Relativistic Outflows from a GRMHD Mean-field Disk Dynamo

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Abstract

In this work, we present simulations of thin accretion disks around black holes, in order to investigate a mean-field disk dynamo, using our resistive GRMHD code, which is able to produce a large-scale magnetic flux. We consider a weak seed field in an initially thin disk, a background (turbulent) magnetic diffusivity, and the dynamo action itself. A standard quenching mechanism is applied to mitigate an otherwise exponential increase in the magnetic field. Comparison simulations of an initial Fishbone–Moncrief torus suggest that reconnection may provide another quenching mechanism. The dynamo-generated magnetic flux expands from the disk interior into the disk corona, becomes advected by disk accretion, and fills the axial region of the domain. The dynamo leads to an initially rapid increase in magnetic energy and flux, while for later evolutionary stages the growth stabilizes. Accretion toward the black hole depends strongly on the type of magnetic-field structure that develops. The radial field component supports extraction of angular momentum, and thus accretion. It also sets the conditions for launching a disk wind, initially from the inner disk area. When a strong field engulfs the disk, strong winds are launched, predominantly driven by the pressure gradient of the toroidal field. For rotating black holes, we identify a Poynting flux-dominated jet, driven by the Blandford–Znajek mechanism. This axial Poynting flux is advected from the disk, and therefore accumulates at the expense of the flux carried by the disk wind, which is itself regenerated by the disk dynamo.

Unified Astronomy Thesaurus concepts: Magnetohydrodynamics (614); Galaxy accretion disks (562); General relativity (641); Black holes (162); Galaxy jets (601)

1. Introduction

One of the fundamental prerequisites for the launching of astrophysical jets is the support of a strong magnetic field. A few mechanisms have been proposed to explain the launching, acceleration, and collimation of these outflows, all of them requiring the additional existence of an accretion disk orbiting a central object (star or black hole). These processes focus either on the jet’s launching from the surface of an accretion disk via magneto-centrifugal acceleration (Blandford & Payne 1982; Uchida & Shibata 1985; Pudritz & Norman 1986) or being driven by a magnetic pressure gradient (Lynden-Bell 1996), or from a black hole ergosphere (Blandford & Znajek 1977; Komissarov 2005), but commonly rely on a substantial magnetic field strength and a favorable magnetic field geometry.

Based on these theoretical ideas, a number of magnetohydrodynamic (MHD) codes have been developed, using relativistic gravity, in order to simulate the rotation of accretion disks around a black hole, and subsequent jet launching (Koide et al. 1999; Gammie et al. 2003; De Villiers & Hawley 2003; Noble et al. 2006; Del Zanna et al. 2007; Porth et al. 2017; Ripperda et al. 2019). Jet-launching simulations in the non-relativistic limit were pioneered by Casse & Keppens (2002) and have subsequently been further developed (Zanni et al. 2007; Murphy et al. 2010; Sheiknazami et al. 2012; Stepanovs & Fendt 2014, 2016). The usual practice in these works is to apply a strong magnetic field as part of the initial conditions. However, the strength and shape of the field may express an unrealistic view of the actual physical case. Only after the disk-field system has evolved can we say that we have approached a more authentic view of a system. Naturally, there is the downside that the origin and the ab initio evolution of the jet-launching magnetic field is practically unknown.

While for protostellar jets, and jets from compact stars, the jet-launching magnetic field could in principle be induced by a strong stellar dipolar field (if not advected from the interstellar medium), this option does not exist for supermassive black holes. In an active galactic nucleus (AGN), the jet-driving magnetic field must therefore be generated by some kind of accretion disk dynamo mechanism, or be advected from the surrounding medium. The situation may be different for stellar mass black holes formed by mergers of magnetized neutron stars. Moreover, the environment of stellar mass black holes resulting from core collapse supernova explosions could be highly magnetized (Cerdá-Durán et al. 2007; Obergaulinger et al. 2018; Matsumoto et al. 2020).

Early studies have derived the structure of black hole magnetospheres via the stationary approach (Fendt 1997; Ghosh 2000; Fendt & Greiner 2001) (but see also Pan et al. 2017; Mahlmann et al. 2018; Pu & Takahashi 2020), as well as via jet acceleration (Fendt & Greiner 2001; Vlahakis & Königl 2001; Tomimatsu & Takahashi 2003). Such steady-state modeling of black hole magnetospheres could be very useful as a basis from which to study particle acceleration and radiation (Takahashi et al. 2018; Rieger 2019). While steady-state solutions are exact solutions of the physical equations, their stability remains unclear. The same holds for the origin of the magnetic field structure under consideration.

Accretion-disk turbulence is strongly believed to be generated by instabilities such as magneto-rotational instability (MRI) (Balbus & Hawley 1991, 1998). At the same time, MRI is known to amplify the magnetic field, giving rise to a turbulent dynamo effect. This has typically been investigated using high-resolution, ideal MHD simulations (see e.g., Stone et al. 1996; Gressel & Pessah 2015), sometimes referred to as direct dynamo simulations, as the dynamo effect is acting ab initio and with no further prescriptions, i.e., solely based on
the turbulent motions of the gas as soon as a weak seed field is present.

Under certain conditions, these small-scale fluctuations of velocity and magnetic field can be subject to a non-linear coupling, which then effectively represents a non-ideal MHD mechanism that amplifies the magnetic field on larger scales. This mechanism is known as the mean-field dynamo (Parker 1955; Steenbeck & Krause 1969a, 1969b), and is introduced in the MHD induction equation as a non-ideal term. Similarly, small-scale turbulence effectively results in a turbulent resistivity (or diffusivity) in the induction equation. These small-scale turbulent motions may be averaged, resulting in a turbulent pattern on larger scales, typically leading to much stronger coefficients for both mean-field dynamo effect and resistivity, as compared to small-scale values.

The mean-field dynamo theory is a powerful tool with which to derive the large-scale field structure of astrophysical objects. In the limit of a kinematic dynamo, the velocity field is prescribed, and remains constant, while for larger magnetic-field strengths, feedback from the field on the flow dynamics is expected, and the non-linear dynamo theory must be considered.

Overall, it is still not fully clear whether such turbulent processes will result in the generation of the well-ordered large-scale magnetic field required to launch an outflow. Direct high-resolution simulations typically demonstrate the saturation of the turbulent dynamo by showing the magnetic energy evolution. However, these do not provide an estimate for the production of large-scale magnetic fields. What is interesting with respect to jet launching, however, is the generation of a large-scale magnetic flux. This has been achieved in simulations applying the mean-field ansatz in non-relativistic studies (see below). This paper aims to extend these studies into the GRMHD context.

With regard to the application of the mean-field dynamo theory for accretion disk and jet simulations, the literature is rather sparse. Dynamos were actually suggested for the purpose of generating the magnetic fields required to launch jets. Pudritz (1981a, 1981b) applied the $\alpha$-dynamo theory for the case of thin accretion disks, demonstrating the growth of a magnetic field by means of the differential rotation of the disk (the $\Omega$-effect), together with the net effect of turbulent motions (the $\alpha$-effect) in a vertically stratified turbulent disk.

Simulations of magnetized shear flows by Brandenburg et al. (1995) demonstrated how a dynamo-generated magnetic field amplifies turbulence in the flow, which, in turn, can amplify the magnetic field via a dynamo process. Simulations of outflow launching from a mean-field dynamo-active accretion disk were first presented by von Rekowski et al. (2003) and von Rekowski & Brandenburg (2004). In Pariev et al. (2007) a possible origin of the dynamo mechanism in the case of AGN accretion flows was suggested, i.e., its being triggered by a passing star, which heats up and perturbs the magnetic field inside the accretion disk, altogether resulting in the induction of a toroidal field into a poloidal magnetic flux.

Stepanovs et al. (2014) and Fendt & Gaßmann (2018) presented simulations of jet launching from an accretion disk, where the jet-driving magnetic field was self-generated by a mean-field disk dynamo. Different magnetic-field structures, and therefore jet parameters, were obtained by exploring mean-field dynamos of different strengths. In addition, by switching the $\alpha$-dynamo mechanism off and on (externally), a periodic ejection of jets could be simulated. Furthermore, researchers found that for mean-field dynamos considering a strong $\alpha$, oscillating dynamo modes may occur, resulting in pulsating ejections into the jet (see also Dyda et al. 2018).

We now turn to mean-field dynamo models in the relativistic context. The existence of a gravito-magnetic dynamo effect was proposed (Khanna & Camenzind 1996a); however, this could not be realized in subsequent studies (Brandenburg 1996; Khanna & Camenzind 1996b). Fully dynamical GRMHD simulations of mean-field dynamos have only recently been considered. Applying GRMHD, Sadowski et al. (2015) parameterized the effect of a mean-field dynamo in combination with radiative transfer. However, their approach was in an ideal MHD regime, which neglected magnetic diffusivity. Moreover, a simplified dynamo model was used, whereby the dynamo action appears as a small numerical correction for the evolution of a poloidal magnetic field, driving it toward a saturated state. They applied their code to investigate the evolution of thick disks at different accretion rates.

The first fully covariant implementation of a dynamo closure in a general relativistic context was presented by Bucciantini & Del Zanna (2013), following the $3+1$ formalism method. A mean-field dynamo was implemented in the $\text{ECHO}$ code (Del Zanna et al. 2007; Bucciantini & Del Zanna 2011), which was later applied to simulations of kinematic dynamo action in tori around black holes, demonstrating the growth of toroidal and poloidal field components (Bugli et al. 2014). Recently, Tomei et al. (2020) have extended these studies by simulating fully dynamical cases, including dynamo quenching.

In this paper, we aim for a broader description of the mean-field dynamo in GRMHD simulations. In comparison to the works mentioned above, we consider the following points in particular:

(i) Compared to Bugli et al. (2014), in our paper we do consider the feedback of the magnetic field on the dynamics of the system. A fully dynamical MHD study offers major advantages over a kinematic study, and is particularly essential with respect to disk accretion and jet launching.

(ii) Compared to Tomei et al. (2020), who consider dynamical feedback in terms of fields, and who also discuss the distribution of poloidal field energy, we will investigate the geometry of the generated poloidal field in detail, including the overall structure of the magnetic-field lines. This type of analysis was not performed by Bugli et al. (2014).

(iii) Compared to Tomei et al. (2020), we also show the evolution of hydrodynamical quantities. This is essential to any discussion of disk accretion and jet launching.

Here, we apply our resistive GRMHD code, $\text{eHARM3D}$ (Qian et al. 2017, 2018; Vourellis et al. 2019), and run a series of fully dynamical simulations with an accretion disk mean-field dynamo. In particular, we aim to demonstrate the growth of the poloidal magnetic field, and thus the generation of the large-scale magnetic flux required to launch winds or jets from the accretion disk.

Our paper is structured as follows: in Section 2 we review the basic GRMHD equations, discuss the mean-field dynamo including its implementation as well as its quenching. In Section 3 we apply the dynamo effect for the setup of a relativistic torus, which allows us to test our results in
comparison with those in the existing literature. In Section 4 we detail the initial conditions for our setup of a thin-disk mean-field dynamo. In Section 5 we present our simulation results, discussing the evolution of the magnetic field and the accretion disk, the mass fluxes, and the launching of disk winds and relativistic jets. In Section 6 we summarize our work.

2. Theoretical Background

In this section, we briefly review the basic equations of resistive GRMHD as a basis for our calculations. For the metric we adopt the signature (-, +, +, +) (Misner et al. 1973), and apply geometrized units, G = c = 1. Thus, length scales are expressed in units of the gravitational radius \( R_g = GM/c^2 \), while time is measured in units of light travel time, \( t_g = GM/c^2 \). Vector quantities are written using bold letters, while vector and tensor components are indicated by their respective indices, with Greek letters running for 0,1,2,3 (\( r, \theta, \phi \)), and Latin letters running for 1,2,3 (\( r, \theta, \phi \)).

The usual 3 + 1 decomposition for GRMHD is used in order to separate the time component from the spatial components (three-dimensional manifolds). Spacetime is described using the metric \( g_{\mu\nu} \) in Kerr–Schild coordinates, with \( g = det(g_{\mu\nu}) \). A zero angular momentum observer frame (ZAMO) exists in the spacelike manifolds, moving only in time with the velocity, \( n_{\mu} = (-\alpha, 0, 0, 0) \), where \( \alpha = 1/\sqrt{1-g^{00}} \) is the lapse function. The gravitational shift is \( \beta^i = \alpha^2 g^{0i} \).

In our simulations we apply our resistive code, rHARM3D (Vourellis et al. 2019), now extended to include a turbulent mean-field dynamo, following the work of Bucciantini & Del Zanna (2013).

2.1. Basic GRMHD Equations

The Maxwell equations in covariant form,
\[
\nabla_\nu F^{\mu\nu} = 0, \quad \nabla_\nu T^{\mu\nu} = J^\mu, \quad (1)
\]
along with the conservation of the stress-energy tensor
\[
\nabla_\nu T^{\mu\nu} = 0, \quad (2)
\]
describe the motion of a magnetized fluid in a general relativistic environment. \( J^\mu \) is the 4-current that satisfies the electric charge conservation \( \nabla_\nu J^\nu = 0 \), \( T^{\mu\nu} \) is the stress-energy tensor, and \( \nabla_\nu \) is the covariant derivative. The electromagnetic field is described by the Faraday and Maxwell tensors,
\[
F^{\mu\nu} = u^\mu e^\nu - e^\mu u^\nu + \epsilon^{\mu\nu\alpha\beta} u_\alpha b_\beta, \\
F^{\mu\nu} = -u^\mu b^\nu + b^\mu u^\nu + \epsilon^{\mu\nu\alpha\beta} u_\alpha e_\beta, \quad (3)
\]
respectively, where \( u^\mu \) is the 4-velocity, and \( e^\mu \) and \( b^\mu \) express the electric and magnetic field in the fluid rest-frame. The Levi-Civita symbol, \( \epsilon^{\mu\nu\alpha\beta} \), is defined as
\[
\epsilon^{\alpha\beta\gamma\delta} = \sqrt{-g} [\alpha \beta \gamma \delta], \\
\epsilon^{\alpha\beta\gamma\delta} = -\frac{1}{\sqrt{-g}} [\alpha \beta \gamma \delta], \quad (4)
\]
where \([\alpha \beta \gamma \delta]\) are the conventional permutation symbols. The magnetic and electric field, as measured by the zero angular momentum observer (ZAMO), are defined as \( B^\mu = -n_\nu F^{\nu\mu} \) and \( \mathcal{E}^i = n_i F^{0\mu} \), with \( \mathcal{B}^i = \alpha F^{i0} \), \( \mathcal{E}^i = -\alpha F^{0i} \), and \( B^0 = \mathcal{E}^0 = 0 \). The stress-energy tensor, \( T^{\mu\nu} \), can be split into a fluid and an electromagnetic component. The fluid component is
\[
T^{\mu\nu}_{\text{fluid}} = (\rho + u + p)u^\mu u^\nu + pg^{\mu\nu}, \quad (5)
\]
where \( \rho \) is the mass density, \( u \) is the internal energy density, and \( p \) is the thermal pressure. Pressure and internal energy are connected via the equation of state for an ideal gas, with
\[
\rho = \frac{p}{\Gamma - 1}, \quad (6)
\]
where \( \Gamma \) is the polytropic exponent. The electromagnetic component of the stress-energy tensor is
\[
T^{\mu\nu}_{\text{EM}} = (b^2 + e^2)(u^\mu u^\nu + g^{\mu\nu}) - b^\mu b^\nu - e^\mu e^\nu - u_\alpha e_\beta (u^\mu e^{\mu\beta\gamma} + u^\nu e^{\nu\beta\gamma}) \quad (7)
\]
Equations (1) and (2), along with (6), are solved by our code as a system of hyperbolic differential equations. For a more detailed analysis of the equations and their numerical implementation and solution we refer the reader to Bucciantini & Del Zanna (2013); Qian et al. (2017), and Vourellis et al. (2019).

2.2. The Mean-field Dynamo

In ideal MHD, the electric field vanishes in the co-moving frame, which is expressed as
\[
E + v \times B = 0, \quad (8)
\]
where \( v, E, \) and \( B \) are the velocity and the electric and magnetic field, respectively. In the more general case of non-ideal MHD, Ohm’s law is given by
\[
\sigma E' = \sigma (E + v \times B) = J, \quad (9)
\]
where \( E' \) is the co-moving electric field, \( J \) is the electric current, and \( \sigma \) is the electric conductivity.

In the mean-field ansatz, the turbulent fluctuations of velocity and magnetic field lead to an averaged, mean electromotive force,
\[
\frac{\partial B}{\partial t} = \nabla \times (v \times B) + \nabla \times \alpha B - \nabla \times \eta J, \quad (10)
\]
which subsequently enters the induction equation. Here, \( \alpha_D \) describes the term responsible for the generation of a magnetic field by turbulent motion, the so-called \( \alpha \)-effect, while the \( \beta_d \) provides an additional resistive term, which enhances the dissipation of the magnetic field by an enhanced turbulent diffusivity, in addition to the ohmic resistivity (Parker 1955; Steinbeck & Krause 1969a, 1969b).

We can thus write the induction equation as
\[
\frac{\partial B}{\partial t} = \nabla \times (v \times B) + \nabla \times \alpha B - \nabla \times \eta J, \quad (11)
\]
where we now have defined \( \alpha \equiv \alpha_D \), and the magnetic diffusivity, \( \eta \), considering both turbulent and ohmic components. Renaming the coefficients also emphasizes the fact that the mean-field dynamo theory starts from the ideal MHD when considering the correlated turbulent fluctuations, but having performed an averaging of the electromotive force term, an extra term for the non-ideal treatment of MHD appears.

The first term on the right-hand side of the induction equation represents a kinematic term that may induce a poloidal
magnetic field from a toroidal motion, the so-called $\Omega$-effect. Combined with the action of the $\alpha$-effect, this may provide a dynamo cycle that induces a poloidal field from a toroidal field (the $\alpha$-effect), which then induces a toroidal field from the poloidal component (the $\Omega$-effect), and so forth—the so-called $\alpha\Omega$-dynamo. Note that also the $\alpha$-effect may induce a toroidal field from a poloidal component. This is particularly important in relation to stellar dynamos with small shear in the toroidal motion, thereby forming an $\alpha^2$ dynamo. However, for strong shear, e.g., in accretion disks following a Keplerian rotation, or in relativistic tori, the $\Omega$-effect will dominate the $\alpha\Omega$-dynamo with respect to the induction of the toroidal field component.

In a more general definition of the mean-field electromagnetic force, $\alpha_D$ and $\beta_D$ will represent tensors (Moffatt 1978). However, for most applications (see e.g., Stepanov et al. 2014; Fendt & Gaßmann 2018; Dyda et al. 2018), including in this paper, they are treated as scalar functions. An exception is the work of Mattia & Fendt (2020a, 2020b) who perform simulations of dynamo action and jet launching considering an anisotropic dynamo alpha. Note that if $\alpha$ and $\eta$ are spatially constant, they can be moved in front of the curl operator in Equation (11).

Here, we briefly refer back to the direct dynamo simulations mentioned in the introduction. Those simulations, applying high resolution, may actually resolve the turbulence pattern in the MHD flow and thus perform the averaging Equation (10), providing an $\alpha_D$ and $\beta_D$ that can be than used for simulations in the mean-field approach. As an example, we cite Gressel (2010), who applied accretion disk shearing-box simulations in order to derive profiles for both magnetic diffusivity and dynamo coefficients. As a complement to the mean-field approach, direct simulations serve as subgrid modeling, in that they can deliver mean-field quantities from a subgrid scale.

2.3. The GRMHD Mean-field Dynamo Equations

The implementation of the mean-field dynamo mechanism builds on our previous works, in which we integrated resistivity, in the form of turbulent diffusivity, into the original HARM code, establishing our resistive GRMHD code, xHARM3D (Qian et al. 2017, 2018; Vourellis et al. 2019).

For our GRMHD treatment, we follow the closure relation introduced by Bucciantini & Del Zanna (2013). Here, the mean-field $\alpha$-dynamo parameter is replaced by $\xi = -\alpha$, to avoid confusion with the gravitational lapse. Thus, in covariant form, and in the fluid frame, Ohm’s law is written as

$$e^\mu = \eta v^\mu + \xi b^\mu,$$

(12)

where $j^\mu$ is the 4-vector of the electric current density. Now, the electric field can no longer be calculated by the cross-product of fluid velocity and magnetic field, and new equations need to be formulated (see Section 2.4).

By setting $\xi = 0$ we get the resistive version of Ohm’s law ($e^\mu = \eta j^\mu$). By setting $\eta = 0$ we get back to the ideal MHD case, $e^\mu = 0$.

In order to derive the equations for the evolution of the magnetic and electric field, we begin by taking each of the temporal and spatial projections in Equation (1) separately. This provides the divergence condition $\partial_t (\gamma^{1/2} B^i) = 0$, and an equation for the time evolution of the magnetic field,

$$\gamma^{-1/2} \partial_t (\gamma^{1/2} B^i) = \gamma^{1/2} \partial_i \mathcal{E} + \gamma^{1/2} \mathcal{E} \partial^i,$$

along with Gauss’ law for the electric field, $\partial_j (\gamma^{1/2} \mathcal{E}^j) = 0$, and the time evolution of the electric field,

$$\gamma^{-1/2} \partial_t (\gamma^{1/2} \mathcal{E}^i) = \gamma^{1/2} \partial_i \mathcal{E}^\alpha + \epsilon_{kAlpha} \mathcal{E}^\beta B^\alpha B^\beta,$$

where $\gamma$ is the determinant of the spatial metric, $\gamma_{\mu\nu} = g_{\mu\nu} + n_{\mu} n_{\nu}$. From Ohm’s law, Equation (12), we obtain

$$E^\mu = \eta i^\mu + \zeta \mathcal{B}^\nu v_i,$$

(16)

and from the spatial projection of Equation (12) we obtain an expression for the electric current

$$\mathcal{E}^i = \eta (J^i - q v^i) + \zeta \mathcal{B}^\nu v_i,$$

(17)

which we can now use to replace the source terms in Equation (14), resulting in the following evolution equation for the electric field:

$$\gamma^{-1/2} \partial_t (\gamma^{1/2} \mathcal{E}^i) = \gamma^{1/2} \partial_i \mathcal{E}^\alpha + \epsilon_{kAlpha} \mathcal{E}^\beta B^\alpha B^\beta,$$

(18)

Discretizing the time evolution of the electric field leads, after some “minor” algebraic calculations, to

$$\mathcal{E} \left[ \frac{\eta + \xi (1 - \frac{1}{\eta})}{\eta + \xi} \right] =$$

$$-e^{-q v_i} \frac{Q^i + (Q^j \tilde{\nabla}_j)}{(\Gamma \tilde{\nabla} + 1)}$$

$$+ \zeta \left( \mathcal{B}^\nu - \frac{\tilde{\nabla}_i}{\Gamma \tilde{\nabla} + 1} \right),$$

$$-\zeta \left( \frac{\eta e^{-q v_i} \tilde{\nabla}_i + (\Gamma^2 - 1) \mathcal{B}^\nu - (\gamma^{1/2} \mathcal{B}^\nu)\tilde{\nabla}_i}{\Gamma + \tilde{\nabla}} \right)$$

$$- \xi \frac{\gamma^{-1/2} \partial_t (\gamma^{1/2} \mathcal{E}^\mu)}{\Gamma + \tilde{\nabla}}$$

$$+ \xi \frac{\mathcal{E}^{\mu} T^\nu}{(\Gamma \tilde{\nabla} + 1)(\Gamma + \tilde{\nabla})},$$

(19)

We note a correction here, i.e., the differently placed parentheses for the term on the right-hand side of the 1st line, and the different sign for the term on the 4th line in comparison to the equation given in Bucciantini & Del Zanna (2013).
where

\[ Q' = \mathcal{E}_0' + \Delta t \left[ -(\alpha v^i - \beta^i) q + e^{\phi^2} \partial_{(\alpha B_k - \epsilon_{kim} \beta^i \mathcal{E}^m)} \right], \]

\[ q = \gamma^{-1/2} \partial_k (\gamma^{1/2} \mathcal{E}^k), \quad \bar{\eta} = \frac{\eta}{\alpha \Delta t}, \quad \bar{\nu}' = \Gamma \nu'. \]

with \( \mathcal{E}_0' \) representing the electric field as calculated in the previous time step.

Unfortunately, Equation (18) appears to be stiff. For low values of diffusivity, the terms on the right-hand side evolve over different times as compared to the time step used for the time discretization of the electric field, resulting in unstable solutions. This problem can be fixed by isolating the stiff terms, and the use of an implicit method to order to calculate them. The non-stiff part of the electric field, i.e., that which does not include diffusive terms, is calculated separately, and is denoted as \( Q' \). For the stiff part we use an iterative method, whereby for every time step we calculate the electric field, until its values have converged to the desired accuracy. A detailed analysis of the numerical implementation is presented in Qian et al. (2017).

We clearly state that we have tested the implementation of resistivity, considering analytical, time-dependent solutions of the induction equation (Qian et al. 2017; Vourellis et al. 2019) and find perfect agreement. Given that the additional implementation of a mean-field dynamo is achieved in exactly the same way as for resistivity, our testing of resistivity also supports the implementation of the dynamo term.

### 2.4. Dynamo Quenching

An exponential increase in magnetic-field strength is a direct result of the mean-field dynamo process. However, a strong magnetic field will suppress the MRI, which is considered to be one of the sources of disk turbulence. Therefore, with the repression of turbulence, the dynamo will also be suppressed, resulting in a saturation level in the magnetic field generated. This has been demonstrated by direct simulations of the MRI (Stone et al. 1996; Gressel 2010; Gressel & Pessah 2015) and has been approved analytically (Ruediger et al. 1995). The suppression of MRI has also recently been considered in GRMHD (Bugli et al. 2018).

In our approach, in the absence of a more detailed turbulence model, dynamo quenching must be enforced by interactively lowering the value of the dynamo parameter, \( \xi \), following certain physical criteria, motivated by models of turbulence.

The usual procedure (which we refer to as “standard quenching”) prescribes a kind of equipartition field strength, beyond which the mean-field dynamo parameter is reduced, until the dynamo becomes effectively inactive. For example, Bardou et al. (2001) have implemented a back-reaction on the \( \alpha \)-dynamo parameter from the magnetic field by making this alpha \( \alpha \) depend on the values of \( B \) (see also von Rekowski et al. 2003; von Rekowski & Brandenburg 2004). A different approach was undertaken by Fendt & Galsmann (2018), who applied a disk diffusivity model where the strength dynamo was quenched by increasing the magnetic diffusivity. This resulted in a smooth and long-term evolution for their jet-launching simulations (up to several 100,000 disk rotations).

However, the rate at which magnetic diffusivity and the mean-field dynamo affect magnetic-field evolution can be very different.

A leading parameter of mean-field dynamo action is the dynamo number. For the case of a thin accretion disk of scale height \( H \), we apply the dynamo parameter, \( \xi \), and magnetic diffusivity \( \eta \) to define the total dynamo number, \( D_\xi \), following previous works in a non-relativistic environment, utilizing a thin accretion disk (Bardou et al. 2001; von Rekowski et al. 2003; Stepanovs et al. 2014),

\[ D_\xi = |R_\xi R_{\Omega}| = \left| \frac{\eta H S H^2}{\eta} \right| \]

(20)

where the two fractions represent the Reynolds numbers, \( R \), from the dynamo action turbulence (mean-field \( \alpha \)-dynamo), and from differential rotation (\( \Omega \)-dynamo), respectively. The function \( S = r d\Omega/dr \) measures the shear due to differential rotation, and is calculated for each cell separately.

Depending on the values of the dynamo number, \( \xi \) will exponentially increase the magnetic field, while \( \eta \) would damp at a rate close to linear. As such, if the dynamo number is too large, the change in diffusivity suggested by Fendt & Galsmann (2018) might not be sufficient, and a standard quenching model must also be employed. Furthermore, by increasing the diffusivity, the numerical time step dramatically decreases, increasing the computational costs of the simulation.

In our models, we follow the standard quenching approach, as applied by e.g., Bardou et al. (2001). We define plasma-\( \beta \) as the ratio between the gas and the magnetic pressure, and magnetization \( \mu \) as its inverse,

\[ \beta = \frac{1}{\mu} = \frac{P_{\text{gas}}}{P_{\text{magn}}}. \]

We implement a quenching prescription that calculates the magnetization, \( \mu \), of the fluid, and when it becomes too large, the actual dynamo parameter, \( \xi_q \), is reduced:

\[ \xi_q = \xi_0 \left( 1 + \frac{\mu_0}{\mu_0 \mu_{\text{eq}}} \right). \]

(22)

where \( \xi_q \) is the quenched dynamo parameter, \( \xi_0 \) is its initial value, \( \mu_0 \) is the disk magnetization averaged vertically over the disk, and \( \mu_{\text{eq}} \) is the equipartition magnetization, expressing the value of the disk magnetization in which the mean-field dynamo is quenched by 50% (i.e., \( \xi_q = \xi_0 / 2 \)).

Note that we initially encountered an issue with the MPI parallelization of the code. In order to find average values for the magnetization over a certain region of cells to calculate the actual quenching, these cells must actually belong in the same processor. Alternatively, we would have to find a way to communicate the average values of certain disk areas between the processors. The second option would require heavy work on the parallelization of rHARM3D, so we follow the first option. The downside of our choice is a restriction to a small number of cores for our simulations, simply due to the need to cover a large area of the disk with a single core.

We compromise by running the simulations with only a small number of cores in the \( \theta \) direction, so that the calculation for quenching is run by the same core in the angular direction. For our 256 \( \times \) 256 grid we use 64 cores with four cells each in the radial direction, and five cores with 51 cells each in the polar direction, resulting in a total of 320 cores.

### 3. The Dynamo Effect in a Relativistic Torus

In this section we investigate the mean-field dynamo effect in a relativistic torus. This setup is essentially different from
that for a thin accretion disk, in the sense that the differential rotation of a torus is stronger compared to a disk, so that the effect of the mean-field dynamo plays a stronger role. This becomes clear if we compare the dynamo numbers for the cases of a disk and a torus. In a torus with a constant specific angular momentum, the angular velocity scales as \( \propto r^{-2} \), while for a Keplerian rotating disk, the scaling follows \( \propto r^{-3/2} \). We assume a scale height of \( H_0 = 0.1R \) for a thin disk as a typical length scale for the disk dynamo action. Similarly, as a typical length scale for the torus dynamo, we take the e-folding length scale, \( H_T \), of the density distribution. Considering the center of the torus \( (r = 15) \), the first contour is at \( z \approx 4.2 \), so that \( H_T / R = 0.285 \). Thus, based on Equation (20), we find the Reynolds numbers from the differential rotation:

\[
\mathcal{R}_T = R \frac{\partial \Omega_T}{\partial R} \frac{H_T^2}{\eta} = 0.16, \\
\mathcal{R}_D = R \frac{\partial \Omega_D}{\partial R} \frac{H_T^2}{\eta} = 0.015 \sqrt{R}. 
\]

Note that the dynamo number of the torus can be higher by an order of magnitude in the inner area; for larger radii, however, the disk dynamo number increases.

We can then write

\[
\frac{\mathcal{R}_T/\mathcal{R}_D}{0.16/0.015 \sqrt{R}} \approx 10R^{-1/2}. 
\]

This relation naturally changes over time as the disks evolve. Compared to those systems with strong shear, stellar dynamos are better described by an \( \alpha^2 \) mean-field dynamo, and are found to be less stable and less symmetric (Kükéer & Rüdiger 1999).

Our simulation setup for this exemplary torus simulation, \( \text{sim}T \), is as follows. We start the simulation with a density and internal energy distribution following the Equation of State for an ideal gas, which reads

\[
\rho = \left( \frac{(h - 1)(\gamma - 1)K}{\gamma} \right)^{1/(\gamma-1)},
\]

where \( h(r, \theta) \) is the specific enthalpy, as calculated by Fishbone & Moncrief (1976). The torus is rotating around a Schwarzschild black hole \( (a = 0) \) with an inner edge at \( r_{\text{in}} = 6 \), with its point of maximum density at \( r_{\text{max}} = 15 \). The internal energy is defined by the polytropic equation of state, \( u = K \rho^{\gamma}/\gamma - 1 \), with \( K = 10^{-3} \) and \( \gamma = 4/3 \).

An initial toroidal magnetic seed field is defined inside the torus, following the density distribution \( (B_t \propto \rho) \) of the standard Fishbone-Moncrief torus, applied using HARM (Gammie et al. 2003). This guarantees that the magnetic field is confined inside the torus. The field direction is positive in both hemispheres, and is normalized by considering a plasma-\( \beta_0 \) = 10^6.

We prescribe a distribution for the magnetic diffusivity that follows the density distribution of the torus, with a maximum value of \( \eta_0 = 10^{-5} \). The diffusivity basically vanishes in the surrounding corona. In addition, the dynamo action is limited to the area inside the torus. It is constrained to an area smaller than the distribution of the initial magnetic field and diffusivity, in order to avoid field generation in the corona of the torus. The dynamo parameter is constant in both hemispheres at a level of \( \xi_0 = 10^{-3} \).

### Table 1

| Run | \( a \) | \( \eta_0 \) | \( \beta_0 \) | \( \xi_0 \) |
|-----|-----|-----|-----|-----|
| \( \text{sim}T \) | 0   | \( 10^{-3} \) | \( 10^6 \) | \( 10^{-3} \) |

**Note.** The ID of the run comprises the following: the application of the Kerr parameter, \( a \), the maximum magnetic diffusivity, \( \eta_0 \), the initial plasma-\( \beta \), and the dynamo parameter, \( \xi_0 \).

![Figure 1](image.png)

**Figure 1.** Initial conditions for our reference simulation. Shown above is the initial distribution of density, including contours separated by one e-folding scale height (log scale; upper left), the toroidal magnetic field, \( B_\phi \) (linear scale; upper right), the diffusivity, \( \eta \) (linear scale; lower left), and the dynamo parameter, \( \xi \) (linear scale; lower right).

The parameters of our torus simulation, \( \text{sim}T \), are summarized in Table 1. The initial conditions for this simulation are shown in Figure 1.

We note that this setup is somewhat different from that chosen by Tomei et al. (2020), as their initial torus is a so-called Polish donut (see e.g., Abramowicz 2013), and their seed-field geometry comprises poloidal loops inside the torus. With our approach we prefer to connect our simulations to the literature of HARM simulations, which typically apply the Fishbone–Moncrief torus as the initial condition, and while our initial setup may look similar to that of Tomei et al. (2020), our results cannot be directly compared.

#### 3.1. Field Induction by a Spatially Constant Dynamo

We now describe the field evolution of a torus induced by a mean-field dynamo in more detail.
When the simulations begins, the poloidal magnetic field appears immediately inside the torus, in the area where \( B_\phi \) and the dynamo coexist. As the simulation continues, the field starts to increase in value due to the dynamo, while advecting toward the black hole, partially following the accretion of the torus, increasing the magnetic field in the inner disk region and toward the black hole. Simultaneously, diffusivity works to dampen the magnetic field. In the end, the \( R_\xi \) determines where the field is amplified or dampened. Since \( \xi \) is constant, the profile of diffusivity determines the value of \( R_\xi \). As an example of their values, in the center of the torus, the numbers are \( R_\xi \sim 15 \), while its maximum value is as high as \( R_\xi \sim 150 \) in the boundaries of the dynamo distribution.

In Figure 2 we present snapshots of the evolution of the poloidal magnetic field. As mentioned above, in the beginning, the field lines are restricted in the area where \( \xi = 0 \). However, they are eventually dragged along with the material being accreted toward the black hole. At time \( t = 2000 \), with the dynamo having been working for approximately 6 rotations of the torus center, we see a low-velocity outflow being launched from the inner part of the torus. The magnetic field lines follow the low-density fluid, showing the first indications of the development of a jet. The launching point of the outflow is barely within the innermost stable circular orbit (ISCO), and can be attributed to the presence of the strong toroidal magnetic field we see in the close atmosphere of the inner part of the torus. This strong toroidal field, which has been amplified by the \( \Omega \) effect of the dynamo, increases the magnetic pressure, and pushes both material and the poloidal field outwards (Lynden-Bell 1996).

The poloidal field continues to grow up to \( t \sim 3800 \), but then the field close to the axis becomes too strong, resulting in the code failing to converge. At that time, both the toroidal and the poloidal components of the field have spread into the greater part of the grid, while the plasma-\( \beta \) has reached values of between 0.01–0.1 inside the ISCO, and around 1–10 in the outflow. Still, the torus is hydrodynamically dominated (see Figure 3).

The material accelerated in the outflow consists mainly of the floor values used by the code as a background environment. We observe outflow velocities larger than 0.1 \( c \) (see Figure 3). The acceleration is supported by magnetic pressure forces. This holds for the high-speed floor material, but for torus material moving with a radial speed \( \approx 0.01 \) (the light red area in Figure 3, left), we expect that the driving is in addition also supported by gas pressure. Note also that along the axis where the toroidal field vanishes we observe infall toward the black hole.

In Figure 4 we show the evolution of the average magnetic energy for both poloidal and the toroidal fields. In both channels the energy increases over orders of magnitude until a saturation level appears around \( t = 1500 \). There is no full saturation, as beyond \( t = 1500 \) the field energy still increases by a factor of 100–1000; however, the strong increase in dynamo action flattens substantially. This flattening is interesting, insofar as we do not apply dynamo quenching in this simulation. As such, the dynamo seems to self-quench. This has also been observed in the literature (see below). We hypothesize that this is due to reconnection of the tangled magnetic field with time, as Figures 2 and 3 show clear evidence of the increasingly turbulent state of the magnetic field. Due to the application of resistivity, this field may physically reconnect, lowering the magnetic energy and heating the plasma.

We think that the process works in two ways: firstly, the dynamo-generated magnetic field can transported out of the dynamo-active area; however, this is still an area with a non-negligible resistivity (basically the light red area in Figure 1 (upper right)). Since the field is tangled (see Figure 2), it will reconnect. However, the same process may also occur inside the torus. The resistivity is largest inside the torus (dark red areas in Figure 1, right), thus, reconnection will be very efficient here. Taking account of this, and also considering that the dynamo-generated field is strongest here (i.e., the \( \xi \) is large), reconnection will work efficiently to lower the net magnetic field energy.

Interestingly, this points toward a new possible channel for dynamo quenching by reconnection. Considering magnetic diffusivity, we may mostly think of diffusing away magnetic flux, and in this way lowering the efficiency of the dynamo. However, reconnection is also a result of magnetic diffusivity, which would naturally lower the field energy.

### 3.2. The Dynamo-generated Torus Magnetic Field

The initial condition of the toroidal component of the magnetic field follows the density profile while keeping a positive sign in both hemispheres. A toroidal field is also...
produced by the $\Omega$ effect by differential rotation of the induced poloidal field, increasing the initial toroidal field.

The induced toroidal field changes sign in the equatorial plane, keeping its positive values in the upper hemisphere (where the radial field is negative). This results in a gradual change in the toroidal field across the torus, when the initial condition is superimposed by the newly generated field.

At time $t = 1000$, the sign has already changed within the borders of the area where dynamo activity is prescribed. The change of sign of the toroidal field then continues, gradually affecting the lower hemisphere as well. At time $t = 2000$ this transformation has almost finished, resulting in a toroidal field distribution that changes sign across the equatorial plane.

The dynamo-generated poloidal field is dominated by the radial component. Note that the direction of the radial field component also affects the direction of the poloidal field. Here we have the problem that the dynamo-generated field, which is initially prescribed as a perfect dipole (with negative values in the upper hemisphere, and positive values in the lower hemisphere), eventually turns into a striped form, exhibiting alternating positive and negative directions, which eventually results in the formation of closed magnetic loops.

This effect of inducing a layered structure of radial magnetic-field components naturally also affects the direction of the toroidal field, where we see a similar behavior in later evolutionary stages. This effect becomes apparent from $t = 500$–$800$. At this time, the initial dipolar field geometry begins to change its topology, as we find a positive $B_t$ in the upper hemisphere, which was initially set to a negative radial field structure.

Similarly, this phenomenon can be seen for all components of the magnetic (as well as electric) field. From $t = 1000$ an additional layer appears in the $B_t$ distribution, now with a negative value which alternates with the previously induced positive $B_t$ in the upper hemisphere.

We observe this process over the whole duration of the simulation run, and eventually find a completely layered magnetic-field structure of alternating field directions. We note that this layered structure shows up preferentially close to the equatorial plane of the torus. Here, we also find high dynamo numbers for the turbulent dynamo, precisely within the layer mentioned above. At later time steps, this region of layered magnetic-field direction spreads out over the whole torus, and can eventually be found in the outflows as well. This dynamical process was also reported in Bugli et al. (2014), and Tomei et al. (2020), despite a different prescription for the symmetry of the dynamo parameter.

The time evolution and the spatial structure of the magnetic-field evolution is strongly reminiscent of the dynamo waves generated during the kinematic phase of the mean-field dynamo process. Dynamo waves are typically discussed as oscillatory solutions of the *kinematic* dynamo problem. The existence of such dynamo waves was investigated decades ago by Parker (1955), as well as in many subsequent studies (see e.g., Weiss et al. 1984; Hughes & Tobias 2010).

Here, we solve for the dynamical dynamo problem, i.e., essentially considering the back-reaction of the magnetic field.
generated in the gas. Nevertheless, in the early stages of the simulations, we are still in the linear regime of dynamo action, and the field is not yet strong enough (high plasma-$\beta$) to act on the gas. As such, the situation is comparable to the kinematic regime, and the excitation of dynamo waves may indeed be expected. More recent studies have also detected dynamo waves in cases of high magnetic Reynolds numbers (Cattaneo & Tobias 2014; Nigro et al. 2017). This is essential to know, as in our simulations the magnetic Reynolds numbers, $R_m \equiv (\nu L)/\eta \approx 1000$, can also be high, assuming as a typical length scale a torus scale height of $L \approx 10$, a typical velocity $v = 0.1$, and a maximum diffusivity of $\eta = 0.001$ (all in code units). We note that much of the work cited here has been conducted in relation to spherical stellar dynamos. However, the case of the relativistic torus is probably not too far from this, in that, compared to our studies of thin disks later in this paper, the shear in the torus is relatively weak.

A characteristic feature of dynamo waves is that the typical timescale of the fluctuations is similar to the diffusion timescale (see e.g., Giesecke et al. 2005 for spherical $\alpha^2$-dynamos). For the simulations presented in this section, this is indeed the case. With a maximum diffusivity of $\eta \approx 10^{-3}$ in the torus, and a typical length scale in the torus of $\Delta R \approx 1$, we calculate a global diffusion timescale of $t_{\text{diff}} = (\Delta R)^2/\eta \approx 10^3$ in code units. This is indeed similar to the timescale we measure for global variations in the dynamo-induced field structure. Although the values considered comprise some rough estimates, and vary dramatically over the torus as a whole, we may see this correlation as a hint of dynamo waves in our simulations.

Even at later stages in our simulation, the plasma-$\beta$ in the torus is of the order of 100–1000; thus the dynamo still acts in the kinematic regime. Nevertheless, the generated magnetic field has expanded, and fills the space between the rotational axis and the torus. Here, the plasma-$\beta$ is lower, at $\beta \approx 0.1$, and its evolution is dominated by the magnetic field.

Overall, we see indications that dynamo waves are excited during the initial stages of magnetic-field evolution in our torus model simulation. This is particularly evident in Figure 5, which shows the generation of a layered structure in an anti-aligned field (as indicated by the radial field component). This figure shows striking similarities to Figure 3 in Tomei et al. (2020); they, however, plot the magnitude of the field only, not its direction.

### 3.3. Comparison with Previous Literature Studies

At this point it is interesting to compare our results to those of the existing literature, particularly with respect to the simulations of Bucciantini & Del Zanna (2013) on whose work the mean-field dynamo closure of our code is based, together with follow-up papers (Bugli et al. 2014; Tomei et al. 2020). As mentioned above, while the geometry of the latter two approaches looks similar, the initial conditions are in fact quite different. Our initial torus structure follows the Fishbone–Moncrief solution (Fishbone & Moncrief 1976), as applied in previous HARM simulations, while their torus follows a different prescription. Overall, our initial torus extends to $r \approx 45$, while Bugli et al. (2014) consider a smaller structure with $r < 10$. The Polish donut torus solution applied in Tomei et al. (2020) appears to be much larger; however, of the full grid of $r < 100$, only the innermost radii, $r < 20$, are shown.

Moreover, the initial field structure and the diffusivity distributions are different.

Given the differences in initial setup, a quantitative comparison to our results is not possible. However, we observe similar structures evolving from our dynamo. Like Tomei et al. (2020), we also observe the generation of bipolar magnetic “arms” around the inner edge of the torus. In addition, the authors claimed to be able to detect dynamo waves being dragged toward the black hole by accretion.

We show the 2D magnetic-field evolution at a somewhat longer time step (in code units $t = 3500$ versus 1608 in Tomei et al. 2020). We are therefore able to follow the expansion of the generated magnetic flux away from the torus and toward the rotational axis. At $t = 3500$ we do indeed see a strongly magnetized axial region, which may give rise to Blandford–Znajek jets in the case of a rotating black hole.

We note however that the dynamical evolution of the torus, and also of the dynamo action, is more rapid in their smaller-sized torus setup. The 6 periods of torus orbits they show in their 2D images (and the 12 orbits they actually simulated) are calculated based on a torus center radius of $r_c = 12$, while the center of our torus is further out, at $r_c = 15$, resulting in rotational periods approximately a factor of 2 longer for our torus center, and a commensurately longer evolution time. The advection time of magnetic flux toward the axis is not affected by this, however.

---

2 Of course this value is subject to change over the area of the torus.
Interestingly, Tomei et al. (2020) find a (second) saturation phase of field amplification, even without quenching their mean-field dynamo, probably due to accretion dynamics, together with an equipartition state in the fluid system.

In our simulations we also find a transition to a saturation state, or, more accurately, a strong break in the growth rate of both field components (see Figure 4 and our corresponding discussion above). Tomei et al. (2020) argue that the change occurs at the time when local equipartition is reached. We may confirm this as close to the inner edges of the torus, as we also find equipartition at this stage (see Figure 3 upper right for a latter evolutionary stage). However, we hypothesize that the change in the slope of the increase in magnetic energy may also be due to the reconnection of the tangled magnetic field produced by the dynamo action (see discussion above).

We do not find the second saturation phase postulated by Tomei et al. (2020). The timescale of our simulation would be sufficiently long; however, since our simulation setup is different, particularly in relation to the dynamics of the initial torus, a proper comparison cannot be made here. We once again note that, measured in orbital periods of the torus, the evolutionary time of their torus is somewhat longer, and we might therefore not yet have reached the second saturation phase (if this phase were present at all in our setup).

The poloidal field structure derived by Tomei et al. (2020) is visualized by means of the magnetic energy distribution. Here, we also show poloidal field lines that can better trace the small-scale structure of the field, as well as the field direction. An observation of the very structure of the vector field, i.e., the geometry of the magnetic field lines and the distribution of the magnetic flux, is essential for an understanding of the jet-launching process. For example, considering magnetocentrifugally driven disk winds, the field inclination is important (Blandford & Payne 1982).

So far, with respect to the test simulations provided in the literature for GRMHD dynamos (Bucciantini & Del Zanna 2013; Bugli et al. 2014; Tomei et al. 2020), field lines for the resulting magnetic-field structure are only given for the case of a neutron star dynamo (see Section 5.2.4 of Bucciantini & Del Zanna 2013). We cannot therefore compare our field structure in detail with the existing literature. This would have been interesting, as the essential effect of the dynamo action is to generate poloidal magnetic flux along the disk.

4. Initial Setup for a Thin-disk Dynamo

Here we describe the initial conditions for our dynamo simulations, when considering a thin accretion disk. The simulation grid extends from just inside the event horizon to an outer radius of $r = 80$. The radial and polar dimensions are based on the original grid given by HARM. The numerical grid size used is $256 \times 256$.

The simulation is initiated with a thin accretion disk, whose density distribution is similar to that used by Vourellis et al. (2019):

$$\rho(r, \theta) = \left[ \frac{\Gamma - 1}{\Gamma} \frac{r_{in}}{r} \frac{1}{\epsilon^2} \left( \sin \theta + \sqrt{\Gamma - 1} \right) \right]^{{\Gamma}/({\Gamma - 1})},$$ (27)

and which is connected with the gas pressure by an ideal equation of state, $P = K \rho^\gamma$. The aspect ratio of the disk is $a \approx 0.1$. The disk is surrounded by a corona, with an initial density and pressure given by

$$\rho_{cor} \propto r^{1/(1-\Gamma)}, \quad \rho_{cor} = K_{cor} \rho_{cor}^r,$$ (28)

We choose $K = 0.001$ for the disk, and $K_{cor} = 1$ for the corona, and force an initial pressure equilibrium between the disk and the corona, implying a density jump between corona and disk surface. When the simulation begins, the initial corona starts collapsing, and is then replaced by the floor values for density and pressure, described via a broken power law (Vourellis et al. 2019). The disk is given an initial rotation profile, following Paczyński & Wiita (1980), as

$$\tilde{w} = r^{-3/2} \left( \frac{r}{r_{PW}} \right)^{1/2},$$ (29)

where $r_{PW}$ is a constant, here set to be equal to the gravitational radius, $R_g$.

The initial seed magnetic field is purely radial, and is prescribed only inside the initial disk. The choice of a radial field has the advantage of a vanishing shear between the rotating disk and the disk corona. We have also run simulations starting with a purely toroidal initial field; this results in a similar long-term magnetic-field evolution. The initial magnetic-field prescription involves purely radial magnetic field lines that converge on the black hole horizon, implemented numerically via the magnetic vector potential,

$$A_\phi(r, \theta) \propto \exp \left[ -\frac{1}{2} \left( \frac{1}{0.05 \tan \theta} \right)^2 \right].$$ (30)

The strength of the seed field is defined by the choice of plasma-$\beta = 10^6$.

In the simulations we apply a profile of magnetic diffusivity that is somewhat modified as compared to Vourellis et al. (2019), but is similar to the dynamo simulations of Stepanovs et al. (2014). This profile has a constant plateau across the equatorial plane, but quickly drops in polar direction for $\theta < 85^\circ$ and $\theta > 95^\circ$,

$$\eta(r, \theta) = \eta_0 \exp \left( -100 \frac{(\pi/2 - \theta)^4}{\arctan(\chi_\eta \cdot \varepsilon)} \right),$$ (31)

where $\eta_0$ is the (initial) maximum value, and $\chi_\eta = 3$ characterizes the scale height of the diffusivity distribution. The profile of the mean-field dynamo follows Stepanovs et al. (2014),

$$\xi(r, \theta) = \xi_0 \frac{1}{\sqrt{R/r_{in}}} \sin \left( \frac{\pi}{\chi_\xi \cdot \varepsilon \cdot \tan \theta} \right),$$ (32)

where $\chi_\xi = 1$ is a characteristic scale height for the dynamo. The latter is an important constraint, as by choosing $\chi_\xi < \chi_\eta$ we ensure that the dynamo action is always enclosed in the resistive area. Otherwise we may consider arbitrarily large dynamo numbers (see below). Furthermore, the purely resistive outer layers of the disk allow for easier launching and mass-loading of disk winds (Vourellis et al. 2019). In Figure 6 we

Note that while the poloidal magnetic flux, $|B_r|dA$, may vanish on average, the magnetic energy, $\Sigma B_r^2$, may still be substantial.
show the distribution of the magnetic diffusivity $\eta$, the dynamo parameter $\xi$, and the Reynolds number $\mathcal{R}_\xi$.

5. The Accretion Disk Dynamo

Our paper focuses on the generation of a strong poloidal flux, anchored to a thin accretion disk. As a reference simulation, we consider the case of a Schwarzschild black hole. A very low radial magnetic field with $\beta = 10^6$ is applied as a seed field for the dynamo action, following the (non-relativistic) approach of Stepanovs et al. (2014), and Fendt & Gaßmann (2018). Note that our initial disk structure is not in complete equilibrium with the black hole (in contrast with non-relativistic simulations). Thus, during its initial evolution, the disk will adjust on the relativistic potential, causing deviations in the seed field from the perfectly radial structure. These deviations, however, are minimal, and are significantly smaller than the magnetic field generated by the dynamo later on. Simulations with a different initial field structure (e.g., a toroidal field) lead to the same magnetic structure in the saturation phase. In total we run a set of eight simulations parameterized with the black hole rotation, the initial dynamo strength, and the level of quenching (see Table 2).

Figure 7 shows exemplary evolution of the poloidal magnetic field, together with the evolution of the density for reference simulation sim1. When the simulation starts, the initial radial structure of the magnetic field inside the disk evolves via the action of the dynamo. The geometry of this dynamo-generated field does not follow a clear dipolar structure.

In addition to the dynamo action, the hydrodynamic turbulence that develops in the disk amplifies the state of the magnetic field. We note here that this turbulence is a consequence of the existence of an anti-aligned poloidal field structure, which develops further when the poloidal field is advected toward the horizon. The field geometry close to the horizon (say for $r < 5$) resembles that of the Wald solution (Wald 1974; Komissarov 2005). With our resistive approach, this leads to strong reconnection events, affecting the mass-loading of the outflow, as well as the accretion of material, and also the foot point of the magnetic field lines guiding the outflow (see also Vourellis et al. 2019).

We do not expect MRI to be detected in our simulations. In fact our resolution would be almost sufficient to resolve the some of the large wave modes. As such, while MRI might potentially be observed in an ideal MHD study, for resistive MHD, the excitation of MRI modes is severely damped. Here, we refer to Fleming et al. (2000), and Longaretti & Lesur (2010), who investigated in great detail the effect of resistivity on the evolution of MRI. Qian et al. (2017), investigating the role of MRI in a general relativistic torus in resistive MHD, came to the conclusion that MRI is increasingly damped for an increasing level of resistivity (although applying a lower resolution).

As soon as the dynamo-induced magnetic field is strong enough to remove angular momentum from the disk, accretion sets in. With accretion occurring, the newly generated field is advected toward the black hole. In the last stage of the simulation, the environment close to the black hole is strongly magnetized by a poloidal field with a dipolar configuration.

| run     | $\alpha$ | $\eta_0$ | $\xi_0$ | $\beta_{eq}$ |
|---------|----------|----------|---------|--------------|
| sim0    | 0        | 0.001    | 0.004   | 1000         |
| sim1    | 0        | 0.001    | 0.004   | 100          |
| sim2    | 0        | 0.001    | 0.004   | 10           |
| sim3    | 0        | 0.001    | 0.004   | 1            |
| sim0.1  | 0.9      | 0.001    | 0.004   | 1000         |
| sim1.1  | 0.9      | 0.001    | 0.004   | 100          |
| sim4    | 0        | 0.001    | 0.004   | ...          |
| sim5    | 0        | 0.001    | 0.004   | no quenching |

Note. The ID of the run comprises the following: the applied Kerr parameter, $\alpha$, the maximum magnetic diffusivity, $\eta_0$, typically located at the inner disk radius, the maximum (initial) dynamo parameter, $\xi_0$, and the quenching parameter, $\beta_{eq}$, corresponding to an equipartition field strength, beyond which dynamo action is strongly quenched.
However, at larger radii, and particularly inside the disk, at the timescales investigated, the dynamo has not yet generated a clear structure capable of providing a large-scale magnetic flux. Note that these simulations are quite CPU-intensive, and the timescales we are reaching here of about $t = 10,000$ corresponds to about 150 inner disk rotations. In contrast, the non-relativistic disk dynamo simulations (Stepanovs et al. 2014; Fendt & Gaßmann 2018) extend to several 100,000 disk rotations.

5.1. The Evolution of the Dynamo-generated Magnetic Field Structure

We now discuss the magnetic-field structure generated by the disk dynamo (see Figure 7), and how its geometry and field strength depends on the parameters triggering the dynamo process, i.e., the strength of the dynamo, $\xi$, the disk diffusivity, $\eta$ (in combination the dynamo number), and the quenching parameter, $\mu_{\text{eq}}$.

Figure 8 shows the evolution of the integrated poloidal and toroidal magnetic energy in three different regions (distinguished by their inner and outer radii). We emphasize that, for later times, the dynamo-induced poloidal field remains substantially higher than the toroidal field component. This again points toward an efficiently working $\alpha$-dynamo, since only the $\alpha$-process can amplify the poloidal field component.

For the innermost part of the accretion stream between the black hole horizon and the initial inner disk radius ($2 < r < 10$), the initial magnetic energy vanishes (since the seed field is confined inside the disk only), then rapidly increases, due to the dynamo-generated field. When the disk starts evolving, meaning that accretion of disk material commences, the magnetic field is advected toward the black hole. Due to the unsteady
accretion process, the advected magnetic energy also varies. Indeed, we find that the initial variations in magnetic energy correlate with spikes in the accretion rate (see top panel of Figure 9). After $t \approx 2000$, the increase in magnetic energy continues, but with a smaller slope, and after $t \approx 6000$ we see indications of a saturation of dynamo action as the slope of the magnetic energy increase decreases even further, to the point where it is almost constant. Nonetheless, we do not reach a constant value, as at $t \approx 10,000$ there is another sudden increase in the magnetic energy, coinciding with another strong accretion event.

At larger disk radii, the time evolution of the magnetic energy looks somewhat different. The initial variations do not appear, as the accretion process operates more smoothly in these areas. However, the overall behavior of the magnetic energy is similar to that of the innermost disk. Again, at the beginning the energy increases much faster for the middle part ($10 < r < 30$), but after $t \approx 3000$ the disk’s magnetic energy is almost the same for both regions, and slightly higher than the energy in the innermost part. After $t \approx 10,000$, with a sudden energy increase from the inner part, the energy in all three radii is approximately the same. The comparison of the magnetic energy content of these disk areas of course depends on the choice of the integration areas. What we learn from this comparison is that the disk dynamo works with similar efficiency over almost the whole disk. Furthermore, the slope of magnetic energy growth is comparable between these areas, indicating a well-constructed setup for the disk dynamo, and, in particular, a direct coupling between the processes of accretion, ejection, and dynamo action.

Figure 10 shows the distribution of the plasma-$\beta$ for poloidal and toroidal magnetic-field components at two different time steps in simulation $\text{sim1}$. Clearly, due to the dynamo activity, the plasma-$\beta$ decreases with time—first in the inner disk area, and later in the main disk body as well. Moreover, the disk wind that is launched at later time carries a low plasma-$\beta$. Overall, the toroidal magnetic-field component predominates. This was also observed by Vourellis et al. (2019) for a magnetic field that is initially prescribed, and is consistent with the literature of GRMHD disk simulations. For the timescales considered here, this tells us that differential rotation ($\Omega$ effect) still dominates the turbulent dynamo ($\alpha$ effect). At $r \sim 15$ along the equatorial plane, the dynamo number characterizing differential rotation is $R_\Omega \approx 59$, while the maximum $\xi$ dynamo number, $R_\xi$, does not exceed 13 (see Equation (20)).

However, we see that the plasma-$\beta$ measured in the inner part of the disk, and in the accretion funnel, slightly increase during the later stages of the simulation. This is a direct effect of the quenching mechanism we implemented. In addition, the magnetic field becomes quite entangled in the later stages of the simulation (see the last panels in Figure 7). As a consequence, further effects are resent which may lead to the dissipation of the magnetic field, magnetic reconnection in particular. What can be clearly seen from the figure is that the areas of low plasma-$\beta$ are also precisely the areas in which strong outflows develop from the inner disk (see also Section 5.4 below).

An essential condition for jet launching is a sufficient magnetic flux of the jet source, synonymous with a large-scale open magnetic field structure. As briefly mentioned above, although we may measure a substantial (poloidal) magnetic energy, $\propto B_\theta^2$, in the disk, the poloidal magnetic flux, $\int B_\phi dA$, may vanish, on average, if the field is strongly tangled.

It is therefore interesting to observe the evolution of the magnetic flux generated by our disk dynamo. This is shown in Figure 11 for simulations with different Kerr parameters and quenching thresholds. Here, we choose two different representations of the absolute integrated magnetic flux. One option is to integrate along a circle of constant radius (lower panel) and the other is to integrate along the disk surface (upper panel), using the $B_\theta$ and $B_\phi$ components of the magnetic field, respectively. Note that a radius of $r = 11$ is chosen for the
circle, such that it measures the magnetic flux close to the black hole, which may potentially launch a Blandford–Znajek jet. The angle of integration is chosen so as to avoid having to account for flux from inside the disk, where the field is more entangled.

The increase in magnetic flux is similar to the time evolution of the magnetic energy (see Figure 8). Differences are due to the fact that when integrating the cumulative magnetic flux, $|B_\phi d\Omega|$, we effectively average over fluxes in opposite directions, as discussed above.

This clearly demonstrates that our disk dynamo also generates a substantial magnetic flux when generating magnetic energy (see Figure 7, which indeed shows large-scale open poloidal field lines). Interestingly, we find that all dynamo simulations evolve with an initially very similar behavior. At the later evolutionary stages, however, they diverge, due to the different quenching thresholds they obey. Simulation sim0 maintains an almost-constant magnetic flux during its final part, while for simulations with a higher quenching threshold, the flux increases with a steeper slope. Simulations sim2 and sim3 end earlier, due to the higher magnetization levels permitted by the quenching. Simulation sim0.1, which considers a rapidly spinning black hole, also provides an ever-increasing magnetic flux over the simulation time; however, the dynamo effect is somewhat delayed.

For comparison, we have also performed test simulation sim4 without any dynamo action, and sim5 without quenching. As expected, in simulation sim4 the initial radial field structure is conserved, while slightly expanding in the surrounding disk corona. Minor weak outflows can be detected from the disk surface, which can largely be attributed to the local force balance between the disk’s vertical pressure gradient (magnetic pressure is negligible here) and the vertical gravitational force (induced by the black holes metric). In simulation sim5 the magnetic field develops as in simulation sim3; however, the lack of a quenching mechanism generates a magnetic field that becomes extremely strong, leading the code to crash rather early. A similar behavior was also observed by Tomei et al. (2020). Both simulations, sim4 and sim5, may serve as reference material for further test simulations of our implementation of the dynamo, and of dynamo quenching.

5.2. Disk Evolution—Magnetic Field and Mass Fluxes

We now investigate the evolution of the accretion disk, in particular the mass accretion rates, in more detail. Figure 9 (top) show the normalized mass flux through surfaces of constant radius $r = (3, 6, 11)$. We have selected a slice between $75^\circ < \theta < 105^\circ$ in order to account for the initial disk structure and the evolution of the disk.

Due to the weak seed field, the initial evolution is basically hydrodynamic. While the initial radial structure of the magnetic field is ideal for angular momentum removal from the disk material, its strength is too low in order to have a big impact.

We find that right in the beginning of the simulation, the accretion rate shows large variations. This is due to the lack of a perfect vertical hydrodynamic equilibrium in the disk. This
leads to distortions in the disk structure, and to episodic accretion events of material that has moved inside the ISCO (note the initial position of the inner disk radius is at \( r = 10 \), well outside the ISCO). The inner part of the disk is quickly advected and when it crosses the marginally stable orbit it disconnects from the disk and falls in the black hole leaving a “gap” behind it which quickly fills with material from the remaining inner part of the disk. The initial radial shape of the initial magnetic field supports the advection of disk material toward the black hole, however, as mentioned above, this field is not dynamically important. On the other hand, the motion parallel to the poloidal field also implies that no magnetic flux is advected initially.

The episodic accretion is repeated several times depending on the disk radius (see the accretion spikes in Figure 9, top). Further out along the disk, the accretion peaks correspond to disruptions that appear at the disk surface displacing disk material (see Figure 7, top middle panel). This is a common consequence of the dynamics of the disk in the relativistic environment (Vourellis et al. 2019).

Around time \( t \approx 2000 \) the disk structure settles just outside the marginally stable orbit resulting in a more continuous accretion rate. At this time, the initial seed magnetic field starts being modified by the dynamo leading to initial weak outflows launched from the accretion funnel between the marginally stable orbit and the black hole. The outflows drag the magnetic flux with them, developing a vertical magnetic-field component which then contributes to establish a smoother accretion rate. We will later discuss the onset and evolution of these outflows in more detail (see Section 5.4).

We find the strongest accretion rates after \( t \approx 10,000 \). We understand this is essentially caused by the interplay between the dynamo activity and dynamo quenching (see Section 5.3). The weak outflows mentioned above remain present also during the second evolutionary phase, however, showing a time variation in velocity and mass flux. Within the accretion disk the magnetic field is substantially entangled, forming loops that reconnect with each other. The disk starts expanding in the vertical direction, but not yet developing any strong disk wind.

After \( t \approx 10,000 \) we notice an increase in the accretion rate that coincides with an increase in the magnetic-field strength close to the black hole. This is the point in time when the dynamo quenching is not sufficient to stop the magnetic field from increasing further mainly due to the radial limits within which it is defined. Note that the dynamo action and its quenching is defined only for \( r > 10 \), which is located inside the accretion disk. When the magnetic field is advected, the magnetic flux in the black hole magnetosphere increases along with the magnetization, but this area is neither dynamo-active nor affected by quenching. The strong field of the black hole magnetosphere results in divergences in the integration of the equations which eventually lead to the termination of the simulation.

When the disk field becomes advected, these field lines also populate the area along the rotational axis resulting in a strong axial magnetic field that could potentially lead into a Blandford–Znajek-driven outflow (in the case of a rotating black hole, see Section 5.5).

### 5.3. Dynamo Quenching

For completeness, we briefly show the evolution of dynamo quenching in our simulation. After an initial evolutionary phase, the magnetic field continues to increase in both poloidal and toroidal components. However, the toroidal component undergoes an extra amplification due to the differential disk rotation (the \( \omega \)-effect). This later evolution is not as rapid as the initial boost, due to dynamo quenching beginning to mitigate the dynamo effect. The dynamo quenching follows Equation (22), which for simulation \( sim1 \) has \( \mu_{eq} = 1/\beta_{eq} = 0.01 \). This implies that as the plasma-\( \beta \) inside the disk decreases from the initial value of \( 10^4 \) to an actual value of 100, the quenching becomes stronger.

Even for a plasma-\( \beta = 1000 \), in our parameter setup the dynamo action is already quenched by \( \sim 11\% \), and by the time plasma-\( \beta = 100 \), the dynamo parameter is at half of its initial value. Note that the dynamo quenching works locally, defined by the local vertically averaged disk magnetization. Depending on this, the actual strength of the dynamo tensor is quenched.

This phenomenon is illustrated in Figure 12, where we show the distribution of the \( \xi \)-dynamo for two exemplary time steps for simulation run \( sim0.1 \), based on a rapidly rotating black hole. While at \( t = 1000 \) the dynamo still works at its initial strength, at \( t = 12,000 \), quenching is clearly observed in certain disk areas. The quenching is locally different, implying that the dynamo operates at different strengths in different positions along the disk, and that while it smooths the exponential amplification of the field, at the same time it restricts a stronger accretion rate.

### 5.4. Generating a Disk Wind

It is well-known that a substantial disk magnetic field is essential for driving a strong disk wind (Blandford & Payne 1982; Casse & Keppens 2002; Ferreira 1997). In addition, field inclination plays a role (Blandford & Payne 1982). In contrast to simulations in the literature considering jet launching from a magnetized disk, which either apply a pre-defined disk field (Zanni et al. 2007; Sheikhnezami et al. 2012; Stepanovs & Fendt 2016) or self-generate the disk...
field from a non-relativistic dynamo (Stepanovs et al. 2014; Fendt & Gaßmann 2018), in our GRMHD dynamo simulations the resulting disk’s magnetic field lacks the smooth large-scale structure to support a Blandford–Payne disk wind, due to the symmetry of the initial field.

However, we can still detect the launching of strong outflows. From our reference simulation sim1 (see Figure 7) we see the evolution of small-scale outflows from the disk surface which merge overall into a broad disk wind. The initially weak winds are launched by a combination of the increased magnetic-field pressure gradient (mainly, but not only, generated by the toroidal component) and the rearrangement of the accretion disk structure in the gravitational field of the black hole (advection of magnetic flux).

In Figure 13 we show the distribution of the radial velocity for the reference simulation at times $t = 8000$ and 12,000. We also display the density contour $\rho \approx 0.003$, which could be understood as a proxy for the disk’s surface. A better choice would be the transition from inflow to outflow; however, for most parts of the disk corona close to the disk, there is a large-sized area with a constantly positive outward velocity. Instead, we find patches of both outflowing and infalling material, with velocities somewhat below ($<0.02c$). This is the clear signature of a turbulent outflow structure. We note that here we are still dealing with the initial stages of outflow from the inner parts of the disk’s main body. Note that the magnetic field and the disk accretion is still evolving, as time $t = 10,000$ corresponds to only 50 inner disk orbits, far fewer than the number achieved in non-relativistic simulations.

An exception to this is a high-speed outflow rooted in the accretion funnel between the black hole and the ISCO. Here, the gas into which the magnetic field is frozen in orbits with high speeds, resulting in radial outflow velocities of up to $0.2c$.

In the last evolutionary stages the picture changes, however. The disk wind is now dominated by outflowing patches moving at $\sim 0.05c$. The infalling patches have almost disappeared, and we see a substantial disk wind.

In the polar area above and below the black hole we see a constant infall of material. Note that this reference simulation is for a Schwarzschild black hole; as such, no Blandford–Znajek driving is possible (see below for rotating black holes). This axial area remains almost free of magnetic flux until the very final stages of the simulation. At this point in time, the accumulation of magnetic flux also leads to an increasing strength in outflows from the inner part of the disk. These outflows are rooted in the accretion disk and outside the accretion funnel, as shown in Figure 13.

With regard to disk dynamics, we observe a mixture of positive and negative velocities, overall pointing toward the turbulent nature of the accretion disk. Strong accretion is only detected inside the marginally stable orbit.

We now want to quantify the fluxes carried by the outflows. In Figure 14 we show the (normalized) radial mass flux, integrated along the polar angle at radius $r \sim 32$, for simulations sim0.1 and sim1.1. These values express the rate at which material flows outwards (positive values) or inwards (accretion, collapse) toward the black hole (negative values).

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4 As the disk evolves constantly, this choice is somewhat arbitrary. Nonetheless, the location of the disk surface is essential to a study of the disk-outflow interrelation, in particular the mass-loading of the wind.
We then compare three different regions along the polar angle. The first region is approximately $0^\circ < \theta < 25^\circ$, which is the funnel region close to the rotational axis. The second region is within $25^\circ < \theta < 65^\circ$ representing the area where a high-speed outflow may appear. The third region is within $65^\circ < \theta < 80^\circ$, which is the area of a potential massive disk wind. The respective areas of the lower hemisphere are also added to the integration of the mass fluxes.

The figure shows only the evolution of the mass flux for the later stages of the simulations, this is the time when a disk wind is established. When comparing the numerical values, we see that simulation sim0.1 shows stronger winds, with an average mass flux in the second and third regions of $5.7 \times 10^{-6}$ and $1.1 \times 10^{-5}$. In contrast, in simulation sim1.1 we find mass fluxes that are only about 50% of that, i.e., $2.5 \times 10^{-6}$, and $8.8 \times 10^{-6}$, respectively. Note that the only difference between these simulations is the quenching threshold (see Table 2). For sim1.1 the dynamo is quenched only at a higher magnetization level, therefore producing a correspondingly stronger flux (compare also to Figure 11). We learn from this comparison that an accretion disk carrying a stronger magnetic flux will eventually generate an outflow of lower mass flux. We emphasize, however, that simulation sim1.1 terminates far earlier than simulation sim0.1 due to a strong magnetic field close to the black hole.5 We suspect that if simulation sim1.1 had not crashed, a correspondingly stronger wind would have been formed.

We close this section by noting that wind is strong enough to affect the disk mass evolution, thereby changing the slope of the disk’s mass evolution toward a larger mass loss, i.e., a more rapidly decreasing disk mass (see Figure 9, bottom).

5.5. Generating a Blandford–Znajek Jet

Simulations with rotating black holes show the launching of strong outflows from the black hole’s magnetosphere, particularly in their later stages, when the magnetic field has engulfed the axial area. This is a clear signature of Blandford–Znajek jets driven by the black hole’s ergosphere. In Figure 15 we show the angular profile of mass flux, Poynting flux, and Lorentz factor, at radii $r = 8, 44$, and at $t = 7000$ for simulation sim0.1.

For the inner radius, the mass flux has negative values around the equatorial plane, which change into positive values for slightly higher and lower angles. Accretion toward the black hole is determined by the negative mass flux, while the mass outflow in the radial direction is attributed to a low-velocity wind from the inner disk, and is expressed by positive values. For angles closer to the polar axis, the mass flux decreases significantly. In this area, the mass flux is determined by the floor density chosen for the simulation, a feature that is common to all GRMHD simulations in the literature. Note, however, that in our case only the axial outflow is affected, and not the disk wind, which is loaded with disk material, and which carries a gas density substantially higher than the floor values. The Poynting flux remains largely negative for polar angles close to the equatorial plane. It is positive for angles closer to the axis, indicative of a Poynting-dominated Blandford–Znajek jet. Similarly, this is accompanied by an increasing Lorentz factor toward the axis, again implying the launching of a relativistic jet.

The picture becomes even more clear for the respective profiles derived for the larger radius. The Poynting flux is predominantly positive in all directions, with higher values detected for the relativistic jet; the case is similar for the Lorentz factor. Note that this Poynting flux is essentially generated by the disk dynamo, and is either launched directly from the disk’s surface, or, through advection of magnetic flux along the accretion disk, is further amplified by the black hole’s rotation.

The mass flux, however, gives a somewhat different picture. It exhibits variations across the equatorial plane, with quite large positive and negative deviations from an average value (more than 4 times lower, approximately, than the variations in the inner disk). At this point in time, at radius $r = 44$ the disk has completed almost 4 orbits around the black hole. This has to be compared to almost 45 disk orbits at $r = 8$. This far slower evolution of the outer parts of the disk, in combination with the entangled magnetic field, results in a far more turbulent accretion pattern in the outer disk regions. Reconnection also plays a role, as the entangled magnetic field may be anti-aligned (see also Vourellis et al. (2019) for a discussion of resistive effects in GRMHD jet launching). Note, however, that the evolutionary time of the outflow is more rapid than the disk evolution, meaning that the observed outflow dynamics provide an instant tracer of the launching conditions at the foot point of the outflow.

In Figure 16 we show the evolution of integrated Poynting flux (integrated along circles of constant radius, $r = 32$) for simulations sim0.1 and sim1.1, hosted by a rotating black hole where $a = 0.9$. We concentrate on two regions. One is the axial funnel area of $0^\circ < \theta < 25^\circ$, where the relativistic jet launched.

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5 This was possible due to the higher magnetization threshold selected for sim1.1.
by the black hole’s magnetosphere is propagating. The other is the disk wind area of $25^\circ < \theta < 65^\circ$, where massive outflows from the disk are being launched. The values of the symmetrical areas in the lower hemisphere are also taken into account. We focus on the later stages of the simulations, where the Poynting flux increases substantially.

The phase of strong Poynting flux begins approximately at the same time for both simulations. However, the higher threshold in plasma-$\beta$ for dynamo quenching allows simulation sim0.1 to establish a high Poynting flux for a much longer time. In both simulations, the Poynting flux measured in the jet funnel and in disk wind start out with similar strengths. Magnetic flux is advected, and when the area close to the black hole and the axial region is more and more encompassed by the dynamo-generated magnetic field, we see that the flux within the funnel increases, while the flux of the disk wind decreases (see the diverging lines in Figure 16, top panels). Our interpretation is that the magnetic flux, which is first anchored in the disk, is advected to the funnel, thereby increasing the funnel flux, and reducing the disk flux. In simulation sim1.1 in particular, this reconfiguration process is more obvious. In Figure 16 (bottom) we show the evolution of the averaged absolute value of the disk dynamo parameter, $\xi$, for the same two simulations. For simulation sim1.1, in which the quenching of the dynamo starts at around $\beta_{eq} = 100$, we notice that the flat profile for the dynamo coincides with the absence of Poynting flux until the sudden increase in Poynting flux. Later, when the flux from the disk wind decreases ($t \approx 7250$), we find a slight increase in the average dynamo parameter, especially where it includes the outer part of the disk.

These considerations are also supported by the time evolution of the magnetic flux of the disk and the black hole, as discussed above (see Figure 11). From the poloidal flux advected to the ergosphere, the black hole rotation will induce a toroidal magnetic field, thereby also increasing the Poynting flux. Note that at this point in time the dynamo action has been reduced significantly, due to quenching (see Section 5.3), and only a little magnetic flux can be generated. Thus, the dynamo action is saturated and in balance with quenching.

6. Summary

In this paper, we have extended our resistive GRMHD code rHARM3D (Qian et al. 2017; Vourellis et al. 2019), implementing a mean-field dynamo in order to study the generation of a large-scale magnetic field, and its effect on the launching of outflows. We have focused on a setup considering a thin accretion disk around a black hole, running a number of simulations with various thresholds of dynamo quenching, and also applying different Kerr parameters for black hole rotation.

Our mean-field dynamo simulations for a thin accretion disk are initiated with a weak seed field in the radial direction, which is confined to the disk. The diffusivity profile follows a plateau profile within the disk, and quickly drops across the disk surface toward an ideal MHD corona. Similarly, the profile for the mean-field dynamo parameter, $\xi$, follows a sinusoidal profile in the vertical direction from the equatorial plane, and a $1/\sqrt{r}$ profile along the radius.

We summarize our results below.
(1) Our implementation of the mean-field dynamo is based on the work of Bucciarelli & Del Zanna (2013), and is an extension of the purely resistive version of our code (Vourellis et al. 2019). The dynamo parameter, $\xi$ (corresponding to the common dynamo-alpha in the literature), is inserted into Ohm's law as a source of the magnetic field, and via Maxwell's equations into an equation for the evolution of the electric field. We also apply a quenching mechanism to mitigate the exponential increase in the magnetic field.

(2) We first construct an initial torus setup for our simulations. When comparing our results to those of Bugli et al. (2014), and Tomei et al. (2020), we find, in general, good agreement, in the sense that the dynamo produces similar field structures within the inner torus. In addition to the published literature we (i) continue the torus magnetic-field evolution to longer times scales, thereby being able to follow the advection of magnetic flux toward the black hole's rotational axis, and (ii) provide the magnetic field lines of the field generated by the torus dynamo.

(3) The magnetic-field evolution of the torus shows a saturation without applying a classical dynamo quenching for the $\alpha$-effect in the code, as reported by Tomei et al. (2020). We hypothesize that in this particular case, the saturation of the dynamo process was established by reconnection of the dynamo-produced tangled magnetic field. We propose this as another channel of dynamo quenching that works self-consistently in a turbulent state, as long as physical resistivity is considered.

(4) Overall, the dynamo works as expected based on non-relativistic simulations, with the magnetic field lines emerging from the disk interior, and expanding into the disk corona as the disk's magnetic energy increases rapidly from the beginning of the simulation. Dynamo action slows primarily due to the quenching function we apply.

The poloidal magnetic field predominates over the toroidal component, particularly in the outer parts of the disk, where the difference can be as much as an order of magnitude. The outer parts of the disk remain weakly magnetized (at the time steps investigated), but the disk magnetization still slowly increases until the final simulation times, with the toroidal field component once again predominating.

(5) We have also examined the evolution of large-scale magnetic flux from the black hole disk system for simulations applying different quenching levels for the dynamo, as well as for rotating and non-rotating black holes, as flux is an essential quantity for jet launching. The time evolution of the magnetic flux basically follows the evolution of magnetic energy, and is naturally affected by the quenching threshold. Even though the first stage is almost identical in all simulations, those with a lower quenching threshold in plasma-$\beta$ increase their magnetic field more rapidly in the later stages, to the point where they also terminate faster. Black hole rotation appears to induce a delay in the increase of magnetic flux, and also leads to higher levels of magnetization.

(6) The hydrodynamic evolution of the disk changes with the growth of the magnetic field. Initially, the radial structure of the seed field allows for episodic accretion until the strong, dynamo-generated field has evolved. The existence of a dynamo-generated, strong vertical field component allows for the gradual appearance of disk winds, which in turn lead to a smoothing of the otherwise quite peaky accretion rate. At the final stages of the simulations, the interplay between dynamo activity and dynamo quenching, together with the diffusion of magnetic flux, facilitates the development of a stronger disk wind, together with a relativistic axial jet.

(7) We have identified small-scale disk outflows, launched during the earlier stages of the simulations from the inner part of the accretion disk, which are interrelated with the presence of a strong magnetic field. These small-scale outflows maybe considered as disk flares. In the later stages of the simulations, strong disk winds are ejected along the poloidal magnetic field, being mainly supported by the magnetic pressure of the toroidal field component. We do not find any indication of Blandford–Payne-driven disk winds in our setup for the timescales considered, as disk winds are kinematically dominated (as per the findings in the simulations by Vourellis et al. 2019).

(8) In our simulations, considering rotating black holes, we observe an additional outflow structure in the form of a highly relativistic jet, apparently driven by the Blandford–Znajek mechanism. This jet carries a large electromagnetic energy flux when it leaves the ergosphere into the axial regions above the poles of the black hole (jet funnel). By measuring the Poynting flux, both in the jet funnel and in the disk wind, we find that the flux in the jet funnel increases over time, at the expense of the magnetic flux in the disk wind. While the dynamo is quenched during later simulation times, and the disk’s magnetic flux therefore becomes saturated, advection of magnetic flux along the disk toward the black hole leads to the high levels of Poynting flux we observe in the funnel.

In summary, we have applied, for the first time, the mean-field dynamo theory in the GRMHD context of thin accretion disks around black holes. We have investigated the evolution of the dynamo-generated magnetic field, the accretion disk, and the outflows launched with the help of the evolved magnetic field. In particular, we discuss the structure and evolution of the poloidal magnetic flux.

We find that the dynamo-generated magnetic field does not follow a clear dipolar structure, in contrast with non-relativistic studies in the literature. Future work is required to understand the mean-field dynamo action in GRMHD in more detail, in particular simulations running for longer timescales. The lack of a well-aligned, large-scale dipolar field does not permit the launch of strong disk winds via magneto-centrifugal driving. However, when the dynamo-generated magnetic field reaches a critical level, it is clearly capable of launching magnetic pressure-driven disk winds, as well as highly relativistic jets, from the black hole ergosphere.

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