Adaptive Aggregation Networks for Class-Incremental Learning

Yaoyao Liu 1 Bernt Schiele 1 Qianru Sun 2

1 Max Planck Institute for Informatics, Saarland Informatics Campus
2 School of Computing and Information Systems, Singapore Management University
{yaoyao.liu, schiele}@mpi-inf.mpg.de qianrusun@smu.edu.sg

Abstract

Class-Incremental Learning (CIL) aims to learn a classification model with the number of classes increasing phase-by-phase. An inherent problem in CIL is the stability-plasticity dilemma between the learning of old and new classes, i.e., high-plasticity models easily forget old classes, but high-stability models are weak to learn new classes. We alleviate this issue by proposing a novel network architecture called Adaptive Aggregation Networks (AANets) in which we explicitly build two types of residual blocks at each residual level (taking ResNet as the baseline architecture): a stable block and a plastic block. We aggregate the output feature maps from these two blocks and then feed the results to the next-level blocks. We adapt the aggregation weights in order to balance these two types of blocks, i.e., to balance stability and plasticity, dynamically. We conduct extensive experiments on three CIL benchmarks: CIFAR-100, ImageNet-Subset, and ImageNet, and show that many existing CIL methods can be straightforwardly incorporated into the architecture of AANets to boost their performances 1.

1. Introduction

AI systems are expected to work in an incremental manner when the amount of knowledge increases over time. They should be capable of learning new concepts while maintaining the ability to recognize previous ones. However, deep-neural-network-based systems often suffer from serious forgetting problems (called “catastrophic forgetting”) when they are continuously updated using new coming data. This is due to two facts: (i) the updates can override the knowledge acquired from the previous data [19, 27, 28, 33, 40], and (ii) the model can not replay the entire previous data to regain the old knowledge.

To encourage solving these problems, [34] defined a class-incremental learning (CIL) protocol for image classification where the training data of different classes gradually come phase-by-phase. In each phase, the classifier is re-trained on new class data, and then evaluated on the test data of both old and new classes. To prevent trivial algorithms such as storing all old data for replaying, there is a strict memory budget due to which a tiny set of exemplars of old classes can be saved in the memory. This memory constraint causes a serious data imbalance problem between old and new classes, and indirectly causes the main problem of CIL – the stability-plasticity dilemma [29]. In particular, higher plasticity results in the forgetting of old classes [27], while higher stability weakens the model from learning the data of new classes (that contain a large number of samples). Existing CIL works try to balance stability and plasticity using data strategies. For example, as illustrated in Figure 1 (a) and (b), some early methods train their models on the imbalanced dataset where there is only a small set of exemplars for old classes [23, 34], and recent methods include a fine-tuning step using a balanced subset of exemplars sampled from all classes [4, 11, 16]. However, these data strategies are still limited in terms of effectiveness. For example, when using the models trained after 25 phases, LUCIR [16] and Mnemonics [25] “forget” the initial 50 classes by 30% and 20%, respectively, on the ImageNet dataset [37].

In this paper, we address the stability-plasticity dilemma by introducing a novel network architecture called Adaptive Aggregation Networks (AANets). Taking the ResNet [14] as an example of baseline architectures, we explicitly build two residual blocks (at each residual level) in AANets: one for maintaining the knowledge of old classes (i.e., the stability) and the other for learning new classes (i.e., the plasticity), as shown in Figure 1 (c). We achieve these by allowing these two blocks to have different levels of learnability, i.e., less learnable parameters in the stable block but more in the plastic one. We apply aggregation weights to the output feature maps of these blocks, sum them up, and pass the result maps to the next residual level. In this way, we are able to dynamically balance the usage of these blocks by updating...
ing their aggregation weights. To achieve auto-updating, we take the weights as hyperparameters and optimize them in an end-to-end manner [12, 25, 48].

Technically, the overall optimization of AANets is bilevel. Level-1 is to learn the network parameters for two types of residual blocks, and level-2 is to adapt their aggregation weights. More specifically, level-1 is the standard optimization of network parameters, for which we use all the data available in the phase. Level-2 aims to balance the usage of the two types of blocks, for which we optimize the aggregation weights using a balanced subset (by downsampling the data of new classes), as illustrated in Figure 1 (c). We formulate these two levels in a bilevel optimization program (BOP) [41] that solves two optimization problems alternatively, i.e., update network parameters with aggregation weights fixed, and then switch. For evaluation, we conduct CIL experiments on three widely-used benchmarks, CIFAR-100, ImageNet-Subset, and ImageNet. We find that many existing CIL methods, e.g., iCaRL [34], LUCIR [16], Mnemonics Training [25], and PODNet [11], can be directly incorporated in the architecture of AANets, yielding consistent performance improvements. We observe that a straightforward plug-in causes memory overheads, e.g., 26% and 15% respectively for CIFAR-100 and ImageNet-Subset. For a fair comparison, we conduct additional experiments under the settings of zero overhead (e.g., by reducing the number of old exemplars for training AANets), and validate that our approach still achieves top performance across all datasets.

Our contribution is three-fold: 1) a novel and generic network architecture called AANets specially designed for tackling the stability-plasticity dilemma in CIL tasks; 2) a BOP-based formulation and an end-to-end training solution for optimizing AANets; and 3) extensive experiments on three CIL benchmarks by incorporating four baseline methods in the architecture of AANets.

2. Related Work

Incremental learning aims to learn efficient machine models from the data that gradually come in a sequence of training phases. Closely related topics are referred to as continual learning [10, 26] and lifelong learning [2, 7, 22]. Recent incremental learning approaches are either task-based, i.e., all-class data come but are from a different dataset for each new phase [5, 6, 8, 17, 23, 35, 40, 54], or class-based i.e., each phase has the data of a new set of classes coming from the identical dataset [4, 16, 18, 25, 34, 48, 53]. The latter one is typically called class-incremental learning (CIL), and our work is based on this setting. Related methods mainly focus on how to solve the problems of forgetting old data. Based on their specific methods, they can be categorized into three classes: regularization-based, replay-based, and parameter-isolation-based [9, 30].

Regularization-based methods introduce regularization terms in the loss function to consolidate previous knowledge when learning new data. Li et al. [23] proposed the regularization term of knowledge distillation [15]. Hou et al. [16] introduced a series of new regularization terms such as for less-forgetting constraint and inter-class separation to mitigate the negative effects caused by the data imbalance between old and new classes. Douillard et al. [11] proposed an effective spatial-based distillation loss applied throughout the model and also a representation comprising multiple proxy vectors for each object class. Tao et al. [44] built the framework with a topology-preserving loss to maintain the topology in the feature space. Yu et al. [51] estimated the drift of previous classes during the training of new classes. Replay-based methods store a tiny subset of old data, and replay the model on them (together with new class data) to reduce the forgetting. Rebuffi et al. [34] picked the nearest neighbors to the average sample per class to build this subset. Liu et al. [25] parameterized the samples in the subset, and then meta-optimized them automatically in an end-to-end manner taking the representation ability of the whole set as the meta-learning objective. Belouadah et al. [3] proposed to leverage a second memory to store statistics of old classes in rather compact formats.

Parameter-isolation-based methods are used in task-based incremental learning (but not CIL). Related methods dedicate different model parameters for different incremental phases, to prevent model forgetting (caused by parameter overwritten). If no constraints on the size of the neural network is given, one can grow new branches for new tasks while freezing old branches. Rusu et al. [38] proposed “progressive networks” to integrate the desiderata of different tasks directly into the networks. Abati et al. [1] equipped each convolution layer with task-specific gating modules that select specific filters to learn each new task. Rajasegaran et al. [31] progressively chose the optimal paths for the new task while encouraging to share parameters across tasks. Xu et al. [49] searched for the best neural network architecture for each coming task by leveraging reinforcement learning strategies. Our differences with these methods include the following aspects. We focus on class-incremental learning, and more importantly, our approach does not continuously increase the network size. We validate in the experiments that under a strict memory budget, our approach can surpass many related methods and its plug-in versions on these related methods can bring consistent performance improvements.

Bilevel Optimization Program can be used to optimize hyperparameters of deep models. Technically, the network parameters are updated at one level and the key hyperparameters are updated at another level [13, 21, 24, 45, 46, 52]. Recently, a few bilevel-optimization-based approaches have emerged for tackling incremental learning tasks. Wu et
al. [48] learned a bias correction layer for incremental learning models using a bilevel optimization framework. Rajasegaran et al. [32] incrementally learned new tasks while learning a generic model to retain the knowledge from all tasks. Riemer et al. [36] learned network updates that are well-aligned with previous phases, such as to avoid learning towards any distracting directions. In our work, we apply the bilevel optimization program to update the aggregation weights in our AANets.

3. Adaptive Aggregation Networks (AANets)

Class-Incremental Learning (CIL) usually assumes \((N + 1)\) learning phases in total, i.e., one initial phase and \(N\) incremental phases during which the number of classes gradually increases \([11, 16, 18, 25]\). In the initial phase, data \(\mathcal{D}_0\) is available to train the first model \(\Theta_0\). There is a strict memory budget in CIL systems, so after the phase, only a small subset of \(\mathcal{D}_0\) (exemplars denoted as \(\mathcal{E}_0\)) can be stored in the memory and used as replay samples in later phases. Specifically in the \(i\)-th \((i \geq 1)\) phase, we load the exemplars of old classes \(\mathcal{E}_{0:i-1} = \{\mathcal{E}_0, \ldots, \mathcal{E}_{i-1}\}\) to train model \(\Theta_i\) together with new class data \(\mathcal{D}_i\). Then, we evaluate the trained model on the test data containing both old and new classes. We repeat such training and evaluation through all phases.

The key issue of CIL is that the models trained at new phases easily “forget” old classes. To tackle this, we introduce a novel architecture called AANets. AANets is based on a ResNet-type architecture, and each of its residual levels is composed of two types of residual blocks: a plastic one to adapt to new class data and a stable one to maintain the knowledge learned from old classes. The details of this architecture are elaborated in Section 3.1. The steps for optimizing AANets are given in Section 3.2.

3.1. Architecture Details

In Figure 2, we provide an illustrative example of our AANets with three residual levels. The inputs \(x^{[0]}\) are the images and the outputs \(x^{[3]}\) are the features used to train classifiers. Each of our residual “levels” consists of two parallel residual “blocks” (of the original ResNet [14]): the orange one (called plastic block) will have its parameters fully adapted to new class data, while the blue one (called stable block) has its parameters partially fixed in order to maintain the knowledge learned from old classes. After feeding the inputs to Level 1, we obtain two sets of feature maps respectively from two blocks, and aggregate them after applying the aggregation weights \(\alpha^{[1]}\). Then, we feed the resulted maps to Level 2 and repeat the aggregation. We apply the same steps for Level 3. Finally, we pool the resulted maps obtained from Level 3 to train classifiers. Below we elaborate the details of this dual-branch design as well as the steps for feature extraction and aggregation.

**Stable and Plastic Blocks.** We deploy a pair of stable and plastic blocks at each residual level, aiming to balance between the plasticity, i.e., for learning new classes, and stability, i.e., for not forgetting the knowledge of old classes. We achieve these two types of blocks by allowing different levels of learnability, i.e., less learnable parameters in the stable block but more in the plastic. We detail the operations in the following. In any CIL phase, Let \(\eta\) and \(\phi\) represent the learnable parameters of plastic and stable blocks, respectively. \(\eta\) contains all the convolutional weights, while \(\phi\) contains only the neuron-level scaling weights \([43]\). Specifically, these scaling weights are applied on the model \(\theta_{\text{base}}\).
obtained in the 0-th phase\(^2\). As a result, the number of learnable parameters \(\phi\) is much less than that of \(\eta\). For example, when using \(3 \times 3\) neurons in \(\theta\)\text{base}, the number of learnable parameters \(\phi\) is only \(\frac{1}{27}\) of the number of full network parameters (while \(\eta\) has the full network parameters). We further elaborate on these in the following paragraph.

**Neuron-level Scaling Weights.** For stable blocks, we learn its neuron parameters in the 0-th phase and freeze them in the other \(N\) phases. In these \(N\) phases, we apply a small set of scaling weights \(\phi\) at the neuron-level, i.e., each weight for scaling one neuron in \(\theta\)\text{base}. We aim to preserve the structural pattern within the neuron and slowly adapt the knowledge of the whole blocks to new class data. Specifically, we assume the \(q\)-th layer of \(\theta\)\text{base} contains \(R\) neurons, so we have \(R\) neuron weights as \(\{W_q, r\}_{r=1}^R\). For conciseness, we denote them as \(W_q\). For \(W_q\), we learn \(R\) scaling weights denoted as \(\phi_q\). Let \(X_{q-1}\) and \(X_q\) be the input and output feature maps of the \(q\)-th layer, respectively. We apply \(\phi_q\) to \(W_q\) as follows,

\[
X_q = (W_q \odot \phi_q)X_{q-1},
\]

where \(\odot\) denotes the element-wise multiplication. Assuming there are \(Q\) layers in total, the overall scaling weights can be denoted as \(\phi = \{\phi_q\}_{q=1}^Q\).

**Feature Extraction and Aggregation.** We elaborate on the process of feature extraction and aggregation across all residual levels in the AANets, as illustrated in Figure 2. Let \(F^{[k]}(\cdot)\) denote the transformation function of the residual block parameterized as \(\mu\) at the Level \(k\). Given a batch of training images \(x^{[0]}\), we feed them to AANets to compute the feature maps at the \(k\)-th level (through the stable and plastic blocks respectively) as follows,

\[
x^{[k]}_{\phi} = F^{[k]}_{\phi \odot \theta\text{base}}(x^{[k-1]}); \quad x^{[k]}_{\eta} = F^{[k]}_{\eta}(x^{[k-1]}).
\]

The transferability (of the knowledge learned from old classes) is different at different levels of neural networks [50]. Therefore, it makes more sense to apply different aggregation weights for different levels of residual blocks. Let \(\alpha^{[k]}_{\phi}\) and \(\alpha^{[k]}_{\eta}\) denote the aggregation weights of the stable and plastic blocks, respectively, at the \(k\)-th level. Then, the weighted sum of \(x^{[k]}_{\phi}\) and \(x^{[k]}_{\eta}\) can be derived as follows,

\[
x^{[k]} = \alpha^{[k]}_{\phi} \cdot x^{[k]}_{\phi} + \alpha^{[k]}_{\eta} \cdot x^{[k]}_{\eta}.
\]

In our illustrative example in Figure 2, there are three pairs of weights to learn at each phase. Hence, it becomes increasingly challenging to choose these weights manually if multiple phases are involved. In this paper, we propose an learning strategy to automatically adapt these weights, i.e., optimizing the weights for different blocks in different phases, see details in Section 3.2.

### 3.2. Optimization Steps

In each incremental phase, we optimize two groups of learnable parameters in AANets: (a) the neuron-level scaling weights \(\phi\) for the stable blocks and the convolutional weights \(\eta\) on the plastic blocks; (b) the feature aggregation weights \(\alpha\). The former is for network parameters and the latter is for hyperparameters. In this paper, we formulate the overall optimization process as a bilevel optimization program (BOP) [13, 25].

**The Formulation of BOP.** In AANets, the network parameters \([\phi, \eta]\) are trained using the aggregation weights \(\alpha\) as hyperparameters. In turn, \(\alpha\) can be updated when temporarily fixing network parameters \([\phi, \eta]\). In this way, the optimality of \([\phi, \eta]\) imposes a constraint on \(\alpha\) and vise versa. Ideally, in the \(i\)-th phase, the CIL system aims to learn the optimal \(\alpha_i\) and \([\phi_i, \eta_i]\) that minimize the classification loss on all training samples seen so far, i.e., \(D_0 \cup D_{0:i-1}\), so the ideal BOP can be formulated as,

\[
\min_{\alpha_i} \mathcal{L}(\alpha_i, \phi_i^*, \eta_i^*; D_0 \cup D_{0:i-1} \cup D_i) \quad (4a)
\]

\[
\text{s.t. } [\phi_i^*, \eta_i^*] = \arg \min_{[\phi_i, \eta_i]} \mathcal{L}(\alpha_i, \phi_i, \eta_i; D_0 \cup D_{0:i-1} \cup D_i), \quad (4b)
\]

where \(\mathcal{L}(\cdot)\) denotes the loss function, e.g., cross-entropy loss. Please note that for the conciseness of the formulation, we use \(\phi_i\) to represent \(\phi_i \odot \theta\text{base}\) (same in the fol-
lowing equations). We call Problem 4a and Problem 4b the upper-level and lower-level problems, respectively.

**Data Strategy.** To solve Problem 4, we need to use $D_{0:i−1}$. However, in the setting of CIL [11, 16, 34], we cannot access $D_{0:i−1}$ but only a small set of exemplars $E_{0:i−1}$, e.g., 20 samples of each old class. Directly replacing $D_{0:i−1} \cup D_i$ with $E_{0:i−1} \cup D_i$ in Problem 4 will lead to the forgetting problem for the old classes. To alleviate this issue, we propose a new data strategy in which we use different training data splits to learn different groups of parameters: 1) in the upper-level problem, $\alpha_i$ is used to balance the stable and the plastic blocks, so we use the balanced subset to update it, i.e., learning $\alpha_i$ on $E_{0:i−1} \cup E_i$ adaptively; 2) in the lower-level problem, $[\phi_i, \eta_i]$ are the network parameters used for feature extraction, so we leverage all the available data to train them, i.e., base-training $[\phi_i, \eta_i]$ on $E_{0:i−1} \cup D_i$. Based on these, we can reformulate the ideal BOP in Problem 4 as a solvable BOP as follows,

\[
\min_{\phi_i, \eta_i} \mathcal{L}(\alpha_i, \phi_i^*, \eta_i^*; E_{0:i−1} \cup E_i) \tag{5a}
\]

s.t. $[\phi_i^*, \eta_i^*] = \arg \min_{[\phi_i, \eta_i]} \mathcal{L}(\alpha_i, \phi_i, \eta_i; E_{0:i−1} \cup D_i), \tag{5b}
\]

where Problem 5a is the upper-level problem and Problem 5b is the lower-level problem we are going to solve.

**Updating Parameters.** We solve Problem 5 by alternatively updating two groups of parameters ($\alpha_i$ and $[\phi, \eta]$) across epochs, e.g., if $\alpha_i$ is updated in the $j$-th epoch, then $[\phi, \eta]$ will be updated in the $(j + 1)$-th epoch, until both of them converge. Taking the $i$-th phase as an example, we initialize $\alpha_i$, $\phi_i$, $\eta_i$ with $\alpha_{i−1}$, $\phi_{i−1}$, $\eta_{i−1}$, respectively. Please note that $\phi_0$ is initialized with ones, following [42, 43]; $\eta_0$ is initialized with $\theta_{base}$; and $\alpha_0$ is initialized with 0.5. Based on our Data Strategy, we use all available data in the current phase to solve the lower-level problem, i.e., training $[\phi_i, \eta_i]$ as follows,

\[
[\phi_i, \eta_i] \leftarrow [\phi_i, \eta_i] - \gamma_1 \nabla_{[\phi_i, \eta_i]} \mathcal{L}(\alpha_i, \phi_i, \eta_i; E_{0:i−1} \cup D_i). \tag{6}
\]

Then, we use a balanced exemplar set to solve the upper-level problem, i.e., training $\alpha_i$ as follows,

\[
\alpha_i \leftarrow \alpha_i - \gamma_2 \nabla_{\alpha_i} \mathcal{L}(\alpha_i, \phi_i, \eta_i; E_{0:i−1} \cup E_i), \tag{7}
\]

where $\gamma_1$ and $\gamma_2$ are the lower-level and upper-level learning rates, respectively.

**3.3. Algorithm**

In Algorithm 1, we summarize the overall training steps of the proposed AAANets in the $i$-th incremental learning phase (where $i \in [1, \ldots, N]$). Lines 1-4 show the pre-processing including loading new data and old exemplars (Line 1), initializing the two groups of learnable parameters (Lines 2-3), and selecting the exemplars for new classes (Line 4). Lines 5-12 optimize alternatively between the network parameters and the Adaptive Aggregation weights. In specific, Lines 6-8 and Lines 9-11 execute the training for solving the upper-level and lower-level problems, respectively. Lines 13-14 update the exemplars and save them to the memory.

**4. Experiments**

We evaluate the proposed AAANets on three CIL benchmarks, i.e., CIFAR-100 [20], ImageNet-Subset [34] and ImageNet [37]. We incorporate AAANets into four baseline methods and boost their model performances consistently for all settings. Below we describe the datasets and implementation details (Section 4.1), followed by the results and analyses (Section 4.2) which include a detailed ablation study, extensive comparisons to related methods, and some visualization of the results.

**4.1. Datasets and Implementation Details**

**Datasets.** We conduct CIL experiments on two datasets, CIFAR-100 [20] and ImageNet [37], following closely related work [11, 16, 25]. CIFAR-100 contains 60,000 samples of $32 \times 32$ color images for 100 classes. There are 500 training and 100 test samples for each class. ImageNet contains around 1.3 million images of $224 \times 224$ color images for 1000 classes. There are approximately 1,300 training and 50 test samples for each class. ImageNet is used in two CIL settings: one based on a subset of 100 classes...
we deploy a 32-sample from ImageNet in the same way as \[11, 16\].

10 residual blocks (in a single branch). Each block has

This ResNet consists of 1 (based on which we build the AANets) for CIFAR-100.

(indicators the number of learnable parameters. “Memory” denotes the peak memory for storing the exemplars and the learnable & frozen

Ablation settings. Adapted \( \alpha \) are applied on Rows 3-8. “all”+“scaling” is the default setting of Rows 6-8. “\#Param” indicates the number of learnable parameters. “Memory” denotes the peak memory for storing the exemplars and the learnable & frozen network parameters during the model training through all phases. Please refer to more results in the supplementary materials.

Table 1. Ablation study. The baseline (Row 1) is LUCIR \[16\]. “all”, “scaling”, and “frozen” denote three types of blocks and they have different numbers of learnable parameters, e.g., “all” means all convolutional weights and biases are learnable. If we name them as A, B, and C, we use A+B in the table to denote the setting of using A-type and B-type blocks respectively as plastic and stable blocks. See more details in Section 4.2 Ablation settings. Adapted \( \alpha \) are applied on Rows 3-8. “all”+“scaling” is the default setting of Rows 6-8. “\#Param” indicates the number of learnable parameters. “Memory” denotes the peak memory for storing the exemplars and the learnable & frozen network parameters during the model training through all phases. Please refer to more results in the supplementary materials.

(32 bytes/image)
Table 2. Average incremental accuracies (%) of four state-of-the-art methods \(w/\) and \(w/o\) our AANets as a plug-in architecture. In the upper block, we present some comparable results reported in some other related works. Please note 1) [11] didn’t report the results for \(N=25\) on the ImageNet, and we produce the results using their public code; 2) [25] updated their results on arXiv (after fixing a bug in their code), different from its conference version; 3) for “\(w/\) AANets”, we use “all+“scaling” blocks corresponding to Row 3 of Table 1; and 4) if applying “strict memory budget”, there is little performance drop. Corresponding results are given in Table 1 and Table S2 in the supplementary materials.

| Method         | CIFAR-100  | ImageNet-Subset | ImageNet |
|----------------|------------|-----------------|----------|
|                | \(N=5\)   | 10             | 25       | 5         | 10         | 25         | 5         | 10         | 25         |
| LwF [23]       | 49.59      | 46.98           | 45.51    | 53.62     | 47.64      | 44.32      | 44.35     | 38.90      | 36.87      |
| BiC [48]       | 59.56      | 54.20           | 50.00    | 70.07     | 64.96      | 57.73      | 62.65     | 58.72      | 53.47      |
| TPCIL [44]     | 65.34      | 63.58           | –        | 76.27     | 74.81      | –          | 64.89     | 62.88      | –          |
| iCuRL [34]     | 57.12 ±0.50 | 52.66 ±0.49    | 48.22 ±0.76 | 65.44 ±0.35 | 59.88 ±0.83 | 52.97 ±1.02 | 51.50 ±0.43 | 46.89 ±0.35 | 43.14 ±0.67 |
| \(w/\) AANets (ours) | 64.22 ±0.42 | 60.26 ±0.73    | 56.43 ±0.81 | 73.45 ±0.51 | 71.78 ±0.64 | 69.22 ±0.83 | 63.91 ±0.50 | 61.28 ±0.49 | 56.97 ±0.80 |
| LUCIR [16]     | 63.17 ±0.87 | 60.14 ±0.73    | 57.54 ±0.43 | 70.84 ±0.60 | 68.32 ±0.81 | 61.44 ±0.91 | 64.45 ±0.32 | 61.57 ±0.23 | 56.56 ±0.36 |
| \(w/\) AANets (ours) | 67.56 ±0.37 | 65.59 ±0.43    | 63.50 ±0.61 | 72.85 ±0.67 | 69.22 ±0.72 | 67.60 ±0.39 | 64.94 ±0.25 | 62.39 ±0.61 | 60.68 ±0.50 |
| Mnemonics [25] | 63.34 ±0.62 | 62.28 ±0.43    | 60.96 ±0.72 | 72.58 ±0.85 | 71.37 ±0.56 | 69.74 ±0.39 | 64.54 ±0.49 | 63.01 ±0.57 | 61.00 ±0.71 |
| \(w/\) AANets (ours) | 67.35 ±0.54 | 65.66 ±0.61    | 63.35 ±0.72 | 72.91 ±0.53 | 71.93 ±0.37 | 70.70 ±0.45 | 65.23 ±0.62 | 63.60 ±0.71 | 61.53 ±0.20 |
| PODNet-CNN [11] | 64.83 ±1.11 | 63.19 ±1.31    | 60.72 ±1.54 | 75.54 ±1.20 | 74.33 ±1.05 | 68.31 ±2.77 | 66.95     | 64.13      | 59.17      |
| \(w/\) AANets (ours) | 66.31 ±0.87 | 64.31 ±0.90    | 62.31 ±1.02 | 76.45 ±0.53 | 75.58 ±0.74 | 71.78 ±0.81 | 67.73 ±0.71 | 68.45 ±0.53 | 61.78 ±0.61 |

In Table 1, the pattern of combining blocks is A+B where A and B stands for the plastic and the stable blocks, respectively. Rows 1 is the baseline method LUCIR [16]. Row 2 is a double-branch version for LUCIR without learning any aggregation weights. Rows 3-5 are our AANets using different combinations of blocks. Row 6-8 use “all+“scaling” under an additional setting as follows. 1) Row 6 uses imbalanced data \(\varepsilon_{0:1} \cup \mathcal{D}_i\) to train \(\alpha_i\) adaptively. 2) Row 7 uses fixed weights \(\alpha_\eta = \alpha_\phi = 0.5\) at each residual level. 3) Row 8 is under the “strict memory budget” setting, where we reduce the numbers of exemplars to 14 and 17 for CIFAR-100 and ImageNet-Subset, respectively.

**Ablation Results.** In Table 1, if comparing the second block (ours) to the first block (single-branch and double-branch baselines), it is obvious that using AANets can clearly improve the model performance, e.g., “scaling”+“frozen” gains an average of 4.8% over LUCIR for the ImageNet-Subset, by optimizing 1.4M parameters during CIL — only 12.6% of that in LUCIR. Among Rows 3-5, we can see that for the ImageNet-Subset, models with the fewest learnable parameters (“scaling”+“frozen”) work the best. We think this is because we use shallower networks for learning larger datasets (ResNet-32 for CIFAR-100; ResNet-18 for ImageNet-Subset), following the Benchmark Protocol. In other words, \(\theta_{base}\) is quite well-trained with the rich data of half ImageNet-Subset (50 classes in the 0-th phase), and can offer high-quality features for later phases. Comparing Row 6 to Row 3 shows the efficiency of using a balanced subset to optimize \(\alpha\). Comparing Row 7 to Row 3 shows the superiority of learning \(\alpha\) (which is dynamic and optimal) over manually-choosing \(\alpha\).

**About the Memory Usage.** By comparing Row 3 to Row 1, we can see that AANets can clearly improve the model performance while introducing small overheads for the memory, e.g., 26% and 14.5% on the CIFAR-100 and ImageNet-Subset, respectively. If comparing Row 8 to Row 3, we find that though the numbers of exemplars are reduced (for Row 8), the model performance of AANets has a very small drop, e.g., only 0.3% for the 5-Phase CIL models of CIFAR-100 and ImageNet-Subset. Therefore, we can conclude that AANets can achieve rather satisfactory performance under strict memory control — a desirable feature needed in class-incremental learning systems.

**Comparing to the State-of-the-Art.** Table 2 shows that taking our AANets as a plug-in architecture for 4 state-of-the-art methods [11, 16, 25, 34] consistently improves their model performances. E.g., for CIFAR-100, LUCIR \(w/\) AANets and Mnemonics \(w/\) AANets respectively gains 4.9% and 3.3% improvements on average. From Table 2, we can see that our approach of using AANets achieves top performances in all settings. Interestingly, we find that AANets can boost more performance for simpler baseline methods, e.g., iCaRL. iCaRL \(w/\) AANets achieves mostly better results than those of LUCIR on three datasets, even though the latter method deploys various regularization techniques.

**Visualizing Activation Maps.** Figure 3 demonstrates the activation maps visualized by Grad-CAM for the final model (obtained after 5 phases) on ImageNet-Subset \((N=5)\). The visualized samples from left to right are picked from the classes coming in the 0-th, 3-rd and 5-th phases, respectively. For the 0-th phase samples, the model makes the prediction according to foreground regions (right) detected by the stable block and background regions (wrong) by the
classes seen in phase 0

goldfinch

goldfinch

quail

classes seen in phase 3

terrier

terrier

dhole

classes seen in phase 5

Arctic fox

Arctic fox

can opener

Figure 3. The activation maps using Grad-CAM [39] for the 5-th phase (the last phase) model on ImageNet-Subset (N=5). Samples are selected from the classes coming in the 0-th phase (left), the 3-rd phase (middle), and the 5-th phase (right), respectively. Green tick (red cross) means the discriminative features are activated on the object regions successfully (unsuccessfully). $\bar{\alpha}_\eta = 0.428$ and $\bar{\alpha}_\phi = 0.572$.

Figure 4. The values of $\alpha_\eta$ and $\alpha_\phi$ adapted for each residual level and in each incremental phase. All curves are smoothed with a rate of 0.8 for better visualization.

Aggregation Weights ($\alpha_\eta$ and $\alpha_\phi$). Figure 4 shows the values of $\alpha_\eta$ and $\alpha_\phi$ learned during training 10-phase models. Each row displays three plots for three residual levels of AANets respectively. Comparing among columns, we can see that Level 1 tends to get larger values of $\alpha_\phi$, while Level 3 tends to get larger values of $\alpha_\eta$. This can be interpreted as lower-level residual blocks learn to stay stabler which is intuitively correct in deep models. With respect to the learning activity of CIL models, it is to continuously transfer the learned knowledge to subsequent phases. The features at different resolutions (levels in our case) have different transferabilities [50]. Level 1 encodes low-level features that are more stable and shareable among all classes. Level 3 nears the classifiers, and tends to be more plastic such as to fast to adapt to new classes.

5. Conclusions

We introduce a novel network architecture AANets specially for CIL. Our main contribution lies in addressing the issue of stability-plasticity dilemma in CIL by a simple modification on plain ResNets — applying two types of residual blocks to respectively and specifically learn stability and plasticity at each residual level, and then aggregating them as a final representation. To achieve efficient aggregation, we adapt the level-specific and phase-specific weights in an end-to-end manner. Our overall approach is generic and can be easily incorporated into existing CIL methods to boost their performance.

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Supplementary Materials

These supplementary materials include the results for different CIL settings ($\S A$), “strict memory budget” experiments ($\S B$), additional ablation results ($\S C$), additional plots ($\S D$), more visualization results ($\S E$), and the execution steps of our source code with PyTorch ($\S F$).

A. Results for Different CIL Settings.

We provide more results on the setting with the same number of classes at all phases [32] in the second block (“same # of cls”) of Table S1. For example, $N=25$ indicates 100 classes evenly come in 25 phases, so 4 new classes arrive in each phase (including the 0-th phase). Further, in this table, each entry represents an accuracy of the last phase (since all-phase accuracies are not comparable to our original setting) averaged over 3 runs; and “update $\theta_{\text{base}}$” means that $\theta_{\text{base}}$ is updated as $\theta_{\text{base}} \leftarrow \phi_i \odot \theta_{\text{base}}$ after each phase. All results are under “strict memory budget” and “all”+“scaling” settings, so $\phi_i$ indicate the meta-learned weights of SS operators. The results show that 1) “w/ AANets” performs best in all settings and brings consistent improvements; and 2) “update $\theta_{\text{base}}$” is helpful for CIFAR-100 but harmful for ImageNet-Subset.

| Last-phase acc. (%) | CIFAR-100 | ImageNet-Subset |
|---------------------|-----------|-----------------|
|                     | N=5 | 10 | 25 | 5 | 10 | 25 |
| LUCIR (50 cls in Phase 0) | 54.3 | 50.3 | 48.4 | 60.0 | 57.1 | 49.3 |
| w/ AANets | **58.6** | **56.7** | **53.3** | 64.3 | 58.0 | 56.5 |
| LUCIR (same # of cls) | 52.1 | 44.9 | 40.6 | 60.3 | 52.5 | 53.3 |
| w/ AANets, update $\theta_{\text{base}}$ | 54.3 | 47.4 | 42.4 | 61.4 | 52.5 | 48.2 |
| w/ AANets | 52.6 | 46.1 | 41.9 | **68.8** | **60.8** | **56.8** |

Table S1. Supplementary to Table 1. Last-phase accuracies (%) for different class-incremental learning (CIL) settings.

B. Strict Memory Budget Experiments

In Table S2, we present the results of 4 state-of-the-art methods w/ and w/o AANetss as a plug-in architecture, under the “strict memory budget” setting which strictly controls the total memory shared by the exemplars and the model parameters. For example, if we incorporate AANetss to LUCIR [16], we need to reduce the number of exemplars to balance the additional memory introduced by AANetss (as AANetss take around 20% more parameters than the plain ResNets used in LUCIR [16]). As a result, we reduce the numbers of exemplars for AANetss from 20 to 13, 16 and 19, respectively, for CIFAR-100, ImageNet-Subset, and ImageNet, in the “strict memory budget” setting. For CIFAR-100, we use 530k additional parameters, so we need to reduce 530k floats $\times$ 4 bytes/float $\div$ $(32 \times 32 \times 3$ bytes/image $) \div 100$ classes $\approx$ 6.9 images/class, and $[6, 9] = 7$ images/class. For ImageNet-Subset, we use 12.6M additional parameters, so we need to reduce 12.6M floats $\times$ 4 bytes/float $\div (224 \times 224 \times 3$ bytes/image $) \div 100$ classes $\approx$ 3.3 images/class, and $[3, 3] = 4$ images/class. For ImageNet, we use 12.6M additional parameters, so we need to reduce 12.6M floats $\times$ 4 bytes/float $\div (224 \times 224 \times 3$ bytes/image $) \div 100$ classes $\approx$ 0.3 images/class, and $[0, 3] = 1$ image/class. From Table S2, we can see that our approach of using AANetss still achieves the top performances in all CIL settings even if the “strict memory budget” is applied.

C. More Ablation Results

In Table S3, we supplement the ablation results obtained in more settings. “$x \times$” denotes that we use 4 same-type blocks at each residual level. Comparing Row 7 to Row 2 (Row 5) shows the efficiency of using different types of blocks for representing stability and plasticity.

D. Additional Plots

In Figures S2, we present the phase-wise accuracies obtained on CIFAR-100, ImageNet-Subset and ImageNet, respectively. “Upper Bound” shows the results of joint training with all previous data accessible in every phase. We can observe that our method achieves the highest accuracies in almost every phase of different settings. In Figures S3 and S4, we supplement the plots for the values of $\alpha$ and $\alpha_3$ learned on the CIFAR-100 and ImageNet-Subset ($N=5, 25$). All curves are smoothed with a rate of 0.8 for a better visualization.

E. More Visualization Results

Figure S1 below shows the activation maps of a “goldfinch” sample (seen in Phase 0) in different-phase models (ImageNet-Subset, $N=5$). Notice that the plastic block gradually loses its attention on this sample (i.e., forgets it), while the stable block retains it. AANetss benefit from its stable blocks.

F. Source Code in PyTorch

We provide our PyTorch code on [https://class-il.mpirmf.de/](https://class-il.mpirmf.de/). To run this repository, we kindly advise you to install Python 3.6 and PyTorch 1.2.0 with Anaconda.
| Method                  | N=5  | 10   | 25   | N=5  | 10   | 25   | N=5  | 10   | 25   |
|------------------------|------|------|------|------|------|------|------|------|------|
| iCaRL [34]             | 57.12 ± 0.50 | 52.66 ± 0.89 | 48.22 ± 0.76 | 65.44 ± 0.35 | 59.88 ± 0.83 | 52.97 ± 0.02 | 51.50 ± 0.33 | 46.89 ± 0.35 | 43.14 ± 0.67 |
| w/ AANetss (ours)      | 63.91 ± 0.52 | 57.65 ± 0.81 | 52.10 ± 0.87 | 71.37 ± 0.57 | 66.34 ± 0.61 | 61.87 ± 0.01 | 63.65 ± 0.02 | 61.14 ± 0.59 | 55.91 ± 0.95 |
| LUCIR [16]             | 64.22 ± 0.42 | 60.26 ± 0.73 | 56.43 ± 0.81 | 73.45 ± 0.51 | 71.78 ± 0.64 | 69.22 ± 0.83 | 63.91 ± 0.59 | 61.28 ± 0.49 | 56.97 ± 0.86 |
| w/ AANetss (ours)      | 63.17 ± 0.87 | 60.14 ± 0.73 | 57.54 ± 0.43 | 70.84 ± 0.69 | 68.32 ± 0.81 | 64.44 ± 0.91 | 64.45 ± 0.32 | 61.57 ± 0.23 | 56.56 ± 0.36 |
| Mnemonics [25]         | 66.46 ± 0.45 | 65.38 ± 0.53 | 61.79 ± 0.51 | 72.21 ± 0.87 | 69.10 ± 0.90 | 67.10 ± 0.54 | 64.83 ± 0.50 | 62.34 ± 0.65 | 60.49 ± 0.78 |
| w/ AANetss (ours)      | 66.74 ± 0.37 | 65.29 ± 0.43 | 63.50 ± 0.61 | 72.55 ± 0.67 | 69.22 ± 0.72 | 67.60 ± 0.39 | 64.94 ± 0.25 | 62.39 ± 0.61 | 60.68 ± 0.58 |
| PODNet-CNN [11]        | 63.34 ± 0.62 | 62.28 ± 0.43 | 60.96 ± 0.72 | 72.58 ± 0.85 | 71.37 ± 0.56 | 69.74 ± 0.39 | 64.54 ± 0.49 | 63.01 ± 0.57 | 61.00 ± 0.71 |
| w/ AANetss (ours)      | 66.12 ± 0.00 | 65.10 ± 0.00 | 61.83 ± 0.00 | 72.88 ± 0.00 | 71.50 ± 0.00 | 70.49 ± 0.00 | 65.21 ± 0.76 | 63.36 ± 0.67 | 61.37 ± 0.80 |
| AANetss (ours)         | 67.59 ± 0.34 | 65.66 ± 0.61 | 63.35 ± 0.72 | 72.91 ± 0.53 | 71.93 ± 0.37 | 70.70 ± 0.45 | 65.23 ± 0.62 | 63.60 ± 0.71 | 61.53 ± 0.29 |

Table S2. Supplementary to Table 2. Using “strict memory budget” setting. Average incremental accuracies (%) of four state-of-the-art methods w/ and w/o our AANetss as a plug-in architecture. The red lines are the corresponding results in Table 2 of the main paper.

| Row | Ablation Setting | CIFAR-100 (acc. %) | ImageNet-Subset (acc. %) |
|-----|-----------------|---------------------|--------------------------|
|     | Memory | FLOPs | #Param | N=5  | 10   | 25   | Memory | FLOPs | #Param | N=5  | 10   | 25   |
| 1   | single-branch “all” [16] | 7.64MB | 70M | 469K | 63.17 | 60.14 | 57.54 | 330MB | 1.82G | 11.2M | 70.84 | 68.32 | 61.44 |
| 2   | “all” + “all” | 9.43MB | 140M | 938K | 64.49 | 61.89 | 58.87 | 372MB | 3.64G | 22.4M | 69.72 | 66.69 | 63.29 |
| 3   | 4× “all” | 13.01MB | 280M | 1.9M | 65.13 | 64.08 | 59.40 | 456MB | 7.28G | 44.8M | 70.12 | 67.31 | 64.00 |
| 4   | single-branch “scaling” | 7.64MB | 70M | 60K | 62.48 | 61.53 | 60.17 | 334MB | 1.82G | 1.4M | 71.29 | 68.88 | 66.75 |
| 5   | “scaling” + “scaling” | 9.43MB | 140M | 120K | 65.13 | 64.08 | 62.50 | 382MB | 3.64G | 2.8M | 71.71 | 71.07 | 66.69 |
| 6   | 4× “scaling” | 13.01MB | 240M | 280K | 66.00 | 64.67 | 63.16 | 478MB | 3.64G | 5.6M | 72.01 | 71.23 | 67.12 |
| 7   | “all” + “scaling” | 9.66MB | 140M | 530K | 66.74 | 65.29 | 63.50 | 378MB | 3.64G | 12.6M | 72.55 | 69.22 | 67.60 |
| 8   | “all” + “frozen” | 9.43MB | 140M | 469K | 65.62 | 64.05 | 63.67 | 372MB | 3.64G | 11.2M | 71.71 | 69.87 | 67.92 |
| 9   | “scaling” + “frozen” | 9.66MB | 140M | 60K | 64.71 | 63.65 | 62.89 | 378MB | 3.64G | 1.4M | 73.01 | 71.65 | 70.30 |

Table S3. Supplementary to Table 1. More ablation study. “4×” denotes that we use 4 same-type blocks at each residual level.
Figure S2. **Supplementary to Table 2.** Phase-wise accuracies (%). Light-color ribbons are visualized to show the 95% confidence intervals. Comparing methods: Upper Bound (the results of joint training with all previous data accessible in each phase); PODNet (2020) [11]; Mnemonics (2020) [25]; LUCIR (2019) [16]; BiC (2019) [48]; iCaRL (2017) [34]; and LwF (2016) [23].

(a) CIFAR-100 (100 classes). In the 0-th phase, $\theta_{base}$ is trained on 50 classes, the remaining classes are given evenly in the subsequent phases.

(b) ImageNet-Subset (100 classes). In the 0-th phase, $\theta_{base}$ is trained on 50 classes, the remaining classes are given evenly in the subsequent phases.

(c) ImageNet (1000 classes). In the 0-th phase, $\theta_{base}$ on is trained on 500 classes, the remaining classes are given evenly in the subsequent phases.
Figure S3. Supplementary to Figure 4. The changes of values for $\alpha_\eta$ and $\alpha_\phi$ on CIFAR-100.

Figure S4. Supplementary to Figure 4. The changes of values for $\alpha_\eta$ and $\alpha_\phi$ on ImageNet-Subset.