TWIST-INDUCED MAGNETOSPHERE RECONFIGURATION FOR INTERMITTENT PULSARS

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ABSTRACT

We propose that the magnetosphere reconfiguration induced by magnetic twists in the closed field line region can account for the mode switching of intermittent pulsars. We carefully investigate the properties of axisymmetric force-free pulsar magnetospheres with magnetic twists in closed field line regions around the polar caps. The magnetosphere with twisted closed lines leads to enhanced spin-down rates. The enhancement in spin-down rate depends on the size of the region with twisted closed lines. Typically, it is increased by a factor of \( \sim 2 \), which is consistent with the intermittent pulsars’ spin-down behavior during the “off” and “on” states. We find that there is a threshold of maximal twist angle \( \Delta \phi_{\text{thres}} \sim 1 \). The magnetosphere is stable only if the closed line twist angle is less than \( \Delta \phi_{\text{thres}} \). Beyond this value, the magnetosphere becomes unstable and gets untwisted. The spin-down rate would reduce to its off-state value. The quasi-periodicity in spin-down rate change can be explained by long-term activities in the star’s crust and the untwisting induced by MHD instability. The estimated duration of on-state is about 1 week, consistent with observations. Due to the MHD instability, there exists an upper limit for the spin-down ratio \( f \sim 3 \) between the on-state and the off-state, if the Y-point remains at the light cylinder.

Key words: instabilities – pulsars: general – stars: magnetars – stars: magnetic field – stars: neutron

1. INTRODUCTION

Intermittent pulsars are a special kind of transient radio pulsars. They showed the correlation between timing and radiation of pulsars \citep[e.g.,][]{Kramer2006, Wang2007, Zhang2007}. The first discovered intermittent pulsar, PSR B1931+24, cycled through the sequence of “on”-states and “off”-states around once a month \citep{Kramer2006}. In the off-states, the pulsar is radio-quiet and has a relatively low spin-down rate. In the on-states, the radio emission is produced and the spin-down rate increases by a factor of about 1.5, i.e., \( \dot{\Omega}_{\text{on}}/\dot{\Omega}_{\text{off}} \sim 1.5 \). The on-state and off-state of PSR B1931+24 are observed to recur quasi-periodically. During each cycle, it remains in the off-state for \( \sim 80\% \) of the time. When it switches to the on-state, it lasts for less time, \( \sim 20\% \) of one cycle, i.e., about 1 week, and then switches off again \citep{Kramer2006, Young2013}. Two more intermittent pulsars, PSR J1832+0029 \citep{Lorimer2012} and PSR J1841-0500 \citep{Camilo2012}, have been discovered. PSR J1841-0500 shows an even higher spin-down rate ratio between on- and off-states, \( \dot{\Omega}_{\text{on}}/\dot{\Omega}_{\text{off}} \sim 2.5 \). The mode-switching timescale for the latter two intermittent pulsars is on the order of years.

The changes in the spin-down rates may suggest that dramatic reconfiguration takes place in the intermittent pulsar magnetosphere. However, the specific physical process in the magnetosphere is still poorly understood. In general, the global configuration of the pulsar magnetosphere is not pure dipole. It is believed that the field lines that have footpoints at the star’s surface on the polar caps and cross the light cylinder (LC) are opening up. The last closed line crosses the equatorial plane at a Y-point, which is located exactly at the LC \citep{Contopoulos1999}. The relativistic particles can be accelerated through these open lines. In previous works, relativistic particles are considered to form currents, or, winds, which contribute to the radio emission and high spin-down rate for the on-state of an intermittent pulsar \citep[e.g.,][]{Xu2001, Li2014, Kou2015}. In its off-state, the particle winds stop acceleration somehow so that the pulsar shows low luminosity and low spin-down rate. \cite{Li2012a, Li2012b} modeled the on-states using a force-free condition in the global magnetosphere, but modeled the off-states using a vacuum condition only in the open-line region. Spin-down rate ratios \( f \lesssim 2.9 \) could be obtained with inclination angles \( \gtrsim 30^\circ \). In other work, the mode-switching behavior of intermittent pulsars is thought to be caused by shifting the Y-point inside the LC \citep{Timokhin2006, Timokhin2010}.

In another kind of neutron stars with ultrastrong magnetic field \( B \gtrsim 10^{14} \text{G} \), magnetars, the crustal motions are widely believed to correspond to many activities in observations \citep{Woods2006, Kaspi2007, Mereghetti2008, Beloborodov2009, Yu2012, Yu2013, Huang2014a, Huang2014b}. There is no doubt that the fields in magnetars are strong enough to cause crustal motions. The average field of an ordinary pulsar is only \( \sim 10^{12} \text{G} \). However, observations in X-ray emission from old radio pulsars imply hot spots in the polar cap region. Research on pair creation proposes a Partially Screened Gap model to describe this inner acceleration region \citep{Szary2013, Szary2015}, references therein). The hot spots are found in small scale, compared to the size of the polar cap from the purely dipolar field. The local surface magnetic field is estimated as of the order of \( 10^{14} \text{G} \), much stronger than the average field of the pulsar. Spot-like regions with ultrastrong fields are also suggested in other studies on neutron stars of nearly all ages \citep[e.g.,][]{Geppert2003, Medin2006, Storch2014}. Hence, we assume that a locally ultrastrong field can exist in polar regions of an intermittent pulsar, which is much older than a magnetar. It causes crustal motions and leads naturally to the magnetic twists in the magnetosphere.
In this paper, we propose a twist-induced intermittent pulsar model caused by local crustal motions around the polar caps. For simplicity, we consider the neutron star as an aligned rotator. The crustal motions deform the magnetosphere by twisting a bundle of closed lines around the polar caps. We quantitatively investigate the effects of the twisted closed lines on the stationary configuration of the magnetosphere. We interpret the mode switching of intermittent pulsars by the twist-induced reconfiguration of the magnetosphere. The surface magnetic field lines are supposed to be highly nondipolar in the polar cap region of a pulsar. Charged particles are accelerated along these curved field lines so that the radio emission is generated. However, it is still a puzzling problem that the radio emission is observed to cease after an intermittent pulsar switches off. In our model, an on-state magnetosphere with twisted closed lines possesses more open lines, i.e., larger emission beam, than its off-state. The generation and nulling of radio emission may be related to the orientation of the line of sight of observers (Timokhin 2010).

The paper is arranged as follows. The basic model is described in Section 2. In Section 3, we describe two types of magnetospheres representing the on-state and off-state, respectively. The state transitions are discussed in Section 4. We further explore the physical constraint on the spin-down rate enhancement of the on-state magnetospheres in Section 5. Discussions are provided in Section 6. Throughout this paper, we set the magnetic dipole moment $\mu$, the pulsar’s rotational frequency $\Omega$, and the speed of light $c$ to unity, i.e., $\mu = \Omega = c = 1$.

2. TWISTING-INDUCED FORCE-FREE PULSAR MAGNETOSPHERE

We assume that the force-free condition $j \times B + c\rho E = 0$ is satisfied everywhere in the stationary axisymmetric pulsar magnetosphere. Here $j$ is the electric current density, $B$ and $E$ are the magnetic field and electric field, and $\rho_e = \nabla \cdot E/4\pi$ is the electric charge density in the pulsar magnetosphere. The force-free condition provides a simple yet accurate approximation to the neutron star’s magnetosphere (Contopoulos et al. 1999; Gruzinov 2005). We adopt cylindrical coordinates $(R, z, \phi)$, where both $R$ and $z$ are in units of $R_{\text{LC}} \equiv c/\Omega$. For typical rotating frequency of intermittent pulsars $\Omega \sim 10^{12}$ s$^{-1}$, the value of $R_{\text{LC}}$ is about $3 \times 10^4$ km, measured from the center of the star. The magnetic field of a standard magnetosphere described above reads

$$B = -\frac{\Psi_z}{R} \hat{R} + \frac{\Psi_R}{R} \hat{\theta} + \frac{2i}{R} \hat{\phi},$$

where $\Psi$ is the magnetic stream function. In previous work (e.g., Contopoulos et al. 1999; Ogura & Kojima 2003; Gruzinov 2005; Takamori et al. 2014), the field-line configuration of a standard rotating pulsar magnetosphere is divided into a closed-line region and an open-line region, separated by a magnetic separatrix that contains a current sheet on the equatorial plane. A Y-point structure is formed at the tip of current sheet. The Y-point is the intersection of the last closed line and the equatorial plane. The distance from the Y-point to the center of star is equal to the cylindrical radius of the LC, $R_{\text{LC}} \equiv c/\Omega$. The electric currents $I(\Psi)$, consisting of relativistic particles, are flowing along the open field lines. The Poynting power $L$ can be calculated as (Gruzinov 2005; Timokhin 2006)

$$L = \int_0^{\Psi_{\text{last}}} 2|I(\Psi)| \ d\Psi,$$

where $\Psi_{\text{last}}$ is the value of $\Psi$ at the last closed field line. In the force-free limit, the spin-down power, which describes the radiation that originated from the star’s rotational energy, is assumed to be comparable to the Poynting power, i.e., $L \sim -I_{\text{rot}} \Omega / \Omega$, where $I_{\text{rot}}$ is the neutron star moment of inertia (Lorimer & Kramer 2005). Therefore, the spin-down rate ratio $f = \Omega_{\text{on}}/\Omega_{\text{off}}$, is approximately measured by $L_{\text{on}}/L_{\text{off}}$. We also call $L$ the spin-down power in this paper.

In the standard pulsar magnetosphere, magnetic twist exists only in the open-line region. The closed-line region is entirely untwisted. In other words, the electric currents $I$ exist only in a range from $\Psi = 0$ to $\Psi = \Psi_{\text{last}}$ (Contopoulos et al. 1999; Gruzinov 2005). However, an ultrastrong field in the region around the polar cap would cause active crust motion. In these crustal active regions, the footpoints of the closed lines on the star’s surface are displaced. As a result, the magnetic twist is generated by the active crust motion in the closed-line region. We measure the twist by the twist angle

$$\Delta \phi(\Psi) = 2|I_{\text{closed}}(\Psi)| \int_\Psi^\infty \frac{d\theta}{r^2 \sin^2 \theta \partial_\theta}, \quad \Psi_{\text{last}} \leq \Psi \leq \Psi_{\text{m}}.$$  

Here $(r, \theta)$ are spherical coordinates and $\Psi_{\text{m}}$ is the upper boundary of twisted closed lines. Basically, $\Psi_{\text{m}}$ should be chosen as small values since the crustal active region cannot be too large. In practice, the star radius of a pulsar is $R_{\text{NS}} \sim 10$ km $\sim 3 \times 10^4 R_{\text{LC}}$. We choose a typical value for the upper boundary as $\Psi_{\text{m}} = 2.1$ (see the next section). Since the magnetosphere near the star’s surface is roughly in a dipolar pattern (Contopoulos et al. 1999), one can find that the footprint on the star’s surface of this boundary field line is at $\theta_m = \arcsin \sqrt{\Psi_{\text{m}} R_{\text{NS}}} = 1^2.44$, in the poloidal direction measured from the star’s pole. Thus, the typical length of the crustal active region on the star’s surface can be estimated as $2R_{\text{NS}} \sin \theta_m \sim 0.5$ km, which is consistent with the length scale of ultrastrong field spots proposed for neutron stars (-Shabaltas & Lai 2012). The global configuration of the magnetosphere should satisfy the following equation:

$$1 - R^2(\Psi_{RR} + \Psi_{zz}) - \frac{1}{R^2} \Psi_{,R} + 4I(\Psi) \frac{dI(\Psi)}{d\Psi} = 0.$$  

Different from the standard magnetosphere (Contopoulos et al. 1999; Gruzinov 2005), the electric currents $I$ become nonzero in the closed-line region due to active crustal motions around polar caps. The twist triggered by local crustal motion is expected to induce reconfiguration of the global magnetosphere.

3. MAGNETOSPHERES DURING ON-STATE AND OFF-STATE

Theoretically, the distribution of poloidal current can be divided into two parts, $I_{\text{closed}}(\Psi_{\text{last}} \leq \Psi \leq \Psi_m)$ in the closed-line region and $I_{\text{open}}(\Psi_m \leq \Psi)$ in the open-line region. While $I_{\text{open}}$ can be consistently determined by iteration, $I_{\text{closed}}$ requires...
a reasonable trial function. We set a trial of \( I_{\text{closed}}(\Psi) \) as
\[
I_{\text{closed}}(\Psi) = A (\Psi - \Psi_{\text{last}})(\Psi - \Psi_{\text{m}}), \quad \Psi_{\text{last}} \leq \Psi \leq \Psi_{\text{m}}. \tag{5}
\]
This trial function provides a simple but reasonable description to the current in the closed-line region induced by crustal motions. We assume a stationary magnetosphere during the on-state so that the electric circuit in the open-line region is close, which requires \( I_{\text{closed}}(\Psi_{\text{last}}) = 0 \). We also require that the magnetic twist be locally confined within the polar region inside \( \Psi_{\text{m}} \), which indicates that \( I_{\text{closed}}(\Psi > \Psi_{\text{m}}) \equiv 0 \). The extent of crustal motions can be described by the amplitude \( A \). The greater the amplitude \( A \), the more twisted the magnetosphere. Notice that we adopt a simple parabolic profile for \( I_{\text{closed}} \).

In practice, it is an effective method in searching the solution empirically. One more complexity in solving Equation (4) is a singularity at the LC, where
\[
\frac{\partial \Omega}{\partial \Psi} \left( \Psi_{\text{off}} \right) \sim \infty.
\]
In order to determine the current distribution in the open-line region \( I_{\text{open}}(\Psi) \), we use L'Hôpital's rule to treat the terms with \( \Psi_{\text{m}} \) that contain singularities in the equation. The boundary conditions on the LC are taken as \( \Psi_{\text{r}}(\Psi_{\text{r}} = 1) = 2(\Psi_{\text{open}}|_{\Psi_{\text{r}} = 1}) \).

We adopt an appropriate trial function of \( I_{\text{open}} \) chosen, we correct the distribution of \( I_{\text{open}} \) across the LC iteratively with (Contopoulos et al. 1999)
\[
(\Psi')_{\text{open}} = \mu_1 (\Psi'_{\text{open}}|_{\Psi_{\text{r}} = 1}) + \mu_2 (\Psi'_{\text{open}}|_{\Psi_{\text{r}} = 1}) + \mu_3 (\Psi_{\text{r}} = 1 - \Psi_{\text{r}} = 1),
\]
\[
\Psi = \frac{1}{2} (\Psi_{\text{r}} = 1 + \Psi_{\text{r}} = 1), \tag{6}
\]
where the weight factors \( \mu_1 + \mu_2 + \mu_3 = 1 \) and \( \mu_3 \) are chosen empirically. One more complexity in solving Equation (4) has to do with the nonlinearity of this equation. Let us define a linear operator \( \mathcal{L} \) to represent the differentiations of \( \Psi \); then Equation (4) becomes \( \mathcal{L}(\Psi) + (II')(\Psi) = 0 \). Obviously, the term \( II' \) contains the complicated nonlinear behavior of the equation. After each correction of \( II' \) by Equation (6), we calculate its differentiation \( d(II')/d\Psi \). We adopt the Newton iteration (Press et al. 1992) to calculate a new \( \Psi \) as
\[
\Psi_{\text{new}} = \Psi - \frac{\mathcal{L} + (II')}{\partial \mathcal{L}/\partial \Psi + d(II')/d\Psi} . \tag{7}
\]
In practice, it is an effective method in searching the solution accurately.

In each step of the iteration, the poloidal current \( I_{\text{open}}(\Psi) \) is obtained by integrating \( (II')_{\text{open}} \). It should be pointed out that there is a constraint \( I_{\text{open}}(\Psi_{\text{last}}) = 0 \), which requires the outgoing current return to the star. In this paper, we assume that an ideal current sheet forms on the equatorial plane outside the LC, i.e., \( \Psi(R \geq 1, 0) = \Psi_{\text{last}} \). Then we adopt a return current \( I_{\text{ret}} \) in a narrow range \( [\Psi_{\text{last}}, \Psi_{\text{last}} + \delta] \) (Contopoulos et al. 1999; Gruzinov 2005), to close the electric circuit. Alternative treatments for closing the circuit without the current sheet assumed can be found in other works (e.g., Takamori et al. 2014).

In Figure 1, we show two kinds of magnetosphere for intermittent pulsars. The upper panels represent the off-state, and the lower panels represent the on-state. The field-line configuration of the off-state magnetosphere is shown in the top left panel, which indicates a magnetosphere without active crustal motions. The dashed line denotes the LC. The thick line designates the magnetic separatrix, which contains a current sheet on the equatorial plane (marked in red). The open-line region (red shaded area) and closed-line region (non-shaded area) are separated by this thick line. The \( \Psi \)-point structure is located at \( (R, z) = (R_{\text{LC}}, 0) \). The value of \( \Psi \) at this last closed line is \( \Psi_{\text{last}} \approx 1.32 \), which agrees with the standard results (Contopoulos et al. 1999; Gruzinov 2005). Only the open-line region contains the electric currents, the distribution of which is shown in the solid red line in the top right panel. The spin-down power of the off-state magnetosphere is \( L_{\text{off}} \approx 1.07 \mu^2 \Omega^4 c^{-3} \).

The bottom panels of Figure 1 show the field configuration and the current distribution of the on-state, where crustal motions give rise to a twisted magnetosphere within the closed field line region around the polar cap, with the upper boundary in poloidal direction \( \theta_\text{m} = 1.44 \), corresponding to \( \Psi_{\text{m}} = 2.1 \). We show the twisted field configuration caused by local crustal motions in the bottom left panel of Figure 1. As expected, global reconfiguration of the magnetosphere is induced by magnetic twist in the closed-line region. Compared with the off-state magnetosphere, the primary feature of the global reconfiguration is that more field lines are opening up, i.e., the open-line region (red shaded area) is enlarged. In the on-state, the magnetosphere enters a new state with electric currents flowing in closed lines (blue shaded area). The current distributions of the on-state are shown with solid lines (red and blue) in the bottom right panel. The value of \( \Psi \) for the last closed line shifts to \( \Psi_{\text{last}} \approx 1.63 \). Correspondingly, the spin-down power of the on-state to the off-state one is approximately \( f \approx 1.55 \), which is consistent with the intermittent spin-down behavior of PSR B1931+24. The steady current distribution in the closed-line region is shown with the solid blue line. The footprints of the twisting closed lines are distributed on the star’s surface in the poloidal direction within a range of \( 1.27 \leq \theta_d \leq 1.44 \), where \( \theta_d = \arcsin \sqrt{\Psi_R R_{\text{NS}}} \). The distribution of twist angle in the closed-line region \( \Delta \phi \) is shown as the thick dashed line. The maximum of twist angle \( \Delta \phi_{\text{max}} \) is about 1. In the following section, we will see that this on-state magnetosphere is a critical state that is expected to get untwisted and transit to off-state again.

4. STATE TRANSITIONS OF TWIST-INDUCED MAGNETOSPHERES

In our model, the on-state and off-state correspond to two different kinds of pulsar magnetospheres with and without crustal motion induced twist, respectively. However, its periodicity of mode switching, or state transition, remains an unresolved issue. Interestingly, we find that the on-state magnetosphere is in a critical state, which may lead to the on-off state transition.
4.1. Critical States of Magnetospheres

The spin-down rate enhancement of the on-state and off-state magnetosphere mentioned above is approximately $f \approx 1.55$, which is consistent with the observational spin-down behavior of the intermittent pulsar, PSR B1931+24, during the on- and off-states. We confine the crustal motions in a small active region around the polar cap, within the specific boundary $\theta_m = 1.54$. Then we investigate the magnetosphere’s response to the different extent of crustal motion, which leads to various global magnetospheric configurations with different levels of twist in the closed-line region.

To measure the magnetic twist, we adopt the maximum of the twist angle, $\Delta \phi_{\text{max}}$, in the closed-line region. The variation of the spin-down rate ratio $f$ with the maximal twist angle $\Delta \phi_{\text{max}}$ is shown as a solid line in Figure 2. The left end of the solid line, marked by a star, represents the standard magnetosphere. Each point on the curve represents a twisted magnetosphere in steady state, which indicates that the reconfigured magnetosphere due to a certain level of twist is achievable. We find that there exists a threshold for the maximum twist angle, $\Delta \phi_{\text{thres}} \approx 1$, marked by a filled circle at the right end of the solid line. If we increase the twists beyond this threshold, we are not able to find equilibrium field configurations any more. Note that a steady-state magnetosphere with $\Delta \phi_{\text{thres}} > 1$ is not achievable because it becomes kink-unstable. Physically, our result is consistent with the MHD instability properties. The growth of twists would be inhibited by the MHD instability (Uzdensky 2002; Beloborodov 2009). The threshold $\Delta \phi_{\text{thres}}$ corresponds to a magnetosphere in the critical state, field lines of which would get untwisted and bring the magnetosphere back into a lower energy state.

A higher value of the spin-down rate ratio, i.e., $f = 2.5$, is observed in PSR J1841-0500. It can be interpreted by a slightly larger crustal active region. We adopt $\theta_m = 2^{\circ}$, corresponding to $\Psi_m = 4$. The results are shown in the dashed line in Figure 2. Similarly, there is a threshold of the maximum of the twist angle, $\Delta \phi_{\text{thres}} \approx 1$, marked by an open circle at the right end of the dashed line.
4.2. State Transitions

The on-state and off-state of PSR B1931+24 are observed to recur quasi-periodically. The latter discovered intermittent pulsar, PSR J1832+0029, shows mode-switching behaviors similar to PSR B1931+24, but with a longer period of ~600 days. We propose that the periodicity of state transitions might be due to the long-term active crustal motions around the star’s polar cap. The closed lines there undergo repeated twisting and untwisting, leading to sequent reconfiguration of the magnetosphere.

The quasi-periodic behavior of the intermittent pulsars, especially for PSR B1931+24, can be naturally explained within the framework of twist-induced reconfiguration. As mentioned above, the state with the standard magnetosphere and the critical state can account for the off-state and on-state of the pulsar, respectively. The locally ultrastrong magnetic field stresses the polar crust. The subsequent crustal motions twist the closed lines significantly. The reconfiguration induced by the crustal motion pushes the magnetosphere to the critical state. Consequently, more field lines are opened, which leads to the enhancement in the spin-down rate. However, the kink instability prevents the magnetosphere from sustaining a stable state with further magnetic twists. The time for which the magnetosphere retains the on-state is limited since the magnetosphere would get untwisted. At the end of the untwisting process, the currents in closed lines are dissipated and the magnetosphere switches back to the off-state, i.e., the standard configuration. As a result, the spin-down rate decreases. With the existence of a locally ultrastrong magnetic field, the polar crusts remain active in the long term. The pulsar would have periodic behavior of the magnetosphere reconfiguration, i.e., switch to on-state, due to twist induced by crustal motions, and switch back to off-state due to untwisting induced by dynamical instabilities. The spin-down rate during on-states is enhanced by a specific factor, compared to that during off-states, which may imply that the active region is locally confined.

The generation or nulling of the radio emission is still an unresolved issue. We interpret the observation of radio emission during on-state with the increasing of opening angle of the emission beam. Specifically, the bundle of open lines in the standard magnetosphere form an opening angle of ~1.2◦. This angle increases to ~1.27 in the twisted magnetosphere established for PSR B1931+24. The radiation area is estimated to increase by ~24%. For PSR J1841-0500, the opening angle increases to ~1.224 during on-state. The corresponding radiation area is estimated to increase by ~50%.

The duration of the on-state of an intermittent pulsar can be estimated by the untwisting timescale. By analogy, we consider the untwisting in the polar cap region to be similar to the untwisting of the global magnetosphere of a magnetar (Beloborodov 2009). The duration of on-state is $t_{on} \sim 15 \sqrt{V_{rad}B_{pol}R_{pol}} / \psi_d$ year. Here $V$ is the voltage in the twisted region, $\psi$ is the twist angle, and $u_d$ is the boundary of the region where $|\psi'(\psi)| > 0$. We calculate $u_d = \psi_d / \psi_R$ by definition, where $\psi_R$ is the field line that touches the star’s surface at cylindrical coordinates $(R_{NS}, 0)$. $\psi_R \sim 10^3$ for a typical pulsars’ rotating frequency $\Omega \sim 10^{-1}$ s$^{-1}$. In the model for PSR B1931+24, $\psi_d = 2.1$, so that $u_d \sim 10^{-3}$. We adopt $V = 10^7$ V and $B = 10^{14}$ G, since the field environment of the crustal active polar cap of an intermittent pulsar is expected to be comparable to a magnetar. $\psi$ is assumed to be equal to $\Delta \phi_{thres} \approx 1$. The estimate of $t_{on}$ is ~6 days. This is consistent with the duration of about 1 week of the on-state in observations. Another two intermittent pulsars stay in on-states for months to 1 yr. If the polar region possesses a locally ultrastrong magnetic field, $B \sim 10^{15}$ G, the relevant timescale could provide an appropriate interpretation. It should be noted that the timescale we estimate is from the pulsar’s switch-on to its switch-off. The spin-down rate is not expected to decrease immediately after the pulsar switches on. The on-state magnetosphere would keep the high spin-down rate for a certain period. Time-dependent calculations are required to describe the details in the state transitions, which will be reported elsewhere.

5. SPIN-DOWN RATE ENHANCEMENT OF ON-STATE MAGNETOSPHERES

The three discovered intermittent pulsars have spin-down rate ratios $f \lesssim 2.5$. This leaves the interesting question whether the spin-down rate ratio could be higher. Theoretically, there exists an upper limit for the spin-down rate enhancement of the on-state magnetosphere since the active region is locally confined and the closed lines cannot twist beyond $\Delta \phi_{thres}$. In the left panel of Figure 3, we show the theoretical expectation of the spin-down rate ratio $f$ in relation to the size of the crustal active polar cap. The lower abscissa shows $\theta_m$, the boundary of the active region in the poloidal direction measured from the star’s pole. The upper abscissa shows the corresponding length of the active region estimated as $2R_{NS}\sin \theta_m$. Obviously, a pulsar with a larger crustal active region is expected to have a higher spin-down rate enhancement in its on-state.
example, a size \( \sim 0.5 \) km corresponds to \( f \approx 1.5 \), and a size \( \sim 0.8 \) km corresponds to \( f \approx 2.5 \). The size of the crustal active region may be less than 1 km, i.e., the crust thickness of the neutron star (Lattimer & Prakash 2004). One can find that high spin-down rate ratios \( f \gtrsim 3 \) are not expected due to the MHD instability.

However, it should be noted that our model assumes that the location of the Y-point is fixed at the LC when the magnetosphere switches to the on-state. If the Y-point could be shifted inside the LC during the on-state, higher values of \( f \) would be achieved. The investigation of states of pulsars purely affected by Y-point shifts can be found in Timokhin (2006). Numerically, drawing the location of the Y-point is achieved by changing the boundary condition on the equatorial plane with \( \Psi(R \gtrsim y_0/R_{LC}, 0) = \Psi_{last} \), where \( y_0(<R_{LC}) \) represents the current sheet tip extended inside the LC. The rest of the numerical treatments are similar to those in searching solutions with Y-points at the LC. We specify the crustal active region with \( \theta_m = 1.72^\circ \), i.e., length of \( \sim 0.65 \) km, and \( \Delta \phi_{max} = 1 \). In the right panel of Figure 3, we show the theoretical expectation of the spin-down rate ratio \( f \) in relation to the shift of the Y-point, \( y_0/R_{LC} \). Spin-down rate ratios higher than current observations, \( f \gtrsim 2.5 \), can be obtained with \( y_0 \lesssim 0.8R_{LC} \). Even higher spin-down rate ratios \( f \gtrsim 3 \) can be obtained with \( y_0 \lesssim 0.7R_{LC} \).

6. DISCUSSIONS

The physics of the unique phenomena of intermittent pulsars is still poorly understood. We propose a simple model to interpret the periodic mode switching of intermittent pulsars. The pulsars possess a modest magnetic field \( \sim 10^{12} \) G on average. However, ultrastrong field \( \gtrsim 10^{14} \) G may exist on stars’ polar caps within a spatially confined region (Geppert et al. 2003). The polar crusts of a star are active in the long term due to the ultrastrong field. The crustal motions trigger magnetic twists in the closed field lines. Global reconfiguration in the pulsar magnetosphere is induced so that the pulsar switches to the on-state. More field lines are open, resulting in an enhancement in spin-down rate. The maximal twist angle in the closed-line region of the on-state magnetosphere reaches a threshold \( \Delta \phi_{thres} \sim 1 \). The on-state magnetosphere stays in a critical steady state. Twisting in the closed-line region beyond \( \Delta \phi_{thres} \) would induce instability, so that the magnetosphere would get untwisted and reconfigure to an off-state pattern. As a result, the spin-down rate decreases. The long-term crustal motions continually trigger magnetic twists. Hence, the magnetosphere would undergo periodic off-on and on-off reconfigurations due to twisting and untwisting in the closed-line region. As long as the crustal active region is finite, the spin-down rate ratio of on-state and off-state magnetospheres, \( f = \Omega_{on}/\Omega_{off} \), is a specific value during each period of mode switching. The crustal active region on a star’s surface with a boundary in the poloidal direction \( \theta_m = 2^\circ \) corresponds to \( f \approx 2.5 \). This value is adequate to interpret the spin-down rate enhancement of all discovered intermittent pulsars. This model may also interpret the spin-down rate change, \( f \approx 1.36 \), observed in PSR B0540-69 (Marshall et al. 2015).

The model proposed in this paper is based on stationary solutions of an axisymmetric magnetosphere. It focuses on the origin of mode switching of the intermittent pulsars. The off-state of a pulsar is interpreted by a standard magnetosphere (Contopoulos et al. 1999; Gruzinov 2005). The on-state is interpreted by a critical steady magnetosphere with threshold twisting in the closed-line region, i.e., \( \Delta \phi_{max} = \Delta \phi_{thres} \). Considering the limitation of stationary solutions, the detailed temporal process of state transitions is beyond the scope of this paper. However, by analogy with the untwisting process in magnetars (Beloborodov 2009), we can roughly estimate the timescale of duration when the magnetosphere is in its on-state.

Recent work provides a new type of magnetosphere with different boundary conditions on the current sheet (Contopoulos et al. 2014). We also consider twisting-induced reconfiguration based on this new boundary treatment on the current sheet. With the boundary of the crustal active region, \( \theta_m = 1.72^\circ \), the magnetosphere reaches its critical state with...
The spin-down rate ratio $f \approx 1.57$, and the corresponding untwisting timescale is 8 days. The results are consistent with the model based on the standard magnetosphere described above. Additionally, the results provide a good interpretation for observations of PSR B1931+24.

We further study the theoretical constraint of the spin-down rate ratio $f$. We find that the value of $f$ is confined by the length scale of the crustal active region and the MHD kink instability. If the length scale is comparable to the crust thickness, the MHD kink instability would prevent high enhancement in spin-down with $f \gtrsim 3$. However, theoretical values of $f \gtrsim 3$ can be obtained if the Y-point shifts inside the LC during the on-off transition induced by untwisting process. The detailed investigation of on-state transitions would be achieved by time-dependent calculations in future work. Moreover, consideration of physics that determine the twist angle distribution in the closed-line region would improve this model in future work.

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