Detectability of a subdominant density component of cold dark matter

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Abstract

Here we examine the detectability of collisionless dark matter candidates that may constitute not all but only a subdominant component of galactic cold dark matter. We show that current axion searches are not suited for a subdominant component, while direct WIMP searches would not be severely affected by the reduced density. In fact, the direct detection rates of neutralinos stay almost constant even if neutralinos constitute 1% of the halo dark matter. Only for lower densities do the rates decrease with density. Even neutralinos accounting for only $10^{-4}$ of the local dark halo density are within proposed future discovery limits. We comment also on indirect WIMP searches.

Claims for the need of collisional cold dark matter as the main form of dark matter in the Universe have led us to consider the observability of collisionless cold dark matter (CCDM) when it is merely a subdominant component of the cold dark matter (CDM). Namely, if the previously favored CDM candidates, such as axions or Weakly Interacting Massive Particles (WIMPs), constitute only a fraction, say 1% or less, of the local dark matter density, would these particles still be observable in the current and proposed direct and indirect...
dark matter searches? This is a valid question even if non-CCDM is proven not to be necessary. In fact there is always the possibility of the CDM consisting of several populations, the one we are searching for not being the dominant one. We could even reverse our question in the following manner. If we see a CDM signal in any of our searches, could we be observing a subdominant component of the total CDM?

Naively one may claim that if the local CDM density is 1%, say, of the local halo density, the expected rates in CDM detectors, being proportional to the local number density, should decrease by the same amount. However, we note that a reduction in the relic CDM density implies in general an increase in the probability of interaction of CDM with the detector, for example an increase in the WIMP–nucleus cross section or an increase in the axion–photon coupling constant. Since the detection rate depends on the product of the interaction probability and the local CDM density, the increase in interaction probability may compensate the decrease in CDM density, and the detection rate would remain unchanged.

For axions, this argument is new; for WIMPs, it is not. It has been mentioned implicitly or explicitly in many papers on WIMP detectability since the inception of the subject. It is timely, we believe, to pinpoint, emphasize and update this argument, because it clearly points to the value of continuing WIMP searches even if WIMPs constitute only a small fraction of the dark matter.

We now present arguments that the compensation between interaction probability and local density occurs for axions and WIMPs, and point out some exceptions.

Unless there is segregation for different types of dark matter, the ratio of CCDM to total DM should be the same locally in the Galaxy and globally in the whole Universe. Thus in the following we assume that the local fraction of CCDM $f_{CCDM}$ is related to the CCDM relic density $\Omega_{CCDM}$ through

$$f_{CCDM} = \frac{\rho_{CCDM}}{\rho_{\text{local}}} = \frac{\Omega_{CCDM}}{\Omega_{DM}},$$  

(1)

where $\rho_{CCDM}$ is the local density of a particular CCDM candidate, $\rho_{\text{local}} \simeq 0.3$ GeV/cm$^3$ is the local halo density (at the location of the Earth), $\Omega_{CCDM}$ is the relic density of our particular CCDM candidate, and $\Omega_{DM} \simeq 0.3$ is the total contribution of DM to the total energy density of the Universe.

Because the relic density of axions is directly related to its mass, and axion searches are tuned to the axion mass, current searches are not suited to look for a subdominant axion component. The axion relic density is directly related to its mass $m_a$. The usual relation (which has its caveats, see for example and references therein) between the axion relic density and its mass $m_a$ is, for a QCD constant of 200 MeV,

$$m_a \simeq \frac{0.6 \times 10^{-5}\text{eV}}{(\Omega_a h^2)^{1/2}},$$  

(2)

where $h$ is the reduced Hubble constant, $h \simeq 0.7$. A dominant component of axions with $\Omega_a = 0.3$ corresponds, according to this relation, to $m_a = 3 \times$
10^{-5}\text{eV. Thus, we could decrease the density at most to } \Omega_a = 0.003, \text{ so that axions contribute 1\% of the total DM density, before encountering the upper bound of } 3 \times 10^{-3}\text{eV on the axion mass derived from the observed duration of the Supernova 1987A neutrino signal (and other bounds which exclude all heavier axions, see for example [3] and references therein).}

The power $P$ from axion to photon conversion in an electromagnetic cavity used for axion dark matter searches is proportional to the product $\rho_a m_a$ of the local axion density and the axion mass. In absence of segregation, eq. (1) shows that the power is also proportional to $\Omega_a m_a$, which using eq. (2) for the axion relic density gives

$$P \propto \Omega_a^{1/7},$$

that is the power is proportional only to the 1/7th power of the axion relic density. For a decrease in $\Omega_a$ by a factor of 100, the power decreases only by a factor of 2. Of course, because the axion mass has shifted to keep relation (2) valid, this power is now at a frequency which is 500 times larger and one would need resonant cavities consequently smaller. The limiting factor of axion dark matter searches with electromagnetic cavities is not the axion to photon conversion power, but the size of the necessary cavities.

The relic density of WIMPs $\Omega_\chi$ is determined by their annihilation cross section $\sigma_a$ by the relation

$$\Omega_\chi h^2 \simeq \frac{1 \times 10^{-37}\text{cm}^2}{\langle \sigma_a v \rangle},$$

where $\langle \sigma_a v \rangle$ is the thermal average of the annihilation cross section times the relative velocity of the WIMPs at freeze-out. A reduction in the relic WIMP density requires an increase in their annihilation cross section in the early Universe. This increase is often associated with an increase in the scattering cross section $\sigma_s$ of WIMPs off atomic nuclei. Since the interaction rate in detectors depends on the product $\sigma_s \rho_\chi$, if the scattering cross section increases as much as the annihilation cross section, the rate would be unchanged even if $\rho_\chi$ has decreased. Concerning indirect detection, the flux of rare cosmic rays and of gamma-rays produced in halo annihilations depends on the product of the square of the density and the annihilation cross section into a particular channel, $\sigma_a \rho_\chi^2$. Thus, even if an increase in the cross section would compensate the decrease in one of the powers of the density, the fluxes would still decrease linearly with the halo WIMP density. However, the intensity of the high-energy neutrino emission from the Sun and the Earth would in many cases decrease only slightly, because, to the extent that capture and annihilation of WIMPs in the Sun and the Earth have the time to equilibrate, the neutrino intensity depends only on the capture rate which in turn depends on the product $\sigma_s \rho_\chi$.

We can understand the relation between the scattering and annihilation cross sections $\sigma_s$ and $\sigma_a$ as follows. The scattering cross section of a WIMP of mass $m_\chi$ with a nucleus of mass $m_N$ is of the form

$$\sigma_s \simeq \frac{m_\chi^2 m_N^2}{(m_\chi + m_N)^3} |A_s|^2,$$

where

$$\langle A_s \rangle$$

is the matrix element of the transition operator.
where \( A_s \) is a reduced amplitude which depends on the dynamics of the collision. The annihilation cross section of WIMPs into light particles is

\[
\sigma_a \simeq N_a m_{\chi}^2 |A_a|^2 ,
\]

where \( A_a \) is the corresponding reduced amplitude and \( N_a \) is the number of annihilation channels. In the case of interactions of weak order, the amplitudes are of the order,

\[
|A_a|^2 \simeq \alpha^2 \frac{M^2}{M^2} , \quad |A_s|^2 \simeq A^2 \alpha^2 \frac{M^2}{M^2} ,
\]

where \( \alpha \) is a coupling constant of weak order \( \alpha \simeq 10^{-2} \), \( M \) is a mass of the particles mediating the interaction, typically \( M \simeq 100\text{GeV} \) and \( A \) is the atomic number of the interacting nucleus. Our expression for the scattering amplitude includes the nuclear coherent enhancement factor \( A^2 \) valid for spin-independent scattering; for spin-dependent scattering the factor \( A^2 \) should be dropped. Also, our expression for the annihilation cross section is valid for \( m_{\chi} < M \), while in the opposite range, \( m_{\chi} > M \), we expect \( \sigma_a \simeq N_a m_{\chi}^2 \).

The simplest case to consider is that of WIMPs lighter than the nuclei they interact with. From the above equations it is obvious that for these WIMPs

\[
\frac{\sigma_s}{\sigma_a} \simeq \frac{|A_s|^2}{|A_a|^2} \simeq \text{const} \quad (8)
\]

the ratio of cross sections is approximately constant. In fact, provided the main annihilation channel is into fermions, quarks in particular, crossing arguments insure that the reduced amplitudes of annihilation and scattering with nucleons are similar.

Heavier WIMPs may have other annihilation channels, such as Higgs bosons or vector boson pairs. The crossing argument then does not apply and we don’t expect the scattering amplitude to grow as much as the annihilation amplitude. Moreover, for WIMPs heavier than the nuclei they scatter from, the scattering cross section becomes largely independent of the WIMP mass, while the annihilation cross section always depends on \( m_{\chi} \). In this case, while the annihilation cross section could be made larger by considering lighter (if \( m_{\chi} > M \)) or heavier (if \( m_{\chi} < M \)) WIMPs, the scattering cross section would remain largely unchanged.

Therefore, for relatively light WIMPs, and to a lesser extent for heavy WIMPs, we expect the scattering cross section to grow by the same factor \( \Omega_{DM}/\Omega_{\chi} \) the annihilation cross section needs to grow to reduce the local CDM density by \( \Omega_{\chi}/\Omega_{DM} \). So the rate, which is proportional to the product of the local CDM density and the scattering cross section, remains unchanged.

This argument ceases to be applicable at some small enough WIMP densities, because the necessary increase in cross sections is due to larger couplings and/or smaller mediator masses, which, at some point, encounter accelerator limits which exclude the model. In fact Fig. 1 (described below) shows that for neutralinos constituting 10% of the halo or more the direct detection rates are...
largely maintained (as evidenced by the behavior of the envelope of the highest rates), and for densities as low as 1% of the halo density, the highest rates only decrease by a factor of about three, showing that there is compensation in the interaction rates while densities decrease by a factor of up to 100. As mentioned, the compensation ceases to work for smaller densities, and for these (as can be seen in Figs. 1 and 2) the envelope of highest rates decreases linearly with the density.

To substantiate the general arguments presented so far, we have analyzed the concrete case of the lightest neutralino in usual variations of the Minimal Supersymmetric Standard model. We used a table of models allowed by all accelerator limits, produced with the DarkSUSY code [4] over the last few years for other purposes, i.e. having in mind other issues which were addressed in the papers of Ref [5] for which the models were originally computed. We have, therefore, not done any particular sampling of the models to favor lower densities and higher detection rates. We restricted our attention to models with \( \Omega_\chi \leq \Omega_{DM} = 0.3 \) (\( \Omega_\chi h^2 \leq 0.15 \)) for which we found about 45,000 points in parameter space. For these models, using the spin-dependent and spin-independent neutralino-nucleon cross sections provided in the table, we computed the integrated interaction rates on Ge, following L. Bergström and P. Gondolo in ref. [5]. We plot the resulting integrated rates (in units of events per kg-day) in the first two figures of this paper.

Figs. 1 and 2 show the expected integrated rates in Ge detectors as function of the lightest neutralino relic density. Fig. 1 shows only a part of Fig. 2 (the part with the highest rates and densities) displaying the original points in the table of models. Fig. 2 shows the whole range of densities (which reach up to \( \Omega_\chi h^2 \simeq 10^{-6} \)) using a regular grid of points covering the region with models.

In Fig. 1 the change of the slope of the envelope of the points with maximal rate as the density diminishes is clearly evident. There is approximately no change in maximal rates in the first decade of decrease of density, from \( \Omega_\chi h^2 = 0.15 \) (for which neutralinos constitute the whole halo, \( f_{CCDM} = 1 \)) to 0.015 (for which neutralinos constitute 10% of the halo, \( f_{CCDM} = 0.1 \)). There is only about a factor of 3 decrease in the next decade, from \( \Omega_\chi h^2 = 0.015 \) to 0.0015 (\( f_{CCDM} \) from 0.1 to 0.01). For smaller densities the slope of the envelope clearly changes, and as evidenced by Fig. 3, the maximal rates decrease linearly with \( \Omega_\chi h^2 \) up to the smallest densities. Some of the points shown in Figs. 1 and 2, mostly among with the smallest densities in Fig. 3, should correspond to resonances in the annihilation cross section.

The compensation in the rates can be largely understood just by looking at the spin-independent neutralino-proton cross section \( \sigma_{\chi-p} \) as a function of the lightest neutralino relic density, shown in Fig. 3, again with a regular grid showing the allowed region where points were found. Also from this figure, looking at the envelope of the highest cross sections, it is evident that for \( \Omega_\chi h^2 \) decreasing from 0.15 to 0.0015, i.e. in the first two decades of decrease in neutralino density, \( \sigma_{\chi-p} \) increases with decreasing densities; this leads to a compensation in the direct rates. On the other hand, for smaller densities, \( \sigma_{\chi-p} \) is about constant or decreases slightly with decreasing densities; this effect is
due to accelerator bounds.

Since experimental upper bounds and discovery regions are at present given in terms of $\sigma_{\chi-p}$, Fig. 3 shows the approximate level of the claimed signal and present bounds (by the DAMA, CDMS, COSME-IGEX, and Heidelberg-Moscow collaborations [6]) and conceivable future discovery level (by the GENIUS proposal [7]) which are of order $10^{-5}$ pbarns and $10^{-9}$ pbarns, respectively, for neutralinos which account for the whole local halo density, i.e. with $f_{CDM}=1$. (These values depend on the neutralino mass, but to simplify the presentation we only take the most conservative bounds in our range of masses.) In our case these values must be understood as levels of $f_{CDM}\sigma_{\chi-p}$, which are shown in Fig. 3 (with short-dashed and long dashed lines respectively). The present level of discovery lightly touches the boundary of the highest rates for densities reduced by up to a factor of about 10. This suggests the possibility that the DAMA claimed signal may correspond to subdominant neutralinos. It is very interesting to see that many models of subdominant neutralinos even with $10^{-4}$ of the total dark matter density, enter in the discovery limit proposed by Genius.

In conclusion, the main point of this paper is that the direct detection rates of neutralinos remain about constant for neutralino densities between 100% and 1% of the halo dark matter and only decrease linearly with the density for lower densities. Thus if a signal is found in direct detection experiments the question of which component of dark matter was found, the primary or a sub-dominant one, may remain open. We also note that neutralinos with density as small as $10^{-4}$ of the local dark halo density are within the discovery limits of proposed future experiments.

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Figure Captions

Fig. 1 Integrated interaction rates of neutralinos in Ge detectors (computed as in L. Bergström and P. Gondolo Ref. [5]) in units of events per kg-day, as function of the neutralino relic density, for $\Omega \chi h^2 \leq 0.15$. Each point represents an actual model.

Fig. 2 Integrated interaction rates of neutralinos on Ge extended to the whole range of densities. A regular grid of points shows the region covered with models.

Fig. 3 Spin-independent neutralino-proton cross section $\sigma_{\chi-p}$ as function of the lightest neutralino relic density. As in Fig.2, a regular grid of points shows the region where models were found. The short-dashed and long dashed lines of $f_{CDM}\sigma_{\chi-p}=10^{-5}$ pbarns and $10^{-9}$ pbarns show the approximate level of DAMA claimed signal and the current bounds, and the conceivable future discovery level, respectively ($f_{CDM}$ is the fraction of the local halo density consisting of neutralinos).
Figure 1.
Figure 2.
Figure 3.