Quantum information processing with single photons and atomic ensembles in microwave coplanar waveguide resonators

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We show that pairs of atoms optically excited to the Rydberg states can strongly interact with each other via effective long-range dipole-dipole or van der Waals interactions mediated by their non-resonant coupling to a common microwave field mode of a superconducting coplanar waveguide cavity. These cavity mediated interactions can be employed to generate single photons and to realize in a scalable configuration a universal phase gate between pairs of single photon pulses propagating or stored in atomic ensembles in the regime of electromagnetically induced transparency.

Ensembles of trapped atoms or molecules are promising systems for quantum information processing and communications [1]. They can serve as convenient and robust quantum memories for photons, providing thereby an interface between static and flying qubits [2], using e.g. stimulated Raman techniques, such as electromagnetically induced transparency (EIT) [3]. However, controlled interactions realizing universal quantum logic gates and entanglement in a deterministic and scalable way are difficult to achieve with photonic qubits propagating or stored in atomic ensembles.

A promising scheme for deterministic logic operations between stored photonic qubits was proposed in [4]. The proposal exploits the so-called dipole blockade mechanism, wherein strong resonant dipole-dipole interaction (DDI) between Rydberg atoms suppresses multiple excitations within a certain interaction volume. First proof-of-principle experiments have impressively demonstrated the blockade effect for related van der Waals interactions (VdWIs) between Rydberg atoms [5]. The achieved blockade radius of only a few $\mu$m is, however, not yet sufficient for implementing logic gates. Furthermore, the Rydberg blockade of [4] has two principle drawbacks. (i) In free-space, the DDI scales with interatomic distance $r_{ij}$ as $r_{ij}^{-3}$. The blockade gap in the Rydberg excitation spectrum of an atomic cloud is determined by the smallest DDI between pairs of atoms at opposite ends of the cloud. Yet, for closely spaced atoms, the DDI can be very large, which may lead to level crossings with other Rydberg states opening detrimental loss channels. (ii) Complete excitation blockade in an atomic ensemble requires spherical symmetry of the resonant DDI, which severely restricts the choice of suitable Rydberg states.

Here we put forward an alternative, scalable and efficient approach untainted by the above difficulties. We first show that superconducting coplanar waveguide (CPW) resonators [6, 7], operating in the microwave regime, can mediate long-range controlled interactions between neutral atoms optically excited to the Rydberg states. By appropriate choice of the system parameters, effective resonant DDI or VdWI between pairs of atoms located near the CPW surface [8] can be achieved. These interactions can then be employed to generate single photons and to realize a universal phase gate between pairs of single photon pulses propagating or stored in cold trapped atomic ensembles in the EIT regime.

Consider a pair of atoms $i$ and $j$ optically excited to the Rydberg states $|r_i\rangle$. The atoms interact non-resonantly with a certain mode of CPW cavity with frequency $\omega_c$, via transitions to adjacent Rydberg states $|a\rangle$ and $|b\rangle$, respectively, above and below $|r_i\rangle$ (Fig. 1). All the other cavity modes are far detuned from the atomic transition frequencies $\omega_{ar}$ and $\omega_{rb}$ and do not play a role. In the frame rotating with $\omega_c$, the Hamiltonian is given by

$$H = \hbar \sum_{l=i,j} \left[ (\Delta_a \hat{\sigma}_{aa}^l - \Delta_b \hat{\sigma}_{bb}^l) + (g_{ab} \hat{c}^l \hat{\sigma}_{bb}^l + g_{ab}^l \hat{c}^l \hat{\sigma}_{aa}^l + H.c.) \right],$$

(1)

where $\Delta_{\mu} = |\mu_i\rangle \langle \mu_j|$ is the transition operator of the $l$th atom, $\Delta_a = \omega_{ar} - \omega_c$ and $\Delta_b = \omega_{rb} - \omega_c$ are the corresponding detunings, $\hat{c}^l$ and $\hat{c}^l$ are the creation and annihilation operators for the cavity field, and $g_{\mu \nu}^l = -\langle \varphi_{\mu \nu} / \hbar \rangle \hat{c}^l \hat{a}^l (r_l)$ is the atom-field coupling rate, which is determined by the dipole matrix element $\varphi_{\mu \nu}$ of the

FIG. 1: (a) CPW cavity with strip-line length $L$ and electrode distance $d$. Ensembles of ground state atoms are trapped near the CPW surface within antinodes of the standing wave field. (b) Level scheme of a pair of excited Rydberg atoms $i$ and $j$ interacting with each other via non-resonant coupling to a common cavity mode.

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atomic transition, the field per photon \( \varepsilon_c = \sqrt{\hbar \omega_c / e_0 c^2} \) within the effective cavity volume \( V_c = 2 \pi d^2 L \), and the cavity mode function \( u(r) \) at atomic position \( r \).

Given an initial configuration \( |r_i \rangle |r_j \rangle |0_c \rangle \), with both atoms in state \( |r \rangle \) and zero cavity photons \( |0_c \rangle \), and large detunings \( \Delta_{a,b} \gg g_{br}^s, g_{br}^l, \kappa \), where \( \kappa \) is the cavity mode linewidth, we can use second order perturbation theory to eliminate the non-resonant states \( |r_{ij} \rangle |b_{ij} \rangle |1_c \rangle \) with a single photon in the cavity. We then obtain that each atom in state \( |r_i \rangle \) experiences a cavity-induced level shift

\[
s_i^l = \sqrt{g_{br}^s / 4 - \Delta_b / 2} \approx g_{br}^l / \Delta_b \quad (\text{ac Stark shift})
\]

and small level broadening \( \gamma_j^l = \kappa |g_{br}^l|^2 / \Delta_b^2 \ll \kappa \). In addition, state \( |r_i \rangle |r_j \rangle \) couples to states \( |b_{ij} \rangle |a_{ij} \rangle \) with rate \( D_{ij} = g_{br}^s g_{br}^l / \Delta_b = g_{br}^s g_{br}^l / \Delta_b \) via virtual photon exchange between the atoms in the cavity [4, 5, 7].

The corresponding interaction Hamiltonian reads

\[
V_{ij}^{(2)} = \hbar D_{ij} (\hat{\sigma}_{br}^l \hat{\sigma}_{br}^l + \hat{\sigma}_{ar}^l \hat{\sigma}_{br}^l) + \text{H.c.}
\]

(2)

Note that states \( |r_i \rangle |r_j \rangle |0_c \rangle \) and \( |b_{ij} \rangle |a_{ij} \rangle |0_c \rangle \) are also coupled to state \( |b_i \rangle |b_j \rangle |2_c \rangle \) with two photons in the cavity. But due to the large detunings \( 2 \Delta_b \), these transitions yield only small (fourth order) level shifts accounted for below. In second order in \( g_{\mu \nu}^l \), the energy offsets of states \( |a_{ij} \rangle |b_{ij} \rangle \), relative to \( |r_i \rangle |r_j \rangle \), is \( \hbar (\delta \omega + s_{ij}^l - s_{ij}^s) \), where \( \delta \omega = \Delta_a - \Delta_b = \omega_{ar} - \omega_{rb} \) and \( s_{ij}^l = |g_{ij}^l|^2 / \Delta_b^2 \).

If, by an appropriate choice of \( \delta \omega \) (with \( s_{ij}^l = s_{ij}^s \)), the transitions \( |r_i \rangle |r_j \rangle \leftrightarrow |b_{ij} \rangle |a_{ij} \rangle \) are made resonant, the Hamiltonian (2) would describe an effective resonant DDI, or Förster process, between a pair of Rydberg atoms [6, 7, 8, 9].

Before proceeding, let us estimate the relevant parameters achievable in a realistic experiment. For atoms placed in the vicinity of CPW field antinodes, the coupling constants \( g_{\mu \nu}^l \) are approximately the same, \( g_{\mu \nu}^l \approx g_{br} \) (see below). Setting \( \Delta_{a,b} \gg g_{r} f (f \gg 1) \)

\[ \delta \omega = s_r = g_r f^{-1}, \]

and \( \Delta_b = \omega_{rr}, \) the DDI coefficient is \( D_{ij} \approx D = g_r f^{-1} \). On the other hand, with \( \Delta_b \approx g_r f \) and \( \delta \omega = -g_r (\Delta_a \approx g_r f = 1) \), the VdWI strength is \( W_{ij} \approx W = 4g_r f^{-3} \). The total relaxation rate of a Rydberg state \( |\mu \rangle (\mu = r, a, b) = \Gamma_{\mu} + \gamma_{\mu} \)

\[ \gamma_{\mu} = \kappa f^{-2}, \]

tantalum to the strong coupling regime of cavity QED [10].

For a CPW cavity with strip-line length \( L \approx 1 \) cm and electrode distance \( d \approx 15 \mu m \) [Fig. 1(a)], the effective cavity volume \( V_c \approx 1.4 \times 10^{-11} \text{m}^3 \). The mode functions are 1D standing waves \( u(z) = \cos(m \pi z / L) \) or \( \sin(m \pi z / L) \)

\[ m \text{ is an even or odd integer and } z \in [-L/2, L/2] \]

Choosing e.g. \( m = 5 \), the mode wavelength is \( \lambda_c = 2L / m \approx 4 \) mm and there are \( m + 1 \) field antinodes. With effective dielectric constant \( \epsilon_r \approx 6 \), the mode frequency \( \omega_c = 2 \pi c / \lambda_c \sqrt{\epsilon_r} \approx 2 \pi \times 30 \text{GHz} \). For properly selected Rydberg states \( |r \rangle, |a \rangle, |b \rangle \), the transition frequencies \( \omega_{ar}, \omega_{rb} \approx \omega_c \) can be adjusted with high precision using static electric and magnetic fields [11].

The dipole matrix element between neighboring Rydberg states with principal quantum number \( n \) scales as \( \propto n^2 a_0 \), which for \( n \approx 50 \) yields \( g_r \approx 2 \pi \times 10 \text{MHz} \). In a cavity with quality factor \( Q \approx 10^6 \), the photon decay rate is \( \kappa = \omega_c / Q \approx 200 \text{ KHz} \), while \( \Gamma_{\mu} \lesssim 1 \text{ KHz} \).

Thus the strong-coupling regime with the above stringent condition can be realised for \( f < \min \left[ \sqrt{g_r / \Gamma_{\mu}}, g_r / \kappa \right] \approx 40. \)
Employing a master equation approach \cite{1}, we have performed numerical simulations of the dissipative dynamics of the full system with above parameters and \( f = 10,20 \). Results for the cases realizing the effective DDI and VdWI are shown in Fig. 2 and compared to simulations with the corresponding effective Hamiltonians \( \hat{H}_d \) and \( \hat{H}_r \). As expected, the agreement between the exact and effective models is good for \( f = 10 \) and excellent for \( f = 20 \). In the case of DDI, for both values of \( f \), the decoherence and population losses are very small on the time scale of several oscillation periods \((\sqrt{2D})^{-1} \) between states \(|r_i\rangle \langle r_j|\) and \(|b_{i,j}\rangle \langle a_{j,i}|\). In the case of VdWI, the population loss is appreciable on the much longer time scale of \( W^{-1} \): At time \( T_x = \pi/W \), when state \(|r_i\rangle \langle r_j|\) acquires phase shift \( \phi_{rr} = \pi \), its population \( p_{rr} \approx 0.92 \) for \( f = 10 \), and \( p_{rr} \approx 0.74 \) for \( f = 20 \). Thus, with realistic experimental parameters and \( f = 10 \) \((D \approx 2\pi \times 1 \text{ MHz}, W \approx 2\pi \times 40 \text{ KHz} \text{ and } \gamma \approx 2 \text{ KHz})\), conditional phase shift of \( \pi \) for a pair of Rydberg atoms can be achieved with fidelity \( F > 90\% \). With CPW cavity improvements and parameter optimization, the above fidelity may be further increased.

We envision several quantum information protocols utilizing Hamiltonians \( \hat{H}_d \) and \( \hat{H}_r \) in ensembles of alkali atoms in the ground state. Trapping cold atoms at a distance of 10-20 \( \mu \)m from the surface of a superconducting chip, incorporating the CPW cavity \cite{11}, is possible with presently available techniques \cite{12,13,14}. A cigar shaped volume \( V_a \approx d \times d \times \lambda_c/20 \) would contain \( N \approx 10^6 \) atoms at density \( \rho_0 \approx 2 \times 10^{13} \text{ cm}^{-3} \). Each atomic ensemble should be positioned near the cavity field antinode, so that the mode function \(|u(r)| \approx 1 \) is approximately constant throughout the atomic cloud. The corresponding coupling constants \( g_{\mu \nu} \) can then be assumed the same for all the atoms in the CPW cavity.

Employing light storage techniques based on EIT \cite{2,3}, the atomic ensembles in the setup of Fig. 1(a) can serve as reversible quantum memories for single photon qubits. Briefly, in a typical EIT setup, atoms in the ground state \(|g\rangle\) resonantly interact with a weak (quantum) field \( \hat{E} \) on the transition \(|g\rangle \leftrightarrow |e\rangle\), while a coherent driving field with Rabi frequency \( \Omega_d \) (and wavevector \( \mathbf{k}_d \)) couples the excited state \(|e\rangle\) to the long-lived (metastable) state \(|s\rangle\). When the light pulse \( \hat{E} \) (with wavevector \( \mathbf{k} \)) enters the EIT medium, it is transformed into the so-called dark-state polariton \( \Psi = \cos \theta \hat{E} - \sin \theta \sqrt{N} \sigma_{gs} \), which propagates in the medium with reduced group velocity \( v_g = c \cos \theta \), where \( \tan \theta = g_{gs}\sqrt{N}/\Omega_d \). The slowing down of the pulse upon entering the medium leads to its spatial compression by a factor of \( c \sin \theta \approx 1 \) \((0 < \theta < \pi/2)\). Once the pulse has been fully accommodated in the medium, by turning off \( \Omega \) \((\theta = \pi/2)\), the photonic excitation is adiabatically mapped onto the collective long-lived atomic excitation represented by state \(|s^{(1)}\rangle = 1/\sqrt{N} \sum_{i=1}^N e^{i\mathbf{k}_i \cdot \mathbf{r}} \langle g_1, \ldots, s_i, \ldots, g_N|\) which involves a single Raman (spin) excitation, i.e., atom in the metastable state \(|s\rangle\). At a later time, the photon can be retrieved on demand by turning \( \Omega_d \) on. Importantly, in order to accommodate the pulse in the medium with negligible losses, the optical depth of the atomic cloud should be large \cite{3,11}. With a typical resonant absorption cross-section for the alkali atoms \( \sigma_0 \approx 10^{-10} \text{ cm}^2 \), and the above cited density \( \rho_0 \) and medium length \( L_a = \lambda_c/20 \approx 0.2 \text{ mm} \), we have large optical depth \( 2\sigma_0 \rho_0 L_a \approx 80 \).

Using the cavity-mediated DDI \( \hat{H}_d \), we can implement the dipole blockade \cite{4} of multiple Rydberg excitations in an atomic ensemble. This, in turn, can be used to prepare the collective state \(|s^{(1)}\rangle\) and subsequently generate single photon pulses, as summarized below. Consider the level scheme of Fig. 3(a), where coherent laser fields with Rabi frequencies \( \Omega_{gr} \) and \( \Omega_{sr} \) and wavevectors \( \mathbf{k}_{gr} \) and \( \mathbf{k}_{sr} \) resonantly couple the lower atomic states \(|g\rangle\) and \(|s\rangle\) to the Rydberg state \(|r\rangle\). Initially all atoms are in state \(|g\rangle\), and only \( \Omega_{gr} \) is on. Then the laser field induces the transition from the ground state \(|g_1, g_2, \ldots, g_N\rangle\) to \(|s^{(0)}\rangle\) to the collective state \(|r^{(1)}\rangle\) \(\equiv 1/\sqrt{N} \sum_i e^{i\mathbf{k}_{gr} \cdot \mathbf{r}^i} |g_1, \ldots, s_i, \ldots, g_N\rangle\) representing a symmetric single Rydberg excitation of the atomic ensemble. The collective Rabi frequency for transition \(|s^{(0)}\rangle \rightarrow |r^{(1)}\rangle\) is \( \sqrt{N} \Omega_{gr} \). Once an atom \( i \in \{1, \ldots, N\} \) is transferred to state \(|r\rangle\), the excitation of a second atom \( j \neq i \) is suppressed by the resonant DDI between the atoms, provided that \( D_{ij} \approx D > \Omega_{gr} \). Indeed, out of the three eigenstates of \( \hat{H}_d \), the unshifted eigenstate \(|\psi_{ij}^{(0)}\rangle\) is not coupled to state \(|r_i\rangle |g_j\rangle\) by \( \Omega_{gr} \), while transitions \(|r_i\rangle |g_j\rangle \rightarrow |\psi_{ij}^{(1)}\rangle\) are shifted away from resonance by \( \pm \sqrt{2} D_{ij} \) and therefore are inhibited. Hence, a laser pulse of area \( \sqrt{N} \Omega_{gr} T_1 = \pi/2 \) (an effective \( \pi \) pulse) prepares the state \(|r^{(1)}\rangle\). The probability of error due to populating the doubly-excited states \(|\psi_{ij}^{(1)}\rangle\) is found by adding the

![FIG. 3: (b) Cavity-mediated DDI between atoms in Rydberg states \(|r\rangle\) facilitates generation of single collective spin excitation, via sequential application of \( \Omega_{gr} \) and \( \Omega_{sr} \) pulses. This excitation can then be converted into a single photon pulse \( \hat{E} \) by applying \( \Omega_d \). (b) Pair of atoms from ensembles \( A \) and \( B \) excited to states \(|r\rangle\) interact via the cavity-mediated VdWI \( W \) resulting in a conditional phase shift for the two ensemble qubits. (b) Two single-photon fields \( \hat{E}_A \) and \( \hat{E}_B \) upon entering the corresponding atomic ensembles in the EIT regime are converted into two dark-state polaritons which interact with each other via VdWI and acquire a dispersive phase shift.](image-url)
by applying \( \Omega \) according to pair of Rydberg atoms in the cavity, the above technique would be implemented between \( D_i \) and \( D_j \) is a short-range interaction, and \( \Phi \) is precisely the state we need for generating a single photon, as described above and illustrated in Fig. 3(a).

For the present parameters and choosing the optimal \( \Omega \) is precisely the state we need for generating a single collective spin excitation, will be produced. The total error probability is minimized by choosing \( \Omega_r \) \( \propto \sqrt{(1 + \gamma_r)} \) (3), as detailed above, atomic ensembles can serve as reversible quantum memories for photonic qubits. Conversely, individual ensembles can represent qubits storing any state of the form \( |\psi\rangle = \alpha |0\rangle + \beta |1\rangle \) in the corresponding superposition \( |\psi\rangle = \alpha |s(0)\rangle + \beta |s(1)\rangle \) of collective states. Consider two such ensemble qubits \( A \) and \( B \) in the cavity of Fig. 3(a). A resonant \( \pi \)-pulse applied to the transition \( |s\rangle \rightarrow |r\rangle \) in both atomic ensembles, \( \Omega_r T_1 = \pi/2 \), will then convert state \( |s(1)\rangle \) of each ensemble to the state \( |r(1)\rangle \) with single Rydberg excitation [see Fig. 3(b)]. Since any two atoms in state \( |r\rangle \) interact via VdWI with strength \( \gamma_d \), during time \( T_\pi = \pi/\gamma \) they will accumulate a phase shift \( \phi \). Thus the two-photon input state \( (|A\rangle |1\rangle) \) acquires a conditional phase shift of \( \pi \), and more generally, the CPHASE gate \( |A\rangle |y\rangle_B \rightarrow (\cos \theta |x\rangle_A |y\rangle_B (x,y \in [0,1]) \) between two photonic qubits is realized.

To summarize, ensembles of cold atoms trapped in the vicinity of a microwave CPW cavity can strongly interact with each other via cavity mediated virtual photon exchange between optically excited atomic Rydberg states. This system can serve as efficient and scalable platform to realize various quantum information processing protocols with ensemble qubits and single photons. This work was supported by the EC Marie-Curie Research Training Network EMALI.

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