Reconstruction and profile analysis of a used straight spur gear

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Abstract. The topic of this publication is the reconstruction and measuring process of a used straight spur gear. The tooth number, the tip circle diameter and the tooth length have to be measured for the reconstruction. Based on these parameters the type of the straight spur gear has to be determined. For the designing of the gear a new computer program is created. Based on the calculated parameters the CAD model of the gear could be designed. The profile of the real spur gear is measured by a microscope having computer controlled. The real and the theoretical involute profile curve will be analysed.

1. Introduction

The aim of the publication is the reconstruction and measuring process of a straight spur gear. In universal case a straight spur gear is given.

Figure 1. The parameters of the spur gear pair having normal straight teeth
The profiles are worn that is why the tooth thickness is changing and different for every tooth. The first step of the reconstruction is the determination of the standard module. The module is the basic parameter of a spur gear [2, 4, 5, 6, 7].

The main parameters of a connecting gear pairs could be seen on Figure 1 and Table 1.

### Table 1. Main parameters of the spur gear pair

| Notation  | Nomination                        | Notation  | Nomination                        |
|-----------|-----------------------------------|-----------|-----------------------------------|
| $d_1$, $d_2$ | Pitch circle diameter             | $h_f$     | Dedendum                          |
| $d_1$, $d_2$ | Root circle diameter              | $h_a$     | Addendum                          |
| $d_{n1}$, $d_{n2}$ | Tip circle diameter              | $h$       | Whole depth                       |
| $d_{b1}$, $d_{b2}$ | Base circle diameter             | $t_o$     | Circular pitch                    |
| $d_{w1}$, $d_{w2}$ | Working pitch circle diameter    | $S_{xa}$  | Tooth thickness                   |
| $c$       | Bottom clearance                  | $m$       | Modulee                           |
| $h_w$     | Working depth                     | $z$       | Number of teeth                   |
| $a$       | Centre distance                   | $j_s$     | Backlash                          |
| $d_{b1}$, $d_{b2}$ | Involute base circle diameter    | $\alpha_0$ | Base profile angle               |
| $t_g$     | Circular pitch along working pitch circle | $S_g$ | Tooth thickness along working pitch circle |

The number of teeth could be counted. The tip circle diameter ($d_a$) of the spur gear is not worn because it is not a connecting surface. This diameter could be measured by a vernier caliper. Knowing of this dimension the module could be calculated [2, 5]:

$$m_{calc} = \frac{d_a}{z + 2}$$

(1)

The $m_{calc}$ calculated module has to be rounded for the nearest and higher module ($m$) from the module standard [6].

![negative profile shift](image1.png) ![positive profile shift](image2.png)

**Figure 2.** Mating of the tool base profile with the tooth of the gear [5]

The addendum modification is given as a function of the module ($xm$), where $x$ is the profile shift coefficient. This ($x$) is positive if the base profile is shifted outward from the shaft, it is negative when the base profile is shifted towards the shaft (Figure 2) [2, 5].

The sum of profile shift coefficients ($\Sigma x$):

$$\Sigma x = x_1 + x_2 = \frac{z_1 + z_2}{2} \cdot \frac{inv \alpha_s - inv \alpha_0}{tg \alpha_0}$$

(2)

Where: $\alpha_0$ – base profile angle, $\alpha_s$ – drive connection angle, which could be selected by the function of the base profile angle from the standard.
The $x_1$ profile shift coefficient:

$$x_1 = \frac{d_{e1} - h_{f1}}{2m} - \frac{z_1}{2} + \frac{h_{f1}}{m} + e$$

(3)

The $x_2$ profile shift coefficient:

$$x_2 = \sum x - x_1$$

(4)

The backlash between the teeth could be calculated by the following suggested formula [5]:

$$j = \frac{t_0}{20}$$

(5)

2. Calculation of the main parameters of a spur gear

After the determination of the standard module all parameters of the spur gear could be calculated by the mathematical formulas [1, 2, 5, 6, 7].

![Figure 3. Creation of the mathematical model of the measured spur gear](image)

**Table 2.** The calculated parameters of the created spur gear

| Parameter            | Nomination | Value   |
|----------------------|------------|---------|
| Module               | m          | 1.75 mm |
| Number of teeth      | z          | 25      |
| Addendum             | $h_a$      | 1.75 mm |
| Clearance            | c          | 0.437 mm|
| Dedendum             | $h_f$      | 2.187 mm|
| Circular pitch       | $t_0$      | 5.497 mm|
| Backlash             | $j_s$      | 0.274 mm|
| Whole depth          | h          | 3.937 mm|
| Working depth        | $h_w$      | 3.5 mm  |
| Tooth thickness      | $S_{ax}$   | 2.611 mm|
| Pitch circle diameter| $d_0$      | 43.75 mm|
| Tip circle diameter  | $d_a$      | 47.25 mm|
| Root circle diameter | $d_f$      | 39.375 mm|
| Basic circle diameter| $d_b$      | 41.111 mm|
A computer software was worked out to calculate the necessary parameters of a connecting gear pairs [1]. The following parameters are given: the $m$ module, the $z_1, z_2$ number of teeth, $\alpha_0$ base profile angle and $\alpha_c$ angle of contact.

The program calculates the significant parameters of the drive pair (centre distance, typical circles etc.) and it draws the spur gears. The profile points of the wheel can be saved in .txt file format and it could be imported into 3 dimensional SolidWorks engineering design software. B – spline surface was fitted on the received tooth surface points [3].

The analysed spur gear having concrete geometry was created by this software (Figure 3, Table 2.). After the calculation the computer aided model (CAD) of the spur gear was designed by SolidWorks designer software (Figure 4).

![Figure 4. The CAD model of the spur gear](image)

### 3. Profile measurement of the used gear

The first problem that we have to solve is to transform the measured parameters to a CAD model. We have to measure with high precision. For that reason we started the measurements on a Mitutoyo Industrial tool which is a combination of a microscope and a profile projector. We have an X-Y moveable base for the measurement with 1µm accuracy.

Originally these microscopes not fitted for digital measurement, so we have to install an HD resolution camera for the measurement. The camera was placed to the original viewing slot as it presented on the figure below (Figure 5).

![Figure 5. Measuring device](image)
After we have the right measurement tool we have to create the method for rebuild the curvature of the spur gear. Our basic conception was that to create sequential images of the object with overlapping and then we can rebuild the image in really good quality and accuracy.

We have to fix the spur gear to the table we used graphite plastic for that. It’s important to avoid image processing problems because we have to handle 2070 pictures for one profile. To get the base point of the image creation process we have to define a base frame on the table. Currently we solved this with tapes. Basically we started the image creation process from the left upper corner of the imaginary matrix.

We have reset the spindles that move the tray as they are equipped with digital tripometers with µm accuracy. We determined the scale by 1 mm. This is sufficient distance to avoid being too large or too small overlap between the images to be made. We have saved the pictures numbered to make it easier and safer. Due to the size of the captured frame and the size of the scale, 45 lines and 46 columns were taken. So we thought it was expedient to use not only numbers but also have to apply letters when naming pictures. The lines are low case letters in the English alphabet, and columns are numbered. Later there are not only one letter, but two, so the line "z" follows the "z" line, enabling the computer to simply sort the images.

These conditions make the images simple, systematizing and later assisting in the compilation. Each photo has useful resolution of 640x480 pixels. Overlapping between images in pixels is 180x135, so the useful interface that I can use for each image is the 460x345 pixel. We cut this area out of each photo taken from the bottom right corner. After that, only the first and the last row or column had to be put together next to each other. The figure below represent a part of the assembly of the spur gear (Figure 6).

![Figure 6. Partial image of the assembly of the spur gear profile](image)

4. Reconstruction of used gear in CREO

As a result of the measurement we get the profiles of the spur gear from both side. From this images the 3D reconstruction of the spur gear was made in CREO 3D designing system which is available at our university by the support of BOSCH group. The result of the reconstruction is represents on the figure below. At the bottom image the mass of the spur gear can see in kg. The weight of the real part
was measured later and the differences between the reconstructed and real data was 0.002 kg (Figure 7).

5. Evaluation of the measurement
After the measurement and the reconstruction process the theoretical and the reconstructed 3D model of the spur gear were fitted (Figure 8).

Table 3. Evaluation of one involute profile difference

| Theoretical profile | Measured profile | Difference |
|---------------------|------------------|------------|
| x       | y       | z      | x       | y       | z      | x       | y       | z      |
| 20,556  | 0.000   | 0      | 20,556  | -0.010  | 0      | -0.010  |         |         |        |
| 20,558  | 0.000   | 0      | 20,558  | -0.020  | 0      | -0.020  |         |         |        |
| 20,574  | 0.001   | 0      | 20,574  | -0.019  | 0      | -0.020  |         |         |        |
| 20,588  | 0.001   | 0      | 20,588  | -0.029  | 0      | -0.030  |         |         |        |
| 20,598  | 0.002   | 0      | 20,598  | -0.013  | 0      | -0.015  |         |         |        |
| 20,600  | 0.002   | 0      | 20,600  | -0.018  | 0      | -0.020  |         |         |        |
| 20,643  | 0.005   | 0      | 20,643  | -0.015  | 0      | -0.020  |         |         |        |
| 20,662  | 0.007   | 0      | 20,662  | -0.008  | 0      | -0.015  |         |         |        |
| 20,713  | 0.013   | 0      | 20,713  | -0.002  | 0      | -0.015  |         |         |        |
| 20,745  | 0.017   | 0      | 20,745  | 0.012   | 0      | -0.005  |         |         |        |
| 20,824  | 0.029   | 0      | 20,824  | 0.014   | 0      | -0.015  |         |         |        |
| 20,865  | 0.036   | 0      | 20,865  | 0.016   | 0      | -0.020  |         |         |        |
| 20,987  | 0.060   | 0      | 20,987  | 0.030   | 0      | -0.030  |         |         |        |
| 21,038  | 0.071   | 0      | 21,038  | 0.056   | 0      | -0.015  |         |         |        |
| 21,159  | 0.099   | 0      | 21,159  | 0.079   | 0      | -0.020  |         |         |        |
| 21,229  | 0.117   | 0      | 21,229  | 0.087   | 0      | -0.030  |         |         |        |
| 21,368  | 0.156   | 0      | 21,368  | 0.136   | 0      | -0.020  |         |         |        |
| 21,437  | 0.176   | 0      | 21,437  | 0.146   | 0      | -0.030  |         |         |        |
| 21,607  | 0.231   | 0      | 21,607  | 0.216   | 0      | -0.015  |         |         |        |
| 21,697  | 0.262   | 0      | 21,697  | 0.237   | 0      | -0.025  |         |         |        |
| 21,859  | 0.321   | 0      | 21,859  | 0.316   | 0      | -0.005  |         |         |        |
| 21,944  | 0.353   | 0      | 21,944  | 0.338   | 0      | -0.015  |         |         |        |
| 22,120  | 0.425   | 0      | 22,120  | 0.410   | 0      | -0.015  |         |         |        |
| 22,212  | 0.464   | 0      | 22,212  | 0.434   | 0      | -0.030  |         |         |        |
| 22,418  | 0.557   | 0      | 22,418  | 0.532   | 0      | -0.025  |         |         |        |
| 22,533  | 0.612   | 0      | 22,533  | 0.582   | 0      | -0.030  |         |         |        |
| 22,737  | 0.713   | 0      | 22,737  | 0.698   | 0      | -0.015  |         |         |        |
| 22,860  | 0.777   | 0      | 22,860  | 0.772   | 0      | -0.005  |         |         |        |
| 23,094  | 0.906   | 0      | 23,094  | 0.881   | 0      | -0.025  |         |         |        |
| 23,206  | 0.970   | 0      | 23,206  | 0.960   | 0      | -0.010  |         |         |        |
| 23,414  | 1.094   | 0      | 23,414  | 1.064   | 0      | -0.030  |         |         |        |
| 23,511  | 1.154   | 0      | 23,511  | 1.139   | 0      | -0.015  |         |         |        |
The differences between the theoretical profile and the real profile could be determined from this fitting (Figure 8, Table 3). On Table 1 one concrete profile measurement could be seen. The differences between the real and the theoretical profile was determined. The prescribed profile tolerance is ±0,003 mm [6]. Based on this tolerance value the profile of the worn spur gear is not on the prescribed tolerance. That is why the analyzed spur gear is too worn. It could not be used further.

Figure 8. Fitting of the theoretical (grey) and the real (green) spur gear by SolidWorks software

6. Further development of measurement
The possible development of the measurement could be the multi-focus imaging, which means that we have to measure i.e. five different focus to get more complex result and more accurate image of the profile. To get the data from five layer we need to take 5x2070 images. If we want to be more accurate we have to increase this number! Because of that we have to build up an automated measuring system with motoric drive for tripometers. To solve this task and apply the image creation and processing task we have to create a complex measurement software. This could be the possible way to improve the measurement accuracy.

7. Conclusion
We had a used spur gear for the examination. Our purpose was to prepare the theoretical model of this gear and the determination of the differences between the real and the theoretical tooth profile. If we have a used spur gear the first step is to determine the standard module. Knowing of this the calculation of the main parameters and the computer aided model (CAD) could be generated by our developed computer program. Using of the profile microscope the profile shape of the worn tooth profile could be measured particularly. After the measurement the CAD model of the real spur gear could be generated. Finally we could evaluate the profile differences between the real and the theoretical profile. Based on our results we could decide the other utility of the given gear by the profile standard. The overall reconstruction and measuring process was introduced a spur gear having concrete geometry. Certainly this process is used and adapted for many type of gears.

8. Tooth contact analysis researches
We do tooth contact analysis (TCA) researches for different gears in the function of the geometric parameters and the material type. We analyse the contact zone between the teeth which is established by the effect of the load. Some results could be seen on Figure 9.
Figure 9. TCA analysis of a given geometric spur gear (m=5 mm, z₁=25, z₂=35)

9. References

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