USING $B^0_d$, $B^0_s$ DECAYS AT HADRONIC $B$-FACTORIES TO DETERMINE CP ANGLES $\alpha$ AND $\beta$\[1\]

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Abstract

At the first PPPP workshop, I gave a review talk on physics of $B$ and CP-phases. In my talk, I explained DRG method and our extension to extract CP angles $\alpha$ and $\gamma$ from measurements of the decay rates of $B^0_{d,s} \rightarrow \pi \pi$, $\pi K$, $K K$. There are several advantages to this extension: discrete ambiguities are removed, fewer assumptions are necessary, and the method works even if all strong phases vanish. I also talked on several other topics.

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1 Introduction

Here I review an extension of DGR method [1] which avoids most of the problems with the DGR method. In addition to the $B^0_d$ and $B^+$ decays used by DGR, it requires the measurement of their $SU(3)$-counterpart $B^0_s$ decays: $B^0_s \to \pi^+ K^-$, $B^0_s(t) \to K^+ K^-$, and $B^0_s \to K^0 \bar{K}^0$. This overconstrains the system, which eliminates most discrete ambiguities in the extraction of $\alpha$ and $\gamma$.

Although challenging experimentally, this method can probably be carried out at hadron-collider experiments such as BTeV and LHC-B [2]. All branching ratios are $\mathcal{O}(10^{-5})$, and all final states involve only charged pions and $K$‘s – there are no $\pi^0$’s. It will be necessary to resolve $B^0_s$-$\bar{B}^0_s$ oscillations, but this should not be be a problem at hadron colliders: for example, BTeV expects to be able to measure values of the mixing parameter $x_s \sim 50$. The principal obstacle to obtaining precise values of $\alpha$ and $\gamma$ is the fact that 12 measurements must be combined to extract these CP angles, as well as other quantities. If the experimental errors are large, then all sensitivity to the CP angles may be washed out. However, preliminary studies indicate that the errors will in fact be relatively small. A BTeV simulation of the decay $B^0_d(t) \to \pi^+ \pi^-$ has been carried out, with the result that the CP asymmetry in this decay can be measured with an error of $\pm 0.05 [4]$. This error includes the background, which consists of the decays $B^0_d \to \pi K$, $B^0_s \to \pi K$ and $B^0_s \to K K$. Since these are precisely the decays involved in our extension of the DGR method, this suggests that these decays can be experimentally separated from one another, and their branching ratios measured with reasonable precision, say $\sim 5\%$ or less. If this is so, then we can expect that $\alpha$ and $\gamma$ can also be extracted fairly precisely.
2 Using $B_s^0$ decays to determine the CP angles $\alpha$ and $\beta$

The DGR method involves the decays $B_d^0 \to \pi^+\pi^-$, $B_d^0 \to \pi^-K^+$, and $B^+ \to \pi^+K^0$. The amplitudes for these decays can be written

\begin{align}
A_{\pi\pi} &\equiv A\left(B^0 \to \pi^+\pi^\pm\right) = -(T + P), \\
A_{\pi K} &\equiv A\left(B^0 \to \pi^-K^+\right) = -(T' + P'), \\
A_{\pi K}^+ &\equiv A\left(B^+ \to \pi^+K^0\right) = P'.
\end{align}

(1)

Note that, since only tree and penguin terms are involved, EWP contributions are negligible.

The weak phase of $T$ is Arg($V_{ud}V_{ub}^\ast$) = $\gamma$, and similarly for $T'$: Arg($V_{us}V_{ub}^\ast$) = $\gamma$. The $b \to s$ penguin $P'$ is dominated by the internal $t$-quark, so its weak phase is Arg($V_{ts}V_{tb}^\ast$) = $\pi$. As for the $b \to d$ penguin $P$, if it also is dominated by the $t$-quark, its weak phase is Arg($V_{ts}V_{tb}^\ast$) = $-\beta$. This is the assumption made by DGR, but we can relax it.

If $SU(3)$ were unbroken, the amplitudes $T$ and $T'$ would be related simply by the ratio of their CKM matrix elements: $|T'/T| = |V_{us}/V_{ud}|$. However, if one includes first-order $SU(3)$ breaking, there is an additional factor involving the ratio of $K$ and $\pi$ decay constants if factorization is assumed: $|T'/T| = |V_{us}|f_K/|V_{ud}|f_\pi \equiv r_u$. On the other hand, since factorization is unlikely to hold for penguin amplitudes, $P$ and $P'$ are not related in a simple way. However, DGR do assume that the strong phase of the penguin diagram, $\delta_P$, is unaffected by $SU(3)$ breaking.

With these assumptions, the amplitudes in Eqs. (1) can be written

\begin{align}
A_{\pi\pi} &= Te^{i\delta_T}e^{i\gamma} + Pe^{i\delta_P}e^{-i\beta}, \\
A_{\pi K} &= ruTe^{i\delta_T}e^{i\gamma} - P'e^{i\delta_P}, \\
A_{\pi K}^+ &= P'e^{i\delta_P},
\end{align}

where $T \equiv |T|$, $P \equiv |P|$, and $P' \equiv |P'|$. 

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There are thus six unknown quantities in the above 3 amplitudes: \( \alpha \equiv \pi - \beta - \gamma, \gamma, T, P, P', \) and \( \delta \equiv \delta_T - \delta_P \). The time-dependent, tagged \( B_d^0 \) and \( \bar{B}_d^0 \) decay rates to \( \pi^+\pi^- \) are given by

\[
\Gamma \left( B_d^0(t) \to \pi^+\pi^- \right) = e^{-\Gamma t} \left[ |A_{\pi\pi}|^2 \cos^2 \left( \frac{\Delta m t}{2} \right) + |A_{\pi\pi}'|^2 \sin^2 \left( \frac{\Delta m t}{2} \right) + \text{Im} \left( e^{2i\beta} A_{\pi\pi} \bar{A}_{\pi\pi}' \right) \sin(\Delta mt) \right],
\]

\[
\Gamma \left( \bar{B}_d^0(t) \to \pi^+\pi^- \right) = e^{-\Gamma t} \left[ |A_{\pi\pi}|^2 \sin^2 \left( \frac{\Delta m t}{2} \right) + |A_{\pi\pi}'|^2 \cos^2 \left( \frac{\Delta m t}{2} \right) - \text{Im} \left( e^{2i\beta} A_{\pi\pi} \bar{A}_{\pi\pi}' \right) \sin(\Delta mt) \right].
\]

From these measurements one can determine the three quantities \( |A_{\pi\pi}|^2, |A_{\pi\pi}'|^2, \) and \( \text{Im} \left( e^{2i\beta} A_{\pi\pi} \bar{A}_{\pi\pi}' \right) \). The rates for the self-tagging decays \( B_d^0 \to \pi^- K^+ \) and \( \bar{B}_d^0 \to \pi^+ K^- \) are

\[
|A_{*K}|^2 = r_u^2 T^2 + P^2 - 2r_u T P' \cos(\delta + \gamma),
\]

\[
|\bar{A}_{*K}|^2 = r_u^2 T^2 + P^2 - 2r_u T P' \cos(\delta - \gamma).
\]

Finally, the rates for \( B^+ \to \pi^+ K^0 \) and its CP-conjugate decay give

\[
|A_{*K}^+|^2 = |A_{*K}^-|^2 = P'^2.
\]

Thus, from the above measurements, one can obtain the following six quantities:

\[
A \equiv \frac{1}{2} \left( |A_{\pi\pi}|^2 + |\bar{A}_{\pi\pi}'|^2 \right) = T^2 + P^2 - 2T P \cos \delta \cos \alpha,
\]

\[
B \equiv \frac{1}{2} \left( |A_{\pi\pi}|^2 - |\bar{A}_{\pi\pi}'|^2 \right) = -2T P \sin \delta \sin \alpha,
\]

\[
C \equiv \text{Im} \left( e^{2i\beta} A_{\pi\pi} \bar{A}_{\pi\pi}' \right) = -T^2 \sin 2\alpha + 2T P \cos \delta \sin \alpha,
\]

\[
D \equiv \frac{1}{2} \left( |A_{*K}|^2 + |\bar{A}_{*K}'|^2 \right) = r_u^2 T^2 + P'^2 - 2r_u T P' \cos \delta \cos \gamma,
\]

\[
E \equiv \frac{1}{2} \left( |A_{*K}|^2 - |\bar{A}_{*K}'|^2 \right) = 2r_u T P' \sin \delta \sin \gamma,
\]

\[
F \equiv |A_{*K}^+|^2 = P'^2.
\]

These give 6 equations in 6 unknowns, so that one can solve for \( \alpha, \gamma, T, P, P', \) and \( \delta \). However, because the equations are nonlinear, there are discrete ambiguities in extracting
these quantities. In fact, a detailed study shows that, depending on the actual values of the phases, there can be up to 8 solutions.

If $\delta = 0$, the quantities $B$ and $E$ vanish, so that one is left with 4 equations in 5 unknowns. In this case one must use additional assumptions to extract information about the CP phases. Furthermore, even if $\delta \neq 0$, if one relaxes any of the assumptions described above, the method breaks down. For example, if one allows the strong phase of the $P'$ diagram to be different from that of the $P$ diagram, as might be the case in the presence of $SU(3)$ breaking, then one has 6 equations in 7 unknowns. And if one relaxes the assumption that the $b \to d$ penguin is dominated by the $t$-quark, then once again additional parameters are introduced, and the method breaks down.

All of these potential problems can be dealt with by considering additional $B_s^0$ decays. The problems with the DGR method can be resolved by adding amplitudes which depend on the same 6 quantities, thus overconstraining the system. In this case, if one adds a parameter or two, perhaps by relaxing certain assumptions, the method will be less likely to break down.

Within $SU(3)$ symmetry, the obvious decays to consider are the $SU(3)$ counterparts to the DGR decays, namely $B_s^0 \to \pi^+K^-$, $B_s^0(t) \to K^+K^-$, and $B_s^0 \to K^0\bar{K}^0$. The amplitudes for these decays are completely analogous to those in Eqs. (1):

\[
B_{\pi K} \equiv A(B_s^0 \to \pi^+K^-) = -\left(\tilde{T} + \tilde{P}\right),
\]

\[
B_{KK} \equiv A(B_s^0 \to K^+K^-) = -\left(\tilde{T}' + \tilde{P}'\right),
\]

\[
B_{KK}^s \equiv A(B_s^0 \to K^0\bar{K}^0) = \tilde{P}'.
\]  

(12)

Here we have denoted the tree and penguin diagrams involving a spectator $s$ quark by $\tilde{T}$ and $\tilde{P}$, respectively. As before, the unprimed and primed quantities denote $\Delta S = 0$ and $\Delta S = 1$ processes, respectively.

The weak phase of $\tilde{T}$ and $\tilde{T}'$ is $\gamma$, and that of $\tilde{P}'$ is $\pi$. As for $\tilde{P}$, as a first step we make the same assumptions as DGR, namely that it is dominated by the $t$-quark, so that its weak phase is $-\beta$. Turning to $SU(3)$ breaking, we assume factorization for the tree amplitudes, so that $|\tilde{T}'/\tilde{T}| = r_u$. The magnitudes of the $\tilde{P}$ and $\tilde{P}'$ amplitudes are
unrelated to one another. However, again as a first step, like DGR we assume that they have the same strong phase, \( \delta_p \). The one new assumption that we make is that the relative strong phase between the tree and penguin amplitudes is independent of the flavor of the spectator quark. Thus we have \( \delta_s = \delta \), where \( \delta_s \equiv \delta_T - \delta_p \) and \( \delta \equiv \delta_T - \delta_p \). (The most likely way for this to occur is if \( \delta_T = \delta_T \) and \( \delta_p = \delta_p \)). This assumption is motivated by the spectator model.

Under these assumptions, the amplitudes in Eqs. (12) can be written

\[
B_{\pi K} = \tilde{T} e^{i\delta_T} e^{i\gamma} + \tilde{P} e^{i\delta_P} e^{-i\beta},
\]

\[
B_{KK} = r_u \tilde{T} e^{i\delta_T} e^{i\gamma} - \tilde{P}' e^{i\delta_P},
\]

\[
B_{KK}^s = \tilde{P}' e^{i\delta_P},
\]

where \( \tilde{T} \equiv |\tilde{T}|, \tilde{P} \equiv |\tilde{P}|, \) and \( \tilde{P}' \equiv |\tilde{P}'| \).

The important point here is that three new parameters have been introduced in the above amplitudes: \( \tilde{T}, \tilde{P}, \) and \( \tilde{P}' \). However, as in the DGR method, 6 quantities can be extracted from measurements of the rates for these decays. Here, the self-tagging decays are \( B_s^0 \to \pi^+ K^- \) and \( \overline{B_s^0} \to \pi^- K^+ \), whose rates are

\[
|B_{sK}|^2 = \tilde{T}^2 + \tilde{P}^2 - 2\tilde{T} \tilde{P} \cos(\delta - \alpha),
\]

\[
|\overline{B}_{sK}|^2 = \tilde{T}^2 + \tilde{P}^2 - 2\tilde{T} \tilde{P} \cos(\delta + \alpha).
\]

The time-dependent, tagged \( B_s^0 \) and \( \overline{B_s^0} \) decay rates to \( K^+ K^- \) are given by

\[
\Gamma \left[ B_s^0(t) \to K^+ K^- \right] = e^{-\Gamma t} \left[ |B_{KK}|^2 \cos^2 \left( \frac{\Delta m_s t}{2} \right) + |\overline{B}_{KK}|^2 \sin^2 \left( \frac{\Delta m_s t}{2} \right) + \text{Im} \left( B_{KK} \overline{B}_{KK}^* \right) \sin(\Delta m_s t) \right],
\]

\[
\Gamma \left[ \overline{B_s^0}(t) \to K^+ K^- \right] = e^{-\Gamma t} \left[ |B_{KK}|^2 \sin^2 \left( \frac{\Delta m_s t}{2} \right) + |\overline{B}_{KK}|^2 \cos^2 \left( \frac{\Delta m_s t}{2} \right) - \text{Im} \left( B_{KK} \overline{B}_{KK}^* \right) \sin(\Delta m_s t) \right],
\]

from which the quantities \( |B_{KK}|, |\overline{B}_{KK}|, \) and \( \text{Im} \left( B_{KK} \overline{B}_{KK}^* \right) \) can be extracted. Finally, we turn to \( B_s^0(t) \to K^0 \overline{K}^0 \). In principle there can be indirect CP violation in these decays. However, within the SM, this CP violation is zero to a good approximation, since both
$B_s^0$-$\overline{B}_s^0$ mixing and the $b \to s$ penguin diagram, which dominates this decay, are real. Thus, measurements of the rates for these decays yield

$$|B_{KK}^s|^2 = |\overline{B}_{KK}^s|^2 = \tilde{P}'^2 .$$

(16)

Obviously, any violation of this equality will be clear evidence for new physics.

Therefore the above measurements yield 6 new quantities:

$$\tilde{A} \equiv \frac{1}{2} \left( |B_{KK}^s|^2 + |\overline{B}_{KK}^s|^2 \right) = \tilde{T}^2 + \tilde{P}^2 - 2\tilde{T}\tilde{P} \cos \delta \cos \alpha ,$$

(17)

$$\tilde{B} \equiv \frac{1}{2} \left( |B_{KK}^s|^2 - |\overline{B}_{KK}^s|^2 \right) = -2\tilde{T}\tilde{P} \sin \delta \sin \alpha ,$$

(18)

$$\tilde{C} \equiv \text{Im} \left( B_{KK}^s \overline{B}_{KK}^s \right) = r_u^2 \tilde{T}^2 \sin 2\gamma - 2r_u \tilde{T}\tilde{P}' \cos \delta \sin \gamma ,$$

(19)

$$\tilde{D} \equiv \frac{1}{2} \left( |B_{KK}^s|^2 + |\overline{B}_{KK}^s|^2 \right) = r_u^2 \tilde{T}^2 + \tilde{P}'^2 - 2r_u \tilde{T}\tilde{P}' \cos \delta \cos \gamma ,$$

(20)

$$\tilde{E} \equiv \frac{1}{2} \left( |B_{KK}^s|^2 - |\overline{B}_{KK}^s|^2 \right) = 2r_u \tilde{T}\tilde{P}' \sin \delta \sin \gamma ,$$

(21)

$$\tilde{F} \equiv |B_{KK}^s|^2 = \tilde{P}'^2 .$$

(22)

Combined with the 6 quantities in Eqs. (6-11), we have 12 equations in 9 unknowns. As shown below, this allows us to solve for the CP angles, as in the DGR method, but greatly reduces the discrete ambiguities.

The CP angles can be obtained as follows. First, one finds the ratios $\tilde{T}/T$, $\tilde{P}/P$, and $\tilde{P}'/P'$:

$$a \equiv \frac{\tilde{T}}{T} = \frac{\tilde{E}}{\overline{B}E} \sqrt{F} , \quad b \equiv \frac{\tilde{P}}{P} = \frac{\tilde{B}E}{\overline{B}E} \sqrt{F} , \quad c \equiv \frac{\tilde{P}'}{P'} = \sqrt{F} .$$

(23)

Using these, we can find the values of all the magnitudes of the amplitudes. The amplitudes $\mathcal{T}$ and $\mathcal{P}$ are obtained from

$$\mathcal{T}^2 = \frac{(acD - \tilde{D}) - c(a - c)F}{a(c - a)r_u^2} , \quad \mathcal{P}^2 = \frac{abA - \tilde{A}}{b(a - b)} + \frac{a}{b} \frac{(acD - \tilde{D}) - c(c - a)F}{a(c - a)r_u^2} ,$$

(24)

and the remaining amplitudes can be found using Eq. (23). Note that all magnitudes are positive, by definition.

We now turn to the angles. Using our knowledge of the magnitudes of the amplitudes, we have

$$\cos(\delta - \alpha) = \frac{\mathcal{T}^2 + \mathcal{P}^2 - A - B}{2\mathcal{T}\mathcal{P}} ,$$

6
\[
\begin{align*}
\cos(\delta + \alpha) &= \frac{T^2 + P^2 - A + B}{2TP}, \\
\cos(\delta - \gamma) &= \frac{r_u T^2 + F - D + E}{2r_u T \sqrt{F}}, \\
\cos(\delta + \gamma) &= \frac{r_u T^2 + F - D - E}{2r_u T \sqrt{F}}.
\end{align*}
\]  

These equations can be solved to give the phases \( \alpha, \gamma \) and \( \delta \) up to a fourfold ambiguity. That is, if \( \alpha_0, \gamma_0 \) and \( \delta_0 \) are the true values of these phases, then the following four sets of phases solve the above equations: \( \{ \alpha_0, \gamma_0, \delta_0 \} \), \( \{-\alpha_0, -\gamma_0, -\delta_0\} \), \( \{\alpha_0 - \pi, \gamma_0 - \pi, \delta_0 - \pi\} \), and \( \{\pi - \alpha_0, \pi - \gamma_0, \pi - \delta_0\} \). Note, however, that we still haven’t used the \( C \) and \( \tilde{C} \) measurements. Their knowledge eliminates two of the four sets, leaving

\[
\begin{align*}
\{ \alpha_0, \gamma_0, \delta_0 \} , \\
\{ \alpha_0 - \pi, \gamma_0 - \pi, \delta_0 - \pi \}.
\end{align*}
\]  

These two solutions correspond to two different orientations of the unitarity triangle, one pointing up, the other down. This final ambiguity cannot be resolved by this method alone. However, within the SM it can be removed by using other measurements such as \( \epsilon \) in the kaon system or the third CP angle \( \beta \).

## 3 More Comments

For more details on this part of talk, please look the paper by Kim, London and Yoshikawa [3].

I also talked on several other topics: determinations of \( |V_{td}/V_{ts}| \) from \( B \to X_{s,d}\bar{l}\bar{l} \), \( (\sin \gamma/\sin \beta) \) from \( B \to \rho(\pi)\nu\bar{\nu} \) and \( |V_{ub}/V_{cb}| \) from invariant hadronic mass distribution of \( B \to Xl\bar{\nu} \):

1. We propose [4] a new method to extract \( |V_{td}/V_{ts}| \) from the ratio of the decay distributions \( B \to X_{d}\bar{l}\bar{l}/B \to X_s\bar{l}\bar{l} \). This ratio depends only on the KM ratio \( |V_{td}/V_{ts}| \) within 15% theoretical uncertainties, if dilepton invariant mass-squared is away from the peaks of the possible resonance states, \( J/\psi, \psi' \), and etc. We also give a detailed analytical and numerical analysis on \( B \to X_d\bar{l}\bar{l} \).
2. We propose[5] a new method for precise determination of $\left| \frac{V_{td}}{V_{ub}} \right|$ from the ratios of branching ratios $\frac{B(B\to\rho\bar{\nu})}{B(B\to\rho\nu)}$ and $\frac{B(B\to\pi\bar{\nu})}{B(B\to\pi\nu)}$. As is well known, $\left| \frac{V_{td}}{V_{ub}} \right|$ equals to $\left( \frac{\sin\gamma}{\sin\beta} \right)$ for the CKM version of CP-violation within the Standard Model.

3. In order to determine the ratio of CKM matrix elements $|V_{ub}/V_{cb}|$ (and $|V_{ub}|$), we propose[6] a new model-independent method based on the heavy quark effective theory. In the forthcoming asymmetric $B$-experiments with microvertex detectors, BABAR and BELLE, the total separation of $b\to u$ semileptonic decays from the dominant $b\to c$ semileptonic decays would be experimentally viable.

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