Polarons in extremely polarized Fermi gases: 
The strongly interacting $^6\text{Li}-^{40}\text{K}$ mixture

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We study the extremely polarized two-component Fermi gas with a mass imbalance in the strongly interacting regime. Specifically we focus on the experimentally available mixture of $^6\text{Li}$ and $^{40}\text{K}$ atoms. In this regime spin polarons, i.e., dressed minority atoms, form. We consider the spectral function for the minority atoms, from which the lifetime and the effective mass of the spin polaron can be determined. Moreover, we predict the radio-frequency (rf) spectrum and the momentum distribution for the spin polarons for experiments with $^6\text{Li}$ and $^{40}\text{K}$ atoms. Subsequently we study the relaxation of the motion of the Fermi polaron due to spin drag.

PACS numbers: 03.75.-b, 67.40.-w, 39.25.+k

I. INTRODUCTION

In many condensed-matter systems the response to a single impurity determines the low-temperature behavior of the system. Probably the most famous example hereof is a single electron moving in a lattice. Local lattice distortions, i.e., phonons, interact with the electron and together they form a quasiparticle that is known as the polaron because of the local change in polarization [1]. Another well-known impurity problem is that of an immobile magnetic impurity in a metal causing an enhanced resistance below a certain temperature due to the Kondo effect [2]. The multichannel version of this effect has especially received much interest in the past, because it leads to the formation of a non-Fermi liquid [3].

Here we study an impurity problem in a two-component atomic Fermi gas. An important motivation to use ultracold atoms is the unprecedented experimental control in these systems. They offer the interesting possibility of not only changing, for instance, particle numbers and temperature, but also the interaction strength. Via a Feshbach resonance the bare interaction can be tuned all the way from being weakly attractive (BCS-regime) to strongly attractive (BEC-regime), where in the intermediate regime the scattering length is much larger than the average interparticle distance. This so-called unitarity or strongly interacting limit, is the regime we focus on in this paper.

We consider a mixture at zero temperature consisting of two (spin)species of fermions, where there is one minority particle immersed in a non-interacting sea of majority particles. The mass-balanced Fermi gas with high spin polarization has been studied extensively, both experimentally [4–6] and theoretically [7–14]. At the unitarity limit the minority particle gets dressed by a cloud of majority particles forming a quasiparticle similar to the polaron. This quasiparticle is often referred to as a spin polaron, because its formation is due to interactions between particles in different spin states, or as a Fermi polaron, because it consists of fermionic atoms. Recently, the imbalanced spin-dipole mode [6], the radio-frequency (rf) spectrum of the spin polaron [4], and its energy and effective mass [3] that are different from those of the bare minority particle, have all been measured in this case.

An intriguing new possibility for experiments is having a mass imbalance between the minority and the majority particles by mixing two different atom species. A very promising mixture in this respect is the one of $^6\text{Li}$ and $^{40}\text{K}$ atoms. These atoms together have already been trapped and cooled to quantum degeneracy [15], and moreover, several Feshbach resonances were identified [16]. Theoretically, the phase diagram of the $^6\text{Li}$-$^{40}\text{K}$ mixture has been determined [17, 18], and it differs greatly from the phase diagram of a spin-imbalanced mixture by having not only a superfluid but also a supersolid region, depending on the sign of the polarization. We show here that already the two limiting cases of this mixture, i.e., a single light impurity in a sea of heavy atoms and vice versa, turn out to differ qualitatively in a manner that reflects the underlying asymmetry of the phase diagram.

Indeed, in the solely spin-imbalanced case, having a $|\sigma\rangle$ or a $|-\sigma\rangle$ minority particle results in the same impurity problem, while with two different atom species there are two fundamentally different impurity problems. Thus, by introducing a mass imbalance, not only does the question whether dressed impurities still represent the ground state of the system arise, but so does the question of what is the difference between a heavy and a light impurity. Here, because the different atom species act as a pseudospin, the same many-body mechanism causes the dressing of the minority atom as for the solely spin-imbalanced case. Therefore, we also call this quasiparticle a spin polaron. In this paper we study for the two mass-imbalanced cases both a molecular bound state and the spin polaron. We show that although it does not form the ground state, the molecular bound state virtually plays an important role in the system. In addition,
we study dissipation of kinetic energy of the minority particle due to interactions with the majority cloud that lead to spin drag \[19\].

In this paper we consider a homogeneous gas of atoms, while experiments are always done in a trap. Still, when \(1/k_F\) is much smaller than the size of the cloud, where \(k_F\) is the Fermi momentum of the majority atoms, the gas can locally be considered homogeneous and all our results apply. In this manner the appropriate averaging over the trap can be fully taken into account.

**II. MOLECULAR BOUND STATE**

In the unitarity limit the minority particle interacts strongly with the Fermi sea of majority particles. Due to the low densities in ultracold atomic systems two-body processes represent the dominant scattering mechanism, where the minority particle can scatter off a majority particle an arbitrary number of times. Taking this into account in diagrammatic language results in an infinite sum of ladder diagrams, the so-called ladder sum. For the extremely imbalanced case at unitarity the bare interaction with the complete ladder sum added, i.e., the many-body \(T\) matrix, obeys

\[
\frac{1}{\hbar p, \hbar \Omega} = \int \frac{d k}{(2\pi)^3} \left[ \frac{N(\xi_{\uparrow}, k) - 1}{\hbar \Omega - \xi_{\uparrow}, k - \xi_{\downarrow}, k + p} - \frac{1}{2\varepsilon_{k}} \right],
\]

with \(\hbar p\) and \(\hbar \Omega\) denoting the total momentum and energy of the two incoming particles, and \(\uparrow (\downarrow)\) denoting a majority (minority) particle. The distribution for the majority particles is the Fermi-Dirac function \(N(x) = 1/\cosh(x/k_B T + 1)\), with \(T\) the temperature and \(k_B\) Boltzmann’s constant. The distribution for a single minority atom can be taken equal to zero in the integrand. The energy of an atom in state \([\sigma]\) is \(\varepsilon_{\sigma, k} = \varepsilon_{\sigma, k} - \mu_{\sigma}\), where \(\varepsilon_{\sigma, k} = \hbar^2 k^2/2m_{\sigma}\) and \(\mu_{\sigma}\) are the kinetic energy and the chemical potential. The kinetic energy \(\varepsilon_k = (\varepsilon_{\uparrow, k} + \varepsilon_{\downarrow, k})/2\) is associated with twice the reduced mass. Throughout this paper we take \(\mu_{\uparrow}\) equal to the Fermi energy \(\varepsilon_F\), while the chemical potential of the minority atom is determined self-consistently from the self-energy \[10\], \(\mu_{\downarrow} = \hbar \Sigma_{\downarrow}(0, 0)\), defined later in Eq. (2).

A pole in the \(T\) matrix corresponds to a bound state, where the real part of the location of the pole is its energy and the imaginary part is inversely proportional to its lifetime. In the above many-body \(T\) matrix the pole physically corresponds to a Feshbach molecule dressed by the Fermi sea, which we here refer to as a molecular bound state. The energy \(E_M(p)\) of this bound state at zero temperature, divided by the majority particles Fermi level \(\varepsilon_F\), is shown as a solid line in Fig. I. Up to some momentum \(p_{\text{max}}\) it is a stable molecular state, while for larger momenta the imaginary part is non-zero and thus the bound state acquires a finite lifetime. A minority and a majority atom cannot scatter off each other if their combined energy lies below a certain level due to Pauli blocking of the Fermi sea. Above this energy level there is a continuum of scattering states. This continuum of particle-particle excitations is also depicted in Fig. II.

From the molecular dispersions it already becomes clear that a light impurity is very different from a heavy one. Fitting the dispersion of the molecular state for small momentum to \(E_M(p) = \hbar^2 p^2/2m_{\sigma} + E_M(0)\), shows that for the light impurity the stable molecule has a negative effective mass, \(m_M \simeq -0.13 m_{\sigma}\), and has an energy \(E_M(0) \simeq 4.4 \varepsilon_F\). The dispersion is qualitatively the same as for the mass-balanced case, where the stable molecule also has a negative mass, namely \(m_M \simeq -3.9 m_{\sigma}\). In contrast to the light impurity, with a heavy impurity the stable molecular state has a positive effective mass \(m_M \simeq 0.96 m_{\sigma}\) and has an energy \(E_M(0) \simeq -0.2 \varepsilon_F\). Interestingly, it is also the part of the phase diagram with a minority of heavy particles that differs qualitatively from the mass-balanced case and contains a supersolid phase \[17, 18\]. In all cases the continuum of particle-particle excitations pushes the molecular state down, which is a consequence of level repulsion as in the more simple case of an avoided crossing of two energy levels. For the light impurity this repulsion results in a negative effective mass for the molecule. For the heavy impurity the effective mass is positive, but smaller than one would obtain in the absence of the continuum.

**III. SPIN POLARON**

The presence of a molecular bound state does not mean necessarily that a molecule is the ground state of the sys-
tem, because some other state can have a lower energy than the molecule. We therefore now consider the dressed impurity, the spin polaron, and compare its energy with the molecule to determine the ground state of the system. The energy and lifetime of the quasiparticle can be obtained from the spectral function $\rho_\uparrow(\varepsilon) = -\text{Im}[G_\uparrow]/\pi$, where $G_\uparrow$ is the Green’s function describing the minority particle in the presence of the Fermi sea. To obtain the latter a self-energy is added to the bare inverse Green’s function via

$$
\Sigma_\downarrow(q, \omega^+) = \int \frac{dk}{(2\pi)^3} T_{k+q, \omega^+ + \xi_\uparrow, k} N(\xi_\uparrow, k),
$$

with $\omega^+ = \omega + i0$. Because the relevant momentum of the minority particle at zero temperature is much smaller than the Fermi momentum of the Fermi sea, we take its momentum equal to zero first. Then the spectral function, for both impurity problems, has at the energy $E_P$ a delta-function peak (see Fig. 2), which corresponds to the energy of a stable quasi-particle, i.e., the spin polaron. After comparing this energy with the energy of the molecular state $E_M(0)$ we conclude that for both cases the quasi-particle has lower energy and thus forms the ground state of the system.

Apart from the energy of the dressed particle, also the quasi-particle residue $Z_P$ and the effective mass $m^*$ can be determined from the spectral function. The quasi-particle residue is the weight of the delta peak and the effective mass can be obtained from the momentum dependence of its location. For the light polaron, a dressed $^6$Li atom, we find $E_P \simeq -2.2\varepsilon_F$, $Z_P \simeq 0.8$ and $m^* \simeq 1.25m_\downarrow$, while for the dressed $^{40}$K atom $E_P \simeq -0.44\varepsilon_F$, $Z_P \simeq 0.64$ and $m^* \simeq 1.16m_\downarrow$. The energies and effective masses are in good agreement with previous theoretical results and Monte-Carlo calculations [10, 21] that do not consider the full spectral function.

The presence of the molecular pole is very important for the spectral functions $\rho_\uparrow(k, \omega)$. In particular, the threshold of the continuum of $\rho_\uparrow(0, \omega)$ is at zero energy when the molecular state always has a positive energy, as for the light impurity, see Fig. 2(a). In contrast, for the heavy impurity the molecular state can have a negative energy and this causes the threshold of the continuum to lie at a negative energy, see Fig. 2(b). The spectral function at zero temperature can be approximated by $\rho_\uparrow(k, \omega) \simeq Z_P \delta(\varepsilon_k^+ + E_P - \mu_\downarrow - \hbar\omega)$, with $\varepsilon_k^+ = \hbar^2k^2/2m^*$. For both impurity problems, however, it does not capture all the features of $\rho_\uparrow(k, \omega)$, as we will see next.

A direct probe for the quasiparticle residue $Z_P$ is the momentum distribution of the minority particles, which can be obtained experimentally by a time-of-flight experiment. From the spectral function it can be calculated by means of $N_\downarrow(k) = \int d\omega \rho_\uparrow(k, \omega)/N(\omega)$. In Fig. 3 the results are shown, both for the full spectral function and for the delta peak only, at zero temperature and for polarization $P = 0.9$. Also depicted are the ideal gas momentum distributions for comparison. The quasiparticle residue can be read off easily in both figures. It can also be seen that the delta peak is a good approximation for the heavy impurities, while for the light impurities $Z_P$ depends more strongly on the external momentum, which is not captured by this approximation.

The energy of the spin polaron can be directly obtained from the rf spectrum, which was recently measured for the mass-balanced case. In an rf experiment incoming photons with frequency $\omega_{rf}$ induce transitions from an occupied hyperfine state to an empty state. The fraction of transferred atoms as a function of the photon frequency is the rf spectrum, where the threshold of the spectrum is the polaron energy. Theoretically, the spectrum can be obtained directly from the spectral function by using the Kubo formula, $I(\omega_{rf}) \propto \int d\varepsilon \rho_\uparrow(k, \varepsilon - \omega_{rf})/\pi$. When using the low-temperature spectral function the integral can be performed, yielding

$$
I(\omega_{rf}) \propto Z_P \sqrt{2(\omega_{rf} + E_P)/N \left( \frac{m_\downarrow\omega_{rf} + m^*E_P}{m^* - m_\downarrow} \right)}.
$$

The rf spectra for the two mass-imbalanced impurity problems are shown in Fig. 3 for $P = 0.99$ and the temperature of the experiment with mass balance, $T = 0.14T_F$. For the light impurity the analytic result from Eq. 3 reproduces the full spectral function result almost exactly.

### IV. SPIN-DRAG RELAXATION RATE

At zero temperature the spin polaron corresponds to a delta-function peak in the spectral function as we have just seen. At nonzero temperatures we expect this peak to broaden and to obtain a width that is proportional to $T^2$ at low temperatures. An immediate consequence of this non-zero width is that the polaron acquires interesting transport properties. In particular it leads to a non-zero spin-drag relaxation rate $1/\tau_{sd}$ of the polaron moving in a Fermi sea of majority particles. The friction of the spin polaron and the out-of-phase dipole mode
are examples of properties determined by $1/\tau_{sd}$. For the mass-balanced case the latter has been studied experimentally \[5\,6\], and the transport properties of the mass-imbalanced impurity problem have been studied theoretically using thermodynamic arguments to calculate the effective interaction \[10\].

We derive here a general expression for the relaxation rate of one polaron moving with velocity $v$ through a cloud of majority particles with which it interacts, where its velocity is small compared to the Fermi velocity of the majority particles \[|v| \ll k_F/m_{\uparrow}\hbar\]. The equation of motion of the spin polaron then reads

$$\frac{dv}{dt} = \frac{Z_P}{m^* n_\downarrow} \Gamma(v) \simeq \frac{v}{\tau_{sd}},$$

where $\Gamma(v)$ is the Boltzmann collision integral, which was linearized in the last step. For the spin-drag relaxation rate for the impurity problem we obtain in this manner

$$\frac{1}{\tau_{sd}} = \frac{-\beta \hbar}{6m^*} \frac{1}{(2\pi)^6} \int dq d|q|^2 |(V_{k,\uparrow 0,\downarrow} q^2 + |V_{k,\downarrow -q,\uparrow}|^2)\sinh^2(\beta \varepsilon_{kF}/2)| \times \Im \left[ \frac{N_\uparrow(k) - N_\downarrow(k - q)}{\varepsilon_{kF} - i0 + \varepsilon_{kF} - \varepsilon_{kF} - q} \right],$$

where $\beta$ is $1/k_B T$ and $N_\uparrow(k)$ is the distribution function of the majority particles. The on-shell effective interaction $V_{k,k',\downarrow}$ in general depends on the incoming momenta $k$ and $k'$ and on the transferred momentum $q$ of the scattering particles. From the linearized collision integral, the above expression is obtained by using as the distribution function for the dressed impurity a delta function. The result in Eq. (5) is generic for any impurity, fermionic or bosonic, in any environment, fermionic or bosonic.

In the impurity problem at hand we take for $N_\uparrow(k)$ the Fermi-Dirac distribution function. At low temperatures only small $q$ contribute and the difference between the two distributions becomes strongly peaked around the Fermi level \[21\]. For the interaction we take the many-body $T$ matrix from Eq. (1), with an additional factor $Z_P$ to account for the wavefunction renormalization, and then ultimately obtain

$$\frac{1}{\tau_{sd}} \simeq \frac{\beta m^2_{\downarrow}}{12\pi \hbar m^* (2\pi)^3} Z_P^2 |T(k_F, \varepsilon_F)|^2 \int dq \frac{q^3}{\sinh^2(\beta \varepsilon_{kF}/2)} \gamma \left( \frac{m_\downarrow}{m_\uparrow} \right) \frac{\varepsilon_F}{\hbar} \left( \frac{T}{T_F} \right)^2,$$

where $\gamma(m_\downarrow/m_\uparrow)$ is a dimensionless function depending on the mass ratio of the minority and majority particles. For the light impurity we find $\gamma(0.15) \simeq 8.58$, while we find $\gamma(6.7) \simeq 1.96$ for the heavy impurity. The temperature dependence for only one minority particle in a fermionic environment is the same as for the spin-drag relaxation rate for equal densities of fermions, namely $1/\tau_{sd} \propto T^2$. This quadratic temperature dependence is expected for a Fermi liquid and recently it was verified experimentally for the mass-balanced case that $1/\tau_{sd}$ indeed decreases as the temperature decreases \[6\,22\]. The result in Eq. (6) implies that at $T = 0$ there is no spin drag relaxation, which in turn implies that the spin polaron is a stable quasiparticle in that case. As mentioned above, the latter is consistent with the delta-peaks in the spectral functions in Fig. 1 and confirms that the ladder approximation captures the relevant physics for these mass-imbalanced mixtures.

**FIG. 4**: RF spectra at $T = 0.14 T_F$ and polarization $P = 0.99$ in arbitrary units. The solid lines are obtained from the full spectral functions, the dashed line from Eq. (3). In panel (a) the results for the light impurity are depicted and in panel (b) for the heavy impurity.

**FIG. 3**: Momentum distributions $N(k)$ for the spin polarons at zero temperature and $P = 0.9$, where the polarization $P = (n_\uparrow - n_\downarrow)/(n_\uparrow + n_\downarrow)$. The solid, black (gray) lines are the momentum distributions of the polarons obtained from the full (delta-peak) spectral function, while the dashed lines are the distributions for an ideal gas. Panel (a) depicts the light spin polaron and panel (b) the heavy one.

**V. CONCLUSION**

We calculated a number of important observables of the extremely polarized $^6\text{Li-}^{40}\text{K}$ mixture. We showed that at the unitarity limit, although virtually the molecular state plays an important role, polarons form at low temperatures and dominate all physical properties of the mixture. Apart from its equilibrium properties we also looked at the transport properties of the spin polaron and found that the spin-drag relaxation rate takes a universal form and scales with the square of the temperature, as expected for a Fermi liquid.
VI. ACKNOWLEDGEMENTS

We thank Martijn Mink for useful discussions. This work is supported by the Stichting voor Fundamenteel Onderzoek der Materie (FOM), the Nederlandse Organisatie voor Wetenschappelijk Onderzoek (NWO), and by the European Research Council (ERC).

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