Effect of partial slip and chemical reaction on convection of a viscoelastic fluid over a stretching surface with Cattaneo-Christov heat flux model

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Abstract. This article explores the effect of homogeneous-heterogeneous chemical reaction and partial slip on convective flow of a viscoelastic fluid with Cattaneo-Christov heat flux model in the presence of suction/injection and convective boundary condition. The governing system of non-linear partial differential equations are reformed into ordinary differential equations with the help of similarity variables and then they are solved using homotopy analysis method. It is found that the surface heat transfer rate enhances on increasing the thermal relaxation time parameter and the surface mass transfer rate improved by increasing the slip parameter and homogeneous chemical reaction parameter.

1.Introduction
The study of viscoelastic fluid flow over a stretching surface has attracted many researchers because of its practical applications in engineering and industrial areas. Such applications are paper production, stretching of plastic film, glass fiber, crystal growing, wire drawing, hot rolling, etc. The heat transfer of a viscoelastic fluid over a stretching surface was studied by Cortell[1]. He found that the skin friction coefficient reduces with enhancing the viscoelastic parameter. Other related studies in this direction was found inRushi Kumar and Sivaraj [2], Eswaramoorthi et al. [3&4] and Sathish Kumar et al.[5].

The heat transfer is essential process in many engineering and industrial applications. Fourier [6] was initially studied the heat transfer phenomenon in the form of parabolic energy equation for temperature field. One of the significant inadequate of this model is that the whole system is initially affected by the initial disturbance. So, Cattaneo[7] modified the Fourier's model including the relaxation time to avoid the paradox of heat conduction. Then, Christov [8] improved the Cattaneo model, namely, Cattaneo-Christov heat flux model to thermal relaxation time with Oldroyd’s upper-convected derivatives to accomplish the material invariant formulation. Boundary layer flow of a
viscoelastic fluid with Cattaneo-Christov heat flux model was investigated by Hayat et al. [9]. Study of convective flow and heat transfer with chemical reaction is used in many chemical engineering/industrial processes, such as, fog formation, food processing, manufacturing of ceramics, drying, evaporation, groves of fruit trees and crops damage via freezing. Some of the studies in this directions are found in Karthikeyan et al. [10], Ruhaila et al. [11], Sivasankaran et al. [12].

The main purpose of this paper is to investigate the convective heat and mass transfer of a viscoelastic fluid flow over a stretching sheet with Cattaneo-Christov heat flux model. Effects of convective boundary condition, partial slip and suction/injection are examined.

2. Mathematical formulation

Let us presume the steady two-dimensional boundary layer flow of a viscoelastic fluid with Cattaneo-Christov heat flux model over a linear stretching sheet with partial slip and convective boundary condition. Assume that the sheet is moving with velocity \( u = U_w = ax \), \( a \) is the positive constant. The sheet is kept at a constant temperature \( T_w \) which is higher than the ambient temperature of the fluid, \( T_\infty \). The effect of homogeneous and heterogeneous chemical reactions are considered in the fluid flow. The homogeneous chemical reaction for the cubic auto catalysis is \( A + 2B \rightarrow 3B \) at rate \( k_c ab^2 \) and the simple first order heterogeneous chemical reaction at the surface is \( A \rightarrow B \) at rate \( k_s A \), where \( a \) and \( b \) are the constants of the chemical species \( A \) and \( B \), \( k_c \) and \( k_s \) are the rate constants of the chemical species \( A \) and \( B \). Also, we assume that both reaction processes are isothermal and there is no change in temperature profile. The bottom of the sheet is heated by convection from hot fluid with temperature \( T_f \) which yields the heat transfer coefficient \( h_c \). The fluid experiences a first-order slip at the sheet surface. Under these assumptions, the governing boundary layer equations of this problem can be written in the following form

\[
\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0
\]

\[
u \frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} = \frac{\partial^2 u}{\partial y^2} + k_0 \left( \frac{u}{\partial x} \frac{\partial^2 u}{\partial y^2} + \frac{\partial u}{\partial x} \frac{\partial^2 u}{\partial y^2} + \frac{v}{\partial y} \frac{\partial^2 u}{\partial y^2} \right)
\]

\[
\rho c_p v. \nabla T = -\nabla q
\]

\[
u \frac{\partial a}{\partial x} + \frac{\partial a}{\partial y} = D_A \frac{\partial^2 a}{\partial y^2} - k_c ab^2
\]

\[
u \frac{\partial b}{\partial x} + \frac{\partial b}{\partial y} = D_B \frac{\partial^2 b}{\partial y^2} + k_c ab^2
\]

where \( u \) and \( v \) are the velocity components in \( x \) and \( y \) directions, \( v \) is the kinematic viscosity, \( k_0 \) is the viscoelastic parameter, \( c_p \) is the specific heat, \( q \) is the heat flux, \( D_A \) and \( D_B \) are the respective diffusion species coefficients of \( A \) and \( B \). The heat flux \( q \) satisfy the following equation

\[
q + \lambda_1 \left( \frac{\partial q}{\partial t} + V. \nabla q - q. \nabla V \right) = -k \nabla T
\]

where \( \lambda_1 \) and \( k \) are the relaxation time of heat flux and thermal conductivity of the fluid, respectively. For \( \lambda_1 = 0 \), the above equation reduces to classical Fourier’s law. The fluid is incompressibility when \( \nabla V = 0 \) and we have

\[
q + \lambda_1 \left( \frac{\partial q}{\partial t} + V. \nabla q - q. \nabla V \right) = -k \nabla T
\]

Eliminating \( q \) between (3) and (7), we have

\[
u \frac{\partial r}{\partial x} + \frac{\partial r}{\partial y} + \lambda_1 \left( u^2 \frac{\partial^2 r}{\partial x^2} + v^2 \frac{\partial^2 r}{\partial y^2} + \left[ \frac{u}{\partial x} + v \frac{\partial u}{\partial y} \right] \frac{\partial r}{\partial x} + 2uv \frac{\partial^2 r}{\partial x \partial y} + \left[ \frac{u}{\partial x} + v \frac{\partial v}{\partial y} \right] \frac{\partial r}{\partial y} \right) = \frac{k}{\rho c_p} \frac{\partial^2 T}{\partial y^2}
\]

The boundary conditions of the above model are

\[
u = U_w = ax + L \frac{\partial u}{\partial y}, \quad v = V_w, \quad -k \frac{\partial T}{\partial y} = h_c (T_f - T_\infty), \quad D_A \frac{\partial a}{\partial y} = k_c a, \quad D_B \frac{\partial b}{\partial y} = -k_s a \text{ at } y = 0
\]
Define the following similarity variables
\[ \eta = \sqrt{\frac{v}{v}} y, \quad u = ax f'(\eta), \quad v = -\sqrt{\alpha \nu} f(\eta), \quad \theta = \frac{T - T_{\infty}}{T_f - T_{\infty}}, \quad a = a_0 \phi, \quad b = a_0 h \]  
\[ (9) \]
Substituting Equation (9) into the Equations (2), (4), (5) and (8), we have
\[ f'' + f f' - f^2 + K \left( 2f f'' - f'' - f f' \right) = 0 \]
\[ \frac{1}{Pr} \theta'' + f \theta' - \gamma (f f' \theta' + f^2 \theta'') = 0 \]
\[ \frac{1}{Sc} \phi'' + f \phi' - k_1 \phi h^2 = 0 \]
\[ \frac{1}{\delta} \phi'' + f \phi' + k_1 \phi h^2 = 0 \]
\[ (10) \]
\[ (11) \]
\[ (12) \]
\[ (13) \]
The boundary conditions become,
\[ f(0) = f_0, \quad f(\infty) = 1 \]
\[ f'(\infty) = 0, \quad f'(0) = 0, \quad \theta'(0) = -Bi \theta(0), \quad \theta(\infty) = 0 \]
\[ \phi'(0) = k_2 \phi(0), \quad \delta h'(0) = -k_2 \phi(0), \quad \phi'(\infty) = 0, \quad h(\infty) = 0 \]
\[ (14) \]
where \( K = \frac{k a}{v} \) is the viscoelastic parameter, \( \lambda = L \frac{\sqrt{\alpha}}{\sqrt{\nu}} \) is the slip parameter, \( Pr = \frac{\nu \rho c_p}{k} \) is the Prandtl number, \( \gamma = \lambda_1 a \) is the thermal relaxation time parameter, \( Sc = \frac{\nu}{D_A} \) is the Schmidt number, \( k_1 = \frac{k a z_1}{\alpha} \) is the strength of homogeneous chemical reaction, \( f_0 = \frac{v}{\sqrt{\alpha \nu}} \) is the suction/injection parameter, \( Bi = \frac{h \sqrt{\alpha}}{k} \) is the Biot number, \( k_2 = \frac{k a z_1}{D_A} \) is the strength of heterogeneous chemical reaction and \( \delta = \frac{D_B}{D_A} \) is the ratio of diffusion coefficient. The diffusion coefficients are comparable size and we assume that these coefficient are equal, i.e. \( \delta = 1 \). Thus
\[ \phi(\eta) + h(\eta) = 1 \]
Then the equation becomes
\[ \frac{1}{Sc} \phi'' + f \phi' - k_1 \phi (1 - \phi)^2 = 0 \]
\[ (15) \]
with the boundary conditions \( \phi'(0) = k_2 \phi(0) \)and \( \phi'(\infty) \rightarrow 1 \).

The skin friction coefficient, local Nusselt number and local Sherwood number are important physical parameters. The reduced skin friction coefficient, local Nusselt number and local Sherwood number are given by
\[ \frac{1}{2} C_f \sqrt{Re} = (1 + 3K) f''(0), Nu/\sqrt{Re} = -\theta'(0) \text{and} Sh/\sqrt{Re} = -\phi'(0). \]

3. HAM solutions

The initial approximations for homotopy analysis solutions are chosen as \( f_0 = f_0 + \frac{1 + e^{-\eta}}{1 + \lambda}; \theta_0 = \frac{Bi e^{-\eta}}{1 + Bi}; \phi_0 = 1 - \frac{1}{2} e^{-k_2 \eta} \), the auxiliary linear operators \( L_f, L_{\theta} \) and \( L_{\phi} \) are defined as \( L_f = f''' - f' ; L_{\theta} = \theta'' - \theta ; L_{\phi} = \phi''' - \phi \) with satisfying the following properties
\[ L_f [C_1 + C_2 e^{\eta} + C_3 e^{-\eta}] = 0; L_{\theta} [C_4 e^{\eta} + C_5 e^{-\eta}] = 0; L_{\phi} [C_6 e^{\eta} + C_7 e^{-\eta}] = 0, \quad \text{where} C_i, (i = 1 - 7) \]
denote the arbitrary constants.

The general solution of the Equations (10)-(11) and (15) are
where \( f_m^*(\eta) \), \( \theta_m^*(\eta) \) and \( \phi_m^*(\eta) \) are the special solutions. These general solutions contain the auxiliary parameters \( h_f, h_\theta \) and \( h_\phi \). These parameters are adjusting and controlling the convergence of the desired solutions. The \( h_f, h_\theta \) and \( h_\phi \) curves are plotted in Figure 1. It is observed from the range values of \( h_f, h_\theta \) and \( h_\phi (-1.8 \leq h_f \leq -0.3\&-2 \leq h_\theta, h_\phi \leq -0.1) \) that the HAM solution convergence in the region of \( \eta \) when \( h_f = h_\theta = h_\phi = -1 \).

Table 1 presents the comparison of skin friction coefficient between our results and published results. It is seen that from the table that our results are in good agreement with previous results.

4. Correlation

The correlation equations of the skin friction, Nusselt number and Sherwood number are derived as:

\[
\frac{1}{2}C_f\sqrt{Re} = -0.580474 - 1.210071K - 0.175144f w + 0.156614\lambda - 0.000515Bi - 0.000527y
\]

\[
Nu/\sqrt{Re} = 0.236642 + 0.019823K + 0.181523f w - 0.017403\lambda + 0.036940Bi + 0.018367y
\]

\[
Sh/\sqrt{Re} = -0.177937 - 0.035976K - 0.247830f w + 0.021479\lambda - 0.000670Bi - 0.000686y
\]

5. Results and Discussion

In this section, we present the velocity, temperature, concentration, skin friction coefficient, local Nusselt number and local Sherwood number for different values of pertinent parameters with the fixed values of the Prandtl number \( (Pr = 0.9) \) and Schmidt number \( (Sc = 0.9) \). Figures 2(a and b) illustrate the influence of slip parameter \( \lambda \) on velocity profile for Newtonian and viscoelastic fluids with suction/injection. It is found that the fluid velocity and its momentum boundary layer thickness suppress on increasing the values of slip parameter. Variations of the slip parameter on temperature profile for Newtonian and viscoelastic fluids are shown in Figures 3(a and b). It observed that the thermal boundary layer thickness enlarges with increasing the slip parameter. Figures 4(a and b) show the effect of the Biot number and thermal relaxation time parameter on temperature profile. It is concluded that the thermal boundary layer thickness boost up on increasing the Biot number and it supresses with increasing the thermal relaxation time parameter.

Concentration profile for different values of slip parameter for Newtonian and viscoelastic fluids are displayed in Figures 5(a and b). It is found that the concentration suppresses by rising the values of the slip parameter. In addition, the boundary layer thickness of viscoelastic fluid is higher than that of the Newtonian fluid. Figure 6(a) depict the skin friction coefficient for various values on \( K, f w \) and \( \lambda \). It is found that the skin friction reduces with increasing the values of viscoelastic parameter and suction/injection parameter. It enhances on increasing the slip parameter. The local Nusselt number for different values of thermal relaxation time parameter, suction/injection parameter and slip parameter are shown in Figure 6(b). It is found that the surface heat transfer rate diminishes on growing the slip parameter and suction/injection parameter. However, it is an increasing function of thermal relaxation time parameter. Figures 7(a and b) explore the local Sherwood number for different values of slip, suction/injection, homogeneous and heterogeneous chemical reaction parameters. It is seen that the surface mass transfer rate drops with increasing the suction/injection parameter and heterogeneous chemical reaction parameter. It augments on increasing the slip parameter and homogeneous chemical reaction parameter.

6. Conclusion

In this article, the effects of homogeneous and heterogeneous chemical reactions and partial slip on convective flow of viscoelastic fluid with Cattaneo-Christov heat flux model in the presence of convective boundary conditions and suction/injection are explored. The velocity reduces by increasing
the values of slip parameter, suction/injection parameter. The thermal boundary layer thickness enlarges with increasing the slip parameter, Biot number and it reduces on increasing the thermal relaxation time parameter and viscoelastic parameter. The solutal boundary layer thickness improves with increasing viscoelastic parameter and suction/injection parameter. The skin friction coefficient falls with increasing viscoelastic parameter and suction/injection parameter. The heat transfer rate enhances on increasing the thermal relaxation time parameter. The mass transfer rate improves by increasing the slip parameter and homogeneous chemical reaction parameter and it reduces by increasing the suction/injection parameter and heterogeneous chemical reaction parameter.

Table 1. Computations showing the comparison with Hayat et al. [9] for different values of $K$ with $fw = 0.2$ and $\lambda = 1.0$.

| $K$ | -0.3 | -0.2 | -0.1 | 0   | 0.1 | 0.2 | 0.3 |
|-----|------|------|------|-----|-----|-----|-----|
| Present study | -0.11952 | -0.44721 | -0.73787 | -1 | -1.23950 | -1.46059 | -1.66641 |
| Hayat et al. [9] | -0.11952 | -0.44721 | -0.73786 | -1 | -1.23950 | -1.46059 | -1.66641 |

Figure 1. Curves of $f''(0), \theta'(0)$ and $\phi'(0)$ with $K = 0.2, fw = 0.2, \lambda = 1.0, \gamma = 0.5, Bi = 0.5k_1=0.5$ and $k_2 = 0.5$.

Figure 2. Velocity profile for different values of $\lambda$ with (a) $K = 0$(dashed line), $K = 0.2$(solid line) and $fw = 0.2$ and (b) $fw = -0.2$(dashed line), $fw = 0.2$(solid line) and $K = 0.2$. 
Figure 3. Temperature profile for different values of \( \lambda \) with (a) \( K = 0.0 \) (dashed line), \( K = 0.2 \) (solid line), \( f_w = 0.2 \), \( \gamma = 0.5 \) and Bi=0.5 and (b) \( f_w = -0.2 \) (dashed line), \( f_w = 0.2 \) (solid line), \( K = 0.2 \), \( \gamma = 0.5 \) and Bi = 0.5.

Figure 4. Temperature profile for different values (a) Bi with \( \lambda = 0.0 \) (dashed line), \( \lambda = 1.0 \) (solid line), \( f_w = 0.2 \), \( K = 0.2 \) and \( \gamma = 0.5 \) and (b) \( \gamma \) with \( \lambda = 0.0 \) (dashed line), \( \lambda = 1.0 \) (solid line), \( f_w = 0.2 \), \( K = 0.2 \) and Bi = 0.5.

Figure 5. Concentration profile for different values of \( \lambda \) with (a) \( K = 0 \) (dashed line), \( K = 0.2 \) (solid line) with \( f_w = 0.2 \), \( k_1=0.5 \) and \( k_2 = 0.5 \) and (b) \( f_w = -0.2 \) (dashed line), \( f_w = 0.2 \) (solid line) \( K = 0.2 \), \( k_1=0.5 \) and \( k_2 = 0.5 \).
Figure 6. Skin friction coefficient for $\lambda = 0.0$(dashed line), $\lambda = 1.0$(solid line) with various values of $K$ and $fw$ (b) local Nusselt number for $fw = -0.2$(dashed line), $fw = 0.0$(solid line) $fw = 0.2$(dotted line) for various values of $\lambda$ and $\gamma$ and $Bi = 0.5$.

Figure 6. Local Sherwood number (a) $\lambda = 0.0$(dashed line), $\lambda = 1.0$(solid line) with various values of $fw$ and $k_2$ for $K = 0.2$ and$k_1 = 0.5$ (b) $fw = -0.2$(dashed line), $fw = 0.0$(solid line) $fw = 0.2$(dotted line) for various values of $k_1, \lambda$ for $K = 0.2$ and $k_2 = 0.5$.

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