Neutrino Phenomenology – the case of two right handed neutrinos.

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Abstract

We make a general analysis of neutrino phenomenology for the case neutrino masses are generated by the see-saw mechanism with just two right handed neutrinos. We find general constraints on leptogenesis and lepton flavour violating processes. We also analyse the predictions following from a nontrivial texture zero structure.

1 Introduction

The see-saw mechanism \cite{1} for generating neutrino masses remains the most plausible one to describe the observed neutrino masses. It relates the smallness of the observed masses to the inverse of the mass scale at which the strong weak and electromagnetic couplings unify. Such a structure is natural if one extends the Standard Model to include Standard Model singlet right handed neutrino states, something that recovers the quark lepton symmetry.

However the analysis of the neutrino phenomenology coming from the see-saw mechanism is made more difficult because it involves many more parameters than can be measured from the neutrino masses and mixings. One may hope that the observed structure will reveal a simplification and that the full range of parameters is not necessary. For example it may be that an underlying symmetry relates the fundamental Yukawa couplings or elements of the Majorana mass matrix or forbids the appearance of one or more of them – “texture zeros”. Another possibility that has been explored recently in \cite{2} is that only two of the right-handed neutrinos (2RHN) play a role in the determination of neutrino properties \cite{3}. This also reduces the number of free parameters.
In order to look for the phenomenological implications of such structure it is necessary to systemetize the parameterisation of the see-saw mechanism. In [4] it was shown how this could be done using a parameterisation suggested by [5]. The method was applied to the case that there were additional texture zeros limiting the number of free parameters. In this paper we extend this analysis by exploring the phenomenological implications of the 2RHN case. We start in Section 2 with a discussion of the number of free parameters in this case and in Section 3 we determine how the general parameterisation of [4] is modified. Section 4 shows that, despite the fact that there are still two undetermined parameters, there are general constraints on leptogenesis and lepton flavour violation in the 2RHN case. To go further requires some model dependent reduction in the number of free parameters. In Section 5 we discuss how this comes about if the Yukawa couplings have one or more texture zero, presumably originating from an underlying symmetry. Section 6 determines the predictions that result if there is one texture zero and Section 7 does the same for the two texture zero case. Finally Section 8 gives our conclusions.

2 Parameter counting for the two right handed neutrino case.

If only two right handed neutrinos play a role in the see-saw mechanism there is a reduction in the number of parameters needed. To see this we consider first the case of three generations of left-handed $SU(2)$ doublet neutrinos, $\nu_{L,i}$, and three generations of right-handed Standard Model singlet neutrinos, $\nu_{R,i}$ (3RHN). The Lagrangian responsible for lepton masses has the form

$$L_{lep} = \nu^c_R Y_\nu \nu_L \langle H^0 \rangle + l^c_R Y_l l_L \langle H^0 \rangle - \frac{1}{2} \nu_R^c M_\nu \nu_R,$$

(1)

where $Y_\nu$ and $Y_l$ are the matrices of Yukawa couplings which give rise to the neutrino and charged lepton Dirac mass matrices respectively and $M_\nu$ is the neutrino Majorana mass matrix. The light neutrino mass matrix, $\mathcal{M}$, is given by the see-saw form

$$\mathcal{M} = Y_\nu^T M_\nu^{-1} Y_\nu,$$

(2)

Consider the basis in which the Majorana masses and the charged leptons are diagonal and real. In this case there are 3 Majorana masses together with 18 real parameters (9 angles and 9 phases) needed to specify $Y_\nu$. Of these, 3 phases are unphysical and can be eliminated by a redefinition of the left handed lepton doublet. However not all of the remaining parameters are independent in the way they determine $\mathcal{M}$. In particular, in the basis in which $M_\nu$ is diagonal, $D_{M_\nu} = diag(M_1, M_2, M_3)$, a simultaneous rescaling of $(Y_\nu)_ij, j = 1, 2, 3$ with $I$ fixed by $\lambda$ can be absorbed by a rescaling of $M_I$ by $\lambda^2$ so that the Majorana masses do not introduce additional parameters. As a result there are only 15 effective parameters determining the light neutrino mass matrix via the see-saw mechanism. The measureable parameters associated with the light neutrino mass matrix consist of 3 masses plus three mixing angles and 3 phases, a total of
9 measureables. That means 6 parameters associated with the see-saw mechanism are not determined by the neutrino masses, mixing angles and phases. In [5] a general parameterisation of these parameters was given. In it the most general neutrino Yukawa coupling which is compatible with low energy data, written in the basis where the charged lepton Yukawa coupling and the right-handed mass matrix are diagonal, is given by

$$Y_\nu = D_{\sqrt{M_\nu}} RD_{\sqrt{m}} U^1 / \langle H^0 \rangle,$$  \hspace{1cm} (3)

where $D_{\sqrt{M_\nu}}$ is the diagonal matrix of the square roots of the eigenvalues of $M_\nu$, $D_{\sqrt{m}}$ is the diagonal matrix of the roots of the physical masses, $m_i$, of the light neutrinos, $U$ is the Maki-Nakagawa-Sakata (MNS) matrix [6] and $R$ is a $3 \times 3$ orthogonal matrix which parameterises the information that is lost in the decoupling of all three right-handed neutrinos. Notice that we have included all the low energy phases in the definition of the matrix $U$, i.e. we have written the MNS matrix in the form $U = V \text{ diag } (e^{-i\phi/2}, e^{-i\phi'/2}, 1)$, where $\phi$ and $\phi'$ are the CP violating phases and $V$ has the form of the CKM matrix. It is important to note that $R$ can be complex as long as $R^T R = R R^T = 1$. Thus $R$ has 6 real parameters corresponding to the 6 undetermined parameters discussed above. The mass matrix, $M$, is determined by $Y_\nu D_{\sqrt{M_\nu}}^{-1}$ and, as expected, it is not separately dependent on $M_\nu$.

If only 2RHN contribute to the see-saw mechanism the number of parameters is reduced. In this case only 12 real parameters (6 moduli and 6 phases) are needed to specify $(Y_\nu)_{ij}, i = 1, 2, j = 1, 2, 3$. Allowing for 3 redundant phases and the rescaling of the 2 Majorana masses the effective number is reduced to 7 plus, of course, the 2 Majorana masses. The number of measureable parameters is only reduced by the 2 corresponding to one mass and one phase. The conclusion is that in the 2RHN case there are only $(9-7)=2$ real parameters determining the light neutrino mass matrix via the see-saw mechanism.

### 3 The 2RHN model as the decoupling limit of the 3RHN model

The 2RHN model can be regarded as the limiting case of the three right-handed neutrino model in which one of the right-handed neutrinos has an infinite mass, while all the Yukawa couplings remain perturbative. As we have discussed, the 2RHN model depends on 4 fewer parameters than the 3RHN model, and this should be reflected in the number of parameters of the matrix $R$. This may be seen by taking the limit in which one of the right handed neutrinos has an infinite mass, say $M_3$. In this case two angles in the matrix $R$ are determined. The reason is the following. From eq.(3) one finds that the elements of the third row of $R$ are given by

$$R_{3i} = \frac{(Y_\nu U)_{3i}}{\sqrt{M_3 m_i}} \langle H^0 \rangle.$$  \hspace{1cm} (4)

Since the numerator is finite and $m_2$ and $m_3$ are different from zero, $R_{32}$ and $R_{33}$ have to vanish as $M_3$ goes to infinity. On the other hand, $m_1 \to 0$ as $M_3 \to \infty$, so the
limit of $R_{31}$ is not well defined and might be non-zero. However, the orthogonality of $R$ requires $R_{31}$ to be unity and $R_{i1} = 0$, $i = 1, 2$. Therefore, in the limit $M_3 \to \infty$, the matrix $R$ takes the form:

$$R = \begin{pmatrix} 0 & \cos z & \pm \sin z \\ 0 & -\sin z & \pm \cos z \\ 1 & 0 & 0 \end{pmatrix},$$  \tag{5}$$

where $z$ is a complex angle and the $\pm$ in the third column has been included to account for the possible reflections in the orthogonal matrix $R$. It is clear that in the 2RHN model, the corresponding matrix $R$ is simply given by the first two rows of eq.(5).

Using this form for $R$ the different elements of the neutrino Yukawa matrix read:

$$Y_{\nu 1i} = \sqrt{M_1} (\sqrt{m_2} \cos z U_{2i}^* \pm \sqrt{m_3} \sin z U_{3i}^*) / \langle H^0 \rangle, \tag{6}$$

$$Y_{\nu 2i} = \sqrt{M_2} (-\sqrt{m_2} \sin z U_{2i}^* \pm \sqrt{m_3} \cos z U_{3i}^*) / \langle H^0 \rangle,$$

where $i = 1, 2, 3$. The unknown complex parameter $z$ encodes the real parameter and the phase necessary to match the total number of parameters at high energies and at low energies in the 2RHN model.

A word of caution is in order concerning the relation between the decoupling limit of the 3RHN case and the 2RHN case. In the 3RHN case the Yukawa couplings $Y_{\nu 3i}$ are not necessarily negligible due to the factor $\sqrt{M_3}$ appearing in eq.(3), and could produce some effect at low energies through the radiative corrections. Whether or not they are, depends on the magnitude of the elements $R_{3j}$, $j = 2, 3$ which vanish like $1/\sqrt{M_3}$. Moreover, the decoupling limit corresponds to the case that the third Majorana mass is at, or above, the cutoff scale so that the third neutrino does not contribute, even via radiative corrections.

### 4 Leptogenesis and lepton flavour violation in the 2RHN case.

Without making assumptions about the parameters relevant at high energy scales there are no definite predictions for the low energy neutrino parameters. However, the reduction in the number of unknown parameters with respect to the 3RHN model makes it possible to extract some general features of this case, in particular, for thermal leptogenesis [7] and for rare lepton decays induced by radiative corrections [8].

#### 4.1 Thermal leptogenesis

First, we derive some general constraints on the thermal leptogenesis scenario, assuming that the decay of the lightest right-handed neutrino is the only source of lepton asymmetry [9]. If this is the case, the CP asymmetry produced by heavy lepton decay can be written as (we will drop small correction terms of $O(m_2/m_3)$)

$$\epsilon \simeq -\frac{Im(\sin^2 z)}{|\sin^2 z| + \frac{m_2}{m_3} |\cos^2 z|}, \tag{7}$$
where $|\epsilon_{\text{max}}| \approx \frac{3}{8\pi} \frac{M_1 m_{\nu}}{(M_{\tilde{m}})^2}$ [10]. It is clear that for a suitable choice of $z$ it is possible to maximise $\epsilon$. However thermal leptogenesis can only lead to an acceptable level of baryogenesis if the subsequent washout effects are not too large. They can be conveniently characterized by [11] the parameter $\tilde{m}_1$, the parameter being bounded above if washout processes are to be limited. In the 2RHN case,

$$\tilde{m}_1 = m_2 \left| \cos^2 z \right| + m_3 \left| \sin^2 z \right|.$$  

(8)

Notice that $\tilde{m}_1 \geq m_2$ and so there is a potential conflict with the upper bound following from the condition that washout is acceptable [4], [12]. To quantify this we note that from eqs.(7) and (8) one finds

$$\left| \frac{\epsilon}{\epsilon_{\text{max}}} \right| \lesssim (1 - \frac{m_2}{\tilde{m}_1}).$$  

(9)

The problem is now obvious because, if the washout effects are minimal (minimum $\tilde{m}_1$ implying from eq.(8) that $\tilde{m}_1 = m_2$), the CP asymmetry vanishes. Physically, taking $\tilde{m}_1 \simeq m_2$ corresponds to the case in which the heaviest light neutrino state is dominated by the lightest right-handed neutrino state ($\cos z \simeq 1$). On the other hand, when the CP asymmetry is close to maximal, $\tilde{m}_1$ must be very large. In this case, to avoid washout [11], we must have a large right handed neutrino mass, $M_1 \geq 10^{11}$ GeV. In particular, this is the case for the limit in which the heaviest light neutrino state is dominated by the heaviest right-handed neutrino state ($\cos z \simeq 0$). In a supersymmetric model with gravity mediated supersymmetry breaking, the reheat temperature which is related to $M_1$ is high and could be in conflict with the gravitino overproduction constraints. However, this problem can be circumvented in other supersymmetry breaking mediation scenarios, such as gauge mediation, where the gravitino can be much lighter or through a weakening of the constraints on the reheat temperature [13].

4.2 Rare processes

The general 2RHN structure also has implications for rare processes in supersymmetric scenarios. We concentrate in the most conservative case from the point of flavour violation\(^1\), namely the class of scenarios where supersymmetry is broken in a hidden sector, and the breaking is transmitted to the observable sector by a flavour blind mechanism, like gravity. If this is the case, all the soft breaking terms are diagonal at the high energy scale, and the only source of flavour violation in the leptonic sector are the radiative corrections to the soft terms, through the neutrino Yukawa couplings. In this class of scenarios, the rate for the process $l_i \to l_j \gamma$ is given by [5]

$$BR(l_i \to l_j \gamma) \simeq \frac{\alpha^3}{G_F^2 m_S^2} \left[ \frac{1}{8\pi^2} \left( 3m_0^2 + A_0^2 \right) \right]^2 |C_{ij}|^2 \tan^2 \beta,$$  

(10)

where $C_{ij} = (Y_\nu^\dagger \log \frac{M_c}{M} Y_\nu)_{ij}$ is the crucial quantity to determine the size of these rates. In this formula, $m_S$ represents supersymmetric leptonic masses, and $m_0$ and $A_0$.

\(^1\)Therefore, our results should be understood as lower bounds on the rates for these rare processes, barring cancellations among different contributions.
Figure 1: Maximum value of the parameter $|C_{12}| = |(Y^\dagger_v \log \frac{M_X}{M} Y_v)_{12}|$ for all the see-saw scenarios that reproduce the observed masses and mixing angles, when there is no CP violation (dotted line) and when the CP violation is consistent with the baryon asymmetry of the Universe through the mechanism of leptogenesis. In this plot we have set $U_{13} = 0$ and $M_2 = 10M_1$.

are, respectively, the universal scalar soft mass and the trilinear term at the GUT scale. If all the parameters are real, we estimate these quantities to be $|C_{12}| \lesssim \mathcal{O}(0.1) \frac{\sqrt{m_2 m_3}}{(H)} M_2 \log \frac{M_X}{M_2}$ and $|C_{13,23}| \sim \sqrt{\frac{m_3}{m_2}} C_{12}$. The predicted rates for the rare processes are therefore rather small, unless $M_2$ is large. For instance, from the present bound on the process $\mu \to e\gamma$ one can set the bound $M_2 \leq 10^{14}$ GeV. This bound could be improved by one order of magnitude if the next generation of experiments reach the projected sensitivity, $\text{BR}(\mu \to e\gamma) \leq 10^{-13}$.

However, if leptogenesis is the correct mechanism to generate the baryon asymmetry of the Universe, the parameter $z$ has to be complex and this could enhance the rates for the rare processes. We concentrate on the combination $C_{12}$ that is related to the process $\mu \to e\gamma$. In Figure 1 we show the maximum value of $|C_{12}|$ as a function of the lightest right-handed neutrino mass, $M_1$, assuming $U_{13} = 0$ and $M_2 = 10M_1$. For other hierarchies of right-handed masses, the results scale roughly as $(M_2/M_1)^2$. In this plot, we take random values for $\phi'$ and $z$, and we fix the lightest right-handed neutrino mass by requiring a correct baryon asymmetry $\eta_B \simeq 6 \times 10^{-10}$, as reported by the WMAP collaboration. To compute the baryon asymmetry, we used the approximate treatment of the dilution effects described in [14]. In the plot we have also used the best fit points for the solar angle and the neutrino masses reported in [15], and we have assumed a hierarchical spectrum of neutrino masses. The enhancement in the rates after including the constrains from leptogenesis illustrates the interesting interplay between low energy lepton flavour violation and leptogenesis in supersymmetric scenarios.
5 Parametrization of the see-saw mechanism in the texture zero basis

As was discussed in [4], further assumptions about the physics relevant at a high energy scale can reduce the number of parameters and even lead to relations among the low energy observables. In the remainder of this paper we consider the possibility that one or more elements of the Yukawa coupling matrix are anomalously small and can be ignored, the so-called “texture zeros”. For example, one texture zero in the neutrino Yukawa matrix would fix the matrix $R$ up to “reflections”, and two texture zeros would lead to relations among the mixing angles and the neutrino masses. One reason for the interest in texture zeros is that they may indicate the presence of a new family symmetry which require certain matrix elements be anomalously small. Thus identification of texture zeros may be an important step in unravelling the origin of the fermion masses and mixings. It is already known that the measured quark masses and mixing angles are consistent with such texture zeros [16],[17]. In this and the following sections we will explore the different consequences at low energies of the 2RHN model assuming that there are texture zeros.

In general, texture zeros do not appear in the basis where the charged lepton Yukawa couplings and the right-handed mass matrix are simultaneously diagonal. To allow for this we write the Lagrangian responsible for lepton masses in the texture zero basis as:

$$L^{TZ}_{lep} = \nu^c_R Y_{\nu}^{TZ} \nu_L \langle H^0 \rangle + \nu^c_R Y_{l}^{TZ} l_L \langle H^0 \rangle - \frac{1}{2} \nu^c_R M^{TZ}_\nu \nu_R .$$  \hspace{1cm} (11)

We can diagonalize the charged lepton Yukawa matrix by $Y_{\nu}^{TZ} = V_l D Y_{\nu} U_l^\dagger$ and the right-handed mass matrix by $M^{TZ}_\nu = V_{\nu}^* D M_{\nu} V_{\nu}^\dagger$. Then, the most general neutrino Yukawa coupling that is compatible with the low-energy data, written in the texture zero basis, is given by:

$$Y_{\nu}^{TZ} = V_{\nu}^* D \sqrt{M_{\nu}} R D \sqrt{m} W^\dagger / \langle H^0 \rangle, \hspace{1cm} (12)$$

where $W = U_l U_l$. Here, $R$ is

$$R = \begin{pmatrix} 0 & \cos z & \pm \sin z \\ 0 & -\sin z & \pm \cos z \end{pmatrix} . \hspace{1cm} (13)$$

In the texture zero basis the right-handed mass matrix can be diagonalized by a unitary matrix that in general depends on three phases and one rotation angle. One of these phases cannot be removed by redefinitions of the right-handed neutrino fields, due to the Majorana nature of these particles. On the other hand, the other two can be removed without altering the texture zero structure of the Yukawa matrices. Hence, in our texture zero basis, the unitary matrix $V_{\nu}$ can be parametrized by

$$V_{\nu} = \begin{pmatrix} \cos \omega & \sin \omega \\ -\sin \omega & \cos \omega \end{pmatrix} \cdot \begin{pmatrix} 1 \\ e^{-i\alpha} \end{pmatrix} , \hspace{1cm} (14)$$

where $\omega$ is the mixing angle, and $\alpha$ is the Majorana phase.
There remains the question of the form of the lepton mixing matrix in the texture basis, needed to determine $W$. The form of this has been discussed in [4]. For the case the lepton mass matrix has off diagonal elements whose magnitude is approximately symmetric and that, like the quarks, the hierarchy of lepton masses is due to an hierarchical structure in the matrix elements and not due to a cancellation between different contributions one has the bounds

$$
|(U_l)_{23}| \leq \sqrt{\frac{m_\mu}{m_\tau}},
$$
$$
|(U_l)_{12}| \leq \sqrt{\frac{m_e}{m_\mu}},
$$
$$
|(U_l)_{13}| \leq \sqrt{\frac{m_e}{m_\tau}}.
$$

(15)

In addition, it is necessary to determine the phases in $U_l$. There is a residual phase ambiguity because the basis in which the MNS matrix has the standard form can be different from the “symmetry” basis in which the texture zero appears. This corresponds to the simultaneous redefinition of the phase of the left- and right-handed states such that the Dirac structure is invariant. With this we have $W = |U_l|P$ where $P = \text{diag}(e^{i\alpha_1}, e^{i\alpha_2}, e^{i\alpha_3})$. In practice the magnitudes of $(U_l)_{23}$ and $(U_l)_{13}$ are so small that they do not affect the mixing coming from the neutrino sector. However $(U_l)_{12}$ close to the upper bound given in eq.(15) does give a significant contribution to the CHOOZ angle. Its effect is considered below. If the magnitude of the charged lepton mass matrix elements departs significantly from the symmetric form there is no constraint on the magnitude of the matrix elements of $U_l$. In this case the contributions to the MNS matrix coming from the neutrino sector should be considered as an indication of the lower bound on the MNS matrix elements, assuming there is no delicate cancellation between the contributions of $U_l$ and $U_\nu$.

With this parametrization of the see-saw mechanism, it is straightforward to compute predictions at low energies from texture zeros. One texture zero in the texture zero basis would fix the angle, $z$, in $R$; two texture zeros would yield relations among physical parameters (light neutrino masses, mixing angles in $W$ and right handed sector parameters). Although some of these physical parameters are not measurable, for instance the right handed sector parameters, this approach will allow us to predict ranges for the measurable quantities, in particular the CHOOZ angle.

## 6 Predictions from models with one texture zero

One texture zero in the neutrino Yukawa coupling fixes the unknown parameter $z$. From eq.(6), we obtain

$$
\tan z = \pm \sqrt{\frac{m_2}{m_3}} W_{i2}^* \pm \sqrt{\frac{M_2}{M_3}} \sqrt{\frac{m_2}{m_3}} W_{i3}^* e^{i\alpha} \tan \omega
$$

$$
\sqrt{\frac{M_2}{M_1}} \sqrt{\frac{m_2}{m_3}} W_{i3}^* e^{i\alpha} \tan \omega,
$$

(16)
Figure 2: Allowed regions in the plane $\tilde{m}_1 - |\frac{e}{e_{\text{max}}}|$, for the neutrino Yukawa couplings with one texture zero that are compatible with all the available data at low energies. The shaded area applies to the case $Y_{21} = 0$ and the hatched area to the case $Y_{11} = 0$. The dotted line shows the allowed area for the cases $Y_{12} = 0$ or $Y_{13} = 0$ and the dashed line shows the allowed area for the cases $Y_{22} = 0$ or $Y_{23} = 0$. In this plot we have assumed that the texture zero appears in the basis where the charged lepton Yukawa coupling and the right-handed mass matrix are both diagonal.

when $Y_{1i}^{TZ} = 0$, and

$$\tan z = \pm \sqrt{\frac{m_3}{m_2}} \frac{W_{i3}^*}{W_{i2}} \sqrt{\frac{M_1}{M_2}} \frac{m_3}{m_2} W_{i2}^* e^{-i\alpha} \tan \omega$$

(17)

when $Y_{2i}^{TZ} = 0$. With only these hypotheses, there are no predictions for the low energy parameters. However, fixing $z$ imposes further restrictions on the leptogenesis parameters $\epsilon$ and $\tilde{m}_1$. These are summarized in Figure 2, where we show the allowed regions in the plane $\tilde{m}_1 - |\frac{e}{e_{\text{max}}}|$. For the (1,1) and (2,1) texture zeros, $\tan z$ depends on the CHOOZ angle, which has not been measured. So, in the plot we have taken the CHOOZ angle between zero and 0.23, which is the 3$\sigma$ bound from the global analysis [15].

From the figure, we see that Yukawa couplings with zeros in the (2,2) and (2,3) positions are very disfavoured from the point of view of leptogenesis. They yield small CP asymmetries, $|\frac{\epsilon}{\epsilon_{\text{max}}}| \lesssim \frac{m_2}{m_3}$, and large washout effects, $\tilde{m}_1 \simeq m_3$. Similarly for the Yukawa coupling with a texture zero in the (1,1) position. On the other hand, for Yukawa couplings with a texture zero in the (2,1) position the bound is almost identical to the model independent bound $|\frac{\epsilon}{\epsilon_{\text{max}}}| \lesssim 1 - \frac{m_2}{m_3}$. However, a stronger bound on the CHOOZ angle will make the allowed region smaller at high values of $\tilde{m}_1$. Yukawa matrices with a (1,2) or (1,3) texture zero are also favoured from the point of view of leptogenesis. They yield almost minimal washout effects, $\tilde{m}_1 \simeq m_2(1 + \cos^2 \theta_{12})$ and
relatively large CP asymmetries, \(|\frac{\epsilon_1}{\epsilon_{\text{max}}}| \lesssim \frac{\cos^2 \theta_{12}}{1 + \cos^2 \theta_{12}}\).

In Figure 2 we have neglected the contributions to the mixing from the charged lepton sector and the right-handed Majorana sector. These mixings do not qualitatively change the results. For example, for the case for the texture zero in the (2,2) position we still find most of the points concentrated in the region around the dashed line in Figure 2. There are only a few points saturating the model independent bound, eq.(9), that correspond to special choices of the right-handed parameters.

An interesting issue that has been discussed extensively in the literature concerns the connection of leptogenesis and low energy observables [18]. In particular the correlation between the sign of the baryon asymmetry and the CP violation at low energies has been discussed [2]. This connection is clear only when \(U_l \simeq 1\) and \(V_\nu \simeq 1\), otherwise some unmeasurable parameters in the charged-lepton sector or the right-handed sector enter into play. We find that one texture zero is enough to establish such connection, since the sign of the CP asymmetry is determined by minus the argument of \(\tan^2 \varphi\), which in turn is fixed with one single texture zero. For instance, when the texture zero appears in the (1,2) or (1,3) position,

\[
\frac{\epsilon_1}{\epsilon_{\text{max}}} \simeq -\frac{\sin \phi'}{1 + |W_{i2}|^2},
\]

\[
\tilde{m}_1 \simeq m_2(1 + |W_{i2}|^2),
\]

(18)

where \(i = 2, 3\). On the other hand, when it appears in the (2,2) or (2,3) position,

\[
\frac{\epsilon_1}{\epsilon_{\text{max}}} \simeq +\sin \phi', \frac{m_2 m_3}{m_3} |W_{i2}|^2,
\]

\[
\tilde{m}_1 \simeq m_2.
\]

Finally, the case when the texture zero appears in the first column deserves some more careful analysis, since it involves the CHOOZ angle which has not been measured. The expressions are rather complicated, but can be readily computed from eqs.(7, 8) and eqs.(16, 17). We just show the numerical results in Figure 3, where we plot the CP asymmetry divided by \(\sin(\phi' - 2\delta)\), to show better the connection between the sign of the CP asymmetry and the low energy CP violation. The lepton asymmetry and \(\sin(\phi' - 2\delta)\) have the same sign for the (2,1) texture zero and opposite for the (1,1) texture zero (the sign of the baryon asymmetry is opposite to the sign of the lepton asymmetry).

7 Predictions from models with two texture zeros

We can write now the predictions from two texture zeros in the neutrino Yukawa matrix. When the two texture zeros appear in the same row, \(Y_{1i}^{TZ} = 0, Y_{1j}^{TZ} = 0\) then

\[
\epsilon_{ijk}W_{k1}(e^{-i\alpha}M_1 \cos^2 \omega + e^{i\alpha}M_2 \sin^2 \omega) = 0.
\]

(19)

or if \(Y_{2i}^{TZ} = 0, Y_{2j}^{TZ} = 0\) then

\[
\epsilon_{ijk}W_{k1}(e^{-i\alpha}M_1 \sin^2 \omega + e^{i\alpha}M_2 \cos^2 \omega) = 0.
\]

(20)
On the other hand, when the texture zeros appear in different rows, $Y_{11}^{TZ} = 0$, $Y_{2j}^{TZ} = 0$, the following relation holds:

$$W_{i3}W_{j3} + \frac{m_2}{m_3}W_{i2}W_{j2} \pm \sqrt{\frac{m_2}{m_3}} \sin \omega \cos \omega (e^{i\alpha} \sqrt{\frac{M_1}{M_2}} - e^{-i\alpha} \sqrt{\frac{M_2}{M_1}}) \epsilon_{ijk}W_{k1}^* \det W = 0, \quad (21)$$

where the ± comes from the two possible reflections in $R$, see eq.(5).

Let us first discuss the results for the case where $U_l \simeq 1$ and $V_\nu \simeq 1$, i.e. when all the mixing in the neutrino sector comes from the neutrino Yukawa matrix, and later on the general case, when there are also contributions coming from the right-handed Majorana sector and the charged-lepton sector.

### 7.1 The case with $U_l \simeq 1$ and $V_\nu \simeq 1$

Under these assumptions it is straightforward to compute the predictions at low energies for all the fifteen possible Yukawa matrix with two texture zeros$^2$. These are summarized in Table 1, where we have set the atmospheric angle to the experimentally favoured maximal value. Only five of them are allowed by present experiments, namely textures IV, VII, and VIII. The matrix with texture zeros in the same column leads to the prediction for the CHOOZ angle $s_{13} \simeq \sqrt{\frac{m_2}{m_3}} \sin \theta_{sol} \simeq 0.22$, which is marginally

$^2$Some related analyses for this case can also be found in [2].
Table 1: Predictions following from the various two texture zero structures.

| Texture for $Y_{\nu}$ | Predictions |
|-------------------------|--------------|
| I
| \((0 \ 0 \ \times)\times \ (\times \ \times \ \times)\) \((\times \ \times \ \times)\times \ (0 \ \times \ \times)\) | $U_{31} = 0$ |
| II
| \((0 \ \times \ \times)\times \ (\times \ \times \ \times)\) \((\times \ \times \ \times)\times \ (0 \ \times \ \times)\) | $U_{21} = 0$ |
| III
| \((\times \ 0 \ \times)\times \ (\times \ \times \ \times)\) \((\times \ \times \ \times)\times \ (0 \ \times \ \times)\) | $U_{11} = 0$ |
| IV
| \((0 \ \times \ \times)\times \ (\times \ \times \ \times)\) \((\times \ \times \ \times)\times \ (0 \ \times \ \times)\) | $U_{13} \simeq \pm i \sqrt{\frac{m_2}{m_3}} \sin \theta_{sol} e^{-i\phi'/2}$ |
| V
| \((\times \ 0 \ \times)\times \ (\times \ \times \ \times)\) \((\times \ \times \ \times)\times \ (0 \ \times \ \times)\) | $U_{23} \simeq \pm \sqrt{\frac{m_2}{2m_3}} \cos \theta_{sol} e^{-i\phi'/2}$ |
| VI
| \((\times \ \times \ \times)\times \ (\times \ \times \ \times)\) \((\times \ \times \ \times)\times \ (0 \ \times \ \times)\) | $U_{33} \simeq \pm \sqrt{\frac{m_2}{2m_3}} \cos \theta_{sol} e^{-i\phi'/2}$ |
| VII
| \((0 \ \times \ \times)\times \ (\times \ 0 \ \times)\) \((\times \ \times \ \times)\times \ (0 \ \times \ \times)\) | $U_{13} \simeq -\frac{m_2}{2m_3} \sin 2\theta_{sol} e^{-i\phi'}$ |
| VIII
| \((\times \ 0 \ \times)\times \ (\times \ \times \ \times)\) \((\times \ \times \ \times)\times \ (0 \ \times \ \times)\) | $U_{13} \simeq \frac{m_2}{2m_3} \sin 2\theta_{sol} e^{-i\phi'}$ |
| IX
| \((\times \ 0 \ \times)\times \ (\times \ 0 \ \times)\) \((\times \ \times \ \times)\times \ (0 \ \times \ \times)\) | $U_{23} \simeq \frac{1}{\sqrt{2}} \frac{m_2}{m_3} \cos^2 \theta_{sol} e^{-i\phi'}$ |

allowed, and a CP violating phase $\delta \simeq \phi'/2$. On the other hand, the other four possibilities yield $s_{13} \simeq \frac{m_2}{2m_3} \sin 2\theta_{sol} \simeq 0.08$ and a phase $\delta \simeq \phi'$, for the textures with zeros in the first and third columns, or $\delta \simeq \phi' + \pi$, for the textures with zeros in the first and second columns.

These four textures cannot be discriminated only with neutrino oscillation experiments. However, if the prediction for the CHOOZ angle is confirmed it would be interesting to determine which is the actual Yukawa matrix at high energies. As we discussed in the introduction, under some well motivated hypothesis on the theory at high energies, rare processes and leptogenesis provide additional information about the neutrino Yukawa matrices. To quantify this we will assume, as is usually done, that all the soft SUSY breaking terms are flavour diagonal at high energies, and that the only particles present in the spectrum are the MSSM particles and the two right-handed neutrino superfields. Under these assumptions, those four allowed textures could be discriminated through their predictions for rare processes. The texture VII yields, at leading order, a vanishing rate for $\mu \rightarrow e\gamma$. On the other hand, the matrix with texture VII gives different predictions for $\tau \rightarrow e\gamma$ and $\tau \rightarrow \mu\gamma$ than the one with texture VII. Analogously, neutrino Yukawa matrices with texture VIII yield vanishing rates for $\tau \rightarrow e\gamma$, but different predictions for $\mu \rightarrow e\gamma$ and $\tau \rightarrow \mu\gamma$. All these results are summarized in Table 2.

The analysis for the remaining allowed texture, texture IV, is more complicated, since the matrix with strict texture zeros would require non perturbative Yukawa couplings to reproduce the low energy masses and mixing angles. If one keeps track of
these texture zeros in the analysis, one can see that the combination $|\langle Y_\nu \rangle \log \frac{M_X}{M} Y_{\nu 23}\rangle_12$ diverges as $M_1/Y_{\nu 11}^2$ or $M_2/Y_{\nu 21}^2$, depending on which of them is larger. However, this texture would still predict $U_{13} \simeq \pm i \sqrt{\frac{m_3}{m_2}} U_{12}$ as long as $\frac{Y_{\nu 11}^2}{M_1} + \frac{Y_{\nu 21}^2}{M_2} \ll \frac{m_3}{(H^0)^2}$, and all the Yukawa couplings would remain perturbative provided the right-handed masses are not too large ($M_2 \lesssim \frac{(H^0)^2}{m_3}$). It is interesting to note that the possibility $|\langle Y_\nu \rangle \log \frac{M_X}{M} Y_{\nu 23}\rangle_12 \simeq 1$, is not excluded, and would yield rates for $\tau \rightarrow \mu \gamma$ at the reach of future experiments, while preserving the prediction for the CHOOZ angle. Finally, one can check that when $\frac{Y_{\nu 21}^2}{M_1}$ and $\frac{Y_{\nu 21}^2}{M_2}$ are sufficiently small to keep the prediction $U_{13} \simeq \pm i \sqrt{\frac{m_3}{m_2}} U_{12}$ approximately valid, one obtains

$$|\langle Y_\nu \rangle \log \frac{M_X}{M} Y_{\nu 23}\rangle_12 \simeq |\langle Y_\nu \rangle \log \frac{M_X}{M} Y_{\nu 13}\rangle_1 \simeq \frac{M_k \sqrt{m_2 m_3}}{(H^0)^2} \frac{\sin \theta_{sol}}{\sqrt{2}} \log \frac{M_X}{M_k},$$

(22)

where $k = 2$ if $Y_{\nu 21}^2 \gg Y_{\nu 11}^2$, and $k = 1$ otherwise.

The only unknown parameters in $|\langle Y_\nu \rangle \log \frac{M_X}{M} Y_{\nu i}\rangle|$, the combination relevant for radiative corrections, are the right-handed neutrino masses. Therefore, for each texture, one could set constraints on these masses from the bounds on the rates of rare decays. Unfortunately, the constraints are very weak: for typical values of the soft SUSY masses and $\tan \beta = 3$ one obtains, from the present bounds on $\mu \rightarrow e \gamma$, that $M_1 \lesssim 9 \times 10^{13}$ GeV for the texture VIII, and $M_2 \lesssim 9 \times 10^{13}$ GeV for VIII. These bounds could be improved by one order of magnitude with the next generation of experiments, that expect to reach a sensitivity on the branching ratio of $10^{-13}$. From $\tau \rightarrow \mu \gamma$ or $\tau \rightarrow e \gamma$ the upper bounds on the right-handed masses are even weaker, of the order of $10^{15}$ GeV. Moreover, theoretical guesses of the right-handed neutrino masses, coming from scenarios of thermal leptogenesis or particular models, would yield rates for the rare processes which are too small to be observed in the next generations of experiments, if radiative corrections induced by right-handed neutrinos are the only source of flavour violation in the slepton sector.

The high predictivity of these textures also allows us to obtain information about the CP asymmetry generated in the decay of the lightest right-handed neutrino and the parameter $m_1$, which are important for thermal leptogenesis and some models of
Table 3: Leptogenesis parameters written in terms of low energy observables. The proportionality constant that relates $|\epsilon_1|$ with the branching ratios for the rare processes depends on supersymmetric parameters, and can be read from eq.(10).

non-thermal leptogenesis. For the texture IV, leptogenesis is not likely to occur since washout effects are very large (in this case, $\tilde{m}_1$ diverges as $\nu_{11}^{-2}$ or $\nu_{21}^{-2}$). The results for the remaining four allowed textures are shown in Table 3. These expressions for the CP asymmetry are all written in terms of parameters that are in principle measurable at low energies (the solar angle, neutrino masses and the Majorana phase), plus the lightest right-handed neutrino mass. As we have just discussed, this mass is related to some matrix elements of $(Y^{\dagger}_\nu M X Y^\nu)$ and consequently to the rates for rare decays in certain scenarios of supersymmetry breaking. Therefore, leptogenesis parameters can be written only in terms of quantities that are, in principle, measurable at low energies. The explicit expressions are shown in Table 3. As before, given the expected range of values for the CP asymmetry in thermal leptogenesis, one could obtain bounds on the rates of the rare decays. Again the constraints are very weak: for example, for the value of the CP asymmetry hinted at by thermal leptogenesis, $|\epsilon_1| \sim 10^{-6}$, one obtains branching ratios of the order of $10^{-18}$. If the only source of lepton flavour violation are the neutrino Yukawa couplings, the observation of any of these rare processes at rates larger than this would imply an overproduction of baryon asymmetry in the Universe in the two right-handed neutrino scenario.

7.2 The general case

We consider now the effect of $U_l$ and $V_\nu$. Under the hypotheses on the charged lepton sector explained in section 5, it is clear from eqs.(19) and (20) that in the general case the textures I, II and III with texture zeros in the same row are still excluded. Similarly the textures V, VI and IX are also excluded. On the other hand texture IV leads to the relation

$$U_{13} \simeq -\epsilon_{13}^{10} \sqrt{\frac{m_\mu}{m_\nu}} U_{23} + i \sqrt{\frac{m_2}{m_3}} U_{12}$$

yielding $0.028 \lesssim |U_{13}| \lesssim 0.13$. 

Textures VII yield the prediction for the CHOOZ angle

$$U_{13} \simeq -e^{i\alpha_1} \sqrt{\frac{m_\mu}{m_\mu}} U_{23} - \frac{m_2}{m_3} U_{12} U_{23} - e^{i\beta} \sqrt{\frac{m_3}{m_3}} \frac{M_2}{M_1} \sin \omega \cos \omega \frac{U_{31}^*}{U_{23}},$$

(24)

where $\beta$ is an unknown phase, in which definition we have also absorbed the indeterminacy coming from the two possible “reflections” in the matrix $R$. On the other hand, the textures VIII give

$$U_{13} \simeq -e^{i\alpha_1} \sqrt{\frac{m_\mu}{m_\mu}} U_{23} - \frac{m_2}{m_3} U_{32} U_{33} + e^{i\beta} \sqrt{\frac{m_3}{m_3}} \frac{M_2}{M_1} \sin \omega \cos \omega \frac{U_{21}}{U_{33}},$$

(25)

The predictions for the CHOOZ angle are very similar for textures VII and VIII, because of the large angles in the atmospheric and the solar sector. In Figure 4 we show the predictions for the Yukawa matrix with texture VII as a function of $\sqrt{M_2/M_1} \sin \omega \cos \omega$. In this plot, we assign random numbers to the unmeasured phase $\phi'$ and to the unknown phase $\beta$, and show the regions at 1\(\sigma\) (darkest) and 2\(\sigma\) (lightest) from the main value. We prefer to leave $\omega$ as a completely free parameter, since we do not have any hint about the Majorana sector and we do not know whether the mixing angle $\omega$ is large or small. If the right-handed mass matrix is hierarchical, one expects this mixing angle to be small, unless some fine-tuning is taking place. To be precise, one expects $\sin \omega \cos \omega \leq \sqrt{M_1/M_2}$, being the bound saturated when there is a zero in the (1,1) position of the right-handed mass matrix. So, in this case the allowed parameter space is $0 \leq \sqrt{M_2/M_1} \sin \omega \cos \omega \leq 1$. On the other hand, it could happen that the eigenvalues in the Majorana mass matrix are quasi-degenerate and the mixing angles could be large without any fine-tuning. If this is the case, it also happens that $0 \leq \sin \omega \cos \omega \leq \sqrt{M_1/M_2} \leq 1$. The plot shown in Figure 4 covers all the possible natural structures in the right-handed mass matrix.

Finally, for the texture IX the prediction is

$$U_{23} \simeq -\frac{m_2}{m_3} U_{22} U_{32} U_{33} + e^{i\beta} \sqrt{\frac{m_3}{m_3}} \frac{M_2}{M_1} \sin \omega \cos \omega \frac{U_{11}}{U_{33}}.$$

(26)

The numerical results for this case are shown in Figure 5. The inclusion of the right-handed mixing effects is not enough to reproduce the experimental value $|U_{23}| \simeq 1/\sqrt{2}$. Therefore, this texture is disfavoured at the 2\(\sigma\) level.

The conclusions about the rates of rare decays and leptogenesis are qualitatively similar to the case in which $U_l \simeq 1$ and $V_{\nu} \simeq 1$, namely that the rare processes are observable and the CP asymmetry is large enough only when the right-handed masses are large.

8 Conclusions

In the case that there are only two right-handed neutrinos or, in the three neutrino case, that the heaviest one decouples, leads to relations amongst observable properties of neutrinos [4]. Here we have presented a general analysis of this possibility. In
Figure 4: Prediction for the CHOOZ for textures VII and VIII, allowing contributions to the mixing from the right-handed sector (encoded in $\sqrt{M_2/M_1} \sin \omega \cos \omega$) and the charged lepton sector.

Figure 5: Prediction for the element (2,3) of the MNS matrix for the textures IX, allowing contributions to the mixing from the right-handed sector (encoded in $\sqrt{M_2/M_1} \sin \omega \cos \omega$) and the charged lepton sector.
this case the number of parameters involved in the see-saw mechanism is significantly reduced from 6 to 2 leading to some general phenomenological implications. In particular, adequate baryogenesis through thermal leptogenesis is problematic due to large washout processes and we give an upper bound on the magnitude of the asymmetry. Inhibiting these, requires a large right handed neutrino mass which causes problems in supersymmetric theories due to excessive gravitino production. In the 2RHN case there are also strong constraints between the level of leptogenesis and lepton flavour violating processes. Requiring that these latter processes be acceptable puts an upper bound on the heaviest of the right-handed neutrino masses.

Further phenomenological implications apply if the two remaining parameters are constrained due to some underlying symmetry of the theory. Such symmetries can give rise to anomalously small elements, “texture zeros”, in the matrix of Yukawa couplings. We have made a complete study of the cases that there are one or two such texture zeros in neutrino sector of Yukawa couplings. For the case of one texture zero we find constraints on the thermal leptogenesis parameters. The case of texture zeros in the (1,1), (2,2) and (2,3) positions give small asymmetries, below the model independent bound found for the general case. The case of texture zeros in the (2,1), (1,2) or (1,3) is more promising, nearly saturating the upper bound. An interesting issue in these cases is the connection between leptogenesis and the low energy phases and we have determined this in the cases that the contribution to mixing from the charged lepton and Majorana sectors are small.

For the case of two texture zeros we find a prediction for the CHOOZ angle and we have classified all possible cases and identified the viable ones. The various viable possibilities cannot be distinguished on the basis of the CHOOZ angle alone but we point out that it may be possible to do so from lepton flavour violating processes. For the viable cases we have also determined the connection between leptogenesis and the low energy phases, again under the assumption that the contribution to mixing from the charged lepton and Majorana sectors are small. Finally we considered the more general case in which the contribution to mixing from the charged lepton and Majorana sectors is non-negligible. Although less predictive we are still able to eliminate several possibilities. For the remainder we determined the expected range of the CHOOZ angle consistent with the texture zero structure.

Acknowledgements

This work was partly funded by the PPARC rolling grant PPA/G/O/2002/00479 and the EU network “Physics Across the Present Energy Frontier”, HPRV-CT-2000-00148.

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