The interplay between spin-orbit interactions and a time-dependent electromagnetic field in monolayer graphene

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We apply a circularly and linearly polarized terahertz field on a monolayer of graphene taking into account spin-orbit interactions of the intrinsic and Rashba type. It turns out that the field can not only be used to induce a gap in the energy spectrum, but also to close an existing gap due to the different reaction of the spin components on circularly polarized light. Signatures of spin-orbit coupling on the density of states of the driven system can be observed even for energies where the static density of states is independent of spin-orbit interactions. Furthermore it is shown that the time evolution of the spin polarization and the orbital dynamics of an initial wave packet can be modulated by varying the ratio of the spin-orbit coupling parameters. Assuming that the system acquires a quasi stationary state, the optical conductivity of the irradiated sample is calculated. Our results confirm the multi step nature of the conductivity obtained recently, where the number of intermediate steps can be changed by adjusting the spin-orbit coupling parameters and the orientation of the field.

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I. INTRODUCTION

Since a monolayer of graphite has been isolated and detected for the very first time\cite{Novoselov04} many theoretical and experimental studies on this remarkable and surprising material have been published\cite{Neto09}. Even several years after its discovery graphene remains one of the most intense research topics in solid state physics. This expresses the high expectations and hopes physicists have on graphene as being a building block for novel electronic devices.

While in the beginning the focus of graphene research was mainly set on spin-independent phenomena, as it was claimed that spin-orbit interactions (SOIs) are virtually unimportant in graphene\cite{Louie05}, recent experimental and theoretical works have demonstrated that spin-orbit coupling (SOC) effects might be important as the characteristic parameters can be enlarged significantly\cite{Neto09,Novoselov05}. This, in principle, opens up the possibility of using spin-related phenomena in this outstanding material with exceptional electronic properties. Moreover, and indeed fortunately, graphene is not the only promising two-dimensional hexagonal system and thus many of the findings of spin-related research in graphene can also be applied to other systems. As an example we mention a monolayer of MoS$_2$\cite{Kloch09,Novoselov11} that can effectively be described as two uncoupled gapped graphene systems, where both the band gap and the SOIs turn out to be large\cite{Neto09,Novoselov11} and Silicene, a two-dimensional honeycomb lattice made of silicon atoms\cite{Novoselov11,Migash11}.

In recent works the effects of an external time-dependent field on two-dimensional materials such as a monolayer\cite{Kloch09,Novoselov11} or bilayer\cite{Kloch09} of graphene, HgTe quantum wells\cite{Novoselov11} or n- and p-doped electron gases\cite{Novoselov11,Neto09} have been discussed. It was shown that an electromagnetic field can induce gaps in the energy spectrum of graphene which might be interesting for future applications such as transistors. Furthermore, in Ref\cite{Kloch09} the possibility of changing the topology of a HgTe sample by applying linearly polarized light leading to so-called Floquet topological insulators has been reported.

In this work we study how SOIs of the intrinsic and Rashba type manifest themselves in a monolayer of graphene under the influence of a time-dependent field whose energy is on the terahertz (THz) regime. The SOC coupling constants are chosen to be of the order of the photon energy. This work is organized as follows. In Sec. [II] we introduce the model Hamiltonian and briefly summarize the main properties of the solution of Schrödinger’s equation according to Floquet’s theorem. In Sec. [III] the energy spectrum and the density of states (DOS) of the driven system is discussed and signatures arising from the interplay of SOC and the THz field are pointed out. In Sec. [IV] the dynamics of physical observables such as the spin polarization and the position operators are studied. In Sec. [V] the optical conductivity of the irradiated sample is calculated for various combinations of the SOC parameters. Finally, Sec. [VI] summarizes the main results of this work.

II. THE MODEL

We use the Kane-Mele model\cite{Kane97} (setting $\hbar = 1$ throughout this work)

$$
\hat{H}_0 = v_F k \cdot \sigma s_0 + \lambda_I \sigma_z s_z + \lambda_R (\sigma \times s) e_z \tag{1}
$$

to describe a monolayer of graphene including SOIs of the intrinsic ($\lambda_I$) and Rashba ($\lambda_R$) type at one K point. The Pauli matrices $\sigma$ ($s$) and the unit matrix $s_0$ ($s_0$) act on the pseudospin (real spin) space. The other K point can be described by the above Hamiltonian with $\sigma_x \rightarrow -\sigma_x$ and $\sigma_z \rightarrow -\sigma_z$. The effect of the electromagnetic field can be incorporated by the minimal coupling scheme $k \rightarrow k + eA(t)$. As the vector potential does not depend on...
the position operators, the Hamiltonian remains diagonal in momentum space and we can treat $k$ as a number instead of a differential operator. The time-dependent contribution to the Hamiltonian,

$$\hat{H}_1(t) = ev_F \mathbf{A}(t) \cdot \mathbf{\sigma} s_0,$$

is assumed to be periodic in time, i.e., $\hat{H}_1(t+T) = \hat{H}_1(t)$, where $T = 2\pi/\Omega$ and $\Omega$ is the frequency of the radiation field.

The vector potential describing a monochromatic wave propagating perpendicular to the graphene plane can either be assumed to be classical,

$$\mathbf{A}(t) = \frac{\sqrt{2}E_0}{\Omega} \left[ \cos \theta_p \cos \Omega t \ \mathbf{e}_x + \sin \theta_p \sin \Omega t \ \mathbf{e}_y \right],$$

or quantized,

$$\mathbf{A}(t) = \mathbf{A} \left[ \cos \theta_p \left( \hat{a} e^{-i\Omega t} + \hat{a}^\dagger e^{i\Omega t} \right) \ \mathbf{e}_x + i \sin \theta_p \left( \hat{a} e^{-i\Omega t} - \hat{a}^\dagger e^{i\Omega t} \right) \ \mathbf{e}_y \right],$$

where, among obvious notation, the parameter $\mathbf{A}$ contains geometric information about the cavity surrounding the system. The field is either circularly ($\theta_p = 45^\circ$) or linearly polarized (e.g., along the $x$ direction for $\theta_p = 0^\circ$). Quantizing the electromagnetic field adds a degree of freedom described by the bosonic operators $\hat{a}(t)$, which comes along with a new conserved quantity given by the helicity $\hat{\varepsilon} = \hat{j} + \hat{a}^\dagger \hat{a}$, where the angular momentum, $\hat{j} = x \hat{k}_y - y \hat{k}_x + \sigma_z / 2$, generates rotations of the carrier degrees of freedom in real and pseudospin space. To treat the electromagnetic field as a quantized operator is important in situations where the charge carrier have significant back-action on the field which in turn can alter the particle dynamics itself.

To analyze this aspect further, let us consider the case of a vanishing field and neglect SOIs for the moment. Now assuming a wave packet with initial momentum along the $y$ axis and the pseudospin initially in the $x$ direction, the dynamics of the system in Heisenberg representation is given by

$$\frac{d^2}{dt^2} \hat{P}_H(t) = -2v_F^2 k \sin (2v_F k t)$$

and $d^2 \hat{y}_H(t)/dt^2 = 0$. From the classical expression for the radiative power of dipolar radiation

$$P = (\epsilon \varepsilon_0 c^3)^2 / 6\pi \epsilon_0 c,$$

we find the time-averaged energy loss per time as

$$\bar{P} = \frac{\epsilon^2 v_F^4 k^2}{3\pi \epsilon_0 c^3} \approx 7.12 \times 10^{-2} \ \text{meV} \ \text{nm}^{-2} \ \text{ps}^{-1},$$

i.e., for a wave vector of $k = 0.1 \text{nm}^{-1}$ the radiative power is of order $10^{-4} \text{meV}/\text{ps}$. Due to the very large Fermi velocity of $v_F = 10^6 \text{m/s}$ in graphene, a time scale of 1ps corresponds, for appropriate initial conditions, to a distance of $1 \mu\text{m}$ traveled by the wave packet. Therefore, the above loss rate should be seen as a small effect. Hence the energy loss due to dipolar radiation induced by Zitterbewegung can be neglected compared to other energy scales in typical experimental situations. Accordingly, we will treat in what follows the electromagnetic field as a classical quantity and not as an operator. For convenience we introduce the dimensionless quantity $\alpha = v_F e E_0 / \Omega^2$.

Due to the periodicity of $\hat{H}(t) = \hat{H}_0 + \hat{H}_1(t)$, the solution of Schrödinger’s equation,

$$\left[i \partial / \partial t - \hat{H}\right] \Psi_{\kappa,\mu\nu} = 0,$$

and thus is of the form

$$|\Psi_{\kappa,\mu\nu}(t)\rangle = e^{-i\varepsilon_{\kappa,\mu\nu} t} |\psi_{\kappa,\mu\nu}(t)\rangle,$$

where $\mu, \nu = \pm 1$ are band indices. The Floquet states $|\psi_{\kappa,\mu\nu}(t)\rangle$ have the same periodicity as the Hamiltonian and can be expanded in a Fourier series

$$|\psi_{\kappa,\mu\nu}(t)\rangle = \sum_{n=-\infty}^{\infty} e^{in\Omega t} |\chi_{\kappa,\mu\nu}^n\rangle.$$

The eigenproblem can now be reduced to the diagonalization of the time-independent Floquet Hamiltonian whose components are defined by

$$\left(\hat{H}_F\right)_{nm} = \frac{1}{T} \int_0^T dt \hat{H}(t) e^{i(n-m)\Omega t} - n\Omega \delta_{nm}.$$

The time evolution of an arbitrary state with respect to an initial time $t_0$ is captured by the operator

$$\hat{U}_k(t, t_0) = \sum_{\mu', \nu'} e^{-i\varepsilon_{\kappa,\mu'\nu'}(t-t_0)} \langle \psi_{\kappa,\mu'\nu'}(t) | \langle \psi_{\kappa,\mu\nu}(t_0) |.$$

However, the energies and wave functions entering Eq. (7) are not uniquely defined as $|\Psi_{k,\mu\nu}^n\rangle = e^{in\Omega t} |\Psi_{k,\mu\nu}\rangle$ (with $n \in \mathbb{Z}$) is a solution of $\hat{H}(t)$ as well. The corresponding quasienergy, $\varepsilon_{k,\mu\nu}^n = \varepsilon_{k,\mu\nu} + n\Omega$, differs only by a multiple of the THz energy. Hence the choice of the eigenenergies is ambiguous as they describe the same physical situation. In order to get a well-defined quantity being the same for all $\varepsilon_{k,\mu\nu}$, we furthermore introduce the time averaged (or quasi-stationary) energy

$$\bar{\varepsilon}_{k,\mu\nu} = \frac{1}{T} \int_0^T dt \langle \Psi_{k,\mu\nu}(t)| \hat{H}(t)|\Psi_{k,\mu\nu}(t)\rangle.$$

In general, there is a non trivial relation between the quasienergies and the mean energies. Notice that in the absence of the driving Eq. (11) reproduces the energies of the unperturbed system (see below).
III. ENERGY SPECTRUM AND DENSITY OF STATES

A. Energy bands

The energy bands of the static problem ($\alpha = 0$) can readily be obtained:

$$E_{\mu\nu}(k) = \mu \lambda_R + \nu \sqrt{v_F^2 k^2 + (\lambda_R - \mu \lambda_I)^2}. \quad (12)$$

For a finite driving, the eigensystem is calculated numerically by diagonalization of the Floquet Hamiltonian in Eq. (9). As mentioned above, this leads to an infinite number of eigenenergies where only four of them are physically independent (corresponding to the dimension of the problem), while all others can be obtained by adding or subtracting a multiple of the energy of the electromagnetic field $E_{\text{em}} = \Omega$.

In Fig. 1 and Fig. 2, the quasienergies within the first and second Brillouin zone (BZ) are shown as red lines for different combinations of the SOC parameters for a fixed field strength of $\alpha = 0.3$. The black dashed line in Fig. 1 shows, for comparison, Eq. (12) projected to the BZ. The SOC parameters are chosen to be of the order of the THz energy, e.g., in the present case $\lambda_R/\Omega, \lambda_I/\Omega = (a) (0, 0), (b) (0, 1), (c) (0.1, 0)$ (c), and (d) (0.1, 0.1). The field strength was set to $\alpha = 0.3$.

Circular polarization The unperturbed energy spectrum of Eq. (12) consists of twofold spin-degenerate bands if $\lambda_R = 0$, while for a finite Rashba coefficient structure inversion symmetry is broken and the bands split up; see dashed lines in Fig. 1. Once $\hat{H}(t)$ is turned on, in (a), (c) and (d) a gap opens up right the Dirac point separating the valence and conduction bands. Here the bands are parabolic around the $K$ point but closely follow the linear behavior of the unperturbed result for $v_F k \gg \Omega$. Similar, a finite gap also appears in the mean energies lifting the $K$ point degeneracy, e.g., in Fig. 3(a) with a gap of $\delta_0 = 4\Omega\alpha^2/\sqrt{1 + 4\alpha^2}$. For finite SOIs the bands react differently on the THz field and hence the degeneracy present in the static case of Fig. 1(b), where $\lambda_I = 0.1\Omega$ and $\lambda_R = 0$, disappears. Right at the Dirac point the quasienergy gap vanishes, while a new gap opens up between the conduction (or valence) bands with different spin orientations. Two of the four bands are now linear and not parabolic as in the case of $\alpha = 0$. Similar, the gap in the time-averaged energies shown in Fig. 3(b) is closed. For larger momenta, $v_F k > 0.5\Omega$, the spin splitting in (b) eventually becomes so small that the bands are virtually degenerate again.

From Fig. 1 we can see that besides the gap at the Dirac point, additional gaps appear at $v_F k \approx n\Omega/2$ ($n \in \mathbb{Z}$). While these gaps are quite large for $v_F \Omega \approx \Omega/2$ and $\Omega$, its value strongly decreases for larger momenta and seems to vanish for $v_F \Omega \gtrsim 1.5\Omega$. The reason for these gaps is the existence of photon resonances, i.e., the absorption and emission of photons, similar to the ac Stark effect in semiconductors. Here transitions might occur at the resonant points $E_{\mu\nu} - E_{\mu'\nu'} \approx nE_{\text{em}}$. In the vicinity of the resonances, $v_F k \approx 0.5n\Omega$, the average energies drop to zero. For large enough momenta the dips eventually become so narrow that they seem to disappear. In case spin degeneracy is broken (i.e., $E_{+\pm} \neq E_{-\pm}$), the above resonant condition can be fulfilled for multiple values of $k$ and hence we observe not one but several nearby dips in the average energy spectrum as shown in Figs. 3(c) and (d).

Linear polarization If the field is linearly polarized, in the following along the $x$ direction, the energy spectrum is expected to be strongly anisotropic. Contrary to the circular case spin degeneracy is broken only if $\lambda_R \neq 0$.

From Fig. 2(a) we can see that for $\lambda_R = 0$ and $\lambda_I = 0$ the quasienergy spectrum for an in plane angle of $\phi_k = 0^\circ$, where $\tan \phi_k = k_y/k_x$, exactly follows the unperturbed spectrum, i.e., the field has no influence. However, if at least one of the SOC parameters is finite, the valence and conduction bands no longer touch at $v_F k \approx 0.5\Omega$, where deviations from the static results are largest, and the THz field induces a gap in Fig. 2(b). The corresponding time-averaged energies, shown in Figs. 2(b)-(d), exhibit characteristic dips at $v_F k \approx 0.5n\Omega$, as for circularly polarized light. However, from Fig. 3(a) and (d) we can see that only those bands are affected by the THz field that are (in the static limit) not linear but parabolic in momentum while the linear bands remain unchanged and in particular
FIG. 2: (Color online) Quasienergy spectrum under linearly polarized light ($\theta_p = 0^\circ$) for various combinations of the SOC parameters and momentum in plane orientations ($\tan \phi_k = k_y/k_x$): ($\lambda_R/\Omega, \lambda_I/\Omega$) = (0, 0) (left column), (0, 0.1) (second from left), (0.1, 0) (second from right), and (0.1, 0.1) (right). The field strength was set to $\alpha = 0.3$. Ungapped. Notice that contrary to the above case where $\theta_p = 45^\circ$ the positions of the dips in the average energies are nearly the same for both spin orientations.

For $\phi_k = 45^\circ$, see middle panel of Fig. 2, we observe remarkable gaps in all quasienergy spectra at $v_F k \approx 0.5\Omega$ and $v_F k \approx \Omega$. Besides that for finite SOIs an additional small gap opens up at the $K$ point separating the valence and conduction bands; see Figs. 2(g) and (h). The time-averaged energies as shown in the middle panel of Fig. 4 resemble the circular result of Fig. 3. The important differences, however, are the absence ((e) and (h)) or reduction ((f) and (g)) of the gap at the Dirac point and the fact that the THz does not cause an additional spin splitting of the bands; compare Fig. 1(b) and Fig. 2(f). Contrary to the case of $\phi_k = 0^\circ$ the positions of the resonant dips clearly split up for $\lambda_R \neq 0$.

Finally, for an in plane angle perpendicular to the polarization direction, i.e., $\phi_k = 90^\circ$, again in all four cases a distinct gap opens up at $v_F k \approx 0.5\Omega$. While for $\lambda_R = 0$ the $K$ point energies do not change, a small gap opens up in the quasienergies in Figs. 2(k) and (l) where $\lambda_R \neq 0$. Furthermore, the dips in the mean energies of cases Figs. 3(i) and (j) are suppressed for $v_F k = \Omega$ but they are clearly present in (k) and (l).

FIG. 3: (Color online) Mean energies derived from Eq. 11 under circularly polarized light ($\theta_p = 45^\circ$) for various combinations of the SOC parameters: ($\lambda_R/\Omega, \lambda_I/\Omega$) = (a) (0, 0), (b) (0, 0.1), (c) (0.1, 0), and (d) (0.1, 0.1). The field strength was set to $\alpha = 0.3$. Ungapped. Notice that contrary to the above case where $\theta_p = 45^\circ$ the positions of the dips in the average energies are nearly the same for both spin orientations.
FIG. 4: (Color online) Mean energies derived from Eq. (11) under linearly polarized light ($\theta_p = 0^\circ$) for various combinations of the SOC parameters and momentum in plane orientations ($\tan \phi_k = k_y/k_x$): ($\lambda R/\Omega, \lambda I/\Omega$) = (0, 0) (left column), (0, 0.1) (second from left), (0.1, 0) (second from right), and (0.1, 0.1) (right). The field strength was set to $\alpha = 0.3$.

B. Density of states

In Fig. 5 and Fig. 6 the time-averaged DOS,

$$D(E) = g_v \sum_{k,\mu\nu} \sum_{n=-\infty}^{\infty} \left\langle \chi_{k,\mu\nu}^n | \chi_{k,\mu\nu}^n \right\rangle \delta[E - \varepsilon_{k,\mu\nu} + n\Omega],$$  \hspace{1cm} (13)

is shown for various combinations of the SOC parameters with (red solid line) and without (black dashed) electromagnetic field for circularly and linearly polarized light, respectively. The field amplitude was set to $\alpha = 0.3$. The prefactor $g_v = 2$ is due to the valley degeneracy.

The static DOS for zero energy, shown as the dashed lines in Figs. 5 and 6, is zero in (a) and (b) and finite in (c) and (d). In the latter case ($\lambda R = \lambda I$) the charge neutrality point is shifted to $\lambda R/I$. The electromagnetic field yields a finite weight $\left\langle \chi_{k,\mu\nu}^n | \chi_{k,\mu\nu}^n \right\rangle$ to the subbands in the first BZ even for momenta $v_F k > 0.5\Omega$. This leads to a distinct increase of the DOS for small energies compared to the field-free situation. In Fig. 6(b), for example, the DOS becomes peaked, with $D(E)$ roughly being linear around $v_F k \approx 0.5\Omega$ and $\Omega$. This is in clear contrast to the case of circularly polarized light, where the decrease of the DOS is much smoother and the DOS becomes peaked, with $D(E)$ roughly being linear around $v_F k \approx 0.5\Omega$ and $\Omega$. Moreover, if spin degeneracy is lifted, the DOS shows additional dips in between neighboring Van Hove singularities. This is also true in Fig. 5(b), where the splitting is caused by the THz field and not by the Rashba term.

In the static limit signatures of SOIs in the DOS can only be seen in a narrow region with $E \lesssim 0.25\Omega$, while for larger energies it is virtually the same in all cases (see dashed lines in Figs. 5 and 6). This changes once the field is switched on. Here SOC manifests itself even for larger energies. Comparing, e.g., Fig. 5(a) and Fig. 5(c), we see a remarkable difference even for energies $E \approx \Omega$ due to the additional dips and peaks in the DOS. This can be understood from the quasiequilibrium spectrum, e.g., in Fig. 1(c), where due to the breaking of spin degeneracy several nearby points with a horizontal dispersion exist. For circularly polarized light qualitatively the same happens also for the case of a purely intrinsic coupling.

While due to the isotropy of the quasienergy spectrum in the case of circularly polarized light these singularities occur for arbitrary angles of $\phi_k$, for a linearly polarized field not all angles lead to Van Hove singularities. As a consequence, the associated peaks rise much stronger for $\theta_p = 45^\circ$ compared to $\theta_p = 0^\circ$. In the former, the DOS drops down almost vertically and remains roughly constant around $v_F k \approx 0.5\Omega$ and $\Omega$. This is in clear contrast to the linearly polarized case, where the decrease of the DOS is much smoother and the DOS becomes peaked, with $D(E)$ roughly being linear around $v_F k \approx 0.5\Omega$ and $\Omega$. Moreover, if spin degeneracy is lifted, the DOS shows additional dips in between neighboring Van Hove singularities. This is also true in Fig. 5(b), where the splitting is caused by the THz field and not by the Rashba term.
(λ_R = 0) as the bands split up for \( \alpha \neq 0 \). However, this splitting is significant only for small momenta and hence the multiple dips in Fig. 5(b) can be seen only for energies around \( E \approx 0.5 \Omega \).

IV. SPIN POLARIZATION AND WAVE PACKET DYNAMICS

We now discuss the dynamics of the real spin expressed by the operator \( \hat{S}_{H,j}(t) = \sigma_{H,0} \hat{s}_{H,j}(t) \) (\( j \in \{x,y,z\} \)). We restrict ourselves to an initial state described by a Gaussian wave packet for a single momentum \( \langle \Phi_{in}| \rangle \) which is appropriate for a sufficiently broad initial wave packet.\(^{35}\)

\[
\langle r|\Phi_{in}(t_0)\rangle = \frac{1}{\sqrt{\pi d^3}} e^{-\frac{r^2}{2d^2}} \begin{pmatrix} \eta_1 \\ \eta_2 \\ \eta_3 \\ \eta_4! \end{pmatrix}
\]  

In the following the spinor components in Eq. (14) are chosen as \( \eta_1 = -i\eta_2 = \eta_3 = \eta_4 = 0.5 \), i.e., the initial state is in general a linear combination of the static eigenvectors. Because of

\[
\frac{d}{dt} \hat{S}_{H,z}(t) = -2\lambda_R \left[ \sigma_{H,x} \hat{s}_{H,x}(t) + \sigma_{H,y} \hat{s}_{H,y}(t) \right],
\]

changes in the initial out of plane spin polarization (SP)

\[
\langle S_z(t_0) \rangle = |\eta_1|^2 + |\eta_2|^2 - |\eta_3|^2 - |\eta_4|^2,
\]

where \( \langle \cdot \rangle := \langle \Phi_{in}|\Phi_{in}\rangle \), can only be induced if the Rashba contribution is finite. Similar, for the other two spin directions,

\[
\langle S_x(t_0) \rangle = 2\text{Re} \left\{ \bar{\eta}_1 \eta_3 + \bar{\eta}_2 \eta_4 \right\}
\]

and

\[
\langle S_y(t_0) \rangle = 2\text{Im} \left\{ \bar{\eta}_1 \eta_3 + \bar{\eta}_2 \eta_4 \right\},
\]

whose dynamics are described by

\[
\frac{d}{dt} \hat{S}_{H,z}(t) = 2 \left[ \lambda_R \sigma_{H,x} \hat{s}_{H,x}(t) + \lambda_I \sigma_{H,y} \hat{s}_{H,y}(t) \right],
\]

at least one of the SOC coefficients has to be non zero in order to get a non trivial time evolution. In the following we set without loss of generality \( t_0 = 0 \).

In Fig. 7 we fix the intrinsic parameter \( \lambda_I = 0.25 \Omega \) and vary the Rashba constant at the Dirac point. The field strength is set to \( \alpha = 0.5 \). While for \( \lambda_R \neq \lambda_I \) the in plane SP of the static system, exemplarily shown for the \( x \) component in Fig. 7(a), shows fast oscillations around zero, right at the point \( \lambda_R = \lambda_I \) the expectation values \( \langle S_x(t) \rangle \) and \( \langle S_y(t) \rangle \) oscillate around a finite value. The field is linearly polarized along the \( x \) direction, \( S_y \) is finite only for \( \lambda_R = \lambda_I \), with \( S_y = -0.5 \) at that point, as in the static case. However, this is no longer true for \( S_x \) as can be seen from Fig. 8(b) where the peak for the \( x \) component disappears. For circularly polarized light both the peak for \( S_x \) and \( S_y \) at \( \lambda_R = \lambda_I \) vanishes and a significantly reduced dip at \( \lambda_R \approx 0.34 \Omega \) appears. The out of plane SP \( \langle \hat{S}_z \rangle \) of the static system oscillates around zero, where the period of the oscillations increases for larger \( \lambda_R \); see Fig. 7(b). Hence contrary to \( \hat{S}_{x/y} \) the mean polarization \( \hat{S}_z \) vanishes for arbitrary \( \lambda_R \),

FIG. 5: (Color online) Time-averaged density of states calculated from Eq. (13) under circularly polarized light \( \theta_p = 45^\circ \) for various combinations of the SOC parameters: \( (\lambda_R/\Omega, \lambda_I/\Omega) = (a) \) (0, 0), (b) (0.1), (c) (0.1, 0), and (d) (0.1, 0.1). The field strength was set to \( \alpha = 0.3 \).

FIG. 6: (Color online) Time-averaged density of states calculated from Eq. (13) under linearly polarized light \( \theta_p = 0^\circ \) for various combinations of the SOC parameters: \( (\lambda_R/\Omega, \lambda_I/\Omega) = (a) \) (0, 0), (b) (0, 1), (c) (0.1, 0), and (d) (0.1, 0.1). The field strength was set to \( \alpha = 0.3 \).
In this section the optical conductivity of irradiated graphene is calculated. As we are not interested in processes that appear right after or before the THz field is turned on and off, we consider the system in a quasi stationary state and assume the probability distribution to be of the form \( P(\mathbf{E}, \mu, \nu) \propto e^{-\beta \mathbf{E} \cdot \mu, \nu} \), where \( \mathbf{E} \) are the average energies introduced in Eq. (11) and \( \beta = 1/T \) the inverse temperature. The quasi equilibrium density matrix in the basis of the Floquet states then reads

\[
\langle \chi_{k,\mu,\nu} | \rho_{\text{eq}} | \chi_{k',\mu',\nu'} \rangle = \delta_{k,k'} \delta_{\mu,\mu'} f(\bar{\mathbf{E}}(\mathbf{k})).
\]

In the following, we restrict ourselves to zero temperature such that the Fermi distribution function reads \( f(\mathbf{E}) = \theta(\mathbf{E} - E_F) \), with \( E_F \) being the chemical potential. The expression for the dissipative part of the time-averaged longitudinal optical conductivity, obtained from

\[
\chi_{\text{opt}} = \int_0^\infty \langle \mathbf{v} \cdot \mathbf{E} \rangle dt
\]

and two different values of the Rashba SOC parameters for a total simulation time of \( \Omega t = 1000 \). While for \( \theta_p = 45^\circ \) and \( \lambda_R \neq 0 \) the trajectory becomes localized, as exemplarily shown in the red curve in Fig. 9(a) for \( \lambda_R = 0.5\Omega \), the basic propagation is along the \( y \) direction if the Rashba contribution vanishes and, compared to \( \langle y \rangle \), only moderate deviations from the initial position in \( x \) direction can be seen.

For \( \theta_p = 0^\circ \) and \( \lambda_R = 0.5\Omega \), see red curve in Fig. 9(b), the main dynamics is along the \( x \) axis with small oscillations around \( \langle y \rangle = 0 \), while in the other case of \( \lambda_R = 0.6\Omega \) (green line) the trajectory is again bounded in a finite region around \( \langle x \rangle = \pm 5v_F/\Omega \) and \( \langle y \rangle = \pm 25v_F/\Omega \), respectively.

V. OPTICAL CONDUCTIVITY

The time evolution of the position operators in Heisenberg representation is given by

\[
\frac{d}{dt} \hat{\mathbf{r}}_H(t) = i \left[ \hat{\mathbf{H}}, \hat{\mathbf{r}}_H \right] = v_F \mathbf{\sigma} \mathbf{r}_H \langle 0 \rangle (t).
\]

Note that contrary to electron and hole gas systems, the dissipative term proportional to momentum is missing in Eq. (15) due to the Dirac-like nature of the charge carriers in graphene. By calculating the usual velocity operator, \( \mathbf{v}_H(t) = v_F \mathbf{\sigma} \mathbf{H} \langle 0 \rangle (t) \), it is thus possible to extract the orbital dynamics of the system

\[
\langle \mathbf{r}(t) \rangle := \langle \Phi_{in} | \hat{\mathbf{r}}_H(t) | \Phi_{in} \rangle
\]

with respect to an initial wave packet given in Eq. (14). In Fig. 8 this is shown for circularly (a) and linearly (b) polarized light of strength \( \alpha = 0.5 \) for fixed \( \lambda_I = 0.5\Omega \) and \( \theta_p = 0^\circ \), as can be seen from the green line in Fig. 8(a). This remains true for linearly polarized light where \( \bar{\mathbf{r}} = 0 \). In Fig. 8(b) the main dynamics is along the \( x \) axis with small oscillations around \( \langle y \rangle = 0 \). For \( \alpha = 0.5^\circ \) and \( \lambda_R = 0.6\Omega \) (green line) the trajectory is again bounded in a finite region around \( \langle x \rangle = \pm 5v_F/\Omega \) and \( \langle y \rangle = \pm 25v_F/\Omega \), respectively.

\[
S_\| = 0 \quad \text{for linearly polarized light without electric field (top row), under a linearly polarized field (} \theta_p = 0^\circ, \alpha = 0.5 \text{) (middle), and for circular polarization (} \theta_p = 45^\circ, \alpha = 0.5 \text{) (bottom), as a function of the Rashba coefficient. Parameters: } \lambda_I = 0.25\Omega, k = 0.
\]

\[
\langle \chi_{k,\mu,\nu} | \rho_{\text{eq}} | \chi_{k',\mu',\nu'} \rangle = \delta_{k,k'} \delta_{\mu,\mu'} f(\bar{\mathbf{E}}(\mathbf{k})).
\]
the nonequilibrium Green's function method derived in Ref. [22], then reads
\[
\text{Re} \left\{ \sigma_{xx}(\omega) \right\} = \frac{\epsilon_0 e^2}{\omega} \sum_{k,m,j,\mu,\nu,\mu',\nu'} \sum_{n} \left| \chi_{k,\mu,\nu}^{n} \right|^{2} \times \left( f \left[ \varepsilon_{k,\mu,\nu} \right] - f \left[ \varepsilon_{k,\mu',\nu'} \right] \right) \delta \left[ \omega + \varepsilon_{k,\mu,\nu} - \varepsilon_{k,\mu',\nu'} - j\Omega \right],
\]
(16)

The quasienergies and states entering Eq. (16) are chosen to be in the first BZ, though any other choice is possible as well. From the \( \delta \)-function in Eq. (16) we can see that, in principle, transitions between all kinds of subbands are possible. In the static limit only those subbands that correspond to the energies of Eq. (12) have a non-zero weight and Eq. (16) reproduces previous results [15,50].

For a finite driving the weight of the other subbands becomes non-zero, whereas it increases for larger driving amplitudes, and hence additional transitions become possible.

In Figs. 10 and 11 we show the optical conductivity calculated for a fixed Fermi energy of \( E_F = 3\Omega \) under the influence of circularly and linearly polarized light, respectively. The field strength is \( \alpha = 0, 0.5, \) and 1.0. The main feature of the static conductivity, as shown e.g., in the dashed curve in Fig. 10(a), is its steplike behavior at \( \omega = 2E_F \), where transitions from the valence to the conduction band become possible. Switching on the time-dependent field leads to several additional steps in \( \sigma_{xx} \) due to photon-assisted processes. By comparing e.g., Figs. 10(a) and (c), it becomes clear that the number of steps increases for larger coupling strengths \( \alpha \) as the weight is distributed over a broader range of subbands. The effect of the Rashba term, which leads to a distinct breaking of the spin degeneracy of each subband, furthermore induces several intermediate steps as the number of possible transitions in the \( \delta \)-function of Eq. (16) becomes much larger. From Figs. 10(b) and 11(b) we can see that the basic structure of \( \sigma_{xx} \) is the same for \( \theta_p = 45^\circ \) and \( \theta_p = 0^\circ \), but in the latter the conductivity turns out to be slightly smoother. By increasing the field strength to \( \alpha = 1.0 \) we observe dips in the conductivity at \( \omega = n\Omega \) where the effect is clearly larger for \( \theta_p = 45^\circ \) than \( \theta_p = 0^\circ \), cf. e.g., Figs. 10(c) and 11(c). The inclusion of the Rashba term creates further dips for slightly smaller and larger frequencies, respectively. These dips are due to the appearance of gaps in the quasienergy spectrum, see discussion in Sec. III, as some transitions are no longer possible. From Figs. 10(c) and (d) one can see that while the static conductivities (dashed curves) are quite similar in both cases, i.e., the effect of \( \lambda_R \) is only little, remarkable differences occur in the driven case, and hence SOC effects are largely enhanced.

**VI. CONCLUSIONS**

In this work the effect of a time-dependent electric field on a monolayer of graphene including SOIs of the intrinsic and Rashba type has been studied.

We have demonstrated that a circularly polarized THz field can not only be used to induce a gap at the Dirac point, which transforms graphene from a semimetal to an insulator, but also to close an existing gap in the quasienergies. In the opposite case of a linear polarization the spectrum turned out to be highly anisotropic and, depending on the strength of the SOC parameters and on the orientation of the field, gaps in the spectrum might appear at the \( K \) point and at the photon resonances \( \nu_F K \approx 0.5n\Omega \) or become suppressed.

While the effect of SOIs on the DOS of the static sample could only be seen for energies \( E \lesssim 0.25\Omega \), due to the existence of a multiple number of dips, signatures of SOC in the DOS of irradiated graphene appear even at much larger energies.

By introducing a time-dependent field it turned out to be possible to induce a finite net spin polarization in the sample. The sign and magnitude, e.g., of the out of plane
FIG. 11: (Color online) Optical conductivity under linearly polarized light (θp = 0°) for various field strengths α = 0 (black dashed curves), 0.5 (red lines in (a) and (b)), and 1.0 ((c) and (d)), and SOC parameters in units of σ0 = e2/4.

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The SOC parameters which can be done experimentally by adjusting the Rashba coefficient via an electric gate.

Finally, in the last part of this work the longitudinal optical conductivity was calculated. As reported already in Ref.[22], the conductivity of irradiated graphene exhibits a multi step structure as transitions between a variety of subbands become possible. The number of steps depends not only on the coupling strength, but also on the magnitude of the Rashba parameter and the polarization direction. Furthermore, for large enough coupling strengths the conductivity drops down for frequencies around the photon energy.[22] As for the DOS, compared to the static result the effect of SOIs on the optical conductivity is largely enhanced for α ≠ 0, which is mainly caused by the Rashba contribution.

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It is instructive to compare this result with the situation of Zitterbewegung in a two-dimensional electron gas with SOC of, say, the Rashba type. Here the Hamiltonian reads $\hat{H}_R = \frac{p^2}{2m} + \alpha [p_x \sigma_y - p_y \sigma_x]$, where $m$ is an effective band mass, and the Rashba parameter $\alpha$ is of order $10^{-11}$eVm. Performing analogous steps as above, it is easy to see that the radiative power here is given by Eq. (6) when replacing $v_F$ with $\alpha/\hbar \approx 1.5 \cdot 10^4$ m/s and hence $P_{\text{2DEG}}/P_{\text{gr}} = (\alpha/\hbar v_F)^4 \approx 5 \cdot 10^{-8}$. The energy loss due to the dipolar radiation in a two-dimensional electron gas is several orders of magnitude smaller than in graphene and thus a vanishingly small effect.