Deconfinement from action restriction

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The effect of restricting the plaquette to be greater than a certain cutoff value is studied. The action considered is the standard Wilson action with the addition of a plaquette restriction, which should not affect the continuum limit of the theory. In this investigation, the strong coupling limit is also taken. It is found that a deconfining phase transition occurs as the cutoff is increased, on all lattices studied (up to $20^4$). The critical cutoff on the infinite lattice appears to be around 0.55. For cutoffs above this, a fixed point behavior is observed in the normalized fourth cumulant of the Polyakov loop, suggesting the existence of a line of critical points corresponding to a massless gluon phase, not unlike the situation in compact U(1). The Polyakov loop susceptibility also appears to be diverging with lattice size at these cutoffs. A strong finite volume behavior is observed in the pseudo-specific heat. It is discussed whether these results could still be consistent with the standard crossover picture which precludes the existence of a deconfining phase transition on an infinite symmetric lattice.

1. INTRODUCTION

In SU(2) lattice gauge theory the plaquette, $P \equiv \frac{1}{4} \text{tr} U_\square$ takes values from $-1$ to 1. The action studied is $S_\square = S_W = (1 - P)$ if $P \geq c$ and $\infty$ if $P < c$, where $c =$ cutoff parameter, $-1 \leq c \leq 1$. For instance, the positive plaquette action\textsuperscript{[1]} has $c = 0$. Since the continuum limit is determined only by the behavior of the action in an infinitesimal region around its minimum which occurs around $P = 1$, this action should have the same continuum limit as the Wilson action for all values of $c$ except $c = 1$. This study was primarily designed to determine whether a sufficiently strong cutoff would cause the system to deconfine. To give the lattices the maximum chance to confine, the strong coupling limit, $\beta \to 0^+$, was taken. This removes the Wilson part of the action completely, leaving only the plaquette restriction. The study was carried out on symmetric lattices from $6^4$ to $20^4$ in an attempt to study the zero-temperature system. However, as is well known, symmetric lattices are not really at zero temperature until one takes the limit of infinite lattice size. Finite size lattices are at a finite temperature corresponding to their inverse time extent. In this sense, finite symmetric lattices can be thought of as small spatial-volume finite temperature systems. The big question is whether the transition observed here as a function of the cutoff is a bulk transition which will remain on the infinite lattice, or the remnant of the usual finite temperature transition which will disappear by moving to $c = 1$ on the infinite lattice.

2. MOTIVATION

The purpose of an action restriction is to eliminate lattice artifacts which occur at strong coupling, and which may give misleading results that have nothing to do with the continuum limit. For instance, the original motivation of the positive plaquette action was to eliminate single-plaquette $Z_2$ monopoles and strings, which were possibly thought to be responsible for confinement \textsuperscript{[2]}. However, for strong enough couplings the positive-plaquette action was shown to still confine \textsuperscript{[3]}. Eliminating these artifacts, does, however, appear to remove the “dip” from the beta-function in the crossover region, possibly improving the scaling properties of the theory, and suggesting that this dip is indeed an artifact of non-universal strong coupling aspects of the Wilson action.

For the U(1) theory, all monopoles are eliminated for $c \geq 0.5$, since in this case six plaquettes cannot carry a visible flux totaling $\pm 2\pi$, $(\cos(\pi/3) = 0.5)$. Therefore a Dirac string cannot end and there are no monopoles. The theory will be deconfined for all $\beta$ for $c \geq 0.5$, since
confinement is due to single-plaquette monopoles in this theory. SU(2) confinement may be related to U(1) confinement through abelian projection. It is an interesting question whether a similar action restriction which limits the non-abelian flux carried by an SU(2) plaquette will eliminate the associated abelian monopoles causing deconfinement, or whether surviving large monopoles can keep SU(2) confining for all couplings. A related question is how can the SU(2) average plaquette go to unity in the weak coupling limit, but the corresponding effective U(1) average plaquette stay in the strong coupling confining region (≤ 0.6)? Chernodub et. al. have found $<P_{U(1)}> \geq 0.82 <P_{SU(2)}>$ over a wide range of couplings.

3. Results

A deconfining phase transition is observed on all lattices studied ($6^4$ to $20^4$). The behavior of the Polyakov loop, $L$, shows the normal symmetry breaking behavior and is smooth, suggesting a second or higher order transition (Figs 1,2). The transition point has a clear lattice size ($N$) dependence (Figs 1,3). The normalized fourth cumulant, $g_4 \equiv 3 - \frac{<L^4>}{<L^2>^2}$, shows fixed point behavior (no discernible lattice size dependence) for $c \geq 0.6$ at a non-trivial value around $g_4 = 1.6$ (Fig 3). This suggests either a line of critical points for $c \geq 0.6$, e.g. from a massless gluon phase, or that correlation lengths are so large that finite lattice size dependence is hidden. For the standard interpretation to hold the normalized fourth cumulant should go to zero as lattice size approaches infinity for all values of $c$. The Polyakov loop susceptibility shows signs of diverging with lattice size ($N$) for $c \geq 0.6$ and converging to a finite value for $c \leq 0.5$ (Fig. 4). This supports the idea of a different behavior in these regions in the infinite volume limit. Extrapolations of finite lattice “critical points” to infinite lattice size are consistent with infinite lattice critical points ranging from around $c^\infty = 0.55$ to $c^\infty = 1.0$ depending on scaling function assumed, so are basically inconclusive (Fig 5). Although a straightforward extrapolation gives infinite lattice critical cutoff around $c^\infty = 0.55$, a logarithmic scaling function can be made consistent with $c^\infty = 1.0$ in which case the transition would no longer exist on the infinite lattice. Finally, the pseudo-specific heat from plaquette fluctuations shows a definite signal of a broad peak or shoulder beginning to form.

Figure 1. Polyakov Loop Modulus. The lower curves are reduced by the value that a Gaussian of the same width would be expected to have.

Figure 2. Polyakov loop histograms from $8^4$ lattice showing typical symmetry breaking behavior.
for the largest lattice sizes (Fig 6), near the location of deconfinement. Interestingly, no noticeable effect was seen here on the smaller lattices, but the $16^4$ and $20^4$ results are definitely shifted up. This indicates a rather large scaling exponent. Preliminary analysis shows an exponent near 4, i.e. a peak height diverging according to the space-time volume (if indeed it is a peak). This would indicate a possible weak first-order transition. Since this is a bulk (local in space-time) quantity, a divergence here would be a surprise for a finite-temperature transition. Clearly larger lattices and better statistics are needed to get the full picture behind this rich and interesting system.

4. CONCLUSIONS

The imposition of an action restriction clearly induces a deconfining “phase transition” on a finite symmetric lattice, even in the strong coupling limit (of course there really is no phase transition on a finite lattice). The extrapolation to infinite lattice size is, as always, difficult. There are two possibilities. Evidence is consistent with deconfinement on the infinite lattice for all $c > 0.55$, representing possibly a massless gluon phase in the continuum limit, similar to the U(1) continuum limit. It should be remembered that the compact U(1) theory itself undergoes a similar deconfining transition at $c = 0.5$. The mechanism in both cases may be the same, namely that limiting the flux that can be carried by an individual plaquette prevents a Dirac string from ending, and spilling its flux out in all directions in the form of a point monopole. Although it has proven hard to identify monopoles in the SU(2) theory, the identification of them with abelian monopoles appearing in the Maximal Abelian Gauge suggests a possibly similar behavior in SU(2) and U(1).

It is also possible, however, that all signs of critical behavior will disappear on large enough lattices, nothing will diverge, and the theory will confine for all values of $c$, (i.e. $c^\infty = 1.0$), the apparent critical behavior being due to the finite temperature associated with finite $N$. However, as a small 3-volume usually masks critical behavior by smoothing would-be criticalities it is difficult to see how this could explain the large divergent-looking finite size effects being seen here. It is especially difficult to explain the large-lattice behavior of the pseudo-specific heat. Due to the small spatial volume and the fact that
one is very far from the scaling region, however, it is rather difficult to make definite predictions from this scenario.

One also must remember that our parameter is the cutoff, \( c \), and not the coupling constant. Although \( c \) does seem to play the role of an effective coupling, (with \( c = -1 \) corresponding to \( \beta = 0 \) in the Wilson theory, and \( c = 1 \) corresponding to \( \beta = \infty \)), it certainly differs in some respects. Incidentally, the average plaquette at \( c = 0.5 \) is 0.7449, corresponding to a Wilson \( \beta \) of around 3.2. This relatively weak coupling means that rather large lattices will be needed to disentangle the finite temperature effects, from any possible underlying bulk transition.

It is likely that further light could be shed on this subject by gauge transforming configurations from restricted action simulations to the maximum abelian gauge and extracting abelian monopole loops. This would answer the question of what effect the SU(2) action restriction has on the corresponding abelian monopoles. By studying the strength of the monopole suppression it may be possible to tell whether a sufficient density of monopoles will survive the continuum limit to produce a confining theory. Such a study is underway.

It is clear that gauge configurations from the restricted action will be relatively smoother on the smallest scale. It should be pointed out, however, that anything can (and does) still happen at larger scales. For instance with \( c = 0.5 \), even the 2 \( \times \) 2 Wilson loop can take on any value. The maximum flux carried by a plaquette is only limited to roughly 1/3 of its unrestricted value, i.e. around the value expected on a lattice twice as fine as that in a typical unrestricted simulation. From this point of view, one should only need lattices twice as large in linear extent to see the same physics in a \( c = .5 \) restricted simulation as in an unrestricted simulation. The smoother configurations produced by restrictions of this strength may also eliminate or at least suppress dislocations, possibly enabling one to study instantons without the need for cooling.

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