125 GeV Higgs Boson From Gauge-Higgs Unification: A Snowmass white paper

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Abstract

In certain five dimensional gauge theories compactified on the orbifold $S^1/Z_2$ the Standard Model Higgs doublet is identified with the zero mode of the fifth component of the gauge field. This gauge-Higgs unification scenario is realized at high energies, and the Standard Model as an effective theory below the compactification scale satisfies the boundary condition that the Higgs quartic coupling vanishes at the compactification scale (gauge-Higgs condition). This is because at energies above the compactification scale, the five dimensional gauge invariance is restored and the Higgs potential vanishes as a consequence. We consider scenario where top quark Yukawa and weak gauge coupling unification can be realized and identify the compactification scale as one at which this two coupling couplings have the same value. Taking into account the experimental uncertainties in measurements of the top quark mass and the QCD coupling constant, the Higgs mass prediction of 119-126 GeV from the gauge-Higgs unification scenario is consistent with the experimentally measured value of 125-126 GeV. More precise measurements of the top quark mass and the QCD coupling constant are crucial to reduce the interval of the Higgs mass prediction and thereby test the feasibility of the gauge-Higgs unification scenario.

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A major goal of the physics program at the Large Hadron Collider is to confirm the origin of the electroweak symmetry breaking and the mechanism of particle mass generation. Both the ATLAS [1] and CMS [2] experiments have discovered, it seems, the Higgs(-like) boson with a mass of around 125 GeV through a variety of Higgs boson decay modes. With the discovery of the Higgs boson, the experimental tests of the Higgs sector of the Standard Model (SM) have just started.

In the SM, the Higgs boson mass is determined by the quartic coupling of the Higgs doublet, so that the measured Higgs boson mass provides us information of the Higgs quartic coupling constant at the electroweak scale. This information has a great impact on the SM as well as new physics beyond the SM which make predictions of the Higgs boson mass. For example, a mass of around 125 GeV is very interesting from the viewpoint of the vacuum stability bound on the Higgs boson mass [3]. If the Higgs quartic coupling corresponding to 125 GeV is extrapolated to high energies according to the renormalization group equations of the SM, the coupling becomes negative at an intermediate scale of around $10^9 - 10^{11}$ GeV, depending on experimental uncertainties in top quark mass and the QCD coupling constant. Thus, the electroweak vacuum is not stable.

This vacuum instability may indicate that new physics beyond the SM appears at or below the intermediate scale, which modifies the renormalization group equations in the SM and prevents the running Higgs quartic coupling from becoming negative up to the Planck mass [4]. In the following, we consider a five dimensional gauge-Higgs unification scenario [5] (see also [6]), compactified on the orbifold $S^1/Z_2$. Through compactification and non-trivial parity assignments for the five dimensional gauge fields, the original gauge group is broken to the SM gauge group, with the Higgs doublet identified with the fifth component of the gauge field. In the four dimensional effective theory, the five dimensional gauge invariance requires a vanishing Higgs potential at tree level. The Higgs potential of the SM is generated below the compactification scale through quantum effects associated with the Kaluza-Klein modes of bulk fields.

Below the compactification scale, all Kaluza-Klein modes are decoupled and the SM is realized as an effective theory at low energies. Although some effort is needed to construct a realistic scenario (see for example [7]), the most important property of the gauge-Higgs unification scenario is encoded in the Higgs quartic coupling. Namely, the Higgs quartic coupling should vanish once the higher dimensional gauge invariance gets restored. In fact, it has been explicitly shown [8] that the effective Higgs quartic coupling calculated in a given gauge-Higgs unification model coincides with the one generated through the renormalization group equations of the SM with a boundary condition that the Higgs quartic coupling vanishes at the compactification scale. Applying this gauge-Higgs condition, we can predict the Higgs boson

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mass as a function of the compactification scale [9].

Our strategy to evaluate the Higgs boson mass is practically the same as the calculation of the instability bound on the Higgs boson mass if we identify the compactification scale with a cutoff for the SM. However, considering that the gauge and Yukawa interactions are unified in the gauge-Higgs unification scenario, we naturally determine the compactification scale at which the running SU(2) gauge and top quark Yukawa couplings are unified. As we will see later, such a compactification scale is found to be around $10^9$ GeV, and therefore the prediction of the gauge-Higgs unification scenario for the Higgs boson mass is compatible with the observed Higgs boson mass of around 125 GeV.

We first give a brief review of the gauge-Higgs unification scenario. Since our purpose is to show the basic structure of the gauge Higgs unification scenario, we only consider a simple toy model based on an SU(3) gauge group with one SU(3) triplet bulk fermion ($\Psi$). The fifth dimension is compactified on the $S^1/Z_2$ orbifold. The model Lagrangian is expressed as

$$\mathcal{L} = -\frac{1}{2} \text{Tr}(F_{MN}F^{MN}) + i\bar{\Psi}\Gamma^MD_M\Psi,$$

where $M, N = 0, 1, 2, 3, 5$, the gamma matrix in 5-dimensional theory is defined as $\Gamma^M = (\gamma^\mu, i\gamma^5)$, $F_{MN} = \partial_M A_N - \partial_N A_M - ig_5 [A_M, A_N]$ with $A_M = A_M^a \lambda^a$ ($\lambda^a$: Gell-Mann matrices), the covariant derivative $D_M = \partial_M - ig_5 A_M$, and $\Psi = (\psi_1 \psi_2 \psi_3)^T$. While the periodic boundary condition along $S^1$ is imposed for all fields, non-trivial $Z_2$ parities are assigned for each field using the orbifolding matrix $P = \text{diag}(-, -, +)$ such as

$$A_\mu(-y) = PA_\mu(y)P^{-1}, \quad A_5(-y) = -PA_5(y)P^{-1}, \quad \Psi(-y) = P\gamma^5\Psi(y),$$

where $y$ is the fifth coordinate, and ± means even/odd $Z_2$ parities. With these boundary conditions, the SU(3) gauge symmetry is broken to the electroweak gauge group SU(2)×U(1).

The zero mode of the SU(3) gauge bosons is decomposed into the electroweak gauge boson of the SM and the Higgs doublet such that

$$A^{(0)}_\mu = \frac{1}{2} \begin{pmatrix} W^3_\mu + \frac{1}{\sqrt{3}} B_\mu & \sqrt{2} W^+_\mu & 0 \\ \sqrt{2} W^-_\mu & -W^3_\mu + \frac{1}{\sqrt{3}} B_\mu & 0 \\ 0 & 0 & -2\sqrt{3}B_\mu \end{pmatrix}, \quad A^{(0)}_5 = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 & 0 & h^+ \\ 0 & 0 & h^0 \\ h^- & h^{0*} & 0 \end{pmatrix},$$

where $W^3_\mu$, $W^\pm_\mu$ and $B_\mu$ are respectively the SU(2) and U(1) gauge bosons of the SM and $h = (h^+, h^0)^T$ is the Higgs doublet field.\footnote{Note that the toy model predicts an unrealistic weak mixing angle, $\sin^2 \theta_W = \frac{4}{3}$. This problem is ameliorated by introducing an additional U(1) gauge interaction and extending the toy model to a model based on SU(3)×U(1)$'$ [7], realizing the Z boson as a combination of $B_\mu$ and the extra U(1)$'$ gauge boson.} Substituting the zero mode expressions in the first
term in Eq. (1), we obtain the kinematic terms for the $SU(2) \times U(1)$ gauge bosons and the Higgs doublet with the correct covariant derivative, but no Higgs potential.

The zero mode of the bulk fermion is decomposed into an $SU(2)$ doublet left-handed fermion and an $SU(2)$ singlet right-handed fermion. Their $U(1)$ charges are the same as those of the quark $SU(2)$ doublet and the $SU(2)$ singlet down-type quark in this toy model. The gauge and Yukawa interactions of the fermion originate from the higher dimensional gauge interaction and as a result, the $SU(2)$ gauge coupling constant is the same as the Yukawa coupling constant at the compactification scale. This relation may be suitable for the weak boson and top quark masses, since their masses are of the same order of magnitude. As we will see below, the $SU(2)$ gauge and top Yukawa couplings are unified at an intermediate scale and this scale is naturally identified with the compactification scale [9].

![Graph](image1.png)

**Figure 1:** (Left panel) Renormalization group running of the $SU(2)$ gauge coupling (shaded in black) and top Yukawa coupling (shaded in red) for various values of inputs in the range of $0.1179 \leq \alpha_3(m_Z) \leq 0.1193$ and $172.24 \text{ GeV} \leq M_t \leq 174.12 \text{ GeV}$. They unify at $\mu \simeq 10^9 \text{ GeV}$, which is identified with the compactification scale. (Right panel) Predicted Higgs boson mass in the gauge-Higgs unification scenario for various values of the top quark pole mass ($172.24 \leq M_t/\text{GeV} \leq 174.12$) and three different values of the QCD coupling constant, $\alpha_3(m_Z) = 0.1179$ (solid line), 0.1186 (dashed line) and 0.1193 (dotted line).

In our analysis we employ the renormalization group equations of the SM at the two loop level [10]. Since the resultant Higgs boson mass is sensitive to the top quark pole mass ($M_t$) and the QCD coupling constant $\alpha_3(M_Z)$ at the $Z$ boson mass scale, we take into account experimental uncertainties in measuring these values: $M_t = 173.18 \pm 0.94$ [11] and $\alpha_3(m_Z) = 0.1186 \pm 0.0007$ [12]. For the $SU(2)$ and $U(1)$ gauge couplings of the SM, we have used $\alpha_1(m_Z) = 0.01618$ and $\alpha_2(m_Z) = 0.03354$. For various values of inputs in the range of $5^\text{th}$ order in the $SU(3) \times U(1)'$ extension, the fermion zero mode can be identified with the top quark by a suitable $U(1)'$ charge assignment.

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5 In the $SU(3) \times U(1)'$ extension, the fermion zero mode can be identified with the top quark by a suitable $U(1)'$ charge assignment.
\[172.24 \leq M_t/\text{GeV} \leq 174.12\] and \[0.1179 \leq \alpha_3(m_Z) \leq 0.1193\], the running top Yukawa coupling (shaded in red) and SU(2) gauge coupling (shaded in black) are depicted in Fig. 1 (left panel). The two couplings unify at \(\mu \approx 10^9\) GeV. As \(M_t\) is raised (\(\alpha_3(m_Z)\) is lowered), the unification scale becomes slightly larger.

For fixed values of \(M_t\) and \(\alpha_3(m_Z)\), we first determine the compactification scale as the merger point of the running top Yukawa coupling and SU(2) gauge coupling. Imposing the gauge-Higgs condition at this compactification scale, we run the Higgs quartic coupling down to low energies. The pole mass of the Higgs boson is evaluated as a solution to the matching condition between the running mass and physical mass [13]. For various values of \(M_t\) and \(\alpha_3(m_Z)\), the Higgs boson mass is shown in Fig. 1 (right panel). Three lines from top to bottom corresponds to input values of QCD coupling as \(\alpha_3(m_Z) = 0.1179\) (solid), 0.1186 (dashed) and 0.1193 (dotted), respectively. As shown in the figure, the Higgs boson mass prediction by the gauge-Higgs unification scenario is compatible with the measured Higgs boson mass of around 125 GeV within the experimental uncertainties in measurements of the top quark pole mass and the QCD coupling constant. Larger (smaller) values for \(M_t\) (\(\alpha_3(m_Z)\)) are preferable. More precise measurement of the Higgs boson mass along with top quark pole mass and the QCD coupling constant is therefore crucial for testing the Higgs boson mass prediction by the gauge-Higgs unification scenario.

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