First evidence that non-metricity \( f(Q) \) gravity could challenge \( \Lambda \)CDM

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We propose a novel model in the framework of \( f(Q) \) gravity, which is a gravitational modification class arising from the incorporation of non-metricity. The model has General Relativity as a particular limit, it has the same number of free parameters to those of \( \Lambda \)CDM, however at a cosmological framework it gives rise to a scenario that does not have \( \Lambda \)CDM as a limit. Nevertheless, confrontation with observations at both background and perturbation levels, namely with Supernovae type Ia (SNIa), Baryonic Acoustic Oscillations (BAO), cosmic chronometers (CC), and Redshift Space Distortion (RSD) data, reveals that the scenario, according to AIC, BIC and DIC information criteria, is in some datasets slightly preferred comparing to \( \Lambda \)CDM cosmology, although in all cases the two models are statistically indiscernible. Finally, the model does not exhibit early dark energy features, and thus it immediately passes BBN constraints, while the variation of the effective Newton’s constant lies well inside the observational bounds.

Introduction

Although General Relativity (GR) is the well-established theory for the description of the gravitational interaction, there are two main motivations that justify the large amount of research devoted to its modification and extension. The first arises from cosmological grounds, since modified gravity is very efficient in describing the universe’s two phases of accelerated expansion [1, 2]. The second, and chronologically older, motivation is purely theoretical, and aims towards the improvement of the renormalizability of General Relativity with the so-called symmetric teleparallel theories, which is an extension of the recently constructed \( f(Q) \) modified gravity, which is a gravitational modification of General Relativity (and thus General Relativity) [4].

In this Letter we propose a specific model in the framework of the recently constructed \( f(Q) \) modified gravity, as which we show is very efficient in fitting the observational data. In this class of modification, one starts from the so-called symmetric teleparallel theories, which is an equivalent description of gravity using the non-metricity scalar \( Q \) [4], and extends it to an arbitrary function \( f(Q) \). \( f(Q) \) gravity leads to interesting applications [5–13], and trivially passes the constraints arising from gravitational wave observations [14]. By confronting our new model with data from Supernovae type Ia (SNIa), Baryonic Acoustic Oscillations (BAO), Hubble parameter cosmic chronometers (CC) and Redshift Space Distortions (RSD) for \( \delta T \) observations, we deduce that the scenario at hand may be, in some cases, slightly statistically preferred than \( \Lambda \)CDM, although it does not include it as a particular limit.

A novel model in \( f(Q) \) gravity

The action of non-metricity-based modified gravity is [4]

\[
S = -\frac{1}{16\pi G} \int d^4 x \sqrt{-g} f(Q),
\]

where the nonmetricity scalar

\[
Q = -\frac{1}{4} Q_{\alpha\beta\gamma} Q^{\alpha\beta\gamma} + \frac{1}{2} Q_{\alpha\beta\gamma} Q^{\gamma\alpha\beta} + \frac{1}{4} Q_\alpha Q^\alpha - \frac{1}{2} Q_\alpha \tilde{Q}^\alpha, \tag{2}
\]

with \( Q_\alpha \equiv Q_{\alpha\mu} \) and \( \tilde{Q}^\alpha = Q^\mu_\alpha \), arises from contractions of the non-metricity tensor \( Q_{\alpha\mu\nu} = \nabla_\alpha g_{\mu\nu} \).

From these it is apparent that Symmetric Teleparallel Equivalent of General Relativity (and thus General Relativity) is obtained in the case of \( f(Q) = Q \).

Varying the total action \( S + S_m \), with \( S_m \) the matter sector action, one obtains the field equations as [5, 7]:

\[
\frac{2}{\sqrt{-g}} \nabla_\alpha \left\{ \sqrt{-g} g_{\beta \mu} f_Q \left[ \frac{1}{2} L^{\alpha\beta\gamma} + \frac{1}{4} g^{\beta\gamma} \left( Q^\alpha - \tilde{Q}^\alpha \right) \right] \right. \\
+ f_Q \left[ -\frac{1}{2} L^\mu_\alpha_\beta + \frac{1}{8} (g^\mu\alpha Q_\beta + g^\mu\beta Q_\alpha) \right. \\
+ \frac{1}{4} g^{\alpha\beta} \left( Q^\mu - \tilde{Q}^\mu \right) \right. Q_\mu_\alpha_\beta + \frac{1}{2} \delta_\mu^\nu f = T^\nu_{\nu}, \tag{3}
\]

with \( L^\mu_\nu = \frac{1}{2} Q^\alpha_\mu_\nu - Q^{\alpha}_\mu_\nu \) the disformation tensor, \( T^\mu_{\nu} \) the energy-momentum tensor, and \( f_Q = \partial f / \partial Q \). In order to apply it in a cosmological framework we impose a flat Friedmann-Robertson-Walker (FRW) metric of the form \( ds^2 = -dt^2 + a(t)^2 \delta_{ij} dx^i dx^j \), in which case equa-
tions (3) give rise to the two Friedmann equations [7]

\[ 6f_QH^2 - \frac{1}{2}f = 8\pi G(\rho_m + \rho_r), \]

\[ (12H^2f_{QQ} + f_Q)\dot{H} = -4\pi G(\rho_m + p_m + \rho_r + p_r), \]

where \( H \equiv \dot{a}/a \) is the the Hubble function and with \( \rho_m, \rho_r \) and \( p_m, p_r \) the energy densities and pressures of the matter and radiation perfect fluids. Additionally, note that the nonmetricity scalar \( Q \) in an FRW background becomes \( Q = 6H^2 \). Finally, the equations close by the consideration of the matter and radiation conservation equations

\[ \dot{\rho}_m + 3H(\rho_m + p_m) = 0 \]

\[ \dot{\rho}_r + 3H(\rho_r + p_r) = 0. \]

Proceeding to the perturbation level, elaborating the full perturbation equations [7] one can extract the evolution equation for the matter overdensity, \( \delta \equiv \delta\rho_m/\rho_m \), at sub-horizon scales in terms of the scale factor as

\[ \delta'' + \left( \frac{H'}{H} + \frac{3}{a} \right)\delta' = \frac{3Gm0}{2H^2a^5}G_{\text{eff}}\delta, \]

with primes denoting derivatives with respect to the scale factor, and where we have introduced the density parameters through \( \Omega_i \equiv \frac{8\pi G}{3M^2} \), with the subscript “0” denoting the value at present time. Finally, \( G_{\text{eff}} \) is the effective Newton’s constant in \( f(Q) \) gravity, which is given as

\[ G_{\text{eff}} \equiv \frac{G}{f_0}. \]

In this work we propose the following model

\[ f(Q) = Qe^{\lambda\frac{Q_0}{Q}}, \]

where \( \lambda \) is the sole model free parameter, and \( Q_0 = 6H_0^2 \) with \( H_0 \) the current value of the Hubble parameter. For \( \lambda = 0 \) GR is recovered but not \( \Lambda \)CDM since the cosmological constant in absent, thus this model alleviates the cosmological constant problem. Note that in certain periods of cosmic history, as the term \( Q_0/Q \) decreases, our model effectively reduces to the polynomial case \( f(Q) = Q^n \). Thus, in a sense, it could be thought as an encapsulation of many \( f(Q) \) models, where at a given cosmic time a term with a particular \( n \) becomes dominant. Using (8) and (4), and considering \( w_m \equiv p_m/\rho_m = 0 \) and \( w_r \equiv p_r/\rho_r = 1/3 \), the corresponding normalized Hubble parameter \( E^2 \equiv H^2/H_0^2 \) is written as

\[ (E^2 - 2\lambda)e^{\lambda/E^2} = \Omega_{m0}a^{-3} + \Omega_{r0}a^{-4}. \]

Applying the above equation at present, the parameter \( \lambda \) can be expressed as

\[ \lambda = 0.5 + W_0\left( \frac{\Omega_{m0} + \Omega_{r0}}{2e^{\lambda/2}} \right), \]

with \( W_0 \) the principal branch of the Lambert function. Hence, the scenario at hand has exactly the same number of free parameters with \( \Lambda \)CDM, and as we will see this is a key reason for its statistical preference over the latter. Lastly, the effective Newton’s constant becomes

\[ G_{\text{eff}} = \frac{G}{e^{\lambda\frac{Q_0}{Q}}(1 - \lambda\frac{Q_0}{Q})}. \]

**Data, Methodology and Results**

We employ Bayesian analysis and the likelihood function \( \mathcal{L}_{\text{tot}} \sim \exp(-\chi^2_2/2) \), where \( \chi^2_2 \) is obtained from different sums of \( \chi^2_{\text{SNIA}}, \chi^2_{\text{BAOs}}, \chi^2_{\text{CC}}, \chi^2_{\text{RSD}} \), to be specified below. The quantities \( \chi^2_{\text{SNIA}}, \chi^2_{\text{RSD}} \) and \( \chi^2_{\text{CC}} \) are defined and described in [15]. In contrast with the latter, we use the full SNIa dataset in order to avoid biases induced via the binning procedure. Moreover, as explained at [16], we employ only a subset of the CC dataset. Since RSD data were extracted by imposing \( \Lambda \)CDM as a reference model, we use the simple correction factor described in [17]. Additionally, we utilize the BAOs dataset of [18], as they employ fiducial cosmology corrections (i.e the term \( r_d/r_{\text{fid}} \)). We sample the posterior distribution of the parameters applying the MCMC method as implemented within the open-source Python package emcee [19], for the following cases:

1. SNIa+CC datasets, with free parameters \( \Omega_{m0}, h, M \).
2. SNIa+CC+BAOs, with free parameters \( \Omega_{m0}, h, r_d, M \).
3. SNIa+CC+RSD, with free parameters \( \Omega_{m0}, h, \sigma_8, M \).

Note that \( M \) is the intrinsic free parameter of Pantheon dataset (see [15] and references therein) and we neglect \( \Omega_{r0} \). We apply the aforementioned setup for our \( f(Q) \) model (8), along with \( \Lambda \)CDM to allow for a direct comparison. In all cases 1000 walkers and 2500 states are employed along with flat priors on the parameters. Although the particular \( f(Q) \) model considered here has the same number of free parameters with \( \Lambda \)CDM, the functional form of the Hubble rate is different and thus the relative fitting quality could only be compared in the context of information criteria differences.

We present the results on the parameters in Table I. Additionally, in Fig. 1 we show the corresponding contour plots for the extracted model parameters for all considered datasets. Finally, in Fig. 2 we depict the contour plots for both the \( f(Q) \) and \( \Lambda \)CDM models, for the case of Pantheon+CC+BAOs datasets. Concerning the parameters values, in all cases \( \Omega_{m0} \) is close to the Planck value, while the deviations from the concordance model is evident. Moreover, as a consistency check, we compare the sound horizon at baryon drag epoch, \( r_d \), with the model-independent one from [20], and we observe 1σ
one of ΛCDM cosmology, through the use of the Akaike quality of the present dataset the parameter corresponding parameter values with the current ones. Lastly, analyze the full CMB spectrum and compare the correlations, in order to be able to extract safe results we should H

| Model          | Ω_{m0} | h     | r_d  | σ_8  | M    | χ^2_{min} | χ_{min}/dof |
|----------------|--------|-------|------|------|------|-----------|-----------|
| Q^{\lambda Q_0}_{CDM} | 0.349 ± 0.021 | 0.6828 ± 0.0203 | –     | –     | –    | −19.412 ± 0.062 | 1033.37   | 0.968     |
| ΛCDM          | 0.299 ± 0.021 | 0.6825 ± 0.0201 | –     | –     | –    | −19.406 ± 0.061 | 1033.267  | 0.968     |

| Model          | Ω_{m0} | h     | r_d  | σ_8  | M    | χ^2_{min} | χ_{min}/dof |
|----------------|--------|-------|------|------|------|-----------|-----------|
| Q^{\lambda Q_0}_{CDM} | 0.353 ± 0.020 | 0.6800 ± 0.0199 | 148.747 ± 4.135 | –     | −19.419 ± 0.066 | 1035.613  | 0.966     |
| ΛCDM          | 0.304 ± 0.0202 | 0.6794 ± 0.0199 | 148.141 ± 4.350 | –     | −19.413 ± 0.060 | 1035.597  | 0.966     |

| Model          | Ω_{m0} | h     | r_d  | σ_8  | M    | χ^2_{min} | χ_{min}/dof |
|----------------|--------|-------|------|------|------|-----------|-----------|
| Q^{\lambda Q_0}_{CDM} | 0.339 ± 0.020 | 0.6864 ± 0.0204 | –     | 0.703 ± 0.0292 | –    | −19.405 ± 0.061 | 1048.392  | 0.964     |
| ΛCDM          | 0.292 ± 0.0202 | 0.6852 ± 0.0201 | –     | 0.742 ± 0.0322 | –    | −19.400 ± 0.061 | 1046.640  | 0.963     |

TABLE I. Observational constraints and the corresponding χ^2_{min} for the new f(Q) gravity model (8), where “dof” stands for degrees of freedom (defined as the number of the used data points minus the number of fitted parameters). In order to allow direct comparison the concordance ΛCDM model is also included.

FIG. 1. The 1σ and 2σ iso-likelihood contours for the f(Q) model (8), for the 2D subsets of the parameter space (Ω_{m0}, h, M), using graphic package getdist [21]. We have used joint analysis of various datasets (see text).

FIG. 2. The 1σ and 2σ iso-likelihood contours for the f(Q) model (8), as well as for the ΛCDM scenario, for the 2D subsets of the parameter space (Ω_{m0}, h, M, r_d), using graphic package getdist [21]. We have used the joint analysis of SNIa+CC+BAOs datasets (see text).

compatibility. Note that concerning the H_0 and σ_8 tensions, in order to be able to extract safe results we should analyze the full CMB spectrum and compare the corresponding parameter values with the current ones. Lastly, we mention that for the case of Pantheon+CC+BAOs dataset the parameter λ is found as λ = 0.371 ± 0.008 at 1σ confidence level.

Proceeding forward, in Table II we compare the fitting quality of the present f(Q) model with the corresponding one of ΛCDM cosmology, through the use of the Akaike Information Criterion (AIC), the Bayesian Information Criterion (BIC), and the Deviance Information Criterion (DIC), see [15] and references therein. For SNIa + CC datasets, we find that ΔIC is almost 0, and thus the two models are statistically compatible. For SNIa + BAOs + CC, our f(Q) model is slightly more preferred by the data. In contrast, for SNIa + BAOs + RSD the f(Q) model is deemed inferior by the data, however still statistically indistinguishable from ΛCDM. We mention here
(without considering the possibility that the BAOs data may include numerical relics) that the fact that these BAOs employ a free parameter to correct for the imposed cosmology, while the $f(z)_{\delta}$ data do not, could be an indication for circularity problem with the RSD data. Hence, although prominent, the present $f(Q)$ model, since it has no $\Lambda\text{CDM}$ limit, gets “punished” by those datasets that incorporate $\Lambda\text{CDM}$ as a reference model. Finally, we mention that our model includes many models as sub-cases (i.e polynomial) at particular times within the cosmic history, however it maintains the same number of free parameters with $\Lambda\text{CDM}$ cosmology.

\begin{table}[h]
\centering
\begin{tabular}{|c|c|c|c|c|c|c|}
\hline
Model & AIC & $\Delta$AIC & BIC & $\Delta$BIC & DIC & $\Delta$DIC \\
\hline
SNi/CC & $Qe^{3\Sigma}$ & 1036.390 & 0.1 & 1054.30 & 0.106 & 1039.320 & 0.115 \\
$\Lambda$CDM & 1039.290 & 0 & 1054.194 & 0.0 & 1039.205 & 0 \\
\hline
SNi/CC/BAOs & $Qe^{3\Sigma}$ & 1043.650 & 0 & 1063.537 & 0 & 1043.542 & 0 \\
$\Lambda$CDM & 1043.994 & 0.344 & 1063.881 & 0.344 & 1043.888 & 0.346 \\
\hline
SNi/CC/RSD & $Qe^{3\Sigma}$ & 1056.430 & 1.753 & 1076.3749 & 1.751 & 1056.3198 & 1.750 \\
$\Lambda$CDM & 1054.677 & 0 & 1074.624 & 0 & 1054.570 & 0 \\
\hline
\end{tabular}
\caption{The information criteria AIC, BIC and DIC for the examined cosmological models, alongside the corresponding differences $\Delta$IC $\equiv$ IC $-$ IC$_{\text{min}}$.}
\end{table}

\textbf{Conclusions}

We proposed a novel $f(Q)$ model and we confronted it against observational data (SNi, BAOs, CC and RSD). For CC + SNi datasets the two models are statistically compatible, however for CC + SNi + BAOs datasets the $f(Q)$ model is slightly statistically preferred comparing to $\Lambda$CDM one. On the other hand, in the case of RSD + CC + SNi data, the $\Lambda$CDM paradigm is slightly preferred by the data, although the two models remain statistically equivalent.

In the large redshift limit (i.e at large $E^2(z) \equiv H^2(z)/H_0^2$) the proposed $f(Q)$ tends to $Q$ and thus the scenario at hand tends to GR, hence it trivially passes the early universe constraints and in particular the BBN ones. Additionally, knowing the observational bounds of $E^2(z)$ throughout the evolution, and using (10), from (11) we deduce that throughout the evolution $|g_{\text{eff}}^{\Lambda\text{CDM}} - 1|$ remains smaller than 0.1 and therefore it satisfies the observational constraints [22].

In summary, our results could serve as motivation for further study of the present model, as well as $f(Q)$ gravity in general, as it constitutes one of the first alternatives to the concordance model that apart from the fact that it might be preferred by the data (at least by some datasets), it does not face the cosmological constant problem since it does not include a “hidden” cosmological constant inside the $f(Q)$ form. Further studies on this model, using the full CMB and LSS spectra, weak lensing data and other datasets, could enlighten our findings and verify whether the present $f(Q)$ model outperforms the concordance one or not.

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