Lens-term- and edge-effect in X-ray grating interferometry

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Abstract: X-ray grating interferometry requires gratings with periods in the micrometer range and allows the acquisition of the dark-field contrast. The analyzer grating is designed to match the period of the interference pattern in order to translate it into a measurable intensity modulation. In this study, we explore the influence of a sample-induced mismatch between the interference pattern and the analyzer grating on the dark-field contrast. We propose a formula for the calculation of the signal due to a period mismatch and present estimations varying periods and detector pixel size. Furthermore, numerical simulations of the X-ray wave-front demonstrate that the wave-front curvature, described by the lens-term, e.g. behind a parabolic lens or edges of a sample can change the period of the interference pattern. Our results give a concrete explanation for the formation of a dark-field contrast from object edges and thus allow a better understanding of the dark-field signal obtained with a grating interferometer.

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Introduction

In recent years, imaging based on X-ray grating interferometry (XGI) has been an active field of research. The introduction of a Talbot-Lau interferometer into an otherwise standard setup has proven to enrich the information obtainable with X-ray imaging [1–3]. Besides the conventional X-ray absorption, XGI allows the extraction of two additional contrasts that rely on the wave characteristics of the radiation. First, the differential-phase contrast (DPC) represents a measure for the refraction of the wave. DPC is proportional to the electron density of the material and may permit a better discrimination between e.g. different soft tissues [4, 5]. The second additional contrast is the so-called dark-field contrast (DFC), which quantifies the visibility of the interference pattern. The interference pattern can be smeared out by ultra-small-angle scattering from structures that have a size of only a few micrometers [6–10]. Directly resolving these structures is either not possible as they are generally smaller than the resolution limit of conventional detectors, or it would lead to typical problems like the need for small detector pixels, very long scanning times, high photon flux, a high noise level etc. Dark-field imaging avoids very long scanning times, high photon flux, a high noise level etc. Dark-field imaging avoids...
these problems and yet gives access to quantitative information about micron-sized scattering structures in a large sample.

First studies in biomedical imaging have already been reported to show very promising results. For example, lung tissue with its numerous air-tissue interfaces of the alveoli scatters strongly. Thus, a reduction of the DFC signal may allow not only the diagnosis but also the grading of certain lung diseases that change the alveolar structure [11–13]. Furthermore, detection of a DFC signal in the breast tissue may hint at the presence of microcalcifications, an indicator for breast cancer [14], and enable an earlier diagnosis. Even the distinction of different kidney stone types was demonstrated and may provide a clinical value for the radiologists [15]. And also other fields like e.g. materials science [16,17] could potentially benefit from DFC imaging.

In most DFC applications it is important to get a quantitative DFC signal in order to draw conclusions. The respective signal that is of interest in the given examples originates from small-angle scattering by structures in the size of micrometers. However, not every detected DFC signal can be explained by the presence of small scatterers. Also other influences can lead to a DFC signal when using XGI, namely beam hardening [18], the second derivative of the object [19], and unresolvable sharp edges [20]. Thus, overlaying structures such as the ribcage surrounding the lung or the ducts in breast tissue can contribute to the DFC signal and hinder a clear analysis. In order to obtain correct quantitative values it is important to know and understand all possible sources of a DFC signal.

The objective of this work is to elucidate the origin of the DFC signal from a curved wavefront and behind edges of an object. The local curvature of the wave-front can be described by the so-called lens-term, which is the second-order term of the irradiance transport equation [21]:

$$\nabla_{xy} I \cdot \nabla_{xy} \Phi + I \nabla^2_{xy} \Phi + \frac{\partial I}{\partial z} = 0. \tag{1}$$

The equation defines the intensity variation due to the tilt and curvature of the wave-front, with \(\nabla_{xy}\) indicating the gradient in the directions perpendicular to the beam direction \(z\) and \(I\) is the intensity of the beam. \(\Phi\) is the phase of the wave-front. The lens-term has a nonzero value for a converging or diverging beam [21]. In this study we focus on the effect of a nonzero lens-term on the DFC signal measured with XGI. We present an explanation based on the changes in the period of the interference pattern and with respect to the way of signal acquisition with an XGI setup.

2. X-ray grating interferometry

The phase shift imprinted on an X-ray wave passing through an object can be measured by different methods [22], of which XGI is the most promising with regard to its requirements for beam coherence and detector resolution, and its potential scalability. A further advantage of this technique is, as mentioned above, the simultaneous detection of three signals, including the DFC signal.

The basic principle of imaging with XGI is to introduce a periodic interference pattern on the wave-front and measure this pattern without sample - flat-field or reference scan - and with sample in the beam. The three image modalities are extracted by comparing both recorded patterns. Figure 1 shows a Talbot-Lau interferometer consisting of three gratings. The bars of the phase grating (G1) typically shift the phase of the wave by a value of \(\pi\) or \(\pi/2\). For sufficiently coherent radiation, such a phase grating creates stripes of high and low intensity at certain distances downstream (fractional Talbot distances) [23]. To achieve the required degree of spatial coherence with a standard laboratory X-ray source, an absorption grating (G0, "source grating") is placed near the source [3]. As the periods of gratings and interference pattern are in
Fig. 1. Sketch of an X-ray grating interferometry (XGI) imaging setup with a three-grating Talbot-Lau interferometer. While G0 enables the use of a standard X-ray source by increasing the spatial coherence, a defined phase-shift introduced by G1 creates a periodic interference pattern at certain distances downstream. Generally, the period of this interference pattern is much smaller than the size of the detector pixels and therefore cannot be resolved directly. The analyzer grating G2 has typically a period matching the interference pattern. It is moved in a number of equidistant steps over one period of G2 perpendicular to the grating bars. For each step the intensity at the detector is recorded and, thus, the pattern is translated into a intensity curve ("stepping curve") in each detector pixel.

The signal extraction from the stepping curves can be performed using Fourier analysis. The transmission signal $T$ is derived as

$$T = \frac{a_i^r}{a_i^s},$$

and the differential phase signal $P$ as

$$P = \arg \left( \frac{a_i^1}{a_i^0} \right),$$

where $a_i$ is the $i$th Fourier component and the superscripts $s$ and $r$ correspond to the sample and the reference scan, respectively. Finally, the dark-field signal $D$ is calculated as

$$D = \frac{V^s}{V^r} = \frac{|a_i^r||a_i^0|}{|a_i^0||a_i^1|},$$

which sets in relation the visibility of the interference pattern $V = (I_{\text{max}} - I_{\text{min}})/(I_{\text{max}} + I_{\text{min}})$ in the scans acquired with and without sample. Thus, a DFC value of unity indicates no signal, while a value of zero means a strong signal.
Fig. 2. Effect of period mismatch between interference pattern and G2 on the stepping curve. In XGI the use of an analyzer grating relies on the fact that the grating period (black bars) matches the period of the interference pattern (light gray bars). Then in an optimal case (a), there is a step of maximum intensity (step 0) and a step of minimum intensity (step 4, remaining intensity depends on blocking capacity and height of grating material). This leads to a maximum visibility value (blue stepping curve plotted over two grating periods (c)). If the periods of the grating and the interference pattern do not match (b), no perfect overlap is possible and the visibility is reduced (red curve). (Note: For cases with detector pixels covering only very few periods even an increased visibility is possible.)

The visibility can be reduced for example by small scatterers, which otherwise cannot be resolved by the detector. The scattered photons smear out the interference pattern and, consequently, lead to a reduction of the amplitude of the stepping curve [9]. Thus, the DFC allows to conclude on the microstructure or even the material of a sample [11, 15]. A further origin for a DFC signal is the filtering of the X-ray spectrum by a strongly absorbing sample. During such a scan the mean energy of the filtered spectrum differs from the mean energy during the flat-field scan. Due to the energy-dependence of the visibility of the interference pattern [18], the stepping curve may change even without the presence of scattering structures, which results in a DFC signal.

3. Effect of mismatch between interference pattern and grating

Imaging with XGI requires that the periods of interference pattern and G2 are equal so that the highest amplitude of the stepping curve can be achieved. If the two periods do not match then the amplitude of the stepping curve is reduced. This behavior is illustrated in Fig. 2, which shows the interference pattern and G2 in one detector pixel for different steps of G2. The presented exemplary case visualizes a binary interference pattern, a perfect grating, and the resulting stepping curves. Only if the periods match, stepping positions of G2 exist in which the grating bars either block or let pass the maxima of the interference pattern. These positions mark the points of maximum and minimum intensity of the stepping curve. In case of a mismatch of periods, these positions of perfect overlap do not exist. Therefore, the maximum measured intensity decreases and the minimum intensity is augmented, which leads to a reduced amplitude of the stepping curve and a reduced visibility value. The mismatch of two periods $p_1$ and $p_2$ leads to the creation of a superimposed moiré pattern, which has a period of [24]

$$p_{\text{moire}} = \frac{p_1 p_2}{p_2 - p_1}.$$
For equal periods $p_1$ and $p_2$ the period of the moiré pattern becomes infinite (Fig. 2(a)). In Fig. 2(b) the moiré pattern is visible in the form of an overlaying frequency in the graphic representations of the grating steps. The position of the moiré pattern relative to the pixel is shifted by the stepping of G2.

The effect of a mismatch between periods of a binary interference pattern and G2 on the intensity recorded in one detector pixel can be calculated by the following equations. The relevant parameters are defined in Fig. 3. The intensity for one step in one pixel is described by

$$I_{\text{pixel}} = i_{\text{factor}} \sum_{n=-1}^{n_{\text{max}}} s_n \quad \text{with } n \text{ odd},$$

(6)

where $s_n$ is the width of a pattern maximum that is not blocked by a grating bar and $i_{\text{factor}}$ is a conversion factor to calculate an intensity from the width. $n$ enumerates the pattern maxima as shown in Fig. 3 with

$$n_{\text{max}} = \frac{2(w_{\text{pix}} - l_{\text{pix}})}{p_1},$$

(7)

where $w_{\text{pix}}$ is the width of the detector pixel and $l_{\text{pix}}$ denotes the distance between the first pattern minimum and the pixel border. $p_1$ stands for the smaller period, in this case of the interference pattern. Discussion of the case where $p_1$ is the larger period can be found in the appendix. $s_n$ can be calculated as

$$s_n = \left| \frac{p_1}{2} - d_n \right| - \frac{p_2 - p_1}{2} m \quad \text{with } m = \begin{cases} 0 & \text{if } d_n \leq \frac{p_1}{2} \\ 1 & \text{if } d_n > \frac{p_1}{2} \end{cases},$$

(8)

where $p_2$ is the larger period and

$$d_n = \left\lfloor (n - c) \frac{p_2 - p_1}{2} \right\rfloor + l_{\text{offset}} \mod p_2,$$

(9)

with

$$l_{\text{offset}} = \begin{cases} (i_{\text{step}} \frac{p_2}{N_{\text{steps}}} + l_{\text{diff}}) \mod p_2 & \text{initially}, \\
 s_{\hat{n}} - \frac{p_1}{2} & \text{for } n > \hat{n} \
 s_{\hat{n}} > \frac{p_1}{2} \land \hat{n} \leq \frac{p_1}{2},
\end{cases}$$

(10)
Fig. 4. Effect of period mismatch between interference pattern and G2 on the dark-field contrast (DFC). (a) shows calculations based on Eq. (6), where the period of G2 was set to 2.4 μm and the period of the interference pattern was varied. Both, smaller and larger periods of the interference pattern lead to decreased visibility and consequently a DFC signal, relative to the case of matching periods. The influence of the detector pixel size is shown in (b) for the specific case with periods of 2.37 and 2.4 μm. Both, in (a) and (b), points of zero visibility occur when the resulting moiré pattern period has the same size as the detector pixel or a multiple thereof. The error bars indicate the range of possible DFC values for variation of the positions of interference pattern and grating.

and

\[ c = \begin{cases} 
0 & \text{initially,} \\
\hat{n} + 1 & \text{for } n > \hat{n} \text{ if } s_{\hat{n}} > \frac{p_{\text{diff}}}{2} \land s_{\hat{n}} \leq \frac{p_{\text{pix}}}{2}.
\end{cases} \]  

(11)

Here, \( N_{\text{steps}} \) is the number of grating steps used to sample the interference pattern in order to obtain the stepping curve, and \( \hat{l}_{\text{step}} \) is the index of the current step. \( \hat{l}_{\text{offset}} \) describes the offset of the interference pattern and the grating for \( n=0 \) and is equal to \( l_{\text{diff}} \) for the zeroth grating step (Fig. 3). The exact treatment of the pixel borders is given in the appendix.

Based on Eq. (6) stepping curves can be calculated for different periods. To analyze the influence of a change of the interference pattern period, the stepping curves were evaluated for matching periods in the reference scan and a mismatch of periods in the object scan. Based on these curves a DFC signal was calculated.

The DFC signal depends on the pattern mismatch and the pixel size (Fig. 4). In the given example the grating period was kept constant at 2.4 μm and a change of the interference pattern period in the object scan was assumed. As can be seen in Fig. 4(a) a larger difference between the periods leads to a stronger signal. This is true until the moiré period coincides exactly with the size of the pixel. In this case the intensity recorded by the detector remains constant, independent of the grating position. This leads to a stepping curve amplitude of zero and consequently to a zero DFC signal. For even larger differences in the periods the signal generally becomes stronger and reaches zero for every multiple of \( p_{\text{moire}} \). The error bars visualize the possible maximum and minimum DFC values when \( l_{\text{pix}} \) and \( l_{\text{diff}} \) of the object scan as well as \( l_{\text{pix}} \) of the flat-field are varied.

In a further simulation both periods were kept constant but the pixel size was varied (Fig. 4(b)). The resulting DFC signal strongly depends on the size of the detector pixel. The plot shows an example for an interference pattern of 2.37 μm period and a 2.4 μm G2 period. Larger pixel sizes cause a stronger dark-field signal, until the pixel size is equal to the moiré period, where the DFC value is zero. The random absolute position of the pattern within the pixel in reference or object scan and the undefined relative position of pattern and grating in the object scan cause a range of possible stepping curves and, therefore, DFC values. These are indicated by the error bars. The variation is stronger for smaller pixel sizes because one pixel averages over fewer periods and the influence of the pixel margins is more prominent. A notable result is that for very small detector pixels even a DFC signal greater than unity is possible.
Fig. 5. Simulations for a parabolic beryllium X-ray lens with a radius of 50 µm [25]. Panel (a) presents a line plot of the wave-front intensity at the detector. The wave-front shows the characteristic interference pattern of high and low intensity (b), which is visible at fractional Talbot distances. In the simulated configuration the interference has a period of 2.4 µm outside of the lens. The lens causes strong distortions at its edges (b). The parabolic shape of the lens leads to a constant compression of the interference pattern. The distances between the maxima (marked by red triangles in (a)) behind the lens therefore measure only 2.37 µm, as shown in (c). Binning of the wave-front into detector pixels of 50 µm size (indicated by dashed lines) leads to an almost constant DFC signal behind the lens (d).

4. Period mismatch caused by objects

A period mismatch can occur if the period of G2 is not chosen correctly for the magnification factor of the setup. In this case, the visibility is reduced in both, reference and object scan and would not necessarily lead to a DFC signal. However, also the curvature of a sample can cause a nonzero value of the lens-term, i.e. a converging or diverging beam leading to a compression or augmentation of the interference pattern period. Then, the two scans have different visibilities due to the effect described above. In the following sections we present two cases, in which the shape of the sample causes a DFC signal, without the presence of micro-scatterers or beam hardening.

4.1. Effect of X-ray lenses on the interference pattern

Parabolic refractive X-ray lenses are used as optical elements to focus X-rays [25]. Their parabolic shape leads to a homogeneous focusing over the whole aperture. The lenses are able to change the interference pattern such that the period is still constant but decreased. Consequently, when measuring a lens as a sample with XGI the period of G2 will match the pattern in the reference scan but not in the object scan. This causes a DFC signal over the area covered by the lens.

Figure 5 shows the simulation of a wave-front behind a parabolic Beryllium lens with a curvature radius \( R \) of 50 µm for a 23 keV monochromatic beam. The focal length of such a lens can be calculated as [25]

\[
f = \frac{R}{2\delta},
\]
where \( \delta \) is the decrement of the real part of the refractive index \( n = 1 - \delta + i\beta \). For the presented case, the focal length is 38.8 m. The wave-optical simulation was carried out with the pyXSFW framework [9] using Fresnel propagation [26]. G1 was chosen to introduce a phase shift of \( \pi \) and to have a period of 4.8 \( \mu \)m. At a distance of 48.08 cm (9th fractional Talbot distance) this leads to an interference pattern with a period of 2.4 \( \mu \)m, equal to the period of G2. The wave-front spacing was chosen to sample the period with 1000 points. The detector pixels had a size of 50 \( \mu \)m. Furthermore, Gaussian smoothing (sigma 0.48 \( \mu \)m) was applied to account for grating imperfections.

Figure 5 shows the simulated intensity, interference pattern period, and DFC signal for this case. The parabolic sample can be recognized by the shape of the wave-front (Fig. 5(a)). The lens edges cause a strong disturbance of the periodic sinusoidal shape of the interference pattern (Fig. 5(b)). An analysis of the distances between the maxima of the wave-front (red triangles) reveals that behind the curvature of the lens the interference pattern period is decreased from 2.4 \( \mu \)m to approx. 2.37 \( \mu \)m (Fig. 5(c)). This period is in agreement with the formula [24]

\[
p = \frac{p_1 f - d}{2f},
\]

where \( d \) denotes the distance between G1 and G2. Binning of the wave-front into detector pixels of 50 \( \mu \)m as indicated by the vertical dashed lines results in a nearly constant DFC signal over the whole width of the lens (Fig. 5(d)). In the two pixels showing the strongest DFC signal (at positions -200 and 200 \( \mu \)m) the effects of the pattern disturbance and the decreased period are combined. Comparison to Fig. 4 shows that the formula is in good accordance with the simulation, despite the assumption of a perfectly rectangular interference pattern.

4.2. Signal from edges

Focusing of X-rays cannot only be observed for parabolic objects but also for objects with a strong curvature like e.g. at the margins of a cylinder. In contrast to the lens a cylinder does not cause a constant nonzero lens-term, i.e. no constant change of the interference pattern period, over the whole width of the object. The period of the pattern is only noticeably changed at the edges and is less affected near the cylinder axis. Yang et al. [19] describe the formation of a DFC signal behind edges based on the second derivative of a sample. Here, we demonstrate the underlying mechanism of the signal creation, which is specific to the grating interferometry setup.

Figure 6 shows a comparison between the experimentally measured (Fig. 6(a)) and the simulated (Fig. 6(b)) DFC signal from the right edge of an epoxy cylinder with a diameter of 6 mm oriented parallel to the grating bars. In this configuration the interferometer is most sensitive to the phase shifts created at the edge. The measurement was conducted with a tungsten X-ray source at a voltage of 60 kV, thus resulting in a mean energy of 38.7 keV. The setup consisted of a \( \frac{\pi}{2} \) phase-shifting G1 grating with 5 \( \mu \)m period and a G2 grating with a period of 10 \( \mu \)m, i.e. the magnification factor of the setup was 2. The sample was placed 66.2 cm in front of G1, and the distance between G1 and G2 was 92.5 cm. The G0 grating had a period of 10 \( \mu \)m and the projected pixel size was 18.1 \( \mu \)m. The experimental measurements were confirmed by simulations as described above. The system point spread function (PSF) was modeled by Gaussian smoothing with a sigma of 3.5 times the pixel length. The noise-free simulation looks very similar to the result of the measurement (Fig. 6(b)). The signal profile shown in a line plot in Fig. 6(c) (blue line) is a mean over image (a) and is well reproduced by the simulation, apart from a small error presumably introduced by beam hardening. A monochromatic simulation with an energy of 38.73 keV (red line) gives strong evidence that the difference in the area of the cylinder (left side) is indeed caused by beam hardening, which is only noticeable in the perfect setup.
Fig. 6. DFC signal from the edge of an epoxy cylinder. The cylinder covers only the left part of the image. Experimental measurement (a) and simulation (b) demonstrate that a DFC signal can be detected over several pixels at the edge of the cylinder. As reported in [20] the signal decreases slower towards the center of the object than at the other side of the edge (c, arrows). The signal larger than unity in the area behind the cylinder in the simulation with polychromatic radiation is most likely caused by beam hardening, as simulations with monochromatic radiation and a purely phase-shifting cylinder do not show this behavior. Simulations of the wave-front reveal that two effects are responsible for the signal: the strong disturbance of the pattern directly behind the edge (d), and the increased period of the interference pattern due to the curvature of the cylinder acting as a diverging lens for the X-rays (e). The point spread function smears the signal out over a few pixels.
of the simulation. Moreover, if a purely phase-shifting and non-absorbing cylinder material is used in a polychromatic simulation the DFC value stays smaller than unity. The signal plot is in accordance to the one described by Yashiro et al. [20] and shows a signal at the edge and slower decrease of the signal towards the center of the cylinder than to the outside.

With the wave-optical simulations of the wave-front it can be shown that the interference pattern directly behind the projected edge of the cylinder is disturbed, which leads to a strong DFC signal at this position (Fig. 6(d)). Outside of the object, the period is restored directly next to the disturbance region (Fig. 6(e), right side). However, near the edge of the cylinder the interference pattern period is increased because the curvature of the cylinder is strong enough to act as a diverging lens for the X-rays. The period gradually goes back to the normal value towards the cylinder axis. The system PSF smears the signal over a few pixels left and right of the projected edge. The two effects - the strong disturbance and the increased period of the pattern - together lead to the characteristic shape of the DFC signal near rounded edges.

5. Summary and discussion

X-ray DFC imaging is a recently developed imaging modality that is promising for applications in different fields. Most interesting is its sensitivity to scattering from microstructures, which are below the resolution of commonly used detectors. In some applications, e.g. the detection of microcalcifications in mammography, only qualitative information about the DFC signal may already be sufficient [14]. However, for other purposes such as lung imaging or materials science quantitative data may be of greater importance [11, 12]. Therefore, it is necessary to understand all sources that contribute to the obtained DFC signal.

Different effects have been reported to generate a DFC signal even in the absence of micron-scaled scattering structures, namely beam hardening [18], the second derivative of an object [19], and unresolvable sharp edges [20]. Here, we have shown examples for all three effects. The polychromatic simulation of an epoxy cylinder reveals a small DFC signal over the area of its projection. This signal is most likely caused by beam hardening as it is not visible for a monochromatic simulation or a simulation of a purely phase-shifting cylinder. The numerical investigation of a parabolic lens demonstrated the (almost) constant dark-field signal due to the constant lens-term of the focused beam. Furthermore, the DFC signal from the edge of a cylinder showed a good consistency between results of measurement and simulation.

Additionally, in this study, we presented another effect leading to the formation of a DFC signal that is related to the XGI imaging system and the acquisition mechanism. Yang et al. [19] and Yashiro et al. [20] both develop general formulae to account for the DFC signal from the second-order phase-shift or behind unresolvable sharp edges, respectively. Both publications offer analytic descriptions of the separated problems. This study, however, provides a unifying explanation for both cases by taking into account the scan process and geometric parameters of the imaging setup. A mismatch between the period of interference pattern and analyzer grating caused by the focusing capacities of a sample reduce the visibility in the object scan and lead to a DFC signal. To our best knowledge this effect has not been reported so far and we believe it provides an accurate characterization for the influence of a nonzero lens-term as well as object edges on the DFC signal.

We started with a theoretical, qualitative description of the impact of a mismatch between the interference pattern and the analyzer grating. In order to investigate the effect quantitatively we proposed a formula that calculates the intensity in a detector pixel based on the part of the interference maxima that is not blocked by the analyzer grating. The presented formula allows a time-efficient test of a large number of parameters. Thus, it allowed us to examine the theoretical behavior of the DFC signal as a function of period mismatch and detector pixel size. Although the formula involves certain assumptions like perfectly rectangular grating and...
pattern, constant periods, and perfectly sharp pixel edges, the results showed good agreement with the simulations that included imperfections modeled by Gaussian smoothing.

We showed that for a perfectly boxlike PSF changes of the absolute and relative position of grating and interference pattern in the detector pixel and to each other lead to a range of possible DFC values. This range calculated by the formula will not be reproducible in a real XGI setup, as it becomes smaller if a Gaussian PSF smears out the signal. However, the variance of the signal due to the position shifts reduces the predictability of the DFC signal to a degree that makes e.g. metrology of lenses impractical. X-ray lenses typically have diameters of hundreds of micrometers and, consequently, quality control would require small detector pixel sizes of a few microns. In this range the strong variation of the DFC signal would make conclusions about the lens shape very difficult.

The simulations of an epoxy cylinder revealed that the DFC signal at the projected edge originates from two effects. In the pixel directly downstream the edge a strong disturbance of the periodicity of the interference pattern is visible. This can most likely be explained by the influence of Fresnel fringes, which are intensity modulations that occur behind object edges due to the propagation of the wave-front. The second effect is based on a period mismatch. The strong curvature near the edge leads to an increased period of the interference pattern. In the presented example the diameter of the cylinder is relatively large so that only few pixels are affected. The system PSF, however, smears the signal over a larger area and leads to the characteristic slower signal decrease towards the cylinder center. The simulation tool was verified with a good agreement with the theory [9]. However, direct comparison between the simulations and measurements of period changes in the interference pattern, unfortunately, is not possible due to the lack of a detector with a sufficiently fine resolution in the nanometer range.

Most applications exploiting the DFC signal aim at the detection of microstructures. Truly quantitative DFC values origination from microstructures can be difficult to obtain if e.g. object edges overlay these structures. Due to the strong dependence on the detector pixel size it may be possible to estimate the signal component caused by a period mismatch by binning the image, as reported in [19]. Thus, the quantitative accuracy of the measurement might be improved in some cases. For cases, in which bone overlays the structure of interest, an attribution of the signal to e.g. the lung and the ribcage may be difficult because also osseous structures scatter X-rays. For the distinction between ducts, glandular tissue and microcalcifications in breast tissue this approach may be more beneficial.

6. Conclusion

Through calculations and simulations we showed that a mismatch between the periods of the interference pattern and the analyzer grating causes a visibility reduction. Additional to scattering from microstructures and beam hardening, a period mismatch can thus be the origin of a DFC signal. Furthermore, we demonstrated that a nonzero lens-term, i.e. a locally converging or diverging wave-front, induced by an object or the edges of samples can influence the period of the interference pattern. We thereby propose a coherent explanation for the DFC signal behind object edges and from a curved wave-front.

Appendix

The calculation of the range of possible DFC values with Eq. (6) requires the variation of the parameters defining the absolute and relative position of interference pattern and G2 grating. Therefore, the margins of the evaluated pixel have to be considered separately. Whenever a value marked with asterisk (*) is calculated, it has to be used in the further context instead of the value without.

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1. Left margin \((n = -1)\):

\(l_{\text{pix}}\) is defined in the half-open interval \(]-\frac{p_1}{2}, \frac{p_1}{2}\]. If \(l_{\text{pix}} > 0\) and \(s_{-1} > 0\), then the following cases have to be considered:

\[
s_{-1}^* = \begin{cases} 
  l_{\text{pix}}, & \text{if } d_{-1} \leq \frac{p_1}{2} \text{ and } s_{-1} > l_{\text{pix}}, \\
  s_{-1} - (\frac{p_1}{2} - l_{\text{pix}}), & \text{if } d_{-1} > \frac{p_1}{2}.
\end{cases}
\] (14)

2. Right margin \((n = n_{\text{max}})\):

For \(n_{\text{max}}\) the portion of last interference maximum that still lies within the pixel can be calculated as

\[
r_{n_{\text{max}}} = (\left(\text{w}_{\text{pix}} - l_{\text{pix}}\right) \mod \frac{p_1}{2})/\frac{p_1}{2}.
\] (15)

If \(s_{n_{\text{max}}} > 0\), then the following cases have to be considered:

\[
s_{n_{\text{max}}}^* = \begin{cases} 
  s_{n_{\text{max}}} - \frac{p_1}{2} (1 - r_{n_{\text{max}}}), & \text{if } d_{n_{\text{max}}} \leq \frac{p_1}{2}, \\
  \frac{p_1}{2} r_{n_{\text{max}}}, & \text{if } d_{n_{\text{max}}} > \frac{p_1}{2} \text{ and } s_{n_{\text{max}}} > \frac{p_1}{2} r_{n_{\text{max}}}.
\end{cases}
\] (16)

Generally, as no negative length is possible, \(s_i \geq 0\). In case \(s_i < 0\), it is intrinsically set to \(s_i^* = 0\). All regarded cases assume that \(G2\) is stepped. Furthermore, so far the interference pattern period has been considered to be smaller than the period of \(G2\). If the period of \(G2\) is chosen to be smaller, the general calculation using Eqs. (6)-(11) remains unchanged. Only the initial values have to be adapted for \(i\)th grating step:

\[
l_{\text{pix},i} = l_{\text{pix}} + i_{\text{step}} \frac{p_1}{N_{\text{steps}}},
\] (17)

If \(l_{\text{pix},i} > \frac{p_2}{2}\), then

\[
l_{\text{pix},i}^* = l_{\text{pix},i} - p_1,
\] (18)

\[
l_{n_{\text{pix}}}^* = l_{\text{pix}} - p_1,
\] (19)

and

\[
l_{\text{diff},i}^* = l_{\text{diff},i} - (p_2 - p_1).
\] (20)

Furthermore,

\[
l_{\text{diff},i} = (l_{\text{diff}} - i_{\text{step}} \frac{p_1}{N_{\text{steps}}}) \mod p_2,
\] (21)

and

\[
l_{\text{offset},i} = l_{\text{diff},i}.
\] (22)

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