Amplitude analysis of the $\bar{N}N \rightarrow K^-K^+$ reaction

W.M. Kloet

Department of Physics and Astronomy, Rutgers University,
Piscataway, New Jersey 08855, USA

F. Myhrer

Department of Physics and Astronomy, University of South Carolina,
Columbia, South Carolina 29208, USA

(September 10, 2018)

Abstract

A simple partial wave amplitude analysis of $\bar{p}p \rightarrow K^-K^+$ has been performed for data in the range $p_{\text{lab}} = 360 - 1000$ MeV/c. In this low momentum interval only partial wave amplitudes with $J$ equal to 0, 1 and 2 are needed to obtain a good fit to the experimental data. This maximal $J = 2$ value is smaller than what is required for the data of the reaction $\bar{p}p \rightarrow \pi^-\pi^+$ in the same momentum interval.

13.75.Cs, 13.75.Jz, 13.88.+e, 25.43.+t
The reaction $\ov{NN} \to K^-K^+$ differs from the reaction $\ov{NN} \to \pi^-\pi^+$ in that at least two pairs of valence quarks and antiquarks must be annihilated and a strange quark antiquark pair must be created. We expect the $\ov{pp} \to K^-K^+$ reaction to take place in a smaller interaction volume than the $\ov{NN} \to \pi^-\pi^+$ reaction. A reason for this expectation is illustrated by the analogy with the atomic annihilation processes of $\mu^+e^-$ and $\mu^-e^+$ into three possible final states which are respectively $\mu^+\mu^- + e^+e^-$ (rearrangement), $\mu^+\mu^- + \gamma$ (one lepton pair annihilation) and $\gamma + \gamma$ (two lepton pair annihilation). In this atomic case there is a sharp decrease in interaction volume for the three respective mechanisms of annihilation, as mentioned for example in Ref. [1]. Since such arguments are based on QED and not on strong interactions this analogy should only be taken as an indication of what might be occurring when comparing the two annihilation channels of $\ov{NN}$ into $K^-K^+$ and $\pi^-\pi^+$, which is the topic of the present paper. If this analogy has merit, we expect that, in terms of a partial wave analysis of $\ov{pp} \to K^-K^+$, fewer partial waves will be active in the $K^-K^+$ annihilation channel as compared to the $\pi^-\pi^+$ channel. This feature is not readily apparent in the data for $\ov{pp} \to \pi^-\pi^+$ and $\ov{pp} \to K^-K^+$ from the CERN Low Energy Antiproton Ring (LEAR) [2]. At the lowest energies the measured $d\sigma/d\Omega$ and analyzing power $A_{an}$ show a rather rich angular dependence in both reactions. This dependence changes rapidly with increasing energy. A detailed analysis of these very good LEAR data may shed some light on the microscopic annihilation and subsequent hadronization processes involved and guide us in obtaining a model understanding of these elementary annihilation reactions.

In this paper we report on a simple amplitude analysis of the $\ov{pp} \to K^-K^+$ data in the restricted momentum range, $p_{lab} = 360 - 988$ MeV/c. The method of analysis and the assumptions are the same as were used in the reaction $\ov{NN} \to \pi^-\pi^+$ [7]. Because we know of no reliable model for any of these two annihilation processes to guide or to restrict this analysis, we reduce the theoretical input of this analysis to a minimum. We assume only that very few partial waves contribute to this annihilation reaction. This means that only partial waves with $J$ smaller or equal to $J_{max}$ ($J_{max}$ is one of the parameters in this analysis)
contribute to the observables. In a previous paper we analyzed the $\bar{N}N \to \pi^-\pi^+$ reaction based on a $\chi^2$ fit and we found that all the amplitudes with $J \leq J_{max} = 3$ were necessary and sufficient to fit the data below 1 GeV/c.

Similar to the reaction $\bar{N}N \to \pi^-\pi^+$ there are also two independent helicity amplitudes $f_{++}$ and $f_{+-}$ for the annihilation reaction $pp \to K^-K^+$. The two measured observables are the differential cross section

$$d\sigma/d\Omega = (|f_{++}|^2 + |f_{+-}|^2)/2,$$

and the analyzing power $A_{on}$, defined by

$$A_{on} d\sigma/d\Omega = Im(f_{++} f_{+-}^*).$$

The convention is used where $\hat{n}$ is the spin direction normal to the scattering plane. The unit-vector $\hat{n}$ is along $\vec{p} \times \vec{q}$, where $\vec{p}$ is the antiproton center-of-mass (c.m.) momentum and $\vec{q}$ is the c.m. momentum of $K^-$. Additional spin observables are the spin-correlations $A_{ss}$ and $A_{ls}$, which are expressed by the following helicity amplitude combinations:

$$A_{ss} d\sigma/d\Omega = (|f_{++}|^2 - |f_{+-}|^2)/2,$$

and

$$A_{ls} d\sigma/d\Omega = Re(f_{++} f_{+-}^*).$$

However, there are as yet no data on spin-correlations for this reaction.

The two helicity amplitudes are expanded in $J \neq L$ spin-triplet partial wave amplitudes

$$f_{++} = \frac{1}{p} \sum_{J} \sqrt{J + \frac{1}{2}} \left( \sqrt{J} f_{J-1}^J - \sqrt{J+1} f_{J+1}^J \right) P_J(cos\theta)$$

and

$$f_{+-} = \frac{1}{p} \sum_{J} \sqrt{J + \frac{1}{2}} \left( \frac{1}{\sqrt{J}} f_{J-1}^J + \frac{1}{\sqrt{J+1}} f_{J+1}^J \right) P_J'(cos\theta) sin\theta.$$
where \( p \) is the \( \overline{p}p \) center of mass momentum and where \( P'_J \) denotes the first derivative of the Legendre polynomial \( P_J \).

In the data analysis of \( \overline{p}p \to K^-K^+ \), we parameterize the partial wave amplitudes at each energy as

\[
f^J_L = R^J_L e^{i \delta^J_L}.
\]

(7)

where \( R^J_L \) and \( \delta^J_L \) are free parameters. At each energy we choose the \( J_{\text{max}} \) to be included in our \( \chi^2 \) search and find the best fit to both \( d\sigma/d\Omega \) and \( A_{\text{on}} \) by minimizing the \( \chi^2 \) sum. In our fits we choose \( \delta_{10} = 0 \) for the \( ^3P_0 \) partial wave whereas \( R_{10} \) is a free parameter. For all other \( LJ \) values both phase and amplitude in Eq. (7) are allowed to vary to obtain the best fit. In our analysis we did not try to correlate the amplitudes at the different energies by a smoothness procedure. We did however use the set of amplitude values, determined at one energy, as start values in the \( \chi^2 \) search at the neighboring energy. Due to the incompleteness of the set of measured observables, we do not find a unique solution, i.e., a unique set of partial wave amplitudes in our \( \chi^2 \) search. However, the minimal values of \( \chi^2 \) found in the various possible fits are the same. As discussed in our analysis of \( N\overline{N} \to \pi^-\pi^+ \) [7], if we had available data on other spin observables it would be possible to restrict the choice among the various amplitude-sets with equally good \( \chi^2 \).

The data for all measured energies starting from \( p_{\text{lab}} = 360 \) MeV/c up to 1 GeV/c can be fitted with partial wave amplitudes with total angular momentum \( J \leq 2 \). We have also fitted the data with a maximal \( J = 3 \) using the same procedure. It appears that for \( \overline{p} \) momenta, \( p_{\text{lab}} \), above 886 MeV/c the total \( \chi^2 \) improves when we include the \( J = 3 \) partial wave amplitudes, but below 886 MeV/c the improvement is marginal. As examples we show three fits in Figs. 1-3 for respectively \( p_{\text{lab}} = 360, 585, \) and 988 MeV/c. By including the \( J = 3 \) amplitude the \( \chi^2 \) per degree of freedom hardly improves except for \( d\sigma/d\Omega \) at 988 MeV/c as seen in Fig. 3. It is remarkable that so few partial waves with \( J_{\text{max}} = 2 \) are sufficient in order to get a satisfactory \( \chi^2 \) fit to the data in such a large energy range. On the other hand, we note that the \( J = 2 \) partial wave amplitude is essential already at the three lowest
measured energies due to the presence of two minima in the angular behaviour of both the differential cross section \(d\sigma/d\Omega\) and of the asymmetry \(A_{\text{on}}\).

In Table I we show the \(\chi^2\) per degree of freedom for one set of partial wave amplitude parameters with \(J_{\text{max}} = 2\) as well as the case where \(J_{\text{max}} = 1, 3\) or 4. Listed are the ten momenta between \(p_{\text{lab}} = 360\) and 988 MeV/c, where there are available LEAR data. From Table I one notes that \(\chi^2\) for \(J_{\text{max}} = 2\) is much lower than the corresponding \(\chi^2\) for \(J_{\text{max}} = 1\). On the other hand an increase of the value of \(J_{\text{max}}\) to 3 or to 4 does not improve the \(\chi^2\) significantly except at \(p_{\text{lab}} = 988\) MeV/c. For comparison we give in Table II a similar list of \(\chi^2\) for the process \(p\bar{p} \rightarrow \pi^-\pi^+\) \([7]\). In that case the preferred maximum angular momentum is clearly \(J_{\text{max}} = 3\) at all momenta. The values of \(\chi^2\) for \(p\bar{p} \rightarrow K^-K^+\) are less smooth than for \(p\bar{p} \rightarrow \pi^-\pi^+\), because for the \(K^-K^+\) final state there are only half as many data points and in addition the error bars in the data are larger than for annihilation into \(\pi^-\pi^+\). However, given our simple amplitude assumptions and the large experimental errors for \(A_{\text{on}}\), we did not make a serious error analysis of our \(\chi^2\) fit.

In Tables III and IV we give an example of a set of values for the partial wave amplitudes \(R_{LJ}\) and their phases \(\delta_{LJ}\) found by our \(\chi^2\) fit to the data \(p\bar{p} \rightarrow K^-K^+\), with \(J_{\text{max}} = 2\). The normalization of \(R_{LJ}\) is such that, if the momentum \(p\) in Eqs. (5) and (6) is expressed in GeV/c, the cross section defined in Eq. (1) is in \(\mu b/\text{srad}\). The corresponding \(\chi^2\) values are those of Table I for \(J_{\text{max}} = 2\). From these tables one notes that the two \(J = 2\) partial wave amplitudes give very significant contributions to the cross section at all momenta. Since we have not used any energy smoothing procedure in our analysis, and since the ambiguities do not permit a unique solution, the amplitudes necessarily carry substantial uncertainties. However, independent of ambiguities, all fits require an important contribution from the \(J = 2\) partial wave amplitudes at all energies and show a need for adding \(J = 3\) amplitudes for \(p_{\text{lab}}\) above 0.9 GeV/c. This statement can be made, while no theoretical bias as to the energy behaviour of the amplitudes has been imposed on our fits.

These results are consistent with the simple model analysis at higher energies \([8]\). This earlier work \([8]\) used a diffractive scattering model from a simple black or grey sphere which
could explain most of the features of the higher energy data \( (p_{lab} \text{ above } 1.5 \text{ GeV/c for } \overline{p}p \rightarrow \pi^- \pi^+, \text{ and above } 1.0 \text{ GeV/c for } \overline{p}p \rightarrow K^-K^+) \). In that model description of the data, the spin dependent forces were assumed to act in the surface region only. The idea was that since the central region was “black” no detailed information would escape from the central interaction region. Only the more transparent surface region would provide the spin-forces giving the asymmetries of this annihilation reaction.

We interpret the fact that very few partial waves are needed in the analysis to mean that the annihilation reaction \( \overline{p}p \rightarrow K^-K^+ \) is a very central process as was previously found also for the reaction \( \overline{p}p \rightarrow \pi^-\pi^+ \). Moreover, this analysis clearly shows that \( \overline{p}p \rightarrow K^-K^+ \) requires even less partial waves than \( \overline{p}p \rightarrow \pi^-\pi^+ \). It is possible that part of this effect is related to the annihilation of an additional quark-antiquark pair and the creation of strange quarks needed to obtain a \( K^-K^+ \) final state. We note that this effect cannot be explained by the lower final momenta in the \( K^-K^+ \) system versus the \( \pi^-\pi^+ \) system. A check of the kinematics shows that corresponding momenta in \( K^-K^+ \) and \( \pi^-\pi^+ \) systems at these energies are not very different. At the lowest antiproton momentum, 360 MeV/c, the kaon momentum is a factor 0.86 of the pion momentum. As the antiproton energy increases, this factor approaches 1. Therefore the small difference in the final meson momenta does not explain that \( J_{\text{max}} = 2 \) for \( K^-K^+ \) and \( J_{\text{max}} = 3 \) for \( \pi^-\pi^+ \). Finally, it is difficult to interpret annihilation ranges for these reactions since both reactions are strongly influenced by final state meson-meson interactions as well as by effects of coupling to other annihilation channels. Therefore, statements about annihilation ranges will necessarily be strongly model dependent.

In conclusion, the present experimental data and analyses of these data for both \( \overline{p}p \rightarrow \pi^-\pi^+ \) and \( \overline{p}p \rightarrow K^-K^+ \) reactions may help to build better models of these simple, but very fundamental annihilation reactions. Two properties are essential for a successful model description of these two reactions. The annihilation model should be of short range and the model should allow for substantial \( J = 2 \) partial wave contributions to \( \overline{p}p \rightarrow K^-K^+ \) while still allowing a significant \( J = 3 \) partial wave for the \( \overline{p}p \rightarrow \pi^-\pi^+ \) reaction.
The analysis described in this paper suggests the following future experiments necessary in order to clarify further the understanding of the $\bar{p}p \rightarrow K^-K^+$ annihilation reaction:

(i) Measurements of this annihilation reaction should be made at antiproton momenta closer to threshold. At very low energies we expect that even fewer partial wave amplitudes would contribute and the ambiguities of an analysis would be reduced.

(ii) A further constraint on the analysis of this reaction would come from data on the other spin observables for $\bar{p}p \rightarrow K^-K^+$ with longitudinal and/or transverse polarized beam and target, for example data on the spin-correlations $A_{ss}$ or $A_{ls}$. No spin-correlation observables have as yet been measured. We propose the use of polarized antiprotons to obtain data on the observables $A_{ls}$ or $A_{ss}$ for $\bar{p}p \rightarrow K^-K^+$. This would allow to determine more accurately the various angular momentum amplitudes and contribute further to our understanding of this fundamental annihilation and hadronization process.

We thank F. Bradamante for providing us with the LEAR data, and R. Timmermans for many useful discussions on the art of phase shift analyses. One of the authors (W.M.K.) is grateful to the University of South Carolina for its hospitality during his stay, when this work was initiated. This work was supported in part by NSF Grant Nos. PHYS-9310124 and PHYS-9504866.
REFERENCES

[1] J-M Richard, Nucl. Phys. (Proc. Suppl.) B8, 128 (1989); J. Carbonell, G. Ihle, J.-M. Richard, Zeit. Phys. A334, 329 (1989).

[2] A. Hasan et al., Nucl. Phys. B378, 3 (1992).

[3] E. Eisenhandler et al., Nucl. Phys. B98, 109 (1975).

[4] A.A. Carter et al., Phys. Lett. B67, 117 (1977).

[5] T. Tanimori et al., Phys. Rev. Lett. 55, 185 (1985) and Phys. Rev. D41, 744 (1990).

[6] A.A. Carter, Phys. Lett. B67, 122 (1977).

[7] W.M. Kloet and F. Myhrer, Phys. Rev. D53 (in press).

[8] S. Takeuchi, F. Myhrer and K. Kubodera, in Proceedings of Intersection between Particle and Nuclear Physics, AIP Conf. Proc. 243, 358 (1992); and Nucl. Phys. A556, 601 (1993).

[9] V. Mull and K. Holinde, Phys. Rev. C51, 2360 (1995); V. Mull, K. Holinde and J. Speth, Phys. Lett. B275, 12 (1992).
FIGURES

FIG. 1. $d\sigma/d\Omega$ and $A_{on}$ at $p_{lab} = 360$ MeV/c for $\overline{p}p \rightarrow K^- K^+$. The solid curves give the fit for $J_{max} = 2$ and the dashed curves are the fit for $J_{max} = 3$.

FIG. 2. $d\sigma/d\Omega$ and $A_{on}$ at $p_{lab} = 585$ MeV/c for $\overline{p}p \rightarrow K^- K^+$. The solid curves give the fit for $J_{max} = 2$ and the dashed curves are the fit for $J_{max} = 3$.

FIG. 3. $d\sigma/d\Omega$ and $A_{on}$ at $p_{lab} = 988$ MeV/c for $\overline{p}p \rightarrow K^- K^+$. The solid curves give the fit for $J_{max} = 2$ and the dashed curves are the fit for $J_{max} = 3$. For $p_{lab} = 988$ MeV/c it is clear from $d\sigma/d\Omega$ that $J = 3$ is necessary.
### TABLE I. Examples of $\chi^2$ per degree of freedom for $K^-K^+$ at each energy.

| $p_{lab}$ (MeV/c) | $\chi^2(J_{max} = 1)$ | $\chi^2(J_{max} = 2)$ | $\chi^2(J_{max} = 3)$ | $\chi^2(J_{max} = 4)$ |
|------------------|----------------------|----------------------|----------------------|----------------------|
| 360              | 2.26                 | 0.93                 | 0.97                 | 0.95                 |
| 404              | 2.32                 | 1.23                 | 1.40                 | 1.44                 |
| 467              | 1.90                 | 1.00                 | 1.13                 | 0.92                 |
| 497              | 3.81                 | 1.40                 | 1.02                 | 1.03                 |
| 523              | 4.70                 | 1.03                 | 0.72                 | 0.80                 |
| 585              | 3.25                 | 0.95                 | 0.98                 | 0.79                 |
| 679              | 3.42                 | 1.44                 | 1.53                 | 1.52                 |
| 783              | 6.92                 | 2.45                 | 2.41                 | 2.22                 |
| 886              | 5.08                 | 1.59                 | 1.37                 | 1.30                 |
| 988              | 4.73                 | 2.80                 | 1.06                 | 1.20                 |

### TABLE II. Examples from Ref. [7] of $\chi^2$ per degree of freedom for $\pi^-\pi^+$ at each energy.

| $p_{lab}$ (MeV/c) | $\chi^2(J_{max} = 2)$ | $\chi^2(J_{max} = 3)$ | $\chi^2(J_{max} = 4)$ |
|------------------|----------------------|----------------------|----------------------|
| 360              | 1.96                 | 1.77                 | 1.74                 |
| 404              | 1.38                 | 1.12                 | 1.12                 |
| 467              | 1.98                 | 1.31                 | 1.18                 |
| 497              | 3.04                 | 1.50                 | 1.45                 |
| 523              | 2.63                 | 1.45                 | 1.43                 |
| 585              | 1.96                 | 1.51                 | 1.57                 |
| 679              | 2.17                 | 1.50                 | 1.53                 |
| 783              | 2.50                 | 1.49                 | 1.47                 |
| 886              | 3.21                 | 1.23                 | 1.13                 |
| 988              | 4.39                 | 1.85                 | 1.55                 |
TABLE III. Energy dependence of parameters of $R_{LJ}$ and $\delta_{LJ}$ for $J_{\text{max}} = 2$ for $K^-K^+$.  

| $p_{\text{lab}}$ (MeV/c) | $R_{10}$ | $\delta_{10}$ | $R_{01}$ | $\delta_{01}$ | $R_{21}$ | $\delta_{21}$ | 
|--------------------------|---------|--------------|---------|--------------|---------|--------------|
| 360                      | 0.94    | 0            | 0.26    | 35           | 0.30    | 75           |
| 404                      | 0.94    | 0            | 0.22    | 50           | 0.24    | 40           |
| 467                      | 1.38    | 0            | 0.20    | 5            | 0.30    | 70           |
| 497                      | 0.94    | 0            | 0.22    | 30           | 0.26    | 50           |
| 523                      | 1.08    | 0            | 0.30    | 20           | 0.40    | 60           |
| 585                      | 1.14    | 0            | 0.20    | -50          | 0.44    | 50           |
| 679                      | 1.26    | 0            | 0.56    | -85          | 0.40    | 35           |
| 783                      | 1.38    | 0            | 0.58    | -95          | 0.36    | 35           |
| 886                      | 1.70    | 0            | 0.66    | -95          | 0.20    | 45           |
| 988                      | 1.62    | 0            | 0.36    | -100         | 0.18    | 10           |

TABLE IV. Energy dependence of parameters $R_{LJ}$ and $\delta_{LJ}$ for $J_{\text{max}} = 2$ for $K^-K^+$.  

| $p_{\text{lab}}$ (MeV/c) | $R_{12}$ | $\delta_{12}$ | $R_{32}$ | $\delta_{32}$ | 
|--------------------------|---------|--------------|---------|--------------|
| 360                      | 0.14    | 75           | 0.38    | -90          |
| 404                      | 0.14    | 95           | 0.42    | -85          |
| 467                      | 0.22    | 125          | 0.38    | -65          |
| 497                      | 0.30    | 105          | 0.56    | -55          |
| 523                      | 0.28    | 130          | 0.42    | -50          |
| 585                      | 0.38    | 125          | 0.40    | -60          |
| 679                      | 0.58    | 130          | 0.38    | -55          |
| 783                      | 0.50    | 130          | 0.46    | -50          |
| 886                      | 0.44    | 110          | 0.46    | -40          |
| 988                      | 0.44    | 95           | 0.48    | -65          |
FIG. 1
FIG. 2
FIG. 3