Beyond the Standard Model with Effective Lagrangians

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ABSTRACT

An Effective Lagrangian description is useful for describing potential physics beyond the Standard Model. The method is illustrated by reference to interactions among the electroweak bosons ($W^\pm$ & $Z^0$). The resulting estimates of the magnitude of these corrections suggest that they would be at best marginally detectable at high energy $e^-e^+$ colliders or hadron colliders. In loop calculations, contrary to lore, such deviations from Standard Model self-couplings never give observable corrections that grow as a power of the scale $\Lambda$ at which new physics enters, so there are never effects proportional to powers of $\Lambda/M_W$.

* Invited talk at the “Conference on Unified Symmetry in the Small and in the Large,” Coral Gables, Jan. 25–27, 1993; to be published in the Proceedings.
1. INTRODUCTION

We hope that new hadron colliders such as LHC and SSC or future high-energy linear $e^-e^+$ colliders (generically referred to as the NLC) will cross a threshold for producing particles not included in the Standard Model (SM.) Often, the hope is also expressed that the existence and nature of physics beyond the SM might be inferred from deviations from the SM couplings, and it is this topic of how virtual phenomena may influence observables that I shall focus on in this talk. I shall discuss how one may address “The Discovery Potential of Future Colliders” theoretically without prejudice as to the particular kind of new physics to be discovered.

In this regard, the interactions among the weak vector bosons are of special interest, since nearly all theorists feel that the present theory is but an approximation to a more fundamental description of the origin of electroweak symmetry breaking. One might well think that the composite nature of longitudinal helicity states of the vectors would be revealed by non-canonical interactions, revealing their origin in a Higgs field or a more complicated underlying mechanism. Thus, the study of $WW$ scattering is a primary goal for the SSC, and $e^-e^+ \rightarrow W^-W^+$ is, in any case, the largest electron-positron annihilation channel. In this talk, I will focus on deviations of the triple-vector boson vertices and two-body scattering of vector bosons.

Quite generally, such new effects can be represented as new effective vertices involving the particles we know, but unfortunately, the language we use depends on whether the underlying physics is decoupling or nondecoupling. In the former case, these are represented simply by higher dimensional operators, while in the latter case, one must resort to an expansion in powers of momenta, in the manner familiar from chiral perturbation theory. [1] Such a description has the advantage of being model independent and process independent. Indeed, a useful way to compare the phenomenological implications of different models is to compare the values implied for the parameters of the effective Lagrangian. This method is advantageous compared, for example, to parameterizing a “form factor” associated with a particular process. The effective Lagrangian can be taken to respect the gauge invariance of the Standard Model. Its disadvantages are that it involves many parameters, since it represents all possible models, and kinematic thresholds are not reflected, so that the expansion will break down as one nears the threshold for new particle production. Such a formalism has found applications to composite fermions, $WW$ scattering, especially interactions involving longitudinal $W$’s and, it can be applied to hadrons or to quarks and leptons. It is in fact so general that one may wonder whether this is, in fact, a useful approach. I will begin by discussing the triple-vector boson interactions and later discuss $WW$ scattering.

2. TRIPLE VECTOR BOSON VERTICES

To describe the possibilities, it has become conventional in recent years to write down the most general Lorentz invariant Lagrangian that describes the interaction of a photon or $Z^0$ with the $W^\pm$: For pedagogical simplicity, we will restrict our attention to the CP-
invariant terms:

\[ \mathcal{L}_{WWV}/g_{WWV} = ig_{V1}^\mu (W_{[\mu\nu]}^\dagger W^{\mu\nu} - h.c.) + i\kappa_W W_{\mu} W_{\nu} V^{\mu\nu} \]

\[ + i\frac{\lambda_W}{M_W^2} W_{[\mu]}^\dagger W^{[\mu\nu]} V_{\nu}^\lambda - g_{W1}^\mu W_{\mu} W_{\nu} V^{\mu\nu} + \partial^\nu V^{\mu} \]

where \( W_{\mu} \) is the \( W^- \) field, \( V_{\mu} \) represents either the photon \( V = A \) or the \( Z^0 \)-boson \( V = Z \), \( V_{\mu\nu} = \partial_\mu V_\nu - \partial_\nu V_\mu \) and \( W_{[\mu\nu]} = \partial_\mu W_\nu - \partial_\nu W_\mu \) are the “Abelian field strengths.” Electromagnetic gauge invariance, generally assumed, requires \( g_{WWV} = e, g_{W1}^1 = 1, g_{W4}^1 = 0 \). With the choice \( g_{WWZ} = \cot \theta_w \), the Standard Model values are \( g_{W1}^Z = \kappa_A = \kappa_Z = 1, \lambda_A = \lambda_Z = g_{W1}^Z = 0 = 0 \). The sensitivity of various facilities are often quoted in terms of the limits they can place on quantities such as \( \kappa_V - 1 \) and \( \lambda_V \). For example, LEP-2 is expected to restrict \( \kappa_A - 1 \) and \( \lambda \) to less than about 10%. [3] Similarly, NLC studies [4,5] estimate a sensitivity to deviations on the order of a few per cent at 500 GeV and perhaps a few tenths of a percent at 1 TeV, depending on assumptions about achievable luminosities.

This raises the question of what is the order of magnitude to be expected for the deviations of these couplings from their SM values, the phenomenological implications of those estimates, and the consistent use of non-SM vertices in loop calculations. I will argue that

(a) the deviations of anomalous triple-vector-boson couplings from the SM are necessarily small, since they must be associated with loop corrections in the underlying theory,

(b) divergences of loop integrals are either fictitious or not observable and, hence, do not constrain these deviations.*

3. EFFECTIVE LAGRANGIANS

Quite generally, effective Lagrangians are non-renormalizable by virtue of higher-dimensional interactions, such as the \( \lambda_V \) term, but \( \mathcal{L}_{WWV} \) is also non-renormalizable because it introduces dimension-four vertices, such as \( \kappa_V \), in a manner that explicitly breaks the \( SU_2^L \otimes U_1^Y \) electroweak gauge invariance. Indeed, it is regarded as an experimental goal to establish or disprove that these couplings are equal to those of the SM. So long as one is not imposing \( SU_2^L \otimes U_1^Y \) gauge invariance, one ought to include in the effective Lagrangian explicit mass terms for the vector fields,

\[ M_W^2 W_{\mu}^\dagger W_{\mu} + \frac{M_Z^2}{2} Z_{\mu} Z_{\nu}. \]

Thus, the first question for such a general approach should be why the SM provides such a good prediction for the relation between these masses (given the Fermi constant \( G_F \) and the fine structure constant \( \alpha \)). The usual answer, at least in technicolor models, is that

* For discussion of the phenomenology of CP-violating terms, see, e.g., Ref. 2.
* I have discussed these points previously, especially at the Yale Workshop last October.[6]
the Higgs sector respects an approximate global, weak isospin symmetry, called custodial $SU_2$, broken only by hypercharge and quark masses, but, especially in the aftermath of the LEP-1 results, we are confident of the SM at accuracies at the level of SM one-loop corrections, so a discussion of custodial symmetry is somewhat beside the point of this talk.

But the general question raised is how can one understand the success of the SM, including the one-loop corrections important for interpreting LEP experiments, if one does not have $SU_2^L \otimes U_1^Y$ gauge invariance.\* This is not quite the right or meaningful question, since one may, without loss of generality, assume that the effective Lagrangian, Eqs. (01) and (02), is an expression of a gauge invariant effective Lagrangian in a unitary gauge.\† This observation, while correct, is somewhat off the mark, since one may regard a Lagrangian that appears to be non-gauge-invariant as the unitary gauge expression of a gauge-invariant Lagrangian corresponding to gauge symmetries other than $SU_2^L \otimes U_1^Y$. The electroweak $SU_2^L \otimes U_1^Y$ is not singled out by this observation, so we must probe more deeply to discover the significance of the SM gauge symmetry.

Unfortunately, the discussion necessarily bifurcates, depending on the nature of the underlying physics.\* In general, the most conservative approach, which can always be used for energies small compared to the scale of new physics, is to expand Green’s functions in powers of momenta, so-called chiral perturbation theory.[10, 12, 13.]

The prototypical underlying model is technicolor, in which the natural scale of the expansion is at most $4\pi v$, where $v \approx 250$ GeV is the weak scale. In such a case, the new physics is strongly interacting, and its scale is set by the ratio of this strong interaction to the gauge interactions. Inevitably, it seems, there are technifermions and/or scalar pseudo-Goldstone bosons in the hundreds of GeV mass range, so this is not to say that thresholds for new physics are necessarily in the multi-TeV range. There could also be new physics at higher scales, such as the extended technicolor interactions, or a fourth fermion generation of fermions (which, because of the invisible $Z^0$ width, would necessarily involve a heavy neutrino.)

If, on the other hand, there is a relatively light Higgs boson ($m_H \lesssim 800$ GeV), then a more appropriate description of electroweak symmetry-breaking is, as in the SM, with an explicit Higgs field treated perturbatively. Although there may be other relatively light particles, the intrinsic scale of new physics may not be directly associated with electroweak symmetry breaking and may even be higher than the weak scale, such as the scale of supersymmetry (SUSY) breaking in “low-energy” SUSY models.

Finally, of course, both kinds of new physics may be present, as in technicolor models with extended technicolor interactions at much higher scales responsible for fermion masses.

In any case, the SM may be understood as the leading behavior in a systematic expan-

\* This has been discussed at some length in Ref. 8.

\† This point has been made previously in Ref. 6 and emphasized in Ref. 9.

\* Often times, these are referred to as the strongly interacting, nondecoupling scenario and the weakly interacting, decoupling scenario.
sion of a more comprehensive theory applicable at higher energy scales. Familiar examples are the manner in which QED emerges from a unified electroweak theory below 80 GeV or the way chiral perturbation theory for pions emerges from QCD below 1 GeV. As a result, contrary to what you might think, the behavior of vector boson interactions below the weak scale will not be wildly different in theories in which $W$ bosons are in some sense composite. Light vector bosons cannot behave other than gauge bosons.\footnote{This point of view has been elaborated more than ten years ago by Veltman\cite{14} and is closely related to the notion of naturalness.} The so-called delicate gauge theory cancellations are intimately linked with the fact that the vector masses are small compared to the scale of their compositeness. So, in a sense, gauge invariance is not really the relevant issue; the question is whether we may regard the weak vector bosons as elementary gauge particles over a range of momenta large compared to their masses. This is what is tacitly assumed in performing radiative corrections in the SM, and these calculations and their consequences will break down if the cutoff on momentum integrals is not much greater than the vector boson masses. This is illustrated by technicolor models, which are elegant and reasonably unambiguous expressions of the notion of a composite Higgs field, where the scale of new physics is expected to be around 1 TeV, an order of magnitude larger than the vector masses.

Let me first describe non-SM triple-vector-boson couplings. Because the description in terms of an elementary Higgs is somewhat more familiar to most people than chiral perturbation theory, I will use the linear language, but the general conclusions that I will draw for triple-vector-boson interactions are not qualitatively different from the nonlinear scenario, to which I shall return in discussing $WW$ scattering. In this case, the corrections to the SM Lagrangian are simply gauge-invariant, Lorentz-invariant, higher dimensional operators involving the Higgs scalar, vector, or fermion fields. In principle, there can be more than one Higgs doublet, as in SUSY models, but it is not necessary to consider that alternative for our present purposes. The first corrections coming from physics at higher scales that modify the vector boson properties are dimension six operators,\cite{15} such as:

$$\mathcal{O}^{(3)}_{\phi} = |\phi^\dagger D_\mu \phi|^2,$$
$$\mathcal{O}_W = \epsilon_{IJK} W^I_\mu W^K_\nu W^J_\lambda W^K_\mu,$$
$$\mathcal{O}_{WB} = (\phi^\dagger \tau^I \phi) W^I_\mu B^{\mu \nu},$$
$$\mathcal{O}_{\phi W} = \frac{1}{2} (\phi^\dagger \phi) W^I_\mu W^I_{\mu \nu},$$
$$\mathcal{O}_{\phi B} = \frac{1}{2} (\phi^\dagger \phi) B_{\mu \nu} B^{\mu \nu},$$

where $\phi$ is the usual Higgs doublet. These add to the SM Lagrangian terms of the form

$$\frac{1}{\Lambda^2} \sum \alpha_k \mathcal{O}_k.$$  (04)

Here, $\Lambda$ represents a generic scale of new physics and each $\alpha_k$ may be thought of a coupling...
constant associated with the corresponding operator. For convenience, we use the same scale $\Lambda$ for each operator, but, of course, this is simply a matter of convention, and the threshold for new particle production could be different from $\Lambda$.

These operators contribute a variety of corrections to the SM. To make contact with the non-gauge invariant formalism of Eq. (01), one may go to the unitary gauge

$$\phi = \frac{1}{\sqrt{2}} \left( \begin{array}{c} 0 \\ v + \sigma \end{array} \right),$$

(05).

Obviously, if we can replace the Higgs field by its VEV $v$, the last two operators are simply wave function renormalizations and have no observable consequences. This means that, until Higgs bosons are observed, these operators can be ignored at tree level. Similarly, unless one observes Higgs boson interactions directly, the first operator, $O^{(3)}_\phi$, simply contributes to $M^2_Z$, altering the well-confirmed relationship expressed by the $\rho$ parameter. So this contribution from new physics must be small, less than about 1% of $M_Z$. Other effects can be easily worked out, for example,

$$\kappa_A - 1 = \frac{\alpha_W B}{\Lambda^2} \frac{4M^2_W}{gg'},$$

$$\frac{\lambda_A}{M^2_W} = \frac{6 \alpha_W}{g \Lambda^2},$$

(06)

with similar expressions for $g^2$, $\kappa_Z$ and $\lambda_Z$.

As a theoretical aside, there are several technical subtleties associated with the elaboration of higher dimensional operators. One is whether the effective Lagrangian may be assumed to be gauge invariant, given that the effective action is gauge dependent in general. Fortunately, the answer is in the affirmative.[17]

Another point has to do with the fact that not all higher dimensional operators are physically distinct, since one may use the classical equations of motion to relate some operators to others. Generally, this application of the classical equations of motion has been said to be limited to tree approximation, but in fact, one may use them even though the resulting effective Lagrangian is to be used in loop calculations.[17] Certain operators, such as $O_W$, that contribute new three-point and higher vertices may be related to others that modify the two-point function (vacuum polarization tensor.) A further complication is precisely how the equations of motion are to be applied in spontaneously broken theories.

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For simplicity, we have restricted our attention here to CP-conserving operators, but the CP-violating operators have also been delineated.[15]

One can also make contact with chiral perturbation theory by setting $\Lambda = v$, in which case these operators are a subset of those that arise from the chiral-dimension two and four operators. See Section 6 below.

Here $\rho \equiv M^2_W/M^2_Z \cos^2 \theta_w$. For a review of its significance, see Chapter 1 of Ref. [16].

Once the top quark mass is known, the limit will drop by almost another order of magnitude.
In the final analysis, the answer is quite simple: one may assume that the form of the effective Lagrangian is gauge invariant and ignore spontaneous symmetry breaking in the application of the classical equations of motion. The bottom line is that one may simply drop operators involving scalar or vector fields on which a d’Alembertian \((\square)\) acts or involving fermion fields on which \(\bar{D}\) acts. Thus, one may arrive at a basis set of operators that are independent under application of the classical equations of motion.

De Rújula et.al.[8] have emphasized that, depending on the order in which experiments are done, some basis sets may be better than others. They have conjectured that it would be unnatural for new physics to yield only those operators that contribute anomalous vertices without also modifying the vacuum polarization tensor. Thus, this seemingly technical issue has physics content, inasmuch as some of the operators unrestricted by LEP-1 measurements are not independent of operators that are restricted in the absence of fine tuning. For the present, one must be clear that the conclusion of Ref. 8, that LEP-1 restrictions obviate LEP-2 measurements of triple vector boson vertices, is a direct consequence of this assumption. We do not know whether this conjectured extension of the notion of naturality is correct, but we have not found a model that provides a counterexample and believe it is very likely true. On general grounds, we shall argue below that the deviations from the SM will be too small to be observed at LEP-2.

4. ORDERS OF MAGNITUDE

Assuming that \(\Lambda\) does represent a new heavy particle mass, how large would we expect the various couplings \(\alpha_k\) to be? There is certainly one power of \(g\) or \(g'\) for each power of the corresponding field operator \(W^I\) or \(B_\mu\), but an essential question governing their magnitude is whether we can expect contributions to these operators at tree level in some models or whether they invariably represent loop corrections. For the operators discussed in the preceding section, the only tree diagram contributions would be from heavy vectors or scalars, and one can show that there can be no tree contributions compatible with SU\(_L^2\) \(\otimes\) U\(_Y^1\) symmetry.[18] Therefore, all these operators arise from one-loop corrections at best, so that it is natural to associate a factor of \(1/16\pi^2\) should be associated with each. As a result, so long as \(\Lambda \gtrsim v\), the presence of such terms are not really troublesome for the SM. Consider for example, \(\mathcal{O}_\phi^{(3)}\). This can get a contribution from essentially any one loop correction that breaks the custodial SU\(_2\) symmetry; for example, the top quark makes an important contribution to \(\alpha_\phi^{(3)}\). Because this is proportional to the square of the Yukawa coupling of the top to the Higgs, the limits on its contribution to \(M_Z\) place an upper limit of about 180 GeV on \(m_t\).

Our interest for collider physics has more to do with the contributions of the two other operators \(\mathcal{O}_W\) and \(\mathcal{O}_{WB}\) in Eq. (03). The preceding reasoning leads to the estimates

\[
\alpha_W \sim \frac{g^3}{16\pi^2}, \quad \alpha_{WB} \sim \frac{gg'}{16\pi^2}.
\]

Correspondingly, one finds from Eq. (06) that, taking \(\Lambda \approx v\), \(|\kappa - 1| \sim 3 \times 10^{-3}\) and \(|\lambda| \sim \ldots\)

* For a discussion of the usual SM radiative corrections in the language of effective field theory, see Ref. 19.
2 \times 10^{-3}. These estimates are an order of magnitude smaller than the sensitivity claimed for an NLC at 500 GeV and are borderline for an $e^-e^+$ collider at 1000 GeV! It should be said that, if one takes QCD as a prototype of technicolor,[20] it turns out that these sort of back-of-the-envelope estimates of couplings are in agreement with some and are larger than most of the experimental values for coefficients in the chiral perturbation expansion.†

In some models, it may also be the case that there are many similar contributions at approximately the same mass scale that add constructively, enhancing the one-loop estimate. But, even allowing an order of magnitude increase, these estimates remain at the borderline of the anticipated sensitivity of a 500 GeV $e^-e^+$ collider.

A higher energy $e^-e^+$ collider may achieve greater sensitivity, but, at 1 TeV, if one has not crossed a threshold for new particle production, then surely the relevant scale of new physics $\Lambda$ will also be larger and, correspondingly, $\kappa-1$ and $\lambda$ smaller. This illustrates one problem with probing virtual effects by going to higher energy: your figure of merit is a moving target. Not that I am against going to higher energy; the best way to study new physics is to cross its threshold and look directly at it. While an NLC at 500 GeV may be a fine facility for many things, it seems unlikely to be very useful for probing anomalous vector boson couplings.

With regard to the sensitivity of LHC or SSC to anomalous triple vector boson couplings, I am frankly uncertain. A recent study in the context of chiral perturbation theory, performed by Falk, Luke, and Simmons,[21] suggests that the sensitivity to “anomalous couplings” will be no greater than at an NLC at 500 GeV, a somewhat surprising conclusion. This results from the assumption that, because of QCD jet backgrounds, hadronic decay modes of $W^\pm$ or $Z^0$ will not be identifiable, about which there is some dispute.[22] If the hadronic decays were visible, then the SSC sensitivity could be increased. This is a topic requiring further detailed study.

5. LOOP CORRECTIONS

The contributions of anomalous vector boson interactions to radiative corrections have often been used to constrain these vertices.[23] The point is that, when one performs loop integrals starting from the non-gauge invariant form, Eq. (01), one often finds that they diverge as a power of momentum. The origin of these divergences are threefold: (1) the factor of $k^\mu k^\nu/M_W^2$ in the vector boson propagators, (2) the lack of “gauge cancellations” when vertices such as $\kappa_V$ do not take their SM values, and (3) the higher-dimensional vertices such as $\lambda_V$ that contain additional powers of momenta. As I shall explain, all claims concerning the observable sensitivity of radiative corrections to powers of a cutoff are wrong!* Unfortunately, such analyses have frequently been cited as one justification for LEP-2,[3] SSC, or NLC, which might reduce limits by another order of magnitude or more. (See the discussions in Refs. 4 and 5.)

First of all, we have remarked and illustrated that the non-gauge-invariant expression Eq. (01) may be regarded as the unitary gauge result of a general gauge-invariant

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† For a detailed analysis of $e^-e^+ \rightarrow W^-W^+$ in the language of chiral perturbation theory, see Ref. 12.

* This has been emphasized in Ref. 6 and again recently in Ref. 24.
effective Lagrangian. Therefore, those power-law divergences that are associated with the \( k^\mu k^\nu / M_W^2 \) terms in the propagators are unitary gauge artifacts, since in any renormalizable gauge, these are replaced by terms behaving as \( k^\mu k^\nu / k^2 \) for large momenta. This dispenses with (1) and (2) above. Secondly, while vertices associated with higher-dimensional operators will lead to power divergences, these are compensated by corresponding powers of \( 1/\Lambda \) extracted in front. So, higher-dimensional operators do not lead to net increases in the dependence on the cutoff. The only quantities that remain sensitive to the scale of new physics are scalar mass parameters; that is the usual naturalness problem. Finally, such divergences, even logarithmic ones, are not in principle observable; power-counting arguments similar to proofs of renormalizability show that these divergences are the coefficients of local operators, so they simply renormalize the couplings associated with other local operators already present. (The effective Lagrangian already contains all possible operators consistent with the symmetries of the theory.) So, as usual, such divergences can only be used to argue about the natural size of such couplings, just as the quadratic divergence in the scalar mass of the Higgs field restricts the scale \( \Lambda \) of new physics to be naturally less than about \( 4\pi v \).[25] Concerning the logarithmic divergences, they may be interpreted as contributing to the \( \beta \) functions for other renormalized operators and so, the way in which operators mix and coupling constants evolve from the cutoff \( \Lambda \) down may be of interest. In practice, however, these are invariably small effects for the range of \( \Lambda \) to which experiments are actually sensitive.†

6. V V-SCATTERING IN HADRON COLLIDERS

Now let me turn to a discussion of two-body scattering of weak-vector-bosons in hadron colliders. Given that triple-vector boson vertices are so difficult to observe, one would not think that anomalous quadruple vector boson vertices would be any easier. The answer is in fact rather complicated and depends on a number of factors. Quite generally, at energies large compared to their mass, longitudinal vector bosons retain the memory of their origin in the Higgs mechanism and behave more like scalar particles than massless vector particles. This behavior is embodied in the equivalence theorem,[27] one consequence of which can be stated by comparison with the situation when the gauge coupling is set zero: The equivalence theorem tells us that the interactions of longitudinal vector bosons go smoothly over to those of the Goldstone bosons that were eaten when gauge interactions were switched on.

First of all, we should remind ourselves of the situation within the SM. At energies large compared to the Higgs mass \( m_H \), the scattering amplitude for longitudinal vectors is that of four scalars, which in tree approximation is simply their self-coupling \( \lambda = m_H^2 / 2v^2 \). Thus, even for \( \lambda / 16\pi^2 \) not large, i.e. even within the perturbative regime, the interaction strength can be much larger than its naive value \( g^2 \), characteristic of massless or massive but transverse vector bosons. And, if there is a Higgs boson or other thresholds in the

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† We have illustrated the correct use of loop corrections in the case of past and future measurements of \( g_\mu - 2 \), a truly high precision experiment.[26]

* The breakdown of perturbation theory occurs for \( m_H \approx 1 \) TeV.
hundreds of GeV, it is likely that the SSC or, possibly, the LHC will find evidence for the production of new particles.[28]

If, on the other hand, the threshold associated with the physics of electroweak symmetry breaking is very large (sometimes described as the heavy Higgs limit) compared to accessible energies, then the question becomes whether its effects can be detected at lower energies. In this extreme, the amplitude is described by chiral perturbation theory, to be reviewed below. In lowest order, the amplitude for longitudinal vector boson scattering is proportional to $(E/v)^2$, where $E$ is the center-of-mass energy. In fact, this reaches the unitarity limit on the real part of an S-wave scattering amplitude at $E \approx 1.2 \, TeV$. This result can be obtained by direct calculation in the SM for $E \ll m_H$, but, given that $\rho = 1$, this behavior below the scale $v$ is universal, a kind of low-energy theorem resulting from the $SU_2^L$ symmetry of the SM.[30]

To sum up, it is generally believed that the SSC will either produce and detect a Higgs boson or discover a strongly interacting Higgs sector.[31] Of interest to us here is the latter case in which we must infer the properties of the Higgs sector from virtual effects. The expression of an appropriate effective Lagrangian requires a nonlinear representation of the underlying $SU_2^L \otimes U_1^Y$. To this end, one defines a field $\Sigma$ in terms of three would-be Goldstone bosons $w_i$ according to

$$\Sigma \equiv \exp \left( i \bar{\tau} \cdot \vec{w} / v \right), \quad V_\mu \equiv \Sigma^\dagger (D_\mu \Sigma),$$

where $D_\mu \Sigma = \partial_\mu \Sigma - i g W^a_\mu \tau^a / 2 \Sigma$ and $B_\mu$ is the hypercharge field. By construction, $\Sigma$ transforms linearly under an $SU_2^L \otimes U_1^Y$ transformation as

$$\Sigma \rightarrow e^{-i \theta L / 2} \Sigma e^{i \theta R \tau_3 / 2}.$$  

The effective field theory, called chiral perturbation theory,[32, 1] is simply a momentum expansion that, in coordinate space, corresponds to an expansion in powers of the covariant derivative $D_\mu$. Note that the gauge fields are to be counted as dimension 1, the same as the ordinary derivative, which is correct, at least for the longitudinal degree of freedom. The “chiral-dimension-two” and “-four” terms take the following form:

$$L_{eff} = L_2 + L_4$$

$$L_2 = \frac{\nu^2}{4} \text{Tr} (V_\mu V^\mu) + \delta \rho \frac{\nu^2}{8} \left[ \text{Tr} (\tau_3 V_\mu) \right]^2,$$

$$L_4 = \frac{L_1}{16 \pi^2} \left[ \text{Tr} (V_\mu V^\mu) \right]^2 + \frac{L_2}{16 \pi^2} \left[ \text{Tr} (V_\mu V_\nu) \right]^2$$

$$- i g \frac{L_{9W}}{16 \pi^2} \text{Tr} [W^{\mu \nu} D_\mu \Sigma D_\nu \Sigma^\dagger] - i g' \frac{L_{9B}}{16 \pi^2} \text{Tr} [B^{\mu \nu} D_\mu \Sigma^\dagger D_\nu \Sigma]$$

$$- i g g' \frac{L_{10}}{16 \pi^2} \text{Tr} [B^{\mu \nu} \Sigma^\dagger W_{\mu \nu} \Sigma].$$

* It must be understood that the maximum energy of the SSC is not accessible, either because the parton distribution functions fall as the parton momentum increases or because the relevant cross sections are small, or both.
In these expressions, the field strength tensors are given by $B_{\mu\nu} \equiv (\partial_\mu B_\nu - \partial_\nu B_\mu)\tau_3$ and $W_{\mu\nu} \equiv \partial_\mu W_\nu - \partial_\nu W_\mu - ig[W_\mu, W_\nu]$. In some ways, the physical significance of the various terms may be interpreted most directly in the “unitary gauge” where $\Sigma = 1$. Thus, it would appear that the first term of $L_2$, the “kinetic energy,” contributes only to the vector masses. However, this term contains more information than that, since, in the limit that the gauge couplings are zero, it also contributes to the scattering of Goldstone bosons which, by the equivalence theorem, is equal to the scattering of longitudinal gauge bosons. Thus, this term includes the low-energy theorem we referred to earlier, even though paradoxically, in the unitary gauge, it appears to describe no scattering at all. The second term in $L_2$ term with coefficient $\delta \rho$ is obviously like the term we called $O(3)\phi$ in the linear case. Thus we know that $\delta \rho$ is experimentally restricted to be $\lesssim 1\%$, which is evidence of the approximate $SU_R$ custodial symmetry of the strongly interacting Higgs sector. The terms $L_9$ and $L_{10}$ involve triple vector boson couplings, contributing to $\kappa - 1$, while $L_1$ and $L_2$ contribute to the scattering of longitudinal vectors. As this latter is our present interest, we will ignore all but $L_1$ and $L_2$ for now. We have extracted appropriate powers of $16\pi^2$ according to the usual prescription. [1, 33] However, in principle, it is not obvious that $4\pi v$ sets the scale of the chiral perturbation expansion and that these terms as small as indicated. However, as mentioned earlier, it does work in hadron physics, i.e., the scale of the momentum variation is set by the vector mesons $\rho, \omega, \phi$, and is the reason why the vector-meson-dominance model works so well. Thus the scale of the breakdown of the chiral perturbation expansion is on the order of $m_\rho$, which is only about half of $4\pi f_\pi$. Indeed, the famous fits of Gasser and Leutwyler[32] confirm that the $L_i$ are of order one, in particular, $L_1 = 0.9 \pm 0.3$ and $L_2 = 1.7 \pm 0.7$. Thus, this “naive dimensional analysis” does work.

If one can measure $L_1$ and $L_2$, one can discriminate somewhat between alternate underlying dynamical mechanisms.[11, 13] For example, a heavy scalar (Higgs) gives $L_1 = (64\pi/3)(v^4\Gamma_H/m_H^5)$, $L_2 = 0$, whereas a heavy vector meson (techni-$\rho$) gives $-L_1 = L_2 = (192\pi)(v^4\Gamma_\rho/m_\rho^5)$, a very different pattern. For example, for the SM Higgs with $m_H = 1$ TeV, we find $L_1 \approx 0.13$, with the others quite negligible. For a heavier Higgs, it may be even larger, but this is a strongly interacting regime about which we cannot speak with confidence. As another example, for a technirho with mass 1.5–2.0 $TeV$, typically $|L_1|$ and $|L_2|$ are on the order of a few tenths, whereas $L_9$ and $L_{10}$ are of $O(1)$. In fact, the situation may be slightly better, since our order of magnitude estimates apply at the scale of new physics $\Lambda$ and, at a lower scale $\mu$, they may be logarithmically enhanced by a factor of $\ln(\Lambda/\mu)$.

The question is whether numbers such as these are large enough to be detectable. Just how well SSC experiments can determine $L_1$ and $L_2$ is not known, because it is not now understood how efficiently weak vector bosons can be observed in their non-leptonic decay modes. A rough estimate of the sensitivity[21, 13] suggests that, relying solely on leptonic decay modes, the triple vector boson couplings can be detected only if $L_9$ or $L_{10}$ are larger than about 3. This is, as we have said before, quite inauspicious.

Similarly, $L_1$ and $L_2$ must be greater than about 1 or so to be detectable, so these
are slightly more interesting limits. In this regard, it seems that the $W^+W^+$ mode is particularly attractive,[34, 13] inasmuch as the backgrounds are less and the sensitivity to these two parameters greatest. The conclusion is that, while the SSC may be good enough to determine whether the Higgs sector is strongly interacting, unless a new threshold is crossed, it will be very difficult to ascertain the nature of the underlying dynamics.

7. CONCLUSIONS

I want to emphasize once again that there is content to saying that the weak vector bosons are associated with gauge fields, and the equivalence of Eq. (01) to the unitary gauge expression of a gauge-invariant Lagrangian may be misinterpreted. This equivalence does not imply that every vector particle in the world is necessarily associated with a gauge field. While the effective interactions of composite particles may be written in gauge invariant form, if the Lagrangian can only be used in tree approximation, that is irrelevant. The essence of a gauge particle is that it may be treated as elementary over a momentum range large compared to its mass, so that its composite structure, if any, is not revealed up to some scale of new physics. The gauge symmetry is irrelevant if this new scale is comparable to the vector masses. Contrast the situation we find with the $W^\pm$ and $Z^0$ weakly bounds systems such as orthopositronium or the $J/\psi$, whose substructure is revealed at momenta small compared to their masses. Similarly, I would argue that the $\rho$ meson is not a gauge particle. For a gauge particle, loop calculations may be reliably performed up to the scale at which its substructure must be taken into account. People who wish to entertain that the weak vector bosons are composite and suggest that larger deviations from SM values are possible than those suggested here are obliged to provide a framework for understanding why the SM works at all, including its radiative corrections.

To sum up, I would reiterate my main conclusions:

- Deviations of anomalous gauge boson couplings from the SM may be estimated on general grounds and are related to the scale of underlying new physics. They are invariably small.

- The divergences of loop corrections are not observable and so are without phenomenological consequences. There are fine-tuning questions, but the only naturalness problem arising from loop divergences is, other than the cosmological constant, the well-known quadratic divergences of scalar particle masses in nonsupersymmetric theories.

To the extent that the preceding considerations are typical, our best hope to discover new physics is probably to detect it directly through the production of new particles or resonances and not, given the anticipated accuracy of future colliders, via virtual effects. The possibilities are better, and the constraints more significant and more easily interpreted, for those higher order effective interactions (such as four-fermion interactions) that can arise in tree approximation in an underlying theory, but, unfortunately, the anomalous triple vector boson couplings are not among them. As for the four-vector-boson interactions, their leading behavior is determined solely by global symmetries and the association of longitudinal vector bosons with Goldstone bosons. Since these are derivatively-coupled, their description involves some scale for the momentum variations which is, if experience
in hadron physics is any guide at all, characteristically also of the order of 1 TeV or more.

ACKNOWLEDGEMENTS

I have enjoyed collaborations with C. Arzt and J. Wudka concerning much of the work discussed herein.

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