Pseudo-Majoron as Dark Matter

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We consider the singlet Majoron model with softly broken lepton number. This model contains three right-handed neutrinos and a singlet scalar besides the standard model fields. The real part of the singlet scalar develops a vacuum expectation value to generate the lepton number violation for seesaw and leptogenesis. The imaginary part of the singlet scalar becomes a massive pseudo-Majoron to be a dark matter candidate with testability by colliders, direct detection experiments and neutrino observations.

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I. INTRODUCTION

The existence of non-baryonic dark matter [1] indicates the necessity of supplementing the $SU(3)_c \times SU(2)_L \times U(1)_Y$ Standard Model (SM) with new ingredients. There have been many interesting proposals for the dark matter. The simplest dark matter candidate seems to be a scalar phantom [2,4]. It is a stable SM-singlet and has a quartic coupling with the SM Higgs doublet. Through its annihilation into the SM particles, the dark matter scalar can contribute a desired relic density to our universe. The coupling of the dark matter scalar to the SM Higgs also opens a window to verify the dark matter property by colliders and direct detection experiments.

Observations on solar, atmospheric, reactor and accelerator neutrinos have established the phenomenon of neutrino oscillations [3]. The required massive and mixing neutrinos also implies the need for new physics beyond the SM, where the neutrinos are massless. Currently the seesaw [6] mechanism and its variants [7,8] are the most popular scheme for generating the small neutrino masses. If the neutrinos are Majorana particles, lepton number must be broken by the neutrino masses. For example, in the canonical seesaw model [6] the lepton number is explicitly broken by the Majorana masses of right-handed neutrinos, which have Yukawa couplings with the SM lepton and Higgs doublets. The seesaw models also accommodate the famous leptogenesis [9] mechanism to explain the matter-antimatter asymmetry in our universe. In this scenario, a lepton asymmetry is first produced by the lepton number violating interactions for generating the neutrino masses and then is partially converted to a baryon asymmetry through sphaleron [10].

In order to understand the lepton number violation for the seesaw and the leptogenesis, we can consider more fundamental theories, where the lepton number violation is induced by spontaneous breaking of a local or global symmetry. The simplest proposal is to consider the singlet Majoron model [11], where the global lepton number will be spontaneously broken after a complex singlet scalar develops a vacuum expectation value (VEV). The right-handed neutrinos can obtain Majorana masses through their Yukawa couplings with this singlet scalar. Associated with the spontaneous symmetry breaking of the global lepton number, there is a massless Goldstone–Majoron. The Majoron could become a massive pseudo-Majoron in some variants of the singlet Majoron model. For example, in the presence of the Yukawa couplings of new colored fermions to the singlet scalar, the Majoron could get a tiny mass through the color anomaly [12,13]. This Majoron thus becomes a pseudo-Majoron, which provides a solution for the strong CP problem, as the global lepton number symmetry now is identified with the $U(1)_{PQ}$ symmetry [14]. In this paper, we extend the singlet Majoron model with softly broken lepton number. We find the induced pseudo-Majoron can account for the dark matter. Compared with the dark matter model of the scalar phantom [2,4], the pseudo-Majoron as the dark matter will not provide new implications on the collider phenomenology and the direct detection experiments. However, the neutrino fluxes from the decay of the pseudo-Majoron will induce distinguishable signals in the future neutrino experiments [15].

II. THE MODEL

We extend the SM with two types of singlets: one is a scalar and the other contains three right-handed neutrinos. As the SM leptons carry a lepton number $L = 1$, we assign $L = 1$ for the right-handed neutrinos and $L = -2$ for the singlet scalar. Within the renormalizable context, we can write down the lepton number conserving interactions involving the neutrinos,

$$\mathcal{L}_Y \supset -y_\nu \bar{\psi}_L \phi N_R - \frac{1}{2} h \xi N_R^c N_R + \text{H.c.}. \quad (1)$$

Here $\psi_L$ and $\phi$, respectively, are the SM lepton and Higgs doublets, $N_R$ denotes the right-handed neutrinos and $\xi$
is the singlet scalar. The lepton number conserving potential of the scalar fields should be
\[ V(\xi, \phi) = -\mu_1^2 \xi^2 + \lambda_1 (\xi^2 \xi)^2 - \mu_2^2 \phi^2 + \lambda_2 (\phi^2 \phi)^2 + 2\lambda_3 \xi^2 \phi^2 \phi, \]  
(2)
where \( \lambda_{1,2} > 0 \) and \( \lambda_3 > -\sqrt{\lambda_1 \lambda_2} \) to guarantee the potential bounded from below.

In the Yukawa interaction (11) and the scalar potential (12), the lepton number is flexible to be either a local or a global symmetry. In the present work, we are interested in the global case. It is well known that the spontaneous symmetry breaking of the global lepton number will induce a massless Majoron. Alternatively, we take into account the soft breaking of the global lepton number to make the massless Majoron become a massive pseudo-Majoron. For simplicity, we only introduce the following soft term,
\[ V_{\text{soft}} = -\frac{1}{2} \mu_3^2 (\xi^2 + \text{H.c.}) \]  
(3)
Other soft terms such as the Majorana masses of the right-handed neutrinos can be forbidden by appropriate discrete symmetries. For example, we can impose a \( Z_4 \) symmetry under which
\[ \phi \rightarrow \phi, \quad \xi \rightarrow -\xi, \quad f \rightarrow if. \]  
(4)
Here \( f \) stands for the SM fermions and the right-handed neutrinos.

It is convenient to expand the singlet scalar by two real scalars,
\[ \xi = \frac{1}{\sqrt{2}} (\sigma + i\chi), \]  
(5)
and then rewrite the full potential (12) and (13) in a new form,
\[ V = -\frac{1}{2} (\mu_1^2 + \mu_2^2) \sigma^2 + \frac{1}{4} \lambda_1 \sigma^4 - \frac{1}{2} (\mu_1^2 - \mu_2^2) \chi^2 + \frac{1}{4} \lambda_1 \chi^4 - \mu_2^2 \phi^2 + \lambda_2 (\phi^2 \phi)^2 + \frac{1}{2} \lambda_3 \sigma^2 \chi^2 + \lambda_3 \sigma^2 \phi^2 + \lambda_3 \chi^2 \phi^2 \phi. \]  
(6)

**III. VACUUM EXPECTATION VALUES AND MASSES OF SCALARS**

In order to break the lepton number and the electroweak symmetry, the singlet scalar \( \xi \) and the SM Higgs doublet \( \phi \) will develop VEVs. For example, with the following parameter choice,
\[ \frac{\lambda_2 (\mu_1^2 + \mu_2^2) - \lambda_3 \mu_2^2}{\lambda_1 \lambda_2 - \lambda_3^2} > 0, \]  
(7a)
\[ \frac{\lambda_1 \lambda_2^2 - \lambda_2 (\mu_1^2 + \mu_2^2)}{\lambda_1 \lambda_2 - \lambda_3^2} > 0, \]  
(7b)
\[ \frac{\lambda_1 \lambda_2 \mu_2^2 - \lambda_2 \mu_2^2}{\lambda_1 \lambda_2 - \lambda_3^2} > 0, \]  
(7c)
we find two nonzero and one zero VEVs,
\[ u = \sqrt{2} (\sigma) = \sqrt{\frac{\lambda_2 (\mu_1^2 + \mu_2^2) - \lambda_3 \mu_2^2}{\lambda_1 \lambda_2 - \lambda_3^2}}, \]  
(8a)
\[ w = \sqrt{2} (\chi) = 0, \]  
(8b)
\[ v = \sqrt{2} (\phi) = \sqrt{\frac{\lambda_1 \lambda_2^2 - \lambda_2 (\mu_1^2 + \mu_2^2)}{\lambda_1 \lambda_2 - \lambda_3^2}}. \]  
(8c)

In consequence, there are three massive scalars,
\[ \mathcal{L}_m \supset -\frac{1}{2} (\Phi, H, \chi) \begin{bmatrix} 2 \lambda_1 u^2 & 2 \lambda_3 uv & 0 \\ 2 \lambda_3 uv & 2 \lambda_2 v^2 & 0 \\ 0 & 0 & 2 \mu_3^2 \end{bmatrix} \begin{bmatrix} \Phi \\ H \\ \chi \end{bmatrix}, \]  
(9)
where the scalars \( \Phi \) and \( H \) are defined by
\[ \sigma = \frac{1}{\sqrt{2}} (u + \Phi) \quad \text{and} \quad \phi = \frac{1}{\sqrt{2}} \begin{bmatrix} v + H \\ 0 \end{bmatrix}. \]  
(10)

The mass term (11) clearly indicates that \( \chi \) has no mixing with \( \Phi \) and \( H \) because of its zero VEV. The VEV \( v \simeq 246 \text{ GeV} \) has been determined for the electroweak symmetry breaking. We will clarify later the VEV \( u \) should be near the GUT scale as the pseudo-Majoron \( \chi \) is expected to be an attractive dark matter candidate. For such a huge hierarchy between \( u \) and \( v \), the mixing between \( \Phi \) and \( H \) is extremely small so that \( H \) can be identical to the SM Higgs boson. We thus simplify the mass term (11) to be
\[ \mathcal{L}_m \supset -\frac{1}{2} m_\phi^2 \Phi^2 - \frac{1}{2} m_H^2 H^2 - \frac{1}{2} m_\chi^2 \chi^2 \]  
(11)
with
\[ m_\phi^2 \simeq 2 \lambda_1 u^2, \quad m_H^2 \simeq 2 (\lambda_2 - \frac{\lambda_3^2}{\lambda_1}) v^2, \quad m_\chi^2 = 2 \mu_3^2. \]  
(12)

**IV. LEPTOGENESIS AND SEESAW**

Before the spontaneous symmetry breaking of the lepton number, the right-handed neutrinos don’t have Majorana masses. Instead, they have Yukawa couplings with the singlet scalar as shown in Eq. (14). After the singlet scalar develops its VEV, we can obtain the Majorana masses of the right-handed neutrinos. At this stage, we can conveniently derive
\[ \mathcal{L} \supset -y_{iR} \tilde{N}_R \phi N_R - \frac{1}{2} M_N \tilde{N}_R N_R + \text{H.c.}, \]  
(13)
with the definition
\[ M_N = \frac{1}{\sqrt{2}} m_h u. \]  
(14)
Clearly, with the Yukawa and mass terms \( \mathbf{13} \), the decays of the right-handed neutrinos can generate a lepton asymmetry stored in the SM lepton doublets as long as CP is not conserved \( \mathbf{3, 10} \). Subsequently the sphaleron \( \mathbf{10} \) process will partially transfer the produced lepton asymmetry to a baryon asymmetry which accounts for the matter-antimatter asymmetry in the universe. We should keep in mind that through the Yukawa interaction \( \mathbf{11} \), the right-handed neutrinos not only obtain Majorana masses \( M_N \) but also couple to the scalars \( \Phi \) and \( \chi \). The couplings to \( \Phi \) and \( \chi \) will result in some annihilations of the right-handed neutrinos \( \mathbf{17} \). For a successful leptogenesis, these annihilations should go out of equilibrium before the out-of-equilibrium decays of the right-handed neutrinos. Fortunately, in the presence of a huge lepton number breaking scale \( u \sim 10^{16} \) GeV, the Yukawa coupling \( h \) is much smaller than unit for the desired Majorana masses \( M_N < 10^{14} \) GeV so that the annihilations can be highly suppressed.

The interactions \( \mathbf{13} \) also accommodates the seesaw solution to the small neutrino masses. After the electroweak symmetry breaking, the SM neutrinos \( \nu_L \) can obtain Majorana masses,

\[
\mathcal{L}_m \supset -\frac{1}{2} m_\nu \bar{\nu}_L \nu_L + \text{H.c.} .
\]

The neutrino mass matrix \( m_\nu \) is given by the seesaw \( \mathbf{6} \) formula,

\[
m_\nu = -m_D \frac{1}{M_N} m_D^T \quad \text{with} \quad m_D = \frac{1}{\sqrt{2}} y_v v ,
\]

where \( m_D \) is the Dirac mass matrix between the left- and right-handed neutrinos. For \( M_N < 10^{14} \) GeV, the neutrino masses can be naturally small with \( m_D \) being at the weak scale.

\[ V \supset \lambda_3 \chi^2 \phi^\dagger \phi \Rightarrow \lambda_3 v \chi^2 H + \frac{1}{2} \lambda_3 \chi^2 H^2 .
\]

This means the unstable pseudo-Majoron \( \chi \) with a very long lifetime definitely can play the role of the dark matter. Note for a sizable \( \lambda_3 \), the present symmetry breaking pattern, i.e. \( u \sim 10^{16} \) GeV \( \gg v\sim 246 \) GeV, requires a large cancelation between \( \lambda_3 u^2 \) and \( \mu_2^2 \) so that \( \lambda_3 u^2 - \mu_2^2 \) can be of the order of \( v^2 \).

The coupling of the pseudo-Majoron \( \chi \) to the SM Higgs \( H \) not only determines the dark matter relic density but also opens a window for the dark matter detection \( \mathbf{2-4} \). For example, the Higgs \( H \) can mostly decays into the dark matter \( \chi \) so that the dark matter is possible to find as a missing energy at colliders such as the CERN LHC. Furthermore, through the t-channel exchange of the Higgs \( H \), the dark matter \( \chi \) will result in an elastic scattering on nucleons. Therefore its property can be verified by the dark matter direct detection experiments. Actually, such a type of scalar dark matter can well explain the recent CDMS-II \( \mathbf{18} \) discovery \( \mathbf{4} \). On the other hand, the pseudo-Majoron \( \chi \) couples to the right-handed neutrinos \( N_R \). This makes the dark matter \( \chi \) unstable. Although the dark matter decays are highly suppressed by the huge lepton number breaking scale, it can produce sizable neutrino fluxes which are sensitive in the next generation of neutrino experiments \( \mathbf{13} \).

\[ \mathcal{L}_{\text{eff}} = \frac{i m_\nu}{2u} \chi \bar{\nu}_L \nu_L \left( 1 + \frac{H}{v} \right)^2 + \text{H.c.} .
\]

\[ \Gamma_\chi = \frac{1}{16\pi} \text{Tr}(m_\nu^2 m_\nu^* m_\chi) = \frac{1}{16\pi} \frac{\Sigma m^2_{\nu_i}}{u^2} m_\chi ,
\]

which, in turn, gives the lifetime,

\[
\tau_\chi = \frac{1}{\Gamma_\chi} = \left( \frac{0.01 \text{eV}^2}{\Sigma m^2_{\nu_i}} \right) \left( \frac{u}{5.5 \times 10^{15} \text{GeV}} \right)^2 \left( \frac{1 \text{TeV}}{m_\chi} \right) \times 10^{26} \text{sec} .
\]

It is straightforward to see the lifetime can be very long for a huge lepton number breaking scale.

As a successful dark matter candidate, its relic density should be consistent with the cosmological observations \( \mathbf{1} \). It has been studied by many people \( \mathbf{2-4} \) that a stable SM-singlet scalar with a quartic coupling to the SM Higgs doublet can serve as the dark matter because it contributes a desired relic density through the annihilations into the SM particles. In the present model, the pseudo-Majoron \( \chi \) also has a quartic coupling with the SM Higgs.

\[ \mathcal{L}_m \supset \frac{1}{2} m_\nu \bar{\nu}_L \nu_L + \text{H.c.} .
\]

If the pseudo-Majoron is in the TeV region, its decay width will be dominated by the two-body decay into the left-handed neutrinos,

\[ \Gamma_\chi = \frac{1}{16\pi} \text{Tr}(m_\nu^2 m_\nu^* m_\chi) = \frac{1}{16\pi} \frac{\Sigma m^2_{\nu_i}}{u^2} m_\chi ,
\]

and right-handed neutrinos.

\[ \tau_\chi = \frac{1}{\Gamma_\chi} = \left( \frac{0.01 \text{eV}^2}{\Sigma m^2_{\nu_i}} \right) \left( \frac{u}{5.5 \times 10^{15} \text{GeV}} \right)^2 \left( \frac{1 \text{TeV}}{m_\chi} \right) \times 10^{26} \text{sec} .
\]

\[ \mathcal{L}_{\text{eff}} = \frac{i m_\nu}{2u} \chi \bar{\nu}_L \nu_L \left( 1 + \frac{H}{v} \right)^2 + \text{H.c.} .
\]

\[ \Gamma_\chi = \frac{1}{16\pi} \text{Tr}(m_\nu^2 m_\nu^* m_\chi) = \frac{1}{16\pi} \frac{\Sigma m^2_{\nu_i}}{u^2} m_\chi ,
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\]

\[ V \supset \lambda_3 \chi^2 \phi^\dagger \phi \Rightarrow \lambda_3 v \chi^2 H + \frac{1}{2} \lambda_3 \chi^2 H^2 .
\]

VI. SUMMARY

In the singlet Majoron model, there would be a massless Majoron associated with the spontaneous symmetry breaking of the global lepton number. In this paper, we
extend the singlet Majoron model with the softly broken lepton number so that the massless Majoron can become a massive pseudo-Majoron. We demonstrate that this pseudo-Majoron can be a dark matter candidate with interesting implications on the collider phenomenology, the dark matter direct detection experiments and the future neutrino observations.

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