Synthesis of the control loop for the coordinate of the horizontal plane of the quadcopter information-measuring and control system

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Abstract. The article discusses the translational motion of a quadcopter on a horizontal plane. Based on the analysis of this movement, a control loop for the horizontal plane coordinate of the information-measuring and control system of quadcopters was synthesized. The results obtained provide a stable flight of the quadcopter, a decrease in the influence of wind disturbances and monitoring of various objects when performing special tasks.

1. Introduction
Due to its compactness, high stability and maneuverability, quadcopters (QC) are currently widely used [1,2]. Control of the quadcopter is carried out by changing the rotation speed of one or more engines and, therefore, the corresponding torques and traction and lifting characteristics change [2]. Despite the advantages of quadcopters, control is a difficult problem. The main problems when controlling a quadcopter are: loss of balance of movement in case of damage to any engine; lack of high quality control loops; lack of appropriate sensors; relatively high power consumption and significant inertia in control.

To solve the above problems, an information-measuring and control system of a quadcopter (IMS QC) has been developed. This system is a set of technical means for processing measurement information and solving control problems [3]. Since the quadcopter has 6 degrees of freedom, including 3 for displacements (X, Y, Z) and 3 for orientation (R, I, P), to improve the IMS QC it is necessary to synthesize a control loop along the height Z, horizontally X with subordinate contour R (in Y it is similar to subordinate contour P).

In [3], the synthesis of the control loop for the Z-coordinate of the IMS QC is considered, and in this paper, the peculiarity of the synthesis of the control loop for the X coordinate in the horizontal plane with a subordinate contour along R of the information-measuring and control system of the quadcopter will be considered.

2. Research results
To synthesize the control loop of a quadcopter along the X axis, we will consider its motion only along the horizontal OXY plane. In this case, the quadcopter needs to rise to a certain height Z0, and then its movement along the X axis is carried out by changing the speeds of the two engines (ASD1 and ASD2) and the angles...
between the axes of these vectors and the local coordinate system. We assume that at the beginning of movement at altitude Z0 the quadcopter has zero velocity and the centre of mass coordinate is located at point O (0, 0, Z0) with orientation angles (0,0,0), and after a certain time dt the quadcopter is at point O1 (X1, 0, Z0) with angles (dr,0,0). Figure 1 shows a kinematic diagram of a quadcopter with ASD1 and ASD2 engines involved.

Figure 1. Kinematic diagram of a quadcopter with 2 engines involved.

Here ОXYZ is the coordinate system of the centre of gravity of the spacecraft on the ground; ОkXkYkZk – QC coordinate system at height Z0 at the initial time of movement along X; О1X1Y1Z1 – fixed coordinate system of the spacecraft is rigidly connected to the body; R, I, P – orientation angles; ASD1–ASD4 – speed-controlled electric drives; V1–V4 – rotation speeds respectively of electric motors ASD1 – ASD4; dv1, dv2 – additional speeds to ASD1 – ASD2; dr – is the deviation of the orientation angles when the spacecraft body is rotated relative to the position of the centre of mass; dX – distance between point O1 and point Ok.

Let us analyze the linearized differential equations of quadcopter motion along the X axis and get [3,4,5]:

\[
\frac{d^2x}{dt^2} = Kmx \times I - \frac{dx}{dt} \times Kx
\]

(1)

\[
\frac{d^2r}{dt^2} = Km \times du
\]

(2)

where Kmx is the coefficient of mutual influence of the quadcopter coordinates; I is the pitch parameter; Kx – coefficient of aerodynamic resistance along X; du is the difference in force rise between ASD1 and ASD3. Based on this equation, it will be possible to construct a structural diagram of the control loop of the IMS QC along the X coordinate with a subordinate contour R, which is shown in figure 2.

Figure 2. Block diagram of the control loop of the IMS QC along the X coordinate.

The following designations are adopted:
Wрпx (S) - transfer function of the position controller along the X coordinate; Fрпx - nonlinearity of the position controller; Wkr(S) - transfer function of the slave control loop along R; K1, K2 - respectively, the coefficients of thrust, counteraction along the X coordinate; Wдx(S) - X position sensor transfer function; Uзx(S), Uдx(S) - Laplace images, respectively, of position setting signals and from position sensors along the X coordinate; Fсx(S) - Laplace image of the resistance force along the X coordinate. Let us synthesize the slave contour R with the transfer function Wkr(S). The block diagram of the control loop of the IMS QC along the R coordinate is shown in figure 3.

Figure 3. Block diagram of the control loop of the IMS QC along the coordinate R.

where: Wрпr (S) is the transfer function of the position controller along the R coordinate; Fрпr - nonlinearity of the coordinate position controller; Wpc(S) - transfer function of the speed regulator according to R; Fpc - nonlinearity of the speed regulator; Km - ASD torque transmission ratio; Tm is the time constant of the ASD torque loop; J is the moment of inertia of the ASD motor; Wдr(S), Wдсr(S) - transfer functions, respectively, of position and speed sensors along the R coordinate; Uзr(S), Uдr(S) - Laplace images, respectively, of signals for setting signals for the position and from position sensors along the R coordinate; Uзvr(S) - Laplace images from speed sensors along the R coordinate; Uzm(S) - Laplace image of the torque setting signal in ASD; Md(S) - Laplace image of the torque developed by the ASD electric motor; Fтx(S), Fсx(S) - Laplace image, respectively, of thrust and drag force along the X coordinate; KT, K2r - respectively the coefficients of thrust, counteraction along the X coordinate and along the R coordinate, K4 = $\frac{1}{K2}\frac{1}{1+\left(\frac{K1}{K2}\right)}$.

In order to reduce the influence of the wind load on the QC movement (reflected on the model by the force Ff), when determining the transfer function Wрпr(S), the position contour should be adjusted to a symmetric optimum [6]. In this case:

$$W_{рпr}(S) \times W_{вср}(S) \times K_T \times \frac{K4}{S} \times \frac{1}{S} \times W_{дп}(S) = \frac{1+4x1xS}{8x1^2xSx(1+1xS)}$$ (3)

Where Wвср(S) is the closed-loop transfer function for the coordinate velocity R. When using a low-inertia position sensor, Wвср(S) = $\frac{1}{K2 + 1.5xTm}$ and $r1 = 1.5xTm$ [9]

Then:

$$W_{рпr}(S) = \frac{0.05xK2x(1+6xTm)}{K2xK1xK4xTm2} = K_{рпr}(1 + T_{рпr} \times S)$$ (4)

This is the transfer function of the PD controller. In the case of using an inertial position sensor, we have:
\[ W_{ks}(S) = \frac{1}{1+1.5TmS} (1+T_{ac} \times S) \]  \hspace{1cm} (5)

\[ W_{npr}(S) = \frac{0.05 \times K_{dc} \times (1+6TmS) \times (1+T_{dmr} \times S)}{K_{dmr} \times K_{r} \times K_{4} \times Tm^{2}} = K_{mp} \times (1 + T_{mp} \times S) \times (1 + T_{dmr} \times S) \]  \hspace{1cm} (6)

This is the transfer function of the PD controller and PD controller combination, which is practically impossible to implement. Hence follows a practically important conclusion: in the control loop along the R coordinate, it is desirable to use a low-inertia position sensor.

In order to verify the results obtained, mathematical modelling of the control loop of the IMS QC with inertial and low-inertia sensors along the R coordinate was carried out with the following parameters: \( K_{mp} \approx 250000 \); \( T_{mp} = 0.006 \text{ s}; \) \( K_{dc} = 0.08 \); \( T_{m} = 0.001 \text{ s}; \) \( K_{r} = 0.005 \frac{\text{rad}}{\text{sec}^{2}}; \) \( K_{4} = 0.5 \frac{\text{rad}}{\text{sec}^{2}}; \) \( K_{dm} = 1 \); \( T_{dm} = 0.03 \text{ s}. \)

Simulation results for positional control, i.e. at \( U_{mpr} = 1(t) \) and disturbing influence \( F_{cr} = 10(t-1) \) are presented in figures 4 and 5.

**Figure 4.** Transient processes in the control loop of the IMS QC along the R coordinate with a low-inertia position sensor.

**Figure 5.** Transient processes in the control loop of the IMS QC along the R coordinate with an inertial position sensor.

Analysis of the obtained transient processes shows that the control loop with an inertial position sensor can be described by a transfer function of the form:

\[ W_{kr}(S) = \frac{U_{mp}(S)}{U_{pr}(S)} \approx \frac{K_{r}}{1+\tau_{r}S} \]  \hspace{1cm} (7)

The transfer function \( W_{kr}(S) \) is obtained to allow synthesizing the control loop along the X coordinate. In order to reduce the influence of the wind load \( F_{c} \) on the motion of the QC, when determining the transfer function \( W_{npr}(S) \), the position contour should also be adjusted to the symmetric optimum \[9\]

\[ W_{npx}(S) \times W_{kr}(S) \times K_{1} \times \frac{1}{1+\left(\frac{1}{\tau_{6}}\right) \times S} \times \frac{1}{S} \times W_{dm}(S) = \frac{1+4 \times 6 \times S}{8 \times 6^2 \times S^2 \times (1+6 \times S)} \]  \hspace{1cm} (8)

When using a low-inertia position sensor \( (W_{dm}(S) = K_{dm}) \) we have \( (\tau_{6} = \tau_{r}) \):

\[ W_{npr}(S) = \frac{0.12 \times K_{2} \times (1+4 \times \tau_{r} \times S) \times (1+T_{mp} \times S)}{K_{dm} \times K_{r} \times K_{1} \times \tau_{r} \times S} = \frac{K_{mp} \times (1+T_{mp} \times S) \times (1+T_{dmr} \times S)}{S} \]  \hspace{1cm} (9)
This is the transfer function of the PID controller.

In order to check the results obtained, mathematical modelling of the control loop of the IMS QC was carried out along the X coordinate when moving by 1m with the following parameters: $K_{pr}= 400$; $T_{pr1}=0.16 \text{ s}$; $T_{pr2}=0.1 \text{ s}$; $K_2= 10 \text{ 1/c}$; $K_1=2 \frac{M}{\text{rad}^2}$; $K_{dp}=1$; $T_{dp}=0.03 \text{ s}$; $K_{rm}=30 \frac{\text{rad} \times M}{\text{rad}^2}$. The simulation results for positional control, i.e., with $U_{зп}= 1(t)$ и $F_{c}= 10(t-1)$ are presented in figures 6 and 7.

![Figure 6. Transient processes in the control loop of the X coordinate in the IMS QC with a low-inertia position sensor.](image)

![Figure 7. Transient processes in the control loop of the X coordinate in the IMS QC with an inertial position sensor.](image)

It can be seen that the inertia of the position sensor along the X and R coordinates significantly worsens the quality of transient processes - the oscillation increases from 30% to 76%. Consider a control loop with a linearized parabolic controller shown in figure 8.

![Figure 8. Static characteristic of the linearized parabolic position controller.](image)
The simulation results for positional control, i.e., with \( U_{\text{in}} = 1(t) \) and \( F_c = 10(t-1) \) with a linearized parabolic controller are presented in figures 9 and 10.

**Figure 9.** Transient processes in the control loop in the IMS QC with a linearized parabolic controller with a low-inertia position sensor.

**Figure 10.** Transient processes in the control loop in the IMS QC with a linearized parabolic controller with an inertial position sensor.

In this case, it can be seen that a control loop with a linearized parabolic controller has good transients.

### 3. Conclusions

The inertial position sensor significantly degrades the characteristics of the control loop of the spacecraft along the X coordinate and along the R coordinate, making it ineffective, especially under external disturbances.

In the X control loop, fast position and speed sensors must be used in the ASD.

In the control loop along the R coordinate, it is necessary to use low-inertia position and speed sensors in the ASD.

The use of a control loop with a linearized parabolic controller allows providing the necessary characteristics and application of systems with a PID - position controller in the control loop along the X coordinate and with a PD - position regulator in the control loop along the R coordinate. This allows to ensure an acceptable quality of transient processes.

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