The dependence of the image edge detection directivity by Brewster effect on the gradient of inhomogeneities of objects

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Abstract. Optical image edge detection for amplitude and phase objects can be performed using Brewster effect. We demonstrate experimentally that the direction of the amplitude and phase gradients in the objects affects the image transformation directivity. In the theoretical study, we consider transfer function for in-plane and out-of-plane rays. The results of this work can be used for design and optimization of optical systems for image and information processing.

1. Introduction

During the last decade, the field of analog image processing has recently received a considerable attention. Thus, the implementation of optical differentiation based on the excitation of surface plasmon polariton modes in the Kretschman configuration was described in [1]. In the previous works, image edge detection using resonant structures was studied for monochromatic light [2-7]. The feasibility of implementation of a wide class of mathematical operations of light signals including integration, differentiation, and Laplace operator was demonstrated. Recently, the spatial differentiation of polarized optical signals based of the Brewster effect was suggested in the theoretical work [8]. In contrast to metals, small dispersion of dielectrics suggests that this effect can be applied to a broadband polarized light. Experimental feasibility of implementation of the Brewster effect to the optical edge detection of images of both phase and amplitude objects was performed in our previous work [9]. Based on the obtained experimental data, we noted that the directivity of edge detection by Brewster effect depends on the direction of the gradient of inhomogeneities of objects. However, the transfer function in [9] was derived for the beams that propagate in the plane of incidence only.

In this paper, the influence of the direction of the gradient of the two-dimensional distribution of the object’s inhomogeneity on the direction of optical differentiation based on the Brewster effect is studied theoretically and experimentally.

2. Optical schema

It is known that for a plane $p$ polarized wave, which is incident at a certain angle $\theta_n$ on the interface between two dielectrics with refractive indices $n_1$ and $n_2$, the reflection coefficient tends to zero, and the transmittance reaches its maximum value. The angle $\theta_n$ is referred to as Brewster angle, where $\tan(\theta_n) = n_2 / n_1$, and this effect is referred to as Brewster effect.
The scheme of the optical system used in the experimental study is shown in Fig. 1, where ND is a neutral optical density filter, P is a polarizer, L\textsubscript{1}, L\textsubscript{2}, L\textsubscript{3} are lenses, A is an aperture, O is an optical structure, Pr is a prism, PD is a photodetector matrix. The lens L\textsubscript{3} is at the same distance 2f from the optical structure O and the photodetector matrix PD, where f is the focal length of the lens L\textsubscript{3}. The imaging of the object O on the photodetector matrix PD is performed by the lens L\textsubscript{3}, which focuses the rays scattered from the object. A detailed description of the operation of the optical scheme is given in [3].

Assume that the field distributions in the coordinate system of the beam of the incident and reflected beams are $S_{\text{in}}(x, y)$ and $S_{\text{out}}(x, y)$, respectively. In the coordinate systems of the beams, x axis is perpendicular to the directions of the beam propagation and the magnetic field vector, and y axis is parallel to the direction of the magnetic field vector. Using the spatial Fourier transform, the incident (reflected) beam can be written as a superposition of plane waves $S_{\text{in(out)}}(x, y) = \int \int s_{\text{in(out)}}(k_x, k_y) \exp(ik_x x + ik_y y) dk_x dk_y$, where $k_x$ and $k_y$ are the projections of the wave vector of the plane wave along the directions x and y, respectively, and $s_{\text{in(out)}}(k_x, k_y)$ are the corresponding amplitudes of the plane waves. Consequently, the transform of the profile of the incident field into the reflected one is described by the spatial spectral transfer function $H(k_x, k_y)$, where $H(k_x, k_y) = s_{\text{out}}(k_x, k_y) / s_{\text{in}}(k_x, k_y)$.

When the angle of incidence approaches the Brewster angle $\theta_B$, a minimum in the spectra of the transfer function $H(k_x, k_y) = 0$ is observed at $k_x, k_y \rightarrow 0$ due to the total transmission. To analyse $H(k_x, k_y)$, the reflection coefficient $r_{ij}$ of a TM polarized plane wave for the interface between media with dielectric constants $\varepsilon_1$ and $\varepsilon_2$ can be written as:

$$r_{ij} (\alpha, \beta) = \frac{\varepsilon_1 - \varepsilon_2}{2\alpha_\beta^3} (\varepsilon_1 - \varepsilon_2) + \frac{n_1 - n_2}{n_1 + n_2} \frac{\beta}{\alpha_\beta^3} \cos \theta_B,$$

(1)

where $\alpha = n_1 \sin \theta$ and $\beta$ are the propagation constants in the coordinate system of prism. In this coordinate system, $\alpha k_0$ is the projection of the free-space wave vector $k_0 = 2\pi / \lambda$ of a plane wave with the wavelength $\lambda$ on the prism surface in the plane of incidence at the angle $\theta$ to the normal of the surface, $\beta k_0$ is the projection of the wave vector to the normal of the incidence plane. For $\alpha_\beta = \sqrt{\varepsilon_1 \varepsilon_j / (\varepsilon_1 + \varepsilon_j)}$ and $\beta = 0$, the reflection coefficient is $r_{ij} = 0$.

Expressing the transfer function in terms of the reflection coefficient $H(k_x, k_y) = r_{ij} \left( \alpha_\beta + k_x / k_0 \cos \theta_B, k_y / k_0 \right)$, we get the following approximation

$$H(k_x, k_y) = \left[ (\varepsilon_1 - \varepsilon_2) / 2\alpha_\beta^3 + \frac{n_1 - n_2}{n_1 + n_2} \frac{k_y}{\alpha_\beta^3} \right] \cos \theta_B.$$

(2)
The spatial function (2) of spectral transfer can be represented near \( k_x, k_y = 0 \) as

\[
H(k_x, k_y) = ik_x A + ik_y B ,
\]

where \( A \) and \( B \) are complex constants. Eq. (3) is the transfer function of the first-order differentiator. Accordingly, in the spatial domain, the profile of the reflected field is described by approximation

\[
S_{out} = A(S_{in})_x + B(S_{in})_y .
\]

Thus, spatial differentiation can be realized by Brewster effect for p-polarized two-dimensional light distribution. For air \((n_1 = 1)\) and BK7 prism \((n_2 = 1.515)\), \( |A|/|B| > 20 \), in this case the intensity of the transformed image along the \( x \) axis is higher than the intensity of differentiation along the \( y \) axis by more than 20 times.

3. Results and discussion

Amplitude masks in an optically opaque film of chromium oxide on a glass substrate were used as objects of imaging. Figures 2(a) and 2(c) show the images of the masks with curvilinear logo and a cross consisting of horizontal and vertical lines. Figures 2(b) and 2(d) show the corresponding transformed images. The figures show that the edge detection is mainly pronounced along the vertical direction, which is parallel to the normal of the plane of incidence of the beam. To observe the effect of edge detection the beam divergence from the Brewster angle should not exceed 0.5°. The intensity of the \( p \)-polarized source is attenuated by 4 orders of magnitude in this angular region. Detection of the transformed image requires implementation of light source of high intensity. A deviation of the main incidence angle from the Brewster angle results in the transition from the edge highlighting to the normal image.

![Figure 2](image_url)

**Figure 2.** The demonstration edge detection for the non-rotated amplitude samples on a transparent substrate with dimensions of 1200×1200 \( \mu m^2 \): (a), (c) original images; (b), (d) corresponding transformed images.

To demonstrate the dependence of the directivity of the image edge detection, we performed experiments with the rotated sample. Figures 3(a) and 3(c) show images of the samples oriented at 45°
relative to the original position. The converted images of the rotated masks are shown in Figures 3(b) and 3(d). The pronounced bright vertical contours of the curvilinear logo highlight another area. The logo text consists mainly of lines located at the angle of 45°. In the transformed images, the text is almost completely highlighted. The transformed image of the mask "cross" is shown in Figure 3(d). In this case, edges of the cross exhibit approximately uniform intensity as all the mask gradients are inclined by the angle of 45°. A small difference in the intensities of the ‘cross’ lines can be explained by a certain intensity nonuniformity of the Gaussian beam and by a shift in the position and rotation of the objective lens.

![Figure 3](image_url)

**Figure 3.** The edge detection for the amplitude samples on a transparent substrate with dimensions of 1200x1200 µm² rotated by 45°: (a), (c) original images; (b), (d) corresponding transformed images.

For the gradients of mask, which are oriented horizontally, rays are mainly scattered in the plane of propagation of the incident beam. The spatial differentiation of these rays in the direction x occurs with amplitude A. For gradients of mask, which are oriented vertically, scattering of rays occurs in the plane of the incident beam. The spatial differentiation of these rays in the y direction occurs with amplitude B. Thus, intensity of spatial differentiation for mask gradients, which are oriented horizontally, is 20 times higher than that for the gradients of the mask, which are oriented vertically. For mask gradients oriented at 45°, we observe equal intensity, which is 6 times lower than mask gradients oriented horizontally.

### 4. Conclusion

In this work, we experimentally and theoretically demonstrated the edge detection of two-dimensional images of objects based on the Brewster effect is mainly fulfilled for object inhomogeneities, with gradients parallel to the plane of incidence of the beam. The considered effect can be used for selective image processing for amplitude and phase objects with linear dimensions exceeding the wavelength of the light source used in microscopy of biological and medical objects.
5. References

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Acknowledgments
This work was partly funded by Ministry of Science and Higher Education within the State assignment FSRC “Crystallography and Photonics” RAS under agreement 007ГЗ/Ч3363/26 and the Russian Foundation for Basic Research under project No. 18-07-00613.