Wavelet and Statistical Analysis of Dissolved Oxygen and Biological Oxygen Demand of Ramganga River Water

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Author's contribution

The sole author designed, analysed, interpreted and prepared the manuscript.

ABSTRACT

Ramganga river is the main tributary of holy river Ganga and navigates through various cities of Uttarakhand and Uttar Pradesh of India. Its water quality is very important because a lot of population is directly connected to this river. Wavelet transforms is a new analytical tool to analyze non-stationary signals/data because it captures the localized time frequency information of a signal. In wavelet transforms, the Approximation gives the low frequency terms and average behaviour of any data, while Detail gives the high frequency terms and differential behaviour of any data. The trend represents the slowest part of the signal and corresponds to the greatest scale value. As the scale increases, the resolution decreases, producing a better estimate of the unknown trend of the signal. The dissolved oxygen and biological oxygen demand data of station Kannauj, Uttar Pradesh from October 2015 to June 2020 are studied and processed by Haar wavelet transforms. The statistical parameters like skewness, kurtosis and correlation coefficient are determined and discussed. The strong agreement between wavelet analytical and statistical results is obtained.

Keywords: Approximation; detail; Ramganga; trend; water; wavelet.
1. INTRODUCTION

We are living in an era of urbanization and industrialization development, but this development should be planned, controlled and with the coexistence of environment. We know that civilizations have been developed near the rivers and we can say that rivers have been the lifeline of humans and animals. River water has been and is still the primary and vital part of the environment. The river water quality is generally measured in terms of its dissolved oxygen (DO) and biological oxygen demand (BOD). The BOD is the gaseous oxygen which is mixed with the water through atmosphere and plant photosynthesis. Aquatic animals breath this oxygen because they are not capable to assimilate the oxygen of water and other oxygen containing compounds. The BOD is the amount of oxygen in milligram in per litre of water needed by biological organisms to decompose the organic material at a particular temperature in a particular time interval [1]. Generally the particular temperature is 20°C and time interval is 5 days. Ramganga river is originated from Dudhatoli hills of Gairsain, Pauri Garhwal, Uttarakhand and navigates through different parts of hills of Uttarakhand and planes of Uttar Pradesh and in last at Kannauj, Uttar Pradesh, it meets to Holy river Ganga after covering 373 mile distance. Several cities are established at its bank in which Chaukhutiya, Masri, Bhikiyasen, Marchula, Kalagarh, Seohara, Moradabad, Bareilly, Budaon, Shahjahanpur, Hardoi and Kannauj are the main. Data of DO and BOD of Ramganga river water from October 2015 to June 2020 are taken from website of Uttar Pradesh Pollution Control Board belonging to City Kannauj (Code-1064) of Uttar Pradesh.

Fourier transforms (FT) has been an important tool to analyse the functions/signals in Mathematics, Physics and Engineering. It extracts the frequency information of a signal by converting amplitude - time plane in to amplitude - frequency plane. But there is a limitation on FT, that is, it is not suitable to analyse non-stationary signals. The wavelet theory has been introduced in 1980 to analyze specially non-stationary and transient signals. In wavelet transforms, the inner product of signal and wavelets are taken. These wavelets are generated by dilation and translation of a basic function called mother wavelet. The WT extracts localized time – frequency information of a signal/data. Therefore, the WT has become a new analytical tool for analyzing chaotic data for the Mathematicians, Physicists and Engineers [2-4].

The whole family of wavelets is obtained using dilation and translation of mother wavelet as follows:-

\[ \psi_{a,b}(t) = \frac{1}{\sqrt{a}} \psi \left( \frac{t-b}{a} \right) = T_b D_a \psi \]  

(1)

where \( a \) is the dilation or scaling parameter, \( b \) is the translation parameter and \( \psi(t) \) is the mother wavelet. By taking \( a = 2^{-j} \) and \( b = k \) with \( j, k \in Z \), the discrete wavelets are defined as [5]:-

\[ \psi_{j,k}(t) = 2^j \psi(2^j t - k) \]  

(2)

2. DISCRETE WAVELET TRANSFORMS

The continuous wavelet transform of a function \( f \) is defined as:-

\[ W_{a,b} = \int f(t) \psi_{a,b}(t) dt \]

\[ = \int f(t) \frac{1}{\sqrt{a}} \psi \left( \frac{t-b}{a} \right) dt \]

The discrete wavelet transforms by using the set of discrete dilation and discrete translation \( a = 2^{-j} \) and \( b = k \) with \( j, k \in Z \), is defined as [6]:

\[ W_{j,k} = \int f(t) 2^j \psi(2^j t - k) dt \]  

(3)

2.1 Haar Wavelet

Haar discovered the simplest wavelet called Haar wavelet. It is also represented by Daubechies1 wavelet. Haar wavelet function is equal to 1 on \([0, 1/2] \) and -1 on \([1/2, 1] \), and 0 outside the interval \([0, 1] \), while its scaling function is equal to 1 on the interval \([0, 1] \) and outside this interval its value is 0.

Multi-resolution analysis (MRA) is a technique introduced by Mallat in which the scale of resolution changes as per requirement of the signal [7-8]. Haar wavelet is constructed from the MRA generated by scaling function

\[ \phi(t) = \chi_{[0,1]}(t). \]  

Since,

\[ \phi(t) = \phi(2t) + \phi(2t - 1) = \chi_{[1/2,1]} + \chi_{[0,1/2]} \]  

and

\[ \psi(t) = \phi(2t) - \phi(2t - 1) = \chi_{[1/2,1]} - \chi_{[0,1/2]} \]
3. DECOMPOSITION OF DATA AND METHODOLOGY

With help of MRA, any vector subspace can be represented by combination of two orthogonal subspaces as following manner [9]:

\[ V_j = V_{j+1} \oplus W_{j+1} \]  \hspace{1cm} (4)

where subspace

\[ V_j = \text{span} \left( \phi_{j,k}(x) \right) \]

\[ V_{j+1} = \text{span} \left( \phi_{j+1,k}(x) \right) \]

\[ W_{j+1} = \text{span} \left( \psi_{j+1,k}(x) \right) \]

With help of above theory of MRA, any function or signal can be expressed by first order decomposition as follows:

\[ f_1(x) = \sum_k a_{j=1,k} \phi_{j+1,k}(x) + \sum_k d_{j=1,k} \psi_{j+1,k}(x) \]  \hspace{1cm} (5)

By second order decomposition, it can be expressed as:

\[ f_2(x) = \sum_k a_{j=2,k} \phi_{j+2,k}(x) + \sum_k d_{j+2,k} \psi_{j+2,k}(x) + \sum_k d_{j=1,k} \psi_{j+1,k}(x) \]

In general, for \( p \)th order decomposition, a signal can be expressed as:

\[ f_p(x) = \sum_k a_{j=p,k} \phi_{j+p,k}(x) + \sum_{p=1}^\infty \sum_k d_{j+p,k} \psi_{j+p,k}(x) \]  \hspace{1cm} (6)

Where \( p \in \mathbb{Z}^+ \) represents to the order of decomposition of signal or level of the wavelet transforms.

Where

\[ a_{j=p,k} = \langle f, \phi_{j+p,k} \rangle = \int f(x) \phi_{j+p,k}(x) \, dx, \forall k \in \mathbb{Z} \]  \hspace{1cm} (7)

and

\[ d_{j=p,k} = \langle f, \psi_{j+p,k} \rangle = \int f(x) \psi_{j+p,k}(x) \, dx \]  \hspace{1cm} (8)

are collectively known as approximation and detailed coefficients of the given signal [10]. Thus a given signal takes place a new version such as:

\[ f = A_j = A_{j+p} + \sum_{p=1}^\infty D_{j+p} \]  \hspace{1cm} (9)

where

\[ A_{j+p} = \sum_k a_{j+p,k} \phi_{j+p,k}(x) \]  \hspace{1cm} (10)

and

\[ D_{j+p} = \sum_k d_{j+p,k} \psi_{j+p,k}(x) \]  \hspace{1cm} (11)

Here \( A_{j+p} \) is approximation and \( D_{j+p} \) is detail of signal at \( p \)th level or time frames. Taking \( f = A_0 \), i.e. putting \( j = 0 \) in equation (5), a signal \( f \) can be expressed as:

\[ f_1 = \sum_k a_{1,k} \phi_{1,k}(x) + \sum_k d_{1,k} \psi_{1,k}(x) \]
A signal $f$ can be decomposed as in simplest form (level 1) [11]:

$$f_1 = A_{j+1} + D_{j+1}$$

Taking $j = 0$, we can write,

$$f_1 = A_1 + D_1$$
$$f_2 = A_2 + D_2 + D_1$$
$$f_3 = A_3 + D_3 + D_2 + D_1$$
$$f_p = A_p + \sum_{p=1}^{\infty} D_p$$

To perform the spectral analysis of any signal by wavelet transforms, we use dyadwaves software of wavelet analysis. The data is decomposed up to level 4 using Haar wavelet. The approximation is the low frequency part of the signal and hence provides coarse information or average behaviour of the signal, while detail is the high frequency part of signal and provides the fine information or differential behaviour of signal. As the level of decomposition increases, the resolution decreases and better estimate of the unknown trend of signal is obtained. That is, the highest level of decomposition is the slowest part of the signal and represents to its trend. Some statistical parameters like skewness, kurtosis and correlation are also determined and discussed.

4. RESULTS AND DISCUSSION

In Figs. 2 and 3, the quantitative behaviour of DO and BOD of Ramganga river water corresponding to station Kannauj, Uttar Pradesh from October 2015 to June 2020 and its decomposition up to level 4 with help of Haar wavelet transforms are shown. The original signal represents the plot of raw data. Both original signals indicate the fluctuations from time to time depending upon weathers.

Maximum 04 decomposition levels are allowed (maximum value of $p$ is 4), therefore the original signal represents the superposition of 1 approximation component $A_4$ and 4 detail components $D_1$, $D_2$, $D_3$ and $D_4$. The approximation component $A_4$ is the zero frequency component of the signal at level 4 and represents to the average behaviour of the signal, while detail components $D_1$, $D_2$, $D_3$ and $D_4$ are the components having frequencies at level 1, 2, 3 and 4 respectively and represent the time of fluctuations in the data. The approximation $A_4$ represents to the trend of the signal corresponding to the greatest scale value. The trend of dissolved oxygen shows slight increasing order while that of biological oxygen demand shows slight decreasing order with time. Some statistical parameters are shown in the Table 1.

![Fig. 2. Dissolved oxygen (DO) in mg/lit. and its wavelet decomposition](image-url)
The skewness represents the asymmetry of the probability distribution of a real valued random variable about its mean, while the kurtosis represents their peakedness [12]. The skewness for DO is negative while for BOD is positive with small low value. The negative and positive value of skewness represent that the data are skewed left and right respectively. Negative value of kurtosis indicated the low intermittency of the data. The correlation between two quantities describes that how they are related to each other. The value of correlation coefficient is low and negative, which indicates that the DO and BOD data of Ramganga river water are weakly and inversely correlated.

5. CONCLUSION

The original signals of DO and BOD show fluctuations time to time. With help of Haar wavelet transforms slightly increasing trend of DO and slightly decreasing trend of the BOD are observed from the Figs. 2 and 3 respectively. The fluctuations in data are more cleared by their detail components. The DO and BOD are skew left and right respectively. The negative and low values of kurtosis indicate that both data are less peaked from mean value. The correlation coefficient indicates that DO and BOD data are weakly and inversely correlated. It is clear that wavelet analysis provides a simple and accurate framework for modelling the quantitative behaviour of the DO and BOD of Ramganga river water. In Figs. 2 and 3, the plot of approximation coefficients at level $j = 4$ using wavelet transforms represent the average behaviour or trend of the both DO and BOD. The interpretation of statistical results and spectrum of wavelet analytical results prove consistency between both. Obviously, the statistical parameters of the given data provide the strong agreement in favour of wavelet analytical results.

COMPETING INTERESTS

Author has declared that no competing interests exist.

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