Shape Analysis of the Level Spacing Distribution
around the Metal Insulator Transition
in the Three Dimensional Anderson Model

Imre Varga\textsuperscript{1}, Etienne Hofstetter\textsuperscript{2}*, Michael Schreiber\textsuperscript{3}, and János Pipek\textsuperscript{1}

\textsuperscript{1}Quantum Theory Group, Institute of Physics, Technical University of Budapest, H–1521 Budapest, Hungary

\textsuperscript{2}Institut für Physikalische Chemie, Johannes-Gutenberg-Universität, Jakob-Welder-Weg 11, D-55099 Mainz, Germany

\textsuperscript{3}Institut für Physik, Technische Universität Chemnitz–Zwickau, Postfach 964, D-09009 Chemnitz, Germany

PACS numbers: 71.30.+h, 05.45.+b, 64.60.Cn

Abstract - We present a new method for the numerical treatment of second order phase transitions using the level spacing distribution function $P(s)$. We show that the quantities introduced originally for the shape analysis of eigenvectors can be properly applied for the description of the eigenvalues as well. The position of the metal–insulator transition (MIT) of the three dimensional Anderson model and the critical exponent are evaluated. The shape analysis of $P(s)$ obtained numerically shows that near the MIT $P(s)$ is clearly different from both the Brody distribution and from Izrailev’s formula, and the best description is of the form $P(s) = c_1 s \exp(-c_2 s^{1+\beta})$, with $\beta \approx 0.2$. This is in good agreement with recent analytical results.
I. INTRODUCTION

Recently a novel method has been introduced for the location of the critical point and the determination of the critical exponent in the three dimensional (3D) Anderson model exhibiting a metal-insulator transition (MIT). It has been demonstrated by Shklovskii et al.\textsuperscript{1} and by Hofstetter and Schreiber\textsuperscript{2,3} that random matrix theory (RMT)\textsuperscript{4} may serve as a tool for a surprisingly accurate calculation. This time the necessary information is derived using the spectrum rather than the wave functions of the system. It is also expected that this new method is easily applicable for other types of second order phase transitions.

The novel approach is based on the study of the statistical properties of the eigenvalues of the Hamiltonian on both sides of the MIT. Based on an analogy between the kicked rotator and the Anderson model\textsuperscript{5} the MIT can be considered as a transition from the chaotic regime to the non-chaotic one, or in other words using the terminology of the RMT, from the Gaussian orthogonal ensemble (GOE) to the Poisson ensemble (PE) of random matrices. The level spacing distributions for both ensembles are known: for the GOE it is very well described by the Wigner surmise

\[ P_{\text{GOE}}(s) \approx \frac{\pi}{2} s \exp \left( -\frac{\pi}{4} s^2 \right), \quad (1) \]

that shows linear level repulsion for low \( s \). For the PE one has

\[ P_{\text{PE}}(s) = \exp(-s), \quad (2) \]

i.e. in this case the energy levels of localized states may be arbitrarily close.

The model under consideration is described by the usual tight-binding Hamiltonian

\[ \mathcal{H} = \sum_i \epsilon_i |i><i| + \sum_{i \neq j} V|i><j|, \quad (3) \]
where \( i \) labels the sites of a simple cubic \( M \times M \times M \) lattice. In the second sum only the nearest neighbor interactions are considered, and for sake of simplicity we chose \( V = 1 \) as the unit of the energy scale. The potentials \( \varepsilon_i \) are the site energies taken from a uniform distribution \(-W/2 \leq \varepsilon_i \leq W/2\). Therefore the disorder \( W \) will be the critical parameter.

One expects that for small disorder the spectrum of the Hamiltonian should be described by the level statistics of the GOE where due to hybridization level repulsion occurs and states become delocalized, while for large enough disorder the eigenvalues will tend to be uncorrelated random numbers and the corresponding eigenstates will be localized. Therefore as disorder increases the MIT is accompanied with a transition from \( P_{\text{GOE}}(s) \) to \( P_{\text{PE}}(s) \) with some unknown spacing distribution \( P_{\text{CE}}(s) \) at the MIT. (The index CE stands for the critical ensemble occurring, as demonstrated below, at the MIT.) \( P_{\text{CE}}(s) \) may at the same time show characteristics of both the GOE and the PE as suggested by Shklowskii et al.\(^1\) In infinite systems this transition is discontinuous;\(^2\) however, simulations in finite systems show a continuous variation of the level spacing distribution. In fact there is a scaling property\(^1-^3\) of these \( P(s) \) as \( M \) changes for any fixed value of \( W \). The sign of this scaling is clearly seen as a fixed ensemble in the \( P(s) \), namely the CE obtained for different values of \( M \). Moreover, there appears a fixed point \( s_0 \approx 2 \) in the \( P(s) \) curves for different disorders \( W \). Therefore one may divide the interval \([0, \infty)\) into \([0, 2]\) and \([2, \infty)\). The first part has been studied in Refs. 2, 3 and the latter (which is equivalent due to the normalization of \( P(s) \)) in Ref. 1. This time we will use all the numerically obtained \( P(s) \) functions over a wide interval \( s \in [0, 5] \).

The transition between the GOE and the PE can be approximated by several interpolation formulas. One of them is due to Brody\(^6\)

\[
P_B(s) = c_1 s^\beta \exp(-c_2 s^{1+\beta}),
\]  

(4)
where \( c_1 \) and \( c_2 \) are determined according to the conditions of normalization

\[
\int_0^\infty P(s) \, ds = 1 \tag{5}
\]

and that the mean spacing is unity

\[
\int_0^\infty sP(s) \, ds = 1. \tag{6}
\]

Any spacing distribution \( P(s) \) should satisfy Eqs. (5) and (6). Therefore we have \( c_2 = [\Gamma((\beta + 2)/(\beta + 1))]^{1+\beta} \) and \( c_1 = (1 + \beta) c_2 \). Another interpolation formula was given by Izrailev

\[
P_1(s) = As^\beta(1 + B\beta s)^Ce^{-Ds^2-Es}, \tag{7}
\]

where

\[
C = \frac{2^\beta}{\beta} \left( 1 - \frac{\beta}{2} \right) - 0.16874, \quad D = \frac{\pi^2}{16}\beta, \quad E = \frac{\pi}{2} \left( 1 - \frac{\beta}{2} \right), \tag{8}
\]

and the constants \( A \) and \( B \) are to be calculated numerically according to conditions (5) and (6). Both of these interpolations give back the limiting cases: for \( \beta = 1 \) the GOE distribution and for \( \beta = 0 \) the PE one.

Concerning the \( P(s) \) close to the MIT, in a recent publication Aronov et al.\(^8\) have shown analytically that the distribution at the transition may well be described by

\[
P_A(s) = c_1 s \exp(-c_2 s^{1+\beta}), \tag{9}
\]

where constants \( c_1 \) and \( c_2 \) are fixed according to conditions (5) and (6) and for parameter \( \beta \) they obtained \( 0 < \beta < 1 \). Furthermore \( \beta \) is related to the correlation length exponent \( \nu \) by\(^9\)

\[
\beta = \frac{1}{d\nu}. \tag{10}
\]
In this paper we wish to show a numerical analysis that confirms the form of Eq. (9) and at the same time provides a critical exponent that satisfies relation (10).

II. SHAPE ANALYSIS OF THE LEVEL SPACING DISTRIBUTION

Since we expect to see a transition from the GOE to the PE statistics as disorder increases we propose to study such quantities that describe the shape of the calculated \( P(s) \) and compare them to the known limiting cases. If one parameter scaling holds the plot of these quantities versus disorder obtained for different system sizes \( M \) should show a fixed point, yielding the approximate position of the critical point as well as the approximate value of the critical exponent. For such a calculation Shklovskii et al.\(^1\) used the tail, \( s \in [2, \infty) \) of \( P(s) \), while Hofstetter and Schreiber\(^2\) employed the numerical fit of Eq. (7) with Eq. (8) on the other part, \( s \in [0, 2] \), of the \( P(s) \). The latter authors have also analyzed\(^3\) the integrated level statistics and the Dyson–Mehta statistics, as well, and shown that these quantities enable an even better finite size scaling then \( P(s) \). In this contribution we introduce a different approach for the characterization of the level statistics. We will use all of our numerically obtained \( P(s) \) functions and in contrast to previous methods we will not introduce special parameters other than those that are uniquely related to the shape of the distribution function of a set of random numbers.

It has already been shown\(^10\) that it is advantageous to characterize a set of non-negative random numbers by certain moments of their distribution. This problem may arise studying e.g. noisy wave functions. The quantities introduced in Ref. 10 are the spatial filling factor or participation ratio which is calculated as

\[
q = \frac{\mu_1^2}{\mu_2}
\]  

(11)
and the structural entropy

\[ S_{str} = \frac{\mu_S}{\mu_1} + \ln \frac{\mu_2}{\mu_1}, \]  \hspace{1cm} (12)

where \( \mu_1 \) and \( \mu_2 \) are the usual first and second moments of the distribution \( p(x) \) of the random variables

\[ \mu_k = \int_0^\infty x^k p(x) dx, \] \hspace{1cm} (13a)

and \( \mu_S \) is calculated as

\[ \mu_S = -\int_0^\infty x \ln(x) p(x) dx. \]  \hspace{1cm} (13b)

As Eq. (6) ensures \( \mu_1 = 1 \) when using the level spacing distribution we will have simply \( q = \mu_2^{-1} \) and \( S_{str} = \mu_S + \ln \mu_2 \). Note that in practical calculations Eqs. (13a) and (13b) are approximated by finite sums. The shape analysis resides on the comparison of points plotted on the \((q, S_{str})\) plane with curves calculated with known \( p(x) \) functions.\(^\text{11}\) Note that for a trivial distribution \( p(x) = \delta(x-x_0) \), one obtains \( q = 1 \) and \( S_{str} = 0 \). For any other distribution one will have \( q \leq 1 \) and \( S_{str} \geq 0 \) and the relations \( 0 \leq q \leq 1 \) and \( 0 \leq S_{str} \leq -\ln q \) always hold.

The above characteristics will be employed here as well. Eqs. (11) and (12) should be calculated for every value of disorder parameter \( W \) and system size \( M \) replacing \( x \) by \( s \) and \( p(x) \) by \( P(s) \). The calculated values \((q, S_{str})\) will be compared to the continuous curves obtained using the interpolation formulas due to Brody (4) and to Izrailev (7) as well as with other possible \( P(s) \) functions. We will show that the \( P_{CE}(s) \) is qualitatively different from the ones obtained for GOE and PE.

We would like to emphasize that the calculation of \( q \) and \( S_{str} \) is a method which is not affected by the position of the fixed point in the \( P(s) \) at \( s \approx 2 \).
Therefore we are rather using all the obtained level distribution. At the same time no fitting procedure is necessary. We should note that one of the quantities, q, originally used as the participation ratio of a wave function serves as a measure of the skewness (or peakedness) of the distribution in our context. The structural entropy has no previously known meaning in our formalism. Additionally we have to mention that these quantities contain information about many-level correlations.

III. ANALYTICAL CALCULATIONS

First we give the \((q, S_{str})\) relations for the interpolating distributions as we wish to compare the numerical results with these phenomenological functions. As \(\beta\) runs from zero to unity the application of definitions (9) and (10) using \(P_B(s)\) from Eq. (4) yields explicitly

\[
q_B(\beta) = \left[ \frac{\Gamma(\beta + 2 \beta + 1)}{\Gamma(\beta + 3 \beta + 1)} \right]^2, \quad (14)
\]

and

\[
S_{str}^B = \ln \Gamma \left( \frac{\beta + 3}{\beta + 1} \right) - \ln \Gamma \left( \frac{\beta + 2}{\beta + 1} \right) - \frac{1}{\beta + 1} \psi \left( \frac{\beta + 2}{\beta + 1} \right). \quad (15)
\]

Here \(\psi(x)\) is the digamma function. On the other hand the \(q^I(\beta)\) and \(S_{str}^I(\beta)\) functions obtained using \(P_I(s)\) from Eqs. (7) and (8) can be calculated numerically with sufficient accuracy. Both of the cases are shown in Fig. 1, where the quantities, \(q\) and \(S_{str}\) are plotted as a function of \(\beta\).

As one can observe following the transition from PE to GOE the form of the \(P(s)\) changes in two ways: first the low-\(s\) behavior changes from a constant to linear level repulsion and at the same time the large-\(s\) tail changes from \(\exp(-s)\) to \(\exp(-s^2)\). These two changes are accounted for by both of the interpolation functions (4) and (7). However, at the transition in our
physical system one might expect that $P_{\text{CE}}(s)$ shows characteristics of both of the two limiting ensembles. Keeping this in mind we introduce further possible interpolations between the exponential $P_{\text{PE}}(s)$ and the Wigner $P_{\text{GOE}}(s)$, e.g. such an intermediate distribution (IM1) may look like

$$P_{\text{IM1}}(s) = c_1 s^\beta \exp(-c_2 s),$$  \hspace{1cm} (16)$$

where $c_1 = 1 + \beta$ and $c_2 = c_1 / \Gamma(1 + \beta)$. The parameter $\beta$ runs in the $[0, 1]$ interval. This distribution is that of the PE for $\beta = 0$ and at $\beta = 1$ it has the low-$s$ behavior of the GOE. Similarly another intermediate distribution (IM2) may look like (see also Eq. (9))

$$P_{\text{IM2}}(s) = c_1 s \exp(-c_2 s^{1+\beta}),$$  \hspace{1cm} (17)$$

where $c_1 = (1+\beta)[\Gamma(3/(1+\beta))]^2/\Gamma(2/(1+\beta))]^3$ and $c_2 = [\Gamma(3/(1+\beta))/\Gamma(2/(1+\beta))]^{1+\beta}$. In this formula the parameter $\beta$ also runs in the $[0, 1]$ interval. At $\beta = 1$ this distribution is just the Wigner surmise and for $\beta = 0$ it coincides with $P_{\text{IM1}}(s)$ with $\beta = 1$, i.e. the two functions meet with

$$P_{\text{IM}}(s) = 4s e^{-2s},$$  \hspace{1cm} (18)$$

which for $s \ll 1$ is a GOE and for $s \gg 1$ is a PE distribution. The quantities $q$ and $S_{\text{str}}$ for these new intermediate $P(s)$ distribution as a function of their parameters are

$$q_{\text{IM1}}(\beta) = \frac{\beta + 1}{\beta + 2},$$  \hspace{1cm} (19)$$

$$S_{\text{str}}^{\text{IM1}}(\beta) = \ln(\beta + 2) - \psi(\beta + 2)$$  \hspace{1cm} (20)$$

for $P_{\text{IM1}}(s)$ and

$$q_{\text{IM2}}(\beta) = \frac{[\Gamma(3/(1+\beta))]^2}{\Gamma(2/(1+\beta))\Gamma(4/(1+\beta))},$$  \hspace{1cm} (21)$$
\[ S_{\text{str}}^{\text{IM2}}(\beta) = \ln \Gamma \left( \frac{4}{1+\beta} \right) - \ln \Gamma \left( \frac{3}{1+\beta} \right) - \frac{1}{1+\beta} \psi \left( \frac{3}{1+\beta} \right) \]  

for \( P_{\text{IM2}}(s) \). We have plotted Eqs. (19), (20) and Eqs. (21), (22) in Fig. 1.

The combination of the two intermediate forms can be given as a two-parameter form

\[ P_{\text{IM3}}(s) = c_1 s^\delta \exp(-c_2 s^\alpha), \]  

but for sake of simplicity we restrict ourselves to the one-parameter versions of either Eq. (16) or (17).

**IV. NUMERICAL CALCULATIONS AND RESULTS**

In our investigation we have used the results of the numerical simulation presented and described in detail in Ref. 2. We have taken the data of the \( P(s) \) histograms and calculated the quantities \( q \) and \( S_{\text{str}} \) as a function of \( W \) around the critical disorder \( 15 \leq W \leq 18 \) for different system size \( M \) ranging from 13 up to 21. The states were obtained at the band center \((E = 0)\) for which the critical disorder is expected\(^1\) to be around \( W_c = 16.5 \).

First we present our results concerning the position of the critical point and the critical exponent. In Fig. 2 we show for \( M = 21 \) how the calculated \( q \) and \( S_{\text{str}} \) values change with the increase of disorder interpolating between the PE and GOE values. For an infinite system one expects a step function–like behavior, here it is smeared out by the finite size of the system. In Fig. 3 we have plotted our results for both quantities for different system sizes. The dashed line in both figures shows the expected position of the critical disorder. It is clear that a fixed point exists around \( W_c = 16.75 \) for both quantities. In this work we have calculated the fixed point from second order polynomial fits to the data and averaged over the different pairs of \( M \) and \( M' \). The value one obtains in this way is \( W_c = 16.87 \pm 0.52 \) for \( q \) and \( W_c = 16.77 \pm 0.63 \) for \( S_{\text{str}} \).
The critical exponent can be determined in a similar way. The approximate value is given by
\[ \nu_{M,M'} = \frac{\ln(M/M')}{\ln(\Lambda_M/\Lambda_{M'})}, \]  
(24)
where
\[ \Lambda_M = \frac{\partial X}{\partial W} \bigg|_{W_{c,M,M'}^c}. \]  
(25)

\( W_{c,M,M'}^c \) is the approximate value for the critical disorder obtained for the pair of \( M \) and \( M' \). The averaged results for the critical exponents are \( \nu = 1.27 \pm 0.29 \) for \( X = q \) and \( \nu = 1.30 \pm 0.38 \) for \( X = S_{str} \).

From these calculations we may conclude that this method does indeed give quantitatively correct results. The value of \( \nu \approx 1.34 \) for the critical exponent is obtained in Ref. 3. Note that the resulting value for the critical disorder \( W_c \) is slightly higher than 16.5 as in recent calculations of the multifractal properties of the wave functions at the MIT.\(^{13}\) But noting the accuracy in both cases, we point out that \( W_c = 16.5 \) is well within the error bars.

Now we analyze the calculated \( S_{str} \) values as a function of \( q \) as \( W \) changes around the critical point. Such a relation may be compared with the continuous ones obtained in the previous section. In Fig. 4 we display the data in the \( S_{str} \) vs \( q \) diagram. The symbols denote the simulation and the curves the analytical results. The numerical data show a remarkable trend in that respect that they fall onto a common line independent of the system size for a wide range of disorder \( 15 \leq W \leq 18 \). It is also clear that this trend is different from the \( S_{str}(q) \) curve observed for the Brody or the Izrailev distribution. In Fig. 5 we have enlarged the most important part of Fig. 4. As the numerical simulation leads to \( (q,S_{str}) \) values close to that of the approximation IM2 we conclude that the empirical \( P(s) \) function should have very similar properties as the \( P_{IM2}(s) \) has. In fact choosing the calculated \( P(s) \) function for \( M = 21 \) and \( W = 16.75 \),
the corresponding quantities are $q \approx 0.703$ and $S_{\text{str}} \approx 0.156$. These values can be obtained with the choice of $\beta = 0.18 \pm 0.02$ in Eq. (17), while for $W = 16.5$ we compute $q \approx 0.708$ and $S_{\text{str}} \approx 0.153$ which can be reproduced with $\beta = 0.21 \pm 0.02$ in Eq. (17). Hence we conclude that the intermediate distribution $P_{\text{IM2}}(s)$ with a parameter $\beta \approx 0.20$ gives a good approximation of the $P_{\text{CE}}(s)$ at the MIT. In Fig. 6 we show that the numerical histogram at $W = 16.75$ for $M = 21$ is well approximated by the distribution of the form of Eq. (17) with $\beta = 0.2$.

We note that this distribution shows the GOE characteristics for small level spacing $s$. For large $s$, however, does not follow the PE statistics (2) so that our data do not support the expectation that the CE shows characteristics of both limiting ensembles.

The visible discrepancy between the curve for $P_{\text{IM2}}$ and the calculated data in Figs. 4 and 5 is still unknown. We have performed preliminary calculations with the two–parameter distribution $P_{\text{IM3}}$ given in Eq. (23). This function can approximate the numerical points with a better accuracy, e.g. with the choice $\delta \approx 1.3$ and $\alpha \approx 1.1$. Such situation with $\delta > 1$, however, would violate a general symmetry theorem by Dyson.\(^{14}\) We have also performed a very accurate analysis of small–$s$ data considering the integrated level spacing distribution and obtained a value $\delta \approx 0.97$ for the best fit at the MIT.\(^{15}\)

The result presented is at the same time capable to explain the strange behavior of the normalization parameter $A$ (see e.g. Eq. (7)) observed in Ref. 2, where at the transition Hofstetter and Schreiber reported indications of a discontinuous change of $A$ as a function of $W$. The normalization constant in Eq. (17) with $\beta = 0.2$ is $c_1 \approx 2.28$ which is larger than for the Poisson and Wigner distributions. In the limit $M \to \infty$ the normalization constant is expected to be $A = \pi/2$ on the metallic side and $A = 1$ on the insulating side.
while at the transition it is larger than both values. Similar arguments for the parameter $B$ in Eq. (7) follow from Eqs. (7) and (8) and the above arguments for $A$, explaining the respective observations in Ref. 2.

V. CONCLUSIONS

We have presented a general method for the analysis of the spacing distribution around the critical point of a second order phase transition. As an example we have calculated the position of the MIT and the critical exponent in the 3D Anderson model. Although with lower accuracy, this method does indeed give the correct answer. On the other hand the main result of our paper is that for the MIT in the 3D Anderson model we have found a possible shape of the spacing distribution $P_{CE}(s)$ as given by Eq. (17). We have also presented an explanation for the strange behavior of the normalization constant observed in Ref. 2. Our results are in agreement with recent theoretical expectations derived by Aronov et al.\textsuperscript{8} which yield\textsuperscript{9} the relation (10) between $\beta$ and the correlation length exponent $\nu$. For our numerical value $\nu \approx 1.3$ in $d = 3$ the relation (10) yields a value $\beta = 0.26$, which is close to the value $\beta \approx 0.2$, obtained from the shape analysis above.

ACKNOWLEDGEMENT

One of the authors (I.V.) is grateful for the warm hospitality at the Institut für Physikalische Chemie, Johannes-Gutenberg-Universität where part of this work has been completed. Financial support from Országos Tudományos Kutatási Alap (OTKA), Grant Nos. 517/1991, T7238/1993 and T014413/1994 is gratefully acknowledged, as well as from the Deutscher Akademischer Austauschdienst.
References

* Present address: Blackett Laboratory, Imperial College, London SW7 2BZ, UK

1 D. I. Shklovskii, B. Shapiro, B. R. Sears, P. Lambrianides, and H. B. Shore, Phys. Rev. B 47, 11487 (1993).

2 E. Hofstetter and M. Schreiber, Phys. Rev. B 48, 16979 (1993).

3 E. Hofstetter and M. Schreiber, Phys. Rev. B 49 (1994) in press.

4 M. L. Mehta, Random Matrices (Academic Press, Boston, 1991).

5 D. R. Grempel, R. E. Prange, S. Fishman, Phys. Rev. A 29, 1639 (1984).

6 T. A. Brody, Lett. Nuovo Cimento 7, 482 (1973).

7 G. Casati, F. Izrailev, and L. Molinari, J. Phys. A: Math. Gen. 24, 4755 (1991).

8 A. G. Aronov, V. E. Kravtsov, I. V. Lerner, JETP Lett. 59, 39 (1994).

9 V. E. Kravtsov, I. V. Lerner, B. L. Altshuler, and A. G. Aronov, Phys. Rev. Lett. 72, 888 (1994).

10 J. Pipek and I. Varga, Phys. Rev. A 46 3148, (1992).

11 I. Varga, Ph. D. Thesis, Technical University of Budapest (1993), unpublished.

12 B. Bulka, K. Broderix, A. MacKinnon, and M. Schreiber, Physica A 167, 163 (1990).

13 H. Grussbach, private communication.

14 F. J. Dyson, J. Math. Phys. 3, 140 (1962).

15 E. Hofstetter and M. Schreiber, unpublished.
Figure Captions

Fig. 1. Quantities $q$ and $S_{str}$ as functions of the free parameter $\beta$ for the Brody (dashed line) the Izrailev distribution (solid line), the first intermediate (IM1) (dashed–dotted line), and the second intermediate (IM2) distribution (dotted line).

Fig. 2. Calculated $q$ and $S_{str}$ for $M = 21$ as a function of disorder $W$. The limiting values for the GOE and PE are shown with horizontal dashed–dotted lines. The expected position of the MIT is shown by a vertical dashed line.

Fig. 3. Calculated $S_{str}$ (a) and $q$ (b) for $M = 13, 15, 17, 19, 21$ as a function of disorder $W$. The expected position of the MIT is shown by a vertical dashed line.

Fig. 4. Calculated $S_{str}$ vs $q$ for $M = 13, 15, 17, 19, 21$ in the range of $15 \leq W \leq 18$. The solid circles are the points representing the PE, the GOE and the IM (Eq. (18)) cases. Solid line represents the relation for the Izrailev, dashed line the one for the Brody distribution. The dashed–dotted line reflects the intermediate distribution IM1 (between PE and IM) given in Eqs. (19), (20), and IM2 (between IM and GOE) given in Eqs. (21), (22).

Fig. 5. A part of Fig. 4 enlarged. The result of the simulation at $W = 16.75$ for $M = 21$ is plotted bold and marked with the text MIT. The solid line is obtained from the interpolation with the Izrailev, dashed line with the Brody, and dashed–dotted line with the intermediate (IM2) distribution.

Fig. 6. The numerically obtained histogram of $P(s)$ for $W = 16.75$ and $M = 21$ (solid line) and the distribution given in Eq. (17) with $\beta = 0.2$ (dashed line).