Growth of power spectrum due to decrease of sound speed during inflation

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Abstract

We study the amplification of the curvature perturbations due to a small sound speed and find that its origin is different completely from that due to the ultraslow-roll inflation. This is because when the sound speed is very small the enhancement of the power spectrum comes from the fact that the curvature perturbations at the scales smaller than the cosmic microwave background (CMB) scale becomes scale-variant, rather than growing that leads to the amplification of the curvature perturbations during the ultraslow-roll inflation. At large scales the power spectrum of the curvature perturbations remains to be scale invariant, which is consistent with the CMB observations, and then it will have a transient $k^2$ growth and finally approach a $k^4$ growth as the scale becomes smaller and smaller. Thus the power spectrum can be enhanced to generate a sizable amount of primordial black holes. Furthermore, when the high order correction in the dispersion relation of the curvature perturbations is considered the growth of the power spectrum of the curvature perturbations has the same origin as that in the case without this correction.
I. INTRODUCTION

Black holes, as one of the most mysterious objects in nature, can be formed from the supernova explosion of massive stars. It can also be generated in the very early Universe from the matter collapse resulting from large density perturbations and such a black hole is dubbed a primordial black hole (PBH) [1–3]. These density perturbations originate from the curvature fluctuations during inflation [4–7], which are stretched to outside the Hubble horizon by the exponential cosmic expansion with the amplitudes frozen at certain values. When these superhorizon fluctuations reenter the Hubble horizon during the radiation- or matter-dominated era, they will lead to the formation of large scale cosmic structures and at the same time may also result in the PBH production.

To generate a sizable amount of PBHs, it is required that the amplitude $P_R$ of the power spectrum of primordial curvature perturbations reaches the $\mathcal{O}(10^{-2})$ order, as has been demonstrated by extensive studies carried out in, for example, Refs. [8–11]. However, the cosmic microwave background (CMB) observations have limited the primordial curvature perturbations to be about $10^{-5}$ with the amplitude of the corresponding power spectrum being about $10^{-9}$ at the CMB scale [12]. Thus, to be consistent with the CMB observations and at the same time to generate a sizable amount of PBHs, the primordial curvature perturbations can only be enhanced at scales smaller than the CMB scale during inflation. Since $P_R \propto \frac{1}{c_s^4}$, where $\epsilon = -\frac{\dot{H}}{H^2}$ is the slow-roll parameter, with $H$ being the Hubble parameter and $\dot{H} = \frac{dH}{dt}$, and $c_s$ is the sound speed of the curvature perturbations, the enhancement of the curvature perturbations during inflation can be achieved by reducing the rolling speed of the inflaton to obtain a phase of ultraslow-roll inflation [13–43] or the sound speed of the curvature perturbations [44–49].

Slowing down the inflaton can be naturally realized by flattening the inflationary potential as well as by increasing the friction [36, 50–52] or introducing a noncanonical kinetic term [23]. When the cosmic evolution changes from the slow-roll inflation to the ultraslow-roll one, the slow-roll parameter $\eta$, which is defined as $\eta = \frac{\epsilon}{\epsilon H}$, will change from $\sim 0$ to $-6$. Solving the equation of motion for the curvature perturbations, one can find that the solution for modes outside the Hubble horizon during the slow-roll inflation consists of a constant term and a decaying one, which leads to a nearly scale-invariant spectrum. Assuming that the Universe transitions suddenly at a time from the slow-roll inflation to the
ultraslow-roll one, we can match the solution of the curvature perturbations and its first derivative at this time by the Israel junction conditions [53, 54]. After this transition, the term, which is decaying with time before the transition, becomes “growing”. At first, the solution is dominated by the constant term and the power spectrum of the curvature perturbations remains to be scale invariant \([P_{\mathcal{R}}(k) \propto k^0]\). If the ultraslow-roll inflation lasts sufficiently long, the growing term will become dominant, which results in a \(k^4\)-dependent power spectrum [10, 55, 56]. Because the constant and growing terms cancel each other, the power spectrum has a dip preceding the \(k^4\) dependence. If an \(\eta = -1\) middle phase is added between the slow- and ultraslow-roll inflations, it has been found that the power spectrum has a \(k^5(\log k)^2\) growth [55, 57].

Similar to the case of a tiny \(\epsilon\), a smaller value of the sound speed can also lead to an amplification of the primordial curvature perturbations. For a scalar field with a standard kinetic term, the sound speed equals the light speed \((c_s = 1)\). A varying \(c_s\) can be realized in many inflationary models including the inflation with a noncanonical kinetic term [46, 58–60], the Dirac-Born-Infeld inflation [61], the multifield inflationary model [62, 63], the inflationary models in modified gravity theories [64, 65], and so on. Using the effective field theory of inflation, the generation of PBHs due to a small \(c_s\) has been studied in [44–47]. However, whether the growth of the power spectrum due to decreasing sound speed is similar to that due to ultraslow-roll inflation and whether this growth results from the appearance of a growing term in the solution of the curvature perturbations remains unclear. Recently, it was pointed out that when the sound speed is very small the high order \(k^4\) term in the dispersion relation will become dominant [44, 45]. Although this high order term leads to a new solution of the curvature perturbations, the power spectrum is enhanced by small \(c_s\) and it has a \(k^4\) growth, which is the same as that in the case of the ultraslow-roll inflation. Can both \(k^4\) growths of the power spectrum have the same origin? The purpose of the present paper is to answer the above mentioned questions by studying in detail the growth of the power spectrum of the curvature perturbations due to the decrease of the sound speed.

The paper is organized as follows: In Sec. II, we investigate the growth of the power spectrum due to a sudden decrease of the sound speed. In Sec. III, the effect of the high order \(k^4\) term in the dispersion relation is analyzed. Finally, we give our conclusions in Sec. IV. Throughout this paper, we set \(c = \hbar = M_{\text{Pl}} = 1\).
II. THE GROWTH OF POWER SPECTRUM FROM A SMALL SOUND SPEED

To study the growth of the power spectrum of the curvature perturbations due to a sudden decrease of the sound speed, we need to consider the Sasaki-Mukhanov equation

\[ v''_k + \left( c_s^2 k^2 - \frac{z''}{z} \right) v_k = 0 \]  

(1)

in Fourier space, where a dash denotes a differentiation with respect to the conformal time \( \tau \), \( v_k = z R_k \), \( R \) is the curvature perturbation, and \( z \) is defined to be

\[ z^2 \equiv \frac{2a^2 \epsilon}{c_s^2} \]  

(2)

with \( a \) being the cosmic scale factor. From Eq. (2), one can obtain that

\[ \frac{z''}{z} = (aH)^2 \left( 2 - \epsilon + \frac{3}{2} \eta - 3s + s^2 + s \epsilon - s \eta + \frac{1}{4} \eta^2 - \frac{1}{2} \epsilon \eta \right) \]  

(3)

where \( s = \frac{\dot{c}_s}{c_s H} \). Equation (3) can be reexpressed to be

\[ \frac{z''}{z} = \nu^2 - \frac{1}{4} \left( -\tau \right)^2 \]  

(4)

where \( aH = \frac{-1}{\tau} \) has been used, and \( \nu \simeq \frac{3}{2} + \frac{1}{3} \epsilon + \frac{1}{2} \eta - s \). If \( c_s \) is a constant, the general solution of Eq. (1) has the form

\[ v_k(\tau) = A \sqrt{-\tau} H^{(1)}_{\nu}(-c_s k \tau) + B \sqrt{-\tau} H^{(2)}_{\nu}(-c_s k \tau) \]  

(5)

Here \( A \) and \( B \) are two constants. Thus, the solution of the curvature perturbations can be obtained through the relation \( R_k = \frac{v_k}{z} \).

Now we consider that during the slow-roll inflation the sound speed \( c_s \) will change suddenly from 1 to a very small value denoted by \( 1/A_s \) at conformal time \( \tau_1 \), where \( A_s \) is a constant and \( A_s \gg 1 \). Since \( \nu \simeq -3/2 \) for the slow-roll inflation with a constant sound speed, we find that when \( c_s = 1 \) the solution of the curvature perturbations has the form

\[ R^{(1)}_k(\tau) = i \frac{H}{2 \sqrt{\pi A_s k^3}} e^{-ik\tau} (1 + i k \tau) \]  

(6)

after choosing the adiabatic Bunch-Davis vacuum condition.

For \( c_s = 1/A_s \), from Eq. (5), we obtain the general solution of the curvature perturbations

\[ R^{(2)}_k(\tau) = \frac{H}{2 \sqrt{\pi A_s k^3}} \left[ A_2 e^{-i\frac{k\tau}{A_s}} (A_s + i k \tau) - B_2 e^{i\frac{k\tau}{A_s}} (A_s - i k \tau) \right] \]  

(7)
where \( A_2 \) and \( B_2 \) are two constants. Matching \( R_k^{(1)} \) and \( R_k^{(2)} \) at \( \tau = \tau_1 \) by using the conditions \( R_k^{(1)}(\tau_1) = R_k^{(2)}(\tau_1) \) and \( R'_k^{(1)}(\tau_1) = R'_k^{(2)}(\tau_1) \), one can obtain that

\[
A_2 = \frac{\sqrt{A_s \pi}}{2k} (1 + A_s)(-1 + ik + A_s)e^{ik(1 - \frac{1}{2})}, \\
B_2 = \frac{\sqrt{A_s \pi}}{2k} (-1 + A_s)(1 - ik + A_s)e^{ik(1 + \frac{1}{2})},
\]

where we have set \( \tau_1 = -1 \), which simplifies the expressions but does not change any physical result. Substituting \( A_2 \) and \( B_2 \) into Eq. (7) gives the expression of the curvature perturbations during the phase of \( c_s = 1/A_s \). Expanding this expression in the infrared limit \( (c_s k \tau \to 0) \), we arrive at

\[
R_k^{(2)}(\tau) \sim \frac{iH A_s e^{ik}}{2\sqrt{c^3 k}} C_2 - \frac{iH A_s e^{ik}}{2\sqrt{c^5 k}} D_2 - \frac{iHe^{ik}}{4A_s \sqrt{c^3 k}} (-\tau)^2 D_2 + \cdots. 
\]

where

\[
C_2 = A_s \cos \left( \frac{k}{A_s} \right) - i \sin \left( \frac{k}{A_s} \right) \\
D_2 = \left( A_s^2 - 1 \right) \sin \left( \frac{k}{A_s} \right) .
\]

For modes that are superhorizon at \( \tau_1 \), we can further expand Eq. (11) at \( k\tau_1 = -k \to 0 \) and then obtain

\[
R_k^{(2)}(\tau) \sim \frac{iHe^{ik}}{2\sqrt{c^3 k}} + \frac{He^{ik}}{2\sqrt{c^3 k}} - \frac{iH(2A_s^2 + 1)e^{ikk^{1/2}}}{12A_s^2 \sqrt{\epsilon}} - \frac{iH(A_s^2 - 1)e^{ikk^{1/2}}}{4A_s^2 \sqrt{\epsilon}} (-\tau)^2 + \cdots . 
\]

It is easy to see that the leading part is constant independent of \( \tau \), which contains three different \( k \)-dependent terms and the subleading part decays with time since \( |\tau| \) decreases during inflation. These characters are different from that in the case of the transition from the slow-roll inflation to the ultraslow-roll one, where there is a growing part and the constant part only has a \( k^{-3/2} \)-dependent term [44].

Equation (12) gives that the power spectrum of the curvature perturbations has the form

\[
\mathcal{P}_{R_k^{(2)}} = \frac{k^3}{2\pi^2} |R_k^{(2)}|^2 \sim \frac{H^2}{8\pi^2 \epsilon} + \frac{(A_s^2 - 1)H^2}{24\pi^2 A_s^2 \epsilon} k^2 + \frac{(2A_s^2 + 1)^2 H^2}{2(12\pi A_s^2)^2 \epsilon} k^4 .
\]

Apparently, the power spectrum of the curvature perturbations consists of a \( k \)-independent term and two \( k \)-dependent ones. At the CMB scale, the first term dominates, which leads to a scale-invariant spectrum consistent with the CMB observations. Going to the scales that
are smaller than the CMB scale, the second term begins to play a role. The power spectrum becomes scale dependent and has a short era with a $k^2$ growth. Then, the third term finally dominates and the power spectrum displays a $k^4$ growth. These results are shown clearly in Fig 1. This figure indicates that there is no dip in the power spectrum, although the dip appears in the power spectrum in the ultraslow-roll inflation. This is because the second term in Eq. (13) does not cancel the first term. From Fig. 1, one can see that the power spectrum oscillates at the small scales. The reason is that although these small scales are superhorizon at $\tau$, they are sub-horizon at $\tau_1$. For subhorizon scales at $\tau_1$, Eq. (11) shows clearly that $C_2$ and $D_2$ are oscillating functions, which results in an oscillating power spectrum.

![FIG. 1: The evolution of power spectrum $P_R$ as functions of wave number $k$ with $A_s = 50$. The dashed red line indicates the $k^4$ slope.](image)

### III. THE EFFECT OF HIGHER ORDER CORRECTION IN DISPERSION RELATION

When $c_s$ is very small, the high order correction in the dispersion relation of the curvature perturbations needs to be considered [44–46]. When the $k^4$ corrected term in the dispersion relation is included, the numerical calculation shows that the power spectrum has a $k^4$ growth [44, 45]. In this section, we discuss the origin of this growth. We find that $v_k$ satisfies the following equation

$$v_k'' + \left[ c_s^2 k^2 + \alpha k^4 (-\tau)^2 - \frac{\nu^2 - 1/4}{(-\tau)^2} \right] v_k = 0 .$$

(14)
where $\alpha$ is a constant. When $c_s = 1$, this $k^4$ corrected term can be neglected and $v_k$ has a general solution shown in Eq. (5). Thus, choosing the adiabatic Bunch-Davis vacuum condition the solution of $R_k^{(1)}$ is given in Eq. (6).

For a very small $c_s$, which implies a very large $A_s$ after setting $c_s = \frac{1}{A_s}$, the $k^4$ corrected term in Eq. (14) will dominate over $c_s^2 k^2$ and the latter can be neglected. Solving Eq. (14) and using $R_k = \frac{v_k}{2}$ and $\nu = -3/2$, we find that the curvature perturbation has the general solution

$$R_k^{(2)}(\tau) = -\frac{\alpha^{1/8}}{2A_s\sqrt{\epsilon}} k^{1/2}(-\tau)^{3/2} \left[ A_3 H_{3/4}^{(1)} \left( \frac{1}{2} \sqrt{\alpha k^2} \tau \right) + B_3 H_{3/4}^{(2)} \left( \frac{1}{2} \sqrt{\alpha k^2} \tau \right) \right], \quad (15)$$

where $A_3$ and $B_3$ are two constants. Using the matching condition: $R_k^{(1)}(\tau_1) = R_k^{(2)}(\tau_1)$ and $R_k^{(1)}(\tau_1) = R_k^{(2)}(\tau_1)$ at time $\tau_1$ which is set to be $-1$, one can obtain that

$$A_3 = \frac{A_s e^{ik} C}{\alpha^{5/8} k^4 E},$$

$$B_3 = \frac{A_s e^{ik} D}{\alpha^{5/8} k^4 E}, \quad (16)$$

where

$$C = i \sqrt{\alpha(i + k)} k^2 H_{-1/4}^{(2)} \left( \frac{1}{2} \sqrt{\alpha k^2} \right) - (3 - 3i k - 2k^2) H_{3/4}^{(2)} \left( \frac{1}{2} \sqrt{\alpha k^2} \right)$$

$$+ \sqrt{\alpha}(1 - ik) k^2 H_{t/4}^{(2)} \left( \frac{1}{2} \sqrt{\alpha k^2} \right),$$

$$D = \sqrt{\alpha}(1 - ik) k^2 H_{t/4}^{(1)} \left( \frac{1}{2} \sqrt{\alpha k^2} \right) + (3 - 3i k - 2k^2) H_{3/4}^{(1)} \left( \frac{1}{2} \sqrt{\alpha k^2} \right)$$

$$- \sqrt{\alpha}(1 - ik) k^2 H_{t/4}^{(1)} \left( \frac{1}{2} \sqrt{\alpha k^2} \right),$$

$$E = H_{3/4}^{(2)} \left( \frac{1}{2} \sqrt{\alpha k^2} \right) \left[ H_{-1/4}^{(1)} \left( \frac{1}{2} \sqrt{\alpha k^2} \right) - H_{t/4}^{(1)} \left( \frac{1}{2} \sqrt{\alpha k^2} \right) \right]$$

$$+ H_{3/4}^{(1)} \left( \frac{1}{2} \sqrt{\alpha k^2} \right) \left[ H_{t/4}^{(2)} \left( \frac{1}{2} \sqrt{\alpha k^2} \right) - H_{-1/4}^{(2)} \left( \frac{1}{2} \sqrt{\alpha k^2} \right) \right]. \quad (18)$$

In the infrared region, we have

$$R_k^{(2)}(\tau) \simeq \frac{He^{ik}}{2\sqrt{ek^3}} - \frac{He^{ik} k^{1/2}}{6\sqrt{\epsilon}} + \frac{He^{ik} k^{1/2}}{6\sqrt{\epsilon}} (-\tau)^3 + \cdots. \quad (19)$$

This solution consists of three constant terms and the decaying terms, but there is no growing term. This is similar to the case studied in the previous section. Then, we obtain the power spectrum during the small sound speed phase,

$$P_{R_k^{(2)}} = \frac{k^3}{2\pi^2} |R_k^{(2)}|^2 \simeq \frac{H^2}{8\pi^2 \epsilon} + \frac{H^2}{24\pi^2 \epsilon} k^2 + \frac{H^2}{72\pi^2 \epsilon} k^4. \quad (20)$$
This is the same as that given in Eq. (13) since $A_s \gg 1$. So, the high order correction in the dispersion relation has no contribution to the growth of the power spectrum of the curvature perturbations.

IV. CONCLUSION

A generation of abundant PBHs in the early Universe requires that the amplitude of the power spectrum of the primordial curvature perturbations is enhanced by about 7 orders during inflation, which can be realized by the ultraslow-rolling of inflaton, a very small sound speed of the curvature perturbations, and even some other mechanisms. It has been found that in the ultraslow-roll inflation the amplification of the power spectrum can be attributed to the appearance of a growing solution of the curvature perturbations. In this paper, we find that the origin of the amplification of the curvature perturbations due to a small sound speed is different completely from that in the case of the ultraslow-roll inflation. This is because when the sound speed is very small the enhancement of the power spectrum comes from the fact that the curvature perturbations at the scales smaller than the CMB scale becomes scale variant, rather than growing. The power spectrum of curvature perturbations remains to be scale invariant at large scales, and then it has a short time $k^2$ growth and finally approaches a $k^4$ growth as the scale becomes smaller and smaller. Thus, the curvature perturbations during inflation with a decrease of the sound speed can be consistent with the CMB observations and at the same time enhanced to generate a sizable amount of PBHs. Furthermore, we find that when the high order correction in the dispersion relation of the curvature perturbations is considered, the growth of the power spectrum of the curvature perturbations has the same origin as that without this correction.

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