Experimental observation of the bifurcation dynamics of an intrinsic localized mode in a driven 1-D nonlinear lattice

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Linear response spectra of a driven intrinsic localized mode in a micromechanical array are measured as it approaches two fundamentally different kinds of bifurcation points. A linear phase mode associated with this autoresonant state softens in frequency and its amplitude grows as the upper frequency bifurcation point is approached, similar to the soft mode kinetic transition for a single driven Duffing resonator. A lower frequency bifurcation point occurs when the four-wave-mixing partner of this same phase mode intercepts the top of the extended wave branch, initiating a second kinetic transition process.

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A general property of a driven nonlinear oscillator is that given sufficient starting amplitude it will stay in resonance as the frequency is changed adiabatically. In this autoresonant (AR) state, where feedback is not required, the oscillator phase is locked to the driver. A variety of applications have been described in Ref.[1, 2]. Recent theoretical work has focused on controlling solitons,[3] and excitations in discrete and continuous nonlinear Schrödinger equations[4] while experimental studies have occurred for micromechanical arrays,[5] optical guided waves,[6] and superconducting Josephson resonators.[7] It has been predicted theoretically for a periodically driven single nonlinear oscillator and examined with analog electrical experiments near the location of its bifurcation region that critical phenomena arise in the density of fluctuations,[8, 9], and the bifurcation transition is characterized by slow dynamics,[10, 11] associated with a soft mode[12], which goes to zero frequency at the transition.

Micro and nano-electrical-mechanical systems (MEMS and NEMS) provide a platform with which to study the intrinsic dynamical localization of vibrations in driven nonlinear lattices. Both experimental[5, 13] and theoretical[5, 14, 19] studies have appeared that showcase the properties of such intrinsic localized modes (ILMs), the fundamental strongly localized excitation that appears when both lattice discreteness and nonlinearity are important.[20–23]. A feature overlooked until recently is the bifurcation properties of this autoresonant (AR) state and the concomitant back reaction of the driven ILM on the dynamics of the lattice. For a 1-D monatomic lattice with hard quartic anharmonicity it has been predicted that linear local modes (LLMs) would appear nearby an ILM and that four-wave mixing between the ILM and the LLM would give rise to additional spectral features.[24] Missing are any experiments or discussion of the existence of a linear ILM soft phase mode or the influence of LLMs as the driven ILM approaches a bifurcation frequency.

In this work the first experimental observations of the dynamical properties of two separate bifurcation transitions from the AR-ILM state of a driven 1-D micromechanical array are presented and analyzed. Linear response measurements are used to probe the small signal dynamics of AR state. Our measurements and associated simulations show that the high frequency bifurcation transition is very similar to that found for a single driven Duffing-like oscillator in that the associated linear soft phase mode goes to zero frequency at this transition.[25]. The low frequency transition, on the other hand, is characterized by a nonlinear interaction between the same soft mode and the highest frequency plane wave mode of the lattice.

Figure 1 shows the experimental setup both for the AR amplitude measurement and for the associated linear response measurement. The driven micromechanical array contains 152 Si$_3$N$_4$ cantilevers coupled together by a common overhang. A cw pump feeds energy to the array maintaining the ILM in the large amplitude AR state. For linear response measurements an additional weak probe driver is used to perturb the array. The output of the probe is combined with the strong pump and connected to the PZT so the perturbation is applied uniformly across the lattice. First, the driver frequency is chirped up to generate the ILM at an arbitrary lattice site. Its position is monitored by a combination of a line focused laser beam and a 1-D CCD camera (not shown). The probe laser beam is then adjusted to the next short cantilever of the ILM to operate in the small signal regime. The motion of this cantilever is monitored by a position sensitive detector(PSD) with a large deflection range. A lock-in amplifier is used to selectively analyze the cantilever motion that is caused by the probe oscillating at a given frequency. The response spectrum is measured by scanning the probe frequency, while the
pump frequency is held fixed. By changing the pump frequency in a stepwise fashion, the linear mode properties can be monitored as a kinetic transition is approached. Once the driver frequency passes one of the bifurcation points, it takes many tries until an ILM is generated at the same lattice site for additional measurements with different pump frequencies. This uniform drive method should only couple to odd symmetry modes; however, heating from the probe laser produces an asymmetric perturbation near the ILM so that an even LLM of the AR region is 140.46 kHz to 144.85 kHz, or 0.148 to 1.57 by the normalized difference frequency. (b) Simulated nonlinear response of the AR state in the hard nonlinear lattice with driver appropriate to the experimental level. Top trace: AR region, identifying its two transitions. Middle trace: F= fast frequency rate required to reach the AR state. Bottom two traces: S= slow up and down scanning, no AR state occurs. The stable frequency region is 137.56 to 146.54 kHz, or 0.102 to 2.29 by the normalized difference frequency. The top of the band frequency is 137.14 kHz and the band width is 4.1 kHz. Curves are shifted up or down for clarity.

These experimental results are compared to simulations incorporating previously applied lumped element lattice model equations
\[ m_i \ddot{x}_i + m_i \dot{x}_i / \tau + k_{2O} x_i + \sum_j k_{2I} (2x_i - x_{i+j} - x_{i-j}) \\
+ k_{4O} x_i^4 + k_{4I} \left\{ (x_i - x_{i+1})^3 + (x_i - x_{i-1})^3 \right\} \\
= m_i \alpha_{pump} \cos \Omega t + m_i \alpha_{probe} \cos \omega t \]
with an obvious notation. Specific array parameters are described in the Supplemental Material[26]. The top curve in Fig. 2(b) is for a driver with amplitude similar to the experimental level. Once the AR-ILM state is generated, its amplitude changes smoothly with a slow variation in the driver frequency. Both AR transitions are evident although the lower frequency one is not as marked as observed experimentally in Fig. 2(a). The next lower trace in Fig. 2(b) shows the result of a relatively rapid increase in the driver frequency necessary to
produce the AR state. The bottom two traces illustrate the small amplitudes that appear for slow up and down frequency scanning where no AR state is produced.

The experimentally measured linear response spectra for the AR-ILM state at different pump frequencies are presented in Fig. 3. As will be demonstrated via simulations below, the two strong sidebands shown here are due to the soft phase mode within the AR state. (For clarity the actual pumped ILM amplitude is deleted in this figure.) The probe spectra are displayed with the pump varying from 140.5 kHz to 144.8 kHz in 100 Hz intervals from bottom to top. (This range corresponds to 0.161-1.55 in terms of the difference frequency normalized by the band width.) The probe spectra are magnified 20 times.

The dynamics behind the lower bifurcation transition is quite different from the upper one. The experimentally measured response spectra for these different excitations is shown in Fig. 5. The sine component of the probe response corresponds to the entire stable pump frequency region (0.112-2.26) for the AR-ILM state is shown. As the upper bifurcation point is approached, the phase mode frequency softens and its response diverges. The associated LLM (off the horizontal scale) has no such dramatic behavior. As the lower bifurcation point is approached the four wave mixing partner of the soft phase mode approaches the top of this extended wave branch. The optic branch modes and their four wave mixing partners appear as weak low frequency satellite features in the figure. The lower transition occurs when these two intersect, presenting a completely different dynamical signature from that of the upper transition. The detailed properties of the modes in Fig. 4 are identified by checking their shapes numerically. We find that the odd symmetry of the soft phase mode is the same as that of the driven ILM. The other small peaks in Fig. 4 are odd symmetry band modes.

The dynamics behind the lower bifurcation transition is quite different from the upper one. The experimentally measured response spectra for these different excitations is shown in Fig. 5. The sine component of the probe response spectra is displayed as the lower bifurcation transition is approached. The large positive and negative peaks are
FIG. 5. Sine component of the experimental probe response near by the lower bifurcation point. Spectra are ordered by the pump frequency from 0.355 to 0.161 in 0.016 step. The large peak at the center is the pump signal. The lower bifurcation takes place at 0.148. The large positive amplitude identifies the soft phase mode oscillation of the ILM, the corresponding negative peak is its four wave mixing partner. The even LLM, activated by asymmetric heating of the ILM, appears as an amplitude modulation response. As the driver frequency is decreased the highest frequency band mode of the array, identified with an arrow, approaches the partner of the soft phase mode. (Lower frequency band modes appear as a sequence of positive and negative peaks.) Near the transition the even LLM frequency crosses the soft phase mode partner, dashed curve, second from the bottom. The transition takes place when the soft mode partner intersects the lower frequency odd symmetry band mode.

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[26] Parameters are in 2nd column of Table II in Ref. [2]. See Supplemental Material at [URL will be inserted by publisher] for details.