Calculation and Characteristic Analysis on Different Types of Shock Response Spectrum

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Abstract. Shock Response Spectrum (SRS) has been proved to be an effective standard tool for analyzing and quantifying shock environment in engineering, which is also the simplest tool for comparing shock severity and potential damage to structures. The calculation formulas of absolute acceleration spectrum, relative displacement spectrum, relative velocity spectrum and pseudo velocity spectrum are given respectively. The responses of different spectrum under typical semi-sinusoidal shock excitation are compared and analyzed, and some enlightening conclusions are obtained.

1. Introduction
In design of launch vehicles and satellites, a large amount of pyroshock devices are often used to achieve separation of loads, deployment of solar panels and other appendages, as well as activation of propellant valves and other systems [1]. These pyroshock devices can generate a pyroshock environment with extremely high acceleration amplitude and wide frequency range. The acceleration amplitude can be as high as 10⁴-10⁵g within 20 milliseconds while the frequency range is 100Hz-100KHz [2-3].

Shock response spectrum (SRS) has proven to be an effective standard tool for engineering analysis and quantification of shock environment [2, 4-5]. SRS describes the relationship between maximum absolute response and natural frequency of a series of single degree-of-freedom (SDOF) oscillators under given base excitation.

SRS transforms time domain shock excitation into frequency domain representation, which can be defined in the form of displacement, velocity or acceleration. On the basis of literature review, the calculation methods of different types of SRS are systematically summarized, and the calculation flow of SRS is defined by typical shock excitation. According to different types of SRS expressions, the characteristics of SRS are analyzed through typical excitation comparison, and some enlightening conclusions are obtained.

2. Calculation Model of Shock Response Spectrum
Shock response spectrum is a representation of transient shock acceleration signal in frequency domain. SRS, as an effective tool to describe shock environment, has been widely used in the fields of national defense, aviation and earthquake.
SRS is a calculation function based on acceleration time history, which applies acceleration time-domain shock excitation to a series of SDOF oscillators with different natural frequencies. Consider a series of linearly damped SDOF oscillators with L different natural frequencies, all mounted on a common fixed base, as shown in Fig. 1 [6].

In Fig. 1, \( \ddot{Y} \) is the input acceleration of shock excitation signal, \( \dddot{X}_i \) is the acceleration response of each subsystem, \( M_i, C_i, K_i \) and \( f_{ni} \) is the mass, damping coefficient, stiffness and natural frequency of each subsystem, respectively. The two main parameters of SDOF system that specifies free oscillation are the natural frequency \( f_{ni} \) and damping of substructure. There are many ways to describe damping, damping ratio \( \xi \) and quality factor \( Q \) are selected in this paper. The relationship between \( \xi \) and \( Q \) is shown in Eq.1.

\[
Q = 1/(2\xi), \quad \text{where} \quad \xi = C/(2\sqrt{K\cdot M})
\]

(1)

For the normalized value of quality factor \( Q \), the natural frequency of substructure plays a key role. As mentioned above, SRS assumes the shock load as a common basic input for a set of independent hypothetical SDOF, these systems differ only in the value of the natural frequency, thus, \( M_i, C_i \) and \( K_i \) have the same values while \( f_{ni} \) has different values in Fig. 1. Damping ratio \( \xi \) is usually a fixed value and is generally set at 5%, which is equivalent to quality factor \( Q = 10 \).

The most commonly used numerical calculation methods for SDOF responses include Runge-Kutta method, implicit integration method, Kelly-Richman digital filtering method and improved slope-invariant digital filter recursive algorithm[7]. The improved slope-invariant digital filter recursive algorithm proposed by Smallwood[8] has become a commonly used numerical algorithm and Engineering standard for SRS[9-10] because of its clear physical meaning, concise algorithm, fast calculation speed and high computational accuracy.

3. Types of Shock Response Spectrum

SRS can be used to define shock excitation and corresponding frequency response in the form of displacement, velocity or acceleration. Table 1[11] gives definition of several different types of SRS.

| Shock response spectrum       | Definition            |
|-------------------------------|-----------------------|
| Absolute Acceleration SRS, AASRS | \( A_a = \max\left(|\dddot{x}|\right) \) |
| Relative Displacement SRS, RDSRS | \( D = \max\left(|\dot{z}|\right) \) |
| Relative Velocity SRS, RVRSRS | \( V = \max\left(|\dot{z}|\right) \) |
| Pseudo Velocity SRS, PVRSRS | \( V_p = \max\left(|a\dot{z}|\right) \) |
Absolute Acceleration SRS (AASRS). Within a given frequency range, the corresponding relation between the absolute value $|\ddot{x}_{\text{max}}|$ of the maximum acceleration response of all frequency points under the specified bandwidth and the frequency is the absolute acceleration spectrum-AASRS.

The transfer function connecting the response acceleration and the input acceleration is:

$$H(s) = \frac{\ddot{X}(s)}{Y(s)} = \frac{2\zeta\omega_n s + \omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2}$$

Later, an improved recursive algorithm of slope invariant digital filtering will be adopted. The key of this algorithm is to consider sampling the response of the continuous system using the slope invariant method. Using $H(z)$ to describe the digital filtering system will obtain the same result as the discrete sequence.

$$z \left\{ L^{-1} \left[ \frac{H(s)}{s^2} \right] \right\}_{s=\omega} = H(z) \cdot \frac{T_z}{(z-1)^2}$$

where, $T = 1/f_s$ is the sampling period and $f_s$ is the sampling frequency. It is recommended to use a sampling frequency at least 10 times the target highest frequency in SRS calculation standard.

$$H(z) = \frac{b_0 + b_1 \cdot z^{-1} + b_2 \cdot z^{-2}}{1 + a_1 \cdot z^{-1} + a_2 \cdot z^{-2}}$$

The detailed process of calculating the derivation of digital filter parameters is omitted here. The values of various parameters of the digital filter are calculated by the following formulas, which are given in ISO/WD 18431-4 standard [10].

$$\begin{align*}
    b_0 &= 1 - \exp(-A) \cdot \sin(B) / B \\
    b_1 &= 2 \exp(-A) \cdot \{ \sin(B) / B - \cos(B) \} \\
    b_2 &= \exp(-2A) - \exp(-A) \cdot \sin(B) / B \\
    a_1 &= -2 \exp(-A) \cdot \cos(B) \\
    a_2 &= \exp(-2A)
\end{align*}$$

Then the difference equation between the response acceleration and the input acceleration expressed by the digital filter coefficient can be obtained:

$$\ddot{x}_i = b_0 \ddot{y}_i + b_1 \ddot{y}_{i-1} + b_2 \ddot{y}_{i-2} - a_1 x_{i-1} - a_2 x_{i-2}$$

where, $\ddot{x}_i$ is the acceleration response of the system at sampling time point $i$, $\ddot{y}_i$ is the acceleration input signal of the system at the sampling time point $i$. Thus, the acceleration response of the system at a certain frequency point can be obtained.

Relative Displacement SRS (RDSRS). Within a given frequency range, the corresponding relation between the absolute value $|z_{\text{max}}|$ of the maximum relative displacement response of all frequency points under the specified bandwidth and the frequency is the relative displacement spectrum-RDSRS. RDSRS itself is not often used in shock analysis, however, the expected extreme displacement of shock can be deeply understood through RDSRS, which is very important to quantify shock isolation requirements and equipment swing space requirements.

The transfer function of the connection between the relative displacement response and the input acceleration is:
The digital filter transfer function corresponding to RDSRS is:
\[ H(z) = \frac{d_0 + d_1 \cdot z^{-1} + d_2 \cdot z^{-2}}{1 + c_1 \cdot z^{-1} + c_2 \cdot z^{-2}} \] (8)

The digital filter coefficients are as follows:
\[
\begin{align*}
    d_0 &= \frac{1}{\omega_0 T} \left( 1 - \exp(-A) \cdot \cos(B) \right) \\
    d_1 &= \frac{1}{\omega_0 T} \left( 2 \exp(-A) \cdot \cos(B) \cdot \omega_0 T - \frac{1 - \exp(-2A)}{Q} + 2q \cdot \exp(-A) \cdot \sin(B) \right) \\
    d_2 &= \frac{1}{\omega_0 T} \left( -\exp(-2A) \cdot \left( \frac{\omega_0 T + 1}{Q} \right) + \frac{\exp(-A) \cdot \cos(B)}{Q} - q \cdot \exp(-A) \cdot \sin(B) \right) \\
    c_1 &= -2 \exp(-A) \cdot \cos(B) \\
    c_2 &= \exp(-2A)
\end{align*}
\]

where, \( q = \frac{1 - \exp(-2A)}{Q} \)...

Relative Velocity SRS (RVSRS). Within a given frequency range, the corresponding relation between the absolute value of the maximum relative velocity response of all frequency points under the specified bandwidth and the frequency is the relative velocity spectrum-RVSRS. RVSRS is not often used for shock analysis, however, due to its certain relationship with pressure, it becomes a very important method for presenting shock response data. On the other hand, RVSRS is also a very effective intermediate step for calculating the energy response spectrum.

The transfer function of the connection between the relative velocity response and the input acceleration is:
\[ H(s) = \frac{\dot{Z}(s)}{Y(s)} = \frac{-s}{s^2 + \omega_0^2 + \omega_n^2} \] (10)

The digital filter transfer function corresponding to RVSRS is:
\[ H(z) = \frac{f_0 + f_1 \cdot z^{-1} + f_2 \cdot z^{-2}}{1 + e_1 \cdot z^{-1} + e_2 \cdot z^{-2}} \] (11)

The digital filter coefficients are as follows:
\[
\begin{align*}
    f_0 &= \frac{1}{\omega_0 T} \left( -\exp(-A) \cdot \cos(B) + \frac{\exp(-A) \cdot \sin(B)}{\sqrt{Q^2 - 1}} \right) \\
    f_1 &= \frac{1}{\omega_0 T} \left( 1 - 2 \exp(-2A) - \frac{2 \exp(-A) \cdot \sin(B)}{\sqrt{Q^2 - 1}} \right) \\
    f_2 &= \frac{1}{\omega_0 T} \left( \exp(-2A) - \exp(-A) \cdot \cos(B) + \frac{\exp(-A) \cdot \sin(B)}{\sqrt{Q^2 - 1}} \right) \\
    e_1 &= -2 \exp(-A) \cdot \cos(B) \\
    e_2 &= \exp(-2A)
\end{align*}
\]

Pseudo Velocity SRS (PVSRS). Another commonly used SRS is PVSRS. At a certain frequency point, the product \( |\dot{z}_{max} \cdot \omega_n| \) of the absolute value \( |\dot{z}_{max}| \) of the maximum relative displacement...
response and the natural frequency $\omega_n$ is the pseudo velocity at that frequency point. Therefore, the corresponding relation between the maximum pseudo velocity response of all frequency points and the frequency is the PVSRS within a given frequency range and a specified bandwidth.

The transfer function of the connection between the pseudo velocity response and the input acceleration is:

$$H(s) = \frac{Z(s)}{Y(s)} = \frac{-\omega_n}{s^2 + \frac{\omega_n}{Q} + \omega_n^2}$$  \hspace{1cm} (13)

The digital filter transfer function corresponding to PVSRS is:

$$H(z) = \frac{h_0 + h_1 \cdot z^{-1} + h_2 \cdot z^{-2}}{1 + g_1 \cdot z^{-1} + g_2 \cdot z^{-2}}$$  \hspace{1cm} (14)

The digital filter coefficients are as follows:

$$h_0 = \frac{1}{\omega_n^2} \left( \frac{1 - \exp(-A) \cdot \cos(B)}{Q} - q \cdot \exp(-A) \cdot \sin(B) - \omega_n^2 \right)$$
$$h_1 = \frac{1}{\omega_n^2} \left( 2 \exp(-A) \cdot \cos(B) \cdot \omega_n T - \frac{1 - \exp(-2A)}{Q} + 2q \cdot \exp(-A) \cdot \sin(B) \right)$$
$$h_2 = \frac{1}{\omega_n^2} \left( -\exp(-2A) \cdot \omega_n T + \frac{1}{Q} \cdot \exp(-A) \cdot \cos(B) \cdot \omega_n T - q \cdot \exp(-A) \cdot \sin(B) \right)$$
$$g_1 = -2 \exp(-A) \cdot \cos(B)$$
$$g_2 = \exp(-2A)$$  \hspace{1cm} (15)

4. Comparative analysis of AASRS, RDSRS, RVSRS and PVSRS

Under the MATLAB environment, the above-mentioned digital filtering method is used to compile the calculation programs of AASRS, RDSRS, RVSRS and PVSRS respectively, and the simple semi-sinusoidal pulse shown in Fig. 2 (the amplitude is 10g and the waveform duration is 11ms) is selected as input excitation, AASRS, RDSRS, RVSRS and PVSRS under this excitation are calculated respectively, and the results are shown in Fig. 3.

![Fig. 2 Semi-sinusoidal pulse](image1)

![Fig. 3 Comparison results of AASRS, RDSRS, RVSRS and PVSRS under semi-sinusoidal shock](image2)

By analyzing the results in Fig. 3, the following conclusions can be drawn:

1. In these four SRS, the values of AASRS are higher than those of the other three spectrum in the whole frequency band. The AASRS shows a trend of gradually increasing at first, reaching the maximum value around 80Hz and then gradually decreasing in the frequency range of 1-1000Hz.
2. RDSRS, RVSRS and PVSRS all show the same trend of decreasing values in the whole calculation frequency band.
3. RDSRS decreases fastest with frequency.
4. RVSRS is slightly higher than PVSRS at very low frequencies and then slightly lower than PVSRS. After that, RVSRS dropped faster than PVSRS.
5. Compared with RVSRS at low frequencies, PVSRS will be close to but slightly lower. Due to its frequency dependence, PVSRS will show higher values at high frequencies than RVSRS.

5. Summary
The classification and calculation methods of several different shock response spectrum are given, and the characteristics of AASRS, RDSRS, RVSRS and PVSRS under classical semi-sinusoidal pulse excitation are compared and analyzed through typical calculation examples, the change trends of these four response spectrum in frequency are different. Some enlightening conclusions are obtained.

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