The Spin Symmetry of Heavy Baryons in the Framework of the B.S. Equation

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Abstract

We construct the B.S. equation for baryons. The most general form of the B.S. wave function is given. In the heavy quark limit we show clearly that the spin symmetry exists in heavy baryon states.

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1. Introduction

It’s known the physics of heavy hadrons is important. The weak decays of heavy quark are employed for the tests of the standard model and the measurements of its parameter. For example, they offer the most direct way to determine the weak mixing angles and to test the unitarity of Kobayashi-Maskawa matrix. In this paper we study baryons composed of a heavy quark and other light constituents in the framework of B.S. equation [1].

The typical four-momentum of the light constituent is small compared with the heavy quark mass, and the typical momentum exchanged between the heavy quark and light constituents are order of $\Lambda_{QCD}$, therefore, the strong interaction physics of such system is mainly nonperturbative. In this situation, it’s suitable to carry out the systematic expansions in $1/M_Q$ based on QCD, then the so called heavy quark effective theory (HQET) is obtained [2-8]. In the leading order of HQET ($M \rightarrow \infty$), the strong interactions of the heavy quark are independent of its mass and spin. Consequently, for $N$ heavy quarks, there will be $SU(2N)$ spin-flavor symmetry in the leading order of HQET. This symmetry directly leads to many predictions. As one of the important results, this symmetry is responsible for some mass degeneracy in hadron spectrum.

In fact, the the systematic expansions in $1/M_Q$ can also be carried out in the B.S. formalism. For general case, the B.S. wave function of hadrons have many components, and the numerical calculation of the relevant integral equation is very difficult. In the case of mesons, it has been shown that the number of components of the general relativistic covariant B.S. wave function is reduced greatly in the leading order of $1/M_Q$ expansion, and the spin-flavor symmetry is shown clearly [9]. In this paper, we extend this method to the system of baryons. The paper is arranged as follows. In section two, we construct the B.S. equation for baryons, and we give the general relativistic B.S. equation. In section three, we study the spin symmetry in heavy quark limit in the framework of B.S. equation. In section three results and discussion are presented.

2. The B.S. equation and the B.S. wave function

Let $\psi_1(x_1)$, $\psi_2(x_2)$ and $\psi_3(x_3)$ be the quark fields at points $x_1$, $x_2$ and $x_3$, $|B\rangle$ the baryon state with momentum $P$
and mass $M$. The B.S. wave function of the baryons is defined as

$$ \chi_P(x_1, x_2, x_3) = \langle |T \psi_1(x_1)\psi_2(x_2)\psi_3(x_3)|B \rangle $$  \hspace{1cm} (1)

In order to separate the center of mass coordinates and the internal relative coordinates, we use the following variables:

$$ X = \frac{1}{m_1 + m_2 + m_3} (m_1 x_1 + m_2 x_2 + m_3 x_3) $$ \hspace{1cm} (2)

$$ x = x_2 - x_3, \quad x' = x_1 - \frac{1}{m_2 + m_3} (m_2 x_2 + m_3 x_3) $$  \hspace{1cm} (3)

Then we have

$$ \chi_P(x_1, x_2, x_3) = e^{-iPX} \chi_P(x, x') $$ \hspace{1cm} (4)

In momentum space, we define the B.S. amplitude as

$$ \chi(P, q, k) = \int d^4x' d^4x e^{iqx} e^{ikx'} \chi_P(x, x') $$ \hspace{1cm} (5)

With a standard method, we obtain the B.S. equation for baryons in momentum space

$$ \chi(P, q, k) = S^{(1)}_F(p_1) S^{(2)}_F(p_2) S^{(3)}_F(p_3) \int \frac{d^4q'}{(2\pi)^4} \frac{d^4k'}{(2\pi)^4} G(P, q, q', k, k') \chi(p, q', k') $$ \hspace{1cm} (6)

where $S_F(p_i)$'s are the propagators of the quark with momentum $p_i$, and $G(P, q, q', k, k')$ is the kernel defined as all irreducible three-particle graphs.

Now we construct the B.S. wave functions of baryons. Generally, the B.S. wave functions of baryons are complicated. The spin-$(j + \frac{1}{2})$ wave functions can be written as the sum of the terms $u^\mu_1\mu_2...\mu_j \Gamma_{\mu_1\mu_2...\mu_j} \beta_\gamma$, where $u^\mu_1\mu_2...\mu_j$ is a tensor spinor, and $\Gamma$ a tensor matrix made of $P_\mu$, $q_\mu$ and $k_\mu$ and $\gamma$-matrices. The tensor spinor $u^\mu_1\mu_2...\mu_j$ is symmetric about the Lorentz indices $\mu_1$, $\mu_2$, ..., $\mu_j$, and satisfies the subsidiary conditions

$$ (1 - v) u^\mu_1\mu_2...\mu_j = 0, \quad u^\mu_1\mu_2...\mu_j = 0 $$ \hspace{1cm} (7)

$$ u_{\mu_1} u^\mu_1\mu_2...\mu_j = 0, \quad \gamma_{\mu_1} u^\mu_1\mu_2...\mu_j = 0. $$ \hspace{1cm} (8)

where $v = P/M$.

For convenience, following we define some tensor matrices

$$ \Gamma = f_1 + f_2 \hat{k} + f_3 \hat{k} + f_4 \hat{k} + f_5 \hat{k} + f_6 \hat{k} + f_7 \hat{k} $$ \hspace{1cm} (9)
\[
\Gamma^\mu = q^\mu_1 \Gamma_1 + k^\mu_1 \Gamma_2 + \gamma_\perp^\mu \Gamma_3 \tag{10}
\]
\[
\Gamma^{\mu\nu} = q^\mu_1 \Gamma'_1 + k^\mu_1 \Gamma'_2 + \gamma_\perp^\mu \Gamma'_3 \tag{11}
\]
\[
\Gamma^{\mu\nu\lambda} = q^\mu_1 \Gamma'^\lambda_1 + k^\mu_1 \Gamma'^\lambda_2 + \gamma_\perp^\mu \Gamma'^\lambda_3 \tag{12}
\]

where \( q_\perp, k_\perp, \gamma_\perp^\mu \) (and \( \sigma^{\mu\nu}_\perp \) and \( g^{\mu\nu}_\perp \) to be used later) are transverse quantities defined as

\[
q^\mu_\perp = q^\mu - v \cdot q^\mu, \quad k^\mu_\perp = q^\mu - v \cdot k^\mu \tag{13}
\]
\[
\gamma_\perp^\mu = \gamma^\mu - \frac{\nu}{\gamma^\mu} v^\mu, \quad \sigma^{\mu\nu}_\perp = \frac{1}{2}[\gamma^\mu, \gamma^\nu] \tag{14}
\]
\[
g^{\mu\nu}_\perp = g^{\mu\nu} - \nu^\mu v^\nu \tag{15}
\]

where \( f_i (i=1,2,\ldots,8) \) are scalar functions of \( P \cdot q, P \cdot k, q \cdot k \ldots \), \( \Gamma_i \) (or \( \Gamma^{\mu}_i, \Gamma^{\mu\nu}_i \)) have the same structure as \( \Gamma \) (or \( \Gamma^\mu_\mu, \Gamma^\mu_\mu \)) but with independent scalar functions (ISF). So \( \Gamma^\mu \) has 24 ISF and \( \Gamma^{\mu\nu} \) has 72 ISF, and so on. The B.S. wave function can be constructed now. For example, the wave function of spin-\( \frac{1}{2} \) baryons can be written as

\[
\chi^{(\frac{1}{2})+}(p, q, k) = u^\dagger \Gamma^{(\frac{3}{2})+}_1 C + \gamma_5 u^\dagger \Gamma^{(\frac{3}{2})+}_2 C + \gamma_\perp^\mu u^\dagger \Gamma^{(\frac{3}{2})+}_{1 \mu} \gamma_5 C \\
+ \gamma_5 \gamma_\perp^\mu u^\dagger \Gamma^{(\frac{3}{2})+}_2 C + \sigma^{\mu\nu}_\perp u^\dagger \Gamma^{(\frac{3}{2})+}_{\mu\nu} \gamma_5 C \tag{16}
\]

For spin-\( \frac{3}{2} \) baryons, the B.S. wave function can be written as

\[
\chi^{(\frac{3}{2})+}(p, q, k) = u^\dagger \Gamma^{(\frac{3}{2})+}_{1 \mu} C + \gamma_5 u^\dagger \Gamma^{(\frac{3}{2})+}_2 C + \gamma_\perp^\nu u^\dagger \Gamma^{(\frac{3}{2})+}_{1 \mu} \gamma_5 C \\
+ \gamma_5 \gamma_\perp^\nu u^\dagger \Gamma^{(\frac{3}{2})+}_2 C + \sigma^{\lambda\nu}_\perp u^\dagger \Gamma^{(\frac{3}{2})+}_{\mu\nu} \gamma_5 C \tag{17}
\]

where \( C \) is the charge conjugation matrix defined as

\[
C^T = -C, \quad C^{-1} \gamma^\mu C = -\gamma^{\mu T} \tag{18}
\]

For the convenience of the following discussion, we rewrite the wave function \( \chi^{(\frac{3}{2})+}(p, q, k) \) according to their symmetric property as

\[
\chi^{\frac{3}{2}+}(p, q, k) = u^\dagger \Gamma^{(\frac{3}{2})+}_{1 \mu} C + \gamma_5 u^\dagger \Gamma^{(\frac{3}{2})+}_2 C \\
\gamma_5 \gamma_\perp^\mu u^\dagger \Gamma^{(\frac{3}{2})+}_{1 \mu} C + \gamma_5 \gamma_\perp^\nu u^\dagger \Gamma^{(\frac{3}{2})+}_2 C \\
\gamma_5 \gamma_\perp^\nu u^\dagger \Gamma^{(\frac{3}{2})+}_2 C + \gamma_5 \gamma_\perp^\mu u^\dagger \Gamma^{(\frac{3}{2})+}_2 C \\
\sigma^{\lambda\nu}_\perp u^\dagger \Gamma^{(\frac{3}{2})+}_{\mu\nu} \gamma_5 C \\
= [\sigma^{\lambda\nu}_\perp u^\dagger \Gamma^{(\frac{3}{2})+}_{\mu\nu} \gamma_5 C + \gamma_5 (\gamma_\perp^\mu u^\dagger \Gamma^{(\frac{3}{2})+}_{1 \mu}) \gamma_5 C \\
+ \gamma_5 (\gamma_\perp^\nu u^\dagger \Gamma^{(\frac{3}{2})+}_2) \gamma_5 C]
\tag{19}
\]
It should be noticed that, in the last term of Eq.(16), $\sigma^\mu_\perp$ is anti-symmetry, so $\Gamma^{(\frac{3}{2})}_\mu\nu$ has 32 ISF instead of 72. Similarly, in Eq(19), $\Gamma^{(\frac{3}{2})}_1\mu\nu$ has 32 ISF, $\Gamma^{(\frac{3}{2})}_2\mu\nu$ has 40 ISF and so on. Therefore, wave functions (17) and (19) have the same number of ISF. One may also note that all tensors in front of $\Gamma^{(\frac{1}{2})}_i\mu\nu (i=1,2,3)$ and $\Gamma^{(\frac{3}{2})}_\mu\nu\lambda$ in (19) vanish after contraction of any two Lorenz indices. So we neglect the terms proportional $g^{\mu\nu}_\perp$ in Eq.(11) and Eq.(12), which is also unnecessary in Eq.(17), since $\sigma^\mu_\perp u^\mu g_{\perp\mu\nu} = -u^\lambda$.

Similarly, the wave function of spin-$\frac{5}{2}$ baryon state is written as

$$
\chi^{(\frac{5}{2})+} (P, q, k) = u^{\mu\nu}\Gamma^{(\frac{5}{2})}_1\mu\nu \gamma_5 C + \gamma_5 u^{\mu\nu}\Gamma^{(\frac{3}{2})}_2\mu\nu C
$$

$$
+ (\gamma^\mu_\perp u^\nu - \gamma^\nu_\perp u^\mu)\Gamma^{(\frac{5}{2})}_1\mu\nu \gamma_5 C + (\gamma^\mu_\perp u^\nu + \gamma^\nu_\perp u^\mu + \gamma^\mu_\parallel u^{\mu\nu})\Gamma^{(\frac{3}{2})}_2\mu\nu\lambda \gamma_5 C
$$

$$
+ \gamma_5 (\gamma^\mu_\perp u^\nu - \gamma^\nu_\perp u^\mu)\Gamma^{(\frac{5}{2})}_3\mu\nu\lambda C + \gamma_5 (\gamma^\mu_\perp u^\nu + \gamma^\nu_\perp u^\mu + \gamma^\mu_\parallel u^{\mu\nu})\Gamma^{(\frac{3}{2})}_4\mu\nu\lambda C
$$

$$
+ [\sigma^\mu_\perp u^\nu - \frac{1}{2}(g^\perp_\mu u^\nu + g^\perp_\nu u^\mu)]\Gamma^{(\frac{3}{2})}_5\mu\nu\lambda \gamma_5 C
$$

(20)

If we adopt that the tensor spinor $u$ is the positive eigenstate of operator $\gamma_5$, the above B.S. wave equations have positive parity. The wave function with negative parity can be constructed easily by multiplying a $\gamma_5$. For example, the wave function of spin-$\frac{1}{2}$ baryon with negative parity can be written as

$$
\chi^{(\frac{1}{2})-} (P, q, k) = u\Gamma^{(\frac{1}{2})}_1\perp - \gamma_5 u\Gamma^{(\frac{3}{2})}_2\perp \gamma_5 C + \gamma_5 u\Gamma^{(\frac{3}{2})}_1\perp C
$$

$$
+ \gamma_5 \gamma^\mu_\perp u\Gamma^{(\frac{3}{2})}_2\perp \gamma_5 C + \sigma^\mu_\perp u\Gamma^{(\frac{3}{2})}_1\perp C
$$

(21)

and for spin-$\frac{5}{2}$ state, the wave function with negative parity is written as

$$
\chi^{(\frac{5}{2})-} (P, q, k) = u^\mu\Gamma^{(\frac{5}{2})}_1\mu \gamma_5 C + \gamma_5 u^\mu\Gamma^{(\frac{3}{2})}_2\mu C
$$

$$
+ (\gamma^\nu_\perp u^\nu - \gamma^\nu_\perp u^\nu)\Gamma^{(\frac{5}{2})}_1\mu\nu \gamma_5 C + (\gamma^\nu_\perp u^\nu + \gamma^\nu_\perp u^\nu)\Gamma^{(\frac{3}{2})}_2\mu\nu \gamma_5 C
$$

$$
+ \gamma_5 (\gamma^\nu_\perp u^\nu - \gamma^\nu_\perp u^\nu)\Gamma^{(\frac{5}{2})}_3\mu\nu\lambda C + \gamma_5 (\gamma^\nu_\perp u^\nu + \gamma^\nu_\perp u^\nu)\Gamma^{(\frac{3}{2})}_4\mu\nu\lambda C
$$

$$
+ [\sigma^\mu_\perp u^\nu - \frac{1}{2}(g^\perp_\mu u^\nu - g^\perp_\nu u^\mu)]\Gamma^{(\frac{3}{2})}_5\mu\nu\lambda \gamma_5 C
$$

(22)

3. The Spin Symmetry of the Heavy baryons

When $m_1$ goes to infinity, the momentum of the heavy quark can be written as

$$
p'^\mu_1 = \frac{m_1}{m_1 + m_2 + m_3} P^\mu + k^\mu = (m_1 + E) v^\mu + k^\mu
$$

(23)
where $E = M - (m_1 + m_2 + m_3)$. The propagator of the heavy quark is then simplified to

$$ S_F^{(1)} = \frac{1}{E + k \cdot v} \frac{1 + \not{\gamma}}{2} \quad (24) $$

Then equation (6) is reduced to

$$ \chi(P, q, k) = \frac{1}{E + k \cdot v} \frac{1 + \not{\gamma}}{2} S_F^{(2)}(p_2) S_F^{(3)}(p_3) \int \frac{d^4 q'}{(2\pi)^4} \frac{d^4 k'}{(2\pi)^4} G(P, q', k, k') \chi(p, q', k') \quad (25) $$

From equation above, we have

$$ \chi(P, q, k) = \frac{1 + \not{\gamma}}{2} \chi(P, q, k) \quad (26) $$

Therefore, all the components of the wave function, which are the sum of the forms

$$ (\gamma_5 u^{\mu_1 \mu_2 \ldots} \Gamma_{\mu_1 \mu_2 \ldots}) \text{ or } (\gamma_\mu^{\mu_1 \mu_2 \ldots} \Gamma_{\mu_1 \mu_2 \ldots}), $$

will be reduced to zero.

Since \( \frac{1 + \not{\gamma}}{2} \Gamma_{\mu} C = v_{\mu} \), in general the kernel can be written as the form

$$ G(P, q, q', k, k') = I \otimes \tilde{G} \quad (27) $$

where $I$ belongs to the heavy quark spin space, and $\tilde{G}$ belongs to the spin space of the light quarks. For spin-$\frac{1}{2}$ baryon with positive parity, substituting the wave function (16) into equation (25), we obtain three decoupled equations

$$ \Gamma_{\mu}^{(1)+} \gamma_5 C = \frac{1}{E + k \cdot v} S_F^{(2)} S_F^{(3)} \int \frac{d^4 q'}{(2\pi)^4} \frac{d^4 k'}{(2\pi)^4} \tilde{G} \Gamma_{\mu}^{(1)+} \gamma_5 C \quad (28) $$

$$ \Gamma_{\mu}^{(2)+} C = \frac{1}{E + k \cdot v} S_F^{(2)} S_F^{(3)} \int \frac{d^4 q'}{(2\pi)^4} \frac{d^4 k'}{(2\pi)^4} \tilde{G} \Gamma_{\mu}^{(2)+} C \quad (29) $$

$$ (\Gamma_{\mu}^{(3)+} - \Gamma_{\nu\mu}^{(3)+}) \gamma_5 C = \frac{1}{E + k \cdot v} S_F^{(2)} S_F^{(3)} \int \frac{d^4 q'}{(2\pi)^4} \frac{d^4 k'}{(2\pi)^4} \tilde{G} (\Gamma_{\mu}^{(3)+} - \Gamma_{\nu\mu}^{(3)+}) \gamma_5 C \quad (30) $$

Similarly, for spin-$\frac{3}{2}$ baryon with positive parity, substituting the wave function (19) into equation (25), we can obtain four decoupled equations. One notice that the tensors

$$ (\gamma_\mu^\mu w^\nu + \gamma_\nu^\nu w^\mu) \text{ and } w^{\mu\nu} \text{ have the same symmetry character and so do the tensors } [\sigma_\mu^\lambda u^\mu - \frac{1}{2}(g_\mu^\lambda u^\nu - g_\nu^\mu u^\lambda)] \text{ and } (\gamma_\mu^\mu w^{\mu\nu} - \gamma_\nu^\nu w^{\mu\lambda}). $$

It’s easy to see that the equation about the component $w^{\mu\nu} \Gamma_{1\mu}^{(4)+} C$ is just the same as equation (29), and the equation about the component $\gamma_5 (\gamma_\mu^\mu w^{\mu\nu} - \gamma_\nu^\nu w^{\mu\lambda}) \Gamma_{3\mu}^{(4)+} \gamma_5 C$ is just the same as equation (30). This
implies that the spin-$\frac{1}{2}$ baryon states represented by wave functions $\gamma_5^{(\frac{1}{2})+}\gamma_\mu u\Gamma_2\mu C$ and $\sigma_\mu^\nu u\Gamma_1^{(\frac{1}{2})+}\gamma_5 C$ degenerate with the spin-$\frac{3}{2}$ states represented by $u^\mu\Gamma_1^{(\frac{3}{2})+}C$ and $\gamma_5(\gamma_\mu^\nu u^\mu - \gamma_\mu^\nu u^\nu)\Gamma_3^{(\frac{3}{2})+}\gamma_5 C$ respectively. Similarly, the spin-$\frac{3}{2}$ baryon states represented by the components, $\gamma_5(\gamma_\mu^\nu u^\mu + \gamma_\mu^\nu u^\nu)\Gamma_1^{(\frac{3}{2})+}\gamma_5 C$ and $[\sigma_\mu^\nu u^\mu - \frac{1}{2}(g_\mu^\nu u^\nu - g_\mu^\nu u^\lambda)]\Gamma^{(\frac{3}{2})+}_{\mu\lambda}C$ will degenerate with the spin-$\frac{5}{2}$ baryon states represented by the components $u^\mu\Gamma_1^{(\frac{5}{2})+}\gamma_5 C$ and $\gamma_5(\gamma_\mu^\nu u^\mu - \gamma_\mu^\nu u^\nu)\Gamma_3^{(\frac{5}{2})+}\gamma_5 C$ respectively. The case for baryons with negative parity is just the same. Such a conclusion is the direct result of heavy quark limit, and can be generalized to any given spin baryon states.

Generally, when $j$ is even, the wave function of spin-$(j + \frac{1}{2})$ baryons with positive parity can be written as

$$\chi^{(j+\frac{1}{2})+}(P, q, k) = u^{\mu_1\mu_2...\mu_j}\Gamma_1^{(j+\frac{1}{2})+}_{\mu_1\mu_2...\mu_j}(\gamma_5 C + \gamma_5 u^{\mu_1\mu_2...\mu_j}\Gamma_2^{(j+\frac{1}{2})+}_{\mu_1\mu_2...\mu_j}C$$

$$+ (\gamma_1^{\mu_1} u^{\mu_2\mu_3...\mu_{j+1}} + \gamma_2^{\mu_1} u^{\mu_2\mu_3...\mu_{j+1}} + ... + \gamma_j^{\mu_1} u^{\mu_2\mu_3...\mu_{j+1}})\Gamma_1^{(j+\frac{1}{2})+}_{\mu_1\mu_2...\mu_{j+1}}\gamma_5 C$$

$$+ (\gamma_1^{\mu_1} u^{\mu_2\mu_3...\mu_{j+1}} - \gamma_2^{\mu_1} u^{\mu_2\mu_3...\mu_{j+1}})\Gamma_{\frac{1}{2}}^{(j+\frac{1}{2})+}_{\mu_1\mu_2...\mu_{j+1}}\gamma_5 C$$

$$+ \frac{1}{j+1}(-g^{\mu_1\mu_2} u^{\mu_3\mu_4...\mu_{j+2}} + g^{\mu_1\mu_2} u^{\mu_3\mu_5...\mu_{j+2}} + ... + g^{\mu_1\mu_2} u^{\mu_3\mu_5...\mu_{j+1}} u^{\mu_1\mu_3...\mu_{j+1}})\Gamma_{\frac{1}{2}}^{(j+\frac{1}{2})+}_{\mu_1\mu_2...\mu_{j+2}}\gamma_5 C)$$

(31)

The spin-$(j - \frac{1}{2})$ wave function with positive parity is written as

$$\chi^{(j-\frac{1}{2})+}(P, q, k) = u^{\mu_1\mu_2...\mu_{j-1}}\Gamma_2^{(j-\frac{1}{2})+}_{\mu_1\mu_2...\mu_{j-1}}(\gamma_5 C + \gamma_5 u^{\mu_1\mu_2...\mu_{j-1}}\Gamma_1^{(j-\frac{1}{2})+}_{\mu_1\mu_2...\mu_{j-1}}C$$

$$+ (\gamma_1^{\mu_1} u^{\mu_2\mu_3...\mu_{j-1}} + \gamma_2^{\mu_1} u^{\mu_2\mu_3...\mu_{j-1}} + ... + \gamma_j^{\mu_1} u^{\mu_2\mu_3...\mu_{j-1}})\Gamma_1^{(j-\frac{1}{2})+}_{\mu_1\mu_2...\mu_{j-1}}C$$

$$+ (\gamma_1^{\mu_1} u^{\mu_2\mu_3...\mu_{j-1}} - \gamma_2^{\mu_1} u^{\mu_2\mu_3...\mu_{j-1}})\Gamma_{\frac{1}{2}}^{(j-\frac{1}{2})+}_{\mu_1\mu_2...\mu_{j-1}}C$$

$$+ \frac{1}{j}(g^{\mu_1\mu_2} u^{\mu_3\mu_4...\mu_{j+2}} + g^{\mu_1\mu_2} u^{\mu_3\mu_5...\mu_{j+2}} + ... + g^{\mu_1\mu_2} u^{\mu_3\mu_5...\mu_{j+1}} u^{\mu_1\mu_3...\mu_{j+1}})\Gamma_{\frac{1}{2}}^{(j-\frac{1}{2})+}_{\mu_1\mu_2...\mu_{j+2}}\gamma_5 C)$$

(32)

We obtain the spin-$(j + \frac{3}{2})$ wave function with positive parity by replacing $j$ with $j + 2$ in the expression (32).
In the heavy quark limit, the second, the third and the fourth components of the wave function (31) are reduced to zero. The components left are decoupled. Instituting the wave function of spin-(\( j + \frac{1}{2} \)), spin-(\( j - \frac{1}{2} \)) and spin-(\( j + \frac{3}{2} \)) into equation (25), we find the following degenerate doublets.

\[
\left\{
\begin{align*}
& u^{\mu_1 \mu_2 \ldots \mu_j} \Gamma_{1 \mu_1 \mu_2 \ldots \mu_j}^{(j+\frac{1}{2})+} \gamma_5 C , \\
& \gamma_5 \left( \gamma_\perp^{\mu_1} u^{\mu_2 \mu_3 \ldots \mu_j} + \gamma_\perp^{\mu_2} u^{\mu_1 \mu_3 \ldots \mu_j} + \ldots + \gamma_\perp^{\mu_j} u^{\mu_1 \mu_2 \ldots \mu_{j-1}} \right) \Gamma_{3 \mu_1 \mu_2 \ldots \mu_{j-1}}^{(j-\frac{1}{2})+} \gamma_5 C ; \\
& \gamma_5 \left( \gamma_\perp^{\mu_1} u^{\mu_2 \mu_3 \ldots \mu_{j+1}} - \gamma_\perp^{\mu_2} u^{\mu_1 \mu_3 \ldots \mu_{j+1}} \right) \Gamma_{4 \mu_1 \mu_2 \ldots \mu_{j+1}}^{(j+\frac{1}{2})+} C , \\
& \left[ \sigma_\perp^{\mu_1 \mu_2} u^{\mu_3 \mu_4 \ldots \mu_{j+1}} - \frac{1}{j} \left( g^{\mu_1 \mu_3 u^{\mu_2 \mu_4 \ldots \mu_{j+1}} + g^{\mu_1 \mu_4 u^{\mu_2 \mu_3 \mu_5 \ldots \mu_{j+1}} + \ldots + g^{\mu_1 \mu_{j+1} u^{\mu_2 \mu_3 \ldots \mu_j}} \right) \\
& + \frac{1}{j} \left( g^{\mu_2 \mu_3 u^{\mu_1 \mu_4 \ldots \mu_{j+1}} + g^{\mu_2 \mu_4 u^{\mu_1 \mu_3 \mu_5 \ldots \mu_{j+1}} + \ldots + g^{\mu_2 \mu_{j+1} u^{\mu_1 \mu_3 \ldots \mu_j}} \right) \right) \Gamma_{5 \mu_1 \mu_2 \ldots \mu_{j+1}}^{(j+\frac{1}{2})+} C ; \\
& \gamma_5 \left( \gamma_\perp^{\mu_1} u^{\mu_2 \mu_3 \ldots \mu_{j+1}} + \gamma_\perp^{\mu_2} u^{\mu_1 \mu_3 \ldots \mu_{j+1}} + \ldots + \gamma_\perp^{\mu_{j+1}} u^{\mu_1 \mu_2 \ldots \mu_j} \right) \Gamma_{6 \mu_1 \mu_2 \ldots \mu_{j+1}}^{(j+\frac{1}{2})+} C , \\
& u^{\mu_1 \mu_2 \ldots \mu_{j+1}} \Gamma_{7 \mu_1 \mu_2 \ldots \mu_{j+1}}^{(j+\frac{1}{2})+} ; \\
& \left[ \sigma_\perp^{\mu_1 \mu_2} u^{\mu_3 \mu_4 \ldots \mu_{j+2}} - \frac{1}{j+1} \left( g^{\mu_1 \mu_3 u^{\mu_2 \mu_4 \ldots \mu_{j+2}} + g^{\mu_1 \mu_4 u^{\mu_2 \mu_3 \mu_5 \ldots \mu_{j+2}} + \ldots + g^{\mu_1 \mu_{j+2} u^{\mu_2 \mu_3 \ldots \mu_{j+1}}} \right) \\
& + \frac{1}{j+1} \left( g^{\mu_2 \mu_3 u^{\mu_1 \mu_4 \ldots \mu_{j+2}} + g^{\mu_2 \mu_4 u^{\mu_1 \mu_3 \mu_5 \ldots \mu_{j+2}} + \ldots + g^{\mu_2 \mu_{j+2} u^{\mu_1 \mu_3 \ldots \mu_{j+1}}} \right) \right) \Gamma_{8 \mu_1 \mu_2 \ldots \mu_{j+2}}^{(j+\frac{1}{2})+} C ; \\
& \gamma_5 \left( \gamma_\perp^{\mu_1} u^{\mu_2 \mu_3 \ldots \mu_{j+2}} - \gamma_\perp^{\mu_2} u^{\mu_1 \mu_3 \ldots \mu_{j+2}} \right) \Gamma_{9 \mu_1 \mu_2 \ldots \mu_{j+2}}^{(j+\frac{1}{2})+} C .
\right.
\end{align*}\]

The corresponding wave function with negative parity have the similar degeneracy.

4. Conclusion  We construct the B.S equation for baryons. In this framework we discuss the spin symmetry of heavy baryon system. The most general form of the B.S. wave function of a baryon is obtained. It is complicated, and the B.S. equations are some coupled integral equations. In the heavy quark limit, the freedom of the heavy quark spin is decoupled from the light ones, and consequently the spin symmetry takes place. This symmetry results in some degeneracy in the spectrum of heavy baryons. We verified this point clearly in the framework of B.S. equation. Meanwhile, in the heavy quark limit, some components of the B.S. wave function becomes to zero, and some others decoupled. Therefore, the B.S. equations of baryons are simplified greatly.
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