ABSTRACT

The possible conversion of neutron matter to strange matter and its relevance to astrophysical problems are discussed. Particular attention is paid to the form in which this hypothetical "burning" process should be propagated in dense matter and to the moment when it should start.

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1. INTRODUCTION

Since first stated by Witten [1] the hypothesis of strange matter (hereafter SQM) being the true ground state of cold hadronic matter has attracted considerable attention [2]. Among the several aspects studied within the strange matter hypothesis we may mention the problem of the conversion of (cold) neutron matter to strange matter in astrophysical objects. This has been first studied in [3,4] and refined in [5,6] and all these works have assumed that the process is analogous to an ordinary (slow) combustion. This is of course very reasonable in a first approximation, but unfortunately things seem not to be that simple. Not only the actual way in which the conversion develops is subject to considerable doubt, but the moment when it can start is also controversial. This point turns out to be very important because it can alter dramatically the fate of the host star. We shall attempt to summarize in this work what is known (and unknown) about this problem, which appears to us far from being closed.

2. HOW AND WHEN DOES THE BURNING START?

A variety of mechanisms for the conversion to start have been discussed in [7], tentatively classified as ”primary” (in which SQM is produced somehow inside the dense environment) and ”secondary” (SQM or a large amount of energy to produce it are seeded from the outside of the star). Clearly, the secondary mechanisms, even though perfectly viable, are much difficult to assess and and we shall not discuss them here.

Perhaps the most likely primary mechanism discussed in the literature is the conversion via two-flavor quark matter. The scenario relies on the possibility of deconfining the light quarks from ordinary nucleons as the stellar matter is compressed/cooled in a supernova aftermath. This is sketched in Fig.1; the transition point is achieved at a density $n_c$ and the quark region relaxes to the SQM state on a timescale typical of weak interaction $\tau_w$ lowering its energy per baryon number unit. From the ”macroscopic” point of view, and since $\tau_w \ll \tau_{\text{dynamical}}$ the process may be modelled as isocore. The released energy difference builds up an overpressure $\Delta P$ whose value can be found for a given model calculation. In [8] we have employed a free massless quark gas + bag
pressure and the Bethe-Johnson I [9] equation of state for neutron matter to calculate
\[ \Delta P \simeq 70^{+10}_{-10} \text{ MeV fm}^{-3}, \]
depending somewhat on the parameters. This result shows that the overpressure is huge and its decay may be very important dynamically (see below). Note that, if at all, this process must occur shortly after the gravitational collapse itself since a significative compression is achieved on a timescale \( \tau \simeq 1 \text{ s} \) [10].

**Fig.1.** A possible route for SQM appearance viewed in the free-energy vs. density plane. Deconfinement is achieved at a value \( n_c \) (dashed line) and relaxation to SQM is further achieved on a weak-interaction timescale (pictured with a wavy line).

If SQM is not produced as the result of two-flavor quark matter “relaxation”, it is possible that it could arise because of thermal nucleation of SQM bubbles inside hot nuclear matter. The idea is that if a single bubble of the fluctuation distribution grows beyond a critical size [11] it will convert the surroundings afterwards. A simple calculation using a kinetic Fokker-Planck-Zel’dovich approach has been presented in [12]. The growth of the bubbles has been described by the expression

\[
\frac{\partial f}{\partial t} = -\frac{\partial \xi}{\partial r}
\]  

where \( f(r, t) \) is the time-dependent size distribution of bubbles and \( \xi \) is the nucleation rate. The appropriate solution for this problem may be written as
\[ \xi = 2.2 \times 10^{-2} \left( \frac{T}{\sigma} \right)^{\frac{1}{2}} N_{qc}^{\frac{3}{2}} \tau_w^{-1} \exp(-3.1 N_{qc}^{\frac{3}{2}} \sigma/T) \]  \hspace{1cm} (2)

with \( N_{qc} \) the number of quarks of the critical size bubble (\( \leq 300 \)), \( \tau_w \) a weak-interaction timescale and \( \sigma \) the surface tension coefficient. A lower bound to the temperature at which at least one bubble larger than the critical size will nucleate can be found by imposing

\[ \xi \Delta t V_{nuc} \geq 1 \]  \hspace{1cm} (3)

where \( \Delta t \) is the time available for prompt nucleation (\( \sim 1 \) s) and \( V_{nuc} \sim (1 \) km\(^3\)) is the volume in which the nucleation can take place. As \( \sigma \leq (70 \) MeV\(^3\)) from detailed calculations [13] and fittings to phenomenological models [14], the condition (3) can be re-written equivalently as

\[ T \geq 2 \) MeV \]  \hspace{1cm} (4)

This bound can be certainly satisfied in the first seconds of the neutron star life but not after that since the initial cooling phase is dominated by neutrino emission, which is in turn a strong function of \( T \). Therefore, we again conclude that the conversion is likely to start immediately after the gravitational collapse itself. The same conclusion holds if the conversion is triggered by ”dormant” strangelets [15] which become active as soon as a full neutronization of the core is possible (\( \tau \sim 1 \) s).

3. WHICH IS THE PREFERRED COMBUSTION MODE?

As we have discussed in section 2, a short timescale \( \sim 1 \) s for SQM to be ”operative” inside a proto-neutron star appears to be a general feature of any triggering mechanism. This is, of course, not enough to address the aftermath, since the form in which the conversion front propagates remains unclear. For a given chemical potential difference \( \Delta \mu \), the conversion \( n \rightarrow SQM \) releases energy much in the same way as a chemical reaction
does. Therefore, the well-known theory of combustions [16] can be applied to study the process. Physically, steady state combustions can occur in two modes: deflagrations and detonations. Deflagrations (or slow combustions) are subsonic burnings in which quasi-equilibrium among the burnt and unburnt fluids holds. Detonations (or rapid combustions) are supersonic and can be described as a shock wave followed by the combustion front.

Assuming that a slow combustion occurs, the velocity of the flame has been calculated to be $v_s \sim \text{few km/s}$ at most [3,4]. These works have focused mainly on the reaction kinetics of the conversion, but the actual boundary conditions in a star and the gas motion should play a role as well. To study these effects, a linear stability analysis of this propagation mode has been performed [17] in which the stability condition was found to be

$$j^4 < 4\, \sigma \, g \, \rho_1^2 \rho_2^2 \frac{1}{(\rho_2 - \rho_1)}$$

with $j$ the mass flux onto the flame, $\sigma$ the surface tension, $g$ the local gravitational acceleration and $\rho_1, \rho_2$ the fluid densities of the unburnt and burnt sides respectively. On the other hand, by definition [16]

$$j^2 = \frac{(P_2 - P_1) \rho_1 \rho_2}{(\rho_2 - \rho_1)}$$

from which either $P_2 > P_1$ and $\rho_2 > \rho_1$ (detonations) or $P_2 < P_1$ and $\rho_2 < \rho_1$ (deflagrations). The important point is that if we impose the latter conditions [18], then eq.(5) can never be satisfied. We conclude that deflagrations are always Rayleigh-Taylor unstable in dense matter for any perturbation wavenumber at the linear level.

These results indicate that, unless the instability can be controlled in some way (e.g. by non-linear effects), the flame should become turbulent and accelerate. Laboratory observations show that, in such a case, the foldings of the front increase, thus increasing the area and the local burning rate. The heat transfer mechanism switches from thermal conduction to turbulent convection, which gives a much larger velocity $v_s$. An estimate of the latter can be given in terms of a length scale, taken to be the radius of the spherical
front $r$ (the only available length of the problem) divided by the timescale for the instability to grow $\tau_{RT}$. Given that $\tau_{RT}$ must necessarily be some multiple of the dynamical time $(G\rho)^{-1/2}$ we find

$$v_{turb} \simeq \frac{r}{\tau_{RT}} = 10^9 \left(\frac{r}{1 \text{ km}}\right) \text{ cm s}^{-1}$$

which probably contains an uncertainty of a factor of 10 in either direction. The point here is that not only $v_{turb} \gg v_s$ but also it is likely to be comparable to the speed of sound $c_s$ at some finite radius. However, since the turbulent convection burning must also be subsonic, its description is also contained in the lower branch of the Hugoniot adiabatic. Note also that, due to the supranuclear density values of the stellar matter, $\tau_{RT} \simeq 1 \text{ ms}$ and thus the acceleration process will be very fast. Moreover, a further change of the combustion mode is to be expected, as we shall now discuss.

We have previously noted that boundary conditions must be taken into account for a reliable and consistent description of the flow. As is well-known [16], these boundary conditions (analogous to the ”closed pipe” ones) force the existence of a shock wave propagating ahead of the flame. The strength of the shock is uniquely determined by the velocity of the flame and the state of the unburnt material. Now, if the flame accelerates the shock becomes stronger, which in turn means that the neutrons are more and more pre-heated and pre-compressed before meeting the flame. Such pre-heating and pre-compression effects on the matter entering the combustion zone help to further accelerate the burning rate and establishes a positive feedback loop [19].

The crucial point is therefore to establish which is the velocity of the flame for which the shock is strong enough to ”ignite” the neutron matter by itself. This type of calculations have been longly performed for laboratory gases (for planar and spherical geometries) and it has been repeatedly found [19] that for $v_{turb} \simeq 0.1 \text{ c}_s$ the so-called auto-ignition condition is met and thus the only possible propagation is the one corresponding to the Chapman-Jouget point on the Hugoniot adiabat [16].

Very recent work [20] on this problem has revealed another important feature of the $n \rightarrow SQM$ combustion: it has been found that deflagrations are not allowed unless
$\rho_1 < 1.63 \rho_o$ ($\rho_o$ being the nuclear saturation density). The reason for this is simply that for higher densities the velocity of the flame becomes larger than the velocity of the pre-compression shock. Physically this means that for this regime the flame and the shock coalesce in a single region, a feature that characterizes a detonation wave [19]. However, given the short timescale for the flame to become turbulent, it is very likely that the detonation develops a width $\delta \sim \tau_w c_s \leq 1 \text{ m}$. So, even though the kind of combustion is limited by the the weak interactions rate (producing $s$ quarks), the heat transport mechanism must be recognized as a fundamental ingredient in the problem. It appears from the above results that we should consider \textit{ab initio} a (turbulent) detonation mode.

To conclude this section two remarks are in order. First, some doubt has been expressed about the ability of ”slow” weak interactions to sustain an energy release fast enough for a detonation to propagate. We note that, even for the longest reasonable timescale of $\sim 10^{-8} \text{ s}$ the front thickness is always negligible with respect to the dimension of the burning region ($\sim \text{ km}$) and, much more important, with respect to the lengthscale of significative variations of the pressure, etc. estimated as $L = (\partial \ln P / \partial r)^{-1}$. In fact $\delta \ll L$ is a sufficient condition for the stability of a propagating detonation front [21], or in other words, ”slow” weak reactions are fast enough to sustain a detonation unless (say) $L < 10 - 100 \text{ m}$ . To be more quantitative, however, a complete numerical simulation, including the (strong) dependence of the weak interactions rate with $T$ [6] should be carefully performed. The second remark is that it is by no means necessary
that the burning passes through an initial slow combustion stage and then accelerates. On the contrary, a ”direct” initiation is more likely if SQM is produced by the decay of a two-flavor plasma in bulk (section 2). In this case, the overpressure $\Delta P$ decays into a shock wave with strength $\delta P \approx \Delta P/2$, which should be enough to ignite the matter by itself (see [22] for a through discussion of this and related points).

4. CONCLUSIONS

We have discussed in this talk several topics related to the possible appearance and propagation of SQM inside dense neutron matter. We have argued that

a) SQM is expected to form/begin to ”operate” after $\sim 1$ s following the collapse at most, and we are not aware of any mechanism to postpone the conversion until, say, ages $\sim yr$ (so that all neutron stars must be strange stars [1,2,7,12,18]).

b) Even if a deflagration may develop first [3,4], it should become turbulent and switch to a detonation shortly after. Alternatively, a detonation may be directly initiated.

The conclusions (to which a factor $0 \leq f \leq 1$ gauging the uncertainties of the problem may be added) point to the need for a better understanding of compact star interiors, specially for the role of few-quark states. Preliminary steps have been taken in [23]. It is also very important to improve the schematic model in which the sudden energy release from a detonation affects the supernovae events (hopefully in a positive way !) [8,24]. The conceptual similarity of the latter with the ”delayed detonation model” for type I supernovae [21] is quite remarkable and may merit a closer look. Note, however, that even if a detonation is not formed after all, the very presence of a strong turbulent flame propagating at high (but subsonic) velocity may be more than enough to add substantial energy to a stalled prompt shock.

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6. REFERENCES

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