Thermomagnetic conversion of low-grade waste heat into electrical power

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Abstract. A theoretical study relying on the thermal modelling of a Curie wheel, used for the conversion of low-grade waste heat into electrical power, is presented in this paper. It allows understanding the thermal behaviour of a Curie wheel operating in steady state in order to optimise its design. To this end, a stationary one-dimensional analytical thermal model, based on a Lagrangian approach, was developed. It allows determining the local distribution over time of the temperature in the magnetocaloric material exposed to a periodic sinusoidal heat source. Thanks to this model, the effects of different parameters (nature of the magnetocaloric material, nature and temperature of the fluid) were highlighted and studied.

Keywords: waste heat conversion, Curie wheel, magnetocaloric material, thermal and magnetic coupling, modelling

Nomenclature

\[a\] thermal diffusivity, \(m^2.s^{-1}\)
\[A\] temperature amplitude, °C
\[c\] specific heat capacity, J.kg\(^{-1}\).K\(^{-1}\)
\[d\] diameter of the fluid source, m
\[D\] diameter of the Curie wheel, m
\[F\] motor force, N
\[h\] heat transfer coefficient, W.m\(^{-2}\).K\(^{-1}\)
\[k\] number of magnetocaloric poles, –
\[L\] distance from the fluid source to the wheel, m
\[M\] magnetisation, A.m\(^{-1}\)
\[N\] rotation speed, rpm
\[Nu\] Nusselt number, –
\[Re\] Reynolds number, –
1. Introduction

The magnetocaloric effect corresponds to an adiabatic change of the temperature $\Delta T_{ad}$ or to an isothermal change of the magnetic entropy $\Delta S_m$ of a magnetic material subjected to a variable magnetic field. This effect is in general maximum at the magnetic transition temperature (Curie temperature) of the magnetocaloric materials.

The waste heat resulting from the industrial processes constitutes an important energy storage still untapped. The exploitation, even partial, of this energy source could enable France to achieve its goals of reducing by a factor 4 the emissions of greenhouse effect gases in 2050 compared to 2002. In France, the deposit of the low-grade waste heat is estimated to exceed 40 TWh [1]. This includes for example the incinerators smokes ($110\text{–}180\, ^\circ\text{C}$), whose potential is estimated at 1-2.3 TWh, or the vapour sent to the condensers (tank of 15.3 TWh). One solution to valorise this heat consists in converting it into another type of energy (electrical, mechanical). The majority of electricity generation technologies by heat conversion (Rankine cycle, thermoelectric conversion) can hardly valorise the low-grade waste heat ($T\leq150\, ^\circ\text{C}$). The thermomagnetic conversion of heat constitutes in contrast a promising platform [2].

At the instigation of the CEA Tech Lorraine, five french laboratories (IJL, LS2T, LEMTA, LMA and ICMPE) aim to develop the technology of thermomagnetic conversion of heat on the one hand, by optimising the performant magnetocaloric materials in the range $50\text{–}250\, ^\circ\text{C}$ and on the other hand, by designing two demonstrators of heat conversion. The first one is a static system (thermomagnetic generator) allowing to convert directly the heat into electrical power [3]. The second one is a dynamic system called also Curie wheel (thermomagnetic motor) allowing to convert the heat into mechanical power, so indirectly into electrical power [4, 5].

The operation principle of these two thermomagnetic systems involves a notable variation of magnetisation of the magnetocaloric material constituting them. This magnetisation variation is obtained by cycling the temperature of the magnetocaloric material from both sides of its Curie
temperature, with the waste heat used as hot source. The achievement of these thermomagnetic systems requires thus, inter alia, the optimisation of the heat transfers and the selection of the suitable magnetocaloric materials.

In this paper, a theoretical study relying on the thermal modelling of a Curie wheel is conducted in order to characterise its thermal behaviour according to the operating conditions.

2. modelling

2.1. Configuration

The configuration of the Curie wheel considered in this study is schematically represented in the figure 1a. This wheel consists mainly of a mobile magnetocaloric material (rotor), of a series of fixed permanent magnets (magnetic circuits) and of a pair of hot and cold sources from both sides of each magnet. A portion of the Curie wheel corresponding to a thermomagnetic pole is schematically represented in the figure 1b.

![Figure 1. Representative diagrams (a) of the total and (b) of a thermomagnetic pole of the Curie wheel considered in this study.](image)

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![Figure 2. Evolution of the fluid temperature seen by the magnetocaloric material as a function of the time for a Curie wheel having a thermomagnetic poles number k and a rotation speed N.](image)

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2.2. Assumptions

In order to simplify the mathematical formulation and the resolution of the developed thermal model, many assumptions were done:
• The reference frame chosen is Lagrangian, related to the magnetocaloric material and rotating at the rotor speed.

• The conductive heat transfer in the magnetocaloric material is unidimensional in the direction \( y \), and neglected in the directions \( x \) (because the Peclet number in this direction is much higher than 2) and \( z \) (because the temperature gradient in this direction is assumed to be zero).

• The temporal variation of the fluid temperature seen by the magnetocaloric material is supposed to be periodic sinusoidal (Figure 2), with an average equal to the Curie temperature of the magnetocaloric material:

\[
T_f(t) = T_f + Asin(\omega t)
\]

\[
T_f = \frac{T_{hf} + T_{cf}}{2} = T_C
\]

\[
A = \frac{T_{hf} - T_{cf}}{2}
\]

\[
w = \frac{k\pi N}{30}
\]

Where \( T_{hf} \) and \( T_{cf} \) are the temperatures of the hot and cold fluids, respectively, \( A \) is the amplitude of the fluid temperature, \( T_C \) is the Curie temperature of the magnetocaloric material, \( w, k \) and \( N \) are the pulsation, the number of thermomagnetic poles and the rotation speed of the Curie wheel, respectively, and \( t \) is the time.

• The thermophysical properties of the fluid are calculated at its average temperature and assumed to be invariant over time.

2.3. Thermal model

Based on the previous assumptions, by considering the relative temperature of the magnetocaloric material with respect to its Curie temperature \( \theta(y,t) = T(y,t) - T_C \) and by neglecting any volumetric production of heat by magnetic effects, the heat equation is written:

\[
\frac{\partial^2 \theta(y,t)}{\partial y^2} = \frac{1}{a} \frac{\partial \theta(y,t)}{\partial t}
\]

\[
a = \frac{\lambda}{\rho c}
\]

Where \( a, \lambda, \rho \) and \( c \) are the thermal diffusivity, the thermal conductivity, the density and the specific heat capacity of the material, respectively. The thermal boundary conditions are written:

\[
-\lambda \frac{\partial \theta}{\partial y}(0,t) = h [\theta_f(t) - \theta(0,t)]
\]

\[
\lambda \frac{\partial \theta}{\partial y}(\delta,t) = h [\theta_f(t) - \theta(\delta,t)]
\]

Where \( \theta_f(t) = T_f(t) - T_C \) is the relative temperature of the fluid with respect to the Curie temperature of the material, \( \delta \) is the thickness of the material, and \( h \) is the convective heat transfer coefficient between the fluid and the surface of the material, defined by:

\[
h = \frac{\lambda N u}{D}
\]
Where $Nu$ is the global Nusselt number calculated as follows [6]:

$$Nu^m = Nu_r^m + Nu_j^m$$  \hspace{1cm} (10)

$$Nu_r = 0.226 Re_r^{0.607}$$  \hspace{1cm} (11)

$$Nu_j = 0.995 Re_j^{0.56} (L/d)^{-0.344} (D_o/d)^{-0.768} (L/d)^{-0.616}$$  \hspace{1cm} (12)

$$Re_r = \frac{\rho f \pi N D_o^2}{120 \mu_f}$$  \hspace{1cm} (13)

$$Re_j = \frac{\rho f U_j D_o}{\mu_f}$$  \hspace{1cm} (14)

Where $Nu_r$ and $Re_r$ are the Nusselt and Reynolds numbers corresponding to the pure rotation of the wheel (without fluid flow), respectively, and $Nu_j$ and $Re_j$ are the Nusselt and Reynolds numbers corresponding to the pure jet of the fluid (without wheel rotation), respectively. The value of the global Nusselt number is effectively reliable for $Re_r = 1975−7899$, $Re_j = 655−60237$, $L/d = 1−16$ and $D_o/d = 2−16$. The solution of the heat equation (5) is thus written:

$$\theta(y, t) = \theta_1(y, t) + \theta_2(y, t)$$  \hspace{1cm} (15)

Where $\theta_1(y, t)$ is the solution for a convective heat transfer at $y = 0$ and an adiabatic condition at $y = \delta/2$, and $\theta_2(y, t)$ is the solution for a convective heat transfer at $y = \delta$ and an adiabatic condition at $y = \delta/2$, expressed as follows:

$$\theta_1(y, t) = \Omega C_2 \exp \left( \sqrt{\frac{\omega}{2a}} (2\delta - y) \right) \sin \left[ \omega t - \left( \Phi_1 + \sqrt{\frac{\omega}{2a}} y \right) \right]$$  \hspace{1cm} (16)

$$+ \Omega C_1 \exp \left( \sqrt{\frac{\omega}{2a}} (\delta - y) \right) \sin \left[ \omega t - \left( \Phi_2 - \sqrt{\frac{\omega}{2a}} (\delta - y) \right) \right]$$  \hspace{1cm} (17)

$$+ \Omega C_1 \exp \left( \sqrt{\frac{\omega}{2a}} y \right) \sin \left[ \omega t - \left( \Phi_2 - \sqrt{\frac{\omega}{2a}} y \right) \right]$$  \hspace{1cm} (18)

$$+ \Omega C_2 \exp \left( \sqrt{\frac{\omega}{2a}} (\delta + y) \right) \sin \left[ \omega t - \left( \Phi_1 + \sqrt{\frac{\omega}{2a}} \delta - y \right) \right]$$  \hspace{1cm} (19)

$$\theta_2(y, t) = \Omega C_2 \exp \left( \sqrt{\frac{\omega}{2a}} (\delta + y) \right) \sin \left[ \omega t - \left( \Phi_1 + \sqrt{\frac{\omega}{2a}} (\delta - y) \right) \right]$$  \hspace{1cm} (20)

$$+ \Omega C_1 \exp \left( \sqrt{\frac{\omega}{2a}} y \right) \sin \left[ \omega t - \left( \Phi_2 - \sqrt{\frac{\omega}{2a}} y \right) \right]$$  \hspace{1cm} (21)

$$+ \Omega C_1 \exp \left( \sqrt{\frac{\omega}{2a}} (\delta - y) \right) \sin \left[ \omega t - \left( \Phi_2 - \sqrt{\frac{\omega}{2a}} (\delta - y) \right) \right]$$  \hspace{1cm} (22)

$$+ \Omega C_2 \exp \left( \sqrt{\frac{\omega}{2a}} (2\delta - y) \right) \sin \left[ \omega t - \left( \Phi_1 + \sqrt{\frac{\omega}{2a}} y \right) \right]$$  \hspace{1cm} (23)

$$\Omega = \frac{AC_1 C_2}{C_1^2 + C_2^2 \exp \left( 2\delta \sqrt{\frac{\omega}{2a}} \right) + 2 C_1 C_2 \exp \left( \delta \sqrt{\frac{\omega}{2a}} \right) \cos (\Phi_1 - \Phi_2 + \delta \sqrt{\frac{\omega}{2a}})}$$  \hspace{1cm} (24)
\[ C_1 = \frac{1}{\sqrt{1 + 2\beta + 2\sqrt{\beta}}} \]  \hspace{1cm} (25) \\
\[ C_2 = \frac{1}{\sqrt{1 + 2\beta - 2\sqrt{\beta}}} \]  \hspace{1cm} (26) \\
\[ \Phi_1 = \arctan \left( \frac{1}{1 + \sqrt{\beta}} \right) \]  \hspace{1cm} (27) \\
\[ \Phi_2 = \arctan \left( \frac{1}{1 - \sqrt{\beta}} \right) \]  \hspace{1cm} (28) \\
\[ \beta = \frac{\omega}{2a} \left( \frac{\lambda}{h} \right)^2 \]  \hspace{1cm} (29)

An analytical magnetic model, complementary to this thermal model, is under development. It allows to determine the local distribution over time of the magnetisation and of the motor force in the magnetocaloric material subjected to a variable magnetic field (Figure 3), relying on the temperature profiles obtained by the thermal model, as shown previously. Hence, the resultant motor force turning the rotor could be calculated and a thermomagnetic characterisation of the Curie wheel could be realised.

Figure 3. Representative diagram of two motor forces generated by a temperature gradient between two positions \( x_1 \) and \( x_2 \) in the magnetocaloric material subjected to a magnetic field.

2.4. Results and discussion
The results presented in this paragraph correspond to a Curie wheel having a rotor of outer diameter \( D_o = 550 \) mm and of thickness \( \delta = 20 \) mm, and a fluid source of diameter \( d = 50 \) mm remote from the rotor of \( L = 100 \) mm. The figure 4 shows the evolution of the relative temperature of gadolinium (\( T_C = 20 \) °C, \( \rho = 7901 \) kg.m\(^{-3} \), \( \lambda = 10.6 \) W.m\(^{-1}.K^{-1} \), \( c = 230 \) J.kg\(^{-1}.K^{-1} \)) as a function of the position \( y \) and the time for a rotation speed \( N = 6 \) rpm, a number of thermomagnetic poles \( k = 3 \), and a velocity and a temperature amplitude of the air \( U_f = 1.5 \) m.s\(^{-1} \) and \( A = 20 \) °C, respectively. The temperature of the material is obviously symmetrical with respect to the position \( y = \delta/2 \), thanks to the identical heat sources on both sides of the material (Figure 4a). A damping and a dephasing of the relative temperature along the thickness of the material were highlighted (Figure 4b), because of the thermal resistance and the thermal diffusion time of the material. Furthermore, the temperature amplitude of the material surface is very low compared to the temperature amplitude of the air. This is mainly due to the low convective heat transfer coefficient between the air and the material surface.
Thus, by substituting the air by the water (with a lower velocity in order to respect the range of $Re_r$), the temperature amplitude of the material surface is clearly greater (Figure 5), resulting from the fact that the convective heat transfer coefficient of water is much greater than that of air. So, there is an interest to use water as fluid to intensify the heat transfer and to obtain a greater temperature gradient (greater resultant motor force) within the material. The figure 6 shows the evolution of the relative temperature of $Y_2Fe_{16.4}Co_{0.6}$ ($T_C = 140 \ ^\circ C$, $\rho = 6745 \ kg.m^{-3}$, $\lambda = 80.2 \ W.m^{-1}.K^{-1}$, $c = 479 \ J.kg^{-1}.K^{-1}$) as a function of the position $y$ and the time for the same conditions as in the figure 4. This magnetocaloric material, synthesised at the IJL for the conversion of low-grade waste heat, presents thermophysical properties different from those of gadolinium, which explains the difference with respect to the results of the figure 4. So, the choice of the magnetocaloric material must take into consideration, besides the appropriate Curie temperature for the intended application, the best thermophysical properties with regard to the heat transfer. Finally, the increase of the variation amplitude of the fluid temperature contributes to the intensification of the convective heat transfer between the fluid and the material, as shown in the figure 7. Thus, a significant temperature difference between the fluid and the material is recommended, of course without degrading the material or the other parts of the Curie wheel by the maximum fluid temperature.

![Figure 4](image)

**Figure 4.** Evolution of the relative temperature of Gd as a function of the position $y$ and the time with air used as fluid, for $N = 6$ rpm, $k = 3$, $U_f = 1.5 \ m.s^{-1}$ and $A = 20 \ ^\circ C$.

![Figure 5](image)

**Figure 5.** Evolution of the relative temperature of Gd as a function of the position $y$ and the time with water used as fluid, for $N = 1$ rpm, $k = 3$, $U_f = 0.1 \ m.s^{-1}$ and $A = 20 \ ^\circ C$. 


Figure 6. Evolution of the relative temperature of Y$_2$Fe$_{16.4}$Co$_{0.6}$ as a function of the position $y$ and the time with air used as fluid, for $N = 6$ rpm, $k = 3$, $U_f = 1.5$ m.s$^{-1}$ and $A = 20$ °C.

Figure 7. Evolution of the relative temperature of Y$_2$Fe$_{16.4}$Co$_{0.6}$ as a function of the position $y$ and the time with air used as fluid, for $N = 6$ rpm, $k = 3$, $U_f = 1.5$ m.s$^{-1}$ and $A = 100$ °C.

3. Conclusions and perspectives
A stationary one-dimensional analytical thermal model, developed on a Lagrangian approach, was presented in this paper. It allowed characterising the Curie wheel from a thermal point of view. Thanks to this model, the effects of the magnetocaloric material nature and the fluid nature and temperature were studied and analysed, and conclusions regarding their choice were drawn. An analytical magnetic model, complementary to the thermal model, is under development in order to determine the resultant motor force in the magnetocaloric material. The coupling of these two models could allow to understand the thermomagnetic behaviour of the Curie wheel in the aim to optimise its design.

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