Spontaneous symmetry breaking (SSB) is responsible for structure formation in scenarios ranging from condensed matter to cosmology. SSB is broadly understood in terms of perturbations to the Hamiltonian governing the dynamics or to the state of the system. We propose an alternative mechanism that results from the quantum monitoring of a system via continuous quantum measurements. The acquisition of information during the measurement process induces a measurement back-action that we identify as the crucial ingredient for SSB. An observer can thus tailor the topology of the vacuum manifold and the pattern of symmetry breaking by monitoring different observables.

Spontaneous symmetry breaking (SSB) occupies a central stage in modern physics [1]. It governs physical mechanisms such as BCS superconductivity, the existence of Nambu-Goldstone bosons [2, 3], and the generation of mass via the Higgs mechanism [4–6]. It is also considered to play a key role in the early history of the universe [7, 8], and may have contributed to structure formation in this context [9]. Symmetry breaking arises in Nature when observable phenomena lack the symmetry of the underlying physical laws dictating them. A system can thus be found in configurations in conflict with the invariance manifested by its equations of motion.

SSB is characteristic of phase transitions from a high-symmetry phase to a lower symmetry phase in which a new kind of macroscopic order emerges, that can be detected by an order parameter [10]. In a specific system, SSB is analyzed through the symmetries of its free energy landscape or field equations [9]. An intuitive understanding of the spontaneous rupture of symmetry is acquired by picturing a spin chain initially prepared in a paramagnetic phase that is driven by a Hamiltonian \( H \propto \sum_j \sigma_j^x \sigma_{j+1}^z \), which favors ferromagnetic order. The ensuing dynamics preserves the symmetries of the Hamiltonian, and in the absence of external perturbations nothing in the evolution biases the choice between the degenerate ground states \( |\uparrow\uparrow\ldots\uparrow\rangle \) and \( |\downarrow\downarrow\ldots\downarrow\rangle \). However, in the thermodynamic limit any infinitesimal external magnetic field is enough to break the tie, and single out one of these states as the observed ground state. In this case the symmetry is explicitly broken. In physical systems of finite size, the breaking of the symmetry can be generally understood as a consequence of random fluctuations, either in the Hamiltonian governing the evolution, or in the state of the system at a given time. In the example of the spin chain driven from the symmetric paramagnetic phase, a local perturbation of the magnetization is enough to implant a seed that leads to a symmetry breaking along the whole system. Whenever present, spatial fluctuations of the magnetization can thus favor the local growth of regions where spins align in a certain direction. This example serves to illustrate the general physical mechanism of domain formation.

A characteristic feature of the canonical description of SSB is its focus on static features: it is understood via the properties of the state of the system in thermal equilibrium. This approach is frequently pursued using elements of homotopy theory to characterize the topology of the vacuum manifold, spanned by the degenerate ground states in the broken-symmetry phase [9, 11]. At the same time, while the consequences of quantum dynamics and decoherence on SSB have been studied [12, 13], the quantum measurement process has never been proposed as the cause behind SSB.

In this proof-of-concept article we propose a novel mechanism for spontaneous symmetry breaking, induced by the continuous monitoring of a quantum system. Unlike the standard picture, where SSB happens due to somewhat ad-hoc perturbations to the Hamiltonian or to the state of the system, our approach is dynamical and purely quantum mechanical in nature. During the monitoring by continuous quantum measurements, symmetry breaking is induced by the quantum measurement back-action [14] associated with the acquisition of information by the observer. The latter can thus alter the topology of the vacuum manifold and the pattern of symmetry breaking by selecting different kinds of measurements.

**Results**

For the sake of illustration, we consider \( N \) 1/2-spins prepared in the ground state of a paramagnetic Hamiltonian \( H_0 = -\Lambda \sum_{j=1}^N \sigma_j^z \), where \( \Lambda \) represents a global energy scale. The initial state of the chain is then

\[
|\Psi(0)\rangle = \bigotimes_{j=1}^N |\rightarrow\rangle_j,
\]

where \( |\rightarrow\rangle_j \) denotes the eigenstate of \( \sigma_j^z \) with eigenvalue 1. Following a sudden quench at \( t = 0 \), the system evolves...
for $t > 0$ according to the ferromagnetic Hamiltonian

$$H = -A \sum_{j=1}^{N} \sigma_j^z \sigma_{j+1}^z,$$

(2)

taking periodic boundary conditions $\sigma_{N+1}^z \equiv \sigma_1^z$ for concreteness.

The initial state of the spin chain shares a symmetry of the system Hamiltonian, and such symmetry is preserved in the case of unitary evolution. This can be easily seen, for instance, by noting that the Hamiltonian commutes with the magnetization $M = \sum_j \sigma_j^z$ of the chain, i.e. $[H, M] = 0$. Since the initial state satisfies $\langle \Psi(0) | M | \Psi(0) \rangle = 0$, the subsequent evolved state necessarily has equal weights on the states $|\uparrow\uparrow \ldots \uparrow\uparrow\rangle$ and $|\downarrow\downarrow \ldots \downarrow\downarrow\rangle$, and is therefore incapable of selecting between them. However, experience tells us that in practice a particular direction for the chain’s magnetization is spontaneously chosen by the system, even if Hamiltonian and initial state are symmetric with respect to the $z$ direction. In the canonical approach to symmetry breaking this is explained by assuming a small perturbation to the Hamiltonian or to the state of the system, locally breaking the symmetry ‘by hand’.

We consider an alternative SSB scenario that results from monitoring a quantum system during time evolution. Such quantum monitoring can be modeled by continuous quantum measurements [15–17], which can be thought of as a sequence of infinitesimally weak measurements. In contrast to strong projective measurements, which can drastically perturb the state of the system, a weak measurement provides only partial information of the state. In doing so, this process induces a mild back action on the state of the system at any given time. The collective information obtained from the continuous measurement record over a period of time can provide full information of the system though [18], and can thus serve as a way to perform full state tomography [19, 20], parameter estimation [21, 22], quantum error correction [23], and quantum control [24]. Note that the action of the observer is in practice tantamount to the coupling to a monitoring environment, whenever the latter weakly interacts with the system of interest in such a way that information of a physical quantity is probed and registered [25].

To start with, assume that independent observers continuously monitor the single-spin components $\{\sigma_j^z\}$ of individual spins along the chain, as illustrated in Fig. 1(a). The output of such continuous measurements over an interval $dt$ is given by $d\rho_j(t) = \langle \sigma_j^z(t) \rangle dt + dW_j^z$, which provides information of the spin components, hidden by additive white noise, i.e. independent zero-mean real Gaussian random variables $dW_j^z$ with variance $dt$ [15–17]. The dynamics of the system undergoing continuous monitoring of the set of observables $\{\sigma_j^z\}$ is well described by a stochastic master equation dictating the change in the state,

$$d\rho_t = L [\rho_t] dt + \sum_j I_j [\rho_t] dW_j^z,$$

(3)

when expressed in Itô form [16, 17]. Here, $L(\rho_t)$ takes the standard Lindblad form for the set of measured operators, which includes the evolution due to the Hamiltonian and dephasing due to the monitoring process:

$$L(\rho_t) = -i [H, \rho_t] - \frac{1}{8\tau_m} \sum_j [\sigma_j^z, [\sigma_j^z, \rho_t]].$$

(4)

In turn, the ‘innovation terms’

$$I_j [\rho_t] = \sqrt{\frac{1}{4\tau_m} \left( \{\sigma_j^z, \rho_t\} - 2 \text{Tr} (\sigma_j^z \rho_t) \rho_t \right)}$$

(5)

account for the change in the state of the system due to the acquisition of information during the measurement.
process, which occurs over a ‘characteristic measurement time’ $\tau_m$. These innovation terms, nonlinear in $\rho_t$, encompass the effect of the back-action on the state of the system due to the quantum measurement. For simplicity we have taken units such that $\hbar = 1$.

Note the connection between the dynamics under continuous measurements and that of an open system in contact with an environment. Observers without access to the measurement output, who need to average over the unobserved measurement outcomes, obtain an averaged description of the state of the system $\rho^m_{\text{obs}}$. The latter evolves according to $d\rho^m_{\text{obs}} = L[\rho^m_{\text{obs}}] dt$, and its evolution is thus identical to that of the system coupled to an environment through the spin components $\{\sigma_j^z\}$. Importantly, such density matrix does not show signs of symmetry breaking, given that the evolution commutes with the magnetization. That is, without registering the measurement outcomes, and in the absence of any further perturbations to Hamiltonian or state, symmetry is fully preserved. In particular, the spin components $\langle \sigma_j^z(t) \rangle$ remain constant.

By contrast, the measurement process does break the symmetry, forcing individual spins to collapse to one of the eigenstates of $\sigma_j^z$. Indeed, when conditioning the state to the observed outcomes the measurement back-action breaks the symmetry in individual realizations. To prove this, let us focus on the evolution of the spin component of spin $j$ in the ferromagnetic part of the quench, with $t \geq 0$. The evolution of the expectation value of the spin component $\sigma_j^z$ is dictated by

$$d\langle \sigma_j^z(t) \rangle = -\frac{1}{\tau_m} \Delta_{\sigma_j^z}(t) dW^j_t,$$

(6)

where the trace $\langle \cdot \rangle(t) \equiv \text{Tr}(\rho_t)$ is taken with respect to the state $\rho_t$ and $\Delta_{\sigma_j^z}(t) = \sqrt{\langle (\sigma_j^z)^2 \rangle(t) - \langle \sigma_j^z \rangle^2(t)}$ is the corresponding standard deviation. This means that, due to quantum monitoring, the spin component evolves whenever its quantum uncertainty is non-zero. Such uncertainty is zero if and only if the spin is found in one of the eigenstates of $\sigma_j^z$, $|\uparrow\rangle_j$ or $|\downarrow\rangle_j$. Therefore, only states with definite values of the spin component are stable under the continuous monitoring. SSB is thus a consequence of the quantum measurement back-action encoded in the ‘innovation terms’ $I_j[\rho_t]$ in equation (5).

Effectively, monitoring the spin components breaks the symmetry in the spin chain, in the basis selected by the measurement process, as we illustrate in Fig. 2. The characteristic measurement time $\tau_m$ dictates the rate at which such symmetry breaking occurs and individual spins collapse ‘up’ or ‘down’. The stochastic dynamics naturally leads to the formation of localized topological defects due to the monitoring of the spin chain.

FIG. 2. Dynamics of symmetry breaking induced by monitoring of each spin for different measurement strengths. (a) A weak monitoring of the individual spins slightly affects their spin components, barely perturbing the otherwise symmetric evolution. (b) As the measurement strength is increased, symmetry is broken by the measurement process, leading to spins collapsing to the stable ‘up’ or ‘down’ configurations. (c) A strong monitoring process rapidly leads to the formation of localized topological defects in individual realizations of the stochastic evolution.

The measured observable is also crucial in determining the nature of the symmetry breaking, and in particular, in governing structure formation in the end state. To illustrate this, we analyze the case in which a coarse-grained local magnetization is probed on the chain. Let us then consider that, instead of continuously monitoring each individual spin component as in Fig. 1(a), the local magnetization over clusters of $N_c$ consecutive spins is continuously measured, as depicted on Fig. 1(b). We denote local magnetization observables by

$$m_\alpha = \sum_{j \in I_\alpha} \sigma_j^z, \quad \alpha = \{1, \ldots, N/N_c\},$$

(7)

where $I_\alpha = [N_c(\alpha - 1) + 1, N_c\alpha]$.

Once again, symmetry is broken by the monitoring process, given that $d\langle m_\alpha(t) \rangle = -\sqrt{\tau_m} \Delta_{m_\alpha}(t) dW^\alpha_t$, where $\Delta_{m_\alpha}(t)$ denotes the standard deviation of the monitored coarse-grained magnetization. In this case, the stable states to which the measurement process leads to, eigenvectors of the set of measured observables $\{m_\alpha\}$, are starkly different, given that each spin cannot be singled out by the measurement process. Such eigenstates have definite values of the magnetization, $\lambda_m = (-N_c, -N_c + 1, \ldots, N_c - 1, N_c)$, on each of the coarse-grained measured regions. As a result, this causes a symmetry breaking with non-uniform magnetization along
shows, the final state in the broken phase has very different properties than the case in which spin clusters are monitored, resulting in homogeneous magnetization along the chain, facilitating the growth of a single domain, and a complete suppression of topological defects. Thus, an observer can control the patterns of symmetry breaking by a choice of the measurement observables. This choice determines the nature of the final state in the broken symmetry phase, including the size of the domains, and the statistics of the magnetization. Conversely, the nature of the domains produced in this setting provides information about the monitoring process. In a broader context, the measurement-back action is expected to govern pattern formation. Notably, this mechanism for symmetry breaking is amenable to experimental test in superconducting qubit platforms with current technology [26, 27].

Methods

The evolution of a system under continuous measurement of an arbitrary set of observables \( \{ A_j \} \) is given by the master equation

\[
d\rho_t = L(\rho_t) dt + \sum_j I_j[\rho_t] dW^j_t, \tag{8}
\]

with

\[
L(\rho_t) = -i[H, \rho_t] - \sum_j \frac{1}{8\tau_m j} [A_j, [A_j, \rho_t]], \tag{9}
\]

and

\[
I_j[\rho_t] = \sqrt{\frac{1}{4\tau_m^2}} (\{ A_j, \rho_t \} - 2 \text{Tr}(A_j \rho_t) \rho_t). \tag{10}
\]

Here \( dW^j_t \) denote independent Gaussian random variables of mean 0 and width \( dt \), while \( \tau_m^j \) is the ‘characteristic measurement time’ with which observable \( A_j \) is monitored, i.e., it provides the timescales over which information of the expectation value of the observable is acquired in the measurement process [17].

During ideal continuous measurement experiments, the output over a time step of length \( dt \) depends on the state of the system via \( dr_j = \text{Tr}(\rho_t A_j) dt + dW^j_t \). That is, the output follows the expectation value of the observable, hidden by additive white noise.

The time evolution is unravelled by modeling continuous measurements with a sequence of infinitesimally weak measurements of the local, coarse-grained and global magnetization as a choice of monitored observables. Such measurements can in turn be modeled by Kraus operators acting on the state at every time step \( dt \) of the evolution, see for example [15]. In the limit of \( dt \) smaller than any other relevant timescales, such approach gives the same dynamics as equation (8). All simulations where performed in QuTiP [28, 29].
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Author Contributions
LPGP and DT implemented the numerical simulations. All authors contributed to initiate and develop the project, analyze the results, and write the manuscript.

Competing Financial Interests
The authors declare no competing financial interests.