Estimation and compensation of motion blur for the reduction of uncertainty in DIC measurements of flexible bodies

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Abstract. Digital image correlation (DIC) is a useful technique to measure displacement/strain fields both for static and dynamic problems in experimental mechanics. When monitoring moving objects with digital cameras, motion blur may occur if the shutter time reaches the time scale of the motion of the measurand. Consequently, motion blur is one of the most relevant problems in those dynamic DIC applications where shutter time cannot be set arbitrarily. This work deals with the problem of compensating motion blur effects on a generic DIC image. The problem of motion blur compensation to reduce DIC uncertainty is discussed in literature in the case of rigid target, where the amount of motion blur is the same for the whole image area. Deformable targets, instead, pose the additional problem when motion blur is variable within a single frame. In this paper a subset-based technique is proposed to estimate and compensate the motion blur for each image region. The approach is tested on synthetically deformed and blurred images of a notched beam specimen.

1. Introduction

This paper proposes a new approach to mitigate the effects of motion blur on 2D Digital Image Correlation (DIC) measurements. DIC is an optical method that employs tracking and image registration techniques for accurate optical flow measurements [1]. This method is often used to measure full-field displacement and strains, being indeed widely applied in many areas of experimental mechanics. Whenever possible the motion blur is reduced during image acquisition by reducing the exposure time and increasing the illumination level. However, this is not always possible since lighting conditions cannot be easily controlled, such as in outdoor conditions or in the case of a camera mounted on a flying drone [2]. The uncertainty of DIC measurements is composed by a correlation bias (namely bias error) [3, 4] and a random error [5] that generates dispersion of measured displacements. When the images are affected by motion blur, random error is the main uncertainty component [6, 7]. Conversely, the contribution of the bias error is less relevant in presence of motion blur [8, 9, 10]. In previous works [10, 11], the authors modeled the physical nature of motion blur phenomenon and how this results into uncertainty. Given that knowledge, a further step is to try to compensate the undesired effects of motion blur, through image deconvolution.

To remove motion blur, it is of primary importance to estimate its intensity in a reliable and appropriate way. Inaccurate blur estimation leads to the generation of artifacts in compensated
images. In this paper, the estimation of motion blur transformation is carried in the frequency domain. In brief, DIC always require an image of the measurand in reference position, hence it is possible to estimate the Optical Transfer Function (OTF) between the reference texture and the deformed one. In presence of motion blur, the OTF is represented by a bi-dimensional sinc kernel. In principle, it is possible to recover the blur transformation by fitting the OTF data with a sinc kernel model. This approach is developed at subset level, to deal with variable motion blur.

Once the motion blur is estimated in a robust way, the paper discusses motion blur removal, using the well known Wiener filter method. This choice is justified by the presence of noise in digital images. Noise itself should be modeled correctly, hence this issue is discussed within the body of the paper.

Eventually, the whole approach is tested and validated against a synthetic experiment on a notched cantilever beam. Data shows that the algorithm is able to remove motion blur from DIC images, also in those situations where it is highly variable inside the acquired frame. At the same time, the compensation of motion blur is able to reduce the uncertainty in DIC measurements of about 80% in cases where motion blur exceeds 7 pixels.

2. State of the art

The phenomenology of motion blur on digital images has been described first by Potmesil et al. [12]. Their work dates back to the 80s and they solved the problem of describing motion blur in analytic way. Nonetheless it enabled the research of methods to remove motion blur in digital images.

The problem of motion blur compensation is relevant in many fields: from general photography [13], to image stabilization in smartphones [14], coming up to the niche of scientific imaging [15]. In any case motion deblurring involves two separate problems:

- determining the motion blur transformation parameters
- removing the motion blur transformation from the acquired image

The two problems can be solved in different ways and each of them can be formulated in separate domains [16], depending on the approach of the proposed solution. In the next paragraphs it is highlighted a review of the literature.

2.1. Estimating motion blur

A sound review of the state of the art of motion blur estimation techniques can be found in [16] and [17]. To focus better the work further presented this paper, it is necessary to highlight the two possible ways which the motion blur estimation problem can be formulated:

- referenced estimation
- blind estimation

Referenced estimation is the problem of estimating motion blur in an acquired image, given that an unblurred image of the scene is available. Blind estimation, instead, is the estimation of motion blur transformation when only the motion blurred scene is available. Even if they deal with the same phenomenon, the two problems are very different from a mathematical point of view. For what concerns referenced deconvolution it is always possible to formulate estimator for the Point Spread Function (PSF) (or OTF in the frequency domain) given that the original scene is available. Conversely, the blind estimation problem is ill-posed since there are not data describing what the scene would appear without blur. Hence solving a blind problem [18] will always involve statistical tools [19, 20] and a-priori hypothesis on what the scene would appear like [21].
If the reader considers the subject of this paper, the problem is indeed a referenced one: for a vibrating target it is usually possible to have a still image of it, as long as mechanical excitation is off. The main idea of referenced estimation lies on the estimation of the OTF starting from reference and blurred images. Example of uses of referenced deconvolution procedure can be found in [22, 23, 24].

The authors tried to focus on an approach dedicated to vision based measurement systems. The main idea of the work is to fit the OTF estimator with a frequency domain model of motion blur, trying to preserve all metrologic information nested in the image (similarly to what documented in [24]). Another goal is to estimate a motion blur which is variable inside a given image. A preliminary study on the feasibility of a monodimensional motion blur estimator for DIC images has been presented in [25]. This work will extend those concepts to the generic bi-dimensional problem.

2.2. Removing motion blur

The task of removing motion blur is instead a well defined problem in all cases where the motion blur PSF (or the OTF) is retrieved. As described in [12], the motion blur is a convolutive operator. Consequently its removal is a problem of signal deconvolution once the convolution kernel is known. Signal deconvolution is a trivial task when working with deterministic signals. Unfortunately, digital images do not fall in the category of deterministic signals, since a moderate level of noise is unavoidable during actual image acquisition. Noise introduces spectral components in the blurred images which are uncorrelated with the reference one. Consequently, the deconvolution problem becomes ill-posed.

For this class of problems, the Wiener filter [26] is considered the best solution. In fact, both the Wiener causal filter and its finite impulse response (FIR) counterpart are optimal filters for the signal deconvolution problem in presence of additive noise. A full description about modern digital implementation of Wiener filters can be found in [27]. To apply the Wiener filter it is necessary to estimate the Optical Transfer Function due to motion blur and the noise-to-signal ratio (NSR) to provide a model of the uncorrelated noise. This paper will also provide a detailed description of the NSR estimation procedure.

3. Motion blur estimation procedure

It is not possible to formulate a universal model able to describe the generation of motion blur in an image of a vibrating target. In fact, even in the ideal case of noise-less image formation, there are phenomenon such as optical distortions or sensor non-linearity, that limit the development of universal models. Stated the impossibility to retrieve a generic universal formulation, it is necessary to limit the field of investigation with a suitable set of hypothesis. In particular:

- existence of a reference image $R(m, n)$ of the vibrating target standing still in equilibrium position
- the non-blurred image $Q(m, n)$ of the vibrating target for a given position is supposed to be equal to the reference one rigidly translated of a quantity $A_0$ in the direction $\phi$, so that $Q(m, n) = R(m - A_0 \cos \phi, n - A_0 \sin \phi)$
- exposure time is a fraction of vibration period and small enough to describe motion during exposure as a uniform rectilinear one.

When the fore mentioned hypothesis are verified, it is possible to state the following corollaries:

(i) the motion blurred image $P(m, n)$ is generated from $Q(m, n)$ after the convolution with a simple 2D rectangular window of length $W_0$ pixels
(ii) motion blur appears into the image in the form of rectilinear swipe trails as shown in figure, orientated in the motion direction

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in the direction of swipe trails the problem is mono dimensional, since each line of the acquired frame is generated from its equivalent line in the reference frame after rigid translation and convolution.

As a consequence the blur phenomenon exemplified in figure 1 is described by the length of convolution window $W_0$ and the direction of swipe trails $\phi$. In particular the length (in pixels) of convolution window is linearly dependent to instantaneous speed of marker during exposure, as described in [11].

![Figure 1: Swipe trails descriptions as for linear blur model](image)

It is possible to highlight that the model is a 2D sinc surface kernel as shown in figure 2a. It requires to identify 3 parameters in order to describe the OTF:

- motion blur direction $\phi$
- rigid translation between reference image and acquired one $A_0$
- blurring window size $W_0$

To sum up, a multi-parametric model that is able to describe the transformation between a reference image and a generic one has been formulated. Now it is necessary to describe how it is possible to estimate those parameters, starting from a reference image and a motion blurred one.

If the reference image $R(m,n)$ and the blurred one $P(m,n)$ are available, it is possible to estimate the OTF $H_{\mathcal{E}}(k,l)$ between the two as in eq. (2), where the $\mathcal{F}$ operator indicates the 2D Fourier transform:

$$H_{\mathcal{E}}(k,l) = \mathcal{F}(P) \mathcal{F}(R)$$  \hspace{1cm} (2)

Given that, the transformation parameters can be retrieved by minimizing the error between the estimated OTF $H_{\mathcal{E}}(k,l)$ and the theoretical one $H(k,l)$, as in equation (3):

$$\begin{bmatrix} \phi \\ W_0 \\ A_0 \end{bmatrix} = \arg \min \left\{ H_{\mathcal{E}} - W_0 \cdot \text{sinc} \left[ W_0 \cdot (\sin \phi \cdot k + \cos \phi \cdot l) \right] \cdot e^{-j2\pi A_0 \cdot (\sin \phi \cdot k + \cos \phi \cdot l)} \right\}^2$$  \hspace{1cm} (3)
Unfortunately, the fitting procedure cannot be performed directly in one step. In fact a set of problems limits the robustness of a direct fitting approach:

- image digitalization deals to relevant spectrum leakage problem [28]
- as a consequence of the previous point, the estimation of OTF $H(k,l)$ should be performed with adequate processing
- noise introduces uncorrelated components in the spectral representation
- background effect: even without blur and noise it’s not in general true that the acquired image is a simple translation of the previous one, as long as there may be elements on the scene background that does not move or vibrate.
- motion blur may be variable inside an image if the measured target is deforming
- fitting the sinc kernel is itself a badly conditioned problem, since the fitting error gradient is much higher for the amount of motion blur than for the direction angle

As a consequence, a dedicated procedure to retrieve a good and robust estimation of motion blur has been developed. The main idea is to focus on the morphology of the OTF to retrieve a first estimate of the motion blur parameters. Then the fitting procedure becomes a better defined bounded minimization problem.

The last step has been to deal with the problem of having variable motion blur inside an acquired image. The solution has been found in mocking the DIC behavior. In this sense, the reference image is divided into subsets. Then a template matching algorithm is run to find the position of the subsets in the generic deformed and blurred image. Then OTF estimation is run at subset level, letting us retrieve the mean value of motion blur inside the subset. For sure this procedure has some limitation as the subset size becomes smaller: in fact the smaller the subset, the less the frequency resolution of OTF is. Therefore the uncertainty in the parameter estimation becomes larger. To validate the approach, the algorithm has been tested in 5.1, to understand what are the limit in shrinking subset size.

4. Motion blur restoration with Wiener filter
As explained in section 2.2, the knowledge of both the Point Spread Function PSF due to motion blur and the noise-to-signal ratio NSR is necessary to implement Wiener filter. Even though the PSF is completely identified once the blurring window size $W_0$ and direction $\phi$ are estimated, an accurate definition of the NSR is key to achieve a good and robust deconvolution. During
the image grabbing process various noises (e.g. heteroscedastic noise, shot noise, thermal noise, cut-off noise) and illumination lighting fluctuations are unavoidably present and most of them are characterized by a peculiar spectral distribution. The authors decided to use as support for Wiener deconvolution the function deconvwnr.m, which is already implemented in Matlab. It considers the noise as an additive and stochastic process, allowing to express the NSR as a scalar constant which approximates the NSR in every point of frequency coordinates.

4.1. NSR estimation

The algorithm adopted to estimate the noise-to-signal ratio to be used for Wiener deconvolution is discussed and presented in this section. It is based on the analysis of a sequence of static images of the target. In detail:

- The first image of the series (I_REF) is chosen as reference and its Root Mean Square (σ_S) is calculated, according to eq.(4), where \( \bar{p} \) is the mean value of the image, \( p_{ij} \) is the pixel intensity and \( m, n \) are the number of rows and columns of the image, respectively.

\[
\sigma_S = \sqrt{\frac{\sum_{j=1}^{n} \sum_{i=1}^{m} (\bar{p} - p_{ij})^2}{m \cdot n}}
\]  

- The mean value \( \bar{p} \) is subtracted from all the still images to compensate any type of variations or fluctuations in lighting conditions.

- The reference image is subtracted from each one of the remaining images (I_NOISE), obtaining a matrix difference called I_DIFF. The standard deviation of I_DIFF is computed and the results are averaged for the number of involved images, obtaining \( \sigma_N \).

In this way it is possible to estimate the NSR as the ratio between the averaged standard deviation of the image series \( \sigma_N \), which represents an index for the noise level, and the RMS of the reference image \( \sigma_S \), which represents an index for the signal level (5):

\[
NSRE = \frac{\sigma_N}{\sigma_S}
\]  

5. Validation on synthetic images

5.1. Estimation results by changing the image size

To address the problem of motion blur at subset level, it is of utmost importance to understand how the estimation results are influenced by the image size. Since blur may change inside the image of a moving deformable body, it is necessary to divide the frame itself into small regions where motion blur is almost homogeneous. This assures the hypothesis of applicability of the deconvolution algorithms. Therefore, a simulation activity is developed involving a synthetic pattern. Four squared images of different size are considered and motion blur, having a length of 6 px and orientation of 45°, is numerically simulated in each of them according to the sub-pixel accurate technique described in [6, 29]. To resemble operating conditions, a zero mean Gaussian noise with standard deviation equal to 1% of the full dynamic range of the image (i.e. 1% of 255) is also added to the images. This is because modern cameras usually operate with a noise level between 0.5% and 1% of the dynamic range [30]. Then, motion blur estimation algorithm is applied on the four images, bringing to the results reported in Tab.1. It can be confirmed the goodness of the estimation technique even in the case of the smallest images. However, it was not possible to decrease the image size below 70 x 70 px. This physical limit is dictated by the lacking of points necessary to perform a robust fitting of the sinc function.
Table 1: Motion blur estimation tests on images having different size. Imposed blur is 6 px on a 45 degrees direction

| Image size [px] | \( w_{est} \) [px] | \( \theta_{est} \) [°] | Blur error [px] | Angle error [deg] |
|-----------------|---------------------|------------------------|-----------------|-------------------|
| 70x70           | 5.80                | 44.6                   | 0.20            | 0.4               |
| 100x100         | 5.97                | 45.1                   | 0.03            | 0.1               |
| 200x200         | 5.98                | 44.9                   | 0.02            | 0.1               |
| 400x400         | 5.98                | 45.0                   | 0.02            | 0.0               |

Figure 3: Motion blur simulation \( (w_{nom} = 6px, \theta_{nom} = 45°) \) on synthetic images having different size

Figure 4: Synthetic experiment of a dynamic beam shear: the images are labelled with the subsets used for motion blur compensation
5.2. Simulation of dynamic beam shear

The effectiveness of the proposed algorithms is tested on the synthetic experiment represented in Fig. 4. To resemble an actual experimental activity, a static image of a notched beam is used as the reference one for DIC. Then, motion blur is numerically simulated in the vertical direction, linearly increasing from 0 px on the left side of the image (simulated clamped constraint) until reaching 10 px on the right hand side. The post-processing activity starts considering the reference image and dividing the beam surface into adjacent subsets along its longitudinal axis. Then, the template matching identifies the $i^{th}$ subset location in the respective blurred image. A subset size of 165 px guarantees homogeneous blurring conditions inside the same region, which is a requirement that must be satisfied for the correct working of the deconvolution algorithms. For each subset the data fitting with a 2D sinc allows to estimate both the intensity and the direction of motion blur (Fig. 5). As soon as those parameters are known, it is possible to compensate the blur with a Wiener filter.

![Figure 5: Estimation of blur parameters at subset level](image)

5.3. DIC analysis after motion blur compensation

In the context of DIC, uncertainty analysis is often carried out by means of synthetic experiments [31, 32], where image transformations are numerically imposed. In the current study, Ncorr [33] software is used as support to perform DIC analysis on the images presented in section 5.2. DIC processing returns as output the displacement of the subsets in which the image has been divided by the software. Each subset has been selected to have a radius of 21 px and a spacing of 2 px. Since a vertical and variable motion along the longitudinal beam axis has been simulated, the displacement matrices obtained for each image are used to calculate both the mean displacement $v_{\text{mean}}$ and the standard deviation of displacement $\sigma_v$ of a column of subsets. The mean displacement returned by DIC estimates the motion of the beam axis. The standard deviation, instead, represents a viable index to express the uncertainty of the data. DIC results before and after blur compensation are reported in Fig. 6. In agreement to what has been previously found in [25], standard deviation of displacement field $\sigma_V$ (the random error) is the most relevant component of uncertainty when motion blur phenomena occur. After the compensation of motion blur (dashed line in Fig. 6) standard deviation is strongly reduced. This demonstrates how deconvolution becomes a viable solution to improve DIC accuracy in motion blurred images. It is important to highlight the presence of a slight bias in the reconstruction of vertical displacement when motion blur exceeds 7 px. This issue will be investigated in future.
works, however it doesn’t affect the overall judgment on the algorithm since the amount of motion blur is higher than the usual range of DIC experiments (where it is mostly below 4 px).

![Graph showing DIC results for the synthetic blur experiment before (solid line) and after (dashed line) motion blur compensation: the upper graph represents the mean value of vertical beam displacement, the lower graph its uncertainty.](image)

**Figure 6:** DIC results for the synthetic blur experiment before (solid line) and after (dashed line) motion blur compensation: the upper graph represents the mean value of vertical beam displacement, the lower graph its uncertainty.

6. Conclusion

In this work the authors demonstrated a generic procedure to detect a generic amount of motion blur in a generic spatial direction. This procedure has been implemented at subset level, thus enabling to work with images having variable motion blur (such as the ones coming from vibrating deformable bodies). The validation of the method is performed by means of a synthetic experiment. However the authors already performed experimental tests which will be documented in future works. The results of the synthetic experiment show a sensible reduction of DIC random uncertainty due to the application of motion blur compensation. Data show that also a slight bias is introduced. This issue will be better investigated in future works.

References

[1] M. Sutton, J. Orteu, and H. Schreier, *Image Correlation for Shape, Motion and Deformation Measurements: Basic Concepts, Theory and Applications*. Springer US, 2009.

[2] D. Reagan, A. Sabato, and C. Niezrecki, “Feasibility of using digital image correlation for unmanned aerial vehicle structural health monitoring of bridges,” *Structural Health Monitoring*. Published online on October 10, 2017. DOI:10.1177/1475921717735326.

[3] B. Pan, “Bias error reduction of digital image correlation using gaussian pre-filtering,” *Optics and Lasers in Engineering*, vol. 51, no. 10, pp. 1161–1167, 2013.

[4] H. Schreier, J. Braasch, and M. Sutton, “Systematic errors in digital image correlation caused by intensity interpolation,” *Optical Engineering*, vol. 39, no. 11, pp. 2915–2921, 2000.

[5] Y. Wang, P. Lava, S. Coppieeters, M. De Strycker, P. Van Houtte, and D. Debruyne, “Investigation of the uncertainty of dic under heterogeneous strain states with numerical tests,” *Strain*, vol. 48, no. 6, pp. 453–462, 2012.

[6] E. Zappa, A. Matinmanesh, and P. Mazzoleni, “Evaluation and improvement of digital image correlation uncertainty in dynamic conditions,” *Optics and Lasers in Engineering*, vol. 59, pp. 82–92, 2014.
[7] E. Hack, X. Lin, E. Patterson, and C. Sebastian, “A reference material for establishing uncertainties in full-field displacement measurements,” *Measurement Science and Technology*, vol. 26, no. 7, 2015.

[8] D. Wang, Y. Jiang, W. Wang, and Y. Wang, “Bias reduction in sub-pixel image registration based on the anti-symmetric feature,” *Measurement Science and Technology*, vol. 27, no. 3, p. 035206, 2016.

[9] P. Mazzoleni, F. Matta, E. Zappa, M. Sutton, and A. Cigada, “Gaussian pre-filtering for uncertainty minimization in digital image correlation using numerically-designed speckle patterns,” *Optics and Lasers in Engineering*, vol. 66, pp. 19–33, 2015.

[10] A. Lavatelli and E. Zappa, “A displacement uncertainty model for 2-d dic measurement under motion blur conditions,” *IEEE Transactions on Instrumentation and Measurement*, vol. 66, pp. 451–459, March 2017.

[11] A. Lavatelli and E. Zappa, “Modeling uncertainty for a vision system applied to vibration measurements,” *IEEE Transactions on Instrumentation and Measurement*, vol. 65, pp. 1818–1826, Aug 2016.

[12] M. Potmesil and I. Chakravarty, “Modeling motion blur in computer-generated images,” *Computer graphics*, vol. 17, no. 3, 1983.

[13] D. Dansecreau, A. Eriksson, and J. Leitner, “Richardson-lucy deblurring for moving light field cameras,” in *IEEE Computer Society Conference on Computer Vision and Pattern Recognition Workshops*, vol. 2017–July, pp. 1783–1794, 2017.

[14] R. Fergus, B. Singh, A. Hertzmann, S. T. Roweis, and W. T. Freeman, “Removing camera shake from a single photograph,” *ACM Trans. Graph.*, vol. 25, pp. 787–794, July 2006.

[15] M. Bertero, P. Boccacci, G. Desidera, and G. Vicidomini, “Image deblurring with poisson data: from cells to galaxies,” *Inverse Problems*, vol. 25, no. 12, 2009.

[16] T. Chan, B. Amizic, R. Molina, S. D. Babacan, T. Bishop, and A. Katsaggelos, “Blind image deconvolution,” in *Blind Image Deconvolution*, pp. 1–41, CRC Press, May 2007.

[17] S. Tiwari, V. Shukla, A. Singh, and S. Biradar, “Review of motion blur estimation techniques,” *Journal of Image and Graphics*, vol. 1, no. 4, pp. 176–184, 2013.

[18] A. Levin, Y. Weiss, F. Durand, and W. Freeman, “Understanding blind deconvolution algorithms,” *IEEE Transactions on Pattern Analysis and Machine Intelligence*, vol. 33, no. 12, pp. 2354–2367, 2011.

[19] R. Nelwin and S. Athira, “Adaptive blind deconvolution and denoising of motion blurred images,” in *2016 IEEE International Conference on Recent Trends in Electronics, Information and Communication Technology*, RTEICT 2016 - Proceedings, pp. 1171–1175, 2017.

[20] T. Chan and C.-K. Wong, “Total variation blind deconvolution,” *IEEE Transactions on Image Processing*, vol. 7, no. 3, pp. 370–375, 1998.

[21] Y. Liu and S. Hu, “Blind restoration based on detection and segmentation of motion blurred image,” *Xi Tong Gong Cheng Yu Dian Zi Ji Shu/Systems Engineering and Electronics*, vol. 39, no. 3, pp. 662–667, 2017.

[22] H. Sun, M. Desvignes, and Y. Yan, “Motion blur adaptive identification from natural image model,” in *2009 16th IEEE International Conference on Image Processing (ICIP)*, pp. 137–140, Nov 2009.

[23] X. Feng and B. Thierry, “Sure-based motion blur estimation,” pp. 373–377, 2012.

[24] A. Stern and N. S. Kopeika, “Analytical method to calculate optical transfer functions for image motion and vibrations using moments,” *J. Opt. Soc. Am. A*, vol. 14, pp. 388–396, Feb 1997.

[25] S. Turrisi, E. Zappa, and A. Lavatelli, “Image deconvolution techniques for motion blur compensation in DIC measurements,” in *2018 IEEE International Instrumentation and Measurement Technology Conference (I2MTC) (I2MTC 2018)*, (Houston, USA), May 2018.

[26] N. Wiener, *Extrapolation, interpolation and smoothing of stationary time series*. The MIT Press, 1st M.I.T. paperback ed., 1964.

[27] T. Kailath, A. Sayed, and B. Hassibi, *Linear Estimation*. Prentice Hall Information and, Prentice Hall, 2000.

[28] J. Goodman, *Introduction to Fourier Optics*. Roberts & Company, 2006.

[29] M. Potmesil and I. Chakravarty, “Modeling motion blur in computer-generated images,” in *Proceedings of the 10th Annual Conference on Computer Graphics and Interactive Techniques*, SIGGRAPH ’83, (New York, NY, USA), pp. 389–399, ACM, 1983.

[30] P. Bing, Q. Kemao, X. Huimin, and A. Anand, “On errors of digital image correlation due to speckle patterns,” vol. 7375, pp. 7375 – 7375 – 7, 2009.

[31] R. Balcaen, P. Reu, P. Lava, and D. Debruyne, “Stereo-di c uncertainty quantification based on simulated images,” *Experimental Mechanics*, vol. 57, pp. 939–951, Jul 2017.

[32] M. Bornert, F. Brémand, P. Doumalin, J.-C. Dupré, M. Fazzini, M. Grédiac, P. Hild, S. Mistou, J. Molimard, J.-J. Orteu, L. Robert, Y. Surrel, P. Vacher, and B. Wattrisse, “Assessment of digital image correlation measurement errors: Methodology and results,” *Experimental Mechanics*, vol. 49, no. 3, pp. 353–370, 2009.

[33] J. Blaber, B. Adair, and A. Antoniou, “Ncorr: Open-source 2d digital image correlation matlab software,” *Experimental Mechanics*, vol. 55, pp. 1105–1122, Jul 2015.