FRESH PRODUCE PRICE-SETTING NEWSVENDOR WITH BIDIRECTIONAL OPTION CONTRACTS

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Abstract. This paper examines a newsvendor problem for fresh produce with bidirectional option contracts, in which the stochastic demand is price-dependent. The bidirectional option, which may be exercised as either a call or put option, provides the newsvendor the flexibility to increase or decrease the initial order after real demand is realized, respectively. The condition of the fresh produce may deteriorate during circulation. The optimal order and pricing decisions for the newsvendor are analytically derived with the bidirectional option and circulation loss. Comparative statics analysis show that the optimal total order quantity and optimal retail price of the newsvendor decrease with the option price but increase with the exercise price. In addition, numerical examples show that the optimal total order quantity and optimal retail price of the newsvendor increase with the circulation loss. The optimal option order quantity first decreases then increases with the exercise price. The optimal firm order quantity first increases then decreases with the circulation loss. The maximum profit of the newsvendor decreases with the option price and circulation loss but increases with the exercise price. Furthermore, the values of bidirectional option contracts are more significant when the demand uncertainty and the circulation loss become more volatile.

1. Introduction. Fresh produce, such as meats, fruits, and vegetables, plays a significant role in our daily life. The production of fresh produce is regional and seasonal, whereas the consumption is global and regular. Thus, efficient supply chains are desired to transport fresh produce to different destinations from distant origins. However, fresh produce supply chains are distinct from well-studied supply chains of industrial products, consumption electronic devices, and so on. Fresh produce easily decays in terms of quantity and quality during transportation, leading to significant loss. For example, in developed western economics, grocery retailers can incur almost 15% loss because of spoilage and damage of fresh produce. In China, the circulation loss in fruits and vegetables is approximately 30% of the annual output [41]. Moreover, fresh produce supply chains are challenging to manage.
because of the random yield, stochastic demand, and supply price volatility of the products.

Option derivatives have been implemented to hedge risks associated with supply chain uncertainties [10, 28]. There are three types of options, namely call, put and bidirectional options. A buyer can reserve a certain quantity of options from a supplier by first paying a reservation fee before demand is realized, and after demand is realized, the buyer has the right, but not the obligation, to exercise the options according to the reservation quantity or the actual demand whichever is smaller at a pre-negotiated price. Call options provide the buyer the right to reorder products from the supplier when the realized demand is high. Put options allow the buyer to return unsold units to the supplier when the realized demand is low. Bidirectional options can be exercised as either call or put options. Thus, bidirectional options guarantee supply to the buyer and provide it with flexibility [8]. To the supplier, the reservation fee reduces its financial pressure. Furthermore, the option purchasing of the buyer close to customers provides the supplier with accurate market information, which reduces the supply chain’s bullwhip effect.

Option purchasing has been adopted by many companies. Hewlett-Packard launched a procurement risk management program, including call and put options, and obtained a total of 35% of the procurement value [9]. Weather derivatives (including put options) have also been extensively used by firms to hedge adverse weather conditions and achieved an annual $45 billion industry in the U.S. [43]. The Council of European Energy Regulators adopted bidirectional option contracts in its new regulation on the security of gas supply to develop a stronger collective response to future supply and demand risks [39]. Moreover, option derivatives have been widely used in various industries, such as fast consumption, fashion apparel, container shipping, and fresh produce [3, 21, 29, 34].

In this paper, we aim to integrate option contracts into the newsvendor (NV) framework. Specifically, we consider an NV who sells fresh produce and faces price-dependent demand. An example is the agricultural super-docking model of fresh produce selling in China. The NV, namely the supermarket, can obtain fresh produce from the farmer by a wholesale price contract and a bidirectional option contract simultaneously. In the production season, the supermarket orders a certain quantity of products (which are called firm orders) through the wholesale price contract and pays a reservation fee for another certain amount of bidirectional options (which are named option orders) through the bidirectional option contract from the farmer. During the selling season, the supermarket can buy up to the actual demand or the option quantities, whichever is smaller, when the actual demand is higher than firm orders. The supermarket can also return unsold units to the farmer when the actual demand is lower than firm orders. Due to the circulation loss of the fresh produce, the supermarket’s net salvage value is greater than the farmer’s net salvage value. Thus, the supermarket does not return unsold units, but salvages them and the farmer credits the supermarket for those units at the end of the selling season. The supermarket decides how many firm orders of fresh produce to place and how many option orders to purchase while considering the circulation loss. The supermarket also decides the selling price of fresh produce at the beginning of the selling season to maximize the expected profit.

When bidirectional option contracts are available, the NV should consider not only the tradeoff between overage cost and underage cost, but also the tradeoff between flexibility and expense cost. With less firm orders and more option orders,
the overage and underage costs will be low, but the expense cost will be high. Otherwise the overage and underage costs will be high, but the flexibility will be low. Setting the retail price further presents a challenge to the fresh produce NV. The following questions are explored.

1. How to make the optimal decisions for the fresh produce price-setting NV with bidirectional option contracts and circulation loss?

2. How the optimal decisions and performance of the fresh produce price-setting NV are affected by bidirectional option contracts and circulation loss?

3. What are the values of bidirectional option contracts, especially in a high-risk scenario (i.e., the demand uncertainty fluctuates in a wide range, in addition to high circulation loss)?

Section 2 reviews the related literature, and Section 3 details the setting and assumptions of our model. Section 4 presents the base model without bidirectional option contracts and derives the optimal joint order and pricing decisions. Section 5 focuses on the NV's optimal joint order and pricing decisions with bidirectional option contracts. Section 6 conducts numerical examples to provide additional insights regarding the proposed model. Section 7 concludes the paper and gives future research directions.

2. Literature review. Our work is relevant to the vast literature on supply chain management for perishable products and supply chains with option contracts.

We first briefly review related literature on supply chain management for perishable products. Lowe and Preckel [23] report that the supply chain management of fresh produce is characterized by significant risks associated with supply and demand uncertainties; these risks can be potentially addressed using modern decision technology tools. Blackburn and Scudder [4] investigate supply chain design strategies for fresh produce by using the marginal value of time. Results show that minimal coordination is necessary to maximize profit by appropriately segmenting the chain. Cai et al. [5] evaluate a fresh produce supplier selling products to a retailer who makes an effort to preserve the product from quality and quantity deterioration; this study characterizes the supplier's optimal wholesale price, the retail's optimal order quantity, effort level and retail price, and supply chain's coordination. Zhang et al. [44] construct a supply chain model for deteriorating items with price-sensitive demand and develop algorithms to obtain the optimal deterioration-preservation effort and pricing policies; they show that the chain can be coordinated by a cooperative investment and revenue sharing contract. Li et al. [19] apply cooperative game to examine the cost allocation problem for the collaboration of less-than-truckload of perishable products among retailers. Wang et al. [37] consider the price fairness perception of fresh food consumers and develop a markdown pricing model to jointly optimize the retailer's revenue and consumers' aggregated utility. Hou and Liu [15] construct a supply chain model for a fresh product by using product quality information asymmetry and design a cooperation mechanism to coordinate the chain. Additional information can be found in the literature [1, 31]. However, none of these studies pursues option contracts for the supply chain.

We then investigate the literature on supply chains with option contracts. According to differences in rights, options can be classified into three types, namely, call option, put option, and bidirectional option. The call option provides the buyer
the right to increase its initial order after additional demand information is realized. This option has various applications. For example, Liu et al. [22] examine overloading problems in the express delivery service industry by using the option contract. Wang et al. [38] employ the option contract for risk hedging in a relief supply chain. Chen et al. [9] expand the NV problem with option contracts to a loss-averse NV situation. Wu and Kleindorfer [40] analyze B2B supply chain issues by using option contracts in the presence of a spot market. Xu [42] study a case in which a manufacturer can obtain goods from a supplier by both call options and an instance order with random yield. Inderfurth et al. [17] extend the portfolio problem with options and spot market to a multi-period scenario. However, the classical contract, i.e., wholesale price contract, is not considered in these works. Barnes-Schuster et al. [3] examine the role of options in a two-period model with correlated demand. Fang and Whinston [11] use option contracts as tool for price discrimination in a one-supplier and two-customer situation. Chen and Shen [10] consider a supplier-retailer supply chain with option contracts and a service requirement. Feng et al. [12] investigate a buyer’s optimal procurement strategy with firm orders, call options, capital constraint, and credit financing. Luo and Chen [24] construct an option model in a two-party supply chain with random yield and spot market under deterministic demand. Wang and Chen [33] investigate the optimal ordering policy for a price-setting NV by using the wholesale price and call option contracts, and they prove that the NV with call options will be beneficial. Merzifonluoglu [26] develops novel models to optimally determine the selection of customer demands and procurement quantity with option contracts and spot market. Lee et al. [18] examine the optimal portfolio procurement strategy of an NV by using option contract and multiple option suppliers.

In contrast to the call option, the put option provides the buyer the right to decrease the initial order. This option has been rarely investigated. Chen and Parlar [6] consider the put option in a single-period inventory problem with a risk-averse price-setting NV; results show that the optimal order quantities for the NV are the same with or without the put option. Treville et al. [32] use the put option to optimize production and sourcing decisions with endogenous lead time and prove that demand volatility increases the relative value of the put option. Xue et al. [43] examine an NV by using CVaR with put options and illustrate that the value of the put options increases as the demand becomes more uncertain.

This paper focuses on the bidirectional option that provides the buyer the right to increase or decrease the initial order. Milner and Rosenblatt [27] perform a pioneer to study bidirectional option contracts in an NV framework and illustrate that the flexible contract effectively reduces the potentially negative effect of the two-period demand. Wang and Tsao [36] investigate a buyer’s optimal procurement policy with uniformly distributed demand; in this model, exercise prices for increasing or decreasing the initial order are separately set. Liu et al. [21] examine the optimal order policy for a container leasing carrier under capacity and minimum order constraints. Zhao et al. [45] investigate coordination of a supply chain with bidirectional option contracts. Nosoohi and Nookabadi [29] derive the optimal ordering and exercising decisions for a manufacturer with a stochastic final processing cost by considering wholesale price and put, call and bidirectional option contracts. Li et al. [20] consider a manufacturer-retailer supply chain with stochastic yield and an instant procurement. Gomez Padilla and Mishina [14] consider bidirectional options in a multi-supplier and one retailer supply chain and focus on analyzing the
value of options to the chain and its members. When there are multi-retailers, bidirectional option contracts allow retailers to ship to and from other locations to match demand, which is similar to trans-shipments. Marzban et al. [25] study a multi-period portfolio optimization problem by using a linear model with call and put options; this model is similar to that proposed by Wang and Tsao [36]. However, the selling price of the product has been exogenously set in the aforementioned studies.

Two studies on joint order and pricing policy with option contracts are considered the most relevant to our paper. Fu et al. [13] analyze the joint optimal procurement and pricing policy of a firm in a multi-period setting with the call option contract and spot market; they illustrate that values of the option contracts are important when the final demand is volatile. Wang and Chen [35] study the joint order and pricing strategy of a fresh produce NV with put option contracts. This study reports that the value of the put option increases with demand uncertainty. Nevertheless, the bidirectional option and its impacts on the NV’s decisions and profit are not investigated. The present paper focuses on the bidirectional option, which differs from the call or put option essentially.

This paper contributes to the literature by combining price-setting into an NV problem with bidirectional option contracts, considering the circulation loss of the fresh produce. The formulation of joint ordering and pricing decisions with bidirectional option contracts differs, in our work, from the basic building block formed by Petruzzi and Dada [30]. When bidirectional option contracts are involved, more decision variables are involved, which consequently make the problem difficult to solve due to high dimensionality. We analytically derive the optimal decisions for a fresh produce price-setting NV with bidirectional option contracts and circulation loss. More managerial insights are provided under the specific scenario. We report the values of the bidirectional option in the scenario with high circulation loss and market demand uncertainty. With bidirectional option contracts, the fresh produce NV not only orders more and can consequently provide the customers with a higher service level but also gains more profits and is more capable to deal with risks coming from high demand uncertainty. The fresh produce price-setting NV suffers a lot from the circulation loss of fresh produce; with bidirectional option contracts, the NV can well deal with the risks coming from the circulation loss.

3. Model formulation and assumptions. We consider a fresh produce NV who faces a random price-dependent demand and has the choice of purchasing bidirectional options. The NV decides the selling price and order quantities (including firm orders through the wholesale price contract and option orders through the bidirectional option contract) simultaneously to maximize the expected profit. To evaluate the performance of the NV with bidirectional options, we use a benchmark model where no bidirectional options are offered, that is the NV orders only through the wholesale price contract. We designate the NV only have firm orders as an F-NV, and the NV mixes firm and option orders as an M-NV. Table 1 presents the notation used in this article.

Figure 1 concludes the sequence of events in our model. Before the selling season (from time $t_0$ to $t_1$), the fresh produce price-setting M-NV decides the firm order quantity ($q_c$) at the wholesale price ($c$) and the option order quantity ($q_o$) at the option price ($o$) from a contract provider, as well as the selling price of the fresh produce ($p$). The provider guarantees to supply $q$ units of fresh produce. At the
beginning of the selling season (time $t_1$), the NV receives firm orders. During the selling season (from time $t_1$ to $t_2$), the NV first fulfills demand with firm orders, and then can reorder products when the actual demand is higher than the firm orders or return unsold units to the supplier otherwise at the exercise price ($e$). Specifically, the NV can reorder according to the realized demand during the whole selling season. At the end of the selling season (time $t_2$), the NV can return unsold firm orders to the supplier. The reorder/return products cannot exceed the option order quantity ($q_o$). The unmatched demand will incur a penalty cost ($s$). After the selling season, any excess fresh produce will have no salvage value.

### Table 1. Notation

| Symbol | Description |
|--------|-------------|
| $\epsilon$ | The random demand, $\epsilon \in [A, B]$, and $E(\epsilon) = \mu$; |
| $f(x)$ | The PDF function of $\epsilon$; |
| $F(x)$ | The CDF function of $\epsilon$; |
| $D(p, \epsilon)$ | The demand function; |
| $\beta$ | The circulation loss of fresh produce, and $0 < \beta < 1$; |
| $o$ | The per unit charge for purchasing bidirectional options (option price); |
| $c$ | The per unit charge for firm orders (wholesale price); |
| $e$ | The per unit charge/compensation for exercising bidirectional options (exercise price); |
| $s$ | The per unit penalty cost for unfilled demand; |
| $p$ | The per unit selling price of product; |
| $q_c$ | The firm order quantity of the M-NV; |
| $q_o$ | The option order quantity of the M-NV; |
| $q$ | The total order quantity of the M-NV, note $q = q_c + q_o$; |
| $\pi(\cdot)$ | The profit of the M-NV; |
| $N$ | Subscript, denoting the case where no bidirectional options are offered for the F-NV. |

**Figure 1. Sequence of events and decisions**

Similar to Wang and Chen [34], we introduce $\beta$ ($0 < \beta < 1$) into our model to determine the condition deterioration of the fresh produce during circulation. For simplicity, we name $\beta$ as circulation loss, which is taken by the NV. That is, the NV can only obtain $(1 - \beta)$ times of the order quantity after the fresh produce is transported from the supplier. Besides the loss during circulation, there is decay in quantity and quality during the period, including the selling season (Cai et al., [5]). Since the circulation loss takes up majority decay of the fresh produce during the whole period, the decay during the selling season is beyond the scope of this paper.

We consider the stochastic demand as additive [30]. We further define $D(p, \epsilon) = y(p) + \epsilon$, and let $y(p) = a - bp$. We assume the following: $p(1 - \beta) > o + e(1 - \beta) > c$ and $p(1 - \beta) > o + c$ to ensure a profit for the NV; $c > e(1 - \beta) - o$ to prevent the NV from arbitraging by exercising bidirectional options when the actual demand is lower than the firm orders; $o + e(1 - \beta) > c > o$ to ensure that the NV places both firm orders and option orders; and $q_o < q_c$ to avoid the quantity of returned unsold
products to exceed the initial order quantity of the NV. Notation $[x]^+ = \max(0, x)$, $\bar{F}(x) = 1 - F(x)$ and $r(x) = f(x)/\bar{F}(x)$ is the failure rate. Furthermore, we assume that $F(x)$ has an increasing failure rate (IFR). Many distributions, such as normal, uniform, exponential and chi-squared have an IFR (Bagnioli and Bergstrom [2]).

4. Base model without bidirectional option contracts. We first model the benchmark problem, i.e., the F-NV without a bidirectional option contract and only a wholesale price contract. In this problem, the fresh produce price-setting NV decides $(q_N, p_N)$. The profit function of the F-NV is

$$\pi_N(q_N, p_N) = p_N \min\{q_N(1 - \beta), D(p_N, \varepsilon)\} - cq_N - s[D(p_N, \varepsilon) - q_N(1 - \beta)\] + .$$

Defining $z_N(1 - \beta) = q_N(1 - \beta) - y(p_N)$, where $z_N(1 - \beta)$ is the stocking factor. If the realized demand of value $\varepsilon$ is not bigger than the decision of $z_N$ considering the circulation loss $\beta$, i.e., $\varepsilon \leq z_N(1 - \beta)$, then leftovers will occur; otherwise, if $\varepsilon > z_N(1 - \beta)$, then shortages will occur. We can rewrite $\pi_N(q_N, p_N)$ as

$$\pi_N(z_N, p_N) = p_N \min\{y(p_N) + z_N(1 - \beta), y(p_N) + \varepsilon\} - c[y(p_N) + \varepsilon] - s[\varepsilon - z_N(1 - \beta)\] + .$$

Defining $\Lambda(z_N) = \int_A^{z_N(1 - \beta)} [z_N(1 - \beta) - x] f(x) dx$ and $\Theta(z_N) = \int_{z_N(1 - \beta)}^B [x - z_N(1 - \beta)] f(x) dx$, we can write the expected profit of the F-NV as

$$E[\pi_N(z_N, p_N)] = \Psi(p_N) - L(z_N, p_N), \quad (1)$$

where

$$\Psi(p_N) = [p_N(1 - \beta) - c] \frac{y(p_N) + \mu}{1 - \beta}, \quad (2)$$

$$L(z_N, p_N) = \frac{\Lambda(z_N)^{1 - \beta}}{1 - \beta} + [(p_N + s)(1 - \beta) - c] \frac{\Theta(z_N)}{1 - \beta}. \quad (3)$$

Equation (2) is the riskless profit function. The unit profit of the fresh produce is equal to unit sales revenue considering the circulation loss $(p_N(1 - \beta))$ minus the unit order cost $(c)$. Equation (3) represents the loss function. When $z_N$ is decided too high, $\frac{\Lambda(z_N)}{1 - \beta}$ expected leftovers appear, and each overage cost is the wholesale price $c$. When $z_N$ is taken too low, $\frac{\Theta(z_N)}{1 - \beta}$ expected shortages appear, and each underage cost is the sum of the unit potential sales profit $(p_N(1 - \beta) - c)$ and unit penalty cost $(s(1 - \beta))$, i.e., $(p_N + s)(1 - \beta) - c$.

For a fixed selling price $p_N$ or a fixed $z_N$, the optimal order policy or optimal pricing policy of the fresh produce F-NV is listed in Lemma 1.

**Lemma 1.** For a given $p_N$, the unique optimal firm order quantity of the fresh produce F-NV is $q_N^* = \frac{y(p_N)}{1 - \beta} + z_N^*$; b) For a given $z_N$, its unique optimal selling price is $p_N^* = \frac{\Theta(z_N)}{1 - \beta}$, where $p_N = \frac{a + \frac{y(p_N)}{1 - \beta} + \mu}{2b}$.

**Proof.** When $p_N$ is given, $E[\pi_N(z_N)] = [p_N(1 - \beta) - c] \frac{y(p_N) + \mu}{1 - \beta} - c \frac{\Lambda(z_N)}{1 - \beta} - [(p_N + s)(1 - \beta) - c] \frac{\Theta(z_N)}{1 - \beta}$. As $\frac{dE[\pi_N(z_N)]}{dz_N} = -c + (p_N + s)(1 - \beta)(1 - F[z_N(1 - \beta)])$ and $\frac{d^2E[\pi_N(z_N)]}{dz_N^2} = -(p_N + s)(1 - \beta)^2 f[z_N(1 - \beta)] < 0$, $E[\pi_N(z_N)]$ is concave in $z_N$, the unique optimal $z_N^*$ is set by $\frac{dE[\pi_N(z_N)]}{dz_N} = 0$, i.e., the unique optimal $q_N^* = \frac{y(p_N)}{1 - \beta} + z_N^*$. 
When \( z_N \) is given, \( E[\pi_N(p_N)] = \left[ p_N(1 - \beta) - c \right] \frac{\mu}{1 - \beta} - c \frac{\mu}{1 - \beta} - [(p_N + s)(1 - \beta) - \alpha](\frac{\Theta(z_N)}{1 - \beta}) \). As \( \frac{dE[\pi_N(p_N)]}{dp_N} = 2b(p_N - p_N) - \Theta(z_N) \) and \( \frac{d^2E[\pi_N(p_N)]}{dp_N^2} = -2b < 0 \), \( E[\pi_N(p_N)] \) is concave in \( p_N \). For a given \( z_N \), the optimal \( p^*_N \) is set by \( \frac{dE[\pi_N(p_N)]}{dp_N} = 0. \)

The term \( \tilde{p}_N \) is the optimal riskless price that maximizes \( \Psi(p_N) \). Since \( \Theta(z_N) \) is nonnegative, \( \tilde{p}_N < \tilde{p}_N \). Then the fresh produce F-NV’s optimal selling price with demand risk will not be higher than without. From this Lemma, a higher demand risk (demand uncertainty) will result in a lower selling price to the fresh produce F-NV. The Lemma will be further discussed in the “Numerical examples” section.

With Lemma 1, the optimal joint order and pricing policy of the F-NV is presented in Proposition 1.

**Proposition 1.** If \( a - b(\frac{e}{1 - \beta} - 2s) + A(1 - \beta) > 0 \), then the unique optimal policy of the fresh produce price-setting F-NV is to order \( q^*_N = \frac{\mu}{1 - \beta} + z^*_N \) products to sell at the unit price \( p^*_N \), in which \( p^*_N \) is given by Lemma 1, and \( z^*_N \) is uniquely determined in the region \([A, B]\) by solving \( \frac{dE[\pi_N(z_N,p_N(z_N))]}{dz_N} = 0 \), i.e., \( \tilde{p}_N - \frac{\Theta(z_N)}{2b} + s \cdot (1 - F[z_N(1 - \beta)]) = \frac{c}{1 - \beta}. \)

**Proof.** By substituting \( p^*_N \equiv p_N(z_N) \) into \( E[\pi_N(z_N,p_N)] \), we obtain \( \frac{dE[\pi_N(z_N,p_N(z_N))]}{dz_N} = -c + [\tilde{p}_N - \frac{\Theta(z_N)}{2b}] + s(1 - \beta)[1 - F(z_N(1 - \beta)] \). Let \( R(z_N) = \frac{dE[\pi_N(z_N,p_N(z_N))]}{dz_N} \). Then \( \frac{dR(z_N)}{dz_N} = -(1 - \beta)^2 \frac{f(z_N(1 - \beta)]}{2b} \left[ 2b(\tilde{p}_N + s) - \Theta(z_N) \right] - \frac{1 - F(z_N(1 - \beta)]}{f(z_N(1 - \beta)]} \right) \) and \( \frac{d^2R(z_N)}{dz_N^2} = \frac{dR(z_N)}{dz_N} \). Then, we have \( \frac{d^2R(z_N)}{dz_N^2} |_{dz_N^2 = 0} = - (1 - \beta)^2 \frac{f(z_N(1 - \beta)]}{2b} \left[ 2r(\tilde{p}_N(1 - \beta)] \right) \right) \).

As \( f(\cdot) \) has an increasing failure rate, \( 2r(\tilde{p}_N(1 - \beta)] \right) \right) > 0 \), then \( \frac{d^2R(z_N)}{dz_N^2} |_{dz_N^2 = 0} < 0 \). Thereby, \( R(z_N) \) obtains its maximum when \( \frac{dR(z_N)}{dz_N} = 0. \) Thus, \( R(z_N) \) either is monotone or unimodal. Then, \( R(z_N) = 0 \) has at most two roots. Further, \( R(B) = -c < 0 \). Hence, \( R(z_N) = 0 \) has only one root if \( R(A) > 0 \), which requires that \( a - b(c/(1 - \beta) - 2s) + A(1 - \beta) > 0. \)

In Proposition 1, the condition \( a = b(\frac{e}{1 - \beta} - 2s) + A(1 - \beta) > 0 \) ensures that the expected profit of the fresh produce price-setting F-NV is unimodal in \( z_N \) in the region \([A, B]\). \( a = b(\frac{e}{1 - \beta} - 2s) + A(1 - \beta) > 0 \) is equivalent to \( R(A) = \frac{dE[\pi_N(p_N(A))]}{dz_A} > 0. \) As \( R(B) = -c < 0 \), \( R(A) > 0 \) indicates a change of sign for \( R(z_N) \) from positive to negative. Thus, \( R(z_N) \) is monotone and has only one root to maximize \( E[\pi_N(z_N,p_N(z_N))] \). Without \( R(A) > 0 \), \( R(z_N) \) is either monotone or unimodal. There are at most two roots for \( R(z_N). \) The larger \( z_N \) corresponds to the maximum of \( E[\pi_N(z_N,p_N(z_N))] \). Thereby, \( a = b(\frac{e}{1 - \beta} - 2s) + A(1 - \beta) > 0 \) is the sufficient condition for unimodality of \( E[\pi_N(z_N,p_N(z_N))] \). Proposition 1 also suggests that, except the contract parameter wholesale price \( c \), the circulation loss \( (\beta) \) and especially demand uncertainty (which is characterized by \( \Theta(z_N) \)) will significantly affect the optimal decisions of the fresh produce F-NV.

5. **Price-setting newsvendor with bidirectional option contracts.** We now consider the fresh produce M-NV problem with bidirectional options. In this problem, the NV decides \( (q_A, q_B, p) \). Thus, the profit function of the fresh produce M-NV
is

\[ \pi(q_c, q_o, p) = \min \{ q_c(1 - \beta), D(p, \varepsilon) \} \]
\[ + (p - c) \min \{ [D(p, \varepsilon) - q_c(1 - \beta)]^+, q_o(1 - \beta) \} \]
\[ + \min \{ [q_c(1 - \beta) - D(p, \varepsilon)]^+, q_o(1 - \beta) \} - c q_c - o q_o \]
\[ - s [D(p, \varepsilon) - q_c(1 - \beta) - q_o(1 - \beta)]^+. \]

In the right hand of the profit function, the first term is the revenue of selling products. The second and third terms are revenues when bidirectional options are exercised as call and put options, respectively. The fourth and fifth terms are purchasing costs for firm orders and bidirectional options, respectively. The last term is the shortage cost.

We define \( z(1 - \beta) = q_c(1 - \beta) - y(p) \), in which \( z(1 - \beta) \) is the stocking factor. If \( \varepsilon \leq (z - q_o)(1 - \beta) \), then leftovers for products will occur; if \( (z - q_o)(1 - \beta) < \varepsilon \leq z(1 - \beta) \), then leftovers for options will occur, and unexercised options are left as put options; if \( z(1 - \beta) < \varepsilon \leq (z + q_o)(1 - \beta) \), then leftovers for options will also occur, while the unexercised options are left as call options; and if \( \varepsilon > (z + q_o)(1 - \beta) \), then shortages will occur. By substituting \( D(p, \varepsilon) = y(p) + \varepsilon \), we rewrite \( \pi(q_c, q_o, p) \) as

\[ \pi(q_o, z, p) = \min \{ y(p) + z(1 - \beta), y(p) + \varepsilon \} \]
\[ + (p - c) \min \{ \varepsilon - z(1 - \beta)^+, q_o(1 - \beta) \} \]
\[ + \min \{ [z(1 - \beta) - \varepsilon]^+, q_o(1 - \beta) \} - c [y(p)/(1 - \beta) + z] - o q_o \]
\[ - s [\varepsilon - (z + q_o)(1 - \beta)]^+. \]

We define \( \Lambda(q_o, z) = \int_A^{(z-q_o)(1-\beta)} [(z-q_o)(1-\beta)-x] f(x) dx, \Omega(q_o, z) = \int_A^{z(1-\beta)} [x-(z-q_o)(1-\beta)] f(x) dx \), \( \Delta(q_o, z) = \int_A^{z(1-\beta)} [z+q_o](1-\beta)-x] f(x) dx \), and \( \Theta(q_o, z) = \int_A^{z(1-\beta)} [x-(z+q_o)(1-\beta)] f(x) dx \). The expected profit of the fresh produce M-NV can be written as

\[ E[\pi(q_o, z, p)] = \Psi(p) - L(q_o, z, p), \]

where

\[ \Psi(p) = [p(1 - \beta) - c] y(p) + \mu \]
\[ L(q_o, z, p) = c \frac{\Lambda(q_o, z)}{1 - \beta} + \varepsilon (1 - \beta) - c \frac{\Omega(q_o, z)}{1 - \beta} + [c - \varepsilon (1 - \beta)] \frac{\Delta(q_o, z)}{(1 - \beta)} \]
\[ + [(p + s)(1 - \beta) - c] \frac{\Theta(q_o, z)}{1 - \beta} + [o + c - \varepsilon (1 - \beta)] q_o \int_A^{z(1-\beta)} f(x) dx \]
\[ + [o + \varepsilon (1 - \beta) - c q_o \int_A^B f(x) dx]. \]

Equation (5) represents the riskless profit function. The unit profit of the fresh produce is equal to the unit sales revenue considering the circulation loss \( p(1 - \beta) \) minus unit order cost \( c \). Equation (6) represents the loss function. Equation (6) assesses an average cost of \( \frac{\Lambda(q_o, z)}{1 - \beta} \) expected leftovers for products; an average cost of \( \frac{\Omega(q_o, z)}{1 - \beta} \) expected leftovers for bidirectional options (in this situation, bidirectional options are left as put options); and an average cost of \( \frac{\Delta(q_o, z)}{1 - \beta} \) expected leftovers for
bidirectional options (in this situation, bidirectional options are left as call options) when $z$ and $q_o$ are set too high, in addition to the underage cost of $\frac{\Theta(q_o, z)}{1-\beta}$ expected shortages when $z$ and $q_o$ are set too low. When leftovers are products, the unit loss for unsold products is the wholesale price $c$. When leftovers for bidirectional options are put options, the unit loss for unexercised options is the option purchasing cost $o$ plus the potential profit when the options are exercised as put options $(e(1-\beta) - (o+c))$, i.e., $e(1-\beta) - c$. When leftovers for bidirectional options are call options, the unit loss for unexercised options is the option price $o$ plus the potential profit when the options are exercised as call options $(o + e(1-\beta) - c)$, i.e., $e(1-\beta) - c$. When shortages occur, the unit loss for the unfulfilled demand is the potential profit of the products $(p(1-\beta) - c)$ plus the unit penalty cost $s(1-\beta)$, i.e., $(p+s)(1-\beta) - c$. Exercising bidirectional options also results in loss of profit. When bidirectional options are exercised as put options, the loss is the difference between obtaining and returning the fresh produce by put options $((o+c - e(1-\beta))q_o)$ multiplied by the event probability $\int_{A} z^{(1-\beta)} f(x)dx$. When bidirectional options are exercised as call options, the unit profit of the product should be $p(1-\beta) - o - e(1-\beta)$. However, in Equation (5), the unit profit is formulated as $p(1-\beta) - c$. Thus, we must minus $(o + e(1-\beta) - c)$ for unit profit which is brought by bidirectional options that are exercised as call options. Consequently, when bidirectional options are exercised as call options, the loss is $(o + e(1-\beta) - c)q_o$ times the event probability $\int_{A} z^{(1-\beta)} f(x)dx$.

The expected profit is represented by Equation (4), i.e., the riskless profit minus the expected loss.

**Lemma 2.** a) For a given $p$, the expected profit of the fresh produce $M-NV$ is jointly concave in $q_o$ and $z$, and $q_o^* \equiv q_o(p) = \frac{\psi(p)}{1-\beta} + z^*$; b) for a given $q_o$ and $z$, the unique optimal selling price is $p^* = p(q_o, z) = \bar{p} - \frac{\Theta(q_o, z)}{2\bar{b}}$, where $\bar{p} = \frac{a + \frac{\psi(p)}{1-\beta} + \mu}{2\bar{b}}$.

**Proof.** When $p$ is given, $E[\pi(q_o, z)] = [p(1-\beta) - c] \frac{\psi(p)}{1-\beta} - c \frac{\Theta(q_o, z)}{1-\beta} - [e(1-\beta) - c] \frac{\Theta(q_o, z)}{1-\beta} - o + e(1-\beta)]q_o z^{(1-\beta)} f(x)dx - o + e(1-\beta) - c q_o \int_{A} z^{(1-\beta)} f(x)dx. \frac{\partial E[\pi(q_o, z)]}{\partial q_o} = [(p+s)(1-\beta) - e(1-\beta)F[(z-q_o)(1-\beta)] - (p+s-e)(1-\beta)F[(z+q_o)(1-\beta)], \frac{\partial^2 E[\pi(q_o, z)]}{\partial q_o \partial z} = -e(1-\beta)^2 f[(z-q_o)(1-\beta)] - (p+s-e)(1-\beta)^2 f[(z+q_o)(1-\beta)], \frac{\partial^2 E[\pi(q_o, z)]}{\partial q_o \partial z} = e(1-\beta)^2 f[(z-q_o)(1-\beta)] - (p+s-e)(1-\beta)^2 f[(z+q_o)(1-\beta)].$ Since $\frac{\partial^2 E[\pi(q_o, z)]}{\partial q_o \partial z} = e(1-\beta)^2 f[(z-q_o)(1-\beta)] - (p+s-e)(1-\beta)^2 f[(z+q_o)(1-\beta)] < 0$, the Hessian matrix of $E[\pi(q_o, z)]$ is negative definite. Thus, $E[\pi(q_o, z)]$ is jointly concave in $q_o$ and $z$, i.e., the unique optimal $q_o^*$ and unique optimal $z^*$ are set by $\frac{\partial^2 E[\pi(q_o, z)]}{\partial q_o \partial z} = 0$ and $\frac{\partial^2 E[\pi(q_o, z)]}{\partial z \partial q_o} = 0$, respectively.

When $q_o$ and $z$ are given, $E[\pi(p)] = [p(1-\beta) - c] \frac{\psi(p)}{1-\beta} - c \frac{\Theta(q_o, z)}{1-\beta} - [e(1-\beta) - c] \frac{\Theta(q_o, z)}{1-\beta} - o + e(1-\beta)]q_o z^{(1-\beta)} f(x)dx - o + e(1-\beta) - c q_o \int_{A} z^{(1-\beta)} f(x)dx. \frac{\partial E[\pi(p)]}{\partial p} = 2b(\bar{p} - p) - ...
\( \Theta(q_0, z) \) and \( \frac{d^2E[\pi(p)]}{dp^2} = -2b < 0 \), \( E[\pi(p)] \) is concave in \( p \). The unique optimal \( p^* \) is set by \( \frac{dE[\pi(p)]}{dp} = 0 \).

The term \( \bar{p} \) is the optimal riskless price which maximizes \( \Psi(p) \). Since \( \Theta(q_0, z) \) is nonnegative, \( p^* \leq \bar{p} \). Thus, similar to that of the fresh produce F-NV, the optimal selling price of the fresh produce M-NV with demand risk will not be higher than that without. From this Lemma, a higher demand risk will result in a lower selling price to the fresh produce M-NV. This hypothesis will be discussed in the “Numerical examples” section.

Remember that the optimal riskless price of the fresh produce F-NV is \( \bar{p}_N = \frac{o - \epsilon}{2b} + \frac{e}{1 - \beta} \). Then \( \bar{p}_N = \bar{p} \), which indicates that if the ordering policies of the NVs include firm orders, the optimal riskless prices of the NVs will be the same.

With Lemma 2, the jointly optimal policy of the M-NV is given by Proposition 2.

**Proposition 2.** The unique optimal policy of the fresh produce price-setting M-NV is to order \( q^* = q^*_o + q^*_z \) products to sell at the unit price \( p^* \), in which \( q^*_o = \frac{\Theta(q_0, z)}{1 - \beta} + z^* \), \( p^* \) is given by Lemma 2. \( q^*_o \) and \( z^* \) \((z^* \in [0, 1]) \) are uniquely determined by solving \( \frac{\partial E[\pi(q_o, p(q_o, z))]}{\partial q_o} = 0 \) and \( \frac{\partial E[\pi(q_o, p(q_o, z))]}{\partial z} = 0 \) simultaneously, i.e.,

\[
\begin{align*}
q_o &= \frac{1}{2(\bar{p} - \Theta(q_0, z)) + s} \left( p - \Theta(q_0, z) + s - \frac{o}{1 - \beta} - e \right), \\
z &= \frac{1}{2(\bar{p} - \Theta(q_0, z)) + s - e} \\
&= \frac{1}{2(\bar{p} - \Theta(q_0, z)) + s - e} \left( -\frac{o}{1 - \beta} - e \right).
\end{align*}
\]

**Proof.** By substituting \( p^* = p(q_o, z) \) into \( E[\pi(q_o, z, p)] \), we obtain

\[
\begin{align*}
\frac{\partial E[\pi(q_o, z, p(q_o, z))]}{\partial q_o} &= \left[ \bar{p} - \Theta(q_o, z) + s - \frac{o}{1 - \beta} - e \right](1 - \beta) + e(1 - \beta)F[(z - q_o)(1 - \beta)] - \left[ \bar{p} - \Theta(q_o, z) + s - \frac{o}{1 - \beta} - e \right]F[(z + q_o)(1 - \beta)], \\
\frac{\partial E[\pi(q_o, z, p(q_o, z))]}{\partial z} &= \left[ \bar{p} - \Theta(q_o, z) + s - \frac{o}{1 - \beta} - e \right](1 - \beta) - e(1 - \beta)F[(z - q_o)(1 - \beta)] - \left[ \bar{p} - \Theta(q_o, z) + s - \frac{o}{1 - \beta} - e \right]F[(z + q_o)(1 - \beta)].
\end{align*}
\]

From Lemma 2, let \( \frac{\partial E[\pi(q_o, z, p(q_o, z))]}{\partial q_o} = 0 \), we obtain

\[
\begin{align*}
\bar{p} - \Theta(q_o, z) + s - e = 0, \\
&= \bar{p} - \Theta(q_o, z) + s - e.
\end{align*}
\]

With (8) minus (7), we have

\[
\begin{align*}
z - q_o &= \frac{1}{1 - \beta} F^{-1} \left[ \frac{e - \frac{o}{1 - \beta} - e}{2e} \right].
\end{align*}
\]

With (7) plus (8), we have

\[
\begin{align*}
z + q_o &= \frac{1}{1 - \beta} F^{-1} \left[ \frac{2 \left( \bar{p} - \Theta(q_o, z) + s - \frac{o}{1 - \beta} - e \right)}{2 \left( \bar{p} - \Theta(q_o, z) + s - e \right)} \right].
\end{align*}
\]
With (10) minus (9), we have
\[ q_o = \frac{1}{2(1-\beta)} \left\{ F^{-1} \left[ \frac{2 \left( \tilde{p} - \frac{\beta(q_o,z)}{2b} + s \right) - \frac{a+\beta}{1-\beta} - e}{2 \left( \tilde{p} - \frac{\beta(q_o,z)}{2b} + s - e \right)} \right] - F^{-1} \left[ \frac{e - \frac{\beta}{1-\beta}}{2e} \right] \right\}. \] (11)

With (9) plus (10), we have
\[ z = \frac{1}{2(1-\beta)} \left\{ F^{-1} \left[ \frac{2 \left( \tilde{p} - \frac{\beta(q_o,z)}{2b} + s \right) - \frac{a+\beta}{1-\beta} - e}{2 \left( \tilde{p} - \frac{\beta(q_o,z)}{2b} + s - e \right)} \right] + F^{-1} \left[ \frac{e - \frac{\beta}{1-\beta}}{2e} \right] \right\}. \] (12)

Then \( q_o^* \) and \( z^*(z^* \in [A,B]) \) is uniquely determined that satisfies (11) and (12) simultaneously.

Proposition 2 illustrates that the optimal decision of the fresh produce M-NV is extra affected by the option prices compared to that of the F-NV. This observation suggests that in contrast to the F-NV, the M-NV needs to consider not only the tradeoff between shortage cost and overage cost but also the tradeoff between expense costs and flexibility. However, the expressions of \( q_o \) and \( z \) are complicated. Thus an efficient computation algorithm is needed to solve the problem.

From Proposition 2, we can obtain Corollary 1.

**Corollary 1.** The existence condition for a unique optimal price \( (p^*) \) is \( bf[(z^* + q_o^*)(1-\beta)](\frac{a+\beta}{2b} + e - 1 - F[(z^* + q_o^*)(1-\beta)])^3 > 0. \)

**Proof.** Through Proposition 2, we have \( z^*(p) = \frac{1}{2(1-\beta)} \{ F^{-1}[m] + F^{-1}[n] \} \) and \( q_o^*(p) = \frac{1}{2(1-\beta)} \{ F^{-1}[m] - F^{-1}[n] \} \), where \( m = \frac{2(p+s) - \frac{a+\beta}{1-\beta} - e}{2(p+s-e)} \) and \( n = \frac{e - \frac{\beta}{1-\beta}}{2e} \).

Substituting \( z^*(p) \) and \( q_o^*(p) \) into \( \pi \) (q, z, p), from some algebra, we obtain \( E[\pi(p)] = E[\pi_A(p)] + [p + s - e] \int_A^{F^{-1}(n)} \xi f(\xi)d\xi - s\mu - e[F^{-1}(n) - e] + [o + e(1 - \beta) - \frac{a+\beta}{2}] F^{-1}(n), \) where \( \pi_d(p) = [p - \frac{e}{2}] g(p) \).

Then, we have \( \frac{dE[\pi(p)]}{dp} = \frac{dE[\pi_A(p)]}{dp} + \int_A^{F^{-1}(n)} \xi f(\xi)d\xi + (z^* + q_o^*)(1-\beta) \frac{e}{2(p+s-e)} \) and \( \frac{d^2E[\pi(p)]}{dp^2} = -2b + 2(z^* + q_o^*)(1-\beta) \int_A^{F^{-1}(n)} \frac{\xi f(\xi)}{(p+s-e)}d\xi \). The existence condition for a unique optimal price \( (p^*) \) is \( \frac{d^2E[\pi(p)]}{dp^2} < 0 \), that is \( bf[(z^* + q_o^*)(1-\beta)](\frac{a+\beta}{2b} + e - 1 - F[(z^* + q_o^*)(1-\beta)])^3 > 0. \)

The condition in Corollary 1 ensures an optimal of \( E[\pi(p,q_o),(z,p)] \). The optimal joint order and pricing policy is given by Proposition 2. When the condition in Corollary 1 is not satisfied, feasible \( p, q_o, (p) \) and \( z(p) \) that maximize \( E[\pi(p,q_o),(z,p)] \) can be computed through numerical examples. For a given \( p \) or \( (q_o,z) \), analytical solutions for \( (q_o^*, z^*) \) or \( p^* \) are able to derive, respectively.

With Corollary 1, we can obtain Proposition 3 through comparative statics analysis to give more insights.

**Proposition 3.** For the M-NV, \( p^* \) and \( q_o^* \) decrease with o but increase with e.

**Proof.** From Proposition 2, we have \( p^* \equiv p \left( q_o^*, z^* \right) = \tilde{p} - \frac{\beta(q_o^*, z^*)}{2b} \). Then \( \frac{dp^*}{do} = \frac{dp}{d\pi^*} \frac{dz^*}{do} + \frac{dp}{dq_o^*} \frac{dq_o^*}{do} = \frac{dp}{d\pi^*} \frac{dz^*}{do} \frac{d(\pi^*)}{d(z^*)} \frac{dz^*}{do} \). From the proof of Proposition 2, we have \( z^* - q_o^* = F^{-1}[m] \), where \( m = 1 - \frac{a+\beta}{2(p+s-e)} \). Then \( \frac{d(z^* + q_o^*)}{do} = \frac{2(1-\beta) \xi f(\xi)}{(p+s-e)} \left\{ - \frac{1}{2(1-\beta)(z^* + q_o^*)(1-\beta)} \right\} \left\{ - \frac{1}{2(1-\beta)(z^* + q_o^*)(1-\beta)} + \frac{dp^*}{do} \right\}. \) Remember
that \( \Theta(q^*_o, z^*) = \int B [x-(z^*+q^*_o)(1-\beta)] f(x)dx \). Then we obtain \( \frac{\partial p^*}{\partial (z^*+q^*_o)} = \frac{\partial p^*}{\partial (z^*+q^*_o)} \cdot d(z^*+q^*_o) \). Thus,

\[
\frac{dp^*}{do} = \frac{\partial p^*}{\partial (z^*+q^*_o)} \cdot d(z^*+q^*_o) = -\frac{1}{2(1-\beta) b f[(z^*+q^*_o)(1-\beta)](\frac{\beta z^*}{\beta z^*}-e)} \{1 - F[(z^*+q^*_o)(1-\beta)]\}^2 \cdot \{1 - F[(z^*+q^*_o)(1-\beta)]\}\).
\]

From Corollary 1, we have \( b f[(z^*+q^*_o)(1-\beta)](\frac{\beta z^*}{\beta z^*}-e) < \{1 - F[(z^*+q^*_o)(1-\beta)]\}\) > 0. Then \( \frac{dp^*}{do} < 0 \). Similarly, we have \( \frac{dp^*}{do} = \frac{\partial p^*}{\partial (z^*+q^*_o)} \cdot d(z^*+q^*_o) \). With

\[
\frac{d(z^*+q^*_o)}{de} = \frac{2(1 - F[(z^*+q^*_o)(1-\beta)])^2}{(1-\beta) b f[(z^*+q^*_o)(1-\beta)](\frac{\beta z^*}{\beta z^*}-e)} \{1 - F[(z^*+q^*_o)(1-\beta)]\} + (\frac{dp^*}{do} - 1) \}
\]

we obtain \( \frac{dp^*}{do} = \frac{d(z^*+q^*_o)}{de} \cdot \{1 - F[(z^*+q^*_o)(1-\beta)]\}^2 \). Since \( F[(z^*+q^*_o)(1-\beta)] = m \) and \( m = 1 - \frac{1}{2(1-\beta) b f[(z^*+q^*_o)(1-\beta)](\frac{\beta z^*}{\beta z^*}-e)} > \frac{1}{2} \), \( F[(z^*+q^*_o)(1-\beta)] > \frac{1}{2} \). Then \( \frac{dp^*}{do} > 0 \).

Next, with \( q^*_o = \frac{y(p^*)}{1-\beta} + \frac{e}{1-\beta} \), \( y(p^*) = a-bp^* \), and \( q^* = q^_*+q^*_o \), we have \( q^* = \frac{a-bp^* + z^* + q^*_o}{1-\beta} \). Then \( \frac{dp^*}{do} = \frac{d(z^*+q^*_o)}{de} \cdot \{1 - F[(z^*+q^*_o)(1-\beta)]\}^2 \). We already have \( \frac{d(z^*+q^*_o)}{de} = \frac{1}{(1-\beta)^2 f[(z^*+q^*_o)(1-\beta)](\frac{\beta z^*}{\beta z^*}-e)} \).

With

\[
\frac{dp^*}{do} = -\frac{1}{2(1-\beta) b f[(z^*+q^*_o)(1-\beta)](\frac{\beta z^*}{\beta z^*}-e)} \{1 - F[(z^*+q^*_o)(1-\beta)]\}^2 \cdot \{1 - F[(z^*+q^*_o)(1-\beta)]\}\).
\]

and \( p+s-e < \frac{\beta z^*}{\beta z^*} - e \), we have \( \frac{dp^*}{do} < \frac{1}{(1-\beta)^2 f[(z^*+q^*_o)(1-\beta)](\frac{\beta z^*}{\beta z^*}-e)} \cdot \frac{1 - F[(z^*+q^*_o)(1-\beta)]}{(1-\beta)^2 f[(z^*+q^*_o)(1-\beta)](\frac{\beta z^*}{\beta z^*}-e)} \).

Then \( \frac{dp^*}{do} < 0 \). Similarly,

\[
\frac{dp^*}{de} = \frac{-b \cdot dp^*}{1-\beta} + \frac{d(z^*+q^*_o)}{de} \cdot \{1 - F[(z^*+q^*_o)(1-\beta)]\}^2 \cdot \{1 - F[(z^*+q^*_o)(1-\beta)]\}\).
\]

and \( p+s-e > \frac{\beta z^*}{\beta z^*} - e \), we have \( \frac{dp^*}{do} > \frac{1}{(1-\beta)^2 f[(z^*+q^*_o)(1-\beta)](\frac{\beta z^*}{\beta z^*}-e)} \cdot \frac{1 - F[(z^*+q^*_o)(1-\beta)]}{(1-\beta)^2 f[(z^*+q^*_o)(1-\beta)](\frac{\beta z^*}{\beta z^*}-e)} \).

Then \( \frac{dp^*}{do} > 0 \).
in $o$ and $e$ will not influence the optimal selling price of the M-NV in a large scale. Proposition 3 also shows that the increase in the unit option price $o$ will decrease the total order quantity of the M-NV. This is mainly because $o$ is a cost parameter, as ordering cost increases, the M-NV will order fewer products. However, the increase in the unit exercise price $e$ will increase the total order quantity of the M-NV. As $e$ increases, the M-NV will obtain more compensation for one unsold product. Thus, the M-NV orders more products.

6. Numerical examples. In this section, several numerical examples are conducted to illustrate the effect of bidirectional option contracts and parameters on the optimal decisions of the NV. We suppose $D(p) = 350 - 10p + \varepsilon$, where $\varepsilon$ follows a normal distribution within the IFR class. Let $A = -300$, $B = 700$, $c = 9$ and $s = 25$. The demand uncertainty CV (defined as $\sigma(\varepsilon)/\mu(\varepsilon)$) and a high CV leads to a high risk; Hua et al., [16]), option price $o$, exercise price $e$, and circulation loss of the fresh produce $\beta$ are altered for sensitive analysis to gain additional insights.

6.1. Effect of bidirectional option contracts. This subsection primarily aims to explore the effect of bidirectional option contracts, especially under different scenarios that CV varies in a wide range. First, we set $o = 3$, $e = 11$, $\beta = 0.2$ and $CV = 0.3$ to show the effect of bidirectional option contracts. The results are presented in Table 2. Table 2 illustrates that the optimal joint order and pricing policies for an NV with or without bidirectional option contracts are unique. Compared with the F-NV without bidirectional option contracts, the M-NV with bidirectional option contracts will (i) order more (i.e., $q^* > q^*_N$), (ii) price higher (i.e., $p^* > p^*_N$), and (iii) be significantly better off (i.e., $E[\pi(q^*_c, q^*_o, p^*)] > E[\pi_N(q^*_N, p^*_N)]$).

|                | Optimal total order quantity | Optimal option order quantity | Optimal price | Maximum expected profit |
|----------------|-------------------------------|--------------------------------|---------------|------------------------|
| Fresh produce  | 342                           | 32.80                          | 3829          |                        |
| Fresh produce  | 403                           | 102                            | 33.07         | 4370                   |

From Table 2, we also get $q^* > q^*_N > q^*_c$ since $403 > 342 > (403 - 102)$ for given parameters. This result indicates that with bidirectional option contracts, the M-NV can purchase less firm orders but with a higher total order quantity than the F-NV without bidirectional option contracts. When the actual demand is less than 301, i.e. the actual demand is lower than the firm order quantity, the M-NV can return unsold products to the supplier. In this case, the bidirectional option is triggered as a put option. When the stochastic demand is more than 301, i.e. the actual demand is higher than the firm order quantity, the M-NV can reorder products from the supplier. In this case, the bidirectional option is triggered as a call option. Nevertheless, the F-NV will encourage an overage cost when the actual demand is less than 342, while will encourage an underage cost when the actual demand is more than 342. Thus, bidirectional option contracts can enhance the supply flexibility for the M-NV.

Chen and Parlar [6] show that put options will not affect the NV’s optimal quantities. They assume that the NV is risk-averse and it attempts to minimize the profit variance. Nevertheless, our model sets the NV to be risk-neutral and it aims to maximize the expected profit. The M-NV can return the leftovers to the supplier when the actual demand is lower than the firm order quantity or obtain additional products from the supplier. To maintain high supply flexibility, the M-NV must
pay additional option reservation fees. Thus, the unit average order cost of the M-NV is higher than that of the F-NV; as such, the M-NV sets a higher retail price than the F-NV. Nevertheless, bidirectional option contracts still benefit the fresh produce M-NV.

Eleven scenarios are formed by changing CV from 0.1 to 0.6, as shown in Figures 2 and 3.

**Figure 2.** Effect of CV on the optimal decisions and maximum expected profits of the NV

![Graph showing the effect of CV on optimal decisions and maximum expected profits of NVs.](image1)

**Figure 3.** Effect of CV on the optimal order quantities of the M-NV

![Graph showing the effect of CV on optimal order quantities of M-NV.](image2)

Figure 2 shows the effect of demand uncertainty on the optimal decisions and maximum expected profits of the fresh produce price-setting NVs. The optimal order quantities of the NVs increase with demand uncertainty because the stochastic demand expands in a wider range as the demand uncertainty rises. The NVs must order more to match the stochastic demand. For the M-NV, both the optimal firm order quantity ($q^*_c$) and optimal option order quantity ($q^*_o$) increase with
demand uncertainty (Figure 3). On the one hand, the M-NV orders more products to deal with the demand uncertainty; on the other hand, the M-NV places more bidirectional options to hedge the risks coming from demand uncertainty.

According to Figure 2, the optimal retail prices and maximum expected profits of the fresh produce price-setting NVs decrease with demand uncertainty. As demand uncertainty increases, the NVs decrease the selling price to gain more deterministic demand \( (y(p) = a - bp) \). Although demand uncertainty severely harms the NVs, the M-NV is significantly better off due to bidirectional option contracts. As demand uncertainty increases from 0.1 to 0.6, the maximum expected profit of the F-NV is reduced by 35.63%, while the maximum expected profit of the M-NV is reduced only by 14.91%. In accordance with Figure 2, \( q^* > q^*_N \), \( p^* > p^*_N \) and \( E[\pi(q^*_c, q^*_o, p^*)] > E[\pi_N(q^*_N, p^*_N)] \) still hold.

According to the above analysis, we have Remark 1.

Remark 1. With bidirectional option contracts, the NV not only orders more and can consequently provide the customers with a higher service level but also gains more profits and is more capable to deal with risks coming from high demand uncertainty.

Remark 1 depends on the benchmark where the NV orders only through the wholesale price contract. It’s interesting to discuss whether the bidirectional option contract still benefits the NV when he purchases through a returns policy or a second replenishment policy. The returns policy enables the NV to return units remaining at the end of the selling season. The second replenishment policy gives the NV the right to have an instantaneous purchase during the selling season. Thus, the returns policy can only deal with the uncertainty when actual demand is lower than firm orders and the second replenishment can only tackle the uncertainty when actual demand is higher than firm orders. Nevertheless, the bidirectional option contract can deal with uncertainties whether the actual demand is higher or lower than the NV’s firm orders. Thus, the retailer is more willing to purchase through bidirectional option contracts. Furthermore, as the NV must pay a reservation fee for purchasing and exercising bidirectional option contracts, consequently provides the supplier with more accurate market information, the supplier is more willing to provide the bidirectional option contract than the returns policy and the second replenishment policy.

6.2. Sensitivity of parameters. In this subsection, we aim to examine the effects of contract parameters as option prices \((o)\) and exercise price \((e)\) on the optimal decisions and corresponding expected profits of the M-NV. Moreover, the effects of the circulation loss of fresh produce \((\beta)\) on both the NVs are also discussed. For this, we set \(CV = 0.3\).

(1) Impacts of option price

To study the impacts of option price, we set \(e = 11, \beta = 0.2\), and constitute eleven scenarios by changing the option price \((o)\) from 1 to 5. The results are presented in Figures 4 and 5.

As shown in Figure 4, the optimal order quantity, optimal retail price, and maximum expected profit of the fresh produce M-NV decrease with \(o\). As \(o\) increases, the cost for the M-NV to purchase a bidirectional option increases. Therefore, the optimal option order quantity \((q^*_o)\) of the M-NV decreases dramatically, which outweighs the slight increase in the optimal firm order quantity \((q^*_c)\) (Figure 5). Although the optimal retail price \((p^*)\) of the M-NV decreases with \(o\), it does not
fluctuate a lot. The effect of \( o \) on \( p^* \) is negligible. The M-NV orders less and thus gains fewer profit. This numerical result also verifies Proposition 3.

Figure 4. Effect of \( o \) on the M-NV’s optimal decisions and maximum expected profits

Figure 5. Effect of \( o \) on the optimal order quantities of the M-NV

(2) Impacts of exercise price
To investigate the impacts of exercise price (\( e \)), we assume \( o = 3 \) and \( \beta = 0.2 \). We formulate eleven scenarios by altering \( e \) from 9 to 13. The results are presented in Figures 6 and 7.

According to Figure 6, the optimal order quantity, optimal retail price, and maximum expected profit of the fresh produce M-NV all increase with \( e \). From Figure 7, the optimal firm order quantity (\( q^*_{c} \)) of the M-NV increases with \( e \); interestingly, it’s the optimal option order quantity (\( q^*_{o} \)) first decreases with \( e \) then increases with \( e \). The bidirectional options can be exercised as either call or put options, depending on the actual demand. When \( q^*_{c} \) is relatively low, the high probability event is
that bidirectional options are exercised as call options. In this situation, \( e \) is a cost similar to \( o \). Then, \( q^*_o \) decreases with \( e \). However, when \( q^*_c \) is relatively high, the high probability event is that bidirectional options are exercised as put options. In this situation, \( e \) is a revenue. Then \( q^*_o \) increases with \( e \). The effect of \( e \) on \( p^* \) is also negligible. The M-NV orders more and thus gains more profit. This numerical result also verifies Proposition 3.

![Figure 6](image6.png)

**Figure 6.** Effect of \( e \) on the optimal decisions and maximum expected profits of the M-NV

![Figure 7](image7.png)

**Figure 7.** Effect of \( e \) on the optimal order quantities of the M-NV

3) Impacts of circulation loss

To investigate the impacts of circulation loss of fresh produce (\( \beta \)), we assume \( o = 3 \) and \( e = 9 \). We formulate eleven scenarios by altering \( \beta \) from 0 to 0.45. The results are illustrated in Figures 8 and 9.

According to Figure 8, the optimal order quantities and optimal retail prices of the fresh produce price-setting NVs all increase with \( \beta \), and the maximum expected
profits decrease with $\beta$. As $\beta$ increases, the NVs must order more to fulfill customer demand. For the M-NV, interestingly, its optimal firm order quantity ($q^*_c$) first increases then decreases with $\beta$, whereas the optimal option order quantity ($q^*_o$) always increases with $\beta$ (Figure 9). This finding is mainly due to the fact that when $\beta$ is relatively low, the loss of fresh produce is not significant, and the M-NV can bear the loss. However, when $\beta$ is relatively high, the loss of fresh produce is significantly high. The M-NV chooses to decrease the firm order and increase the option order remarkably. For practical application, the specific condition of $\beta$ that reduces the firm order quantity can be computed through numerical examples. In the current setting, when $\beta = 0.35$, the firm order quantity reaches its maximum, i.e. $q^*_c = 320$. As $\beta$ increases, the unit cost of a product delivered to the customer expands to $\frac{1}{1-\beta}$ times. Therefore, the NVs increase the retail price. Nevertheless, the NVs still bear significant loss of profit. For example, as $\beta$ increases from 0 to
0.45, the maximum expected profit of the F-NV is reduced by 43.32%, whereas the maximum expected profit of the M-NV is reduced only by 36.80%. Moreover, in accordance with Figure 8, \( q^* > q_N^* \), \( p^* > p_N^* \) and \( E[\pi(q^*_c, q^*_o, p^*)] > E[\pi_N(q_N^*, p_N^*)] \) still hold.

Based on the above analysis, we have Remark 2.

**Remark 2.** The fresh produce price-setting NV suffers a lot from the circulation loss of fresh produce; with bidirectional option contracts, the NV can well deal with the risks coming from the circulation loss.

Combining Remarks 1 and 2 shows that the values of bidirectional option contracts are more significant when the demand uncertainty and the circulation loss become more volatile.

7. **Concluding remarks.** This paper examines a newsvendor problem for fresh produce with bidirectional option contracts and circulation loss, in which the stochastic demand is price-dependent. The optimal decisions for the fresh produce price-setting NV are analytically derived. Comparative statics analysis and numerical simulations are performed to give more managerial insights. This study suggests several interesting observations. (1) The optimal total order quantity of the NV decreases with the option price but increases with the exercise price and circulation loss; the optimal option order quantity first decreases then increases with the exercise price; the optimal firm order quantity first increases then decreases with the circulation loss. (2) The optimal selling price of the NV decreases with the option price but increases with the exercise price and circulation loss. (3) The maximum expected profit of the NV decreases with the option price and circulation loss but increases with the exercise price. (4) The values of bidirectional option contracts are more significant when the demand uncertainty and the circulation loss become more volatile.

Several future research directions are worth exploring. The problem should be remodeled with both quantity and quality losses, which are time-dependent, because their losses involve not only quantity but also quality. Another direction is to extend the model to a multi-supplier or multi-period scenario (Chen and Wan [7]), which is practical but difficult to tackle.

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