Transfer learning-based surrogate-assisted design optimisation of a five-phase magnet-shaping PMSM

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Abstract
Multi-phase permanent-magnet synchronous machines (MPMSMs) with high reliability due to sufficient fault-tolerant capability have considerable potential for transportation electrification applications. Here, an efficient surrogate-assisted design optimisation method is proposed based on analytical model transfer learning for torque characteristic optimisation of a five-phase magnet-shaping PMSM. By employing transfer learning of the source domain analytical model data and the target domain finite element analysis (FEA) data in surrogate model training, the proposed method can achieve both high accuracy and high efficiency from the merits of FEA- and analytical-based optimisations, respectively. The studied machine with five-phases and harmonic injected surface-mounted PMs to enable harmonic injection for torque capability improvement is introduced and the analytical model is built based on the segmented PM and the complex conformal mapping methods. Besides, the optimal Latin hypercube design (LHD) and Taguchi methods are used to form the source and target domain datasets, respectively, so that data features can be efficiently captured over a wide range of optimisation variables. An optimal design is obtained by multi-objective optimisation using the trained surrogate model, which is prototyped and measured to validate the proposed method.

1 | INTRODUCTION

With increasing concerns on energy crisis and environmental pollution, great efforts have been put into the development of transportation electrification [1], which further raises the growing demand for advanced traction machines in electrified propulsion applications [1–3]. Due to high power/torque density, high reliability, and fault-tolerant operation capability, multiphase permanent magnet synchronous machines (MPMSMs) are widely considered as potential candidates in applications that require high reliability and safety, that is, electric vehicles, aircraft, and ships [4–6]. In related applications, 5-phase, 6-phase, 9-phase, 12-phase, and 15-phase motors have been applied to replace the traditional 3-phase motors as the preferred choices [7].

To increase the torque density, the harmonic injection technique is employed for MPMSMs in the literature [8–10]. By injecting harmonic currents to interact with the corresponding order of back electromotive force (EMF) harmonics, additional torque components are generated and the average torque is enhanced [10]. In some studies, harmonic-injected PMs with shaped profiles are utilised to further enhance the average torque. The saddle-shaped (sine + third) PM was proposed in [11–13] and applied both on the five-phase surface-mounted and interior PM motors for average torque improvement, respectively. Kang et al. [14] proposed the quasi-trapezoidal (sine + third + fifth) PM and applied it on the five-phase PM motors, which produced a higher torque density than the saddle-shaped design. Higher-order harmonics-injected PMs were analysed in [15]. Determining the optimal harmonics injection ratios on PMs is critical to the design optimisation of such motors, however, the special shape of PMs makes the optimisation more difficult than conventional PMSMs. In [15], when the edge thickness of PMs is ignored, the analytical solutions of the optimal injected harmonic ratios for maximising the output torque are given, but when the...
minimisation of torque ripple is also considered, the ratio of injected harmonics is hard to be obtained analytically.

Modern machine design optimisation methods can be generally divided into two categories: the analytical model-based methods and the FEA-based methods. The former is based on the accurate prediction of the motor's open-circuit and on-load air-gap magnetic field to further calculate and optimise performance indexes, such as the conformal mapping [16, 17] and the subdomain model [18]. However, due to the simplifications, the analytical models are less accurate despite their efficiency. The FEA-based models usually have high accuracy but are more time-consuming due to massive high-quality meshing [19]. Many research studies have been devoted to solving the computational efficiency problem of FEA-based optimisations, including the methods that combine FEA with various types of the stochastic algorithm [20, 21], the computationally efficient (CE) FEA methods [21–23], and the design of experiment (Doc)–based methods [24, 25]. However, the stochastic algorithm-based methods still use the time-consuming time-stepping (TS) FEA model as evaluating functions for multiple calculations; and the Doc methods cut the originally continuous search space into discrete points, which results in issues of local optimums. To reduce the required number of FEA cases in optimisations, surrogate-assisted methods are proposed and widely applied in machine optimisations [26–28]. Surrogate models are trained to replace the expensive-to-evaluate FEA models through machine learning and are used for subsequent optimisation. In [28], overviews of surrogate model-based methods are given while they also have good prospects in other industrial fields related to expensive-to-evaluate problems [29, 30]. Training high-performance surrogate models with a small expensive dataset is what researchers strive for; generally, a common approach is to train surrogate models with the data sampling by the efficient Doc methods, such as the Taguchi method [27]. Another tool to solve the lack of original data, transfer learning, has gradually been applied to the machine design optimisation field, especially in topology optimisations [31].

This paper proposes a novel surrogate-assisted design optimisation method based on analytical model transfer learning to significantly improve the optimisation efficiency while maintaining the high accuracy of surrogate models, which is applied for torque characteristics optimisation of a five-phase PMSM with quasi-trapezoidal PMs and full-pitched fractional slot concentrated winding (FSCW). The segmented PM [32] and the conformal mapping methods [16] are used to build analytical models of optimisation objectives. Then, using the high similarity between the analytical model and the FEA model, the computationally efficient analytical model data in the source domain is used to assist in the surrogate modelling for the FEA data in the target domain through a transfer learning support vector regression (TL-SVR) algorithm. In this way, the required number of FEA cases for training high generalisation capability surrogate models is greatly reduced, thus improving the optimisation efficiency. The optimal Latin hypercube design (LHD) and the Taguchi method are used to form the source and target domain datasets, respectively. The multi-objective optimisation is performed by the non-dominated sorting genetic algorithm II (NSGAII) [33].

The rest of this paper is organised as follows: Section 2 introduces the studied five-phase PMSM, the variation ranges of optimisation parameters are given. In Section 3, analytical models for optimisation objectives are built. Section 4 proposes the transfer learning-based surrogate-assisted method. Section 5 provides the results and analysis of the whole optimisation process. Besides, the optimal design is selected, prototyped, and tested for validation. Section 6 gives the conclusions.

2 | INITIAL DESIGN AND DESIGN PARAMETERS

Figure 1 shows a 20-slot-22-pole external-rotor five-phase PMSM and its five-phase FSCW arrangement. Offset teeth with 36° crossed electrical angle are introduced so that the stator coils can be full-pitched, which means that the winding factors for all harmonics are equal to 1 and the torque density enhancement from injected harmonic currents is benefited [9, 14]. Besides, this tooth-isolated structure has little mutual inductance between phases, which is conducive to the fault-tolerant operation [34]. The PMs with shaped profiles are used to improve torque characteristics, which consists of first, third, and fifth harmonic components. The maximum thickness of the PM is defined as $H_m$ and the edge thickness as $H_p$; the PM thickness $H_p(0)$ is the sum of $H_p$ and the variable thickness $H_v(0)$. Let $l = H_p/H_m$, $H_p (0)$ be expressed as (1). The fixed parameters are given in Table 1.

![Figure 1](image_url)  
**Figure 1** Five phase magnet shaping SPMSM
The following assumptions are made:

- The analytical model is to generate the source domain data for training the surrogate model, the requirements for its accuracy are not overemphasised. The following assumptions are made:

| TABLE 1 | Initial design parameters |
|----------|---------------------------|
| Fixed parameters | Values |
| Speed (r/min) | 600 |
| Rated power/current (kW/A) | 6.5/22 |
| Stator inner/outer radius (Rsi/Rso, mm) | 90/160.7 |
| Air-gap length (g, mm) | 1 |
| Rotor inner/outer radius (Rri/Rro, mm) | 166.4/180 |
| Pole arc coefficient | 1 |
| Offset/Coil tooth (elec. deg) | 36°/90° |
| Slot opening depth (mm) | 2 |
| Slot depth (mm) | 37 |
| Axial length (l_{a}, mm) | 30 |

\[
H_i(\theta) = H_m \left[ l + (1 - l) \cdot \left[ m_1 \sin(p\theta) + m_3 \sin(3p\theta) + m_5 \sin(5p\theta) \right] \right] 
\]

(1) where \( m_1, m_3, \) and \( m_5 \) denote the amplitude coefficients of the variable thickness of the first, third, and fifth harmonics, respectively. \( \theta \in [0, \pi/p] \), \( p \) denotes the number of pole-pairs.

Although the full-pitch FSCW helps to increase the torque density, it leads to higher harmonics of the back-EMF and larger cogging torque than conventional FSCWs [14, 27], which causes a high torque ripple. To comprehensively improve torque characteristics, three targets, including the average torque, the cogging torque percentage, and back-EMF total harmonic distortion (THD), are selected. The third harmonic current is injected to enhance the torque density; therefore, THD can be represented by:

\[
THD = \sqrt{\sum_{i=1/3}^{5/7} E_i^2} / \sqrt{E_1^2 + E_3^2} 
\]

3.1 Open-circuit slotless air-gap field solution

To analytically model the PMs with shaped profiles, the segmented PM method [32] is used as shown in Figure 2. PMs are segmented into pieces when the number of pieces \( N \) is large enough, each PM piece is considered as a rectangular magnet with a height of \( H_i(\theta_i) \) and corresponding to an arc of \( \pi/(Np) \), where \( \theta_i \) is the \( i \)th PM piece’s starting position angle and \( p \) is the number of pole pairs.

The open-circuit air-gap flux density generated by the \( i \)th PM piece can be expressed as [32],

\[
B_{ri} = \sum_{n} B_{rmi} \cos np\theta + \sum_{n} B_{rni} \sin np\theta 
\]

(3)

\[
B_{Bi} = \sum_{n} B_{bmi} \cos np\theta + \sum_{n} B_{bni} \sin np\theta 
\]

(4)

where \( n = 1, 3, 5, \ldots \). As PMs are radially magnetised, the parameters in Equations (3)–(4) can be expressed by

\[
\begin{align*}
B_{rmi} &= K_{Brc} f_{Br}, \\
B_{rni} &= K_{Brc} f_{Br} \\
B_{bmi} &= -K_{Brc} f_{B0}, \\
B_{bni} &= K_{Brc} f_{B0}
\end{align*}
\]

(5)

\[
\begin{align*}
f_{Br} &= (r/R_{mi})^{n_{p}-1} + (R_{so}/r)^{n_{p}+1}(R_{so}/R_{mi})^{n_{p}-1} \\
f_{B0} &= -(r/R_{mi})^{n_{p}-1} + (R_{so}/r)^{n_{p}+1}(R_{so}/R_{mi})^{n_{p}-1}
\end{align*}
\]

(6)

\[
\begin{align*}
K_{Brc} &= A_n y(n) \frac{4B_r}{n \pi \mu_0} \sin \frac{n \pi}{2N} \cos (n\theta_i) \\
K_{Brs} &= A_n y(n) \frac{4B_r}{n \pi \mu_0} \sin \frac{n \pi}{2N} \sin (n\theta_i)
\end{align*}
\]

(7)

where

\[
R_{mi} = R_{ri} - H_i(\theta_i)
\]

(8)

\[
A_n = -\mu_0 np/2
\]

(9)
\( \gamma(n) = \frac{2}{\rho(n^2 - 1)}(\frac{R_m}{R_m})^{np-1} \cdot \left[ (1 - np)(\frac{R_m}{R_r})^{2np} - 2(\frac{R_m}{R_r})^{np} + (np + 1) \right] \)

\( \rho = (\mu_r + 1)[(\frac{R_i}{R_r})^{2np} - 1] - (\mu_r - 1)[(\frac{R_m}{R_r})^{2np} - (\frac{R_i}{R_m})^{2np}] \)

where \( B_r \) and \( \mu_r \) denote the remanence and relative permeability of PMs, respectively. Thus, the open-circuit air-gap flux densities are

\[
B_r = \sum_{n=1}^{N} B_{rni} \cos np\theta + \sum_{n=1}^{N} B_{rni} \sin np\theta \\
B_\theta = \sum_{n=1}^{N} B_{\theta ni} \cos np\theta + \sum_{n=1}^{N} B_{\theta ni} \sin np\theta \tag{12}
\]

\[
B_{rni} = \sum_{i=1}^{N} B_{rni}, \quad B_{rni} = \sum_{i=1}^{N} B_{rni} \tag{13}
\]

Taking \([W, l, m_1, m_3, m_3] = [4, 0, 1.2, 0.28, 0.08]\) as an example and \( N \) is set to 200, the analytical and FEA field solutions in the slotless air-gap at radius \( r = R_c = R_m + g/2 \) are given in Figure 3.

### 3.2 Open-circuit slotted air-gap field solution

The slotted air gap magnetic field can be obtained by first calculating the complex relative air-gap permeance of the slotted air gap from the conformal mapping and then combining it with the slotless air gap magnetic field \([16, 17]\). Although the slot and tooth distribution of this motor is not uniform, its stator can be divided into several units, as shown in the red box in Figure 1. Therefore, the complex relative air-gap permeance forms a periodic distribution with the stator units as the span. The complex relative permeance is obtained by transforming the slotted air gap (\( S \) plane) into a slotless air gap (\( K \) plane) using conformal transformations, as shown in Figure 4, this process had been described in detail in \([16, 17]\).

If there is only one slot in a tooth pitch, for an external rotor motor, the four conformal transformations required in Figure 4 (c) are given in Equation (14), where \( a \) and \( b \) represent the values of \( w \) at the slot corner points \([16, 17]\). When multiple slots are considered, among the four transformations, it is only the Schwarz–Christoffel (SC) transformation (T2) from the \( Z \) plane to the \( W \) plane for which the analytical expression cannot be obtained. Therefore, the slotted air-gap permeance can be calculated in two ways, one is to assemble point by point using the solution for a single slot, the other one is to calculate the SC transformation numerically using MATLAB SC transformation toolbox \([35]\). In the optimisation, the air-gap permeance under multiple parameter combinations needs to be calculated. Therefore, to ensure computational efficiency, the former method is used to perform the SC transformation analytically.

\[
\begin{align*}
T1 : \quad \frac{\partial z}{\partial s} &= \frac{1}{s} \\
T2 : \quad \frac{\partial w}{\partial z} &= -j \frac{\pi}{\ln(R_i/R_m)} \frac{(w - 1)w}{(w + a)^{1/2}(w + b)^{1/2}} \\
T3 : \quad \frac{\partial l}{\partial w} &= j \frac{\ln(R_i/R_m)}{\pi} \frac{1}{w} \\
T4 : \quad \frac{\partial k}{\partial t} &= \theta = e^{\ln k} = k
\end{align*} \tag{14}
\]

The link between flux density in the \( S \) and \( K \) planes can be expressed as \([16]\),

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**Figure 2** Segmented PM

**Figure 3** Open-circuit field solutions in the slotless air-gap. (a) \( B_r \); (b) \( B_\theta \), FEA, finite element analysis.
\[ B_s = B_k \left( \frac{\partial k}{\partial s} \frac{\partial t}{\partial w} \frac{\partial z}{\partial s} \right)^* = B_k \lambda^* \]

where \( \lambda \) is the complex relative air-gap permeance with \( \lambda_a \) and \( \lambda_b \), as its real and imaginary parts. \( B_r \) and \( B_\theta \) can be obtained from the slotless air-gap field solution.

Since the period of \( \lambda \) is \( 4\pi/Q_\alpha \), \( \lambda \) can be expressed in the form of Fourier series in the entire air-gap as follows; the Fourier coefficients are calculated using the discrete Fourier transform.

\[ \lambda_a = \lambda_0 + \sum_{n=1}^{N_\lambda} \lambda_{an} \cos \left( \frac{nQ_\alpha}{2} \theta \right) \]

\[ \lambda_b = \sum_{n=1}^{N_\lambda} \lambda_{bn} \sin \left( \frac{nQ_\alpha}{2} \theta \right) \]

When using the same parameter combination as in the slotless air-gap magnetic field calculation, the complex relative permeance \( \lambda \) and the radial/tangential flux density \( (B_r, B_\theta) \) in the slotted air-gap at \( R_c \) are given in Figure 5 and Figure 6, respectively. From Figure 6, the radial component has almost no error, while the tangential component has a certain error [17, 18]. The main reason for this error is that when calculating the complex relative air-gap permeance, the deformation of the magnetic density calculation path caused by the conformal transformation of the slot opening is ignored [36].

### 3.3 The analytical model of three targets

Since the open-circuit slotted air-gap flux density distribution is obtained, three optimisation targets can be calculated. First, according to the Maxwell stress tensor method, the cogging torque can be expressed as.

\[ T_{\text{cog}} = \frac{R^2_{\text{eff}}}{\mu_0} \int_0^{2\pi} B_r B_\theta d\theta \]

Then, according to Faraday's law, the voltage induced in a single coil is equal to the negative derivative of the flux linked by the coil.

**Figure 4** Conformal mapping. (a) Slotted air-gap; (b) Slotless air-gap; (c) Steps required for finding the field solution in the slotted air gap based on conformal mapping.

**Figure 5** Complex air-gap permeance.

**Figure 6** No-load field solutions in the slotted air-gap. (a) \( B_r \); (b) \( B_\theta \).
where $N_c$ is the number of coil turns. For calculation of the flux linkage $\phi_c$, it is sufficient to know the only radial component of the air-gap flux density.

$$E_c(t) = -N_c \frac{d\phi_c(t)}{dt}$$

Finally, consider this five-phase motor is powered by the fundamental plus third harmonic currents ($\sin + \text{third currents}$), the average torque $T_{avg}$ is given by

Then, the phase back-EMF $E_{pb}$ is

$$E_{pb}(t) = N_{pb} \omega_c l_d R_c \sum_{n=1,3,5,\ldots} \frac{1}{\sin \left( \frac{np \gamma_c}{2} \right)} \left\{ 2\lambda_0 B_{ren} \sin \left( \frac{np \gamma_c}{2} \right) \right.$$
FIGURE 7 Analytical calculation results. (a) Cogging torque; (b) Back-EMF. EMF, electromotive force; FEA, finite element analysis.

\[ T_{\text{avg}} = \frac{5}{2\omega_r}(E_1 I_1 + E_3 I_3) \]  

(22)

where \( E_1, E_3 \) and \( I_1, I_3 \) denote the amplitudes of the fundamental and third harmonic of the phase back-EMF and the control current. When the effective value of the current is given, the ratio of \( I_3 \) to \( I_1 \) is set the same as the ratio \( E_3 \) to \( E_1 \) to get the maximum average torque [9].

The calculated phase back-EMF and cogging torque are given in Figure 7. The error in the calculation of \( B_{\text{avg}} \) leads to the error of calculated cogging torque [17, 18, 36]. Meanwhile, the error in the back-EMF is small, since the saturation is neglected, the analytical values of back-EMF harmonics are higher than those of the FEA and the value of THD is slightly larger. Therefore, the calculated \( T_{\text{avg}} = 101.5 \) Nm is higher than the FEA value of 100.3 Nm. A global sensitivity analysis (GSA) is performed using Sobol’s method within the above parameter space shown in Table II [37]. 1000 sets of samples are acquired by the analytical model with the Saltelli sample method. Figure 8 presents the GSA results characterised by the total sensitivity coefficient (TSC), which shows that \( I \) has the greatest effect on the three optimisation objectives while \( m_3 \) has the least effect.

4 | TRANSFER LEARNING-BASED SURROGATE ASSISTED DESIGN OPTIMISATION METHOD

In this section, a novel surrogate-assisted optimisation method based on analytical model transfer learning is proposed to train a surrogate model with strong generalisation capability using only a small training dataset within the Taguchi orthogonal array (OA) by FEAs.

FIGURE 8 Global sensitivity analysis results. THD, total harmonic distortion.

4.1 | Taguchi method

The Taguchi method is a typical DoE method, it is used here to capture the data feature between factors and targets with a minimum number of experiments [26]. FEAs are only performed within the OAs generated by the Taguchi method to acquire the values of optimisation targets. Therefore, the difficulty of obtaining training datasets is significantly reduced. As there are five optimisation parameters, if their variable levels are all set to 5, the size of the OA is \( L_{25} \), which means 25 open-circuit and on-load FEA simulation cases are required respectively to obtain the values of optimisation targets.

4.2 | Surrogate model training with transfer learning

The Taguchi method permits to effective capture of the complex relationship between optimisation parameters and
targets with a minimum number of FEA simulations, however, a small training dataset will lead to insufficient generalisation capability of surrogate models, and then cause overfitting problems. Overlearning of discrete data is the main reason for the overfitting, which prevents surrogate models from obtaining the relationship between the parameters and objectives that satisfy the real physical laws.

Transfer learning is an effective tool to solve the overfitting problem caused by insufficient data in the target domain [38]. To implement transfer learning, the analytical model is applied as the source domain that is data-rich and has high similarity with the target domain model, which can correctly reflect the influence law of the optimisation parameters on objectives despite its relatively low accuracy.

For example, when only \( I \) changes, the variation curves of cogging torque obtained by the analytical model and the FEA model are similar, as shown in Figure 9 (a). Due to this similarity, the data generated by the analytical model can help train surrogate models with high generalisation capability through transfer learning (Figure 9 (b)) [38].

In Figure 9, the expensive-to-evaluate FEA data within the Taguchi OAs forms the target domain while the analytical model data forms the source domain. The v-SVR, which has a good regression effect on a small sample dataset, is selected as the base learner for transfer learning [39]. Given a training set \( \{(x_{i1}, y_{i1}), \ldots, (x_{iN}, y_{iN})\} \), where \( x_i = [x_{i1}, x_{i2}, \ldots, x_{id}]^T \), \( d \) is the dimension, the optimisation objective of SVR is to minimise its structural risk term [39], and the optimisation function of the standard v-SVR model is

\[
\min \frac{1}{2} w^T w + C \left( \sum_{i=1}^{N} (\xi_i + \xi_i^*) \right)
\]

\[
\text{s.t.} \quad w^T \phi(x_i) - b - y_i \leq \varepsilon + \xi_i
\]

\[
y_i - w^T \phi(x_i) - b \leq \varepsilon + \xi_i^*
\]

\[
\xi_i, \xi_i^* \geq 0; \quad \varepsilon \geq 0
\]

where \( w \) and \( b \) are the coefficient matrix and the bias matrix, they are used to characterise the regression hyperplane of the SVR model; \( \varepsilon \) is the tolerance error, \( \xi_i, \xi_i^* \) are slack variables, corresponding to the two error bounds of SVR, respectively; \( \phi(x) \) is the kernel function. This complex problem can be transformed into a convex quadratic programming dual problem to solve using the Lagrange multiplier method [39],

\[
\max -\frac{1}{2}(\alpha^* - \alpha)^T K(x_i, x_j)(\alpha^* - \alpha) + \sum_{i=1}^{N} \varepsilon
\]

\[
\text{s.t.} \quad \sum_{i=1}^{N} (\alpha^* - \alpha) = 0, \quad \sum_{i=1}^{N} (\alpha^* + \alpha) = C \nu
\]

\[
0 \leq \alpha, \alpha^* \leq C/N
\]

where \( \alpha \) is the Lagrange multiple, \( K \) is the kernel matrix.

To overcome the overfitting problem, a TL-SVR is proposed. The principle optimisation objective of this algorithm can be expressed as

\[
\min \text{risk}(f_t) + \text{risk}(f_s) + \lambda \text{diff}(f_t, f_s)
\]

where the subscript \( t \) and \( s \) denote the target domain and source domain, respectively. \( \text{risk}(f_t) \) and \( \text{risk}(f_s) \) are the structural risk term when training the target and source domain model, respectively, they correspond to (23). \( \text{diff}(f_t, f_s) \) denotes the difference between the target domain model and the source domain model, this difference can be characterised by the difference of the regression hyperplane. Thus, \( \text{diff}(f_t, f_s) = \|w_t - w_s\|_2^2 + (b_t - b_s)^2 \). \( \lambda \) is a weight factor, it is used to balance structural risks and model difference. This optimisation function Equation (25) is equivalent to the following two requirements: (1). The surrogate models in the target and source domains can obtain good regression results on the training sets corresponding to their respective domains. (2). The differences between the hyperplanes corresponding to the source and target domain data are as small as possible.

Then, assuming the source domain data \( \{(x_{is}, y_{is}), 1 \leq i \leq N_s\} \) and the target domain data \( \{(x_{it}, y_{it}), N_t \leq i \leq N_1\} \), where \( N_t \) and \( N \) denote the number of data in the source domain and the total number of data in the target and source domains,

\[
\text{FIGURE 9} \quad \text{Analytical model transfer learning. (a) finite element analysis and analytical analysis of } I \text{ on the cogging torque. (b) Principle of the transfer learning}
\]
respectively. Add up the expressions corresponding to each of the three terms in Equation (25), the final optimisation function is,

\[
\min_{w, b, \xi_i^t, \eta_i^t} \frac{1}{2}\|w\|^2 + \frac{1}{2}\|w_i\|^2 + C \left( \varphi_{\lambda} + \frac{1}{N} \sum_{i=1}^{N} (\xi_i + \xi_i^t) \right)
\]

\[
+ \frac{1}{N} \sum_{i=1}^{N} \left( (\xi_i + \xi_i^t) \right) + \frac{1}{2} \left( \|w_i - w_i\|^2 + (b_i - b_i^t) \right)
\]

s.t. \( w_i^T \phi(x_i) + b_i - y_i \leq \varepsilon + \xi_i^t; \quad i = 1, 2, \ldots, N \)

\( y_i - w_i^T \phi(x_i) - b_i \leq \varepsilon + \xi_i^t; \quad i = 1, 2, \ldots, N \)

\( w_i^T \phi(x_i) + b_i - y_i \leq \varepsilon + \xi_i^t; \quad i = 1 + N_s, 2 + N_s, \ldots, N \)

\( y_i - w_i^T \phi(x_i) - b_i \leq \varepsilon + \xi_i^t; \quad i = 1 + N_s, 2 + N_s, \ldots, N \)

\( \xi_i, \xi_i^t \geq 0; \quad \varepsilon \geq 0 \)

(26)

Lagrange multipliers are applied to each restriction condition of Equation (26), then, set the partial derivative of each parameter equal to zero, its dual problem can be obtained as Equation (27). The derivation process is given in the Appendix.

\[
\max -\frac{1}{2}(\alpha^* - \alpha)^T H(x_i, x_j) (\alpha^* - \alpha) + (\alpha^* - \alpha)^T y
\]

\[
s.t. \sum_{i=1}^{N} (\alpha^* - \alpha) = 0, \sum_{i=1}^{N} (\alpha^* + \alpha) = C v
\]

\[0 \leq \alpha, \alpha^* \leq C/N\]

where

\[
H = [K_{ij}]_{N \times N} + [Q_{ij}]_{N \times N}
\]

\[
K_{ij} = P_{ij} \cdot \phi(x_i)^T \phi(x_j)
\]

\[
P_{ij} = \begin{cases} 
1 + \frac{\lambda}{1 + 2\lambda}, & 1 \leq i, j \leq N_i \text{ or } 1 + N_i \leq i, j \leq N \\
1 + \frac{\lambda}{1 + 2\lambda}, & \text{else}
\end{cases}
\]

(30)

\[
Q_{ij} = \begin{cases} 
\frac{1}{4\lambda}, & 1 \leq i, j \leq N_i \text{ or } 1 + N_i \leq i, j \leq N \\
-\frac{1}{4\lambda}, & \text{else}
\end{cases}
\]

(31)

The dual form of Equation (26) is equivalent to another ordinary SVR problem in the kernelised space. It can be solved

**Figure 10** Flowchart of optimisation process. LHD, Latin hypercube design
in the same way as the traditional SVR problem, such as the sequential minimal optimisation (SMO) method [39] or with the help of the well-known libSVM [40].

Through transfer learning, the SVR is still able to make accurate predictions for the target domain data, while the overfitting problem that would occur when the target domain data volume is small can be mitigated using the data in the highly similar source domain. Determination coefficient $R^2$ is used to measure the regression effect,

$$R^2 = 1 - \frac{\sum_{i=1}^{n} (y_i - \bar{y})^2}{\sum_{i=1}^{n} (y_i - \bar{y})^2}$$

where $y_i$, $\bar{y}$, $\bar{y}_i$ denote the measured data, the mean value of the measured data, and the predicted data, respectively, and $n$ is the number of data.

### 4.3 Multi-objective optimisation

The multi-objective optimisation is performed using the NSGAII with the three optimisation objectives obtained by the surrogate model,

- **maximize**: $T_{\text{avg}}$
- **minimize**: THD
- **minimize**: $100\% \cdot \frac{T_{\text{cog}}(p_k-p_k)}{T_{\text{avg}}}$

$$s.t. : H_m = \frac{R_{\text{tr}} - R_{\text{so}} - g}{l + (1 - l) \cdot \max \left\{ m_1 \sin(p\theta), m_3 \sin(3p\theta), m_5 \sin(5p\theta) \right\}}$$

**Table 3** Taguchi array and S/N ratios

| Parameters ([W, L, m1, m3, m5]) | AT (Nm) | S/N  | CT (%) | S/N  | THD (%) |
|---------------------------------|---------|------|--------|------|---------|
| [1, 1, 1, 1, 1]                 | 95.6    | 39.61| 0.729  | 2.75 | 2.62    | −8.38  |
| [1, 2, 2, 2, 2]                 | 98.7    | 39.89| 0.886  | 1.05 | 2.27    | −7.13  |
| [1, 3, 3, 3, 3]                 | 101.4   | 40.12| 0.590  | 4.58 | 0.923   | 0.69   |
| [1, 4, 4, 4, 4]                 | 103.5   | 40.29| 0.676  | 3.40 | 2.61    | −8.32  |
| [1, 5, 5, 5, 5]                 | 105.7   | 40.48| 2.883  | −9.20| 6.92    | −16.8  |
| [2, 1, 2, 3, 4]                 | 95.1    | 39.56| 1.846  | −5.32| 5.21    | −14.3  |
| [2, 3, 4, 4, 5]                 | 98.3    | 39.85| 0.967  | 0.29 | 3.95    | −11.9  |
| [2, 3, 5, 5, 1]                 | 98.9    | 39.90| 0.473  | 6.50 | 0.521   | 5.66   |
| [2, 4, 5, 1, 2]                 | 101.7   | 40.15| 1.124  | −1.01| 2.88    | −9.19  |
| [2, 5, 1, 2, 3]                 | 106.0   | 40.51| 2.007  | −6.05| 6.39    | −16.1  |
| [3, 1, 3, 5, 2]                 | 93.4    | 39.41| 4.375  | −12.8| 4.17    | −12.4  |
| [3, 2, 4, 1, 3]                 | 92.0    | 39.27| 0.063  | 23.9 | 1.02    | −0.21  |
| [3, 3, 5, 2, 4]                 | 97.0    | 39.73| 0.946  | 0.48 | 0.483   | 6.33   |
| [3, 4, 1, 3, 5]                 | 103.0   | 40.25| 2.888  | −9.21| 2.57    | −8.19  |
| [3, 5, 2, 4, 1]                 | 106.3   | 40.53| 3.944  | −11.9| 5.72    | −15.2  |
| [4, 1, 4, 2, 5]                 | 86.2    | 38.71| 0.339  | 9.41 | 2.82    | −9.01  |
| [4, 2, 5, 3, 1]                 | 99.8    | 39.98| 1.276  | −2.12| 1.62    | −4.19  |
| [4, 3, 1, 4, 2]                 | 101.0   | 40.08| 0.302  | 10.4 | 0.535   | 5.43   |
| [4, 4, 2, 5, 3]                 | 103.1   | 40.26| 3.517  | −10.9| 2.17    | −6.74  |
| [4, 5, 3, 1, 4]                 | 106.5   | 40.54| 6.194  | −15.8| 4.99    | −13.9  |
| [5, 1, 5, 4, 3]                 | 99.3    | 39.94| 5.560  | −15.0| 3.68    | −11.3  |
| [5, 2, 1, 5, 4]                 | 95.0    | 39.55| 3.110  | −9.85| 2.30    | −7.22  |
| [5, 3, 2, 1, 5]                 | 94.1    | 39.47| 3.111  | −9.86| 0.621   | 4.143  |
| [5, 4, 3, 2, 1]                 | 104.7   | 40.40| 4.288  | −12.6| 1.87    | −5.45  |
| [5, 5, 4, 3, 2]                 | 106.6   | 40.55| 8.421  | −18.5| 4.25    | −12.6  |
To improve the computational efficiency of obtaining a large source domain dataset by the analytical model, the space-filling LHD is used [41]. Analytical calculations are only performed within the optimal LHD, the size of LHD is set to 500. Figure 10 shows the optimisation process.

5 | RESULTS AND VALIDATION

5.1 | FEA and analytical data collection

To obtain the target domain training dataset, an $L_{25}$ Taguchi OA is built in the parameter space and FEA simulations are only carried out within the OA. Table 3 gives the values of objectives and their signal-to-noise (S/N) ratios, average torque and cogging torque are abbreviated as AT and CT, respectively. The S/N ratios under different levels of each parameter are shown in Figure 11(a) to Figure 11(e). The influence degree of different parameters on the same optimisation objective can be compared by the variation range of their S/N ratios. Define the impact factor $IF$ of parameter $i$ on objective $j$ as Equation (34), where $range_{ij}$ indicates the S/N ratio variable range of parameter $i$ on objective $j$. The total impact factor of parameters is also given, which is close to the GSA result.

$$IF_{ij} = \frac{range_{ij}}{\sum \limits_{k=A,B,C,D,E} range_{kj}}$$ (34)

Analytical calculations are performed within an LHD $(500,5)$ (Figure 12(a)), where 500 and 5 denote the number of data and dimension, respectively. The Morris-Mitchell index $\phi_L$ of this LHD is optimised with the iterated local search method to enhance its spatial uniformity [41], the optimisation iteration is set to 300 (Figure 12(b)). The source domain data obtained using the analytical model is given in Table 4.

The state with the thinnest edge of PMs, that is, $l = 0$, is selected for irreversible demagnetisation investigation, a no-load sudden short-circuit simulation is performed, the flux density $B$ when the phase current reaches its maximum ($60A$) is given in Figure 13. The knee point of PMs is $-0.2$ T ($120^\circ$C), which is much lower than the minimum $B$. Thus, irreversible demagnetisation will not occur.

![Figure 11](image.png)

**Figure 11** S/N ratios under different factor levels. (a) Average torque. (b) Cogging torque. (c) THD. (d) Impact factor. THD, total harmonic distortion.
Irreversible demagnetisation

**FIGURE 12** Latin hypercube design (LHD). (a) Parallel coordinates of optimal LHD. (b) Curve of $\phi_p$ during the optimisation

**TABLE 4** Analytical data within the optimal LHD

| Parameters ([W, $l$, $m_x$, $m_y$, $m_z$]) | AT  | CT  | THD |
|-------------------------------------------|-----|-----|-----|
| [5.212, 0.365, 1.266, 0.231, 0.189]       | 96.15 | 1.703 | 1.738 |
| [5.224, 0.302, 1.241, 0.222, 0.120]       | 98.76 | 1.162 | 2.015 |
| [3.818, 0.024, 1.175, 0.265, 0.178]       | 94.88 | 2.566 | 3.942 |
| [3.595, 0.443, 1.276, 0.211, 0.082]       | 101.4 | 0.558 | 0.938 |
| [5.278, 0.445, 1.265, 0.301, 0.121]       | 105.1 | 0.613 | 1.376 |
| ...                                       | ...  | ...  | ... |
| [4.443, 0.451, 1.225, 0.368, 0.191]       | 104.3 | 0.820 | 2.263 |
| [3.511, 0.142, 1.189, 0.349, 0.094]       | 101.8 | 3.125 | 3.888 |
| [4.256, 0.355, 1.126, 0.296, 0.186]       | 101.7 | 1.370 | 2.060 |
| [3.631, 0.636, 1.207, 0.307, 0.127]       | 106.8 | 1.799 | 2.220 |
| [3.271, 0.623, 1.129, 0.347, 0.184]       | 105.6 | 0.712 | 1.764 |

Abbreviations: LHD, Latin hypercube design.

5.2 Surrogate model establishment

Surrogate models for each objective are trained using the method given in Section 3-B, the hyperparameter settings in
the TL-SVR can use the existing toolkit in libSVM [40], and $\lambda$ for three surrogate models are all set to 5. A new test set
(25 sets of data) is obtained, these data are randomly chosen from the 5 parameters, 5 levels dataset, but differ from the
available Taguchi data. The optimisation objective values of the test dataset are obtained by the FEA model and the
analytical model. Figure 14 shows the regression results on the test set, SVR 1 denotes the traditional SVR trained with
the target domain Taguchi dataset, SVR 2 denotes the traditional SVR trained with the source domain analytical
model dataset, and TL-SVR denotes the analytical model-based transfer learning SVR. ELM denotes the extreme
learning machine (ELM) model in [27], it is trained with a total of 125 sets of FEA data in the main space and four
subspaces. $R^2$s of these surrogate models on the test set are given in Figure 14(d), in which SVR 2 is validated on both
the test set generated by the FEA model and the analytical model.

All these surrogate models can achieve good regression results on their corresponding training sets. As shown in
Figure 14, the $R^2$s of SVR 1 is the lowest; although it can accurately predict the training set data, it appears to overfit,
the surrogate model performs poorly on the test set. The SVR 2 has high $R^2$s on the test set generated by the analytical
model, but the prediction results are not accurate enough on the FEA test set due to the deviation between the analytical
model and the FEA model, however, the variation trend is basically correct. TL-SVR has a significantly better regression
effect on the FEA test set, it proves that through transfer learning of the data generated by the analytical model, the
overfitting is mitigated and the generalisation capacity of the surrogate model is enhanced. Compared with SVR 1, the $R^2$s
of TL-SVR are increased by 0.144 (average torque), 0.181 (cogging torque), and 0.169 (THD). Since the surrogate
model is trained entirely with FEA data, ELM achieves...
comparable $R^2$ to TL-SVR for three optimisation objectives. However, ELM requires more FEA data for training and therefore will consume more time.

5.3 Multi-objective optimisation and robustness assessment

As surrogate models with high generalisation capacity are obtained, the multi-objective optimisation is performed using the NSGAII, taking the outputs of the surrogate model as the evaluate functions. The number of populations and generations are set to 400 and 2000. Figure 15 shows the final optimal 3D and 2D Pareto frontiers, an optimal design is selected. Two methods are used to validate the proposed method, one is the surrogate-assisted method given in [27], it uses the Taguchi data within four subspaces and an extreme learning machine (ELM) to train surrogate models; the other one is the FEA-based parametric scanning method and 5 levels are selected for each factor. The required number of FEA cases for the two methods are 125 and 3125, respectively.

Table 5 gives the comparison of these three methods, Method 1 is the proposed method, Method 2 is the method given in [27], and Method 3 is the FEA-based parametric scanning method. FEA cases and surrogate model training processes are performed on a computer with 4 cores, 16G RAM, and a 4.2 GHz CPU. For Method 1 and 2, the values of three optimisation objectives are calculated by surrogate models, and for Method 3, they are obtained by FEA. It should be noted that despite the simple structure of ELM and its faster training speed compared with SVR, this has a negligible impact on the total time spent on the optimisation process and the difference in time consumption is mainly reflected in the number of FEAs required. Method 1 has the best computational efficiency, Method 2 and 3 require a larger number of FEA cases, they are more time consuming. Meanwhile, the surrogate model-based optimisation method guarantees a complete and continuous parameter space and thus does not fall into local optima as easily as parametric scanning methods that discretise the search space.

Referring to [42], a robustness assessment of the optimisation result is performed to consider the effect of manufacturing tolerances (MTs) on optimisation objectives to
4.05
eq 1.6e-2

\[ 0.508 \quad 0.01 \quad 3.3e-3 \]

\[ 1.213 \quad 5e-3 \quad 1.6e-3 \]

\[ 0.298 \quad 5e-3 \quad 1.6e-3 \]

\[ 0.095 \quad 5e-3 \quad 1.6e-3 \]

TABLE 6 Parameter setting for robustness assessment

| Parameter | \( \mu \) | \( g \) | \( \sigma \) |
|-----------|-----------|-----------|-----------|
| \( W \) (mm) | 4.05 | 0.05 | 1.6e-2 |
| \( l \) | 0.508 | 0.01 | 3.3e-3 |
| \( m_1 \) | 1.213 | 5e-3 | 1.6e-3 |
| \( m_3 \) | 0.298 | 5e-3 | 1.6e-3 |
| \( m_5 \) | 0.095 | 5e-3 | 1.6e-3 |

analyse which objective is more vulnerable to MTs. In the assessment, MTs consist of a bilateral deviation of the rated value of a dimension of the type \( x \pm g \). It is assumed that the MTs of each parameter satisfies a normal distribution \( N(\mu, \sigma^2) \) in the robustness assessment, according to the six-sigma rule, the probability that the dimension \( x \) is in the range \([\mu - 3\sigma, \mu + 3\sigma]\) is 99.7% [42]. The probability density distribution \( D_x \) of this normal distribution is given as follows,

\[
D_x = \frac{1}{\sigma \sqrt{2\pi}} e^{-\frac{(x-\mu)^2}{2\sigma^2}}
\]

where \( x \) is the value of the random variable, \( \mu \) is the mean value, \( \mu \) of each optimisation variable is set to its optimisation result; \( \sigma \) is the standard deviation, \( \sigma = g/3 \).

Tolerance \( g \) needs to be determined by the specific manufacturing process; in this work, \( g = 0.05 \text{ mm} \) is assumed to be a representative value for the MTs of the PM total thickness \( H_m \) and the slot opening width \( W \). Considering that there are no existing tolerance reference values for \( l, m_1, m_3, \) and \( m_5 \), the approximate \( g \) of these parameters are determined using the \( g \) of \( H_m \). Taking \( m_1 \) as an example to explain this process, assume the MT of \( H_m \) is only caused by the MT of \( m_1 \), the following equation can be yielded,

\[
H_m + g_{Hm} = H_m \left( l + (1 - l) \cdot \max \left( \frac{m_1 + g_1}{H_m(1-l)} = \max \left[ \left( m_1 + g_1 \sin(3\theta) \right), \left( m_1 + g_1 \sin(5\theta) \right) \right] \right) \right)
\]

where \( g_{Hm} \) and \( g_1 \) denote the MT of \( H_m \) and \( m_1 \), respectively. \( g_{Hm} = 0.05 \text{ mm} \). \( g_1 \) is determined numerically. The \( g \) of \( l, m_3, \) and \( m_5 \) are determined in the same way, as shown in Table 6.

Under this setting given in Table 6, the Monte Carlo method provides the selection of different sets of optimisation

FIGURE 15 Pareto frontiers. (a) 3D frontier. (b) 2D frontier

TABLE 5 Comparison of optimisation results

| Parameters and Objectives | Method 1 | Method 2 | Method 3 |
|---------------------------|----------|----------|----------|
| \( W \) (mm) | 4.05 | 4.03 | 4.5 |
| \( l \) | 0.508 | 0.511 | 0.5 |
| \( m_1 \) | 1.213 | 1.204 | 1.2 |
| \( m_3 \) | 0.298 | 0.293 | 0.3 |
| \( m_5 \) | 0.095 | 0.091 | 0.08 |
| AT (Nm) | 104.6 | 104.4 | 103.8 |
| CT (%) | 0.31 | 0.29 | 0.57 |
| THD (%) | 0.76 | 0.75 | 1.02 |
| Time-cost (h) | 3.4 | 7.1 | 117 |
| \( R^2 \) (AT) | 0.989 | 0.991 | 0.991 |
| \( R^2 \) (CT) | 0.974 | 0.982 | 0.994 |

Abbreviations: THD, total harmonic distortion.
variables [42]. Three optimisation objectives are computed by FEA and 1000 sets of data are used to calculate the mean value and the variance of each objective.

Figure 16 shows the probability density distribution and Gauss fitting results of three objectives. The Shannon Entropy [43] and signal-to-noise ratio (Nominal is best) are used to check the robustness of three objectives; Shannon Entropy and signal-to-noise ratio (Nominal is best) can be calculated as Equation (36) and Equation (37), where \( \mu \) and \( \sigma \) are the mean value and the standard deviation, respectively. The robustness comparison results are given in Table 7.

\[
H = -\sum P_i \ln P_i \quad (36)
\]

where \( P_i \) represents the mass density of a parameter. A more complex, and less predictable parameter carries higher entropy and vice versa.

\[
S/N = 20 \log_{10} \left( \frac{\mu}{\sigma} \right) \quad (37)
\]

From Table 7, since the optimisation objective with larger Shannon entropy and signal-to-noise ratio has better robustness, the robustness can be ranked from better to worse, that is, average torque > THD > cogging torque. This result can also be explained by the GSA result (Figure 8), where the average torque is mainly influenced by \( \mathcal{W}_r / \bar{I} \) and \( m_2' \); while all five optimisation variables have a great influence on THD and coggng torque. Therefore, THD and coggng torque are more influenced by MTs.

5.4 Test validation

The five optimisation parameters were taken as \( [4, 0.51, 1.21, 0.3, 0.09] \) to make a prototype, the rotor and stator of the prototype were given in Figure 17(a) and Figure 17(b). The back-EMF and its fast Fourier transformation (FFT) result are given in Figure 17(c) and Figure 17(d), the tested value of THD is 0.82%. Using the weighted method [44], the cogging torque (Figure 17(e)) was tested in the open-circuit condition. The tested cogging torque is 0.44 Nm, which is 1.4 times the value of FEA (0.31 Nm).

Set the effective value of input current to 22 A, under different \( R = I_2 / I_1 \), the motor's average torque was tested. Figure 18(a) to Figure 18(e) shows the input currents under \( R = 0.05, 0.1, 0.238, 0.3 \), and 0.4, in which \( E_s / E_1 = 0.238 \).

From Figure 18(f), when \( I_2 / I_1 = E_s / E_1 \), the average torque \( T_{\text{avg}} \) takes its maximum, which validates the conclusion given in [9]. Meanwhile, due to the manufacturing deviations and various losses, the tested values of average torque are lower than those of the FEA values.

The values of the three optimisation objectives are summarised in Table 8. The surrogate model outputs match well with the results of the FEA and the test, which proves its high generalisation capacity and validates the proposed optimisation method.

6 CONCLUSION

This paper proposes an improved surrogate-assisted design optimisation method using the transfer learning technique for the torque characteristics optimisation of a five-phase surface-mounted PMSM with harmonic-injected PMs. Using the high similarity between the analytical model and the FEA model, the highly computationally-efficient analytical model data in the

**TABLE 7** Comparison results of robustness assessment

| Objectives | AT  | CT   | THD |
|------------|-----|------|-----|
| Mean value | 104.4 | 0.302 | 0.751 |
| Standard deviation | 0.057 | 0.012 | 0.024 |
| Shannon entropy | 1.43 | 2.91 | 2.77 |
| Signal-to-noise | 65.1 | 28.0 | 29.9 |

**FIGURE 16** Probability density distribution and Gauss fitting results of three objectives. (a) Average torque. (b) Cogging torque. (c) THD. THD, total harmonic distortion.
source domain data is used to assist in the surrogate modelling for the FEA data in the target domain by transfer learning. The trained surrogated model using the proposed method can achieve similar accuracy and significantly higher optimisation efficiency compared with the FEA-based optimisation. Moreover, the issues of local optimums can be avoided.

In the proposed method, the analytical model of the SPMSM with external rotor and shaped PM profiles is developed. Besides, the Taguchi and optimal LHD, and the TL-SVR methods are utilised for generating datasets and surrogate model training, respectively, in the surrogated model while using transfer-learning. The NSGA II is employed for multi-objective optimisation. The validity of the proposed method is verified by experimental results.

The method is also applicable for design optimisation of other electric machines, and the transfer learning technique can be applied to the optimisation of other expensive-to-evaluate problems or deal with the problem of insufficient historical data when training surrogate models.

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FIGURE 18 On-load test. (a) $R = 0.05$. (b) $R = 0.1$. (c) $R = 0.238$. (d) $R = 0.3$. (e) $R = 0.4$. (f) Average torque

TABLE 8 Prototype test results

| Targets                  | TL-SVR | FEA   | Analytical | Test  |
|--------------------------|--------|-------|------------|-------|
| THD (%)                  | 0.76   | 0.74  | 0.86       | 0.82  |
| Cogging torque (%)       | 0.31   | 0.30  | 0.51       | 0.43  |
| Average torque (Nm)      | 104.6  | 104.4 | 105.7      | 101.9 |

Abbreviations: THD, total harmonic distortion; FEA, finite element analysis.

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REFERENCES

1. Bilgin, B., et al.: Making the case for electrified transportation. IEEE Trans. Transp. Electrific. 1(1), 4–17 (2015)
2. Chasiotis, I.D., Karnavas, Y.L.: A generic multi-criteria design approach towards high power density and fault-tolerant low-speed PMSM for pod applications. IEEE Trans. Transp. Electrific. 5(2), 356–370 (2019)
3. Liang, J., et al.: Optimisation of air-gap profile in interior permanent-magnet synchronous motors for torque ripple mitigation, IEEE Trans. Transp. Electrific. 5(1), 118–125 (2019)
4. Barrero, F., Duran, M.J.: Recent advances in the design, modelling, and control of multiphase machines—part I, IEEE Trans. Ind. Electron. 63, (1), 449–458 (2016)
5. Sun, Y., et al.: Torque improvement in dual m-phase permanent-magnet machines by phase shift for electric ship applications, IEEE Trans. Veh. Technol. 69(9), 9601–9612 (2020)
6. Cao, W., et al.: Overview of electric motor technologies used for more electric aircraft (MEA). IEEE Trans. Ind. Electron. 59(9), 3523–3531 (2012)
7. Zhao, T., Wu, S., Cai, S.: Multiphase PMSM with asymmetric windings for more electric aircraft. IEEE Trans. Transp. Electrific. 6(4), 1592–1602 (2020)
8. Parsa, L., Toliyat, H.A.: Five-phase permanent magnet motor drives. IEEE Trans. Ind. Appl. 41(1), 30–37 (2005)
9. Wang, K., et al.: Design and analysis of five phase SPM machine considering third harmonic current injection. IEEE Trans. Energy Convers. 33(3), 1108–1117 (2018)
10. Cervone, A., et al.: Optimal third-harmonic current injection for asymmetrical multiphase permanent magnet synchronous machines, IEEE Trans. Ind. Electron. 68(4), 2772–2783 (2021)
11. Li, Y., Zou, J., Lu, Y.: Optimum design of magnet shape in permanent-magnet synchronous motors, IEEE Trans. Magn. 39(6), 3523–3526 (2003)
12. Wang, K., Zhu, Z.Q., Ombach, G.: Torque improvement of five-phase surface-mounted permanent magnet machine using third-order harmonic, IEEE Trans. Energy Convers. 29(3), 735–747 (2014)
13. Wang, K., et al.: Average torque improvement of interior permanent-magnet machine using 3rd harmonic in rotor shape, IEEE Trans. Ind. Electron. 61(9), 5047–5057 (2014)
14. Kang, H., et al.: Magnet shape optimisation of five-phase surface-mounted permanent magnet machine to improve torque performance based on equivalent permanent magnet method. IEEJ Trans. Electr. Electron. Eng. 10, S133–S143 (2015)
15. Wang, K., et al.: Optimum injected harmonics into magnet shape in multiphase surface-mounted PM machine for maximum output torque, IEEE Trans. Ind. Electron. 64(6), 4434–4443 (2017)
16. Zariko, D., Ban, D., Lipo, T.A.: Analytical calculation of magnetic field distribution in the slotted air gap of a surface permanent-magnet motor using simple complex relative air gap permeance, IEEE Trans. Magn. 42(7), 1828–1837 (2006)
17. Zariko, D., Ban, D., Lipo, T.A.: Analytical solution for cogging torque in surface permanent-magnet motors using conformal mapping, IEEE Trans. Magn. 44(1), 52–65 (2008)
18. Oner, Y., et al.: Analytical on-load subdomain field model of permanent-magnet vernier machines, IEEE Trans. Ind. Electron. 63(7), 4105–4117 (2016)
19. Bramerdorfer, G., et al.: Modern electrical machine design optimisation: techniques, trends, and best practices, IEEE Trans. Ind. Electron. 65(10), 7672–7684 (2018)
20. Vidanalage, B.D.S.G., Toulabi, M.S., Filizadeh, S.: Multimodal design optimisation of V-shapped magnet IPM synchronous machines. IEEE Trans. Energy Convers. 33, (3), 1547–1556 (2018)
21. Sizov, G.Y., et al.: Automated multi-objective design optimisation of PM AC machines using computationally efficient FEA and differential evolution, IEEE Trans. Ind. Appl. 49(5), 2086–2096 (2013)
22. Wu, T., et al.: Calculation of eddy current loss in a tubular oscillatory LPM NSM using computationally efficient FEA, IEEE Trans. Ind. Electron. 66(8), 6200–6209 (2019)
23. Chen, H., et al.: Computationally efficient optimisation of a five-phase flux-switching PM machine under different operating conditions. IEEE Trans. Veh. Technol. 68(7), 6495–6508 (2019)
24. Song, J., et al.: Optimal design of permanent magnet linear synchronous motors based on taguchi method, IET Electr. Power. Appl. 11(1), 41–48 (2017)
25. Shi, Z., et al.: Robust design optimisation of a five-phase PM hub motor for fault-tolerant operation based on taguchi method, IEEE Trans. Energy Convers. 35(4), 2036–2044 (2020)
26. Song, J., et al.: An efficient multiobjective design optimisation method for a PMSL based on an extreme learning machine, IEEE Trans. Ind. Electron. 66(2), 1001–1011 (2019)
27. Ma, Y., et al.: Surrogate-assisted optimisation of a five-phase SPM machine with quasi-trapezoidal PMs, IEEE Trans. Ind. Electron. (2021) (Early Access). https://doi.org/10.1109/TIE.2020.3048279
28. Duan, Y., Ionel, D.M.: A review of recent developments in electrical machine design optimisation methods with a permanent-magnet synchronous motor benchmark study, IEEE Trans. Ind. Appl. 49(3), 1268–1275 (2013)
29. Jiang, C., Bui, T.B.: Reinforcement neural fuzzy surrogate-assisted multiobjective evolutionary fuzzy systems with robot learning control application. IEEE Trans. Fuzzy Syst. 28(3), 434–446 (2020)
30. Xia, Y., Xu, Y., Gou, B.: A data-driven method for IGBT open-circuit fault diagnosis based on hybrid ensemble learning and sliding-window classification, IEEE Trans. Ind. Informat. 16, (8), pp. 5223–5233 (2020)
31. Asanuma, J., Doi, S., Igarashi, H.: Transfer learning through deep learning application to topology optimisation of electric motor, IEEE Trans. Magn. 56(3), 1–4 (2020)
32. Wu, L.J., Zhu, Z.Q.: Analytical modelling of surface-mounted PM machines accounting for magnet shaping and varied magnet property distribution. IEEE Trans. Magn. 50(7), 1–11 (2014)
33. Deb, K., et al.: A fast and elitist multiobjective genetic algorithm: NSGA-II, IEEE Trans. Evol. Comput. 6(2), 182–197 (2002)
34. Sui, Y., et al.: A novel five-phase fault-tolerant modular in-wheel permanent-magnet synchronous machine for electric vehicles, J. Appl. Phys. 117(17), 17B521–17B521-4 (2015)
35. Driscoll, T.A.: Schwarz–Christoffel toolbox user's guide: Version 2.3, DE Dept. Math. Sci., Univ. Delaware, Newark (2005)
36. Wu, L.J., et al.: Comparison of analytical models of cogging torque in surface-mounted PM machines, IEEE Trans. Ind. Electron. 59(6), 2414–2425 (2012)
37. Sobol, I.M.: Global sensitivity indices for nonlinear mathematical models and their Monte Carlo estimates, Math. Comput. Simul. 55, 271–280 (2001)
38. Pan, S.J., Yang, Q.: A survey on transfer learning, IEEE Trans. Knowl. Data Eng. 22(10), 1345–1359 (2010)
39. Scholkopf, B., et al.: New support vector algorithm, Neural Comput. 12(5), 1207–1245 (2000)
40. Chang, C.-C., Lin, C.-J.: LIBSVM: a library for support vector machines, ACM Trans. Intel. Syst. Tec. 2(3), Article 27 (2011)
41. Grosso, A., Jamali, A.R.M.J.U., Locatelli, M.: Finding maxmin Latin hypercube designs by iterated local search heuristics, Eur. J. Oper. Res. 197, 541–547 (2009)
42. Credo, A., Fabri, G., Villani, M.: A robust design methodology for synchronous reluctance motors, IEEE Trans. Energy Convers. 35(4), 2095–2105 (2020)
43. Zou, Y., et al.: Automatic image thresholding based on Shannon entropy difference and dynamic synergetic entropy, IEEE Access. 8, 171218–171239 (2020)
44. Dutta, R., Chong, L., Rahman, M.: Design and experimental verification of an 18-slot/14-pole fractional-slot concentrated winding interior permanent magnet machine, IEEE Trans. Energy Convers. 28(1), 181-190 (2013)
APPENDIX

The minimisation problem in Equation (26) can be solved by transforming it into the following dual problem [39, 40],

$$\max \min L_{\alpha, \xi} = \max \min L_{\alpha, \xi}$$  \hspace{1cm} (A1)

where $L$ is the Lagrange function, which can be expressed as,

$$L = \frac{1}{2} (\|w_i\|^2 + \|w_j\|^2) + C \left( v_0 + \frac{1}{N} \sum_{i=1}^{N} (\xi_i + \xi_j) \right)$$

$$\frac{\partial}{\partial w_i} = \frac{1}{2} \left( \|w_i\|^2 + (b_i - b_j)^2 \right) - \sum_{i=1}^{N} \alpha_i \left( \epsilon + \xi_i + y_i \right)$$

$$- w_i^T \phi(x_i) - b_i - \sum_{i=1}^{N} \alpha_i \left( \epsilon + \xi_i + y_i - w_i^T \phi(x_i) \right) - b_i$$

$$- \sum_{i=1}^{N} \alpha_i \left( \epsilon + \xi_i + y_i - w_i^T \phi(x_i) \right) - b_i$$

$$- \sum_{i=1}^{N} \alpha_i \left( \epsilon + \xi_i + y_i - w_i^T \phi(x_i) \right) - b_i$$

$$- \sum_{i=1}^{N} \alpha_i \left( \epsilon + \xi_i + y_i - w_i^T \phi(x_i) \right) - b_i$$

Substitute (A2) to (A8) into (A1),

$$L = -\frac{1}{2} (\alpha^* - \alpha)^T K (\alpha^* - \alpha)$$

$$\frac{1}{2} (\alpha^* - \alpha)^T Q(x_i, x_j) (\alpha^* - \alpha) + \sum_{i=1}^{N} (\alpha^* - \alpha) y_i$$

$$\frac{1}{2} (\alpha^* - \alpha)^T K (\alpha^* - \alpha) + y^T (\alpha^* - \alpha)$$

From (A8) and (A9),

$$\sum_{i=1}^{N} (\alpha^* - \alpha) = 0$$  \hspace{1cm} (A11)

Based on (A10) and combined with condition (A3) to (A5) and (A11), the dual problem of Equation (26) can be written as,

$$\max -\frac{1}{2} (\alpha^* - \alpha)^T H(x_i, x_j) (\alpha^* - \alpha) + (\alpha^* - \alpha)^T y$$

s.t. \hspace{1cm} $\sum_{i=1}^{N} (\alpha^* - \alpha) = 0, \sum_{i=1}^{N} (\alpha^* + \alpha) = C v$

$$0 \leq \alpha, \alpha^* \leq C/N$$  \hspace{1cm} (A12)