Conditional generation of path-entangled optical NOON states

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We propose a measurement protocol to generate path-entangled NOON states conditionally from two pulsed type II optical parametric oscillators. We calculate the fidelity of the produced states and the success probability of the protocol. The trigger detectors are assumed to have finite dead time, and for short pulse trigger fields they are modeled as on/off detectors with finite efficiency. Continuous-wave operation of the parametric oscillators is also considered.

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I. INTRODUCTION

Nonclassical states of light have many applications, and a number of different protocols exist for the generation of various classes of states. The two-mode maximally entangled N-photon states

\[ |\text{NOON}\rangle = \frac{1}{\sqrt{2}} \left( |N, 0\rangle + e^{i\chi} |0, N\rangle \right), \tag{1} \]

the so-called NOON states, are particularly interesting because a single-photon phase shift of \( \chi \) induced in one of the two components changes the relative phase of the two terms by \( N\chi \). This special property of NOON states may be utilized to enhance spatial resolution in (quantum) microscopy and lithography [1], and in interferometry it has been shown that a certain measurement strategy, using NOON states, leads to a phase estimation error scaling as \( L^{-1/4}N^{3/4} \) if the phase to be estimated is known to lie within an interval from \(-\pi/L\) to \(\pi/L\), where \( N \) is the total number of photons used in the measurements [2]. This is better than the classical shot noise limit of \( N^{-1/2} \), and NOON states are thus useful to perform accurate measurements and may be a valuable field resource in sensors. NOON states are also a source of entanglement with applications in quantum information protocols and in fundamental studies such as tests of Bells inequality [3].

It is thus of great interest to be able to produce NOON states, and various NOON state generation schemes have been suggested theoretically [4] and studied in experiments [5, 6, 7, 8, 9] and theoretically [10, 11]. The \( N = 1 \) and \( N = 2 \) NOON states may be generated by combining either a single photon and a vacuum state or two single-photon states on a 50 : 50 beam splitter, but this simple approach is not directly extendable to \( N > 2 \), and we shall thus mainly be concerned with generation of \( N = 3 \) NOON states in the present paper, even though the suggested protocol is, in principle, applicable for all \( N \). Mitchell, Lundeen, and Steinberg have generated NOON states with \( N = 3 \) from a pair of down converted photons and a local oscillator photon using certain polarization transforming components and post-selection [10]. In this experiment, however, the successful generation of the NOON state is witnessed by a destructive detection of the state. In the present paper we propose and analyze in detail a nondestructive generation protocol, which conditions the successful generation of the \( N \)-photon NOON state on the registration of \( N \) photo detection events in other field modes, and which uses as resource only linear optics and the output from two optical parametric oscillators (OPOs). The protocol does not rely on efficient photo detection. The analysis is carried out in terms of Wigner function formalism, and effects of finite detector efficiency and finite detector dead time are considered.

Conditional generation of nonclassical states occupying a single mode has been investigated both experimentally and theoretically [12, 13, 14, 15, 16, 17, 18, 19, 20]. With the correlated output from a single nondegenerate OPO it is, for instance, possible to generate \( n \)-photon Fock states of light in the signal beam conditioned on \( n \) photo detections in the idler (trigger) beam [16, 19, 20], and in principle the entanglement of a highly squeezed two-mode field from an OPO makes it possible to prepare any state in the signal beam that can be either measured as an eigenstate of a suitable observable of the idler beam or produced as the final state of a generalized measurement. The basic idea of the protocol proposed in the present paper is to mix the output from two OPOs and employ the entanglement to prepare a two-mode state in two of the output beams by detection of the desired output state in the remaining beams.

In Sec. II we explain the NOON state generation protocol in detail. In Sec. III we analyze the performance of the protocol quantitatively for pulsed OPO sources. We provide the fidelity of the generated states and the success probability. In Sec. IV we consider production of NOON states from continuous-wave OPO sources, and Sec. V concludes the paper.

II. EXPERIMENTAL SETUP FOR NOON STATE GENERATION

The experimental setup is illustrated in Fig. 1. Two pulses of two-mode squeezed states are generated by two identical OPOs via type II parametric down conversion. The field mode operators of the modes generated by the
first OPO are denoted \( \hat{a}_+ \) and \( \hat{a}_- \), respectively, while the field mode operators of the modes generated by the second OPO are denoted \( \hat{b}_+ \) and \( \hat{b}_- \), respectively. For definiteness, we assume that the plus modes are vertically polarized and that the minus modes are horizontally polarized. The modes are separated spatially by the first two polarizing beam splitters, and the third polarizing beam splitter combines the \( \hat{a}_- \) and \( \hat{b}_+ \) modes, which are subsequently subjected to the NOON state measurement proposed in [21] and illustrated for \( N = 3 \) in Fig. 1. The idea behind this measurement is to apply the highly nonlinear operator \( A_N = \hat{a}_+^N - \left( \hat{b}_+ e^{i\theta} \right)^N \) to the state. The result is only nonzero if either the \( \hat{a}_- \) mode or the \( \hat{b}_+ \) mode contains at least \( N \) photons. On the other hand, if the squeezing is sufficiently small, it is unlikely to have more than a total of \( N \) photons in the two trigger modes, and by conditioning on the successful application of \( A_N \), we select the pulses of the system where \( N \) photon pairs are generated in one OPO and zero photon pairs in the other. It is equally probable that the photons originate from the first OPO or from the second OPO, and, as we shall see in detail below, the result is that a NOON state is generated conditionally in the output modes \( \hat{a}_+ \) and \( \hat{b}_- \).

As stated in [21], \( A_N \) can be rewritten as a simple product of single photon annihilation operators

\[
\hat{a}_+^N - \left( \hat{b}_+ e^{i\theta} \right)^N = \prod_{n=1}^{N} \left( \hat{a}_- - \left( \hat{b}_+ e^{i\theta} \right) e^{i2\pi n/N} \right), \tag{2}
\]

and it is thus possible to implement \( A_N \) by means of beam splitters and photo detectors. We first consider odd values of \( N \). Beam splitters are used to divide the input into \( N \) distinct spatial modes labeled by \( n = 1, \ldots, N \). The beam splitter reflectivities are chosen to obtain the same expectation value of the intensity in each of the modes. The vertically polarized modes are then phase shifted by the factor \( e^{i2\pi n/N+\pi} \) relative to the horizontally polarized modes, i.e., \( \hat{b}_+ \to -\hat{b}_+ e^{i2\pi n/N} \), and finally polarizing beam splitters with principal planes oriented at 45° relative to the horizontal polarization transform \( \hat{a}_- \) and \( -\hat{b}_+ e^{i2\pi n/N} \) into \( \left( \hat{a}_- - \hat{b}_+ e^{i2\pi n/N} \right) / \sqrt{2} \) (the transmitted mode) and \( \left( \hat{a}_- + \hat{b}_+ e^{i2\pi n/N} \right) / \sqrt{2} \) (the reflected mode) [22]. The annihilation of a photon in each of the modes transmitted by the beam splitters witnesses the overall application of the operator \( A_N \). If one observes both reflected and transmitted modes simultaneously, one conditions on detection events in all the transmitted modes and no detection events in all the reflected modes. If detection events are instead observed in all the reflected modes and in none of the transmitted modes, an operator of the form [2] is also obtained, but \( \theta \) is effectively transformed into \( \theta + \pi \) due to the phase shift at the polarizing beam splitter, and the value of \( \phi \) of the generated NOON states is changed by \( N \pi \) (see below). The success probability is thus increased by a factor of two if both outcomes are accepted.

For even values of \( N \) a similar measurement scheme is applicable, but it is sufficient to divide the field into \( N/2 \) spatial modes initially, and in this case the NOON state generation is conditioned on detection events in both transmitted and reflected modes (see [21]).

### III. PERFORMANCE OF THE PROTOCOL

After this presentation of the basic idea and the physical setup we now consider the actual outcome of the detection process. For short pulse OPO output the dead time of the photo detectors may typically be longer than the pulse duration, and we shall thus assume that it is impossible to obtain more than a single detection event per detector per pulse, i.e., if the detector efficiency is unity, the detectors are only able to distinguish between vacuum and states different from the vacuum state. Such detectors are denoted on/off detectors, and they are discussed in detail in Ref. [23]. The finite dead time of the detectors is not severe to the measurement procedure described in [21] because the on/off detector model and the conventional photo detector model, represented by the annihilation operator, lead to identical signal states if the total number of photons in the idler modes is guaranteed to be less than or equal to the number of conditioning detection events, i.e., \( N \).

We analyze the performance of the setup using Gaussian Wigner function formalism [13, 19, 20], which is applicable because the squeezed states generated by the OPOs and the vacuum states coupled into the system via the beam splitters are all Gaussian. In general, the Wigner function of an \( n \)-mode Gaussian state with zero mean field amplitude takes the form

\[
W_V (x_1, p_1, \ldots, x_n, p_n) = \frac{1}{\pi^n \sqrt{\text{det}(V)}} e^{-y^T V^{-1} y}, \tag{3}
\]
where \( y = (x_1, p_1, \ldots, x_n, p_n)^T \) and \( V \) is the \( 2n \times 2n \) covariance matrix. If \( c_i \) denotes the field mode annihilation operator of mode \( i \), the elements of \( V \) are given in terms of the real and imaginary parts of the expectation values \( \langle c_i^* c_j \rangle \) and \( \langle c_i c_j \rangle \). Note that for a multi-mode Gaussian state we are free to include only the modes of interest because the partial trace operation over unobserved modes is equivalent to integration over the corresponding quadrature variables. A unit efficiency ‘on’ detection in mode \( i \) projects mode \( i \) on the subspace of Hilbert space that is orthogonal to the vacuum state, i.e., the Wigner function is multiplied by \( (1 - 2\pi W_0(x_i, p_i)) \), where \( W_0(x, p) = \exp(-x^2 - p^2)/\pi \) is the Wigner function of the vacuum state, the variables \( x_i \) and \( p_i \) are integrated out, and the state is renormalized. Since the Gaussian nature of a state is preserved under linear transformations, and since a detector with single-photon efficiency \( \eta \) is equivalent to a beam splitter with transmission \( \eta \) followed by a unit efficiency detector [23], effects of non-unit detector efficiency are easily included in the covariance matrix.

To calculate \( \langle c_i c_j \rangle \) and \( \langle c_i^* c_j \rangle \) explicitly we note that the state generated by the OPOs is [24]

\[
|\psi_i\rangle = (1 - r^2) \sum_{n=0}^{\infty} \sum_{m=0}^{\infty} r^{n+m} |n, n, m, m\rangle, \tag{4}
\]

where \( r \) is the squeezing parameter and the modes are listed in the order: \( \hat{a}_+, \hat{a}_-, \hat{b}_+, \hat{b}_- \). We assume that \( N \) is odd and consider the transmitted trigger modes (which we number from 1 to \( N \)), the \( \hat{a}_+ \) mode (mode \( N + 1 \)), and the \( \hat{b}_- \) mode (mode \( N + 2 \)). By expressing the field operators of the trigger modes (those observed by the unit efficiency detectors) in terms of \( \hat{a}_-, \hat{b}_+ \), and field operators representing vacuum states we find

\[
\langle c_j^* c_k \rangle = \langle \psi_i | \sqrt{\eta} (\hat{a}_- + e^{-2\pi i j/N} \hat{b}_+ e^{-i\theta}) \langle \psi_i \rangle
\]

\[
\langle c_j c_k \rangle = \langle \psi_i | \sqrt{\eta} (\hat{a}_+ - e^{-2\pi i k/N} \hat{b}_- e^{-i\theta}) \langle \psi_i \rangle
\]

\[
= \left( 1 + e^{2\pi i (k-j)/N} \right) \frac{\lambda}{2N}, \tag{5}
\]

where \( j \in \{1, 2, \ldots, N\} \), \( k \in \{1, 2, \ldots, N\} \), \( \lambda \equiv \eta r^2/(1 - r^2) \), and we allow of a constant phase shift \( \theta \) of \( \hat{b}_+ \) relative to \( \hat{a}_- \). Furthermore

\[
\langle \hat{c}_N^+ \hat{c}_{N+1} \rangle = \langle \psi_i | \hat{a}_+^* \hat{a}_+ | \psi_i \rangle = r^2/(1 - r^2), \tag{6}
\]

\[
\langle \hat{c}_N^+ \hat{c}_{N+2} \rangle = \langle \psi_i | \hat{a}_-^* \hat{b}_- | \psi_i \rangle = r^2/(1 - r^2), \tag{7}
\]

\[
\langle \hat{c}_k \hat{c}_{N+1} \rangle = \langle \psi_i | \sqrt{\eta} \sqrt{\frac{N}{2}} (\hat{a}_- - e^{2\pi i k/N + i\theta} \hat{b}_+) \hat{a}_+ | \psi_i \rangle
\]

\[
= \sqrt{\frac{\eta}{2N}} \frac{r}{1 - r^2}, \tag{8}
\]

\[
\langle \hat{c}_k \hat{c}_{N+2} \rangle = \langle \psi_i | \sqrt{\eta} \sqrt{\frac{N}{2}} (\hat{a}_- - e^{2\pi i k/N + i\theta} \hat{b}_+) \hat{b}_- | \psi_i \rangle
\]

\[
= -\sqrt{\frac{\eta}{2N}} \frac{r}{1 - r^2} e^{2\pi i k/N + i\theta}, \tag{9}
\]

and

\[
\langle \hat{c}_j \hat{c}_k \rangle = \langle \hat{c}_{N+1} \hat{c}_{N+1} \rangle = \langle \hat{c}_{N+2} \hat{c}_{N+2} \rangle = \langle \hat{c}_{N+1} \hat{c}_{N+2} \rangle = \langle \hat{c}_N^+ \hat{c}_{N+1} \rangle = \langle \hat{c}_N^+ \hat{c}_{N+2} \rangle = 0. \tag{10}
\]

For even values of \( N \) the factors \( \sqrt{\eta/(2N)} \) are replaced by \( \sqrt{\eta/N} \). Note that loss in the signal beam may be taken into account by performing the transformations \( \hat{a}_+ \rightarrow \sqrt{\eta_a} \hat{a}_+ \) and \( \hat{b}_- \rightarrow \sqrt{\eta_b} \hat{b}_- \) in the above expressions, where \( 1 - \eta_a \) is the loss.

The NOON state fidelity \( F_N \) of the signal state conditioned on \( N \) photo detection events in the transmitted trigger modes is

\[
F_N = \frac{4\pi^2}{PN} \int W_{\text{NOON}}(x_{N+1}, p_{N+1}, x_{N+2}, p_{N+2}) \left( \prod_{i=1}^{N} (1 - 2\pi W_0(x_i, p_i)) \right) \left( \prod_{i=1}^{N+2} dx_i dp_i \right), \tag{11}
\]

where \( W_{\text{NOON}} \) is the Wigner function of the NOON state [11], and

\[
P_N = \int \left( \prod_{i=1}^{N+2} (1 - 2\pi W_0(x_i, p_i)) \right) \left( \prod_{i=1}^{N+2} dx_i dp_i \right), \tag{12}
\]

is the success probability, i.e., the probability to obtain the conditioning detection events and produce the NOON state in a given pulse of the OPO system. We expand the product

\[
\prod_{i=1}^{N} (1 - 2\pi W_0(x_i, p_i)) = \sum_d \prod_{i=1}^{N} (-2\pi W_0(x_i, p_i))^{d_i}, \tag{13}
\]

where the sum is over all \( d \equiv (d_1, d_2, \ldots, d_N) \) with \( d_i \in \{0, 1\} \), and define the diagonal matrix \( J_d = \).
\text{diag}(d_1, d_1, d_2, d_2, \ldots, d_N, d_N) \text{ and the } n \times n \text{ identity matrix } I_n. \text{ Furthermore, we divide the covariance matrix into four parts } \begin{equation}
abla = \begin{bmatrix} V_{tt} & V_{ts} \\ V_{ts}^T & V_{ss} \end{bmatrix}, \end{equation}

where \( V_{tt} \) is the \( 2N \times 2N \) covariance matrix of the trigger modes, \( V_{ss} \) is the \( 4 \times 4 \) covariance matrix of the signal modes, while \( V_{ts} \) contains the correlations between the trigger and the signal modes, and we define the vector \( y_s = (x_{N+1}, p_{N+1}, x_{N+2}, p_{N+2})^T \) and the matrix

\begin{equation} U_d = V_{ss} - V_{ts}^T J_d (J_d V_{tt} J_d + I_{2N})^{-1} J_d V_{ts}. \end{equation}

This allows us to write Eqs. (11) and (12) in the following compact forms [25]

\begin{equation}
F_N = \frac{4\pi^2}{F_N} \sum_d \frac{(-2)^{\sum_{i=1}^{d} d_i}}{\sqrt{\det(I_{2N} + J_d V_{tt})}} \int W_{\text{NOON}}(y_s) W_{U_d}(y_s) dy_s,
\end{equation}

and

\begin{equation}
P_N = \sum_d \frac{(-2)^{\sum_{i=1}^{d} d_i}}{\sqrt{\det(I_{2N} + J_d V_{tt})}}.
\end{equation}

Since \( W_{\text{NOON}} \) is a product of a polynomial and a Gaussian, the integral in Eq. (13) may be evaluated analytically for \( N = 3 \) and \( \eta = 1 \) we find

\begin{equation}
F_{\eta=1}^{\eta=1} = (1 - r^2)^2 (2 - r^2)^2 (3 - 2r^2)(6 - 5r^2),
\end{equation}

where the optimal value \( \phi = N\theta + \pi + 2\pi n, n \in Z, \) is assumed. Expressions for \( P_N \) are given in Table I for \( N = 1, 2, 3, \) and \( 4, \) and \( F_N \) and \( P_N \) are plotted for \( N = 3 \) in Figs. 2 and 3, respectively. We observe that high probabilities are only found in the parameter regime, where the fidelity is low. If, for instance, we want a NOON state fidelity of at least 0.9, we choose \( r = 0.14, \) and \( \eta = 0.25, \) \( P_3 \) is of order \( 10^{-8}. \) With a repetition rate of order \( 10^6 \) s\(^{-1} \) (see [10]) one state is produced every second minute on average. The production rate is very dependent on detector efficiency, and if \( \eta \) is increased to unity, the rate is increased by approximately a factor of 60.

For odd values of \( N \) we may observe both reflected and transmitted trigger modes and condition on detection events in all the transmitted trigger modes and no detection events in all the reflected trigger modes, or, vice versa. In this case we also include the reflected trigger modes in the covariance matrix, which we now denote by \( V^+. \) By a similar analysis as above we obtain the success probability

\begin{equation}
P_N^+ = 2 \sum_d \frac{2^N (-2)^{\sum_{i=1}^{d} d_i}}{\sqrt{\det(I_{4N} + J_d^+ V_{tt}^+)}}.
\end{equation}

\text{TABLE I: Success probabilities calculated from Eqs. (17) and (18).} \lambda \equiv \eta^2/(1 - r^2).
model becomes equivalent to the photo detector model. In this case the density operator of the output state is obtained as

$$
\rho = M \sum_{p=0}^{\infty} \sum_{q=0}^{\infty} \langle p | (\hat{a}_-^N - (\hat{b}_+ e^{i\theta})^N) | q \rangle \langle q | p \rangle 
$$

$$
= \frac{(1 - r^2)^{N+2}}{2N!r^{2N}} \left( \sum_{n=N}^{\infty} \sum_{m=0}^{\infty} (r^2)^{n+m} \left( \frac{n!}{(n-N)!} \right) |n,m\rangle \langle n,m| 
- e^{-iN\theta} \sqrt{\frac{n!(m+N)!}{(n-N)!m!}} |n,m\rangle \langle n-N,m+N| 
+ \sum_{n=0}^{\infty} \sum_{m=N}^{\infty} (r^2)^{n+m} \left( \frac{m!}{(m-N)!} \right) |n,m\rangle \langle n,m| 
- e^{iN\theta} \sqrt{\frac{n!(m-N)!}{n!(m-N)!}} |n,m\rangle \langle n+N,m-N| \right),
$$

(21)

where $M$ is a normalization constant and the traces are over the $\hat{a}_-$ and $\hat{b}_+$ modes. This leads to the NOON state fidelity

$$
P_N^{\eta<1} = \langle \text{NOON} | \rho | \text{NOON} \rangle = (1 - r^2)^{N+2},
$$

(22)

where again $\phi = N\theta + \pi + 2\pi n$, $n \in Z$, is assumed. It is interesting to compare this result with the fidelity $(1 - r^2)^{N+1}$ obtained for production of $N$-photon states from a single two-mode squeezed state by conditioning on $N$ detection events in the idler beam and using detectors with very small efficiency. If a single-photon state is produced by this method and transformed into an $N = 1$ NOON state as explained in the Introduction, the NOON state fidelity is $F_{1,s} = (1 - r^2)^2$, and the success probability is $P_{1,s} = \lambda/\lambda + 1$. Choosing squeezing parameters such that $F_{1,s} = F_1$, we find that $P_1^+ = (4/3)P_1,s$ in the high fidelity limit. It is thus possible to achieve a higher success probability using the scheme with two OPOs, but the price to pay is a more technically involved setup, and NOON states with two different values of $\phi$ are produced. For $N = 2$ the present protocol and combination of two single-photon states on a 50:50 beam splitter, each produced conditionally from a single OPO, lead to identical fidelities and success probabilities. Finally we note that the photo detector model underestimates $F_N$ for $\eta > 0$ because $1 - (1 - \eta)^n = \eta \sum_{i=0}^{n-1} (1 - \eta)^i < n\eta$ for $n = 2, 3, \ldots$ while $1 - (1 - \eta)^n = n\eta$ for $n = 0, 1, \ldots$ The ‘wrong’ terms containing more than $N$ photons are given a too large weight. This is also what we observe in Fig. 2.

In the limit of small $r$ and for odd values of $N$ the success probability is given approximately by the simple expression

$$
P_N \approx \left( \frac{\eta}{2N} \right)^N \left( \frac{(\hat{a}_-^N - (\hat{b}_+ e^{i\theta})^N) |\psi\rangle \langle \psi|}{2N^N} \right).
$$

(23)

Again $\eta/(2N)$ must be replaced by $\eta/N$ to obtain $P_N$ for even values of $N$. The approximation to $P_3$ is shown in Fig. 3.

**IV. NOON STATES FROM CONTINUOUS-WAVE OPO SOURCES**

Our protocol is not limited to pulsed fields, and for completeness we now briefly consider NOON state generation from continuously driven OPOs. We assume $N = 3$. For continuous-wave fields each of the three detected trigger beams and the two signal beams are described by time dependent field operators $\hat{c}_i(t)$. The trigger detections take place in particular modes localized around the three detection times $t_{c1}$, $t_{c2}$, and $t_{c3}$, and we want to determine the NOON state fidelity of an output state occupying one temporal mode in each signal beam. Following the general multimode formalism in Refs. [12-20], the five relevant modes are specified by the mode functions $f_i(t)$, and the corresponding five single mode operators are $\hat{c}_i = \int f_i(t) \hat{c}_i(t) dt$. In general, we are free to choose the two output mode functions at will, and in the present case it is natural to choose the mode function which gives rise to the largest three-photon state fidelity when three-photon states are generated from a single type II continuous-wave OPO. Since we are mainly interested in the parameter region where the squeezing is small and the NOON state fidelity is large, we use the optimal three-photon state mode function derived for very low beam intensity in [20], i.e.,

$$
f_x(t) = f_5(t) = \sum_{k=1}^{3} c_k \sqrt{7/2} \exp(-\gamma|t - t_{ck}|/2),
$$

where
the coefficients $c_k$ are functions of the intervals between the detection times and $\gamma$ is the leakage rate of the OPO output mirror. We furthermore assume that the trigger mode functions are nonzero only in an infinitesimal time interval centered at the detection time, which is valid if the trigger detections take place on a time scale much shorter than $\gamma^{-1}$. Since we consider a low intensity continuous beam, and since we formally assume that the trigger detectors only register the light field in infinitesimal time intervals around the detection times, the annihilation operator detector model is perfectly valid in this case and detector dead time is insignificant.

We may now proceed as above and eliminate all the irrelevant modes from the analysis by writing down the Gaussian Wigner function of the five interesting modes. The only difference is that this time $(\hat{c}_i^\dagger \hat{c}_j)$ and $(\hat{c}_i \hat{c}_j)$ are expressed in terms of the two time correlation functions of the OPO output. Also, the operators applied to the Wigner function to take conditioning into account are different because the annihilation detector model is used. The reader is referred to Refs. [15, 20] for details.

The resulting fidelity is shown as a function of $\epsilon/\gamma$ in Fig. 4, where $\epsilon$ is the nonlinear gain in the OPO, and as a function of the temporal distance between the conditioning detection events in Fig. 5. As in the pulsed case the fidelity decreases when the degree of squeezing increases. The fidelity also decreases when the temporal distance between the conditioning detection events increases from zero, but it is permissible to have a small time interval between the trigger detection events. We note that the curves represent a lower limit to the theoretically achievable fidelity since a better fidelity may be obtained for another choice of output state mode functions.

V. CONCLUSION

In conclusion we have analyzed a method to generate path entangled NOON states from the output from two optical parametric oscillators. The method relies on the joint detection of photons in a number of trigger beams, and we presented a theoretical analysis of the role of detector efficiency and dead time for the fidelity of the states obtained and the success probability of the protocol. Our specific NOON state protocol applies for general photon numbers of the states, but in practice it is not realistic to go beyond the case of $N = 3$, studied here. This is due to the reduction of the fidelity due to unwanted contributions from higher number states, when the OPO output power gets too high, combined with the severe reduction of the probability to obtain the number of conditioning detection events needed when the OPO output power is too low. The $N = 3$ NOON states, which can be produced at 90% fidelity at the rate of one state produced every $10 - 100$ seconds, seem to be at the limit of realistic experiments of the proposed kind. Finally, we also determined the NOON state fidelity for continuous-wave fields, where the best NOON state occupies a pair of suitably selected temporal mode functions, and where we find high fidelities as long as the trigger events occur within a short time window compared to the lifetime of the OPO cavity field.

We presented this analysis for the production of optical NOON states, but we note that recent theoretical proposals and experiments with four wave mixing of matter waves [26], engineered quadratic interactions among trapped ions [27], and entanglement between field and atomic degrees of freedom [28, 29] bring promise for similar conditional generation of atomic and interspecies atom-field NOON states.

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[1] A. N. Boto, P. Kok, D. S. Abrams, S. L. Braunstein, C. P. Williams, and J. P. Dowling, Phys. Rev. Lett. 85, 2733 (2000).
[2] L. Pezzè and A. Smerzi, quant-ph/0508158.
[3] M. D’Angelo, A. Zavatta, V. Parigi, and M. Bellini, Phys. Rev. A 74, 052114.
[4] P. Kok, H. Lee, and J. P. Dowling, Phys. Rev. A 65, 052104 (2002).
[5] J. Fiurasek, Phys. Rev. A 65, 053818 (2002).
[6] H. F. Hofmann, Phys. Rev. A 70, 023812 (2004).
[7] P. Walther, J. Pan, M. Aspelmeyer, R. Ursin, S. Gasparoni, and A. Zeilinger, Nature (London) 429, 158 (2004).
[8] H. S. Eisenberg, J. F. Hodelin, G. Khoury, and D. Bouwmeester, Phys. Rev. Lett. 94, 090502 (2005).
[9] N. M. VanMeter, P. Lougovski, D. B. Uskov, K. Kieling, J. Eisert, and J. P. Dowling, quant-ph/0612154.
[10] M. W. Mitchell, J. S. Lundeen, and A. M. Steinberg, Nature (London) 429, 161 (2004).
[11] A. Ourjoumtsev, A. Dantan, R. Tualle-Brouri, and P. Grangier, Phys. Rev. Lett. 98, 030502 (2007).
[12] M. Dakna, T. Anhut, T. Opatrný, L. Knöll, and D.-G. Welsch, Phys. Rev. A 55, 3184 (1997).
[13] A. B. URen, C. Silberhorn, J. L. Ball, K. Banaszek, and I. A. Walmsley, Phys. Rev. A 72, 021802(R) (2005).
[14] A. Ourjoumtsev, R. Tualle-Brouri, J. Laurat, and P. Grangier, Science 312, 83 (2006).
[15] K. Mølmer, Phys. Rev. A 73, 063804 (2006).
[16] A. Ourjoumtsev, R. Tualle-Brouri, and P. Grangier, Phys. Rev. Lett. 96, 213601 (2006).
[17] J. S. Neergaard-Nielsen, B. M. Nielsen, C. Hettich, K. Mølmer, and E. S. Polzik, Phys. Rev. Lett. 97, 083604 (2006).
[18] K. Wakui, H. Takahashi, A. Furusawa, and M. Sasaki, Opt. Express 15, 3568 (2007).
[19] A. E. B. Nielsen and K. Mølmer, Phys. Rev. A 75, 023806 (2007).
[20] A. E. B. Nielsen and K. Mølmer, Phys. Rev. A 75, 043801 (2007).
[21] F. W. Sun, Z. Y. Ou, and G. C. Guo, Phys. Rev. A 73, 023808 (2006).
[22] P. Hariharan, N. Brown, and B. C. Sanders, J. Mod. Opt. 40, 1573 (1993).
[23] P. P. Rohde and T. C. Ralph, J. Mod. Opt. 53, 1589 (2006).
[24] A. Ekert and P. Knight, Am. J. Phys. 63, 415 (1995).
[25] J. Eisert and M. B. Plenio, Int. J. Quantum Inf. 1, 479 (2003).
[26] G. K. Campbell, J. Mun, M. Boyd, E. W. Streed, W. Ketterle, and D. E. Pritchard, Phys. Rev. Lett. 96, 020406 (2006).
[27] D. Porras and J. I. Cirac, Phys. Rev. Lett. 93, 263602 (2004).
[28] B. B. Blinov, D. L. Moehring, L. M. Duan, and C. Monroe, Nature 428, 153 (2004).
[29] J. Volz, M. Weber, D. Schlenk, W. Rosenfeld, J. Vrana, K. Saucke, C. Kurtsiefer, and H. Weinfurter, Phys. Rev. Lett. 96, 030404 (2006).