The X-shaped, localized field generated by a Superluminal electric charge

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Abstract – It is now wellknown that Maxwell equations admit of wavelet-type solutions endowed with arbitrary group-velocities (0 < v_g < ∞). Some of them, which are rigidly moving and have been called localized solutions, attracted large attention. In particular, much work has been done with regard to the Superluminal localized solutions (SLS), the most interesting of which resulted to be the “X-shaped” ones. The SLSs have been actually produced in a number of experiments, always by suitable interference of ordinary-speed waves. In this note we show, by contrast, that even a Superluminal charge creates an electromagnetic X-shaped wave. Namely, on the basis of Maxwell equations, we are able to evaluate the field associated with a Superluminal charge (under the approximation of pointlikeness): it results to constitute a very simple example of true X-wave.

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1. – Introduction

It is well-known that Maxwell equations have been shown to admit of wavelet-type solutions endowed with arbitrary \(0 < v_g < \infty\). Some of them, which are rigidly moving and have been called “localized solutions”, attracted large attention. In particular, much work has been done with regard to the Superluminal localized solutions (SLS), the most interesting of which —as predicted by Special Relativity (SR) itself— resulted to be the “X-shaped” ones. Such X-shaped SLSs have been actually produced in a number of experiments.

The theory of SR, when based on the ordinary postulates but not restricted to subluminal waves and objects, i.e., in its extended version, predicts the simplest X-shaped wave to be the one corresponding to the electromagnetic field created by a Superluminal charge. It is of the utmost importance evaluating the field associated with a Superluminal electric charge, not only as a contribution to the theory of X-shaped waves, but also as a starting point for predicting the electromagnetic interaction of a charged “tachyon” with ordinary matter (and planning, maybe, the construction of a suitable detector).

2. – The toy-model of a pointlike Superluminal charge

Let us first start by considering, formally, a pointlike Superluminal charge, even if the hypothesis of pointlikeness (already unacceptable in the subluminal case) is totally inadequate in the Superluminal case, as it was thoroughly shown in refs.

Then, let us consider the ordinary vector-potential \(A^\mu\) and a current density \(j^\mu \equiv (0,0,j_z;\phi)\) flowing in the \(z\)-direction. On assuming the fields to be generated by the sources only, one has that \(A^\mu \equiv (0,0,A_z;\phi)\), which, when adopting the Lorentz gauge, obeys the equation \(\Box A^\mu = j^\mu\). Such non-homogeneous wave equation, in cylindrical coordinates \((\rho,\theta,z;t)\) and for axial symmetry [which requires a priori that \(A^\mu = A^\mu(\rho,z;t)\)],

\(^{\ast}\)Incidentally, let us recall that the *luminal* case was successfully examined by Bonnor[7], who showed the Maxwell equations to admit of finite-energy solutions even in the limiting case of a (mass-free) “particle” carrying equal amounts of positive and negative electric charge.

\(^{1}\)For simplicity, we are here adopting the standard language, but it should be recalled that we must rather think in terms of an “electromagnetic charge”, which behaves as an electric charge when subluminal and as a magnetic pole when Superluminal: cf. refs.[9].
\[ \frac{\partial^2 A^\mu}{\partial \tau^2} - \frac{1}{\rho} \frac{\partial}{\partial \rho} \left( \rho \frac{\partial A^\mu}{\partial \rho} \right) - \frac{\partial^2 A^\mu}{\partial z^2} = j^\mu, \]  

(1)

where \( \tau \equiv ct \), and \( \mu \) assumes the two values \( \mu = 3, 0 \) only.

We shall now choose the “\( V \)-cone variables”\(^{[11]} \), with \( V^2 > c^2 \),

\[
\begin{align*}
\zeta & \equiv z - Vt \\
\eta & \equiv z + Vt
\end{align*}
\]  

(2)

and rewrite eq.(1) as

\[
\left[ -\rho \frac{\partial}{\partial \rho} \left( \rho \frac{\partial}{\partial \rho} \right) + \frac{1}{\gamma^2} \frac{\partial^2}{\partial \zeta^2} + \frac{1}{\gamma^2} \frac{\partial^2}{\partial \eta^2} - 4 \frac{\partial^2}{\partial \zeta \partial \eta} \right] A^\mu(\rho, \zeta, \eta) = j^\mu(\rho, \zeta, \eta),
\]  

(3)

where\(^{[6]} \) \( A^\mu \equiv (0, 0, A_z; \phi) \) and

\[
\gamma^2 = \frac{1}{V^2 - 1}.
\]  

(3')

Let us now suppose \( A^\mu \) to be actually independent of \( \eta \), namely, \( A^\mu = A^\mu(\rho, \zeta) \). Due to eq.(3), we shall have \( j^\mu = j^\mu(\rho, \zeta) \) too; and therefore \( j_z = V j^0 \) (from the continuity equation), and \( A_z = V \phi/c \) (from the Lorentz gauge). Then, by calling

\[
\psi \equiv A_z,
\]

eq(3) yields the two equations

\[
\left[ -\frac{1}{\rho} \frac{\partial}{\partial \rho} \left( \rho \frac{\partial}{\partial \rho} \right) + \frac{1}{\gamma^2} \frac{\partial^2}{\partial \zeta^2} \right] \psi(\rho, \zeta) = j_z(\rho, \zeta)
\]  

(4a)

and

---

\(^{†}\)As a further check of our calculations, we started also from the so-called scalar Bromwich–Borgnis\(^{[10]} \) potential \( u \), under the hypothesis that \( j = (0, 0, j_z) \), in which case it is \( E_\rho = \partial^2 u/\partial \rho \partial z \); while \( E_z = -\partial^2 u/\partial \tau^2 + \partial^2 u/\partial z^2 \); and \( B_\phi = \partial^2 u/\partial \rho \partial \tau \), where \( \tau = ct \). On defining the function \( \psi \equiv A_z \equiv \partial u/\partial \tau \), we showed by Maxwell equations that \( \psi \) has to obey the same non-homogeneous (axially symmetric) wave equation (1), with \( \mu = 3 \).
\[ \phi = \frac{c}{V} \psi \quad (4b) \]

One can notice that the procedure leading to eqs.(4) constitutes a simple generalization of Lu et al.’s theorem[12] for non-homogeneous equations, i.e., for the case with sources.[13]

As announced above, let us finally analyse the possibility and consequences of having a Superluminal pointlike charge, \( e \), traveling with constant speed \( V \) along the \( z \)-axis (\( \rho = 0 \)) in the positive direction:

\[ j_z = e V \frac{\delta(\rho)}{\rho} \delta(\zeta) . \quad (5) \]

Equation (4a) becomes, then, the hyperbolic equation

\[ \begin{bmatrix} -\frac{1}{\rho} \frac{\partial}{\partial \rho} \left( \rho \frac{\partial}{\partial \rho} \right) + \frac{1}{\gamma^2} \frac{\partial^2}{\partial \zeta^2} \end{bmatrix} \psi = e V \frac{\delta(\rho)}{\rho} \delta(\zeta) . \quad (6) \]

To solve it, let us apply (with respect to the variable \( \rho \)) a Fourier-Bessel (FB) transformation, carrying by definition a function \( f(x) \) into the function \( F(\Omega) \) as follows

\[ f(x) = \int_0^\infty \Omega f(\Omega) J_0(\Omega x) \, d\Omega \quad (7) \]

\[ f(\Omega) = \int_0^\infty x f(x) J_0(\Omega x) \, dx , \quad (7') \]

\( J_0(\Omega x) \) being the ordinary zero-order Bessel function. Equation (6) gets transformed, after some calculations, into

\[ \begin{bmatrix} 1 \frac{\gamma^2}{\partial^2} + \Omega^2 \end{bmatrix} \Psi(\Omega, \zeta) = e V \delta(\zeta) . \quad (8) \]

By applying subsequently the ordinary Fourier transformation with respect to the variable \( \zeta \) (going on, from \( \zeta \), to the variable \( \omega \)), after some further manipulations we obtain

\[ \Psi(\Omega, \omega) = e V \frac{\gamma^2}{(\gamma^2 \Omega^2 - \omega^2)} . \quad (9) \]
Finally, the solution of eq.(6) is got by performing the corresponding inverse Fourier and FB transformations:

$$\psi(\rho, \zeta) = e^{V\gamma^2} \int_{-\infty}^{\infty} \int_{0}^{\infty} d\omega \int_{0}^{\infty} d\Omega \frac{\Omega J_0(\Omega \rho) e^{-i\omega \zeta}}{(\gamma^2\Omega^2 - \omega^2)} ,$$

(10)

which, on using formulae (3.723.9) and (6.671.7) of ref.[14], yields\[\mathbb{I}[\zeta \equiv z - Vt]::

$$\left\{
\begin{array}{ll}
\psi(\rho, \zeta) = 0 & \text{for } 0 < |\zeta| < \rho \\
\psi(\rho, \zeta) = e^{V \sqrt{\zeta^2 - \rho^2(V^2 - 1)}} & \text{for } 0 \leq \rho < |\zeta| .
\end{array}
\right.$$ 

(11)

In Fig.1 we show our solution \(A_z \equiv \psi\), as a function of \(\rho\) and \(\zeta\), evaluated for \(\gamma = 1\) (i.e., for \(V = c\sqrt{2}\)). Of course, we skipped the points in which \(A_z\) must diverge, namely the vertex and the cone surface.

For comparison, one may recall that the classical X-shaped solution\[\mathbb{I}[4]\] of the homogeneous wave-equation —which is shown, e.g., in Fig.2— has the form\[\mathbb{I}[11]\] (with \(a > 0\)):

$$X = \frac{V}{\sqrt{(a - i\zeta)^2 + \rho^2(V^2 - 1)}}$$

(12)

In the second one of eqs.(11) it enters expression (12) with the spectral parameter\[\mathbb{I}[11]\] \(a = 0\), which indeed corresponds to the non-homogeneous case [the fact that for \(a = 0\) these equations differ for an imaginary unit will be discussed elsewhere].

It is quite important, at this point, to notice that such a solution, (11), represents a wave existing only inside the (unlimited) double cone \(C\) generated by the rotation around the \(z\)-axis of the straight lines \(\rho = \pm \gamma \zeta\): This is in full agreement with the predictions\[\mathbb{I}[15]\] of the “extended” theory of Special Relativity\[\mathbb{I}[6]\].

3. – Evaluating the fields generated by the Superluminal charge

Once the solution (11) for the “potential” \(\psi\) has been found, we can evaluate the corresponding electromagnetic fields. The standard relations \(E = -\nabla \phi - \partial A/\partial t\) and

\[\text{In the following we shall put } c = 1, \text{ whenever convenient.}\]
\( H = \nabla \wedge A \) imply in the present case \( [\psi = \psi(\rho, \zeta) \equiv A_z; \text{ and } \phi = \psi/V] \):

\[ H = -\frac{\partial \psi}{\partial \rho} \hat{e}_\phi \]

\[ E = -\frac{1}{V} \frac{\partial \psi}{\partial \rho} \hat{e}_\rho + (V^2 - 1) \frac{\partial \psi}{\partial \zeta} \hat{e}_z . \]

Then, from eqs.(11), when \( 0 \leq \rho < \gamma | \zeta | \) (i.e., inside the cone \( C \)), the fields result to be:

\[ E_\rho = -\pi e \rho \frac{V^2 - 1}{\sqrt{[\zeta^2 - \rho^2(V^2 - 1)]^3}} \]  
(13)

\[ E_z = -\pi e \zeta \frac{V^2 - 1}{\sqrt{[\zeta^2 - \rho^2(V^2 - 1)]^3}} \]  
(14)

\[ H_\phi = -\pi e V \rho \frac{V^2 - 1}{\sqrt{[\zeta^2 - \rho^2(V^2 - 1)]^3}} \]  
(15)

where, let us recall, \( \zeta \equiv z - Vt \), with \( V^2 > c^2 \). We show in Fig.3 the direction of the various field components in our co-ordinates; while the behavior of \( E_z \), as a function of \( \rho \) and \( \zeta \), is shown in Fig.4.

However, outside the cone \( C \), i.e., for \( 0 < \gamma | \zeta | < \rho \), one gets as expected that

\[ E_\rho = E_z = H_\phi = 0 . \]  
(16)

One meets therefore a field discontinuity when crossing the double-cone surface, since the field is zero outside it. Nevertheless, the boundary conditions imposed by Maxwell equations[15] are satisfied by our solution (11), or (13-15), since at each point of the cone surface the electric and the magnetic field are both tangent to the cone: We shall discuss this point below.

Here, let us emphasize that, when \( V \to \infty; \gamma \to 0 \), the electric field tend to vanish, while the magnetic field tends to the value \( H_\phi = -\pi e/\rho^2 \). This does agree with what expected from Extended Relativity[9], which predicts Superluminal charges to behave, in a sense, as magnetic monopoles. In the present note we can only mention such a

\[ \text{It should be noticed that the same results are obtained when starting from the fourpotential associated with a subluminal charge (e.g., an electric charge at rest), and then applying to it the suitable Superluminal Lorentz “transformation”[6].} \]
circumstance, and refer to refs.[3,9]: Where it is shown that, if one calls electric the “electromagnetic charge” when it is subluminal, then he should call it magnetic when Superluminal (cf. Fig.46 at page 155 of the first one of refs.[6]). Actually, result (11) can be obtained in a quicker way just by applying a Superluminal Lorentz “transformation”[6] to the fields generated by a subluminal (in particular, at rest) electric point-charge.

Let us add, more in general, that—as mentioned at the end of the previous Section—extended relativity predicts, e.g., that the spherical equipotential surfaces of the electrostatic field created by a charge at rest get transformed (by a Superluminal Lorentz “transformation”) into two-sheeted rotation-hyperboloids, contained inside an unlimited double-cone[6,8]: see Fig.5. One ought to notice, incidentally, that this double cone does not have much to do with the Cherenkov cone. In fact the double cone is associated with a constant-speed Superluminal charge even in the vacuum, while Cherenkov radiation emission is induced by a fast electric charge only out of a material medium. Moreover (cf. also Fig.27 at page 80 of the first one of refs.[6]) a Superluminal charge traveling at constant speed, in the vacuum, e.g., does not lose energy[8].

We go eventually back to the problem that one meets a field discontinuity across the double-cone surface [see eqs.(13-15) and eqs.(16)], since the field is zero outside C; for \( \rho \to \gamma \mid \zeta \mid \) the fields (13)-(15) even diverge. Nevertheless, one can straightforwardly verify that our solution (11), or (13-16), satisfies the following boundary conditions, required by Maxwell equations (in the present case of a moving boundary)[17,18]:

\[
\begin{align*}
(E_{\text{ext}} - E_{\text{int}}) \cdot \hat{n} &= \sigma \\
(H_{\text{ext}} - H_{\text{int}}) \cdot \hat{n} &= 0 \\
(E_{\text{ext}} - E_{\text{int}})_{\tan} &= -(\hat{n} \cdot V) \hat{n} \wedge (H_{\text{ext}} - H_{\text{int}}) \\
[1 - (\hat{n} \cdot V)^2] \hat{n} \wedge (H_{\text{ext}} - H_{\text{int}}) &= j.
\end{align*}
\]

\(\frac{\text{We have shown elsewhere}[16,9]}{\text{We have shown elsewhere}[16,9]}\) that a Superluminal charge \( e \) and a Superluminal current \( j^\mu \) are a pseudoscalar and a pseudovector, respectively: like in the case of a magnetic charge and a magnetic current; so that they should rather be written as \( \gamma_5 e \) and \( \gamma_5 j^\mu \). But in this preliminary note we shall forget about the symmetry properties of those quantities.
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FIGURE CAPTIONS

Fig.1 – Behaviour of $A_z \equiv \psi$, as a function of $\rho$ and $\zeta$, evaluated for $\gamma = 1$ (i.e., for $V = c\sqrt{2}$). [Of course, we skipped the points in which $A_z$ must diverge: namely, the vertex and the cone surface].

Fig.2 – Illustration of the real part of a classical X-shaped wave (as a function of $\rho$ and $\zeta$), evaluated for $V = 5c$ and $a = 5 \times 10^{-7}$ m.

Fig.3 – The direction is here depicted of the various field components, in our coordinates.

Fig.4 – Behavior of the $z$-component of the electric field generated by a Superluminal (pointlike) charge as a function of $\rho$ and $\zeta$, with the same parameters used for Fig.1. [Once more, we skipped the points in which $E_z$ must diverge: namely, the vertex and the cone surface].

Fig.5 – The spherical equipotential surfaces of the electrostatic field created by a charge at rest get transformed into two-sheeted rotation-hyperboloids, contained inside an unlimited double-cone, when the charge travels at Superluminal speed (cf. refs.[6,8]). This figures shows, among the others, that a Superluminal charge traveling at constant speed, in a homogeneous medium like the vacuum, does not lose energy[8]. Let us mention, incidentally, that this double cone has nothing to do with the Cherenkov cone (see the text). The present picture is a reproduction of our Fig.27, appeared in 1986 at page 80 of the first one of refs.[6].
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