Study on Influence of Micro-tool Tip Arc Waviness on Cutting Micro Aspheric Surface

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Abstract. In this paper, the influence of the waviness of tip arc profile on the surface quality of micro transition arc workpiece is studied. Firstly, the contour curve of the surface of the micro tool was measured by using the radius amplitude measuring instrument, and the characteristic parameters of the waviness in the contour curve were found. A simulation study of the effect of feed rate on the quality of the machined surface of a sine wave model with different characteristics was performed by Matlab. It can be known from the simulation results that when a sine wave of 0.05\sin(10t) exists in the contour of the tool tip arc, when the feed rate is small, the impact on the surface accuracy of the workpiece is greater, and a sine wave of 0.02\sin(30t) locally affects the regional surface roughness is greater. With the increase of feed rate, a sine wave of 0.02\sin(30t) has no effect on the surface roughness of the partial area, while a sine wave of 0.05\sin(10t) still has a greater effect on the surface roughness of the partial area.

1. Introduction
Micro aspheric element has been widely used in optics, aerospace, and laser fusion experiments [1-2]. The surface roughness of nanometer scale and the surface shape precision of submicron scale can be obtained by means of ultra-precision turning. However, when machining some aspheric array microstructures or modulation structures, it is necessary to select micro-arc single-point diamond turning tool for cutting to avoid interference. Micro-arc tool refers to the tip of the arc radius of 0.1–0.001mm (even smaller) of the micro tool, at present micro-arc tool tip of the arc waviness can achieve less than 0.1\mu m accuracy with long production cycle and so many production difficulties. Therefore, Xia et al proposed to obtain a high-precision micro-arc tool by mechanically grinding the tool with a large radius of arc at the tip of the tool, and finally obtained a high-precision micro-arc tool with R8.2\mu m and a waviness less than 0.1\mu m by grinding the arc tool with R0.2mm [3]. In the cutting process, if the cutting position of the tool always keeps the curvature radius unchanged, the workpiece will obtain excellent surface shape accuracy, while in the cutting process, the profile error of the tool tip arc will "copy" on the workpiece surface, thus affecting the surface quality of the processed workpiece [4]. Therefore, the accurate measurement of the profile of the micro arc tool is very important for the error compensation of the actual machining. It is a mature and universal method in industrial production to measure the radius of the tool tip by optical microscope.
and obtain the roundness error and waviness error of the tool tip by processing the image data [5]. He C et al. established a mathematical model for predicting the surface contour of turning plane workpiece by measuring the profile of tool tip and combining machining parameters and material parameters of the workpiece, verified the correctness of the model through experiments, and analyzed the influence of tool profile error on the surface quality of processing workpiece [6-7]. Zhang R used sinusoidal wave to simulate the waviness of tool tip arc to theoretically deduce the influence of waviness on the surface shape accuracy of machining workpiece, and experimentally studied the influence of different tool waviness on the surface shape accuracy of convex circular workpiece with machining radius of 50mm and diameter of 45mm [8].

At present, there are few reports about the influence of tool waviness on the surface quality of the workpiece with small arc shape. In this paper, the characteristic parameters of the waviness in the profile curve of micro arc tool are measured, and the influence of the waviness of the profile of micro arc tool tip on the surface shape accuracy and surface roughness of micro aspheric workpiece is studied by the simulation of Matlab.

2. Numerical simulation model
As shown in figure 1, the profile error curve of tool tip arc measured by the radius amplitude measuring instrument. In this paper, the sine wave (It can be expressed like this: $A \sin(2\pi \lambda \cdot t)$) is used to simulate the ripple existing in the tool profile, which is the wavelength of the whole period of the sine wave. Through the analysis of the corrugation error curve in figure 1, there are two kinds of typical corrugations. One is the wavelength $\lambda = 2\pi / 10$, the amplitude is 0.05μm, it is a medium frequency and large amplitude waviness, the other is the wavelength $\lambda = 2\pi / 30$, the amplitude is 0.02μm, it is a medium frequency and small amplitude waviness, and the sine wave can be expressed as respectively 0.05sin(10t) and 0.02sin(30t).

As shown in figure 2, it is the theoretical derivation diagram of micro transition arc turning. In the turning process, only the tool arc profile "copy" on the workpiece to generate the workpiece surface profile, without considering other factors. In the figure 2, the blue line segment represents the surface profile generated by the ideal tool turning, and the red line segment represents the surface profile generated by the turning when there is a certain waviness on the arc profile of the tool. The rectangular coordinate system is established with the center of the workpiece arc as the origin, R1 as the radius of the workpiece arc, the cutter feeds along the workpiece arc contour and O1, O2, O3, O4...It is the center of the tool tip arc for each feeding of the tool, a is the length of half of the center distance, Rt is the radius of the tool tip arc, and α is the rotation angle of the tool for each feeding relative to the workpiece center.

![Figure 1](image-url)

Figure 1. Profile of tool cutting edge and tool edge waviness.
Figure 2. Schematic diagram of theoretical derivation.

\[ \alpha = \frac{f}{2\pi (R1 + Rt)} \times 360^\circ \]  
where the \( f \) is feed rate.

\[ a = 2(R1 + Rt)\sin \frac{\alpha}{2} \approx \frac{f}{2} \]  

So in this coordinate system, the center of tool arc \( (X_i, Y_i) \) can be expressed as follows:

\[
\begin{align*}
X_i &= (R1 + Rt)\sin (i\alpha) \\
Y_i &= (R1 + Rt)\cos (i\alpha)
\end{align*}
\]  

Where \( i \) is the natural number starting from zero.

\[ \gamma = \arccos \frac{f}{2Rt} \]  

\[ \beta = 90^\circ - \frac{\alpha}{2} - \gamma \]  

It can be seen from the geometric analysis that the contour of the ideal tool "copy" on the workpiece surface is the same radian \( \theta \) every time it is feed, and \( \beta \) is the radian corresponding to half of the contour. So \( \theta \) can be expressed as follows:

\[ \theta = 2\beta = 180^\circ - \frac{f}{2\pi (R1 + Rt)} \times 360^\circ - 2\arccos \frac{f}{2Rt} \]  

Under the rectangular coordinate system established with the workpiece center as the origin, if
the second coordinate system is established with the cutter center, the cutter outline of each section "copy" on the workpiece surface shall be in the third and fourth quadrants of the second coordinate system, then the value range of $\theta_i$ shall be:

$$\theta_i = \left(270^\circ - i\alpha - \beta\right) - \left(270^\circ - i\alpha + \beta\right)$$  \hspace{1cm} \text{(7)}$$

The contour of each feeding tool "copy" on the workpiece can be expressed as

$$\begin{align*}
x_i &= (R_1 + R_t)\sin(i\alpha) + R_t\cdot \cos\theta_i \\
y_i &= (R_1 + R_t)\cos(i\alpha) + R_t\cdot \sin\theta_i
\end{align*}$$  \hspace{1cm} \text{(8)}$$

As shown in figure 2, the blue mark points are the intersection point of the "copy" contour of the ideal tool in two feeds, and the red mark points are the intersection point of the "copy" contour in two feeds when the tool has certain corrugations. The blue mark point will be regularly displayed in the arc position with radius $R$, and the value expression of $R$ is equation.9, while the red mark point will not be regularly displayed each time.

$$R = \sqrt{(R_t + R_1)^2 - \left(\frac{f}{2}\right)^2} - \sqrt{R_t^2 - \left(\frac{f}{2}\right)^2}$$  \hspace{1cm} \text{(9)}$$

The coordinates of the blue mark point $(R_x, R_y)$ can be expressed as

$$\begin{align*}
R_x &= \sqrt{(R_t + R_1)^2 - \left(\frac{f}{2}\right)^2} - \sqrt{R_t^2 - \left(\frac{f}{2}\right)^2} \cdot \sin\left(i\alpha - \frac{\alpha}{2}\right) \\
R_y &= \sqrt{(R_t + R_1)^2 - \left(\frac{f}{2}\right)^2} - \sqrt{R_t^2 - \left(\frac{f}{2}\right)^2} \cdot \cos\left(i\alpha - \frac{\alpha}{2}\right)
\end{align*}$$  \hspace{1cm} \text{(10)}$$

The surface profile generated by the ideal tool cutting, and the blue mark point shall be the highest point of the generated surface profile. After the profile generated by the ideal tool is flattened along the arc radius, the PV value of the generated surface profile can be expressed as follows:

$$R_{th} = \sqrt{(R_t + R_1)^2 - \left(\frac{f}{2}\right)^2} - \sqrt{R_t^2 - \left(\frac{f}{2}\right)^2} - R_1$$  \hspace{1cm} \text{(11)}$$

Through the above derivation, each formula can be input into Matlab for numerical calculation. Since the calculated values in Matlab are all discrete points, the intersection points in Matlab need to be obtained by solving the equation to get the minimum value, then the value of $\theta_i$ increased appropriately $\phi$, and the value of $\phi$ only needs to ensure that the simulation can calculate the intersection point of the contour left by the two feed "copy". $\phi_i$ can be expressed as follows:

$$\phi_i = \left(270^\circ - i\alpha - \beta - \phi\right) - \left(270^\circ - i\alpha + \beta + \phi\right)$$  \hspace{1cm} \text{(12)}$$

When there is a sine wave on the tool profile, the coordinate $(x_{i}, y_{i})$ value of part of the tool profile in Matlab can be expressed as follows:
where the $\varepsilon$ is error factor.

According to the criterion of Equation.14, two groups of coordinate points which satisfy the error factor $\varepsilon$ in the profile of two feed tools can be obtained, they are regarded as intersection points. In order to ensure the calculation efficiency $\varepsilon$ is taken as 0.0005 $\mu$m, the coordinate value between the intersection points is the surface profile of the workpiece obtained by "copy" each feed tool on the workpiece.

Equation.13 is used as the unified expression formula of tool profile in Matlab program for simulation calculation, and the unit is $\mu$m. When $R_1 = 1000$, $R_t = 10$, $f = 5$, $A = 0$, the simulation results are shown in figure 3(a) after the removal of the workpiece radius from the contour surface in the range of $i\alpha = 20$-70 degrees and the angle flattening, and the simulation results of partial $44$-$45$ degrees are shown in figure 3(b).

![Figure 3. Simulation results of ideal tool.](image)

The surface roughness $Ra$ is used to evaluate the surface quality. Since the data calculated by Matlab are all discrete points, the calculation formula of surface roughness is expressed as follows:

$$Ra = \frac{1}{n} \sum_{n=1}^{n} |Z_n|$$

(15)

According to the theoretical derivation, the surface roughness and PV value of the simulation results of the ideal tool in a large area range of 20-70 degrees are consistent with those of the simulation results of the partial area. The surface roughness $Ra$ of partial area is 80.6nm, and the PV value of partial area is 318.87nm

3. Results and discussion

3.1. Influence of medium frequency and small amplitude waviness on workpiece surface

When there is a sine wave with an amplitude of 0.02 $\mu$m and a wavelength $\lambda = 2\pi/30$, that can be expressed as $0.02\sin(30t)$, and the radius of the tool tip arc are 10 $\mu$m, the simulation is carried out at different feed rates of 2, 3, 4 and 5 $\mu$m/r. The simulation results obtained from the range of $i\alpha = 30$-50 degrees are shown in figure 4.
Figure 4. Simulation results of tool with 0.02\sin(30t) under different feed rates.

As shown in figure 4, the radius of the tool tip arc is 10\,\mu m, and the feed rate is 2, 3, 4 and 5\,\mu m/r respectively. The simulation results are obtained. It can be seen from the figure 4 that with the increase of the feed rate, the trend of the upper edge curve is in the range of 33-45 degrees, from the trend of first increasing and then decreasing when the feed rate is 2\,\mu m/r to the trend of first decreasing and then increasing when the feed rate is 4\,\mu m/r to the trend of flattening when the feed rate is 5\,\mu m/r.

The main reason for the change trend of the contour curve of the upper surface is that with the increase of the feed rate, the depth of the tool contour under the "copy" of the single feed on the workpiece increases, as shown in figure 5. As shown in Figure 5(a)(b) are the comparison between the contour generated by the partial area and the contour generated by the ideal tool when the feed rate is 2\,\mu m/r, and figure 5(c)(d) are 5\,\mu m/r respectively. When the feed rate is 2\,\mu m/r, the depth of the contour curve generated by the minimum partial area is obviously smaller than that generated by the ideal tool, while that generated by the maximum partial area is significantly smaller than that generated by the wheel. When the feed rate is 5\,\mu m/r, it can be seen that the contour generated by the partial area almost coincides with the contour generated by the ideal tool, and the depth difference is very small. This shows that with the increase of the feed rate, the influence of the corrugation on the surface roughness is smaller and smaller, so the curve trend of the upper surface profile tends to be flat. Since the contour curve of the lower surface always keeps decreasing first and then increasing, and the values of discrete points in the curve will not change, the difference between the PV value of the contour formed in the partial area and the PV value of the contour formed by the ideal tool will be smaller and smaller with the increase of the feed rate, as shown in figure 6.

As shown in figure 7, the suffixes of Max and min respectively represent the maximum and minimum surface roughness values of partial areas, the suffixes of whole curve represent the surface roughness values within the range of 30-50 degrees, and the suffixes of ideal represent the surface roughness values of ideal tools. It can be seen from the figure 7 that when the feed rate is less than 4\,\mu m/r, the surface roughness values of whole > Max > ideal > min, and the whole roughness values are greater than others. The reason is that when the feed rate is too small, the surface profile of the workpiece is greatly affected by the amplitude of sine wave on the tool, so it
will lead to the calculation of the roughness too large. When the feed rate is greater than or equal to 4μm/r the whole ≈ Max ≈ ideal ≈ min, which can be seen from Figure 8, at this time, the difference of contour PV value obtained is very small, and the influence brought by sine wave on the tool is very small, so when calculating the roughness, the difference between them will be very small.

Through the above analysis, it can be seen that when there is a sine wave with an amplitude of 0.02μm and a wavelength $\lambda=2\pi/30$ on the tool, such a sine wave will have a greater impact on the surface shape accuracy of the generated workpiece surface when the feed rate is small, and at the same time, it will affect the surface profile generated in the partial area. In some areas, the surface quality can be better or even better than that of the ideal tool, but in some areas, the surface quality of the tool is relatively poor. When the feed rate is large, the influence of such sine wave on the surface roughness of workpiece is small.

### 3.2. Influence of medium frequency and large amplitude waviness on workpiece surface

When there is a sine wave with an amplitude of 0.05μm and a wavelength $\lambda=2\pi/10$, that can be expressed as $0.05\sin(10t)$, and the radius of the tool tip arc are 10μm, the simulation is carried out at different feed rates of 2, 3, 4 and 5μm/r. The simulation results obtained from the range of $i\alpha=20$-70 degrees are shown in figure 8.
Figure 8. Simulation results of tool with 0.02sin(30t) under different feed rates.

As shown in figure 8, it can be seen that the trend of the upper edge curve in the range of 27-63 degrees decreases first and then increases with the increase of feed rate at 2μm/r, increases first and then decreases at 3μm/r, increases first and then decreases at 4μm/r, and decreases at 5μm/r, but the slope increases. The lower edge curve is formed at the lowest point of each feed, and these points are approximately equal to the value of the corresponding cutting angle on the sine wave 0.05sin (10t). Therefore, the main reason for the change of the trend of the upper edge curve is that with the increase of the feed rate, the depth of the “copy” contour on the workpiece surface will be different in different areas due to the change of the cutter contour participating in the cutting position in the partial area, and there is a maximum and minimum value of the contour depth in the partial area, so PV is carried out for the maximum and minimum value areas of 44.6-45.6 degrees and 26.6-27.6 degrees respectively Calculation of values and surface roughness.

As shown in figure 9, that is the comparison of PV values in different regions. The suffix is ideal, which indicates the PV value of the workpiece surface profile generated by the ideal tool, and the whole, which indicates the PV value in the range of 20-70 degrees. It can be seen in the figure 9 that there is always the maximum and minimum value of the partial area, and there is a certain difference. At the same time, it can be seen that with the increase of the feed rate, the maximum value of PV value of the partial area is the PV value of the whole area. The difference between the maximum value of PV value in the partial area and the contour PV value formed by the ideal tool is from + 42%, + 41%, 37.5% to + 27.5%; the difference between the minimum value of PV value in the partial area and the contour PV value formed by the ideal tool is from - 48%, - 50%, - 35%, to - 29%; the difference between the PV value of the contour formed by the overall area and the ideal tool is from + 146%, + 49%, + 37.5 to + 27.5%. The percentage of difference decreases with the increase of the feed rate.

As shown in figure 10, that is the comparison of surface roughness in different areas, and the suffix of the curve in the figure is the same as that in figure 9. It can be seen from the figure 10 that when the feed rate is 2μm/r, the surface roughness is whole > Max > ideal > min, when the feed rate is 3μm/r, the surface roughness is Max > whole > ideal > min, and when the feed rate is 3μm/r, the surface roughness is Max > whole ≈ ideal > min.
Four kinds of corrugations with typical characteristics on the corrugation error curve are simulated and studied. The two corrugation parameters are amplitude 0.02, wavelength $\lambda=2\pi/10$, and another is amplitude 0.05, wavelength $\lambda=2\pi/30$, they can be expressed as $0.02\sin(30t)$ and $0.05\sin(10t)$ respectively. Through the theoretical formula derivation, and then the derivation process and formula using Matlab software to write a simulation program for numerical simulation calculation. It can be seen from the simulation results that when there is a sine wave of $0.05\sin(10t)$ in the arc profile of the tool tip, the influence on the surface accuracy of the workpiece is greater when the feed rate is smaller, while the influence of a sine wave of $0.02\sin(30t)$ on the surface roughness of the partial area of the workpiece surface is greater. With the increase of feed rate, a sine wave of $0.02\sin(30t)$ has no effect on the surface roughness of partial area, while a sine wave of $0.05\sin(10t)$ still has great effect on the surface roughness of partial area.

4. Conclusion

In this paper, the sine wave is used to simulate the corrugation error in the circular arc profile of the micro tool tip. Two kinds of corrugations with typical characteristics on the corrugation error curve are simulated and studied. The two corrugation parameters are amplitude 0.02, wavelength $\lambda=2\pi/10$, and the other is amplitude 0.05, wavelength $\lambda=2\pi/30$, they can be expressed as $0.02\sin(30t)$ and $0.05\sin(10t)$ respectively. Through the theoretical formula derivation, and then the derivation process and formula using Matlab software to write a simulation program for numerical simulation calculation. It can be seen from the simulation results that when there is a sine wave of $0.05\sin(10t)$ in the arc profile of the tool tip, the influence on the surface accuracy of the workpiece is greater when the feed rate is smaller, while the influence of a sine wave of $0.02\sin(30t)$ on the surface roughness of the partial area of the workpiece surface is greater. With the increase of feed rate, a sine wave of $0.02\sin(30t)$ has no effect on the surface roughness of partial area, while a sine wave of $0.05\sin(10t)$ still has great effect on the surface roughness of partial area.

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