Exploring the QCD phase diagram with compact stars

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We investigate a nonlocal chiral quark model with separable 4-fermion interaction for the case of U(3) flavor symmetry and show that strange quark matter is unlikely to occur in a large enough volume of a compact star to entail remarkable observational consequences. The phase diagram in the two-flavor sector of such model has a critical end point of the line of first order chiral/deconfinement phase transitions on which a triple point marks the junction with the critical line for second order phase transitions to two-flavor color superconductivity (2SC) below \(T \sim 80\) MeV. Stable hybrid star configurations with large quark matter core in a color superconducting phase can exist. A consistent cooling phenomenology requires that all quark species be gapped, the minimal pairing gap of the order of 10 – 100 keV.

1. Introduction

The investigation of the QCD phase diagram has become a research topic of highest priority. Relativistic heavy ion collisions in the RHIC era have provided results which confirm previous information about the critical temperature \(T_\text{c} \sim 170\) MeV of the hadronization transition in the approximately baryon-free regime \(^1\), which parallels the situation a few microseconds after the big bang. This value is in agreement with the deconfinement transition temperature calculated in 2+1 flavor Lattice QCD \(^2\). These simulations are now extended into the region of (small) finite chemical potentials where one of the characteristic features is a critical endpoint of first order phase transitions (tricritical point). Future heavy-ion collision experiments planned at GSI Darmstadt (FAIR) will explore the phase diagram in the finite density domain and hope to find experimental signatures of the tricritical point. In dense quark matter at temperatures below \(\sim 50\) MeV, due to attractive interaction channels, the Cooper pairing instability is expected to occur which should lead to a variety of possible quark pair condensates corresponding to color superconductivity (CSC) phases, see \(^3\) for a review. Since it is difficult to provide low enough temperatures for CSC phases in heavy-ion collisions, only precursor phenomena \(^4\) are expected under these conditions. CSC phases may occur in neutron star interiors \(^6\) and could manifest themselves, e.g., in the cooling behavior \(^7\).

However, the domain of the QCD phase diagram where neutron star conditions are met is not yet accessible to Lattice QCD studies and theoretical approaches have to rely on nonperturbative QCD modeling. The class of models closest to QCD are Dyson-Schwinger equation (DSE) approaches which have been extended recently to finite temperatures and densities \(^7\). Within simple, infrared-dominant DSE models early studies of quark stars \(^8\) and diquark condensation \(^9\)
have been performed.

The present contribution will be based on results which have been obtained within a nonlocal chiral quark model (NCQM) using a covariant, separable representation of the gluon propagator [12] with formfactors suitable for an extension to finite temperatures and chemical potentials [13,14,15]. Alternative approaches to separable chiral quark models are based on the instanton approach [10,11,15]. As a limiting case, the well-known Nambu–Jona-Lasinio (NJL) model is obtained for a cutoff formfactor. For recent applications of the NJL model to QCD at high density in compact stars, see [13,14,15] and references therein. A different class of QCD-models is of the bag-model type. These models do not describe the chiral phase transition in a self-consistent way and will not be considered here. We will consider within a NCQM the following questions: (i) Is strange quark matter relevant for structure and evolution of compact stars? (ii) Are stable hybrid stars with quark matter interior possible? (iii) What can we learn about possible CSC phases from neutron star cooling?

2. Nonlocal chiral quark model, \(N_f = 3\)

In order to answer the question whether strange quark matter phases should be expected in the neutron star interiors, we consider here a three-flavor generalization of a NCQM with the action

\[
S[q, \bar{q}] = \int_k \left\{ \bar{q}(k)(k - \hat{m})q(k) + D_0 \int_{k'} \sum_{a=0}^8 \left[ j^a_\alpha(k) j^{\alpha}_\beta(k') + j^{\alpha}_\beta(k) j^a_\alpha(k') \right] \right\},
\]

where the constraint of baryon number conservation is realized by the diagonal matrix of chemical potentials \(\hat{\mu}\) (Lagrange multipliers). Further details concerning the notation and the parametrization of this model can be found in Ref. [20].

In order to perform the functional integrations over the quark fields \(\bar{q}\) and \(q\) we use the formalism of bosonisation which is based on the Hubbard-Stratonovich transformation of the four-fermion interaction. The resulting transformed partition function in terms of bosonic variables will be considered in the mean-field approximation

\[
\mathcal{Z}[T, \hat{\mu}] = e^{-2N_c \sum_f \left[ \frac{\beta_f}{2\pi^2} + \int_k \ln(M_f^2 - k_f^2) \right]},
\]

with the 4-vector \(\tilde{k}_f = (k_0 - \mu_f, \vec{k})\) and the effective quark masses \(M_f = M_f(\tilde{k}) = m_f + \phi_f f(\tilde{k})\) containing the flavor dependent mass gaps \(\phi_f\). In the mean field approximation, the grand canonical thermodynamical potential is

\[
\Omega(T, \{\mu\}) = \frac{T}{V} \ln \left\{ \mathcal{Z}[T, \{\mu_f\}] / \mathcal{Z}[0, \{0\}] \right\},
\]

where fluctuations are neglected and the divergent vacuum contribution has been subtracted.

The quark mass gaps \(\phi_f\) are determined by solving the gap equations which correspond to the minimization conditions \(\partial\Omega / \partial \phi_f = 0\) and read

\[
\phi_f = 8D_0 N_c \int_0^{\tilde{k}_f} \frac{d\tilde{k}_f}{(2\pi)^4} \frac{2M_f f(\tilde{k})}{M_f^2 - \tilde{k}_f^2}.
\]

As can be seen from (5), for the chiral \(U(3)\) quark model the three gap equations for \(\phi_u, \phi_d, \phi_s\) are decoupled and can be solved separately.

In what follows we consider the case \(T = 0\) only. All thermodynamical quantities can now be derived from Eq. (4), for details see [20]. For instance, pressure, density and the chiral condensate are given by

\[
p = -\Omega, \quad n_f = -\frac{\partial\Omega}{\partial \mu_f}, \quad <\bar{q}q> = -\frac{\partial\Omega}{\partial m_q},
\]

the energy density follows from \(\varepsilon = -p + \sum_f \mu_f n_f\). For most of the numerical investigations within the NCQM described above we
will employ a simple Gaussian formfactor $f(k) = \exp(k^2/A^2)$, which has been used previously for studies of meson and baryon properties in the vacuum \cite{17} as well as quark deconfinement and mesons at finite temperature \cite{13,15}. A systematic extension to other choices of formfactors can be found in \cite{18}. The Gaussian model has four free parameters to be defined: the coupling constant $K_0$, the interaction range $\Lambda$, and the current quark masses $m_u = m_d$, $m_s$. These are fixed by the pion mass $m_\pi = 140$ MeV, kaon mass $m_K = 494$ MeV, pion decay constant $f_\pi = 93$ MeV and the chiral condensate $- < \bar{q}q >= (240$ MeV$)^3$. Fig. 1 shows the behavior of the thermodynamical potential as a function of the light quark mass $\mu$ for different values of the chemical potential $\mu_c = 330$ MeV for (light) quark deconfinement. A systematic study of the NCQM as a function of the chemical potential for the separable model (solid line) compared to a three-flavor (dotted line) and a two-flavor (dashed line) bag model. All models have the same critical chemical potential $\mu_c = 330$ MeV for (light) quark deconfinement.

3. Color superconductivity, $N_f = 2$

The quark-quark interaction in the color antitriplet scalar diquark channel is attractive driving the pairing with a large zero-temperature pairing gap $\Delta \sim 100$ MeV for the quark chemical potential $\mu_q \sim 300 - 500$ MeV. Therefore, we consider now a NCQM described by the effective action

\begin{equation}
S_{2sc}[\bar{q},q] = S[\bar{q},q] + \frac{3D_0}{4} \int_k j_{d}(k) j_{d}(k'), (7)
\end{equation}

where $j_{d}(k) = \bar{q}^T(k) C i \gamma_5 \gamma_2 \lambda_2 f(k) q(k)$ is the scalar diquark current with the charge conjugation matrix $C = i \gamma_0 \gamma_2$. After bosonization, the mean field approximation introduces a new order parameter: the diquark gap $\Delta$, which can be seen as the gain in energy due to diquark condensation. The mass gaps $\phi_u, \phi_d$ indicate dynamical chiral symmetry breaking. The grand canonical thermodynamic potential per volume can be obtained as

\begin{equation}
\Omega_q(\phi_u, \phi_d, \Delta; \mu_u, \mu_d, T) = \frac{\phi_u^2 + \phi_d^2}{8 D_0} + \frac{|\Delta|^2}{3 D_0} - \frac{T}{2} \sum_n \int \frac{d^3 k}{(2\pi)^3} \text{Tr} \ln \left[ \frac{1}{4 G^{-1}(i\omega_n, \vec{k})} \right], (8)
\end{equation}

\begin{figure}[h]
\centering
\includegraphics[width=0.5\textwidth]{figure1.png}
\caption{Pressure of the NCQM as a function of the chemical potential for the separable model (solid line) compared to a three-flavor (dotted line) and a two-flavor (dashed line) bag model.}
\end{figure}

where $\omega_n = (2n + 1)\pi T$ are the Matsubara frequencies for fermions and the inverse Nambu-
Gorkov quark propagator is
\[ G^{-1} = \begin{pmatrix} \hat{k} - \hat{M} - \hat{\mu} \gamma_0 & \Delta \gamma_5 \tau_2 \lambda_2 f(k) \\ -\Delta^* \gamma_5 \tau_2 \lambda_2 f(k) & \hat{k} - \hat{M} + \hat{\mu} \gamma_0 \end{pmatrix}. \] (9)

Figure 3. Phase diagram for \( N_f = 2 \) quark matter in the NCQM.

The resulting phase diagram is shown in Fig. 3. It includes a 2-flavor color superconductivity (2SC) phase for which quarks of one color, say blue, remain unpaired. The color-flavor locking (CFL) phase \[21\] requires approximate SU(3) flavor symmetry and can be excluded from our discussion since strange quarks remain confined up to the highest densities occurring in a compact star configuration \[20\].

4. Structure and cooling of hybrid stars

In describing the hadronic shell of a hybrid star we exploit a parametrized form \[22\] of the Argonne V18 + \( \delta v + UIX^* \) model of the EoS given in \[23\], which is based on the most recent models for the nucleon-nucleon interaction with the inclusion of a parameterized three-body force and relativistic boost corrections. All details concerning the EoS and the hadronic cooling processes are given in \[24\]. As we have seen in the previous section, it may be sufficient to discuss only two-flavor quark matter for applications to compact stars. We will focus on the model of the quark EoS developed in \[25\] as an instantaneous approximation to the covariant NCQM (INCQM).

In Refs. \[26\] it has been demonstrated on the example of the hadron-quark mixed phase that finite size effects might play a crucial role by substantially narrowing the region of the mixed phase or even preventing its appearance. Therefore we omit the possibility of the hadron-quark mixed phase in our model where the quark phase arises by the Maxwell construction. For the case of the instantaneous Gaussian formfactor with 2SC phase the quark core appears for \( M > 1.21 \ M_\odot \).

Without 2SC phase or for the Lorentzian or NJL formfactor no stable hybrid stars are obtained, see Fig. 4.

In order to describe the cooling of stable hybrid star configurations, we use the approach to the hadronic cooling developed recently in Ref. \[24\].
and add the contribution due to the quark core, see [8]. Once in the quark matter core of a hybrid star unpaired quarks are present the dominant quark direct Urca (QDU) process is operative and cools the star too fast in disagreement with the observational data [8]. If one allows for a residual weak pairing channel as, e.g., the CSL one with typical gaps of $\Delta \sim 10$ keV - 10 MeV, see [27], the hybrid star configuration will not be affected but the QDU cooling process will be efficiently suppressed as is required for a successful description of compact star cooling. Since we don’t know yet the exact pairing pattern for which also the residual (in 2SC unpaired) quarks are paired in the 2-flavor quark matter, we will call this hypothetical phase “2SC+X”. In such a way all the quarks may get paired, either strongly in the 2SC channel or weakly in the X channel. If now the X-gap is of the order of 1 MeV, then the QDU process will be effectively switched off and the cooling becomes too slow, again in disagreement with the data [8]. This seems to be a weak point and we would like to explore whether a density-dependent X-gap could allow a description of the cooling data within a larger interval of compact star masses. For such a density-dependent X-gap function we employ the ansatz

$$\Delta_X(\mu) = \Delta_c \exp\left[-\alpha(\mu - \mu_c)/\mu_c\right].$$ (10)

In Fig. 4 we show the resulting cooling curves for the parameters $\Delta_c = 1.0$ MeV and $\alpha = 10$. We observe that the mass interval for compact stars which obey the cooling data constraint ranges now from $M = 1.32 \, M_\odot$ for slow coolers up to $M = 1.75 \, M_\odot$ for fast coolers such as Vela.

5. Conclusion

We have presented the QCD phase diagram obtained within the NCQM approach. We find that in the framework of our model for neutron star applications it is sufficient to consider only two quark flavors since the critical densities of strange quark deconfinement are not accessible in typical compact star interiors. We investigate the phase transition between superconducting quark matter and hadronic matter under neutron star constraints and find that stable hybrid stars are possible with large quark matter cores in the 2SC phase. The solution of the cooling evolution shows that the presence of ungapped quark species entails too fast cooling due to the QDU process in disagreement with modern neutron star cooling data. A successful description of hybrid star cooling requires a pairing pattern in the quark matter core (if it exists) where all quarks are gapped and a weak pairing channel is present with a gap which does not exceed about $10-50$ keV with a decreasing density dependence. A precise microscopic model for this conjectured "2SC+X" phase is still lacking. Possible candidate for a weak pairing channel is the color- spin-locking (CSL) phase. We have demonstrated on the example of hybrid star cooling how astrophysical observations could provide constraints for microscopic approaches to the QCD phase diagram in the domain of cold dense matter not accessible to heavy-ion collision experiments and lattice QCD simulations.

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