Higgs boson decay to lepton pair and photon and possible non-Hermiticity of the Yukawa interaction

1. Introduction

In 2012 the collaborations ATLAS and CMS at the Large Hadron Collider (LHC) discovered the spinless particle $h$ with the mass about 125 GeV [1, 2]. The study of processes of production of $h$ boson and its decay channels allowed to conclude [3, 4], that its characteristics are consistent with properties of the Higgs boson of the Standard model (SM). In particular, the analysis of the angular correlations in the decays $h \rightarrow ZZ^\ast$, $Z\gamma^\ast$, $\gamma^\ast\gamma^\ast \rightarrow 4\ell$, $h \rightarrow WW^\ast \rightarrow \ell\nu\ell\nu$ ($\ell = e, \mu$), and $h \rightarrow \gamma\gamma$ revealed that all the data agree with predictions for the Higgs boson with the quantum numbers $J^{PC} = 0^{++}$ [3, 4].

The masses of the fermions in the SM arise due to the Yukawa interaction between the Higgs field and the fermion fields. The investigation of this interaction is necessary for identification of the particle $h$ with the Higgs boson of the SM. Particularly, it is important to check whether the Higgs boson interaction with fermions is Hermitian or not. Moreover, it is necessary also to verify the Hermiticity of the Yukawa interaction Lagrangian [5].

Presently for the decay modes $h \rightarrow \tau^+\tau^-$ and $h \rightarrow bb$ the Higgs signal strength parameter $\mu(X)$ is determined. This is the ratio of the experimentally measured cross section of the Higgs boson production with subsequent decay into certain final state $X$, to the corresponding value calculated in the SM:

$$\mu(X) = \frac{\sigma(pp \rightarrow h)_{exp} BR(h \rightarrow X)_{exp}}{\sigma(pp \rightarrow h)_{SM} BR(h \rightarrow X)_{SM}}.\ (1)$$

There are ATLAS and CMS measurements of the production cross section and decay rate of the Higgs boson, as well as the constraints on the coupling constants of the interaction with vector bosons and fermions. The result of this analysis are the values $\mu(\tau^+\tau^-) = 1.12 \pm 0.23$ and $\mu(bb) = 0.82 \pm 0.30$ [5].

Some aspects of possible non-Hermiticity of the Higgs boson interaction with the top quark have been studied in [10, 12]. Namely, in [10, 11], the polariza-
tion characteristics of the photon in the decay processes $h \to \gamma\gamma$ and $h \to \gamma Z$ were studied. The photon circular polarization in these decays arises due to presence of the $\mathcal{CP}$-even and $\mathcal{CP}$-odd components in the $ht\bar{t}$ interaction, small imaginary loop contribution in the SM, and non-Hermiticity of the $ht\bar{t}$ interaction. In (12), it was shown that the lepton forward-backward asymmetry $A_{FB}$ in the processes $h \to \gamma\ell^+\ell^-$ (for $\ell = e, \mu, \tau$) is sensitive to a possible non-Hermiticity of the Higgs interaction with the top quark. We emphasize that measurement of any observable which is sensitive to non-Hermiticity of Lagrangian can be used at the same time for testing the $\mathcal{CPT}$ theorem, since Hermiticity of Hamiltonian (or Lagrangian) is the necessary condition in the proof of the $\mathcal{CPT}$ theorem in the quantum field theory (see, e.g., (13)).

In the present paper, we consider the influence of a possible non-Hermiticity of the Higgs boson interaction with fermions (leptons and quarks), which contains both scalar and pseudoscalar parts, on the decay $h \to \gamma\ell^+\ell^-$, where $\ell = (e, \mu, \tau)$. We calculate the differential decay rate and lepton forward-backward asymmetry as functions of the invariant mass of the lepton pair, and discuss the integrated over the invariant mass decay rate and the asymmetry.

2. Decay amplitudes and angular distribution

We assume that the interaction of the $h$-boson with the fermion fields, $\psi_f$, is described by the Lagrangian

$$\mathcal{L}_{hf} = - \sum_{f = \ell, q} \frac{m_f}{v} \bar{\psi}_f (a_f + i b_f \gamma_5) \psi_f,$$

(2)

which includes both scalar and pseudoscalar parts. Here $v = (\sqrt{2}G_F)^{-1/2} \approx 246$ GeV is the vacuum expectation value of the Higgs field, $G_F = 1.1663787(6) \times 10^{-5}$ GeV$^{-2}$ is the Fermi constant, $m_f$ is the fermion mass, and $a_f, b_f$ are complex-valued parameters (of course, $a_f = 1$ and $b_f = 0$ correspond to the SM). One can consider (2) as a phenomenological parameterization of effects of “new physics” (NP) beyond the SM. At the same time, the interaction with the $W^\pm$ and $Z$ bosons is taken as in the SM. For real values of parameters $a_f$ and $b_f$ the interaction (2) is Hermitian. However, it is straightforward to verify (8) that the width of the $h$-boson decay to the fermion pair in lowest order is the same for Hermitian and non-Hermitian Lagrangian (2).

We consider the $h$-boson decay

$$h(p) \to \gamma(k, \epsilon(k)) + \ell^+(q_+) + \ell^-(q_-),$$

(3)

where the 4-momenta of the $h$ boson, photon and leptons are, respectively, $p, k, q_+, q_-$, and $\epsilon(k)$ is the 4-vector of the photon polarization.

The differential decay width can be written as

$$\frac{d\Gamma}{dq^2 d\cos \theta} = \frac{\beta_\ell (m_h^2 - q^2)}{(8\pi)^3 m_h^3} \sum_{pol} |\mathcal{M}|^2,$$

(4)

where $m_h$ is the mass of the $h$ boson, $q \equiv q_+ + q_-$, $q^2$ is the invariant mass squared of the lepton pair, $\beta_\ell = \sqrt{1 - 4m_L^2/q^2}$ is the velocity of lepton in the rest frame of the lepton pair. The polar angle $\theta$ is determined in this frame; it is the angle between the momentum of $\ell^+$ lepton and the axis which is opposite to the direction of motion of the Higgs boson.

The decay amplitude is

$$\mathcal{M} = \mathcal{M}_{tree} + \mathcal{M}_{loop},$$

(5)

where the tree-level amplitude (see Fig. 1) is

$$\mathcal{M}_{tree} = \epsilon_\mu(k) \bar{u}(q_-)(a_f + i b_f \gamma_5) \times \left( \frac{2q^\mu + k\gamma^\mu}{2k \cdot q_+} - \frac{2q^\mu + \gamma^\mu}{2k \cdot q_-} \right) v(q_+),$$

(6)

with

$$e = \sqrt{4\pi G_F}$$

is the positron electric charge, $Q_\ell = -1$ (lepton charge in units of $e$) and $m_\ell$ is the lepton mass. The electromagnetic coupling constant in the $G_F$ scheme is

$$\alpha_{G_F} = \sqrt{2}G_F m_W^2 (1 - m_W^2/m_Z^2)/\pi,$$

where $m_W$ is the mass of the $W$ boson.

The one-loop contributions to the $h \to \gamma\gamma^*/Z^* \to \gamma\ell^+\ell^-$ decay (see Fig. 1) can be written in the form

$$\mathcal{M}_{loop} = \epsilon_\mu(k) \left[ (q^\mu k^\nu - q^\mu k \cdot q) \times \bar{u}(q_-) (c_1 \gamma_\nu + c_2 \gamma_\nu \gamma_5) v(q_+) \ight.$$

$$\left. - \epsilon_{\mu\nu\alpha\beta} k_\alpha q_\beta \bar{u}(q_-) (c_3 \gamma_\nu + c_4 \gamma_\nu \gamma_5) v(q_+) \right]$$

(8)

with the coefficients $c_1, ..., c_4$ which have been obtained in (12) and are collected for convenience in Appendix A. Besides, $\epsilon_{0123} = +1$.

In the present work we do not include the box-type loop contributions to the process $h \to \gamma\ell^+\ell^-$. The contribution from these diagrams in the SM is very...
small [13, 14]. In addition, there are other mechanisms, \( h \to \gamma V \to \gamma \ell^+ \ell^- \), where \( V \) is intermediate vector resonance decaying to the \( \ell^+ \ell^- \) pair, which can contribute to the decay \( h \to \gamma \ell^+ \ell^- \). Particularly, the production of charmonium \( J/\psi (c \bar{c}) \) and bottomonium \( \Upsilon(1S) (b \bar{b}) \) is interesting for studying the \( hq \bar{q} \) interaction (see, e.g., [15, 20]). However, these processes lie beyond the scope of the present work.

Calculating the amplitude (5) squared and summed over the polarizations of leptons and photon we obtain in the model (2)

\[
\sum_{\text{pol}} |\mathcal{M}|^2 = c_2^2 \left[ |a_t|^2 A + |b_t|^2 \bar{A} \right] + 2 c_0 \left[ \text{Re}(c_1 a_t^*) B + \text{Im}(c_2 b_t^*) \bar{B} \right] + \text{Im}(c_4 a_t^*) C + \text{Re}(c_3 b_t^*) \bar{C} \right] + \left( |c_1|^2 + |c_3|^2 \right) D + \left( |c_2|^2 + |c_4|^2 \right) E + 2 \text{Im}(c_1 c_4^* + c_2 c_3^*) F ,
\]

where \( A, \bar{A}, B, \bar{B}, C, \bar{C}, D, E, F \) have the same form as in (12) and are given in Appendix A.

The forward-backward (FB) asymmetry is defined (see, e.g., [12])

\[
A_{\text{FB}}(q^2) = \left( \frac{d\Gamma_F}{dq^2} - \frac{d\Gamma_B}{dq^2} \right)/\left( \frac{d\Gamma_F}{dq^2} + \frac{d\Gamma_B}{dq^2} \right) ,
\]

where

\[
\frac{d\Gamma_F}{dq^2} = \int_0^1 \frac{d\Gamma}{dq^2} d\cos \theta \, d\cos \theta ,
\]

\[
\frac{d\Gamma_B}{dq^2} = \int_{-1}^0 \frac{d\Gamma}{dq^2} d\cos \theta \, d\cos \theta ,
\]

and also the \( q^2 \) integrated FB asymmetry reads

\[
\langle A_{\text{FB}} \rangle = \frac{\langle d\Gamma_F dq^2 \rangle - \langle d\Gamma_B dq^2 \rangle}{\langle d\Gamma_F dq^2 \rangle + \langle d\Gamma_B dq^2 \rangle} ,
\]

with the notation

\[
\langle J \rangle = \int \frac{q^2_{\text{max}}}{q^2_{\text{min}}} dq^2 J(q^2)
\]

for the integration limits \( q^2_{\text{min}} \geq 4m_t^2 \) and \( q^2_{\text{max}} \leq m_h^2 \).

In Eqs. (4), only the coefficients \( B, C \) and \( F \) are linear in \( \cos \theta \), therefore, as can be seen from (4) and (10), the numerator of the FB asymmetry (10) is determined by the imaginary combination of the terms \( c_2 b_t^* + 4c_4 a_t^* \) and \( c_1 c_4^* + c_2 c_3^* \):

\[
\frac{d\Gamma_F}{dq^2} - \frac{d\Gamma_B}{dq^2} = \frac{2(m_h^2 - q^2)^2}{(8\pi^3)^2 m_h^4}
\]

\[
\times \left[ 8m_t c_0 \text{Im}(c_2 b_t^* + c_4 a_t^*) \log \left( \frac{q^2}{4m_t^2} \right) \right.
\]

\[
+ \text{Im}(c_1 c_4^* + c_2 c_3^*) (q^2 - 4m_t^2) (m_h^2 - q^2) \right] .
\]

In framework of the SM the differential decay width (4), (9) takes a simpler form. In this case, \( a_t = 1, b_t = 0 \) and \( c_{4, \text{SM}} = c_{4, \text{SM}} = 0 \). Thus the FB asymmetry vanishes,

\[
A_{\text{FB}}(q^2)_{\text{SM}} = 0 .
\]

Therefore, nonzero values of this asymmetry can arise only in certain models of new physics.

3. Results of calculations and discussion

First, we discuss the choice of parameters \( a_f \) and \( b_f \), that determine the interaction of the Higgs boson with fermions (2). In the leading order, the rate of the \( h \)-boson decay to fermions, except the top quarks, has the form

\[
\Gamma(h \to f \bar{f}) = \frac{N_f G_F}{4\sqrt{2}\pi} m_f^2 m_h \beta_f (|a_f|^2 \beta_f^2 + |b_f|^2) ,
\]

(16)
where $\beta_f = \sqrt{1 - 4m_f^2/m_h^2}$ is the fermion velocity in the rest frame of $h$, $N_f = 1$ (3) for leptons (quarks). Apparently, $\beta_f$ is equal to one with a good accuracy. We assume further that the $h \to f\bar{f}$ decay rate in the model (2) is the same as in the SM, i.e.,

$$|a_f|^2 + |b_f|^2 = 1. \quad (17)$$

In this case, to search for effects of new physics in the decay $h \to f\bar{f}$, it will be necessary to measure the polarization characteristics of the fermions, which clearly complicates the identification of particle $h$ with the Higgs boson of the SM.

Let us calculate the predictions of the model (2) with the constraint $|a_f|^2 + |b_f|^2 = 1$ for the decay $h \to \gamma\ell^+\ell^-$ and check how much these predictions differ from the SM. In the calculations we use the parameters of NP presented in Table 1.

**Table 1. Parameters of the $hff$ interaction in the SM, and in several models of NP.** Here, $\ell$ denote leptons, $q$ denote quarks of all flavors, except the top quark, for which the couplings are taken from [21]. In the NP1 model, the interaction (2) is non-Hermitian. Finally, in the model NP3, the interaction for the leptons is non-Hermitian, while for all quarks the interaction (2) is taken as in the SM.

The new physics model NP1 is described by the parameters $a_f = 1/\sqrt{2}$, $b_f = 1/\sqrt{2}$ for all fermions, except the top quark for which the couplings are taken from [21]. In the NP1 model, the interaction (2) is Hermitian. In the model NP2, $a_f = 1/\sqrt{2}$, $b_f = i/\sqrt{2}$, while for the top quark we choose $a_t = 1.20$, $b_t = 0.37i$, so that the interaction (2) becomes non-Hermitian. Finally, in the model NP3, the interaction for the leptons is non-Hermitian, while for all quarks the interaction (2) is taken as in the SM.

The numerical values of other parameters of the SM are taken from [14], in particular, the masses of the $W^\pm$ and $Z$ bosons, the decay widths and couplings for $Zff$ interaction. The quark masses are chosen as in [14, 22], and $\sin^2 \theta_W = 1 - m_W^2/m_Z^2$. In Fig. 2 we present the differential width of the $h \to \ell^+\ell^-\gamma$ decay for various leptons $l = (e, \mu, \tau)$ calculated in the SM and in the model (2) with the couplings in Table 1. The minimal photon energy in the rest frame of $h$ boson is chosen $E_{\gamma_{\min}} = 1$ GeV to cut off the infrared divergence at $E_{\gamma} \to 0$. This leads to maximal value of the dilepton invariant mass.
The forward-backward asymmetry in the decay $h \to \gamma\ell^+\ell^-$ for various lepton pairs as a function of $x$, where $x \equiv \sqrt{q^2}/m_h$. The dotted lines correspond to NP1, dashed lines – NP2, dash-dotted lines – NP3. As for the $\tau^+\tau^-$ pair, the behavior is different from the case of the light leptons. In the process $h \to \tau^+\tau^-\gamma$, the dominant contribution comes from the tree-level diagram in Fig. 1 and therefore the

\[ q_{\text{max}} = \left( m_h^2 - 2m_h E_{\gamma}^{\text{min}} \right)^{1/2} \approx m_h - E_{\gamma}^{\text{min}} = 124 \text{ GeV} \]

for $m_h = 125.09 \text{ GeV}$.

As one can see from Fig. 2 there are deviations from the SM predictions with the chosen parameters $a_f$, $b_f$. In Table 2 we also show the decay widths integrated over the invariant mass within the limits $[q_{\text{min}}, q_{\text{max}}]$.

Table 2. The width of the decay $\Gamma(h \to \gamma\ell^+\ell^-)$ (in keV) for various lepton pairs in the invariant mass limits $[q_{\text{min}}, q_{\text{max}}]$ (in GeV)

| $\ell^+\ell^-$ | $q_{\text{min}}$ | $q_{\text{max}}$ | SM | NP1 | NP2 | NP3 |
|---------------|-----------------|-----------------|----|-----|-----|-----|
| $e^+e^-$      | 1.0             | 124.0           | 0.34 | 0.32 | 0.32 | 0.34 |
|               | 1.0             | 30.0            | 0.11 | 0.10 | 0.10 | 0.11 |
|               | 37.5            | 75.0            | 0.02 | 0.02 | 0.02 | 0.02 |
| $\mu^+\mu^-$  | 1.0             | 124.0           | 0.53 | 0.52 | 0.52 | 0.53 |
|               | 1.0             | 30.0            | 0.11 | 0.10 | 0.10 | 0.11 |
|               | 37.5            | 75.0            | 0.03 | 0.03 | 0.03 | 0.03 |
| $\tau^+\tau^-$| 4.0             | 124.0           | 31.0 | 31.1 | 31.1 | 31.1 |
|               | 12.5            | 75.0            | 1.77 | 1.80 | 1.80 | 1.79 |

In the interval of invariant masses from 1.0 GeV to 124.0 GeV, the effect of new physics does not exceed 5% in the decays $h \to \gamma e^+e^-$ and $h \to \gamma\mu^+\mu^-$. However, at small invariant masses below 30 GeV, this effect reaches 10%, although the decay rate in this interval is very small compared, for example, with the rate of the Higgs boson decay to two photons $\Gamma(h \to \gamma\gamma) = 9.28 \text{ keV}$ [14].

In Fig. 3 we show the forward-backward asymmetry [11]. As was previously mentioned, the FB asymmetry takes zero value in the SM, and non-zero values can arise only in models beyond the SM. Of course, not all models of NP lead to non-zero FB asymmetry.

In Table 3 we also present the integrated FB asymmetry [12].

As it is seen from Fig. 3 the FB asymmetry for the $e^+e^-$ and $\mu^+\mu^-$ pairs for the real parameters $a_f$ and $b_f$ is very small, of the order of 1%. For only the pair of the $\tau$-leptons, this asymmetry reaches 2.5% at the invariant mass near the $Z$-boson mass.

The FB asymmetry for the $e^+e^-$ and $\mu^+\mu^-$ pairs increases considerably for the non-Hermitian $hf\bar{f}$ interaction in the model NP2. Namely, the FB asymmetry can reach 15% for the electron-positron pair and 10% for the muon-antimuon pair. In the model NP3, in which only interaction $h\ell^+\ell^-$ is non-Hermitian, the FB asymmetry is still very small. Thus, the most important contribution comes from non-Hermiticity of the Higgs boson interaction with the quarks, mainly with the top quark. This $hf$ interaction enters in the loop diagrams in Fig. 1.

As for the $\tau^+\tau^-$ pair, the behavior is different from the case of the light leptons. In the process $h \to \tau^+\tau^-\gamma$, the dominant contribution comes from the tree-level diagram in Fig. 1 and therefore the
structure of the interaction of Higgs boson with tau lepton is crucial. This explains the observed tendency in Fig. 3 in which the models NP2 and NP3 give the close results, though the absolute value of the FB asymmetry does not exceed 1.5%.

It is seen from Fig. 3 that the FB asymmetries change sign as functions of the variable $x = \sqrt{q^2}/m_h$. Therefore, integration of the asymmetries over the whole interval of invariant masses gives small values. This is demonstrated in Table 3, the obtained values for the $e^+e^-$ and $\mu^+\mu^-$ pairs are less than 1%, and for $\tau^+\tau^-$ pair – less than 0.1%. However, choosing the suitable intervals of integration increases the corresponding values to 13.1% for the $e^+e^-$ pair, and 8.5% for the $\mu^+\mu^-$ pair. For the $\tau^+\tau^-$ pair, the integrated FB asymmetry does not exceed 1%.

4. Conclusions

We studied the decay of the Higgs boson to a photon and a lepton-antilepton pair, $h \rightarrow \gamma \ell^+\ell^-$, where $\ell = (e, \mu, \tau)$. The differential decay width and lepton forward-backward asymmetry are calculated as functions of the dilepton invariant mass.

These observables are calculated in the Standard model and in the model in which the Higgs boson interaction with the fermions consists of scalar and pseudoscalar terms, which imply the $CP$ violation. Moreover, we assume a possible non-Hermiticity of this interaction. The tree-level amplitudes and the one-loop $h \rightarrow Z^* \rightarrow \gamma \ell^+\ell^-$ and $h \rightarrow \gamma^* \rightarrow \gamma \ell^+\ell^-$ diagrams are included. The main emphasis is put on studying effects of possible non-Hermiticity of the $hff$ interaction on the observables.

The observables are calculated with the model parameters $a_f, b_f$ chosen in such a way that the $h \rightarrow f\bar{f}$ decay rate (where $f = (\ell, q)$) coincides with the rate in the SM. For the couplings with the top quark, $a_t, b_t$, we choose the values from Ref. 21 which are constrained from all available data.

The calculations show that the differential decay width for $h \rightarrow \gamma e^+e^-$ and $h \rightarrow \gamma \mu^+\mu^-$ is not very sensitive to effects of NP. Only at small values of the dilepton invariant mass below 30 GeV, the corrections to the SM prediction reach 10%, though the decay rate in this interval is rather small, about 0.1 keV.

The lepton forward-backward asymmetry $A_{FB}$ is more sensitive to effects of NP, because this observable vanishes identically in the SM. However, for real parameters of the $hff$ interaction, $A_{FB}$ is small, of the order of 1% for the $e^+e^-$ and $\mu^+\mu^-$ pairs, and 2.5% for the $\tau^+\tau^-$ pair. This asymmetry becomes sizable for the complex parameters $a_f, b_f$, i.e., for the non-Hermitian interaction. In particular, for the $e^+e^-$ and $\mu^+\mu^-$ pairs, $A_{FB}$ rises to 15% for the $e^+e^-$ pair and to 10% for the $\mu^+\mu^-$ pair. The main contribution to $A_{FB}$ comes from the non-Hermitian interaction of the Higgs boson with the top quark in the loop diagrams.

At the same time, for the $\tau^+\tau^-$ pair, the tree-level diagrams are dominant, and thus the asymmetry depends on the Higgs interaction with the tau leptons. It turns out that effect of a non-Hermiticity in $A_{FB}$ is of the order of 1%, which may be difficult for experimental studies.

Our consideration of the decays $h \rightarrow \gamma \ell^+\ell^-$ demonstrates that a possible non-Hermiticity of the $h\ell\bar{\ell}$ interaction has big impact on the forward-backward asymmetry for the light leptons. The non-Hermiticity of the $h\ell\bar{\ell}$ interaction does not show up in this observable. In summary, the forward-backward asymmetry in the $h \rightarrow \gamma e^+e^-$ and $h \rightarrow \gamma \mu^+\mu^-$ decays is the informative and important observable for experimental studies at the LHC in the search for effects of new physics.

Acknowledgments

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Appendix A: Definition of coefficients $c_1, \ldots, c_4$ and $A, \tilde{A}, B, \tilde{B}, C, \tilde{C}, D, E, F$.

In this Appendix we present the coefficients $c_1, \ldots, c_4$ in Eq. (3), which are determined from the loop diagrams in Fig. 1. They read [12]:

$$
c_1 = \frac{1}{2} \frac{g_{V_L}}{q^2 - m_W^2 + im_W \Gamma_Z} \Pi_Z + \frac{Q_f}{q^2} \Pi_\gamma,
$$

$$
c_2 = -\frac{1}{2} \frac{g_{A_L}}{q^2 - m_Z^2 + im_Z \Gamma_Z} \Pi_Z,
$$

$$
c_3 = \frac{1}{2} \frac{g_{V_L}}{q^2 - m_Z^2 + im_Z \Gamma_Z} \Pi_Z + \frac{Q_f}{q^2} \Pi_\gamma,
$$

$$
c_4 = -\frac{1}{2} \frac{g_{A_L}}{q^2 - m_Z^2 + im_Z \Gamma_Z} \Pi_Z,
$$

with

$$
\Pi_Z = \frac{e g^3}{16 \pi^2 m_W^2} \left[ a_f \frac{g_{V_L}}{c_W} N_f Q_f A_f(\lambda_f', \lambda_f) + A_W(\lambda_W, \lambda_W) \right],
$$

$$
\Pi_\gamma = \frac{e g}{16 \pi^2 m_W^2} \left[ a_f 4 Q_f^2 N_f A_f(\lambda_f', \lambda_f) + A_W(\lambda_W, \lambda_W) \right],
$$

$$
\tilde{\Pi}_Z = \frac{e g^3}{16 \pi^2 m_W^2} b_f \frac{g_{V_L}}{c_W} N_f Q_f I_2(\lambda_f', \lambda_f),
$$

$$
\tilde{\Pi}_\gamma = \frac{e g}{16 \pi^2 m_W^2} b_f 4 Q_f^2 N_f I_2(\lambda_f', \lambda_f). \quad \text{(A5)}
$$

Here, $\Gamma_Z$ is the total decay width of the $Z$ boson, $c_W = \cos \theta_W$, where $\theta_W$ is the weak angle, $g = 2m_W(\sqrt{2}G_F)^{1/2}$, $Q_f$ is the charge of the fermion in units of $e$, $g_{V,L} = t_{3L,f} - 2Q_f s_W^2$, $g_{A,L} = t_{3L,f}$ is the $Zff$ vector coupling (axial-vector one), where $t_{3L,f}$ is projection of the weak isospin. The sum over all leptons and quarks in (A2)- (A5) is implied.

The loop integrals for fermions, $A_f(\lambda_f', \lambda_f)$, and $W$ bosons, $A_W(\lambda_W', \lambda_W)$, are expressed via the loop functions $I_1(\lambda', \lambda)$ and $I_2(\lambda', \lambda)$ introduced in [22] and given explicitly in [12]. The arguments of these functions are

$$
\lambda_{f,W} \equiv 4m_{f,W}^2/q^2, \quad \lambda_{f,W}' \equiv \lambda_{f,W}|_{q^2=m_f^2}. \quad \text{(A6)}
$$

Further, the coefficients in Eq. (3) are defined as follows:

$$
A = \frac{16}{(1 - \beta z^2)^2(1 - m_h^2 - q^2)^2} \left[ (m_h^2 + q^4) - 8m_h^2 q^2 (1 - \beta z^2) + 32m_h^4 - 8m_h^2 m_h^2 \right], \quad \text{(A7)}
$$

$$
\tilde{A} = \frac{16}{(1 - \beta z^2)^2(1 - m_h^2 - q^2)^2} \left[ (m_h^2 + q^4) \times (1 - \beta z^2) - 8m_h^2 m_h^2 \right],
$$

$$
B = -8m_h (1 - \beta z^2) \left[ m_h^2 - q^2 + q^2 \beta z (1 - z^2) \right],
$$

$$
\tilde{B} = -8m_h (1 - \beta z^2) \left[ m_h^2 - q^2 \right] \beta z,
$$

$$
C = -8m_h (1 - \beta z^2) \left[ m_h^2 - q^2 \right] \beta z,
$$

$$
\tilde{C} = -8m_h (1 - \beta z^2) \left[ (m_h^2 - q^2) \beta z \right],
$$

$$
D = \frac{1}{2} (m_h^2 - q^2)^2 \left[ q^2 (1 + \beta z^2) + 4m_h^2 \right],
$$

$$
E = \frac{1}{2} (m_h^2 - q^2)^2 q^2 \beta^2 (1 + z^2),
$$

$$
F = - (m_h^2 - q^2)^2 q^2 \beta z
$$

with the notation $z \equiv \cos \theta$.

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