Finsler space-time can explain both parity asymmetry and power deficit seen in CMB temperature anisotropies

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ABSTRACT

We propose a framework of Finsler space-time to explain the observed parity asymmetry and the power deficit in the low-ℓ (2 ⩽ ℓ ⩽ 29) multipole range of cosmic microwave background (CMB) temperature anisotropies. In the 3 + 1 dimensional space-time, the three-dimensional space is described by a Randers-Finsler space, which is spatially irreversible, inducing the parity asymmetry in the CMB angular power spectrum. We estimate the constraints on the two parameters introduced by Finsler space-time via analyzing the low-ℓ angular power spectrum in PLANCK 2015 CMB temperature data. We see that the low-ℓ power suppression in the CMB temperature anisotropies can also be resolved in this scenario. Our study shows that the two low-ℓ anomalies, i.e., parity asymmetry and power deficit, may have a common origin.

Key words: cosmic background radiation – cosmological parameters

1 INTRODUCTION

The cosmological principle says that the Universe is statistically homogeneous and isotropic on large scales. Based on it, a standard cosmological model has been well established, all the six base parameters of which have been constrained to a few percent level via performing data analysis of the Cosmic Microwave Background (CMB) temperature anisotropies and polarizations (Bennett et al. 2013; Ade et al. 2016a). However, several large-scale anomalies has been reported by WMAP (Hinshaw et al. 2013) and PLANCK (Ade et al. 2016b) team. For the CMB temperature anisotropies, the anomalies include the power suppression (Efstathiou 2003; Bonga & Gupt 2016; Cai et al. 2015; Chang et al. 2012; Zhao et al. 2015; Chang et al. 2018), the parity asymmetry (Hansen et al. 2012; Aluri & Jain 2012; Liu et al. 2013; Zhao 2014; Land & Magueijo 2005a; Kim & Naselsky 2010b; Gruppuso et al. 2011), an alignment of quadrupole and octopole (Chang et al. 2015; Copi et al. 2015a; Land & Magueijo 2005b; Copi et al. 2004; Abramo et al. 2006), the hemispherical power asymmetry (Chang & Wang 2013; Rath & Jain 2013; Rath et al. 2015; Jain & Rath 2015; Eriksen et al. 2007), and the lack of angular power on angular scales larger than 60’ (Spergel et al. 2003; Copi et al. 2009, 2015b).

Among these anomalies, the first two are related with this study. The power suppression of the CMB temperature anisotropies shows that the low-ℓ angular power spectrum, especially the quadrupole, was found to be lower than what predicted by the standard cosmological model (Hinshaw et al. 1996, 2013; Adam et al. 2016). The PLANCK data confirmed the power suppression of the CMB temperature anisotropies with 2σ-3σ confidence level (CL) in the multipole range 2 ⩽ ℓ ⩽ 29 (Aghanim et al. 2016). The parity asymmetry implies that the odd multipoles of CMB has excess power compared to the even multipoles. In refs. Kim & Naselsky (2010b,a), the authors investigated WMAP 7-year temperature data, and found odd parity preferences at low multipoles (ℓ ⩽ 22) with high statistical significance (99.6% CL). PLANCK 2015 results confirmed the parity asymmetry with 2σ CL in the multipole range ℓ < 20 and with 3σ CL in the mutipole range 20 ⩽ ℓ ⩽ 30 (Ade et al. 2016b).

The connections between these anomalies are still open questions, and the physics behind them is worthy of study. Several works towards this direction have been done, and three examples are as follows. The study in ref. Zhao (2014) suggested that the parity asymmetry and the alignment of quadrupole and octopole may stem from the same source. In ref. Kim & Naselsky (2010b), the authors showed that the low quadrupole anomaly may be correlated with the parity asymmetry, and the low quadrupole anomaly is part of the parity asymmetry rather than an independent anomaly. The ref. Chang et al. (2015) showed that the quadrupole-octopole alignment may be related with the hemispherical power asymmetry.

In this paper, we will propose a new approach to explain the parity asymmetry in the framework of Finsler space-time, and show that the power suppression of the CMB temperature anisotropies can be also resolved in this model. As a generalization of Riemann geometry, Finsler geometry does not have the quadratic restriction on the metric (Bao et al. 2000). Finsler spacetime admits less sym-

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metrics than the Riemann one does, and thus privileged directions can exist in Finsler spacetime (Li & Chang 2012). In this work, we adopt the 3+1 dimensional Finsler spacetime, whose spatial part is described by a Randers-Finsler space (Randers 1941). The three dimensional space is thus irreversible under the parity transformation. This property would induce the parity asymmetry in the CMB temperature anisotropies. In the low-l range \(2 \leq l \leq 29\), the power of odd multipoles remains unchanged while the power of even ones are suppressed. As an additional result, we will show that the power of low-l multipoles in the CMB temperature anisotropies is suppressed in the Finslerian model.

The rest of the paper is organized as follows. In Section 2, we briefly review the Finsler geometry and the derivation of comoving curvature perturbation in Finsler space-time. In Section 3, we calculate the angular power spectrum of CMB temperature anisotropies in Finsler space-time and introduce two new Finslerian parameters. As expected, the obtained angular power spectrum has an obvious odd-multipole preference. In Section 4, we use the chi-square statistic of the parity asymmetry estimator to infer the constraints on the Finslerian parameters using PLANCK CMB temperature data. We also study the power suppression in the CMB temperature anisotropies using this model. Conclusions are given in Section 5.

2 THE COMOVING CURVATURE PERTURBATION IN FINSLER SPACE-TIME

Finsler geometry is a natural generalization of Riemann geometry in absence of quadratic restriction (Bao et al. 2000). The foundation of Finsler geometry is the Finsler structure, denoted by \(F(x, y)\), which is defined on the tangent bundle of a manifold \(M\), satisfying \(F(x, Ay) = AF(x, y)\) for any \(A > 0\). Here \(x\) denotes a position on \(M\), and \(y = dx/dt\) denotes a velocity. The Finslerian metric is defined as

\[
g_{\mu\nu} = \frac{\partial \tilde{F}^2}{\partial y^\mu \partial y^\nu} = \frac{F^2}{2} .
\]

(1)

The Finslerian metric and its inverse are used to lower and raise the space-time indices. The geodesic spray coefficients \(G^\mu\) are given as

\[
G^\mu = \frac{1}{4} g^{\mu \nu \lambda \rho} \left( \frac{\partial \tilde{F}^2}{\partial y^\nu} \frac{\partial \tilde{F}^2}{\partial y^\lambda} - \frac{\partial \tilde{F}^2}{\partial y^\rho} \frac{\partial \tilde{F}^2}{\partial y^\lambda} \right) .
\]

(2)

The Ricci scalar in Finsler geometry can be expressed in terms of \(G^\mu\), namely (Bao et al. 2000)

\[
Ric = \frac{1}{\tilde{F}^2} \left( \frac{\partial G^\mu}{\partial y^\mu} - y^\mu \frac{\partial \tilde{G}^\mu}{\partial y^\nu} y^\nu + 2G^\mu \frac{\partial \tilde{G}^\nu}{\partial y^\nu} \frac{\partial G^\mu}{\partial y^\nu} - \partial G^\mu \partial G^\nu \right) .
\]

(3)

Correspondingly, the Ricci tensor \(Ric_{\mu\nu}\) and the scalar curvature \(S\) are, respectively, defined as (Arkab-Zadeh 1988)

\[
Ric_{\mu\nu} = \frac{\partial (\frac{1}{2} \tilde{F}^2 Ric)}{\partial y^\mu \partial y^\nu} ,
\]

(4)

\[
S = g^{\mu\nu} Ric_{\mu\nu} .
\]

(5)

In Finsler space-time, the gravitational field equation takes the form (Li & Chang 2014; Li et al. 2015b; Li & Lin 2017)

\[
Ric_{\mu\nu} - \frac{1}{2} g_{\mu\nu} S = 8 \pi G T^\mu_{\nu} ,
\]

(6)

where \(T^\mu_{\nu}\) is the energy-momentum tensor. Finsler space-time is fully described by \(F\). The Finslerian metric is reduced to a Riemannian metric if and only if \(\tilde{F}^2\) is quadratic in \(y\).

As a special case of Finsler geometry, Randers space has been used to study the anisotropic inflation (Li et al. 2015b; Li & Wang 2016; Li & Lin 2017) and the anisotropic accelerating expansion of the universe (Li et al. 2015a). For the background space-time, the Finsler structure in this work is given by

\[
\tilde{F}^2 = y^\nu y^\rho - a^2(t) F_{\mu\nu}^2 ,
\]

(7)

where \(F_{\mu\nu}\) denotes the Finsler structure of Randers space (Randers 1941) and is given as

\[
F_{\mu\nu} = \sqrt{\delta_{ij} y^i y^j + \delta_{ij} b^i y^j} .
\]

(8)

Here, for simplicity the spatial vector \(b\) is of the form \(b^i = (0, 0, b(x))\) and \(b(x)\) depends only on the spatial coordinates \(x\). In this work, we consider the slow-roll inflation. The action of the inflaton field is given by

\[
S = \int d^4x \sqrt{-g} \left( \frac{1}{2} g^{\mu\nu} \partial_\mu \phi \partial_\nu \phi - V(\phi) \right) ,
\]

(9)

where \(g = -a^4 F_{\mu\nu}^2 / (\delta_{ij} y^i y^j)^{1/2}\) (Chang & Wang 2013; Li et al. 2015b; Li & Lin 2017). The equation of motion for the first order perturbation of inflaton field is given as (Li et al. 2015b)

\[
\delta \phi + 3H \delta \phi - a^2 \frac{\partial F_{\nu\rho}}{\partial y^\nu} \partial_\rho \delta \phi = 0 ,
\]

(10)

where \(H = \dot{a}/a\), \(\dot{a}\) represents the Finslerian metric of the Rander space and \(\delta \phi\) is the first order perturbation of inflaton field. For simplicity, the direction of \(y\) is taken to be parallel with the wave vector \(k\) in the momentum space (Li et al. 2015b; Li & Wang 2016; Li et al. 2017). Expanding \(\delta \phi\) in Fourier modes, we can write the Eq.(10) as

\[
\delta \phi_k + 3H \delta \phi_k - a^2 k^2 \delta \phi_k = 0 ,
\]

(11)

where \(\delta \phi_k\) represents the Fourier expansion of \(\delta \phi\) in Finsler space-time and \(k^2\) is the effective wavenumber, given by

\[
k^2 = \tilde{g}^i j k_i k_j = k^2(1 + b(k) \hat{k} \cdot \hat{n})^2 ,
\]

(12)

Working in conformal time \(dt' = dt/a\), we can obtain an exact solution of Eq.(11) as follows

\[
\delta \phi_k = \frac{e^{-ikr}}{a \sqrt{2k}} \left( 1 - \frac{i}{kt} \right) \delta \phi_{k0} = \frac{3H^2}{2 \sqrt{2k}} e^{-ikr} \hat{k} \cdot \hat{n} b(k) \hat{k} \cdot \hat{n} ,
\]

(13)

where \(\delta \phi_{k0}\) represents the Fourier modes of \(\phi\) in the standard inflation model, taking the form (Riotto 2003)

\[
\delta \phi_k = \frac{e^{-ikr}}{a \sqrt{2k}} \left( 1 - \frac{i}{kt} \right) .
\]

(14)

The comoving curvature perturbation in Finsler spacetime is given as (Li et al. 2015b; Li & Lin 2017)

\[
R_c = H \frac{\delta \phi_k}{\phi_0} - \Phi ,
\]

(15)

where \(\Phi\) is the scalar perturbation of the Finsler structure F. Plugging the Eq.(13) into Eq.(15), we obtain

\[
R_c = H \frac{\delta \phi_{k0}}{\phi_0} - \Phi - i \frac{3H^2}{2 \sqrt{2k}} e^{-ikr} \hat{k} \cdot \hat{n} b(k) \hat{k} \cdot \hat{n} ,
\]

(16)

where \(R = H \frac{\delta \phi_k}{\phi_0} - \Phi\) denotes the comoving curvature perturbation in the standard inflation model and \(A(k, \tau) = i \frac{3H^2}{2 \sqrt{2k}} e^{-ikr} \hat{k} \cdot \hat{n}\). In Finslerian space-time, the extra part \(A(k, \tau) = \hat{k} \cdot \hat{n}\) will induce anisotropic terms in the comoving curvature perturbation. In next section, we will show that the anisotropic terms can explain the parity asymmetry and the power suppression of the CMB temperature anisotropies.
3 THE ANGULAR POWER SPECTRUM OF TEMPERATURE ANISOTROPIES IN THE CMB

The spherical harmonic coefficients of the CMB temperature fluctuation are given as

$$a_{lm} = 4\pi(-i)^l \int \frac{d^3k}{(2\pi)^{3/2}} R_F \Delta(k)Y_{lm}^*(\hat{k})$$

where $R_F$ and $R$ represent the comoving curvature perturbation in Finsler spacetime and standard inflation model, respectively. We have used the Eq. (16) to get the second equality. The first term at the right hand side of Eq. (17) corresponds to the usual FRW space-time, and its two-point correlation leads to the isotropic angular power spectrum of CMB temperature anisotropies, whereas the second term is the correction term due to the Finsler space-time.

In fact, the Finslerian correction term can be represented as

$$a_{lm} = 4\pi(-i)^l \int \frac{d^3k}{(2\pi)^{3/2}} R_F \Delta(k)Y_{lm}^*(\hat{k})$$

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(17)

To obtain the second equality in the above derivation, the polar coordinates $(\theta, \phi)$ change to $(\theta' = \pi - \theta, \phi' = \phi - \pi)$, and the wave vector $k$ changes to $-k$ for the second term at the right hand side. If the $b(k)$ in Eq. (18) satisfies $b(-k) = -b(k)$, which is an ansatz of this work, the $(a_{lm})$ has an obvious odd-multipole preference. In this case, the Finslerian correction term reduces to

$$a_{lm} = 4\pi(-i)^l \int \frac{d^3k}{(2\pi)^{3/2}} R_F \Delta(k)Y_{lm}^*(\hat{k})$$

(18)

$$\times \int d\phi_0 A(k, \tau)b(k)(\hat{k} \cdot \hat{n})_A(k)Y_{lm}^*(\hat{k})$$

$$+ \frac{1}{2} \int d\phi_0 A(k, \tau)b(k)(\hat{k} \cdot \hat{n})_A(k)Y_{lm}^*(\hat{k})$$

$$\times \int \frac{d^3k}{(2\pi)^{3/2}} \frac{1}{2} \left\langle A(k, \tau)b(k)(\hat{k} \cdot \hat{n})_A(k)Y_{lm}^*(\hat{k}) \right\rangle$$

$$\times \Delta(k)Y_{lm}^*(\hat{k})Y_{lm}^*(\hat{k})$$

(19)

The equation shows that the Finslerian correction term works only for the even multipoles, while it always vanishes for the odd multipoles. Therefore, we can rewrite the Eq. (17) as follows

$$a_{lm} = 4\pi(-i)^l \int \frac{d^3k}{(2\pi)^{3/2}} R_F \Delta(k)Y_{lm}^*(\hat{k})$$

(20)

(17)

Given the expression of spherical harmonic coefficients $a_{lm}$, the correlation of two spherical harmonic coefficients, namely $C_{ll'} = \langle a_{ll'}a_{ll'}^* \rangle$, can be given as

$$C_{ll'} = 4\pi \int \frac{d^3k}{k} P_R(k) \Delta(k)\Delta(k')$$

$$+ 16\pi^2 (1 + (-1)^l)^2 \int \frac{d^3k}{(2\pi)^{3/2}} \int \frac{d^3k'}{(2\pi)^{3/2}}$$

$$\times \left\langle \frac{1}{2} \left\langle A(k, \tau)b(k)(\hat{k} \cdot \hat{n})_A(k)Y_{lm}^*(\hat{k}) \right\rangle \right\rangle$$

$$\times \left\langle \frac{1}{2} \left\langle A(k', \tau)b(k')(\hat{k} \cdot \hat{n})_A(k')Y_{lm}^*(\hat{k}') \right\rangle \right\rangle$$

$$\times \Delta(k)\Delta(k')Y_{lm}^*(\hat{k})Y_{lm}^*(\hat{k})$$

(21)

where $P_R(k)$ denotes the isotropic power spectrum of comoving curvature perturbation in standard inflation model. In Finsler spacetime, the cross term of $R$ and $b(k)(\hat{k} \cdot \hat{n})_A(k)$ can generate anisotropic effect in the primordial power spectrum. As discussed in Ref. Li et al. (2015); Li & Lin (2017), the anisotropic effect induced by the cross term only contributes to the off-diagonal part of $C_{ll'} (l' = l + 1)$. In this work, we mainly consider the effect of quadrupolar modulation on the diagonal part of $C_{ll'} (l' = 0)$, which is generated by the correlation between $b(k)(\hat{k} \cdot \hat{n})_A(k)$ and $b(k')(\hat{k}' \cdot \hat{n})_A(k')$. Here $P_R(k)$ takes the nearly scale-invariant form as $P_R(k) = A_k/(k/\ell)^{n_s-1}$, where $A_k$ and $n_s$ denote the amplitude and spectral index, respectively. We set a pivot scale as $k_0 = 0.05 \text{Mpc}^{-1}$ throughout this work.

As an ansatz, we assume a simple form of the correlation between $b(k)(\hat{k} \cdot \hat{n})_A(k)$ and $b(k')(\hat{k}' \cdot \hat{n})_A(k')$ as follows

$$\langle [A(k, \tau)b(k)(\hat{k} \cdot \hat{n})_A(k)] [A(k', \tau)b(k')(\hat{k}' \cdot \hat{n})_A(k')] \rangle = \frac{2\pi^2}{k^3} f(k) \delta(k - k')$$

(22)

where the term $f(k)$ is a dimensionless quantity and depends only on $k$. The effect of $b(k)$ can be regarded as the first-order correction to the power spectrum in the $\Lambda$CDM model. It is possible to derive the negative spectra from the fluctuations in quantum field theory (Hsiang et al. 2011). The contribution of the vacuum fluctuation in Finsler space-time is sub-dominant so that $f(k)$ in Eq. (22) could be negligible.

The angular power spectrum of CMB temperature anisotropies in Finsler space-time can be written as

$$C_{ll'} = 4\pi \int \frac{d^3k}{k} P_R(k) \left(1 + \frac{1 + (-1)^l}{2} f(k) \right) \Delta(k)\Delta(k')$$

(23)

For simplicity, we can parameterize $f(k)$ to be a power-law function as follows

$$f(k) = a_b \left( \frac{k}{k_0} \right)^{b_y}$$

(24)

where $a_b$ and $b_y$ denote the magnitude and the spectral index, respectively. Therefore, we can rewrite the angular power spectrum of CMB temperature anisotropies as

$$C_{ll'} = 4\pi \int \frac{d^3k}{k} P_R(k) \left[ 1 + B_0 \left( \frac{k}{k_0} \right)^{-\gamma} \right] \Delta(k)\Delta(k')$$

(25)

where we define two new parameters, namely, $B_0 = a_b/A_k$ and $\gamma = n_s - b_y - 1$. In the next section, we will infer the constraints on these parameters through analyzing the low-multipole angular power spectrum of CMB temperature anisotropies in the PLANCK data.
4 Data Analysis and Results

We fit the two parameters $B_0$ and $\alpha$ to the low-multipole ($2 \leq l \leq 29$) TT angular power spectrum in the PLANCK CMB data. To describe the parity asymmetry, we use an estimator $g(l)$ which is defined as

$$g(l) = \frac{\sum_{l=1}^{l+2} \Omega(l + 1)C_l^T}{\sum_{l=1}^{l+2} \Omega(l + 1)C_l^T},$$

where $l = 4n + 2$ and $0 \leq n \leq 6$, and $C_l^T$ are defined as

$$C_l^T = \frac{1 + (-1)^l}{2}C_l^\Lambda, \quad C_l^\Lambda = \frac{1 + (-1)^{l+1}}{2}C_l.$$

To calculate the CMB angular power spectrum, the six base cosmological parameters (i.e., the baryon density today [$\Omega_b h^2$], the cold dark matter density today [$\Omega_c h^2$], the angular scale of the sound horizon at last-scattering [$d_{ls}$]), the reionization optical depth [$\tau$], the amplitude of scalar power spectrum [$A_s$], and the spectral index of scalar power spectrum [$n_s$]) are fixed to be their central values ($\Omega_b h^2 = 0.22222, \Omega_c h^2 = 0.1197, d_{ls} = 0.0104085, \tau = 0.078, A_s = 2.195 \times 10^{-9}, n_s = 0.9655$), which were given by the PLANCK TT data (Ade et al. 2016a).

To infer the parameter constraints in this work, we employ the $\chi^2$ statistic as follows

$$\chi^2 = \sum_{n=0}^{l} \frac{(g_n^\Lambda(l) - g_n^\Lambda(l))^2}{\sigma_{l_F}^2},$$

where $g_n^\Lambda(l)$ and $g_n^\Lambda(l)$ are given by the Finslerian model and the PLANCK data, respectively. The $\sigma_{l_F}$ is the uncertainty of $g_n^\Lambda(l)$. Given the uncertainty $\sigma_l$ for each $C_l$, we use the formula of propagation of error to calculate the $\sigma_{l_F}^2$. Due to difference between the upper deviation $\sigma_{l_F}^{\text{up}}$ and lower deviation $\sigma_{l_F}^{\text{low}}$ of $C_l$, we consider an approach to take account of $\sigma_l$ in our analysis, namely

$$\sigma_l = \sqrt{\left(\frac{\sigma_{l_F}^{\text{up}}}{2}\right)^2 + \left(\frac{\sigma_{l_F}^{\text{low}}}{2}\right)^2},$$

which is used for the calculation of $\sigma_{l_F}$. In our fitting, the prior probability distribution functions of the two Finslerian parameters are assumed to be uniform, namely, $B_0 \in [-1, 1]$ and $\alpha \in [0, 5]$.

Our results are as follows. At 68% confidence level, the constraints on the two Finslerian parameters are given by

$$B_0 = (-1.903 \pm 1.430) \times 10^{-3},$$

$$\alpha = 1.149^{+0.110}_{-0.284}. $$

Here $B_0$ is constrained to be of order $10^{-3}$, and deviates from zero by around 1.3 standard deviation.

Using the best-fit values of the Finslerian parameters, we depict the estimator $g(l)$ versus the seven multipole bins within $2 \leq l \leq 29$ in Fig. 1. The black points represent $g_n^\Lambda(l)$ obtained from PLANCK temperature data, and the error bars represent the relating $1\sigma$ deviations. The prediction of the Finslerian model, denoted by the solid line, is well consistent with the PLANCK temperature data within $1\sigma$ uncertainty. Therefore, we conclude that the Finslerian spacetime can account for the parity asymmetry in the CMB temperature anisotropies to some extent. In addition, we also depict the prediction of the best-fit $\Lambda$CDM model, denoted by the dashed line, for comparison.

Using the best-fit values of the Finslerian parameters, we also depict the angular power spectrum of CMB temperature anisotropies in Fig. 2. For comparison, we depict the PLANCK temperature data and the prediction of best-fit $\Lambda$CDM model. We can see that the Finslerian model is well consistent with PLANCK temperature data within $1\sigma$ uncertainty. In addition, we infer that the low-$l$ power suppression of CMB temperature anisotropies can be also resolved in the Finslerian model. Therefore, we suggest that the parity asymmetry and the power deficit in the CMB temperature power spectrum may not be independent anomalies, and they may have a common origin.

5 Conclusions

In this paper, we proposed to explain the parity asymmetry and the power deficit of CMB temperature anisotropies in the framework of Finsler space-time. We first reviewed the derivation of comoving curvature perturbation in Finslerian spacetime, and then calculated the angular power spectrum of temperature anisotropies in the CMB. We showed that the Finslerian correction to the CMB temperature fluctuations, i.e., the quadrupolar modulation, can account for the parity asymmetry phenomenologically. Finally, we estimated the cosmological constraints on the two Finslerian parameters using the CMB temperature power spectrum within $2 \leq l \leq 29$ in PLANCK 2015 data. Our results showed that the Finslerian model can nicely account for the observed parity asymmetry of the CMB temperature anisotropies. In addition, we noticed that the power of low-$l$ multipoles in the CMB temperature fluctuations is suppressed in the Finslerian model. This power suppression predicted by the Finslerian model can account for the observed power deficit. Therefore, this study suggested that the parity asymmetry and the power deficit of the CMB temperature anisotropies may
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This conclusion is compatible with the existing one in ref. Kim & Naselsky (2010b), which showed that the low quadrupole anomaly could be related with the parity asymmetry.

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