Power Grid Partition Method for Black Start Based On Complex Network Theory

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Abstract. The paper proposes one power grid partition method for black start based on complex network theory. The method is composed of splitting algorithm and agglomerative algorithm. The former algorithm divided grid to some temp zone according to the number of black-start unit, which determines the basic structure of the partition and reduces the number of iterations in agglomerative process. The latter algorithm redefines the line weight and weighted modularity based on flow distribution and network topology, then merges neighbouring temp zone to maximum weighted modularity. The method can reflect the close degree of the partition, accurately assess the partition quality and determine the optimal number of partitions. The simulation results on IEEE 39-bus test system illustrate the validity of this methodology.

1. Introduction
When power grid subjected to large-scale blackout after large disturbance or failure, system recovery can be realized by black-start scheme [1]-[3]. The units with self-start ability in grid can be used to bring others to start up and recover the system gradually. Finally, the whole power system can be restored to normal operation state. In order to speed up the recovery process, the network can be divided into several sub zones in which black start unit can recover corresponding load. When some sub zones run in a stable state, Quasi-synchronizing parallel devices can achieve the grid connection between two sub zones. There are two main types of power system partition methods: the fixed partition method based on the administrative division or jurisdiction scope, and the dynamic partition method based on the power failure information after the accident [4]. Due to the especially serious power outage, earthquake and typhoon, the communication system cannot receive sufficient information which lead s to the invalidation of dynamic partition method. So power grid should keep a watchful eye on predetermined fixed partition scheme. The traditional partition method makes a mechanical partition and cannot describe the tightness of inner zone [5].
Complex network theory studies the areal network in social, natural and engineering systems, aiming to reveal the connection between network function and structural features [6]-[9]. With the establishment of small world network and scale-free network model, this theory has been paid more attention in many fields of scientific research. A large number of researches show that, as one of the large scale and complex artificial networks, the power system also has the characteristics of small world network, community structure and other complex networks. Community structure is generally properties in complex network topology properties [10]-[13], it automatically divided network into several subnets. The structure has typical characteristic that there is a close connection within the community and relatively sparse connection between communities. The characteristic is highly similar
to the requirement of grid partition. This result provides a new way for the research of power system partition, which can help overcome lack of traditional partition method.

The paper proposes a novel power grid partition algorithm for black start based on complex network theory. The method redefines the line weight and weighted modularity based on flow distribution and network topology. The algorithm divided partitioning process into two phases. The first stage applies splitting algorithm to reasonably determine the maximum partition number in black start stage on the basis of the number of black start units. The second stage uses the agglomerative algorithm to combine the adjacent area according to the weighted modularity, ultimately determine the optimal partitioning scheme.

2. Community structure

Community structure is most important branch of the complex network theory, has been widely used in the social relations, the Internet and business conduct clustering [14]. It also has been gradually introduced into the complex behaviour study of power grid [15]. Some apexes constitute special set which has a close contact between vertices in set and relatively spars contact between vertices external set. so the network can be divided into some community structures consist of special set, it is shown as figure 1.

![Figure 1. Community structure.](image)

At present, the most commonly used description indicators of community structure is module degree defined by Girvan and Newman, it is usually called Q function. module degree is the fraction of the edges that fall within the given communities minus the expected fraction if edges were distributed at random [16]. For a given division of the network's vertices into some modules, modularity reflects the concentration of edges within modules compared with random distribution of links between all nodes regardless of modules [17], [18].

$$Q = \frac{1}{2m} \sum_{ij} \left( A_{ij} - \frac{k_i k_j}{2m} \right) \delta(c_i, c_j)$$

(1)

Where $A_{ij}$ is the weight of line between node $i$ and node $j$, $k_i = \sum_j A_{ij}$ is the sum weight of all line connected to node $i$; $m = 0.5 \sum_j A_{ij}$ is the sum weight of all line. $c_j$ is serial number of community structure within node $i$. If the node $i$ and node $j$ are in same community structure, then $c_i = c_j$ and $\delta(c_i, c_j) = 1$, otherwise $\delta(c_i, c_j) = 0$. If the $k_i$ of all node keep unchanged and form the random network, $\frac{k_i k_j}{2m}$ is the weight expectation of line between node $i$ and node $j$.

The range of module degree is from 0 to 1, if the sum weight in community structure is not greater than the expectation weight of the random network, $Q=0$ [19]. The closer to upper limit $Q$ is, the more
obvious the community structure is. Generally, when Q is around 0.32 or greater than 0.32, it indicates that the partitioning method is a more appropriate approach. According to above characteristics of module degree, the maximum module degree of different networks can be compared to evaluate its structure separability. The local peak of Q is usually not more than 2, so it is easy to carry out the partition based on Q. In conclusion, according to the peak of module degree, the optimal partition number can be selected to determine the optimal partition mode.

3. Power grid partition method

3.1. Line weight

Complex network theory uses node set $V$, edge set $S$ and edge weights $w_{ij}$ to describe complex network. The model of complex power system usually considers generator and load as node, considers line as edge. The double or multiple circuits are usually merged into one edge. The traditional method uses the reciprocal of the line reactance as weight. Based on flow distribution and network topology, this paper defines weight of line $(i, j)$ as:

$$w_{ij} = \sum_{m,n \in G \cup L} \sqrt{W_m W_n} |I^m(i, j)|$$

(2)

Where $(m, n)$ is a plant-load pairs; $|I^m(i, j)|$ is the current of line $ij$ caused by the injection of per unit current in; $W_m$ and $W_n$ are weighted coefficients of generator $m$ and load $n$, corresponding to the rated capacity or actual output of the generator and the actual or peak value of the load, respectively; $G$ and $L$ are sets of generators and loads, respectively. According to circuit equation, the proportion of power from each plant-load pair in line can be clearly described. The method quantifies the role of each line in entire grid power flow.

3.2. Weight module degree

The line weight can consider the connection between the nodes and the strength of the connection in community partition process, it is able to partition the grid more accurately. According to the line weight, this paper redefines the weight module degree as:

$$Q = \sum_s (\alpha_s - \beta_s)^2$$

$$\alpha_s = \frac{1}{2m} \sum_{ij} w_{ij} \delta(c_i, s) \delta(s, c_j)$$

$$\beta_s = \frac{1}{2m} \sum_i w_{ij} \delta(c_i, s)$$

(3)

Where $\alpha_s$ is ratio between sum line weight in community $s$ and sum line weight in network; $\beta_s$ is ratio between sum weight of line connected to node $i$ in community $s$ and sum line weight in network.

3.3. Partition method

Based on the similarity or intensity of connections between nodes, a certain algorithm can be used to divide the network into multiple sub-groups. The algorithm can be divided into splitting algorithm and agglomerative algorithm according to whether the process of partition is to add edge to network or remove edge from network. Single aggregation algorithm and splitting algorithm are not sufficient [20], because the two algorithms are used in this paper. The two algorithms all have certain advantages and shortcomings, so the paper combines two algorithms at different stage. At black start initial phase, the method uses splitting algorithm to set up the initial zones, then uses aggregation algorithm to
compete zone merging. The method calculates the weight module degree of initial zone which is written as $Q^0$. Further, the method assumes merge any two contiguous zone $p$ and $q$ as zone $(p, q)$, calculates the module degree of the zone $(p, q)$ which is written as $Q_{\text{temp}}(p, q)$. If zone $m$ and $n$ are merged as zone $(m, n)$ and zone $(m, n)$ has maximal module degree $\max(Q_{\text{temp}})$, then the method merges zone $m, n$ and defines $Q' = \max(Q_{\text{temp}})$, where $r$ is times of iteration. If the module degree $Q^r$ after $g$ times of iteration is equal to the maximum module degree in the iteration process, which corresponds to the optimal partition scheme. The specific process of power grid partition method is as follows:

(1) Initialize parameter, times of iteration $r = 0$, the initial partition number $N = n_g$, where $n_g$ is number of black start unit,

(2) Calculate the module degree of initial partition $Q^0$,

(3) Construct the edge weight matrix $W_{n \times n}$, $n$ is number of line.

\[ w_{ij} = \sum_{m \in G, n \in L} \sqrt{W_m W_n} f_{mn}(i, j) \]  

(4) Calculate module degree of any two zone $p, q$ as $Q_{\text{temp}}$

\[ Q_{\text{temp}}(p, q) = \begin{cases} 0 & \text{\text{if } } p, q \text{ is not connected} \\ \frac{1}{2m} \sum_{ij} A_{ij} - \frac{k_i k_j}{2m} \delta(c_i, c_j) & \text{\text{if } } p, q \text{ is connected} \end{cases} \]  

(5) Merge two zone $p, q$

\[ \text{IF } Q_{\text{temp}}(p, q) = \max(Q_{\text{temp}}) \Rightarrow \begin{cases} Q_i^{r+1} = Q' \cup Q_k' \\ Q_k^{r+1} = \emptyset \end{cases} \]  

(6) Update the variable

\[ N = N - 1 \]

\[ r = r + 1 \]

\[ Q' = \max(Q_{\text{temp}}) \]  

(7) Return to step (4) until initial partition number $N = 1$

(8) Determine the optimal partition scheme. If $Q' = \max(Q^r)$, the partition scheme is final result.

3.4. Partition result evaluation

If any node $i$ and community $s$ satisfy $Q(i \in s) > Q(i \notin s)$, any node of $s$ connection with all other nodes inside the community is closer than connection with all other nodes outside the community, the community $s$ has strong community structure. The partition method should ensure all zones have strong community structure. The paper introduces the index such as $A$ and $B$ to evaluate the result of partition method. $A$ is the ratio of number of edges in community and total number of edges in the rule
network with same nodes, it reflects the connection tightness inside community. \( B \) is the ratio of number of edges between communities and total number of edges in the network, it reflects the connection tightness between communities. If the \( A \) large value and \( B \) has little value, the partition method is more effective.

4. **Study case**

The paper applies IEEE 39-bus test system to validate the validity of the partition method, the parameters of test system can be found in reference [21]. The betweenness defined in paper and traditional impedance are used to describe weight of line respectively, the corresponding partition results are shown in figure 2 and figure 3. The \( Q \), \( A \) and \( B \) are used to evaluate the result of two partition methods, the comparison is listed in Table 1.

![Figure 2. Partition result with impedance weight.](image1)

![Figure 3. Partition result with betweenness weight.](image2)
Table 1. The comparison between two method

| Method   | Line Weight | A    | B    | Q    |
|----------|-------------|------|------|------|
| Method 1 | Impedance   | 0.3545 | 0.0319 | 0.6235 |
| Method 2 | Betweenness | 0.4713 | 0.0168 | 0.76545 |

With the same grid, the two methods divide test network into six zones. The figure 2 and figure 3 shows the two weight algorithm forms some similar areas, which shows that two models can reflect the local characteristics of the power grid and present the locally community structure. According to the above constraint condition of strong community structure, the six zones in figure 3 are all strong community structure, the zone 2 and zone 6 in figure 2 is weak community structure, it shows the method 1 has worse division effect, the betweenness weight can more effectively measure strength of connection between nodes.

The data in table 1 shows method 2 has larger A and Q than method 1, the nodes are more closely connected inside community. The less B of method 2 shows that the connection between communities is more spars, the flow coupling relationship between communities is weaker.

5. Conclusion
The paper proposes one power grid partition method for black start based on complex network theory. The method is composed of splitting algorithm and agglomerative algorithm. The former algorithm divided grid to some temp zone according to the number of black-start unit, which determines the basic structure of the partition and reduces the number of iterations in agglomerative process. The latter algorithm redefines the line weight and weighted modularity based on flow distribution and network topology, then merges neighbouring temp zone to maximum weighted modularity. The method can reflect the close degree of the partition, accurately assess the partition quality and determine the optimal number of partitions. The simulation results on IEEE 39-bus test system illustrate the validity of this methodology.

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