Flavor ordering of elliptic flows at high transverse momentum

Zi-wei Lin and C.M. Ko
Cyclotron Institute and Physics Department, Texas A&M University, College Station, Texas 77843-3366

Based on the quark coalescence model for the parton-to-hadron phase transition in ultra-relativistic heavy ion collisions, we relate the elliptic flow \( (v_2) \) of high \( p_T \) hadrons to that of high \( p_T \) quarks. For high \( p_T \) hadrons produced from an isospin symmetric and quark-antiquark symmetric parton matter, magnitudes of their elliptic flows follow a flavor ordering as \( (v_{2,p} = v_{2,n}) > (v_{2,\Lambda} = v_{2,\Sigma}) > v_{2,K} > v_{2,\Xi} > (v_{2,\phi} = v_{2,\Omega}) \) if strange quarks have a smaller elliptic flow than light quarks.

The elliptic flows of high \( p_T \) hadrons further follow a simple quark counting rule if strange quarks and light quarks have same high \( p_T \) spectrum and coalescence probability.

PACS numbers: 25.75.Ld, 25.75.-q, 24.10.Lx

Elliptic flow in heavy ion collisions is a measure of the azimuthal asymmetry of particle momentum distributions in the plane perpendicular to the beam direction. It results from the initial spatial asymmetry in the transverse plane in non-central collisions and is thus sensitive to the properties of the dense matter formed during the initial stage of heavy ion collisions \cite{1,5}. There have been extensive experimental \cite{1,2,3,4,11} and theoretical \cite{3,4,5,6,7} studies of elliptic flow in heavy ion collisions at various energies. For heavy ion collisions at the Relativistic Heavy Ion Collider (RHIC), the elliptic flow has been measured as functions of the centrality of collisions \cite{11,12,13,14}, as well as the particle transverse momentum \cite{11,12,13} and pseudo-rapidity \cite{13}. Theoretical studies indicate that these experimental results provide not only information on the equation of state of nuclear matter at high density and temperature \cite{6,5} but also on the scattering cross section of partons produced in the collisions \cite{10,13}.

The elliptic flow has also been measured at RHIC for different hadron species, such as pions, kaons, nucleons and \( \Lambda \) \cite{20,21}. The experimental data show that at low \( p_T \) the elliptic flow of heavier particles is smaller than that of lighter particles. In the hydrodynamical model, this mass ordering of elliptic flow at low \( p_T \) is attributed to the mass dependence of radial flow \cite{6}. For high \( p_T \) hadrons, we expect the flavor dependence to be different from that at low \( p_T \), since high transverse momentum hadrons originate from hard processes while low transverse momentum particles are mostly produced from soft non-perturbative processes and are much closer to thermal equilibrium. Indeed, the observed saturation of hadron elliptic flow at \( p_T > 2 \) GeV/c \cite{20,21} contradicts the predictions from the hydrodynamical model \cite{6}, but is roughly consistent with the results expected from a large parton transport opacity \cite{17} or energy loss \cite{23} in the partonic matter.

In this Letter, we shall study the flavor dependence of the elliptic flow of high \( p_T \) hadrons in ultra-relativistic heavy ion collisions, using a quark coalescence model to describe the phase transition from the partonic matter to the hadronic matter. In this model, the elliptic flow of high \( p_T \) hadrons can be expressed in terms of the elliptic flow of high \( p_T \) quarks. As a result, several relations between the elliptic flow of hadrons of different flavors are obtained. We further discuss some special cases where these relations become more transparent. Throughout this study, we limit the discussions to hadrons made of \( u, d, s \) quarks and antiquarks.

In the quark coalescence model, one assumes that quarks and antiquarks are the effective degrees of freedom in the parton phase near the phase transition, and they combine to form hadrons according to the valence quark structure of hadrons. A meson is thus formed from the coalescence of a quark and an antiquark, while a baryon is due to the coalescence of three quarks. The idea of quark coalescence has been used in models such as the ALCOR \cite{24} or MICOR model \cite{25} to describe hadron abundance and the AMPT model with string melting \cite{19} to describe the elliptic flow at RHIC.

In ultra-relativistic heavy ion collisions, high \( p_T \) partons are produced from initial hard scatterings between nucleons for which the perturbative QCD is applicable. From the leading-order calculation, the parton transverse momentum spectrum from the subprocess of a two-parton hard scattering is given by:

\[
\frac{d\sigma}{dt} \propto \frac{1}{p_T^4}.
\]

The high \( p_T \) parton spectrum thus follows an inverse power law modulo the corrections from the parton distribution function in the nucleus and higher-order effects. On the other hand, low \( p_T \) partons, that are produced from initial soft processes and dominate the dynamics of partonic evolution in heavy ion collisions at RHIC, typically have an exponential spectrum close to a thermal distribution. The parton \( p_T \) spectrum can thus be represented by an exponential function below a certain momentum scale \( p_0 \) and an inverse power law above \( p_0 \).

Let us consider via the quark coalescence model the formation of a high \( p_T \) meson with transverse momentum \( \vec{p}_T \) from one parton with \( \vec{p}_H \) and one parton with zero \( p_T \), or from two partons with equal high \( p_T \) of \( \vec{p}_H/2 \). The ratio of the probabilities for forming a high \( p_T \) meson in these two cases is then proportional to \((e^{p_T/4p_0})^n\), where \( n \) represents the exponent of the inverse power law.
for final high \( p_T \) partons. Since this ratio is much greater than one for \( p_T \gg p_0 \), a high \( p_T \) meson is dominantly formed from the coalescence of one high \( p_T \) parton and one soft parton. Similarly, a high \( p_T \) baryon is mainly formed from the coalescence of one high \( p_T \) parton and two soft partons.

The transverse momentum distribution \( F_H(p_T) = dN/(dp_T dp_T) \) of initial high \( p_T \) mesons formed after the phase transition can thus be expressed in terms of that of final high \( p_T \) partons as

\[
F_H(p_T) = F_i(p_T) c_j F_j(p_T) c_i, \tag{2}
\]

where \( i \) and \( j \) denote the flavor of the valence quark and antiquark of meson \( H \). The coefficient \( c_i \) represents the capture probability for a soft parton \( i \) by a high \( p_T \) parton to form a high \( p_T \) meson; it is thus related to the density of soft quarks near the phase transition.

For high \( p_T \) baryons or antibaryons, one can write down a similar expression, involving the product of two \( c_i \), for their transverse momentum distribution.

The elliptic flow is generated during the early stage of heavy ion collisions when the pressure gradient and the spatial azimuthal asymmetry are the largest. In transport model studies, it has been found that the elliptic flow in heavy ion collisions at RHIC develops mostly in the initial partonic phase, with later hadronic interactions having negligible effects on its final value. We expect that the elliptic flow of high \( p_T \) hadrons are even less affected by hadronic interactions, as the proper formation time from a high \( p_T \) parton is considered

\[
T_{\text{formation}} \gg T_{\text{hadronization}}.
\]

The transverse momentum distribution \( F_H(p_T) \) of initial high \( p_T \) mesons is mostly in the initial partonic phase, with later hadronic interactions having negligible effects on its final value at high \( p_T \). We can thus use Eq. (2) to relate the final elliptic flow of high \( p_T \) hadrons to that of high \( p_T \) partons. For mesons, we have

\[
v_{2,H}(p_T) = \frac{1}{2} \int F_H(p_T) d\phi' = \frac{v_{2,i}(p_T)f_i(p_T)c_j + v_{2,j}(p_T)f_j(p_T)c_i}{f_i(p_T)c_j + f_j(p_T)c_i}, \tag{3}
\]

where \( \phi' \) is the azimuthal angle with respect to the reaction plane, and \( f(p_T) = dN/(2\pi p_T dp_T) \) denotes the transverse momentum distribution after averaging over the azimuthal angle. In the following, we omit the label \( p_T \) in the variables \( v_2(p_T) \) and \( f(p_T) \) but keep in mind that they are evaluated at a given high \( p_T \).

For SU(3) baryons consisting of \( u, d, s \) quarks and antiquarks, their \( v_2 \) values at high \( p_T \) are then given by:

\[
v_{2,\pi^+} = \frac{v_{2,u} f_u c_d + v_{2,d} f_d c_u}{f_u c_d + f_d c_u}, \quad v_{2,K^+} = \frac{v_{2,u} f_u c_s + v_{2,s} f_s c_u}{f_u c_s + f_s c_u}, \quad v_{2,\phi} = \frac{v_{2,s} f_s c_u + v_{2,s} f_s c_s}{f_s c_u + f_s c_s}, \quad v_{2,p} = \frac{v_{2,u} f_u c_d + v_{2,d} f_d c_u}{f_u c_d + f_d c_u/2},
\]

where \( v_{2,u} = v_{2,\pi^+}, \quad v_{2,\pi^0} = v_{2,\pi^0} = \frac{v_{2,u} f_u c_d + v_{2,d} f_d c_u + v_{2,s} f_s c_u}{f_u c_d + f_d c_u + f_s c_u}, \quad v_{2,\Xi} = \frac{v_{2,u} f_u c_d/2 + v_{2,s} f_s c_u}{f_u c_d/2 + f_s c_u}, \quad v_{2,\Omega} = v_{2,s}, \tag{4}
\]

with similar expressions for isospin partners and antiparticles.

The above relations become simpler if the quantities \( v_{2,i}, f_i, \) and \( c_i \) are independent of isospin and are also the same for strange and antistrange quarks, i.e., \( u = d = q, \quad u = d = \bar{q}, \) and \( s = \bar{s} \). These conditions are approximately satisfied in heavy ion collisions at RHIC as the \( \pi^+ / \pi^- \) ratio is almost one around central rapidity. In this isospin symmetric and strangantistrange symmetric limit, the \( v_2 \) values for hadrons at a given high \( p_T \) are given by:

\[
v_{2,N} = v_{2,q}, \quad v_{2,K} = v_{2,q}, \quad v_{2,\phi} = v_{2,\Omega} = v_{2,s}, \quad v_{2,\pi^+} = v_{2,\pi^0} = v_{2,\pi^-} = \frac{v_{2,q} + v_{2,q} v_{2,\bar{q}}}{1 + r_{\bar{q}}}, \tag{5}
\]

with \( N \) denoting a nucleon. In the above, the \( p_T \)-dependent variables \( r_{\bar{q}} \) and \( r_s \) are defined as

\[
r_{\bar{q}} = \frac{f_{\bar{q}} c_q}{f_{\bar{q}} c_q}, \quad r_s = \frac{f_s c_q}{f_s c_q}.
\]

From Eq. (3), we see that, e.g., \( r_{\bar{q}} \) can be determined from the \( v_2 \) of high \( p_T \) pion, proton and antiproton, while \( r_s \) can be determined from the \( v_2 \) of high \( p_T \) proton, kaon, and \( \phi \) meson.

Since the \( K^+/K^- \) ratio is close to one and \( \bar{p}/p \) ratio is about 0.7 in heavy ion collisions at RHIC, and they should be closer to one in heavy ion collisions at LHC, we consider the case where the variables \( v_{2,i}, f_i, \) and \( c_i \) are the same for quarks and antiquarks, i.e., \( q = \bar{q} \) and thus \( r_{\bar{q}} = 1 \). For such a quark-antiquark symmetric partonic matter, Eq. (6) simplifies to:

\[
v_{2,\pi} = v_{2,N} = v_{2,q}, \quad v_{2,\phi} = v_{2,\Omega} = v_{2,s}, \tag{7}
\]

\[
v_{2,K} = v_{2,q} + v_{2,q} v_{2,\bar{q}} \quad v_{2,\Lambda} = v_{2,\Sigma} = \frac{v_{2,q} + v_{2,q} v_{2,\bar{q}}}{1 + r_{\bar{q}}}, \quad v_{2,\Xi} = v_{2,q} + 2v_{2,q} v_{2,\bar{q}} \quad v_{2,\Omega} = \frac{v_{2,q} + 2v_{2,q} v_{2,\bar{q}}}{1 + 2r_{\bar{q}}}, \tag{8}
\]

Eliminating the variable \( r_{\bar{q}} \) in Eq. (8), we obtain two relations involving the \( v_2 \) values of four different hadron species, and they can be any two of the following three relations:

\[
\begin{align*}
v_{2,\pi^+} &= \frac{v_{2,u} f_u c_d + v_{2,d} f_d c_u + v_{2,s} f_s c_u}{f_u c_d + f_d c_u + f_s c_u}, \\
v_{2,\pi^0} &= \frac{v_{2,u} f_u c_d/2 + v_{2,s} f_s c_u}{f_u c_d/2 + f_s c_u}, \\
v_{2,\Omega} &= v_{2,s},
\end{align*}
\]
values of hadrons at high $p_T$ and $v_s$ regardless of the value of $v_s$ determined by the relative magnitude of the elliptic flow of $p_T$ flows of high $p_T$ hadrons. It is thus possible that the elliptic flow of strange quarks is greater than that of light quarks in a partonic matter. In the parton transport model, this would imply that high $p_T$ strange quarks will decay to stable hadrons at different $p_T$. The relations derived in Eqs. (7) and (10) show that the dependence of the elliptic flows of high $p_T$ hadrons on their flavor composition is determined by the relative magnitude of the elliptic flow of high $p_T$ strange quarks to that of high $p_T$ light quarks. If strange quarks have the same elliptic flow as light quarks at high $p_T$, i.e., $v_{2,s} = v_{2,q}$, then $v_{2,H} = v_{2,q}$ for all SU$(3)$ hadrons regardless of the value of $r_s$. This is true even if the $p_T$ spectrum for strange quarks is different from that for light quarks. We note that in the present study we are only concerned with the relative magnitude of the elliptic flow of different hadrons at high $p_T$, not their absolute magnitudes or shape.

On the other hand, fast moving heavy quarks have been shown to suffer less energy loss in a thermalized parton plasma than fast moving light quarks. In the parton transport model, this would imply that high $p_T$ strange and heavier quarks may have smaller scattering cross sections than light quarks in a partonic matter. It is thus possible that the elliptic flow of strange quarks is smaller than that of light quarks, i.e., $v_{2,s} < v_{2,q}$. In this case, we obtain from Eqs. (7) and (10) the following flavor ordering of the $v_2$ values for hadrons at a given high $p_T$:

$$(v_{2,s}=v_{2,q}) > (v_{2,Λ}=v_{2,Ξ}) > v_{2,K} > v_{2,Ξ} > (v_{2,φ}=v_{2,Ω}). \quad (11)$$

In Fig. 1, we illustrate the flavor ordering of hadron elliptic flows at high $p_T$ for the case of $v_{2,s} < v_{2,q}$. The spacings between different curves correspond to the case of $r_s = 1$ and thus follow the quark counting relations of Eqs. (9) and (10). We note that the vertical scale for $v_2$ is in arbitrary units, and the shape of $v_2$ as a function of $p_T$ is also arbitrary. The scale $p_0$ denotes the typical transverse momentum above which the $p_T$ spectra of final partons changes from soft to hard, and its value should probably be a few GeV/$c$. All curves are shown well above $p_0$, reflecting the fact that the relations derived in the present study only apply to hadron elliptic flows at high $p_T$. In general, $r_s$ can take any finite positive value, but the flavor ordering of hadron elliptic flows remains similar to that shown in Fig. 1 as long as $v_{2,s} < v_{2,q}$. However, the spacings between different curves can be different, while still being constraint by the two relations given in Eqs. (9) and (10). Since the $v_2$ magnitudes of hadrons follow the mass ordering at low $p_T$ and the flavor ordering at high $p_T$, the curve for kaon $v_2(p_T)$, which is above those for proton and $Λ$ at low $p_T$, will cross and become lower than the latter two curves as $p_T$ increases. A similar relation exists between the curve for $φ$ meson $v_2(p_T)$ and those for $Λ$ and $Ξ$.

We have not included the effects of resonance decays in this study. The relations shown in Eq. (11) can be extended to resonances such as $η, ρ, ω, K^*$, and $Δ$. These resonances at high $p_T$ will decay to stable hadrons at different transverse momenta, thus complicating the relations we have thus derived for hadrons which are directly formed from the quark coalescence. Since the transverse momentum of a decay product is usually small compared to that of the parent hadron, and the inverse power law spectrum shows a rapid decrease with $p_T$, we expect that the resonance contribution to hadron elliptic flow at high $p_T$ is small compared to the contribution from directly formed hadrons.

In summary, using a parton coalescence model to describe the formation of hadrons from the initial partonic matter in ultra-relativistic heavy ion collisions, we have
studied the dependence of the elliptic flows of hadrons at high $p_T$ on their flavor composition. Since the elliptic flow is generated mostly in the early partonic phase, and high $p_T$ hadrons are mainly formed from the coalescence of a high $p_T$ quark or antiquark produced from the initial hard processes and low $p_T$ quarks or antiquarks from the soft processes, the magnitudes of the hadron elliptic flows at high $p_T$ are determined by that of high $p_T$ quarks. The relations between hadron and parton elliptic flows at high $p_T$ also depend on the final quark spectrum at high $p_T$ ($f_i(p_T)$) and the capture probability of a soft quark ($c_i$) by a high $p_T$ quark to form a high $p_T$ hadron. If strange quarks have a smaller elliptic flow than the light quarks, then the quark coalescence model leads to the flavor ordering in the elliptic flows of the hadrons formed from an isospin symmetric and quark-antiquark symmetric partonic matter, i.e., $(v_{2,\pi} = v_{2,\Lambda}) > (v_{2,A} = v_{2,\Sigma}) > (v_{2,K} > (v_{2,\phi} = v_{2,\Omega})$.

We have also obtained two relations which are independent of $f_i(p_T)$ and $c_i$ and involve the elliptic flows of four hadron species at high $p_T$. In the special case that $f_i(p_T)$ and $c_i$ are the same for strange quarks and light quarks, values of the elliptic flows for high $p_T$ hadrons of different flavors are found to be close to the quark counting rule. It will be very interesting to test these predictions in current and future heavy ion collisions. Such studies will provide valuable information on whether a partonic matter is formed in the collisions and the subsequent formation of hadrons can be described by the quark coalescence model.

We appreciate useful discussions with H. Huang, P. Sorensen, and A. Tai. This paper is based on work supported by the U.S. National Science Foundation under Grant Nos. PHY-9870038 and PHY-0098805, the Welch Foundation under Grant No. A-1358, and the Texas Advanced Research Program under Grant No. FY99-010366-0081.

[1] J.Y. Ollitrault, Phys. Rev. D 46, 229 (1992).
[2] H. Sorge, Phys. Rev. Lett. 78, 2309 (1997) arXiv:nucl-th/9610026.
[3] H. Sorge, Phys. Lett. B 402, 251 (1997) arXiv:nucl-th/9701012; Phys. Rev. Lett. 82, 2048 (1999) arXiv:nucl-th/9812057.
[4] P. Danielewicz, R. A. Lacey, P. B. Gossiaux, C. Pinkenburg, P. Chung, J. M. Alexander and R. L. McGrath, Phys. Rev. Lett. 81, 2438 (1998) arXiv:nucl-th/9803047.
[5] D. Teaney, J. Lauriat and E. V. Shuryak, Phys. Rev. Lett. 86, 4783 (2001) arXiv:nucl-th/0011058; arXiv:nucl-th/0110037.
[6] P. F. Kolb, P. Huovinen, U. W. Heinz and H. Heiselberg, Phys. Lett. B 500, 232 (2001) arXiv:hep-ph/0012137.
[7] P. Huovinen, P. F. Kolb, U. Heinz, P. V. Ruuskunen and S. A. Voloshin, Phys. Lett. B 503, 58 (2001) arXiv:hep-ph/0101139.
[8] P. F. Kolb, U. W. Heinz, P. Huovinen, K. J. Eskola and K. Tuominen, Nucl. Phys. A 696, 197 (2001) arXiv:hep-ph/0103234.
[9] J. Barrette et al. [E877 Collaboration], Phys. Rev. Lett. 73, 2532 (1994) hep-ex/9405003.
[10] H. Appelshauser et al. [NA49 Collaboration], Phys. Rev. Lett. 80, 4136 (1998) nucl-ex/9711001.
[11] K. H. Ackermann et al. [STAR Collaboration], Phys. Rev. Lett. 86, 402 (2001) nucl-ex/0000001.
[12] R. A. Lacey [PHENIX Collaboration], Nucl. Phys. A 698, 559 (2002) arXiv:nucl-ex/0105001.
[13] B. B. Back et al. [PHOBOS Collaboration], arXiv:nucl-ex/0205021.
[14] C. Adler et al. [STAR Collaboration], arXiv:nucl-ex/0206001.
[15] V. Zheng, C. M. Ko, B. A. Li and B. Zhang, Phys. Rev. Lett. 83, 2534 (1999) nucl-th/9906075.
[16] B. Zhang, M. Gyulassy and C. M. Ko, Phys. Lett. B 455, 45 (1999) arXiv:nucl-th/9902014.
[17] D. Molnar and M. Gyulassy, Nucl. Phys. A 698, 379 (2002) arXiv:nucl-th/0104018; Nucl. Phys. A 697, 495 (2002) Erratum-ibid. A 703, 893 (2002) arXiv:nucl-th/0104073.
[18] E. E. Zabrodin, C. Fuchs, L. V. Bravina and A. Faessler, Phys. Lett. B 508, 184 (2001) nucl-th/0104054.
[19] Z. W. Lin and C. M. Ko, Phys. Rev. C 65, 034904 (2002) arXiv:nucl-th/0108039.
[20] C. Adler et al. [STAR Collaboration], Phys. Rev. Lett. 87, 182301 (2001) arXiv:nucl-ex/0107003.
[21] C. Adler et al. [STAR Collaboration], Phys. Rev. Lett. 87, 2089 (2002) arXiv:hep-ex/0205072.
[22] C. Adler et al. [STAR Collaboration], arXiv:nucl-ex/0206006.
[23] M. Gyulassy, I. Vitev and X. N. Wang, Phys. Rev. Lett. 86, 2537 (2001) nucl-th/0112092.
[24] T. S. Biro, P. Levai and J. Zimanyi, Phys. Lett. B 347, 6 (1995).
[25] P. Csizmadia, P. Levai, S. E. Vance, T. S. Biro, M. Gyulassy and J. Zimanyi, J. Phys. G 25, 321 (1999) arXiv:hep-ph/9809456.
[26] We caution that Eq. (2) cannot be used for observables such as the final hadron $m_T$ spectra, as they are significantly modified by interactions in the hadronic phase.
[27] K. Adcox et al. [PHENIX Collaboration], Phys. Rev. Lett. 88, 242301 (2002) arXiv:nucl-ex/0112006.
[28] B. B. Back [PHOBOS Collaboration], Phys. Rev. Lett. 87, 102301 (2001) arXiv:hep-ex/0104032; arXiv:nucl-ex/0206012.
[29] We note that both the isospin symmetry and quark-antiquark symmetry will break down for quarks with $p_T$ above the scale of the order of $\sqrt{s}/6$, with $\sqrt{s}$ being the center-of-mass energy, since the valence quark distribution function of nucleons starts to dominate the flavor of the produced jets.
[30] E. Braaten and M. H. Thoma, Phys. Rev. D 44, 2625 (1991).
[31] M. H. Thoma and M. Gyulassy, Nucl. Phys. B 351, 491 (1991).
[32] Y. L. Dokshitzer and D. E. Kharzeev, Phys. Lett. B 519, 199 (2001) arXiv:hep-ph/0106202.