Incorrect sample classification in “Electron localization induced by intrinsic anion disorder in a transition metal oxynitride”

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I. INTRODUCTION

With their experimental study of temperature and composition dependences of transport properties of the disordered crystalline solid SrNbO$_3-x$N$_x$, Ref. 1 Daichi Oka and co-workers aimed to contribute to a better understanding of electron localization. Interpreting their measured data according to the currently prevailing theories, they came across a gold nugget, but ignored it. That is, they obtained some data which clearly contradict these theories. Although those data are presented graphically in Ref. 1, that serious inconsistency is not mentioned at all in its text. – This is surprising since the authors cite a review and a comment, their Refs. 30 and 32 (here Refs. 2 and 3), to which discrepancies of this type are central. – Those data are very interesting because they challenge the widely presumed continuity of the metal-insulator transition (MIT) in three-dimensional disordered solids, and thus the applicability of the very highly cited scaling theory of localization, Ref. 4. In detail:

In identifying the MIT, Daichi Oka and co-workers rely on the theoretical ideas of Anderson localization and of the interference of electron-electron interaction and elastic impurity scattering of electrons. In particular, they do so when analysing the temperature dependence of the conductivity, $\sigma(T)$, of the sample with $x = 0.96$, referred to as sample 0.96 in the following. According to the $T \to 0$ extrapolation of a $\sigma = a + b T^{1/2}$ fit, this sample is classified as metallic in Ref. 1; thus the authors conclude the MIT to happen within the $x$ interval (0.96,1.02). That data evaluation, however, is invalid for several reasons as the present reconsideration from three different perspectives shows.

II. THE LOGARITHMIC DERIVATIVE OF $\sigma(T)$

Consider first the logarithmic derivative $w(T) = \mathrm{d} \ln \sigma / \mathrm{d} \ln T$. If measured data really obey $\sigma = a + b T^{p}$ with positive $a$, $b$, and $p$, then the slope of $w(T)$ is positive – as claimed in the text of Ref. 1 to be the case for sample 0.96 at low $T$ – and $w(T) \to 0$ as $T \to 0$. The validity of these statements on $w(T)$ should be verifiable by looking at Fig. 1d of Ref. 1. Unfortunately, however, evaluating the low-$T$ part of the $w(T)$ data for sample 0.96 shown therein is complicated by the wide width of the $w$ range presented. Nevertheless, a thorough inspection of this diagram, enlarged by means of the PDF viewer, yields information on sample 0.96 that differs qualitatively from the above conclusions drawn from the ansatz $\sigma = a + b T^{p}$: (i) With decreasing $T$, between about 8 K and the lowest $T$ value considered, roughly 2 K, $w(T)$ no longer decreases as for $25 \, \mathrm{K} > T \gtrsim 11$ K, but even increases slightly; this finding contradicts the above mentioned claim of Ref. 1. (ii) In the whole $T$ range shown, $w(T) > 0$. Taking these two points together, one naively expects that $w(T)$ tends to some finite positive value $w_0$ as $T \to 0$, so that $\sigma(T)$ vanishes as $T^{-w_0} – or that $w(T)$ even diverges in this limit, so that $\sigma(T)$ vanishes in some exponential manner –. Therefore, the electric transport in sample 0.96 is most likely activated instead of metallic.

To shed more light on the contradiction between this conclusion and the interpretation in Ref. 1, we digitized the information on sample 0.96 given in Figs. 1b, 1c, and 1d of Ref. 1 by means of WinDIG 2.5. Furthermore, for comparison, data for the sample with $x = 1.02$, presented in Figs. 1c, 1d and in Suppl. Fig. 5 of Ref. 1, were read out. The latter sample is referred to as sample 1.02 from now on.

The left panel of Fig. 1 of our reconsideration makes the above described observations easily comprehensible. Therein, the $w(T)$ data points of sample 0.96 given in Fig. 1d of Ref. 1 are redrawn, where a roughly tenfold zoomed in $w$ scale is used. These data are contrasted with the analytically calculated logarithmic derivative of the $\sigma = a + b T^{1/2}$ fit shown in Fig. 1c of Ref. 1; for related details see below. Three characteristics of this set of redrawn data points catch the eye: (1) When $T$ decreases from 25 K to about 10.5 K, $w(T)$ decreases too, though more and more slowly in this $w$ versus $T^{1/2}$ representation. (2) Between about 10.5 K and about 4.7 K, $w(T)$ exhibits a peculiar feature, that is a pronounced hump and a subsequent swale. (3) Finally, below about 4.7 K, with further decreasing $T$, $w(T)$ slowly rises again; $w(T)$ increases by about 9 % from 4.7 K to 2.0 K – whereas the logarithmic derivative of the fit function (red curve) decreases by 31 % over this interval –.

It is instructive to compare the behaviour of $w(T)$ for sample 0.96 to that of $w(T)$ for sample 1.02, nominally SrNbO$_2$N. For the latter sample, two data sets are available from Ref. 1 that is from Fig. 1d and from Suppl. Fig. 5, in the following denoted as data sets A and B, respectively. Surprisingly, they differ considerably from each other, as is obvious upon first look at the respective $T$ regions around 10 K in both the diagrams. These two $w(T)$ data sets are reproduced in the right panel of our Fig. 1; they are contrasted with the $w(T)$ data for sample 0.96 therein.

According to the right panel of Fig. 1, in case of data set A, $w(T)$ smoothly runs through a minimum when $T$
FIG. 1: Temperature dependence of the logarithmic derivative $w(T) = d \ln \sigma / d \ln T$ of the conductivity $\sigma(T)$ for the sample with $x = 0.96$, marked by black +. These data points were read out from Fig. 1d of Ref. 1. In the left panel, they are compared to the logarithmic derivative of the fit function shown in Fig. 1c of Ref. 1, given as red line here, and, additionally, to the logarithmic derivative of the power law interpolation between the $\sigma(4.7 \, \text{K})$ and $\sigma(10.7 \, \text{K})$ from Fig. 1c of Ref. 1, shown as dashed magenta line. The right panel contrasts the $w(T)$ data points for the sample with $x = 0.96$ with $w(T)$ data points for the sample with $x = 1.02$, where different scale (zero) was used. The latter data points, read out from Fig. 1d of Ref. 1 and Suppl. Fig. 5 of Ref. 1, are marked by blue $\times$ and orange $\bullet$, respectively; in the text, these data sets are referred to as set A and set B, respectively.

decreases from 25 K to 2 K. This minimum is passed at about 15 K. Below it, $w(T)$ increases more and more rapidly with decreasing $T$. In contrast, when going from high to low $T$, the graph of set B exhibits a pronounced hump and a subsequent swale in the intermediate $T$ region, that is between 10.5 K and 4.7 K, similarly as $w(T)$ for sample 0.96. Above 10.5 K and below 4.7 K, however, the graphs of sets A and B roughly agree with each other.

The small shift between these graphs at low $T$ would have been further diminished by ascribing the difference quotients to the respective interval centres, instead of to the lower interval boundaries as done in Ref. 1. This way, by utilizing the symmetric difference quotients, the numerical error would have been considerably reduced; see also Appendix C of Ref. 2.

There is a further inconsistency in Ref. 1: the $T$ values of the $w(T)$ data points of sample 1.02 shown in its Fig. 1d (set A) are considerably closer together than the $T$ values of the $\sigma(T)$ data points of this sample presented in its Fig. 1c, although the $w_i$ should have been calculated from the $\sigma_i$ via difference quotients. – The reader can easily check our statement by enlarging Figs. 1c and 1d of Ref. 1 by means of the PDF viewer. – Remarkably, however, those $\sigma(T)$ data points in Fig. 1c relate to the same $T$ values as the $w(T)$ points for sample 1.02 presented in Suppl. Fig. 5 of Ref. 1 (set B).

The most plausible hypothesis on the origin of the inconsistencies uncovered above is the following. Worried by the strange peculiarity obvious in Suppl. Fig. 5, Daichi Oka and co-workers may have remeasured $\sigma(T)$ for sample 1.02 following an improved procedure. Unfortunately, however, they may have only updated Fig. 1d, letting Fig. 1c and Suppl. Fig. 5 untouched.

We now turn back to sample 0.96. To render the comparison of samples 0.96 and 1.02 as informative as possible, we use different $w$ scales for the individual samples in the right panel of Fig. 1. The left scale is valid for sample 0.96; it agrees with the $w$ scale of the left panel of Fig. 1. The right scale applying to data for sample 1.02 has the same zero as the left one. Its unit, however, is chosen such that, at $T = 10.7 \, \text{K}$, the $w(T)$ data points for both the samples roughly fall together in this diagram.

Two striking similarities between the peculiar features of the $w(T)$ for sample 0.96 and for set B of sample 1.02 are clearly recognizable in the right panel of Fig. 1: (i) Both humps appear in the same $T$ region. (ii) Both humps have approximately the same shape and the same relative magnitude. These two findings, together with the absence of such a peculiar feature in the data set A for sample 1.02, suggest the following interpretation. Both the peculiar features are probably artifacts. Presumably, they originate from similar errors of the related $T$ values, caused by any shortcoming of the measuring procedure. Since Daichi Oka et al. apparently took values with slowly sliding temperature, insufficient equilibration together with a change of the cooling / heating
rate may have been the origin of the peculiar features of the \( w(T) \) for sample 0.96 and for set B of sample 1.02.

Together, the above discussed two comparisons in the right panel of Fig. 1 enable the following conclusion: the peculiar feature, that is hump plus swale in the intermediate \( T \) range, does not disprove the validity of the \( w(T) \) data points for \( T \leq 4.7 \) K. With respect to sample 0.96, the trustworthiness of these low-\( T \) data points is further supported by the logarithmic derivative of the power law interpolation between \( \sigma(10.7 \) K) and \( \sigma(4.7 \) K); this function provides a plausible estimate of \( w(T) \) for the intermediate \( T \) range, \( w = 0.090 \), see the left panel of Fig. 1.

Therefore, it seems highly probable that, for sample 0.96, the slow increase of \( w(T) \) with decreasing \( T \) below about 4.7 K is real, that it is not an artifact of temperature or resistivity measurements.

Apart from the mysterious feature hump plus swale, the graphical representations of the \( w(T) \) data sets of sample 0.96 and of sample 1.02 in our Fig. 1 have qualitatively similar shapes – though they differ in the position of the minimum and in the associated \( w \) value –. This resemblance provides additional support for the above reclassification of sample 0.96.

In this context, we stress that, apart from hump and swale, the appearance of both the \( w(T) \) data sets of the samples 0.96 and 1.02 is rather typical for disordered solids, even if the minimum value of \( w(T) \) is as small as roughly 0.1. The reader may compare them with the \( w(T) \) data sets for compensated crystalline Si:(P,B) in Fig. 1 of Ref. 8, an unusually unbiased and constructive reply to a comment. Furthermore, he/she may compare with data for crystalline Si:As and CdSe:In in Figs. 4 and 13, respectively, of the review Ref. 2 as well as with the ones for the amorphous solids a-Si\(_{1-x}\)Cr\(_x\) and a-Si\(_{1-x}\)Mn\(_x\) in Figs. 7 and 8, respectively, of the review Ref. 9. Correspondingly, having the four qualitative scenarios analysed in Section 5 of Ref. 2 in mind, under certain conditions, one can consider the \( w(T) \) data of sample 0.96 as an indication of the existence of a finite minimum metallic conductivity.

Nevertheless, alternatively, it might be speculated that, for sample 0.96, the negative slope of \( w(T) \) below about 4.7 K could arise from only a part of the sample exhibiting metallic conduction, means from the sample being significantly inhomogeneous; compare Ref. 8.

However, due to a scaling law discovered roughly four decades ago, \(^{10}\) it seems not unlikely that the observed negative slope of \( w(T) \) for sample 0.96 is a generic feature: for a-Si\(_{1-x}\)Cr\(_x\) films deposited by means of e-beam evaporation, the \( T \) and \( x \) dependences of \( \sigma \) were observed to satisfy \( \sigma(T, x) = \sigma_{sc}(T/T_0(x)) \) if \( \sigma(T, x) < \sigma_0 = 270 \pm 50 \) Q \( \cdot \) cm\(^{-1}\) and 100 mK \( < T \) \( < 50 \) K. In words: under these conditions, \( \sigma(T, x) \) is completely determined by the quotient of \( T \) and an empirically introduced characteristic temperature \( T_0(x) \). Since \( \sigma_{sc}(T/T_0) \) vanishes in an exponential manner as \( T/T_0 \to 0 \), this behaviour is related to activated conduction. – Concerning exponential \( T \) dependences, such scaling relations were observed to hold in several other homogeneous solids too.\(^{12}\)

Note: that scaling law implies the existence of a finite minimum metallic conductivity, in contradiction to Ref. 10. The scaling behaviour, however, breaks down in consequence of annealing, that is in consequence of the formation of nanocrystallites, see Subsection 3.3 of Ref. 11. Nevertheless, the appearance of \( \sigma(T, x) \) varies only gradually in this process, not abruptly.

When we have the possibility of such a scaling in mind, what do we have to look out for when studying other solids? For the reconsideration at hand particularly important: within the validity range of this scaling law, the slope of \( w(T) \) is always negative. In various cases, this finding has motivated the study of \( w(T) \) as cross-check of the interpretation of \( \sigma(T) \) in terms of metallic conduction, see Refs. 3,8 as well as the review Ref. 2 and references therein.

For the reconsideration at hand similarly important: the validity range of the scaling law mentioned above encompasses not only a conductivity region in which fast, exponential \( T \) dependences are observed, but, furthermore, also a conductivity region where \( \sigma(T) \) varies only slowly, in some non-exponential manner with \( T \).

This arises from the nature of the \( x \) dependence of the characteristic temperature \( T_0(x) \), which is defined by \( \sigma(T, x) \propto \exp(-T_0(x)/T)^{1/2} \) in the low-temperature limit. Apparently, \( T_0(x) \to 0 \) as \( x \) tends to its critical value, at which the nature of conduction changes from activated to metallic, compare Fig. 3 of Ref. 10. This behaviour of \( T_0(x) \) has a fundamental consequence: the closer the composition of the sample to that at the MIT, the higher is the lowest experimentally accessible temperature. Our only at first glance absurd statement relates to the point that it is not the ratio of \( T \) to the absolute unit Kelvin that is crucial here, but that it is the ratio of \( T \) to the characteristic hopping temperature, \( T_0(x) \): for \( T \gg T_0(x) \), \( \sigma(T/T_0(x)) \) varies in some non-exponential manner, even if, for example, \( T = 1 \) K.

Therefore, the following interpretation difficulty may arise in any experiment: prior to the transition from activated to metallic conduction, there is some finite \( x \) range in which only rather flat, non-exponential \( T \) dependences of \( \sigma \) can be observed; compare the schematic diagram Fig. 2 of Ref. 8. Without a really unbiased, very thorough analysis – as the evaluation of \( w(T) \) –, such \( \sigma(T) \) data may easily be misinterpreted to indicate metallic conduction.

III. APPROXIMATION BY \( \sigma(T) = a + bT^{1/2} \)

Assume now, a reader is not aware of this principal difficulty and has no information on \( w(T) \). Then Fig. 1c of Ref. 1 in particular the seemingly rather good quality of the \( \sigma = a + bT^{1/2} \) fit to the data of sample 0.96, may mislead him/her: it could make him/her feel sure about this sample being clearly metallic. So the following questions arise. Is the quality of the approximation shown
Insight into these interpretation issues is gained by studying under which conditions significant deviations of the measured data points of sample 0.96 from the adjusted function \( \sigma = a + b T^{1/2} \) occur. For such an analysis, one needs as precise as possible \((T_i, \sigma_i)\) data. Thus, first, we checked the precision of the data read out from Fig. 1c of Ref. 1 by comparing the \( w_i \) values calculated from the digitized \((T_i, \sigma_i)\) to the \( w_i \) data read out from Fig. 1d of Ref. 1. Then, we used the digitized \( T_i \) and \( w_i \) to refine the \( \sigma_i \) values. This precision improvement bases on the digitized \( w_i \) having a smaller discretization error than the differences between neighbouring \( \sigma_i \) as read out. Additionally, a tiny constant shift of all \( w_i \) was allowed to minimize the sum of the squares of the small corrections of the \( \sigma_i \) values in the refinement; the optimum constant shift corresponds to roughly half a pixel in Fig. 1d of Ref. 1.

The results are presented in our Fig. 2. Its left panel displays \( \sigma \) vs. \( T^{1/2} \) with much better \( \sigma \) resolution than Fig. 1c of Ref. 1. Two data sets are shown therein: the set of \( \sigma(T) \) points obtained directly by means of digitization of Fig. 1c of Ref. 1, as well as the refined set obtained as described in the previous paragraph. Both these data sets deviate only slightly from each other. Completing, the right panel of Fig. 2 presents an overview over a considerably wider \( T \) range, from 0 K to 49 K, though with somewhat lower \( \sigma \) resolution. It combines the refined set and data read out from Fig. 1b of Ref. 1.

The left panel of Fig. 2 compares three fits of the function \( \sigma = a + b T^{1/2} \) to different subsets of the set of the refined digitized data: (1) The solid red line corresponds to the approximation shown in Fig. 1c of Ref. 1: the parameters \( a \) and \( b \) are adjusted to optimally reproduce the measured points within the \( T \) interval 2 K to 14 K. This fit yields \( \sigma(0 \text{ K}) = 75.7 \Omega^{-1}\text{cm}^{-1} \), in very good agreement with the value read out from Fig. 1c of Ref. 1: 75.8 \( \Omega^{-1}\text{cm}^{-1} \). (2) The dashed green line refers to a fit to the data points within an intermediate \( T \) region, that is 7 K < \( T < 19 \) K. According to this approximation, \( \sigma(0 \text{ K}) = 78.6 \Omega^{-1}\text{cm}^{-1} \). (3) The dashed-dotted blue line represents a fit to the data points for the three lowest \( T \) values, 2 K < \( T < 4 \) K; it yields \( \sigma(0 \text{ K}) = 70.2 \Omega^{-1}\text{cm}^{-1} \).

Three conclusions can be drawn from this comparison. First, substantial deviations from the \( \sigma = a + b T^{1/2} \) approximation are clearly recognizable. Second, these deviations have primarily a systematic, non-random character: apparently, the experimental \( \sigma(T^{1/2}) \) crosses the solid red line only twice. Third, the result of the extrapolation to \( T = 0 \) K depends considerably on which \( T \) interval is taken into account in the fit: the lower the upper boundary of this \( T \) interval, the smaller is the estimated value of \( \sigma(0 \text{ K}) \).

In spite of the above demonstrated low-temperature
extrapolation uncertainty, the fit of \( \sigma = a + b T^{1/2} \) to the measured \( \sigma(T) \) data given in Fig. 1c of Ref. 1 seems to yield a quite reasonable approximation within a rather wide interval. The question is why. Inclusion of data of sample 0.96 from Fig. 1b of Ref. 1 into the comparison solves this puzzle: in the right panel of our Fig. 2 the set of refined data of sample 0.96 is supplemented with data from Fig. 1b of Ref. 1 for \( 15 K < T < 49 K \). Contrasting this composed graph with the \( \sigma = a + b T^{1/2} \) fit to all refined data points within the range \( 7 K < T < 19 K \), marked by the dashed green line, suggests the following explanation.

Overall, the experimental relation \( \sigma(T^{1/2}) \) shown in the right panel of Fig. 2 has an inverse S shape and, correspondingly, an inflection point within the considered \( T \) range; this characteristic temperature amounts to (very) roughly 10 K. So the dashed green line represents a linear fit focusing on the surroundings of an inflection point. Therefore, the quadratic term of the Taylor expansion of \( \sigma(T^{1/2}) \) around the fit interval centre is almost negligible so that the region of approximately linear behaviour is comparably wide. Thus, simply for numerical reasons, it is not surprising that the dashed green line provides a quite reasonable description over the rather wide \( T \) range from 4 K to 25 K. As cross-check of this interpretation, consider the fit focusing on the interval 2 K to 4 K - dashed-dotted blue line in the left panel of Fig. 2. It concerns a region considerably away from the inflection point. As expected, it seems to have a far smaller applicability range; in this case, the judgement bases on the estimation of the upper half width of this range.

FIG. 3: Temperature dependences of the deviations between measured data and their \( \sigma = a + b T^{1/2} \) approximation, relating to the interval \( 2 K < T < 14 K \), for the sample with \( x = 0.96 \). The left panel presents the absolute conductivity misfit, while the right panel shows the relative temperature misfit, see text.

IV. SAMPLE AS SECONDARY THERMOMETER

Now, we have a closer look at the deviations of the \( \sigma(T) \) data points of sample 0.96 from their \( \sigma = a + b T^{1/2} \) approximation. The left panel of our Fig. 3 shows the differences between the refined \( \sigma \) data and the corresponding values \( a + b T^{1/2} \) of the fit for the region 2 K to 14 K; this approximation focuses on the same \( T \) interval as considered in Ref. 1. Note: the resolution of the vertical axis of the left panel of Fig. 3 is by a factor 13 higher than that of the \( \sigma \) axis of the left panel of Fig. 2.

Loosely speaking, the left panel of Fig. 3 presumes the \( T \) values to be exact and shows how the deviation of the \( \sigma \) value from its estimate depends on \( T^{1/2} \). Complementing this presentation, the right panel of Fig. 3 relates to the opposite perspective: the \( \sigma \) values are presumed to be exact, and we ask for the deviation of the measured \( T \) value from the \( T \) value estimated on the basis of the same \( \sigma = a + b T^{1/2} \) approximation. In other words, in the right panel of Fig. 3 the sample is considered as secondary thermometer: to each value of \( \sigma_i \), a secondary temperature value, \( T_i^{\text{sec}} \), is ascribed. The corresponding relative misfit, \( (T_i - T_i^{\text{sec}})/T_i \), is depicted versus the square root of the actual temperature, \( T_i^{1/2} \), in this diagram.

The left panel of Fig. 3 confirms the presence of substantial systematic, non-random deviations between measured data and fit. Furthermore, it provides the following two interesting messages. (i) The very systematic \( T \) dependence of the deviation between measured data and fit present above 10 K testifies the rather high pre-
cision of the resistivity measurements by Daichi Oka and co-workers. (ii) Simultaneously, however, this $T$ dependence exhibits a strange feature between about 5 K and about 10 K, marked by a red ellipse; that presumable inconsistency correlates with the peculiar feature hump plus swale illuminated in our analysis of the logarithmic derivative of $\sigma(T)$ in Section 2. Both these statements are confirmed by the right panel of Fig. 3.

Finally, we emphasize two quantitative conclusions which can be drawn from the right panel of Fig. 3. First, above 3 K, the relative misfit, $(T_i - T_\text{fit})/T_i$, varies between as much as $-9\%$ and $+8\%$. Second, below 3 K, where the relative misfit is positive, it increases rapidly with decreasing $T$ and reaches a value of 24\% at 2 K! Of course, this sharp increase should be checked by additional measurements at intermediate $T$ values. Furthermore, an extension of the considered $T$ range down to 1.8 K – presumably in reach of the equipment used in Ref. 1 – should be very helpful in clarifying the situation.

Both these findings are further strong arguments against the significance of the $T \to 0$ extrapolation of the $a + b T^{1/2}$ fit to the measured $\sigma(T)$ data. This extrapolation, however, is very important to Ref. 1.

V. CONCLUSIONS

We summarize: regarding the sample with $x = 0.96$, Daichi Oka and co-workers comprehensively reported on their quite careful, albeit not problem-free determinations of $\sigma(T)$ and $w(T)$ in Figs. 1b, 1c, and 1d of Ref. 1. In doing so, unfortunately, they used inappropriate $\sigma$ and $w$ scales. Here, zooming into Figs. 1c and 1d of Ref. 1, we have uncovered a fundamental contradiction between the authors’ theoretical interpretation of these $\sigma(T)$ measurements, on the one hand, and their $w(T)$ data, on the other hand. The authors claim in their text that, for $x = 0.96$, the $w(T)$ dependence, determined in an ansatz-free manner, would have a positive slope at low temperatures so that this sample would be metallic. In fact, however, below 4.7 K, $w(T)$ does not decrease with decreasing $T$, but even increases slightly, indicating activated transport.

Motivated by this contradiction, we have checked in detail the quality of the $\sigma = a + b T^{1/2}$ fit for the sample with $x = 0.96$ shown in Fig. 1c of Ref. 1. In our Fig. 2, the uncertainty of corresponding $T \to 0$ extrapolations is demonstrated. Simultaneously, it is shown, that it is the vicinity of an inflection point of $\sigma(T^{1/2})$ that pretends a high significance of this approximation. Finally, focusing on the deviations between this fit and the measured data, we have visualized, in our Fig. 3, the doubtfulness of this approximation of the data points taken at the lowest temperatures.

It is surprising that the contradiction highlighted above was not mentioned at all in the text of Ref. 1. It was not mentioned though, concerning the interpretation of $w(T)$ data, the authors cite Ref. 2 that recent critical review discusses observations and consequences of similar behaviour of $w(T)$ for various other disordered solids in great detail.

Thus an unusual question suggests itself: to what extent is the narrative that the scaling theory of localization qualitatively correctly describes the MIT of various three-dimensional disordered systems a result of a strong confirmation bias? Serious doubts about the applicability of this theory have been on the table since the identification of a scaling law for the temperature dependence of the conductivity in the hopping region of various three-dimensional disordered systems roughly four decades ago. This empirical finding implies the existence of a finite minimum metallic conductivity, which in contrast is denied by the scaling theory of localization. The mentioned doubts were strongly enhanced by the surprising discovery of a metallic phase in two-dimensional disordered systems by Sergey Kravchenko and co-workers in 1994; the existence of such a phase is ruled out by the scaling theory of localization.

Hence, further thorough and unbiased experimental studies of the MIT in disordered solids are urgently needed. So it will certainly be highly valuable if Daichi Oka and co-workers repeat the $\sigma(T)$ measurement of their sample with $x = 0.96$ with increased precision – provided this sample is sufficiently long-term stable –. In doing so, compared to Ref. 1, a greater part of the data should be taken close to the lower end of the $T$ region considered: constant quotient of subsequent $T$ values is appropriate. Precision and significance of such measurements will benefit from carefully thermalizing the setup of sample and thermometer before each resistivity measurement, instead of measuring with slowly sliding temperature. Including into the proposed study additional samples with slightly different compositions will presumably be fruitful.

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Ethics declaration

The author declares no competing interests.

Data availability

All data obtained in this analysis are available from the author upon reasonable request.
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