Forecasting characteristics of flood effects

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Abstract: The article presents the development of a mathematical model of the system dynamics. Mathematical model allows forecasting the characteristics of flood effects. Model is based on a causal diagram and is presented by a system of nonlinear differential equations. Simulated characteristics are the nodes of the diagram, and edges define the functional relationships between them. The numerical solution of the system of equations using the Runge-Kutta method was obtained. Computer experiments to determine the characteristics on different time interval have been made and results of experiments have been compared with real data of real flood. The obtained results make it possible to assert that the developed model is valid. The results of study are useful in development of an information system for the operating and dispatching staff of the Ministry of the Russian Federation for Civil Defence, Emergencies and Elimination of Consequences of Natural Disasters (EMERCOM).

1. Introduction
Floods are among Earth's most common and most destructive natural hazards. To improve the efficiency of elimination of effects of flood, it is necessary to improve the management system and operational service of forecasting of the EMERCOM. The existing forecasting models do not allow one to obtain the set of characteristics of flood effects with many nonlinear feedbacks between them. These circumstances produce a decrease in forecast accuracy, which adversely affects the processes of elimination of effects of flood during the emergency situation.

The paper studies a process of developing a mathematical model of forecasting characteristics of flood effects. The development of a mathematical model is based on the system dynamics approach in which causal relationships between the simulated characteristics are taken into account. The results of computational experiments confirm the validity of the mathematical model.

2. Mathematical model
According to [1], the following flood effects characteristics were chosen as a variables of the model:
\( X_1 \) - the number of forces involved in emergency-and-rescue operations; \( X_2 \) - the number of houses destroyed and damaged in the floods; \( X_3 \) - the number of people evacuated from the flooded area; \( X_4 \) - number of deaths (human life losses); \( X_5 \) - the length of railways and roads in the flood zone;
$X_6$ - the number of industrial enterprises in the flood zone; $X_7$ - the number of facilities involved in emergency-and-rescue operations; $X_8$ - the population in the flood zone; $X_9$ - area of agricultural land in the flood zone; $X_{10}$ - the number of dead farm animals.

On the basis of the system dynamics, a mathematical model of forecasting characteristics of flood effects was developed. The model is represented as a system of nonlinear first-order differential equations. In the system dynamics methodology, a system may be represented as a causal diagram:

$$\frac{dX_i}{dt} = X_i^+ + X_i^-, \ i = 1, n, \ (1)$$

where $X_i^+, X_i^-$, $i = 1, n$ are continuous or piecewise continuous functions, defining positive and negative rate of change of variable $X_i$ [2,3]. In turn, $X_i^- = f_i^-(F_1, F_2, \ldots, F_m)$, $X_i^+ = f_i^+(F_1, F_2, \ldots, F_m)$ are functions, its arguments $F_j$, $j = 1, m$ affecting the rate of change of variable $X_i$. $F_j$ can be functions whose arguments are simulated variables $X_i$, $i = 1, n$.

The casual diagram, based on the analysis of causal links between the simulated characteristics and presented in figure 1, was developed. Simulated characteristics are the nodes of the diagram, and outgoing and incoming edges define the functional relationships between them.

![Causal Diagram](image)

**Figure 1.** The causal diagram.

The mathematical model of forecasting characteristics of flood effects has form (2) considering presented diagram. Here, $A(t)$ is the density of transport networks in the flooding area; $D(t)$ is the density of the population; $F(t), G(t), T(t)$ are the average daily values of flow rate, the water level and the temperature respectively; $I(t)$ is the fraction (share) of agricultural land, and $S(t)$ is the flooding area.
Functions are defined on the basis of analysis statistical data and, as a rule, are approximated by polynomials. If there are no statistical data, then functions are determined from the physical meaning [4]. Flooding area $S(t)$ and average water level $G(t)$ are calculated by experts according to known methods [5,6]. The system of differential equations (2) is solved by the fourth-order Runge-Kutta method with initial values $t_0 = 1, X_i(t_0) = X_{i0}, i = 1,10$.

3. Verification of validity of the mathematical model

Verification of validity of the mathematical model was performed by comparison of the characteristics defined by model (2) with the actual characteristics of flood occurred in August 2001 in Primorye [7].

In table 1, there are functions $f_i^-$ and $f_i^+$ of system (2). The functions was defined for the flood in Primorye. Let us consider the process of constructing function $f_4^+$ by the example of differential equation for variable $X_4$, which has form $\frac{dX_4}{dt} = X_4^+ + X_4^-$. Variable $X_4$ is affected by average daily values of water flow rate $F$, of water level $G$ and water temperature $T$. In addition, the rate of change of variable $X_4$ depends on the number of forces involved in emergency-and-rescue operations, the number of facilities involved in emergency-and-rescue operations $X_7$ and the population in flood zone $X_8$ (figure 2). The rate of change of variable $X_4$ does not decrease, therefore, $X_4^-$ equals to zero. The result is the following functions: $X_4^+ = f_4^+(F(t),G(t),T(t),X_8(t),X_7(t),X_1(t)), X_4^- = 0$.

Analysis of function $f_4^+$ suggests that the rate of change $X_4$ is directly proportional to $F$, $G$, $T$, $X_8$ and inversely proportional to $X_1$, $X_7$, therefore $\frac{dX_4}{dt} = k_4 \frac{F(t)G(t)T(t)X_8}{X_1X_1X_8}$. Coefficients $k_i, i = 1,14$ are determined by experiments on a stage adaptation of the model to the research object.
The solution of the system of equations (2) normalized relative to the maximum values of simulated characteristics $X_i^N = \frac{X_i}{X_i^{max}}$ is presented in figure 3. In particular, the analysis of the results allows us to conclude that the maximum loss among the population falls off at first day of flood and later the growth of characteristic $X_4(t)$ is small. The increase of the other simulated characteristics is conditioned growth of the flooding area, peak of which falls on the 4th day of floods.

Lagrange polynomials have been constructed on the basis of an actual data of flood characteristics for visualization the results of validity of the mathematical model. For example, the results of the solution of system (2) for $X_4(t)$ have been approximated by polynomial $L(X_4^N(t)) = 0.09t^2 - 0.86t^2 + 2.6t - 1.85$. For a given set of points $(t_i, Y_4^N(t_i))$ of an actual values of

| $f_1^+$ | $k_1(S(t)X_4)^{0.5}, S(t) > \epsilon$ | $f_6^+$ | $k_6S(t)^{0.5}X_8^{0.1}, S(t) > \epsilon$ |
| --- | --- | --- | --- |
| $f_2^+$ | $k_2F(t)G(t)S(t)X_8^{1/3}, S(t) > \epsilon$ | $f_7^+$ | $k_7X_1$ |
| $f_3^+$ | $k_3\frac{X_4X_7}{X_7}$ | $f_8^+$ | $k_8D(t)S(t), S(t) > \epsilon$ |
| $f_4^+$ | $k_4\frac{F(t)G(t)T(t)X_8}{X_7X_1}$ | $f_9^+$ | $k_9I(t)S(t), S(t) > \epsilon$ |
| $f_5^+$ | $k_5A(t)S(t), S(t) > \epsilon$ | $f_{10}^+$ | $k_{10}\frac{F(t)G(t)T(t)S(t)}{X_1X_7}, S(t) > \epsilon$ |
| $f_6^-$ | $k_{11}X_1X_7$ | $f_7^-$ | $k_{12}X_4X_7$ |
| $f_8^-$ | $k_{13}X_4$ | $f_9^-$ | $k_{14}X_1X_7$ |

Table 1. Functions $f_i^+, f_i^-$.  

| $f_1^+$ | $k_1(S(t)X_4)^{0.5}, S(t) > \epsilon$ | $f_6^+$ | $k_6S(t)^{0.5}X_8^{0.1}, S(t) > \epsilon$ |
| --- | --- | --- | --- |
| $f_2^+$ | $k_2F(t)G(t)S(t)X_8^{1/3}, S(t) > \epsilon$ | $f_7^+$ | $k_7X_1$ |
| $f_3^+$ | $k_3\frac{X_4X_7}{X_7}$ | $f_8^+$ | $k_8D(t)S(t), S(t) > \epsilon$ |
| $f_4^+$ | $k_4\frac{F(t)G(t)T(t)X_8}{X_7X_1}$ | $f_9^+$ | $k_9I(t)S(t), S(t) > \epsilon$ |
| $f_5^+$ | $k_5A(t)S(t), S(t) > \epsilon$ | $f_{10}^+$ | $k_{10}\frac{F(t)G(t)T(t)S(t)}{X_1X_7}, S(t) > \epsilon$ |
| $f_6^-$ | $k_{11}X_1X_7$ | $f_7^-$ | $k_{12}X_4X_7$ |
| $f_8^-$ | $k_{13}X_4$ | $f_9^-$ | $k_{14}X_1X_7$ |
simulated characteristic $X^N(t)$, also normalized with respect to maximum values, the Lagrange polynomial $L(Y^N(t)) = 0.095t^3 - 0.89t^2 + 2.7t - 1.92$ has been constructed. Figure 4 shows the comparison of polynomials $L(X^N(t))$ and $L(Y^N(t))$. An analysis of the obtained curves shows that the values determined by model (2) differ very little from the actual data of the characteristic,

$$\Delta_{\text{av}} = \frac{1}{k+1} \sum_{j \in [1;4]} \left| \frac{L(X^N(t_j)) - L(Y^N(t_j))}{L(X^N(t_j))} \right| \times 100\% = 3\% , j = 0, k, t \in [1;4].$$

![Figure 3. The results of forecasting.](image)

![Figure 4. The comparison of polynomials $L(X^N(t))$ and $L(Y^N(t))$.](image)

By comparing other simulated characteristics calculated by the model, with their actual values, the average value of relative errors for each of the characteristics does not exceed 10%. This suggests that the developed mathematical model is valid.

It is planned to adapt the mathematical model for prevent critical events related to flooding at industrial enterprises according to recommendations [8-10].
4. Conclusion
The mathematical model of forecasting the main flood effects characteristics affecting the amount of damage was developed on the basis of a system dynamics. The casual diagram was used to develop the model. The authors verified the validity of mathematical models by comparing the simulated characteristics with the corresponding actual values of the flood occurred in 2001 in Primorye. The results can be used in development of the information and advising systems for operating and dispatching staff of the EMERCOM.

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