Dynamic Buckling of Elastic Cylindrical Shell under Axial Impact Load

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Abstract. In this paper, the original and dual variables for the dynamic buckling of elastic cylindrical shell under the axial impact load are constructed by means of the energy of the system, and the Hamiltonian system of the system is established. In symplectic geometry space, the critical buckling loads and buckling modes of cylindrical shells are reduced to symplectic eigenvalues and eigensolutions.

1. Introduction
Stability mainly includes two aspects, one is the stability of the material itself, the other is the balance and stability of the structural system. The static problems have been studied deeply, but the dynamic problems, especially the dynamic buckling problems, are difficult to study because of the basic model and the high order partial differential equation.

The study of structural dynamic buckling can be divided into two kinds of problems: one is parametric buckling, which means that the structure with periodic loading function produces structural resonance in a certain buckling mode, and the loading function is displacement parameter in the differential equation of motion [1-3]. Another main characteristic of parametric buckling is that the load amplitude required for collapse of structure is lower than that of the corresponding static buckling collapse; the other is the impact buckling caused by non periodic impact load. At present, the theoretical and experimental researches on the impact buckling are mainly focused on the buckling problems caused by the ideal pulse load and step load. The buckling problem caused by the ideal pulse load is called the ideal pulse buckling, and the buckling caused by the step load with constant amplitude and infinite duration mainly refers to the dynamic jump buckling under the action of the step load [4-6].

Impact dynamics is an important research direction in solid mechanics, which is developing rapidly at present. The significance of the research on the dynamic buckling characteristics of structures under impact load has become more and more obvious with the development of aviation, aerospace and atomic energy utilization. For example, when a long rod-shaped bullet hits a target plate, it will not cause material instability and structural buckling [7].

Cylindrical shell is one of the most commonly used structures in engineering. The buckling of cylindrical shell plays an important role in the theory of structural stability. In the early years, people found that the experimental results are only one-fifth to one-half of the predicted values of linear theory, and the experimental data are not repeatable to a certain extent. The great difference between
the experiment and the theory has attracted the attention of many researchers, and also greatly promoted the in-depth development of the theory of continuous system stability.

2. The symplectic solutions
For an elastic cylindrical shell under the axial impact load in the coordinate system \((x, \theta, r)\), the internal force and strain, bending moment and curvature satisfy the following relations

\[
N_x = K(\varepsilon_x + \nu \varepsilon_\theta) \quad M_x = D(\kappa_x + \nu \kappa_\theta) \\
N_\theta = K(\varepsilon_\theta + \nu \varepsilon_x) \quad M_\theta = D(\kappa_\theta + \nu \kappa_x) \\
N_{s\theta} = K(1-\nu)\varepsilon_{s\theta} / 2 \quad M_{s\theta} = D(1-\nu)\kappa_{s\theta}
\]

According to the theory of small deformation, the relationship between the surface strain, curvature and displacement in the shell is

\[
\kappa_x = \frac{\partial^2 w}{\partial x^2}, \quad \kappa_\theta = -\frac{1}{r^2} \frac{\partial^2 w}{\partial \theta^2}, \quad \kappa_{s\theta} = -\frac{1}{r} \frac{\partial}{\partial x} \frac{\partial w}{\partial \theta}, \\
\varepsilon_x = \frac{\partial u}{\partial x}, \quad \varepsilon_\theta = \frac{1}{r} \frac{\partial v}{\partial \theta} - \frac{w}{r}, \quad \varepsilon_{s\theta} = \frac{\partial u}{\partial \theta} + \frac{\partial v}{\partial x}
\]

The deformation potential energy density of the shell

\[
\Pi_x = \frac{1}{2} K(\varepsilon_x^2 + \varepsilon_\theta^2 + 2\nu \varepsilon_x \varepsilon_\theta + \frac{1-\nu}{2} \varepsilon_{s\theta}^2)
\]

Because the thickness diameter ratio of the shell is very small and the cross section bears uniform load, the Lagrangian function is

\[
L = \int \left[ \frac{1}{2} \rho h \left(\frac{\partial u}{\partial t}\right)^2 + \frac{1}{2} \rho h \left(\frac{\partial v}{\partial t}\right)^2 + \frac{1}{2} \rho h \left(\frac{\partial w}{\partial t}\right)^2 - \frac{1}{2} K(\varepsilon_x^2 + \varepsilon_\theta^2 + 2\nu \varepsilon_x \varepsilon_\theta + \frac{1-\nu}{2} \varepsilon_{s\theta}^2) \right] \, dt \, dx
\]

\[
- \frac{1}{2} D(\kappa_x^2 + \kappa_\theta^2 + 2\nu \kappa_x \kappa_\theta + 2(1-\nu)\kappa_{s\theta}^2) - \frac{N}{2} (\partial_\theta w)^2 \, r \, d\theta \, dx
\]

According to the variational principle, one has

\[
\delta \int \left[ \frac{\rho h}{2} \left(\partial_\theta u\right)^2 - \frac{E h}{2r^2} (\partial_\theta w)^2 - \frac{E h}{2} (\partial_r w)^2 - \frac{D}{2} (\partial_r^2 w)^2 + \frac{1}{r^2} (\partial_{\theta r} w)^2 + 2\nu (\partial_\theta^2 w)^2 + \frac{1}{r} \frac{\partial}{\partial \theta} \left(\partial_\theta^2 w\right) \right] \, r \, d\theta \, dx \, dt = 0
\]

The Hamiltonian function

\[
H(q, p) = p^T q - L(q, p) = -\frac{E h}{2r^2} w^2 + \frac{1}{2D} p^2 - p_\theta \phi_\theta + p_\phi \phi_\phi - \frac{N}{2} (w')^2
\]

By solving Eq. (6), we get

\[
\eta_n^{(\gamma)} = \begin{cases} 1 \\
- \frac{ni}{r} \\
is D(\gamma_n^2 + n^2 / r^2) / r \\
dD(\gamma_n^2 + n^2 / r^2)
\end{cases}
\]

and

\[
(\gamma_n = \alpha_n, \beta_n)
\]
\[
\alpha_{in} = \frac{N_i}{D} - n^2 + \left[ \left( \frac{N_i}{D} - n^2 \right)^2 - 4\left( \frac{Eh}{r^2D} + \frac{n^4}{r^2} \right) \right]^{1/2} / \sqrt{2}
\]
\[
\beta_{in} = \frac{N_i}{D} - n^2 - \left[ \left( \frac{N_i}{D} - n^2 \right)^2 - 4\left( \frac{Eh}{r^2D} + \frac{n^4}{r^2} \right) \right]^{1/2} / \sqrt{2}
\]

The boundary condition of reflection end should satisfy
\[
\begin{align*}
w &= 0 \quad \text{or} \quad \partial_x p_2 + N_i \partial_x w = 0 \\
\partial_x w &= 0 \quad \text{or} \quad \nu p_2 - D(1-\nu)\partial_x^2 w = 0
\end{align*}
\]

\[(x = l) \quad (9)\]

The continuity condition is
\[
A \cdot C = 0
\]

where
\[
A = \begin{bmatrix}
A_1(0) & A_2(0) & A_3(0) & A_4(0) \\
A_{21}(0) & A_{22}(0) & A_{23}(0) & A_{24}(0) \\
A_{31}(x_c) & A_{32}(x_c) & A_{33}(x_c) & A_{34}(x_c) \\
A_{41}(x_c) & A_{42}(x_c) & A_{43}(x_c) & A_{44}(x_c)
\end{bmatrix}
\]

\[(11)\]

and
\[
C = \{C_1, C_2, C_3, C_4\}^T
\]

\[(12)\]

3. Results and discussion
Fig.1 shows the first ten critical buckling load curves. It can be seen from the figure that at the beginning of the impact load, the critical buckling load of the shell decreases rapidly with the propagation of the axial stress wave.

Figure 1. The branches of critical buckling loads with the wave propagation.

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References
[1] Gordienko, B.A. (1972) Buckling of inelastic cylindrical shells under axial impact. Arch. Mech., 24: 383-394.
[2] Jamal, M., Lahlou, L. (2004) A semi-analytical buckling analysis of imperfect cylindrical shells under axial compression. Int. J. Solid. Struct., 40: 1311-1327.
[3] Greiner, R., Guggenberger, W. (1998) Buckling behavior of axially loaded steel cylinders on local supports—with and without internal pressure. Thin Wall. Struct., 31: 159-167.
[4] Abrahamson, G.R, Goodier, J.N. (1996) Dynamic flexural buckling of rods within an axial plastic compressive wave. J. Appl. Mech., 33: 241-247.
[5] Symodns, P.S. (1973) Approximation techniques for impulsively loaded structures of rate sensitive plastic behavior. J. Appl. Math., 25: 462-473.
[6] Priza, K., Wijeyewickrema, A.C., Kikuo, K. (2011) Wave propagation along a non-principal direction in a compressible pre-stressed elastic layer. Int. J. Solid. Struct., 48: 2141-2153.
[7] Shouetsu, I. (2013) Effect of couple-stresses on the Mode I dynamic stress intensity factors for two equal collinear cracks in an infinite elastic medium during passage of time-harmonic stress waves. Int. J. Solid. Struct., 50: 1597-1604.