Performance Analysis of Joint Transmission Schemes in Ultra-Dense Networks – A Unified Approach

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Abstract—Ultra-dense network (UDN) is one of the enabling technologies in the fifth generation (5G) wireless communications, and the application of joint transmission (JT) is extremely important to deal with severe inter-cell interferences in UDNs. However, most of the current works done in performance analysis on JT schemes in the literature were based largely on simulation results due to the difficulties in quantitatively identifying the number of desirable and interfering transmitters in a UDN setup. In this work, we are motivated to propose an analytical approach to investigate the performance of JT schemes with a unified approach based on stochastic geometry, which is in particular useful for studying different JT schemes as well as conventional transmission schemes without JT. Using the proposed approach, we can unveil the statistical characteristics (i.e., expectation, moment generation function, and variance) of desirable signal and interference powers of a given user equipment (UE), and thus the system performances, such as average signal-to-interference-plus-noise ratio (SINR) and area spectral efficiency, can be evaluated analytically. The simulation results are illustrated to verify the effectiveness of the proposed approach.

Index Terms—Ultra-dense network, joint transmission, stochastic geometry, area spectral efficiency.

I. INTRODUCTION

A FAST-GROWING demand for high-speed wireless communications has exerted a great pressure on the existing wireless communication systems, and thus the fifth generation (5G) communications have started to be deployed in some countries in the world to meet the demand. Downlink peak data rate and downlink user experienced data rate in the 5G communication systems can go up as high as 20 Gbps and 100 Mbps, respectively, as specified by the ITU Focus Group of IMT 2020 [1]. In order to meet such a requirement, the current wireless systems should be improved in terms of three aspects, i.e., spectral efficiency enhancement, the use of new spectrum resources, and area reuse factor improvement [2].

The emerging ultra-dense network (UDN) has been viewed as one of the key enabling technologies to improve the area reuse factor. The UDN is a network topology, where different types of small-cell base stations (SBSs) are deployed densely within the coverage of a macro-cell base station (MBS) [3]. The density of SBSs can be even higher than that of user equipments (UEs) [4]. For example, the density of SBSs can be as high as 103/km2, while the density of UEs in an urban scenario is less than 600/km2 [5]. The deployment of UDNs can effectively offload the traffic from MBSs to meet the communication requirements in service-intensive areas, such as apartments, enterprises, and downtown areas. It has been shown that with a cell size of less than 10 m2, the capacity target of 1 Gb/s/m2 can be achieved in a 5G system with the help of UDNs [6].

However, the inter-cell interference may significantly impair UDN performance due to very short distances amongst adjacent cells. If the density of small cells goes high beyond a certain threshold, network throughput starts to decline and may approach to zero, as shown in the literatures [7], [8]. In order to mitigate the inter-cell interferences, coordinated multipoint (CoMP) transmission schemes were proposed to explore the cooperation among adjacent cells to mitigate the interferences [9] [10]. In general, there are three different types of downlink CoMP techniques, i.e., coordinated beamforming (CB), coordinated scheduling (CS), and joint transmission (JT) [11]. In the CB scheme, beamforming schemes should be designed jointly among cooperative SBSs [12]. In the CS scheme, time and frequency resources are allocated jointly to mitigate inter-cell interference [13]. In the JT scheme, with the help of space-time coding the cooperative SBSs are scheduled to use the same resource block to transmit same messages to a UE simultaneously, and thus the inter-cell interference can be reduced. A systematic analysis on JT can be found in Chapter 6 in [14]. If band-pass signals from cooperative SBSs are aligned at an intended destination, this transmission scheme is called coherent JT [15], [16]. Even though the implementation complexity of phase synchronization in the coherent JT may be high, multiple solutions were proposed...
to make the application of coherent JT possible, such as the use of stable GPS-referenced local oscillators or a two-way open-loop carrier synchronization scheme [14]. If a perfect synchronization cannot be achieved, the synchronization error in frequency domain can be estimated and compensated [17], [18]. In this paper, we will focus only on the performance analysis of the coherent JT in UDNs.

Stochastic geometry has been widely used to analyze large-scale wireless networks, and numerous stochastic geometry models were developed based on different network topologies or application scenarios [19], [20]. In [21], ergodic capacity was calculated for $L$-branch coherent diversity combiners, and a unified moment generation function (MGF) of the envelope was derived in a Gamma-shadowed generalized Nakagami-$m$ fading channel. Outage probability was acquired based on the MGF of interferences under Nakagami-Lognormal fading and time-shared shadowed/unshadowed fading channels [22]. A closed-form MGF expression of SNR was derived in [23] in Beckmann fading channels. In addition to the MGF, the other performance metrics, such as outage probability and link capacity, can also be obtained with the help of incomplete MGF (IMGF) [24]. In [25], the authors summarized mathematical theorems, such as Campbell-Mecke theorem, Campbell’s theorem, and probability generating functional (PGF), to calculate different stochastic properties of received signals in a homogeneous network, where base stations are distributed following a Poisson point process (PPP). The authors extended the method using a general formula to calculate the sum-product functionals for PPP. The application of JT scheme is appealing in both homogeneous and heterogeneous networks, and its theoretical analysis was carried out accordingly [26]. In [27], the performance of JT scheme based on carrier aggregation (CA) was analyzed. The involvement of CA guarantees that cooperative SBSs use different frequency resources, and thus the analysis can be made very similar to that in a conventional transmission scheme without JT. Another method to analyze the performance of JT was proposed in [28], where the authors ignored spatial randomness of SBSs and established a relationship between area spectral efficiency (ASE) and SINR with a given target bit error rate constraint. With a total transmit power constraint at each user, the average user throughput was obtained for user-centric JT schemes, where the cooperative SBSs were assumed to be located within a known distance [29]. Under an assumption that cell-edge UEs are served by two nearest SBSs, the outage probability and effective throughput in the JT schemes were analyzed in [30], where the spatial randomness of SBSs was ignored. If all UEs are assumed to be served by $n$ nearest SBSs, successful transmission probability bound for JT schemes was derived in [31].

To the best of our knowledge, this work is the first research effort to analyze the system performance of various coherent JT schemes based on stochastic geometry. The main contribution of this paper is to propose a unified analytical approach suitable to analyze aggregate desirable signal and interference powers for different transmission schemes with or without coherent JT. By dividing an entire area into two regions in each transmission scheme, the problem of calculating received desirable and interfering powers can be transformed to the calculation of aggregate powers transmitted from SBSs inside each region. The boundary of each region is determined by the distribution of the distances between a UE and its serving SBS(s). Using the proposed method, different stochastic characteristics (i.e., expectation, MGF, and variance) of the aggregate desirable signal and interference powers can be obtained. The other performance indicators, i.e., average SINR, average spectral efficiency, and average area spectral efficiency, can also be calculated easily. Note that the randomness of received power is due mainly to the spatial distribution of SBSs and channel variation. We first analyze the effect of spatial randomness of SBSs by assuming a constant channel coefficient, followed by illustrating the results obtained in Rayleigh fading and Nakagami-$m$ fading channels.

The rest of this paper can be outlined as follows. Section II introduces the system model briefly, including the description on different JT schemes and the main idea of the proposed unified analytical approach. In Section III, we characterize the properties of aggregate received desirable signal and interference powers, such as their expectations, MGFs, and variances. Then, with the help of the results obtained in Section III, the system performance metrics are derived in Section IV, including SINR, spectral efficiency, and area spectral efficiency. Simulation results are presented in Section V, followed by the conclusions made in Section VI.

II. SYSTEM MODEL

A. Transmission Schemes

The very basic idea of JT lies on the fact that a set of base stations should transmit the same message cooperatively to a UE simultaneously, and the choice of cooperative set is critical to successful JT operation. There are three different schemes to choose a cooperative set, which are summarized as follows.

1) 2-nearest-SBSs (2NS) scheme: This is a conventional JT scheme working in a way that a UE chooses $N$ nearest SBSs, and $N = 2$ is the most common choice of the scheme, which will be analyzed in this paper.

2) Constant-distance (CD) scheme: Each SBS compares its distance to a UE with a predetermined value $R_0$. If the distance is no longer than $R_0$, it is a cooperative SBS [32], [33]. This scheme can be slightly altered for its use in another JT scheme, which works based on the average received power. More specifically, if the received power from a given SBS is above a given threshold, the SBS is a cooperative SBS [34], [35].

3) Fixed-power-difference (FPD) scheme: The third scheme works in a way based on the difference in received powers. To be more specific, if the highest average received power is $P_{max}$ dBm, other SBSs, whose received power are higher than $P_{max} - \eta$ dBm, are selected as the cooperative SBSs, where $\eta$ dB is a predetermined value [16], [36].

Of course, there may exist more than two cooperative SBSs in each cooperative set, and thus we can select some or all of SBSs from the set for their participations in JT based on
the QoS requirement or network traffic condition. To simplify analytical process, we assume that all SBSs in a cooperative set are involved in the transmission process, and the scenarios with selective SBSs will be considered in our future works. In the rest of this paper, we use the terms, 2NS, CD, and FPD, to denote the aforementioned three JT schemes, respectively. Meanwhile, the traditional transmission scheme, i.e., a UE is served by its nearest SBS, is also analyzed in this paper as a benchmark, and it is denoted by NoJT scheme.

B. Analytical Approach

Nearly all existing works in the literatures focused on the first two JT schemes, and there is no papers published to consider theoretical analysis of the third scheme. In most existing works, the analytical approaches of the first two JT schemes are different. To be more specific, in the 2NS scheme, the distances between a UE and two nearest SBSs are calculated, respectively, and then the aggregate received power is obtained by adding the received powers from these two SBSs. In the CD scheme, the boundary distance between a UE and its serving SBSs is known, and then the aggregate received power can be obtained based on Campbell’s theorem.

Even though there exist obvious differences in the aforementioned four schemes (i.e., three JT schemes plus NoJT scheme), we aim to present a unified approach suitable to study all of these schemes. As shown in Fig. 1, SBSs are distributed following a Poisson point process (PPP). Using Slivnyak’s theorem, here we investigate the performance from the viewpoint of a typical UE located at the center without loss of generality, and this assumption has been used in many other related works [37]. If NoJT scheme is used, the UE is served by its nearest SBS, as illustrated in the Voronoi diagram.

The distance between the UE and its nearest SBS is $r_1$, and a circle is drawn with its radius $r_1$. We can claim that SBSs that are located within the circle are the serving SBSs of a given UE. Similarly, we can also determine the circles for the aforementioned three JT schemes, and their radii are $r_2$, $R_0$, and $r_\eta$ for 2NS, CD, and FPD, respectively. $r_2$ is the distance between a UE and the second nearest SBS, and $R_0$ is a predefined constant. $r_\eta$ can be obtained as follows.

$$\eta = 10\log \left( K_s \mathbb{E}[|h|] r_1^{-\alpha_s} P_s \right) - 10\log \left( K_s \mathbb{E}[|h|] r_\eta^{-\alpha_s} P_s \right). \quad (1)$$

where $K_s$, $h$, $P_s$, and $\alpha_s$ are the constants owing to antenna and average channel attenuation, multipath and shadowing fading coefficient, transmit power of SBS, and a large-scale fading factor ($\alpha_s > 0$), respectively. Thus, we have $r_\eta = \frac{\eta}{\eta_1} r_1$.

As shown in Fig. 1, the whole area can be divided into two regions for each transmission scheme. SBS(s) inside a circle are the desirable transmitters, and SBSs outside the circles are the interferers. Thus, the problem to calculate received desirable signal and interference powers can be transformed to the calculation of aggregate powers transmitted from SBSs inside and outside the circle, respectively. Take NoJT scheme as an example. The desirable and interfering powers are calculated respectively by

$$P_1 = \sum_{r \leq r_1} K_s h r^{-\alpha_s} P_s, \quad I_1 = \sum_{r > r_1} K_s h r^{-\alpha_s} P_s.$$  

Therefore, various mathematical properties of PPP can be explored directly to obtain different statistical characteristics (such as expectations, MGFs, and variances) for both desirable signal and interference powers. Then, the other system performance metrics can also be evaluated accordingly.

C. Coverage Radius

The first step here is to determine the coverage radius in different transmission schemes. Assume that SBSs are distributed following a PPP with its density $\lambda_s$, which is defined as the number of SBSs per square meter. The probability distribution function (PDF) of the radius can be calculated as follows.

1) NoJT scheme: The radius $r_1$ is the distance between a UE and its nearest SBS, and thus the PDF of $r_1$ is [38]

$$f_1(r) = 2\pi \lambda_s r \exp \left( -\pi \lambda_s r^2 \right). \quad (2)$$

2) 2-nearest-SBSs (2NS) scheme: The radius $r_2$ is the distance between a UE and the second nearest SBS, and similarly its PDF is [38]

$$f_2(r) = 2(\pi \lambda_s)^2 r^3 \exp \left( -\pi \lambda_s r^2 \right). \quad (3)$$

3) Constant-distance (CD) scheme: The radius is a pre-defined constant, which is denoted by $R_0$.

4) Fixed-power-difference (FPD) scheme: The relationship between $r_\eta$ and $r_1$ is shown in Eqn. (1). Let $\eta_1 = 10\log \eta_\eta^2 r_\eta^2$. We have $f_\eta(r) = \eta_1 f_1(\eta_1 r)$. Thus, the PDF of $r_\eta$ is

$$f_\eta(r) = 2\pi \lambda_s \eta_\eta^2 r \exp \left( -\pi \lambda_s \eta_\eta^2 r^2 \right). \quad (4)$$
fading channel is written as
\[ f_{h,n}(h) = \frac{m^m}{\Omega m^m \Gamma(m)} h^{m-1} \exp(-\frac{m}{\Omega}), \]
where \( m > \frac{1}{2} \) and \( \Omega = \mathbb{E}[h]. \)

A. Expectations

The average aggregate desirable signal and interference powers can be calculated based on Campbell’s theorem of a PPP [41], i.e.,

\[ \mathbb{E}\left[\sum_{x \in \Phi} f(x)\right] = 2\pi \lambda \int_{\Phi} f(x) x dx, \quad (5) \]

for any non-negative measurable function \( f \) over a PPP \( \Phi \). Therefore, for all SBSs, whose distances to a given UE are within the range \([R_1, R_2]\), the average aggregate power received by the UE is

\[ \mathbb{E}[P] = 2\pi \lambda_b \int_{R_1}^{R_2} K_s h_P r^{-\alpha_s} r dr \]
\[ = \left\{ \begin{array}{ll}
2\pi \lambda_b K_s h_P r_{1}^{-\alpha_s+2} - R_1^{-\alpha_s+2}, & \alpha_s < 2, \\
2\pi \lambda_b K_s h_P \ln \frac{R_2}{R_1}, & \alpha_s = 2.
\end{array} \right. \quad (6) \]

Eqn. (6) is useful to calculate both desirable signal and interference powers. Assume that the minimum distance between an SBS and a UE is \( R_1 \) and the radius of a network is \( R_m \). To calculate the desirable signal power, the lower limit in Eqn. (6) is \( R_1 \) (i.e., \( R_1 = R_l \)) and the upper limit is the radius, i.e., \( r_1, r_2, R_0 \), and \( r_n \) for NoJT, 2NS, CD, and FPD schemes, respectively. Except for CD scheme, the radii in the other three schemes are random variables with their PDFs given in Section II-C. Thus, the average aggregate desirable signal power can be obtained by integrating the radius (i.e., \( r_1, r_2, R_0 \), and \( r_n \)) over the range \([R_1, R_m]\). Similarly, when calculating the interference power, the lower limit is the variable of radius (i.e., \( r_1, r_2, R_0 \), and \( r_n \)), and the upper limit is \( R_m \). Except for CD scheme, an integral over \([R_1, R_m]\) yields the aggregate interference power for the other three transmission schemes.

Meanwhile, \( \alpha_s = 2 \) is for the scenario of free space propagation, and \( \alpha_s \neq 2 \) is a more general case. When \( h \) is a constant, the results for \( \alpha_s \neq 2 \) and \( \alpha_s = 2 \) are given in Eqns. (7) and (8), as shown at the bottom of next page, respectively, where \( P_x \) and \( I_x \) are the aggregate received desirable signal and interference powers for \( x \) transmission scheme, and the subscripts 1, 2, \( R_0 \), and \( n \) represent NoJT, 2NS, CD, and FPD schemes, respectively.

In Rayleigh and Nakagami-\( m \) fading channels, similar results can be obtained by substituting the expectation of the channel (i.e., 1 and \( \Omega \), respectively) for \( h \) in Eqns. (7) and (8). Due to the page limit of this paper, the results are made available in an uploaded file [40].

B. Moment Generating Functions

MGF is an alternative representation of probability distribution function (PDF), which is defined as \( M(z) = \mathbb{E}[e^{-zX}] \) for variable \( X \). The relationship between MGF and PDF is

\[ M(z) = \mathbb{E}[e^{-zX}] = \int_{-\infty}^{\infty} e^{-zx} f_X(x) dx. \]
In other words, a MGF is the Laplace transform of a PDF. In this paper, the MGF is used for three purposes, i.e., to calculate the variance in Subsection III-C, to evaluate spectral efficiency and area spectral efficiency in Section IV-B, and to determine the numerical value of a PDF in Section V-A.

With the help of the Laplace function of Campbell theorem [41] (which is also referred to as a probability generating
function for a PPP), the MGF of the aggregate power in the range \([R_1, R_2]\) can be written as

\[
\mathcal{M}^C(z) = \exp \left\{ -2\pi \lambda_0 \int_{R_1}^{R_2} \left[ 1 - \exp \left( -zK_shP_sr^{-\alpha_s} \right) \right] rd\rr \right\}
\]

\[
= \exp \left\{ 2\pi \lambda_0 \left[ \frac{(K_shP_s)^{2\alpha_s}}{\alpha_s} \right] \right\}
\times \Gamma \left\{ \left( -\frac{2}{\alpha_s}, zK_shP_sR_2^{-\alpha_s}, zK_shP_sR_1^{-\alpha_s} \right) - \frac{R_2^2}{2} + \frac{R_1^2}{2} \right\},
\]

(9)

where the channel coefficient \(h\) is assumed to be a constant.

In a Rayleigh fading channel, the result becomes

\[
\mathcal{M}^R(z) = \mathbb{E}_r \left( \prod_{R_1<r<R_2} \frac{1}{1 + zK_s r^{-\alpha_s} P_s} \right)
\]

\[
= \exp \left\{ -2\pi \lambda_0 \int_{R_1}^{R_2} \frac{zK_shP_}\alpha_s}{1 + zK_s r^{-\alpha_s} P_s} dr \right\}.
\]

(10)

If \(\alpha_s\) is a positive integer, a closed form expression can be obtained. For example, if \(\alpha_s\) is two, the result is

\[
\mathcal{M}^R(z) = \exp \left\{ -2\pi \lambda_0 \left[ (\frac{R_2}{R_1^2} + zK_s P_s) - \pi \lambda_0 zK_s P_s \right] \right\}.
\]

(11)

If \(\alpha_s\) is four, the result becomes

\[
\mathcal{M}^R(z) = \exp \left\{ -2\pi \lambda_0 \left[ (\frac{R_2}{R_1^2} + zK_s P_s) - \pi \lambda_0 zK_s P_s \right] \right\}.
\]

(12)

If there is no restriction for \(\alpha_s\), a closed form expression can also be obtained based on the equations [3.194] in [42] when \(R_2 \to \infty\) or \(R_1 \to 0\).

If \(R_2 \to \infty\), substituting \(t = \frac{zK_s P_s R_2^{2\alpha_s}}{\alpha_s - 2}\) into Eqn. (10), we obtain

\[
\mathcal{M}^R(z) = \exp \left\{ -2\pi \lambda_0 \left[ \frac{R_2}{R_1^{2\alpha_s}} - \frac{zK_s P_s}{\alpha_s - 2} \right] \right\}.
\]

(13)

If \(R_1 \to 0\), we get

\[
\mathcal{M}^R(z) = \exp \left\{ -2\pi \lambda_0 \left[ \frac{R_2}{R_1^{2\alpha_s}} - \frac{zK_s P_s}{\alpha_s - 2} \right] \right\}.
\]

(14)

In a Nakagami-\(m\) fading channel, we have

\[
\mathcal{M}^N(z) = \mathbb{E}_r \left( \prod_{R_1<r<R_2} \int_0^\infty e^{zK_s P_s hr^{-\alpha_s} f_{h,N}(h) dh} \right)
\]

\[
= \exp \left\{ -2\pi \lambda_0 \int_{R_1}^{R_2} \left[ 1 - \left( \frac{\Omega}{m} zK_s P_s r^{-\alpha_s} \right)^{-m} \right] r dr \right\}.
\]

(15)

If \(\alpha_s = 2\) and \(m\) is a positive integer, a closed form expression can be obtained based on the equations [2.117] in [42]. Here, let us take \(m = 2\) as an example to have

\[
\mathcal{M}^N(z) = \exp \left\{ \pi \lambda_0 \left[ \frac{(\Omega zK_s P_s)^2 (R_2^2 - R_1^2)}{(\Omega zK_s P_s + R_2^2) (\Omega zK_s P_s + R_1^2)} \right] \right\}.
\]

(16)

Thus, the MGFs for different JT schemes in different channels can be calculated and their results are shown in Appendix.

C. Variances

Variance of a random variable measures how much the random variable is spread over from its average value, which can be obtained from MGF. Substitute \(s = -z\) to all MGFs to obtain the expression \(\mathcal{M}(s)\). Then, we can use the property of MGF to calculate the expectation of the \(n\)th moment, i.e.,

\[
\mathbb{E}[x^n] = \Delta^n \mathcal{M}(s) \bigg|_{s=0}.
\]

Thus, the variance can be calculated by

\[
\mathbb{V}[x] = \mathbb{E}[x^2] - \mathbb{E}[x]^2.
\]

To apply the aforementioned method to calculate the variances of aggregate desirable signal and interference powers, first we need to compute the expectation of the square, i.e., \(\mathbb{E}[P^2]\) and \(\mathbb{E}[I^2]\), for different JT schemes. Then, we combine them with the average value in Subsection III-A to give the variance. An example is shown below, which is the variance of the aggregate desirable power in the NoJT scheme when the channel coefficient \(h\) is a constant. Due to the space limit, the calculations for other cases are given in an uploaded file [40].

If \(h\) is assumed to be a constant, the first order derivative of the MGF of the desirable signal power for NoJT scheme is

\[
\frac{\Delta \mathcal{M}^S_{P_i}(s)}{\Delta s} \bigg|_{s=0} = \int_{R_1}^{R_2} \frac{\Delta b(s)}{\Delta s} f_1(r) dr
\]

\[
+ \int_{R_1}^{R_2} 2\pi \lambda_0 \int_{R_1}^{r} K_s h P_s x^{1-\alpha_s} f_1(r) dr,
\]

(17)

where (a) is established based on the Leibniz integral rule

\[
b(s) = \exp \left\{ -2\pi \lambda_0 \int_{R_1}^{r} \left[ 1 - \exp(sK_s h P_s x^{-\alpha_s}) \right] x dx \right\},
\]

and \(a(s) = \exp(sK_s h P_s x^{-\alpha_s})\).

The second order derivative of the MGF is

\[
\frac{\Delta^2 \mathcal{M}^S_{P_i}(s)}{\Delta s^2} = 2\pi \lambda_0 K_s h P_s \int_{R_1}^{R_2} \{ \int_{R_1}^{r} x^{1-2\alpha_s} K_s h P_s a(s) dx \} b(s) f_1(r) dr,
\]

(18)
Thus, the expectation of the second order moment is given in Eqn. (19), as shown at the bottom of this page, where (a) is held based on the relation

\[ \Gamma(s + 1, a, b) = \Gamma(s, a, b) + a^s e^{-a} - b^s e^{-b}. \]

The variance of the desirable power is calculated in Eqn. (20), as shown at the bottom of this page.

IV. PERFORMANCE EVALUATION

Various statistical characteristics of the received signal power can be used to evaluate system performance of a UDN, such as SINR, spectral efficiency, and area spectral efficiency. In this section, we will show how to calculate them, respectively.

A. Expectation of SINR

It is difficult to obtain an exact value of SINR expectation, denoted as \( T \), and thus different approximation methods were proposed in the literature. For example, the expectation of SINR was calculated in [43], [44] as

\[ E[T] = \frac{E[P]}{E[I] + N_0}, \]

and the expectation of the \( n \)th order moment of SINR is

\[ E[T^n] = E[P^n] \int_0^{\infty} \frac{z^{n-1}}{1(n)} \Psi_I(z) e^{-zN_0} dz, \quad n > 0, \]

where \( N_0 \) is the noise power.

The average (i.e., \( E[P] \) and \( E[I] \)), the MGF of interference power (i.e., \( \Psi_I(z) \)), and the expectation of its square (i.e., \( E[P^n] \)) have been calculated in Subsections III-A, III-B, and III-C, respectively. Thus, we can obtain the expectations of the first and second order moments of SINR for different transmission schemes.

B. Spectral Efficiency

Spectral efficiency is defined as

\[ S = \log(1 + T). \]

The authors in [45] used the relation \( \ln(1 + x) = \int_0^1 \frac{x}{1 + \eta} e^{-\eta} d\eta \) to transform the expression into

\[ S = \frac{1}{\ln 2} \int_0^\infty \frac{1}{s} [1 - M_T(s)] e^{-s} ds, \]

where \( M_T(s) = E[\exp(-sT)] \), which is the MGF of SINR. Thus, we have

\[ S = \frac{1}{\ln 2} \int_0^\infty \frac{1}{s} \left[ 1 - \exp \left( -s \frac{P}{T + N_0} \right) \right] e^{-s} ds \]

and

\[ \Psi_I(z) = e^{-zN_0} \int_0^\infty \Psi_I(z) e^{-sN_0} dz, \]

where \( \Psi_I(z) = E[\exp(-zT)] \) is the MGF of the aggregate received signal power from both desirable and interfering transmitters, which is the same for different transmission schemes, as shown in Eqn. (25), as shown at the bottom of this page.
Therefore, we can evaluate the spectral efficiency for different JT schemes using the corresponding MGFs of aggregate interference.

C. Area Spectral Efficiency

Area spectral efficiency (ASE) was proposed first by Alouini et al., which was defined as the capacity achieved by all users per spectrum within a unit area [46]. Compared to conventional spectral efficiency, ASE can depict the spatial reuse efficiency within a given area, which is in particular relevant for UDNs [47], [48].

Assume that all SBSs serve at least one UE and the same spectral resource is shared by all those SBSs. The ASE can be calculated approximatively by

\[
C = \sum_{n=1}^{\infty} \mathbb{P}(|\Phi| = n) \frac{n S}{\pi R_m^2},
\]

where \( \mathbb{P}(|\Phi| = n) \) is the probability that there are \( n \) SBSs distributed following a PPP \( \Phi \), which is

\[
\mathbb{P}(|\Phi| = n) = \frac{1}{n!} e^{-\lambda_s \pi R_m^2} (\lambda_s \pi R_m^2)^n, \quad n \in \mathbb{Z}^+.
\]

Substituting spectral efficiency obtained in Eqn. (24) to the above expression, we can obtain the approximate ASE in Eqn. (26).

V. NUMERICAL RESULTS

In this section, we will verify the proposed analytical approach and show the performances of different transmission schemes. The transmit power \( P_t \) is set to be 17 dBm, and the constant channel attenuation factor \( K_s \) is one. Meanwhile, we assume that the minimum distance \( R_0 \) between a UE and an SBS is 0.1 m, and the radius of the network is 60 m. The results are presented from Figs. 2 to 5.

A. Aggregate Received Power

1) Average: The average aggregate desirable signal and interference powers are shown in Figs. 2(a) and 2(b), respectively. Simulation results match to the analytical results very well, and thus the effectiveness of the proposed model is verified. Compared to NoJT scheme, the use of JT helps increase the desirable signal power and suppress interference power. Using different JT schemes results in different performance gains. If \( R_0 = 3 \) and \( \eta = 10 \), the CD scheme offers the highest average desirable signal power and the lowest average interference power when the density is higher than 0.12/m². If the density of SBSs goes below 0.12/m², the performance of FPD is better than CD scheme, and the scenarios with less SBSs are not discussed in detail in this paper. Meanwhile, both desirable signal and interference powers increase with SBSs density. It can be clearly observed from Eqn. (7) that both the average desirable signal and interference powers are linear functions of SBS density for CD scheme. As shown in Fig. 2(a), a linearly increasing trend of the desirable signal power is observed for other transmission schemes. Meanwhile, the interference power grows faster with the SBS density, especially in NoJT scheme. In other words, the use of JT may slow down the growth rate of interference power, and thus JT is proved to be more effective in the scenarios with densely distributed SBSs. As shown in Subsection III-A, the same result can be observed when \( h = 1 \) or in a Rayleigh fading channel, due to the fact that the expectation of \( h \) in a Rayleigh fading channel is one. In other words, the shape of the curve depends only on the spatial randomness of SBSs, and the fading channel affects only the magnitude scale of the curve. In order to distinguish these two channel conditions, we assume that \( h \) is two if it is a constant in Fig. 2. Thus, a higher value can be observed if \( h = 2 \).

2) Probability Density Functions: Theoretical probability density functions (PDFs) were depicted based on the inverse
Laplace transform of MGFs, and the PDFs of the interference power for different transmission schemes are shown in Fig. 3. Fig. 3(a) depicts a scenario, in which channel coefficient $h$ is a constant and thus the statistical characteristics of the aggregate power depends only on the spatial distribution of SBSs. Fig. 3(b) shows the PDFs for a Rayleigh fading channel, and thus both the randomness of spatial distribution and the channel condition contribute to the distribution of aggregate power. PDF curves in Fig. 3(b) appear wider than that in Fig. 3(a), and thus the channel randomness results in a more scattered distribution of the aggregate power in all schemes, and the most obvious example is the CD scheme. That is because, based on the chosen parameters, the average desirable signal power is the largest for CD scheme, as shown in Fig. 2(a). Thus, the number of cooperative SBSs is likely to be the largest in the CD scheme and there is more likely a nearby SBS in a large cooperative set. If the value of $r$ is sufficiently small, i.e., a nearby SBS exists, the received power from a single SBS will be much higher than the sum of received powers from several far-away SBSs. Thus, a scattered distribution is observed obviously in the CD scheme.

Meanwhile, we also note that the shape of each curve resembles that of a Gamma distribution. The authors in [20] and [49] analyzed the distribution of the aggregate interference power in NoJT scheme in a Rayleigh fading channel, where the received interference can be viewed as the sum of multiple random variables following Rayleigh distributions, resulting in a Gaussian distribution. Thus, they used a Gaussian distribution to approximate the aggregate power to simplify the calculation process of other system parameters. The square of a Gaussian random variable follows a Gamma distribution. In addition, we can also conclude that the distribution of the aggregate interference power resembles a Gamma distribution, regardless of the fading channel, as shown in Fig. 3(a). Therefore, we can expect that the distribution of the aggregate desirable signal power is similar to that of the interference power, and thus an identical approximation method can be used to further simplify the calculation of system performance metrics.

3) Variances: The variances of the aggregate desirable signal and interference powers for different JT schemes are given in Figs. 4(a) and 4(b), respectively. The shape of the curve is similar to that of Fig. 2. In other words, the relationship between the expectation of aggregate power and SBS density resembles that between its variance and SBS density. Comparing Fig. 4(b) with Fig. 4(a), we can observe a higher variance in a Rayleigh channel due to the existence of extra randomness. Meanwhile, an interesting phenomenon can be observed when comparing Figs. 2, 3, and 4, that is, the expectation is much larger than the value of the highest PDF, and the variance is even larger. In other words, an extremely large aggregate power can be received with an extremely small probability, and the majority of the received power is much lower than its average value. Thus, the aggregate power follows a long-tail distribution. This phenomenon is a direct result due to the use of the large-scale path loss models we assumed in the analysis. If $\alpha$ is larger than two, such a long-tail distribution can be observed easily. Recently, numerous new path loss models were proposed for UDNs, such as multi-slope model [50] and exponential model [51]. If a different path loss model is used, a different distribution, rather than a long-tail distribution, can be observed.

In [52], the authors used an alpha-stable distribution, which is a type of long-tail distributions, to depict the aggregate interference for NoJT scheme in a Rayleigh fading channel, which is consistent to the results obtained here. Moreover, we
can also foresee that a similar approach can be applied to other JT schemes as well.

To sum up, both alpha-stable and Gamma distributions can be used. An alpha-stable distribution can precisely model the tail portion of a PDF, while a Gamma distribution is better to capture the head portion of a PDF. The investigations to find a better approximation method to depict the aggregate power in UDNs will be one of our future works.

B. System Performance Analysis

The average SINR for different JT schemes are shown in Fig. 5(a). The analytical average SINR obtained by Eqn. (21) is the same as that for \( h = 1 \) constant and Rayleigh fading channels. As shown in Fig. 5(a), the analytical result can approximately reveal the actual value in these two channel conditions, and the difference between the analytical and simulation results is within an acceptable range (a similar
mismatch can be seen in [43]). Comparing to the NoJT scheme, we see that using JT gives a better performance, especially in the areas with a high density of SBSs.

The average spectral efficiency is shown in Fig. 5(b). The analytical results match to the simulation results well, and thus the correctness of the proposed model is verified. Using JT schemes can improve the average spectral efficiency, and different JT schemes have different performance gains.

The performances of different schemes in terms of received power can be ranked as CD > FPD > 2NS > NoJT, which means that using the CD scheme results in the highest desirable signal power and the lowest interference power. Their performances in terms of SINR can be ranked as CD > 2NS > FPD > NoJT. The reason is that SINR is calculated approximately by Eqn. (21), which is the ratio between the average desirable signal power and the average interference power. Thus, the CD scheme, with the highest desirable power and the lowest interference power, is ranked the first. Even though FPD scheme performs better than 2NS scheme when considering interference power, is ranked the first. Even though FPD scheme performs better than 2NS scheme when considering interference power, not necessarily yield the highest spectral efficiency. Based on the chosen parameters, the FPD scheme is ranked the first in terms of spectral efficiency. When choosing a different value of $R_0$ or $\eta$, we can generate a different ranking order for these schemes. Thus, different schemes should be chosen based on different requirements. Even though the ranking may vary, we can still conclude that no matter which parameters are chosen, the performance of JT exceeds that of NoJT in terms of the received power, SINR and spectral efficiency.

VI. CONCLUSIONS

This paper proposed a unified approach to analyze the performance of different joint transmission schemes in UDNs. Dividing a UDN into two regions, we transformed the calculation of aggregate desirable signal and interference powers into an integral of received power over each region. Thus, the statistical characteristics of aggregate power, such as expectations, MGFs, and variances, can be obtained. Note that the randomness of the received power is caused by both spatial distribution of SBSs and multipath/shadowing effects. First, we identified the impact of spatial randomness by assuming a constant channel. Then, the results in Rayleigh and Nakagami-\(m\) channels were derived. As shown in Fig. 2, the shapes of the average power curves depend only on the spatial randomness of SBSs, and the expectation of channel gain (i.e., $\mathbb{E}[h]$) affects the scale of the curve. Channel variation increases the span of the aggregate power. A long-tail distribution was observed with or without channel fading. In other words, an extremely high aggregate power may be received at a small probability, and the majority of received power is much lower than the average value. The other system performance metrics, such as SINR, spectral efficiency, and ASE, were calculated based on the expectation, MGF, and variance of aggregate power. A good match between simulation and analytical results justified the proposed model. As a matter of fact, the proposed unified analytical method can also be applied to many other applications (such as ad hoc networks) as long as the nodes are distributed following a PPP.

APPENDIX

DERIVATIONS OF MGFs

Assume that $h$ is a constant and the MGFs of the desirable signal and interference powers in NoJT scheme are

\[
M_{P_d}^{\text{NoJT}}(z) = \frac{2 \pi \lambda_b}{\pi \lambda_b} \int_{R_l}^{R_m} r \exp \left\{ 2 \pi \lambda_b \left[ \frac{(z K_s h P_s) \frac{2}{\alpha_s}}{\alpha_s} \times \Gamma\left(-\frac{2}{\alpha_s}, z K_s h P_s R_m^{-\alpha_s}, z K_s h P_s R_l^{-\alpha_s}\right) - r^2 + \frac{R_l^2}{2} \right] \right\} dr,
\]

\[
M_{I_d}^{\text{NoJT}}(z) = \frac{2 \pi \lambda_b}{\pi \lambda_b} \int_{R_l}^{R_m} r \exp \left\{ 2 \pi \lambda_b \left[ \frac{(z K_s h P_s) \frac{2}{\alpha_s}}{\alpha_s} \times \Gamma\left(-\frac{2}{\alpha_s}, z K_s h P_s R_m^{-\alpha_s}, z K_s h P_s R_l^{-\alpha_s}\right) - \frac{R_m^2}{2} \right] \right\} dr.
\]

(28)

In a Rayleigh channel, the MGFs for NoJT scheme are

\[
M_{P_d}^{\text{NoJT}}(z) = 2 \pi \lambda_b \int_{R_l}^{R_m} r \exp \left\{ -2 \pi \lambda_b \int_{R_l}^{R_m} \frac{z K_s h P_s x^{-\alpha_s}}{\alpha_s} dx dr, \right\}
\]

\[
M_{I_d}^{\text{NoJT}}(z) = 2 \pi \lambda_b \int_{R_l}^{R_m} r \exp \left\{ -2 \pi \lambda_b \int_{R_l}^{R_m} \frac{z K_s h P_s x^{-\alpha_s}}{\alpha_s} dx dr, \right\}
\]

(29)

In a Nakagami-\(m\) channel, the MGFs for NoJT scheme are

\[
M_{P_d}^{\text{NoJT}}(z) = 2 \pi \lambda_b \int_{R_l}^{R_m} r \exp \left\{ -2 \pi \lambda_b \int_{R_l}^{R_m} \frac{z K_s h P_s x^{-\alpha_s}}{\alpha_s} dx dr, \right\}
\]

(30)

Similarly, in a constant channel the MGFs of desirable signal and interference powers in 2NS scheme are

\[
M_{P_d}^{\text{2NS}}(z) = 2 \pi^2 \lambda_b^2 \int_{R_l}^{R_m} r^3 \exp \left\{ 2 \pi \lambda_b \left[ \frac{(z K_s h P_s) \frac{2}{\alpha_s}}{\alpha_s} \times \Gamma\left(-\frac{2}{\alpha_s}, z K_s h P_s R_l^{-\alpha_s}, z K_s h P_s R_l^{-\alpha_s}\right) - r^2 + \frac{R_l^2}{2} \right] \right\} dr,
\]

\[
M_{I_d}^{\text{2NS}}(z) = 2 \pi^2 \lambda_b^2 \int_{R_l}^{R_m} r^3 \exp \left\{ 2 \pi \lambda_b \left[ \frac{(z K_s h P_s) \frac{2}{\alpha_s}}{\alpha_s} \times \Gamma\left(-\frac{2}{\alpha_s}, z K_s h P_s R_l^{-\alpha_s}, z K_s h P_s R_l^{-\alpha_s}\right) - \frac{R_l^2}{2} \right] \right\} dr.
\]
In a Rayleigh channel, the MGFs for CD scheme are

\[ M_{F_{00}}^R(z) = 2\pi^2 \lambda_b^2 \int_{R_l}^{R_m} r^3 \exp \left\{ -2\pi \lambda_b \left[ \frac{(zK_s h P_s)^{\frac{2}{\alpha_s}}}{\alpha_s} \right] \right\} \times \int_{R_l}^{r} \frac{x z K_s h P_s}{x^{\alpha_s} + z K_s h P_s} dx - \pi \lambda_b r^2 \right\} dr. \] (31)

In a Nakagami-\( m \) channel, the MGFs for CD scheme are

\[ M_{F_{00}}^N(z) = \exp \left\{ -2\pi \lambda_b \int_{R_l}^{R_m} \left[ 1 - \left( 1 + \frac{\Omega_m}{m z K_s h P_s r_m^{-\alpha_s}} \right)^{-m} \right] r dr \right\}, \]

\[ M_{I_{00}}^N(z) = \exp \left\{ -2\pi \lambda_b \int_{R_l}^{R_m} \left[ 1 - \left( 1 + \frac{\Omega_m}{m z K_s h P_s r_m^{-\alpha_s}} \right)^{-m} \right] r dr \right\}. \] (36)

Similarly, in a deterministic channel, the MGFs of desirable signal and interference powers in the FPD scheme are

\[ M_{F_{00}}^C(z) = 2\pi \lambda_b \eta_t^2 \int_{R_l}^{R_m} r \exp \left\{ 2\pi \lambda_b \left[ \frac{(zK_s h P_s)^{\frac{2}{\alpha_s}}}{\alpha_s} \right] \right\} \times \int_{R_l}^{r} \frac{x z K_s h P_s}{x^{\alpha_s} + z K_s h P_s} dx - \pi \lambda_b \eta_t^2 r^2 \right\} dr, \]

\[ M_{I_{00}}^C(z) = 2\pi \lambda_b \eta_t^2 \int_{R_l}^{R_m} r \exp \left\{ 2\pi \lambda_b \left[ \frac{(zK_s h P_s)^{\frac{2}{\alpha_s}}}{\alpha_s} \right] \right\} \times \int_{R_l}^{r} \frac{x z K_s h P_s}{x^{\alpha_s} + z K_s h P_s} dx - \pi \lambda_b \eta_t^2 r^2 \right\} dr. \] (37)

In a Rayleigh channel, the MGFs for FPD scheme are

\[ M_{F_{00}}^R(z) = 2\pi \lambda_b \eta_t^2 \int_{R_l}^{R_m} r \exp \left\{ -2\pi \lambda_b \right\} \times \int_{R_l}^{r} \frac{x z K_s h P_s}{x^{\alpha_s} + z K_s h P_s} dx - \pi \lambda_b \eta_t^2 r^2 \right\} dr, \]

\[ M_{I_{00}}^R(z) = 2\pi \lambda_b \eta_t^2 \int_{R_l}^{R_m} r \exp \left\{ -2\pi \lambda_b \right\} \times \int_{R_l}^{r} \frac{x z K_s h P_s}{x^{\alpha_s} + z K_s h P_s} dx - \pi \lambda_b \eta_t^2 r^2 \right\} dr. \] (38)

In a Nakagami-\( m \) channel, the MGFs for FPD scheme are

\[ M_{F_{00}}^N(z) = \exp \left\{ -2\pi \lambda_b \int_{R_l}^{R_m} \left[ 1 - \left( 1 + \frac{\Omega_m}{m z K_s h P_s r_m^{-\alpha_s}} \right)^{-m} \right] r dr \right\}, \]

\[ M_{I_{00}}^N(z) = \exp \left\{ -2\pi \lambda_b \int_{R_l}^{R_m} \left[ 1 - \left( 1 + \frac{\Omega_m}{m z K_s h P_s r_m^{-\alpha_s}} \right)^{-m} \right] r dr \right\}. \] (39)

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