MINIMAL LEPTON FLAVOUR VIOLATION AND LEPTOGENESIS

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We study the role of leptogenesis in the framework of minimal lepton flavour violation.

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1. Outline

If neutrino masses are generated by a seesaw mechanism with heavy Majorana neutrinos, the hypothesis of minimal flavour violation (MFV) in the lepton sector cannot be a simple replica of MFV in the quark sector (sec.2): more parameters are involved (sec.3) and high energy processes like leptogenesis (sec.4) can have a role in the predictions for low energy experiments (sec.5). We conclude summarizing the main points in sec.6.

The introduction in sec.2 is mainly based on Refs.1,2, the rest on Ref.3. Aspects of MFV in the lepton sector have been discussed also in Refs.4,5,6,7.

2. A definition of minimal lepton flavour violation

In the Standard Model (SM) flavour changing neutral currents are absent at tree level and mixing between different families is suppressed by small fermion masses and small CKM angles. Any new physics (NP) that fails to reproduce these special features of the SM is likely to give visible contributions to flavour changing rates. Experiments indicate that this is not the case, so that or flavour violating NP intervenes at a very high energy scale or its flavour structure is somehow hidden behind the SM one. The second option is more appealing. It looks more compatible with the hierarchy problem and offers realistic chances to discover NP directly. A natural and general proposal in this direction is the MFV hypothesis, where the Yukawa matrices are assumed to be the only sources of flavour violation also beyond the SM.

More precisely, we assume that there is flavour violating NP at some high energy scale \( \Lambda_{FV} \) which enters the SM lagrangian with non-renormalizable operators suppressed by inverse powers of \( \Lambda_{FV} \). In the limit of vanishing Yukawa couplings, the Standard Model lagrangian is invariant under the flavour symmetry group \( G = SU(3)_{Q_L} \times SU(3)_{u_R} \times SU(3)_{d_R} \times SU(3)_{L_L} \times SU(3)_{e_R} \). The invariance of the SM Lagrangian under \( G \) can be formally recovered elevating the Yukawa matrices to spurion fields with appropriate transformation properties under \( G \). The hypothesis of MFV states that this is sufficient to make also NP operators invariant under \( G \).

At the moment, the main motivation for extending the hypothesis of MFV to leptons is the analogy with the quark sector, where it is supported by a large amount of experimental data. We would like to understand whether MFV is a general principle that acts also on leptons. The exact analogy would imply that neutrinos are Dirac particles. In this case, the exceptional smallness of neutrino masses would remain unexplained and flavour changing rates like \( \mu \to e \gamma \) and \( \mu - e \) conversion in nuclei would be safely below
current and future experimental limits.

For these reasons, it is more interesting to include in the MFV hypothesis lepton number violation (LNV), that explains the smallness of neutrino masses with the high scale of LNV, $\Lambda_{LNV} \gg \Lambda_{MFV}$. Once LNV is introduced, the concept of analogy with quarks is ambiguous. A possible choice is to assume that right-handed neutrinos exist (the counterpart of right-handed up-quarks) and that their Majorana mass matrix is proportional to the identity matrix, so that the only flavour non-diagonal objects are the Yukawa couplings, like in the quark sector. However this does not automatically guarantee that flavour changing rates are completely specified in terms of the masses and mixings measured in experiments, as it happens with quarks.

3. Parameter counting

The basic flavour changing unit we obtain from the spurion analysis of NP operators is $\lambda^\dagger_{\nu} \lambda_\nu$. In the quark sector the analogue quantity $\lambda^\dagger_{\nu} \lambda_\nu$ can be expressed in terms of quark masses and CKM angles, so that the only unknown in the predictions for flavour changing branching ratios is the overall factor $1/\Lambda^2_{MFV}$. In a model-independent approach we cannot fix the coefficients in (6) but in a perturbative scenario we expect $\epsilon_v \sim g_{eff}/4\pi$ and $\epsilon_{\nu} \sim g_{eff}/4\pi$.

Let us count the parameters in eq.(4): there are 9 parameters in principle measurable at low energy (MNS angles, Dirac and Majorana phases, neutrino masses) and 4 unknown parameters: the normalization $M_R$ and $\phi_{1,2,3}$ in the matrix $H$, which disappear in the see-saw relation (2). With non degenerate right-handed neutrinos we would have 5 unknown parameters more (the 2 mass splittings of $M_R$ and the 3 angles in $O$).

Assuming that all the baryon asymmetry $\eta_B$ of the universe has been generated through sphalerons effects by a lepton asymmetry, we can use the observed value of $\eta_B = (6.3 \pm 0.3) \times 10^{-10}$ to get some information on the high energy parameters $M_R$ and $\phi_{1,2,3}$. In fact, at first order leptogenesis depends on the combination

$$\lambda^\dagger_{\nu} \lambda_\nu = \frac{M_R}{v^2} H m_{\text{diag}} H.$$  

4. Analysis of leptogenesis

A necessary condition for generating a lepton asymmetry is the non-degeneracy of heavy neutrinos. We in general expect that the tree-level degeneracy of heavy neutrinos is lifted by radiative corrections. In MFV models the most general form of the allowed mass-splittings is

$$\frac{\Delta M_R}{M_R} = c_{\nu} \left[ \lambda_{\nu} \lambda^\dagger_{\nu} + (\lambda_{\nu} \lambda^\dagger_{\nu})^T \right]$$

$$+ c^{(1)}_{\nu\nu} \left[ \lambda_{\nu} \lambda^\dagger_{\nu} \lambda_{\nu} \lambda^\dagger_{\nu} + (\lambda_{\nu} \lambda^\dagger_{\nu} \lambda_{\nu} \lambda^\dagger_{\nu})^T \right]$$

$$+ c^{(2)}_{\nu\nu} \left[ \lambda_{\nu} \lambda^\dagger_{\nu} (\lambda_{\nu} \lambda^\dagger_{\nu})^T \right] + c^{(3)}_{\nu} \left[ (\lambda_{\nu} \lambda^\dagger_{\nu})^T \lambda_{\nu} \lambda^\dagger_{\nu} \right]$$

$$+ c_{\nu \nu} \left[ \lambda_{\nu} \lambda^\dagger_{\nu} \lambda_{\nu} \lambda^\dagger_{\nu} + (\lambda_{\nu} \lambda^\dagger_{\nu} \lambda_{\nu} \lambda^\dagger_{\nu})^T \right] + \ldots$$

In a model-independent approach we cannot make the coefficients in (6) but in a perturbative scenario we expect $c_{\nu} \sim g_{eff}/4\pi$ and
\begin{align} 
c_{\nu}, \ c_{\nu} \sim g^2_{\text{eff}}/(4\pi)^2. \end{align}

From eq. (6) we can derive some general properties of leptogenesis in MFV models: (i) the term proportional to \( c_{\nu} \) does not generate an asymmetry by itself, but (ii) sets the order of magnitude of the mass splitting and naturally gives the condition for resonant leptogenesis: the mass splitting of right-handed neutrinos is comparable to the decay width,

\begin{align} 
\Delta M_{R,i} = M_R \frac{|\lambda_i |}{8\pi}. 
\end{align}

(iii) The right amount of leptogenesis can be generated even with \( \lambda_\nu = 0 \), if all the three parameters \( \phi_{1,2,3} \neq 0 \). (iv) Since \( \lambda_\nu \sim \sqrt{M_R} \), for low values of \( M_R \left( \lesssim 10^{12} \text{ GeV} \right) \) the asymmetry generated by the \( c_{\nu} \) term dominates but is typically too small to match the observed value of \( \eta_B \). In this regime we find the flat dependence on \( M_R \) typical of resonant leptogenesis. At \( M_R \gtrsim 10^{12} \text{ GeV} \) the quadratic terms \( c_{\nu}^{(i)} \) dominate the generation of the asymmetry, which grows linearly with \( M_R \). These specific features of resonant leptogenesis in MFV can be derived with a general analysis of CP-invariants and reproduced analytically. Properties (i),(iv) explain the characteristic behaviour of \( \eta_B \) as function of \( M_R \), shown in fig. 1 for reference values of the parameters. Varying all the parameters in a wide range range we find the result in fig. 2. In deriving this result we used the analytic formulae for leptogenesis without flavour effects of Ref.\textsuperscript{9} and assuming a loop hierarchy between the coefficients of the mass splittings. Under these assumptions, the right size for \( \eta_B \) can be reached for \( M_R \gtrsim 10^{12} \text{ GeV} \). The regime \( M_R \gg 10^{12} \text{ GeV} \) is particularly interesting since in this case the CP-violating parameters \( \phi_i \) are very small and we recover the predictive scheme of Ref.\textsuperscript{2}.

A MFV model is for instance the Minimal Supersymmetric Standard Model with degenerate right-handed neutrinos at the GUT scale. This scenario has been recently analyzed in Ref.\textsuperscript{7}. The same behaviour shown if fig. 2 was found, but the use of a different analysis of leptogenesis \textsuperscript{10} and the inclusion of flavour effects slightly enhanced the average value of \( \eta_B \) in the low \( M_R \) region so that successful leptogenesis could be achieved for some points of the parameter space down to \( 10^6 \text{ GeV} \).
5. Effect of high energy parameters on $l_i \rightarrow l_j \gamma$

In the MLFV framework, the effective Lagrangian relevant for the radiative decays $l_i \rightarrow l_j \gamma$ is

$$L_{\text{eff}} = \frac{1}{\Lambda_{LFV}^2} \left(c_{RL}^{(1)} O_{RL}^{(1)} + c_{RL}^{(2)} O_{RL}^{(2)} \right),$$

where

$$O_{RL}^{(1)} = g' H^\dagger \epsilon_R \sigma^{\mu\nu} \lambda_e \Delta_{FCNC} L L B_{\mu\nu},$$
$$O_{RL}^{(2)} = g H^\dagger \epsilon_R \sigma^{\mu\nu} \tau^a \lambda_e \Delta_{FCNC} L L W_{\mu\nu}^a,$$

and $g'$ ($g$) and $B_{\mu\nu}$ ($W_{\mu\nu}^a$) are the coupling constant and the field strength tensor of the $U(1)_Y$ ($SU(2)_L$) gauge group. This effective Lagrangian leads to

$$BR_{l_i \rightarrow l_j \gamma} \equiv \frac{\Gamma(l_i \rightarrow l_j \gamma)}{\Gamma(l_i \rightarrow \ell_j \nu_{\ell_j} \bar{\nu}_{\ell_j})} = 384 \pi^2 e^2 v^4 \lambda^2_{\text{LFV}} \left| c_{RL}^{(2)} - c_{RL}^{(1)} \right|^2 \sim \left( \frac{\Lambda_{LFV}}{\Lambda_{\text{LFV}}^2} \right)^2 f \left( m_{\nu_i}, m_{l_j}, \frac{U_{\PMNS} \phi_i}{\phi_i} \right).$$

(10)

6. Summary

In this talk we studied leptogenesis in a generic MFV scenario with right-handed Majorana neutrinos degenerate in mass. Radiative corrections lift the tree-level degeneracy of right-handed neutrinos and induce mass-splittings proportional to the neutrino and charged lepton Yukawa couplings. We showed that leptogenesis is viable and most efficient at high values of right-handed neutrino masses ($\gtrsim 10^{12}$ GeV), where it is driven by the mass-splittings quartic in the neutrino Yukawa couplings. As a consequence, predictions for $\mu \rightarrow e \gamma$ are enhanced and should be observable in next experiments for natural values of the scale of NP. High energy CP-violating parameters, that disappear in the see-saw relation but take part into leptogenesis, have a significative impact on low-energy processes. The expectation $BR(\mu \rightarrow e \gamma)/BR(\tau \rightarrow \mu \gamma) \ll 1$, valid in the CP limit, is recovered in the regime of very heavy right-handed neutrinos ($M_R \gg 10^{12}$ GeV).

References

1. G. D’Ambrosio, G. F. Giudice, G. Isidori and A. Strumia, Nucl. Phys. B 645 (2002) 155
2. V. Cirigliano, B. Grinstein, G. Isidori and M. B. Wise, Nucl. Phys. B 728 (2005) 121 [arXiv:hep-ph/0507001].
3. V. Cirigliano, G. Isidori and V. Porretti, arXiv:hep-ph/0607068.
4. V. Cirigliano and B. Grinstein, Nucl. Phys. B 752 (2006) 18 [arXiv:hep-ph/0601111].
5. S. Davidson and F. Palorini, arXiv:hep-ph/0607329.
6. B. Grinstein, V. Cirigliano, G. Isidori and M. B. Wise, arXiv:hep-ph/0608123.
7. G. C. Branco, A. J. Buras, S. Jager, S. Uhlig and A. Weiler, arXiv:hep-ph/0609067.
8. J. A. Casas and A. Ibarra, Nucl. Phys. B 618, 171 (2001) [hep-ph/0103065]; S. Pascoli, S. T. Petcov and C. E. Yaguna, Phys. Lett. B 564, 241 (2003) [hep-ph/0301095].
9. S. Blanchet and P. Di Bari, arXiv:hep-ph/0603107.
10. A. Pilaftsis and T. E. J. Underwood, Phys. Rev. D 72, 113001 (2005); T. E. J. Underwood, arXiv:hep-ph/0605232.