Competition between Two Kinds of Correlations in Literary Texts

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A theory of additive Markov chains with long-range memory is used for description of correlation properties of coarse-grained literary texts. The complex structure of the correlations in texts is revealed. Antipersistent correlations at small distances, \( L \lesssim 300 \), and persistent ones at \( L \gtrsim 300 \) define this nontrivial structure. For some concrete examples of literary texts, the memory functions are obtained and their power-law behavior at long distances is disclosed. This property is shown to be a cause of self-similarity of texts with respect to the decimation procedure.

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INTRODUCTION

The problem of long-range correlated stochastic dynamic systems (LRSCS) has been under study for a long time in many areas of contemporary physics [1, 2, 5, 7, 8, 9, 11, 15], biology [5, 8, 9, 10, 11, 12], economics [8, 11, 12, 14, etc. 8, 11]. An important examples of complex LRSCS are literary texts [10, 15, 16, 18, 19, 20].

One of the ways to get a correct insight into the nature of correlations in a symbolic system consists in constructing a mathematical object (for example, a correlated sequence of symbols) possessing the same statistical properties as the initial dynamic system. There exist many algorithms for generating long-range correlated sequences: the inverse Fourier transformation [13, 21], the expansion-modification Li method [22], the Voss procedure of consequent random additions [23], the correlated Levy walks [24], etc. [13]. We believe that, of the above-mentioned methods, the use of the many-step Markov chains is one of the most important because it allows constructing random sequences with prescribed correlation properties in the most natural way. This was demonstrated in Ref. [25], where the Markov chains with the step-like memory function (MF) were studied. It was shown that there exist some dynamical systems (coarse-grained sequences of the Eukarya’s DNA and dictionaries) with correlation properties that can be properly described by this model.

The many-step Markov chain is the sequence of symbols of some alphabet constructed using a conditional probability function, which determines the probability of occurring some definite symbol of sequence depending on \( N \) previous ones. The property of additivity of Markov chain means the independent influence of different previous symbols on generated one. The concept of additivity, primarily introduced in paper [20], was later generalized for the case of binary non-stationary Markov chains [26]. Another generalization was based on consideration of Markov sequences with a many-valued alphabet [27, 28].

The efficient method for investigating into the LRSCS systems consists in decomposing the space of states into a finite number of parts labelled by definite symbols, which are naturally ordered according to the dynamics of the system. The most frequently used decomposition procedure is based on the introduction of two parts of the phase space. In other words, the approach presupposes mapping two kinds of states into two symbols, say 0 and 1. This procedure is often referred to as coarse graining. Thus, the problem is reduced to investigating the statistical properties of binary sequences.

It might be thought that the coarse graining could result in losing, at least, the short-range memory in the sequence. The authors of Ref. [25] argued that the mapping of a given sequence into a small-alphabet sequence does not necessarily imply that the long-range correlations presented in the initial text would be preserved. However, as was shown in Ref. [20], the statistical properties of coarse-grained texts depend, but not significantly, on the kind of mapping. This implies that only the small part of all possible kinds of mapping can destroy the initial correlations in the system. Below, we demonstrate that the coarse graining retains, although not completely, the correlations at all distances. This means that there is no point in coding every symbol (associating every part of the phase space of the system with its binary code) to analyze the correlation properties of the dynamic systems, as it is done, for example, in Ref. [18], but it is sufficient to use the coarse-graining procedure.

In the present work, we study the coarse-grained literary texts examining them as additive Markov chains. A recently obtained equation [24] connecting mutually-complementary characteristics of these sequence, the memory and correlation functions, is used. Once the
memory function of the original sequence is found from the analysis of the correlation function, we construct the corresponding Markov chain with the same statistical properties. This method for constructing the sequence of elements with a given correlation function seems to be very important for other applications, e.g., it can be employed to fabricate the effective filters of electrical or optical signals.

We show that the memory function of any coarse-grained literary text is characterized by a complex structure because of the competition between two kinds of correlations. One type of correlations works at short distances, $L \lesssim 300$. The corresponding MF is negative, which reflects the anti-persistent nature of such correlations. Other type of correlations with the positive memory function acts at long distances, $L \gtrsim 300$. The strength of these persistent correlations decreases as a power-law function. We demonstrate that the power-law decrease of the memory function results in the self-similarity phenomenon in the coarse-grained texts with respect to the decimation procedure.

The paper is organized as follows. In the next Section, we introduce some general relations for the additive Markov chains and present an equation connecting the correlation and memory functions. Section III contains the application of the concept of additive Markov chains to literary works. In Conclusion, we summarize the obtained results.

**MATHEMATICAL MODEL**

**Markov Processes**

Let us consider a homogeneous binary sequence of symbols, $a_i = \{0, 1\}$. To determine the $N$-step Markov chain we have to introduce the conditional probability $P(a_i \mid a_{i-N}, a_{i-N+1}, \ldots, a_{i-1})$ of occurring the definite symbol $a_i$ (for example, $a_i = 1$) after $N$-word $T_{N,i}$, where $T_{N,i}$ stands for the sequence of symbols $a_{i-N}, a_{i-N+1}, \ldots, a_{i-1}$. Thus, it is necessary to define $2^N$ values of the $P$-function corresponding to each possible configuration of the symbols $a_i$ in $N$-word $T_{N,i}$. Since we will apply our theory to the sequences with long memory lengths of the order of $10^6$, some special restrictions to the class of $P$-functions should be imposed. We consider the MF of the additive form,

$$P(a_i = 1 \mid T_{N,i}) = \sum_{r=1}^{N} f(a_{i-r}, r). \quad (1)$$

Here the function $f(a_{i-r}, r)$ describes the additive contribution of the symbol $a_{i-r}$ to the conditional probability of occurring the symbol unity, $a_i = 1$, at the $i$th site. The homogeneity of the Markov chain is provided by independence of the conditional probability $f$ of the index $i$. It is possible to consider Eq. (1) as the first term in expansion of conditional probability in the formal series, where each term corresponds to the additive (unary), binary, ternary, and so on functions up to the $N$-ary one.

Let us rewrite Eq. (1) in an equivalent form,

$$P(a_i = 1 \mid T_{N,i}) = \bar{a} + \sum_{r=1}^{N} F(r)(a_{i-r} - \bar{a}), \quad (2)$$

with

$$\bar{a} = \frac{\sum_{r=1}^{N} f(0, r)}{[1 - \sum_{r=1}^{N} (f(1, r) - f(0, r))]}$$

and

$$F(r) = f(1, r) - f(0, r).$$

We refer to $F(r)$ as the memory function (MF). It describes the strength of influence of previous symbol $a_{i-r}$ ($r = 1, \ldots, N$) upon a generated one, $a_i$. It can be shown that $\bar{a}$ coincides with the value of $a_i$ averaged over the whole sequence. To the best of our knowledge, basically the concept of the memory function for many-step Markov chains was originally used in Refs. 20, 22, where they are well suited to describe the LRSCS.

The memory function $F(r)$ contains complete information about the correlation properties of the Markov chain. Usually, the correlation function and other moments are employed as the input characteristics describing the correlated random systems. However, the correlation function describes not only the direct connection of elements $a_i$ and $a_{i+r}$, but also takes into account their indirect interaction via other intermediate elements. Our approach operates with the "origin" characteristics of the system, specifically with the memory function. This allows one to disclose the fundamental intrinsic properties of the system which provide the correlations between the elements.

A sequence of symbols in a Markov chain can be thought of as the sequence of states of some particle, which participates in a correlated Brownian motion. Thus, every $L$-word (a set of consequent symbols of the length $L$) can be considered as one of the realizations of the ensemble of correlated Brownian trajectories in the "time" interval $L$. The positive values of the MF result in persistent diffusion where previous displacements of the Brownian particle in some direction provoke its consequence displacement in the same direction. The negative values of the MF correspond to the antipersistent diffusion where the changes in the direction of motion are more probable. Another physical system, the Ising chain of spins with long-range interactions, could also be associated with the Markov sequence for which the positive
values of the MF correspond to the attraction of spins whereas the negative ones conform to the repulsion.

Below we will use some more statistical characteristics of the random sequences. We consider the distribution $W_L(k)$ of the words of definite length $L$ by the number $k$ of unities in them, $k_i(L) = \sum_{l=1}^{L} a_{i+l}$, and the variance $D(L)$,

$$D(L) = \langle k - \bar{k} \rangle^2, \quad (3)$$

where the average $\overline{g(k)}$ is defined as $\overline{g(k)} = \sum_{k=0}^{L} g(k) W_L(k)$. Another important value is the correlation function,

$$K(r) = \overline{a_i a_{i+r}} - \bar{a}^2, \quad K(0) = \bar{a} (1 - \bar{a}). \quad (4)$$

By definition, the correlation function is even, $K(r) = K(|r|)$. It is connected with the above mentioned variance by the equation [2],

$$K(r) = \frac{1}{2} (D(r-1) - 2D(r) + D(r+1)), \quad (5)$$

or

$$K(r) = \frac{1}{2} \frac{d^2 D(r)}{dr^2} \quad (6)$$

in the continuous limit.

The memory function used in Refs. [20, 21] is characterized by a step-like behavior and is defined by two parameters only: the memory depth $N$ and the strength $f$ of symbol's correlations. The value of $f$ was assumed to be independent of the distance $r$ between the symbols at $r < N$. This memory function was employed to describe the long-range persistent properties of the coarse-grained literary texts, specifically, the super-linear dependence of the variance $D(L)$. However, it does not reflect the antipersistent behavior of $D(L)$ (observed in Refs. [22]) at short distances. Obviously, we need a more complex memory function for detailed description of the both short-range and long-range properties of the coarse-grained texts.

**Equation for the memory function**

We suggest two methods for finding the memory function $F(r)$ of the Markov chain $a_i$ that possess the same correlation function as a given random sequence $b_i$. The first one is based on the minimization of a "distance", $Dist$, between the Markov chain generated by means of a sought-for MF and the initial sequence $b_i$. This distance is determined by a formula,

$$Dist = (b_i - P(b_i = 1 \mid T_{N,i}))^2 \quad (7)$$

with $P$-function [2]. Equating the variational derivative $\delta Dist/\delta F(r)$ to zero, we get the following relation between the memory function $F(r)$ and the correlation one $K(r)$:

$$K(r) = \sum_{r'=1}^{N} F(r') K(r - r'), \quad r \geq 1. \quad (8)$$

Equation (8) can also be obtained by a straightforward calculation of expression $a_i a_{i+r}$ in Eq. (4) using definition $P$ of the memory function.

The second method resulting from the first one establishes a relationship between the memory function $F(r)$ and the variance $D(L)$,

$$M(r, 0) = \sum_{r'=1}^{N} F(r') M(r, r'), \quad r \geq 1, \quad (9)$$

$$M(r, r') = D(r - r') - (D(-r') + r[D(-r' + 1) - D(-r')]).$$

It is a set of linear equations for $F(r)$ with coefficients $M(r, r')$ determined by $D(r)$. Equations (5) and $D(-r) = D(r)$ are used here.

The function $K(r)$, being a second derivative of $D(r)$, is less manageable and robust in computer simulations. It is the reason why we prefer to use the second method $[2]$. This is our instrument for finding the memory function $F(r)$ of a sequence using the known variance $D(L)$. The robustness of the method in the numerical simulations was demonstrated in Ref. [23].

**LITERATURE TEXTS VIEWED AS THE MARKOV CHAINS**

Variance and correlation function

Let us apply our method to the investigation into the correlation properties of the coarse-grained literary texts. At the outset, we examine the variance $D(L)$ of the coarse-grained text of the King James Version of the Bible [31]. The result of the numerical simulation is presented by solid line in Fig. 1. The straight dotted line describes the variance $D_0(L) = L \bar{a} (1 - \bar{a})$, which corresponds to the non-correlated biased Brownian diffusion. One of the typical coarse-graining procedure was used for mapping the letters of the text onto the symbols zero and unity, ($(a - m) \rightarrow 0, (n - z) \rightarrow 1$). It is clearly seen that the diffusion is antipersistent at small distances, $L \leq 300$, (see inset) whereas it is persistent at long distances $[32]$. The deviation of the solid line from the dotted one testifies to the existence of the correlations in the text of the Bible. To confirm this statement we break down the original text into subsequences of a given length $L_0 = 3000$ and randomly shuffle them. The results from the calculation of the variance for the coarse-grained initial and
The values of $L$ coarse-grained text of the Bible we perform the numerical process being studied. To verify the stationarity of the coherence of these curves proves the robustness of our method of the MF reconstruction. The dotted straight line describes the non-correlated Brownian diffusion, $D_0(l) = L\bar{a}(1 - \bar{a})$. The inset demonstrates the antipersistent dependence of the dimensionless ratio $D(L)/D_0(L)$ upon $L$ at short distances.

The shuffled texts of the Bible are given in Fig. 2. For $L \ll L_0$, the difference in $D(L)$ is negligible [33]. At $L \sim L_0$, the variance and correlation function of the shuffled sequence are less than the original ones. At $L > L_0$, the correlations in the shuffled text vanish. In this region, the variance $D(L)$ is a linear function, and the correlation function being the second derivative of variance equals to zero.

It is easy to show that the correlation function of the shuffled sequence can be written as,

$$K(r) = \begin{cases} K_0(r)(1 - \frac{r}{L_0}), & r < L_0, \\ 0, & r \geq L_0, \end{cases}$$

where $K_0(r)$ is the correlation function of the original non-shuffled sequence. The corresponding variance obtained by double numeric integration (see Eq. 10) of the function $K(r)$ given by Eq. 11 is shown in Fig. 2 by solid line.

Along with the global characteristic $D(L)$, it is interesting to study its local analogue,

$$D_l(L) = \langle (k - < k >_{L_0})^2 >_{L_0},$$

where $L_0$ is the interval of local averaging and $l$ is the coordinate of the left border of this interval. An existence of a trend in the dependence $D_l(L)$ on $l$ would be clearly indicative of non-stationarity of the stochastic process being studied. To verify the stationarity of the coarse-grained text of the Bible we perform the numerical simulation of the $D_l(L)$ dependence on $l$ at different fixed values of $L$. As an example, the result of this simulation for $L = 10$ is shown in Fig. 3. It is clearly seen that there exist regular fluctuations without a pronounced trend. The fluctuations result from the finiteness of interval $L_0$ of averaging. This fact allows us to make a conclusion about stationarity of the coarse-grained text of the Bible. The similar analysis of many other texts gave the same result. It is expedient to study the global characteristics $D(L)$, Eq. 10, of the sequence instead of the local one, $D_l(L)$.

### Memory function

According to Eqs. 10 and 11, the memory function can be restored using the variance or the correlation func-
tion. The MF thus obtained for the coarse-grained text of the Bible at \( r < 300 \) is given in Fig. 4. At long distances, \( r > 300 \), the memory function can be nicely approximated by the power function \( F(r) = 0.27r^{-1.1} \), which is shown by the solid line in the inset in Fig. 4. Note that the persistent part of the MF, \( F(r > 300) \leq 0.0008 \), is much less than its typical magnitude 0.02 in the antipersistent region \( r < 40 \).

![FIG. 4: The memory function \( F(r) \) for the coarse-grained text of the Bible at short distances. The power-law decreasing portion of the \( F(r) \) plot for the Bible is presented by filled circles in the inset. The solid line corresponds to the power-law fitting.](image)

It should be emphasized that the short-range part of the memory function at \( r \lesssim 40 \), as well as the \( D(L) \) function at \( L \lesssim 300 \), is essentially dependent on the method of coarse-graining. Nevertheless, the antipersistent correlations exist for practically all kinds of the coarse-graining procedure. An interesting feature is that the region \( r \lesssim 40 \) of negative antipersistent memory function provides much longer distances \( L \sim 300 \) of antipersistent behavior of the variance \( D(L) \).

In order to prove the universal character of the power-law decrease of the memory function at long distances, we compare the MF of the coarse-grained texts for more than fifty different literary works. The texts are coarse-grained by mapping the letters from the first and second halves of the alphabet into zero and unity, respectively. Subsequently, using Eq. 4, we first calculate the variances and then the memory functions. All curves for the memory functions can be well fitted by the power-law functions \( F(r) = cr^{-b} \). The results of the fitting for eight texts written or translated into Russian \( \{2, 3, 4, 5, 6, 7, 8\} \) are shown in Fig. 5. The exponents in all curves vary over the interval between \( b_{\text{min}} = 1.02 \) for “War and Peace” and \( b_{\text{max}} = 1.56 \) for the Koran. Thus, the constants \( c \) and \( b \) can be used for linguistic classification of different literary works. It is interesting to see that the memory functions for the texts of the English- and Russian-worded Bible, as well as the texts of the Old and New Testaments are practically coincident.

![FIG. 5: The memory function at long distances for the coarse-grained texts of eight literary works: 1. The Bible, 2. “Oliver Twist” by Charles Dickens, 3. “War and Peace” by Leo Tolstoy, 4. The Tora, 5. “Master and Margarita” by Mikhail Bulgakov, 6. “Don Quixote” by Miguel de Servantes, 7. “Oblomov” by Ivan Goncharov, 8. The Koran.](image)

The existence of two characteristic regions having different behavior of the memory function and, correspondingly, of the persistent and antipersistent portions in the \( D(L) \) dependence appears to be a prominent feature of all texts in any language. Note that the antipersistent portion of the memory function corresponds to the region where the grammatical rules are in use. Therefore, we call this kind of correlations the “grammatical” ones. The persistent correlations in a text at very long distances can be related to a general idea of the literary work. Thus, this kind of correlations is referred to as the “semantic” ones.

Two fundamentally different portions in the MF plots result from a peculiar competition between the two above-mentioned kinds of correlations. We would like to stress that both portions of the MF are equally important to gain an insight into the correlation properties of the literary texts. To support this statement we generate two special sequences. In both of them, only one kind of the memory function for the coarse-grained text of the Bible is taken into account, and the memory function in another region is assumed to be zero. The variance \( D(L) \) for these two sequences is given in Fig. 6. The lower (dashed) line corresponds to the case where only the negative antipersistent portion, \( r < 40 \), of the memory function is allowed for. The upper (dash-dot-dotted) curve corresponds to the sequence, which is generated by means of the long-range persistent memory, \( F(r) = 0.27r^{-1.1}, r > 100 \). It is evident that the generated sequence with the antipersistent memory function displays the sub-diffusion only, whereas the sequence that corresponds to the persistent memory function is characterized by the super-diffusion behavior of the variance.
$D(L)$. The difference between the variances for two generated sequences and for the original coarse-grained text of the Bible, shown by the solid line in the same figure, corroborates our assumption about the significance of both kinds of the memory function.

![FIG. 6: The variance $D(L)$ for the coarse-grained text of the Bible (solid line), and for the sequences constructed with using the persistent part of the MF (dash-dot-dotted line) and the antipersistent one (dashed line). The dotted line describes the non-correlated Brownian diffusion, $D_0(L) = L\bar{a}(1 - \bar{a})$.](image)

**Self-similarity of the coarse-grained texts**

The power-law decrease (without characteristic scale) of the memory function at long distances leads to quite an essential property of self-similarity of the coarse-grained texts with respect to the decimation procedure discussed in Ref. 27. This procedure implies the deterministic or random removal of some part of symbols from a sequence and is characterized by the decimation parameter $\lambda < 1$ which represents the fraction of symbols kept in the chain. For example, under the random decimation each symbol is eliminated with probability $1 - \lambda$. It can be shown that both of these procedures, deterministic and stochastic, are equivalent for a Markov chain. The sequence is self-similar if its variance $D(L)$ does not change after the decimation up to a definite value of $L$ (which is dependent on the memory length of the original sequence and the decimation parameter). The model of the additive binary many-step Markov chain with the step-like MF (which was discussed in Ref. 27) offers the exact property of self-similarity at the length shorter than the memory length $N$. The coarse-grained literary texts have the self-similarity property as well. It is indicated in Fig. 4 where three $D(L)$ curves correspond to different values of the parameters of the regular decimation. Note that the decimation procedure leads to a decrease in the effective memory length. As a result, the variance curves coincide up to the effective memory depth, which is proportional to the decimation parameter. A similar phenomenon occurs in the case of random decimation as well.

![FIG. 7: Numerically calculated variance $D(L)$ for the coarse-grained text of the Bible (solid line) and for the sequences obtained after their regular decimation. Circles, triangles, and dots correspond to the decimation parameters 2, 4, and 8, respectively. The dotted line describes the non-correlated Brownian diffusion, $D_0(L) = L\bar{a}(1 - \bar{a})$. The similar curves obtained for the sequence constructed by using the long-range part of Bible’s memory function only are shown in the inset.](image)

A question arises: what particular property of the memory function is crucial for the self-similarity of the coarse-grained literary texts. It is natural to assume that the persistent long-range scale-free portion of the memory function affords this property because the self-similarity is specifically manifest at long distances. To verify this supposition we carry out the decimation procedure with different $\lambda$ for the Markov chain constructed by using the long-range part of the Bible memory function only and then plot the correspondent $D(L)$ dependence. The curves are shown in the inset in Fig. 7. It is seen that the property of self-similarity for this sequence appears to be much more pronounced than for the original coarse-grained text of the Bible. Moreover, the antipersistent part of the MF disappears very fast after the decimation procedure. This is clearly observed as a disappearance of the antipersistent sub-linear portion of the $D(L)$ curves in Fig. 7 where after decimation the solid line transforms into the wholly persistent super-linear curve, which goes above the curve $D_0 = L\bar{a}(1 - \bar{a})$. The conclusion about the invariance of the statistical properties of studied sequence with respect to the decimation procedure is an additional argument in favor of coarse-graining efficiency. The decimation can be considered as additional coarse-graining of the initial random sequence.
CONCLUSION

Thus, we have demonstrated that the description of the literary works is suitable in terms of the Markov chains with complex memory functions. Actually, the memory function appears to be a convenient informative "visiting card" of any symbolic stochastic process. We have studied the coarse-grained literary texts and shown the complexity of their organization in contrast to a previously discussed simple power-law decrease of correlations. We have proved that the competition between the two kinds of correlations govern the statistical properties of the coarse-grained texts. The antipersistent correlations exist at short distances, $L \lesssim 300$, in the region of grammatical rules efficiency. Another kind of correlations, persistent one, plays the main role at long distances, $L \gtrsim 300$. It can be related to the general idea of a literary work. Therefore, the first kind of correlations may be referred to as the grammatical one, whereas the second kind may be named as semantic correlations. However, the nature of the correlations should be clarified by linguists.

If our supposition about the nature of both kinds of correlations in the literary texts is correct, several important questions will be of great interest, e.g.:

- Does the lack of the antipersistent portion in the memory function (and in the $D(L)$ dependence [20]) in the DNA texts mean that the "grammatical rules" are absent in the "DNA language"?

- If we consider the variance $D(L)$ as a measure of information redundancy, can we explain the equality $D(L)_{DNA} \simeq 10 \cdot D(L)_{Text}$ resulting from the comparison between literary and DNA texts at $L \sim 3 \times 10^5$ [20] in the following way: the Nature is more careful about the conservation of the information stored in the DNA sequences than the Writer in his literary works?

We have examined the simplest examples of random sequences, the dichotomic one. However, our preliminary consideration shows that the presented concept of additive Markov chains can by generalized to a larger class of random Markov processes with the finite or infinite number of states in the discrete or continuous "time". The suggested approach can be used for the analysis of other correlated systems in different fields of science.

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