CHARACTERIZATIONS OF $(m,n)$-JORDAN DERIVATIONS ON SOME ALGEBRAS

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Abstract. Let $\mathcal{R}$ be a ring, $\mathcal{M}$ be a $\mathcal{R}$-bimodule and $m,n$ be two fixed nonnegative integers with $m+n\neq 0$. An additive mapping $\delta$ from $\mathcal{R}$ into $\mathcal{M}$ is called an $(m,n)$-Jordan derivation if $(m+n)\delta(A^2) = 2mA\delta(A) + 2n\delta(A)A$ for every $A$ in $\mathcal{R}$. In this paper, we prove that every $(m,n)$-Jordan derivation from a $C^*$-algebra into its Banach bimodule is zero. An additive mapping $\delta$ from $\mathcal{R}$ into $\mathcal{M}$ is called a $(m,n)$-Jordan derivable mapping at $W$ in $\mathcal{R}$ if $(m+n)\delta(AB+BA) = 2mA\delta(A)B + 2m\delta(B)A + 2nA\delta(B) + 2nB\delta(A)$ for each $A$ and $B$ in $\mathcal{R}$ with $AB = BA = W$. We prove that if $\mathcal{M}$ is a unital $\mathcal{A}$-bimodule with a left (right) separating set generated algebraically by all idempotents in $\mathcal{A}$, then every $(m,n)$-Jordan derivable mapping at zero from $\mathcal{A}$ into $\mathcal{M}$ is identical with zero. We also show that if $\mathcal{A}$ and $\mathcal{B}$ are two unital algebras, $\mathcal{M}$ is a faithful unital $(\mathcal{A},\mathcal{B})$-bimodule and $U = \left[ \begin{array}{cc} \mathcal{A} & \mathcal{M} \\ \mathcal{N} & \mathcal{B} \end{array} \right]$ is a generalized matrix algebra, then every $(m,n)$-Jordan derivable mapping at zero from $U$ into itself is equal to zero.

1. Introduction

Let $\mathcal{R}$ be an associative ring. For an integer $n \geq 2$, $\mathcal{R}$ is said to be $n$-torsion-free if $nA = 0$ implies that $A = 0$ for every $A$ in $\mathcal{R}$. Recall that a ring $\mathcal{R}$ is prime if $ARB = (0)$ implies that either $A = 0$ or $B = 0$ for each $A, B$ in $\mathcal{R}$; and is semiprime if $ARA = (0)$ implies that $A = 0$ for every $A$ in $\mathcal{R}$.

Suppose that $\mathcal{M}$ is a $\mathcal{R}$-bimodule. An additive mapping $\delta$ from $\mathcal{R}$ into $\mathcal{M}$ is called a derivation if $\delta(AB) = \delta(A)B + A\delta(B)$ for each $A, B$ in $\mathcal{R}$; and $\delta$ is called a Jordan derivation if $\delta(A^2) = \delta(A)A + A\delta(A)$ for every $A$ in $\mathcal{R}$. Obviously, every derivation is a Jordan derivation, the converse is, in general, not true. A classical result of Herstein [8] proves that every Jordan derivation on a 2-torsion-free prime ring is a derivation; Cusack [6] generalizes [8, Theorem 3.1] to 2-torsion-free semiprime rings.

In [4], Brešar and Vukman introduce the concepts of left derivations and Jordan left derivations. Suppose that $\mathcal{M}$ is a left $\mathcal{R}$-module. An additive mapping $\delta$ from $\mathcal{R}$ into $\mathcal{M}$ is called a left derivation if $\delta(AB) = A\delta(B) + B\delta(A)$ for each $A, B$ in $\mathcal{R}$; and $\delta$ is called a Jordan left derivation if $\delta(A^2) = 2A\delta(A)$ for every $A$ in $\mathcal{R}$. Brešar and Vukman [4] prove that if $\mathcal{R}$ is a prime ring and $\mathcal{M}$ is a 6-torsion free
left $\mathcal{R}$-module, then the existence of nonzero Jordan left derivations from $\mathcal{R}$ into $\mathcal{M}$ implies that $\mathcal{R}$ is a commutative ring. Deng [7] shows that [4, Theorem 2.1] is still true when $\mathcal{M}$ is only 2-torsion free.

In [12], Vukman introduces the concept of $(m, n)$-Jordan derivations. Suppose that $\mathcal{M}$ is a $\mathcal{R}$-bimodule and $m, n$ are two fixed nonnegative integers with $m + n \neq 0$. An additive mapping $\delta$ from $\mathcal{R}$ into $\mathcal{M}$ is called an $(m, n)$-Jordan derivation if

$$(m+n)\delta(A^2) = 2mA\delta(A) + 2n\delta(A)A$$

for every $A$ in $\mathcal{R}$. It is easy to show that the concept of $(m, n)$-Jordan derivations covers the concept of Jordan derivations as well as the concept of Jordan left derivations. By Vukman [11, Theorem 4] and Kosi-Ulbl [9, Theorem 8], we know that if $m, n$ are two nonnegative integers with $m \neq n$, then every $(m, n)$-Jordan derivation from a complex semisimple Banach algebra into itself is identically equal to zero.

In Section 2, we prove that if $m, n$ are two positive integers with $m \neq n$, then every $(m, n)$-Jordan derivation from a $C^*$-algebra into its Banach bimodule is identically equal to zero.

Suppose that $\mathcal{M}$ is a $\mathcal{R}$-bimodule and $m, n$ are two fixed nonnegative integers with $m + n \neq 0$. An additive mapping $\delta$ from $\mathcal{R}$ into $\mathcal{M}$ is called an $(m, n)$-Jordan derivable mapping at an element $W$ in $\mathcal{R}$ if

$$(m+n)\delta(AB + BA) = 2m\delta(A)B + 2m\delta(B)A + 2nA\delta(B) + 2nB\delta(A)$$

for each $A, B$ in $\mathcal{A}$ with $AB = BA = W$.

Let $\mathcal{J}$ be an ideal in $\mathcal{R}$. $\mathcal{J}$ is said to be a left separating set of $\mathcal{M}$ if for every $N$ in $\mathcal{M}$, $N\mathcal{J} = \{0\}$ implies $N = 0$; and $\mathcal{J}$ is said to be a right separating set of $\mathcal{M}$ if for every $M$ in $\mathcal{M}$, $\mathcal{JM} = \{0\}$ implies $M = 0$. When $\mathcal{J}$ is a left separating set and a right separating set of $\mathcal{M}$, we call $\mathcal{J}$ a separating set of $\mathcal{M}$. Denote by $\mathfrak{J}(\mathcal{R})$ the subring of $\mathcal{R}$ generated algebraically by all idempotents in $\mathcal{R}$.

In Section 3, we assume that $\mathcal{A}$ is a unital algebra, $\mathcal{M}$ is a unital $\mathcal{A}$-bimodule with a left (right) separating $\mathcal{J} \subseteq \mathfrak{J}(\mathcal{A})$, and $\delta$ is an $(m, n)$-Jordan derivable mapping at zero from $\mathcal{A}$ into $\mathcal{M}$ such that $\delta(I) = 0$, we show that if $m, n$ are two positive integers with $m \neq n$, then $\delta$ is identically equal to zero. As applications, we study the $(m, n)$-Jordan derivable mappings at zero on some non self-adjoint operator algebras.

A Morita context is a set $(\mathcal{A}, \mathcal{B}, \mathcal{M}, \mathcal{N})$ and two mappings $\phi$ and $\varphi$, where $\mathcal{A}$ and $\mathcal{B}$ are two algebras, $\mathcal{M}$ is an $(\mathcal{A}, \mathcal{B})$-bimodule and $\mathcal{N}$ is a $(\mathcal{B}, \mathcal{A})$-bimodule, $\phi : \mathcal{M} \otimes_\mathcal{B} \mathcal{N} \to \mathcal{A}$ and $\varphi : \mathcal{N} \otimes_\mathcal{A} \mathcal{M} \to \mathcal{B}$ are two homomorphisms satisfying the following commutative diagrams:

\[
\begin{array}{ccc}
\mathcal{M} \otimes_\mathcal{B} \mathcal{N} \otimes_\mathcal{A} \mathcal{M} & \xrightarrow{\phi \otimes I_M} & \mathcal{A} \otimes_\mathcal{A} \mathcal{M} \\
I_M \otimes \varphi & \Downarrow & \cong \\
\mathcal{M} \otimes_\mathcal{B} \mathcal{B} & \cong & \mathcal{M}
\end{array}
\]
and

\[
\begin{array}{c}
\mathcal{N} \otimes_A \mathcal{M} \otimes_B \mathcal{N} \\
\xrightarrow{\varphi \otimes I_B} \\
\xrightarrow{I_N \otimes \phi} \\
\mathcal{N} \otimes_A \mathcal{A} \\
\xrightarrow{\cong} \\
\mathcal{N}.
\end{array}
\]

These conditions insure that the set

\[
\left[ \begin{array}{c}
A \\
N \\
B
\end{array} \right] = \left\{ \left[ \begin{array}{c}
A \\
N \\
B
\end{array} \right] : A \in \mathcal{A}, B \in \mathcal{B}, M \in \mathcal{M}, N \in \mathcal{N} \right\}
\]

forms an algebra under the usual matrix addition and the matrix multiplication. We call it a \textit{generalized matrix algebra}.

Let \(\mathcal{A}, \mathcal{B}\) be two algebras and \(\mathcal{M}\) be an \((\mathcal{A}, \mathcal{B})\)-bimodule, the set

\[
\left[ \begin{array}{c}
A \\
0 \\
B
\end{array} \right] = \left\{ \left[ \begin{array}{c}
A \\
0 \\
B
\end{array} \right] : A \in \mathcal{A}, B \in \mathcal{B}, M \in \mathcal{M} \right\}
\]

under the usual matrix addition and matrix multiplication is called a \textit{triangular algebra}.

\(\mathcal{M}\) is called a \textit{left faithful unital} \(\mathcal{A}\)-module if for every \(A\) in \(\mathcal{A}\), \(AM = \{0\}\) implies \(A = 0\); \(\mathcal{M}\) is called a \textit{right faithful unital} \(\mathcal{B}\)-module if for every \(B\) in \(\mathcal{B}\), \(MB = \{0\}\) implies \(B = 0\). When \(\mathcal{M}\) is a left faithful unital \(\mathcal{A}\)-module and a right faithful unital \(\mathcal{B}\)-module, we call \(\mathcal{M}\) a faithful unital \((\mathcal{A}, \mathcal{B})\)-bimodule.

In Section 4, we suppose that \(\mathcal{A}, \mathcal{B}\) are two unital algebras, \(\mathcal{M}\) is a faithful unital \((\mathcal{A}, \mathcal{B})\)-bimodule, and we prove that if \(m, n\) are two positive integers with \(m \neq n\), then every \((m, n)\)-Jordan derivable mapping at zero from a generalized matrix algebra \(\mathcal{U} = \left[ \begin{array}{c}
\mathcal{A} \\
\mathcal{N} \\
\mathcal{B}
\end{array} \right] \) into itself is identically equal to zero.

Throughout this paper, \(\mathcal{A}\) denotes an algebra over the complex field \(\mathbb{C}\), and \(\mathcal{M}\) denotes an \(\mathcal{A}\)-bimodule.

\section*{2. \((m, n)\)-Jordan Derivations on \(C^*\)-Algebras}

In [1], we prove that if \(n = 0\) and \(m \neq n\), then every \((m, n)\)-Jordan derivation from a \(C^*\)-algebra into its Banach bimodule is zero. In this section, we assume that \(m, n\) are two positive integers with \(m \neq n\) and study the \((m, n)\)-Jordan derivations on \(C^*\)-algebras.

The following lemma will be used repeatedly in this section.

\begin{lemma}
[12, Proposition 1] Let \(\mathcal{A}\) be an algebra, \(\mathcal{M}\) be an \(\mathcal{A}\)-bimodule and \(m, n\) be two nonnegative integers with \(m \neq n\). If \(\delta\) is an \((m, n)\)-Jordan derivation from \(\mathcal{A}\) into \(\mathcal{M}\), then for each \(A, B\) in \(\mathcal{A}\), we have that

\[
(m+n)\delta(AB+BA) = 2m\delta(A)B + 2m\delta(B)A + 2nA\delta(B) + 2nB\delta(A).
\]
\end{lemma}

\begin{proposition}
Let \(\mathcal{A}\) be a commutative \(C^*\)-algebra, \(\mathcal{M}\) be a Banach \(\mathcal{A}\)-bimodule and \(m, n\) be two positive integers with \(m \neq n\). If \(\delta\) is an \((m, n)\)-Jordan derivation from \(\mathcal{A}\) into \(\mathcal{M}\), then \(\delta\) is automatically continuous.
\end{proposition}
Proof. Let \( \mathcal{J} = \{ J \in \mathcal{A} : D_J(T) = \delta(JT) \text{ is continuous for every } T \text{ in } \mathcal{A} \} \). Since \( \mathcal{A} \) is a commutative algebra and by Lemma 2.1, we have that
\[
nJ\delta(T) + m\delta(T)J = (m + n)\delta(JT) - m\delta(J)T - nT\delta(J)
\]
for every \( T \) in \( \mathcal{A} \) and every \( J \) in \( \mathcal{J} \). Then
\[
\mathcal{J} = \{ J \in \mathcal{A} : S_J(T) = nJ\delta(T) + m\delta(T)J \text{ is continuous every } T \text{ in } \mathcal{A} \}.
\]

In the following we divide the proof into four steps.

First, we show that \( \mathcal{J} \) is a closed two-sided ideal in \( \mathcal{A} \). Clearly \( \mathcal{J} \) is a right ideal in \( \mathcal{A} \). Moreover, for each \( A, T \) in \( \mathcal{A} \) and every \( J \) in \( \mathcal{J} \), we have that
\[
(m + n)\delta(AJT) = m\delta(A)JT + m\delta(JT)A + nA\delta(JT) + nJT\delta(A).
\]
Thus \( D_{AJ}(T) \) is continuous for every \( T \) in \( \mathcal{A} \) and \( \mathcal{J} \) is also a left ideal in \( \mathcal{A} \).

Suppose that \( \{ J_k \}_{k \geq 1} \subseteq \mathcal{J} \) and \( J \in \mathcal{A} \) such that \( \lim_{k \to \infty} J_k = J \). Then every \( S_{J_k} \) is a continuous linear operator; hence we obtain that
\[
S_J(T) = nJ\delta(T) + m\delta(T)J = \lim_{k \to \infty} nJ_k\delta(T) + m\delta(T)J_k = \lim_{k \to \infty} S_{J_k}(T)
\]
for every \( T \) in \( \mathcal{A} \). By the principle of uniform boundedness, we have that \( S_J \) is norm continuous and \( J \in \mathcal{A} \). Thus, \( \mathcal{J} \) is a closed two-sided ideal in \( \mathcal{A} \).

Next, we show that the restriction \( \delta_{|\mathcal{J}} \) is norm continuous. Suppose the contrary. We can choose \( \{ J_k \}_{k \geq 1} \subseteq \mathcal{J} \) such that
\[
\sum_{k=1}^{\infty} \| J_k \|^2 \leq 1 \text{ and } \| \delta(J_k) \| \to \infty, \text{ when } k \to \infty.
\]
Let \( B = (\sum_{k=1}^{\infty} J_kJ^*_k)^{1/4} \). Then \( B \) is a positive element in \( \mathcal{J} \) with \( \| B \| \leq 1 \). By [10, Lemma 1] we know that \( J_k = BC_k \) for some \( \{ C_k \} \subseteq \mathcal{J} \) with \( \| C_k \| \leq 1 \), and
\[
\| D_B(C_k) \| = \| \delta(BC_k) \| = \| \delta(J_k) \| \to \infty, \text{ when } k \to \infty.
\]
This leads to a contradiction. Hence \( \delta_{|\mathcal{J}} \) is norm continuous.

In the following, we show that the \( C^* \)-algebra \( \mathcal{A}/\mathcal{J} \) is finite-dimensional. Otherwise, by [13] we know that \( \mathcal{A}/\mathcal{J} \) has an infinite-dimensional abelian \( C^* \)-subalgebra \( \hat{\mathcal{A}} \). Since the carrier space \( \mathcal{X} \) of \( \hat{\mathcal{A}} \) is infinite, it follows easily from the isomorphism between \( \hat{\mathcal{A}} \) and \( C_0(\mathcal{X}) \) that there is a positive element \( H \) in \( \hat{\mathcal{A}} \) whose spectrum is infinite. Hence we can choose some nonnegative continuous mappings \( f_1, f_2, \ldots \), defined on the positive real axis such that
\[
f_jf_k = 0 \text{ if } j \neq k \text{ and } f_j(H) \neq 0 \text{ (} j = 1, 2, \ldots \).
\]
Let \( \varphi \) be a natural mapping from \( \mathcal{A} \) into \( \mathcal{A}/\mathcal{J} \). Then there exists a positive element \( K \) in \( \mathcal{A} \) such that \( \varphi(K) = H \). Denote \( A_j = f_j(K) \) for every \( j \). Then we have that \( A_j \in \mathcal{A} \) and
\[
\varphi(A^2_j) = \varphi(f_j(K))^2 = [f_j(\varphi(K))]^2 = f_j(H)^2 \neq 0.
\]
It follows that \( A^2_j \notin \mathcal{J} \) and \( A_jA_k = 0 \) if \( j \neq k \). If we replace \( A_j \) by an appropriate scalar multiple, we may suppose that \( \| A_j \| \leq 1 \). By \( A^2_j \notin \mathcal{J} \), we have that \( D_{A^2_j} \) is unbounded. Thus, we can choose \( T_j \in \mathcal{A} \) such that
\[
\| T_j \| \leq 2^{-j} \text{ and } (m + n)\| \delta(A^2_jT_j) \| \geq (m + n)K\| \delta(A_j) \| + j,
\]
where $K = \max\{M, N\}$, $M$ is the bound of the linear mapping
$$(T, M) \rightarrow MT : \mathcal{A} \times \mathcal{M} \rightarrow \mathcal{A}$$
and $N$ is the bound of the linear mapping
$$(T, M) \rightarrow TM : \mathcal{A} \times \mathcal{M} \rightarrow \mathcal{A}.$$ Let $C = \sum_{j \geq 1} A_jT_j$. Then we have that $\|C\| \leq 1$ and $A_jC = A_j^2T_j$, and so
$$\|nA_j\delta(C) + m\delta(C)A_j\| = \|(m + n)\delta(A_jC) - m\delta(A_j)C - nC\delta(A_j)\|$$
$$\geq \|(m + n)\delta(A_j^2T_j)\| - mM\|\delta(A_j)\|\|C\| - nN\|\delta(A_j)\|\|C\|$$
$$\geq (m + n)K\|\delta(A_j)\| + j - (m + n)K\|\delta(A_j)\| = j.$$ However, this is impossible because, in fact, $\|A_j\| \leq 1$ and the linear mapping
$$T \rightarrow nT\delta(C) + m\delta(C)T : \mathcal{A} \rightarrow \mathcal{M}$$
is bounded. Thus we prove that $\mathcal{A}/\mathcal{J}$ is finite-dimensional.

Finally we show that $\delta$ is a continuous linear mapping from $\mathcal{A}$ into $\mathcal{M}$. Since $\mathcal{A}/\mathcal{J}$ is finite-dimensional, we can choose some elements $A_1, A_2, \cdots, A_r$ in $\mathcal{A}$ such that $\varphi(A_1), \varphi(A_2), \cdots, \varphi(A_r)$ is a basis for the Banach space $\mathcal{A}/\mathcal{J}$, and let $\tau_1, \tau_2, \cdots, \tau_r$ be continuous linear functional on $\mathcal{A}/\mathcal{J}$ such that
$$\tau_j(\varphi(A_k)) = 1 \text{ when } j = k \text{ and } \tau_j(\varphi(A_k)) = 0 \text{ when } j \neq k.$$ For every $A$ in $\mathcal{A}$, we have that
$$\varphi(A) = \sum_{k=1}^{r} c_k\varphi(A_k),$$
and the scalars $c_1, c_2, \cdots, c_r$ are determined by $c_j = \tau_j(\varphi(A)) = \rho_j(A)$, where $\rho_j$ is continuous linear functional $\tau_j \circ \varphi$ on $\mathcal{A}$. Since
$$\varphi(A) = \sum_{j=1}^{r} \rho_j(A)\varphi(A_j),$$
we can obtain that $\varphi(A - \sum_{j=1}^{r} \rho_j(A)A_j) = 0$. It follows that $A \rightarrow A - \sum_{j=1}^{r} \rho_j(A)A_j$ is a continuous mapping from $\mathcal{A}$ into $\mathcal{J}$. Since $\delta|\mathcal{J}$ is continuous and $\rho_1, \rho_2, \cdots, \rho_r$ are continuous, it implies that
$$A \rightarrow [\delta(A) - \sum_{j=1}^{r} \rho_j(A)\delta(A_j)] + \sum_{j=1}^{r} \rho_j(A)\delta(A_j) = \delta(A)$$
is continuous from $\mathcal{A}$ into $\mathcal{M}$. \hfill \Box

Given an element $A$ of the algebra $B(\mathcal{H})$ of all bounded linear operators on a Hilbert space $\mathcal{H}$, we denote by $\mathcal{G}(A)$ the $C^*$-algebra generated by $A$. For any self-adjoint subalgebra $\mathcal{A}$ of $B(\mathcal{H})$, if $\mathcal{G}(B) \subseteq \mathcal{A}$ for every self-adjoint element $B \in \mathcal{A}$, then we call $\mathcal{A}$ locally closed. Obviously, every $C^*$-algebra is locally closed and we have the following result.
Lemma 2.3. [5, Corollary 1.2] Let $A$ be a locally closed subalgebra of $B(H)$, $Y$ be a locally convex linear space and $\psi$ be a linear mapping from $A$ into $Y$. If $\psi$ is continuous from every commutative self-adjoint subalgebra of $A$ into $Y$, then $\psi$ is continuous.

By Proposition 2.2 and Lemma 2.3, we can obtain the following corollary.

Corollary 2.4. Let $A$ be a $C^*$-algebra, $M$ be a Banach $A$-bimodule and $m, n$ be two positive integers with $m \neq n$. If $\delta$ is an $(m, n)$-Jordan derivation from $A$ into $M$, then $\delta$ is automatically continuous.

Proof. By Lemma 2.3, it is sufficient to prove that $\delta$ is continuous from every commutative self-adjoint subalgebra $B$ of $A$ into $M$. It is clear that the norm closure $\bar{B}$ of $B$ is a commutative $C^*$-algebra. Thus, we only need to show that the restriction $\delta|_{\bar{B}}$ is continuous.

By Proposition 2.2 we know that $\delta|_{\bar{B}}$ is automatically continuous. Hence $\delta$ is continuous on $B$. □

By Corollary 2.4 and [2, Theorem 2.3], we have the following theorem immediately.

Theorem 2.5. Let $A$ be a $C^*$-algebra, $M$ be a Banach $A$-bimodule and $m, n$ be two positive integers with $m \neq n$. If $\delta$ is an $(m, n)$-Jordan derivation from $A$ into $M$, then $\delta$ is identically equal to zero.

3. $(m, n)$-Jordan derivable mappings at zero on some algebras

In [3], we give a characterization of $(m, n)$-Jordan derivable mappings at zero on some algebras when $n = 0$ and $m \neq 0$. In this section, we assume that $m, n$ are two positive numbers and study the propositions of $(m, n)$-Jordan derivable mappings at zero.

Let $A$ be an algebra. $\mathfrak{z}(A)$ denotes the subalgebra of $A$ generated algebraically by all idempotents in $A$.

Lemma 3.1. [3, Lemma 2.2] Let $A$ be a unital algebra and $X$ be a vector space. If $\phi$ is a bilinear mapping from $A \times A$ into $X$ such that for each $A, B$ in $A$,

$$AB = BA = 0 \Rightarrow \phi(A, B) = 0,$$

then we have that

$$\phi(A, J) + \phi(J, A) = \phi(AJ, I) + \phi(I, JA)$$

for every $A$ in $A$ and every $J$ in $\mathfrak{z}(A)$.

By Lemma 3.1, we have the following result.

Proposition 3.2. Let $A$ be a unital algebra, $M$ be a unital $A$-bimodule and $m, n$ be two positive integers with $m \neq n$. If $\delta$ is a $(m, n)$-Jordan derivable mapping at zero from $A$ into $M$ such that $\delta(I) = 0$, then for every $A$ in $A$ and every idempotent $P$ in $A$, we have the following two statements:

1. $\delta(P) = 0$;
2. $\delta(PA) = \delta(AP) = \delta(A)P = P\delta(A)$. 

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Proof. Let $P$ be an idempotent in $\mathcal{A}$, since $P(I - P) = (I - P)P = 0$, we have that $m\delta(P)(I - P) + m\delta(I - P)P + nP\delta(I - P) + n(I - P)\delta(P) = 0$. By $\delta(I) = 0$, we can easily show that

$$(m + n)\delta(P) - 2m\delta(P)P - 2nP\delta(P) = 0. \tag{3.1}$$

Multiply $P$ from the both sides of (3.1) and by $m + n \neq 0$, we have that $P\delta(P)P = 0$. Then multiply $P$ from the left side of (3.1) and by $m \neq n$, we can obtain that $P\delta(P) = 0$. Similarly, we can prove that $\delta(P)P = 0$. By (3.1) and $m + n \neq 0$, it is easy to show that $\delta(P) = 0$.

For each $A, B$ in $\mathcal{A}$, define a bilinear mapping $\phi$ from $\mathcal{A} \times \mathcal{A}$ into $\mathcal{M}$ by

$$\phi(A, B) = m\delta(A)B + m\delta(B)A + nA\delta(B) + nB\delta(A).$$

It is clear that

$$AB = BA = 0 \Rightarrow \phi(A, B) = 0,$$

by Lemma 3.1, we have that

$$\phi(A, P) + \phi(P, A) = \phi(AP, I) + \phi(I, PA)$$

for every $A$ in $\mathcal{A}$ and every idempotent $P$ in $\mathcal{A}$. By the definition of $\phi$ and $\delta(I) = 0$, it follows that

$$(m + n)\delta(AP + PA) = 2m\delta(A)P + 2nP\delta(A). \tag{3.2}$$

Replace $A$ by $AP$ in (3.2), we have that

$$(m + n)\delta(AP) + (m + n)\delta(PAP) = 2m\delta(AP)P + 2nP\delta(AP). \tag{3.3}$$

Multiply $P$ from the both sides of (3.3) and by $m + n \neq 0$, it implies that

$$P\delta(AP)P = P\delta(PAP)P.$$

Similarly, we have that

$$P\delta(PA)P = P\delta(PAP)P.$$

Multiply $P$ from the both sides of (3.2), we can obtain that

$$P\delta(A)P = P\delta(AP)P = P\delta(PA)P = P(PAP)P. \tag{3.4}$$

Next we prove that $P\delta(A) = P\delta(AP)$, $\delta(A)P = \delta(AP)P$ and $\delta(PA) = \delta(PAP)$. Replace $A$ by $A - AP$ in (3.2), we have that

$$(m + n)\delta(PA - PAP) = 2m\delta(A - AP)P + 2nP\delta(A - AP), \tag{3.5}$$

multiply $P$ from the left side of (3.5) and by (3.4), we can obtain that

$$(m + n)P\delta(PA - PAP) = 2nP\delta(A) - 2nP\delta(AP), \tag{3.6}$$

replace $A$ by $PA$ in (3.6), we have that

$$(m + n)P\delta(PA - PAP) = 2nP\delta(PA) - 2nP\delta(PAP)$$

$$= 2nP\delta(PA - PAP), \tag{3.7}$$

by $m \neq n$ and (3.7), it implies that

$$P\delta(PA) = P\delta(PAP), \tag{3.8}$$
by (3.6) and (3.8), we can obtain that
\[ P\delta(A) = P\delta(AP). \] (3.9)

Multiply \( P \) from the right side of (3.5) and by (3.4), it follows that
\[ (m + n)\delta(PA - PAP)P = 2m\delta(A)P - 2m\delta(AP)P, \] (3.10)

replace \( A \) by \( PA \) in (3.10), we have that
\[ (m + n)\delta(PA - PAP)P = 2m\delta(PA)P - 2m\delta(PAP)P \]
\[ = 2m\delta(PA - PAP)P, \] (3.11)

by \( n \neq m \) and (3.11), we can obtain that
\[ \delta(PA)P = \delta(PAP)P, \] (3.12)

by (3.10) and (3.12), we have that
\[ \delta(A)P = \delta(AP)P. \] (3.13)

By (3.5), (3.9) and (3.13), it follows that
\[ \delta(PA) = \delta(PAP). \] (3.14)

Similarly, it is easy to obtain three identities as follows:
\[ P\delta(A) = P\delta(AP), \; \delta(A)P = \delta(PA)P \; \text{and} \; \delta(AP) = \delta(PAP). \] (3.15)

Multiply \( P \) from the left side of (3.2), we have that
\[ (m + n)P\delta(PA + AP) = 2mP\delta(A)P + 2nP\delta(A), \] (3.16)

by (3.9), (3.15) and (3.16), we can obtain that
\[ P\delta(PA) = P\delta(AP) = P\delta(A)P. \] (3.17)

Multiply \( P \) from the right side of (3.2), we have that
\[ (m + n)\delta(PA + AP)P = 2m\delta(A)P + 2nP\delta(A)P, \] (3.18)

by (3.13), (3.15) and (3.18), it follows that
\[ \delta(PA)P = \delta(AP)P = P\delta(A)P. \] (3.19)

By (3.15), (3.17) and (3.19), we have that
\[ P\delta(A) = \delta(A)P. \] (3.20)

Finally, by (3.2), (3.15) and (3.20), it implies that \( \delta(PA) = \delta(AP) = \delta(A)P = P\delta(A) \).

By the definition of \( \mathfrak{J}(\mathcal{A}) \) and by Proposition 3.2, it is easy to show the following result.

**Corollary 3.3.** Let \( \mathcal{A} \) be a unital algebra, \( \mathcal{M} \) be a unital \( \mathcal{A} \)-bimodule and \( m, n \) be two positive integers with \( m \neq n \). If \( \delta \) is an \((m,n)\)-Jordan derivable mapping at zero from \( \mathcal{A} \) into \( \mathcal{M} \) such that \( \delta(I) = 0 \), then for every \( S \) in \( \mathfrak{J}(\mathcal{A}) \) and every \( A \) in \( \mathcal{A} \), we have that
\[ \delta(SA) = \delta(AS) = \delta(A)S = S\delta(A). \]
Recall the definition of separating set. For an ideal $\mathcal{J}$ of an algebra $\mathcal{A}$, we say that $\mathcal{J}$ is a right separating set of $\mathcal{A}$-bimodule $\mathcal{M}$ if for every $M$ in $\mathcal{M}$, $\mathcal{J}M = \{0\}$ implies $M = 0$; and we say that $\mathcal{J}$ is a left separating set of $\mathcal{M}$ if for every $N$ in $\mathcal{M}$, $N\mathcal{J} = \{0\}$ implies $N = 0$.

**Theorem 3.4.** Let $\mathcal{A}$ be a unital algebra, $\mathcal{M}$ be a unital $\mathcal{A}$-bimodule with a right or a left separating set $\mathcal{J} \subseteq \mathfrak{J}(\mathcal{A})$ and $m, n$ be two positive integers with $m \neq n$. If $\delta$ is an $(m, n)$-Jordan derivable mapping at zero from $\mathcal{A}$ into $\mathcal{M}$ such that $\delta(I) = 0$, then $\delta$ is identically equal to zero.

**Proof.** Let $A, B$ be in $\mathcal{A}$ and every $S$ be in $\mathcal{J}$. By Corollary 3.3, we have that

$$\delta(SAB) = S\delta(AB)$$

and

$$\delta(SAB) = \delta((SA)B) = SA\delta(B).$$

It follows that $S(\delta(AB) - A\delta(B)) = 0$. If $\mathcal{J}$ is a right separating set of $\mathcal{M}$, we can obtain that $\delta(AB) = A\delta(B)$. Take $B = I$ and by $\delta(I) = 0$, we have that $\delta(A) = A\delta(I) = 0$.

Similarly, if $\mathcal{J}$ is a left separating set of $\mathcal{M}$, then we also can show that $\delta(A) = \delta(I)A = 0$. $\square$

By Theorem 3.4, it is easy to show the following result.

**Corollary 3.5.** Let $\mathcal{A}$ be a unital algebra with $\mathcal{A} = \mathfrak{J}(\mathcal{A})$, $\mathcal{M}$ be a unital $\mathcal{A}$-bimodule and $m, n$ be two positive integers with $m \neq n$. If $\delta$ is an $(m, n)$-Jordan derivable mapping at zero from $\mathcal{A}$ into $\mathcal{M}$ such that $\delta(I) = 0$, then $\delta$ is identically equal to zero.

Let $X$ be a complex Banach space and $B(X)$ be the set of all bounded linear operators on $X$. In this paper, every subspace of $X$ is a closed linear manifold. By a **subspace lattice** on $X$, we mean a collection $\mathcal{L}$ of subspaces of $X$ with $(0)$ and $X$ in $\mathcal{L}$ such that, for every family $\{M_r\}$ of elements of $\mathcal{L}$, both $\cap M_r$ and $\cup M_r$ belong to $\mathcal{L}$, where $\cup M_r$ denotes the closed linear span of $\{M_r\}$.

For every subspace lattice $\mathcal{L}$ on $X$, we use $\text{Alg}\mathcal{L}$ to denote the algebra of all operators in $B(X)$ that leave members of $\mathcal{L}$ invariant.

For a subspace lattice $\mathcal{L}$ on $X$ and for every $E$ in $\mathcal{L}$, we denote by

$$E_- = \lor\{F \in \mathcal{L} : F \not\supset E\}, \quad (0)_- = (0);$$

and

$$E_+ = \land\{F \in \mathcal{L} : F \not\subset E\}, \quad X_+ = X.$$

A totally ordered subspace lattice $\mathcal{N}$ is called a **nest**, $\mathcal{N}$ is called a **discrete nest** if $L_- \neq L$ for every nontrivial subspace $L$ in $\mathcal{N}$, and $\mathcal{N}$ is called a **continuous nest** if $L_- = L$ for every subspace $L$ in $\mathcal{N}$.

By [16] and Theorem 3.4, we have the following two corollaries.

**Corollary 3.6.** Let $\mathcal{L}$ be a subspace lattice in a von Neumann algebra $\mathcal{B}$ on a Hilbert space $\mathcal{H}$ such that $\mathcal{H}_- \neq \mathcal{H}$ or $(0)_+ \neq (0)$, and $m, n$ be two positive integers with $m \neq n$. If $\delta$ is an $(m, n)$-Jordan derivable mapping at zero from $\mathcal{B} \cap \text{Alg}\mathcal{L}$ into $\mathcal{B}$ such that $\delta(I) = 0$, then $\delta$ is identically equal to zero.
Corollary 3.7. Let $\mathcal{N}$ be a nest in a von Neumann algebra $\mathcal{B}$ and $m, n$ be two positive integers with $m \neq n$. If $\delta$ is an $(m, n)$-Jordan derivable mapping at zero from $\mathcal{B} \cap \operatorname{Alg}\mathcal{N}$ into $\mathcal{B}$ such that $\delta(I) = 0$, then $\delta$ is identically equal to zero.

Let $\mathcal{L}$ be a subspace lattice on $X$. Denote by $\mathcal{J}_{\mathcal{L}} = \{L \in \mathcal{L} : L \neq (0), L_+ \neq X\}$ and $\mathcal{P}_{\mathcal{L}} = \{L \in \mathcal{L} : L_- \nsubseteq L\}$. $\mathcal{L}$ is called a $\mathcal{J}$-subspace lattice on $X$ if it satisfies $E \vee E_- = X$ and $E \cap E_- = (0)$ for every $E$ in $\mathcal{J}_{\mathcal{L}}$; $\bigvee \{E : E \in \mathcal{J}_{\mathcal{L}}\} = X$ and $\bigcap \{E_- : E \in \mathcal{J}_{\mathcal{L}}\} = (0)$. $\mathcal{L}$ is called a $\mathcal{P}$-subspace lattice on $X$ if it satisfies $\bigvee \{E : E \in \mathcal{P}_{\mathcal{L}}\} = X$ or $\bigcap \{E_- : E \in \mathcal{P}_{\mathcal{L}}\} = (0)$.

The class of $\mathcal{P}$-subspace lattice algebras is very large, it includes the following:
(1) $\mathcal{J}$-subspace lattice algebras;
(2) discrete nest algebras;
(3) reflexive algebras $\operatorname{Alg}\mathcal{L}$ such that $(0)_+ \neq (0)$ or $X_- \neq X$.

In Y. Chen and J. Li [14], if $\mathcal{L}$ satisfies $\bigvee \{L : L \in \mathcal{P}_{\mathcal{L}}\} = X$ or $\bigcap \{L_- : L \in \mathcal{P}_{\mathcal{L}}\} = (0)$, then the ideal $\mathcal{T} = \text{span}\{x \otimes f : x \in E, f \in E_+, E \in \mathcal{P}_{\mathcal{L}}\}$ in $\operatorname{Alg}\mathcal{L}$ is generated by the idempotents in $\operatorname{Alg}\mathcal{L}$ and $\mathcal{T}$ is right separating set of $B(X)$. It follows the following result.

Corollary 3.8. Let $\mathcal{L}$ be a $\mathcal{P}$-subspace lattice on a Banach space $X$ and $m, n$ be two positive integers with $m \neq n$. If $\delta$ is an $(m, n)$-Jordan derivable mapping at zero from $\operatorname{Alg}\mathcal{L}$ into $B(X)$ such that $\delta(I) = 0$, then $\delta$ is identically equal to zero.

Let $\mathcal{L}$ be a subspace lattice on $X$. $\mathcal{L}$ is said to be completely distributive if its subspaces satisfy the identity
$$\bigwedge_{a \in I} \bigvee_{b \in J} L_{a,b} = \bigvee_{f \in J'} \bigwedge_{a \in I} L_{a,f(a)},$$
where $J'$ denotes the set of all $f : I \to J$.

Suppose that $\mathcal{L}$ is a completely distributive subspace lattice on $X$ and $\mathcal{A} = \operatorname{Alg}\mathcal{L}$. By D. Hadwin [15], we know that $\mathcal{T} = \text{span}\{T : T \in \mathcal{A}, \text{rank } T = 1\}$ is an ideal of $\mathcal{A}$ and by C. Laurie [17], we have that $\mathcal{T}$ is a separating set of $\mathcal{M}$.

Corollary 3.9. Let $\mathcal{L}$ be a completely distributive subspace lattice on a Hilbert space $\mathcal{H}$, $\mathcal{M}$ be a dual normal Banach $\operatorname{Alg}\mathcal{L}$-bimodule and $m, n$ be two positive integers with $m \neq n$. If $\delta$ is an $(m, n)$-Jordan derivable mapping at zero from $\operatorname{Alg}\mathcal{L}$ into $\mathcal{M}$ such that $\delta(I) = 0$, then $\delta$ is identically equal to zero.

Corollary 3.10. Let $\mathcal{A}$ be a unital subalgebra of $B(X)$ such that $\mathcal{A}$ contains $\{x_0 \otimes f : f \in X^*\}$, where $0 \neq x_0 \in X$, and $m, n$ be two positive integers with $m \neq n$. If $\delta$ is an $(m, n)$-Jordan derivable mapping at zero from $\mathcal{A}$ into $B(X)$ such that $\delta(I) = 0$, then $\delta$ is identically equal to zero.

Similar to the corollary 3.10, we have the following result.

Corollary 3.11. Let $\mathcal{A}$ be a unital subalgebra of $B(X)$ such that $\mathcal{A}$ contains $\{x \otimes f_0 : x \in X\}$, where $0 \neq f_0 \in X^*$, and $m, n$ be two positive integers with $m \neq n$. If $\delta$ is an $(m, n)$-Jordan derivable mapping at zero from $\mathcal{A}$ into $B(X)$ such that $\delta(I) = 0$, then $\delta$ is identically equal to zero.
4. \((m, n)\)-JORDAN DERIVABLE MAPPINGS AT ZERO ON GENERALIZED MATRIX ALGEBRAS

In this section, we give a characterization of \((m, n)\)-Jordan derivable mappings at zero on generalized matrix algebras.

**Theorem 4.1.** Suppose that \(A, B\) are two unital algebras, \(m, n\) be two positive integers with \(m \neq n\), and \(U = \begin{bmatrix} A & M \\ N & B \end{bmatrix}\) is a generalized matrix ring. If one of the following four statements holds:

(1) \(M\) is a faithful unital \((A, B)\)-bimodule;
(2) \(N\) is a faithful unital \((B, A)\)-bimodule;
(3) \(M\) is a faithful unital left \(A\)-module, \(N\) is a faithful unital left \(B\)-module;
(4) \(N\) is a faithful unital right \(A\)-module, \(M\) is a faithful unital right \(B\)-module,

then every \((m, n)\)-Jordan derivable mapping from generalized matrix algebra \(U\) into itself satisfies \(\delta \left( \begin{bmatrix} I_A & 0 \\ 0 & I_B \end{bmatrix} \right) = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}\) is identically equal to zero.

**Proof.** Since \(\delta\) is a linear mapping, for each \(A \in A, B \in B, M \in M\) and \(N \in N\), we have that

\[
\delta \left( \begin{bmatrix} A & M \\ N & B \end{bmatrix} \right) = \begin{bmatrix} a_{11}(A) + b_{11}(M) + c_{11}(N) + d_{11}(B) & a_{12}(A) + b_{12}(M) + c_{12}(N) + d_{12}(B) \\ a_{21}(A) + b_{21}(M) + c_{21}(N) + d_{21}(B) & a_{22}(A) + b_{22}(M) + c_{22}(N) + d_{22}(B) \end{bmatrix},
\]

where \(a_{ij}, b_{ij}, c_{ij}\) and \(d_{ij}\) are linear mappings, \(i, j \in \{1, 2\}\).

Let \(I_A\) be a unit element in \(A\) and \(I_B\) be a unit element in \(B\). For every \(M\) in \(M\), suppose that \(T = \begin{bmatrix} 0 & M \\ 0 & 0 \end{bmatrix}\) and \(S = \begin{bmatrix} I_A & 0 \\ 0 & 0 \end{bmatrix}\). By Proposition 3.2, we have that \(\delta(TS) = \delta(ST)\), that is

\[
\delta \left( \begin{bmatrix} 0 & M \\ 0 & 0 \end{bmatrix} \begin{bmatrix} I_A & 0 \\ 0 & 0 \end{bmatrix} \right) = \delta \left( \begin{bmatrix} I_A & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} 0 & M \\ 0 & 0 \end{bmatrix} \right),
\]

it follows that

\[
0 = \delta \left( \begin{bmatrix} 0 & M \\ 0 & 0 \end{bmatrix} \right) = \begin{bmatrix} b_{11}(M) & b_{12}(M) \\ b_{21}(M) & b_{22}(M) \end{bmatrix}.
\]

Thus, for every \(M\) in \(M\), we can obtain that

\[
b_{11}(M) = b_{12}(M) = b_{21}(M) = b_{22}(M) = 0. \tag{4.1}
\]

Similarly, for every \(N\) in \(N\), we can show that

\[
c_{11}(N) = c_{12}(N) = c_{21}(N) = c_{22}(N) = 0. \tag{4.2}
\]

For every \(A\) in \(A\), suppose that \(T = \begin{bmatrix} A & 0 \\ 0 & 0 \end{bmatrix}\) and \(S = \begin{bmatrix} I_A & 0 \\ 0 & 0 \end{bmatrix}\). By Proposition 3.2, we have that \(\delta(TS) = S\delta(T) = \delta(T)S\), that is

\[
\delta \left( \begin{bmatrix} A & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} I_A & 0 \\ 0 & 0 \end{bmatrix} \right) = \begin{bmatrix} I_A & 0 \\ 0 & 0 \end{bmatrix} \delta \left( \begin{bmatrix} A & 0 \\ 0 & 0 \end{bmatrix} \right) = \begin{bmatrix} I_A & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} A & 0 \\ 0 & 0 \end{bmatrix},
\]
it follows that
\[
\begin{bmatrix}
a_{11}(A) & a_{12}(A) \\
a_{21}(A) & a_{22}(A)
\end{bmatrix}
= \begin{bmatrix}
I_A & 0 \\
0 & 0
\end{bmatrix}
\begin{bmatrix}
a_{11}(A) & a_{12}(A) \\
a_{21}(A) & a_{22}(A)
\end{bmatrix}
= \begin{bmatrix}
a_{11}(A) & a_{12}(A) \\
a_{21}(A) & a_{22}(A)
\end{bmatrix}
\begin{bmatrix}
I_A & 0 \\
0 & 0
\end{bmatrix},
\]
it implies that
\[
\begin{bmatrix}
a_{11}(A) & a_{12}(A) \\
a_{21}(A) & a_{22}(A)
\end{bmatrix}
= \begin{bmatrix}
a_{11}(A) & a_{12}(A) \\
0 & 0
\end{bmatrix}
= \begin{bmatrix}
a_{11}(A) & a_{12}(A) \\
a_{21}(A) & a_{22}(A)
\end{bmatrix}.
\]
Thus, for every \(A\) in \(A\), we can obtain that
\[
a_{12}(A) = a_{21}(A) = a_{22}(A) = 0. \tag{4.3}
\]
Similarly for every \(B\) in \(B\), we can show that
\[
d_{12}(B) = d_{21}(B) = d_{11}(B) = 0. \tag{4.4}
\]
In the following, we prove that \(a_{11}(A) = d_{22}(B) = 0\).

Suppose that \(M\) is a faithful unital \((A, B)\)-bimodule. For every \(A\) in \(A\), suppose that \(T = \begin{bmatrix} A & -AM \\
0 & 0 \end{bmatrix}\) and \(S = \begin{bmatrix} 0 & M \\
0 & I_B \end{bmatrix}\). It is clear that \(TS = ST = 0\). By the definition of \(\delta\), we have that
\[
m\delta(T)S + m\delta(S)T + nT\delta(S) + nS\delta(T) = 0.
\]
By (4.1) and (4.3), we can obtain that
\[
m \begin{bmatrix} a_{11}(A) & 0 \\
0 & 0 \end{bmatrix} \begin{bmatrix} 0 & M \\
0 & I_B \end{bmatrix} + n \begin{bmatrix} 0 & M \\
0 & I_B \end{bmatrix} \begin{bmatrix} a_{11}(A) & 0 \\
0 & 0 \end{bmatrix} = 0.
\]
It follows that \(m \begin{bmatrix} 0 & a_{11}(A)M \\
0 & 0 \end{bmatrix} = 0\), it means that \(ma_{11}(A)M\) for every \(M\) in \(M\), since \(m > 0\) and \(M\) is a left faithful unital \(A\)-module, we can obtain that \(a_{11}(A) = 0\). Similarly, since \(n > 0\) and \(M\) is a right faithful unital \(B\)-module, we can obtain that \(d_{22}(B) = 0\).

Suppose that \(N\) is a faithful unital \((B, A)\)-bimodule. Similar to the above method, we also can prove that \(a_{11}(A) = d_{22}(B) = 0\).

Suppose that \(M\) is a faithful unital left \(A\)-module and \(N\) is a faithful unital left \(B\)-module. For every \(A\) in \(A\), we have prove that \(a_{11}(A) = 0\). For every \(B\) in \(B\), suppose that \(T = \begin{bmatrix} 0 & 0 \\
-NB & B \end{bmatrix}\) and \(S = \begin{bmatrix} I_A & 0 \\
N & 0 \end{bmatrix}\), it is clear that \(TS = ST = 0\). By the definition of \(\delta\), we have that
\[
m\delta(T)S + m\delta(S)T + nT\delta(S) + nS\delta(T) = 0.
\]
By (4.1) and (4.3), we can obtain that
\[
m \begin{bmatrix} 0 & 0 \\
0 & d_{22}(B) \end{bmatrix} \begin{bmatrix} I_A & 0 \\
N & 0 \end{bmatrix} + n \begin{bmatrix} I_A & 0 \\
N & 0 \end{bmatrix} \begin{bmatrix} 0 & 0 \\
0 & d_{22}(B) \end{bmatrix} = 0.
\]
It follows that \(m \begin{bmatrix} 0 & 0 \\
0 & d_{22}(B) \end{bmatrix} = 0\), it means that \(md_{22}(B)N\) for every \(N\) in \(N\), since \(m > 0\) and \(N\) is a left faithful unital \(N\)-module, we can obtain that \(d_{22}(B) = 0\). \(\square\)


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