Tetraneutron: Rigorous continuum calculation

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Abstract

The four-neutron system is studied using exact continuum equations for transition operators and solving them in the momentum-space framework. A resonant behavior is found for strongly enhanced interaction but not a the physical strength, indicating the absence of an observable tetraneutron resonance, in contrast to a number of earlier works. As the transition operators acquire large values at low energies, it is conjectured that this behavior may explain peaks in many-body reactions even without a resonance.

Key words: Four-body scattering, transition operators, resonance, universality

1. Introduction

The four-neutron (4n) system is an exotic few-body system challenging experimental techniques as well as theoretical understanding of the nuclear force and methods for the description of the few-particle continuum. It has attracted a great interest in the last few years [1–5], but, nevertheless, remains highly controversial. An experimental observation of few events in the double charge-exchange reaction \(^{4}\)He\(^{(8}\)He, \(^{8}\)Be), that were interpreted as a formation of a tetraneutron resonance with the energy \(E_r = 0.83 \pm 0.65\) (stat) \(\pm 1.25\) (syst) MeV and width \(\Gamma \leq 2.6\) MeV [1], still awaits a confirmation in the analysis of further experiments. The theoretical predictions for the tetraneutron are even more contradictory: They range from a narrow near-threshold resonance with \(E_r \approx 0.8\) MeV and \(\Gamma \approx 1.4\) MeV [2] or \(E_r \approx 2.1\) MeV [3] to a broad resonance with \(E_r \approx 7.3\) MeV and \(\Gamma \approx 3.7\) MeV [4] while other authors [5,6] predict no observable tetraneutron resonance at all, i.e., negative \(E_r\) and very large \(\Gamma\). Despite these differences, all above works concluded that tetraneutron properties are insensitive to the details of the neutron-neutron (\(nn\)) and three-neutron (\(3n\)) interaction models as long as they remain realistic. Thus, those very different predictions cannot be explained by differences in employed potentials but raise question on the reliability of at least some of the above calculations. Indeed, the 4n system resides in the continuum whose exact treatment is much more complicated as compared to bound states.

However, among the above-mentioned works only the solution of the complex-scaled Faddeev-Yakubovsky (FY) equations [5,6] treats the continuum rigorously; if no further simplifications are made unlike in Ref. [4] this applies also the no-core Gamow shell model. In contrast, the harmonic oscillator representation of the continuum [2] and the bound-state quantum Monte Carlo with the extrapolation to the continuum [3] approaches are not natural methods for a rigorous description of the four-particle continuum. In fact, none of the approaches from Refs. [2–4] has been applied successfully to other four-nucleon scattering processes, in contrast to FY equations [7]. However, the only method that so far provided reliable results for all four-nucleon reactions above the complete breakup threshold, i.e., for elastic, charge-exchange, transfer, and breakup processes in nucleon-trinucleon and deuteron-deuteron collisions, is the momentum-space transition operator method [8,9]. Furthermore, it provided the most accurate results in the field of the universal four-fermion [10] and four-boson [11] physics, including the properties of resonant (unstable) four-particle states. The method is an exact integral version of FY equations [12] proposed by Alt, Grassberger, and Sandhas (AGS) [13,14]. Its application to the 4n problem is highly desirable, since being a rigorous continuum method it should provide reliable conclusions regarding the tetraneutron resonance, much like in the case of the 3n system [15], where it clearly supported the earlier conclusion [5,16] on the 3n resonance unobservability in contrast to Ref. [3]. Another very important advantage of the transition operator method is its ability to determine not only the resonance position and width but also the nonreso-
nant (background) contribution to scattering amplitudes, thereby making solid conclusions regarding the resonance observability in physical processes.

2. Theory

AGS equations for four-nucleon transition operators have been applied to the study of reactions initiated by all possible two-cluster collisions [8,9]. The situation is different in the 4n system that has no bound subsystems and the only possible reaction is the elastic scattering of four free particles. Starting from Ref. [17], the operator for this 4 → 4 process can be split into two-, three-, and four-particle components, i.e.,

\[ T_{4\rightarrow 4} = \sum_j t_j + \sum_{j \beta} t_j G_0 U_{j \beta}^{ji} G_0 t_i + \sum_{j \beta \alpha} T_{j \beta \alpha}^{ji} \]

(1)

Here Latin (sub)superscripts denote pairs while Greek sub- scripts denote two-cluster partitions (subsystems) that can be of 3+1 or 2+2 type. Furthermore, \( G_0 \) is the free resolvent at the available energy \( E \), \( t_j = v_j + v_j G_0 t_j \) are the pair transition operators with pair potentials \( v_j \), and \( U_{j \beta}^{ji} = G_0^{-1} \delta_{ji} + \sum_k \delta_{jk} t_k G_0 U_{\gamma k}^{ji} \) are the subsystem transition operator where \( \delta_{ji} = 1 - \delta_{ji} \). The four-particle transition operators obey the system of integral equations

\[ T_{j \beta \alpha}^{ji} = \sum_k t_j G_0 U_{j \beta k}^{ji} \delta_{\beta \alpha} G_0 t_k G_0 U_{\gamma k}^{ji} G_0 t_i + \sum_{\gamma k} t_j G_0 U_{j \beta k}^{ji} G_0 \delta_{\beta \gamma} T_{\gamma \alpha}^{ki} \]

(2)

Taking into account identity of neutrons the equations (2) can be symmetrized, reducing the number of \( j \beta \) components from 18 to just two, one being of the 3+1 type and another of the 2+2 type; in the following they will be abbreviated by subscripts 1 and 2, respectively. For example, four-neutron operators \( T_{j \beta}^{ji} \) are obtained from integral equations

\[ T_{12} = t G_0 U_1 G_0 t G_0 U_2 G_0 t + t G_0 U_1 G_0 (T_{22} - P_{34} T_{12}), \]

(3a)

\[ T_{22} = t G_0 U_2 G_0 (1 - P_{34}) T_{12}, \]

(3b)

where \( P_{34} \) is the permutation operator of particles 3 and 4, while \( t \) and \( U_j \) are symmetrized pair and subsystem operators, respectively [18]. Kernel of Eqs. (3) is built from the same operators (just in a different order) as in Refs. [8,9,18] for two-cluster reactions. Thus, the solution technique to a large extent can be taken over from Refs. [8,9,18]. It is performed in the momentum-space partial-wave representation [18], whereas kernel singularities arising from \( G_0 \) are treated by the complex-energy method with special integration weights [19]. As the four-matrix elements exhibit stronger dependence on the imaginary part \( \varepsilon \) of the energy, smaller values 0.1 MeV ≤ \( \varepsilon \) ≤ 1 MeV as compared to Refs. [8,9,19] have to be used, which implies larger number (around 80) of grid points for the discretization of Jacobi momenta \( k_x, k_y, \) and \( k_z \) in the notation of Refs. [11,19].

A pure 4n scattering experiment is practically impossible, with presently available experiment techniques one may only indirectly observe 4n as a final subsystem in a more complicated reaction such as \( ^{4}\text{He}(^{8}\text{He}, ^{8}\text{Be}) \). It is complicated many-body process that cannot be described rigorously by presently available methods, however, half-shell matrix elements of \( T_{j \beta \alpha} \) that determine the 4n wave function, together with some simplified reaction model, may provide estimation for the properties of the final 4n subsystem, e.g., its energy distribution. Therefore it is important to evaluate also half-shell matrix elements of \( T_{j \beta \alpha} \).

3. Results

In the following I consider the 4n state with total angular momentum \( J^P = 0^+ \); namely in this state Refs. [2–4] predict the 4n resonance. In order to make comparison with those works, I use chiral effective field theory (χEFT) potential at next-to-leading order (NLO) [20], an improved version of the local NLO potential used in Ref. [3], and a low-momentum potential that should have similar behavior as those used in Refs. [2,4]. It is based on a realistic Argonne V18 potential [21] evolved using the similarity renormalization group (SRG) transformation [22] with the flow parameter \( \lambda = 1.8 \text{ fm}^{-1} \). It is important that this is one of few models able to reproduce quite well not only the \(^3\text{H} \) binding energy but also the cross section for \(^3\text{H} \) scattering in the energy regime with pronounced four-nucleon (3n + proton) resonances [23]. For this reason it can be considered as a well suited model for the 4n resonance study.

In order to follow the evolution of the 4n resonance, I also perform calculations enhancing the \( nn \) potential by a factor \( f > 1 \) in \( nn \) partial waves with the total angular momentum \( j_x < 3 \) except for the \(^{1}\text{S}_0 \) wave where the physical potential strength is kept, ensuring that there are no bound dineutrons. The calculations include \( nn \) partial waves with \( j_x < 3 \) and \( 3n \) partial waves with the total angular momentum \( J_y < \frac{7}{2} \), while subsystem orbital angular momenta are \( l_y, l_z < 5 \). With these cutoffs the results appear to be well converged.

Since the 4n resonance corresponds to the pole of the transition operators \( T_{j \beta \alpha} \) in the complex energy plane at \( E_r - i\Gamma/2 \), these values are extracted from the energy dependence of calculated \( T_{j \beta \alpha} \) matrix elements much like in Ref. [15] for the 3n resonance. In general they are functions of six Jacobi momentum variables, but since all of them exhibit the same resonant behavior, they are shown for few initial and final on-shell and off-shell states only, abbreviated by \( |k_n \rangle \) and \( |k_o \rangle \), respectively. They are chosen with \( l_x = l_y = l_z = 0 \) and two vanishing Jacobi momenta \( k_0 = 0 \) for \( j \neq a \), the remaining one being \( k_a = \sqrt{2\mu_a E} \) and \( k_o = \sqrt{2\mu_o (E + \epsilon_{off})} \), respectively. For example, the state \( |k_z \rangle \) in the 2+2 configuration corresponds to two pairs of neutrons.
with vanishing relative nn momentum, that can be interpreted as a two (unbound) dineutron state. In the above relations \( \mu_a \) is the associated reduced mass while \( \epsilon_{\text{eff}} \) measures how much off-shell the system is. A typical value in the shown results is \( \epsilon_{\text{eff}} = 2 \) MeV that roughly corresponds to \( ^8\text{He} \) binding with respect to the \(^4\text{He} + 4n\) threshold.

4\( n \) transition operators \( T_{\beta\alpha} \) in the \( \mathcal{J}^\Pi = 0^+ \) wave calculated using the physical \( nn \) potential, i.e., \( f = 1 \), show no indications of resonance, but for sufficiently large \( f \) a resonant behavior is clearly seen in all matrix elements; two examples for the SRG potential with \( f = 1 \) and \( f = 5 \) are presented in Fig. 1. The results indicate that nonresonant contributions are very important even at \( f = 5 \) with \( E_r - i\Gamma/2 \approx (5.9 - 0.6i) \) MeV. The \( \mathcal{J}^\Pi = 0^+ \) resonance position and width extracted at different \( f \) values are displayed in Fig. 2. The 4\( n \) system becomes bound at \( f = 5.29 \). Thus, the tetraneutron is lower in energy than the tricenter that in the SRG model becomes bound only at \( f > 6 \) [15]. The results for \( f \geq 5.3 \) obtained solving the standard bound state FY equations connect to \( f \leq 5.3 \) results indicating the consistency between the simpler bound state and much more complicated continuum calculations. Surely, the resonance trajectory depends on the particular enhancement scheme used and therefore is not identical with those in Refs. [5,6]. Nevertheless it exhibits a typical behavior [5,6,16,15]: decreasing the enhancement factor \( f \) the pole first moves to higher energy and away from the real axis until the turning point where \( E_r \) starts to decrease while \( \Gamma \) continues increasing rapidly. For \( f < 4.3 \) the pole of \( T_{\beta\alpha} \) becomes too far from the real axis to be discernible from the nonresonant continuum, which results in increasing theoretical error bars estimated as in Ref. [15]. Thus, an unrealistically large enhancement of the higher-wave potential is needed to support an observable 4\( n \) resonance, which strongly suggests that at the physical interaction strength there is no observable 4\( n \) resonance, in agreement with Refs. [5,6] and in contradiction with Refs. [2–4].

The absence of an observable 4\( n \) resonance with a physical \( nn \) interaction is shown in Fig. 3 over a broader energy range on the example of still another matrix elements of \( T_{\beta\alpha} \). Also predictions with the NLO potential are presented. In fact, the results are almost independent of the force model, as observed also in previous works. Calculations using the CD Bonn potential [24], not shown in Fig. 3, provide an additional confirmation. Furthermore, the results appear to be insensitive to \( P^- \) and higher-wave interaction: SRG calculations including only the \(^1S_0 \) \( nn \) force agree quite well with full SRG results. The dominance of the \( S \)-wave interaction may indicate a manifestation of the four-fermion universality where observables are governed by a large \( nn \) scattering length. This point of view also supports the absence of an observable 4\( n \) resonance since the universal four-fermion system is very far from being bound: a positive scattering length for two difermions in-
nounced peak without a resonance. Nevertheless, to illustrate the possibility of the low-energy enhancement, in the inset of Fig. 3 the squared matrix element of $T_{\beta\alpha}$ multiplied with $k_z$ due to the phase-space factor is plotted; this product would be a factor in the integrand determining the cross section for two (unbound) dineutrons. Indeed, this quantity exhibits a two-peak shape: a sharp and narrow one around 0.25 MeV and a broad one around 4.5 MeV. Note, that peaks are possible even in repulsive systems; a textbook example is given in Ref. [27].

Finally, it is important to understand the difference to Refs. [2–4] that predicted a tetraneutron resonance. Among the approaches used in those works there is also one based on the $nn$ force enhancement by a factor $f$ and subsequent extrapolation of the obtained bound-state energy to the $f = 1$ limit in the continuum. However, Refs. [2–4] apply the same factor $f$ in all $nn$ waves thereby generating a bound $S_0$ dineutron once $f$ exceeds roughly 1.1. Thus, $4n$ states interpreted in Refs. [2–4] as bound tetraneutrons are in fact above the two-dineutron threshold. Strictly speaking, no stable $4n$ bound state is possible above the two-dineutron threshold, only scattering states. Thus, a calculation of $4n$ bound states in the regime above the two-dineutron threshold and extrapolation of their energies is meaningless. A similar situation arises for the $4n$ system in an external trap where a tetraneutron “bound” at the $4n$ threshold [3] is above the dineutron threshold. These simple considerations indicate serious shortcomings in the calculations of Refs. [2–4] and question the reliability of their results.

4. Conclusions

The $4n$ system was studied using one of the most reliable four-nucleon continuum methods. The integral equations for transition operators were solved in the momentum space leading to well-converged results. Strongly enhancing the $nn$ force in higher partial waves the $4n$ model system in the $f^{11} = 0^+$ state was made bound or resonant. In the latter case the resonant behavior was seen in all transition matrix elements, their energy dependence was used to extract the resonance position and width. However, reducing the enhancement factor the resonant behavior disappears well before the physical strength is reached. This indicates the absence of an observable $4n$ resonance, in agreement with Refs. [5,6] and in contradiction with Refs. [2–4], a possible reason being the neglect of the dineutron threshold in the latter works.

Even without an observable $4n$ resonance the transition operators exhibit pronounced low-energy peaks. It is conjectured that they may be seen also in more complicated reactions such as $^4\text{He}^8\text{He}, ^8\text{Be}$ of Ref. [1] with the $4n$ sub-system in the final state. The present calculation of half-shell matrix elements of $4n$ transition operators is a first step toward understanding of those reactions.

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