The emptiness of voids: yet another over-abundance problem for the LCDM model.

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ABSTRACT
We study the luminosity function, the peculiar velocities, and the sizes of voids in the Local Volume (LV) in observational samples of galaxies which contain galaxies down to $M_B = -10$ and to $M_B = -12$ within the distance $4 - 8$ Mpc. When we compare the results with the predictions of the standard cosmological LCDM model, we find that the theory faces a sever problem: it predicts a factor of ten more dwarf haloes as compared with the observed number of dwarf galaxies. In the LV we identify voids with sizes ranging from 1 to 4.5 Mpc and compare the observational distribution of void sizes with the voids in very high resolution simulations of the LCDM model with WMAP1 and WMAP3 parameters. The theoretical void function matches the observations remarkably well only if we use haloes with circular velocities $V_c$ larger than $40 - 45$ km/s ($M_{\text{vir}} = (1 - 2) \times 10^{10} M_{\odot}$) for models with with $\sigma_8 = 0.9$ and $V_c > 35$ km/s ($M_{\text{vir}} = (6 - 8) \times 10^9 M_{\odot}$) for $\sigma_8 = 0.75$. We exclude the possibility that in the LCDM model haloes with circular velocities < 35 km/s can host galaxies as bright as $M_B = -12$: there are too many small haloes in the LCDM model resulting in voids being too small as compared with the observations. The problem is that many of the observed dwarf galaxies have HI rotational velocities below 25 km/s that strictly contradicts the LCDM predictions. Thus, the LCDM model faces the same overabundance problem, which it had with the number of satellites in the LG. We also estimate the rms deviations from the Hubble flow $\sigma_H$ for galaxies at different distances from the Local Group and find that in most of our model LV-candidates the rms peculiar velocities are consistent with observational values: $\sigma_H = 50$ km/s for distances less than 3 Mpc and $\sigma_H = 80$ km/s for distances less than 8 Mpc. At distances 4 (8) Mpc, the observed overdensities of galaxies are 3.5-5.5 (1.3-1.6) – significantly larger than typically assumed.

Key words: cosmology: large-scale structure of Universe, voids, dark matter; galaxies: luminosity function, kinematics and dynamics, galaxy formation.

1 INTRODUCTION

Voids in the distribution of galaxies are an important ingradient of the distribution of light and mass in the Universe. They constitute a natural outcome of structure formation via gravitational instability, and, thus, can be used to constrain theories of galaxy formation. Emptiness of voids – the number of small galaxies in the voids – is an interesting question for both the observations and the theory to tackle (Peebles 2001, Gottloeber et al. 2003, Patiri et al. 2006, Tinker & Conroy 2008). Cosmological simulations predict (e.g., Gottloeber et al. 2003) that many small DM haloes should reside in voids. There seems to be no disagreement between the ΛCDM theory and the observations regarding the giant voids defined by $M_*$ galaxies or by $\sim 10^{12} M_{\odot}$ haloes (Patiri et al. 2006). The situation is less clear on smaller scales. In the region of $\sim 10$ Mpc around the Milky Way, where observations go to remarkably low luminosities, small voids look very empty: dwarf galaxies do not show a tendency to fill the voids and voids are still relatively large. The theory predicts that many dwarf dark matter haloes should be in the voids, which puts it on a collision course with observations. Yet, below some mass the haloes are expected to stop producing galaxies inside them. There are different arguments for that: stellar feed-
back (Dekel & Silk 1986) or photoionization (Bullock et al. 2004) may play a significant role in quenching star formation in too small haloes. For example, Loeb (2008) made a simple estimation of the limiting circular velocity below which haloes have essentially no gas infall due to increase of Jeans mass caused by UV background at the epoch of reionization: \( V_{\text{lim}} = 34 \times \left( \frac{T_{\text{IGM}}}{1.5 \times 10^{4} \text{K}} \right)^{1/2} \text{km/s}, \) where \( T_{\text{IGM}} \) is the temperature of intergalactic medium gas ionized by stars. Hoefnagel et al. (2006) studied formation of dwarf DM haloes in cosmological void regions using high-resolution hydrodynamic simulations and assuming that cosmological UV-background photo-evaporates baryons out of haloes of dwarf galaxies, and thereby limits their cooling and star formation rate. Hoefnagel et al. (2006) give characteristic mass \( M_c = 6 \times 10^5 h^{-1} M_{\odot} \) below which haloes start to fail accreting gas.

Theoretical estimates for the least massive luminous halo are still uncertain. It is difficult to get a definite answer because the physics of dwarf galaxies at high redshifts is quite complicated. Star formation histories of some isolated dwarfs indicate that starbursts may produce enough power to throw gas away, but not sufficient for galaxy to get rid of it – gas again returns to potential well of the DM halo, hosting the galaxy (Quillen & Bland-Hawthorn 2008).

Satellites of the Local Group shed light on the problem from a different direction. The LCDM model predicts that thousands of dwarf DM haloes should exist in the Local Group (Klypin et al. 1999; Moore et al. 1999; Madau et al. 2002), while only \( \sim 50 \) are observed. Recent discoveries of very low luminosity dwarfs (Simon & Geha 2007) and careful analysis of incompleteness effects in SDSS (Strigari et al. 2008; Simon & Geha 2007; Tollerud et al. 2008) bring the theory and observations a bit closer, but the mismatch still seems to be present. The currently favored explanation of the overabundance of the dark matter subhaloes (Bullock et al. 2001; Kravtsov et al. 2004) assumes that dwarf haloes above \( V_c \approx 30 - 50 \text{ km/s} \) were forming stars before they fall into the Milky Way or M31 and that smaller haloes never formed any substantial amount of stars. Once the haloes above the limit fall into the halo of the Milky Way or M31, they get severely stripped and may substantially reduce their circular velocity producing galaxies such as Draco or Fornax with the rms line-of-sight velocities only a few \( \text{km/s}. \) The largest subhaloes retain their gas and continue to form stars, while smaller ones may lose the gas and become dwarf spheroidals. Haloes below the limit never had substantial star formation. They are truly dark. This scenario implies that circular velocity before the infall \( V_c \approx 30 - 50 \text{ km/s} \) is the limit for star formation in haloes. Moore et al. (2006) give additional arguments in favor of this scenario.

If this picture is correct, it can be tested using the abundance and the distribution of dwarf galaxies outside of the Local Group. Because dwarfs can only be detected at small distances, useful observational samples are limited to distances less than 10 Mpc.

While the Local Volume sample of galaxies is not as deep as the sample of satellite galaxies in the Local Group, we argue that the Local Volume dwarfs provide a unique opportunity to study the smallest and the darkest galaxies. The problem with using the LG satellites is related with the fact that the satellites have been tidally stripped by the Milky Way or by M31. Indeed, theoretical estimates indicate that a very substantial mass loss occurs even at very small distances from the centres of the dwarfs (e.g., Madau et al. 2008). Thus, we really do not know whether we deal with a truly low mass and low circular velocity satellite or with a satellite, which was much more massive in the past and was later severely stripped by its parent galaxy. The Local Volume dwarfs represent a more pristine sample in this respect.

The problem of the emptiness of voids was recently revisited by Tinker & Conroy (2008) with the conclusion that voids are not a problem for the LCDM model. Most of the observational results used in Tinker & Conroy (2008) are based on the void probability function in the SDSS DR6 sample and on the nearest neighbor statistics in the ORS catalog as analyzed by Peebles (2001). The void probability function was estimated for relatively bright galaxies with \( M_r < -17 \). The nearest neighbor statistics goes slightly deeper: the ORS sample is formally complete to \( m_B = 14.5 \), which gives \( M_B \approx -15 \) at the distance of 8 Mpc. Our sample is 3 magnitudes deeper, which is essential for the "void phenomenon". We also note that the quality of our sample is much better than that of the ORS sample. For example, we use real distances to galaxies, not redshifts. Special effort was made to ensure that the sample is complete for LSB galaxies and for galaxies in the zone of avoidance.

Tikhonov & Karachentsev (2006) presented results on statistics of nearby voids in the Local Volume. Here we continue the analysis using an updated list of galaxies (Karachentsev, private communication). We characterize the spatial distribution of galaxies in the LV mostly by studying the distribution of sizes of quasi-spherical regions — voids. The voids may still contain gas and small dark matter haloes. For our purpose void statistic is reasonably robust since 3d void maps are not very sensitive to a total number of objects in a sample. The distribution of void sizes have the advantage that they are sensitive to appearance of galaxies in very low density environments: an important property for studying the smallest existing galaxies.

We compare the spectrum of void sizes in the LV with the distribution of voids in high-resolution cosmological simulations. The simulations give us detailed information (positions, velocities, masses, circular velocities, and so on) for dark matter haloes and their satellites. However, they do not provide luminosities of galaxies. Theoretical predictions of the luminosity of a galaxy hosted by a halo with given mass, circular velocity and merging history are quite uncertain and cannot be used for our analysis. Instead, we ask a more simple question: what luminosity a halo or subhalo with given circular velocity should have in order to reproduce the observed spectrum of void sizes. When doing this, we assume that haloes with larger circular velocities should host more luminous galaxies. We will later see that matching of the void spectrum in simulations and with the observations puts significant constraint on relation of the halo circular velocity and the luminosity of a galaxy hosted by the halo. If we take too large circular velocity, there are too few galaxies and sizes of voids become too large. Instead, if very small haloes host galaxies, the number of large voids declines well below what is observed in the Local Volume.

In addition to the analysis of distribution of voids in the Local Volume we also re-visit the problem of the deviations from the Hubble flow. The flow of field galaxies in LV
appeared to be rather “cold”: deviations from the Hubble velocity are rather small. For example, using the Tully-Fisher distances and applying error correction via quadrature subtraction (Schlegel et al. 1994) have found for 15 galaxies within 500 km/s and outside Local Group the rms peculiar velocity $\sim$60 km/s. They noted that such values are very rare in the CDM models, but are not uncommon in MDM models that include massive neutrinos. Comparable values have been derived by Karachentsev & Makarov (1996) for galaxies within 7 Mpc from the Local Group (Maccio et al. 2003) based on their galaxy sample obtained $\sigma_h \sim 52$ km/s within 3 Mpc.

Karachentsev & Makarov (2001) found evidence of anisotropy of Hubble flow in LV. There is a dipole component, which is due to the LG motion relative to galaxies in the LV. This is typically described as apex motion of the LV. There is also a quadrupole component of the deviations from the Hubble flow interpreted as anisotropic expansion of Local Volume. Karachentsev & Makarov (2001) removed the apex motion and the quadrupole component from estimates of $\sigma_H$. They also removed galaxies with large velocity deviations, which were considered due to infall of galaxies onto large groups inside the LV. Galaxies inside groups were also not considered. Using only the galaxies with accurate measurements of distances and assuming that errors in distances increase the rms peculiar velocities, Karachentsev & Makarov (2001) estimate $\sigma_H$ in distance range 1-3 Mpc from the centre of LG may be as low as $\sim$30 km/s. Karachentsev et al. (2003) found $\sigma_H \sim 40$ km/s inside a sphere of 5 Mpc using distances from the luminosity of the tip of the red giant branch stars of about 20 dwarf galaxies. Their procedure of $\sigma_H$ estimation include determination of the “local” Hubble constant, which is slightly different from universal expansion rate; the true Hubble constant.

All these effects and corrections are valid, but they have a tendency to systematically underestimate the deviations from the Hubble flow. Some of the corrections can be mimicked in simulations, but it is difficult to do all of them. At the same time, one should not use these corrections; any deviation from the global Hubble flow must be included in the estimates. We make only one exclusion. We still consider the apex motion because it is related with the selection of the reference frame in which the whole sample does not have a net velocity.

There were several attempts to study $\sigma_H$ in simulations. Governato et al. (1997) emphasized that the dispersion of random motions of field galaxies and the centres of groups allow to discriminate between models with different values of matter density $\Omega_M$. Klypin et al. (2003) obtained in their constrained “Local Supercluster” simulation the peculiar line-of-sight velocity dispersion within $7h^{-1}$ Mpc of the model LG $\sigma_H \sim 60$ km/sec comparable to the observed velocity dispersion of nearby galaxies. They emphasize that there is no need in exotic explanations of the “coldness” of Hubble flow. Maccio et al. (2003) have found that in their simulations the Hubble flow is significantly colder around model LG-candidates selected in a CDM cosmology than around LG-candidates in open or critical models. Their estimation of $\sigma_H$ was much simpler then Karachentsev & Makarov (2001) approach – they calculate rms around mean value of peculiar velocity with local Hubble constant (best fit to data).

| Radial bin (Mpc) | $M_B < -15$ | $M_B = -12 - 14.5$ | Ratio |
|-----------------|------------|------------------|-------|
| 1 – 2           | 1          | 3                | 3     |
| 2 – 3           | 10         | 8                | 0.8   |
| 3 – 4           | 28         | 34               | 1.21 ±0.3 |
| 4 – 5           | 22         | 28               | 1.27 ±0.35 |
| 5 – 6           | 16         | 22               | 1.37 ±0.45 |
| 6 – 7           | 18         | 24               | 1.33 ±0.4 |
| 7 – 8           | 25         | 24               | 0.96 ±0.3 |
| 2 – 5           | 60         | 70               | 1.17 ±0.2 |
| 5 – 8           | 59         | 70               | 1.18 ±0.2 |

Table 1. Test of sample completeness: Counts of galaxies with different absolute magnitudes $M_B$ in radial shells.

A key question for our kind of investigation is to define what an “LV-candidate” is. In other words, it is important to define our local environment. Usually the conditions for selection of LG-candidates are rather simple and include distances of the LG-candidate to the nearest Virgo-type cluster (e.g., Hoffman et al. 2008) and the overdensity of a sphere of 8 Mpc radius (Maccio et al. 2003). Schlegel et al. (1994) used the overdensity 0.25. Maccio et al. (2003) used 0.1 < $\delta \rho / \rho$ < 0.6. Karachentsev et al. (2004) noted that inside the radius of 8 Mpc centred on the Local Group the luminosity density in the B-band is 1.8 – 2 times larger than the average luminosity density.

2 DATA: LOCAL VOLUME

Over the past few years searches for galaxies with distances less than 10 Mpc have been undertaken using numerous observational data including searches for LSB galaxies, blind HI surveys, and NIR and HI observations of galaxies in the zone of avoidance (Karachentsev et al. 2004, 2007). At present, the sample contains about 550 galaxies. The distances to the galaxies are not measured using the redshifts because the perturbations of the Hubble flow in the Local Volume are large and significantly distort the spatial distribution of galaxies. The distances are mostly measured with the tip of the red giant branch (TRGB) stars, cepheids, the Tully-Fisher relation, and some other secondary distance indicators. For most of galaxies the distances have been measured with the accuracy of 8-10% (Karachentsev et al. 2004).

Karachentsev et al. (2004, Section 4) discuss completeness of the earlier sample and conclude that within 8 Mpc radius the sample was 70-80 percent complete: about 100 galaxies were estimated to be missed in that sample. We use the updated sample, which has almost 100 more galaxies and, thus, it is expected to be nearly complete. We estimate the completeness of our updated sample using two methods. In both methods we use the ratio of the number of dwarf galaxies to the number of bright galaxies as an indicator of completeness: the ratio should not depend on the distance.

First, we count the number of bright galaxies ($M_B < -15$) and the number of dwarf galaxies ($M_B = -12 - 14.5$) inside radial shells of 1 Mpc width. If the sample is not complete, we would expect a decline with the distance of the number of dwarf galaxies. The ratio of the number of dwarf to large
Figure 1. The luminosity function of galaxies in the the Local Volume. Circles with errors show results for the 8 Mpc sample (complete to $M_B = -12$). The full curve is for the 4 Mpc sample (complete to $M_B = -10$). The dashed curve is for the Schechter approximation of 2dFGRS scaled to $h = 0.72$. The faint-end slope of the approximation $\alpha = 1.2$ fits well the slope of the luminosity function of galaxies in the Local Volume. For illustrative purposes we plot the approximation well outside of the $M_B = -17.2$ completeness limit of 2dFGRS.

The numbers indicate that there is no large incompleteness. However, we cannot exclude a very likely possibility that the sample still misses few galaxies. This is why we do not use statistics (e.g., the size of the largest void), which are sensitive to small effects such as missed few galaxies or few galaxies with wrong distances. Results presented in section 3 demonstrate stability of our statistics to variations in the sample and to errors in distances.

Galaxies in the Local Volume were detected down to extremely low luminosities. This gives us a unique chance to detect voids, which may be empty of any galaxies. We use two volume-limited samples. The main sample is complete for galaxies with absolute magnitudes $M_B < -12$ within 8 Mpc radius. Another volume limited sample is $M_B < -10$ within 4 Mpc.

We use three N-body simulations: two are done with the Adaptive Refinement Tree code [Kravtsov et al. (1997), and one simulation, which is done with the GADGET2 code Springel (2005). Parameters of simulations are presented in the Table 2. The simulation $S_1$ has a high-resolution spherical region of radius $10 h^{-1}$ Mpc = 14 Mpc, which is resolved with ~150 million particles each having mass $5 \times 10^{9} h^{-1} M_{\odot}$. This high-resolution region is embedded into box of $80 h^{-1}$ Mpc were mass and force resolutions are lower. In the other two simulations the whole computational box was resolved with equal-mass 1024$^3$ particles particles.

We use halo finders, which detect both distinct haloes and their subhaloes. The halo finders provide us with different parameters of (sub)haloes. As a measure of how large is a halo we typically use the maximum circular velocity $V_c$, which is easier to relate to observations as compared with the virial mass. For reference, haloes with $V_c = 50$ km/s have virial mass about $10^{10} M_{\odot}$ and haloes with $V_c = 20$ km/s have virial mass about $10^{9} M_{\odot}$.

The simulations are for a spatially flat cosmological LCDM model with following parameters. The simulation $S_1$ has $\Omega_0 = 0.3, \Omega_\Lambda = 0.7; \sigma_8 = 0.9; h = 0.7$ (WMAP1 parameters). Simulations $S_2$ and $S_3$ are done for $\Omega_0 = 0.24, \Omega_\Lambda = 0.76; \sigma_8 = 0.75; h = 0.73$ (WMAP3 parameters).

We re-scaled all data (coordinates and masses of haloes) to “real” units assuming $H_0 = 72$ km/s/Mpc, which is close to recent WMAP results.

4 RESULTS

4.1 Luminosity function and global parameters of LV

Figure 1 shows the luminosity function of LV galaxies. The Figure also shows the Schechter approximation for the 2dFGRS catalog [Norberg 2002], with parameters $\alpha = -1.21, M_\nu = -19.66 + 5 \log h, \Phi_* = 1.61 \times 10^{-22} h^3$ Mpc$^{-3}$ scaled for $h = 0.72$. The approximation gives the average luminosity function of galaxies in the $B$-band in the Universe scaled to the $B$-band by means $b_j = B - 0.28(B - V)$ [Norberg 2002], and by assuming the mean B-V = 0.5 color for giant spiral galaxies dominating the Local Volume. Note that the 2dFGRS luminosity function extends only down to $M_B \approx -17.2$. We plot it well below its completeness limit just for reference. We construct the LF of the LV using magnitudes corrected for the internal extinction. The LF for uncorrected magnitudes from tables of Karachentsev et al. (2004) doesn’t change the LF for most of the luminosity bins except for the very brightest one.

The luminosity function of the 8 Mpc sample is larger than the universal LF for all luminosities. With the exception of the very brightest galaxies, the LF in the Local Volume exceeds the universal LF by a factor of 1.3. The LF of the 4 Mpc sample gives significantly larger excess over the universal LF: factor 3.6 for $M_B > -19$. Another interesting feature of LV $M_B < -12$ luminosity function is the prominent peak at $M_B = -14$. It is mostly produced by dwarf irregular isolated galaxies, which have large gas masses, specific SFRs, and total mass-to-light ratios. The overall shape of the LF in the Local Volume is surprisingly robust. For example, the excess of the bright galaxies with $M_B < -20$ is reproduced in both subsamples. The slope of the LF at the faint end $\alpha \approx -1.2$ is also the same for the two samples.

In addition to the overall normalization of the LF, we also estimate the overdensity for the LV sample using a
4.2 Selecting model LV-candidates

To mimic the main features of the real Local Volume galaxy sample we use a number of criteria to select from our simulations spheres of radius 8 Mpc that we can consider as “LV-candidates”. Criteria differ slightly from one simulation box to another to select at least some candidates. In all simulations the candidates must be centered on a halo with the virial mass in the range \(1.5 \times 10^{12} M_\odot < \text{Mass} < 3 \times 10^{13} M_\odot\). We do not require that the LG-candidates should have two comparable in mass haloes. Appearance of two haloes instead of one halo of the same total mass does not change the dynamics of matter outside the LG-candidate. We also do not require that the candidates should be at the same distance from a Virgo-type cluster as the real LG. About a half of our LG-candidates are at close (15 – 25Mpc) distance from a cluster. We do not find any substantial difference between those candidates. Instead, we impose detailed constraints on distribution of mass in the LG-candidates, which we take from the real LV sample of galaxies. Below is the list of conditions, which were used for different simulations:

| Simulation | \(S_1\) | \(S_2\) | \(S_3\) |
|------------|------|------|------|
| Box Size (h^{-1}\text{Mpc}) | 80   | 160  | 64   |
| \(\sigma_8\) | 0.90 | 0.75 | 0.75 |
| Mass of a high resolution particle (h^{-1}M_\odot) | \(4.91 \times 10^6\) | \(3.18 \times 10^6\) | \(1.6 \times 10^7\) |
| Spatial Resolution (h^{-1}\text{kpc}) | 0.52 | 1.2  | 1.6  |
| Number of high resolution particles | \(1.6 \times 10^4\) | \(1024^3\) | \(1024^3\) |
| Circular velocity of the smallest resolved halo (km/s) | 9    | 27   | 15   |

Simulation \(S_1\): (1) no haloes with mass\(>4 \times 10^{11} M_\odot\) inside a 8Mpc sphere. Thus, no large groups and clusters in a sample.; (2) The number density of haloes found inside 8Mpc sphere with \(V_c > 100 \text{km/s}\) exceeds the mean value in the whole box by factor in the range 1.3 – 1.7; (3) There are no haloes more massive than \(8 \times 10^{11} M_\odot\) with distances in the range (1-3Mpc). In total, there are 3 LG-candidates in the simulation \(S_1\), which satisfy these conditions.

Simulation \(S_2\): (1) no haloes with Mass\(> 2 \times 10^{13} M_\odot\) inside a 8Mpc sphere; (2) The number density of haloes found inside 8Mpc sphere with \(V_c > 100 \text{km/s}\) exceeds the mean value in the whole box by factor in the range 1.5 – 1.7; (3) There are no haloes more massive than \(1.0 \times 10^{12} M_\odot\) with distances in the range 1-3Mpc. (4) Central haloes of different LG-candidates are located at distance more then 5Mpc one from the other. There are 14 samples with above criteria in the simulation \(S_2\).

Simulation \(S_3\): The volume of the simulation is large enough to allow us to select candidates, that mimic LV features more closely than in the previous simulations. Conditions are: (1) no haloes with Mass\(> 2 \times 10^{13} M_\odot\) inside a 8Mpc sphere; (2) The number density of haloes found inside 8Mpc sphere with \(V_c > 100 \text{km/s}\) exceeds mean value in the whole box by factor in the range 1.5 – 1.7; (3) The number density of haloes found inside 4.5Mpc sphere with \(V_c > 100 \text{km/s}\) exceeds the mean value in the whole box by factor greater then 3.5; (4) There are no haloes more massive than \(5.0 \times 10^{11} M_\odot\) with distances in the range (1-3Mpc). (5) Central haloes of different LG-candidates are separated by distances larger than 8Mpc. We’ve found 7 such candidates in the simulation \(S_3\).

We want to note that our LV-candidates look quite similar to the LV galaxy sample. They have the same environment, and they have the same mixture of groups (or lack of those). The number of large galaxies is also very similar. In the observed sample there are about 20 galaxies with rotational velocities larger than 120 km/s. On average, there are about 20 haloes with circular velocity above 100 km/s in our LV-candidates in the simulations with the WMAP3 parameters. Taking into account the increase in rotational speed produced by baryonic infall into DM haloes, we expect the models produce the same number of large “galaxies” as observed in the real LV. For all selected LV-candidates we estimate the rms velocity deviations from the Hubble flow and find that they are reasonably consistent with values obtained from LV galaxy sample.
The Hubble flow from the observed galaxies in the LV underestimates of groups – from statistics. Removing these velocities from the LV: we do not exclude any galaxies – such as members do not correct results for virial motions due to groups inside Milky Way and M31. We assume that the Hubble flow is (placing the centre of our samples just in the middle between Milky Way and M31). We use computational scheme that differs from that used by Karachentsev’s group. In order to have a simple and clear interpretation of obtained values and direct and unbiased comparison with simulated LV-candidates, we take all galaxies with known radial velocities in the frame of Local Group (placing the centre of our samples just in the middle between Milky Way and M31). We assume that the Hubble flow is universal and equal to the universal cosmic expansion. We do not correct results for virial motions due to groups inside the LV: we do not exclude any galaxies – such as members of groups – from statistics. Removing these velocities from the estimates of peculiar velocities and finding the Hubble flow from the observed galaxies in the LV underestimates the value of \( \sigma_H \).

We assume a 10% error as the mean error of distance measurements. We start our estimation just outside 1 Mpc radius from the centre of the Local Group and calculate \( \sigma_H \) for galaxies with distances \( D \) from 1 Mpc to the distance \( D_{out} \). We apply a correction for the apex motion – the mean motion of galaxies in surrounding volume with respect to the Local Group in simple dipole approximation (making our \( \sigma_H \) estimation in the frame where surrounding galaxies have no dipole component motion relative to LG). The vector of apex motion \( \{A_x, A_y, A_z\} \) is estimated by minimization of the sum:

\[
\sum_{i=1}^{N} \left( v_i - H_0 D_i - A_x x_i + A_y y_i + A_z z_i \right),
\]

where \( H_0 = 72 \text{ km/s/Mpc} \) and \( v_i, D_i, x_i, y_i, z_i \) are velocity, distance and Cartesian coordinates of galaxies in the Local Group frame. After subtracting the apex and the Hubble flow \((H_0 D_i)\) from velocities, we calculate the residual rms peculiar radial velocity in a certain distance range. By subtracting apex from velocities (converted into LG frame) we remove the dipole component of motions of galaxies. Now we need to correct rms peculiar velocities for the errors in distances of galaxies. We subtract in quadratures the estimated rms velocity \( \sigma_m \) due to the errors:

\[
\sigma_m = \frac{1}{N} \sum_{i=1}^{N} (\Delta v_i + H_0 \delta D_i)^2 = \frac{1}{N} \sum_{i=1}^{N} \Delta v_i^2 + \frac{\sigma_H^2}{N} \sum_{i=1}^{N} D_i^2 + \frac{2H_0^2}{N} \sum_{i=1}^{N} (\Delta v_i \delta D_i),
\]

where we assume that the error in the peculiar radial velocity \( \sigma_m \) is produced by errors in measurements of velocities \( \Delta v_i \) and distances \( \delta D_i \). Here the parameter \( \alpha \) is the rms error in distance measurements: \( \delta D_i = \alpha D_i, \alpha \approx 0.1 \). We assume that the third term in the last equation is small: no correlation of errors of peculiar velocities and distance errors. Our Monte-Carlo modeling of the errors shows that this is the case. We also neglect the first term in the right-hand side of the equation (2) since velocity measurements are accurate enough (\( \Delta v_i \approx 5 \text{ km/s} \)).

Values of \( \sigma_H \) on different scales for galaxies in the Local Volume and complementary parameters are presented in Table 2. The table gives the rms velocity in different overlapping regions of LV for distances from 1 Mpc to \( D_{out} \). The second column gives the number of galaxies \( N \) in a region limited by \( D_{out} \) and with distance larger than 1 Mpc. The parameter \( \sigma_H^0 \) is the radial rms deviation from Hubble flow with no corrections for either apex or distance errors. The parameter \( \sigma_H' \) is the rms deviations corrected for distances errors: \( \sigma_m \) (last column) is estimated using the equation (2) and subtracted in quadratures from \( \sigma_H' \). Column 5 gives the rms velocity corrected for apex only. Parameter \( \sigma_H^{\text{final}} \) (column 6) is the final estimate of the rms radial deviations: corrected for apex motion and for distance errors. Two last columns give the velocity of apex motion and the error estimated by eq.(2). Figure 2 shows \( \sigma_H^{\text{final}} \) in graphical form (the red curve with circles).

When analyzing LV-candidates in the simulations, we subtract 3d velocity of the central halo from the rest of sample velocities, find radial velocities of haloes and apply the above procedure of apex correction and Hubble flow subtraction and then obtain \( \sigma_H^{\text{model}} \). In this case the apex motion is just the dipole component of motions of haloes with respect to the central halo – LG-analog. Results for the simulation \( S_2 \) are shown in Figure 2 with black circles. Results of the simulation \( S_3 \) are very similar.

As can be seen from Figure 2, the theoretical predictions are close to the observed values. We note the importance of criterion that between 1 Mpc and 3 Mpc there are no relatively large DM haloes. With such a condition we get the

### Table 3. Velocity scatter around the Hubble flow in Local Volume as the function of the distance from the Local Group.

| Outer bin radius \( D_{out} \) (Mpc) | Number of galaxies \( N \) | Uncorrected rms velocity \( \sigma_H^0 \) | Corrected for distance errors \( \sigma_H' \) | Corrected for apex motion \( \sigma_H'' \) | Corrected for all effects \( \sigma_H^{\text{final}} \) | Apex velocity \( V_{\text{apex}} \) (km/s) | \( \sigma_m \) |
|-------------------------------------|-----------------------------|--------------------------------|--------------------------------|--------------------------------|--------------------------------|--------------------------------|-------------------------|
| 3.0                                 | 43                          | 73.7                           | 71.7                           | 56.2                           | 53.5                           | 65.5                           | 17.4                    |
| 4.0                                 | 106                         | 84.6                           | 81.6                           | 83.4                           | 80.3                           | 62.7                           | 22.4                    |
| 5.0                                 | 162                         | 84.3                           | 80.1                           | 83.1                           | 78.8                           | 21.9                           | 26.3                    |
| 6.0                                 | 214                         | 83.6                           | 78.1                           | 81.2                           | 75.4                           | 32.0                           | 29.9                    |
| 7.0                                 | 273                         | 96.8                           | 90.4                           | 90.6                           | 83.8                           | 54.9                           | 34.4                    |
| 8.0                                 | 335                         | 106.6                          | 99.3                           | 98.2                           | 90.2                           | 68.8                           | 38.8                    |
| 9.0                                 | 360                         | 107.5                          | 99.6                           | 99.4                           | 90.7                           | 68.5                           | 40.7                    |
mean rms velocity $\sigma_{H}^{\text{model}} \sim 50\,\text{km/s}$ for 8 Mpc samples and for the distances less than 3 Mpc in the simulation $S_2$. This is very close to the observational value. For some samples (not shown on the plot) $\sigma_{H}^{\text{model}}$ is as low as about 20 km/s. If we don’t apply this condition and allow large haloes with mass $\sim 10^{12} M_\odot$ to reside between 1 and 3 Mpc, then the mean $\sigma_{H}^{\text{model}}$ in this region is as high as 72.3 km/s. So massive haloes heat the “halo gas” leading to large rms velocities.

### 4.4 Detecting Voids

In order to detect voids, we place a 3d mesh on the observational or simulation volume. We then find initial centres of voids as the mesh centres having the largest distances to nearest objects. In the next iteration, an initial spherical void may be increased by adding additional off-centre empty spheres with smaller radius. The radius of the spheres is limited to be larger than 0.9 of the initial sphere radius $R_{\text{seed}}$ and their centres must stay inside the volume already connected to void. The process is repeated until $R_{\text{seed}} = 1$ Mpc. It produces voids which are slightly aspherical. Mean ellipticity of our voids is about 0.7. Artificial objects are placed on the boundaries of the sample to prevent voids getting out of the boundaries of the sample. We define the cumulative void function (VF) as the fraction of the total volume occupied by voids with effective radius larger than $R_{\text{eff}} = (3V_{\text{void}}/4\pi)^{1/3}$ (further $R_{\text{void}}$). As we already mentioned, for our purpose – to match observational VF by model one with a certain limit on circular velocity – the voids volume statistic (VF) is a robust one since it is not very sensitive to the total number of objects in a sample.

### 4.5 Void Functions

We use two samples to construct VF of the Local Volume:

1. Galaxies brighter than $M_B = -12$ inside sphere of radius 8 Mpc; the number of galaxies is 315.
2. All galaxies

Figure 2. The rms radial velocity deviations from the Hubble flow $\sigma_H$ for galaxies in the Local Volume with distances from 1 Mpc up to R (full red curve with open circles). The estimates are corrected for the apex motion and for distance errors. Black filled circles show theoretical predictions for 7 LV candidates in the simulation $S_2$.

Figure 3. Fraction of volume $\Delta V/V$ in voids with radius larger than $R_{\text{void}}$. Observational data (the complete sample, open circles) are compared with the distribution of voids in samples of haloes with different limits on halo circular velocity in the simulation $S_1$ with $\sigma_8 = 0.9$. VF for $V_c = 45$ km/s provides a remarkably good fit to observations. Note that the Local Volume has very large empty regions with about 1/4 of the whole volume occupied by one void.

Figure 4. The void function for two observational samples. The full curve with open circles are for a complete volume limited sample with $M_B < -12$ and $R < 8$ Mpc. 1σ errors obtained by Monte Carlo re-sampling of distances from catalog by means of addition of Gaussian radial displacements with typical distance error of 10% of distance measurements. The filled circles are for all observed galaxies inside 7.5 Mpc. Comparison of the samples shows reasonable stability of the void function.
inside 7.5 Mpc; the number of galaxies is 376. Results are present in the Figure 3. There are about 30 voids in the observational samples with radii $1 - 4.5$ Mpc. We limit the radius of voids to be more than 1 Mpc. The two subsamples indicate some degree of stability: inclusion of a number of low-luminosity galaxies does not change the void function significantly.

The Figure 3 shows the mean VF for 3 different samples of haloes from the simulation $S_1$ and the observed VF. Results indicate that voids in the distribution of haloes with $V_c > 45$ km/s give the best fit to the observed VF: the spectrum of voids in the most valid range of $R_{void}$ is reproduced by the theory. The theoretical VF goes above the observational data, if we use circular velocities larger than 60 km/s. If we use significantly lower limits, than the theory predicts too few large voids. The theoretical results match the observations, if we use $V_{circ} = 45 \pm 5$ km/s.

The Figure 5 and Figure 6 show the mean VF for 14 and 7 different samples of haloes from simulations $S_3$ and $S_2$ respectively and the observed VF. Here voids in the distribution of haloes with $V_c > 35$ km/s match the observed VF. The difference in limiting value of $V_c$ between the simulations is related to different values of $\sigma_8$ used in simulations. The theoretical results match the observations, if we use $V_c = 35 \pm 5$ km/s. Note that 7 LV-candidates used here are those whose $\sigma_H$ values plotted on Figure 2; these samples mimic LV environment more closely.

We study the distribution of very small haloes inside voids defined by larger haloes. We use 8 largest voids in the simulation $S_1$, which are defined by haloes with $V_c > 45$ km/s. There are smaller haloes inside the voids. We characterize the haloes by their distance $R_{border}$ to the border of a void. (Note that $R_{border} = 0$ is for a halo at the boundary of a void, not at its centre). We count the number of the haloes in shells with width 300 kpc. The Figure 6 shows the number density profile of haloes as the function of $R_{border}$. The number density of haloes is very low close to the centres of voids and increases very substantially when we get closer to the void boundary. In this respect the small voids, which we study in this paper, behave very similar to giant voids found in simulations of Gottloeber et al. (2003). To a large degree, the small and giant voids are similar. For example, there are very small filaments made of tiny haloes inside our small voids. Altogether we have a kind of self-similarity of voids properties on different levels of their detection both in observations and in simulations (Patiri et al. 2006; Tikhonov 2007; Gottloeber et al. 2003).

### Figure 5. The observational void function (the complete sample with $M_B < -12$) is compared with the distribution of voids in 14 samples in the simulation $S_3$ for haloes with different limits on halo circular velocity. In this case VF for $V_c = 35$ km/s (shown with 1σ scatter) provides a better fit to observations.

### Figure 6. Observational data (the complete sample $M_B < -12$) are compared with the distribution of voids in 7 samples from $S_2$ simulation for haloes with different limits on circular velocity. VF for $V_c = 35$ km/s (shown with 1σ scatter) provides a remarkably good fit to observations. Because of resolution limitations in this simulation, we do not present results for smaller $V_c$ limits.

## 5 DISCUSSION AND CONCLUSIONS

We use an updated version of the Karachentsev et al. (2004) sample of galaxies to study the distribution and motions of galaxies in the Local Volume. There are about 30 voids, which range in radius from 1 Mpc to 4 Mpc. We demonstrate that the spectrum of void sizes is relatively stable for variations of the sample and for uncertainties of distances to individual galaxies. Estimates of the cosmic variance, which we get from cosmological simulations, also show stability of the void statistics.

When making the theoretical predictions for the LCDM model, we carefully select Local Volume candidates. In many
respects the candidates look very similar to the reality: they have similar number of large haloes, similar density contrasts at different scales, and have similar rms velocity deviations from the Hubble flow. The spectrum of void sizes in simulations traces the observed spectrum remarkably well, if we assume that haloes with circular velocities \( V_c > 35 \text{ km/s} \) for \( \sigma_8 = 0.75 \) and \( V_c > 45 \text{ km/s} \) for \( \sigma_8 = 0.90 \) host galaxies brighter than \( M_B = -12 \). The mass limits are quite consistent with the theoretical expectations for the mass of smallest halo, which can host a galaxy (e.g., Hoefy et al. 2006; Loeb 2008).

At the same time, if much smaller haloes with \( V_c > 20 \text{ km/s} \) host galaxies with the observed absolute magnitudes \( M_B = -12 \), voids in the LCDM model would be too small and their spectrum of sizes would strictly contradict the observations. This is hardly an unexpected conclusion: in the hierarchical scenario any void is filled with small haloes. The only question is what is the mass of the haloes. For the Local Volume with the completeness limit \( M_B = -12 \) this appears to be \( V_c > 35 - 45 \text{ km/s} \). If this is true, haloes with \( V_c \approx 20 \text{ km/s} \) should not host galaxies. The problem is that in reality they do: in the Local Volume many luminous galaxies with these absolute magnitudes rotate with velocities \( V_{rot} \approx 20 \text{ km/s} \) or smaller.

In order to demonstrate this, we select all isolated galaxies with \(-13.2 < M_B < -11.8\). The isolation criteria are very strict because we would like to be sure that the rotational velocities of galaxies were not reduced by stripping or by any other interaction with large neighboring galaxies. We select the galaxies, which are more than 1 Mpc away from any galaxy brighter than \( M_B < -19 \) and do not have companions within 200 kpc, which are brighter than the galaxy itself. Note that the galaxies are so small that the

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**Table 4. Properties of isolated dwarf galaxies with \( M_B = -11.8 - 13.2 \)**

| Name            | \( M_B \) | axial ratio | \( W_{50} \) | \( V_{rot} \) | Distance | reference            |
|-----------------|-----------|-------------|--------------|--------------|----------|----------------------|
| E349-031,SDIG   | -12.10    | 0.82        | 20.0         | 17.5         | 3.21     | Karachentsev et al. 2004 |
| KKH5            | -12.27    | 0.62        | 37.0         | 23.6         | 4.26     | Karachentsev et al. 2004 |
| KKH6            | -12.38    | 0.60        | 31.0         | 19.4         | 3.75     | Karachentsev et al. 2004 |
| KK16            | -12.65    | 0.37        | 24.0         | 12.9         | 5.40     | Karachentsev et al. 2004 |
| KKH18           | -12.39    | 0.57        | 34.0         | 20.7         | 4.43     | Karachentsev et al. 2004 |
| KKH34,Mai13     | -12.30    | 0.56        | 24.0         | 14.5         | 4.61     | Karachentsev et al. 2004 |
| E489-56,KK54    | -13.07    | 0.53        | 33.8         | 19.9         | 4.99     | Doyle et al. 2005     |
| KKH46           | -11.93    | 0.86        | 25.0         | 24.5         | 5.70     | Karachentsev et al. 2004 |
| U5186           | -12.98    | 0.23        | 42.0         | 21.6         | 6.90     | Karachentsev et al. 2004 |
| E321-014        | -12.70    | 0.43        | 39.8         | 22.0         | 3.19     | Doyle et al. 2005     |
| KK144           | -12.59    | 0.33        | 44.0         | 23.3         | 6.30     | Karachentsev et al. 2004 |
| E443-99,KK170   | -12.03    | 0.75        | 29.0         | 21.9         | 5.78     | Karachentsev et al. 2004 |
| KK182,Centri    | -11.89    | 0.60        | 16.0         | 10.0         | 5.78     | Karachentsev et al. 2004 |
| DDO181,U8651    | -12.97    | 0.57        | 42            | 23.7         | 3.02     | Karachentsev et al. 2004; Springob et al. 2005 |
| DDO183,U8760    | -13.13    | 0.32        | 30.0         | 15.8         | 3.18     | Karachentsev et al. 2004 |
| HIPASS1351-47   | -11.88    | 0.60        | 38.8         | 24.2         | 5.65     | Doyle et al. 2005     |

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**Figure 7.** The number density profiles \( N/\Delta V \) of haloes with \( V_c < 45 \text{ km/s} \) inside 8 largest voids (different symbols) found in the \( S_1 \) simulation. The distance \( R_{border} \) is the distance of a halo from the void boundary. The haloes are binned in 300 kpc shells.

**Figure 8.** The velocity-magnitude relation for galaxies in the Local Volume (open circles with error bars) is compared with predictions of the LCDM model. Two other observational estimates are also shown as full and dashed lines. The theory (filled circles and diamonds) makes reasonable predictions for bright galaxies with \( M_B < -17 \). At smaller luminosities the theoretical curves are systematically above the observations. At \( M_B = -12 \) the disagreement is a factor of two in circular velocities implying a factor of \( \sim 10 \) disagreement in the number of haloes.
expected virial radius is smaller than 100 kpc. All the galaxies are dwarf irregulars, and for half of them there are measurements of the 21 cm HI line width. Using the HI full-width-half-maximum \( W_{20} \) measurements, we estimate the rotational velocity of a galaxy: \( v_{\text{rot}} = W_{20}/2\sqrt{1 - (b/a)^2} \), where \( b/a \) is the axial ratio. A large fraction of the HI line width is likely produced by random 8–10 km/s velocities. We do not subtract those because we compare the results with the circular velocities of dark matter haloes. Table 4 presents the results for galaxies with detected HI emission and with \( v_{\text{rot}} < 25 \) km/s. Galaxies in the Table 4 have rotational velocities well below those required by the LCDM model. Half of the galaxies rotate slower than 20 km/s. We also studied galaxies, which are not so isolated and galaxies, which are slightly brighter than those presented in the Table 4. Results are qualitatively the same.

The disagreement between the theory and observations also shows up in the Tully-Fisher (TF) relation for galaxies in the Local Volume. In order to construct the relation, we use all galaxies in the Karachentsev et al. (2004) sample with measured HI line width \( W_{20} \), which have morphological type > 0 (i.e. spirals and irregulars). The rotational velocities are corrected for inclinations. In order to allow the comparison with theory, we do not make any corrections for internal gas motions. Open circles with error bars in Figure 3 show the TF relation of galaxies in the Local Volume. Two lines in the plot also show the TF relation from Tully & Pierce (2000) and Epinat et al. (2008), which we extrapolate down to small magnitudes (GHASP results extend down to \( M_B \approx -16 \)). The observational TF estimates agree reasonably well for bright galaxies with \( M_B < -16 \). At smaller luminosities the LV results go slightly above the extrapolations from brighter samples.

When assigning luminosities to dark matter haloes we follow the prescription of Conroy et al. (2006). Specifically, we rank by luminosity all the galaxies in the LV sample and we rank by circular velocity all the haloes in our LV-candidate samples in simulations. We then take the luminosity of the brightest galaxy and assign it to the halo with the largest circular velocity. Then we take the second brightest galaxy and give its luminosity to the second halo and so on. According to Conroy et al. (2006), this prescription reproduces clustering properties of galaxies in the SDSS sample. We add a small (20%) correction to the circular velocity of haloes to accommodate the effect of adiabatic contraction due to infall of baryons. Figure 5 shows that the LCDM model gives a good match to observations at the bright end of the luminosity function (\( M_B < -17 \)). The model with lower normalization produces a better fit, but even the high normalization model cannot be excluded: a more accurate treatment of the adiabatic inflow may slightly improve the situation. At low luminosities the theory and observations gradually diverge, and at \( M_B = -12 \) the differences are quite substantial: a factor of two in circular velocities. This is the same problem, which we found using the spectrum of voids: haloes with \( V_c \approx 35 - 45 \) km/s should have luminosities \( M_B = -12 \) in order to match the observational data.

We would like to emphasize that the disagreement with the theory is staggering. The observed spectrum of void sizes disagrees at many sigma level from the theoretical void spectrum if haloes with \( V_c > 20 \) km/s host galaxies brighter than \( M_B = -12 \). We can look at the situation from a different angle. In the LCDM model with \( \sigma_0 = 0.9 \) there are \( \sim 320 \) haloes with \( V_c > 45 \) km/s – the same number as the number of galaxies in the Local Volume with the \( M_B = -12 \) limit. In the same volume in the LCDM model there are \( \sim 3500 \) haloes with \( V_c > 20 \) km/s. If all these haloes host galaxies brighter than \( M_B = -12 \), the theory predicts a factor of ten more haloes as compared with the observations.

The problem has the same roots as the overabundance of substructure in the Local Group: the LCDM model predicts too many dwarf dark matter (sub)haloes as compared with the observed dwarf galaxies (Klypin et al. 1999, Moore et al. 1999, Madau et al. 2008). We suggest that the solution of the problem of the overabundance of the dwarfs in the Local Volume may be similar to current explanations of the substructure problem in the LG:

- The observational sample is not complete: there are ten times more dwarf galaxies down to limiting magnitude \( M_B = -12 \) than listed in the Karachentsev et al. (2004) sample. The “missed” dwarfs are unlikely to be dwarf irregulars because they would have HI emission and would have been detected by blind HI surveys such as HIPASS (Doyle et al. 2005). Dwarf spheroidal galaxies is a possibility. They do not have gas and cannot be detected in HI. They have very low surface brightness, which makes it difficult to detect them on photographic plates. So, it is likely that many of the galaxies were missed. Still, we do not know whether a large population of dSph galaxies exists in the LV. If this is so, we will have another problem: how to form thousands of dwarf spheroidals in very low density environments without any tidal stripping or interaction with massive parent galaxy. The slope of the luminosity function also will be much steeper: \( \alpha \approx 2 - 2.5 \).
- The observed galaxies with \( v_{\text{rot}} \approx 20 \) km/s are hosted by significantly more massive haloes. The overabundance problem would be solved, if the circular velocity of a dark matter halo is \( V_c \approx 2V_{\text{rot}} \). This is somewhat similar to the solution of the overabundance problem in the LG (e.g., Peñaarrubia et al. 2008).
- Most of the dwarf haloes with \( V_c < 35 \) km/s in local voids failed to form stars because they collapsed after the epoch of reionization (Bullock et al. 2000).

We also estimate the rms deviations from the Hubble flow \( \sigma_H \) for galaxies at different distances from the Local Group and find that in most of our model LV-candidates the rms peculiar velocities are consistent with observational values: \( \sigma_H = 50 \) km/s for distances less than 3 Mpc and \( \sigma_H = 80 \) km/s for distances less than 8 Mpc. At the distances 4 (8) Mpc the observed overdensities of galaxies are 3.5-5.5 (1.3-1.6) – significantly larger than typically assumed.

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REFERENCES

Bullock J.S., Kravtsov A.V., Weinberg, D.H., 2000, *ApJ*, 539, 517
Conroy, C., Wechsler, R. H., & Kravtsov, A. V. 2006, *ApJ*, 647, 201
Dekel A., Silk J., 1986, *ApJ*303, 39
Doyle M.T., et al., 2005, *MNRAS*, 361, 34
Epinat, B., Amram, P., & Marcelin, M. 2008, *MNRAS*, 390, 466
Fisher J.R., Tully R.B., 1975, A&A, 44, 151
Governato F., Moore B., Cen R., Stadel J., Lake G., Quinn T., 1997, NewA, 2, 91
Gottlöber S., Lokas E.L., Klypin A., Hoffman Y., 2003, *MNRAS*, 344, 715
Hoefl M. et al., *MNRAS*, 2006, 371, 401
Hoffman Y., Martinez-Vaquero L. A., Yepes G., Gottloeber S., 2008, *MNRAS*, 386, 390
Karachentsev I.D., Makarov D.I., 1996, *Af*, 111, 535
Karachentsev I.D., Makarov D.I., 2001, Astrofizika, 44, 5
Karachentsev I.D. et al., 2003, A&A, 389, 479
Karachentsev I.D., Karachentseva V.E., Huchtmeier W.K., Makarov D.I., 2004 *Af*, 127, 2031
Karachentsev I.D., Karachentseva V., Huchtmeier W., Makarov D., Kaisin S., Sharina M., Mining the Local Volume, 2007, [arXiv:0710.0520](http://arxiv.org/abs/0710.0520)
Klypin, A., Kravtsov, A.V., Valenzuela, O., Prada, F., 1999, *ApJ*, 522, 82
Klypin A., Hoffman Y., Kravtsov. A.V., Gottlöber S., 2003, *ApJ*, 596, 19
Kravtsov A.V., Klypin A., Khokhlov A.M., 1997, *ApJ Suppl.*, 111, 73
Kravtsov A.V., Gnedin O.Y., Klypin A., 2004, *ApJ*, 609, 482
Loeb A., 2008, [arXiv:0804.2258](http://arxiv.org/abs/0804.2258)
Maccio A.V., Governato F., Horellou C., 2005, *MNRAS*, 359, 941
Madau P., Diemand J., Kuhlen M., 2008, *ApJ*, 679, 1260
Moore B., Ghigna S., Governato F., Lake G., Quinn T., Stadel J., Tozzi P., 1999, *ApJ Lett.*, 524, L19
Moore B., Diemand J., Madau P., Zemp M., Stadel J., 2006, *MNRAS*, 368, 563
Norberg P., 2002, *MNRAS*, 336, 907
Peebles P.J.E., The Large-Scale Structure of the Universe, Princeton University Press, 1980
Peebles P.J.E., 2001, *ApJ*, 557, 495
Patiri S.G., et al., 2006, *MNRAS*, 372, 1710
Peñarrubia J., McConnachie A.W., Navarro J.F., 2008, *ApJ*, 672, 904
Quillen A.C., Bland-Hawthorn J., 2008, submitted to *MNRAS*, [arXiv:0801.3469](http://arxiv.org/abs/0801.3469)
Schlegel D., Davis M., Summers F., Holtzman J.A., 1994, *ApJ*, 427, 527
Simon J.D., Geha M., 2007, *ApJ*, 670, 313
Springel V., 2005, *MNRAS*, 364, 1105
Springob C.M., Haynes M.P., Giovanelli R., Kent B.R., 2005, *ApJ Suppl.*, 160, 149
Strigari L.E., Bullock J.S., Kaplinghat M., Diemand J., Kuhlen M., Madau P., 2007, *ApJ*, 669, 676
Tikhonov A.V., Karachentsev I.D., 2006, *ApJ*, 653, 969
Tikhonov A.V., 2007, Astronomy Letters, Vol.33, No.8, p. 499
Tollerud, E. J., Bullock, J. S., Strigari, L. E., & Willman, B. 2008, [arXiv:0806.4381](http://arxiv.org/abs/0806.4381)
Tully, R. B., & Pierce, M. J. 2000, *ApJ*, 533, 744