On Direct and Indirect Scattering Approaches for Homogenization of Particulate Composites

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Abstract: Theoretical formalisms for the homogenization of particulate composites are identified as following either the direct scattering approach (DSA) or the indirect scattering approach (ISA). Both approaches can take inclusion size-dependence and distribution statistics into account. However, the DSA is generally limited to mediums with direction-independent constitutive properties and inclusions with simple shapes, but the ISA is not hobbled thus.

Keywords: Particulate Composites; Homogenization; Scattering; Electrically Small Particles

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1 Introduction

Random dispersions of identical, electrically small inclusions in a homogeneous host medium can be equivalently considered as effectively homogeneous, after using various homogenization formalisms [1]. Two broad approaches can be discerned from the homogenization literature for particulate composites, spanning almost two centuries. Any inclusion is considered as a scatterer, in either approach. In the direct scattering approach, DSA for short, the relevant scattering problem is treated as a boundary value problem. This is exemplified by the multiple scattering theory [2, 3] and the structural unit model [4]. In the indirect scattering approach (ISA), on the other hand, a scatterer is treated as a source of an excess electromagnetic field in some embedding medium; and the field induced in an electrically small, exclusion region of the same shape (and, possibly, size) as the inclusion has to be estimated. Derivations of the Maxwell Garnett formula and its size-dependent variants from integral equations [5, 6] furnish suitable examples.

Both approaches may be equivalent for the homogenization of simple composites. For example, Maxwell Garnett [7] applied a DSA to obtain his eponymous formula for isotropic, dielectric–in–dielectric, particulate composites with spherical inclusions — towards which formula an ISA was provided later by Faxén [8]. Likewise, for the same type of composites, Bruggeman [9] adopted the DSA to get the Bruggeman formula, which also emerges naturally from an ISA involving the bilocal approximation in the context of the strong property fluctuation theory (SPFT) [10]. When the DSA is adopted, the original Maxwell Garnett and the Bruggeman formulas can be extended to cover chiral–in–chiral, particulate composites with spheroidal inclusions (as will become clear in the next section). But in a framework of the ISA, extensions of the same formulas have been made for much more complex — i.e., anisotropic and bianisotropic — composites with inclusions of aciculate, discotic, ellipsoidal and other convex shapes [11, 12].

The foregoing delineation of the two scattering approaches to homogenization has never been published earlier, to the author’s knowledge. Consequently, no comparison of their different

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2 The homogenization literature is vast, and only representative works are cited here. Unless mentioned otherwise, attention is focused here on 3D composites.
capabilities has every been reported. That situation is remedied in this brief review, mostly with reference to particulate composites with complex constitutive properties.

2 Direct Scattering Approach

Adoption of the DSA requires that a boundary value problem be solved for scattering by an inclusion in some embedding medium. The embedding medium is, e.g., the host medium in the Maxwell Garnett formalism, and the homogenized composite in the Bruggeman formalism.

The incident field and the scattered field must be expanded in terms of appropriate eigen-solutions of the frequency–domain Maxwell postulates applied to the embedding medium. In practical terms, the field induced inside the inclusion must also be expanded in terms of appropriate eigensolutions of the Maxwell postulates applied to the inclusion medium, which is best seen in the T–matrix method and its simplifications for inclusions of special geometries [13, 14]. Boundary conditions across the surface of the scatterer are then enforced, and the scattered field coefficients are related to the incident field coefficients. Then, either the scattered field is made to satisfy some condition [15, 16], or the total field everywhere is averaged in some fashion [2, 3, 14].

Clearly, if appropriate eigensolutions are not available, the DSA turns out be barren. At this time, the following remarks on the utility of the DSA are in order:

- Both the inclusion and the embedding mediums must have direction–independent constitutive properties [13]. Consistently with the Post constraint [17], this means that the most complex 3D homogenization problem thus solvable can have isotropic chiral inclusions embedded in an isotropic chiral medium [18, 19].

- For 2D problems wherein the inclusions are parallel, infinitely long cylinders with convex cross–sections, both the inclusion and the embedding mediums are allowed to have uniaxial constitutive dyadics, provided the crystallographic axes of both mediums are parallel to the inclusion axis. Although no results appear to have been reported, the situation described
should be mathematically tractable [20]. Thus uniaxial bianisotropic materials may fall within the purview of the DSA, to a limited extent.

- The DSA is most tractable when the inclusions are either spheres (3D problems) or infinitely long cylinders with circular cross-sections (2D problems). These restrictions can be relaxed somewhat because analytic continuation is invoked in the T-matrix method [13, 21, 22], so that spheroids and cylinders with elliptical cross-sections may also be entertained to a limited degree. The T-matrix method for scatterers of other shapes is not generally practical. Alternately, by virtue of the helicity properties of Beltrami fields [23], the use of spheroidal eigenfunctions (3D problems) [24] and elliptical eigenfunctions (2D problems) [25] may be helpful if both the inclusion and the embedding mediums have direction-independent properties.

A major advantage of DSA is that it allows the use of inclusion distribution statistics, such as the pair-correlation function [2, 3, 14, 26]. Furthermore, as exemplified by Doyle [16] and Ma et al. [14], the DSA can take the actual size of each inclusion into account, in addition to the volumetric fraction of the inclusion medium. But, by no means are both advantages unique to the adoption of the DSA.

3 Indirect Scattering Approach

The ISA is less concerned with the scattering response of a solitary inclusion as with the estimation of the source-region field, i.e., the field induced inside a so-called exclusion region which is of the same shape (and size) as the inclusion. This estimate can be obtained, once an embedding medium is decided upon. The embedding medium can be conveniently chosen, e.g., in the SPFT [27].

When the dyadic Green function for the embedding medium is explicitly known, the source-region field can be estimated for any convex-shaped exclusion region, and the exclusion region need not be of infinitesimal dimensions [28, 29]. Otherwise, the source-region field can be
estimated for an arbitrary bianisotropic medium, provided the exclusion region is ellipsoidal and is treated as infinitesimally small \[30\]. Thus, the ISA enables the homogenization of composites that have far more complex properties — in terms of constitutive properties as well as inclusion shapes — than the DSA does.

Whereas the Maxwell Garnett and the Bruggeman formalisms take only the volumetric fraction of the inclusion medium into account, higher-order statistical measures of the distribution of inclusions can be incorporated in the SPFT \[10, 31, 32\]. In this attribute, the SPFT is at par with the multiple scattering theory \[2, 3, 14, 26\].

### 4 Concluding Remarks

To conclude, two scattering approaches for the homogenization of particulate composites have been identified here. Both approaches have been useful in the past and will continue to be so. Although inclusion size–dependence as well as inclusion distribution statistics can be addressed in both approaches, the scope of the ISA is much more vast than of the DSA. To a large degree, the DSA is confined to mediums with direction–independent constitutive properties and inclusions with simple shapes; but the ISA is not hobbled in that manner. Let us note, in this connection, that the scope of either approach can be enhanced somewhat by using such mathematical devices as the affine transformation \[33, 34\] and the addition of gyrotropic–like magnetoelectric components to the constitutive dyadics \[35\]. The variety of nonlinear composites that can be treated with the DSA \[36, 37\] is considerably limited than those treatable with the ISA \[38, 39\], because nonlinearity normally appears hand–in–hand with anisotropy but rarely with isotropy \[40, 41\]. Finally, nonlocality in homogenized composites can also be tackled using the ISA \[27, 32\].
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