ALMOST BI-HYPERIDEALS AND THEIR FUZZIFICATION
OF SEMIHYPERGROUPS

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Abstract. In this paper, we introduce the concept of almost bi-hyperideals of semihypergroups which is a generalization of bi-hyperideals, and we give some properties of them. Moreover, we consider the connections between almost bi-hyperideals and their fuzzification of semihypergroups.

Keywords: bi-hyperideal; almost bi-hyperideal; fuzzy almost bi-hyperideal; semihypergroup.

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1. INTRODUCTION

The concepts of left, right, two-sided almost ideals of semigroups were introduced by Grosek and Satko [7] in 1980. They studied the characterization of these ideals when a semigroup contains no proper left, right, two-sided ideals. Later in 1981, Bogdanovic [1] introduced the notion of almost bi-ideals in semigroups as a generalization of bi-ideals. The concept of fuzzy subsets was first introduced by Zadeh [18] as a function from a nonempty set $X$ to the unit interval $[0, 1]$. The fuzzy subset theory is a generalization of traditional mathematics set theory. In 2018, Wattanatripop, Chinram and Changphas [17] introduced the notion of fuzzy almost

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bi-ideals in semigroups and discussed some relationships between almost bi-ideals and fuzzy almost bi-ideals of semigroups. Then, Simuen, et al. [14] investigated some properties of fuzzy almost bi-$\Gamma$-ideals of $\Gamma$-semigroups.

Algebraic hyperstructure was introduced in 1934, by Marty [10], as the $8^{th}$ Congress of Scandinavian Mathematicians. In a classical algebraic structure, composition of two elements is an element, while in an algebraic hyperstructure, the composition of two elements is a nonempty set. There are many authors expanded the concept of hyperstructure, see, e.g., [3], [4], [6], [11], [12], [16]. In this work, the authors focus on semihypergroups. Semihypergroups are studied by many authors, for instance, [2], [5], [8], [9], [13]. In 2020, Suebsung, Kaewnoi and Chinram [15] studied the concept of almost hyperideals in semihypergroups and gave some interesting properties.

In this paper, we introduce the concept of almost bi-hyperideals of semihypergroups as a generalization of bi-hyperideals and investigate some properties of them. Then, we discuss the connections between almost bi-hyperideals and fuzzy almost bi-hyperideals of semihypergroups.

2. Preliminaries

Let $H$ be a nonempty set. A hyperoperation on $H$ is a mapping $\circ : H \times H \rightarrow \mathcal{P}^*(H)$, where $\mathcal{P}^*(H)$ denotes the set of all nonempty subsets of $H$. Then, the structure $(H, \circ)$ is called a hypergroupoid. If $A, B \in \mathcal{P}^*(H)$ and $x \in H$, then we denote

$$A \circ B = \bigcup_{a \in A, b \in B} a \circ b, \quad A \circ x = A \circ \{x\} \quad \text{and} \quad x \circ B = \{x\} \circ B.$$ 

A hypergroupoid $(H, \circ)$ is called a semihypergroup if for every $x, y, z \in H$, $(x \circ y) \circ z = x \circ (y \circ z)$, which means that

$$\bigcup_{u \in x \circ y} u \circ z = \bigcup_{v \in y \circ z} x \circ v.$$ 

A nonempty subset $A$ of a semihypergroup $(S, \circ)$ is called a subsemihypergroup of $S$ if $A \circ A \subseteq A$. A subsemihypergroup $B$ of a semihypergroup $(S, \circ)$ is called a bi-hyperideal of $S$ if $B \circ S \circ B \subseteq B$. For more convenient, we write $S$ instead of a semihypergroup $(S, \circ)$ and $AB$ instead of $A \circ B$, for any nonempty subsets $A$ and $B$ of $S$. 
A fuzzy subset \cite{18} of a nonempty set \( X \) is a mapping \( f : X \to [0, 1] \). Let \( f \) and \( g \) be any two fuzzy subsets of a nonempty set \( X \). Then, \( f \subseteq g \) if and only if \( f(x) \leq g(x) \) for all \( x \in X \). The intersection and the union of two fuzzy subsets \( f \) and \( g \) of a nonempty set \( X \), denoted by \( f \cap g \) and \( f \cup g \), respectively, are defined by letting \( x \in X \),

\[
(f \cap g)(x) = \min\{f(x), g(x)\},
\]

\[
(f \cup g)(x) = \max\{f(x), g(x)\}.
\]

Let \( X \) be a nonempty set. For a fuzzy subset \( f \) of \( X \), the support of \( f \) is defined by \( \text{supp}(f) := \{x \in X \mid f(x) \neq 0\} \). Let \( A \) be a nonempty subset of \( X \). The characteristic mapping \( \chi_A \) of \( A \) is a fuzzy subset of \( X \) defined by

\[
\chi_A(x) = \begin{cases} 
1 & \text{if } x \in A, \\
0 & \text{otherwise},
\end{cases}
\]

for all \( x \in X \).

For any element \( s \) of \( X \) and \( t \in (0, 1] \), a fuzzy point \( s_t \) of \( X \) defined by

\[
s_t(x) = \begin{cases} 
t & \text{if } x = s, \\
0 & \text{otherwise},
\end{cases}
\]

for all \( x \in X \).

**Lemma 2.1.** Let \( A \) and \( B \) be nonempty subsets of a nonempty set \( X \) and let \( f \) and \( g \) be fuzzy subsets of \( X \). Then the following statements hold:

(i) \( \chi_{A \cap B} = \chi_A \cap \chi_B \);

(ii) \( A \subseteq B \) if and only if \( \chi_A \subseteq \chi_B \);

(iii) \( \text{supp}(\chi_A) = A \);

(iv) if \( f \subseteq g \), then \( \text{supp}(f) \subseteq \text{supp}(g) \).

**Proof.** The proof is straightforward. \( \square \)

Let \( f \) and \( g \) be fuzzy subsets of a semihypergroup \( S \). A product \( f \circ g \) is defined by

\[
(f \circ g)(x) = \begin{cases} 
\sup_{x \in yz} \{\min\{f(y), g(z)\}\} & \text{if } \exists y, z \in S \text{ such that } x \in yz, \\
0 & \text{otherwise},
\end{cases}
\]
for all $x \in S$.

**Lemma 2.2.** If $A$ and $B$ are subsets of a semihypergroup $S$, then $\chi_A \circ \chi_B = \chi_{AB}$.

**Proof.** Let $x \in S$. If $\chi_{AB}(x) = 0$, then $x \notin AB$. This means that $x \notin ab$ for all $a \in A$ and $b \in B$. Thus, $(\chi_A \circ \chi_B)(x) = 0$. That is, $\chi_{AB}(x) = (\chi_A \circ \chi_B)(x)$. If $\chi_{AB}(x) = 1$, then $x \in AB$. This implies that $x \in ab$ for some $a \in A$ and $b \in B$. Hence, $(\chi_A \circ \chi_B)(x) = \sup_{x \in ab} \{\min\{\chi_A(a), \chi_B(b)\}\} = 1$. So, $\chi_{AB}(x) = (\chi_A \circ \chi_B)(x)$. Therefore, $\chi_A \circ \chi_B = \chi_{AB}$. □

### 3. Almost Bi-Hyperideals

In this section, we introduce the concept of almost bi-hyperideals of semihypergroups and give some of its properties.

**Definition 3.1.** A nonempty subset $B$ of a semihypergroup $S$ is called an almost bi-hyperideal of $S$ if $BxB \cap B \neq \emptyset$ for all $x \in S$.

**Example 3.2.** Let $S = \{a, b, c\}$. Define a hyperoperation $\cdot$ on $S$ by the following table:

| $\cdot$ | $a$ | $b$ | $c$ |
|--------|----|----|----|
| $a$    | $\{a\}$ | $\{b,c\}$ | $\{b,c\}$ |
| $b$    | $\{b,c\}$ | $\{b,c\}$ | $\{b,c\}$ |
| $c$    | $\{c\}$ | $\{c\}$ | $\{c\}$ |

Then, $(S, \cdot)$ is a semihypergroup. Let $B = \{b, c\}$. Hence,

$$BxB \cap B = \{b, c\} \neq \emptyset,$$

$$BbB \cap B = \{b, c\} \neq \emptyset,$$

$$BcB \cap B = \{b, c\} \neq \emptyset.$$

Therefore, $B$ is an almost bi-hyperideal of $S$.

**Proposition 3.3.** Every bi-hyperideal of a semihypergroup $S$ is an almost bi-hyperideal.

**Proof.** Let $B$ be a bi-hyperideal of a semihypergroup $S$. Then, $BSB \subseteq B$. It follows that for any $x \in S$, $BxB \subseteq BSB \subseteq B$. That is, $BxB \cap B = BxB \neq \emptyset$ for all $x \in S$. Hence, $B$ is an almost bi-hyperideal of $S$. □
In general, an almost bi-hyperideal of a semihypergroup need not to be a bi-hyperideal as the following example.

**Example 3.4.** Consider \( S = \{a, b, c\} \) together with the hyperoperation \( \cdot \) on \( S \) defined in Example 3.2. Let \( B = \{a, b\} \). By routine computations, \( B \) is an almost bi-hyperideal of \( S \), but \( B \) is not a bi-hyperideal of \( S \) because \( BSB = S \nsubseteq B \).

Next, we discuss some properties of almost bi-hyperideals of semihypergroups.

**Theorem 3.5.** Let \( B \) be an almost bi-hyperideal of a semihypergroup \( S \). If \( A \) is any subset of \( S \) containing \( B \), then \( A \) is also an almost bi-hyperideal of \( S \).

*Proof.* Assume that \( A \) is a subset of \( S \) containing \( B \). Since \( B \) is an almost bi-hyperideal of \( S \) and \( B \subseteq A \), we have that \( BxB \cap B \neq \emptyset \) and \( BxB \cap B \subseteq AxA \cap A \) for all \( x \in S \). It turns out that \( AxA \cap A \neq \emptyset \) for all \( x \in S \). Hence, \( A \) is an almost bi-hyperideal of \( S \). \( \square \)

**Corollary 3.6.** The union of any two almost bi-hyperideals of a semihypergroup \( S \) is also an almost bi-hyperideal of \( S \).

*Proof.* Let \( A \) and \( B \) be any two almost bi-hyperideals of a semihypergroup \( S \). Since \( A \subseteq A \cup B \) and by Theorem 3.5, we get that \( A \cup B \) is an almost bi-hyperideal of \( S \). \( \square \)

**Example 3.7.** Let \( S = \{a, b, c\} \). Then the hyperoperation \( \cdot \) on \( S \) defined by the following:

| \( \cdot \) | \( a \) | \( b \) | \( c \) |
|---|---|---|---|
| \( a \) | \( \{a\} \) | \( \{b, c\} \) | \( \{c\} \) |
| \( b \) | \( \{b, c\} \) | \( \{b, c\} \) | \( \{c\} \) |
| \( c \) | \( \{b, c\} \) | \( \{b, c\} \) | \( \{c\} \) |

Hence, \((S, \cdot)\) is a semihypergroup. Let \( B_1 = \{a, b\} \) and \( B_2 = \{a, c\} \). By routine calculations, \( B_1 \) and \( B_2 \) are almost bi-hyperideals of \( S \). However, \( B = B_1 \cap B_2 = \{a\} \) is not an almost bi-hyperideal of \( S \) because \( BbB \cap B = \emptyset \).

By Example 3.7, we have that the intersection of any two almost bi-hyperideals of a semihypergroup \( S \) need not to be an almost bi-hyperideal of \( S \).
**Theorem 3.8.** Let $S$ be a semihypergroup. Then $S$ contains a proper almost bi-hyperideal if and only if there exists an element $x$ of $S$ such that $S \setminus \{x\}$ is an almost bi-hyperideal of $S$.

**Proof.** Assume that $S$ contains a proper almost bi-hyperideal. Let $A$ be a proper almost bi-hyperideal of $S$. Then, there exists $x \in S$ such that $x \notin A$. So, $A \subseteq S \setminus \{x\}$. By Theorem 3.5, $S \setminus \{x\}$ is an almost bi-hyperideal of $S$. Conversely, consider $S \setminus \{x\}$ for some $x \in S$. Then, $S \setminus \{x\}$ is a proper subset of $S$. By assumption, we have that $S \setminus \{x\}$ is an almost bi-hyperideal of $S$. Therefore, $S$ contains a proper almost bi-hyperideal. \hfill \Box

**Theorem 3.9.** Let $S$ be a semihypergroup and $|S| > 1$. Then $S$ has no proper almost bi-hyperideals if and only if for every $x \in S$ there exists $a \in S$ such that $(S \setminus \{x\})a(S \setminus \{x\}) = \{x\}$.

**Proof.** Assume that $S$ has no proper almost bi-hyperideals. Let $x \in S$. Then, $S \setminus \{x\}$ is not an almost bi-hyperideal of $S$. Thus, there exists $a \in S$ such that

$$[(S \setminus \{x\})a(S \setminus \{x\})] \cap (S \setminus \{x\}) = \emptyset.$$  

We obtain that

$$(S \setminus \{x\})a(S \setminus \{x\}) \subseteq S \setminus (S \setminus \{x\}) = \{x\}.$$  

This implies that $(S \setminus \{x\})a(S \setminus \{x\}) = \{x\}$.

Conversely, suppose that $S$ contains a proper almost bi-hyperideal $B$. Let $x \in S \setminus B$. By assumption, there exists $a \in S$ such that $(S \setminus \{x\})a(S \setminus \{x\}) = \{x\}$. Since $B \subseteq S \setminus \{x\}$ and by Theorem 3.5, we get that $S \setminus \{x\}$ is an almost bi-hyperideal of $S$. It follows that

$$\emptyset = \{x\} \cap (S \setminus \{x\}) = [(S \setminus \{x\})a(S \setminus \{x\})] \cap (S \setminus \{x\}) \neq \emptyset.$$  

This is a contradiction. Therefore, $S$ has no proper almost bi-hyperideals. \hfill \Box

**4. Fuzzy Almost Bi-Hyperideals**

In this section, we introduce the concept of fuzzy almost bi-hyperideals of semihypergroups, and we study the connections between almost bi-hyperideals and their fuzzification of semihypergroups.
**Definition 4.1.** Let $f$ be a fuzzy subset of a semihypergroup $S$ such that $f \neq 0$. Then $f$ is called a *fuzzy almost bi-hyperideal* of $S$ if for every fuzzy point $s_t$ of $S$, $(f \circ s_t \circ f) \cap f \neq 0$.

**Theorem 4.2.** Let $f$ be a fuzzy almost bi-hyperideal of a semihypergroup $S$. If $g$ is a fuzzy subset of $S$ such that $f \subseteq g$, then $g$ is a fuzzy almost bi-hyperideal of $S$.

*Proof.* Assume that $g$ is a fuzzy subset of $S$ such that $f \subseteq g$. By assumption, $(f \circ s_t \circ f) \cap f \subseteq (g \circ s_t \circ g) \cap g$ and $(f \circ s_t \circ f) \cap f \neq 0$ for all fuzzy points $s_t$ of $S$. It follows that $(g \circ s_t \circ g) \cap g \neq 0$ for all fuzzy points $s_t$ of $S$. Hence, $g$ is a fuzzy almost bi-hyperideal of $S$. □

**Corollary 4.3.** Let $f$ and $g$ be fuzzy almost bi-hyperideals of a semihypergroup $S$. Then $f \cup g$ is also a fuzzy almost bi-hyperideal of $S$.

*Proof.* It follows from Theorem 4.2. □

**Example 4.4.** Consider $S = \{a, b, c\}$ together with the hyperoperation $\cdot$ on $S$ defined in Example 3.7. Let $f$ and $g$ be fuzzy subsets of $S$ defined by

$$f(a) = 0, \ f(b) = 0, \ f(c) = 0.3$$

and

$$g(a) = 0, \ g(b) = 0.7, \ g(c) = 0.$$ 

It not difficult to show that

$$[\ (f \circ s_t \circ f) \cap f](c) \neq 0 \ \text{and} \ \ [\ (g \circ s_t \circ g) \cap g](b) \neq 0$$

for all fuzzy points $s_t$ of $S$. Then, $f$ and $g$ are fuzzy almost bi-hyperideals of $S$. Moreover, for a fuzzy point $a_t$ of $S$, $f \cap g$ is not a fuzzy almost bi-hyperideal of $S$ because $[\ ((f \cap g) \circ a_t \circ (f \cap g)) \cap (f \cap g)](x) = 0$ for all $x \in S$.

**Theorem 4.5.** Let $B$ be a nonempty subset of a semihypergroup $S$. Then $B$ is an almost bi-hyperideal of $S$ if and only if $\chi_B$ is a fuzzy almost bi-hyperideal of $S$.

*Proof.* Assume that $B$ is an almost bi-hyperideal of $S$. Then, $BsB \cap B \neq \emptyset$ for all $s \in S$. So, there exists $x \in S$ such that $x \in BsB$ and $x \in B$. Thus, $x \in b_1b_2$ for some $b_1, b_2 \in B$. It follows that

$$(\chi_B \circ s_t \circ \chi_B)(x) = \sup_{x \in b_1b_2} \{\min\{\chi_B(b_1), s_t(s), \chi_B(b_2)\}\} \neq 0 \ \text{and} \ \chi_B(x) = 1.$$
This implies that \((\chi_B \circ s_t \circ \chi_B) \cap \chi_B \neq 0\) for all fuzzy points \(s_t\) of \(S\). Hence, \(\chi_B\) is a fuzzy almost bi-hyperideal of \(S\).

Conversely, assume that \(\chi_B\) is a fuzzy almost bi-hyperideal of \(S\). Let \(s \in S\). Then, \((\chi_B \circ s_t \circ \chi_B) \cap \chi_B \neq 0\). Thus, there exists \(x \in S\) such that \((\chi_B \circ s_t \circ \chi_B)(x) \neq 0\) and \(\chi_B(x) \neq 0\). Then, there exist \(b_1, b_2 \in B\) such that \(x \in b_1 s b_2\) and \(x \in B\). This means that \(x \in BsB\) and \(x \in B\). That is, \(BsB \cap B \neq \emptyset\). Therefore, \(B\) is an almost bi-hyperideal of \(S\).

\[\square\]

**Theorem 4.6.** Let \(f\) be a fuzzy subset of a semihypergroup \(S\). Then \(f\) is a fuzzy almost bi-hyperideal of \(S\) if and only if \(\text{supp}(f)\) is an almost bi-hyperideal of \(S\).

**Proof.** Assume that \(f\) is a fuzzy almost bi-hyperideal of \(S\). Let \(s \in A\) and \(s_t\) be a fuzzy point of \(S\). Then, \((f \circ s_t \circ f) \cap f \neq 0\). Thus, there exists \(x \in S\) such that \([\{(f \circ s_t \circ f) \cap f\}(x) \neq 0\). That is, \((f \circ s_t \circ f)(x) \neq 0\) and \(f(x) \neq 0\). We obtain that there exist \(y_1, y_2 \in S\) such that \(x \in y_1 s y_2\),

\[0 \neq (f \circ s_t \circ f)(x) = \sup_{x \in y_1 s y_2} \{\min\{f(y_1), s_t(s), f(y_2)\}\} \] 

So, \(f(y_1) \neq 0\) and \(f(y_2) \neq 0\). Hence, \(x, y_1, y_2 \in \text{supp}(f)\). It follows that \((\chi_{\text{supp}(f)} \circ s_t \circ \chi_{\text{supp}(f)})(x) \neq 0\) and \(\chi_{\text{supp}(f)}(x) \neq 0\). Also, \((\chi_{\text{supp}(f)} \circ s_t \circ \chi_{\text{supp}(f)}) \cap \chi_{\text{supp}(f)} \neq 0\). This means that \(\chi_{\text{supp}(f)}\) is a fuzzy almost bi-hyperideal of \(S\). By Theorem 4.5, \(\text{supp}(f)\) is an almost bi-hyperideal of \(S\).

Conversely, assume that \(\text{supp}(f)\) is an almost bi-hyperideal of \(S\). By Theorem 4.5, \(\chi_{\text{supp}(f)}\) is a fuzzy almost bi-hyperideal of \(S\). Let \(s_t\) be any fuzzy point of \(S\). Then, \((\chi_{\text{supp}(f)} \circ s_t \circ \chi_{\text{supp}(f)}) \cap \chi_{\text{supp}(f)} \neq 0\). Thus, there exists \(x \in S\) such that \([\{(\chi_{\text{supp}(f)} \circ s_t \circ \chi_{\text{supp}(f)})(x) \neq 0\) and \(\chi_{\text{supp}(f)}(x) \neq 0\). That is, there exist \(y_1, y_2 \in S\) such that \(x \in y_1 s y_2\),

\[0 \neq (\chi_{\text{supp}(f)} \circ s_t \circ \chi_{\text{supp}(f)})(x) = \sup_{x \in y_1 s y_2} \{\min\{\chi_{\text{supp}(f)}(y_1), s_t(s), \chi_{\text{supp}(f)}(y_2)\}\}, \]

which implies that, \(\chi_{\text{supp}(f)}(y_1) \neq 0\) and \(\chi_{\text{supp}(f)}(y_2) \neq 0\). It turns out that \(f(y_1) \neq 0\), \(f(y_2) \neq 0\) and \(f(x) \neq 0\). Hence, \((f \circ s_t \circ f) \cap f \neq 0\). Consequently, \(f\) is a fuzzy almost bi-hyperideal of \(S\).

\[\square\]

An almost bi-hyperideal \(M\) of a semihypergroup \(S\) is minimal if for any almost bi-hyperideal \(A\) of \(S\) such that \(A \subseteq M\) implies that \(A = M\).
**Definition 4.7.** Let $S$ be a semihypergroup. A fuzzy almost bi-hyperideal $f$ of $S$ is called *minimal* if for any fuzzy almost bi-hyperideal $g$ of $S$ such that $g \subseteq f$, we get $\text{supp}(f) = \text{supp}(g)$.

Next, we investigate the minimality of fuzzy almost bi-hyperideals of semihypergroups.

**Theorem 4.8.** Let $B$ be a nonempty subset of a semihypergroup $S$. Then $B$ is a minimal almost bi-hyperideal of $S$ if and only if $\chi_B$ is a minimal fuzzy almost bi-hyperideal of $S$.

**Proof.** Assume that $B$ is a minimal almost bi-hyperideal of $S$. By Theorem 4.5, $\chi_B$ is a fuzzy almost bi-hyperideal of $S$. Let $g$ be a fuzzy almost bi-hyperideal of $S$ such that $g \subseteq \chi_B$. By Lemma 2.1, $\text{supp}(g) \subseteq \text{supp}(\chi_B) = B$. By Theorem 4.6, $\text{supp}(g)$ is an almost bi-hyperideal of $S$. By the minimality of $B$, we have $\text{supp}(g) = B = \text{supp}(\chi_B)$. Therefore, $\chi_B$ is a minimal fuzzy almost bi-hyperideal of $S$.

Conversely, assume that $\chi_B$ is a minimal fuzzy almost bi-hyperideal of $S$. Then, $B$ is an almost bi-hyperideal of $S$. Let $A$ be any almost bi-hyperideal of $S$ such that $A \subseteq B$. By Theorem 4.5, $\chi_A$ is a fuzzy almost bi-hyperideal of $S$ such that $\chi_A \subseteq \chi_B$. Since $\chi_B$ is minimal, we get that $\text{supp}(\chi_A) = \text{supp}(\chi_B)$. We obtain that $A = \text{supp}(\chi_A) = \text{supp}(\chi_B) = B$ by Lemma 2.1. Consequently, $B$ is a minimal almost bi-hyperideal of $S$. □

**Corollary 4.9.** Let $S$ be a semihypergroup. Then $S$ has no proper almost bi-hyperideals if and only if for every fuzzy almost bi-hyperideal $f$ of $S$, $\text{supp}(f) = S$.

**Proof.** Assume that $S$ has no proper almost bi-hyperideals. Let $f$ be a fuzzy almost bi-hyperideal of $S$. By Theorem 4.6, $\text{supp}(f)$ is an almost bi-hyperideal of $S$. By assumption, we have that $\text{supp}(f) = S$. Conversely, let $B$ be any almost bi-hyperideal of $S$. Then, $\chi_B$ is a fuzzy almost bi-hyperideal of $S$ by Theorem 4.5. It follows that $B = \text{supp}(\chi_B) = S$. This shows that $S$ has no proper almost bi-hyperideals. □

Let $S$ be a semihypergroup. An almost bi-hyperideal $P$ of $S$ is *prime* if for any almost bi-hyperideals $A$ and $B$ of $S$ such that $AB \subseteq P$ implies that $A \subseteq P$ or $B \subseteq P$. An almost bi-hyperideal $P$ of $S$ is *semiprime* if for any almost bi-hyperideal $A$ of $S$ such that $AA \subseteq P$ implies that $A \subseteq P$. An almost bi-hyperideal $P$ of $S$ is *strongly prime* if for any almost bi-hyperideals $A$ and $B$ of $S$ such that $AB \cap BA \subseteq P$ implies that $A \subseteq P$ or $B \subseteq P$. 
Definition 4.10. A fuzzy almost bi-hyperideal $h$ of a semihypergroup $S$ is called a fuzzy prime almost bi-hyperideal of $S$ if for any two fuzzy almost bi-hyperideals $f$ and $g$ of $S$,
\[ f \circ g \subseteq h \text{ implies that } f \subseteq h \text{ or } g \subseteq h. \]

Definition 4.11. A fuzzy almost bi-hyperideal $h$ of a semihypergroup $S$ is called a fuzzy semiprime almost bi-hyperideal of $S$ if for any fuzzy almost bi-hyperideal $f$ of $S$,
\[ f \circ f \subseteq h \text{ implies that } f \subseteq h. \]

Definition 4.12. A fuzzy almost bi-hyperideal $h$ of a semihypergroup $S$ is called a fuzzy strongly prime almost bi-hyperideal of $S$ if for any two fuzzy almost bi-hyperideals $f$ and $g$ of $S$,
\[ (f \circ g) \cap (g \circ f) \subseteq h \text{ implies that } f \subseteq h \text{ or } g \subseteq h. \]

We note that every fuzzy strongly prime almost bi-hyperideal of a semihypergroup is a fuzzy prime almost bi-hyperideal, and every fuzzy prime almost bi-hyperideal of a semihypergroup is a fuzzy semiprime almost bi-hyperideal, but the converse is not true in general.

Finally, we study the relationships between prime (resp., semiprime, strongly prime) almost bi-hyperideals and their fuzzification of semihypergroups.

Theorem 4.13. Let $P$ be a nonempty subset of a semihypergroup $S$. Then $P$ is a prime almost bi-hyperideal of $S$ if and only if $\chi_P$ is a fuzzy prime almost bi-hyperideal of $S$.

Proof. Assume that $P$ is a prime almost bi-hyperideal of $S$. By Theorem 4.5, $\chi_P$ is a fuzzy almost hyperideal of $S$. Let $f$ and $g$ be fuzzy almost bi-hyperideals of $S$ such that $f \circ g \subseteq \chi_P$. Suppose that $f \not\subseteq \chi_P$ and $g \not\subseteq \chi_P$. Then, there exist $x, y \in S$ such that $f(x) \neq 0$ and $g(y) \neq 0$, but $\chi_P(x) = 0$ and $\chi_P(y) = 0$. It follows that $x \not\in P$ and $y \not\in P$. Since $f(x) \neq 0$ and $g(y) \neq 0$, we get that $x \in supp(f)$ and $y \in supp(g)$. Thus, $supp(f) \not\subseteq P$ and $supp(g) \not\subseteq P$. By Theorem 4.6, we have that $supp(f)$ and $supp(g)$ are almost bi-hyperideals of $S$. Since $P$ is prime, $supp(f)supp(g) \not\subseteq P$. Then, there exists $t \in ab$ for some $a \in supp(f)$ and $b \in supp(g)$ such that $t \not\in P$. So, $\chi_P(t) = 0$, and then $(f \circ g)(t) = 0$ because $f \circ g \subseteq \chi_P$. Since $a \in supp(f)$ and $b \in supp(g)$, we have that $f(a) \neq 0$ and $g(b) \neq 0$. Hence, $\min\{f(a), g(b)\} \neq 0$, which implies that $(f \circ g)(t) = \sup_{t \in ab} \{\min\{f(a), g(b)\}\} \neq 0$. This is a contradiction to the fact that $(f \circ g)(t) = 0$. Therefore, $f \subseteq \chi_P$ or $g \subseteq \chi_P$. Consequently, $\chi_P$ is a fuzzy prime almost bi-hyperideal of $S$. 
Conversely, assume that $\chi_P$ is a fuzzy prime almost bi-hyperideal of $S$. By Theorem 4.5, $P$ is an almost bi-hyperideal of $S$. Let $A$ and $B$ be any two almost bi-hyperideals of $S$ such that $AB \subseteq P$. By Lemma 2.1 and Lemma 2.2, we get that $\chi_A \circ \chi_B = \chi_{AB} \subseteq \chi_P$. Again by Theorem 4.5, $\chi_A$ and $\chi_B$ are fuzzy almost bi-hyperideals of $S$. By hypothesis, $\chi_A \subseteq \chi_P$ or $\chi_B \subseteq \chi_P$. That is, $A \subseteq P$ or $B \subseteq P$. Hence, $P$ is a prime almost bi-hyperideal of $S$.

The proof of the following theorem is similar to Theorem 4.13.

**Theorem 4.14.** Let $P$ be a nonempty subset of a semihypergroup $S$. Then $P$ is a semiprime almost bi-hyperideal of $S$ if and only if $\chi_P$ is a fuzzy semiprime almost bi-hyperideal of $S$.

**Theorem 4.15.** Let $P$ be a nonempty subset of a semihypergroup $S$. Then $P$ is a strongly prime almost bi-hyperideal of $S$ if and only if $\chi_P$ is a fuzzy strongly prime almost bi-hyperideal of $S$.

**Proof.** Assume that $P$ is a strongly prime almost bi-hyperideal of $S$. By Theorem 4.5, $\chi_P$ is a fuzzy almost bi-hyperideal of $S$. Let $f$ and $g$ be any two fuzzy almost bi-hyperideals of $S$ such that $(f \circ g) \cap (g \circ f) \subseteq \chi_P$. Suppose that $f \not\subseteq \chi_P$ and $g \not\subseteq \chi_P$. Then, there exist $x, y \in S$ such that $f(x) \neq 0$ and $g(y) \neq 0$, but $\chi_P(x) = 0$ and $\chi_P(y) = 0$. So, $x \in supp(f)$ and $y \in supp(g)$ such that $x \not\in P$ and $y \not\in P$. It follows that $supp(f) \not\subseteq P$ and $supp(g) \not\subseteq P$. By Theorem 4.6 and the hypothesis, we have that $[supp(f)supp(g)] \cap [supp(g)supp(f)] \not\subseteq P$. Hence, there exists $t \in [supp(f)supp(g)] \cap [supp(g)supp(f)]$ such that $t \not\in P$. Also, $\chi_P(t) = 0$, and then $[(f \circ g) \cap (g \circ f)](t) = 0$ because $(f \circ g) \cap (g \circ f) \subseteq \chi_P$. Since $t \in supp(f)supp(g)$ and $t \in supp(g)supp(f)$, we have that $t \in a_1b_1$ and $t \in b_2a_2$ for some $a_1, a_2 \in supp(f)$ and $b_1, b_2 \in supp(g)$. It turns out that

$$(f \circ g)(t) = \sup_{t \in a_1b_1} \{\min\{f(a_1), g(b_1)\}\} \neq 0 \quad \text{and} \quad (g \circ f)(t) = \sup_{t \in b_2a_2} \{\min\{g(b_2), f(a_2)\}\} \neq 0.$$

This implies that $\min\{(f \circ g)(t), (g \circ f)(t)\} \neq 0$, that is, $[(f \circ g) \cap (g \circ f)](t) \neq 0$. This is a contradiction with the fact that $[(f \circ g) \cap (g \circ f)](t) = 0$. Hence, $f \subseteq \chi_P$ or $g \subseteq \chi_P$. Therefore, $\chi_P$ is a fuzzy strongly prime almost bi-hyperideal of $S$.

Conversely, assume that $\chi_P$ is a fuzzy strongly prime almost bi-hyperideal of $S$. Then, $P$ is an almost bi-hyperideal of $S$ by Theorem 4.5. Let $A$ and $B$ be any two almost bi-hyperideals of $S$ such that $(AB) \cap (BA) \subseteq P$. By Theorem 4.5, $\chi_A$ and $\chi_B$ are fuzzy almost bi-hyperideals of $S$. 
By Lemma 2.2, $\chi_{AB} = \chi_A \circ \chi_B$ and $\chi_{BA} = \chi_B \circ \chi_A$. By Lemma 2.1, we have that $(\chi_A \circ \chi_B) \cap (\chi_B \circ \chi_A) = \chi_{AB} \cap \chi_{BA} = \chi_{(AB)\cap(BA)} \subseteq \chi_P$. By assumption, we get that $\chi_A \subseteq \chi_P$ or $\chi_B \subseteq \chi_P$ implies that $A \subseteq P$ or $B \subseteq P$. Consequently, $P$ is a strongly prime almost bi-hyperideal of $S$. □

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CONFLICT OF INTERESTS

The author(s) declare that there is no conflict of interests.

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