Effective nucleon mass in relativistic mean field theory

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In the $\sigma$-$\omega$-$\rho$ model of the relativistic mean field theory with nonlinear $\sigma$-meson self-interaction, the effective nucleon mass $M^*$ is discussed with relation to the symmetry incompressibility $K_s$ of nuclear matter, based on the model parameters fitted to nuclear matter properties. It is shown that $M^*$ is larger than 0.73$M$, if $K_s$ is assumed to be negative and the nuclear matter incompressibility $K_0$ is kept less than 300$MeV$. Furthermore, the field system is shown to be stable, as the $\sigma$-meson self-interaction energy is lower bounded in this parameter region.

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As starting point for the relativistic microscopic description of the nuclear many-body system, within the framework of quantum hadrodynamics, the well-studied $\sigma$-$\omega$-$\rho$ model with nonlinear $\sigma$-meson self-interaction has been proved to be able to describe the saturation and other properties of nuclear matter ¹. However, there is some controversy about the stability of the field system specified by the fitted parameters, as the fitted coefficient of the fourth power term of the $\sigma$-meson field in the Lagrangian density is negative, and thus the energy of the field system diverges for large $\sigma$-field ² ³. In addition, the symmetry incompressibility $K_s$ of nuclear matter determined by the parameter sets existing in the market is positive, which is opposite to that given by the nonrelativistic models of nuclei ³. Furthermore, the effective nucleon mass $M^*$ is around 0.6$M$ which seems uncomfortably low. Then, the relevant question is: are these features manifestation of intrinsic properties of the model itself, or do they depend on the input data used for fitting the model parameters? The purpose of this rapid communication is to show that the stable result with negative symmetry incompressibility $K_s$ and larger effective nucleon mass $M^*$ can be obtained, if the model parameters are fitted to reasonable nuclear matter properties.

The $\sigma$-$\omega$-$\rho$ model of the relativistic mean field theory is specified by the following Lagrangian density ¹ (we use natural units with $\hbar = c = 1$):

$$\mathcal{L} = \overline{\psi} [i \gamma_{\mu} (\partial^\mu - g_{\omega} \omega^\mu - g_{\rho} \tau \cdot \mathbf{b}^\mu) - (M - g_{\sigma} \phi)] \psi$$

$$+ \frac{1}{2} (\partial^\mu \phi \partial_{\mu} \phi - m_{\sigma}^2 \phi^2) - \frac{1}{3} Mb(g_{\sigma} \phi)^3 - \frac{1}{4} c(g_{\sigma} \phi)^4$$

$$- \frac{1}{4} F_{\mu\nu} F^{\mu\nu} + \frac{1}{2} m_{\omega}^2 \omega_{\mu} \omega^\mu$$

$$- \frac{1}{4} B_{\mu\nu} B^{\mu\nu} + \frac{1}{2} m_{\rho}^2 b_{\mu} b^\mu,$$

(1)

where $F^{\mu\nu} = \partial^\mu \omega^\nu - \partial^\nu \omega^\mu$, $B^{\mu\nu} = \partial^\mu b^\nu - \partial^\nu b^\mu$; $\psi$, $\phi$, $\omega$ and $\mathbf{b}$ are the nucleon, $\sigma$, $\omega$ and $\rho$ meson fields with masses $M$, $m_{\sigma}$, $m_{\omega}$ and $m_{\rho}$, respectively, while $g_{\sigma}$, $g_{\omega}$ and $g_{\rho}$ are the respective meson-nucleon coupling constants; $b$ and $c$ are the nonlinear term coefficients, and $\tau$ are isospin matrices. As $M$, $m_{\omega}$ and $m_{\rho}$ are taken from experiment, the model parameters are $g_{\sigma}$, $g_{\omega}$, $g_{\rho}$, $b$, $c$ and $m_{\sigma}$.

The nuclear matter equation of state derived from this Lagrangian density can be expressed in terms of the nuclear energy density $\mathcal{E}$ as $\epsilon = \mathcal{E}/\rho_N - M$, where $\rho_N$ is the baryonic density, and

$$\mathcal{E} = \mathcal{E}_k + \mathcal{E}_\sigma + \mathcal{E}_\omega + \mathcal{E}_\rho,$$

(2)

$$\mathcal{E}_k = \frac{M^4 \xi^4}{\pi^2} \sum_{i=p,n} F_i(k_i/\xi M),$$

(3)

$$\mathcal{E}_\sigma = M^4 \left[ \frac{1}{2C^2_{\sigma}} (1-\xi)^2 + \frac{1}{3} b (1-\xi)^3 + \frac{1}{4} c (1-\xi)^4 \right],$$

(4)
\[ \epsilon_\omega = \frac{C_\omega \rho_N^2}{2M^2}, \]

\[ \epsilon_\rho = \frac{C_\rho \rho_N^2}{2M^2} \delta^2, \]  

where \( \delta = (\rho_p - \rho_n)/\rho_N \) is the nuclear matter asymmetry; \( k_p \) and \( k_n \) are the proton and neutron Fermi momenta respectively,

\[ \xi = \frac{M^*}{M} = 1 - \frac{g_\sigma}{M} \phi, \]

\[ C_i = g_i \frac{M}{m_i}, \ i = \sigma, \omega, \rho, \]  

and the function \( F_m(x) \) is defined as (see Ref. [3] for details):

\[ F_m(x) = \int_0^x dx \, x^{2m} \sqrt{1 + x^2}. \]

The reduced effective nucleon mass \( \xi \) and thus the field \( \phi \) is determined by

\[ (1 - \xi) + bC_\sigma^2(1 - \xi)^2 + cC_\sigma^2(1 - \xi)^3 = \frac{C_\sigma^2}{M^3} \rho_s, \]

where the scalar density \( \rho_s \) can be expressed as

\[ \rho_s = \frac{M^3 \xi^3}{\pi^2} \sum_{i=p,n} f_1(k_i/\xi M), \]

and the function \( f_m(x) \) is defined as (see Ref. [3] for details)

\[ f_m(x) = \int_0^x dx \, x^{2m} \sqrt{1 + x^2}. \]

The model parameters related to the nuclear equation of state are only \( C_\sigma^2, C_\omega^2, C_\rho^2, b \) and \( c \). As \( m_\sigma \) is the inverse Compton wavelength of the \( \sigma \)-meson, it is related only to finite size effects, such as surface energy and shell effects of nuclei.

Knowing the equation of state, the following formula for pressure \( p \) can be obtained:

\[ p = -\epsilon + \rho_N \frac{\partial \epsilon}{\partial \rho_N} = \frac{1}{3} \epsilon_k - \frac{1}{3} M \xi \rho_s - \epsilon_\sigma + \epsilon_\omega + \epsilon_\rho. \]

The standard state \( (\rho_N = \rho_0, \delta = 0) \) is defined by the equilibrium condition, \( p(\rho_0, 0) = 0 \). The standard density \( \rho_0 \) can be written in terms of nuclear radius constant \( r_0 \) or nucleon Fermi momentum \( k_F \) as

\[ \rho_0 = \frac{1}{4\pi r_0^3/3} = \frac{2k_F^3}{3\pi^2}. \]

As the finite nuclei are in states near the standard one, the equation of state can be written approximately as

\[ \epsilon(\rho_N, \delta) = -a_1 + \frac{1}{18} (K_0 + K_\omega \delta^2) \left( \frac{\rho_N - \rho_0}{\rho_0} \right)^2 + \left[ J + \frac{L}{3} \left( \frac{\rho_N - \rho_0}{\rho_0} \right) \right] \delta^2, \]

which is specified by the standard density \( \rho_0 \), volume energy \( a_1 \), symmetry energy \( J \), incompressibility \( K_0 \), density symmetry \( L \) and symmetry incompressibility \( K_\sigma \). These 6 quantities can be expressed in terms of 5 model parameters \( C_\sigma^2, C_\omega^2, C_\rho^2, b \) and \( c \). Reversely, the 5 model parameters can be fixed if 5 of those quantities are known. In this case, we will choose \( r_0 \), \( a_1 \), \( K_0 \), \( J \) and \( K_\sigma \) as input data, where \( r_0 \) is equivalent to \( \rho_0 \). Specifically, the procedure is as follows.
At the standard state \((\rho_0, 0)\), \(c(\rho_0, 0) = -a_1\), and \(E_\rho = 0\), we have
\[
E_k + E_\sigma + E_\omega = \rho_0(M - a_1).
\] (16)

In addition, the equilibrium condition, \(p(\rho_0, 0) = 0\), can be written as
\[
\frac{1}{3}(E_k - M\xi\rho_s) - E_\sigma + E_\omega = 0.
\] (17)

The following formulas can be derived from above two equations:
\[
C_\omega^2 = \frac{2M^2}{\rho_0^2}E_\omega = \frac{M^2}{\rho_0^2}\left[\rho_0(M - a_1) - \frac{1}{3}(4E_k - M\xi\rho_s)\right],
\] (18)
\[
E_\sigma = \frac{1}{2}\left[\rho_0(M - a_1) - \frac{1}{3}(2E_k + M\xi\rho_s)\right].
\] (19)

Furthermore, Eqs. (18) and (1) can be combined to solve the parameters \(b\) and \(c\) as
\[
b = \frac{12E_\sigma}{M^4(1 - \xi)^3} - \frac{3\rho_s}{M^3(1 - \xi)^2} - \frac{3}{C_\omega^2(1 - \xi)},
\] (20)
\[
c = -\frac{12E_\sigma}{M^4(1 - \xi)^4} + \frac{4\rho_s}{M^3(1 - \xi)^3} + \frac{2}{C_\omega^2(1 - \xi)^2}.
\] (21)

Finally, the equation for symmetry energy \(J\) is solved to give
\[
C_\rho^2 = \frac{2M^2}{\rho_0^2}\left(J - \frac{1}{6}\frac{k^2_F}{k^2_F + M^2\xi^2}\right).
\] (22)

For given \(\rho_0, a_1, \xi, C_\sigma^2\) and \(J\), the above formulas can be used to calculate \(C_\omega^2, C_\rho^2, b\) and \(c\), and thus to calculate \(K_0, K_s\) and \(L\). According to the calculated \(K_0\) and \(K_s\), we can fix \(\xi\) and \(C_\omega^2\). The following experimentally acceptable values are used as input data in our calculation:
\[
r_0 \approx 1.14\text{ fm}, \quad a_1 \approx 16\text{ MeV}.
\] (23)

Fig.1 plots the calculated \(K_s\) versus \(K_0\) for given \(\xi\). The solid curves from top to bottom correspond to \(\xi = 0.5, 0.6, 0.7, 0.8\) and 0.85, respectively. It can be seen that \(K_s\) is negative only for \(\xi\) larger than about 0.7 - 0.8, if \(K_0 \leq 300\) MeV is assumed. This is shown more clearly in Fig.2, where the calculated \(K_s\) versus \(\xi\) is displayed for given \(K_0\). The solid curves from top to bottom correspond to \(K_0 = 200, 300, 400\) and 500 MeV, respectively. It is shown that \(K_s\) is negative when \(\xi\) is larger than about 0.73, for \(K_0 \leq 300\) MeV.

Fig.3 is the calculated \(C_\omega^2\) versus \(K_0\) for given \(\xi\). The solid curves from top to bottom correspond to \(\xi = 0.5, 0.6, 0.7, 0.8\) and 0.85, respectively. Fig.4 gives the calculated \(C_\sigma^2\) versus \(\xi\). It can be seen from Eq.(18) that \(C_\omega^2\) depends only on \(\xi\), for given \(r_0\) and \(a_1\). The calculation of \(C_\rho^2\) depends, besides \(\xi\), also on the input value of \(J\) and Fig.5 displays the calculated \(C_\rho^2\) versus \(\xi\) for given \(r_0, a_1\) and \(J = 30\) MeV. On its turn, \(L\) depends, besides \(\xi\) and \(J\), also on the input value of \(K_0\), and Fig.6 shows the calculated \(L\) versus \(\xi\) for given \(r_0, a_1, J = 30\) MeV and \(K_0\). The solid curves from top to bottom correspond to \(K_0 = 200, 300, 400\) and 500 MeV, respectively.

Fig.7 presents the nonlinear coefficient \(b\) versus \(K_0\) for given \(\xi\). On the right hand side of the plot, the first three curves correspond to \(\xi = 0.5, 0.6, 0.7\) from top to bottom, respectively. In the middle of the plot, the lower two curves correspond to \(\xi = 0.8\) and 0.85 from top to bottom, respectively, being the first curve scaled by \(\times 1/2\) and the second one by \(\times 1/10\).

Fig.8 displays the nonlinear coefficient \(c\) versus \(K_0\) for given \(\xi\). On the right hand side of the plot, the solid curves correspond to \(\xi = 0.5, 0.6, 0.7, 0.8\) and 0.85 from bottom to top, respectively. The value of \(c\) should be scaled by \(\times 1/10\) for the curve of \(\xi = 0.8\), while by \(\times 1/50\) for the curve of \(\xi = 0.85\). It can be seen from Fig.8 that \(c\) is positive if \(\xi\) is larger than about 0.7 - 0.8, for \(K_0 \leq 300\) MeV.

In addition to these results, it is worthwhile to see what could be obtained, if the realistic nuclear matter properties, extracted from measured data of finite nuclei by nonrelativistic models, are used as input data. In this case, the results
given by Myers-Swiatecki phenomenological nucleon-nucleon interaction \[7\], Skyrme interaction \[8\] as well as Tondeur interaction \[9\] are employed.

The results of the calculation is presented in Table \[\] The input data set \((r_0, a_1, K_0, J, K_s)\) is taken from the compilation of Ref. \[10\]. MS is for the Myers-Swiatecki interaction, SIII, Ska, SkM, SkM* and RATP for the Skyrme interaction, and Tondeur for the Tondeur interaction. It is worthwhile to note that the input value of \(K_0\) for all of these interactions. This is in agreement with the most expectations based on the nonrelativistic model \[11\]. As it can be seen from Eq.(15), physically, \(K_0 = 0\) means that the compression modulus of nuclear matter will decrease if the asymmetry \(\delta\) of nuclear matter increases. Experimentally, \(K_0\) extracted from the isoscalar giant monopole resonance energy is between \(-566 \pm 1350\) to \(34 \pm 159\) MeV \[12\]. It is also interesting to note that the effective nucleon mass \(M^*/M\) is around 0.89, which reproduces nicely the value given by nonrelativistic models of nuclei. In addition, the nonlinear coefficient \(c\) is positive, which means the field system is stable for all of these parameter sets.

In summary, the effective nucleon mass \(M^*/M\) is discussed with relation to the symmetry incompressibility \(K_s\) of nuclear matter, based on the model parameters fitted to nuclear matter properties, in the \(\sigma-\omega-\rho\) model of the relativistic mean field theory with nonlinear \(\sigma\)-meson self-interaction. It is shown that \(M^*/M\) is larger than 0.73\(M\), if \(K_s\) is assumed to be negative and nuclear matter incompressibly \(K_0\) is kept less than 300 MeV. Furthermore, it is shown also that the field system is stable, as there is a lower limit for the \(\sigma\)-meson self-interaction energy in this parameter region. Our conclusion is: a stable result can be obtained by using an appropriate model parameter set altogether with a judicious selection of nuclear properties, within the \(\sigma-\omega-\rho\) model of the relativistic mean field theory with nonlinear \(\sigma\)-meson self-interaction, at least for nuclear matter properties.

[1] B.D. Serot and J.D. Walecka, Int. J. Mod. Phys. **E6**, 515 (1997).
[2] P.G. Reinhard, Rep. Prog. Phys. **52**, 439 (1989).
[3] K.C. Chung, C.S. Wang, J.A. Santiago and J.W. Zhang, Nuclear matter properties and relativistic mean field theory, to be published in *Eur. Phys. Journ. A*.
[4] W.D. Myers and W.J. Swiatecki, Ann. Phys. (N.Y.) **55**, 395 (1969).
[5] K.C. Chung, C.S. Wang, A.J. Santiago and J.W. Zhang, Phys. Rev. **C61**, 047303 (2000).
[6] W.D. Myers and W.J. Swiatecki, Phys. Rev. **C57**, 3020 (1998).
[7] W.D. Myers and W.J. Swiatecki, Nucl. Phys. **A601**, 141 (1996).
[8] M. Brack, C. Guet, and H.-B. Häkansson, Phys. Rep. **123**, 275 (1985).
[9] F. Tondeur, Nucl. Phys. **A315**, 353 (1978).
[10] K.C. Chung, C.S. Wang, A.J. Santiago and J.W. Zhang, Effective nucleon-nucleon interactions and nuclear matter equation of state, to be published in *Eur. Phys. Journ. A*.
[11] B.A. Li, C.M. Ko and W. Bauer, Int. J. Mod. Phys. **E7**, 147 (1998).
[12] S. Shlomo and D.H. Youngblood, Phys. Rev. **C47**, 529 (1993).

FIG. 1. Calculated \(K_s\) versus \(K_0\) for given \(r_0 = 1.14\) fm, \(a_1 = 16\) MeV and \(\xi\). The solid curves from top to bottom correspond to \(\xi = 0.5, 0.6, 0.7, 0.8\) and 0.85, respectively.

FIG. 2. Calculated \(K_s\) versus \(\xi\) for given \(r_0 = 1.14\) fm, \(a_1 = 16\) MeV and \(K_0\). The solid curves from top to bottom correspond to \(K_0 = 200, 300, 400\) and 500 MeV, respectively.

FIG. 3. Calculated \(C^2_\rho\) versus \(K_0\) for given \(r_0 = 1.14\) fm, \(a_1 = 16\) MeV and \(\xi\). The solid curves from top to bottom correspond to \(\xi = 0.5, 0.6, 0.7, 0.8\) and 0.85, respectively.

FIG. 4. Calculated \(C^2_\rho\) versus \(\xi\) for given \(r_0 = 1.14\) fm and \(a_1 = 16\) MeV.

FIG. 5. Calculated \(C^2_\rho\) versus \(\xi\) for given \(r_0 = 1.14\) fm, \(a_1 = 16\) MeV and \(J = 30\) MeV.
FIG. 6. Calculated $L$ versus $\xi$ for given $r_0 = 1.14$ fm, $a_1 = 16$ MeV, $J = 30$ MeV and $K_0$. The solid curves from top to bottom correspond to $K_0 = 200, 300, 400$ and 500 MeV, respectively.

FIG. 7. Nonlinear coefficient $b$ versus $K_0$ for given $r_0 = 1.14$ fm, $a_1 = 16$ MeV and $\xi$. The solid curves correspond to $\xi = 0.5, 0.6, 0.7, 0.8$ and 0.85, respectively.

FIG. 8. Nonlinear coefficient $c$ versus $K_0$ for given $r_0 = 1.14$ fm, $a_1 = 16$ MeV and $\xi$. The solid curves correspond to $\xi = 0.5, 0.6, 0.7, 0.8$ and 0.85, respectively.

TABLE I. The nuclear matter properties $r_0$(fm), $a_1$(MeV), $J$(MeV), $K_0$(MeV), $L$(MeV), and the parameters $C^2_\sigma$, $C^2_\omega$, $C^2_\rho$, $b$, $c$ of nonlinear $\sigma$-$\omega$-$\rho$ model in the relativistic mean field theory. See text for details.

|         | $r_0$ | $a_1$ | $K_0$ | $J$   | $K_0$ | $L$   | $M^*/M$ | $C^2_\sigma$ | $C^2_\omega$ | $C^2_\rho$ | $b$    | $c$    |
|---------|-------|-------|-------|-------|-------|-------|---------|---------------|---------------|-------------|--------|--------|
| MS      | 1.140 | 16.24 | 234.4 | 32.65 | -147.1| 85.553| 0.893371| 92.728        | 30.908        | 27.729     | -0.09203| 1.1137 |
| SIII    | 1.180 | 15.86 | 355.5 | 28.16 | -393.9| 72.866| 0.877399| 77.041        | 47.982        | 24.665     | -0.15264| 1.0935 |
| Ska     | 1.154 | 15.99 | 263.1 | 32.91 | -78.45| 86.766| 0.885116| 96.522        | 38.415        | 29.434     | -0.08115| 0.8458 |
| SkM     | 1.142 | 15.77 | 216.6 | 30.75 | -148.8| 79.898| 0.897339| 95.423        | 28.962        | 25.294     | -0.08287| 1.1499 |
| SkM*    | 1.142 | 15.77 | 216.6 | 30.03 | -155.9| 77.733| 0.897526| 94.984        | 28.842        | 24.267     | -0.08441| 1.1655 |
| RATP    | 1.143 | 16.05 | 239.6 | 29.26 | -191.3| 75.430| 0.893634| 89.460        | 31.269        | 23.183     | -0.10318| 1.1808 |
| Tondeur | 1.145 | 15.98 | 235.8 | 19.89 | -39.78| 47.603| 0.886214| 107.753       | 36.378        | 9.705      | -0.04911| 0.6885 |
\[ \xi = 0.5 \]

\[ K_S (\text{MeV}) \]

\[ K_\theta (\text{MeV}) \]


\[ C_\sigma^2 = 0. \]

\[ K_0 (\text{MeV}) \]

\[ \xi = 0.5 \]

\[ \xi = 0.6 \]

\[ \xi = 0.7 \]

\[ \xi = 0.8 \]

\[ \xi = 0.85 \]
\[ K_0 = 200 \text{ MeV} \]

\[ 300 \text{ MeV} \]

\[ 400 \text{ MeV} \]

\[ 500 \text{ MeV} \]
\[ b = 0.5, 0.6, 0.7 \]

\[ \xi = 0.8, \times 1/2 \]

\[ 0.85, \times 1/10 \]
\[ c = 0.5, \quad \xi = 0.5 \] 

\[ c = 0.8, \times 1/10 \] 

\[ c = 0.85, \times 1/50 \]