Magnetic properties of ground-state mesons

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Starting with the bag model a method for the study of the magnetic properties (magnetic moments, magnetic dipole transition widths) of ground-state mesons is developed. We calculate the M1 transition moments and use them subsequently to estimate the corresponding decay widths. These are compared with experimental data, where available, and with the results obtained in other approaches. Finally, we give the predictions for the static magnetic moments of all ground-state vector mesons including those containing heavy quarks. We have a good agreement with experimental data for the M1 decay rates of light as well as heavy mesons. Therefore, we expect our predictions for the static magnetic properties (i.e., usual magnetic moments) to be of sufficiently high quality, too.

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I. INTRODUCTION

The magnetic moments are among the fundamental properties of every hadron. They play an important role in the understanding of the hadronic structure. Theoretically the magnetic moment is related to the magnetic form factor of the hadron. For instance, it can be obtained by the extrapolation of the magnetic form factor \( G_M(Q^2) \) to zero momentum transfer. In the face of their importance, the magnetic moments of mesons have not received much interest except for the \( \rho^+ \) meson whose properties were studied quite intensively using various approaches [1–19]. The \( K^* \) mesons have received less attention [2–4, 6, 7, 9, 19], and we have found only three attempts [19–21] to give some estimates for the magnetic moments of heavy mesons. We think it is timely to pay the debt. Therefore, the study of the magnetic moments of heavy mesons was the initial object of our investigation. Because of the short lifetime the direct measurement of the magnetic moments of vector mesons seems to be hardly possible. The indirect estimate is possible [22], but it still suffers from the large uncertainties. So we need some other observable to check the reliability of our predictions. To this end we study the magnetic dipole (M1) transitions of these mesons. In this case the experimental situation is better [23]. In addition, there are also plenty of theoretical predictions obtained using various approaches comprising the quark model and the vector dominance model, nonrelativistic QCD, the potential model, various relativistic or semirelativistic models, the bag model (including chiral extensions), the light front quark model, models based on Bethe-Salpeter equation, QCD sum rules, numerical simulations of lattice QCD, chiral models, the statistical model, the Nambu-Jona-Lasinio model, the constituent quark-meson model, and the dispersion relation approach.

From a theoretical standpoint a static magnetic moment of a single quark is the long wavelength limit of the M1 transition (spin-flip) moment of this quark. Hence, these magnetic properties of hadrons are closely related, and, if we succeeded in predicting M1 decay rates, we would get some confidence that the predictions for magnetic moments were also reliable. We are going to implement this plan with the help of the modified bag model which was used earlier to calculate masses of light and heavy hadrons, magnetic moments, and M1 decay widths of heavy baryons.

The remainder of the paper is as follows. In sect. II the short description of our version of the bag model is given, and the formalism we use to treat the magnetic properties of the hadrons is presented. In sect. III the predictions for the M1 transitions moments and partial decay widths are given. They are compared with the results obtained in other approaches and with experimental data. Our predictions for the magnetic moments of ground-state vector mesons are presented in sect. IV. The last section serves for the summary and discussion.

II. BAG MODEL AND MAGNETIC OBSERVABLES

The MIT bag model in the static cavity approximation is a simple intuitive approach to hadron structure (see also the excellent review [123]). We use the modified version designed to reconcile the initial ultrarelativistic model with the heavy quark physics. Here we review the main features of this model (for details we refer to ref. [118]).

It is assumed that quarks are confined in the sphere of...
radius $R$, within which they obey the free Dirac equation. The four-component wave function of the quark in the $s$-mode is given by

$$
\Psi_{1/2}^s(r) = \frac{1}{\sqrt{4\pi}} \left( G(r) - i(\sigma \cdot \hat{r})F(r) \right) \Phi_{1/2}^s,
$$

(1)

where $\Psi_{1/2}^s$ is two-component spinor, $\sigma$ are usual Pauli matrices, and $\hat{r}$ is unit radius-vector. Solutions of the free Dirac equation in the spherical coordinate system are simple Bessel functions, so that

$$
G(r) = N j_0(pr),
$$

(2)

$$
F(r) = -N \sqrt{\frac{\varepsilon - m}{\varepsilon + m}} j_1(pr),
$$

(3)

where $\varepsilon^2 = p^2 + m^2$, and

$$
j_0(x) = \frac{\sin x}{x}, \quad j_1(x) = \frac{\sin x}{x^2} - \cos x.
$$

(4)

The normalization factor $N$ is

$$
N = \frac{p^2}{\sin(pR) \sqrt{2\varepsilon(\varepsilon R - 1) + m}}.
$$

(5)

The eigenenergy of the quark is determined by the matching condition at the bag boundary

$$
G(R) = -F(R),
$$

(6)

from which using eqs. (2)–(4) one obtains

$$
\tan(pR) = \frac{pR}{1 - mR - \varepsilon R}.
$$

(7)

The energy of the bag associated with a particular hadron is given by

$$
E = \frac{4\pi}{3} BR^3 + \sum_i \varepsilon_i + E_{\text{int}},
$$

(8)

where $R$ denotes the bag radius, and $B$ is the bag constant. The entries on the right-hand side of this expression are the bag volume energy, the sum of single-particle eigenenergies, and the quark-quark interaction energy due to one-gluon-exchange. Minimization of the energy determines the bag radius $R_H$ of particular hadron. $E_{\text{int}}$ represents the interaction energy of quarks in the Abelian approximation to QCD and is comprised of color-electric and color-magnetic parts as described in ref. [118] in detail. Equation (8) differs from the usual MIT bag energy in that we have omitted the Coulomb-type self-energy and Casimir energy terms. The interaction energy $E_{\text{int}}$ depends on the running coupling constant, for which we employ the parametrization proposed in ref. [124].

$$
\alpha_e(R) = \frac{2\pi}{9 \ln(A + R_0/R)}.
$$

(9)

In eq. (9) $R_0$ is the scale parameter analogous to the QCD constant $\Lambda$. The parameter $A$ helps to avoid divergences when $R_0 \to R$. Other differences from the original MIT bag model are the effective (running) quark mass

$$
m_f(R) = m_f + \alpha_e(R) \cdot \delta_f,
$$

(10)

and the corrections for the center-of-mass motion (CMM). Parameters $m_f$ and $\delta_f$ are necessary to define the mass functions $m_f(R)$ for each quark flavor.

The CMM corrected bag energy $M$ is identified with the mass of the hadron. It is related to the uncorrected energy $E$ by

$$
M^2 = E^2 - P^2,
$$

(11)

where

$$
P^2 = \gamma \sum_i p_i^2
$$

(12)

is the effective momentum square, $\gamma$ is the parameter governing the CMM prescription, $p_i = \sqrt{\varepsilon_i^2 - m_i^2}$ represent momenta of individual quarks, and $m_i$ is the effective quark mass given by eq. (10).

To fix the model parameters $B$, $\gamma$, $A$ and $R_0$ the experimentally observed masses of the light hadrons $N$, $\Delta$, $\pi$ and $\rho$ were chosen. To fix the mass function parameters $m_s$, $\delta_s$, $m_c$, $\delta_c$, $m_b$, $\delta_b$ we have employed the masses of vector mesons ($\phi$, $J/\psi$, $Y$) and the mass values of the corresponding lightest baryons ($\Lambda$, $\Lambda_c$, $\Lambda_b$). The lightest (up and down) quarks are assumed to be massless. The fitted numerical values are $B = 7.301 \times 10^{-14}$ GeV$^4$, $\gamma = 1.785$, $A = 0.7719$, $R_0 = 3.876$ GeV$^{-1}$, $m_s = 0.2173$ GeV, $\delta_s = 0.1088$ GeV, $m_c = 1.456$ GeV, $\delta_c = 0.1003$ GeV, $m_b = 4.746$ GeV, and $\delta_b = 0.0880$ GeV.

The magnetic moment of a hadron is obtained from the usual definition

$$
\mu = \frac{1}{2} \int d^3x [x \times j_{\text{em}}],
$$

(13)

where $j_{\text{em}}$ is the Dirac electromagnetic current (see ref. [125]). After some simple algebra one obtains

$$
\mu = \sum_i \mu_i \langle h \uparrow | e_i \sigma_i | h \uparrow \rangle,
$$

(14)

where $e_i$ is the charge of the corresponding quark and $\mu_i$ is it’s reduced (charge-independent) magnetic moment

$$
\mu_i = \int d^3r \frac{2r}{3} G_i(r) F_i(r).
$$

(15)

Upon evaluating the above matrix element for the reduced magnetic moment of a quark confined in the bag of radius $R_H$ we obtain

$$
\mu_i = \frac{1}{6} 2R_H (2\varepsilon_i + m_i) - \frac{1}{6} 2\varepsilon_i (\varepsilon_i R_H - 1) + m_i.
$$

(16)
The calculation of spin-flavor matrix elements is somewhat lengthy but straightforward task. Using the technique described in ref. [123] for the vector meson made of quark \( q_a \) and antiquark \( \bar{q}_b \) we obtain
\[
\mu = e_a \mu_a + \bar{e}_b \mu_b, \tag{17}
\]
where \( e_a (\bar{e}_b) \) is the charge of the corresponding quark (antiquark). Note that \( e_a \mu_a (\bar{e}_b \mu_b) \) in the last equation represents the magnetic moment of the individual quark (antiquark).

For the M1 decay widths of vector mesons we use the expression derived in ref. [72], though written in a slightly different fashion
\[
\Gamma_{V \to PS} = \frac{4\alpha k^3}{3} (\mu_{V \to PS})^2, \tag{18}
\]
where \( k = (M_V^2 - M_{PS}^2)/2M_V \) is the photon momentum in the rest frame of decaying a particle, \( \alpha \approx \frac{1}{137} \) is the fine structure constant, and the transition moments connecting vector and pseudoscalar mesons are
\[
\mu_{V \to PS} = e_a \mu_a^{TR} - \bar{e}_b \mu_b^{TR}. \tag{19}
\]

The reduced transition moment \( \tilde{\mu}_{i}^{TR}(k) \) of an individual quark depends on the momentum of the emitted photon \( k \) and is given by
\[
\tilde{\mu}_{i}^{TR}(k) = \int r^2 dr j_1(kr) [G_{i1}(r) F_{2i}(r) + G_{2i}(r) F_{1i}(r)]. \tag{20}
\]

Indices 1 and 2 denote the initial and final particle. In the limit \( k \to 0 \), with \( R_1 = R_2, G_i(r) = G_2(r) \), and \( F_1(r) = F_2(r) \), eq. (20) reproduces the expression (15) for the static magnetic moment.

For the magnetic dipole decay of the pseudoscalar meson instead of eq. (15) we have the expression
\[
\Gamma_{PS \to V} = 4\alpha k^3 (\mu_{PS \to V})^2, \tag{21}
\]
with \( k = (M_{PS}^2 - M_V^2)/2M_{PS} \). In this case the transition moment \( \mu_{PS \to V} \) is given by eqs. (19) and (20), but with the indices 1 and 2 interchanged.

In evaluating the integral in (20), it is necessary to choose the value of the upper limit of the integral. It is not a trivial procedure because, in general, the bag radii of the particles under transition are different. The choice in ref. [75] was to take \( R = (R_1 + R_2)/2 \). In the present treatment we prefer to use the smaller of the two in order to take approximately into account the overlap of bags.

So far we have ignored any dynamical recoil effects and CMM corrections to the magnetic (or magnetic transition) moments and decay rates. Unfortunately, because of the fundamental difficulties, there is no rigorous formalism available to account for the CMM and recoil corrections in the relativistic constituent particle models such as the bag model. In general, these corrections are model dependent and plausibly should be treated simultaneously. Strictly speaking, any treatment of these corrections is nothing more than a possible approximate method. In the case of light hadrons the CMM correction is expected to play the main role. It is known that in the bag model calculation of nucleon magnetic moment the recoil correction is relatively small (\( \leq 10\% \)) and negative [126, 127]. In the calculations of the form factors and magnetic moments of octet baryons using cloudy bag model [128] the typical recoil corrections also do not exceed 10%. If one decides to ignore the recoil, for the CMM corrections of the static magnetic moments one can use the simple approximate expression proposed by Halprin and Kerman [129]
\[
\mu = \frac{E}{M}\mu^0, \tag{22}
\]
where \( \mu \) is the CMM corrected magnetic moment, \( \mu^0 \) is the usual uncorrected one, \( M \) is the mass of the hadron, and \( E \) represents the uncorrected bag energy. However, for the transition moments this approach can not be applied literally because the ratios \( E_i/M_i \) for the initial and final mesons differ (sometimes vastly). It could be interesting to test the model using more sophisticated CMM correction schemes, such us suggested by Peierls and Yoccoz [130], for example. But such treatment is rather complicated and for the moment is outside the scope of present investigation. Instead, we resort to the rescaling procedure adopted in some bag model calculations [74, 79]. It was also suggested [128] that for the light baryon octet the effect of the CMM correction on the magnetic form factors can be simulated reasonably well by multiplying them with an overall constant. It is evident that we can not use a single scale factor for all hadrons. In what follows we assume that the integrated effect of the CMM, recoil, and possibly other corrections on the magnetic observables (such as magnetic moments or M1 transition moments) can be simulated by a simple rescaling of the quark level quantities. In practice as minimal prescription we have used the simplest possible ansatz
\[
\mu_L = C_L \mu_L^0, \quad \mu_H = C_H \mu_H^0, \tag{23}
\]
where \( \mu_L^0 \) and \( \mu_H^0 \) represent the initial bag model quantities (magnetic or M1 transition moments) for the light \( (u, d, \text{or} s) \) and heavy \( (c \text{or} b) \) quarks, respectively. In short, our prescription is to calculate various magnetic observables using usual expressions (such as eqs. (17)–(21)), but with magnetic observables of individual quarks replaced by the corrected ones. The scale factor \( C_L \) has been chosen to reproduce the experimental value of the magnetic moment of the proton. Note that the same prescription was used in Ref. [76]. In our model we have \( C_L = 1.43 \). The scale factor for the heavy quarks \( C_H \) is essentially a new parameter. It was adjusted to reproduce the experimental values of the transition moments \( \mu_{D^0 \to D^+} \) and \( \mu_{J/\psi \to \eta_\gamma} \) simultaneously. We have found that acceptable values of \( C_H \) span the range 0.82 – 0.92 and have chosen as optimal the central value \( C_H = 0.87 \).
TABLE I: Spin-flavor content of ground-state mesons.

| Flavor content | $J = 0$ | $J = 1$ |
|---------------|-----|-----|
| $-ud$         | $\pi^+$ | $\rho^+$ |
| $(u\bar{u} - d\bar{d})/\sqrt{2}$ | $\eta$ | $\omega_1$ |
| $(u\bar{d} + d\bar{u})/\sqrt{2}$ | $\eta_0$ | $\omega_0$ |
| $a\bar{s}$    | $K^+$ | $K^{*+}$ |
| $d\bar{s}$    | $K^0$ | $K^{*0}$ |
| $-s\bar{c}$   | $D^+$ | $D^{*+}$ |
| $c\bar{s}$    | $D^0$ | $D^{*0}$ |
| $c\bar{c}$    | $D^*_s$ | $D^{*+}_s$ |
| $s\bar{b}$    | $B^+$ | $B^{*+}$ |
| $d\bar{b}$    | $B^0$ | $B^{*0}$ |
| $\bar{b}\bar{b}$ | $\eta_b$ | $\Upsilon$ |

As we see, the scale factor for the heavy quarks $C_H$ is appreciably smaller than $C_L$. This could mean that in the case of heavy quarks the CMM correction tends to vanish and the recoil plausibly gains an advantage. Actually we do not know if we can use for the strange quarks the same scale factor that was adjusted for the lightest ($u$ and $d$) quarks and for the bottom quarks the same scale factor that was adjusted for the charmed quarks. At present it seems to be a reliable choice. Either way, the magnetic moment of the bottom quark is relatively small, and therefore tiny changes in it do not affect other predictions greatly.

### III. MAGNETIC DIPOLE TRANSITIONS

First, for convenience, we present in table 1 the quark-antiquark structure of $s$-state mesons. Conjugate particles can be obtained from those listed in the table by the substitution $q_a \rightarrow \bar{q}_a, q_b \rightarrow q_b$.

We assume the physical states of pseudoscalar $(\eta, \eta')$ and vector $(\omega^0, \phi)$ mesons to be the mixtures of the $(\eta, \eta_s)$ and $(\omega_1, \phi_s)$ states

\[
\eta = -\eta_1 \sin \alpha_P + \eta_s \cos \alpha_P ,
\]

\[
\eta' = \eta_1 \cos \alpha_P + \eta_s \sin \alpha_P ,
\]

\[
\omega^0 = \omega_1 \cos \alpha_V + \phi_s \sin \alpha_V ,
\]

\[
\phi = -\omega_1 \sin \alpha_V + \phi_s \cos \alpha_V .
\]

Definitions of the mixed states and phase systems of the wave functions used by various authors may differ. Ours are the same as in ref. [133].

Strictly speaking, the physical states $(\eta, \eta')$ and $(\omega^0, \phi)$ can also contain the admixtures of radial excitations, heavier quarkonium states, and glue [132,134]. Because the state mixing problem is not our main subject of interest, we confine ourselves with the simplified picture given by eqs. (24) and (25). If not explicitly stated otherwise, we will determine the mixing angle $\alpha_V$ empirically from the fit to the data. In our model setting $\alpha_V = -4.1^\circ$ we have $\mu_{\phi \rightarrow \pi^0} = 0.124 \mu_N$ consistent with the experimental value $0.125 \pm 0.004 \mu_N$. Here and in what follows $\mu_N = e/(2M_P)$ is the nuclear magneton, and $M_P$ is the mass of the proton. For the pseudoscalars we take the widely accepted $\mu_{\pi^0} = 1.31$ perfect mixing angle $\alpha_P = -45^\circ$. This choice is useful if we want to compare our predictions with the results obtained using other approaches in which the same perfect mixing angle was used.

With these preliminaries by using eqs. (19), (24), and (25) we can write down the detailed expressions for the transition moments, i.e.,

\[
\mu_{\rho^+ \rightarrow \pi^+}(k) = \mu_{\rho^0 \rightarrow \pi^0}(k) = \frac{1}{3} \tilde{\mu}_1^{TR}(k),
\]

\[
\mu_{K^{*+} \rightarrow K^0}(k) = \frac{2}{3} \tilde{\mu}_1^{TR}(k) - \frac{1}{3} \tilde{\mu}_s^{TR}(k),
\]

\[
\mu_{K^{*+} \rightarrow K^0}(k) = -\frac{1}{3} \tilde{\mu}_1^{TR}(k) - \frac{1}{3} \tilde{\mu}_s^{TR}(k),
\]

\[
\mu_{\omega^0 \rightarrow \pi^0}(k) = \tilde{\mu}_1^{TR}(k) \cos \alpha_V ,
\]

\[
\mu_{\phi \rightarrow \pi^0}(k) = -\tilde{\mu}_1^{TR}(k) \sin \alpha_V ,
\]

\[
\mu_{\rho^0 \rightarrow \eta}(k) = -\tilde{\mu}_1^{TR}(k) \sin \alpha_P ,
\]

\[
\mu_{\eta' \rightarrow \rho^0}(k) = \tilde{\mu}_1^{TR}(k) \cos \alpha_P ,
\]

\[
\mu_{\omega^0 \rightarrow \eta}(k) = -\frac{1}{3} \tilde{\mu}_1^{TR}(k) \cos \alpha_V \sin \alpha_P - \frac{2}{3} \tilde{\mu}_s^{TR}(k) \sin \alpha_V \cos \alpha_P ,
\]

\[
\mu_{\omega^0 \rightarrow \eta}(k) = \frac{1}{3} \tilde{\mu}_1^{TR}(k) \cos \alpha_V \cos \alpha_P - \frac{2}{3} \tilde{\mu}_s^{TR}(k) \sin \alpha_V \sin \alpha_P ,
\]

\[
\mu_{\phi \rightarrow \eta'}(k) = -\frac{1}{3} \tilde{\mu}_1^{TR}(k) \sin \alpha_V \cos \alpha_P - \frac{2}{3} \tilde{\mu}_s^{TR}(k) \cos \alpha_V \sin \alpha_P .
\]
\[
\mu_{\phi \to \gamma}(k) = \frac{1}{3} \mu^{TR}_{\phi}(k) \sin \alpha_V \sin \alpha_P - \frac{2}{3} \mu^{TR}_{\mu}(k) \cos \alpha_V \cos \alpha_P, \quad (36)
\]

\[
\mu_{D^{*-} \to D^{*}}(k) = -\frac{1}{3} \mu^{TR}_{s}(k) + \frac{2}{3} \mu^{TR}_{c}(k), \quad (37)
\]

\[
\mu_{D^{*0} \to D^{0}}(k) = \frac{1}{3} \mu^{TR}_{s}(k) + \frac{2}{3} \mu^{TR}_{c}(k), \quad (38)
\]

\[
\mu_{D^{*+} \to D^{+}}(k) = -\frac{1}{3} \mu^{TR}_{s}(k) + \frac{2}{3} \mu^{TR}_{c}(k), \quad (39)
\]

\[
\mu_{J/\psi \to \eta_c}(k) = \frac{4}{3} \mu^{TR}_{c}(k), \quad (40)
\]

\[
\mu_{B^{*-} \to B^{-}}(k) = \frac{2}{3} \mu^{TR}_{s}(k) - \frac{1}{3} \mu^{TR}_{b}(k), \quad (41)
\]

\[
\mu_{B^{*0} \to B^{0}}(k) = -\frac{1}{3} \mu^{TR}_{s}(k) - \frac{1}{3} \mu^{TR}_{b}(k), \quad (42)
\]

\[
\mu_{B^{*+} \to B^{+}}(k) = \frac{1}{3} \mu^{TR}_{s}(k) - \frac{1}{3} \mu^{TR}_{b}(k), \quad (43)
\]

\[
\mu_{B^{*0} \to B^{0}}(k) = -\frac{1}{3} \mu^{TR}_{s}(k) - \frac{1}{3} \mu^{TR}_{b}(k), \quad (44)
\]

\[
\mu_{\Upsilon \to \eta_c}(k) = -\frac{2}{3} \mu^{TR}_{b}(k). \quad (45)
\]

\(\mu^{TR}_i\) in the expressions above denote the reduced transition moments of the lightest (up or down) quarks. Note that isospin symmetry implies \(\mu^{TR}_i = \mu^{TR}_u = \mu^{TR}_d\).

To proceed with the calculations we must specify the photon momenta \(k\). In order to reduce possible uncertainties these momenta have been calculated using experimental mass values of the mesons under consideration. The only exception is the vector meson \(B^*_c\) whose experimental mass is still missing. In this case the theoretical estimate is necessary. For heavy mesons the photon momentum \(k\) approximately coincides with the corresponding mass difference. The typical quark model estimate of the hyperfine splitting for the \(B_c\) meson is \(68 \pm 8\) MeV, with an alternative method giving \(84\) MeV \(\[131\]\). The lattice QCD prediction is \(M_{B^*_c} - M_{B_c} = 54 \pm 3\) MeV, very close to the hyperfine splittings of the heavy-light mesons \(M_{B_c} - M_B = 52 \pm 3\) MeV and \(M_{B_c} - M_{B_s} = 50 \pm 3\) MeV \(\[130\]\). This is the definite indication that the \(B_c\) system behaves in some ways more like a heavy-light than a heavy-heavy one \(\[137\]\). Nevertheless, we know that lattice QCD predictions, as a rule, tend to underestimate the masses and hyperfine splittings of heavy mesons. On the other hand, the well elaborated relativistic potential model \(\[72\]\), which provides a good agreement with the data for the hyperfine splittings of other heavy mesons, predicts the splitting \(M_{B_c} - M_B\) a few MeV above the splitting \(M_{B_c} - M_{B_s}\) in bottomonium. We follow this suggestion, take the experimental value \(M_{\Upsilon} - M_{\eta_c} = 62.3 \pm 3.2\) MeV, add 2 MeV, and obtain the estimate \(M_{B^*_c} - M_{B_c} \approx 64\) MeV in rough agreement with the modern quark model prediction \(68 \pm 8\) MeV \(\[135\]\). The dependence of the transition moments on \(k\) in the case of heavy hadrons is relatively slow, therefore for the calculation of these observables the exact value of \(k\) is not very important. On the other hand, the transition rates behave as \(k^3\). Therefore, in the calculation of the decay width of \(B^*_c\) meson the main source of uncertainty is the ambiguity in the choice of the photon momentum.

The predictions obtained using our extended bag model are listed in the column of table III denoted as Our. It could be useful to check what would happen in the long wavelength limit \(k = 0\). The predictions obtained in such simplified version of the model with the same other parameters \((C_L = 1.43, C_H = 0.87, \alpha_V = -41^\circ, \alpha_P = -45^\circ)\) are listed in the column LWL of this table. In the column denoted as NR we present predictions of the simple nonrelativistic model, in which the quark transition moments are replaced by the static magnetic moments. We take the input values of the quark magnetic moments from ref. \(\[138\]\). They are \(\mu_u = 1.86 \mu_N, \mu_d = -0.93 \mu_N, \mu_s = -0.61 \mu_N, \mu_c = 0.39 \mu_N, \mu_b = -0.06 \mu_N\). Note that these values are adjusted to reproduce the magnetic moments of light baryons.

The vector mixing angle obtained from the fit to the experimental data for the hyperfine splittings of \(\Upsilon\) is \(\alpha_V = -2.6^\circ\), and the pseudoscalar mixing angle \(\alpha_P = -45^\circ\) is left unchanged. Evidently these non-relativistic results are not acceptable for a serious comparison with experiment. Of course, they could serve as a kind of reference point revealing some quark model problems, but there is one useful exception. For the bottomonium the nonrelativistic description is undoubtedly a good approximation, and therefore we expect the transition moment \(\mu_{\Upsilon \to \eta_c}\) obtained in the nonrelativistic quark model to be of sufficiently high accuracy.

In table III we also compare our predictions with the results obtained using other approaches, such as:

- the semi-relativistic (relativized) potential model (SRPM) \(\[54\]\),
- relativistic potential models (RPM) \(\[62, 70\]\),
- the statistical model (SM) \(\[112\]\).

The experimental values of M1 transition moments have been deduced from the partial decay widths given in Particle Data Tables \(\[23\]\) with the help of relations inverse to eqs. \(\[13\]\) and \(\[21\]\).

From table III we see that the results obtained in our present model are of similar quality as those obtained using other approaches. Moreover, apart from several exceptions, our predictions are close to the predictions obtained in the relativistic potential model \(\[62\]\).
TABLE II: Transition moments (in nuclear magnetons) of ground-state vector mesons.

| Transition | Experimenta | NR | LWL | Our SRPM RPMa RPMa | SM |
|------------|-------------|----|-----|-------------------|----|
| \(\rho^+ \to \pi^+\) | 0.68 ± 0.04 | 0.93 | 0.82 | 0.68 | 0.69 | 0.68 | 0.62 | 0.69 |
| \(\rho^0 \to \pi^0\) | 0.78 ± 0.05 | 0.93 | 0.82 | 0.68 | 0.69 | 0.68 | 0.61 | 0.69 |
| \(\omega^0 \to \pi^0\) | 2.15 ± 0.04 | 2.79 | 2.47 | 2.01 | 2.07 | 2.02 | 1.82 | 2.07 |
| \(\omega^0 \to \eta\) | 0.42 ± 0.02 | 0.70 | 0.64 | 0.60 | 0.50 | 0.47 | 0.49 | 0.50 |
| \(\rho^0 \to \eta\) | 1.49 ± 0.05 | 1.97 | 1.75 | 1.66 | 1.53 | 1.67 | 1.66 | 1.50 |
| \(K^{*+} \to K^+\) | 0.78 ± 0.04 | 1.25 | 0.97 | 0.85 | 0.91 | 0.84 | 0.80 | ⋯ |
| \(K^{*0} \to K^0\) | 1.19 ± 0.05 | −1.54 | −1.35 | −1.20 | −1.20 | 1.17 | 1.23 | ⋯ |
| \(\phi \to \pi^0\) | 0.125 ± 0.004 | 0.125 | 0.18 | 0.124 | 0.06 | 0.12 | 0.092 | 0.45 |
| \(\phi \to \eta\) | 0.65 ± 0.01 | −0.83 | −0.74 | 0.65 | 0.71 | 0.72 | ⋯ |
| \(\phi \to \eta'\) | 0.69 ± 0.02 | 0.89 | 0.83 | 0.82 | −0.66 | 0.71 | 0.88 | −0.67 |
| \(\eta' \to \omega^0\) | 0.40 ± 0.02 | 0.62 | 0.53 | 0.50 | 0.63 | 0.58 | ⋯ | 0.49 |
| \(\eta' \to \rho^0\) | 1.23 ± 0.01 | 1.97 | 1.75 | 1.68 | 1.85 | 1.69 | ⋯ | 1.48 |
| \(D^{*+} \to D^+\) | 0.44 ± 0.05 | −0.54 | −0.45 | −0.40 | −0.35 | 0.49 | 0.44 | ⋯ |
| \(D^{*0} \to D^0\) | ⋯ | 2.25 | 1.82 | 1.68 | 1.78 | 1.64 | 1.2 | ⋯ |
| \(D_T^{*+} \to D_T^+\) | −0.22 | −0.26 | −0.23 | −0.13 | 0.25 | 0.26 | ⋯ |
| \(J/\psi \to \eta_b\) | 0.65 ± 0.09 | 0.78 | 0.60 | 0.59 | 0.69 | ⋯ | 0.59 |
| \(B^{*+} \to B^+\) | ⋯ | 1.80 | 1.36 | 1.31 | −1.37 | 1.39 | 1.63 | ⋯ |
| \(B^{*0} \to B^0\) | ⋯ | −0.99 | −0.76 | −0.74 | −0.78 | 0.72 | 0.92 | ⋯ |
| \(B_T^{*0} \to B_T^0\) | ⋯ | −0.67 | −0.59 | −0.58 | −0.55 | 0.47 | 0.71 | ⋯ |
| \(B_T^{*+} \to B_T^+\) | ⋯ | 0.33 | 0.24 | 0.24 | ⋯ | 0.33 | 0.31 | ⋯ |
| \(\Upsilon \to \eta_b\) | ⋯ | −0.12 | −0.11 | −0.11 | −0.13 | ⋯ | −0.09 | ⋯ |

a Only absolute values |\(\mu| are presented.

The comparison with experimental data shows that in the light meson sector for the transitions without most problematic mesons \(\eta\) and \(\eta'\) the agreement with data is satisfactory. In more detail, for the transitions \(\rho^+ \to \pi^+\) and \(K^{*0} \to K^0\) the agreement width the data is excellent. For the decay \(\omega^0 \to \pi^0\) a possible deviation of the calculated transition moment from the experimental value can be 5−7%, for the decay \(K^{*+} \to K^+\) about 10%, and for the transition \(\rho^0 \to \pi^0\) the possible deviation from the experiment can be as large as 18%. The latter uncertainty is the largest. Note that this is a common problem of all quark model based approaches. The isospin symmetry implies \(\mu_{\rho^+ \to \pi^+} = \mu_{\rho^0 \to \pi^0}\), however, we see that the experimental values of these quantities do not coincide. This could mean that in this case the isospin symmetry breaking should be taken into account, and, as a consequence, the \(\rho^0, \omega^0\) mixing is possible. If we ignore this largest uncertainty, then typical deviation from experimental data does not exceed 10%. We think this is a reasonable measure of the accuracy of the method. We can also conclude that the typical accuracy of the calculated magnetic (transition) moments of the light quarks is about 5%.

Because we have used a rather crude description of pseudoscalars \(\eta\) and \(\eta'\), in these cases we do not expect a very good agreement with experiment. Our predictions for the transition moments of these mesons are to some extent similar to the predictions obtained in other relativistic models. In all cases the agreement of the predictions with the experimental data is far from being perfect. Moreover, the predictions strongly depend on the pseudoscalar mixing angle \(\alpha_P\). We have analyzed the dependence of the ratios \(\mu_{\phi \to \eta' \to \eta} / \mu_{\phi \to \eta} \mu_{\omega \to \eta' \to \eta} / \mu_{\omega \to \omega} \), and \(\mu_{\rho^0 \to \eta' / \mu_{\rho^0 \to \rho^0}\)} on the angle \(\alpha_P\). The experimental values are \(\mu_{\phi \to \eta' / \mu_{\phi \to \eta} = 1.06 \pm 0.03, \mu_{\omega \to \eta / \mu_{\omega \to \omega}} = 1.05 \pm 0.09, \mu_{\rho^0 \to \eta / \mu_{\rho^0 \to \rho^0}} = 1.21 \pm 0.05\). The first two are compatible with the predictions obtained with \(\alpha_P = −40^\circ\), while \(\mu_{\rho^0 \to \eta / \mu_{\rho^0 \to \rho^0}} = 1.21\) requires \(\alpha_P = −51^\circ\). So we can not fit all three ratios simultaneously, and the perfect mixing angle \(\alpha_P = −45^\circ\) seems to be an acceptable compromise.

In the heavy meson sector our predictions are in satisfactory agreement with the available experimental data (transition moments \(\mu_{D^{*+} \to D^+}\) and \(\mu_{J/\psi \to \eta_b}\)). Moreover, we have one additional reliable entry to compare with, i.e., the nonrelativistic value of the transition moment \(\mu_{\Upsilon \to \eta_b}\). We see that our prediction \(\mu_{\Upsilon \to \eta_b} = −0.11\) is close enough to the nonrelativistic one. Thus, adjusting one model parameter (scale factor \(C_H\)) we have achieved the reliable predictions for three transition moments, and this gives us some confidence that our further predictions for the heavy mesons are more or less reliable, too.
TABLE III: M1 decay widths (in keV) of ground-state vector mesons.

| Transition | Experiment | Our Bag | CBM | QM | RQM | RPM | RPM | LFQM |
|------------|------------|---------|-----|----|-----|-----|-----|-------|
| \(\rho^+ \to \pi^+\) | 67 ± 7 | 66.7 ± 43.45 | 124.74 | 76.84 | 64.78 | 53.3 | 69 |
| \(\rho^0 \to \pi^0\) | 90 ± 12 | 66.7 ± 43.45 | 124.74 | 76.84 | 65.45 | 53.7 | 69 |
| \(\omega^+ \to \pi^0\) | 703 ± 24 | 616 ± 398.7 | 1180.716 | 730 | 613.3 | 498.6 | 667 |
| \(\omega^+ \to \eta\) | 3.9 ± 0.4 | 7.96 | 6.36 | 2.3 | 8.1 | 8.0 | 4.90 | 5.28 | 6.3 ± 0.3 |
| \(\rho^0 \to \eta\) | 45 ± 3 | 55.7 | 58.33 | 23 | 57.5 | 59 | 51.96 | 51.6 | 54 |
| \(K^{*+} \to K^+\) | 50 ± 5 | 59.8 | 7.71 | 47 | 82.3 | 50 | 58.10 | 52 | 71.4 |
| \(K^{*0} \to K^0\) | 116 ± 10 | 118.9372 | 98 | 114.117 | 112.3 | 125 | 116.6 |
| \(\phi \to \pi^0\) | 5.4 ± 0.3 | 5.31 | 0 | 4.7 | 5.8 | 5.6 | 4.89 | 2.93 | 2.5 ± 8.7 |
| \(\phi \to \eta\) | 55.8 ± 1.4 | 54.7 | 43.72 | 43 | 31.3 | 55.3 | 68.23 | 78 | 47.6 ± 1.5 |
| \(\phi \to \eta'\) | 0.267 ± 0.011 | 0.384 | 2.39 | 0.29 | 0.34 | 0.57 | 0.30 | 0.46 | 0.34 ± 0.01 |
| \(\eta' \to \omega^0\) | 5.4 ± 0.5 | 8.72 | 0 | 6.0 | 4.8 | 4.8 | 11.53 | 13 | 7.0 ± 0.4 |
| \(\eta' \to \rho^0\) | 57.6 ± 1.0 | 108 | 0 | 53 | 67.3 | 67.5 | 117.6 | 117 | 62 |
| \(D^+ \to D^0\) | 1.33 ± 0.33 | 1.10 | 0.82 | 1.7 | 1.42 | 0.56 | 1.63 | 1.36 | 0.90 ± 0.02 |
| \(D^0 \to D^0\) | 19.7 | 22.57 | 18.2 | 21.7 | 21.69 | 19.48 | 10.25 | 20.0 ± 0.3 |
| \(D^{*+} \to D^+_s\) | 0.40 | 0.12 | 0.10 | 0.21 | ⋯ | 0.44 | 0.48 | 0.18 ± 0.01 |
| \(J/\psi \to \eta_c\) | 1.58 ± 0.42 | 1.31 | 21.00 | 2.0 | 1.27 | ⋯ | ⋯ | 1.69 ± 0.05 |
| \(B^{*+} \to B^+\) | 0.459 | ⋯ | 0.62 | ⋯ | 0.429 | 0.52 | 0.71 | 0.40 ± 0.03 |
| \(B^{*0} \to B^0\) | 0.146 | ⋯ | 0.28 | ⋯ | 0.142 | 0.14 | 0.22 | 0.13 ± 0.01 |
| \(B_{s1}^{*0} \to B_{s1}^0\) | 0.102 | ⋯ | 0.10 | ⋯ | ⋯ | 0.06 | 0.15 | 0.068 ± 0.017 |
| \(B_{s2}^{*+} \to B_{s2}^+\) | 0.041 | ⋯ | ⋯ | ⋯ | 0.03 | 0.032 | ⋯ |

TABLE IV: M1 decay widths (in keV) of heavy vector mesons.

| Transition | Experiment | Our HB | HB | QCDSSR | LCSR | BSLT | RPM | \(\chi_{RQM}\) | \(\chi_{EFT}\) | PM |
|------------|------------|-------|-----|--------|------|------|-----|--------|-------|-----|
| \(D^{*+} \to D^+\) | 1.33 ± 0.33 | 1.10 | 0.9 | 1.72 | 0.23 ± 0.10 | 1.50 | 1.10 | 1.04 | 1.5 | 1.63 | 2.4 |
| \(D^{*0} \to D^0\) | ⋯ | 19.7 | 20 | 7.18 | 12.9 ± 2.0 | 14.40 | 1.25 | 11.5 | 32 ± 1 | 33.5 | 35.2 |
| \(D^{*+} \to D^+_s\) | 0.40 | 0.5 | ⋯ | 0.13 ± 0.05 | ⋯ | 0.337 | 0.19 | 0.32 ± 0.01 | 0.43 | 0.32 |
| \(B^{*+} \to B^+\) | 0.46 | 1.3 | 0.272 | 0.38 ± 0.06 | 0.63 | 0.0674 | 0.19 | 0.74 ± 0.09 | 0.78 | 1.7 |
| \(B^{*0} \to B^0\) | 0.15 | 0.5 | 0.064 | 0.13 ± 0.03 | 0.16 | 0.0096 | 0.070 | 0.23 ± 0.03 | 0.24 | 0.5 |
| \(B_{s1}^{*0} \to B_{s1}^0\) | 0.10 | 0.3 | 0.051 | 0.22 ± 0.04 | ⋯ | 0.148 | 0.054 | 0.14 ± 0.02 | 0.15 | 0.2 |

Heavy meson sector the serious source of uncertainty is the ambiguity in the choice of the scale factor \(C_H\). Most sensitive to this ambiguity are small transition moments. The uncertainties are estimated to be 4% for \(\mu_{D^{*+} \to D^+}\), 9% for \(\mu_{D^{*+} \to D^+_s}\), 5% for \(\mu_{J/\psi \to \eta_c}\), 7% for \(\mu_{B_{s1}^{*0} \to B_{s1}^0}\), and 6% for \(\mu_{T \to \eta_c}\). The influence of this ambiguity on other transition moments (\(\mu_{D^{*0} \to D^0}\), \(\mu_{B^{*+} \to B^+}\), \(\mu_{B^{*0} \to B^0}\), and \(\mu_{B_{s1}^{*0} \to B_{s1}^0}\)) does not exceed 1%, however, the real uncertainties for these transitions could be larger (up to 6%) due to the error in the values of transition moments of the light quarks.

From the comparison of our predictions for the transition moments of light mesons with the results obtained in the long wavelength limit (LWL) and with the results obtained using nonrelativistic quark model (NR) we see that it is essential to take into account the k-dependence of these quantities in order to achieve the satisfactory agreement with experimental data. In the case of the mesons made of one charmed and one light quark the k-dependence of the transition moments is also important. For the heaviest mesons (\(J/\psi, B, B_s, B_c, \) and \(T\)) the results obtained in the long wavelength limit are similar to our present predictions, and we conclude that for the M1 transitions of these hadrons the long wavelength limit can be a reasonable approximation.

So far we were concentrated on the M1 transition moments, however, the really measurable quantities are the transition rates. In order to have a complete picture we have used the values of the transition moments presented above as inputs to calculate the partial decay widths. They are listed in table III and compared with the experimental data. We also compare our predictions with the...
Our predictions in most cases agree with the available experimental data and are similar to the results obtained in the earlier versions of bag model [75, 76, 78–82]. For the heavy mesons similar results are obtained in our model and in ref. [76] in the charm sector, but in the bottom sector the predictions differ substantially. We stress that our predictions in most cases agree with available experimental data and are similar to the results obtained in the framework of the relativistic potential model [62]. They are also more or less compatible with the predictions obtained in the light front quark model [85, 86]. For the transitions of heavy-light mesons, in tables IV–VII we continue the comparison of our predictions with the results for the heavy meson sector obtained using the following methods:

- the bag model (Bag) [75];
- the cloudy bag model (CBM) [78–80]. The presented results for $D$ mesons correspond to the value of their scale factor $\lambda = 0.7$;
- the simple quark model (QM) [27];
- the relativistic quark model (RQM) [29, 30];
- relativistic potential models (RPM) [62, 70];
- the light front quark model with linear confining potential (LFQM) [85, 86].

Given our special interest in the magnetic properties of heavy mesons, in tables IV–VII we continue the comparison of our predictions with the results for the heavy meson sector obtained using the following approaches:

- bag models for heavy hadrons (HB) [76, 82];
- usual QCD sum rules (QCDSR) [92, 96, 98];
- light cone QCD sum rules (LCSR) [95];
- potential models (PM) [35, 43, 45, 47];
- the formalism based on the Blankenbecler-Sugar equation (BSLT) [21, 80];
- the Bethe-Salpeter formalism (BSF) [87];
- the emi-relativistic (relativized) potential model (SRPM) [48, 55];
- relativistic potential models (RPM) [58–61, 71, 72];
- the elativistic chiral quark model ($\chi$RQM) [108];
- chiral effective field theory ($\chi$EFT) [109];
- lattice QCD calculations (Latt) [102];
- the framework of the spectral integral equations (SIE) [117] (the fit with retarded interactions).

The inspection of the predictions obtained using our approach and various other methods shows that the overall agreement is not bad, nevertheless, some mess-up is present especially for the transitions of heavy-light mesons. We see the serious improvement in the predictions of the decay widths as compared with the results obtained in the earlier versions of bag model [75, 76, 78–82]. For the heavy mesons similar results are obtained in our model and in ref. [76] in the charm sector, but in the bottom sector the predictions differ substantially.

### IV. MAGNETIC MOMENTS

As we have seen, the method developed above is capable to provide reasonable predictions for M1 transition moments and corresponding partial decay rates of ground-state vector mesons. Thus we can expect that the predictions for usual magnetic moments obtained using the same method also should be reliable. Below we list detailed expressions for these magnetic moments (they are extremely simple and are presented only for convenience).

$$
\mu_{\rho^+} = \bar{\mu}_4,
$$

(46)
\[ \mu_{K^{*+}} = \frac{2}{3} \mu_l + \frac{1}{3} \bar{\mu}_s, \quad (47) \]
\[ \mu_{K^{*0}} = -\frac{1}{3} \bar{\mu}_l + \frac{1}{3} \bar{\mu}_s, \quad (48) \]
\[ \mu_{D^{*+}} = \frac{1}{3} \bar{\mu}_l + \frac{2}{3} \bar{\mu}_c, \quad (49) \]
\[ \mu_{D^{*0}} = -\frac{2}{3} \bar{\mu}_l + \frac{2}{3} \bar{\mu}_c, \quad (50) \]
\[ \mu_{D^\pi^\pm} = \frac{2}{3} \bar{\mu}_c + \frac{1}{3} \bar{\mu}_s, \quad (51) \]
\[ \mu_{B^{*+}} = \frac{2}{3} \bar{\mu}_l + \frac{1}{3} \bar{\mu}_b, \quad (52) \]
\[ \mu_{B^{*0}} = -\frac{1}{3} \bar{\mu}_l + \frac{1}{3} \bar{\mu}_b, \quad (53) \]
\[ \mu_{B_{s}^*0} = -\frac{1}{3} \bar{\mu}_s + \frac{1}{3} \bar{\mu}_b, \quad (54) \]
\[ \mu_{B_{s}^*+} = \frac{2}{3} \bar{\mu}_c + \frac{1}{3} \bar{\mu}_b. \quad (55) \]

Note that \( \mu_{\rho^0} = \mu_{\omega^0} = \mu_{\phi^0} = \mu_T = 0. \)

The results of our calculations for the light mesons are presented in tables VIII and IX together with the estimates obtained in nonrelativistic quark model (NR). These predictions are compared with the estimates obtained in other approaches. These are:

- the relativistic Hamiltonian model (RH) \([6]\);
- various models based on the Dyson-Schwinger equation (DSE) \([7, 9]\);
- lattice QCD calculations (Latt) \([3, 4, 18]\);
- approaches based on standard QCD sum rules (QCDSR) \([1]\) and its light cone modification (LCSR) \([2]\);
- the extended Nambu-Jona-Lasinio model with heavy quarks (NJL) \([15]\);
- the effective field theory (EFT) \([17]\);
- light front quark models (LFQM) \([11, 13]\).

Also we include in the comparison (column denoted as Experiment in table IX) the attempt to extract the magnetic dipole moment of the \( \rho^+ \) meson using preliminary experimental data from the BaBar Collaboration.

In many papers the values of magnetic moments are presented in natural particle’s magnetons \( e/(2M_i) \), where \( M_i \) represents the mass of corresponding meson. We have converted them to the values expressed in nuclear magnetons by multiplying with the factor \( M_P/M_i \). The exception is lattice QCD predictions \([3, 4, 18]\), where they have used a specific extrapolation procedure.

From tables VIII and IX we see that our predictions are somewhere between the predictions obtained within the relativistic Hamiltonian formalism (RH) and results obtained in the simple nonrelativistic quark model (NR), but closer to the RH predictions. They are also more or less compatible with the theoretical results obtained in ref. \([7] \) using the approach based on the Dyson-Schwinger equation (DSE) and with quenched lattice predictions (ref. \([3]\)). Within the error bars our predictions for the \( \rho^+ \) and \( K^{*+} \) mesons also agree with the results obtained using light cone QCD sum rules (LCSR) and are close to the predictions obtained in the framework of the extended Nambu-Jona-Lasinio model (NJL). On the other hand, predictions obtained using another version of Dyson-Schwinger formalism (ref. \([4]\)) and lattice predictions obtained in ref. \([4]\) differ substantially from ours.

For the \( \rho^+ \) meson our prediction is slightly above the effective field theory tree level value 2.42 \( \mu_N \) (2.0 in natural magnetons) and agrees within the error bars with the estimate extracted from experimental data (ref. \([22]\)). The recent full QCD prediction \([18]\) is also in good agreement with our result. Regarding the light front quark models (LFQM), the predictions obtained for the \( \rho^+ \) using various variants of LFQM cover rather large range from 2.24 \( \mu_N \) up to 2.86 \( \mu_N \). Our prediction is closest to the results obtained in the covariant versions of LFQM \([11, 13]\).

We finish our investigation with the predictions for the magnetic moments of ground-state vector mesons containing heavy quarks. The results are listed in table X. They are compared with the results obtained in the nonrelativistic quark model (NR), the predictions obtained in the framework of extended Nambu-Jona-Lasinio model (NJL), and with the prediction for \( B_{c}^{*+} \) meson obtained using the formalism based on the covariant Blankenbecler-Sugar equation (BSLT) \([21]\). There also exists the old bag model prediction for the ratios of these magnetic moments to the magnetic moment of the proton \([20]\). For comparison we have presented these ratios together with ours in the two last columns of table X. Magnetic moments deduced from these ratios by multiplying them with the magnetic moment of proton \( \mu_P \) are given in the column named as Bag.

We see that the values of the magnetic moments predicted using our extended version of the bag model are appreciably smaller than nonrelativistic results. Our prediction for the \( B_{c}^{*+} \) meson is also smaller than the corresponding prediction obtained using BSLT formalism, the latter being similar to the nonrelativistic one. For \( D^{*+} \) and \( D_s^{*+} \) mesons the values obtained in the NJL model are between the NR predictions and ours. For \( B^{*+} \) our and NJL predictions coincide. We have found with some surprise that our predictions for \( D^{*+}, B^{*+}, B^{*0}, \) and \( B_s^{*0} \)
TABLE VIII: Magnetic moments of light mesons in nuclear magnetons (values in natural magnetons are given in parenthesis).

| Particle | Our | NR | RH | DSE | DSE | Latt | Latt | LCSR | NJL |
|----------|-----|----|----|-----|-----|------|------|------|-----|
| \( \rho^+ \) | 2.50 | 2.79 | 2.37 | 2.43 | 3.28 | 3.25 ± 0.03 | 2.3 | 2.9 ± 0.5 | 2.54 |
| \( \pi^+ \) | (2.06) | (2.31) | (1.96) | (2.01) | (2.69) | (2.39 ± 0.01) | (2.2) | (2.4 ± 0.4) | (2.09) |
| \( K^{*+} \) | 2.21 | 2.47 | 2.19 | 2.34 | 2.49 | 2.81 ± 0.01 | 2.1 | 2.1 ± 0.4 | 2.26 |
| \( \pi^{*+} \) | (2.10) | (2.35) | (2.09) | (2.23) | (2.37) | (2.38 ± 0.01) | (2.0) | (2.0 ± 0.4) | (2.14) |
| \( K^{*0} \) | −0.216 | −0.32 | −0.183 | −0.27 | −0.42 | ⋮ | −0.07 | 0.29 ± 0.04 | ⋮ |
| \( \pi^{*0} \) | (−0.206) | (−0.31) | (−0.175) | (−0.26) | (−0.40) | ⋮ | (−0.07) | (0.28 ± 0.04) | ⋮ |

TABLE IX: Magnetic moment of \( \rho^+ \) meson in nuclear magnetons (values in natural magnetons are given in parenthesis).

| Our | Experiment | Latt | EFT | QCDSR | DSE | LFQM | LFQM | LFQM | LFQM | LFQM | LFQM |
|-----|------------|------|-----|-------|-----|------|------|------|------|------|------|
| 2.50 | 2.54 ± 0.61 | 2.61 ± 0.10 | 2.71 | 2.4 ± 0.4 | 2.95 | 2.75 | 2.55 | 2.34 | 2.61 | 2.24 | 2.86 |
| (2.05) | (2.1 ± 0.5) | (2.21 ± 0.08) | (2.24) | (2.0 ± 0.3) | (2.44) | (2.26) | (2.1) | (1.92) | (2.14) | (1.83) | (2.35) |

TABLE X: Magnetic moments (in nuclear magnetons) of heavy mesons and ratios of these magnetic moments to that of the proton.

| Particle | Our BSJT | NJL | NR | Bag \( \mu_h/\mu_P \) \( \mu_2/\mu_P \) |
|----------|---------|-----|----|--------------|----------|
| \( D^{*+} \) | 1.06 | ⋮ | 1.16 | 1.32 | 1.17 | 0.38 | 0.42 |
| \( D^{*0} \) | −1.21 | ⋮ | ⋮ | −1.47 | −0.89 | −0.43 | −0.32 |
| \( D_1^{*+} \) | 0.87 | ⋮ | 0.98 | 1.00 | 1.03 | 0.31 | 0.37 |
| \( B^{*+} \) | 1.47 | ⋮ | 1.47 | 1.92 | 1.54 | 0.53 | 0.55 |
| \( B^{*0} \) | −0.65 | ⋮ | ⋮ | −0.87 | −0.64 | −0.23 | −0.23 |
| \( B_2^{*0} \) | −0.48 | ⋮ | ⋮ | −0.55 | −0.47 | −0.17 | −0.17 |
| \( B_2^{*+} \) | 0.35 | 0.426 | ⋮ | 0.45 | 0.56 | 0.13 | 0.20 |

mesons are similar to the rescaled old bag model results. However, for other mesons (\( D^{*0} \), \( D_1^{*+} \), and \( B_2^{*+} \)) the difference is evident.

The uncertainties for the magnetic moments are estimated to be of the same order as for transition moments. In the light meson sector the reasonable estimate of possible error could be about 5% for \( \mu_{\rho^+} \), and up to 10% for \( \mu_{K^{*+}} \) and \( \mu_{K^{*0}} \). In the heavy meson sector the largest uncertainty (≈6%) is expected for the \( \mu_{B_2^{*+}} \). For all other magnetic moments of heavy mesons the possible uncertainty is expected to be smaller than 5%.

V. DISCUSSION AND SUMMARY

We have developed a method to treat the magnetic observables (i.e., magnetic moments, M1 transition moments, and partial M1 decay widths) of ground-state vector mesons. The method is based on slightly modified bag model [118]. The main difference from our earlier approach [118, 121] is the recipe how to take into account the CMM corrections of the magnetic observables. In our previous approach following the usual procedure these corrections were applied to the magnetic observable as a whole. The present investigation has shown that a more reliable approach is to apply these corrections at the quark level.

We have used this extended bag model to calculate magnetic moments and partial M1 decay widths of all ground-state vector mesons. To our knowledge, our current predictions for the magnetic moments of neutral mesons \( D^{*0} \), \( B^{*0} \), and \( B_2^{*0} \) are the first reliable theoretical estimates of these properties.

In order to test the method we have compared our predictions for M1 transition moments and partial decay widths with the experimental data and with the results obtained using other approaches. We have found a satisfactory agreement with experiment and, to some extent, with other theoretical predictions. Nevertheless, some aspects concerning the heavy meson sector are not completely clear. Theoretical predictions obtained in various approaches are somewhat dispersed, and so far only two M1 decay widths of heavy mesons have been measured experimentally. The accuracy of data also is not very high. It is not utterly clear if with such a small amount of data the reliability of any model in heavy meson sector can be tested with sufficiently high accuracy. At present the agreement of our predictions with available data is good. In addition, our prediction for the transition moment \( \mu_{\Upsilon \to \eta_b} \) is similar to the more or less realistic result obtained in the nonrelativistic quark model. As concerns the magnetic moments, our prediction for the \( \rho^+ \) meson agrees within the error bars with
the estimate extracted from experimental data (ref. [22]) and is in good agreement with the recent full QCD prediction [18]. Agreement with the predictions obtained using other approaches for the light mesons is also satisfactory. We see that our method is capable to provide the reasonable predictions for various magnetic properties, such as M1 transition moments (together with the partial decay widths) of light and heavy mesons and magnetic moments of light mesons. Encouraged by this success we expect our predictions for the magnetic moments of heavy mesons to be also trustworthy.

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