On the vacuum energy in Bohmian mechanics

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Abstract
We consider a universe consisting of a finite number of electrons on Bohmian trajectories. We derive a quasi-vacuum solution for the Schrödinger equation of the electron-system and establish a corresponding invariant vacuum energy density $\Lambda$. The result sheds light on some fundamental issues regarding the vacuum and the cosmological constant.

1. Introduction

The theory of quantum fields in curved space-time is in the absence of a full-fledged theory of quantum gravity only available in a semi-classical formulation. It is governed by the Einstein equations

$$R_{\mu\nu} - \frac{1}{2} R g_{\mu\nu} + \Lambda g_{\mu\nu} = \frac{8\pi G}{c^4} T_{\mu\nu}. \quad (1)$$

$\langle T_{\mu\nu} \rangle$ stands for the energy (density) expectation of the relevant quantum fields. The quantity $\Lambda$ is the cosmological constant, which is proportionate to the expectation value of the vacuum-energy density, which in cosmology is deemed responsible for the accelerated expansion of the Universe

$$\Lambda \cdot g_{\mu\nu} = \frac{8\pi G}{c^4} \langle 0 | T_{\mu\nu} | 0 \rangle$$

$R_{\mu\nu}, 0 \leq \mu, \nu \leq 3,$ and $R$ denote as usual the Ricci-tensor and Ricci-scalar, respectively. There is a significant discrepancy between the value of $\Lambda$, which is experimentally determined from the measured expansion of the Universe and from data of the cosmic-microwave background, and the one resulting from theoretical calculations in quantum field theory. Whereas the experimental data suggest $\Lambda \approx 1 \cdot 10^{-121}$ Planck Units [1], the theoretical value, by whatever method it is calculated, is much larger [2]. Possible explanations for the discrepancy are that either not all the fields play a role or that the contributions cancel each other.

Because of the symmetry of the vacuum, the expectation term $\langle 0 | T_{\mu\nu} | 0 \rangle$ in Minkowski space must have the form

$$\langle 0 | T_{\mu\nu} | 0 \rangle = \varphi_0 \cdot \eta_{\mu\nu}, \quad (3)$$

with a constant $\varphi_0 \in \mathbb{R}$ [3]. This implies Lorentz-invariance of $\langle 0 | T_{\mu\nu} | 0 \rangle$ and energy-momentum conservation $\partial_\nu T_{\mu\nu} = 0$. The generalization to arbitrary metric fields $\varphi_0 \cdot g_{\mu\nu}$ still locally preserves energy [4], because of the choice of an affine connection, but it is no longer clear, whether $\langle 0' | T_{\mu\nu}' | 0' \rangle$ still is the energy expectation of a vacuum state, since for general metric fields $g_{\mu\nu}$ the state $| 0' \rangle$ does no longer necessarily represent a state with no particles, as the Davis-Unruh effect demonstrates [4, 5]. There arises the question what the vacuum really signifies [3]. Einstein introduced the constant $\Lambda_0$ as a bare cosmological energy, without any radiation or matter fields present. Adding quantum field vacuum-energy would then amount to the

1 By a factor $10^{121 - 10^{125}}$ [2].
2 $\eta_{\mu\nu}$ is the Minkowski metric.
3 Of course, in QFT $\langle 0 | T_{\mu\nu} | 0 \rangle$ is Lorentz-invariant by construction.
4 In contrast to the general non-vacuum case.

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effective cosmological constant

\[ \Lambda = \Lambda_0 + \eta \langle E_{\text{vac}} \rangle \]  

(4)

with a suitable constant \( \eta \). As mentioned above, there are two immediate strategies\(^5\) to explain the measured contribution \( \eta \langle E_{\text{vac}} \rangle \) to \( \Lambda \). Either one shows that some contributions cancel each other, or that some fields do not play a role. Supersymmetry would do the job along the first strategy, but it is clearly broken in nature. We take a path along the second one. In order to approach the question of \( \Lambda = \eta \langle E_{\text{vac}} \rangle \) we can look at the ontology of quantum physics. There is one interpretation, which allows to select some fields. Our choice for the ‘beables’ is persistent fermionic particles on Bohmian trajectories [6]. We make this choice for different reasons. First of all, because it is a simple ontology, which clearly decides that the primary reality is matter and by taking the fermion sectors of the standard model we cover the bulk of it. Second, because it clarifies the status of space, which is a property, a relation between pieces of matter. Finally, by taking the particle-ontology we omit the fallacies of the field-ontology [7]. We will in this paper rigorously develop the model for the electron sector.

Very few rigorous results of the Bohmian evolution of quantum fields are known due to the complicated nature of the equations [6]. We can, however, make qualitative statements subsuming e.g. their interaction with the electron sector effectively in a time dependent ‘external’ potential. We will thus work in a universe with a number of \( N \) electrons, moving on Bohmian trajectories. We consider electromagnetic and gravitational forces acting on them in such a way, that the guiding equation evolves in a, yet to be determined, quasi-vacuum and then derive \( \Lambda \) in this model, by bridging in a suitable manner from Minkowski space to general manifolds and showing that the resulting energy tensor is of the form (3).

2. The electron gas

2.1. The setting

We will use the Bohmian picture, as done in [6], and think of the electron gas as a large number \( N \in \mathbb{N} \) of electrons, which move along trajectories defined by a spinor guiding-field \( \Psi(t) \in \mathcal{H}^{1,N} \), \( \Psi_{\text{t}}(t) \in \mathcal{H} = L^2(B_R, \mathbb{C}^N), B_R = \{ x \in \mathbb{R}^3 | |x| \leq R \} \), satisfying

\[ v^k(x_k, t) = \frac{\psi_k^* \alpha_k \psi_k}{\psi_k^* \psi_k} \]  

(5)

for the velocity of the \( k \)-th particle and with guiding equation for the field \( \Psi(t) \)

\[ i\hbar \partial_t \Psi = H^N \Psi. \]  

(6)

The Hamiltonian is

\[ H^N = H_0 + H_{EM} + H_G = \sum_{k=1}^{N} (H^k_0 + H^k_{EM} + H^k_G). \]  

(7)

The single components of \( H^N \) are: first the free Dirac Hamiltonian \( H_0 = \sum_{k=1}^{N} H^k_0 \), where

\[ H^k_0(x_k) = -im \nabla_{x_k} + \beta mc^2. \]  

(8)

The matrices \( \alpha^l, 0 \leq l \leq 3 \), and \( \beta \) are \( \mathbb{C}^{4 \times 4} \)-matrices, which can be expressed in terms of the Dirac matrices \( \gamma^l \). With \( \gamma^0 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \) and \( \gamma^l = \begin{pmatrix} 0 & \sigma^l \\ -\sigma^l & 0 \end{pmatrix} \), where \( \sigma^l \) are the Pauli \( \mathbb{C}^{2 \times 2} \)-matrices, there holds \( \beta = \gamma^0 \) and \( \alpha^l = \gamma^0 \gamma^l \). The Dirac matrices satisfy the known commutator-relations

\[ \gamma^k \gamma^l + \gamma^l \gamma^k = 2 \eta^{kl}. \]

The second component in (7) is the electromagnetic interaction \( H_{EM} = \sum_{k=1}^{N} H^k_{EM} \), with \( H^k_{EM} = \mathbb{I}^{3(k-1)} \otimes H_{EM}(x_k) \otimes \mathbb{I}^{2(N-k)}. \) The Hamiltonian is

\[ H_{EM}(x_k) = \frac{1}{\alpha} \sum_{j=1}^{3} \sum_{\mu=0}^{3} \gamma^\mu \epsilon A^j_\mu. \]  

(9)

\(^5\) Many others are of course thinkable, like new fields, extra dimensions or simple fine-tuning by renormalization.
The zero-component of $H_{EM}^k$ in (9) is
\[ H_C(x_k) = \frac{1}{2} \sum_{j=k}^3 \gamma^0 e A_j^k(x_k) = \frac{1}{2} \sum_{j=k}^3 \gamma^0 e \Phi^j(x_k), \tag{10} \]
with Coulomb-potential $\Phi^j(x_k) = \frac{e}{4\pi \varepsilon_0 |x_k - x_j|}$. The space-like components of $H_{EM}^k$ constitute the magnetic interaction induced by the charges, which move by (5) with velocity $v^j$, $1 \leq j \leq N^k$,
\[ H_B(x_k) = \frac{1}{2} \sum_{j=k}^3 \gamma^i e A_i^j(x_k) = \frac{1}{2} \sum_{j=k}^3 \gamma^i \frac{e v^j}{4\pi |x_k - x_j|}. \tag{11} \]
Finally there is the gravitational interaction $H_G = \sum_{k=1}^N H_G^k$, which we model like the Coulomb interaction $H_G^k = \sum_{k=1}^N H_G^k = \sum_{k=1}^N$ with Hamiltonian
\[ H_G(x_k) = \frac{1}{2} \sum_{j=k}^3 \gamma^0 m_e U^j(x_k), \tag{12} \]
and Newtonian potential $U^j(x_k) = -\frac{m_e G}{|x_k - x_j|}$, where $m_e$ is the electron mass. In order to ensure that all the components of the Hamiltonian $H^N$ are well defined, we must as usual assume an ultraviolet cut-off, implying for the momenta $|p_i| \leq P$, $0 \leq j \leq N$ and an infrared cut-off, implying for position $|x_i| \leq R$. Finally, we can model the effects of the other sectors of the standard model by use of an effective potential
\[ H_N = \sum_{k=1}^N V_k(t, x_k), \tag{13} \]
with $V_k(t, x) = \sum_{k=1}^N V(t, x) \otimes \sum_{k=1}^N$. We will begin by making the assumption that $V(t, x) = 0$ and work with product states $\Psi(t) = \wedge_{i=1}^N \Psi_i$. To justify this we will construct an approximate vacuum-type solution $\Omega_t = \wedge_{i=1}^N \omega_i(t)$ of (5), (6) such that no mixing occurs.

2.2. Approximate solution
It is a useful strategy [6] to try to simplify the system of equations (5), (6) by choosing states $\Psi \in \mathcal{H}^{N^k}$, satisfying for all interactions $H_i^k$, $i = B, C, G$, and corresponding constants $E_i$
\[ \langle \Psi | H_i^k | \Psi \rangle \approx E_i. \tag{14} \]
These $\Psi$ are then solutions of a much simpler, effective guiding equation, which with $E_i = E_B + E_C + E_G$ and $\tilde{H}^N = \sum_{k=1}^N (H_B^k + E_i)$ is
\[ i\hbar \partial_t \Psi = \tilde{H}^N \Psi. \tag{15} \]
To find a candidate-solution of (15), we first search for $\Omega_0$ such that
\[ H_0 \Omega_0 = \sum_{k=1}^N H_0^k \Omega_0 = E_0 \Omega_0. \tag{16} \]
Since the Dirac operators $H_0^k$ commute pairwise, we find $\Omega_0$ and $E_0$, as explained in [6], by filling successively the $N$ lowest negative eigenstates $\omega_i$, $i \leq N$, of $H_0(x)$. Since $\frac{\pm E_p}{c} = \frac{\sqrt{p^2 + m_e^2 c^2}}{c}$, and since there are two states (spin up and down) for each energy level, we will have a minimum for $N = 2|\mathcal{P}_B|$, where $\mathcal{P}_B = \{ p = (p_1, p_2, p_3) \in \mathbb{R}^3 \mid p \leq P \}$, $\Omega_0 = \omega_1 \wedge \ldots \omega_k \wedge \ldots \omega_N$ is called the vacuum². In out context the vacuum is not a situation with no particles but a situation, where a system of $N$ free particles does have minimum energy by filling all the negative energy-states available and the particles are equally distributed within the region $B_R$.
This corresponds to the original intuition of Dirac [8]. If we can show that for $i = B, C, G$ and fixed $t = t_0$ there holds
\[ \langle \Omega_0 | H_i^k \Omega_0 \rangle \approx E_i, \tag{17} \]
then the states
\[ \Omega_t = e^{-\frac{i}{\hbar} (E_B + E_G) t} \Omega_0 \tag{18} \]
are a vacuum-like solution of equation (15).

6 We work in the Lorentz-gauge.
7 We consider $\Omega_0$ to be normalized $\langle \Omega_0 | \Omega_0 \rangle = 1$. 

3
Let us calculate (17) for each interaction term in (7) individually. Due to the normalization constants and the definition of the inner product on $\mathcal{H}^N$, it follows for each $k$, that

$$\langle \Omega_0 | H^k | \Omega_0 \rangle = \frac{1}{N} \sum_{n=1}^{N} \langle \omega_n | H^k | \omega_n \rangle.$$  

(19)

For the Coulomb interaction we get

$$\langle \omega_n | H^k | \omega_n \rangle = \frac{1}{2} \sum_{j<k} \int_{B_R} \omega_n^* \gamma^0 \gamma^j \phi_j(x-y) \omega_n \, d^3y = \frac{1}{2} \sum_{j<k} \int_{B_R} e^2 \frac{e^2}{4\pi \varepsilon_0 |x-y|} |\omega_n|^2 \, d^3y.$$  

(20)

Therefore we have with (19)

$$E_C(x) = \langle \Omega_0 | H^k | \Omega_0 \rangle = \frac{1}{2} \frac{1}{N-1} \sum_{n=1}^{N} \int_{B_R} e^2 \frac{e^2}{4\pi \varepsilon_0 |x-y|} |\omega_n|^2 \, d^3y.$$  

(21)

Since $|\omega_n|^2 = \frac{1}{|B_R|}$ and because of the law of large numbers, (21) is (approximately) a constant $E_C$ for $N$ large enough. The potential of a uniformly distributed charge $e$ $|\omega_n|^2$ over a sphere of radius $R$ at distance $|x| < R$ is known to be

$$\varphi(|x|) = \frac{e}{4\pi \varepsilon_0} \left( \frac{3R^2 - |x|^2}{2R^2} \right).$$  

(22)

From here we can calculate the corresponding expectation value $E_C$ directly to get with $Q_e = (N - 1)e$

$$E_C = \frac{3}{5} \cdot \frac{Q_e}{4\pi \varepsilon_0 R}.$$  

(23)

The structure of the gravitational Hamiltonian $H_G$ in (12) immediately suggests that by exactly the same arguments we arrive with $M_G = (N - 1)m_e$ at

$$E_G = \frac{3}{5} \cdot \frac{M_e m_e G}{R}.$$  

(24)

We will later include the factor $\frac{3}{5}$ with suitable other factors into the definition of $Q_e$ and $M_G$ for the sake of simplicity of notation.

For the interaction between matter and the induced magnetic fields we have

$$\langle \omega_n | H^k_B | \omega_n \rangle = \frac{1}{2} \sum_{j<k} \int_{B_R} \omega_n^* \gamma^0 \gamma^j \omega_n \, d^3y = \frac{1}{2} \sum_{j<k} \int_{B_R} e^2 \frac{e^2}{4\pi |x-y|} |\omega_n|^2 \, d^3y.$$  

(25)

Therefore

$$E_B^k(x) = \langle \Omega_0 | H^k_B | \Omega_0 \rangle = \frac{1}{2} \sum_{j<k} \frac{1}{N-1} \sum_{n=1}^{N} \int_{B_R} e^2 \frac{e^2}{4\pi |x-y|} |\omega_n|^2 \, d^3y.$$  

(26)

The terms in the number $E_B^k$ are depending on $n$ and and hence there is no straightforward application of the law of large numbers at the level of an individual component $k$. It is, however, possible to control $E_B^k$ if we go back to the definition of the currents by the spinor-fields. For the Bohmian trajectories we have by construction from the Dirac equation $|\psi|^2 = \left| \frac{\omega_k^\dagger \omega_k}{\omega_k^\dagger \omega_k} \right| < c$. Therefore there are real numbers $\{\lambda_{ij}, |\lambda_{ij}| < 1\}$, such that

$$\langle \psi^i | \psi^j \rangle = \lambda_{ij} e^2.$$  

(27)

Together with the fact that $\varepsilon_0 = \frac{1}{\mu_0 e^2}$ we can rewrite (26)

$$E_B^k(x) = \langle \Omega_0 | H^k_B | \Omega_0 \rangle = \frac{1}{2} \sum_{j<k} \frac{1}{N-1} \sum_{n=1}^{N} \int_{B_R} \lambda_{ij} e^2 \frac{e^2}{4\pi \varepsilon_0 |x-y|} |\omega_n|^2 \, d^3y.$$  

(28)

In order to continue we make the plausible assumption, that we can replace the individual numbers $\lambda_{ij}$ by the average $\bar{\lambda} = \frac{1}{N^2} \sum_{j,n=1}^{N} \lambda_{ij}$, which stays bounded, $|\bar{\lambda}| < 1$, for all $N$, to get
Having this in mind, we will seek to bridge from the vacuum-light clock8 with energy \( E_{\nu} \) to the vacuum-energy of the matter-field \( E_0 \), which amounts to

\[
\sum_{\nu} \frac{\hbar \nu}{2} \quad \text{per mode } \nu.
\]

Having this in mind, we will seek to bridge from quantum mechanics to general relativity in the second part of the paper and find an invariant \( \Lambda \).

3. Vacuum energy density

3.1. Energy terms and coupling-constants

Based on the results in section 2, we calculate the acceleration of a test-electron, which is positioned at the spatial-edge \( |x| = R \) of the model-universe. In [9] a universal observer is introduced, who measures the duration until an event has happened by means of a thermal clock. In order to have consistent duration-intervals, accelerated observers have to synchronize their clocks and in [10] it is shown that the synchronization of a thermal clock in the local rest-frame of an observer with acceleration \( \kappa \), and a single-mode vacuum light clock9 with energy \( E_{\nu} = \frac{\hbar \nu}{2} \) leads independently of \( \nu \) to the relation

\[
\frac{4}{\hbar} k_B T_\kappa \frac{e^2}{\kappa} = \frac{1}{\pi^2}. \tag{31}
\]

In (31) \( k_B \) and \( h \) denote, as usual, the Boltzmann and Planck-constants. The temperature \( T_\kappa \) is hence

\[
T_\kappa = \frac{h c}{2 \pi k_B}, \tag{32}
\]

which is the Unruh-Davies temperature. Relations (31) and (32) will now serve as the link between quantum mechanics and general relativity.

Assume that we work in a local Minkowski space, namely the local rest frame of a test-electron at distance \( |x| = R \). In paragraph 2 we calculated the energies \( E_0, E_C \) and \( E_G \) of \( \Omega_0 \), from which we can now derive the corresponding accelerations

\[
\bar{g}_R = \frac{2 \partial_R E_G}{m_e c^2} = \frac{2 M_e G}{c^2 R^2}, \quad \bar{a}_R = \frac{2 \partial_R E_C}{m_e c^2} = \frac{Q_e e}{2 \pi \varepsilon_0 m_e c R^2}. \tag{33}
\]

Considering equations (23), (24) and (29), we define for the rest of the paper \( Q_e = N \left( 1 + \frac{3}{5} \right) \frac{e}{2} \) and

\[
M_e = N \left( \frac{2}{5} m_e \right). \tag{34}
\]

The vacuum-energy \( E_0 \) of the matter field can be considered as a constant of integration. We now plug the accelerations (33) into relations (31), (32) individually10. Defining \( k_C = \frac{1}{4 \pi \varepsilon_0} \) we obtain the following chain of expressions involving \( a_R \)

\[
\frac{2}{\hbar} k_B T_{\kappa_0} \frac{R^2 m_e c^3}{k_C Q_e} = \frac{1}{\pi^2}, \tag{34}
\]

8 We mean in this context rather the core of a clock, since we do not need devices to indicate time.

9 In order to later on define a four metric by (33), we multiply the accelerations for dimensional reasons by the Lorentz-scalar \( \frac{1}{c^2} \).

10 This approach is consistent, since it ensures \( \frac{4}{\hbar} k_B k \frac{e^2}{a_{\text{rel}}} = \frac{1}{\pi^2} \), where \( a_{\text{rel}} = a_m + a_e \).
and arranged differently
\[ \frac{k_B T_{sa} A_R e^2}{4} \cdot \frac{m_e c}{\hbar} \cdot \frac{1}{k_C Q_e} e = 1. \]  

(35)

Here \( A_R = 4\pi R^2 \) is the surface-area of a sphere of radius \( R \). The term \( \lambda_C = \frac{m_e c}{\hbar} \) is the (reduced) Compton wave-length and by multiplying both sides of (35) by \( \frac{1}{\lambda_C} \) we arrive at
\[ k_B T_{sa} A_R e^2 = \frac{\lambda_C}{k_C Q_e} e = E_C(\lambda_C). \]  

(36)

Making use of (32) and simplifying notation \( E_C(\lambda_C) = \frac{k_C Q_e}{\lambda_C} e = E_C^\lambda \), we finally get
\[ g_R \cdot A_R = \frac{8\pi \lambda_C^2}{\hbar c} E_C^\lambda. \]  

(37)

Analogously we get for \( g_R \) the following chain of expressions
\[ \frac{2}{k_B T_{sa}} R^2 \frac{e^2}{M_G} = \frac{1}{\pi^2}. \]  

(38)

Arranged differently and with the Planck length \( l_p = \sqrt{\frac{G\hbar}{c^3}} \)
\[ k_B T_{sa} \frac{4\pi R^2}{4 M_e c} \cdot \frac{e^2}{\hbar G} = k_B T_{sa} \frac{A_R}{4 l_p^2} = \frac{E_M}{c^2}, \]  

(39)

where \( E_M = M_e c^2 \). By again making use of (32) we finally get in analogy to (37)
\[ g_R \cdot A_R = \frac{8\pi l_p^2}{\hbar c} E_M. \]  

(40)

We have found the two coupling- constants \( \alpha_C, \alpha_G \)
\[ \alpha_C = \frac{8\pi \lambda_C^2}{\hbar c}, \quad \alpha_G = \frac{8\pi l_p^2}{\hbar c} = \frac{8\pi G}{c^4}. \]  

(41)

With \( \Sigma_0 \) denoting the spatial volume of the Universe we define the energy densities \( e_0 = \frac{E_0}{\Sigma_0}, \quad e_\lambda = \frac{E_C^\lambda}{\Sigma_0} \)
and \( e_M = \frac{E_M}{\Sigma_0} \) and are now in a position that the tensor
\[ T_{\mu \nu} = (\alpha_G (|e_0| - e_M) + \alpha_C e_\lambda^2) g_{\mu \nu}, \]  

(42)

has the necessary features of an energy tensor (3). It is invariant and the divergence is zero. The energy terms (and coupling-constants) were explicitly constructed by considering the Unruh effect.

3.2. Cosmological constant

The scalar factor in equation (42) can be considered as a cosmological constant
\[ \Lambda = \alpha_G (|e_0| - e_M) + \alpha_C e_\lambda^2 > 0. \]  

(43)

Although we describe a universe with matter, the structure of the corresponding Einstein equations
\[ R_{\mu \nu} - \frac{1}{2} R g_{\mu \nu} + \Lambda g_{\mu \nu} = 0 \]  

(44)

resembles the one of a vacuum. One component of \( \Lambda \) is indeed the vacuum-energy density \( e_0 \) of the electron matter-field. The vacuum energy of the radiation-field, however, is not explicitly present, right in accordance with the Bohmian ontology. It was fundamental though in deriving relationship (31)/(32) and enters indirectly in a fundamental way. Moving observers synchronize their clocks with the vacuum light clock, which is the Unruh effect. Since the energy terms, which gauge the duration, are invariant, there results a curved geometry. The electromagnetic energies contribute to \( \Lambda \) directly with a specific coupling-constant \( \alpha_C \), which is larger than the gravitational coupling-constant \( \alpha_G \). Of course, there is no general equivalence principle in electromagnetism like in gravity. In our model-universe it results from the universality of \( m_e \).

11 Note that the two coupling constants coincide for the Planck mass \( m_p = \sqrt{\frac{\hbar c}{G}} \).
12 The derivation of Einstein equations from (36) and (39) in a static metric is shown in [10–12].
4. Discussion

We made the assumption in paragraph 2.1 that the effective potential $V(t, x) = 0$. If this is not the case, then it is show in [6] that what happens is, that some components of the vacuum state $\Omega_l$ develop into excited states above the ground state $\omega_1 \ldots \omega_N \rightarrow \theta \wedge \omega_2 \ldots \wedge \omega_N$. In the Einstein equations the energy contribution by $\theta$ will enter the ordinary, matter-induced energy term $\langle T_{\mu\nu} \rangle$. So we can expect over time that the number of particles following a vacuum trajectory $\omega_i$ is going to decrease, $N(t) < N$, and hence $\Lambda = \Lambda(t)$ is time-dependent and decreasing. We developed the equations in the intuition of the Bohmian ontology, where permanent particles move in space and distances are relational properties [13]. If during this process the spatial distances increase and hence $\Sigma_1 > \Sigma_0$, then this fact is another source for a decreasing cosmological factor (30). The explicit contributions of the evolution of other fermionic sectors to the vacuum energy is shown in [11]. In the Einstein equations the constant of the vacuum sector to the vacuum energy $\Lambda$ is beyond mathematical reach at the moment, simply because the system of equation (5) is very complicated. Due to the structure of the underlying forces (e.g. asymptotic freedom), we would however conjecture that there might be similar vacuum contributions by other leptons, behaving similarly to electrons, but that any vacuum states for quarks very quickly develop into ordinary matter contributions above a ground state, which is indeed in confirmation with recent results [14]. Observed data today are interpreted as the result of an accelerated, expanding space-time, making a cosmological term necessary. The model we have chosen helps to show why there is a cosmological factor $\Lambda$ at the first place and what its sources might be, namely some of the fermion sectors in quantum field theory. In addition we saw that this cosmological factor, being a percentage of the number of excited particles $N(t)$, is time-dependent and decreasing, which can help to explain the small value of $\Lambda$ today. In fact, in the theory of an expanding universe the constancy of $\Lambda$ has to be assumed a priori and can only be explained in our model universe by the introduction of a bare cosmological constant $\Lambda_{0}(4)$, which has no relation to any known fields.

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References

[1] Barrows J D and Shaw D J 2011 The value of the cosmological constant *Gen. Relativ. Gravit.* 43 2555–60
[2] Perlmutter S 2003 Supernovae, dark energy, and the accelerating universe *Phys. Today* 56 53–60
[3] Rugh S E and Zinkernagel H 2002 The quantum vacuum and the cosmological constant problem *Studies in History and Philosophy of Modern Physics* 33 663–705
[4] Unruh W G 1976 Notes on black hole evaporation *Phys. Rev.* D14 870
[5] Davies P C W 1975 Scalar production in Schwarzschild and Rindler metrics *Journal of Physics A: Math. Gen.* 8 609
[6] Deckert D-A, Esfeld M and Oldofredi A 2017 Persistent particle ontology for quantum field theory *Phys. Rev. Lett.* 116 201101
[7] Synthese *International Journal of Quantum Foundations* 3 65–77
[8] Schlatter A 2017 Duration and its relationship to the geometry of space-time *International Journal of Quantum Foundations* 3 119–25
[9] Verlinde E 2010 On the origin of gravity and the laws of newton *J. High Energy Phys.* 4 29
[10] Jacobson T 2016 Entanglement equilibrium and the einstein equation *Phys. Rev. Lett.* 116 201101
[11] Esfeld M, Deckert D-A and Oldofredi A 2017 What is matter? The fundamental ontology of atomism and structural realism *Synthese* 194 2329–44
[12] Brodsky S J and Shrock R 2011 Condensates in quantum chronodynamics and the cosmological constant problem *PNAS* 108 34–50