Controllable Entanglement of Lights in a Five-Level System

Yun Li,¹ Chao Hang,¹ Lei Ma,¹ and Guoxiang Huang¹†

¹Department of Physics, East China Normal University, Shanghai 200062, China
(Dated: April 1, 2022)

Abstract

We analyze the nonlinear optical response of a five-level system under a novel configuration of
electro-magnetically induced transparency. We show that a giant Kerr nonlinearity with a relatively
large cross-phase modulation coefficient that occurs in such system may be used to produce an
efficient photon-photon entanglement. We demonstrate that such photon-photon entanglement is
practically controllable and hence facilitates promising applications in quantum information and
computation.

PACS numbers: 03.67.Mn, 42.65.-k, 42.50.Gy

*Electronic address: lma@phy.ecnu.edu.cn
†Electronic address: gxhuang@phy.ecnu.edu.cn
I. INTRODUCTION

Entanglement is one of the most profound features of quantum mechanics[1]. A system consisting of two subsystems is said to be entangled if its quantum state cannot be described by a product of the quantum states of the two subsystems[2]. In past decades, entanglement has been the focus of a large amount of study on the foundation of quantum mechanics, associated particularly with quantum nonseparability and violation of Bell’s inequalities[3]. It has also been viewed as a potential resource that can be used for many practical applications, especially for quantum communication[4], cryptography[5], and computation[6].

It is important for quantum information processing to be able to create entangled states in a controllable way. In recent years, many novel methods have been proposed to generate controllable entangled states[7, 8, 9]. Some of them are based on quantum interference effects associated with electromagnetically induced transparency (EIT)[10], including atom-atom, atom-photon and photon-photon entangled states[12, 13, 14, 15, 16, 17, 18, 19]. Comparing with conventional nonresonant media[20, 21], atomic systems under an EIT configuration possess many striking features, such as very low absorption, ultraslow group velocity, and enhanced Kerr nonlinearity under weak driving conditions[22]. It has also been shown recently that these properties can be used to produce a new type of optical solitons, i.e. ultraslow optical solitons[23, 24].

As is well known, Kerr nonlinearity is important for producing an interaction between light fields. It is also crucial to get an efficient photon-photon entanglement[25]. In an EIT medium, a giant enhancement of Kerr nonlinearity can be obtained by a slightly disturbance of resonance condition. Up to now, different schemes has been proposed for obtaining such enhancement through a cross-phase modulation (CPM) effect, including “N” configuration[26, 27], chain-A configuration[28, 29, 30] and tripod configuration[31]. It has also been suggested to achieve a large nonlinear mutual phase shift[12, 26] and to construct all-optical two-qubit quantum phase gates under weak driving conditions[32, 33].

In this paper, we investigate the photon-photon entanglement in a five-level atomic system under a novel EIT configuration. Our study exhibits several important features. Firstly, the self-phase modulation (SPM) effect can be detached and suppressed, which is different from the case with a tripod configuration[33], where the effect of SPM counteracts that of CPM and impairs the formation of the entanglement. Secondly, an enhanced CPM effect
can be obtained in our system because the EIT configuration in our system has a better symmetry, which makes not only the group velocities of both the probe and trigger pulses decrease several orders of magnitude but also the group-velocity matching can be achieved much easier than the case using a chain-Λ configuration\cite{32}. In addition, in our approach the effect of absorption and dispersion is taken into account. When the binary information is encoded in the polarization degree of freedom of the probe and the trigger pulses, the entanglement between the probe and trigger pulses relies on the large nonlinear mutual phase shift contributed by the enhanced CPM effect. We shall show that such photon-photon entanglement is controllable and hence may facilitate more applications in in quantum information and computation. The paper is arranged as follows. The following section (Sec. II) describes the model under study. In Sec. III linear and nonlinear susceptibilities of the system are calculated based on Bloch equations. The group-velocity matching and the deformation of the probe and trigger pulses due to the dispersion and absorption effects are also discussed. In Sec. IV we investigate the degree of entanglement of two-qubit states with realistic parameters by using entanglement of formation. Finally, Sec. V contains a discussion and a summary of our results.

II. THE MODEL

We consider a life-time broadened five-state atomic system, shown in Fig. 1. The system interacts with a weak, pulsed probe field of center frequency $\omega_P/(2\pi)$ ($|0\rangle \rightarrow |4\rangle$ transition), a weak, pulsed trigger field $\omega_T/(2\pi)$ ($|2\rangle \rightarrow |3\rangle$ transition) and two strong, continuous-wave (cw) coupling fields of frequencies $\omega_B/(2\pi)$ ($|1\rangle \rightarrow |3\rangle$ transition) and $\omega_C/(2\pi)$ ($|1\rangle \rightarrow |4\rangle$ transition). The electric-field can be written as $E = \frac{1}{2}(\mathcal{E}_P e^{-i\omega_P t} + \mathcal{E}_B e^{-i\omega_B t} + \mathcal{E}_C e^{-i\omega_C t} + \mathcal{T} e^{-i\omega_T t}) + \text{c.c.}$, where c.c. represents complex conjugate.

In Schrödinger picture the Hamiltonian of the system is given by $H = H_0 + H_1$, with

$$H_0 = \hbar \omega_0 |0\rangle \langle 0| + \hbar \omega_1 |1\rangle \langle 1| + \hbar \omega_2 |2\rangle \langle 2| + \hbar \omega_3 |3\rangle \langle 3| + \hbar \omega_4 |4\rangle \langle 4|$$

$$H_1 = -\hbar (\Omega_P^* e^{i\omega_P t} |0\rangle \langle 1| + \Omega_B^* e^{i\omega_B t} |1\rangle \langle 3| + \Omega_C^* e^{i\omega_C t} |1\rangle \langle 4|$$

$$+ \Omega_T^* e^{i\omega_T t} |2\rangle \langle 3| + \text{h.c.}),$$

where $\Omega_l$ ($l=P,B,C,T$) are the half Rabi frequencies of relevant fields with $\Omega_P = D_{40} \mathcal{E}_P/(2\hbar)$, $\Omega_T = D_{32} \mathcal{E}_T/(2\hbar)$, $\Omega_B = D_{31} \mathcal{E}_B/(2\hbar)$ and $\Omega_C = D_{41} \mathcal{E}_C/(2\hbar)$.

$D_{ij}$ are
the system is described by the Bloch equations for density-matrix elements

\[ \sigma \]

the electric dipole matrix elements. Within a rotating-wave approximation the evolution of the system is described by the Bloch equations for density-matrix elements

\[ \dot{\sigma}_{00} = -\gamma_{00}\sigma_{00} + i\Omega_p^*\sigma_{40} - i\Omega_P\sigma_{04}, \]

\[ \dot{\sigma}_{11} = -\gamma_{11}\sigma_{11} + i\Omega_B^*\sigma_{31} - i\Omega_B\sigma_{13} + i\Omega_C^*\sigma_{41} - i\Omega_C\sigma_{14}, \]

\[ \dot{\sigma}_{22} = -\gamma_{22}\sigma_{22} + i\Omega_T^*\sigma_{32} - i\Omega_T\sigma_{23}, \]

\[ \dot{\sigma}_{33} = -\gamma_{33}\sigma_{33} + i\Omega_B^*\sigma_{13} - i\Omega_B\sigma_{31} + i\Omega_T^*\sigma_{23} - i\Omega_T\sigma_{32}, \]

\[ \dot{\sigma}_{44} = -\gamma_{44}\sigma_{44} + i\Omega_p^*\sigma_{04} - i\Omega_P^*\sigma_{40} + i\Omega_C^*\sigma_{14} - i\Omega_C\sigma_{41}, \]

\[ \dot{\sigma}_{01} = -(\gamma_{01} + i\Delta_C - i\Delta_P)\sigma_{01} + i\Omega_P^*\sigma_{41} - i\Omega_B\sigma_{03} - i\Omega_C\sigma_{04}, \]

\[ \dot{\sigma}_{02} = -(\gamma_{02} + i\Delta_C + i\Delta_T - i\Delta_P - i\Delta_B)\sigma_{02} + i\Omega_P^*\sigma_{42} - i\Omega_T\sigma_{03}, \]

\[ \dot{\sigma}_{03} = -(\gamma_{03} + i\Delta_C - i\Delta_P - i\Delta_B)\sigma_{03} + i\Omega_P^*\sigma_{43} - i\Omega_B^*\sigma_{01} - i\Omega_T^*\sigma_{02}, \]

\[ \dot{\sigma}_{04} = -(\gamma_{04} - i\Delta_P)\sigma_{04} + i\Omega_P^*(\sigma_{44} - \sigma_{00}) - i\Omega_C^*\sigma_{01}, \]

\[ \dot{\sigma}_{12} = -(\gamma_{12} + i\Delta_T - i\Delta_B)\sigma_{12} + i\Omega_B^*\sigma_{32} + i\Omega_C^*\sigma_{42} - i\Omega_T\sigma_{13}, \]

\[ \dot{\sigma}_{13} = -(\gamma_{13} - i\Delta_B)\sigma_{13} + i\Omega_B^*(\sigma_{33} - \sigma_{11}) + i\Omega_C^*\sigma_{43} - i\Omega_T^*\sigma_{12}, \]

\[ \dot{\sigma}_{14} = -(\gamma_{14} - i\Delta_C)\sigma_{14} + i\Omega_B^*\sigma_{34} + i\Omega_C^*(\sigma_{44} - \sigma_{11}) - i\Omega_P^*\sigma_{10}, \]

\[ \dot{\sigma}_{23} = -(\gamma_{23} - i\Delta_T)\sigma_{23} + i\Omega_T^*(\sigma_{33} - \sigma_{22}) - i\Omega_B^*\sigma_{21}, \]

\[ \dot{\sigma}_{24} = -(\gamma_{24} + i\Delta_B - i\Delta_C - i\Delta_T)\sigma_{24} + i\Omega_T^*\sigma_{34} - i\Omega_P^*\sigma_{20} - i\Omega_C^*\sigma_{21}, \]

\[ \dot{\sigma}_{34} = -(\gamma_{34} + i\Delta_B - i\Delta_C)\sigma_{34} + i\Omega_B\sigma_{14} + i\Omega_T\sigma_{24} - i\Omega_P^*\sigma_{30} - i\Omega_C^*\sigma_{31}, \]

where \( \sigma_{ij} = \rho_{ij} \exp(-i\omega_{ij}t) \) (\( i, j = 0 \) to 4). \( \gamma_{ij} \) describe decay of populations (\( i=j \)) and coher-
ences \(i \neq j\). \(\Delta_P = \omega_{40} - \omega_P\), \(\Delta_B = \omega_{31} - \omega_B\), \(\Delta_C = \omega_{41} - \omega_C\) and \(\Delta_T = \omega_{32} - \omega_T\) are frequency detunings.

**III. LARGE CROSS-KERR NONLINEARITY AND GROUP-VELOCITY MATCHING**

For solving the Bloch Eq. (3) we assume that the temporal duration of the probe and trigger fields is longer enough so that a steady state approximation can be employed. When the intensity of the probe and trigger fields is much weaker than the intensity of both coupling fields, i.e. \(|\Omega_P|^2, |\Omega_T|^2 \ll |\Omega_B|^2, |\Omega_C|^2\), the population in the ground states \(|0\rangle\) and \(|2\rangle\) is not depleted and symmetric with respect to \(0 \leftrightarrow 2\) exchange, thus \(\sigma_{00} \approx \sigma_{22} \approx 1/2\) with the population of other three levels vanishing, i.e. \(\sigma_{11} \approx \sigma_{33} \approx \sigma_{44} \approx 0\). Furthermore, \(\sigma_{13}, \sigma_{14}\) and \(\sigma_{34}\) vanish also due to small population. We solve Eq. (3) under these consideration and obtain the following expressions for the susceptibilities of the probe and trigger fields

\[
\chi_P = n_a|D_{04}|^2\sigma_{40}/(2\epsilon_0 \hbar \Omega_P) = \chi_P^{(1)} + \chi_P^{(3)}|E_P|^2 + \chi_P^{(3)}|E_T|^2, \quad (4a)
\]

\[
\chi_T = n_a|D_{23}|^2\sigma_{32}/(2\epsilon_0 \hbar \Omega_T) = \chi_T^{(1)} + \chi_T^{(3)}|E_T|^2 + \chi_T^{(3)}|E_P|^2, \quad (4b)
\]

where \(\chi_P^{(1)}\) and \(\chi_T^{(1)}\) are respectively the linear susceptibilities of the probe and trigger fields, and \(\chi_P^{(3)}\) and \(\chi_T^{(3)}\) (\(\chi_P^{(3)}\) and \(\chi_T^{(3)}\)) are respectively the nonlinear susceptibilities characterizing the effect of self-Kerr (cross-Kerr) nonlinearity. Since we are interested in getting a large interaction between the probe and trigger fields that favors the generation of photon-photon entanglement (discussed in Sec. IV below), it is necessary to have a relatively large cross-Kerr nonlinearity, which can be realized in our system if one has \(|\Omega_B|^2 \approx \Delta_T(\Delta_T - \Delta_B)\) and \(|\Omega_C|^2 \approx \Delta_P(\Delta_P - \Delta_C)\). Under these conditions, the susceptibilities related to SPM effect, \(\chi_P^{(3)}\) and \(\chi_T^{(3)}\), are suppressed and can thus be neglected. Thus, we have \(\chi_P \approx \chi_P^{(1)} + \chi_P^{(3)}|E_P|^2\) and \(\chi_T \approx \chi_T^{(1)} + \chi_T^{(3)}|E_T|^2\), with

\[
\chi_P^{(1)} = \frac{n_a|D_{04}|^2|\Omega_B|^2}{2\epsilon_0 \hbar} \left(2(\Delta_P - \Delta_C)(\Delta_P + \Delta_B - \Delta_C) + i\gamma(\Delta_P - \Delta_C)\right), \quad (5a)
\]

\[
\chi_P^{(3)} = \frac{n_a|D_{04}|^2|D_{23}|^2}{2\epsilon_0 \hbar^3} \frac{4|\Omega_B|^2|\Omega_C|^2 + (\Delta_C - \Delta_P)N_1^2}{(\Delta_T + \Delta_C - \Delta_P - \Delta_B)N_1^2N_2^*}, \quad (5b)
\]

\[
\chi_T^{(1)} = \frac{n_a|D_{23}|^2|\Omega_C|^2}{2\epsilon_0 \hbar} \left(2(\Delta_T - \Delta_B)(\Delta_T + \Delta_C - \Delta_B) + i\gamma(\Delta_T - \Delta_B)\right), \quad (5c)
\]

\[
\chi_T^{(3)} = \frac{n_a|D_{23}|^2|D_{04}|^2}{2\epsilon_0 \hbar^3} \frac{4|\Omega_C|^2|\Omega_B|^2 + (\Delta_B - \Delta_T)N_1}{(\Delta_P + \Delta_B - \Delta_T - \Delta_C)N_1N_2}. \quad (5d)
\]
where \( n_a \) is the atomic density, \( N_1 = 4|\Omega_B|^2\Delta_P + 4|\Omega_C|^2 - \Delta_P(\Delta_P - \Delta_C)(\Delta_P + \Delta_B - \Delta_C) + \gamma^2(\Delta_P - \Delta_C) + 2i\gamma|\Omega_B|^2 + |\Omega_C|^2 - (\Delta_P - \Delta_C)(2\Delta_P + \Delta_B - \Delta_C) \) and \( N_2 = 4|\Omega_C|^2\Delta_T + 4|\Omega_B|^2 - \Delta_T(\Delta_T - \Delta_B)(\Delta_T + \Delta_C - \Delta_B) + \gamma^2(\Delta_T - \Delta_B) - 2i\gamma|\Omega_C|^2 + |\Omega_B|^2 - (\Delta_T - \Delta_B)(2\Delta_T + \Delta_C - \Delta_B) \).

In above derivation, we have assumed \( \Omega_P \) and \( \Omega_T \) are sufficiently weak, i.e. \( |\Omega_P|^2 \) and \( |\Omega_T|^2 \ll |\Omega_B|^2, |\Omega_C|^2 \) and the decay rates \( \gamma_{ii} \approx 0 \) (i=0 to 2), \( \gamma_{33} = \gamma_{44} = \gamma, \gamma_{ij} = \gamma_{34} \approx 0 \) (i, j=0 to 2, i \neq j) and \( \gamma_{i3} = \gamma_{j4} = 0.5\gamma \) (i, j=0 to 2). Due to the symmetry of our system configuration, Eqs. (5a), (5b) and Eqs. (5c), (5d) are symmetric under the exchange \( P \leftrightarrow T \), \( B \leftrightarrow C \) and \( D_{04} \leftrightarrow D_{23} \). We stress that the imaginary parts of the linear and nonlinear susceptibilities given above are much smaller than their relevant real parts under the (EIT) conditions \( |\Omega_P|^2 \) and \( |\Omega_T|^2 \ll |\Omega_B|^2, |\Omega_C|^2 \), which result in quantum interferences between the states \(|0\rangle\) and \(|1\rangle\) and the states \(|1\rangle\) and \(|2\rangle\), making the population in the states \(|2\rangle\) and \(|3\rangle\) be small thus very low absorption for the probe and trigger fields [34].

In addition to a relatively large cross-Kerr nonlinearity, group velocity matching is another necessary condition for achieving a large mutual phase shift because only in this way can the probe and trigger optical pulses interact for a sufficiently long time [11, 12]. Note that the group velocity of an optical pulse is given by \( v_g = c/(n_0 + \omega \partial n_0/\partial \omega) \), where \( n_0 = \sqrt{1 + \chi^{(1)}(\omega)} \) is linear index of refraction. Using the expressions of linear susceptibilities given in Eq. (6), we obtain

\[
v_g^P = \frac{1}{c} + \frac{n_a|D_{04}|^2\omega_P}{2\epsilon_0\hbar c (|\Omega_C|^2 + |\Omega_B|^2)} \left[ \frac{1}{4} - \frac{|\Omega_B|^2}{4(|\Omega_B|^2 + |\Omega_C|^2)} + \frac{|\Omega_B|^2}{(2\Delta - i\gamma)^2} \right], \tag{6a}
\]

\[
v_g^T = \frac{1}{c} \frac{n_a|D_{23}|^2\omega_T}{2\epsilon_0\hbar c (|\Omega_C|^2 + |\Omega_B|^2)} \left[ \frac{1}{4} - \frac{|\Omega_C|^2}{4(|\Omega_B|^2 + |\Omega_C|^2)} + \frac{|\Omega_C|^2}{(2\Delta - i\gamma)^2} \right], \tag{6b}
\]

for the probe and trigger pulses, respectively. Actually, the group velocities are the real part of \( \tilde{v}_g^P \) and \( \tilde{v}_g^T \), which are denoted by \( \tilde{v}_g^P \) and \( \tilde{v}_g^T \), used in the next section. The imaginary parts of the group velocities result also in a damping for wave propagation. For obtaining the above relatively simple expressions, we have assumed that all frequency detunings are nearly, but not exactly, equal (\( \approx \Delta \)). By Eq. (6) the group velocity matching can be achieved under the condition \( \Omega_B \approx \Omega_C \).

Although under the EIT configuration of Fig. 1 the absorption can be made very small but it is not vanishing and its presence may result in an attenuation for the propagation of the probe and trigger pulses. In addition, the dispersion effect existing in the system will also
result in a distortion of the probe and trigger fields. For example, for a Gaussian input of the probe pulse with the form \( \Omega_P(0,0) \exp(-t^2/\tau_P^2) \), the initial amplitude \( \Omega_P(0,0) \) decreases to \( \Omega_P(0,0)/\sqrt{1 - i2zG_P/\tau_P^2} \) while the initial duration \( \tau_P \) increases to \( \tau_P\sqrt{1 - i2zG_P/\tau_P^2} \), where \( z \) means the distance of the pulse passing through the medium, \( G_P \) denotes the group velocity dispersion (GVD) of the probe field. The same analysis applies for the trigger field by just taking \( P \rightarrow T \). The GVD of the probe and trigger fields can be obtained by the expressions of linear susceptibilities given in Eqs. (5), which read

\[
G_P = \frac{n_i D_{04}}{\epsilon_0 \hbar c} \left( |\Omega_B|^2 + |\Omega_C|^2 \right) \left[ -|\Omega_C|^2(2\Delta - i\gamma) \right. \\
\left. + \frac{2|\Omega_B|^2}{8(|\Omega_B|^2 + |\Omega_C|^2)^2} \right] \left( 2\Delta - i\gamma \right)^3,
\]

(7a)

\[
G_T = \frac{n_i D_{23}}{\epsilon_0 \hbar c} \left( |\Omega_C|^2 + |\Omega_B|^2 \right) \left[ -|\Omega_B|^2(2\Delta - i\gamma) \right. \\
\left. + \frac{2|\Omega_C|^2}{8(|\Omega_C|^2 + |\Omega_B|^2)^2} \right] \left( 2\Delta - i\gamma \right)^3.
\]

(7b)

### IV. CONTROLLABLE ENTANGLEMENT BETWEEN PROBE AND TRIGGER LIGHTS

A significant interaction is key ingredient for the realization of the entanglement between the probe and trigger fields. In our system, such interaction can be realized by the giant CPM effect, discussed above, by which an optical field acquires a large phase shift conditional to the state of another optical field. We choose two orthogonal light polarizations \( |\sigma^-\rangle \) and \( |\sigma^+\rangle \) as the basis of the entanglement state. Assuming that the input probe and trigger polarized single photon wave packets can be expressed as a superposition of the circularly polarized states, i.e.

\[
|\psi_i\rangle = \frac{1}{\sqrt{2}} |\sigma^-\rangle_i + \frac{1}{\sqrt{2}} |\sigma^+\rangle_i, \quad i = \{P,T\}
\]

(8)

where \( |\sigma^\pm\rangle_i = \int d\omega \xi_i(\omega)a^\dagger_\pm(\omega)|0\rangle \), with \( \xi_i(\omega) \) being a Gaussian frequency distribution of incident wave packets, centered at frequency \( \omega_i \). The input-output relations \( |\alpha\rangle_P|\beta\rangle_T \rightarrow \exp(i\phi_{\alpha\beta})|\alpha\rangle_P|\beta\rangle_T \) can be satisfied. Here \( \alpha, \beta = 0, 1 \) denote two-qubit basis.

We assume the five-state system shown Fig. 1 is implemented only when the probe has \( \sigma^+ \) polarization and the trigger has \( \sigma^- \) polarization. When either (both) the probe or (and) the trigger polarizations are changed, the phase shifts acquired by two pulses do not involve the nonlinear susceptibilities and lead to a difference. In fact, when both of them have the “wrong” polarization (probe \( \sigma^- \) polarized and trigger \( \sigma^+ \) polarized) there is no sufficiently close level which the atoms can be driven to and the field acquires the trivial vacuum phase
shift \( \phi_0^j = k_j L \) (\( j = P, T \)), where \( L \) is the length of the medium. Instead, when only one of them have the wrong polarization, the right one acquires a linear phase shift \( \phi_0^j + \phi_{\text{lin}}^j \), where \( \phi_{\text{lin}}^j = 2\pi k_j L \Re(\chi_j^{(1)}) \). Actually, the imaginary parts of the linear susceptibilities result in an effect of linear absorption, which makes the output states decoherent. When none of them have the wrong polarization, each pulse acquires a nonlinear phase shift \( \phi_{\text{nlin}}^j \), where \( \phi_{\text{nlin}}^j \) denotes the probe or trigger nonlinear phase shift when the five-state configuration is realized. For a Gaussian trigger pulse of time duration \( \tau_T \) and Rabi frequency \( \Omega_T \), moving with group velocity \( \tilde{v}_g \), the shift of the probe field reads

\[
\phi_{\text{nlin}}^P = k_T \frac{\pi \hbar^2 |\Omega_T|^2}{2|D_{04}|^2} \Re(\chi_{XX}^{(3)}) \int_0^L \frac{1}{\beta_T^2} \exp \left( -\frac{\omega_T z}{c} \Im(\chi_T^{(1)}) - \left[ \frac{(1/\tilde{v}_g - 1/\tilde{v}_g)^2}{\tau_T \beta_T} \right]^2 \right) dz,
\]

where \( \beta_T = \sqrt{1 - i2zG_T/\tau_T^2} \). The phase shift of the trigger field can be obtained upon interchanging \( P \leftrightarrow T \) and \( D_{04} \leftrightarrow D_{23} \) in Eq. (9):

\[
\phi_{\text{nlin}}^T = k_T \frac{\pi \hbar^2 |\Omega_P|^2}{2|D_{04}|^2} \Re(\chi_{XX}^{(3)}) \int_0^L \frac{1}{\beta_P^2} \exp \left( -\frac{\omega_P z}{c} \Im(\chi_T^{(1)}) - \left[ \frac{(1/\tilde{v}_g - 1/\tilde{v}_g)^2}{\tau_P \beta_P} \right]^2 \right) dz
\]

with \( \beta_P = \sqrt{1 - i2zG_P/\tau_P^2} \). We should point out that the imaginary parts of the nonlinear susceptibilities may result in a nonlinear absorption. However, this absorption is much weaker than the linear one and thus can be neglected. Note that the GVD effect, reflected by the parameters \( G_P \) and \( G_P \), has been taken into account in the formulas of the phase shifts (9) and (10).

Using the results given above the the density matrix of the output state is expressed as

\[
\hat{\rho} = \frac{1}{Z} \begin{pmatrix}
e^{i(\phi_{\text{lin}}^P - \phi_{\text{lin}}^P)} & e^{i(\phi_{\text{lin}}^P - \phi_{\text{lin}}^P - \phi_{\text{lin}}^T)} & e^{i(\phi_{\text{lin}}^P + \phi_{\text{lin}}^P + \phi_{\text{lin}}^T)} & e^{i(\phi_{\text{lin}}^P - \phi_{\text{lin}}^T)} \\
e^{i(\phi_{\text{lin}}^T - \phi_{\text{lin}}^P)} & e^{i(\phi_{\text{lin}}^P - \phi_{\text{lin}}^P + \phi_{\text{lin}}^T)} & e^{i(\phi_{\text{lin}}^P + \phi_{\text{lin}}^P + \phi_{\text{lin}}^T)} & e^{i(\phi_{\text{lin}}^P + \phi_{\text{lin}}^P - \phi_{\text{lin}}^T)} \\
e^{-i(\phi_{\text{lin}}^P + \phi_{\text{lin}}^P + \phi_{\text{lin}}^T)} & e^{-i(\phi_{\text{lin}}^P + \phi_{\text{lin}}^P + \phi_{\text{lin}}^T)} & 1 & e^{-i(\phi_{\text{lin}}^P + \phi_{\text{lin}}^P + \phi_{\text{lin}}^T)} \\
e^{-i(\phi_{\text{lin}}^P + \phi_{\text{lin}}^P)} & e^{-i(\phi_{\text{lin}}^P + \phi_{\text{lin}}^P - \phi_{\text{lin}}^T)} & e^{i(\phi_{\text{lin}}^P + \phi_{\text{lin}}^P + \phi_{\text{lin}}^T)} & e^{i(\phi_{\text{lin}}^P - \phi_{\text{lin}}^T)}
\end{pmatrix}
\]

in the computational basis \( \{|\sigma^-\rangle_P |\sigma^-\rangle_T, |\sigma^-\rangle_P |\sigma^+\rangle_T, |\sigma^+\rangle_P |\sigma^-\rangle_T, |\sigma^+\rangle_P |\sigma^+\rangle_T\} \), where \( Z = [1 + e^{i(\phi_{\text{lin}}^P - \phi_{\text{lin}}^P)}] \cdot [1 + e^{i(\phi_{\text{lin}}^P - \phi_{\text{lin}}^T)}] \) is a normalized coefficient. Actually, the linear phase shift counts no contribution on the entanglement of the probe and trigger lights because when \( \phi_{\text{nlin}}^P = \phi_{\text{nlin}}^T = 0 \), the space spanned by the two qubits can be expressed as a product \( \rho_P \otimes \rho_T \) of pure state of its parts.
There is a variety of measures known for quantifying the degree of entanglement in a bipartite system, including the entanglement of distillation\cite{2}, the relative entropy of entanglement\cite{36}, the entanglement of formation\cite{2} and the entanglement witnesses\cite{37}. Here, we use the entanglement of formation as the measure of the purity and degree of entanglement of our two-qubit state. For an arbitrary two-qubit system, it is given by\cite{38}

\[
E_F(C) = h \left( \frac{1 + \sqrt{1 - C^2}}{2} \right),
\]  

(11)

where \( h(x) = -x \log_2(x) - (1-x) \log_2(1-x) \) is Shannon’s entropy function, and \( C \), called “concurrence”, is given by \( C(\hat{\rho}) = \max \{0, \lambda_1 - \lambda_2 - \lambda_3 - \lambda_4\} \)

(12)

with the \( \lambda_i \)s being the square roots of the eigenvalues, in a decreasing order, of the Hermitian matrix \( \hat{\rho}_y = \hat{\rho} \sigma_y \otimes \hat{\sigma}_y^* \otimes \hat{\sigma}_y \), here \( \hat{\rho}^* \) denotes the complex conjugation of \( \hat{\rho} \) in the computational basis, and \( \hat{\sigma}_y \) is the \( y \)-component of the Pauli matrix. Since \( E_F(C) \) is a monotonic function of \( C \), thus we can also use the concurrence directly as our measure of entanglement.

Now we consider a typical system working with alkali atoms to estimate the physical parameters and the degree of the entanglement. We take \( \gamma = 10^6 \text{ s}^{-1} \). The detunings are chosen as \( \Delta_P = 80.2 \gamma, \Delta_T = 80.1 \gamma, \Delta_B = 79.9 \gamma \) and \( \Delta_C = 80.0 \gamma \). The Rabi frequencies are taken as \( \Omega_P = 0.6 \gamma, \Omega_T = 0.8 \gamma, \Omega_B = 3.6 \gamma \) and \( \Omega_C = 4.1 \gamma \). The density of the atomic gas \( n_a = 1.0 \times 10^{13} \text{cm}^{-3} \) and the length of the medium \( l = 0.1 \text{ mm} \). The probe and trigger have a mean amplitude of about one photon when the beams are tightly focused and has a time duration about five microseconds. With the above parameters, we get \( \phi_{lin}^P = -23.42 + 0.68i, \phi_{lin}^T = 12.00 + 0.27i, \phi_{nlin}^P = 0.26 \) and \( \phi_{nlin}^T = 0.05 \). Note that in this case \( |\text{Re}(\chi^{(3)}_{XP})/\text{Re}(\chi^{(1)}_P)| \approx 40.0, |\text{Re}(\chi^{(3)}_{XT})/\text{Re}(\chi^{(1)}_T)| \approx 56.6 \). The group velocities of the probe and trigger fields read \( v_g^P = 9.7 \text{ m/s} \) and \( v_g^T = 18.9 \text{ m/s} \), very small indeed in comparison with the light speed in vacuum. Using these results we obtain the degree of entanglement \( E_F = 3.5\% \).

Because there are many physical parameters which can be adjusted in a fairly large extent, the entanglement of the probe and trigger fields in our system is practically controllable. For example, we choose the detunings as \( \Delta_P = 40.2 \gamma, \Delta_T = 40.1 \gamma, \Delta_B = 39.9 \gamma \) and \( \Delta_C = 40.0 \gamma \) and the Rabi frequencies as \( \Omega_P = 0.6 \gamma, \Omega_T = 0.8 \gamma, \Omega_B = 2.5 \gamma \) and \( \Omega_C = 2.7 \gamma \). The density
of the atomic gas is retained as above but the length of the medium \( l = 0.03 \text{mm} \). By a similar calculation we obtain \( \phi_{lin}^P = -7.24 + 0.65i \), \( \phi_{lin}^T = 4.13 + 0.32i \), \( \phi_{nlin}^P = 0.36 \) and \( \phi_{nlin}^T = 0.08 \) (where \(|\text{Re}(\chi^{(3)}_{XP})/\text{Re}(\chi^{(1)}_P)| \approx 148.6\), \(|\text{Re}(\chi^{(3)}_{XT})/\text{Re}(\chi^{(1)}_T)| \approx 210.2\)). We get the group velocities \( v_g^P = 4.6 \text{ m/s} \) and \( v_g^T = 8.1 \text{ m/s} \). The degree of entanglement in this case is given by \( E_F = 6.3\% \).

We have made a numerical computations on the concurrence versus the Rabi frequencies of two control fields using the two sets of parameters, given above (except for the control fields). The results are shown in Fig. 2(a) and (b), respectively. The range of frequency of the control fields are chosen as \( 1.5\gamma < \Omega_B < 6.0\gamma \) and \( 2.0\gamma < \Omega_C < 6.5\gamma \). From the results shown in Fig. 2(a) and Fig. 2(b) we see that there is a significant enhancement of concurrence for some values of the control fields \( \Omega_P \) and \( \Omega_T \) when the group velocity matching condition is approximately satisfied. Thus the entanglement of the probe and trigger fields can indeed be controlled in our system.

In Fig. 3, we plot the concurrence versus the medium length \( L \) under the two sets of parameters (the same as those in Fig. 2(a) and Fig. 2(b), respectively). The solid line corresponds to the first set of parameters while the dash line corresponds to the second ones. From the figure we find that although a long medium permits a more sufficiently interaction between the two optical pulses and favor to the entanglement, it also leads to larger effects of absorption and dispersion, which impair the entanglement when \( L \) becomes
V. CONCLUSION

We have proposed a scheme to create a pair of entangled photons by using a novel five-level EIT configuration. We have shown that, under suitable conditions, a large cross-phase modulation appears when the group velocities of two pulses, the probe and the trigger, are both small and comparable because of the symmetry of the system. In our approach the effect of absorption and dispersion of the system has been taken into account, which contribute to the deformation of pulse shapes. The binary information is encoded in the polarization degree of freedom of the probe and the trigger pulses and the entanglement of the probe and trigger pulses comes from the cross-Kerr nonlinearity. The entanglement can be practically controlled by adjusting the parameters of the system, such as the Rabi frequencies of two coupling fields. The controllable entanglement suggested here may facilitate promising applications for quantum information and computation. The results presented in this work may be useful for guiding an experimental finding of the photon-photon entanglement in atomic systems.
Acknowledgments

The authors thank Dr. Weiping Zhang and Dr. Jinming Liu for useful discussions. The work was supported by the Key Development Program for Basic Research of China under Grant Nos. 2001CB309300 and 2005CB724508, and NSF-China under Grant Nos. 10434060 and 90403008.

[1] A. Einstein, B. Podosky and N. Rosen, Phys. Rev. 47, 777 (1935).
[2] C. H. Bennett, D. P. DiVincenzo, J. A. Smolin and W. K. Wootters, Phys. Rev. A 54, 3824 (1996).
[3] For a guide to some of the literature, see L. E. Ballentine, Am. J. Phys. 55, 875 (1986).
[4] D. Bouwmeester, J.-W. Pan, K. Mattle, M. Eibl, H. Weinfurter and A. Zeilinger, Nature (London) 390, 575 (1997).
[5] N. Gisin, G. Ribordy, W. Tittle and H. Zbinden, Rev. Mod. Phys. 74, 145 (2002).
[6] C. Williams and S. Clearwater, Explorations in Quantum Computing (Springer-Verlag, New York, 1998).
[7] R. G. Unanyan, B. W. Shore and K. Bergmann, Phys. Rev. A 63, 043405 (2001).
[8] R. G. Unanyan, N. V. Vitanov and K. Bergmann, Phys. Rev. Lett. 87, 137902 (2001).
[9] C. Emary, B. Trauzettel and C. W. J. Beenakker, Phys. Rev. Lett. 95, 127401 (2005).
[10] S. E. Harris, Phys. Today 50, 36 (1997).
[11] M. D. Lukin and A. Imamoglu, Nature (London) 413, 273 (2001).
[12] M. D. Lukin and A. Imamoglu, Phys. Rev. Lett. 84, 1419 (2000); M. D. Lukin and P. R. Hemmer Phys. Rev. Lett. 84, 2818 (2000); M. D. Lukin, S. F. Yelin and M. Fleischhauer, Phys. Rev. Lett. 84, 4232 (2000).
[13] M. Paternostro, M. S. Kim and B. S. Ham, Phys. Rev. A 67, 023811 (2003).
[14] L. M. Kuang and L. Zhou, Phys. Rev. A 68, 043606 (2003).
[15] M.G. Payne and L. Deng, Phys. Rev. Lett. 91, 123602 (2003).
[16] M. Kiffner and K.-P. Marzlin, Phys. Rev. A 71, 033811 (2005).
[17] A. Peng, M. Johnsson, W. P. Bowen, P. K. Lam, H.-A. Bachor and J. J. Hope, Phys. Rev. A 71, 033809 (2005).
When discussing the entanglement between the probe and trigger fields, realistic parameters for these conditions have been chosen, see Sec. IV.

The effects of the absorption and dispersion on the entanglement of the probe and trigger fields will be discussed in the next section.

V. Vedral, M. B. Plenio, M. A. Rippin and P. L. Knight, Phys. Rev. Lett. 78, 2275 (1997).
Fernando G. S. L. Brandão, Phys. Rev. A 72, 022310 (2005).
W. K. Wootters, Phys. Rev. Lett. 80, 2245 (1998).