Load Distribution Analysis for The Plastic Helical Gear Meshing with Steel Worm

YUN Yonghu¹² HU Hong¹ TA Jingning²

¹ Shenzhen Graduate School, Harbin Institute of Technology, Shenzhen Guangdong, 518005, China
² Johnson Electric Company Corporate Engineering, Shenzhen Guangdong, 518125, China

Abstract. This paper presents an approach for the analysis of load distribution of plastic helical gear meshing with steel worm. Based on the deformation compatibility assumption in the gear mesh process, the load on the contact point of each gear is calculated. By calculating the applied load on each gear tooth in a meshing cycle, the load distribution on the gear tooth is obtained. As compared with the finite element method, the presented method result correlates well with the simulation result with the ANSYS Workbench. The method presented in this paper realizes the fast calculation of meshing force between plastic helical gear and steel worm, which can provide some references for gear design and gear loading capacity improved in engineering.

1. Introduction

The helical gear and worm drive pairs are commonly used in the speed reduction mechanism of automotive motor. And the helical gears are made of engineering plastic. Compared with the metal gears, the plastic gears have the advantages of low transmission noise, vibration absorption, self-lubrication, etc. and they can be molded by injection molding, with high production efficiency and low manufacturing cost, so they are widely used in automotive motors[1].

As the number of meshing teeth and contact position changes periodically, the normal force acting on the tooth surface also varies periodically with time. Contact stress and bending stress are two main factors that cause the gear tooth failures. Thus it is important to understand the time-varying load distribution of gears.

In recent years, the gear tooth load distribution has been studied by many researchers. Pedrero[2] introduced a model of non-uniform load distribution along the line of contact by considering the change of meshing stiffness of gear pairs, and an approximate, accurate equation which allows analytic calculations of the load per unit of length at any point of the line of contact and any position of the cycle of meshing is presented; SABINIAK[3] presented a method of determination of load distribution between two plates being in contact along an arbitrary line. For the low modulus and large contact ratio of non-metallic gears, WALTON[4] proposed a finite element method to calculate the deformation and load distribution of gears; LETZELTER [5] presented a fast and efficient computational method to predict the mechanical behaviour of plastic cylindrical gear made of polymeric material.

In this study, instead of focusing on the complex model computation, a new approach, which is more efficient and more accurate than the traditional methods, is presented. This method takes into account the varying meshing stiffness and tooth elastic deformations due to bending, shear,
compression and foundation. The deformation of gear is based on the assumption of deformation compatibility. And the helical gear and worm are sliced along the contact line. Therefore, the 3D model of worm and gear can be efficiently approximated replaced by 2D model. The fundamental geometry of the helical gear and worm pair can then be represented by its transverse section, and a 2D model that ensures the computational precision. Finally, the correctness of the model is verified by finite element analysis.

2. Meshing model of helical gear worm transmission pair

The helical gear and worm pair was shown in Fig.1. The axes of helical gear and worm are crossed and perpendicular to each other in space. The meshing pair usually has a large contact ratio, which was between three or four teeth. It means that on the meshing period of one pitch, the number of meshing teeth alternates between three and four. The normal load on the meshing pair was transmitted by three pair of teeth for part of the period of engagement, and by four pair of teeth during rest of the period.

In order to determine the distribution of a given load which transmitted by the each tooth pair, the meshing models of three-tooth meshing and four-tooth meshing are established and analyzed respectively[6].
If the meshing gear pair proceeds further, another tooth pair on the right side will come into mesh in the contact line, and the state of four-tooth pairs contact is appear again. So the meshing period of one pitch is end, and the next meshing period which contain the three-tooth and four-tooth meshing will be repeated again. Thus, the meshing zones of three-tooth and four-tooth on the surface of the helical gear can be calculated by the geometry parameters from Fig.2 and 3 and the result is presented in Fig.4. The points of contact on the meshing zone of tooth profile are shown in the Fig.4.

Fig.4 Meshing position distribution on helical gear

3. Calculation of gear load distribution

In order to simplify the analysis procedure and increase the computational efficiency, it is assumed that the gears pair has no manufacturing error and assembly error, the gears pair has small contact deformation, and the friction between the gears pair is not considered.

In Fig. 2 and 3, the normal contact force at each contact point of the meshing gears pair is expressed as $F_i (i = 1, 2, ..., j)$, and at any instant of the gear engagement, the total normal load $F_n$ of gears pair is equal to the sum of normal force of each contact point ($F_i$), whereas

$$\sum_{i=1}^{j} F_i = F_n (i = 1, 2, ..., j)$$

Where

$$F_n = \frac{2T}{d \cos \beta \cos \alpha_n}$$

Where

- $T$= the load of helical gear;
- $d$= reference diameter of helical gear;
- $\beta$= helix angle of helical gear;
- $\alpha_n$= normal pressure angle of helical gear.

It is clear that, at any instant of gear engagement, the contact stiffness of the meshing pair is difference because the number of meshing teeth is different with time-vary, resulting in different load distribution for each meshing tooth.

In order to accurately calculate the load distribution on the tooth surface, it is necessary to introduce the assumption of gear meshing deformation compatibility. This is because in order to ensure the continuity of the contact between the two meshing gear and to avoid interference of the meshing gear, it is considered that at any instant of gear engagement, the deformation of each contact point on the contact line should be equal.

Tooth deflection under unit load $n_i$ may be found from the equation given in reference [9-10]. The total deflection consists of bending deflection $\delta_b$, shearing deflection $\delta_s$, contact deflection $\delta_c$, foundation deflection $\delta_f$, the total deflection is

$$\delta_i = \delta_{bi} + \delta_{si} + \delta_{ci} + \delta_{fi} (i = 1, 2, ..., j)$$
The expression of $\delta_{bi}$, $\delta_{gi}$, $\delta_{pl}$ are introduced in reference[9].

The tooth stiffness $k_i$ then becomes:

$$ k_i = \frac{n_i}{\delta_i} \quad (i = 1,2, \ldots, j) $$  \hspace{1cm} (4)

It is clear that the tooth stiffness $k_i$ depends upon the meshing point of load acting as well as the gear geometry. The contact stiffness of the each gear pair is:

$$ k_{eq,i} = \frac{k_{w,i}k_{g,i}}{k_{w,i}+k_{g,i}} \quad (i = 1,2, \ldots, j) $$  \hspace{1cm} (5)

Where,

$k_{w,i}$ = the worm stiffness of the meshing gear pair $i$;

$k_{g,i}$ = the gear stiffness of the meshing gear pair $i$;

According to the deformation compatibility assumption and considering the pitch error of each gear, the follow formula is obtained by combing the Eq.(1):

$$ f_{i1} + \gamma_1 = \frac{f_{i2}}{k_{eq,1}} + \gamma_2 = \cdots = \frac{f_{i1}}{k_{eq,i}} + \gamma_i \quad (i = 1,2, \ldots, j) $$  \hspace{1cm} (6)

where,

$\gamma_i$ = pitch error of each meshing gear pair;

By transforming Eq.(6) and substituting Eq.(1), the meshing force expression of contact point at any instant of gear engagement can be obtained, as shown in Eq.(7):

$$ F_1 = \frac{k_{eq,1}}{k_{eq,1}+k_{eq,2}+\cdots+k_{eq,i}} \left( F_n - k_{eq,i} \gamma_i - k_{eq,i} \gamma_{i-1} \right) $$

$$ F_2 = \frac{k_{eq,2}}{k_{eq,1}+k_{eq,2}+\cdots+k_{eq,i}} \left( F_n - k_{eq,i} \gamma_i - k_{eq,i} \gamma_{i-1} \right) \quad (i = 1,2, \ldots, j) $$

$$ \cdots $$

$$ F_i = \frac{k_{eq,i}}{k_{eq,1}+k_{eq,2}+\cdots+k_{eq,i}} \left( F_n - k_{eq,i} \gamma_i - k_{eq,i} \gamma_{i-1} \right) \quad (i = 1,2, \ldots, j) $$

(7)

The number of Eq.(7) depends on the meshing gear pairs $i$. Therefore, when $i=3$, the tooth surface load distribution of the meshing gear pair in three-tooth meshing can be obtained by solving Eq.(7). Similarly, when $i=4$, the tooth surface load distribution of the meshing gear pair in four-tooth meshing also obtained by Eq.(7).

4. Numerical example and discussion

In this section, we will compare the numerical result of finite element model with the calculation method of this paper in order to verify our proposed methodology.

The relevant data for the worm gear pair are listed in Table1 side. According to the data in Table1, the finite element model of the plastic involute helical gear and the steel cylindrical worm with a load torque of 1579 Nmm on the gear and a forced rotation on the worm was built in ANSYS Workbench.

In order to reduce the number of grids and improve the computational efficiency, the finite element model of worm gear pair is shown in Fig.5. To obtain the precise meshing force of tooth surfaces, the engagement teeth pairs are fine meshed with small elements for computation.

In Fig.6, it can be observed from the load distribution on the plastic involute helical gear that the meshing area is an ellipse area. And the movement path of the meshing point is from the tooth tip to the tooth root. The location of meshing points of each gear tooth surface is consistent with the results obtained by the present approach.
Table 1. Design parameters and material properties of worm gear pair

| Parameter                        | Worm | Helical gear |
|----------------------------------|------|--------------|
| Module $m_2$/mm                  | 0.8  | 0.8          |
| Pressure angle $\alpha_2$(°)     | 10   | 10           |
| Helix angle $\beta$(°)           | 84.5 | 5.5          |
| Base circle helix angle $\beta_b$(°) | 78.601 | 4.591        |
| Number of teeth $z$              | 1    | 48           |
| Young’s modules $E$/MPa          | 210000 | 3600         |
| Poisson’s ratio $\gamma$         | 0.3  | 0.4          |
| Outside diameter $d_a$/mm        | 10.86 | 41.791       |
| Base circle $d_b$/mm             | 3.986 | 37.938       |
| Reference pitch diameter $d_o$/mm| 8.347 | 38.527       |
| Face width $b$/mm                | 17.68 | 9.7          |

Fig.5 FEM model of helical gear and worm

Fig.6 Meshing force distribution on the tooth surface of helical gear

Fig.7 show the comparison of meshing force on the gear tooth surface between the Finite element method (FEM) and the proposed approach. The meshing force on the tooth calculated by proposed approach is agreed with the FEM result very well. The maximum meshing force computed by the present method is about 40.37 N, while the number calculated by FEM is about 37.158 N. The difference between the present method and FEM is about 8.9%.

In Fig.7, the X axis represents the tooth height direction of the helical gear, and the Y axis represents the meshing force value on the tooth. The curves in the figure represent the distribution of the meshing force along the tooth profile.

From the Fig.7, the meshing force on the three-tooth meshing region is greater than that on the four-tooth meshing region. This is because the number of teeth in the three-tooth meshing is less, and the load distribution to each tooth is greater.

Fig.7 Comparison of load distribution on tooth surface

Fig.8 Deformation on the difference position of helical gear

The deformation curve of the helical gear meshing point at different meshing position is shown in Fig.8. As shown in Fig.7, the gear has a greater load distribution in the three-tooth meshing position. Thus, the three-tooth meshing position of gear also has a larger deformation than that on the four-tooth
meshing position. As the number of mesh teeth changes alternately, the gear deformation also changes
alternately.

From Fig. 2, 3 and 8, it can be concluded that in the three-tooth meshing position, the contact points
of three pairs of meshing tooth begin to simultaneously meshing-in at A', C' and D', and then
simultaneously meshing-out at C, D and E. Thus, at each moment of the three-tooth meshing process,
the deformation of each contact point on contact line is qual. Similarly, the process is also the same in
the four-tooth meshing zone.

5. Conclusion
In this study, a method for the analysis of load distribution of the helical gear and worm pair has been
developed. The meshing force acting on helical gears increases first and then decreases, and fluctuates
with the number of meshing teeth alternately. The deformation of the helical gear varies with the
number of meshing teeth. In the position of less teeth meshing, the deformation of the gear is large,
and in the position of more teeth meshing, the deformation of gear is small. The results of this method
are in good agreement with those of finite element simulation. Thus, the load distribution and the
deformation of the tooth surface can be obtained by this method. The presented method in this paper
can be further extended for the determination of the mesh stiffness of the worm gear pair.

References
[1] LIU, Z. Y., et al. (2014) The mesh property of the steel involute cylindrical worm with a plastic
involute helical gear. Journal of Mechanics, 30(2): 185-192.
[2] J.I. Pedrero, M. Pleguezuelos, M. Artés, J.A. Antona. (2010). Load distribution model along the
line of contact for involute external gears, Mech. Mach. Theory, 45 (5), 780-794
[3] SABINIAK, Henryk G. (2016) Load distribution in the worm meshing. Journal of Theoretical and
Applied Mechanics, 54(4), 1169-1181.
[4] WALTON, D., et al. (1994) Load sharing in metallic and non-metallic gears. Proceedings of the
Institution of Mechanical Engineers, Part C: Journal of Mechanical Engineering Science,
208(2), 81-87.
[5] LETZELTER, Eric, et al. (2009) Quasi-static load sharing model in the case of Nylon 6/6
cylindrical gears. Materials & Design, 30(10), 4360-4368.
[6] FALAH, A. H.; ELKHOLY, A. H. Load sharing and stress analysis of single enveloping
worm gearing considering transmission errors. The International Journal of Advanced
Manufacturing Technology, 37(3), 211-220.
[7] Hasl, C. Liu, H. Oster, P. Tobie, T. & Stahl, K. (2017) Method for calculating the tooth root stress
of plastic spur gears meshing with steel gears under consideration of deflection-induced load
sharing. Mechanism and Machine Theory, 111, 152-163.
[8] SHUAI, Mo; YIDU, Zhang; QIONG, Wu. (2015) Research on multiple-split load sharing of two-
stage star gearing system in consideration of displacement compatibility. Mechanism and
Machine Theory, 88, 1-15.
[9] SHIGLEY, Joseph Edward. (2011) Shigley's mechanical engineering design. Tata McGraw-Hill
Education.
[10] Falah, A. H, Elkholy, A. H. (2006) Load and stress analysis of cylindrical worm gearing using
tooth slicing method. Transactions of the Canadian Society for Mechanical Engineering,
30(1), 97-112.