A robust nonlinear position observer for synchronous motors with relaxed excitation conditions

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\textbf{ABSTRACT}

A robust, nonlinear and globally convergent rotor position observer for surface-mounted permanent magnet synchronous motors was recently proposed by the authors. The key feature of this observer is that it requires only the knowledge of the motor's resistance and inductance. Using some particular properties of the mathematical model it is shown that the problem of state observation can be translated into one of estimation of two constant parameters, which is carried out with a standard gradient algorithm. In this work, we propose to replace this estimator with a new one called dynamic regressor extension and mixing, which has the following advantages with respect to gradient estimators: (1) the stringent persistence of excitation (PE) condition of the regressor is not necessary to ensure parameter convergence; (2) the latter is guaranteed requiring instead a non-square-integrability condition that has a clear physical meaning in terms of signal energy; (3) if the regressor is PE, the new observer (like the old one) ensures convergence is exponential, entailing some robustness properties to the observer; (4) the new estimator includes an additional filter that constitutes an additional degree of freedom to satisfy the non-square integrability condition. Realistic simulation results show significant performance improvement of the position observer using the new parameter estimator, with a less oscillatory behaviour and a faster convergence speed.

\textbf{1. Introduction}

The use of permanent magnet synchronous motors (PMSMs) in various technical systems is associated with the benefits of this type of drives: high power and payload, maintaining a constant speed under shock loads and fluctuations of supply voltage, high efficiency, etc. Field-oriented control of PMSM requires rotor position information and, due to this, position sensors (Hall sensors, optical encoders) are often installed on PMSM to provide high performance of different applications. However, design features of some devices (vacuum pumps, cranes, elevators) obstruct the mounting of such sensors. Also, for mass-produced applications (refrigerators, air conditioners, washing machines), the use of sensorless control techniques instead of measuring equipment can reduce production costs and expand the sales market. Moreover, if the motor is controlled using measured position, addition of sensorless algorithms may increase the reliability of the entire technical system in the event of sensor failure.

In view of the above problems, many papers devoted to sensorless control algorithms for PMSMs have been published. See Acarnley and Watson (2006), Nam (2010) for a recent review of the literature. Since the angular position of the rotor can be easily derived from the stator flux via some simple trigonometric relations, many sensorless approaches are based on the estimation of flux. Successful implementation of such methods has become available only in the last two decades due to the progress of CPU power. The key idea of the position sensing based on the flux linkage is very simple and consists in integrating the phase-voltage equation

\[ v = Ri + \frac{d\lambda}{dt}, \]

where \( v \) is the phase voltage, \( i \) is the phase current, \( R \) is the phase resistance and \( \lambda \) is the phase flux linkage. Precise estimation of rotor position requires the knowledge of the flux initial conditions, which are uncertain. Therefore, a large amount of work has been devoted to the compensation of the bias in the flux estimation (Dib, Ortega, & Malaize, 2011; Henwood, Malaize, & Praly, 2012; Ortega, Praly, Astolfi, Lee, & Nam, 2014; Shah, Espinosa, Ortega, & Hilairet, 2014; Tomei & Varrelli, 2011).
Recently, a robust, nonlinear and globally convergent position observer for surface-mounted PMSMs was proposed by Bobtsov et al. (2015). Only the knowledge of stator currents, voltages, resistance and inductance is required for the rotor position estimation – obviating the need to know other uncertain motor parameters like magnetic flux constant and rotor inertia. The key observation made in Bobtsov et al. (2015) is that the position observation problem can be recast in terms of classical parameter identification – a result that has been recently generalized in Ortega, Bobtsov, Pyrkin, and Aranovskiy (2015), where a class of systems for which this problem reformulation is possible has been identified. This reformulation is achieved for the PMSM in Bobtsov et al. (2015) via a new reparameterisation of the motor dynamics and some signal manipulation to obtain a classical linear regression form with two unknown constant parameters. A fundamental step in all parameter identification problems is, obviously, the selection of a suitable adaptation algorithm. For the sake of simplicity, in Bobtsov et al. (2015), it is proposed to use a basic gradient observer, which requires the usual, hardly verifiable persistent excitation (PE) condition (Ljung, 1987).

To improve the performance of the observer of Bobtsov et al. (2015), we propose in this paper to replace the gradient adaptation algorithm with the new dynamic regressor extension and mixing (DREM) estimator, which was recently proposed in Aranovskiy, Bobtsov, Ortega, and Pyrkin (2015). Our motivation to use DREM stems from the fact that, in contrast with standard gradient (or least-squares) identifiers, the convergence of DREM estimators is established without the stringent PE condition on the regressor vector – instead, it imposes a non-square-integrability assumption, which has a clear physical interpretation in terms of signal energy. The new DREM-based observer, as the one with gradient (or least squares) estimators, ensures parameter convergence is exponentially fast if its regressor is PE. If $\phi(t)$ is only non-square integrable (but not PE), then the convergence of the DREM-based observer is still guaranteed, however, convergence is not exponential. For the gradient estimator, in its turn, there is no analytical proof of convergence if its regressor $m(t) \in \text{PE}$. In this regard, the DREM estimator is superior because, even if $\phi(t)$ is not PE convergence is ensured, provided $\phi(t)$ is non-square integrable. Simulation results presented in the paper illustrate the advantages of the proposed observer. Namely, convergence speed of the new estimator is higher and its transient behaviour is significantly better. Due to this, the position and flux observers with a DREM estimator are more accurate and faster than the gradient-based estimator used in Bobtsov et al. (2015).

The remainder of the paper is organized as follows. Section 2 presents the classical model of the PMSM. In Section 3, we recall the main result of Bobtsov et al. (2015). The DREM procedure for parameter estimation is briefly recalled in Section 4. Section 5 contains the new position and flux observer design obtained from the application of the DREM approach. Simulation results are presented in Section 6. The paper is wrapped up with concluding remarks in Section 7.

2. PMSM Motor Model

We consider the classical, two-phase $\alpha\beta$ model of the unsaturated, non-salient, PMSM given by Krause (1986) and Nam (2010)

\[
\begin{align*}
\dot{\lambda} & = v - Ri \\
\dot{\omega} & = -f\omega + \tau_e - \tau_L \\
\dot{\theta} & = \omega,
\end{align*}
\]

where $\lambda \in \mathbb{R}^2$ is the stator flux, $i \in \mathbb{R}^2$ are the currents, $v \in \mathbb{R}^2$ the voltages, $R > 0$ is the stator windings resistance, $f > 0$ the rotor inertia, $\theta \in \mathbb{S} := [0, 2\pi)$ is the rotor phase, $\omega \in \mathbb{R}$ is the mechanical angular velocity, $f \geq 0$ is the viscous friction coefficient, $\tau_L \in \mathbb{R}$ is the – possibly time-varying – load torque, $\tau_e$ is the torque of electrical origin, given by

$$
\tau_e = n_p i^T J \lambda,
$$

with $n_p \in \mathbb{N}$ the number of pole pairs and $J \in \mathbb{R}^{2 \times 2}$ is the rotation matrix

$$
J = \begin{bmatrix}
0 & -1 \\
1 & 0
\end{bmatrix}.
$$

The total flux of surface-mounted PMSM verifies

$$
\lambda = Li + \lambda_mC(\theta),
$$

where $L > 0$ is the stator inductance, $\lambda_m$ is the constant flux generated by permanent magnets and, to simplify the notation, we defined

$$
C(\theta) := \text{col}(\cos(n_p\theta), \sin(n_p\theta)).
$$

Hence, a state-space model of the PMSM is given as

\[
\begin{align*}
L \frac{di}{dt} & = -Ri - \lambda_mC'(\theta) + v \\
\dot{\omega} & = -f\omega + \lambda_m i^T C'(\theta) - \tau_L \\
\dot{\theta} & = \omega,
\end{align*}
\]

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where \( C'(\theta) := \frac{dC}{d\theta} \) and we have used (1) and the fact that
\[
C'(\theta) = n_p J \dot{C}(\theta),
\]
to obtain the expression for \( \tau_e \).

From (1), it is clear that \( \theta \) can be directly reconstructed from knowledge of \( \lambda \) and \( i \) via
\[
\theta = \frac{1}{n_p} \arctan \left\{ \frac{\lambda_2 - Li_2}{\lambda_1 - Li_1} \right\}. \tag{4}
\]

3. The robust position observer of Bobtsov et al. (2015)

To make the paper self-contained, we recall below the standing assumptions and the robust observer proposed in Bobtsov et al. (2015) to estimate the angle \( \theta \).

3.1 Assumptions

In the design of the position observer (Bobtsov et al., 2015), the following assumptions are imposed.

A1 The measurable signals are currents \( i(t) \) and voltages \( v(t) \) of the stator windings.
A2 The control signal \( v(t) \) and the unknown external load torque \( \tau_\ell(t) \) are such that trajectories of the PMSM model (1)–(3) exist for all \( t \geq 0 \) and are bounded.
A3 The only known parameters of the PMSM are stator resistance \( R \) and inductance \( L \). All other parameters, i.e. \( \lambda_m, f, j \), are unknown.
A4 The measurable signals \( i(t) \) and \( v(t) \) are integrable.
A5 The signals \( v(t) \) and \( \tau_\ell(t) \) are such that the rotor speed is PE, that is, there exists constants \( T > 0 \) and \( \delta > 0 \)
\[
\int_t^{t+T} \omega^2(\tau) d\tau \geq \delta, \quad \forall t \geq 0. \tag{5}
\]

Assumption A1 is satisfied in the usual operation mode for most motors. Even though in some modern industrial motors, voltages of the stator windings are not measured directly, but estimated, the precision of such voltage observer is rather high. Hence, voltage signals \( v(t) \) are considered as known quantities. Assumption A2 is standard in all observer designs. Assumption A3 defines the knowledge about motor parameters that is required for the nonlinear position observer (Bobtsov et al., 2015). It should be mentioned that only two electrical parameters \( R \) and \( L \) are used in the observer construction. In practice, these stator parameters are uncertain and can change their values during PMSM operation by reasons of heating, mechanical wear, and external impacts. Nevertheless, relaxing Assumption A3 results in a difficult adaptive estimation problem with products of unknown states and uncertain parameters, which is essentially open-even for linear systems. Assumption A4 is a technical condition required to prove the observer boundedness.

The PE condition of Assumption A5 is necessary and sufficient for exponential convergence of standard gradient (or least-squares) estimators (Anderson et al., 1986; Ljung, 1987; Sastry & Bodson, 1989). Therefore, if it is not satisfied, the estimation will be biased. Moreover, if the excitation conditions are ‘weak’, the performance will be degraded. The main contribution of this paper is to propose an estimator that overcomes these shortcomings.

It should be noted that in this paper we study open-loop observer design problem; therefore, the conditions of Assumption A5 are imposed on the external signals \( v(t) \) and \( \tau_\ell(t) \). In closed-loop identification, the conditions should be given on the reference signals and are, in principle, harder to verify. This important issue was considered in Khatounian, Moreau, Monmasson, Janot, and Louveau (2006) and will be investigated in the future.

3.2 Convergence properties of the observer of Bobtsov et al. (2015)

Proposition 3.1 contains the main stability result of Bobtsov et al. (2015), which is derived via a reparametrization of the PMSM model, linear filtering of the new signals and a standard gradient-descent-based estimator of \( \theta \). Interested readers are referred to Bobtsov et al. (2015) for the proof of the proposition and additional technical details.

**Proposition 3.1:** Consider the PMSM model given by (2) and (3). Define the ninth-dimensional position observer
\[
\begin{align*}
\dot{\xi} &= F(\xi, i, v) \\
\dot{\theta} &= H(\xi, i, v),
\end{align*}
\]
where the mappings \( F : \mathbb{R}^9 \times \mathbb{R}^2 \times \mathbb{R}^2 \rightarrow \mathbb{R}^9 \) and \( H : \mathbb{R}^9 \times \mathbb{R}^2 \times \mathbb{R}^2 \rightarrow \mathbb{R} \) are defined as
\[
\begin{align*}
\dot{\xi}_{14} &= \begin{bmatrix} v \\ i \end{bmatrix} \\
q &= \xi_{12} - R \xi_{34} - Li \\
\dot{\xi}_5 &= -\alpha (\xi_5 + |q|^2) \\
\dot{\xi}_{67} &= -\alpha (\xi_{67} - 2q)
\end{align*}
\]

\[\]
where \( \alpha > 0 \) and \( \Gamma = \Gamma^\top > 0 \) are design parameters.

(i) If Assumptions A1–A3 hold, the observation error \( \tilde{\theta} := \hat{\theta} - \theta \) satisfies

\[
|\tilde{\theta}(t)| \leq c_1 |\tilde{\theta}(0)| + c_2, \quad \forall t \geq 0,
\]

for some constants \( c_1, c_2 > 0 \).

(ii) If, moreover, Assumptions A4 and A5 hold, then all signals are bounded and

\[
\lim_{t \to \infty} |\tilde{\theta}(t)| \to 0, \quad \tilde{\theta}(t) \in \mathbb{S}.
\]

The rationale of the observer above may be explained as follows. As shown in Bobtsov et al. (2015), the signals defined above satisfy the relationships

\[
\lambda - Li = q + \eta + \epsilon_i
\]

where \( \eta \) is a constant, unknown vector satisfying

\[
y = m^\top \eta + \epsilon_i,
\]

and \( \epsilon_i \) is a (generic) exponentially decaying signal. Since \( y \) and \( m \) are known – given by (10) and (11), respectively – equation (15) is a linear regression from which we can estimate the unknown parameters \( \eta \) via the gradient estimator (12), where \( \xi_{89} \) are the online estimates of \( \eta \) and \( \Gamma \) is the adaptation gain. This, together with (4) and (14), explains the expression for the estimated phase (13).

In Bobtsov et al. (2015), it is shown that Assumption A5 ensures that \( m \) is PE that, as is well known (Ljung, 1987; Sastry & Bodson, 1989), is the necessary and sufficient condition for (exponential) convergence of the error equation

\[
\dot{\xi}_{89} = -\Gamma m m^\top \xi_{89},
\]

obtained replacing (15) in the gradient estimator (12) and defining the parameter error \( \xi_{89} := \xi_{89} - \eta \). In order to relax the PE condition, we propose, in Section 5, to replace the gradient estimator (12) by a DREM estimator, whose construction is explained below.

4. Dynamic regressor extension and mixing procedure

Here we give a brief description of the DREM procedure used for online estimation of unknown constant parameters. We consider a \( q \)-dimensional linear regression

\[
y = m^\top \eta,
\]

where the signals \( y : \mathbb{R}_+ \to \mathbb{R} \) and \( m : \mathbb{R}_+ \to \mathbb{R}^q \) are known and bounded and \( \eta \in \mathbb{R}^q \) is the vector of unknown, constant parameters.

Application of the DREM technique leads to generation of \( q \) new, one-dimensional, regression models and independent estimation of each parameter, whose convergence does not require the PE condition (5). The procedure consists of two main stages:

(a) the application of a dynamic operator to the original regression model that leads to the generation of new regression forms

(b) obtaining the final desired regression form via a suitable mixing of the new regressors.

The first step in DREM is then to select \( q - 1 \) linear dynamic operators \( H_k : \mathcal{L}_\infty \to \mathcal{L}_\infty \) and defining its zero initial condition response

\[
u_k(t) := [H_k(u)](t).
\]

The second step is to apply these operators to the regressor form (16). Neglecting the decaying terms, we obtain \( q - 1 \) filtered regression forms

\[
y_{fi} = m_{fi}^\top \eta.
\]

Next, we add the new \( q - 1 \) regressions to the original one yielding the extended regressor model

\[
Y_e = M_e \eta,
\]

where \( Y_e : \mathbb{R}_+ \to \mathbb{R}^{q+q} \) and \( M_e : \mathbb{R}_+ \to \mathbb{R}^{q\times q} \) are defined as

\[
Y_e := \begin{bmatrix} y \\ y_{fi} \\ \vdots \\ y_{f_{i-1}} \end{bmatrix}, \quad M_e := \begin{bmatrix} m^\top \\ m_{fi}^\top \\ \vdots \\ m_{f_{i-1}}^\top \end{bmatrix}.
\]

It should be noted, that \( Y_e \) and \( M_e \) are bounded since the operators \( H_k \) are \( \mathcal{L}_\infty \)-stable.

Define the determinant of \( M_e \) as

\[
\phi := \det\{M_e\},
\]
and the vector \( Y : \mathbb{R}^+ \rightarrow \mathbb{R}^q \)
\[
Y := \text{adj}(M_c) Y_c, \quad (22)
\]
where \( \text{adj}(M_c) \) stays for adjoint matrix. Now, multiplying (19) by \( \text{adj}(M_c) \) yields \( q \) scalar regressions of the form
\[
Y_k = \phi \eta_k. \quad (23)
\]
To estimate the parameters \( \eta_k \) from (23), we use \( q \) standard, scalar gradient estimators
\[
\hat{\eta}_k = \gamma_k \phi(Y_k - \phi \hat{\eta}_k), \quad (24)
\]
where \( \gamma_k > 0 \) are the adaptation gains. Thus, the error model is a scalar differential equation
\[
\dot{\hat{\eta}}_k = -\gamma_k \phi^2 \hat{\eta}_k, \quad k \in \hat{q}, \quad (25)
\]
where \( \hat{\eta}_k = \eta_k - \hat{\eta}_k \). Solving (25), one can conclude that
\[
\phi(t) \notin \mathcal{L}_2 \iff \lim_{t \to \infty} \dot{\hat{\eta}}_k(t) = 0. \quad (26)
\]
The aforementioned derivations establish the following proposition.

**Proposition 4.1:** Consider the linear regression form (16) with known, bounded functions of time \( y : \mathbb{R}^+ \rightarrow \mathbb{R} \) and \( m : \mathbb{R}^+ \rightarrow \mathbb{R}^q \), and the vector of unknown parameters \( \eta \in \mathbb{R}^q \). Introduce \( q - 1 \) linear and \( \mathcal{L}_\infty \)-stable operators \( H_k \) and define the filtered signals (17). Introduce the vector \( Y_c \) and the matrix \( M_c \) defined in (20). The estimator (24), where \( \phi \) and \( Y_k \) are defined in (21) and (22), respectively, verifies (26).

Notice that the \( q - 1 \) filtered regressors of (18) are derived neglecting exponentially decaying terms stemming from the initial conditions of the operators \( H_k \). Taking these terms into consideration, the error model (25) takes the form
\[
\dot{\hat{\eta}}_k = -\gamma_k \phi^2 \hat{\eta}_k + \epsilon_k.
\]

Lemma 1 of Aranovskiy, Bobtsov, Pyrkin, Ortega, and Chaillot (2015) contains the investigation of this equation, that allows to establish (26) (see also Aranovskiy, Bobtsov, Ortega, & Pyrkin, 2015).

## 5. New Position Observer with DREM

In this section, we propose to apply DREM to the linear regression (15) that results from (6)–(11) of the flux observer of Proposition 3.1. More precisely, in the new observer, we will keep (6)–(11) and replace (12) by a DREM estimator and, subsequently, redefine the position observer (13).

As mentioned in Section 4, the first step in DREM is the choice of the operators \( H_k \). Given the fact that two parameters need to be estimated, \( q = 2 \) and only one operator is required, which is selected as a simple exponentially stable, linear time-invariant filter
\[
H(s) = \frac{\beta}{s + \beta},
\]
where \( \beta > 0 \) is a design parameter. The next step is to define the filtered signals
\[
\tilde{m}_f = -\beta m_f + \beta m, \quad \gamma_f = -\beta \gamma_f + \beta \gamma.
\]

Following the procedure described in Section 4, we introduce the new regressor
\[
\phi := m_2 m_1 - m_1 m_2 f. \quad (28)
\]
The following replaces Assumption A5 of the gradient estimator.

**A5** Consider the PMSM model given by (2) and (3). The signals \( v(t) \) and \( t(t) \) are such that the new regressor \( \phi \) defined by (6)–(11), (27) and (28) is not square-integrable.

**Proposition 5.1:** Consider the PMSM model given by (2) and (3) verifying Assumptions A1–A4 and A5. Define the ninth-dimensional position observer (6)–(11) with the DREM estimator
\[
\dot{\hat{\eta}}_k = \gamma_k \phi(Y_k - \phi \hat{\eta}_k), \quad k = 1, 2 \quad (29)
\]
where the extended measurement vector is defined as
\[
Y := \begin{bmatrix} m_2 y - m_2 y_f \\ m_1 y_f - m_1 y_f \end{bmatrix},
\]
the adaptation gains \( \gamma_k > 0 \) and the observed position is given by
\[
\hat{\theta} = \frac{1}{\eta_p} \arctan \left\{ \begin{array}{l} q_2 + \hat{q}_2 \\ q_1 + \hat{q}_1 \end{array} \right\}. \quad (30)
\]

All signals are bounded and \( \lim_{t \to \infty} |\hat{\theta}(t)| \to 0, \quad \tilde{\theta}(t) \in \mathcal{S} \)

**Proof:** The proof is a direct application of Proposition 4.1 noting that the extended regressor model (19) takes, in this case, the form
\[
Y_c = \begin{bmatrix} y \\ y_f \end{bmatrix} = M_c \eta = \begin{bmatrix} m_1 & m_2 \\ m_1 f & m_2 f \end{bmatrix} \begin{bmatrix} \eta_1 \\ \eta_2 \end{bmatrix}.
\]
the adjoint matrix of $M_e$ is
$$\text{adj}\{M_e\} = \begin{bmatrix} m_{f2} & -m_2 \\ -m_{f1} & m_1 \end{bmatrix},$$
and $\phi = \det\{M_e\}$.

6. Simulation Results

In this section, we present some simulations that illustrate the performance improvement of the new position observer – that uses DREM estimator – compared with the gradient-based one of Bobtsov et al. (2015). For both observers, simulation results include the parameter estimates $\hat{\xi}$ and $\hat{\eta}$, estimation errors of rotor position $\tilde{\theta}$, and speed $\tilde{\omega}$. In addition, we show transients for the observed flux, which are defined as $\hat{\lambda} = Li + q + \xi_{89}$ for the observer of Bobtsov et al. (2015) and $\dot{\lambda} = Li + q + \hat{\eta}$ for the new observer.

Following standard engineering practice (Nam, 2010), the rotor velocity is reconstructed from the observed position $\dot{\theta}$ using a PLL.

Table 1. Parameters of the first motor BMP0701F (BMP manual, 2012) and the second one taken from SimPowerSystems toolbox.

| Parameter (units)       | Motor 1 | Motor 2 |
|------------------------|---------|---------|
| Inductance $L$ (mH)    | 40.03   | 8.5     |
| Resistance $R$ (Ω)     | 8.875   | 0.2     |
| Drive inertia $J$ (kgm²) | $60 \times 10^{-6}$ | 0.089   |
| Pairs of poles $n_p$  | 5       | 8       |
| Magnetic flux $\lambda_m$ (Wb) | 0.2086 | 0.175   |

Figure 1. Parameter estimates, errors for position, speed and flux for the new observer (I – blue) and the one of Bobtsov et al. (2015) (II – red) with $\alpha = 50$, $\beta = 100$, $\Gamma = 0.31$, $\gamma_{1,2} = 0.3$, $\xi_{34}(0) = (3, -2)$, $\xi_{89}(0) = (0, 0)$ and $\hat{\eta}(0) = (0, 0)$. (a) Transients for the parameter estimates $\hat{\xi}$ and $\hat{\eta}$, (b) transients for the parameter estimates $\hat{\xi}$ and $\hat{\xi}$, (c) transients for the position error $\tilde{\theta}$, (d) transients for the speed error $\tilde{\omega}$, (e) transients for the flux error $|\tilde{\lambda}|$, (f) transients for the flux error $\hat{\lambda}_1$, (g) transients for the flux error $\hat{\lambda}_2$. (To view this figure in colour, please see the online version of this journal.)
The observer of Bobtsov et al. (2015) generate the estimates of the unknown parameters $\xi_{89}$ and position $\hat{\theta}$ using (12) and (13), respectively. Design parameters for this observer are $\alpha$ and $\Gamma$. The new nonlinear observer with DREM uses (29) and (30) to obtain the estimates of the parameter $\hat{\eta}$ and the position $\hat{\theta}$, respectively. Its design parameters are $\alpha$, $\beta$ and $\gamma_i$, $i = 1, 2$. In order to provide similar conditions, we use the same values of design parameter $\alpha$ and initial conditions $\xi_{12}(0)$, $\xi_{34}(0)$, $\hat{\eta}(0) = \xi_{89}(0)$ for both observers in each simulation scenario.

### 6.1 Simplified model

In the simplified type of modelling, which is realised in continuous time, we assume that current and voltage sensors are ideal and there is no power switching devices in the control system. Such simulation has two objectives. The first one is to verify the performance and robustness of the new observer for different gain settings and initial conditions. The second objective is to compare the efficiency of the proposed observer with the one from Bobtsov et al. (2015).
In this section, we use motor 1, the parameters of which are listed in Table 1, and apply the external load torque \( \tau_L = 0 \). For all tests, both Assumptions A5 and A5* are fulfilled.

**Figure 1** demonstrates the transients of both nonlinear observers, when the motor is driven by the voltage \( v(t) = (100\sin(50t), 100\cos(50t)) \). The initial conditions of the observer states \( \xi_{14}(0) \) are taken different from zero, while \( \xi_{89}(0) \) and \( \hat{\eta}(0) \) have zero values. As seen from the figure, the convergence of the DREM estimator is faster and is less oscillatory than the standard gradient estimator.

In **Figure 2**, the motor is driven in speed control mode. The speed reference is shown in **Figure 3**. The parameter \( \alpha \) and adaptation gains are increased and the initial conditions of the observer states \( \xi_{14}(0) \) are

**Figure 3.** Reference speed.
changed. In comparison with the previous test, unknown parameters and flux errors of the gradient estimator converge to zero with stronger oscillations, while the DREM-based estimator has an overshoot only at the beginning. The position and speed errors of the new observer also show a better transient behaviour.

In Figure 4, motor control mode and speed reference are the same as in the previous test. This simulation illustrates that all errors for both observers converge to zero starting with non-zero initial conditions and increased design parameters $\alpha$ and $\beta$. As expected, the new observer shows better accuracy and performance than the one utilizing the standard estimator.

### 6.2 Detailed model

Realistic type of modelling is implemented using Simscape Power Systems. This Matlab toolbox contains simulators with electronic components, which are common for PMSM control systems (see also Wang, Chai, Yoo, Gan, & Ng, 2015). The main objective of such simulation is to examine efficiency and robust properties of the proposed observer for the case, when the control system contains noises due to current sensors and three-phase IGBT inverter. Three phases of current are mixed with bandlimited white noise separately impairing the useful signal by 10%–15%. The model is fed by a three-phase source (220 V, 50 Hz), and also includes a three-phase diode rectifier and a braking chopper.

The PMSM used in the simulation is the motor 2 from Table 1 driven in speed control mode. Simulations are carried out in discrete time (sampling time is 2 $\mu$s).

Figure 5 demonstrates the robustness of the new observer, when both Assumptions A5 and A5* hold. The load is zero at the beginning, then after 6 s, $\tau_L = -1.5$ N m and $\tau_L = 1$ N m starting at $t = 12$ s. All initial conditions of the observer were set to zero. One can see that the proposed observer is more efficient than the one of Bobtsov et al. (2015).

Figure 6 shows transients of both observers, when Assumption A5 is not satisfied, but A5* holds. The reference speed is chosen as

$$\omega^*(t) = \frac{10}{\sqrt{20t + 1}},$$

which is not PE and not square-integrable. The load torque $\tau_L = 0$. Notice that both nonlinear observers are robust but DREM-based observer gives better performance.

In Figure 7, speed reference and adaptation parameters are the same as in the previous test. The load torque is zero.
at $t = 0$, $\tau_L = -1.5 \text{ N m}$ starting at $t = 6 \text{s}$ and then $\tau_L = 1 \text{ N m}$ at $t = 12 \text{s}$. One can see that the position error of the observer proposed by Bobtsov et al. (2015) has lower value. In contrast to this, the new algorithm shows better transients of the speed error.

7. Conclusions

The nonlinear position and flux observer for PMSM proposed in Bobtsov et al. (2015) belongs to the class of parameter estimation-based observers reported in Ortega
et al. (2015). A key component of this observers is, obviously, the parameter adaptation algorithm itself. In Bobtsov et al. (2015), this algorithm is taken, for simplicity, as a simple gradient descent that, as is well known, requires stringent PE conditions for parameter convergence. In this paper, we proposed to replace this estimator by a DREM one, whose convergence does not rely on PE (Aranovskiy et al., 2015). Instead, a non-square-integrability assumption on the determinant of the extended regressor matrix is imposed. As thoroughly discussed in Aranovskiy, Bobtsov, Ortega, and Pyrkin (2015), the two conditions are different and neither one of them implies the other. It should, however, be underscored that to satisfy the convergence condition of the DREM estimator, we have large degrees of freedom, for instance, in the choice of the operators $H_k$ and the arrangement of the filtered regressions in the extended regression matrix $M_e$. On the other hand, PE of the original regressor $m$ is solely determined by the external signals $v$ and $\tau_L$, which are not available to the designer.

Realistic simulations demonstrate that the DREM estimator yields a faster convergence speed and, at the same time, a less oscillatory transient. As a result, more efficient estimation of constant parameters leads to significant performance improvement of the position and flux observation.

Notes

1. For brevity, we adopt here the notation $\xi_{ij} := col(\xi_i, \xi_{i+1}, ..., \xi_j$, for all $i, j \in \mathbb{N}, i < j$.
2. To simplify the presentation, this terms are neglected in the sequel, referring the reader to Bobtsov et al. (2015) and Aranovskiy, Bobtsov, Ortega, and Pyrkin (2015) for the analysis including these terms.
3. That is, the output of the operator when it is applied with zero initial conditions.
4. It is essential to emphasize that for any matrix $A \in \mathbb{R}^{n \times q}$, we have that $\text{adj}(A)A = \det(A)I_q$, even if $A$ is not full rank (Lancaster & Tismenetsky, 1985).

Disclosure statement

No potential conflict of interest was reported by the authors.

Funding

This article is supported by Government of Russian Federation (grant number 074-U01, GOSZADANIE 2014/190 (project 2118)), the Ministry of Education and Science of Russian Federation (project 14.Z50.31.0031).

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