Probabilistic thinking level of junior high school 2 Rembang students who have not formally learnt probability material

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Abstract. This study aimed to describe the level of probabilistic thinking of students with high mathematics abilities and have not formally learned about probability material in Junior High School of 2 Rembang. The research subjects were 2 seventh graders with high mathematics abilities. This research was qualitative research that employed the case study. The main instrument of the research was the researcher and the supporting instruments were the task in the form of probabilistic problems and interview guidelines. The data had been validated using the triangulation of the method. The results showed that the level of probabilistic thinking of junior high school students with high mathematics abilities who have not formally learned the probability material are as follows: (a) In the construct of sample space, the students were at the level of informal quantitative level (level 3) with the characteristics of being able to list the results of stage two experiment using part of the generative strategy. (b) In the construct of experimental abilities of an event, the students were at the numerical level (level 4) with the characteristics of being able to collect appropriate data to determine the numerical value of the experimental probabilities. (c) In the construct of probabilities comparison, students were at the numerical level (level 4) with the characteristics of being able to determine the size numerically and compare the probabilities.

Keywords: High Mathematics Abilities, Probabilistic Thinking, Probabilities.

1. Introduction

Everyone will certainly face several situations in life. One of which is the situation that will occur in the future. It is a situation that contains an element of uncertainty. The situation that involves uncertainty refers to a random activity or experiment that contains some probable results that cannot be determined accurately in advance. [1] The situation involved uncertainty is called a probabilistic situation. [2] Many aspects of life are related to probabilistic situations, for example, predicting the size of the success of a
treatment method, predicting the weather condition, predicting the result of school entrance exams, and so on.

Thinking in a probabilistic situation is called probabilistic thinking. A mental activity related to a context containing the element of uncertainty is called probabilistic thinking [1,17,18,19]. If the probabilistic thinking is a person's mental activity which contains elements of uncertainty, the probability is a branch of mathematics that studies the problems containing uncertain elements [22]. Probabilistic thinking will be used to describe children's thinking in response to various kinds of probabilistic situations [1]. Probabilistic thinking in this research is a way for students to process information to respond to various situations in a context that contains elements of uncertainty.

Probabilistic thinking involves informal and formal knowledge of probabilities [8]. The informal knowledge of probabilities is students' knowledge built-in non-academic settings while the formal knowledge of probability is students' knowledge built-in academic settings. Students' informal knowledge is influenced by the culture which includes language, beliefs (religion), and experiences (example: games) [9,10]. This assumes that students who have not formally learned about probabilities already could think probabilistically. In fact, in everyday games such as dice games, rock paper scissor games, and so on. The terms of probability such as possible and impossible must be known to students before they learned about probability.

Students' have different levels of thinking skills in answering probabilistic problems [11]. This is known as the probabilistic level [2]. The probabilistic level of thinking describes the level of thinking of students in responding to various probabilistic problems. There are 4 levels of probabilistic thinking. The first level is level 1 (subjective) where students' thinking is tied to subjective reasons. The second level is level 2 (transitional). It is the transition period between subjective thinking and quantitative thinking characterized by naive students' thinking and often changes in quantifying probabilities. The third level is level 3 (informal quantitative) where students use generative strategies in applying the results of 2-stage experiments and have the abilities to align and quantifies their thoughts on the sample space of probabilities. The fourth level is level 4 (numerical) where students can make appropriate relationships about the sample space and its probabilities and can use numerical measures appropriately to describe the probabilities of an event [1,12].

The research results of [13] showed that students around the age of 9 had basic probabilities concepts and were more likely to respond to lessons that help them to develop simple numerical strategies into proportional thinking. Based on the results of research [1], third graders of elementary school who had been formally taught about probability material in advance with the topic of the sample space, the probabilities of an event, the comparison of probabilities, the conditional probabilities, and the free probabilities used probabilities assignments in the context of selecting class leaders. These students were at levels 1 to 4. Meanwhile, the results of the research [20] of kindergarten students in Athens public schools had obtained a transitional level (level 2) in response to probabilities assignments. The results of the research [21] showed that kindergarten students who had not formally learned about probabilities in advance were successful in solving problems related to several probabilities concepts. According to the results of the research [19], there was a difference between the fifth graders with high-abilities and low-abilities in responding to probabilities assignments. The results of the study [2] showed that 237 eighth-graders of high school students aged 13-15 years were at the subjective and transitional levels (level 1 and 2). After being taught about the probabilities level, some students improved to the numerical level (level 4). Also, the results of research [25] in eleventh-graders of Senior High School 9 Bengkulu City showed that 3 students were at the subjective level (level 1), 16 students who were at the transitional level (level 2), 4 students who were at the informal quantitative level (level 3), and none of them were at the numerical thinking level (level 4).
The level of probabilistic thinking of junior high school students who had not formally learned about probabilities was different from the level of probabilistic thinking of junior high school students who had formally learned about probabilities. This happened since the culture, language, beliefs, daily experiences, and school (games on probabilities theory) influenced their probabilistic knowledge and probabilistic thinking [1,11,15]. This research had been conducted on students who had not formally learned about probabilities by giving a structured activity to study material related to probabilities. This research was focused on students with high mathematics abilities in Junior High School of 2 Rembang. The students had not formally learned about probabilities.

In the world of education in Indonesia, especially in mathematics, there is a branch of science that studies predicting possible outcomes from statistical experiments, namely probability. Probability tries to measure uncertainty as a tool for making decisions [3,23]. Therefore, the probability is an increasingly important concept in scientific thought [4] so that it is essential in everyday life. The importance of probability makes it a part of the school curriculum in most countries of the world [5,6,7]. In America, kindergarten students have been given treatment about probabilities ideas informally [8] whereas in Indonesia, based on the 2013 revised curriculum, probability material is studied when students are in eighth-grade of the second semester. It focuses on solving problems in real-life related to the possibility of an event. Probabilities problem solving is a part of probabilistic thinking. Probability is not only important in everyday life but also needed in other subjects [24].

2. Methods
This research employed the descriptive-qualitative approach. Qualitative research is research that explores and understands individual meanings related to social problems [14]. The main data in this research was the information regarding the probabilistic thinking of Junior High School 2 Rembang students with high mathematics abilities and had not formally learned about probability. The main sources of the data were the students' answers and responses during the interview. The subjects in this study were 2 eighth-grade students with high Mathematics abilities of Junior High School of 2 Rembang in the second semester of the 2019/2020 academic year. The sampling technique used was purposive sampling with the following criteria: (1) the student had not received the probability material, (2) based on the high, medium, and low initial abilities (2 students in each category), (3) the teacher recommendations and students' abilities to express ideas verbally or in writing to affirm the communication fluency between the researchers and research subjects.

The process of analyzing data in this research was as follows: (1) transcribing verbal data, (2) examining the data from various sources, the results of written assignments, interviews, field notes, (3) reducing the data to create abstractions to obtain relevant data by removing irrelevant data, (4) categorizing the data, (5) validating the data, and (6) drawing conclusions. The data were validated using the triangulation method. There were two strategies applied, namely checking the confidence level of research findings using data collection techniques and checking the confidence level of data sources using the same method [16]. The steps of the research are presented in Figure 1.
3. Result and Discussion

This research used three constructs of probabilistic thinking level framework since it had been conducted in junior high school. The three constructs of the probabilistic framework were the sample space, the experimental probabilities of an event, and the comparison of the probabilities. The researchers used several initials to make it easier to describe and analyze the data, namely the researcher was symbolized by P, the subject was symbolized by SX where X is the sequence of subjects {1,2}.

To show the level of probabilistic thinking, the framework built [1] was implemented on two seventh-graders with high mathematics abilities (Subject S1 and Subject S2). Based on the results of the data analysis, interview data triangulation was carried out based on the task-based probabilistic problems I and II. This was done to obtain valid data results.

3.1 The Sample Space Construct

Table 1 and Table 2 present the summary of the task-based interview based on probabilistic problems I and II with the subjects S1 and S2 in the sample space construct.

Table 1. The Results of Interview Analysis Based on Task I and II of Subjects S1 in Sample Space Constructs.

| Experiment | Stage I Interview | Stage II Interview |
|------------|-------------------|--------------------|
| Stage one  | The subject was able to fully name the members of the stage one experiment shown by stating all the | The subject was able to fully name the members of the stage one experiment indicated by mentioning |
Experiment | Stage I Interview | Stage II Interview
---|---|---
Stage one | numbers on the tops, namely 1, 2, 4, and 5. | all the colors found on the tops, namely red, yellow, blue, and green.
Stage two | The subject was able to fully name the members of the stage two experiment indicated by listing all the pairs that arise from the spinning the top A and B. | The subject was able to fully name the members of stage two experimental indicated by listing all the pairs of top colors that may appear from spinning tops A and B.

The valid data of S1 are as follows:
The subject was consistently able to fully describe the results of stage one and stage two experiments.

### Table 2. The Results of Interview Analysis Based on Task I and II with Subject S2 in Sample Space Constructs.

| Experiment | Stage I Interview | Stage II Interview |
|---|---|---|
| Stage one | The subject was able to fully name the members of the stage one experiment shown by stating all the numbers on the tops, namely 1, 2, 4, and 5. | The subject was able to fully name the members of the stage one experiment indicated by mentioning all the colors found on the tops, namely red, yellow, blue, and green. |
| Stage two | The subject was able to fully name the members of the stage two experiment indicated by listing all the number pairs that arise from the spinning the top A and B. | The subject was able to fully name the members of stage two experimental indicated by listing all the pairs of top colors that may appear from spinning tops A and B. |

The valid data of S2 are as follows:
The subject consistently described the results of stage one and stage two experiments.

Based on the triangulation of the results of the interview data analysis, there were similarities in the results of data collection. The same data was considered valid while different data was used as another finding in the research. Based on the results of the data analysis of the level of probabilistic thinking in stage one experiment, the two subjects completely named all members of the experiments. Subjects S1 and S2 mentioned that all the numbers on the tops were possible to obtain.

In stage two, both subjects were able to fully name the members of the sample space. This can be seen from the answers of the two subjects who can completely state all the number pairs that appear from the spinning tops A and B as well as all the color pairs that appear from spinning tops A and B. Based on these results, both subjects, for the sample space construct, were at the informal quantitative level (level 3). Both subjects can list the result of second-level experiments consistently using some of the generative strategies.
3.2 The Construct of Experimental Probabilities of an Event

3 and Table 4 present the summary of interview analysis based on task I and II with subject S1 and S2 on the construct of experimental probabilities of an event.

Table 3 The Results of Interview Analysis Based on Task I and II with the Subject S1 in the Constructs of Experimental Probabilities of an Event.

| Experiment | Stage I Interview                                                                 | Stage II Interview                                                                                      |
|------------|----------------------------------------------------------------------------------|---------------------------------------------------------------------------------------------------------|
| Stage one  | - The subjects began to realize that determining the most likely events requires a wider sampling. It indicates that the subject answering to number 2 may appear when the top is rotated repeatedly even though the number 1 five times appear before. | - The subjects began to realize that determining the most unlikely events requires a wider sampling. It is shown that the subject answered white color may not appear when the top is rotated repeatedly. |
|            |  - Subjects were able to determine the numerical value of experimental probabilities. The subject answered 2/5 when asked “how many estimates show the number 5 when the top is rotated once.” | - The subjects were able to determine the numerical value of experimental probabilities. The subject answered 1/4 when asked “how many times the red probably appears when the top is rotated once.” |
| Stage two  | - The subjects were able to recognize the most uncertain events in stage two experiments. The subject stated that number 4 does not necessarily appear when top A and top B were played together. | - The subjects were able to recognize the most uncertain events in stage two experiments. The subject stated an indeterminate green color appears when top A and top B are played together. |
|            |  - The subjects were able to determine numerical values in the two-step experiment precisely. The subjects were able to predict the appearance of the number 3 when the two tops were played simultaneously. | - The subjects were able to determine the numerical value of the two-stage experiment precisely. The subject was able to predict the appearance of red when the two tops were rotated simultaneously. |

The valid S1 data are as follows:

1. Stage one Experiment
   Subjects began to realize that to determine the most likely or least likely events a wider sampling is required, and the subjects were able to determine the numerical value of experimental probabilities.

2. Stage two Experiment
   Subjects were able to recognize the most uncertain events and determine a numerical value in a two-stage experiment.
Table 4. The Results of Interview Analysis Based on Task I and II with Subject S2 in the Constructs of Experimental Probabilities of an Event.

| Experiment | Stage I Interview                                                                 | Stage II Interview                                                                 |
|------------|-----------------------------------------------------------------------------------|-----------------------------------------------------------------------------------|
| Stage one  | - The subject began to realize that determining the most likely events requires a wider sampling. The subject answered that the number 2 may appear when the top is played many times even though the number 1 has appeared five times. However, S2 felt a little doubtful. - The subject was able to determine the numerical value of experimental probabilities. The subject answered 2/5 when asked "how many times the number 5 will appear when the top is rotated once. | - The subject began to realize that determining the most unlikely events requires a wider sampling. It is shown that the subject answered white color may not appear when the top is rotated repeatedly. - The subject was able to determine the numerical value of experimental probabilities. Subjects answered 1/4 when asked “how many times red will appear when the top is turned once. |
| Stage two  | - The subject was able to recognize the most uncertain events in stage two experiments. The subject stated that number 4 should not necessarily appear when top A and top B were played together. - The subject was able to determine the numerical value of the stage two experiments precisely. The subject was able to predict the appearance of the number 3 when the two tops are played simultaneously. | - The subject was able to recognize the most uncertain events in stage two experiments. The subject stated that green color should appear when top A and top B were played together. - The subject was able to determine the numerical value of the stage two experiments precisely. The subject was able to predict the appearance of red when the two tops were played simultaneously. |

The valid data of S2 are as follows:

1. Stage One
The subjects began to realize that to determine the most likely or least likely events a wider sampling is required, and the subject can determine the numerical value of experimental probabilities.

2. Stage Two
The subjects were able to recognize that the most uncertain events can determine a numerical value in a stage two experiment.

Based on the triangulation of the results of the interview data analysis above, there were similarities in the results of data collection. The same data was considered valid while different data was used as another finding in the research. Based on the results of data analysis at the level of probabilistic thinking,
the two subjects began to realize that to determine the most likely or least likely event, a wider sampling was needed. This can be seen from the answers in stage I and stage II, although at first, the S2 subject answered hesitantly when interviewed. However, when it was reconfirmed through almost the same questions, the S2 subject began to believe in the answer. Subjects S1 and S2 were also able to determine the numerical value of experimental probabilities. Both subjects can explain the reasons for answering to determine the numeric value.

In the construction of the two stages of experimental probabilities of an event, both subjects were able to recognize the most uncertain events. However, the reasons expressed at stage one of the data collection either subject S1 and S2 were less accurate. Then at the time of data collection in the second stage, S1 and S2 subjects were able to recognize the most uncertain events by providing the appropriate reasons. In other cases, subjects S1 and S2 can determine numerical values in a stage two experiment for good reason. This can be seen from the answers of the two subjects who can completely state all the number pairs that appear from the spinning tops A and B as well as all the color pairs that appear from spinning tops A and B. Based on these results, it appears that the S1 and S2 subjects for the sample space construct are at the numerical level (level 4). Both subjects S1 and S2 can collect appropriate data to determine numerical values for experimental probabilities.

3.3 The Construct of Probabilities Comparison

The following Table 5 and Table 6 present the summary of the results of the analysis of the task-based interview with probabilistic problems I and II subject S1 and S2 are presented in the construct of probabilities comparison.

| Experiment | Stage I Interview | Stage II Interview |
|------------|-------------------|--------------------|
| Stage one  | The subjects were able to numerically determine the probabilities of comparing two events in one sample space. Subjects answering the number 5 were more likely to appear. The most likely it will appear is 2/5. | The subjects were able to numerically determine the probabilities of comparing two events in one sample space. Subjects answering the color blue were more likely to appear. Likely to emerge that is 2/3. |
| Stage two  | The subjects were able to numerically determine the probabilities of comparing two events for two different sample spaces. The subjects answered that top B is more likely to appear the number 3 because there was a high probability for top B, which was 1/3, while top A was ¼. | The subjects were able to numerically determine the probabilities of comparing two events for two different sample spaces. The subjects answered spinning top A was more likely to appear green because most likely to top A of 1/2, while top A was ½. |

The valid data of S1 are as follows:
Subjects can numerically determine the probabilities of comparing events in either one sample space or two different sample spaces.
Table 6. The Analysis Results of Task-Based Interview with Subject 2 in the Construct of Probabilities Comparison.

| Experiment | Stage I Interview                                                                 | Stage II Interview                                                                 |
|------------|----------------------------------------------------------------------------------|-----------------------------------------------------------------------------------|
| Stage one  | The subjects were able to numerically determine the probabilities of comparing two events in one sample space. The subjects believed that number 5 should have been more likely to appear with the probability of 2/5. | The subjects were able to numerically determine the probabilities of comparing two events in one sample space. The subjects believed that the color blue should have been more likely to appear with the probability of 2/3. |
| Stage two  | The subjects were able to numerically determine the probabilities of comparing two events for two different sample spaces. The subject answered that top B should have been more likely to appear because there was a high probability for top B which was 1/3, while top A was ¼. | The subjects were able to numerically determine the probabilities of comparing two events for two different sample spaces. The subjects answered top A should have been more likely to appear because there was a high probability for top A which was 1/2, while top A was ½. |

The valid data of S2 are as follows:
Subjects were able to numerically determine the probabilities of comparing events in either one sample space or two different sample spaces.

Based on the triangulation of the results of the interview data analysis above, there were similarities in the results of stage I and II data collection in the construct of probabilities comparison. The same data was considered valid while different data was used as another finding in this research. Based on the results of the data analysis of the level of probabilistic thinking in the construct of probabilities comparison, the two subjects can numerically determine the probabilities in comparing events. It can be seen from the answers of the two subjects that they can determine the event that has the greatest chance among other events appropriately. Furthermore, the subject can determine the probabilities of each event and use it to compare the probabilities of occurrence in either one sample space or two different sample spaces. Based on the results of this research, the construct of probabilities comparison level of the two subjects was at the numerical level (level 4). Both S1 and S2 subjects can determine the size of the probabilities and compare the events. The level of probabilistic thinking of S1 and S2 subjects in each construct is presented in Figure 2.
The level of probabilistic thinking of students with high mathematics abilities and have not formally learned the probability material in the construct of the sample space was at the informal quantitative level. Students with high mathematics abilities were able to list consistently the results of two-tier experiments using some of the generative strategies. They were also able to list a complete set of experimental results. This was relevant with the level 3 of probabilistic thinking [1,10,11,16].

In the construct of experimental probabilities of an event, students with high mathematical abilities were at the numerical level. They were able to collect the right data to determine the numerical value of experimental probabilities. Also, they had realized that broader sampling was needed to determine the most likely or least likely events. This was relevant to level 4 of probabilistic thinking [1,10,11,16].

In the construct of probabilities comparison, the students were at the numerical level. They were able to choose events that provide greater probabilities among other events. Students with high mathematics abilities can quantify the probabilities numerically and compare events in stage one and stage two experiments. This was relevant to level 4 of probabilistic thinking [1,10,11,16].

The results of this research indicated that junior high school students who have not formally learned probability material and have high mathematics abilities had met the numerical probabilistic thinking indicator, namely level 4. The results were different from the results of Kurniasih and Sujadi's research (2017) which explains that eighth-grade junior high school students aged 13 to 15 were at the subjective and transitional levels (levels 1 and 2).

4. Conclusion

Based on the result of the analysis, the level of probabilities thinking of students with high mathematics abilities who have not formally learned probability material in Junior High School of 2 Rembang are as follows: (a) In the construct of sample space, the students were at the level of informal quantitative level (level 3) with the characteristics of being able to list the results of stage two experiment using part of the generative strategy. (b) In the construct of experimental probabilities of an event, the students were at the numerical level (level 4) with the characteristics of being able to collect appropriate data to determine the numerical value of the experimental probabilities. (c) In the construct of probabilities comparison, students were at the numerical level (level 4) with the characteristics of being able to determine the size numerically and compare the probabilities.

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