Effects of the SO(10) D-Term on Yukawa Unification and Unstable Minima of the Supersymmetric Scalar Potential

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Abstract

We study the effects of SO(10) D-terms on the allowed parameter space (APS) in models with $t - b - \tau$ and $b - \tau$ Yukawa unification. The former is allowed only for moderate values of the D-term, if very precise ($\leq 5\%$) unification is required. Next we constrain the parameter space by looking for different dangerous directions where the scalar potential may be unbounded from below (UFB1 and UFB3). The common trilinear coupling $A_0$ plays a significant role in constraining the APS. For very precise $t - b - \tau$ Yukawa unification, $-m_{16} \lesssim A_0 \lesssim m_{16}$ can be probed at the LHC, where $m_{16}$ is the common soft breaking mass for the sfermions. Moreover, an interesting mass hierarchy with very heavy sfermions but light gauginos, which is strongly disfavoured in models without D-terms, becomes fairly common in the presence of the D-terms. The APS exhibits interesting characteristics if $m_{16}$ is not the same as the soft breaking mass $m_{10}$ for the Higgs sector. In $b - \tau$ unification models with D-terms, the APS consistent with Yukawa unification and radiative electroweak symmetry breaking, increases as the UFB1 constraint becomes weaker. However for $A_0 \lesssim 0$, a stronger UFB3 condition still puts, for a given $m_{16}$, a stringent upper bound on the common gaugino mass ($m_{1/2}$) and a lower bound on $m_{16}$ for a given $m_{1/2}$. The effects of sign of $\mu$ on Yukawa unification and UFB constraints are also discussed.

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1 Introduction

It is quite possible that the Standard Model (SM), is not the ultimate theory of nature, as is hinted by a number of theoretical shortcomings. One of the most popular choices for physics beyond SM is supersymmetry (SUSY) [1]. However, the experimental requirement that SUSY must be a broken symmetry introduces a plethora of new soft breaking parameters. There are important constraints on this large parameter space from the negative results of the sparticle searches at colliders like LEP[2] and Tevatron[3]. In addition there are important theoretical constraints which are often introduced for aesthetic reasons. From practical point of view, however, the most important effect of such constraints is to reduce the number of free parameters. For example, the assumption that the soft breaking terms arise as a result of gravitational interactions leads to the popular minimal supergravity (mSUGRA) model with five free parameters only, defined at a high energy scale where SUSY is broken. They are the common scalar mass ($m_0$), the common gaugino mass ($m_{1/2}$), the common trilinear coupling ($A_0$), the ratio of vacuum expectation values of two Higgs field ($\tan \beta$) and the sign of $\mu$, the higgsino mass parameter. In this paper we shall restrict ourselves to variations of this basic framework.

A very useful way to further constrain the allowed parameter space (APS) of softly broken SUSY models is to consider the dangerous directions of the scalar potential, where the potential may be unbounded from below (UFB) or develop a charge and/or color breaking (CCB) minima [4]. Different directions are chosen by giving vacuum expectation value (VEV) to one or more coloured and/or charged scalar fields, while the VEVs of the other scalars are taken to be zero.

In a very interesting paper which revived interest in UFB and CCB constraints, Casas et al [5] investigated the effects of such constraints on SUSY models. Though their formulae are fairly model-independent, they had carried out the numerical analysis within the framework of mSUGRA for moderate values of $\tan \beta$ only, when one can ignore the effects of $b$ and $\tau$ Yukawa couplings in the relevant renormalization group equations (RGEs). Their main result was that a certain UFB constraint known as UFB3 with VEVs given in the direction of the slepton fields puts the tightest bound on the SUSY parameter space that they considered (see eq. (93) of [5] and the discussions that follow).
In an earlier paper [6], we had extended and complemented the work of [5] by looking at the APS subject to such ‘potential constraints’ for large values of \( \tan \beta \), motivated by partial \( b-\tau \) [7, 8] or full \( t-b-\tau \) Yukawa unification [9]. Such unifications are natural consequences of an underlying Grand Unified Theory (GUT). We considered a popular model in which the GUT group \( \text{SO}(10) \) breaks directly into the SM gauge group \( \text{SU}(3) \times \text{SU}(2) \times \text{U}(1) \). All matter fields belonging to a particular generation is contained in a 16 dimensional representation of \( \text{SO}(10) \). With a minimal Higgs field content (one \( 10 \)-plet containing both the Higgs doublets required to give masses to u and d type quarks) all three Yukawa couplings related to the third generation fermions must unify at the GUT scale. If one assumes more than one \( 10 \)-plet, at least the bottom and the tau Yukawa couplings should unify.

In [6] we assumed a common soft breaking (SB) mass \( m_{16} \) at \( M_G \) for all sfermions of a given generation. Similarly a common mass parameter \( m_{10} \) was chosen for both the Higgs fields. We then studied the stability of the potential for two sets of boundary conditions: i) the mSUGRA motivated universal scenario \( m_{16} = m_{10} \), and ii) a nonuniversal scenario \( m_{16} \neq m_{10} \). The second condition is motivated by the fact that a common scalar mass at the Planck scale, generated, \textit{e.g.}, by the SUGRA mechanism, may lead to nonuniversal scalar masses at \( M_G \) due to different running of \( m_{10} \) and \( m_{16} \), as they belong to different GUT multiplets [10].

In this paper we shall extend the work of [6] by considering the APS due to Yukawa unification and UFB constraints in the presence of \( \text{SO}(10) \) breaking D-terms. The group \( \text{SO}(10) \) contains \( \text{SU}(5) \times \text{U}(1) \) as a subgroup. It is well known that the breaking of \( \text{SO}(10) \) to the lower rank SM group may introduce nonzero D-terms at the GUT scale[11]. We further assume that the D-terms are linked to the breaking of \( \text{U}(1)_X \) only. It should be noted that if one assumes the existence of additional \( \text{U}(1) \)’s at high energies, it is quite natural to assume that the D-term contributions to scalar masses are non-zero[11]. The only uncertainty lies in the magnitude of the D-terms which may or may not be significant. The squark - slepton and Higgs soft breaking masses in this case can be parametrized as

\[
\begin{align*}
    m^2_Q &= m^2_E = m^2_U = m^2_{16} + m^2_D \\
    m^2_D &= m^2_L = m^2_{16} - 3m^2_D \\
    m^2_{H_d,u} &= m^2_{10} \pm 2m^2_D
\end{align*}
\]
where $\tilde{Q}$ and $\tilde{L}$ are SU(2) doublets of squarks and sleptons, $\tilde{E}$, $\tilde{U}$ and $\tilde{D}$ are SU(2) singlet charged sleptons, up and down type squarks respectively. The unknown parameter $m^2_D$ (the D-term) can be of either sign. The mass differences arise because of the differences in the $U(1)$ quantum numbers of the sparticles concerned. As can be readily seen from the above formula for $m^2_D > 0$, the left handed sleptons and right handed down type squarks (belonging to the $\bar{5}$ representation of SU(5)), are lighter than the members of the 10 plet of SU(5). In recent times the phenomenology of the D-terms has attained wide attention\[12, 13\].

D-terms acquire particular significance in the context of $t-b-\tau$ unification as has already been noted in the literature [12]. A new result of this work is that while moderate values of D-terms facilitate very accurate unification, high values of this parameter spoil it.

The UFB and CCB constraints depend crucially on the particle spectra at the properly chosen scale where the true minimum and the dangerous minimum can be reliably evaluated from the tree level potential ($V_{\text{tree}}$) [5, 14]. Such spectra, in turn, depend on the boundary conditions at the GUT scale. The SO(10) breaking D-terms alter the sparticle spectra at the GUT scale and may affect the stability of the potential. In this paper we focus our attention on the impact of such D-terms on the APS restricted by Yukawa unification and the stability of the potential in both universal and nonuniversal scenarios.

Throughout the paper we ignore the possibility that nonrenormalizable effective operators may stabilise the potential [15]. The dangerous minima that we encounter in our analysis typically occur at scales $\lesssim 10^8$ GeV where the effects induced by the nonrenormalizable operators, which in principle can be significant in the vicinity of the GUT scale, are not likely to be very serious.

It has been pointed out in the literature that the standard vacuum, though metastable, may have a lifetime longer than the age of the universe [16], while the true vacuum is indeed charge and colour breaking. If this be the case, the theory seems to be acceptable in spite of the existence of the unacceptable UFB minima that we have analysed. However, the lifetime calculation, which is relatively straightforward for a single scalar field, is much more uncertain in theories where the potential is a function of many scalar fields. Thus it is difficult to judge the reliability of these calculations. Moreover, the constraints obtained by us does not loose their significance even if the false vacuum idea happens to be the correct theory. If these constraints are violated by future experimental data then that would automatically
lead to the startling conclusion that we are living in a false vacuum and charge and colour symmetry may eventually breakdown.

It has been known for quite some time that while $\mu > 0$ (in our sign convention which is opposite to that of Haber and Kane [1]) is required by Yukawa unification, the opposite sign is preferred by the data on the branching ratio of $b \rightarrow s \gamma$ and that on $g_{\mu} - 2$ (see [17, 18] for some of the recent analyses and references to the earlier works).

It has been shown in [17] and also in the first paper of [18] that in a narrow region of the parameter space there is no conflict between data and Yukawa unification. We have analysed the parameter space found in [17] in the light of the stability of the vacuum and the results are given in the next section (see Table 1 in particular). The above conflict may also be resolved by introducing non-universal gaugino masses (see Chattopadhaya and Nath in [18]).

In section 2 we discuss the effects of $\tan \beta$, $m_D$ and sign of $\mu$ on Yukawa unification and stability of the potential. In subsection 2.2 and 2.3 we study the APS for both $t - b - \tau$ and $b - \tau$ unification in conjunction with the UFB constraints. In the last section we summarise and conclude.

2 Results

2.1 General Discussions

The methodology of finding the spectra is the same as in [6], which is based on the computer program ISASUGRA, a part of the ISAJET package, version 7.48[19]. The parameters $\mu$ and $B$ are fixed by radiative electroweak symmetry breaking (REWSB) [20] at a scale $M_S = \sqrt{m_{\tilde{t}_L} m_{\tilde{t}_R}}$. We further require that the lightest neutralino ($\tilde{\chi}^0$) be the lightest supersymmetric particle (LSP). The above two constraints will also be used to obtain the allowed parameter space (APS) although their use may not be mentioned explicitly everywhere. We then fix $\tan \beta$ to its lowest value required by Yukawa unification. Next we check the experimental constraints on sparticle masses. Finally we impose the UFB constraints.

Before discussing the basic reasons of how Yukawa unification plays a significant role in restricting the APS, we will review the different uncertainties of Yukawa unification. The
effectiveness of Yukawa unification as a restrictor of the APS diminishes, as expected, as the accuracy with which we require the unification to hold good is relaxed. There are several reasons why the unification may not be exact. First, there may be threshold corrections [21], both at the SUSY breaking scale (due to nondegeneracy of the sparticles) and at $M_G$, of which no exact estimate exist. Secondly, we have used two-loop RGEs for the evolutions of gauge couplings as well as Yukawa couplings and one loop RGEs for the soft breaking parameters, but higher order loop corrections may be important at a few percent level at higher energy scales. Finally the success of the unification program is also dependent on the choice of $\alpha_s(M_Z)$ which is not known as precisely as $\alpha_1$ or $\alpha_2$. To take into account such uncertainties, one relaxes the Yukawa unification condition to a finite amount (5%, 10% or 20%) which should indirectly take care of the above caveats. The demand of very accurate Yukawa coupling unification at $M_G$ puts severe constraint on $\tan \beta$ restricting it to very large values only.

The accuracy of the $t - b - \tau$ unification is usually relaxed since there are more elements of uncertainty, e.g., the choice of the Higgs sector. To quantify this accuracy, one can define three variables $r_{br}$, $r_{tb}$ and $r_{tr}$ where generically $r_{xy} = \max(Y_x/Y_y, Y_y/Y_x)$. For example, to check whether the couplings unify, one should select only those points in the parameter space where, e.g., $\max(r_{br}, r_{tb}, r_{tr}) < 1.10$ (for 10% $t - b - \tau$ unification) and $r_{br} < 1.05$ (for 5% $b - \tau$ unification).

Now we will focus on the basic reasons which lead to upper and lower bounds on the APS in the $m_{16} - m_{1/2}$ plane, if partial ($b - \tau$) or full ($t - b - \tau$) unification is required. It is wellknown that for precise Yukawa unification one should have $\mu > 0$. The partial Yukawa unification can be accommodated at relatively low values of $\tan \beta$ when the phenomenologically interesting small $m_{16}, m_{1/2}$ region of the parameter space is allowed (viz. for $m_{16}, m_{1/2} \sim 200$ GeV, the required minimum value of $\tan \beta \sim 30$, and for $m_{16}, m_{1/2} \sim 800$ GeV, $(\tan \beta)_{\text{min}} \sim 41$). On the otherhand $\tan \beta$ cannot be arbitrarily increased due to the REWSB. This basic trend, which often makes the two constraints incompatible, remains unaltered irrespective of the choice of the other parameters.

The constraints due to Yukawa unification and REWSB are relatively weak for large negative values of $A_0$ and becomes stronger as this parameter is algebraically increased $^1$.

\footnote{This is due to the fact that unification holds at relatively lower values of $\tan \beta$ as one goes to larger}
On the other hand, the UFB constraints are very potent for large negative values of $A_0$. The expanded APS allowed by Yukawa unification, is eaten up by the UFB constraints. In this sense the two sets of constraints are complementary[6].

$Y_t$ varies relatively slowly with respect to $\tan \beta$ compared to $Y_\tau$ and $Y_b$. For very accurate (5 %) $t - b - \tau$ unification, we, therefore, need high values of $\tan \beta \sim 47 - 51$. In this case the low $m_{16} - m_{1/2}$ region is excluded by the REWSB condition, leading to lower bounds much stronger than the experimental ones and the resulting APS is restricted to phenomenologically uninteresting high $m_{16}, m_{1/2}$ region. For example, with $\tan \beta = 49.5$ the lowest allowed values are $m_{16} = 600\text{GeV}$, $m_{1/2} = 1000\text{GeV}$ leading to rather heavy sparticles.

In the presence of D-terms a larger APS is obtained even if very accurate full unification is required[12]. A new finding of this paper is that though moderate values of $m_D$ leads to better Yukawa unification, somewhat larger values spoil it. Although the D-terms do not affect the evolution of the Yukawa couplings directly through the RGEs, they change the initial conditions through SUSY radiative corrections to $m_b(m_Z)$ [22]. This is illustrated in figs. 1–3, where approximate unification is studied for three different values of $m_D$. The choice of other SUSY parameters for these figures are as follows:

$$m_{10} = m_{16} = 1500\text{GeV}, \quad m_{1/2} = 500\text{GeV}, \quad \tan \beta = 48.5, \quad A_0 = 0 \text{ and } \mu > 0.$$  

From fig 1 ($m_D = 0$), we see that the accuracy of unification is rather modest ($\sim 15\%$). As $m_D$ is further increased to $m_{16}/5$ (fig. 2), the $b\bar{g}$ loop corrections (see eq. (8) of [22]) to $m_b(m_Z)$ increases and leads to better unification. However, if we increase $m_D$ further to $m_{16}/3$, the accuracy of unification deteriorates (fig. 3) since $m_b(m_Z)$ suffers a correction which is too large. We have checked that this feature holds for a wide choice of SUSY parameters.

Quite often the APS expanded due to the presence of D-terms is significantly reduced by the UFB constraints. As discussed in our earlier work[6], the variation of $m_{H_u}^2$ and $m_{H_d}^2$, the soft breaking masses of the two Higgs bosons, with respect to the common trilinear coupling $A_0$ is of crucial importance in understanding this. Here we extend the discussion for non-zero values of the D-term, $m_D = m_{16}/5$ and $m_{16}/3$. The effects are illustrated in negative values of $A_0$. There is, therefore, more room for increasing $\tan \beta$, if required, without violating REWSB condition. This point was not elaborated in our earlier work[6].
fig. 4. As we increase the magnitude of the D-term, the UFB3 becomes more potent though UFB1 loses its restrictive power for a fixed value of tan \( \beta \). To clarify this result we examine two important expressions of Casas et al.[5]. The first one is

\[
m^2_{H_u} + m^2_{H_d} + 2\mu^2 \geq 2|\mu B|,
\]

which is known as the UFB1 condition and should be satisfied at any scale \( \hat{Q} > M_S \), in particular at the unification scale \( \hat{Q} = M_G \). The second one is the UFB3 constraint,

\[
V_{UFB3} = [m^2_{H_u} + m^2_{L_i}]|H_u|^2 + \frac{|\mu|}{\lambda_{E_j}}[m^2_{L_j} + m^2_{E_j} + m^2_{L_i}]|H_u| - \frac{2m_{L_i}^4}{g'^2 + g^2},
\]

where \( g' \) and \( g \) are normalised gauge couplings of \( U(1) \) and \( SU(2) \) respectively, \( \lambda_{E_j} \) is a Yukawa coupling and \( i, j \) are generation indices.

We find that larger \( m_D \) drives \( m^2_{H_u} \) to more negative values, while \( m^2_{H_d} \) is driven to positive values (see fig. 4). In addition, it follows from REWSB condition that as the difference \( m^2_{H_d} - m^2_{H_u} \) increases, the higgsino mass parameter \( \mu \) increases. As a result the UFB1 constraint becomes weaker for large \( m_D \) values (see eq. 1). On the other hand at the GUT scale, \( m^2_{L_i} \) becomes smaller for larger \( m_D \). From eq. 2 it can be concluded that the parameter space where \( m^2_{H_u} + m^2_{L_i} \) is negative increases and the model is more susceptible to the UFB3 condition. These effects will be reflected in \( b-\tau \) unification as well.

For precise (\( \leq 5\% \)) \( b-\tau \) unification the required \( \tan \beta \) is very high (\( \sim 49 \)) and the allowed \( m_{16} \) values are large. Here the magnitude of \( \mu \) as determined by the REWSB becomes very low even for moderate values of the D-term (\( m_D \approx m_{16}/5 \)). Consequently UFB1 still disallows a significant part of the enlarged APS obtained with introduction of the D-term. However, the effectiveness of the UFB1 constraint depends crucially on \( A_0 \). We see that \( A_0 \lesssim m_{16} \) is ruled out by UFB1 if the D-term is zero. In presence of the D-terms the UFB1 constraints become weaker but still have some restrictive power for \( A_0 \lesssim 0 \). Moreover UFB3 becomes weaker for large \( m_{16} \) in general.

Now we shall discuss the impact of the sign of \( \mu \) on both UFB constraints and Yukawa unification. Yukawa unification generally favours \( \mu > 0 \). The sign of \( \mu \) affects unification through loop corrections[22] to the bottom Yukawa coupling, which are incorporated at the weak scale. These corrections lower the bottom Yukawa coupling significantly, consequently the GUT scale value becomes very low, which tends to spoil Yukawa unification. Baer et al.
al.[17] showed that for $\mu < 0$ full unification with low accuracy ($\sim 30\%$) is possible. This is interesting since approximate unification then becomes consistent with the constraints from $b \to s\gamma$ and $g - 2$ of the muon. It was shown in [17] that the Yukawa unified APS favours $A_0 \approx -2m_{16}$ and $m_{10} \approx \sqrt{2}m_{16}$ (see fig. 1 of [17]). We have extended the analysis of ref[6] for $\mu < 0$ and have found that the UFB1 condition loses it effectiveness for $\mu < 0$. The bottom Yukawa coupling affects the value of $m_{H_d}^2$ through renormalization group (RG) running and cannot make $m_{H_d}^2$ large negative as in the $\mu > 0$ case. This is why UFB1 is weakened (see eq. 1). We have studied the APS obtained in [17] and found that UFB1 can disallow certain negative values of $m_D^2$ depending on the magnitudes of $m_{16}$ and $m_{1/2}$. If $m_{16}, m_{1/2}$ are increased, relatively small negative values of $m_D^2$ make the potential unstable under UFB1 condition. Some representative regions of APS are shown in Table 1. In obtaining Table 1, $A_0$, $m_{10}$ and $\tan \beta$ are varied within the ranges indicated by ref[17].

Table 1: Representative D-terms allowed by UFB1 for $\mu < 0$.

| $m_{16}$ (GeV) | $m_{1/2}$ (GeV) | allowed $m_D^2$ (GeV$^2$) |
|---------------|----------------|--------------------------|
| 600           | 300            | $\gtrsim -(m_{16}/4.3)^2$ |
| 1000          | 300            | $\gtrsim -(m_{16}/4.5)^2$ |
| 1000          | 500            | $\gtrsim -(m_{16}/5.0)^2$ |

We have also checked that precise $b - \tau$ Yukawa unification is not possible for $\mu < 0$ except for very low $\tan \beta (\sim 1)$. As $\tan \beta$ is increased, $Y_\tau$ at $M_G$ increases rapidly compared to $Y_b$. This is clear from $\tilde{t}\tilde{\chi}^+$ loop correction (see eqn. 15 of [22]).

2.2 $t$-$b$-$\tau$ Unification

Through out this section we shall restrict ourselves to unification within 5%. For the sake of completeness and systematic analysis, we start our discussion for large negative values of $A_0$ (say, $A_0 = -2m_{16}$), though it is not interesting from the point of view of collider searches. We first consider moderate values of the D - term ( e.g., $m_D = m_{16}/5$). Though at low values of $\tan \beta$ the large negative values of $A_0$ are favoured by REWSB (see, e.g., the following
section on $b - \tau$ unification), they are strongly disfavoured at large $\tan \beta$ ($\sim 49$) which is
required by full unification. Only a narrow band of $m_{1/2}$ is allowed. However, the APS
 corresponds to rather heavy sparticles (e.g., $m_{16}(m_{1/2}) \gtrsim 1100(1300)$GeV) which are of little
interest even for SUSY searches at the LHC. Non-universality affects the APS marginally;
no significant change can be obtained. Thus no squarks - gluino signal is expected at LHC
for $A_0 \lesssim -2m_{16}$ irrespective of the boundary conditions (universal or non-universal) on the
scalar masses. Over a small region of the APS somewhat lighter sleptons ($m_{\tilde{l}} \sim 1000$GeV)
are still permitted. Moreover, the tiny APS allowed by the unification criterion is ruled
out by the UFB1 condition. For $A_0 \gtrsim 2m_{16}$, the APS is qualitatively the same as that for
$A_0 \lesssim -2m_{16}$, with the only difference that the UFB constraint does not play any role.

Relatively large APSs with phenomenologically interesting sparticle masses open up for
$-m_{16} \lesssim A_0 \lesssim m_{16}$, which is favourable for both Yukawa unification and REWSB. The
common feature of the APS is that gluino masses almost as low as the current experimental lower
bound with much heavier squark and slepton masses ($\gtrsim 1$TeV) can be obtained irrespective
of universality or non-universality of scalar masses. It should be stressed that this mass
pattern cannot be accommodated without the D-terms. In the presence of D-term this mass
hierarchy becomes a distinct possibility.

In fig. 5 we present the $m_{16} - m_{1/2}$ plane for $A_0 = -m_{16}$ in the universal model. A large
APS is obtained by the unification criterion alone. For each $m_{16}$ there are lower and upper
bounds on $m_{1/2}$. For $m_{16} < 1200$GeV relatively low values of $m_{1/2}$ are excluded by REWSB
while very high values are excluded by the requirement that the neutralino be the LSP. The
value of $m_{16}$ can be as low as 700 GeV, which corresponds $m_{1/2} \gtrsim 1100$ GeV, yielding $m_{\tilde{g}} \gtrsim
2422$ GeV, $m_{\tilde{q}} \approx 2200$ GeV, $m_{\tilde{l}} \approx 829$ GeV. For $m_{16} \gtrsim 1200$GeV, low values of $m_{1/2}$ are quite
common. Scanning over the APS we find that the lowest allowed gluino mass is just above
the experimental lower bound. Corresponding to this gluino mass the minimum sfermion
masses are $m_{\tilde{g}} \approx 1200$ GeV, $m_{\tilde{l}} \approx 1200$ GeV.

As $A_0$ is further increased algebraically from $-m_{16}$, the APS slightly decreases due to
unification and REWSB constraints. For $A_0 = 0$, we obtain an upper limit $m_{16} \leq 2400$
GeV. However, the lower limits on $m_{16}$ is relaxed by $\sim 200$ GeV in comparison to the
$A_0 = -m_{16}$ case. As we further increase the value of $A_0$ to $A_0 = m_{16}$, the APS is almost the
same as that for $A_0 = -m_{16}$. This trend is observed in all cases irrespective of universality
or non-universality of the scalar masses and even for $b - \tau$ unification. We find that as the absolute value of $A_0$ increases, Yukawa unification is less restricted, while REWSB is somewhat disfavoured. When both act in combination, we get a relatively large APS for $|A_0| = m_{16}$ and a somewhat smaller one for $A_0 = 0$.

As the potential constraints are switched on for $A_0 = -m_{16}$, an interesting upper bound on $m_{1/2}$ for each given $m_{16}$ is imposed by the UFB1 constraint (fig. 5). As a result practically over the entire APS, the gauginos are required to be significantly lighter than the sfermions. Moreover, the allowed gaugino masses are accessible to searches at the LHC.

We next focus on the impact of a particular type of non-universality ($m_{10} < m_{16}$) for the negative $A_0$ scenario. The shape of the APS is affected appreciably. As $m_{10}$ decreases, $Y_b$ gets larger SUSY threshold corrections than $Y_\tau$ and $Y_t$; this disfavours Yukawa unification. On the other hand $m_{H_u}^2$ and $m_{H_d}^2$ becomes more negative for even smaller values of $m_{16}$ and $m_{1/2}$, which disfavors REWSB. The overall APS is somewhat smaller compared to the universal case, which is illustrated in fig. 6 for $m_{10} = 0.8m_{16}$ (compare with fig. 5). The UFB1 constraint still imposes an upper bound on the gaugino mass for a given $m_{16}$ as in the universal case. As a result the gauginos are within the striking range of LHC practically over the entire APS. We also note from fig. 6 that $m_{16} \gtrsim 1600$ GeV over the entire APS.

For a different pattern of non-universality ($m_{10} > m_{16}$), Yukawa unification alone narrows down the APS considerably. However, it is seen that regions with simultaneously low values of $m_{16}$ and $m_{1/2}$ are permitted. This happens in this specific nonuniversal scenario only. On the otherhand, the parameter space with large $m_{16}$ and small $m_{1/2}$, preferred by the earlier scenarios, is disfavoured. With $m_{10} = 1.2m_{16}$ (fig. 7), it is found that $m_{1/2} \gtrsim 300$ GeV. The unification allowed parameter space, however, is very sensitive to the UFB conditions which practically rules out the entire APS for negative $A_0$. No major change is noted in the APS for $A_0 = 0$ and $A_0 = m_{16}$ apart from the fact that the UFB constraints get weaker.

We now discuss the impact of larger D-terms on the parameter space. For example, with $m_D = m_{16}/3$, the APS reduces drastically in the universal as well as non-universal scenario with $m_{10} < m_{16}$, irrespective of $A_0$. This is illustrated in fig. 8. and is in complete agreement with our qualitative discussion in the earlier section.

Only in the specific nonuniversal scenario with $m_{10} > m_{16}$, slightly larger $m_D$ is preferred. However, $m_D$ cannot be increased arbitrarily. For $m_{10} = 1.2m_{16}$, the APS begins to shrink
again for \( m_D \gtrsim m_{16}/3 \) and we find no allowed point for \( m_D = m_{16}/2 \).

As \( m_{10} \) is increased further, Yukawa unification occurs in a narrower APS. This, nevertheless, is a phenomenologically interesting region where lower \( m_{16} - m_{1/2} \) values can be accommodated. For example, \( m_{16} (m_{1/2}) = 400(300) \text{GeV} \) is allowed with \( m_{10} = 1.5m_{16}, m_D = m_{16}/3, A_0 = 0 \) and \( \tan \beta \sim 51 \), leading to \( m_{\tilde{g}} = 742 \text{GeV}, m_{\tilde{q}} \approx 700 \text{ GeV}, m_{\tilde{t}} \approx 400 \text{GeV} \) and \( m_{\tilde{f}_1} = 274 \text{GeV} \). However, we cannot increase \( m_{10} \) arbitrarily either, the APS reduces drastically for \( m_{10} \gtrsim 1.5m_{16} \) irrespective of the value of \( m_D \). This trend qualitatively remains the same even if \( A_0 \) is changed. This effect can be seen in \( b-\tau \) unification as well.

### 2.3 \( b-\tau \) Yukawa unification

In our earlier work[6] without D-terms, we had shown that the APS is strongly restricted due to Yukawa unification and UFB constraints. The minimum value of \( \tan \beta \) required for unification is \( \approx 30 \). If D-terms are included, Yukawa unification and REWSB occur over a larger region of the parameter space. This is primarily due to two reasons: i) Yukawa unification can now be accommodated for lower values of \( \tan \beta \) (\( \sim 20 \)) and ii) REWSB is allowed at somewhat higher values of \( \tan \beta \) than the values permitted in \( m_D = 0 \) case. This reduces the conflict between Yukawa unification and REWSB. As a result \( m_{1/2} \) almost as low as that allowed by the LEP bound on the chargino mass is permitted over a wide range of \( m_{16} \). In some cases the upper bound on \( m_{16} \) for a given \( m_{1/2} \) is also relaxed. Similarly for a fixed \( m_{16} \), the upper bound on \( m_{1/2} \) is sometimes relaxed by few hundred GeVs. Throughout this work we require this partial unification to an accuracy of \(< 5\%\).

Now we will focus our attention on large negative values of \( A_0 \) (\( A_0 = -2m_{16} \)) with \( m_D = m_{16}/5 \) in the universal scenario. The unification allowed APS, as shown in fig. 9, expands compared to the \( m_D = 0 \) scenario (compare with fig. 6 of [6]). Moreover, the phenomenologically interesting scenario with light gauginos but very heavy sleptons and squarks beyond the reach of LHC, which was rather disfavoured without the D-terms (see [6]), is now viable. Without the D-term, the APS was severely restricted by the UFB conditions for large negative values of \( A_0 \). As discussed earlier, inclusion of the D-term increases the value of \( \mu \). As a result UFB1 looses its constraining power; lower values of \( m_{1/2} \) are allowed for large \( m_{16} \) by UFB1. On the other hand as the value of D-term increases,
UFB3 becomes more powerful and the upper bounds on $m_{1/2}$ for relatively low values of $m_{16}$ get stronger (e.g., for $m_{16} = 600(1000)$GeV, $m_{1/2} < 300(600)$GeV). For $A_0 > 0$, the APS again expands. However, the UFB constraints are found to be progressively weaker as $A_0$ is increased from $A_0 = -2m_{16}$.

We next consider the non-universal scenario $m_{10} \neq m_{16}$. If we take $m_{10} < m_{16}$ (say, $m_{10} = .6m_{16}$) and $m_D = m_{16}/5$, the unification allowed parameter space for $A_0 = -2m_{16}$, as shown in fig. 10, is more or less the same as in the universal scenario. The entire APS is, however, ruled out due to a very powerful constraint obtained from the UFB3 condition. This conclusion obviously holds for larger values of $m_D$.

For $m_{10} > m_{16}$ and large negative $A_0$ ($A_0 = -2m_{16}$), the unification allowed APS (fig. 11) is smaller compared to that in the universal case (fig. 9). The same trend was also observed with $m_D = 0[6]$. The APS, however, is significantly larger than that for $m_D = 0$. For a given $m_{1/2}$ ($m_{16}$) the upper-bound on $m_{16}$ ($m_{1/2}$) gets weaker for non-zero D-terms. Relatively light gluinos consistent with current bounds are allowed over a larger region of the parameter space. The UFB constraints restrict the APS further and put rather strong bounds on $m_{1/2}$ and $m_{16}$. A large fraction of this restricted APS is accessible to tests at LHC energies. The usual reduction of the APS due to unification constraints as $A_0$ is increased from $A_0 = -2m_{16}$ also holds in this nonuniversal scenario.

If we increase $m_D$ further, the APS due to Yukawa unification reduces for reasons already discussed. The UFB1 constraint also gets weaker. On the other hand the UFB3 constraints become rather potent. For example, i) with $A_0 = -2m_{16}$ and $m_D \gtrsim m_{16}/3$ the entire APS for $m_{10} = m_{16}$ or $m_{10} < m_{16}$ is ruled out. ii) $A_0 = -m_{16}$ and $m_D \gtrsim m_{16}/2$ the entire APS corresponding to $m_{10} = m_{16}$ or $m_{10} < m_{16}$ is ruled out. On the other hand, for $m_{10} > m_{16}$ the APS further reduces as $m_D$ is increased.

3 Conclusion

For moderate values of the D-terms ($m_D \approx m_{16}/5$), the APS expands in general compared to the $m_D = 0$ case for both $b - \tau$ and $t - b - \tau$ Yukawa unification. A large fraction of the enlarged APS is, however, reduced by the requirement of vacuum stability and the predictive power is not lost altogether. D-terms with much larger magnitudes, however, are
not favourable for unification. In the $t-b-\tau$ Yukawa unified model (accuracy $\leq 5\%$), a band of very low gaugino mass close to the current experimental lower limit is a common feature in the presence of D-terms. For a given $m_{16}$ there is an upper bound on $m_{1/2}$ from unification and stability of the potential constraints. This happens for $-m_{16} \lesssim A_0 \lesssim m_{16}$. Outside this range of $A_0$, the APS is very small with sparticle masses of the first two generations well above 1 TeV. In $b-\tau$ unification, UFB3 strongly restricts the APS while UFB1 becomes less potent in the presence of D-terms.

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Figure 1: The variation of Yukawa couplings with renormalization scale $Q$ (GeV). From above the lines are for top, bottom and $\tau$ Yukawa couplings respectively. We have used $m_{16} = m_{10} = 1.5$ TeV, $m_{1/2} = 0.5$ TeV. $A_0 = 0$, $\tan \beta = 48.5$ and $m_D = 0$. 

Figure 2: The same as fig. 1, with $m_D = \frac{m_{16}}{5}$.

Figure 3: The same as fig. 1, with $m_D = \frac{m_{16}}{3}$.
Figure 4: The variation of the Higgs mass parameters $m_{H_d}^2$ and $m_{H_u}^2$, evaluated at the scale $M_S = \sqrt{m_{t_L}m_{t_R}}$, with the trilinear coupling $A_0$. The solid (dotted) lines are for $m_D = m_{16}/5(m_{16}/3)$. The top two lines are for $m_{H_d}^2$ while the lower pair is for $m_{H_u}^2$. We have used $m_{16} = m_{10} = m_{1/2} = 1$ TeV, $\tan\beta = 45$. 
Figure 5: The allowed parameter space in the universal scenario with $t - b - \tau$ unification $\leq 5\%$. All the points are allowed by the Yukawa unification criterion; the asterisks are ruled out by UFB1. We set $m_D = m_{16}/5$ and $A_0 = -m_{16}$.

Figure 6: The same as fig. 5, with $m_{10} = 0.8m_{16}$.
Figure 7: The same as Fig. 5, with $m_{10} = 1.2m_{16}$.

Figure 8: The same as fig. 5, with $m_D = m_{16}/3$. 
Figure 9: The allowed parameter space in the universal scenario with $b - \tau$ unification $\leq 5\%$. All the points are allowed by the Yukawa unification criterion; the asterisks are ruled out by UFB1 and the boxes are ruled out by UFB3. We set $m_D = \frac{m_{16}}{5}$ and $A_0 = -2m_{16}$.

Figure 10: The same as Fig. 9, with $m_{10} = 0.6m_{16}$. 
Figure 11: The same as Fig. 9, with $m_{10} = 1.2m_{16}$. 