Current-Voltage Dependencies in Ultra-Narrow Superconducting Nanowires in the Regime of Quantum Fluctuations

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Abstract. We have studied current-voltage characteristics of ultra-narrow superconducting aluminium nanowires governed by quantum fluctuations of the order parameter. In this regime a quasi-one-dimensional superconducting channel is in a resistive state well below the critical temperature. It has been found that the $V(I)$ characteristics show-up non-monotonous dependency which is very strongly affected by temperature and/or external magnetic field. The origin of the effect is not clear.

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1. Introduction
Zero resistance is known to be a fundamental attribute of superconductivity. One might naively expect that this property is retained down to nanoscales. Unfortunately, there are indications [1] - [9] that below a certain level $\sim 10 \text{ nm}$ quantum fluctuations of the order parameter $|\Delta|e^{i\phi}$ suppress dissipationless electric current in quasi-one-dimensional superconducting channels. Approaching this ultra-narrow limit the $R(T)$ dependencies become very wide and the zero state resistance is not reached even well below the critical temperature $T_c$. The topic is of a vital importance for numerous nanoelectronic applications utilizing superconducting components.

Various approaches to model the contribution of the phenomenon (also often called quantum phase slip) have been proposed [10]-[20]. The commonly accepted simplified picture accounts for tunneling through a potential barrier of the macroscopically coherent system between two local minima of the free energy in the phase space $\phi$. It has been shown [21] that application of a finite current breaks the symmetry between transitions $\Delta \phi = \pm 2\pi$ causing energy dissipation and, hence, resulting in non-zero resistance. The corresponding contribution of finite measuring current is expected to provide a trivial monotonous dependence: the higher the current, the higher the effective resistance. In present paper we report observation of non-monotonous $V(I)$ dependencies in ultra-narrow aluminium nanowires.

2. The model
Without going into details not necessary for the needs of the present subject [10]-[21], one may show that the effective resistance of a quasi-one-dimensional superconductor as function of temperature can be written as:
If quantum effects are important (e.g. at temperatures below the onset of superconductivity), 1D crystals (whiskers) with effective diameter, $d_{eff}$, have been experimentally confirmed by measuring the shape of the current-voltage characteristics. The critical current, $I_c$, is determined by the finite current, $I$, and the penetration depth of the superconducting flux quantum, $\lambda_c$.

If fluctuations of the superconducting order parameter are solely determined by classical (thermal) effects, then $\mathcal{E} = k_B T$, where $k_B$ is the Boltzmann constant [21]. Predictions of the model [21] have been experimentally confirmed by measuring the shape of $R(T, I)$ dependencies of extremely homogeneous superconducting 1D crystals (whiskers) with effective diameter, $d_{eff} \approx 0.5 \mu m$ [22].

If quantum effects are important (e.g. at temperatures $T < T_c$ [1] - [9]), then $\mathcal{E} = \hbar \Gamma_{QPS}$, where $\Gamma_{QPS}$ is the rate of quantum fluctuations [10]-[20]. The exact form of the pre-factor $\Omega(T, I, L)$ has been determined for thermal activation mechanism [21]. For quantum fluctuations functional dependence $\Omega(T, I, L)$ is currently under intensive debates [10]-[20]. However, though the pre-factor $\Omega(T, I, L)$ contains temperature- and current-dependent terms, it contributes negligible compared to the strong exponential dependence. In both cases (thermal and quantum) the current impact is determined by the $\sinh(I/2I_0)$ term and the current-dependent terms under the exponent. The latter one is responsible for the effective reduction of the potential barrier $\Delta F$ by the finite current $I$.

\[ R(T, I) \equiv V(T, I)/I = \frac{\Omega(T, I, L)}{I} \exp \left[ -\frac{\Delta F}{\mathcal{E}} - \left( \frac{2}{3} \right)^{1/2} \frac{I^2}{3\pi I_0 I_c} \sinh \left( \frac{I}{2I_0} \right) \right], \quad (1) \]

where $I_c$ is the temperature-dependent critical current, $I_0 = k_B T / \phi_0$ with $\phi_0 = \hbar / 2e$ being the superconducting flux quantum.

The most intriguing parameter is the energy $\mathcal{E}$ in the denominator under the exponent. If fluctuations of the order parameter are solely determined by classical (thermal) effects, then $\mathcal{E} = k_B T$, where $k_B$ is the Boltzmann constant [21]. Predictions of the model [21] have been experimentally confirmed by measuring the shape of $R(T, I)$ dependencies of extremely homogeneous superconducting 1D crystals (whiskers) with effective diameter, $d_{eff} \approx 0.5 \mu m$ [22].

3. Experiment and discussion

We have used e-beam lithography to fabricate aluminium nanowires on doped Si substrates. Typical cross-section of the samples just after lift-off was about 100 nm × 80 nm. Three different lengths were used: 1 µm, 5 µm and 10 µm. An AFM image of a typical nanowire is shown in Fig. 1. We used our proprietary method of re-shaping a nanostructure by low energy ion bombardment [23]. The method enables progressive reduction of a nanowire cross-section $\sigma$ between sessions of electric transport measurements. Using this approach we were able to obtain structures with effective diameter $\sigma^{1/2} \approx 10$ nm. An additional feature of the method is 'polishing' of the surface of the ion beam treated samples [6]. Inevitable roughness of the lift-off fabricated structures has been reduced down to $\pm 2$ nm, which is comparable to the penetration depth of $Ar^+$ ions inside aluminium matrix and is of the same order as the thickness of naturally grown oxide on the surface. All measurements were made using 4-probe configuration using carefully rf filtered lines. The samples were immersed into directly pumped helium bath. The temperature of the superfluid $^4He$ was stabilized using PID controlled with accuracy $\pm 0.1$ mK.

As reported in our earlier studies [6], [9] with progressive reduction of cross-section of an aluminium nanowire, the $R(T, I)$ dependencies become very wide. An example of such a broad 'superconducting' transition is presented in inset of Fig. 2. The effect is associated with quantum fluctuations of the superconducting order parameter, which start to dominate over the thermal mechanism at temperatures well below the $T_c$. Manifestation of the phenomenon is observable only in the very thin quasi-one-dimensional aluminium structures below $\sim 15$ nm [6], [9].

We have measured the current - voltage dependencies $V(I)$ and their derivatives $dV/dI(V)$ using modulation technique at constant temperatures (Fig. 2). At temperatures above the onset of superconductivity (critical temperature) the current-voltage characteristics show a trivial Ohmic behavior with no signs of Coulomb blockade. The observation supports the results of AFM analysis: at $\sim 10$ nm scales the nanowires are still continuous and do not consist of weakly coupled metallic grains. At temperatures slightly below the $T_c$ the $V(I)$’s deviate from linear dependency and qualitatively can be fitted by the $\sim \sinh(I)$ term from Eq. (1). However, at temperatures well below the onset of superconductivity one can clearly see the
non-monotonous behaviour (Fig. 2b). The positions of the 'bumps' on \(dV/dI(V)\)’s strongly depend on temperature and external magnetic field. Application of perpendicular magnetic field \(B \sim 50 \text{ mT}\) completely removes the 'wiggling', while the normal state is recovered only at fields few times higher. The non-monotonous dependencies are observed only in the thinnest nanowires with effective diameter \(\sigma^{1/2} < 15 \text{ nm}\).

Currently we do not have a solid explanation of the phenomenon. Hypothetically one may imagine that the energy spectrum of an ultra-narrow superconducting wire in a resistive state is quantized. The quantization might originate from size phenomena [24] or/and QPS-mediated excitation of an electromagnetic disturbance (e.g. plasmon propagation [25], [19]) forming a ‘standing’ wave inside the wire. If the spectrum is quantized, then the quantum tunneling of the phase might have a resonant nature resulting in non-monotonous \(V(I)\) dependencies. Further experiments and development of theory are required to make more conclusive statements.

4. Conclusions
In conclusion, we have studied electron transport properties of ultra-narrow superconducting aluminium nanowires in the resistive state governed by quantum fluctuations of the order parameter. It has been found that in the thinnest structures the \(V(I)\) characteristics show-up non-monotonous dependency which is very strongly affected by temperature and external magnetic field much smaller than the critical one. The origin of the effect is not clear.

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**Figure 1.** Atomic force microscope (AFM) image of a typical aluminum nanowire just after lift-off fabrication.
Figure 2. Inset: resistance vs. temperature for $\sigma^{1/2} \simeq 12 \text{ nm} \pm 2 \text{ nm}$ and $10 \mu \text{m}$ long aluminium nanowire measured with 1 nA AC current. (a) The current - voltage $V(I)$ dependencies and (b) the derivatives $dV/dI(V)$ measured at various temperatures for the same nanowire as in the inset. Data have been taken in zero magnetic field.
6. References

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