Study of Giant Pairing Vibrations with neutron-rich nuclei.

Lorenzo Fortunato
Dip. di Fisica 'G.Galilei' and INFN,
v. Marzolo 8, 35100 Padova, Italy
E-mail: fortunat@pd.infn.it

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Abstract

We investigate the possible signature of the presence of giant pairing states at excitation energy of about 10 MeV via two-particle transfer reactions induced by neutron-rich weakly-bound projectiles. Performing particle-particle RPA calculations on $^{208}$Pb and BCS+RPA calculations on $^{116}$Sn, we obtain the pairing strength distribution for two particles addition and removal modes. Estimates of two-particle transfer cross sections can be obtained in the framework of the 'macroscopic model'. The weak-binding nature of the projectile kinematically favours transitions to high-lying states. In the case of ($^6$He, $^4$He) reaction we predict a population of the Giant Pairing Vibration with cross sections of the order of a millibarn, dominating over the mismatched transition to the ground state.

1 Pairing field and reaction mechanisms.

1.1 Introduction.

Nuclei in interaction with external fields display a wide variety of collective vibrations known as giant resonances, associated with various degrees of freedom and multipolarities. The giant isovector dipole resonance and the giant isoscalar quadrupole resonance are the most studied examples in this class of phenomena. A particular mode, that is associated with vibrations in the number of particles, has been predicted in the 70's and discussed, under the name of Giant Pairing Resonance, in the middle of the 80's in a number of papers. This phenomenon, despite some early efforts aimed to resolve some broad bump
in the high-lying spectrum in (p,t) reactions\cite{3}, is still without any conclusive experimental confirmation. For a discussion, in particular in connection with two-particle transfer reactions, on many aspects of pairing correlations in nuclei we refer to a recent review\cite{4}.

We have studied the problem of collective pairing modes at high excitation energy in two neutron transfer reactions with the aim to prove the advantage of using unstable beam as a new tool to enhance the excitation of such modes \cite{5}. The main point is that with standard available beams one is faced with a large energy mismatch that strongly hinders the excitation of high-lying states and favours the transition to the ground state of the final system. Instead the 'optimum' Q-value condition in the \(^{\alpha}\text{He},^{\beta}\text{He}\) stripping reaction suppresses the ground state and should allow the transition to 10-15 MeV energy region. We have performed particle-particle RPA calculations on lead and BCS+RPA on tin, as paradigmatic examples of normal and superfluid systems, evaluating the response to the pairing operator. Subsequently the two-neutron transfer form factors have been constructed in the framework of the 'mesoscopic model'\cite{6} and used in DWBA computer codes. We have estimated cross-sections of the order of some millibarns, dominating over the mismatched transition to the ground state. Recently we added similar calculations on other much studied targets to give some guide for experimental work.

1.2 The Giant Pairing Vibrations.

The formal analogy between particle-hole and particle-particle excitations is very well established both from the theoretical side\cite{7} and from the experimental side for what concern low-lying pairing vibrations around closed shell nuclei and pairing rotations in open shells. The predicted concentration of strength of a \(L = 0\) character in the high-energy region (8-15 MeV for most nuclei) is understood microscopically as the coherent superposition of 2\(p\) (or 2\(h\)) states in the next major shell above the Fermi level. We have roughly depicted the situation in Fig. 1. In closed shell nuclei the addition of a pair of particles (or holes) to the next major shell, with a total energy \(2\hbar\omega\), is expected to have a high degree of collectivity. Also in the case of open shell nuclei the same is expected for the excitation of a pair of particles with \(2\hbar\omega\) energies.

2 Details of calculations.

For normal nuclei the hamiltonian with a monopole strength interaction reads:

\[
H = \sum_j \epsilon_j a_j^\dagger a_j - 4\pi G P \dagger P, \tag{1}
\]

where \(P\) annihilates a pair of particles coupled to \(0\) total angular momentum. Getting rid of all the technicalities of the solution of the pp-RPA equations (that may be found in the already cited work by the author) we merely state that the pairing phonon may be expressed as a superposition of 2\(p\) (or 2\(h\)) states.
with proper forward and backward amplitudes ($X_n$ and $Y_n$). The pair transfer strength, that is a measure of the amount of collectivity of a each state $n$, is given by:

$$\beta_{P_n} = \sum_j \sqrt{2j+1}[X_n(j) + Y_n(j)].$$

(2)

This quantity is plotted in the first column of fig. 3 for the removal (upper panel) and addition mode (lower panel). In the same figure are reported the pairing strength parameters for the states of $^{116}\text{Sn}$. To obtain these last quantities for superfluid spherical nuclei one has to rewrite the hamiltonian according to the BCS transformation and has to solve more complex RPA equations. In this case the pairing strength for the addition of two particles is given, for each state $n$, by:

$$\beta_{P}^{(2p)} = \sum_j \sqrt{2j+1} \langle n| [a_j^\dagger a_j^\dagger]_0 |0\rangle = \sum_j \sqrt{2j+1}[U_j^2 X_n(j) + V_j^2 Y_n(j)]$$

(3)

where the $U$ and $V$ are the usual occupation probabilities. The amount of collectivity is a clear signal of the structural existence of giant pairing vibrations in the high-lying energy region. We also report here a number of analogous results for other commonly studied targets with the aim of giving some indications to experimentalists on the reasons why we think that lead and tin are some of the most promising candidates. We have studied two isotopes of calcium with closed shells. Even if the absolute magnitudes of the $\beta_P$ is lower, it is worthwhile to notice that some enhancement is seen in the more neutron-rich $^{48}\text{Ca}$ with respect to $^{40}\text{Ca}$. An important role in this change is certainly due to the different shell structure of the two nuclei as well as to the scheme that we implemented.
to obtain the set of single particle levels. The latter is responsible for the collectivity of the removal modes in both Ca isotopes and also for the difficulty in finding out a collective state in the addition modes. We display also results for $^{90}$Zr where the strength is much more fragmented and the identification of the GPV is more difficult. In the work of Broglia and Bes estimates for the energy of the pairing resonance are given as $68/A^{1/3}$ MeV and $72/A^{1/3}$ MeV for normal and superfluid systems respectively. Our figures follow roughly these prescriptions based on simple arguments (and much more grounded in the case of normal nuclei) as evident from Table 1.

![Table 1: Comparison of position of GPV between our calculation and the Broglia and Bes estimate.](image)

| Nucleus | Our calculation | Broglia & Bes estimate |
|---------|-----------------|------------------------|
| Sn      | 12.68 MeV       | 14.76 MeV              |
| Pb      | 11.81 MeV       | 11.47 MeV              |

3 Macroscopic model for two-particle transfer reactions.

The starting point of the 'macroscopic model' for two particle transfer reactions is to push further the analogy of the vibrations of the nuclear surface with the 'vibrations' across different mass partitions. If one imagine an idealized space in which a discrete coordinate (the number of particles of the system) labels different sections of the space, it is plausible to give an interpretation of pairing modes as back and forth oscillations in the number of particles. The role of macroscopic variable in this game is played by the quantity $\Delta A$, that is the difference in mass from the initial mass partition. Exploiting the analogy with inelastic modes lead us to construct a macroscopic guess for the pairing transition density $\delta \rho_p$ modeled on the surface transition density $\delta \rho_s$:

$$\delta \rho_s = \frac{\partial \rho}{\partial \alpha} \alpha = \frac{\partial \rho}{\partial r} R_0 \alpha$$  \hspace{1cm} (4)

$$\delta \rho_p = \frac{\partial \rho}{\partial \Delta A} \Delta A = \left( \frac{R_0}{3A} \right) \frac{\partial \rho}{\partial r} \Delta A$$  \hspace{1cm} (5)

One usually identifies $\alpha$ with the deformation parameter $\beta_s$, and the formal analogy suggests the correspondence with a 'pairing deformation' parameter $\beta_s \leftrightarrow \beta_p/(3A)$. This scheme implies the assumption that nuclear density is saturated and that a change in the number of particles is strictly related to a change of volume. The two-particle transfer form factors may then be connected.
to the ion-ion potential $U(r)$ as:

$$F_p(r) = \left(\frac{\beta_p}{3A}\right) R_0 \frac{\partial U(r)}{\partial r}$$  \hspace{1cm} (6)

This formalism has been applied to many low-energy aspects of two-particle transfer reactions\cite{8,9}. Certainly the macroscopic approach is liable of improvements when one turns to a microscopic description, but the predictions may be considered robust for giving order of magnitude evaluations.

4 Results for Pb, Sn and other targets.

DWBA calculations have been performed for two-neutron transfer reactions on the two cited targets either with usually available beams ($^{14}$C, $^{12}$C) either with new unstable ones ($^6$He, $^4$He). The last reaction has been chosen since it has optimal matching conditions: the Q-values for the transition to the ground states of both targets strongly positive, with the consequence of Q-values to the GPV close to the optimum Q-value ($Q_{opt} \sim 0$). This should favour the excitation of the pairing mode, while the situation with carbon beam is reversed, having large (and negative) Q-values for the high-lying energy region and small Q-values for the low-lying region. In table 2 we report the angle-integrated cross-sections obtained with standard DWBA computer codes. These cross-sections have been derived for sharp states, and we refer to the numbers in the last table when speaking of order of magnitude estimates. Obviously cross-section in the high-lying energy region have a finite (and large) width that should be inserted for a more realistic description of the spectrum. We have chosen a simple scheme that gives a lorentzian distribution with a width that grows quadratically with the excitation energy, $\Gamma = kE_x^2$, with $k$ adjusted to give a width of 4 MeV for the GPV. This could seem rather arbitrary since there is no reason for an a priori assignment of this quantity. We have been brought to this simple prescription because other collective states (of different nature) lying in the same energy region display similar values for their width, and it is reasonable to assume some rule to narrow the low-energy states and to broaden the high-energy ones.

|         | $^{14}$C → $^{12}$C | 6He → $^4$He |
|---------|-------------------|-------------|
| $^{116}$Sn → $^{118}$Sn$_{gs}$ | 19.4 mb | 0.4 mb |
| $^{208}$Pb → $^{210}$Pb$_{gs}$ | 15.3 mb | 1.8 mb |
| $^{116}$Sn → $^{118}$Sn$_{GPV}$ | 0.14 mb | 2.4 mb |
| $^{208}$Pb → $^{210}$Pb$_{GPV}$ | 0.04 mb | 3.1 mb |

Table 2: Cross-sections for ground-state and GPV transitions obtained with the DWBA code Ptolemy. The target (column) and projectile (row) are specified.
5 Final remarks.

The final achievements for the four reactions studied in detail are presented in Figure 4 where the areas corresponding to the cross-sections given above have been shaded to give a feeling of the relative magnitudes of the transition to the ground states and to the GPV’s. It is worthwhile to note that in the case of Pb there is a considerable gain in using unstable beams, while in Sn is much less evident. One sees the need for unstable helium when compares the magnitude for the pairing resonance in the right a) and b) panels with the peak at zero energy: in the first panel the transition to the ground state is extremely hindered.

A $^6$He beam is currently available (or it will be available in the very near future) in many radioactive ion beams facilities around the world and the calculations that we have presented could allow a planning for future experiments aimed to study the not yet completely unraveled role of pairing interaction in common nuclei, using exotic weakly bound nuclei as useful tools.

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Figure 2: Pairing response for removal and addition mode in $^{208}$Pb and $^{116}$Sn. The ground-state transition and the candidate for the GPV are marked.
Figure 3: Pairing response for removal and addition mode in $^{40}$Ca, $^{48}$Ca and $^{90}$Zr.
Figure 4: Differential cross-sections as function of the excitation energy. The shaded areas for the $^6$He-$^4$He reactions allow a comparison between the transition to the ground states and to the GPV’s. Notice that vertical scale is changed in Sn with respect to Pb.