Generalized space time autoregressive with exogenous variable model and its application

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Abstract. In this paper we proposed the Generalized Space Time Autoregressive with variable Exogenous, abbreviated GSTARX as GSTAR development with the addition of exogenous variables. GSTARX not only involves the element of time and location, but also the influence of exogenous variables in the model. GSTARX equation can be written as a linear model, so we can estimate parameters of GSTARX model using Ordinary Least Squares (OLS) method. For our case study, we use GSTARX model with uniform and inverse distance weights to predict an export volume of Crude Palm Oil (CPO) in several locations on the island of Sumatera, where X is the international CPO prices.

1. Introduction

Time series data were recorded simultaneously in multiple locations called space-time data, data with time and location simultaneously observation. We can use multivariate time series model [5] for space-time phenomena, such as VAR, STAR, and GSTAR models [1]. Nowadays, space-time model, not only influenced by previous observations at the same location and previous observations in different location, or there are not only time and location dependencies, but also there are some other things that affect, which can be expressed as an exogenous variable. The development of multivariate time series model for the space time data involving exogenous variables are VARX, STARX and GSTARX. GSTARX model itself is a special form of VARX model that combine time and location and involves an exogenous variables [3].

CPO export volume data recorded simultaneously in multiple locations, especially at province in Sumatera island, Indonesia. In this study, the space time data, CPO export volume in North Sumatera, Riau and West Sumatera, are not only influenced by the previous data of CPO export volume in the same province, and the previous ones in different provinces, but there are also other factors that influence, expressed in an exogenous variable, that is international CPO prices, we called it X.

2. Literature Study

2.1. GSTARX Model

*Generalized Space Time Autoregressive with Exogenous Variable* (GSTARX) model is the development of GSTAR model [4] with the addition of an exogenous variables in the model.
GSTARX model with order time \( p \) and order location \( \lambda_k \), GSTARX\((p, \lambda_k)\) for \( k = 1, 2, ..., N \) could be written as:

\[
Z_{(N \times 1)}(t) = (\sum_{k=1}^{N} \sum_{l=0}^{\lambda_k} \Phi_{kl(N \times N)} W_{(N \times N)}^{(l)} Z_{(N \times 1)}(t-k)) + \gamma X(t) + a_{(N \times 1)}(t)
\]

where \( \Phi_{kl(N \times N)} \) and \( \gamma_{(N \times N)} \) where are the parameters of GSTARX model, and \( a_{(N \times 1)}(t) \sim iid N(0, \sigma^2 I) \).

For example, GSTARX model with Autoregressive’s order is 1 and spatial’s order is also 1, GSTARX(1,1), with location \( N = 3 \), in matrix notation could be written as:

\[
\begin{bmatrix}
\dot{Z}_1(t) \\
\dot{Z}_2(t) \\
\dot{Z}_3(t)
\end{bmatrix} = \begin{bmatrix}
\phi_{10} & 0 & 0 \\
0 & \phi_{20} & 0 \\
0 & 0 & \phi_{30}
\end{bmatrix} \begin{bmatrix}
\dot{Z}_1(t-1) \\
\dot{Z}_2(t-1) \\
\dot{Z}_3(t-1)
\end{bmatrix} + \begin{bmatrix}
\phi_{11} & 0 & 0 \\
0 & \phi_{21} & 0 \\
0 & 0 & \phi_{31}
\end{bmatrix} \begin{bmatrix}
0 & w_{12} & w_{13} \\
w_{21} & 0 & w_{23} \\
w_{31} & w_{32} & 0
\end{bmatrix} \begin{bmatrix}
\dot{Z}_1(t-1) \\
\dot{Z}_2(t-1) \\
\dot{Z}_3(t-1)
\end{bmatrix} + \begin{bmatrix}
\gamma_1 & 0 & 0 \\
0 & \gamma_2 & 0 \\
0 & 0 & \gamma_3
\end{bmatrix} \begin{bmatrix}
X_1(t) \\
X_2(t) \\
X_3(t)
\end{bmatrix} + \begin{bmatrix}
a_1(t) \\
a_2(t) \\
a_3(t)
\end{bmatrix}
\]

(1)

The parameter of GSTARX(1,1) model could be estimated using ordinary least square method (OLS). In order to satisfy the equation of a linear model, so the GSTARX(1,1) restructured in to the equation[4]:

\[
\begin{bmatrix}
\dot{Z}_1(t) \\
\dot{Z}_2(t) \\
\dot{Z}_3(t)
\end{bmatrix} = \begin{bmatrix}
\dot{Z}_1(t-1) \\
\dot{Z}_2(t-1) \\
\dot{Z}_3(t-1)
\end{bmatrix} + \begin{bmatrix}
\phi_{10} \\
\phi_{20} \\
\phi_{30}
\end{bmatrix} \begin{bmatrix}
V_1(t-1) \\
V_2(t-1) \\
V_3(t-1)
\end{bmatrix} + \begin{bmatrix}
\phi_{11} \\
\phi_{21} \\
\phi_{31}
\end{bmatrix} \begin{bmatrix}
0 & w_{12} & w_{13} \\
w_{21} & 0 & w_{23} \\
w_{31} & w_{32} & 0
\end{bmatrix} \begin{bmatrix}
\dot{Z}_1(t-1) \\
\dot{Z}_2(t-1) \\
\dot{Z}_3(t-1)
\end{bmatrix} + \begin{bmatrix}
\gamma_1 \\
\gamma_2 \\
\gamma_3
\end{bmatrix} \begin{bmatrix}
X_1(t) \\
X_2(t) \\
X_3(t)
\end{bmatrix} + \begin{bmatrix}
a_1(t) \\
a_2(t) \\
a_3(t)
\end{bmatrix}
\]

2.2. Uniform Weight for GSTARX (1,1) Model

Uniform weight gives equal weight to each location. Therefore, this weight often used on data that have same distance between locations. The uniform weight is defined as:

\[
w_{ij} = \frac{1}{n_i}
\]

(2)

where \( n_i \) is the number of the location where are located near to location \( i \) [4].

For example, for three different locations uniform weight matrix could be written as:

\[
\begin{bmatrix}
0 & w_{12} & w_{13} \\
w_{21} & 0 & w_{23} \\
w_{31} & w_{32} & 0
\end{bmatrix}
\]

Because there are three locations, so the number of location where are located near to location \( i \) are two locations. Thus, the uniform weighting matrix \( w_{ij} \) could be elaborated into:

\[
\begin{bmatrix}
0 & \frac{1}{2} & \frac{1}{2} \\
\frac{1}{2} & 0 & \frac{1}{2} \\
\frac{1}{2} & \frac{1}{2} & 0
\end{bmatrix}
\]
2.3. Inverse Distance Weight for GSTARX (1,1) Model
Inverse Distance weight for GSTARX model refers to the actual distance between location at the same region, and then normalized. Normalization is conducted to satisfy the condition the condition \( \sum_{i \neq j} w_{ij}^{(1)} = 1 \). Suppose the distance between three locations are defined as follows:

- \( r_1 \) : the distance between location 1 and 2
- \( r_2 \) : the distance between location 1 and 3
- \( r_3 \) : the distance between location 2 and 3

so that the matrix of inverse distance weight is obtained [1]:

\[
\begin{bmatrix}
0 & w_{12} & w_{13} \\
0 & 0 & w_{23} \\
w_{31} & w_{32} & 0
\end{bmatrix}
= \begin{bmatrix}
0 & \frac{1}{r_1} & \frac{1}{r_1+1/r_2} \\
\frac{1}{r_2} & 0 & \frac{1}{r_1+1/r_3} \\
\frac{1}{r_2+1/r_3} & \frac{1}{r_3} & 0
\end{bmatrix}
= \begin{bmatrix}
0 & \frac{r_2}{r_1+r_2} & \frac{r_1}{r_1+r_2} \\
\frac{r_3}{r_1+r_3} & 0 & \frac{r_1}{r_1+r_3} \\
\frac{r_3}{r_2+r_3} & \frac{r_2}{r_2+r_3} & 0
\end{bmatrix}
\]

3. Main Result
3.1. Descriptive Analysis
In case study, we use CPO export volume (HS 1511100000) in three locations in Sumatra, namely North Sumatera, Riau, and West Sumatra, from January 2004 until August 2015. From the plot of time series data [2], the CPO export volume movement in these three locations tend to be similar.

![Plot of time series data, CPO export volume, January 2004 - August 2015](image)

Identification process of CPO export volume data in these three locations by using the Pearson correlation between observation at three locations. Table 1 presents the correlation between three of location of the CPO export volume. We can see in Table 1 that the CPO export volume data between locations are correlated. The correlation coefficient were high enough.
Table 1. Correlation of CPO Export Volume between Locations

|       | North Sumatera | Riau     | West Sumatera |
|-------|----------------|----------|---------------|
| North Sumatera | 1              | 0.740 (0.000) | 0.620 (0.000) |
| Riau   | 0.740 (0.000) | 1        | 0.663 (0.000) |
| West Sumatera | 0.620 (0.000) | 0.663 (0.000) | 1            |

Time series data analysis performed by dividing the data into two part, in sample data and out sample data. For each province, the data from January 2004 until April 2013 is used as the in sample data, 112 observations. Meanwhile the out sample data is the monthly data from May 2013 until August 2015, 28 observations.

In further analysis, to get the stationary data in mean level, so the data has been centralized, it means that the initial data is reduced by the average ($Z_t - \mu$) for each variable.

3.2. The GSTARX Model for CPO Export Volume

Before performing the analysis of time series data, both univariate and multivariate, the main requirements that must be fulfilled is stationarity. Visually, the stationarity of the data in the mean can be seen from ACF plot. ACF plot rapidly towards zero indicates that the data is already stationary. To make sure of the assessment stationarity data visually, we perform stationarity test with Augmented Dickey Fuller (ADF) test for CPO export volume data at each location. The ADF test results of the CPO export volume data in three locations, in Table II, shows that the p-value is less than 0.01. So it was decided to reject $H_0$. It can be concluded that the data did not contain a unit root, or the data is already stationary.

Table 2. ADF Test of Centralized CPO Export Volume Data

| Location          | ADF   | p-value |
|-------------------|-------|---------|
| 1. North Sumatera | -6.1278 | < 0.01  |
| 2. Riau           | -7.7209 | < 0.01  |
| 3. West Sumatera  | -8.9654 | < 0.01  |

For determining the order of GSTARX model can be performed by the smallest AIC value. The AIC minimum criteria in order selection are generally used in multivariate time series model, because the order of each univariate variables can be different. The AIC minimum criteria gives AIC value of each order so that it can be compared, and then determined the lag time that is appropriate for the model GSTARX.

Table 3. AIC Value of CPO Export Volume Data

| Lag | MA (0)  | MA (1)  | MA (2)  | MA (3)  | MA (4)  | MA (5)  |
|-----|---------|---------|---------|---------|---------|---------|
| AR(0)| 25.9649 | 26.1296 | 26.1759 | 26.0679 | 26.1022 | 25.9553 |
| AR(1)| 25.6925 | 25.6900 | 25.7019 | 25.7312 | 25.8558 | 25.7579 |
| AR(2)| 25.7295 | 25.7103 | 25.8613 | 25.8804 | 25.9711 | 25.8974 |
| AR(3)| 25.6925 | 25.7339 | 25.8986 | 26.0571 | 26.1685 | 26.0727 |
| AR(4)| 25.6517 | 25.7339 | 25.8986 | 26.0571 | 26.1685 | 26.0727 |
| AR(5)| 25.8115 | 25.8399 | 25.9797 | 26.1059 | 26.2075 | 26.1999 |
| AR(6)| 25.8379 | 25.8427 | 26.0064 | 26.2842 | 26.1022 | 25.9553 |
3.3. GSTARX(3,1) Model with Uniform Weight Matrix

The uniform weight matrix in modeling GSTARX on CPO export volume data assumes that the CPO export volume in one location have the same effect on the CPO export volume in other locations. In this study, the weight applied uniformly, because the three locations are in one area. Uniform weight matrix in this study can be written as follows:

\[
W_{ij} = \begin{bmatrix}
0 & 0.5000 & 0.5000 \\
0.5000 & 0 & 0.5000 \\
0.5000 & 0.5000 & 0 \\
\end{bmatrix}
\]

GSTARX(3,1) model for CPO export volume data in the three locations has the equation as follows:

\[
\begin{align*}
\hat{Z}_1(t) & = \phi_{10}^k 0 0 + \phi_{11}^k 0 0 + \phi_{12}^k 0 0 + 0.0002* \gamma_1 X_1(t) \\
\hat{Z}_2(t) & = \phi_{20}^k 0 0 + \phi_{21}^k 0 0 + \phi_{22}^k 0 0 + 0.0573* \gamma_2 X_2(t) \\
\hat{Z}_3(t) & = \phi_{30}^k 0 0 + \phi_{31}^k 0 0 + \phi_{32}^k 0 0 + 0.0618* \gamma_3 X_3(t)
\end{align*}
\]

| Location       | Parameter | Estimation | Standard Error | t-value | p-value | Variable |
|----------------|-----------|------------|----------------|---------|---------|----------|
| North Sumatera | \(\phi_{10}^k\) | 0.1268     | 0.1340         | -0.94   | 0.3467  | Z_2(t-1) |
|                 | \(\phi_{11}^k\) | 0.7628     | 0.2993         | -2.55   | 0.0123* | V_2(t-1) |
|                 | \(\phi_{12}^k\) | -0.1333    | 0.1346         | -0.84   | 0.4021  | Z_3(t-2) |
|                 | \(\phi_{20}^k\) | 0.4608     | 0.3076         | 1.50    | 0.1372  | V_1(t-2) |
|                 | \(\phi_{21}^k\) | -0.0887    | 0.1376         | -0.65   | 0.5195  | Z_2(t-3) |
|                 | \(\phi_{22}^k\) | 0.6895     | 0.3191         | 2.19    | 0.0309* | V_2(t-3) |
|                 | \(\gamma_1\)  | 0.1221     | 0.0540         | 2.26    | 0.0250* | X(t)     |
| Riau            | \(\phi_{10}^k\) | -0.1236    | 0.1250         | -0.99   | 0.3252  | Z_2(t-1) |
|                 | \(\phi_{11}^k\) | 0.1216     | 0.0562         | 2.17    | 0.0326* | V_3(t-1) |
|                 | \(\phi_{12}^k\) | 0.1441     | 0.1249         | 1.15    | 0.2514  | Z_3(t-2) |
|                 | \(\phi_{20}^k\) | 0.1410     | 0.0578         | 0.36    | 0.7371  | V_1(t-2) |
|                 | \(\phi_{21}^k\) | -0.0206    | 0.1229         | -0.17   | 0.8671  | Z_3(t-3) |
|                 | \(\phi_{22}^k\) | 0.0554     | 0.0554         | 1.89    | 0.3194  | V_3(t-3) |
|                 | \(\gamma_1\)  | 0.0451     | 0.0198         | 2.28    | 0.0247* | X(t)     |

Note: * Significant with \(\alpha = 10\%\)

Based on the parameters in Table VI, the GSTARX(3,1) with uniform weight on the CPO export volume data can be written in matrix form as follows:

\[
\begin{bmatrix}
\hat{Z}_1(t) \\
\hat{Z}_2(t) \\
\hat{Z}_3(t)
\end{bmatrix} = \begin{bmatrix}
0.5173 & 0 & 0 \\
0 & -0.1268 & 0 \\
0 & 0 & -0.1236
\end{bmatrix} \begin{bmatrix}
0 & -0.1050 & -0.1050 \\
0 & 0.3814 & 0.3814 \\
0.0618 & 0.0618 & 0
\end{bmatrix} \begin{bmatrix}
\hat{Z}_1(t-1) \\
\hat{Z}_2(t-1) \\
\hat{Z}_3(t-1)
\end{bmatrix}
\]
each of these locations, then the inverse distance weight matrix can be notified as follows: coordinates of the location of each study are presented in Table V. Based on the distance of closeness with other locations. This model assumes that the CPO export volume in one province is affected by distance or the between the CPO export volume in three locations based on the distance between locations. GSTARX(3,1) model with an inverse distance weight is another way to see the relationship 3.4. GSTARX(3,1) Model with Inverse Distance Weight Matrix GSTARX(3,1) model with an inverse distance weight is another way to see the relationship between the CPO export volume in three locations based on the distance between locations. This model assumes that the CPO export volume in one province is affected by distance or the closeness with other locations.

| Location          | Latitude  | Longitude  |
|-------------------|-----------|------------|
| North Sumatera    | 3.580592  | 98.671687  |
| Riau              | 0.517062  | 101.445981 |
| West Sumatera     | -0.937687 | 100.361094 |

We calculate the distances between locations using the Euclidean distance formula. The coordinates of the location of each study are presented in Table V. Based on the distance of each of these locations, then the inverse distance weight matrix can be notified as follows:

$$ W_{ij} = \begin{bmatrix} 0 & 0.5386 & 0.4614 \\ 0.3051 & 0 & 0.6949 \\ 0.2734 & 0.7266 & 0 \end{bmatrix} $$

$$ \begin{align*} 
\hat{Z}_1(t) & = \sum_{k=1}^{2} \begin{bmatrix} \phi_{10}^k & 0 & 0 \\ 0 & \phi_{20}^k & 0 \\ 0 & 0 & \phi_{30}^k \end{bmatrix} + \begin{bmatrix} \phi_{11}^k & 0 & 0 \\ 0 & \phi_{21}^k & 0 \\ 0 & 0 & \phi_{31}^k \end{bmatrix} \begin{bmatrix} 0 & 0.5386 & 0.4614 \\ 0.3051 & 0 & 0.6949 \\ 0.2734 & 0.7266 & 0 \end{bmatrix} 
\end{align*} $$

$$ \begin{bmatrix} \gamma_1 & 0 & 0 \\ 0 & \gamma_2 & 0 \\ 0 & 0 & \gamma_3 \end{bmatrix} \begin{bmatrix} X_1(t) \\ X_2(t) \\ X_3(t) \end{bmatrix} + \begin{bmatrix} a_1(t) \\ a_2(t) \\ a_3(t) \end{bmatrix} $$

Parameter estimation GSTARX(3,1) model with the inverse distance weight on the CPO export volume data in three locations with OLS method produces 21 parameters (Table 6).
The assumption of the model is residual diagnostic check model is an examination of whether the model assumptions fulfilled. The basic model is significantly affected by the CPO export volume in North Sumatera and Riau at 1 month international CPO price in the same month. As for the CPO export volume in West Sumatera price in the same month. CPO export volume in Riau province significantly affected CPO export volume in Riau and West Sumatera 2 months before as well as the international CPO by the CPO export volume in the same province at 1 month and 3 months before, the CPO interpreted CPO export volume in each province. Model GSTARX(3,1) with the inverse distance weight on the CPO export volume data as follows

\[
\begin{align*}
Z_1(t) &= [0.5186 \ 0 \ 0 \ 0] + [0 \ -0.1118 \ -0.095702025 \ 0.06120.0278 \ 0.0739 \ 0] [Z_1(t-1) \ Z_2(t-1)] \\
Z_2(t) &= [0.2052 \ 0 \ 0 \ 0] + [0 \ 0 \ -0.1941 \ 0] [Z_1(t-2) \ Z_2(t-2) \ Z_3(t-2)] \\
Z_3(t) &= [0.3695 \ 0 \ 0 \ 0] + [0 \ 0 \ -0.0235 \ 0] [Z_1(t-3) \ Z_2(t-3) \ Z_3(t-3)] \\
&+ [0.0654 \ 0 \ 0 \ 0] [X_1(t-1) \ X_2(t-1) \ X_3(t-1)] + [a_1(t) \ a_2(t) \ a_3(t)]
\end{align*}
\]

Furthermore, from GSTARX(3,1) model with the distance inverse space weight also can be interpreted CPO export volume in each province. Model GSTARX(3,1) with the inverse distance weight indicates that the CPO export volume in North Sumatera province is significantly affected by the CPO export volume in the same province at 1 month and 3 months before, the CPO export volume in Riau and West Sumatera 2 months before as well as the international CPO price in the same month. CPO export volume in Riau province significantly affected CPO export volume in North Sumatera and West Sumatera at 1 month and 3 months before and international CPO price in the same month. As for the CPO export volume in West Sumatera is significantly affected by the CPO export volume in North Sumatera and Riau at 1 month before, as well as international CPO price in the same month.

### 3.5 Diagnostic Checking Model

Diagnostic check model is an examination of whether the model assumptions fulfilled. The basic assumption of the model is residual \( \{a_t\} \) white noise. The detection of the residual white noise

### Table 6. Parameter Estimation of GSTARX(3,1) Model with Inverse Distance Weight Matrix

| Location   | Parameter | Location | Parameter | Standard Error | t-value | p-value | Variable |
|------------|-----------|----------|-----------|----------------|---------|---------|-----------|
| North Sumatera | \( \phi_{10} \) | 0.1184 | 0.1327 | 3.91 | 0.0062* | \( Z_1(t-1) \) |
|             | \( \phi_{11} \) | -0.2075 | 0.1380 | -1.50 | 0.1358 | \( Z_1(t-1) \) |
|             | \( \phi_{12} \) | 0.2053 | 0.1417 | 1.45 | 0.1505 | \( Z_1(t-2) \) |
|             | \( \phi_{13} \) | -0.2374 | 0.1365 | -1.72 | 0.0876* | \( Z_1(t-3) \) |
|             | \( \psi_{10} \) | 0.3695 | 0.1410 | 2.62 | 0.0101* | \( Z_1(t-3) \) |
|             | \( \psi_{11} \) | -0.0714 | 0.1352 | -0.53 | 0.5986 | \( Z_1(t-3) \) |
|             | \( \gamma_{11} \) | 0.0854 | 0.0376 | 2.14 | 0.0847* | \( Z_1(t) \) |

Riau

| Location | Parameter | Location | Parameter | Standard Error | t-value | p-value | Variable |
|----------|-----------|----------|-----------|----------------|---------|---------|-----------|
| \( \phi_{20} \) | -0.0370 | 0.1330 | -0.28 | 0.7815 | \( Z_2(t-1) \) |
| \( \phi_{21} \) | 0.6636 | 0.3393 | 1.96 | 0.0532* | \( Z_2(t-1) \) |
| \( \phi_{22} \) | -0.0865 | 0.1324 | -0.65 | 0.5151 | \( Z_2(t-2) \) |
| \( \phi_{23} \) | 0.5384 | 0.3377 | 1.59 | 0.1140 | \( Z_2(t-2) \) |
| \( \phi_{24} \) | -0.0752 | 0.1345 | -0.56 | 0.5776 | \( Z_2(t-3) \) |
| \( \phi_{25} \) | 0.7530 | 0.3543 | 2.13 | 0.0017* | \( Z_2(t-3) \) |
| \( \gamma_{12} \) | 0.9444 | 0.5544 | 1.74 | 0.0851* | \( Z_2(t) \) |

West Sumatera

| Location | Parameter | Location | Parameter | Standard Error | t-value | p-value | Variable |
|----------|-----------|----------|-----------|----------------|---------|---------|-----------|
| \( \phi_{30} \) | -0.0939 | 0.1233 | -0.76 | 0.4479 | \( Z_3(t-1) \) |
| \( \phi_{31} \) | 0.1017 | 0.0489 | 2.08 | 0.0401* | \( Z_3(t-1) \) |
| \( \phi_{32} \) | 0.0191 | 0.0299 | 1.58 | 0.1168 | \( Z_3(t-2) \) |
| \( \phi_{33} \) | 0.0032 | 0.00499 | 0.06 | 0.9489 | \( Z_3(t-2) \) |
| \( \phi_{34} \) | 0.0235 | 0.1219 | 0.19 | 0.8476 | \( Z_3(t-3) \) |
| \( \phi_{35} \) | 0.0289 | 0.0488 | 0.59 | 0.5549 | \( Z_3(t-3) \) |
| \( \gamma_{23} \) | 0.1412 | 0.0199 | 2.07 | 0.0408* | \( X(t) \) |

Note: * Significant with \( \alpha = 10\% \)
can be done by modeling the residual obtained from the model for each of the space weights. If the value of the smallest AIC currently on lag-0, it can be said that there is no correlation between residual, meaning that the residuals are white noise.

Table 7. AIC Value of Residual GSTARX(3,1) Model (Uniform Weight)

| Lag | MA(0)   | MA(1)   | MA(2)   | MA(3)   | MA(4)   | MA(5)   |
|-----|---------|---------|---------|---------|---------|---------|
| AR (0) | 25.4792 | 25.6175 | 25.6701 | 25.6860 | 25.7968 | 25.7726 |
| AR (1) | 25.5244 | 25.5518 | 25.6927 | 25.7414 | 25.8696 | 25.9529 |
| AR (2) | 25.6053 | 25.6666 | 25.8021 | 25.8808 | 26.0431 | 26.1375 |
| AR (3) | 25.5857 | 25.7238 | 25.8497 | 26.0139 | 26.1734 | 26.2829 |
| AR (4) | 25.7084 | 25.8408 | 25.9724 | 26.1571 | 26.3440 | 26.2949 |
| AR (5) | 25.7851 | 26.9475 | 26.1439 | 26.3272 | 26.5118 | 26.5626 |

Table 8. AIC Value of Residual GSTARX(3,1) Model (Inverse Distance Weight)

| Lag | MA(0)   | MA(1)   | MA(2)   | MA(3)   | MA(4)   | MA(5)   |
|-----|---------|---------|---------|---------|---------|---------|
| AR (0) | 25.5732 | 25.6866 | 25.7474 | 25.7355 | 25.8268 | 25.8096 |
| AR (1) | 25.6327 | 25.6504 | 25.7993 | 25.8109 | 25.9136 | 26.0083 |
| AR (2) | 25.7237 | 25.7457 | 25.8724 | 25.9412 | 26.0955 | 26.1861 |
| AR (3) | 25.6809 | 25.7886 | 25.9266 | 26.0918 | 26.2084 | 26.2946 |
| AR (4) | 25.7642 | 25.8331 | 26.0208 | 26.1935 | 26.3639 | 26.2789 |
| AR (5) | 25.8398 | 26.0061 | 26.1982 | 26.3587 | 26.5330 | 26.5687 |

Furthermore, from GSTARX(3,1) model with the distance inverse space weight also can be interpreted CPO export volume in each province. Model GSTARX(3,1) with the distance inverse space weight indicates that the CPO export volume in North Sumatera province is significantly affected by the CPO export volume in the same province at 1 month and 3 months before, the CPO export volume in Riau and West Sumatera 2 months before as well as the international CPO price in the same month. CPO export volume in Riau province significantly affected CPO export volume in North Sumatera and West Sumatera at 1 month and 3 months before and international CPO price in the same month. As for the CPO export volume in West Sumatera is significantly affected by the CPO export volume in North Sumatera and Riau at 1 month before, as well as international CPO price in the same month.

3.6. Diagnostic Checking Model

Diagnostic check model is an examination of whether the model assumptions fulfilled. The basic assumption of the model is residual $a_t$ white noise. The detection of the residual white noise can be done by modeling the residual obtained from the model for each of the space weights. If the value of the smallest AIC currently on lag-0, it can be said that there is no correlation between residual, meaning that the residuals are white noise.

Table VII and VIII presents the AIC value of residual GSTARX(3,1) model with uniform and inverse distance weight. The table shows that for each weight, the smallest AIC value are in lag-0, or in AR (0) and MA (0) to model GSTARX (3,1) with uniform and inverse distance weight. This indicates that the residuals of the model GSTARX (3,1) with two weights have fulfilled the assumption of white noise.

3.7. The Accuracy of Forecasting GSTARX(3,1) Model

Comparison of the accuracy of the estimation is done through the value of Mean Square Error (MSE) in-sample data from the model GSTARX (3,1) using two weight matrix. The result
are presented in Table IX. Table IX shows that the smallest MSE value for in-sample data GSTARX(3,1) model is on uniform weight. So, it can conclude that GSTARX(3,1) model with uniform weight matrix is the fit model for CPO export volume data in three location at Sumatera island.

Table 9. The Comparison of MSE Value in Sample Data of GSTARX(3,1) Model using Two Weight Matrix

| Space Weight     | MSE Value for In-Sample Data |
|------------------|-----------------------------|
|                  | North Sumatera | Riau | West Sumatera | Three Location |
| Uniform Weight   | 7.96 x 10³     | 17.33 | 2.62 x 10³   | 9.30 x 10³ |
| Inverse Distance Weight | 7.92 x 10³ | 18.34 | 2.71 x 10³ | 9.66 x 10³ |

Otherwise, in order to determine the most appropriate space weight for forecasting the CPO export volume in the three locations, the forecast data should validate with the actual one in out sample data. The comparison of the accuracy of forecasting can be shown through value Mean Square Error (MSE) of out-sample data. Table 9 shows that the smallest MSE value in the out-sample data GSTARX (3.1) models on the average obtained from the inverse distance weighting locations.

4. Conclusion
Based on the analysis and the discussion above, it can be concluded that the best model for CPO export volume data in three provinces above are GSTARX(3,1) model. CPO export volume in one location affected by the CPO export volume in the past in the same location and the volume of exports of CPO in the past in other location as well as in international CPO prices. GSTARX(3,1) model with uniform space weight is the fit model for CPO export volume data in three location. Otherwise, for forecasting the CPO export volume data in three location, the GSTARX(3,1) model with inverse distance space weight is the most appropriate model, which the smallest MSE value in out sample data.

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