Imaging submilliarcsecond stellar features with intensity interferometry using air Cherenkov telescope arrays

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ABSTRACT
Recent proposals have been advanced to apply imaging air Cherenkov telescope arrays to stellar intensity interferometry (SII). Of particular interest is the possibility of model-independent image recovery afforded by the good (u, v)-plane coverage of these arrays, as well as recent developments in phase retrieval techniques. The capabilities of these instruments used as SII receivers have already been explored for simple stellar objects, and here the focus is on reconstructing stellar images with non-uniform radiance distributions. We find that hot stars (T > 6000 K) containing hot and/or cool localized regions (∆T ≳ 500 K) as small as ∼ 0.1 mas can be imaged at short wavelengths (λ = 400 nm).

Key words: instrumentation: interferometers – techniques: high angular resolution – techniques: image processing – techniques: interferometric – stars: imaging.

1 INTRODUCTION

Stellar intensity interferometry (SII) has seen a revival due to the extraordinary (u, v) coverage that future air Cherenkov telescope arrays (CTA) will provide (Holder & LeBohec 2006). The angular resolution that can be achieved is as fine as 0.06 mas at the longest baselines (1.4 km) and the shortest optical wavelengths (∼ 400 nm) (Nuñez et al. 2012). The possible improvement in angular resolution by an order of magnitude and increased sensitivity for hot (> 6000 K) stellar objects (Dravins et al. 2010) has motivated the exploration of SII capabilities to investigate several science topics. These include diameter measurements, stellar rotation, gravity darkening, mass-loss and mass transfer (see Dravins et al. 2010 for more details).

Intensity interferometry consists in measuring the squared modulus of the complex mutual degree of coherence between detector pairs from the cross-correlation between the light intensity fluctuations. Therefore, only the magnitude of the Fourier transform of the stellar radiance distribution is accessible. This poses a challenge for performing model-independent imaging. However, recent developments in phase recovery techniques from simulated SII data will make model-independent imaging possible (Holmes, Nuñez & LeBohec 2010; Nuñez et al. 2012). In previous work, we have shown that overall shapes and dimensions can be reconstructed with sub-per cent accuracies by using a Cauchy–Riemann (CR) phase reconstruction algorithm (Nuñez et al. 2012). The use of this phase reconstruction algorithm along with iterative post-processing routines also allows for further details to be imaged.

This paper is organized as follows. Section 2 outlines the generation of simplified pristine stellar images and the simulation of intensity interferometry data. Then, phase recovery is briefly outlined and image post-processing is presented in Section 3. Results are then presented in Section 4 for images with increasing degrees of pristine image complexity.

2 PRISTINE STELLAR IMAGES AND DATA SIMULATION

Pristine images of disc-like stars are first generated. These images correspond to blackbodies of a specified temperature containing an arbitrary number of ‘star spots’ of variable size, temperature and location at the surface of the spherical star in this three-dimensional model. The simulated stellar surface is then projected on to a plane, so that spots located near the edge of the visible half-sphere appear more elongated than those located near the centre. Additionally, limb darkening is included by assuming that the stellar atmosphere has a constant opacity. More details of the simulated images are presented in Section 4.

From these pristine images, simulated SII array data are obtained by computing the Fourier transform of the ‘pristine’ image so that the squared modulus of the degree of coherence can be found between telescope pairs in a large imaging air Cherenkov telescope.

1 The original ‘pristine’ image consists of 2048 × 2048 pixels corresponding to ∼ 10 mas × 10 mas of angular extension and a wavelength of λ = 400 nm.
(IACT) array.\(^2\) Gaussian noise is then added in the simulations. The signal-to-noise ratio depends on the squared modulus of the degree of correlation, the area of each of the light receivers, the spectral density (number of photons per unit area per unit time, per unit frequency), the quantum efficiency, the electronic bandwidth and the observation time (see Nuñez et al. 2012 for more details). More detailed simulations which include finite detector size, photodetector pulse shape and excess noise are currently under development (Rou, Núñez, LeBohec in preparation).

The simulations used here do not include the change in baseline due to Earth’s rotation, and yet exposure times are typically of the order of 10 h. These simulations can be regarded as corresponding to the accumulation of a sequence of short (< 1 h) observations taken over several (∼10) nights. It is also possible to save the recorded signal histories in sequences as short as 5 min or less, and then these data can be correlated in post-processing to avoid smearing of information in the (\(u\), \(v\)) plane. The simulated stellar images have corresponding temperatures of ∼6000 K, and the total exposure time can be significantly reduced if the source has a higher temperature. In fact, most stars for which photon correlations can be detected within ∼100 h with a large IACT array have temperatures greater than ∼6000 K (see Section 2 in Nuñez et al. 2012 for details).

3 CR PHASE RECOVERY AND POST-PROCESSING ROUTINES

The CR phase recovery algorithm consists in using the theory of analytic functions to relate the magnitude and the phase of the Fourier transform (Holmes & Belen’kii 2004). That is, in one dimension, the CR equations relate the magnitude and the phase differentials along the real and imaginary directions in the complex plane. For details on the two-dimensional CR phase recovery algorithm, see Nuñez et al. (2012, section 5). The resulting reconstructed image with this estimated phase is sometimes not ideal, and so is taken as a first guess for the iterative algorithms that are now described.

The Gerchberg–Saxton algorithm (GS; Gerchberg & Saxton 1972), also known as the error-reduction algorithm, is an iterative procedure. Starting from a reasonable guess of the image, the algorithm consists in going back and forth between image and Fourier space, each time imposing general restrictions in each domain. Fig. 1 describes the GS algorithm. Starting from an image \(\mathcal{O}_k\), the first step consists in taking the Fourier transform to obtain something of the form \(\mathcal{M}_k e^{i\phi_k}\). Next, Fourier constraints can then be applied, i.e. the magnitude is replaced by that given by the data, and the phase of the Fourier transform is maintained. Next, the inverse Fourier transform is calculated and constraints can be imposed in image space.

The constraints in image space can be very general. The image constraint that we impose comprises applying a mask, so that only pixels within a certain region are allowed to have positive non-zero values. For the images presented below, the mask is a circular region whose radius is typically found by measuring the radius of the first guess obtained from the CR approach. In all reconstructions where the GS is used, we perform 50 iterations, and found that more iterations typically do not produce significant changes in the reconstruction.

Another post-processing application that has been utilized is the multi-aperture image reconstruction algorithm (MIRA; Thiebaut 2009). MIRA has become a standard tool for image reconstruction in amplitude (Michelson) interferometry. MIRA is an iterative procedure which slightly modifies image pixel values so as to maximize the agreement with the data. In the image reconstruction process, additional constraints such as smoothness or compactness can be applied simultaneously, but this is something that we have not yet experimented with, i.e. the regularization parameter is set to zero for all reconstructions presented here. In the results presented below, the number of iterations is set by the default stopping criterion of the optimizer. The MIRA software only uses existing data in the \((u, v)\) plane, as opposed to using the fit of an analytic function to the data as is done in the CR and GS routines. This results in removing artefacts in the reconstruction that can be caused by the fit of an analytic function to the data. Renard, Thiebaut & Malbet (2011) have recently investigated imaging capabilities with MIRA in the context of amplitude interferometry, and the results shown below are presented in such a way that they can be compared with amplitude interferometry capabilities.

4 RESULTS

Pristine images are generated with varying complexity. We first generate pristine images of stars with limb-darkened atmospheres and investigate the reconstruction capabilities. Then, we introduce a localized bright or dark feature, and finally increase the number of features and explore some of the parameter space, i.e. spot size, location, etc.

4.1 Limb darkening

Image reconstruction is actually not necessary for the study of limb darkening, which can be studied with the knowledge of the squared degree of coherence only. A direct analysis of the data, with no image reconstruction, is likely to yield better results than the ones presented below. However, it is instructive to first see this effect in reconstructed images before adding stellar features to the simulated pristine images. Limb darkening can be approximately modelled with a single parameter \(\alpha\) as \(I(\phi)/I_0 = (\cos \phi)^\alpha\) (Hestroffer 1997), where \(\phi = 0\) refers to light being emitted from the centre of the stellar disc, and directed radially to the observer. The values of \(\alpha\) depend on the wavelength and can be found by assuming hydrostatic equilibrium. At a wavelength of 400 nm, \(\alpha \approx 0.7\) (Hestroffer & Magnan 1998) for sun-type stars, and deviations from this value may be indicative of stellar mass-loss. An example of the reconstruction of a limb-darkened star with \(\alpha = 5\) is shown in Fig. 2; such a large value is chosen so that the effect is clearly visible in a two-dimensional image with linear scale. More realistic values are considered below.

To obtain Fig. 2, a first estimate of the phase was obtained from the CR algorithm and used to generate a raw image. Then, the GS post-processing loop (Fig. 1) was performed several (50) times. In Fig. 3, the ratio of the average radius at half-maximum \(R_{1/2}\) and the
The half-height is found by calculating the average intensity at each radial position. The radius and \( m \) is apparent visual magnitude parameter shown.

The CR phase reconstruction was performed to produce a raw image, and then the GS routine was implemented to produce the post-processed image shown.

From Fig. 3 we can see that we are sensitive to changes of the ratio of the angular size of the image and the region over which the MSE is calculated (0.6 mas/1.76 mas) is comparable.

4.2 Stars with single features

Stars were simulated as blackbodies with a localized feature of a higher or lower temperature as described in Section 2. In the simulated images, the effect of limb darkening is included as described in the previous section. Here the full reconstruction analysis was used, which consists in first recovering a raw image from the CR algorithm. The raw image is then used as a starting point for several iterations of the GS loop (see Fig. 1), and finally the output of the GS algorithm is the starting image for the MIRA algorithm. Examples can be seen in Figs 4 and 5, corresponding to the post-processed reconstructions of bright stars of \( m_v = 3 \), 10 h of observation time, and a temperature of \( T = 6000 \) K. In Fig. 4, the temperature of the spot is \( T_{\text{spot}} = 6500 \) K, and in Fig. 5 the temperature of the spot is \( T_{\text{spot}} = 5500 \) K.

We can estimate the smallest temperature contrast that can be detected by varying the parameters in the model producing the pristine image. The performance in terms of temperature contrast obviously depends on several variables such as the size, location and shape of the spot. To quantify the smallest detectable spot temperature contrast, we calculate the mean square error (MSE)\(^4\) comparing the reconstructed image and the pristine image convolved with the array PSF. The ratio of the angular size of the image and the region over which the MSE is calculated (0.6 mas/1.76 mas) is comparable.

\[ \text{MSE} = \frac{\sum_{i,j} \{ A_{i,j} - B_{i,j} \}^2}{N \times N \times \sum_{i,j} B_{i,j}} \]

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\[^4\] For two images \( A_{i,j} \) and \( B_{i,j} \) of dimension \( N \times N \), the MSE = \( \sum_{i,j} (A_{i,j} - B_{i,j})^2 / N \).
and found the statistical standard deviation of the MSE. To estimate the uncertainties, we performed several reconstructions to the same ratio in the work of Renard, Thiébaut & Malbet (2011) (34 mas/100 mas), so that the MSEs presented below are directly comparable to those obtained by Renard et al. (2011). The MSE is still difficult to interpret by itself, so it is compared with the MSE between reconstructed images and their corresponding pristine images containing spots of temperature \( T + \Delta T \), where \( T = 6000 \text{K} \) (\( m_v = 3 \) and 10 h of observation time). The top curve and data points are the MSE comparing the reconstructed image and a uniform disc of the same size as the pristine image. To estimate the uncertainties, we performed several reconstructions and found the statistical standard deviation of the MSE.

Figure 5. Reconstructed dark spot. This simulated reconstruction corresponds to a star of \( m_v = 3, 10 \) h of observation time, \( T = 6000 \text{K} \) and spot temperature of \( T_{\text{spot}} = 5500 \text{K} \).

Figure 6. The bottom curve and data points correspond to the MSE between reconstructed images and their corresponding pristine images containing spots of temperature \( T + \Delta T \), where \( T = 6000 \text{K} \) (\( m_v = 3 \) and 10 h of observation time). The top curve and data points are the MSE comparing the reconstructed image and a uniform disc of the same size as the pristine image. To estimate the uncertainties, we performed several reconstructions and found the statistical standard deviation of the MSE.

To test whether the full chain of algorithms is needed to produce Figs 4 and 5, and to investigate algorithm performance, we reconstruct Figs 4 and 5 with different algorithms and combinations of algorithms. Then, we calculate the MSE of the reconstructions with the pristine image (convolved with the PSF), and also compare it to the MSE with a uniform disc. The MSE is found for reconstructions using combinations of CR\(^5\), GS and \( \text{MIRA} \). When only \( \text{MIRA} \) or GS is used directly (not preceded by CR), the initial guess of the image is a uniform disc. The results are shown in Table 1. The resulting reconstruction for each of the algorithm combinations is shown in Figs 7 and 8. The single most effective algorithm for these reconstructions is \( \text{MIRA} \), and the lowest MSE is obtained by using CR, followed by GS and \( \text{MIRA} \). According to Table 1, the GS algorithm by itself does better than the CR algorithm by itself. This is because the pristine images used here have a high degree of symmetry. This preferentially aids the GS algorithm. Also, the GS algorithm does better on these cases because the support constraint, given by the size of the star, is easy to compute. When \( \text{MIRA} \) or the GS algorithm is used directly, the reconstructed image is usually symmetric. The CR stage provides sensitivity to asymmetries such as with the bright or dark spot displayed in Figs 4 and 5. The role of GS is more to improve the phase reconstruction. \( \text{MIRA} \) plays the important role of removing artefacts, caused for example by the data fitting in the CR phase, and improving overall definition. Even though non-symmetric images can be reconstructed, the final product still is somewhat more symmetric than the pristine image for reasons that are still under investigation. When the MSE with the pristine image is compared to the MSE with a uniform disc, we can again see that the bright spot (Fig. 4) is more easily detected than the dark spot (Fig. 5).

A better estimate of the smallest temperature difference that can be imaged requires an exhaustive exploration of parameter space, but temperature differences of less than a few hundred Kelvin do not seem to be possible to image when the same brightness, temperature, exposure time, angular diameter, spot size and spot position as above are used. Results are likely to improve for hotter stars than those simulated above since the signal-to-noise ratio is higher and also the brightness contrast is higher for the same relative temperature differences (\( \Delta T/T \)). Another question related to imaging single spots is that of finding the smallest spot that can be reconstructed. In previous work (Nuñez et al. 2012), we show that the smallest possible spot that can be reconstructed is given by the PSF of the IACT array used in the simulations, namely 0.06 mas.

\( B(T + \Delta T)/B(T) > 1.2 \). This asymmetry can be partly understood in terms of the brightness ratio between blackbodies \( B(T + \Delta T)/B(T) \), whose rate of change is higher when \( \Delta T > 0 \) than when \( \Delta T < 0 \). This however does not fully account for the asymmetry between cool and hot spots. Most of the asymmetry is due to the fact that all the simulated stars in Fig. 6 have the same integrated brightness, and the radiance per solid angle is larger for a star containing a bright spot than for an annular region in a star containing a dark spot. The same analysis can be performed by simulating stars with different integrated brightness, but the estimate becomes unnecessarily cumbersome and implies knowledge that we would not have access to prior to performing an image reconstruction.\(^5\) We should also not forget that this is an estimate, and in a more precise calculation we would need to consider additional variables such as spot size, position, etc.

\( 5 \) For example, we would need to have information on the radiance per solid angle in an annular region in a star containing a dark spot.

\( 6 \) It only makes sense to use the CR algorithm first, since this is not an iterative algorithm relying on a first guess.
Table 1. MSEs for different combinations of reconstruction algorithms starting from simulated data corresponding to the pristine image of Figs 4 (top part of the table) and 5 (bottom part of the table). The algorithms are CR, GS and MIRA. The MSE between the reconstructed image and the pristine image (convolved with the PSF of the array) is shown in the second column and is compared to the MSE comparing the reconstructed image and a uniform disc in the third column. The uncertainty in the MSE is $\Delta \text{MSE} = 4 \times 10^{-11}$.

| Algorithm       | MSE (Fig. 4) | MSE (Fig. 5) |
|-----------------|--------------|--------------|
| CR              | $1.5 \times 10^{-8}$ | $1.4 \times 10^{-8}$ |
| GS              | $1.1 \times 10^{-9}$ | $1.6 \times 10^{-9}$ |
| MIRA            | $4.8 \times 10^{-10}$ | $6.7 \times 10^{-10}$ |
| CR $\rightarrow$ GS | $1.7 \times 10^{-8}$ | $1.7 \times 10^{-8}$ |
| CR $\rightarrow$ MIRA | $4.4 \times 10^{-10}$ | $6.9 \times 10^{-10}$ |
| GS $\rightarrow$ MIRA | $5.4 \times 10^{-10}$ | $6.5 \times 10^{-10}$ |
| MIRA $\rightarrow$ GS | $6.0 \times 10^{-10}$ | $8.4 \times 10^{-10}$ |
| CR $\rightarrow$ MIRA $\rightarrow$ GS | $5.2 \times 10^{-10}$ | $8.6 \times 10^{-10}$ |
| CR $\rightarrow$ GS $\rightarrow$ MIRA | $4.7 \times 10^{-10}$ | $7.6 \times 10^{-10}$ |

4.3 Multiple features

As a natural extension to the simulations presented above, data were simulated corresponding to stars with two or more recognizable features. In Figs 9 and 10, reconstructions of stars containing several hotspots are shown. The brightness and exposure time are the same as those used to simulate single-spot stars ($m_v = 3$, $T = 10 h$). A detailed investigation of reconstruction of two-spot stars was not performed, but the general behaviour is similar to that presented in Section 4.2. The reconstructions improve significantly when the pristine image has a higher degree of symmetry, e.g. when both spots lie along a line that splits the star into two. This is expected since phase reconstruction is not really necessary for centro-symmetric images. For this reason, we tested reconstructions with non-symmetric pristine images. Even though the shape of the spots is usually not well reconstructed, the approximate position and size are reasonably accurate.

The reconstruction and identification of features degrade as the number of features in the pristine image is increased. A common characteristic of reconstructing stars with several features is that the larger features in the pristine image are better reconstructed. This is more so in the case of stars containing darker regions. Nevertheless, information of positions, sizes and relative brightness of star spots can still be extracted. In Fig. 10, a reconstruction of a star containing three hotspots of different sizes and relative brightness is shown. This simulated reconstruction corresponds to the same brightness and exposure parameters as those of Fig. 9.

5 CONCLUSIONS

The capabilities of future IACT arrays for reconstructing complex images were demonstrated via simulations. Stars were simulated as blackbodies with temperatures of 6000 K with localized hot or cool regions, and then data were simulated as would be obtained...
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Figure 10. Reconstructed star with three hotspots. The pristine image has a temperature of 6000 K, and the spots have temperatures of 6500 K (top-right and left spots) and 6800 K (lower spot). The simulated data correspond to $n_0 = 3$ and $T = 10^4$ h.

with an IACT array of $\sim 100$ telescopes separated by up to 1.4 km. A first raw image obtained with the CR phase reconstruction algorithm was performed, followed by post-processing with the GS and MIRA routines. Post-processing significantly improves the image reconstruction, but the post-processing routines by themselves are usually not sufficient for performing reconstructions, especially when the pristine image is not centro-symmetric. Features with sizes of $\sim 0.05$ mas and temperature differences of a few hundred Kelvin were reconstructed. Some statistical tests were performed to quantify the confidence with which these features were reconstructed. The resulting MSEs of the simulated reconstructions are comparable to simulated results obtained by Renard et al. (2011) in the context of amplitude interferometry, but in a different angular resolution regime.

Results suggest that a variety of physical phenomena can be imaged. For example, a hot star that is losing mass radiatively can have localized bright and dark regions corresponding to a higher or lower local mass-loss rate (Lépine & Moffat 2008). The origin of these localized regions is unknown, but is likely related to magnetic activity. An order of magnitude estimate\(^7\) shows that stars with a local radiative flux that is roughly twice as high as the rest of the star, such as those shown in the figures above, undergo a localized mass-loss rate that is three times as high as the rest of the star. A related effect that is within imaging reach is the von Zeipel effect, in which the local radiative flux is proportional to the surface gravity, so that rotationally distorted stars have brighter poles. The analysis of featured oblate star observations would be very similar to the one done here. From previous results obtained in Nuñez et al. (2012), oblateness ratios (ratio of semimajor and semiminor axes) of 10–20 per cent are within reach. These in turn correspond to flux ratios of the order of 20–40 per cent, which have also been simulated and reconstructed herein. A more comprehensive study on the imagability of these physical phenomena will be the subject of future research. It will be interesting to see how well radiative-transfer models can be tested with the proposed approach.

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\(^7\) This can be done within the CAK (Castor, Abbott & Klein 1975) formalism for radiatively driven mass-loss.

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