Experimental investigations on secondary structures in a fully developed turbulent jet

A Capone\textsuperscript{1,3}, G P Romano\textsuperscript{2}

\textsuperscript{1} Department of Energetics, University of Udine, Via delle Scienze, Udine, 33100, Italy
\textsuperscript{2} Department of Mechanics and Aeronautics, University of Rome La Sapienza, Via Eudossiana, Rome, 00184, Italy
\textsuperscript{3} Present address: Department of Mechanics and Aeronautics, University of Rome La Sapienza, Via Eudossiana, Rome, 00184, Italy

E-mail: alessandro.capone@uniud.it

Abstract. The effect of streamwise vorticity on entrainment in jet flows has been the subject of various works and is related to many applications involving effective fluid mixing. In this work the near-field of a fully developed turbulent round jet was investigated at different Reynolds numbers ranging from \( \approx 3000 \) to \( \approx 30000 \) by means of time-resolved particle image velocimetry. Acquisitions were carried out on transverse planes at different downstream positions, in order to visualize the evolution of vortical structures and to assess their features. Results from average velocity fields and rms show a strong impact of crosswise vortical structures on radial velocity distributions as downstream distance increases. Velocity correlation functions analysis exhibits a decrease of crosswise structures size as the Reynolds number increases, reaching an asymptotic behaviour which may be also observed in TKE distributions, which feature high levels in the outer radial area. Entrainment rates calculations supported by spectral analysis confirm the decisive role of streamwise vorticity in the control of the entrainment process as well as its dependence on Reynolds number. Entrainment rates approaching an asymptotic state versus Reynolds number suggests a connection between the effectiveness of this process and crosswise vortical structures size.

1. Introduction

The dynamics of large vortical structures development in turbulent flows has been the subject of many works (Bernal & Roshko (1986), Hussain (1986), Hussain & Clark (1981), Yule (1978), Sirovich (1987) among others) which investigated various shear layer configurations. Of particular interest for the current work’s purposes is the interaction between primary and secondary flow instabilities in the development of fully three-dimensional vortical structures in axisymmetric jet flows and their impact on important global processes such as fluid entrainment. The mechanism of development of secondary instability, characterized by streamwise vorticity, has been analysed in various works such as Klaasen & Peltier (1988) and Liepmann & Gharib (1992). Typical cross-planes double counter-rotating structures stemming from this phenomenon are described in Liepmann & Gharib (1992) and Romano (2002). Such counter-rotating formations give rise to three-dimensional structures named ribs described first in Hussain (1986), which connect adjacent vortex rings and are characterized by high crosswise vorticity. The
The persistence of streamwise vorticity has quite a relevant effect on flow dynamics, enhancing the flow mixing both locally and globally, as shown in Liepmann & Gharib (1992), taking control of the entrainment process. The effect of Reynolds number on streamwise vorticity and entrainment has been investigated in Liepmann & Gharib (1992) where the number of such streamwise structures as well as their size has been analysed. In particular the authors found that the number of identifiable structures grows up to \( Re \approx 10000 \) when it drops suddenly. As far as structures’ size is concerned, authors found that it appears to be inversely proportional to Reynolds number up to \( Re \approx 10000 \) and it is nearly constant for higher values. The experiments carried out in this work help shed light on the connection between Reynolds number, streamwise vortical structures and entrainment rate, setting a relation between structures’ size and entrainment efficiency, by means of spectral and correlation functions analysis.

2. Experimental set-up, data acquisition and analysis

The facility consists of an axisymmetric water jet flowing downstream through a long pipe ending nearly 7 cm inside the observation tank. The jet’s diameter is \( D = 2.2 \text{ cm} \) and the pipe is circa 100\( D \) long, which is generally enough for a fully developed turbulent flow to establish. Being the pipe nozzle not flush with the tank’s wall the flow is characterized by a free-slip condition. Downstream of the nozzle the flow enters into a tank with height and width \( \approx 37D \), whereas the length \( \approx 60D \). Due to the limited size of the tank, the jet has then to be considered as confined rather than free. The Reynolds number based on pipe diameter and jet’s bulk velocity \( U_0 \) ranges from 3200 to 32000. The X axis is assumed parallel to the jet’s main flow whereas the Y and Z axis form the jet’s cross-planes. Two types of acquisition have been carried out in this work, based on the position of the camera. The first one has been performed on longitudinal (X,Y) planes and reached an area as far as 18\( D \) in the far-field. The second data set has been acquired on transverse (Y,Z) or (r,\( \theta \)) planes at six downstream positions, from 0.75 \( x/D \) up to 4.5 \( x/D \). Crosswise images have been collected over a square area with 3\( D \) side.

A high-speed PIV system has been set up by means of a high-speed 8-bit BW, Photron APX CMOS camera with 1,024 times 1,024 pixels resolution and up to 2\( kHz \) frame rate. Spatial resolution was in both acquisition strategies nearly 0.1 mm, about 0.0045\( D \). The camera objective used for all the acquisitions was a Nikon F 50 mm focal length with maximum aperture of 1.2 and the data acquisition rate was set to 500\( Hz \) or 1000\( Hz \) depending on flow speed. The illumination was provided by a continuous Spectra Physics Ar-ion laser, 488—514 nm in wavelength, with a maximum power equal to 7 W, and flow seeding was attained with 10\( \mu m \) diameter hollow glass spheres.

As described in detail in Falchi & Romano (2009), data collected via high-speed PIV systems have a high degree of correlation in time. Comparison of temporal resolution and acquisition times to the integral time scale derived by correlation functions confirms that data may be used to derive correlation and spectral functions as well as statistical quantities.

A commercial PIV software, that is DaVis by LaVision Gmbh, has been employed for instantaneous vector field computation. The advanced image deformation multi-pass PIV cross-correlation algorithm with window offset, adaptive window deformation and Gaussian sub-pixel approximation, is thoroughly described in Stanislas et al. (2008). The window size adopted was 32 and 16 pixels for streamwise and crosswise measurements respectively and spacing between velocity vectors was 8 and 4 pixels.

3. Results on streamwise decay and validation

In Figure 1 the horizontal decay of average axial velocity is reported and compared to that provided by Djeridane et al. (1993) obtained by means of Laser Doppler Anemometry at a Reynolds number of 32000. As Reynolds number increases a good agreement to LDA data is
noticeable. Decay exhibits the expected \((x/D)^{-1}\) behaviour described in Liepmann & Gharib (1992) and a decrease with respect to Reynolds number, as found in Choi & Kim (2009). Present data are also validated with respect to radial velocity profile very close to the nozzle. Comparison to LDA data and to empirical fully developed pipe flow power law \((1 - 2r/D)^{-1/n}\) with \(n = 6.5\) described in Mi, Nobes & Nathan (2001) are shown in Figure 2. The smooth ends at the jet’s boundaries \((r/D=0.5)\) are a consequence of the free-slip condition of the flow as described in Romano (2002) and the limited spatial resolution of high velocity gradients at the pipe outlet. These findings confirm the hypothesis of completely developed turbulent flow at the nozzle exit.

4. Results on crosswise acquisitions

As described previously, one of the aims of the current work is to investigate cross-planes vortical structures dependence on Reynolds number. Thus acquisitions were carried out on planes perpendicular to the jet’s main velocity, covering a range of Reynolds number of 3000-32000
The mean velocity field obtained from 5000 images over about 10 seconds at \(x/D = 1.5\) and \(Re = 4800\) and \(Re = 19300\) are displayed respectively in Figure 3 and 4. The actual pipe outlet shape is depicted as the gray circle. In the far-field area, for both Reynolds numbers, the entrainment enhancing effect stemming from secondary instabilities investigated in Liepmann & Gharib (1992) is evident in the average velocity and is clearly visible up to the border of the imaged area, namely as far as \(2D\).

The effect of the crosswise vortical structures on the cross-plane average field is highlighted in Figure 5, where the mean velocity field is shown at three downstream distances \(x/D\). For the purpose of the current work, the velocity vector has been decomposed in radial and azimuthal components, respectively \(U_r\) and \(U_\theta\). The mean \(U_r\) field displayed on the left side of Figure 5, normalized by the flow velocity \(U_0\), reveals a strong dependence on the downstream distance: up to \(x/D = 1\) the field’s shape is still unaffected by secondary structures, and vortex rings action is predominant. From \(x/D = 1.5\) up to \(2.5\), instability of the potential core begins to be visible and consequently the velocity field shows an increasing dependence of the radial velocity on the azimuthal angle. In the last downstream position, \(x/D = 4.5\), the presence of secondary vortex structures is substantial and modifies the velocity field’s shape up to \(r/D = 2\). On the other hand, the mean \(U_\theta\) distribution is quite independent from \(x/D\) as may be seen in the right side of Figure 5. In particular, for high \(x/D\), the field retains its shape and no sign of far-field modification is evident. For different Reynolds numbers average velocity fields results are very similar with maximum radial velocity ranging from \(+0.4U_0\) for \(Re \leq 4800\), to \(+0.3U_0\) for \(Re = 9600\) and \(+0.25U_0\) for \(Re \geq 14300\) whereas azimuthal velocity showed to be quite independent from Reynolds number, with values in the range \(+0.005U_0\).

The behaviour of velocity fluctuations provides further insight on the phenomenon. In Figure 5, normalized \(U_r\) and \(U_\theta\) mean field for \(Re = 14500\) at various \(x/D\) (1, 2.5, 4.5 starting from top). The grey circle represents the pipe outlet’s rim.
6, radial profiles of rms values for radial and azimuthal velocities for different Reynolds numbers are presented. Radial rms were obtained by averaging four data sets lying on two mutually perpendicular axis passing by the pipe outlet’s geometric center.

For all Reynolds numbers, the normalized rms level decreases along the radius from about 0.7-1.0 $U_0$ to zero, which is attained at $r = D$ (but of course this also changes with streamwise distance). This reflects the fluctuations of radial motions due to crosswise structures. Considering the data at different Reynolds numbers, a decrease of normalized fluctuations with Reynolds number at each radial distance appears. However, a careful inspection reveals that the rms levels decrease up to $Re = 9600$ and then increase for higher Reynolds numbers in particular for $r = D < 0.5$.

Figure 6. Rms of $u_r$ (top) and $u_\theta$ (bottom) at $x/D = 1.5$ for different Reynolds numbers. Figure 7. Velocity correlation functions at $x/D = 3.5$ for different Reynolds numbers.

4.1. Velocity correlation and TKE distribution

The effect of Reynolds number on the evolution of streamwise vortical structures has been investigated thoroughly in Liepmann & Gharib (1992) where the number of such streamwise structures as well as their size has been analysed. In the present work, the average size of cross-planes vortical structures was assessed by means of radial and azimuthal velocity correlation functions defined respectively as

$$R_{rr}(x_0, r) = \langle u_r(x_0) u_r(x_0 + r) \rangle, R_{\theta\theta}(x_0, r) = \langle u_\theta(x_0) u_\theta(x_0 + r) \rangle$$

(1)

Where the reference position $x_0$ was set right inside the pipe’s outlet rim (at $r/D \approx 0.45$) and $r$ direction represents the radial position with respect to pipe’s center. Correlation functions were then normalized to their initial value and were subsequently spatially averaged from four data sets as described before. Figure 7 shows such correlation functions calculated at $x/D = 3.5$ for different Reynolds numbers. The radial correlation profiles show a dependance on Reynolds number with decreasing levels of correlation as Reynolds number increases. For $Re \leq 4800$, $R_{rr}$ retains 50% of its starting value up to $r/D \approx 0.65$ whereas at the same radial distance for the $Re = 32200$ case it is as low as 25%. Furthermore, $R_{rr}$ is above 10% of its initial value up to $r/D \approx 1$ for lower Reynolds numbers, whereas such relative value is reached already at $r/D \approx 0.7$ at $Re = 32200$. These findings confirm that the average spatial extension in the radial direction of counter-rotating, streamwise vortical structures decreases with growing
Reynolds numbers suggesting the presence of an asymptotic behaviour. Differently from the radial correlation, the azimuthal correlation shows negative values due to the vortical structures which roll up inwards and is related to the spatial extension of such structures in the azimuthal direction. For increasing Reynolds number the radial position at which the anti-correlation peak is attained decreases confirming again the shrinking of vortical formations.

Turbulent kinetic energy, defined as $u'^2 + u_y'^2$, was calculated at cross-plane $x/D = 4.5$ for different Reynolds numbers as shown in Figure 8. For low Reynolds numbers, namely $Re \leq 3200$, relevant TKE values are observed up to $r/D \approx 1.5$ having streamwise structures a relevant impact on the flow field. The region affected by high TKE levels shrinks progressively as the Reynolds number raises, remaining nearly constant from $Re \geq 29100$. These findings, together with the results from velocity correlations, suggest the existence of an initial regime, for $Re < 10000$, where the size of the cross-planes vortical structures decreases having an effect on turbulent energy distribution which do not scale with Reynolds number.

![Figure 8](image1.png)

**Figure 8.** Normalized Turbulent Kinetic Energy distributions at $x/D = 4.5$ versus Reynolds number.

![Figure 9](image2.png)

**Figure 9.** Normalized average coherency spectra for $u_r$ at $x/D = 1$ for different Reynolds numbers.

### 4.2. Spectral analysis

As described by Stanley et al. (2002) and Sbrizzai et al. (2004) the use of the coherency spectrum reduces the influence of the broad-band background energy accentuating the discrete frequencies at the side of the jet. Following the definition given in Stanley et al. (2002), the spectrum was thus defined as

$$Coh_{u_r,u_r}(f) = \left| \int_{-\infty}^{+\infty} C_{u_r,u_r}(\tau) \exp(-2\pi j f \tau) \, d\tau \right|$$

(2)

where $C_{u_r,u_r}$ is the two-times correlation of the radial velocity component of two diametrically opposing points, $A_i$ and $B_i$, with respect to pipe nozzle’s center $C_{u_r,u_r}(\tau) = u_{A_i}(t + \tau)u_{B_i}(t)$. The coherency spectrum was calculated for 32 couples of points from $A_1$, $B_1$ to $A_{32}$, $B_{32}$ positioned on a circle centered on the nozzle’s outlet and their spectra were averaged finally obtaining an average coherency spectrum. In Figure 9 a comparison between spectra calculated from points close the pipe’s outlet is given for four different Reynolds number at a downstream position of $x/D = 1$. Coherency spectra were normalized and plotted versus Strouhal number $St = fD/U_0$. At $Re = 3200$ a strong peak is noticeable which corresponds to a high degree
of spatial correlation at $St \approx 0.54$. The peak at this value of Strouhal number is in the range [0.24-0.64] given by Gutmark & Ho (1983) for the preferred mode of various jet configurations. As the Reynolds number increases, a damping effect on the spectrum’s peak is visible, although the frequency of the peak is not changing, being correlated to the jet’s vortex rings shedding frequency. Nonetheless, as the peak’s magnitude decreases, the energy correlated to the spectrum’s high-frequency components grows accordingly. Further insight is given by the average coherency spectra variation with streamwise distance, as presented in Figure 10 for $Re = 4800$. In cross-planes close to nozzle ($x/D \leq 1.5$) the strong correlation caused by vortex rings is clearly noticeable by a strong peak in the spectra. Differently from Figure 9 where spectrum peak is attenuated as the Reynolds number increases, in this case the peak becomes less visible as the downstream position increases, already at $x/D = 2.5$, where the spectrum shows a different shape which is no longer featuring a well-distinct peak. These modifications develop fully for higher downstream positions, where the original peak merges to low-frequency part of the spectrum and the energy level increases having nearly the same value for $x/D = 3.5$ and $x/D = 4.5$. As pointed out before, the high temporal resolution attained is adequate, with respect to the integral time scale, to carry out spectral analysis.

Figure 10. Averaged normalized coherency spectra at $r/D = 0.5$.

Figure 11. Normalized entrainment rate at $x/D = 0.75$ for different Reynolds numbers.

4.3. Instantaneous and mean entrainment rates
For the purposes of the present work, it is of interest to focus on the effect of Reynolds number on the evolution of streamwise vorticity and how this, in the end, affects fluid entrainment efficiency. Given the jet’s axial volumetric flow rate

$$Q = \int_0^{2\pi} \int_0^\infty u_e r dr d\theta$$

the definition of entrainment rate is given in Wygnanski & Fiedler (1969) and Crow & Champagne (1971) as the spatial derivative $\frac{dQ}{dx}$. Assuming constant density, the volume flow rate balance requires that the net axial volume flux be equalled by the radial volume flux across a circular path of radius $r$, thus leading to:
with $Q_0$ derived by the flow bulk velocity $U_0$ times pipe’s nozzle area. The above formulation states that the entrainment rate at a specific downstream distance $x/D$ will be derived by integrating the radial velocity $u_r$ along a circle path centered on the pipe’s outlet. For large values of $r$, the entrainment rate will reach a state where it is independent from the radius of the circle from which it is calculated. The entrainment rate derived by the average radial velocity field $U_r$ is displayed in Figure 11 at downstream position $x/D = 0.75$ for different Reynolds numbers. The calculation of the entrainment rate at growing radial distances $r/D$ allows a better understanding of how different types of instability concur to this process. For $r/D < 0.5$ the curves associated to $Re = 3200$ and $Re = 4800$ feature a negative entrainment rate. The primary vortex rings structures are characterized in fact by a strong outward fluid ejection phase which on average is not compensated by the subsequent roll-up, leading to a negative entrainment rate in the area where the fluid projection occurs. As the circle upon which the calculation is based is made larger, entrainment rate level becomes positive and increases considerably. The effect of primary instability structures is predominant at this downstream position and vortex rings roll-up determines a noticeable increase of entrainment. The peak achieved between $r/D$ range [0.8-1] confirms the role of vortex rings passing, whose effect is most relevant in a limited flow region and settles in the cross-plane far field ($r/D > 1$). Profiles for $Re > 4800$ feature lower levels of negative entrainment in the $r/D < 0.5$ range whereas do not show a well-distinct peak. This behaviour is similar to that noticed in Figure 9 for spectral data, where a dampening effect was reported for higher Reynolds numbers. The asymptotic entrainment rate level, achieved quite early, shows to be decreasing with growing Reynolds numbers. The steady entrainment rate is in agreement with the observed linear behaviour of $Q$ according to equation 3 derived by means of longitudinal acquisitions.

As investigated cross-planes move farther from the nozzle, crosswise vorticity becomes more efficient in entraining fluid and, when the potential core ends, it takes full control of the entraining process. In Figures 12 and 13 two $x/D$ positions are analysed shedding light on this process. Normalized entrainment rate at $x/D = 1.5$ displays interesting features. First of all no visible peak is noticeable as it may be expected, since the primary instability effect in cross-planes weakens as the potential core is approaching its end. The onset of radial distance independence in entrainment rate calculation moves towards higher $r/D$ values; this is more evident as $x/D$ increases and has been reported in similar studies (Hassan & Meslem (2010)). A possible explanation to these findings may be found in the number of streamwise vortical structures which develop in cross-planes and raise with downstream distance as described by Liepmann & Gharib (1992). For the entrainment rate to reach a steady state with respect to radial distance, the streamwise structures should not be in their early formation stage, characterized by strong outwards bursts of fluid, but in their final phase of their life cycle, during which counter-rotating mushrooms formations get stretched and convected within the flow. As $x/D$ increases, this state is attained at higher $r/D$, as confirmed by the present experimental data. Looking at entrainment rate asymptotic values, a significant increase is observed for $x/D = 4.5$, where for all Reynolds numbers values twice as high as those at $x/D = 0.75$ are reported. Dependence on Reynolds number shows to be strong in the lower range [3200-4800] whereas it proves to be very weak for higher values approaching an asymptotic state. Such behaviour is not noticeable for $x/D = 0.75$ where crosswise structures have not yet developed. This outcome is a supporting evidence as to the role of streamwise structures in controlling the entrainment process for higher $x/D$. In fact, the size of such structures has shown a similar dependance on Reynolds number as detailed in 4.1, and despite the number of such structures it could be deemed to be the
controlling mechanism affecting entrainment rate.

\[ \frac{dQ}{d(x/D)} \]

**Figure 12.** Normalized entrainment rate for \( x/D = 1.5 \).

**Figure 13.** Normalized entrainment rate for \( x/D = 4.5 \).

5. Conclusions

In this work, the evolution of streamwise vorticity in a turbulent jet, and its effect on entrainment rate have been investigated, with a focus on Reynolds number dependence. Mean velocity fields calculated in planes within a range of different downstream distances confirm the strong effect of streamwise vorticity on the radial average velocity field, whose distribution scale with respect to flow reference velocity for \( Re > 14000 \). Azimuthal velocity maps are not sensitive to Reynolds variations suggesting to be linked to vortex rings induced dynamics. Rms of velocity confirmed the azimuthal component to bear little fluctuations compared to radial one as Reynolds number grows. Correlation functions give a good insight as to how vortical structures’ size is affected by Reynolds regime: both radial and azimuthal functions indicate a stark decrease in structures’ dimensions reaching an asymptote at high Reynolds numbers and suggesting a strong link to global phenomena induced by such formations. TKE and entrainment rate results show in fact a similar asymptotic behaviour, which is more evident for downstream positions farther from nozzle. This confirms not only that streamwise vorticity takes control of the entrainment process as it develops fully downstream, as reported by Liepmann & Gharib (1992), but that size is the leading factor in such phenomena. Coherency spectra findings add to this perspective making clear that, as Reynolds number grows, spatial coherency induced by vortex rings is progressively lost and so their effect on global processes is weakened. At the same time, as downstream distance increases, the spectra display an increasing level of energy which is transferred from primary, vortex rings structures, to secondary streamwise vortical formations. The impact of the latter phenomena on entrainment rate is a progressive shift towards higher radial distances of entrainment peak which suggests the entrainment process to be stronger within the outer field area, where the vortical structures undergo stretching and convection within the flow.

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