Can leptophilic-ALP be a solution to the muon \((g - 2)\) anomaly?

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In the light of recent measurement of muon \((g - 2)\), we investigate the phenomenological implications of an axion-like particle (ALP) which only couples to the standard model charged leptons. We find that in a narrow mass range of ALP, it can alleviate the tension between the theoretical prediction and experimental observation of \((g - 2)\mu\) once we consider all possible one and two-loop diagrams. In particular, ALP can either explain the muon \((g - 2)\) anomaly for \(5 \text{ GeV} \lesssim m_a \lesssim 6 \text{ GeV}\) while satisfying the other experimental constraints, or be restricted by it.

I. Introduction

The precise measurement of the anomalous magnetic moment of muon reveals the possibility of existing physics beyond Standard Model (BSM). Combining the old measurement of muon \((g - 2)\) by BNL E821 experiment [1] along with the recent one by Fermilab [2], it was observed that the experimental observation of \(a_\mu (= (g - 2)/2)\) differs with the Standard Model (SM) prediction by more than 5\(\sigma\) and the deviation is given by [2]

\[\Delta a_\mu = (249 \pm 48) \times 10^{-11}.\]

The apparent mismatch between theoretical prediction [3–23] and experimental observation may be a pointer towards physics beyond the standard model (BSM). People have proposed many new ideas to address this issue [24–26].

It should, however, be mentioned that the above discrepancy is based on the estimate of the hadronic vacuum polarisation (HVP)/light-by-light (HLBL) contribution in data-driven approaches [23]. On the other hand, estimates based on lattice calculation exhibit closer statistical agreement [27], although this may also in principle be due to theoretical uncertainties in such estimates. It should also be mentioned that a stand-alone data-driven estimate, namely that by the CMD3 Collaboration [28], has recently reported a rather satisfactory agreement with observations.

The discrepancy between the observation and theoretical prediction of \((g - 2)_\mu\) is thus an issue which is still quite alive. Therefore, any theoretical scenario beyond the standard model which has potential contribution to \((g - 2)_\mu\), remains a topic of active interest. On the one hand, it may be looked upon as a way of explaining \((g - 2)_\mu\) if the discrepancy persists. On the other side, such a scenario gets subjected to serious constraints, if indeed the discrepancy turns out to be insignificant.

Another, prima facie unrelated, issue of the SM is the appearance of a CP violating phase \((\theta_{\text{QCD}})\) in the quantum chromodynamics (QCD) Lagrangian [29, 30] which has to be tightly constrained from the measurement of neutron electric dipole moment and the upper bound of \(\theta_{\text{QCD}}\) is \(10^{-10}\) [31]. The problem of tiny value of \(\theta_{\text{QCD}}\) has been resolved by proposing a new axial \(U(1)\) global symmetry, known as Pecci-Quinn (PQ) symmetry [32, 33]. In this scenario, \(\theta_{\text{QCD}}\) parameter is replaced by a dynamical field, known as axion [34, 35]. The axion field generates tiny mass from the QCD non-perturbative effects and it can act as a dark matter (DM) candidate [36–40]. It is to be noted that the mass of axion and its coupling with SM are not free parameters because both of them are functions of the PQ breaking scale.

Generalising the idea of axion, one can also consider axion-like particles (ALP) which is a pseudo Nambu-Goldstone boson, appearing in the theory due to the spontaneous breaking of an approximate global symmetry. An ALP can be motivated, for example, in terms of some high-scale scenarios such as string theories [41–43]. The phenomenology of ALP is rich since its mass and couplings with the SM fields are independent parameters, in contrast to a PQ axion [44–49]. An ALP, according to some conjectures, may also serve as a DM [50–52] as well as a DM portal [53]. The couplings of ALP with the SM fields are similar to that of an axion which implies that the ALP couples with two photon via a dimension five operator and it can decay into two photon. The coupling of ALP with photon has very rich phenomenology and it can be constrained from various laboratory, astrophysical observations as discussed in [44, 51, 54–56].

In this work, we primarily focus on leptophilic ALP scenario where the ALP couples with the charged SM leptons. The interaction of ALP with the SM is governed by the following effective Lagrangian.

\[
\mathcal{L}_a = \frac{1}{2} \partial^\mu a \partial_\mu a - \frac{1}{2} m_a^2 a^2 - \sum_{\ell} \frac{ie a}{f_\ell} \bar{\ell} \gamma_5 \ell a , \quad (1)
\]
where the ALP $a$ of mass $m_a$ couples with the charged SM leptons $\ell$ of mass $m_\ell$ with coupling $c_{\ell\ell}/f_a$ and $f_a$ is the ALP decay constant. In our analysis, we consider $c_{\ell\ell}/f_a$ coefficients are same for all the SM charged leptons. Let us note that although the ALP does not couple with SM quarks and photon at tree level but these couplings can be generated at loop level.

Considering Eq. 1 as our starting point, we consider all possible one and two loop diagrams for the calculation of muon $(g-2)$. We found that in this set-up, muon $(g-2)$ anomaly can be explained for $5 \text{ GeV} \lesssim m_a \lesssim 6 \text{ GeV}$ while satisfying the other constraints. We also discuss the phenomenological consequences of loop-induced ALP-photon and ALP-quark couplings.

The rest of the article is organised as follows. In Section II, ALP interpretation of muon $(g-2)$ has been thoroughly discussed. The constraints in the leptophilic-ALP scenario has been discussed in Section III and finally we conclude in Section IV.

II. ALP interpretation of muon $(g-2)$

The coupling of ALP with the muon opens up the possibility to ameliorate the tension between SM prediction and experimental observation of anomalous magnetic moment of muon. The one loop contribution of ALP to the $\Delta a_\mu$ (see Fig. 1) is given by [57, 58]

$$\Delta a_\mu^{1-\text{loop}} = - \left( \frac{c_{\ell\ell}m_\mu}{f_a} \right)^2 \frac{r}{8\pi^2} \int_0^1 dx \frac{x^3}{1-x+rx^2},$$  \hspace{1cm} (2)

where $m_\mu$ is the mass of the muon, $r = m_\mu^2/m_a^2$ and the integral is always positive (since $r > 0$ always).

As one can see from Eq. 2, $\Delta a_\mu^{1-\text{loop}}$ is always negative irrespective of the sign of $c_{\ell\ell}$ because of the presence of $\gamma_5$ in the ALP muon coupling. Thus, by considering Fig. 1 alone, it is not possible to improve the theoretical prediction of $\Delta a_\mu$.

However, in our framework, ALP-lepton couplings can contribute to the muon $(g-2)$ at two-loop level. The two loop diagrams are shown in Fig. 2 and its contribution to $\Delta a_\mu$ is given by the following expression [59],

$$\Delta a_\mu^{2-\text{loop}} = \frac{\alpha_{\text{em}} m_\mu^2}{8\pi^3} \left( \frac{c_{\ell\ell}}{f_a} \right)^2 \sum_\ell q_\ell^2 F \left( \frac{m_\mu^2}{m_\ell^2}, \frac{m_\mu^2}{m_a^2} \right),$$  \hspace{1cm} (3)

where $q_\ell$ is the electromagnetic charge of the charged SM lepton $\ell$ running in the loop and the loop function $F(a, b)$ is given by

$$F(a, b) = \int_0^1 dx dy dz \frac{ax}{a(1-x) + abyz(1-z) + bz(1-z)x^2(1-y)^2}.$$  \hspace{1cm} (4)

Let us note in passing, Eq. 3 always gives positive contribution to $\Delta a_\mu$ since $F(a, b) > 0$, $\forall a, b > 0$. It is to be noted, we have not considered the diagrams in which the photon propagator is replaced by the $Z$ boson propagator. This is because, those contributions are suppressed due to the massive $Z$ boson propagator and $Z$ boson couplings with SM fermions.

Thus the total ALP contribution to $\Delta a_\mu$ is given by

$$\Delta a_\mu = \Delta a_\mu^{1-\text{loop}} + \Delta a_\mu^{2-\text{loop}},$$  \hspace{1cm} (5)

where individual expressions are given in Eq. 2 and Eq. 3.

In Fig. 3, we show the effect of individual loop diagrams as a function of ALP mass for a fixed value of $c_{\ell\ell}/f_a$. As one can see, for $m_a \lesssim 1 \text{ GeV}$, $\Delta a_\mu < 0$ and $\Delta a_\mu > 0$ as we increase $m_a$ above a few GeV. This is because ALP contribution to $\Delta a_\mu$ is a linear combination between a negative term, coming from Eq. 2 and a positive term,
coming from Eq. 3 and the interplay between these two contributions highly dependent on the ALP mass. One can clearly see from the figure that the negative contribution gradually increases (decreases) as we decrease (increase) $m_a$. Thus for small $m_a$ the positive contribution coming from Eq. 3 will be negated by the large negative contribution. However, the effect of one loop contribution to $\Delta a_\mu$ becomes smaller as we increase the ALP mass $m_a$ and as a result, positive contribution of Eq. 3 dominates over the negative contribution of Eq. 2. Finally, in Fig. 4, we show the region of parameter space in $m_a - |c_{\ell\ell}/f_a|$ plane where the muon $(g - 2)$ anomaly is satisfied within 2$\sigma$ limit (red points). 

In the next section, we will discuss the collider constraints in $m_a - c_{\ell\ell}/f_a$ plane.

![FIG. 3. Variation of $\Delta a_\mu$ (dashed line), $\Delta a_\mu^{1\text{-loop}}$ (dotted line), and $\Delta a_\mu$ (solid line) as a function of ALP mass $m_a$ for $c_{\ell\ell}/f_a = 0.129 \text{GeV}^{-1}$. The grey band denotes the 2$\sigma$ limit of $\Delta a_\mu$.](image)

**III. Existing constraints and result**

In our proposed scenario, ALP-photon and ALP-quark couplings are absent at tree-level but these couplings can be generated radiatively, as mentioned in the introduction. Thus the constraints on the ALP-photon and ALP-quark coupling can be translated to the ALP-lepton coupling. The constraints on the ALP-lepton coupling from various observations are discussed below.

- Constraint from ALP-photon coupling: The ALP-photon coupling is severely constrained from various astrophysical, cosmological, and laboratory observations [44]. In our parameter space of interest, the constraints mainly arise from the searches of $\gamma + \gamma$ and tri-photon final states at LEP, LHC, and CDF [60, 61]. In our scenario, the ALP-photon coupling can be generated at one-loop level and the effective ALP-photon coupling is given by [44]

$$\frac{c_{\gamma\gamma}}{f_a} = \frac{c_{\ell\ell}}{16\pi^2 f_a} \sum \mathcal{B}(\tau_\ell)$$

where $\tau_\ell = 4m_{H}^2/m_a^2$ and the loop function $\mathcal{B}(\tau_\ell)$ is given by

$$\mathcal{B}(\tau_\ell) = 1 - \tau_\ell f(\tau_\ell)^2$$

with

$$f(\tau_\ell) = \arcsin \left( \frac{1}{\sqrt{\tau_\ell}} \right) \quad \text{for } \tau_\ell \geq 1$$

$$= \frac{\pi}{2} + \frac{i}{2} \ln \left( \frac{1 + \sqrt{1 - \tau_\ell}}{1 - \sqrt{1 - \tau_\ell}} \right) \quad \text{for } \tau_\ell < 1 \, .$$

Using Eq. 6, we translate the constraints from $m_a - c_{\gamma\gamma}/f_a$ plane to $m_a - c_{\ell\ell}/f_a$ plane and these constraints are shown by the light green region in Fig. 4.

- Constraint from ALP-lepton coupling: In our scenario, the tree-level ALP-lepton coupling can be constrained from $e^+e^- \rightarrow \mu^+\mu^- a \rightarrow 2\mu^+2\mu^-$ search at BaBar [62] and the relevant constraint is shown by the light blue region of Fig. 4. The constraint on the ALP parameter space from the recently released data of $e^+e^- \rightarrow \mu^+\mu^- a \rightarrow 2\mu^+2\mu^-$ search at Belle II [63] requires a dedicated analysis which is beyond the scope of this work.

- Constraint from ALP-quark coupling: Existence of ALP-quark coupling can induce flavor-violating decay modes and thus the ALP-quark coupling can be constrained from the upper limit of these decay modes, as discussed in [44, 47, 64].

![FIG. 4. ALP parameter space in $m_a - |c_{\ell\ell}/f_a|$ plane. The red points are the $\Delta a_\mu$ satisfied points within 2$\sigma$ limit. The light blue region is excluded from BaBar and the combined constraint from LEP, LHC, and CDF is depicted by light green region.](image)
In our model, the ALP-quark coupling is absent at tree-level but it can be generated at two-loop level [65]. Parametrizing the ALP-quark coupling as $-ig_Y(\sqrt{2}m_q/v)\bar{q}q$, the estimated value of the ALP-top quark coupling for $m_a = 5$ GeV is $g_Y \approx 10^{-2} \times c_{\ell} / f_a$ GeV. Thus for $c_{\ell} / f_a \sim 0.1$ GeV$^{-1}$, the value of $g_Y$ is two-orders of magnitude smaller than the existing constraint from $B_s \to \mu^+\mu^-$. Due to these loop induced effects, the constraints of loop-induced ALP-photon and ALP-quark couplings are absent.

**IV. Summary and Conclusions**

In this work we have studied the prospect of ALP to satisfy the muon $(g-2)$ anomaly. In order to investigate this, we consider leptophilic-ALP model where the ALP couplings with quarks and photon are absent at tree-level. In this formulation, we consider all possible one and two-loop diagrams for the calculation of muon $(g-2)$ anomaly. We found that in our model it is possible to explain the muon $(g-2)$ anomaly due to presence of two-loop diagrams because of its positive contribution to $\Delta a_\mu$. We also discuss the phenomenological implications of loop-induced ALP-photon and ALP-quark couplings. Due to these loop induced effects, the constraints on the ALP-photon and ALP-quark couplings can be translated into the ALP-lepton interaction. We found that for leptophilic-ALP scenario, most of the parameter space is excluded by the existing collider bounds. However, in a narrow region of parameter space between $m_a \approx 5$ GeV to $m_a \approx 6$ GeV, muon $(g-2)$ anomaly can be explained by ALP while satisfying the other existing constraints. Since this picture regarding the “estimate-observation” issue on $(g-2)_\mu$ is still somewhat unclear, it is fair to say that the above statements apply to the situation where this shortfall in theoretical prediction persists. On the other hand, if the deficit finally disappears, and suggested by lattice estimates on the CMD3 analysis, then the game gets turned around. In such an eventuality, one might expect the ‘favoured’ and ‘disfavoured’ regions in the ALP parameter space to be swapped, subject to all other phenomenological constraints.

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