EFFECT OF THE DRAG FORCE ON THE ORBITAL MOTION OF THE BROAD-LINE REGION CLOUDS

Fazeleh Khajenabi
Department of Physics, Faculty of Sciences, Golestan University, Gorgan 49138-15739, Iran; fkhajenabi@gu.ac.ir

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Abstract

We investigate the orbital motion of cold clouds in the broad-line region of active galactic nuclei subject to the gravity of a black hole, a force due to a non-isotropic central source, and a drag force proportional to the velocity square. The intercloud is described using the standard solutions for the advection-dominated accretion flows. The orbit of a cloud decays because of the drag force, but the typical timescale of clouds falling onto the central black hole is shorter compared to the linear drag case. This timescale is calculated when a cloud moves through a static or rotating intercloud. We show that when the drag force is a quadratic function of the velocity, irrespective of the initial conditions and other input parameters, clouds will generally fall onto the central region much faster than the age of whole system, and since cold clouds present in most of the broad-line regions, we suggest that mechanisms for the continuous creation of the clouds must operate in these systems.

Key words: galaxies: active – galaxies: nuclei

1. INTRODUCTION

The emission of the broad-line region (BLR) of active galactic nuclei (AGNs) is explained by models which propose continuous steady flows or the presence of a very large number of clouds which exhibit some bulk motion (e.g., Osterbrock & Mathews 1986; Rees 1987; Rees et al. 1989; Netzer 2013). In the early models of the BLRs the clouds move inward or outward through a static or slowly moving background gas (e.g., Mathews 1974; Blumenhal & Mathews 1975, 1979). Subsequent studies showed that the clouds may exhibit orbital motion under the strong gravitational potential of a central object. Kwan & Carroll (1982) constructed a kinematic model in which the BLR clouds orbit the central object in nearly parabolic orbits, and this model has been extended by Carroll & Kwan (1985) to include the effects of a finite infalling cloud number and size. There are considerable uncertainties about the formation of the BLR clouds and their dynamical stability (e.g., Mathews 1986; Mathews & Veilleux 1989); however, many authors have successfully produced emission of BLR systems based on the discrete cloud concept (e.g., Capriotti et al. 1979, 1980, 1981; Fromerth & Melia 2001; Netzer & Marziani 2010).

Most of these models assume that the BLR clouds are pressure-confined, although a few authors argue that the clouds are transient rather than stable long-lived objects. The orbital motions of the BLR clouds are a rich source of information for estimating the mass of the central black hole. While early models consider the gravitational force of the central black hole as a dominant force, it has been argued that BLR clouds are also subject to a force due to the intense radiation of a central source (e.g., an accretion disk) and the mass of the central black hole is underestimated if radiation pressure is neglected (e.g., Marconi et al. 2008; Namekata et al. 2014). Moreover, it has been suggested that the central radiation is non-isotropic (Liu & Zhang 2011) and the orbits of BLR clouds are significantly modified when this feature of radiation is considered (Plewa et al. 2013; Khajenabi 2015).

Clouds embedded in a hot gaseous medium have also been discovered near the Galactic center (Gillessen et al. 2012). These low-mass gas clouds, known as G1 and G2, are moving on highly eccentric orbits through a gaseous medium around a central black hole. Using the orbital motions of these clouds, one can probe the accretion flow feeding Sgr A* (McCourt & Madigan 2015; Pfuhl et al. 2015). There are considerable uncertainties about the true nature of the intercloud medium and the physical mechanisms that may lead to the formation of these clouds. Although various processes have been proposed for the formation of G1 and G2 or BLR clouds, we do not yet know for sure if these clouds are formed as a result of such mechanisms. The intercloud medium of G1 and G2 is successfully described, however, using a kind of accretion flow which is known as Advection-Dominated Accretion Flow (ADAF; Narayan & Yi 1994). It is a good idea to assume that BLR clouds are also moving through this type of accretion flows (Krause et al. 2011). Nevertheless, most of the previous semi-analytical studies of the BLR clouds’ dynamics prescribe a pressure profile of the intercloud medium as a simple power-law function of the radial distance (e.g., Netzer & Marziani 2010; Krause et al. 2011, 2012; Plewa et al. 2013; Khajenabi 2015).

Since each BLR cloud is assumed to be in a pressure-confined state, its radius is determined by a balance between the interior pressure of a cloud and its ambient pressure. Then, the orbital motion of a BLR cloud is treated like a classical two-body problem where a cloud with a fixed mass is subject to the central gravity and a force due to the radiation. Most of the previous analytical studies of the BLR clouds’ dynamics actually follow this approach.

Netzer & Marziani (2010) studied the orbital motion of pressure-confined BLR clouds in AGNs considering the combined influence of central gravity and radiation pressure. A modified estimate for the mass of the central black hole is presented according to their orbital analysis. Krause et al. (2011) addressed the stability of the orbits using analytical calculations for both isotropic and anisotropic light sources, and found that stable orbits may exist under certain circumstances. Although it is unlikely that analytical solutions for the orbital motion of BLR clouds could be obtained under general conditions, an interesting analytical solution for the orbit of BLR clouds with a fixed column density has been obtained by Plewa et al. (2013). In all these works, the intercloud is a simple power-law prescription not based on a
physically supported model. Moreover, variations of the intercloud’s pressure profile with the polar angle has been neglected. These issues motivated Khajenabi (2015) to study the orbital motion of BLR clouds through an ADAF atmosphere where the pressure profile is based on a two-dimensional self-similar analytical solution for ADAFs (Shadmehri 2014). Under these conditions it was shown that the stability of the orbits implies that the ensemble of clouds tends to have a disk-like configuration.

None of the above studies of the orbital motion of BLR clouds considered the interaction of the clouds with the surrounding gas via a drag force. As for the G1 and G2 clouds, recent studies show that the drag force has a vital role in the orbits of these clouds (e.g., McCourt & Madigan 2015; Pfuhl et al. 2015). Just recently, Shadmehri (2015) studied the orbits of BLR clouds subject to a drag force proportional to the velocity. For a particular set of the input parameters, Shadmehri (2015) presented an analytical solution for the orbits of the clouds which reduces to the analytical solution of Plewa et al. (2013) in the absence of the drag force. In the presence of the drag force, irrespective of the input parameters, the orbit of a BLR cloud will decay in such a way that it will eventually fall onto the central region. Shadmehri (2015) argues that if the time it takes for a cloud to travel from its initial position to the central region (time-of-flight) becomes less than the lifetime of the whole system, then BLR clouds should be considered transient structures rather than long-lived objects. Therefore, mechanisms for the continuous formation of BLR clouds would be needed. In other words, drag force implies a physical constraint for analyzing the orbits of the clouds. Shadmehri (2015) found that the time-of-flight of a BLR cloud is proportional to the inverse of the dimensionless drag coefficient and, using this relation, showed that the time-of-flight is indeed shorter than the lifetime of the whole system for a wide range of the input parameters. This interesting finding implies the existence of mechanisms for the continuous formation of these clouds.

However, there are caveats in regards to the analysis of Shadmehri (2015). First of all, in his study the drag force is proportional to the velocity, which is valid as long as the intercloud is laminar. Although he argues that the Reynolds number is less than one, which confirms the adopted drag force, for some other input parameters one can easily show that the Reynolds number could be much larger than one. Introducing the Reynolds number as \( \text{Re} = \frac{\rho L}{\mu} \), it can be rewritten as \( \text{Re} = 2(\frac{v}{\bar{u}})(\frac{L}{\lambda}) \), where \( \rho \), \( v \), \( L \), \( \mu \), \( \bar{u} \), and \( \lambda \) are the density of gas, the mean velocity of the cloud relative to the gas, the characteristic length, the dynamic viscosity, the average molecular speed, and the mean free path, respectively. If the number density of the intercloud gas is \( 10^8 \text{ cm}^{-3} \) and its average temperature is \( 10^6 \text{ K} \), then we have \( x \approx 2 \times 10^6 \text{ m s}^{-1} \) and \( \lambda \approx 10^{10} \text{ m} \). The Keplerian velocity at the radial distance \( 1 \text{ pc} \) from the central mass, which is \( 10^8 \text{ solar mass} \), is around \( 6.5 \times 10^4 \text{ m s}^{-1} \). If we adopt the velocity of a cloud approximately equal to this Keplerian velocity and for a typical length of \( 10^{13} \text{ cm} \), the Reynolds number becomes around 0.65. Obviously, if the typical length is larger, for example \( 10^{15} \text{ cm} \), then we have \( \text{Re} \approx 65 \). Also, for a more massive central mass, the Reynolds number is larger than unity. It means that the intercloud medium is turbulent and the drag force should be taken in proportion to the velocity square. In the present work we plan to study the orbits of BLR clouds with a quadratic drag force. At variance with previous work, nevertheless, the intercloud is prescribed using the standard ADAF solutions. Under these circumstances we calculate the time-of-flight of the clouds to see if the main finding of Shadmehri (2015) is still valid when the drag force is a quadratic function of the velocity. Moreover, in most previous studies, the background gas is assumed to be in a static configuration. We also consider the rotation of the medium as a cloud moves through it. In the next section we present basic assumptions and the orbital equations. In Section 3, time-of-flight is calculated numerically. We then conclude with our main findings in Section 4.

2. GENERAL FORMULATION

2.1. Basic Assumptions

We study the orbital motion of a BLR cloud with mass \( m \) subject to three main forces: the gravitational force of a central black hole with mass \( M \), a non-isotropic force due to the radiation of a central accretion disk (Liu & Zhang 2011), and a drag force in the opposite direction of the BLR’s orbital motion. Under these circumstances the direction of the cloud’s angular momentum is conserved, although its magnitude gradually decreases because of the resistive force. Therefore, the motion of a BLR cloud will be in a plane where its inclination, \( i \), is fixed by the initial angular momentum, and it is an input parameter in our model. A system of coordinates \((x, y, z)\) is constructed so that the central black hole locates at its origin and the central radiating thin accretion is at \( z = 0 \) plane (Figure 1). Thus, the location of a cloud in its orbit with an inclination angle, \( i \), with respect to the \( x-y \) plane is uniquely determined by its distance, \( r \), from the origin and the polar angle, \( \theta \). The cloud orbit intersects the \( x-y \) (disk) plane at the ascending node \( A \) so that we define the angle \( \angle AOC = \psi \).

![Figure 1. The central black hole locates at origin, \( O \), and the position of the cloud, \( C \), is determined by the radial distance, \( r \), the polar angle, \( \theta \), and the inclination angle of the orbital plane. Since the direction of the angular momentum is conserved, the orientation of the orbital plane is fixed and its intersection with the \( x-y \) plane is denoted by \( OA \). Thus, position of the cloud, \( C \), is equivalently determined by the radial distance, \( r \), and the angle \( \angle AOC = \psi \).](image-url)
Having the inclination angle \( i \), the position of a cloud is determined by \( r \) and \( \psi \).

For the sake of simplicity, the geometrical shape of a BLR cloud is assumed to be spherical. Moreover, the cloud is considered to be optically thick. But since BLR clouds are pressure-confined by definition, the physical properties of the ambient gaseous medium, like its pressure profile, determine how the radius of a cloud varies depending on its position in orbital motion. There are considerable uncertainties about the true nature of the intercloud medium. In other words, irrespective of the confinement mechanisms, the internal pressure of a cloud is in balance with the ambient pressure.

Since each cloud is in pressure equilibrium with the hot background gas, its radius becomes \( R_{\text{cl}} \propto P_{\text{gas}}^{-1/3} \), where \( P_{\text{gas}} \) is the intercloud gas pressure. On the other hand, according to the standard similarity solutions for ADAFs (Narayan & Yi 1994), the pressure distribution is proportional to a power-law function of the radial distance as \( P_{\text{gas}} \propto r^{-2} \), where \( s \) is 5/2. Therefore, we can re-write the radius of a single cloud as a function of its location, i.e.,

\[
R_{\text{cl}}(r) = \frac{r_{0}^{5/6}}{r^{5/6}},
\]

where \( r_{0} \) is the initial radial distance of the cloud and \( R_{\text{cl}0} \) is the radius of the cloud at \( r_{0} \). We can calculate the column density \( N_{\text{cl}} = 3m/2\mu_{\text{m}}m_{\text{p}}A \), where \( \mu_{\text{m}} \) is the mean molecular weight and \( A \) is the cross-section of a cloud. Having the above relations for \( R_{\text{cl}} \) and \( P_{\text{gas}} \), the column density of a pressure-confined cloud becomes \( N_{\text{cl}} \propto P_{\text{gas}}^{2/3} \) or \( N_{\text{cl}} = N_{0}r^{-5/3} \), where \( N_{0} = 3m/(2\mu_{\text{m}}m_{\text{p}}\pi R_{\text{cl0}}^{3}) \) is a constant column density. Also, the density of gas in the standard ADAF similarity solution is written as a power-law function of the radial distance, i.e.,

\[
\rho = \rho_{0}\left(\frac{r}{r_{0}}\right)^{-3/2},
\]

where \( \rho_{0} \) is the mass density of the intercloud gas at radius \( r_{0} \). We adopt the outer radius of the system as \( r_{0} = 10^{4} \) cm, which is the cross-section of a cloud. Having the above relations for \( R_{\text{cl}} \) and \( P_{\text{gas}} \), the column density of a pressure-confined cloud becomes \( N_{\text{cl}} \propto P_{\text{gas}}^{2/3} \) or \( N_{\text{cl}} = N_{0}r^{-5/3} \), where \( N_{0} = 3m/(2\mu_{\text{m}}m_{\text{p}}\pi R_{\text{cl0}}^{3}) \) is a constant column density.

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\]

where \( \rho_{0} \) is the mass density of the intercloud gas at radius \( r_{0} \). We adopt the outer radius of the system as \( r_{0} = 10^{4} \) cm, and the number density is \( n_{0} = 10^{7} \) cm\(^{-3} \) according to observations (e.g., Rees et al. 1989; Marconi et al. 2008; Netzer 2013; Plewa et al. 2013).

The radial velocity, \( v_{r} \), and the rotational velocity, \( v_{\psi} \), of an ADAF are also power-law functions of the radial distance. The similarity solutions of Narayan & Yi (1994) are written as \( v_{r} = v_{0r} vK \) and \( v_{\psi} = v_{0\psi} vK \), where \( vK = \sqrt{GM/r} \) is the Keplerian velocity, and the coefficients \( v_{0r} \) and \( v_{0\psi} \) are obtained as

\[
v_{0r} = \frac{(5 + 2\varepsilon')}{3\alpha} \frac{g(\alpha, \varepsilon')}{\gamma},
\]

and

\[
v_{0\psi} = \sqrt{\frac{2\varepsilon'(5 + 2\varepsilon')}{9\alpha^{2}} \frac{g(\alpha, \varepsilon')}}{\gamma}.
\]

Here we have

\[
g(\alpha, \varepsilon') = \sqrt{1 + \frac{18\alpha^{2}}{(5 + 2\varepsilon')^{2}} - 1},
\]

where \( \alpha \) is the standard Shakura–Sunyaev viscosity parameter for modeling ADAF’s turbulence. Moreover, the parameter \( \varepsilon' \) is written as \( \varepsilon' = \varepsilon/f \) where \( \varepsilon = (5/3 - \gamma)/\gamma \).}

2.2. Equations of Motion in a Static Atmosphere

We can now write equations for the motion of a cloud which is under the influence of the three main forces: the gravitational force of the central mass, \( F_{\text{grav}} \), a force due to the non-isotropic radiation of the central accretion disk (Liu & Zhang 2011), \( F_{\text{rad}} \), and a drag force proportional to the velocity square in the opposite direction of the cloud’s motion, \( F_{\text{drag}} \). These forces can be written as

\[
F_{\text{grav}} = -\frac{Gm_{\text{cl}}}{r^{2}}e_{r},
\]

\[
F_{\text{rad}} = \frac{A}{c} \frac{L_{a}}{2\pi r^{2}} \cos \theta |e_{r}|,
\]

\[
F_{\text{drag}} = -\frac{1}{2} \rho C_{D}A|v|v_{r},
\]

where \( A \) is the cross-sectional area of a cloud and \( L_{a} \) is the luminosity of the central source. In the above equation for the drag force, \( C_{D} \) is the drag coefficient which depends on the shape of the cloud and an even Reynolds number (e.g., McCormick 1979). For a sphere, the value of \( C_{D} \) may vary from large values for laminar flow to 0.47 for turbulent flow (McCormick 1979).

It is more convenient to re-write the force due to the radiation in terms of the column density, \( N_{\text{cl}} \), and the Eddington ratio, \( l = La/L_{\text{edd}} \), where \( L_{\text{edd}} = 4\pi GMm_{\text{p}}c/\sigma_{T} \) is the Eddington luminosity. Here, \( \sigma_{T} \) is the Thompson cross-section. Thus, the force due to the radiation becomes

\[
F_{\text{rad}} = \frac{Gm_{\text{cl}}}{r^{2}} \frac{3l}{\mu_{\text{m}}N_{\text{cl}}\sigma_{T}} |\cos \theta| e_{r},
\]

or

\[
F_{\text{rad}} = \frac{Gm_{\text{cl}}}{r^{2}} k |\sin \psi| e_{r},
\]

where the dimensionless parameter \( k \) is defined as

\[
k = \frac{3l}{\mu_{\text{m}}N_{\text{cl}}\sigma_{T}} \sin(i).
\]

Substituting radial dependence of the column density, the parameter \( k \) becomes \( k_{0}(r/r_{0})^{5/3} \) where \( k_{0} = 3l \sin(i)/\mu_{\text{m}}N_{0}\sigma_{T} \). Most of the previous authors assume that the intercloud is static, which means that the background gas does not move. Thus, the relative velocity of a cloud with respect to its ambient medium is the cloud’s velocity itself. We first consider this simplified situation. Thus, the equations for the orbital motion are written as

\[
r - r_{0}^{2} = \frac{GM}{r^{2}} (k |\sin \psi| - 1) - \frac{\rho C_{D}A}{2} \sqrt{r^{2} + r_{0}^{2}},
\]
\[ r \ddot{\psi} + 2 \dot{r} \dot{\psi} = -\frac{\rho C_D A}{2} \dot{r} \sqrt{\dot{r}^2 + \dot{\psi}^2}, \quad (13) \]

where \( \dot{r} = dr/dt, \dot{\psi} = d\psi/dt \). Note that the temperature of a cloud during its orbital motion is almost constant.

The above orbital Equations (12) and (13) are now written in the non-dimensional forms that are more convenient for the numerical integration. Thus, we use the initial radial distance, \( r_0 \), as a reference length scale. Then the Keplerian velocity at this radial distance is written as \( v_K(r_0) = \sqrt{GM/r_0} \) and our unit time becomes \( t_0 = r_0/v_K(r_0) \). We now change the variables as \( r = r_0 \tilde{r} \) and \( t = t_0 \tau \). Thus, Equations (12) and (13) become

\[ \ddot{\psi} - \dot{\psi}^2 = \frac{1}{\tilde{r}^2} \left( k_0 \tilde{r}^5 / \sin \psi \right) - 1 - \beta \tilde{r} \dot{\psi} \sqrt{\tilde{r}^2 + \dot{\psi}^2}, \quad (14) \]

and

\[ \ddot{r} + 2 \dot{r} \dot{\psi} = -\beta \dot{\psi} \tilde{r} \sqrt{\tilde{r}^2 + \dot{\psi}^2}, \quad (15) \]

where \( \tilde{r} = dr/d\tau, \tilde{\psi} = d\psi/d\tau \). The dimensionless drag coefficient is denoted by \( \beta \), i.e.,

\[ \beta = \frac{3}{8} C_D \left( \frac{r_0}{R_{c0}} \right) \left( \frac{\rho_0}{\rho_{c0}} \right). \quad (16) \]

In writing the above equations we assume that the mass of a cloud is conserved during its orbital motion. We note that the radius of a cloud and its density at the distance \( r_0 \) are denoted by \( R_{c0} \) and \( \rho_{c0} \), respectively. Equations (14) and (15) are our main equations for determining the orbit of a cloud in a static atmosphere when the drag force is proportional to the velocity square. In Section 3 we solve these equations to analyze the orbits of a cloud.

2.3. Equations of Motion in a Rotating Atmosphere

We now consider a more realistic situation where the ambient gas is rotating and has a radial velocity according to Equations (3) and (4). In writing the drag force, then the relative velocity is considered. Thus the orbital equations become

\[ \ddot{\psi} - \dot{\psi}^2 = \frac{GM}{r^2} (k_0 \tilde{r}^5 / \sin \psi) - 1 - \frac{\rho C_D A}{2} \dot{r} \sqrt{\dot{r}^2 + \dot{\psi}^2} \times \left( \dot{\psi} + v_0 v_K \right) \sqrt{\left( \dot{\psi} + v_0 v_K \right)^2 + \left( \dot{r} - v_0 v_K \right)^2}, \quad (17) \]

\[ \ddot{r} + 2 \dot{r} \dot{\psi} = -\frac{\rho C_D A}{2} \left( \dot{r} \dot{\psi} - v_0 v_K \right) \times \sqrt{\left( \dot{\psi} + v_0 v_K \right)^2 + \left( \dot{r} - v_0 v_K \right)^2}. \quad (18) \]

Again, it is more convenient to use non-dimensional equations instead of the above orbital equations. So we transform Equations (17) and (18) into the following non-dimensional forms:

\[ \ddot{\psi} - \dot{\psi}^2 = \frac{1}{\tilde{r}^2} \left( k_0 \tilde{r}^5 / \sin \psi \right) - 1 - \beta (\dot{\psi} + v_0 \tilde{r}^{-1/2}) \frac{\dot{\psi}}{\tilde{r}^{3/2}} \times \sqrt{\left( \dot{\psi} + v_0 \tilde{r}^{-1/2} \right)^2 + \left( \dot{r} - v_0 \tilde{r}^{-1/2} \right)^2}, \quad (19) \]

\[ \ddot{r} + 2 \dot{r} \dot{\psi} = -\beta (\dot{\psi} - v_0 \tilde{r}^{-3/2}) \frac{\dot{r}}{\tilde{r}^{3/2}} \times \sqrt{\left( \dot{\psi} + v_0 \tilde{r}^{-1/2} \right)^2 + \left( \dot{r} - v_0 \tilde{r}^{-1/2} \right)^2}. \quad (20) \]

\[ \beta = 0.01 \text{ pc} \times 10^{10} \text{ cm}. \]

Table 1

| \( r_0 \) | \( n_0 \) | \( R_{\infty} \) | \( n_{a0} \) | \( \beta/C_D \) |
|---------|---------|-------|-------|---------|
| 1 pc    | 106 cm^-3 | 10^4 cm | 10^{10} cm | 1.15 \times 10^{-2} |
| 0.01 pc | 10^6 cm^-3 | 10^4 cm | 10^{10} cm | 1.15 \times 10^{-4} |
| 0.01 pc | 10^6 cm^-3 | 10^12 cm | 10^{10} cm | 1.15 \times 10^{-2} |

Note. Each row corresponds to a cloud with a certain mass.

Table 2

| \( r_0 \) | \( n_0 \) | \( R_{\infty} \) | \( n_{a0} \) | \( \beta/C_D \) |
|---------|---------|-------|-------|---------|
| 1 pc    | 10^4 cm^-3 | 10^4 cm | 4.6 \times 10^6 cm | 25 |
| 0.01 pc | 10^6 cm^-3 | 10^4 cm | 4.6 \times 10^{12} cm | 2.5 \times 10^{-3} |
| 0.01 pc | 10^6 cm^-3 | 10^10 cm | 10^{10} cm | 0.25 |

\[ \ddot{r} + 2 \dot{r} \dot{\psi} = -\beta (\dot{\psi} - v_0 \tilde{r}^{-3/2}) \frac{\dot{r}}{\tilde{r}^{3/2}} \times \sqrt{\left( \dot{\psi} + v_0 \tilde{r}^{-1/2} \right)^2 + \left( \dot{r} - v_0 \tilde{r}^{-1/2} \right)^2}. \quad (20) \]

where dimensionless parameter \( \beta \) is defined in Equation (16). The above equations are solved subject to the appropriate initial conditions in the next section.

3. ANALYSIS

We now examine the orbits of the BLR clouds in the plane of motion by solving the orbital equations. The background gas is rotating according to ADAF solutions. Describing the results is easier if the same initial conditions are used for all the considered cases. Thus we assume that a cloud starts its journey from the initial location \( \tilde{r} = 1 \) (note that all variables are dimensionless). The rest of the initial conditions are \( \tilde{r} (\tau = 0) = 1, \tilde{\psi} (\tau = 0) = 0, \) and \( \psi (\tau = 0) = 1 \). We found that the shape of the orbits is qualitatively similar to when a linear drag is used, i.e., the orbit of a BLR cloud decays due to the resistive nature of the drag force. But we can calculate the timescale of this orbital decay when the drag force is quadratic. In doing so, the time-of-flight, \( \tau_f \), is defined as the time needed for a cloud to travel from its initial location to the center. In order to determine the orbital shape of a BLR cloud and its time-of-flight we have to adopt the input parameters consistent with the observational data. According to the observations, we have \( r_0 \sim 0.01 \text{ pc}, R_{\infty} \sim 10^{14} \text{ cm}, n_0 \sim 10^4 \text{ cm}^{-3}, \) and \( n_{a0} \sim 10^4 \text{ cm}^{-3} \) (e.g., Rees et al. 1989; Marconi et al. 2008; Netzer 2013; Plewa et al. 2013). Tables 1 and 2 summarize our input parameters; however, in Table 2 the mass of a BLR cloud is assumed to be 10^{-8} \( M_\odot \).

Figure 2 displays the orbital shape of a BLR cloud in the plane of motion (i.e., the XY-plane where the X-axis is along OA in Figure 1) with inclination angles \( \iota = \pi/18 \) (top) and \( \iota = \pi/3 \) (bottom) for the dimensionless drag coefficient \( \beta = 0.01 \). Input parameters are \( \alpha = 0.1, \epsilon = 1, \) and \( k_0 = 0.2 \sin \iota \). Also, the initial conditions are \( \tilde{r}(\tau = 0) = 1, \)
The radial distance of a cloud gradually decreases when considering the drag force. The non-isotropic nature of the central radiation becomes more significant with an increase in the inclination angle, \(i\). In Figures 3 and 4 the orbital motion of clouds with the same initial and input parameters are explored, but with larger values for the drag coefficient.

In our model the effect of the radiation force on the orbit of a cloud appears through the dimensionless parameter \(k_0\), which is directly proportional to the Eddington ratio \(l\) and \(\sin i\). Thus the radiation force operates more significantly in cases with a high inclination angle or a large Eddington ratio. In order to have bound orbits the radiation force cannot be arbitrarily large; however, for a given set of the input parameters there is always a maximum critical value of parameter \(k_0\) such that beyond this value the gravitational force is not able to keep a cloud in a bound orbit. In Figures 2–4 the orbits are shown for two values of inclination. Since the radiation force pushes a cloud toward larger radii, one can expect cases with a larger inclination angle to exhibit wider orbits in comparison to a case with a smaller inclination angle. This speculation has been confirmed in Figures 2–4. The effects of the Eddington ratio on the shape of

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**Figure 2.** Orbit of a BLR cloud in the plane of motion with inclination angles \(i = \pi/18\) (top) and \(i = \pi/3\) (bottom) for \(\beta = 0.01\). The other input parameters are \(\alpha = 0.1, \nu' = 1, \mu_m N_0 r_T = 3/2,\) and \(l = 0.1\). The initial conditions are \(\dot{r}(\tau = 0) = 1, \dot{\theta}(\tau = 0) = 0, \dot{\psi}(\tau = 0) = 0,\) and \(\psi(\tau = 0) = 1\).

**Figure 3.** Same as Figure 2 but with inclination angles \(i = \pi/18\) (solid) and \(i = \pi/3\) (dashed), and a larger dimensionless drag coefficient, i.e., \(\beta = 0.1\).

**Figure 4.** Same as Figure 2 but with inclination angles \(i = \pi/18\) (solid) and \(i = \pi/3\) (dashed), and dimensionless drag coefficient, i.e., \(\beta = 0.5\).
 orbits are explored in Figure 5 for different values of the Eddington ratio, \( l \). Here we have \( \beta = 1 \) and \( i = \pi/3 \), and the remaining input parameters are similar to Figure 2. Having all the parameters fixed, we found that the orbits are no longer bound once the Eddington ratio exceeds a value around 0.3. Nevertheless, the shape of the orbits is not modified significantly so long as the ratio \( l \) is roughly less than 0.1. Corresponding to the cases with \( l = 0.01, 0.1, \) and 0.3, the dimensionless time-of-flight is found to be 7.47, 7.78 and 12.75, respectively.

We explored various cases with different sets of input parameters and the corresponding dimensionless time-of-flight, \( \tau_\beta \), is obtained. Interestingly, we found that the time-of-flight is in proportion to the inverse of the dimensionless drag coefficient, \( \beta \), so the constant of the proportionality depends on the input parameters. For static intercloud gas, we found that \( \tau_\beta(i = 0) \approx 2.13/\beta^{0.89} \), \( \tau_\beta(i = \pi/6) \approx 2.20/\beta^{0.90} \), and \( \tau_\beta(i = \pi/3) \approx 2.52/\beta^{0.01} \). When the intercloud is rotating, the time-of-flight is \( \tau_\beta(i = 0) \approx 7.38/\beta^{0.97} \), \( \tau_\beta(i = \pi/6) \approx 8.00/\beta^{0.96} \), and \( \tau_\beta(i = \pi/3) \approx 9.54/\beta^{1.20} \). However, cases above the fitted time-of-flight functions are not valid for the whole range of the dimensionless drag coefficient, \( \beta \), except for cases with a zero inclination angle where the radiation force does not operate. For a static intercloud gas, a BLR cloud will not be in bound orbit once the parameter \( \beta \) drops to values less than 0.005 for \( i = \pi/6 \) and 0.1 for \( i = \pi/3 \). For rotating intercloud gas these critical values are larger, so that we do not observe bound orbits when \( \beta \) is less than 0.02 for \( i = \pi/6 \) and 0.43 for \( i = \pi/3 \). Figure 6 shows the ratio \([\tau_\beta(i) - \tau_\beta(i = 0)]/\tau_\beta(i = 0)\) as a function of the parameter \( \beta \) when the intercloud is static (solid) or rotating (dashed) for different inclination angles.

Thus we can write \( \tau_\beta \approx \tau_0/\beta \), where \( \tau_0 \) depends on the input parameters. Although the constant of proportionality \( \tau_0 \) depends on the input parameters, we found that its variations from the input parameters do not significantly affect the main conclusion in our subsequent discussions. The time-of-flight for a linear drag is also proportional to the inverse of the dimensionless drag coefficient, although the definition of this coefficient is different from ours (see Equation (8) in Shadmehri 2015). One can easily confirm our approximate relation for the time-of-flight using dimensional analysis. A cloud loses its kinetic energy \( 1/2mv^2 \) due to the dissipative nature of the drag force with a rate equal to \( F \dot{v} \), where \( v \) is the velocity of the cloud, \( m \) is its mass, and \( F \) is the drag force. Thus, time-of-flight can be written as \((1/2mv^2)/(1/2\rho C_d \pi R_i^2 v^3)\) which implies the dimensionless time-of-flight to be proportional to \( \beta^{-1} \), i.e., \( \tau_\beta \propto \beta^{-1} \).

For a cloudy BLR system around a black hole with mass \( 10^8 M_\odot \), our time unit becomes \( t_0 \approx 1.5 \) year if we set \( \tau_0 = 0.01 \) pc. Tables 1 and 2 show that the dimensionless drag coefficient varies from \( 2.5 \times 10^{-3} C_d \) to \( 25 C_d \) depending on the background gas density and the properties of a cloud, such as its density and radius. Obviously, the longest cloud flight times occur when the parameter \( \beta \) is as small as permissible and the radiation force is as large as it can be. The effect of the radiation force does not appear for clouds with zero inclination angle, and, considering the above fitted functions for the time-of-flight, this timescale will be between \( \tau_\beta(\beta = 10^{-5}) \approx 3 \times 10^6 \) year and \( \tau_\beta(\beta = 10) \approx 0.2 \) year for static intercloud gas. These estimates are modified for rotating background gas as \( \tau_\beta(\beta = 10^{-5}) \approx 10^7 \) year and \( \tau_\beta(\beta = 10) \approx 2
0.7 year. For clouds with non-zero inclination angles, however, the radiation pressure force increases $\tau_1$ as we confirmed in Figures 5 and 6. But, in these cases, there are always lower limits for $\beta$, so that, for the drag coefficient less than this critical value, clouds will be pushed outward due to the strong radiation force. When the background gas is static, for example in the cases shown in Figure 6, and the critical value of $\beta$ is 0.005 and 0.1 for inclination angles $\pi/6$ and $\pi/3$, respectively, then the time-of-flight becomes $\tau_1(\beta = 0.005) \approx 390$ yr and $\tau_1(\beta = 0.1) \approx 39$ yr. These are considerably shorter than the estimated $\tau_1$ for the clouds with zero inclination angle. The critical value of $\beta$ is larger in a system with rotating intercloud gas, and the corresponding time-of-flight is found to be $\tau_1(\beta = 0.02, i = \pi/6) \approx 500$ year and $\tau_1(\beta = 0.43, i = \pi/3) \approx 40$ year. Using this approximate relation for the time-of-flight, we can discuss the nature of BLR clouds by comparing it to the lifetime of the whole system, $\tau_{\text{life}}$.

4. CONCLUSION

We note that our analysis is based on an assumption which states that the clouds are in pressure equilibrium with their ambient medium. This constraint should not lead to unphysical values for the ratio of the density of cloud to the ambient density, i.e., $\rho_{cl}/\rho$, which scales as the ratio of intercloud medium to cloud temperature. Since the mass of the cloud, $m$, is conserved during its journey, we obtain $\rho_{cl} = \rho_{\text{cl0}} \rho_0 (r/r_0)^{-5/2}$, where $\rho_{\text{cl0}} = 3 m/(4\pi R_{\text{cl0}}^3) \sim 10^{10} \text{ cm}^{-3}$. Having Equation (2) for the intercloud density, we obtain $\rho_{cl}/\rho = \rho_{\text{cl0}} \rho_0 (r/r_0)^{-1}$ or $\rho_{cl}/\rho \sim 10^6 (r/r_0)^{-1}$, which means the ratio of the densities cannot be arbitrarily large so long as a cloud is not very close to the central parts.

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