Optimization of gear ratio of variable mechanical gearboxes

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Abstract. The effectiveness of the coordination process of the “engine-transmission” system is largely determined by the functional purpose of the car being designed. The solution to this problem is possible by choosing the optimal transmission parameters. Using a continuously variable transmission will make this process manageable. However, optimization is always performed according to a certain criterion. You can use its dynamic performance and fuel economy as an optimization criterion. The article presents a mathematical model for determining the current gear ratios of a continuously variable transmission of a vehicle depending on its mode of movement. The dependence of the transmission ratio of the vehicle on speed allowed us to find the mode of movement of the car with the minimum acceleration time. The dependence of the transmission ratio of the vehicle on the engine speed allowed us to find the driving mode with minimal fuel consumption. The recommendations received by the authors on the choice of transmission ratios can be used in the design of the control system. Automation of control will allow optimizing the pattern of change in the gear ratio of a continuously variable transmission depending on the mode of movement of the vehicle.

1. Introduction

Consumer requirements for cars are constantly growing. This leads to a variety of existing designs. Ensuring the required performance of the car becomes the main task of design. This can be achieved only with full coordination of the joint work of the engine and transmission [1, 2, 3, 4].

The effectiveness of the process of coordinating the joint work of the engine and transmission is largely determined by the functional purpose of the vehicle. For example, some want to have minimal fuel consumption, and others want to have the best dynamic performance. In this regard, of great interest are studies aimed at unlocking the potential of the “engine-transmission” system.

Mechanical step transmissions have practically exhausted their possibilities of technical improvement today [5]. The emergence on the market of continuously variable transmissions provides the ability to control the movement depending on the priorities set [6, 7, 8, 9]. A variety of designs of modern variable transmission allows [10, 11, 12, 13, 14] making the control process optimal [15, 16, 17, 18]. The result of optimization depends on the selected criterion determined by the mode of movement of the vehicle [19].

The purpose of this study was to develop a mathematical model to determine the optimal gear ratios of the variable transmission, depending on the selected criterion. The article presents the results of optimization of the gear ratio of a continuously variable transmission according to the criterion of dynamism and fuel efficiency.
Mathematical model

The maximum acceleration of the car when the engine is running with full fuel supply is equal to [20, 21].

\[ j = \frac{dv}{dt} = (D - f) \frac{g}{\delta}, \quad (1) \]

where \( j \) is acceleration of the car, \( v \) is speed of the car, \( t \) is acceleration time, \( D \) is the dynamic factor, \( f \) is the road drag coefficient, \( g \) is acceleration of gravity, \( \delta \) is the coefficient of accounting for rotating masses.

The dynamic factor is equal to

\[ D = \frac{P_k - P_w}{mg}, \quad (2) \]

where \( P_k \) is traction force on the wheels, \( P_w \) is air resistance force; \( m \) is the mass of the car.

The traction force on wheels is equal to

\[ P_k = \frac{M_k \eta_{tr}}{R_k}, \quad (3) \]

where \( M_k \) is the engine torque, \( i_{tr} \) is gear ratio of continuously variable transmission, \( \eta_{tr} \) is coefficient of efficiency of transmission, \( R_k \) is the rolling radius of the wheel.

The air resistance force is equal to

\[ P_w = kF v^2, \quad (4) \]

where \( kF \) is the streamlining factor, \( v = \frac{\pi n N}{30 R_k} \) is speed of the car.

The system “Eqs. 1 – 4” is a mathematical model of vehicle movement during acceleration, where the vehicle speed depends on the transmission ratio and engine speed. The task of optimizing the gear ratio of the continuously variable transmission was solved by finding the extremum of the acceleration function “Eq. 1” by some criterion.

2.1. Optimization of vehicle dynamics

Exploring the function find of the vehicle acceleration changes by the speed of its movement at the extremum expression the expression [22, 23] to determine the ratio of the transmission values, depending on the vehicle speed was obtained.

\[ i_{tr} = -2 \sqrt{\frac{0.0628 R_k^2 n_N^2}{\nu^2}} \cos \left( \phi + \frac{2}{3} \pi \right) + \frac{0.1253 R_k n_N}{\nu}, \quad (5) \]

where \( R_k \) is the rolling radius of the wheel, \( n_N \) is rated engine speed.

The quantity \( \phi \) in equation “Eq. 5” is equal to

\[ \phi = \frac{1}{2} \arccos \left( \frac{0.000469 m g v}{N_m \eta_{tr} + k_f (1 + k_1 v^2)} - 0.6875 \right), \quad (6) \]

where \( N_m=9550M_k n_m \) is maximum engine power, \( f_0 \) is coefficient of rolling resistance at low speeds, \( k_1 \) is the empirical coefficient equal to \((4...5) \times 10^{-5}\).

Figure 1 represents the dependence “Eq. 5” of the transmission ratio on the speed of movement on the example of a Subaru Impreza 2.0R car.

Using patterns “Eq. 5” when choosing the gear ratio of a continuously variable transmission allowed the car to achieve maximum speed in the shortest time, i.e. it provides the best dynamics of vehicle acceleration.
2.2. Optimization of vehicle fuel economy
Exploring the function of change of car acceleration through the speed of the motor shaft “Eq. 1” at the extremum [22, 23] the expression to determine the ratio of the transmission values, depending on the engine speed, was obtained.

\[ i_{tr} = \frac{0.1414 R^3 m g n_m^2 (kF/13mg + f_0 k_1)n}{9550 n_m \eta_t (0.5 - n/n_m)} \]

(7)

where \( n_m \) is the maximum engine speed, \( n \) is the current value of the number of revolutions of the engine shaft.

The area of existence of the resulting function “Eq. 7” to determine the ratio of the transmission \( i_{tr} \) is limited by the values of engine speed \( n < 0.5n_m \).

Figure 2 shows, as an example, the proposed regularity of the change of the gear ratio of the variable speed drive depending on the speed of the motor shaft of Subaru Impreza 2.0R.

Using the patterns “Eq. 7” for selecting the gear ratio of a continuously variable transmission, depending on the speed of the motor shaft will allow the car to move with the least fuel consumption.

3. Calculation results
Comparative calculations confirmed the assumptions. We made calculations of acceleration time and vehicle fuel consumption of Subaru Impreza 2.0R with serial mechanical transmissions and serial variable speed drive, whose gear ratios changed during acceleration in the first case, according to the expression “Eq. 5”, and in the second – according to the expression “Eq. 7”. The results are presented respectively in figures 3 and 4.
It is seen from these diagrams that, if during a vehicle acceleration with a continuously variable transmission its gear ratios vary according to the patterns “Eq. 5” and “Eq. 7”, in the first case the vehicle will accelerate to a predetermined velocity in the shortest amount of time, and in the second the fuel flow rate will be minimal. And in fact, in both cases the goal is reached: the acceleration process is controlled and optimal.

4. Conclusion
The pattern of change in the gear ratio of the continuously variable transmission is controlled and optimal depending on the mode of movement of the car. The results of the presented research can be used in the design of modern gearboxes. The modern level of development of automation tools makes it easy to implement the process of choosing the optimal gear ratios in practice.

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