CKM matrix and FCNC suppression
in $SO(5) \times U(1) \times SU(3)$ gauge-Higgs unification

Shuichiro Funatsu$^1$, Hisaki Hatanaka$^2$, Yutaka Hosotani$^3$, Yuta Orikasa$^4$ and Naoki Yamatsu$^5$

$^1$Institute of Particle Physics and Key Laboratory of Quark and Lepton Physics (MOE), Central China Normal University, Wuhan, Hubei 430079, China
$^2$Osaka, Osaka 536-0014, Japan
$^3$Department of Physics, Osaka University, Toyonaka, Osaka 560-0043, Japan
$^4$Institute of Experimental and Applied Physics, Czech Technical University in Prague, Husova 240/5, 110 00 Prague 1, Czech Republic
$^5$Department of Physics, Kyoto University, Kyoto 606-8502, Japan

Abstract

The Cabibbo-Kobayashi-Maskawa (CKM) mixing matrix and flavor-changing neutral currents (FCNC’s) in the quark sector are examined in the GUT inspired $SO(5) \times U(1) \times SU(3)$ gauge-Higgs unification in which the 4D Higgs boson is identified with the Aharonov-Bohm phase in the fifth dimension. Gauge invariant brane interactions play an important role for the flavor mixing in the charged-current weak interactions. The CKM matrix is reproduced except that the up quark mass needs to be larger than the observed one. FCNC’s are naturally suppressed as a consequence of the gauge invariance, with a factor of order $10^{-6}$. It is also shown that induced flavor-changing Yukawa couplings are extremely small.
1 Introduction

The standard model (SM), $SU(3)_C \times SU(2)_L \times U(1)_Y$ gauge theory, has been firmly established at low energies. Yet it is not clear what the observed Higgs boson is. All of the Higgs couplings to other fields and to itself need to be determined with better accuracy in the coming experiments. The fundamental problem is the lack of a principle which regulates the Higgs interactions.

One possible answer is the gauge-Higgs unification in which the Higgs boson is identified with the zero mode of the fifth dimensional component of the gauge potential.[1]–[6] It appears as a fluctuation mode of the Aharonov-Bohm (AB) phase $\theta_H$ in the fifth dimension. As a concrete model, the $SU(3)_C \times SO(5) \times U(1)_X$ gauge theory in the Randall-Sundrum (RS) warped space has been proposed.[7]–[10] It gives nearly the same phenomenology at low energies as the standard model (SM).[10]–[12] Deviations of the gauge couplings of quarks and leptons from the SM values are less than $10^{-3}$ for $\theta_H \sim 0.1$. Higgs couplings of quarks, leptons, $W$ and $Z$ are approximately the SM values times $\cos \theta_H$, the deviation being about 1%. The Kaluza-Klein (KK) mass scale is about $m_{KK} \sim 8 \text{ TeV}$ for $\theta_H \sim 0.1$. The KK excited states contribute, for instance, in intermediate states of the two $\gamma$ decay of the Higgs boson. Their contribution is finite and very small. The signal strengths of various Higgs decay modes are approximately $\cos^2 \theta_H$ times the SM values. The branching fractions of those decay modes are approximately the same as in the SM.

Gauge-Higgs unification predicts $Z'$ bosons, which are the first KK modes of $\gamma$, $Z$, and $Z_R$ ($SU(2)_R$ gauge boson). Their masses are in the 6 TeV-9 TeV range for $\theta_H = 0.11$-0.07 in the model with quark-lepton multiplets introduced in the vector representation of $SO(5)$, which will be referred to as the A-model below. Those $Z'$ bosons have broad widths and can be produced at 14 TeV LHC. The current non-observation of $Z'$ signals puts the limit $\theta_H \lesssim 0.11$. Recently an alternative model with quark-lepton multiplets introduced in the spinor, vector, and singlet representations of $SO(5)$ (referred to as the B-model below) has been proposed,[13] which can be incorporated in the $SO(11)$ gauge-Higgs grand unification.[14, 15] Other variants of the fermion content have also been proposed.[16] Implications of gauge-Higgs unification to precision electroweak observables have been investigated. It has been shown that the typical models are consistent with the current measurements.

Distinct signals of the gauge-Higgs unification can be found in $e^+e^-$ collisions.[17]–[20] Large parity violation appears in the couplings of quarks and leptons to KK gauge bosons, particularly to the $Z'$ bosons. In the A-model right-handed quarks and charged leptons
have rather large couplings to $Z'$. The interference effects of $Z'$ bosons can be clearly observed at 250 GeV $e^+e^-$ international linear collider (ILC). In the process $e^+e^- \rightarrow \mu^+\mu^-$ the deviation from the SM amounts to $-4\%$ with the electron beam polarized in the right-handed mode by $80\%$ ($P_{e^-} = 0.8$) for $\theta_H \sim 0.09$, whereas there appears negligible deviation with the electron beam polarized in the left-handed mode by $80\%$ ($P_{e^-} = -0.8$).

In the forward-backward asymmetry $A_{FB}(\mu^+\mu^-)$ the deviation from the SM becomes $-2\%$ for $P_{e^-} = 0.8$. These deviations can be seen at 250 GeV ILC even with $250\text{fb}^{-1}$ data.\cite{21,22} In the B-model the pattern of the polarization dependence is reversed.

So far quarks and leptons in the gauge-Higgs unification models have been incorporated generation by generation so that the flavor mixing among quarks and leptons is left unexplained. In this paper we tackle the flavor mixing in the quark sector.\cite{23,24} We will argue in the B-model that the Cabibbo-Kobayashi-Maskawa (CKM) mixing matrix in the charged current interaction is reproduced with brane interactions. These brane interactions generally lead to flavor-changing neutral current (FCNC) interactions. It will be shown that the FCNC interactions are naturally suppressed in the gauge-Higgs unification as a consequence of the gauge invariance. The FCNC interaction is suppressed by a factor of $(m_b/m_{KK})^2 \sim 10^{-6}$ where $m_b$ and $m_{KK}$ are the bottom quark mass and the KK mass scale. It is also shown that induced flavor-changing Yukawa interactions are extremely small.

We stress that the natural suppression of FCNC in the gauge-Higgs unification results from the gauge-invariance and the orbifold structure, without relying on additional symmetry or mechanism. We present rigorous treatment of deriving and evaluating the CKM matrix and $Z$ couplings in the quark sector in the gauge-Higgs unification. We also give simple explanation in the effective theory of quarks and relevant heavy fields to illuminate the mechanism of suppressing FCNC interactions.

In section 2 the minimal GUT inspired $SU(3)_C \times SO(5) \times U(1)_X$ model of gauge-Higgs unification is described with brane interactions. In section 3 mass spectra and wave functions of gauge bosons and quarks are derived. Detailed derivation of the mass spectrum and mixing in the down-type quark sector is given. In section 4 an effective theory in 4D is formulated for quarks and $SO(5)$ singlet heavy fermion fields. We show how mass terms connecting down quarks and singlet fields lead to flavor mixing. It also illuminates how FCNC interactions are naturally suppressed. In section 5 we evaluate $W$ and $Z$ couplings of quarks, using the wave functions obtained in section 3. The gauge couplings turn out very close to those in the SM. It is confirmed that FCNC interactions are naturally suppressed. Section 6 is devoted to summary and discussions. Basis functions used in the text are summarized in the appendix.
2 Model

The GUT inspired $SU(3)_C \times SO(5) \times U(1)_X (\equiv G)$ gauge-Higgs unification has been introduced in ref. [13]. It is defined in the Randall-Sundrum (RS) warped space with metric given by

\[ ds^2 = g_{MN} dx^M dx^N = e^{-2\sigma(y)} \eta_{\mu\nu} dx^\mu dx^\nu + dy^2, \]

(2.1)

where $M, N = 0, 1, 2, 3, 5, \mu, \nu = 0, 1, 2, 3, y = x^5$, $\eta_{\mu\nu} = \text{diag}(-1, +1, +1, +1)$, $\sigma(y) = \sigma(y + 2L) = \sigma(-y)$, and $\sigma(y) = ky$ for $0 \leq y \leq L$. In terms of the conformal coordinate $z = e^{ky}$ ($1 \leq z \leq z_L = e^{kL}$) in the region $0 \leq y \leq L$

\[ ds^2 = \frac{1}{z^2} \left( \eta_{\mu\nu} dx^\mu dx^\nu + \frac{dz^2}{k^2} \right). \]

(2.2)

The bulk region $0 < y < L$ ($1 < z < z_L$) is anti-de Sitter (AdS) spacetime with a cosmological constant $\Lambda = -6k^2$, which is sandwiched by the UV brane at $y = 0$ ($z = 1$) and the IR brane at $y = L$ ($z = z_L$). The KK mass scale is $m_{KK} = \pi k/(z_L - 1) \simeq \pi k z_L^{-1}$ for $z_L \gg 1$.

Let us denote gauge fields of $SU(3)_C$, $SO(5)$, and $U(1)_X$ by $A_{M}^{SU(3)_C}$, $A_{M}^{SO(5)}$, and $A_{M}^{U(1)_X}$, respectively. The orbifold boundary conditions (BC) are given by

\[ \left( \begin{array}{c} A_{\mu} \\ A_{y} \end{array} \right) (x, y_j - y) = P_j \left( \begin{array}{c} A_{\mu} \\ -A_{y} \end{array} \right) (x, y_j + y) P_j^{-1} \]

(2.3)

for each gauge field where $(y_0, y_1) = (0, L)$. In terms of

\[ P_3^{SU(3)} = I_3, \]
\[ P_4^{SO(5)} = \text{diag} (I_2, -I_2), \]
\[ P_5^{SO(5)} = \text{diag} (I_4, -I_1), \]

(2.4)

$P_0 = P_1 = P_3^{SU(3)}$ for $A_M^{SU(3)_C}$ and $P_0 = P_1 = 1$ for $A_M^{U(1)_X}$. $P_0 = P_1 = P_5^{SO(5)}$ for $A_M^{SO(5)}$ in the vector representation and $P_4^{SO(5)}$ in the spinor representation, respectively. $P_4^{SO(5)}$ and $P_5^{SO(5)}$ break $SO(5)$ to $SO(4) \simeq SU(2)_L \times SU(2)_R$. $W$, $Z$ bosons and $\gamma$ (photon) are zero modes in the $SO(4)$ part of $A_\mu^{SO(5)}$, whereas the 4D Higgs boson is a zero mode in the $SO(5)/SO(4)$ part of $A_y^{SO(5)}$.

Matter fields are introduced both in 5D bulk and on the UV brane. They are listed in Table II. Quark multiplets $(3, 4)_5^-$ and $(3, 1)^{\pm}_{-\frac{1}{3}}$ are introduced in the 5D bulk in three
generations. They are denoted as $\Psi^{(3,4)}_{\alpha}(x, y)$ and $\Psi^{\pm \alpha}_{(3,1)}(x, y)$ ($\alpha = 1, 2, 3$). $\Psi^{(3,4)}_{\alpha}$ and $\Psi^{\pm \alpha}_{(3,1)}$ intertwine with each other. These fields obey boundary conditions

$$
\Psi^{(3,4)}_{\alpha}(x, y_j - y) = -P_4^{SO(5)} \gamma^5 \Psi^{(3,4)}_{\alpha}(x, y_j + y), \\
\Psi^{\pm \alpha}_{(3,1)}(x, y_j - y) = \mp \gamma^5 \Psi^{\pm \alpha}_{(3,1)}(x, y_j + y).
$$

(2.5)

With (2.5) the parity of quark fields are summarized in Table 2 with names adopted in the present paper.

Table 1: $\mathcal{G} = SU(3)_C \times SO(5) \times U(1)_X$ content of matter fields is shown in the GUT inspired model (B model) and previous model (A model). In the A model only $SU(3)_C \times SO(4) \times U(1)_X$ symmetry is preserved on the UV brane so that the $SU(2)_L \times SU(2)_R$ content is shown for brane fields. The B model is analyzed in the present paper.

| Field                  | B model                  | A model                  |
|-----------------------|--------------------------|--------------------------|
| quark                 | $(3, 4)_{\frac{1}{6}}$, $(3, 1)_{\frac{1}{3}}^+$, $(3, 1)_{-\frac{1}{3}}^-$ | $(3, 5)_{\frac{1}{6}}$, $(3, 5)_{-\frac{1}{2}}$ |
| lepton                | $(1, 4)_{-\frac{1}{2}}$  | $(1, 5)_0$, $(1, 5)_{-1}$ |
| dark fermion          | $(3, 4)_{\frac{1}{2}}$, $(1, 5)_0^+$, $(1, 5)_0^-$ | $(1, 4)_{\frac{1}{2}}$ |
| brane fermion         | $(1, 1)_0$               | $(3, [2, 1])_{\frac{1}{6}, \frac{1}{3}}$, $(3, [2, 1])_{\frac{1}{3}, \frac{1}{2}}$ |
| brane scalar          | $(1, 4)_{\frac{1}{2}}$   | $(1, [1, 2])_{\frac{1}{2}}$ |
| symmetry of brane interactions | $SU(3)_C \times SO(5) \times U(1)_X$ | $SU(3)_C \times SO(4) \times U(1)_X$ |

Table 2: Parity assignment ($P_0, P_1$) of quark multiplets in the bulk. In the third column $G_{22} = SU(2)_L \times SU(2)_R$ content is shown. Brane scalar field $\Phi_{(1,4)}$ is also listed at the bottom for convenience.

| Field | $\mathcal{G}$ | $G_{22}$ | left-handed | right-handed | name |
|-------|----------------|---------|-------------|--------------|------|
| $\Psi^{\alpha}_{(3,4)}$ | $(3, 4)_{\frac{1}{6}}$ | $[2, 1]$ | $(+, +)$    | $(-, -)$    | $u' c' t'$ |
|       |                | $[1, 2]$ | $(-, -)$    | $(+, +)$    | $d' s' b'$ |
| $\Psi^{\pm \alpha}_{(3,1)}$ | $(3, 1)_{-\frac{1}{2}}$ | $[1, 1]$ | $(\pm, \pm)$ | $(\mp, \mp)$ | $D^d_d D^s_s D^b_b$ |
| $\Phi_{(1,4)}$ | $(1, 4)_{\frac{1}{2}}$ | $[2, 1]$ | $\ldots$    | $\ldots$    | $\Phi_{[2, 1]}$ |
|       |                | $[1, 2]$ | $\ldots$    | $\ldots$    | $\Phi_{[1, 2]}$ |
The action of each gauge field, $A_M^{SU(3)_c}$, $A_M^{SO(5)}$, or $A_M^{U(1)_X}$, is given by

$$S^\text{gauge}_{\text{bulk}} = \int d^5 x \sqrt{-\det G} \left[ - \text{tr} \left( \frac{1}{4} F^{MN} F_{MN} + \frac{1}{2 \xi} (f_{gf})^2 + \mathcal{L}_{gh} \right) \right], \quad (2.6)$$

where $\sqrt{-\det G} = 1/kz^5$. Field strengths are defined by $F_{MN} = \partial_M A_N - \partial_N A_M - i g [A_M, A_N]$ with each 5D gauge coupling constant $g$. The gauge fixing and ghost terms are taken as

$$f_{gf} = z^2 \left\{ \eta^{\mu\nu} D^c_{\mu} A_0^q + \xi k^2 z D^c z \left( \frac{1}{z} A^q_z \right) \right\},$$

$$\mathcal{L}_{gh} = \bar{c} \left\{ \eta^{\mu\nu} D^c_{\mu} D_{\nu} + \xi k^2 z D^c z D_z \right\} c, \quad (2.7)$$

where $A_M = A^c_M + A_q^c$. $D^c_B B = \partial_M B - i g [A^c_M, B]$ and $D^c_{M+B} = \partial_M B - i g [A^c_M, B]$ where $B = A^q_B, A^q_z/z$ and $c$. Only $A_z$ component of $A^c_M$ has non-vanishing classical background $A^c_z$.

Each fermion multiplet $\Psi(x, y)$ in the bulk has its own bulk-mass parameter $c$. The covariant derivative is given by

$$D(c) = \gamma^A e_A^M \left(D_M + \frac{1}{8} \omega_{MBC}[\gamma^B, \gamma^C]\right) - c \sigma'(y),$$

$$D_M = \partial_M - i g_S A^M_{SU(3)} - i g_A A^M_{SO(5)} - i g_B Q_X A^M_{U(1)}. \quad (2.8)$$

Here $\sigma' = d\sigma(y)/dy$ and $\sigma'(y) = k$ for $0 < y < L$. $g_S, g_A, g_B$ are $SU(3)_c, SO(5), U(1)_X$ gauge coupling constants. The bulk part of the action for the quark multiplets are given by

$$S^\text{quark}_{\text{bulk}} = \int d^5 x \sqrt{-\det G} \sum_{\alpha=1}^3 \left\{ \overline{\Psi}_{(3,4)}^\alpha D(c_{\alpha}) \Psi_{(3,4)}^{\alpha} + \overline{\Psi}_{(3,1)}^{+\alpha} D(c_{D^+}) \Psi_{(3,1)}^{+\alpha} + \overline{\Psi}_{(3,1)}^{-\alpha} D(c_{D^-}) \Psi_{(3,1)}^{-\alpha} - m_D \overline{\Psi}_{(3,1)}^{\alpha} \Psi_{(3,1)}^{\alpha} \right\}, \quad (2.9)$$

where $\overline{\Psi} = i \Psi \gamma^0$. The bulk mass parameters of the $SO(5)$ spinor multiplets are denoted as $(c_1, c_2, c_3) = (c_u, c_c, c_t)$ below as each $c_\alpha$ is determined from the mass of each up-type quark. For the $SO(5)$ singlet multiplets we consider the case $c_{D^+} = c_{D^-} \equiv c_{D_\alpha}$ in the present paper. (An alternative choice $c_{D^\pm} = -c_{D^-}$ is also possible. See ref. [13].)

The action for the brane scalar field $\Phi_{(1,4)}(x)$ is given by

$$S^\Phi_{\text{brane}} = \int d^5 x \sqrt{-\det G} \delta(y)$$
\[
\times \left\{ - (D_\mu \Phi_{(1,4)})^\dagger D^\mu \Phi_{(1,4)} - \lambda_{\Phi_{(1,4)}}(\Phi_{(1,4)}^\dagger \Phi_{(1,4)} - |w|^2)^2 \right\},
\]

\[
D_\mu \Phi_{(1,4)} = \left\{ \partial_\mu - ig_A \sum_{\alpha=1}^{10} A^a_\mu T^a - ig_B Q_X B_\mu \right\} \Phi_{(1,4)}.
\]

A spinor 4 of \(SO(5)\) is decomposed to \([2, 1] \oplus [1, 2]\) of \(SO(4) \simeq SU(2)_L \times SU(2)_R\). \(\Phi_{(1,4)}\) develops a nonvanishing vacuum expectation value (VEV);

\[
\Phi_{(1,4)} = \begin{pmatrix} \Phi_{[2,1]}^\dagger \\ \Phi_{[1,2]}^\dagger \end{pmatrix}, \quad \langle \Phi_{[1,2]} \rangle = \begin{pmatrix} 0 \\ w \end{pmatrix},
\]

which reduces the symmetry \(G' = SU(3)_C \times SO(4) \times U(1)_X\) to the SM gauge group \(G_{SM} = SU(3)_C \times SU(2)_L \times U(1)_Y\). It is assumed that \(w \gg m_{KK}\), which ensures that boundary conditions for the 4D components of gauge fields corresponding to broken generators in the breaking \(SU(2)_R \times U(1)_X \rightarrow U(1)_Y\) obey effectively Dirichlet conditions at the UV brane for low-lying KK modes. Accordingly the mass of the neutral physical mode of \(\Phi_{(1,4)}\) is much larger than \(m_{KK}\).

There are brane interactions on the UV brane which are invariant under \(G = SU(3)_C \times SO(5) \times U(1)_X\).

\[
S_{\text{brane}}^{\text{int}} = - \int d^5x \sqrt{-G} \delta(y) \left\{ \sum_{\alpha,\beta} \kappa_{\alpha\beta} \bar{\Psi}_{(3,4),\alpha}^\dagger \Phi_{(1,4),\beta}^\dagger \Phi_{(1,4),\beta}^\dagger + \text{h.c.} \right\},
\]

where \(\kappa_{\alpha\beta}\)'s are coupling constants. If only the gauge invariance under \(G'\) were imposed, there would appear additional brane interactions. Instead of (2.12) one would have

\[
\sum_{\alpha,\beta} \left\{ \kappa^{(1)}_{\alpha\beta} \bar{\Psi}_{(3,2,1)\alpha}^\dagger \Phi_{(1,2,1)\beta}^\dagger \Psi_{(3,1)}^{+\beta} + \kappa^{(2)}_{\alpha\beta} \bar{\Psi}_{(3,1,2)\alpha}^\dagger \Phi_{(1,1,2)\beta}^\dagger \Psi_{(3,1)}^{+\beta} \right\} + \text{h.c.}
\]

in the Lagrangian density. The invariance under \(G\) implies \(\kappa^{(1)}_{\alpha\beta} = \kappa^{(2)}_{\alpha\beta}\). For fermion fields we define \(\tilde{\Psi} = z^{-2} \Psi\). With nonvanishing VEV \(\langle \Phi_{(1,4)} \rangle \neq 0\), (2.12) generates mass terms

\[
S_{\text{brane mass}}^{\text{fermion}} = \int d^5x \sqrt{-G} \delta(y) \left\{ \sum_{\alpha,\beta} 2\mu_{\alpha\beta} \bar{\Psi}_{R}^{\dagger \alpha} \tilde{D}_{L}^{+\beta} \tilde{D}_{L}^{\beta} + \text{h.c.} \right\},
\]

where \(2\mu_{\alpha\beta} = \sqrt{2} \kappa_{\alpha\beta} w\), \((d^1, d^2, d^3) = (d^s, b^s, b^l)\) and \((D^1, D^2, D^3) = (D^{d^s}, D^{b^s}, D^{b^l})\). Only the \(\kappa^{(2)}_{\alpha\beta}\) part in the decomposition (2.13) gives rise to mass terms. Brane interaction of the form \(\bar{\Psi}_{(3,4),\alpha}^\dagger \Phi_{(1,4),\beta}^\dagger \Psi_{(3,1)}^{-\beta}\) is possible, which, however, does not yield a mass term as \(D_L^{-\beta}|y=0 = 0\) due to the BC. It will be shown below that the brane interactions (2.12) lead to the flavor mixing, yielding the CKM matrix in the charged current interactions. We stress that the brane interactions (2.12) respect full \(G = SU(3)_C \times SO(5) \times U(1)_X\) gauge invariance. It may be contrasted to the earlier attempts [23, 24] to incorporate
flavor mixing in higher dimensional theories where only $SU(3)_C \times SU(2)_L \times U(1)_Y$ gauge invariance is respected. We note the same mass terms are generated from (2.13) so that the results obtained below remain valid even with only the $G'$ invariance imposed on the brane so long as $|\kappa^{(1)}_{\alpha\beta}/\kappa^{(2)}_{\alpha\beta}|$ is not extremely large.

Nonvanishing VEV $\langle \Phi_{(1,4)} \rangle$ also breaks $SU(2)_R \times U(1)_X$ to $U(1)_Y$. $U(1)_Y$ gauge field $B^Y_M$ is given in terms of $SU(2)_R$ gauge fields $A^a_M$ and $U(1)_X$ gauge field $B_M$ by

$$B^Y_M = s_\phi A^3_M + c_\phi B_M ,$$

$$c_\phi = \frac{g_A}{\sqrt{g_A^2 + g_B^2}} , \quad s_\phi = \frac{g_B}{\sqrt{g_A^2 + g_B^2}} ,$$

where $g_A$ and $g_B$ are gauge couplings in $SO(5)$ and $U(1)_X$, respectively. The 5D $U(1)_Y$ gauge coupling is given by $g^5_Y = g_A s_\phi$. The 4D $SU(2)_L$ gauge coupling is given by $g_w = g_A/\sqrt{L}$.

The 4D Higgs boson doublet $\phi_H(x)$ is the zero mode contained in the $A_z = (kz)^{-1}A_y$ component;

$$A_z^{(j5)}(x, z) = \frac{1}{\sqrt{k}} \phi_j(x) u_H(z) + \cdots , \quad u_H(z) = \sqrt{\frac{2}{z_L^2 - 1}} z ,$$

$$\phi_H(x) = \frac{1}{\sqrt{2}} \left( \begin{array}{c} \phi_2 + i\phi_1 \\ \phi_4 - i\phi_3 \end{array} \right) .$$

(2.16)

Without loss of generality we assume $\langle \phi_1 \rangle, \langle \phi_2 \rangle, \langle \phi_3 \rangle = 0$ and $\langle \phi_4 \rangle \neq 0$, which is related to the Aharonov-Bohm (AB) phase $\theta_H$ in the fifth dimension by $\langle \phi_4 \rangle = \theta_H f_H$ where

$$f_H = \frac{2}{g_w} \sqrt{\frac{k}{L(z_L^2 - 1)}} .$$

(2.17)

## 3 Mass spectrum and wave functions

Manipulations are simplified in the twisted gauge [26 27] defined by an $SO(5)$ gauge transformation

$$\tilde{A}_M(x, z) = \Omega A_M \Omega^{-1} + \frac{i}{g_A} \Omega \partial_M \Omega^{-1} ,$$

$$\Omega(z) = \exp \left\{ i\theta(z) T^{(45)} \right\} , \quad \theta(z) = \frac{z^2}{z_L^2 - 1} ,$$

(3.1)

where $T^{jk}$’s are $SO(5)$ generators and $A_M = 2^{-1/2} \sum_{1 \leq j < k \leq 5} A_M^{(jk)} T^{jk}$. In the twisted gauge the background field vanishes ($\hat{\theta}_H = 0$) so that all fields satisfy free equations in the RS space in the bulk. Boundary conditions at the UV brane are modified, whereas boundary conditions at the IR brane remain the same as in the original gauge.
3.1 Gauge fields

The masses of $W$ and $Z$ bosons at the tree level, $m_W = k\lambda_W$ and $m_Z = k\lambda_Z$, are determined by

$$
2S(1; \lambda_W)C'(1; \lambda_W) + \lambda_W \sin^2 \theta_H = 0 ,
$$

$$
2S(1; \lambda_Z)C'(1; \lambda_Z) + (1 + s_\phi^2)\lambda_Z \sin^2 \theta_H = 0 ,
$$

(3.2)

where functions $S(z; \lambda)$ and $C(z; \lambda)$ are given in (A.2) and $s_\phi$ is defined in (2.15). The masses are approximately given by

$$
m_W \simeq \sqrt{\frac{k}{L}} z_L^{-1} \sin \theta_H \simeq \frac{\sin \theta_H}{\pi \sqrt{kL}} m_{KK} ,
$$

$$
m_Z \simeq \sqrt{1 + s_\phi^2} m_W .
$$

(3.3)

$s_\phi$ is related to the Weinberg angle at the tree level by

$$
\sin^2 \theta_W^0 = s_\phi^2 / \sqrt{1 + s_\phi^2} .
$$

Let us define

$$
A_{aL}^M = \frac{1}{\sqrt{2}} \left( \frac{1}{2} \epsilon_{abc} A_{M}^{(bc)} + A_{M}^{(a4)} \right) ,
$$

$$
A_{aR}^M = \frac{1}{\sqrt{2}} \left( \frac{1}{2} \epsilon_{abc} A_{M}^{(bc)} - A_{M}^{(a4)} \right) ,
$$

$$
A_{\hat{p}}^M = A_{M}^{(p5)} ,
$$

(3.4)

where $a, b, c = 1 \sim 3$ and $p = 1 \sim 4$. $A_{aL}^M$ and $A_{aR}^M$ are gauge fields of $SU(2)_L$ and $SU(2)_R$. For $W$ and $Z$ bosons and photon $\gamma$ we define

$$
\begin{bmatrix}
\hat{W}_\mu(x, z) \\
\hat{W}_\mu^S(x, z)
\end{bmatrix} = \sqrt{k} W_\mu(x) \frac{1}{\sqrt{r_W}} \begin{bmatrix} C(z, \lambda_W) \\ \hat{S}(z, \lambda_W) \end{bmatrix} ,
$$

$$
\begin{bmatrix}
\hat{Z}_\mu(x, z) \\
\hat{Z}_\mu^S(x, z)
\end{bmatrix} = \sqrt{k} Z_\mu(x) \frac{1}{\sqrt{r_Z}} \begin{bmatrix} C(z, \lambda_Z) \\ \hat{S}(z, \lambda_Z) \end{bmatrix} ,
$$

$$
\hat{A}_\mu^\gamma = \sqrt{k} A_\mu^\gamma(x) \frac{1}{\sqrt{r_L}} , \quad \hat{S}(z, \lambda) = \frac{C(1, \lambda)}{S(1, \lambda)} S(z, \lambda) ,
$$

(3.5)

where

$$
r_W = \int_{z_L}^{z_L} \frac{dz}{z} \left\{ (1 + c_H^2) C(z, \lambda_W)^2 + s_H^2 \hat{S}(z, \lambda_W)^2 \right\} ,
$$

$$
r_Z = \int_{z_L}^{z_L} \frac{dz}{z} \left\{ [c_\phi^2 + (1 + s_\phi^2)c_H^2] C(z, \lambda_Z)^2 + (1 + s_\phi^2)s_H^2 \hat{S}(z, \lambda_Z)^2 \right\} ,
$$

9
Here $W_\mu(x)$, $Z_\mu(x)$ and $A_\mu'(x)$ represent canonically normalized $W$, $Z$, and $\gamma$ fields, respectively. Note that $\lambda_W z_L$, $\lambda_Z z_L \ll 1$. For $\lambda z_L \ll 1$, $C(z, \lambda) \sim z_L$ and $\hat{S}(z, \lambda) \sim z_L(1-z^2/z_c^2)$.

Couplings of $W$, $Z$, and $\gamma$ are obtained by inserting

$$
\begin{bmatrix}
\hat{A}_\mu^L - i\hat{\bar{A}}^2_\mu \\
\hat{A}_\mu^R - i\hat{\bar{A}}^2_\mu \\
\hat{A}_\mu^3 - i\hat{\bar{A}}^2_\mu \\
B_\mu
\end{bmatrix}
= 
\begin{bmatrix}
(1 + c_H)\hat{W}_\mu^L \\
(1 - c_H)\hat{W}_\mu^R \\
-\sqrt{2} s_H \hat{W}_\mu^S \\
\end{bmatrix}, 
$$

$$
\begin{bmatrix}
\hat{A}_\mu^L \\
\hat{A}_\mu^R \\
\hat{A}_\mu^3 \\
B_\mu
\end{bmatrix} = \begin{bmatrix}
(1 + c_H)\hat{Z}_\mu^L \\
(1 - c_H)\hat{Z}_\mu^R \\
-\sqrt{2} s_H \hat{Z}_\mu^S \\
0
\end{bmatrix} + \frac{1}{\sqrt{1 + s_\phi^2}} \begin{bmatrix}
s_\phi \\
s_\phi \\
0 \\
c_\phi
\end{bmatrix} (\hat{A}_\mu^3 - \sqrt{2} s_\phi \hat{Z}_\mu^S) $$

(3.7)

in the $SO(5)$ gauge fields $\hat{A}_\mu$ in the twisted gauge and $U(1)_Y$ gauge field $B_\mu^Y$ in the action.

### 3.2 Up-type quarks

Up, charm, and top quarks are zero modes contained solely in the fields $\Psi^{Q}_{(3,4)}$ and there arises no mixing in generation. The mass spectrum $m_q = k\lambda_q \ (q = u, c, t)$ is determined by

$$
S_L(1; \lambda, c_q)S_R(1; \lambda, c_q) + \sin^2 \frac{1}{2} \theta_H = 0 .
$$

(3.8)

Basis functions for fermions, $S_{L/R}(z, \lambda, c)$ and $C_{L/R}(z, \lambda, c)$, are given by (A.3). For the first and second generation $|c_u|, |c_c| > \frac{1}{2}$, whereas for the third generation $|c_t| < \frac{1}{2}$. The masses are approximately given by

$$
m_{u,c} \sim \pi^{-1} \sqrt{4c_{u,c}^2 - 1} z_L^{|c_{u,c}|+0.5} \sin \frac{1}{2} \theta_H m_{KK},
$$

$$
m_t \sim \pi^{-1} \sqrt{1 - 4c_t^2} \sin \frac{1}{2} \theta_H m_{KK} .
$$

(3.9)

4D fields denoted by $\hat{u}(x)$ appear in the $(u, u')$ components in the 5D fields $\Psi^{Q}_{(3,4)}(x, z)$. (See Table 2) In the twisted gauge,

$$
\begin{bmatrix}
\hat{u} \\
\hat{\bar{z}} \hat{u}'
\end{bmatrix} = \sqrt{\frac{r_u}{r_u}} \left\{ \hat{u}_L(x) \left[ \hat{c}_H C_L(z; \lambda_u, c_u) \right] + \hat{u}_R(x) \left[ \hat{c}_H S_R(z; \lambda_u, c_u) \right] \right\},
$$

$$
r_u = \int_1^{z_L} dz \left\{ \hat{c}_H^2 C_L(z; \lambda_u, c_u)^2 + \hat{s}_H^2 \hat{S}_L(z; \lambda_u, c_u)^2 \right\}
$$

$$
= \int_1^{z_L} dz \left\{ \hat{e}_H^2 S_R(z; \lambda_u, c_u)^2 + \hat{s}_H^2 \hat{C}_R(z; \lambda_u, c_u)^2 \right\},
$$

(3.10)
\[ c_H = \cos \frac{1}{2} \theta_H, \quad s_H = \sin \frac{1}{2} \theta_H, \]
\[ \hat{S}_L(z; \lambda, c) = \frac{C_L(1; \lambda, c)}{S_L(1; \lambda, c)} S_L(z; \lambda, c), \quad \hat{C}_R(z; \lambda, c) = \frac{C_L(1; \lambda, c)}{S_L(1; \lambda, c)} C_R(z; \lambda, c). \] (3.10)

The equality of the two expressions for \( r_u \) is confirmed with the aid of (3.8). Formulas for charm and top quark fields are obtained by substitution \( u \to c, t \).

### 3.3 Down-type quarks

Down, strange and bottom quarks are contained in \( \Psi^\alpha_{(3,4)} \) and \( \Psi^{\pm \alpha}_{(3,1)} \). By the brane interactions (2.12) and (2.14) all three generations mix with each other. In ref. [13] the mass spectrum is determined in each generation separately. Generalization to the case with mixing is straightforward. We consider the case in which both \( \Psi^{+ \alpha}_{(3,1)} \) and \( \Psi^{- \alpha}_{(3,1)} \) have the same bulk mass parameters \( c_{D_\pm} = c_{D^-} \equiv c_{D_\alpha} \). Without loss of generality we assume Dirac masses \( m_{D_\alpha} \) in (2.9) are real.

For the sake of clarity we adopt vector/matrix notation in the generation space. Fermion fields are expressed in terms of “checked” fields; \( \tilde{\psi} = z^{-2 \psi} \). Write

\[ \tilde{\mathbf{d}} = \begin{pmatrix} \tilde{d} \\ \tilde{s} \\ \tilde{b} \end{pmatrix}, \quad \tilde{\mathbf{d}} = \begin{pmatrix} \tilde{d}' \\ \tilde{s}' \\ \tilde{b}' \end{pmatrix}, \quad \tilde{D}^\pm = \begin{pmatrix} D_\pm^d \\ D_\pm^s \\ D_\pm^b \end{pmatrix}, \]

\[ D_\pm^\parallel = \begin{pmatrix} D_\pm(c_u) \\ D_\pm(c_c) \\ D_\pm(c_t) \end{pmatrix}, \quad D_\pm(c) = \pm \frac{\partial}{\partial z} + \frac{c}{z}, \]

\[ D_\pm^D = \begin{pmatrix} D_\pm(c_{D_d}) \\ D_\pm(c_{D_s}) \\ D_\pm(c_{D_b}) \end{pmatrix}, \]

\[ \tilde{\mathbf{m}}_D = \begin{pmatrix} \tilde{m}_{D_d} \\ \tilde{m}_{D_s} \\ \tilde{m}_{D_b} \end{pmatrix}, \quad \tilde{\mathbf{m}}_D = \frac{m_{D_\alpha}}{k}, \]

\[ \mu = \begin{pmatrix} \mu_{11} & \mu_{12} & \mu_{13} \\ \mu_{21} & \mu_{22} & \mu_{23} \\ \mu_{31} & \mu_{32} & \mu_{33} \end{pmatrix}. \] (3.11)

In terms of two-component 4D Lorentz spinors (\( \tilde{d}_L, \tilde{d}_R \) etc.) the equations of motion in the original gauge are given by

\[ (a) : \quad \sigma^\mu \partial_\mu \left( \tilde{d}_L \frac{\partial}{\partial \tilde{d}_L} \right) - k \tilde{D}_+^q \left( \tilde{d}_R \frac{\partial}{\partial \tilde{d}_R} \right) = 0, \]

\[ (b) : \quad \tilde{\sigma}^\mu \partial_\mu \left( \tilde{d}_R \frac{\partial}{\partial \tilde{d}_R} \right) - k \tilde{D}_-^q \left( \tilde{d}_L \frac{\partial}{\partial \tilde{d}_L} \right) = \delta(y) 2\mu \left( \frac{0}{\tilde{D}_L^+} \right), \]
\((e)\) : \(\sigma^\mu \partial_\mu \vec{D}_L^+ - kD_+^D \vec{D}_R^+ - \frac{k\tilde{m}_D}{z} \vec{D}_R^- = \delta(y)2\mu^\dagger \tilde{d}_R\),

\((f)\) : \(\bar{\sigma}^\mu \partial_\mu \bar{D}_L^+ - kD_+^D \bar{D}_R^+ - \frac{k\tilde{m}_D}{z} \bar{D}_R^- = 0\),

\((g)\) : \(\sigma^\mu \partial_\mu \bar{D}_L^- - kD_-^D \bar{D}_R^- - \frac{k\tilde{m}_D}{z} \bar{D}_R^+ = 0\),

\((h)\) : \(\bar{\sigma}^\mu \partial_\mu \bar{D}_R^- - kD_-^D \bar{D}_L^- - \frac{k\tilde{m}_D}{z} \bar{D}_L^+ = 0\).  \(\text{(3.12)}\)

The \(\mu\) terms on the right side of the equations come from the brane interaction \((2.14)\).

The derivative \(\hat{D}_\pm^\alpha\) in Eqs. \((a)-(d)\) represents, in each generation subspace,

\[\hat{D}_\pm(c) = D_\pm(c) \pm i\theta(z)T^{45},\]

\[T^{45} = \frac{1}{2} \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \text{ for } \left( \begin{array}{c} \hat{d} \\ \hat{d}' \end{array} \right), \left( \begin{array}{c} \hat{s} \\ \hat{s}' \end{array} \right), \left( \begin{array}{c} \hat{b} \\ \hat{b}' \end{array} \right),\]

\(\text{where } \theta(z) \text{ is given by (3.11)\}. Note that the mass dimension of each coupling and field is e.g., \([d_{R/L}] = 2, [k] = 1\) and \([\tilde{m}_D] = [\mu] = 0\).

Boundary conditions at the IR brane \((z = z_L)\) are, in the original gauge,

\[
\begin{align*}
\vec{d}_R &= 0, \\
D_+^D \vec{d}_L &= 0, \\
D_-^D \vec{d}_R &= 0, \\
\vec{d}_L &= 0,
\end{align*}
\]

\(\text{Fields in the twisted gauge } (\tilde{\chi}) \text{ are related to those in the original gauge } (\chi) \text{ by}

\[
\chi = \begin{pmatrix} \cos \frac{1}{2}\theta(z) & -i \sin \frac{1}{2}\theta(z) \\ -i \sin \frac{1}{2}\theta(z) & \cos \frac{1}{2}\theta(z) \end{pmatrix} \tilde{\chi},
\]

\[
\chi = \begin{pmatrix} \hat{d} \\ \hat{d}' \end{pmatrix}, \begin{pmatrix} \hat{s} \\ \hat{s}' \end{pmatrix}, \begin{pmatrix} \hat{b} \\ \hat{b}' \end{pmatrix},
\]

\(\text{so that all fields in the twisted gauge obey the same boundary conditions as (3.14).}

In the twisted gauge all fields in the bulk \((1 < z < z_L)\) satisfy free equations with vanishing background field \(\tilde{\theta}_H = 0\). General solutions satisfying BC \((3.14)\) are

\[
\begin{align*}
\vec{d}_R &= \begin{pmatrix} \alpha_d S_R(z; \lambda, c_u) \\ \alpha_\delta S_R(z; \lambda, c_c) \\ \alpha_\beta S_R(z; \lambda, c_t) \end{pmatrix}, \\
\vec{d}_L &= \begin{pmatrix} \alpha_d C_L(z; \lambda, c_u) \\ \alpha_\delta C_L(z; \lambda, c_c) \\ \alpha_\beta C_L(z; \lambda, c_t) \end{pmatrix},
\end{align*}
\]

\[
\begin{align*}
\vec{d}'_R &= \begin{pmatrix} \alpha_d C_R(z; \lambda, c_u) \\ \alpha_\delta C_R(z; \lambda, c_c) \\ \alpha_\beta C_R(z; \lambda, c_t) \end{pmatrix}, \\
\vec{d}'_L &= \begin{pmatrix} \alpha_d S_L(z; \lambda, c_u) \\ \alpha_\delta S_L(z; \lambda, c_c) \\ \alpha_\beta S_L(z; \lambda, c_t) \end{pmatrix},
\end{align*}
\]
\[ \bar{D}^+_R = \left( a_d \mathcal{S}_{R2}(z; \lambda, c_{Dd}, \tilde{m}_{Dd}) + b_d \mathcal{S}_{R1}(z; \lambda, c_{Dd}, \tilde{m}_{Dd}) \right), \]
\[ \bar{D}^+_L = \left( a_d \mathcal{C}_{L2}(z; \lambda, c_{Dd}, \tilde{m}_{Dd}) + b_d \mathcal{C}_{L1}(z; \lambda, c_{Dd}, \tilde{m}_{Dd}) \right), \]
\[ \bar{D}^-_R = \left( a_d \mathcal{C}_{R1}(z; \lambda, c_{Dd}, \tilde{m}_{Dd}) + b_d \mathcal{C}_{R2}(z; \lambda, c_{Dd}, \tilde{m}_{Dd}) \right), \]
\[ \bar{D}^-_L = \left( a_d \mathcal{S}_{L1}(z; \lambda, c_{Dd}, \tilde{m}_{Dd}) + b_d \mathcal{S}_{L2}(z; \lambda, c_{Dd}, \tilde{m}_{Dd}) \right). \] (3.16)

The tilde \( \sim \) above each field indicates that it is in the twisted gauge. Note \( \bar{D}^\pm = \bar{D}^\pm \).

Functions \( \mathcal{S}_{R1}(z; \lambda, c, \tilde{m}) \) etc. are defined in (A.4). The coefficients

\[ \bar{a} = \left( \frac{\alpha_d}{\alpha_s} \right), \quad \bar{a}' = \left( \frac{\alpha_{d'}}{\alpha_{s'}} \right), \quad \bar{a} = \left( \frac{a_d}{a_s} \right), \quad \bar{b} = \left( \begin{array}{c} b_d \\ b_s \end{array} \right) \] (3.17)

are determined such that BC at \( z = 1^+ \) \( (y = +\epsilon) \) be satisfied.

To find BC at \( z = 1^+ \), first note that in the \( y \) coordinate

\[ D_\pm(c) = \frac{e^{-\sigma(y)}}{k} \left\{ \pm \frac{\partial}{\partial y} + c \alpha'(y) \right\}. \] (3.18)

Fields \( \bar{d}_L, \bar{d}_R, D_L^+ \) and \( D_R^- \) are parity even at \( y = 0 \), whereas \( \bar{d}_R, \bar{d}_L, D_R^+ \) and \( D_L^- \) are parity odd. We integrate the equations for parity odd fields, (a), (d), (e) and (h) in (3.12), from \( y = -\epsilon \) to \(+\epsilon\) to find

\[ \bar{d}_R(\epsilon) = 0, \]
\[ \bar{d}_L(\epsilon) + \mu \bar{D}^+_L(0) = 0, \]
\[ \bar{D}^+_R(\epsilon) - \mu^1 \bar{d}_R(0) = 0, \]
\[ \bar{D}^-_L(\epsilon) = 0. \] (3.19)

For parity even fields we evaluate the equations (b), (c), (f) and (g) at \( y = +\epsilon \), by using (3.19), to find

\[ \hat{D}_d \bar{d}_R + \mu \left\{ D^D \bar{d}_R + \tilde{m}_D \bar{D}_R \right\} = 0, \]
\[ \hat{D}_d \bar{d}_L = 0, \]
\[ D^D \bar{D}_R^+ - \mu^1 D^D \bar{d}_L^+ = 0, \]
\[ D^D \bar{D}_R^- + \tilde{m}_D \bar{D}_R^+ = 0. \] (3.20)
Inserting (3.16) into (3.19) and (3.20), one finds equations for the coefficient vectors in (3.17). The conditions (3.19) and (3.20) are split into two sets, one for left-handed components and the other for right-handed components. The two sets yield equivalent conditions. Making use of the relation (3.15) and equations \( D_+ (C_L, S_L) = \lambda (S_R, C_R) \), \( D_+ (C_{Lj}, S_{Lj}) = \lambda (S_{Rj}, C_{Rj}) - (\tilde{m}/z) (S_{Lk}, C_{Lk}) \) \((j, k) = (1, 2), (2, 1)\) etc., one finds for the set of left-handed components that

\[
(p_1) : \bar{c}_H S^q_R \bar{\alpha} - i \bar{s}_H C^q_R \bar{\alpha}' = 0 ,
\]

\[
(p_2) : - i \bar{s}_H C^q_R \bar{\alpha} + \bar{c}_H S^q_L \bar{\alpha}' + \mu \{ C^D_{L2} \bar{a} + C^D_{L1} \tilde{b} \} = 0 ,
\]

\[
(p_3) : S^D_{L1} \bar{a} + S^D_{L2} \tilde{b} = 0 ,
\]

\[
(p_4) : S^D_{R2} \bar{a} + S^D_{R1} \tilde{b} - \mu \{ - i \bar{s}_H S^q_R \bar{\alpha} + \bar{c}_H C^q_R \bar{\alpha}' \} = 0 ,
\]

(3.21)

where

\[
S^q_R = \begin{pmatrix} S_R(1,\lambda,c_u) & \ CR(1,\lambda,c_c) \\ S_R(1,\lambda,c_t) & \ CR(1,\lambda,c_t) \end{pmatrix} ,
\]

\[
S^D_{Rj} = \begin{pmatrix} S_{Rj}(1,\lambda,c_{Dd},m_{Dd}) & \ CR_{Rj}(1,\lambda,c_{Dd},m_{Dd}) \\ S_{Rj}(1,\lambda,c_{Dd},m_{Dd}) & \ CR_{Rj}(1,\lambda,c_{Dd},m_{Dd}) \end{pmatrix} ,
\]

(3.22)

and so on. With the use of \((p_1)\) and \((p_3)\), \(\bar{\alpha}'\) and \(\tilde{b}\) are expressed in terms of \(\bar{\alpha}\) and \(\bar{a}\), respectively. Then \((p_2)\) and \((p_4)\) become

\[
\frac{i}{\bar{s}_H} \{ \bar{s}_H^2 C^q_L + \bar{c}_H^2 S^q_L (C^q_R)^{-1} S^q_R \} \bar{\alpha} - \mu \{ C^D_{L2} - C^D_{L1} (S^D_{L2})^{-1} S^D_{L1} \} \bar{a} = 0 ,
\]

\[
\{ S^D_{R2} - S^D_{R1} (S^D_{L2})^{-1} S^D_{L1} \} \bar{a} + \frac{i}{\bar{s}_H} \mu S^q_R \bar{\alpha} = 0 .
\]

(3.23)

All matrices in (3.23) except for \(\mu\) are diagonal. Eliminating \(\bar{a}\), one finds that

\[
K(\lambda) S^q_R \bar{\alpha} = 0 ,
\]

\[
K(\lambda) = \frac{S^q_L S^q_R + \bar{s}_H^2}{S^q_R C^q_R} + \mu \frac{C^D_{L1} S^D_{L1} - C^D_{L2} S^D_{L2}}{S^D_{R1} S^D_{L1} - S^D_{R2} S^D_{L2}} \bar{\mu} .
\]

(3.24)

The mass spectrum \(m_n = k \lambda_n\) of down-type quarks is obtained by

\[
\det K(\lambda_n) = 0 .
\]

(3.25)

Three lowest roots correspond to \(m_d, m_s, m_b\). In the \(\mu \rightarrow 0\) limit, the down-quark spectrum is given by \(\det (S^q_L S^q_R + \bar{s}_H^2) = 0\), the same formula as for the up-quark spectrum,
and the spectrum of $D^\pm$ fields is given by $\det(S^D_{R_1}S^D_{L_1} - S^D_{R_2}S^D_{L_2}) = 0$. As pointed out in ref. [13], the spectrum for $c_u, c_c > 0$ contains exotic light fermions when $\mu \neq 0$. For this reason we take $c_u, c_c, c_t < 0$. We shall see below that gauge couplings of quarks remain very close to those in the SM for $c_u, c_c, c_t < 0$ as well.

The coefficient vector $S^q_R \vec{\alpha}$ of each down-type quark is an eigenvector of $K(\lambda_n)$ with a zero eigenvalue. Once $\vec{\alpha}$ is determined, $\vec{a}$, and $\vec{\alpha}'$ and $\vec{b}$ are determined. Consequently the wave functions in (3.16) are determined, with which all gauge couplings can be evaluated.

## 4 Effective theory of CKM and FCNC

Before evaluating the $W, Z$ gauge couplings of quarks by using exact wave functions obtained in section 3, it is instructive to write down an effective theory of relevant fields to see how the brane interactions $\mu$ lead to flavor mixing and FCNC. The effective theory illuminates also how FCNC interactions are naturally suppressed.

One crucial ingredient for lifting the degeneracy in the masses of up and down quarks is that right-handed component of down quark is mixture of $d'$ and $D^+_d$. As confirmed in the next section, dominant part of physical down-type quarks, $(\hat{d}_R, \hat{s}_R, \hat{b}_R)$, are contained in $(D^-_{dR}, D^-_{sR}, D^-_{bR})$. It also assures that the $W$ boson barely couples to right-handed components of physical up-type quarks as they are contained solely in $\Psi^{q}_{(3,4)}$.

### 4.1 Mass matrix

To simplify expressions, we use the following vector notation for 4D fermion fields in this section.

$$
\vec{u} = \begin{pmatrix} u \\ c \\ t \end{pmatrix}, \quad \vec{u}' = \begin{pmatrix} u' \\ c' \\ t' \end{pmatrix}, \quad \vec{d} = \begin{pmatrix} d \\ s \\ b \end{pmatrix}, \quad \vec{d}' = \begin{pmatrix} d' \\ s' \\ b' \end{pmatrix}, \quad \vec{D} = \begin{pmatrix} D_d \\ D_s \\ D_b \end{pmatrix}.
$$

The masses of up-type quarks are generated solely by the Hosotani mechanism. The effective mass terms in four dimensions are written as

$$
\mathcal{L}^\text{up}_m = -\left\{ \vec{u}_{L}^t M_{\text{up}} \vec{u}_R + \text{h.c.} \right\},
$$

$$
M_{\text{up}} = \begin{pmatrix} m_u & m_c & m_t \\ m_c & m_c & m_t \\ m_t & m_t & m_t \end{pmatrix}.
$$

For down-type quarks the effective mass terms are written as

$$
\mathcal{L}^\text{down}_m = -\left\{ (\vec{d}_{L}^t, \vec{D}_{L}^t) M_{\text{down}} \left( \vec{d}_R, \vec{D}_R \right) + \text{h.c.} \right\},
$$

15
\[ M_{\text{down}} = \begin{pmatrix} M_{\text{up}} & 0 \\ \bar{\mu} & \bar{m}_D \end{pmatrix} , \]
\[ \bar{\mu} = \begin{pmatrix} \hat{\mu}_{11} & \hat{\mu}_{12} & \hat{\mu}_{13} \\ \hat{\mu}_{21} & \hat{\mu}_{22} & \hat{\mu}_{23} \\ \hat{\mu}_{31} & \hat{\mu}_{32} & \hat{\mu}_{33} \end{pmatrix} , \quad \bar{m}_D = \begin{pmatrix} \bar{m}_{D_d} \\ \bar{m}_{D_s} \\ \bar{m}_{D_b} \end{pmatrix} . \quad (4.3) \]

The Hosotani mechanism generates degenerate masses, the \( M_{\text{up}} \) term in \( M_{\text{down}} \), for the components in \( \Psi_{(3,4)} \). \( D_{\alpha L} \) (\( D_{\alpha R} \)) is approximately \( D_{\alpha L}^+ \) (\( D_{\alpha R}^- \)). \( \bar{m}_{D_\alpha} \) is a mass generated by \( m_{D_\alpha} \) in (2.9). The matrix \( \bar{\mu} \) represents the brane interactions (2.14). Each element \( \hat{\mu}_{\alpha\beta} \) is proportional to \( (\mu^\dagger)_{\alpha\beta} = \mu^*_{\beta\alpha} \). (Note that \( \hat{\mu} \) has dimension of mass and that \( \hat{\mu} \) is not proportional to \( \mu^\dagger \) as a matrix.)

Mass-eigenstates of up-type quarks are gauge-eigenstates. However mass-eigenstates of down-type quarks are not gauge-eigenstates as a result of \( \hat{\mu} \). \( M_{\text{down}} \) can be expressed, in the canonical form, as

\[ M_{\text{down}} = \Omega \begin{pmatrix} M_{\text{down}} & M_D \\ M_D & \bar{\Omega} \end{pmatrix} \tilde{\Omega}^\dagger , \quad \Omega^\dagger = \Omega^{-1} , \quad \tilde{\Omega}^\dagger = \tilde{\Omega}^{-1} , \]
\[ M_{\text{down}} = \begin{pmatrix} m_d \\ m_s \\ m_b \end{pmatrix} , \quad M_D = \begin{pmatrix} m_{D_1} \\ m_{D_2} \\ m_{D_3} \end{pmatrix} . \quad (4.4) \]

Note \( \Omega \neq \tilde{\Omega} \) for \( \hat{\mu} \neq 0 \). Mass-eigenstates denoted by \( \hat{\ } \) are given by

\[ \begin{pmatrix} \tilde{d}_L \\ \bar{D}_L \end{pmatrix} = \Omega^\dagger \begin{pmatrix} \tilde{d}_L \\ \bar{D}_L \end{pmatrix} , \quad \begin{pmatrix} \tilde{d}_R \\ \bar{D}_R \end{pmatrix} = \tilde{\Omega}^\dagger \begin{pmatrix} \tilde{d}_R \\ \bar{D}_R \end{pmatrix} , \]
\[ \mathcal{L}_m^{\text{down}} = \left\{ \tilde{d}_L^\dagger M_{\text{down}} \tilde{d}_R + \bar{D}_L^\dagger M_D \bar{D}_R + \text{h.c.} \right\} . \quad (4.5) \]

All \( m_{D_\alpha} \)'s are of \( O(m_{\text{KK}}) \), and much larger than \( m_d, m_s \) and \( m_b \). Unitary matrices \( \Omega \) and \( \tilde{\Omega} \) are decomposed as

\[ \Omega = \begin{pmatrix} \Omega_q & \Omega_b \\ \Omega_a & \Omega_D \end{pmatrix} , \quad \tilde{\Omega}^\dagger = \begin{pmatrix} \tilde{\Omega}_q & \tilde{\Omega}_b \\ \tilde{\Omega}_a & \tilde{\Omega}_D \end{pmatrix} , \quad (4.6) \]

where all \( \Omega_q, \tilde{\Omega}_q \) etc. are 3-by-3 matrices. The unitarity of \( \Omega \) implies that

\[ \Omega_q \Omega_q^\dagger + \Omega_b \Omega_b^\dagger = I_3 , \quad \Omega_q^\dagger \Omega_q + \Omega_a^\dagger \Omega_a = I_3 , \]
\[ \Omega_a \Omega_a^\dagger + \Omega_D \Omega_D^\dagger = I_3 , \quad \Omega_b^\dagger \Omega_b + \Omega_D^\dagger \Omega_D = I_3 , \]
\[ \Omega_q^\dagger \Omega_a + \Omega_b^\dagger \Omega_D = 0 , \quad \Omega_q \Omega_a^\dagger + \Omega_b \Omega_D^\dagger = 0 , \quad (4.7) \]

where \( I_3 \) is a 3-by-3 unit matrix. Similar relations hold for \( \tilde{\Omega} \).
4.2 \( W \) couplings

The gauge coupling of \( \Psi^\alpha_{(3,4)}(x, z) \) leads to the \( W \) coupling

\[
L_W \simeq \frac{g_W}{\sqrt{2}} W_\mu \tilde{u}_L \Gamma^\mu \tilde{d}_L + \text{h.c.} \quad (4.8)
\]

In the next section we will confirm that \( g_W^L \sim g_w \) and that couplings of right-handed components are tiny, \( g_W^R / g_w \lesssim 10^{-6} \). It follows from (4.5) that the gauge-eigenstate \( \tilde{d}_L \) is related to the mass-eigenstate \( \tilde{d}_L \) by \( \tilde{d}_L = \Omega_q \tilde{d}_L + \Omega_b \tilde{D}_L \). For up-type quarks \( \tilde{u}_L = \tilde{u}_L \).

At low energies \( (\sqrt{s} \ll m_D) \) the \( \tilde{D} \) field may be dropped so that

\[
L_W \simeq \frac{g_W^L}{\sqrt{2}} W_\mu \tilde{u}_L \Gamma^\mu \tilde{d}_L + \text{h.c.} \quad (4.9)
\]

In other words the CKM matrix is given by

\[
V^{\text{CKM}} \simeq \Omega_q \quad (4.10)
\]

It should be noted that \( \Omega_q \) is not unitary in rigorous sense, as \( \Omega_q \Omega_q^\dagger = I_3 - \Omega_b \Omega_b^\dagger \). \( (4.3) \) and \( (4.4) \) lead to

\[
(q_1) : \Omega_q M_{\text{down}} \tilde{\Omega}_b + \Omega_b M_D \tilde{\Omega}_D = 0 ,
\]

\[
(q_2) : \Omega_a M_{\text{down}} \tilde{\Omega}_q + \Omega_D M_D \tilde{\Omega}_a = \bar{\mu} ,
\]

\[
(q_3) : \Omega_q M_{\text{down}} \tilde{\Omega}_q + \Omega_b M_D \tilde{\Omega}_a = M_{\text{up}} ,
\]

\[
(q_4) : \Omega_a M_{\text{down}} \tilde{\Omega}_b + \Omega_D M_D \tilde{\Omega}_D = \bar{m}_D ,
\]

or equivalently

\[
(r_1) : \Omega_q M_{\text{down}} = M_{\text{up}} \tilde{\Omega}_q^\dagger ,
\]

\[
(r_2) : \Omega_b M_D = M_{\text{up}} \tilde{\Omega}_a^\dagger ,
\]

\[
(r_3) : \Omega_a M_{\text{down}} = \bar{\mu} \tilde{\Omega}_q^\dagger + \bar{m}_D \tilde{\Omega}_b^\dagger ,
\]

\[
(r_4) : \Omega_D M_D = \bar{\mu} \tilde{\Omega}_a^\dagger + \bar{m}_D \tilde{\Omega}_D^\dagger .
\]

From the relation \( (q_1) \) and \( (r_2) \) above one finds

\[
\Omega_b = -\Omega_q M_{\text{down}} \tilde{\Omega}_b \tilde{\Omega}_D^{-1} M_D^{-1} = -M_{\text{up}} \tilde{\Omega}_a^\dagger M_D^{-1} .
\]

In other words the magnitude of each matrix element of \( \Omega_b \), denoted as \( ||\Omega_b|| \), is

\[
||\Omega_b|| = O\left(\frac{m_a}{m_D}\right)||\tilde{\Omega}_b|| \ll 1 \quad (4.14)
\]
where \( m_q = m_d, m_s, m_b \) and \( m_D = m_{D'} \). As \( m_b/m_D \sim 10^{-3} \), \( \Omega_q \) is nearly unitary. As \( \Omega_a = -(\Omega_D^T)^{-1}\Omega_b\Omega_q \), one sees that \( ||\Omega_a|| = O(m_q/m_D) \) as well.

Further \((q_2)\) and \((q_4)\) in (4.11) imply that

\[
\bar{\mu} \sim \Omega_D M_D \tilde{\Omega}_a, \quad \bar{m}_D \sim \Omega_D M_D \tilde{\Omega}_D .
\]

The relation \((r_1)\) in (4.12) gives a severe constraint on the mass spectrum. Recall (4.10), which implies that

\[
\frac{m_{d(k)}}{m_{u(j)}} |V^\text{CKM}_{jk}| \sim |(\tilde{\Omega}_q^T)_{jk}| < 1
\]

where \((m_{d1}, m_{d2}, m_{d3}) = (m_d, m_s, m_b)\) and \((m_{u1}, m_{u2}, m_{u3}) = (m_u, m_c, m_t)\). The observed mean value (magnitude) of \( V^\text{CKM}_{\text{obs}} \) is

\[
V^\text{CKM}_{\text{obs}} \sim \begin{pmatrix} 0.974 & 0.224 & 0.004 \\ 0.218 & 0.997 & 0.042 \\ 0.008 & 0.030 & 1.019 \end{pmatrix} .
\]

The observed \( m_u \sim 1.3 \text{ MeV} \) is too small, and the inequality (4.16) is not satisfied for the 11, 12 and 13 elements. Rigorous treatment presented in the previous and next sections also confirms this behavior. In the present paper we tentatively suppose that \( m_u \sim 20 \text{ MeV} \). The issue of small \( m_u \) is left for future investigation.

### 4.3 Z couplings

For up-type quarks one finds

\[
L^\text{up}_Z \sim -\frac{g_w}{\cos \theta_W} Z_{\mu} \left\{ \frac{1}{2} \tilde{\bar{u}}_L \bar{u}_L \Gamma^\mu \bar{u}_L - \frac{2}{3} \sin^2 \theta_W \left( \tilde{\bar{u}}_L \Gamma^\mu \bar{u}_L + \tilde{\bar{u}}_R \Gamma^\mu \bar{u}_R \right) \right\} .
\]

Recall that \( D_\alpha \) fields are \( SO(5) \) singlet. \( Z \) couplings of down-type quarks are given by

\[
L^\text{down}_Z \sim -\frac{g_w}{\cos \theta_W} Z_{\mu} \left\{ -\frac{1}{2} \tilde{\bar{d}}_L \Gamma^\mu \bar{d}_L ight.
\]

\[
\left. + \frac{1}{3} \sin^2 \theta_W \left( \tilde{\bar{d}}_L \Gamma^\mu \tilde{\bar{d}}_L + \tilde{\bar{D}}_L \Gamma^\mu \bar{D}_L + \tilde{\bar{d}}_R \Gamma^\mu \bar{d}_R + \tilde{\bar{D}}_R \Gamma^\mu \bar{D}_R \right) \right\} .
\]

In terms of mass-eigenstates in (4.5), \( Z \) couplings at low energies are expressed as

\[
L^\text{down}_Z \sim -\frac{g_w}{\cos \theta_W} Z_{\mu} \left\{ -\frac{1}{2} \left( \tilde{\bar{d}}_L \Gamma^\mu \tilde{\bar{d}}_L + \tilde{\bar{D}}_L \Gamma^\mu \bar{D}_L \right) \Gamma^\mu \left( \Omega_q \tilde{\bar{d}}_L + \Omega_b \tilde{\bar{D}}_L \right) ight.
\]

\[
\left. + \frac{1}{3} \sin^2 \theta_W \left( \tilde{\bar{d}}_L \Gamma^\mu \tilde{\bar{d}}_L + \tilde{\bar{D}}_L \Gamma^\mu \bar{D}_L + \tilde{\bar{d}}_R \Gamma^\mu \bar{d}_R + \tilde{\bar{D}}_R \Gamma^\mu \bar{D}_R \right) \right\}
\]
\[ \sim - \frac{g_w}{\cos \theta_W} Z \mu \left\{ - \frac{1}{2} \bar{d}_L \Gamma^\mu \Omega_\alpha^\dagger \Omega_\alpha \bar{d}_L + \frac{1}{3} \sin^2 \theta_W \left( \bar{d}_L \Gamma^\mu \bar{d}_L + \bar{d}_R \Gamma^\mu \bar{d}_R \right) \right\}. \quad (4.20) \]

In the first term \( \Omega_\alpha^\dagger \Omega_\alpha = I_3 - \Omega_{\alpha a} \), and the \( \Omega_{\alpha a} \) term gives rise to FCNC. However, with the use of the last two relations in (4.7) and the relation (4.13) one sees

\[ \Omega_{\alpha a}^\dagger \Omega_a = \Omega_{\alpha a b} \Omega_b^\dagger (\Omega_a^\dagger)^{-1} = O\left( \frac{m_q^2}{m_D^2} \right) \lesssim 10^{-6} . \quad (4.21) \]

FCNC interactions are naturally suppressed. The FCNC suppression will be confirmed by rigorous treatment in the next section as well.

### 5 Evaluation of gauge couplings

In section 3 we have obtained wave functions of gauge bosons and quarks, with which gauge couplings of quarks can be evaluated. Given the parameters \( \mu_{\alpha \beta} \) of the brane interaction (2.14) and the Dirac masses \( m_{D_\alpha} \) for the \( D_\alpha^\pm \) fields, the bulk mass parameters \( c_{D_\alpha} \) are chosen such that the mass spectrum of down-type quarks are reproduced by the condition (3.25). Then the wave functions of all quarks are unambiguously determined. The parameters \( \mu_{\alpha \beta} \) need be chosen such that the observed CKM mixing matrix is reproduced.

This process, however, is not so trivial. As inferred in the effective theory formulated in the previous section, consistent solutions are available only when \( m_d < m_u \). This behavior has been already recognized in the case of no-mixing in ref. [13]. In this section we present the detailed results for the \( W \) and \( Z \) couplings of quarks with typical \( \mu_{\alpha \beta} \). It will be seen that a simple form of \( \mu \) matrix leads to reasonable CKM mixing matrix, though it may not be perfect.

\[ m_Z, z_L = 10^{10}, m_t, m_b, m_c, m_s, m_u, \text{ and } m_d \text{ are inputs. The bare Weinberg angle } \sin^2 \theta_W = \frac{s^2}{1 + s^2} \text{ with a given } \theta_H \text{ is determined to fit the LEP1 data for } e^+e^- \rightarrow \mu^+\mu^- \text{ at } \sqrt{s} = m_Z. \quad [28] \]

It will be seen below that evaluated gauge couplings turn out very close to those in the SM with \( \sin^2 \theta_W = 0.2312 \). The values for \( m_{KK}, c_u, c_c, c_t \) etc. with given \( \theta_H \) are summarized in Table 3.

In general nine elements of the brane interaction matrix \( \mu \) can be complex. Six out of nine phases can be absorbed by redefinition of the fields \( \bar{d}_R \) and \( \bar{D}_L^\dagger \). Three of them remain as CP violation phases. When all heavy fields such as \( \bar{D}^\pm \) are integrated out, only one complex phase survives at the CKM matrix level. In the present paper we consider a real matrix \( \mu \), which is parametrized as

\[ \mu = U_{12}(\phi_{12})U_{13}(\phi_{13})U_{23}(\phi_{23}) \begin{pmatrix} \mu_1 \\ \mu_2 \\ \mu_3 \end{pmatrix} U_{23}(\omega_{23})^\dagger U_{13}(\omega_{13})^\dagger U_{12}(\omega_{12})^\dagger. \quad (5.1) \]
Table 3: Values of $m_{\text{KK}}$, $k$, $\sin^2 \theta_W = s^2_\phi/(1+s^2_\phi)$, $c_u$, $c_c$, $c_t$ are tabulated for $\theta_H = 0.10$, 0.15 and $z_L = 10^{10}$. We set $m_Z = 91.1876$ GeV, $\alpha_{\text{EM}}(m_Z) = 1/128$ and $(m_u, m_c, m_t) = (0.020, 0.619, 171.17)$ GeV. The value $m_u > m_d$ has been used for a reason explained in the text.

| $\theta_H$ | $m_{\text{KK}}$ (TeV) | $k$ (GeV) | $\sin^2 \theta_W$ | $c_u$ | $c_c$ | $c_t$ |
|------------|----------------|----------|----------------|------|------|------|
| 0.10       | 12.08          | 3.84 x 10^{13} | 0.2306           | -0.9169 | -0.7545 | -0.2274 |
| 0.15       | 8.07           | 2.57 x 10^{13} | 0.2299           | -0.9170 | -0.7546 | -0.2294 |

Here $U_{jk}(\phi)$ is a rotation matrix in the $jk$ subspace:

$$U_{12}(\phi) = \begin{pmatrix}
\cos \phi & \sin \phi & 0 \\
-\sin \phi & \cos \phi & 0 \\
0 & 0 & 1
\end{pmatrix}.
$$

As typical values we set $\tilde{m}_{D_d} = \tilde{m}_{D_s} = \tilde{m}_{D_b} = 1$. For the $\mu$ matrix, we take $(\mu_1, \mu_2, \mu_3) = (0.1, 0.1, 1)$ as reference values suggested in ref. [13]. Among the rotation angles in (5.1), $\omega_{12}$ is most responsible for the Cabibbo angle. We have explored the parameter space $(\omega_{12}, \omega_{23})$, while keeping $\phi_{jk} = \omega_{13} = 0$. Given $\mu$, the bulk mass parameters of $D$ fields, $(c_{D_d}, c_{D_s}, c_{D_b})$, are determined so as to reproduce $(m_d, m_s, m_b)$. This turns out possible only for appropriate $\mu$. With all the parameter sets, wave functions of down-type quarks are determined, and gauge couplings of quarks are evaluated. Sets of typical values of these parameters are tabulated in Table 4. We note that the masses of the first KK excited states of $d, s, b$ quarks turn out around 0.6 $m_{\text{KK}}$.

Table 4: Sets of parameters which yield a reasonable CKM matrix. $(c_{D_d}, c_{D_s}, c_{D_b})$ is determined to give $(m_d, m_s, m_b) = (0.0029, 0.055, 2.89)$ GeV by (3.25). We set $\phi_{jk} = \omega_{13} = 0$ in (5.1) and $\tilde{m}_{D_d} = \tilde{m}_{D_s} = \tilde{m}_{D_b} = 1$.

| $\theta_H$ | $(\mu_1, \mu_2, \mu_3)$ | $(\omega_{12}, \omega_{23})$ | $c_{D_d}$ | $c_{D_s}$ | $c_{D_b}$ |
|------------|----------------|----------------|----------|----------|----------|
| (a) 0.10   | (0.1, 0.1, 1) | (0.1055, 0.0018) | 0.520074 | 0.751360 | 0.951239 |
| (b) 0.15   | (0.1, 0.1, 1) | (0.1055, 0.00198) | 0.478059 | 0.751545 | 0.955367 |

Wave functions of each down-type quark consist of 12 components, $(d, d', D^+_d, D^-_d)$, $(s, s', D^+_s, D^-_s)$, $(b, b', D^+_b, D^-_b)$. Coefficient vectors, $\vec{\alpha}, \vec{\alpha}', \vec{a}$ and $\vec{b}$ in (3.16) and (3.17) for $\theta_H = 0.15$ with the parameter set (b) in Table 4 are tabulated in Table 5. With these coefficients wave functions of left- and right-handed components, $f_{jL}(z)$ and $f_{jR}(z)$, are
Table 5: Coefficient vectors in (3.17) for wave functions of down-type quarks for \( \theta_H = 0.15 \) with the parameter set (b) in Table 4 are listed. (\( \hat{d}, \hat{s}, \hat{b} \)) represent mass-eigenstates.

|   | \( \hat{d} \) | \( \hat{a} \) | \( \hat{a}' \) | \( \hat{a}'' \) | \( \hat{b} \) |
|---|---|---|---|---|---|
| \( \hat{d} \) | 1.640 | \(-8.692 \times 10^{-6} i\) | 0.007734 i | 2.207 \times 10^{-9} i |
| \( \hat{d} \) | -0.3588 | 2.148 \times 10^{-6} i | -0.4697 i | -1.178 \times 10^{-7} i |
| \( \hat{d} \) | 1.476 \times 10^{-5} | -1.520 \times 10^{-10} i | 0.005361 i | 1.232 \times 10^{-9} i |
| \( \hat{s} \) | 0.3812 | -3.832 \times 10^{-5} i | 0.03452 i | 1.868 \times 10^{-7} i |
| \( \hat{s} \) | 1.542 | -1.752 \times 10^{-4} i | 0.04968 i | 2.362 \times 10^{-7} i |
| \( \hat{s} \) | -0.02291 | 4.474 \times 10^{-6} i | -0.4376 i | -1.908 \times 10^{-6} i |
| \( \hat{b} \) | 0.007211 | -3.809 \times 10^{-5} i | 0.03431 i | 9.756 \times 10^{-6} i |
| \( \hat{b} \) | 0.02927 | -1.746 \times 10^{-4} i | 0.05525 i | 1.380 \times 10^{-5} i |
| \( \hat{b} \) | 1.208 | -0.01239 i | 0.4389 i | 1.005 \times 10^{-4} i |

The \( W \) couplings can be expanded as

\[
A_M = \sum_{a=1}^{3} \left\{ A_{M}^{aL} T^{aL} + A_{M}^{aR} T^{aR} + A_{M}^{\hat{a}} T^{\hat{a}} \right\} + A_{M}^{4} T^{4},
\]

where \( T^{aL} \) and \( T^{aR} \) are \( SU(2)_L \) and \( SU(2)_R \) generators, respectively. \( \{T^{p}; p = 1, \cdots, 4\} \) are generators of \( SO(5)/SO(4) \). In the spinor representation, for instance,

\[
T^{aL} = \frac{1}{2} \begin{pmatrix} \sigma^{a} & 0 \\ 0 & 0 \end{pmatrix}, \quad T^{aR} = \frac{1}{2} \begin{pmatrix} 0 & 0 \\ 0 & \sigma^{a} \end{pmatrix},
\]

\[
T^{\hat{a}} = \frac{1}{2\sqrt{2}} \begin{pmatrix} 0 & i\sigma^{a} \\ -i\sigma^{a} & 0 \end{pmatrix}, \quad T^{4} = \frac{1}{2\sqrt{2}} \begin{pmatrix} 0 & I_{2} \\ I_{2} & 0 \end{pmatrix}
\]
Here the expression (3.7) has been inserted.

Table 6: Norm of each component of down-type quarks for $\theta_H = 0.15$ with the parameter set (b) in Table 4 is listed. $(d, s, b)$ represent mass-eigenstates. In this table $10^{-13}$, for instance, implies order of $10^{-13}$.

|     | $d_L$   | $s_L$  | $b_L$  | $d_R$   | $s_R$  | $b_R$  |
|-----|---------|--------|--------|---------|--------|--------|
| $d$ | 0.9487  | 0.0513 | $10^{-5}$ | 0.0001  | 0.0022 | 0.0022 |
| $s$ | 0.0513  | 0.9484 | 0.0003 | $10^{-8}$  | 0.0022 | 0.0022 |
| $b$ | $10^{-10}$ | 0.0004 | 0.9996 | $10^{-22}$  | $10^{-13}$  | $10^{-6}$  |
| $d'$ | $10^{-23}$  | $10^{-19}$ | $10^{-15}$ | 0.0198  | 0.3856 | 0.3810 |
| $s'$ | $10^{-24}$  | $10^{-18}$ | $10^{-14}$ | $10^{-6}$  | 0.0074 | 0.0074 |
| $b'$ | $10^{-32}$  | $10^{-21}$ | $10^{-10}$ | $10^{-20}$  | $10^{-11}$  | 0.0003 |
| $D^+_d$ | $10^{-17}$  | $10^{-13}$ | $10^{-9}$ | 0.0023  | 0.0465 | 0.0459 |
| $D^+_s$ | $10^{-13}$  | $10^{-12}$ | $10^{-9}$ | 0.3113  | 0.0035 | 0.0043 |
| $D^+_b$ | $10^{-17}$  | $10^{-10}$ | $10^{-7}$ | $10^{-5}$  | 0.1177 | 0.1184 |
| $D^-_d$ | $10^{-17}$  | $10^{-13}$ | $10^{-9}$ | 0.0025  | 0.0489 | 0.0484 |
| $D^-_s$ | $10^{-13}$  | $10^{-13}$ | $10^{-9}$ | 0.6639  | 0.0074 | 0.0092 |
| $D^-_b$ | $10^{-17}$  | $10^{-11}$ | $10^{-7}$ | 0.0001  | 0.3808 | 0.3830 |

where $\sigma^a$’s and $I_2$ are Pauli matrices and a 2-by-2 unit matrix. $W$ boson is contained, in the twisted gauge, in

$$
\tilde{A}_\mu \Rightarrow \frac{1}{2} \left\{ (\tilde{A}_\mu^L - i \tilde{A}_\mu^R)(T^{1L}_L + i T^{2L}_L) + (\tilde{A}_\mu^R - i \tilde{A}_\mu^L)(T^{1R}_L + i T^{2R}_L) \\
+ (\tilde{A}_\mu^1 - i \tilde{A}_\mu^2)(T^1 + i T^2) \right\} + \text{h.c.}
$$

$$
\Rightarrow \frac{1}{2} \left\{ (1 + c_H) \tilde{W}_\mu (T^{1L} + i T^{2L}) + (1 - c_H) \tilde{W}_\mu (T^{1R} + i T^{2R}) \\
- \sqrt{2} s_H \tilde{W}_\mu (T^1 + i T^2) \right\} + \text{h.c.} \ .
$$

(5.5)

Here the expression (3.7) has been inserted. $W$ couplings of quarks come solely from the couplings of $\Psi_\alpha^{(3.4)}$,

$$
\mathcal{L}^{d=4}_{W} = -i g_A \int_1^{z_L} \frac{dz}{k} \left\{ \tilde{W}_\mu \left( \frac{1 + c_H}{2} \bar{\tilde{u}} \Gamma^\mu \tilde{d} + \frac{1 - c_H}{2} \bar{\tilde{u}}' \Gamma^\mu \tilde{d}' \right) \\
+ \tilde{W}^S_\mu \left( - \frac{s_H}{2} \bar{\tilde{u}} \Gamma^\mu \tilde{d} + \frac{s_H}{2} \bar{\tilde{u}}' \Gamma^\mu \tilde{d}' \right) \right\} + \text{h.c.} \ .
$$

(5.6)

Here, as in (3.11), we have denoted as

$$
\bar{u} = \begin{pmatrix} \bar{u} \\ \bar{u}' \end{pmatrix} , \quad \bar{u}' = \begin{pmatrix} \bar{u}' \\ \bar{u} \end{pmatrix} .
$$

(5.7)
We use the following notation for wave functions of quarks. 4D quark fields are denoted by hat \( \hat{\cdot} \):

\[
\begin{pmatrix}
\hat{u}_1(x) \\
\hat{u}_2(x) \\
\hat{u}_3(x)
\end{pmatrix} = \begin{pmatrix}
\hat{u}(x) \\
\hat{c}(x) \\
\hat{t}(x)
\end{pmatrix},
\begin{pmatrix}
\hat{d}_1(x) \\
\hat{d}_2(x) \\
\hat{d}_3(x)
\end{pmatrix} = \begin{pmatrix}
\hat{d}(x) \\
\hat{s}(x) \\
\hat{b}(x)
\end{pmatrix}.
\] 

(5.8)

For up-type quarks 5D fields in the twisted gauge are expanded as

\[
\tilde{u}_j(x, z) = \sqrt{k} \left\{ \hat{u}_jL(x) f^\hat{u}_{Lj}(z) + \hat{u}_jR(x) f^\hat{u}_{Rj}(z) \right\},
\]

\[
\tilde{d}_j(x, z) = \sqrt{k} \left\{ \hat{d}_jL(x) f^\hat{d}_{Lj}(z) + \hat{d}_jR(x) f^\hat{d}_{Rj}(z) \right\}.
\] 

(5.9)

With the expression in (3.10), for instance,

\[
f^\hat{u}_{L1}(z) = \bar{c}_H C_L(z; \lambda_u, c_u)/\sqrt{r_u},
\]

\[
f^\hat{u}_{L2}(z) = i\bar{s}_H \hat{S}_L(z; \lambda_c, c_c)/\sqrt{r_c}.
\] 

(5.10)

For down-type quarks 5D fields in the twisted gauge are expanded as

\[
\tilde{d}_j(x, z) = \sqrt{k} \sum_{m=1}^{3} \left\{ \hat{d}_mL(x) f^\hat{d}_{Ld_j}(z) + \hat{d}_mR(x) f^\hat{d}_{Rd_j}(z) \right\},
\]

\[
\tilde{d}_j(x, z) = \sqrt{k} \sum_{m=1}^{3} \left\{ \hat{d}_mL(x) f^\hat{d}_{Ld_j}(z) + \hat{d}_mR(x) f^\hat{d}_{Rd_j}(z) \right\},
\]

\[
\tilde{D}_j^+(x, z) = \sqrt{k} \sum_{m=1}^{3} \left\{ \hat{d}_mL(x) f^\hat{d}_{LD_j^+}(z) + \hat{d}_mR(x) f^\hat{d}_{RD_j^+}(z) \right\},
\]

\[
\tilde{D}_j^-(x, z) = \sqrt{k} \sum_{m=1}^{3} \left\{ \hat{d}_mL(x) f^\hat{d}_{LD_j^-}(z) + \hat{d}_mR(x) f^\hat{d}_{RD_j^-}(z) \right\}.
\] 

(5.11)

With the expression in (3.16), one finds, for instance,

\[
f^\hat{d}_{Ld}(z) = \alpha^\hat{d}_s C_L(z; \lambda_d, c_c),
\]

\[
f^\hat{d}_{Rd}(z) = \alpha^\hat{d}_b C_R(z; \lambda_s, c_t),
\]

\[
f^\hat{d}_{RD}^+(z) = \alpha^\hat{b}_s S_R(z; \lambda_b, c_{Dd}, \bar{m}_{Dd}) + \alpha^\hat{b}_s S_R(z; \lambda_b, c_{Dd}, \bar{m}_{Dd}) + \alpha^\hat{b}_b S_R(z; \lambda_b, c_{Dd}, \bar{m}_{Dd}).
\] 

(5.12)

Here \( \alpha^\hat{d}_j, \alpha^\hat{d}_j, \alpha^\hat{d}_j, \) and \( \alpha^\hat{d}_j \) are the coefficient vectors determined for \( \hat{d}_j \).

W couplings of quarks are defined by

\[
\mathcal{L}_W^{d=4} = \frac{i}{\sqrt{2}} W_\mu \sum_{j,k} \left\{ g^W_{Ljk} \bar{u}_j L \Gamma \mu \hat{d}_k L + g^W_{Rjk} \bar{u}_j R \Gamma \mu \hat{d}_k R \right\} + \text{h.c.}.
\] 

(5.13)
Inserting (5.9) and (5.11) into (5.6), one finds
\[
\begin{bmatrix}
g^W_{Ljk} \\
g^W_{Rjk}
\end{bmatrix} = -ig_w \sqrt{\frac{kL}{f_W}} \int_1^{z_L} dz \times \left\{ C(z, \lambda_W) \left( \frac{1 + c_H}{2} \left[ f^\dagger_{Lu_j}(z) f_{Ld_j}(z) \right] + \frac{1 - c_H}{2} \left[ f^\dagger_{Ru_j}(z) f_{Rd_j}(z) \right] \right) + S(z, \lambda_W)(-i) \frac{s_H}{2} \left[ f^\dagger_{Lu_j}(z) f_{Ld_j}(z) - f^\dagger_{Lu_j}(z) f_{Ld_j}(z) \right] \right\}.
\]

Let us denote the couplings in the matrix form; \((\hat{g}^W_L)_{jk} = g^W_{Ljk}\) and \((\hat{g}^W_R)_{jk} = g^W_{Rjk}\). \(\hat{g}^W_L\) is parametrized as
\[
\hat{g}^W_L = g^W_L \hat{V}_{CKM}, \quad \det V_{CKM} = 1.
\]

\(\hat{g}^W_L\) and \(\hat{g}^W_R\) are evaluated for the two sets of parameters in Table 4.

(a) \(\theta_H = 0.10\) :
\[
g^W_L = 0.9978 g_w, \quad \hat{V}_{CKM} = \begin{pmatrix} 0.9744 & 0.2245 & 0.0031 \\ -0.2245 & 0.9743 & 0.0134 \\ 9 \times 10^{-6} & -0.0138 & 1.0002 \end{pmatrix},
\]
\[
\hat{g}^W_R = g_w \begin{pmatrix} 2 \times 10^{-12} & 8 \times 10^{-12} & 6 \times 10^{-12} \\ -1 \times 10^{-11} & 9 \times 10^{-10} & 7 \times 10^{-10} \\ 1 \times 10^{-13} & -3 \times 10^{-9} & 1 \times 10^{-5} \end{pmatrix},
\]

(b) \(\theta_H = 0.15\) :
\[
g^W_L = 0.9950 g_w, \quad \hat{V}_{CKM} = \begin{pmatrix} 0.9737 & 0.2264 & 0.0043 \\ -0.2264 & 0.9736 & 0.0185 \\ 1 \times 10^{-5} & -0.0190 & 1.0004 \end{pmatrix},
\]
\[
\hat{g}^W_R = g_w \begin{pmatrix} 4 \times 10^{-12} & 1 \times 10^{-11} & 2 \times 10^{-11} \\ -3 \times 10^{-11} & 2 \times 10^{-9} & 2 \times 10^{-9} \\ 4 \times 10^{-13} & -1 \times 10^{-8} & 3 \times 10^{-5} \end{pmatrix}.
\]

We have checked remarkable cancellation among four terms in the right-handed couplings \(g^W_{Rjk}\) in (5.14). The resultant \(\hat{V}_{CKM}\) is reasonably close to the observed CKM matrix, although the 31 element is still too small. We have evaluated the \(W\) couplings of leptons as well. The couplings of left-handed leptons \((e, \mu, \tau)\) are \((0.997665, 0.997662, 0.997659)g_w\) for \(\theta_H = 0.10\), and \((0.994756, 0.994748, 0.994743)g_w\) for \(\theta_H = 0.15\). The relative coupling \(g^W_L\) to \(g^W_{L\text{lepton}}\) is \(g^W_L/g^W_{L\text{lepton}} = 1.00013\) and 1.00028 for \(\theta_H = 0.10\) and 0.15, respectively. The universality holds to high accuracy. The \(W\) couplings of right-handed leptons are typically of order \(10^{-20} g_w\).
5.2 \( Z \) couplings

Photon \( \gamma \) and \( Z \) boson are contained in

\[
\hat{A}_\mu + \frac{g_B}{g_A} Q_X B_\mu \Rightarrow (\hat{A}_\mu T^3_L + \hat{A}_\mu T^3_R + \hat{A}_\mu T^3) + \frac{g_B}{g_A} Q_X B_\mu
\]

\[
\Rightarrow \sqrt{1 + s_\phi^2} \left\{ \left[ (1 + c_H)T^3_L + (1 - c_H)T^3_R \right] \hat{Z}_\mu - \sqrt{2} s_H T^3 \hat{Z}^S \right\}
\]

\[
+ \frac{s_\phi}{\sqrt{1 + s_\phi^2}} Q_{EM}(\hat{A}_\mu - \sqrt{2}s_H \hat{Z}_\mu).
\]

Here (5.17) and the relation \( Q_{EM} = T^3_L + T^3_R + Q_X \) have been used. Photon couplings are given by

\[
\mathcal{L}_{\gamma}^{d=4} = -i g_A \frac{s_\phi}{\sqrt{1 + s_\phi^2}} \hat{A}_\mu(x) \int_1^{2L} dz \, J_\gamma^\mu(x, z),
\]

\[
J_\gamma^\mu(x, z) = \frac{2}{3} \sum_{j=1}^3 \left[ \bar{u}_{jL} \Gamma^\mu u_{jL}(x) \left\{ f_{L\mu j}(z)^* f_{L\nu j}(z) + f_{L\nu j}^*(z) f_{L\mu j}'(z) \right\} + (L \rightarrow R) \right]
\]

\[
- \frac{1}{3} \sum_{l, m=1}^3 \left[ \bar{d}_{lL} \Gamma^\mu \bar{d}_{mL}(x) \left\{ f_{Ld j}(z)^* f_{Ld j}'(z) + f_{Ld j}^*(z) f_{Ld j}^*(z) \right\} + (L \rightarrow R) \right].
\]

By making use of orthonormality relations, the \( z \) integration can be done to lead to

\[
\mathcal{L}_{\gamma}^{d=4} = -i e A_\mu(x) \sum_{j=1}^3 \left\{ \frac{2}{3} \bar{u}_j(x) \Gamma^\mu u_j(x) - \frac{1}{3} \bar{d}_j(x) \Gamma^\mu d_j(x) \right\},
\]

\[
e = g_w \sin \theta^0_W, \quad \sin \theta^0_W = \frac{s_\phi}{\sqrt{1 + s_\phi^2}}.
\]

\( Z \) couplings are given by

\[
\mathcal{L}_{Z}^{d=4} = -i g_A \sqrt{1 + s_\phi^2} \int_1^{2L} dz \, k
\]

\[
\times \left\{ \hat{Z}_\mu \left[ \frac{1}{2} \left( c_H \bar{u} \Gamma^\mu u - \bar{d} \Gamma^\mu \bar{d} - c_H \bar{u} \Gamma^\mu u - \bar{d} \Gamma^\mu \bar{d} \right) \right] \right]\]

25
\begin{equation}
Z^2 \mu \frac{iS_H}{2} \left[ \tilde{u}^\dagger \Gamma^\mu \tilde{u} - \tilde{u}^\dagger \Gamma^\mu \tilde{u} - \tilde{d}^\dagger \Gamma^\mu \tilde{d} + \tilde{d}^\dagger \Gamma^\mu \tilde{d} \right]
\end{equation}

\begin{equation}
+i g_A \frac{\sqrt{2} s_\phi}{\sqrt{1 + s_\phi^2}} \int_1^{z_L} \frac{dz}{k} \dot{Z}_\mu J^\mu_\gamma
\end{equation}

where \(J^\mu_\gamma\) is given in (5.19). Let us denote \(Z\) couplings of quarks as

\begin{equation}
L^d_{Z} = -\frac{i}{\cos \theta_W} Z^\mu \sum_j \left( g^Z_{L_{u_j u_j}} \tilde{u}_{jL} \Gamma^\mu \tilde{u}_{jL} + g^Z_{R_{u_j u_j}} \tilde{u}_{jR} \Gamma^\mu \tilde{u}_{jR} \right)
\end{equation}

\begin{equation}
+ \sum_{j,k} \left( g^Z_{L_{d_j d_k}} \tilde{d}_{jL} \Gamma^\mu \tilde{d}_{kL} + g^Z_{R_{d_j d_k}} \tilde{d}_{jR} \Gamma^\mu \tilde{d}_{kR} \right)
\end{equation}

The couplings of up-type quarks are diagonal in flavor, but there appear off-diagonal couplings (FCNC) for down-type quarks. Insertion of (3.5), (5.9) and (5.11) into (5.21) leads to

\begin{equation}
g^Z_{L_{u_j u_j}} = g_w \frac{\sqrt{2 k_L}}{\sqrt{T_Z}} \int_1^{z_L} dz \left\{ C(z, \lambda_Z) \left( \frac{1 + c_H}{4} \tilde{f}_{L_{u_j u_j}}(z)^* f_{L_{u_j u_j}}(z) + \frac{1 - c_H}{4} f_{L_{u_j u_j}}(z)^* f_{L_{u_j u_j}}(z) \right.ight.
\end{equation}

\begin{equation}
- \frac{2}{3} \sin^2 \theta_W \left[ \tilde{f}_{L_{u_j u_j}}(z)^* f_{L_{u_j u_j}}(z) + f_{L_{u_j u_j}}(z)^* \tilde{f}_{L_{u_j u_j}}(z) \right]
\end{equation}

\begin{equation}
- \frac{i S_H}{2} \tilde{S}(z, \lambda_Z) \left[ \tilde{f}_{L_{u_j u_j}}(z)^* f_{L_{u_j u_j}}(z) - f_{L_{u_j u_j}}(z)^* \tilde{f}_{L_{u_j u_j}}(z) \right] \right\}
\end{equation}

\begin{equation}
g^Z_{L_{d_j d_k}} = g_w \frac{\sqrt{2 k_L}}{\sqrt{T_Z}} \int_1^{z_L} dz \sum_{\ell=1}^3 \left\{ C(z, \lambda_Z) \left( - \frac{1 + c_H}{4} \tilde{f}_{L_{d_j d_k}}(z)^* f_{L_{d_j d_k}}(z) - \frac{1 - c_H}{4} f_{L_{d_j d_k}}(z)^* f_{L_{d_j d_k}}(z) \right.ight.
\end{equation}

\begin{equation}
+ \frac{1}{3} \sin^2 \theta_W \left[ \tilde{f}_{L_{d_j d_k}}(z)^* f_{L_{d_j d_k}}(z) + f_{L_{d_j d_k}}(z)^* \tilde{f}_{L_{d_j d_k}}(z) \right.
\end{equation}

\begin{equation}
+ \tilde{f}_{L_{d_j d_k}}(z)^* f_{L_{d_j d_k}}(z) + f_{L_{d_j d_k}}(z)^* \tilde{f}_{L_{d_j d_k}}(z) \right]\left. \right]\right\}
\end{equation}

Formulas for \(g^Z_{R_{u_j u_j}}\) and \(g^Z_{R_{d_j d_k}}\) are obtained by the replacement \(L \rightarrow R\) in each expression.

The \(Z\) couplings of down-type quarks are written in the matrix form; \((\hat{g}_{L_{d_j}})_{jk} = g^Z_{L_{d_j d_k}}\) and \((\hat{g}_{R_{d_j}})_{jk} = g^Z_{R_{d_j d_k}}\). One find for the two sets of parameters in Table 4:

(a) \(\theta_H = 0.10\) :
effective theory developed in section 4. The orbifold boundary condition breaks $m_{\ref{2.12}}$, which yield the specific mass terms of the form $\ref{2.14}$. The resultant FCNC's are $SO_{\ref{4}}$ brane. As explained earlier, the above conclusion remains valid even with the $G_{\ref{10}}$ invariance on the $SU_{\ref{10}}$ so that one might expect only $G_{\ref{10}}, K, B_{\ref{3}}, d_{\ref{4}}, L_{\ref{3}}, U_{\ref{10}}$. Although flavor-changing neutral currents (FCNC's) emerge for the down-type quarks, their magnitude is naturally suppressed. FCNC's induce the mixing of neutral mesons $(M = K, B_d, B_s)$ at the tree level, yielding $\Delta m_M \sim (m_M f_M^2/3m_Z^2)(\tilde{g}_d|_M)^2$ where $m_M$ and $f_M$ are the meson mass and decay constant and $\tilde{g}_d^2|_M$ is the relevant coupling in $\tilde{g}_L^2$ or $\tilde{g}_R^2$. Making use of $(m_K, m_{B_d}, m_{B_s}) \sim (0.498, 5.280, 5.367) \text{GeV}$ and $(f_K, f_{B_d}, f_{B_s}) \sim (0.156, 0.191, 0.274) \text{GeV}$, one finds, for $\theta_H = 0.10$, $(\Delta m_{K}, \Delta m_{B_d}, \Delta m_{B_s}) \sim (7 \times 10^{-20}, 5 \times 10^{-19}, 6 \times 10^{-17}) \text{GeV}$, which are much smaller than the experimental values $(3.48 \times 10^{-15}, 3.36 \times 10^{-13}, 1.17 \times 10^{-11}) \text{GeV}$. \[29, 30\]

The gauge invariance guarantees natural suppression of FCNC interactions. This should be contrasted to the previous approaches of refs. \[23, 24\], in which only $SU(3)_C \times SU(2)_L \times U(1)_Y$ invariance is imposed on the brane. The requirement of the gauge invariance under $G = SU(3)_C \times SO(5) \times U(1)_X$ restricts the form of brane interactions to $(\ref{2.12})$, which yield the specific mass terms of the form $(\ref{2.14})$. The resultant FCNC's are suppressed with a factor of order $(m_q/m_D)^2$ $(m_q = m_d, m_s, m_b)$ as anticipated from the effective theory developed in section 4. The orbifold boundary condition breaks $SO(5)$ to $SO(4)$ so that one might expect only $G' = SU(3)_C \times SO(4) \times U(1)_X$ invariance on the brane. As explained earlier, the above conclusion remains valid even with the $G'$ gauge invariance alone being imposed.

\[
\begin{pmatrix}
g^Z_{Luu} \\
g^Z_{Lcc} \\
g^Z_{Lt}\end{pmatrix} = \begin{pmatrix}0.3451 & 0.3451 & 0.3455 \end{pmatrix} g_w, \quad \begin{pmatrix}g^Z_{Ruu} \\
g^Z_{Rcc} \\
g^Z_{Rt}\end{pmatrix} = \begin{pmatrix}-0.1538 & -0.1538 & -0.1534 \end{pmatrix} g_w,
\]

\[
\tilde{g}_{L_d}^Z = g_w \begin{pmatrix}-0.4220 & -3 \times 10^{-7} & -4 \times 10^{-9} \\ -3 \times 10^{-7} & -0.4220 & -1 \times 10^{-7} \\ -4 \times 10^{-9} & -1 \times 10^{-7} & -0.4220 \end{pmatrix},
\]

\[
\tilde{g}_{R_d}^Z = g_w \begin{pmatrix}0.0769 & -6 \times 10^{-7} & -4 \times 10^{-7} \\ -6 \times 10^{-7} & 0.0769 & -3 \times 10^{-6} \\ -4 \times 10^{-7} & -3 \times 10^{-6} & 0.0769 \end{pmatrix},
\]

(b) $\theta_H = 0.15$ :

\[
\begin{pmatrix}g^Z_{Luu} \\
g^Z_{Lcc} \\
g^Z_{Lt}\end{pmatrix} = \begin{pmatrix}0.3441 & 0.3441 & 0.3449 \end{pmatrix} g_w, \quad \begin{pmatrix}g^Z_{Ruu} \\
g^Z_{Rcc} \\
g^Z_{Rt}\end{pmatrix} = \begin{pmatrix}-0.1533 & -0.1533 & -0.1524 \end{pmatrix} g_w,
\]

\[
\tilde{g}_{L_d}^Z = g_w \begin{pmatrix}-0.4208 & -7 \times 10^{-7} & -1 \times 10^{-8} \\ -7 \times 10^{-7} & -0.4208 & -4 \times 10^{-7} \\ -1 \times 10^{-8} & -4 \times 10^{-7} & -0.4207 \end{pmatrix},
\]

\[
\tilde{g}_{R_d}^Z = g_w \begin{pmatrix}0.0767 & -1 \times 10^{-6} & -1 \times 10^{-6} \\ -1 \times 10^{-6} & 0.0767 & -7 \times 10^{-6} \\ -1 \times 10^{-6} & -7 \times 10^{-6} & 0.0767 \end{pmatrix}.
\]
We remark that the relative couplings to $g_{L\text{lepton}}^W$ are

$$
\frac{1}{g_{L\text{lepton}}^W} \begin{pmatrix}
g_{L\text{uu}}^Z \\
g_{R\text{uu}}^Z \\
g_{L\text{dd}}^Z \\
g_{R\text{dd}}^Z
\end{pmatrix} = \begin{pmatrix}
0.34588 \\
-0.15413 \\
-0.42295 \\
0.07707
\end{pmatrix}, \quad \begin{pmatrix}
0.34591 \\
-0.15411 \\
-0.42298 \\
0.07706
\end{pmatrix}
$$

(5.25)

for $\theta_H = 0.10$, and 0.15, respectively. The values in the SM with $\sin^2 \theta_W = 0.2312$ are 0.3458, $-0.1541$, $-0.4229$ and 0.0771. The values (5.25) in the gauge-Higgs unification are very close to those in the SM.

### 5.3 Yukawa couplings

The flavor mixing in the down-type quarks induces flavor-changing Yukawa couplings. We show that its effect is extremely tiny. The 4D Higgs field $H(x)$ is contained in $A_\pm^4$ in the expansion (5.3):

$$
\tilde{A}_z = \tilde{H}(x, z) T^4 + \cdots,
$$

$$
\tilde{H}(x, z) = \frac{1}{\sqrt{k}} H(x) h_H(z) + \cdots, \quad h_H(z) = \sqrt{\frac{2}{z_L^2 - 1}} z.
$$

(5.26)

Inserting (5.26) into the gauge interaction part of the action, one obtains

$$
-ig_A \int_1^{z_L} dz \tilde{H} \sum_{\alpha=1}^3 \bar{\tilde{\Psi}}_{(3, 4)}^{\alpha} \Gamma^5 T^4 \tilde{\Psi}_{(3, 4)}^{\alpha}
$$

$$
= -\frac{g_A}{2\sqrt{2}} \int_1^{z_L} dz \tilde{H} \left\{ \bar{\tilde{u}}_L^j \tilde{u}_R^j + \bar{\tilde{u}}_R^j \tilde{u}_L^j + \bar{\tilde{d}}_R^j \tilde{d}_L^j + \bar{\tilde{d}}_L^j \tilde{d}_R^j \right\}
$$

(5.27)

where the notation (5.7) has been used. We insert (5.9) and (5.11) into (5.27) and integrate over $z$. In terms of mass eigenstates (5.8) the Yukawa interactions are written as

$$
-iH(x) \left\{ \sum_{j=1}^3 y_{u_{j\mu}} \left( \tilde{u}_{jL}^\dagger \tilde{u}_{jR} - \tilde{u}_{jR}^\dagger \tilde{u}_{jL} \right) + \sum_{j,k=1}^3 y_{d_{j\mu k}} \left( \tilde{d}_{jL}^\dagger \tilde{d}_{kR} - \tilde{d}_{kR}^\dagger \tilde{d}_{jL} \right) \right\}
$$

(5.28)

where the Yukawa couplings are given by

$$
y_{u_{j\mu}} = -i g_w \sqrt{\frac{kL}{2\sqrt{2}}} \int_1^{z_L} dz h_H(z) \left\{ f_{L\mu j}(z)^* f_{R\mu j}(z) + f_{L\mu j}(z)^* f_{R\mu j}(z) \right\},
$$

$$
y_{d_{j\mu k}} = -i g_w \sqrt{\frac{kL}{2\sqrt{2}}} \int_1^{z_L} dz h_H(z) \sum_{m=1}^3 \left\{ f_{L\mu m}(z)^* f_{R\mu m}(z) + f_{L\mu m}(z)^* f_{R\mu m}(z) \right\}.
$$

(5.29)
Note that the Yukawa couplings in the up-type quark sector are diagonal in the generation space, whereas those in the down-type quark sector have nonvanishing off-diagonal elements.

For the two sets of parameters in Table 4 one finds

\[(a) \quad \theta_H = 0.10 :\]

\[
(y_{uu}, y_{cc}, y_{tt}) = (8.1376 \times 10^{-5}, 2.5186 \times 10^{-3}, 0.69693),
\]

\[
\hat{y}_d = \begin{pmatrix}
1.1800 \times 10^{-5} & -1 \times 10^{-16} & -2 \times 10^{-13} \\
-2 \times 10^{-18} & 2.2378 \times 10^{-4} & 1 \times 10^{-11} \\
-9 \times 10^{-17} & 4 \times 10^{-13} & 1.1759 \times 10^{-2}
\end{pmatrix}.
\]

\[(b) \quad \theta_H = 0.15 :\]

\[
(y_{uu}, y_{cc}, y_{tt}) = (8.1222 \times 10^{-5}, 2.5138 \times 10^{-3}, 0.69620),
\]

\[
\hat{y}_d = \begin{pmatrix}
1.1777 \times 10^{-5} & -3 \times 10^{-16} & -9 \times 10^{-13} \\
-6 \times 10^{-18} & 2.2336 \times 10^{-4} & 2 \times 10^{-11} \\
-4 \times 10^{-16} & 8 \times 10^{-13} & 1.1737 \times 10^{-2}
\end{pmatrix}.
\]

Here \((\hat{y}_d)_{jk} = y_{dj,dk}\). Note that in the evaluation we have used the values \((m_u, m_d) = (20, 2.90)\) MeV for the reason described earlier. The flavor-changing Yukawa couplings are exceedingly small. Splitting of mass \(\Delta m_M\) of neutral mesons \((M = \bar{d}_jd_k, d_j\bar{d}_k, j \neq k)\) due to \(y_{dj,dk}\) is estimated to be at most \[\left(\frac{m_M}{(m_{d_j} + m_{d_k})^2} (m_M f_M^2 / m_H^2) (y_{dj,dk})^2\right)\], which is much smaller than the observed \(\Delta m_M\).

The values of the diagonal part of the Yukawa couplings can be understood from the effective theory as well. Recalling that the 4D Higgs field \(H(x)\) is the fluctuation mode of the AB phase \(\theta_H\), the effective interactions of \(W, Z\) and fermion field \(\psi_f\) with the Higgs field can be written as

\[
\mathcal{L} \sim -\bar{m}_W(\hat{\theta}_H)^2 W_\mu^i W^\mu_i - \frac{1}{2} \bar{m}_Z(\hat{\theta}_H)^2 Z_\mu Z^\mu - \bar{m}_f(\hat{\theta}_H) \bar{\psi}_f \psi_f ,
\]

\[
\hat{\theta}_H(x) = \theta_H + \frac{H(x)}{f_H} .
\]

The mass functions are, in good approximation, given by

\[
\bar{m}_W(\hat{\theta}_H) \sim a_W \sin \hat{\theta}_H ,
\]

\[
\bar{m}_Z(\hat{\theta}_H) \sim a_Z \sin \hat{\theta}_H ,
\]

\[
\bar{m}_f(\hat{\theta}_H) \sim \begin{cases}
a_f \sin \hat{\theta}_H & \text{in the A model} \\
a_f \sin \frac{1}{2} \hat{\theta}_H & \text{in the B model}
\end{cases}.
\]
where $a_W$, $a_Z$ and $a_f$ are constants. At the tree level $m_W = m_W(\theta_H) = \frac{1}{2} g_w f_H \sin \theta_H$, $m_Z = m_Z(\theta_H) = m_W / \cos \theta_W^0$ and $m_f = m_f(\theta_H)$. Expanding the mass functions in (5.31) around $\theta_H$, one find the Higgs couplings to be
\[
g_{WWH} = \frac{2 m_W^2 \cos \theta_H}{f_H \sin \theta_H} = g_w m_W \cos \theta_H ,
\]
\[
g_{ZZH} = \frac{2 m_Z^2 \cos \theta_H}{f_H \sin \theta_H} = \frac{g_w m_Z}{\cos \theta_W^0} \cos \theta_H ,
\]
\[
y_f = \begin{cases} 
m_f \cos \theta_H = \frac{m_f}{v_{SM}} \cos \theta_H \quad & \text{in the A model} \\
\frac{m_f \cos \frac{1}{2} \theta_H}{2 f_H \sin \frac{1}{2} \theta_H} = \frac{m_f}{v_{SM}} \cos^2 \frac{1}{2} \theta_H \quad & \text{in the B model} .
\end{cases}
\] (5.33)

Here $v_{SM} = f_H \sin \theta_H$. In other words, compared to the couplings in the SM, the Higgs couplings of $W$ and $Z$ in the gauge-Higgs unification are suppressed by a factor $\cos \theta_H$. The Yukawa couplings of quarks and leptons are suppressed by a factor $\cos \theta_H$ in the A model and by a factor $\cos^2 \frac{1}{2} \theta_H$ in the B model.

The diagonal part of the evaluated Yukawa couplings (5.30) are well described by the formula in (5.33). Denoting the couplings in the SM by $y_{f,SM}^f = m_f / v_{SM}$, one finds

(a) $\theta_H = 0.10 : \cos^2 \frac{1}{2} \theta_H = 0.99750$
\[
\left( \frac{y_{uu}}{y_{SM}^u}, \frac{y_{cc}}{y_{SM}^c}, \frac{y_{tt}}{y_{SM}^t} \right) = (0.99758, 0.99758, 0.99826) ,
\]
\[
\left( \frac{y_{dd}}{y_{SM}^d}, \frac{y_{ss}}{y_{SM}^s}, \frac{y_{bb}}{y_{SM}^b} \right) = (0.99758, 0.99758, 0.99758) .
\]

(b) $\theta_H = 0.15 : \cos^2 \frac{1}{2} \theta_H = 0.99439$
\[
\left( \frac{y_{uu}}{y_{SM}^u}, \frac{y_{cc}}{y_{SM}^c}, \frac{y_{tt}}{y_{SM}^t} \right) = (0.99456, 0.99456, 0.99607) ,
\]
\[
\left( \frac{y_{dd}}{y_{SM}^d}, \frac{y_{ss}}{y_{SM}^s}, \frac{y_{bb}}{y_{SM}^b} \right) = (0.99456, 0.99456, 0.99456) .
\] (5.34)

The deviation from the SM is rather small.

We would like to add a comment. As explained in Section 2, the neutral physical scalar of $\Phi_{(1,4)}$ has a large mass ($\gg m_{KK}$) so that its couplings to quarks and leptons at low energies are negligible, playing no role in flavor changing processes.

6 Summary and discussions

In this paper we have shown that the flavor mixing in the quark sector can be incorporated in the GUT inspired $SU(3)_C \times SO(5) \times U(1)_X$ gauge-Higgs unification. The
brane interactions on the UV brane are responsible both for splitting the mass spectrum between the up-type quarks and down-type quarks and for generating flavor mixing in the charged current ($W$) interactions. Quite reasonable form of the CKM matrix has been obtained. The mixing, in general, induces FCNC interactions in the $Z$ couplings of quarks. It is shown that the FCNC interactions are naturally suppressed, with a suppression factor of order $10^{-6}$. The suppression is a result of the $SU(3)_C \times SO(5) \times U(1)_X$ or $SU(3)_C \times SO(4) \times U(1)_X$ gauge invariance which allows only a certain class of interactions on the UV brane. In addition to presenting rigorous evaluation of the gauge couplings, we have also given an explanation in terms of the effective theory which illustrates how the natural suppression of the FCNC interactions results in the gauge-Higgs unification. The flavor-mixing induces flavor-changing Yukawa couplings as well. We have confirmed that those couplings are extremely small.

There remains an issue to be clarified. In the present model we could obtain a consistent spectrum and mixing only if the up quark mass $m_u$ were larger than the down quark mass $m_d$. With the minimal matter content in the GUT inspired gauge-Higgs unification, $m_d$ necessarily becomes smaller than $m_u$. One may have an additional field which affects $m_u$, or may consider the running of quark masses which reverses the order of $m_u$ and $m_d$ at low energies. We leave the issue for future investigation.

In the GUT inspired gauge-Higgs unification we have chosen negative bulk mass parameters. With positive bulk mass parameters there arise exotic light fermions with the same quantum numbers as the down-type quarks. Although negative bulk mass parameters imply that left-handed (right-handed) light quarks are localized near the IR (UV) brane, we have shown that the $W$ and $Z$ couplings of all quarks are very close to those in the SM. This is one of the remarkable properties in the gauge-Higgs unification in the RS space. Similarly negative bulk mass parameters of leptons are preferred to positive ones, as positive ones yield additional light neutral fermions.

The sign of the bulk mass parameters of quarks and leptons can be investigated by $e^+e^-$ collider experiments, as the couplings of quarks and leptons to $Z'$ bosons, namely KK excited states of $Z$, $\gamma$ and $Z_R$, have large parity violation. It has been shown in the previous A-model of $SO(5) \times U(1)$ gauge-Higgs unification that right-handed quarks and leptons have much larger couplings to $Z'$ bosons so that in the process $e^+e^- \to \mu^+\mu^-$, for instance, significant deviation from the SM appears even at 250 GeV ILC with 250 fb$^{-1}$ data. If the $e^-$ beam is polarized in the left-handed mode, there would be no deviation from the SM, whereas, if the $e^-$ beam is polarized in the right-handed mode, then there appears large deviation. By changing the polarization of the $e^-$ beam, one can see a
distinct pattern of deviation. Similar effects are seen in the forward-backward asymmetry in various processes as well. In the present B-model left-handed leptons and quarks have much larger couplings to $Z'$ bosons than right-handed ones. As a consequence the pattern of the dependence on the $e^-$ polarization is reversed in comparison with that in the A-model. ILC experiments can provide rich information on underlying physics.

Gauge-Higgs unification is formulated in five or higher dimensions in which the running of gauge couplings is much more rapid than in four dimensions.\[32\] In this paper we have analyzed the $W$ and $Z$ couplings of quarks below the KK mass scale $m_{KK}$. All relations presented in this paper should be understood as those for the energy scale below $m_{KK}$. Above $m_{KK}$ effects of KK modes need to be properly incorporated. Gauge-Higgs unification is a new approach to physics beyond the SM. It may provide a key to solve the problems of dark matter, gauge hierarchy, neutrinos, Higgs couplings, and grand unification as well.\[33]-[36] We will come back to these issues in the future.

**Acknowledgements**

This work was supported in part by European Regional Development Fund-Project Engineering Applications of Microworld Physics (No. CZ.02.1.01/0.0/0.0/16-019/0000766) (Y.O.), by the National Natural Science Foundation of China (Grant Nos. 11775092, 11675061, 11521064, 11435003 and 11947213) (S.F.), by the International Postdoctoral Exchange Fellowship Program (IPEFP) (S.F.), and by Japan Society for the Promotion of Science, Grants-in-Aid for Scientific Research, No. 19K03873 (Y.H.) and Nos. 18H05543 and 19K23440 (N.Y.).

**A Basis functions**

We summarize basis functions in the RS space. We define

$$ F_{\alpha,\beta}(u, v) \equiv J_\alpha(u)Y_\beta(v) - Y_\alpha(u)J_\beta(v) \quad (A.1) $$

where $J_\alpha(x)$ and $Y_\alpha(x)$ are Bessel functions of the 1st and 2nd kind, respectively. For gauge bosons $C = C(z; \lambda)$ and $S = S(z; \lambda)$ are defined by

$$ C(z; \lambda) = \frac{\pi}{2} \lambda z z_L F_{1,0}(\lambda z, \lambda z_L), $$
$$ C'(z; \lambda) = \frac{\pi}{2} \lambda^2 z^2 z_L F_{0,0}(\lambda z, \lambda z_L), $$
$$ S(z; \lambda) = -\frac{\pi}{2} \lambda z F_{1,1}(\lambda z, \lambda z_L), $$
$$ S'(z; \lambda) = -\frac{\pi}{2} \lambda^2 z F_{0,1}(\lambda z, \lambda z_L). \quad (A.2) $$
We note that $CS' - SC' = \lambda z$.

For massless fermions we define

\[
\begin{pmatrix}
  C_R \\ S_R
\end{pmatrix}(z; \lambda, c) = \pm \frac{\pi}{2} \sqrt{zz_F} \frac{e^{-z \lambda \sqrt{zz_F}}}{\sqrt{zz_F}} \left( \lambda z, \lambda z_L \right),
\]
\[
\begin{pmatrix}
  C_L \\ S_L
\end{pmatrix}(z; \lambda, c) = \pm \frac{\pi}{2} \lambda \sqrt{zz_F} \frac{e^{z \lambda \sqrt{zz_F}}}{\sqrt{zz_F}} \left( \lambda z, \lambda z_L \right).
\]

(A.3)

These satisfy $C_L C_R - S_L S_R = 1$, $C_L(z; \lambda, -c) = C_R(z; \lambda, c)$, and $S_L(z; \lambda, -c) = -S_R(z; \lambda, c)$.

For massive fermions such as $D^\pm$ fields with $m_D \neq 0$ we define basis functions

\[
\begin{pmatrix}
  C_{R1} \\ C_{L1}
\end{pmatrix}(z; \lambda, c, \tilde{m}) = \begin{pmatrix}
  C_R \\ C_L
\end{pmatrix}(z; \lambda, c + \tilde{m}) + \begin{pmatrix}
  C_R \\ C_L
\end{pmatrix}(z; \lambda, c - \tilde{m}),
\]
\[
\begin{pmatrix}
  C_{R2} \\ C_{L2}
\end{pmatrix}(z; \lambda, c, \tilde{m}) = \begin{pmatrix}
  S_R \\ S_L
\end{pmatrix}(z; \lambda, c + \tilde{m}) - \begin{pmatrix}
  S_R \\ S_L
\end{pmatrix}(z; \lambda, c - \tilde{m}),
\]
\[
\begin{pmatrix}
  S_{R1} \\ S_{L1}
\end{pmatrix}(z; \lambda, c, \tilde{m}) = \begin{pmatrix}
  S_R \\ S_L
\end{pmatrix}(z; \lambda, c + \tilde{m}) + \begin{pmatrix}
  S_R \\ S_L
\end{pmatrix}(z; \lambda, c - \tilde{m}),
\]
\[
\begin{pmatrix}
  S_{R2} \\ S_{L2}
\end{pmatrix}(z; \lambda, c, \tilde{m}) = \begin{pmatrix}
  C_R \\ C_L
\end{pmatrix}(z; \lambda, c + \tilde{m}) - \begin{pmatrix}
  C_R \\ C_L
\end{pmatrix}(z; \lambda, c - \tilde{m}).
\]

(A.4)

These functions satisfy various relations which are summarized in Appendix B of ref. [13].

References

[1] Y. Hosotani, “Dynamical mass generation by compact extra dimensions”, Phys. Lett. B126, 309 (1983).
[2] Y. Hosotani, “Dynamics of nonintegrable phases and gauge symmetry breaking”, Ann. Phys. (N.Y.) 190, 233 (1989).
[3] A. T. Davies and A. McLachlan, “Gauge group breaking by Wilson loops”, Phys. Lett. B200, 305 (1988); “Congruency class effects in the Hosotani model”, Nucl. Phys. B317, 237 (1989).
[4] H. Hatanaka, T. Inami, C.S. Lim, “The gauge hierarchy problem and higher dimensional gauge theories”, Mod. Phys. Lett. A13, 2601 (1998).
[5] H. Hatanaka, “Matter representations and gauge symmetry breaking via compactified space”, Prog. Theoret. Phys. 102, 407 (1999).
[6] M. Kubo, C.S. Lim and H. Yamashita, “The Hosotani mechanism in bulk gauge theories with an orbifold extra space $S^1/Z_2$”, Mod. Phys. Lett. A17, 2249 (2002).
[7] K. Agashe, R. Contino and A. Pomarol, “The minimal composite Higgs model”, Nucl. Phys. B719, 165 (2005).
[8] A. D. Medina, N. R. Shah and C. E. M. Wagner, “Gauge-Higgs unification and radiative electroweak symmetry breaking in warped extra dimensions”, Phys. Rev. D76, 095010 (2007).

[9] Y. Hosotani, K. Oda, T. Ohnuma and Y. Sakamura, “Dynamical electroweak symmetry breaking in SO(5)×U(1) gauge-Higgs unification with top and bottom quarks”, Phys. Rev. D78, 096002 (2008); Erratum-ibid. D79, 079902 (2009).

[10] S. Funatsu, H. Hatanaka, Y. Hosotani, Y. Orikasa and T. Shimotani, “Novel universality and Higgs decay H → γγ, gg in the SO(5)×U(1) gauge-Higgs unification”, Phys. Lett. B722, 94 (2013).

[11] S. Funatsu, H. Hatanaka, Y. Hosotani, Y. Orikasa, and T. Shimotani, “LHC signals of the SO(5)×U(1) gauge-Higgs unification”, Phys. Rev. D89, 095019 (2014).

[12] S. Funatsu, H. Hatanaka, Y. Hosotani and Y. Orikasa, “Collider signals of W’ and Z’ bosons in the gauge-Higgs unification”, Phys. Rev. D95, 035032 (2017).

[13] S. Funatsu, H. Hatanaka, Y. Hosotani, Y. Orikasa and N. Yamatsu, “GUT inspired SO(5)×U(1)×SU(3) gauge-Higgs unification”, Phys. Rev. D99, 095010 (2019).

[14] Y. Hosotani and N. Yamatsu, “Gauge-Higgs grand unification”, Prog. Theoret. Exp. Phys. 2015, 111B01 (2015).

[15] A. Furui, Y. Hosotani, and N. Yamatsu, “Toward realistic gauge-Higgs grand unification”, Prog. Theoret. Exp. Phys. 2016, 093B01 (2016); Y. Hosotani and N. Yamatsu, “Electroweak symmetry breaking and mass spectra in six-dimensional gauge-Higgs grand unification”, Prog. Theoret. Exp. Phys. 2018, 023B05 (2018).

[16] J. Yoon and M.E. Peskin, “Dissection of an SO(5)×U(1) gauge-Higgs unification model”, Phys. Rev. D100, 015001 (2019).

[17] S. Funatsu, H. Hatanaka, Y. Hosotani and Y. Orikasa, “Distinct signals of the gauge-Higgs unification in e+e− collider experiments”, Phys. Lett. B775, 297 (2017).

[18] J. Yoon and M.E. Peskin, “Fermion pair production in SO(5)×U(1) gauge-Higgs unification models”, arXiv:1811.07877 [hep-ph].

[19] S. Funatsu, “Forward-backward asymmetry in the gauge-Higgs unification at the International Linear Collider”, arXiv:1905.10007 [hep-ph].

[20] Y. Hosotani, “Gauge-Higgs unification at e+e− linear colliders”, arXiv:1904.10156 [hep-ph].

[21] S. Bilokin, R. Pöschl and F. Richard, “Measurement of b quark EW couplings at ILC”, arXiv:1709.04289 [hep-ex]; F. Richard, “Bhabha scattering at ILC250”, arXiv:1804.02846 [hep-ex]; A. Irles, R. Pöschl, F. Richard and H. Yamamoto, “Complementarity between ILC250 and ILC-GigaZ”, arXiv:1905.00220 [hep-ex].
[22] K. Fujii et al., “Physics case for the 250 GeV stage of the International Linear Collider”, arXiv:1710.07621 [hep-ex]; ILC Collaboration (H. Aihara et al.), “The International Linear Collider: A Global Project”, arXiv:1901.09829 [hep-ex]; Philip Bambade et al., “The International Linear Collider: A Global Project”, arXiv:1903.01629 [hep-ex];

[23] Y. Adachi, N. Kurahashi, C.S. Lim and N. Maru, “Flavor mixing in gauge-Higgs unification”, JHEP 1011, 150 (2010); Y. Adachi, N. Kurahashi and N. Maru, “$D^0$-$\bar{D}^0$ mixing in gauge-Higgs unification”, JHEP 1201, 047 (2012); Y. Adachi, N. Kurahashi, N. Maru and K. Tanabe, “$B^0$-$\bar{B}^0$ mixing in gauge-Higgs unification”, Phys. Rev. D85, 096001 (2012).

[24] G. Cacciapaglia, C. Csaki, J. Galloway, G. Marandella, J. Terning and A. Weiler, “A GIM mechanism from extra dimensions”, JHEP 0804, 006 (2008).

[25] T. Gherghetta and A. Pomarol, “Bulk fields and supersymmetry in a slice of AdS”, Nucl. Phys. B586, 141 (2000).

[26] A. Falkowski, “Holographic pseudo-Goldstone boson”, Phys. Rev. D75, 025017 (2007).

[27] Y. Hosotani and Y. Sakamura, “Anomalous Higgs couplings in the $SO(5) \times U(1)_{B-L}$ gauge-Higgs unification in warped spacetime”, Prog. Theoret. Phys. 118, 935 (2007).

[28] M. Tanabashi et al. (Particle Data Group), “Review of Particle Physics”, Phys. Rev. D98, 030001 (2018), Chap. 10. “Electroweak Model and Constraints on New Physics”.

[29] K.A. Olive et al. [Particle Data Group], Review of Particle Physics, Chin. Phys. C38, 090001 (2014).

[30] A. Crivellin, G. DAmbrosio, M. Hoferichter, and L.C. Tunstall, “Violation of lepton flavor and lepton flavor universality in rare kaon decays, Phys. Rev. D93, 074038 (2016); F. Gabbiani, E. Gabrielli, A. Masiero, and L. Silvestrini, “A complete analysis of FCNC and CP constraints in general SUSY extensions of the standard model”, Nucl. Phys. B477, 321 (1996); P.T.P. Hutauruk, T. Nomura, H. Okada and Y. Orikasa, “Dark matter and $B$-meson anomalies in a flavor dependent gauge symmetry”, Phys. Rev. D99, 055041 (2019).

[31] Y. Hosotani and Y. Kobayashi, “Yukawa couplings and effective interactions in gauge-Higgs unification”, Phys. Lett. B674, 192 (2009).

[32] N. Yamatsu, “Gauge coupling unification in gauge-Higgs grand unification”, Prog. Theoret. Exp. Phys. 2016, 043B02 (2016); A. Abdalgabar, M.O. Khojali, A.S. Cornell, G. Cacciapaglia, A. Deandrea, “Unification of gauge and Yukawa couplings”, Phys. Lett. B776, 231 (2018).
[33] G. Burdman and Y. Nomura, “Unification of Higgs and gauge fields in five dimensions”, Nucl. Phys. B656, 3 (2003); N. Haba, Masatomi Harada, Y. Hosotani and Y. Kawamura, “Dynamical rearrangement of gauge symmetry on the orbifold \(S^1/Z_2\)”, Nucl. Phys. B657, 169 (2003); Erratum-ibid. B669, 381 (2003); N. Haba, Y. Hosotani, Y. Kawamura and T. Yamashita, “Dynamical symmetry breaking in gauge Higgs unification on orbifold”, Phys. Rev. D70, 015010 (2004); C.S. Lim and N. Maru, “Towards a realistic grand gauge-Higgs unification”, Phys. Lett. B653, 320 (2007); K. Kojima, K. Takenaga and T. Yamashita, “Grand gauge-Higgs unification”, Phys. Rev. D84, 051701(R) (2011); “Gauge symmetry breaking patterns in an SU(5) grand gauge-Higgs unification”, Phys. Rev. D95, 015021 (2017); M. Frigerio, J. Serra and A. Varagnolo, “Composite GUTs: Models and expectations at the LHC”, JHEP 1106, 029 (2011).

[34] S. Funatsu, H. Hatanaka, Y. Hosotani, Y. Orikasa and T. Shimotani, “Dark matter in the \(SO(5) \times U(1)\) gauge-Higgs unification”, Prog. Theoret. Exp. Phys. 2014, 113B01 (2014); N. Maru, N. Okada and S. Okada, “\(SU(2)_L\) doublet vector dark matter from gauge-Higgs unification”, Phys. Rev. D98, 075021 (2018); G. Cacciapaglia, H. Cai, A. Deandrea, A. Kushwaha, “Composite Higgs and dark matter model in \(SU(6)/SO(6)\)”, arXiv:1904.09301 [hep-ph].

[35] Y. Adachi and N. Maru, “Revisiting electroweak symmetry breaking and the Higgs boson mass in gauge-Higgs unification”, Phys. Rev. D98, 015022 (2018); K. Hasegawa and C.S. Lim, “Majorana neutrino masses in the scenario of gauge-Higgs unification”, Prog. Theoret. Exp. Phys. 2018, 073B01 (2018); C.S. Lim, “The implication of gauge-Higgs unification for the hierarchical fermion masses”, Prog. Theoret. Exp. Phys. 2018, 093B02 (2018); N. Maru and Y. Yatagai, “Fermion mass hierarchy in grand gauge-Higgs unification”, arXiv:1903.08359 [hep-ph]; K. Hasegawa and C.S. Lim, “\(t\) Hooft-Polyakov monopole and instanton-like topological solution in gauge-Higgs unification”, arXiv:1908.07156 [hep-ph].

[36] J. Hisano, Y. Shoji and A. Yamada, “To be, or not to be finite? The Higgs potential in gauge Higgs unification”, arXiv:1908.09155 [hep-ph]; Y. Hosotani, N. Maru, K. Takenaga and T. Yamashita, “Two loop finiteness of Higgs mass and potential in the gauge-Higgs unification”, Prog. Theoret. Phys. 118, 1053 (2007).