Estimation of the mass outflow rates from viscous accretion discs

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ABSTRACT
We study the viscous accretion disc around black holes, and all possible accretion solutions, including shocked as well as shock-free accretion branches. Shock-driven bipolar outflows from a viscous accretion disc around a black hole have been investigated. One can identify two critical viscosity parameters, $\alpha_{cl}$ and $\alpha_{cu}$, within which the stationary shocks may occur, for each set of boundary conditions. Adiabatic shock has been found for up to viscosity parameter $\alpha = 0.3$, while in the presence of dissipation and mass loss we have found stationary shock up to $\alpha = 0.15$. The mass outflow rate may increase or decrease with the change in disc parameters, and is usually around a few to 10 per cent of the mass inflow rate. We show that for the same outer boundary condition, the shock front decreases to a smaller distance with the increase in $\alpha$. We also show that the increase in dissipation reduces the thermal driving in the post-shock disc, and hence the mass outflow rate decreases up to a few per cent.

Key words: accretion, accretion discs – black hole physics – hydrodynamics – ISM: jets and outflows.

1 INTRODUCTION
One of the curious things about the galactic as well as extragalactic black hole (BH) candidates is that they show moderate to strong jets. Since BHs are compact and do not have hard surfaces, outflows and jets can originate only from the accreting material. Moreover, observations have indeed shown that the persistent jet or outflow activities are correlated with the hard spectral states of the disc, i.e. when the power radiated maximizes in the hard non-thermal part of the spectrum (Gallo, Fender & Pooley 2003). Moreover, there seems to be enough evidence that these jets are produced within 100 Schwarzschild radius of the central object (junor, Biretta & Livio 1999). So accretion disc models that are to be considered for persistent jet generation should allow the formation of jet close to the central object, and that the spectra of the disc should be in the hard state.

One of the most widely used accretion disc models, the Keplerian disc (Novikov & Thorne 1973; Shakura & Sunyaev 1973) is very successful in explaining the multi-coloured blackbody part of the spectrum. However, the presence of the Keplerian disc alone is inadequate in explaining the existence of the non-thermal tail of the spectrum. Moreover, the advection term in the equation of motion is poorly handled in this model, where the inner disc was chopped off ad hoc, and cannot automatically explain jet generation mechanism, neither can it explain the relatively small size of the jet base. Hence, one has to consider accretion models that have significant advection. An elegant summary of all types of viscous accretion disc model has been provided by Lee, Ryu & Chattopadhyay (2011). For the sake of completeness, we present a very brief account of advective discs in the following.

Accretion on to BH is necessarily transonic since radial velocity can only be small far away from the BH, but must cross the horizon with velocity equal to the speed of light (c). Hence, accretion solutions around BHs should consider the advection term self-consistently. Accretion solution, like the Bondi flow or, radial flow (Bondi 1952; Chattopadhyay & Ryu 2009), although satisfies the transonicity criteria, but it is of very low luminosity. Therefore, there was a need to consider rotating flow which are transonic too, such that the infall time-scale would be long enough to generate the photons observed from microquasars and active galactic nuclei (AGNs). The accretion model with advection which got wide attention was Advection Dominated Accretion Flows (ADAF) (Ichimaru 1977; Narayan & Yi 1994). Initially, ADAF was constructed around a Newtonian gravitational potential, where the viscously dissipated energy is advected along with the mass, and the momentum of the flow. The original ADAF solution was self-similar and wholly subsonic, thus violating the inner boundary condition around a BH. This inadequacy was rectified when global solutions of ADAF in the presence of strong gravity showed that the flow actually becomes transonic at around a few Schwarzschild radii ($r_{s}$), while remains subsonic elsewhere. The self-similarity of such a solution may be maintained far away from the sonic point (Chen, Abramowicz & Lasota 1997). Moreover, the Bernoulli parameter is positive, making the solutions unbound. This prompted the introduction of self-similar outflows in the ADAF abbreviated as ADIOS, which would carry away mass, momentum and energy (Blandford & Begelman 1999). Although this is an interesting addition to the generalization...
of ADAF-type solutions, no physical mechanism was identified except the positivity of the Bernoulli parameter of the accretion flow that would drive these outflows.

Simultaneous to the research on ADAF class of solutions, a lot of progress has been made in the research of general advective, rotating solutions. For rotating advective flow, Liang & Thompson (1980) showed that with the increase in angular momentum of the inviscid flow, the number of physical critical points increases from one to two, and later it was shown that a standing shock can exist in between the two critical points (Fukue 1987; Chakrabarti 1989; Fukumura & Kazanas 2007; Chattopadhyay 2008; Chattopadhyay & Chakrabarti 2011). Since the accretion shock is centrifugal pressure mediated, there was apprehension about the stability and formation of such shocks in the presence of processes such as viscosity, which transports angular momentum outwards. All doubts about the stability of such shocks in the presence of viscosity was subsequently removed (Lanzafame, Molteni & Chakrabarti 1998; Chakrabarti 1996; Chakrabarti & Das 2004; Chattopadhyay & Das 2007; Das & Chattopadhyay 2008, Lee et al. 2011). Furthermore, Fukumura & Tsuruta (2004) conjectured about the existence of multiple shocks and, later, independent numerical simulations serendipitously found the existence of transient multiple shocks in the presence of Shakura–Sunyaev type viscosity (Lanzafame et al. 2008; Lee et al. 2011). Although, general advective and ADAF solutions both start with the same set of equations, it was intriguing that there can be two mutually exclusive class of solutions, and without any knowledge under which condition these solutions may arise. Lu, Gu & Yuan (1999) later showed that the global ADAF solution is a subset of general advective solutions. In other words, the models that concentrate only on the regime where the gravitational energy is converted mainly to the thermal and rotational energy may either be cooling dominated (e.g. Keplerian disc; Shakura & Sunyaev 1973; Novikov & Thorne 1973) or advection dominated (Narayan, Kato & Honma 1997). Either way, these models remain mainly subsonic (except very close to the BH) and hence do not show shock transition. However, if the entire parameter space is searched, one can retrieve solutions that have enough kinetic energy to become transonic at distances of a few \( 100 \, r_g \). A subset amongst these solutions admit shock transitions when shock conditions are satisfied. This is the physical reason why some disc models show shock transitions and others do not (Lu et al. 1999; Das, Becker & Le 2009). Whether a flow will follow an ADAF solution or some kind of hybrid solution with or without shock will depend on the outer boundary condition and the physical processes dominant in the disc.

The shock model was later used to explain observations. Chakrabarti & Titarchuk (1995), Chakrabarti & Mandal (2006) and Mandal & Chakrabarti (2008, 2010) showed that the post-shock region being hotter can produce the hard power-law tail by inverse-Comptonizing soft photons from pre-shock and post-shock parts of the accretion disc. The soft state and hard states are automatically explained depending on the existence or non-existence of the shock.

The presence of such a transonic advective flow has also been suggested by observations (Smith et al. 2001; Smith, Heindl & Swank 2002). In fact, in the ‘hardness–intensity diagram’, or HID, a hysteresis like behaviour seen in microquasars has been reproduced by this model (Mandal & Chakrabarti 2010). In other words, the post-shock region is the elusive Compton cloud.

Interestingly enough, the shock fronts are stable in a limited region of energy-angular momentum parameter space and naturally give rise to time-dependent solutions whenever the exact momentum balance across the shock front is not achieved. This might be due to different rates of cooling (Molteni, Sponholz & Chakrabarti 1996b), or different rates of viscous transport (Lanzafame et al. 1998; Lee et al. 2011). These oscillations were also confirmed by pure general relativistic simulations (Aoki et al. 2004; Nakamura & Yamada 2008, 2009). Molteni et al. (1996b) suggested that if the post-shock region oscillates, then the hard radiation produced by the post-shock region would oscillate as well, and hence explain the Quasi Periodic Oscillation (QPO). In fact, the evolution of QPO frequencies during the outburst states of various micro-quasars like XTEJ1550−654, GRO 1655−40, etc. was explained by this model (Chakrabarti, Dutta & Pal 2009) by assuming inward drift of the oscillating shock due to increased viscosity of the flow.

Another interesting consequence of accretion shock is that it automatically explains the formation of outflows. The unbalanced pressure gradient force in the axial direction drives matter in the form of outflows and may be considered as the precursor of jets (Molteni, Lanzafame & Chakrabarti 1994; Chakrabarti 1999; Chattopadhyay & Das 2007; Das & Chattopadhyay 2008). Various accelerating mechanisms can accelerate these outflows to relativistic terminal speeds (Chattopadhyay, Das & Chakrabarti 2004; Chattopadhyay 2005). If the shock conditions are properly considered, the mass outflow should reduce the pressure of the post-shock region and hence the shock would move inwards, which in turn would modify the shock parameter space in the presence of mass loss (Singh & Chakrabarti 2012). Another model of non-fluid bipolar outflows has been enthusiastically pursued by Becker and his collaborators, which involves Fermi acceleration of particles in isothermal shocks (Le & Becker 2005; Becker, Das & Le 2008; Das et al. 2009). The advantage of shock in accretion model is that the presence or absence of shock is good enough to broadly explain many varied aspects of BH candidates such as spectral states, QPOs, jets and outflows, etc. and the correlation between these aspects (Chakrabarti et al. 2009). Such correlations were also reported in observations (Smith et al. 2001, 2002; Gallo et al. 2003).

There are other jet generation models too. Apart from ADIOS, magnetically driven outflows were also proposed by many authors (Blandford & Znajek 1977; Blandford & Payne 1982; Proga 2005; Hawley, Beckwith & Krolik 2007). These magnetic bipolar outflows may be powered by the extraction of rotation energy of the BH in the form of the Poynting flux. However, recent observations find weak or no correlation of BH spin with the jet formation around microquasars and are conjectured to be similar for AGNs as well (Fender et al. 2010). Bipolar outflows may even be powered by centrifugal or magnetic effect, and has shown that for low angular momentum even weak magnetic fields can produce equatorial inflow, bipolar outflow, polar funnel inflow and polar funnel outflow, and the magnetic effect was identified as the main driver of such outflows. Interesting as it may be, however, its connection with spectral and radiative state of BH candidates is not well explored. It is well known that the spectra of the BH candidates extend to high energy domain (\( \sim 10^0 \) MeVs), and one of the ways to generate such high energy non-thermal spectra is by shock acceleration of electrons, and has been used to explain spectra from BH candidates (Chakrabarti & Mandal 2006; Mandal & Chakrabarti 2008). Not only hydrodynamic calculations, even magnetohydrodynamic investigations bear the possibility of shock in accretion (Nishikawa et al. 2005; Takahashi et al. 2006). Therefore we are looking for solutions of outflow generation which incorporates shocks, but only investigating the effect of viscosity in formation of such outflows in the present paper.

Various simulations with the Paczynski–Wiita potential (Molteni et al. 1994, 1996b, Molteni, Ryu & Chakrabarti 1996a, Lanzafame et al. 1998, 2008), as well as GRMHD simulations (Nishikawa et al. 2005) showed the presence of post-shock bipolar...
outflows, which are far from isothermal approximation. Therefore, presently we relax the strict isothermality condition, and concentrate on conservation of fluxes across the shock front. Theoretical framework of thermally driven outflows has been done either for inviscid discs (Chakrabarti 1999; Das, Chattopadhyay & Chakrabarti 2001; Singh & Chakrabarti 2012), or for viscous discs where the viscous stress was assumed to be proportional to the total pressure (Chattopadhyay & Das 2007; Das & Chattopadhyay 2008). No theoretical investigations have thus far been made to study the thermally driven outflows from discs where the viscous stress is proportional to the shear, a form of viscosity which is probably more realistic for the BH system (Becker & Subramaniam 2005). In this paper we solve viscous accretion disc equations for the contentious viscosity prescription to compute the solution topologies of thermally driven outflows and the mass–outflow rates. We compute the energy-angular-momentum parameter space for the accretion flows which will produce such outflows, and its dependence on viscosity parameter. In numerical simulation with the Shakura–Sunyaev viscosity prescription, shock location seems to increase with the increase in the viscosity parameter (Lanza et al. 1998; Lee et al. 2011), while theoretical investigations with the Chakrabarti–Molteni viscosity prescription showed the opposite phenomenon (Chattopadhyay & Das 2007; Das & Chattopadhyay 2008). We address this issue and attempt to remove any ambiguity. The post-shock disc could well be the elusive Compton cloud, while the Shakura-Sunyaev viscosity might be more physical since it satisfies proper inner boundary conditions around black holes (Becker & Le 2003). Therefore, investigations of shock driven outflows and its dependence on viscosity would throw light in understanding the radio X-ray correlation from X-ray binaries, especially, whether the jet becomes stronger or weaker with the increasing viscosity parameter; this would have an interesting connotation in interpreting observations. Since the post-shock disc can produce high-energy photons, some part of the thermal energy gained through shocks will be dissipated as radiations. We study the issue of the origin of such outflows in the presence of dissipative shocks too.

In the next section, we present the simplifying assumptions and equations of motion. In Section 3, we present the methodology of the solution. In Section 4, we present the solutions, and in the last section we present discussion and the concluding remarks.

2 Model Assumptions and Equations of Motion

It has recently been stressed that BH rotation plays no, or little, part in generating or powering these jets (Fender et al. 2010). So it is expected that plasma processes or fluid properties of the disc would generate jets. In this paper, we consider a non-rotating BH and focus only on the fluid properties of accretion disc, which may be considered to be responsible for jet generation. We have assumed the axis-symmetric disc–jet system to be in steady state. The space–time properties around a Schwarzschild BH is described by the pseudo-Newtonian potential introduced by Paczyński & Wiita (1980). The viscosity prescription in the disc is described by the Shakura–Sunyaev prescription. We ignore any cooling mechanism in order to focus on the effect of viscosity. The jets are tenuous and should have less differential rotation than the accretion disc; as a result the viscosity in jets can be ignored. To negate any resulting torque, the angular momentum at the jet base is assumed to be the same as that of the local value of angular momentum of the disc. The accretion disc occupies the space on or about the equatorial plane. But the jet flow geometry is described about the axis of symmetry. We first present the equations of motions of the accretion disc and the jet separately in the subsequent part of this section, and then in the next section, we describe the method to obtain the self-consistent accretion–ejection solution. In this paper we have used the geometric unit system where \(2G = M = c = 1\) (\(M\) is the mass of the BH and \(G\) is the Gravitational constant). Therefore, in this representation the units of length, mass and time are the Schwarzschild radius or \(r_s = 2GM/c^2\), \(M\) and \(r_e/c = 2GM/c^2\), respectively; consequently the unit of speed is \(c\).

2.1 Equations of motion for accretion

The equations of motion for viscous accreting matter around the equatorial plane, in cylindrical coordinates \((r, \phi, z)\), are given by the radial momentum equation:

\[
\frac{d}{dx} u + \frac{1}{\rho} \frac{dp}{dx} + \frac{1}{2(x-1)^2} - \frac{\lambda^2(x)}{x^3} = 0.
\]

The mass-accretion rate equation

\[
\dot{M} = 2\pi \Sigma u x.
\]

The mass-accretion rate \(\dot{M}\) is a constant of motion, except at the regions from where the mass may be lost into the jets. We present the exact form of the conservation of \(\dot{M}\) later in the paper. The angular momentum distribution equation:

\[
\frac{d}{dx} W_{\phi} + \frac{1}{x} \frac{d}{dx} \left( x^2 W_{\phi} \right) = 0.
\]

The entropy generation equation:

\[
\Sigma u T \frac{d}{dx} = Q^+ - Q^-.
\]

The local variables \(u, a, p, \rho\) and \(\lambda\) in the above equations are the radial bulk velocity, sound speed, isotropic pressure and specific angular momentum of the flow, respectively. Here, \(\Sigma = 2\rho h\) and \(W_{\phi}\) are the vertically integrated density and the viscous stress tensor (Matsumoto et al. 1984). Other quantities like the entropy density, the local temperature and the local half height of the disc are given by \(s, T\) and \(h\), respectively. The local heat gained and lost by the flow are given by \(Q^+ = W_{\phi}^+/\eta(x)\) and \(Q^-\).

The constant of motion of the flow is obtained by integrating equation (1) with the help of equations (2)–(4); we find that the energy per unit mass is given by

\[
E = \frac{u^2}{2} + \frac{a^2}{\gamma - 1} - \frac{\lambda^2}{2x^2} + \frac{\lambda \Phi(x)}{x^2} + \Phi(x),
\]

and is also called the specific grand energy of the flow, and is conserved throughout the flow even in the presence of dissipative dissipation (Gu & Lu 2004), except across a dissipative shock (see Section 3.1.1). In equation (5), \(\Phi(x) = -0.5/(x - 1)\) is the pseudo-Newtonian gravitational potential.

The viscous stress is given by

\[
W_{\phi} = \eta \frac{d\Omega}{dx},
\]

where \(\eta = \rho v h\) is the dynamic viscosity coefficient, \(v = a a'^2/\gamma q\Omega\) is the kinematic viscosity, \(a\) is the Shakura–Sunyaev viscosity parameter, \(q\) and \(\Omega\) are the local angular velocity and local Keplerian angular velocity, respectively. Considering hydrostatic equilibrium in vertical direction, the local disc half height is obtained as

\[
h(x) = \frac{2}{\gamma} ax^{1/2}(x - 1).
\]
The adiabatic sound speed is defined as
\[ a = \sqrt{\gamma p / \rho}, \]  
where \( \gamma \) is the adiabatic index. The expression of the entropy-accretion rate is given by
\[ \dot{M}(x) = \frac{d(2e+1)u x}{\Omega_k}. \]  
(9)

If there is no viscosity, i.e. \( \alpha = 0 \), then \( \dot{M} \) is a constant except at the shock. At shock the entropy accretion rate will suffer discontinuous jump. The immediate pre-shock and post-shock entropy-accretion rate denoted as \( \dot{M}_{\pm} \) and \( \dot{M}_{\mp} \) is related as \( \dot{M}_{\pm} > \dot{M}_{\mp} \). But for \( 0 < \alpha < 1 \) or viscous flow, \( \dot{M} \) varies continuously in the disc since viscosity dissipates and increases the entropy. If shock exists in viscous flow then, similar to the inviscid case, \( \dot{M}_{\pm} > \dot{M}_{\mp} \).

The gradient of the angular velocity can be obtained by integrating equation (3) and also by utilizing equation (2) and the expression of \( W_0 \).
\[ \frac{d\Omega}{dx} = -\frac{\gamma u \Omega_k(\lambda - \lambda_0)}{\alpha a^2 x^2}, \]  
where \( \lambda_0 \) is the specific angular momentum at the horizon obtained by considering vanishing torque at the event horizon (Weinberg 1972; Becker et al. 2008). Since \( \lambda = x^2 \Omega \), the radial derivative of \( \lambda \) is given by
\[ \frac{d\lambda}{dx} = 2x \Omega + x^2 \frac{d\Omega}{dx}. \]  
(11)

Moreover, \( \Omega_k \) denotes the Keplerian angular velocity and is defined as
\[ \Omega^2(x) = \frac{1}{2x(x-1)^2}. \]  
(12)

The Keplerian specific angular momentum is defined as
\[ \lambda_k = \Omega_k x^2 = \frac{\left[ \frac{x^3}{2(x-1)^2} \right]^{1/2}}{x^2}. \]  
(13)

Manipulating equations (1)–(4) with the help of equations (7)–(13) we obtain
\[ \frac{du}{dx} = \frac{N}{D}, \]  
(14)

where
\[ N = \frac{2}{\gamma + 1} \frac{(5x - 3)u}{2x(x-1)} + \frac{(\lambda^2 - \lambda_0^2)a^2}{x^3}u \]
\[ + \gamma Y \left( \frac{1}{\gamma + 1} \right) \frac{\lambda^2 \lambda_k(\lambda - \lambda_0)^2}{\alpha a^2 x^4} \]

and
\[ D = \frac{u^2}{a^2} - \frac{2}{\gamma + 1}. \]

The gradient of the sound speed is
\[ \frac{d\alpha}{dx} = \left( \frac{a - \gamma u}{a} \right) \frac{du}{dx} + \frac{(5x - 3)a}{2x(x-1)} + \frac{\gamma (\lambda^2 - \lambda_0^2)}{\alpha x^3}. \]  
(15)

Therefore, the accretion disc problem in vertical equilibrium is solved by integrating equations (11), (14) and (15).

2.1.1 Critical point conditions for accretion

At large distances away from the horizon, the inward velocity is very small and therefore the flow is subsonic, but matter enters the BH with the speed of light and therefore it is supersonic close to the horizon. Hence, accreting matter around BHs must be transonic, since it makes a transition from subsonic to supersonic. Therefore, at some location the denominator \( D \) of equation (14) will go to zero, and hence the numerator \( N \) goes to zero too. Such a location is called the sonic point or critical point. The critical point conditions are given as
\[ M^2_c = \frac{u_c^2}{a_c^2} = \frac{2}{\gamma + 1}. \]  
(16)

\[ \frac{(5x_c - 3)M_c^2}{2x_c(x_c - 1)} + \frac{\left( \lambda^2 - \lambda_0^2 \right)a_c}{x_c^3} + \gamma (\lambda^2 - \lambda_0^2) \frac{M_c \lambda_k(\lambda - \lambda_0)^2}{\alpha x_c^4} = 0. \]  
(17)

where \( M_c, u_c, a_c, x_c \) and \( \lambda_c \) are the Mach number, the bulk velocity, the sound speed, the radial distance and the specific angular momentum at the critical point, respectively.

The radial velocity gradient at the critical point is calculated by employing the Hospital rule.
\[ \frac{du}{dx} = \frac{(dN/dx)_{r=r_c}}{(dD/dx)_{r=r_c}} \]  
(18)

and by combining equations (15) and (18) we get
\[ \frac{du}{dx} = \left( \frac{a_c}{u_c} - \frac{\gamma u_c}{\alpha} \right) \frac{du}{dx} \]
\[ + \frac{(5x_c - 3)a_c}{2x_c(x_c - 1)} + \frac{\gamma (\lambda^2 - \lambda_0^2)}{a_c x_c^4}. \]  
(19)

So, the solution of equations (11), (14) and (15) can only be obtained if we know the sonic point and its conditions (equations 16–19).

2.2 Equations of motion for outflows

The flow geometry for accretion is about the equatorial plane; however, the jet or outflow geometry is about the axis of symmetry. If the outflow possess some angular momentum, then the outflow geometry should be hollow. Indeed, numerical simulations by Molteni et al. (1996a) suggest that the outflowing matter tends to emerge out between two surfaces, namely the funnel wall (FW) and centrifugal barrier (CB). In Fig. 1, the schematic diagram of the jet geometry is shown. The CB surface is defined as the pressure maxima surface and is expressed as
\[ x_{CB} = \left[ 2 \lambda^2 \lambda_{CB}(r_{CB} - 1) \right]^{1/2}, \]  
(20)

where \( r_{CB} = \sqrt{x_{CB}^2 + y_{CB}^2} \), spherical radius of CB. Here, \( x_{CB} \) and \( y_{CB} \) are the cylindrical radius and axial coordinate (i.e. height at \( r_{CB} \)) of CB. We compute the jet geometry with respect to \( y_{CB} \), i.e. \( y_{FW} = y_1 = y_{CB} \), where \( y_{FW} \) and \( y_1 \) are the height of FW and the jet at \( r_{CB} \), respectively. The FW is obtained by considering null effective potential and is given by
\[ x_{FW} = \frac{\lambda^2 (k^2 - 2) + \sqrt{\lambda^2 (k^2 - 2) - 4(1 - \lambda_{CB}^2)}}{2}, \]  
(21)

where \( x_{FW} \) is the cylindrical radius of FW. We define the cylindrical radius of the outflow as
\[ x_j = x_{FW} + x_{CB}. \]  
(22)
The spherical radius of the jet is given by \( r_j = \sqrt{x_1^2 + y_1^2} \). In Fig. 1, \( \text{OB}(x_r) \) defines the streamline (solid) of the outflow. The total area function of the jet is defined as

\[
A = \frac{2\pi x_1^2}{\sqrt{1 + (\text{d}x_1/\text{d}y_1)^2}},
\]

where the denominator is the projection effect of the jet streamline on its cross-section. The integrated radial momentum equation for jet is given by

\[
\mathcal{E}_j = \frac{1}{2} v_{j0}^2 + n a_j^2 + \frac{\lambda_j}{2\gamma_j} - \frac{1}{2(r_j - 1)},
\]

where \( \mathcal{E}_j \) is the specific energy, \( \lambda_j \) is the angular momentum of the jet and \( n = 1/(\gamma - 1) \) is the polytropic index. The integrated continuity equation is

\[
\dot{M}_{\text{out}} = \rho_j v_j A.
\]

and the entropy generation equation is integrated to obtain the polytropic equation \( (p = K_j \rho_j^{\gamma_j}) \) of the state for the jet. The entropy accretion rate for the jet is given by

\[
\dot{M}_s = a_j v_j A.
\]

In equations (24) and (25), the suffix \( j \) indicates jet variables, where \( v_j, a_j \), and \( \rho_j \) are the velocity, the sound speed and the density of the jet.

Equations (24) and (25) are differentiated with respect to \( r(= r_{\text{CB}}) \) to obtain

\[
\frac{\text{d}v_j}{\text{d}r} = \frac{\mathcal{N}}{\mathcal{D}},
\]

where

\[
\mathcal{N} = \frac{1}{2(r_j - 1)^2} \frac{\text{d}r_j}{\text{d}r} - \frac{\lambda_j}{\gamma_j} \frac{\text{d}x_j}{\text{d}r} - \frac{a_j^2}{A} \frac{\text{d}A}{\text{d}r},
\]

and

\[
\mathcal{D} = \frac{a_j^2}{v_j} - v_j.
\]

The critical point \( r_{j_c} \) condition for jet is

\[
v_{j_c}^2 = a_j^2 = \frac{1}{2(r_j - 1)^2} \left( \frac{\text{d}r_j}{\text{d}r} \right)_{r_c} - \frac{\lambda_j}{\gamma_j} \left( \frac{\text{d}x_j}{\text{d}r} \right)_{r_c},
\]

\[
\times \left[ \frac{1}{a_j} \left( \frac{\text{d}A}{\text{d}r} \right)_{r_c} \right]^{-1}.
\]

The outflow solution is obtained by integrating equations (27) with the help of equation (30). The outflow equations can be determined uniquely if \( \mathcal{E}_j \) and \( \lambda_j \) are known because the value of \( \mathcal{E}_j \) and \( \lambda_j \) determines \( r_{j_c} \). However, for consistent accretion–ejection solution, both accretion and jet part have to be solved simultaneously, a technique previously used by Chattopadhyay & Das (2007), and further refined in this paper.

3 Methodology

The accretion–ejection solutions are self-consistently and simultaneously solved, and we present the methodology to find such solutions in this section. We start with the solution of the accretion disc. It has been mentioned before that equations (11), (14) and (15) can be integrated, if we know the sonic point \( x_s \). One of the longstanding problems in accretion physics is to determine the sonic point of the flow in the presence of a viscous stress of the form equation (6), which keeps the angular momentum equation in a differential form (equation 11) rather than in an algebraic form (Chakrabarti 1996; Gu & Lu 2004). The problem is compounded by the fact that although quantities on the horizon are known, the coordinate singularity on the horizon makes it difficult to solve the equations by taking those as the starting values. The problem could be circumvented if the asymptotic values of the flow variables close to the horizon are known.

With a proper use of conservation equations, Becker & Le (2003) found the asymptotic distribution of the specific angular momentum and the radial velocity close to the horizon, and they are

\[
\lambda_j(x) = \lambda_{j0} \left[ 1 + \frac{2\alpha}{\gamma_j r_j} \left( \frac{2}{r_j^2} \right)^{1/2} \times \left( \frac{\mathcal{N}^2}{2r_j^3} \right)^{1/4} (x - r_j)^{3/4} \right], \quad x \to 1
\]

and

\[
u(x) = v_{j0}(x) \left[ 1 + \frac{2E_x^2 - \lambda_{j0}^2}{x^2 u_{j0}(x) - (\gamma - 1) f(x)} \right]^{1/2}, \quad x \to 1
\]

where the function \( f(x) \) is \( f(x) = \frac{x_j^2}{2\gamma_j - 1} \left[ \frac{\mathcal{N}^2}{2r_j^3} \right]^{1/4} \) and the free fall velocity in the pseudo-Newtonian potential geometry is given by

\[
u_{j0}(x) = \frac{1}{\sqrt{(x - 1)}}.
\]

3.1 Integration procedure of accretion solution

A position very close to the horizon \( x = x_{i0} = 1.01 \) is chosen. For the given values of the parameters \( \alpha, E, \lambda_{j0} \) at the horizon and \( \mathcal{M}_{i0} \) at \( x_{i0} \),
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3.1.1 Shock equations

The Rankine–Hugoniot (RH) shock conditions are obtained by conservation of the mass, momentum and energy flux across the shock. In the presence of mass loss and energy loss, the shock conditions are given by the modified mass conservation

$$M_+ = M_- - M_{\text{out}} = M_-(1 - R_m).$$

(34)

This equation effectively divides the accretion rate of pre-shock accretion disc ($M_-$) into two channels, namely the post-shock accretion disc (represented by $M_+$) and the jet (represented by $M_{\text{out}}$). The modified momentum conservation

$$W_+ + \Sigma_+ u_+^2 = W_- + \Sigma_- u_-^2,$$

(35)

and the third shock condition is the modified energy conservation

$$E_+ = E_- - \Delta E,$$

(36)

where $R_m$ is the relative mass outflow rate given by

$$R_m = \frac{M_{\text{out}}}{M_-}.$$  

(37)

Here subscripts minus (−) and plus (+) denote the quantities before and after the shock. $W$ is the vertically integrated pressure. In the absence of mass loss ($R_m = 0$) or dissipation (i.e. $\Delta E = 0$), equations (34)–(36) reduce to the standard RH shock conditions. Since the dissipation is expected at the shock location, it is assumed that energy dissipation takes place mostly through the thermal Comptonization (Chakrabarti & Titarchuk 1995; Mandal & Chakrabarti 2010) and is likely to be very important within a distance $dx$ inside the shock where the optical depth is around unity. So this energy dissipation in the post-shock flow reduces the temperature of the flow and the loss of energy is proportional to the temperature difference between the post-shock and the pre-shock flows, i.e.

$$\Delta E = f_e n (a_+^2 - a_-^2),$$

where $f_e$ is the fraction of the difference in thermal energy dissipation across the shock transition and $n$ is the polytropic index. We use $f_e$ as a constant parameter, but in the presence of detailed radiative processes $f_e$ can be self-consistently determined. Since shock width is infinitesimally thin, so we assume that $d\Omega/\text{d}x$ is continuous across the shock. The angular momentum jump condition is calculated by considering the conservation of angular momentum flux, and is given by

$$\lambda_+ = \lambda_+ + C_{\text{sh}} \left[ \frac{a_+^2}{u_+} - \frac{a_-^2}{u_-} \right],$$

(39)

where

$$C_{\text{sh}} = -a_- (\lambda_+ - \lambda_0)/a_+^3.$$

Equation (39) can be re-written as

$$(\lambda_+ - \lambda_0) = \frac{a_-^2 u_+ (\lambda_+ - \lambda_0)}{a_+^2 u_-}.$$  

(40)

Since at shock, $a_+ > a_-$ and $u_+ > u_-$, therefore, $\lambda_+ > \lambda_-$. Using shock condition equations (34) and (35), the pre-shock sound speed and bulk velocity can be written as

$$a_-^2 = k_1 u_- - \gamma u_-^2,$$

(41)

where

$$k_1 = (a_+^2 + \gamma u_+^2)/(f u_+)$$

and $f = 1/(1 - R_m)$. Now substituting for $a_-$ and $\lambda_-$ in equation (36), we find a quadratic equation of $u_-$ as

$$C_2 u_-^2 + C_1 u_- + C_0 = 0,$$

(42)

where

$$C_2 = \left[ 1 - n(1 + f_e) - \frac{\gamma^2 C_{\text{sh}}^2}{2x_+^4} \right],$$

$$C_1 = \left[ n(1 + f_e) k_1 + \frac{\gamma \lambda_0 C_{\text{sh}}}{x_+^2} + \frac{\gamma \lambda_0 C_{\text{sh}}}{x_+^2} \right],$$

$$C_0 = \left[ \lambda_0 C_{\text{sh}} - \frac{C_2^2}{2x_+^4} - k_2 \right],$$

and

$$k_2 = E_+ - \Phi(x_+) + f_e n a_+^3.$$

In terms of the shock quantities, the mass outflow rate is given by

$$R_m = \frac{M_{\text{out}} - M_-}{M_-} = \frac{R V (x_+) A(x_+)}{4 \pi \sqrt{2 \pi} \Gamma^{3/2} (x_+ - 1) a_+ u_-} = RVG,$$

(43)

where $R = \Sigma_+ / \Sigma_-$ is the compression ratio across the shock, $V = v_j (x_+) / u_-$ is the ratio of the jet base velocity and the pre-shock velocity of the disc ($u_-$), and $G = A(x_+) / (4 \pi \sqrt{2 \pi} \Gamma^{3/2} x_+^{3/2} (x_+ - 1) a_+ u_-)$ is the ratio of the jet cross-sectional area at $r = x_+$ and the post-shock accretion disc cross-sectional area.

3.2 Accretion–ejection solution

The accretion–ejection is computed self-consistently. We have set $\gamma = 1.4$ and $x_0 = 1.01$ throughout the paper. The method to find the accretion–ejection solution is as follows.
4 SOLUTIONS

We present every possible way in which matter may dive into the BH. In this section, we start with accretion solutions without considering mass loss or dissipation at the shock front and study the effect of viscosity on accretion solution. Then we present the accretion–ejection solution and study how the viscosity can affect the mass outflow rates. Finally we present accretion–ejection solutions in the presence of dissipative shocks and show the effect of both the viscosity and dissipation at the shock front.

4.1 All possible accretion solutions in the advective regime

Since BH accretion is necessarily transonic, at first, we present the simplest rotating transonic solutions, i.e. inviscid global solutions (global solution = which connect horizon and large distances) in Figs 2(a)–(d). The inviscid solutions are presented as an attempt to recall the simplest accretion case (Chakrabarti 1989; Das et al. 2001). For inviscid flow, the constants of motions are $E$, $\lambda$ (i.e. $\lambda_0$), and if outflows are not present then $\dot{M}$ is also a constant of motion. Moreover, in the absence of viscosity, it straightforward follows from equation (5) that $E = E = u^2/2 + na^2 + \lambda^2/(2x^2) = 0.5/(x - 1)$, where $E$ is the Bernoulli parameter or the specific energy of the flow. In Figs 2(a)–(d), accretion solutions in terms of the Mach number $M = u/a$ distribution are presented, and in Fig. 2(e), the $E - \lambda$ parameter space for multiple sonic point and the shock is
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Figure 3. Variation in $M$ with log $(x)$ for the accretion solutions with different viscosity parameter $\alpha$. In a, c, i, m we present inviscid solutions corresponding to the O, A, W, I type of solutions from Figs 2(a)–(d). Towards right $E$, $\lambda_0$ is kept the same but $\alpha$ is increased. The flow parameters for which these plots are generated $E = 0.001$, $\lambda = 1.5$ (a, b, c, d); $E = 0.001$, $\lambda = 1.68$ (e, f, g, h); $E = 0.001$, $\lambda = 1.75$ (i, j, k, l) and $E = 0.005$, $\lambda = 1.75$ (m, n, o, p). The viscosity parameter $\alpha$ is mentioned in the figure. The vertical long–short dashed line shows the location of sonic points.

Presented too. Depending on $E$ and $\lambda_0$ of the flow, the solutions are also different. If the $\lambda_0$ is low, there is only one outer sonic point $x_{co}$, and solution type is O-type or Bondi type ($E = 0.001$, $\lambda_0 = 1.5$; Fig. 2a). As $\lambda_0$ is increased, the number of physical sonic points increases to two and the accretion flow that becomes supersonic through $x_{co}$ can enter the BH through $x_{ci}$ if a shock condition is satisfied. Although the shock-free solution is possible, in this part of the parameter space a shocked solution will be preferred because a shocked solution is of higher entropy (or in other words of higher $\dot{M}$). Such a class of solution is called A-type ($E = 0.001$, $\lambda_0 = 1.68$, Fig. 2b). For even higher $\lambda_0$ only one sonic point is possible ($E = 0.001$, $\lambda_0 = 1.75$ for Type W; and $E = 0.005$, $\lambda_0 = 1.75$ for Type I shown in Figs 2c and d), and the solutions are wholly subsonic until $x_{co}$ and then dives on to BH supersonically. W type solutions are different from I type in the sense that W type is still within the MCP domain while I type is not. Moreover, I type is a smooth monotonic solution, although W is smooth and shock free but is not monotonic and has an extremum at around $x_{co}$. The parameter space $E - \lambda_0$, bounded by solid line (mmo), shows the RH shock parameter space, while the dotted one (PQR) shows the MCP domain (Fig. 2e). It is to be noted that in the inviscid limit, accretion is only possible if $\lambda_0 < \lambda_{K}$.

Figs 3(a), (e), (i) and (m) represent the inviscid solutions corresponding to the O type solutions ($E$, $\lambda_0 = 0.001$, 1.5), A type ($E$, $\lambda_0 = 0.001, 1.68$), W type ($E$, $\lambda_0 = 0.001, 1.68$) and I type ($E$, $\lambda_0 = 0.006, 1.75$), and are also depicted in Figs 2(a)–(d). Keeping $E$ and $\lambda_0$ same, we increase the viscosity parameter in the right direction. Figs 3(a)–(d) have the same $E$ and $\lambda_0$, but progressively increasing $\alpha = 0.06$ (Fig. 3b), 0.068 (Fig. 3c) and 0.07 (Fig. 3d). Similarly for Figs 3(e)–(h), $E - \lambda_0$ is same but has different $\alpha$, so is the case for Figs 3(i)–(l), and Figs 3(m)–(p). Interestingly, the viscous I type is in principle the much vaunted ADAF type solution presented in Figs 3(d), (h), (k)–(l) and (n)–(p). The ADAF solution is monotonic, wholly subsonic except very close to the horizon, and has also been shown to be a subset of the general advective solution (Lu et al. 1999; Becker et al. 2008; Das et al. 2009). It is evident from Figs 3(a)–(p) that the effect of viscosity is to create additional sonic points in some parts of the parameter space, opening up of closed topologies and might trigger shock formation where there was no shock, while removing both shock and MCPs in other regimes of the parameter space. All of this is achieved by removing angular momentum outwards while increasing the entropy inwards. In this connection one may find two kinds of critical viscosity parameters in the advective domain. If the inviscid solution is O type (Fig. 3a), then there would be a lower bound of critical viscosity $\alpha_{cl}$ which would transport angular momentum in a manner that would trigger the standing shock. Moreover, there would be another upper bound of viscosity parameter $\alpha_{cu}$ that would quench the standing shock. While if the inviscid solution has a shock to start with (Fig. 3e), there could only be $\alpha_{cu}$. For the case presented in Figs 3(a)–(d), $\alpha_{cl} = 0.0465$ and $\alpha_{cu} = 0.065$. For the case presented in Figs 3(e)–(h), $\alpha_{cu} = 0.0126$. 

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For any viscosity parameter \( \alpha \), the effect of \( \alpha \) on shock location \( x_s \) is an interesting issue. Numerical simulations show that for a fixed outer boundary condition, \( x_i \) expands to larger distances with the increase of \( \alpha \) (Lanzafame et al. 1998; Lee et al. 2011), while analytically Chattopadhyay & Das (2007) showed that for the same outer boundary condition, \( x_s \) shifts closer to the horizon with the increase in \( \alpha \). Although the viscosity prescription of Chattopadhyay & Das (2007) and the simulations are different, namely the former chose the stress to be proportional to total pressure, while in simulations the stress is proportional to the shear, still viscosity reduces angular momentum, and we know for lower angular momentum if the shock forms, it should form closer
to the BH! Since the viscosity prescription in this paper is similar to the simulations, we should be able to answer the dichotomy. So a concrete question may arise that if viscosity is increased, does the \( x_b \) expand to larger distances or contracts to a position closer to the horizon? If the viscosity acts in a way such that the \( \lambda^+ \) (immediate post-shock \( \lambda \)) is less than its inviscid value at the shock then \( x_b \) will move closer to the horizon. However, such simple-minded reasoning may fail if shocks exist, then the post-shock flow being hotter would transport more efficiently than the immediate pre-shock flow. So although \( \lambda_- < \lambda(x_{inj}) \), it is not necessary that \( \lambda_+ \) will be less than \( \lambda(x_{inj}) \). We scoured the parameter space to find the answer, and in the following we present the explanation.

Let \( x_b \) be the distance at which the \( \lambda \) distribution of the viscous solution is coincident with the \( \lambda \) value of the inviscid solution, let us further assign \( \lambda_b = \lambda(x_b) \). It is to be remembered that \( \lambda_0 = \lambda_b \) for the inviscid (\( \alpha = 0 \)) solution, but \( \lambda_0 < \lambda_b \) for viscous solution because viscosity reduces the angular momentum. In all the simulations done on the viscous flow in the advective regime referred in this paper, one starts with an inviscid solution (since analytical solutions are readily available), and then the viscosity is turned on, keeping the values at the outer boundary fixed. In other words, it is this \( x_b \) that is called the outer boundary in numerical simulations, and generally, \( x_b \lesssim a \times 100 \) since it is computationally expensive to simulate a large domain from just outside the horizon to a few \( \times 1000 r_\odot \) and still retain required resolution to achieve intricate structures in the accretion disc. In Figs 5(a)–(c), we compare the \( \lambda(x) \) of shocked accretion flows starting with the same \( E = 10^{-5} \), and \( \lambda_b = 1.68 \) for various viscosity parameters such as \( \alpha = 0 \) (solid), \( \alpha = 0.0075 \) (dotted), \( \alpha = 0.015 \), but for different points of coincidence, e.g. \( x_b = 2220 \) (5a), \( x_b = 3660 \) (5b) and \( x_b = 5230 \) (5c). The vertical solid line and dash–dotted line show the shock location for \( \alpha = 0 \) curve and outer boundary location \( (x_b) \), respectively. The variation of limiting \( x_b \) with \( E \) for \( \lambda_b = 1.68 \) (d) and \( \lambda_b = 1.65 \) (e). Domain 1 represents all \( x_b \) for which \( x_b \) decreases with the increase in \( \alpha \), but for any \( x_b \) in 2, \( x_b \) will increase with the increase in \( \alpha \).

![Figure 5](https://academic.oup.com/mnras/article-abstract/430/1/386/985563)
present the self-consistent inflow–outflow solutions. In the presence of mass loss, the mass conservation equation across the shock will be modified in the form of equation (34). All the steps mentioned in Section 3.2 are followed to compute the self-consistent inflow–outflow solution. In Figs 7(a)–(d), we present a case of global accretion–ejection solution. In Fig. 7(a), the accretion solution in terms of $M(x)$ or the Mach number is presented; in Fig. 7(b), the jet Mach number $M_j$ is plotted with the height $y_1$ of the jet from the equatorial plane of the disc, where the radial coordinate of the jet is $r_1 = \sqrt{x_1^2 + y_1^2}$. The sonic points are marked with open circles. The inner boundary for the accretion solution is $E = 0.001$, $\lambda_0 = 1.542$. The specific energy and the angular momentum at the shock are $E_s = 1.084 \times 10^{-3}$, $\lambda_s = 1.699$. The collimation parameter of the jet (see Chattopadhyay 2005) at its sonic point is $x_\psi/y_\psi \sim 0.24$, and hence the spread is quite moderate. The dotted vertical line is the position of the shock when $R_n = 0$, and the solid vertical line is the position of the shock after the mass-outflow rate has been computed. Clearly, since excess thermal gradient force in the post-shock disc drives bipolar outflows, it reduces the pressure, and hence to maintain the total pressure balance across the shock, the shock front moves closer to the horizon. The relative mass outflow rate computed for the particular case depicted in Figs 7(a)–(d) is $R_m = 0.1044$. In Fig. 7(c), we plot the density $\rho_s$ of the jet derived from equation (34), and non-dissipative or RH shocks. In this section we present the essential quantities that would contribute in driving a part of the post-shock matter as outflows. The shock location $x_\psi$ decreases with the increase in $E$, and for increasing $x_\psi$, $R$ should decrease. Since $R_m$ is the combination of $R$, $V$ and $\mathcal{G}$, even for falling $R$ and $\mathcal{G}$, the mass outflow rate increases since it is being compensated by the increase in $V$. Interestingly, none of the quantities shows a monotonic variation. It is important to note that all the three parameters $R$, $V$ and $\mathcal{G}$ represent the jet to disc connection and not the actual driving. The real drivers are, however, the post-shock specific energy $E_\psi$, and the jet base cross-section $A_s(=A(x_s))$. Higher $E_\psi$ means hotter flow at the jet base, and therefore the thermal driving will be more. This is complemented by the cross-section $A_s$ of the jet base. Higher $E_\psi$ would drive more matter into the jet channel but will be limited by the cross-sectional area. In Fig. 8(f), we plot the pre-shock entropy–accretion rate $\dot{M}_s$ (solid), the post-shock entropy–accretion rate $\dot{M}_p$ (dashed) and the jet entropy–accretion rate $\dot{M}_j$ (dotted), as a function of $E$, and it is quite evident that the pre-shock entropy is less than both $\dot{M}_s$ and $\dot{M}_j$. Moreover, when the value of $\dot{M}_j$ is high, $R_m$ is found to be high too, which indicates that matter would prefer to flow through channels with higher entropy.

From equation (34), we know that $\dot{M}_s < \dot{M}_p$ if $R_m \neq 0$. Therefore, the post-shock pressure would be reduced. This would cause $x_\psi$ to decrease as shown in Fig. 7(a) [also see Chattopadhyay & Das (2007)]. Furthermore, the mass loss from the post-shock flow and consequent reduction in pressure would also modify shock...
parameter space. In Fig. 9, the bounded regions in the $E - \lambda_0$ space represent the shock parameter space for $\alpha = 0, 0.05, 0.1, 0.15$ and 0.2, marked in the figure. Compared to the shock parameter space in the absence of mass loss (Fig. 6), the parameter space in the presence of mass loss gets reduced and beyond $\alpha = 0.2$ the standing shock seems to vanish. Moreover, the shock parameter space shows that the standing shock does not seem to exist for very low $E$. Non-existence of standing shock, of course, does not imply the non-existence of non-steady or oscillatory shocks.

4.3 Mass loss from the dissipative shocks

Chakrabarti & Titarchuk (1995) and later Mandal & Chakrabarti (2010) have shown that the post-shock hot flows can produce the high energy photons easily by inverse-Comptonizing the soft photons and reproduced the observed spectra from a variety of objects. If indeed the observed spectra and luminosity can be reproduced from the post-shock flow, then the grand energy will not be conserved across the shock and will be given by equation (36), and the dissipated energy can be radiated away. In Figs 10(a)–(f), various flow variables $M$ (a), $u$ (b), $\lambda$ (c), $E$ (d), $a$ (e) and $M$ (f) are plotted for $f_i = 0$ (solid) and $f_i = 0.1$ (dotted). For $f_i = 0$ (solid), the solutions are plotted for flow parameters $E, \lambda_0 = 7.373 \times 10^{-4}, 1.527$. For $f_i = 0$, we have $E = E_i = E_\infty$. However, for $f_i > 0, E_i < E_\infty$. So for $f_i = 0.1$ (dotted), the inner boundary is represented by $E_i, \lambda_0 = 2.5 \times 10^{-4}, 1.527$, while the pre-shock $E_\infty = 7.373 \times 10^{-4}$. The post-shock specific energy or the Bernoulli parameter, temperature, angular momentum and entropy of the solution with dissipative shock (dotted) are lower than those corresponding to the non-dissipative shock (solid). Since a part of the thermal energy gained through shock is spent in powering jets and to produce radiation, the shock front moves closer to the BH. The relative mass outflow rate or $R_m$ is lower for dissipative shocks because part of the thermal energy of the post-shock disc is lost through radiation. However, the thermal energy lost as radiation can still contribute to jet power if those photons deposit momentum on to the jets (Chattopadhyay & Chakrabarti 2002; Chattopadhyay et al. 2004; Chattopadhyay 2005).

If $f_i \neq 0$ and shocks are present, then the accretion solutions are obtained either for $E_\infty, \lambda_0, \alpha, f_i$, or, equivalently, $E_\infty, \lambda_0, \alpha, f_i$, or, $E_\infty, \lambda_{ini}, \alpha, f_i$. If $f_i = 0$ then $E = E_i = E_\infty$; therefore any of the following two sets would suffice $E, \lambda_0, \alpha, \text{or}, E, \lambda_{ini}, \alpha$. Since the outflows are launched from the post-shock flow, the mass outflow rates are computed only from the shocked accretion flows, and we are interested in finding the dependence of $R_m$ on each of the above-mentioned parameters. In Fig. 11(a), we plot $R_m$ as a function of $E_\infty$. When the other parameters are fixed at $\alpha = 0.02, \lambda_0 = 1.65$ where each of the curve is for $f_i = 0$ (solid), $f_i = 0.05$ (dotted) and $f_i = 0.1$ (dashed). With the increase in $E_\infty$, the post-shock-specific energy $E_\infty$ increases which drives more matter as outflow, and hence $R_m$ increases. However, for any given $E_\infty$, $R_m$ decreases with the increase in $f_i$, i.e. the mass outflow decreases with the increase in the thermal energy dissipation. Although for very high $E_\infty$, the separation of the curves decreases, this shows that flows
starting with higher energy would have enough thermal energy to drive significant outflows even for \( f_e \neq 0 \). Now, let us fix \( E_e \) but vary \( \lambda_0 \) in Fig. 11(b). \( R_m \) is plotted with \( \lambda_0 \) for \( f_e = 0 \) (solid), \( f_e = 0.05 \) (dotted) and \( f_e = 0.1 \) (dashed). The fixed parameters are \( E_e = 0.001 \), and \( \alpha = 0.02 \), and clearly \( R_m \) is not a monotonic function of \( \lambda_0 \). Increasing \( \lambda_0 \) would increase \( \lambda_+ \) and therefore increase \( x_s \) but would decrease \( E_\alpha \). The increase in \( x_s \) increases the jet base cross-section. The competition between \( E_\alpha \) and \( A_\alpha \) causes a dip in \( R_m \) for moderate values of \( \lambda_0 \).

However, \( R_m \) decreases with the increase in \( f_e \). In Fig. 11(c), we fix the outer boundary condition, namely, \( E_e = 0.001, \lambda_{maj} = 70.6 \) at \( x_{maj} = 10^3 \), and \( R_m \) is plotted as a function of \( \alpha \), where each of the curve is for \( f_e = 0 \) (solid), \( f_e = 0.05 \) (dotted) and \( f_e = 0.1 \) (dashed). Since for flows starting with such high \( \lambda_{maj} \), there would be no accretion shock solution for low \( \alpha \), therefore one observes outflows only beyond a critical value of \( \alpha \). We know (see Fig. 5c) that for flow starting with the same outer boundary condition, the shock location decreases with the increasing \( \alpha \). Decrease in \( x_s \) would mean decrease in \( A_\alpha \). Since \( x_s \) decreases, it means the post-shock energy increases due to viscous dissipation, i.e. \( E_\alpha \) increases. Increase in \( E_\alpha \) would drive more matter into the outflow channel. Therefore, the decrease in \( A_\alpha \) is dominated by the increase in \( E_\alpha \), and so \( R_m \) increases with \( \alpha \) for a fixed outer boundary condition.

However, \( R_m \) decreases with the increase in \( f_e \), which also shows that these jets are thermally driven. In Fig. 11(d), \( R_m \) is plotted with \( f_e \) for \( \alpha = 0 \) (solid), \( \alpha = 0.01 \) (dotted) and \( \alpha = 0.02 \) (dashed). The fixed parameters are \( E_e = 0.001 \) and \( \lambda_0 = 1.65 \). It was also found out that there are critical \( f_e \) beyond which no standing shock conditions are satisfied, and they are \( f_{ec} = 0.135 \) for \( \alpha = 0 \) (solid), \( f_{ec} = 0.22 \) for \( \alpha = 0.05 \) (dotted) and \( f_{ec} = 0.645 \) for \( \alpha = 0.1 \) (dashed). It is obvious that \( f_{ec} \) increases with the increase in \( \alpha \).

It is to be noted that \( R_m \) is the fraction of matter which is shock heated and ejected out as bipolar outflows. To properly understand the role of shock in driving bipolar outflow, we consider accretion solutions with the same outer boundary condition, or the case presented in Fig. 11(c). In Fig. 12(a), we plot various quantities across the shock, e.g. \( R_m \) (solid), \( E_\alpha \) (dotted), \( x_s \) (dashed) and \( A_\alpha \) (long dashed), as a function of \( \alpha \), and for \( f_e = 0 \), i.e. corresponding to the solid plot of Fig. 11(c). In Fig. 12(b), we plot \( R_m \) (solid), \( E_\alpha \) (dotted), \( x_s \) (dashed) and \( A_\alpha \) (long dashed), as a function of \( \alpha \), and for \( f_e = 0.1 \), i.e. corresponding to the dashed plot of Fig. 11(c). Increasing \( \alpha \) in accretion with the same outer boundary condition will decrease \( \lambda_+ \) and therefore decrease \( x_s \) (dashed). Decrease in \( x_s \) implies that the shock is formed closer to the BH, where the viscous dissipation would be more, i.e. \( E_\alpha \) will be higher (dotted). The jet velocity is small at the jet base, so large \( E_\alpha \) means hotter flow and stronger driving of the outflow, i.e. higher \( R_m \). Fig. 12(b) shows that, as the shock dissipation \( f_e \) is increased, at certain \( \alpha \), \( E_\alpha \) is reduced while the decrease in \( A_\alpha \) is marginal; this causes \( R_m \) initially to decrease with \( \alpha \), but eventually starts to increase as \( E_\alpha \) increases appreciably.

In Fig. 13(a), we plot the \( E - \lambda_0 \) shock parameter space for flows with \( f = 1, f_e = 0 \) (solid), \( f > 1, f_e = 0 \) (dotted), \( f > 1, f_e = 0.05 \) (dashed) and \( f > 1, f_e = 0.2 \) (long dashed). The viscosity parameter

![Figure 8](https://academic.oup.com/mnras/article-abstract/430/1/386/985563/10.1093/mnras/staa2449/108398563)
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5 DISCUSSION AND CONCLUSION

Quasars and micro-quasars may show strong jets, and these outflows are correlated with the spectral state of the object. Therefore, the quantitative estimate of the generation of these outflows is required, especially the relation between the mass outflow rates and the viscosity parameter needs to be ascertained; this is because the viscosity will determine the disc spectral states. Most of these estimates are available for inviscid flow, or special viscosity prescription or shock condition, but not for the most general shock condition, i.e. partially dissipative shock.

In this paper, our main concern has been to estimate the thermally driven bipolar outflows from shocked accretion discs around BHs. For solutions with the same inner boundary conditions (i.e. Figs 3 a–p), we found two critical viscosity parameters, the first one being the onset of shock (αcl), and the other being the one above which standing shock disappears (αcu), and generates a shock-free global solution which is wholly subsonic except close to the horizon, or the ADAF type solution. Simultaneous existence of both αcl and αcu will depend on whether λs is small enough to produce a corresponding O-type inviscid solution. If inviscid solution is already shocked then only αcu will exist, while if the inviscid solution is O or W type then neither αcl nor αcu exists. For solutions with the same outer boundary condition, i.e. flow starting with same E, λs = λs(xinj) at some injection radius xinj (e.g. Figs 4a–e), there would be an additional critical viscosity parameter α1, which would allow for this figure is α = 0.05. It is to be remembered that f = 1 means no mass loss and f > 1 means the presence of bipolar outflows. Therefore, f = 1, f = 0 (solid), implies that the shock parameter space for RH shocks with no mass loss and no dissipation at the shock, while f > 1, f = 0 (dotted) implies non-dissipative shock but mass loss is present. Similarly, f > 1, f = 0.05 (dashed) and f > 1, f = 0.2 (long dashed) shock parameter space for dissipative shocks of various strength and in the presence of mass loss. The parameter space for standing shock shrinks with the increase in f. In Fig. 13(b), we plot the shock parameter space for fixed f = 0.1 and various values of α = 0 (solid), α = 0.05 (dotted), α = 0.1 (dashed) and α = 0.15 (long dashed). We know from Fig. 6 that with the increase in α the shock parameter space shrinks; however in the presence of mass loss, flows with low E seem to show no standing shock. In Fig. 13(b), we see that although the parameter space for shock shrinks but low E flow again exhibits standing shock, which signifies that the mass outflow rate decreases with the increase in f. Since accretion disc should possess some amount of viscosity, a proper understanding of the viscous accretion disc is required. As outflows are generated from the post-shock disc and high energy photons should also emerge from the post-shock disc, an investigation of estimating the outflow rate and correlating it with the dissipation parameters is a must requirement. In order to estimate the outflows correctly, a proper understanding of the accretion process has to be undertaken, and we presented all possible accretion solutions without mass loss in Section 4.1, including the shocked and shock-free accretion solutions and the dependence of these solutions on viscosity parameter. While doing so, we have compared solutions with the same inner and outer boundary conditions, and have shown that these differences produce a significant difference in interpreting the results.

For solutions with the same inner boundary conditions (i.e., f_e = 0 (solid), and dissipative shock f_e = 0.1 (dotted)). The inner boundary condition for f_e = 0 are E, λ = 7.373 × 10^{-4}, 1.527 (solid), and for f_e = 0.1 are E = 2.5 × 10^{-4}, 1.527 (dotted). In case of the dotted curve E = 7.373 × 10^{-4}. For f_e = 0, x_s = 20.15, and R_{inj} = 0.1133 (solid), and for f_e = 0.1, x_s = 18.04 and R_{inj} = 0.0735 (dashed). Various flow variables are M(a), u(b), λ(c), E(d), a(e) and M(f).

Figure 9. E − λ parameter space of the standing adiabatic shock with the Shakura–Sunyaev viscosity parameter α = 0.0, 0.05, 0.1, 0.15, 0.2 marked in the figure, and in the presence of mass loss.

Figure 10. Comparison of accretion solutions with non-dissipative shock, i.e., f_e = 0 (solid), and dissipative shock f_e = 0.1 (dotted). The inner boundary condition for f_e = 0 are E, λ = 7.373 × 10^{-4}, 1.527 (solid), and for f_e = 0.1 are E = 2.5 × 10^{-4}, 1.527 (dotted). In case of the dotted curve E = 7.373 × 10^{-4}. For f_e = 0, x_s = 20.15, and R_{inj} = 0.1133 (solid), and for f_e = 0.1, x_s = 18.04 and R_{inj} = 0.0735 (dashed). Various flow variables are M(a), u(b), λ(c), E(d), a(e) and M(f).
Figure 11. Relative mass outflow rate $R_m$ as a function of (a) $E_-$ for fixed values of $\alpha = 0.02, \lambda_0 = 1.65$; (b) $\lambda_0$ for fixed values of $E_- = 0.001$ and $\alpha = 0.02$; (c) $\alpha$ for fixed values of $E_- = 0.001$ and $\lambda_{inj} = 70.6$ at $x_{inj} = 10^4$; where all the plots are for $f_e = 0$ (solid), $f_e = 0.05$ (dotted) and $f_e = 0.1$ (dashed). (d) $R_m$ as a function of $f_e$ for fixed values of $E_- = 0.001$ and $\lambda_0 = 1.65$ and each curve is for $\alpha = 0$ (solid), $\alpha = 0.01$ (dotted) and $\alpha = 0.02$ (dashed).

Figure 12. Relative mass outflow rate $R_m$ (solid), $x_s$ (dashed), $E_+$ (dotted) and $A_s$ (long-dashed) as a function of $\alpha$ for $f_e = 0$ (a) and $f_e = 0.1$ (b) for fixed values of $E_- = 0.001$ and $\lambda_{inj} = \lambda_{K(x_{inj})} = 70.6$ at $x_{inj} = 10^4$. 
a global solution connecting the horizon and the outer boundary $x_{o0}$. We have also confirmed that fluids in such cases, $x_{s}$, would decrease with the increase in $\alpha$. The decrease in $x_{s}$ with the increase in $\alpha$ is interesting. Chakrabarti & Titarchuk (1995) for the first time argued that the post-shock disc is the elusive Compton cloud, which inverse-Comptonizes the pre-shock soft photons to produce power-law tail. If the shock remains strong then we have the canonical hard state and when the shock becomes weak or disappear we have the canonical soft state. Moreover, Molteni et al. (1996b) showed that if the shock oscillates, it does with a frequency $\omega \sim x_{s}^{-\beta}$, where $\beta = 1 \rightarrow 3/2$. If the shock oscillates then the hard radiation from it would oscillate with the same frequency and could explain the mHz to a few tens of Hz QPO observed in stellar mass BH candidates. Outburst phase in microquasars starts with low-frequency QPOs in hard state, but as the source moves to intermediate states the QPO frequency increases to a maximum and then goes down in the declining phase, a fact well explained by approaching oscillating $x_{s}$ with the increase in viscosity (Chakrabarti et al. 2009). Our steady-state model also shows that if every other conditions at the outer boundary are same, then $x_{s}$ decreases with the increase in $\alpha$, so we expect with the increase in viscosity the shock oscillation too will increase, and therefore the shock oscillation model of QPO seems to follow observations.

We have computed the mass outflow rate from both non-dissipative and dissipative shocks. The mass outflow rate is always of higher entropy than the pre-shock disc, which shows that mass-outflow rate is a natural consequence of a shocked accretion disc, a fact readily supported by various multi-dimensional simulations. In this connection we would like to comment that unlike Chattopadhyay & Das (2007) and Das & Chattopadhyay (2008), we have corrected the jet cross-sectional area with the projection effect on to the jet streamline. We showed that $R_{J}$ generally decreases with $f_{e}$. We also showed that the parameter space is significantly modified due to the presence of mass loss and dissipation at the shock. When dissipative shocks are included we do see that the relative mass outflow rate decreases which is also to be expected. Increasing dissipation would make the shocks weaker, which can be identified with the softer spectral state, and the accretion disc in the soft state will give weaker or no outflow (Gallo et. al. 2003). Comparison of steady shock parameter space (Figs 6 and 9) suggests that mass loss may trigger an instability. It shows that parts of parameter space that produced steady shocks in the absence of mass loss do not show steady shocks in the presence of mass loss. This is because the post-shock pressure gets reduced as it loses mass, and the shock moves closer to the BH (Fig. 7) to regain the momentum balance. However, this is not always possible, and the shock may oscillate or get disrupted altogether; therefore, there should be a mass loss induced instability as well. We have also shown that the main driver for the bipolar outflow is the energy gained through shock. Interestingly, the mass outflow rate generally increases with the increasing viscosity parameter. Since the shock location also decreases with the increasing $\alpha$ for identical outer boundary condition, there is a possibility that the QPOs and mass outflow rates be correlated – an issue we will pursue in fully time-dependent studies.

It is interesting to note that the dissipation parameter used in this work has been assumed constant; however this would actually depend on accretion rates and the size of the post-shock region. As the accretion rate changes the total photons generated would change, and similarly as $x_{s}$ changes the fraction of photons intercepted by post-shock disc as well as its optical depth would change; this would render $f_{e}$ variable.

It has been observationally established that steady jets are observed from BH candidates, when the spectrum of the disc is in the hard state (Gallo et al. 2003). In our model, the presence of strong shock is similar to the hard state. We have shown that with the increasing $\alpha$, we get stronger jet; however the evolution of jet states with spectral states is a time-dependent phenomenon. Moreover, the phenomena of QPO and the growth or decay of QPO are time-dependent phenomenon too. Our conjecture that the QPO will be correlated with the jet states can only be vindicated through a

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**Figure 13.** $E - \lambda_{0}$ shock parameter space for (a) $\alpha = 0.05$, and each curve is for $f_{s} = 1$, $f_{e} = 0$ (solid), $f_{s} > 1$, $f_{e} = 0$ (dotted), $f_{s} > 1$, $f_{e} = 0.05$ (dashed), and $f_{s} > 1$, $f_{e} = 0.2$ (long dashed), and (b) $f_{s} > 1$, $f_{e} = 0.1$, and each curve is for $\alpha = 0.0$ (solid), 0.05 (dotted), 0.1 (dashed) and 0.15 (long dashed).
fully time-dependent study, which is beyond the scope of this paper. Furthermore, since we concentrated on the effect of viscosity on accretion disc and outflows, therefore, magnetic field and other realistic cooling processes have been ignored. If cooling processes are considered then a direct comparison with the observation will be possible. As has been noted that a little bit of magnetic field will have an important effect on the dynamics of the flow (Proga 2005), however transonicity criteria will be important too. Because a magnetized flow may possess fast, slow or Alfvénic waves, the number of sonic points may increase (Takahashi et al. 2006). If magnetized flows admit shocks, then the shock produced may be even more robust. This hydrodynamic model might well act as the simpler version of the magneto fluid model. We are working on dynamics of magnetized flows and would be reported elsewhere.

The concrete conclusions we draw from this paper is the following. Viscosity is important and affects the accretion solutions both quantitatively and qualitatively. Shock in accretion can be obtained for fairly high viscosity parameter. Shocks naturally produce outflows, and for fixed outer boundary conditions of the disc, shock location decreases but mass outflow rate generally increases. This augurs well for the model as this is exactly observed in hard to intermediate hard spectral transitions. However, in the presence of dissipative shocks the mass outflow rate decreases. Over all, we see that $\dot{m}$ may vary between a few per cent to more than 10 per cent, although in the presence of dissipative shocks, the estimate of $\dot{m}$ is about a few per cent. We have also computed the shock parameter space for accreting flows without mass loss and dissipation, with mass loss but no dissipation and with mass loss and dissipation and have shown that the parameter space for steady shocks shrinks with increasing dissipation.

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