Control synthesis for tasks with a given output function
decentralized SISO-systems

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Abstract. For the control synthesis of multiply connected objects with many inputs and many outputs (MIMO) by importance of reducing computational costs by reducing accuracy, the method of decentralized control is used by dividing the original system into systems with one input and one output (SISO). The paper proposes an approach to the control synthesis of a decentralized SISO system, which is a part of a multi-connected MIMO object in conditions of impossibility of identifying model parameters. The paper considers the problem of control synthesis, which provides a given desired form of the output process.

1. Introduction
Large technical control systems in industry are described based on models with many inputs and outputs (MIMO), sometimes they have very large dimensions. Although there are modeling techniques and approaches to control synthesis, the computational costs of optimization problems are enormous. An alternative approach is a decentralized management strategy, splitting in some way into systems with one input and one output (SISO) [1]. The loss of accuracy and other efficiency criteria in decentralized control versus full multi-parameter control is compensated by the simplicity of the equipment, ease of development and use. The basic level of regulatory control in MIMO installations mainly consists of SISO decentralized PID controllers [2] due to their simple structure, fewer tunable parameters and loop fault tolerance, since it is easier to identify and eliminate sensor or actuator failure in a decentralized structure, although it includes a significant number of specialized tasks [3].

An important feature of splitting into SISO systems is the complexity of identification, since it is difficult to send test signals directly to a selected object, for example, if the object is part of a large mechanism or automatic line and is not directly controlled by the input parameters of the entire MIMO system. However, in practice, the projector of the control system can formulate the requirements for a specific decentralized SISO-system in the form of the desired schedule, which the system should issue in the specified modes. In practice, there have been cases when, due to the design features and technical requirements of the technological process, it is impossible to send a test signal for object identification, and only smooth signals are available (for example, in heating systems; in systems with sensitive to sudden changes in parameters), while the desired type of the output function is set. In these cases, it is needed to find the kind of input function that provides the required output. Control kind found can be provided by Iterative Learning Control [4], repetitive control [5] or fuzzy controller technologies.

It should be noted that the approach of searching for transformations that leave the required characteristics invariants is widely used in radio electronics [6]. Solving the problems of searching for
mapping admitted by differential equations allows to obtain methods for modeling objects with nonlinear elements [7], generating robust chaos [8] and systems with hidden attractors [9], and others. In this paper, it is proposed to replace the identification of a SISO object of a multiply connected MIMO system with a search for the type of control action of this SISO object, which provides the required specified type of output signal. It is proposed to look for a solution in the form of graph transformation.

2. Method
Let there be a SISO LTI system described by the differential equation:

\[
\begin{align*}
\dot{x} &= Ax(t) + Bu(t), \\
y(t) &= Cx(t),
\end{align*}
\]

where \(x(t)\) — \(n\)-dimensional vector of systems states from a variety \(X\); \(u(t)\) — control from \(\mathbb{R}\); \(y(t)\) — output measured process from \(\mathbb{R}\); \(X\) — variety, usually identified with \(\mathbb{R}^n\), or an invariant torus; \(\dot{x} = (dx)/(dt) \in T_xX\); \(T_xX\) — tangent space to \(X\) in \(x\), determined by control; \(A \in \mathbb{R}^{nn}\); \(B \in \mathbb{R}^{nr}\); \(C \in \mathbb{R}^{mn}\).

Let for fixed \(u^1(t)\) and \(u^2(t)\) corresponding processes are observed \(y^1(t)\) and \(y^2(t)\). Then if the symmetry group \(G\) is allowed by system (1), then the transformation \(g:\)

\[g : y^1(t) \rightarrow y^2(t),\]

is at the same time a transformation between control processes:

\[g : u^1(t) \rightarrow u^2(t).\]

As an illustration of the proof, the communication scheme is presented:

\[
\begin{array}{c}
u^1 \xrightarrow{\xi} y^1 \\
\downarrow \quad \downarrow \\
u^2 \xrightarrow{\xi} y^2
\end{array}
\]

Here \(\xi\) is transformation that transfers the control process to the observable output process. It is given by the system of equations 2 with the same initial conditions.

In other words, in the system under control \(u^1(t)\), output processes \(y^1(t)\) are observed in accordance with the system of equations (1). By controlling \(u^2(t)\), output processes \(y^2(t)\) are observed respectively. Since the solutions of system (1) form a Lie group, that is, a group of transformations with a commutation operation, then there is a transformation that transforms one solution into another. By the Grobman-Hartman theorem, such a transformation is linear.

The solution to stationary system (1) is defined as following

\[y(t) = \int_{t_0}^t C\Phi(t - \tau)Bu(\tau)d\tau,\]

where \(\Phi(t - \tau) = \exp\{A(t - \tau)\}\).

For the output process

\[y^1(t) = \int_{t_0}^t C\Phi(t - \tau)Bu^1(\tau)d\tau,\]
by condition

\[ y^2(t) = g(y^1(t)). \]

from

\[ y^2(t) = g \left( \int \Phi(t - \tau)Bu^1(\tau) d\tau \right), \]

Considering the \( G \) group properties, following is found

\[ y^2(t) = \int_{t_0}^t C\Phi(t - \tau)Bu^1(\tau) d\tau, \quad (2) \]

on the other hand, the solution for the output process \( y^2 \):

\[ y^2(t) = \int_{t_0}^t C\Phi(t - \tau)Bu^2(\tau) d\tau, \quad (3) \]

Thus, from the uniqueness of the process \( y^2 \) defined by equations (2) and (3), it follows that

\[ u^2(t) = g(u^1(t)). \]

Thus, it is shown that to find the desired function \( u^2 \) that provides a given form of output \( y^2 \), it is sufficient to find the transformation \( g \) and apply it to the observed form \( u^1 \). The conversion \( g \) is looked for as the conversion of the desired function to the given one.

3. Numerical experiment

Figure 1 shows two output processes: system characteristic (blue), desired process (orange)

![Figure 1. Observed and Desired System Output Functions.](image1.jpg)

An approximation of the transformation \( g \) in the form of a polynomial was found [10]. When executing the program control scheme shown in Figure 2, a view of the output process corresponding to the given one is obtained. The discrepancy is due to the error of the numerical methods used.

![Figure 2. System output with found control (blue — given output, orange — observed system output).](image2.jpg)
4. Conclusions
The paper proposes an approach to the synthesis of control of a decentralized SISO system, which is a part of a multi-connected MIMO object in conditions of impossibility of identifying the parameters of an LTI object. The problem of such a control synthesis, which provides a given desired form of the output process, is considered. It is shown theoretically and numerically, that to find the desired input function that provides a given type of output, it is enough to find a transformation and apply it to the observed type of the input process.

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