Purcell effect in GaN-based waveguiding structures

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Abstract. We provide an analysis of Purcell coefficient dependence on frequency, wavevector and emitter position in nitride-based waveguide structures. Was shown that spontaneous emission of emitter placed in one-dimensional slab waveguide structure could lead to a modification of spontaneous emission rate in the both considered structures. Results for symmetric and asymmetric nitride-based waveguides in TE polarization case was demonstrated.

1. Introduction
Using nitride material system allows to utilize some important properties, like a wide transparency range \[1\], and suitability for harsh environments in optoelectronic devices. Various optical components have already been demonstrated based on the nitride material system like directional couplers \[2\], all-optical modulators, waveguide gratings \[3,4\]. In the nitride materials system highlights Gallium Nitride (GaN). GaN is perspective, high refractive index material to using in integrated optics (fabricate low loss waveguides in ultraviolet region and other components to optical circuits systems). In the last decade improvement in technological methods allow to fabricate nanoscale waveguide structures with high quality of interfaces with different geometries, like slab, ridge and rib on different kind of substrates \[5,6\].

Modification of spontaneous emission rate of emitters, coupled with different types of resonant modes has attracted attention of researchers due to perspectives of fabrication of high efficiency compact light sources, single-photon generators etc. The rate of spontaneous emission can be estimated by a number of various methods. Here, for the analysis of Purcell effect in layered waveguide structures, was used approach, called S-quantization method. S-quantization formalism allows to calculate the spontaneous emission rate in arbitrary layered dielectric structure for states inside and outside the light cone \[7\]. These rigorous and self-consistent method is based on analysis of scattering matrix eigenvalues and don't require to solve integral-differential equations and apply perturbation theory methods.

The aim of that paper is to calculate the modification of the spontaneous emission rate of known GaN core-based waveguide structures by S-quantization method and to get analysis of the results with compare to analytics.
2. Results and discussion

Was considered the well-known slab waveguide model. As model structures was chosen slab GaN waveguide layer \((n_{\text{GaN}}=2.33)\) of thickness 400 nm in two different cladding semi-infinite materials: in two semi-infinite AlN claddings \((n_{\text{AlN}}=2.075)\) and semi-infinite sapphire substrate layer \((n_{\text{Al}_2\text{O}_3}=1.76)\) with air on the top. Schemes of two waveguide structures are pictured on figure 1.

As known, both cladding and core materials of first structure (GaN and AlN) has birefringence phenomenon and describes by ordinary and extraordinary refractive indexes. In our paper we limit consideration only on transfer electric (TE) waveguide modes that describes only with ordinary refractive index for simplicity.

![Figure 1](image)

Figure 1. (color online) Schemes of considered waveguide structure. (a) Slab GaN waveguide structure in AlN claddings. (b) Asymmetric GaN waveguide layer in air and sapphire claddings.

S-quantization method uses transfer and scattering matrix formalism in the traveling-wave basis and in case of TE-polarized field, \(E_y\) component can be written as:

\[
E_y(x, z) = E(z) \exp(i K_x x).
\]  

(1)

Scattering matrix \(\hat{S}\) interrelate incident and runaway waves at boundary of quantization box as:

\[
\begin{pmatrix}
E_{K_x}^+(L) \\
E_{K_x}^-(0)
\end{pmatrix} = \hat{S} \begin{pmatrix}
E_{K_x}^+(0) \\
E_{K_x}^-(L)
\end{pmatrix}.
\]  

(2)

Field in S-quantization formalism quantizes by equating \(\hat{S}\) matrix eigenvalues to unity:

\[
\beta^{(1,2)} = 1.
\]  

(3)

In waveguide regime, when the wavevector component \(K_x\) is outside the light cone, equation (3) becomes:

\[
(1 \pm i \sqrt{M_{12}M_{21}}) = 0,
\]  

(4)

where \(M_{12}, M_{21}\) - the components of transfer matrix through the quantization box and “+” sign corresponds to symmetric eigenvector, and sign “-” corresponds to antisymmetric eigenvector. Equations (4) are easily covered to known form of dispersion equations for even and odd waveguide modes. Mode Purcell factor could get from Fermi Golden rule and reduces to relation between squared dipole matrix elements, in waveguide case to homogeneous media case respectively.
Figure 2. (color online) Dependence of the mode Purcell coefficient for the dipole oriented along the \( y \) axis and (a) placed at the centre of a 400-nm-thick GaN slab waveguide, limited by semi-infinite AlN layers, and (b) moved by 150 nm with respect to the centre, on the emission frequency and wave-vector component \( k_x \) parallel to the layer interfaces (case of TE polarization).

The patterns of the modal Purcell factor in case of TE polarization are shown on figure 2. Figure 2 (a) shows distribution of mode Purcell coefficient on energy and wave vector component \( k_x \) when the dipole is placed at the centre of GaN layer and oriented along \( Oy \) axis. It can be seen, that in area with Purcell factor much more than zero (inside the light cone) are interleaving of local maximums and minimums of Purcell factor. The maximums are corresponded to Fabry-Perot modes of the slab. On the edge of light cone (between light cones of core and cladding materials), shows two branches of considered even waveguide TE modes. Figure 2 (b) shows the case of dipole emitter placed at 150 nm from the structure centre. In comparison with (a) there are more waveguide modes: both even and odd. Because in this case, emitter is placed in area where odd modes electric field nonzero. The maximum value of mode Purcell factor in current structure is close to 1 and corresponded to fundamental mode (TE\(_0\)) and dipole position in the centre of waveguide (where placed maximum magnitude of electric field too).

It is known, that difference between core and cladding refractive indexes defines the confinement of the field in waveguide. To compare values of mode Purcell factors was considered the situation of known waveguide system GaN core layer in sapphire and air claddings. Figure 3 shown the pattern of modal Purcell factor dependence on wave vector component \( k_x \) and energy, when dipole is placed at centre of waveguide layer. So as the second structure is the asymmetric waveguide, there are three light cone curves, that corresponding to each layer. Three even waveguide modes are shown in area 3. Areas 1 and 2 are areas within light cone for air and \( Al_2O_3 \) layers, where as in previous case, figured interleaving of Fabry-Perot modes. In both considered structures, maximum value of mode Purcell factor, that corresponds to waveguide modes, does not exceed to value of 2. It can be explained by parameters of structure – there are not huge difference between core and cladding refractive indexes, as analytical equation for maximum mode Purcell factor from [7] says: \( F_p \approx 1.3n_{core}^2(n_{core}^2 - n_{cladding}^2) \).
Figure 3. (color online) Dependence of the mode Purcell coefficient for the dipole oriented along the y axis and placed at the centre of a 400-nm-thick GaN slab waveguide, limited by semi-infinite \( \text{Al}_2\text{O}_3 \) and air layers on the emission frequency and wave-vector component \( k_x \) parallel to the layer interfaces (case of TE polarization).

3. Conclusions
Calculation of spontaneous emission rate for emitter placed in two different slab GaN-based waveguide structures using S-quantization formalism was carried out. Cases of symmetric and asymmetric waveguide structures were considered. Maximum calculated value of the mode Purcell factor in both considered structures doesn’t exceed value of 2, that could be explained by low refractive indexes difference between core and cladding layers.

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References

[1] Chowdhury A, Ng H M, Bhardwaj M, Weimann N G 2003 Appl. Phys. Lett. 83, 1077
[2] Zhang Y, McKnight L, Engin E, Watson I M, Cryan M J, Gu E, Thompson M G, Calvez S, O’Brien J L, Dawson M D 2011 Appl. Phys. Lett. 99, 161119
[3] Hui R, Wan Y, Li J, Jin S, Lin J, Jiang H 2005 IEEE J. Quantum Electron. 41, 100
[4] Gromovyi M et al. 2014 Journal of the European Optical Society - Rapid publications, Europe, v. 9
[5] Westreich O, Katz M, Paltiel Y, Ternyak O, Sicron N. 2015 Phys. Status Solidi A, 212, 1043-1048
[6] Chen H, Fu H, Huang X et al. 2017 Opt Express 25(25):31758-31773
[7] Ivanov KA, Gubaydullin A R, Morozov K M, Sasin M E, Kaliteevski M A 2017 Opt. and Spectrosc. 122.5: 864-872