Experimental investigation of dynamic characteristics of material applied to reduced scale physical models in the elastic state

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Abstract
Dynamic and vibration difficulties have begun to play a major part in structural analysis as a result of the increasing complexity and slimness of bridges. Nonetheless, creating numerical models that accurately capture the true behaviour of structures remains a challenge. It can be caused by the lack of references that characterise the dynamic and vibration properties of the material. For that reason, the main goal of this work is to investigate experimental methods to characterise materials that are commonly used to build reduced scale physical models. We tested four types of materials in order to achieve this goal, as well as evaluating the major influence of reinforcement on the material’s elastic stiffness. That presents a novel approach to support engineers to suitably select scaled physical models materials. In addition to identifying the basic static properties of the studied materials, material damping and material relaxation were also characterised through the following: General logarithmic decrement method, Half-power band, modal damping and random decrement. In order to evaluate the material damping, we analysed the acceleration and displacement of cantilever specimens using a data acquisition module. As a result, the confidence level of each approach in contrast to the experimental data could be determined. Finally, the material behaviour of each material was examined in order to determine the best material for producing reduced scaled models of reinforced concrete bridges. As a result, it was easy to see that modal damping indicates higher confidence, whereas logarithmic decrement indicates lower confidence. It is also important to keep in mind that most methods for determining damping ratios are based on viscous damping. As a result, non-linear regression and modal damping were deemed the best methods for characterising vibration and dynamic responses of materials.

Keywords
Material damping, dynamic behaviour, experimental methods, numerical modelling, reduced scale physical models

Introduction
During the last few years, structures have become more complex and slimmer because of advances in construction engineering and materials science. As a result, the cost and weight of structures have been decreasing. In contrast with that, external dynamic loads caused by tropical climate change have started playing an important role in structural design and simulation.¹ In addition, several studies show that the main cause of bridge collapses was by dynamic conditions.² In spite of that, dynamic loads are poorly considered in several standards, such as ABNT 61183. In most cases, standards only consider the analysis of natural frequencies in order to add safety factors or quasi-static magnification.

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Reduced scale research and simulation processes, on the other hand, are usually linked with those standards, with Eigen analyses indicating natural frequencies and natural damped frequencies. As a result, in addition to the general viscous damping imparted to the entire structure, the dynamic behaviour of the structure takes into account the modal form obtained by the resonant frequencies. One of the reasons of that is the fact that information about the most suitable method to determine material damping is not widely studied.

For that reason, the main goal of this work is to experimentally investigate the dynamic behaviour of reduced scale physical models materials in the elastic state. As a result, divergences between damping models and experimental data might be identified.

This research offers a unique perspective in the sector, assisting engineers in designing and selecting the best materials for lower scaled physical models of reinforced concrete bridges.

Among the most used materials for reduced scale prototypes (physical models), it is possible to find fiber glass, composite resins, plastic (3D printing), micro concrete and gypsum concrete.3–5

Nevertheless, plastic materials and 3D printing technologies are known to have high anisotropic and viscous-elastic behaviour.6,7 Therefore, those materials were ignored in this work. Future studies are still needed to be conducted in this subject in order to better understand the correlation between dynamic properties, anisotropy and additive manufacturing.

Chen8 also claims that material behaviour is influenced by the environment, with material damping increasing underwater.

In the current study, four materials were investigated in order to characterise density, ultimate flexural strength, flexural stiffness, fluency and damping. The materials are based on composite resin, reinforced composite resin, gypsum concrete and reinforced gypsum concrete.

It is important to highlight that those materials were selected because they are the most used materials in the state of art related to reduced scale prototypes.4,5

As the scaled physical models are designed for low frequencies which must be correlated to real structures, those materials are usually designed for low and intermediate resonant frequencies. Therefore, the effect of high frequencies on material behaviour was ignored in this work.

In addition, we also evaluated four methods to determine material damping: General logarithmic decrement method, Half-power band, modal damping and random decrement. In Table 1, we present the formulation of the main damping models used in this work. Where \( \delta \) is the logarithmic decay, \( A \) is the Amplitude, \( n \) is the number of wave peaks, \( \xi \) is the damping ratio, \( f_u \) is the upper frequency of \( -30 \) dB, \( f_l \) is the lower frequency of \( -30 \) dB, \( f_p \) is the peak frequency, \( x \) is the displacement, \( m \) is the fitting coefficient and \( f_n \) is the fitted frequency.

It’s worth noting that all of the methods mentioned are based on viscous damping. Other nonlinear and linear damping models can be utilised to quantify material damping with advanced geometries, such as lattices. For example, Coulomb, dry frictional, cracking frictional and frictional-viscous-elastic damping provide different effects on structures from the point of view of vibration and dynamic responses.9 The characteristic behaviour of these types of damping might even generate either new resonant frequencies or response noises.10

The fractal vibration model for a concrete beam, on the other hand, takes into account the heterogeneity of material and shape, resulting in material damping variations in different regions of the beam.11,12

This type of behaviour is also found in complex and advanced structures, such as lattices and scaffolds.13

Moreover, the current work focuses on comparing the materials and methods that are the most used worldwide. Therefore, further studies are still needed to be conducted in order to identify and compare the efficiency and confidence of other methods against the four methods presented in this work.

Another point that is also interesting to be considered in the analysis of reduced scale models is that the material damping is necessary when dynamic responses are required. For studies which are majorly static and quasi-static, material damping might be ignored.14 Therefore, other materials, such as 3D printed plastics, are also used for fast evaluation of reduced scale models.

In the end, the analysis of the general inaccuracy of each model caused by the simulation revealed the most appropriate way for defining material damping.

**Material and methods**

For this study, we analysed four of the most common materials which are applied to reduced scale experiments.4 We considered reinforced and non-reinforced materials based on composite resin and gypsum concrete, as it is possible to see in Table 2.

In this table, the formulation of the four materials is presented, while four samples of each material were tested in each experiment. The basic dimensions of all specimens were \( 200 \times 50 \times 10 \) mm, and the fabrication method was based on RTV (Room temperature vulcanisation) moulding, as presented in Figure 1.
In this method, the following steps are considered: (1) fabrication of pattern (3D printing), (2) mould fabrication, (3) reinforcement fabrication, (4) placement of reinforcement, (5) material pouring and (6) the removal of specimen from mould.

In order to analyse flexural strength and elasticity modulus, we used ASTM C78 standard and the universal test machine Instron Series 23. The displacement resolution of machine was 0.01 mm, while the speed was 500 mm/min. In addition, strain-gauges were applied in order to identify material strain, Figure 2.

**Table 1.** Main types of damping models.

| Method                | Formula | Advantages                                                                 | Disadvantages                                                                 |
|-----------------------|---------|-----------------------------------------------------------------------------|-------------------------------------------------------------------------------|
| Logarithmic decrement method | $\delta = (1/n) \log [A_1/A_n + 1]$ | - Time-domain analysis  
- Good for predominant viscous damping  
- Good for $x > 0.1$  
- Good for systems with I predominant resonant frequency  
- Good for systems with constant damping for low frequencies | - Not indicated for MDOF  
- Not indicated for non-linear damping (Coulomb, frictional, wet friction, visco-elastic,..)  
- Not indicated for high resonant frequencies |
| Half-power method     | $\zeta = (f_{01} - f_{02})/2 f_{02}$ for $\zeta < 0.05$ | - Easy calculation requires to define $x$  
- Good for systems with I predominant resonant frequency  
- Recommended for $x < 0.05$  
- Good for viscous damping  
- Recommended for first resonant frequencies | - Requires definition of frequency response function (FRF) and Fourier analysis  
- Not indicated for MDOF  
- Not indicated for non-linear damping (Coulomb, frictional, wet friction, visco-elastic,..)  
- Not indicated for high resonant frequencies  
- Not indicated for $x > 0.1$ |
| Modal damping         | $\zeta_n = (f_{01} - f_{0n})/2 f_{0n}$ | - Defines a individual $x$ for each resonant frequency  
- Good for systems MDOF systems  
- Recommended for $x < 0.1$  
- Good for viscous damping  
- Good for low and high resonant frequencies  
- Recommended for frequency proportional damping  
- Useful to estimate Rayleigh damping coefficients | - Requires definition of frequency response function (FRF) and Fourier analysis  
- Not indicated for non-linear damping (Coulomb, frictional, wet friction, visco-elastic,..)  
- Not indicated for high resonant frequencies  
- Not indicated for $x > 0.1$ |
| Random decrement method | Fitting of: $x (t) = \sum_n f_n \cdot \sin(f_n \cdot t)$ | - Time-domain analysis  
- Good for predominant viscous damping  
- Good for systems with MDOF  
- Good for systems with frequency proportional damping  
- Good for low resonant frequencies | - Not indicated for non-linear damping (Coulomb, frictional, wet friction, visco-elastic,..)  
- Not indicated for high resonant frequencies |

**Table 2.** Formulation for the four types of materials analysed in this work.

| Formulation                              | Material Name                  |
|------------------------------------------|-------------------------------|
|                                          | Gypsum concrete | Reinforcement gypsum concrete | Composite polyester resin | Reinforcement composite polyester resin |
| Water/Cement ratio (per weight)          | 0.3 | 0.3 | — | — |
| Aggregate/Cement ratio (per weight)      | 0.8 | 0.8 | — | — |
| Catalyst/Resin ratio (per weight)        | — | — | 2% | 2% |
| Calcite/Polyester ratio (per weight)     | — | — | 1.5 | 1.5 |
| Reinforcement                            | No | Yes (3%) | No | Yes (3%) |
For density, we applied standard ISO 1920-5 2018, where the precision of scale was 0.001 g and the volume meter was 0.001 dm³.

The fluency and damping tests were characterised by a cantilever system, as presented in Figure 3. In this case, the specimen was fixed and anchored in a concrete block wall. This concrete block was also anchored in a concrete floor.

In order to see the effect of foundation on the specimen, we preliminarily evaluated the vibration of foundation as a function of the cantilever load. This test implies no relevant vibrations. The initial deflection was measured by 0.005 mm resolution dial gauge while the force was acquired by force transducer (composed by pressure sensors and electronic transducer with filtered amplification). In this case, force and acceleration were independently acquired by two channels where an anti-aliasing filter was installed between sensors and data acquisition system.

The acceleration was acquired by a capacitive Freescale accelerometer, whose sensitivity is 800 mV/g. For data acquisition, we used National instrument NI-6009, which has a 14 bits analogue-digital resolution and sampling/s equal to

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Figure 1. Fabrication of mould and specimen dimensions.

Figure 2. Schematic of flexural test and strain gauges.
48kS/s. It is important to note that, despite this hardware is low cost in comparison with piezo sensors, it is suitable to measure low frequencies in multi-axial directions. It is also possible to be seen in the autocorrelation analysis.

For frequency test, the cantilever specimens were submitted to constant load and monitored for 10 min, according to ASTM D 2990. Therefore, it was possible to identify the permanent and non-permanent coefficients of generalised Maxwell–Kelvin–Voigt viscous-elastic model (Burger Model) by nonlinear goodness fitting equation (1). In order to identify all permanent and non-permanent coefficients of this model, each material (cantilever) was submitted to two constant loads and monitored for 10 min. The schematic model is also presented in Figure 4.

\[
\varepsilon = \sigma \left( \frac{1}{kE} + \frac{t}{\eta_s} + \frac{1 - \phi_s T/\eta}{k_1} \right)
\]

(1)

In contrast, damping test adopts step relaxation transient load in order to analyse decay of force, displacement and acceleration. Therefore, it was possible to identify the four damping models coefficients and their error: General logarithmic decrement method, Half-power band, modal damping and random decrement.

For half-power band and modal damping, we identified frequency response function of Accelerance in order to analyse the damping correspondent to each resonance frequency. For that, we obtained the FFT of force and acceleration in order to find Frequency Response Function (FRF) of Accelerance function \((A(\omega)/F(\omega))\), as presented in Figure 5.

On the other hand, logarithmic decrement, random decrement and non-linear goodness fitting were performed in the time domain. Logarithmic decrement was acquired by the average decrement of the first 3 oscillation peaks. With regards to random decrement, we considered 3 frequencies and 3 decrements as variables of frequency-damping model. In this case, the nonlinear goodness fitting method was used to identify the frequencies and decrements of this model. The statistical analyses were performed in Minitab while nonlinear regressions and model investigations were performed in MATLAB.

**Results and discussions**

With respect to the general results, Table 3 presents statistical results of study material of density. It is possible to see that reinforced gypsum concrete has the highest mean density among the studied materials (2.215 g/cm³). PES composite resulted in the lowest density (1.612 g/cm³). The Shapiro–Wilk test indicated that the density of all materials was normal, while the maximum standard deviation was 0.074 g/cm³. In addition, the T-test indicated that only the composite resin material and the reinforced composite material were equal. All the other materials were statistically different among each other.

Similarly, the main results of mechanical strength are presented in Table 4, where the descriptive statistics are presented. In this table, the ultimate strength, the maximum deflection and the elasticity modulus are indicated, where composite resin presented the highest strength (36.7 MPa). On the other hand, reinforced gypsum concrete was found to be the weakest material (2.5 MPa).
In the flexural test, shown in Figure 6, we can evidence that the mechanical behaviour of gypsum concrete is more brittle than composite resin, even though the elasticity modulus of all studied materials varied from 1.5 to 1.85 GPa. It might indicate similarity between materials for vibration analysis, whereas natural frequencies are mainly controlled by mass and stiffness.

Considering that deformation as a function of time directly affects the dynamic behaviour of structures, we analysed the fluency of the materials by the relaxation method and non-linear fitting of material Maxwell–Kelvin–Voigt model. Each sample was tested with two different loads along the time, as presented in the displacement–time diagram of Figure 7.

In this study, we analysed the deformation as a function of time, fitting the data to Maxwell–Kelvin–Voigt viscous-elastic model (Burger Model). Therefore, the model coefficients and $r^2$ are presented in Table 5. In this table, we can identify that composite resin presents high viscous-elastic behaviour, being unsuitable for reduced scale physical models which evaluate dynamic responses and vibration.

With regards to material damping, we analysed the data of cantilever under step relaxation load. Using this data, we identified the damping ratio and decay according to four damping models: General logarithmic decrement method, Half-power band, modal damping and random decrement.

After identifying the main coefficients of general logarithmic decay, we compared model results with experimental data in addition to analysing the goodness fitting of model-data. The summary of the general logarithmic decrement results is presented in Table 6. The mean decay varied from 0.107 (reinforced gypsum concrete) to 0.177
Using the half power band method, it was possible to analyse the average damping of the most significant frequency. In other words, this method is designed to systems that only have one predominant resonant frequency.

Table 7 presents the damping ratio found by this method in addition to the coefficient of determination ($r^2$) of this model against the experimental data. In this method, it is possible to see that all $r^2$ are below 0.90, expressing the lack of confidence of this method in comparison with other methods, such as general logarithmic decay.
Nevertheless, it was also possible to identify that the values of damping ratios fluctuate around 0.0551 and 0.0993 for gypsum-based materials, while PES composites fluctuate from 0.0071 to 0.1718.

On the other hand, the modal damping model was compounded by 3 frequencies and 3 damping ratios, whereas 3 resonant frequencies were identified in the Accelerance FRF. Table 8 presents the summary of these results, where the damping ratio varied from 0.0511 to 0.1829 for low frequencies.

In contrast, higher frequencies caused damping ratio variation from 0.0019 to 0.0109. It is possible to see that this method implies on $r^2$ up to 0.85 (gypsum concrete), indicating low confidence, even though it is more confident than half power band method.

Table 5. Fluency test data and goodness fitting coefficients.

| Material                              | Load time (s) | Strain (mm) | Load (g) | $R^2$  | A     | B       |
|---------------------------------------|---------------|-------------|----------|--------|--------|---------|
| 1 Gypsum concrete                     | 100.000       | 0.230       | 200      | 1      | 0      | 0       |
| 2 Reinforced gypsum concrete          | 100.000       | 0.230       | 200      | 1      | 0      | 0       |
| 3 PES composite resin                 | 100.000       | 1.300       | 200      | 0.97904140273576 | 0.420018625436272 | 0.002032120625913 |
| 4 Reinforced PES composite            | 100.000       | 0.880       | 200      | 0.986893962413734 | 0.10495261305856 | 0.002447353024076 |

Table 6. Results of damping considering general logarithmic decay method.

| Material                              | Maximum $\delta$ | Minimum $\delta$ | Mean $\delta$ | Mean $\xi$ | Mean $r^2$ | Mean resonant frequency (Hz) |
|---------------------------------------|------------------|-----------------|---------------|------------|-------------|-----------------------------|
| 1 Gypsum concrete                     | 0.262            | 0.064           | 0.114         | 0.018      | 0.870       | 68.280                      |
| 2 Reinforced gypsum concrete          | 0.167            | 0.097           | 0.107         | 0.017      | 0.927       | 71.422                      |
| 3 PES composite resin                 | 0.191            | 0.109           | 0.177         | 0.028      | 0.924       | 68.032                      |
| 4 Reinforced PES composite            | 0.195            | 0.111           | 0.170         | 0.027      | 0.934       | 68.850                      |
The random decrement method was obtained by the goodness fitting of model and the time-domain experimental data, indicating 3 resonant frequencies and 3 damping ratios. Table 9 sums up these results, where the gypsum concrete damping ratio fluctuated from 0.001809 to 0.8314. In contrast, composite resin damping ratio varied from 0.00068 to 0.9632.

This method also implies on $r^2$ around 0.96 for all studied materials, apart from reinforced composite resin. As a consequence, it indicates that the reinforced composite resin damping might not be viscous. Therefore, frictional coefficients might cause distortions in small displacements. Moreover, this subject needs to be detailed in future studies.

With regards to Figure 8, a comparison between methods, materials, decay and confidence ratio is presented. It is possible to see that half power band method presented the lowest confidence level among the studied methods. Similarly, gypsum concrete was shown to have the highest divergence and standard deviation among the studied materials. We can also highlight that composite resin also presented high frequency, which make the use of such material unsuitable to build reduced scale models of reinforced concrete bridges.

It was also evidenced that random decrement presented the highest coefficient of determination ($r^2$) among the studied methods.

Considering that the divergence between methods used to determine the damping ratio is high, further studies are needed to better understand the application of each method in accordance with materials, structures and excitation methods.
It is also important to note that the material damping is a function of temperature, environment conditions, geometry, cracking ratio and material fluency. Nonetheless, this work kept those parameters constant in order to establish the comparison among type of material and methods to determine material damping.

Therefore, further studies that aim to detail the correlation between the damping material and other variables that affect damping material still need to be conducted in the future.

By the end, Table 10 shows a comparison among materials in order to identify benefits and disadvantages of materials in accordance with the application for reduced scale physical models.

Note that materials based on composite resin imply on high viscous-elastic behaviour, which affects dynamic behaviour of material and structure.

This sort of material is very interesting for static loads applied for reduced scale models, whereas they are easy to fabricate in addition to having good homogeneity, strength and elastic modulus that are excellent for 1:10 to 1:50 reduction scales.

On the other hand, gypsum concrete materials have shown the behaviour that is the most likely to real scale concrete materials, considering 1:50 scale factor. Although the Young’s modulus and strength are similar to PES composite resin, the damping ratios are significantly lower. The viscous-elastic behaviour of this kind of material also tends to zero.

### Table 9. Results of damping considering random logarithmic decay method.

| Material                        | 1 Gypsum concrete | 2 Reinforced gypsum concrete | 3 PES composite resin | 4 Reinforced PES composite |
|---------------------------------|-------------------|------------------------------|-----------------------|---------------------------|
| Maximum δ1                      | 0.822             | 0.213                        | 0.225                 | 2.489                     |
| Minimum δ1                      | 0.109             | 0.155                        | 0.059                 | 1.918                     |
| Mean δ1                         | 0.460             | 0.191                        | 0.142                 | 2.203                     |
| Mean δ2                         | 0.073             | 0.030                        | 0.023                 | 0.331                     |
| Mean f1 (Hz)                    | 69.5379591978042  | 70.7354584224014             | 68.7539479785961      | 61.6950241330854          |
| Maximum δ2                      | 4.811             | 6.217                        | 0.264                 | 0.237                     |
| Minimum δ2                      | 0.109             | 3.006                        | 0.264                 | 0.213                     |
| Mean δ2                         | 2.531             | 5.071                        | 0.264                 | 0.225                     |
| Mean δ2                         | 0.374             | 0.628                        | 0.042                 | 0.036                     |
| Mean f2 (Hz)                    | 132.904644271258  | 157.833170558829             | 76.9566610145881      | 68.4831959671468          |
| Maximum r²                      | 0.952574981028814 | 0.964278033743129           | 0.962754831442065     | 0.946469912439407         |
| Minimum r²                      | 0.861108773343221 | 0.93251935788597            | 0.914677953853465     | 0.925862877005488         |
| Mean r²                         | 0.921             | 0.952                        | 0.939                 | 0.936                     |
It is interesting to see that non-reinforced gypsum concrete is easier to fabricate because the reinforced copper is more difficult to produce and place into mould. In contrast, those materials are fragile and small geometries tend to break easily during demoulding process. For that reason, the reinforced gypsum concrete has been shown to be the most suitable material in this work.

**Conclusions**

The key dynamic properties of materials that are commonly applied to reduced scale models were compared in this study. Furthermore, the damping ratio was calculated and assessed using four distinct ways, revealing the applicability and reliability of each approach in depicting material damping.

Even though stiffness and density were comparable to materials made of gypsum concrete, structures made of PES composite resin had a strong viscous-elastic effect and fluency. As a consequence, composites based on plastic resins have been shown to be unsuitable for the construction of reduced scale models.

Half-band and modal damping presented the lowest coefficient of determination ($r^2$) among the studied methods. The material damping ratio was found to be 10 times lower than the standardized global damping (0.1–5%), indicating that further studies are still needed in the topic in order to better understand the implications caused by the divergence between standards, real scale materials/structures and reduced scale materials/structures.

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