MMSE Symbol Level Precoding Under a Per Antenna Power Constraint for Multiuser MIMO Systems With PSK Modulation

Erico S. P. Lopes®, Graduate Student Member, IEEE, and Lukas T. N. Landau®, Senior Member, IEEE

Abstract—This letter proposes two MMSE symbol-level precoding approaches considering a strict per antenna power constraint and PSK modulation for perfect and imperfect channel state information scenarios. The proposed designs are formulated as second-order cone programs, allowing for an optimal solution via the interior point method. Numerical results confirm that, for the scenario with perfect channel state information, the proposed designs outperform the existing techniques in terms of bit-error-rate for low and intermediate signal-to-noise-ratio. Numerical evaluations also confirm the superiority of the proposed robust MMSE design in the presence of channel state information imperfection.

Index Terms—MMSE, symbol-level precoding, strict per antenna power constraint, MIMO systems, second-order cone programming.

I. INTRODUCTION

MASSIVE multiuser multiple-input multiple-output (MU-MIMO) systems are considered as a promising technology and are expected to be vital for future wireless communications networks [1]. For MU-MIMO systems a fundamental problem is the design of low-complexity precoding algorithms that attain the high reliability constraints of future wireless communications networks.

Linear techniques such as zero forcing (ZF) and matched filtering [2], [3] are known to be asymptotically optimal [4] for massive MIMO systems due to the favorable propagation effects that rise for infinitely large arrays. However, when considering linear precoding, an established assumption in the literature [5], [6] is that the transmit symbols are constrained by an average total power constraint (TPC). This yields a system that is easier to model, yet, according to [7], in a realistic scenario each base station (BS) antenna is connected to its own power amplifier (PA) and thus has to meet its specific power constraints.

With this, several precoding techniques arose considering per antenna power constraints (PAPC). Linear channel-level precoding strategies considering an average PAPC are well studied in the literature [8], [9], [10], [11]. However, according to [12], the consideration of a strict PAPC (SPAPC) yields a more realistic scenario since the transmit power at each antenna is upper bounded by a threshold to avoid severe distortion at the PA due to clipping. With this, different linear precoding techniques have been developed considering SPAPCs [12], [13]. More recently, the symbol-level precoding (SLP) strategy has been receiving increasing attention since it allows for a higher degree of reliability. In [14] a SLP method is devised considering a per antenna transmit power minimization under the condition of attaining quality of service constraints for M-PSK modulations. In [15] SLP is considered with a SPAPC and two novel strategies based on the concept of strict and non strict rotation for constructive interference (CI) based precoding are proposed. SLP is also considered with other constraints, e.g., low-resolution [16], [17], [18] and constant envelope [19].

Besides the aforementioned concepts, one of the most prominent design criteria in the literature is the minimum mean squared error (MMSE). The MMSE utilization ranges from the established channel-level linear precoding strategy presented in [5] to a SLP design considering coarse quantization [20]. Although prominent in the literature to the best of the authors’ knowledge the MMSE objective has not been considered for SLP under a SPAPC.

In this context, by considering a SPAPC this letter proposes two SLP techniques for PSK modulation. While the first method utilizes the MMSE criterion considering perfect channel state information (CSI) at the transmitter, the second approach allows for imperfect CSI scenarios by exploiting knowledge about second order statistics of the CSI mismatch. The proposed approaches are formulated in the standard second-order cone programming (SOCPs) form and are readily solved with polynomial complexity using the interior points method (IPM). Numerical results indicate that the proposed MMSE methods are superior to the existing techniques in terms of BER for the low and medium SNR regime. Moreover, regarding CSI imperfection the proposed robust MMSE (RMMSE) design outperforms the examined state-of-the-art algorithms for all values of CSI mismatch.

The remainder of this letter is organized as follows: Section II describes the system model. Section III exposes the MMSE and RMMSE optimization problems, formulates them as SOCPs and provides the complexity analysis of the proposed algorithms. Section IV presents and discusses numerical results, while Section V gives the conclusions.

Regarding the notation, bold lower case and upper case letters indicate vectors and matrices, respectively. Non-bold letters express scalars. The operator $(\cdot)^T$ denotes transposition. $S_n^+$ denotes the set of symmetric positive semidefinite matrices of dimension $n \times n$. The operator $R(\cdot)$ converts a complex-valued vector into the equivalent real-valued notation. For a given column vector $a \in \mathbb{C}^M$ the equivalent real-valued vector $a_r = R(a)$ reads as

$$a_r = \left[ \text{Re}\{a_1\} \; \text{Im}\{a_1\} \; \cdots \; \text{Re}\{a_M\} \; \text{Im}\{a_M\} \right]^T.$$ (1)
For a given matrix \( A \in \mathbb{C}^{K \times M} \) the equivalent real-valued matrix \( \tilde{A}_r = R(A) \) is given by
\[
\tilde{A}_r = \begin{bmatrix}
\text{Re}\{a_{11}\} & -\text{Im}\{a_{11}\} & \cdots & -\text{Im}\{a_{1M}\} \\
\text{Im}\{a_{11}\} & \text{Re}\{a_{11}\} & \cdots & \text{Im}\{a_{1M}\} \\
\vdots & \vdots & \ddots & \vdots \\
\text{Re}\{a_{K1}\} & -\text{Im}\{a_{K1}\} & \cdots & -\text{Im}\{a_{KM}\} \\
\text{Im}\{a_{K1}\} & \text{Re}\{a_{K1}\} & \cdots & \text{Im}\{a_{KM}\}
\end{bmatrix}.
\]
(2)

The inverse operation is denoted as \( C(\cdot) \) which converts equivalent real-valued notation into complex-valued notation.

II. SYSTEM MODEL

The system model consists of a single-cell MU-MIMO scenario where the BS is equipped with \( M \) transmit antennas serving \( K \) single antenna users. A symbol level transmission is considered where \( s_k \) represents the data symbol to be delivered for the \( k \)-th user. Each symbol \( s_k \) is considered to belong to the set \( S \) that represents all possible symbols of a \( \alpha_s \)-PSK modulation and is given by
\[
S = \left\{ s : s = e^{j\pi(2i+1)/\alpha_s}, \text{for } i = 1, \ldots, \alpha_s \right\}.
\]
(3)
The symbols of all users are described in a stacked vector notation as \( s = [s_1, \ldots, s_K] \in \mathbb{C}^K \). Based on \( s \) the precoder computes the transmit vector \( x = [x_1, \ldots, x_M]^T \). The entries of \( x \) are constrained by a SPAPC, meaning \( |x_m|^2 \leq P_A \) for \( m \in \{1, \ldots, M\} \), where \( P_A \) represents the maximum per antenna transmit power. A frequency flat fading channel described by the matrix \( H \in \mathbb{C}^{K \times M} \) is considered. The BS is considered to receive the CSI coefficients from the users which corresponds to the matrix \( \tilde{H} = \text{Re} \{ H \} \), which implies spatial correlation
\[
E\{ \tilde{H}^H \tilde{H} \} = K R_s.
\]
It is considered that the spatial correlation matrix \( R_s \) can be estimated and is known at the BS. It is considered that the spatial correlation is modeled by the Kronecker model [21], which implies that the entries of \( R_s \) are in the form of \( r_{ij} = \rho^{|i-j|^2} \) for \( i, j \in \{1, \ldots, M\} \). The factor \( \rho \in [0,1] \) is the correlation index of neighboring antennas. In this letter the channel model is described by
\[
H = N \tilde{H} + \sqrt{I - N^2} \Phi R_s^{1/2}.
\]
(4)
The matrix \( N = \text{diag}(\eta) \), with \( \eta = [\eta_1, \ldots, \eta_K] \in [0,1]^K \) describes the user-specific quality of the CSI which can also be interpreted as the temporal correlation factor. It is considered that \( N \) can be estimated and is known at the BS. The matrix \( \Phi \), with \( \Phi_k \sim \mathcal{C}_\mathcal{N}(0, I) \) being the \( k \)-th row of \( \Phi \) for \( k \in \{1, \ldots, K\} \), describes the random part of the channel model. The received signal for all users \( z \) can be described as
\[
z = H x + w,
\]
(5)
with its \( k \)-th entry \( z_k \) being the received signal from the \( k \)-th user. The vector \( w \sim \mathcal{C}_\mathcal{N}(0, \sigma_w^2 I) \) represents additive white Gaussian noise (AWGN). Each received symbol \( z_k \) is detected as \( \hat{s}_k = D(z_k) \), where \( D(\cdot) \) denotes the detected symbol for the \( k \)-th user and \( D(\cdot) \) the hard detection operation.

III. PROPOSED MMSE PRECODING DESIGNS UNDER A STRICT PER ANTENNA POWER CONSTRAINT

In this section, we propose SLP designs based on the MMSE objective under a SPAPC for two different scenarios. In the first scenario perfect CSI is considered, meaning \( \hat{H} = H \). In the second scenario, it is considered that the BS has imperfect CSI and knowledge about the matrices \( \hat{H}, N \) and \( R_s \). The MMSE objective, similar as proposed in [5], can be utilized under a SPAPC with the following problem
\[
\min_{x, \beta} E\{\|\beta z - s\|^2\}
\]
\[\text{s.t. } |x_m|^2 \leq P_A, \text{ for } m \in \{1, \ldots, M\}, \beta \geq 0.\]
(6)
Note that the real-valued factor \( \beta \) represents a theoretical automatic gain control which is part of the established MMSE objective as proposed in [5]. The factor \( \beta \) is computed by the precoder alongside the transmit vector \( x \). Yet, since in this letter PSK modulation is considered, knowledge of \( \beta \) is not required for hard detection.

A. Proposed MMSE SPAPC Design

The MMSE optimization problem from (6) can be rewritten by substituting \( z \) in the objective which yields
\[
\min_{x, \beta} E\{\|\beta H x + \beta w - s\|^2\}
\]
\[\text{s.t. } |x_m|^2 \leq P_A, \text{ for } m \in \{1, \ldots, M\}, \beta \geq 0.\]
(7)
Note that \( x \) is a complex-valued vector and \( \beta \) a real-valued scaling factor. Since most established optimization algorithms consider real-valued variables, the problem from (7) is in the following rewritten in a real-valued notation which yields
\[
\min_{x, \beta} E\{\|\beta H x + \beta w - s\|^2\}
\]
\[\text{s.t. } |x_m|^2 \leq P_A, \text{ for } m \in \{1, \ldots, M\}, \beta \geq 0.\]
(8)
where \( w_r = R(w), s_r = R(s) \) and \( H_r = R(H) \), with the operator \( R(\cdot) \) introduced in (1) and (2). Considering that perfect CSI is available at the BS, i.e., \( H = \hat{H} \), the problem from (8) can be expressed as an equivalent problem with
\[
\min_{x, \beta} \beta^2 x_r^T H_r^T H_r x_r - 2\beta x_r^T H_r s_r + \beta^2 K\sigma_w^2
\]
\[\text{s.t. } \{ |x_m|^2 \leq P_A, \text{ for } m \in \{1, \ldots, M\}, \beta \geq 0.\]
(9)
If \( \beta \geq 0 \) would be constant, the objective would be a convex quadratically constrained quadratic program (QCQP), since \( H_r^T H_r \in \mathbb{S}_{++}^M \). Yet, the objective is in general not jointly convex in \( x \) and \( \beta \). Nevertheless, it can be rewritten as an equivalent convex function by substituting the optimization variable \( x_r \). In this context, we introduce a new optimization variable \( x_m = \beta x_r \), similar as done in [20], [24]. With this, the optimization problem described in (9) can be rewritten as
\[
\min_{x, \beta} x_r^T H_r^T H_r x_r - 2x_r^T H_r s_r + \beta^2 K\sigma_w^2
\]
\[\text{s.t. } |x_m|^2 \leq P_A, \text{ for } m \in \{1, \ldots, M\}, \beta \geq 0.\]
(10)
The problem can be written in matrix form as
\[
\min_{v} v^T U v + p^T v
\]
\[\text{s.t. } \|E_m v\|_2 \leq g^T v, \text{ for } m \in \{1, \ldots, M\}, a^T v \leq 0.\]
(11)
where \( v = [\beta, x_s^T]^T \), \( a = [-1, 0^T]^T \), \( g = [\sqrt{P_L}, 0^T]^T \),
\( p = [0, -2a^T R_s^T H_f]^T \),
\( U = \begin{bmatrix} K \sigma_w^2 & 0 \\ \frac{1}{2} & H_f^2 H_f^T \end{bmatrix} \),
\( E_m = \begin{bmatrix} 0 & 0 \\ 0^T & \text{diag}(d_m) \end{bmatrix} \),
with \( d_m \in \mathbb{R}^{2M \times 1} \) being a vector of zeros with ones at entries \( 2m-1 \) and \( 2m \). Note that, since the root of the quadratic constraint the problem can be rewritten as
\( \min_{x_r, \beta} \quad \beta^2 x_r^T (\hat{H}_f^T N_r^2 H_f + \gamma R_{s,t}) x_r - 2 \beta x_r^T \hat{H}_f^T N_r s_t + \beta^2 K \sigma_w^2 \)
\text{s.t.} \{ |x_r|_{2m-1}^2 + |x_r|_{2m}^2 \leq P_L, \quad \text{for } m \in \{1, \ldots, M\}, \quad \beta \geq 0 \},
where \( \gamma = \text{trace}(I - N_f^2) \). As before, this proposed objective is not jointly convex in \( x_r \) and \( \beta \). Yet, an equivalent convex problem can be cast by substituting \( x_r = \beta x_r' \), which yields
\( \min_{x_r, (x_r')^T} \quad x_r^T (\hat{H}_f^T N_r^2 H_f + \gamma R_{s,t}) x_r - 2 x_r^T \hat{H}_f^T N_r s_t + \beta^2 K \sigma_w^2 \)
\text{s.t.} \{ |x_r|_{2m-1}^2 + |x_r|_{2m}^2 \leq \beta^2 P_L, \quad \text{for } m \in \{1, \ldots, M\}, \quad \beta \geq 0 \}
\text{The problem can be written in matrix form as}
\( \min_v \quad v^T \tilde{U} v + \tilde{p}^T v \)
\text{s.t.} \| E_m v \|_2 \leq g^T v, \quad \text{for } m \in \{1, \ldots, M\},
\| a^T v \|_2 \leq 0. \quad \text{(17)}
\text{where}
\( \tilde{U} = \begin{bmatrix} K \sigma_w^2 & 0 \\ 0^T & \hat{H}_f^T N_r^2 H_f + \gamma R_{s,t} \end{bmatrix} \),
\( \tilde{p} = \begin{bmatrix} 0 \\ -2 \hat{H}_f^T N_r s_t \end{bmatrix} \),
\text{and the other quantities are defined in (12). Note that, since}
\( \tilde{U} \in \mathbb{S}_+^{2M + 1} \text{ the problem is convex. By following the same steps utilized in the Section III-A one can write the problem described in (17) as the following SOCP} \)
\( \min_{u} \quad \tilde{r}^T u \)
\text{s.t.} \| F_m u \|_2 \leq l^T u, \quad \text{for } m \in \{1, \ldots, M\},
\| o^T u \|_2 \leq 0, \quad \text{(18)}
\text{where}
\( \tilde{r} = \begin{bmatrix} \tilde{p}^T 0 \\ 0^T \end{bmatrix}, \quad \tilde{G} = \begin{bmatrix} L & 0 \\ 0^T & 1 \end{bmatrix}, \quad \tilde{L} = \tilde{U}^2 \quad \text{(19)}
\text{and the other quantities are defined in (16). As before the problem described in (18) is a SOCP, cf. [22, Sec. 4.4.2], and can be readily solved with IPM. The solution can be converted back to complex-valued notation by extracting \( x_s \) and \( \beta \) from} \( u_{\text{opt}} \) \text{and applying} \( x = C(\tilde{x}_{\tilde{p}}) \).
\text{Note that, the MSE associated to the solution of (17) is lower bounded by} \( \text{MSE}(s_t) = K - s_t^2 [N_f^2 H_f^2 + \gamma R_{s,t})^{-1} H_f^T N_r s_t] \).
\text{This MSE bound, which is greater than zero due to the CSI imperfection, is computed by considering the unconstrained version of (17).}

**C. About the Complexity of the Proposed Designs**

As mentioned the MMSE and the RMMSE optimization problems are SOCPs and thus can be solved via IPM. According to [25], the number of iterations of the primal-dual IPM can be upper bounded by \( \sqrt{n} \log(n/e_{tol}) \) where \( n \) is the number of variables and \( e_{tol} \) is the predefined optimality tolerance. Note that, the complexity of the iterations is dominated by solving a linear system needed to compute the primal-dual search direction. With this, considering that the linear systems can be solved with complexity \( O(n^3) \) via Gauss-Jordan elimination, the total complexity of the proposed approaches can be upper bounded by \( O(M^{3.5} \log(M/e_{tol})) \).
IV. NUMERICAL RESULTS

In this section, the proposed precoders are evaluated in terms of BER and computational complexity and compared with other state-of-the-art designs. To this end, the SNR is defined as \( \text{SNR} = \frac{M \cdot P_A}{\sigma_w^2} \). The proposed methods are evaluated against the following state-of-the-art approaches: 1- The ZF SPAPC precoder [15]; 2- The CVX-CIO precoder [7] designed for constant envelope; 3- The Strict CI SPAPC precoder [15]; 4- The Non-Strict CI SPAPC precoder [15] and 5- The LMMSE precoder [5] (average TPC).

A. BER Evaluation Under Perfect CSI

In this section, a BER \( \times \text{SNR} \) evaluation is considered assuming no spatial correlation, i.e., \( \rho = 0 \) and perfect CSI. In this context, the considered MIMO scenario consists of a BS with \( M = 15 \) antennas serving \( K = 15 \) users with QPSK user symbols, meaning that \( \alpha_s = 4 \). As can be seen in Fig. 1, the proposed methods outperform the existing approaches in terms of BER for the low and intermediate SNR regimes. For high-SNR, the proposed MMSE precoders outperform all investigated approaches except for the Non-Strict CI-based precoder [15]. This is expected since it is known that CI is nearly optimal for high SNR [26] and the MMSE criterion is favorable for low and medium SNR [20].

B. BER Evaluation Under Imperfect CSI

In this section, the proposed approaches are evaluated in terms of BER with CSI imperfection. The evaluated MIMO scenario consists of a BS with \( M = 50 \) antennas which serves \( K = 5 \) users with \( \alpha_s = 8 \). To facilitate the analysis during this section it is considered \( \eta = \xi_1 \), meaning that all channels have the same CSI quality. The CSI imperfection is then expressed in terms of \( \lambda^2 = \sqrt{1 - \xi^2} \).

The first experiment consists of a BER performance evaluation for different levels of CSI imperfection under an SNR of 12 dB. For this experiment no spatial correlation is considered, meaning \( \rho = 0 \). As can be seen in Fig. 2 the proposed RMMSE SPAPC design outperforms all other examined SPAPC state-of-the-art approaches for \( \lambda^2 > 0 \). Moreover, the proposed RMMSE approach yields similar performance in terms of BER as the LMMSE [5] (average TPC) design for very low CSI quality (\( \lambda^2 > 0.8 \)).

The second experiment consists of a BER performance evaluation against the spatial correlation factor \( \rho \) for SNR = 12 dB and \( \lambda^2 = 0.2 \). As shown in Fig. 3, the proposed RMMSE approach outperforms all other SPAPC designs for all examined \( \rho \). Moreover, it also outperforms the LMMSE design for \( \rho > 0.5 \).

Finally, the third experiment consists of a BER \( \times \text{SNR} \) evaluation considering both imperfect CSI and spatial correlation with the parameters \( \lambda^2 = 0.2 \) and \( \rho = 0.15 \). As can be seen in Fig. 4, the proposed RMMSE precoder outperforms all other SPAPC approaches in terms of BER. Note that, both proposed MMSE and RMMSE approaches yield similar performance for low SNR. Starting from medium SNR, as the SNR grows, the proposed RMMSE approach deviates in performance from the proposed MMSE counterpart. Finally, for very high SNR...
(SNR > 27.5 dB) the proposed RMMSE approach shows significant advantage and outperforms also the LMMSE method (average TPC).

C. Complexity Analysis

As discussed in Section III-C the complexity of the proposed methods is upper bounded by $O(\epsilon^3)$, where $\epsilon$ can be solved via the primal-dual IPM with complexity $CI SPAPC$ [15]). Table I summarizes the complexity of the considered approaches.

Note that, the optimization based state-of-the-art algorithms (namely CVX-CIO [7], Strict CI SPAPC [15] and Non-Strict CI SPAPC [15]) can be transformed in standard form SOCPs which can be solved via the primal-dual IPM with complexity $O(\epsilon^3)$. With this, it can be concluded that these approaches yield similar complexity as the proposed methods.

V. CONCLUSION

This letter proposes two symbol-level precoding approaches considering a SPAPC and PSK modulation for perfect and imperfect CSI. The proposed precoding designs are formulated as SOCPs and are solved using the IPM in polynomial time. Numerical results confirm that for the perfect CSI scenario the proposed designs are superior to the existing techniques in terms of BER for low and intermediate SNR. Moreover, when considering imperfect CSI numerical evaluations underline the superiority of the proposed robust MMSE design.

REFERENCES

[1] L. U. Khan, I. Yaqoob, M. Imran, Z. Han, and C. S. Hong, “6G wireless systems: A vision, architectural elements, and future directions,” *IEEE Access*, vol. 8, pp. 147029–147044, 2020.
[2] A. Kammoun, A. Müller, E. Björnson, and M. Debbah, “Linear precoding based on polynomial expansion: Large-scale multi-cell MIMO systems,” *IEEE J. Sel. Areas Commun.*, vol. 8, no. 5, pp. 861–875, Oct. 2014.
[3] J. Hoydis, S. ten Brink, and M. Debbah, “Massive MIMO in the UL/DL of cellular networks: How many antennas do we need?” *IEEE J. Sel. Areas Commun.*, vol. 31, no. 2, pp. 160–171, Feb. 2013.
[4] F. Rusek et al., “Scaling up MIMO: Opportunities and challenges with very large arrays,” *IEEE Signal Process. Mag.*, vol. 30, no. 1, pp. 40–60, Jan. 2013.
[5] M. Joham, W. Utschick, and J. A. Nossek, “Linear transmit processing in MIMO communications systems,” *IEEE Trans. Signal Process.*, vol. 53, no. 8, pp. 2700–2712, Aug. 2005.
[6] C. B. Peel, B. M. Hochwald, and A. L. Swindlehurst, “A vector-perturbation technique for near-capacity multi-antenna multiuser communication-part I: Channel inversion and regularization,” *IEEE Trans. Commun.*, vol. 53, no. 1, pp. 195–202, Jan. 2005.
[7] P. V. Amadori and C. Masouros, “Constant envelope precoding by interference exploitation in phase shift keying-modulated multiuser transmission,” *IEEE Trans. Commun.*, vol. 16, no. 1, pp. 538–550, Jan. 2017.
[8] W. Yu and T. Lan, “Transmitter optimization for the multi-antenna downlink with per-antenna power constraints,” *IEEE Trans. Signal Process.*, vol. 55, no. 6, pp. 2646–2660, Jun. 2007.
[9] F. Boccardi and H. Huang, “Zero-forcing precoding for the MIMO broadcast channel under per-antenna power constraints,” in *Proc. IEEE 7th Workshop Signal Process. Adv. Wireless Commun.* (SPAWC), 2006, pp. 1–5.
[10] K. Karakayali, R. Yates, G. Foschini, and R. Valenzuela, “Optimum zero-forcing beamforming with per-antenna power constraints,” in *Proc. IEEE Int. Symp. Inf. Theory*, 2007, pp. 101–105.
[11] C. Feng and Y. Jing, “Modified MRT and outage probability analysis for massive MIMO downlink under per-antenna power constraint,” in *Proc. IEEE 17th Workshop Signal Process. Adv. Wireless Commun.* (SPAWC), 2016, pp. 1–6.
[12] C.-E. Chen, “MSE-based precoder designs for transmitter-preprocessing-aided spatial modulation under per-antenna power constraints,” *IEEE Trans. Veh. Technol.*, vol. 66, no. 3, pp. 2879–2883, Mar. 2017.
[13] Z. Pi, “Optimal transmitter beamforming with per-antenna power constraints,” in *Proc. IEEE Int. Conf. Commun.*, (ICC), 2012, pp. 3779–3784.
[14] D. Sano, M. Alodeh, S. Chatzinotas, and B. Ottersten, “Per-antenna power minimization in symbol-level precoding,” in *Proc. IEEE Global Commun. Conf.* (GLOBECOM), 2016, pp. 1–6.
[15] C.-E. Chen, “Computationally efficient constructive interference precoding for PSK modulations under per-antenna power constraint,” *IEEE Trans. Veh. Technol.*, vol. 69, no. 8, pp. 9206–9211, Aug. 2020.
[16] H. Jeddah, A. Mezghani, A. L. Swindlehurst, and J. A. Nossek, “Quantized constant envelope precoding with PSK and QAM signaling,” *IEEE Trans. Wireless Commun.*, vol. 17, no. 12, pp. 8022–8034, Dec. 2018.
[17] L. T. N. Landau and R. C. de Lamare, “Branch-and-bound precoding for multiuser MIMO systems with 1-bit quantization,” *IEEE Wireless Commun. Lett.*, vol. 6, no. 6, pp. 770–773, Dec. 2017.
[18] E. S. P. Lopes and L. T. N. Landau, “Optimal precoding for multiuser MIMO systems with phase quantization and PSK modulation via branch-and-bound,” *IEEE Wireless Commun. Lett.*, vol. 9, no. 9, pp. 1393–1397, Sep. 2020.
[19] F. Liu, C. Masouros, P. V. Amadori, and H. Sun, “An efficient manifold algorithm for constructive interference based constant envelope precoding,” *IEEE Signal Process. Lett.*, vol. 24, no. 10, pp. 1542–1546, Oct. 2017.
[20] E. S. P. Lopes and L. T. N. Landau, “Discrete MMSE precoding for multiuser MIMO systems with PSK modulation,” *IEEE Trans. Wireless Commun.*, early access, May 3, 2022, doi: 10.1109/TWC.2022.3170277.
[21] D.-S. Shiu, G. J. Foschini, M. J. Gans, and J. M. Kahn, “Fading correlation and its effect on the capacity of multielement antenna systems,” *IEEE Trans. Commun.*, vol. 48, no. 3, pp. 502–513, Mar. 2000.
[22] S. Boyd and L. Vandenberghe, *Convex Optimization*. New York, NY, USA: Cambridge Univ. Press, 2004.
[23] E. S. P. Lopes and L. T. N. Landau, “Optimal and suboptimal MMSE precoding for multiuser MIMO systems with phase quantization and PSK modulation,” in *Proc. 24th Int. ITG Workshop Smart Antennas (WSA)*, 2020, pp. 1–6.
[24] S. Jacobsson, G. Durisi, T. Oldroyd, T. Goldstein, and C. Studer, “Quantized precoding for massive MU-MIMO,” *IEEE Trans. Commun.*, vol. 65, no. 11, pp. 4670–4684, Nov. 2017.
[25] J. Peng, C. Roos, and T. Terlaky, “New complexity analysis of the primal—Dual Newton method for linear optimization,” *Ann. Oper. Res.*, vol. 99, pp. 23–39, Dec. 2000.
[26] H. Jeddah, J. A. Nossek, and A. Mezghani, “Minimum BER precoding in 1-bit massive MIMO systems,” in *Proc. IEEE Sensor Array Multichannel Signal Process. Workshop (SAM)*, Jul. 2016, pp. 1–5.