THE POSSIBILITY OF THERMAL INSTABILITY IN EARLY-TYPE STARS DUE TO ALFVÉN WAVES

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ABSTRACT

The importance of Alfvén waves to explain the winds of Wolf-Rayet stars was shown by dos Santos and coworkers. We investigate here the possible importance of Alfvén waves in the creation of inhomogeneities in the winds of early-type stars. The observed infrared emission (at the base of the wind) of early-type stars is often larger than expected. The clumping explains this characteristic in the wind, increasing the mean density and hence the emission measure, making it possible to understand the observed infrared, as well as the observed enhancement in the blue wing of the Hα line. In this study, we investigate the formation of these clumps via thermal instability. The heat-loss function used, \( H(T, n) \), includes physical processes such as emission of (continuous and line) recombination radiation, resonance line emission excited by electron collisions, thermal bremsstrahlung, Compton heating and cooling, and damping of Alfvén waves. As a result of this heat-loss function we show the existence of two stable equilibrium regions. At high temperature the stable equilibrium region is the diffuse medium, and at low temperature it is the clumps. Using this reasonable heat-loss function, we show that the two stable equilibrium regions can coexist over a narrow range of pressures describing the diffuse medium and the clumps.

Subject headings: instabilities — radiative transfer — stars: early-type — stars: mass-loss — waves

1. INTRODUCTION

As demonstrated first by Lucy & Solomon (1970), the radiative momentum absorbed by UV spectral lines is able to initiate stellar winds, since the radiative line acceleration exceeds the gradient by a large factor. The first model to derive mass-loss rates (\( M \)) and flow speeds in good agreement with observations was that of Castor, Abbott, & Klein (1975, hereafter CAK). One of the major difficulties presented by the radiation-driven wind theory is the momentum problem in W-R stars, which can be described using the ratio \( \eta = (Mv_x)/(L_\star/c) \), where \( v_x \) is the terminal velocity and \( L_\star \) the star luminosity. Barlow, Smith, & Willis (1981) found that, in W-R stars, \( \eta \) ranges from 3 to 30. This means that there is about an order of magnitude more momentum in the wind than in the radiation field. It was assumed that every stellar photon transfers its momentum, \( h\nu/c \), only once (single scattering), but even with multiple scattering of the photons one obtains \( \dot{M}v_x > 5L_\star/c \). To get around the momentum problem, one cannot simply appeal to a larger luminosity, because the values that are used cannot be near the Eddington limit (Cassinelli & van der Hucht 1987).

Following the suggestion that there may be appreciable magnetic fields in W-R stars larger than 1000 G (Maheswaran & Cassinelli 1988; Poe, Friend, & Cassinelli 1989), it was suggested that the wind in a WN5 star, for instance, can be driven by Alfvén waves (see Hartmann & Cassinelli 1981). They assumed \( B = 20,000 \) G and a mechanical flux of Alfvén waves of \( \Phi_\nu = 1.1 \times 10^{-14} \) ergs cm\(^{-2}\) s\(^{-1}\) (this work did not take into account the contribution of the radiation pressure on the lines).

As implied by the work of Willis (1991), a mechanism in addition to radiation pressure may be required to initiate the high W-R mass loss, although thereafter the winds may be radiatively accelerated. In this context, dos Santos et al. (1993a, 1993b) proposed a model for mass loss in W-R stars where both a flux of Alfvén waves and radiation pressure are considered. The model is a fusion of the Alfvén wave wind model of Jatenco-Pereira & Ophe (1989a, 1989b) and the radiation pressure CAK model. In the model an effective escape velocity is used that takes into account the CAK power index expressing the effect of all lines, possible non-solar abundances, and the finite size of the star disk. Their work indicates that Alfvén waves, acting jointly with radiation pressure, provide the necessary energy and momentum for the wind, with reasonable Alfvén fluxes and magnetic fields.

Early-type stars show superionization lines O vi, N v, Hα, and X-rays, which cannot be explained by the high-temperature star. Applying the coronal zone model to the winds of early-type stars, Cassinelli & Olson (1979) derived the ionization conditions expected in the wind of ζ Pup. The results of this study explain very well the persistence of low effective temperatures of the strong lines of O vi, N v, C iv, and Si iv.

Since Abbott, Telesco, & Wolf (1984), one knows that the observed IR emission is often larger than expected from a homogeneous wind. From that time it was pointed out that the clumping in the wind increases the mean density and hence the emission measure. Clumping can also explain an observed enhancement in the blue wing of the Hα line. The narrow absorption components are likely to be direct manifestations of dense clumps. Now, the existence of these clumps is largely known in many individual hot stars, and clumping may be important in all hot stars with winds (Hillier 1991; Robert 1994; Moffat & Robert 1994; Massa et al. 1995; Brown et al. 1995; Moffat 1994, 1996; Eversberg, Lépine, & Moffat 1998).

In principle, it can be said that the series of papers by Owocki, Rybicki, and Castor (Owocki & Rybicki 1984, 1985, 1986, 1991; Owocki, Castor, & Rybicki 1988; Rybicki & Owocki 1990), which contains numerical hydrodynamic calculations, shows that the radiation-driven winds are violently unstable and that the consequent shocks can explain the X-ray emission of early-type stars; moreover, the clumping explains the infrared emission excess and the formation

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the two rates are
\[ \frac{\Gamma_r}{\Gamma_c} = \frac{\int_{v_0}^{\infty} [4\pi B_0(T_r)/h^2] a_r dv}{n_e \langle \sigma_r v_r \rangle}, \]
where \( \sigma_r \) is the cross section for collisional ionizations, \( v_r \) is the electron velocity, and \( a_r \) is the photoionization cross section. Böhm (1960) has given approximations for these quantities, from which one derives
\[ \frac{\Gamma_r}{\Gamma_c} \approx 6 \times 10^{10} \frac{T_r^{1/2} \chi}{n_e}, \]
where \( T_r = T_c, hv/kT_r \geq 1 \), approximately, and the units of \( \chi \) are electron volts.

Applying these results to the ionization of C III ions, for instance, \( \chi_r = 47.9 \) eV with \( \log T_{\text{eff}} = 4.5 \). Taking \( T_r \) (in the hot atmosphere) \( \approx 10^3 T_{\text{eff}} \), and \( n_e = 2 \times 10^{13} \) cm\(^{-3} \), one obtains \( \Gamma_r/\Gamma_c = 3 \times 10^5 \) (for lower density the ratio is even higher), so that collisional ionization may be completely neglected.

Considering a thermal instability in an isobaric regime (e.g., Field 1965; internal pressure balanced by the external pressure), we looked for a set of physical parameters that, at equilibrium \([H(T, n) = 0]\), show three equilibrium regions: one stable region representing the diffuse medium; one unstable region; and another stable region representing the condensations. The following energy gains are considered: heating by photoionization-recombination, \( H_r \); Compton heating, \( H_c \); and Alfven wave heating, \( H_A \). These gains are balanced by the following radiative loss processes: cooling via thermal bremsstrahlung, \( H_B \); inverse Compton cooling (this term is computed jointly with \( H_B \)); and collisional excitation followed by resonance line emission, \( H_I \).

2.1. Bremsstrahlung Losses

The total amount of energy radiated in free-free transitions, per cubic centimeters per second, in the case of a Maxwellian distribution of velocities, is
\[ H_B = -\left(2\pi kT_e^{1/2} \frac{2^5 \rho e^6}{3^3 h m_e c^3} Z^2 n_e n_i \bar{g}_{\text{ff}}, \right. \]
\[ H_B = -1.42 \times 10^{-27} Z^2 n_e n_i T_e^{1/2} \bar{g}_{\text{ff}}, \]
\[ H_b = -\lambda_b T_e^{1/2} n_i^2. \]

The quantity \( \bar{g}_{\text{ff}} \) appearing above is a correction factor required for precise results. Its value is generally about unity (Spitzer 1978), \( \lambda_b = 2.4 \times 10^{-27} \). Hereafter \( n_e \equiv n \), the number density.

2.2. Resonance Line Emission

Raymond, Cox, & Smith (1976) calculated a radiative cooling coefficient for a low-density gas—optically thin with cosmic abundances—between the temperatures of \( 10^4 \) and \( 10^8 \) K, which we adopt in this work. A good fit to radiative losses in this temperature range, because of electron excitation of resonance transitions in common metal ions (ergs cm\(^{-3} \) s\(^{-1} \)), is
\[ H_{n_i} = -\frac{a T_p^p}{1 + b T_p^q} n_i^2 \]

(Raymond et al. 1976; Mathews & Doane 1990), with \( a = 1.53 \times 10^{-27}, b = 1.25 \times 10^{-9}, p = 1.2, \) and \( q = 1.85 \).
2.3. Photoionization-Recombination Heating

An approximate equation to express the residual heating due to radiative ionization followed by recombination, in ergs cm\(^{-3}\) s\(^{-1}\), is

\[ H_r = \sigma_a(T) \text{ max} \left(0, \langle hv \rangle_i - \nu_v - f3k_B T/2\right)n^2, \]

where \(\langle hv \rangle_i\) is the mean energy of ionizing photons, is

\[ \langle hv \rangle_i = \frac{\sum n_j h\nu_j}{\sum n_j} = \frac{n_H k_B T_H + n_L k_B T_L}{n_H + n_L}, \]

\(n_L\) and \(n_H\) are the numerical densities of photons of the low- and high-temperature regions, respectively, given by

\[ n_L = \frac{\sigma T_L^4}{k_B T_L c} \quad \text{with} \quad T_L = T_*; \]

and

\[ n_H = \left[H_s(T_H, n_H) + H_s(T_H, n_H)\right]tR_*, \]

where \(\sigma_a = 2.60 \times 10^{-13} (T/10^4)^{-0.8} \text{ cm}^3 \text{ s}^{-1}\) is the recombination coefficient and \(\nu_v\) is the ionization potential of hydrogen. In the above equations \(n_H\) and \(T_H\) are the density and temperature of a high-temperature region, \(n_L\) and \(T_L\) are the same for a low-temperature region, and \(tR_*\) is the region thickness. Each recombination results in a loss of energy \(f3k_B T/2\) from the thermal energy of the plasma, with \(f \approx 0.43\) (Mathews & Doane 1990).

The recombination expression, equation (3), is an approximated one. Equation (3) states that we have recombination only when the average energy of the photons, \(\langle hv \rangle_i\), is sufficiently high such that ionization can occur, that is, when \(\langle hv \rangle_i\) is greater than \(\nu_v + f3k_B T/2\) (i.e., the sum of the ionization potential plus the average energy of the electron that is liberated).

2.4. Compton Heating-Cooling

We have to estimate the number and frequency of the photons acting in the immediate neighborhood of the star surface and anywhere in the star atmosphere. Taking into account the interaction between thermal electrons and the radiation field, photons with lower frequency come from a cooler, optically thick region, the stellar continuum. Their flux is \(\sigma T_*^4\), and then for these photons we have

\[ \frac{L_{BL}}{4\pi R_*^2} = \frac{\sigma T_*^4}{c}. \]

On the other hand, a hot region of thickness \(tR_*\) (optically thin), at temperature \(T_H\), causes heating in the medium, principally via thermal bremsstrahlung and resonance line emission \(\left[H_s(T, n) + H_s(T, n)\right]tR_*\). Hence, for these photons,

\[ \frac{L_{BL}}{4\pi R_*^2} = \left[H_s(T_H, n_H) + H_s(T_H, n_H)\right]tR_* . \]

The complete expression for Compton heating and cooling is then

\[ H_c = \frac{4k_B \sigma_T n}{mc} \left\{ \left[ (T_H - T) \frac{L_{BL}}{4\pi R_*^2} \right] + \left[ (T_L - T) \frac{L_{BL}}{4\pi R_*^2} \right] \right\}, \]

which is similar to the expression usually adopted, for instance, by Mathews & Doane (1990). In the above equations \(k_B\) is the Boltzmann constant, \(\sigma_T\) the Thomson cross section, \(\sigma\) the Stefan-Boltzmann constant, \(n\) the number density, \(m\) the electron mass, \(T\) the temperature, \(L_b\) the bolometric luminosity, \(c\) the light speed, \(T_*\) the stellar temperature, and \(R_*\) the stellar radius.

3. DAMPING AND HEATING FROM ALFVÉN WAVES

Alfvén waves in a early-type star, whose winds are primarily radiatively driven, are subject to damping (or amplification) as described, for example, by MacGregor (1996). In this case, the dispersion relation for Alfvén waves in a radiatively driven wind is \(k^2v_A^2 = \omega^2 - io\omega_o\) (instead of \(k^2v_A^2 = \omega^2\)), where

\[ \omega_o = \frac{2\pi k_L v_L dI_a(v)}{3c^2 \frac{dv}{dI_a}}, \]

\(I_a(v)\) is the intensity of the photospheric radiation field, \(v_L\) is a line rest frequency, and \(\kappa_L\) is the line mass absorption coefficient. If \(\frac{dI_a(v)}{dv} > 0\), then the Alfvén wave is amplified, while if \(\frac{dI_a(v)}{dv} < 0\) the Alfvén wave is damped. However, as MacGregor (1996, p. 334) noted, “the presence of such radiatively modified Alfvén waves in the flow has yet to be explored.” Therefore, in the present investigation, we apply the dampings described below (§§ 3.1 and 3.2) with their heatings (§ 3.2).

The damping mechanisms that we assumed here were used before in many astrophysical objects: protostellar, late-type stars and solar winds (Jatenco-Pereira & Opher 1989a, 1989b); galactic and extragalactic jets (Opher & Pereira 1986; Gonçalves et al. 1993b); early-type stars (dos Santos et al. 1993a, 1993b); broad-line regions of quasars (Gonçalves et al. 1993a, 1996); cooling flows of galaxy clusters (Friaça et al. 1997); and others.

3.1. Nonlinear Damping

Parallel Alfvén waves are purely transverse, and there is no important linear damping. The damping that does occur is not linear, and it arises from a beat wave (two circularly polarized, parallel propagating waves), which contains a longitudinal field component and a longitudinal gradient in the magnetic field. This results in a nonlinear damping of both electrostatic and magnetostatic components, i.e., transient time damping.

Völk & Cesarsky (1982) derived an equation that represents the unsaturated Landau damping, in the case of nonlinear two-wave interaction, that can be written as

\[ \Gamma(k) = \frac{1}{4} \sqrt{2 \xi k v_s J}, \]

where \(J\) is the energy density in waves, normalized to the ambient magnetic energy density, \(B_0^2/8\pi\) (Lagage & Cesarsky 1983). Using \(J = \rho \langle \delta v^2 \rangle/(B_0^2/8\pi)\) and \(k = \sigma v_s A\), we obtain:

\[ \Gamma_{nl} = \frac{1}{4} \sqrt{ \frac{\pi}{2} \xi \left( \frac{v_s}{v_A} \right)^2 \rho \langle \delta v^2 \rangle \frac{B_0^2}{8\pi} }, \]

where \(\xi = 5–10\) and \(v_s\) is the sound velocity.

3.2. Turbulent Damping

There is strong evidence favoring anisotropic, supersonic, and compressible turbulence in W-R winds. Since all W-R stars intensively observed so far behave similarly, and W-R
stars are extreme manifestations of winds in hot luminous stars, it is possible or even likely that all hot-star winds show the same basic phenomenon (Moffat et al. 1994 and references therein). A necessary (but not sufficient) condition to show that one is dealing with turbulence is that the Reynolds number, Re, be \( \gg 1 \). For an expanding wind, with \( v_a(r) \) the expansion speed and \( r \) the distance from the star, one has

\[
\text{Re} \approx \frac{v_a(r)}{\nu_{\text{thermal}}} \approx \frac{r}{l_{\text{mfp}}} \frac{v_a(r)}{\nu},
\]

where \( \nu \) is the viscosity and \( l_{\text{mfp}} \) the mean free path of the average particle in the medium. Thus, with typical W-R wind values, where the observed lines form, Re is much higher than 1, so turbulence is likely to exist if there is a driving force.

Hollweg (1986) considered a new hypothesis for the nonlinear wave dissipation of Alfvén waves. The hypothesis is that the wave dissipates via turbulent cascade; that is, this hypothesis concerns the consequences of the Alfvén wave dissipation in terms of wave-particle interactions, where the required power at high frequencies is presumably supplied via turbulent cascade. Then, exploiting the similarity of the magnetic field power spectra, \( P_B \propto k^{-5/3} \), and Kolmogorov turbulence in ordinary fluids, the plasma volumetric heating rate associated with the cascade is given by

\[
E_H = \frac{\rho \langle v^2 \rangle^{3/2}}{L_{\text{corr}}},
\]

where \( \rho \) is the mass density, \( \langle v^2 \rangle \) is the velocity variance associated with the wave field, and \( L_{\text{corr}} \) is a measure of the transverse correlation length. A subhypothesis is that the correlation length scales as the distance between magnetic field lines,

\[
L_{\text{corr}} \propto B^{-1/2}.
\]

In spite of \( L_{\text{corr}} \) being a free parameter, the model comes close to the notion of a Kolmogorov-like cascade to small scales. The waves themselves are here regarded as the source of the heating. In this case \( \langle v^2 \rangle \) only includes the power associated with Alfvénic fluctuations. Similarly, \( L_{\text{corr}} \) concerns the correlation length of the Alfvénic fluctuations. Finally, in terms of damping length, we have

\[
L_d = L_{\text{corr}} v_A \langle v^2 \rangle^{-1/2}
\]

(Hollweg 1986, 1987).

3.3. Alfvén Wave Heating

Data of the last 10 years show us that early-type stars can be separated into two groups: magnetic stars, with surface strengths of a dipole or quadrupole magnetic field of \( B \approx n(10^{-10} - 10^3) \) G, \( n = 2, 3, \ldots, 7 \); and normal stars, with \( B \approx 1 - 100 \) G. The magnetic field strength increases toward the center of the star and is \( \approx 0.1 - 10 \times 10^6 \) G in the core, depending on the stellar mass (Moffat et al. 1994; Bohm 1994). The origin of these fields is an open question, and two theories compete to explain it: dynamo and fossil theories (Moss 1994).

Consider now a collapsing cloud. For the collapsing cloud the cross-sectional area, \( A \), perpendicular to a magnetic field is \( \propto \rho^{-2/3} \) and \( B \propto A^{-1} \propto \rho^{2/3} \), where \( \rho \) is the mass density of the gas and \( B \) is the magnetic field. The damping length in each case is \( L = v_A/B \) (i.e., the ratio between the Alfvén velocity and the damping rate). Knowing that \( v_A = B/(4\pi\rho)^{1/2} \), \( \rho \langle v^2 \rangle \propto \Phi_w/v_A \), where \( \Phi_w \) is the wave flux and \( \Phi_w \propto \rho^{2/3} \), we write the nonlinear Alfvénic heating as

\[
H_{\text{nl}} = \frac{\Phi_w}{L_{\text{nl}}} = \frac{\Phi_w}{v_A} \Gamma_{\text{nl}} \propto \frac{\rho \langle v^2 \rangle}{B^2} \propto \rho^{-1/2}
\]

and the turbulent Alfvénic heating as

\[
H_t = \frac{\Phi_w}{L_t} \propto \Phi_w B^{1/2} \left( \frac{\langle v^2 \rangle}{v_A} \right)^{1/2} \propto \rho^{7/12},
\]

following equations (5) and (7).

The sum of the contributions from Compton and inverse Compton, photoionization-recombination, bremsstrahlung, and resonance line emission is about \( 10^{-22} \text{ergs cm}^{-3} \). We are adopting \( n_H \) (the density of the hot atmosphere) = \( 2 \times 10^{13} \) \text{cm}^{-3}, resulting in \( 10^{-22}n_H^2 \approx 4 \times 10^4 \) \text{ergs cm}^{-3} \text{s}^{-1}. We then normalize the Alfvénic heatings using \( F_{\text{nl}}, F_t = 10^2 - 10^5 \text{ergs cm}^{-3} \text{s}^{-1} \). So,

\[
H_{\text{nl}} = F_{\text{nl}} \left( \frac{n}{2 \times 10^{13}} \right)^{-1/2},
\]

and

\[
H_t = F_t \left( \frac{n}{2 \times 10^{13}} \right)^{7/12}.
\]

3.4. The Overall Heating/Cooling Behavior

In order to make the relevance of each heating/cooling process in the overall balance clear, in Figure 1 we plot the module of the heating or cooling—due to resonance line emission, photoionization-recombination, thermal bremsstrahlung, Alfvénic turbulent and nonlinear heating, and Compton interactions—as a function of temperature, in units of \text{ergs cm}^{-3} \text{s}^{-1}. 

![Figure 1](image-url)

FIG. 1.—Module of the heating/cooling processes, for \( \log P_H = 4.74 \) (\( P_H = 5.5 \times 10^4 \text{dynes cm}^{-2} \)), \( F_{\text{nl}} = F_t = F_A = 10^5 \), and \( T = 0.2 \) (as in Figs. 2-5, where the thermal instability process occurs for \( T_L < T < T_H; T_L = 3 \times 10^4 \text{K} \) and \( T_H = 10^7 \text{K} \).
The first characteristic we note from Figure 1 is the fact that resonance line emission is the most important cooling. In fact, it dominates over all other processes in this range of temperature. Another aspect is that we are using only the contribution of the photoionization-recombination processes that produces heating. This mechanism is not considered at temperatures higher than \( \log T \approx 6.3 \). At these high temperatures it appears as cooling (see also eq. [3]).

From the physics of Alfvén wave heating, it is clearly not temperature dependent (eqs. [10] and [11]), beyond a dependence on the density that implicitly scales inversely to temperature, keeping \( P \) fixed. Then, as Alfvén heating is proportional to \( n^\alpha (\alpha = -1/2, 7/12) \) at a given pressure, we have this heating proportional to \( T^{-\alpha} \), as can be seen from Figure 1.

4. RESULTS

The complete heating-cooling function, \( H(T, n) \), including the physical processes discussed above, is

\[
H(T, n) = -\lambda_0 T^{1/2} n^2 - \frac{a T^p}{1 + b T^q} n^2 + \frac{4 k_B \sigma T n}{m c^2} \\
\times \left\{ (T_H - T) [H_{\text{nl}}(T_H, n_H) + H_{\text{t}}(T_H, n_H)] + R_* \right\} + \alpha_0 T^{4/3} + \alpha_0(T) \times \max \left( 0, \langle h\nu \rangle_i - \langle h\nu_0 \rangle - \frac{f 3 k_B T}{2} \right) n^2 + H_A ,
\]

with \( H_A \) assuming the form of \( H_{\text{nl}} \) and \( H_{\text{t}} \), given by equations (10) and (11). All the constants in equation (12) are in

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Fig. 2.—Diagram of \( \log T \) vs. \( \log P \) \([H(T, n) \approx 0]\), with \( F_A = 10^2 \): nonlinear Alfvénic heating and turbulent Alfvénic heating.

Fig. 3.—Diagram of \( \log T \) vs. \( \log P \) \([H(T, n) \approx 0]\), with \( F_A = 5 \times 10^3 \): nonlinear Alfvénic heating and turbulent Alfvénic heating.

Fig. 4.—Diagram of \( \log T \) vs. \( \log P \) \([H(T, n) \approx 0]\), with \( F_A = 10^4 \): nonlinear Alfvénic heating and turbulent Alfvénic heating.

Fig. 5.—Diagram of \( \log T \) vs. \( \log P \) \([H(T, n) \approx 0]\), with \( F_A = 10^5 \): nonlinear Alfvénic heating and turbulent Alfvénic heating.
cgs units.

Figures 2–5 show the balance between energetic gains and radiative losses (from eq. [12]), i.e., the equilibrium of $H(T, n_H)$ in a log $P$–log $T$ diagram. For this calculation we assume $n_H = 2.0 \times 10^{13}$ cm$^{-3}$, $T_H = 1.0 \times 10^7$ K, and $T_L = T_b = 3.0 \times 10^4$ K; $R_\star = 12R_\odot$ and $t = 0.2$.

As we were forming clouds, via thermal instability, from the hot atmosphere ($n_H = 2.0 \times 10^{13}$ cm$^{-3}$ and $T_H = 1.0 \times 10^7$ K), we performed calculations in order to find, for each temperature, the density that corresponds to the balance $[H(n, T) \approx 0]$. From the isobaric instability criterion, we need the clouds and the hot medium coexisting at the same pressure. In the hot atmosphere the pressure is $P_H = 2n_H k T_H \approx 5.5 \times 10^4$ dynes cm$^{-2}$ s$^{-1}$. We then want to verify the possibility of forming clouds ($\approx 10^4$ K, cool and dense atmosphere) at pressures near the characteristic pressure of the hot medium.

In Figures 2–5 we assume a single value of $F_\lambda$ for the two Alfvénic heating terms, nonlinear ($F_{nl}$) and turbulent ($F_t$). In Figure 2, we have the equilibrium diagram in terms of pressure and temperature, adopting $F_\lambda = 10^5$. This figure shows us the coexistence of the two-phase equilibrium at pressures lower than the characteristic pressure of the hot atmosphere (log $P_H = 4.74$). For this value of $F_\lambda$, the nonlinear heating (in the case in which the Alfvénic heating contribution to the overall heating-cooling function is the nonlinear one) as well as the turbulent heating are insufficient to permit the coexistence of the hot and cool atmospheres at appropriate pressures. Analyzing Figure 3 ($F_\lambda = 5 \times 10^3$), we observe that both cases (nonlinear and turbulent) are satisfactory to permit cloud formation via thermal instability, since they reach the pressure desired. Finally, in Figure 4 ($F_\lambda = 10^5$) and Figure 5 ($F_\lambda = 10^3$), the cool and hot stable solution can be found at the pressure desired (the reference pressure and higher pressures) in the two cases. In addition, it is clear that turbulent Alfvénic heating does a better job than nonlinear heating, since the pressure range of the coexistence of both stable equilibria is bigger (see the bottom panel in Figs. 4 and 5) in the case of this Alfvénic heating.

5. DISCUSSION AND CONCLUSIONS

Our results can be discussed in terms of the efficacy of the Alfvénic heatings in forming condensations near the surface of W-R stars because of thermal instability. This efficiency is included in the scaling factor, $F_\lambda$, that is equal to $F_{nl}$ or $F_t$ in Figures 2–5.

We consider in this study a heuristic derivation of the expressions for Alfvénic heatings. The nonlinear and turbulent Alfvénic heatings represent extreme opposite dependencies of these heatings on density. Comparing this behavior, we have, on one hand, turbulent heating ($H_t \propto n^{1/3}$), which deposits more energy when the density is higher, and, on the other hand, nonlinear heating ($H_{nl} \propto n^{-1/2}$), which deposits more energy when the density is lower. These behaviors also can be seen from Figure 1, in which cooler regions are denser than hotter ones (that figure was plotted for a fixed pressure $P_H$; thus, the nonlinear heating curve is a decreasing function of density, opposite to the case of turbulent heating). Because of the completely different behavior of these types of heating, the results from each one are very different (see Figs. 2–5, with the equilibrium solution for our models). Despite the fact that these types of Alfvénic heating do not work in the cool solution as well as in the hot one, the results with nonlinear heating produce the cool condensations at about $2.2 \times 10^4$ K, while the results with turbulent heating show cooler low-temperature solutions ($\sim 1.0 \times 10^4$ K). Noting also the way that each process scales with $P$ at fixed $T$, one can understand why the stable hot solution has a narrower range in pressure than the stable cool solution in all figures of equilibrium (Figs. 2–5).

In the work of dos Santos et al. (1993a), the principal emphasis was on determining the terminal velocity of the Wolf-Rayet star winds, using a model that had radiation pressure and Alfvén waves driving the wind. The initial Alfvén wave flux, $\Phi_w$, required was $\approx 5.6 \times 10^{12}$ ergs cm$^{-2}$ s$^{-1}$. Using this value for the wave flux, we can estimate the damping length for the Alfvén waves. As in the derivation of Alfvénic heatings (§3.3), $\Phi_w$ is equal to $H_t L_A$. Adopting, for example, the maximum pressure in which the stable two-phase equilibrium exists, in Figure 5b, $P_H \approx 5 \times 10^5$ dynes cm$^{-2}$, the turbulent Alfvénic heating for the formed clouds is $\sim 1.9 \times 10^7$ ergs cm$^{-3}$ s$^{-1}$. Then

$$L_{t(min)} = \frac{\Phi_w}{H_t} = \frac{5.6 \times 10^{12}}{1.9 \times 10^7} \approx 3 \times 10^5 \text{ cm}.$$ 

Taking now the minimum value for the pressure in the cloud in Figure 5b, $P \approx 3.9 \times 10^4$ dynes cm$^{-2}$, the turbulent Alfvénic heating is $\sim 4 \times 10^6$ ergs cm$^{-3}$ s$^{-1}$. We then have

$$L_{t(max)} = \frac{\Phi_w}{H_t} = \frac{5.6 \times 10^{12}}{4 \times 10^6} \approx 1.4 \times 10^6 \text{ cm}.$$ 

These values for the Alfvénic damping lengths can be understood as limits to the size of the formed clouds, since the cloud diameters ($d_c$) must be smaller than the damping lengths ($3 \times 10^5 \leq d_c \leq 1.4 \times 10^6$ cm) in order to have turbulent damping of the Alfvén waves effective in this cloud formation process.

The Alfvén flux adopted here was the necessary value in order to accelerate the wind to the observed velocity and obtain the necessary momentum in the wind with minimum magnetic field. The ratio between this flux and the total one at the star surface is

$$\xi = \frac{\Phi_w}{L_\odot/4\pi R_\odot^2}.$$ 

Assuming typical values for $L_\odot (10^{5.5} L_\odot)$ and $R_\star (12 R_\odot)$, we obtain $\xi \approx 0.04$, which means that $\Phi_w$ is only a few percent of the total stellar flux at the star surface. (This flux of waves is lower than others used in the literature, for instance, Hartmann & Cassinelli 1981.)

It is also interesting to estimate the magnetic field before (in the diffuse medium, $B_0$) and after (in the clouds, $B_c$) the condensation process. During the collapse, the density increases $10^3$–$10^4$ times. Since $B \propto \rho^{1/3}$, the magnetic field increases $10^2$–$10^2.7$ times during the cloud formation (we are considering the magnetic field frozen in the plasma). For a pressure of $\sim 5.5 \times 10^6$ dynes cm$^{-2}$, the maximum magnetic field in the cloud, in order to have the magnetic field not dominating the pressure, is

$$\frac{B^2}{4\pi} \leq 5.5 \times 10^4 \text{ or } B_c \leq 8.3 \times 10^2 \text{ G}.$$ 

In this way, the ambient magnetic field is then $B_0 \leq (1.65–8.3) \text{ G}$. 

Our treatment can be understood in light of the model of dos Santos et al. (1993a, 1993b), which focuses on the mass loss from W-R stars due to radiation pressure and Alfvén waves. As radiation pressure line-driven models have difficulties in explaining some observational characteristics of these stars (such as the disagreement between the observational mass-loss rates and the maximum predicted mass loss by radiation pressure), they proposed that a flux of Alfvén waves and radiation pressure act jointly. This fusion of an Alfvén wave wind model (Jatenco-Pereira & Opher 1989a) and the radiation pressure CAK model resulted in good agreement with observations. In the present model, we adopted the Alfvén wave flux from the model of dos Santos et al. (1993a, 1993b) and showed that the heating due to the Alfvén wave flux can cause a thermal instability that results in cold, dense clumps coexisting with a hot, diffuse gas. It is also important to note that near the star surface the magnetic force is more efficient than elsewhere (this is seen, for instance, in Figs. 7–9 of dos Santos et al. 1993a). The thermal instability processes that we propose occur just near the stellar surface.

Another important aspect to discuss here is the competition between the instabilities acting in the wind. In principle, any thermal instability of the wind material would have to compete against the intrinsic line-driven instability of the flow. Then, consider that the cooling time, or thermal instability time, in the unperturbed medium is

\[ t_{\text{cool}} \approx \frac{k_B T}{n \Delta} \approx 1.6 \text{ s} \quad (\text{at } P_H = 5.5 \times 10^4 \text{ dynes cm}^{-2}) , \]

where \( T = T_H \) and \( \Delta \) is the cooling rate dominated by radiative processes \( (H_s + H_{10}) \). Consider also that the dynamical, or the line-driven instability time, at the base of the wind (up to 1 stellar radius), is

\[ t_{\text{dyn}} \approx \frac{R}{\langle \phi \rangle} \approx 1.6 \times 10^3 \text{ s} \ , \]

where \( R \) is the distance from the ionization source \((\sim 0.1 R_*\) and \( \langle \phi \rangle \approx 5.04 \times 10^7 \text{ cm s}^{-1} \) (Owocki 1994; dos Santos et al. 1993a) is the wind velocity in this region. The above estimates show that the cooling time is much smaller than the dynamical time, as is necessary in order to have the thermal instability predominate in this region (see Krolik 1988; Mathews & Doane 1990). Moreover, the analysis of what instability is the most relevant for the thermodynamic state of the wind is related to the region in which each instability operates. For instance, following Owocki (1994), we know that in the line-driven instability a minor part of the material is actually accelerated to high speed; for most of the mass, the dominant effect is clumping. Diffuse radiation plays an important role in reducing the line-driven instability, especially near the wind base. The competition between these two instabilities may be important far in the wind, but at the base it is not. In this region thermal instability works alone.

Thinking about the mass quantity involved in this condensation process, we can also note that if, at a given pressure, a thermal instability is indicated—namely, matter at a high temperature and a low temperature is permitted—little can be said about the fraction of the matter that is at the high or low temperature. For example, for the Crab Nebula observations we have that almost all the matter is found to be contained in the filaments (Wilson 1971). These filaments are generally held to have formed by a thermal instability (e.g., Gouveia Dal Pino & Opher 1989).

In spite of the presence of a gradient in the temperature, we did not consider the effect of thermal conduction in our model. We proceeded in this way because thermal conduction is extremely reduced in the gas, because of the presence of the magnetic field. This reduction occurs in the direction perpendicular to the field lines (Field 1965), and it is so important that even small magnetic fields are sufficient to eliminate this component of the thermal conduction (for some applications see Begelman & McKee 1990; McKee & Begelman 1990).

In conclusion, our principal goal in this study was to explore whether or not a thermal instability, assisted by Alfvénic heatings, can play a role in the base of early-star winds. In spite of the fact that a number of simplifications were adopted in this first investigation, our results limit the pressure range for the existence of the two-phase equilibrium at the base of these winds. Our calculations indicate that a thermal instability may be a viable mechanism to form clumps in the winds of early-type stars.

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