The superradiance phenomena of massive bosons and fermions in the Kerr spacetime are studied in the Bargmann-Wigner formulation. In case of bi-spinor, the four independent components spinors correspond to the four bosonic freedom: one scalar and three vectors uniquely. The consistent description of the Bargmann-Wigner equations between fermions and bosons shows that the superradiance of the type with positive energy (0 < \omega) and negative momentum near horizon (p_H < 0) is shown not to occur. On the other hand, the superradiance of the type with negative energy (\omega < 0) and positive momentum near horizon (0 < p_H) is still possible for both scalar bosons and spinor fermions.

PACS numbers:

I. INTRODUCTION

One of standard radiation problems of matter fields around rotating black holes is the superradiance problem in which the reflected intensity becomes stronger than the incident intensity. The successive occurrence of the superradiance could cause a serious problem on the stability of black holes known as the blackhole bomb \cite{1-4}. The superradiance phenomena have still new interests in theoretical and numerical analysis \cite{5}.

Under Kerr space-time background, Klein-Gordon, Dirac, Maxwell, Rarita-Shwinger and Einstein equations for massless fields are known to reduce to the separable one component field equations, which are called as the Teukolsky equations \cite{6, 7}. The analytic perturbative solutions of the Teukolsky equations show that the reflected intensity can become stronger than the incident intensity for bosons, which tells that the superradiance occurs for strongly rotating (or light mass) black holes.

We list up some special features of the superradiance phenomena to rotating black holes as follows:
1. The superradiance may occur for bosonic fields to the strongly rotating (or light mass) black holes but not to the weakly rotating (or heavy mass) black holes in four dimensional space-time \cite{1, 8, 13}.
2. The superradiance does not occur for massive as well as massless fermionic fields to any rotating black holes in four dimensional space-time \cite{9, 10, 14, 15}.
3. In the three dimensional case, the superradiance phenomena of the type with positive energy (0 < \omega) and negative momentum near horizon (p_H < 0) have been shown not to occur \cite{16, 18}.

Although this difference between bosons and fermions has been attributed to the exclusion principle, we need a better understanding. The purpose of this paper is to resolve this problem using the Bargmann-Wigner (BW) formulation which connects massive fermions and bosons consistently in the unquantized field theory. Bosons can be considered as composite particles of even number of fermions, and the BW formulation realizes the consistent relation among them.

We extend the original BW equations to include scalars (lower spin state) as well as vectors (highest spin state) in the bi-spinor case. It should be noted that the boson states can be described by spinor from one side and by boson from the other side in BW formulation. Then we apply it to the current conservation law among massive scalars and spinors in asymptotic region of Kerr space-time. It is noted that the BW equations relate, in this case, bi-spinor with bosons kinematically contrasting with the Bethe-Salpeter equations which relate them dynamically.

Our result shows that the momentum near horizon cannot be negative, therefore the superradiance for bosons does not occur in case of 0 < \omega and p_H = \omega - m\Omega_H < 0 (where m is the magnetic quantum number of fields and \Omega_H is the angular velocity of black hole). This tells that there is no difference in superradiance between fermions and bosons. This article is partially based on the preliminary work \cite{19}.
The organization of this paper is as follows. In section 2, BW equations for bi-spinor in flat space-time are extended to include scalar part as well as vector parts. The Lagrangian and the conserved currents are also studied. In section 3, BW formulation is applied to the asymptotically flat space-time and try to solve the fermion and boson puzzle in superradiance phenomena in Kerr geometry. The summary is given in the final section.

II. BARGMANN-WIGNER FORMULATION IN FLAT SPACETIME

From the representation theory of Lorentz group, the general spin states can be formulated by the direct product of the fundamental representation (spin 1/2 spinors). This is realized in the relativistic field equations for arbitrary spin [20]. According to the original formulation of Bargmann and Wigner [21, 22], bosonic fields of mass \(\mu\) and spin \(s\) are represented by a completely symmetric multi-spinor of rank \(2s\)

\[
\Psi(x) = \Psi(x)^{(BW)} C^{-1}\gamma_5,
\]

satisfying Dirac-type equations in all index:

\[
\begin{align*}
(\gamma^\lambda \partial_\lambda + \mu)_{\alpha\alpha'} \Psi(x)^{(BW)}_{\alpha'...\tau} &= 0 \\
(\gamma^\lambda \partial_\lambda + \mu)_{\beta\beta'} \Psi(x)^{(BW)}_{\alpha\beta'...\tau} &= 0 \\
&\cdots
\end{align*}
\]

(1)

In the following we do not require the completely symmetry for the multi-spinor index of rank \(2s\) in order to include any bosonic spin states \(0, 1, \ldots s\).

Let us start with the Bargmann-Wigner equations for scalar and vector fields. For spin 0 and 1 states, we introduce the modified second rank BW field by

\[
\Psi(x) = \Psi(x)^{(BW)} C^{-1}\gamma_5,
\]

(2)

where \(C\) denotes the charge conjugation operation. With this the BW equations (1) are expressed as

\[
\begin{align*}
(\gamma^\lambda \partial_\lambda + \mu)\Psi(x) &= 0, \\
\Psi(x)(\gamma_5 \partial_\lambda \gamma^\lambda + \mu) &= 0.
\end{align*}
\]

(3)

(4)

We expand the bi-spinor fields with a set of bosons as

\[
\Psi(x) = \sqrt{\mu}(S(x) + \gamma_5 P(x) - \gamma^\lambda V_\lambda(x) + \gamma_5 \gamma^\lambda A_\lambda(x)) + \frac{1}{2\sqrt{\mu}} \gamma_5 \Sigma^\lambda\tau F_{\lambda\tau}(x),
\]

(5)

where \(S, A_\lambda\) denote scalar and vector fields respectively, and the spin matrix is defined by \(\Sigma^\lambda\tau := (\gamma_\lambda \gamma_\tau - \gamma_\tau \gamma_\lambda)/2\). Other fields \(P, V_\lambda, F_{\lambda\tau}\) are auxiliary fields.

Now we apply BW equations (3)-(4) to boson expansion form (5). Adding these equations, we find the set of relations among boson fields

\[
\begin{align*}
\mu S(x) &= \partial^\lambda V_\lambda(x) \\
\mu V_\lambda(x) &= \partial_\lambda S(x) \\
P(x) &= 0 \\
\mu^2 A_\lambda(x) &= \partial_\tau F_{\tau\lambda}(x) \\
F_{\lambda\tau}(x) &= \partial_\lambda A_\tau(x) - \partial_\tau A_\lambda(x).
\end{align*}
\]

(6)

(7)

(8)

(9)

(10)

With this we obtain the Klein-Gordon type field equations for independent spin 0 and 1 fields,

\[
\begin{align*}
(\partial^\lambda \partial_\lambda - \mu^2) S(x) &= 0, \\
\partial^\lambda (\partial_\lambda A_\lambda - \partial_\tau A_\lambda) - \mu^2 A_\tau(x) &= 0.
\end{align*}
\]

(11)

(12)
The supplementary condition can be derived from (12)
\[ \partial^\lambda A_\lambda(x) = 0, \] (13)
which guarantees that the independent freedom of vector fields is three. Now, inserting the relations among bosonic fields (6)-(10) in (5) we obtain the bi-spinor solution
\[ \Psi(x) = \frac{1}{\sqrt{\mu}}(\mu - \gamma^\lambda \partial_\lambda)(S(x) + \gamma_5 \gamma^\lambda A_\lambda(x)). \] (14)
Notice that this satisfies the BW equations (3)-(4) automatically.

Here we stress that the relations between fermionic solutions and bosonic solutions via bi-spinor field. The bi-spinor field is considered as bosonic fields in one side and as four spinor fields in the other side of (14):
\[ \Psi(x) = (\psi^{(1)}(x), \psi^{(2)}(x), \psi^{(3)}(x), \psi^{(4)}(x)), \] (15)
where each spinor satisfies Dirac equation. Among four spinors \( \psi^{(i)}(x) \) (\( i = 1, 2, 3, 4 \)), four components are independent which can be selected as
\[ \left( \begin{array}{c}
\psi_{1}^{(1)} \\
\psi_{1}^{(2)} \\
\psi_{2}^{(1)} \\
\psi_{2}^{(2)}
\end{array} \right), \] (16)
and other components are determined using BW equations. Explicitly, these four independent components in (16) are written as \( I \) and \( \sigma \), which correspond to anti-symmetric and symmetric parts in the original BW field \( \Psi(BW) = \Psi_\gamma C \) as \( \sigma_y \) and \( \sigma \sigma_y \). And these four components correspond to bosonic freedom: one for scalar \( S \) and three for vectors \( A_k \). Then the correspondence between fermions and bosons is uniquely established.

Now, we write down the Lagrangian for the bi-spinor fields which gives us the BW equations (3)-(4).
\[ L = -\frac{1}{8} \text{Tr} \{ \bar{\Psi}(x)(\gamma^\lambda \partial_\lambda + \mu)\Psi(x) + \bar{\Psi}(x)\bar{\gamma}^\lambda \gamma_5 \gamma^\lambda + \mu)\bar{\Psi}(x) \}, \] (17)
where the adjoint bi-spinor is defined as
\[ \bar{\Psi} = (-i\gamma_0)\Psi^\dagger(-i\gamma_0). \] (18)
In terms of the bosonic fields \( S(x) \) and \( A_\mu(x) \), this Lagrangian is written as
\[ L = -\mu^2 S^\dagger(x)S(x) - \partial^\lambda S^\dagger(x)\partial_\lambda S(x) \\
-\mu^2 A^\dagger(\gamma^\lambda A_\lambda(x) - \frac{1}{2} F^{\lambda\tau\dagger}(x)F_{\lambda\tau}(x), \] (19)
which reproduces the correct equations of motion (11)-(12) for the bosonic fields.

The invariance of the Lagrangian (17) under the phase transformation \( \Psi(x) \rightarrow e^{i\alpha(x)}\Psi(x) \) leads the conserved current in the bi-spinor expression
\[ J_{\tau} = -\frac{\delta L}{\delta \partial_{\tau}\alpha(x)} = \frac{i}{4} \text{Tr} \bar{\Psi} \gamma_\tau \Psi. \] (20)
In the bosonic expression we have
\[ J_{\tau} = -i(S^\dagger \partial_\tau S - \partial_\tau S^\dagger S) + i(A^\dagger F_{\lambda\tau} - F_{\lambda\tau} A^\lambda). \] (21)
Clearly the current satisfies the conservation law
\[ \partial^\tau J_{\tau} = 0. \] (22)
III. SUPERRADIANCE PUZZLE BETWEEN BOSONS AND FERMIONS IN KERR SPACE-TIME

We now apply BW formulation to the asymptotic flat space-time in Kerr black hole and try to solve the superradiance puzzle between bosons and fermions. Consider the case for scalar bosons and the corresponding bi-spinor in flat space-time. We first express the scalar boson solution of Klein-Gordon equation (11) in the polar coordinate as

\[ S(x) = Y^m_\ell(\theta, \phi) R_\ell(r) \exp(-i\omega t), \]  

where \( Y^m_\ell, R_\ell \) and \( \omega \) denote the spherical harmonics, the radial wave function and the frequency respectively. The corresponding bi-spinor solution is obtained using (14) and (23).

\[ \Psi^{(\text{scalar})}(x) = \frac{1}{\sqrt{\mu}} (\mu - \gamma^\lambda \partial_\lambda) S(x) \]

\[ = \frac{1}{\sqrt{\mu}} \left( \frac{\mu + \omega}{-i\sigma \cdot \nabla, \mu - \omega} \right) Y^m_\ell(\theta, \phi) R_\ell(r) \exp(-i\omega t). \]  

We write this as a set of four spinors,

\[ \Psi^{(\text{scalar})}(x) = \sqrt{\frac{\mu + \omega}{\mu}} (\psi^{(\omega \uparrow)}(x), \psi^{(\omega \downarrow)}(x), \psi^{(-\omega \uparrow)}(x), \psi^{(-\omega \downarrow)}(x)), \]

where each suffix \((\omega \uparrow), \cdots (\omega \downarrow)\) denotes the frequency and spin direction respectively.

Among four spinors in (25) we consider one solution \( \Psi^{(\omega \uparrow)} \) because the net freedom is one \([26]\). The solution \( \Psi^{(-\omega \uparrow)} \) is recombined in eigen-states of total angular momentum. For this purpose, the angular wave functions \( Y^m_\ell \) can be written by the combination of normalized spin-angular functions \( \mathcal{Y} \) as

\[ Y^m_\ell(\theta, \phi) \begin{pmatrix} 1 \\ 0 \end{pmatrix} = \sqrt{\frac{\ell + m + 1}{2\ell + 1}} \mathcal{Y}^{j_3}_{j,\ell}(\theta, \phi) - \sqrt{\frac{\ell - m}{2\ell + 1}} \mathcal{Y}^{j_3}_{-j,\ell}(\theta, \phi), \]

where the total angular momenta are \( j = \ell + 1/2 \) and \( j' = \ell - 1/2 \) with their azimuthal component \( j_3 = m + 1/2 \). Correspondingly the frequency \( \omega \) with spin up spinor can be written as \([24]\)

\[ \psi^{(\omega \uparrow)} = \sqrt{\frac{\ell + m + 1}{2\ell + 1}} \psi^{(\omega \uparrow)}(x) - \sqrt{\frac{\ell - m}{2\ell + 1}} \psi^{(\omega \downarrow)}(x), \]

where \( \psi^{(\omega \uparrow)} \) and \( \psi^{(\omega \downarrow)} \) denote the spin parallel and antiparallel spinors defined by

\[ \psi^{(\omega \uparrow)} = \frac{1}{r} \left( \frac{F(r)}{iG^{(+)}(r)} \mathcal{Y}^{j_3}_{j,\ell}(r) \right) e^{-i\omega t}, \]

\[ \psi^{(\omega \downarrow)} = \frac{1}{r} \left( \frac{F(r)}{iG^{(-)}(r)} \mathcal{Y}^{j_3}_{-j,\ell}(r) \right) e^{-i\omega t}, \]

\[ F(r) = \sqrt{\frac{\mu + \omega}{\mu}} r R_\ell(r), \]

\[ G^{(+)}(r) = \frac{(\partial_r - (\ell + 1)/r) F(r)}{\omega + \mu}, \]

\[ G^{(-)}(r) = \frac{(\partial_r + \ell/r) F(r)}{\omega + \mu}. \]

This establishes the relation between scalar wave function \( S \) and spinor wave function \( \psi^{(\omega \uparrow)} \) or \( \psi^{(\pm)} \) using the bi-spinor field \( \Psi \) explicitly.

Next we consider the radial component of the conserved current for scalar boson \([25]\),

\[ J^{(\text{scalar})}_r = \int \frac{r^2 \sin \theta d\theta d\phi (-i)(S^{(0)} \ast \partial_r S^{(0)} - \partial_r S^{(0)} \ast S^{(0)})}{N_B} \]

\[ = r^2 i(\partial_r R_\ell \partial_\ell - R_\ell \partial_r \partial_\ell). \]  

(29)

For spinors we have \([26]\)

\[ J^{(k)}_r = \int r^2 \sin \theta d\theta d\phi \left( \mathcal{Y}_k^{(\gamma)} \right) \cdot \frac{\gamma}{r} \psi^{(k)} / N_F = r^2 i(\partial_r R_\ell \partial_\ell - R_\ell \partial_r \partial_\ell), \]

(30)
where suffix $k$ denotes $(\omega \uparrow), (\omega \uparrow)$ or $(\omega \downarrow)$ respectively. Then we obtain the current relation between scalar boson and spinor (spin up, spin parallel or spin anti-parallel) as

$$J_{r}^{(\text{scalar})} = J_{r}^{(\omega \uparrow)} = J_{r}^{(\omega \uparrow)} = J_{r}^{(\omega \downarrow)}.$$  \hfill (31)

Note that the expression for each current does not depend on the wave function normalization factors (see footnotes $b$ and $c$).

In order to solve the superradiance puzzle applying the BW formulation, we study the scattering problems for fermionic and bosonic fields in Kerr black hole space-time in Boyer-Lindquist coordinates [27]

$$ds^2 = \frac{\Delta}{\Sigma}(dt - a\sin^2 \theta d\phi)^2 + \frac{\Sigma}{\Delta}dr^2 + \Sigma d\theta^2 + \frac{\sin^2 \theta}{\Sigma}[(r^2 + a^2)d\phi - adt]^2,$$

$$\Delta = r^2 - 2Mr + a^2, \quad \Sigma = r^2 + a^2 \cos^2 \theta,$$  \hfill (32)

where $M$ and $a$ denote the mass and angular momentum of the Kerr black hole respectively. For the scattering problem for the spin 0 scalar filed, we let

$$S(x) = R(r)Y(\theta, \phi) \exp(-i\omega t),$$

where $R(r)$ and $Y(\theta, \phi)$ denote the radial and angular field functions respectively. They obey the field equations

$$\left(\frac{1}{\sin \theta}\partial_\theta \sin \theta \partial_\theta - (a\omega \sin \theta - \frac{m}{\sin \theta})^2 - \mu^2 a^2 \cos^2 \theta + \nu\right)Y(\theta, \phi) = 0,$$

$$\left(\partial_r \Delta \partial_r + \frac{(\nu^2 + a^2)\omega - am)^2}{\Delta} - \mu^2 r^2 - \nu\right)R(r) = 0,$$  \hfill (34)

where $\mu$ and $\nu$ denote the mass of particle and separation parameter.

To study the behavior of radial wave function near infinity and event horizon we introduce a new radial coordinate $r_*$,

$$\frac{dr_*}{dr} = \frac{r^2 + a^2}{r^2 - 2Mr + a^2}.$$  \hfill (36)

Using the new coordinate, the radial field solutions in Kerr space-time become free waves near the spatial infinity $r \to \infty (r_* \to \infty)$ and event horizon $r \to r_H = M + \sqrt{M^2 - a^2} (r_* \to -\infty)$,

$$\sqrt{r^2 + a^2} R(r) \sim \begin{cases} \frac{A_B^{(\text{inc})}}{\sqrt{\omega}} \exp(-ip_\infty r_*) + \frac{A_B^{(\text{ref})}}{\sqrt{\omega}} \exp(ip_\infty r_*) & (r \to \infty), \\ \frac{A_B^{(\text{trans})}}{\sqrt{|p_H|}} \exp(-ip_H r_*) & (r \to r_H), \end{cases}$$

where $A_B^{(\text{inc})}, A_B^{(\text{ref})},$ and $A_B^{(\text{trans})}$ denote the incident, reflected, and transmitted waves respectively. Momenta near the infinity and event horizon are denoted by

$$p_\infty = \sqrt{\omega^2 - \mu^2} \quad \text{and} \quad p_H = \omega - m\Omega_H,$$  \hfill (38)

where particle mass $\mu$, magnetic quantum momentum $m$ and angular velocity $\Omega_H = a/(r_H^2 + a^2)$ respectively. Note that momentum $p_H$ is equivalent to the effective energy because particles are treated as massless near horizon. Using the asymptotic solution for scalar boson [27], the current relation of the radial component can be derived for the infinity and event horizon region,

$$\frac{p_\infty}{\omega} (\frac{|A_B^{(\text{inc})}|^2}{|A_B^{(\text{ref})}|^2} - \frac{|A_B^{(\text{ref})}|^2}{|A_B^{(\text{trans})}|^2}) = \frac{p_H}{|p_H|} |A_B^{(\text{trans})}|^2.$$  \hfill (39)

The momentum at infinity $p_\infty$ is positive definite by definition of incident and reflected waves. From this we have the superradiance condition for scalar bosons

$$\omega p_H < 0.$$  \hfill (40)
This superradiance condition taking account only of scalar bosons is consistent with previous works on the second quantized method by Unruh [10], general analytical method using the Teukolsky equation in the massless limit by Mano and Takasugi [12], and Hawking’s thermodynamic theorem of black hole area by Bekenstein [28, 29]. The condition (40) is realized in the cases (type 1) \( \omega > 0 \) and \( p_H < 0 \) or (type 2) \( \omega < 0 \) and \( p_H > 0 \).

Next we consider the scattering problem for the spin 1/2 spinor field in Kerr space-time. Write the spinor wave function in the form,

\[
\psi(x) = 1/ \Delta \Sigma \left( F(r) Y(\theta, \phi) + iG(r) Y'(\theta, \phi) \right),
\]

where \( Y(\theta, \phi), Y'(\theta, \phi) \) stand for normalized spin-angular functions. Asymptotic solutions of radial components are obtained

\[
F(r) \sim \sqrt{\frac{\omega + \mu}{\omega}} \left( A_F^{(\text{inc})} \exp(-ipHr) + A_F^{(\text{ref})} \exp(ipHr) \right),
\]

\[
G(r) \sim 1/ \left( \omega + \mu \right) \frac{d}{dr} F(r),
\]

for infinity \((r \rightarrow \infty)\) and

\[
F(r) \sim A_F^{(\text{trans})} \exp(-ipHr),
\]

\[
G(r) \sim iF(r),
\]

for near event horizon \((r \rightarrow r_H)\), where \( A_F^{(\text{inc})}, A_F^{(\text{ref})}, \) and \( A_F^{(\text{trans})} \) denote fermionic amplitudes of incident wave, reflected, and transmitted waves respectively. The radial current relation for spinors between the infinity and event horizon is given by

\[
\frac{p_\infty}{\omega} (|A_F^{(\text{inc})}|^2 - |A_F^{(\text{ref})}|^2) = |A_F^{(\text{trans})}|^2.
\]

Note that the normalization factors of spinor solution in (44) for near horizon is one, and as a result the factor in front of \(|A_F^{(\text{trans})}|^2\) is also one (the speed of light), which is consistent with the massless neutrino theory. From this we conclude that the superradiance cannot occur for \( \omega < 0 \) and \( p_H < 0 \) (type 1). The result is completely consistent with the previous works on the second quantization method with respect to the exclusion principle which states that all states with \( p_H < 0 \) are filled by Unruh and others [10, 14, 15].

In order to relate the bosonic and fermionic current, we apply the BW formulation for spacial infinity, which can be treated as flat space-time. Using (31) we obtain the relation at spacial infinity \((r \rightarrow \infty)\),

\[
|A_B^{(\text{inc})}|^2 - |A_B^{(\text{ref})}|^2 = |A_F^{(\text{inc})}|^2 - |A_F^{(\text{ref})}|^2.
\]

Combining three conserved current relations (39), (46), and (47), we obtain the current relation near the horizon as

\[
\frac{p_H}{\omega} |A_B^{(\text{trans})}|^2 = |A_F^{(\text{trans})}|^2,
\]

which shows that the momentum near the event horizon is positive,

\[
0 < p_H = \omega - m\Omega_H.
\]

This means that the superradiance of \(0 < \omega \) and \( p_H < 0 \) (type 1) can not occur for both bosons and fermions. It should be noted that the superradiance of \( \omega < 0 \) and \( p_H > 0 \) (type 2) is consistent with all current relations of (39), (46), and (47).

### IV. SUMMARY

We have studied the superradiance puzzle between bosons and fermions in Kerr black hole space-time using the Bargmann-Wigner formulation, and established the direct wave function relation between massive scalar bosons and
spinors via bi-spinor fields. With this we have demonstrated that, just as the fermionic case, the superradiance for scalar bosons of the type $0 < \omega$ and $p_H = \omega - m\Omega_H < 0$ does not occur in Kerr space-time. This result is consistent with that in $(2+1)$-dimensional analysis.\cite{17,18}.

However, it should be emphasized that the superradiance of $\omega < 0$ and $0 < p_H$ (type 2) is still possible (the magnetic quantum number $m$ should be negative in this case).\cite{30} In type 2 case, the negative energy particle goes into black hole with positive momentum near horizon ($p_H > 0$), which corresponds to the interpretation to get energy from black hole and can occur the superradiance phenomena.

In case of $\omega < 0$ and $0 < p_H$, we use $\psi(-\omega)$ instead of $\psi(\omega)$ as an independent spinor among four spinors in (25) and even in this case the current relations (39), (40), and (47) hold in the same way. These current relations are consistent with the superradiance of (type 2) $\omega < 0$ and $0 < p_H$. In this case the incoming particles with negative frequency get enough energy from black hole and are scattered backward strongly when they have opposite angular momentum to that of black hole.

This is very important because in quantum field theory the negative frequency component becomes an essential part of the field. Moreover, the physical vacuum is made of infinite number of virtual particle-antiparticle pairs, so that the incoming particle is accompanied by a large number of virtual particle-antiparticle pairs. So near the horizon, the virtual antiparticles (the negative frequency component) can be absorbed by the blackholes. Physically this could make the reflected flux larger than the incident flux at the horizon, resulting in the superradiance.\cite{31}.

This naturally relates our result to the Hawking radiation.\cite{32,33}. Of course the Hawking radiation is a quantum effect, and the superradiance is a classical phenomenon. Nevertheless the above discussion tells that our result is not inconsistent with the Hawking radiation. It suggests that the Hawking radiation could be interpreted as a superradiation of the virtual particle-antiparticle pairs.

We can extend our method for the superradiance problem of vector bosons using similar analysis to scalar bosons which will be done in a separate paper.

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The current is normalized by the total scalar density, $N_B = \int d^3x (-i)(S(0)^\ast \partial_t S(0) - \partial_t S(0)^\ast S(0))$. By this expression of current, the wave function normalization becomes irrelevant.

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The current is normalized by the total scalar density, $N_B = \int d^3x (-i)(S(0)^\ast \partial_t S(0) - \partial_t S(0)^\ast S(0))$. By this expression of current, the wave function normalization becomes irrelevant.

The current is normalized by the total spinor density, $N_F = \int d^3x \bar{\psi}(k)\gamma_0 \psi(k)$.

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Bekenstein derives the relation $dM(1 - m\Omega_H/\omega) > 0$ from the Hawking’s thermodynamical theorem with the conservation law of energy and angular momentum, and derives the superradiance condition for $dM < 0$ as $1 - m\Omega_H/\omega < 0$. This is coincident with our result (30) for both positive and negative $\omega$.

The theory is invariant under the reflection symmetry with respect to frequency and azimuthal number $(\omega, m) \leftrightarrow (-\omega, -m)$; one of which is physical ($\omega < 0, p_H > 0$: type 2) and the other is unphysical ($\omega > 0, p_H < 0$: type 1).

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