Inflation in string-inspired cosmology and suppression of CMB low multipoles

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We construct a string-inspired nonsingular cosmological scenario in which an inflaton field is driven up the potential before the graceful exit by employing a low-energy string effective action with an orbifold compactification. This sets up an initial condition for the inflaton to lead to a sufficient amount of e-foldings and to generate a nearly scale-invariant primordial density perturbation during slow-roll inflation. Our scenario provides an interesting possibility to explain a suppressed power spectrum at low multipoles due to the presence of the modulus-driven phase prior to slow-roll inflation and thus can leave a strong imprint of extra dimensions in observed CMB anisotropies.

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String theory has been regarded as one of the promising candidate theories of quantum gravity over the past 20 years. However it is unfortunately not easy to detect the signature of extra dimensions and any stringy effect in accelerators in the foreseeable future. In this sense string cosmology presumably provides the only means of testing string or M-theory concretely. It is therefore very important to investigate the viability of string theory by exploring possible cosmological implications.

From the observational side, the recent WMAP measurements showed a lack of power on the largest scales and the running of the spectral index in temperature anisotropies. This is difficult to be explained by standard slow-roll inflation, thus may have roots in esoteric Planck scale physics. One can consider an exciting possibility where the suppression of power originates from some stringy effect which may be present prior to standard inflation (see e.g., Refs. for a number of attempts to explain this suppression).

String theory provides an interesting possibility to avoid the big-bang singularity due to its underlying symmetries. The construction of nonsingular cosmological models based on the low-energy string effective action was pioneered by Veneziano and Gasperini, whose model is called the Pre-Big Bang (PBB) scenario. According to this scenario, the universe exhibits a super-inflation when the dilaton field, which characterizes the strength of the string coupling parameter, evolves toward the strong coupling regime. The curvature singularity can be avoided by implementing the dilatonic loop and higher-derivative corrections to the tree-level action, which is followed by a post-big bang phase with a decreasing Hubble rate.

While the PBB scenario is an intriguing attempt to merge string theory and cosmology, the spectrum of curvature perturbations is highly blue-tilted \((n \simeq 4)\) in its simplest form, which is incompatible with observations. If we take into account the contribution of an axion field, the spectrum of the axion perturbation can be scale-invariant depending on the expansion rate of an internal dimension. Although there remains a possibility to explain the observationally supported flat spectra if the axion plays the role of the curvaton, the PBB scenario still suffers from the problem of the requirement of an exponentially large homogenous region in order to solve the major cosmological problems.

If standard slow-roll inflation occurs after a graceful exit, this solves the homogeneity problem above as long as we have sufficient amount of e-foldings \((N \gtrsim 60)\). In this paper we shall investigate a scenario in which the PBB phase is followed by slow-roll inflation with an assumption that there exists a light scalar field (inflaton) other than the dilaton. Note that a similar idea was proposed in Ref. where slow-roll inflation is preceded by a contracting phase. In this work we implement the tree-level \(\alpha'\) Gauss-Bonnet correction to the string effective action in order to construct viable nonsingular solutions. We will show that the inflaton can be driven up the potential hill due to a nontrivial background dynamics prior to the graceful exit, thereby setting up sufficient initial conditions for following slow-roll inflation. This has a similarity to the inflation in loop quantum gravity, but the observational signatures are different. In fact we shall see that the presence of the kinematically driven super-inflation stage before the graceful exit can lead to a stronger suppression of the power spectrum on largest scales, as favoured from observations.

We start with the following 4-dimensional action that appears for the orbifold compactification in low-energy effective string theory

\[
S = \int d^4x \sqrt{-g} \left[ \frac{R}{2} - \frac{1}{4} (\nabla \phi)^2 - \frac{3}{4} (\nabla \sigma)^2 + L_{\text{inf}} + L_c \right],
\]

which is written in the Einstein frame. Here \(R\) is the...
scalar curvature and the fields $\phi$ and $\sigma$ correspond to the dilaton and the modulus, respectively. We shall consider a minimally coupled inflaton field, $\chi$, with lagrangian
\[ L_{\text{int}} = -(1/2)(\nabla \chi)^2 - V(\chi). \tag{2} \]
Hereafter we assume that the inflaton has a mass $m_\chi$ with potential $V(\chi) = (1/2)m_\chi^2 \chi^2$.

The correction term $L_c$ is given by
\[ L_c = (1/16) [\lambda e^{-\phi} - \delta \xi(\sigma)] R_{\text{GB}}^2, \tag{3} \]
where $R_{\text{GB}}^2 = R^2 - 4R^{\mu\nu}R_{\mu\nu} + \epsilon^{\mu\nu\alpha\beta}R_{\mu\nu\alpha\beta}$ is a Gauss-Bonnet term. Note that we are considering the correction up to the first order in $\alpha'$. The coefficients, $\lambda$ and $\delta$, are determined by the inverse string tension $\alpha'$ and the 4-dimensional trace anomaly of the $N = 2$ sector. $\lambda$ is a positive constant, whereas $\delta$ can be positive or negative. The function, $\xi(\sigma)$, is written in terms of the Dedekind $\eta$-function and is well approximated as $\xi(\sigma) \simeq -(2\sigma/3)\cos\sigma$. The dilatonic higher-order loop and derivative corrections may be included as in Ref. [6], but we do not take them into account in this work. In fact singularity-free cosmological solutions can be constructed if the modulus-dependent term dominates over the dilatonic one in Eq. (3).

With the flat Friedmann-Robertson-Walker (FRW) background described by the scale factor $a$, the variation of the action (4) yields the following background equations
\[ 12H^2 = \dot{\phi}^2 + 2\chi^2 + 3\dot{\sigma}^2 + 2m_\chi^2 \chi^2 - 96H^3 \dot{f}, \tag{4} \]
\[ 8(1 + 8Hf)(\dot{H} + H^2) + 4(1 + 8f)H^2 \]
\[ + \dot{\phi}^2 + 2\chi^2 + 3\dot{\sigma}^2 - 2m_\chi^2 \chi^2 = 0, \tag{5} \]
\[ \ddot{\phi} + 3H \dot{\phi} + 2f_\phi R_{\text{GB}}^2 = 0, \tag{6} \]
\[ \ddot{\sigma} + 3H \dot{\sigma} - (2/3)f_\sigma R_{\text{GB}}^2 = 0, \tag{7} \]
\[ \ddot{\chi} + 3H \dot{\chi} + m_\chi^2 \chi = 0, \tag{8} \]
where a dot denotes the derivative in terms of cosmic time $t$, $H \equiv \dot{a}/a$ is the Hubble parameter and $f = (1/16) [\lambda e^{-\phi} - \delta \xi(\sigma)]$. The Gauss-Bonnet term is simply given as $R_{\text{GB}}^2 = 24H^2(H^2 + H^2)$.

The universe starts out from a weak coupling regime with $v^\phi \ll 1$. In the absence of the modulus, the dynamics of the system in the weak coupling region is characterized by the dilaton-driven phase in PBB cosmology [3]. During this phase the universe contracts as $a \propto (-t)^{1/3}$ in the Einstein frame. This solution does not connect to our expanding branch in a nonsingular way in the tree-level action. We can construct singularity-free bouncing solutions if higher-order loop and derivative corrections are taken into account [8]. Nevertheless it is required to fine-tune the coefficients of the correction terms. In addition the perturbations inside the Hubble radius exhibit a negative instability [14], which casts doubt for the validity of linear perturbation theory.

The situation is different when the modulus field is present. If the modulus dynamically controls the system rather than the dilaton in Eqs. (4)- (8), one gets the super-inflationary solution with a growing Hubble rate in the Einstein frame. In particular we can construct non-singular solutions by taking into account the correction $\delta$ for negative $\delta$. For positive $\delta$, the first-order $\alpha'$ corrections do not help to lead to a successful graceful exit, as analyzed in Ref. [15].

When the modulus-type $\alpha'$ corrections dominate the system, we get the background evolution for $t < 0$ as
\[ a \simeq a_0, \quad H = H_0/t^2, \quad \dot{\sigma} = 5/t, \tag{9} \]
where $a_0$ and $H_0 (> 0)$ are constants. When one starts in an expanding branch, the background is characterized by the super-inflationary solution [9] until the graceful exit ($t \simeq 0$). The Universe expands slowly initially with a nearly constant scale factor. Eventually the Hubble rate reaches its peak value $H = H_{\text{max}}$, after which the system connects to a Friedmann-like branch. In the numerical simulation of Fig. 1, $H_{\text{max}}$ corresponds to of order 0.1. To ensure the validity of the theory, this maximum value should be less than of order unity (i.e., less than the Planck energy). In fact there is a negative instability for the perturbations if $H$ is larger than unity [16].

We include the dilaton in our simulations by generalizing the analysis of Refs. [12, 13, 16]. As we find in Fig. 1, it is possible to obtain singularity-free solutions even when the dilaton is present as long as the modulus $\alpha'$ correction dominates over the dilatonic one. If the super-inflationary modulus-driven phase leads to a large number of e-foldings and also generates a nearly scale-invariant density perturbation, this can be an alternative to standard slow-roll inflation. Unfortunately the scale factor evolves very slowly during this modulus-driven phase (see Fig. 1). In addition the spectrum of the density perturbation generated in modulus-driven inflation is highly blue-tilted, as we see later.

We are interested in whether the presence of the modulus-driven phase can set the sufficient initial conditions for inflaton to drive slow-roll inflation after the graceful exit. We shall consider a natural situation where the inflaton is initially located around the potential minimum, $\chi = 0$. If the contribution of the potential term is dropped in Eq. (3), one has $\ddot{\chi} \propto a^{-3} \sim \text{const}$. Therefore $\chi$ grows linearly during the super-inflationary phase, which continues up to the Hubble peak [see Eq. (9)]. In Fig. 1 we show one example of the evolution for $\chi$. The inflaton is driven up its potential hill during the modulus-driven stage even in the presence of the inflaton potential. The maximum value of $\chi (= \chi_{\text{max}})$ reached at the Hubble peak is $\chi_{\text{max}} = 2.9$ in a Planck unit in this case. This maximum value increases if the initial values of $\dot{\chi}$ get larger.

We wish to analyze the case in which slow-roll inflation occurs after the graceful exit. If the large-scale power
The Planck mass). Since this value is much smaller than a Planck unit, it is possible to have more than 60 e-foldings on the largest scales in CMB, corresponding to the value of $\chi_{\text{max}}$ that is slightly less than $3m_{\text{Pl}}$.

Let us consider scalar perturbations about the FRW background with a single-field, $\psi$. Note that our system involves multiple scalar fields, but it is sufficient to study the perturbations of one dynamically important field in estimating the spectrum of metric perturbations. Then it is convenient to introduce a gauge-invariant variable,

$$R \equiv 2(H/\psi)\psi,$$

where $\varphi$ is the scalar perturbation in the perturbed metric $g_{ij}$. In the uniform-field gauge each Fourier mode of the perturbed Einstein equation with the correction (3) is written as

$$\ddot{R}_k + (2/t) \dot{R}_k - \beta(k^2/a_0^2)t \dot{R}_k = 0,$$

where $k$ is a comoving wavenumber and $\beta$ and $a_0$ are constants. The solution of this equation is written in terms of the Bessel functions, $J_{\pm 1/3}$, and the spectrum of the large-scale curvature perturbation is given by

$$P_R \equiv k^3(|R_k|^2)/2\pi^2 \propto k^{n_1-1},$$

with $n_1 = 10/3$, which is a highly blue-tinted spectrum [11]. Note that when the kinematic term dominates over the potential term the blue-tinted spectra are generic features even in generalized Einstein theories [12, 20]. Around the graceful exit, $\beta$ can be negative when the Hubble rate grows larger than the Planck energy [11], which means that perturbations on the scales inside the Hubble radius exhibit a negative instability. However, we can
avoid this instability as long as $H_{\text{max}}$ is smaller than of order the Planck energy. This is contrasted with the dilaton-driven inflation in which inclusion of the $\alpha'$ correction is accompanied by a similar negative instability even for $H_{\text{max}} \lesssim m_{\text{Pl}}^{1/2}$. Therefore perturbations in the modulus-driven case which crossed the Hubble radius around the graceful exit is more stable than in the dilaton-driven case.

The spectral index of curvature perturbations generated in slow-roll inflation after the graceful exit is close to scale-invariant and is given as $n_S \approx 1 - (1/\alpha)(m_{\text{Pl}}/\chi)^2$ [14]. When $\chi_{\text{max}} \gtrsim 3m_{\text{Pl}}$, corresponding to the $\epsilon$-folds $N \gtrsim 60$, the CMB spectrum on cosmologically relevant scales is determined by the spectral index $n_S$. Meanwhile, when $\chi_{\text{max}} \lesssim 3m_{\text{Pl}}$, the spectrum [12] is within an observational range on the scales larger than a cut-off which connects two power spectra. Then we simply model the power spectrum to be $P_{R} = A_{1}(k/k_0)^{n_1-1}$ for $k \leq k_c$ and $P_{R} = A_{2}(k/k_0)^{n_2-1}$ for $k \geq k_c$, where $k_c$ is a cut-off wavenumber. As shown in Fig. 2 it is possible to explain the loss of power at low multipoles by employing the highly blue-tilted spectrum [12], as long as $\chi_{\text{max}}$ is slightly less than $3m_{\text{Pl}}$. We have carried out the likelihood analysis by varying 7 cosmological & inflationary parameters using the recent WMAP [2] and 2dF [21] data sets, and found that the best-fit value of the cut-off scale corresponds to $k_c = 3.5 \times 10^{-4}\text{Mpc}^{-1}$ (see the inset of Fig. 2). The smaller cut-off corresponding to $\chi_{\text{max}} \gtrsim 3m_{\text{Pl}}$ is within the $2\sigma$ contour bound, thus not ruled out observationally. We also analyzed the smoothly connected power spectrum, $P_{R} = A(k/k_0)^{n_2-1}[1 - \exp\{-1/(k/k_0)^{n_1-1}\}]$, around the cut-off $k_c$, which may be a possible case when higher-order loop corrections become important around the graceful exit. We obtained a similar likelihood value of the cut-off as shown in Fig. 2, which is also consistent with the past related works in Refs. [3, 22].

In summary, we studied a nonsingular cosmological scenario based on the orbifold compactification in low-energy effective string theory and showed that the inflation is driven up the potential hill during the modulus-driven stage before the graceful exit, thus setting up sufficient initial conditions for following slow-roll inflation. The presence of the modulus-driven phase generates a blue-tilted primordial power spectrum, which can explain the loss of power observed in CMB low multipoles. The spectral index [12] is much larger than in the case of noncommutative inflation [23] or loop quantum gravity [12], thus leading to a stronger suppression. Our work provides an exciting possibility to pick up the signature of extra dimensions from high-precision observations.

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