Modelling material dependent parameters of layer type straight coils for fast transient pulses

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Abstract. The behavior of coils in case of fast transient pulses is different from that they show at low frequencies. If the dimensions of the coil, i.e. mainly the length of the winding is much shorter than the wavelength of the signal on the coil, a lumped element model can be effectively used taking the capacitances of windings into consideration. In this study a different type of straight, layered coil have been investigated in order to determine parameters of a lumped circuit model of the windings. The frequency dependent parameters are modeled by analytical and finite element calculations and the results are compared to the results of measurements on coils. The finite element method can improve the accuracy of parameter estimation.

1. Introduction
Modelling of coils in case of transient pulses has high importance because transient overvoltages can cause non-uniform voltage distribution between turns, which overloads the turn to turn insulation in electrical equipments e.g. transformers. For measuring high frequency currents and current pulses of partial discharges occurring in insulations simple air-core (Rogowski) coils are used as a current transducer, whose bandwidth is limited by the capacitance of windings [1, 2]. The behavior of coils in case of fast transient pulses is different from their low frequency behavior. While at low frequencies the coil can be modeled as an inductance and a resistance (simple L-R model), at higher frequencies this model have to be improved. If the dimensions of the coil, i.e. mainly the length of the wire is much shorter than the wavelength of the signal on the coil, a lumped element model can also be used to model the coil, however this simple model have to be complemented by a lumped capacitor, which is the result of the stray capacitances. If the wavelength of the signal is shorter than the length of the coil, distributed model has to be used to investigate the behavior of the coil. Treating the lumped parameter model seems much easier than the distributed one, nevertheless the calculation of model parameters is suffer from more difficulty. There are several analytical forms to calculate the inductance of straight coils but the results provided by these formulas are strongly dependent on the shape of the coil and the skin-effect cannot be negligible at the calculation of the inductance. Similarly, the calculation of stray capacitances by analytical method shows high variations in the result depending upon the used formulas. In this study, a single- and multi-layer straight coils are modeled by analytical and finite element methods (FEM) and the results are compared to the results of the measurements.
2. Model parameters
The coil is modeled with a lumped parameter model, which consists of a parallel inductance and capacitance. The resistance of the coil is not calculated because it has no effect on the values of either the capacitance or the inductance. The tested coils are straight single- and multi-layered coil. The diameter ($D$) of the coil is 22 mm, the height ($h$) is 17 mm. The insulated diameter ($d_{ins}$) of the wire is 0.65 mm and the diameter of the conductor ($d$) is 0.63 mm.

2.1. Turn capacitances
In the analytical approach the turn capacitance of two consecutive turns is modeled by the capacitance of two parallel wires far from ground potential [1]. The turn capacitance can be expressed:

$$C_t = \epsilon \cdot \pi \frac{D \cdot \pi}{\ln \frac{\sqrt{d_{ins}^2 - d^2 + (d_{ins} - d)^2}}{\sqrt{d_{ins}^2 - d^2 - (d_{ins} - d)^2}}},$$  

(1)

where $d_{ins}$ is the insulated diameter of the wire, $d$ is the diameter of the conductor, and $D$ is the mean diameter of a given turn. In this equation (1) the fringe effects and related stray capacitances are not taken into consideration. Practically, these effects can be estimated by the increased $d_{ins}$ distance [3] [4]. More general and accurate solution can be achieved by a finite element technique (FEM) from an electrostatic field simulation. The simulation has been made by Agros2D [5]. The turn and layer capacitances can be calculated from the sum of the electrostatic energy:

$$W_m = \frac{1}{2} \sum C_i \cdot V_i^2$$

(2)

To obtain the turn capacitances $C_i$, one calculation is enough since the difference between the turn capacitances along a layer can be negligible. The inner $i^{th}$ turn potential is set to $V_i = 1$ V, all the others to 0 V, and the turn capacitance is obtained from (2). The field distribution of the simulation are in Figure 1.

2.2. Layer capacitances
To obtain the layer capacitances, the most common technique is used: the layers are replaced to two fictitious parallel plates [3] [4]:

$$C_l = \epsilon \cdot l \cdot \frac{h + 2 \cdot (d_{ins} - d)}{d_{ins} - d},$$

(3)

where $h$ is the height of the layer, $d_{ins} - d$ is the assumed distance between the two parallel layers and $l$ is the length of the coil. The height of the winding is increased considering the fringe effects [3].

2.3. Equivalent capacitance
The equivalent capacitance is calculated in the following way [1, 4, 6]:

$$C_{eq} = \frac{C_t}{N_l \cdot N_t^2} \cdot (N_t - 1) + \frac{4 \cdot (N_l - 1)}{N_t^2} \cdot C_l,$$

(4)

where $C_{eq}$ is the equivalent capacitance of the winding, $N_l$ means the layer number, $N_t$ is the turn number.
Figure 1. Finite element models of single-layered test coil, the a) and b) images for the turn capacitance calculation, and modeling of the inductance is shown in the c) image.

2.4. Inductance calculation
The inductance of the coil at low frequencies is calculated by the following analytical formula:

\[ L = F^2 \cdot N \cdot R, \]  

(5)

where \( N \) is the turn number, \( R \) is the mean radius of the coil and \( F \) is an empirical correction factor which depends on the ratio of the coil height and mean diameter [7] [8].

3. Measurement of parameters of coils
Measuring of coil parameters have been based on the determination of the resonance frequency of coils. From the resonance frequency the value of lumped capacitance can be calculated if the inductance is the coil is known. The inductance of the coil is measured at a relative low frequency (75 kHz), when the effect of capacitances can be negligible. The measurements have been carried out by a HP4285A precision impedance analyzer. Table 1. shows the results of measurements and the calculated capacitance values.

4. Discussion of the results
The Table 2. contains the result of calculations. Analytical and FEM calculations of the inductance show a good agreement with measured results; however, the difference of analytical calculations from the measurement is higher. The results of turn capacitance calculations are lower than the measured value in both cases, but the FEM calculation provides better results. The explanation is that the turn to turn capacitance in analytical calculations is simplified to the capacitance of two parallel wires [1], therefore this approach neglects the stray capacitances to the farther turns. Comparison of the results of FEM calculations to measurements the values
are also lower. However, this accuracy can be enough for designing in most common cases (e.g., Rogowski-coil current transducer), the accuracy can be improved if not only one turn capacitance is calculated in a given layer and the layer to layer capacitances are also calculated.

Table 1. The measured inductance and resonance frequency parameters and the calculated capacitances of the test coils.

| Number of layers | Inductance [mH] | Resonance frequency [MHz] | Capacitance [pF] |
|------------------|-----------------|---------------------------|------------------|
| 1                | 11.75           | 30                        | 2                |
| 2                | 99.5            | 1.75                      | 83               |
| 3                | 169             | 1.572                     | 61               |
| 4                | 255             | 1.335                     | 56               |

Table 2. The calculated parameters of the test coils with analytical and finite element method.

| Number of layers | $L_{eq}$ FEM [mH] | $L_{eq}$ Analytical [mH] | $C_t$ FEM [pF] | $C_t$ Analytical [pF] | $C_{eq}$ FEM [pF] | $C_{eq}$ Analytical [pF] |
|------------------|-------------------|--------------------------|----------------|------------------------|-------------------|--------------------------|
| 1                | 11.3              | 9.96                     | 41.3           | 15.3                   | 1.65              | 0.64                     |
| 2                | 98.3              | 81.2                     | 41.3           | 15.3                   | 64.3              | 17.3                     |
| 3                | 166.9             | 165.8                    | 41.3           | 15.3                   | 57.0              | 15.3                     |
| 4                | 258.4             | 246.6                    | 41.3           | 15.3                   | 48.1              | 12.9                     |

5. Conclusion
The high frequency behavior of coils is important in case of fast transient pulses. In this study, straight, single- and multi-layer, air-core coils were modeled by analytical and FEM calculations and the results were compared to measurements on test coils. The results shows the simple analytical formulas do not provide correct results especially at the capacitance calculations. The results of the FEM model are much closer to the measured ones; however, if more precise capacitance values are required for a given application. The improvement of the model is necessary.

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