Solution of Matrix Games with Generalised Trapezoidal Fuzzy Payoffs

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ABSTRACT

In this paper, it is shown that once linear ranking function is chosen, then the fuzzy matrix game (FMG) is converted into crisp one and then it is solved easily by the proposed method. The main aim of this paper is to reduce the computational complexity of the FMG using the ranking function. The applicability and feasibility of the FMG is illustrated with a numerical example. This paper finalises with the conclusion and to include an outlook for future study in this direction.

1. Introduction

Non-cooperative game theory considers conflict and competition between two decision makers. The decision makers are treated here as the players. A two-person zero-sum game is the simplest game involving two players, where the payoffs of the matrix games are precisely known. Then, player I wins and hereby the player II losses when two players choose the pure strategies $S_1 = \{\alpha_i : i = 1, 2, \ldots, m\}$ and $S_2 = \{\beta_j : j = 1, 2, \ldots, n\}$. Non-cooperative game theory is mainly used in economics, political science, business model, sports competitions and different types of contexts [1]. In practical situations, there are available some matrix games in which the payoffs are not known in precisely and have to be calculated even though two players unchanged their strategies. Hence, to tackle the imprecise payoff elements in matrix games, we incorporate the fuzzy numbers in our proposed study. Fuzzy sets are sets whose elements have the degree of membership function. The concept of fuzzy set theory was initiated by Zadeh [2]. It is widely used in various fields such as decision theory, expert systems, management science, computer science and operations research. Dubois and Prade [3] gave a summary on fuzzy sets. Bector and Chandra [4], Nishizaki and Sakawa [5, 6] made a good contribution on fuzzy matrix games and linear programming models and also updated research of this area. Bector et al. [7, 8] defined the fuzzy linear programming duality for the matrix games with fuzzy goal and fuzzy payoffs. Recently, Li and Nan [9] discussed about matrix games with payoffs of triangular intuitionistic fuzzy numbers. Li [10] studied on an effective methodology for solving matrix game with fuzzy payoffs. Campos [11], Campos and Gonzalez [12] and Campos et al. [13] discussed...
for solving the matrix games by using the proposed ranking function-based methods. Liu and Kao [14] viewed on the solution of fuzzy matrix games. Aggarwal et al. [15] studied on solving matrix game with l-fuzzy payoffs. Also, Roy [16] investigated on game theory under multi-criteria decision making and fuzzy set theory. Roy et al. [17] proposed a new solution concept in a credibilistic game. Roy and Mula [18–20] also analysed the matrix game in bi-fuzzy rough environment and bimatrix game in rough set environment. Mula et al. [21] and Roy [22] discussed on birough programming approach for solving bi-matrix games with birough payoff elements and fuzzy programming approach to two-person multicriteria bimatrix games, respectively. Roy and Mondal [23] studied on an approach to solve fuzzy interval valued matrix game. Also, Roy and Mula [24] discussed for analysing matrix games with rough payoffs using genetic algorithm. More recently, Bhaumik et al. [25] analysed a triangular intuitionistic fuzzy matrix game using a robust ranking technique. Deli and Cagman [26] discussed in detail about the probabilistic equilibrium solution of soft games.

Nowadays, we observe that in fuzzy decision making, ranking of the fuzzy numbers is a very important concept in research areas. From the recent literature survey, we see that many researchers, such as Chen and Chen [27–29], have attached their exertions by using generalised fuzzy numbers for solving real-life problem but to the first rate of our experience, to date no one has used generalised trapezoidal fuzzy numbers for solving the fuzzy matrix game (FMG).

In this paper, we present the ranking function which is converted to a matrix game with generalised trapezoidal fuzzy payoffs to reduce into crisp payoffs. Then, the FMG is solved by the methods such as Concept of dominance, Graphical method, Algebraic method and Simplex method. Here, it is pointed out that the methods are used for solving FMG by using linear ranking function which is simple and computationally more feasible. We apply the proposed ranking function in practical situations of matrix game as it is more comfortable to understand and to apply in decision-making problems.

The remainder of this paper is organised as follows. In Section 2, we briefly review some definitions and arithmetic operations of generalised trapezoidal fuzzy numbers. In Section 3, we incorporate a ranking function for converting generalised trapezoidal fuzzy numbers to the real numbers. In Section 4, we discuss about the matrix games with fuzzy payoffs with the applicability of the proposed ranking function are displayed using a numerical example. The accepted results are discussed in Section 5. Finally, conclusions and suggestions for future work are described in Section 6.

2. Preliminaries

Fuzzy set theory was introduced by Zadeh [2]. It is widely used in almost all areas to mathematically describe uncertainty and vagueness and to give formalised tools for dealing with the imprecision intrinsic to many problems. Here, we present some basic definitions of fuzzy set, fuzzy number, trapezoidal fuzzy number, generalised trapezoidal fuzzy number and the arithmetic operations of generalised trapezoidal fuzzy numbers.

**Definition 2.1 ([2]):** A fuzzy set $\tilde{A}$, defined on the universal set $X$ is the family $\tilde{A} = \{(x, \mu_{\tilde{A}}(x)) : x \in X\}$ where $\mu_{\tilde{A}} : X \rightarrow [0, 1]$ is the membership function such that $\mu_{\tilde{A}}(x) = 0$ if $x$ does not belong to $\tilde{A}$, $\mu_{\tilde{A}}(x) = 1$ if $x$ strictly belongs to $\tilde{A}$. 
**Definition 2.2 ([30]):** A fuzzy set \( \tilde{A} \), defined on the universal set of real numbers \( \mathbb{R} \), is said to be fuzzy number if its membership function has the following characteristics:

(i) \( \mu_{\tilde{A}}(x) : \mathbb{R} \rightarrow [0, 1] \) is continuous,
(ii) \( \mu_{\tilde{A}}(x) = 0 \) for all \( x \in (-\infty, a] \cup [d, \infty) \),
(iii) \( \mu_{\tilde{A}}(x) \) is strictly increasing on \([a,b]\) and strictly decreasing on \([c,d]\),
(iv) \( \mu_{\tilde{A}}(x) = w \), for all \( x \in [b, c] \), where \( 0 < w \leq 1 \).

**Definition 2.3 ([30]):** A fuzzy number \( \tilde{A} = (a, b, c, d) \) is said to be a trapezoidal fuzzy number if its membership function is defined as follows:

\[
\mu_{\tilde{A}}(x) = \begin{cases} 
\frac{x-a}{b-a}, & \text{if } a \leq x \leq b, \\
1, & \text{if } b \leq x \leq c, \\
\frac{x-d}{c-d}, & \text{if } c \leq x \leq d, \\
0, & \text{if elsewhere.}
\end{cases}
\]

**Definition 2.4 ([29]):** A fuzzy number \( \tilde{A} \), defined on the universal set of real numbers \( \mathbb{R} \), is said to be a generalised fuzzy number if its membership function has the following characteristics:

(i) \( \mu_{\tilde{A}}(x) : \mathbb{R} \rightarrow [0, w] \) is continuous,
(ii) \( \mu_{\tilde{A}}(x) = 0 \) for all \( x \in (-\infty, a] \cup [d, \infty) \),
(iii) \( \mu_{\tilde{A}}(x) \) is strictly increasing on \([a,b]\) and strictly decreasing on \([c,d]\),
(iv) \( \mu_{\tilde{A}}(x) = w \), for all \( x \in [b, c] \), where \( 0 < w \leq 1 \).

**Definition 2.5 ([29]):** A fuzzy number \( \tilde{A} = (a, b, c, d; w) \) is said to be a generalised trapezoidal fuzzy number if its membership function is stated as shown below:

\[
\mu_{\tilde{A}}(x) = \begin{cases} 
\frac{w}{b-a} \cdot \frac{x-a}{b-a}, & \text{if } a \leq x \leq b, \\
w, & \text{if } b \leq x \leq c, \\
\frac{w}{c-d} \cdot \frac{x-d}{c-d}, & \text{if } c \leq x \leq d, \\
0, & \text{if elsewhere.}
\end{cases}
\]

Here, \( w \) is the weight function and \( 0 < w \leq 1 \). The graphical view of generalised trapezoidal fuzzy number is shown in Figure 1.

**Arithmetic operations:**

Here, the arithmetic operations between two generalised trapezoidal fuzzy numbers, defined on universal set of real numbers \( \mathbb{R} \), are presented. Let \( \tilde{A}_1 = (a_1, b_1, c_1, d_1; w_1) \) and \( \tilde{A}_2 = (a_2, b_2, c_2, d_2; w_2) \) be the generalised trapezoidal fuzzy numbers, then

(i) \( \tilde{A}_1 \oplus \tilde{A}_2 = (a_1 + a_2, b_1 + b_2, c_1 + c_2, d_1 + d_2; \min\{w_1, w_2\}) \),
(ii) \( \tilde{A}_1 \oplus \tilde{A}_2 = (a_1 - d_2, b_1 - c_2, c_1 - b_2, d_1 - a_2; \min\{w_1, w_2\}) \) for all \( x \in (-\infty, a] \cup [d, \infty) \).

(iii) \( \tilde{A}_1 \otimes \tilde{A}_2 \simeq (u, v, r, s; \min\{w_1, w_2\}) \), where \( u = \min(a_1 a_2, a_1 d_2, a_2 d_1, d_1 d_2) \), \( v = \min(b_1 b_2, b_1 c_2, c_1 b_2, c_1 c_2) \), \( r = \max(b_1 b_2, b_1 c_2, c_1 b_2, c_1 c_2) \), \( s = \max(a_1 a_2, a_1 d_2, a_2 d_1, d_1 d_2) \).

(iv) \( \lambda \tilde{A}_1 = \begin{cases} \langle \lambda a_1, \lambda b_1, \lambda c_1, \lambda d_1; w_1 \rangle & \text{if } \lambda \geq 0, \\ \langle \lambda d_1, \lambda c_1, \lambda b_1, \lambda a_1; w_1 \rangle & \text{if } \lambda < 0. \end{cases} \)

Here, the symbols \( \oplus, \ominus, \otimes \) and \( \simeq \) are denoted as addition, subtraction, multiplication and similar or equal to of the fuzzy numbers, respectively.

### 3. Ranking function

Ranking of fuzzy numbers is one of the fundamental problems of fuzzy arithmetic and fuzzy decision making. In fuzzy multi-attribute decision making, the ratings and decision-maker’s risk performance are calculated based on fuzzy numbers. When the information is disclosed as a fuzzy number, it is necessary to calculate and compare fuzzy data before a decision is made. Therefore, ranking of fuzzy numbers is a useful approach for tackling decision making problem. Many methods have been introduced to deal with the ranking of fuzzy numbers. However, the methods have never been simplified and tested in real-life applications. In the recent decades, many researchers have investigated various ranking functions. So, we can see that the task in ranking of fuzzy numbers is very important in the research topic of fuzzy decision making. Fuzzy number must be ranked before an action is taken by a decision maker. By ranking of fuzzy numbers, we can rank alternatives and find the best one from them. In this paper, we define a ranking function, \( \mathcal{R}(\tilde{A}) \) for generalised trapezoidal fuzzy number, \( \tilde{A} = (a, b, c, d; w) \) as follows:

\[
\mathcal{R}(\tilde{A}) = w \cdot \frac{2a_1 + b_1 + c_1 + 2d_1}{6}.
\]

Since ranking function \( \mathcal{R} \) is an approach for comparing two generalised trapezoidal fuzzy numbers \( \tilde{A}_1 = (a_1, b_1, c_1, d_1; w_1) \) and \( \tilde{A}_2 = (a_2, b_2, c_2, d_2; w_2) \) is defined as \( \mathcal{R} : F(\mathbb{R}) \to \mathbb{R} \).
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where \( F(\mathbb{R}) \) is a set of fuzzy numbers defined on the universal set of real numbers which map each fuzzy number into the real line. Thus specific ranking of fuzzy numbers is an important procedure for decision making in a fuzzy environment and generally has become one of the main problems in fuzzy set theory. Then

(i) \( \tilde{A}_1 \preceq \tilde{A}_2 \), if and only if \( \Re(\tilde{A}_1) = w \cdot \left( (2a_1 + b_1 + c_1 + 2d_1) / 6 \right) \leq \Re(\tilde{A}_2) = w \cdot \left( (2a_2 + b_2 + c_2 + 2d_2) / 6 \right) \), \( w = \min\{w_1, w_2\} \),

(ii) \( \tilde{A}_1 \succeq \tilde{A}_2 \), if and only if \( \Re(\tilde{A}_1) = w \cdot \left( (2a_1 + b_1 + c_1 + 2d_1) / 6 \right) \geq \Re(\tilde{A}_2) = w \cdot \left( (2a_2 + b_2 + c_2 + 2d_2) / 6 \right) \), \( w = \min\{w_1, w_2\} \),

(iii) \( \tilde{A}_1 \cong \tilde{A}_2 \), if and only if \( \Re(\tilde{A}_1) = w \cdot \left( (2a_1 + b_1 + c_1 + 2d_1) / 6 \right) = \Re(\tilde{A}_2) = w \cdot \left( (2a_2 + b_2 + c_2 + 2d_2) / 6 \right) \), \( w = \min\{w_1, w_2\} \).

where the symbols \( \preceq, \succeq \), and \( \cong \) are chosen as fuzzy relations.

4. Matrix games with fuzzy payoffs

In the traditional game problem, it is assumed that the decision makers have an idea of how payment should be made at the end of the table. In real-world problems, the payoffs may not be known precisely by the decision makers. Then some sorts of uncertainty arise about the payoffs. Therefore, the fuzzy set theory is applied to accommodate such types of matrix game problems. Here, we consider a matrix game \( A = (a_{ij})_{m \times n} \) and assume that \( S_1 = \{\alpha_i : i = 1, 2, \ldots, m\} \) and \( S_2 = \{\beta_j : j = 1, 2, \ldots, n\} \) are the sets of pure strategies of the players I and II, respectively. Also let \( X \) and \( Y \) be the mixed strategies for the players I and II, respectively, where \( X = \{x_i : i = 1, 2, \ldots, m\} \) and \( Y = \{y_j : j = 1, 2, \ldots, n\} \). Then a simple two-person zero-sum matrix game can be expressed with the triplet \( G = < X, Y, A > \).

Here, the existing methods, such as Concept of dominance, Graphical method, Algebraic method and Simplex method, are used to calculate the optimal strategy and the value of the matrix game when the payoffs are generalised trapezoidal fuzzy numbers. Assume that a matrix game with generalised trapezoidal fuzzy payoffs is given in Table 1.

First, we define the rank of each generalised trapezoidal fuzzy number by using the proposed ranking function. This supports us to reduce the payoff matrix with generalised trapezoidal fuzzy payoffs into an equivalent crisp game problem and we solve it by the existing methods. So, the corresponding crisp game problem is defined in Table 2.

### Table 1. Generalised trapezoidal fuzzy payoff matrix.

|       | \( B_1 \)          | \( B_2 \)          | \( B_3 \)          |
|-------|-------------------|-------------------|-------------------|
| \( A_1 \) | (1,3,8,18;0.5)    | (1,2,4,8;0.4)     | (2,4,6,18;0.5)    |
| \( A_2 \) | (7,8,11,25;0.5)   | (3,4,7,11;0.3)    | (6,8,12,26;0.4)   |
| \( A_3 \) | (10,11,19,26;0.5) | (0,5,9,14;0.8)    | (3,4,7,10;0.6)    |

### Table 2. Reduced crisp payoff matrix.

|       | \( B_1 \) | \( B_2 \) | \( B_3 \) |
|-------|-----------|-----------|-----------|
| \( A_1 \) | 2.45      | 1.20      | 2.50      |
| \( A_2 \) | 4.15      | 1.95      | 4.20      |
| \( A_3 \) | 5.10      | 2.10      | 1.85      |
Let A be the maximising player and B be the minimising player. Here, max (row min) = 1.95 and min (column max) = 2.10. So, the game problem has no saddle point because the maximin is not equal to the minimax. Therefore, we find the optimal strategy and the value of the game by calculating the mixed strategies. Existing methods, such as Concept of dominance, Graphical method, Algebraic method and Simplex method, are chosen to the reduced payoff matrix (i.e. shown in Table 2) for obtaining the optimal strategies and value of the game.

**Concept of dominance:**
By applying the concept of dominance to diminish payoff matrix, i.e. in Table 2 which becomes a $2 \times 2$ payoff matrix and is shown in Table 3.

If $x_2$ and $x_3$ are the probabilities of player I, i.e. the mixed strategy of player I in which $A_2$ and $A_3$ are played and similarly $y_2$ and $y_3$ are the probabilities in which $B_2$ and $B_3$ are played for the optimal solution, then we derive the mixed strategies and the value of the game by using the procedure for solving $2 \times 2$ rectangular game. Hence, the optimal solution to the original game problem is for player I (0.00, 0.10, 0.90), for player II (0.00, 0.94, 0.06) and the value of the game is 2.085.

**Graphical method of solution:**
Solving the reduced crisp payoff matrix by graphically, i.e. Table 2 converts to the corresponding $2 \times 2$ payoff matrix which is shown in Table 4.

Hence, we get the points which maximise the minimum expected gain of player II. The highest point M (maximum) of this bound refers to the maximum of minimum gains. We obtain maximin points using the thick line which is depicted in Figure 2. At this point, it is obvious, player II has used his course of actions and they are $B_2$ and $B_3$ as it is the point of intersection of $L_2$ and $L_3$. Therefore, the $2 \times 2$ submatrix which provides the optimal strategies and the value of the game. Now, solving this $2 \times 2$ subgame, we obtain the optimal strategies for player I as (0.00, 0.10, 0.90) and for player II as (0.00, 0.94, 0.06) and the value of the game is 2.085.

**Algebraic method of solution:**
Here, we use an algebraic method to shorten payoff matrix (i.e. in Table 2) to find the optimal strategies and the value of the game. Let $(x_1, x_2, x_3)$ and $(y_1, y_2, y_3)$ be the probabilities of the optimal strategy of players I and II, respectively, of the reduced payoff matrix whose value is supposed to be $v$. Player I expects at least a gain $v$, so we have

\[ 2.45x_1 + 4.15x_2 + 5.10x_3 \geq v, \]  

\[ (2) \]

| Table 3. Dominated payoff matrix. |
|-----------------------------------|
|       | $B_2$ | $B_3$ |
| $A_2$ | 1.95  | 4.20  |
| $A_3$ | 2.10  | 1.85  |

| Table 4. Corresponding payoff matrix. |
|--------------------------------------|
|       | $B_2$ | $B_3$ |
| $A_2$ | 1.95  | 4.20  |
| $A_3$ | 2.10  | 1.85  |
Figure 2. Graphical view of maximin point.

\[ 1.20x_1 + 1.95x_2 + 2.10x_3 \geq v, \quad (3) \]

\[ 2.50x_1 + 4.20x_2 + 1.85x_3 \geq v, \quad (4) \]

and for player II to expect at most a loss \( v \), therefore,

\[ 2.45y_1 + 1.20y_2 + 2.50y_3 \leq v, \quad (5) \]

\[ 4.15y_1 + 1.95y_2 + 4.20y_3 \leq v, \quad (6) \]

\[ 5.10y_1 + 2.10y_2 + 1.85y_3 \leq v. \quad (7) \]

Furthermore,

\[ x_1 + x_2 + x_3 \leq 1, \quad (8) \]

\[ y_1 + y_2 + y_3 \leq 1, \quad (9) \]

\[ x_1, x_2, x_3 \geq 0, \quad (10) \]

\[ y_1, y_2, y_3 \geq 0. \quad (11) \]

Without loss of generality, all inequations (2)–(9) are considered as equations to be stated as below.

\[ 2.45x_1 + 4.15x_2 + 5.10x_3 = v, \quad (12) \]
1.20x_1 + 1.95x_2 + 2.10x_3 = v, \quad (13)
\quad 2.50x_1 + 4.20x_2 + 1.85x_3 = v, \quad (14)
\quad 2.45y_1 + 1.20y_2 + 2.50y_3 = v, \quad (15)
\quad 4.15y_1 + 1.95y_2 + 4.20y_3 = v, \quad (16)
\quad 5.10y_1 + 2.10y_2 + 1.85y_3 = v, \quad (17)
\quad x_1 + x_2 + x_3 = 1, \quad (18)
\quad y_1 + y_2 + y_3 = 1. \quad (19)

Now, we have eight equations (12)– (19) with seven unknowns \( x_1, x_2, x_3, y_1, y_2, y_3 \) and \( v \). These linear equations can be solved to find the values of the variable. In Equations (16) and (17), we set \( y_1 = 0 \) then solving the equation, we have \( y_1 = 0.00, y_2 = 0.94, y_3 = 0.06 \). Similarly we have \( x_1 = 0.00, x_2 = 0.10, x_3 = 0.90 \). Therefore, the optimal solution of the game problem is given as for player I (0.00, 0.10, 0.90) and for player II (0.00, 0.94, 0.06) and the value of the game is 2.085.

**Simplex method of solution:**

It is well known that every two-person zero-sum matrix game is reduced to two linear programming problems which are dual with one another. If any one of these solved, then the solution of other can be calculated by the method of duality. If \( (y_1, y_2, y_3) \) are the probabilities of the optimal strategies of player II that are played. We are to find the optimal quantities by the following problem as follows:

**Model 1**

\[
\begin{align*}
\text{minimise} \quad & v \\
\text{subject to} \quad & 2.45y_1 + 1.20y_2 + 2.50y_3 \leq v, \\
& 4.15y_1 + 1.95y_2 + 4.20y_3 \leq v, \\
& 5.10y_1 + 2.10y_2 + 1.85y_3 \leq v, \\
& y_1 + y_2 + y_3 = 1, \\
& y_j \geq 0 \quad j = 1, 2, 3.
\end{align*}
\]

Putting \( Y_j = \frac{y_j}{v} \), \( j = 1, 2, 3 \), in Model 1, we can state the Linear Programming Problem (LPP) is indicated as given below.

**Model 2**

\[
\begin{align*}
\text{maximise} \quad & \frac{1}{v} = Y_1 + Y_2 + Y_3 \\
\text{subject to} \quad & 2.45Y_1 + 1.20Y_2 + 2.50Y_3 \leq 1, \\
& 4.15Y_1 + 1.95Y_2 + 4.20Y_3 \leq 1, \\
& 5.10Y_1 + 2.10Y_2 + 1.85Y_3 \leq 1, \\
& Y_j \geq 0 \quad j = 1, 2, 3.
\end{align*}
\]

Solving Model 2 by the simplex method, we obtain \( Y_1 = 0.000, Y_2 = 0.450, Y_3 = 0.028, v = 2.085 \). From the relation \( y_j = \frac{v}{v}Y_j \) for \( j = 1, 2, 3 \), we have \( y_1 = 0.00, y_2 = 0.94, y_3 = 0.06 \).
Again, we assume that $(x_1, x_2, x_3)$ are the probabilities of the optimal strategies with which player I plays. We are to calculate the optimal quantities by the following problem as follows.

**Model 3**

\[
\begin{align*}
\text{maximise} & \quad v \\
\text{subject to} & \quad 2.45x_1 + 4.15x_2 + 5.10x_3 \geq v, \\
& \quad 1.20x_1 + 1.95x_2 + 2.10x_3 \geq v, \\
& \quad 2.50x_1 + 4.20x_2 + 1.85x_3 \geq v, \\
& \quad x_1 + x_2 + x_3 = 1, \\
& \quad x_i \geq 0 \quad i = 1, 2, 3.
\end{align*}
\]

Dividing by $v$ and substituting $X_i = x_i/v$, $i = 1, 2, 3$, in Model 3, we can define the LPP as follows.

**Model 4**

\[
\begin{align*}
\text{minimise} & \quad \frac{1}{v} = X_1 + X_2 + X_3 \\
\text{subject to} & \quad 2.45X_1 + 4.15X_2 + 5.10X_3 \geq 1, \\
& \quad 1.20X_1 + 1.95X_2 + 2.10X_3 \geq 1, \\
& \quad 2.50X_1 + 4.20X_2 + 1.85X_3 \geq 1, \\
& \quad X_i \geq 0 \quad i = 1, 2, 3.
\end{align*}
\]

Using the simplex method in Model 4, we obtain $X_1 = 0.000$, $X_2 = 0.047$, $X_3 = 0.431$, $v = 2.085$. From the relation $x_i = vX_i$ for $i = 1, 2, 3$, we have $x_1 = 0.00$, $x_2 = 0.10$, $x_3 = 0.90$. Therefore, the optimal strategy for player I is $(0.00, 0.10, 0.90)$ and for player II $(0.00, 0.94, 0.06)$ and value of the game is 2.085.

It is expensive to note that the same optimal solution can be found by using all the existing methods by considering the proposed ranking function.

5. Results and discussions

In this paper, the main improvements of ranking function by using existing methods are experimented. If we can use any other linear ranking function, and even though the accepted solution may not be the same, but the results are at rest valid for the new solution. In fact, building on the ranking index to be applied by the decision maker, the algorithm can return a set of optimal solutions or a unique solution. For example, we adopt that a linear ranking function, $\mathcal{R}^\lambda$ is applied for the generalised trapezoidal fuzzy number $\tilde{A} = (a, b, c, d; w)$ as follows:

$$\mathcal{R}^\lambda(\tilde{A}) = w \left[ (1 - \lambda)a + \lambda d + \frac{1}{3}((1 - \lambda)(b - a) + \lambda(c - d)) \right], \quad (20)$$

where $\lambda \in [0, 1]$ is the optimism index reflecting the optimism degree of a decision maker. The larger $\lambda$ is more optimistic. The two extreme cases are $\lambda = 0$, the decision maker is
completely pessimistic; and \( \lambda = 1 \), the decision maker is completely optimistic. It can be easily confirmed that the ranking function (1) is a particular case of the modified linear ranking function (20) when \( \lambda = \frac{1}{2} \). Solving the FMG based on modified linear ranking function, given for \( \lambda = 1 \), by using the existing methods, we have the optimal solution for player I (0.00, 0.22, 0.78) and that for player II (0.00, 0.82, 0.18) and the value of the game is 3.52.

This solution occurs based on the point of view of a completely optimistic decision maker. However, the accepted optimal solution based on the linear ranking function is not same with the optimal solution given by the ranking function. But the result is at rest valid for this solution yet. This result ensures that if we want to solve an FMG depending on linear ranking function, then the existing methods are easily applied to solve the real-life problems which are considered by decision makers.

Let us experiment the main advantages of the proposed ranking function.

(i) The proposed ranking function is used to solve FMG using existing methods. Because, the proposed ranking function is very easy to understand and to apply for solving the fuzzy game problems appearing in real-life situations.

(ii) There are several papers available in the literature to solve the FMG, where the researchers used arithmetic operations on generalised fuzzy numbers. But, if the linear ranking function is used for the same, then there is no necessity to use arithmetic operations of real numbers.

(iii) In addition, it is feasible to take a specific ranking index for comparing the fuzzy numbers engaged in the FMG, in such a way that every time in which the decision maker needs to solve FMG under consideration he/she can propose the ranking index that excellent case in FMG.

6. Conclusion and future work

Non-cooperative game theory can be provided an idea for modelling and analysing the problems that the strategy of one player depends on the strategy of another player. In the literature, of the various types of games, matrix games with crisp payoffs and fuzzy payoffs have been studied widely. Although few of these problems have been fixed in real-world applications, the traditional game problems in generally assume crisp payoffs. Inversely to the traditional game problems, we have discussed imprecise payoffs in the real-life problems and have displayed another way that is easy and yet addressed these drawbacks in existing papers. Here, we have considered a matrix game with generalised trapezoidal fuzzy payoffs; and we have solved the FMG with the help of ranking function. We have also accommodated more information on FMG in practical situations by considering generalised trapezoidal fuzzy numbers than the fuzzy numbers. To explain the proposed ranking function, we have solved the FMG then the value of the game will be increased as compared to any other raking functions. Hence, the maximising player gains the maximum profits and the minimising player minimises his maximum losses. Another advantage of the proposed study is that it is more easy to capture with less computational efforts for solving the FMG.

The proposed ranking function can be used to find soft saddle points, soft lower and soft upper values, soft dominated strategies and soft value of the game when the matrix games consider the soft elements from soft set in connection with the practical situations.
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