Some New Multiplicative Geometric-Arithmetic Indices

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Abstract

In this paper, we propose some new topological indices: second, third, fourth and fifth multiplicative geometric-arithmetic indices of a molecular graph. A topological index is a numeric quantity from the structural graph of a molecule. Here, we compute the fifth multiplicative geometric arithmetic index of line graphs of subdivision graphs of 2D-lattice, nanotube and nanotorus of $TUC_4C_8[p,q]$.

Key words: molecular graph, fifth multiplicative geometric-arithmetic index, nanostructures.

Mathematics Subject Classification: 05C05, 05C12, 05C35.

1. Introduction

In this paper, we consider only finite, simple and connected graph with a vertex set $V(G)$ and an edge set $E(G)$. A molecular graph or a chemical graph is a simple graph related to the structure of a chemical compound. Each vertex of a chemical graph represents an atom of the molecule and its edges to the bonds between atoms. A topological index is a numerical parameter mathematically derived from the graph structure. These indices are useful for establishing correlation between the structure of a molecular compound and its physico-chemical properties, see $^1$.

The degree $d_G(v)$ of a vertex $v$ is the number of vertices adjacent to $v$. Let $S_G(v)$ denote the sum of degrees of all vertices adjacent to a vertex $v$. The line graph $L(G)$ of a graph $G$ is the graph whose vertex set corresponds to the edges of $G$ such that two vertices of $L(G)$ are adjacent if the corresponding edges of $G$ are adjacent. The subdivision graph $S(G)$ of a graph $G$ is the graph obtained from $G$ by replacing each of its edges by a path of length two. We refer to $^2, ^3$ for undefined term and notation.

We need the following results

Lemma 1$^3$. Let $G$ be a $(p, q)$ graph. Then $L(G)$ has $q$ vertices and $\frac{1}{2} \sum_{i=1}^{p} d_G(u_i)^2 - q$ edges.

Lemma 2$^3$. Let $G$ be a $(p, q)$ graph. Then $S(G)$ has $p+q$ vertices and $2q$ edges.

One of the well-known and widely used topological index is the product connectivity index or Randić index
introduced by Randić in\textsuperscript{4}.

Motivated by the definition of the product connectivity index and its wide applications, Kulli [5] introduced the first multiplicative geometric-arithmetic index of a graph $G$ and it is defined as

$$GA_1 II (G) = \prod_{uv \in E(G)} \frac{2 \sqrt{d_G(u)d_G(v)}}{d_G(u) + d_G(v)}.$$ 

Recently many other multiplicative indices were studied, for example, in \textsuperscript{6,7,8,9,10,11,12,13,14}.

Motivated by the definition of the first multiplicative geometric-arithmetic index and by previous research on topological indices, we now propose the second, third, fourth and fifth multiplicative geometric-arithmetic indices of a graph as follows:

The second multiplicative geometric-arithmetic index of a graph $G$ is defined as

$$GA_2 II (G) = \prod_{uv \in E(G)} \frac{2 \sqrt{n_u n_v}}{n_u + n_v}$$

where the number $n_u$ of vertices of $G$ lying closer to the vertex $u$ than to the vertex $v$ for the edge $uv$ of a graph $G$.

The third multiplicative geometric-arithmetic index of a graph $G$ is defined as

$$GA_3 II (G) = \prod_{uv \in E(G)} \frac{2 \sqrt{m_u m_v}}{m_u + m_v}$$

where the number $m_u$ of edges of $G$ lying closer to the vertex $u$ than to the vertex $v$ for the edge $uv$ of a graph $G$.

The fourth multiplicative geometric-arithmetic index of a graph $G$ is defined as

$$GA_4 II (G) = \prod_{uv \in E(G)} \frac{2 \sqrt{\varepsilon(u) \varepsilon(v)}}{\varepsilon(u) + \varepsilon(v)}$$

where the number $\varepsilon(u)$ is the eccentricity of all vertices adjacent to a vertex $u$.

The fifth multiplicative geometric-arithmetic index of a graph $G$ is defined as

$$GA_5 II (G) = \prod_{uv \in E(G)} \frac{2 \sqrt{S_G(u) S_G(v)}}{S_G(u) + S_G(v)}$$

where $S_G(u) = \sum_{uv \in E(G)} d_G(v)$.

In \textsuperscript{14}, Todeshine \textit{et al.} introduced the first and second multiplicative Zagreb indices of a graph $G$ and they are defined as

$$II_1 (G) = \prod_{u \in V(G)} d_G(u)^2, \quad II_2 (G) = \prod_{u \in V(G)} d_G(u)d_G(v)$$

In \textsuperscript{15} the first multiplicative Zagreb index is defined as

$$II^*_1 (G) = \prod_{uv \in E(G)} \left[ d_G(u) + d_G(v) \right].$$

We now define a new version of multiplicative Zagreb indices as follows.

$$II_1^2 (G) = \prod_{u \in V(G)} n_u^2, \quad II_2^2 (G) = \prod_{u \in V(G)} n_u n_v, \quad II^*_1 (G) = \prod_{uv \in E(G)} (n_u + n_v).$$

Also we define a new version of multiplicative Zagreb indices as follows:

$$II_1^3 (G) = \prod_{u \in V(G)} m_u^2, \quad II_2^3 (G) = \prod_{u \in V(G)} m_u m_v, \quad II^*_1 (G) = \prod_{uv \in E(G)} (m_u + m_v).$$

We define another version of multiplicative Zagreb indices as follows:
Some New Multiplicative Geometric-Arithmetic Indices.

\[ II^G_1 = \prod_{u \in V(G)} e(u)^2, \quad II^G_2 = \prod_{u \in V(G)} e(u) e(v), \quad II^G_v = \prod_{u \in E(G)} \left[ e(u) + e(v) \right]. \]

We also define another version of multiplicative Zagreb indices as follows:

\[ II^S_1 = \prod_{u \in V(G)} S_G(u)^2, \quad II^S_2 = \prod_{u \in V(G)} S_G(u) S_G(v), \quad II^S_v = \prod_{u \in E(G)} \left[ S_G(u) + S_G(v) \right]. \]

In this paper, we determine the fifth multiplicative geometric arithmetic index of line graphs of subgraph graphs of 2D-lattice, nanotube and nanotorus of TUC[C_4,p,q].

2. 2D-lattice, nanotube, nanotorus of TUC[C_4,p,q]:

We consider the graph of 2D-lattice nanotube and nanotorus of TUC[C_4,p,q] where p and q denote the number of squares in a row and the number of rows of squares respectively. These graphs are shown in Figure 1.

![Figure 1](image1.png)

(a) 2D-lattice of TUC[C_4,p,q] (b) TUC[C_4,p,q] nanotube (c) TUC[C_4,p,q] nanotorus

By algebraic method, we get |V(G)| = 4pq, |E(G)| = 6pq - p - q; |V(H)| = 4pq, |E(H)| = 6pq - p, |V(K)| = 4pq, |E(K)| = 6pq.

3. Results for 2D-Lattice of TUC[C_4,p,q]:

The line graph of the subdivision graph of 2D-lattice of TUC[C_4,p,q] is shown Figure 2(b).

![Figure 2](image2.png)

(a) subdivision graph of 2D-lattice of TUC[C_4,p,q] (b) line graph of the subdivision graph of TUC[C_4,p,q]

**Theorem 1.** Let G be the line graph of the subdivision graph of 2D-lattice of TUC[C_4,p,q]. Then

\[ GA, II(G) = \left( \frac{4\sqrt{5}}{9} \right)^4 \times \left( \frac{4\sqrt{10}}{13} \right)^4 \times \left( \frac{12\sqrt{2}}{17} \right)^4 \quad \text{if } p > 1, q > 1, \]

\[ = \left( \frac{4\sqrt{5}}{9} \right)^4 \times \left( \frac{4\sqrt{13}}{10} \right)^4 \times \left( \frac{12\sqrt{2}}{17} \right)^4 \quad \text{if } p > 1, q = 1. \]

**Proof:** The 2D-lattice of TUC[C_4,p,q] is a graph G with 4pq vertices and 6pq - p - q edges. By Lemma 2, the subdivision graph of 2D-lattice of TUC[C_4,p,q] is a graph with 10pq - p - q vertices and 2(6pq - p - q) edges. Thus by Lemma 1, G has 2(6pq - p - q) vertices and 8pq - 5p - 5q edges. It is easy to see that the vertices of G are either of degree 2 or 3, see Figure 2. Therefore we have partition of the edge set of G as follows.
Table 1. Edge partition of $G$ with $p>1$ and $q>1$.

| S$_G(u), S_G(v)$uv $\in E(G)$ | (4, 4) | (4, 5) | (5, 5) | (5, 8) | (8, 9) | (9, 9) |
|---------------------------------|-------|-------|-------|-------|-------|-------|
| Number of edges                 | 4     | 8     | 2(p+q–4) | 4(p+q–2) | 8(p+q–2) | 2(9pq+10)–19(p+q) |

Table 2. Edge partition of $G$ with $p>1$ and $q=1$.

| S$_G(u), S_G(v)$uv $\in E(G)$ | (4, 4) | (4, 5) | (5, 5) | (5, 8) | (8, 8) | (8, 9) | (9, 9) |
|---------------------------------|-------|-------|-------|-------|-------|-------|-------|
| Number of edges                 | 6     | 4     | 2(p–2) | 4(p–1) | 4(p–1) | p–1   |

Case 1. Suppose $p>1$ and $q>1$.

By algebraic method, we obtain $|V_4|=8$, $|V_5|=4(p+q–2)$, $|V_8|=4(p+q–2)$, and $|V_9|=2(6pq+10)–19(p+q)$ in $G$. Thus the edge partition based on the degree sum of neighbor vertices of each vertex is obtained as given in Table 1.

Case 2. Suppose $p>1$ and $q=1$.

The edge partition based on the degree sum of neighbor vertices of each vertex is obtained as given in Table 2.

4. Results for $TUC_4[p,q]$ nanotube:

The line graph of the subdivision graph of $TUC_4[p,q]$ nanotube is shown in Figure 3(b)
Theorem 2. Let $H$ be the line graph of the subdivision graph of $TUC_4C_8[p,q]$ nanotube. Then

$$GA_{II}(H) = \left(\frac{4\sqrt{10}}{13}\right)^{4p} \times \left(\frac{12\sqrt{2}}{17}\right)^{8p} \times \left(\frac{4\sqrt{10}}{13}\right)^{4p} \times \left(\frac{12\sqrt{2}}{17}\right)^{4p} \times \frac{18pq - 19p}{2}$$

if $p > 1$ and $q > 1$.

Proof: The $TUC_4C_8[p,q]$ nanotube is a graph $H$ with $4pq$ vertices and $6pq - p$ edges. By Lemma 2, the subdivision graph of $TUC_4C_8[p,q]$ nanotube is a graph with $10pq - p$ vertices and $12pq - 2p$ edges. Thus by Lemma 1, $H$ has $12pq - p$ vertices and $18pq - 5p$ edges. We see that in $H$, there are $4p$ vertices are of degree 2 and remaining all vertices are of degree 3. Therefore we have partition of the edge set of $H$ as follows:

| $S_H(u), S_H(v) \in E(H)$ | (5, 5) | (5, 8) | (8, 9) | (9, 9) |
|---------------------------|-------|-------|-------|-------|
| Number of edges           | 2p    | 4p    | 8p    | 18pq - 19p |

Table 3. Edge partition of $H$ with $p > 1$ and $q > 1$.

| $S_H(u), S_H(v) \in E(H)$ | (5, 5) | (5, 8) | (8, 8) | (8, 9) | (9, 9) |
|---------------------------|-------|-------|-------|-------|
| Number of edges           | 2p    | 4p    | 2p    | 4p    |

Table 4. Edge partition of $H$ with $p > 1$ and $q = 1$

Case 1: Suppose $p > 1$ and $q > 1$.

By algebraic method, we obtain $|V_u| = 4p, |V_v| = 4p$ and $|V_e| = 2(6pq - 5p)$ in $H$. Thus the edge partition based on the degree sum of neighbor vertices of each vertex is obtained as given in Table 3.

Case 2. Suppose $p > 1$ and $q = 1$.

The edge partition based on the degree sum of neighbor vertices of each vertex is obtained as given in Table 4.
5. Results for $TUC_C[p,q]$ nanotorus:

The line graph of the subdivision graph of $TUC_C[p,q]$ nanotorus is shown in Figure 4 (b).

**Theorem 3.** Let $K$ be the line graph of the subdivision graph of $TUC_C[p,q]$ nanotorus. Then $GA_{I,II}(K) = 1$.

**Proof:** Let $K$ be the line graph of subdivision graph of $TUC_C[p,q]$ nanotorus with $4pq$ vertices and $6pq$ edges. Then Lemma 2, the subdivision graph of $TUC_C[p,q]$ nanotorus is a graph with $10pq$ vertices and $12pq$ edges. Thus by Lemma 1, $K$ has $12pq$ vertices and $18pq$ edges. We see easily that in $K$, $|V| = 12pq$ and we have edge partition based on the degree sum of neighbor vertices of each vertex as given in Table 5.

| $u$ | $v$ | $uv \in E(K)$ | 
|-----|-----|-----------------|
| $S_K(u)$ | $S_K(v)$ | $2\sqrt{9 \times 9}$ |

Thus $GA_{I,II}(K) = \prod_{uv \in E(K)} \frac{S_K(u)S_K(v)}{S_K(u) + S_K(v)} = (9,9) = 1$.

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