Probing resonance decays to two visible and multiple invisible particles

Won Sang Cho,1 Doojin Kim,2 Konstantin T. Matchev,1 and Myeonghun Park3

1Physics Department, University of Florida, Gainesville, FL 32611, USA
2Department of Physics, University of Maryland, College Park, MD 20742, USA
3CERN, Theory Division, CH-1211 Geneva 23, Switzerland

(Dated: March 5, 2014)

We consider the decay of a generic resonance to two visible particles and any number of invisible particles. We show that the shape of the invariant mass distribution of the two visible particles is sensitive to both the mass spectrum of the new particles, as well as the decay topology. We provide the analytical formulas describing the invariant mass shapes for the nine simplest topologies (with up to two invisible particles in the final state). Any such distribution can be simply categorized by its endpoint, peak location and curvature, which are typically sufficient to discriminate among the competing topologies. In each case, we list the effective mass parameters which can be measured by experiment. In certain cases, the invariant mass shape is sufficient to completely determine the new particle mass spectrum, including the overall mass scale.

PACS numbers: 13.85.Rm, 14.60.Lm, 14.80.Ly, 95.35.+d

![FIG. 1](image)

FIG. 1. The generic decay topology under consideration.

The dark matter problem and the mystery of the feeble neutrinos greatly motivate the ongoing LHC searches for new physics in channels with missing energy. Alas, at hadron colliders like the LHC, deciphering events with invisible particles in the final state is notoriously difficult.

The problem is schematically illustrated in Fig. 1 which depicts the generic decay of some new heavy resonance $A$ into $N_v$ visible particles $v_i$ and $N_{\chi}$ “invisible” particles $\chi_i$ (neutrinos or dark matter candidates) which leave no trace in the detector. A priori we have no way of knowing the underlying physics behind Fig. 1 and thus we are missing the answers to some very basic questions: 1) How many invisible particles are in the final state? 2) What are their masses? 3) What is the exact topology (i.e. Feynman diagram) of Fig. 1 are there any intermediate resonances, and if so, what are their masses?

Historically, the topic of mass measurements has attracted the most attention in the literature (for a review, see [1]). Unfortunately, virtually all proposed methods suffer from two drawbacks. First, one must typically assume the correct decay topology for Fig. 1 including the correct number $N_{\chi}$ of invisible particles. If this guess is incorrect, the method does not apply. This motivates us to address the issue of the correct decay topology and number of invisibles concurrently with (perhaps even prior to) the more traditional question of mass measurements. Second, most methods for mass measurements utilize kinematic endpoints, where the available statistics can be rather poor (in the sense that the most populated bins are rarely near the kinematic endpoint). Here we shall instead concentrate on the region near the peak rather than the endpoint of the kinematic distribution. (The exact shape in the vicinity of the endpoint does contain information about $N_{\chi}$ but is difficult to measure precisely in the presence of backgrounds and detector effects.) Our main result will be the derivation of the analytical formulas necessary to analyze the full shape of the invariant mass distributions of the visible particles in Fig. 1 including the location of the peak. We shall then demonstrate how those results can be used to determine: 1) the number of missing particles; 2) their masses; and 3) the associated event topology.

Our setup is as follows. We consider the generic decay from Fig. 1 without any prior assumptions about the decay topology or the number of invisibles. As seen in Table I, the number of inequivalent event topologies as a function of $1 \leq N_v \leq 4$ and $1 \leq N_{\chi} \leq 5$.

| $N_v$ | $N_{\chi}$ | 1 | 2 | 3 | 4 | 5 |
|------|-----------|---|---|---|---|---|
| 1    |           | 1 | 2 | 4 | 8 | 16 |
| 2    |           | 2 | 7 | 20| 55| 142|
| 3    |           | 4 | 20| 78| 270| 860|
| 4    |           | 8 | 55| 270|1138|4294|

TABLE I. The number of inequivalent event topologies as a function of $1 \leq N_v \leq 4$ and $1 \leq N_{\chi} \leq 5$. The exact shape in the vicinity of the endpoint does contain information about $N_{\chi}$ but is difficult to measure precisely in the presence of backgrounds and detector effects. Our main result will be the derivation of the analytical formulas necessary to analyze the full shape of the invariant mass distributions of the visible particles in Fig. 1 including the location of the peak. We shall then demonstrate how those results can be used to determine: 1) the number of missing particles; 2) their masses; and 3) the associated event topology.
The parameters $E$, $P$ and $R_n$ are in principle all experimentally measurable from the distribution (11). Traditional studies [9] have always concentrated on measuring just the endpoint $E$, failing to utilize all of the available information encoded in the distribution $f(m)$. The endpoint approach gives a single measurement (12), which is clearly insufficient to determine the full spectrum of resonances involved in the decay chain of Fig. 1. Here we propose to invoke the full shape (11) in the analysis [7]. We envision that in practice this will be done by performing unbinned maximum-likelihood fits of (11) to the observed data. In order to illustrate the power of the method here, it is sufficient to consider just the additional individual measurements of $P$ and $R_2$. Since they are obtained from the most populated bins near the peak, we can expect that they will be rather well measured. More importantly, the additional information about $P$ and $R_2$ might be sufficient to completely determine the mass spectrum (see eqs. (13,14) below). But first we need to present our results for (11,12) in each of the nine cases in Fig. 2.

The topology of Fig. 2(a). For a three body decay to massless visible particles, one has

$$f(m; M_A, M_\chi) \sim m^{3/2} (m^2, M_A^2, M_\chi^2),$$

where

$$\lambda(x, y, z) \equiv x^2 + y^2 + z^2 - 2xy - 2yz - 2xz.$$  

In this case

$$E = M_A - M_\chi, \quad P = 2M_A M_\chi \left[ 2 - \sqrt{1 + 3\alpha^2} \right] / (3\alpha) \right]^{1/2},$$

$$R_2 = 6 \left[ 1 + (1 + 3\alpha^2)^{-1/2} \right]^{-1},$$

where

$$\alpha \equiv 2M_A M_\chi / (M_A^2 + M_\chi^2).$$

Contrary to popular belief, one can now solve for both masses $M_A$ and $M_\chi$, given two of the three measurements (7-9). For example, using the peak location $P$ and the endpoint $E$, we find

$$M_A = \frac{E}{2} \left( \frac{P}{E} \sqrt{\frac{2 - 3(P/E)^2}{1 - 2(P/E)^2}} + 1 \right),$$

$$M_\chi = \frac{E}{2} \left( \frac{P}{E} \sqrt{\frac{2 - 3(P/E)^2}{1 - 2(P/E)^2}} - 1 \right).$$

Eqs. (11,12) offer a new method of determining both $M_A$ and $M_\chi$, which is a simpler alternative to the $M_{T2}$ kink method of [11], since here we do not rely on the $E_T$ measurement at all, and do not require to reconstruct the decay chain on the other side of the event.

---

1 We note that the resonance $A$ is in general allowed to be produced fully inclusively, with an arbitrary number of additional visible or invisible particles recoiling against $A$ in the event. This precludes us from using the $E_T$ measurement, since it will be corrupted by the invisible recoils, which leaves us with $m_{v1v2}$ as the only viable observable to study. The related combinatorial problem of partitioning the visibles in the event was addressed in [12,13].

2 Note that some of the event topologies in Fig. 2 involve effective higher-dimensional interactions [14,15], which we assume to be point-like, otherwise effects of their mediators can be seen in other processes at the LHC.
In fact, one does not even need an endpoint measurement, since the peak location $P$ and the curvature $R_2$ are sufficient for this purpose:

$$M_A = \frac{P}{\sqrt{2}} \left( \frac{6 - R_2}{4 - R_2} + \sqrt{\frac{12 - R_2}{4 - R_2}} \right)^{1/2},$$  

(13)

$$M_\chi = \frac{P}{\sqrt{2}} \left( \frac{6 - R_2}{4 - R_2} - \sqrt{\frac{12 - R_2}{4 - R_2}} \right)^{1/2}. $$  

(14)

Note that, in analogy to the matrix element method [11], eqs. (13, 14) are capable of determining the complete mass spectrum in a short SUSY-like decay chain, without relying on any kinematic endpoint measurements.

In order to get a rough idea of the precision of these mass determinations, in Fig. 3 on the left (right) we show the results from 10,000 pseudo-experiments with 100 (1000) signal events each. In each pseudo-experiment, the two masses $M_A$ and $M_\chi$ are extracted from a maximum-likelihood fit of the simulated data to the full distribution [5]. Fig. 3 shows that, as expected, the mass difference is measured quite well, at the level of \( \sim 1\% \) with just 100 events. At the same time, the mass sum (or equivalently, the absolute mass scale) is also being determined, albeit less precisely: at the level of \( \sim 30\% \) (\( \sim 10\% \)) with 100 (1000) events.

The topology of Fig. 2(b). Here one obtains the celebrated triangular shape

$$f(m) \sim m,$$  

(15)

$$E = P = \sqrt{(M_A^2 - M_B^2)(1 - M_\chi^2/M_B^2)},$$  

(16)

$$R_2 = \infty.$$  

(17)

Unfortunately, the masses enter the shape [13] only through the combination [10], which is the single effective mass parameter accessible experimentally.

The topology of Fig. 2(c). The shape is more conveniently given in integral form, which is easy to code up:

$$f(m) \sim m \int_{(M_\chi + M_\chi^2)^2}^{(M_A - M_\chi^2)^2} \frac{d s}{s} \sqrt{\lambda(M_A^2, M_B^2, M_\chi^2)} \lambda(s, M_\chi^2, M_\chi^2),$$  

(18)

where

$$X_\pm \equiv M_A \pm M_\chi, \quad K_\pm \equiv \sqrt{X_\pm^2 - K^2(m)},$$  

(21)

$$K^2(m) \equiv M_B^2 \left( 1 + \frac{m^2}{M_B^2 - M_\chi^2} \right),$$  

(22)

$$E = \sqrt{((M_A - M_\chi^2)^2 - M_B^2)(1 - M_\chi^2/M_B^2)}.$$  

(23)

The explicit formulas for $P$ and $R_2$ will be shown in [5]. The important point is that in principle all three masses $M_A, M_\chi, \text{ and } M_\chi^2$ can be simultaneously determined from a fit of eq. (15) to the data, just like in Fig. 3 [3].

The topology of Fig. 2(d). The invariant mass distribution of the visible particles $v_1$ and $v_2$ is not affected by the emission of invisible particles upstream and so this case is equivalent to the topology of Fig. 2(a). The corresponding results can be obtained from [5, 6] with the substitution $A \to B$, since now the role of the parent resonance is played by the intermediate particle $B$. One would then be able to determine independently $M_B$ and $M_\chi^2$, while $M_A \text{ and } M_\chi$ would remain unknown.

The topology of Fig. 2(e). Similarly, this case is equivalent to Fig. 2(b), with the substitutions $A \to B_1, B \to B_2$ and $\chi \to \chi_2$. Once again, the emission of the invisible particle $\chi_1$ upstream is not observable. The only measurable parameter in this case will be the endpoint $E$.

The topology of Fig. 2(f). We find

$$f(m) \sim m \int_{(m^2 + m^2)^2}^{(M_A - M_\chi^2)^2} \frac{d s}{s} \sqrt{\lambda(s, M_A^2, M_\chi^2)} \lambda(s, M_\chi^2, M_\chi^2),$$  

(24)
In this case, out of the 4 input masses entering the topology of Fig. [2(f), one can measure three independent degrees of freedom, e.g., $M_A/M_B$, $M_{X1}/M_B$ and $M_B^2 - M_{X2}^2$.

The topology of Fig. [2(g)]. The shape is described by

$$f(m) \sim m \int_s^{M_B^2(1 - M_A^2/M_B^2)} ds \frac{1}{2} \sqrt{\lambda(s, M_{X1}^2, M_{X2}^2)}$$

and it is easy to see that the results are obtained from [20] with the substitution $M_A \leftrightarrow -M_{X2}$. In particular, the three measurable parameters in this case can be taken as $M_{X1}/M_B$, $M_{X2}/M_B$ and $M_A^2 - M_B^2$.

The topology of Fig. [2(h)]. This is the “sandwich” topology studied in [12]. The shape is given by

$$f(m) \sim \begin{cases} \eta m, & 0 \leq m \leq e^{-\eta}, \\ m \ln (E/m), & e^{-\eta} \leq m \leq E, \\ \eta \equiv \cosh^{-1}\left(\frac{M_{B1}^2 + M_{B2}^2 - M_{X1}^2}{2M_{B1}M_{B2}}\right), & \end{cases}$$

and

$$E = [e^{\eta}(M_A^2 - M_{B1}^2)(M_{B2}^2 - M_{X1}^2)/(M_{B1}M_{B2})]^{1/2},$$

$$P = \begin{cases} E\ln \eta, & \eta < 1; \\ E\eta^{-1}, & \eta \geq 1; \end{cases}$$

$$R_2 = \begin{cases} \infty, & \eta < 1; \\ 1, & \eta \geq 1. \end{cases}$$ (27)

The distribution [26] exhibits a cusp at the non-differentiable point $m = e^{-\eta}E$. In this case, there are 5 mass inputs: $M_A$, $M_{B1}$, $M_{B2}$, $M_{X1}$, and $M_{X2}$, but only two independent measurable parameters: $\eta$ and $E$.

The topology of Fig. [2(i)]. This is the “antler” topology which was studied in [13] for the symmetric case of $M_{B1} = M_B$ and $M_{X1} = M_{X2}$. Here we generalize the result in [13] to arbitrary masses and find that $f(m)$ is given by the same expression [28], only this time

$$\eta \equiv \cosh^{-1}\left(\frac{M_A^2 - M_{B1}^2 - M_{X1}^2}{2M_{B1}M_{B2}}\right),$$

$$E = [e^{\eta}(M_{B1}^2 - M_{X1}^2)(M_{B2}^2 - M_{X1}^2)/(M_{B1}M_{B2})]^{1/2}$$ (29)

and identical expressions [28] for $P$ and $R_2$. Just like the case of Fig. [2(h)], out of the 5 mass inputs, $\eta$ and $E$ are the only two measurable mass parameters. Table II summarizes the final tally of input particle masses and independent measurable parameters for each topology.

Each topology from Fig. [2] also maps onto a restricted region in the $(R_2, P/E)$ plane, as shown in Fig. [3] (for convenience, instead of $R_2 \in (0, \infty)$, in the figure we plot $2/\pi \tan^{-1} R_2 \in (0, 1)$). For example, the cyan circle at $(1,1)$ marks the prediction for the two topologies of Fig. [2(b,e)], while the magenta dot at $(0.5,0.37)$ and the magenta vertical line correspond to the two topologies of Fig. [2(h,i)]. The blue (red, green, black) points refer to the topologies of Fig. [2(a,d)] (Fig. [2(g)], Fig. [2(f)], Fig. [2(c)]). Fig. [3] demonstrates that with the three measurements $E$, $P$ and $R_2$, one can already begin to constrain qualitatively the allowed event topologies.

In fact, one can do even better by fitting to the full invariant mass shapes derived here. For illustration, we consider a scenario where particle $A$ is a vector boson (V), $B$ is a fermion (F) and $C$ is another vector boson (V), and study two representative event topologies. The blue squares in Fig. [5] correspond to the antler topology case of Fig. [2(i)] for which the $m$ distribution exhibits a cusp at $m = e^{-\eta}E$ (see Eq. [26]). This example was considered in [13] for the purpose of measuring the masses, which were chosen as $M_A = 1500$ GeV, $M_{B1} = 730$ GeV and $M_{X1} = 100$ GeV. We also consider one cusp-less case, namely, the topology of Fig. [2(a)] with a mass spectrum $M_A = 550$ GeV, $M_X = 400$ GeV (red circles in Fig. [5]).

Fig. [4] shows the average $p$-values ($\bar{P}$) obtained in 200 pseudo-experiments, with 500 events each. For each example, the filled symbols represent the case in which spin correlations are absent, i.e., the “data” is sampled from the phase space distributions derived earlier. We see that the fit clearly prefers the correct topologies from Fig. [2(i)] and Fig. [2(a)] (and their identical twins from Figs. [2(h) and 2(d)]), while the wrong topologies are disfavored.

In models in which the fermions have chiral couplings, the invariant mass shapes considered here will be slightly distorted due to spin correlations [14]. In order to study the effect of spins, we repeat the two exercises for the case of purely left-handed (L) or purely right-handed (R) couplings of the fermions $B$ to the vector bosons $A$ and $C$. The results are displayed in Fig. [4] with open symbols.

### Table II

| Topology | (a,d) | (b,e) | (c) | (f,g) | (h,i) |
|----------|------|------|-----|-------|-------|
| $N_m$    | 2    | 3    | 3   | 4     | 5     |
| $N_p$    | 2    | 1    | 3   | 3     | 2     |

![FIG. 5. Results from a quantitative topology disambiguation exercise using $\chi^2$ as a test statistic.](image-url)
We see that, even though we were fitting to pure phase space formulas\footnote{Following the procedure in \cite{14}, one could derive analytical formulas for the invariant mass distributions in the presence of spin effects, but that is beyond the scope of the current paper.}, the correct topologies are still singled out, as they provide the best fit to the data.

ACKNOWLEDGMENTS

WSC thanks the US National Science Foundation, grant NSF-PHY-0969510, the LHC Theory Initiative. DK acknowledges support from the LHC Theory Initiative graduate fellowship (NSF Grant No. PHY-0969510). MP is supported by the CERN-Korea fellowship through National Research Foundation of Korea. Work supported in part by U.S. Department of Energy Grant DE-FG02-97ER41029

[1] A. Barr and C. Lester, J. Phys. G 37, 123001 (2010).
[2] Y. Bai and H. -C. Cheng, JHEP 1106, 021 (2011).
[3] E. Byckling and K. Kajantie, “Particle Kinematics”, John Wiley & Sons, 1973.
[4] K. Agashe, D. Kim, D. G. E. Walker and L. Zhu, Phys. Rev. D 84, 055020 (2011); G. F. Giudice, B. Gripaios and R. Mahbubani, Phys. Rev. D 85, 075019 (2012).
[5] W. Cho, D. Kim, K. Matchev and M. Park, to appear.
[6] M. Blanke, D. Curtin and M. Perelstein, Phys. Rev. D 82, 035020 (2010); A. Rajaraman and F. Yu, Phys. Lett. B 700, 126 (2011); P. Baringer, K. Kong, M. McCaskey and D. Noonan, JHEP 1110, 101 (2011).
[7] A. Birkedal, R. Group and K. Matchev, hep-ph/0507002.
[8] D. J. Phalen and A. Pierce, Phys. Rev. D 76, 075002 (2007); L. Edelhauser, W. Porod and R. K. Singh, JHEP 1008, 053 (2010); C. -Y. Chen and A. Freitas, JHEP 1201, 124 (2012).
[9] I. Hinchliffe, F. Paige, M. Shapiro, J. Soderqvist and W. Yao, Phys. Rev. D 55, 5520 (1997); K. Matchev, F. Moortgat, L. Pape and M. Park, JHEP 0908, 104 (2009).
[10] W. S. Cho, K. Choi, Y. Kim and C. Park, Phys. Rev. Lett. 100, 171801 (2008).
[11] J. Alwall, A. Freitas and O. Mattelaer, AIP Conf. Proc. 1200, 442 (2010) [arXiv:0910.2522 [hep-ph]].
[12] K. Agashe, D. Kim, M. Toharia and D. Walker, Phys. Rev. D 82, 015007 (2010).
[13] T. Han, I.-W. Kim and J. Song, Phys. Lett. B 693, 575 (2010).
[14] M. Burns, K. Kong, K. T. Matchev and M. Park, JHEP 0810, 081 (2008); L. Edelhauser, K. T. Matchev and M. Park, JHEP 1211, 006 (2012).