Large $N_c$ Expansion and the Parity Violating $\pi, N, \Delta$ Couplings

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In the limit of large $N_c$ we first consider the $N_c$ ordering of the various parity violating $\pi, N, \Delta$ couplings. Then we derive the relations among these couplings and consistency relations from the stability of these couplings under the chiral loop corrections with and without the mass splitting between $N$ and $\Delta$. Especially we find that $h_\Delta = -\frac{3}{\sqrt{5}} h_\pi$ in the large $N_c$ limit, which correctly reproduces the relative sign and magnitude of the "DDH" values for these PV couplings.

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I. INTRODUCTION

The long range parity violating (PV) force between nucleons are mediated by pions with one vertex being parity violating. These PV $\pi, N, \Delta$ couplings play an important role in the various hadronic PV experiments and polarized $\vec{e}p$ scattering experiments. For example, these PV couplings may contribute to the nucleon anapole moment and PV nuclear forces. They may be classified into: the Yukawa coupling $h_\pi, h_\Delta$, the vector couplings $h^{0,1,2}_V$ and the axial vector couplings $h^{1,2}_A$ etc.

The Yukawa coupling $h_\pi$ was estimated using $SU(6)$ symmetry, inputs from hyperon decay phenomenology and quark model. The one loop chiral corrections to $h_\pi$ up to order $1/\Lambda^3$ were presented in where the corrections from $h^{1,2}_A, h_\Delta$ etc found to be significant. Despite these efforts, both theoretical and experimental information of these PV couplings is still rather scarce.

In this work we want to explore these PV coupling constants in the framework of the large $N_c$ expansion. We first review the large $N_c$ expansion formalism and collect some useful results from literature in Section II. We present the $N_c$ ordering of $h_\pi, h^i_V, h^i_A$ in Section III. Then we relate the $\pi NN$ PV couplings to the $\pi \Delta \Delta$ and $\pi N \Delta$ couplings since the nucleon and delta resonance are in the same band in the large $N_c$ limit. Several interesting relations are found such as

$$h_\Delta = -\frac{3}{\sqrt{5}} h_\pi.$$  \hspace{1cm}(1)

In Section IV we discuss the $N_c$ orderings of the chiral loop corrections. The last section is a short summary.

II. LARGE $N_c$ FORMALISM

Quantum Chromodynamics (QCD) is very complicated at the hadronic scale. In order to explore the hadron structure, various theoretical frameworks are proposed such as lattice QCD, the chiral perturbation theory ($\chi$PT), QCD sum rule (QSR) etc. QSR works at the typical hadronic scale. The expansion parameter of $\chi$PT is $p/\Lambda_\chi$ where $p$ is the pion momentum. The convergence of the chiral expansion series requires that the typical momentum of the process be small. In contrast, the expansion parameter of perturbative QCD is the strong coupling constant $\alpha_s(Q^2)$ where $Q^2$ should be large.

't Hooft first suggested an alternative expansion scheme in terms of $N_c^{-1}$ with $N_c$ the color number. Such an expansion is valid in the whole momentum range so long as $N_c$ is large. Some salient features of $N_c = 3$ theory (QCD) like asymptotic freedom, confinement, chiral symmetry and its spontaneous breaking have to be kept if the large $N_c$ theory is similar to QCD. The requirement that the large $N_c$ theory is a nontrivial asymptotic one leads to the scaling behavior $g_s \sim O(N_c^{-2})$ from the gluon self energy diagram.

In the large $N_c$ limit the planar diagrams are dominant and the internal quark loops are suppressed while the gluon loops are not. Mesons are stable with its mass $\sim O(1)$ and decay width $\sim O(N_c^{-1})$. The pion decay constant

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FIG. 1: Feynman diagrams for parity conserving pion baryons scattering process. The solid and dashed line corresponds to the nucleon and pion respectively.

\( F_\pi \sim O(\sqrt{N_c}) \). A nice review of meson properties in the large \( N_c \) limit was presented in Ref. [9]. Witten extended the \( N_c \) counting rules to the baryons based on the mean field theory picture [9]. We quote some results which we need below. Baryon masses are \( \sim O(N_c) \) and the meson baryon scattering amplitude is \( \sim O(1) \) at most.

### A. Operator formalism and large \( N_c \) expansion

Several groups considered the pion baryon scattering process in Fig. 1(a)-(b) to derive the consistency conditions [10, 12, 14]. Fig. 1(c) is suppressed by \( 1/N_c \) compared to Fig. 1(a)-(b). The amplitude reads

\[
A \sim \frac{N_c^2 q_i q_j}{\ell^2} g_A \omega [X^{ia}, X^{jb}]
\]

where \( q_{i,j} \) is the initial and final pion three-momentum, \( g_A \) is the axial charge, \( \omega \) is the pion energy, \( X^{ia} \) is the operator for the pion baryon interaction vertex with \( I = J = 1 \), \( i, a \) is the spin and isospin indices respectively. A factor \( N_c \) has been extracted since pions can couple to \( N_c \) quarks at each vertex. The coefficient in (2) is \( \sim O(N_c) \). Unless the commutator vanishes the amplitude is \( \sim O(N_c) \), which violates Witten’s large \( N_c \) counting rules that the meson baryon scattering amplitude is \( \sim O(1) \) at most. Naturally we have the consistency conditions:

\[
[X^{ia}, X^{jb}] = 0
\]

in the leading order of \( 1/N_c \) expansion. However if the nucleon is the only intermediate state, the above commutator definitely does not hold. In other words, other states are needed to fulfill Eq. (3). In fact the consistency condition requires the presence of a band of states with \( I = J = 1/2, 3/2, \ldots \) [11, 12]. From now on summation over all possible intermediate states are assumed in the commutator.

For the phenomenological application it’s convenient to use the formalism of operator expansion and operator algebra generated by SU(4) spin flavor symmetry group [12, 13]. In such an approach the large \( N_c \) counting of a specific process is made explicit by relevant operators and their reduction for the low lying baryons in the band. For the two flavor case, the commutation relations for the generators of SU(4) algebra are

\[
[J^i, T^a] = 0, \\
[J^i, J^j] = i \epsilon^{ijk} J^k, \\
[T^a, T^b] = i \epsilon^{abc} T^c, \\
[J^i, G^{ja}] = i \epsilon^{ijk} G^{ka}, \\
[T^a, G^{ib}] = i \epsilon^{abc} G^{ic}, \\
[G^{ia}, G^{jb}] = \frac{2}{4} \delta^{ij} \epsilon^{abc} T^c + \frac{1}{4} \delta^{ab} \epsilon^{ijk} J^k
\]

where \( X^{ia} = G^{ia}/N_c \) [11, 12].

In the large \( N_c \) limit the baryons form a band with \( I = J = 1/2, 3/2, \ldots \) [11, 12]. For baryons lying in the upper part of the band the matrix elements of the generators are naturally of the order of \( N_c \),

\[
\langle N^* | T^a | N^* \rangle \sim O(N_c), \\
\langle N^* | J^i | N^* \rangle \sim O(N_c), \\
\langle N^* | G^{ia} | N^* \rangle \sim O(N_c)
\]

(5)
where we have denoted these highly lying states by $N^*$. However these physically interesting states, which correspond to the nucleon and delta baryon in the $N_c = 3$ world, do lie at the bottom of the band. In other words,

\[
\langle N, \Delta| T^a |N, \Delta \rangle \sim O(1) \\
\langle N, \Delta| J^i |N, \Delta \rangle \sim O(1) \\
\langle N, \Delta| G^{ia} |N, \Delta \rangle \sim O(N_c)
\]  

(6)

For the two flavor case the spin and isospin of the nucleon and delta baryon remains to be order of unity even in the large $N_c$ limit. This fact is very useful and important, which can be used to expand operators in terms of $N_c$. For example the baryon mass operator $\hat{H}$ must respect the rotation and isospin symmetry. The lowest order few terms of $\hat{H}$ are \[11, 12, 13\]

\[
\hat{H} = N_c m_0 \hat{1} + m_1 \frac{J^2}{N_c} + \cdots
\]  

(7)

where we have used the fact that $I^2 = J^2$ in the large $N_c$ limit for the two flavor case. For those $N^*$ states every operator in Eq. (7) contributes at the same $N_c$ order, $\sim O(N_c)$. If we focus on the nucleon and delta resonances instead, we notice that their mass splitting is

\[
\frac{m_\Delta - m_N}{m_\Delta + m_N} \sim \frac{1}{N_c^2}
\]  

(8)

The above equation seems to hold even for the physical value $N_c = 3$.

Another example is the pion nucelon scattering amplitude in Fig. 1(a)-(b).\[4, 5\]

\[
A \sim \frac{g_A^2 q_i q_j}{F_\pi^2} \omega [G^{ia}, G^{jb}] \\
= \frac{i g_A^2 q_i q_j}{4 F_\pi^2} \omega \{G^{[ia}, \epsilon^{abc} T^c + \epsilon^{ijk} J^k\} \\
\sim O(N_c^{-1})
\]  

(9)

where we have used Eq. (4) and (6). However if we take into account the mass splitting between the intermediate state and initial and final states, we have \[15\]

\[
A \sim \frac{g_A^2 q_i q_j}{F_\pi^2} \omega [G^{ia}, [\hat{M}, G^{jb}]] \\
\sim \frac{g_A^2 q_i q_j}{F_\pi^2} \omega [G^{ia}, \frac{J^2}{N_c}, G^{jb}] \\
\sim \frac{g_A^2}{F_\pi^2} \frac{1}{N_c} \{G^{ia}, G^{jb}\} \\
\sim O(1)
\]  

(10)

since $\{G^{ia}, G^{jb}\} \sim O(N_c^2)$. A simpler way to understand the above relation is to expand the baryon propagator $v \frac{-i}{k^2 + \delta + i\epsilon}$ in Fig. 1(a) and $v \frac{-i}{k^2 + \delta + i\epsilon}$ in Fig. 1(b) to the first order in $\delta$, the correction to the amplitude due to the mass splitting reads,

\[
A \sim \frac{g_A^2 q_i q_j}{F_\pi^2} \delta \{G^{ia}, G^{jb}\} \\
\sim \frac{1}{N_c^2} \{G^{ia}, G^{jb}\} \\
\sim O(1)
\]  

(11)

where we have used $\delta \sim O(N_c^{-1})$.

**B. Loop corrections in the large $N_c$ expansion**

Note the chiral loop corrections are not always suppressed in the large $N_c$ counting. For example the dominant piece from the chiral correction to the baryon mass is $\sim O(N_c)$ and does not break the degeneracy of the spectrum
FIG. 2: The parity conserving (a) and violating (b) pion baryons scattering process at the quark level. The dashed line in (b) denotes the vector bosons $W^\pm, Z$. The curly line is the gluon.

FIG. 3: Feynman diagrams for the parity violating pion baryons scattering process. The filled circle denotes parity violating vertex.

The subleading symmetry breaking term from the loops is $\sim \mathcal{O}(N_c^{-1})$. The correction from the mass splitting $\delta$ reads

$$A \sim \frac{g^2}{F^2} \delta \{G^{ia}, G^{ia} \} \sim \mathcal{O}(1)$$  \hspace{1cm} (12)

Generally the correction from mass splitting is suppressed by $1/N_c$ compared the leading term.

Consider a general operator $V^{\cdots}$ where $\cdots$ denotes the isospin and spin indices. The one loop chiral correction arises from the vertex and self energy diagrams

$$\sim \frac{N^2 g^2}{F^2 \pi} [X^{ia}, [X^{ia}, V^{\cdots}]]$$ \hspace{1cm} (13)

where the sum over the spin index arises after finishing the momentum integral. The coefficient in Eq. (13) is $\sim \mathcal{O}(N_c)$. The stability of the operator $V^{\cdots}$ under the chiral loop corrections leads to the consistency condition

$$[X^{ia}, [X^{ia}, V^{\cdots}]] = 0$$ \hspace{1cm} (14)

Eq. (14) is valid for any operator. If we include the mass splitting, we have a correction term to (13)

$$\sim \frac{N^2 g^2}{F^2 \pi} \{X^{ia}, [V^{\cdots}, [M, X^{ia}]]\}$$ \hspace{1cm} (15)

From (15) we have the following consistency condition

$$\{X^{ia}, [V^{\cdots}, [J^2, X^{ia}]]\} \sim \mathcal{O}(1)$$ \hspace{1cm} (16)

III. LARGE $N_c$ COUNTING FOR PV COUPLING CONSTANTS

Now we move on to the parity violating meson baryon scattering in Fig. 2(b). Since the exchange of the $W$ or $Z$ bosons does not change the color flow, the diagram Fig. 2(b) is of the same order in the $N_c$ counting as Fig. 2(a) where there is no vector boson exchange. From this observation we know that the parity violating pion nucleon scattering process $\pi^a + N \rightarrow \pi^b + N$ shown in Fig. 3 is $\leq \mathcal{O}(1)$ in the $N_c$ counting and $\mathcal{O}(G_F)$.

The scattering amplitude of Fig. 3(c) is $\sim \frac{h_I^i}{F^2} \leq \mathcal{O}(1)$. We have

$$h_I^i \leq \mathcal{O}(N_c).$$ \hspace{1cm} (17)
We will derive a more rigorous constraint below that
\[ h_A^1 \leq O(1), \]
\[ h_A^2 \leq O(N_c^{-1}) \]
from the consistency conditions for the chiral loop corrections in Section V.

Consider Fig. 3 (a1)-(a2) and (b1)-(b2) with the PV Yukawa insertions. The scattering amplitude at the leading order of the \( N_c \) counting is
\[ A \sim h_{\pi N c} g_A \frac{q_i X_i^0 - q'_i X_i^0}{\omega} \]
\[ \sim h_{\pi N c} g_A \frac{q_i + q'_i}{\omega} X_i^0 \]
(20)
The commutator in the first line of (20) sums all the intermediate states. But the PV Yukawa interaction does not change spin. Hence the intermediate state must have the same spin and isospin as the initial or final state. So the commutator does not vanish, which is strong contrast with the parity conserving pion nucleon scattering where the summation is over the entire \( I = J \) band. In the present case the summation is over a particular state only due to the specific Lorentz structure of the PV Yukawa vertex. Since \( A \sim O(1) \), we arrive at
\[ h_{\pi} \sim O\left(\frac{1}{\sqrt{N_c}}\right). \]
(21)

If we insert the PV vector coupling, only the time component is relevant after the non-relativistic reduction. The discussion is similar to (20). We have
\[ h^V \sim O(1). \]
(22)

IV. THE RELATIONS BETWEEN PV COUPLING CONSTANTS

There exist several interesting relations between various PV coupling constants in the large \( N_c \) limit.

(i) PV Yukawa coupling

In the \( N_c \to \infty \) limit only baryons with \( I = J = 1/2, 3/2, \ldots \) are relevant. The operator for the PV Yukawa pion baryon interaction does not change the spin, i.e., \( \Delta J = 0 \). The initial and final baryons have the same spin and isospin. That’s exactly what we have for the PV \( \pi N N \) Yukawa interaction. There does not exist the PV Yukawa \( \pi N \Delta \) coupling as required by the large \( N_c \) argument. Let’s denote the PV Yukawa operator by \( Y^{i=0,a} \) with the isospin index \( a = \pm \). The PV Yukawa couplings are determined by the following matrix elements
\[ \langle I_f, m_f; J_f, j_f | Y^{i=0,a} | I_i, m_i; J_i, j_i \rangle = \frac{2 I_i + 1}{2 I_f + 1} \frac{1}{2} \left( \begin{array}{cc} I_i & 1 \\ m_i & a \\ \end{array} \right) \left( \begin{array}{cc} I_f & 0 \\ j_i & 0 \end{array} \right) c_Y \]
(23)
where \( I_i, j_i \) is the isospin (spin) of the initial and final baryons, \( m_i, j_i \) etc is the third component. \( c_Y \) is a constant which can not be determined with the \( N_c \) argument only.

The tree level operator \( Y^{i=0,a} \) in the PV Yukawa Lagrangians is \( \tau^\pm \). So for the PV \( \pi NN \) Yukawa coupling we have
\[ h_{\pi} = c_Y \left( \frac{1}{-1} \frac{1}{1} \frac{1}{2} \right) = \frac{\sqrt{2}}{3} c_Y \]
(24)
while for the PV \( \pi \Delta \Delta \) Yukawa coupling we have
\[ h_{\Delta} = \frac{c_Y}{\sqrt{3}} \left( \frac{3}{2} \frac{1}{3} \frac{1}{2} \right) = -\frac{\sqrt{2}}{5} c_Y \]
(25)
where the factor \( \frac{1}{\sqrt{3}} \) arises from the DDH convention for \( h_{\Delta} \), which is a linear combination of \( h_{\pi \Delta}^1 \) and \( h_{\pi \Delta}^2 \) in (AN).

Combining (24) and (25) we have
\[ h_{\Delta} = -\frac{3}{\sqrt{5}} h_{\pi} \]
(26)
It’s very interesting to note $h_\Delta$ and $h_N$ has opposite sign in the large $N_c$ limit, which is consistent with the DDH phenomenological analysis based on $SU(6)$ symmetry and inputs from hyperon decay data and quark model. The DDH ranges are $(0 \rightarrow 17)g_\pi$ for $h_\pi$ and $(-51 \rightarrow 0)g_\pi$ for $h_\Delta$ where $g_\pi = 3.8 \times 10^{-8}$ \[1, 7\]. Our large $N_c$ argument correctly reproduces the relative sign and magnitude.

(ii) PV vector couplings

Similar relations hold for the PV vector and axial vector pion baryon couplings. For the vector PV coupling case, only the time component of the vector operator $V^{i=0, a}$ remains after the non-relativistic reduction, i.e., $\Delta J = 0$. Therefore we do not expect the vector-like $\pi NN$ PV tree level interaction from the large $N_c$ argument. Analysis in \[2 \text{[6]}\] explicitly shows that such a PV coupling is suppressed at the leading order of the heavy baryon expansion.

(1) $\Delta I = 0$ case:

Now let’s move on to the PV coupling $h_V^0$ in \[A1\] and $j_0$ in \[A7\] respectively. The relevant operator has spin zero and isospin one.

\[
h_V^0 = c^0_V \left( \frac{1}{2} \frac{1}{2} \frac{1}{2} \frac{1}{2} \right) = -\sqrt{\frac{1}{3}} c^0_V
\]

(27)

\[
j_0 = c^0_V \left( \frac{3}{2} \frac{1}{2} \frac{1}{2} \right) = \sqrt{\frac{3}{5}} c^0_V
\]

(28)

Hence $j_0 = -\frac{3}{\sqrt{5}} h_V^0$.

(2) $\Delta I = 1$ case:

For $h_V^1$ in \[A2\] and $j_1$ in \[A8\] the isospin violation arises solely from the trace part $\text{Tr}(A_\mu X^\mu)$ of the relevant operator does not carry isospin or spin. So we simply have $h_V^1 = j_1$. Note $j_{2,3,4}$ has no corresponding terms in the PV $\pi NN$ Lagrangian.

(3) $\Delta I = 2$ case:

$h_V^2$ in \[A3\] and $j_2$ in \[A9\] involve $\Delta I = 2, \Delta I_z = 0$. The relevant operator is $\text{Tr}(A_\mu X^\mu)$ at the tree level. Hence, $j_2 = -\frac{3}{\sqrt{5}} h_V^2$. In contrast, $j_3$ has not any similar term in the PV $\pi NN$ Lagrangian.

(iii) PV axial vector couplings

Due to our vector spinor formalism for the $\Delta$ field the operator for the PV axial vector $\pi N \Delta$ couplings in \[A5\]-\[A6\] have a rather different form as those in \[A2\]-\[A3\] and \[A8\]-\[A9\]. However after the non-relativistic reduction, they have the same isospin and spin structure since the nucleon and delta lie in the same tower. Hence, $I = J = 1/2, 3/2, \cdots$ tower.

(1) $\Delta I = 1$ case:

For $h_A^1$ in \[A2\] and $k_1$ in \[A8\] the isospin violation arises solely from $\text{Tr}(A_\mu X^\mu)$ of the relevant operator has spin one and isospin zero. Similarly we have $k_1 = -\frac{3}{\sqrt{5}} h_A^1$. $k_{2,3,4}$ has no similar terms in \[A2\].

(2) $\Delta I = 2$ case:

$h_A^2$ in \[A3\] and $k_5$ in \[A9\] involve operator with $I = J = 1$ at the tree level. This operator is similar to $X^{ia}$ associated with the strong pionic vertex. So we have,

\[
-\frac{h_A^2}{2} = \left( \frac{1}{2} \frac{1}{2} \frac{1}{2} \frac{1}{2} \right)^2 c_A^2 = \frac{1}{3} c_A^2
\]

(29)

\[
k_5 = \left( \frac{1}{3} \frac{1}{3} \frac{1}{3} \right)^2 c_A^2 = \frac{3}{5} c_A^2
\]

(30)

where the factor $-\frac{1}{2}$ is our convention in the definition of $h_A^2$. Clearly,

\[
\begin{align*}
\frac{h_A^2}{g_\pi} & = \frac{g_{\pi^0 \Delta^{++} \Delta^{++}}}{g_{\pi^0 pp}} = \frac{g_\pi}{g_A} = \frac{9}{5}
\end{align*}
\]

(31)

in the large $N_c$ limit. Hence

\[
h_A^2 = -\frac{10}{9} k_5.
\]

(32)

The above analysis can be extended to the PV axial vector $\pi NN\Delta$ couplings.
FIG. 4: The chiral loop corrections to $h_\pi$ from the nucleon intermediate states.

FIG. 5: Chiral loop corrections to $h_\pi$ from delta intermediate states. The double line is the delta resonance.

V. THE CHIRAL LOOPS

In this section we consider the large $N_c$ counting of the various chiral loop corrections to the bare PV Yukawa coupling $h_\pi$ in Fig. 4-5 which was calculated in [6]. The relevant operator for $h_\pi$ is $T^\pm$.

Fig. 4(a) arises from expanding the PV Yukawa $\pi NN$ vertex to the third order. Its contribution is

$$\sim h_\pi T^\pm \sim O \left( N_c^{-1} \right) h_\pi$$ (33)

which is suppressed by $1/N_c$ compared to the tree-level term $h_\pi$.

The vertex correction and wave function renormalization corrections in Fig. 4(b), 4(c1)-(c2) and Fig. 5(a), 5(c1)-(c2) yield the sum

$$\sim g_\pi^2 h_\pi \frac{1}{F_\pi^2} \left( [G^{ia}, [G^{ia}, T^+]] \right) \sim g_\pi^2 h_\pi \frac{1}{F_\pi^2} T^+$$ (34)

where we have used the commutators in (31) twice. Clearly its contribution is also $\sim O(N_c^{-1})h_\pi$. The above relation holds in the limit of large $N_c$, i.e., when the nucleon and delta states are degenerate in mass. If we explicitly take into account the corrections of $\delta$ as done in [6], the loop correction reads

$$\sim \frac{h_\pi g_\pi^2}{F_\pi^2} \frac{1}{N_c} \left( [G^{ia}, [T^+, [J^2, G^{ia}]]] \right) \sim O(1) h_\pi$$ (35)

which has the same $N_c$ order as the tree-level term! Fig. 4(d1)-(d2) arises from the Yukawa vertex and expansion of the chiral connection. Its contribution reads

$$\sim \frac{h_\pi}{F_\pi^2} [T^3, T^+] \sim \frac{h_\pi}{F_\pi^2} T^+$$ (36)
It is also $\sim O(N_c^{-1})h_\pi$. The correction from mass splitting from these two diagrams is $\sim O(N_c^{-2})h_\pi$.

The contribution from the PV $\pi\pi NN$ and $\pi\pi N\Delta$ vertices are more subtle. This kind of contribution is very similar to the chiral loop correction to the baryon mass. There are two types of operators corresponding to the $\Delta I = 1, 2$ two pions axial vertex respectively. In the leading order of the large $N_c$ expansion, these operators can be read from Eq. (A2)-(A3). They are $J^i$ for the $h_\Delta^1$ type and $G^{ia}$ for the $h_\Delta^2$ type. For the $h_\Delta^1$ type, the sum of Fig. 4(c1)-(c2) and Fig. 5(b1)-(b2) yields the anti-commutator:

$$\sim \frac{g_A h_\Delta^1}{F_\pi} \{G^{ia}, J^i\}$$

$$\sim \frac{g_A h_\Delta^1}{F_\pi^2} \frac{N_c + 2}{2} T^a$$  \hspace{1cm} (37)$$

where we have used the operator identity \[12\]

$$2\{G^{ia}, J^i\} = (N_c + 2)T^a$$  \hspace{1cm} (38)$$

From the stability of the Yukawa coupling $h_\pi$ under the chiral correction, we conclude the loop correction is $\sim O(N_c^{-\frac{1}{2}})$ at most. Since $T^a \sim O(1)$, $F_\pi \sim O(\sqrt{N_c})$, we get

$$h_\Delta^1 \leq O(1) .$$  \hspace{1cm} (39)$$

For the $h_\Delta^2$ type correction, the sum of Fig. 4(c1)-(c2) and Fig. 5(b1)-(b2) yields

$$\sim \frac{g_A h_\Delta^2}{F_\pi^2} G^{ia} G^{ib}$$

$$\sim \frac{g_A h_\Delta^2}{2F_\pi^2} \{[G^{ia}, G^{ib}]_+ + \{G^{ia}, G^{ib}\}_-\}$$

$$\sim N_c h_\Delta^2$$  \hspace{1cm} (40)$$

where we have used the identity \[12\]

$$\{G^{ia}, G^{ib}\} = \frac{1}{4}\{T^a, T^b\} + \frac{\delta^{ab}}{3}\{G^{ic}, G^{ic}\}_+ - \frac{\delta^{ab}}{4} T^2 .$$  \hspace{1cm} (41)$$

From the same stability requirement we have

$$h_\Delta^2 \leq O(N_c^{-1}) .$$  \hspace{1cm} (42)$$

VI. SUMMARY

In short summary, we have investigated the $N_c$ ordering of the parity violating $\pi, N, \Delta$ couplings in the limit of large $N_c$. There exists several interesting relations between these couplings. For the PV Yukawa type couplings $h_\Delta = -\frac{1}{\sqrt{6}}h_\pi$. Both the sign and magnitude are consistent with the widely used "DDH" values for these PV coupling constants. The stability of these couplings under the chiral loop corrections leads to rather rigorous constraint on the $N_c$ ordering of the PV axial vector type couplings. The loop correction of the PV axial two pion vertex is of the same $N_c$ order as $h_\pi$, which partly explains why $h_\pi$ receives a large radiative correction from these couplings as noted in Ref. [6].

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APPENDIX A: PV πNN, π∆∆ AND πN∆ LAGRANGIANS

We collect the PV pion nucleon delta Lagrangians below. Details can be found in [2, 3, 6]. The PV πNN Lagrangian reads

\[ \mathcal{L}_{\Delta I=0}^{\pi N} = h_{V}^{0} \bar{N} A_{\mu} \gamma^{\mu} N \]  

(A1)

\[ \mathcal{L}_{\Delta I=1}^{\pi N} = \frac{h_{V}^{1}}{2} \bar{N} \gamma^{\nu} N \text{Tr}(A_{\mu}X_{R}^{3}) - \frac{h_{V}^{2}}{2} \bar{N} \gamma^{\mu} \gamma_{5} N \text{Tr}(A_{\mu}X_{L}^{3}) - \frac{h_{\pi}}{2\sqrt{2}} F_{\pi} \bar{N} X_{R}^{3} N \]  

(A2)

\[ \mathcal{L}_{\Delta I=2}^{\pi N} = \frac{h_{V}^{2}}{2} X_{b}^{\alpha} \bar{N} \left[ X_{R}^{b} A_{\mu} X_{R}^{b} + X_{L}^{b} A_{\mu} X_{L}^{b} \right] \gamma^{\mu} N - \frac{h_{V}^{3}}{2} X_{R}^{3} \bar{N} \left[ X_{R}^{b} A_{\mu} X_{R}^{b} - X_{L}^{b} A_{\mu} X_{L}^{b} \right] \gamma^{\mu} \gamma_{5} N \]  

(A3)

The analogues of Eqs. (A1A3) for πN∆ are

\[ \mathcal{L}_{\Delta I=0}^{\pi N \Delta} = f_{1} \epsilon^{abc} \bar{N} i \gamma_{5} \left[ X_{L}^{a} A_{\mu} X_{L}^{b} + X_{R}^{a} A_{\mu} X_{R}^{b} \right] T_{\mu}^{c} + g_{1} \bar{N} [A_{\mu}, X_{c}] + T_{\mu}^{c} + g_{2} \bar{N} [A_{\mu}, X_{c}] - T_{\mu}^{c} + \text{h.c.} \]  

(A4)

\[ \mathcal{L}_{\Delta I=1}^{\pi N \Delta} = f_{2} \epsilon^{abc} \bar{N} i \gamma_{5} \left[ A_{\mu}, X_{a}^{c} \right] T_{\mu}^{c} + f_{3} \epsilon^{abc} \bar{N} i \gamma_{5} \left[ A_{\mu}, X_{a}^{c} \right] T_{\mu}^{c} + \frac{g_{3}}{2} \bar{N} \left[ X_{L}^{a} A_{\mu} X_{L}^{b} - X_{L}^{a} A_{\mu} X_{L}^{b} \right] - \left( T_{\mu}^{c} A_{\mu} X_{L}^{c} X_{L}^{c} A_{\mu} X_{L}^{c} \right) \]  

(A5)

\[ \mathcal{L}_{\Delta I=2}^{\pi N \Delta} = f_{4} \epsilon^{abc} T^{cd} \bar{N} i \gamma_{5} \left[ X_{L}^{a} A_{\mu} X_{L}^{b} + X_{R}^{a} A_{\mu} X_{R}^{b} \right] T_{\mu}^{c} + f_{5} \epsilon^{abc} \bar{N} i \gamma_{5} \left[ X_{L}^{a} A_{\mu} X_{L}^{b} + X_{L}^{a} A_{\mu} X_{L}^{b} \right] + \text{h.c.} \]  

For the pv π∆∆ effective Lagrangians we have

\[ \mathcal{L}_{\Delta I=0}^{\pi \Delta \Delta} = j_{0} \bar{T}^{a} A_{\mu} \gamma^{\mu} T_{i}, \]  

(A7)

\[ \mathcal{L}_{\Delta I=1}^{\pi \Delta \Delta} = \frac{j_{1}}{2} \bar{T}^{a} \gamma^{\mu} T_{i} T_{j} T_{i} A_{\mu} X_{\nu}^{\pm} - \frac{k_{1}}{2} \bar{T}^{a} \gamma^{\mu} \gamma_{5} T_{i} T_{j} A_{\mu} X_{\nu}^{\pm} - \frac{h_{1}^{2}}{2\sqrt{2}} F_{\pi} \bar{T}^{a} X_{3}^{3} T_{i} \]  

\[ - \frac{h_{1}^{2}}{2\sqrt{2}} F_{\pi} \left\{ 3T^{3}(X_{L}^{3} T^{1} + X_{R}^{3} T^{2}) + 3(T_{j}^{2} X_{L}^{3} T^{1} + T_{j}^{2} X_{R}^{3} T^{2} + 2T^{3} X_{3}^{3} T^{3}) \right\} \]  

\[ + j_{2} \left\{ 3((T_{3}^{3} \gamma_{5} T^{1} + T_{1}^{2} \gamma_{5} T^{2} + T_{2}^{2} \gamma_{5} T^{3}) T_{3} A_{\mu} X_{\nu}^{\pm} + (T_{3}^{3} \gamma_{5} T^{1} + T_{1}^{2} \gamma_{5} T^{2} + T_{2}^{2} \gamma_{5} T^{3}) T_{3} A_{\mu} X_{\nu}^{\pm} \right\} \]  

\[ + k_{2} \left\{ 3(T_{3}^{3} \gamma_{5} T^{1} + T_{1}^{2} \gamma_{5} T^{2} + T_{2}^{2} \gamma_{5} T^{3}) T_{3} A_{\mu} X_{\nu}^{\pm} + (T_{3}^{3} \gamma_{5} T^{1} + T_{1}^{2} \gamma_{5} T^{2} + T_{2}^{2} \gamma_{5} T^{3}) T_{3} A_{\mu} X_{\nu}^{\pm} \right\} \]  

\[ + j_{3} \left\{ T^{a} \gamma^{\mu} A_{\mu} X_{\nu}^{a} + T^{3} \gamma^{\mu} A_{\mu} X_{\nu}^{a} \right\} + j_{4} \left\{ T^{a} \gamma^{\mu} A_{\mu} X_{\nu}^{a} - T^{3} \gamma^{\mu} A_{\mu} X_{\nu}^{a} \right\} \]  

\[ + k_{3} \left\{ T^{a} \gamma_{5} A_{\mu} X_{\nu}^{a} + T^{3} \gamma_{5} A_{\mu} X_{\nu}^{a} \right\} + k_{4} \left\{ T^{a} \gamma_{5} A_{\mu} X_{\nu}^{a} - T^{3} \gamma_{5} A_{\mu} X_{\nu}^{a} \right\} \]  

\[ + k_{5} \left\{ T^{a} \gamma_{5} A_{\mu} X_{\nu}^{a} + T^{3} \gamma_{5} A_{\mu} X_{\nu}^{a} \right\} \]  

(A8)

\[ \mathcal{L}_{\Delta I=2}^{\pi \Delta \Delta} = j_{5} \bar{T}_{i}^{a} T_{i} T_{i} A_{\mu} T_{b} + j_{6} \bar{T}_{i}^{a} T_{i} \left[ X_{R}^{a} A_{\mu} X_{R}^{b} + X_{L}^{a} A_{\mu} X_{L}^{b} \right] \gamma^{\mu} T_{i} \]  

\[ + k_{5} \bar{T}_{i}^{a} T_{i} \left[ X_{R}^{a} A_{\mu} X_{R}^{b} - X_{L}^{a} A_{\mu} X_{L}^{b} \right] \gamma^{\mu} T_{i} + k_{6} \epsilon^{abc} [T^{3} i \gamma_{5} X_{3}^{3} T^{a} + T^{a} i \gamma_{5} X_{3}^{3} T^{3}] \]  

(A9)