Universal multi-mode linear optical quantum operation in the time domain

Kazuma Yonezu, Yutaro Enomoto, Takato Yoshida, and Shuntaro Takeda*

Department of Applied Physics, School of Engineering,
The University of Tokyo, 7-3-1 Hongo, Bunkyo-ku, Tokyo 113-8656, Japan
(Dated: October 31, 2022)

Universal multi-mode linear optical operations are essential for almost all quantum information protocols (QIPs) for both qubits and continuous variables. Thus far, large-scale implementation of such operations has been pursued mainly by developing photonic chips that contain large interferometers for path-encoded optical modes. However, such encoding requires larger circuits for larger-scale operations, possibly limiting scalability. Here, we realize a scalable dual-loop optical circuit that can programmably perform universal three-mode linear optical operations in the time domain. The programmability, validity, and quantum feature of our circuit are demonstrated by performing nine different three-mode operations on squeezed-state pulses, fully characterizing their output states via homodyne detection, and confirming their entanglement. Our circuit can be straightforwardly scaled up by making the outer loop longer and also extended to universal quantum computers by incorporating measurement and feedforward systems. Thus, our work paves the way to large-scale QIPs which exhibit quantum advantage.

Optics has been crucial in implementing various quantum information protocols (QIPs), such as quantum computing\textsuperscript{1,2}, quantum networking\textsuperscript{3}, and quantum simulation\textsuperscript{4}. A key technology common to almost all optical QIPs for both qubits and continuous variables is multi-mode linear optical quantum operations, which linearly transform creation operators of photons\textsuperscript{5}. Such operations are not only central to the implementation of boson sampling\textsuperscript{6,7} and quantum walk\textsuperscript{8,9}, but also essential for preparing entangled resource states for various QIPs\textsuperscript{10}. Furthermore, such operations enable universal quantum computation when combined with appropriate measurement and feedforward operations\textsuperscript{1,2}. Thus, realizing large-scale universal linear optical operations holds great promise for achieving quantum advantages in various QIPs.

Thus far, universal multi-mode linear optical operations have been implemented and scaled up to 20 modes by developing multi-mode linear interferometers on programmable photonic chips\textsuperscript{11–14}. In such implementations, one optical path represents one mode, and spatial arrays of phase shifters (PSs) and beam splitters (BSs) perform the desired operations. In this path encoding, increasing the number of modes requires quadratically growing numbers of BSs and PSs. This makes the interferometer larger and the alignment of all the interferometric points more difficult, possibly limiting scalability.

A more scalable option to realize large-scale linear optical operations is to use temporal encoding, where unlimited number of modes can be defined as sequential optical pulses on a single optical path\textsuperscript{15}. High scalability of the temporal encoding has already been shown in recent optical demonstrations of quantum supremacy\textsuperscript{15}, large-scale entanglement generation\textsuperscript{16,17}, and multi-mode multi-step quantum gates\textsuperscript{18–20}. The temporal encoding is also useful to scale up universal linear optical operations by adopting a dual-loop optical circuit proposed in Ref.\textsuperscript{21}. Several experiments have adopted this idea to scale up boson sampling\textsuperscript{2} and quantum walk\textsuperscript{8,9}. However, these experiments were designed for the specific protocols and thus were unable to perform universal linear optical operations due to the lack of phase locking or full dynamic controllability of the loops.

Here, we realize a scalable dual-loop optical circuit that can programmably perform universal three-mode linear optical operations in the time domain. Our system completes all the functionalities for the original dual-loop circuit\textsuperscript{21} by combining an arbitrarily controllable variable beam splitter (VBS) and variable phase shifter (VPS) with fully phase-stabilized dual loops. The programmability, validity, and quantum feature of our circuit are demonstrated by performing nine different three-mode operations on squeezed-state pulses, fully characterizing their output states via homodyne detection, and confirming their entanglement. Our circuit can be straightforwardly scaled up just by making the outer loop longer and storing more modes in the loops. Moreover, our circuit can be extended to universal quantum computing architectures by incorporating extra measurement and feedforward systems into the loops\textsuperscript{22,23}. Thus, our work paves the way to large-scale QIPs in the time domain, possibly leading to quantum advantages in various QIPs.

**RESULTS**

**Working principle of the dual-loop circuit**

In the typical path encoding, universal N-mode linear optical operations can be performed by a linear optical network consisting of BSs and PSs, as Fig. 1a shows\textsuperscript{24}. In the temporal encoding, the same operations can be done by the dual-loop circuit in Fig. 1b\textsuperscript{21}. The working principle of the dual-loop circuit is the following. First, N sequential pulsed optical modes with time in-
FIG. 1. Optical circuits for universal $N$-mode linear optical operations. a A circuit in the path encoding with a linear optical network consisting of arrays of BSs and PSs. b A circuit in the temporal encoding with dual loops. The inner-loop round-trip time corresponds to the time interval between the neighboring temporal modes ($\tau$). Universal $N$-mode linear optical operations require the outer-loop round-trip time of $(N-1)\tau$. VBS, variable beam splitter; VPS, variable phase shifter.

FIG. 2. Dynamics of the dual-loop circuit for universal three-mode linear optical operations. a One of the possible configurations to perform universal three-mode linear optical operations in the path encoding. The sides of the BSs that invert the phase of the reflected modes are colored light blue. A phase shift of $180^\circ$ is added to the circuit to make it completely equivalent to our dual-loop circuit (see Methods). b–e Dynamics of the dual-loop circuit to perform universal three-mode operations in the temporal encoding. f Temporal control sequence for the dual-loop circuit. The transmissivity $T(t)$ of the VBS and the phase $\theta(t)$ of the VPS are synchronously controlled every $\tau$ in the time domain.

terval $\tau$ are injected and stored in the dual-loop circuit via optical switches (Switch-1, 2). Here, $N-1$ modes are stored in the outer loop whose round-trip time is $(N-1)\tau$, while the remaining mode is stored in the inner loop whose round-trip time is $\tau$. The inner loop includes a VBS with transmissivity $T(t)$ and a VPS with phase $\theta(t)$, where $t$ denotes time. This inner loop repeatedly performs two-mode BS interactions between the pulsed modes in the inner and outer loops while dynamically changing $T(t)$ and $\theta(t)$ for each pulse. It can be shown that such operations enable an arbitrary linear optical operation between the $N$ modes (see Methods). After the desired operations, Switch-2 sequentially exports the output modes. This dual-loop circuit is highly scalable since it can process an arbitrary number of modes with a constant number of optical components just by making the outer loop appropriately long. Furthermore, operations are fully programmable since they are determined by the electric control sequence of $T(t)$ and $\theta(t)$.

Figure 2 exemplifies a more concrete sequence to perform an arbitrary linear optical operation for $N=3$ modes, which we adopt in our experiment. Figure 2a illustrates one of the possible configurations to perform an arbitrary three-mode linear optical operation in the path encoding. The same operation can be done in the dual-loop circuit as shown in Figs. 2b–e based on the control sequence in Fig. 2f. Here, after storing the first mode in the inner loop by setting $T_0 = 1$ (Fig. 2b), the inner loop sequentially performs three BS operations $T_1$, $T_2$, $T_3$ while adjusting the relative phases $\theta_1$, $\theta_2$, $\theta_3$ (Figs. 2c–e). Finally, the transmissivity of the VBS is kept 1 and the VPS adds individual phase shifts $\theta_4$, $\theta_5$, $\theta_6$ to these modes to finalize the operation. A more general procedure to perform $N$-mode linear optical operations are shown in Methods.

Experimental setup

We develop the dual-loop circuit with $N=3$ that can programmably perform universal three-mode linear optical quantum operations, as shown in Fig. 3 (see Methods for details). Our setup achieves all the functionalities in the original proposal of the dual-loop circuit\textsuperscript{21}. In our setup, we choose the time interval of $\tau = 66$ ns and the corresponding inner and outer loop lengths of 19.8 m ($\tau$) and 39.6 m ($(N-1)\tau$), respectively. Both the inner and outer loops are phase locked. Two switches, one VPS, and one VBS are incorporated in the loops and synchronously controlled every $\tau = 66$ ns. To evaluate the performance of the operations in the dual-loop circuit,
three-mode squeezed-state pulses are injected and each output pulse is measured by a homodyne detector (HD) with a variable measurement basis \( \hat{x} \cos \phi(t) + \hat{p} \sin \phi(t) \), where \( \hat{x} \) and \( \hat{p} \) are the quadrature operators of the light field and \( \phi \) is called a homodyne angle. Our control sequence is based on Figs. 2b-f, but the final unimportant local phase shifts \( (\theta_4, \theta_5, \theta_6) \) in the VPS are omitted and equivalently performed by shifting the measurement bases at the HD. This reduces the number of round trips of optical pulses in the loops and thus minimizes the optical loss during the operations.

**Three-mode linear optical operations**

As a demonstration of programmable multi-mode linear optical operations in the time domain, we perform nine different three-mode operations on the input \( p \)-squeezed state pulses using our dual-loop circuit. It is known that appropriate linear operations can transform such squeezed states into various multi-mode continuous-variable entangled states\(^{10,17} \). Thus our overall system can also be regarded as a general multi-mode continuous-variable photonic entanglement synthesizer\(^{17} \). We mainly adopt such operations for the demonstration and quantitatively evaluate the covariance matrices of the output states to verify the validity of the operations. Note that the covariance matrices fully characterize the output states which are always Gaussian states with zero-mean quadratures in this experiment. In addition, we evaluate the degree of entanglement of the generated entangled states to show that the operations are performed in the quantum regime.

As shown in Fig. 2, three-mode linear operations are composed of three two-mode BS interactions. First, we run our dual-loop circuit in the simplest setting where all these BS interactions are switched off by always setting the VBS transmissivity to 1 (Operation 1). The equivalent circuit in the path encoding is shown in the left panel of Fig. 4a. This operation only rearranges the order of the input modes and thus each output mode becomes a \( p \)-squeezed state. The experimental output covariance matrix is shown in the middle panel of Fig. 4a. As expected, it shows (anti-)squeezed variances in the \( p \) \( (x) \) quadratures for all modes, while not showing correlation between these modes. The theoretical covariance matrix including estimated optical losses (see Methods) is also plotted in the right panel of Fig. 4a, which reasonably well agrees with the experimental one. As can be seen from the covariance matrix, the output modes are slightly asymmetric and the squeezed state in mode 1 seems to suffer from more loss than the others. This is because, in our sequence, the squeezed state coming to mode 1 suffers from an extra round-trip loss in the outer loop compared to the other modes.

Next, we perform three-mode linear optical operations that generate various continuous-variable entangled states. Specifically, we choose eight different operations and generate four types of entangled states: Einstein-Podolsky-Rosen (EPR) states generated by switching on one BS interaction (Operation 2-i, ii, iii), Greenberger-Horne-Zeilinger (GHZ) states generated by switching on two BS interactions (Operation 3-i, ii, iii), and two shapes of cluster states generated by switching on all three BS interactions (Operation 4-i, ii). Figures 4b–d are the representative results for each case, showing the equivalent path-encoding circuits and the output covariance matrices. As opposed to Operation 1, the experimental covariance matrices show non-zero off-diagonal elements for all cases, which implies that some of the modes are entangled. In addition, the experimental covariance matrices agree well with the theoretical ones, demonstrating that the dual-loop circuit performs the three-mode operations as expected. The covariance matrices of all the other operations are summarized in Supplementary Information.

Finally, we quantitatively evaluate the performance of all the above nine operations. We calculate and summarize the fidelities between the experimental covariance matrices and the theoretical ones including losses in Ta-
FIG. 4. **Representative results of three-mode linear operations in the dual-loop circuit.** The left column shows the equivalent circuits in the path encoding based on Fig 2a. The middle and right columns show the experimental and theoretical output covariance matrices, respectively. The matrix elements represent covariances $\langle \hat{\xi}_i \hat{\xi}_j + \hat{\xi}_j \hat{\xi}_i \rangle / 2 - \langle \hat{\xi}_i \rangle \langle \hat{\xi}_j \rangle$, where $\langle \cdots \rangle$ denotes the mean value and $\hat{\xi} = (\hat{x}_1, \hat{p}_1, \hat{x}_2, \hat{p}_2, \hat{x}_3, \hat{p}_3)^T$. The vacuum variance is set to 1 ($\hbar = 2$). Note that the phase-inverting side of one of the three BSs is flipped in d since the phase-inverting side of the VBS is flipped when $T < 0.5^{17}$. 
Table I. Fidelities and inseparability parameters for the output modes of various three-mode linear operations. The first and second columns show the operation number and the corresponding output states, respectively. The third column shows fidelities between the experimental covariance matrices and the theoretical ones including optical losses. The fourth column shows the expressions and measured values of the inseparability parameters. When the inseparability parameter is below 4 \( (\hbar = 2) \), the modes are inseparable.

| Operation | Output state | Fidelity | Inseparability parameter |
|-----------|--------------|----------|--------------------------|
| 1         | Individual squeezed vacuum states (\(1, 2, \) and \(3 \)) | 0.992 ± 0.002 | \(-\) |
| 2-i       | EPR state (1 and 3), squeezed vacuum state (2) | 0.958 ± 0.007 | \(|\Delta(\hat{x}_1 - \hat{x}_3)|^2 + |\Delta(\hat{x}_1 + \hat{x}_3)|^2 = 2.38 ± 0.05\) |
| 2-ii      | EPR state (2 and 3), squeezed vacuum state (1) | 0.966 ± 0.008 | \(|\Delta(\hat{x}_2 - \hat{x}_1)|^2 + |\Delta(\hat{x}_2 + \hat{x}_1)|^2 = 2.09 ± 0.03\) |
| 2-iii     | EPR state (1 and 2), squeezed vacuum state (3) | 0.965 ± 0.004 | \(|\Delta(\hat{x}_1 - \hat{x}_2)|^2 + |\Delta(\hat{x}_1 + \hat{x}_2)|^2 = 2.56 ± 0.03\) |
| 3-i       | GHZ state (1, 2, and 3) | 0.947 ± 0.012 | \(|\Delta(\hat{x}_1 - \hat{x}_2)|^2 + |\Delta(\hat{x}_1 + \hat{x}_2)|^2 = 2.91 ± 0.06\) |
| 3-ii      | GHZ state (1, 2, and 3) | 0.896 ± 0.007 | \(|\Delta(\hat{x}_1 - \hat{x}_2)|^2 + |\Delta(\hat{x}_1 + \hat{x}_2)|^2 = 3.39 ± 0.04\) |
| 3-iii     | GHZ state (1, 2, and 3) | 0.888 ± 0.008 | \(|\Delta(\hat{x}_1 - \hat{x}_2)|^2 + |\Delta(\hat{x}_1 + \hat{x}_2)|^2 = 3.34 ± 0.07\) |
| 4-i       | Triangle cluster state (1, 2, and 3) | 0.909 ± 0.019 | \(|\Delta(\hat{p}_1 - \hat{x}_2 - \hat{x}_1)|^2 + |\Delta(\hat{p}_1 + \hat{x}_2 - \hat{x}_1)|^2 = 3.21 ± 0.05\) |
| 4-ii      | Linear cluster state (1, 2, and 3) | 0.976 ± 0.007 | \(|\Delta(\hat{x}_1 - \hat{x}_3)|^2 + |\Delta(\hat{x}_1 + \hat{x}_3)|^2 = 2.77 ± 0.05\) |

DISCUSSION

In conclusion, we developed a scalable dual-loop circuit that can perform universal multi-mode linear optical quantum operations in the time domain. This was achieved by developing fully phase-locked dual loops that include a dynamically and synchronously controllable VBS and VPS. We showed a general sequence to perform universal linear optical operations in the dual-loop circuit. Based on this sequence, we programmably performed nine different three-mode operations on squeezed-state pulses, demonstrating the validity and quantum feature of these operations by evaluating the output covariance matrices and entanglement.

Our dual-loop circuit adopts the temporal encoding and thus offers higher scalability than the conventional path-encoding circuits; we can increase the number of processable modes just by making the outer loop appropriately long without increasing the number of optical components. Moreover, our dual-loop circuit for universal linear optical operations can be further extended to loop-based universal optical quantum computers for both qubits and continuous variables by incorporating measurement and feedforward operations, as proposed in Refs.\(^{22,23}\). Therefore, our demonstration is a crucial step toward large-scale optical QIP in the time domain, including quantum computing, quantum networking, and quantum simulation.

Note added. – We have recently become aware of a paper\(^{26}\) in which a loop circuit with a different configuration performed universal linear optical operations on input squeezed pulses in the time domain, although the output was measured by a photon detector to acquire data samples for Gaussian boson sampling.

METHODS

Experimental setup

Figure 3 illustrates our experimental setup for the dual-loop circuit with \(N = 3\). This setup is extended from our previous single-loop circuit, which is described in detail in Refs.\(^{17,20}\). A squeezed-vacuum beam at 860 nm is
generated from a continuously-pumped optical parametric oscillator (OPO), and the input squeezed-state pulses are defined at the time interval of $\tau = 66$ ns. These pulses are then sent to the dual-loop circuit. The inner and outer loops are constructed by Herriott-type optical delay lines and have round-trip times of $\tau = 66$ ns (19.8 m) and $(N - 1)\tau = 132$ ns (39.6 m), respectively. This dual-loop circuit includes four dynamically controllable elements: Switch-1, Switch-2, VPS, and VBS. For example, the VBS is composed of an electro-optic modulator (EOM) named EOM-2 and two polarizing BSs (PBSs). Additionally, a quarter-wave plate (QWP) is inserted between the PBSs to set the VBS transmissivity to 0.5 by default for phase locking of the loops. We then apply appropriate voltages to EOM-2 to dynamically control the transmissivity. Switch-1 (Switch-2) works in the same way by using EOM-1 (EOM-3). The VPS is also realized by EOM-4. The output pulses from the dual-loop circuit are finally measured by the HD, whose homodyne angle is controlled by EOM-5. All the above EOMs are synchronously and dynamically controlled every $\tau = 66$ ns by the timing controller. The switching time of these EOMs is $\sim 10$ ns.

**Data analysis**

The quadratures of the output modes are extracted by applying temporal mode functions to the acquired homodyne signal. The function for the $k$-th mode ($k = 1, 2, 3$) is defined as

$$f_k(t) \propto e^{-\gamma^2(t-t_k)^2} (t - t_k)^{(2|t - t_k| \leq \Delta t)} (\text{otherwise}) \quad (1)$$

and normalized to be $\int_{-\infty}^{\infty} |f_k(t)|^2 dt = 1^{17,20}$. Here we choose $\Delta t = 46$ ns, $\gamma = 6 \times 10^7$ $/s$, and $t_k = t_0 + (k - 1)\tau$. To calculate the output covariance matrices and inseparability parameters for each operation, we acquire 5000 samples of the output quadratures for each of the following five different measurement bases: $(\hat{p}_1, \hat{x}_2, \hat{x}_3)$, $(\hat{x}_1, \hat{p}_2, \hat{x}_3)$, $(\hat{x}_1, \hat{x}_2, \hat{p}_3)$, $(\hat{p}_1, \hat{p}_2, \hat{p}_3)$, and $(\hat{x}_1 + \hat{p}_2 + \hat{x}_3 \sqrt{2}, \hat{x}_2 + \hat{x}_3 \sqrt{2})$. Here, $\hat{x}$, $\hat{p}$, and $\hat{x} + \hat{p}$ for each mode can be measured by adding phase shifts of $0^\circ$, $45^\circ$, and $90^\circ$ to the local oscillator (LO) beam, respectively. All the elements of the output covariance matrices as well as the values of the inseparability parameters can be calculated from the above five sets of measurement results.

**Dual-loop phase locking**

Let us here describe the way to lock the phases of the dual loops that can be regarded as coupled resonators.

For the phase locking, a reference light is injected into the OPO after being phase-modulated at two frequencies of 300 kHz and 4.5 MHz. This light is then detected at an unused port of PBS-2 in Fig. 3. We demodulate the detection signal at 300 kHz and 4.5 MHz, thereby obtaining the error signals for the outer and inner loops, respectively. The outer and inner loops can be simultaneously phase-locked by feeding back these error signals. Here, since the Pound–Drever–Hall error signal of a resonator behaves differently for lower and higher frequencies than the linewidth of the resonator$^{28}$, the demodulated signals contain the error signals of the inner and outer loops in different proportions. These specific frequencies are chosen so that the error signals of the two loops are separated the most based on numerical simulations.

When the inner loop is locked in our setting, the outer-loop path suffers from a 180° phase shift at the reflecting point of the VBS. The outer loop is also locked so that the phase returns to the original value during the round trip. This means that the outer-loop path except for the VBS part also has a 180° phase shift to compensate for the 180° phase shift at the VBS. This phase shift is introduced in the equivalent path-encoding circuits of Figs. 2 and 4.

**Optical loss**

Optical losses in our setup are estimated from two measurement results. First, optical losses in the generation and measurement part of the squeezed light are estimated by the input covariance matrix in Fig. 5. This matrix is measured by keeping the VBS transmissivity 0 and thereby directly transmitting the input squeezed modes to the output port without letting them go around the loops. In this case, the squeezed modes suffer from the same amount of optical losses, including the OPO internal loss, the propagation loss from the OPO to the HD, and the readout loss of the HD. From this input covariance matrix, the total loss is estimated to be 23%, and the initial pure squeezing level before suffering from any optical losses is estimated to be 7.4 dB. Next, the round-trip optical losses in the loops are estimated from the output covariance matrix of Operation 1 in Fig. 4a. Here the squeezed states coming to mode 2 and 3 suffer from the round-trip loss in the inner loop in addition to the losses in the generation and measurement part. Furthermore, the squeezed state coming to mode 1 suffers from...
the extra round-trip loss in the outer loop. From these facts, the round-trip losses in the inner and outer loops are estimated to be 15\% and 20\%, respectively. The estimated losses and squeezing level are taken into account to calculate the theoretical output covariance matrices.

**Multi-mode linear optical operation**

Here we define general multi-mode linear operations and explain how covariance matrices are transformed by such operations. In the Heisenberg picture, the input-output relation of $N$-mode linear operations are defined in a matrix form as

$$
\begin{pmatrix}
\hat{a}_{1\text{out}}
\hat{a}_{2\text{out}} \\
\vdots \\
\hat{a}_{N\text{out}}
\end{pmatrix} = U_N
\begin{pmatrix}
\hat{a}_{1\text{in}}
\hat{a}_{2\text{in}} \\
\vdots \\
\hat{a}_{N\text{in}}
\end{pmatrix},
$$

(2)

where $\hat{a}_i^k$ ($k = \text{in, out}$) is an input or output annihilation operator of mode $i$ and $U_N$ is a unitary matrix. This relation can be reformed by using quadratures $\hat{x}_i^k = \hat{a}_i^k + (\hat{a}_i^k)^\dagger$ and $\hat{p}_i^k = -i\hat{a}_i^k + i(\hat{a}_i^k)^\dagger$ as

$$
\hat{\xi}_\text{out} = (WA)^{-1} \begin{pmatrix} U_N & 0 \\ 0 & U_N^{-1} \end{pmatrix} WA\hat{\xi}_\text{in} = S\hat{\xi}_\text{in},
$$

(3)

where $\hat{\xi}_k = (\hat{x}_1^k, \hat{p}_1^k, \ldots, \hat{x}_N^k, \hat{p}_N^k)^T$, $A$ and $W$ are 2$N$-dimensional matrices defined by

$$
A_{ij} = \begin{cases} 
1 & (j \text{ even and } i = N + j/2) \\
1 & (j \text{ odd and } i = (j+1)/2) \\
0 & \text{(otherwise)} 
\end{cases},
$$

(4)

and $I$ is an $N$-dimensional identity matrix. This quadrature transformation also changes the corresponding covariance matrix $\Gamma_k$, whose elements are defined as $\Gamma_{ij} = \langle \hat{\xi}_i^k|\hat{\xi}_j^k \rangle$. The input-output relation of the covariance matrices can be written as

$$
\Gamma_{\text{out}} = S\Gamma_{\text{in}}S^T.
$$

(5)

This transformation rule is used to derive the theoretical covariance matrices.

**N-mode linear optical operation in dual loops**

We show the general sequence to perform an arbitrary $N$-mode linear optical operation in the dual-loop circuit. In general, an arbitrary linear optical operation $U_N$ can be decomposed into sequences of BS and PS operations. Several ways for the decomposition are known, but here we drive a slightly modified decomposition that is compatible with the dual-loop circuit. When the $N$-dimensional unitary matrix $U_N$ is multiplied with appropriate unitary matrices $T_{l,m}^{(1)}$, the effective dimension of the unitary matrix can be reduced as

$$
U_N T_{1,2}^{(1)} T_{1,3}^{(1)} \cdots T_{1,N}^{(1)} = \begin{pmatrix} U_{N-1} & 0 \\ 0 & e^{i\alpha_N} \end{pmatrix},
$$

(6)

where $\alpha_N$ is a real constant, $U_{N-1}$ is a certain $(N - 1)$-dimensional unitary matrix, and $T_{l,m}^{(1)}$ is an $N$-dimensional identity matrix whose $(1,1)$, $(l,m)$, $(m,l)$, and $(m,m)$ elements are replaced with $e^{i\alpha_1}$, $e^{i\alpha_2}$, $e^{i\alpha_3}$, and $e^{i\alpha_4}$, respectively. The matrix $U_N$ can be diagonalized by repeating this procedure as

$$
U_N T_{1,2}^{(1)} T_{1,3}^{(1)} \cdots T_{1,N}^{(1)} T_{1,2}^{(2)} T_{1,3}^{(2)} \cdots T_{1,N}^{(2)} \cdots T_{1,2}^{(N-1)} = D,
$$

(7)

where $D = \text{diag}(e^{i\alpha_1}, e^{i\alpha_2}, \ldots, e^{i\alpha_N})$ for real constants $\alpha_1, \alpha_2, \ldots, \alpha_N$. As a result, $U_N$ can be decomposed as

$$
U_N = D T_{1,2}^{(1)} T_{1,3}^{(1)} \cdots T_{1,N}^{(1)} T_{1,2}^{(2)} T_{1,3}^{(2)} \cdots T_{1,N}^{(2)} \cdots T_{1,2}^{(N-1)}
$$

$$
= D T_{1,2}^{(N-1)} T_{1,3}^{(N-1)} \cdots T_{1,N}^{(N-1)}
$$

$$
\cdots T_{1,2}^{(1)} T_{1,3}^{(1)} \cdots T_{1,N}^{(1)} T_{1,2}^{(1)} T_{1,3}^{(1)} \cdots T_{1,N}^{(1)}.
$$

(8)

This decomposition has clear correspondence with the dual-loop circuit and thus is suitable for the implementation in the circuit. In the dual-loop circuit, mode 1 is stored in the inner loop, while the other modes are stored in the outer loop, as shown in Fig. 6. Then mode 1 is sequentially phase-shifted and interfered with mode 2, 3, \ldots, $N$ in the outer loop. Such sequential operations can perform $(T_{1,2}^{(1)})^{-1} \cdots (T_{1,3}^{(1)})^{-1} (T_{1,2}^{(2)})^{-1}$. During the next round trip, we can perform $(T_{1,2}^{(2)})^{-1} \cdots (T_{1,3}^{(2)})^{-1} (T_{1,2}^{(3)})^{-1}$ (the unnecessary term $(T_{1,2}^{(2)})^{-1}$ can be skipped by setting the VBS transmissivity to 0). By repeating this sequence, the dual-loop circuit can perform $(T_{1,2}^{(1)})^{-1} \cdots (T_{1,3}^{(1)})^{-1} \cdots (T_{1,2}^{(2)})^{-1} (T_{1,2}^{(3)})^{-1} (T_{1,2}^{(4)})^{-1}$, \ldots, $(T_{1,2}^{(1)})^{-1}$, $(T_{1,2}^{(2)})^{-1}$, $(T_{1,2}^{(3)})^{-1}$, $(T_{1,2}^{(4)})^{-1}$. Finally, individual phase shifts are applied to all modes by keeping the VBS transmissivity 1, which implements the diagonal matrix $D$ and completes the $N$-mode linear optical operation in Eq. (8). We perform three-mode linear optical operations based on the above decomposition.
DATA AVAILABILITY

The data that support the findings of this study are available from the corresponding author upon reasonable request.

CODE AVAILABILITY

The code that supports the findings of this study is available from the corresponding author upon reasonable request.

ACKNOWLEDGMENTS

This work was partly supported by JSPS KAKENHI Grant Numbers 20H01833 and 21K18593, MEXT Leading Initiative for Excellent Young Researchers, Toray Science Foundation (19-6006), and the Canon Foundation. The authors thank Akira Furusawa for providing space for the experiment. The authors also thank Takahiro Mitani for the careful proof-reading of the manuscript.

AUTHOR CONTRIBUTIONS

K. Y. developed the experimental setup and acquired the data. Y. E. and K. Y. devised and implemented the dual-loop phase locking system. K. Y. and T. Y. analyzed the data. S. T. conceived, planned, and supervised the project. All authors contributed to writing the manuscript.

[1] Knill, E., Laflamme, R. & Milburn, G. J. A scheme for efficient quantum computation with linear optics. Nature 409, 46–52 (2001).
[2] Takeda, S. & Furusawa, A. Toward large-scale fault-tolerant universal photonic quantum computing. APL Photonics 4, 060902 (2019).
[3] Wehner, S., Elkouss, D. & Hanson, R. Quantum internet: A vision for the road ahead. Science 362, eaam9288 (2018).
[4] Flamini, F., Spagnolo, N. & Sciarrino, F. Photonic quantum information processing: a review. Rep. Prog. Phys 82, 016001 (2018).
[5] Kok, P. et al. Linear optical quantum computing with photonic qubits. Rev. Mod. Phys. 79, 135 (2007).
[6] Aaronson, S. & Arkhipov, A. The computational complexity of linear optics. In Proceedings of the forty-third annual ACM symposium on Theory of computing, 333–342 (2011).
[7] He, Y. et al. Time-bin-encoding boson sampling with a single-photon device. Phys. Rev. Lett. 118, 190501 (2017).
[8] Aharonov, Y., Davidovich, L. & Zagury, N. Quantum random walks. Phys. Rev. A 48, 1687 (1993).
[9] Schreiber, A. et al. A 2D quantum walk simulation of two-particle dynamics. Science 336, 55–58 (2012).
[10] van Loock, P., Weedbrook, C. & Gu, M. Building Gaussian cluster states by linear optics. Phys. Rev. A 76, 032321 (2007).
[11] Carolan, J. et al. Universal linear optics. Science 349, 711–716 (2015).
[12] Qiang, X. et al. Large-scale silicon quantum photonics implementing arbitrary two-qubit processing. Nat. Photonics 12, 534–539 (2018).
[13] Taballione, C. et al. 20-mode universal quantum photonic processor. Preprint https://arxiv.org/abs/2203.01801 (2022).
[14] Taballione, C. et al. A universal fully reconfigurable 12-mode quantum photonic processor. Mater. Quantum Technol. 1, 035002 (2021).
[15] Madsen, L. S. et al. Quantum computational advantage with a programmable photonic processor. Nature 606, 75–81 (2022).
[16] Yokoyama, S. et al. Ultra-large-scale continuous-variable cluster states multiplexed in the time domain. Nat. Photonics 7, 982–986 (2013).
[17] Takeda, S., Takase, K. & Furusawa, A. On-demand photonic entanglement synthesizer. Sci. Adv. 5, eaaw4530 (2019).
[18] Asavanant, W. et al. Time-domain-multiplexed measurement-based quantum operations with 25-MHz clock frequency. Phys. Rev. Appl. 16, 034005 (2021).
[19] Larsen, M. V., Guo, X., Breum, C. R., Neergaard-Nielsen, J. S. & Andersen, U. L. Deterministic multimode gates on a scalable photonic quantum computing platform. Nat. Phys. 17, 1018 (2021).
[20] Enomoto, Y., Yonezu, K., Mitsuhashi, Y., Takase, K. & Takeda, S. Programmable and sequential Gaussian gates in a loop-based single-mode photonic quantum processor. Sci. Adv. 7, eabj6624 (2021).
[21] Motes, K. R., Gilchrist, A., Dowling, J. P. & Rohde, P. P. Scalable boson sampling with time-bin encoding using a loop-based architecture. Phys. Rev. Lett. 113, 120501 (2014).
[22] Rohde, P. P. Simple scheme for universal linear-optics quantum computing with constant experimental complexity using fiber loops. Phys. Rev. A 91, 012306 (2015).
[23] Takeda, S. & Furusawa, A. Universal quantum computing with measurement-induced continuous-variable gate sequence in a loop-based architecture. Phys. Rev. Lett. 119, 120504 (2017).
[24] Reck, M., Zeilinger, A., Bernstein, H. J. & Bertani, P. Experimental realization of any discrete unitary operator. Phys. Rev. Lett. 73, 58 (1994).
[25] van Loock, P. & Furusawa, A. Detecting genuine multipartite continuous-variable entanglement. Phys. Rev. A 67, 052315 (2003).
[26] Yu, S. et al. A universal programmable Gaussian boson sampler for drug discovery. Preprint
https://arxiv.org/abs/2210.14877 (2022).
[27] Herriott, D., Kogelnik, H. & Kompfner, R. Off-axis paths in spherical mirror interferometers. Appl. Opt. 3, 523–526 (1964).
[28] Black, E. D. An introduction to Pound–Drever–Hall laser frequency stabilization. Am. J. Phys. 69, 79–87 (2001).
[29] Simon, R., Mukunda, N. & Dutta, B. Quantum-noise matrix for multimode systems: U (n) invariance, squeezing, and normal forms. Phys. Rev. A 49, 1567 (1994).
[30] Folland, G. B. Harmonic analysis in phase space.(am-122), volume 122. In Harmonic Analysis in Phase Space.(AM-122), Volume 122 (Princeton university press, 2016).
[31] de Guise, H., Di Matteo, O. & Sánchez-Soto, L. L. Simple factorization of unitary transformations. Phys. Rev. A 97, 022328 (2018).
[32] Clements, W. R., Humphreys, P. C., Metcalf, B. J., Kolthammer, W. S. & Walmsley, I. A. Optimal design for universal multiport interferometers. Optica 3, 1460–1465 (2016).
Supplementary information for
Universal multi-mode linear optical quantum operation in the time domain

Kazuma Yonezu, Yutaro Enomoto, Takato Yoshida, and Shuntaro Takeda∗
(Dated: October 31, 2022)

The experimental results of the three-mode linear optical operations not shown in the main text are summarized in Fig. S1.

---

**Operation 2-ii**

**a** Equivalent circuit

**b** Equivalent circuit

**c** Equivalent circuit

---

* takeda@ap.t.u-tokyo.ac.jp
FIG. S1. **Experimental results of three-mode linear operations in the dual-loop circuit.** The left column shows the equivalent circuits in the path encoding. The middle and right columns show the experimental and theoretical output covariance matrices, respectively. The matrix elements represent covariances $\langle \hat{\xi}_i \hat{\xi}_j + \hat{\xi}_j \hat{\xi}_i \rangle / 2 - \langle \hat{\xi}_i \rangle \langle \hat{\xi}_j \rangle$, where $\langle \cdots \rangle$ denotes the mean value and $\hat{\xi} = (\hat{x}_1, \hat{p}_1, \hat{x}_2, \hat{p}_2, \hat{x}_3, \hat{p}_3)^T$. The vacuum variance is set to 1 ($\hbar = 2$).