Enhancement of evanescent waves in a planar waveguides with an anisotropic metamaterial layer

Wenqiang Qiu¹, Min Cheng², Rong Chen¹*, Hongxin Lin¹, Zufang Huang¹,
¹Key Laboratory of Optoelectronic Science and Technology for Medicine, Ministry of Education, Fujian Normal University, Fuzhou 350007, China
²Department of Physics and Electronic Information Engineering, Minjiang University, Fuzhou 350108, China

*Corresponding author: chenr@fjnu.edu.cn

Abstract. The enhancement of evanescent waves in waveguides consisting of an anisotropic metamaterials layer with partially negative permittivity and permeability is investigated for both TE and TM modes. The amplitudes of evanescent waves increase exponentially with the thickness of the metamaterials. The enhancement factor increases with the thickness of the metamaterial but reaches its maximum value when complete surface polaritons are established. Numerical results confirm our analysis. The enhancement effect may have potential applications in integrated optical devices.

1. Introduction

As early as 40 years ago artificial materials, called as the left-handed materials (LHMs) or metamaterials, were first analyzed theoretically by Veselago.[1] The novel materials exhibit simultaneous negative permittivity and permeability in certain frequencies and thus would possess blinking properties, such as negative refraction,[1,2] inverse Doppler shift,[1,2,3] reverse Goos-Hänchen shift[4] and reversed Cerenkov radiation[1,2,5]. Periodical arrays of metallic rods and split ring resonators (SRRs) were used to fabricate the LHMs by Shelby et al. in 2001. [6] Since that many potential applications of the LHMs have been proposed. Up to now almost all of the LHMs having been realized in experiments are actually anisotropic in nature and easier to implement in practical applications. Recently, the properties of evanescent field in the LHMs slab have attracted much attention.[7-11] The enhancement of evanescent waves in waveguides using isotropic metamaterials has been discussed [11]. However, the properties of the enhancement of evanescent waves using anisotropic metamaterials is still not clear.

In this paper, we investigate the enhancement of evanescent waves in an anisotropic planar waveguides. The characteristic equations are derived. According to different combinations of parameters of the anisotropic medium, four distinct cases for oscillating and evanescent modes are analyzed. Then we present the transverse profiles for TE₀ and TE₁ modes and examine the enhancement factor in detail. This treatment reveals that the enhancement factor increases exponentially with the thickness of the metamaterial but saturates when complete surface polaritons mode are established at the interface between the metamaterial and the cladding. The numerical results
confirm our analysis that the evanescent waves can be drastically enhanced by metamaterials, as long as we choose the appropriate parameters.

2. Method

We consider a structure of multilayer-integrated waveguides, adding an anisotropic metamaterial layer into the conventional dielectric three-layer waveguides, as illustrated in Fig.1. A substrate has positive permittivity $\varepsilon_s$ and permeability $\mu_s$. The width of the guiding layer with positive $\varepsilon_g$ and $\mu_g$ is $d_1$. A cladding layer with positive $\varepsilon_c$ and $\mu_c$ was coated onto the guiding layer. However, in the working region there is a metamaterials layer which is a biaxial anisotropic medium with permittivity tensor $\bar{\varepsilon} = \varepsilon_0 \left( \varepsilon_{xx}, \varepsilon_{yy}, \varepsilon_{zz}, \varepsilon_{xy}, \varepsilon_{yz}, \varepsilon_{xz} \right)$ and permeability tensor $\bar{\mu} = \mu_0 \left( \mu_{xx}, \mu_{yy}, \mu_{zz}, \mu_{xy}, \mu_{yz}, \mu_{xz} \right)$. The thickness of metamaterials layer is $d_2$.

![Figure 1](image1.png)

**Figure 1.** Schematic structure of the multilayer integrated waveguides with an anisotropic metamaterial layer.

For simplicity, we assume all the materials are lossless and in order to analytically express the electromagnetic fields in such structures, our discussions are limited to planar structures. We begin by focusing on the transverse electric (TE) waves. An analogous result can be obtained for transverse magnetic (TM) waves by a dual analysis, i.e., by making the following corresponding transformations: $\left( \mu \rightarrow \varepsilon \right)$ and $\left( E \rightarrow H \right)$. Suppose that the electromagnetic waves propagate along the $x$ direction, and the electric field $E$ is polarized along the $z$-axis (see Fig.1). The time dependence of the monochromatic field is expressed as $E(x,y,z,t) = E(x,y) \exp \left( \omega t - \beta z \right)$, where $\omega$ is the angular frequency of the field and $\beta$ is the propagation constant. From Maxwell’s equations, we can get the scalar wave equations of a waveguide with a coordination system shown in Fig.1 as follows:

$$\frac{d^2}{dy^2} F(x,y) + \mu_0 \left( \frac{\omega^2}{c^2} \varepsilon_s(y) \mu_s(y) - \beta^2 \right) E_s(y) = 0,$$

$$\frac{d^2 E_z(y)}{dy^2} + \left( \frac{\omega^2}{c^2} \varepsilon_g(y) \mu_g(y) - \beta^2 \right) E_z(y) = 0, \quad (i=g,c,s).$$

Eq.(1) refers to the anisotropic metamaterials layer, and Eq.(2) corresponds to other regions, that is the cladding, the substrate and the guiding layer. The dispersion relations in normal isotropic media and anisotropic metamaterials can be written as
When $\beta$ is large enough, there will be a case that only evanescent waves exist in the substrate, the cladding and the metamaterial layer while guided waves oscillate in the guiding layer. Under this condition, in the working region the electric field in each layer of the waveguides has the following solutions for TE modes:

\begin{equation}
E_z(y) = A \exp[k_x(y + d_1)](-\infty < y < -d_1),
\end{equation}

\begin{equation}
E_z(y) = B \cos(k_y y + \phi),(-d_1 < y < 0),
\end{equation}

\begin{equation}
E_z(y) = C \exp(-k_m y) + C_2 \exp(k_m y),(0 < y < d_2),
\end{equation}

\begin{equation}
E_z(y) = D \exp[-k_x(y - d_2)],(d_2 < y < \infty),
\end{equation}

Where $k_x,k_m$ and $k_y$ correspond to the evanescent coefficients of the claddings, the metamaterial and the substrate, respectively, and $k_g$ is the transverse wave number in the guiding layer.

Due to only evanescent waves existing in the metamaterial layer, $k_m$ should be imaginary i.e., $k_m = i k_m$, where $k_m = \sqrt{(\beta^2 - \varepsilon \mu \omega^2/c^2)} \mu_i / \mu_s$, and the following inequality is valid:

\begin{equation}
\frac{\mu_s}{\mu_i} \beta^2 - \varepsilon \mu \omega^2/c^2 > 0.
\end{equation}

After matching the boundary conditions that require the tangential components $E_x(y)$ and $H_y(y)$ to be continuous at $y = -d_1, 0$ and $d_2$, respectively, we get the characteristic equation of the multi-layer waveguides:

\begin{equation}
k_g d_1 = n \pi + \arctan \left( \frac{\mu_x k_x}{\mu_y k_y} \right) + \arctan \left( \frac{k_m \mu_x}{k_g \mu_y} \left( k_m \mu_x + k_m \mu_y \right) - (k_m \mu_x - k_m \mu_y) \cdot \exp(-2k_m d_2) \right) + \arctan \left( k_c / k_g \right) + \arctan \left( k_c / k_g \right).
\end{equation}

Where $n$ is the mode number, and takes the value of 0, 1, 2, 3 and so on.

If we choose a metamaterial in a way that $k_x + k_m + k_y = 0$, according to Eqs.(7) and (8) and also the boundary conditions we get $C_1 = 0, D = C_2 \exp(k_m d_2) = B \cos(\phi) \exp(k_m d_2)$. In the cladding of the conventional dielectric three-layer waveguides (the nonworking region, see Fig.1), it is easy to obtain that $D = B \cos(\phi)$. Here, to one’s interest, it is found that the amplitude of evanescent wave is enhanced by $\exp(k_m d_2)$ due to the existence of metamaterials layer. Provided that $\mu_1 = \mu_2 = \mu_x = \mu_y$, Eq.(10) reduces to

\begin{equation}
k_g d_1 = n \pi + \arctan \left( k_x / k_g \right) + \arctan \left( k_c / k_g \right).
\end{equation}

Equation (11) is exactly identical to the characteristic equation of the conventional dielectric three-layer waveguide without the metamaterials layer. Therefore, the same characteristic equation holds for both the working region and the nonworking region, and consequently the propagation constants for them are identical.

3. Numerical analyses and results

In our coordinate system (see Fig.1), the time-averaged Poynting vector in the metamaterials layer can be developed as follows:

\begin{equation}
\langle S \rangle = \frac{\beta}{2 \mu_\omega} \Re \left[ C_1 \epsilon e^{-2k_x y} + C_2 \epsilon e^{2k_x y} + 2C_1 C_2 \right].
\end{equation}

The boundary conditions require that the $x$ component of the wave vectors in each layer should be continuous and equal to the same vector $\beta$. Since the wave is propagating along the direction of $x$ axis, parallel to the interface, we have $\beta > 0$. By further analyzing the time-averaged Poynting vector, we
can easily find that the following inequality is satisfied: $\beta \cdot \mathbf{S} < 0$ when $\mu_\varepsilon < 0$. And thus the wave vector and the Poynting vector are antiparallel in the $x$ direction for $\mu_\varepsilon < 0$. Similar conclusions were also achieved in Ref.10, 13 and 14.

The dispersion relation of the anisotropic medium, given in equation (4), may be elliptic or hyperbolic according to different combinations of parameters. Since the metamaterials exhibit the negative group refraction for $\mu_\varepsilon < 0$, in what follows we will discuss four distinct cases where $\mu_\varepsilon$ is always negative.

**Case (I):** $\mu_\varepsilon < 0$ and $\varepsilon_\varepsilon < 0$. The tensor elements are just opposite in sign to those of the conventional anisotropic media. The metamaterials satisfy the elliptic dispersion relations, given in equation (4), and can support both oscillating and evanescent modes. The inequality (9) can be extended as follows: $\frac{\omega_0}{c} \sqrt{\varepsilon_\varepsilon \mu_\varepsilon} \geq \beta \geq \max \left\{ \frac{\omega_0}{c} \sqrt{\varepsilon_\varepsilon \mu_\varepsilon}, \frac{\omega_0}{c} \sqrt{\varepsilon_\varepsilon \mu_\varepsilon}, \frac{\omega_0}{c} \sqrt{\varepsilon_\varepsilon \mu_\varepsilon} \right\}$.

**Case (II):** $\mu_\varepsilon < 0$ and $\varepsilon_\varepsilon > 0$. It is apparent from Eq.(4) that $k_{my}$ is purely imaginary, i.e., the inequality (9) always holds, therefore the metamaterials only support the surface modes. No other addition condition is required upon the metamaterials region in this case, but the condition $\frac{\omega_0}{c} \sqrt{\varepsilon_\varepsilon \mu_\varepsilon} \geq \beta \geq \max \left\{ \frac{\omega_0}{c} \sqrt{\varepsilon_\varepsilon \mu_\varepsilon}, \frac{\omega_0}{c} \sqrt{\varepsilon_\varepsilon \mu_\varepsilon} \right\}$ is also needed.

**Case (III):** $\mu_\varepsilon > 0$ and $\varepsilon_\varepsilon < 0$. The dispersion relation of the metamaterial is a hyperbolic one. Such a metamaterial layer can support both oscillating and evanescent modes. The inequality (9) can be expressed as $\frac{\omega_0}{c} \sqrt{\varepsilon_\varepsilon \mu_\varepsilon} \geq \beta$, while $\frac{\omega_0}{c} \sqrt{\varepsilon_\varepsilon \mu_\varepsilon} \geq \beta \geq \max \left\{ \frac{\omega_0}{c} \sqrt{\varepsilon_\varepsilon \mu_\varepsilon}, \frac{\omega_0}{c} \sqrt{\varepsilon_\varepsilon \mu_\varepsilon} \right\}$ are the necessary conditions for such a waveguide. To sum up, in this case $\beta$ must satisfy the conditions: $\min \left\{ \frac{\omega_0}{c} \sqrt{\varepsilon_\varepsilon \mu_\varepsilon}, \frac{\omega_0}{c} \sqrt{\varepsilon_\varepsilon \mu_\varepsilon} \right\} \geq \beta \geq \max \left\{ \frac{\omega_0}{c} \sqrt{\varepsilon_\varepsilon \mu_\varepsilon}, \frac{\omega_0}{c} \sqrt{\varepsilon_\varepsilon \mu_\varepsilon} \right\}$.

**Case (IV):** $\mu_\varepsilon > 0$ and $\varepsilon_\varepsilon > 0$. The metamaterials satisfy the hyperbolic dispersion relations. Due to the real component $k_{my}$, the inequality (9) is invalid, i.e., $k_{my}$ is always imaginary, so it can only support the oscillating modes in the anisotropic media. Thus in this case it won’t enhance the evanescent waves with the metamaterial layer.

Here, we take the case I as an example. The guidance condition curve and the dependence of the electric field distribution on the thickness are shown in Figures 2 and 3, respectively. For simplicity, we choose small refractive index contrast, namely, assume that $\varepsilon_\varepsilon \mu_\varepsilon = 1.55$, $\varepsilon_\varepsilon \mu_\varepsilon = 1.2$, $d_1 = d_2$, $b = k_0 d_1 \sqrt{(\varepsilon_\varepsilon \mu_\varepsilon - \varepsilon_\varepsilon \mu_\varepsilon)} = 14$, $c = k_0 d_1 \sqrt{(\varepsilon_\varepsilon \mu_\varepsilon - \varepsilon_\varepsilon \mu_\varepsilon)} = 25$ and $\mu_\varepsilon = -0.4$. We use a graphical method to determine the solution to $k_{my} d_1$ determined by Eq.(11), as illustrated in Fig.2, where the intersections show the existence of the guided modes. The transverse profiles corresponding to the two guided modes in Fig.2 are shown in Fig.3. As illustrated in Fig.3, the electric fields are oscillating in the guiding layer but evanescent outside the guiding layer. The fundamental mode $\text{TE}_0$ exists in this case, as shown in Fig.3(a). The field amplitudes have a critical enhancement in the metamaterial layer and decay exponentially in cladding layer for both $\text{TE}_0$ and $\text{TE}_1$ modes, while decaying slowly in the substrate, as Fig.3(a) and Fig.3(b) show. Both of these electric fields distribution make us believe that metamaterials can indeed enhance the evanescent wave in this multi-layer waveguides. However, as stressed in Ref.11, the enhancement effect is the result of redistribution of the electromagnetic field intensity profile due to the existence of the metamaterial, and does not contradict the energy-conservation law.
Figure 3 Transverse profiles of the guided modes corresponding to the intersections in Fig.2. (a) TE0: $k_g d_1 = 2.82515$; (b) TE1: $k_g d_1 = 5.64088$. Parameters are $\mu_\epsilon = -0.4$, $\varepsilon_\mu = 1.2$, $\varepsilon_g \mu_g = 1.55$, $\varepsilon_\mu = 1.45$ and $d_1 = 2d_2$. The vertical dot-lines denote the interface of two slabs. Amplitudes are in arbitrary units.

Secondly, comparing the power fraction in the cladding without and with a metamaterial will help us to evaluate the enhancement effect. Where no metamaterial exists (the nonworking region in Fig.1), the power fraction $P_1$ in the cladding for TE modes can be written as\(^{12}\)

$$P_1 = \frac{k_g^2}{k_c (k_g^2 + k_c^2) \left( d_1 + \frac{1}{k_g} + \frac{1}{k_c} \right)}$$

Where there is a metamaterials layer, the power fraction in the cladding $P_2$ is

$$P_2 = \frac{k_g^2 \cdot \exp(2k_m d_2)}{k_c (k_g^2 + k_c^2) \left( \left| k_m \right| \mu_\epsilon + k_g^2 \cdot \exp(2k_m d_2) - k_g^2 k_c^2 \mu_\epsilon \right) + d_1 + \frac{1}{k_g}}^{-1}.$$  \hspace{1cm} (14)

The enhancement factor $Q$ is defined by $Q = P_2 / P_1$. Here, we take the case 1 as an example and plot how $Q$ responds to $d_2$ and $\mu_\epsilon$ in Fig.4.

Figure 4. Enhancement factor $Q$ as the function of the thickness of metamatals $d_2$ for TE$_0$ (case I: $\mu_\epsilon < 0$, $\varepsilon_\mu < 0$). Parameters are the same as in Fig.2 except that $\mu_\epsilon$ changes for several other values.
As shown in Fig.4, one could see that for various parameters of \( \mu \), there are different values of enhancement factor. The enhancement factor for \( \mu = -0.4 \) in Fig.4 corresponds to the case displayed by the electric field component in Figs.2 and 3. To one’s interest, the enhancement factor increases with the thickness of metamaterials, but this does not last forever. Instead, there exists a saturation value of the enhancement factor. As Qing et al. have pointed out in Ref.11, this can be interpreted by surface polaritons effects and is actually reasonable: when the thickness becomes large enough, a complete surface polaritons mode is established at the interface between the metamaterial and the cladding. Owing to the surface polaritons effect, all the energy flux is confined closely at the two sides of the interface, and thus the power fraction in the cladding will saturate.

4. Conclusions

In this paper, we have demonstrated that metamaterials with partially negative permittivity and permeability can enhance the amplitude of the evanescent waves in waveguides for both TE and TM modes, provided that the surface polaritons conditions are satisfied. After the characteristic equations having been derived, we present the transverse profile for each layer. It is showed that the amplitude of evanescent waves is enhanced exponentially. We also examine the enhancement factor and find that it does not always increase with the thickness but saturates when complete surface polaritons have been established at interface between the metamaterial and the cladding. The enhancement effect may have potential applications in integrated optical devices, such as optical direction couplers and waveguide sensors. With the metamaterials, it may enhance the coupling effect and reduce the dimensions of these devices.

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