First results from the Very Small Array – III. The cosmic microwave background power spectrum

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ABSTRACT

We present the power spectrum of the fluctuations in the cosmic microwave background detected by the Very Small Array (VSA) in its first season of observations in its compact configuration. We find clear detections of first and second acoustic peaks at $\ell \approx 200$ and $\approx 550$, plus detection of power on scales up to $\ell = 800$. The VSA power spectrum is in very good agreement with the results of the BOOMERanG, DASI and MAXIMA telescopes despite the differing potential systematic errors.

Key words: cosmic microwave background – cosmology: observations.

1 INTRODUCTION

Anisotropies in the cosmic microwave background (CMB) have now been detected by many experiments [most recently those of Netterfield et al. (2002), Lee et al. (2001), Padin et al. (2001) and Halverson et al. (2002)]. At present the most successful model for explaining the origin of these fluctuations postulates that they are seeded in the very early universe by quantum perturbations with random phase, followed by a period of inflationary expansion. The fluctuations in the CMB are predicted to be Gaussian in nature, and hence can be completely characterized through their power spectrum. A further prediction is that the power spectrum will show acoustic peaks arising from plasma oscillations on scales smaller than the sound horizon at the surface of last scattering.

In this paper we present the power spectrum of the CMB fluctuations detected by the Very Small Array (VSA) on spherical harmonic modes $\ell \approx 150–900$. We outline how the fully calibrated time-stream data are converted into a power spectrum, and the various data checks that we have performed to confirm the validity of our analysis. This paper is the third in a series of four papers which report the results of the first season of observations made using the VSA in its compact configuration. Paper I (Watson et al. 2003) describes the design of the VSA and our experimental method; the observational strategy, foreground removal and reduction techniques for the data analysed in this paper are described in Paper II (Taylor et al. 2003); finally, the cosmological implications of the VSA power spectrum are discussed by Rubiño-Martín et al. 2003 (Paper IV).

2 OBSERVATIONS AND INITIAL DATA PROCESSING

2.1 The observations

The VSA is a 14-element interferometer array for CMB observations; it operates at a frequency between 26 and 36 GHz with a receiver bandwidth of 1.5 GHz. In its compact configuration, used here, the instrument is most sensitive to angular structure in the range $\ell \approx 150–900$. As well as the 14-element array, there is also a single-baseline interferometer used for radio source flux measurements. A fuller description of the instrument is given in Paper I.
The present observations were made at a frequency of 34 GHz in the period 2000 September to 2001 September and were centred on three separate areas of sky. Each VSA observation maps a region with a FWHM of 4:6. The process of field selection is discussed in Paper II; selection was based primarily on low Galactic (synchrotron, free–free and dust) emission and an absence of known bright foreground sources. Overlapping fields were observed in each area in order to reduce the sample variance, to increase the resolution in ℓ-space and to allow for direct assessment of data reliability and the detection of any residual instrumental effects (Section 4.2). The array configuration used was designed to provide an approximately uniform spread of interferometer baselines while retaining a reasonable aperture filling factor.

The amplitude and phase calibrations of the individual interferometer baselines were checked both by frequent short measurements of a number of secondary calibration sources, and by regular longer observations of a small number of primary calibration sources. The overall calibration procedure is described fully in Paper I; the overall accuracy of the calibration in flux density and temperature is 3.5 per cent.

An important aspect of the VSA is the inclusion of a separate interferometer, comprising two 3.7-m dishes on a 9-m north–south baseline, for determining the flux densities of foreground sources (radio galaxies and quasars) falling within the observed fields. The positions of all sources that might affect the VSA observations were obtained from survey observations using the Ryle Telescope (Walram et al. 2003); these positions were subsequently observed concurrently with the main VSA observations in a series of regular pointed observations, and the measured flux densities subtracted from the visibilities observed by the main array (Paper II).

2.2 Initial data processing

The data were calibrated and processed as described in Papers I and II. Early tests of the telescope revealed an unwanted local spurious signal, particularly evident on the shorter baselines. The procedures for removing this signal from the data are described fully in Paper II; the tests presented in Section 4.2 below indicate that the removal of this signal is complete. Similar filtering processes were used to eliminate signals arising from the Sun and the Moon.

3 DERIVATION OF THE POWER SPECTRUM

Since the three sets of mosaicked fields are widely separated from one another on the sky, the data corresponding to each field were analysed individually. The derivation of the CMB power spectrum for each mosaic was performed using the maximum-likelihood method presented by Hobson & Maisinger (2002), which we summarize here. The results from each mosaic were then combined, as outlined below, to produce the final estimate of the power spectrum.

Since the number of individual visibility measurements for each mosaic is very large (∼800 000), it is first necessary to compress these data in some way. For each separate field in the mosaic, the visibilities are binned into cells in the uv-plane; for the case in which the instrumental noise covariance matrix is diagonal, this corresponds to the maximum-likelihood solution for the value of the binned signal visibility in each cell. This is analogous to the ‘map-making’ step in the analysis of single-dish CMB observations, in which time-ordered data are binned into pixels on the sky (see e.g. Borrill 1999). Since we are not interested here in making accurate CMB maps from the binned visibility data, the uv-plane is simply divided into equal-area cells of size Δu = 3 wavelengths.

As the aperture function of the compact VSA is well modelled by a Gaussian with a FWHM of 12 wavelengths, this cell size ensures that the uv-plane is comfortably oversampled, while reducing the total number of visibility measurements for each field significantly (to ∼2500).

The binned visibilities are thus the basic input to the likelihood analysis for the CMB power spectrum. The compact VSA is sensitive to the multipole range ℓ ~ 150–900, and the effective aperture function (after mosaicking) has a FWHM of Δℓ = 83. We thus divide the total ℓ-range into 10 spectral bands each of width Δℓ, in order to limit the correlation between adjacent bands. In each band, we assume ℓ(ℓ + 1)Cℓ to have a constant value ĈB (B = 1, 2, . . . , 10). These flat power levels in each band are then the parameters to be determined in our likelihood analysis of the data for each mosaic. We denote these band powers collectively by the parameter vector a.

Assuming the CMB emission and the instrumental noise to be Gaussian random fields, the log-likelihood of obtaining the binned visibility data vector v (which contains the real and imaginary parts separately of each binned visibility), given some set of flat band powers a, is given by

\[ \ln L = \text{constant} - \frac{1}{2} [\ln (\mathbf{C}(a)) + v^T \mathbf{C}^{-1}(a) v], \]

where \( \mathbf{C}(a) = \mathbf{S}(a) + \mathbf{N} \) is the sum of the predicted signal covariance matrix and the noise covariance matrix. The maximum-likelihood CMB power spectrum \( \hat{a} \) is calculated using a simple iterative numerical maximization algorithm. Starting from an initial guess \( a_0 \) (which is unimportant), independent line maximizations are performed for each band power \( \hat{C}_B \) in turn, while keeping the others fixed. The whole solution vector \( \hat{a} \) is then updated and the process repeated until convergence is obtained. This typically requires around five iterations.

The well-defined correlation structure of visibility data in the uv-plane allows each line maximization to be performed using only the subset of visibilities that are sensitive to the band power being varied, thereby speeding up the evaluation of the likelihood function, while keeping the calculation exact. For a single VSA mosaic, the maximum-likelihood solution can be obtained in around 1 h on an eight-node Beowulf Cluster with 1.8-GHz AMD Athlon processors.

The uncertainties in the derived maximum-likelihood CMB power spectrum are estimated in two complementary ways. Assuming the likelihood function in the parameter space \( a \) to be well approximated by a multivariate Gaussian near its peak, the covariance matrix of the parameter uncertainties is given simply by (minus) the inverse of the curvature (or Hessian) matrix at the peak \( \hat{a} \). This matrix is easily evaluated numerically in a few hours of CPU time. The square-root of the diagonal elements of the resulting covariance matrix gives the standard error \( \Delta \hat{C}_B \) on each band power, whereas the off-diagonal elements provide a measure of the correlation between the band power estimates in different spectral bins. We find, typically, that adjacent bins are anti-correlated at around the 5–20 per cent level, and for more widely separated bins the correlation is negligible.

Using the covariance matrix clearly produces symmetric error bars on each band power \( \hat{C}_B \), which may be a poor representation of the uncertainty, especially for poorly constrained band powers. An alternative approach is to make use of the fact that the band power estimates in different spectral bins are only weakly correlated, so that the off-diagonal elements of the covariance matrix are small compared with those on the diagonal. In this case, a better representation of the uncertainty in each estimate may be obtained by directly evaluating the likelihood function through the peak \( \hat{a} \),
along each parameter direction in turn. In the ideal case, where the band power estimates are independent, the resulting curves would be the marginal distributions of each band power. Since the likelihood function can be evaluated very quickly along each direction in parameter space, the resulting ‘marginal’ distributions can be calculated for each VSA mosaic in around 2 h, and provide a useful cross-check of the standard errors obtained from the covariance matrix at the peak. We find that for spectral bins in which the flat band power is tightly constrained, the shape of the likelihood function is very close to Gaussian. However, for bins in which the power level is less well constrained, the likelihood function is better described by an offset log-normal distribution; this is discussed further in Paper IV.

Once the likelihood functions for the flat band power \( C_B \) in each spectral bin have been obtained separately for each VSA mosaic, the mosaics are combined simply by multiplying together the respective likelihood functions in each bin. This assumes that each mosaic provides an independent measurement of the CMB power spectrum in each spectral bin, which is valid given that the three sets of mosaicked fields are widely separated on the sky. The correlation between the resulting band power estimates in different bins is easily obtained by calculating the covariance matrix, as described above, at the new joint optimum \( \tilde{a} \).

All the above functions are implemented using the MADCOW analysis package (Hobson & Maisinger 2002).

### 3.1 Window functions

Because of various instrumental effects (e.g. non-uniform \( uv \)-coverage and the finite size of the primary beam of the telescope), a bin samples the underlying power spectrum \( C_{\ell} \) through a window function \( W(\ell) \). For the \( B \)th bin, the measured power corresponds to

\[
p_B = \sum_{\ell} \frac{W_B(\ell)}{\ell} C_\ell,
\]

where \( C_\ell = (\ell + 1)C_{\ell}/(2\pi) \).

The window function for a given bin is determined as follows. The range of Fourier modes to which a given visibility is sensitive are given by the Fourier transform of the primary beam for a single field observation (see e.g. Hobson & Maisinger 2002). For a Gaussian primary beam,

\[
\bar{A}(u) = 2\pi\sigma^2 \exp(-2\pi^2 \sigma^2 |u|^2),
\]

where \( u \) is the vector in the visibility plane (measured in wave-lengths). For mosaicked observations, the effective beam is a superposition of displaced primary beams. We can think of this superposition as a convolution of a centred primary beam with a sum of delta functions at the beam centres. In the Fourier domain this corresponds to a multiplication of the aperture function with the ‘Fraunhofer diffraction pattern’ of the beam centres

\[
\bar{A}_{\text{eff}}(u) = \sum_j \bar{A}(u) e^{2\pi i u x_j},
\]

where \( x_j \) are the positions of the beam centres from the origin, measured in radians. Note that in general this aperture function is complex.

For a given bin, the weighted complex sensitivity map is

\[
S(u) = \sum w_k \bar{A}_{\text{eff}}(u - u_k),
\]

where index \( k \) runs over visibilities in a given bin. These visitibilities have instrumental weights \( w_k \) and positions on the visibility plane \( u_k \).

![Figure 1. Window functions for the combined data set. The functions are normalized to unit area, and different bins are plotted with different line styles to allow easier visual differentiation.](image)

The un-normalized window function is then given by

\[
W(\ell) = \mathcal{N} \int_{\phi=0}^{\phi=2\pi} |S(\ell, \phi)|^2 \, d\phi,
\]

where \( \ell = 2\pi|u| \). The normalization constant \( \mathcal{N} \) can be trivially found using \( \sum_{\ell} (2\ell + 1) W(\ell)/[2(\ell + 1)] = 1 \). The window functions for the combined VSA data set are plotted in Fig. 1, showing the small degree of correlation between adjacent bins. These functions are used in Paper IV in the estimation of cosmological parameters.

### 4 DATA CHECKS

The complete process of editing and filtering the data and also the subsequent stages of data reduction were carried out independently by the Cambridge group and, jointly, by the combined Instituto de Astrofísica de Canarias (IAC) and Jodrell Bank Observatory (JBO) teams. A comparison of the two sets of results showed good agreement, the effect of any differences being small compared with the intrinsic uncertainties on the final power spectra.

#### 4.1 Test of data reduction procedure

Aspects of our data reduction procedure, such as filtering and calibration, could potentially have introduced systematic errors into the VSA data. In order to test this, we produced a realization of the CMB sky and simulated a mock VSA observation including such instrumental effects as visibility quadrature errors, phase steps owing to path compensation and thermal noise. We analysed these simulated data using our standard reduction procedure and then produced a power spectrum as described in Section 3. We found this to be entirely consistent with our input model and so concluded that our method of data reduction did not introduce any significant systematic errors. In particular, it demonstrates that any correlations in the visibilities introduced by the filtering process have no observable effect on the derived power spectrum.

#### 4.2 The local spurious signal

The local spurious signal, fully discussed in Paper I, was found to depend only on the antenna tracking angle, and not on the table elevation. Therefore we would expect the spurious signal to be
identical for different fields with the same declination observed over the same hour angle range. This is easily confirmed by combining the (unfiltered) baseline time series of two such fields by both addition and subtraction. We find that adding the two fields enhances the spurious signal, whilst subtraction entirely removes it. Whilst this technique of addition and subtraction is adequate for detecting the presence of spurious signal in unfiltered data, it is insufficiently sensitive to detect possible low-level residual signal once filtering has been applied.

The increase in detection sensitivity that we require in order to test the filtered data for residual spurious signal can, however, be obtained using a modified version of a maximum entropy method (MEM) algorithm used for extraction of CMB signal from VSA data (Majinger, Hobson & Lasenby 1997).

We add an extra term to the MEM reconstruction which is the signal that is identical in the CMB data sets. We then consider the case of two fields at identical declination. As the CMB signals in the two fields will not be identical, and the noise is random in each case separately, any common component to the two fields will be spurious signal.

We tested this algorithm by applying it to pairs of simulated observations of CMB fields assuming cold dark matter (CDM) primordial fluctuations, to which we added an identical component with an rms level such that it was not the dominant signal. We used a variety of common components, including a scaled-down version of the unfiltered spurious signal. We found that the MEM algorithm was able to reconstruct the common signals well, recovering the structure excellently, and recovering the amplitude of the signal to within 10 per cent. As we are primarily interested in whether or not the spurious signal is still present, as opposed to any accurate quantification, this is perfectly adequate. Note that, even for entirely independent fields, the reconstructed shared signal power spectra are not zero (Fig. 2), but rather show a value increasing as $\ell^2$, consistent with the correlation between two white noise signals.

The two VSA2 fields had identical declination to allow testing of our filtering procedures. The MEM algorithm was applied to these fields, and the results compared with CDM realizations with identical $uv$-coverage and thermal noise in which no shared signal is present. The results (Fig. 2) show that the common signal found from the real data fields and the simulations agree within the errors, the difference between the two sets of points being less than 4 per cent of the measured CMB for values of $\ell < 750$. In the same manner, we find no evidence for residual spurious signal in the pairs of VSA1,3 fields at similar declinations.

Non-Gaussianity analysis of the binned visibilities allows us to locate and remove the few remaining visibilities contaminated with spurious signals down to a low level. The removal of these points has a negligible effect on the final power spectrum, giving us confidence that we are subtracting the spurious signal to a level well below that which could affect our results. The full details of this analysis will be published by Savage et al. (in preparation).

4.3 Foreground source subtraction

Radio galaxies and quasars are a significant contaminant of the CMB at microwave frequencies and in the higher $\ell$-ranges will dominate the CMB signal, making it essential to remove their contribution. Tests with simulated fields have been used to assess the potential contribution, before subtraction, of these sources to the final CMB spectrum. We generated 10 realizations of the CMB sky using a particular CDM model and added to these the sources that we have observed to be present in the VSA1 field. We then compared the power spectra recovered from simulated VSA observations with and without sources. Two of these simulations, representing the range of results obtained from the 10 simulations, are shown in Fig. 3. It is apparent that, with the fairly small number of sources in any one VSA field (typically 12 sources), the impact on the power spectrum is unpredictable. Although the main contribution tends to be in the higher $\ell$-bins (with errors potentially reaching $\pm 100$ per cent at $\ell = 900$), changes of up to $\pm 10$ per cent can occur in the lowest $\ell$-bin; these are due to the chance superposition of interference fringes.

To determine the effect of residual sources on the VSA results, two further simulations without a CMB contribution have been carried out. We based these simulations on the 15-GHz source counts from Taylor et al. (2001) and extrapolated up to 34 GHz using a mean spectral index of $\alpha = 0.55$. The first (Fig. 4, upper plot) comprises the contributions of the known point sources and a statistical distribution of weaker sources; the second (lower plot) includes only the statistical contribution of weaker sources and gives an indication of the possible residual contribution to the VSA power spectrum after subtraction of known sources. It is clear that, with no source subtraction, the CMB data are significantly compromised for $\ell$-values $\geq 600$. After subtraction of known sources, the contribution is reasonably small for $\ell$-values up to about 1000. As demonstrated by the results of Fig. 3, the contribution of sources to the observed spectra cannot generally be predicted from a simple combination of simulated power spectra, although such an approach becomes feasible in the limiting case of many weak sources per synthesized beam.

We can also estimate the residual source contribution to the power spectrum using the preliminary 34-GHz source count derived from the source-subtractor observations in Paper II. Integrating the count from zero flux to our complete source-subtraction limit of 80 mJy and converting to units of $\Delta T/T_0$, the source power is given by $C_{\ell} = 7.7 \times 10^{-16}$, corresponding to a power spectrum value of

\[
(\ell+1)C_{\ell} = \frac{580 \mu K^2}{2\pi}
\]

at $\ell = 800$. For the case with effectively no source subtraction, taking the upper flux limit to be 0.5 Jy, we find $C_{\ell} = 3.7 \times 10^{-15}$, equivalent

Figure 2. Power spectra of the recovered common signal from the two VSA2 fields (squares) and from a pair of simulated data sets with no common signal (i.e. independent noise only) (crosses). The pairs of points have been separated laterally for clarity. Coverage of the $uv$-plane and the thermal noise level are identical for both the real data and the simulation.
to 2800 \( \mu K \) at \( \ell = 1000 \); these results are in good agreement with the extrapolations from the 15-GHz counts.

The precise residual contribution arising from sources is subject to uncertainties in our knowledge of the true weak-source distribution; the values shown in Fig. 4 are likely to be overestimates since some sources with flux densities less than 80 mJy were actually subtracted. In order to assess the possible impact of these residual sources on derived parameters (Paper IV), the cosmic parameter analysis was repeated after subtracting the residual contributions shown in Fig. 4 from the observed power spectrum. The changes in the fitted parameters are small, the largest effect being a reduction of 0.05 in \( n_s \), the initial power-law spectrum of scalar perturbations, representing a change of about 0.5 s.d.

4.4 Galactic foregrounds

The diffuse Galactic foregrounds are discussed in Paper II, where it is suggested that the total contribution from the three components of the Galactic foreground (synchrotron, free–free and possibly spinning dust) amounts to no more than about \( \Delta T = 5 \mu K \) at an angular scale of 1° (\( \ell \approx 200 \)). Here we discuss their effect on the results presented in this paper. The free–free and synchrotron components are relatively well-known and are each expected to contribute about \( \Delta T = 1–2 \mu K \). The spinning dust component is more uncertain, but may be the dominant component, perhaps contributing up to \( \Delta T \approx 5 \mu K \). This is based on a dust-correlated component with a correlation coefficient of 10 \( \mu K \) (MJy sr\(^{-1}\)) at 100 \( \mu m \). This coefficient is still to be clearly demonstrated observationally. The power spectrum of the diffuse foregrounds falls with increasing \( \ell \) (for example, Giardino et al. 2001) and is likely to be half these values at \( \ell \approx 500 \).

Any Galactic contribution adds in quadrature with the CMB signal, and hence at the position of the first CMB peak (\( \ell \approx 200 \)) which has \( \Delta T_{\text{CMB}} \approx 75 \mu K \), 5 \( \mu K \) of foreground signal will increase the observed signal by 0.17 \( \mu K \). Similarly, at \( \ell \approx 500 \) where \( \Delta T_{\text{CMB}} \approx 45 \mu K \), the increase will be 0.28 \( \mu K \) at the most. We see therefore, for the VSA fields, and at a frequency of 34 GHz, that the contribution from Galactic foregrounds is likely to be negligible (<1 per cent). A similar conclusion, albeit based on observations of different regions of sky, was reached by Halverson et al. (2002) who found that Galactic emission made a negligible contribution to the observed CMB power spectrum. A more complete cross-correlation analysis to investigate the contribution from dust-correlated emission is in progress (Dickinson, Davies & Davis 2003).

4.5 Noise estimation

The likelihood analysis used for power spectrum estimation requires an accurate estimate of the rms noise level. Since on individual baselines the contribution of the CMB to the individual data samples is
very small, the noise level can be obtained directly from the standard deviation of the data, after the filtering and flagging processes have been completed. The noise level associated with each of the binned visibilities (Section 3) is obtained from an appropriately weighted combination of the noise levels of the data in each bin. As an additional check, the scatter in the data points contributing to each bin has also been used to provide a noise estimate. Consistent estimates of the noise level were obtained by the two methods. The overall noise estimate is accurate to 2.5 per cent.

The sensitivity of the likelihood analysis to errors in the noise level has been tested by analysing the same data set with different estimates of the noise level. The overall consistency of the power spectra derived from each of the three VSA mosaicked fields (VSA1M, VSA2M and VSA3M) was also compared by forming the \( \chi^2 \) statistic on pairs of power spectra. In this case the \( \chi^2 \) value is given by

\[
\chi^2 = (a_1 - a_2)^T \left( H_1^{-1} + H_2^{-1} \right)^{-1} (a_1 - a_2),
\]

where \( a_1 \) and \( a_2 \) are the two sets of bandpowers and \( H_1^{-1} \) and \( H_2^{-1} \) are the corresponding inverse Hessians. Since the \( \chi^2 \) statistic assumes that the likelihoods are Gaussian, we use the Hessians calculated using an offset log-normal approximation (see Paper IV for further details). The \( \chi^2 \) values (and significances) for the VSA1M/VSA2M, VSA1M/VSA3M and VSA2M/VSA3M power spectrum comparisons are 7.6 (0.67), 9.82 (0.46) and 5.03 (0.89) respectively. In each case there are 10 degrees of freedom in the power spectrum analysis.

5 THE POWER SPECTRUM

The filtered and source-subtracted data for each of the VSA fields have been analysed using the MADCOW software package (Hobson & Maisinger 2002) as described in Section 3. The combination of the three mosaicked spectra is shown in Fig. 5. The bin width used is \( \Delta \ell = 83 \), which gives weakly correlated errors in each bin. The actual correlations between bins are given by the correlation matrix (Table 2). To reduce the bias in assessing features in the power spectrum caused by the settings of the bin centres, we have also calculated the power spectrum with bin centres shifted by one-half a bin-width to the right of the original bin centres. These results are shown in Fig. 5 with dotted error bars. Adjacent ‘double-binned’ points are highly correlated but do sensibly sample the power spectrum of our data. Numerical values for both binnings are given in Table 3.

The plotted error bars contain the contributions from both thermal noise and sample variance, but not calibration errors, which introduce a completely correlated uncertainty in all the points of ±7 per cent. Errors from pointing and primary beam uncertainties are negligible. Since the temperature sensitivity of the VSA compact configuration falls off dramatically after \( \ell = 800 \), all data above this have been binned together.

Table 1. The \( \chi^2 \) values for data splits on each of the VSA fields.

| Field     | d.o.f. | \( \chi^2 \) | Significance |
|-----------|--------|--------------|--------------|
| VSA1      | 977    | 1033.7       | 0.10         |
| VSA1A     | 1884   | 1947.7       | 0.15         |
| VSA1B     | 1387   | 1410.3       | 0.33         |
| VSA2      | 915    | 984.2        | 0.06         |
| VSA2-OFF  | 1384   | 1420.2       | 0.24         |
| VSA3      | 1287   | 1356.2       | 0.09         |
| VSA3A     | 2003   | 2094.2       | 0.08         |
| VSA3B     | 1584   | 1660.1       | 0.09         |

Figure 5. Combined CMB power spectrum from the three mosaicked VSA fields. The error bars represent 1\( \sigma \) limits; the two sets of data points correspond to alternate interleaved binnings of the data.


6 DISCUSSION

6.1 The VSA power spectrum

The power spectrum shown in Fig. 5 shows a clear detection of the first peak at \( \ell \simeq 220 \), and power at the level of about 2000 \( \mu K^2 \) between \( \ell = 300 \) and 900. We have attempted to quantify the detection of a second peak at \( \ell \simeq 550 \), as this is the region of the power spectrum with the largest anti-correlations between adjacent bins (see Table 2); bin 6 centred at \( \ell = 556 \) is anti-correlated with its neighbours at the \( \simeq 20 \) per cent level. We made Monte Carlo simulations of the five \( C_\ell \) points between \( \ell = 390 \) and 722. Points were drawn from the five-dimensional Gaussian distribution described by the actual correlation matrix of these points, but with mean values equal to the weighted mean of the five actual points. Following Hobson & Magueijo (1996), who define a normalized convexity about a power spectrum point, we calculated the change in normalized convexity of the power spectrum about the inner three points, defined by

\[
\Sigma = \frac{C_i}{\sigma},
\]

where \( C = [C_4 + C_6 - 3(C_5 + C_7)]/2 + 2C_6 \) and \( \sigma \) is the overall error in \( C \) given from the errors in the individual points \( \sigma_i \) by \( \sigma^2 = [\sigma_4^2 + \sigma_5^2 + 9(\sigma_5^2 + \sigma_7^2)]/4 + 4\sigma_6^2 \). This effectively compares the hypothesis that the power spectrum in this region is flat with the one that it is described by ‘trough–peak–trough’.

In 1000 realizations, we found only 27 instances of \( \Sigma \) being larger than the value observed in the real data of \( \Sigma = 2.2 \). We therefore conclude that the observed second peak is detected at 97 per cent confidence. The power spectrum is completely consistent with the adiabatic inflationary models, fits to which are discussed in Paper IV.

6.2 Comparison with other experiments

In Fig. 6 we compare the new VSA power spectrum plotted with those from BOOMERanG (Netterfield et al. 2002), DASI (Halverson et al. 2002) and MAXIMA (Lee et al. 2001). Only single-binned (weakly correlated) points are shown. We have attempted to compare the random and correlated errors on the various experiments in a consistent way, difficult though this is on a single plot. Two sets of error bars are shown for each plot; the smaller bars indicate 68 per cent confidence limits from the random (thermal and sample variance) errors, while the larger error bars represent systematic (calibration and beam) errors as reported – all the points from a single experiment are able to move up or down within the larger error bars. It is significant that, although the overall errors of the different experiments are comparable, the relative contribution of systematic errors (including beam uncertainty) is much smaller for the interferometric data.

The agreement between the experiments on the existence, heights and positions of two peaks and of power at higher \( \ell \) is evident. This is particularly significant given the very different experimental techniques involved and the different foregrounds and systematic errors faced by the different experiments. The points in Fig. 6 have been obtained over a frequency range of 26–150 GHz, and by ground-based interferometers and balloon-borne scanned total-power telescopes. They are all from different regions of the sky, the calibrations are all independent and based on different absolute calibration sources, and for the two low-frequency experiments foreground sources have been subtracted in different ways, yet the agreement of the power spectra is striking.

A detailed comparison between the experiments is difficult to do from the data points alone because of the correlated errors between points for each experiment. To make a meaningful comparison it is necessary to fit the underlying power spectrum to each data set, taking into account the correlations, and to compare the parameters that describe the power spectrum. In principle many parametrizations of the power spectrum would suffice for this comparison, but in practice it is obviously sensible to use the standard

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Table 2. The correlation matrix \( C_{ij} \) for the combined VSA data set (main binning only). These are calculated by normalizing the covariance matrix of errors. Note that the correlation is only significant for adjacent bins. The values of the matrix for which \( C_{ij} \) is not reported can be assumed to be zero. The final column gives the diagonal elements of the covariance matrix in units of \( 10^5 \times \mu K^2 \).

| \( B \) | \( C_{B,B-2} \) | \( C_{B,B-1} \) | \( C_{B,B} \) | \( C_{B,B+1} \) | \( C_{B,B+2} \) | \( \text{Cov}_{B,B} \) |
|---|---|---|---|---|---|---|
| 1 | 1.00 | -0.06 | 0.00 | 18.78 |
| 2 | -0.06 | 1.00 | -0.13 | 0.00 | 15.53 |
| 3 | 0.00 | -0.13 | 1.00 | -0.07 | 0.02 | 4.86 |
| 4 | 0.00 | -0.07 | 1.00 | -0.18 | 0.06 | 1.98 |
| 5 | 0.02 | -0.18 | 1.00 | -0.18 | 0.01 | 2.46 |
| 6 | 0.06 | -0.18 | 1.00 | -0.15 | 0.05 | 6.19 |
| 7 | 0.01 | -0.15 | 1.00 | -0.22 | 0.02 | 5.96 |
| 8 | 0.05 | -0.22 | 1.00 | -0.16 | 0.03 | 10.06 |
| 9 | 0.02 | -0.16 | 1.00 | -0.21 | 0.12 | 12.41 |
| 10 | 0.03 | -0.21 | 1.00 | 35.88 |

Table 3. The power spectrum from combining the three VSA fields. The two sets of bin numbers (1, 1A etc.) refer to the main and alternate binning, the latter being shifted by half a bin-width. All the bins have \( \Delta \ell = 83 \). The reported error bars correspond to 68 per cent confidence limits and were calculated by enclosing 68 per cent area under a likelihood curve assuming independent bins. For further data analysis it is necessary to use full window functions and covariance matrices which can be downloaded from our website (http://www.mrao.cam.ac.uk/telescopes/vsa/results.html). In addition to these errors, there is an overall 7 per cent calibration uncertainty in power.

| \( B \) | \( \ell \) | \( \ell (\ell + 1)C_\ell /2\pi (\mu K^2) \) |
|---|---|---|
| 1 | 142 | 3953^{+1709}_{-1248} |
| 1A | 184 | 5246^{+1493}_{-1211} |
| 2 | 224 | 6200^{+1382}_{-1122} |
| 2A | 266 | 6494^{+1233}_{-1040} |
| 3 | 307 | 3496^{+713}_{-661} |
| 3A | 349 | 2080^{+460}_{-416} |
| 4 | 390 | 2124^{+416}_{-416} |
| 4A | 432 | 1954^{+497}_{-371} |
| 5 | 473 | 1498^{+497}_{-360} |
| 5A | 515 | 2452^{+624}_{-335} |
| 6 | 556 | 3246^{+278}_{-705} |
| 6A | 598 | 1998^{+750}_{-705} |
| 7 | 639 | 1207^{+795}_{-662} |
| 7A | 681 | 2162^{+876}_{-787} |
| 8 | 722 | 203^{+1003}_{-689} |
| 8A | 764 | 666^{+917}_{-665} |
| 9 | 806 | 499^{+1292}_{-499} |
| 9A | 847 | 1954^{+1664}_{-1413} |
| 10 | 888 | 1914^{+1873}_{-1543} |
| 10A | 930 | 541^{+2832}_{-541} |
adiabatic CDM power spectrum models, and fits to these models for the VSA and other experiments are considered in detail in Paper IV.

7 CONCLUSIONS

We have derived the power spectrum of the CMB anisotropies from the first year of VSA observations, made using its compact array configuration. We measure the flat band power in 10 weakly correlated bins of width $\Delta \ell = 83$ between $\ell \approx 150$ and $\ell \approx 900$. The results are subject to a calibration uncertainty of $\pm 7$ per cent in power, with negligible beam uncertainty. The contribution to the power spectrum from diffuse Galactic emission and residual radio sources is also negligible. Our results are in excellent agreement with other recent measurements as regards the amplitude and position of two peaks in the power spectrum; power is also detected out to the resolution limit of the experiment at $\ell \approx 900$.

Band powers, correlation matrices and window functions are available from http://www.mrao.cam.ac.uk/telescopes/vsa/results.html.

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