EFFECTIVE FIELD THEORY OF
ANOMALOUS GAUGE-BOSON COUPLINGS
AT HIGH-ENERGY $pp$ COLLIDERS

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Abstract

We compute the effects of anomalous gauge-boson couplings at high-energy hadron colliders using next-to-leading order $SU(2) \times SU(2)$ chiral perturbation theory. By comparing the yields from the universal $p^2$ terms with those arising from new physics at order $p^4$, we estimate the sensitivity of the SSC and LHC to the indirect effects of electroweak symmetry breaking.

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1. Introduction

The study of electroweak symmetry breaking is a major reason for building new high energy hadron colliders. At present, not much is known about the mechanism of symmetry breaking, except for the fact that the $W$ and $Z$ bosons have mass, or longitudinal degrees of freedom. This leads one to expect that longitudinally-polarized gauge bosons will provide an important probe of this new physics.

Over the past few years, the theory of electroweak symmetry breaking has received much attention. The case that has been studied most is, of course, the standard model with an elementary Higgs boson [1]. If the Higgs boson is light, standard perturbative calculations give a reliable indication of its prospects for discovery. If it is heavier than about 800 GeV, however, the symmetry-breaking sector is strongly coupled and naive perturbation theory breaks down [2–4].

The problem with a strongly-interacting symmetry-breaking sector is that one cannot make firm predictions. For example, the strong dynamics might give rise to a resonance, with the same quantum numbers as the standard-model Higgs boson, whose properties are not simply related to the parameters of the standard model [3,5]. Alternatively, the symmetry-breaking sector might not be at all like the standard model, and many possibilities have been suggested. Typically, these theories give rise to a rich spectrum of resonances in the TeV region, as in technicolor models [6].

On general grounds, one knows that some of the resonances associated with electroweak symmetry breaking must couple to the $V_LV_L$ final state. If the resonances are light enough to be produced at the SSC or LHC, their discovery prospects depend on their widths. For example, the relatively narrow techni-rho should be easy to see [7]. In contrast, a heavy Higgs is so broad that it is difficult to isolate the signal from the background [1].

Of course, it is also possible for the new resonances to be too heavy to be produced at the SSC or LHC. In this case one would expect an enhancement in

\#1 We generically denote the longitudinal $W$ and $Z$ bosons by $V_L$. 

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the yield of $V_LV_L$ pairs. Such an enhancement would signal the onset of a new strongly-interacting symmetry-breaking sector [8], with new resonances that lie out of reach [9,10].

It is this second scenario that we will study in this paper. We will use the formalism of effective field theory to describe the symmetry breaking, and take the effective Lagrangian to contain all the fields of the standard model except the Higgs boson. We will assume that the resulting theory has an effective $SU(2) \times SU(2)$ chiral symmetry, and replace the Higgs by an infinite set of non-renormalizable operators whose coefficients parametrize the low-energy effects of the electroweak symmetry breaking [11,12]. In unitary gauge, these operators reduce to anomalous couplings of the standard-model gauge bosons.

The formalism of effective Lagrangian provides a well-defined computational framework for investigating the physics of anomalous couplings and electroweak symmetry breaking. The infinite set of terms in the effective Lagrangian can be organized in an energy expansion. At low energies, only a finite number of terms contribute to any given process. At higher energies, more and more terms become important, until the whole process breaks down at the scale of the symmetry breaking [13]. A similar procedure gives an acceptable description of $\pi\pi$ scattering amplitudes up to energies of about 500 MeV [14], so we expect the effective Lagrangian for $V_LV_L$ scattering to be reasonable up to the TeV scale.

Thus, in this paper we shall study the sensitivity of the SSC and LHC to new physics beyond the standard model. We will assume that all new resonances lie out of reach, and that the physics of electroweak symmetry breaking is described by a model-independent low-energy effective Lagrangian. We will use this Lagrangian to carry out a complete, order $p^4$, calculation of $V_LV_L$ pair production in $pp$ colliders, for invariant masses up to of order 1.0 TeV. Our results extend previous studies by including all initial states and all next-to-leading corrections to the processes of interest [15–17].

Throughout this paper we will use the electroweak equivalence theorem to aid
our analysis [18]. We will work in Landau gauge,\(^\#2\) and calculate our amplitudes to leading order in \(M_W^2/s\). To any order in the loop expansion, this amounts to keeping only those terms of “enhanced electroweak strength” [19]. Therefore our amplitudes are valid for the SSC and LHC, but not for lower-energy machines.

We shall simplify our calculations by assuming factorization of the production, scattering and decay of the \(V_L V_L\) pairs. For initial states with vector bosons, we will use the effective \(W\) approximation to compute the luminosities of the transverse and longitudinal polarizations. We then fold these luminosities with the scattering sub-processes to find the \(pp\) cross sections [20].

In what follows we will assume that all of the new physics associated with electroweak symmetry breaking is contained in the vector boson self-couplings. In particular, we shall take the couplings of the fermions to be the same as in the standard model, we will ignore the possibility of extra pseudo-Goldstone bosons [21]. We will not discuss the detection issues that go into analyzing the decay of the longitudinal vector bosons, nor will we try to define or study realistic experimental signatures.

2. Effective Lagrangians

2.1. Global Symmetries

In this paper we will assume that the effective Lagrangian for electroweak symmetry breaking is determined by new physics outside the reach of the SSC or LHC. Since we do not know the full theory, we must build the effective Lagrangian out of all operators consistent with the unbroken symmetries. In particular, we must include operators of all dimensions, whether or not they are renormalizable. In this way we construct the most general effective Lagrangian that describes electroweak symmetry breaking.

\(^\#2\) In this gauge, the would-be Goldstone bosons are massless and decouple from the ghost sector.
To specify the effective Lagrangian, we must first fix the pattern of symmetry breaking. In the standard model, the gauge group is $SU(2)_L \times U(1)_Y$, spontaneously broken to the $U(1)$ of electromagnetism. The minimal global symmetry consistent with this gauge group is $G = SU(2) \times U(1)$, spontaneously broken to $H = U(1)$. Of course, the global symmetry group can also be larger. For example, it could be $G = SU(2) \times SU(2)$, broken to $H = SU(2)$, as in the minimal standard model. In this case, there is a "custodial" $SU(2)$ symmetry which ensures that $\rho = 1$, up to radiative corrections.\(^3\) Experimentally, we know that $\rho \simeq 1$, so we will adopt the second group, and assume that the custodial $SU(2)$ symmetry\(^4\) is broken only by terms that vanish as the hypercharge coupling\(^5\) $g' \to 0$.

Let us start by constructing the effective Lagrangian associated with breaking $SU(2) \times SU(2) \to SU(2)$. We introduce the would-be Goldstone boson fields $w^+$, $w^-$ and $z$, as well as the gauge fields $W_i$ and $B$, through the matrices

$$
\Sigma = \exp \left( \frac{i w^i \tau^i}{v} \right)
$$

$$
W^i_\mu \equiv W^i_\mu \tau^i
$$

$$
B_{\mu\nu} = \frac{1}{2} \left( \partial_\mu B_\nu - \partial_\nu B_\mu \right) \tau^3
$$

$$
W_{\mu\nu} = \frac{1}{2} \left( \partial_\mu W_\nu - \partial_\nu W_\mu - i \frac{2}{g} g[W_\mu, W_\nu] \right),
$$

where the $\tau^i$ are Pauli matrices, normalized so that $\text{Tr}(\tau^i \tau^j) = 2 \delta^{ij}$. The derivative

$$
D_\mu \Sigma = \partial_\mu \Sigma + \frac{i}{2} g W_\mu \Sigma - \frac{i}{2} g' B_\mu \Sigma \tau^3
$$

---

\(^3\) In analogy with QCD, we call the unbroken $SU(2)$ "isospin."

\(^4\) Note that this is a very strong assumption. The constraint $\rho \simeq 1$ affects only one term in the full effective Lagrangian.

\(^5\) The custodial $SU(2)$ symmetry is also broken by the mass splittings in the fermion doublets. We shall ignore this symmetry breaking in what follows.
transforms covariantly under global $SU(2) \times SU(2)$ transformations,

$$\Sigma \rightarrow L \Sigma R^\dagger,$$  \hspace{1cm} (2.3)

and $g$ and $g'$ are the coupling constants of the gauged $SU(2)_L$ and $U(1)_Y$ respectively.

The lowest-order term in the effective Lagrangian contains two derivatives,

$$\mathcal{L}^{(2)} = \frac{v^2}{4} \text{Tr} \, D^\mu \Sigma^\dagger D_\mu \Sigma.$$  \hspace{1cm} (2.4)

The couplings of the would-be Goldstone bosons to the $SU(2)_L \times U(1)_Y$ gauge fields are fixed by the covariant derivative. To this order, the effective Lagrangian is unique. The full Lagrangian is the sum of the lowest-order effective Lagrangian, together with the usual gauge-boson kinetic energy, gauge-fixing and Fadeev-Popov terms.

The next-to-leading order terms in the effective Lagrangian contain six free parameters:\#6

$$\mathcal{L}^{(4)} = \frac{L_1}{16\pi^2} \left[ \text{Tr} \left( D^\mu \Sigma^\dagger D_\mu \Sigma \right) \right]^2 + \frac{L_2}{16\pi^2} \text{Tr} \left( D_\mu \Sigma^\dagger D_\nu \Sigma \right) \text{Tr} \left( D^\mu \Sigma^\dagger D^\nu \Sigma \right)$$

$$- ig \frac{L_{9L}}{16\pi^2} \text{Tr} \left( W^{\mu \nu} D_\mu \Sigma D_\nu \Sigma^\dagger \right) - ig' \frac{L_{9R}}{16\pi^2} \text{Tr} \left( B^{\mu \nu} D_\mu \Sigma^\dagger D_\nu \Sigma \right)$$

$$+ gg' \frac{L_{10}}{16\pi^2} \text{Tr} \left( \Sigma B^{\mu \nu} \Sigma^\dagger W_{\mu \nu} \right) + \frac{1}{8} \Delta \rho v^2 \left[ \text{Tr} \left( \tau^3 \Sigma^\dagger D_\mu \Sigma \right) \right]^2.$$  \hspace{1cm} (2.5)

To this order, these are the only terms when $G = SU(2) \times SU(2)$, broken only by the hypercharge coupling $g'$. One can think of other terms, such as $\text{Tr} \left( D^2 \Sigma^\dagger D^2 \Sigma \right)$,\#6 We have normalized the coefficients as in Ref. 17, so they are “naturally” $\mathcal{O}(1)$. Note that we have included one term that is formally of order $p^2$. This term is induced at one loop by hypercharge gauge-boson exchange. Therefore $\Delta \rho$ is proportional to $g'^2/16\pi^2$, and the term can be considered to be of order $p^4$.\#6
but they can all be absorbed in Eq. (2.5) by using the lowest-order equations of motion, \( \Sigma D^2 \Sigma^\dagger = (D^2 \Sigma) \Sigma^\dagger \). Therefore we construct our \( \mathcal{O}(p^4) \) amplitudes by using Eq. (2.4) at the tree and one-loop levels, and Eq. (2.5) at tree level only.

### 2.2. Renormalization Scheme

As usual with effective Lagrangians, we renormalize our amplitudes using a mass-independent renormalization scheme. To order \( p^4 \), the infinities that appear at one loop can all be absorbed by defining renormalized parameters \( L^r_i(\mu) \). Therefore we use dimensional regularization and adopt the following renormalization scheme:

\[
\begin{align*}
L^r_1(\mu) &= L_1 + \frac{1}{24} \left( \frac{1}{\hat{\epsilon}} + \frac{5}{3} \right), \\
L^r_2(\mu) &= L_2 + \frac{1}{12} \left( \frac{1}{\hat{\epsilon}} + \frac{13}{6} \right), \\
L^r_{9L}(\mu) &= L_{9L} + \frac{1}{12} \left( \frac{1}{\hat{\epsilon}} + \frac{8}{3} \right), \\
L^r_{9R}(\mu) &= L_{9R} + \frac{1}{12} \left( \frac{1}{\hat{\epsilon}} + \frac{8}{3} \right), \\
L^r_{10}(\mu) &= L_{10} - \frac{1}{12} \left( \frac{1}{\hat{\epsilon}} + \frac{8}{3} \right),
\end{align*}
\]

where

\[
\frac{1}{\hat{\epsilon}} = \frac{2}{4-n} - \gamma + \log(4\pi) - \log(\mu^2).
\]

These definitions remove extraneous constants that can be absorbed into redefinitions of the \( L^r_i(\mu) \) in our amplitudes.

As mentioned earlier, we work to lowest order in the electroweak couplings, and compute the leading corrections of enhanced electroweak strength. Effectively this means that when we compute one-loop diagrams, we allow only would-be Goldstone bosons in the loops. This implies that we do not need to renormalize the usual gauge-boson sector of the theory.
To one-loop order, the $\Delta \rho$ term in the effective Lagrangian is renormalized by diagrams with a hypercharge gauge boson in the loop. It is not renormalized at all if we only consider the terms of enhanced electroweak strength.\footnote{Recall that we assume $H = SU(2)$.} Therefore, we do not need to specify a renormalization for the coefficient $\Delta \rho$.

For studies at energies on the order of the $W$ mass, it is not possible to separate the terms into those of electroweak and enhanced electroweak strength. One must calculate beyond leading order in $g$ or $g'$, and introduce a renormalization scheme for the usual gauge sector of the electroweak interactions. One also needs an additional counterterm for $\Delta \rho$.

All these complications become necessary in studies for lower energy machines, such as LEP2 and the Tevatron. We are able to avoid these issues because we concentrate on a kinematic regime where the only relevant terms are those of enhanced electroweak strength.

### 2.3. Present Constraints

We can gain some insight into the constraints on our $SU(2) \times SU(2)$ effective Lagrangian by reducing it to unitary gauge, with $\Sigma = 1$. In this gauge the new physics appears in the form of anomalous gauge-boson couplings.

Let us first consider the term with $L_{10}$, which reduces to

$$\frac{g g'}{2} \frac{L_{10}}{16\pi^2} \left( \partial_\mu B_\nu - \partial_\nu B_\mu \right) \left[ \partial_\mu W_3^3 - \partial_\nu W_3^3 - i g (W_\mu^+ W_\nu^- - W_\mu^- W_\nu^+) \right]. \quad (2.8)$$

This term contains a three-gauge-boson coupling, as well as an “oblique” correction to the gauge-boson self-energies [22]. This correction is related to the $S$ parameter that occurs in electroweak radiative corrections for $Z$-physics, $L_{10}^r(M_Z) = -\pi S$ [23,24]. A recent best fit to the data gives $L_{10}^r(M_Z) = 0.0 \pm 1.6$ [25], which we translate into

$$L_{10}^r(\mu) = 0.5 \pm 1.6,$$

for $\mu = 1.5$ TeV.
The terms with $L_0$ contain anomalous three- and four-gauge-boson couplings. The standard notation for anomalous three-gauge-boson couplings is given by [26,27,28]

$$-ie\, \kappa_\gamma \, W^+_{\mu} W^-_{\nu} A^{\mu\nu} - ie \cot \theta \, \kappa_Z \, W^+_{\mu} W^-_{\nu} Z^{\mu\nu},$$

where [17,28]

$$\kappa_Z - 1 \simeq \kappa_\gamma - 1 \equiv \Delta \kappa \simeq O \left( g^2 \frac{L_{0L,R}}{16\pi^2} \right).$$

An analysis by Kane, Vidal and Yuan [29] finds that the SSC will be sensitive to values of $|\Delta \kappa| \gtrsim 0.15$, which we translate to $|L_{0L,R}(\mu)| \gtrsim 25$, for $\mu \simeq 1.5$ TeV. \#8

This is compatible with the results of Falk, Luke and Simmons [17], who studied $pp \to W^\pm Z$ and $pp \to W^\pm \gamma$ and found that the SSC will not be sensitive to the values

$$-16 \lesssim L_{0L}(\mu) \lesssim 7$$

$$-119 \lesssim L_{0R}(\mu) \lesssim 113,$$

evaluated at $\mu \simeq 1.5$ TeV. The present bounds are of order [30]

$$-2.2 \leq \kappa_\gamma - 1 \leq 2.6,$$

although the Tevatron is expected to reach a sensitivity of about 1.3 for $|\kappa_\gamma - 1|$ [33]. This is similar to the expected sensitivity of LEP2 [26,29,31,32].

The terms with $L_1$ and $L_2$ give rise to anomalous four-gauge-boson couplings. These couplings are not constrained by experiment. They are limited, however, by perturbative unitarity. As we will see in the following sections, $L^r_1(\mu)$ and $L^s_2(\mu)$ contribute to the $V_L V_L \to V_L V_L$ scattering amplitudes. These amplitudes grow with energy, which allows us to place “unitarity bounds” on the parameters. The procedure is simple: one first computes the partial wave amplitudes $a^0_0$, $a^2_0$, $a^1_1$, $a^2_0$ and $a^2_2$ to one loop, and then demands that they do not violate the elastic partial

\#8 We interpret the results of Refs. 17 and 29 in terms of running couplings evaluated at $\mu \simeq 1.5$ TeV.
wave unitarity condition $|Re(a_f^j)| \leq 1/2$ below $M_{VV} = 1.0$ TeV (or $M_{VV} = 1.5$ TeV). This gives the “allowed” regions shown in Fig. 1. By considering inelastic unitarity constraints on the $q\bar{q} \to V_LV_L$ amplitudes, one can perform a similar exercise to bound $L_{0L}(\mu)$ and $L_{0R}(\mu)$. It turns out, however, that this does not significantly constrain $L_{rL,R}(\mu)$.

The final term in (2.5) is proportional to $\Delta \rho$. By power counting, we assign a factor of $g^2/16\pi^2$ to this coupling,#9

$$\Delta \rho \equiv g^2 \frac{L_{12}}{16\pi^2}. \quad (2.13)$$

Combining LEP data and low energy data from deep inelastic scattering and parity violation in Cesium, Altarelli [25] finds $\Delta \rho = 0.0016 \pm 0.0032$, which gives $L_{12} \simeq 2.0 \pm 4.0$. We shall set $\Delta \rho = 0$ in what follows.

With these counting rules, all other terms in Ref. 12 with four derivatives are actually of order $p^6$. In particular, this includes a term recently discussed by Holdom [34],

$$K \text{Tr} \left( \tau_3 \Sigma^\dagger D_\mu D_\nu \Sigma \right) \text{Tr} \left( \tau_3 (D_\mu D_\nu \Sigma^\dagger) \Sigma \right). \quad (2.14)$$

This term is related to the observable $U$ that has been used in the study of electroweak radiative corrections. Other anomalous three-gauge-boson couplings considered in the literature include#10

$$\frac{\lambda_\gamma}{M_W^2} W^+_{\lambda\mu} W^-_{\nu\mu} A^{\nu\lambda} + \frac{\lambda_Z}{M_W^2} W^+_{\lambda\mu} W^-_{\nu\mu} Z^{\nu\lambda}. \quad (2.15)$$

These couplings are of order $p^6$ and are suppressed within the framework of our discussion.

#9 As mentioned before, this term arises at one loop from hypercharge gauge-boson exchange. It is not renormalized by loops containing only would-be Goldstone bosons. Since we are only computing the corrections of enhanced electroweak strength, any “bare” value of $L_{12}$ is not renormalized, and $L_{12}$ is independent of $\mu$.

#10 Note that power counting indicates that these terms should not be suppressed by $M_W^2$, but by $\Lambda^2 \lesssim 16\pi^2 v^2$. 

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2.4. Typical Coefficients

In order to understand the bounds on the coefficients \( L_i^r(\mu) \), it is instructive to estimate the size of the coefficients in typical theories. Using the effective Lagrangian approach, this can be done in a consistent way. We first consider a model with three would-be Goldstone bosons, interacting with a scalar, isoscalar resonance like the Higgs boson. We assume that the \( L_i^r(\mu) \) are dominated by tree-level exchange of the scalar boson. If one integrates out the scalar, and matches coefficients at the scale \( M_H \), one finds [12,13,35]:

\[
L_1^r(\mu) = \frac{64\pi^3}{3} \frac{\Gamma_H v^4}{M_H^5} + \frac{1}{24} \log \left( \frac{M_H^2}{\mu^2} \right)
\]

\[
L_2^r(\mu) = \frac{1}{12} \log \left( \frac{M_H^2}{\mu^2} \right)
\]

\[
L_{9L}^r(\mu) = L_{9R}^r(\mu) = \frac{1}{12} \log \left( \frac{M_H^2}{\mu^2} \right)
\]

\[
L_{10}^r(\mu) = -\frac{1}{12} \log \left( \frac{M_H^2}{\mu^2} \right),
\]

where \( \Gamma_H \) is the width of the scalar into Goldstone bosons. If we naively take

\[
\Gamma_H = \frac{3M_H^3}{32\pi v^2},
\]

as in the standard model, we find the values quoted in Table 1, assuming \( \mu = 1.5 \) TeV.

Let us now consider a second model for the \( L_i^r(\mu) \), and assume that the coefficients are dominated by tree-level exchange of a rho-like particle with spin and isospin one. Integrating out the rho, and matching coefficients at the scale \( M_\rho \),
Table 1

Coefficients induced by a scalar, isoscalar particle like the Higgs, with $\mu = 1.5$ TeV.

| $M_H$ (TeV) | $L_1^r(\mu)$ | $L_5^r(\mu)$ | $L_6^r(\mu)$ | $L_{10}^r(\mu)$ |
|------------|--------------|--------------|--------------|----------------|
| 2.0        | 0.33         | 0.01         | 0.01         | −0.01          |
| 1.5        | 0.55         | 0.00         | 0.00         | 0.00           |

one finds: \(^{11}\)

\[
L_1^r(\mu) = \frac{1}{24} \left[ -96\pi^2f^2 \frac{2}{M_\rho^2} + \log\left(\frac{M_\rho^2}{\mu^2}\right) \right]
\]

\[
L_5^r(\mu) = \frac{1}{12} \left[ 48\pi^2f^2 \frac{2}{M_\rho^2} + \log\left(\frac{M_\rho^2}{\mu^2}\right) \right]
\]

\[
L_{9L}^r(\mu) = L_{9R}^r(\mu) = \frac{1}{12} \left[ 96\pi^2fF_\rho \frac{2}{M_\rho^2} + \log\left(\frac{M_\rho^2}{\mu^2}\right) \right]
\]

\[
L_{10}^r(\mu) = -\frac{1}{12} \left[ 48\pi^2F_\rho^2 \frac{2}{M_\rho^2} + \log\left(\frac{M_\rho^2}{\mu^2}\right) \right]
\]

where the constant $f$ is related to the width $\Gamma_\rho$,

\[
\Gamma_\rho = \frac{1}{48\pi} \frac{f^2}{v^4} M_\rho^3 , \tag{2.19}
\]

and $F_\rho$ is defined by:

\[
\langle 0|V_{\mu}^i|\rho^k(p)\rangle = \delta^{ik}\epsilon_\mu F_\rho M_\rho . \tag{2.20}
\]

To estimate these parameters, we will model the resonance by a techni-rho whose properties are fixed by the ordinary QCD rho. Using large-$N$ scaling arguments

\(^{11}\) See the second paper of Ref. 13. The contribution to $\Delta_\rho$ is computed in Ref. 36. Related issues are discussed in Ref. 37.
Table 2

Coefficients induced by a vector, isovector particle like the techni-rho, with \( \mu = 1.5 \) TeV.

| \( M_\rho \) (TeV) | \( L_1^r(\mu) \) | \( L_2^r(\mu) \) | \( L_0^r(\mu) \) | \( L_{10}^r(\mu) \) |
|-------------------|-----------------|-----------------|-----------------|-----------------|
| 2.0               | -0.31           | 0.38            | 1.4             | -1.5            |
| 1.5               | -0.60           | 0.60            | 2.4             | -2.5            |

for the mass and width of the resonance, we find the values quoted in Table 2, for \( \mu = 1.5 \) TeV.

In either case, the \( L_i^r(\mu) \) are numbers of order one, which implies that \( \Delta \kappa \simeq O(g^2/16\pi^2) \). Values much larger than these are associated with light resonances (or other light particles not present in the minimal standard model), in which case chiral perturbation theory breaks down at a very low energy. Since one would presumably see the new particles directly, it does not make sense to take the \( L_i^r(\mu) \) much larger than one.

3. Vector Boson Fusion

In this section we will consider the production of longitudinal vector bosons by the process of vector boson fusion. Previous studies have examined the case with two longitudinal vector bosons in the initial state. At high energies, this process dominates the scattering of vector bosons with two transverse (or one transverse and one longitudinal) polarizations.

This is illustrated in Figs. 2 and 3, where we show the counting rules that isolate the terms of enhanced electroweak strength. The counting rules indicate that the most important diagrams are those with longitudinal polarizations in the external states. They also imply that the most important radiative corrections
come from diagrams with Goldstone bosons in the loops. These are the radiative corrections of enhanced electroweak strength, as in standard model calculations [19].

This power counting is completely correct for final states, so we will restrict our attention to final states of purely longitudinal polarization. For initial states, however, the power counting is too naive: it ignores the fact that the luminosity of transverse pairs is much larger than that of longitudinal pairs [39]. The large transverse luminosity can compensate for the relatively smaller subprocess cross sections. Therefore we shall study all production mechanisms, including those with transverse vector bosons in the initial state.

3.1. Tensor structure

We begin by decomposing the amplitude into five different tensor forms that contain “transverse,” “longitudinal,” and “mixed” polarization pieces. We ignore the possibility of epsilon tensors because they do not appear at the order to which we are working. Therefore we write:

$$M = \epsilon_\mu(q_1)\epsilon_\nu(q_2)M_{\mu\nu}$$

$$M_{\mu\nu} = T_{\mu\nu} + L_{\mu\nu} + X_{\mu\nu},$$

where we adopt the notation

$$V_\mu(q_1)V_\nu(q_2) \rightarrow w(p)w(p').$$

In the final state, we invoke the equivalence theorem and denote the longitudinal vector particles by their corresponding would-be Goldstone bosons. For charged particles, we take $q_1 = q^+$ and $p = p^+$. #12

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#12 One might think that contributions from the heavy top quark would play an important role because they are enhanced by factors of $M_t^2/M_W^2$. In Ref. 38 it was shown that these contributions are small for the expected values of the top quark mass. They could be important, however, for a heavy fourth generation of fermions.

#13 Note that $M(V_\mu V_\nu \rightarrow ww) = -M(V_\mu V_\nu \rightarrow V_L V_L)$. 
In Eq. (3.1), $T_{\mu\nu}$ denotes the tensor for transverse gauge bosons. It satisfies the conditions

$$T_{\mu\nu} q_1^\mu = 0 \quad \text{and} \quad T_{\mu\nu} q_2^\nu = 0 ,$$  \hspace{1cm} (3.3)

and contains two form factors

$$T_{\mu\nu} = T_s(s, t, u) \left( -\frac{s}{2} g_{\mu\nu} + q_2^\mu q_1^\nu \right)$$

$$+ T_d(s, t, u) \left( \frac{ut}{2} g_{\mu\nu} + up_\mu q_1^\nu + sp_\mu p_\nu + t q_2^\mu p_\nu \right)$$ \hspace{1cm} (3.4)

When the initial or final states contain identical particles, Bose symmetry requires that

$$T_s(s, t, u) = T_s(s, u, t)$$

$$T_d(s, t, u) = T_d(s, u, t) .$$ \hspace{1cm} (3.5)

Note that gauge invariance implies that amplitudes involving two photons must reduce to this form. Eq. (3.4) is also, of course, the form that is found for the $gg \rightarrow V_L V_L$ amplitudes in the gluon fusion process.

In Eq. (3.1), $L_{\mu\nu}$ denotes the part of the amplitude that describes the purely longitudinal polarizations. In particular, this amplitude must vanish when contracted with a transverse polarization vector, which implies

$$L_{\mu\nu} = T_l(s, t, u) q_1^\nu q_2^\mu .$$ \hspace{1cm} (3.6)

Then the $V_L V_L \rightarrow V_L V_L$ amplitudes are given by

$$- \frac{s^2}{4 M_{V_1} M_{V_2}} T_l(s, t, u) .$$ \hspace{1cm} (3.7)

We have checked our results by comparing with the amplitudes computed directly from the equivalence theorem.
The remaining tensor structure corresponds to states of mixed polarization, with one transverse and one longitudinal vector boson. There are two form factors for this part of the amplitude:

\[
X_{\mu\nu} = T_{m1}(s, t, u) \left( q_2\mu p_{\nu} + p_\mu q_1\nu - q_2\mu q_1\nu \right) + T_{m2}(s, t, u) \left( tq_2\mu p_{\nu} - up_\mu q_1\nu \right). \tag{3.8}
\]

As above, we have checked our results by contracting with one longitudinal polarization vector and comparing with the results obtained from the equivalence theorem.

### 3.2. Independent Amplitudes

Since our chiral Lagrangian preserves an approximate \( SU(2) \) symmetry, we can use isospin arguments to reduce the number of independent form factors. In this section we work in the \( (W_3, B) \) basis because it simplifies the isospin properties of the vector bosons.

For amplitudes involving two \( W \)'s in the initial state, all the particles are isospin triplets. The reaction is then characterized by four isovector indices \( ij \to kl \). Ignoring the tensor structure (but remembering that interchanges of \( s, t, u \) also imply interchanges of momenta and Lorentz indices), we write:

\[
M^{ijkl}(s, t, u) = A(s, t, u) \delta^{ij} \delta^{kl} + B(s, t, u) \delta^{ik} \delta^{jl} + C(s, t, u) \delta^{il} \delta^{jk}. \tag{3.9}
\]

Since the particles in the initial and final states are typically not the same, we can only use \( t \leftrightarrow u \) crossing to simplify this expression. We find

\[
A(s, t, u) = A(s, u, t) \tag{3.10} \\
C(s, t, u) = B(s, u, t),
\]

16
which implies:

\[ M^{ijkl}(s, t, u) = A(s, t, u)\delta^{ij}\delta^{kl} + B(s, t, u)\delta^{ik}\delta^{jl} + B(s, u, t)\delta^{il}\delta^{jk}. \quad (3.11) \]

We see that there are two independent functions for amplitudes involving only the W bosons. They are:

\[ M(W^+W^- \rightarrow zz) = A(s, t, u) \]
\[ M(W^+W^3 \rightarrow w^+z) = B(s, t, u). \quad (3.12) \]

The other W amplitudes can then be reconstructed from the relations,

\[ M(W^3W^3 \rightarrow w^+w^-) = A(s, t, u) \]
\[ M(W^-W^3 \rightarrow w^-z) = B(s, t, u) \]
\[ M(W^+W^- \rightarrow w^+w^-) = A(s, t, u) + B(s, t, u) \]
\[ M(W^\pm W^\pm \rightarrow w^\pm w^\pm) = B(s, s, t) + B(s, u, t) \]
\[ M(W^3W^3 \rightarrow zz) = A(s, t, u) + B(s, t, u) + B(s, u, t). \quad (3.13) \]

Note that for purely longitudinal scattering, \( s \leftrightarrow t \) crossing implies \( B(s, t, u) = A(t, s, u) \).

The amplitudes involving two hypercharge bosons are a-priori independent. From our Lagrangian, it is not hard to see that

\[ M(BB \rightarrow zz) = \left( \frac{g'}{g} \right)^2 M(W^3W^3 \rightarrow zz) \]
\[ M(BB \rightarrow w^+w^-) = \left( \frac{g'}{g} \right)^2 M(W^3W^3 \rightarrow w^+w^-). \quad (3.14) \]

The amplitudes involving one W and one B cannot be simplified with isospin arguments and we must compute them explicitly.

In the appendix, we present explicit results for the independent scattering amplitudes. The physical amplitudes can be reconstructed using the relationships of this and the previous section.
4. Quark Anti-quark Annihilation

Light $q\bar{q}$ annihilation is the most important mechanism for vector boson pair production in hadronic colliders [41,42]. This process tends to produce transversely-polarized vector bosons and has been typically considered as a background to new physics. In traditional studies of a heavy standard-model Higgs boson, one tries to suppress this mechanism with appropriate cuts [43].

Quark anti-quark annihilation also produces a smaller number of longitudinal vector boson pairs in an $I = 1$ state (to the extent that quark masses can be ignored). This production mechanism must be considered when searching for new physics with isotriplet resonances like the techni-rho [28,44].

For our calculation we assume standard couplings of the gauge bosons to the light fermions. Nonetheless, the next-to-leading terms in the chiral Lagrangian (2.5) affect the production of $V_LV_L$ pairs through $q\bar{q}$ annihilation. This has been discussed in Refs. 15 and 17. In Ref. 15 an estimate of the rescattering of the $V_LV_L$ pair was made by considering its absorptive part. Since the $V_LV_L$ pair is produced in an $I = 1$ state, the rescattering is sensitive to $L^r_{1,2}(\mu)$ through the $I = 1, J = 1$ partial wave. This effect is $\mathcal{O}(p^6)$ in the energy expansion.

At order $p^4$, $q\bar{q}$ annihilation is sensitive to the parameters $L^r_{9L,R}(\mu)$, as discussed\#14 in Ref. 17. Our calculations differ from those of Ref. 17 in two ways. First, the authors of Ref. 17 did not include loop effects, which enter the amplitude at the same order as the $L^r_{9L,R}(\mu)$. Second, we did not compute any $\mathcal{O}(g^4)$ contributions, which are suppressed by $M_W^2/s$ with respect to the leading terms. In our calculations, we include the full set of $\mathcal{O}(p^4)$ terms. These contributions are of enhanced electroweak strength; they dominate the scattering amplitudes at high energies.

We obtain the terms of enhanced electroweak strength from the diagrams depicted schematically in Fig. 4. If we define form factors for the $\gamma w^+w^-$ and $Zw^+w^-$

\#14 It is also sensitive to $L^r_{10}(\mu)$, but not through terms of enhanced electroweak strength.
vertices by

\[ A(\gamma \rightarrow w^+w^-)_\mu = -e(p^+ - p^-)_\mu f_\gamma(s) \]
\[ A(Z \rightarrow w^+w^-)_\mu = -\frac{g}{2\cos\theta_W} (p^+ - p^-)_\mu \cos(2\theta_W) f_Z(s) , \]  

we find the following one-loop result in Landau gauge:

\[ f_\gamma(s) = 1 - \frac{s}{96\pi^2 v^2} \log \left( \frac{s}{\mu^2} \right) + \frac{s}{16\pi^2 v^2} \left[ L^r_{0L}(\mu) + L^r_{0R}(\mu) \right] \]
\[ f_Z(s) = f_\gamma(s) + \frac{1}{\cos(2\theta_W)} \left[ \frac{s}{16\pi^2 v^2} (L^r_{0L}(\mu) - L^r_{0R}(\mu)) \right] . \]  

(4.1)

In the helicity basis, the amplitude for \( q(k^+, \lambda)\bar{q}(k^-, \lambda') \rightarrow w^+(p)w^-(p') \) can be written as

\[ A_{\lambda\lambda'} = C_{\lambda\lambda'} \sin \theta , \]  

(4.3)

where \( \theta \) is the angle between \( \vec{k}^+ \) and \( \vec{p} \) in the \( q\bar{q} \) center of mass frame. Using Eq. (4.2), and assuming that the couplings of the gauge bosons to the light quarks are the same as in the standard model, we find

\[ C_{+-} = g^2 \left[ \sin^2(\theta_W) Q_q f_\gamma(s) - \frac{\sin^2(\theta_W) \cos(2\theta_W) Q_q}{2 \cos^2(\theta_W)} f_Z(s) \right] \]
\[ C_{-+} = -g^2 \left[ \sin^2(\theta_W) Q_q f_\gamma(s) - \frac{\sin^2(\theta_W) Q_q - T_3}{2 \cos^2(\theta_W)} \cos(2\theta_W) f_Z(s) \right] , \]  

(4.4)

to one-loop order, with \( T_3 = \pm 1/2 \). In a similar way, for \( u(k^+, \lambda)\bar{d}(k^-, \lambda') \rightarrow w^+(p)z(p') \), we find

\[ A_{\pm\pm} = -\frac{g^2 \sin \theta V_{ud}}{4\sqrt{2}} \left\{ 1 - \frac{s}{96\pi^2 v^2} \left( \log \left( \frac{s}{\mu^2} \right) - 12L^r_{0L}(\mu) \right) \right\} , \]  

(4.5)

where \( V_{ud} \) is the Kobayashi-Maskawa mixing angle. Note that the amplitude for \( q\bar{q} \rightarrow zz \) vanishes in the limit of massless quarks.
5. Numerical Results

In this section we present the complete cross section for the process \( pp \to V_L V_L \), to order \( p^4 \) in \( SU(2) \times SU(2) \) chiral perturbation theory. We include the contributions from all production mechanisms: \( q\bar{q} \) annihilation, vector boson fusion and gluon fusion through a top-quark loop [21]. Our results contain all terms of enhanced electroweak strength, and are correct to order \( p^4 \) in chiral perturbation theory.

For initial states with quarks and antiquarks, we find the hadronic cross sections in the usual way, by convoluting the subprocess cross sections with the EHLQ [41] structure functions (set 1), evaluated at \( Q^2 = s \), where \( s \) is the squared center of mass energy for the scattering subprocess. For initial states with vector bosons, we use the effective \( W \) approximation to find effective luminosities for the initial particles. We evaluate the EHLQ structure functions at \( Q^2 = M_W^2 \), which has been shown to be a reasonable approximation for the standard-model Higgs [45]. We then compute the full \( pp \) cross sections by folding these luminosities with the subprocess cross sections,

\[
\frac{d\sigma}{dM_{VV}}(pp \to V_L V_L) = 2M_{VV} \frac{d\mathcal{L}}{d\tau} \bigg|_{pp/VV} \hat{\sigma}(M_{VV}^2). \tag{5.1}
\]

In this way we find the phenomenological cross sections for \( V_L V_L \) production at hadronic colliders. It is important to note, however, that in the \( W^+ W^- \), \( ZZ \), and \( W^\pm Z \) channels, the longitudinal final states are dominated by configurations with transverse polarizations. The transverse background must be suppressed if we are to have any hope of observing the new physics associated with the longitudinal final states\(^{15} \) [43, 45-48].

There are several ways this can be done. One possibility is to separate the longitudinally-polarized gauge bosons from transverse background, as has been

\(^{15} \) As discussed above, new physics also affects the transverse states. The effects are small, however, because the terms are not of enhanced electroweak strength.
studied in Ref. [46]. A second proposal is to use forward jet tagging to eliminate the states produced by $q\bar{q}$ annihilation and gluon fusion [43,47,48]. A third suggestion is to study the $W^\pm W^\pm$ channel because it receives no contribution from the $q\bar{q}$ or $gg$ initial states [45,48].

In Figs. 5 − 8, we show the cross sections for $pp \to V_LV_L$, using our $O(p^4)$ amplitudes with $L_\mu^L(\mu) = 0$, for $\mu = 1.5$ TeV. The figures include all production mechanisms, and describe a universal background that is always present in theories with no light resonances. From the figures we see that $q\bar{q}$ annihilation provides the most important contribution to the $W_L^+W_L^-$ and $W_L^\pm Z_L$ final states.\textsuperscript{#16} The $W_L^+W_L^-$ and $Z_L Z_L$ final states also receive contributions from gluon fusion through loops of heavy quarks. The figures indicate that the contribution to $Z_L Z_L$ is of the same order as that from vector boson fusion. We have not illustrated the $W_L^+W_L^-$ rate here; a complete calculation for the standard model shows that it is significantly smaller than the contribution from $q\bar{q}$ annihilation [49].

In Figs. 5 − 8, we also show the contributions from all polarizations of vector boson fusion, $V_LV_L \to V_LV_L$, $V_LV_T \to V_LV_L$, and $V_TV_T \to V_LV_L$ (where the two transverse polarizations are summed). At low energy, we have

$$\sigma(V_TV_T \to V_LV_L) > \sigma(V_TV_L \to V_LV_L) > \sigma(V_LV_L \to V_LV_L), \quad (5.2)$$

which is entirely due to the magnitudes of the vector boson luminosities. We see that at $M_{VV} \simeq 400$ GeV, the transversely-polarized vector bosons increase the rate by a factor of about two; above this energy they become less important. At high energy, we have

$$\sigma(V_LV_L \to V_LV_L) \simeq \frac{s}{v^4},$$

$$\sigma(V_TV_L \to V_LV_L) \simeq \frac{1}{v^2}, \quad (5.3)$$

$$\sigma(V_TV_T \to V_LV_L) \simeq \frac{1}{s},$$

\textsuperscript{#16} This process vanishes for the $Z_L Z_L$ and $W_L^+W_L^+$ channels.
where $s$ is the squared center of mass energy in the vector boson scattering sub-system. In this regime, the dominant contribution comes from initial states with longitudinally-polarized particles. Our results indicate that the cross-over occurs at approximately 400 GeV. Note that the $V_LV_T$ cross sections for $W^+W^+$ and $ZZ$ vanish at $O(p^2)$, so they do not satisfy Eq. (5.3).

As discussed above, the major contribution to the $W_L^+W_L^-$ and $W_L^\pm Z_L$ final states comes from $q\bar{q}$ annihilation. In the $W_L^+W_L^-$ channel, this is sensitive to $L_{9L,R}(\mu)$. In contrast, the $q\bar{q}$ contribution to the $W_L^\pm Z_L$ channel depends on $L_{9L}(\mu)$ only [17]. The $Z_LZ_L$ and $W_L^\pm W_L^\pm$ final states do not receive contributions from $q\bar{q}$ annihilation. They probe new physics through vector boson fusion, which depends primarily on $L_1^R(\mu)$ and $L_2^R(\mu)$. In the $Z_LZ_L$ case, however, the cross section must be disentangled from the background from gluon fusion through a top-quark loop.

In Figs. 9 − 12, we show the total contribution of vector boson scattering to $V_LV_L$ production for the SSC and the LHC. The solid curves were computed with all the $L_i^R(\mu) = 0$, for $\mu = 1.5$ TeV. The dotted curves correspond to $L_1^R(\mu) = -0.6$, $L_2^R(\mu) = 0.6$, $L_9^L(\mu) = L_9^R(\mu) = 2.4$, and $L_{10}^R(\mu) = -2.5$, which result from a spin-one, isospin-one resonance of mass 1.5 TeV. From the figures we see that the differences are very small. Whether or not they can be detected is a question that requires a full phenomenological analysis of potential signals, backgrounds and cuts, which is far beyond the scope of this paper. In what follows, we will attempt to make an initial rough estimate, and leave a more detailed analysis to later work.

In Fig. 13 we plot the total rate of $W_L^+Z_L$ pairs, integrated over the region $0.5 < M_{WZ} < 1.0$ TeV, as a function of $L_{9L}(\mu)$ with $\mu = 1.5$ TeV. Since this channel is particularly sensitive to $L_{9L}(\mu)$, we have set all the other coefficients $L_i^R(\mu) = 0$. Assuming that it will be possible to measure the polarization of the final state, and defining $L_{9L}(\mu)$ as being observable if it induces a 50% change in the integrated cross section, we see that the SSC and LHC will be sensitive to $L_{9L}(\mu) \lesssim -3.5$ and $L_{9L}(\mu) \gtrsim 2.5$. If we assume that the polarization measurement

\#17 The number of events is much larger at the SSC than the LHC, however, so the statistical significance of the results will be larger at the SSC.
is not possible, the change in the rate is always less than about 5%. Our results are consistent with those of Ref. 17, and indicate that polarization measurements will be necessary for the SSC and LHC to place meaningful constraints on the three-gauge-boson vertices $L^9_{\varphi L}(\mu)$ and $L^9_{\varphi R}(\mu)$.

In Fig. 14 we plot the total rate of $W^+_L W^-_L$ pairs, integrated over the region $0.5 < M_{WZ} < 1.0$ TeV, as a function of $L^r_{\varphi L}(\mu) = L^r_{\varphi R}(\mu)$ with $\mu = 1.5$ TeV. All other coefficients have been set to zero. Using the above assumptions, we estimate the SSC and LHC will be sensitive to $L^9_{\varphi}(\mu) \lesssim -4.0$ and $L^9_{\varphi}(\mu) \gtrsim 3.0$, modulo the question of backgrounds. The results are similar to those of Fig. 13.

In Fig. 15 we plot the total rate of $W^+_L W^+_L$ pairs in the range $0.5 < M_{WW} < 1.0$ TeV, as a function of $L^r_1(\mu)$, with $L^r_2(\mu) = 0$ and $\mu = 1.5$ TeV. The values of $L^r_1(\mu)$ are those that preserve unitarity up to 1 TeV (see Fig. 1). We have set all the other coefficients $L^r_i(\mu) = 0$. With the previous assumptions, we estimate that the SSC and LHC will be sensitive to $L^r_1(\mu) \lesssim -0.75$. A similar figure with $L^r_1(\mu) = -L^r_2(\mu)$ in shown in Fig. 16. The corresponding limits are $-4.0 \lesssim L^r_1(\mu) \lesssim -1.0$ and $L^r_1(\mu) \gtrsim 0.8$. The SSC and LHC will be the first machines to seriously constrain the four-gauge-boson vertices $L^r_1(\mu)$ and $L^r_2(\mu)$.

Of course, we are well aware that searching for new physics by measuring deviations in absolute rates requires a firm understanding of the uncertainties involved. Our estimates are subject to substantial corrections because of detection issues that we have ignored. In addition, we have not addressed the dependence of our results on the choice of structure functions or on the scale at which the structure functions are evaluated. Nevertheless, the calculations presented here provide a consistent framework for more detailed phenomenological studies of electroweak symmetry breaking. Clearly, more work is required before definitive answers can be found.
6. Summary and Conclusions

In this paper we computed the cross section for producing longitudinal vector boson pairs at hadron colliders. We included all production mechanisms (except $gg \rightarrow W_L^+W_L^-$), and evaluated our amplitudes to next-to-leading order in $SU(2) \times SU(2)$ chiral perturbation theory. The formalism describes electroweak symmetry breaking in terms of five free parameters, and is appropriate for theories with no new resonances below the TeV scale. Our results should give a satisfactory representation of the scattering amplitudes below one TeV. Since they were obtained using the equivalence theorem, they are valid only to leading order in the electroweak gauge couplings. Our amplitudes contain the leading corrections from terms of enhanced electroweak strength.

In the $W^+W^-$, $ZZ$ and $W^\pm Z$ channels, we saw that $q\overline{q}$ annihilation gives the most important contribution to the hadronic cross section. The rate is dominated by the transverse modes, whose dependence on new physics is not of enhanced electroweak strength. Clearly, if we wish to study the physics of electroweak symmetry breaking, we must isolate the longitudinal final states. Even in this case, there is an important model-independent background from gluon fusion through heavy quark loops. This provides a major contribution to the $W_L^+W_L^-$ and $Z_LZ_L$ channels.\footnote{To $\mathcal{O}(p^4)$, the $gg \rightarrow Z_LZ_L$ and $gg \rightarrow W_L^+W_L^-$ cross sections are sensitive to anomalous top couplings \cite{50}. In the $W_L^+W_L^-$ channel, the signal is masked by $q\overline{q}$ annihilation.} In contrast, the $W_L^\pm W_L^\mp$ channel is particularly clean because it receives no contribution from $q\overline{q}$ annihilation or gluon fusion.

The sensitivity to new physics is shown in Table 3, where the rows are labelled by production mechanism, and the columns by the final state. Since $q\overline{q}$ annihilation provides a significant source of longitudinal pairs in the $W_L^+W_L^-$ and $W_L^\pm Z_L$ channels, we see that these contributions are sensitive to the parameters $L^r_9L(\mu)$ and $L^r_{9R}(\mu)$ in the effective Lagrangian. These parameters describe anomalous three-gauge-boson couplings.

From Table 3 we see that $V_LV_L$ fusion contributes to all final states. This
process is sensitive to $L_1^r(\mu)$ and $L_2^r(\mu)$ in the effective Lagrangian, and probes anomalous four-gauge-boson couplings. In the $W_L^+W_L^-$, $Z_LZ_L$ and $W_L^\pm Z_L$ channels, the $V_LV_L$ rate must be isolated from the other production mechanisms. In the $W_L^\pm W_L^\mp$ channel, however, $V_LV_L$ fusion gives the most important contribution. This process can be used to bound $L_1^r(\mu)$ and $L_2^r(\mu)$.

In this paper we have used chiral perturbation theory to explore the mechanism of electroweak symmetry breaking in the absence of light resonances. Clearly, our calculations should be extended to include realistic backgrounds, cuts and detector simulations. Only then can one determine the full capability of the SSC and LHC for exploring the physics of electroweak symmetry breaking.

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APPENDIX A

In this appendix, we present explicit results for vector boson scattering into longitudinal gauge-boson pairs. We work at one-loop order in the high-energy limit, and assume that the chiral symmetry group is $SU(2) \times SU(2)$, spontaneously broken to $SU(2)$. All the relevant amplitudes can be extracted using the relationships of sections 3.1 and 3.2.

I. $W^3_\mu(q_1)W^3_{\nu_1}(q_2) \rightarrow w^+(p)w^-$

$$T_s(s,t,u) = g^2 \left[ \frac{1}{s} + \frac{1}{2\pi^2 v^2} \left( L_1^r(\mu) + \frac{tu}{s^2} L_2^r(\mu) + \frac{1}{16} \right) \right]$$

$$- \frac{g^2}{96\pi^2 v^2} \left[ 3 \log \left( -\frac{s}{\mu^2} \right) + \frac{u-t}{s^2} \left( t \log \left( -\frac{t}{\mu^2} \right) - u \log \left( -\frac{u}{\mu^2} \right) \right) \right]$$

$$T_d(s,t,u) = g^2 \left[ \frac{1}{tu} + \frac{1}{2\pi^2 v^2 s} L_2^r(\mu) \right] - \frac{g^2}{48\pi^2 v^2 s} \left[ \log \left( -\frac{t}{\mu^2} \right) + \log \left( -\frac{u}{\mu^2} \right) \right]$$

$$T_l(s,t,u) = -g^2 \left[ \frac{1}{s} + \frac{1}{4\pi^2 v^2} \left( 2L_1^r(\mu) + L_2^r(\mu) - \frac{2tu}{s^2} L_2^r(\mu) \right) \right]$$

$$+ \frac{g^2}{96\pi^2 v^2} \left[ 3 \log \left( -\frac{s}{\mu^2} \right) + \frac{u-t}{s^2} \left( u \log \left( -\frac{u}{\mu^2} \right) - t \log \left( -\frac{t}{\mu^2} \right) \right) \right]$$

$$T_{m1}(s,t,u) = \frac{g^2}{4\pi^2 v^2} \left[ 4 \frac{tu}{s^2} - 1 \right] L_2^r(\mu)$$

$$+ \frac{g^2}{48\pi^2 v^2} \frac{u-t}{s^2} \left[ u \log \left( -\frac{u}{\mu^2} \right) - t \log \left( -\frac{t}{\mu^2} \right) \right]$$

$$T_{m2}(s,t,u) = \frac{g^2}{2\pi^2 v^2} \frac{t-u}{s^2} L_2^r(\mu)$$

$$+ \frac{g^2}{24\pi^2 v^2 s^2} \left[ u \log \left( -\frac{u}{\mu^2} \right) - t \log \left( -\frac{t}{\mu^2} \right) \right] \tag{A.1}$$
II. $W^+_{\mu}(q_1)W^3_{\nu}(q_2) \rightarrow w^+(p)z$

\[ T_s(s, t, u) = -g^2\left[\frac{1}{s} + \frac{t}{s^2} - \frac{1}{4\pi^2v^2}\left(\frac{tu}{s^2}(2L_1^r(\mu) + L_2^r(\mu)) + L_2^5(\mu)\right)\right.\]
\[ + \frac{1}{24}\left(1 - \frac{u}{s}\right)\] \[\left. - \frac{g^2}{96\pi^2v^2}\left(1 - \frac{t}{s}\right)\left\{\log\left(-\frac{s}{\mu^2}\right)\right.\right.\]
\[ - \frac{u}{s}\log\left(-\frac{u}{\mu^2}\right) + 3\frac{ut}{s^2}\log\left(-\frac{t}{\mu^2}\right)\right]\]

\[ T_d(s, t, u) = g^2\left[\frac{1}{su} + \frac{1}{4\pi^2v^2s}(2L_1^r(\mu) + L_2^r(\mu))\right]\]
\[ - \frac{g^2}{96\pi^2v^2s}\left[\log\left(-\frac{u}{\mu^2}\right) + 3\log\left(-\frac{t}{\mu^2}\right)\right]\]

\[ T_l(s, t, u) = -g^2\left[\frac{t}{s^2} + \frac{1}{4\pi^2v^2s^2}(2L_1^r(\mu) + (u^2 + s^2)L_2^r(\mu))\right]\]
\[ + \frac{g^2}{96\pi^2v^2}\left[\left(1 - \frac{u}{s}\right)\log\left(-\frac{s}{\mu^2}\right) + 3\frac{t^2}{s^2}\log\left(-\frac{t}{\mu^2}\right)\right.\]
\[ + \frac{u(u - s)}{s^2}\log\left(-\frac{u}{\mu^2}\right)\right]\]

\[ T_{m1}(s, t, u) = g^2\left[\frac{u - t}{s^2} + \frac{tu}{2\pi^2v^2s^2}(2L_1^r(\mu) + L_2^r(\mu))\right]\]
\[ + \frac{1}{4\pi^2v^2s}\left(2L_1^r(\mu) + uL_2^r(\mu)\right)\] \[+ \frac{g^2}{96\pi^2v^2s}\left[\log\left(-\frac{s}{\mu^2}\right)\right.\]
\[ + 3\frac{t}{s}\log\left(-\frac{t}{\mu^2}\right) - \frac{u}{s}\log\left(-\frac{u}{\mu^2}\right)\right]\]
\[ T_{m2}(s,t,u) = g^2 \left[ \frac{2}{s^2} + \frac{1}{4\pi^2 v^2 s}(-2L_1^r(\mu) + L_2^r(\mu)) \right. \]
\[ \left. + \frac{t - u}{4\pi^2 v^2 s^2}(2L_1^r(\mu) + L_2^r(\mu)) \right] \]
\[ + \frac{g^2}{96\pi^2 v^2 s} \left[ -2 \log \left( -\frac{s}{\mu^2} \right) - 6 \frac{s}{s} \log \left( -\frac{t}{\mu^2} \right) + 2 \frac{u}{s} \log \left( -\frac{u}{\mu^2} \right) \right] \]

(A.2)

III. \( W_3^\pm(q_1)B_\nu(q_2) \rightarrow w^+(p)w^- \)

\[ \mathcal{M} = - \frac{g g'}{8\pi^2 v^2} \left( \frac{L_{9L}^r(\mu) + L_{9R}^r(\mu) + 2L_{10}^r(\mu)}{2} - \frac{1}{2} \right) \left( -\frac{s}{2} g_{\mu\nu} + q_{2\mu q_{1\nu}} \right) \]
\[ + 2 \frac{gg'}{ut} \left( \frac{u t}{2} g_{\mu\nu} + u p_{\mu q_{1\nu}} + s p_{\mu p_{\nu}} + t q_{2\mu p_{\nu}} \right) \]
\[ - \left( \frac{g'}{g} \right) \mathcal{M}(W_3^\pm(q_1)W_\nu^3(q_2) \rightarrow w^+(p)w^-) \]

(A.3)

IV. \( W_\mu^+(q_1)B_\nu(q_2) \rightarrow w^+(p)z \)

\[ \mathcal{M} = \frac{g g'}{16\pi^2 v^2} \left( \frac{L_{9L}^r(\mu) + L_{9R}^r(\mu) + 2L_{10}^r(\mu) + s - u}{3s} \right) \left( -\frac{s}{2} g_{\mu\nu} + q_{2\mu q_{1\nu}} \right) \]
\[ + 2 \frac{gg'}{us} \left( \frac{u t}{2} g_{\mu\nu} + u p_{\mu q_{1\nu}} + s p_{\mu p_{\nu}} + t q_{2\mu p_{\nu}} \right) \]
\[ - \left( \frac{g'}{g} \right) \mathcal{M}(W_\mu^+(q_1)W_\nu^3(q_2) \rightarrow w^+(p)z) \]

(A.4)

V. \( W_\mu^3(q_1)B_\nu(q_2) \rightarrow z(p)z \)

\[ \mathcal{M} = 2 \frac{g g'}{16\pi^2 v^2} \left( -\frac{s}{2} g_{\mu\nu} + q_{2\mu q_{1\nu}} \right) \]
\[ - \left( \frac{g'}{g} \right) \mathcal{M}(W_\mu^3(q_1)W_\nu^3(q_2) \rightarrow z(p)z) \]

(A.5)

We have checked that the photon amplitudes, \( \gamma\gamma \rightarrow w^+w^- \), \( \gamma\gamma \rightarrow zz \) and
$W^+\gamma \rightarrow w^+z$ are all gauge invariant.\textsuperscript{#19}

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\textsuperscript{#19} We have also extracted the $\gamma\gamma \rightarrow \pi\pi$ amplitudes from our results and checked that they reduce to the amplitudes of Bijnens and Cornet [51] in their $SU(2)$ and chiral limits, with $v \rightarrow f_\pi \simeq 93$ MeV. We are grateful to J. Bijnens for providing us with the $SU(2)$ limit of his result to check our amplitudes.
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FIGURE CAPTIONS

1) Values of $L_1^r(\mu)$ and $L_2^r(\mu)$ at $\mu = 1.5$ TeV allowed by the unitarity constraints described in the text. The solid (dashed) line is the boundary of the allowed region when one requires the partial waves with $J \leq 2$ to satisfy $|Re(a_J^l)| < 1/2$ for $M_{VV} \leq 1.0$ TeV ($M_{VV} \leq 1.5$ TeV).

2) Counting rules for the production of $V_LV_L$ pairs through vector boson fusion:
   a) Tree-level $\mathcal{O}(p^2)$ diagrams; b) Tree-level $\mathcal{O}(p^4)$ diagrams. The dashed lines represent longitudinally-polarized gauge bosons; the wavy lines denote their transversely polarized partners. The heavy dots represent vertices from Eq. (2.5), and the scale $\Lambda \lesssim 4\pi v$.

3) One-loop $\mathcal{O}(p^4)$ diagrams from Eq. (2.4) that contribute to $V_LV_L$ pair production via vector boson fusion. The terms of enhanced electroweak strength are those with Goldstone bosons in the loop.

4) Counting rules for the production of $V_LV_L$ pairs through $q\bar{q}$ annihilation: a) Tree and one-loop amplitudes from Eq. (2.4). The diagrams with Goldstone
bosons in the loop are enhanced at high energies; b) Tree-level terms from Eq. (2.5); c) Possible new physics contributions that are not included. The first are oblique corrections that are not of enhanced electroweak strength. The second are nonstandard fermion couplings that are excluded by assumption.

5) $d\sigma/dM^{WW}$ for $pp \rightarrow W^+_L W^-_L$ at $\sqrt{S} = 40$ TeV. The amplitudes include only the universal contributions at order $p^4$, that is, we have set $L_i^r(\mu) = 0$ at $\mu = 1.5$ TeV. The long-dashed line is the contribution from $q\bar{q}$ annihilation. The solid, dotted, and dashed lines are the contributions from $V_L V_L$, $V_L V_T$, and $V_T V_T$ initial states, respectively. The dot-dashed line is the total contribution from vector boson scattering. The vector boson curves include both the $W^+_L W^-_L$ and $Z_L Z_L$ initial states.

6) $d\sigma/dM^{ZZ}$ for $pp \rightarrow Z_L Z_L$ at $\sqrt{S} = 40$ TeV. The amplitudes include only the universal contributions at order $p^4$, that is, we have set $L_i^r(\mu) = 0$ at $\mu = 1.5$ TeV. The long-dashed line is the contribution from $gg$ scattering through a top quark loop with $M_{t_{top}} = 200$ GeV. The solid and dashed lines are the contributions from $V_L V_L$ and $V_T V_T$ initial states, respectively. The dot-dashed line is the total contribution from vector boson scattering. The vector boson curves include both the $W^+_L W^-_L$ and $Z_L Z_L$ initial states.

7) $d\sigma/dM^{WZ}$ for $pp \rightarrow W^+_L Z_L$ at $\sqrt{S} = 40$ TeV. The amplitudes include only the universal contributions at order $p^4$, that is, we have set $L_i^r(\mu) = 0$ at $\mu = 1.5$ TeV. The long-dashed line is the contribution from $q\bar{q}$ annihilation. The solid, dotted, and dashed lines are the contributions from $V_L V_L$, $V_L V_T$, and $V_T V_T$ initial states, respectively. The dot-dashed line is the total contribution from vector boson scattering.

8) $d\sigma/dM^{WW}$ for $pp \rightarrow W^+_L W^+_L$ at $\sqrt{S} = 40$ TeV. The amplitudes include only the universal contributions at order $p^4$, that is, we have set $L_i(\mu) = 0$ at $\mu = 1.5$ TeV. The solid and dashed lines are the contributions from $V_L V_L$ and $V_T V_T$ initial states, respectively. The dot-dashed line is the total contribution from vector boson scattering.
9) The total vector boson scattering contribution to $d\sigma/dM_W$ for $pp \to W_L^+W_L^-$. The upper curves have $\sqrt{S} = 40$ TeV, while the lower curves have $\sqrt{S} = 17$ TeV. The solid curves have all $L^r_i(\mu) = 0$ and the dotted curves have $L^r_1(\mu) = -0.6$, $L^r_2(\mu) = 0.6$, $L^r_{9L}(\mu) = L^r_{9R}(\mu) = 2.4$, and $L^r_{10}(\mu) = -2.5$, at $\mu = 1.5$ TeV.

10) The total vector boson scattering contribution to $d\sigma/dM_{ZZ}$ for $pp \to Z_LZ_L$. The upper curves have $\sqrt{S} = 40$ TeV and the lower curves have $\sqrt{S} = 17$ TeV. The solid curves have all $L^r_i(\mu) = 0$ and the dotted curves have $L^r_1(\mu) = -0.6$, $L^r_2(\mu) = 0.6$, $L^r_{9L}(\mu) = L^r_{9R}(\mu) = 2.4$, and $L^r_{10}(\mu) = -2.5$, at $\mu = 1.5$ TeV.

11) The total vector boson scattering contribution to $d\sigma/dM_{WZ}$ for $pp \to W_L^+Z_L$. The upper curves have $\sqrt{S} = 40$ TeV and the lower curves have $\sqrt{S} = 17$ TeV. The solid curves have all $L^r_i(\mu) = 0$ and the dotted curves have $L^r_1(\mu) = -0.6$, $L^r_2(\mu) = 0.6$, $L^r_{9L}(\mu) = L^r_{9R}(\mu) = 2.4$, and $L^r_{10}(\mu) = -2.5$, at $\mu = 1.5$ TeV.

12) The total vector boson scattering contribution to $d\sigma/dM_{WW}$ for $pp \to W_L^+W_L^-$. The upper curves have $\sqrt{S} = 40$ TeV and the lower curves have $\sqrt{S} = 17$ TeV. The solid curves have all $L^r_i(\mu) = 0$ and the dotted curves have $L^r_1(\mu) = -0.6$, $L^r_2(\mu) = 0.6$, $L^r_{9L}(\mu) = L^r_{9R}(\mu) = 2.4$, and $L^r_{10}(\mu) = -2.5$, at $\mu = 1.5$ TeV.

13) The number of $W^+_LZ_L$ events per year with $0.5 < M_{WZ} < 1.0$ TeV, as a function of $L^r_{9L}(\mu)$ with $\mu = 1.5$ TeV, assuming an integrated luminosity of $10^{40}$ cm$^{-2}$ at the SSC (solid line) and LHC (dotted line). With no anomalous couplings, the total number of $W^+_Z$ events per year is expected to be 88,500 at the SSC and 34,400 at the LHC. Of these, the number of $W^+_LZ_L$ events is about 10,000 for the SSC and 4,000 for the LHC.

14) The number of $W^+_LW^-_L$ events per year with $0.5 < M_{WW} < 1.0$ TeV, as a function of $L^r_{9L}(\mu) = L^r_{9R}(\mu)$ with $\mu = 1.5$ TeV, assuming an integrated luminosity of $10^{40}$ cm$^{-2}$ at the SSC (solid line) and LHC (dotted line). With no
anomalous couplings, the total number of $W^+Z$ events per year is expected to be 174,000 at the SSC and 60,000 at the LHC.

15) The number of $W_L^+W_L^+$ events per year with $0.5 < M_{WW} < 1.0$ TeV, as a function of $L_1^r(\mu)$, with $L_2^r(\mu) = 0$ and $\mu = 1.5$ TeV, assuming an integrated luminosity of $10^{40}$ cm$^{-2}$ at the SSC (solid line) and LHC (dotted line). The values of $L_1^r(\mu)$ preserve unitarity up to 1.0 TeV.

16) The number of $W_L^+W_L^+$ events per year with $0.5 < M_{WW} < 1.0$ TeV, as a function of $L_1^r(\mu)$, with $L_1^r(\mu) = -L_2^r(\mu) = 0$ and $\mu = 1.5$ TeV, assuming an integrated luminosity of $10^{40}$ cm$^{-2}$ at the SSC (solid line) and LHC (dotted line). The values of $L_1^r(\mu)$ preserve unitarity up to 1.0 TeV.