Structure and Pinning of the Moving Vortex Matter in Type II Superconductors

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Abstract

The Martin - Siggia - Rose method is applied to the time dependent Ginzburg - Landau model in the presence of both disorder and thermal fluctuations beyond linear response. This allows calculation of correlators of the order parameter, the critical current as function of magnetic field and temperature and the I-V curves. The static or moving flux line lattice in type II superconductors undergoes a transition into three disordered phases: moving vortex liquid (not pinned), homogeneous vortex glass (pinned) and crystalline Bragg glass (pinned) due to both thermal fluctuations and random quenched disorder. The location of the glass transition line is determined and compared to experiments. The line is clearly different from both the melting line and the second peak line describing the translational and rotational symmetry breaking at high and low temperatures respectively. Applications to thermal transport and Nernst effect are considered.
Introduction and Summary. In type II superconductors for which the penetration depth $\lambda$ exceeds the correlation length $\xi$ the magnetic field penetrates the sample in a form of Abrikosov vortices, which strongly interact thereby creating an elastic "vortex matter". Impurities always present in a sample lead to inhomogeneities which greatly affect the thermodynamic and especially dynamic properties of the vortex matter. In addition, thermal fluctuations also significantly influence the vortex matter, either directly by melting the vortex lattice into a vortex liquid or by reducing the efficiency of the disorder. Once electric current $J$ is injected into a sample, one is faced with a problem of description of the dynamical phase diagram which should be drawn in the three dimensional space $T - H - J$. This makes the analysis essentially more complicated, since one has to go beyond linear response.

In this note we investigate dynamics of the vortex matter beyond linear response using the disordered Ginzburg - Landau model (GL). In statics the replica method of handling disorder in the framework of GL model was utilized [1] to obtain the irreversibility line along with other properties of the disordered vortex matter. Dynamics in the presence of thermal fluctuations and disorder is phenomenologically described using the time dependent Ginzburg - Landau (TDGL) model in which the coefficients have random components. Such an attempt was made by Dorsey, Huang, and Fisher [2] in the homogeneous (liquid) phase using a dynamic Martin-Siggia-Rose (MSR) formalism. They obtained the irreversibility line and formulated the linear response theory of the vortex matter.

In strongly type II superconductors magnetic and electric fields inside the superconductor are homogeneous over a wide range of parameters since the mixed state originates from superposition of many ($B/H_{c1} \gg 1$) vortices. The same argument is valid for electric field which arises due to vortex motion. Indeed, the electric field is related by Lorentz transformation to the homogeneous magnetic field appearing in the frame moving with vortices. Finite electric fields without disorder have been considered within framework of TDGL in [3].

We study the dynamic correlation function and the response functions of the
order parameter $\psi$ within an appropriately generalized gaussian approximation in both the flux flow and the pinned phases. We consider the stationary case only, namely when the correlation function depends on the time difference and is therefore characterized by the spectrum $C_\omega$. The critical surface $T^g (H, J)$ in the three dimensional space $T - H - J$, separating the pinned and unpinned phases is obtained as a surface at which $C_\omega \to \infty$ for $\omega \to 0$. Above this surface the real space correlator decays exponentially, while below it is a constant at large time scales. The constant is proportional to the Edwards - Anderson (EA) order parameter characterizing transition to a glassy state. Approaching criticality in the parameter space $T \to T^g$ various quantities diverge power-wise in $(T - T^g)$ with critical exponents calculated in mean field. The static glass transition line, namely the line at zero electric field, coincides with the one obtained using the replica method [1].

**Basic equations and methods.** Our starting point is the TDGL equation in the presence of thermal fluctuations which on the mesoscopic scale are represented by a white noise $\zeta$:

$$\frac{\hbar^2 \gamma}{4m} D_r \psi = - \frac{\delta}{\delta \psi} F + \zeta.$$  

The covariant time derivative is $D_r \equiv \frac{\partial}{\partial r} + \frac{i e^r}{\hbar c} \Phi$, where $\Phi$ is the scalar electric potential describing the driving force and the inverse diffusion constant is $\gamma/2$. The variance of the thermal noise $\zeta$ determines the temperature $T = t T_c$. The static GL free energy including the $T_c$ disorder is:

$$F = \int d^3r \frac{\hbar^2}{2m^*} \left| \nabla + \frac{i e^r}{\hbar c} A \right|^2 + \frac{\hbar^2}{2m^*} m^* J^2 \xi^2 - \alpha T_c (1 - t) (1 + U (r)) |\psi|^2 + \frac{1}{2} |\psi|^4.$$  

The random component of $T_c$, $U (r)$, is modeled by a white noise characterized by variance depending on the pinning, $U (r) U (r') = \delta (r - r') \xi^2 \xi_n \gamma_p$. The dimensionless pinning strength $n_p$ is proportional to the density of pinning centers in units of the coherence volume $\xi^2 \xi_z $, where $\xi_z = \gamma_p \xi$ with anisotropy parameter $\gamma_a = \sqrt{\frac{m^*}{m^*}}$.

A Langevin type dynamics can be formulated as a functional problem with the dynamical "partition function", defined by the MSR functional integral over the order parameter $\psi$ and an additional "ghost" field $\phi$. The ghost field allows exact integration over the white noise $\zeta$:

$$Z = \int D\psi^* D\psi D\phi^* D\phi \exp \{- A_{MSR} [\psi, \phi] \}.$$  

The explicit form is given elsewhere[4]. The functional approach enable us to calculate both the dynamics correlators and response function of the system in close analogy to calculation of the static correlators in statistical physics. For example dynamical correlator is

$$C (r, \tau, r', \tau') = Z^{-1} \int D\psi^* D\psi D\phi^* D\phi \psi_r (\tau) \psi_{r'}^* (\tau') \exp \{- A_{MSR} [\psi, \phi] \}.$$  

(3)
The system has two dimensionless couplings \(n = \frac{\alpha_p}{4\pi \sqrt{2G_i}} \frac{(1-t)^3}{t}\) characterizing relative strength of disorder compared to interactions and \(g = 8\pi b \sqrt{2G_i}\) characterizing interactions compared to thermal fluctuations with 
\(\sqrt{2G_i} = \frac{\delta}{3\pi e^2 T \xi^2 \zeta}\) and units of \(H_G\) and \(E_0 = \frac{4h}{e^2 \gamma}\) are used for magnetic \(b = \frac{B}{H_0}\) and electric \(E = \frac{E_0}{\xi}\) fields.

Since the model is highly nontrivial even in the simplest cases, one has to use an approximation scheme. We utilize a method which evolved from the gaussian variational approach to quantum mechanics called here gaussian approximation. Since higher correlators are needed, a certain generalization of the gaussian approximation introduced in detail in [4] is used. An additional advantage of this approach over the resummation of diagram technique used in [2] (borrowed from the physics of weak localization) is that it is a systematic and unambiguous without any reference to the "large number of components" limit.

Correlators near the glass line and critical exponents. It is convenient to rescale the correlation functions as \(c_\omega = \left(\frac{\pi}{4}\right)^{2/3} C_\omega\) and \(\rho_\omega = \left(\frac{\pi}{4}\right)^{2/3} R_\omega\).

An appropriate units of time is \(\tau_{GL} = \gamma \xi^2\) and we use the dynamical scaled temperature \(a_T = (8\pi/g)^{2/3} a_h\), \(a_h = -\frac{1}{2} (1 - t - E^2/b^2)\). In the "liquid" phase in which disorder does not alter the vacuum structure significantly, the solution can be found assuming the validity of the dissipation - fluctuation theorem (DFT), which subsequently can be checked by substitution back into the gap equation. A closed form cubic equation for the frequency dependent response function \(\rho_\omega\) is

\[
4n\rho_\omega^{3/2} - 4\rho_0^{1/2} \rho_\omega - a_T \left(1 - \frac{i\omega}{4a_h}\right) \rho_\omega + 1 = 0, \quad \rho_0 = -\frac{a_T^2 + a_T^2 d^{-1/3} + d^{1/3}}{12 (1 - n)^{4/3}} \tag{4}
\]

\[
d = a_T^3 + 12 (1 - n) \sqrt{324 (1 - n)^2 - 3a_T^3} - 216 (1 - n)^2,
\]

which determines the correlator \(c_\omega\) due to DFT:

\[
c_\omega = \left(\frac{8\pi}{g}\right)^{2/3} \frac{\rho_0^{1/2} \rho_\omega \left(\rho_\omega^{1/2} + \rho_\omega^{1/2}\right)}{\rho_\omega^{1/2} + \rho_\omega^{1/2} - 4n \rho_0^{1/2} \rho_\omega} \tag{5}
\]

The critical surface in the "space" of dimensionless scaled parameters \((t, b, E)\) is defined as a set of values of the parameters for which the correlator \(C_\omega\) at \(\omega = 0\) diverges:

\[
\frac{\epsilon^2}{g b^2} = 1 - t - b + \left(\frac{ng}{4\pi}\right)^{2/3} \left(3 - \frac{2}{n}\right) \tag{6}
\]

The critical electric field \(\epsilon_g\) which destroys the "glassy" state forcing vortices to move in a direction perpendicular to the field. The expansion of the correlation functions near the glass line in a small parameter \(\Delta_T = a_T - a_T^b = (8\pi/g)^{2/3} \Delta\) and leads to the following critical behavior:

\[
c_\omega = \frac{\tau_{rel} \Delta_T}{\sqrt{2 (2n)^{1/3}}} \left[1 + \left(1 + (\tau_{rel})^2\right)^{1/2}\right]^{-1/2}, \tag{7}
\]
where the characteristic time, \( \tau_{rel} = \frac{8}{3 \Delta T} \left( \frac{2 \pi}{ng} \right)^{2/3} \), determines a long scale decay of the correlator \( C(\tau) \propto e^{-\tau/\tau_{rel}} \). On the critical surface, where \( \Delta T = 0 \) and \( \tau_{rel} \) diverges, the correlator and response function,

\[
\rho_\omega^g = \frac{1}{(2n)^{2/3}} \left[ 1 - \frac{2e^{-i\pi/4} \left( \frac{4\pi}{ng} \right)^{1/3}}{\sqrt{6}} \omega^{1/2} \right], \quad c_\omega^g = \sqrt{\frac{T}{3}} \left( \frac{8\pi}{g} \right)^{1/3} \frac{1}{n\sqrt{\omega}}. \quad (8)
\]

both have a fractional power dependence on \( \omega \). One therefore observes criticality with exponent \( \frac{1}{2} \). In the static limit near transition, the response function \( \rho_0 \) is continuous, while the correlator \( c_0 = \frac{8}{3\sqrt{2}} \left( \frac{8\pi}{g} \right)^{2/3} \frac{1}{(2n)^{2/3} \Delta T} \) diverges with critical exponent 1.

In the glass phase, namely when \( a_T < a_g^2 \),

\[
c_\omega = \left( \frac{8\pi}{g} \right)^{2/3} \frac{\rho_\omega^g \rho_\omega \left( \rho_\omega^{1/2} + \rho_\omega^{1/2} \right)}{\rho_\omega^{1/2} + \rho_\omega^{1/2} - 4n\rho_\omega \rho_\omega^*} + \lambda \delta(\omega), \quad \lambda = -\frac{\pi}{(2n)^{1/3}} \Delta T. \quad (9)
\]

where the constant \( \lambda \) is the EA order parameter which vanishes on the dynamical glass line and increases below it.

The expression for \( \rho_\omega \) is given by a solution of

\[
4n\rho_\omega^{3/2} - 4(2n)^{-1/3} \rho_\omega - a_T \left( 1 - \frac{i\omega}{4a_h} \right) \rho_\omega + 1 = 0. \quad (10)
\]

Indeed, in addition to a regular part obeying the DFT, there is a singular (at zero \( \omega \)) contribution expressing the persistent correlation \( c_0(\tau) \rightarrow \lambda \). Near the glass line it diverges as \( \Delta^{-1/2} \) due to HLL part. Nernst coefficient is derived in a similar way.

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References

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