Couette flow of an incompressible fluid in a porous channel with mass transfer

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Abstract. The present discussion deals with the study of couette flow through a porous medium of a viscous incompressible fluid between two infinite horizontal parallel porous flat plates with heat and mass transfer. The stationary plate and the plate in uniform motion are subjected to transverse sinusoidal injection and uniform suction of the fluid. Due to this type of injection velocity, the flow becomes three dimensional. The analytical solutions of the nonlinear partial differential equations of this problem are obtained by using perturbation technique. Expressions for the velocity, temperature fields and the rate of heat and mass transfers are obtained numerically and depicted through graphs. The rate of heat and mass transfer are also analyzed.

1. Introduction
The flows of fluid through porous media initiated the attention of a number of researchers because of their abundant applications in various fields of science and technology. Porous media are widely used in high temperature heat exchangers, turbine blades, jet nozzles etc. Cooling of porous structure is achieved by forcing the liquid or gas through capillaries of solid. They are used to insulate a heated body to maintain its temperature. Porous media are considered to be useful in diminishing the natural free convection which would otherwise occur intensely on a vertical heated surface. In order to make heat insulation of surface more effective it is mandatory to study the free convection flow through a porous medium and to estimate its effect in heat and mass transfer.

Mass transfer is applied in ablative cooling, sudden decrease in the temperature of space vehicles during their re-entry into the atmosphere, transpiration and film cooling of rocket and jet engines. A porous material containing the fluid is a non-homogeneous medium.

Singh et al [1] analyzed about an oscillatory free convective flow through porous channel. Guria et al [2] studied about three dimensional fluctuating couette flows through the porous plates with heat transfer. Govindarajan et al [3] discussed 3D couette flow of dusty fluid with transpiration cooling. Jain et al [4] reported on 3D radiative heat and mass transfer in a periodic flow through a porous
channel in a vertical position with slip boundary conditions and cooled transpiration. Vidhya et al [5]
discussed about laminar convection through porous medium between two vertical parallel plates with heat
source. Govindarajan et al [6] analyzed Chemical reaction effects on unsteady magneto hydrodynamic free
convective flow in a rotating porous medium with mass transfer. Chaudhary et al [7] studied about three
dimensional couette flow and heat transfer through a porous medium with variable permeability. Singh et al [8]
discussed about MHD three dimensional couette flows with transpiration cooling. Gireesha et al [9] studied
about three dimensional couette flow of a dusty fluid with heat transfer. Ahmed et al [10] analyzed about analytical
and numerical solution of three dimensional channel flows in presence of a sinusoidal fluid injection and chemical reaction.

The main aim of this paper is to study the three dimensional couette flow of a clean fluid with heat and
mass transfer through porous medium.

2. Mathematical Analysis
Consider couette flow of a viscous incompressible fluid through porous medium bounded between two
infinite parallel flat porous plates with mass transfer. A co-ordinate system is introduced with the
origin at the lower stationary plate lying horizontally x* - z* plane, and the upper plate at a distance ‘d’
from it is subjected to a uniform motion U. The y* axis is taken perpendicular to the plane of plates.
The lower and the upper plates are assumed to be at constant temperature T0 and T1 respectively with
T1 > T0.

It is assumed that the concentration c* of the diffusing species is very less in comparison to other
chemical species. This leads to the assumption that Soret and Dufour effects are negligible. It is also
assumed that the effect of viscous dissipation is negligible in the energy equation. The upper plate is
subjected to a constant suction V whereas the lower plate to the transverse sinusoidal injection
velocity distribution of the form:

\[ v^* (z^*) = v (1 + \epsilon \cos \pi z^* / d) \]  

where \( \epsilon \) \((<<1)\) is a positive constant quantity, without any loss of generality, the distance ‘d’ between
the plates is taken equal to the wavelength of the injection velocity. All physical quantities are
independent of x* for this problem of fully developed laminar flow but the flow remains three
dimensional due to the periodic injection velocity (1). Denoting the velocity components u*, v*, w*
in the x*, y*, z* directions respectively and the temperature by T*. The problem is governed by the
following equations:

Continuity equation:

\[ \frac{\partial v^*}{\partial y^*} + \frac{\partial u^*}{\partial z^*} = 0 , \]  

Fig. 1: Couette flow with periodic injection and constant suction at the porous plate.
Momentum equations:

\[
v^* \frac{\partial u^*}{\partial y^*} + w^* \frac{\partial u^*}{\partial z^*} = g \beta^* (C_w^* - C_w^*) + \nu \left( \frac{\partial^2 u^*}{\partial y^*^2} + \frac{\partial^2 u^*}{\partial z^*^2} \right) - \frac{\nu u^*}{k^*},
\]

(3)

\[
v^* \frac{\partial v^*}{\partial y^*} + w^* \frac{\partial v^*}{\partial z^*} = -\frac{1}{\rho} \frac{\partial p^*}{\partial y^*} + \nu \left( \frac{\partial^2 v^*}{\partial y^*^2} + \frac{\partial^2 v^*}{\partial z^*^2} \right) - \frac{\nu w^*}{k^*},
\]

(4)

\[
v^* \frac{\partial w^*}{\partial y^*} + w^* \frac{\partial w^*}{\partial z^*} = -\frac{1}{\rho} \frac{\partial p^*}{\partial z^*} + \nu \left( \frac{\partial^2 w^*}{\partial y^*^2} + \frac{\partial^2 w^*}{\partial z^*^2} \right) - \frac{\nu w^*}{k^*},
\]

(5)

Energy equation:

\[
v^* \frac{\partial T^*}{\partial y^*} + w^* \frac{\partial T^*}{\partial z^*} = \alpha \left( \frac{\partial^2 T^*}{\partial y^*^2} + \frac{\partial^2 T^*}{\partial z^*^2} \right),
\]

(6)

Mass concentration equation:

\[
v^* \frac{\partial C^*}{\partial y^*} + w^* \frac{\partial C^*}{\partial z^*} = D \left( \frac{\partial^2 C^*}{\partial y^*^2} + \frac{\partial^2 C^*}{\partial z^*^2} \right),
\]

(7)

where \( \rho \) the density, \( p^* \) the pressure, \( k^* \) the permeability of the porous medium, \( \nu \) the kinematic viscosity, \( \alpha \) the thermal diffusivity, \( g \) the acceleration due to gravity, \( \beta^* \) the coefficient of volume expansion for heat transfer, \( g^* \) the volumetric coefficient of expansion with species concentration, \( T \) the temperature of the fluid, \( T_0 \) the fluid temperature at \( \infty \), \( C^* \) the dimensional species concentration, \( C_w^* \) the species concentration at \( \infty \), \( D \) the chemical molecular diffusivity. * stands for dimensional quantities. The last terms on the right hand side of the equations (3), (4) and (5) account for the pressure drop across the porous material.

The boundary conditions of the problem are:

\[
y^* = 0 : u^* = 0, v^* (x^*) = v (1 + \epsilon \cos \pi z^*/d) w^* = 0, T^* = T_0 ,
\]

\[
y^* = d : u^* = U, v^* = V, w^* = 0, T^* = T_1
\]

(8)

Non-dimensional parameters are introduces as follows:

\[
y = \frac{y^*}{d}, z = \frac{z^*}{d}, u = \frac{u^*}{U}, w = \frac{w^*}{V}, p = \frac{p^*}{\rho \nu^2},
\]

(9)

\[
\theta = \frac{T^* - T_0}{T_1 - T_0}, \phi = \frac{C^* - C_w^*}{C_w^* - C_w^*}
\]

In equations (2) – (6), we get

Continuity equation:

\[
\frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0.
\]

(10)

Momentum equations:

\[
\frac{\partial u}{\partial y} + w \frac{\partial u}{\partial z} = \frac{G e Re \phi}{1 + \frac{1}{Re \left( \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2} \right)}} - \frac{u}{Re k_0},
\]

(11)

\[
\frac{\partial v}{\partial y} + w \frac{\partial v}{\partial z} = -\frac{p}{\rho} + \frac{1}{Re \left( \frac{\partial^2 v}{\partial y^2} + \frac{\partial^2 v}{\partial z^2} \right)} - \frac{v}{Re k_0},
\]

(12)

\[
\frac{\partial w}{\partial y} + w \frac{\partial w}{\partial z} = -\frac{p}{\rho} + \frac{1}{Re \left( \frac{\partial^2 w}{\partial y^2} + \frac{\partial^2 w}{\partial z^2} \right)} - \frac{w}{Re k_0},
\]

(13)

Energy equation:
\[
\nu \frac{\partial \theta}{\partial y} + w \frac{\partial \theta}{\partial z} = \frac{1}{\text{Re} \text{Pr}} \left( \frac{\partial^2 \theta}{\partial y^2} + \frac{\partial^2 \theta}{\partial z^2} \right),
\]

(14)

Mass concentration equation:

\[
\nu \frac{\partial \phi}{\partial y} + w \frac{\partial \phi}{\partial z} = \frac{1}{\text{Re} \text{Sc}} \left( \frac{\partial^2 \phi}{\partial y^2} + \frac{\partial^2 \phi}{\partial z^2} \right),
\]

(15)

where

\[
\text{Re} = \frac{\nu d}{\nu}, \text{ the injection / suction parameter,}
\]

\[
\text{Pr} = \frac{\nu}{\alpha}, \text{ the Prandtl Number,}
\]

\[
k_0 = \frac{\nu^2}{d^2}, \text{ the permeability parameter}
\]

\[
\text{Ge} = \frac{\nu \beta (C_y^+ - C_y^-)}{u V^+}, \text{ the modified Grashoff number}
\]

The boundary conditions (8) reduces to

\[
y = 0: u = 0, v(y) = 1 + \varepsilon \cos \pi z, \quad w = 0, \theta = 0, \phi = 1
\]

\[
y = 1: u = 1, \quad v = 1, \quad w = 0, \quad \theta = 1, \quad \phi = 0
\]

In order to solve these non-linear partial differential equations, we assume the solution of the following form because the amplitude’\varepsilon’ (<<1) of the injection velocity is very small. Regular perturbation technique is used in this paper. Muthucumaraswamy used finite difference method using Crank Nicholson scheme. He used numerical method whereas the solution given here is a closed form solution. The solution is defined for any region of the plate. But when numerical methods are employed stability and convergence of the (profiles) solution have to be checked whereas, that is not required in perturbation method.

\[
F(y,z) = f_0(y) + \varepsilon f_1(y,z) + O(\varepsilon^2),
\]

(17)

Here \(f\) stands for \(u, v, w, \theta\) and \(c\). When, \(\varepsilon = 0\), the problem reduces to the two dimensional couette flow through porous medium with constant injection and suction at the respective plates with heat and mass transfer. The solution of this two-dimensional problem is

\[
u_0(y) = c_1 e^{\nu y} + c_2 e^{-\nu y} + L e^{\nu y} + L_0,
\]

(18)

\[
\theta_0(y) = \frac{1}{1 - e^{\nu y}}, \quad \phi_0(y) = \frac{e^{\nu y} - e^{-\nu y}}{1 - e^{2\nu y}}.
\]

(19)

with \(V_0 = 1, w_0 = 0, P_0 = \text{constant,}

\[
m_1 = \frac{1}{2} \left[ \text{Re} + \left( \text{Re}^2 + \frac{4}{k_0} \right)^{1/2} \right] \quad \text{and} \quad m_2 = \frac{1}{2} \left[ \text{Re} - \left( \text{Re}^2 + \frac{4}{k_0} \right)^{1/2} \right],
\]

(21)

\[
L_1 = \frac{Gm R^2}{h}, \quad h = \left( 1 - e^{2\nu y} \right) \left( \text{Re} \nu \text{Sc}^2 - \text{Re} \nu \text{Sc} - \frac{1}{k_0} \right), \quad L_0 = \frac{Gm \text{Re}^2 e^{\nu y} k_0}{1 - e^{2\nu y}},
\]

suppose \(\varepsilon \neq 0\), using (17) in equations (10) to (15) and estimating the similar power of \(\varepsilon\), neglecting those of \(\varepsilon^2\), we get the following equations as the coefficients of \(\varepsilon\) with the help of equations (18) to (21).

\[
\frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0,
\]

(22)
\[
\v_1 \frac{\partial u_0}{\partial y} + \frac{\partial u_1}{\partial z} = \text{Gr} \, \text{Re} \, \phi_1 + \frac{1}{\text{Re}} \left( \frac{\partial^2 u_0}{\partial y^2} + \frac{\partial^2 u_1}{\partial z^2} \right) - \frac{u_1}{\text{Re} \, k_0}, \tag{23}
\]

\[
\frac{\partial v_1}{\partial y} = \frac{\partial p_1}{\partial y} \left( \frac{1}{\text{Re}} \left( \frac{\partial^2 v_1}{\partial y^2} + \frac{\partial^2 v_1}{\partial z^2} \right) - v_1 \right), \tag{24}
\]

\[
\frac{\partial w_1}{\partial y} = \frac{\partial p_1}{\partial z} \left( \frac{1}{\text{Re}} \left( \frac{\partial^2 w_1}{\partial y^2} + \frac{\partial^2 w_1}{\partial z^2} \right) - w_1 \right), \tag{25}
\]

\[
\frac{\partial \theta_1}{\partial y} + \frac{\partial \theta_1}{\partial z} = \frac{1}{\text{Re} \, \text{Pr}} \left( \frac{\partial^2 \theta_1}{\partial y^2} + \frac{\partial^2 \theta_1}{\partial z^2} \right), \tag{26}
\]

\[
\frac{\partial \theta_1}{\partial y} + \frac{\partial \theta_1}{\partial z} = \frac{1}{\text{Re} \, \text{Sc}} \left( \frac{\partial^2 \phi_1}{\partial y^2} + \frac{\partial^2 \phi_1}{\partial z^2} \right), \tag{27}
\]

The corresponding boundary conditions become
\[
y = 0: \quad u_1 = 0, \quad v_1 = \cos \pi z, \quad w_1 = 0, \quad \theta_1 = 0, \quad \phi_1 = 0 \tag{28}
\]

\[
y = 1: \quad u_1 = 0, \quad v_1 = 0, \quad w_1 = 0, \quad \theta_1 = 0, \quad \phi_1 = 0
\]

These are the linear partial differential equations which describe the three-dimensional flow through porous medium with heat and mass transfer. To solve these equations, we shall consider the equations (24) and (25), the main flow element \( u_1 \) and the temperature field \( \theta_1 \) are not dependent.

Consider \( v_1, w_1 \) and \( p_1 \) as follows:
\[
v_1(y, z) = v_{11}(y) \cos \pi \pi z, \tag{29}
\]

\[
w_1(y, z) = -\frac{1}{\pi} v_{11}(y) \sin \pi z, \tag{30}
\]

\[
p_1(y, z) = p_{11}(y) \cos \pi z, \tag{31}
\]

where, the prime in \( v_{11}(y) \) denotes differentiation with respect to \( y \). Equations (29) and (30) have been chosen such that the continuity equation (22) is satisfied. Substituting these equations into equations (24) and (25) and applying the corresponding transformed boundary conditions we get the solutions of \( v_1, w_1 \) and \( p_1 \) as:
\[
v_1(y, z) = \left( c_1 e^{\lambda_1 y} + c_2 e^{\lambda_2 y} - L_e c_3 e^{\lambda_3 y} \right) \cos \pi \pi z, \tag{32}
\]

where, \( L_e = \frac{\text{Re} \pi}{\text{Re} \pi + 1} \), \( \lambda_1 = \frac{\text{Re} \pi}{\text{Re} \pi + 1} \), \( \lambda_2 = \frac{\text{Re} \pi}{\text{Re} \pi + 1} \),
\[
w_1(y, z) = \left[ c_3 \lambda_3 e^{\lambda_1 y} + c_4 \lambda_3 e^{\lambda_2 y} - L_e \pi c_3 e^{\lambda_3 y} + L_e \pi c_4 e^{\lambda_3 y} \right] \sin \pi \pi z, \tag{33}
\]

where
\[
\lambda_1 = \frac{\text{Re} \pi^2}{4 + \left( \frac{\pi^2}{1} \right)} \lambda_2 = \frac{\text{Re} \pi^2}{4 + \left( \frac{\pi^2}{1} \right)}, \tag{34}
\]

\( c_1, c_2, c_3, c_4 \) are known constants but whose expressions are not given due to the sake of brevity.

To solve (27), assume that \( \phi_1(y, z) = \phi_{11}(y) \cos \pi z \) in equation (27) and applying the corresponding boundary conditions we get,
\[
\phi_1(y, z) = c_5 e^{m_3 y} + c_6 e^{m_4 y} + \left( \frac{\text{Re} \, \text{Sc}^2}{1 - e^{m_3 y}} \right) \times
\]

\[
\left[ \frac{c_7 e^{(\lambda_3 + \text{Re} \, \text{Sc}^2) y}}{\lambda_3^2 + \lambda_3 \text{Re} \, \text{Sc} - \pi^2} + \frac{c_8 e^{(\lambda_4 + \text{Re} \, \text{Sc}^2) y}}{\lambda_4^2 + \lambda_4 \text{Re} \, \text{Sc} - \pi^2} - \frac{c_9 e^{(\lambda_5 + \text{Re} \, \text{Sc}^2) y}}{\pi \text{Re} \, \text{Sc}} + \frac{c_{10} e^{(\lambda_6 + \text{Re} \, \text{Sc}^2) y}}{\pi \text{Re} \, \text{Sc}} \right] \cos \pi \pi z \tag{35}
\]

where \( m_3 = \frac{\text{Re} \, \text{Sc}^2}{4 + \pi^2}, m_4 = \frac{\text{Re} \, \text{Sc}^2}{4 + \pi^2} \).
\[ A_1 = -c_5 - c_7, A_2 = -c_7 e^{m_0} - c_5 e^{m_0}, \]
\[ c_7 = \frac{A_2 - A_1 e^{m_0}}{e^{m_0} - e^{m_0}}, c_9 = \frac{e^{m_0} A_1 - A_2}{e^{m_0} - e^{m_0}}, \]

In order to solve the differential equations (23) and (26) for \( u_1 \) and \( \theta_1 \) respectively, we assume
\[ u_1 = u_{11} (y) \cos \pi z, \] (36)
\[ \theta_1 = \theta_{11} (y) \cos \pi z, \] (37)

Substituting these equations into equations (23) and (26) we obtain the following equations
\[ u_{11}' - u_{11} \operatorname{Re} u_{11} \left( \frac{1}{k_0} + \pi^2 \right) = -Gc Re^2 \theta_{11} + \operatorname{Re} v_{11} \mu_{10}', \] (38)
\[ \theta_{11}' - \operatorname{Pr} \mu_{10} \theta_{11} - \pi^2 \theta_{11} = \operatorname{Pr} v_{11} \theta_{10}', \] (39)

with corresponding boundary conditions:
\[ y = 0: \quad u_{11} = 0, \quad \theta_{11} = 0, \]
\[ y = 1: \quad u_{11} = 0, \quad \theta_{11} = 0, \] (40)

where, the parameters noted as the differentiation with respect to \( y \). Apply boundary conditions and solving the equations (38) and (39) and using in equations (32) and (35) we get,
\[ \left\{ \begin{array}{l}
\frac{c_5 e^{m_0}}{(1 - e^{pr Re})} + \frac{c_7 e^{m_0}}{(1 - e^{pr Re})} - Gc Re^2 \frac{1}{k_0} + \pi^2 \theta_{11} + \operatorname{Re} v_{11} \mu_{10}' \\
\frac{c_5 e^{m_0}}{(1 - e^{pr Re})} + \frac{c_7 e^{m_0}}{(1 - e^{pr Re})} - \pi^2 \theta_{11} = \operatorname{Pr} v_{11} \theta_{10}'
\end{array} \right. \] (41)
\[ \left\{ \begin{array}{l}
\frac{c_5 e^{m_0}}{(1 - e^{pr Re})} + \frac{c_7 e^{m_0}}{(1 - e^{pr Re})} - Gc Re^2 \frac{1}{k_0} + \pi^2 \theta_{11} + \operatorname{Re} v_{11} \mu_{10}' \\
\frac{c_5 e^{m_0}}{(1 - e^{pr Re})} + \frac{c_7 e^{m_0}}{(1 - e^{pr Re})} - \pi^2 \theta_{11} = \operatorname{Pr} v_{11} \theta_{10}'
\end{array} \right. \] (42)

If the velocity field suggested, then find the skin-friction elements \( T_x \) in the main flow and \( T_z \) in the transverse directions
\[ T_x = \frac{\partial u}{\partial y} \bigg|_{y=0} = \left( \frac{\partial u}{\partial y} \right)_{y=0} \cos \pi z, \]
\[ = \left[ c_1 m_1 + c_2 m_2 + L_1 \operatorname{Re} \sin \right] \left( \begin{array}{c}
\frac{c_5 e^{m_0}}{(1 - e^{pr Re})} + \frac{c_7 e^{m_0}}{(1 - e^{pr Re})} - Gc Re^2 \frac{1}{k_0} + \pi^2 \theta_{11} + \operatorname{Re} v_{11} \mu_{10}' \\
\frac{c_5 e^{m_0}}{(1 - e^{pr Re})} + \frac{c_7 e^{m_0}}{(1 - e^{pr Re})} - \pi^2 \theta_{11} = \operatorname{Pr} v_{11} \theta_{10}'
\end{array} \right), \] (43)
\[
T_z = \frac{\partial w}{\partial y} = \left( \frac{\partial u_1}{\partial y} \right)_{y=0} \sin z, \\
= -\frac{\epsilon}{\pi} \left[ c_1 \lambda_1^2 + c_2 \lambda_2^2 + c_3 \lambda_3^2 - c_4 \lambda_4^2 - c_5 \lambda_5^2 \right] \sin \lambda, \\
\] (44)

where \( c_1, c_2, c_3, c_4, c_5, c_6, c_7, c_8, c_9, c_{10}, c_{11}, c_{12}, c_{13}, c_{14}, c_{15}, c_{16}, c_{17}, \) and \( c_{18} \) are known constants. The values of these constants are not given due to the sake of brevity.

The rate of heat transfer in terms of Nusselt number is given by

\[
Nu = \left( \frac{\partial \theta}{\partial y} \right)_{y=0} = \left( \frac{\partial \theta_1}{\partial y} \right)_{y=0} = \left( \frac{\partial \theta_{11}}{\partial y} \right)_{y=0} \cos \lambda, \\
\] (45)

\[
Nu = -\frac{\epsilon}{\pi} \left[ c_1 \lambda_1^2 + c_2 \lambda_2^2 + c_3 \lambda_3^2 - c_4 \lambda_4^2 - c_5 \lambda_5^2 \right] \left[ \frac{\epsilon^2 \pi^2}{1 - e^{\epsilon \pi \lambda}} \right] \\
\times \left[ c_1 \left( \lambda_1 + \epsilon \pi \right) + c_2 \left( \lambda_2 + \epsilon \pi \right) + c_3 \left( \lambda_3 + \epsilon \pi \right) + c_4 \left( \lambda_4 + \epsilon \pi \right) + c_5 \left( \lambda_5 + \epsilon \pi \right) \right] \\
\times \left[ \frac{\epsilon^2 \pi^2}{1 - e^{\epsilon \pi \lambda}} \right] \\
\times \frac{1}{\epsilon \pi \lambda} \\
\times \left[ \frac{c_1 \lambda_1^2 + c_2 \lambda_2^2 + c_3 \lambda_3^2 - c_4 \lambda_4^2 - c_5 \lambda_5^2}{\epsilon \pi \lambda} \right] \cos \lambda, \\
\] (46)

3. Results and Discussions

The main flow velocity profiles are shown graphically in figure 2(a). It is clear from figure 2(a) that the velocity profiles increase with the increase of injection / suction parameter \( Re \). It is observed from figure 2(b) that the main flow velocity profiles increase due to an increase in Grashof number for mass transfer.

Fig 2(a) Velocity profile for various values of Re

Fig 2(b) Velocity profile for various values of GC
The effect of permeability and Re on main flow profiles are drawn in figure 3. It is seen that an increase in the permeability of the porous medium leads to an increase in the main flow profiles. The straight line in the figure 3 represents couette flow in an ordinary medium when there is neither injection nor suction in the plate. Here also all the profiles increase steadily near the lower plate and reach the maximum value at the other plate.

![Fig. 3 Main flow velocity profiles of u(y, z) against y for various values of k0 and Re, Pr = 0.71, GC = 3, SC = 0.22, ε = 0.2](image)

The effect of Schmidt number on concentration species profiles are shown in figure 4. The concentration profiles increase due to an increase in Schmidt number.

![Fig. 4 Concentration profiles against y for various values of SC, Pr = 0.71, Re = 1.5, GC = 3, k0 = 0.5, ε = 0.2. Values of SC for curves are A = 1, B = 1.5, C = 2, D = 2.5, E = 3, F = 3.5, G = 4](image)

In figure 5 the species concentration profiles increase with an increase in injection / suction parameter.

![Fig. 5 Concentration profiles against y for various values of Re, Pr = 0.71, GC = 3, k0 = 0.5, SC = 2, ε = 0.2. Values of Re for curves A = 0.5, B = 1, C = 1.5](image)
The porous plate is at rest. The cross flow velocity component $w_1$ applied through the porous plate due to the transverse sinusoidal injection. This secondary flow component is shown in figure 6.

It is interesting to note that in the lower half of the channel, the cross flow component $w_1$ decreases with the increase of the permeability $k_0$ of the porous medium or the injection / suction parameter $Re$, whereas in the upper half of the channel the effect of permeability or the injection / suction parameter on $w_1$ is reversed. This is due to the fact that there is injection at the stationary plate and suction at the plate in uniform motion which are two exactly opposite processes.

![Fig. 6 Cross flows velocity profiles against y for various values of Re and k0 and Z = 0.5, SC = 0.22, Pr = 0.71, GC = 1, $\varepsilon = 0.2$](image)

The variation of skin friction component $T_x$ in the main flow direction is shown in figure 7(a), 7(b) and 8(a). It is evident from figure 7(a), 7(b) and 8(a) that the skin friction component $T_x$ increases with an increase of either permeability of the porous medium (or) modified Grashoff number (or) Schmidt number.

![Fig. 7(a) Variations of skin friction with k0](image)  
![Fig. 7(b) Variations of skin friction with GC](image)
The variation of skin friction component $T_z$ in the cross flow direction or in the transverse direction is shown graphically in figure 8(b). It is clear that the skin friction component $T_z$ decreases with an increase in permeability of the porous medium (or) an increase in injection / suction parameter.

![Variations of skin friction with SC](image1)

![Variations of skin friction component Tz in cross flow direction with k0](image2)

Figure 9(a) and 9(b) shows the variation Nusselt number with permeability of porous medium for the cases of air ($Pr = 0.71$) and water ($Pr = 7.0$). It is found that when the permeability of the porous medium and injection / suction parameter for both air and water increases, the rate of heat transfer decreases.

![Nusselt number for various values of k0 and Pr = 0.71](image3)

![Nusselt number for various values of k0 and Pr = 7.0](image4)
4. Conclusion
   (1) The effect of injection/suction parameter, modified Grashoff number for mass transfer Schmidt number and permeability parameter on main flow velocity profiles and skin friction in the main flow direction are same.
   (2) Injection / suction Re and permeability parameter to have got the influence of increasing main flow velocity profiles, species concentration and skin friction in the main flow direction, whereas they have got an opposite influence on skin friction in cross flow direction and heat transfer coefficient.
   (3) Modified Grashoff number Gm and Schmidt number Sc, show a significant increase in the profiles of main flow velocity, skin friction in main flow direction and species concentration.
   (4) As far as the cross flow velocity profiles are concerned, permeability parameter K₀ and injection/suction parameter Re have the same effect of increasing the profiles upto the midpoint of the channel thereafter they have reverse effect on the cross flow profiles.
   (5) The presence of foreign species increases the profiles of main flow velocity, species concentration and shear stress.
   (6) The porosity of the medium has considerable effect on velocity and skin friction in main flow direction. Both the profiles increase with increases in permeability parameter.
   (7) The thickness of concentration layer decreases both in magnitude and extent in the presence of thicker diffusing species.

References
[1] Singh K D 2000 An Oscillatory Hydromagnetic Couette Flow in a Rotating System ZAMM 80(6) 429–432
[2] Guria M and Jana R N 2006 Three dimensional fluctuating couette flow through the porous plates with heat transfer International Journal of Mathematics and Mathematical Sciences 1–18
[3] Govindarajan A, Ramamurthy V and Sundarammal K 2007 3D couette flow of dusty fluid with transpiration cooling Journal of Zhejiang University SCIENCE A 8(2) 313–322
[4] Jain N C, Chaudhary D and Hoshiyar Singh 2013 Three Dimensional Radiative Heat and Mass Transfer Periodic Flow through a Vertical Porous Channel with Transpiration Cooling and Slip Boundary Conditions Applied Mathematics 3(3) 71–92
[5] Vidhya M and Sundarammal Kesavan 2010 Laminar Convection through porous medium between two vertical parallel plate with heat source IEEE Transaction 195–200
[6] Govindarajan A, Ali chamkha, Sundarammal kesavan and Vidhya M 2014 Chemical reaction effects on unsteady magneto hydrodynamic free convective flow in a rotating porous medium with mass transfer Thermal Science 18(2) 515–526
[7] Chaudhary R C and Sharma Pawan Kumar 2003 Three dimensional couette flow and heat transfer through a porous medium with variable permeability Indian Journal of Pure and Applied Mathematics 4(2) 181–185
[8] Singh K D and Rakesh Sharma 2001 MHD three dimensional couette flows with transpiration cooling ZAMM 81(10) 715–720
[9] Gireesha B J, Chamkha A J, Visalakshi C S and Bagawade C S 2012 Three dimensional couette flow of a dusty fluid with heat transfer Applied Mathematical Modelling 36(2) 683–701
[10] Sahin Ahmed, Karabi Kalitha and Chamkha A J 2015 Analytical and numerical solution of three dimensional channel flows in presence of a sinusoidal fluid injection and chemical reaction Ains Shams Engineering Journal 6(2) 691–701