Control of a nonlinear continuous stirred tank reactor
via event triggered sliding modes

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Abstract

Continuous Stirred Tank Reactors (CSTR) are the most important and central
equipment in many chemical and biochemical industry that exhibit second order
complex nonlinear dynamics. The nonlinear dynamics of CSTR poses many de-
sign and control challenges. The proposed controller guarantees a stable closed
loop behavior over multiple operating points even in presence of disturbances
and parametric uncertainties. An event driven sliding mode control is presented
in this work to regulate the temperature and concentration very close to the
equilibrium points of CSTR. The control is executed only when a predefined
condition gets violated and hence the controller is relaxed when the system is
operating under tolerable limits in terms of closed loop performance. A novel
dynamic event triggering rule is presented to maintain desired performance with
minimum computational cost. The inter event execution time is shown to be
lower bounded by a finite positive quantity to exclude Zeno behavior. Sliding
mode control (SMC) combined with event triggering scheme retains the inherent
robustness of traditional SMC and aids in reducing computational load on the
controller involved. Simulation results validate the efficiency of the proposed
controller.

Keywords: CSTR, event based sliding mode control, Riemann sampling,
Lebesgue sampling, event conditions, triggering rule, inter event time.

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1. Introduction

A Continuous Stirred Tank Reactor (CSTR) exhibits complex nonlinear dynamics and is a benchmark equipment in many process industries\cite{1} that require continuous addition and withdrawal of reactants and products. These reactors are used to get optimal productivity in chemical plants by maintaining very high conversion rate to maximize economy. A CSTR may be assumed to be somewhat opposite of an idealized well-stirred batch and tubular plug-flow reactors. The set of operating points should exhibit a stable steady state behavior under the influence of disturbances as well. Linear controllers designed for such process fail to deliver optimal performance outside the linear operating range. The PID controller is the most commonly used controller in industries due to its easy design and tuning properties. Feedback linearization has been also extensively used to control CSTR \cite{2} wherein the controller fails to deliver under varying transient behavior of the plant model due to the non-adaptive nature of the controller. A linear controller using Taylor’s linearization has been designed assuming bounded uncertainty in \cite{3} and in \cite{4}, which are again based on operation of CSTR in a limited regime. It has been discussed in \cite{5} that use of local linearization cannot ensure global stability. Input-output feedback linearization method proposed in \cite{6} also failed because it requires continuous measurement of states, which is quite expensive and impractical in practical scenario. In \cite{6}, a method based on state coordination transformation has also been studied for linear input state behavior. Observer based designs for state measurements have been discussed in \cite{7, 8, 9, 10, 11, 12, 13} to name a few. While this method ensured asymptotic stability, the effects of disturbance was overlooked. Observer based design failed to deliver desired response under varying process conditions. A high gain controller used in \cite{14} exhibited quicker response but resulted in unwanted control effort saturation. While performing linearization of a plant model, there remains a part of transformed system which is non-linearizable \cite{15} and has zero dynamics which cannot be ignored \cite{6}. In spite of
being a non-model based control, techniques such as adaptive and fuzzy do not yield optimal performance under widely varying and fast process dynamics.

In this work, we propose a controller based on paradigms of event based sliding mode control. Sliding Mode Control (SMC) [16], [17], [18], [19], [20], [21], [22], [23] is a control scheme which guarantees finite time convergence and provides robust operation over the entire regime with complete rejection to matched perturbations that may creep in the system from input channel. The advantage of using this control is that we can tailor the system dynamical behavior by a particular choice of sliding function. SMC, used in conjunction with event triggered control, retains its robustness as well as event triggering approach aids in saving energy expenditure. When measured variables of a system do not deviate frequently, event based control offers numerous advantages over time triggered control. The control is executed only when needed, thus computational complexity is also reduced.

2. Plant Dynamic Model

Chemical reactions in a reactor can be characterized as endothermic or exothermic. In order to maintain the temperature of the reactor at a desired reference, a finite amount of energy is required to be added to or removed from the reactor. Usually a CSTR operates at steady state with contents well mixed, so modelling does not involve significant variations in concentration, temperature or reaction rate throughout the vessel. Since the system internal states under consideration, i.e, temperature and concentration are identical everywhere within the reaction vessel, they are the same at the exit as they are anywhere else in the tank. Consequently, the temperature and concentration at the exit are modelled as being the same as those inside the reactor. In situations where mixing is highly nonideal, the well mixed model fails and nonideal CSTR model must be formulated.
In this work, we are concerned with a dynamic description of the reactor in which mixing is adequate [1]. Thus, an ideal CSTR model as provided in Ray, et. al [24] has been adopted in our study. Presence of exponential terms in the modelling equations make the description a nonlinear one. A complex chemical reaction occurs in CSTR, e.g. conversion of a hazardous chemical waste (reactant) into an acceptable and tolerable chemical (product). Under the assumption of complete mixing, the reactor gets cooled in a continuous manner. The volume of the chemical product $B$ is equal to the volume of the input reactant $A$. The reactor is assumed to be non isothermal and exhibiting an irreversible exothermic first order chemical reaction $A \rightarrow B$.

The dynamic model is then given as

$$\frac{dC_A}{dt} = \frac{F}{V}(C_{Af} - C_A) - r$$

$$\frac{dT}{dt} = \frac{F}{V}(T_f - T) + \frac{(-\Delta H)}{\rho C_p}r - \frac{hA}{V \rho C_p}(T - T_c)$$

Model parameters of significance are given in table (1) and a schematic diagram of CSTR is shown in figure (1).
Table 1: CSTR model parameters

| Meaning                             | Symbol | Unit       |
|-------------------------------------|--------|------------|
| $1^{st}$ order reaction rate constant | $k_0$  | $min^{-1}$ |
| inlet concentration of $A$          | $C_{Af}$ | $kmol/m^3$ |
| steady state flow rate of $A$       | $F$    | $m^3/min$  |
| density of the reagent $A$          | $\rho$ | $g/m^3$    |
| specific heat capacity of $A$       | $C_p$  | $cal/^\circ C g$ |
| heat of reaction                    | $\Delta H$ | $cal/kmol$ |
| density of coolant                  | $\rho_c$ | $g/m^3$    |
| specific heat capacity of coolant   | $C_{p_c}$ | $cal/^\circ C g$ |
| volume of the CSTR                  | $V$    | $m^3$      |
| coolant flow rate                   | $F_c$  | $m^3/min$  |
| reactor temperature                 | $T$    | $K$        |
| reactor concentration of $A$        | $C_A$  | $kmol/m^3$ |
| activation energy                   | $E$    | $J/mol$    |
| universal ideal gas constant        | $R$    | $J/molK$   |

\[
r = k_0 \exp\left(-\frac{E}{RT}\right)C_A
\]

(2)

\[
hA = \frac{aF_c^{b+1}}{F_c + aF_c^b} \rho_c C_{p_c}
\]

(3)

$a, b$ are CSTR model parameters and $hA$ is the heat transfer term.

A computationally more convenient form of the above modelling equations are presented in state space formulation below. For original convention and nomenclature, the reader is suggested to refer [24].

\[
\dot{x}_1 = -x_1 + D_a(1 - x_1)\exp\left(\frac{x_2}{1 + x_2/\gamma}\right) - d_2
\]

(4)

\[
\dot{x}_2 = -x_2 + BD_a(1 - x_1)\exp\left(\frac{x_2}{1 + x_2/\gamma}\right) - \beta(x_2 - x_{2_c}) + \beta u_T + d_1
\]

(5)
This formulation utilizes dimensionless modelling of CSTR for which the parameters are tabulated in table (2). Parameters $d_1$ and $d_2$ are bounded and measurable disturbances. The nominal coolant temperature is $x_{2,0}$. Also, there are two ways to manipulate the observed states (outputs)- coolant temperature and input feed flow. In this study, we have used the coolant temperature as our control input to regulate the temperature of the CSTR. This is because in industrial environments, temperature becomes more critical to be controlled in order to avoid any secondary reaction in the reactor. It should be noted that while designing a controller to regulate temperature, the other state, i.e., composition should not be allowed to enter the region of instability. Furthermore, design of a second controller to regulate composition is also discussed in this work.
Table 2: Dimensionless parameters in CSTR modelling

| Dimensionless Parameters for CSTR | Dimless Time |
|----------------------------------|--------------|
| ratio of activation energy to average kinetic energy | $\gamma = \frac{E}{RT_f}$ |
| adiabatic temperature rise | $B = \frac{(\Delta H)C_{A_f}}{\rho C_p T_f}$ |
| Damkohler number | $D_a = k_0 \exp(-\gamma) V/F_0$ |
| heat transfer coefficient | $\beta = hA/\rho C_p F_0$ |
| dimensionless time | $t = t'(F_0/V)$ |
| dimensionless composition | $x_1 = (C_{A_f} - C_A)/C_{A_f}$ |
| dimensionless temperature | $x_2 = \gamma(T - T_f)/T_f$ |
| dimensionless control input | $u_T = \gamma(T - T_c)/T_f$ |
| dimensionless control input | $u_T = (F - F_0)/F_0$ |
| feed temperature disturbance | $d_1 = \gamma(T_f - T_f)/T_f$ |
| feed composition disturbance | $d_2 = (C_{A_f} - C_{A_f})/C_{A_f}$ |

For ease of controller synthesis, let us formulate a functional description of the state equations given by (4) and (5).

$$\dot{x}_1 = f_1(x_1, x_2) - d_2 \quad (6)$$

$$\dot{x}_2 = f_2(x_1, x_2) + \beta u_T + d_1 \quad (7)$$

3. Controller Synthesis

The controller in this study is synthesized without any linearization of the dynamics. An event based sliding mode control has been used to implement the controller.

3.1. Control Objective

Primary objective of the control scheme is to maintain the states to a desired reference value with minimum computational expense. For computational purposes, it is desirable to define the following error candidates. The deviation
from the desired temperature is given by

\[ e_{x_2}(t) = x_2(t) - x_{2_{ref}}(t) \]  

(8)

Similarly, error variable for the state representing composition is given by

\[ e_{x_1}(t) = x_1(t) - x_{1_{ref}}(t) \]  

(9)

The control effort must be designed robust enough to achieve accurate desired reference tracking, reject disturbances and deliver acceptable results quickly. Stated alternatively, the error variables are required to vanish or at least settle in close vicinity of zero after a transient of acceptable duration.

3.2. Traditional sliding mode controller

Sliding Mode Control (SMC) [19], [25] is long known for its inherent robustness. The switching nature of the control is used to nullify exogeneous bounded disturbances and matched uncertainties. The switching happens about a manifold in state space known as sliding manifold. The control forces the state trajectories monotonically towards the sliding manifold and this phase is regarded as reaching phase. When state trajectories reach the manifold, they remain there for all future time, thereby ensuring that the system dynamics remain independent of bounded disturbances and matched uncertainties. This phase is regarded as sliding phase. Thus, the controller has reaching phase (trajectories in phase plane emanate and move towards the sliding manifold) and sliding phase (trajectories in the phase plane that reach the sliding manifold try to remain there).

3.2.1. Reaching Phase

Let the manifold discussed above be described mathematically as \( \sigma(x) \). In order to drive state trajectories onto this manifold, a proper discontinuous control effort \( u(t, x) \) needs to be synthesized for which the following inequality is respected.

\[ \sigma^T(x)\dot{\sigma}(x) \leq -\eta \| \sigma(x) \| \]  

(10)
with $\eta$ being positive and is called the reachability constant.

\[
\dot{\sigma}(x) = \frac{\partial \sigma}{\partial x} \dot{x} = \frac{\partial \sigma}{\partial x} f(t, x, u) \quad (11)
\]

\[
\sigma^T(x) \frac{\partial \sigma}{\partial x} f(t, x, u) \leq -\eta \|\sigma(x)\| \quad (12)
\]

It is clear that the control $u$ can be synthesized from the above equation.

### 3.2.2. Sliding Phase

The motion of state trajectories confined on the switching manifold is known as *sliding*. A sliding mode is said to exist in close vicinity of the manifold if the state velocity vectors are directed towards the manifold in its neighbourhood [22], [19]. Under this circumstance, the manifold is called attractive [22], i.e., trajectories starting on it remain there for all future time and trajectories starting outside it tend to it in an asymptotic manner.

\[
\dot{\sigma}(x) = \frac{\partial \sigma}{\partial x} f(t, x, u) \quad (13)
\]

Then $u = u_{eq}$ (say) be a solution and is generally referred to as the equivalent control. This $u_{eq}$ is not the actual control applied to the system but can be thought of as a control that must be applied on an average to maintain sliding motion. It is mainly used for analysis of sliding motion [16].

By the theory of sliding modes, let us formulate the sliding manifold as

\[
\sigma(t) = \lambda_1 x_1(t) + \lambda_2 x_2(t) \quad (14)
\]

where $\lambda_1$ and $\lambda_2$ are the coefficient weights which can be tuned as per performance needs. In design, it is not the actual weights that matter, rather relative weights are of significance. The sliding manifold can alternatively be written in
error dynamical form as

$$\sigma(t) = \lambda_1 e_{x_1}(t) + \lambda_2 e_{x_2}(t)$$  \hspace{1cm} (15)

During sliding, $\dot{\sigma}(t) = 0$ and the corrective term used to force the trajectories onto the sliding surface is chosen as $\mu \text{sign}(\sigma(t))$, where $\mu$ is the adjustable gain.

We now proceed to design the control law for the discussed case.

**Theorem 3.2.1.** Given plant dynamics (6–7), errors (8–9) and the sliding manifold (15), the control law due to traditional sliding mode controller is given by

$$u_T(t) = -\lambda_2^{-1} \beta^{-1}(\lambda^T f(x(t)) + \mu \text{sign}(\sigma(t)))$$  \hspace{1cm} (16)

where $\lambda^T = [\lambda_1 \lambda_2]$ and the function $f(x)$ is given as

$$f(x) = \begin{bmatrix} f_1(x_1, x_2) - d_2 - \dot{x}_{1_{\text{ref}}} \\ f_2(x_1, x_2) + d_1 - \dot{x}_{2_{\text{ref}}} \end{bmatrix}$$  \hspace{1cm} (17)

**Proof.** Before proceeding towards proof, we make an assumption on the nature of the function $f(x)$.

**Assumption 3.2.1.** The function $f(x)$ satisfies Lipschitz conditions. Hence, we can write $\|f(x) - f(y)\| \leq \bar{L}\|x - y\|$ for some $x$ and $y$ in the domain $\mathbb{D}_L \subset \mathbb{R}^n$ with $\bar{L}$ as the Lipschitz constant.

We now proceed to a formal proof. From (15), we have

$$\sigma(t) = \lambda_1 e_{x_1}(t) + \lambda_2 e_{x_2}(t)$$
$$\Rightarrow \dot{\sigma}(t) = \lambda_1 \dot{e}_{x_1}(t) + \lambda_2 \dot{e}_{x_2}(t)$$
$$\Rightarrow \dot{\sigma}(t) = \lambda_1 (\dot{x}_1(t) - \dot{x}_{1_{\text{ref}}}) + \lambda_2 (\dot{x}_2(t) - \dot{x}_{2_{\text{ref}}})$$
$$\Rightarrow \dot{\sigma}(t) = \lambda_1 (f_1(x_1, x_2) - d_2 - \dot{x}_{1_{\text{ref}}}) + \lambda_2 (f_2(x_1, x_2) + d_1 - \dot{x}_{2_{\text{ref}}})$$
$$\Rightarrow \dot{\sigma}(t) = \lambda^T f(x(t)) + \lambda_2 \beta u_T(t)$$  \hspace{1cm} (18)

$$\therefore u_T(t) = -\lambda_2^{-1} \beta^{-1}(\lambda^T f(x(t)) + \mu \text{sign}(\sigma(t)))$$

where $\lambda^T = [\lambda_1 \lambda_2]$ and $f(x) = \begin{bmatrix} f_1(x_1, x_2) - d_2 - \dot{x}_{1_{\text{ref}}} \\ f_2(x_1, x_2) + d_1 - \dot{x}_{2_{\text{ref}}} \end{bmatrix}$

This completes the theorem along its proof. $\square$
3.3. Event based control

Recently, there has been a tremendous growth of interest in the area of event based systems due to reduced computational cost. The challenge, however, in this type of control is to maintain performance, stability, optimality, etc. in the presence of uncertainties and reduced computation/communication. A modern control system consists of a computer and the signal under consideration is sampled periodically to cater the needs of a classic sampled data control system. Under such scheme, the interval between two successive clock pulses is predetermined and fixed. The sampling takes place along the horizontal axis, also known as Riemann sampling. An alternate, more natural and efficient way is to sample along the vertical axis, also known as Lebesgue sampling [26]. In the latter case, the sampling is not periodic rather it depends on the value of previous sample or certain conditions that need to be violated to bring forth the next clock pulse. These conditions are some noticeable changes (events or event conditions) on which the next sampling instant depends.

This type of control seems to be a reasonable choice in applications where signal of interest slowly varies. In chemical process industries that contain many production units, primary units are separated by buffer units. Each change in the unit can cause upset and hence it is desirable to keep the change in process variables less frequent. Event based control comes handy in such applications. No action is taken unless there is a huge upset. It is also advantageous to use event based control when the steady state value of a process variable needs to be fixed irrespective of the manner in which the states evolve. Early contributions on event based control, readers are requested to refer [27], [28], [29], [30], [31], [32], [33], [34], [35] to name a few.

3.3.1. Event based sliding mode control

Since, next sample instant is dependent on the previous sampling information, the control (17) is held constant between successive events or sampling instants. The control is not updated periodically and is held at the previous
value in the interval \([t_k, t_{k+1}]\). This, however, introduces a discretization error between the states of the system.

\[
\epsilon(t) = x(t) - x(t_k)
\]  

(19)
such that at \(t = t_k\), \(\epsilon(t)\) vanishes. The term \(t_k\) is the triggering instant at \(k^{th}\) sampling instant. Control gets updated \(t_k\) instants only. The sampling is not periodic and hence \(t_{k+1} - t_k \neq constant\).

Hence, the control signal from (16) modifies to yield the event triggered sliding mode control law

\[
u_T(t) = -\lambda_2^{-1} \beta^{-1}(\lambda^T f(x(t_k)) + \mu \text{sign}(\sigma(t_k)))
\]  

(20)

**Theorem 3.3.1.1.** Consider the system described by (6 – 7), error candidates (8 – 9) and (19), sliding manifold (15) and control law of (20). Then, the event triggered control law (20) makes the system stable in the sense of Lyapunov and sliding mode is said to exist in the vicinity of the manifold (15). The manifold is an attractor if reachability to the surface is ascertained for some reachability constant \(\eta > 0\).

**Proof.** Let us consider a Lyapunov candidate \(V\) such that

\[
V = \frac{1}{2} \sigma^T(t) \sigma(t)
\]  

(21)

Time derivative of the candidate given in (21) for \(t \in [t_k, t_{k+1}]\) along the state trajectories yield

\[
\dot{V} = \sigma(t) \dot{\sigma}(t)
\]  

(22)

It can be written from (18),

\[
\dot{V} = \sigma(t)(\lambda^T f(x(t)) + \lambda_2 \beta u_T(t))
\]  

(23)

Thus \(\forall t \in [t_k, t_{k+1}],\) it can be written as

\[
\dot{V} = \sigma(t)(\lambda^T f(x(t)) - \lambda^T f(x(t_k)) - \mu \text{sign}(\sigma(t_k)))
\]

\[
\dot{V} \leq -\sigma(t) \mu \text{sign}(\sigma(t_k)) + ||\sigma(t)|| \lambda^T ||f(x(t)) - f(x(t_k))||
\]

\[
\dot{V} \leq -\sigma(t) \mu \text{sign}(\sigma(t_k)) + ||\sigma(t)|| \lambda^T ||\bar{L}|| ||x(t) - x(t_k)||
\]

\[
\dot{V} \leq -\sigma(t) \mu \text{sign}(\sigma(t_k)) + ||\sigma(t)|| \lambda^T ||\bar{L}|| ||\epsilon(t)||
\]  

(24)
As long as \( \sigma(t) > 0 \) or \( \sigma(t) < 0 \), the condition \( \text{sign}(\sigma(t)) = \text{sign}(\sigma(t_k)) \) is strictly met \( \forall t \in [t_k, t_{k+1}] \). Hence, when trajectories are just outside the sliding surface,

\[
\dot{V} \leq -\|\sigma(t)\|\mu + \|\sigma(t)\|\lambda^T \|\bar{L}\|\epsilon(t)\|
\]

\[
\Rightarrow \dot{V} \leq -\|\sigma(t)\|\left(\mu + \|\lambda^T \|\bar{L}\|\epsilon(t)\|\right)
\]

\[
\Rightarrow \dot{V} \leq -\eta\|\sigma(t)\|\quad (25)
\]

with \( \eta > 0 \). This completes the proof of reachability. \( \square \)

For stability, it is required to be shown that \( \dot{V} < 0 \).

At \( t = t_k \), \( \|\epsilon(t)\| \to 0 \) and the control signal is updated. Thus,

\[
\dot{V} \leq -\|\sigma(t)\|\left(\mu + \|\lambda^T \|\bar{L}\|\epsilon(t)\|\right)
\]

\[
\therefore \|\epsilon(t)\| \to 0 \Rightarrow \dot{V} < 0
\]

This completes the proof of stability. \( \square \)

For time instants between \( [t_k, t_{k+1}] \) the states show a tendency to deviate from the sliding manifold but remain bounded within a band near the manifold. The triggering instant \( t_k \) is completely characterized by a triggering rule. Next sampling instant is by virtue of this criterion. As long as this criterion is respected, next clock pulse is not called upon and the control signal is maintained constant at the previous value. The triggering rule used in this work is given by

\[
\delta = \|\zeta e_{x_i} + \xi^2 \xi z_i^2\| - \psi(m_1 + m_2 e^{-\varsigma t})
\]

with \( i = 1, 2 \) for respective error in states, \( \zeta > 0 \), \( \xi > 0 \), \( \psi \in (0, 1) \), \( m_1 \geq 0 \), \( m_2 \geq 0 \), \( m_1 + m_2 > 0 \) and \( \varsigma \in (0, 1) \). The term \( (m_1 + m_2 e^{-\varsigma t}) \) ensures a finite lower bound on inter event execution time and avoids accumulation of samples at same instant, known as Zeno behaviour in literature. The following relation completely determines the triggering instants in an iterative manner

\[
t_{k+1} = \inf\{t \in ]t_k, \infty[ : \delta \geq 0\}
\]

(28)
The inter event time is given by

\[ T_k = t_{k+1} - t_k \]  \hfill (29)

**Assumption 3.3.1.1.** A finite but not necessarily constant delay \( \Delta \) might occur during sampling and is unavoidable due to hardware characteristics. In such cases the control is maintained constant \( \forall t_i \in [t_i^k + \Delta, t_i^{k+1} + \Delta] \). It has been assumed that \( \Delta \) is negligible and has been neglected innocuously. Hence for our case, control is constant in the interval \( [t_i^k, t_i^{k+1}] \).

**Theorem 3.3.1.2.** Consider the system described by (6-7), the control signal given in (20) and the discretization error (19). The sequence of triggering instants \( \{t_k\}_{k=0}^\infty \) respects the triggering rule given in (28). Consequently, Zeno phenomenon is not exhibited and the inter event execution time \( T_k \) is bounded below by a finite positive quantity such that

\[ T_k \geq \frac{1}{L} \ln \left( \frac{\bar{L}\|\epsilon\|_\infty}{L(1 + \|B\lambda_2^{-1}\beta^{-1}\lambda_T\|)(\|x(t_k)\| + \|B\mu\| + 1)} \right) \]  \hfill (30)

where \( \|\epsilon\|_\infty \) is the maximum discretization error.

**Proof.** Without loss of generality, the system described in (6-7) is recalled here as

\[ \dot{x}(t) = f(x) + \bar{B}u_T(t) \]  \hfill (31)

where \( f(x) \) is same as (17) and \( \bar{B} = [0 \quad \beta]^T \). Between \( k^{th} \) and \( (k+1)^{th} \) sampling instant in the execution of control, the discretization error (19) is non zero. \( T_k \) is the time it takes the discretization error to rise from 0 to some finite value. Thus,

\[ \frac{d}{dt}\|\epsilon(t)\| \leq \|\frac{d}{dt}\epsilon(t)\| \leq \|\frac{d}{dt}x(t)\| \]  \hfill (32)

\[ \Rightarrow \|\frac{d}{dt}\epsilon(t)\| \leq \|f(x(t)) + \bar{B}u_T(t)\| \]  \hfill (33)
Substituting the control input (20) in the above inequality, we get

\[
\| \frac{d}{dt} \epsilon(t) \| \leq \| f(x(t)) - \bar{B}\lambda_2^{-1}\beta^{-1}\lambda^T f(x(t_k)) - \bar{B}\mu \text{sign}(\sigma(t_k)) \|
\]

\[
\leq \bar{L}\| x(t) \| + \| \bar{B}\lambda_2^{-1}\beta^{-1}\lambda^T \| \bar{L}\| x(t_k) \| + \| \bar{B}\| \mu
\]

\[
\leq \bar{L}(\| \epsilon(t) \| + \| x(t_k) \|) + \| \bar{B}\lambda_2^{-1}\beta^{-1}\lambda^T \| \bar{L}\| x(t_k) \| + \| \bar{B}\| \mu
\]

\[
\leq \bar{L}(\| \epsilon(t) \| + \bar{L}(1 + \| \bar{B}\lambda_2^{-1}\beta^{-1}\lambda^T \| )\| x(t_k) \| + \| \bar{B}\| \mu
\]

\[ (34) \]

The solution to the differential inequality (34) \( \forall t \in [t_k, t_{k+1}] \) can be understood by using Comparison Lemma [36] with initial condition \( \| \epsilon(t) \| = 0 \) and is given as

\[
\| \epsilon(t) \| \leq \frac{\bar{L}(1 + \| \bar{B}\lambda_2^{-1}\beta^{-1}\lambda^T \| )\| x(t_k) \| + \| \bar{B}\| \mu}{\bar{L}} \left( e^{\bar{L}(t - t_k)} - 1 \right) \]

\[ (35) \]

Comparison Lemma [36], [37] is particularly useful when information on bounds on the solution is of greater significance than the solution itself. For triggering time instant \( t_{k+1} \),

\[
\| \epsilon \|_{\infty} = \| \epsilon(t_{k+1}) \| \leq \frac{\bar{L}(1 + \| \bar{B}\lambda_2^{-1}\beta^{-1}\lambda^T \| )\| x(t_k) \| + \| \bar{B}\| \mu}{\bar{L}} \left( e^{\bar{L}T_k} - 1 \right) \]

\[ (36) \]

\[
\therefore T_k \geq \frac{1}{\bar{L}} \ln \left( \frac{\bar{L}\| \epsilon \|_{\infty} + \| \bar{B}\| \mu}{\bar{L}(1 + \| \bar{B}\lambda_2^{-1}\beta^{-1}\lambda^T \| )\| x(t_k) \| + \| \bar{B}\| \mu} + 1 \right) \]

\[ (37) \]

\[
\therefore \text{the right hand side of (37) is always positive, it is, therefore concluded that inter event execution time is bounded below by a finite positive quantity [38].}
\]

This concludes the proof. \( \square \)

4. Numerical Simulation

The efficacy of the proposed control scheme is demonstrated by computer simulation of the given model for two scenarios, i.e., operation of CSTR under no disturbances and time varying disturbances. Following parametric values are used in the experiment.

\( \mu = 25, \beta = 0.3, D_a = 0.078, \gamma = 20, B = 8 \) and \( x_{2c} = 0. d_1 \) and \( d_2 \) are exogeneous disturbances of magnitude 0.026 \( \sin(0.1t) \) and 0.037 \( \sin(0.1t) \)
respectively. Moreover these disturbances are fixed by a positive upper bound, i.e., $|d_1| < |d_2| < |d|_\infty$. Surface coefficient weights $\lambda_1$ and $\lambda_2$ are chosen to be 1 and 2 respectively. $\zeta = \xi = 0.8$, $m_1 = 10^{-4}$, $m_2 = 0.2025$, $\psi = 0.5$ and $\varsigma = 0.97$ are taken to be parameters of the triggering rule. The startup reference trajectory to be tracked is taken as

$$y_{ref} = x_{2ss}(1 - k_1 e^{-k_2 t})$$

where $x_{2ss} = 2.6516$ is close to an equilibrium point of the system. $k_2$ and $k_2$ are parameters that are dependent on practical restrictions on the reactor. Here $k_1 = 1 = k_2$. In [6], it has been shown that the system has multiple steady state equilibrium points, one of which is $(x_1, x_2) = (0.4472, 2.7517)$. Hence, in our experiment, we have provided $x_{2ss}$ very close to this equilibrium point. Therefore, $x_1$ must remain close to 0.4472 to ensure that the proposed control is worthy and robust.

4.1. System operating in absence of disturbances

Figures (2) and (3) show the states of the system under the influence of control signal (20) when no disturbance affect the system. Figure (4) is the plot of inter event execution time $T_k$ which depicts the sampling interval. Clearly the sampling is non uniform. Figure (5) shows sampling instants.
Figure 2: Composition ($x_1$)

Figure 3: Temperature ($x_2$)
4.2. System operating in presence of disturbances

Figures (6) and (7) show the states of the system under the influence of control signal (20) when time varying disturbances $d_1$ and $d_2$ are also taken into
Similarly, figures (8) depict inter event execution time $T_k$ for this case.

Figure 6: Composition ($x_1$)

Figure 7: Temperature ($x_2$)
5. Conclusion

A novel nonlinear controller based on archetype of event triggered sliding mode has been designed to control a continuous stirred tank reactor. It has been shown that the controller is sturdy and provides stability to the system in a very short span of time. State trajectories have been maintained in close proximity of equilibrium points with minimum computation by the controller. Event triggering technique is one practical control application wherein resource utilization is minimal but optimal closed loop performance is not compromised. The proposed controller based on event triggering SMC provides stability to the system in the sense of Lyapunov. The inter event time is separated by a finite discrete time interval to ascertain no Zeno behavior results. Numerical simulations are presented to confirm the effectiveness of the proposed event driven sliding mode control.
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Highlights

1. Sliding mode control has been used to obtain the desired closed loop performance. The control rejects any matched disturbance.
2. The controller is an event-based controller that significantly reduces the computation required and energy expenses to execute the task.
3. The novel triggering rule is constructed on error and the squared rate of change of error, introducing anticipatory action into the system and making the tolerable limits for satisfactory closed loop performance a dynamic one.
4. The controller is synthesized considered system in nonlinear form. No linearization of dynamics is done.
5. The system delivers desired closed loop performance with minimal computations of the control signal even in the presence of disturbances.
6. The inter event execution time is lower bounded by a finite positive value that completely excludes any Zeno behavior.