Non-stationary filtration through a homogeneous rectangular closing dike

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Abstract. A hydraulic structure is an object for the use of water resources, as well as for combating the harmful effects of water. Knowledge of the main parameters of the filtration flow is necessary for solving problems related to the design and operation of hydraulic structures. The article considers filtration calculations of a homogeneous closing dike, which is: determination of instantaneous filtration flow rate and seepage area. It is proved that when the length of the closing dike increases, the height of the seepage area monotonically decreases; the depression curve is constructed; critical time value after which the flow rate takes a constant value equal to the Dupuis flow rate is found.

1. Introduction

Closing dikes are temporary pressure structures designed to protect pits from flooding during the construction or repair of hydraulic structures. Since closing dikes accept hydrostatic loads, they must meet special requirements for stability, strength, and water permeability [1–4]. This is especially typical for the city of St. Petersburg, adjacent to the Finland Gulf (Figure 1). The permanent construction of new ports and berths requires the construction of temporary pressure structures-bridges. Their main task is to protect them from the impact of water flow and prevent flooding. Weather change can significantly affect the upstream water level, and therefore the nature of filtration (flow rate, velocity, seepage level). Filtration is extremely unstable for many factors, it affects the construction conditions; it dictates the choice of structure and design technologies. The further operation of man-made structures is also constantly influenced by the changing behavior of filtration [5, 6].

Water filtration through closing dikes plays a very important role. Statistics show that more than half of all earthen dam accidents were caused by water filtration. Therefore, it is necessary to perform correct filtration calculation: to set the position of the depression curve; to determine the seepage velocity; to determine the filtration flow rate through the dam body and position of seepage point during the design and construction period of closing dikes [7, 8].
Depression curve and seepage area. During the drainage pit water is filtered from the upstream to the downstream [9]. In the case of non-pressure water movement in the pores of the soil the depression curve is formed as a line separating the saturated and unsaturated porous medium (fig. 1). Depression curve is the contact surface coinciding with the atmospheric pressure line in non-pressure motion (free surface). Below the depression curve, the dam's soil is saturated with water (fig. 2).

The determination of the depression curve configuration and break point of the depression curve (seepage area) is necessary to evaluate the groundwater level at any time. This task is important for pumping and water–lowering technology, essentially non-stationary processes.

Seepage area ($\Delta h$) – is a section in a porous body where water is seed into the atmosphere. It is important to take into account the presence of the seepage area, especially for relatively short closing dikes. The removal of soil particles from the porous medium (suffusion) by the filtration stream can occur within the seepage area [10, 11].

However, it is not always possible to define visually the water leak in the soil. This raises the challenge of mathematical definition the value of the seepage area. It is assumed that the seepage area has a "non-stationary nature".

The purpose of this work is to determine the critical value of the length of the closing dike, at which the height of the seepage area is negligible. To archive this goal, the following tasks have to be solved:
- To determine the instantaneous flow rate;
- To determine the seepage area;
- To find the position of the depression curves.

2. Methods
Filtration in ground dams is smoothly changing in most cases, which allows using the hydraulic method based on Darcy’s law and Dupuy’s formulas in filtration calculations. The starting point is the continuity condition for fluid flow [12–14]

\[ m \frac{\partial h}{\partial t} = k \frac{\partial}{\partial x} \left( h \frac{\partial h}{\partial x} \right), \quad (1) \]

where \( h = h(t, s) \) is the depth of the seepage flow (\( t = \) the time, \( s = \) horizontal coordinate); \( k \) = the filtration coefficient; \( q \) = filtration flow; \( m \) is the porosity coefficient, \( 0 < m < 1 \);

At the moment \( t=0 \) tailwater decreases from the value \( h=H \) to the certain value \( h(t,0)=h_0(t) \), \( h_e < h_0(t) < H \). Depression curve drops from the value \( h(0,0)=H \) to a certain value \( h(0,0)=h_0(0) = \text{level of the seepage flow} \).

Set the boundary conditions:

\[ h(0, x) = H, \quad h(t, 0) = h_0. \]

If we start using dimensionless coordinates:

\[ u : h / H; s := x / H, \tau := k t / (m H), u = (\tau, s) \]

Original Boussinesque equation (1) will take the form:

\[ \frac{\partial u}{\partial \tau} = \frac{\partial}{\partial s} \left( u \frac{\partial u}{\partial s} \right), D(u) = (\tau, s : 0 < \tau < \infty, 0 < s < \infty), \]

\[ \ln(u) = (u : 0 \leq u_e \leq u_0 \leq u \leq 1), \quad (2) \]

\[ u(0, s) - 1 = u(\tau, 0) - u_0 = 0. \]

where \( u_e := h_e / H, u_0 := h_0 / H, u_0 \geq u_e \geq 0 \).

Seepage area can be determined as:

\[ \delta h = \frac{\Delta}{H} = e^{-\lambda}, \lambda = \frac{L}{H} \quad (3) \]

where \( L \) = length of the closing dike, \( m \); \( H \) = water pressure, \( m \); \( \Delta \) = geometric height of the seepage area, \( m \).

The water flow through the cross-section \( s=\Lambda \) (where \( \Lambda \) = length of the closing dike) coincides with the Dupuy flow rate, which is equal in the accepted values: \( \theta = \frac{1 - u_e^2}{2 \Lambda} ; \)

Configuration of the depression curve is equal to:

\[ u = \sqrt{u_0^2 + (1 - u_e^2) \frac{s}{\lambda}}, \]

where \( u_0 = u_e + (1 - u_e) \exp(-\lambda) \). Since \( u_e = 0 \), therefore

\[ u_e = \exp(-\lambda). \]

Therefore, the calculation formula takes the form:

\[ u = \sqrt{\exp(-2\lambda) + (1 - \exp(-2\lambda)) \frac{s}{\lambda}}. \]

Dupuis flow rate can be found as:

\[ \theta = \frac{1 - \exp(-2\lambda)}{2\lambda}. \quad (4) \]

Time of establishment of the stationary regime (setting time of seepage area):
Projected instant length of the depression curve on the horizontal axis:

$$l = \sqrt{\frac{1}{3k} \cdot t \cdot H}, 0 < t < T.$$  

3. Results and discussion

Consider the closing dike from the porous medium that separates headwater and tailwater with the length $L=5$ m (fig. 2). Infiltration or evaporation are absent.

Changing the depth of the upstream ($H$) we can consider both short and long closing dikes. For some construction works the water area is drained and tailwater became equal zero. Water filtration occurs due to the difference in the water level. The breakwater is made of sand with filtration coefficient $k=5$ m/day. Let us define the seepage area that occurs due to the water filtration for cases then $\lambda=L/H=0.8, 1, 1.3, 2, 4, 8$. Results for different water pressure ($H$) are presented in Table 1.

| $H$  | $\lambda$ | $\delta h$ | $\theta_d$ | $t=T$ | $L$ (after 24 hours) |
|------|-----------|------------|------------|-------|---------------------|
| 6.25 | 0.8       | 0.4500     | 0.49       | 60    | 3.16                |
| 5    | 1         | 0.3700     | 0.43       | 75    | 2.83                |
| 3.76 | 1.33      | 0.2600     | 0.35       | 100   | 2.45                |
| 2.5  | 2         | 0.1300     | 0.24       | 150   | 2                   |
| 1.25 | 4         | 0.0200     | 0.13       | 300   | 1.41                |
| 0.625| 8         | 0.0003     | 0.06       | 600   | 1                   |

As the value $L/H$ increases, the length of the depression curve increases and the angle decreases (Fig. 3).

![Figure 3. Position of the depression curve.](image)

According to the results, we can see the longer closing dike, the smaller seepage area. This means, that the seepage area problem is actual only for short closing dikes.
A sharp change in a flow rate (0.1-1) is observed than value $L/H$ changes from 0 to 4. After the value $L/H = 4$, the flow rate is almost stationary.

Studies of unsteady filtration are one of the most difficult problems of filtration theory. The main idea of the work is that the Dupuy’s theory fixes the final steady-state values of the filtration flow rate and the configuration of the depression curve [15, 16]. The seepage area, the orthogonal intersection of the depression curve to the upstream side, the tangency of the depression curve to the downstream side in the output point are absent in the Dupuy’s theory.

4. Conclusions

1. The value of the filtration flow rate in the downstream section is monotonically reduced during stabilization from the initial value to the flow rate according to the Dupuy’s formula. In this case, the stabilization time of the flow rate is proportional to the length of the closing dike;
2. The depression curve decreases monotonously over time and the volume of dry soil increases.
3. The height of the seepage area is monotonously reduced from the initial value $u=1$ to the limit value $u=\exp(-\lambda)$ as time passes.
4. The critical value of the closing dike length is approximately $L/H=4$. With value $L/H>4$ the height of the seepage area is negligible.

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