Generation of surface-plasmon-polariton like resonance mode on microwave metallic gratings

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Abstract. In this work, transmission resonance modes in microwave metallic gratings are discussed with focus on the physical origins of the surface-plasmon-polariton (SPP)-like resonance mode and its relationship with the enhanced transmission phenomenon. A pseudoanalytical theoretical model using the surface impedance boundary conditions was introduced to characterize the electromagnetic (EM) diffraction of the grating, which was examined by a numerical calculation based on a finite element method. The numerically predicted near-field diffraction pattern as well as the photonic band diagram showed that the SPP-like mode in the microwave grating was of a hybrid mode character represented by the coupled resonance of bounded mode on the grating surface and localized standing wave inside the grating slit. Redistribution of the diffraction energy associated with the mode coupling would give rise to enhanced transmission efficiency when the standing wave inside the grating slit was strengthened. The results were compared with SPP-assisted light transmission over metallic optical aperture arrays. The present work is believed to contribute towards extending our understanding of the EM interaction in microwave photonic crystals.

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1. Introduction

In the past few years electromagnetic (EM) interactions of metallic subwavelength aperture arrays have regained great research interest due to the extraordinary light transmission efficiency or light focusing effects [1]–[5]. Generally, excitation of surface-plasmon-polaritons (SPP), collective oscillation of surface electrons, has been thought to play a key role in these effects, which originate from the unique dielectric response of a metal. According to the Drude’s mode which treats the metal classically as a gas of electrons, a metal’s relative permittivity $\varepsilon_m$ is a function of frequency

$$\varepsilon_m = 1 - \frac{\omega_p^2 \tau^2}{1 + \omega^2 \tau^2} + i \frac{\omega_p \tau}{\omega (1 + \omega_p^2 \tau^2)}, \quad (1)$$

where $\omega_p$ is the plasmon frequency, $\tau$ is the relaxation time related to the electron mean free path before collision, and $\omega$ is the angular frequency. $\omega_p$ and $\tau$ are related to the dc conductivity $\sigma_0$ of the metal by $\omega_p^2 \tau = \sigma_0 / \varepsilon_0$. For aluminium, $\omega_p = 2.28 \times 10^{16}$ rad s$^{-1}$ and $\tau = 4.13 \times 10^{-14}$ s.

At the optical or far-infrared frequency, the real part of $\varepsilon_m$ will be dominant by a large negative value at $\omega < \omega_p$. According to Maxwell’s equations, the propagation constant $k_{spp}$ of the surface wave at the dielectric/metal interface can be expressed by

$$k_{spp} = \frac{\omega}{c} \sqrt{\frac{\varepsilon_d \varepsilon_m}{\varepsilon_d + \varepsilon_m}}, \quad (2)$$

where $\varepsilon_d$ is the relative permittivity of the dielectric part space. From equations (1) and (2), it can be seen that $k_{spp}$ should be highly dispersive at the frequency near $\omega_p$ and its real value is larger than the wavevector $k_0(=\omega/c)$ of the incident light. Thus, additional momentum is needed to excite the SPPs on a metal surface. This difference can be compensated by introducing an artificial lattice vector by periodically modulating the metal surface into corrugated structures such as a hole array [6] or grating [7]. This modulation will physically fold the SPP’s band into the light radiation cone.

But at low frequencies ($\omega \ll \omega_p$) like microwave or infrared, of interest in this work, $\omega \tau \ll 1$. The metal permittivity is dominated by its dc conductivity and it has a huge imaginary part. In this case equation (2) can be simplified into $k_{spp} \approx k_0$. The surface wave is very easily excited at microwave wavelengths since $(k_{spp} - k_0)$ is very small. The dielectric response of a metal is better approximated by the surface current flow rather than the free electron oscillation. At microwave wavelength, the surface mode decays quite slowly from the dielectric/meta interface into the dielectric space with a typical decay length from several to a hundred

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wavelengths. It can penetrate into the metal by several microns as typically characterized by the skin depth \( \delta = \sqrt{2/\omega \mu \sigma} \). Their EM response can still possess SPP-like characteristics in periodical aperture arrays, as theoretically predicted by Pendry and Garcia-Vidal on metals perforated by subwavelength hole arrays or slits \([8, 9]\). Experimental work has been done by Hibbins et al \([10]\) to verify the existence of the SPP-like mode on the surface of brass tube arrays periodically disturbed by metal strips. Direct characterization of the field distribution of the surface mode on a metallic hole array was even performed by Hou et al \([11]\) at normal microwave illumination. In addition, the out-of-phase resonance in adjacent slits recently found on microwave compound metallic gratings also confirms the existence of a surface mode which could mediate the interslit mode coupling \([12]–[14]\).

The microwave metallic grating of subwavelength period is a kind of photonic crystal. Although the SPP-like EM response has been widely reported, there is no systematic work to illustrate the mechanism for generation of the SPP-like mode on a metallic structure and its relation to the transmission efficiency. Answers to these questions should be of great interest for extending our understanding of the EM interactions in microwave photonic crystals. In this work, we will give a comprehensive analysis of the SPP-like surface wave on a microwave metallic grating from the transmittance spectrum, the diffraction field pattern and the photonic band diagram. We will show that a hybrid resonance mode from the coupling of the bounded surface wave and the localized cavity mode is responsible for the SPP-like mode observed on the microwave metallic grating. It will also be shown that the cavity mode excited inside the grating slit plays a key role to enhance the overall transmission efficiency of the grating. The results are believed to be instrumental in improving our understanding in microwave photonics.

The discussion of the paper is organized as follows: section 1 gives the simple introductions of the dielectric response of a metal/air interface and the research history on the SPP-like mode as well as our research motivations; in section 2, a pseudoanalytic theoretical model using the surface impedance boundary condition is introduced to give an analytical prediction of the transmission spectrum; section 3 describes the transmittance curves calculated by both analytical and numerical methods; in section 4, the E-field diffraction patterns of the resonance modes numerically predicted are given and discussed; and section 5 gives the photonic band structures of the metallic gratings calculated by the analytical model. The main conclusions of the paper are summarized in section 6.

2. Analytical model

Figure 1 shows a cross-sectional structure of a transmission metallic grating in one period consisting of aluminium slats and air-fillers, which is infinite in the \( y \)-axis direction. The parameters \( a \), \( d \) and \( h \) denote slit width, grating period and thickness, respectively. p-type polarized (transverse magnetic (TM) mode) plane wave is radiated on the grating surface at an incident angle \( \theta \) with the incident plane coincident with the \( xz \)-plane. The EM wave diffraction behavior on the rectangular grating is modeled by a pseudoanalytical formalism, which was first presented by Chen \([15]\) to solve the transmittance of a periodically perforated conducting screen and subsequently developed by Lochbihler and Depine \([16]\) and Garcia-Vidal et al \([17]\) to calculate the scattering of optical metallic gratings. This model is based on the assumptions that: (i) the surface-impedance boundary condition can be imposed on the dielectric/metal interface, and (ii) the fundamental eigenmode in the modal expansion of the EM field inside the slits of the grating can be considered. The first condition is well satisfied at microwave wavelength where...
Figure 1. Cross-sectional schematic diagram of the metallic grating in one period in the xz-plane together with their structural parameters (slit width \(a\), grating period \(d\), and grating thickness \(h\)) and the \(p\)-polarized wave incident direction at an angle \(\theta\).

The metal can be regarded as a perfect conductor, and the second condition is satisfied by the subwavelength slit width considered in this work. The EM fields in the free space above the top as well as below the bottom of the grating can be expressed in terms of plane-wave or Rayleigh expansions. The \(x\)-component of the normalized electric field above the grating (\(z \geq h/2\)) can be written as

\[
E_x(x, z) = e^{ik_0[x \sin \theta - (z-h/2) \cos \theta]} + \sum_{n=-\infty}^{\infty} R_n e^{ik_0[x \alpha_n + (z-h/2) \beta_n]},
\]

and those below the grating (\(z \leq -h/2\)) can be written as

\[
E_x(x, z) = \sum_{n=-\infty}^{\infty} T_n e^{-ik_0[x \alpha_n + (z+h/2) \beta_n]},
\]

where \(R_n\) and \(T_n\) are the complex reflection and transmission coefficients, respectively. \(\alpha_n = \sin \theta + n \lambda/d\) and \(\beta_n = \sqrt{1 - \alpha_n^2}\) are the normalized wavevectors of the \(n\)th-order diffraction waves in the \(x\)- and \(z\)-axes, respectively. For the fields inside the grating slits, they can be expressed as the summation of their eigenmodes according to the modal expansion. In this case, the \(x\)-component of the normalized electric field inside the slits (\(|z| \leq h/2\)) can be written as

\[
E_x(x, z) = \sum_{m=0}^{\infty} \left(A_m e^{-ik_0v_{mz}} + B_m e^{ik_0v_{mz}}\right) \cos(k_0 \mu_m x),
\]

where \(A_m\) and \(B_m\) are the complex coefficients of the forth and back propagating waves inside the slits. \(\mu_m = m \lambda/2a\) and \(v_m = \sqrt{1 - \mu_m^2}\) are the normalized wavevectors of the \(m\)th-order mode in the \(x\)- and \(z\)-axes, respectively.

The magnetic field component in free space can be derived from Maxwell’s equation that gives \(H_y(x, z) = -i\omega \mu_0 \partial E_x/\partial z\). Using the surface impedance condition on the dielectric/metal interface (i.e. \(E_x = Z_s n \times H_y\), where the surface impedance \(Z_s = (1 + i)\sqrt{\omega \mu_0 / 2 \sigma}\) and \(n\) denotes the normal direction of the interface) and the tangential continuity boundary condition for the fields at the ends of the air slits, one can construct a series of equations consisting of the above unknown coefficients. In principle, the analytical expressions for these coefficients can be obtained by solving the eigenvalues of the matrices consisting of these equations [16]. In the present case, we are mainly interested in the wavelength range larger than the grating period.
Figure 2. Transmittance spectra of the metallic gratings under normal microwave incidence at selected thicknesses of $h = 0.1, 0.25, 0.5$ and 0.75 cm in (a) and of $h = 1, 1.5, 2$ and 4 cm in (b). The grating period and slit width are fixed at 1.6 and 0.1 cm, respectively. The black and the red curves respectively represent the analytical and the numerical results. The dashed line at $\lambda = d$ denotes the Rayleigh anomaly. In the diagram, the transmittance values for the gratings have been subsequently added by different integers. The inset of figure 2(a) shows the zoom-out peak region for the grating of $h = 0.1$ cm.

Thus, studying the zeroth diffraction order will be enough in considering the total transmittance, which is solved to be

$$T_0 = \frac{4aS_n^2 \cos \theta}{d(c \cos \theta + Z_s)^2} \frac{1}{1 - \rho^2 e^{2ikh}}.$$

Here $S_n = \sin(k_0\alpha_n a/a)/\sin(k_0\alpha_n a/2)$, which describes the coupling of the grating surface mode with the eigenmode of the grating slit, and $f = a/d \sum_{n=-\infty}^{\infty} S_n^2/[(1 - \alpha_n^2)^{1/2} + Z_s]$, which can be regarded as the impedance of the fundamental slit eigenmode in the approximation of a perfect conductor (or $Z_s = 0$). $\rho$ characterizes the scattering coefficient of the fundamental eigenmode reflected from the top/bottom slit end, which can be expressed by $\rho = -[1 - (1 + Z_s)f]/[1 + (1 - Z_s)f]$. Substituting the structural parameters into equation (6), one can obtain the analytical transmittance spectrum of the one-dimensional metallic grating in the wavelength range beyond the grating period ($\lambda > d$).

3. Transmission spectra

Figures 2(a) and (b) show the transmission spectra of the metallic gratings under normal microwave incidence at different thicknesses $h$. The period and slit width are fixed at 1.6 and 0.1 cm, respectively, for all the gratings discussed in this paper. The black curves represent the analytical results obtained by equation (6) and the red curves are the numerical results calculated using the Ansoft HFSS solution solver with driven modal solution type which is based on a
finite element method. At \( \lambda < d \), it is seen that the numerical calculation gives much larger transmittance values than those analytically predicted. This is because equation (6) does not include the transmission contribution from the first diffraction order which becomes propagating in this wavelength range. When \( \lambda \) is larger than \( d \), the spectra obtained by the two methods can excellently agree with each other. This indicates the validity of the pseudoanalytical model for solving transmittance questions in the long wavelength range. Their difference becomes manifest only when \( \lambda \) is reduced close to \( d \) where the surface modes will be excited. Because of the large decaying length of the surface mode in the free space, which will be discussed in detail later, a certain amount of bounded surface waves might be taken into account in the transmitted signals by the solution solver confined by the finite physical volume of the simulation model. This leads to small bloated increments for the transmittance compared to the analytical values, which become more serious in thinner gratings where surface modes have larger decaying lengths. But this effect only causes small data drifts along the \( y \)-axis in the transmittance spectrum and has negligible influence on the wavelength positions and the near-field diffraction patterns. As shown in figure 2(a), for the gratings of small thicknesses like \( h = 0.1 \) cm, the transmittance at \( \lambda > d \) is small and featureless. The numerical solution gives a tiny peak of value \( \sim 0.2 \) at \( \lambda = 1.605 \) cm. In thicker gratings, this peak value is quickly increased and the position is shifted toward lower frequency. For the 0.5 and 0.75 cm thick gratings, large transmittance of values of 0.9 and 1 are achieved, respectively, at \( \lambda = 1.643 \) and 1.905 cm. The broadness and symmetry of the transmission peak is also improved as the resonance wavelength gradually deviates from the point \( \lambda = d \) (the dashed line). When the grating thickness is further increased, more transmission peaks with unity transmittance appear at larger wavelengths as shown in figure 2(b) for the gratings of \( h = 1.5, 2 \) and 4 cm.

The additional peak appearing in thicker gratings is contributed by the excitation of localized standing wave inside the narrow slit of the grating, which is normally called a Fabry–Perot (FP)-like waveguide mode [18]. In the case of infinitesimal slit width and a perfect conductor, the FP-like modes can be predicted at \( \lambda = nh/2 \), where \( n \) is an integer. The origin of the peak in thinner gratings as shown in figure 2(a) needs more consideration, although an ‘SPP-like mode’ has generally been claimed in previous works [8]–[13]. Its thickness-dependent character is different from a pure bounded surface mode which has a dispersion only related to the dielectric constants at the air/metal interface (see equation (2)). To realize this question, first we see that at \( \lambda = d \) the transmittance always corresponds to a dip irrespective of the grating thickness. This dip is normally identified as the Rayleigh anomaly resulting from the redistribution of EM energy when the propagating diffraction order becomes evanescent. Their wavevectors satisfy the relationship that \( \beta_n = 0 \) and \( \alpha_n = 1 \), i.e. the parallel wavevector \( k_1 (\equiv k_0\alpha_n) = k_0 \). This wavevector relationship is coincident with that required for the excitation of bounded surface waves on a flat metal at microwave wavelength. This means that on ultrathin metallic gratings (\( h \ll d \)) the surface mode is coincident with the Rayleigh anomaly, and thus its excitation actually results in a transmittance dip rather than a peak. However, the situation can be different in the optical range where the SPP mode can be highly dispersive and have no coincidence with the Rayleigh anomaly in the photonic band diagram [19]. As a result, the SPP-assisted enhanced light transmission or absorption could be observed over thin subwavelength optical apertures [1], [20]–[22]. From these analyses, it is reasonable to assume that in thicker gratings the dispersion of the surface mode is disturbed by the inner slit cavity, which in turn leads to the SPP-assisted-like enhanced transmission.
Figure 3. Calculated electric field configurations at the transmittance peaks ($\lambda > d$) for the gratings at selected thicknesses of (a) 0.1, (b) 0.25, (c) 0.5 and (d) 0.75 cm at normal incidence. The arrows represent the field directions at one snapshot. The arrow lengths and the color difference denote the amplitudes of the fields. $e_{\text{Max}}$, the ratio of the amplitudes of the maximum diffraction field to the incident field, is used to characterize the field intensification effect.

4. Electric field patterns

In order to visualize the surface wave configurations, figure 3 gives the predictions of the calculated electric field distributions at the transmittance peaks for the gratings at selected thicknesses of $h = (a) 0.1$, (b) 0.25, (c) 0.5 and (d) 0.75 cm at normal incidence. The field directions at one snapshot are represented by the arrow directions, and their amplitudes are represented by the different colors and the arrow lengths. The field intensification effect due to the resonance is characterized by a factor $e_{\text{Max}}$, defined by the ratio of the amplitudes of the strongest diffracted field to the incident field. In figure 3(a), a surface wave with a long decaying length is clearly shown on the 0.1 cm thick grating. Basically, it is the superposition of all the diffraction orders analyzed from the plane-wave expansions. But the contribution from the first order is dominant, while the fundamental propagating mode or higher evanescent orders are too weak to be shown in this near-field pattern. Evanescence of the first order can be characterized by a decaying length $\delta_1 = \lambda / 2\pi |\beta_1|$. For extremely thin gratings ($h \ll d$), the surface mode almost superposes the Rayleigh anomaly and $|\beta_1| \approx 0$, which corresponds to an infinite decaying length. For the grating of finite thickness, the surface mode shifts away from the Rayleigh anomaly due to the perturbation of the inner cavity which results in the redistribution of the diffraction energy around the grating and $|\beta_1|$ has a certain value. The decaying length will be a finite number dependent on the grating thickness. For the 0.25 cm...
thick grating, as shown in figure 3(b), the surface wave is suppressed into a narrower free space around the gratings, indicating an expected reduction in the decaying length. The field magnitude factor ($e_{\text{Max}} = 5.6$) is nearly three times higher than that ($e_{\text{Max}} = 1.8$) on the 0.1 cm thick grating. These trends become more obvious as $h$ further increases. As shown in figures 3(c) ($h = 0.5$ cm) and 3(d) ($h = 0.75$ cm), the $E$-field is concentrated near the slit ends and the surface wave is localized more strongly close to the surface in general. Meanwhile, a strong localized standing wave is formed inside the slit of the grating. This implies that as the grating becomes thicker, the resonance mode evolves from the original pure bounded surface wave into a hybrid pattern with the fields simultaneously resonating outside and inside the gratings [23, 24]. Graphically, it looks like two dipolar resonations around the corners of the metal slats in a manner like a quadrupolar resonance as previously predicted in an optical nanoslit grating [25]. Correspondingly, the diffraction energy will undergo a redistribution process from the grating surface into the inner slit as the grating thickness increases.

It should be noted that the resonance wavelengths for these hybrid modes are much larger than those required to excite the FP-like waveguide modes in thicker gratings (for the gratings of $h = 0.5$ and 0.75 cm, the first FP-like harmonics are at $\lambda \sim 1$ and 1.5 cm, respectively) [26]. Hence, strong couplings between the bounded surface waves and the localized slit standing waves can be assumed to support such hybrid resonance patterns. The capacitive responses of the narrow slits will concentrate the incident energy into a two-dimensional space, which gives the cavity modes much stronger intensities than those of the three-dimensionally distributed surface waves. The higher efficiency to forward transmit EM energy for the cavity mode is closely related to their large real forward propagation wavevectors (equal to $k_0$ in the case of subwavelength slit width), while the surface mode has low efficiency in transmission due to its decaying character in the wave propagation direction besides the wavelength proximity to the Rayleigh anomaly. Putting the transmittance spectra and the field patterns together, one can realize that the SPP-like mode in microwave gratings is represented by a picture with collective field resonance around the metal slabs and the intensification of the cavity mode in between adjacent slabs is a key factor to induce the enhanced transmittance.

Figure 4 predicts the electric field configurations of the gratings at some specific wavelengths at normal incidence. First, figure 4(a) gives the typical field pattern at the transmittance peak ($\lambda = 1.545$ cm) just before the Rayleigh anomaly for a 0.5 cm thick grating, as denoted by $\lambda_1$ in the numerical spectrum curve shown in figure 4(d). This peak can be normally identified in the transmittance spectra for the gratings at different thicknesses (see figures 2(a) and (b)). A complex vortex-like field pattern is shown, which can infinitely extend into the free space. At this wavelength ($\lambda < d$), the diffracted propagating waves are the superposition of the TEM-type fundamental mode and the TM$^{10}$-type first diffraction order. Propagation of the first order mainly contributes to the enhanced transmittance as $\lambda$ is reduced to a value smaller than $d$.

In figure 4(b), the typical electric field pattern at the Rayleigh anomaly is shown for the grating of thickness of 0.5 cm. The diffracted wave above the top of the grating also shows a vortex-like feature, but that below the grating is obviously a TEM-type propagating wave. This means that at this specific wavelength the first diffraction order, which carries the most energy, mainly forms in the reflected signals, i.e. in the incident free half space. Such a diffraction pattern is consistent with the zero transmittance predicted at the Rayleigh wavelength. Figure 4(c) shows the butterfly-like surface wave and a localized slit mode at the transmission peak of $\lambda = 1.643$ cm for the same grating as addressed in figure 3(c). From the
Figure 4. Calculated electric field configurations of the gratings at some specific wavelengths at normal incidence. (a), (b) and (c) give the patterns respectively at the transmittance peak before the Rayleigh anomaly, at the Rayleigh anomaly, and at the peak after the Rayleigh anomaly with $h = 0.5$ cm. (e) and (f) give the patterns at the two transmittance peaks with $h = 1.5$ cm. (d) shows the wavelength locations of the patterns in (a), (b), (c), (e) and (f) on the numerically calculated transmittance spectra of the gratings. $e_{\text{Max}}$ has the same definition as that in figure 3.

comparison of the wave patterns shown in figures 4(a) and (b), it is evident that the distribution of the bounded surface wave is of symmetric characters along the central line of a grating. This indicates the intrinsic inter-coupling of surface modes through the slit. Figures 4(e) and (f) show the electric field patterns of the 1.5 cm thick grating at the two transmittance peaks indicated in figure 4(d) by $\lambda'_1$ and $\lambda'_2$. Strong standing waves inside the slits are observed at both wavelengths, which can be attributed to the second and the first FP-like harmonics. Besides the slit waves, figure 4(e) also shows the existence of certain broad and strong surface waves in the free space near the grating surface. These surface waves combine with the standing wave in the slit to form a typical hybrid field pattern. At normal incidence, such a hybrid pattern can be generally identified for the transmittance peaks near to the position of Rayleigh wavelength in thick gratings. Figure 4(f) represents the typical field pattern for the transmittance peak arising from the waveguide resonance which traps strong energy inside the slit cavity.
Figure 5. Correspondence relationship between the analytical transmittance spectra (black lines) and the accumulated phase shift $\phi_T$ (blue lines) of the fundamental wave inside the slits for the gratings at selected thicknesses of (a) 0.1, (b) 0.5, (c) 1.5 and (d) 4 cm with $\phi_T = 2\text{arg}(\rho) + 2k_0h$. The black dashed line denotes the transmittance peak position and the dotted line denotes the phase point $\phi_T = 2n\pi$. The red dashed line at $\lambda = d$ represents the Rayleigh anomaly.

5. Photonic band diagrams

From the field patterns, the hybrid resonance character for the SPP-like mode is understood. In addition, it is known that the formation of the slit mode plays a key role in enhancing the transmission efficiency for microwave metallic gratings. To further understand and verify these arguments, figure 5 plots the analytical transmittance spectra (black lines) for the gratings at selected thicknesses of (a) 0.1, (b) 0.5, (c) 1.5 and (d) 4 cm. The curves (blue lines) representing the accumulated phase shift $\phi_T$ of the fundamental propagating mode reflected back and forth inside the slits is also plotted. $\phi_T$ is defined by $\phi_T = 2\text{arg}(\rho) + 2k_0h$. In an ultrathin grating ($h \ll d$), the total phase shift is solely determined by the scattering phase of the propagating mode at the slit ends, i.e. $\phi_T \approx 2\text{arg}(\rho)$. The calculated $\phi_T$ (not given here, but quite similar to that shown in figure 5(a)) has a small value below 1 which slowly increases as $\lambda$ is reduced. It dramatically increases to a peak of value $2\pi$ at $\lambda = d$. This singularity point is coincident with
Figure 6. Photonic band diagrams up to 25 GHz for the gratings at selected thicknesses of (a) 0.1, (b) 0.5, (c) 1.5 and (d) 4 cm. The dashed lines in (c) denote the sweeping directions of the incident wave at different angles.

the position of the Rayleigh anomaly and the bounded surface mode in a thin grating. Thus it corresponds to a transmittance dip. Raising the grating thickness will shift the $\phi_T$ curves upward. As shown in figure 5(a) for the 0.1 cm thick grating, $\phi_T$ has a sharp peak of value $2.2\pi$ at $\lambda = d$. Bounded surface waves can be identified at wavelengths near this point, as characterized in figure 3(a). For thicker gratings, as shown in figures 5(b)–(d), the transmittance peaks (denoted by the black dashed lines) always correspond to $\phi_T$ which have integer times $2\pi$ values (as denoted by the black dotted lines in figure 5). A corresponding relationship between $\phi_T = 2n\pi$, i.e. the condition to excite a cavity resonance inside the slit, and the possible transmission resonance mode can be established in the EM interaction of metallic gratings. It also implies that in thick gratings there is no pure surface mode and a cavity mode is always involved in a resonance mode.

The above corresponding relationship gives us a way to construct the photonic band diagram of the metallic grating by studying the conditions $\phi_T = 2n\pi$ at different incident angles $\theta$. Figure 6 plots the obtained band structures up to 25 GHz for the gratings at selected thicknesses of (a) 0.1, (b) 0.5, (c) 1.5 and (d) 4 cm. These results have been verified by numerical calculations on the bands using HFSS solution solver in Eigenmode which can search all the
possible resonance modes for a given simulation model. The dotted straight lines represent the parallel incident light lines which are coincident with the Rayleigh anomaly in position. As shown in figure 6(a), the bands for the 0.1 cm thick grating have no obvious dispersion except for the tiny gaps appearing at the Brillouin zone boundaries \((k_x = 0 \text{ or } \pi / d)\), indicating the character of the pure surface mode in thin microwave gratings. In thicker gratings, as shown in figures 6(b)–(d), the dispersions of the surface modes are highly disturbed with larger deviations from the light lines when \(k_x\) approaches the boundary. These are similar to those for the SPPs on optical sinusoidal gratings [18], but in the present case they are jointly contributed by the resonances of the surface wave and cavity mode. In addition, the band has flat dispersions for thicker gratings, which are the typical characteristics of the FP-like harmonics. These modes are independent of the incident angle. For the gratings of specific structural parameters, these flatbands split and couple with a SPP-like band as they approach the light lines. The band continuity is realized with mode transition, as typically represented by the second band shown in figure 6(d). Therefore, the photonic band diagrams for the metallic gratings are assemblies of the flatbands from the waveguide mode and the SPP-like curved bands from the couplings of the surface mode and its nearest cavity (waveguide) mode.

The band diagrams shown in figure 6 are quite similar to those predicted on optical gratings, as previously reported in [27], where the authors discussed in depth and comprehensively the dispersion of the SPPs on short-pitch gratings. They found very flat SPP bands were formed for gratings with depths greater than their pitch in the zero-order region of the spectrum. These bands and their anticrossing with the light line produced the complex dispersion curves of the SPPs on optical gratings with the opening of very large band gaps due to the interactions of the SPP bands. The localized SPP wave formed inside the grating groove responsible for the flatband is analogous to the FP-like waveguide mode formed in our grating slit, while its band anticrossing with the light line graphically gives the same shaped dispersion curves as our SPP-like mode. This means the band diagrams of metallic gratings from optical to microwave wavelength experience no obvious variations although the dielectric response of the metal has huge changes. This may be understandable if one contrasts the oscillation of surface electrons in the optical spectrum with the oscillation of surface currents in the microwave spectrum when considering the interface dielectric response.

For microwave gratings, because the surface and the cavity modes have different dispersions, the diffraction energy for the SPP-like mode will be redistributed between the grating’s outer surfaces and inner slits as one sweeps the parallel wavevector \(k_x\). The transmission efficiency will be changed as a result of the energy redistribution. This effect can be realized by varying the incident angle. Figure 7 shows the analytical transmittance spectra of the 1.5 cm thick grating at the incident angles \(\theta = 0^\circ, 5^\circ, 10^\circ\) and \(15^\circ\), respectively. Their incident directions are indicated in figure 6(c) by the \(y\)-axis and the dashed lines. As shown in figure 7, the first transmittance peak at 9.01 GHz which is from the first-order FP-like harmonic has the least change as \(\theta\) is varied, while the second peak from the SPP-like mode is greatly reduced in both amplitude and broadness. The third peak at 18.35 GHz from the second FP-like harmonic appears only as \(\theta > 0^\circ\). These results confirm the conclusion that the farther the mode deviates away from the Rayleigh anomaly, the higher the transmittance will be. Energy conversion from the grating surfaces into the inner slits will largely strengthen the forward wave transmission efficiency. The intensification of the cavity resonance inside the slits should be the principal driving force to induce a large transmittance for the SPP-like mode on metallic gratings.
Figure 7. Analytical transmittance spectra of the 1.5 cm thick grating at the incident angles $\theta = 0^\circ$, $5^\circ$, $10^\circ$ and $15^\circ$.

6. Conclusions

The EM interaction in the metallic grating of subwavelength period has been comprehensively discussed from the transmission spectrum, the field pattern and the photonic band diagram. The generation of the SPP-like mode in microwave grating has been established as a result of the coupling of the surface and cavity modes. For ultrathin gratings, the surface mode with large decaying length can be excited. It corresponds to a minimum transmittance due to its wavelength coincidence with the Rayleigh anomaly. As the grating becomes thicker, a cavity mode with field resonance inside the grating slit is excited. It combines with the surface mode to form a hybrid resonance pattern. In the process, the diffraction energy will be redistributed from the outer surface into the inner slits. The mode position will be shifted away from the Rayleigh wavelength and a transmittance peak will be induced. The greatly enhanced transmission efficiency in a metallic grating results from the intensification of the cavity mode, which is physically different from the metallic optical hole arrays where the enhanced transmission observed was caused by the excitation of the SPPs under the cutoff frequency [28]–[30]. Due to the high efficiency of the cavity mode to forward emit energy, unity transmittance can be obtained by microwave gratings with the additional advantage of negligible absorption loss. The dispersions of the SPP-like mode on microwave metallic gratings are dependent on the structural parameters, rather than the material parameters. Frequency-selective transmission gratings with different band widths can be designed for specific application purposes.
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