Worldvolume Theories, Holography, Duality and Time

Chris M. Hull\textsuperscript{1} and Ramzi R. Khuri\textsuperscript{2,3}

\textsuperscript{1}Department of Physics
Queen Mary and Westfield College
Mile End Road
London E1 4NS UK

\textsuperscript{2}Department of Natural Sciences
Baruch College, CUNY
17 Lexington Avenue
New York, NY 10010
and
The Graduate School and University Center, CUNY
33 West 42nd Street
New York, NY 10036-8099

\textsuperscript{3}Center for Advanced Mathematical Sciences
American University of Beirut
Beirut, Lebanon

\textsuperscript{a} C.M.Hull@qmw.ac.uk
\textsuperscript{b} khuri@gursey.baruch.cuny.edu
\textsuperscript{c} Associate member
Abstract

Duality transformations involving compactifications on timelike as well as spacelike circles link M-theory, the 10+1-dimensional strong coupling limit of IIA string theory, to other 11-dimensional theories in signatures 9+2 and 6+5 and to type II string theories in all 10-dimensional signatures. These theories have BPS branes of various world-volume signatures, and here we construct the world-volume theories for these branes, which in each case have 16 supersymmetries. For the generalised D-branes of the various type II string theories, these are always supersymmetric Yang-Mills theories with 16 supersymmetries, and we show that these all arise from compactifications of the supersymmetric Yang-Mills theories in 9+1 or 5+5 dimensions. We discuss the geometry of the brane solutions and, for the cases in which the world-volume theories are superconformally invariant, we propose holographically dual string or M theories in constant curvature backgrounds. For product space solutions $X \times Y$, there is in general a conformal field theory associated with the boundary of $X$ and another with the boundary of $Y$.

November 1999
1. Introduction

Duality maps relate the five distinct perturbative string theories in 9+1 dimensions [1,2], and these are now understood as different limits of a theory in 10+1 dimensions, the so-called M-theory. For the purposes of this paper, we will define M-theory as the 10+1 dimensional theory arising as the strong-coupling limit of the IIA string theory. In [3,4], this picture was expanded to include dualities involving compactification on timelike circles as well as spacelike ones. In [4], it was shown that T-duality on a time-like circle takes the IIA theory into a IIB* theory and the IIB theory into a IIA* theory. The strong coupling limit of the IIA* theory is a theory in 9+2 dimensions, denoted the M* theory in [4], which can also be obtained by compactifying M-theory on a Lorentzian 3-torus $T^{2,1}$ with 2 spacelike circles and one timelike one, and taking the limit in which all three circles shrink to zero size. Compactifying the M* theory on a Euclidean 3-torus $T^3$ and shrinking the torus to zero size then gives an M' theory in 6+5 dimensions. Compactifying the M, M*, M' theories on spacelike or timelike circles gives rise to IIA-like string theories in signatures 10+0, 9+1, 8+2, 6+4 and 5+5, and T-dualities relate these to IIB-like string theories in signatures 9+1, 7+3 and 5+5. Each of these theories has 32 local supersymmetries, which in some cases satisfy a twisted supersymmetry algebra [1], and each has a supergravity limit. These theories are linked by an intricate web of duality transformations which can change the number of time dimensions as well as the number of space dimensions. As all of these theories are linked by dualities, they should all be regarded as different limits of a single underlying theory. For a review, see [5].

In [4], the generalised brane-type solutions of these theories were constructed, and found to have various world-volume signatures. For example, the 9+2 dimensional M* theory has membrane-type solutions with world-volumes of signature (3,0) and (1,2) and a solitonic fivebrane-type solution with signature (5,1). The rules for determining which signature branes appear in which theories were given in [4], as well as the duality transformations relating the various generalised branes. These were found to be consistent with
considerations of dimensional reduction from 11 to 10 dimensions.

There are of course many issues arising concerning the interpretation of these theories with multiple times, some of which are discussed in [3,4,5]. Here we will proceed formally and continue to map out the structure of the theories that arise.

The plan of this paper is as follows: in section 2, we review the D-brane and E-brane solutions of the type II and type II* string theories and discuss their singularities. In section 3, we review some of the main results of [6], emphasizing the various brane solutions obtained in 10 and 11 dimensions. In section 4, we interpret the branes as interpolating solutions, summarizing the results in eleven dimensions found in [6] and explicitly writing the interpolating geometries for the family of four-dimensional brane solutions of the ten-dimensional type IIB theories. In section 5, we discuss world-volume actions for these generalized branes, noting that each can be linked to a standard brane of M-theory. In sections 6, 7 and 8 we discuss the world-volume theories of the D-branes of the various type II theories, and the M-branes of the M-type theories. Finally, in section 9 we discuss the dualities between conformal field theories and de Sitter space theories following the arguments of Maldacena [7].

Various generalised de Sitter spaces of various signatures arise in solutions of these theories [8]; all of which are coset spaces \( SO(p, q)/SO(p - 1, q) \) with the \( SO(p, q) \)-invariant metric and signature \( (p - 1, q) \); when these have two sheets, we take one connected component. These include \( d \)-dimensional de Sitter space

\[
dS_d = \frac{SO(d, 1)}{SO(d - 1, 1)}, \tag{1.1}
\]

\( d \)-dimensional anti-de Sitter space

\[
AdS_d = \frac{SO(d - 1, 2)}{SO(d - 1, 1)}, \tag{1.2}
\]

the \( d \)-sphere

\[
S^d = \frac{SO(d + 1)}{SO(d)}, \tag{1.3}
\]
the $d$-hyperboloid

$$H^d = \frac{SO(d,1)}{SO(d)}$$  \hfill (1.4)

(which has a Euclidean metric and was referred to in [8] as Euclidean anti-de Sitter space) and the space

$$AAdS_d = \frac{SO(d-1,2)}{SO(d-2,2)},$$  \hfill (1.5)

with two-timing signature $(d-2,2)$.

2. D-Branes and E-Branes

The IIA* and IIB* theories in 9+1 dimensions contain E-branes [3], which are the images of D-branes under timelike T-duality. Whereas D-branes are timelike planes on which strings can end, the E-branes are spacelike surfaces on which strings can end. The strings ending on the E-branes govern the behaviour of the E-branes, and the zero-slope limit of this string theory gives world-volume theories which are Euclidean super-Yang-Mills theories obtained by reducing 9+1 dimensional super-Yang-Mills on Lorentzian tori. In particular, the E4-brane solution of the IIB* theory leads, following [7], to a duality between the large $N$ limit of Euclidean 4-dimensional $U(N)$ super-Yang-Mills theory and the IIB* string on the product of the 5-dimensional de Sitter space $dS_5$ and the hyperbolic 5-space $H_5$ [3].

The bosonic part of the IIA supergravity action is

$$S_{IIA} = \int d^{10}x \sqrt{-g} \left[ e^{-2\Phi} \left( R + 4(\partial\Phi)^2 - \frac{H^2}{12} \right) - \frac{G_2^2}{4} - \frac{G_4^2}{48} \right] + \frac{4}{\sqrt{3}} \int G_4 \wedge G_4 \wedge B_2 + \ldots$$  \hfill (2.1)

while that of IIB supergravity is

$$S_{IIB} = \int d^{10}x \sqrt{-g} \left[ e^{-2\Phi} \left( R + 4(\partial\Phi)^2 - \frac{H^2}{12} \right) - \frac{G_1^2}{2} - \frac{G_3^2}{12} - \frac{G_5^2}{240} \right] + \ldots$$  \hfill (2.2)

Here $\Phi$ is the dilaton, $H = dB_2$ is the field strength of the NS-NS 2-form gauge field $B_2$ and $G_{n+1} = dC_n + \ldots$ is the field strength for the RR $n$-form gauge field $C_n$. The
field equations derived from the IIB action (2.2) are supplemented with the self-duality constraint $G_5 = *G_5$. Our conventions are that in signature $S + T$, the metric has $S$ positive spacelike eigenvalues and $T$ negative timelike ones, so that a Lorentzian metric has signature $(S, 1)$, i.e. $(+ + \ldots + -)$.

The type II supergravity solution for a $Dp$-brane ($p$ is even for IIA and odd for IIB) is given by $\mathbb{I}$

$$ ds^2 = H^{-1/2}(-dt^2 + dx_1^2 + \ldots + dx_p^2) + H^{1/2}(dy_{p+1}^2 + \ldots + dy_9^2) $$

$$ e^{-2\Phi} = H^{(p-3)/2}, \quad C_{012..p} = -H^{-1} + k, $$

where $H$ is a harmonic function of the transverse coordinates $y_{n+1}, \ldots, y_9$, $k$ is a constant and here and throughout in the paper we denote longitudinal spatial coordinates by $x_a$ and transverse spatial coordinates by $y_i$. The simplest choice for $H$ is

$$ H = c + \frac{Q}{y^{7-p}}, $$

where $c$ is a constant (which can be taken to be 0 or 1), $y$ is the radial coordinate defined by

$$ y^2 = \sum_{i=p+1}^{9} y_i^2 $$

and $Q$ is proportional to the D-brane tension, and is taken to be positive (taking $Q < 0$ gives an unphysical negative-tension brane with a naked singularity where $H = 0$). When $c \neq 0$, it is conventional to set $k = c^{-1}$, so that as $y \rightarrow \infty$, $C_{012..p} \rightarrow 0$. However, for convenience we will henceforth set $k = 0$ and usually take $c = 1$.

The bosonic part of the type IIA* and type IIB* supergravity actions are given by reversing the signs of the RR kinetic terms in (2.1), (2.2) to give $\mathbb{I}$

$$ S_{IIA^*} = \int d^{10}x \sqrt{-g} \left[ e^{-2\Phi} \left( R + 4(\partial\Phi)^2 - \frac{H^2}{12} \right) + \frac{G_2^2}{4} + \frac{G_4^2}{48} \right] + \ldots $$

and

$$ S_{IIB^*} = \int d^{10}x \sqrt{-g} \left[ e^{-2\Phi} \left( R + 4(\partial\Phi)^2 - \frac{H^2}{12} \right) + \frac{G_1^2}{2} + \frac{G_3^2}{12} + \frac{G_5^2}{240} \right] + \ldots, $$
where the field equations from (2.7) are supplemented by the constraint $G_5 = *G_5$. The full theories are invariant under a twisted $N = 2$ supersymmetry [3].

The $E^p$-brane solutions to (2.6) and (2.7) are given by

$$
ds^2 = H^{-1/2}(dx_1^2 + \ldots + dx_p^2) + H^{1/2}(-dt^2 + dy_{p+1}^2 + \ldots + dy_9^2),
$$

$$
e^{-2\phi} = H^{(p-4)/2}, \quad C_{12...p} = -H^{-1}. \quad (2.8)
$$

In this case, $H$ is a harmonic function of $t, y_{p+1}, \ldots, y_9$ (i.e. it is a solution of the wave equation $\nabla^2 H = 0$), and $H$ can depend on time as well as the spatial transverse coordinates.

There are a number of different possibilities for $H$. First, we can take the time-independent $H$ given by (2.4). These are the solutions that arise from the D-brane supergravity solutions on performing a timelike T-duality, using a generalisation [4] of the usual T-duality rules [12],[13]. Secondly, we can consider the solution [3]

$$
H = c + \frac{Q}{\tau^{8-p}} \quad (2.9)
$$

where $\tau, \sigma$ are the proper time and distance defined by

$$
\tau^2 = -\sigma^2 = t^2 - y^2. \quad (2.10)
$$

This corresponds to a source located at a point in the transverse space-time. For odd $p$, taking

$$
H = c + \frac{Q'}{\sigma^{8-p}} \quad (2.11)
$$

gives a different real solution, related to (2.9) by taking $Q' = iQ$.

The solutions have potential singularities on the light-cone $t^2 = y^2$, where $H$ diverges, and on the hyperboloid where $H = 0$. There are in general two distinct solutions, one in which the coordinates are restricted to the interior of the light cone, $t^2 \geq y^2$, and one in which they are restricted to the exterior of the light cone, $t^2 \leq y^2$. For example, for the $E^4$-brane, it was shown in [3] that the geometry near $t^2 = y^2$ is non-singular and
approaches $dS_5 \times H^5$, the product of 5-dimensional de Sitter space and hyperbolic 5-space with constant negative-curvature and positive definite metric. Taking $Q$ to be positive (with $c = 1$) so that $H$ is non-vanishing then avoids the potential singularity at $H = 0$. The region $t^2 \leq y^2$ then defines a non-singular solution which interpolates between flat space (the region in which $\sigma$ is large) and $dS_5 \times H^5$ (where $\sigma$ is small), and is geodesically complete \cite{3}. The interior of the light-cone $t^2 \geq y^2$ also defines a non-singular geodesically complete solution, with the future and past regions $t \geq 0$ and $t \leq 0$ each giving a coordinate patch covering half the space. The region near $t^2 = y^2$ is again $dS_5 \times H^5$, with the future-cone $t > y > 0$ covering one half of $dS^5$ and the past-cone covering the other half \cite{3}. The situation is similar for the other E-branes, with the interior and exterior solutions defining two different solutions \cite{14}. For the solution with timelike interpolation in which $t^2 \geq y^2$, we take the harmonic function to be given by (2.9) with $Q > 0$, while for the solution with spacelike interpolation in which $t^2 \leq y^2$, we take the harmonic function to be given by (2.11) with $Q' > 0$, so that in each case $H > 0$.

The solution (2.8) is an extended object associated with a spacelike $p$-surface with coordinates $x^1, \ldots, x^p$ located at $t = y^{p+1} = \ldots = y^9 = 0$. This is to be compared with a D-brane, which is associated with a timelike $p + 1$-surface with coordinates $t, x^1, \ldots, x^p$ located at $y^{p+1} = \ldots = y^9 = 0$. A D-brane arises in perturbative type II string theory from imposing Dirichlet boundary conditions in the directions $y^{p+1}, \ldots, y^9$ and Neumann conditions in the remaining directions, and the D-brane solution (2.3) describes the supergravity fields resulting from such a D-brane source. Similarly, the E-brane in perturbative type II* string theory arises from imposing Dirichlet boundary conditions in the directions $t, y^{p+1}, \ldots, y^9$, including time, and Neumann conditions in the remaining directions, and the E-brane solution (2.8) describes the supergravity fields resulting from such an E-brane source. In the perturbative string theory, the timelike T-duality taking the type II theory to the type II* theory changes the boundary condition in the time direction from Dirichlet to Neumann, and so takes a D$p$-brane to an E$p$-brane. The E-branes preserve 16 of the
32 supersymmetries of the type II* theories. Smearing these solutions in the time direction gives the time-independent solutions given by (2.8) with $H$ given by (2.4), and other solutions can be obtained by smearing in spacelike directions.

3. Brane Solutions of Arbitrary Signature

For theories in spacetimes of signature $(S, T)$, we are interested in generalised brane solutions with metric of the form

$$ds^2 = H^{-\alpha} \eta_{ab} dX^a dX^b + H^\beta \tilde{\eta}_{ij} dY^i dY^j,$$

(3.1)

where $\eta_{ab}$ is a flat metric of signature $(s, t)$ and $\tilde{\eta}_{ij}$ is a flat metric of signature $(\tilde{s}, \tilde{t}) = (S - s, T - t)$. We will require $H$ to be a function of the transverse coordinates $Y^i$ and find that the field equations imply that $H(Y)$ has to be a harmonic function, satisfying

$$\tilde{\eta}^{ij} \partial_i \partial_j H = 0,$$

(3.2)

and determine the constants $\alpha, \beta$. The longitudinal space has signature $(s, t)$, and we refer to it as an $(s, t)$-brane, so that a conventional $p$-brane of a Lorentzian theory with $(S, T) = (D - 1, 1)$ is a $(p, 1)$-brane. These solutions also have a non-vanishing $n$-form gauge field $C_n$ with $n = s + t$

$$C_n = H^\gamma dX^1 \wedge \ldots \wedge dX^n$$

(3.3)

for some constant $\gamma$.

Different types of solutions arise for different choices of harmonic function. A simple choice is the wave-type solution

$$H = A \sin(K_i Y^i + c), \quad \tilde{\eta}^{ij} K_i K_j = 0$$

(3.4)

with $K$ a constant null vector. For the Euclidean transverse space (as in $p$-branes) there are no non-trivial null vectors $K$, but there are non-trivial solutions if the transverse space
has both spacelike and timelike dimensions. Here we shall concentrate on solutions of the form

\[ H = c + \frac{Q}{\tau^m} \] (3.5)

or

\[ H = c + \frac{Q'}{\sigma^m} \] (3.6)

where \( m = S + T - s - t - 2 \), \( Q, Q' \) are real constants and \( \tau, \sigma \) are the proper time and distance defined by

\[ \sigma^2 = -\tau^2 = \tilde{\eta}_{ij} Y^i Y^j. \] (3.7)

These solutions correspond to a source located at a point in the transverse space-time. The two solutions (3.5),(3.6) are related by \( Q' = i^m Q \) and so are distinct real solutions only for odd \( m \). There are also multi-centre generalisations with

\[ H = c + \sum_I \frac{Q_I}{[\tilde{\eta}_{ij}(Y^i - Y^i_I)(Y^j - Y^j_I)]^{m/2}} \] (3.8)

corresponding to sources at the points \( Y^i_I \) in the transverse space.

The single-centre solutions have potential singularities on the ‘light-cone’ \( \sigma^2 = 0 \), where \( H \) diverges, and on the hyperboloid where \( H = 0 \). As in the case of the E-branes, the regions \( \sigma^2 \geq 0 \) and \( \sigma^2 \leq 0 \) corresponding to the exterior and interior of the light-cone define two distinct solutions and in each case we take the sign of \( Q \) or \( Q' \) so that \( H > 0 \) in the appropriate region. We will sometimes refer to the \((s,t)\)-brane solution with metric (3.1) with (3.5) or (3.6) and coordinates restricted to \( \sigma^2 \geq 0 \) as an \((s,t,+)\) brane and that for the region \( \sigma^2 \leq 0 \) as an \((s,t,-)\) brane. Then for the \((s,t,-)\) solution with \( \tau^2 \geq 0 \), we take (3.5) with \( c \geq 0, Q > 0 \) while for the \((s,t,+)\) solution with \( \sigma^2 \geq 0 \) we take (3.6) with \( c \geq 0, Q' > 0 \); for \( m \) even, this is of course equivalent to taking (3.5) with \( c \geq 0, Q < 0 \).

We will be particularly interested in the asymptotic form of the geometries near \( \sigma^2 = 0 \). Note that this asymptotic form can be different for the two solutions \((s,t,\pm)\). We will focus on the cases in which this is a product of constant curvature spaces (generalising
the usual $p$-brane case in which it is a product of a sphere and an anti-de Sitter space) in which the brane solution interpolates between this asymptotic geometry and flat space.

M-theory is the strong coupling limit of the type IIA string \[2\] and is a theory in $10+1$ dimensions whose low-energy effective field theory is 11-dimensional supergravity \[15\] with bosonic action

$$S_M = \int d^{11}x \sqrt{-g} \left( R - \frac{G_4^2}{48} \right) - \frac{1}{12} \int C \wedge G \wedge G. \quad (3.9)$$

The M2-brane solution of the 10+1 supergravity action (3.9) is given by \[16\]

$$ds^2 = H^{-2/3}(-dt^2 + dx_1^2 + dx_2^2) + H^{1/3}(dy_3^2 + \ldots + dy_5^2 + dy_10^2),$$

$$C_{012} = H^{-1}, \quad (3.10)$$

where $H(y_3, \ldots, y_{10})$ is a harmonic function in the transverse space. For a single membrane at $y = 0$, we take

$$H = 1 + \frac{Q}{y_6}, \quad (3.11)$$

where here and throughout $y^2 = \sum_i y_i^2$, where $i$ runs over the spatial indices in the transverse space. The world-volume has signature $(2,1)$. The M2-brane solution has bosonic symmetry $ISO(2,1) \times SO(8)$. It is nonsingular at $y = 0$ with near-horizon geometry $AdS_4 \times S^7$ \[17\].

The M5-brane \[18\] is given by

$$ds^2 = H^{-1/3}(-dt^2 + dx_1^2 + \ldots + dx_5^2) + H^{2/3}(dy_6^2 + dy_7^2 + dy_8^2 + dy_9^2 + dy_{10}^2),$$

$$C_{t12345} = H^{-1}, \quad (3.12)$$

where

$$H = 1 + \frac{Q}{y^3} \quad (3.13)$$

and $y^2 = y_6^2 + y_7^2 + \ldots + y_{10}^2$. This solution has bosonic symmetry $ISO(5,1) \times SO(5)$ and interpolates between $AdS_7 \times S^4$ and flat space \[17\].

We will now consider the analogues of the M2-brane and M5-brane that occur in the M* theory. The M* theory \[4\] is the strong coupling limit of the IIA* theory and is a
theory in 9+2 dimensions whose field theory limit is a supergravity theory with bosonic action
\[ S_M = \int d^{11}x \sqrt{g} \left( R + \frac{G_4^2}{48} \right) - \frac{1}{12} \int C \wedge G \wedge G. \] (3.14)

Note that the sign of the kinetic term of $G_4$ is opposite to that of the action (3.9); as was shown in [6], the sign of the kinetic term is intimately related with the world-volume signatures that can occur. The M* theory with action (3.14) has brane solutions with world-volume signatures (3,0) and (1,2) [6], while if the sign of the kinetic term of $G_4$ had been the opposite of that in (3.14) to give a Lagrangian $R - \frac{G_4^2}{48} + ...$ in 9+2 dimensions, there would have been a membrane solution with 2+1 dimensional world-volume. The sign of the $G_4$ kinetic term in actions (3.9) and (3.14) is determined by supersymmetry [4].

The (1,2)-brane of the M*-theory is given by
\[ ds^2 = H^{-2/3}(-dt^2 - dt'^2 + dx^2) + H^{1/3}(dy_2^2 + ... + dy_9^2), \]
\[ C_{tt'x} = H^{-1}, \] (3.15)

where $H(y_3, \ldots, y_{10})$ is again a harmonic function in $\mathbb{R}^8$, which we can take to be (3.14). The world-volume has signature (1,2), with two times. This solution has bosonic symmetry $ISO(1,2) \times SO(8)$. The transverse space is Euclidean, so there is only one solution, the (1,2,+) solution. Near $y = 0$, the metric takes the form
\[ ds^2 = \frac{U^2}{R^2}(-dt^2 - dt'^2 + dx^2) + \frac{R^2dU^2}{U^2} + 4R^2d\Omega^2_7, \] (3.16)

which is the metric on $AAdS_4 \times S^7$, where $d\Omega^2_n$ is the metric on the $n$-sphere of unit volume, $AAdS_4$ is the de Sitter-like space of signature 2+2 given by the coset $SO(3,2)/SO(2,2)$ [6], $U = Q^{-1/6}y^2/2$ and $R = Q^{1/6}/2 = R_{S7}/2$. The (1,2)-brane interpolates between the flat space $\mathbb{R}^{9,2}$ and $AAdS_4 \times S^7$.

The second membrane-type solution of the M* theory is the (3,0)-brane given by
\[ ds^2 = H_2^{-2/3}(dx^2_1 + dx^2_2 + dx^2_3) + H_2^{1/3}(-dt^2 - dt'^2 + dy_4^2 + ... + dy_9^2), \]
\[ C_{123} = H_2^{-1}, \] (3.17)
where $H$ is a harmonic function on the transverse space. The world-volume is Euclidean, with signature $(3,0)$, and has isometry group $ISO(3) \times SO(6,2)$. There are two distinct complete solutions, the $(3,0,\ +)$ brane given by (3.17) in the region $\sigma^2 = y^2 - t^2 - t'^2 \geq 0$ and the $(3,0,\ -)$ brane given by (3.17) in the region $\sigma^2 \leq 0$. Near $y = 0$ with $\sigma^2 \geq 0$ the metric approaches $H^4 \times AAdS_7$ as $y \to 0$, while for $\sigma^2 \leq 0$ it approaches $dS_4 \times AdS_7$.

Hence the $(3,0,\ +)$-brane solution interpolates between the flat space $R^{9,2}$ and $H^4 \times AAdS_7$ while the $(3,0,\ -)$ brane solution interpolates between the flat space $R^{9,2}$ and $dS_4 \times AdS_7$.

The M* theory has a $(5,1)$-brane solution (analogous to the M5-brane of M-theory) which is given by

$$ds^2 = H^{-1/3}(-dt^2 + dx_1^2 + \ldots + dx_5^2) + H^{2/3}(-dt'^2 + dy_6^2 + dy_7^2 + dy_8^2 + dy_9^2),$$

$$C_{t12345} = H^{-1},$$

(3.18)

Here $H$ is a harmonic function and there are two solutions. In the $(5,1,\ -)$ solution

$$H = 1 + \frac{Q}{\tau^3},$$

(3.19)

where $Q > 0$ and the transverse coordinates are restricted to the region $\tau^2 = y^2 - t'^2 > 0$, where $y^2 = y_6^2 + y_7^2 + \ldots + y_9^2$. In the $(5,1,\ +)$ solution

$$H = 1 + \frac{Q}{\sigma^3},$$

(3.20)

where the transverse coordinates are restricted to the region $\sigma^2 = t'^2 - y^2 > 0$, with $Q > 0$. Both solutions have bosonic symmetry $ISO(5,1) \times SO(4,1)$ and world-volume signature $(5,1)$.

There is a IIA string theory in a spacetime with signature $5+5$ whose strong coupling limit is the M' theory with signature $6+5$ [4]. The field theory limit of M' theory is a supergravity theory in $6+5$ dimensions with bosonic action

$$S_{M'} = \int d^{11}x \sqrt{-g} \left(R - \frac{G_4^2}{48}\right) - \frac{1}{12} \int C \wedge G \wedge G.$$  

(3.21)
This has branes of world-volume signature 2+1, 0+3, 5+1, 3+3 and 1+5, and in each case there are two solutions $\sigma^2 \geq 0$ or $\sigma^2 \leq 0$, except for the (1,5)-brane which has a transverse space of Euclidean signature.

The various brane solutions in eleven dimensions are summarised in Table 1.

|     | $C_3$, $s + t = 3$ | $C_6$, $s + t = 6$ |
|-----|--------------------|--------------------|
| $M_{10,1}$ | (2,1)               | (5,1)              |
| $M_{9,2}$       | (3,0,±),(1,2)       | (5,1,±)            |
| $M_{6,5}$       | (2,1,±),(0,3,±)     | (5,1,±),(3,3,±),(1,5) |

**Table 1** The M-branes with world-sheet signature $(s, t)$ coupling to the 3-form gauge field $C_3$ or its 6-form dual $\tilde{C}_6$ in the various M-theories with signature $(S, T)$. For Lorentzian transverse spaces, there are two solutions, $(s, t, \pm)$

The brane solutions of the IIA-type and IIB-type theories are summarised in Tables 2 and 3 respectively. The type II branes all arise from solutions of the various 11-dimensional theories [6]. The generalised fundamental strings have two-dimensional world-sheets of signature $(s, 2 - s)$ and couple to the NS-NS 2-form gauge field $B_2$, while the generalised NS 5-branes couple to its dual $\tilde{B}_6$. The branes coupling to the RR $n$-form gauge fields $C_n$ or their duals $\tilde{C}_m$ are D-branes on which fundamental strings can end and the solutions are of the form (3.1) with $\alpha = \beta = 1/2$.

We have included the D8-brane family of branes with $s + t = 9$ that are obtained by T-duality from the results of [3]. Note that the $IIA_{5,5}^*$ theory, defined as the spacelike T-dual of the $IIB_{5,5}^*$ theory or the timelike T-dual of the $IIB_{5,5}$ theory [4], is related to the $IIA_{5,5}$ theory by $g_{\mu\nu} \rightarrow -g_{\mu\nu}$, interchanging space and time, and so every $(s, t)$ brane of the $IIA_{5,5}$ theory corresponds to a $(t, s)$ brane of the $IIA_{5,5}^*$ theory. These branes are needed in checking the T-duality relations of the various D-branes. For example, a timelike T-duality takes the (3,3) brane of the $IIB_{5,5}$ theory to a (3,4) brane or a (3,2) brane of the $IIA_{5,5}^*$ theory.
Note that each entry in tables 1,2,3 defines two solutions whenever the transverse space has indefinite signature, one with $\sigma^2 \geq 0$ and one with $\sigma^2 \leq 0$.

|     | $C_1$ | $B_2$ | $C_3$ | $C_5$ | $B_6$ | $C_7$ | $C_9$ |
|-----|-------|-------|-------|-------|-------|-------|-------|
| $IIA_{10,0}$ | (1,0) | (2,0) | –     | (5,0) | –     | –     | –     |
| $IIA_{9,1}$   | (0,1) | (1,1) | (2,1) | (4,1) | (5,1) | (6,1) | (8,1) |
| $IIA^*_{9,1}$ | (1,0) | (1,1) | (3,0) | (5,0) | (5,1) | (7,0) | (9,0) |
| $IIA_{8,2}$   | (0,1) | (2,0), (0,2) | (3,0), (1,2) | (4,1) | (5,1) | (7,0), (5,2) | (8,1) |
| $IIA_{6,4}$   | (1,0) | (2,0), (0,2) | (2,1), (0,3) | (5,0), (3,2), (1,4) | (5,1), (3,3) | (6,1), (4,3) | (5,4) |
| $IIA_{5,5}$   | (0,1) | (1,1) | (2,1), (0,3) | (4,1), (2,3), (0,5) | (5,1), (3,3), (1,5) | (4,3), (2,5) | (4,5) |

**Table 2** The branes with world-sheet signature $(s, t)$ of the various $IIA_{S,T}$ theories with signature $(S, T)$, coupling to RR $n$-forms $C_n$, the NS-NS 2-form $B_2$ or its dual $\tilde B_6$.

|     | $C_0$ | $B_2$ | $C_2$ | $C_4$ | $B_6$ | $C_6$ | $C_8$ |
|-----|-------|-------|-------|-------|-------|-------|-------|
| $IIB_{0,1}$ | –     | (1,1) | (1,1) | (3,1) | (5,1) | (5,1) | (7,1) |
| $IIB^*_{0,1}$ | (0,0) | (1,1) | (2,0) | (4,0) | (5,1) | (6,0) | (8,0) |
| $IIB'_{0,1}$ | (0,0) | (2,0) | (1,1) | (4,0) | (6,0) | (5,1) | (8,0) |
| $IIB_{7,3}$   | –     | (2,0), (0,2) | (2,0), (0,2) | (3,1), (1,3) | (6,0), (4,2) | (6,0), (4,2) | (7,1), (5,3) |
| $IIB_{5,5}$   | –     | (1,1) | (1,1) | (3,1), (1,3) | (5,1), (3,3), (1,5) | (5,1), (3,3), (1,5) | (5,3), (3,5) |
| $IIB^*_{5,5}$ | (0,0) | (1,1) | (2,0), (0,2) | (4,0), (2,2), (0,4) | (5,1), (3,3), (1,5) | (4,2), (2,4) | (4,4) |
| $IIB'_{5,5}$ | (0,0) | (2,0), (0,2) | (1,1) | (4,0), (2,2), (0,4) | (4,2), (2,4) | (5,1), (3,3), (1,5) | (4,4) |

**Table 3** The branes with world-sheet signature $(s, t)$ of the various $IIB_{S,T}$ theories with signature $(S, T)$.

4. Branes, Interpolations and De Sitter Spaces

The M2-brane can be viewed as a soliton interpolating between 11-dimensional Minkowski space and the solution $AdS_4 \times S^7$, while the M5-brane interpolates between
11-dimensional Minkowski space and the solution $AdS_7 \times S^4$ [17]. There are two M-brane solutions corresponding to each entry in table 1 whenever the transverse space has indefinite signature, one with $\sigma^2 \geq 0$ and one with $\sigma^2 \leq 0$, and the asymptotic form of the geometry near $\sigma^2 = 0$ was found to be a coset-space solution of signature $S + T$ for each case [3]. In each case, the metric is given by (3.1) with the harmonic function given by (3.6) with vanishing constant piece, $c = 0$. The coset spaces that arise are as follows:

I-Solutions of M-theory

a-(2,1)-brane: $AdS_4 \times S^7 = \frac{O(3,2)}{O(3,1)} \times \frac{O(8)}{O(7)}$.

b-(5,1)-brane: $AdS_7 \times S^4 = \frac{O(6,2)}{O(6,1)} \times \frac{O(5)}{O(4)}$.

II-Solutions of M*-theory

a-(1,2)-brane: $AAdS_4 \times S^7 = \frac{O(3,2)}{O(2,2)} \times \frac{O(8)}{O(7)}$.

b-(5,1,−)-brane and (3,0,+)-brane: $AAdS_7 \times H^4 = \frac{O(6,2)}{O(5,2)} \times \frac{O(4,1)}{O(4)}$.

c-(5,1,+)-brane and (3,0,−)-brane: $AdS_7 \times dS^4 = \frac{O(6,2)}{O(6,1)} \times \frac{O(4,1)}{O(3,1)}$.

III-Solutions of M’-theory

a-(2,1,−)-brane and (3,3,+)-brane: $AAdS_4 \times S^7 = \frac{O(4,4)}{O(4,3)} = \frac{O(3,2)}{O(2,2)} \times \frac{O(4,4)}{O(4,3)}$.

b-(2,1,+)-brane and (3,3,−)-brane: $AdS_4 \times \frac{O(4,4)}{O(4,3)} = \frac{O(3,2)}{O(3,1)} \times \frac{O(4,4)}{O(4,3)}$.

c-(5,1,−)-brane and (0,3,+)-brane: $AAdS_7 \times -dS^4 = \frac{O(6,2)}{O(5,2)} \times \frac{O(4,1)}{O(1,3)}$.

d-(5,1,+)-brane, (0,3,−)-brane: $AdS_7 \times -H^4 = \frac{O(6,2)}{O(6,1)} \times \frac{O(1,4)}{O(4)}$.

e-(1,5)-brane: $-AAdS_7 \times S^4 = \frac{O(2,6)}{O(2,5)} \times \frac{O(5)}{O(4)}$.

For a given space $N$ of signature $(S,T)$, we denote by $-N$ the space of signature $(-S,-T)$ given by multiplying the metric by $-1$, so that whereas $AdS_n$ is a space of signature $(n-1,1)$, $-AdS_n$ is a space of signature $(1,n-1)$. In the cases IIIb,c and IIIa-d, the same geometry arises as the asymptotic limit for two different brane solutions. We will discuss some of the implications of this in section 9.

We will now extend this to the brane solutions of the IIB theories with $s + t = 4$ that correspond to the D3-brane that are listed in the $C_4$ column of table 3, and find their
interpolating geometries. The \((s, t)\) \(D\)-brane solutions are all of the form

\[
ds^2 = H^{-1/2} \eta_{ab} dX^a dX^b + H^{1/2} \tilde{\eta}_{ij} dY^i dY^j,
\]

(4.1)

where \(\eta_{ab}\) is a flat metric of signature \((s, t)\) and \(\tilde{\eta}_{ij}\) is a flat metric of signature \((S-s, T-t)\), with \(H\) a function of the transverse coordinates \(Y^i\). For the D3-brane family for which \(s + t = 4\), the dilaton is zero and the four-form antisymmetric tensor is given by \(C_4 = -H^{-1}\epsilon\), where \(\epsilon\) is the volume-form on the longitudinal part of the spacetime. We define \(y^2 = \Sigma y_i^2\) and \(\tilde{t}^2 = \Sigma \tilde{t}_j^2\) and let \(\sigma^2 = y^2 - \tilde{t}^2 = \tilde{\eta}_{ij} Y^i Y^j\) and \(\tau^2 = \tilde{t}^2 - y^2 = -\tilde{\eta}_{ij} Y^i Y^j\). In this case, we take the harmonic function to be

\[
H = 1 + \frac{Q^2}{\sigma^4}
\]

(4.2)

so that \(H \geq 1\) and

\[
H^{1/2} = \pm \frac{Q}{\sigma^2} + ...
\]

(4.3)

when \(\sigma^2\) is small. When the transverse space has indefinite signature, there are two distinct solutions, one defined for \(\sigma^2 \geq 0\) and one for \(\sigma^2 \leq 0\). In each of the two cases, \(\sigma^2 \geq 0\) and \(\sigma^2 \leq 0\), we choose the sign so that \(H^{1/2}\) is positive in (4.3). The \((s, t)\)-brane solution interpolates between flat space and the asymptotic geometry as \(\sigma \to 0\), given by (4.1) with \(H = Q^2/\sigma^4\). We list these asymptotic geometries below.
| String Theory | $(s, t, \pm)$ | Asymptotic Geometry |
|---------------|---------------|---------------------|
| $IIB_{9,1}$   | (3, 1)        | $O(4, 2)/O(4, 1) \times O(6)/O(5) = AdS_5 \times S^5$ |
| $IIB^*_{9,1}, IIB'^*_{9,1}$ | (4, 0, ±)     | $O(5, 1)/O(5) \times O(5, 1)/O(4, 1) = dS_5 \times H^5$ |
| $IIB_{7,3}$   | (3, 1, ±)     | $O(4, 2)/O(4, 1) \times O(4, 2)/O(3, 2) = AdS_5 \times AAdS_5$ |
| $IIB_{7,3}$   | (1,3,+)       | $O(2, 4)/O(2, 3) \times O(6)/O(5) = -AAdS_5 \times S^5$ |
| $IIB_{5,5}$   | (3,1,+)       | $O(4, 2)/O(4, 1) \times O(2, 4)/O(1, 4) = AdS_5 \times -AdS_5$ |
| $IIB_{5,5}$   | (3, 1, −)     | $O(4, 2)/O(3, 2) \times O(2, 4)/O(2, 3) = AAdS_5 \times -AAdS_5$ |
| $IIB_{5,5}$   | (1,3,+)       | $O(4, 2)/O(3, 2) \times O(2, 4)/O(2, 3) = AAdS_5 \times -AAdS_5$ |
| $IIB^*_{5,5}, IIB'^*_{5,5}$ | (4, 0, +)     | $O(5, 1)/O(5) \times O(1, 5)/O(5) = H^5 \times -H^5$ |
| $IIB^*_{5,5}, IIB'^*_{5,5}$ | (4, 0, −)     | $O(5, 1)/O(4, 1) \times O(1, 5)/O(1, 4) = dS^5 \times -dS^5$ |
| $IIB^*_{5,5}, IIB'^*_{5,5}$ | (2, 2, ±)     | $O(3, 3)/O(3, 2) \times O(3, 3)/O(2, 3)$ |
| $IIB^*_{5,5}, IIB'^*_{5,5}$ | (0, 4, +)     | $O(5, 1)/O(4, 1) \times O(1, 5)/O(1, 4) = dS^5 \times -dS^5$ |
| $IIB^*_{5,5}, IIB'^*_{5,5}$ | (0, 4, −)     | $O(5, 1)/O(5) \times O(1, 5)/O(5) = H^5 \times -H^5$ |

Table 4 Asymptotic geometries near $\sigma = 0$ of the D-branes with world-volume signature $(s, t)$ with $s + t = 4$ for the $(s, t, +)$ solutions with $\sigma^2 \geq 0$ and the $(s, t, −)$ solutions with $\sigma^2 \leq 0$.

5. World-Volume Actions

An $(s, t)$-brane is invariant under the Poincaré group $ISO(s, t)$ and the world-volume action should be a superPoincaré-invariant theory in $(s, t)$ dimensions with 16 supersymmetries. In some cases the Poincaré symmetry will be enhanced to the conformal group $SO(s + 1, t + 1)$, in which case the world-volume theory is invariant under the corresponding superconformal group with 32 supersymmetries. If the $(s, t)$-brane is embedded in $(S, T)$ dimensions, the world-volume action should also have an R-symmetry that contains $SO(\bar{s}, \bar{t})$, where

$$\bar{s} = S - s, \quad \bar{t} = T - t.$$  \hfill (5.1)

Here and in the following sections, we will find the field theories in $(s, t)$ dimensions with 16 supersymmetries that are the unique candidates for world-volume actions for the various
branes, and postpone until section 9 a discussion of the differences between \((s, t, +)\) branes and \((s, t, -)\) branes.

The \((s, t)\)-branes that couple to the Ramond-Ramond gauge fields in a string theory in \((S, T)\) dimensions with \(S + T = 10\) are generalised D-branes and the dynamics are governed by fundamental strings ending on the brane. The low-energy effective action for these strings is a world-volume super-Yang-Mills (SYM) theory (with a Born-Infeld-type action) in \((s, t)\) dimensions. In 10 dimensions, the only signatures \((S, T)\) that allow an \(N = 1\) superalgebra with 16 supersymmetries are those admitting Majorana-Weyl spinors, and these are the signatures \((9,1)\) and \((5,5)\) (together with the reverse signature \((1,9)\)). The \((9,1)\) SYM theory can be dimensionally reduced on a Euclidean \(p\)-torus \(T^p\) to give the usual maximal SYM theory in \((9 - p, 1)\) dimensions with R-symmetry \(SO(p)\) or on a Lorentzian torus \(T^{p,1}\) to give a Euclidean SYM theory in \((9 - p, 0)\) dimensions with R-symmetry \(SO(p, 1)\). Similarly, the \((5,5)\) SYM theory can be reduced on a torus \(T^{p,q}\) with signature \((p, q)\) (with \(p \leq 5, q \leq 5\)) to give a SYM theory in signature \((5 - p, 5 - q)\) with \(SO(p, q)\) R-symmetry.

One apparent problem is that there are many more types of generalised D-branes in tables 2 and 3 than there are types of SYM theories, and it is not clear whether all cases can be covered by the available SYM theories. For example, an \((s, t)\) brane of the IIB theory in \((7,3)\) dimensions has \(ISO(s, t) \times SO(7 - s, 3 - t)\) symmetry, and a natural way that this could arise would be from compactifying a Yang-Mills theory in \((7,3)\) dimensions on a torus \(T^{7-s,3-t}\), but there is no supersymmetric Yang-Mills theory in \((7,3)\) dimensions. This means that the world-volume theory must either come from a Yang-Mills theory in \((7,3)\) dimensions without supersymmetry, or must arise in some other way. We will address this issue in the next section, and show that in each case the world-volume theory is in fact a SYM theory in \(s + t\) dimensions with these symmetries, and explain how this comes about.

The M-theories in \((10,1)\), \((9,2)\) and \((6,5)\) dimensions have generalised membranes with
$s + t = 3$ whose world-volumes are theories with 8 scalars and 8 fermions with Poincaré symmetry $ISO(s,t)$ which is enhanced to the conformal group $SO(s + 1, t + 1)$ and R-symmetry $SO(S - s, T - t)$ at a conformal fixed point. There are also generalised 5-brane solutions with world-volume signatures $(s,t)=(5,1), (3,3), (1,5)$ and these are described by tensor multiplets in $(s,t)$ dimensions with a 2-form gauge-field with self-dual field strength, 5 scalars and 4 fermions with an $SO(S - s, T - t)$ R-symmetry and $SO(s+1, t+1)$ conformal symmetry. The generalised membranes will be discussed in section 7 and the generalised 5-branes in section 8.

In each case, each of these branes can be linked to standard branes of M-theory or string theory by chains of dualities, and this implies that for $N$ coincident branes there is a $U(N)$ gauge symmetry.

6. World-Volume Theories for Generalised D-Branes

In this section, we will discuss the world-volume theories of the D-branes of the various type II string theories. These are the branes that couple to RR gauge fields and on which the fundamental string (which has either Lorentzian or Euclidean world-sheet, depending on the case) can end.

The usual type IIA and IIB theories in $(9,1)$ dimensions have $D_p$-branes with world-volume signature $(p,1)$ (with $p$ even for IIA and odd for IIB) and these have effective world-volume theories which are SYM theories in $(p,1)$ dimensions obtained by reducing $(9,1)$ dimensional SYM on the Euclidean torus $T^{9-p}$. Timelike T-duality takes the type IIA and IIB theories to the IIB* and IIA* theories in $(9,1)$ dimensions respectively, and takes a D-brane with Lorentzian world-volume of signature $(p,1)$ to an E-brane with Euclidean signature $(p,0)$. The effective world-volume theory is Euclidean SYM in $(p,0)$ dimensions obtained by reducing $(9,1)$ dimensional SYM on the Lorentzian torus $T^{9-p,1}$. T-dualities take D-branes or E-branes in $(9,1)$ dimensions to D-branes or E-branes in $(9,1)$ dimensions.
Similarly, the IIA and IIB, IIA* and IIB* theories in (5,5) dimensions have D-branes with world-volume signature \((s, t)\) and these have effective world-volume theories which are SYM theories in \((s, t)\) dimensions obtained by reducing (5,5) dimensional SYM on the torus \(T^{5-s,5-t}\). (The IIA* theory in (5,5) dimensions is obtained from the IIA theory by the mirror transformation \(g_{\mu\nu} \rightarrow -g_{\mu\nu}\) interchanging space and time, and so is equivalent to the IIA theory [4].)

Thus in each of these cases, the world-volume theory for an \((s, t)\) D-brane in \((S, T)\) dimensions (which is (9,1) or (5,5)) is \((s, t)\)-dimensional SYM obtained by reducing from \((S, T)\) SYM on \(T^{\bar{s},\bar{t}}\) to give a theory with \(ISO(s, t) \times SO(\bar{s}, \bar{t})\) symmetry. For example, the (2,1)-brane in (9,1) dimensions is a (2,1)-dimensional SYM theory obtained by reducing from (9,1)-dimensional SYM theory on \(T^7\) to give a theory with \(ISO(2, 1) \times SO(7)\) symmetry, which is enhanced to an \(O(3, 2) \times O(8)\) symmetry at the conformal fixed point.

As mentioned in the previous section, this pattern cannot carry over to the string theories in (8,2), (7,3) or (6,4) dimensions. In each case, the brane is expected to have a symmetry containing \(ISO(s, t) \times SO(\bar{s}, \bar{t})\), and this would be the symmetry obtained by reducing a Yang-Mills theory in \((S, T)\) dimensions on \(T^{\bar{s},\bar{t}}\). But there is no SYM theory for these values of \((S, T)\), so that if this picture is correct, there can be no conventional supersymmetry.

However, there is another way of getting a SYM theory with the right symmetry. Consider, for example, the (3,1) brane of the IIB\(_{7+3}\) theory, which should have \(ISO(3, 1) \times SO(4, 2)\) symmetry, and which could be expected to be governed by a SYM theory. If there were a SYM theory in (7,3) dimensions, this could arise by reducing on \(T^{4,2}\), but there is no such theory. However, reducing the SYM theory in (5,5) dimensions on \(T^{2,4}\) does give a SYM theory in (3,1) dimensions with the required \(ISO(3, 1) \times SO(4, 2)\) symmetry. However, this would give an SYM action in (3,1) dimensions in which 2 scalars have a kinetic term of the ‘right’ sign, and 4 with the ‘wrong’ sign, while the (3,1) brane should have 4 scalars with a kinetic term of the ‘right’ sign, which are the zero-modes for
translations in the transverse spatial dimensions, and 2 with the ‘wrong’ sign, which are
the zero-modes for translations in the transverse temporal dimensions. However, the right
scalar action could be obtained by starting from (5,5) dimensional SYM in which the whole
action is multiplied by $-1$, so that the gauge fields have kinetic terms of the ‘wrong’ sign;
this is clearly consistent with supersymmetry.

This works for all the D-branes of the type II theories in (8,2), (7,3) or (6,4) dimensions.
In each case, there is a candidate world-volume SYM theory in $(s, t)$ dimensions with
$ISO(s, t) \times SO(\tilde{s}, \tilde{t})$ symmetry which is obtained by reducing an SYM theory in $(\tilde{S}, \tilde{T})$
dimensions on $T^{\tilde{t}, \tilde{s}}$, where

$$\tilde{S} = s + \tilde{t}, \quad \tilde{T} = t + \tilde{s} \quad (6.1)$$

For this to work, it is essential that $(\tilde{S}, \tilde{T})$ should be (9,1), (5,5) or (1,9), so that an SYM
theory in $(\tilde{S}, \tilde{T})$ dimensions exists, and remarkably this is the case for each D-brane in these
theories, as is easily checked. The IIB$'$ theories in (9,1) or (5,5) dimensions, which are the
strong-coupling duals of the IIB$^*$ theories, are related to these theories by T-dualities (the
(9,1) IIB$'$ theory is T-dual to the (10,0) and (8,2) IIA theories while the (5,5) IIB$'$ theory
is T-dual to the (6,4) IIA theory), and so the same pattern should persist for these theories
also; indeed, $(\tilde{S}, \tilde{T})$ is (9,1), (5,5) or (1,9) for each of the D-branes of the IIB$'$ theories,
as is required. In each of these cases, the SYM action in $(\tilde{S}, \tilde{T})$ dimensions has a gauge
field kinetic term of the wrong sign, with all other signs following from supersymmetry.
As another example, consider the (1,3)-brane solution of IIB$_{7+3}$. The SYM theory in this
case arises from the reduction of a (1,9) SYM theory with the ‘wrong’ sign on $T^{0,6}$ to
give a $1+3$-dimensional SYM theory with $ISO(1,3) \times O(6)$ symmetry. For completeness,
we note that the (3,1), (2,0), (4,2) and (5,3) solutions of IIB$_{7+3}$ arise from reductions of
‘wrong’ sign (5,5) SYM, the (0,2), (6,0) and (7,1) solutions of IIB$_{7+3}$ arise from reductions
of ‘wrong’ sign (9,1) SYM, and the (1,3) and (0,2) solutions of IIB$_{7+3}$ arise from reductions
of ‘wrong’ sign (1,9) SYM.

Thus we have found the following picture. For the theories in which the fundamental
string has a Lorentzian world-sheet, i.e. the IIA, IIB, IIA*, IIB* theories in (9,1) or (5,5) dimensions, the world-volume theory is \((S,T)\) dimensional SYM compactified on \(T^{\tilde{s},\tilde{t}}\) to give a theory with \(ISO(s,t) \times SO(\tilde{s},\tilde{t})\) symmetry, and the \((S,T)\) dimensional SYM theory has action

\[
S = -\frac{1}{4} \int d^{10} x F^2 + \ldots
\]  

(6.2)

with the ‘right’ sign.

For the theories in which the fundamental brane has 2 Euclidean dimensions, i.e. the IIB’ theories in (9,1) or (5,5) dimensions, the IIA theories in (10,0),(8,2) and (6,4) dimensions, and the IIB theory in (7,3) dimensions, an SYM theory with the right symmetry is obtained by reducing the SYM theory in \((\tilde{S},\tilde{T})\) dimensions on \(T^{\tilde{s},\tilde{t}}\), where the \((\tilde{S},\tilde{T})\)-dimensional SYM action has the ‘wrong’ sign for the gauge field kinetic term.

\[
S = \frac{1}{4} \int d^{10} x F^2 + \ldots
\]  

(6.3)

For the D-branes in (10,0), (8,2), (7,3) or (6,4) dimensions, this is the *unique* SYM theory with \(ISO(s,t) \times SO(\tilde{s},\tilde{t})\) symmetry, and so supersymmetry implies this must be the correct world-volume theory in these cases, and then T-duality implies that this construction must also give the world-volume theory for the D-branes of the IIB’ theories.

As a further example, a \((4,0)\) brane in (9,1) dimensions should be governed by an SYM theory in \((4,0)\) dimensions with \(ISO(4) \times SO(5,1)\) symmetry, and such an SYM theory can be obtained either by reducing \((9,1)\) dimensional SYM on \(T^{5,1}\) or by reducing \((5,5)\) dimensional SYM on \(T^{1,4}\); the former gives the world-volume theory of the \((4,0)\) brane of the IIB*\(_{9,1}\) theory, and the latter gives the world-volume theory of the \((4,0)\) brane of the IIB’\(_{9,1}\) theory.

The D-brane world-volume theories are then always supersymmetric Yang-Mills theories with 16 supersymmetries, and the identifications of these theories above is consistent with T-duality. The theories in which the fundamental string is Lorentzian have SYM theories with lagrangians \(-F^2 + \ldots\) while those in which the fundamental ‘string’ is Euclidean
have SYM theories with lagrangians $+F^2 + ...$.

### 6.1. Superconformal Symmetry

For those D-branes with $s + t = 4$ (including the usual D3-brane) the SYM theory is invariant under the conformal group $SO(s + 1, t + 1)$, containing the Poincaré group $ISO(s, t)$. The bosonic symmetry is then $SO(s + 1, t + 1) \times SO(\tilde{s}, \tilde{t})$ and, as the theory is supersymmetric, there must be a superconformal group for which this is the bosonic part. For the usual D3-brane, this is the group $SU(2, 2/2)$, for the E4-brane of the IIB*$_{9+1}$ theory, this is $SU(2, 2/2)^*$ and for the other cases other real forms of the same group emerge. We summarise the results below (recall that $SU(4) \sim SO(6)$, $SU(2, 2) \sim SO(4, 2)$, $SL(4, \mathbb{R}) \sim SO(3, 3)$).

| Theory | $(s, t)$ | Bosonic Symmetry | Supergroup | SYM on $T^{p,q}$ |
|--------|----------|------------------|------------|------------------|
| $IIB_{3,1}$ | (3,1) | $O(4, 2) \times O(6)$ | $SU(2,2/4)$ | $SYM_{9,1}$ on $T^6$ |
| $IIB_{9,1}^*$ | (4,0) | $O(5, 1) \times O(5, 1)$ | $SU^*(2, 2/4)$ | $SYM_{9,1}$ on $T^{5,1}$ |
| $IIB_{3,1}^*$ | (4,0) | $O(5, 1) \times O(5, 1)$ | $SU^*(2, 2/4)$ | $SYM_{9,1}$ on $T^{5,1}$ |
| $IIB_{7,3}$ | (3,1) | $O(4, 2) \times O(4, 2)$ | $SU(2,2/2,2)$ | $SYM_{5,5}$ on $T^{2,4}$ |
| $IIB_{7,3}$ | (1,3) | $O(4, 2) \times O(6)$ | $SU(2,2/4)$ | $SYM_{1,9}$ on $T^6$ |
| $IIB_{5,5}$ | (3,1) | $O(4, 2) \times O(4, 2)$ | $SU(2,2/2,2)$ | $SYM_{5,5}$ on $T^{2,4}$ |
| $IIB_{5,5}^*$ | (1,3) | $O(4, 2) \times O(4, 2)$ | $SU^*(2, 2/4)$ | $SYM_{5,5}$ on $T^{2,4}$ |
| $IIB_{5,5}^*$ | (4,0) | $O(5, 1) \times O(5, 1)$ | $SU^*(2, 2/4)$ | $SYM_{5,5}$ on $T^{1,5}$ |
| $IIB_{5,5}^*$ | (2,2) | $O(3, 3) \times O(3, 3)$ | $SL(4/4)$ | $SYM_{5,5}$ on $T^{3,3}$ |
| $IIB_{5,5}^*$ | (0,4) | $O(5, 1) \times O(5, 1)$ | $SU^*(2, 2/4)$ | $SYM_{5,5}$ on $T^{5,1}$ |
| $IIB_{5,5}^*$ | (4,0) | $O(5, 1) \times O(5, 1)$ | $SU^*(2, 2/4)$ | $SYM_{5,5}$ on $T^{1,5}$ |
| $IIB_{5,5}^*$ | (2,2) | $O(3, 3) \times O(3, 3)$ | $SL(4/4)$ | $SYM_{5,5}$ on $T^{3,3}$ |
| $IIB_{5,5}^*$ | (0,4) | $O(5, 1) \times O(5, 1)$ | $SU^*(2, 2/4)$ | $SYM_{5,5}$ on $T^{5,1}$ |

**Table 5** The symmetries of the D-branes with world-volume signature $(s, t)$ with $s + t = 4$ coupling to $C_4$ in the various IIB theories. The world-volume theories arise in each case from super-Yang-Mills theory in 9+1, 5+5 or 1+9 dimensions compactified on a torus $T^{p,q}$.
The table also gives the massless sector of the world-volume theories. In each case, it is given by the super-Yang-Mills theory in 9+1, 5+5 or 1+9 dimensions dimensionally reduced on a torus $T^{p,q}$. The sign of the kinetic term of the gauge field is the ‘wrong’ one ($+F^2$) for the branes of the $IIB'$ theories and the $IIB_{7+3}$ theories. For example, the (4,0)-brane of the $IIB_{5,5}'$ has a world-volume theory given by the SYM theory in 5+5 dimensions compactified on $T^{1,5}$ to obtain a theory with manifest $ISO(4) \times O(1,5)$ symmetry which is enhanced to $O(5,1) \times O(1,5)$ by the conformal invariance.

7. The M2-Brane Family

In this section we consider the world-volume theories of the M-branes with $s + t = 3$ in $S + T = 11$ dimensions for $S = 10, 9$ or 6. A vertical dimensional reduction in a space dimension then gives the $(s, t)$ D-brane of a IIA theory in $(S−1, T)$ dimensions while a time reduction gives the $(s, t)$ D-brane of a IIA theory in $(S−1, T)$ dimensions. In either case, we know from the previous section which SYM theory with a vector and 7 scalars gives the world-volume theory of the $(s, t)$ D-brane, and dualising the vector to an eighth scalar then gives the world-volume theory of the M-brane, generalising the relation between the D2-brane and the M2-brane [19]. The sign of the kinetic term of the extra scalar then follows from the analysis of section 8 of [3].

It has been argued [20] that the world-volume theory of the M2-brane has an infra-red fixed point at which the theory becomes superconformally invariant. We shall assume that all the world-volume theories in the M2-brane family have a superconformal fixed point. This assumption is supported by the fact that the near-horizon limit of each of these branes is superconformally invariant.

The results are as follows. The world-volume theory is a theory in $(s, t)$ dimensions $(s + t = 3)$ with 8 scalars and 8 fermions and a bosonic symmetry $ISO(s, t) \times SO(\tilde{s}, \tilde{t})$. The R-symmetry is $SO(\tilde{s}, \tilde{t})$ and there are $\tilde{s}$ scalars with the ‘right’ sign kinetic term, and $\tilde{t}$ with the ‘wrong’ one. The theory is assumed to have a conformal fixed point at which the
ISO(s, t) symmetry is enhanced to the conformal group \( SO(s + 1, t + 1) \), and the theory is invariant under a superconformal group with bosonic subgroup \( SO(s + 1, t + 1) \times SO(\tilde{s}, \tilde{t}) \). For the usual M2-brane, this is the supergroup \( OSp(4/8) \) while for the other cases it is a different real form of this group.

For example, the SYM theory of the (2,1)-brane solution of (10,1) M theory arises from the (9,1) SYM theory by compactifying on \( T^7 \). The \( O(7) \) symmetry is enhanced to \( O(8) \) by dualising the vector \( A_\mu \) to an eighth scalar \( X^8 \). The \( ISO(2, 1) \times O(8) \) symmetry is then enhanced to \( O(3, 2) \times O(8) \) at the conformal fixed point. The SYM theory of the (3,0)-brane solution of the (9,2) M* theory arises from the (9,1) SYM theory by compactifying on \( T^6 \). The \( O(6, 1) \) R-symmetry is enlarged to \( O(6, 2) \) by dualising the vector \( A_\mu \) to an eighth scalar \( X^8 \), which has a kinetic term of the ‘wrong’ sign. The \( ISO(3) \times O(6, 2) \) symmetry is then enhanced to \( O(4, 1) \times O(6, 2) \) at the conformal point.

A similar analysis holds for the (2,1) solution of M* and for the (2,1) and (0,3) solutions of the (6,5) M’. We summarise the results below.

| \((S, T)\)  | \((s, t)\)  | Bosonic Symmetry         | Supergroup          |
|------------|------------|-------------------------|---------------------|
| (10,1)     | (2,1)     | \(O(3, 2) \times O(8)\) | \(OSp(4/8)\)       |
| (9,2)      | (3,0)     | \(O(4, 1) \times O(6, 2)\) | \(OSp^*(4/8)\)     |
| (9,2)      | (1,2)     | \(O(3, 2) \times O(8)\) | \(OSp(4/8)\)       |
| (6,5)      | (2,1)     | \(O(3, 2) \times O(4, 4)\) | \(OSp(4/4,4)\)     |
| (6,5)      | (0,3)     | \(O(4, 1) \times O(6, 2)\) | \(OSp^*(4/8)\)     |

**Table 5** The symmetries of the M-branes with world-volume signature \((s, t)\) with \(s + t = 3\) coupling to \(C_3\) in the various M-theories with signature \((S, T)\).

8. The M5-Brane Family

In this section we consider the world-volume theories of the M-branes with \(s + t = 6\) in \(S + T = 11\) dimensions for \(S = 10, 9\) or \(6\). A spatial double dimensional reduction of an \((s, t)\) M-brane gives an \((s - 1, t)\) D-brane in the IIA theory in signature \((S - 1, T)\) and
a timelike double dimensional reduction gives an \((s, t - 1)\) D-brane in the IIA theory in signature \((S, T - 1)\). In either case, the world-volume theory of the D-brane is the SYM theory in \((s - 1, t)\) dimensions or \((s, t - 1)\) dimensions determined in section 5, with bosonic symmetry \(ISO(s - 1, t) \times SO(\tilde{s}, \tilde{t})\) or \(ISO(s, t - 1) \times SO(\tilde{s}, \tilde{t})\). The M-brane world-volume theory is a self-dual tensor multiplet with 5 scalars and 4 spinor fields that gives the SYM theories on spacelike or timelike dimensional reduction. This tensor multiplet theory has bosonic symmetry containing \(ISO(s, t) \times SO(\tilde{s}, \tilde{t})\), with the \(\tilde{s} + \tilde{t} = 5\) scalars in a vector representation of \(SO(\tilde{s}, \tilde{t})\), so that there are \(\tilde{s}\) scalars with a kinetic term of the right sign and \(\tilde{t}\) with the wrong sign. This theory is in fact conformally invariant, so that the Poincaré symmetry \(ISO(s, t)\) is enhanced to the conformal group \(SO(s + 1, t + 1)\), and the theory is invariant under a superconformal group with bosonic subgroup \(SO(s + 1, t + 1) \times SO(\tilde{s}, \tilde{t})\), and in each case is some real form of \(OSp(8/4)\).

For example, the M5-brane or \((5,1)\)-brane of the \((10,1)\) M theory arises as the strong coupling limit of the D4-brane. The D4-brane world-volume theory is \((4,1)\)-dimensional SYM with \(ISO(4,1) \times O(5)\). At strong coupling an extra dimension emerges, so that the Poincaré symmetry becomes \(ISO(5,1)\) and is a subgroup of the conformal symmetry \(SO(6,2)\), so that the full bosonic symmetry is \(O(6,2) \times O(5)\). On the other hand, the \((5,1)\)-brane of the \((9,2)\) M* theory is the strong-coupling limit of the E5-brane of the IIA* theory in 9+1 dimensions, and the E5-brane symmetry \(O(4,1) \times ISO(5)\) is enhanced to \(O(4,1) \times ISO(5,1)\) in the strong coupling limit in which an extra time dimension emerges, and conformal invariance then increases this to the \(O(4,1) \times O(6,2)\) symmetry of a self-dual tensor theory in 5+1 dimensions with \(O(4,1)\) R-symmetry and twisted supersymmetry with 16 supercharges. A similar analysis holds for the other M5-branes. We summarise the results below:
Table 6 The symmetries of the M-branes with world-volume signature \((s,t)\) with \(s + t = 5\) coupling to \(\tilde{C}_6\) in the various M-theories with signature \((S,T)\).

9. Dualities Between Conformal Field Theories and De Sitter Space Theories.

The massless sector of the D3-brane, M2-brane and M5-brane world-volume theories are field theories in 3+1, 2+1 and 5+1 dimensions which are superconformally invariant and are dual to the IIB string theory in \(AdS_5 \times S^5\), M-theory in \(AdS_4 \times S^7\) and M-theory in \(AdS_7 \times S^4\), respectively [7]. The original argument leading to this holographic duality was based on considering a certain limit of the brane theory. In [7], \(N\) parallel D3-branes separated by distances of order \(\rho\) were considered and the zero-slope limit \(\alpha' \to 0\) was taken keeping \(r = \rho/\alpha'\) fixed, so that the energy of stretched strings remained finite. This decoupled the bulk and string degrees of freedom leaving a theory on the brane which is \(U(N)\) \(N = 4\) super Yang-Mills with Higgs expectation values, which are of order \(r\), corresponding to the brane separations. The corresponding D3-brane supergravity solution is of the form (2.3) and has charge \(Q = a^2 \propto N g_s/\alpha'^2\) where \(g_s\) is the string coupling constant, which is related to the super Yang-Mills coupling constant \(g_{YM}\) by \(g_s = g_{YM}^2\). Then as \(\alpha' \to 0\), \(Q\) becomes large and the background becomes \(AdS_5 \times S^5\). The IIB string theory in the \(AdS_5 \times S^5\) background is a good description if the curvature \(R \sim 1/a^2\) is not too large, while if \(a^2\) is large, the super Yang-Mills description is reliable. In the ’t Hooft limit in which \(N\) becomes large while \(g_{YM}^2 N\) is kept fixed, \(g_s \sim 1/N\), so that as \(N \to \infty\), we get the free string limit \(g_s \to 0\), while string loop corrections correspond to
1/N corrections in the super Yang-Mills theory. Similar arguments apply to the M2 and M5 brane cases.

This was extended to the case of E4-branes in $E_8$, and the two E4-brane solutions, the $(4, 0, \pm)$-branes, correspond to whether the separation between the E4-branes that is kept fixed is spacelike or timelike. Recall that the scalars of the super Yang-Mills theory are in a vector representation of the $SO(5, 1)$ R-symmetry, where those in the $5$ of $SO(5) \subset SO(5, 1)$ have kinetic terms of the right sign and correspond to brane separations in the 5 spacelike transverse dimensions, while the remaining ($U(N)$-valued) scalar is a ghost and corresponds to timelike separations of the E-branes. For the case of $N$ parallel E4-branes of the IIB* string theory separated by distances of order $\rho$ in one of the 5 spacelike transverse dimensions, we take the zero-slope limit $\alpha' \to 0$ keeping $\sigma = \rho/\alpha'$ fixed. This gives a decoupled theory on the brane consisting of the $U(N) \mathcal{N} = 4$ Euclidean super Yang-Mills, with Higgs expectation values of order $\sigma$ for the scalars corresponding to the spacelike separations. The corresponding supergravity background is the $(4, 0, +)$-brane. We again have $Q = a^2 \propto N g_s/\alpha'^2$ and $g_s = g_{YM}^2$, so that for large $N$, the system can be described by the IIB* string theory in $dS_5 \times H^5$ if $a^2$ is large and by the large $N$ Euclidean super Yang-Mills theory when $a^2$ is small. In the 't Hooft limit, string loop corrections again correspond to 1/N corrections in the super Yang-Mills theory. For $N$ E4-branes of the IIB* string theory separated by distances of order $T$ in the timelike transverse dimension, we take the zero-slope limit $\alpha' \to 0$ keeping $\tau = T/\alpha'$ fixed. This gives a decoupled theory on the brane consisting of the $U(N) \mathcal{N} = 4$ Euclidean super Yang-Mills, with Higgs expectation values of order $\tau$ for the scalars corresponding to the timelike separations. The corresponding supergravity background is the $(4, 0, -)$-brane. Again for large $N$, the system can be described by the IIB* string theory in $dS_5 \times H^5$ if $a^2$ is large and by the large $N$ Euclidean super Yang-Mills theory when $a^2$ is small.

Similar arguments can be given for each of the brane solutions in the D3-brane, M2-brane and M5-brane families with various signatures. Considering $N$ parallel $(s, t)$ branes
and taking the Maldacena limit, we formally obtain a duality between the \((s, t)\) brane world-volume superconformal field theory and the IIB or M-theory in \((S, T)\) dimensions in the spacetime given by the asymptotic form of the \((s, t)\) brane solution. Moreover, it is straightforward to check that the superconformal group found for the world-volume theories matches the super-isometry group of the asymptotic geometry, so that the symmetries of the two theories agree.

However, there are some new complications that arise for the cases considered here. The metric (3.1),(3.5) corresponds to two solutions, the \((s, t, +)\) brane and the \((s, t, -)\) brane (unless the transverse space is Euclidean), and each of these has a different asymptotic geometry in general. Although for a given \((s, t)\) there are two brane solutions, there is only one \((s, t)\) worldvolume field theory allowed by supersymmetry. A given spacetime can arise as the asymptotic geometry for several different types of brane. For example, the M*-theory solution \(AdS_7 \times dS_4\) is the asymptotic geometry for both the \((5,1,+)\) brane and the \((3, 0, -)\) brane of M*-theory. The asymptotic geometry is often the product of two non-compact spaces, and so holography might be expected to operate in each factor, with a field theory associated with some surface (e.g. the boundary) for each. Clearly, the dualities between theories will be more complicated than in the cases considered in [7].

Let us consider the example of the \((0,3)\) brane of M’ theory in some detail. The transverse space has signature \((6,2)\) and the world-volume theory has 8 scalars transforming as a vector under the \(O(6,2)\) R-symmetry and 8 fermions transforming as a spinor of \(O(6,2)\). The \(ISO(3)\) Poincaré symmetry is enhanced to \(O(1,4)\) conformal symmetry, and the superconformal group is \(OSp^*(4/8)\). In the usual way, for \(N\) branes the world-volume fields take values in the Lie algebra of \(U(N)\) and giving some of the scalars expectation values corresponds to separating the branes. Two branes at two positions in \(\mathbb{R}^{6,2}\) have a separation which can be spacelike, timelike or null.

The two associated supergravity solutions, the \((0,3,+)\) brane and the \((0, 3, -)\) brane, both have isometry \(ISO(3) \times O(6,2)\). The asymptotic geometry for the \((0,3,+)\) brane is
$AAdS_7 \times -dS_4$ while that for the $(0, 3, -)$ brane is $AdS_7 \times -H_4$, and both have isometry group $O(4,1) \times O(6,2)$, contained in the super-isometry group $OSp^*(4/8)$. If the two solutions, the $(0, 3, \pm)$ branes, are both associated with the same field theory, then this would suggest that $M'$ theory in $AdS_7 \times -H_4$ is dual to $M'$ theory in $AAdS_7 \times -dS_4$. Moreover, precisely the same asymptotic geometries arise for the $(5,1)$ brane – the $(5,1, +)$ brane tends to $AdS_7 \times -H_4$ and the $(5,1, -)$ to $AAdS_7 \times -dS_4$. This would then lead to the suggestion that the $(5, 1, \pm)$ theory should be dual to the $(0, 3, \mp)$ theory. Then there are four theories – the world-volume theories of the $(5, 1)$ and $(0, 3)$ branes, and $M'$ theory in the backgrounds $AdS_7 \times -H_4$ and $AAdS_7 \times -dS_4$ – each with the same symmetry group, $OSp^*(4/8)$, and naive application of standard arguments suggest they might all be related by dualities.

To see what is going on, it will be useful to consider the application of the Maldacena argument to this case. Consider a two-centre $(0,3)$ brane solution with metric (3.1) and harmonic function

$$H = c + \frac{Q}{(\tilde{\eta}_{ij}Y^i Y^j)^3} + \frac{q}{[\tilde{\eta}_{ij}(Y^i - Y^i_0)(Y^j - Y^j_0)]^3}$$

(9.1)

corresponding to a brane of charge $Q$ at $Y = 0$ and one of charge $q$ at $Y = Y_0$. Suppose further that $Q >> q$, corresponding to $N$ branes at $Y = 0$ and $n$ branes at $Y = Y_0$ with $N >> n$. The contribution of the $n$ branes will be small except in a neighbourhood of $Y_0$, and outside this neighbourhood the solution is approximately that of $N$ branes at the origin, so that the $n$ branes can be thought of as probes in the $N$ brane background. If $Y_0$ is spacelike, we should take this $N$ brane geometry to be the $(0,3,+)$ solution while if it is timelike, we should take the $(0,3,-)$ one, and more generally for a multi-centre $(0,3,+)$ solution the positions $Y_0$ in (3.8) should all be spacelike, while for the $(0,3,-)$ solution they should be timelike. These correspond to the spacelike and timelike interpolations of $\mathbb{R}$.

This suggests that the $(0,3)$ world-volume theory has (at least) two ‘branches’, one in which the scalar expectation values are all spacelike, and one in which they are all
timelike. Then the (0,3,+) brane solutions should correspond to the branch of the world-volume theory in which the expectation values of the scalars are all spacelike and the (0,3,−) brane solutions to the timelike branch. However, these two branches are clearly part of the same connected moduli space of one theory, so that presumably one can move continuously between them.

We now take the Maldacena limit in which the Planck length \( l_p \) tends to zero while the brane separation \( Y_0 \) scales so that \( Y_0/l_p^3 \) stays fixed. For the case with \( Y_0 \) spacelike, this gives the (0,3,+) asymptotic geometry, \( AAdS_7 \times -dS_4 \), while for the timelike case this gives \( AdS_7 \times -H_4 \). Then \( M' \) theory in these two asymptotic geometries should each, by the usual arguments, be holographically related to the corresponding branch of the (0,3) world-volume theory, and so \( M' \) theory in these two different backgrounds correspond to different points in the same connected moduli space, and so in this sense are dual to each other.

Consider the space \( AdS_7 \times -H_4 \). The hyperbolic space \( -H_4 \) has a boundary \( -S^3 \) and the (0,3) brane world-volume theory is associated with this boundary, while \( AdS_7 \) has a timelike boundary \( S^5 \times S^1 \) (or \( S^5 \times \mathbb{R} \) for the covering space \( CAdS_7 \), conformal to Minkowski space \( \mathbb{R}^{5,1} \)) and this boundary is associated with the (5,1) brane world-volume theory.

For supersymmetric solutions of any of these theories of the form \( AdS_7 \times Y_4 \) with \( Y^4 \) compact (as in the solution of M-theory with \( Y_4 = S^4 \)), there is a holographic relation between the theory in \( AdS_7 \times Y_4 \) and a 5+1 dimensional CFT on the boundary of the anti-de Sitter space. The relation is that of \([21,22]\); if the fields bulk \( \phi^i \) tend to prescribed functions \( \phi_0(x) \) on the \( AdS \) boundary, the partition function \( Z(\phi_0^i) \) for the bulk theory is identified with (a generating functional for) correlation functions of a conformal field theory on the boundary. Similarly, for solutions \( X_7 \times \pm H_4 \) with \( X_7 \) compact, then the bulk partition function with prescribed boundary values on the boundary \( S^3 \) of \( H^4 \) is identified with correlation functions of a Euclidean CFT on \( S^3 \) (since \( H^4 \) is the analytic continuation of
$AdS_4$). Now the same should apply if $X$ or $Y$ is non-compact, provided trivial boundary conditions are imposed on $X$ or $Y$. In particular, for $AdS_7 \times -H_4$, the $M'$ theory partition function with prescribed boundary conditions on $AdS_7$ and trivial boundary conditions on $-H_4$ should be identified with correlation functions of the $(5,1,+)$ brane world-volume CFT, while the $M'$ theory partition function with prescribed boundary conditions on $-H_4$ and trivial boundary conditions on $AdS_7$ should be identified with correlation functions of the $(0,3,-)$ brane world-volume CFT. When there are non-trivial boundary conditions for both spaces, it seems unlikely that the system could be described by either the 3-dimensional CFT or the 6-dimensional CFT on their own, but that perhaps both would be needed.

This suggests the following picture for the general case. For a given $(s,t)$, the $(s,t,\pm)$ branes will tend to the asymptotic geometries

$$\left( s,t,+, \right) -\text{brane} \rightarrow X^+_{(s+1,t)} \times Y^+_{(\tilde{s}-1,\tilde{t})}, \quad \left( s,t,-, \right) -\text{brane} \rightarrow X^-_{(s,t+1)} \times Y^-_{(\tilde{s},\tilde{t}-1)} \quad (9.2)$$

for spaces $X,Y$, listed in section 4, where $X^+_{(s+1,t)}$ has signature $(s+1,t)$ etc. In some situations, as in M-theory or the IIB string, one of the two cases will be absent. The $(s,t)$ brane world-volume CFT will correspond to the bulk theory on $X^\pm \times Y^\pm$ with prescribed boundary conditions on $X^\pm$ and trivial boundary conditions on $Y^\pm$. The world-volume theory of the $(s,t)$ brane has two branches, the $(s,t,\pm)$ branch with scalar expectation values in a spacelike direction in the moduli space, and the $(s,t,\mp)$ branch with scalar expectation values in a timelike direction. The $(s,t,\pm)$ branch of the field theory is ‘dual’ to the theory on $X^+_{(s+1,t)} \times Y^+_{(\tilde{s}-1,\tilde{t})}$ and is associated with the boundary of $X^+_{(s+1,t)}$, while the $(s,t,\mp)$ branch of the field theory is ‘dual’ to the theory on $X^-_{(s,t+1)} \times Y^-_{(\tilde{s},\tilde{t}-1)}$ and is associated with the boundary of $X^-_{(s,t+1)}$. Further, assuming that it is valid to view the $(s,t,\pm)$ branches as two regions in the moduli space of a single theory, namely the $(s,t)$-brane world-volume theory, then this theory is holographically related to the string or $M$-type theory on the two spaces $X^+_{(s+1,t)} \times Y^+_{(\tilde{s}-1,\tilde{t})}$ and $X^-_{(s,t+1)} \times Y^-_{(\tilde{s},\tilde{t}-1)}$. These two
solutions also have a holographic dual associated with the boundary of $Y$, a conformal field theory in $(\tilde{s} - 1, \tilde{t} - 1)$ dimensions associated with the boundaries of both $Y^+_{(\tilde{s}-1,\tilde{t})}$ and of $Y^-_{(\tilde{s},\tilde{t}-1)}$, with the two branches of the field theory corresponding to the two different solutions. Then there is a quartet of related theories, the string or M-type theory on the two spaces $X^+_{(s+1,t)} \times Y^+_{(\tilde{s}-1,\tilde{t})}$ and $X^-_{(s,t+1)} \times Y^-_{(\tilde{s},\tilde{t}-1)}$, and the conformal field theories in $(s, t)$ dimensions and in $(\tilde{s} - 1, \tilde{t} - 1)$ dimensions. The relation between these theories that is suggested is intriguing and clearly deserves further investigation.

**Acknowledgements:**

CMH is supported by an EPSRC Senior Fellowship and would like to thank Rockefeller University and Baruch College for hospitality. RRK is supported by NSF Grant 9900773 and by a Eugene Lang Junior Faculty Research Scholarship.
References

[1] C.M. Hull and P.K. Townsend, Nucl. Phys. B438 (1995) 109, hep-th/9410167.
[2] E. Witten, Nucl. Phys. B443 (1995) 85.
[3] C. M. Hull, JHEP 9807:021, 1998, hep-th/9806146.
[4] C. M. Hull, JHEP 9811:017, 1998, hep-th/9807127.
[5] C. M. Hull, hep-th/9911080.
[6] C. M. Hull and R. R. Khuri, Nucl. Phys. B536 (1999) 219, hep-th/9808069.
[7] J. Maldacena, hep-th/9711200.
[8] E. Witten, hep-th/9802150.
[9] G. Horowitz and A. Strominger, Nucl. Phys B360 (1991) 197.
[10] See M. J. Duff, R. R. Khuri and J. X. Lu, Phys. Rep. B259 (1995) 213 and references therein.
[11] C.M. Hull, Phys. Lett. B357 (1995) 545, hep-th/9506194.
[12] T. H. Buscher, Phys. Lett. 159B (1985) 127, Phys. Lett. B194 (1987), 51 ; Phys. Lett. B201 (1988), 466.
[13] E. Bergshoeff, C.M. Hull and T. Ortin, Nucl. Phys. B451 (1995) 547, hep-th/9504081.
[14] C. M. Hull and R. R. Khuri, in preparation.
[15] E. Cremmer, B. Julia and J. Scherk, Phys. Lett.B76 (1978) 409.
[16] M.J. Duff and K.S. Stelle, Phys. Lett. B253 (1991) 113.
[17] G. W. Gibbons and P. K. Townsend, Phys. Rev. Lett. 71 (1993) 3754; M. J. Duff, G. W. Gibbons and P. K. Townsend, Phys. Lett. B332 (1994) 321; G. W. Gibbons, G. T. Horowitz and P. K. Townsend, Class. Quan. Grav. 12 (1995) 297.
[18] R. Guven, Phys. Lett. B276 (1992) 49.
[19] M. J. Duff, P. S. Howe, T. Inami and K. Stelle, Phys. Lett. B191 (1987) 70.
[20] N. Seiberg, Phys. Lett. B384 (1996) 81.
[21] E. Witten, Adv. Theor. Math. Phys. 2 (1998) 253, hep-th/9802150.
[22] L. Susskind and E. Witten, hep-th/9805114; S.S. Gubser, I.R. Klebanov and A.M. Polyakov, Phys. Lett. B428 (1998) 105, 9801003.