Towards absolute scales of radii and masses of open clusters

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Received 10 January 2007; accepted ...

Abstract

Aims. In this paper we derive tidal radii and masses of open clusters in the nearest kiloparsecs around the Sun.

Methods. For each cluster, the mass is estimated from tidal radii determined from a fitting of three-parametric King’s profiles to the observed integrated density distribution. Different samples of members are investigated.

Results. For 236 open clusters, all contained in the catalogue ASCC-2.5, we obtain core and tidal radii, as well as tidal masses. The distributions of the core and tidal radii peak at about 1.5 pc and 7 - 10 pc, respectively. A typical relative error of the core radius lies between 15% and 50%, whereas, for the majority of clusters, the tidal radius was determined with a relative accuracy better than 20%. Most of the clusters have tidal masses between 50 and 1000 M⊙, and for about half of the clusters, the masses were obtained with a relative error better than 50%.

Key words. Galaxy: open clusters and associations: general – solar neighbourhood – Galaxy: stellar content

1. Introduction

As a minimum characteristics to describe a stellar cluster one needs to specify the position of a cluster centre and its apparent (angular) size. Both parameters serve as a means of identifying a cluster on the sky. Also, these parameters are the most common, present in numerous catalogues of clusters, and presently available for about 1700 galactic open clusters (see e.g. Dias et al. 2002). In the majority of cases, however, they are derived from visual inspection of the area of a cluster on the sky. So, the parameters may be strongly biased due to the size of the detector field, and/or by contamination of field stars, and, hence, present a lower limit of the real size of a cluster. As shown by Kharchenko et al. (2005a) these data from literature are normally smaller by a factor of two with respect to cluster radii drawn from the analysis of a uniform membership based on photometric and spatio-kinematic constraints. These latter data are in turn subject to various biases (see Schilbach et al. 2006) and need to be reduced to a uniform system in order to allow physical insight into structural properties of the population of galactic open clusters.

Besides the morphological description of a cluster itself, the structural parameters carry important information on its basic physical properties like mass, and on the surrounding galactic tidal field (von Hoerner 1957). King (1962) has proposed an empirical set of cluster parameters as a quantifier of the structure of spherical systems, and has shown later (King 1966) that they correspond to theoretical density profiles of quasi-equilibrium configurations and suit well to stellar clusters with masses spanning from those of open clusters to globular ones. King (1962) introduced three spatial parameters r c, r k, and k, hereafter referred to as King’s parameters (r c is the so-called core radius, r k the tidal radius, and k is a profile normalization factor), fully describing the distribution of the projected density in a cluster. Since that time, this parameter set is widely used to quantitatively describe the density laws of globular clusters. Here we mention a number of studies, where King’s parameters were determined both for galactic globular clusters (e.g. Peterson and King 1975, Trager et al. 1995, Lehmann & Scholz 1997), and for extragalactic ones (Kontizas 1984, Hill & Zaritski 2006) in the SMC, or LMC (Elson et al. 1987).

The application of King’s parameters might be useful to open clusters as well, especially from the point of view of establishing a uniform scale of structural parameters and providing independent estimates of cluster masses. However, the literature on the determination of King’s parameters of open clusters is much poorer than that of globulars. We mention here the following studies based on three-parameter fits of selected clusters: King (1962), Leonard (1988), Raboud & Merrilliod (1998, 2002). Among other complicating reasons such as an insufficient stellar population, and heavy and irregular foreground/background, making the study of King’s parameters in open clusters difficult, we emphasise the difficulty of acquiring data in wide-field areas around a cluster. The latter is due to the much larger size of a typical open cluster compared to the fields of view of detectors currently used in studies of individual clusters. Recently, when a number of all-sky catalogues has become available, studies exploring the unlimited neighbourhood of clusters have been published (Adams et al. 2001, 2002, Bica et al. 2005a, 2005b, 2006, Froebrich et al. 2007) have been searching the 2MASS survey for new clusters, and provided spatial parame-
ters for all newly identified cluster candidates derived from the fitting of surface density patterns with King profiles.

Mass is one of the fundamental parameters of star clusters. There are several independent methods to estimate cluster masses. Each of them has its advantages or disadvantages with respect to the other ones. But so far there is no method, which can be regarded as absolutely satisfactory. The most simple and straightforward way is to count cluster members and to sum up their masses. Since there is no cluster with a complete census of members, one always observes only a subset of cluster stars, truncated by the limiting magnitude and by the limited area covered by a study, and, therefore, masses from star counts should be regarded as lower estimates of real mass of a cluster. The extrapolation of the mass spectrum to an unseen lower limit of stellar masses along with some template of the IMF frequently applied in such studies, leads to unjustified and unpredictable modifications of the observed mass and should be avoided. The farther away the cluster is, the larger is the uncounted fraction of faint members, often residing in the cluster periphery. In fact, this method could be applied with reasonable degree of safety to the nearest clusters observed with deep, wide-area surveys, and so providing secure and complete membership. Due to its simplicity the method is currently widely accepted, and possibly it is the only technique which is applied to relatively large samples of open clusters (see Danilov & Seleznov 1994, Tadross et al. 2002, and Lamers et al. 2005).

The second method is the classical one, namely the application of the virial theorem. It gives the mass of a cluster from an estimate of the stellar velocity dispersion, and average interstellar distances. It does not require the observation and membership determination of all cluster stars. The application of the method is, however, limited to sufficiently massive stellar systems (globulars and dwarf spheroidals) with dispersions of internal motions large enough to be measurable. For open clusters with typical dispersions of the order of or less than 1 km s$^{-1}$, present-day accuracies of both proper motions and radial velocities are fairly rough and are marginally available for a few selected clusters only. In spite of this, several attempts have been undertaken for clusters with the most accurate proper motions (Jones 1970, 1977, McNamara & Sanders 1977, 1983, McNamara & Sekiguchi 1986, Girard et al. 1989, Leonard & Merritt 1989), or for clusters with mass determination from radial velocities (Eigenbrod et al. 2004).

The third method uses the interpretation of the tidal interaction of a cluster with the parent galaxy, and requires the knowledge of the tidal radius of a cluster. Considering globular clusters which, in general, have elliptical orbits, King (1962) differentiates between the tidal and the limiting radius of a cluster. For open clusters revolving at approximately circular orbits one can expect the observed tidal radius being approximately equal to the limiting one. Though a probable deviation of the cluster shape from sphericity may have some impact onto the computed cluster mass. Nevertheless, this method gives a mass estimate of a cluster (Raboud & Mermilliod 1984a, 1984b) which is independent from the results of the two methods mentioned above. Due to the cubic dependence on $r_t$, masses drawn from tidal radii are strongly influenced by the uncertainties of $r_t$, however. Taking these circumstances into account, one usually reverses the relation and calculates tidal radii from counted masses.

For our studies of open clusters we use the All-Sky Compiled Catalogue of 2.5 million stars (ASCC-2.5, Kharchenko 2001), including absolute proper motions in the Hipparcos system, $B$, $V$ magnitudes in the Johnson photometric system, and supplemented with spectral types and radial velocities if available. The ASCC-2.5 is complete down to about $V = 11.5$ mag. Based on the ASCC-2.5 we were able to construct reliable combined kinematic-photometric membership probabilities of bright stars ($V \lesssim 12$) for 520 open clusters (Kharchenko et al. 2004, Paper I), to compute a uniform set of astrophysical parameters of clusters, (Kharchenko et al. 2005a, Paper II), as well as to identify 130 new clusters (Kharchenko et al. 2005b, Paper III) in ASCC-2.5. Currently, we have a sample of 650 open clusters, which is complete within a distance of about 1 kpc from the Sun. This sample was used to study the population of open clusters in the local Galactic disk by jointly analysing the spatial and kinematic distributions of clusters (Piskunov et al. 2006, Paper IV), for an analysis of different biases affecting the apparent size of open clusters, and the segregation of stars of different mass in open clusters (Schilbach et al. 2006, Paper V).

In this paper we determine King’s parameters and tidal masses for a large fraction of open clusters from our sample to get an independent basis for the construction of a uniform and objective scale of spatial parameters and masses. In Sec. 2 we briefly discuss our input data, Sec. 3 contains the description of the pipeline we apply for the determination of King’s parameters, in Sec. 4 we construct and discuss our sample, in Sec. 5 we compare our results with published data on King’s parameters and with independent estimates of cluster masses. In Sec. 6 we summarize the results.

Figure 1. Apparent density profiles of two open clusters used for the determination of tidal radii. The Pleiades are shown in the top panel, NGC 129 in the bottom panel. The different colors indicate different samples of stars considered: black for $1\sigma$-members (group (a), see text), magenta for $2\sigma$-members (group (b)), blue for $3\sigma$-members (group (c)), and green for “$4\sigma$-members” (group (d)). Vertical lines show the radii $r_1$ (solid) and $r_2$ (dashed).
2. Data

In this study, we make use of our results on the cluster membership (Paper I), and on the parameter determination (Paper II and Paper III, respectively) for 650 open clusters. Together with other basic parameters like the position of the cluster centre, age, distance, and angular size (the apparent radii of the core \( r_1 \) and the corona \( r_2 \)), stellar density profiles in the wider neighbourhood of each cluster are available from our data. According to the membership probability \( P_{\text{ap}} \), density profiles have been constructed for four groups of stars in each cluster: (a) - the most probable members (\( P_{\text{ap}} > 61\% \)), so called 1σ-members, (b) - possible members (\( P_{\text{ap}} > 14\% \)), or 2σ-members, (c) - stars with \( P_{\text{ap}} > 1\% \), or 3σ-members, and finally, (d) - all stars in the cluster area which, for convenience, we call “4σ-members”.

For the construction of the density profiles, we count stars in concentric rings around the cluster centre up to \( 5 r_2 \), where \( r_2 \) is the apparent radius of the corona. Due to the relatively bright magnitude limit of the ASCC-2.5, the number of cluster members available for the profile construction is rather low (on average, about 45 2σ-members per cluster). In general, the number of identified cluster members decreases with increasing distance modulus of a cluster. In order to have a statistically relevant number of stars per concentric ring, one has to chose steps kinematic and photometric criteria, without spatial selection criteria, computed for each star in a cluster area by taking into account only the membership probability differs from that of \( P_{\text{ap}} \) used in Paper I.

According to the definition, \( r_c \) is the core radius, \( r_t \) is the tidal radius (approximately equal to the limiting radius in case of an open cluster), and \( k \) is a normalization factor which is related to the central density of the cluster. This approach has also been successfully applied for the determination of the King parameters in several nicely populated open clusters (see e.g., Raboud & Merrilliod [1998a, 1998b]). Nevertheless, the direct way of fitting the observed density distribution by the model (eq. (1)) does not work for the majority of clusters of our sample. The main reason is the relatively bright magnitude limit of the ASCC-2.5 and, consequently, the low number of cluster members that causes uncertainties in the observed density profiles. In order to weaken the influence of poor statistics, we use the integrated form of King’s formula:

\[
n(r) = \pi r^2 k \left\{ \ln[1 + (r/r_c)^2] + \frac{(r/r_c)^2}{1 + (r/r_c)^2} \right\}.
\]

(2)

where \( n(r) \) is the number of stars within a circle of radius \( r \). Since the spatial boundaries of observed open clusters are not clearly defined, and the proportion of field stars projected on the cluster area is relatively high, we must take special care of choosing the integration limits and the background level if using eq. (2).

Contrary to globular clusters, where the statistical errors of empirical density profiles are negligibly low, and the data fix a model safely in internal regions of the cluster area, in open clusters a fit based on the inner area is less reliable and can lead to a significant bias in the resulting tidal radius. Therefore, we must consider the behaviour of the density profile in exterior regions of a cluster and even outside the cluster limits which, a priori, we do not know. On the other hand, as one can see from eq. (1), the value of \( f(r) \) increases at \( r > r_t \) and tends to a finite limit \( f(r\to\infty) \), whereas \( n(r) \) goes to infinity for \( r \to \infty \) in eq. (2). This contradicts the physical meaning of \( n(r) \) since the number of cluster members should be finite, independently of how far one expands the counts. Therefore, for a physically correct application of eq. (2) one should complement it by a boundary condition \( n(r \geq r_t) = N \) where \( N \) is the number of cluster stars. Again, \( r_t \) and \( N \) are unknown.

In order to overcome the problem, we tried to find a range \( \Delta r \) where \( n(r) \) is practically flat (see for illustration Fig. 2 left panel). This range includes \( r_t \) and its length depends on the concentration \( c = \log r_t/r_c \). The range \( \Delta r \) degenerates to a point \( r = r_t \) at \( c = 0 \), and increases with increasing \( c \).

For each cluster, a given membership sample may include a number of field stars having, by chance, the proper motions and photometry which are compatible with those of real cluster members. Therefore, before we can try to localize \( \Delta r \) in the empirical integrated density profiles, the residual background contamination has to be removed. If not, the profiles would increase steadily with increasing \( r \). This is especially important for 3σ- and “4σ”-samples which are strongly contaminated by field stars, although, it can be essential for 1σ and 2σ-samples, too.

The background correction was done in a uniform way for all clusters and membership samples. Assuming that the majority of stars at \( r > r_2 \) are not cluster members, we took the average density of a given membership group in a ring \( r_2 < r < 2 r_2 \) to be the initial background level in the internal cluster area (\( r < r_2 \)). Outside \( r_2 \) (\( r_2 < r < 5 r_2 \)), the initial background was first set to the observed stellar density in each radial bin. The final, smooth background profile over the complete area \( 0 < r < 5 r_2 \) was then recomputed as a running average of the initial values.

3. Determination of tidal radii and cluster masses

3.1. Fitting cluster profiles

The method we apply is based on the well-known empirical model of King (1962), describing the observed projected density profiles \( f(r) \) in globular clusters with three parameters \( r_c \), \( r_t \), and \( k \):

\[
f(r) = k \left\{ 1 + (r/r_c)^2 \right\}^{-1/2} - 1 - (r/r_c)^2 \right\}^{-1/2} \right\}.
\]

(1)

According to King’s definition, \( r_c \) is the core radius, \( r_t \) is the tidal radius (approximately equal to the limiting radius in case

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\footnote{\( P_{\text{ap}} \) is a newly defined combined membership probability computed for each star in a cluster area by taking into account only the kinematic and photometric criteria, without spatial selection criteria, \( P_{\text{ap}} = \min\{P_u, P_p\} \). Note that this definition of the combined membership probability differs from that of \( P_{\text{ap}} \) used in Paper I.}

\footnote{ftp://cdsarc.u-strasbg.fr/pub/cats/J/A+A/438/1163/atlas and http://cdsarc.u-strasbg.fr/pub/cats/J/A+A/440/403/atlas}
The squares is allowed to be greater than the probability that the difference between the observations and the empirical parameters together with their Poisson errors. The vertical (blue) solid and dashed lines mark for the background (see text) is indicated by crosses, the bars are shown by the short-dashed line. The empirical profile corrected as indicated. Right panel: the density profiles of the open clusters with a filter size of 0.55 and a step of 0.05. Finally, the background profile was subtracted from the original density distribution. An example of an observed profile before and after background correction is shown in Fig. 2 right panel. Except for a few cases of poor and extended clusters projected on heavy and variable background, we obtained reasonable results.

The general scheme of the profile fitting is shown in Fig. 2(left) and is explained in the following:

**Step 1:** we start with the determination of an initial value \( r_0 \) of the fit radius \( r_{fit} \). Assuming the empirical cluster sizes \( r_1 \) and \( r_2 \) to be \( r_e \) and \( r_f \) respectively, we compute \( r_0 \) from King's profile of the concentration class \( \tilde{c} = \log r_f/r_e \) as the distance from the cluster centre where the profile does not differ from \( n(r) \) by more than an assumed tolerance \( \theta \). The tolerance is chosen depending on the observation quality of a given cluster, and is defined as the average of the Poisson errors of the profile outside \( r_f / \theta = (\sigma_r / r > r_f) \), \( \sigma_r = \sqrt{n(r)} \).

**Step 2:** we now are able to apply a nonlinear fitting routine based on the Levenberg-Marquardt optimization method (Press et al. [1993]) to eq. (2) for \( r = 0...r_f \). As initial-guess parameters we chose \( r_0 = r_1 \) and \( \tilde{c}^0 \), whereas \( \tilde{c}^0 \) is obtained from the solution of eq. (2) at \( r = r_e \). Empty bins in the differential density distribution were omitted from the integrated profile fitting. As a result of successive iterations, we get King's parameters together with their \( \text{rms} \) errors and a \( \chi^2 \)-value. The iterations are stopped when two successive \( \chi^2 \)-values do not differ by more than \( 10^{-3} \) and the solution is accepted when the number of iterations is less than 100. Then we compute the \( \chi^2 \)-probability function \( Q(\chi^2, \nu) \) which can be used as a measure of the goodness of fit. For a given degree of freedom \( \nu \), \( Q(\chi^2, \nu) \) is the probability that the difference between the observations and the fitted function can be considered random, and their sum of the squares is allowed to be greater than \( \chi^2 \). According to Press et al. [1993], the fit can be accepted when \( Q(\chi^2, \nu) > 0.1 \).

**Step 3:** in order to check whether a better convergence can be achieved, we run the fitting routine (i.e. **Step 2**) for different \( r_{fit} \) ranging from \( r_2 \) to \( 3r_2 \). If several acceptable solutions (i.e. \( Q(\chi^2, \nu) > 0.1 \)) were obtained, we selected that one which yielded the smallest \( \text{rms} \) errors in \( r_e \) and \( r_2 \).

The complete pipeline including the background elimination and the profile fitting has been applied to all four membership classes. Although the initial guess \( r_0 \) of the fit radius is usually \( \approx 1.5 \, r_2 \), the final fit radius \( r_{fit} \) turns out to be about \( 2 \, r_2 \), and the best \( \tilde{c}_r \) ranges from \( 1 \, r_2 \) to \( 2 \, r_2 \).

However, we must keep in mind that eqs. (1), (2) assume spherical symmetry in the spatial distribution of cluster stars, whereas a real open cluster is expected to have an elongated form with the major axis directed towards the Galactic centre (Wielen [1974]). Thus, depending on the orientation of the line of sight, an observer measures a projection rather than the real size of a cluster, and the values of \( r_1 \) derived via eqs. (1), (2) give, in general, lower limits of the tidal radii.

### 3.2. Cluster masses

According to King [1962], the mass \( M_c \) of a cluster at the galactocentric distance \( R_G \) follows the relation

\[
M_c = \frac{4 \, A \,(A-B) \, r_t^3}{G},
\]

where \( A, B \) are Oort's constants valid for \( R_G \), and \( G \) is the gravitational constant (see Standish [1995]). Since the bulk of our clusters is located within 2 kpc from the Sun, the linear approximation to the velocity field of the disk seems to be reasonable. Thus, Oort’s constants could be easily expressed by their local values \( A_0 = 14.5 \pm 0.8 \, \text{km/s/kpc}, B_0 = -13.0 \pm 1.1 \, \text{km/s/kpc} \) derived in Paper IV for the open cluster subsystem

\[
A = A_0 - A \delta R_G
\]

\[
A = A_0 - B - B_0 - 2A \delta R_G
\]

where \( \delta R_G = (R_G - R_{G,0})/R_{G,0} \), and \( R_{G,0} = 8.5 \, \text{kpc} \).

A relative random error of the cluster mass \( \delta M_c = \varepsilon M_c / M_c \) can be derived from eq. (3) as

\[
\delta M_c^2 = 9 \, \delta r^2 + \left( \frac{2A - B}{A - B} \right)^2 \delta r^2 + \left( \frac{B}{A - B} \right)^2 \delta B^2
\]

(4)
Table 2. Sample table of King parameters and tidal masses for 236 open clusters. The full table is available in machine readable form only. See text for further explanations.

| COCD Cluster | l | b | d | $E_{B-V}$ | log t | r1 | n1 | n2 | r⊙ | $e_{r⊙}$ | r⊙ | $e_{r⊙}$ | k | $e_{k}$ | log m | $e_{log m}$ |
|---------------|---|---|---|---------|-------|----|----|----|-----|---------|----|----------|---|-------|-------|----------|
| 3 Blanco 1    | 14.14 | -79.02 | 269 | 0.01 | 8.32 | 0.70 | 2.90 | 4 | 2 | 53 | 1.5 | 0.2 | 20.0 | 3.4 | 3.2 | 0.4 | 3.475 | 0.224 |
| 4 Alessi 20   | 117.64 | -3.69 | 450 | 0.22 | 8.22 | 0.12 | 0.30 | 2 | 2 | 14 | 0.5 | 0.3 | 3.4 | 1.3 | 10.3 | 5.7 | 1.152 | 0.504 |
| 8 NGC 129     | 120.27 | -2.54 | 1625 | 0.55 | 7.87 | 0.11 | 0.27 | 4 | 1 | 12 | 2.2 | 1.1 | 8.9 | 1.5 | 1.0 | 0.4 | 2.311 | 0.226 |
| 11 Berkeley 4 | 122.28 | 1.53 | 3200 | 0.70 | 7.08 | 0.08 | 0.20 | 3 | 4 | 19 | 4.3 | 2.0 | 14.6 | 2.7 | 0.5 | 0.2 | 2.815 | 0.244 |
| 13 Alessi 1   | 123.26 | -13.30 | 800 | 0.40 | 8.85 | 0.13 | 0.45 | 4 | 1 | 19 | 1.0 | 0.3 | 9.1 | 2.3 | 2.3 | 1.3 | 3.590 | 0.340 |
| 15 Platais 2  | 128.23 | -30.57 | 201 | 0.05 | 8.54 | 1.70 | 1.57 | 2 | 2 | 18 | 4.4 | 1.2 | 8.9 | 1.1 | 1.2 | 0.4 | 2.411 | 0.166 |
| 17 NGC 457    | 126.63 | -4.37 | 2499 | 0.47 | 7.38 | 0.12 | 0.26 | 4 | 2 | 26 | 1.2 | 0.6 | 13.1 | 3.1 | 2.6 | 1.6 | 2.729 | 0.310 |
| 22 NGC 663    | 129.46 | -0.94 | 1952 | 0.78 | 7.14 | 0.10 | 0.30 | 4 | 1 | 24 | 3.1 | 1.1 | 15.3 | 3.3 | 0.8 | 0.2 | 2.965 | 0.280 |
| 23 Collinder 463 | 127.28 | 9.40 | 702 | 0.30 | 8.35 | 0.22 | 0.72 | 4 | 3 | 77 | 3.2 | 0.4 | 12.4 | 1.1 | 3.3 | 0.3 | 2.803 | 0.124 |

4. King’s parameters: Results

4.1. Construction of the output sample

For 236 out of 290 clusters in the input list, we obtained at least one set of King’s parameters by applying the method described in Sec. 3.1. Depending on the membership group, each of these clusters got from one to four different solutions, and the total number of solutions was 708. Per cluster, we have at least one set of parameters which are larger than their rms errors, i.e. with relative errors of $\delta_{A} > 0.06, \delta_{B} \approx 0.08$. Taking further into account that typically $\delta_{c} > 0.1$ (see Sec. 2), one finds that the relative error of mass determination is dominated strongly by the uncertainties of the tidal radius i.e.,

$$9 \delta_{r}^2 \gg \left( \frac{2A - B}{A - B} \right)^2 \delta_{A}^2 + \left( \frac{B}{A - B} \right)^2 \delta_{B}^2,$$

and therefore

$$\delta_{ML} \approx 3 \delta_{r}, \quad (5)$$

As expected, the relative errors in the determination of the parameters $\delta_{c}, \delta_{r}, \delta_{k}$ are largest for the 1σ-solution. The 1σ-sample contains the most probable cluster members, but their number is relatively low compared to the other membership samples. This is the reason for relatively large Poisson errors and, consequently, for higher rms errors in the fitted parameters.

The goodness of a fit is given by the $Q(\chi^2, \nu)$-probability, an output parameter of the fitting pipeline (see Sec. 3.1). The normalized $Q(\chi^2, \nu)$-parameters averaged over 114 clusters are also given in Table 1. As can be seen, $Q(\chi^2, \nu)$ does not depend on the richness of the sample and indicates a more suitable fitting with 1σ- and 2σ-samples than with $‘4\sigma’$-groups.

Based on the statistics in Table 1, we chose the following ranking of the solutions. We give the highest weight to the solutions with the largest $Q(\chi^2, \nu)$-probability. If, for a given cluster there are more than one solutions of the same quality (i.e., the $Q(\chi^2, \nu)$-probabilities differ by less than 0.1%), we use an additional criterion based on $\chi^2$-values supplied by the fitting pipeline. Since $\chi^2$ does depend on the sample size $n$ and on the degree of freedom $\nu$, a readjusting of the $\chi^2$-estimate is needed when we compare fitting results derived with different membership samples. Therefore, we select a solution with a smaller value $\sum \chi^2_i = \chi^2/(\nu \times n) which is, in fact, an average mean square deviation of the observed from the fitted profiles in units of Poisson errors computed per star. In a few cases when even the $\sum \chi^2_i$-parameters differ insignificantly (by less than 0.001) for two solutions, we give priority to the solution with smaller error $\varepsilon_{\chi^2}$. According to the selection procedure, the solution from the 1σ-samples gets the best ranking in 94 cases out of 236, from 2σ-samples - in 72 cases, from 3σ-samples - in 39 cases, and from $‘4\sigma’$-samples - in 31 cases.

The data on the structural parameters are compiled in a table which is available in machine-readable form only. As an example, Table 4 lists a few entries of the complete data. Column
better than 20%. Therefore, for these clusters, we expect to obtain reasonable estimates of masses from the present data (cf. eqs. (4), (5)). As a rule, clusters which are more distant and/or less populated got relatively large errors in \( r_t \). In these cases the method reaches its limitations. Probably, if applied to deeper photometric data, the method may provide acceptable results for a large portion of these clusters. Nevertheless, there is a number of clusters with “irregular” density profiles, and, fitting them by a model proposed for spherical systems in equilibrium, does not have much prospect of success.

In order to give the reader an idea of typical profiles we show, in Fig. 4 a set of different cases selected from the final sample. Compared to the empirical cluster radius \( r_2 \), the tidal radius \( r_t \) ranges between 1 \( r_2 \) and 2 \( r_2 \). For clusters with relatively accurate radii (\( \delta r_c < 0.33 \) and \( \delta r_t < 0.33 \)), the averaged relation is

\[
{ r_t = (1.54 \pm 0.02) \times r_2. }
\]

Therefore, we conclude that the empirical parameter \( r_2 \) scales the tidal radius reasonably well.

4.3. Cluster masses

Using eq. (3) and the tidal radius \( r_t \), we estimated masses for each of the 236 open clusters. The results are shown in Fig. 5 where the distribution of clusters over mass is given in panel a), whereas panel b) shows the distribution of relative errors in mass. Most of the clusters in our sample have masses in a range \( \log M_c/m_\odot = 1.6 - 2.8 \), though a few clusters have masses as small as \( 10 m_\odot \). Three objects got masses of about \( 10^7 m_\odot \). They are the associations Nor OB5, Sco OB4 and Sco OB5. For 139 clusters, the masses were obtained with a relative accuracy better than 60%. Their distribution shows the same features as the complete sample.

We note that the masses based on \( r_1 \) from eq. (2) do not take into account a possible flattening of clusters which arises due to the tidal coupling with the Milky Way. Because of this, a stellar cluster has a shape of a three-axial ellipsoid with the major axis oriented in the direction of the Galactic centre. In general, we obtain a projection of the tidal radius on the celestial sphere from eq. (2), and the relation between the tidal radius and this projection depends on the mutual position of the Sun and a given cluster. Comparing masses of two clusters with different location in the Galactic disk, the corresponding effect must, therefore, be taken into account.

5. Comparison with other determinations

5.1. King’s radii from a three parameter fit

In contrast to massive spherical systems, there are only a few results reported on the determination of structural parameters of open clusters via direct parameter fits of King’s profiles to the observed density distributions. Some of them consider remote clusters (e.g., King 1962, Leonard 1988), or newly detected cluster candidates (Froebrich et al. 2007) which are absent in our cluster sample. Others are based on a two parameter fit (Keenan 1973, Bica 2006, Bonatto et al. 2005b), and, therefore, these results cannot be compared directly with ours. We found only seven papers where a three parameter fit was applied to open clusters. In these papers only four clusters are in common with our sample. These are two nearby clusters, the Pleiades and Praesepe, NGC 2168 (M35) at 830 pc from the Sun, and the relatively distant cluster NGC 2477, at 1.2 kpc.

Figure 3. Distributions of King’s radii and their relative errors for 236 open clusters. Panels (a) and (b) are for the core radius \( r_c \) and the tidal radius \( r_t \) measured in pc. Distributions of relative errors in core radius \( \delta r_c \) and in tidal radius \( \delta r_t \), are shown in panels (c) and (d), respectively.

1 gives the cluster number in the COCD catalogue, columns 2 through 9 are taken from the COCD, while columns 10 through 20 include the information obtained in this work. For each of the 236 clusters we give: name (2), galactic coordinates (3, 4), the distance from the Sun in pc (5), the reddening (6), the logarithm of the cluster age in years (7), empirical angular radii (in degrees) of the corona \( r_1 \) (8) and of the corona \( r_2 \) (9). Column (10) is the number of the acceptable solutions of King’s parameters from the four membership groups. Column (11) gives the number of the membership sample \( (1, 2, 3, 4, \text{ or } \text{“}4\sigma\text{”}) \) chosen by the selection procedure as providing the best solution, whereas column (12) is the number of cluster members of this sample within \( r_2 \) after background correction. Columns 13 through 18 give the corresponding King parameters \( r_c, r_t, k \) with their \( \text{rms} \) errors. The parameters are taken as obtained from the selected solution. Depending on the applications desired, the reader is advised to take into account the empirical relations between the selected solution for \( r_t \) and \( k \) and the membership sample used to obtain these parameters (cf. Table 1 and eq. (6)). Finally, columns (19, 20) provide the logarithm of the cluster mass and its \( \text{rms} \) error computed from \( r_t \) by use of eq. (3) and discussed in Sec. 4.3.
Figure 4. Examples of radial profiles from our final sample ordered by the priority of the solution. The uppermost row represents the $1\sigma$-sample. The second row is for $2\sigma$-solutions, the third and forth rows illustrate $3\sigma$ and $4\sigma$ samples. The crosses show empirical data corrected for the background, the bars are Poisson errors. The vertical lines indicate the derived radii. The solid lines show $r_2$ and $r_c$, the broken lines $r_2$ and $r_t$. The empirical parameters are shown with blue color, while the fitted data are shown with red color. Horizontal bars indicate the value of the $\text{rms}$ error of the parameter. The solid curve is the fitted King profile, the broken curve (yellow) is the total profile not corrected for background.

In Table 3 we compare the results obtained in this paper with those found in the literature for the four clusters. Column 1 is the cluster identification. In columns 2 through 6 we provide our results: in column 2 we give the number of members within the empirical cluster radius $r_2$, column 3 is for $r_{fit}$, i.e. a radius found as best suited for the profile fitting (cf. Sec. 3.1). The results of the fitting, King’s radii $r_c$ and $r_t$ are listed in columns 4 and 5 together with their $\text{rms}$ errors, whereas the corresponding tidal mass is given in column 6. Columns 7 through 10 show the data from literature: columns 7 and 8 give the number of stars used to construct the density profiles and the radius of the area considered, respectively. The King radii are listed in columns 9 and 10. Note that we are not able to keep a uniform format for these data due to the different presentation of the results in different papers.
Table 3. Comparison of our results with literature data on King’s parameters for the clusters in common

| Cluster       | $n_r$ | $r_{fit}$ | $r_c$ | $r_t$ | $M_C$ | $N^*$ | $r_{area}$ | $r_c$ | $r_t$ | note | ref |
|---------------|-------|-----------|-------|-------|-------|-------|------------|-------|-------|------|-----|
| Pleiades      | 219   | 14        | 1.4 ± 0.1 | 20.5 ± 1.3 | 3107 ± 638 | 270 | 6 | 1.4 ± 0.5 | 16 ± 7 | 1 |
| Praesepe      | 154   | 9         | 0.8 ± 0.1 | 17.1 ± 1.2 | 1806 ± 428 | 1067 | 3 | 0.9 ± 0.9 | 13.1 | a |
| NGC 2168      | 186   | 1.95      | 1.8 ± 0.2 | 17.9 ± 2.3 | 724 ± 677 | 185 | 4 | 1.0 ± 0.5 | 11.1 ± 4.9 | 4 |
| NGC 2477      | 10    | 0.55      | 1.9 ± 1.0 | 6.5 ± 1.2 | 89 ± 51 | 1000 | 3.8 | 3.5 | 16 | c |

Notes: (a) $- r_c$ is computed for member groups of different mass, $r_t$ is derived from the cluster mass via eq. (3); (b) $- r_c$, $r_t$ are obtained from the distribution of low-mass stars ($m < 1 m_\odot$); the original results are given in angular units and are transformed to linear sizes by us; (c) $- r_c$, $r_t$ are obtained from the distribution of low-mass stars ($m < 1 m_\odot$); (d) the area is a square of 0.5 x 0.5 sq.deg.

References: 1– Raboud & Mermilliod (1998a), 2– Pinfield et al. (1998), 3– Adams et al. (2001), 4– Raboud & Mermilliod (1998b), 5– Adams et al. (2002), 6– Leonard & Merritt (1989), 7– Eigenbrod et al. (2004).

Figure 5. Distributions of cluster masses (panel (a)) and their relative rms errors (panel (b)). Clusters with accurate masses ($\delta M < 60\%$) are shown as the dark histogram in panel (a).

We found three different estimates of the tidal radius published for the Pleiades. A direct comparison with our results can be done only with radii obtained by Raboud and Mermilliod (1998a) who consistently applied the 3-parameter fitting technique both to various sub-samples of the Pleiades stellar population and to the total membership sample. From Table 3 we conclude that their findings of $r_c$ and $r_t$ coincide well with our results. A slightly different method was applied by Pinfield et al. (1998) to derive a tidal radius for the Pleiades. They used differential density profiles, which were constructed for cluster members falling in different mass ranges, and tidal radii computed for each of these sub-samples. The results were used to derive the partial and total masses of the cluster. The tidal radius for the cluster as a whole was then computed from the cluster mass via eq. (3). A similar approach was used by Adams et al. (2001) who applied a 3-parameter fit to a sample of low mass ($m < 1 m_\odot$) members of the Pleiades. Again, the tidal radius $r_t$ was derived from stellar mass counts in the cluster by eq. (3). The approach by Pinfield et al. (1998) and Adams et al. (2001) provides a smaller value for $r_t$ and larger value for $r_c$ (for Pleiades members less massive than 1.2 $m_\odot$) than a direct 3-parameter fitting of the density distribution applied to the full sample of cluster members. Nevertheless, taking into account that the three studies and ours are based on observations of different spatial and magnitude coverage, and on an independent membership evaluation, the agreement between the results is quite acceptable.

We arrive at similar conclusions in the case of Praesepe. Our estimate for $r_t$ is compatible with the result by Raboud & Mermilliod (1998b) within the rms errors, it coincides well with the finding of Adams et al. (2002) who, as in the case of the Pleiades, fitted the King profile to low-mass ($m = 0.1 − 1 m_\odot$) cluster members. With respect to $r_c$, we achieved good agreement with the estimate by Raboud & Mermilliod (1998b) whereas $r_c$, from Adams et al. (2002) is significantly higher. Such a systematic difference can possibly be explained by the considerably deeper survey used by Adams et al. (2002), in studies of the Pleiades and the Praesepe. Since their empirical profiles are based on USNO POSS I E and POSS II F plate scans, and on the 2MASS catalogue, the input data are dominated by lower mass stars which, due to the energy equipartition process, show a wider distribution than the more massive stars.

For the last two clusters in Table 3 there are only two papers reporting the determinations of King’s parameters. Leonard & Merritt (1989) studied the central area of the NGC 2168 cluster. Therefore, they were able only to set probable limits for the cluster radii. Our estimates of $r_c$ and $r_t$ fit these limits well. For the relatively distant and rich cluster NGC 2477, the King radii were published by Eigenbrod et al. (2004). The authors defined the membership sample on the basis of radial velocities of numerous red giants and constructed density profiles for groups of stars of various masses. The core and tidal radii of the cluster were determined from the comparison of the structure parameters of single groups. Also in this case, their results are in good agreement with our estimates of $r_c$ and $r_t$, though NGC 2477 belongs to the poorer clusters in our sample due to its large distance from the Sun. With 10 members within $r_2$, NGC 2477 satisfies marginally our constraints. Nevertheless, the fitted parameters coincide well with the corresponding estimates obtained with the much deeper survey ($V < 17$) by Eigenbrod et al. (2004).

5.2. Various scales of cluster masses

Although the adopted values of Oort’s constants and of a distance to a cluster can slightly influence its mass estimate, the tidal radius is the major source of uncertainty in the mass determination via eq. (3), simply due to the cubic power relation between tidal radius and cluster mass. Analysing different groups of the Pleiades members, Raboud & Mermilliod (1998a) concluded that the tidal mass of the Pleiades is about 1400$m_\odot$ with a 1σ confidence interval of [530, 2900]$m_\odot$. With Oort’s
constants $A = 15$ km/s/kpc, $B = -12$ km/s/kpc and a cluster distance of 125 pc adopted by the authors, a strict application of eq. (3) would provide a tidal mass of $1620 m_{\odot}$ ($330, 4600 m_{\odot}$) for the Pleiades. From a similar analysis for Praesepe, Raboud & Mermilliod (1998b) derived a tidal mass of $440 m_{\odot}$ ($157, 987 m_{\odot}$) (a consequent use of eq. 3) would give $520 m_{\odot}$ ($90, 1570 m_{\odot}$)). We conclude that the disagreement between our estimates and those of Raboud & Mermilliod (1998a, 1998b) for the Pleiades and Praesepe are mainly caused by statistical uncertainties in the determination of the tidal radii (cf. Table 3).

In the case of NGC 2477, the situation is less clear. For this cluster, Eigenbrod et al. (2004) estimated the tidal and virial masses to be $M_{\text{tid}} = 5400 m_{\odot}$ and $M_{\text{vir}} = 5500 m_{\odot}$, respectively. This result differs considerably from our estimate. Moreover, it is in contradiction to the value of their tidal radius of $r_{\text{tid}} = 8.1$ pc which according to eq. (3), should give a tidal mass of $M_{\text{tid}} \approx 170 m_{\odot}$. Therefore, the coincidence achieved by Eigenbrod et al. (2004) between the tidal and virial masses should be considered with caution. We note that NGC 2477 is still badly studied, and its membership is poorly established. Therefore, a contamination of the stellar sample from Eigenbrod et al. (2004) is well probable. This could be one reason for an overestimation of the velocity dispersion and, consequently, of the virial mass of the cluster. The presently large uncertainties in kinematical data for determining virial masses can also cause these discrepancies. (see Appendix A for more detail). On the other hand, this large disagreement can be partly explained by an underestimation of the tidal radius: if this relatively distant cluster is subject to strong mass segregation, low mass stars on the cluster edges can be beyond the magnitude limit even in a deep survey.

Since literature data on tidal masses of open clusters are rather scarce, we looked for recent publications on cluster masses estimated with other methods. We omit here a discussion on the determination of virial masses of open clusters and refer the reader to Appendix A where this method is discussed in more detail. In order to compare our results on cluster masses, we consider only those publications where cluster masses are obtained for a relevant number of open clusters rather than for a single cluster. Under these constraints, we found three publications on mass determination for galactic open clusters, Danilov & Seleznev (1994), Tadross et al. (2002), and Lamers et al. (2005). In Fig. 6 we compare the different results on mass estimates.

Danilov & Seleznev (1994) derived masses for 103 compact, distant (>1 kpc) clusters from star counts down to $B = 16$ from homogeneous wide-field observations with the 50-cm Schmidt camera of the Ural university. For each cluster, the authors estimated the average mass of a star observed in a cluster and then computed the total visible cluster mass. The average mass is found either from star counts, or from an extrapolation of the Salpeter IMF down to the magnitude limit $B = 16$. On one hand, cluster IMF should be underestimated due to their magnitude limited survey, and the bias should increase with increasing distance modulus of a cluster. On the other hand, without membership information, the masses could be overestimated for clusters located at relatively low distances from the Sun. In a certain respect, these biases may partly compensate each other.

Based on UBV-CCD observations compiled from the literature, Tadross et al. (2002) redetermined ages and distances for 160 open clusters, and derived cluster masses from counts of photometrically selected cluster members. Since they used observations taken with different telescopes, i.e., for different clusters one expects different limiting magnitudes, it is rather difficult to estimate possible biases. In any case, a large portion of clusters should get underestimated masses due to the relatively small area of the sky usually covered by CCD observations in the past.

Lamers et al. (2005) used data from the COCD for the determination of cluster masses. For each cluster, the authors normalized the Salpeter IMF in the mass range of stars present in COCD ($V < 11.5$), and the normalized Salpeter IMF was extrapolated from large masses down to $m = 0.15 m_{\odot}$. For distant clusters with $V - M_V > 8$, the extrapolation was done over a very large range, from masses larger than $\approx 1.5 m_{\odot}$ (or $M_V < 3.5$) to masses of $0.15 m_{\odot}$ (or $M_V \approx 13$), where the IMF is still not very well known. If the IMF at low masses were flatter than

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{figure6.png}
\caption{Comparison of tidal masses $M_c$ of open clusters derived in the present study with literature values on counted- and IMF-scaled masses as a function of the distance modulus. From top to bottom we show ratios of: counted masses of Danilov & Seleznev (1994) ($M_D$ to tidal masses); counted masses of Tadross et al. (2002) ($M_T$ to tidal masses); and the IMF-scaled masses of Lamers et al. (2005) ($M_L$ to tidal masses). The last panels show intercomparisons of literature data.}
\end{figure}
the Salpeter IMF (see e.g. Kroupa et al. [1993]), the approach by Lamers et al. [2005] would give overestimated cluster masses. Moreover, in a segregated cluster one should expect different forms of the mass function in the central area and at the edges. Integration of the Salpeter IMF over the complete cluster area would, also, result in overestimating the cluster mass. A comparison of our cluster masses with those of Lamers et al. [2005] is especially interesting: since both papers use the same observational basis, we can estimate uncertainties caused by the different approaches.

According to our determination of cluster masses based on tidal radii, we expect possible biases for relatively distant clusters where we observe only a tip of bright stars. Especially in segregated clusters, due to energy equipartition the brightest stars are more concentrated to the cluster centre, and do not reproduce the correct tidal radius. Nevertheless, for distant clusters we can see a distance-dependent effect only in the upper panel of Fig. 6, where we compare our masses with those of Danilov and Seleznev [1994]. Since the $M_D/M_C$-relations decrease with distance modulus, we suppose that our method manages biases better than the approach by Danilov and Seleznev [1994]. Moreover, the dependence of the $M_D/M_C$-relations on distance modulus may be explained alone by the biases in the mass determination by Danilov and Seleznev [1994] described above, i.e. overestimated masses for clusters at low distances and underestimated masses for distant clusters. This interpretation seems to be plausible, but we cannot exclude completely distance-dependent biases in our determinations.

Furthermore, due to a possibly elongated form of open clusters (cf. Sec. 3), our estimates provide a lower limit for cluster masses. This can be one of the reasons for systematic differences to cluster masses from Lamers et al. [2005]. Except for nearby clusters with $V - M_V < 8$, their masses are, on average, larger by a factor of 10. On the other hand, masses from Lamers et al. [2005] are systematically larger than masses from Danilov and Seleznev [1994] and Tadross et al. [2002], too. Therefore, we can not exclude that the approach by Lamers et al. [2005] is, at least partly, responsible for these differences.

### 6. Conclusions

In an ideal case, i.e. if, for a given cluster, the membership is determined with certainty and completeness, the cluster mass could simply be derived from counting masses of individual members. In the future, with deeper surveys and an increasing accuracy of kinematic and photometric data, this primary method will provide sufficiently accurate and uniform mass estimates for a significant number of open clusters in the Galaxy. At present, however, there is no way to measure the masses of open clusters directly. The methods currently applied require a number of assumptions, and depending on the method and assumptions used, the results can differ by a factor of 100 for individual clusters (cf. Fig. 3). Therefore, the determination of cluster masses is still a very challenging task.

Our aim was to estimate masses for a larger number of clusters by applying a uniform and possibly objective method, and to obtain an independent basis for statistical studies of the distribution of cluster masses in the Galaxy. In our work we could benefit from the homogeneous set of cluster parameters derived for 650 open clusters with good membership based on the astrometric and photometric data of the ASCC-2.5. The estimation of cluster masses was done via tidal radii determined from a three-parameter fit of King’s profiles to the observed density distribution (King [1962]). This method is weakly dependent on assumptions and can be applied to all spherical systems in equilibrium with well-defined density profiles. Since these requirements are not always met in the case of open clusters, we could obtain solutions only for 236 clusters, i.e. for less than half of the clusters in our sample. However, this number is considerably larger compared to the small number of clusters with tidal masses determined before.

The main difficulties in the practical application of King’s model to open clusters arise from the relatively poor stellar population (compared to globular clusters) and from the higher degree of contamination by field stars in the Galactic disk. Using an all-sky survey, we could rely on the completeness of data in the selected sampling areas, down to the limiting magnitude of the ASCC-2.5. Further, since we were free in selecting the size of the sampling areas, we were able to optimize the boundary condition for each cluster as well as possible. As a result, we could partly decrease the influence of the above mentioned problems in applying King’s method and improve the solutions by taking into account the outermost regions of the clusters and by excluding residual field stars from the solution.

Together with a realistic membership based on both kinematic and photometric constraints, a good profile fitting could be achieved even for clusters with a relatively low number of members. The highest quality of the fitting (goodness-of-fit) was achieved with the best-determined membership sample (so-called “1σ-members”) and, hence, a low contamination by field stars. However, it turned out that the membership criterion alone did not have very strong impact onto the values of the fitted parameters $r_t$ and $r_e$ themselves. In fact, for compact and relatively distant clusters, we sometimes found the best results without a preliminary membership selection (so called “2σ-members”). In conclusion, this paper could be seen as a justification for a simple application of King’s method to observed brightness profiles of compact open clusters no matter if membership is determined or not, provided that the observed density profiles are properly corrected for the background.

### Acknowledgements

We are grateful to Henny Lamers for providing us with unpublished data on cluster masses. This study was supported by DFG grant 436 RUS 113/757/0-2, and RFBR grant 06-02-16379.

### Appendix A: Virial masses of open clusters: current status

All our attempts failed to compute reasonable cluster masses from the dispersion of proper motions and/or radial velocities taken from the ASCC-2.5 catalogue for cluster members. The main reason is the accuracy of kinematical data, which is still too low in current all-sky surveys.

Up to now, the best published data on velocity dispersions were obtained for a few open clusters from proper motions obtained from long-term observations with the Yerkes 40-inch refractor (F = 19.3 m, a scale of 10.7 arcsec/mm, a typical epoch difference of more than 55 years, and a typical $r_m$-error of the proper motions of a few 0.1 mas/s). The clusters are the Pleiades (Jones [1970], Praesepe (Jones [1971]), NGC 6705 (McNamara & Sanders [1977]), NGC 6494 (McNamara & Sanders [1983]), NGC 2168 (McNamara & Sekiguchi [1986], NGC 2682 (Girard et al. [1989]). In these papers, the internal proper motion dispersions are corrected for different biases, and virial as well as counted masses are determined for four clusters i.e., for the Pleiades, Praesepe, NGC 6494, NGC 6705. The results are shown in Fig.A.1.

Independent of the methods of mass determination, the masses in Fig. A.1 show a correlation with cluster distance.
Since the number of clusters is very low, the correlation does not need to be real, but can appear by chance, due to the small sample. In order to understand the effect, we transformed the proper motion dispersions \( \sigma_\mu \) published for 6 clusters to one-dimensional tangential velocity dispersions \( \sigma_v \) via \( \sigma_v = 4.74 \times d \sigma_\mu \), where \( d \) is the distance of a cluster adopted in the original papers. In the upper panel of Fig. A.1 we show \( \sigma_\mu \) and \( \sigma_v \) as functions of distance \( d \). Whereas \( \sigma_\mu \) is independent from the cluster distance, the tangential velocity dispersion \( \sigma_v \) indicates a strong correlation which is well described by a first-order polynomial. Therefore, we suppose a non negligible random component in \( \sigma_\mu \) resulting rather from residual \( \text{rms} \) errors in proper motions than from the internal velocity dispersion. This is not surprising, because, for all clusters at distances larger than the Pleiades and Praesepe, the dispersions are of the order of the \( \text{rms} \) errors of the proper motion measurements. Assuming that the internal velocity dispersion \( \sigma_v \) were similar for all these clusters, we obtained \( \sigma_v = 0.31 \text{ km/s} \) by extrapolation of the linear regression polynomial to \( d = 0 \). In other words, we need tangential velocities determined with an accuracy better than 0.3 km/s (or proper motions with an accuracy better than 0.06-0.07 mas/yr for a cluster at \( d = 1 \) kpc) for a more or less reliable estimate of its virial mass. Even the Hipparcos proper motions with typical \( \text{rms} \) errors of 1 mas/yr do not meet this requirement.

With respect to radial velocities, one may suppose that these give a better basis for the determination of the internal velocity dispersion since their accuracy does not depend on distance. Recently, radial velocities have been measured in several open clusters for a sufficient number of cluster members (see e.g., Eigenbrod et al. 2004 for NGC 2477, or Fürész et al. 2006 for NGC 2264). The derived velocity dispersions are, however, somewhat too large, 0.93 km/s in NGC 2477, and 3.5 km/s in NGC 2264. This may be a consequence of biases which still affect the data and which are very difficult to take into account. Among them may be the contamination by field stars and by unresolved binaries, or motions within stellar atmospheres.

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