Performance analysis of LDPC decoding algorithm

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Abstract. The LDPC code is the guarantee of reliable information transmission channel coding technology, compared with other coding technology, LDPC code has excellent performance, LDPC decoding algorithm performance is a key, in order to find out the performance of decoding algorithm, compare the BF algorithm and WBF algorithm, BP algorithm and BP algorithm LLR, MS algorithm, NMS algorithm performance, the results show that NMS algorithm has lower complexity and better performance.

1. Introduction
With the development of modern communication technology, LDPC codes have become a hotspot in coding research. Theoretical research and experiments show that LDPC codes have better performance and approach the Shannon limit under certain conditions. More and more people are studying LDPC codes[1]. Although LDPC decoding The algorithm has also been continuously improved. The LDPC decoding algorithm with low complexity and good performance is facing great challenges. Therefore, comparing and analyzing the advantages and disadvantages of different decoding algorithms has an important role in practical applications.

Hard decision decoding algorithms and soft decision decoding algorithms are the current research focus of decoding algorithms. Bit flip (BF) and weighted bit flip (WBF) algorithms belong to hard decision decoding algorithms, and the amount of operations in the decoding process is low[2]. However, the decoding performance is poor. The most traditional soft decision decoding is the belief propagation (BP) algorithm [3]. To solve the problem of high complexity of the BP algorithm, proposed to replace the log of the probability message in the BP algorithm with the likelihood log ratio. Likelihood Ratio Confidence Propagation (LLR BP) algorithm, proposed a minimum sum (MS) decoding algorithm that updates the LLR BP algorithm and checks the nodes, and the minimum sum (MS) algorithm performs a complex sum operation. The approximate value is used instead to reduce the decoding complexity, but the decoding performance will be lost[4]. Jing hu Chen and MPCFossorier use a correction factor to make up the minimum sum decoding algorithm, and propose a normalized minimum sum (NMS) decoding algorithm LDPC representation methods.

2. LDPC representation methods
LDPC is an abbreviation for Low-Density Parity Check Code. The biggest feature of LDPC code is that the "1" element in the check matrix representing this code is far less than the "0" element. $c_{10}, c_9, c_8, c_7, c_6$ represent information bit and $c_5, c_4, c_3, c_2, c_1$ represent the supervision bit. Then the code word is:

\[
\begin{align*}
 c_1 + c_2 + c_3 + c_4 &= 0 \\
 c_1 + c_5 + c_6 + c_7 &= 0 \\
 c_2 + c_5 + c_8 + c_9 &= 0 \\
 c_3 + c_6 + c_8 + c_{10} &= 0 \\
 c_4 + c_7 + c_9 + c_{10} &= 0
\end{align*}
\]

There are two ways to define LDPC codes. One can be defined by a check matrix in mathematics, and the other can be represented by Tanner diagram[5]. The Tanner diagram graphically represents the information in the LDPC code. The relationship between bits and check bits is a grid graph composed of check nodes and variable nodes. The Tanner graph is expressed as $(V_v \cup V_a, E)$. $V_a$ expresses as a set of check nodes, $V_v$ presents the set of Variable nodes. $E$ expresses the set of connecting edges between nodes, which corresponds to the "1" element in the test matrix. The test matrix in the above formula has 5 rows and 10 columns, then there are 5 test nodes and 10 variable nodes. Each row in the test matrix has four elements. The "1" element represents a linear relationship between the information bits and check bits at this position, which is reflected in Tanner. The graph shows that there is a connection between the test node and the information node. The four "1" elements indicate that four edges are connected. The Tanner graph corresponding to the test matrix in the above formula is as follows:

![Tanner diagram](image)

**Figure 1.** Tanner diagram corresponding to matrix $H$

### 3. LDPC decoding algorithms

LDPC decoding algorithms are divided into two categories according to different message passing forms. One is the hard decision decoding algorithm in which the confidence message is in the form of bit value, and the other is the soft decision decoding algorithm in which the confidence message is the posterior probability. Hard decision decoding algorithms commonly include BF algorithm, WBF algorithm, soft decision decoding algorithms include BP decoding algorithm, LLR BP decoding algorithm, MS decoding algorithm, NMS decoding algorithm, Soft decision decoding is an improvement on the basis of BP decoding algorithm[6]. In order to find the best decoding algorithm, compare the performance of different decoding algorithms.

#### 3.1. Hard decision decoding algorithm

Hard decision means that the algorithm determines whether the input signal is 0 or 1 by setting a threshold value. The hard decision decoding algorithm includes BF algorithm and WBF algorithm.

#### 3.1.1 BF algorithm
Bit flip translation (BF) is a classic algorithm of hard decision algorithm. The BF algorithm records the bits that do not satisfy the equation, and considers that the bits are likely to be interfered with to the greatest extent[7]. The bit is flipped, and then the code word after the flip is used to continue the iterative operation. The steps of the bit-flip decoding algorithm are as follows:

1. Set the number of initial iterations $k$ and its upper limit $k_{\text{max}}$, judge the information $y = [y_1, y_2, \cdots, y_n]$ , and obtain a decision sequence is:
   $$z_n = \text{sgn}(y_n) = \begin{cases} 1, & y_n > 0 \\ 0, & y_n \leq 0 \end{cases}, \quad n = 1, 2, \cdots, N$$

2. The decision sequence $z_n$ is calculated with the $h_{m,n}$ elements of the check matrix, and the calculation result is represented $s = [s_1, s_2, \cdots, s_n]$. The formula is calculated as follows:
   $$s_m = \sum_{n=0}^{N-1} z_n h_{m,n} \mod (2), m = 0, 1, \cdots, M - 1$$

3. If indicates that the code word translated is correct, the end of decoding, otherwise indicates that there is a bit error.

3.1.2 WBF algorithm

Weighted bits flip (WBF) calculated the influence of different weights, highlights the importance of weight of decoding algorithm, this method will be involved in each calibration equation of element weight is calculated[8], and then the weighted sum of the corresponding position of the element addition, turnover is one of the largest elements according to the weight of a bit of information, the decoding algorithm steps are as follows:

1. The weighted bit flip decoding algorithm is the same as the first two steps of the bit flip decoding algorithm. The decision decoding sequence $z_n$ is obtained and substituted into $s_m = z_n \cdot H$

The weight of each check formula is:
   $$w_m = \min_{i \in N(m)} |y_i|$$
   (5)

   Where, $w_m$ is determined by the adjacent nodes.

2. Calculate the corresponding weights of all elements:
   $$E = \sum_{m \in N(n)} (2s_m - 1)w_m, \quad n \in [0, M - 1]$$
   (6)

3. Compared with the above formula, if $E$ is the least reliable, then flip the corresponding bit and update the code word sequence, The flipped bit is substituted in $s_m = z_n \cdot H$. If $s_m = 0$, then the iteration operation is stopped. Otherwise, the iteration operation is continued and the above steps are repeated until the condition is met or the maximum number of iterations is reached.

3.2. Soft decision decoding algorithm

The LDPC soft decision decoding algorithm uses the characteristics of the channel. The soft decision decoding algorithm uses the posterior probability of the bit value to continuously pass between the
variable node and the check node. The posterior probability value is continuously modified as the number of iterations increases. Determine the bit value based on the magnitude of the probability value [9]. Soft decision decoding algorithms include BP algorithm, LLR BP algorithm, MS algorithm, NMS algorithm.

### 3.2.1. BP algorithm
BP algorithm is the most classical soft decision decoding algorithm. The decoding steps of BP algorithm are as follows:

1. Initial assignment. \( q_{ba}(0) \) is the probability of the information bit 0, and \( q_{ba}(1) \) is the probability of the information 1, the probability formula is:

\[
q_{ba}(0) = p_b(0) \tag{7}
\]

\[
q_{ba}(1) = p_b(1) \tag{8}
\]

2. Check node update. \( r_{ab}(0) \) is the probability of the check node be 0 and \( r_{ab}(1) \) is probability of the check node be 1, the probability formula is expressed as:

\[
r_{ab}(0) = \frac{1}{2} + \frac{1}{2} \prod_{b' \in R_a \setminus b} (1 - 2q_{b'a}(1)) \tag{9}
\]

\[
r_{ab}(1) = \frac{1}{2} - \frac{1}{2} \prod_{b' \in R_a \setminus b} (1 - 2q_{b'a}(1)) \tag{10}
\]

3. Variable node update. \( q_{ba}(0) \) is probability of the variable node be 0, \( q_{ba}(1) \) is probability of the variable node be 1, the formula is expressed as:

\[
q_{ba}(0) = k_{ba}p_b(0) \prod_{a' \in C_b \setminus a} r_{a'b}(0) \tag{11}
\]

\[
q_{ba}(1) = k_{ba}p_b(1) \prod_{a' \in C_b \setminus a} r_{a'b}(1) \tag{12}
\]

4. Decode the decision to determine the probability of the variable node. The probability expression is:

\[
q_b(0) = k_bp_b(0) \prod_{a \in C_b} r_{ab}(0) \tag{13}
\]

\[
q_b(1) = k_bp_b(1) \prod_{a \in C_b} r_{ab}(1) \tag{14}
\]

### 3.2.2. LLR BP algorithm
The LLR BP decoding algorithm is developed on the basis of the BP decoding algorithm, which reduces the complexity of the algorithm by transforming a large number of multiplication operations into addition operations in the BP decoding algorithm, and the confidence messages to be delivered are expressed by logarithmic likelihood ratio [10]. The workflow of LLR BP decoding algorithm is as follows:

1. Initial assignment. Representing the information received by the channel as a log-likelihood ratio, the log-likelihood ratio of the variable nodes \( L(p_b) \) is:

\[
L(p_b) = \ln \frac{p_b(0)}{p_b(1)} \tag{15}
\]
(2) Check node update. The variable node is passed to the check node after processing, and the log-likelihood ratio of the check node $L(r_{ab})$ is:

$$L(r_{ab}) = 2 \tan^{-1} \prod_{b' \in R_a \setminus b} \tan \left( \frac{1}{2} L(q_{b'a}) \right)$$

(16)

(3) Variable node update. The log-likelihood ratio of the variable nodes $L(q_{ba})$ is:

$$L(q_{ba}) = \ln \frac{q_{ba}(0)}{q_{ba}(1)} = L(p_b) + \sum_{a' \in C_b \setminus a} L(r_{ab})$$

(17)

(4) Decoding judgment. The log-likelihood ratio information of the decision variable node $L(q_b)$ is:

$$L(q_b) = \ln \frac{q_b(0)}{q_b(1)} = L(p_b) + \sum_{a' \in C_b} L(r_{ab})$$

(18)

If $L(q_b) > 0$, then $\hat{c}_b = 0$, otherwise $\hat{c}_b = 1$, the code after the judgment is $\hat{c} = (\hat{c}_1, \hat{c}_2, \ldots, \hat{c}_n)$, if $H \cdot \hat{c}^T = 0^T$, then decoding is over.

3.2.3. MS algorithm

MS algorithm is on the basis of the LLR BP decoding algorithm, MS algorithm will use approximation instead of complicated operation, MS algorithm and LLR BP algorithm steps are the same, decoding steps difference is check node update[11]. The minimum and decoding algorithm verifies that the node is updated as:

$$L(r_{ab}) = \prod_{b' \in R_a \setminus b} \text{sgn}(L(q_{b'a})) \cdot \min(\{L(q_{b'a})\})$$

(19)

3.2.4. NMS decoding algorithm

NMS algorithm takes measures to increase the normalization factor to make up for the performance loss of MS algorithm. NMS decoding algorithm adds the normalization factor in the update of the verification node of MS decoding algorithm. NMS algorithm verifies the node update step is:

$$L(r_{ab}) = \alpha \prod_{b' \in R_a \setminus b} \text{sgn}(L(q_{b'a})) \cdot \min(\{L(q_{b'a})\})$$

(20)

4. Simulation results and analysis

On the basis of the above description of the workflow of BP decoding algorithm, LLR BP decoding algorithm, MS decoding algorithm and NMS decoding algorithm, the performance of the quasi-cyclic LDPC decoding algorithm based on 802.16e is comprehensively compared and analyzed. MATLAB is used as the simulation tool to analyze the performance of the four decoding algorithms from the perspectives of complexity and bit error rate.

In order to compare the performance of the soft decision decoding algorithm and the hard decision decoding algorithm, the code rate is 1/2, the code length is 576, and the channel environment is AWGN channel simulation unencoded, BF, WBF algorithms and soft decisions in the hard decision algorithm. The performance of the decoding algorithm BP algorithm, the bit error rate curves of different decoding algorithms are shown in Figure 2.
Figure 2. Performance curve of soft and hard decision decoding algorithm

The simulation results show that no matter what encoding algorithm is used, the bit error rate performance of the encoded signal is much better than that of the unencoded signal. The hard decision BF algorithm makes a hard decision on the signal with a low calculation amount, but does not take full advantage of the characteristics of the channel. The decoding effect on codewords affected by channel noise is very poor, and the bit error rate is the highest. The WVBF algorithm makes full use of the influence of each signal on the encoding and decoding weights, and improves the BF algorithm with better performance than the BF algorithm. The BP algorithm makes full use of channel characteristics and performing soft decision on signals, the decoding performance is the best.

Soft decision decoding algorithm is better than hard decision decoding performance, thus decoding algorithm selects the soft decision decoding algorithm, further, in order to compare the performance of different soft decision decoding algorithm, the simulation compare BP decoding algorithm, log-domain BP decoding algorithm, the minimum and decoding algorithm, normalized the minimum and the decoding principle of decoding algorithm, in order to find out the best decoding algorithm, the IEEE802.16 e of quasi-cyclic LDPC code, on the basis of the bit rate is 1/2, code length respectively take 576, the simulation contrast four decoding algorithm of bit error rate curve is shown in Figure 3.

Figure 3. Performance curves of four decoding algorithms

The simulation results show that BP decoding algorithm, LLR BP decoding algorithm and NMS decoding algorithm almost coincide in the code length of 576, indicating that the performance of these three decoding algorithms is similar. The bit error rate curve of MS decoding algorithm is higher than that of the three decoding algorithms, indicating that the performance of MS decoding algorithm is the worst.
5. Conclusions
Research in this paper, the LDPC decoding algorithm, compare the hard decision decoding algorithm is analyzed and the performance of the soft decision decoding algorithm, the simulation results show that the soft decision decoding algorithm of BP decoding algorithm is better than hard decision in BF, the WBF performance, on the basis of BP decoding algorithm, the contrast analysis of the BP decoding algorithm, LLR BP decoding algorithm, MS performance and complexity of decoding algorithm, NMS decoding algorithm, combined with the two aspects of complexity and performance considerations, NMS decoding algorithm has the best performance.

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