The use of pumice spatial distribution to determine the initial conditions and the ballistic effects of Samalas eruption

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Abstract. Mount Barujari is an active volcano on Lombok Island located within the Mount Rinjani Complex. This spot is suspected of having had a massive eruption, hence the spread of pumice, with deposits assumed to have buried previously existing royal sites. The phenomenon of the distribution of volcanic sediments, especially pumice deposits, has not been studied qualitatively and quantitatively concerning the eruption dynamics in this complex. Therefore, the purpose of this research is to analyze the spatial distribution of pumice to determine the plume column dynamics and effects of the Samalas eruption 1257, and it was verified by the Kelud eruption in 2014. The results showed that the ballistic terminal velocity without atmospheric effects was 914.09 m/s, while the impact velocity was 955.50 m/s. These are both equivalents to 2.69 Mach (supersonic speed), resulting in a huge eruption explosion, which is heard up to a distance of 3000 km, often caused by the drag coefficient. Furthermore, the calculated drag coefficient shows the dynamics of current plume flow, including the transition zone between stable (Stoke's Law) and turbulent flow (Newton's Law). Simultaneously, the Samalas eruption's kinetic energy was established as equivalent to 7 on the VEI scale. Therefore, it is necessary to examine the magma Chambers and their dynamics in the future.

1. Introduction

Explosive eruptions throw rock fragments, including bombs, ballistic blocks, and magma discharge. Volcanic Ballistic Projectile (VBP) possesses a similar parabolic track, modified by several forces, especially the drag type, before landing on the earth's surface [1, 2, 3, 4, 5, 6, 7].

The interpretation of eruption dynamics parameters, including column height, is an important factor in determining a volcano's eruption strength. These, alongside historical eruptions, are possibly evaluated using the distribution of deposits that fall in the proximal and distal regions [8, 9]. In contrast, VBP poses a major threat in the proximal region, resulting from the influence of kinematic energy and high temperatures on life, the environment, leading to fires [5]. Besides, bombs and ballistic blocks are hazardous to flights and can also penetrate roofs, destroying lodges, humans, and animals, alongside inflicting damage to other infrastructure [9].

The grain size distribution of volcanic and ballistic material is important for hazard assessment and interpretation of eruption dynamics, including column height and the explosion mechanism. Also, there is a possibility of utilizing data on material transportation to determine ballistic discharge, landing speed, and kinetic energy. [10] measured the vulnerability of buildings towards the ballistic...
effects by calculating the projectile velocity, mass, and energy, while [11] evaluated the vulnerability of Kelud eruption in 2014 by calculating the eruption rate (MER: Mass Eruption Rate).

Some researchers, including [12], [13], and [14], reported on the dynamics of VBP, although the initial ejecta and impact velocity was not determined. [15] and [16] examined the possibility of using the initial ejecta velocity to calculate the maximum horizontal and vertical distance achieved by ballistics. In contrast, ejecta kinetic energy and impact are determined after obtaining data on speed. Besides, it is feasible to design buildings with the ability to be impact resistant or built outside the reach of ballistics. Therefore, the VBP method is possibly used to reduce the risk of dangerous bomb drops and ballistic blocks during volcanic eruptions.

The wind controls the explosion column height and distribution of materials emerging from the vent. Moreover, ballistic trajectories are followed by elastic discharges near the column base and situated at certain angles [8]. Therefore, the range achieved by these clusters is determined using data on takeoff speed, exit angle, cluster size, shape, and density.

Acceleration of normal gravity, drag coefficient, Reynolds number, and atmospheric conditions during the eruption incidence is another important basic factor for determining ejecta velocity, ballistic range, and type of fluid flow. The acceleration of gravity is dependent on the latitude and altitude position. The drag coefficient values influence the acceleration of gravity. These fundamental features are quantitatively analyzed for the 1257 Samalas eruption.

This study investigates the eruption of Mount Samalas in 1257, followed by the verification with the Kelud eruption in 2014. Therefore, the purpose of this study was: (1) Analyze gravitational acceleration, height functions, and latitude position. (2) Calculate ballistic terminal velocity, Mach numbers, Reynolds numbers, and ballistic drag coefficients, individually with and without the floating effect. (3) VBP modeling and (4) calculating the kinetic energy values as well as VEI estimations.

Ballistic trajectories are similar to parabolic paths in the air, influenced by the wind and terminal freefall [12]. Bombs and ballistic blocks are injected with high kinetic energy, and sometimes with thermal energy, as the ranges obtained are possibly used to trace and illustrate the eruption dynamics.

2. Method
2.1. Gravitational acceleration of altitude function in atmosphere
The atmospheric terminal velocity is determined by first analyzing the effect of stratigraphy on gravitational acceleration value. This is dependent on the altitude and latitude, which is calculated according to Newton's laws of gravity [17], as follows:

$$\vec{F} = G \frac{M m}{r^2} \hat{r}$$

Where \( F \) is the force of gravity, \( G \) is the general gravitational constant, \( M \) is the mass of the sun. The gravitational acceleration \( g \) at certain height functions above sea level \( r + h \) is obtained by dividing equation (1) with the mass of the Earth, \( m \):

$$g(r + h) = G \frac{M}{(r+h)^2} = g_0 \frac{r^2}{(r+h)^2} \hat{r}$$

where \( g \) (\( r + h \)) is the acceleration due to gravity above the Earth's surface \( h \) and \( r \) is the radius of the Earth. Assuming that the radius of the earth \( r \) is constant, then the value of \( g \) only varies at a function of height \( h \), hence equation (2) is written as \( g(h) \):

$$g(h) = g_0 \frac{r^2}{(r+h)^2}$$

\( g_0 \): Standard gravitational acceleration from the World Geodetic System 1984 (WGS84) of Chebyshev approach [18]:

$$g_0 = 9.780327(1 + 0.0052790414 \sin^2 \phi + 0.000 023 2718 \sin^4 \phi + 0.000 000 1262 \sin^6 \phi + 0.000 000 0007 \sin^8 \phi)$$

\( g_0 \) is a function of the latitude position \( \phi \) where the volcano is. Therefore, the atmospheric terminal velocity is measured as a function of height and latitude position, \( g(h, \phi) \). \( \phi \) is the latitude positions.
2.2. Terminal velocity and ballistic number mach

2.2.1. Terminal velocity without atmospheric effects

Terminal velocity ($v_t$) is an indirect measure of the strength of a volcanic eruption. Without atmospheric effects, this is possibly calculated with the function of height (3) and latitude (4) as follows,

$$v_t = \sqrt{2g(h, \varnothing)h}, \quad \text{(5)}$$

when $h$ is the height above sea level, which is the plume summit of the volcanic eruption. Moreover, the record for Samalas blob was 43 km [19] while 3.73 km was measured at the area peak of Mount Rinjani in the position at the southern latitude. However, Mount Kelud, which erupted in 2014 with column height based on four versions of researchers respectively were: i) 17 km [21], ii) 18 and 20 km [22], iii) 18 and 23 km [23], and iv) 20 and 26 km [24].

2.2.2 Mach Number of Ballistic. In fluid dynamics, Mach numbers (Mach) are dimensionless that represent the speed of fluid flow ($u$), which exceeds the value for local sound ($c$),

$$\text{Mach} = \frac{u}{c} \quad \text{(6)}$$

c is the speed of sound in the medium at the boundary between internal and external flow, and $u$ is the local fluid flow rate.

2.3. Drag coefficient ($Cd$) and ballistic Reynolds number

2.3.1 Ballistic drag coefficient due to air friction. The ballistic drag coefficient ($Cd$) is caused by friction with the surrounding air and fluid during the eruption. This $Cd$ is calculated based on Newton's law which states that the magnitude of the force in the vertical direction on an object at maximum height is zero. Therefore, this occurs due to gravitational force, which moves in the direction is toward the earth center ($W$), while the frictional force ($Fd$) is in the opposite direction of ballistic motion. According to the second Newton law in a balanced state, the amount of force is zero, represent by $W - F_d = 0$, which is further simplified to obtain the drag coefficient ($Cd$) as follows,

$$Cd = \frac{8g \varnothing d \rho_b}{3 \rho_m v_t^2} \quad \text{(7)}$$

2.3.2 Cd due to medium buoyancy effect. The force on an object is the amount of gravitation ($W$), and buoyancy ($F_b$), especially in atmospheric stratigraphy, as well as drag force ($Fd$). Under similar conditions, the total of these forces are $\sum(W - F_b - F_d) = 0$, hence the drag coefficient ($Cd$), which is affected by air buoyancy is further stated with the following equation:

$$Cd = \frac{4g \varnothing d}{3v_t^2} \left( \frac{\rho_b - \rho_m}{\rho_m} \right) \quad \text{(8)}$$

The input parameters of equation (8) are terminal velocity ($v_t$), gravitational acceleration ($g$) at plume height, grain diameter ($d$), density ($\rho_b$), ballistic shape, and atmospheric density ($\rho_m$).

2.3.3 Reynolds Number ($Re_e$). The Reynolds number ($Re$) is estimated after calculating the drag coefficient. This is subsequently divided into three categories of flow zones, according to the following equation [25]:

$$Re = \frac{24}{Cd}, \text{ laminar flow zone (Stokes' Law)} \ (Re \leq 1) \quad \text{(9)}$$

$$Cd = \frac{24}{Re} \left( 1 + 0.15Re^{0.687} \right), \text{transition region} \ (1 < Re < 1000) \quad \text{(10)}$$

$$Re \equiv 0.44, \text{turbulent flow (Newton's Law)} \ (Re > 1000) \quad \text{(11)}$$
2.4. VBP modeling of Samalas eruption in 1257 and Mount Kelud in 2014

Without the effects of drag and wind, the equation of ballistic projectile motion is stated in two components, comprising of the horizontal and vertical, stipulated as \( y(x, \theta) \) [26]. The horizontal distances is \( x \) and \( y \) is the plume height and \( \theta \) is the latitude position. This function is obtained by eliminating \( t \) in the horizontal components \( (x) \) and substituting the result into the vertical component equation and shown below:

\[
y(x, \theta) = y_o + x \tan \theta - \frac{gx^2}{2(v \cos \theta)^2}
\] (12)

This equation (12) is relevant only for ballistic transportation in the air. Therefore, the parameters of maximum plume height \( (H_{\text{max}}) \), ejecta velocity \( (v_i) \), maximum distance \( (D) \), and impact velocity \( (v_f) \) are simultaneously calculated. Conversely, the initial input for ballistic modeling is the mountain height \( (y_o) \), at 3726 m for Mount Rinjani, at the southern latitude 8.25°, while Mount Kelud possesses an altitude of 1731 m, at the southern latitude of 7.93°. The second input value was \( g \) at the peak of each plume. Besides, the eruption column measurement for Mount Samalas in 1257 [19] was 43 km, while the Mount Kelud plume in 2014, based on four researches were: i) 17 km [21], ii) 18 and 20 km [22], iii) 18 and 23 km [23], and iv) 20 and 26 km [24].

2.5. Ballistic kinetic energy of Mount Samalas eruption in 1257 and Mount Kelud in 2014

The kinetic energy of the eruption of Mount Samalas in 1257 was calculated using the law of conservation of mechanical energy. Mechanical energy includes: (1) Potential Energy \( (\text{PE}) \) which depends on height, acceleration due to gravity, and mass of ejecta, and (2) Kinetic Energy \( (\text{KE}) \), which depends on the object's mass and the square of the terminal velocity. Therefore, the ejecta ballistic kinetic energy and impact are both determined by assuming a ball shaped material with radius \( (r) \) and density \( (\rho) \) as follows,

\[
\text{KE} = \frac{1}{2}mv_i^2 = \frac{2}{3} \pi r^3 \rho v_i^2
\] (13)

From the equation, \( m \) denotes mass \((\text{kg})\), \( v \) is velocity, \( v_i \) \((\text{m/s})\) is the material velocity escape from the vent. Therefore, knowledge on the eruption kinetic energy is needed to calculate flux or eruption rate, which is then used to estimate the eruption VEI index value, according to [20].

3. Results and discussion

3.1 Gravitational acceleration of altitude function

Variations in gravitational acceleration \( (g) \) with the altitude have previously been analyzed. The plume column height is assumed to be influenced by atmospheric stratigraphy (Figure 1). Atmospheric stratigraphy which consists of: (i) Troposphere: 0 – 10 km, (ii) Stratosphere: 10 – 50 km, (iii) Mesosphere: 50 – 85 km, (iv) Thermosphere 85 – 500 km, and (v) Exosphere, with altitude of over 500 km (Figure 1a). Besides, a different value for gravitational acceleration was reported as different in each stratigraphy, because of the distinct effects of particle content, temperature, and pressure.

Figure 1a shows the gravitational acceleration profile of the altitude function to the exosphere (600 km). Based on Eq. 3, the normal values are calculated at the latitude position of the location, which is Mount Rinjani area, being the source of a massive eruption in 1257, at the southern direction of 8.42°. Acceleration due to gravity decreases by 8.61% (from 9.80665 m \( / s^2 \) to 8.962583 m \( / s^2 \)) as altitude increases from 0 km to 600 km above sea level.

Figure 1b is the gravitational acceleration profile for altitudes from zero to 50 km above sea level, where 9.831957 \( / s^2 \) was recorded at zero altitudes and \( g = 9.7553986 \ m / s^2 \) at 50 km above sea level. Therefore, a decrease by 0.78% was observed with the rise from zero to 50 km.

From the two examples above, it can be concluded that the higher above sea level, an eruption event will be easier to reach a larger column height. This occurs because the force of gravity reduces in the opposite direction of thrust from inside the eruption source.
Figure 1b shows the special gravity acceleration at the peak of Mount Rinjani (3.73 km) and the plume (43 km), where the value of $g$ changes from $9.826210 \text{ m/s}^2$ to $9.766043 \text{ m/s}^2$ characterized by a 0.61% decline. Likewise, when compared with the value of $g$ in the crater of Mount Kelud (1.7 km) and at the peak of the burst (26 km), it decreased from $9.781313 \text{ m/s}^2$ to $9.741554 \text{ m/s}^2$. The value of $g$ decreased 0.41%. It appears that the value of the earth's gravitational acceleration is strongly influenced by altitude, therefore the value of $g$ needs to be taken into account when analyzing the dynamics of volcanic eruptions.

This parameter is also used to analyze the eruption of Mount Samalas and Mount Kelud. The values obtained for $g$ are used to calculate the terminal velocity, Reynolds number, and drag coefficient of volcanic material spurts during the eruption.

> **Figure 1** a) Analysis of the gravitational acceleration as a function of atmospheric stratigraphy. The normal gravitational acceleration ($g_o$) at an altitude of $H = 0 \text{ m}$, is $9.831957 \text{ m/s}^2$ and at an altitude of 600 km, $g$ drops to $8.985712 \text{ m/s}^2$. b). The value of $g$ above the summit of Mount Rinjani (3.726 km) is $9.826210 \text{ m/s}^2$ and $9.7660435 \text{ m/s}^2$ at the peak of the plume (43 km).

### 3.2 Terminal velocity without the effect of atmosphere and ballistic number Mach

The ballistic terminal velocity of Mount Samalas in 1257 was calculated on the basis of plume height (43 km) [19], while the normal gravitational acceleration was evaluated based on WGS84 [18]. Besides, $g = 9.781427 \text{ m/s}^2$ at the southern latitude of $8.42^0$, while $g$ at an altitude of 43 km above sea level is $9.715852 \text{ m/s}^2$, and the terminal velocity at the plume peak was $v = 914.09 \text{ m/s}$.

For comparison, data from the Kelud eruption in 2014 at the southern latitudes of $7.93^0$ showed $g = 9.7813130 \text{ m/s}^2$ and the values recorded at the peak of each ballistic plume, based on the four researchers above were: 9.755283, 9.753756, 9.750703, and 9.741558 in units of $\text{m/s}^2$. Besides, the calculated terminal velocity were 575.92, 592.57, 624.52, and 711.73 in units of $\text{m/s}$.

The data above demonstrates supersonic speed in both ballistic velocities, exceeding the momentum of sound, which is $340.3 \text{ m/s}$, according to the "International Standard" at a standardized temperature of $15^\circ \text{C}$ [27]. The Kelud eruption in 2014 occurred in Sleman Yogyakarta and was heard at a distance of 218 km, while [29] reported that the voices heard reached 200 km. This is evidence that the sound of the 2014 eruption of Mount Kelud with the VEI index of 5 was a supersonic explosion. The sound of the eruption is similar, namely the sound of the eruption of Mount Tambora in 1815 (VEI 7.0) heard up to a distance of 2600 km [29]. So, how did the sound of the eruption of Mount Samalas in 1257 that occurred with a VEI 7.0 index and a speed of 2.69 Mach and a burst height of 43 km above sea level? Of course the explosion is relatively strong and sounds further away,
and the drag coefficient is higher. The drag coefficient is one of the factors that influence the size of the eruption.

### 3.3 Drag coefficient and ballistic Reynolds number

The drag coefficient \( C_d \) was then determined by analyzing the main forces acting on the object, which include gravity, thickness and drag. These calculations use Newton’s II law of motion, where the total amount of force equates to \( \sum F = 0 \). Furthermore, the friction and buoyancy effects of fluids passed by ballistics during eruption are calculated in this study, and \( C_d \) is related to the effect of viscosity, according to Stokes law, which more suitable for the movement of small particles (diameter less than 2 mm) at low speeds. This momentum is indicated by a small Reynolds number \( R_e \) less 1, as the current research focuses on bomb sizes and ballistic blocks (more than 64 mm diameter), while the effect of viscosity is not calculated.

The value of \( C_d \) is often calculated after determining \( R_e \) first, on the contrary in this paper, the value of \( C_d \) is calculated first. The terminal velocity is estimated as a function of altitude, where the burst height is known. Moreover, the input parameters for calculating \( C_d \) were functions of gravitational acceleration, terminal velocity, grain density and size, observed directly from the field, while variations in the resulting values are indicative of changes in \( R_e \). This is used alongside the Mach number to demonstrate the fluid flow regime limits, where \( R_e \) is defined as \( R_e = v * d / \rho \), \( v \): ballistic terminal velocity, \( d \): ballistic diameter, and \( \rho \): kinematic viscosity of the media. Furthermore, Mach is defined as the ratio between ballistic speed and speed of sound (c). Mach is indirectly influenced by the two parameters, namely the effect of atmospheric buoyancy and without this effect.

The results achieved with and without ballistic float at the Samalas and Kelud eruptions in 1257 and 2014, respectively, show all \( C_d \) values as less than one. This indicates a high terminal velocity, as drag coefficient is inversely proportional to the square of speed. Based on the calculations, \( R_e \) value was recognized in the transition zone (\( R_e \) between 1 and 1000), which is inversely proportional to \( C_d \). The distribution of grain size and buoyancy density is inversely related to the drag coefficient, although density has a greater influence on \( C_d \) than on surface area. Besides, shape and porosity have also been suspected as strongly dependent parameters to be discussed in the future. Therefore, what is the effect of distance on the \( C_d \) value?

Blocks and bombs usually comes out from volcanic thrust zones in large fragments, indicating a relative closeness to the source. Pumice falls from the eruption of Samalas in 1257 are confirmed to have fallen at a distance of 9 km (LN sample), 17.51 km (ML sample) and 26.79 km (IB sample) respectively to the southeast of Sumber. Likewise, the pumice stone was distributed to the south, at distances of 4.37 km and 14.13, respectively (BK and BS samples). Comparatively, the Kelud 2014 samples: Kl-1 was identified at less than one km, while Kl-2 fell at closer than 6 km. The same thing was reported by Andersen (a photographer) who had visited the site on 22 and 23 February 2014 that the volcanic ash deposits and bombs were found at a distance of 2 to 5 km from the Kelud crater (source). Furthermore, [30] demonstrated a similar result, indicating the flight of small blocks and bombs by up to (20 – 80) km at speeds of (75 – 200) m/s.

[31] used \( C_d \) values: 0.2, 0.5, and 0.8 to explain its importance, which is dependent on the rate of expansion and gas clutch or separate discharges. [32] reported that values near the gas thrust region are close to zero, which subsequently increases with a rise in gas, as observed in separate columns, and is followed by an eruption. [13] used \( C_d \) value=1; [33, 34] adopted 0.65 for subsonic, and 1.25 for supersonic flow, while [16] used 0.8.

In relation to variations in object shape, the results of \( C_d \) measurements include: ball: 0.47, half ball: 0.42, Cone: 0.50, cube: 1.05, cube angle: 0.8, long cylinder: 0.82, Short cylinder: 1.15, slim body: 0.04, half-slim body: 0.09. Besides, all structures indicate the falling of larger \( C_d \) value closer to the source. This is usually caused by low density, surface area, shape, roughness and porosity to be learned in the future.

Without atmospheric effects, ballistics is built based on equation (12). This VBP modeling is performed on two object prototypes, including the Samalas eruption in 1257 and Mount Kelud in
2014, using the input parameters of the initial height of the Mountain ($y_i$). Furthermore, the values recorded for Mount Rinjani, and Kelud were $y_i = 3726$ m and $y_i = 1731$ m, respectively, while the angle to the horizontal axis and initial speed are obtained simultaneously by trial and error.

VBP modelling was carried out by considering the influence of the atmosphere and without the influence of the atmosphere for the two locations, namely Mount Samalas and Mount Kelud. VBP modelling is carried out in several main steps, namely the first stage, calculating the variation in the acceleration of gravity as a function of height ($g (h)$). The second stage, calculating the terminal velocity. Finally, perform VBP modelling using the following input parameters: terminal velocity, installation and plume velocity. This is followed by the calculation and analysis of Mach numbers. Furthermore, the outputs obtained include: $g$ value variations with respect to altitude, terminal velocity, VBP model, exit angle, maximum height, maximum ballistic distance, landing speed, and total flight time in the air.

The second stage involves the modeling of VBP, using Newton II's laws of motion, which considers atmospheric effects. The related parameters include drag coefficient, resulting from the effects of air friction and the influence of buoyancy are affiliated with Reynolds numbers and Mach. Due to the accordance of viscosity with Stokes law, the effect is ignored in the movement of small particles (ash less than 2 mm) at low speeds, indicated by small Reynolds numbers ($Re$ less than 1).

### 3.4 VBP modeling of Mount Samalas eruption in 1257 and Mount Kelud in 2014

#### 3.4.1 VBP Model of Mount Samalas eruption in 1257.

Based on the calculation results, the ballistics ejected from Mount Samalas in 1257 possess a velocity of 914.09 m/s and an ejecta angle of $72.25^\circ$. This degree used in exit is in accordance with the report of [35], where the caldera wall of Mount Rinjani was observed to be elliptical, with a slope of $60^\circ – 80^\circ$. Therefore, both parameters collected at the site are possibly used to determine the maximum range of tephra fall, by evaluating the kinetic energy of eruption, falling speed, impact energy, and the duration of ballistics in the air.

Figure 2 shows the VBP model of Mount Samalas, which indicates the flight through the ventilation mouth at a speed of 914.09 m/s into the air, attaining a maximum plume of 42896.09 m above sea level. Furthermore, these ballistics fall from the highest point at a distance of 25076.88 m from the center of the eruption. This decreased to zero speed (at the highest point), followed by ground landing at a distance of 51319.39 m, from the source, with an impact momentum of 955.50 m/s. Conversely, the total time spent during the flight was 18.11663 seconds.

The calculated VBP model demonstrates a maximum horizontal fall distance of 51.32 km from the center of the eruption. [19] recognized some pumice deposits (diameter: 50 mm) at a distance of 46 km east of the source, on Sumbawa Island, denoting the eruption strength of Mount Samalas. Besides, the maximum dispersion of fragment toss depends on the ejecta angle, as well as the grain size, character and distribution, initial velocity and the coefficient of fragment flakes [34].

Several studies have previously explained the VBP movement [12, 14, 32, 34, 35]. These reported on the use of initial ballistic speed during the eruption process to calculate and determine the maximum range, and models based on the main forces, which act on ballistics, including gravitational and drag forces were also used. Besides, the Lapili and bomb particle velocities recorded were 213 m/s and 129 m/s, respectively [15], while that for ballistic tend to reach over 2000 m/s [16].
Figure 2. The Samalas Mountain ballistic model in 1257 with an altitude of 3726 m above sea level. The initial conditions parameter model are: \(v_t = 914.09\) m/s, \(\theta = 72.25^\circ\), \(D = 51319.38\) m, \(H_T = 42896.08\) m, velocity of impact \(v_f = 955.52\) m/s. Flight duration \(t = 18.12\) s.

3.4.2 VBP model of Mount Kelud eruption in 2014. Figure 3 is a ballistic VBP model for the Kelud eruption in 2014, built based on the plume height. These were recorded by four groups of previous researchers, with the following respective codes: VBP_H17, VBP_H18, VBP_H20, and VBP_H26.

Figure 3a represents the VBP_H17 profile of the Kelud eruption, which was based on the plume height of 17 km [21] and other parameters obtained include angle and velocity of the material emerging from the ventilation (Crater) at an altitude of 1731 m asl. Therefore, the model parameters include the ejecta angle of 71.5 degrees, the terminal velocity of 575.92 m/s, and maximum plume height of 17.023 km above sea level. Conversely, the maximum horizontal ballistics landing distance was 21.04 km, with a speed of 606.10 m/s and the flight duration from ventilation to landing was 114.68 seconds.

According to [23], figure 3b is the second version of VBP_H18 with plume height of 18 km. Pyroclastic material exits the vent with a slope angle of 71.89° and an exit speed of 594.22 m/s. Furthermore, pyroclastic material flew into the air reaching an altitude of 18.85 km above sea level, then descending and landing at a distance of 21.84 km from the center of the eruption at a speed of 622.14 m/s. Meanwhile, the differences in height between the observed plume (event riel) and the calculation model was 5.49 m, with flight duration of 118.13 seconds.

Figure 3c shows the third version of plume height, according to [22], which demonstrated pyroclastic flow from a height of 1731 m asl in air, observed at an angle of 71.89° to horizontal, with a speed of 626.31 m/s. These materials fly attain heights of about 20.00 km above sea level, and subsequently fall to the ground at a span of 23.078 km from the source, with 652.86 m/s impact speed. Therefore, the variation in model height (calculation) and observed measurement was 4.29 m (0.02% error), with a time duration of 124.84 seconds.
In figure 3d, the upper limit of the plume reached 26 km [22, 23], while the results on calculation show the attainment of 25.99 km plume heights. However, the difference between the observed and calculated model value was only 2 m (error 0.007%). Besides, the pyroclastic flow rate at an exit angle of 75° to the horizontal was 711.73 m/s, and the materials subsequently landed on the ground at a distance of 26.466 km from the eruption center. This occurred at a fall speed of 737.39 m/s and a flight duration of 143.07 seconds from launch.

The ballistic terminal velocity of the two mountains is included in the supersonic sound speed group, because it has a sound speed of between 412 and 1715 meters per second which is equivalent to Mach 1.2 to Mach 5.0. These values indicate a tremendous eruption, which is possibly adopted in the design of walls and glasses of modern aircraft and also to ensure safety on entry into extreme atmospheric stratigraphy. Furthermore, terminal velocity is also one of the characteristics used to determine the pattern and thickness of a pyroclastic deposit.

This parameter controls the individual pyroclastic deposit characteristics [12, 34], while the terminal velocity recorded for ballistics of Mount Samalas (914.09 m/s) exceeds supersonic speed. This high value tends to be produced by eruptions with magma, with the following chemical composition: SiO₂ (60 – 75)% and Al₂O₃ (12 – 16)%, as well as N glass (main) 85 and the N volatile matrix contents [19]. Furthermore, the compound is assumed to be most responsible for thickening the magma, producing air bubbles and higher gas pressure, and also functioning as a coolant. In addition, the last two compounds are highly explosive, and are involved in producing extremely unstable and very large eruptions characterized by high speed, loud noise, and high kinetic energy.

![Figure 3](image-url)  
**Figure 3.** Mount Kelud ballistic model in 2014 with a height, \( H_o = 1731 \) m in succession: a. VBP_H17, b. VBP_H18, c. VBP_H20, and d. VBP_H26. The source of data: [21, 22, 23, 24]. VBP_H: volcanic ballistic projectile at height of plume.
3.5 Volcanic kinetic energy and Volcanic Eruption Index

3.5.1 Kinetic energy eruption of Mount Samalas in 1257. We have calculated the kinetic and impact energies, as well as the eruption strength of the Samalas volcano in 1257 at the Mount Rinjani Complex. This was calculated based on the grain size distribution, and ballistic volume. Besides, the volume used to calculate eruption kinetic energy were 58.39 km$^3$ from the evaluation and 40.2 km$^3$ DRE [19].

The results of kinetic energy calculations (in logarithms) range between (12.20 – 12.92) joule, which was reported to be (12.04 – 12.76) joule, using the [19] version. Similarly, the impact kinetic energy was between (12.24 – 12.96) joule and, while the [19] version was (12.07 – 12.79) joule. The values for both were equivalent to 7 on the VEI scale, and the differences observed after comparing was only 3.76% in both ejecta and impact energy.

Variations were observed only in terms of terminal velocity and falling speed, as the value recorded for collision (955.50 m/s) was higher than ejecta (914.09 m/s). This happens because both occur in opposite directions, namely in the form of a collision that hits the surface of the earth due to the acceleration of gravity, while the ejecta velocity is opposite to the gravitational force, as a result the speed is slower.

3.5.2 Kinetic energy eruption of Mount Kelud in 2014. This method is also used to calculate the kinetic energy of the Kelud eruption in 2014. Based on the two-speeds version of 575.73 m/s and 711.73 m/s, denoting the plume peak at 17 km and 26 km above sea level, respectively, while the volume used include 1.5 x 10$^8$ m$^3$ and 2.0 x10$^8$ m$^3$.

The results show that both velocity and volume of 1.5 x 10$^8$ m$^3$ (3.49 x10$^{10}$ – 6.67 x 10$^{15}$) joule and the 2.0x10$^8$ m$^3$ versions (4.66 x10$^{10}$ – 8.89 x 10$^{15}$) joule were within the range. Similarly, the impact (fall) kinetic energy for the volume of 1.5 x 10$^8$ m$^3$ was between 3.87 x 10$^{10}$ and 7.17 x10$^{15}$ joule, which was (5.16x10$^{10}$ – 9.56x10$^{15}$) joule for the 2.0 x10$^8$ m$^3$ measurement.

The kinetic energy of the ejecta, as well as the possible effects, appear to be different between both mountains (Samalas and Kelud), although the value for impact was collectively greater than release. The interpretations are made in the order of 10$^{15}$ joule.

Furthermore, kinetic energy is converted to Volcanic Explosivity Index (VEI), according to [20] which was reported to be between 3 and 8 for the Kelud eruption in 2014. Based on the plume height (20 – 26 km) and the dominant grain size (20 – 40 mm) identified at a fall distance of 10 – 15 km from the center of the eruption, the most convincing VEI was estimated as 5. This is in line with the report by [21], where the height (17 – 28) km was equivalent to a VEI of 5. Also, another indication of the Kelud eruption severity in 2014 was the explosion sound heard in Yogyakarta, which was also stated by [22]. A similar case is the Tambora eruption in 1815 with an index of VEI 6.9 which was heard at a distance of 2600 km [23]. Therefore, the sound of the Samalas explosion in 1257 with an index of VEI 7 was possibly heard at a distance of over 2600 km.

4. Conclusions

Our research contributes to the determination of the static and dynamic parameters of a volcanic eruption based on the laws of classical mechanics. So far, these two parameters (static and dynamic) are only assumptions. Based on the results on some characteristics, including pumice deposits distribution, gravitational acceleration analysis, height function velocity, terminal velocity, ballistic impact velocity, Mach number, Reynolds number, drag coefficient, ejecta kinetic energy model, and ballistic impact kinetic energy model, the following was concluded: The parameter of the Samalas eruption ballistic model in 1257 provided a terminal velocity of 914.09 m/s, which is equivalent to the 2.69 Mach (supersonic sound), heard at a distance of over 2600 km. Besides, the tilt angle of the ejecta was 72.25$^0$, with plume column height of 42.25 km above the Stratosphere, while (12.20 – 12.92) joule was recorded as the Log of ejecta kinetic energy. Meanwhile, the ballistic impact parameters include a speed of 955.50 m/s, which extended through a maximum distance of 51.32 km, with flight duration of 18.12 seconds. Also, the Log of impact energy recorded was (12.24 – 12.96) joule, and
both were equivalent to VEI 7. The static parameters that are not directly related to the eruption dynamics parameters, but of high importance before, during, and after the Samalas eruption process in 1257. These include drag coefficient and Reynolds number without buoyancy effects at 0.149 – 0.375 and 0.84 – 256, respectively, while the values reported with the influence of buoyancy were 0.075 – 0.188 and 193 – 2264, respectively.

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