Research on Reservoir Numerical Simulation with Consideration of Time-dependent Physical Properties

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Abstract. Fine reservoir description is an important prerequisite to ensure the scientific and efficient development of oilfields. Reservoir numerical simulation is an important tool for efficient development. The accuracy of reservoir description determines the reliability of numerical simulation results. After long-term water injection development, the physical properties of the reservoir have undergone great changes. How to identify the time-dependent law of reservoir physical properties and finely characterize them during numerical simulation has been the focus and research difficulty in this field in recent years. This paper combines the results of previous studies on the changes of reservoir physical properties in the later stage of water drive development in Area A with numerical simulation methods. Predecessors realized time-dependent numerical simulation methods include staged numerical simulation and self-developed numerical simulation programs. Although the above two types of methods can achieve the purpose of realizing time-dependent numerical simulation, they all have certain defects. The segmented simulation method requires multiple modeling, heavy workload, multiple manual operations, and poor continuity and non-directional calculation results. Although the self-developed numerical simulation program calculation results can have better continuity and directionality, but this requires a lot of manpower and material resources to carry out the research. The research and development cost is high, the cycle is long, and the simulation accuracy and speed are difficult to reach the level of commercial numerical simulation software in the short term. Based on the above analysis, this paper has developed a simulation program TD-SIM (Time Dependent Simulation) that combines the commercial numerical simulation software ECLIPSE and can realize the time-dependent function of the reservoir permeability field. This program has convenient use and low cost. At the same time, it is necessary to take into account faster simulation speed, higher simulation accuracy, and better continuity and directionality of the simulation results.

Key words: reservoir description; time-dependent law of physical properties; time-dependent simulation.
1. Introduction
Long-term waterflooding development will cause more complicated changes in the microstructure of the reservoir and bring about changes in the macroscopic physical properties of the reservoir, which is often referred to as the time-dependent phenomenon of reservoir physical properties. With the development of oilfields, the time-dependent phenomenon of reservoir physical properties has become more and more obvious. This change will affect the oil-water flow law of the entire reservoir and further affect the entire oilfield's prediction of future remaining oil distribution and the formulation of enhanced oil recovery programs. In order to grasp the time-dependent laws of reservoirs, domestic and foreign experts and scholars mainly use indoor experiments, field experiments, four-dimensional geological modeling and reservoir dynamic numerical simulation techniques to analyze and study the influencing factors, change mechanism, change law of layer time-dependent physical properties and their influence on subsequent development of the water drive reservoirs in the later stages of development from both the micro and macro perspectives.

Domestic and foreign scholars research methods on reservoir physical properties changes after water injection development can be summarized into four categories: indoor core experiment, logging parameter interpretation, four-dimensional geological modeling technology and reservoir numerical simulation technology. Among them, reservoir numerical simulation is more mature in describing the remaining oil, but for reservoirs with a long history of water injection, the initial input values of reservoir physical properties have changed significantly. At present, mainstream commercial numerical simulation software believes that reservoir physical properties are fixed initial input values, and it is difficult to ensure the accuracy of calculation results. In order to realize time-dependent numerical simulation, the main methods at home and abroad are: one is to calculate in sections on the basis of existing commercial numerical simulation software. The other is to consider reservoir physical properties, water cut, injection volume and other parameters on the basis of experimental results, compile and solve the new reservoir numerical simulation software. This paper establishes a set of simulation methods combining the commercial numerical simulation software ECLIPSE and the simulation program TD-SIM (Time Dependent Simulation) that realizes the time-dependent function of reservoir permeability field. The simulation method is convenient and easy to operate, and has good accuracy. It has achieved good application effects in oil field practice.

2. The realization of reservoir numerical simulation with consideration of time-dependent physical properties

2.1. Time-dependent simulation realization ideas
The core idea of realizing time-dependent simulation is to perform interpolation calculation for different time steps to obtain a smoother difference function result, and use the cubic spline difference algorithm for calculation.

![Figure 1. Curve obtained by spline interpolation algorithm](image-url)
Figure 2. Comparison of calculation results of spline function and linear interpolation calculation

The permeability field at different calculation time steps is calculated according to the following formula, and the following cubic spline interpolation algorithm is applied according to the time-dependent step length:

Let $T_s$ be the start date, $(X_i, Y_j, Z_k)$ are each grid corresponds to the permeability field values in the XYZ direction at the start date. Set $T_e$ as the end date, $(X_i, Y_j, Z_k)$ are the end date of each grid corresponding permeability field values in the XYZ direction. Let $T_c$ be the current date, $(X_i, Y_j, Z_k)$ are each grid corresponds to the permeability field values in the XYZ direction at the current date.

1. Selection of boundary conditions: natural boundary conditions $S''(x_0) = S''(x_a)$, which is:

$$
KX_v(i, j, k) = KX_v(i, j, k) = 0
$$

2. Use the three-angle equation to solve the system of equations. Set $(i, j, k)$ as $x$, $KX_v(x_0) = y_a$. The value $KX_v(x)$ at node a is $m_a$. Then the piecewise cubic Hermite interpolation can be obtained:

$$
KX_v(x) = \sum_{a=0}^{n} [\alpha_a(x) + m_a \beta_a(x)]
$$

$\alpha_a(x) \beta_a(x)$ is the interpolation basis function;

$$
\alpha_a(x) = \begin{cases} 
\left( \frac{x - x_{a-1}}{x_a - x_{a-1}} \right)^2 
+ 2 \left( \frac{x - x_a}{x_a - x_{a-1}} \right), & x_{a-1} \leq x \leq x_a \\
\left( \frac{x - x_{a+1}}{x_a - x_{a+1}} \right)^2 
+ 2 \left( \frac{x - x_a}{x_a - x_{a+1}} \right), & x_a \leq x \leq x_{a+1}
\end{cases}
$$

$$
\beta_a(x) = \begin{cases} 
\left( \frac{x - x_{a+1}}{x_a - x_{a+1}} \right)^2 
+ 2 \left( \frac{x - x_a}{x_a - x_{a+1}} \right), & x_{a-1} \leq x \leq x_a \\
\left( \frac{x - x_{a-1}}{x_a - x_{a-1}} \right)^2 
+ 2 \left( \frac{x - x_a}{x_a - x_{a-1}} \right), & x_a \leq x \leq x_{a+1}
\end{cases}
$$
Obviously, $KX (x)$ and $KX' (x)$ are continuous on the interval $[a, b]$. Satisfy $KX (x) = y_a$, with $m_a$ unknown. Exploit $KX' (x_a - 0) = KX' (x_a + 0) \quad (a = 1, \ldots, n - 1)$ and boundary conditions $KX' (x) = KX' (x) = 0$. The expression $KX (x)$ in the interval $[x_a, x_{a+1}]$ is:

$$\begin{align*}
KX (x) &= \frac{h_a}{h_a^2} \left[ y_{a+1} - 2(x - x_a) \right] + \frac{h_a}{h_a^2} \left[ y_a + 2(x - x_{a+1}) \right] + \frac{h_a}{h_a^2} \left[ y_{a+1} - (x - x_a) \right] + \frac{h_a}{h_a^2} \left[ y_a + (x - x_{a+1}) \right] + \frac{h_a}{h_a^2} \left[ y_{a+1} - (x - x_a) \right] + \frac{h_a}{h_a^2} \left[ y_a + (x - x_{a+1}) \right] \quad (5)
\end{align*}$$

$$\begin{align*}
h_a &= x_{a+1} - x_a \text{, find the second derivative for } KX (x);
KX' (x) &= \frac{6x - 2x_a}{h_a} m_a + 6x - 4x_a + 2\frac{x_{a+1}}{h_a} m_{a+1} + 6\left( x_a + x_{a+1} - 2x \right) \left( y_{a+1} - y_a \right) \quad (6)
\end{align*}$$

We can get $KX' (x_a + 0) = -\frac{4}{h_a} m_a - \frac{2}{h_a} m_{a+1} - \frac{6}{h_a^2} (y_{a+1} - y_a)$. Similarly, $KX (x)$ can be obtained on $[x_{a-1}, x_a]$:

$$\begin{align*}
KX (x) &= \frac{6x - 2x_{a+1}}{h_{a-1}} m_{a-1} + 6x - 4x_{a+1} + 2\frac{x_a}{h_{a-1}} m_a + 6\left( x_a + x_{a+1} - 2x \right) \left( y_a - y_{a-1} \right) \quad (7)
\end{align*}$$

Which is:

$$\begin{align*}
KX' (x_a - 0) &= \frac{2}{h_{a-1}} m_{a-1} + \frac{4}{h_{a-1}} m_a - \frac{6}{h_{a-1}^2} (y_a - y_{a-1}) \quad (8)
\end{align*}$$

From the conditions $KX' (x_a - 0) = KX' (x_a + 0) \quad (a = 1, \ldots, n - 1)$, we can get,

$$\begin{align*}
\frac{1}{h_{a-1}} m_{a-1} + 2 \left( \frac{1}{h_{a-1}} + \frac{1}{h_a} \right) m_a + \frac{1}{h_a^2} m_{a+1} = 3 \left( \frac{V_{a+1} - y_a}{h_a} + \frac{y_a - y_{a-1}}{h_{a-1}} \right) \quad (a = 1, \ldots, n - 1)
\end{align*}$$

Divide by $\frac{1}{h_{a-1}} + \frac{1}{h_a}$, $y_a = f_a \frac{y_{a+1} - y_a}{h_a} = f_a \left[ x_a, x_{a+1} \right]$, 

$$\begin{align*}
\lambda_a m_{a-1} + 2 m_a + \mu_a m_{a+1} = y_a \quad (a = 1, \ldots, n - 1)
\end{align*}$$

In this, $\lambda_a = \frac{h_a}{h_{a-1} + h_a}$, $\mu_a = \frac{h_{a-1}}{h_{a-1} + h_a} \quad (a = 1, \ldots, n - 1)$

Take $y_a = 3 \left( \lambda_a f_a \left[ x_{a-1}, x_a \right] + \mu_a f_a \left[ x_a, x_{a+1} \right] \right) \quad (a = 1, \ldots, n - 1)$ into boundary conditions,

$$\begin{align*}
2m_0 = 3 f_a \left[ x_0, x_1 \right] = g_0 \\
m_{m-1} + 2m_n = 3 f_a \left[ x_{a-1}, x_a \right] = g_a
\end{align*}$$

(11)

Formula (10) and formula (11) are combined and expressed as a matrix:

$$\begin{align*}
\begin{bmatrix}
2 & 1 & \cdots & 1 & \cdots & 0 \\
\lambda_1 & 2 & \mu_2 & \cdots & \cdots & \vdots \\
0 & \lambda_2 & 2 & \mu_2 & \cdots & \vdots \\
\vdots & \ddots & \ddots & \ddots & \cdots & \vdots \\
\vdots & \ddots & \ddots & \ddots & \ddots & \vdots \\
\vdots & \cdots & \lambda_{n-1} & 2 & \mu_{n-1} & \cdots \\
0 & \cdots & 0 & 1 & 2 & \cdots \\
m_1 & \cdots & \cdots & \cdots & \cdots & \cdots \\
g_0 & \cdots & \cdots & \cdots & \cdots & g_a
\end{bmatrix}
\end{align*}$$

(12)
Use the catch-up method to find the solution: \( m_a \), \((a = 1, \ldots, n - 1)\)

Calculate and find the value of \( K_Y (i, j, k) \); The same can be obtained: \( K_Y (i, j, k) \), \( K_Z (i, j, k) \).

2.2. Time-dependent simulation realization process

The main realization ideas of the software are as follows:

2.3. Time-dependent simulation example

The method is based on the mature commercial numerical simulation software ECLIPSE as a whole to improve, so only need to verify the time-dependent simulation program can successfully simulate the time-dependent process to verify the effectiveness of the method. Take the A oilfield numerical simulation model as an example, use the TD-Sim time-dependent numerical simulation program to perform simulation calculations and analyze the results. The model has 232×204×118 grids, a total of 5,584,704 grids. In the model, there are 14 oil wells and 2 water injection wells. The operation time step is 1 year. A total of 20 years are simulated from 2000 to 2020.

The running result is shown in the figure below:
Figure 4. Time-dependent results demonstration

Table 1. Comparison of operation time

| Simulation method | Operation time/minute |
|-------------------|-----------------------|
| ECLIPSE           | 25                    |
| TD-Sim            | 36                    |

The left figure in the above figure is the calculation result without the time-dependent simulation program, and the right figure is the calculation result using the time-dependent simulation program. It can be clearly compared to draw a conclusion: Compared with no using time-dependent simulation result, the time-dependent simulation result is considered. Taking water injection measures for the water injection wells in the red circle resulted in a faster increase in local water saturation than other locations, and the permeability of the water injection to the production well increased significantly, which was in line with the conclusion that the permeability field changes with the water injection development time. The calculation time has increased after considering the time-dependent simulation, but it is also within the acceptable range.

Considering the time-dependent effect of permeability, the recovery is reduced by 0.9%. This is because water injection in plane direction leads to an increase in the directional permeability, forming a dominant channel in the direction of the main flow line, which aggravates the water injection inrush phenomenon, and the longitudinal direction will also cause fingering. The simultaneous action of the two phenomena leads to the advancement of the water breakthrough time of the production well, the deterioration of the water injection effect, and ultimately the reduction of the recovery. The use of time-dependent simulation can significantly improve the problem of low watercut in the later stage of history matching.

Figure 5. The comparison of recovery
3. Conclusion
This paper has completed the development of a time-dependent numerical simulation program for reservoir physical properties, and introduced the realization ideas, realization process and simulation examples.

The time-dependent simulation is based on the derived empirical formula and calculated by cubic spline interpolation to obtain values at different times to assign values to the permeability field at different time steps. The commercial numerical simulation software ECLIPSE is automatically called for multiple calculations. In the end, it is convenient to use, low research cost, and at the same time, it takes into account faster simulation speed, higher simulation accuracy, and better continuity and directionality of simulation results. The simulation example of this method also shows that the process of reservoir permeability field change can be described finely, and the simulation results are of great significance for the accurate definition of remaining oil in the later stage and optimization of development adjustment programs.

This method can better realize the time-dependent simulation task, but also provides an idea for the development of time-dependent numerical simulation technology.

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