A coherent mechanism of super-resolution by a dielectric microsphere and microcylinder

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January 2020

Abstract. One of the physical mechanisms which allow a microsphere or a microcylinder to operate as a magnifying superlens is unveiled. This mechanism is related to the efficient excitation of creeping waves at the curved interface by a closely positioned dipole oriented normally to the interface. The set of creeping waves after their ejection from the surface creates an imaging beam which is quite close to either a diffraction-free Bessel beam or a Mathieu beam. However, the subwavelength resolution enabled by creeping waves is possible only for a coherent illumination which needs to be asymmetric with respect to imaged objects.
1. Introduction

Nanoimaging of objects in real time – beyond scanning the substantial areas with strongly submicron tips connected to cantilevers – is a very important branch of nanophotonics. A lot of top-level studies has been done in this field recently and pioneering techniques were developed, such as stimulated emission depletion [1], awarded by the Nobel prize in chemistry (2014). Stimulated emission depletion is a technique combining the dark-field microscopy and the use of fluorescent labels. However, in spite of advantages of this method, there are applications, especially in the biomedicine, where fluorescent labels in the object area are prohibited (see e.g. in [2]). Label-free optical nanoimaging still evokes a keen interest, and so-called superlenses (see e.g. in [4]) are still a subject of an intensive research. Superlens is a passive device forming a remote image of the object subwavelength details that does not require a post-processing for visualization. In the present work, we concentrate on a technique which seems to be the most affordable and straightforward type of superlens - dielectric spherical or cylindrical microlens.

In work [3] it was experimentally revealed that a simple glass microsphere operates as a far-field magnifying superlens – a device which creates a far-field magnified image of an object with its subwavelength details detectable by a conventional microscope. The imaged area is rather small (several square microns) and centered by the optical axis of the microscope passing through the microsphere center. In practice, an object either represents a submicron pattern of the substrate itself or (in biomedical applications) a set of subwavelength scatterers such as viruses, proteins or small cells sandwiched between the substrate and the sphere. Even few square microns is an area much larger than the object field of a scanning near-field optical microscope (SNOM). Therefore, this superlens technique promises a much faster imaging of the whole substrate than the use of SNOM. In order to inspect a macroscopic area, one can move the sphere over the substrate and/or collect the partial images offered by arrayed microspheres. Therefore, the discovery of the superlens functionality of a microsphere in [3] was noticed by the scientific community and resulted in a series of articles developing this technique.

However, in order to properly exploit a novel technique, one needs to understand its physics. This physics has not been really understood up to now. Initially, authors of [3] assumed that this imaging is related with the phenomenon of so-called photonic nanojet (PNJ). PNJ is a narrow wave beam revealed experimentally in work [5] by the same team. The PNJ propagates into ambient from the rear side of a dielectric microcylinder or microsphere illuminated by a wave beam with sufficiently flat wavefront. The PNJ maintains a slightly subwavelength $((0.3-0.5)\lambda)$ effective width along a path that extends more than $2\lambda$ behind the dielectric microparticle (MP). Further, the PNJ diverges and transforms into a usual wave beam. PNJ is a non-resonant phenomenon – it appears for a wide range of the MP diameters. Further studies [6, 7, 8] have shown that the PNJ results from the constructive interference of cylindrical or spherical harmonics excited inside the MP by a plane wave. For the (relative to the ambient) refractive index of the MP $n < 2$ a whatever MP radius from $R = \lambda$ to $R = 20\lambda$ corresponds to a sufficient amount of spatial harmonics which experience the constructive interference at the rear extremity of the MP. The set of
harmonics summing up at this point in the range $R = (1 - 20)\lambda$ is sufficient so that the most part of the incident wave power would be concentrated around this point forming the PNJ. This concentration disappears for macroscopic particles ($kR \gg 100$). Then the amount of spatial harmonics essentially contributing into the transmitted wave beam turns huge and strongly exceeds the number of those summing up at the rear point. The studies also have shown that in the near vicinity of the rear edge point the package of evanescent waves is excited that grants to the waist of the wave beam a high intensity significantly exceeding that of the incident wave. This is why the effective beamwidth of a PNJ at the rear interface of the MP is so small (0.3$\lambda$ for glass microspheres in free space, 0.4$\lambda$ for microcylinders). Authors of [3] assumed – if plane waves excite the evanescent waves in a MP, the evanescent waves excited by subwavelength scatterers located near the MP should also convert into propagating waves. This conversion was assumed to be similar to the formation of the subwavelength magnified image by a metamaterial hyperlens [10, 11, 12, 13]. It worth noticing that the topic of hyperlenses was very hot in 2007–2012, but did not result in anything practical. Both magnification and spatial resolution granted by hyperlenses turned out to be fundamentally restricted on a quite modest level, whereas the fabrication costs were huge. Since dielectric MPs are rather inexpensive their hypothetic hyperlens functionality was very intriguing. However, further studies (see e.g. in [15, 14, 9, 17, 18, 16] etc.) have shown that a nearly located scatterer does not produce a PNJ. The role of evanescent waves in the far-field subwavelength imaging remained cryptic. In [14] a direct lateral resolution on the level $\delta = \lambda/6$ was complemented by the interferometric resolution $\delta = \lambda/10$ in the normal direction. Also, in this work it was shown via simulations that two dipoles located at the surface of a glass microsphere of the dimensionless radius $kR = 60$ with the gap $\delta = \lambda/6$ between them can be resolved (in simulations) if and only if they are excited with nearly opposite phases. It is impossible for two small scatterers illuminated by daylight or by a plane waves. However, these scatterers in [14] were resolved experimentally.

In work [9] an explanation of the magnifying superlens operation involving the PNJ was suggested. It was based on the reciprocity theorem. Really, a microscope can be used in one of two ways: (a) focusing light to a small spot in the focal plane, and (b) imaging the light emitted by a small sample. Due to the reciprocity theorem, these two cases should be equivalent to each other. If a broad wave beam creates a PNJ with a subwavelength waist a rear point of the microsphere, a subwavelength emitter or scatterer located on the surface of the sphere should form the broad wave beam exactly in the diametral direction. Thus two point-wise scatterers on the sphere surface separated by a subwavelength gap should form two wave beams in their diametral directions which create two independent images behind the lens. This explanation did not answer the question why the resolution $\delta = \lambda/6$ is achievable for two dielectric scatterers and why $\delta = \lambda/15$ is achievable for plasmonic ones [15] if the waist diameter is as substantial as $\lambda/3$. Moreover, as it was correctly noticed in [16], the reciprocity theorem was applied in [9] wrongly. In presence of evanescent waves, the reciprocity is not reducible to the inversion of the wave propagation. Really, in both focusing and emitting schemes of the microscopy, the evanescent waves decay in the same directions – from the rear point of the
sphere. Also, in [15] the point-spread function of a point dipole located in the near vicinity of the MP in the presence of an objective lens was numerically simulated and no subwavelength imaging was found. In works [17, 18] resonant mechanisms of subwavelength imaging by microspheres were analyzed. One was related to whispering gallery resonances, another – to the Mie resonances. In both cases, the wave packages responsible for subwavelength hot spots inside the particle experience the leakage and partial (quite weak) conversion into propagating waves. However, these resonant mechanisms have nothing to do with the known experiments in which a simple glass microsphere operates as a far-field superlens in a broad frequency range.

In the present paper, we unveil one of the mechanisms of the non-resonant imaging by a microsphere or microcylinder. It arises when the structure is illuminated by a laser light. It is, definitely, not an only mechanism of nanoimaging potentially granted by a simple dielectric MP placed on a dielectric substrate. We believe that there are several mechanisms and we are eager to study all of them in our next papers.

2. Theory

When the unpolarized light impinges a dipole scatterer located near an MP, the scatterer polarizes both tangentially to the particle surface and normally to it. To our knowledge, in all known works aiming to explain the superlens operation of a microsphere only a tangential dipole depicted in Fig. 1(a) was considered. It is difficult to expect the nanoimaging in this case even though \(kR \gg \pi\) and \(kd \ll \pi\). Well, for a subwavelength distance \(d\) there is a near-field interaction resulting in the excitation of some evanescent waves in the vicinity of point \(A\), however, this effect is relatively weak. The near field coupling of the dipole and a microsphere can be qualitatively characterized by the interaction factor of two dipoles – a real dipole \(p\) and its quasi-static image \(p_i\) located at the distance nearly equal to \(2d\) from the center of the real scatterer. This coupling factor for a tangential dipole in the quasi-static approximation is equal \(A_t \approx ik/4\pi\varepsilon_0(2d)^2\).

The absolute value of \(A_t\) in the case \(kd \ll 1\) is much lower than the interaction factor of two collinear dipoles separated by the same gap \(2d\) as it is depicted in Fig. 1(b). For a normally polarized dipole we have \(A_n \approx 1/2\pi\varepsilon_0(2d)^3\), and it is clear that the normal dipole transfers more power to the MP than the similar tangential one. Further, this power propagates from this point in the MP forming a hollow wave beam that being transmitted through the MP can be called an imaging beam.

Fig. 1(b) illustrates a simplistic model of the imaging beam. Within the framework of geometrical optics the radiation created by the dipole \(p\) in the sphere around point \(A\) is a diverging wave beam with the zero intensity along the axis \(x\). The rays strongly tilted to this axis experience multiple total internal reflections and return back leaving the sphere with a negative tilt to \(x\). The rays which refract to free space from the rear side form a weakly divergent radially polarized beam. Wave beams with radial polarization experience a much weaker Abbe diffraction than the conventional wave beams. In accordance to the known theory, a radially polarized beam of the same initial divergence as that of a conventional Gauss beam being focused by a usual lens offers a \(3 - 4\) times smaller radius of the Airy circle [19]. This Airy radius in our case is the same as the effective image size of the normal dipole [21].
two closely located dipoles the image size in accordance to the commonly adopted Rayleigh criterion equals to the spatial resolution $\delta$ – the minimal distance between two point-wise dipole sources reliably distinguished by either a human eye or any device recording the intensity distribution in the image plane [21]. A standard confocal microscope grants a point-wise source image size equal to $\delta \approx \lambda/2$ [22]. In the case illustrated by Fig. 1(a), the sphere does not increase this resolution but hardly worsens it, and we can admit it equal to $\delta \approx \lambda/2$. In the case depicted in Fig. 1(b), the resolution should be $3-4$ times finer i.e. $\delta \leq \lambda/6 - \lambda/8$. At this point we could terminate our paper claiming that we found the reason of the superlens operation of a microsphere in the experiments reported in [3, 9, 14, 15] and several other works. However, we have to continue our study because in reality the formation of the imaging beam is not so simple as that presented in Fig. 1(b).

If a small scatterer modeled as a Hertzian dipole $p$ is located far from the MP ($kd \gg 1$) the near-field coupling is absent. Instead, the radiation of $p$ impinges the MP experiencing a strong diffraction on its surface. Then only a small portion of the energy transfers to the radially polarized beam. If we treat the radially polarized beam depicted in Fig. 1(b) as the imaging one, the large-angle beams scattered by the particle surface and collected by the lens represent the optical noise veiling the image. Since in this paper, we consider namely the diffraction-limited image, we assume that the limitation of $\delta$ by optical noises is finer than the diffraction limit. In accordance to [24], it implies that the optical signal level in the image plane (intensity of the focused imaging beam) must be much higher than the noise level (intensity of the scattered beams collected and focused by the objective in the same plane). In the case ($kd \gg 1$) it is not so – in this case, in accordance to [24], the resolution limit will be determined by noises.

If the dipole scatterer is located closely to the MP ($kd \ll \pi$) the transmission of the radiation through the MP is also different.
from that shown in Fig. 1(b) because the near-field coupling results in a very efficient excitation of creeping waves \[20\]. For a subsurface source (located near the interface inside an optically large particle) a part of the radiated power transferring to the creeping waves (CWs) is so important that the far-field terms corresponding to the beam transmitted across the particle and to the beam ejected from CWs have the same order of magnitude \[20\] and in some specific directions the beam resulting from the ejection of CWs dominates. If the source is located outside the MP the portion of power transferred to CWs should be even larger because the interface hinders the straightforward propagation of the dipole radiation to the half-space \(x > 0\). We see that in both cases (large distances \(d\) and small distances \(d\)), the simplistic model illustrated by Fig. 1(b) is irrelevant. Below we concentrate on the case when the CWs are efficiently excited by a normally polarized dipole.

Though in \[20\] the source is subsurface, the general properties of CWs are the same. These creeping waves are TM-polarized and propagate on the internal side of the MP surface. In the 3D geometry when the MP is a sphere the CWs propagate in polar directions from a normally polarized dipole. In the 2D geometry when the MP is a cylinder the CWs propagate azimuthally from a dipole line. Fig. 2 can be referred to both these cases. For different sizes of the MP the wave numbers of CWs can vary from \(k_m \approx k\) to \(k_m \approx k(n + 1)/2\) \[23\]. For given \(kR\) and \(n\) a given source effectively excites a finite number \(M\) of CWs \[20\] \[23\]. If \(kR \sim 10 - 20\) as in Fig. 2(a), all \(k_m\) have real parts nearly equal to \(k\), whereas the number \(M\) of CWs is comparatively small and the angular paths \(\Psi_m\) for all CWs from the birthplace to the ejection point exceeds \(\pi/2\). If \(kR \sim 50 - 100\) - this case corresponds to Fig. 2(b) - all \(k_m\) have real parts close to \(k(n + 1)/2\), and the path of all CWs is rather short (\(\Psi_m < \pi/2\)), whereas \(M\) is comparatively large. In \[20\] it is stressed, that in the case \(kR \gg \pi\) the region where all CWs are ejected from the boundary is geometrically narrow (for a circular cylinder the angular width is equal to few degrees). As we can see in Fig. 2 in both cases one \(m\)-numbered CW forms two symmetrically tilted partial beams. One can be numbered \(+m\) and another one can be numbered \(-m\). Now, let us see what kind of image of our dipole is formed by such CWs.

2.1. Imaging of a normally polarized dipole

As an example, consider the most interesting case when all \(\Psi_m\) are close to \(\pi/2\). This case is illustrated by Fig. 3. Here we show the dipole creating three CWs and the regions where these CWs are ejected from the MP. Smaller path \(\Psi_m\) corresponds to larger wave numbers \(k_m\). Therefore, the beam ejected from point \(A\) passing near the point \(B\) has nearly the same phase as the beam ejected at \(B\) (lower phase shift of the previously ejected beam due to its propagation along \(AB\) in free space compared to a CW passing the same path in the dielectric is compensated by higher wave number of the CW ejected at point \(A\)). The same refers to the beams ejected at points \(B\) and \(C\). Though the size of the ejection region (the distance \(AC\)) is of the order of \(\lambda\) \[20\] \[23\], three rays \(A, B\) and \(C\) are nearly homocentric as if they were ejected from a subwavelength spatial region centered at \(B\). In other words, the three wave beams resulting from these three CWs have the common wave front (cylindrical in the 2D geometry) as it is shown in Fig. 3(a). The same refers to the rays \(A', B'\) and \(C'\). In the 3D geometry, the wavefront corresponds to the
Figure 2. An \( m \)-numbered creeping wave produced by a dipole after its leakage creates two partial beams (\( \pm m \)). For modest \( kR \) the spectrum of CWs is rather broad whereas all \( M \) CWs have the angular paths \( \Psi \) larger than \( \pi/2 \). (b) The case of large \( kR \) corresponds to all CWs having the paths \( \Psi < \pi/2 \). Positive maxima of \( E \) along the CW trajectory are shown as red squares.

Figure 3. Formation of a Bessel-like imaging beam consisting of \( 2M = 6 \) partial beams resulting from \( M \) creeping waves. (a) Beams ejected from points A,B,C can be treated as three rays (nearly homocentric and emitted from the central point B of the ejection region). The same refers to beams ejected from points A',B',C'. (b) In the Fraunhofer zone, the true phase of the electromagnetic field alternates across the imaging beam (\( \pi \)-jumps versus \( m \)). Vectors \( E \) on every ray are shown in phase (with the interval \( \lambda \)).

Beams ejected from points B and B' have the smallest tilt and can be called partial beams of the first order (\( m = \pm 1 \)), beams ejected from points C and C' are partial beams of the 2d order (\( m = \pm 2 \)) and beams ejected from points A and A' are partial beams of the 3d order (\( m = \pm 3 \)). Now let us take into account that the CW corresponding to the middle of their spectrum has maximal amplitude \( [23] \), i.e. the electromagnetic field in the first-order partial beams (ejected from points B and B') is higher than that of the other partial beams. Assume (this assumption is checked below) that the electromagnetic field in the partial beams C and C' (\( m = \pm 2 \)) is higher than that in the partial beams A and A' (\( m = \pm 3 \)). Then our imaging beam mimics the Bessel function of type \( J_\nu \) of the argument \( \xi \theta \), where \( \theta \) is the tilt angle (\( \nu \) and...
ξ are parameters to be found). The true phase of the electromagnetic field in the partial beams jumps from 0 to π versus $m$ and it can be treated as the oscillation inherent to the Bessel function describing the electromagnetic field ($E$ and $H$) of our imaging beam. The first (positive) maximum of the Bessel function holds in the directions corresponding to the centers of the first-order beams. In these partial beams, the electric field is polarized along the polar vector $\theta_0$ with sign plus. In the middle between beams $m = \pm 1$ and $m = \pm 2$ the field is zero. The centers of the beams $m = \pm 2$ point out the second (negative) maximum of the Bessel function. In these partial beams, the electric field is polarized along the polar vector $\theta_0$ with sign minus. Subtracting $(m - 1)\pi$ from the true phase of each partial beam we may equate the phase of the whole imaging beam and introduce its common phase front. It is evident, that this phase front has a non-uniform curvature versus the polar angle $\theta$. Therefore, the imaging beam produced by a point dipole will not be focused to a single subwavelength spot. Fig. 3(b) allows us to understand that in the 2D geometry (microcylinder) our point dipole will be imaged as two parallel line sources. In the 3D geometry the imaging beam will create a magnified image of the MP perimeter. In the experiments this ring in the image plane is difficult to distinguish from the Airy rings corresponding to the diffraction-limited image of the tangentially polarized dipole. Thus, the radially polarized scatterer in the cases of the oblique incidence of a plane wave and of the non-polarized light does not grant a distinct subwavelength image.

However, realistic Bessel beams used in modern optics though differ from an ideal Bessel beam exactly described by a Bessel function $J_\nu(\xi \theta)$, still grant the suppression of the Abbe diffraction by orders of magnitude [25]. Therefore, the suppression of diffraction in the imaging beam created by a normally polarized scatterer though does not grant its true image leaves the room for a subwavelength resolution of two point sources. This issue will be studied in the next subsection.

To conclude this part, let us notice that the Bessel beam can be mimicked by our imaging beam only if there are CWs having both $\Psi > \pi/2$ and $\Psi < \pi/2$. In the 2D geometry it occurs when $kR = 20 - 30$ and $n = 1.7$. In this case, the distribution of the electromagnetic field across the imaging beam (simulated below) mimics the Bessel functions with the indices $\nu = 0.5 - 1.5$ and an argument proportional to $\theta$. If $\Psi < \pi/2$ or $\Psi > \pi/2$ for all CWs, the true phases of all partial beams tilted upward in the geometry of Fig. 3 will be positive and the true phases of all partial beams tilted downward will be negative on the common phase front of the imaging beam. In our numerical simulations, we observed $\Psi > \pi/2$ when $n = 1.4$, $kR = 10$ and $n = 1.7$, $R = 10 - 20$, whereas $\Psi < \pi/2$ corresponds to $kR = 20 - 30$ and $n = 1.4$. In all these cases, the imaging beam turned out to be a Mathieu-like one. Realistic Mathieu light beams can be also considered as also practically diffraction-free ones at the mm and cm distances [25].

2.2. Imaging of a pair of normally polarized dipoles

Consider a pair of dipoles $p_{1,2}$ separated by a subwavelength gap $\delta$ and located at a subwavelength distance $d$ from the MP. If these dipoles are induced in two identical scatterers 1 and 2 by the incident wave polarized along $x$ as it is shown in Fig. 4 it is basically the same as the polarization of $p_{1,2}$ normal to the surface of the MP, whereas the absolute
value of $p_{1,2}$ of these dipole moments is the same. Two CWs of the same order produced by these two dipoles results in two pairs of symmetrically tilted beams ejected from points $A_{1,2}$ and $A'_{1,2}$. The angle between the beams ejected from points $A_1$ and $A_2$ ($A'_1$ and $A'_2$) equals $\delta/R$. This angle is much smaller than the angles between partial beams of different order $m$ and even smaller than the angular width of a partial beam corresponding to a given $m$. Therefore, if the phase shift $\phi$ is zero between dipole moments $p_1$ and $p_2$ (e.g. scatterers 1 and 2 are excited by a non-coherent light) we have the same fields $E_2 = E_1$ on the effective phase fronts of the partial beams emitted from points $A_1(A'_1)$ and $A_2(A'_2)$. Here we imply the same phase for the vectors oriented so that the true phase of $E_y$ and $H_z$ at two symmetric partial beams differs by $\pi$. In other words, for a Bessel-like imaging beam we mean the Bessel phase that takes into account the $\pi$-jump of the true phase of $E_y$ and $H_z$ between two adjacent particle beams. For a Mathieu-like imaging beams we mean the Mathieu phase that implies $\pi$ subtracted from the true phase of $E_y$ and $H_z$ of all partial beams with $m < 0$. For a sharply focusing lens (objective of a microscope) the $x$-component of the electric field is dominating in the paraxial image formation. Therefore, the notion of the in-phase partial beams refers to namely to the Bessel or Mathieu phases and not to the true phases of the electromagnetic field in the partial beams.

For a symmetric dual source $p_1 = p_2$, a slight birefringence of the imaging beam granted by the gap $\delta$ between two dipoles has no noticeable implications for the imaging. It only results in a slight extension of the single dipole image corresponding to the total dipole
$p_1 + p_2$. However, a coherent illumination illustrated by Fig. 4(a) implies $p_2 = p_1 \exp(i\phi)$, where $\phi = k\delta$. This situation is drastically different because we have $E_2 = E_1 \exp(i\phi)$ for two partial beams ejected from the top and $E_{-1} = E_{-2} \exp(i\phi)$ for two partial beams ejected from the bottom. Two in-phase partial beams (the same color in our drawing) have different tilt angles. The practical absence of the Abbe diffraction means that these partial beams though interfere do not mix up and form an anti-symmetric interference pattern in the top and bottom parts of the imaging beam. For all partial beams emitted from the region $A_1A_2$ and for those emitted from the region $A_1'A_2'$ the phase distribution is anti-symmetric. The phase difference for a given tilt $\theta$ is equal $\phi$.

In Fig. 4(b) two pairs of in-phase partial beams (main maxima of the imaging beam of the Bessel or Mathieu type) meet one another in phase at points $I_1$ and $I_2$, which are, therefore, local maxima of intensity. The coordinate $x_I$ of these points is close to the coordinate of the plane where the top and bottom parts of the imaging beam converge and the aforementioned image of the total dipole is formed. Therefore, around points $I_1$ and $I_2$ all partial beams are though not yet focused, but sufficiently converged so that their intensity at points $I_1$ and $I_2$ would be sufficient for imaging. Thus, we obtain two rather weak but distinguished images centered at points $I_1$ and $I_2$ which are distanced from one another by the macroscopic gap $2\Delta y$. This gap is a magnified distance between two virtual objects $VO_1$ and $VO_2$ and equals to the product of the gap $2h$ between these points by the standard lens magnification factor $\Gamma$. $VO_1$ and $VO_2$ are effective phase centers from which the pairs of the in-phase beams are seemingly emitted. Since $A_1A_2 = \delta$ is easy to see that

$$h = \delta/2\sin \Psi$$

where $\Psi$ is the angular path of the CW from point $p_i$ to point $A_i$ ($A_i'$), $i = 1, 2$. Since for given $kR$ paths $\Psi$ of different CWs differ weakly, in our estimation we may admit that the points $VO_{1,2}$ correspond to the mean angle $\Psi$ of the corresponding spectrum of CWs. Then we may write the result of our model in the form

$$\Delta y = \Gamma \frac{\delta}{2\sin \Psi}. \quad (1)$$

Since $\sin \Psi$ is not very small the magnification of the dual dipole source is of the same order of magnitude as the standard magnification granted by the lens. The same refers to the image seen in a microscope. In the 2D case, points $I_1$ and $I_2$ on the plane $xy$ mean the central lines of the strips of enhanced intensity. The local maximum of intensity at these points is granted by the constructive interference of any partial beam $+m$ with a beam $-m$ created by the same source and having therefore a different tilt. In the 3D case, the points $I_{1,2}$ are namely points of the maximal intensity and not traces of a ring. The ring imaging the total dipole is located in a different plane $x_R \neq x_I$ (distanced by several $\lambda$). The gap between the dipoles 1 and 2 is located in the plane $xy$, the phase shift between symmetrically tilted rays holds namely in this plane and the corresponding maxima of intensity are formed only in this plane. Thus, a conformal magnified image of two dipole scatterers separated by a subwavelength gap $\delta$ arises in both 2D and 3D cases.

Here, it is worth noticing that the virtual objects in Fig. 4(b) arise if and only if the dipoles $p_1$ and $p_2$ are out of phase. If the beams ejected from points $A_1$ and $A_2$ are in phase, the top and bottom parts of the imaging beam are homocentric and we will see only the image of the total dipole in the plane $x = x_R$. Virtual objects $VO_1$ and $VO_2$ shown in Fig. 4(b) arise only if the
homocentricity corresponding to a symmetric source is destroyed and replaced by another homocentricity – that of rays shown by the same color in Fig. 4. Moreover, these objects are located as it is shown in the drawing only in the case $\phi = k \delta$. Note, that for better visibility in Fig. 4 the dipoles $p_1, p_2$ are separated by a substantial gap $\delta$ comparable with $R$ that results in the large angle $\delta/R$ between the rays ejected from points $A_1$ and $A_2$ and therefore in the large distance between the virtual objects and the MP center. In reality, $k \delta \ll \pi$, $kR \gg \pi$, and the angle $\delta/R$ is smaller than the angular width of any partial beam. Therefore, points $VO_{1,2}$ are located near the plane $x = -R - d$ where the sources are located.

It is important to understand that the areas of enhanced intensity centered by points $I_{1,2}$ are not images of separate dipoles $p_1$ and $p_2$. What is shown in Fig. 4(b) is a consolidate image of an asymmetric (phase-shifted) dual source. However, in accordance to formula (1) it is a conformal image. It keeps conformal if the incidence of the illuminating wave is not grazing and the phase difference between the rays ejected from $A_1$ and $A_2$ is lower than $k \delta$. Then $\phi = k \delta \cos \Phi$, where $\Phi$ is the incidence angle counted from the $y$-axis. In this case, beams ejected from $A_1$ and $A_2'$ are not in phase. The in-phase condition holds for beams with closer ejection angles. In this case, we have $h = \delta \cos \Phi / 2 \sin \Psi$ and the right-hand side in (1) should be multiplied by $\cos \Phi$. When $\cos \Phi$ decreases maxima of intensity at points $I_{1,2}$ become weaker and finally overlap. Then the superlens operation disappears.

To conclude the theoretical part: if an object substantial in the plane $yz$ but small compared to $R$ is located closely to a dielectric MP a sufficiently asymmetric illumination by a coherent light allows its magnified image with subwavelength details. This nanoimaging is granted by the polarization of the object normally to the surface of the MP.

3. Calculations: Results and Discussions

In this section we report only the simulations of a 2D structure because for $kR \gg 10$ no one available simulator offers a reliable solution of the 3D problem in a reasonable computation time. Meanwhile COMSOL Multiphysics provides a rapid solver of 2D problems. It allowed us to obtain the color movies of the wave beams, color maps of their intensities and vector maps. Using COMSOL we are capable to study the evolution of the wave beams up to hundreds $\lambda$. On the first stage, we checked the accuracy of the COMSOL solver reproducing the results obtained for a tangentially oriented dipole source in work [16]. We obtained the results depicted in Figs. 7 and 8 of [16] for a glass cylinder with radiuses $kR = 10, 20, 20.382, 30$ for a dipole located at the distance $d = 1/k$. The case of whispering gallery resonance when $kR = 20.382$ corresponds to the maximal image size. We have further analyzed the diffraction spreading of the transmitted beam in the Fraunhofer zone of the MP and can confirm what is claimed in [16] about the properties of a point-spread function. We have seen no CWs excited by a tangential dipole in both glass MP ($n = 1.4$) and in MP of transparent resin ($n = 1.7$).

3.1. Simulations for one normally polarized dipole line

We have performed extended numerical simulations of the structure with a single normally polarized dipole line source varying the radius
Figure 5. Single radially polarized source – wave pictures of the Fresnel zone for a glass microparticle with $kR = 10$ (a) and $kR = 20$ (b). Red arrows show $E$ in some symmetrically located points of the wave picture.

of the MP in the range $kR = 10 - 30$ for two values of the refractive index $n = 1.4$ and $n = 1.7$. In our simulations, the distance $d$ from the sources to the cylinder is equal $1/2k$. For the dual source the gap between the dipole lines was equal $\delta = 2d$. The phase shift in the dual source in our simulations was varying from 0 to $k\delta(n + 1)/2$ (the results for $\phi = k\delta$ are most important).

In Fig. 5 we present two instantaneous pictures of the wave movie for the cases $kR = 10$, $n = 1.4$ and $kR = 20$, $n = 1.4$. The patterns of dominating CWs are clearly seen. In the case $kR = 10$ all CWs eject from the bottom part of the particle i.e. $\Psi > \pi/2$ as it was expected. The vertical ($x$-) component of the electric field has the same true phase at two points symmetrically located with respect to the axis $x$. The intensity is maximal in the partial beams of lowest order $m = \pm 1$. This beam mimics a Mathieu function and our studies of its evolution in the Fraunhofer zone confirm it. In the case $kR = 20$ some CWs have the angular path $\Psi > \pi/2$ and some CWs have $\Psi < \pi/2$ as it was expected. For this case we expected to obtain a Bessel-like imaging beam. However, in this particular case the imaging beam does not mimic any Bessel function – the intensity in the partial beams $m = \pm 1$ is lower than that in the partial beams $m = \pm 2$ for which the intensity is maximal over $m$. The distribution of the magnetic field $H_z$ in this case mimics the derivative of the Bessel function over the index: $\partial J_\nu(\xi\theta)/\partial\nu$, where $\xi \approx 15.1$ rad and $\nu = 1.5$. Such beams, to our knowledge, have never been studied. In our simulations we have not found the features of the Abbe diffraction for these beams, as well for we have found in all other cases when the imaging beams mimic the Mathieu and Bessel beams.

Fig. 6(a) shows that the effective angular width of all partial beams keeps the same at large distances from the MP corresponding to its Fraunhofer zone. There is no typical spread inherent to the Abbe diffraction. We have checked that the Mathieu-like imaging beam (negative coordinates $x$ here correspond to the domain behind the cylinder) keeps practically diffraction-free up to $100 - 200\lambda$. We are sure that the diffraction is not similarly absent at larger distances up to macroscopic ones. In Fig. 6(b) we have shown the phase distribution of the magnetic field vector ($\mathbf{H} = Hz_0$) across
Figure 6. Single source – an intensity map in the optically large area (a) and a phase distribution across partial beams with \( m = \pm 1, \pm 2, \pm 3 \) (b). White dashed line in (a) shows the argument of the phase distribution in (b). Glass microparticle with \( kR = 10, \lambda = 550 \) nm.

Four partial beams \( m = \pm 1, \pm 2 \). The true phase jumps by \( \pi \) at the symmetry axis and subtracting this jump we see the symmetric phase distribution that we called above the Mathieu phase. Partial beams \( m = \pm 1, \pm 2 \) occupy the region \( y = [-7, 7] \lambda \) and we see a small oscillation of the phase corresponding to the dark area between the partial beams \( m = \pm 2 \) and \( m = \pm 3 \). Notice that the Mathieu phase is not constant across a partial beam because the dashed line shown in Fig. 6(a) does not coincide with the phase front. The numerical retrieval of the phase front is doable but difficult and not relevant. Jumps of the Mathieu phase equal to \( 2\pi \) are introduced in Fig. 6(b) to make the plot more compact. Both pictures Fig. 6(a) and (b) correspond to our theoretical expectations.

Similar conclusions refer to all our simulations for a single normally oriented dipole. Two more examples are presented in Fig. 7. Here in both pictures Fig. 7(a) and Fig. 7(b) we can see the mixture of two mechanisms of subwavelength imaging. In the wave picture Fig. 7(a) we observe a weaker impact of CWs. Here a radially polarized imaging beam with low divergence as in Fig. 2(b) is presented together with the Mathieu-like beam resulting from CWs. The mechanism of CWs is more pronounced in Fig. 7(b). The features of the diffraction were not found for both these cases until \( 100 - 200 \lambda \).

In Fig. 8 we present a comparison of the large-area intensity maps for the cases \( kR = 20, n = 1.4 \) and \( kR = 20, n = 1.7 \). In the first case, the imaging beam mimics the function \( \partial J_\nu(15\theta)/\partial \nu \) at \( \nu = 1.5 \) and in the second case – the function \( J_1(\xi \theta) \), where \( \xi = 31 \). In both cases, there are no features of diffraction.

### 3.2. Simulations for a dual normally polarized source

We have performed similar simulations for a symmetric (in-phase) dual source with \( \delta = 2d = 1/k \) and observed no changes but a slight angular extension of the partial beams by the angle close to \( \delta/R \). Below we concentrate on the case of the asymmetric dual source \( p_2 = p_1 \exp(ik\delta) \). The impact of the phase asymmetry is especially spectacular when \( n \) is
larger and $kR$ is smaller. In Fig. 9 we compare the wave pictures obtained for a symmetric dual source and an asymmetric one when $n = 1.7$ and $kR = 10$. Since in this case $\Psi > \pi$ we observe a standing wave pattern in the microparticle that resembles a whispering gallery resonance (though it is not so). In the symmetric case (as well as for a single source) it results in strongly dominating partial beams of the first order. In the asymmetric case, this picture is drastically modified. Even from the wave picture in the Fresnel zone it is clear that the image of the asymmetric source should be qualitatively different from that of a symmetric one.

In Fig. 10 we compare the spatial distributions of the vector $\mathbf{E}$ (shown by arrows on the background of a wave picture) for two cases: (a) a single dipole source and (b) a dual asymmetric source. In these pictures, the bounds of partial beams $m = \pm 1, \pm 2$ are shown by dashed lines. For the dual source we see that the angular width of partial beams is slightly extended (nearly by $\delta/R$, [...]

**Figure 7.** Single source – a wave picture in the large area for $n = 1.4$ (a) and an intensity map in the same area for $n = 1.7$ (b). Here $kR = 30$ and $\lambda = 550$ nm.

**Figure 8.** Single source – intensity maps in the large area for $n = 1.4$ (a) and $n = 1.7$ (b). Here $kR = 20$ and $\lambda = 550$ nm.
Figure 9. Dual source – wave pictures for the symmetric \((\phi = 0)\) case (a) and asymmetric \((\phi = k\delta)\) case (b). Here \(kR = 10\) and \(n = 1.7\).

Figure 10. Single source (a) and asymmetric dual (b) source – vector distributions and wave pictures for the case \(kR = 10, n = 1.4\). Dashed lines show the bounds of partial beams. The color map represents the instantaneous magnetic field and arrows show the direction of total electric field. The color map for a dual source is brighter because the total dipole moment is larger.

as predicted by the theory). The phase of the electric field on the rays propagating with the same tilt \(\theta\) to the \(x\)-axis in the left and right halves of the plot depicted Fig. 10(b) are clearly different. Visually it is impossible to estimate this difference, but qualitatively, these observations confirm the theoretical expectations. Note, that in Fig. 10 the color map shows only positive maxima of \(H\) (those where \(H_z > 0\)). Therefore the distance between the bright areas of picture is here equal \(\lambda\) (not \(\lambda/2\) as in the wave pictures above). This is also the reason why the bright areas in the left \((y < 0)\) and right \((y > 0)\) halves are shifted by \(\lambda/2\).

Finally, in Fig. 11 we present typical distributions of intensity and phase across an imaging beam. The distributions corresponds to the case \(n = 1.4, kR = 10\) (Mathieu beam) and in this example the axis \(y\) crosses the partial beams \(m = \pm 1, \pm 2, \pm 3\) on the same distance as in Fig. 6(a). For
Figure 11. Asymmetric dual source. Intensity (a) and phase (b) distributions across six partial beams for the case $kR = 10, n = 1.4$.

of the distance $2\Delta y$ between points $I_1$ and $I_2$. The reduction of the phase difference (that physically corresponds to the oblique incidence of the illuminating wave to the scatterers 1 and 2) decreases the phase shift between two symmetrically tilted rays that should decrease the magnification.

3.3. Discussion

Our predictions about the superlens operation of a microsphere (microcylinder) were based on the concept of creeping waves excited by a dipole source polarized normally with respect to the microparticle surface. All we needed in order to understand the mechanism of nanoimaging was the knowledge that the spectrum of CWs is discrete and finite and that they eject from the MP surface forming an imaging beam consisting of several partial beams. The polarization of this beam in accordance to the symmetry of the problem is anti-symmetric with respect to the axis drawn between the source and particle center. This polarization promises that the imaging beam does not experience a noticeable Abbe diffraction. The absence of diffraction is a prerequisite of the nanoimaging. These
expectations were fully confirmed by exact numerical simulations.

The imaging in CWs is not straightforward and does not allow us to obtain a true, non-coherent, subwavelength resolution. Though the CWs are ejected from a narrow angular region of a MP, this region is larger than the angular distance between two point-wise scatterers $\delta/R$. Partial beams of the same order created by two sources overlap and a non-coherent image of a dual source cannot be resolved. In the 3D case the image of a single or a dual symmetric source is a ring. However, if the dual source comprises the phase shift, in a plane located rather closely to the plane where the image of the total dipole is formed two maxima of intensity arise. The gap between these maxima is proportional to $\delta$ and their location with respect to the optical axis correspond to the location of the dual source. For it we needed a confirmation of two theoretical expectations: absence of diffraction, and anti-symmetric phase distribution in the partial beams. Both these effects were confirmed by exact numerical simulations.

Now, let us discuss the issue of the subwavelength resolution granted by a MP. The present paper does not aim to calculate this value. This work only qualitatively explains the mechanism of the nanoimaging granted by a microsphere or microcylinder in presence of an object with normally oriented polarization. Of course, this limit is not extraordinary small. The smaller is $\delta$ the weaker are intensity maxima at the image points $I_{1,2}$ and the contrast the intensity at these points with the intensity at the point $(x = x_I, y = 0)$ for too small $\delta$ vanishes. The corresponding study will be a subject of our next papers.

However, it is reasonable to discuss the issue of the spatial resolution in general. The geometry of the MP offers several mechanisms of subwavelength imaging. There are resonant and non-resonant mechanisms. To our opinion, all non-resonant mechanisms granted by the MP in absence of the substrate are related with the excitation of the normal polarization in the object. The simplest mechanism of subwavelength imaging is formation of a radially polarized imaging beam illustrated by Fig. 1(b). The point-wise source image size for this case is well known – it is reduced 3-4 times compared to the image size in absence of a microparticle. For a standard microscope with f-number close to unity, it can be $\delta = (0.12 - 0.16)\lambda$. Numerical simulations have shown that this simple mechanism of subwavelength imaging is not so easy to implement. For a distant source the most part of the power radiated by a dipole is scattered by the surface of the MP. For a closely located source this power is spent to the excitation of CWs. However, look at Fig. 7 and see the evidence of both mechanisms in the wave picture and in the intensity plot. For the case $kR = 30$, $n = 1.4$, $d = 1/2k$ the simple mechanism dominates! For this particular case we expect a subwavelength image of a single source behind the lens with the radius of the Airy circle of the order $\lambda/6$. In our next papers, we will report thus study.

Now, let us compare the result of the present paper with the literature data. In work [26] the superlens operation of a barium titanate microsphere in the case of a coherent illumination was experimentally studied. The illumination was both coaxial (along the $x$ axis in our notations) and nearly grazing ($20^\circ$ to the $y$ axis). For the symmetric excitation the subwavelength ($\delta < \lambda/2$) resolution of two scatterers (representing the grooves and notches in the substrate) was obtained only for scatterers located in the circle of radius $0.8\,\mu\text{m}$.
centered by the point where the sphere touched the patterned substrate. Since the sphere in this experiment had the radius $R = 27 \, \mu m$ it means that the distance between the substrate and the sphere does not exceed 10 nm in this circle. This nanoimaging is not that granted by a microsphere. It is granted by a tiny crevice formed in between barium titanate of the sphere and silicon of the substrate.

The asymmetric incidence for which the normal polarization of the object dominates because the incidence angle is large offers the enlargement of the circle in which the subwavelength resolution is observed. Its radius enhances from 0.8 $\mu m$ to 1.7 $\mu m$. In this enlarged circle the distance between the sphere and the substrate increases up to 40 nm. In this case the spherical profile becomes important, and the mechanism of nanoimaging studied in the present paper can prevail. Also, the comparison of Fig. 8 and supplementary Fig. 4 of [26] allowed us to notice that the mean resolution within the circle of the same radius is sharper in the case of asymmetric excitation. Thus, the claims of our work agree with the literature data.

### 4. Conclusions

In the present work, we discussed some possible non-resonant mechanisms of the superlens operation of a dielectric microsphere or microcylinder. It is important to understand that there are several mechanisms of nanoimaging and that, contrarily to the popular opinion, most of them if not all have nothing to do with the phenomenon of photonic nanojet. There are resonant and non-resonant mechanisms. Besides of the non-resonant mechanisms discussed in the present paper, two resonant mechanisms were recently reported in works [27] and [28]. The first one demands a covering of the structure with a thin shell of a plasmonic metal. Then the image of an object arises first the surface-plasmon polarizations excited in the crevice between the substrate and the microparticle and adiabatically transmitted to the rear surface of the MP with a magnification. The second resonant mechanism results from the effect of so-called superoscillations (also called the Aharnov-Berry effect in the spatial domain). In this case, the subwavelength hot spots can arise inside the microsphere near to its surface due to the super-oscillatory interference of spherical spatial harmonics. The qualitative difference of this hot spot from the waist of a photonic nanojet is much smaller spot size because the PNJ is induced by a plane wave, whereas the superoscillatory interference arises due to a closely located dipole source.

In the present paper, a non-resonant mechanism is granted by the normal (radial) polarization of the object with respect to the microparticle. The impact of the substrate is neglected. In this case, there are two scenarios of nanoimaging – a coherent one and a non-coherent one. A coherent scenario is considered in details. A non-coherent one is discussed briefly and will be studied in our next paper. Also, we plan to study a non-resonant mechanism granted by the coupling between the particle and the substrate.

**ACKNOWLEDGMENTS**

This work was funded through the EMPIR project 17FUN01-BeCOMe. The EMPIR initiative is co-funded by the European Union Horizon 2020 research and innovation programme and the EMPIR participating States.
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