Nuclear reaction cross sections for hadron therapy

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Abstract. Treatment planning in hadron therapy uses Monte Carlo techniques that require total nuclear reaction cross sections for a variety of interacting systems and over a wide range of energies. Current empirical models are tuned to reproduce cross sections for stable isotopes, but in some cases, overestimate measured cross sections by 20%. Here we discuss an alternative approach: the optical limit of Glauber theory, which describes existing measurements better and offers more reliable predictions for the many systems where there is no experimental information. We also suggest how existing implementations of empirical models within GEANT4 could be improved. Ultimately, these calculations should contribute to improvements in planning of hadron therapy treatments.

1. Introduction
Hadron therapy encompasses a new class of treatments for cancer, where a beam of highly energetic protons or heavier ions are used to kill tumour cells. In contrast to the photons used in conventional radiotherapy, charged particles deposit the majority of their energy at the end of their track, making them much more effective at killing tumours whilst causing less damage to healthy tissue. Currently protons and carbon ions are used. Heavy ions such as carbon carry more charge than protons and create more ionisation at the end of the track, increasing the biological efficiency of the dose. The use of even heavier beams such as $^{16}$O, $^{28}$Si, $^{48}$Ti and $^{58}$Fe is currently being explored.

One significant complication of beams of hadrons are the nuclear reactions that can occur en route to the tumour. For the highest energies used for carbon-beam therapy, around 70% of the beam particles undergo a nuclear reaction before reaching the tumour site [1]. These reactions (a) cause angular straggling and energy loss, leading to greater dose prior to the tumour, (b) create a dose tail after the tumour from low-charge reaction products, and (c) create positron emitting isotopes such as $^{11}$C and $^{10}$C, which could be used for in-vivo dose verification using positron emission tomography (PET) [2].

The complications cannot be avoided, and they must be factored into dose planning through Monte Carlo modelling. A basic input for this is the total reaction cross section (i.e. the total reaction probability), which is used to scale the fragment production probabilities from other nuclear reaction models [3]. For carbon beams, the reaction cross section is required for all energies up to 400 MeV/nucleon, for the projectile and all possible secondary fragments interacting with all the isotopes found in the body. It is not feasible to measure all the required projectile and target combinations at all energies, so reaction models must be used. Current models in one of the main Monte Carlo software packages used (GEANT4) [3] are based on empirical parameterizations of cross sections measurements, but in some cases, overestimate measured cross sections by 20%. Modern alternatives offer much better...
accuracy, and here calculations are presented using the optical limit of Glauber theory [4], for several reactions of $^{12}$C. We first briefly introduce the models, and then discuss results for $^{12}$C+$^1$H, $^9$Be, $^{12}$C and $^{27}$Al. This approach, applied systematically to all available projectile and targets, should allow the development of a new modern, reliable empirical model for use in Monte Carlo treatment planning.

2. Nuclear reaction cross sections

2.1 Empirical models
We first briefly discuss the empirical models of nuclear reaction cross sections that are currently implemented in GEANT4 (e.g., by Kox [5], Shen [6], and Tripathi [7]) and often used in heavy ion therapy modelling [8]. At their root, the models parameterize the reaction cross section $\sigma_R$ in a similar way:

$$\sigma_R = \pi R^2 \left(1 - \frac{B}{E}\right)$$

where $R$ is the interaction radius, $B$ is the Coulomb barrier, and $E$ is the centre-of-mass collision energy. The bracketed term simulates Coulomb repulsion and reduces $\sigma_R$ at low energies. The effective interaction radius $R$ depends primarily on the masses (i.e. sizes) of the projectile and target, but also phenomenologically accounts for effects such as the energy dependence of the nucleon-nucleon interaction, neutron excess, and Pauli blocking. Typically, four or five parameters are required, the values of which are tuned to measured cross sections for stable projectiles and targets.

Here we have used the parameterization as implemented in GEANT4 [3]. However, one term $C(E)$, present in both the Kox [5] and Shen [6] models (and which defines the energy dependence of $R$), is not given explicitly in the original articles, and is implemented in GEANT4 using a parameterization taken from a figure. The parameterization of this term is not perfect, and this has some effect on the resulting reaction cross sections, particularly in the energies most relevant to hadron therapy. Here we have developed an improved version of $C(E)$ that better reproduces what was used in Refs. [5] and [6], and the associated cross sections. Results for this improved implementation are also shown.

2.2 Optical limit of Glauber theory and density folding models
The optical limit of Glauber theory (or density folding models) offer much better prospects for systematic, reliable predictions for reaction cross sections, particularly for unstable secondary fragments. The key step is calculation of the interaction potential $U(R)$ between the projectile $p$ and target $p$ using a density folding approach:

$$U(R) = \int_{r_i} dr_i \int_{r_p} dr_p \, \rho_i(r_i) \, \rho_p(r_p) \, v_{NN}(R + r_p - r_i).$$

Here $R$ is the separation between the centres of the projectile and target, with their respective point-nucleon densities are given by $\rho_p$ and $\rho_t$. Here, $\rho_i$ and $\rho_p$ are taken from Hartree-Fock calculations which have been shown to precisely reproduce measured nuclear density distributions [9]. The effective nucleon-nucleon interaction $v_{NN}$ is energy dependent and may be either zero-ranged or finite-ranged. Very often it is parameterized in terms of the nucleon-nucleon total cross section and may separately account for the proton and neutron components of the density distributions. In some parameterizations it may also have some density dependence due to modification of the nucleon-nucleon interaction in-medium, and due to Pauli blocking effects [10]. Here we use the free nucleon-nucleon scattering cross section parameterization of Ref. [10].
Assuming that the projectile and target follow a straight line trajectory (the eikonal approximation) in the beam direction $Z$, which is valid at the high energies discussed here, one may then calculate the elastic scattering amplitude $S(b)$, as function of impact parameter $b$,

$$S(b) = \exp \left( -\frac{i\mu}{\hbar^2 k} \int_{-\infty}^{\infty} dZ U(R) \right),$$

where $R=b+Z$, $\mu$ is the reduced mass and $k$ the wavenumber. The square modulus gives the elastic scattering probability, which, in the context of reaction cross sections, is often called the transparency function $T(b) = |S(b)|^2$. The reaction cross section is then calculated by integrating over $b$,

$$\sigma_R = 2\pi \int_{0}^{\infty} db \ b \left( 1 - |S(b)|^2 \right).$$

### 3. Results

The results for the reactions $^{12}\text{C}+^{1}\text{H}$, $^{9}\text{Be}$, $^{12}\text{C}$ and $^{27}\text{Al}$ are shown in Figure 1. The density folding calculations are shown along with the empirical Shen parameterization [6] as implemented in GEANT4, and the improved version suggested here. For each of the four reactions, the broad energy dependence of the cross section is significantly better when using the density folding model than the empirical models, except at the lowest energies. The density folding model calculations overestimate the cross section.
sections, since the current calculations take no account of Coulomb effects. These Coulomb effects could be incorporated by modifying the assumed trajectory, or phenomenologically [11].

The new implementation of the $C(E)$ term in the Shen parameterization (dotted line) reproduces the energy dependence of the cross section significantly better than the GEANT4 version. The differences are up to 6% in the region between 200 and 300 MeV/nucleon, which is critical in carbon ion therapy. Despite this improvement, the present density folding model calculations perform better across the whole energy range.

There is scope for improving the density folding model calculations. In addition to the Coulomb effects noted above, one should also consider: (a) A more sophisticated implementation of $\nu_{NN}$ which includes an energy dependent range [16]. This will tend to enhance the energy dependence of $\sigma_R$, causing a decrease near 200 MeV/nucleon; (b) cluster structure in $^9$Be and other light isotopes, unaccounted for in the mean-field calculations used, are likely to extend their density distribution and increase $\sigma_R$; (c) Fermi motion effects, which increase cross sections at low energy [15]; (d) in-medium modification of $\nu_{NN}$, giving it an effect density dependence and reducing cross sections; and (e) Pauli blocking [10].

4. Conclusions
We have discussed models for nuclear reaction cross sections, relevant to heavy ion therapy. The implementation of empirical models within GEANT4 could be easily improved to make them closer to the original papers and give a better description of experimental measurements. However, density folding models can offer a superior description. Further effects need to be added into the present calculations, including in-medium modifications of the nucleon-nucleon interaction, Pauli blocking [10], Fermi-motion effects [15], and Coulomb repulsion. However, these preliminary calculations show promise for more robust and reliable reaction cross sections where no measurements exist. This may prove particularly helpful for dose predictions with beams heavier than carbon. The calculations are too computationally intensive to use directly within Monte Carlo simulations, but a comprehensive set of cross sections for all targets and projectiles could provide the basis for a modern empirical parameterization suitable for heavy ion therapy treatment planning.

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