or, briefly, sectors) if $(A\Psi_1, \Psi_2) = 0$ for all $A \in \mathcal{A}'$.

In general, a mixed state $\psi$ on $\mathcal{A}$ may not have a unique extremal decomposition $\psi = \sum_i p_i \psi_i$ into pure states $\psi_i$ (with the coefficients $p_i$ adding up to one). However, we are in the special situation that the state $\psi$ on $\mathcal{A}$ is the restriction to $\mathcal{A} \subset \mathfrak{B}(\mathcal{H})$ of a state defined by a vector $\Psi$ in $\mathcal{H}$. In that case we can write $\Psi = \sum_i c_i \Psi_i$, with all the $\Psi_i$ lying in different sectors, and this decomposition is unique. Moreover, our assumption that $\mathcal{A}'$ be abelian implies that each $\Psi_i$ corresponds to a pure state $\psi_i$ on $\mathcal{A}$. Hence with $p_i = |c_i|^2$ we obtain the unique extremal decomposition $\psi = \sum_i p_i \psi_i$. As will become clear shortly, these coefficients $p_i$ are the Born probabilities. The uniqueness of the central decomposition is crucial for this interpretation.

Indeed, at this point the formalism ought to be connected to observation. The simplest way to do so is to only assign properties to classical observables. Here a self-adjoint operator in $\mathcal{A}$ is called a classical observable if it belongs to the centre $Z(\mathcal{A})$ of the algebra of observables $\mathcal{A}$ - this means that it commutes with all

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4 A state $\psi$ is mixed if it may be decomposed as $\psi = \lambda \psi_1 + (1 - \lambda) \psi_2$ with $0 < \lambda < 1$ and $\psi_1 \neq \psi_2$.
5 This is the set of all bounded operators on $\mathcal{H}$ which commute with all elements of $\mathcal{A}$.
6 Which implies that $\mathcal{A}$ must be a type I von Neumann algebra.
7 See footnote 12 for the general case.
8 Together with our assumption, this implies that the corresponding states $\psi_1, \psi_2$ define inequivalent representations of $\mathcal{A}$.
9 An extremal decomposition of a mixed state is by definition a decomposition into pure states. See Bratteli and Robinson (1987) for an exhaustive mathematical account of the decomposition theory of states on $C^*$-algebras, and cf. sect. 4.5 of Landsman (1991) for the general conditions under which the extremal decomposition is unique.
10 For simplicity we suppress the possibility of a direct integral decomposition of the state; the argument is analogous in that case.
other elements of \( A \). Equivalently, eigenstates with different eigenvalues cannot be coherently superposed. Only classical observables possess values, and all classical observables simultaneously assume (sharp) values. In a pure state \( \psi \), the value of \( A \in \mathcal{Z}(A) \) is simply \( \psi(A) \). To deal with mixed states (crucial in the measurement problem) we adopt the ignorance interpretation of mixed states which have a unique extremal decomposition; as shown above, if the total universe is in a pure state this uniqueness assumption can be justified\(^{11} \). Thus in a state \( \psi = \sum_i p_i \psi_i \) the classical observable \( A \) has value \( \psi_i(A) \) with probability \( p_i \).

We have dropped the eigenvector-eigenvalue link for general operators: for us, an eigenvector \( \Psi_a \) of a given operator \( A \in \mathcal{A} \) corresponds to a possessed (eigen) value \( a \) of \( A \) if and only if \( A \) is classical. Similarly, the Born probability \( |(\Psi, \Psi_a)|^2 \) only has the usual meaning if \( A \) is classical. In this circumstance, each eigenvalue will be highly if not infinitely degenerate; the probability that the observable \( A \) has the value \( a \) in the state \( \Psi \) is the sum over all pertinent Born probabilities. Our main reason for dropping the eigenvector-eigenvalue link is Ockham’s razor: as is well known (van Fraassen, 1991, Healey, 1989) this link is empirically superfluous, and in our opinion it results from misguided attempts to assign properties to observables in a specific way, thus introducing the Born probabilities at the level of the kernel of quantum mechanics. Further arguments in favour of our minimal value attribution will be presented in the course of this paper\(^{12} \).

\(^{11}\) If the universe is in a mixed state (whatever that may mean) we still recover the Born probabilities, as shown in footnote \(^{12} \) below.

\(^{12}\) The first such argument comes from states on \( \mathcal{A} \) which do not admit a unique extremal decomposition. This may happen if the state on \( \mathcal{B}(\mathcal{H}) \) (whose restriction to \( \mathcal{A} \) is \( \psi \)) is itself mixed; alternatively, even a pure state on \( \mathcal{B}(\mathcal{H}) \) may restrict to such a state on \( \mathcal{A} \) if we drop the assumption that the commutant \( \mathcal{A}' \) on \( \mathcal{H} \) is abelian. (Lifting this assumption is necessary to describe certain states of infinite systems, whose algebras of observables always admit representations which are not of type I.)

Any state \( \psi \) has a unique central decomposition \( \psi = \sum_i p_i \omega_i \) into ‘macroscopically pure’ states. A macroscopically pure state \( \omega \) on \( \mathcal{A} \), also called a primary or a factorial state, may be defined by the property that any discrete decomposition \( \omega = \sum_n \lambda_n \omega_n \) consists of states \( \omega_n \) which all lie in the same sector, up to multiplicity (that is, the GNS representations of \( \mathcal{A} \) defined by these states are all unitarily equivalent up to multiplicity). If \( \mathcal{A} \) is of type I then \( \omega \) has discrete extremal decompositions. (If \( \mathcal{A} \) is not of type I the extremal decompositions of \( \omega \) are of direct integral type, and the component states will not lie in the same sector. Such extremal decompositions are useless.)

The main point is now that the states occurring in any further decomposition of a given macroscopically pure component of the central decomposition of \( \psi \) all coincide on the centre \( \mathcal{Z}(\mathcal{A}) \) of
The statistical character of quantum mechanics entirely derives from the coefficients $p_i$ in the unique decomposition of mixed states into pure ones, if this uniqueness applies. Thus it fully sits inside the user interface. We find this most satisfactory, for it shows that probability only emerges if some part of the system is left out of consideration, and therefore has a similar origin as in classical physics: it reflects ignorance of the discarded part. The kernel of quantum mechanics remains fully deterministic.

Carefully note, that this does not lead to a conventional hidden variables interpretation of quantum mechanics, for the part of the world that is declared irrelevant is a quantum system. Also, the elimination of the irrelevant observables should not be confused with incorrect attempts to resolve the measurement problem by finding the apparatus in a mixed state (namely a reduced density matrix) obtained by tracing out either the measured system or the environment; such mixed states do not have a unique extremal decomposition, and even if their orthogonal decomposition is unique, they do not admit a consistent ignorance interpretation (on the latter point see e.g. Beltrametti and Cassinelli (1981), van Fraassen (1991), Busch et al. (1991)).

If this approach can be made to work, it obviates the need for a ‘collapse of the wavefunction’ as a consequence of the act of perception. Recall that in his attempt to resolve the measurement problem von Neumann (1932) introduced an irreversible time-evolution in quantum mechanics, on top of the reversible unitary evolution given by the Schrödinger equation. This was motivated by his idea that these two evolutions would correspond to the system evolving either while being perceived, or autonomously, respectively. In contrast, in approaches based on superselection rules there is a single time-evolution, and the duality solecistically identified by

\[ A. \] In other words, for the classical observables there is no distinction between the central and the extremal decomposition. In view of our value attribution rule, the coefficients of the central decomposition may therefore still be interpreted as Born probabilities. The ignorance interpretation of mixed states is now only applied to the central decomposition. Conversely, the uniqueness of the central decomposition as opposed to the nonuniqueness of the extremal decomposition may be used as an argument in favour of our point of view that only classical observables possess values. For states $\psi$ of the special type considered in the main text, the extremal and the central decompositions coincide.
von Neumann actually corresponds to the system being purely quantal, or (partly) classical, depending on its algebra of observables. The ‘collapse’ does enter through the back door, but plays a different, mind-independent role: as we shall see, it sneaks in through the definition of the time-evolution of the algebra of observables of the environment. We stress that this has nothing to do with acts of measurement of observation.

The main program in this approach is evidently to firstly describe and justify the truncations allowing the ensuing algebra of observables to have a nontrivial centre, and secondly to derive the appropriate mixed ‘post-measurement state’ \( \psi \) having the desired unique extremal decomposition. This leads to considerable difficulties, to which we shall turn first.

## 2 A simple model

The following model of a measurement apparatus was considered by Hepp (also cf. Bona (1980), Bub (1988)). It seems that most points of philosophical interest can be discussed in its context. The apparatus consists of an infinite chain of spin 1/2 particles. The Hilbert space of states \( \mathcal{H} \) (which is non-separable) is the so-called complete\(^{13}\) infinite tensor product of all single-particle Hilbert spaces \( \mathbb{C}^2 \). The set of all beables would correspond to the algebra \( \mathcal{B}(\mathcal{H}) \) of all bounded self-adjoint linear operators on \( \mathcal{H} \). The dynamics of the system is given by a nearest-neighbour coupling of ferromagnetic type. Hence the ground state is doubly degenerate: either all spins are up, or they are all down. Equivalently, one may regard this apparatus as a photo-emulsion (which is what Hepp did). In this interpretation the ‘spin up’ state of each spin 1/2 particle is replaced by an AgBr molecule (assumed to have just one state of interest), while the ‘spin down’ state corresponds to an Ag atom.

The crucial assumption is now that pointer observables are generated by local operators of the type \( A_1 \otimes A_2 \otimes \ldots A_n \ldots \mathbb{I} \ldots \mathbb{I} \ldots \), in which only a finite number of entries \( A_i \) (which are 2x2 matrices) are different from the unit matrix \( \mathbb{I} \). This as-

\(^{13}\)This means that there are no restrictions on the infinite tail. An example of an incomplete infinite tensor product would be the Hilbert space completion of the set of those states in which only a finite numbers of spin are up. See von Neumann (1938).
On this assumption, $\mathcal{H}$ splits up into disjoint superselection sectors. For example, the ground states $\uparrow \ldots \uparrow$ in which all spins are up, and $\downarrow \ldots \downarrow$ in which they are all down, have the property $(A \uparrow \ldots \uparrow, \downarrow \ldots \downarrow) = 0$ for all $A$ in $\mathfrak{A}$. Any two such vectors in $\mathcal{H}$ between which all matrix elements of $\mathfrak{A}$ vanish, are accordingly said to lie in different superselection sectors. Conversely, two vectors which differ only in a finite number of single-spin states are clearly in the same sector. If we form a sum $\Psi = a\Psi_1 + b\Psi_2$ of two vectors lying in different sectors, then the corresponding state $\psi$ on $\mathfrak{A}$ equals $|a|^2\psi_1 + |b|^2\psi_2$, that is, it is mixed.

Now consider the operator $s_n = I \otimes \ldots \sigma_3 \otimes I \ldots$, which has $\sigma_3 = \text{diag}(1, -1)$ in the n-th entry, and unit matrices everywhere else. From the $s_n$ we build $S = \lim_{N \to \infty} 1/N \sum_{n=1}^N s_n$. (This limit exists in the weak operator topology and lies in $\mathfrak{A}$.) The operator $S$ has the remarkable property that its matrix elements between any two vectors in a given sector coincide, so that it only ‘sees’ in which sector a given vector lies. For example, $(S\Psi, \Phi) = 1$ for any two vectors $\Psi, \Phi$ which lie in the same sector as $\uparrow \ldots \uparrow$, and $(S\Psi, \Phi) = -1$ in the sector of $\downarrow \ldots \downarrow$. Indeed, the operator $S$ is a classical observable, as defined in the Introduction.

To apply this setting to the measurement problem, consider a single spin 1/2 particle, whose spin is measured by the pointer. Assuming that the initial state of the pointer is $\uparrow \ldots \uparrow$, this implies that the post-measurement state of the pointer should be in the sector of $\uparrow \ldots \uparrow$ in case the particle spin is up, whereas it should be in the sector of $\downarrow \ldots \downarrow$ in the opposite case. If so, we can simply look at the operator $S$ to see what the spin of the particle was. If the particle is in a superposition $a \uparrow + b \downarrow$ of up and down states, the pointer will not be in the corresponding superposition of $\uparrow \ldots \uparrow$ and $\downarrow \ldots \downarrow$: instead, its state $\psi$ will be mixed on $\mathfrak{A}$, and the unique decomposition into pure states is (with some abuse of notation)

$$\psi = |a|^2 \uparrow \ldots \uparrow + |b|^2 \downarrow \ldots \downarrow.$$
Hence, as promised, the Born probabilities normally associated with the particle whose spin is being measured, emerge as coefficients in the decomposition of the mixed post-measurement state of the apparatus. Moreover, the usual measurement 'paradox' of arriving at never-observed superpositions of pointer states has been obviated\textsuperscript{15}. If the post-measurement emergence of the pointer in a definite state, in which $S$ has either the value 1 or -1, is taken to be an event, then the occurrence of such an event is definitely predicted, although which of the two possibilities is realized is a probabilistic affair, as usual. This statement relies on the ignorance interpretation of mixed states allowing a unique extremal decomposition.

But how is the desired post-measurement state to be arrived at? The problem is that the initial pointer state $\uparrow \cdots \uparrow$ should stay in the same sector if coupled to the state $\uparrow$ of the particle, whereas it should evolve into the $\downarrow \cdots \downarrow$ sector if coupled to the $\downarrow$ state. Hepp showed, in models as well as in a theorem, that this cannot be done in finite time with an automorphic evolution\textsuperscript{16}. Heuristically, what has to happen is that the particle in the spin down state should flip the first spin in the pointer, then the second..., so that it is obvious that one needs an infinitely long time to complete the measurement. In other words, the dilemma is this: superselection sectors are defined by the property that no observable (such as the Hamiltonian) can interpolate between them. Yet the little spin which is measured is supposed to cause precisely such a transition.

The best one can achieve is that
\[
\lim_{t \to \infty} (A(t) \uparrow \cdots \uparrow \otimes \downarrow, \uparrow \cdots \uparrow \otimes \downarrow) = (A(0) \downarrow \cdots \downarrow \otimes \downarrow, \downarrow \cdots \downarrow \otimes \downarrow)
\]
for each fixed $A \in \mathfrak{A}$; here $A(t)$ is the Heisenberg picture time evolution of the operator $A$. This led to (a revival of) the idea, that one ought to regard a measurement

\textsuperscript{15}Cf. Dieks (1991) and Landsman (1991) for two explanations why this 'paradox' is spurious anyway.

\textsuperscript{16}This is the algebraic counterpart of a unitary evolution in the usual formalism. Also cf. Landsman (1991) for a further discussion. The claim in Bub (1988) that this can be accomplished nonetheless is too hasty. It is trivial to find a unitary group $U_t$ which maps any sector in any other one in a finite time; the point is that in addition a corresponding Heisenberg picture time-evolution of the algebra of observables should be defined. This is not done in Bub (1988); in particular, for his $U_t$ the map $A(t) = U_t A U_t^*$ maps $A$ outside the algebra of observables $\mathfrak{A}$. To get back into $\mathfrak{A}$, one would need a conditional expectation from $\mathfrak{B}(\mathcal{H})$ to $\mathfrak{A}$, as in section 4 below. In the present case (and similar ones) such an object is not readily available.
as a process analogous to a scattering event, which officially takes infinite time to be completed, too (in the sense that the particles only approach their on-shell states in the $t \to \infty$ limit), with no one ever complaining about the idealization this implies.

### 3 The disaster of infinite measurement time

Unfortunately, one cannot help feeling uncomfortable with this approach to the measurement problem. States in different superselection sectors may be thought of as being ‘macroscopically’ different: they differ by an infinite number of single spin states in the pointer. If one only monitors a finite number of spins, one cannot detect in which sector the pointer state lies. We are supposed to see instantly what the (final) macroscopic state of the pointer is, for otherwise we would not accept it as a measurement device. However, at any finite time this macroscopic state coincides with the initial state (up to a finite number of spins). (Note, that our alleged ability to see an infinite number of spins as such does not conflict with the assumption that the pointer observables are (quasi-) local. This ability means, that we can monitor observables like $S$. A nonlocal observable, however, would detect the quantum interference between states in different sectors, that is, such an observable would correlate an infinite number of spins. And to see such correlations is surely beyond us).

The achievement of a limit means, that for finite but very large time the quantity that is to have the given limit (namely the matrix element of a fixed local observable $A$), differs from its limit value by an arbitrarily small number. Since in any finite time only a finite number of spins in the pointer have flipped, that is, the pointer is still in its original sector $\uparrow \ldots \uparrow$, we can only have the illusion that the measurement has almost been completed because the fixed operator $A$ above monitors a finite number of spins (or, as in the case of the pointer observable $S$, is a weak limit of such operators). But it is precisely the infinite ‘tail’ of the pointer which determines in which sector it is, and which ‘macroscopic’ properties it has. If we really ‘observe’ the entire pointer, we would at any finite time conclude that the pointer is still in the $\uparrow \ldots \uparrow$ sector, and abandon the hope that any form of measurement of the particle
spin is being performed.

A somewhat confusing variant of this argument was forwarded in Bell (1975), where the poverty of the way the infinite-time limit of the measurement is approached is illustrated by the following observation. Suppose we give the operator \( A \) an explicit time-dependence \( A_t \), which exactly cancels its Heisenberg-picture time-dependence \( A(t) \). Then \( (A_t(t) \uparrow \ldots \uparrow \otimes \downarrow, \uparrow \ldots \uparrow \otimes \downarrow) \) is obviously time-independent, and no limit will ever be reached! Of course, we previously assumed \( A \) to be fixed, and by replacing it with \( A_t \) one inserts an infinite family of operators. To interpret this argument properly, it should be realized that it can be forwarded against the idea of measurement altogether, in that the explicit time-evolution \( A_t \) simply ‘undoes’ the measurement. Thus we briefly discuss this possibility.

The ‘undoing’ of quantum measurements was discussed by Peres (1980, 1986), who claimed its impossibility. The argument is that the construction of the explicit time-dependence \( A_t \) amounts to an inversion of the equations of motion; this would imply that given the pointer state at time \( t \), Bell (1975) would have to calculate its state at \( t = 0 \). For the Heisenberg picture matrix element \( (A_t(t)\Psi_1, \Psi_2) \) equals the Schrödinger picture \( (A\Psi_1(t), \Psi_2(t)) \), which indeed by construction equals \( (A\Psi_1(0), \Psi_2(0)) \). If the pointer is sufficiently large (it is, indeed, assumed to be infinite), that would certainly have been beyond Bell’s abilities, were it not for the fact that the initial pointer state is known to be \( \uparrow \ldots \uparrow \!).

Thus Peres’ counterargument seems to lose its weight. On the other hand, one might argue that the initial pointer state does not need to be exactly \( \uparrow \ldots \uparrow \); it suffices for it to be in the same superselection sector. But to use the type of complexity argument in Peres (1980), a sufficiently large number of pointer spins should be randomly determined in the initial state. But how large is ‘sufficiently large’, compared to the infinite number of spins that are fixed to be up in this sector? Fortunately, irreversibility arguments attempting to make the undoing of the measurement impossible by declaring the initial state to be randomish are very powerful indeed, but need a little extra ingredient, as we shall see in the next section.

A further point is that one should acknowledge that all pointers are actually
composed of a finite number of particles; the infinite system is the ‘exact background theory’ that B-realists always call for. But to what extent does the theory of a finite pointer approximate this exact theory? One usually hears that on account of the Stone-von Neumann uniqueness theorem, superselection rules do not exist in finite systems. Hence one would have to go for approximate superselection rules, in the sense that one tries to find classes of states with the property that matrix elements of ‘relevant’ observables between any two vectors lying in a different class are ‘small’, cf. Lloyd (1988). The idea here is that only a relatively small number of spins determine the relevant algebra of observables, the much larger remainder playing the role of the infinite tail of the exact theory.

This is problematic; for one thing, the ignorance interpretation of mixed states only works if some unique decomposition is applied (such as the central decomposition, cf. footnote [2]). But if the superselection rules are only approximate, the centre is trivial and one does not even have suitable ingredients for a decomposition of an exact mixture, let alone of an approximate one. And what is small and what is large? The relevant scale should be set externally. It all starts to sound pretty vague. (This dilemma has led Bub (1988) to propose that one needs truly infinite apparatuses to resolve the problem; hence only infinite systems can have ‘objective’ properties. While we are sympathetic to the idea of an approximate reality in finite systems, the problem of the infinite-time limit is not resolved in this way (cf. footnote [6] on Bub (1988)). Moreover, the origin of the objectification of properties of infinite systems still lies in the choice of local observables; this idea, however, is much more convincingly implemented if one invokes the environment (see below).)

Apart from this conflict between macroscopic observability and locality of the interaction with the measured system, one may argue that resolutions of the measurement problem involving superselection rules of the apparatus are incomplete.

17The fact that the stuff of the world is quantum fields rather than particles does not affect this discussion, despite the fact that a field theory has an infinite number of degrees of freedom even in a localized region. For the local algebras in quantum field theory are factorial von Neumann algebras of type I (in the non-relativistic case) or type III (in the relativistic case), and these admit only one (normal) representation up to unitary equivalence. Hence they cannot lead to superselection rules.
even if they work, for the ‘algebra of observables’ is not an intrinsic object: by
name alone, it presupposes a theory of the observer and his/her/its coupling to the
apparatus. Hence in any case the choice of the observables, and thereby the supers-
election rules, must be motivated externally. So far, their introduction has (at best)
merely parametrized the classical behaviour.

4 Environmental superselection rules

Let us return to a finite (but large) pointer, into whose state space we still would
like to introduce superselection sectors in some sense. A crucial assumption in
the Stone-von Neumann uniqueness theorem, which is usually quoted as excluding
superselection rules in finite systems, is that one has a simple algebra\(^{18}\). Indeed,
a finite system does have superselection rules if its algebra of observables is not
simple, cf. Landsman (1991). To illustrate this point, consider the algebra \(M_2\) of
\(2 \times 2\) matrices, acting on the Hilbert space \(\mathbb{C}^2\). Let the basis vector \(e_1\) stand for a
photon\(^{19}\) state localized very far away from the photon state \(e_2\). Saying that \(M_2\) is
the algebra of observables of this system amounts to pretending that an operation
exists by which one can determine quantum interference between these basis states.
This not being feasible\(^{20}\), one will have to admit that the actual ‘effective’ algebra
of observables, relevant to a local observer who/which is unable to perform highly
nonlocal measurements, is \(D_2 \simeq \mathbb{C} \oplus \mathbb{C}\), the algebra of diagonal \(2 \times 2\) matrices. This
surely has two inequivalent representations\(^{21}\) and hence two superselection sectors.
The first sector has \(e_1\) as its only normalized vector (up to a phase), and the second
sector contains merely \(e_2\). This is possible, as \(D_2\) is not simple; instead, it is the

\(^{18}\) A simple \(C^*\)-algebra is one without closed 2-sided ideals. Since the kernel of any representation
forms an ideal, it follows that all representations of simple algebras are faithful.

\(^{19}\) Any light particle would do. We ignore helicity, anyway.

\(^{20}\) The claim in Aharonov et al. (1986) that nonlocal states can sometimes be measured is of no
relevance here. To determine quantum interference between localized states \(x\) and \(y\) one needs an
operator \(A\) such that \((Ax, y) \neq 0\). But the observables considered in Aharonov et al. (1986) are
of the form \(A_1 + A_2\), where the localization region \(O_1\) of \(A_1\) contains \(x\) but not \(y\), and \textit{vice versa}
for \(A_2\). Such operators have vanishing matrix elements between \(x\) and \(y\). Moreover, they are not
nonlocal at all in the usual sense (Haag, 1992): they are localized in \(O_1 \cup O_2\).

\(^{21}\) The first one maps the first copy of \(\mathbb{C}\) to itself, and the second copy to zero; the second one
does the opposite.
direct sum of 2 copies of the algebra of the complex numbers.

Our finite pointer can inherit these superselection rules in the following way. If the pointer contains \( N \) spins, its algebra of observables is \( M_{2^N} \). The algebra of the total system pointer + photon is then \( \mathfrak{A}_{\text{tot}} = M_{2^N} \otimes D_2 \), and has 2 sectors. In a first attempt to exploit this, one would like to find an initial photon state \( I_E \) and a pointer-photon interaction such that, any pointer state \( U \) in which an overwhelming majority of the spins is up, coupled to (i.e., tensored with) \( I_E \) evolves into \( U \otimes e_1 \), whereas the analogous states \( D \otimes I_E \) with most spins down evolve into \( D \otimes e_2 \). These two final states are in different superselection sectors, and the same effect has been achieved as in the infinite pointer. The role of the infinite tail of the latter is now played by the photon, which flies away to the Andromeda nebula. (As in the preceding section, this transition cannot be achieved with an automorphic time evolution; see below for the resolution.)

If we now re-introduce the particle whose spin is measured, we see that once again a superposition \( a \uparrow + b \downarrow \) of states of this particle does not lead to a corresponding superposition of the pointer + photon, but to a mixture whose decomposition into pure states has the Born numbers \( |a|^2 \) and \( |b|^2 \) as coefficients. The origin of this ‘collapse of the wavepacket’ now lies in the relative delocalization of the photon states \( e_1 \) and \( e_2 \), combined with the presumed locality of the observer, rather than in the locality of the pointer observables (see below for a more detailed explanation of this point). Also, since the pointer is finite, one only needs a finite time for all the spins in it to flip if necessary, and the photon flies away rather quickly, too.

Unitarity of the time evolution implies that the initial photon state has to be fixed in order to achieve this scenario. This is undesirable, firstly because photons are not part of the construction of pointers (they belong to the ‘environment’), and secondly because the invariability of the initial state leads to the possibility of ‘undoing’ the measurement, as explained earlier. The way out is to consider a very large environment \( E \) (perhaps consisting of an enormous number of photons). Its algebra of observables \( \mathfrak{A}_E \) is still supposed to lead to superselection rules on its state space, by not containing operators whose measurement would involve the detection
of quantum interference between states which are localized lightseconds or more away from each other. The mechanism by which the omission of such operators from the original simple algebra leads to the ensuing algebra being non-simple, is exactly the same as in the example above.

The general strategy to obtain superselection rules (caused by the locality of the observer) is to identify subsystems between which no correlations are observed. For example, one may imagine a sphere with radius $R$ around the observer, and stipulate that no correlations with any region outside this sphere are observed. The precise value of $R$ is rather arbitrary, as long as it is very large compared to the size of the system studied. A measurement is then completed the moment the objects (e.g., photons) carrying the quantum correlations of the system after the interaction with the environment have left the sphere.

The only requirement on the dynamics is that $U \otimes I_E$ and $D \otimes I_E$ evolve into $U' \otimes e_n$ and $D' \otimes e_m$, respectively, where the final environment states $e_n$ and $e_m$ (which may vary with the initial environment state $I_E$) lie in different superselection sectors, and $U'$ and $D'$ are ‘close’ to $U$ and $D$ (in a sense to be made more precise shortly). If the interaction between pointer and photon bath is suitable (see below), this condition will be satisfied by the overwhelming majority of initial states, for the environment has a large number of particles, each of which interacts with the pointer, and it suffices if merely one of these particles has an initial state leading to orthogonal final states after coupling with the pointer states $U$ and $D$, respectively. Even if that is not the case, the phases of the terms in the inner product $(e_n, e_m)$ (which is a sum of products of a gigantic number of single-particle inner products) will be random so that the product vanishes (a point forcefully made by van Kampen (1988), who added that someone who does not accept this doesn’t understand what physics is). Any reasonable assumption on what the algebra of observables of the

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22 This differs from the approach of Wan and Jackson (1984). Their condition for an operator $A$ on $L^2(\mathbb{R}^3)$ to be an observable is that $(A\Psi, \Psi) = 0$ for all $\Psi$ localized outside the sphere. Our condition is that $(A\Psi, \Phi) = 0$ for all $\Psi$ localized outside the sphere and $\Psi, \Phi$ having disjoint (essential) support.

23 Sending $R$ to infinity is not the same as having no dismissed correlations at all; it rather corresponds to ignoring correlations at infinity. The superselection rules of local field theories are a consequence of this.
environment relative to a local observer is will then imply that \( e_n \) and \( e_m \) are not merely orthogonal but lie in different sectors on top of that.

The technical conditions under which the above scenario works were established by Zurek (1981, 1982, 1993) (also cf. Joos and Zeh (1985) and Zeh (1970)), where the idea of introducing the environment in the measurement problem was first proposed. In these papers, the central issue is to identify the family of pointer states \( \{ p_n \} \) which has the property we ascribed to \( U \) and \( D \), that is, that for most environment states \( I_E \), \( p_n \otimes I_E \) evolves into \( p_n \otimes e_n \) for some environmental state \( e_n \), with \( e_n \) and \( e_m \) orthogonal for \( n \neq m \). This is so if the operator \( P \) which has the \( p_n \) as eigenstates commutes with the interaction Hamiltonian describing the coupling between the pointer and the environment.

As a first application of this rule, we can easily see why the presumed locality of an observer excludes the possibility of measuring correlations between two photon states which are delocalized relative to each other. For let \( e_1 \) and \( e_2 \) have their previous meaning. To detect interference between these states, one would need to bring eigenstates of the Pauli matrices \( \sigma_1 \) or \( \sigma_2 \) into correlation with the eigenstates of a certain operator relevant to the observer; the former eigenstates here play the role of the \( p_n \). But a local observer-photon interaction Hamiltonian must have the form \( H_I = [e_1] \Phi_1 + [e_2] \Phi_2 \), where \([e_i]\) is the projector on \( e_i \), and the \( \Phi_i \) are operators on the state space of the observer. Now clearly the commutators \([H_I, \sigma_1]\) and \([H_I, \sigma_2]\) are nonzero, so it follows that the establishment of the desired nonlocal correlation must involve a nonlocal interaction Hamiltonian (an obvious result!).

If the pointer is large, one can relax the stability condition on \( p_n \) under time-evolution, so that one is in an even more comfortable position than Zurek’s. For in that case the only requirement is that \( p_n \otimes I_E \) evolves into \( p'_n \otimes e_n \), where \( p'_n \) is macroscopically close to \( p_n \) (and \((e_n, e_m) \simeq \delta_{nm}\), as before). To illustrate what this means, consider the case where the pointer observable \( P \) is taken to be \( S = 1/N \sum_{n=1}^{N} s_n \), defined as in the previous section, but not taking the limit \( N \to \infty \). The eigenstates of \( S \) are states of the form \( \otimes_{n=1}^{N} u_n \), where each \( u_n \) is an eigenstate of \( \sigma_3 \). The spectrum of \( S \) is \( \{(2N_+ - N)/N \mid N_+ = 0, \ldots, N\} \); here \( N_+ \) is simply
the number of $u_n$'s whose spin is up. This spectrum is highly degenerate, and, more importantly, for large $N$ the eigenvalues are closely packed. To read off the measurement outcome it is sufficient that the expectation value of $S$ in the final state is either close to 1, or to -1. So in the above argument, $p_n$ and $p'_n$ should have almost the same expectation value of $S$. This modified requirement makes Zurek's stability condition far more robust, and easier to satisfy.

The transition $p_n \otimes I_E \rightarrow p_n \otimes e_n$ is evidently a time-dependent process, so let us write $e_n(t)$ to make this clear. It turns out that $(e_n(t), e_m(t)) \simeq \delta_{nm}$ only for $t \rightarrow \infty$ and an infinite environment. For finite environments the inner product typically becomes exponentially and phenomenally small quickly, gets ever smaller, until it returns to sizable values after supersonic timescales (Poincaré recurrence), cf. Zurek (1981, 1982, 1993), Joos and Zeh (1985), and refs. therein. The consequences of this behaviour may be illustrated on our familiar one-photon example. So let the post-measurement state vector be $a U \otimes (e_1 + \epsilon(t)e_2) + b D \otimes (e_2 + \epsilon(t)e_1)$; we ignore terms of order $\epsilon^2$, so that this state is normalized. Under the same assumptions as before, this state is mixed, and the corresponding density matrix has the unique extremal decomposition $\rho(t) = |a|^2[v_1] + |b|^2[v_2]$, where $[v_i]$ denotes the projector on $v_i$, $v_1 = |a|^{-1}(a U + \epsilon(t)b D) \otimes e_1$ and $v_2 = |b|^{-1}(b D + \epsilon(t)a U) \otimes e_2$. Since the decomposition is unique, we are still entitled to apply the ignorance interpretation of such mixed states, which implies that the coupled system is either in the state $v_1$ (with probability $|a|^2$), or in the state $v_2$ (with probability $|b|^2$). Here $\epsilon(t)$ is very small, and gets increasingly smaller, as indicated above.

We see that no difficulty arises. All macroscopic observables (like $S$) will equate $v_1$ with $U \otimes e_1$ and $v_2$ with $D \otimes e_2$ (up to terms of order $\epsilon^2$). To distinguish between $v_1$ and $U \otimes e_1$, one would have to perform interference experiments involving all spins in the chain, and even so one would detect an effect of order $\epsilon(t)$, which, as mentioned before, is astoundingly small. In case one is actually able to do all this, it has to be concluded that the pointer has failed its service as a measurement apparatus: if an undesired superposition can be detected, one should stop looking for arguments why it is actually not there. But if it is there, the measurement problem does not
arise, for no measurement has been performed in that case!

The same would-be difficulty arises in the modal interpretation (see below), and, as pointed out by Dieks (1994a), in practically any measurement theory. His answer is as satisfactory as the one given above; he simply points out that in the modal interpretation the pointer possesses a (macroscopic) spin in the direction determined by $a U + \epsilon(t)b D$; this direction is practically indistinguishable from the $z$-axis, and the problem has been obviated (the second point discussed by Dieks concerns the situation where $a \simeq b$, but the difficulties this leads to are only relevant for small pointers$^{24}$).

The argument above was given in the Schrödinger picture, where the states evolve in time. This conceals a very important aspect of our scenario. In the single-photon example, where the algebra of observables was taken to be $D_2 \subset M_2$, the time-evolution of the state vectors $\Psi \in \mathbb{C}^2$ is simply given by a unitary one-parameter group $U_t$ in $M_2$, i.e., $\Psi(t) = U_t \Psi$. This defines an automorphic time-evolution on $M_2$ by passing to the Heisenberg picture, in which $A(t) = U_t^* A U_t$ for each $A \in M_2$. However, this will generally map elements of $D_2$ outside this algebra, unless each $U_t$ itself lies in $D_2$. But if the initial state is $c_1 e_1 + c_2 e_2$, then such a time-evolution would merely change the phases of the $c_i$, and the whole show would have to be canceled. However, the observed time-evolution $A_{\text{obs}}(t)$ of an observable $A \in D_2$ is obviously not $A(t)$, but merely the part of this operator which lies in $D_2$, that is, its diagonal. Defining the projectors $P_i = [e_i]$ ($i = 1, 2$) we can write this as $A_{\text{obs}}(t) = \sum_i P_i A(t) P_i$.

The proper setting for this discussion is the algebra of observables $\mathfrak{A}$ of the system, the apparatus, and the environment together. In our context its superselection structure only derives from the last component, but this particularity is irrelevant for the following construction. Let $\mathcal{H}$ be the Hilbert space of state vectors of $\mathfrak{A}$; according to our assumptions, this contains a number of superselection sectors $\mathcal{H}_i$ (for simplicity we assume discrete superselection rules). We denote the projector onto $\mathcal{H}_i$ by $P_i$. If $U_t$ is the unitary one-parameter group defining time-evolution on $\mathcal{H}$, 

$^{24}$Also cf. Bacciagaluppi and Hemmo (1994) for an extensive discussion of this controversial issue.
then, as before, one has $A_{\text{obs}}(t) = \sum_i P_i U_t^* A U_t P_i$. For $U_t \notin \mathfrak{A}$ this evolution is non-automorphic and irreversible, and the origin of this irreversibility (namely projection on the relevant degrees of freedom) is the same as in conscientious non-equilibrium statistical mechanics, e.g. Balian et al. (1986).

Also, one is immediately reminded of the collapse of the wavefunction. For if $\rho$ is a density matrix then the expectation value $\text{Tr} \rho A_{\text{obs}}(t)$ equals $\text{Tr} \rho_{\text{collapsed}} A(t)$, where $\rho_{\text{collapsed}} = \sum_i P_i \rho P_i$. But there are two important differences between the above ‘collapse’ of $\rho$ to $\rho_{\text{collapsed}}$, and the traditional (von Neumann) collapse. Firstly, the former does not take place instantaneously; indeed, it has nothing to do with perception and little with measurement. Rather, it is a consequence of the selection of the algebra of observables. Secondly, the collapse does not take place with respect to the spectral projections of the system observable that is measured, but with respect to the projectors on the superselection sectors. Since in our approach the latter are neither related to the system that is measured, nor to the measurement apparatus, we see that the two collapses should not be confused indeed.

A further point concerns the argument why the pointer should be in either the state $U$ or in $D$, given that the total system of pointer + environment has the state vector $a U \otimes e_1 + b D \otimes e_2$. Zurek here mumbles something about “the right of a macroscopic (but ultimately quantum) system to be in its own state” (Zurek, 1993, p. 287), and proceeds with the usual argument of reducing the density matrix of the total system by tracing over the environment. This leaves the mixed state $|a|^2[U] + |b|^2[D]$ of the pointer, which is then construed according to the ignorance interpretation. The last step is justified by Zurek by the idea that the basis relative to which this reduced state is decomposed is precisely the preferred pointer basis (Zurek, 1981).

Although this reasoning leads to the correct result, it is based on a questionable application of the ignorance interpretation (namely to mixtures whose extremal...
decomposition is non-unique). Moreover, it obscures the origin of the ‘factualization’ of the states $U$ and $D$, viz. the assumed superselection rule of the environment, which itself is caused by the delocalization of the quantum coherence (which according to the ‘kernel’ of the theory is always there) and the locality of the observer. Hence in our opinion the correct argument is as follows. Without superselection, i.e., serious restrictions on what the observables of the theory are, there is no collapse of the wavepacket. On the other hand, if the operators interpolating between highly delocalized states of the environment are removed, the state $|a U \otimes e_1 + b D \otimes e_2|_+$ collapses to $|a|^2[U \otimes e_1] + |b|^2[D \otimes e_2]$ in the case at hand. Thus one not only has the collapse of a pure to a mixed state, but on top of that the mixed state has a unique decomposition into the pure states $U \otimes e_1$ and $D \otimes e_2$. These are product states, hence the only state of each subsystem consistent with each of these pure states on the combined system is the one occurring as a factor in the tensor product.

However, this does not mean that the pointer observable $S$ now has a value, at least not in our interpretation. In the Introduction we mentioned that in our interpretation an observable can only have a given value (in other words, possess a property) if it is in the centre of the algebra of observables; in particular, we abandon the usual stipulation that an arbitrary observable possesses a value (namely, the pertinent eigenvalue) if the system is in an eigenstate of the corresponding operator. Hence, if we take the (finite) pointer by itself, we are unable to conclude that the macroscopic spin observable $S$ has the value $+1$ even if all the spins in the pointer are up. Indeed, since the algebra of observables of the finite pointer has no superselection rules, none of its observables has a value in any state: a pure quantum system simply has no properties in the usual sense.

Our illusion that the pointer is ‘up’ is entirely caused by its photon environment. It suffices to illustrate this matter in the model we used before, where the environment consists of a single photon. The total algebra of observables was $\mathcal{A}_{\text{tot}} = M_{2^N} \otimes D_2$; its centre is simply $\mathbb{I}_{2^N} \otimes D_2$, where $\mathbb{I}_{2^N}$ is the identity operator for the pointer. In particular, the photon operator $\mathbb{I}_{2^N} \otimes \sigma_3$ is in this centre, and in our interpretation we are allowed to conclude that it actually has the value $+1$.
in any state of the form $\Psi \otimes e_1$, where $\Psi$ is an arbitrary pointer state. Given the dynamics of the pointer-environment interaction, which brings the states $\uparrow \ldots \uparrow$ and $e_1$ in correlation, we may then conclude that the pointer is in the state $\uparrow \ldots \uparrow$ if we observe the photon in the state $e_1$, and, by implication, that the little particle whose spin was measured by the pointer, is in the state ‘spin up’ as well. Yet none of the pointer observables possess values corresponding to this state, because they do not possess any value at all. This is an entirely satisfactory conclusion, for all we actually see is the photon environment. Of course, this conclusion is really convincing only if the photon environment is huge; the qualitative argument is the same also in that case. And when means of monitoring other than vision apply, an analogous argument hawks about, the photon environment replaced by whatever is being monitored by the observer.

Finally, note that the pointer itself does have properties if it is infinite, and its algebra of observables has a nontrivial center. The point of the discussion in section 3 is that this property does not yet suffice to make it a measurement apparatus.

## 5 Some remarks on the modal interpretation

Many issues concerning environment and superselection are also relevant for the modal interpretation of quantum mechanics. This programme is still under development; there are four different versions, due to van Fraassen (1991), Kochen (1985), Healey (1989), and Dieks (1994a). The essential business of the modal interpretation is to put a subtle form of the projection postulate neither in the mathematical formalism, nor in Nature itself, but in the rules of interpretation of the formalism. As such it is able to provide a realistic interpretation of quantum mechanics (in the sense of ‘A-realism’, cf. the Introduction) without changing its mathematical structure at all. Further using the terminology of the Introduction, the modal reading of quantum mechanics puts the physical interpretation of the formalism into the kernel, and aims at describing observations as special instances of properties of systems

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27 We have cited the most accessible and relevant publications. There are older papers by van Fraassen and by Dieks on this subject, and the four authors have developed their ideas independently.
possessed anyway.

The modal interpretation maintains the rule that an observable of the total system actually possesses a value if the total system is in an eigenstate of it. However, the eigenvector-eigenvalue link is dropped as a bidirectional connection, in that observables of subsystems may have values in states where the von Neumann liturgy would deny that these values are possessed.

The starting point of the modal interpretation (at least in the version of Dieks (1994a,b), on which we will concentrate) is an arbitrary decomposition of the world $S$ into (say) two parts $S_1$, $S_2$, so that (in obvious notation) $\mathcal{H} = \mathcal{H}_1 \otimes \mathcal{H}_2$, and the state of the whole system $\Psi \in \mathcal{H}$ has the Schmidt decomposition $\Psi = \sum_{i=1}^{N} c_i \varphi_i \otimes \chi_i$. We assume that all the $\varphi_i$ and $\chi_i$ are normalized, so that $\sum_{i} |c_i|^2 = 1$; $N$ may be infinite, and we take $\mathcal{H}$ separable. Let us now concentrate on the subsystem $S_1$ taken by itself. Given $\Psi$, the Hilbert space of $S_1$ decomposes as $\mathcal{H}_1 = \oplus_{i'}^{N'} \mathcal{H}_1^{(i')}$. Here firstly $\mathcal{H}_1^{(0)}$ is the subspace orthogonal to all $\varphi_i$, secondly $\mathcal{H}_1^{(i')}$ equals the one-dimensional space spanned by $\varphi_i$ whenever $c_i$ is nondegenerate, and finally all vectors $\varphi_i$ with identical coefficients $c_i$ together span a single summand $\mathcal{H}_1^{(i')}$. Thus apart from 0 the index $i'$ takes the same values as $i$, except that those values of $i$ which correspond to the same numerical value of $c_i$ are combined into a single $i'$.

The crucial step in the modal interpretation is to specify which observables in $\mathfrak{B}(\mathcal{H}_1)$ possess values, given the total state $\Psi$. In a slight modification of the literature we stipulate that each self-adjoint element of the von Neumann algebra $\mathfrak{A}_d$ of all operators which are diagonal w.r.t. the above decomposition possesses a value. An operator $A \in \mathfrak{A}_d$ necessarily has discrete spectrum, and has the form $A = \sum_{i'} a_{i'} P^{(i')}$, where $P^{(i')}$ is the orthogonal projector onto $\mathcal{H}_1^{(i')}$. When all $a_{i'}$'s are different, the value $a_{i'}$ is possessed with probability $p_{i'} = \dim \mathcal{H}_1^{(i')} \cdot |c_i|^2$, where

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28For Healey (1989) there is a preferred decomposition corresponding to elementary particles.

29The choices in Dieks (1994a), Dieks (1994b), and Bacciagaluppi and Hemmo (1994) are all different from each other, as well as from our prescription. Dieks (1994a) excludes any operator acting nontrivially on $\mathcal{H}_1^{(0)}$. Dieks (1994b) includes all such operators as long as they act trivially on the orthogonal complement of $\mathcal{H}_1^{(0)}$, and assigns the possessed value 0 with probability 1 to them. The choice of Bacciagaluppi and Hemmo (1994) does not only depend on $\Psi$, but in addition on the contingent value state.

30In which case $A$ may still have degenerate spectrum, namely whenever the Schmidt decompo-
$i$ is related to $i'$ as explained in the previous paragraph. (We put $p_0 \equiv 0$.) If there are degeneracies among the $a_{i'}$ one merely has to sum over the $p_{i'}$ corresponding to the degenerate subspaces. Note that $\sum_{i'} p_{i'} = \sum_i |c_i|^2 = 1$. A similar assignment of properties holds for $S_2$ taken by itself.

We have written the modal interpretation in the above form in order to stress the analogy with the value attribution for systems with superselection rules discussed in this paper. The point is that both $\mathfrak{A}_d \subset \mathfrak{B}(\mathcal{H}_1)$ and $\mathcal{Z}(\mathfrak{A}) \subset \mathfrak{A}$ are commutative subalgebras of the algebra of beables and of observables of the system in question, respectively. It is clear that at any fixed time one may consistently assign properties to all observables in such subalgebras. In the modal interpretation this subalgebra depends on the choice of subsystem and on the state of the total system. In the superselection approach it originates in the observer and the environment. (Note that $\mathfrak{B}(\mathcal{H}_1)$ and $\mathfrak{A}$ have a rather different structure, e.g., the former does not have a nontrivial centre.)

The modal interpretation is tailor-made to add the finishing touch to the environmental approach in its original formulation (Zurek, 1982, Joos and Zeh, 1985), that is, without superselection rules. For, as we mentioned earlier, the aim of this approach is to interpret ‘pointer’ states of the form $p_n$, which have the property that they couple to the environment $E$ according to $p_n \otimes I_E \rightarrow p_n \otimes e_n$ for ‘arbitrary’ initial states $I_E$ of $E$, and $(e_n, e_m) \simeq \delta_{nm}$, as classical states. The modal interpretation provides exactly the missing link allowing such an interpretation, which helpfully shows that even such an intuitively attractive solution to the measurement problem as the environmental one, requires a double Dutch extra interpretative rule of quantum mechanics.

The value attribution given by the modal interpretation is equivalent to the ignorance interpretation of mixed states in the following sense. If the extremal decomposition of the mixed state obtained by restricting $\Psi$ to (say) $S_1$ is nonunique (which is the usual situation), the modal interpretation precisely leads to the correct interpretation of the orthogonal decomposition (which most often is unique, and cor-

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responds to the spectral decomposition of the reduced density operator). If, on the other hand, one of the subsystems (say $S_2$) possesses superselection rules, then after a short time the ‘pointer basis’ of $S_1$ is singled out by the Schmidt decomposition, and the modal interpretation ascertains that $S_1$ taken by itself is effectively in one of the states $p_n$, quite in accordance with our predilection.

However, this is precisely where we feel that the modal interpretation overshoots its aim: for it applies whether or not $S_1$ or $S_2$ are macroscopic, possess superselection rules, etc. For example, the moon may acquire properties (that it would not have had otherwise) by becoming correlated with a mouse, and this seems almost as bad to us as the moon only existing when the mouse looks at it (as in extreme versions of the Copenhagen interpretation, ridiculed in this way by Einstein). Thus to our mind the modal interpretation has a similar flaw as B-realists’ approaches to the measurement problem: it is made regardless of the situation at hand (system small or large? observer present and/or relevant? what are the relevant degrees of freedom?), and thus provides a universal rule in a problem where the particulars seem to be of prime importance.

In addition, there are two (related) technical difficulties with the modal interpretation, which are absent in our approach. The first problem is that of property composition of subsystems; it appeared in a seminar by R. Clifton and was further analyzed by Bacciagaluppi (1994). The perplexity is that if $S_1$ and $S_2$ possess property $P_1$ and $P_2$, respectively, one cannot conclude that the combined system $S_1 \& S_2$ possesses the combined property $P_1 \& P_2$. This is even true if one of the $P_i$ is the trivial property always possessed (which is represented by the unit operator). Hence an observable $A$ may apply to $S_1$ but $S_1 \otimes I$ may then not apply to the total system. Our approach does not have this problem, because the centre of an algebra is naturally contained in the center of any tensor product in which the algebra appears as a factor.

The second brain-twister concerns the combination of properties of a single sys-

\[31\] Here meant as a value state in the sense of van Fraassen (1991).

\[32\] Healey’s version of the modal interpretation is free of this difficulty, at some other expenses, cf. Bacciagaluppi (1994).
tem at different times. It amounts to the fact that if the system has properties $P_i$ at various times $t_i$, then the combination of $P_1$ at $t_1$ and $P_2$ at $t_2$ and... cannot really be regarded as a property, because the different (contingent) properties of that nature do not combine well under the usual rules of probability theory. In other words, the properties assigned by the modal interpretation at different times do not necessarily form a consistent history in the sense of Omnès (1992), Gell-Mann and Hartle (1990, 1993).

Our proposed value attribution does not suffer from this blemish, for the projection operators defining properties are central. Therefore, any history composed out of them automatically satisfies the so-called consistency conditions, which guarantee that the joint probability distribution of the multi-time properties behaves correctly under taking marginals. Conversely, this could be taken as an argument in favour of our value attribution prescription, which only assigns properties to classical observables. The price we pay is that (as we have seen) the time-evolution on the algebra of observables $\mathfrak{A}$ is generally non-automorphic, so that the centre is not necessarily stable under time-evolution. Hence a given observable may be classical at one time, but not at another.

Fortunately, the modal interpretation and the superselection approach (both crucially amended by environmental ideas; for the former this is explained in Dieks (1994a), for the latter we refer to the present paper) completely agree when applied to everyday situations. And it is precisely the explanation of common or garden life by quantum mechanics that causes B-realists such night thoughts... The essential point of coincidence between the two is that a subsystem may have properties that the total system does not have: objectification is achieved by ignorance. The ignorance in the former relates to the step of taking a subsystem by itself in order to assign properties to it, whereas in the latter the blindness is to the correlations in the discarded part of the world.

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33 This is being studied by (at least) D. Albert, D. Dieks, A. Kent, and P. Vermaas. We learnt of the difficulty from Kent.
6 Discussion

The bravura language used in this paper is not meant to conceal the fact that only the barest outline of a resolution (or, better, dissolution) of the measurement problem along the lines sketched has been given. More detail was not omitted for reasons of space or appropriateness to the journal, but because what has been written is all there is. What more is to be done?

Philosophically, the possibility that the abeyance of a certain class of observables is the origin of stochasticity in quantum mechanics deserves to be investigated. We stress that such an account should not be confused with the kind of ignorance about the state of some discarded subsystem (or environment) that is usually invoked. Both types of ignorance are codified by restricting a state on some algebra to a subalgebra; when the latter is a simple factor in a tensor product this amounts to partial tracing, and leads to the usual sort of nescience. In our proposal, on the other hand, one restricts to an algebra with a centre, which restriction cannot be described by partial tracing, for the original algebra is not the tensor product of the reduced one with some other algebra.

In this light, arguments in favour of the ignorance interpretation of mixed states (that is, only those whose extremal decomposition is unique, of course) would be welcome; such an interpretation is necessary to comply with the empirical predictions of quantum mechanics, but is not at all obvious (cf. van Fraassen (1991, sect. 7.3)). Similarly, it would be welcome to have an interpretation of the notion of a state which goes beyond the minimal definition as a rule to compute expectation values of observables. This definition begs the question of how such expectation values are to be interpreted, and is only unambiguous for the restriction of a state to the centre of an algebra of observables with superselection rules. For in that case, a state is simply a value attribution rule in the sense of classical physics. In the opposite case of a system without superselection sectors, we have seen that no value attribution takes place of any observable, and we should apparently look in the (unsatisfactory) direction of regarding states as preparation procedures.
In this context it is remarkable that the $\uparrow \ldots \uparrow$ state of the pointer (taken by itself) does not ascribe the value +1 to the observable $S$, although it has this very expectation value with mathematical variance (spread) 0. This should not make us feel too uncomfortable, for one always implicitly has to add the clause “if it were observed”, which for us means that it is brought into a suitable correlation with a system (i.e., the environment) having ‘beables’ that are not observed, and accordingly accommodates classical (i.e., central) observables. We have seen that if such a correlation is indeed established, it is the observation of the correlated state of the environment (rather than the pointer state itself) that leads to conclusions about the value of $S$. If, on the other hand, no such correlation is established, the statement that $S$ has the sharp expectation value +1 should be read as part of the mathematical specification of the state, rather than as a value attribution. For the latter would only be defined counterfactually, and we know from the EPR discussion how dangerous it is to confuse counterfactual conditionals with actual measurements in quantum mechanics.\textsuperscript{34}

On the technical side, the models in the literature (see Joos and Zeh (1985), Zurek (1993) and refs. therein; also cf. the discussion with refs. in Busch et al. (1991)) describing how system-environment interaction leads to decoherence, should be extended so as to deal with very large systems (so far, only the environment $E$ has been treated as macroscopic). In that way, the robustness of Zurek’s stability condition can be examined. Also, stochastic Schrödinger equations for the pointer state should be derived from its coupling to $E$, and solved; much of the necessary mathematical technology is under active development, inspired by rather different philosophies, but leading to similar mathematical structures, cf. the contributions of A. Amann, P. Blanchard and A. Jadczyk, P. Bona, N. Gisin, and H. Primas to Busch et al. (1994), as well as Blanchard and Jadczyk (1993) and Jadczyk (1994a,b),\textsuperscript{35}

\textsuperscript{34}Cf. Healey (1989) for arguments not to assign a value to certain observables which are dispersion-free in a given state.

\textsuperscript{35}In the approach of Primas the entire environment is taken to be classical, i.e., its algebra of observables is commutative. This exorbitant truncation of its beables is well-motivated by a deep result (Raggio’s theorem) to the effect that only in that case the quantum system coupled to the environment is free of EPR-correlations with it, and admits a so-called individual description. For our purposes the mere presence of a central subalgebra suffices.
for an up-to-date survey, and the seminal paper Gisin (1984) for older work\textsuperscript{36}. A theory of human consciousness would be helpful, too.

To close the paper, we will now recapitulate by answering some rational objections to the superselection programme we found in the literature. They come from van Fraassen’s book (van Fraassen, 1991), and are attributed to Beltrametti-Cassinelli, Hughes, Leggett, and van Fraassen himself, respectively (in Busch et al. (1991) the second one is ascribed to Piron).

1. Question: what accounts for the superselection rule?
   Answer: Ultimately, the locality of the observer. Under appropriate circumstances the coherence of the coupled system is delocalized, hence ever-present from an absolute point of view, but lost to the observer. This phenomenon causes most instances of ‘objectification’ in quantum mechanics.

2. Question: the fact that the system carrying the superselection rules is able to evolve in finite time from a given initial state into various different sectors (depending on the state of the system it is coupled to), implies that its Hamiltonian is not an observable. How about that? After all, the identification of the Hamiltonian with the observable Energy is one of the cornerstones of quantum mechanics.
   Answer: We have seen that in Hepp’s approach this objection is met by taking the $t \to \infty$ limit (at least when the time-evolution is automorphic). This limit also played a role in the environmental approach, in that the inner product $(e_n(t), e_m(t)) \simeq \delta_{nm}$ only for $t \to \infty$ and an infinite environment (cf. section 4).

As we have seen, the former limit was fatal, but the latter harmless. But that does not answer the question, for even if the inner product above is nonzero for finite times, the initial environment state must still evolve nontrivially ‘through’ the sectors, and that is not possible either, if the Hamiltonian is a function of the observables (cf. the discussion in section 4). The answer to

\textsuperscript{36} It is to be expected that such stochastic equations, derived from the unitary time-evolution of the whole system, reproduce the main features of the so-called GRW theory (Ghirardi et al., 1986), cf. Jadczyk (1994b). Thus it is curious that the spontaneous localization model of GRW is often interpreted as a fundamental modification of quantum mechanics.
the question is that the evolution driving the environment through its various superselection sectors is not Hamiltonian; more importantly, this evolution is generated by operators which do not belong to the algebra of observables. This is possible and consistent because of the special way the superselection rules of the environment arise in our approach. Namely, we start with a simple algebra of ‘beables’ of the environment; the Hamiltonian is in (or, technically, is affiliated to) this algebra. Then we truncate this algebra of beables to a smaller, effective algebra of observables having a nontrivial centre. The Hamiltonian is not in this smaller algebra, but it still drives the time-evolution of its state space. In other words, the reason that the Hamiltonian is not an observable is that we have willy-nilly restricted the set of operators to define the algebra of observables, but the discarded operators still contribute to the Hamiltonian.

3. Question: Do superselection rules add empirical content?
Answer: yes, in the following sense: the inspection of a certain apparatus or the study of some environment coupling to a given system may reveal that certain operators are not monitored, and that a ‘pointer basis’ of the system is singled out. In case that this leads to superselection rules, the theory will predict that certain superpositions do not exist as pure states. See Amann (1991) for the case of chiral molecules, and Wan and Harrison (1993) for Josephson junctions in SQUID rings. Thus far, such ‘predictions’ have not quite run ahead of observations, but as theoretical explanations they are nontrivial. Moreover, certain superselection rules exist in their own right (e.g., parity), in the sense that they are not caused by the limited resources available to physicists and other IGUS’s. Perhaps new such rules may be empirically discovered.

4. Question: Is it claimed that quantum mechanics without superselection rules makes no predictions for what happens in micro processes in the iono-sphere?
Answer: Relative to the algebra of all beables in the world, quantum mechanics indeed fails to predict any concrete event or outcome. Without superselection, nothing ‘happens’ up there in the iono-sphere. But there is still the state of
the total system and its restriction to the algebra of beables of the iono-sphere; this restriction is generically mixed, and its time-evolution is duly given by the theory. However, one does not need a balloon with an observer in it in order to make predictions of events defined relative to any such observer; their algebras of observables will all be compatible with locality, and practically all events in the iono-sphere, with their associated probabilities, will be meaningfully defined for all such local algebras of observables simultaneously. But, once again, these events are not there intrinsically.

We hope that the reader is satisfied with these answers. If so, we trust that (s)he agrees with the author, that the real problem in the interpretation of quantum mechanics is not the explanation of classical ‘reality’ in a quantum world (for the interpretation of objective classical phenomena is rather clear), but the clarification of the physical meaning of the ‘kernel’ of quantum mechanics in situations where no ‘objectification’ in the usual sense takes place. Thus we feel that the Copenhagen interpretation is too limited in its claim that the entire physical meaning of quantum mechanics must be expressed in terms of the properties of classical physics\(^7\); the physical interpretation of non-central operators is as yet merely unknown - not meaningless in principle.

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\(^7\)Hence even the Copenhagen interpretation has B-realist tendencies, for it attempts to relate the subjective quantum world to a putative objective classical world in which we can tell our friends what we have done and what we have learnt. A version of Copenhagen which is more in the spirit of our proposal posits that measurement apparatuses behave as if they were classical.
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