Mathematical modelling of a novel heterojunction SIS front surface and interdigitated back-contact solar cell

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Abstract
In this paper, we propose the design and fabrication of a novel heterojunction semiconductor–insulator–semiconductor (SIS) front surface and interdigitated back-contact (IBC) solar cell. We approximate the performance parameters and loss analysis of the proposed solar cell using MATLAB software programming. Many studies have reported the experimental analysis of amorphous silicon (a-Si) IBC solar cells. A number of silicon heterojunction solar cell designs with promising efficiency have been reported in the past few decades. In this study, a long-lifetime (~ 2 ms) n-Si substrate was considered so that a sufficient number of photogenerated carriers could reach the interdigitated layer and be absorbed. The availability of carriers at the interdigitated back surface was further enhanced by considering a high-low junction created by a ZnO n+ layer at the front surface. A very thin layer of thermally deposited insulator SiO2 was considered between the ZnO and n-Si. This layer reduces the detrimental effects of interface defects. This is the first study in which we have theoretically investigated an IBC solar cell using metal oxide semiconductor layer deposition, thereby avoiding the expensive and complicated doping and diffusion process. In general, a high-concentration n+ layer is doped to create a high-low junction at the front to accelerate the transport of carriers to the back junctions. We propose a cost-effective method using thermal deposition of a SiO2 layer followed by sol–gel ZnO layer deposition, which serves the same purpose as an n+ layer by introducing an SIS junction potential at the front. The interdigitated back surface was designed with sequential n+ a-Si and p+ a-Si vertical junctions.

Keywords Interdigitated back-contact solar cell · SIS solar cell · MATLAB · ZnO · Mathematical modelling

1 Introduction
The optimization of front surface metallization is a continuing challenge in solar cell design. On one hand, less metallic contact on the front surface increases the front surface layer resistivity, causing high internal series resistance, which is an impediment to achieving higher efficiency. On the other hand, we can decrease the series resistance using different grid patterns of closely placed metal contacts, but this causes a greater shading effect. Light entering the cell is reduced, resulting in lower efficiency. To resolve this dilemma, Schwartz et al. designed and fabricated a solar cell in 1975 [1] which had all the metal contacts on the back surface. The back surface was fabricated with sequential n+ and p+ layers parallel to the rays of light falling on the cell. This interdigitated back surface creates multiple junctions at the back. The open-circuit voltage arises from the junction built-in potentials as per connection. The short-circuit current of an interdigitated back-contact (IBC) solar cell largely depends on the bulk lifetime [2]. The charge carrier transport can be modified by creating a front surface field (FSF). This FSF enhances the photogenerated current at the back surface interdigitated junctions. Bao et al. recently reported a theoretical analysis showing the effect of the FSF created by a hydrogenated amorphous silicon (a-Si:H) n+ layer on a crystalline silicon (c-Si) n substrate [3]. In this paper, we adopt a different approach to analyse the overall performance of a c-Si n substrate-based a-Si:H n+ and a-Si:H p+ IBC solar cell with and without an FSF. We investigate the effect of the FSF formed by a semiconductor–insulator–semiconductor (SIS) Schottky junction (FSF1). We present a comparative
performance analysis between the effects of FSF1 and FSF2 (created by an $n^+$-amorphous front layer) on the IBC without a solar FSF.

A suitable mathematical model for the IBC solar cell, as depicted in Fig. 1a–c, is presented using MATLAB programming. MATLAB is useful software for solar cell modelling. We developed a mathematical model for a c-Si and SIS solar cell in our previous studies [4, 5]. Dey et al. used the MATLAB Simulink environment to establish a general model of a solar cell [6]. Nayan et al. solved the standard diode equation to obtain the $I–V$ characteristics of a silicon solar cell [7]. We were motivated by the logic and algorithms used in these works. In this paper, first, a simple IBC structure without a back surface field (BSF) (Fig. 1a) is theoretically designed by solving the second-order transport equation and corresponding energy band diagram. The same procedure is then followed for an IBC with a BSF ($n^+$ ZnO) structure as shown in Fig. 1b, c to describe our novel IBC structure with a Schottky heterojunction SIS FSF ($n^+$ a-Si). We further investigate the effect of the IBC finger width on the solar cell performance. Finger width optimization is an important factor in IBC solar cell performance [8, 9]. We have tried to find a facile solution to optimize the finger width in this mathematical modelling. In this study, an $n$-type c-Si with a long-lifetime substrate was considered. The front surface was considered passivated with an anti-reflective coating (ARC) of a SiN$_x$ layer. The IBC was designed with sequential $n^+$ and $p^+$ hydrogenated amorphous silicon (a-Si:H) fingers. The $n^+$ finger acts as emitter, while the $p^+$ fingers provide the BSF. The IBC cell was modified by a tunnelling SiO$_2$ layer and ZnO layer (deposited by the sol–gel method) to form an FSF. The physical properties and structural parameters are taken from previously published reports and are listed in Table 1.

Here, we consider some ideal conditions which have negligible effects on the actual values. These are listed below.

1. Surface recombination and the fringe effect at the edge are neglected [8, 8].
2. The ideality factor of each $p–n$ junction is considered to be $1$ [20].
3. The cell parameters are considered to be homogeneous two-dimensional parameters; that is, the 2D unit cell structure shown in Fig. 2 is considered to be repeated along the horizontal axis throughout the cell.
4. The FSF is considered to have no effect on the dark saturation current. There are no electrodes at the front surface.
5. The photogeneration of carriers at the FSF layer is considered to be negligible with respect to the bulk layer generation.
6. The effect of the FSF layer is only considered in the bulk region.

Figure 2 shows the unit cell structure of an IBC solar cell. This structure is considered the basic unit. If the total width (W) of the cell is W μm, then there are W/750 numbers of such units in the cell. In this study, we consider three cases for FSF.

Case I: No FSF is considered.

Case II: FSF formed by a ZnO-SiO$_2$/c-Si($n^+$-type) SIS junction.

Case III: FSF formed by an a-Si:H $n^+/c$-Si($n$-type) heterojunction.

We simulated the above cases with MATLAB software and performed a comparative analysis. We solved the basic partial differential equations (PDE) to obtain the values of the collection probability and photogenerated carriers at each point of the 2D model of the cell as shown in Fig. 2. Hence, we evaluated the photogenerated current integrating all the values. The dark saturation current is a junction property. We calculated the dark current from the appropriate band diagram of the IBC heterojunction. The front junction plays no role in photo collection, and hence the dark current density does not depend upon the nature of the FSF.
Energy band diagram and dark saturation current

The IBC is formed by an a-Si:H n+/c-Si(p-type)/a-Si:H p+ heterojunction. The energy band diagram of this heterojunction device is shown in Fig. 3. The built-in potential was estimated from the band bending of the heterojunction. When the device reaches equilibrium, the Fermi level throughout the device must be constant.

The built-in potential of the a-Si:H n+/c-Si(p-type) junction and the c-Si(p-type)/a-Si:H p+ junction contribute to the overall built-in potential of the device. Analytically, if the built-in potential \( \psi_1 \) arises from the band bending of junction 1 (\( E_1 - E_2 \)) and \( \psi_2 \) from the band bending of junction 2 (\( E_2 - E_3 \)), then the total built-in potential \( \psi = \psi_1 + \psi_2 \) is due to the total band bending (\( E_1 - E_3 \)). This implies that the overall built-in potential can be estimated from the equivalent band bending of the junction between a-Si:H n and a-Si:H p+ − Δ\( E_g \), where \( ΔE_g \) is the difference between the band gap energy of amorphous silicon and crystalline silicon. We estimated the value of the resultant built-in potential by an equivalent homojunction of amorphous silicon.

| Material | Dimensions | Properties |
|----------|------------|------------|
| ARC (SiNx) | Thickness \( d_{ARC} = 75 \text{ nm} \) | Refractive index \( n = 2.05 \) [10] |
| FSF2 (n-type ZnO) | Thickness \( d_{FSF} = 400 \text{ nm} \) | Carrier concentration \( N_{ZnO} = 8 \times 10^{18} \text{ cm}^{-3} \) [11] |
| | | Energy band gap \( E_{ZnO} = 3.37 \text{ eV} \) [12] |
| | | Electron affinity \( \chi_{ZnO} = 4.5 \text{ eV} \) [12] |
| | | Effective mass of electron \( m_{e,ZnO} = 0.24 m_0 \) [13] |
| | | Effective mass of hole \( m_{h,ZnO} = 0.59 m_0 \) [13] |
| | | Relative permittivity \( \varepsilon_{ZnO} = 8.5 \) [14] |
| Tunnelling insulator (SiO2) | Thickness \( d_{SiO2} = 5 \text{ nm} \) | Relative permittivity \( \varepsilon_{SiO2} = 3.9 \) |
| Emitter finger (a-Si:H p+) | Width \( W_{BSF} = 500 \mu\text{m} \) | Carrier concentration \( N_{aSi} = 8 \times 10^{18} \text{ cm}^{-3} \) [16] |
| | | Energy band gap \( E_{aSi} = 1.7 \text{ eV} \) [3] |
| | | Electron affinity \( \chi_{aSi} = 3.8 \text{ eV} \) [3] |
| | | Effective density of state conduction band edge \( N_{C aSi} = 4.5 \times 10^{21} \text{ cm}^{-3} \) [17] |
| | | Effective density of state valance band edge \( N_{V aSi} = 6.4 \times 10^{21} \text{ cm}^{-3} \) [17] |
| | | Drift mobility of electron \( \mu_{e,aSi} = 1 \text{ cm}^2/\text{Vs} \) [17] |
| | | Drift mobility of hole \( \mu_{p,aSi} = 0.01 \text{ cm}^2/\text{Vs} \) [17] |
| | | Electron lifetime \( \tau_{aSi} = 900 \mu\text{s} \) [18] |
| | | Back surface recombination velocity \( S_{BSF} = 10 \text{ cm/s} \) [19] |
| BSF finger (a-Si:H n+)/FSF2 | Width \( W_{emitter} = 150 \mu\text{m} \) | Carrier concentration \( N_{aSi} = 5 \times 10^{18} \text{ cm}^{-3} \) [16] |
| | | Energy band gap \( E_{aSi} = 1.72 \text{ eV} \) [3] |
| | | Electron affinity \( \chi_{aSi} = 3.7 \text{ eV} \) [3] |
| | | Relative permittivity \( \varepsilon_{aSi} = 6 \) [16] |
| | | Effective density of state conduction band edge \( N_{C aSi} = 4.5 \times 10^{21} \text{ cm}^{-3} \) [17] |
| | | Effective density of state valance band edge \( N_{V aSi} = 6.4 \times 10^{21} \text{ cm}^{-3} \) [17] |
| | | Drift mobility of electron \( \mu_{e,aSi} = 1 \text{ cm}^2/\text{Vs} \) [17] |
| | | Drift mobility of hole \( \mu_{p,aSi} = 0.01 \text{ cm}^2/\text{Vs} \) [17] |
| | | Hole lifetime \( \tau_{p,aSi} = 700 \mu\text{s} \) [18] |
| | | Back surface recombination velocity \( S_{emitter} = 90 \text{ cm/s} \) [19] |
| c-Si n-type substrate (bulk) | Thickness \( d_{bulk} = 300 \mu\text{m} \) | Carrier concentration \( N_{cSi} = 2 \times 10^{16} \text{ cm}^{-3} \) [16] |
| | | Energy band gap \( E_{cSi} = 1.12 \text{ eV} \) [17] |
| | | Electron affinity \( \chi_{cSi} = 4.05 \text{ eV} \) [17] |
| | | Relative permittivity \( \varepsilon_{cSi} = 11.9 \) [17] |
| | | Effective density of state conduction band edge \( N_{C cSi} = 2.82 \times 10^{19} \text{ cm}^{-3} \) [17] |
| | | Effective density of state valance band edge \( N_{V cSi} = 1.83 \times 10^{19} \text{ cm}^{-3} \) [17] |
| | | Drift mobility of electron \( \mu_{e,cSi} = 1400 \text{ cm}^2/\text{Vs} \) [17] |
| | | Drift mobility of hole \( \mu_{p,cSi} = 480 \text{ cm}^2/\text{Vs} \) [17] |
| | | Best carrier lifetime \( \tau_{bulk} = 2000 \mu\text{s} \) [18] |
| Overall IBC solar cell | Area = 1 cm² | Front surface recombination velocity (FSRV) \( S_F = 500 \text{ cm/s} \) [10] |
| | | Back surface recombination velocity (BSRV) \( S_B = 500 \text{ cm/s} \) [10] |
| | | Contact resistance \( 0.1 \Omega/\text{cm}^2 \) [3] |
silicon. We applied the Fermi–Dirac statistical distribution to obtain the junction’s built-in potential [21].

\[ \Psi = \frac{1}{q} \ln \left( \frac{N_{AaSi} N_{DaSi}}{n_{iaSi}^2} \right) - \left( E_{gaSi} - E_{gcSi} \right) \] (1)

where \( n_{iaSiH} \) is the intrinsic concentration of amorphous hydrogenated silicon which can be derived from the equation as described by Markvart et al. [22].

\[ n_{iaSi} = (N_{CaSi}/N_{VaSi})^{1/2} \exp(-E_{gaSi}/2k_BT) \] (2)

Further, the dark saturation current density was computed as a function of built-in potential:

\[ J_0 = q e^{-q\Psi/k_BT} \left[ \frac{D_{paSi}}{\tau_{paSi}} + \frac{D_{naSi}}{\tau_{naSi}} \right] \] (3)

The diffusion coefficient was calculated from Einstein’s equation

\[ D = \frac{kT}{\mu} \] (4)

3 Collection probability and photogenerated current

The well-accepted photogenerated current density equation is written for the two-dimensional unit cell structure as shown in Fig. 2.

\[ J_L = \frac{J_L}{w} A/cm^2 \] (6)

If the width of the unit cell structure is \( w \) cm (0.075 cm), then the photogenerated current density in \( A/cm^2 \) is expressed by the following equation.

First, we calculated the photogenerated carrier distribution throughout the unit cell structure using the photon flux data for standard AM 1.5 irradiance. The surface roughness plays an important role in determining the reflectance for different wavelengths of light. The front surface is textured to form the microscale pyramidal structure. The front surface texturization is considered as a micro-pyramidal texture of 3 μm to 5 μm created by conventional KOH-IPA alkaline etching. The \( R(\lambda) \) data are extracted from the experimental graph of this texturization [23]. We used the experimental data that were obtained in our laboratory and published previously [24]. The absorption coefficient of Si was taken from data published by Green et al. [25]. Since we neglected the photogeneration at the FSF in our mathematical model, the photogenerated carrier distribution was unchanged for all cases considered.

We show the photogenerated carriers throughout the unit cell structure for total spectra of AM 1.5 irradiance in Fig. 3a, b. Now, the challenge is to obtain the collection probability throughout the unit cell structure. The collection probability largely depends upon the surface recombination and minority carrier lifetime. We consider three cases with different FSF and analyse their effect on the collection probability. The insightful calculation developed by Donolato was used to estimate the collection probability of the photogenerated carriers as shown in Fig. 3a, b. In 1985, Donolato proposed the reciprocity theorem, which states that “the current collected by a \( p-n \) junction as a unit carrier point charge generated at a point \( P(x) \) (where \( x \) is the distance of point P from the surface) is quantitatively the
same as the excess minority charge density at point $P(x)$ due to the unit density of the carrier at the $p$–$n$ junction edge” [26]. Later, in 1994, the theorem was formulated for two-dimensional applications [27]. The two-dimensional PDE for collection probability ($CP(x,y)$) is expressed as

$$D \left( \frac{\partial^2 CP(x,y)}{\partial x^2} + \frac{\partial^2 CP(x,y)}{\partial y^2} \right) - \frac{1}{\tau} CP(x,y) = 0 \quad (7)$$

where $D$ is the coefficient of diffusion and $\tau$ is the lifetime of the minority carriers.

We considered the boundaries of a unit cell structure in which photo-carriers are generated. Different boundary conditions were considered to solve Eq. 7 for these three different regions.

The bulk region boundaries are shown in Fig. 4. When determining the boundary conditions, we followed two fundamental considerations. First, the collection probability at the junction edge is always unity. All the carriers generated at the junction edge were collected immediately. Second, the boundary conditions which define the surface of the cell were set as the Neumann condition determined by the
surface recombination velocity. The different boundary conditions are explained below.

\( a \rightarrow b \) and \( m \rightarrow n \): Neumann condition

\[
- \frac{\partial C_p(x, y)}{\partial n_f} = 0
\]

As per our assumption, the recombination velocity at the edge is zero.

\( b \rightarrow c, f \rightarrow g \) and \( l \rightarrow m \): Neumann condition

\[
- \frac{\partial C_p(x, y)}{\partial n_f} = \left( \frac{S_F}{D_{pcSi}} \right) C_p(x, y)
\]

\( S_B \) is the back surface recombination velocity for c-Si.

\( n \rightarrow a \): Neumann condition

\[
- \frac{\partial C_p(x, y)}{\partial n_f} = \left( \frac{S_F}{D_{pcSi}} \right) C_p(x, y)
\]

\( S_F \) is the front surface recombination velocity for c-Si.

\( c \rightarrow d, d \rightarrow e \) and \( e \rightarrow f \): these are junction boundaries. We considered the Dirichlet condition.

\[
C_p(x, y) = 1
\]

\( c \rightarrow f \): Neumann condition

\[
- \frac{\partial C_p(x, y)}{\partial n_f} = \left( \frac{S_{BSF}}{D_{naSi}} \right) C_p(x, y)
\]

\( S_{BSF} \) is the surface recombination velocity of the BSF back surface.

\( g \rightarrow l \): Neumann condition

\[
- \frac{\partial C_p(x, y)}{\partial n_f} = \left( \frac{S_{emitter}}{D_{pSi}} \right) C_p(x, y)
\]

\( S_{emitter} \) is the surface recombination velocity of the emitter back surface.

We solve the reciprocity theorem (Eq. 7) considering the boundary conditions (Eq. 8–11) for three cases of three different FSF materials.

Case I: No FSF is considered.

We obtained Fig. 5, Fig. 6 and Fig. 7 by solving the PDE in MATLAB. The collection probability profile of the bulk, BSF and emitter regions are shown, respectively. The total collection probability \( C_p(x, y) \) is shown in Fig. 8. Theoretically, the light-generated current density for the unit cell structure is computed by multiplying the mesh point to mesh point values of Fig. 3b (photogenerated carriers) and Fig. 8 (collection probability). The photogenerated current for the unit cell structure is hence calculated as

\[
J_L = 2.329 \text{ mA}
\]

Therefore, the light-generated current density for the IBC was calculated as

\[
J_L = \frac{2.118}{0.075} = 28.24 \text{ mA/cm}^2
\]

Case II: FSF formed by a ZnO/SiO\(_2\)/c-Si(n-type) SIS junction.

In Case II we consider our novel proposition of an FSF induced by the front surface SIS junction with an \( n \)-type ZnO layer. The material properties of ZnO were taken from different previously published reports as listed in Table 1.
The SIS junction energy band diagram was helpful for finding the built-in potential across the FSF.

The potential was calculated from the energy band bending as shown in Fig. 9 [21].

The interface of the SIS structure acts as tunnelling media of the Schottky junction. The efficiency of the tunnelling depends on the tunnelling coefficient. The tunnelling coefficient was estimated from the formula proposed by Pauwels et al. [28].

\[
t_n = \exp \left( -2 \sqrt{\frac{2m^*_{\text{ZnO}} k_B T}{\hbar^2}} x_{\text{ZnO}d} \right) \tag{14}
\]

Here, \(d\) is the thickness of the insulator layer which was optimized to 10 Å [5]. Considering all the values from Table 1, we approximated the tunnelling probability as unity.

The FSF developed by the SIS junction was calculated from the energy band diagram shown in Fig. 9.

\[
\Psi_{\text{FSF1}} = \frac{1}{q} \left[ x_{\text{Si}} - x_{\text{ZnO}} + E_{\text{gSi}} - k_B T \ln \left( \frac{N_{\text{VSi}} N_{\text{CZnO}}}{N_{\text{AxCZnO}}} \right) \right] \tag{15}
\]

Further, the effect of the insulator interface on built-in potential was expressed as explained by Shousha [29].

\[
\Psi_s = \left( \frac{\gamma d_{\text{SiO}_2}/\varepsilon_{\text{SiO}_2}}{\varepsilon_{\text{Si}}} \right) \sqrt{2 q N_{\text{AxCZnO}}} \left[ \sqrt{\Psi_{\text{FSF1}}} - \sqrt{(\Psi_{\text{FSF1}} - \Psi_s)} \right] \tag{16}
\]

where

\[
\gamma = \frac{1}{1 + \left( qD_{ss} d_{\text{SiO}_2}/\varepsilon_{\text{SiO}_2} \right)} \tag{17}
\]

\(D_{ss}\) is the density of states on the interface. The value of the density of states was taken from our previously published work [30]. Hence, we approximated the effect of the interface on the built-in potential as negligible.

The field estimated from Eq. 15 accelerates the carriers to the back surface junction where they are collected. Thus, the probability of a carrier being collected is enhanced in the presence of the FSF. This effect is reflected in the collection probability profile. We used the continuity equation for electrons to understand the effect of this FSF [31].
In Eq. 14, $E$ is the electric field in a quasi-neutral area, and $G_p$ and $R_p$ are the generation and recombination rates. In Case I, as there was no FSF, $E$ was considered to be zero. Hence, the continuity equation was reduced to Eq. 15.

$$\frac{\partial \Delta p}{\partial t} = \mu_p \Delta p \frac{\partial E}{\partial y} + \mu_n E \frac{\partial \Delta p}{\partial y} + D_p \frac{\partial^2 \Delta p}{\partial y^2} + (G_p - R_p)$$  \tag{18}

In the presence of FSF there will be an electric field due to the built-in potential at the SIS junction as given by Eq. 13. Considering a steady state with no generation, Eq. 14 can be written as

$$\mu_p \Delta p \frac{\partial E}{\partial y} + \mu_n E \frac{\partial \Delta p}{\partial y} + D_p \frac{\partial^2 \Delta p}{\partial y^2} = \frac{\Delta p}{\tau_p}$$ \tag{20}

Here, the recombination rate is taken as

$$R_p = \frac{\Delta p}{\tau_p}$$ \tag{21}
We observed that Eq. 18 and Eq. 20 are mutually exclusive and do not satisfy Eq. 19. Hence, we consider the effective lifetime $\tau_{\text{eff}}$ in Eq. 20 for the change in the recombination rate in the presence of FSF. Equation 19 is rewritten taking the values from Eq. 18.

$$\mu_p \Delta p \frac{\partial E}{\partial y} + \mu_p E \frac{\partial \Delta p}{\partial y} + \frac{\Delta p}{\tau_p} \tau_{\text{eff}} = \frac{\Delta p}{\tau_{\text{eff}}}$$  \hspace{1cm} (22)

Further, considering no change in carrier concentration in the bulk, the change in carrier lifetime was established as

$$\frac{1}{\tau_p} - \frac{1}{\tau_{\text{eff}}} = \mu_p \frac{\partial^2 \psi_{\text{FSF1}}}{\partial y^2}$$  \hspace{1cm} (23)

From Poisson’s equation at the junction, we found the derivative of the built-in potential.

Fig. 7 Collection probability profile in the emitter region
Here, we considered that the excess carrier concentration is very high and the electric field does not change with respect to the $y$ axis (Fig. 4). We then approximated the effective carrier lifetime from Eq. 19 and solved the PDE of Eq. 7 using the appropriate boundary conditions of Eqs. 8–11.

Comparing Fig. 5a, b with Fig. 10a, b, we conclude that there is a significant improvement in collection probability in the bulk region. As per our assumption, the collection probability profile in BSF and the emitter region will be the same as in Case I.

In Fig. 11 we show the total collection probability profile of the unit cell structure. Following the same method as in Case I, we obtain the light-generated current density for Case II as 32.67 mA/cm2.

Case III: FSF formed by an a-Si: H $n^+$/c-Si($p$-type) heterojunction.

We follow the same approach to calculate the photogenerated current in Case III. The built-in potential is found from the following equation.

$$
\Psi_{FSF2} = \frac{1}{q} \left[ \chi_{cSi} - \chi_{aSi} + E_{gSi} - k_B T \ln \left( \frac{N_{VcSi} N_{CaSi}}{N_{AcSi} N_{DaSi}} \right) \right]
$$

The effective lifetime was likewise calculated as

$$
\frac{1}{\tau_p} - \frac{1}{\tau_{eff}} = \mu_p \frac{\partial^2 \psi_{FSF2}}{\partial y^2}
$$

$$
\frac{\partial^2 \psi_{FSF2}}{\partial y^2} = \frac{q}{\varepsilon_0} \left( \frac{N_{DaSi}}{\varepsilon_{aSi}} - \frac{N_{DcSi}}{\varepsilon_{cSi}} \right)
$$

The collection probability profile was thus solved for Case III using the effective lifetime from Eq. 21 (Figs. 12 and 13).

The estimated current density for the FSF of $a$-Si was 34.09 mA/cm2.

### 4 Series and shunt resistance

The series resistance of a solar cell is mainly a contact property. It arises from emitter resistance, BSF resistance and contact resistance in the IBC solar cell. The major advantage of IBC is very low series resistance. As all contacts were made at the back, almost 86% of the back surface was utilized for metallization.

$$
R_s = R_{BSF} + R_e + R_c
$$
Fig. 10 Collection probability profile in the bulk region for Case II

Fig. 11 Collection probability profile in the entire unit cell structure for Case II
Fig. 12  Collection probability profile in the bulk region for Case III

Fig. 13  Collection probability profile in the entire unit cell structure for Case III
In Eq. 22, RBSF and Re are the BSF and emitter finger resistance, respectively.

\[ R_e = \frac{d_{\text{emitter}}}{q \mu_{p\text{Si}} N_{A\text{Si}} W_{\text{emitter}}} \]  
(29)

\[ R_{\text{BSF}} = \frac{d_{\text{BSF}}}{q \mu_{n\text{Si}} N_{D\text{Si}} W_{\text{BSF}}} \]  
(30)

The contact resistance Rc was considered as a function of carrier concentration as obtained by Schroder et al. [23, 30].

\[ Rc = 10^{(-5 \times \log_{10} (N_c)+96.6)} \]  
(31)

From Eq. 25, it is evident that Rc is much smaller than RBSF and Re and hence is neglected. Shunt resistance Rsh was considered a bulk property and taken as a standard value of 130 Ω [32].

5 Results and discussion

The basic solar cell voltage current relationship was established from the equivalent circuit.

\[ I = a J_L - a J_0 \exp \left[ \frac{q (V_O + IR_S)}{kT} \right] - \frac{V_O + IR_S}{R_{\text{SH}}} \]  
(32)

The I–V characteristics were obtained for all three cases of FSF.

In Fig. 14, we show the I–V characteristics for all three modelled cases of FSF. FSF1 was the ZnO film-induced FSF, while FSF2 was formed by the a-Si:H layer. Different performance parameters were extracted from the I–V characteristics.

According to the performance indexes as shown in Table 2, the best result was obtained for the FSF2 IBC solar cell. Still, the novel SIS FSF1 solar cell is relevant for its lower thermal budget and less complicated fabrication engineering. ZnO film deposition using simple sol–gel techniques is a well investigated method [33]. There was a significant improvement in efficiency using the novel SIS FSF modification 19.1887–22.3130.

Further, we investigated the effect of finger area modification of both BSF and emitter fingers on the fill factor and efficiency of the IBC with FSF1. We considered the ratio of the area of the emitter and BSF to be FIBC, and we varied the ratio to examine the effect on solar cell performance.

\[ F_{\text{IBC}} = \frac{d_{\text{emitter}} W_{\text{emitter}}}{d_{\text{BSF}} W_{\text{BSF}}} \]  
(33)

FIBC was defined as the finger ratio.

Fill factor is a contact property. A higher finger ratio indicates a smaller BSF area. This results in greater series

| IBC type       | VOC in volts | Jsc in mA/cm² | Fill factor (FF) | % Efficiency (η) (Derived from mathematical modelling) | % Efficiency (η) (Experimental/simulation published earlier) |
|----------------|--------------|---------------|------------------|------------------------------------------------------|---------------------------------------------------------|
| No FSF (Case I)| 0.8225       | 28.24         | 0.8261           | 19.1887                                              | 19.2 [14]                                               |
| FSF1 (Case II) | 0.8263       | 32.67         | 0.8265           | 22.3130                                              | –                                                      |
| FSF2 (Case III)| 0.8274       | 34.09         | 0.8267           | 23.3176                                              | 21.5 [2]                                               |
**Fig. 15** Variation in fill factor with finger ratio

**Fig. 16** Variation in short-circuit current density with finger ratio

**Fig. 17** Variation in efficiency with finger ratio
resistance, and hence there was a drop in fill factor as shown in Fig. 15. Likewise, a very low finger ratio indicates less emitter area. This was also detrimental for the fill factor.

Figure 16 shows the results obtained from a simulation in which the finger ratio was varied and the corresponding short-circuit current density was computed using the method described earlier in this paper. From Fig. 6a, b and Fig. 7a, b, it is clear that the collection probability profile is much better in the emitter region than the BSF. Thus, increasing the finger ratio increases the collection probability. This is reflected in Fig. 16, resulting in an increase in short-circuit current density.

We obtained the efficiency versus finger ratio graph from the change in the fill factor and short-circuit current with respect to the finger ratio. We observed that an increase in the finger ratio affects the Jsc and FF differently. FF decreased with an increased in the finger ratio, while the short-circuit current showed the opposite result. Therefore, efficiency is a product of FF and Jsc. From Fig. 17, it can be concluded that maximum efficiency was achieved at a finger ratio of ~ 3. Lammert et al. predicted maximum efficiency for a finger ratio of 2.5 [1], with a much shorter lifetime.

6 Conclusion

In this paper, we have presented a mathematical model of an IBC solar cell with a novel FSF technique using ZnO and SiO₂ thin film deposition. The ZnO film can be easily deposited with a simple sol–gel method. The SiO₂ is formed by thermal oxidization. The parameters of the ZnO film were taken from previously published experimental reports. ZnO/SiO₂/c-Si formed an SIS which induced an FSF. We compared the performance of our proposed IBC structure with an existing amorphous Si FSF structure. The simulated results of the amorphous Si FSF IBC solar cell were found to be quite comparable to the results published by other scientists [2, 3, 10]. Thus, we can confidently conclude that this new mathematical modelling approach achieves high accuracy. The MATLAB simulation showed that our SIS FSF significantly enhanced the efficiency of the IBC solar cell without any FSF. However, the amorphous Si FSF still had better performance. Here, we wanted to put forth our argument that although amorphous Si FSF had better performance than our proposed SIS FSF, SIS FSF fabrication can be done with a much lower thermal budget, and hence the investigation of such an SIS structure is still relevant.

Appendix

MATLAB program code:

```matlab
% Creating mesh of generated carrier
% Import data from AM 1.5
a=VarName2.';
phi0= VarName3.';
for i=1:951
  y1=0:0.0000001:0.0003;
```
for j=1:3001
    y(j)=0.0003-y1(j);
    z1(i,j)=phi0(i)*exp(-a(i)*y(j))*0.9*0.075;
end
end
z2=z1.1;
S=sum(z2,2);
plot(y,S)
z=S;
x=0:0.00001:0.0075;
[x x y]=meshgrid(x,y);
[z x z]=meshgrid(x,z);
surf(z)
colormap(jet)
shading interp

% Bulk Collection probability (CASE1)
% This script is written and read by pdetool and should NOT be edited.
% There are two recommended alternatives:
% 1) Export the required variables from pdetool and create a MATLAB script
%    to perform operations on these.
% 2) Define the problem completely using a MATLAB script. See
%    http://www.mathworks.com/help/pde/examples/index.html for examples
%    of this approach.
function pdemodel
[pde_fig,ax]=pdeinit;
pdetool('applcy',1);
set(ax,'DataAspectRatio',[1 1 1]);
set(ax,'PlotBoxAspectRatio',[102 8 508.4999999999977 2905714.285714285]);
set(ax,'XLimMode','auto');
set(ax,'YLimMode','auto');
set(ax,'XTickMode','auto');
set(ax,'YTickMode','auto');

% Geometry description:
pdepoly([5.2493438320206113e-07,...
0.00075013123359580049,...
0.00075013123359580049,...
0.00072493438320209982,...
0.00072493438320209982,...
0.0005480314960629917,...
0.0005480314960629917,...
0.00052545931758530185,...
0.00052545931758530185,...
2.5721784776902889e-05,...
2.6771653543307069e-05,...
5.2493438320211534e-07,...],...
[0.00030004921259842518,...
0.0003003818897637794,...
1.0334645669291444e-06,...
3.4448818897643563e-07,...
7.2342519685039556e-06,...
6.5452755905511927e-06,...
3.4448818897643563e-07,...
3.4448818897643563e-07,...
1.2746062992126006e-05,...
1.205708614173241e-05,...
3.4448818897643563e-07,...
3.4448818897643563e-07,...],...
'P1');
set(findobj{get(pde_fig,'Children'),'Tag','PDEEval'},'String','P1')

% Boundary conditions:
pdetool('changemode',0)
pdesetbd(12,...
'neu',...
1,...
'5',...
'0')
pdesetbd(11,...
'neu',...
1,...
'5',...
'0')
pdesetbd(10,...
'neu',...
1,...
'0',...
'0')
pdesetbd(9,...
'dir',...
1,...
'1',....
'1')
pdesetbd(8,...
'dir',....
1,...
'1',....
'1')
pdesetbd(7,...
'dir',....
1,...
'1',....
'1')
pdesetbd(6,...
'dir',....
1,...
'1',....
'1')
pdesetbd(5,...
'dir',....
1,...
'1',....
'1')
pdesetbd(4,...
'dir',....
1,...
'1',....
'1')
pdesetbd(3,...
'neu',....
1,...
'5',....
'0')
pdesetbd(2,...
'neu',....
1,...
'0',....
'0')
pdesetbd(1,...
'neu',....
1,...
'5',....
'0')}

% Mesh generation:
sctappdata(pde_fig,'Hgrad',1.3);
sctappdata(pde_fig,'refinemethod','regular');
sctappdata(pde_fig,'jiggle',char('on','mean',''));
sctappdata(pde_fig,'MesherVersion','preR2013a');
Buck Collection probability (CASE2)
% This script is written and read by pdetool and should NOT be edited.
% There are two recommended alternatives:
% 1) Export the required variables from pdetool and create a MATLAB script
%    to perform operations on these.
% 2) Define the problem completely using a MATLAB script. See
%    http://www.mathworks.com/help/pde/examples/index.html for examples
%    of this approach.

function pdemodel
[pde_fig,ax]=pdefinit;
pdetool('appl_cb',1);
set(ax,'DataAspectRatio',[1 1 1]);
set(ax,'PlotBoxAspectRatio',[1092.8 508.499999999977 2905714.285714285]);
set(ax,'XLimMode','auto');
set(ax,'YLimMode','auto');
set(ax,'XTickMode','auto');
set(ax,'YTickMode','auto');

% Geometry description:
pdepoly([ 5.2493438320206113e-07,...
0.00075013123359580049,...
0.00075013123359580049,...
0.00072493438320209982,...
0.00072493438320209982,...
0.00057480314960629917,...
0.00057480314960629917,...

% PDE coefficients:
pdeseteq(1,...
'0.0036',...
'100',...
'0',...
'1.0',...
'0:10',...
'0.0',...
'0.0',...
'0100');
setappdata(pde_fig,'currparam',...'
'0.003';....
'10 ';....
'0 ';....
'1.0 ']);

% Solve parameters:
setappdata(pde_fig,'solveparam',...'
char('0',',10512',',',0',',',pdeadworst',...'
',',',longest',',',0',',',18-4',',',',',fixed',',',inf');

% Plotflags and user data strings:
setappdata(pde_fig,'plotflags',',',1111117100011000011);
setappdata(pde_fig,'colstring',','');
setappdata(pde_fig,'arrowstring',','');
setappdata(pde_fig,'deformstring',','');
setappdata(pde_fig,'heightstring',','');

% Solve PDE:
pdetool('solve')
0.00052545931758530185,
0.00052545931758530185,
2.5721784776902689e-05,
2.671653543307069e-05,
5.2493438320211534e-07,
},
,...
[ 0.0030004921259842518,
0.00030073818897637794,
1.0334645669291443e-06,
3.4448818897643563e-07,
7.2342519685039556e-06,
6.5452755905511927e-06,
3.4448818897643563e-07,
1.27460629921226006e-05,
1.2057086514173241e-05,
3.4448818897643563e-07,
3.4448818897643563e-07,
],
,'Pl');
set(findobj(get(pde_fig,'Children'),'Tag','PDEEval'),'String','Pl')

% Boundary conditions:
pdetool('changemode',0)
pdesetbd(12,...
'neu',...
1,...
'St',...
'0')
pdesetbd(11,...
'neu',...
1,...
'St',...
'0')
pdesetbd(10,...
'neu',...
1,...
'O',...
'0')
pdesetbd(9,...
'dir',...
1,...
'1',...
'1')
pdesetbd(8,...
'dir',...
1,...
'1',...
'1')
pdesetbd(7,...
'dir',...
1,...
'1',...
'1')
pdesetbd(6,...
'dir',...
1,...
'1',...
'1')
pdesetbd(5,...
'dir',...
1,...
'1',...
'1')
pdesetbd(4,...
'dir',...
1,...
1',...
'1']
pdesetbd(3,...
'neu',...
1,...
'5',...
'0'])
pdesetbd(2,...
'neu',...
1,...
'0',...
'0'])
pdesetbd(1,...
'neu',...
1,...
'5',...
'0'])

% Mesh generation:
setappdata(pde_fig,'Hgrad',1.3);
setappdata(pde_fig,'refinemethod','regular');
setappdata(pde_fig,'jiggle',char('on','mean',''));
setappdata(pde_fig,'MeshVersion','preR2013a');
pdetool('initmesh')
pdetool('refine')
pdetool('refine')
pdetool('jiggle')
pdetool('jiggle')

% PDE coefficients:
pdessteq(1,...
'0.0036',...
'500',...
'0',...
'1.0',...
'0:10',....
'0.0',...
'0.0',...
'[0 100]')
setappdata(pde_fig,'currparam',...[
'0.0036';... 
'500';...
'0';...
'1.0'])

% Solve parameters:
setappdata(pde_fig,'solveparam',... 
char('0','10512','10','pdeadworst',...
'0.5','longest','0','1E-4','''','fixed','Inf'))

% Plotflags and user data strings:
setappdata(pde_fig,'plotflags',[1 1 1 1 1 1 1 0 0 0 1 1 0 0 0 0 1]);
setappdata(pde_fig,'colstring','');
setappdata(pde_fig,'arrowstring','');
setappdata(pde_fig,'deformstring','');
setappdata(pde_fig,'heightstring','');

% Solve PDE:
pdetool('solve')

Bulk Collection probability (CASE3)
% This script is written and read by pdetool and should NOT be edited.
% There are two recommended alternatives:
% 1) Export the required variables from pdetool and create a MATLAB script
% to perform operations on these.
% 2) Define the problem completely using a MATLAB script. See
%  http://www.mathworks.com/help/pde/examples/index.html for examples
% of this approach.

```matlab
function pdemodel
    [pdeFig, ax] = pdeinit;
pdetooll('appi_cb',1);
    set(ax,'DataAspectRatio', [1 1 1]);
    set(ax,'PlotBoxAspectRatio', [1024.8 508.4999999999977 2905714.285714285]);
    set(ax,'XLimMode', 'auto');
    set(ax,'YLimMode', 'auto');
    set(ax,'XTickMode', 'auto');
    set(ax,'YTickMode', 'auto');

    % Geometry description:
    pdepoly([5.2493438320206113e-07,...
        0.0007503123359580049,...
        0.00072493438320209982,...
        0.0002545931758530185,...
        2.5721784776902889e-05,...
        5.2493438320206113e-07,...
        0.00030004921259842518,...
        0.00030073818897637794,...
        1.0334645669291443e-06,...
        3.444818897643563e-07,...
        7.8453446198643563e-07,...
        6.5452759055159356e-06,...
        3.444818897643563e-07,...
        3.444818897643563e-07,...
        1.2746062992126006e-05,...
        1.2057006141732841e-05,...
        3.444818897643563e-07,...
        3.444818897643563e-07,...
        ],
    'P1');
    set(findobj(get(pdeFig, 'Children'), 'Tag', 'PDEeval'), 'String', 'P1')

    % Boundary conditions:
    pdetooll('changeMode', 0)
pdsetbedd(12,...
    'neu',...
    1,...
    's',...
    '0');
    pdsetbedd(11,...
    'neu',...
    1,...
    'S',...
    '0');
    pdsetbedd(10,...
    'neu',...
    1,...
    'O',...
    '0');
    pdsetbedd(9,...
    'dir',...
    1,...
    '1',...
    '1');
```
pdesetbd(8,...
'dir',...
1,...
'1',....
'1')
pdesetbd(7,...
'dir',...
1,...
'1',....
'1')
pdesetbd(6,...
'dir',...
1,...
'1',....
'1')
pdesetbd(5,...
'dir',...
1,...
'1',....
'1')
pdesetbd(4,...
'dir',...
1,...
'1',....
'1')
pdesetbd(3,...
'neu',...
1,...
'5',....
'0')
pdesetbd(2,...
'neu',...
1,...
'0',....
'0')
pdesetbd(1,...
'neu',...
1,...
'5',....
'0')

% Mesh generation:
setappdata(pde_fig,'Hgrad',1.3);
setappdata(pde_fig,'refinemethod','regular');
setappdata(pde_fig,'jiggle',char('on','mean',''));
setappdata(pde_fig,'MeshVersion','preR2013a');
pdetool('initmesh')
pdetool('refine')
pdetool('refine')
pdetool('jiggle')
pdetool('jiggle')

% PDE coefficients:
pdeseq(1,...
'0.004',....
'100',....
'0',....
'1.0',....
'0:10',....
'0.0',....
'0.0',....
'([0 100])')
setappdata(pde_fig,'currparam',...
['0.004';...
'100 ';'....
'0 ';'....

Springer
'1.0 '])

% Solve parameters:
setappdata(pde_fig,'solveparam',... 
char('0','10512','10','pdeadworst',... 
'0.5','longest','0','1E-4','','fixed','Inf'))

% Plotflags and user data strings:
setappdata(pde_fig,'plotflags',[1 1 1 1 1 1 1 0 0 0 1 1 0 0 0 0 1]);
setappdata(pde_fig,'colstring',''');
setappdata(pde_fig,'arrowstring','''');
setappdata(pde_fig,'deformstring','''');
setappdata(pde_fig,'heightstring','''');

% Solve PDE:
pdetool('solve')

% Collection probability emitter
% This script is written and read by pdetool and should NOT be edited.
% There are two recommended alternatives:
% 1) Export the required variables from pdetool and create a MATLAB script
% to perform operations on these.
% 2) Define the problem completely using a MATLAB script. See http://www.mathworks.com/help/pde/examples/index.html for examples
% of this approach.
function pdemodel
[pde_fig,ax]=pdeinit;
pdetool('app1_cb',1);
set(ax,'DataAspectRatio',[1 1 1]);
set(ax,'PlotBoxAspectRatio',[1.3992789254994214 1 10000]);
set(ax,'xLimMode','auto');
set(ax,'yLimMode','auto');
set(ax,'xTickMode','auto');
set(ax,'yTickMode','auto');

% Geometry description:
pderect([0.00014999999999999 0 0 6.00000000000002e-06],'R1');
set(findobj(get(pde_fig,'Children'),'Tag','PDERectVal'),'String','R1')

% Boundary conditions:
pdetool('changenode',0)
pdeshell(4,...
'dir',... 
'1',... 
'1',... 
'1')
pdeshell(3,...
'dir',... 
'1',... 
'1',... 
'1')
pdeshell(2,...
'dir',... 
'1',... 
'1',... 
'1')
pdeshell(1,...
'new',... 
'1',... 
'0.1',... 
'0')

% Mesh generation:
setappdata(pde_fig,'Hgrad',1.3);
setappdata(pde_fig,'refinemethod','regular');
setappdata(pde_fig,'jiggle',char('on',',mean',','));
setappdata(pde_fig,'MeshVersion',,'preR2013a');
pdetool('initmesh')
pdetool('refine')
pdetool('refine')
pdetool('refine')
pdetool('jiggle')
pdetool('jiggle')

% PDE coefficients:
pdeseteq(1,...
'2.5840e-08',...
'1428',....
'0',....
'1.0',....
'0.10',....
'0.0',....
'0.0',....
[0 100])
setappdata(pde_fig,'currparam',...[2.5840e-08];...
'1428 ';...
'0 ';...
'1.0 ']}

% Solve parameters:
setappdata(pde_fig,'solveparam',...char('0',',2688',',10',',pdeadworst',...,
'0.5',',longest',',0',',1e-4',',',',fixed',',inf'))

% Plotflags and user data strings:
setappdata(pde_fig,'plotflags',[1 1 1 1 1 7 1 0 0 1 1 0 1 0 0 1]);
setappdata(pde_fig,'colstring',',');
setappdata(pde_fig,'arrowstring',',');
setappdata(pde_fig,'deformstring',',');
setappdata(pde_fig,'heightstring',',');

% Solve PDE:
pdetool('solve')

% Collection probability BSF
% This script is written and read by pdetool and should NOT be edited.
% There are two recommended alternatives:
% 1) Export the required variables from pdetool and create a MATLAB script
% to perform operations on these.
% 2) Define the problem completely using a MATLAB script. See
% http://www.mathworks.com/help/pde/examples/index.html for examples
% of this approach.

function pdemodel
[pde_fig,ax]=pdeinit;
pdetool('applcb',1);
set(ax,DataAspectRatio',[21.428571428571427 1 285714.28571428574]);
set(ax,PlotBoxAspectRatio',[1 0.5 0.5]);
set(ax,XLimMode',auto');
set(ax,YLimMode',auto');
set(ax,XTickMode',auto');
set(ax,YTickMode',auto');

% Geometry description:
pderect([0.0001499999999999999 0 0 6.000000000000000e-06],',R1');
set(findobj(get(pde_fig,'Children'),',Tag',',PDEeval'),',String',',R1')

% Boundary conditions:
pdetool('changenode',0)
pdesetbd(4,...
'dir', ...
1, ...
'1', ...
'1')
pdesetbd(2, ...
'dir', ...
1, ...
'1', ...
'1')
pdesetbd(1, ...
'neu', ...
1, ...
'0.1', ...
'0')

% Mesh generation:
setappdata(pde_fig, 'Hgrad', 1.3);
setappdata(pde_fig, 'refinemethod', 'regular');
setappdata(pde_fig, 'jiggle', char('on', 'mean', ''));
setappdata(pde_fig, 'MesherVersion', 'preR2013a');
pdetool('initmesh')
pdetool('refine')
pdetool('refine')
pdetool('refine')
pdetool('jiggle')
pdetool('jiggle')

% PDE coefficients:
pdesetq(1, ...
'2.5840e-08', ...
'1428', ...
'0', ...
'1.0', ...
'0:1.0', ...
'0.0', ...
'0.0', ...
'100')
setappdata(pde_fig, 'currparam', ...
['2.5840e-08'; ...
'1428' ...
'0' ...
'1.0']

% Solve parameters:
setappdata(pde_fig, 'solveparam', ...
char('0', '2688', '10', 'pdeadworst', ...
'0.5', 'longest', '0', '1E-4', '', 'fixed', 'Inf'))

% Plotflags and user data strings:
setappdata(pde_fig, 'plotflags', [1 1 1 1 1 1 1 1 0 0 0 1 1 0 1 0 0 1]);
setappdata(pde_fig, 'colstring', '');
setappdata(pde_fig, 'arrowstring', '');
setappdata(pde_fig, 'deformstring', '');
setappdata(pde_fig, 'heightstring', '');

% Solve PDE:
pdetool('solve')

% To obtain I-V characteristics
q = 1.6021765*10.^-19;
k = 1.38*10.^-23;
c = 3*10.^8;
T = 300;
h=6.626*10.^(-34);
mass= \frac{9.109*10.\times (-31)}{\text{Nas}}=8*10.^24; \quad \text{Ndsia}=5*10.^25; \quad \text{Ncsia}=4.5*10.^27; \quad \text{Nvsia}=6.4*10.^27; \\
Egsia=1.72*q; \quad \text{Eqsic}=1.12*q; \\
mupsia=0.000001; \quad \text{munsia}=0.0001; \\
\text{taupsia}=700*10.^{-6}; \quad \text{taunsia}=900*10.^{-6}; \\
\text{Ksia}=3.7*q; \quad \text{Nasic}=2*10.^{20}; \\
\text{Ncsic}=2.62*10.^{25}; \quad \text{Nvscic}=1.83*10.^{25}; \\
\text{kisic}=4.05*q; \\
\text{nisia}=\left(N\text{csia}^{*}(N\text{vsia}^{0.5})\right)\exp\left(-E\text{gsia}/(2*\text{K}^{*}\text{T})\right); \\
\text{phi1}=\left((\text{K}^{*}\text{T})\log\left((\text{Nasia}^{*}\text{Ndsia})/(\text{nisia}^{2})\right)/\text{q}\right)-(1.72-1.12); \\
\text{Dpsia}=\left(\text{K}^{*}\text{T}\text{mupsia}\right)/\text{q}; \quad \text{Dnsia}=\left(\text{K}^{*}\text{T}\text{munsia}\right)/\text{q}; \\
\text{J0}=\left(\exp\left(-\text{q}^{*}\text{phi}/(\text{K}^{*}\text{T})\right)\left((\text{Nasia}^{*}(\text{Dpsia}^{*}\text{taupsia})^{0.5})+(\text{Ndsia}^{*}(\text{Dnsia}^{*}\text{taunsia})^{0.5}))\right)\right)/\text{q}; \\
\text{Rs}=0.38*13; \quad \text{Rsh}=110; \\
\text{m}=(\text{Rs}+\text{Rsh}); \quad \text{JL}=0.01924; \\
\text{IL1}=\text{JL}; \quad \text{I01}=1.3*\text{J0}^{*}(10^{-4}); \\
\text{b1}=(\text{IL1}^{*}\text{I01}); \quad \text{a1}=(\text{IL1}^{*}\text{b1}); \\
\text{IL}=\text{b1}^{*}\text{a1}; \quad \text{x}=(\text{IL}^{*}\text{IL}); \\
\text{y}=(\text{x}^{*}\text{I01}); \quad \text{V010}=\left((\text{k}^{*}\text{T})/\text{q}\right)^{*}\text{log}(\text{y}); \\
\text{V01}=\text{V010}^{*}\text{m}; \quad \text{p}=\text{V01}^{*}\text{IL}; \\
\text{plot} (\text{V01}, \text{IL}) \quad \text{hold on} \\
\text{M}=(\text{max}(\text{p})); \quad \text{Voc}=\left((\text{k}^{*}\text{T})/\text{q}\right)^{*}\text{log}((\text{IL})/\text{I01}) \\
\text{P}=(\text{Voc}^{*}\text{IL}); \quad \text{ff}=\text{M}/\text{P} \\
\text{eff}=\left((\text{ff}^{*}\text{P})/0.1\right)^{100} \quad \% \text{case2} \\
\text{J2}=0.02567; \quad \text{IL2}=\text{J2}; \\
\text{I02}=1.15^{*}\text{J0}^{*}(10^{-4}); \quad \text{b2}=(\text{IL2}^{*}\text{I02}); \\
\text{a2}=\text{inspace}(0, \text{b2}); \quad \text{I2}=\text{b2}^{*}\text{a2}; \\
\text{x2}=(\text{IL2}^{*}\text{I2}); \quad \text{y2}=\text{x2}/\text{I02}; \\
\text{V020}=\left((\text{k}^{*}\text{T})/\text{q}\right)^{*}\text{log}(\text{y2}); \\
\text{V02}=\text{V020}^{*}\text{m}; \quad \text{p2}=\text{V02}^{*}\text{I2}; \\
\text{plot} (\text{V02}, \text{I2}) \quad \text{hold on} \\
\text{M2}=\text{max}(\text{p2}); \quad \text{Voc2}=\left((\text{k}^{*}\text{T})/\text{q}\right)^{*}\text{log}((\text{IL2})/\text{I02}) \\
\text{P2}=\text{Voc2}^{*}\text{IL2}; \quad \text{ff2}=\text{M2}/\text{P2}
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Declaration

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