Research on irregular wear mechanism of planar multi-body mechanical system with multi-clearance joints

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Abstract
An approach for analyzing and quantifying the irregular wear effect of planar multi-body mechanical system with multi-clearances is proposed. The classical planar four-bar linkage is used as a demonstrative example to study the mechanism of irregular wear effect of multi-body system with multi-clearances. The system dynamic equation is established using Lagrange multiplier method, the contact force and friction in the process of contact collision are respectively modeled using the Lankarani-Nikravesh (L-N) nonlinear contact force and LuGre models, and the wear depth prediction model for multi-clearance joints is built based on Archard's equation. The influence of the crank speed, clearance number, clearance position and clearance size on the wear effect of multi-clearances system is comprehensively studied. Numerical simulation results show that the wear depth joint increases with increase of crank speed, and there are significant differences in the wear depth of different clearance joints. When the clearance size changes, the significant nonlinear wear phenomenon appears at different joints, when the clearance size of different joints meets a certain proportion, the system wear phenomenon is the least. The obtained valuable results provide important theoretical support for the optimal design of multi-clearance joints system.

Keywords
Wear, friction, contact force, multiple clearance joints, dynamics of multi-body systems

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Introduction
The existence of clearance is inevitable in practical mechanical systems due to design, manufacturing, assembly and wear factors.¹⁻³ The existence of joint clearance makes the mechanism deviate from the ideal motion trajectory, which intensifies the impact-collision force between the internal components of the system, resulting in serious vibration, friction, noise, and wear of the system.⁴⁻⁵ Classical dynamic analysis ignores the influence of joint clearance, that is, it assumes that the components are ideal links. At the same time, the wear effect caused by clearance joints is also ignored.

The wear effect is the most important and complex phenomenon in the contact collision problem. At the same time, wear is also an important factor that causes the failure of the mechanical system.⁶ In the multi-clearance joint mechanism system, the wear is affected by the geometric size of the contact body, contact temperature, material properties and clearance size, etc.

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The change of any parameter may cause the change of the wear effect. Therefore, in order to study the mechanism of wear failure in multi-clearance joint mechanisms, scholars focused on exploring the wear effect of clearance joints.

In the past few decades, there are many researches on dynamics modeling, improvement of contact force models and wear of mechanisms with single clearance joints. However, the research on the dynamics of multi-clearance joint mechanisms has become a new hotspot. Bai and Sun used a planar four-bar linkage to study the dynamic characteristics of a multi-body system with multi-clearances. Chen et al. studied the dynamic characteristics of a two-degree-of-freedom nine-bar linkage with clearances, and verified the correctness of the dynamic model by ADAMS software. Song et al. proposed a non-smooth strategy for solving a planar rigid body mechanism with multi-clearances based on the variational inequality, and compared the accuracy with the ADAMS software. Li et al. researched the dynamic behaviors of double clearances crank slider system with harmonic drive and flexible linkage, and pointed out that harmonic drive could suppress the output peak better than flexible linkage, but with the increase of clearance size, the suppression effect would decrease. Li et al. investigated the dynamic influence of the rigid-flexible coupling solar panel system considering the clearance, and the research pointed out that the reasonable configuration of the clearance size is an effective way to reduce the contact collision. Wang et al. used an improved nonlinear contact force model to study the dynamic behavior of four-bar linkage with multi-clearances, and discussed the effect of clearance position and number on the dynamic behavior of the system. The multi-body system with multi-clearances presents more complex dynamics and nonlinear dynamic characteristics, and the interaction characteristics and wear effect between multi-clearance to be further studied.

The joint clearances aggravates the wear of multi-body systems. At present, the research on the wear effect of multi-body mechanical with clearances is still insufficient. In the actual contact collision problem, due to the influence of factors such as friction at the clearance joint, clearance size, contact material, and geometric size, it exhibits complex wear behavior. Zhu et al. proposed a nonlinear pressure distribution model that combines dynamic analysis and wear calculation. Taken the crank slider as the research object, the wear behavior of the system under the action of a single clearance was studied. Xu and Han proposed a contact method for the analysis of non-circular clearance revolute joints, studied the obvious non-roundness of the clearance joints caused by non-uniform wear for a long time. This method can be used to effectively analyze the non-circular contact problem. Xiang et al. used on the Archard model, proposed a wear prediction model for the mechanism with clearances, and took the crank-slider system with single clearance as the research object, and studied the influence of clearance size and drive power on the wear phenomenon of joints. Zhao et al. proposed the wear prediction approach for flexible system with clearances by combining multi-body dynamics with wear prediction process, researched the influence of different flexible positions on the wear phenomenon of joints, pointed out that flexible connecting rods could reduce the impact-collision force of joints, and the wear coefficients under different contact conditions were obtained by using neural network. Wang and Liu studied the wear behavior of five-bar system with clearances, the results showed that the multi-clearance joints produced stronger impact-collision forces, leading to more serious wear behavior, but the existence of flexible rods can reduce this effect. With the development of the dynamic theory of multisystems with clearances, the experimental research on clearance dynamics has also made great progress, such as the crank-slider systems with clearances, planar 4 bar mechanism, the three-joint locking mechanism in the cabin door, etc. have conducted experimental research, and the experimental results are consistent with the results of the built wear prediction model. In order to reduce the system wear problem caused by the existence of clearance joints, scholars not only studied the wear effect of the system with clearance, but also reduced the wear behavior of the multi-body system by optimizing system design, local flexibility and lubrication. Research shows that reasonable design of the clearance size, local flexible members, and lubrication can effectively reduce wear.

In summary, the research on the dynamics and wear prediction of multibody mechanism with clearances has achieved fruitful results, but the research on the dynamics and wear failure behavior of multibody mechanism with clearances is still limited. Compared with the single clearance mechanism, the multi-clearance mechanism exhibits more severe impact-collision force during the contact collision process, the wear is more serious, and the mutual coupling effect of the different clearance joints is more obvious. Therefore, it is essential to further research the wear behavior of systems with multi-clearances to understand the wear mechanism between different joint clearances.

To research the dynamics and wear effects of a multi-body system with multi-clearances, the planar four-bar linkage with double clearances is studied, and the system dynamics differential equation with multi-clearances is built using the Lagrange multiplier method. The L-N nonlinear contact force model and LuGre model are used to simulate the contact force and friction in the process of contact collision,
respectively, and the multi-clearances wear prediction model is established based on Archard’s equation. The influence of crank speed, clearance position, clearance size and clearance number on the wear depth of multi-clearance system is studied. The coupling effect of different clearance joints is discussed, which resulted in nonlinear wear behavior at different clearance joints.

The main contribution of this paper is to establish a wear prediction model suitable for irregular clearance joints and to propose a quantitative analysis method for irregular wear effect of plane multi-body mechanical with multi-clearances. To explore the wear effect of multi-clearances system and quantitatively analyze the wear mechanism of multi-clearance joint system. Reveal the nonlinear phenomenon existing between the different clearance joints, and prove through the data simulation results that the reasonable design of the clearance size of the different clearances can effectively suppress the wear of joints, and to provide a theoretical basis for inhibiting joint wear caused by clearances.

Mathematical modeling method of rotary clearance joint

To accurately describe the relative motion state of journal and bearing, a mathematical model of regular joint is established based on the collision hinge model, which includes the eccentricity model of journal center, the deformation model during contact collision and the relative motion velocity model. Figure 1 shows model of joint with clearance, which is established in the whole coordinate system \(xoy\). \(\eta_i, \xi_i\) and \(\eta_j, \xi_j\) are the local reference system fixed by the dependent body on the colliding body \(i\) and \(j\), respectively.

Figure 1 shows that, in the generalized coordinate system, the eccentricity vector \(e\), which connects the journal and the bearing center, is defined as

\[
e = r_j^0 - r_i^0
\]

where \(r_j^0\) and \(r_i^0\) are the position vectors of the bearing and journal center, \(Q_j\) and \(Q_i\) are center of two contact bodies, which is given by

\[
r_j^0 = r_j + A_js_j^0
\]

\[
r_i^0 = r_i + A_is_i^0
\]

\[
A_k = \begin{bmatrix} \sin \beta & \cos \beta \\ -\cos \beta & \sin \beta \end{bmatrix} \quad k = i, j
\]

where \(r_j, r_i\) are the position vectors of the origin of local coordinate \(\eta_j, \xi_j\) and \(\eta_i, \xi_i\), respectively, \(A_j\) and \(A_i\) are the transformation matrices from the local reference system to the global reference system of the bearing and journal, \(\beta\) is the deflection angle, and \(\beta = \arctan \left( \frac{c}{s} \right)\), \(s_j^0\) and \(s_i^0\) are the position vectors of the center point of two contact bodies. Modulus of the eccentricity vector \(e\) is written as

\[
e = \sqrt{e^T \cdot e}
\]

When the contact collision occurs, the bearing and journal will produce different degrees of elastic deformation due to the impact-collision force. The magnitude of deformation can be expressed as

\[
\delta = e - c
\]

the deformation \(\delta\) is the main indicator for judging the motion state of the journal relative to the bearing, the initial regular clearance \(c = R_j - R_i\). If \(\delta < 0\), it is the state of free movement. If \(\delta > 0\), it means that the shaft journal and bearing have contact collision, which is the contact collision motion state or transition state.

Figure 2 shows that when contact collision occurs, the deformation vector \(\delta\), is expressed as

\[
\delta = r_j^d - r_i^d
\]

where \(r_j^d\) and \(r_i^d\) are the position vectors of the contact collision point of two contact bodies, \(q_j\) and \(q_i\) are maximum deformation contact point of the contact bodies when deformation occurs, is written as

\[
r_j^d = r_j + A_js_j^d + R_jn
\]

\[
r_i^d = r_i + A_is_i^d + R_in
\]

where \(R_j\) and \(R_i\) are initial radius of two cylindrical contacting bodies, and \(n\) is the unit normal vector of the contact point.

\[
n = \frac{e}{e}
\]
model considering the damping term is derived. Therefore, the widely used L-N contact force model is selected.

**Normal impact force model**

Scholars Lankarani and Nikravesh\(^3\) comprehensively considered the amount of deformation, geometric dimensions and material properties, and introduced the initial deformation velocity into the model to more clearly described the energy consumption and transfer in the process of contact collision. Similar to Hertz model, the L-N impact force model is also derived from point contact theory, and is only suitable for the condition of low impact velocity, coefficient of recovery close to 1, and small contact area relative to the collider.

According to whether the identified deformation \(\delta\) is greater than 0, the motion state of the clearance joint is judged, if \(\delta \geq 0\), it means that there is collision motion or continuous contact motion at the clearance joints, and the clearance joint is affected by contact force at this time. If \(\delta < 0\), it means that the journal has no contact with the bearing, that is, the mode of free movement, and the contact force is 0. The L-N\(^3\) model is expressed as

\[
F_N = \begin{cases} 
K\delta^2 + \chi\delta\dot{\delta}' & \delta \geq 0 \\
0 & \delta < 0
\end{cases}
\] (15)

The first part of the formula is the completely elastic impact force, and the second part is damping term to calculate the energy dissipation, when contact and collision. The stiffness coefficient \(K\) is defined as the material properties and geometry, \(\chi\) is the damping coefficient, \(\delta'\) is the deformation velocity during the collision. The stiffness coefficient is expressed as

\[
K = \frac{4}{3(\sigma_i + \sigma_j)} \left( \frac{R_i R_j}{R_j - R_i} \right)^2
\] (16)

\[
\sigma_k = \frac{1 - \nu_k^2}{\pi E_k} \quad (k = i, j)
\] (17)

where \(\sigma_k(k = i, j)\) is the material parameter of the collider \(i\) and \(j\), \(\nu_k\) is Poisson’s ratio, \(E_k\) is Young’s modulus, both of which depend on material properties. The viscous damping coefficient \(\chi\) can be expressed as

\[
\chi = \frac{3K(1 - c_r^2)}{46(\delta')^2}
\] (18)

where \(c_r\) is recovery coefficient of the material, \(\delta'\) is deformation velocity at the initial collision. In contact collision, the final formula of the L-N contact force is
The friction effect is very important phenomena in the clearance dynamics problem. The classic Coulomb friction law is simple to use, but when the relative tangential velocity of the two contact bodies is 0, the friction force will change suddenly, which poses challenges to the numerical solution, and the model does not consider the viscous sliding phenomenon. The modified Coulomb’s law can guarantee the stability of the algorithm when the tangential velocity is 0. Therefore, the modified Coulomb’s law is widely used in the dynamic analysis of multi-body systems with clearance. However, the modified Coulomb’s law cannot simulate the viscous sliding phenomenon.

Therefore, the LuGre model is calculated the friction force in the process of contact and collision. The LuGre friction model can easily reflect the change of friction force with slip velocity. It is not only suitable for studying the phenomenon of viscous sliding, but also can be used to study the Stribeck effect in the friction process. The model can be defined as the product of the instantaneous friction coefficient and the impact-collision force, which can be written as

\[ F_T = \mu F_N \]  

where \( \mu \) is the instantaneous friction coefficient in the friction process, which can be expressed as the expression of tangential velocity.

\[ \mu = \eta_0 z + \eta_1 z' + \eta_2 v_t \]

where \( \eta_0 \) is bristle stiffness coefficient, \( \eta_1 \) is microscopic damping coefficient, \( \eta_2 \) is viscous damping coefficient, and \( v_t \) is tangential sliding velocity.

The differential expression of the state variable \( z \) can be written as

\[ z' = v_t - \frac{\eta_0 |v_t|}{\mu_k + (\mu_s - \mu_k)e^{-|z'|/\xi}} z \]

where \( \mu_s \) and \( \mu_k \) represent coefficient of static friction and dynamic friction, respectively, \( v_t \) is Stribeck velocity threshold, and \( \xi \) is friction attenuation gradient coefficient. The expression of the internal state variable \( z \) can be expressed as

\[ z = \frac{|v_t|}{v_t} \left[ \frac{\mu_k + (\mu_s - \mu_k)e^{-|z'|/\xi}}{\eta_0} \right] \]  

Dynamic modeling of mechanism with multi-clearances

The four-bar mechanism with multi-clearances, such as Figure 3, is chosen as an example to demonstrate the methodologies presented. The mechanism consists of crank 1, coupler 2, follower 3, and ground 4, the lengths are \( l_1 \), \( l_2 \), \( l_3 \), and \( l_4 \) respectively. \( O_1 \), \( O_2 \), and \( O_3 \) respectively represent the center of mass of crank, coupler, and follower. \( u_1 \), \( u_2 \), and \( u_3 \) respectively represent the angle between the crank, the coupler, and the follower in the positive direction of the \( x \) axis. There are clearance joints \( C_1 \) and \( C_2 \) at the connection between crank 1 and coupler 2, and the connection between coupler 2 and follower 3 respectively, the clearance value is defined as \( c_1 \) and \( c_2 \), the other joints are all ideally connected. Under the action of the driving torque, the crank rotates counter clockwise periodically at constant speed. According to the theory of multibody system dynamics, the dynamics equations of system with multi-clearances are deduced. The geometrical parameters of the four-bar linkage are listed in Table 1.

The existence of clearance joints increases freedom degree and makes the kinematic characteristics of the mechanism more complex. The generalized coordinates of the system are

\[ q = [x_1 \ y_1 \ \theta_1 \ x_2 \ y_2 \ \theta_2 \ x_3 \ y_3 \ \theta_3]^T \]  

The mechanism with clearance joint is a typical variable topological system, and the dynamic equations...
under different motion states are different. Therefore, the dynamic model is established by using the idea of dynamic piecewise modeling. If there is no contact and collision at the clearance joints, that is, the state of free movement, there is no impact-collision force at joint, the journal moves freely in the bearing depending on the inertial force. Therefore, the dynamic equation and the complete constraint equation of the mechanism is expressed as

\[
\begin{bmatrix}
M & \Phi_q^T \\
\Phi_q & 0
\end{bmatrix}
\begin{bmatrix}
q'' \\
\lambda
\end{bmatrix} = \begin{bmatrix} f \end{bmatrix}
\]

where \( M \), \( \Phi_q \), and \( f \) represent the generalized mass matrix, Jacobian matrix, and generalized force vector respectively, \( \lambda \) is the Lagrange multiplier array. The expressions of the other variables are obtained by the above formulas.

The constraint equation and Jacobian matrix of the system is given as follows

\[
\Phi = \begin{bmatrix}
x_1 - \frac{1}{2} \cos \theta_1 \\
y_1 - \frac{1}{2} \sin \theta_1 \\
x_2 - l_1 \cos \theta_1 - \frac{1}{2} \cos \theta_2 - e_{x1} \\
y_2 - l_1 \sin \theta_1 - \frac{1}{2} \sin \theta_2 - e_{y1} \\
x_3 - \frac{1}{2} \cos \theta_3 - l_4 \\
y_3 - \frac{1}{2} \sin \theta_3 \\
\theta_1 - \theta_0 - \omega t
\end{bmatrix} = 0
\]

where \( \theta_0 \) is the initial phase of the crank and \( \omega \) is the rotational speed of the crank, \( e_{x1} \) and \( e_{y1} \) are the components of the eccentricity at the clearance joint \( C_1 \) in the \( x \)-directions and \( y \)-directions, respectively.

According to the constraint equation, the expression of vector \( \gamma \) in the kinematics equation of multi-body mechanism is deduced.

| Bodies   | Length (m) | Mass (kg) | Moment of inertia (kg m²) |
|----------|------------|-----------|---------------------------|
| Crank    | 0.4        | 1.3053    | 0.01924                   |
| Coupler  | 0.32       | 1.0541    | 0.01026                   |
| Follower | 0.5        | 1.6193    | 0.03644                   |
| Ground   | 0.2        | -         | -                         |

The diagonal matrix of inertia and mass of the four-bar mechanism system can be expressed as

\[
M = \text{diag}(m_1, m_1, m_2, m_2, m_3, m_3, m_3, I_3)
\]

where \( I_1 \), \( I_2 \), and \( I_3 \) are respectively the moment of inertia of member crank, coupler, and follower.

The generalized force vector of the system can be expressed as

\[
f = \begin{bmatrix}
F_{x1}^- & -m_1 g + F_{x1}^v & T_1 & F_{x3}^3 & -m_2 g + F_{x2}^v \\
T_2 & F_{x3}^- & -m_3 g + F_{x3}^v & T_3
\end{bmatrix}^T
\]

where \( T_1 \), \( T_2 \), and \( T_3 \) are the external moment of the crank, the coupler, and the follower, \( g \) is the acceleration of gravity, \( F_{x1}^- \), \( F_{x1}^v \), \( F_{x2}^v \), and \( F_{x3}^v \) are the \( x \) and \( y \) directions of the external force acting on the center of mass of crank 1, coupler 2, and follower 3, respectively. The rest of the generalized forces can be obtained by the above expressions.

In the contact and collision phase, the contact deformation of two contact bodies results in the interaction of impact-collision force, which results in a strong nonlinear response of the system. At this point, the dynamic equation of the mechanism is expressed as

\[
\begin{bmatrix}
M & \Phi_q^T \\
\Phi_q & 0
\end{bmatrix}
\begin{bmatrix}
q'' \\
\lambda
\end{bmatrix} = \begin{bmatrix} f + F_C \end{bmatrix}
\]

where \( F_C \) is the impact-collision force, which includes the contact force \( F_N \) and the friction force \( F_r \), it is calculated by equations (19) and (20).

**Modeling process of wear prediction of rotary joint**

The wear behavior is another important feature of contact impact problem. There are more than 300 methods for the study of wear and friction phenomena. In the dynamics of multibody systems with clearances, the wear prediction model is mainly established by using Archard’s equation. This model links the wear with physical properties such as material hardness, impact load and sliding distance of the contact colliding body. The expression can be written as

\[
\frac{V}{s} = \frac{K_w F_N}{H_w}
\]
(34) can be converted into a differential form.

where $s$ is the relative sliding distance of contact surface, $K_w$ is the dimensionless wear coefficient, $V$ is the wear volume, $H_w$ is the Brinell hardness of the contact material.

In practical engineering, more consideration is given to the size of the wear depth during contact. Equation (32) is divided by the contact area to obtain the wear depth expression (34). $h/s$ is the wear rate at any moment.

$$V = hA_w$$

where $A_w$ is the contact area, $h$ is the depth of wear, $k_w = K_w/H_w$ is the dimensionless coefficient, and $p$ is the load per unit contact area.

In fact, in the study of contact and collision problems, the wear behavior is usually regarded as a dynamic process. Therefore, the wear rate in equation (34) can be converted into a differential form.

$$\frac{dh}{ds} = k_w p(s)$$

For the wear of joint, the wear depth of $i$ can be expressed as

$$h_i = h_{i-1} + k_w p_i \Delta s_i$$

where $h_i$ is the $i$ th cumulative wear depth, which is the sum of the $(i-1)$ th cumulative wear depth and the $i$ th wear amount, and $\Delta s_i$ is the $i$ th sliding distance.

It can be seen from equation (36) that the solution of sliding distance $\Delta s_i$ is crucial to accuracy of the calculation of wear depth. The movement of the journal in the bearing has the characteristics of strong randomness and mutation (Figure 4). In the impact-collision stage, the relative rotation angle of the journal and the bearing will mutate, resulting in errors in the calculation of sliding distance. Therefore, the sliding distance can be expressed as

$$\Delta s_i = (R_i + \delta_{\max})(\varphi_i - \varphi_{i-1})$$

$$\varphi_i = \alpha_i - \alpha_j$$

where $\varphi_i$ and $\varphi_{i-1}$ are the angle difference of rotation of the collider at time $t_i$ and $t_{i-1}$, respectively. $Q_j$ and $Q_i$ are center of two contact bodies. The parameters $\alpha_i$ and $\alpha_j$ are the rotation angles of the collider $i$ and $j$.

The contact area is defined as

$$A_w = |\Delta s_i| \cdot z_w$$

where $z_w$ is the axial length of the contact area.

Assuming that the wear depth of the journal is consistent with that of the bearing, the worn journal and bearing radius can be expressed as

$$R_i^* = R_i - \frac{h}{2}$$

$$R_j^* = R_j + \frac{h}{2}$$

$$c^* = R_j^* - R_i^*$$

Where $h$ is the wear amount of the clearance joint, $c^*$ is the clearance value after wear, $R_i^*$ and $R_j^*$ are the radius of the journal and bearing after wear, respectively.

The key of dynamic simulation is embedded the wear prediction model into the dynamic equation of system. The wear behavior of multi-clearances system involves complex calculation, it is necessary to use a reasonable calculation strategy. Figure 5 shows the algorithm flow chart of wear prediction modeling for the multi-clearance joint system, and describes the main calculation steps and dynamic equations.

**Simulation results and discussion**

Numerical simulation is used to research the characteristics of wear effect in the four-bar linkage system with multi-clearances. The simulation parameters are shown in Table 2.\textsuperscript{35,38} The influence of different crank speed, clearance size, and clearance position on the wear characteristics are emphatically investigated, and the non-linear wear phenomenon between different clearance joints is explored.

**Influence of clearances number on the depth of wear**

To study the effect of clearances number on wear depth, it is analyzed in three cases: (1) only a clearance at $C_1$, (2) only a clearance at $C_2$, (3) there are clearances at both $C_1$ and $C_2$. The crank speed is set to $20\pi$ rad/s and
The clearance value is 0.2 mm. The simulation results are shown in Figures 6 and 7.

Figure 6 shows that in the four-bar mechanism, the increase in the number of clearances aggravates the contact deformation of the joints. In Figure 6(a), the joint position of the same clearance is affected by the number of clearances, and the motion trajectories of the journal center changes significantly, and the position where the maximum deformation occurs and the deformation area are significantly different. In Figure 6(b), the joint $C_2$ is more affected by the change in the number of clearances. The results show that the increase in the number of clearances aggravates the impact and vibration inside the four-bar mechanism, and also aggravates the wear of the clearance joints. Due to the difference of clearance position, the degree of influence of the clearances number are different.

The influence of crank speed on the depth of wear

To study the effect of crank speed on the wear depth of $C_1$ and $C_2$, the crank speed is set at 10, 30, and 50 $\pi$ rad/s respectively, $c_1$ and $c_2$ are 0.2 mm.

Figures 8 and 9 show the movement trajectory of the journal center. It is observed from the figure that the deformation at clearance joint $C_1$ and $C_2$ gradually increases with the crank speed increasing from 10 to 50 $\pi$ rad/s. Figures 8(a) and 9(a) show that when the crank rotates at a low speed, a small amount of deformation is generated at clearance joint $C_1$ and $C_2$, and the journal frequently changes motion state relative to the bearing: free motion state, impact state, and contact deformation state. Figure 8 shows that with the increase of rotational speed, the journal moves in a state of contact deformation relative to the bearing, and the amount of deformation also gradually increases, but the size of deformation presents an obvious non-uniform feature.
By comparing Figures 8 and 9, it can be observed that the amount of deformation of $C_1$ is obviously larger than that of clearance joint $C_2$. Affected by clearance position, the position points where the maximum amount of deformation occurs are greatly different. Figure 8 shows that the maximum amount of deformation appears in the area of $[200^\circ, 270^\circ]$, and the deformation area will also increase with the increase of rotational speed. Figure 9(a)–(c) show that clearance joint $C_2$ produces the largest amount of deformation in the region of $[260^\circ, 290^\circ]$, but with the increase of rotational speed, the deformation region has no obvious change significantly. This is indicated that the joints at different positions present different friction and wear phenomena in the multibody system.

Figure 10 shows that the depth of wear with different crank speeds. Figure 10(a) and (b) show that in the same position, the depth of wear increases with the increase of rotational speed, which is indicated that rotational speed is one of the important factors affecting joint wear. However, the wear depth of $C_1$ is about 10 times that of wear depth of $C_2$, and the wear depth of $C_1$ is within the region of $[200^\circ, 300^\circ]$, which is obviously larger than that of the wear depth $C_2$. This is because the clearance joint $C_1$ is directly connected with the crank, and is affected by greater impact-collision force, resulting in severe wear effect. Compared with clearance joint $C_1$, the impact-impact force at clearance joint $C_2$ is significantly reduced, the amount of deformation generated is reduced, and the wear depth also is decreased.

**Influence of clearance size on the depth of wear**

The crank speed is set at $20\pi$ rad/s, $c_1$ and $c_2$ are 0.1, 0.3, and 0.5 mm, respectively. The movement trajectory chart of the journal center and the wear depth chart are drawn. The influence of the interaction between the clearance joint $C_1$ and $C_2$ on the wear depth are studied, and it is proved that the wear depth of $C_1$ and $C_2$ has obvious nonlinear characteristics.

**Influence of $C_1$ on the depth of wear.** To explore the effect of clearance size of $C_1$ on the depth of wear, $c_2$ is 0.1 mm, and $c_1$ is 0.1, 0.3, and 0.5 mm, respectively. Figures 11 and 12 respectively show the movement track of the journal center of $C_1$ and $C_2$. It is observed from Figures 11 and 12 that the clearance size of $C_1$ increases from 0.1 to 0.5 mm, but the amount of deformation of $C_1$ and $C_2$ does not change significantly. Figure 13(a) shows that the wear depth of $C_1$ is the most serious in the interval of $[160^\circ, 300^\circ]$, and the wear depth in the interval of $[170^\circ, 250^\circ]$ is about two times that in the interval of $[260^\circ, 300^\circ]$, indicating that the wear at the clearance joint is non-uniform. From Figure 13(b), the wear depth of $C_2$ is obvious within the interval $[160^\circ, 290^\circ]$, and the main wear area is within $[260^\circ, 290^\circ]$. Compared with clearance joint $C_1$, the wear depth distribution of $C_2$ is obviously more concentrated.

Figure 11 shows that the increase of clearance size $C_1$ does not cause a significant change in the center trajectory of the journal. According to Figure 11(a), the smaller clearance size will aggravate the contact deformation of $C_1$. Similar to $C_1$, the trajectory diagram of the journal center of $C_2$ also has no obvious change. As is shown in Figure 12, influenced by the clearance size of $C_1$, the trajectory diagram of $C_2$ only changes slightly, which is limited influence on the size of the deformation area, this can be verified by Figure 13(b).

Figure 13(a) shows that the increase of $c_1$ does not cause an increase in the wear depth, and the wear amount outside the interval of $[110^\circ, 300^\circ]$ is very small, which is also proved by the trajectory diagram in...
Figure 11. The wear depth of $C_2$ is very small compared to $C_1$. From Figure 13(b), the wear depth of $C_2$ does not increase with the increase of $c_1$. The wear depth is the highest when $c_1$ is 0.3 mm, but the wear depth when $c_1$ is 0.5 mm is greater than that the clearance size of $C_1$ is 0.1 mm. Therefore, influenced by the change of clearance size $C_1$, the wear depth of $C_1$ and $C_2$ present significant nonlinear characteristic, which is the result of the mutual coupling between multiple clearances.
Influence of $C_2$ on the depth of wear. In the same way, the $c_1$ is set as a constant value of 0.1 mm, and the $c_2$ is 0.1, 0.3, and 0.5 mm, respectively. To study the influence of the $C_2$ on the depth of wear.

Figures 14 and 15 are the motion trajectories of journal center with different clearance size. Figure 14 shows that the increase of $c_2$, the amount of deformation of $C_1$ changes locally. As the value of $c_2$ increases, the contact deformation of $C_1$ gradually increases, but the deformation position does not change significantly. Figure 16(a) shows that the wear depth of $C_1$ does not increase with the increase of $c_2$. When the $c_2$ is 0.1 and 0.3 mm, the influence on the wear depth of $C_1$ is consistent. When $c_2$ is 0.5 mm, the wear depth within the interval $[260^\circ, 290^\circ]$ is more significant.

Figure 15(c) shows that when the $c_2$ is 0.5 mm, the journal and bearing mainly move in the state of contact deformation. By comparison with Figure 15(a) and (c), it is clearly observed that the smaller the $c_2$, the more frequent the contact collisions. Figure 16(b) shows that the wear depth of $C_2$ is mainly concentrated in the interval $[260^\circ, 290^\circ]$. It can be observed that when $c_2$ is 0.5 mm, the wear depth of $C_2$ is the maximum, and when the $c_2$ is 0.1 mm, the wear depth of $C_2$ is the minimum. This indicates that the wear depth of $C_2$ is proportional to the clearance size.

Coupling interaction analysis between $C_1$ and $C_2$

Due to the influence of crank speed and clearance size, the wear depth of $C_1$ and $C_2$ present strong nonlinear characteristics. In order to further study the possible interaction between $C_1$ and $C_2$, the crank speed is set as 20 m rad/s, and the $c_1$ and $c_2$ are set as 0.1, 0.2, 0.3, 0.4, and 0.5 mm respectively. $C_1$ and $C_2$ have 25 kinds of combinations, and 25 times of numerical simulation are required, the simulation time is more than 10 cycles at a time, for solving the maximum depth of 10 cycle average as the analysis parameters, solving the 25 largest wear depth value, and by using a numerical fitting method. The $C_1$ and $C_2$ clearance size deformation wear depth map are drawn, and the nonlinear coupling phenomenon between $C_1$ and $C_2$ are analyzed.

Figure 17 shows that the wear depth of $C_1$ is still far greater than the wear depth of $C_1$, which is caused by the different contact forces and positions of $C_1$ and $C_2$ in the planar four-bar mechanism. Figures 17(a) and 18(a) show when clearance size of $C_1$ is 0.1 mm, the wear depth of $C_1$ becomes more serious with the increase of $c_2$. When the $c_2$ is within the interval $[0.1\text{ mm}, 0.5\text{ mm}]$, the smaller the clearance size of $C_1$, the more serious the wear. There is no obvious rule for the wear depth and clearance size of $C_1$, only a strong rule exists in $C_1$ specific area. When the $c_1$ and $c_2$ are within $[0.35\text{ mm}, 0.5\text{ mm}]$ and $[0.4\text{ mm}, 0.5\text{ mm}]$, respectively, the wear depth of $C_1$ decreases significantly. Figure 18 shows that the wear depth of $C_1$ presents a strong nonlinear behavior under the influence of the $c_2$. However, when the $c_1$ is greater than 0.35 mm, the change of clearance size of $C_1$ has no obvious effect, and the depth of wear is the least. When the clearance size of $C_1$ is within the interval $[0.2\text{ mm}, 0.35\text{ mm}]$, the wear depth of $C_1$ is more insensitive to the change of size of $C_2$. However, when the $c_1$ is less than 0.2 mm, the wear depth of $C_1$ increases rapidly with the increase of $c_2$, it is very sensitive to the change of the clearance value. In general, the smaller the clearance size of $C_1$ clearance, the more serious the wear.

As is shown in Figures 17(b) and 18(b), the wear depth of $C_2$ presents a stronger regularity than that at $C_1$. On the whole, the wear depth of $C_2$ increases with the increase of the $c_1$ and $c_2$. As is shown in Figure 17(b), when the $c_1$ and $c_2$ are 0.5 mm, the wear depth reaches the maximum, when the $c_1$ and $c_2$ are 0.1 mm,
the wear effect is the minimum. From Figure 18(b), the wear depth of \( C_2 \) and the clearance size does not conform to the linear law. Different wear areas show irregular changes, and the boundary value curve obviously shows a strong nonlinear relationship. Due to the interaction between \( C_1 \) and \( C_2 \), the widths of different wear zones in \( C_2 \) are also significantly different. According to Figure 18(b), when the \( c_1 \) and \( c_2 \) are at the lower left of Figure 11.

**Figure 11.** Motion trajectories of journal center of \( C_1 \) with different clearance sizes of \( C_1 \): (a) \( c_1 = 0.1 \) mm, (b) \( c_1 = 0.3 \) mm, and (c) \( c_1 = 0.5 \) mm.

**Figure 12.** Motion trajectories of journal center of \( C_2 \) with different clearance sizes of \( C_1 \): (a) \( c_1 = 0.1 \) mm, (b) \( c_1 = 0.3 \) mm, and (c) \( c_1 = 0.5 \) mm.

**Figure 13.** Wear depth of \( C_1 \) and \( C_2 \) with different clearance sizes of \( C_1 \): (a) wear depth of \( C_1 \) and (b) wear depth of \( C_2 \).
line $c_1 = -5/c_2$, the wear depth of $C_2$ is relatively low. When in the upper right of the straight line $c_1 = -5/c_2$, any increase in clearance size will result in a sharp increase in wear depth at $C_2$. When the $c_1$ and $c_2$ are within [0.4 mm, 0.5 mm], the wear effect of $C_2$ has a strong sensitivity and dependence on the change of clearance. Especially after clearance size of $C_1$ and $C_2$ are greater than 0.45 mm, the wear of $C_2$ increases sharply, which must be avoided in the design and manufacturing process of the mechanism.
It is found through research that in the mechanism system with multi-clearances, the wear depth of different clearances has a strong nonlinear phenomenon due to the influence of clearance size and clearance position. Therefore, in the design and manufacturing process of multibody system, to reduce the wear damage caused by clearances, it is necessary to reasonably design the clearance values at different joints.

For the four-bar linkages, the clearance sizes of $C_1$ and $C_2$ are within the interval of $[0.43 \text{ mm}, 0.5 \text{ mm}]$ and $[0.1 \text{ mm}, 0.35 \text{ mm}]$ respectively to ensure the design needs of minimum wear effect of the mechanism.

Therefore, it is proved that in the process of mechanical design and manufacturing, smaller clearance value at the joints is not the better, and a reasonable design interval should be satisfied between different joints to ensure the overall mechanical wear effect be minimized and the final optimization design of the machinery be achieved.

**Conclusions**

To investigate the problem of irregular wear leading to the failure of multi-body systems with clearances, this paper takes the planar four-bar linkages with multi-clearances as the object of study, and studies the mechanism of crank speed, clearance number, clearance position, and clearance size on the wear effect of multi-clearance system. The following conclusions are drawn

1. A quantitative analysis method for irregular wear effect of plane multi-body system with multi-clearances is proposed, which effectively
solved the problem of irregular joints wear prediction of multi-clearance system. The results show that the higher the rotation speed, the more serious the wear and the greater the impact-collision force. The influence of crank speed, clearance number, clearance position and clearance size on the wear effect presents obvious nonlinear characteristics. Moreover, this analysis method is applicable to planar mechanisms with more than two clearance joints.

(2) The iterative wear prediction process based on Archard’s equation was used to calculate the wear characteristics, and a dynamic irregular wear model considering multi-clearance joints was established. An analytical method for quantifying irregular wear clearances is proposed to analyze the wear characteristics of joints with different clearances, which could provide theoretical support for joint contour reconstruction after wear.

(3) For the first time, the wear characteristics of different clearances under different clearance sizes were analyzed, and the reasonable design interval of the clearance value when the clearance wear is minimal was given. It is found that there are strong nonlinear wear characteristics between different clearances, which provides theoretical support for the design of clearances mechanism.

Therefore, by studying the wear mechanism of multibody system with multi-clearances, appropriate design of different joint clearance sizes is an effective way to reduce the wear failure of multi-body system. In addition, a real prototype will be established based on the subsequent optimal design results and relevant experiment will be conducted to validate further the mechanism of wear effect in future.

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