Abstract

The spin asymmetry parameter $A_\tau$ characterizing the angular distribution of the total hadron momentum in the decay of a polarized tau can be calculated rigorously using perturbative QCD and the operator product expansion. Perturbative QCD corrections to the free quark result $A_\tau = 1/3$ can be expressed as a power series in $\alpha_s(M_\tau)$ and nonperturbative QCD corrections can be expanded systematically in powers of $1/M_\tau^2$. The QCD prediction is $A_\tau = 0.41 \pm 0.02$. In the decay of a high energy tau into hadrons, the value of the hadronic energy distribution $dR_\tau/dz$ evaluated at the maximum hadronic energy fraction $z = 1$ can also be calculated rigorously from QCD.
The spin-dependence of processes involving elementary particles contains a wealth of information about their fundamental interactions. Unfortunately this information is not easily accessible to experiment. It requires either the use of polarized beams and targets, or the measurement of the polarization of final state particles. The tau lepton is one of the few elementary particles whose spin can be effectively analyzed by its decays. It is well known that several of the exclusive decay modes of the tau can be used to analyze its spin \cite{1, 2}. In this Letter, I point out that inclusive decays into hadrons can also be used for this purpose. The polarization of a sample of taus that decay into hadrons can be determined from the angular distribution of the total momentum of the hadrons. The asymmetry parameter that characterizes the angular distribution can be computed systematically using perturbative QCD and the operator product expansion. In a sample of high energy taus that decay into hadrons, the endpoint value of the hadronic energy distribution can also be computed rigorously from QCD, and can be used to measure the average helicity of the taus. Measurements of these spin-dependent observables could provide dramatic confirmation of the applicability of perturbative QCD to the inclusive hadronic decays of the tau.

It is convenient to normalize the inclusive decay rate of the tau lepton into a neutrino plus hadrons to the electronic decay rate by defining the ratio

\[ R_\tau = \frac{\Gamma(\tau^- \to \nu_\tau + \text{hadrons})}{\Gamma(\tau^- \to \nu_\tau e^- \bar{\nu}_e)}. \]  

For a sample of taus with polarization $P$, the angle $\theta$ between the total momentum of the hadrons in the tau rest frame and the spin quantization axis has a distribution proportional to $1 + A_\tau P \cos \theta$, where $A_\tau$ is an asymmetry parameter. This angular distribution can be used to separate $R_\tau$ into “forward” and “backward” components $R_F$ and $R_B$:

\[ \frac{dR_\tau}{d\cos \theta} = R_F \frac{1 + P \cos \theta}{2} + R_B \frac{1 - P \cos \theta}{2}. \]  

The asymmetry parameter $A_\tau$ is then

\[ A_\tau = \frac{R_F - R_B}{R_F + R_B}. \]  

A naive estimate of the asymmetry parameter can be obtained by considering the decay of the tau into hadrons at the quark level, where it proceeds through the processes
\[ \tau^- \to \nu_\tau d\bar{u} \text{ and } \tau^- \to \nu_\tau s\bar{u}. \] The momentum of the \( d\bar{u} \) and \( s\bar{u} \) pairs can be identified with the total momentum of the hadrons. Ignoring the QCD interactions that bind the quarks into color singlet hadrons, the angular distribution of the total hadron momentum is

\[ \frac{dR_\tau}{d \cos \theta} \approx \frac{3}{2} \left( |V_{ud}|^2 + |V_{us}|^2 \right) \left( 1 + \frac{1}{3} P \cos \theta \right). \] (4)

The squares of the Kobayashi-Maskawa matrix elements add up to 1 to high accuracy, so they will be omitted below. The naive estimates for the ratio (4) and the asymmetry parameter (3) are \( R_\tau = 3 \) and \( A_\tau = 1/3 \).

In the case of \( R_\tau \), the QCD corrections to the naive result can be computed systematically using perturbative QCD and the operator product expansion [3, 4, 5]. The perturbative corrections can be expanded as a power series in \( \alpha_s(M_\tau) \) [6] and the nonperturbative corrections can be organized systematically into an expansion in powers of \( 1/M_\tau^2 \). A thorough analysis of the QCD and electroweak corrections to the ratio \( R_\tau \) has recently been carried out [7]. The methods that were used to calculate the ratio \( R_\tau \) can also be used for a rigorous calculation of the asymmetry parameter \( A_\tau \). The starting point is an expression for the angular distribution of the total hadron momentum as an integral over the invariant mass \( s \) of the hadrons:

\[ \frac{dR_\tau}{d \cos \theta} = 6\pi \int_{M_\tau^2}^{M_\tau^2} \frac{ds}{M_\tau^2} \left( 1 - \frac{s}{M_\tau^2} \right)^2 \left( \text{Im}\Pi^{(1)}(s + i\epsilon) \left( 1 + P \cos \theta + 2 \frac{s}{M_\tau^2}(1 - P \cos \theta) \right) \right. \]

\[ \left. + \text{Im}\Pi^{(0)}(s + i\epsilon)(1 + P \cos \theta) \right), \] (5)

where \( \Pi^{(J)}(s) \), \( J = 0, 1 \) are the transverse and longitudinal correlators for the quark current that couples to the virtual \( W \) boson. The notation is the same as in Ref. [7]. The correlators \( \Pi^{(J)}(s) \) are analytic functions of \( s \) except along the positive real \( s \)-axis. This allows the integral in (5) to be expressed as a contour integral in the complex \( s \)-plane. The contour can be deformed so that it runs counterclockwise around the circle \( |s| = M_\tau^2 \). The resulting expressions for the forward and backward components of \( R_\tau \) defined in (2) are

\[ R_F = -\frac{12\pi^2}{2\pi i} \int_{|s|=M_\tau^2} \frac{ds}{M_\tau^2} \left( 1 - \frac{s}{M_\tau^2} \right)^2 \left( \Pi^{(1)}(s) + \Pi^{(0)}(s) \right), \] (6)
\[ R_B = \frac{-12\pi^2}{2\pi i} \int_{|s|=M_T^2} ds \left( 1 - \frac{s}{M_T^2} \right)^2 \left( 2 \frac{s}{M_T^2} \Pi^{(1)}(s) \right). \]  

(7)

The contour integral expressions (6) and (7) reveal that the polarization asymmetry \( A_\tau \), like the ratio \( R_\tau \), is completely determined by correlation functions at the distance scale \( 1/M_\tau \). This implies that the nonperturbative long distance effects of QCD can be expressed in terms of matrix elements of local operators. These matrix elements appear when the operator product expansion is used to expand the correlators \( \Pi^{(f)}(s) \) in (6) and (7) in powers of \( 1/s \). Evaluating the contour integrals, the QCD corrections to the naive predictions \( R_F = 2 \) and \( R_B = 1 \) are obtained as systematic expansions in powers of \( 1/M_\tau^2 \). There is also an important electroweak correction consisting of a multiplicative short distance factor \( S_{EW} = 1.019 \) [8].

The resulting expressions for the forward and backward components of \( R_\tau \) have the form

\[ R_F = 2S_{EW} \left( 1 + \delta_F^{(0)} + \delta_F^{(2)} + \delta_F^{(4)} + \delta_F^{(6)} + \ldots \right), \]

(8)

\[ R_B = S_{EW} \left( 1 + \delta_B^{(0)} + \delta_B^{(2)} + \delta_B^{(4)} + \delta_B^{(6)} + \ldots \right), \]

(9)

where the fractional corrections \( \delta^{(n)}_F \) and \( \delta^{(n)}_B \) are proportional to \( 1/M_\tau^n \) with coefficients that depend logarithmically on \( M_\tau \). For \( R_\tau = R_F + R_B \), the fractional corrections to the free quark value \( 3S_{EW} \) are \( (2\delta^{(n)}_F + \delta^{(n)}_B)/3 \).

The fractional corrections \( \delta^{(n)}_F \) and \( \delta^{(n)}_B \) can be calculated straightforwardly using the operator product expansions for the correlators \( \Pi^{(f)}(s) \) that are collected in Ref. [7]. The dimension-0 corrections, which represent the purely perturbative effects from the interactions of massless quarks and gluons, are

\[ \delta^{(0)}_F = \frac{\alpha_s}{\pi} + 5.765 \left( \frac{\alpha_s}{\pi} \right)^2 + 34.48 \left( \frac{\alpha_s}{\pi} \right)^3 + (d_4 + 165.1) \left( \frac{\alpha_s}{\pi} \right)^4, \]

(10)

\[ \delta^{(0)}_B = \frac{\alpha_s}{\pi} + 4.077 \left( \frac{\alpha_s}{\pi} \right)^2 + 10.13 \left( \frac{\alpha_s}{\pi} \right)^3 + (d_4 - 96.1) \left( \frac{\alpha_s}{\pi} \right)^4, \]

(11)

where \( \alpha_s = \alpha_s(M_\tau) \) is the running coupling constant of QCD in the \( \overline{MS} \) scheme evaluated at the scale \( M_\tau \). The coefficient \( d_4 \) in the \( \alpha_s^4 \) term in (10) and (11) is the fourth coefficient in the perturbative expansion of \( -2\pi^2 s(d/ds)\Pi^{(1)}(s) \) in powers of \( \alpha_s/\pi \) and has not been calculated.
The previous coefficients are 1, 1, 1.64, and 6.37 \[9\]. We assign a very conservative error to this unknown coefficient: \(d_4 = 0 \pm 100\). The corresponding coefficient in the fractional correction to \(R_\tau\) is then 78.0 \(\pm\) 100. The dimension-2 corrections are perturbative corrections due to the running quark masses. The only correction that is numerically significant comes from the strange quark mass \(m_s = m_s(M_\tau)\):

\[
\delta_F^{(4)} = -9 \sin^2 \theta_C \left(1 + \frac{16 \alpha_s}{3 \pi}\right) \frac{m_s^2}{M_\tau^2},
\]

\[
\delta_B^{(4)} = -6 \sin^2 \theta_C \left(1 + \frac{16 \alpha_s}{3 \pi}\right) \frac{m_s^2}{M_\tau^2},
\]

where \(\theta_C\) is the Cabbibo mixing angle: \(\sin^2\theta_c = 0.049\). For the running strange quark mass in the \(\overline{\text{MS}}\) scheme evaluated at the scale \(M_\tau\), we take the value \(m_s(M_\tau) = (0.17 \pm 0.02)\) GeV.

The first nonperturbative corrections appear at dimension 4 in the form of scale invariant matrix elements called the gluon condensate and quark condensates:

\[
\delta_F^{(4)} = 2\pi^2 \left(1 - \frac{11 \alpha_s}{18 \pi}\right) \frac{\langle\alpha_s/\pi)GG\rangle}{M_\tau^4} + 48\pi^2 \frac{\langle m\bar{\psi}\psi\rangle}{M_\tau^4} - \frac{72}{7} \sin^2 \theta_C \frac{\pi}{\alpha_s} \frac{m_s^4}{M_\tau^4},
\]

\[
\delta_B^{(4)} = -4\pi^2 \left(1 - \frac{11 \alpha_s}{18 \pi}\right) \frac{\langle\alpha_s/\pi)GG\rangle}{M_\tau^4}.
\]

Note that the contribution of the gluon condensate \(\langle\alpha_s/\pi)GG\rangle\) cancels to order \(\alpha_s\) in the fractional correction \((2\delta_F^{(4)} + \delta_B^{(4)})/3\) to the ratio \(R_\tau\). This results in a suppression of the gluon condensate contribution to \(R_\tau\) by two orders of magnitude. We take the value of the gluon condensate to be \(\langle\alpha_s/\pi)GG\rangle = (2 \pm 1) \times 10^{-2}\) GeV\(^4\) \[10\]. The matrix element \(\langle m\bar{\psi}\psi\rangle\) in (14) is a weighted average of the quark condensates:

\[
\langle m\bar{\psi}\psi\rangle = \frac{\langle m_u\bar{u}u\rangle + \cos^2 \theta_C \langle m_d\bar{d}d\rangle + \sin^2 \theta_C \langle m_s\bar{s}s\rangle}{2}.
\]

Its value is \(\langle m\bar{\psi}\psi\rangle = (-8 \pm 1) \times 10^{-5}\) GeV\(^4\). The inverse power of \(\alpha_s(M_\tau)\) multiplying the \(m_s(M_\tau)^4\) term in (14) was first understood by Broadhurst and Generalis \[11\]. At dimension 6, there are too many unknown matrix elements for a completely systematic treatment.
Within the vacuum saturation approximation, these corrections are

\[ \delta_F^{(6)} = \frac{256\pi^3 \rho \alpha_s \langle \bar{\psi} \psi \rangle^2}{27 M_T^6}, \]  
\[ \delta_B^{(6)} = -\frac{2048\pi^3 \rho \alpha_s \langle \bar{\psi} \psi \rangle^2}{27 M_T^6}, \]

with \( \rho = 1 \). It has been found empirically that this approximation underestimates the size of the dimension-6 correction, and it is better to treat \( \rho \alpha_s \langle \bar{\psi} \psi \rangle^2 \) as an effective scale-invariant operator of dimension 6, independent of \( \langle \bar{\psi} \psi \rangle \). The best estimate for this parameter is \( \rho \alpha_s \langle \bar{\psi} \psi \rangle^2 = (4 \pm 2) \times 10^{-4} \text{ GeV}^6 \). The dimension-8 and higher corrections are assumed to be completely negligible.

Inserting the fractional corrections given above into (8) and (9), we obtain predictions for the ratio \( R_\tau = R_F + R_B \) and the asymmetry parameter \( A_\tau \) defined in (3) as a function of \( \alpha_s(M_\tau) \) and the five parameters \( d_4, m_s(M_\tau), \langle (\alpha_s/\pi)GG \rangle, \langle m_\bar{\psi}\psi \rangle, \) and \( \rho \alpha_s \langle \bar{\psi} \psi \rangle^2 \). Alternatively, given a value for \( R_\tau \), we can predict both \( \alpha_s(M_\tau) \) and \( A_\tau \). The predictions are shown in Table 1. The uncertainty in \( \alpha_s(M_\tau) \) is dominated by the assumed error of \( \pm 100 \) in the coefficient \( d_4 \). For \( R_\tau = 3.60 \), the uncertainty in \( \alpha_s(M_\tau) \) is 3.6%. After \( d_4 \), the next largest errors are 1.1% from \( \rho \alpha_s \langle \bar{\psi} \psi \rangle^2 \) and 0.4% from \( m_s(M_\tau) \). The uncertainty in \( A_\tau \) is dominated by the gluon condensate, and is 5.4% for \( R_\tau = 3.60 \). The next largest errors are 1.8% from \( \rho \alpha_s \langle \bar{\psi} \psi \rangle^2 \) and 0.7% from \( d_4 \).

There are two independent ways of measuring the ratio \( R_\tau \) experimentally. Using the universality of electron and muon couplings, it can be expressed in terms of the electronic branching fraction \( B_e \) of the tau: \( R_\tau = \frac{1}{B_e} - 1.973 \). Alternatively, using the universality of electron and tau couplings as well, it can be expressed in terms of the masses and lifetimes of the \( \mu \) and \( \tau \): \( R_\tau = \frac{(\tau_\mu/\tau_\tau)(M_\mu/M_\tau)^5}{(\tau_\mu/\tau_\tau)(M_\mu/M_\tau)^5} - 1.973 \). The present world average for the electronic branching fraction is \( B_e = (17.78 \pm 0.15)\% \) [12], and it gives the ratio \( R_\tau = 3.651 \pm 0.047 \). The present world average for the tau lifetime is \( \tau_\tau = (2.96 \pm 0.03) \times 10^{-13} \text{ s} \) [12]. Combined with the recent precise measurement of the tau mass [13], it gives the ratio \( R_\tau = 3.545 \pm 0.056 \). Forming the weighted average of the two independent determinations of \( R_\tau \), we get \( R_\tau = 3.607 \pm 0.036 \). From Table 1, this determines the running coupling constant at the scale...
to be $\alpha_s(M_\tau) = 0.319 \pm 0.017$. We have added in quadrature the error from Table 1 and the error due to the experimental uncertainty in $R_\tau$. Evolving the running coupling constant up to the $Z^0$ mass, it becomes $\alpha_s(M_Z) = 0.1176 \pm 0.0021$. The QCD prediction for the asymmetry parameter is $A_\tau = 0.413 \pm 0.022$. A measurement of $A_\tau$ consistent with this prediction would provide dramatic confirmation of the accuracy of the QCD calculation of the ratio $R_\tau$.

The angular distribution $dR_\tau/d\cos\theta$ that defines the asymmetry parameter $A_\tau$ is most easily measured in low energy experiments. In high energy experiments, such as the decay of the $Z^0$ into taus, the quantity that is most easily measured is the distribution $dR_\tau/dz$ of the hadron energy fraction $z = E_H/E_\tau$, where $E_H$ is the total energy of the hadrons in the rest frame of the $Z^0$ and $E_\tau$ is the energy of the decaying tau. If the spin quantization axis is chosen to lie along the direction of the tau momentum, then the variables $z$ and $\cos\theta$ are related by

$$
\cos\theta = \frac{(2z - 1)M_\tau^2 - s}{M_\tau^2 - s}
$$

for a hadronic state with invariant mass $s$ [2]. Changing variables in (5) from $\cos\theta$ to $z$, we obtain an expression for the hadronic energy distribution $D_P(z) = dR_\tau/dz$ for a tau with polarization $P$ in the helicity basis. At the endpoint $z = 1$, it reduces to

$$
D_P(z = 1) = 12\pi \int_0^{M_\tau^2} \frac{ds}{M_\tau^2} \left(1 - \frac{s}{M_\tau^2}\right) \left(\text{Im}\Pi^{(1)}(s + i\epsilon) \left(1 + P + 2\frac{s}{M_\tau^2}(1 - P)\right) + \text{Im}\Pi^{(0)}(s + i\epsilon)(1 + P)\right).
$$

(20)

We define the energy asymmetry function $A(z)$ by

$$
D_P(z) = D_0(z) \left(1 + A(z)P\right),
$$

(21)

where $D_0(z)$ is the energy distribution for an unpolarized tau.

By the same arguments that were used for $R_\tau$ and $A_\tau$, the quantities $D_0(1)$ and $A(1)$ can be calculated rigorously using QCD. Perturbative corrections can be expanded as a power series in $\alpha_s(M_\tau)$, and nonperturbative corrections can be expanded systematically...
in powers of $1/M^2$. The free quark prediction are $D_0(1) = 5$ and $A(1) = 1/5$. We express the QCD corrections in terms of fractional corrections $\delta_R^{(n)}$ and $\delta_L^{(n)}$ which scale like $1/M^n$:

$$D_{P=+1}(z = 1) = 6 \, S_{EW} \left( 1 + \delta_R^{(0)} + \delta_R^{(2)} + \delta_R^{(4)} + \delta_R^{(6)} + \ldots \right),$$

$$D_{P=-1}(z = 1) = 4 \, S_{EW} \left( 1 + \delta_L^{(0)} + \delta_L^{(2)} + \delta_L^{(4)} + \delta_L^{(6)} + \ldots \right).$$

The fractional corrections due to the perturbative interactions of massless quarks and gluons are

$$\delta_R^{(0)} = \frac{\alpha_s}{\pi} + 5.015 \left( \frac{\alpha_s}{\pi} \right)^2 + 24.50 \left( \frac{\alpha_s}{\pi} \right)^3 + (d_4 + 68.7) \left( \frac{\alpha_s}{\pi} \right)^4,$$

$$\delta_L^{(0)} = \frac{\alpha_s}{\pi} + 3.515 \left( \frac{\alpha_s}{\pi} \right)^2 + 4.54 \left( \frac{\alpha_s}{\pi} \right)^3 + (d_4 - 123.9) \left( \frac{\alpha_s}{\pi} \right)^4.$$  

The fractional corrections from the running strange quark mass are

$$\delta_R^{(2)} = -6 \, \sin^2 \theta_C \left( 1 + \frac{13 \, \alpha_s}{3 \, \pi} \right) \frac{m_s^2}{M_T^2},$$

$$\delta_L^{(2)} = -\frac{9}{2} \, \sin^2 \theta_C \left( 1 + \frac{14 \, \alpha_s}{3 \, \pi} \right) \frac{m_s^2}{M_T^2}.$$  

At the order in $\alpha_s(M_T)$ to which we have calculated the coefficients of the matrix elements, the fractional corrections $\delta_R^{(n)}$ and $\delta_L^{(n)}$, $n = 4, 6$, are related in a simple way to the fractional corrections to $R_F$ and $R_B$. The dimension-4 corrections are $\delta_R^{(4)} = \delta_F^{(4)}/3$ and $\delta_L^{(4)} = \delta_B^{(4)}/2$ and the dimension-6 corrections are $\delta_R^{(6)} = 0$ and $\delta_L^{(6)} = \delta_B^{(6)}/4$. The QCD predictions for $D_0(1)$ and $A(1)$ as a function of $R_T$ are presented in Table 1. The error on $D_0(1)$ is remarkably small, only 0.4% for $R_T = 3.60$. The dominant errors are 0.3% from $<(\alpha_s/\pi)GG>$, 0.3% from $d_4$, and 0.1% from $\rho \alpha_s <\bar{\psi}\psi>^2$. The error in $A(1)$ is dominated by the gluon condensate and is 4.5% for $R_T = 3.60$. The next largest errors are 1.1% from $d_4$ and 0.9% from $\rho \alpha_s <\bar{\psi}\psi>^2$. Taking $R_T = 3.607 \pm 0.036$ and combining the error from Table 1 in quadrature with the error due to $R_T$, the QCD predictions are $D_0(1) = 5.853 \pm 0.051$ and $A(1) = 0.246 \pm 0.011$.

The QCD predictions for $A_T$ and for the high energy parameters $D_0(1)$ and $A(1)$ are sufficiently precise that measurements of these parameters could be used to determine the
polarization of a sample of taus. The polarization of the taus produced in the decay of the 
$Z^0$ has been measured using the energy distributions of the exclusive decay modes $e^-\bar{\nu}_e\nu_\tau$, 
$\mu^-\bar{\nu}_\mu\nu_\tau$, $\pi^-\nu_\tau$, $\rho^-\nu_\tau$, and $a_1^-\nu_\tau$ [14]. The measurement of the endpoint value $D_0(1)(1 + 
A(1)P)$ of the inclusive hadronic energy distribution might be competitive as a method 
for determining the tau polarization $P$, since it is less sensitive to errors due to particle 
identification.

Precise measurements of the asymmetry parameters could also be used to determine 
the nonperturbative matrix elements that arise in the operator product expansion. The 
errors in $A_\tau$ and $A(1)$ are dominated by the gluon condensate $< (\alpha_s/\pi)GG >$, with the other 
errors being smaller by a factor of 3. Thus precision measurements of $A_\tau$ and $A(1)$ could 
improve the determination of the gluon condensate by about a factor of 3.

In this Letter, we have introduced several new observables involving hadronic decays 
of the tau lepton that can be calculated rigorously from QCD. The only ones that were 
known previously were the ratio $R_\tau$ and the moments of the invariant mass distribution 
$dR_\tau/ds$ [4, 15]. The endpoint value of the hadronic energy distribution $dR_\tau/dz$ in the decay 
of a high energy tau can also be calculated rigorously, as can the moments of the energy 
distribution [16]. For each of these quantities, there is also a polarization asymmetry that 
can be calculated. The QCD predictions for the quantities $A_\tau$, $D_0(1)$, and $A(1)$ differ from 
the free quark predictions by about 20%, which is much larger than the errors in the QCD 
predictions. Thus hadronic decays of the tau lepton provide a remarkable laboratory in 
which QCD can be tested at low energies with unprecedented precision.

This work was supported in part by the U.S. Department of Energy, Division of High 
Energy Physics, under Grant DE-FG02-91-ER40684.

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Table Captions

1. QCD predictions for $\alpha_s(M_\tau)$, $A_\tau$, $D_0(1)$, and $A(1)$ as a function of the ratio $R_\tau$. The errors due to variations of $d_4$, $m_s(M_\tau)$, $<(\alpha_s/\pi)GG>$, $<m\bar{\psi}\psi>$, and $\rho\alpha_s<\bar{\psi}\psi>^2$ have been added in quadrature.

| $R_\tau$ | $\alpha_s(M_\tau)$ | $A_\tau$   | $D_0(1)$   | $A(1)$   |
|---------|---------------------|------------|------------|----------|
| 3.50    | 0.287 ± 0.009       | 0.407 ± 0.023 | 5.735 ± 0.022 | 0.241 ± 0.011 |
| 3.52    | 0.294 ± 0.009       | 0.408 ± 0.023 | 5.761 ± 0.022 | 0.242 ± 0.011 |
| 3.54    | 0.301 ± 0.010       | 0.409 ± 0.023 | 5.786 ± 0.023 | 0.243 ± 0.011 |
| 3.56    | 0.308 ± 0.010       | 0.411 ± 0.023 | 5.812 ± 0.023 | 0.245 ± 0.011 |
| 3.58    | 0.315 ± 0.011       | 0.412 ± 0.023 | 5.837 ± 0.024 | 0.246 ± 0.011 |
| 3.60    | 0.321 ± 0.012       | 0.413 ± 0.022 | 5.862 ± 0.025 | 0.247 ± 0.011 |
| 3.62    | 0.328 ± 0.012       | 0.414 ± 0.022 | 5.886 ± 0.026 | 0.248 ± 0.011 |
| 3.64    | 0.334 ± 0.013       | 0.415 ± 0.022 | 5.911 ± 0.027 | 0.250 ± 0.011 |
| 3.66    | 0.340 ± 0.013       | 0.417 ± 0.022 | 5.935 ± 0.028 | 0.251 ± 0.011 |
| 3.68    | 0.346 ± 0.014       | 0.418 ± 0.022 | 5.959 ± 0.029 | 0.252 ± 0.011 |
| 3.70    | 0.352 ± 0.015       | 0.419 ± 0.022 | 5.983 ± 0.031 | 0.254 ± 0.011 |

Table 1