Measuring the speed of gravitational waves with the distorted pulsars

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The measurement of the speed of gravitational waves (GWs) is useful to distinguish general relativity from massive gravity. We propose a new model-independent strategy to measure the speed of GWs with the distorted pulsars. Theoretically, when the GW frequencies from a distorted pulsar are twice the frequencies of EM pulses, they should be emitted at the same time. By measuring the arrival times of these two signals emitted at the same time, the speed of GWs can be calculated with the time difference. Specifically, when the glitches of pulsars are taken into consideration, some pulsars in our Galaxy and nearby galaxies are potential to test our new strategy at high accuracy in the foreseeable future. On the other hand, the new method is meaningful as a motivation for future design of instruments.

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INTRODUCTION

Gravitational waves (GWs) are transverse waves of spatial strain, which were predicted by Albert Einstein in 1916 \cite{1}. In 1975, Hulse and Taylor verified the existence of GWs indirectly through the observation of the binary pulsar of PSR 1913+16 \cite{11}. Recently, GWs from binary black hole (BH) or binary neutron star mergers events, i.e., GW150914, GW151226, GW170104, GW170814 and GW 170817, are detected by the advanced LIGO/Virgo \cite{2,4}. These great direct discoveries have opened the new era of gravitational wave astronomy.

Since the rest mass of gravitons is zero in the general relativity (GR) theory, then the propagation speed of GWs must equal the speed of light. However, in the context of massive gravity \cite{8}, the alternate theory of gravity, the speed of GWs is predicted to be different from the speed of light and energy-dependent. So the measurement of the speed of GWs will be very helpful to distinguish GR from massive gravity theory. Some authors have proposed different methods to derive the speed of GWs \cite{9,11}. Recently, Reference \cite{12} proposed a strategy to measure the speed of GWs directly with the help of suitable and strongly lensed GWs and their EM counterparts. Their method works by comparing arrival times of GWs and their EM counterparts and by comparing the time delays between images seen in GWs and their EM counterparts. With the latest detection of GWs and EM radiation from GW 170817/GRB 170817A, the fractional speed difference between the speed of GW and light are well constrained according to the model of gamma-ray bursts \cite{13}.

In this paper, we propose a new model-independent strategy to measure the speed of GWs. This new method only requires a single distorted pulsar as the source of GWs and EM pulses. As elaborated in the following sections, one can see that our strategy is concise and not related with theoretical model with uncertainties, furthermore, it is hopeful to be tested with some pulsars in Milky way and nearby galaxies.

METHOD

Bell and Hewish detected the first pulsar in 1967 \cite{14}, which was identified as a spinning NS \cite{15}. It is widely believed that the model of the lighthouse effect can explain the pulsating emission of the pulsars \cite{16}. The EM radiation from a pulsar is two beams along its magnetic axis. General speaking, the magnetic axis does not coincide with the axis of rotation, thus external viewers can only see these beams of radiation when the beams point to them during the neutron star rotation. So the EM signals will appear as periodic pulse radiation, whose frequency is the same as the rotational frequency \( f \) of the pulsars \cite{16}. In addition to the EM radiation, the rapid rotating distorted pulsar can also emit GWs due to time variations of the mass quadruple moment. The NS rotates around its net center of mass, so the frequency of GWs is twice the rotation frequency of the pulsar \cite{17}.

It is noticed that the rotation period \( T = 1/f \) of pulsars is not a constant, but a time dependent parameter. When one detects a EM pulse emitted by a pulsar with frequency \( f \), and a GW signal emitted by the same pulsar with frequency \( 2f \), one can conclude that the EM pulse and the GW signal are emitted at the same time, denoted as \( t_0 \). To simplify, we call this relation between GWs and EM pulse the same time (ST) relation hereafter.

Based on this ST relation, if we can measure the arrival time \( t_{GW} \) of the GW and the arrival time \( t_{EM} \) of the EM
pulsar on the Earth, the speed of GW can be expressed as
\[ V_{GW} = \frac{cD}{D + c[t_{GW} - t_0] - (t_{EM} - t_0)} \]
\[ \approx c \left[ 1 - \frac{c}{D} (t_{GW} - t_{EM}) \right], \quad (1) \]
where \( D \) is the distance of the pulsar, \( c \) is the speed of light. One can see that the ST relation does not involve in with any detailed theoretical model with uncertainties.

**MEASUREMENT ERROR**

The ST relation requires that a GW signal and an EM pulse with certain frequencies emit out at the same time \( t_0 \). However, in practice, the rotations of pulsars are usually stable, so the change in frequency \( f \) will be not significant. If the sensitivity of detectors is not enough to distinguish the slight change in frequency, only a single frequency would be received during a time interval \([t_0, t_0 + \delta t]\). In this case, even if an EM pulse with frequency \( f \) and a GW signal with frequency \( 2f \) are detected, it is difficult to find out the exact emission time \( t_0 \). The main experimental error is the uncertainty of the emission time \( t_0 \). Next we will discuss this error.

In our method, the measurement error of the difference of time interval \( \Delta t = t_{GW} - t_{EM} \) contains two parts. The one is the instrumental errors \( \delta t_{GW} \) and \( \delta t_{EM} \), which present the uncertainty from the arrival times of GWs and EM pulses, respectively. And another is the system error \( \delta t \) of the emission time \( t_0 \). For instance, for the five hundred meter aperture spherical radio telescope (FAST) \[18\], the largest radio telescope in the world, \( \delta t_{EM} = 20 \text{ns} \) is adopted. We also assume that GW detectors have the same resolution as astronomical telescopes, i.e., \( \delta t_{EM} = \delta t_{GW} \). Then \( \delta t \) is usually much bigger relative to \( \delta t_{EM} \) and \( \delta t_{GW} \) (see Eq. (3)). Thus the measuring error of the difference of arrival times \( \Delta t = t_{GW} - t_{EM} \) is dominated by \( \delta t \). Then, the uncertainty of the speed of GWs, \( \Delta V_{GW} \) can be expressed as
\[ \frac{\Delta V_{GW}}{c} \approx \frac{1}{c} \frac{\partial V_{GW}}{\partial \Delta t} \delta t \approx \frac{1}{c} \frac{1}{D} \left( \frac{t_{GW} - t_{EM}}{c} \right) \delta t \]
\[ \approx 1 \times 10^{-14} \cdot \left( \frac{1 \text{Mpc}}{D} \right) \cdot \left( \frac{\delta t}{1 \text{ns}} \right). \quad (2) \]

It is worth emphasizing that the detection range of FAST for pulsars is close to the magnitude of 1Mpc, which is also the scale of Local Group of Galaxies. In this scale, the cosmic acceleration can be ignored due to gravitational binding.

**TWO CONDITIONS**

We should note here that two conditions must be satisfied for the application of our method. The first is that the GWs from a pulsar are detectable; the second is that the instrument resolution should be high enough.

The massive gravity theory will reduce to GR when the rest mass of graviton tends to zero \[19\]. Previous works have shown that the rest mass of gravitons must be very small \[8\]. It is reasonable to estimate the detection probability of GWs from the distorted pulsars by using the formulas from the GR theory.

The radiation power of GWs from a pulsar can be written as
\[ P_{GW} = \frac{32GI^2c^2\Omega^6}{5c^5}, \quad (3) \]
where \( G \) is the Newton gravitational constant, \( I \) is the rotation inertia, \( \epsilon \) is the ellipticity, and \( \Omega \) is the angular frequency of the pulsar. Usually, the angular frequency \( \Omega \) of the pulsar is also a function of time, i.e., \( \Omega = \Omega(t) \). For the same pulsar, the corresponding EM radiation power is
\[ P_{EM} = \frac{B_p^2R^6\Omega^4}{6c^3}, \quad (4) \]
where \( B_p \) is the dipolar field strength on the surface of the pulsar, and \( R \) is the radius of the star. From the law of conservation of energy, one has \[20\]
\[ \dot{E} = -I\Omega \dot{\Omega} = \frac{32GI^2c^2\Omega^6}{5c^5} + \frac{B_p^2R^6\Omega^4}{6c^3}, \quad (5) \]
where \( \dot{E} \) is the rotational energy loss rate. According to Eq.(5), one may find that pulsars should be slowing down. The corresponding frequency of GW, \( f_{GW} = \Omega/\pi \) will decrease with \( t \).

The GWs due to the time variations of the mass quadruple moment from a distorted pulsar should have a spin-down strain \[21\]
\[ h = \frac{16\pi^2GItf_{GW}^2}{c^4D}, \quad (6) \]
Young pulsars or new born neutron stars are useful to our method for their fast spin.

The strain sensitivities of the advanced LIGO can exceed \( 10^{-25} \) at the frequency range \( f_{GW} \in [40\text{Hz}, 10^3\text{Hz}] \) \[21\]. In order to detect the spin-down GWs signals emitted from a distorted pulsar by using the advanced LIGO, the following condition must be satisfied
\[ h = \frac{16\pi^2GIt}{c^4DT^2} \geq 10^{-25}. \quad (7) \]
Eq. (7) is a theoretical constraint on \( \epsilon \) and \( T \). It is worth pointing out that a negligible modification will be introduced into the above equation under the frame of the massive gravity theory.

We will discuss the second condition. As mentioned above, the arrival times of the signal from pulsars are
measured as precisely as 20ns, which allows the change of pulsar period $\Delta T > 20$ns, i.e., $\dot{T}\delta t > 20$ ns. The change rate of the rotation period, which can be measured by detectors during time interval $\delta t$, should satisfy

$$\dot{T} \geq 2 \times 10^{-8} \cdot \frac{18}{\delta t} \text{s} \cdot \text{s}^{-1}.$$  \hspace{1cm} (8)

We require $\Delta V_{\text{GW}}/c \leq 10^{-10}$, which is the same order of magnitude as that of Ref. [12]. Substituting this requirement of precision into Eq. (2), we have

$$\frac{\delta t}{18} \leq 10^4 \frac{D}{1\text{Mpc}},$$  \hspace{1cm} (9)

which means that the detectors measure the change of pulsar period in $> 10^4$ circles in the time interval of $T < 1$ s. Maybe it seems to be a little optimistic, but it’s not going too far for the current instruments and pulsars in our Galaxies, see the next section for discussion. Besides, during this time, the signal can accumulate to a high enough signal-to-noise ratio. Ideally, if the detectors measure the period change of millisecond pulsar during 3 circles, the upper limit of precision of our method reaches $\Delta V_{\text{GW}}/c \sim 10^{-17}$, which is the same as the result of Ref. [11].

In addition, we have the following results from Eq. (8)

$$-\dot{\Omega} = \frac{32GIc^2\dot{\Omega}^5}{5c^3} + \frac{B^2 R^6 \rho^3 \Omega^3}{6IC^3}.$$  \hspace{1cm} (10)

Combining the Eq. (8,9,10), we obtain:

$$\dot{T} = \frac{512\pi^4 G I c^2}{5c^3 T^3} + \frac{4\pi^2 B^2 R^6}{6IC^3 T} \geq 2 \times 10^{-12} \cdot \frac{(1\text{Mpc})}{D} \cdot \frac{(1\text{s})}{\text{s} \cdot \text{s}^{-1}}.$$  \hspace{1cm} (11)

Eq. (11) is another theoretical constraint on $\epsilon$ and $T$.

**CANDIDATES**

We obtain two theoretical constraints on the pulsar parameters $\epsilon$ and $T$, i.e., Eq. (11) and Eq. (11). In this section, we will look into potential candidates in the Milky Way and nearby galaxies to test our method.

The frequency range of the advanced LIGO is $f_{\text{GW}} \in [40\text{Hz}, 10^3\text{Hz}]$. The frequency of GW is twice the rotation frequency of the pulsar, the available range of pulsars’ rotation period is $T \in [2 \times 10^{-3}s, 5 \times 10^{-2}s]$. There are more than 2500 pulsars observed in the Milky Way and nearby galaxies from the ATNF pulsar catalog [22]. About 350 pulsars therein, whose periods are in boundary of $T \in [2 \times 10^{-3}s, 5 \times 10^{-2}s]$, fall in the detection range of the advanced LIGO. Unfortunately, we find that pulsars can only satisfy the Eq. (7) or the Eq. (11), no one meet the above two condition at the same time. Pulsars, whose strain intensity are below the sensitivity of the advanced LIGO, are hopeful to be detected by the next generation GW detector, such as Einstein Telescope (ET) (see Figure 3 in Ref. [23]). And pulsars in the states of glitch can break the limits by Eq. (11). Glitches [24,25] are commonly observed sudden spin changes of pulsars. During the process of glitch, the period change of a pulsar, denoted as $(T_a - T_b)/T_b$, can be larger than $10^{-6}$, where $T_b$ and $T_a$ are the period of pulsar before and after the glitch, respectively. The frequencies of EM pulse and GW will respond to this sudden change of pulse period simultaneously, so that it is possible to make sure these two kinds of signals are emitted at the same time. Considering all these factors, some pulsars candidates are selected, listed in Table 1. In Table 1, $T$, $\dot{T}$ and $D$ are direct observations. The rest parameters listed in Table 1 can be obtained by adopting typical values of $I = 10^{35}$g $\cdot$ cm$^2$ and $R = 10^6$cm.

**SUMMARY**

In this work we propose a new model-independent strategy to measure the speed of GWs. The speed of GWs can be calculated through comparing the arrival time difference of the GW signals and the EM emission at a certain frequency, which is in formula of $V_{\text{GW}} = c \left[ 1 - \frac{1}{T} \left( t_{\text{GW}} - t_{\text{EM}} \right) \right]$. Taking into account that the emission time $t_0$ has an uncertainty $\delta t$, and a resolution of $\delta t = 20$ ns for the GW and EM detectors, the uncertainty of the speed of GWs can be expressed as $\Delta V_{\text{GW}}/c \sim 1 \times 10^{-14} \cdot \frac{(1\text{Mpc})}{D} \cdot \frac{(1\text{s})}{\text{s} \cdot \text{s}^{-1}}$. Furthermore, we obtained two theoretical constraints on the two pulsar parameters, i.e., $\epsilon$ and $T$, see Eq. (7) and Eq. (11). Currently, there is no pulsar satisfied the above condition at the level of constraints on the the order of magnitude of $\Delta V_{\text{GW}}/c \leq 10^{-10}$. When the glitches are considered, some pulsars (see Table 1) are potential to test our method.

In our strategy, recognizing the change of pulsar period in a small time interval is a way to improve the accuracy. This requirement can be regarded as a motivation for future design of instruments. Anyway, with the help of FAST and ET, we hope the speed of GWs will be constrained to an accuracy of at least $10^{-9}$ in the foreseeable future.

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TABLE I. The parameters of candidates pulsars.\(^1\)

| Name          | \(T_{\text{ms}}\) | \(T_{\text{s} \cdot \text{s}^{-1}}\) | \(D\) | \(B_p\) (Gs) | \(h\) | \(\epsilon \) |
|---------------|------------------|------------------|------|-------------|-------|-------------|
| J0537-6910    | 16               | \(5.2 \times 10^{-16}\) | 50.0 | \(9.2 \times 10^{11}\) | \(3.8 \times 10^{-26}\) | \(1.2 \times 10^{-4}\) |
| J1952+3252    | 40               | \(1.4 \times 10^{-15}\) | 3.0  | \(4.8 \times 10^{11}\) | \(2.7 \times 10^{-25}\) | \(3.0 \times 10^{-4}\) |
| J1913+1011    | 36               | \(3.4 \times 10^{-15}\) | 4.5  | \(3.2 \times 10^{11}\) | \(1.6 \times 10^{-26}\) | \(2.2 \times 10^{-4}\) |
| J1119-6127    | 408              | \(4.0 \times 10^{-12}\) | 8.4  | \(4.1 \times 10^{13}\) | \(\sim 4.0 \times 10^{-26}\) | \(\sim 10^{-3}\) |
| J1513-5908    | 151              | \(1.5 \times 10^{-12}\) | 4.4  | \(1.5 \times 10^{13}\) | \(\sim 10^{-25}\) | \(\sim 10^{-3}\) |
| J0534+2200    | 33               | \(4.2 \times 10^{-13}\) | 2.0  | \(3.8 \times 10^{12}\) | \(\sim 2.0 \times 10^{-25}\) | \(\sim 10^{-3}\) |

1 The values of \(T, \dot{T}, D,\) and \(B_p\) are taken from \[^{24}\]. The values of \(h\) and \(\epsilon\) are from \[^{21}\] and from \[^{26}\] for the first three pulsars and the last three pulsars, respectively.

2 It seems that \(\epsilon \sim 10^{-4}\) is a little larger than the value of elastic quadrupole deformation, but comparable to the value derived from crystalline colour-superconducting quark matter \[^{27}\], as well as the value of the deformation induced by the toroidal magnetic field for magnetars \[^{28}\].

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