Measuring asymmetries in flavor asymmetric machines

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The LHC offers a unique opportunity to investigate an ample spectra of phenomenons ranging from the Electro-Weak (EW) to the QCD sector of the Standard Model (SM). Among the quantities which can be measured in the LHC experiments are the CP and production asymmetries for several particles in a wide variety of decay modes. In this work we discuss about the interplay between production and CP asymmetries for particles produced in proton-proton interactions and the effects of one on the measurement of the other. This kind of effects are not present in flavor symmetric machines like the Tevatron or $e^+ - e^-$ colliders.

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I. INTRODUCTION

Particle-antiparticle production asymmetries have played an important role in the understanding of the hadronization mechanisms of partons in high energy hadron-hadron interactions. Today there exist copious experimental evidence [1] indicating that produced particles sharing valence quarks with the initial hadrons are produced at a different rate than particles sharing none. In fact, the so called leading particle effect, which is responsible for the particle-antiparticle production asymmetry, has firmly established the role of the recombination mechanism in hadron production [2] in high energy interactions. Those studies have also given important insights on the structure of the initial hadrons [3].

So far, particle-antiparticle asymmetries, both as a function of the transverse momentum, $p_T$, and as a function of the scaled longitudinal momentum, $x_F = 2p_L/\sqrt{s}$, of the produced particles, have been measured in the range of a few tenth of GeV center of mass (c.m.) energies [1], and mostly in the production of strange and charm hadrons. With the advent of LHC, it could be interesting to measure such particle-antiparticle production asymmetries at highest c.m. energies to understand to which extent the ratio between the fragmentation and recombination mechanisms in the hadronization is dependent on the c.m. energy. Furthermore, since beauty hadron production asymmetries have not been measured at all, it could be interesting to investigate also the role of the leading particle effects by itself in this case. However, in the case of beauty meson production, as mesons are detected and measured through their decay products, CP asymmetries and mixing effects can affect the determination of the production asymmetries, thus spoiling the study of the hadronization mechanisms. Conversely, particle-antiparticle production asymmetries can be an important effect, polluting weak interaction effects, in the measurement of quantities involving the comparison of particles decays with their charge conjugate ones in machines such LHC, which are not symmetric with respect to particle and antiparticle production. Thus, the above mentioned effects can be important in the determination of mixing parameters in the $B^\pm_{d/s} - \bar{B}^\mp_{d/s}$ system, CP asymmetries in $B^\pm$ and $B^\pm_{d/s}/\bar{B}^\mp_{d/s}$ decays, etc. Furthermore, the LHCb Collaboration [4] has an extensive program to measure $D^0 - \bar{D}^0$ mixing and possible CP violation asymmetries in the charm sector of the Standard Model. Since those effects are expected to be small, thought much smaller than production effects, then the interplay between the production and CP asymmetries has to be very well understood in order to measure the later with significative precision .

In this work we shall discuss about how the measurement of meson production asymmetries are affected by CP asymmetries and mixing effects and viceversa, for mesons produced in $p - p$ collisions. Along the text the discussion will be focused in B-meson production, but most of it is directly applicable to D-meson production.

II. B-MESON PRODUCTION ASYMMETRIES IN $p - p$ COLLISIONS

A. $B^\pm$ production in $p - p$ collisions

In p-p collisions the production mechanisms of $B^+$ and $B^-$ mesons are expected to be different because of the leading particle effect. In fact, since the $B^+ = (u\bar{b})$ shares a valence quark with the initial protons and the $B^- = (\bar{u}b)$ shares none, it is expected that $B^+$s be produced at a higher rate than $B^-$s. The above differences in the production processes of $B^\pm$ mesons in p-p collisions can be characterized by means of the so called production

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asymmetry, which is defined by

\[ A = \frac{N_{B^+} - N_{B^-}}{N_{B^+} + N_{B^-}}, \]  

(1)

where \( N_{B^\pm} \) are the number of \( B^+ \) and \( B^- \) mesons produced at the interaction point.

In order to measure the production asymmetry of Eq. (1) \( N_{B^\pm} \) have to be determined by reconstructing \( B^\pm \) mesons decaying into a given final state. However, \( B^\pm \) mesons could decay violating the so called CP symmetry, thus spoiling the determination of \( N_{B^+} \) and \( N_{B^-} \) at the production vertex. To be concrete, let us start by considering \( B^\pm \) mesons decaying into an arbitrary final state \( f \),

\[ B^+ \rightarrow f, \]  

(2)

and its charge conjugated (c.c.). If the CP symmetry is violated in the decay, then it follows that

\[ A_{CP} = \frac{BR(B^+ \rightarrow f) - BR(B^- \rightarrow \bar{f})}{BR(B^+ \rightarrow f) + BR(B^- \rightarrow \bar{f})} \neq 0. \]  

(3)

Since

\[ N_{B^+ \rightarrow f} = N_{B^+} \times BR(B^+ \rightarrow f) \]  

\[ N_{B^- \rightarrow \bar{f}} = N_{B^-} \times BR(B^- \rightarrow \bar{f}), \]  

(4)

then the production asymmetry of Eq. (1) has to be modified to

\[ A = \frac{N_{B^+ \rightarrow f} - N_{B^- \rightarrow \bar{f}}}{N_{B^+ \rightarrow f} + N_{B^- \rightarrow \bar{f}}} \quad \text{and} \quad \frac{1}{1 - A_{CP}}, \]  

(5)

where

\[ R = \frac{BR(B^+ \rightarrow f)}{BR(B^- \rightarrow \bar{f})} = \frac{1 + A_{CP}}{1 - A_{CP}}. \]  

(6)

Using Eq. (6) we can rewrite Eq. (5) as

\[ A = \frac{\left[ N_{B^+ \rightarrow f} - N_{B^- \rightarrow \bar{f}} \right]}{\left[ N_{B^+ \rightarrow f} + N_{B^- \rightarrow \bar{f}} \right]} - A_{CP} \]  

\[ 1 - A_{CP} \]  

which reduces to the usual formula of Eq. (1) when \( A_{CP} = 0 \). A similar formula can be obtained for the production asymmetry as a function of \( p_T^2 \) and/or \( x_F \). The only difference with Eq. (7) will be the dependence in \( p_T^2 \) and/or \( x_F \) arising in \( N_{B^\pm \rightarrow f} \), since the \( A_{CP} \) does not depend on the momentum of the produced particle.

In order to give a numerical estimate of the effect, since there is no data on \( B^\pm \) production asymmetries in \( p - p \) collisions and Monte Carlo generators do not provide a meaningful prediction, we will assume that \( B^\pm \) production in \( p - p \) interactions is similar to that of \( D_s^+ \) production in \( \Sigma^- - \text{Nucleus} \) interactions. In \( \Sigma^- \text{(}d, s\text{)}\text{-Nucleus} \) interactions, the \( D_s^+ \) is leading. The SELEX Collaboration [6] has measured \( dN/dx_F \) for both \( D_s^+ \) and \( D_s^- \) and the production asymmetry also as a function of \( x_F \) in the range \([0.15, 0.7]\) in 600 GeV/c beam energy interactions.

The asymmetry as a function of \( x_F \) is well represented by

\[ A_{D_s^+}(x_F) = \frac{(1 - x_F)^{3.8} - 1.5(1 - x_F)^{7.9}}{(1 - x_F)^{3.8} + 1.5(1 - x_F)^{7.9}}. \]  

(8)

With the above assumption, the \( B^\pm \) asymmetry as a function of \( x_F \) and \( A_{CP} \) in \( p - p \) collisions is given by

\[ A_{B^\pm}(x_F) = \frac{2(1 - x_F)^{-1.1} - 3 - A_{CP} \left[ 3 + 2(1 - x_F)^{-1.1} \right]}{2(1 - x_F)^{-1.1} + 3 - A_{CP} \left[ 2(1 - x_F)^{-1.1} - 3 \right]}. \]  

(9)

In Fig. 1 it is shown the effect of the CP asymmetry on the production asymmetry for \( A_{CP} = 0.03 \pm 0.06, 0.038 \pm 0.022 \) corresponding to \( B^\pm \) decaying into \( \pi^+\pi^-\pi^+ \) and \( K^+\pi^-\pi^+ \) respectively, which are well suited modes to study \( B^\pm \) production. The effect can be barely noted, depending however on the error in the measurement of both, the production asymmetry and \( A_{CP} \).

For the integrated production asymmetry in the range \( x_F \in [0.15, 0.7] \) we obtain \( A = 0.385 \pm 0.048, 0.378 \pm 0.018 \) respectively, while for \( A_{CP} = 0 \) one gets \( A = 0.41 \), which is an effect of about 10%, depending on the decay mode in which the \( B^\pm \)s are reconstructed.

However, for real \( B^\pm \) production in \( p - p \) collisions at a c.m. energy of 7 TeV the production asymmetry is expected to be smaller than in interactions at a lower c.m. energy. The leading particle effect will be still operative, but particle production from sea-sea quark recombination is expected to be enhanced. As this mechanism works for both, particle and antiparticle, the production asymmetry should be smaller, thus increasing the effect of the CP asymmetry. Once again, to have a numerical estimate let us assume that the \( B^\pm \) production asymmetry at 7 TeV c.m. energy is \( A = 0.2 \) when measured in a decay mode in which \( A_{CP} = 0 \). The raw asymme-
of measured \(p_t\) positively. In Eq. 10, \(f_\text{mixing}\).

However, \(B\) production is somewhat more complicated than the previous cases because of the asymmetry in \(x_F\) as expected to play a role since \(B_0\) is produced at the interaction point and its c.c. decay. Once a \(B_0\) is produced at the interaction vertex, it can decay into \(\rightarrow 0\) or can oscillate to \(\rightarrow 0\)

### B. \(B^0/\bar{B}^0\) production in \(p-p\) collisions

Let us now consider \(B^0/\bar{B}^0\) production in \(p-p\) collisions. As in \(B^\pm\) production, leading particle effects are expected to play a role since \(B^0 = (db)\) shares valence quarks with the initial protons while \(\bar{B}^0 = (\bar{d}\bar{b})\) does not. However, \(B^0/\bar{B}^0\) production is somewhat more complicated than the previous cases because of the \(B^0 - \bar{B}^0\) mixing.

Once again, for the sake of concreteness, let us assume \(B^0 \rightarrow f\) and its c.c. decay. Once a \(B^0\) is produced at the interaction vertex, it can decay into \(f\), or can oscillate to a \(\bar{B}^0\), thus decaying into \(\bar{f}\). It means that the number of measured \(B^0\) and \(\bar{B}^0\) mesons decaying into \(f\) and \(\bar{f}\) are related by

\[
\begin{align*}
N_{\text{exp}}^{B^0} & = p_0 N_{B^0} + p_2 N_{\bar{B}^0}, \\
N_{\text{exp}}^{\bar{B}^0} & = p_0 N_{\bar{B}^0} + p_1 N_{B^0},
\end{align*}
\]

where \(N_{B^0}, N_{\bar{B}^0}\) are the number of \(B^0\) and \(\bar{B}^0\) mesons produced at the interaction point and \(N_{\text{exp}}^{B^0}, N_{\text{exp}}^{\bar{B}^0}\) are the number of reconstructed \(f\) and \(\bar{f}\) final states, respectively. In Eq. 10, \(p_0, p_1, p_2\) are the transition probabilities defined by [3]

\[
\begin{align*}
p_0 & = \int_0^\infty dt \ P(B^0 \rightarrow B^0) = \int_0^\infty dt \ P(\bar{B}^0 \rightarrow \bar{B}^0), \\
p_1 & = \int_0^\infty dt \ P(B^0 \rightarrow \bar{B}^0), \\
p_2 & = \int_0^\infty dt \ P(\bar{B}^0 \rightarrow B^0).
\end{align*}
\]

Since the solution of the linear system of Eq. 10 is given by

\[
\begin{align*}
N_{B^0} & = \frac{N_{\text{exp}}^{B^0} - (p_2/p_0) N_{\text{exp}}^{\bar{B}^0}}{p_0 - p_1 p_2/p_0}, \\
N_{\bar{B}^0} & = \frac{N_{\text{exp}}^{\bar{B}^0} - (p_1/p_0) N_{\text{exp}}^{B^0}}{p_0 - p_1 p_2/p_0},
\end{align*}
\]

then the \(B^0/\bar{B}^0\) production asymmetry as given by Eq. 1 is now

\[
A = \frac{(1 + r) N_{\text{exp}}^{B^0} - (1 - r) N_{\text{exp}}^{\bar{B}^0}}{(1 - r) N_{\text{exp}}^{B^0} + (1 - r) N_{\text{exp}}^{\bar{B}^0}}.
\]

where \(r, \bar{r}\) are defined as [3]

\[
\begin{align*}
r & = \frac{p_1}{p_0} = q^2 \frac{x^2 + y^2}{2 + x^2 - y^2}, \\
\bar{r} & = \frac{p_2}{p_0} = q \frac{x^2 + y^2}{2 + x^2 - y^2}.
\end{align*}
\]

In the case of mixing with no CP violation, the production asymmetry of Eq. 13 reduces to

\[
A = \frac{(1 + r) N_{\text{exp}}^{B^0} - (1 - r) N_{\text{exp}}^{\bar{B}^0}}{(1 - r) N_{\text{exp}}^{B^0} + (1 - r) N_{\text{exp}}^{\bar{B}^0}}.
\]

Notice also that no tagging needs to be used to know whether a \(B^0\) or \(\bar{B}^0\) is produced at the interaction point since the correct rate for \(B^0/\bar{B}^0\) production is accounted for by the transition probabilities of Eqs. 14. In other words, the number of tagged \(B^0/\bar{B}^0\) mesons is given by Eqs. 12 in terms of the reconstructed \(f\) and \(f\) final states.

Having in mind that \(x_d = 0.774 \pm 0.008, y_d^2 = -0.0003 \pm 0.109\) and \(|q/p| \sim 1\) for the \(B_d^0\) [7], it follows that \(1 + r)/(1 - r) \sim 1.6\), which means an increase, due to mixing, of 60% in the raw asymmetry of Eq. 15.

As in the previous case of \(B^\pm\) production, similar formulas to those of Eqs. 13 and 15 can be obtained for the production asymmetry as a function of \(p_T^2\) and/or \(x_F\).

### III. B-MESON CP ASYMMETRIES \(p-p\) COLLISIONS

In flavor symmetric machines, like the Tevatron or \(e^+ - e^-\) colliders, it is customary to measure CP asymmetries just by counting the number of particles and anti-particles in a given decay mode, making use of

\[
A_{CP} = \frac{N_{B^+ \rightarrow f} - N_{B^- \rightarrow f}}{N_{B^+ \rightarrow f} + N_{B^- \rightarrow f}},
\]
where $B^+$ and $B^-$ are respectively the leading and non-leading particles decaying into a given final state and its c.c. respectively. However, the use of the above equation is incorrect in flavor asymmetric machines like the LHC because $N_{B^+ \rightarrow f} \pm N_{B^- \rightarrow f}$ necessarily contains production effects. In fact, it is rather straightforward to show that the CP asymmetry, corrected by the effect of the production asymmetry, is given by

$$A_{CP} = \frac{N_{B^+ \rightarrow f} - N_{B^- \rightarrow f}}{N_{B^+ \rightarrow f} + N_{B^- \rightarrow f}} - A,$$

(17)

which is formally identical to Eq. 7 once the replacement $A \leftrightarrow A_{CP}$ has been made. Note however that Eq. 17 is of no practical use unless the production asymmetry, $A$, is previously measured independently of the CP asymmetry. The alternative is to use

$$A_{CP} = \frac{\Gamma(B^+ \rightarrow f) - \Gamma(B^- \rightarrow f)}{\Gamma(B^+ \rightarrow f) + \Gamma(B^- \rightarrow f)},$$

(18)

which is equivalent to Eq. 3 since $BR(B^\pm \rightarrow f) = \Gamma(B^\pm \rightarrow f)/\tau_p$. The decay width $\Gamma$ is measured independently of the production asymmetry by means of

$$\Gamma(B^\pm \rightarrow f) = -\frac{1}{N_{B^\pm \rightarrow f}} \frac{dN_{B^\pm \rightarrow f}}{dt},$$

(19)

while the lifetime $\tau_p$ can be measured making use of Eq. 19 and summing over all the decay modes.

The above discussion is not merely academic, but of rather practical consequences. In fact, there exist several CP related quantities which are measured by determining $N_{B^+ \rightarrow f} \pm N_{B^- \rightarrow f}$. As an example let us consider a recently proposed method to search for CP asymmetries in Dalitz analysis [8], which is based on the measurement, bin by bin in the Dalitz plot, of the quantity

$$DpS_{CP}^A = \frac{N_{B^+ \rightarrow f(i)} - N_{B^- \rightarrow f(i)}}{\sqrt{N_{B^+ \rightarrow f(i)} + N_{B^- \rightarrow f(i)}}}. $$

(20)

The quantity of Eq. 20 has the property of being Gaussian distributed with mean $\mu^A = 0$ and width $\sigma^A = 1$ in the limit of large number of events, as can be easily seen by calculating the error in $DpS_{CP}^A$ as a function of the errors in $N_{B^+ \rightarrow f(i)}$ and $N_{B^- \rightarrow f(i)}$ in the limit of $N_{B^+ \rightarrow f(i)} \rightarrow N_{B^- \rightarrow f(i)}$, when no production and CP asymmetries are present. Effects due to either production or CP asymmetries in $DpS_{CP}^A$ reveal through a shift of the center and a modification of the width of the Gaussian.

In $p\bar{p}$ colliders where production effects are at work, the above quantity does not measure the CP asymmetry alone, as discussed at the beginning of this section, but a combined effect of both, the CP and production asymmetry. Thought the production asymmetry is constant all over the Dalitz plot, the formula of Eq. 20 has to be replaced by

$$DpS_{CP}^A = \frac{N_{B^+ \rightarrow f(i)} - N_{B^- \rightarrow f(i)}}{\sqrt{N_{B^+ \rightarrow f(i)} + N_{B^- \rightarrow f(i)}}},$$

(21)

to account for production effects. Eq. 21 can be obtained from Eq. 20 with the replacement (A similar treatment is made in Ref. 9, in a different context.)

$$N_{B^- \rightarrow f(i)} \leftrightarrow N_{B^- \rightarrow f(i)}N_{B^+ \rightarrow f(i)}N_{B^- \rightarrow f(i)} = N_{B^- \rightarrow f(i)} \left[1 + A \right],$$

(22)

where $N_{B^+}(N_{B^-})$ is the total number of particles (antiparticles) produced at the interaction vertex. In the absence of CP violating effects, $DpS_{CP}^A$ is Gaussian distributed with mean $\mu^A = 0$ and width $\sigma^A$ which is a complicated function of $N_{B^+ \rightarrow f(i)}$, $N_{B^- \rightarrow f(i)}$, $A$ and their errors. We refrain to show the mathematical form of $\sigma^A$ because it is of no particular utility and can be easily found by calculating the error on $DpS_{CP}^A$. CP-violating effects can still be seen in $DpS_{CP}^A$ by looking for deviations from $\mu^A = 0$. In addition, it is important to remark that the departure from zero of the mean $\mu^A$ and the value of $\sigma^A$ is dependent on the number of events in the particular bin/region of the Dalitz plot, making absolute measurements of the effect impossible, unless the effect of the statistics on $\mu^A$ and $\sigma^A$ be known. This behavior is due to the presence of the square root in the denominator of Eqs. 20 and 21.

The use of the above method to look for CP violation in the Dalitz plot presupposes that the production asymmetry $A$ has been measured independently of the CP asymmetry, using a control channel free of CP-violating effects. In addition, attention has to be paid to the fact that the asymmetry $A$ has to be measured in the same momentum range in which the decaying particles are selected for the Dalitz analysis.

Of course, any detector effect leading to asymmetries in the measurement of the number of particles and an-
to perform precision measurements of CP asymmetries and asymmetries in the production, while the second has to account for both, differences in the decay widths and asymmetries in the production, where no production asymmetries are expected, quantitatively. Finally, we would like to emphasize that, although along the text the discussion has been focused on the particular decay channel in which particle and antiparticles are being reconstructed, this cannot be avoided, as CP asymmetries and/or mixing effects have to do with the dynamics of the particular decay mode in which particles are reconstructed. Fortunately, CP asymmetries and mixing effects can always be measured independently of production effects.

Finally, we would like to emphasize that, although along the text the discussion has been focused on the production of B-mesons in $p-p$ collisions, it can be extended with almost no changes to the production of particles and antiparticles in any flavor asymmetric machine.

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