Modelling of filtration of liquid binder in the composite textile structures under RTM processes

Yu I Dimitrienko¹ and I O Bogdanov¹

¹Department of Computational Mathematics and Mathematical Physics, Bauman Moscow State Technical University, 5 Baumanskaya 2-ya, Moscow, 105005, Russia
E-mail: dimit.bmstu@gmail.com, biofamily_7394@mail.ru

Abstract. The paper deals with three-scale modelling of the filtration of a weakly compressible liquid binder in the tissue material of the filler in the production of composite materials based on the RTM method. The technique is based on the application of the asymptotic homogenization method for the Navier-Stokes system of equations. Local filtration problems for the threads level and for the monofilaments level are obtained.

1. Introduction

One of the most effective methods for manufacturing products from composite materials is the technology of impregnating fabric reinforcing filler with a liquid binder in a mould (RTM, Resin Transfer Molding). The quality of composite structures obtained using this method largely depends on the parameters of the manufacturing technology. In this regard, it is extremely important to adequately model the flow of liquid binder in the porous space of a tissue filler, which has a complex microstructure of pores formed by interlacing the fibres and their bundles – threads.

It should be noted that in the overwhelming majority of publications, the study of the flow of liquids in porous structures is studied in the framework of the phenomenological theory of filtration, which is based on the Darcy law. In this case, the permeability coefficients of the porous medium are determined either experimentally or by using various empirical and approximate relations for describing local filtration processes. In this case, rather rough estimates of real processes occurring inside pores with complex geometry are obtained, which leads to large deviations in determining the permeability. Therefore, an important part of the filtration study is the analysis of local processes of spatial fluid flow in a single pore using direct analysis of the Navier-Stokes equations. At the same time, a direct solution of the Navier-Stokes equations is not possible due to the complexity of the computational domain and the need to use extremely fine grids. Therefore, in this case, the asymptotic averaging method is used.

Questions of the study of the movement of liquids and gases in porous structures, in particular, in relation to RTM technology, are considered in many papers, for example, in [1–4]. However, it should be noted that the study of the behaviour of mediums within separate pores with a complex geometric shape and a multiscale structure has not been studied enough.

2. Geometric models of computational domains.

We assume that the porous medium (the fabric material of the filler) has the property of periodicity, i.e. it is possible to select geometrically repeating regions, called periodicity cells. In addition, we will assume that the porous medium has several spatial levels. The zero spatial level is, in fact, the porous medium itself, formed by interlacing the filaments (see Figure 1). The first spatial level is the level of...
periodicity cells $V_1^e$ (hereinafter referred to as PC1) formed by fibre bundles (see Figure 2, a). Finally, the second spatial level is the level of the periodicity cells $V_2^e$ (hereinafter referred to as PC2) formed by monofilaments (see Figure 2, b).

3. General formulation of the filtering problem.

The original exact equations of motion of a weakly compressible fluid have the form:

$$
\frac{\partial \rho_i}{\partial t} + \nabla \cdot (\rho_i \mathbf{v}_i) = 0,
$$

$$
\kappa \left( \frac{\partial \rho_i \mathbf{v}_i}{\partial t} + \nabla \cdot (\rho_i \mathbf{v}_i \otimes \mathbf{v}_i) \right) = \nabla \cdot \mathbf{T}_i.
$$

(1)

Here $i = l, f, g, p$, where $l$ – fluid in threads from the area I (see Figure 1); $f$ – fluid between threads from the area I; $g$ – gas in area II; $p$ – gas contained in threads from region I. $\kappa$ – small task parameter.

The equation of state for the liquid phase is:
\[ p_i = p_{0i} + K \left( \frac{\rho_i}{\rho_{0i}} - 1 \right), \]  

where \( \rho_{0i} \) – fluid density at the initial time, \( \text{kg/m}^3 \); \( p_{0i} \) – initial pressure in the fluid, \( \text{Pa} \); \( K \) – bulk compression module, \( \text{Pa} \). For the case of gas, the Mendeleev-Clapeyron equation of state is used:

\[ p_g = \rho_g R \theta_0, \]  

where \( R \) – gas constant, \( \text{J kg K}^{-1} \); \( \theta_0 = \text{const} \) – constant temperature, \( \text{K} \).

We supplement the system with defining relations. For an isotropic linear-viscous fluid (gas), the stress tensor has the form:

\[ \tau_i = p_i E + \kappa \tau_i (D_i) \]  

where \( p_i \) – pressure, \( \text{Pa} \); \( \tau_i (D_i) = \lambda_i (\nabla \cdot \mathbf{v}_i) E + 2 \mu_i D_i \) – viscous stress tensor; \( \lambda_i, \mu_i \) – viscosity coefficients, \( \text{Pa s} \); \( E, D_i \) – metric tensor.

At the interface \( \Sigma_{fg} \) between liquid and gas, the following conditions are set:

\[ -p_i E + \tau_f \cdot \mathbf{n} = -(p_g E + \tau_g) \cdot \mathbf{n}, \quad \mathbf{v}_f = \mathbf{v}_g. \]  

Adhesion conditions are set at the interface between the liquid and the gas and the monofilament:

\[ \mathbf{v}_i = 0. \]  

The initial formulation (1) should also be supplemented with conditions on the external surfaces of the porous medium, as well as with initial conditions.

4. Statement of local problems for the first and second spatial levels.

Let \( \hat{L}_1 \) – the linear size of the periodicity cell \( V_{\xi_1}^1 \) of the medium and \( \hat{L}_2 \) – the linear size of the periodic cell \( V_{\xi_2}^2 \) of the medium. Let us introduce small parameters \( \kappa_i = \hat{L}_i / \hat{L} \ll 1 \) and two types of dimensionless coordinates: global (slow) \( \xi = x / \hat{L} \) (we omit) and local (fast) \( \xi_1 = x / \hat{L}_1 \) and \( \xi_2 = \xi_1 / \kappa_2 \). Wherein \( \kappa = \kappa_1 \kappa_2 \). Differentiation rules for new coordinates will take the form:

\[ \nabla \rightarrow \nabla_x + \kappa_1^{-1} \nabla_{\xi_1} + \nabla_{\xi_2}, \quad \nabla_{\xi_1} \rightarrow \nabla_x + \kappa_2^{-1} \nabla_{\xi_2}. \]  

Taking into account (7) for (1), we obtain:

\[ \frac{\partial \rho_i}{\partial t} + \nabla_x \cdot (\rho_i \mathbf{v}_i) + \kappa_1^{-1} \nabla_{\xi_1} \cdot (\rho_i \mathbf{v}_i) = 0, \]

\[ -\nabla_x p_i - \kappa_1^{-1} \nabla_{\xi_1} p_i + \kappa \left( \nabla_x \cdot \tau_i + \kappa_1^{-1} \nabla_{\xi_1} \cdot \tau_i \right) = 0, \]  

In accordance with the general concept of the asymptotic homogenization method [5–8], the solution of the problem (8) is sought in the form of asymptotic expansions in powers of a small parameter \( \kappa_1 \) in regions I and II:
\[\rho_i(x, \xi, t) = \rho_i^{(0)}(x, \xi, t) + \kappa_i \rho_i^{(1)}(x, \xi, t) + O(\kappa_i^2),\]
\[p_i(x, \xi, t) = p_i^{(0)}(x, \xi, t) + \kappa_i p_i^{(1)}(x, \xi, t) + O(\kappa_i^2),\]
\[v_i(x, \xi, t) = v_i^{(0)}(x, \xi, t) + \kappa_i v_i^{(1)}(x, \xi, t) + O(\kappa_i^2),\]
\[\tau_i(x, \xi, t) = \tau_i^{(0)}(x, \xi, t) + \kappa_i \tau_i^{(1)}(x, \xi, t) + O(\kappa_i^2),\] (9)

After substituting (9) into (8) and grouping the terms with the same powers of the parameter \(\kappa_i\), the desired local problem can be obtained for the first spatial level (PC1):
\[
\begin{align}
\nabla_{\xi_i} \cdot \mathbf{v}_i^{(0)} &= 0, \\
-\nabla_{\xi_i} p_i^{(0)} - \nabla_{\xi_i} p_i^{(1)} + \kappa_2 \nabla_{\xi_i} \tau_i^{(0)} &= 0, \quad \xi_1 \in V_1^1, \\
i &= f, l, g, p.
\end{align}
\] (10)

Further, taking into account the second expression in (7), we obtain from (10):
\[
\begin{align}
-\nabla_{\xi_1} p_i^{(0)} - \nabla_{\xi_2} p_i^{(1)} - \kappa_2^{-1} \nabla_{\xi_2} \tau_i^{(0)} + \kappa_2 \left( \nabla_{\xi_1} \tau_i^{(0)} + \kappa_2^{-1} \nabla_{\xi_2} \tau_i^{(0)} \right) &= 0, \quad \xi_2 \in V_2^1, \\
i &= l, p.
\end{align}
\] (11)

The solution (11) will also be sought in the form of asymptotic expansions:
\[v_i^{(0)}(x, \xi_1, \xi_2) = v_i^{(0)(0)}(x, \xi_1, \xi_2) + \kappa_2 v_i^{(0)(1)}(x, \xi_1, \xi_2) + O(\kappa_2^2),\]
\[p_i^{(1)}(x, \xi_1, \xi_2) = p_i^{(1)(0)}(x, \xi_1, \xi_2) + \kappa_2 p_i^{(1)(1)}(x, \xi_1, \xi_2) + O(\kappa_2^2).\] (12)

Then, similarly can obtain the desired local problem second spatial level (PC2):
\[
\begin{align}
-\nabla_{\xi_1} p_i^{(0)} - \nabla_{\xi_1} p_i^{(1)} - \nabla_{\xi_2} p_i^{(1)(0)} + \nabla_{\xi_2} \tau_i^{(0)(0)} &= 0, \quad \xi_2 \in V_2^2, \\
i &= l, p.
\end{align}
\] (13)

Note that the following approximation for the continuity equation in the problem on the PC2 is:
\[\nabla_{\xi_1} \cdot \mathbf{v}_i^{(0)(0)} + \nabla_{\xi_2} \cdot \mathbf{v}_i^{(0)(1)} = 0.\] (14)

We will seek a solution of the problem (13) in the form:
\[v_i^{(0)(0)} = -V_i^{(0)(0)}(\xi_1, \xi_2) \cdot (\nabla_{\xi_1} p_i^{(0)} + \nabla_{\xi_2} p_i^{(1)(0)}).\] (15)

We introduce the averaging operator over the region \(V_i^2 \subset V_2^2\) occupied by the i-th phase in the region \(V_2^2:\)
\[
\langle \cdot \rangle_i^2 = \frac{1}{|V_i^2|} \int_{V_i^2} (\cdot) dV, \quad i = l, p.
\] (16)
Averaging using (16) equations (14) and (15), we obtain the filtration equations at the second spatial level:

$$\nabla \xi_i \cdot \left( \psi^{(0)(0)} \right)_i = 0,$$

$$\left( \psi^{(0)(0)} \right)_i = -K_{2i} \left( \nabla \psi^{(0)} + \nabla \psi^{(1)(0)} \right), \quad i = l, p. \quad (17)$$

To the problem (17) should add conditions on the interface $\Sigma_{fl}$:

$$\left( \psi_f^{(0)(0)} \right)_l \cdot \left( \psi_f^{(0)} \right)_p^{(1)} = \psi_f^{(1)} p_l^{(1)}. \quad (18)$$

Note that the solution of the problem (17), in turn, can be represented in the form:

$$\left( \psi_f^{(0)} \right)_l = -\nabla \psi^{(0)(1)} \cdot (\xi_1) \nabla \psi_i^{(0)}, \quad i = l, p. \quad (19)$$

and the solution of the problem (10) for $f$ and $g$ can be searched in the form:

$$\psi_i^{(0)} = -\nabla \psi^{(0)} \cdot (\xi_1) \nabla \psi_i^{(0)}, \quad i = f, g. \quad (20)$$

5. Results

In this work, we numerically simulated the flow of medium in porous composite structures using the finite element method [9–10]. Examples of the distribution of micro fields of pressure pulsations and components of the velocity vector, obtained by solving a local problem for a porous structure with a dimensionless filament radius of 0.125, are shown in Figures 3-4. The dimensionless permeability coefficients $K_{11}$ and $K_{33}$ for this structure turned out to be equal, respectively, $2.047 \cdot 10^{-3}$ and $1.385 \cdot 10^{-3}$ with porosity of 0.555.

![Figure 3](image1.png) Figure 3. Distribution of pressure $\bar{P}^{(1)}$ in 1/8 of the tissue structure periodicity cell (PC1)

![Figure 4](image2.png) Figure 4. Distribution of the velocity component $\bar{W}_i^{(1)}$ in 1/8 of the tissue structure periodicity cell (PC1)

6. Conclusions

The paper considers the method of three-scale modelling of filtration, based on the use of the asymptotic homogenization method. Statements of local filtration problems are obtained for two spatial levels: threads and monofilaments. A number of results are presented demonstrating the effectiveness of the considered algorithms.
Acknowledgments
The work was carried out with support of state task of Ministry for Science and Education of the RF № 9.5387.2017/БЧ

References
[1] Shargatov V A, Il'Ichev A T and Tsypkin G G 2015 Dynamics and stability of moving fronts of water evaporation in a porous medium International J. of Heat and Mass Transfer 83 552-61
[2] Borodulin A S, Malysheva G V and Romanova I K 2015 Optimization of rheological properties of binders used in vacuum assisted resin transfer molding of fiberglass Polymer Science, Series D. Glues and Sealing Materials 8(4) 300-3
[3] Golovatov D, Mikhaylov M and Bosov A 2016 Optimization of technological parameters of impregnation of load-bearing rod elements of reflector made of polymer composite materials by transfer molding method Indian J. of Science and Technology 9(46) 11
[4] Loudad R, Saouab A, Beauchene P, Agogue R and Desjoyeaux B 2017 Numerical modeling of vacuum-assisted resin transfer molding using multilayer approach J. of Composite Materials. 51(24) 3441-52.
[5] Hornung U 1997 Homogenization and Porous Media (New York: Springer-Verlag).
[6] Dimitrienko Yu I and Yakovlev D O 2014 Asymptotic theory of thermoelasticity of multilayer composite plates Mechanics of Composite Materials and Structures. 20(2) 260-82.
[7] Dimitrienko Yu I and Dimitrienko I D 2013 Simulation of local transfer in periodic porous media European J. of Mechanics – B-Fluids. 1 174-9.
[8] Dimitrienko Yu I and Bogdanov I O 2018 Two-scale modeling of spatial flows of gas and weakly compressible liquid in porous composite structures J. of Physics: Conference Series. 1141(1) 012099
[9] Dimitrienko Yu I and Bogdanov I O 2016 Finite-element method for three-dimensional problems of elastic structures buckling theory Herald of the Bauman Moscow State Technical University. Series Natural Sciences. 6 73-92.
[10] Zienkiewicz O C, Taylor R L and Zhu J Z 2013 The Finite Element Method: Its Basis and Fundamentals: Seventh Edition (Butterworth-Heinemann).