The Transformation of a Large Gas Bubble When Purging a Metal with a Gas Jet

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Abstract The article «The Transformation of a Large Gas Bubble When Purging a Metal with a Gas Jet» is devoted to the actual problem of the behavior of gases during the injection of powders by gas jets into a liquid metal. One of the possible mechanisms of splitting a gas bubble when it is wrapped with liquid is considered. To solve the problem, the problem of continuous flow around a ball was used. The disintegration process is represented as a transformation of a large gas bubble into a disk formation, which loses its shape and breaks up into spherical shells. The number of gas bubbles that are formed from this with a certain initial radius is presented in the graphs. The consideration is based on the condition of equality of the volumes of the initial large gas bubble and the small bubbles formed. In addition, formulas for the rate of rise of gas bubbles in a liquid are given.

Keywords: bubble, metal melt, ball wrapping, disk volume.

1. Introduction
Currently, the method of injection of various powders into the metal melt by active and inert gases is widely used to produce high-quality steels. Therefore, consideration of the hydromechanics of the behavior of gas bubbles remains an urgent task [1, 3, 4, 18, 21].

2. Research
Consider the process of formation and rise of the gas bubble during the purging of the metal.

When the gas leaves the injection tube, a spherical gas bubble forms. When a certain size is reached, the gas ball comes off and begins its rise. Suppose that at this time the gas ball is flowed around continuously, then by this time we apply the theory of the continuous flow around the ball [2, 13, 16, 22]. We write out the basic formulas for the continuous flow around the ball. The radial component of the velocity of the ball is:

\[ V_r = U \cos \theta \left( 1 - \frac{r_0^3}{r^3} \right) \]  

Tangent component of speed:

\[ V_\theta = -U \sin \theta \left( 1 + \frac{1}{2} \frac{r_0^3}{r^3} \right) \]
On the surface of the ball with \( r = r_0 \):
- the radial component is equal to:

\[
V_r = U \cos \theta \left( 1 - \frac{r_0^3}{r^3} \right) = 0
\]  

(3)

- the tangent component is equal to:

\[
V_\theta = -U \sin \theta \left( 1 + \frac{1}{2} \right) = -\frac{3}{2} U \sin \theta
\]  

(4)

The graph of speed distribution is presented in Figure 1 [6]. This distribution of speeds according to the Euler – Bernoulli equation will correspond to the distribution of pressures shown in Figure 2 [11, 15]. Under the action of tensile pressures, the gas bubble will stretch in the equatorial plane, and the ball will take the form of a disk. For further calculations we will take the following geometrical dimensions of the disk: \( R \) – the main disk size; \( r \) – the radius of rounding. Determine the volume of the disk, its area and the area of the mid-section [8, 14, 23].

**Figure 1.** Distribution of speeds around a sphere at continuous flow

**Figure 2.** Distribution of pressure at flow of a sphere

**Figure 3.** Estimated intermediate form of a sphere

Area of midsection [19]:

\[
S_{mig} = \pi (R + r)^2 = \pi R^2 \left( 1 + \frac{r}{R} \right)^2 = \pi R^2 \left( 1 + 2 \frac{r}{R} + \frac{r^2}{R^2} \right)
\]  

(5)

The lateral area of the disk is determined by the integral [17]:

\[
S = 2 \int_0^{\pi/2} 2\pi (R + r \cos \varphi) r d\varphi = 4\pi \left[ \frac{R^2}{2} r d\varphi + \int_0^{\pi/2} r^2 \cos \varphi \ d\varphi \right] = 4\pi \left( \frac{Rr}{2} \frac{\varphi}{\pi} + r^2 \sin \frac{\varphi}{2} \right) = 2\pi^2 Rr + 4\pi r^2
\]  

(6)

That is, the lateral area of the disk is equal to

\[
S = 2\pi^2 Rr + 4\pi r^2.
\]  

(7)
Similarly, we determine the volume of the disk: \( x = r \cos \varphi, \ y = r \sin \varphi \):

\[
V = 2 \int_0^{\pi/2} \pi (R + r \cos \varphi)^2 \, dy = 2 \int_0^{\pi/2} \pi (R^2 + 2Rr \cos \varphi + r^2 \cos^2 \varphi) \, r \cos \varphi \, d\varphi =
\]

\[
= 2\pi \left( R^2 r \sin \varphi + 2R^2 \int_0^{\pi/2} \cos^2 \varphi \, d\varphi + 3 \int_0^{\pi/2} \cos^3 \varphi \, d\varphi \right) =
\]

\[
= 2\pi \left( R^2 r \sin \varphi + 2R^2 \left[ \frac{1 + \cos 2\varphi}{2} \right]_0^{\pi/2} + 3 \left[ \frac{\sin 3\varphi}{3} \right]_0^{\pi/2} \right) =
\]

\[
= \frac{\pi}{4} - \frac{1}{3} \frac{2}{3}.
\]

Therefore, the disk size is:

\[
V = 2\pi \left( R^2 r + 2R^2 \frac{\pi}{4} + \frac{2}{3} r^3 \right) = 2\pi R^2 \left( 1 + \frac{\pi r}{2R} + \frac{2}{3} \frac{r^2}{R^2} \right).
\]

(9)

Knowing the volume of the disk and the area of the mid-section, we write the steady speed of the disk from the equation of motion of the disk:

\[
M \frac{dU}{dt} = -C_D S_{mg} \rho_{\infty} \frac{U^2}{2} + \rho_{\infty} V_g + (\rho_{\infty} - \rho) W_g.
\]

(10)

\[
U^2 = \frac{2(\rho_{\infty} - \rho) V_g}{C_D S_{mg} \rho_{\infty}} = \frac{2V_g}{C_D S_{mg}} = \frac{2gV}{C_D S_{mg}} - (\text{neglecting gas density})
\]

(11)

\[
U^2 = \frac{2gR^2 \left( 1 + \frac{\pi r}{2R} + \frac{2}{3} \frac{r^2}{R^2} \right)}{C_D \pi R^2 \left( 1 + \frac{r}{R} + \frac{r^2}{R^2} \right)} = \frac{4gR}{C_D} \left( 1 + \frac{\pi r}{2R} + \frac{2}{3} \frac{r^2}{R^2} \right) = \frac{4gR}{C_D}.
\]

(12)

Consider the pressure inside and outside the disk. The pressure inside the disk in the area of rounding: \( p = p_0 + \rho gh - \frac{\varphi}{r} \) [9, 10]. The pressure outside the disk in the area of rounding:

\[
p = p_0 + \rho gh - \frac{5}{4} \left( \frac{\rho U^2}{2} \right).
\]

(13)

Here \( p_0 \) - atmosphere pressure;
\( \rho gh \) - pressure at depth;
\( \varphi \)
(\( \alpha \))/\( r \) - Laplace pressure;
\[ \frac{5}{4} \left( \frac{\rho U^2}{2} \right) \] - pressure loss due to accelerated speed and U.

Equating them, we get:
\[ \frac{\alpha}{r^2} = \frac{5}{4} \left( \frac{\rho U^2}{2} \right) = \frac{5 \rho}{8} \left( \frac{4 g \rho}{C_D} \right) \]
\[ r^2 = \frac{2 C_D \cdot \alpha}{5 \rho g} \]

That is, for the radius of the disc, we get a formula that does not depend on speed [7]:
\[ r = \sqrt{\frac{2 C_D \cdot \alpha}{5 \rho g}} \] (15)

Perform calculations using this formula:
- for liquid metal:
\[ \rho \approx 7 \cdot 10^3 \text{ kg/m}^3; g \approx 10m/c^2; \alpha = 1,2H/m; C_D = 1,1; r = \sqrt{\frac{2 \cdot 1,1 \cdot 1,2}{5 \cdot 7 \cdot 10^3 \cdot 10}} = 2,74 \cdot 10^{-3} = 2,74mm \]
\[ r = \sqrt{\frac{2 \cdot 1,1 \cdot 0,075}{5 \cdot 10^3 \cdot 10}} = 1,8mm \]
- for water:
\[ U_{speed} = \sqrt{\frac{4 \cdot 10 \cdot 2,7 \cdot 10^{-3}}{1,1}} = 0,31m/c \]
\[ U_{water} = \sqrt{\frac{4 \cdot 10 \cdot 1,8}{1,1}} = 0,25m/c \]

The area of the disk is obviously larger than the area of the ball from which it was formed [5], therefore, it will be energetically more advantageous for the disk to decay into spheres.

Let us estimate the radius of the gas spheres obtained, equating the disk volume to the volume of gas spheres.
\[ V = 2 \pi R^2 r \left( 1 + \frac{\pi r}{2} + \frac{2 r^2}{3 R^2} \right) \]

Disk size is
\[ S = 2 \pi^2 R r + 4 \pi r^2 = 2 \pi R \left( \pi + 2 \frac{r}{R} \right) \] (16)

The lateral area of the disk is equal to
\[ S = 2 \pi R \left( \pi + 2 \frac{r}{R} \right) + 2 \pi R^2 \] (17)

The total disk area is equal to
\[ V = \frac{4 \pi r^3}{3} \] (18)

The volume of n-gas spheres is equal to
\[ V = n \cdot \frac{4 \pi}{3} r_0^3 \] (19)

The area of n-gas spheres is equal to
\[ S = n \cdot 4 \pi r_0^2 \] (20)

Equating these expressions, we have:
\[ 2 \pi R^2 r \left( 1 + \frac{\pi r}{2} + \frac{2 r^2}{3 R^2} \right) = n \cdot \frac{4 \pi}{3} r_0^3 \]
\[ 2 \pi R^2 r \left( 1 + \frac{\pi r}{2} + \frac{2 r^2}{3 R^2} \right) = n \cdot \frac{4 \pi}{3} r_0^3 \]
\[2\pi R \left( \pi + \frac{2r}{R} \right) + 2\pi R^2 = n4\pi r_0^2.\]

Dividing the first equation by the second, we get:

\[
\frac{1 + \frac{\pi r}{2R} + \frac{2r^2}{3R^2}}{1 + \frac{\pi r}{R} + \frac{2r^2}{R^3}} = \frac{r_0}{3},
\]

here \(r/R=0\), \(r_0 = 3r\).

An important result was obtained: the radius of the bubbles formed was three times the minimum radius of curvature. Calculate the radius of the formed bubbles:

\(r_0 = 3r = 3 \times 2,74mm = 8,22mm\).

For metal, respectively, the diameter is equal to:

\(d_0 = 2 \times 8,22 = 16,44mm \approx 1,6sm\);

For water: \(d_0 = 6r = 6 \times 1,8 = 10,8mm \approx 1,1sm\).

Substitute the given value of \(r_0 = 3r\) into the system of equations:

\[2\pi R^2 \left( 1 + \frac{\pi r}{2R} + \frac{2r^2}{3R^2} \right) = n4\pi 2r_0^3; \quad \text{or} \quad R^2 \left( 1 + \frac{\pi r}{2R} + \frac{2r^2}{3R^2} \right) = 2n9r^2 = 18nr^2;\]

\[2\pi R^2 \left( 1 + \frac{\pi r}{R} + \frac{2r^2}{R^3} \right) = n4\pi 9r^2.\]

Here \(r/R=0\), \(R^2 = 18nr^2\).

Consider the equality of the volumes of the initial radius \(R0\) and the disk. We get the following equation:

\[2\pi R^2 \left( 1 + \frac{\pi r}{2R} + \frac{2r^2}{3R^2} \right) = n4\pi 2r^3 = \frac{4\pi}{3} R_0^3.\]

Then for the number of gas spheres we get the formula:

\[n = \frac{R_0^3}{27r^3}.\]

Construct a graph of the number of bubbles from the initial radius \(R0\): \(R0=10, 20, 30, 40, 50\) mm, \(r = 2,74mm\).

\(n_{10} = \frac{10^3}{27 \cdot 2,74^3} = 1,8; \quad n_{20} = \frac{20^3}{27 \cdot 2,74^3} = 14,4; \quad n_{30} = \frac{30^3}{27 \cdot 2,74^3} = 48,6; \quad n_{40} = \frac{40^3}{27 \cdot 2,74^3} = 115; \quad n_{50} = \frac{50^3}{27 \cdot 2,74^3} = 251.1; \)

Large disk radius:

\[R = \sqrt{\frac{18^2 r^2 R_0^3}{27r^3}} = \sqrt{\frac{2R_0^3}{3r}}.\]

Graphics large disk sizes:
\[ R_{8,22} = 11.6 \text{mm} \quad ; \quad R_{10} = 15.6 \text{mm} \quad ; \quad R_{20} = 44 \text{mm} \quad ; \quad R_{30} = 81 \text{mm} \quad ; \quad R_{40} = 124.8 \text{mm} \quad ; \quad R_{50} = 174.4 \text{mm} \quad ; \quad R_{60} = 229 \text{mm} \]

**Figure 4.** Schedules of compliance "size of a sphere and disk" and "size of a sphere and quantity of bubbles".

Estimate the time of the collapse of the gas bubble. For a continuous flow around the ball, the equations of motion will look like this:

\[ m_g \frac{dV}{dt} = m_w g = \rho_w g V \quad ; \quad \rho V \frac{dV}{dt} = \rho_w g V \]

or

\[ \rho_s a = \rho_w g \]

here \( V \) – speed, \( a \) – acceleration, \( \rho_s, \rho_w \) - density of gas and liquid, respectively; \( g \) – acceleration of gravity.

Speed \( V = at \). Therefore, the time to reach speed \( V = 0.31 m/s \) is equal to:

\[ t = \frac{V}{a} = \frac{V \rho_s}{\rho_w g} = \frac{0.31*1.8}{7*10^3*10} = 8*10^{-6} \text{c}, \]

that is almost instantly.

Consider this process from an energy point. Suppose that during this time the work of the Archimedean forces is spent on the formation of a new surface. The work of the surface forces will be equal to:

\[ A_{\text{surf}} = \alpha[2\pi R^2 \left( 1 + \frac{\pi r}{2R} + \frac{2r^2}{3R^2} \right) - 4\pi R_0^2]. \tag{23} \]

The work of Archimedean forces \[ [12,20]: \]

\[ A_{\text{arch}} = F_{\text{arch}} S = \left( g \rho \frac{4\pi R_0^3}{3} \right) \frac{V^2}{2a} = g \rho \frac{4\pi R_0^3}{3} \frac{V^2 \rho_s}{2\rho_w g} = \frac{4\pi R_0^3}{3} \left( \frac{\rho_s V^2}{2} \right). \tag{24} \]
Evaluating these jobs, we get:
\[
\alpha[2\pi R^2 \left(1 + \frac{\pi r}{2R} + \frac{2r^2}{3R^2}\right) - 4\pi R_0^2] = \frac{4\pi R_0^3}{3} \left(\frac{\rho_\alpha V^2}{2}\right);
\]
\[
2\pi R^2 \left(1 + \frac{\pi r}{2R} + \frac{2r^2}{3R^2}\right) - 4\pi R_0^2 = \frac{4\pi R_0^3}{3} \left(\frac{\rho_\alpha V^2}{2\alpha}\right);
\]
Equating these jobs, we get:
\[
\frac{4\pi R_0^3}{3} \left(\frac{\rho_\alpha V^2}{2}\right) = \frac{4\pi R_0^3}{3} \left(\frac{1.8 \times 0.0961}{2 \times 1.2}\right) = 0.3R_0^3.
\]

Evaluate the last term in this formula or otherwise. Evaluate the work of the Archimedes forces:
\[
\frac{4\pi R_0^3}{3} \left(\frac{\rho_\alpha V^2}{2}\right) = \frac{4 \times 3.14 R_0^3}{3} \frac{1.8 \times 0.0961}{2 \times 1.2} = 0.3R_0^3.
\]

Consistently produce calculations:
\[
2\pi \frac{2R_0^3}{3r} - 4\pi R_0^2 = \frac{4\pi R_0^3}{3} \frac{\rho_\alpha V}{2\alpha};
\]
\[
\frac{4\pi R_0^3}{3} \frac{\rho_\alpha V}{2\alpha} - 4\pi R_0^2 = \frac{4\pi R_0^3}{3} \frac{\rho_\alpha V}{2\alpha};
\]
\[
\frac{4\pi R_0^3}{3} \frac{\rho_\alpha V}{2\alpha} \left(1 - \frac{3}{r R_0}\right) = \frac{3}{2\alpha} - \frac{\rho_\alpha V^2 r}{2\alpha R_0};
\]
\[
R_0 = \frac{6\alpha}{2\alpha - \rho_\alpha V^2 r} = \frac{6\alpha}{2\alpha \left(1 - \frac{\rho_\alpha V^2 r}{2\alpha}\right)} = \frac{3r}{1 - \frac{\rho_\alpha V^2 r}{2\alpha}} = \frac{3 \times 2.74}{1 - \frac{1.8 \times 0.0961 \times 2.74 \times 10^{-3}}{2 \times 1.2}} = \frac{3 \times 2.74}{1 - 2 \times 10^{-4}} = 3 \times 2.74 = 8.22 \text{ mm}.
\]

The result will coincide with the minimum radius of a bubble formed from a large one. Thus, a bubble with a radius \(R_0 = 8.22 \text{ mm}\) no longer decays with this mechanism of decay.

**Summary**
The proposed mechanism explains the splitting of large gas bubbles in a metal melt. Formulas are obtained for the steady-state rate of rise of gas bubbles.

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