Non-monotonic shallow nucleus-nucleus potential for heavy-ion elastic scattering

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Abstract. ‘Goldberg criterion’ [Goldberg and Smith, Phys. Rev. Lett. 29 (1972) 500] tells that at sufficiently high energies, where pronounced refractive scattering with nuclear rainbow oscillations are followed by an ‘exponential-type falloff’ in the angular distribution, discrete ambiguities are eliminated for the deep monotonic potential. The criterion is also confirmed in the work of Bartnitzky et al. [Phys. Lett. B 365 (1996) 23] on the $^{16}$O+$^{16}$O elastic scattering in the energy range of 250 - 704 MeV. However, their finding ‘using model-independent potentials’ suggests that heavy-ion elastic scattering data unambiguously favour deep potentials. The Goldberg criterion is examined in our work for non-monotonic shallow potentials using the $^{16}$O+$^{16}$O elastic scattering at energy region up to 350 MeV.

1. Introduction
1.1. Atmospheric Rainbow
The atmospheric rainbows are formed by sunlight in suspended water droplets. The most observed rainbows are primary and secondary rainbows correspond respectively, to one and two reflections inside a water droplet with two refractions. Those rays, which are deflected by minimum angle of deflection, carry sufficient intensity and produce rainbows. The corresponding maximum (negative) angle by which the rainbow is seen called ‘rainbow angle’ $\theta_R$ shown in Fig. 1. Classically, the light intensity has a maximum at $\theta_R$ which is also known as a ‘caustic in optics’ [1]. Sometimes other bows, which are observed, are the near edge of the primary bow or near the outer edge of the secondary bow. These are also known as supernumerary bows shown in Fig. 2 (marked as 1, 2, 3, 4 . . . ) and depend on the size of the water drops. In the Airy model [3], the interference of the two components of the incident wave-front produces supernumerary bows corresponding to the amplitude $A(x)$ and intensity $|A(x)|^2$ distributions shown in Fig. 4. The dark regions are termed as ‘Airy minima’.

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Figure 1. Deflection of monochromatic light rays incident on the water droplet with different impact parameters for the rays numbered from \( N=1, 2 \ldots N+x \). The ray following \( N^{th} \) path and emerging with minimum deflection angle forms the rainbows with concentration of emergent rays.

Figure 2. Supernumerary bows (1, 2, 3, 4, \ldots) formed near the lower edge of the primary bow. Illustration is taken from [2]

1.2. Refractive Scattering and Nuclear Rainbow
To understand the refractive scattering and nuclear rainbow in the nucleus-nucleus elastic scattering, the quantum mechanical scattering amplitude \( f(\theta) \) is decomposed into sum of its so-called nearside \( f_N(\theta) \) and farside \( f_F(\theta) \) amplitudes as done by Fuller [6], shown in Fig. 5. To achieve this, the Legendre function \( P_l(\cos \theta) \), in the total scattering amplitude

\[
f(\theta) = f_C(\theta) - \frac{i}{2k} \sum_l (2l + 1)e^{2i\sigma_l}(S_l - 1)P_l(\cos \theta),
\]  

(1)
Figure 3. Different descriptions of the light intensity of the atmospheric rainbow which shows the functional dependence of the light intensity at the maximum deflection angle. The origin of the supernumerary bows cannot be explained in the Descartes’ and Young’s theory. This problem is resolved by Airy’s theory. Illustration is taken from [4] and redrawn with colour.

Figure 4. Production of supernumerary bows corresponding to the Airy amplitudes $A(x)$ and intensities $|A(x)|^2$ resulting from interference of two-component wavefront. The dark lines of the intensity pattern are termed as ‘Airy minima’. Illustration is taken from [5].
Figure 5. The farside and nearside trajectories contributing to the total scattering amplitude at scattering angle $\theta$. Illustration is taken from [7] and redrawn with colour.

is replaced by sum of $\tilde{Q}_I^-$ and $\tilde{Q}_I^+$ traveling waves running from opposite sides of the scattering centre given by

$$\tilde{Q}_L^\pm = \frac{1}{2} \left[ P_l(\cos \theta) \mp \frac{2}{\pi} P_l(\cos \theta) \right]. \quad (2)$$

This approach in Fuller’s method leads to

$$f_F(\theta) = f_{CF}(\theta) - \frac{i}{2k} \sum_l (2l + 1)e^{2i\sigma_l} (S_l - 1)\tilde{Q}_I^-(\cos \theta) \quad (3)$$

and

$$f_N(\theta) = f_{CN}(\theta) - \frac{i}{2k} \sum_l (2l + 1)e^{2i\sigma_l} (S_l - 1)\tilde{Q}_I^+(\cos \theta). \quad (4)$$

Here, $f_{C}(\theta)$’s are the pure Coulomb amplitudes, $\sigma_l$’s are the Coulomb phase-shifts and the partial elastic $S$-matrix $S_l$ is given in terms of the partial nuclear phase-shift $\delta_l$ by

$$S_l = |S_l| e^{2i\delta_l}. \quad (5)$$

The Fraunhofer diffraction scattering, dominant at lower energies, occurs due to absorptive target medium which removes flux and, in Fuller’s picture, is contributed by the nearside scattering with negligible contribution from the nuclear interior. The Airy structure of nuclear refractive scattering arises from the coherent sum of farside amplitudes $f_F(\theta)$ originating from different ranges of angular momenta $l_{<}$ and $l_{>}$ as shown in Fig. 6. The scattering angle as a function of impact parameter $b$ (Fig. 5) or angular momentum $(l + 1/2)\hbar \approx kb$, is called the deflection function $\theta(l)$. Fig. 7 illustrates some typical farside and nearside trajectories for scattering from the effect of nuclear attractive potential and Coulomb effect respectively, of the total potential and the corresponding construction of deflection function $\theta(l)$ with respect to $l$ is shown in the right of the figure. From Fig. 7, it is seen that, the inner minimum in $\theta(l)$ gives rise to nuclear rainbow at an angle $\theta_R = |\theta(l_R)|$. The rainbow maximum is formed in nuclear rainbow pattern due to the maximum concentration of farside trajectories at the position of minimum deflection angle $\theta(l_R)$. The notation $l_R$ means the angular momentum of the corresponding rainbow trajectory with rainbow angle $\theta_R$. The outer maximum $\theta_R(C)$ in $\theta(l)$ gives Coulomb rainbow, originating from the coherent contributions of the grazing partial waves.
Figure 6. Coherent sum of farside amplitudes of the partial waves between the lower $l_<$ and the higher $l_>$ from the lower and higher impact parameters resulting in Airy structure.

Figure 7. Deflection of farside and nearside trajectories, respectively, by the nuclear attractive and the Coulomb potentials of the total optical potential. Right: The corresponding construction of deflection function $\theta(l)$ with respect to $l$. The inner minimum in $\theta(l)$ gives the nuclear rainbow at $\theta_R = |\theta(l_R)|$. Illustration is taken from [7] and redrawn with colour.

The Fraunhofer diffraction pattern shows intensity maxima separated by very sharp minima. The Airy structure in the nuclear refractive scattering is marked by several broad oscillations with Airy minima. The refractive scattering occurs at relatively higher energies where the incident ion can penetrate without a complete absorption. At intermediate energies, a coherent mixture of the Fraunhofer diffraction arising from the nuclear surface and refractive scattering from the nuclear interior is produced. At sufficiently higher energies $E_{CM}$ in the CM frame, the target may become adequately transparent leading to a dominant effect of the refractive scattering as the refractive index $n(R) = \sqrt{1 - V(R)/E_{CM}}$ at a distance $R$ from the scattering centre becomes substantially larger than unity ($n(R) > 1$) for a strong attractive potential $V(R)$. This leads to the appearance of the rainbow pattern, identified by ‘exponential-type falloff’ of cross sections following the first Airy minimum (A1), which makes the nuclear rainbow very attractive as an excellent probe of nuclear potential as first observed by Goldberg and Smith [8]. Their stupendous success in describing the nuclear rainbow structure with an exponential falloff in cross sections of the $\alpha + ^{58}\text{Ni}$ elastic scattering at 139 MeV has led to formulate important criteria regarding the removal of ‘discrete ambiguity’ in potential-families [9]. The nuclear rainbow scattering differs from the
atmospheric rainbow in that the former deals with the refractive index dependent on the position in the nuclear interior as opposed to its constancy in the latter.

The criteria concerning the removal of discrete ambiguity, referred to here as the ‘Goldberg criteria’, can be restated as “at sufficiently high projectile energies where pronounced refractive minimum is followed by an exponential-type falloff in the angular distribution, discrete ambiguities are eliminated for the deep monotonic nuclear potential”. The criterion is also confirmed in the work of Bartnitzky et al. [10] on the $^{16}\text{O} + ^{16}\text{O}$ elastic scattering in the energy range of 250 – 704 MeV, where the exponential-type falloff following the first Airy minimum $A_1$ can be seen at 350 MeV and beyond. Their finding using model-independent potentials suggests that heavy-ion (HI) elastic scattering data unambiguously favour deep HI potentials.

1.3. Objective of the present work
The present investigation aims to examine different families [9] of non-monotonic (NM) nucleus-nucleus potentials on the $^{16}\text{O} + ^{16}\text{O}$ elastic scattering in the 75.0 – 350.0 MeV energy range. In particular, the validity of the Goldberg criteria for the NM shallow potential is reported.

2. Optical Model Analysis and Results
2.1. Procedure and results
The experimental cross-section data for the $^{16}\text{O} + ^{16}\text{O}$ elastic scattering at 75.0, 92.4, 124.0, 145.0, 250.0 and 350.0 MeV are taken from [11, 12, 13, 14]. The data are converted from the ratios to the Mott cross section to those normalized to Rutherford scattering to adapt the data for the OM analysis using the code SFRESCO, which incorporates the coupled-channels code FRESCO2.5 [15] and the $\chi^2$-minimization code MINUIT [16]. The data at the individual energies have been analyzed for the best possible fits using the NM real part of the OM potential. The starting NM potential has been obtained from the energy-density functional (EDF) theory of Brueckner, Coon and Dabrowski (BCD) [17] for the incompressibility $K = 230$ MeV of homogeneous nuclear matter. A detailed discussion on the derivation of NM potential from the EDF theory and the sensitivity of the $^{16}\text{O} + ^{16}\text{O}$ elastic scattering data on $K$ are given in Basak et al. [19]. The reason for picking up $K = 230$ MeV is that the $^{16}\text{O} + ^{16}\text{O}$ elastic scattering data have been shown to favour this value out of the discrete values $K = 188, 211, 230, 240$ and 250 MeV, considered in the work of Basak et al. [19].

The nuclear real part of the OM potential is parametrized with the inclusion of the Gaussian repulsive core as

$$V_N(R) = -V_0 \left[ 1 + \exp \left( \frac{R - R_0}{a_0} \right) \right]^{-1} + V_1 \exp \left( -\frac{R - D_1}{R_1} \right)^2. \quad (6)$$

The imaginary part of the $^{16}\text{O} - ^{16}\text{O}$ potential is taken phenomenologically to be composed of volume and surface terms as

$$W(R) = -W_0 \exp \left( -\frac{R}{R_W} \right)^2 - W_S \exp \left( -\frac{R - D_S}{R_S} \right)^2. \quad (7)$$

The Coulomb potential $V_C(R)$ of a uniformly charged sphere with radius $R_C = 5$ fm is added to obtain the total real part of the OM potential.
Table 1. Nuclear optical potential (OP) parameters for the $^{16}$O+$^{16}$O elastic scattering at the incident energies 75.0, 92.4, 124.0, 145.0, 250.0 and 350.0 MeV with $J_{R}/256$ values for the fits. The Coulomb radius is $R_{C} = 5.0$ fm. The parameters of the EDF set corresponds to $K = 230$ MeV. $V_0$ and $V_1$ are in MeV; $R_0$, $R_1$, $a_0$ and $D_1$, in fm; and $J_{R}/256$, in MeV fm$^3$.

$$
\begin{array}{cccccccc}
E_{\text{lab}} & \text{Set} & V_0 & R_0 & a_0 & V_1 & R_1 & D_1 & J_{R}/256 & \chi^2 \\
\hline
\text{EDF} & 38.42 & 5.40 & 0.603 & 123.8 & 2.281 & 0.00 & -79.20 & - \\
1 & 80.0 & 5.795 & 0.575 & 107.0 & 2.110 & 2.825 & -79.93 & 3.2 \\
2 & 55.8 & 5.793 & 0.560 & 110.4 & 1.580 & 3.063 & -32.62 & 5.0 \\
75.0 & 3 & 98.0 & 5.823 & 0.505 & 107.0 & 1.981 & 2.724 & -239.45 & 4.0 \\
4 & 210.0 & 5.405 & 0.528 & 210.2 & 2.042 & 2.724 & -329.45 & 4.0 \\
5 & 207.0 & 5.346 & 0.497 & 139.4 & 1.796 & 2.919 & -340.52 & 3.1 \\
92.4 & 1 & 37.4 & 6.526 & 0.385 & 104.0 & 0.715 & 3.936 & -74.05 & 7.7 \\
2 & 40.0 & 6.443 & 0.395 & 141.8 & 1.150 & 3.219 & -25.15 & 6.2 \\
3 & 50.0 & 6.198 & 0.500 & 105.0 & 0.312 & 4.491 & -149.31 & 6.5 \\
4 & 60.2 & 6.384 & 0.372 & 60.0 & 0.429 & 4.057 & -227.83 & 4.9 \\
5 & 87.9 & 6.248 & 0.375 & 80.1 & 0.402 & 3.812 & -320.00 & 5.2 \\
124.0 & 1 & 84.8 & 5.291 & 0.639 & 135.1 & 0.828 & 4.187 & -62.20 & 17.5 \\
2 & 55.8 & 5.644 & 0.639 & 190.0 & 0.564 & 4.229 & -18.58 & 14.7 \\
3 & 93.0 & 5.260 & 0.646 & 101.5 & 0.766 & 4.368 & -123.30 & 19.2 \\
4 & 134.9 & 4.888 & 0.659 & 180.1 & 0.274 & 4.849 & -203.28 & 18.7 \\
5 & 153.9 & 4.838 & 0.677 & 170.1 & 0.150 & 4.948 & -285.99 & 16.2 \\
145.0 & 1 & 75.0 & 4.964 & 0.790 & 353.0 & 0.290 & 3.854 & -54.97 & 7.9 \\
2 & 43.5 & 5.475 & 0.760 & 396.1 & 0.408 & 3.854 & -109.30 & 6.4 \\
3 & 100.0 & 4.776 & 0.778 & 300.3 & 0.343 & 3.514 & -180.68 & 6.3 \\
4 & 150.0 & 4.434 & 0.756 & 99.7 & 0.180 & 3.376 & -257.66 & 7.3 \\
5 & 188.3 & 5.087 & 0.644 & 190.0 & 1.714 & 3.815 & -15.88 & 16.8 \\
250.0 & 1 & 188.3 & 5.087 & 0.644 & 190.0 & 1.714 & 3.815 & -15.88 & 16.8 \\
2 & 209.0 & 5.060 & 0.660 & 226.8 & 1.815 & 3.605 & +4.80 & 21.5 \\
3 & 224.3 & 5.054 & 0.642 & 205.0 & 1.744 & 3.815 & -46.25 & 19.2 \\
4 & 180.3 & 5.064 & 0.644 & 140.3 & 1.604 & 4.091 & -78.73 & 16.3 \\
5 & 205.0 & 5.087 & 0.644 & 148.0 & 1.637 & 4.092 & -124.82 & 21.5 \\
350.0 & 1 & 151.0 & 5.04 & 0.670 & 162.0 & 1.783 & 3.742 & +20.30 & 3.5 \\
\end{array}
$$

Figure 8. Variation of volume integrals $J_{R}/256$ with $E_{\text{lab}}$ for the potential parameters of set-1, set-2, set-3, set-4 and set-5 listed in Table 1 with the potential-families indicated by the solid lines.
Table 2. Same as in Table 1 for the imaginary part of OP with the corresponding volume integral per nucleon-pair $J_I/256$. $J_I/256$ is in MeV.fm$^3$ and the depth and geometry parameters are, respectively, in MeV and fm.

| $E_{Lab}$ | Set | $W_S$ | $D_S$ | $R_S$ | $W_0$ | $R_W$ | $J_I/256$ | $\chi^2$ |
|----------|-----|------|------|------|------|------|----------|---------|
| 75.0     | 1   | 1.20 | 7.11 | 0.227| 81.4 | 3.40 | -71.06   | 3.2     |
|          | 2   | 0.33 | 7.71 | 1.448| 411.95| 2.75 | -188.46  | 5.0     |
|          | 3   | 1.20 | 7.11 | 0.176| 59.8 | 3.51 | -57.18   | 4.4     |
|          | 4   | 4.79 | 7.18 | 0.056| 92.8 | 3.21 | -67.54   | 4.0     |
|          | 5   | 4.80 | 6.11 | 0.799| 20.0 | 3.56 | -32.17   | 3.1     |
| 92.4     | 1   | 0.40 | 8.95 | 0.300| 187.0| 3.40 | -160.72  | 7.7     |
|          | 2   | 0.27 | 8.77 | 0.463| 160.3| 3.38 | -135.49  | 6.2     |
|          | 3   | 1.60 | 8.90 | 0.046| 396.6| 3.17 | -275.40  | 6.5     |
|          | 4   | 0.34 | 8.92 | 0.371| 150.0| 3.41 | -130.26  | 4.9     |
|          | 5   | 0.33 | 8.91 | 0.375| 141.2| 3.38 | -119.30  | 5.2     |
| 124.0    | 1   | 0.27 | 7.95 | 0.100| 365.2| 3.27 | -277.93  | 17.5    |
|          | 2   | 0.27 | 7.89 | 0.012| 385.4| 3.31 | -304.21  | 14.7    |
|          | 3   | 0.22 | 7.91 | 0.168| 299.9| 3.36 | -247.67  | 19.2    |
|          | 4   | 0.59 | 7.76 | 0.051| 933.7| 3.07 | -589.27  | 18.7    |
|          | 5   | 0.53 | 7.56 | 0.101| 934.0| 3.07 | -586.42  | 16.2    |
| 145.0    | 1   | 0.50 | 7.14 | 0.100| 413.0| 3.12 | -273.09  | 7.9     |
|          | 2   | 1.20 | 7.14 | 0.025| 319.9| 3.19 | -225.93  | 10.6    |
|          | 3   | 0.50 | 7.14 | 0.100| 162.9| 3.52 | -154.78  | 6.4     |
|          | 4   | 0.57 | 7.23 | 0.101| 145.3| 3.58 | -144.98  | 6.3     |
|          | 5   | 0.56 | 7.26 | 0.087| 112.5| 3.70 | -123.79  | 7.3     |
| 250      | 1   | 0.23 | 6.72 | 0.150| 1035.5| 3.01 | -614.13  | 16.8    |
|          | 2   | 0.24 | 6.73 | 0.171| 1000.0| 3.00 | -587.50  | 21.5    |
|          | 3   | 0.23 | 6.80 | 0.150| 900.0 | 3.07 | -566.62  | 19.2    |
|          | 4   | 0.24 | 6.73 | 0.171| 986.7 | 3.02 | -591.43  | 16.3    |
|          | 5   | 0.23 | 6.72 | 0.150| 1500.0| 2.91 | -802.56  | 21.5    |
| 350.0    | 1   | 0.35 | 6.56 | 0.320| 1258.0| 2.98 | -724.60  | 3.5     |

The parameters in Eq. (6) of the EDF-generated NM potential for $K = 230$ MeV is given in Table 1. These parameters of $V_N(R)$ have been adjusted and the corresponding parameters of $W(R)$ have been searched for the best possible fits the data at each of the incident energies. The final fits have been done visually after taking guidance from the $\chi^2$ fits, since it is more important to reproduce the refractive features, e.g., the positions of the Airy minima, rainbow maximum etc, of the angular distributions than naively minimizing only $\chi^2$ [20]. Successive application of grid and global searches for achieving these high quality fits to the experimental data has led to a multiple set of potential-parameters with distinctly different volume integrals per nucleon-pair $J_R/256$ for the real part of OP at each of the incident energies. These best-fit potential parameters for the real and imaginary potentials at each of the incident energies are listed, respectively, in Tables 1 and 2. The plots of estimated $J_R/256$ versus $E_{lab}$ for the set-1, set-2, set-3, set-4 and set-5 potential parameters are displayed in Fig. 8. The sets are labeled by the potential-families which are identified by the smooth variation of $J_R/256$ values on the same curve per family.

The set-1 parameters correspond to the family of the EDF-derived potentials because (i) at the lowest incident energy $E_{lab} = 75.0$ MeV, the estimated $J_R/256 = -79.93$ MeV.fm$^3$ closely agrees with the EDF value of $J_R/256 = -79.20$ MeV.fm$^3$ and (ii) the latter volume integral for the EDF potential parameters is generated for the nucleonic mean-field i.e. average potential energy per nucleon in homogeneous nuclear
Figure 9. Comparative fits to the experimental data at (a) 75, 93.4, 124, 145, 250 and 350 MeV using the set-1 (solid lines) and set-2 (long-dash lines) potential parameters, and (b) 75, 93.4, 124, 145 and 250 MeV using set-3 (broken lines), set-4 (solid lines) and set-5 (long-dash lines) potential parameters, all listed in Tables 1 and 2.

matter with $K = 230$ MeV. [19]. The OM predictions for the different sets of potential parameters are compared to the experimental data in Fig. 9.

3. Discussion and Conclusion
This presentation reports the results of the OM analysis of the the $^{16}$O+$^{16}$O elastic scattering data at 6 points in the energy range $E_{lab} = 75.0 – 350$ MeV using the NM potentials in 5 discrete potential families. The set-1 belongs to the potential family with its origin from the EDF theory of BCD [13] which generates the potential corresponding to the ground state of the composite nucleus as discussed in [19]. The study reveals for the first time the consecutive potential families through an excellent description of quality experimental data spanning over wide angular ranges. The families involve the shallow potential with $J_{R}/256 = –25.1$ MeV.fm$^3$ for set-2 to the deep potential with $J_{R}/256 = –323.0$ MeV.fm$^3$ for set-5 at $E_{lab} = 92.4$ MeV. As one can note in Fig. 8 that gaps in the $J_{R}/256$-values between the consecutive families decrease with the projectile energy and converge to a unique value of $J_{R}/256 = +21.5$ MeV.fm$^3$ at $E_{lab} = 350$ MeV.

The experimental data in the $75.0 – 350.0$ MeV, considered herein, involve refractive scattering as
observed by Michel et al. [21] but the principal Airy minimum A1 followed by the exponential-type falloff in the angular distribution showing a dominant effect of the nuclear rainbow structure becomes prominent at 350 MeV [10, 14]. The clear indication of the convergence of the potential families at 350 MeV in Fig. 8 conforms to the Goldberg criterion concerning the removal of discrete ambiguities even for the shallow NM potential. However, the requirement of a deep attractive real part of the nuclear potential is found to be ‘non-stringent’ for NM potentials confirming the findings of Hossain et al. [22] in the OM analysis of the $\alpha^{+}\text{Zr}$ elastic scattering data.

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