BRANCHED POLYMERS WITH LOOPS

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Abstract

We propose a classification of critical behaviours of branched polymers for arbitrary topology. We show that in an appropriately defined double scaling limit the singular part of the partition function is universal. We calculate this partition function exactly in the generic case and perturbatively otherwise. In the discussion section we comment on the relation between branched polymer theory and Euclidean quantum gravity.
1 Introduction

The statistical mechanics of branched polymers (BP) is one of the simplest and most tractable models of random geometry. It is a subject of intrinsic interest and has already been studied by several authors [1]-[7]. A further motivation for developing these studies is provided by the recent suggestion [8] that important features of Euclidean quantum gravity can be inferred from those of the ensemble of branched polymers isomorphic to the ensemble of trees of baby universes connected by wormholes.

The main body of this paper is devoted to the discussion of the critical properties of naked BP models, including the possibility of loop formation. Some results have been obtained earlier by other authors and are included here for completeness. We shall comment about quantum gravity in the discussion section.

We formulate the problem as a minifield theory, defining a generating function $W(j)$ by the familiar equation:

$$e^{\Lambda W(j)} = \int d\phi e^{\Lambda[-\frac{1}{2}\phi^2 + V(\phi)+j\phi]}$$ (1)

However, here the integration variable $\phi$ is just a real number. The interaction potential is assumed to have the form

$$V(\phi) = \sum_k (p_k/k)\phi^k, \ k > 2$$ (2)

with positive couplings: $p_k \geq 0$. Thus, strictly speaking, the integral in (1) does not exist. It is introduced in the first place to define a perturbation series. Obviously, one can associate diagrams with the terms of this series. The propagator corresponding to a link is $\lambda^{-1}$, a source $j$ is attached to each external link and the number of loops in the diagram is identical to the power of $\Lambda^{-1}$. The generating function of rooted diagrams (those with one marked external point) is given by $\partial W/\partial j$.

2 The trees

The saddle point $\phi = Z$ is found from the equation

$$Z = \lambda^{-1}[j + \sum_k p_k Z^{k-1}]$$ (3)

When calculated in the saddle point approximation, the functions $W = W_0$ and $\partial W_0/\partial j$ generate the tree and the rooted tree diagrams respectively. Since

$$Z = \partial W_0/\partial j,$$ (4)

it is evident that $Z$ is the partition function of rooted BP. And indeed eq. (3) is generally used as the defining equation for rooted BP. We write here $j$ instead of $p_1$ and we have, for definiteness, incorporated $p_2$ into $\lambda$. But this is just a matter of conventions.

3That is without matter fields living on them.
Introduce the positive definite function
\[ F(\phi) = \frac{j}{\phi} + \sum_k p_k \phi^{k-2} \]  
and rewrite (3) as
\[ \lambda = F(Z) \]  
The positivity of \( p_k \) implies that \( F'(\phi) \) can have at most one zero for \( \phi > 0 \) and that \( F''(\phi) > 0 \). Let \( r \) denote the radius of convergence of the series in (4) and let \( \phi = \bar{\phi} \) be the point in the interval \((0, r]\) where \( F(\phi) \) takes its minimum value. We deduce from (4) that the parameter \( \lambda \) cannot decrease below \( \lambda_c = F(\bar{\phi}) \). Consequently, the partition function \( Z \) must have a singularity at \( \lambda = \lambda_c \).

Write \( \delta \lambda = \lambda - \lambda_c \) for later convenience and assume that in the neighbourhood of \( \delta \lambda = 0 \) the singular part of the partition function behaves as
\[ Z_s \sim \delta \lambda^{1-\gamma}, \]  
This is the conventional definition of the (geometrical) susceptibility exponent \( \gamma \). Another interesting quantity is the two-point correlation function \( C(x) \), where \( x \) is the (integer) distance between two marked points. It has been shown in [2] that at the tree level this function is
\[ C(x) \sim [\lambda^{-1} \frac{\partial}{\partial Z}(ZF(Z))]^x \]  
For large \( x \)
\[ x^{-1} \log C(x) = - \text{const} \delta \lambda \frac{1}{d_H} \]  
where \( d_H \) is the Hausdorff dimension.

Generalizing the considerations of ref. [3] one can present the catalogue of possible critical behaviours:

**The generic case.**

In the generic situation \( \bar{\phi} < r \). One then has \( F'(\bar{\phi}) = 0 \) and
\[ F(Z) = F(\bar{\phi}) + \frac{1}{2} F''(\bar{\phi})(Z - \bar{\phi})^2 + \ldots, \]  
which after inversion yields
\[ Z \simeq \bar{\phi} - [2/F''(\bar{\phi})]^{1/2} \delta \lambda^{1/2}. \]  
Hence \( \gamma = \frac{1}{2} \). One also finds \( d_H = 2 \).

The alternative to the generic case occurs when \( \bar{\phi} = r < \infty \). It follows from the positivity of \( p_k \) that \( \phi = r \) is a singular point of \( F(\phi) \). Since \( F(\phi) \) decreases monotonically towards \( F(r) \geq 0 \) one expects the singularity to be a branch point in all cases of physical interest. Hence (modulo logs)
\[ F(Z) = \sum_0^n \frac{(-1)^k}{k!} F^{[k]}(r)(r - Z)^k + c(r - Z)^{\beta - 1} + \ldots, \ n + 1 < \beta < n + 2 \]  

\(^4\)Possibly infinite, but not infinitesimally small.

\(^5\)We have written a minus sign in front of the 2nd term on the right-hand side because we consider the branch \( Z \leq \bar{\phi} \) as the physical one.

\(^6\)The interested reader can consult [3] and in particular the theorems by Leau, Le Roy and Lindelöf.
Since $F$ is concave, one must have $\beta > 2$. Furthermore, from the relation between $F$ and $V$ and from the positivity of all the derivatives of $V$ one easily deduces that $c(-1)^n < 0$. There are two possibilities:

**The semi-generic case.**

It occurs when $\bar{\phi} = r$ and $F'(r) < 0$. Inverting (12) one finds

$$Z_S \simeq \frac{c}{[-F'(r)]^\beta} \delta \lambda^{\beta-1}$$

(13)

and therefore $\gamma = 2 - \beta$. In this case $d_H = \infty$.

**The marginal case.**

It occurs when $\bar{\phi} = r$ and $F'(r) = 0$. For $2 < \beta < 3$, inverting eq. (12) one finds

$$Z_S \simeq \left(\frac{\delta \lambda}{c}\right)^{\frac{1}{\beta-1}}$$

(14)

so that $\gamma = (\beta - 2)/(\beta - 1)$. The Hausdorff dimension is $d_H = (\beta - 1)/(\beta - 2)$. When $\beta > 3$ the situation is analogous to the generic one: one has (11) with $r$ in place of $\bar{\phi}$ and $\gamma = \frac{1}{2}$ while $d_H = 2$.

To summarize, in the non-generic cases there is a continuum of universality classes, characterized by the exponent $\beta$ and by the vanishing (or non-vanishing) of $F'(r)$. Each class corresponds to an infinity of different choices of the couplings $p_k$.

## 3 The loop expansion

### 3.1 The generic case

The loop expansion is obtained calculating corrections to the saddle-point approximation. This can be done either directly or by using Dyson-Schwinger equations. We employ here the second method, which is more elegant and enables one to write rapidly the BP equation, an analogue of the string equation one finds in the double scaling limit of 2d gravity [10].

Set $U(\phi) = \frac{\lambda}{2} \phi^2 - V(\phi)$. The Dyson-Schwinger equation reads

$$[U'(\Lambda^{-1} \frac{\partial}{\partial j}) - j] e^{AW(j)} = 0$$

(15)

which can be rewritten as

$$U'(Z + \Lambda^{-1} \frac{\partial}{\partial j}) \cdot 1 = j$$

(16)

Here $Z = \partial W / \partial j$ denotes the full partition function, with loop corrections included. Let $\phi_0$ be the point where $U''$ vanishes: $U''(\phi_0) = 0$. Define

$$\Delta = 2[j - U'(\phi_0)]/U''(\phi_0)$$

(17)

and the double scaling limit:

$$\Delta \to 0, \ \Lambda \to \infty, \ t = -\frac{1}{3} U''(\phi_0) \Lambda \Delta^{\frac{2}{3}} = \text{const}$$

(18)
Both $\Delta$ and $t$ are positive definite. Set $Z = \phi_0 - \Delta^{\frac{1}{2}} \chi(t)$ in (18) and expand. In the limit (18) one gets the BP equation promised at the beginning of this section:

$$1 = \chi^2(t) + \frac{1}{3t} \chi(t) + \chi'(t)$$

(19)

From the Riccati equation (19) one can obtain $\chi$ as a universal power series in $t^{-1}$:

$$\chi(t) = 1 - \frac{1}{6t} - \frac{5}{72t^2} + ...$$

(20)

We have assumed that the first term is +1. This is the physical choice (cf. the footnote preceding eq. (11)), which also guarantees that the terms corresponding to higher topologies give a positive contribution to the partition function. The behaviour of the coefficient $\chi_n$ of $t^{-n}$, for $n \gg 1$, can be easily estimated using (19):

$$\chi_n \sim \text{const} \frac{\Gamma(n)}{2^n}$$

(21)

The constant above can be found numerically and is approximately equal to $-0.32$. As one could expect, the series (20) is not Borel summable. The perturbative series does not determine $\chi(t)$ uniquely. The first singularity of the Borel transform occurs when its argument equals 2. Thus the leading non-perturbative contribution to $\chi(t)$ is proportional to $e^{-2t}$.

Let $f(t)$ be the primitive of $\chi(t)$: $f'(t) = \chi(t)$. Setting $\Phi(t) = e^{f(t)}$ and changing the independent variable $t \to z = (\frac{3}{2}t)^{\frac{2}{3}}$ one gets from (19) the Airy equation

$$\Phi'' = z \Phi$$

(22)

Hence

$$\Phi(z) = a \text{Ai}(z) + b \text{Bi}(z)$$

(23)

and

$$\chi(t) = \left(\frac{3}{2}t\right)^{-\frac{1}{4}} \frac{\Phi'(z)}{\Phi(z)}, \quad z = \left(\frac{3}{2}t\right)^{\frac{2}{3}}$$

(24)

The function $\chi(t)$ depends on a single, but arbitrary parameter $a/b$, which measures the strength of the nonperturbative contribution to the solution. In this respect the situation resembles that of string theory.

It remains to find the relation between $\Delta$ and $\delta \lambda$. The position of the point $\phi = \phi_0$ depends on $\lambda$, but not on $j$. One easily finds, however, that as $\lambda$ approaches $\lambda_0$, its $j$-dependent critical value, one has $\phi_0 \to \bar{\phi} + \delta \lambda / V''(\bar{\phi})$, which is also $j$-dependent. One further finds from the definition of $\Delta$ that

$$\Delta = \frac{2\bar{\phi}}{V''(\phi)} \delta \lambda [1 + O(\delta \lambda^2)]$$

(25)

The behaviour of $\chi(t)$ at large $t$ and the above equation imply the following dependence of the susceptibility exponent on the number of loops $L$:

$$\gamma_L = \frac{1}{2} + \frac{3}{2}L$$

(26)

a result obtained by a different method in [3].

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\textsuperscript{7} In the limit $b \to 0$ one obtains the unphysical solution with $\chi(\infty) = -1$. 
3.2 The strong coupling regime

Set $V(\phi) = g \bar{V}(\phi)$ and assume that $g \gg 1$. This defines the strong coupling regime. It is obvious from (5) that in this regime $\bar{\phi} \sim \sqrt{\frac{j}{g}}$ and $V''(\bar{\phi}) \sim g$. Hence, for large enough $g$ one necessarily has $\bar{\phi} < r$ and one is in the generic situation. One easily finds

$$t \sim \frac{j^{\frac{3}{4}}}{g^{\frac{5}{4}}} \Lambda \delta \lambda^{\frac{3}{2}}$$

(27)

Therefore, in the double scaling limit $t$ remains a small parameter. Using (24) one gets

$$Z_{\text{sing}} = -\sqrt{\Delta} \chi(t) \sim -\frac{j^{\frac{1}{4}}}{g^{\frac{5}{4}}} \sqrt{\delta \lambda}$$

(28)

The free parameter $a/b$ is hidden in the coefficient multiplying the right-hand side of (28). The susceptibility exponent $\gamma = \frac{1}{2}$ and the number of loops is zero.$^8$

3.3 The non-generic cases

When the minimum value of $\lambda$ is found at the boundary of the convergence interval of the potential, the equation $U''(\phi_0) = 0$ has no solution with $\phi_0 \leq r$. In this case we combine the Dyson-Schwinger equation (16) with the expansion (12). We then encounter formal expressions of the type $[h(x) + \partial/\partial x]^{\beta-1}$, which must be given a precise meaning. Write

$$[h(x) + \partial/\partial x]^{\beta-1} = \frac{1}{\Gamma(1-\beta)} \int_0^\infty ds \ s^{-\beta} e^{-s[h(x)+\partial/\partial x]}$$

(29)

Define $G$ by

$$e^{-s[h(x)+\partial/\partial x]} = e^G e^{-s\partial/\partial x}$$

(30)

It is easy to check that $e^G$ satisfies the differential equation

$$\frac{\partial}{\partial s} e^G = -e^G e^{s\partial/\partial x} h(x) e^{-s\partial/\partial x} \equiv -e^G h(x-s)$$

(31)

with the boundary condition $G = 1$ at $s = 0$. Hence $G$ is a c-number function:

$$G = H(x-s) - H(x)$$

(32)

where $H(x)$ is the primitive of $h(x)$: $H'(x) = h(x)$. Using this result we obtain

$$[h(x) + \partial/\partial x]^{\beta-1} = \sum_{k=0}^\infty \frac{\Gamma(1-\beta+k)}{\Gamma(1-\beta)} h^{\beta-1-k} R_k(h)$$

(33)

where $R_k(h)$ is defined by the equation

$$R_k(h) = \frac{e^{-H(x)} \partial^k}{k! \partial s^k} [e^{H(x-s)} e^{sh(x)}]_{s=0}$$

(34)

$^8$The average number of loops equals $\partial \ln Z/\partial \ln N$ calculated at fixed $\delta \lambda$. 
and depends on the derivatives $h'(x), \ldots, h^{[n-1]}(x)$. Using Leibniz formula to calculate the derivative in (34) and replacing $\partial/\partial s$ by $\partial/\partial x$ in the appropriate place one can get rid of $s$. After some algebra one obtains the following recursion formula

$$R_k(h) = \frac{1}{k}[h'(x)R_{k-2}(h) - R'_{k-1}(h)]$$

with $R_0 = 1$ and $R_1 = 0$. As one might expect the series (33) would be truncated at $k = \beta$ if $\beta$ were an integer (which it is not!). We shall now use the above results to study the non-generic scaling behaviour.

**The semi-generic case**

We define

$$\Delta = U'(r) - j \equiv r\delta \lambda$$

and define the scaling limit as follows:

$$\Delta \to 0, \Lambda \to \infty, \quad t = \frac{\Lambda \Delta^2}{-r F'(r)} = \text{const}$$

Rewrite the Dyson-Schwinger eq. (16) in terms of $F$, set $Z = r - f$ and use the expansion (12) to get

$$0 = \Delta + r F'(r) f - r c f + \frac{\partial}{\Lambda \partial \Delta} \beta^{-1} \cdot 1 + \ldots$$

where the dots represent terms irrelevant in the scaling limit.

We have $f = f_a - Z_s$, where

$$f_a = \frac{\Delta}{-r F'(r)} + \ldots$$

is analytic in $\Delta$. In the case under consideration, $Z_s \sim \Delta^{\beta-1}$ is subleading, compared to $f_a$, because $\beta > 2$. But we are precisely interested in this subleading term.

In the scaling limit one gets from (38)

$$Z_s \simeq c \frac{\Delta}{-F'(r)} (f + \frac{\partial}{\Lambda \partial \Delta})^{\beta-1} \cdot 1$$

We can now use (33), slightly modified: because of the factor $\Lambda^{-1}$ in front of $\frac{\partial}{\partial \Delta}$ one has $\bar{R}_k(f) = \Lambda^{-k} R_k(\Lambda f)$ instead of $R_k(f)$:

$$Z_s \simeq \frac{c}{-F'(r)} \sum_{k=0}^{\infty} \frac{\Gamma(1 - \beta + k)}{\Gamma(1 - \beta)} f^{\beta-1-k} \Lambda^{-k} R_k(\Lambda f)$$

One finds by inspection that in the scaling limit only the terms with even $k$ contribute to the right-hand side. The leading contribution comes from the first term in

$$R_{2m} = c_{2m} (f')^m + \ldots$$

We find from the recursion equation (35) that

$$c_{2m} = \frac{1}{2^m m!}$$
Using (39), (42) and (43) we finally get

\[ Z_s \simeq cr \frac{\Delta^{\beta-1}}{[-rF'(r)]^\beta} \left[ 1 + \sum_{m=1}^{\infty} \frac{\Gamma(1 - \beta + 2m)}{\Gamma(1 - \beta)\Gamma(m + 1)2^m t^{-m}} \right] \quad (44) \]

From the above equation one reads the susceptibility exponent

\[ \gamma_L = 2 - \beta + 2L \quad (45) \]

**The marginal case**

One again has the expansion (38), but with \( F'(r) \) set to zero. For \( \beta > 3 \) the problem reduces to the generic one: it is sufficient to replace in all formulae \( \bar{\phi} \) by \( r \). When \( 2 < \beta < 3 \) the scaling limit is defined as follows:

\[ \Delta \to 0, \; \Lambda \to \infty, \; t = (rc)^{\frac{1}{\beta - 1}} \Lambda(\Delta)^{\frac{\beta}{\beta - 1}} = \text{const} \quad (46) \]

In this limit the problem reduces to the solution of the equation

\[ \frac{\Delta}{rc} = (f + \frac{\partial}{\Lambda \partial \Delta})^{\beta-1} \cdot 1 \quad (47) \]

with \( f = -Z_s = (\Delta/rc)^{\frac{1}{\beta - 1}} \chi(t) \). We have not succeeded in calculating in a closed form the asymptotic expansion of \( \chi(t) \). What we can offer is a systematic recursive scheme, more efficient than eq. (33). Before entering into further algebra and anticipating on the result to be obtained, let us mention that the expansion is in inverse powers of \( t \), which implies that the susceptibility exponent is

\[ \gamma_L = \frac{\beta - 2}{\beta - 1} + \frac{\beta L}{\beta - 1} \quad (48) \]

Introduce an auxiliary variable \( x = t^{\frac{\beta-1}{\beta}} \) and rewrite (47) as

\[ x = \frac{1}{\Gamma(1 - \beta)} \int_0^\infty ds \, s^{-\beta} e^{G(s,x)} \quad (49) \]

where \( G \) is given by (32) and \( h(x) = t^{\frac{1}{\beta}} \chi(t) \). Define \( u = t^{-1} \) and perform the substitution

\[ s = \sigma u^{\frac{1}{\beta}} \quad (50) \]

\[ x = u^{\frac{1}{\beta - 1}} \quad (51) \]

After simple algebra one obtains a differential equation with analytic coefficients

\[ [(1 - \frac{1}{\beta}) + \frac{\sigma u^{\frac{1}{\beta}}}{\beta}] \frac{\partial G}{\partial \sigma} - u^2 \frac{\partial G}{\partial u} = - (1 - \frac{1}{\beta}) \chi \quad (52) \]

\(^9\)Remember that for \( n + 1 < \beta < n + 2 \) one has \( c(-1)^n < 0 \). This insures that the terms with \( 2m > \beta - 1 \) are positive.
We write $G$ and $\chi$ as power series in $u$ and insert these expansions into eq. (52) to get a hierarchy of differential equations for the coefficients $G_n(\sigma)$. The solution to these equations is $G_0(\sigma) = -\sigma$ (since $\chi_0 = 1$) and

$$G_n(\sigma) = -\sum_{k=1}^{n+1} \frac{\Gamma\left(\frac{n-k}{1-\beta} + k\right)}{\Gamma(k+1)\Gamma\left(\frac{n-k}{1-\beta} + 1\right)} \sigma^k \chi_{n+1-k}, \quad n \geq 1$$

(53)

Notice that $G_n$ depends on $\chi_k$ with $k \leq n$ only. Finally, eq. (49) becomes

$$1 = \frac{1}{\Gamma(1-\beta)} \int_0^{\infty} d\sigma \sigma^{-\beta} e^{-\sigma + \sum_{n=1}^{\infty} G_n(\sigma)t^{-n}}$$

(54)

Equating to zero the coefficient of $t^{-n}$ on the right-hand side yields $\chi_n$ in terms of $\{ \chi_k : k \leq n \}$.

4 Discussion

Let us summarize what has been achieved in this work. We have generalized the discussion of ref. [6], showing that BP models fall into one of three categories. Furthermore, we have extended the discussion to arbitrary topology. For each of the categories in question we have defined an appropriate double scaling limit and written the singular part of the partition function as a universal asymptotic series, each term of the series corresponding to a given topology. In the generic case, we have derived a BP equation, valid also in the non-perturbative regime. In short, we have done in the context of BP models what has been earlier achieved for matrix models.

As in the latter case, one can relax the constraint $p_k \geq 0$ and consider multi-critical model, where there exists a point $\phi = \phi_0$ such that $U''(\phi_0) = \ldots = U^{[m-1]}(\phi_0) = 0$. One obtains an equation analogous to (47). However, now one has the integer $m$ instead of $\beta - 1$, so that the equation is a genuine differential equation of order $m - 1$. Using the techniques developed in the last section one defines a universal expansion of the partition function

$$Z_s \sim -\Delta^\frac{1}{m} (1 - \sum_{n=1}^{\infty} \chi_n t^{-n}), \quad t \sim N\Delta^\frac{m-1}{m}$$

(55)

However, the sign of the coefficients $\chi_n$ is not positive definite, as expected.

Our motivation for studying this problem originated from our involvement in the study of simplicial gravity. Therefore, we would like to end this paper with remarks concerning the hypothetical relevance of the study of branched polymers for quantum gravity.

One of the striking results obtained from computer simulations of random geometries is that a random manifold does not stay smooth during the simulation. It develops a tree structure: the nodes are the baby universes and the links are the bottlenecks (wormholes) that connect them. We shall call this tree the skeleton tree of the manifold. For example, in 4d one observes a transition between two phases. In one of them the skeleton trees resemble generic BP. In the other the manifolds...
are crumpled: there is one big mother universe and a large number of small babies attached to it. Similar phase transitions are encountered in lower dimensions.

It has been suggested in ref. [8] that these phase transitions reflect the dynamics of the skeleton trees. Indeed, in BP theory the position of the minimum of $F(\phi)$ changes when one moves in the coupling space and a phase transition occurs when this minimum hits the boundary of the support of this function: the generic BP turn into crumpled structures i.e. with large or infinite Hausdorff dimension. In sect. 2 the Hausdorff dimension has been calculated at the tree level, but the result seems to hold for any fixed topology.

The hypothetical relation between the physics of 4d manifolds and that of the associated skeletons, if confirmed by further studies, may turn out to be a fruitful idea. The present numerical studies of 4d random manifolds are limited to simplest topologies. Thus the models under study are non-unitary: the wormholes that have been emitted cannot be reabsorbed. These simulations, although limited to skeleton trees without loops, are however sufficient for the determination of the couplings at the nodes (the $p_k$’s). A glimpse at the unitarized theory is provided by an extension to trees with loops, along the lines of this paper.

Notice also, that a phase transition analogous to that observed in 4d occurs in 2d, but only for large enough $c$ (or at least for $c > 1$). But a sensible 2d gravity theory exists for $c \leq 1$ only\(^{10}\). Thus one is led to speculate that a sensible gravity theory in 4d will only be obtained when one does in 4d something analogous to the reduction of the central charge in 2d. One can develop a heuristic argument analogous to that proposed by Cates [13] to explain the $c = 1$ barrier in 2d. Consider the continuum theory with Einstein-Hilbert action and write the metric in the form $g_{ab} = e^{2\sigma} \hat{g}_{ab}$. Assume that the BP phase in 4d is dominated by the dynamics of the conformal factor and use the effective action calculated in [14] to estimate the free energy of a diluted gas of "spikes": $e^{2\sigma} = 1 + \rho^2 / [(x-x_0)^2 + a^2]$, with $a \ll \rho < \text{const} \times a$. As discussed in [13] the parameter $a$ can be taken arbitrarily small, while keeping the invariant cut-off fixed. The free energy is

$$F \simeq \left[ \frac{1411 + N_S + 11N_F + 62N_V - 28}{360} - 4 \right] \ln \frac{1}{a}, \quad (56)$$

where 1411 is the contribution of transverse gravitons, $N_{S,F,V}$ is the number of scalar, fermion and vector fields respectively and $-28$ is the quantum $\sigma$ and ghost contribution. Notice that, as emphasized in [14], the sign of matter and ghost contributions is in 4d opposite to that found in 2d. For pure gravity the coefficient in front of $\ln \frac{1}{a}$ is negative and one expects a condensation of "spikes". However the addition of matter fields can easily change the sign of this coefficient and stabilize the theory.

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\(^{10}\)This is at least true for generic random surface models. Exceptions to this rule have been claimed [11].
References

[1] J. Ambjørn, B. Durhuus, J. Fröhlich and P. Orland, Nucl. Phys. B270 (1986) 457.

[2] J. Ambjørn, B. Durhuus and T. Jónsson, Phys. Lett. B244 (1990) 403.

[3] P. Bialas, Phys. Lett. B373 (1996) 289.

[4] J. Ambjørn, P. Bialas and J. Jurkiewicz, *RG flow in an exactly solvable model with fluctuating geometry*, hep-lat/9602021.

[5] B. Durhuus, Nucl. Phys. B426 (1994) 203.

[6] P. Bialas and Z. Burda, *Phase transition in fluctuating branched geometry*, hep-lat/9605020, to appear in Phys. Lett. B.

[7] P. Bialas, Z. Burda and D. Johnston, *Backgammon and gravity*, cond-mat/9609264.

[8] P. Bialas, Z. Burda, B. Petersson and J. Tabacek, *Appearance of mother universe and singular vertices in random geometries*, hep-lat/9608030.

[9] P. Dienes, *The Taylor series*, Dover (New York 1957).

[10] E. Brézin and V. Kazakov, Phys. Lett. B236 (1990) 144; D. Gross and A.A. Migdal, Nucl. Phys. B340 (1990) 333 ; M. Douglas and S.H. Shenker, Nucl. Phys. B335 (1990) 635.

[11] J.L. Gervais and J.F. Roussel, Phys. Lett. B338 (1994) 437 and references therein.

[12] P. Bialas, Z. Burda, A. Krzywicki and B. Petersson, Nucl. Phys. B472 (1996) 293.

[13] M.E. Cates, Europhys. Lett. 8 (1988) 719; A. Krzywicki, Phys. Rev. D41 (1990) 3086; F. David, Nucl. Phys. B368 (1992) 671.

[14] I. Antoniadis and E. Mottola, Phys. Rev. D45 (1992) 2013; I. Antoniadis, P.O. Mazur and E. Mottola, Nucl. Phys. B388 (1992) 627.