RADIO-TO-TeV PHASE-RESOLVED EMISSION FROM THE CRAB PULSAR: THE ANNULAR GAP MODEL

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ABSTRACT

The Crab pulsar is a quite young, famous pulsar that radiates multi-wavelength pulsed photons. The latest detection of GeV and TeV pulsed emission with an unprecedented signal-to-noise ratio, supplied by the powerful telescopes Fermi, MAGIC, and VERITAS, challenges the current popular pulsar models, and can be a valuable discriminator to justify the pulsar high-energy-emission models. Our work is divided into two steps. First, taking reasonable parameters (the magnetic inclination angle $\alpha = 45^\circ$ and the view angle $\zeta = 63^\circ$), we use the latest high-energy data to calculate radio, X-ray, $\gamma$-ray, and TeV light curves from a geometric view to obtain crucial information on emission locations. Second, we calculate the phase-averaged spectrum and phase-resolved spectra for the Crab pulsar and take a theoretical justification from a physical view for the emission properties as found in the first step. It is found that a Gaussian emissivity distribution with the peak emission near the null charge surface in the so-called annular gap (AG) region gives the best modeled light curves. The pulsed radio, X-ray, $\gamma$-ray, and TeV emission are mainly generated from the emission of primary particles or secondary particles with different emission mechanisms in the nearly similar region of the AG located in the only magnetic pole, which leads to the nearly “phase-aligned” multi-wavelength light curves. The emission of peak 1 and peak 2 originates from the AG region near the null charge surface, while the emission of the bridge primarily originates from the core gap (CG) region. The charged particles cannot co-rotate with the pulsar and escape from the magnetosphere, which determines the original flowing primary particles. The acceleration electric field and potential in the AG and CG are huge enough and are in the several tens of neutron star radii. Thus, the primary particles are accelerated to ultra-relativistic energies and produce numerous secondary particles (pairs) in the inner regions of the AG and CG. We emphasize that there are mainly two types of pairs: one is curvature-radiation induced (CR-induced) and the other is inverse-Compton-scattering induced (ICS-induced). The phase-averaged spectrum and phase-resolved spectra from soft X-ray to TeV bands are produced by four components: synchrotron radiation from CR- and ICS-induced pairs dominates the X-ray band to soft $\gamma$-ray band (100 eV to 10 MeV); curvature radiation and synchrotron radiation from the primary particles mainly contribute to the $\gamma$-ray band (10 MeV to ~20 GeV); ICS from the pairs significantly contributes to the TeV $\gamma$-ray band (~20–400 GeV). The multi-wavelength pulsed emission from the Crab pulsar can be well modeled with the AG and CG model. To distinguish our single magnetic pole model from two-pole models, the convincing values of the magnetic inclination angle and the viewing angle will play a key role.

Key words: acceleration of particles – gamma rays: stars – pulsars: general – pulsars: individual (Crab: PSR B0531+21) – radiation mechanisms: non-thermal – X-rays: individual (PSR B0531+21)

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1. INTRODUCTION

The Crab pulsar (PSR B0531+21 or PSR J0534+2200) is the second most energetic pulsar to date with a spin-down luminosity of $E_{\text{rot}} = 4.6 \times 10^{38}$ erg s$^{-1}$. It is at a distance of $d \sim 2$ kpc and located in the Crab Nebulae, which is a center-filled remnant of a supernova discovered by Chinese "astronomers" in 1054 AD. The Crab pulsar, a very young pulsar with a characteristic age $\tau = 1240$ yr and a characteristic magnetic field of $B_0 = 3.715 \times 10^{12}$ G, has a spin period of $P = 33$ ms (Manchester et al. 2005).

The Crab pulsar radiates multi-wavelength pulsed emission from the radio ($10^{-6}$ eV) to $\gamma$-ray (up to TeV) bands, especially the new $\gamma$-ray results of the sensitive Large Area Telescope (LAT) on board the Fermi Gamma-ray Space Telescope (Fermi) and a discovery of TeV pulsed emission by the powerful VERITAS array of atmospheric Cherenkov telescopes. Fermi LAT presented the high-quality $\gamma$-ray (100 MeV to 20 GeV) light curves and spectral data using eight months of survey data, and predicted an exponential power-law spectrum with cutoff energies of a few GeV (Abdo et al. 2010). To verify the exponential power-law cutoff spectrum, the 25–100 GeV pulsed emission from the Crab pulsar has been precisely measured by the MAGIC telescope (Aleksić et al. 2012). It shows that the observed cutoff spectrum has a large deviation from the inferred exponential one. VERITAS Collaboration et al. (2011) reported that the pulsed emission above 100 GeV from the Crab pulsar has been detected by VERITAS and showed their light curves and spectral data with an excellent signal-to-noise ratio. They declared that current popular pulsar models (e.g., the outer gap and slot gap models) cannot explain the detection, and the observation might not be explained by the curvature radiation (CR) as the origin of the observed emission above 100 GeV. These findings enable us to obtain considerable insights into magnetospheric physics, e.g., the acceleration electric field, emission region, and the relevant emission mechanisms. The multi-wavelength phase-averaged spectrum and phase-resolved spectra can discriminate the various pulsar non-thermal emission models.

There are four major physical or geometrical magnetospheric models that have previously been proposed to explain pulsed $\gamma$-ray emission of pulsars: the polar cap model (Daugherty & Harding 1994, 1996), the outer gap model (Cheng et al. 1986a, 1986b, 2000; Romani & Yadigaroglu 1995; Zhang & Cheng...
1997; Zhang et al. 2004, 2007; Hirotani 2008; Tang et al. 2008; Lin & Zhang 2009), the two-pole caustic (TPC) model or the slot gap model (Dyks & Rudak 2003; Muslimov & Harding 2003, 2004; Harding et al. 2008), and the annular gap (AG) model (Qiao et al. 2004a, 2004b, 2007; Du et al. 2010, 2011).

The distinguishing features of these pulsar models are different acceleration electric field regions for primary particles and relevant emission mechanisms to radiate high-energy photons (Du et al. 2011). One of the key discrepancies between these emission models is the two important geometry parameters: the magnetic inclination angle $\alpha$ and the view angle $\zeta$.

Cheng et al. (2000) used a three-dimensional single-pole outer gap model to present incipient results of light curves, phase-averaged spectrum, and phase-resolved spectra for the Crab pulsar. Eight years later, Tang et al. (2008) improved the outer gap model and used a modified outer gap model that considered the emission from both poles to calculate light curves, phase-averaged spectrum, and phase-resolved spectra from 100 eV to 10 GeV. In the case of the larger viewing angle $\zeta = 78^\circ$ or $\zeta = 83^\circ$ and the intermediate inclination angle $\alpha = 45^\circ$, Zhang & Li (2009) and Li & Zhang (2010) improved the three-dimensional two-pole outer gap model (Tang et al. 2008) to present their best results of light curves, phase-averaged spectrum, and GeV phase-resolved spectra. They also used physical emissivities to calculate the light curves, showed the best-fit phase-averaged spectrum using emission components from the both magnetic poles, and indicated that inverse-Compton-scattering (ICS) from pairs mainly contributed to the $10^3$–$10^6$ keV band. Harding et al. (2008) used a three-dimensional slot gap model developed from the TPC model to calculate optical-to-$\gamma$-ray light curves, phase-averaged spectrum, and phase-resolved spectra for the Crab pulsar by assuming a broken power-law pair energy spectrum. Hirotani (2008) demonstrated that the slot gap model reproduces at most 20% of the observed GeV fluxes owing to the small trans-field thickness.

There are two methods to study pulsed emission from pulsars: (1) one can use a physical model with a realistic accelerating electric field to directly calculate the light curves and spectra; (2) one can also utilize a reasonable assumption of numerical emissivity to calculate light curves from a geometric point of view, then obtain some valuable information of radiation locations, and finally calculate the spectra and take a consistent theoretical justification from a physical point of view. In conventional cases, some scientists choose the first method. Here we choose the second way to study pulsed emission from the Crab pulsar. In this paper, we study multi-wavelength light curves, phase-averaged spectrum, and GeV phase-resolved spectra.

As noted by Du et al. (2011), the open-field-line region of a pulsar magnetosphere is divided into two isolated parts by the critical field lines that denote a set of special field lines that satisfy the condition of $\mathbf{B} \cdot \mathbf{n} = 0$ at the light cylinder. The core region around the magnetic axis is defined by the critical field lines, and the annular region is located between the critical field lines and the last open field lines (see Figure 1). The width of the annular polar region is anti-correlated with the pulsar period; it is therefore larger for pulsars with smaller spin periods. The annular acceleration potential is negligible for older long-period pulsars, but very important for pulsars with a small period, e.g., millisecond pulsars and young pulsars. The acceleration electric field extends from the pulsar surface to the null charge surface or even beyond it. The AG has a sufficient thickness of trans-field lines and a wide altitude range for particle acceleration. In the AG model, the high-energy emission is generated in the vicinity of the null charge surface (Du et al. 2010). This leads to a wide $\gamma$-ray emission beam (Qiao et al. 2007). The radiation components from both the CG and the AG can be observed simultaneously by one observer (Qiao et al. 2004b) if the inclination angle and the viewing angle are suitable.

2. THE ANNULAR GAP AND CORE GAP

2.1. Formation of the Annular Gap and the Core Gap

As noted by Du et al. (2011), the open-field-line region of a pulsar magnetosphere is divided into two isolated parts by the critical field lines that denote a set of special field lines that satisfy the condition of $\mathbf{B} \cdot \mathbf{n} = 0$ at the light cylinder. The core region around the magnetic axis is defined by the critical field lines, and the annular region is located between the critical field lines and the last open field lines (see Figure 1). The width of the annular polar region is anti-correlated with the pulsar period; it is therefore larger for pulsars with smaller spin periods. The annular acceleration potential is negligible for older long-period pulsars, but very important for pulsars with a small period, e.g., millisecond pulsars and young pulsars. The acceleration electric field extends from the pulsar surface to the null charge surface or even beyond it. The AG has a sufficient thickness of trans-field lines and a wide altitude range for particle acceleration. In the AG model, the high-energy emission is generated in the vicinity of the null charge surface (Du et al. 2010). This leads to a wide $\gamma$-ray emission beam (Qiao et al. 2007). The radiation components from both the CG and the AG can be observed simultaneously by one observer (Qiao et al. 2004b) if the inclination angle and the viewing angle are suitable.

2.2. Acceleration Potential in the Annular Gap

Here we will explore the formation mechanism of the acceleration electric field in the AG. To give a simplified picture of the powerful acceleration electric field along an open field line, we attempt to derive a one-dimensional continuous solution for the acceleration potential. We then use this realistic acceleration field to obtain the Lorentz factor $\gamma_p$ of the primary particle that is from the balance of the acceleration and CR reaction (see Equation (5) in Section 2.3). The $\gamma_p$ is not a true maximum Lorentz factor but a valuable criterion. It can be regarded as an upper limit of the real maximum Lorentz factor of the primary particles that shape the observed spectra.

Under the assumption of a fully charge-separated magnetosphere, the outer gap can be formed (Cheng et al. 1986a). However, if abundant pairs are produced, this leads to a large...
neutral charge component in the magnetosphere. In this case, the acceleration mechanism of particles becomes different from the case of the outer gap model.

The pulsar magnetosphere is filled with charge-separated pair plasma, and the charged particles cannot co-rotate with the neutron star near the light cylinder and must escape from the magnetosphere. This is the generation mechanism for the acceleration electric field. The AG and the CG simultaneously export charged particles with opposite signs, which can lead to the circuit closure in the whole magnetosphere. The parallel acceleration electric field \( E_1 \) in the AG and CG regions is opposite. As a result, \( E_1 \) vanishes at the boundary (the critical field lines) between the annular and core regions and also vanishes along the closed field lines. The positive and negative increases are accelerated from the core and annular regions, respectively.

To unveil the acceleration potential in the AG region, we now consider a tiny magnetic tube embedded in an open field line. We assume that the particles flow out of the co-rotating magnetosphere at a radial distance of about \( r_{\text{out}} \sim R_{1.6} \times 10^3 \) km and that the charge density of flowing-out particles \( \rho_b \) is equal to the local Goldreich–Julian (GJ) charge density \( \rho_{\text{GJ}} \) (Goldreich & Julian 1969). For any altitude \( r < r_{\text{out}} \), \( \rho_b(r) < \rho_{\text{GJ}}(r) \). The acceleration electric field therefore exists along the field line, and cannot vanish until approaching the altitude of \( r_{\text{out}} \).

In a static dipole magnetic field configuration, the field components can be described as \( B_r = (2\mu \cos \theta/r^3) n_r \) and \( B_\theta = (\mu \sin \theta/r^3) n_\theta \); here, \( \theta \) is the zenith angle in the magnetic coordinate and \( B_0 \) is the surface magnetic field. Thus, the magnetic field strength at an altitude \( r \) is \( B(r) = (B_0 R^3/2)/(\sqrt{3} \cos^2 \theta + 1)/r^3) \).

In the co-rotating frame, the equation for acceleration potential \( \Psi \) is

\[
\nabla^2 \Psi = -4\pi (\rho_b - \rho_{\text{GJ}}). \tag{1}
\]

Although the same Poisson equation for the acceleration field is used by all of the pulsar emission models (some models also considered the relativistic effect), the key discrepancy among these models is how to obtain and explain the difference between the above angular dependence and the local GJ charge density \( \rho_{\text{GJ}} \). Normally, it is assumed that \( \rho_b - \rho_{\text{GJ}} \sim 0 \) at the star surface. This leads to quite different results of the acceleration electric field.

Using the conservation laws of the particle number and magnetic flux in the magnetic flux tube, the difference between the flowing charge density and local GJ charge density at altitude \( r \) can be written as

\[
\rho_b(r) - \rho_{\text{GJ}}(r) = -\frac{\Omega B(r)}{2\pi c} (\cos \zeta_{\text{out}} - \cos \zeta), \tag{2}
\]

where \( \Omega = 2\pi/P \) is the angular velocity, \( P \) is the rotation period, and \( \zeta \) and \( \zeta_{\text{out}} \) are the angles between the rotational axis and the B field direction at \( r \) and \( r_{\text{out}} \), respectively. It is found that

\[
\cos \zeta = \cos \varphi \cos \theta_{\mu} - \sin \varphi \sin \theta_{\mu} \cos \psi, \tag{3}
\]

where \( \psi \) and \( \theta_{\mu} \) are the azimuthal angle and the tangent angle (half beam angle) in the magnetic field coordinate, respectively. Combining Equations (1), (2), and (3), we then obtain a one-dimensional two-order differential equation for the acceleration potential, i.e.,

\[
\frac{d^2 \Psi}{d\theta^2} - \frac{s''}{s'} \frac{d\Psi}{d\theta} = \frac{\Omega B_0 R^3}{cr^3} \sqrt{3 \cos^2 \theta + 1} (\cos \zeta_{\text{out}} - \cos \zeta) \phi^2, \tag{4}
\]

where \( s' \) and \( s'' \) are given by

\[
s' = \frac{ds}{d\theta}, \quad s'' = \frac{ds'}{d\theta}, \quad r = R_e \sin^2 \theta;
\]

where \( R_e \) is a field line constant that denotes the maximum length of a certain point on a field line. Then, substituting \( \tan \theta_{\mu} = \tan \theta_{\mu}/2 - \tan^2 \theta/2 \) (Qiao & Lin 1998) into Equations (3) and (4), we can solve Equation (4), and achieve the one-dimensional solution for the electric potential \( \Psi \) along a magnetic field line with a magnetic azimuthal of \( \psi = \psi_0 \) for the Crab pulsar, as shown in Figure 2. The maximum potential drop on this field line is comparable to the one generated by the unipolar effect.

\[
\Delta V = \frac{\Omega B R^2}{2c} \times 1.0 \times 10^{14} B_1 R_6^2 P^{-1},
\]

which is generated by the unipolar effect. The acceleration potential is quite huge in the inner region of AG, and the primary particles are therefore accelerated to ultra-relativistic energies with large Lorentz factors of \( \gamma \times 10^6-10^7 \). Simultaneously, the accelerated primary particles emit abundant \( \gamma \)-ray photons through the ICS and CR process, then dense \( e^\pm \) pairs are generated via the \( \gamma - B \) (photon magnetic absorption) process.

2.3. Pair Production

Since ICS and CR are two effective radiation mechanisms to generate high-energy \( \gamma \)-ray photons, \( e^\pm \) pairs can be generated by these ICS and CR photons emitted from the accelerated primary particles in both AG and CG. Thus, three gap modes exist for pair production, namely, CR gap, thermal ICS gap, and
resonant ICS gap (Zhang et al. 1997a, 1997b), which will be briefly introduced below.

In the traditional inner gap model (Ruderman & Sutherland 1975, hereafter RS75), the $γ − B$ process plays a very important role, and two conditions should be satisfied at the same time for pair production: (1) to produce enough high-energy $γ$-ray photons, a strong enough potential drop should be reached; (2) the energy component of $γ$-ray photons perpendicular to the magnetic field must satisfy the condition $E_{γ⊥} \geq 2m_e c^2$.

The accelerated particles are assumed to flow along a field line in a quasi-steady state. Using the calculated acceleration electric field, we can obtain the Lorentz factor $γ_p$ of the primary particle from the CR reaction

$$γ_p = \left( \frac{3ρ_e^2E_1}{2e} \right)^{1/4} = 2.36 \times 10^7 ρ_T^{0.5} E_{||,6}^{0.25},$$

where $e$ is the charge of an electron, $ρ_T$ is the curvature radius in units of $10^7$ cm, and $E_{||,6}$ is the acceleration electric field in units of $10^6$ V cm$^{-1}$.

In the $γ − B$ process, the condition for pair production is that the mean free path of the $γ$-ray photon is equal to the gap height, $l \approx h$. The mean free path of the $γ$-ray photon is given by (Erber 1966)

$$l = \frac{4.4}{c^2/hc m_e c^2 B_⊥} \exp \left( \frac{4}{3h} \right),$$

where $B_c = 4.414 \times 10^{13}$ G is the critical magnetic field, $h$ is the reduced Planck’s constant,

$$χ = \frac{E_γ}{2m_e c^2} \sin \theta \frac{B}{B_c} = \frac{E_γ}{2m_e c^2} \frac{B_⊥}{B_c},$$

and $B_⊥$ is the magnetic field perpendicular to the moving direction of $γ$ photons, which can be expressed as (RS75)

$$B_⊥ \approx \frac{h}{ρ} B \approx \frac{l}{ρ} B.$$  

Here, $l \approx h$ is the condition for gap sparks (pair production) to take place and $ρ$ is the curvature radius of a spot on a magnetic field line. For a dipole magnetic configuration, it can be estimated as

$$ρ \approx \frac{4}{3} (λ Rc/Ω)^{1/2}$$

(Zhang et al. 1997a) if the spot position is near the neutron star surface. Here, $λ$ is a parameter to show the field lines, with $λ = 1$ corresponding to the last open field line. The characteristic energy from the CR process can be written as

$$E_{γ,cr} = \frac{h}{2ρ} \frac{3γ^3c}{2}.$$  

Then the CR gap height $h_{CR}$ is

$$h_{CR} \approx 5 \times 10^3 P^{3/7} B_{12}^{-4/7} \rho_6^{2/7} \text{ cm}$$  

(Zhang et al. 1997a).

The so-called CR-mode gap indicates that the pair production cascades are dominated by the $γ$-ray photons emanated from the CR process of the accelerated primary particles; this gap is somehow like the RS gap (RS75). The resonant ICS gap is formed from the resonant scattering of soft photons, which is a quantum effect with a large electron scattering cross section estimated as $σ \sim 10^6 σ_T$ ($σ_T$ is the Thompson cross section) in strong magnetic fields. Zhang et al. (1997a) obtained the gap height ($h_{res}$) of the resonant ICS mode,

$$h_{res} \approx 1.1 \times 10^3 P^{1/3} B_{12}^{-1} ρ_6^{1/3} \text{ cm},$$

and the Lorentz factor ($γ_{2, res}$) of pairs for the resonant ICS mode,

$$γ_{2, res} = 890 P^{-1/3} B_{12}^{2/3} ρ_6^{2/3}.$$  

The thermal ICS gap is determined by those thermal-peak photons that have the maximum photon number density of the Planck spectrum at a certain temperature. It has a lower gap height $h_{th}$,

$$h_{th} = 2.7 \times 10^2 P^{2/5} B_{12}^{-3/5} ρ_6^{-1/5} T_6^{-1/5} \text{ cm},$$

but leads to a larger Lorentz factor ($γ_{2, th}$) for the secondary particles,

$$γ_{2, th} = 3.7 \times 10^3 P^{-2/5} B_{12}^{3/5} ρ_6^{4/5} T_6^{1/5}.$$  

These two CR and ICS gaps, which have relatively lower gap heights, would dominate the inner gap breakdown (Zhang et al. 1997a; Du et al. 2009). The pairs can also be abundantly generated by the primary particles escaped from the inner gap within a few neutron star radii, and they could have two major energy distributions due to the different types of primary particles. We will therefore use two different pair energy distributions for CR and ICS pairs to calculate the phase-averaged spectrum and phase-resolved spectra for the Crab pulsar.

3. MODELING THE MULTI-WAVELENGTH LIGHT CURVES AND SPECTRA FOR THE CRAB PULSAR

To explain the multi-wavelength light curves with a nearly aligned peak phase for the Crab pulsar, we should obtain the high signal-to-noise data, which are adopted from Figure 2 of Abdo et al. (2010). We also reprocessed the Fermi $γ$-ray data to obtain the three $γ$-ray band light curves (see left panels of Figure 3) in the following steps.

1. Limited by the timing solution for the Crab pulsar$^3$ from the Fermi Science Support Center, we reprocessed the original data observed from 2008 August 4 to 2009 April 8.

2. We selected photons of 0.1–300 GeV in the “Diffuse” event class, within a radius of 2° of the Crab pulsar position (R.A. = 83:63, decl. = 22:01) and the zenith angle smaller than 105°.

3. As done by Abdo et al. (2010), we used “isselect” to select photons of energy $E_{GeV}$ within an angle of $< \max \left[ 6.68 − 1.76 \log_{10}(E_{GeV}), 1.3 \right]$ deg from the pulsar position.

4. We then obtained the rotational phase for each photon using the tempo2 (Hobbs et al. 2006) with the Fermi plug-in.

$^3$ http://fermi.gsfc.nasa.gov/ssc/data/access/lat/ephems/
Figure 3. Multi-wavelength (radio, X-ray, and γ-ray) light curves for the Crab pulsar. The observations are shown in the left panels, and the RXTE, International Gamma-Ray Astrophysics Laboratory, Comptel, and Nancay data are taken from Figure 2 of Abdo et al. (2010). The 25–100 GeV light-curve data are taken from Aleksic et al. (2012), and the TeV (>120 GeV) data are taken from VERITAS Collaboration et al. (2011). The photon sky map (middle panels) for an inclination angle and the corresponding modeled light curves (right panels) for a viewing angle of $\zeta = 63^\circ$ are also presented, using the single-pole AG model. Our AG model can well explain the multi-wavelength light curves with phase-aligned peaks.

(A color version of this figure is available in the online journal.)
5. Finally, we obtained the multi-wavelength $\gamma$-ray light curves with 256 bins, as presented in Figure 3 (left panels). Two sharp phase-aligned peaks have a phase separation of $\delta \phi \sim 0.4$.

An acceptable model should have reasonable input parameters (e.g., magnetic inclination angle $\alpha$ and viewing angle $\zeta$) and consistently produce multi-wavelength light curves with phase-aligned peaks and bridge emission and phase-resolved spectra for the Crab pulsar.

### 3.1. Light-curve Modeling

We adopted the geometrical method (see details in Section 3.1 of Du et al. 2011, including basic formulae and gap physics) to model the light curves. The key idea of modeling light curves is to project the radiation intensity of every spot on each open field line (in either AG or CG) to the “non-rotating” sky, with considerations of physical effects. Here the emissivities are numerically assumed to facilitate calculations. They are, however, consistent with the physically calculated spectra, as noted in Figure 8 of Du et al. (2011). Some model parameters for both the AG and CG should be adjusted for the emission regions where the corresponding waveband emission is generated. The framework of the AG model as well as the details of the coordinate system has been presented in Du et al. (2010), which can be used for simulation of the multi-wavelength light curves of the Crab pulsar. We adopted the inclination angle of $\alpha = 45^\circ$ and the viewing angle $\zeta = 63^\circ$, which were obtained from the Chandra X-ray torus fitting (Ng & Romani 2008). The modeling methods are briefly delineated as follows.

1. We first use the critical field line to separate the polar cap region into the annular and CG regions. Then we use the so-called open volume coordinates $(r_{OVC}, \psi)$ to label the open field lines of the AG and CG, respectively. Here, $r_{OVC}$ is the normalized magnetic colatitude and $\psi$ is the magnetic azimuthal. For the AG, we define the inner rim $r_{OVC, AG} = 0$ for the critical field lines and the outer rim $r_{OVC, AG} = 1$ for the last open field lines, while for the CG, we define the outer rim $r_{OVC, CG} = 1$ for the critical field lines and the inner rim $r_{OVC, CG} = 0$ for the magnetic axis. We also divide both the AG ($0 \lesssim r_{OVC, AG} \lesssim 1$) and the CG ($0.1 \lesssim r_{OVC, CG} \lesssim 1$) into 40 rings for calculation.

2. When calculating the emissivities for modeling light curves, we postulate that the emissivities along one open field line have a Gaussian distribution rather than the frequently used assumption of the uniform emissivity along an open field line (Du et al. 2011). From that figure, one can clearly see that the flux is likely to have a Gaussian distribution against altitude near the peak position. The peak emissivities $I_p(\theta_b, \psi)$ may follow another Gaussian distribution against $\theta$ for a bunch of open field lines (Cheng et al. 2010; Du et al. 2011). As seen above, we use two different Gaussian distributions to describe the emissivities on open field lines for both the AG and the CG. The model parameters are adjusted to maximally fit the observed multi-wavelength light curves.

3. To derive the “photon sky map” in the observer frame, we first calculate the emission direction of each emission spot $\mathbf{n}_b$ in the magnetic frame; then use a transformation matrix $T_\alpha$ to transform $\mathbf{n}_b$ into $\mathbf{n}_{spin}$ in the spin frame; finally use an aberration matrix to transform $\mathbf{n}_{spin}$ into $\mathbf{n}_{observer} = \{n_x, n_y, n_z\}$ in the observer frame. Here $\phi_0 = \arctan(n_y / n_x)$ and $\zeta = \arccos(n_x / \sqrt{n_x^2 + n_y^2 + n_z^2})$ are the rotation phase (without retardation effect) of the emission spot with respect to the pulsar rotation axis and the viewing angle for a distant, non-rotating observer. The detailed calculations for the aberration effect can be found in Lee et al. (2010).

4. We also add the phase shift $\delta \phi_{ret}$ caused by the retardation effect to the emission phase, that is, $\phi = \phi_0 - \delta \phi_{ret}$. There is no minus sign for $\delta \phi$ because our coordinate system is different from other models.

5. The “photon sky map,” defined by the binned projected emission intensities on the $(\phi, \zeta)$ plane, can be plotted in 256 bins (see the middle panel of Figure 3). The corresponding light curves cut by a line of sight with a viewing angle $\zeta = 63^\circ$ are finally obtained. For the viewing angle $\zeta = 63^\circ$, any magnetic inclination angle $\alpha$ between 40$^\circ$ and 65$^\circ$ in the AG model can produce light curves with two peaks and a large peak separation similar to the observed ones. The emission from the single pole is favored for the Crab pulsar in our model.

The modeled light curves from radio to TeV band are presented in Figure 3 (black solid lines), and the key model parameters are listed in Table 1. Emission from peak 1 (P1) and peak 2 (P2) of multi-wavelength light curves originates from the AG region in the vicinity of the null charge surface, while the bridge emission comes from the CG region. We emphasize that all the multi-wavelength emission are originated from only one magnetic pole; our AG is therefore a single-pole magnetospheric model.

| Band          | $k$ | $\lambda$ | $\epsilon$ | $\sigma_A$ | $\sigma_\Lambda$ | $\sigma_C$ | $\sigma_{C1}$ | $\sigma_{C2}$ |
|--------------|-----|-----------|------------|------------|------------------|------------|----------------|----------------|
| 25–100 GeV   | 0.65| 0.6       | 0.8        | 0.4        | 0.0004           | 0.25       | 0.0023         | 0.0035         |
| >120 GeV     | 0.68| 0.8       | 0.5        | 0.3        | 0.0035           | 0.25       | 0.0025         | 0.0032         |
| 2–16 keV     | 0.50| 0.8       | 1.6        | 0.5        | 0.0005           | 0.3        | 0.0022         | 0.0035         |
| 100–200 keV  | 0.50| 0.8       | 0.6        | 0.3        | 0.0006           | 0.35       | 0.0035         | 0.0045         |
| 0.75–30 MeV  | 0.52| 0.8       | 0.5        | 0.3        | 0.0004           | 0.3        | 0.0055         | 0.0060         |
| 0.1–0.3 GeV  | 0.53| 0.8       | 0.6        | 0.5        | 0.0004           | 0.15       | 0.0012         | 0.0012         |
| 0.3–1.0 GeV  | 0.54| 0.8       | 0.7        | 0.5        | 0.0005           | 0.15       | 0.0014         | 0.0014         |
| >1.0 GeV     | 0.57| 0.8       | 0.5        | 0.5        | 0.0005           | 0.35       | 0.0035         | 0.0035         |

Notes. $k$ and $\lambda$ are two geometry parameters used to determine the peak altitude in the AG; $\epsilon$ is a parameter for the peak altitude in the CG; $\sigma_A$ and $\sigma_C$ are length scales for the emission region on each open field line in the AG and the CG, respectively, in units of $R_{LC}$; $\sigma_\Lambda$ is the transverse bunch scale for field lines in the AG; $\sigma_{C1}$ and $\sigma_{C2}$ are the bunch scales for field lines of $-180^\circ < \psi < 90^\circ$ and $90^\circ < \psi < 180^\circ$ in the CG, respectively. A detailed description of these symbols can be found in Du et al. (2011).
of $-98^\circ$ to $-96^\circ$, while P2 is $106^\circ$ to $109^\circ$. It is generated in an intermediate-altitude AG region, which might be due to the coherence condition and propagation effects. It is found that the positions of both the $\gamma$-ray (X-ray) peak and the radio peak (1.4 GHz) are overlapped; this leads to the nearly phase-aligned pulse peaks of multi-wavelength emission, except for several GHz radio emissions due possibly to plasma propagation effects. Nevertheless, the $\gamma$-ray emission altitudes are above the lower bound of the height determined by the $\gamma-B$ absorption (Lee et al. 2010). Based on our model, not all $\gamma$-ray pulsars can be detected in the radio band, and not all radio pulsars can have a $\gamma$-ray beam with sufficiently high flux toward us. The beam shapes and intensities of $\gamma$-ray and radio can evolve with pulsar ages.

### 3.2. Multi-wavelength Spectra for the Crab Pulsar

In this section, we will use the AG model to calculate the multi-wavelength phase-averaged spectrum and phase-resolved spectra for the Crab pulsar. Kuiper et al. (2001) achieved seven band phase-resolved spectra (LW1, P1, TW1, Bridge, LW2, P2, and TW2) and the phase-averaged spectrum of the Crab pulsar with EGRET $\gamma$-ray data. We will also add the new high-quality Fermi, MAGIC, and VERITAS data to the total phase-averaged spectrum. The $\gamma$-ray emission is believed to originate from the CR of primary particles (Tang et al. 2008; Harding et al. 2008; Meng et al. 2008), which generally gives a super-exponential power-law spectrum with the cutoff energy of a few GeV. However, the TeV (20–400 GeV) emission mechanism remains a mystery, and it requires a global phase-averaged spectrum fitting to resolve this problem.

We first discuss the particle dynamics in the AG. After exploring the formation mechanism of the acceleration electric field, the relevant dynamic parameters, acceleration electric field $E_A$, Lorentz factor of primary particles $\gamma_{p}$, characteristic energy of CR emitted from primary particles $E_{C}$, and escape photon energy $E_{es}$, are calculated for two field lines where P1 and P2 mainly originate. The calculated results are shown in Figure 5. As introduced in Section 2.2, the derived acceleration electric field $E_A$ is quite huge in the inner region of AG below the altitude of $R_{es}$, which leads to the generation of numerous pairs via the $\gamma-B$ absorption effect. The Lorentz factor of primary particles $\gamma_{p}$ is derived from the CR reaction using Equation (5). However, the actual Lorentz factor of the primary particles is smaller than the one shown in Figure 5 if other energy loss mechanisms (e.g., ICS loss rate) as well as relativistic and pair screening effects are taken into account. $E_{C}$ denotes the characteristic energy of CR emitted from primary particles, derived using Equations (5) and (10). $E_{es}$, due to the $\gamma-B$ absorption effect based on Equation (26), is the escape (maximum) photon energy for a certain altitude.

We will further consider the emission mechanisms for the Crab pulsar. The photon number spectrum of the synchrotron radiation is given by

$$
\frac{d^2N_{\gamma}}{dE_{\gamma}dt} \bigg|_{\text{syn}} = \frac{\sqrt{3}e^{3}B(r)\sin\phi(r)}{m_e c^2 h} \frac{1}{E_{\gamma}} G(x),
$$

where $\phi(r)$ is the pitch angle at a distance of $r$ on an open field line, $h$ is the Planck constant, $G(x) = x \int_{\xi}^{\infty} K_{5/3}(\xi)d\xi$, $K_{5/3}$ is the modified Bessel function with an order of 5/3, $x = E_{\gamma}/E_{es,c}(r)$, and

$$
E_{es,c}(r) = 1.5 h\gamma_{\perp} v_{\perp} \sin \phi(r) = 4.3 \times 10^{6} h B(r) \gamma_{\perp}^{2} \sin \phi(r)
$$

is the critical synchrotron photon energy. The pitch angle $\phi(r)$ of relativistic primary particles flowing along a magnetic field line could be small, but it cannot be neglected for synchrotron radiation, while the pitch angle $\phi(r)$ of pairs increases due to the cyclotron resonant absorption of the low-energy photons (Harding et al. 2008) and it varies with the emission altitudes. The mean pitch angles of the two types of pairs are different owing to the effect of cyclotron resonant absorption for different particles with different Lorentz factors. The synchrotron radiation from pairs plays an important role in the X-ray band to soft $\gamma$-ray band (e.g., $\lesssim 0.02$ GeV).

The photon number spectrum of the CR can be given by

$$
\frac{d^2N_{\gamma}}{dE_{\gamma}dt} \bigg|_{\text{ICS}} = \frac{3\sigma T_{C}}{4\gamma^{2}} \frac{n_{\gamma}(\varepsilon, r) + n_{X}(\varepsilon, r)}{\varepsilon} E_{\gamma}^{2} G(x),
$$

where $\rho(r)$ is the curvature radius at $r$ and Equation (10) shows the critical curvature photon energy.

Blumenthal & Gould (1970) presented an analytic formula for the photon spectrum of the inverse Compton scattered photons per electron in the case of an extreme Klein–Nishina limit and then Tang et al. (2008) gave a simplified form, i.e.,

$$
\frac{d^2N_{\gamma}}{dE_{\gamma}dt} \bigg|_{\text{ICS}} = \int_{\varepsilon_{1}}^{\varepsilon_{2}} \frac{3\sigma T_{C}}{4\gamma^{2}} \left[2q \ln q + (1 + 2q)(1 - q) + \frac{(\Gamma q)^{2}(1 - q) + (1 + \Gamma q)}{2(1 + \Gamma q)} \right] d\varepsilon,
$$

where $q = E_{1}/\Gamma(1 - E_{1})$, $\Gamma = 4\gamma^{2} / m_{e} c^{2}$, and $E_{1} = E_{\gamma}/E_{c}$. $\varepsilon$ is the energy of soft photons for scattering, and $\varepsilon_{1}$ and $\varepsilon_{2}$ are
the minimum and the maximum energies of the soft photons for integration, respectively. We choose the values of $\epsilon_1$ and $\epsilon_2$ to fulfill the condition of $E_1 < 1$. The lower limit $\epsilon_1$ in Equation (19) is chosen to be around 1 eV, and the upper limit $\epsilon_2$ is adjusted artificially to make a quick convergence of Equation (19).

There are two possible sources of soft photons for the ICS: one is thermal photons and the other is synchrotron photons. The thermal photons, generated by the stellar surface with a typical characteristic energy of CR emitted from primary particles. $E_{\text{esc}}$ (dot-dashed line) is the escape (maximum) photon energy for a certain altitude due to the $γ-B$ absorption.

Therefore, we use an approximate formula to facilitate ICS calculations when we consider the soft seed synchrotron photons. The number density of the soft synchrotron photons $n_{\text{syn}}(\epsilon, r)$ can be given by

$$n_{\text{syn}}(\epsilon, r) = \frac{1}{\pi^2 h^3 c^3} \exp(\epsilon/kT) - 1 \left( \frac{R}{r} \right)^2,$$

where $k$ is the Boltzmann constant. The temperature $T$ is taken to be $2 \times 10^9$ K in our calculations.

The other source of soft photons arises from synchrotron radiation of pairs. Owing to abundant soft synchrotron photons, the scattering of this kind of photon is more significant at higher altitudes near the light cylinder. Although the synchrotron radiation spectrum from a single particle with a Lorentz factor of $γ$ is maintained to a wide energy band (for example, soft X-ray band to $γ$-band), it is more likely to be a spectral line, which is concentrated on the critical energy. Thus, we can rewrite the synchrotron spectral power using the total energy loss rate, i.e.,

$$P_{\text{v, syn}} \simeq \dot{E}_{\text{syn}} \delta(\epsilon - \epsilon_{\text{syn}, c}) \simeq \frac{4}{3} \sigma_T c U_B \gamma^2 \delta(\epsilon - \epsilon_c),$$

where $\sigma_T$ is the Thompson scattering cross section, $U_B$ is the energy density of the local magnetic field, and $\delta(\epsilon - \epsilon_c)$ is a Delta function.

Supposing pairs follow a power-law spectrum $n_0(\gamma) = n_0 \gamma^{-s}$ with a particle energy spectral index of $s$, we can obtain the synchrotron emissivity $j_\nu$ in a simple form

$$j_\nu = \frac{1}{\Delta \Omega} \int P_{\nu, \text{syn}} n_0(\gamma) d\gamma = \frac{2\sigma_T c U_B n_0}{3\Delta \Omega \nu_L} \left( \frac{\nu}{\nu_L} \right)^{1-s},$$

where $\nu_L$ is the Larmor frequency of an electron in the magnetic field, $n_0$ is a constant, and $\Delta \Omega(r)$ is the solid angle of the beam of synchrotron photons and is estimated as

$$\Delta \Omega(r) = \int_0^{2\pi} \int_0^{\pi} d\phi \sin \theta d\theta \approx \pi \psi^2(r).$$

Following the method of Du et al. (2011), we divide the AG region into 40 rings and 360 equal intervals in the magnetic azimuth, i.e., in total $40 \times 360$ small magnetic tubes. A small magnetic tube has an area $A_0$ on the neutron star surface. From Equation (2), the number density of primary particles at an altitude $r$ is $n(r) = (\Omega B(r) / 2\pi c e) \cos \xi_{\text{out}}$. The cross-section area of the magnetic tube at $r$ is $A(r) = B_0 A_0 / B(r)$. The flowing particle number at $r$ in the magnetic tube is

$$\Delta N(r) = A(r) \Delta s \frac{\Omega B(r)}{2\pi c e} \cos \xi_{\text{out}},$$

where $\Delta s$ is the arc length along the field. The energy spectrum $N_{\text{pri}} = dN/d\gamma$ of the accelerated primary particles is not well
understood in the first physical principle. Here we assume the primary particles in the magnetic tube to follow a power-law energy distribution $dN/d\gamma = N_0 \gamma^{-\Gamma}$ with an index of $\Gamma = -2.2$. Using Equation (25), $N_0$ can be derived by integrating the equation above.

Harding et al. (2008) have assumed a broken power-law distribution for pairs with indices of $-2.0$ and $-2.8$ (see their Equation (47)). But in our model, we postulate that the two types of relativistic pairs follow two different power-law energy distributions as noted in Section 2.3, i.e.,

$$dN = dN_{\text{pairs}}(\gamma),$$

where $s_1 = 2.45$ and $s_2 = 2.6$ are the spectral indices, $C_1$ and $C_2$ are two coefficients, and $\gamma_{\text{min}}, \gamma_{\text{max}}, \gamma_{\text{min}}^\text{CR}, \gamma_{\text{max}}^\text{CR}, \gamma_{\text{min}}^\text{ICS}, \gamma_{\text{max}}^\text{ICS}$ are lower limits and upper limits of the Lorentz factors for the CR and ICS pairs.

The pairs can be generated with a large multiplicity $(10^3-10^5)$ via the $\gamma-B$ process in the lower regions of the AG and the CG near the neutron star surface. The pitch angle of pairs increases due to the cyclotron resonant absorption of the low-energy photons. The mean pitch angle of secondary particles is considerable owing to the effect of cyclotron resonant absorption. The synchrotron radiation from secondary particles has some contributions to the low-energy $\gamma$-ray emission, e.g., $\lesssim 0.02$ GeV.

We also had an analytical formula of optical depth $\tau_{\gamma-B}$ due to the $\gamma-B$ absorption (Lee et al. 2010),

$$\tau_{\gamma-B}(r) = \frac{1.55 \times 10^7 r}{E_\gamma} \times \frac{1}{K_{1/3}} \times \left( \frac{2.76 \times 10^6 \theta_2^{-1/2} \rho^{1/2}}{E_\gamma} \right),$$

where $E_\gamma$ is in units of MeV and $B_{0.12}$ is the surface magnetic field in units of $10^{-12}$ G. We found that the photons of the Crab pulsar with an energy of < 100 GeV always satisfy the condition of $\tau_{\gamma-B} \ll 1$ if the emission altitude is greater than a few hundred kilometers. Thus, the final multi-wavelength spectrum emitted by the primary particles and secondary particles can be calculated by

$$F(E_\gamma) = \frac{1}{\Delta \Omega_{\text{eff}}} \int_{E_{\gamma \text{min}}}^{E_{\gamma \text{max}}} \left[ \frac{d^2 N_\gamma}{dE_\gamma dt} \right]_\text{cur} + \left[ \frac{d^2 N_\gamma}{dE_\gamma dt} \right]_\text{syn} + \left[ \frac{d^2 N_\gamma}{dE_\gamma dt} \right]_\text{ICS} \frac{e^{-\tau_{\gamma-B}(r)}}{\Delta \Omega_{\text{eff}}},$$

where $\Delta \Omega_{\text{eff}}$ is the effective solid angle of the emission beam and $D = 2$ kpc is the distance from the Crab pulsar to the Earth.

There are actually six main spectral components for the pulsar total spectrum. Based on our calculations, the synchrotron and CR from primary particles and the synchrotron radiation and ICS from secondary particles are required to calculate the phase-averaged and phase-resolved spectra, while ICS from primary particles and CR from pairs can be ignored for the Crab pulsar. Then, we compute their acceleration electric field $E_2$, and potential $\Psi$, and adjust the minimum and maximum Lorentz factors for both primary particles and pairs ($\gamma_\text{min}, \gamma_\text{max}, \gamma_\text{min}^\text{CR}, \gamma_\text{max}^\text{CR}$, $\gamma_\text{min}^\text{ICS}, \gamma_\text{max}^\text{ICS}$), the pitch angle $\epsilon(r)$, and the $\gamma$-ray beam angle $\Delta \Omega_{\text{eff}}$ to fit the multi-wavelength phase-averaged spectrum and phase-resolved spectra for the Crab pulsar.

We fitted the phase-averaged spectrum and seven phase-band phase-resolved spectra of the Crab pulsar, and the results are shown in Figures 6 and 7, respectively. For the phase-averaged spectrum, we basically used the multi-wavelength data from Kuiper et al. (2001) and combined them with the latest Fermi, MAGIC, and VERITAS $\gamma$-ray spectral data, whereas for phase-resolved spectra, we only used the multi-wavelength data from Kuiper et al. (2001). The best-fit parameters for the phase-averaged spectrum and phase-resolved spectra are listed in Tables 2 and 3, respectively. From spectra fitting, we found that the calculated $\gamma$-ray spectra are not sensitive to $\gamma_\text{min}^\text{CR}$, but quite sensitive to $\gamma_\text{max}^\text{CR}$ which is chosen below the value obtained from the balance of CR and radiation reaction shown in Figure 5. The solid angle $\Delta \Omega_{\text{eff}}$ was always assumed to be 1 by many authors for simplicity. We adjusted it as a free parameter around 1 for different phases.

![Figure 6. Modeled phase-averaged spectrum for the Crab pulsar. The calculated total spectrum (thick black solid line) is obtained from the sum of synchrotron radiation and ICS from two kinds of pairs and CR and synchrotron radiation from primary particles. It is found that the CR and synchrotron radiation from primary particles is mainly contributed to the $\gamma$-ray band (20 MeV to 20 GeV), synchrotron radiation from CR- and ICS-induced pairs dominates the X-ray band and the soft $\gamma$-ray band (100 eV to 10 MeV). ICS from the pairs contributes to the hard TeV $\gamma$-ray band (∼20 GeV–400 GeV). The data (solid circles) are taken from Kuiper et al. (2001). The Fermi $\gamma$-ray data are plotted in purple circles. The $\gamma$-ray spectral data of VERITAS (square) are taken from VERITAS Collaboration et al. (2011), and the MAGIC data (big hollow circle) are taken from Aleksic et al. (2012). (A color version of this figure is available in the online journal.)](https://example.com/spectrum6.png)
Figure 7. Similar to Figure 6, but for modeled seven phase-band phase-resolved spectra of the Crab pulsar. TW1 and LW2 are plotted twice. The MAGIC (50–400 GeV) spectral data are available only for P1 and P2, which are taken from Aleksić et al. (2012), whereas the Fermi and VERITAS data are not included here. (A color version of this figure is available in the online journal.)

Table 2
Best-fit Parameters for the Calculated Phase-averaged Spectrum of the Crab Pulsar

| $\alpha$ | $\zeta$ | $\gamma_{\text{min, pair}}^{\text{CR}}$ | $\gamma_{\text{max, pair}}^{\text{CR}}$ | $\psi_{\text{pair}}^{\text{CR}}$ | $\gamma_{\text{min, pair}}^{\text{ICS}}$ | $\gamma_{\text{max, pair}}^{\text{ICS}}$ | $\psi_{\text{pair}}^{\text{ICS}}$ | $\gamma_{\text{min, pair}}^{\text{pri}}$ | $\gamma_{\text{max, pair}}^{\text{pri}}$ | $\psi_{\text{pair}}^{\text{pri}}$ | $\Delta \Omega_{\text{eff}}$
|----------|--------|---------------------------------|---------------------------------|------------------------------|---------------------------------|---------------------------------|------------------------------|---------------------------------|---------------------------------|------------------------------|------------------|
| 45°      | 63°    | $2.5 \times 10^2$               | $2.18 \times 10^4$               | 0.0092                        | 6.0 $\times 10^2$               | 1.19 $\times 10^3$               | 0.0074                        | 1.0 $\times 10^6$               | 2.76 $\times 10^7$               | 0.0000098

Table 3
Best-fit Parameters for the Calculated Phase-resolved Spectra of the Crab Pulsar

| Phase Band | $\gamma_{\text{min, pair}}^{\text{CR}}$ | $\gamma_{\text{max, pair}}^{\text{CR}}$ | $\psi_{\text{pair}}^{\text{CR}}$ | $\gamma_{\text{min, pair}}^{\text{ICS}}$ | $\gamma_{\text{max, pair}}^{\text{ICS}}$ | $\psi_{\text{pair}}^{\text{ICS}}$ | $\gamma_{\text{min, pair}}^{\text{pri}}$ | $\gamma_{\text{max, pair}}^{\text{pri}}$ | $\psi_{\text{pair}}^{\text{pri}}$ | $\Delta \Omega_{\text{eff}}$
|-----------|---------------------------------|---------------------------------|------------------------------|---------------------------------|---------------------------------|------------------------------|---------------------------------|---------------------------------|------------------------------|------------------|
| LW1       | $2.5 \times 10^2$               | $1.2 \times 10^4$               | 0.0092                        | $4.6 \times 10^2$               | 0.86 $\times 10^5$               | 0.0071                        | $5.0 \times 10^2$               | 0.55 $\times 10^5$               | 0.0070                        | 1.0 $\times 10^6$               | 1.76 $\times 10^7$               | 0.000005 | 1.82
| P1        | $2.5 \times 10^2$               | $0.9 \times 10^4$               | 0.0078                        | $4.6 \times 10^2$               | 0.86 $\times 10^5$               | 0.0071                        | $5.0 \times 10^2$               | 0.55 $\times 10^5$               | 0.0070                        | 1.0 $\times 10^6$               | 2.47 $\times 10^7$               | 0.000009 | 3.03
| TW1       | $2.5 \times 10^2$               | $0.95 \times 10^4$              | 0.0099                        | $3.0 \times 10^2$               | 0.68 $\times 10^5$               | 0.0079                        | $5.0 \times 10^2$               | 0.55 $\times 10^5$               | 0.0070                        | 1.0 $\times 10^6$               | 2.78 $\times 10^7$               | 0.000008 | 3.16
| Bridge    | $3.0 \times 10^2$               | $0.71 \times 10^4$              | 0.0085                        | $5.0 \times 10^2$               | $0.52 \times 10^5$              | 0.0079                        | $5.0 \times 10^2$               | $0.82 \times 10^5$              | 0.0085                        | 1.0 $\times 10^6$               | 2.97 $\times 10^7$               | 0.0000072 | 3.81
| LW2       | $3.5 \times 10^2$               | $1.25 \times 10^4$              | 0.008                         | $5.0 \times 10^2$               | $0.52 \times 10^5$              | 0.0079                        | $5.0 \times 10^2$               | $0.82 \times 10^5$              | 0.0085                        | 1.0 $\times 10^6$               | 2.75 $\times 10^7$               | 0.000003 | 2.53
| P2        | $5.0 \times 10^2$               | $1.22 \times 10^4$              | 0.0078                        | $5.0 \times 10^2$               | $0.82 \times 10^5$              | 0.0085                        | $5.0 \times 10^2$               | $0.82 \times 10^5$              | 0.0085                        | 1.0 $\times 10^6$               | 2.97 $\times 10^7$               | 0.0000072 | 3.81
| TW2       | $5.0 \times 10^2$               | $1.3 \times 10^4$               | 0.008                         | $5.0 \times 10^2$               | $0.82 \times 10^5$              | 0.0085                        | $5.0 \times 10^2$               | $0.82 \times 10^5$              | 0.0085                        | 1.0 $\times 10^6$               | 1.82 $\times 10^7$               | 0.000007 | 2.78
We found that multi-wavelength emission from the phase bands of P1, P2, and the bridge contribute significantly to the total phase-averaged spectrum. The phase-averaged spectrum and phase-resolved spectra are decomposed into four spectral components. Curvature radiation and synchrotron radiation from primary particles are the main origins of the observed $\gamma$-ray emission (10 MeV to $\sim$20 GeV); synchrotron radiation from CR- and ICS-induced pairs dominates the X-ray band and soft $\gamma$-ray band. ICS from the pairs contributes significantly up to the TeV band, while ICS from both primary particles and CR from pairs can be neglected for the spectrum fitting. Owing to the larger emission altitudes (which leads to lower acceleration electric field) for LW1 and TW2, the Lorentz factors $\gamma$ are very low, which results from a lower acceleration electric field; CR and synchrotron radiation have little contribution to the LW1 and TW2 phase band spectra. The TeV emission of ICS from pairs can be also found for P1, bridge, and P2 in our calculated phase-resolved spectra, which are consistent with the modeled TeV light curve. Two types of pairs, CR-induced and ICS-induced, could be therefore confirmed by the spectra of the Crab pulsar; these may also be the origin of the two types of wind pairs in the Crab Nebula.

4. CONCLUSIONS AND DISCUSSIONS

Owing to its strong multi-wavelength emission, the famous Crab pulsar is a crucial astrophysical object to distinguish the emission mechanisms from different magnetospheric models. In this paper, we calculated radio, X-ray, $\gamma$-ray, and TeV light curves, phase-averaged spectrum, and phase-resolved spectra in the framework of the three-dimensional AG and CG model with reasonable emission-geometry parameters ($\alpha = 45^\circ$ and $\zeta = 63^\circ$). It is found that the electric field in the AG is huge ($\sim 10^{16}$ eV) in the several tens of neutron star radii and vanishes near the region of $R_{\text{GC}}$. The primary particles are accelerated to ultra-relativistic energies and produce numerous secondary particles (CR and ICS pairs) in the inner region of the AG via the $\gamma$–B process. The pulsed emission of radio, X-ray, and $\gamma$-ray are generated from the emission of primary particles or secondary particles (pairs) with different emission mechanisms in the nearly similar region of the AG (or CG) in only one pole; this leads to the “phase-aligned” multi-wavelength light curves. The emission of P1 and P2 originates from the AG region near the null charge surface, while the emission of the bridge mainly originates from the CG region.

Assuming power-law energy distributions of primary particles and two types of pairs, the phase-averaged spectrum and phase-resolved spectra of the Crab pulsar are well produced by mainly four components: synchrotron radiation from CR- and ICS-induced pairs dominates the X-ray band to soft $\gamma$-ray band (100 eV to 10 MeV); CR and synchrotron radiation from the primary particles mainly contribute to the $\gamma$-ray band (10 MeV to $\sim$20 GeV); and ICS from the pairs significantly contributes to the TeV $\gamma$-ray band (100–400 GeV). Our fitted phase-averaged spectrum and phase-resolved spectra have similar tendencies varying with the photon energy and are basically consistent with the outer gap model (Tang et al. 2008) and the slot gap model (Harding et al. 2008) at the soft X-ray to a few tens of the GeV band, but quite different in $> 20$ GeV band. This is mainly due to the additional spectral component of ICS from pairs. From the multi-wavelength spectral fitting we emphasize that CR alone emitted from the primary particles cannot explain the TeV band (25–400 GeV) emission of the phase-averaged spectrum; ICS from pairs significantly contributes to this $> 20$ GeV band. In addition, two types of pairs are generated in the magnetosphere, and they may also be the origin of the two types of wind pairs in the Crab Nebula (Abdo et al. 2010).

Radio emission (1.4 GHz) of the Crab pulsar originates from a narrow and high-altitude region with a similar location of the $\gamma$-ray emission, which leads to the phase-aligned peaks. Our model for the radio photon sky map is patch-like; however, the detailed emission mechanism for radio emission needs to be further studied.

The popular outer gap (except for the versions of Romani & Yadigaroglu 1995; Cheng et al. 2000; Romani & Watters 2010) and slot gap models are two-pole models (Tang et al. 2008; Zhang & Li 2009; Li & Zhang 2010; Harding et al. 2008). To model the observed light curves and spectra for the Crab pulsar, they require the emission from both magnetic poles, which results from larger magnetic inclination angle $\alpha$ or larger viewing angle $\zeta$. However, our AG model is an intermediate emission-altitude and single-pole model with reasonable $\alpha$ and $\zeta$ from the X-ray torus fitting (Ng & Romani 2008). Unfortunately, the important parameters $\alpha$ and $\zeta$ for pulsar emission geometry are uncertain so far. Ng & Romani (2008) can only give a reliable viewing angle $\zeta$ for some young pulsars that have X-ray torus configurations, and then combine with the radio rotating vector model (RVM) to obtain the inclination angle $\alpha$ using the radio polarization angle fitting. The simple RVM model is only based on the geometry at a certain low altitude for an assumed circular emission beam and the propagation effects that can change the polarization states that had already been ignored by the RVM model. The derived $\alpha$ by this method is therefore debatable. A better method is strongly desired to obtain the convincing values of $\alpha$ and $\zeta$.

To adequately explain the multi-wavelength pulsed emission from pulsars, the detailed magnetic field configuration and three-dimensional global acceleration electric field distribution with proper boundary conditions for the AG and the CG should be carefully studied. Unfortunately, these two physical aspects are not fully understood. Recently, Romani & Watters (2010) studied pulsar light curves with magnetosphere beaming models and found that the outer gap model and approximating force-free dipole field were preferred at their high statistical significance. However, Harding et al. (2011) also studied high-energy pulse profiles (e.g., the Vela pulsar) using both retarded vacuum dipole and force-free field geometry. They found that the slot gap model with vacuum dipole was more favorable. Therefore, the subject of pulsar magnetic field configuration is still debatable. In addition, the problem of the three-dimensional acceleration field with the general relativistic effect and pair screening effect is more complicated, although many efforts have been made. We just derived the one-dimensional (actually two-dimensional) continuous solution for the acceleration Poisson equation, and the general relativistic and pair screening effects have not been taken into account in our AG model at present. This is our first step to establish our model picture, and will benefit more complicated three-dimensional physical studies with considerations of related effects. We emphasize that some simplified hypotheses considering qualitative physical effects have been used in our model to study pulsar light curves and spectra. This can give us insightful enlightenments to improve our knowledge of pulsar radiation physics. We will further improve our model to give more precise modeled light curves, especially for the phases of LW1 and TW2.

In sum, the multi-wavelength emission from the Crab pulsar can be effectively explained in the AG and CG model, and this
is also done for the Vela pulsar (Du et al. 2011). Our model is a promising model to unveil the multi-wavelength pulsed emission from γ-ray pulsars.

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Facilities: Fermi (LAT), MAGIC, VERITAS

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