Multi-stability and condensation of exciton-polaritons below threshold

Jiun-Yi Lien,1,2 Yueh-Nan Chen,1,* Natsuko Ishida,3 Hong-Bin Chen,1 Chi-Chuan Hwang,2 and Franco Nori3,4

1Department of Physics and National Center for Theoretical Sciences, National Cheng Kung University, Tainan 701, Taiwan
2Department of Engineering Science and Supercomputing Research Center, National Cheng Kung University, Tainan 701, Taiwan
3CEMS, RIKEN, Wako-shi, Saitama 351-0198, Japan
4Physics Department, University of Michigan, Ann Arbor, MI 48104-4313, USA
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Exciton-polaritons can condense to a macroscopic quantum state through a non-equilibrium process of pumping and decay. In recent experiments, polariton condensates are used to observe, for a short time, nonlinear Josephson phenomena by coupling two condensates. However, it is still not clear how these phenomena are affected by the pumping and decay at long times and how the coupling alters the polariton condensation. Here, we consider a polariton Josephson junction pumped on one side and study its dynamics within a mean-field theory. The Josephson current is found to give rise to multi-stability of the stationary states, which are sensitive to the initial conditions and incoherent noises. These states can be attributed to either the self-trapping effect or the parity-time (PT) symmetry of the system. These results can be used to explain the emission spectra and the \( \pi \)-phase locking observed in recent experiments. We further predict that the multi-stability can reduce to the self-trapped state if the PT symmetry is broken. Moreover, the polaritons can condense even below the threshold, exhibiting hysteresis.

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Exciton-polaritons, i.e., quasi-particles composed of cavity photons and quantum-well excitons in semiconducting microcavities, have recently been demonstrated to form a Bose-Einstein condensate (BEC) due to their light effective mass originating from the photonic part [1–5]. The exciton-polariton BEC can be used to investigate macroscopic quantum effects in semiconducting systems, such as superfluidity [6] and quantized vortices [7, 8]. In a polariton bosonic Josephson junction (BJJ) consisting of two adjacent microcavities, the coupling between two exciton-polariton condensates (through photonic tunneling) can induce Rabi oscillations, a.c. and d.c. Josephson phenomena, or macroscopic quantum self-trapping [9]. These phenomena have been demonstrated recently in CdTe microcavities with disordered double wells [10] and semiconductor micropillars [11]. Furthermore, the coupled microcavity array can also allow controlling the phase transition between the superfluid and the Mott insulator in Bose-Hubbard systems [12–15].

Since the lifetime of exciton-polaritons is about tens to hundreds of picoseconds, an optical pumping is necessary to compensate both the radiative and non-radiative decay of the polaritons. Therefore, the condensates are formed in such a non-equilibrium process. At low temperatures, one can use a non-resonant pumping to excite higher-energy excitons, which rapidly relax to form an incoherent polariton reservoir at the bottleneck of the lower-energy polariton band through the exciton-phonon interactions. The polaritons are then stimulatedly scattered (or cooled) into the ground state to achieve condensation with spontaneous coherence, provided that the incoherent polariton density reaches the threshold density [1, 16]. The polariton reservoir drastically changes the Bogoliubov dispersion of the elementary excitations for condensates with conserved particle numbers, leading to a diffusive behavior in the long wavelength regime and the dynamical instability for the reservoir lifetime being comparable to the condensate [17, 18].

While the dynamics of the polariton BJJ for different regimes can be accessed by a pulsed resonant excitation that gives the appropriate initial conditions [11], the short-time behavior of the polariton BJJ is also affected by the reservoir, e.g. the spontaneous coherent oscillations driven by a non-resonant pumping covering the double well [10]. In addition to these short-time oscillations, the non-equilibrium process would also alter the behavior at long times. Intuitively, after the oscillations fade out, the BJJ should reach its stationary states, e.g., bonding or anti-bonding states, for which the corresponding eigen-energies are dependent on the particle numbers of each side owing to the nonlinearity. The pump-decay mechanism leads to a non-Hermitian Hamiltonian, and the states with complex eigenvalues will disappear. Unlike a single condensate for which the spectrum is real, with the stimulated scattering equal to the decay rate, the condition is not necessary for a BJJ because of the Josephson current. It has been analyzed [19] that under equivalent pumping on both sides, the synchronization of the condensates with a single eigen-energy can be destroyed by the potential difference across the junction. A similar synchronization-desynchronization phase transition is also observed [20] in microcavities with in-plane disorder.

In this work, we consider a different situation: a po-
where a pumping laser with small enough spot size has been realized in recent experiments using micropillars [11, 21]. The condensates are always synchronized since the polaritons in the unpumped side are injected from the pumped side by the Josephson current. We will present the multiple stability induced by injected from the pumped side by the Josephson current, as well as its effect on the threshold. We will also find the possibility of condensation even below the threshold. Furthermore, we also find the possibility of condensation even below the threshold.

Our analysis is based on the generalized Gross-Pitaevskii equation (GPE) of the BJJ wave functions \( \Psi \) of condensation even below the threshold. We can ignore the non-condensed solution becomes unstable. For \( J = 0 \), the threshold reduces to the single-BEC case with.

reservoir polaritons \( N_{Rj}(t) \) and is given by

\[
V_j(N_{Rj}) \equiv \tilde{g} / A_j N_{Rj} + i / 2 (R_j(N_{Rj}) - \gamma_j)
\]

where \( \tilde{g} / A_j \) is the interaction between the condensate and reservoir polaritons, \( A_j \) is approximately the distribution area of the reservoir, and \( \gamma_j \) is the decay of the condensates. The term \( R_j(N_{Rj}) \) is the stimulated scattering from the reservoir to the condensate, and, for simplicity, we only consider it as a linear function, i.e. \( R_j(N_{Rj}) \equiv R_j' N_{Rj} \). The rate equation of the reservoir on the pumped site is given by

\[
\frac{d}{dt} N_{R1} = P_1 - \gamma_{R1} N_{R1} - R_1(N_{R1})|\Psi_1|^2,
\]

where the reservoir decay \( \gamma_{R1} \) and scattering loss are balanced by the laser pumping \( P_1 \). We can ignore \( N_{R2} \) due to the weak diffusion of the reservoir polaritons [18]. For the double well or micropillars with diameters about the order of \( \mu \)m in the experiments, the strength of the term \( \tilde{g}/A_j \) is much weaker than the charging energy and can be ignored.

A trivial solution is that no polaritons are condensed and the reservoir-polariton number is proportional to the pumping from (4). By linearizing Eq. (1), the fluctuation spectrum can be derived as

\[
\omega_{\pm}^{(0)} = \frac{1}{2} \left( E_1^{(0)} + E_2^{(0)} \right) \pm \frac{1}{2} \sqrt{\left( E_1^{(0)} - E_2^{(0)} \right)^2 + 4J^2},
\]

where \( E_j^{(0)} \) is given by Eq. (2) with \( \Psi = 0 \). As shown in Fig. 2, the threshold pumping \( P_{th} = \gamma_{R1} N_{R1h} \) can be determined by the first point of \( \text{Im}[^{\omega_{\pm}^{(0)}}] = 0 \), where the non-condensed solution becomes unstable. For \( J = 0 \), the threshold reduces to the single-BEC case with.
$R_1(N_{R1}) = \gamma_1$. It increases with $J$ until $N_{R1}$ reaches a saturation point with $R_1(N_{R1}) = \gamma_1 + \gamma_2$.

In general, the spectrum of the non-Hermitian Hamiltonian $H$ is a complex function of the polariton numbers of the condensates and the reservoir. To solve the nonzero stationary states, $\Psi(t) = \Psi(0)e^{-i\Omega t}$, and $N_{R1}$, we have to search for the real spectrum, i.e.,

$$\Omega_\pm = \frac{1}{2} (E_1 + E_2) \pm \frac{1}{2} \sqrt{(E_1 - E_2)^2 + 4 J^2} \in \mathcal{R}.$$  \hspace{1cm} (6)

The stationary states pumped from one side must possess a finite d.c. Josephson current $2J\sqrt{N_{c1}N_{c2}}\sin(\Delta\varphi)$ to balance the loss from the other side, and the relative phase $\Delta\varphi \equiv \varphi_2 - \varphi_1$ across the BJJ (with respect to $\Omega_-$ and $\Omega_+$) deviates from 0 and $\pi$, corresponding to the usual bonding and anti-bonding states with zero current.

Two analytic solutions can be derived from $R_1(N_{R1}) = \gamma_1 + \gamma_2$; one corresponds to $\Omega_-$, and the other to $\Omega_+$ (see yellow and green curves in Fig. 3). In this case, the Hamiltonian possesses parity-time (PT) symmetry, i.e. $E_1 = E_2^*$, where the injection of the condensate polaritons at site-1, $R_1(N_{R1}) = \gamma_1$, is equal to the decay at site-2 [22, 23]. These stationary states only exist with a real spectrum under the condition

$$J^2 \geq \frac{1}{16} [R_1(N_{R1}) - \gamma_1 + \gamma_2]^2 = \frac{\gamma_2^2}{4}.$$  \hspace{1cm} (7)

Violating this condition by increasing the decay or decreasing the tunneling makes the spectrum complex and leads to spontaneous PT-symmetry breaking [23–25]. The equal sign of Eq. (7) gives the exceptional point, where these two states coalesce.

For both PT-symmetric states, the condensate polaritons are equally populated across the junction,

$$|\Psi_1|^2 = |\Psi_2|^2 = \frac{P_1 - \gamma_{R1}N_{R1}}{R_1(N_{R1})}.$$  \hspace{1cm} (8)

By coincidence, these states are created above the threshold because the reservoir polaritons have to be pumped to the condition $N_{R1} = N_{R1h}$. The reservoir-polariton number is kept constant above the threshold and the total condensate-polariton number ($N_{cT} \equiv N_{c1} + N_{c2}$) increases linearly with the pumping [Fig. 3(b, c)].

In general, there exist two other solutions, both corresponding to $\Omega_+$, with the imbalanced population of the condensate polaritons $\zeta \equiv (N_{c1} - N_{c2})/N_{cT} \neq 0$ [Fig. 3(d)]. They are obtained by numerically finding the roots of Im($\Omega$) = 0. One solution is localized in site-1 (the blue curves in Fig. 3), and this localization is due to the same mechanism of macroscopic quantum self-trapping for short-time oscillations [9], where the interaction between the condensate polaritons shifts the energy difference across the BJJ and reduces the Josephson current. Therefore, increasing the pumping or decreasing the junction tunneling enhances the self-trapping effect. The other solution is more populated in site-2 with weaker condensation (the red curves in Fig. 3) and the self-trapping effect is limited. Increasing the pumping will drive more polaritons tunneling to the unpumped site and further reduce the condensation. Thus, the solution reduces to the zero-condensate state above a critical pumping. Interestingly, the imbalanced states appear even below the threshold. We will show later that the polariton BJJ exhibits bistability below the threshold.

In order to determine which states can be observed experimentally, the stability is analyzed by calculating the complex spectrum of deviation from stationary states, with details given in Appendix A. Figure 3(e) shows the maximum imaginary part of the fluctuation spectrum. One of the PT-symmetric states $\Omega_+$ is always stable and the other $\Omega_-$ becomes unstable when increasing $P_1$. As
for the imbalanced states, the self-trapped state is stable except for a small region below the threshold, and the other one (untrapped state) is generally unstable unless it is close to the PT state with $\Omega_+$. This leads to multi-stability of the condensation resulting from the polaron tunneling.

Below threshold, there exists a bistable regime with a self-trapped condensed state and a non-condensed state. The bistability can be explained by damped oscillations of the polaron BJJ. The self-trapping oscillations occur if the initial polaron number is larger than a critical value $[\bar{U}N_{cT}(0)/2J > \Lambda_{c}]$, with a suitable range of the initial imbalance $\zeta(0)$ [9], and it is eventually damped to the imbalanced equilibrium position $\zeta(t \to \infty) > 0$. Otherwise, the oscillations are damped to a balanced state with $\zeta(t \to \infty) = 0$, and from the condition $N_{cT} = 0$ in Appendix A (A1) the condensation must vanish unless the threshold is reached. Owing to the initial-value dependence, the bistability can be experimentally observed through the hysteresis of condensation, by cyclically increasing and decreasing the pumping near the threshold.

Above the threshold, the multiple stable states are also determined by the initial values. When the self-trapping condition is not satisfied, the condensates evolve to other stable states. In conserved BJJs [9], for $|\Delta \varphi(0)| \leq \pi/2$, the self trapping occurs in the running-phase modes provided $\zeta(0) \geq \zeta_c$, while for $|\Delta \varphi(0)| > \pi/2$, it could be either in the running-phase modes or the $\pi$-phase modes, depending on both $N_{cT}(0)$ and $\zeta(0)$. The suitable range of $\zeta(0)$ is altered by the reservoir and the decay, and a large initial imbalance generally induces the self trapping. We have shown that the self-trapped state must be antibonding-like with a phase difference close to $\pi$, even if the initial phase is zero [Fig. 3(f)]. This can be understood by the running phase being damped to $(2n + 1)\pi$ and eventually reaching the antibonding-like stationary state. One notes that the damped running phase and $\pi$-phase locking have been observed in a recent experiment [11]. This phenomenon seems to be different from our case because the eigen-energy is never real without pumping, i.e., no stationary condensate is achieved. We also derive an oscillator model to explain the $\pi$-phase locking either with or without the stationary condensation (see Appendix B).

Our results can give further insight on the micropillar experiment [21], where several states with different energies are found above the threshold in the spectrally-resolved emission distribution. This phenomenon can be attributed to the coexistence of multiple stable states resulting from incoherent initial conditions or noises. Although the interaction and scattering among these states are ignored in our simplified model, the signatures can be qualitatively captured by the multi-stability (Fig. 4). Close to the threshold, both the PT-symmetric states ($\Omega_{\pm}$) as well as the self-trapped state ($\Omega_+$) are stable, and the self-trapped state has a much stronger condensation with a higher energy than the other two states owing to the localization of the stimulated scattering. At high pumping, the anti-bonding symmetric state becomes unstable, and the emission spectrum is dominated by the self-trapped state and the bonding symmetric state.

It should be noted that the interaction between the reservoir and the condensate polaritons $(V^R_J)$ is ignored and the bottom states remain PT-symmetric. The reservoir polaritons are not accumulated much when increasing the pumping strength, due to the depletion by condensation. Thus, at high pumping, the contribution of the high-energy excitons which are inactive to the polaron condensation should be accounted for the effective potential difference [26]. The potential difference does not affect the self-trapping much but only makes the bonding state more localized to the unpumped site. Although the potential difference seems to assist the polaron scattering to the bottom states in the experiment, we argue that it should break the PT symmetry of $H$ above a critical value and induce a complex eigenenergy where the bonding state vanishes.

In summary, we have analyzed, within mean-field theory, the dynamics of a polaron Josephson junction pumped on one side. The Josephson current induces multiple stable states corresponding to different initial conditions. These states can be attributed to either the self-trapping effect resulting from the nonlinearity or the parity-time symmetry of the system, and the incoherent noises lead to the coexistence of these states. The results can be used to explain recent experiments. Moreover, we also predict the condensation and a hysteresis phenomenon below the threshold.

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Appendix A: Stability analysis

In order to analyze the stability of the stationary states, we do not employ the usual derivation of the elementary excitations by directly linearizing Eq. (1), which includes both the phase terms $\varphi_1$ and $\varphi_2$. This is because only the phase difference $\Delta \varphi \equiv \varphi_2 - \varphi_1$ is physically meaningful without considering the local dispersion. Instead, we derive the equations of motion of the relative phase $\Delta \varphi$ and the population imbalance, $\zeta \equiv (N_{c1} - N_{c2})/N_{cT}$, similar to the conserved BEC systems [9], and take into account the additional degrees of freedom, i.e. the total condensate-polariton number $N_{cT}$ and the reservoir-polariton number $N_{R1}$. The equations of motions are given by

$$\dot{\zeta} = V_{12}^I (1 - \zeta^2) - 2J \sqrt{1 - \zeta^2} \sin(\Delta \varphi)$$

$$\Delta \dot{\varphi} = \epsilon_{12} + V_{12}^R + \left( \frac{U_{12}}{2} + \bar{U} \zeta \right) N_{cT}$$

$$+ 2J \frac{\zeta}{\sqrt{1 - \zeta^2}} \cos(\Delta \varphi)$$

$$\dot{N}_{cT} = [2\bar{V}^I + V_{12}^R \zeta] N_{cT}$$

$$\dot{N}_{R1} = P_1 - \gamma_{R1} N_{R1} - R_1 (N_{R1}) N_{cT} \frac{1 + \zeta}{2},$$

where

$$\bar{U} \equiv \frac{U_1 + U_2}{2}$$

$$U_{12} \equiv U_1 - U_2$$

and similar definitions are applied to $V_j^{R/I}$ and $\epsilon_j$. By linearizing Eq. (A1), we can calculate the fluctuation spectrum and analyze the dynamical stability with respect to the stationary states.

Appendix B: $\pi$-phase locking

Figure 5 shows the $\pi$-phase locking of the self-trapped states in a non-equilibrium polariton BJJ by solving Eq. (A1), either with or without stationary condensation. The non-condensed case agrees well with the experiment [11]. Here, we derive a nonlinear dissipated-oscillator model from (A1) with, $\epsilon_{12} = V_{12}^R = U_{12} = 0$, in order to further understand the mechanism of this phenomenon. Under the self-trapping conditions, $\eta \equiv \sqrt{1 - \zeta^2(t)} \ll 1$ and $\zeta(t) \approx 1 - \eta > 0$, the system can reduce to a second-order differential equation

$$\Delta \ddot{\varphi}(t) \approx \bar{U} \dot{N}_{cT}(t) - 2J \left( \frac{1}{\eta} \right) \sin(\Delta \varphi(t)) \Delta \dot{\varphi}(t)$$

$$- 2J^2 \left( \frac{1}{\eta} \right) \sin(2\Delta \varphi(t)) + O(\eta)$$

corresponding to a pendulum with a position-dependent dissipation and a decaying driving force $\bar{U} \dot{N}_{cT}$ satisfying

$$\bar{U} \dot{N}_{cT} \approx [2\bar{V}^I + V_{12}^R (1 - \eta)] \bar{U} N_{cT}. $$

The angle of the pendulum is defined by $2\Delta \varphi(t)$, and thus the pendulum has minimal potential energy for $\Delta \varphi = n\pi$, with $n$ being either odd or even. However,
the angular velocity given by
\[
\Delta \dot{\varphi} = \bar{U}(1 - \eta)N_{\pi T} + 2J\left(\frac{1 - \eta}{\eta}\right) \cos(\Delta \varphi) \tag{B3}
\]

can be zero only for \(\Delta \varphi \in (\pi/2, 3\pi/2)\), because the first term in (B3) is non-negative. Hence the pendulum is stable for odd \(n\). If the initial angular velocity is small, the pendulum oscillates with small amplitude around the \(\pi\) phase. Otherwise, it moves with an increasing phase. The driving force is able to decelerate the running phase and to lock the \(\pi\) phase before \(\bar{U}N_{\pi T} \approx 0\), as long as \(N_{\pi T}(0)\) is large enough.

This model can unify the \(\pi\)-phase locking phenomenon for both the condensed and non-condensed cases. A main difference between these two cases is the population imbalance after the phase is locked. For the non-condensed case, \(N_{\pi T}\) decays to zero and the self-trapping (\(\eta \ll 1\)) no longer holds. However, the phase is still locked because the angular velocity has been decelerated to be small.

\(^*\) yuehnan@mail.ncku.edu.tw

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