Can neutrino viscosity drive the late time cosmic acceleration?

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Abstract

In this paper it has been shown that the neutrino bulk viscous stresses can give rise to the late time acceleration of the universe. It is found that a number of spatially flat FRW models with a negative deceleration parameter can be constructed using neutrino viscosity and one of them mimics a ΛCDM model. This does not require any exotic dark energy component or any modification of gravity.

1 Introduction

That the present universe is undergoing an accelerated expansion has now been firmly established. The initial indications came from the supernovae data [1] and were soon confirmed by many high precision observations including the WMAP [2]. Theoretical physics has thus been thrust with the challenge of finding the agent, dubbed “dark energy”, which can drive this acceleration. Naturally a host of candidates appeared in the literature which can provide this antigravity effect. However, the dark energy should become dominant only during the later stages of matter era so that nucleosynthesis in the early universe and the large scale structure formation in the matter dominated regime could proceed unhindered and make the universe look the place where we live in now.

Amongst the various dark energy candidates, the cosmological constant Λ is certainly the most talked about one. It matches different observational requirements quite efficiently and has been known in cosmology for quite a long time for the various roles it could play. The insurmountable problem is of course that of the huge discrepancy between the required value of Λ and that predicted theoretically [3].

The quintessence models, where an effective negative pressure generated by a scalar field potential drives the acceleration, work extremely well to fit into various observational constraints. For a very brief review, we refer to Martin’s recent work [4]. But none of the potentials employed for the purpose can boast of any sound theoretical motivation. Non minimally coupled scalar field theories like Brans - Dicke theory can also be used where even a dark energy is not required [5], but the value of the Brans - Dicke parameter ω needs

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to be given a very small value contrary to the observational requirements. Scalar - tensor theories with a dark energy, particularly where an interaction between the dark energy and the geometrical scalar field like the Brans - Dicke scalar field is allowed so as to alleviate the coincidence problem, appear to do well [6]. But the nature of the interaction is hardly well-motivated. Also, a recent work shows that energy has to be pumped in to the dark matter from the dark energy sector as demanded by the second law of thermodynamics [7]. This is indeed counter-intuitive in view of the fact that the dark energy dominates over the dark matter only during the later stages of evolution. Chaplygin gas models [8], modified gravity theories [9] and many other models are proposed. They all have their success stories as well as failures in some way or the other. In the absence of a clear verdict in favour of a particular dark energy candidate, all of them have to be discussed seriously. There are excellent reviews regarding different models and their relative merits [10].

One important general feature is that all these models use either some kind of an exotic field or some modification of the firmly established general relativity - the effect of the modification being hardly required by other branches of well established physical theories, and the possibility of an actual physical detection of them appears to be quite a far-fetched one.

Recently a well known sector of matter, whose existence in abundance has been firmly established, namely the neutrino distribution, has been proposed as a candidate for the dark energy [11]. The motivation comes from particle physics, and for a brief but comprehensive review we refer to [12]. However, in these models, neutrinos are normally the sector which ‘feels’ the existence of dark energy and cannot really solve the problem by itself, i.e, without a quintessence potential. For a review of the neutrino properties in a cosmological context, we refer to [13].

In the present work, we treat the neutrinoes completely classically and show that bulk viscous stresses in the distribution can indeed do the trick.

The advantage of the neutrinoes is that they are real objects, the method of detection being quite well conceived. The neutrinoes were decoupled from the background radiation quite early in the evolution when the temperature was as high as $3 \times 10^{10} K$ [14]. Thus the interaction of the neutrinoes with other forms of matter can be ignored and hence the problem of the “direction of the flow of energy” [7] does not arise and the model becomes much more tractable.

Neutrino viscosity, both in the form of shear [15] or in the form of bulk viscosity [16] had been investigated quite a long time back for various purpose. There has been a renewed interest in the neutrino viscosity quite recently as well [17]. The problem of dissipative effects like viscosity or heat conduction had been that of a parabolic transport equation, which could allow the signals to travel with super-luminal speed resulting in a violation of causality. But the extended irreversible thermodynamics, which modifies the transport equation by including a relaxation time and a further divergence term to avoid this problem, is now quite well understood [18] and the modified transport equation had already been quite extensively used in cosmology [19].

In what follows, we employ a two - component non-interacting matter sector, one is the normal cold dark matter and the other being a neutrino distribution. The latter is endowed with bulk viscosity which produces a negative stress. It is shown that a very simple accelerated model can be constructed from this. The model looks simple as the shear viscosity is neglected in order to be consistent with the isotropic nature of the universe.
2 Field equations and results:

Einstein equations for a spatially flat FRW universe are given by

\[ 3 \frac{\dot{a}^2}{a^2} = \rho_m + \rho_n , \]  
\[ 2 \frac{\ddot{a}}{a} + \frac{\dot{a}^2}{a^2} = -\Pi , \]

where \( a \) is the scale factor for the model given by

\[ ds^2 = dt^2 - a^2(t)(dr^2 + r^2d\Omega^2) ; \]

\( \rho_m \) and \( \rho_n \) are the densities of the normal matter and the neutrino distribution respectively and an overhead dot represents a differentiation with respect to the cosmic time \( t \). Consistent with the present universe, we assume that both the forms of matter are pressureless. The advent of massive neutrinoes provides the possibility that they can be non-relativistic so that the pressure can be neglected as in the case of the cold dark matter. The equations are written in units where \( 8\pi G = 1 \). The bulk viscous stress satisfies the causal transport equation

\[ \Pi + \tau \dot{\Pi} = -3\eta H - \frac{1}{2} \tau \Pi \left[ 3H + \frac{\dot{\tau}}{\tau} - \frac{\ddot{H}}{\eta} - \frac{T}{T} \right] , \]

where \( \tau \) is the relaxation time for the dissipative effects, \( \eta \) is the co-efficient of viscosity and \( T \) is the temperature. Being non-interacting amongst each other, both \( \rho_m \) and \( \rho_n \) satisfy their own conservation equations. For \( \rho_m \), the conservation equation

\[ \dot{\rho}_m + 3H \rho_m = 0 \]

immediately integrates to yield

\[ \rho_m = \frac{\rho_{m0}}{a^3} , \]

\( \rho_{m0} \) being a constant of integration. Equations (1), (2) and (4) combine to give the conservation equation for \( \rho_n \) as

\[ \dot{\rho}_n + 3H (\rho_n + \Pi) = 0 , \]

which indeed is not an independent equation. So, one has three unknowns, namely \( a, \rho_n \) and \( \Pi \) whereas only two equations to solve for them. In order to close the system of equations, another equation will be required. As we require a specific dynamics for the universe, which is accelerating at the present moment but had experienced a more sedate form of a decelerated expansion during a none-too-distant past in the matter dominated regime itself, we take the form of \( a \) as

\[ a = a_0 [\sinh(\alpha t)]^{2/3} , \]

as the third equation. Here \( a_0 \) and \( \alpha \) are positive constants. This simple ansatz gives the required behaviour. For small values of \( 't' \),

\[ a \sim t^{2/3} , \]

which indeed yields a decelerated expansion. In fact the deceleration parameter
\[ q = -\frac{\ddot{a}}{a^2} / \frac{\dot{a}^2}{a^2} = \frac{1}{2}, \]

which is positive and exactly same as that for a matter dominated universe without any other field. For a large \( t \),

\[ a \sim e^{2t/3}. \]

The universe is exponentially expanding and hence accelerated with \( q = -1 \). The evolution of \( q \) with the redshift parameter \( z \) for the choice of \( a \) as in equation (6) is given in figure 1 which indicates that \( q \) has a smooth transition from a positive to a negative phase. As \( z \) is given by

\[ 1 + z = \frac{a_0}{a}, \]

where \( a_0 \) is the present value of \( a \), \( z \) is a measure of the epoch in the past that one is looking at. The present value of \( z \) is zero.

Figure 1: Plot of \( q \) vs. \( z \) for the choice of \( a \) given by equation (6).

Equation (6) can be used in equation (2) to find \( \Pi \) as

\[ \Pi = -\frac{4\alpha^2}{3}, \]

which is a constant. Equation (5) can now be integrated to yield

\[ \rho_n = \frac{\rho_{n0}}{a^3} + \frac{4\alpha^2}{3}, \]

where \( \rho_{n0} \) is a constant of integration. So, now the model is completely solved, it allows for the observed behaviour of \( q \) and is extremely simple.

The great advantage of this model is the constancy of \( \Pi \). This effectively gives rise to a \( \Lambda \)CDM model, as Einstein’s equations now look like

\[ 3 \frac{a^2}{a^2} = \frac{\rho_{m0} + \rho_{n0}}{a^3} + \frac{4\alpha^2}{3}, \]
\[ \frac{2 \ddot{a}}{a} + \frac{\dot{a}^2}{a^2} = \frac{4\alpha^2}{3} . \] (10)

The constant value \( \frac{4\alpha^2}{3} \) of \( \Pi \) serves the purpose of a cosmological constant. So one can have all the virtues of a \( \Lambda \)CDM model, without having to write a cosmological constant by hand. An effective \( \Lambda \) is provided by the bulk viscous stresses of neutrinos, which is already quite well understood. So the model does not require any exotic field or a modification of gravity.

The high degree of non linearity of Einstein’s equations allows us to have other solutions as well. So this beautiful solution is by no means a unique one. In fact there could be other consistent accelerated solutions for equations (1), (2) and (5). One example is

\[ H = \frac{\dot{a}}{a} = A \left( 1 + a^{-3/2} \right), \] (11)

where \( A \) is a constant. This solution also describes the present acceleration quite efficiently as discussed by the present authors [20]. This ansatz also satisfies all the field equations. However, this will not mimic a \( \Lambda \)CDM model as \( \Pi \) comes out to be evolving rather than being a constant. Using equation (11) in equation (2), one can write \( \Pi \) as

\[ \Pi = -3AH . \]

The density perturbation for the “effectively \( \Lambda \)CDM” model given by equation (6) can be studied. The linearized perturbation equation

\[ \ddot{D} + 2H \dot{D} - 4\pi G\rho_mD = 0 \] (12)

for the model can be numerically integrated to yield the growth of the density perturbation as shown in figure 2. The density contrast \( D \) is as usual given by

\[ D = \frac{\rho_m - \bar{\rho}}{\bar{\rho}} \]

where \( \rho_m \) is the perturbed density and \( \bar{\rho} \) is the background density.

Figure 2: Plot of density contrast \( D \) vs. \( a \) for \( \alpha = 1 \) which clearly shows a growing mode for the dark matter distribution.
The temperature profile of the neutrino distribution can also be estimated with the help of the equation (3). For this purpose, the coefficient of viscosity $\eta$ and the relaxation time $\tau$ have to be given in terms of quantities like the density. The popular choice of $\eta = \eta_0 \rho^m$ and $\tau = \frac{\eta}{\rho}$ where $m$ is a constant [18], however, does not seem to work well. With these choices and for $m = \frac{1}{2}$, equation (3) provides an expression for the neutrino temperature $T$ as

$$T = T_0 \left( \frac{X \sqrt{p} \left(2\sqrt{X+1}\sqrt{X+p} + 2X + p + 1\right)}{2\sqrt{p}\sqrt{X+1}\sqrt{X+p} + (p+1)X + 2p} \right)^{-\frac{1}{\eta_0\rho} \frac{\sqrt{2\eta/\rho}}{T_0}} \left( \frac{X}{X+p} \right) e^{\frac{2X}{p}} \quad (13)$$

where $X = \operatorname{cosech}^2(\alpha t) = \left(\frac{\alpha}{a}\right)^3 = (1 + z)^3$ and $p = \frac{4\alpha^2 a_0^3}{3\eta_0 a_0}$. The plot of $T$ vs. $z$ shows that the neutrino temperature given by this ansatz shoots to a very high value at low values of the redshift $z$ ($z \sim 1$) (see figure 3), whereas the neutrino temperature is known to be below that of the cosmic microwave background radiation. Some other choices of $\eta$ and $\tau$ may provide a better temperature profile.

![Figure 3: Plot of neutrino temperature $T$ (in the units $T_0 = 1$) vs. redshift $z$ for $\eta = \eta_0 \rho^{1/2}$ and $\tau = \frac{\eta}{\rho}$.](image)

### 3 Conclusion:

As a conclusion, one can say that a sufficient bulk viscous stress in the neutrino distribution can potentially serve the purpose of a dark energy. This does not require any ill-motivated scalar potential or an otherwise unwarranted modification of general relativity. However, the model is far from being complete. The amount of the bulk viscous stress has to be sufficient to drive the acceleration, and the correct phenomenological connection between the quantities like $\eta$, $\tau$ and $\rho$ has to be found out so that temperature of neutrinoes has a realistic profile. The neutrino viscosity can in fact give a wide range of accelerating models. One of them, the effective $\Lambda$CDM model is discussed here. But the other example mentioned also works, and leaves a possibility of finding many others within the scope of it. It has to
be searched which solution is favoured from the consideration of stability as well as that of observational bounds.

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