Deformed shell model results for two neutrino positron double beta decay of $^{74}$Se

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Half-lives $T_{1/2}^{2ν}$ for two neutrino positron double beta decay modes $β^+EC/ECEC$ are calculated for $^{74}$Se, a nucleus of current experimental interest, using deformed shell model based on Hartree-Fock states employing a modified Kuo interaction in ($^2p_{3/2}$, $^1f_{5/2}$, $^2p_{1/2}$, $^1g_{9/2}$) space. The calculated half-life for the ECEC mode is $\sim 10^{25}$yr and it may be possible to observe this in the near future with improved sensitivity of experiments.

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I. INTRODUCTION

Double beta decay (DBD) is a rare weak interaction process in which two identical nucleons inside the nucleus undergo decay with or without emission of neutrinos. The two neutrino double beta decay ($2νβ^−β^−$) which was first predicted long back by Meyer [1] is fully consistent with standard model and has been observed experimentally in more than 10 nuclei. The neutrinoless double beta decay ($0νβ^−β^−$) which involves emission of two electrons and no neutrinos, has not been observed experimentally and it violates lepton number conservation. The claim for observation of $0νβ^−β^−$ decay of $^{76}$Ge by Heidelberg-Moscow group [2] is controversial and yet to be confirmed by other ongoing experiments. This process is one of the best probes for studying physics beyond the standard model. To extract the mass of neutrino via $0νβ^−β^−$ decay, it is necessary to have good nuclear structure models for calculating the nuclear transition matrix elements (NTME) involved. Large number of theoretical studies for various candidate nuclei, for $2ν$ and $0νβ^−β^−$ decay, using many nuclear models have been carried out so that the calculated NTME can be established to be reliable; see for example [3, 4]. It is important to add that not all models will work for all DBD nuclei due to various reasons. For example for $A >> 64$ nuclei full shell model in ($^2p_{3/2}$, $^1f_{5/2}$, $^2p_{1/2}$, $^1g_{9/2}$) space is still not feasible.

In contrast to the $2νβ^−β^−$ decay, the positron decay modes, i.e. $2νβ^+β^+EC/ECEC$ decay modes (hereafter, all these three combined is called $2ν e^+DBD$) are not yet observed experimentally and hence there are not many theoretical studies of the NTME involved in $2ν e^+DBD$. However, in the last few years serious attempts are made to measure half-lives for $2ν e^+DBD$ modes in the upper ($pfg_{9/2}$) shell nuclei $^{76}$Kr [5], $^{64}$Zn [6] and $^{74}$Se [7] (in the past, attempts are also made for $^{106,108}$Cd [8] and $^{130,132}$Ba [9] nuclei). Prompted by this experimental interest, recently [10] we have carried out calculations for $^{76}$Kr using the so called deformed shell model (DSM) by employing a modified Kuo interaction in ($^2p_{3/2}$, $^1f_{5/2}$, $^2p_{1/2}$, $^1g_{9/2}$) space. It is seen that the predictions of DSM for $2ν e^+DBD$ half-lives are close to those of QRPA and PHFB models. Extending the study in [10] further, we have carried out DSM calculations for $2ν e^+DBD$ half-lives for $^{74}$Se nucleus and the results are reported in this brief report. We did not consider $^{64}$Zn as spherical shell model is well suited [11] for the three nuclei $^{64}$Zn, $^{64}$Cu and $^{64}$Ni due to the fact that they are not well deformed with proton numbers close to the N=28 closed core.

Over the years, we have been using with success the deformed Shell Model (DSM) based on Hartree-Fock states to study the spectroscopic properties, such as band structures, shapes, nature of band crossings, electromagnetic transition probabilities and so on, for medium heavy nuclei [12, 13, 14]. More recently this model is applied to N=Z and N=Z+1 nuclei by including isospin projection
The spectroscopic properties especially electromagnetic transitions like B(E2) and B(M1) values provide a stringent test for the goodness of the nuclear wave functions generated using the model. It is also important to add that DSM results are being used by many groups in the discussion of experimental data for $A \sim 64$–80 nuclei. In addition DSM was used in calculating transition matrix elements for $\mu - e$ conversion in $^{72}$Ge and in the analysis of data for inelastic scattering of electrons from $fp$-shell nuclei. This model has also been used for studying $2\nu$ double beta decay transition matrix elements for $^{76}$Ge $\rightarrow ^{76}$Se with considerable success. More recently in we have applied DSM to study $\beta$-decay half lives, GT distributions, electron capture rates and $2\nu$ $e^+\text{DBD}$ in $^{78}$Kr. All these confirm that DSM generates good nuclear wave functions for nuclei in the mass region $A \sim 64$–80. Now, we will first discuss the DSM formalism briefly and then the results for $^{74}$Se are described.

II. DSM FORMALISM

Half-life for the $2\nu$ $e^+\text{DBD}$ decay modes for the $0^+_1 \rightarrow J^+_F$ transitions, with $J^+_F$ for the daughter nucleus being $J^+_F = 0^+_1$ or $2^+_1$, is given by

$$\left[T_{1/2}^{2\nu}(k,J_F)\right]^{-1} = G_{2\nu}(k,J_F) \left|M_{2\nu}(J_F)\right|^2$$  \hspace{1cm} (1)

where $k$ denotes the modes $\beta^+\beta^+$, $\beta^+\text{EC}$ and ECEC. As, besides the $0^+_1 \rightarrow 0^+_1$ transition, the ECEC mode for $0^+_1 \rightarrow 2^+_1$ is also of experimental interest, we are considering both $J^+_F = 0^+_1$ and $2^+_1$ in Eq. (1). The integrated kinematical factors $G_{2\nu}(k,J_F)$ are independent of nuclear structure (except for the dependence on the excitation energy $E_F$ of the $J_F$ state of the daughter nucleus) and they can be calculated with good accuracy. Further, the nuclear transition matrix elements (NTME) $M_{2\nu}$ are nuclear model dependent and they are given by

$$M_{2\nu}(J_F) = \frac{1}{\sqrt{J_F + 1}} \sum_N \frac{\langle J_F^+|\sigma\tau^-||1^+_N\rangle\langle 1^+_N||\sigma\tau^-|0^+_I\rangle}{[E_0 + E_N - E_I]^{J_F + 1}}$$  \hspace{1cm} (2)

where $|0^+_I\rangle$, $|J_F^+\rangle$ and $|1^+_N\rangle$ are the initial, final and virtual intermediate states respectively and $E_N(E_I)$ is the energy of intermediate (initial) nucleus. Note that $E_0 = \frac{1}{2}(E_I - E_F) = \frac{1}{2}W_0$. Here, $W_0$ is the total energy released for different $2\nu$ $e^+\text{DBD}$ modes. For $0^+_1 \rightarrow 0^+_1$ transitions

$$W_0(\beta^+\beta^+) = Q_{\beta^+\beta^+} + 2m_e,$$  \hspace{1cm} (3)

$$W_0(\beta^+\text{EC}) = Q_{\beta^+\text{EC}} + e_b$$  \hspace{1cm} (4)

$$W_0(\text{ECEC}) = Q_{\text{ECEC}} - 2m_e + e_b + e_{b2}$$  \hspace{1cm} (5)

Note that the $Q$-values are given by the difference of neutral atomic masses of parent and daughter nuclei involved in the positron double beta decay process and $e_b$ is the binding energy of the absorbed atomic electron. For the $0^+_1 \rightarrow 2^+_1$ ECEC transition, denoted by ECEC*, we have $W_0(\text{ECEC}^*) = Q_{\text{ECEC}} - \Delta E - 2m_e + e_{b1} + e_{b2}$ where $\Delta E$ is the excitation energy of the $2^+_1$ state. We have employed DSM to calculate the reduced matrix element appearing in Eq. (2).

In DSM, for a given nucleus, starting with a model space consisting of a given set of single particle orbitals and effective two-body Hamiltonian, the lowest prolate and oblate intrinsic states are obtained by solving the Hartree-Fock (HF) single particle equation self-consistently. Excited intrinsic configurations are obtained by making particle-hole excitations over the lowest intrinsic state. These intrinsic states will not have good angular momentum and good angular momentum states are obtained by angular momentum projection from these intrinsic states. In general the projected states with same $J$ but coming from different intrinsic states will not be orthogonal to each other. Hence they are orthonormalized and then band mixing calculations are performed. DSM is well established to be a successful model for transitional nuclei when sufficiently large number of intrinsic states are included in the band mixing calculations; see and references therein. Performing DSM calculations for the parent, daughter and the intermediate odd-odd nucleus (here we need only the $1^+$ states) and then using the DSM wavefunctions, the $\sigma\tau^-$ matrix elements in Eq. (2) are calculated. For further details see . Now we will discuss the results for $^{74}$Se.
III. RESULTS AND DISCUSSION

In our calculations of $^{74}$Se $2\nu e^+\text{DBD}$ half-lives, for the structure of the nuclei $^{74}$Se, $^{74}$As and $^{74}$Ge we have used a modified Kuo effective interaction $[25]$ in the $(^2p_{3/2}, ^1f_{5/2}, ^2p_{1/2}, ^1g_{9/2})$ space with $^{56}$Ni as the inert core. The single particle energies of these orbitals are taken as 0.0, 0.78, 1.08 and 4.5 MeV respectively. DSM with modified Kuo effective interaction has been quite successfully used by us in describing many important features of nuclei in $A \sim 60-80$ region. In particular, shape coexistence in spectra, observed $B(E2)$ values, band crossings and so on in $^{76,72,74}$Se isotopes are well described by DSM $[26]$. Therefore, just as $^{78}$Kr studied using DSM in $[10]$, for $^{74}$Se $2\nu e^+\text{DBD}$ decay DSM is expected to be good. We have also verified that $^{74}$Ge spectroscopic properties are well described by DSM. For $2\nu e^+\text{DBD}$ half-lives calculations, we have first performed axially symmetric HF calculations and obtained the lowest prolate HF intrinsic states. The lowest HF single particle spectra for $^{74}$Se, $^{74}$Ge and $^{74}$As nuclei are shown in Figs. 1, 2 and 3 respectively. Only prolate intrinsic states are considered in these calculations and the oblate intrinsic states are ignored just as in the previous $^{78}$Kr analysis $[10]$ using DSM. The reason for neglecting the oblate states has been discussed in an earlier publication $[27]$. For these three nuclei, we found that the spectroscopic results obtained with only oblate states compare poorly with experiment and hence we did not include oblate states in the final calculation. We have also seen in the band mixing calculations that oblate states do not mix with prolate states significantly and hence they are not expected to affect our final results. By particle-hole excitations from the lowest intrinsic states shown in Figs. 1-3, excited configurations are generated. For $^{74}$Se ground state $0^+$, 10 intrinsic states with $K = 0^+$ are used for band mixing. Similarly 24 configuration with $K = 0^+$ for $^{74}$Ge and 65 configurations with $K = 1^+$ for $^{74}$As are employed. We have verified that these configurations are sufficient to provide adequate description of $2\nu e^+\text{DBD}$. Further increase in the number of configurations does not change the results significantly.

Using the wavefunctions generated by DSM, $2\nu \beta^+\text{EC}$ and ECEC half-lives for $^{74}$Se $\rightarrow ^{74}$Ge transitions are calculated and the results are shown in Table I. The integrated kinematical factors $G_{2\nu}(k, J_F)$ have been calculated following the prescription given by Doi and Kotani $[23]$. The limits for $\beta^+\text{EC}$ processes in $^{74}$Se were determined only recently, in the SuperNEMO project. Measurements of Se sample consisting of natural selenium powder using a 400 cm$^3$ HPGe detector resulted in the first $T_{1/2}$ limits to be $> 10^{18} - 10^{19}$ yr $[7]$ for $2\nu \beta^+\text{EC}$ and ECEC. The DSM results in Table I are the first theoretical estimates for the half-lives for positron double decay modes of $^{74}$Se and there does not exist any other model calculations. Let us recall here the statement in $[7]$: “It is necessary to stress that $^{74}$Se has never been investigated before and all results here are obtained for the first time. Neither has this isotope been investigated theoretically; thus there are no predictions with which to compare. Nevertheless, we will try to estimate the significance of the obtained results and the possibility to increase the sensitivity of this type of experiments in the future.”

IV. CONCLUSIONS

In this brief report, by extending our recent results for $^{78}$Kr $[10]$, we have presented results for positron double beta decay half lives for $^{74}$Se. They are obtained using the DSM model with a modified Kuo interaction in $(^2p_{3/2}, ^1f_{5/2}, ^2p_{1/2}, ^1g_{9/2})$ space. As spectroscopic properties of Se isotopes are well described by DSM, the half-lives calculated for $2\nu e^+\text{DBD}$ modes of $^{74}$Se, given in Table I can be taken as reliable predictions. The calculated half-life for the ECEC mode is $\sim 10^{28}$ yr and it may be possible to observe this in the near future with improved sensitivity of experiments.
Acknowledgments

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[1] M. Meyer, Phys. Rev. 48, 512 (1935).
[2] H. V. Klapdor-Kleingrothaus et al., Mod. Phys. Lett. A16, 2409 (2001); hep-ph/0103062.
[3] S.R. Elliott and P. Vogel, Annu. Rev. Nucl. Part. Sci. 52, 115 (2002).
[4] F.T. Avignone III, S.R. Elliott, and J. Engel, Rev. Mod. Phys. 80, 481 (2008).
[5] Ju. M. Gavriljuk et al, Phys. At. Nucl. 69, 2124 (2006); C. Sáenz et al, Phys. Rev. C 50, 1170 (1994).
[6] P. Belli et al., Physics Letter B658, 193 (2008); preprint ROM2F/2007/13.
[7] A.S. Barabash, Ph. Hubert, A. Nachab, and V. Umatov, Nucl. Phys. A785, 371 (2007).
[8] P. Belli et al., Astropart. Phys. 10, 115 (1999).
[9] R. Cerulli et al., Nucl. Instrum. Meth. Phys. Res. A525, 535 (2004).
[10] S. Mishra, A. Shukla, R. Sahu, and V.K.B. Kota, Phys. Rev. C 78, 024307 (2008).
[11] E.-W. Grewe et al., Phys. Rev. C 77, 064303 (2008).
[12] R. Sahu and S.P. Pandya, J. Phys. G 14, L165 (1988); Nucl. Phys. A548, 64 (1992); K.C. Tripathy and R. Sahu, J. Phys. G 20, 911 (1994); Int. J. Mod. Phys. E 11, 531 (2002).
[13] K. C. Tripathy and R. Sahu, Nucl. Phys. A597, 177 (1996).
[14] R. Sahu and S.P. Pandya, Nucl. Phys. A571, 253 (1994).
[15] R. Sahu and V.K.B. Kota, Phys. Rev. C 66, 024301 (2002); C 67, 054323 (2003); Eur. Phys. J. A 24, 5 (2005).
[16] S. Mishra, R. Sahu, and V.K.B. Kota, Prog. Theo. Phys. 118, 59 (2007).
[17] D.C. Zheng and L. Zamick, Phys. Lett. B266, 5 (1991); S.L. Tabor and J. Doring, Physica Scripta T56, 175 (1995); H. Sun et al, Phys. Rev. C 59, 655 (1999); B.S. Narasingh et al, Phys. Rev. C 75, 061301(R) (2007).
[18] T.S. Kosmas, A. Faessler, and R. Sahu, Phys. Rev. C 68, 054315 (2003).
[19] R. Sahu, K.H. Bhatt, and D.P. Ahalpara, J. Phys. G 16, 733 (1990).
[20] R. Sahu, F. Simkovic, and A. Faessler, J. Phys. G 25, 1159 (1999).
[21] M. Doi, T. Kotani, and E. Takasugi, Prog. Theor. Phys. 83, 1 (1985).
[22] T. Tomoda, Rep. Prog. Phys. 54, 53 (1991).
[23] M. Doi and T. Kotani, Prog. Theo. Phys. 87, 1207 (1992).
[24] F. Boehm and P. Vogel, Physics of Massive Neutrinos (Cambridge University Press, Cambridge, 1992).
[25] D.P. Ahalpara, K.H. Bhatt, and R. Sahu, J. Phys. G 11, 735 (1985).
[26] R. Sahu, D.P. Ahalpara, and S.P. Pandya, J. Phys. G 13, 603 (1987).
[27] K.C. Tripathy and R. Sahu, Nucl. Phys. A 597, 177 (1996).
[28] G. Audi, A. H. Wapstra, and C. Thibault, Nucl. Phys. A729, 337 (2003).
[29] J. K. Bohr et al., J. Phys. Chem. Ref. Data 34, 57 (2005).
TABLE I: Experimental limit on half-lives $T_{1/2}^{1/2}$ along with theoretical estimates in Deformed Shell Model and corresponding phase space factor $G_{2\nu}$ for possible decay modes for $^{74}\text{Se} \rightarrow ^{74}\text{Ge}$. The Q-values are taken from [28] and Abundance (P) values are from [29]. The range (a-b) given in parenthesis for the theoretical estimate of the half-life is given for $g_A/g_V = 1.261$ and 1 respectively.

| Transition   | Q-value (in keV) | P (in %) | Decay | $G_{2\nu}$ | $T_{1/2}^{1/2}$ (in yrs) | Expt. limit | Theory |
|--------------|------------------|----------|-------|------------|--------------------------|-------------|--------|
| $^{74}\text{Se} \rightarrow ^{74}\text{Ge}$ | 1209.7±0.6       | 0.89     | $\beta^+\text{EC}$ | $2.05 \times 10^{-29}$ | $> 1.9 \times 10^{18}$ [7] | $(14.99 - 37.9) \times 10^{18}$ |
|              |                  |          | $E\text{EC}$     | $2.63 \times 10^{-24}$ |             | $(7.56 - 19.12) \times 10^{25}$ |
|              |                  |          | $E\text{EC}$*    | $3.06 \times 10^{-27}$ | $> 7.7 \times 10^{18}$ [7] | $(15.55 - 39.32) \times 10^{30}$ |

* represents g.s. to $2^+_1$ state transition ($\Delta E = 595.8$keV).
FIG. 1: HF single particle spectrum for $^{74}$Se. In the figure circles represent protons and crosses represent neutrons. The Hartree-Fock energy ($E$) in MeV, mass quadrupole moment ($Q$) in units of the square of the oscillator length parameter and the total $K$ quantum number of the lowest intrinsic state are given in the figure.
FIG. 2: Same as Fig. 1 but for $^{74}$As.

$E = -25.4$
$Q = 25.6$
$K = 3^{-}$
FIG. 3: Same as Fig. 1 but for $^{74}$Ge.

$E = -17.5$
$Q = 27.3$
$K = 0^+$