Observing exceptional-point degeneracy of radiation with electrically pumped photonic crystal lasers

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Non-Hermitian degeneracies with singularities, called exceptional points (EPs), incorporate unconventional functionalities into photonic devices. However, the radiative response under the EP degeneracy has yet to be experimentally elaborated due to both the limited controllability of pumping and the lifting of degeneracy by carrier-induced mode detuning. Here, we report the spontaneous emission of a second-order EP with two electrically pumped photonic crystal lasers. Systematically tuned and independent current injection to our active heterostructure nanocavities enables us to demonstrate the clear EP phase transition of their spontaneous emission, accompanied with the spectral coalescence of coupled modes and reversed pump dependence of the intensity. Furthermore, we find experimentally and confirm theoretically the peculiar squared Lorentzian emission spectrum very near the exact EP, which indicates the four-fold enhancement of the photonic local density of states induced purely by the degeneracy. Our results open a new pathway to engineer the light-matter interaction with optical non-Hermiticity.
Coupled optical cavities and waveguides with imaginary refractive index contrast, i.e. distributed gain and loss, can exhibit peculiar degeneracies called exceptional points\(^1\textsuperscript{-5}\) (EPs), which divide their eigenmodes into two phases. One phase comprises extended supermodes with parity-time (PT) symmetry\(^6\textsuperscript{-9}\). Here, the real parts of their eigenvalues are split, and the imaginary parts are clamped at the average of the imaginary effective potential, canceling its local contribution over the unit cell (symmetric phase). In the other regime, PT symmetry is spontaneously broken, and thus the eigenstates localize at either the amplifying or de-amplifying elements (broken phase). Correspondingly, the imaginary spectrum bifurcates into two branches, after the real-part splitting is cancelled at the degeneracy with a singularity (EP). This EP transition leads to intriguing features, such as reversed pump dependence\(^10\textsuperscript{-12}\), single-mode oscillation\(^13,14\), and enhanced sensitivity\(^15,16\).

There has also been rising interest in the photonic EP itself. Distinct from the accidental degeneracy of eigenvalues with orthogonal modes in Hermitian systems, the EP makes not only some eigenvalues but also corresponding eigenmodes identical. Thus, the effective non-Hermitian Hamiltonian becomes non-diagonalizable. The resultant nonorthogonal eigenstates surrounding the EP can enjoy optical isolation\(^17,18\), coherent absorption\(^19,20\), unidirectional reflectivity\(^21\textsuperscript{-23}\), and chiral mode conversion\(^24,25\).

Although many papers have studied phenomena around the EP, observing optical responses at the EP degeneracy has been a persistent technical challenge, even for basic two-cavity devices\(^11,13,14,17,18,26\textsuperscript{-31}\). The EP degeneracy is predicted to have significant influence on radiation processes\(^32\textsuperscript{-35}\). Ideally, however, it is a single spot in the continuous parameter space for eigenfrequencies; therefore, fine and independent control of gain and loss is required for each cavity, which is demanding for passive scattering or optical pumping\(^8,9,13,14,17,18,23,27\). To this end, preparing
strongly coupled lasers with current injection is desirable. Meanwhile, carrier plasma and thermo-optic effects arising with asymmetric pumping induce detuning of their resonance frequencies. This active mismatch lifts the degeneracy, damps one of the coupled modes\textsuperscript{11,26,27}, and can even spawn spectral hysteresis\textsuperscript{28}, all of which hamper the EP response. Moreover, multiple cavity modes with comparative $Q$ factors\textsuperscript{26,29–31} are subject to carrier-mediated mode competition, which also disrupts the pristine properties of the EP.

Here, we report the observation of spontaneous emission under the EP degeneracy with two electrically pumped photonic crystal lasers. Our buried heterostructure nanocavities\textsuperscript{36,37} enable efficient carrier injection and high heat conductivity, which minimize pump-induced resonance shifts. They also provide high $Q$ factors, especially for the coupled ground modes\textsuperscript{38}, which suppress mode competition. By investigating the system with highly asymmetric pumping, we first clarify that whenever there is non-negligible cavity detuning, it is impossible for the lasing supermodes to reach non-Hermitian coalescence. In contrast, our elaborate measurement and analysis of the spontaneous emission demonstrate the distinct EP transition without severe detrimental effects and identify the fine EP location. Remarkably, we find the squared Lorentzian emission spectrum near the exact EP, which signifies the unconventional enhancement of the photonic local density of states (LDOS)\textsuperscript{32–35,39}. Our results provide a new approach to handle the light-matter interaction and radiation.

**Results**

In this work, to achieve the second-order EP, we consider two identically designed optical cavities with spatial proximity and imaginary potential contrast [Fig. 1(a)]. Their ground cavity modes exchange photons with evanescent waves, and thus the system eigenfrequencies are split by the
mode coupling, $\kappa$. However, the gain and loss can counteract the coupling without shifting the local cavity resonances. The first-order temporal coupled mode equations (CMEs) for the complex cavity-mode amplitudes $\{a_i(t)\}$ are derived as,

$$\frac{d}{dt} \begin{pmatrix} a_1 \\ a_2 \end{pmatrix} = \begin{pmatrix} i(\omega_0 + \delta) - \gamma_1 & i\kappa \\ i(\omega_0 - \delta) - \gamma_2 & -i\kappa \end{pmatrix} \begin{pmatrix} a_1 \\ a_2 \end{pmatrix},$$

(1)

where $\gamma_i$ is the loss (positive) or gain (negative) for each cavity, and $\omega_0$ is the average resonance frequency. Without loss of generality, small cavity detuning to $\omega_0$ is introduced as $\pm \delta \in \mathbb{R}$. The model reduces to the eigenvalue problem with the ansatz $(a_1 \ a_2)^T = (A_1 \ A_2)^T e^{i\omega t}$. The resultant eigen-detuning, $\Delta \omega_i \equiv \omega - \omega_0 = i(\gamma_1 + \gamma_2)/2 \pm \sqrt{\kappa^2 - [(\gamma_1 - \gamma_2)/2 - i\delta]^2}$, turns into the aforementioned EP when its second term vanishes [Fig. 1 (b)]. Here, the two eigenvectors become degenerate and chiral, $(A_1 \ A_2)^T = (1 - i)/\sqrt{2}$.

The EP also exhibits peculiar spectral radiation responses [Fig. 1(c)]. When the system is in the symmetric phase and the eigenfrequency splitting is large, the photonic LDOS of the two coupled modes are Lorentzian functions with ideally the same linewidth. At the non-Hermitian degeneracy, however, the two spectral peaks coalesce and interfere with each other. Thus, the resultant radiation power spectrum, which is directly relevant to the LDOS, takes on a squared Lorentzian shape. Without any net gain, the corresponding peak LDOS is increased purely by the effect of the coalescence. Such enhancement at this passive EP is four-fold, compared with each of the separate peaks in the large coupling limit. Namely, compared to the mere sum of the two Lorentzian modes (i.e. a Hermitian diabolic point of two orthogonal states), the EP with the common loss factor and the same integral intensity has a doubly high peak and $\sqrt{\frac{\sqrt{2} - 1}{\sqrt{2}}} \approx 0.644$ times narrower linewidth [Fig. 1(d); see also Sec. VIII of Supplementary Information]. Active cavities with spontaneous emission, i.e. flat spectral excitation via the pumped gain media, are well
suited for its demonstration. In contrast, EPs with nonlinear processes can generate excess noise and result in their linewidth broadening\textsuperscript{41}. Instead of the theoretical set-up\textsuperscript{34,35} with a completely dark state ($Q = \infty$) and a considerably leaky mode, we apply manageable alternatives for our experiment: pumped transparent (loss-compensated) and absorptive cavity modes.

We prepared a sample comprising two coupled photonic crystal lasers based on buried heterostructure nanocavities\textsuperscript{36,37} [Fig. 2(a) and Methods]. Here, gain media with six quantum wells (colored red) were patterned by electron beam lithography and embedded in an air-suspended InP photonic crystal slab. Two line defects narrower than the lattice-matched width improve the cold $Q$ factors of the incorporated cavities’ ground modes. DC current is applied and controlled for each cavity via independent PIN junctions. Note that a single-laser device with a commensurate electric channel has a lasing threshold $I_{\text{th}}$ of about 37 $\mu$A, at which it has a high $Q$ factor of 14,000 (Supplementary Sec. I). When symmetrically pumped below the threshold with 30 $\mu$A for comparison, the two-laser sample gives spontaneous emission of the two coupled modes with $Q = 4,000$, slightly below that of the single diode (5,000). Their resonance peak splitting of about 1.0 nm in reference to 1529.7 nm, along with our CME fitting, indicates $\kappa = 61$ GHz (Supplementary Sec. III), which agrees well with that of the ground modes simulated by the finite-element method, $\kappa_{\text{sim}} = 65$ GHz [Fig. 2(b) and Methods]. The near-field emission from both lasers is also observed [Fig. 2(b), inset].

We fix the injection current for one channel [$I_2$ for channel 2 here, left in Fig. 2(a)] and sweep that for the other ($I_1$ for channel 1, right), to vary the loss contrast $\gamma_1 - \gamma_2$. As a result, the detected ground-mode power systematically recovers by the reduction of the local current $I_1$ [Fig. 2(c)]. This reversed pump dependence\textsuperscript{10} indicates the EP transition (see also Methods and
Supplementary Sec. II. Here, the retrieval ratio $P_R/P_{\min}$, between the power $P_R$ for zero bias along channel 1 ($V_1 = 0$) and the minimal value $P_{\min}$, is maximized by heavy pumping $I_2 = 800 \ \mu$A, where cavity 2 provides gain. In that case, however, the system is critically affected by the cavity detuning $\delta$ and hence misses the EP degeneracy.

Fig. 2(d) depicts the device emission spectra in the lasing regime for constant $I_2 = 800 \ \mu$A and different $I_1$, measured with an optical spectrum analyzer. Here, although some leakage current from channel 2 induces a negative $I_1 \approx -6 \ \mu$A for $V_1 = 0$, the data and hence loss $\gamma_1$ in cavity 1 consistently change under the reverse current. As $I_1$ decreases from $I_1 = 100 \ \mu$A and $\gamma_1$ hence increases, the blue-side peak $|\lambda_-\rangle$ gradually damps while the other red-side one $|\lambda_+\rangle$ remains bright. This is a direct reflection of finite detuning $\delta$, with which the asymmetric pumping $I_2 \gg I_1$ selectively excites the coupled mode closer to the solitary resonance of cavity 2, $\omega_0 - \delta$. Eventually, the power of $|\lambda_+\rangle$ also drops sharply around $I_1 = 5.4 \ \mu$A, indicating the suppression of oscillation. However, it is $|\lambda_-\rangle$ that undergoes the revival of lasing with a kinked rise in power and linewidth narrowing (Supplementary Sec. XI). Such switching of the dominant mode has been observed in relevant studies$^{37,42}$ and attributed to the pump-induced sign flip of $\delta$, namely the crossing of cavity resonances. The restored peak moves toward the middle of the original supermodes with the growing contrast of pumping and hence evidences the EP process in our device. The near-field patterns for selected $I_1$ [Fig. 2 (e)] not only show the above-mentioned processes in the real space but also exhibit clear mode localization at cavity 2 in the intensity recovery, supporting the PT symmetry breaking.

The steady oscillation condition $\text{Im } \omega_e = 0$ enables us to estimate the eigenfrequencies $\omega_e$ for the lasing spectra$^{42}$, despite that the system here provides an adaptive (variable) gain $\gamma_2 < 0$. 
The numerical curves (black dots) in Fig. 2(d), considering an average effect of detuning $2\delta = -14.1$ GHz and additional thermal and carrier shifts, successfully explain the major portion of the data. Remarkably, one of the eigenmodes manifests itself as two different branches, corresponding to different $\gamma_2$. One is the observable coupled mode $|\lambda_+\rangle$ in the symmetric phase; the other is the virtual middle branch, which is the same eigenstate in the broken phase, requires larger gain and still satisfies $\omega_c \in \mathbb{R}$. They are actually annihilated as a pair with a singularity, which does not represent an EP, and turn into a damping mode ($\text{Im } \omega_c \neq 0$). This destabilization always occurs for finite $\delta$ before the system obtains enough loss for the ideal PT-symmetric EP with $\delta = 0$, i.e. $\gamma_1(I_1) < \gamma_{1,\text{EP}} = \kappa(= -\gamma_2)$. Our analysis hence means that it is infeasible for lasing coupled modes to be coalesced by the non-Hermiticity as long as they have cavity detuning larger than their narrow linewidths. This is why the EP transition with just a single peak is mostly observed in lasing systems$^{11,27,28}$, including our result here with revived $|\lambda_-\rangle$.

Additional data are shown in Supplementary Sec. X.

The spontaneous emission (non-lasing) regime, in contrast, enables us to observe the clear EP transition with spectral coalescence of the two coupled modes, as shown in Fig. 3(a) for $I_2 = 100$ $\mu$A and decreasing $I_1$. Here, the oscillation threshold for the case of pumping only one of them is about 200 $\mu$A, because the other cavity behaves as an additional absorber (Supplementary Fig. S2). The radiation was measured by a spectrometer with a cryogenic InGaAs line detector (see Methods). In Fig. 3(a), the two distinct spectral peaks originally at 1529.3 and 1530.2 nm coalesce when $I_1 \approx 2$ $\mu$A. In addition, the peak count of the merged ground resonance at 1529.9 nm increases back to around the saturation level of about 55,000 for $I_1 = 0$, confirming the reversed pump dependence (Supplementary Sec. IV). Although higher order modes are also found around
1523.4 nm [bottom of Fig. 3(a)] and 1521.5 nm (not shown), they are hardly affected by the change in $I_1$. This means that the mode competition is insignificant, because the ground coupled modes have $Q$ factors sufficiently higher than those of other modes.

To analyze the system response theoretically, we performed the Fourier transform of the CMEs [Eq. (1)] for the spectral cavity amplitudes $a_i(\omega) = \mathcal{F}[a_i(t)] = \int a_i(t)e^{-i\omega t}dt$, together with net excitation terms $\{c_i(\omega)\}$ arising from the pumping. Here, because $I_2$ is sufficiently larger than $I_1$ over the entire measurement, we neglect the excitation $c_1$ of cavity 1 for simplicity. By solving the resultant linear equation (shown in Methods), we reach,

$$\begin{pmatrix} a_1(\omega) \\ a_2(\omega) \end{pmatrix} = \frac{c_2(\omega)}{\kappa^2 + [\gamma_1 + i(\Delta \omega - \delta)][\gamma_2 + i(\Delta \omega + \delta)]} \left( \gamma_1 + i(\Delta \omega - \delta) \right).$$

(2)

If the spontaneous emission has an ideal flat spectrum, the spectral intensity of the indirectly pumped cavity $|a_1(\omega)|^2$ reflects the LDOS of the system, which was derived from a singular perturbation analysis\textsuperscript{34}. In contrast, $|a_2(\omega)|^2$ is additionally affected by the relative resonance of cavity 1, i.e. $\Delta \omega - \delta$ on the numerator in Eq. (2) (see also Methods and Supplementary Sec. VIII).

The theoretical fitting for the spectral data involves the detailed conditions of the optical collection system. Because $a_1$ and $a_2$ hold phase coherence with evanescent coupling, their radiation is expected to have a spatial (directional) intensity distribution due to interference\textsuperscript{28}. The detector signal hence depends on the position of the objective lens controlled by the three-axis nano-positioner. Here, it is aligned so that the out-coupled intensity at the coalescence is maximized. Considering that the degenerate eigenstate is $(1 - i)^T/\sqrt{2}$, we take the analytic power spectrum for our measurement as $P(\omega) = \eta \gamma_{\text{cav}} |a_1(\omega) + ia_2(\omega)|^2$, under the premise that the identically designed cavity modes have the same radiation loss $\gamma_{\text{cav}}$ and collection efficiency $\eta$. Note that our I-L data assure that the system detects the light from both cavity 1 and 2 [Fig. 2(c) and
Supplementary Sec. II]. For other major possibilities like \( \eta_{\text{cav}} |a_1(\omega) \pm a_2(\omega)|^2 \), one of the coupled modes is cancelled out in the symmetric phase, and the other exhibits a Fano resonance with a peculiar dip beside the main peak. We can exclude such cases since none of them were seen in our entire experiment.

Figure 3(b) presents our least-square theoretical fitting for the emission spectra with \( P(\omega) \). Here, because the pumped gain medium in cavity 2 is considered to be nearly transparent, we assume a low \( \gamma_2 \), setting it to 0.1 GHz to avoid any numerical problems like divergence. The data agree well with the experimental result, and the theoretical peak with the shorter wavelength for \( I_1 \geq 5 \mu\text{A} \) is slightly narrower mostly due to the neglected excitation of cavity 1 (Supplementary Sec. V). The analysis enables us to estimate the physical fitting parameters in the model, such as \( \kappa \), \( \gamma_1 \), and \( \delta \), which include the effect of the mode confinement factor. The frequencies \( \text{Re} \Delta \omega_i \) reconstructed with them, depicted by black points, ensure the correspondence between the sharp coalescence of the eigenstates and the measured spectra.

Figure 3(c) and (d) show the \( I_1 \) dependence of estimated \( \gamma_1 \) and \( \delta \). Here, the cavity coupling is found to be about \( \kappa = 58 \text{ GHz} \) for the case of split resonances, and it is fixed as that value in fitting the coalesced peaks for \( I_1 \leq 0.8 \mu\text{A} \), which are of more complexity (Supplementary Sec. VII). The reduction of the current for cavity 1 monotonically enhances its material absorption and hence \( \gamma_1 \). On the other hand, the reduction of the local carrier plasma effect by decreasing \( I_1 \) induces its red shift that continuously diminishes \( \delta \). Ideally, the EP should be near \( \gamma_{1,\text{EP}} = 2\kappa \approx 116 \text{ GHz} \). Our measurement points have an interval of \( \Delta I_1 = 0.2 \mu\text{A} \) when \( I_1 \) is small, and \( I_1 = 1.4 \mu\text{A} \) is considered the closest to the EP. By carrying proper current \( I_2 = 100 \mu\text{A} \) for cavity 2, we can cancel the detuning \( \delta \) around the EP condition, which detrimentally lifts the
degeneracy.

The spontaneous emission spectrum for $I_1 = 1.4 \mu A$ is fit by some distinct trial functions and shown in linear and semi-logarithmic scales [Fig. 4(a)]. Again, our spectral CME analysis reproduces the experimental result well, and the apparent discrepancy between the data is just seen in the region with 10% or less of the peak count. The errors in their skirts can be mostly attributed to the background emission spectrum’s slightly ascending in frequency due to its luminescence peak located at around 1440 nm (Supplementary Sec. VI). Importantly, the experimental data are in accordance with the squared Lorentzian function, $4\pi^{-1}C\{\gamma^2/((\Delta\omega^2 + \gamma^2)\}^2$ with coefficient $C$, rather than with the ordinary Lorentzian function (Supplementary Fig. S6). In addition, the peak count increases by 30% from the apparent mode coalescence ($I_1 = 2.4 \mu A, \gamma_1 = 93.4 \text{ GHz}$) to the estimated EP ($I_1 = 1.4 \mu A, \gamma_1 = 115.6 \text{ GHz}$), despite that the net loss of the eigenstates confined in the symmetric phase without gain is magnified [Fig. 1 (b), Supplementary Fig. S4(b)]. These features evidence the peculiar enhancement of the photonic LDOS by the EP degeneracy (not the reversed pump dependence). Here, we can exclude the Voigt fitting function$^{45}$, i.e. the convolution of the cavity Lorentzian factor and Gaussian noise, because it requires a too small average loss to have the EP ($\approx 26 \text{ GHz} < \gamma_{\text{EP}}/2 = 58 \text{ GHz}$), as well as persistent Gaussian noise ($\approx 27 \text{ GHz}$) inconsistently larger than our lasers’ oscillation linewidths$^{37}$ ($< 4 \text{ GHz}$: our finest measurement resolution; see Supplementary Fig. S1). As $I_1$ further decreases down to $I_1 = 0.2 \mu A$, the experimental and best-fit CME spectra get settled in more Lorentzian shapes [Fig. 4(b), Supplementary Sec. VIII]. This means that the system loses the effect of the degeneracy on the LDOS in the limit of a large imaginary potential contrast.

Discussion
Enhancing the peak LDOS at the passive EP will drastically modulate the photonic responses of quantum emitters\textsuperscript{46}, coherent absorbers\textsuperscript{20} and nonlinear optical devices\textsuperscript{47}. It can also be combined with the effect of gain\textsuperscript{34} and get further enhanced at a higher order EP\textsuperscript{33} with a ratio of $\sqrt{\pi} \Gamma(n + 1)/\Gamma(n - 1/2) \ (n = 4 \ \text{for} \ n = 2)$, where $n$ is its order and $\Gamma(n)$ is the gamma function. Nonlinear effects will even be made more than a hundred times more efficient\textsuperscript{35} by adopting the non-Hermitian degenerate states. Moreover, the reversed power dependence in the EP transition also shows nonlinearity on the pumping. This property provides us with new possibilities for nanophotonic switches and regulators.

Achieving the EP degeneracy is also essential in applications of its topological properties\textsuperscript{48}, such as the chirality of the degenerate eigenstate and the vortex charge around the EP frequency. Larger periodic devices\textsuperscript{23,49,50} in one and two dimensions exhibit more flexibility of parameters for an EP and rings of EPs in the Brillouin zone, respectively. Meanwhile, fabrication-induced defects make it challenging to handle the degeneracy in such systems. Further study of the radiative responses of corresponding cavity arrays will be of great significance.

In conclusion, we showed the clear EP transition of spontaneous emission with our photonic crystal lasers. Our device has strong light confinement only for coupled ground modes and hence suppresses mode competition. The independent and efficient electrical pumping enables the radiation near the exact EP, by limiting detrimental resonance shifts to the minimal level for active devices. Around the accurate EP position elaborated by both our measurement and analysis, we found a squared Lorentzian emission spectrum and loss-induced growth of the peak power. They demonstrate the four-fold peak LDOS enhancement that is intrinsic to the EP degeneracy. Our results represent an important step toward controlling optoelectronic processes via non-Hermitian
Methods

Sample fabrication and design. The sample [Fig. 2(a)] contains an air-suspended InP photonic crystal slab and two InGaAlAs-based buried heterostructure nanocavities (red) with six quantum wells embedded. Here, an InAlAs sacrificial layer, the active layer and an overcladding InP layer were grown by metal-organic chemical vapor deposition (MOCVD). The heterostructures and airholes were patterned by electron-beam lithography with SiO$_2$ and SiN mask layers, respectively. The periodic air holes and narrow trenches (black lines) were opened by inductively coupled plasma reactive etching (ICP-RIE). Selective wet chemical etching was carried out to define the nanocavities. After the regrowth of the intrinsic InP layer over the heterostructures, Si ion implantation followed by activation annealing and Zn thermal diffusion was applied to diagonally pattern n-doped and p-doped layers. Each of the resultant lateral PIN junctions is in contact with an InGaAs contact layer and Au-alloy metal pads at its edges. The diagonal doped layers and current blocking trenches help suppress leakage current between the electric channels.

Seven and five rows of airholes with a lattice constant of $a = 437$ nm are aligned on the outer sides of and between the line defects, respectively. A narrow line defect width of $0.85 \sqrt{a}$ improves the modal confinement. The air hole radius and dimensions of the cavities are designed to be $R_D = 100$ nm and $2.185(5a) \times 0.3 \times 0.15 \mu$m$^3$. The slab thickness is 250 nm.

Finite element simulation. The sample’s eigenmodes were simulated with a commercial electromagnetic solver based on the finite element method (COMSOL Multiphysics). The fine structure of the buried nanocavities including the quantum wells were taken into consideration by
using their effective index of \( n_{\text{BH}} = 3.41 \), and the refractive index of InP is \( n_{\text{InP}} = 3.16 \). No finite imaginary part of the index was assumed over the entire device. The computational domain was halved by a two-dimensional perfect magnetic conductor placed along the middle of the slab \((z = 0)\). An air layer with a height of 3.5 µm was attached to the slab, and radiation loss was captured by the surrounding boundaries with the second-order scattering condition. We calculated the resonance frequencies and \( Q \) factors of the coupled ground modes for different air-hole radii \( R \), which can change depending fabrication conditions. The simulated wavelengths close to our experimental result were found for \( R = 104.4 \) nm (shown in the main text). The corresponding theoretical \( Q \) factors are 60,000 and 62,000, while those of the first-order modes are less than 19,000. A similar device with a single nanocavity and the same parameters had the ground mode with a wavelength of 1530.8 nm and \( Q = 2.9 \times 10^5 \).

**Measurement set-up.** We use a caged measurement system with a device stage, a probe station, and a nano-motion lens revolver. The sample was placed on a device holder with a vacuum contact. The temperature of the holder was maintained at \( 25 ^\circ C \) with its Peltier unit and a feedback controller. Four electric microprobes were put on the metallic pads in contact with the ends of the doped layers. Independent DC currents for the right (channel 1) and left (channel 2) pairs of probes were applied and controlled with a precision source/measure unit. The device radiation was collected at the top with a 20X objective lens with a numerical aperture (NA) of 0.26 and coupled to an optical fiber. The near-field patterns [Fig. 2(b) and (e)] were observed with another 50X lens with \( \text{NA} = 0.42 \) and a near-infrared InGaAs camera.

In the I-L measurement [Fig. 2(c)], the voltage (and hence current) for one channel was swept upward and backward, with the injection current to the other kept constant. The output light passed
through a variable filter with a bandwidth of 3 nm around 1530 nm and was measured with a low-noise power meter. Owing to vibrational and mechanical stability over the measurement system, we have not seen notable hysteresis behavior in the signal. Thus, we show the data without the points in the backward sweep for a better view.

The spontaneous emission (Fig. 3) was resolved with a spectrometer that has a grating with a groove number of 1000 g/mm and a spectral resolution of 0.12 nm. It was then detected by an InGaAs line detector array cooled down to $-95^\circ$C with liquid nitrogen. The detector integration time was 60 s. The measurement was performed in the wavelength range of 1517 to 1542 nm, and the data from 195.5 to 196.7 THz were extracted for analyzing the ground-mode spectra. The minimum count in each curve, detected far away from the ground-mode resonances, was subtracted as the background component.

For the coherent radiation of the sample with the intense local pump (Fig. 4), we used a fiber-coupled optical spectrum analyzer. The wavelength sweep was performed under the lowest video bandwidth of 10 Hz for the maximum sensitivity, and every data point was averaged with 100 measurements. To compensate for negative detection currents (and power levels) caused by the spectral analyzer’s small calibration error, the data were offset with reference to their minimum value in each sweep.

**Spectral coupled-mode analysis.** The Fourier transformation of Eq. (1) with the net external excitation terms

$$i\omega \begin{pmatrix} a_1(\omega) \\ a_2(\omega) \end{pmatrix} = \begin{pmatrix} i(\omega_0 + \delta) - \gamma_1 & i\kappa \\ i(\omega_0 - \delta) - \gamma_2 & -i\kappa \end{pmatrix} \begin{pmatrix} a_1(\omega) \\ a_2(\omega) \end{pmatrix} + \begin{pmatrix} c_1(\omega) \\ c_2(\omega) \end{pmatrix},$$

is solved for $a_1(\omega)$ and $a_2(\omega)$ with $c_1 = 0$ to obtain Eq. (2). Considering our experimental
conditions, the fitting function for the measured spectra is expected to be

\[ P(\omega) = \frac{(\gamma_1 + \kappa)^2 + (\Delta\omega - \delta)^2}{(\kappa^2 + \gamma_1\gamma_2 - \Delta\omega^2 + \delta^2)^2 + [(\gamma_1 + \gamma_2)\Delta\omega + (\gamma_1 - \gamma_2)\delta]^2} \eta_{\gamma_{\text{cav}}} |c_2(\omega)|^2, \]

where \( \Delta\omega = \omega - \omega_0 \), and \( \eta_{\gamma_{\text{cav}}} |c_2(\omega)|^2 \) works as a proportional constant depending on the peak count. Here, \( P(\Delta\omega) \) is affected by the radiation of both the excited and absorptive cavities, including the term \( (\Delta\omega - \delta)^2 \) in the numerator, which results in small deviation from the exact LDOS\(^3\). However, its information is well reflected in the main part of the spectral peaks near the EP, namely \( (\Delta\omega - \delta)^2 < (\gamma_1 + \kappa)^2 \), for the trial function and hence the observed emission.

In the fitting, we set \( \gamma_2 = 0.1 \) GHz because cavity 2 was considered to be transparent but not to provide gain. Note that the peak photon count started saturating, but the I-L data did not show any apparent reversed intensity (viz. oscillation) for \( (I_1, I_2) = (0, 100 \, \mu\text{A}) \). Although we had to reduce the number of parameters due to the complexity of the problem, we were able to find good values of \( \eta_{\gamma_{\text{cav}}} |c_2(\omega)|^2 \) (\( \propto \) peak count) and \( \omega_0 \) (center of the peak structure). Thus, we performed the nonlinear least-square fitting for the rest of the variables, i.e. \( \kappa, \delta \), and \( \gamma_1 \) with OriginPro. Here, the initial value of \( \kappa \) was 60 GHz, considering the experimental and simulation result. Because \( \kappa, \delta \), and \( \gamma_1 \) basically dominated the splitting, level difference and decaying tails of the spectral peaks, respectively, we were able to achieve a reliable estimation of them. Nonetheless, it became hard to have the consistent convergence of the fitting for the data after the EP transition. We hence fixed \( \kappa \) as 58 GHz, as a value close to those obtained otherwise, and estimated \( \gamma_1 \) and \( \delta \) for \( I_1 < 1.0 \, \mu\text{A} \).

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**Author contributions**

K. Takata designed the sample, built the measurement set-up, performed the measurement, analyzed
the data, and wrote the manuscript. K.N. supported the sample design, measurement set-up and fabrication. E.K. supported the device design and discussion. S.M., K. Takeda, and T.F. fabricated the sample. S.K. supported the measurement and discussion. A.S. supported the sample design and organized the project. M.N. conceived the idea and led the project.

**Competing interests**

The authors declare no competing interests.

**Additional information**

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Figure 1 | Spontaneous emission properties of two coupled non-Hermitian nanolasers. a, Schematic of the system. Laser $i$ has frequency detuning $(-1)^{i-1} \delta$ to their average resonance $\omega_0$, local loss $\gamma_i$, and an evanescent coupling $\kappa$ with the other. b, EP transition of the complex eigenfrequency detuning $\Delta \omega_i = \omega_i - \omega_0$ in reference to the coupling, $\kappa = 1$, for $\gamma_2 = 0$. The EP is at $(\gamma_1, \delta) = (2, 0)$, and finite $\delta$ blurs the sharp coalescence of the two branches. c, Comparison of photonic LDOS for the system in the large coupling limit and that at the EP. The spectral LDOS of the EP resonance becomes a squared Lorentzian shape, and its peak is four times higher than that for one of the split Lorentzian supermodes far from the EP. d, Lorentzian and squared Lorentzian spectral functions with the same loss factor $\gamma$ and integrated intensity. By the non-Hermitian degeneracy, its peak power doubles compared to the accidental merging of two orthogonal states with the same linewidth (Hermitian diabolic point).
Figure 2 | Electrically pumped photonic crystal lasers and EP transition of their lasing ground modes. a, False-color laser microscope image of the sample. Each of the two buried InGaAlAs heterostructure nanocavities (red squares) has six quantum wells. Diagonally patterned doping layers (purple: p-doped, green: n-doped) with contact pads (yellow) provide independent electric channels for the cavities (right and left: channel 1 and 2 with current $I_1$ and $I_2$, respectively). b, One of the simulated ground supermodes for $R = 104.4$ nm with a
cavity coupling of $\kappa_{\text{sim}} \approx 65$ GHz and wavelengths of 1529.25 and 1530.27 nm. Inset: near-field image of the device emission for $I_1 = I_2 = 100$ μA. c, Current-in and light-out (I-L) curves of the filtered ground modes’ emission for several fixed $I_2$ values and swept $I_1$. The photodetector signal shows the systematic reversed pump dependence of the coherent device emission when $I_1$ is small. d, Device emission spectra for fixed $I_2 = 800$ μA and varied $I_1$, measured with an optical spectrum analyzer. As $I_1$ drops, the spectrum features continuous decay of the lower branch $|\lambda_-\rangle$, followed by a discrete suppression of the upper branch $|\lambda_+\rangle$ and the revival of $|\lambda_-\rangle$. Black dots: eigenvalue fitting with Ref. 42. e, Corresponding near-field patterns for different $I_1$, showing clear localization of emission at cavity 2 with decreasing $I_1$. The abruptly darkened signal from $I_1 = 5.5$ to 2.0 μA and glare spot with $I_1 = -5.0$ μA support the suppression and revival of lasing by the transition.
Figure 3 | Spectroscopy of the EP transition in the sample’s spontaneous emission. **a,** Color plot of the observed spectra for constant $I_2 = 100 \mu A$ and varied $I_1$. Upper: coupled ground eigenmodes, clearly exhibiting the EP transition with the spectral peak coalescence and reversed pump dependence of the peak intensity. Lower: most clearly visible higher order mode, which is hardly affected by the ground-mode process. **b,** Result of theoretical fitting via the coupled-mode analysis. It reproduces the experimental spectra well and enables parameter estimation within the model. Here, $\kappa \approx 58 \text{ GHz}$ over the entire analysis. Black dots: eigen-wavelengths calculated with the obtained parameters, including their nearly exact coalescence. **c, d,** Estimated (c) loss rate $\gamma_1$ of cavity 1 and (d) cavity detuning $\delta$, dependent
on $I_1$. The EP condition for $\gamma_1$ is $\gamma_1 = 2\kappa \approx 116 \text{ GHz}$, and the corresponding closest measurement point is $I_1 = 1.4 \mu\text{A}$. Note that the detrimental detuning is almost cancelled out there, by the suppression of the carrier plasma effect with decreasing $I_1$. 
Figure 4 | Device emission spectra near and far from the EP degeneracy. a, b, Observed photon count spectra (blue dots) and their theoretical fitting for (a) $I_1 = 1.4 \, \mu A$ and (b) $I_1 = 0.2 \, \mu A$. Left and right: their linear and semi-log plots, respectively. Our CME analysis (red line) explains both data well, and the plot in (a) agrees with a squared Lorentzian trial function (dotted orange curve) clearly better than a least-square Lorentzian trace (dashed purple curve), demonstrating the LDOS enhancement in the proximity of the EP degeneracy. The emission with a smaller $I_1$ (b) comes to have a more Lorentzian component, as it is a state localizing at the heavily pumped cavity in the broken phase.
Supplementary Information:

Observing exceptional-point degeneracy of radiation with electrically pumped photonic crystal lasers

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I. Single-laser sample

To estimate the optical properties of each diode in our two-cavity sample, we fabricated a device comprising a single nanocavity with the same structural parameter set, based on the previously reported technique¹. Its false-color image is shown in Fig. S1(a). To well confine the ground mode of our mode-gap nanocavity, seven rows of triangular-lattice air holes are placed on both the upper and lower sides of the line defects, whose width is $0.85\sqrt{3}a$ ($a = 437$ nm). An active heterostructure designed to $2.185 \times 0.3 \times 0.15 \mu m^3$ (red) is buried at the center of the line defect. Trapezoidal p-doped (purple) and n-doped (green) layers adjacent to the cavity form a PIN junction, and they are in contact with the edges of $50 \times 50 \mu m^2$ metal pads at the top and bottom of the figure. An I-L curve of the device [Fig. S1(b)] indicates that its lasing threshold is $I_{th} \approx 37 \mu A$. In its emission spectra around $I_{th}$ [Fig. S1(c), (d)], we see that, as the injection current $I$ increases, an abrupt blue shift by the carrier plasma effect turns off and a linear red shift due to the thermal effect dominates. The consistent linewidth narrowing beyond the resolution ($0.03$ nm) of our optical spectral analyzer (OSA) together with the bright near-field patterns like ones seen in the two-laser experiment confirm that a high-$Q$ nanolaser is achieved. It is noteworthy that $Q = 14,000$ is obtained at $I = 37 \mu A$ with the corresponding raw spectral data. The optical coherence of the emission from a similar heterostructure nanolaser was assured in Ref. 2. The threshold current here is larger than that seen in Ref. 1, because the doped layers are located closer to the heterostructure. Remarkably, their material absorptions help broaden the range of the loss (i.e. the on-site effective imaginary potential) of the nanocavity.
Figure S1. (a) False-color image of the single-laser sample. (b) I-L curve of the device. The oscillation threshold is $I_{th} \approx 37 \ \mu A$. (c) Spectral peak wavelength (black) and linewidth (red) of the emission, dependent on the injection current $I$. Filled markers: estimations with moving average of the spectra. (d) Device emission spectra around $I_{th}$ measured with the OSA. The data for $I = 37 \ \mu A$ (green), without significant amplification by oscillation, indicate $Q = 14,000$. (e) The linearly scaled spectrum for $I = 42 \ \mu A$ with offset Lorentzian and Gaussian fitting.

Figure S1(e) depicts the spectrum just above the threshold ($I = 42 \ \mu A$) and its fitting on a linear scale. The experimental data (orange points) contain non-negligible white spectral components and hence indicate that spontaneous emission is still effective, as well as low-power stimulated emission. Thus, the data match up well with an offset Lorentzian function, which has a FWHM of 14.0 GHz and a background level of 0.183 pW (dashed purple curve). On the other hand, because its shape is asymmetric to the peak center, a Gaussian function $\propto \exp[-(\omega - \omega_C)^2 / \sigma^2]$, based on 1/f flicker noise, can also explain the plot (dot-dashed curve in green). Importantly, the noise in this case comes from the fluctuation of the active carrier population that is clamped around lasing. Thus, the effect of the Gaussian part on the line shape varies little and is well below the variance of the fitting in Fig. S1(e), $\sigma = 9.92$ GHz. We can hence neglect this factor for fitting Fig. 3 and 4 in the main text, where the loss-biased resonances, including the EP, have linewidths as large as several tens of gigahertz.
II. Current-in light-out curves

The I-L data for broader ranges of pumping under bidirectional sweeps are presented in Fig. S2. Because our measurement cage and equipment are connected with an optical fiber and a connector, mechanical and vibrational fluctuation in the set-up results in small discrepancies between the signal in the upward and downward sweeps. Nonetheless, we do not see any significant hysteresis behavior in the plots, showing that the mode competition and thermal nonlinearity are well suppressed in the sample. When $I_2$ is fixed and $I_1$ is swept [Fig. S2(a), also shown in the main text], we systematically see the steep suppression and revival of the coherent radiation power in diminishing $I_1$ for $I_2 \geq 300 \mu$A. In comparison, the rise in power by pumping cavity 1 after oscillation is gentle. Fig. S2(a) also indicates that the system with $I_2 = 100 \mu$A and $I_1 \lesssim 10 \mu$A is in the spontaneous emission regime. On the other hand, the reversed pump dependence for the opposite case [Fig. S2(b)] is less prominent. This is probably because the cavities are detuned more in this condition; for example, we have a solitary resonance of cavity 1 at 1529.47 nm for $(I_1, I_2) = (300 \mu$A, 0). This mode is away from the coalescence at 1529.9 nm under the constant $I_2 = 100 \mu$A (Fig. 3).

![Figure S2](image)

Figure S2. (a) I-L plots of the sample for swept $I_1$ and fixed $I_2$ and (b) those for varied $I_1$ and constant $I_2$. The signal is filtered around 1530 nm with a bandwidth of 3 nm. The integration time of the power meter is 1 s.
III. Symmetrically pumped two-laser sample

To estimate the cavity coupling $\kappa$, we also analyze the emission spectrum of the symmetrically pumped system with $I_1 = I_2 = 30 \, \mu\text{A}$, which is below the oscillation threshold (Fig. S3). The experimental data (blue dots) show the two coupled-mode peaks with $Q$ factors of about 4,000 based on their 3-dB linewidths. Because both channels have the same injection current, we need to take into consideration the net field excitation $c_1(\omega)$ for cavity 1 that is neglected in the main text. The solution of the coupled mode theory (CMT) is hence modified as,

$$a_1(\omega) = \frac{[\gamma_2 + i(\Delta \omega + \delta)]c_1(\omega) + i\kappa c_2(\omega)}{\kappa^2 + [\gamma_1 + i(\Delta \omega - \delta)][\gamma_2 + i(\Delta \omega + \delta)]}, \quad (S1)$$

$$a_2(\omega) = \frac{i\kappa c_1(\omega) + [\gamma_1 + i(\Delta \omega - \delta)]c_2(\omega)}{\kappa^2 + [\gamma_1 + i(\Delta \omega - \delta)][\gamma_2 + i(\Delta \omega + \delta)]}, \quad (S2)$$

and we again consider $P(\omega) = \eta \gamma_{\text{cav}} |a_1(\omega) + ia_2(\omega)|^2$ as our trial function.

If we assume that the net pumping is identical for the two cavities, we reasonably have $\gamma_1 \approx \gamma_2$ and $|c_1(\omega)| \approx |c_2(\omega)|$. In that case, however, finite $\delta$ does not induce any level margins of the two peaks, despite that the experimental result exhibits a 20% discrepancy between the peak counts. This means that the two cavities have imbalanced pumping possibly due to inhomogeneous electric contact among the four DC probes and metal pads and/or due to a difference in current leak paths of the channels. A best-fit theoretical curve is obtained under a condition that the ratio of the field excitation rates is inversely proportional to that of the on-site field loss rates, namely

$$r \equiv \left| \frac{c_2(\omega)}{c_1(\omega)} \right| = \frac{\gamma_1}{\gamma_2}, \quad (S3)$$

which is a rational indication of the linear loss model.

Another important factor is that, $c_1(\omega)$ and $c_2(\omega)$ are not expected to have any phase coherence, since they are spontaneous emissions from distinct quantum wells and cavities (in sharp contrast to the interference between the two cavity sources $a_1$ and $a_2$ under the single point excitation). Thus, we take the unweighted average of $P(\omega)$ for their phase differences spanning over $[0, \pi)$ at even intervals. We eventually find theoretical data that agree with the experiment in the peak counts, the ridge between the peaks, and their skirts outside (Fig. S3), which also suggests a good estimation of the parameters, $\kappa = 61 \, \text{GHz}$, $\delta = 22 \, \text{GHz}$, $\gamma_2 = 18.3 \, \text{GHz}$ and $r = 0.82$.

It is noteworthy that a small change in the coupling $\Delta \kappa$ up to a few gigahertz, together with a compensation of the detuning by $\Delta \delta = -2\Delta \kappa$, little affects the theoretical curve. We also see broader linewidths of the experimental peaks than those of the theory. This indicates small extra Gaussian noise, which becomes negligible in the case of asymmetric pumping and high local loss.
Figure S3. The spectral photon counts for $I_1 = I_2 = 30 \, \mu A$ in (a) linear and (b) semi-logarithmic plots. Blue dots: experimental data. Red curve: best CMT fitting with Eqs. (S1)-(S3) for $\kappa = 61 \, \text{GHz}$, $\delta = 22 \, \text{GHz}$, $\gamma_2 = 18.3 \, \text{GHz}$ and $r = 0.82$. The discrepancy between the experiment and analysis is attributed to small Gaussian random noise, which becomes minor in the spontaneous emission measurement with the asymmetric pumping and high local loss factors (Fig. 3 and 4).

IV. Peak count in spontaneous emission regime

The peak counts of the spontaneous emission ($I_2 = 100 \, \mu A$) for different $I_1$ are plotted in Fig. S4(a). Here, we see the reversed pump dependence for $I_1 \leq 2.4 \, \mu A$, and it is clearly steeper than the intensity reduction before the reversal ($I_1 > 2.4 \, \mu A$). This property evidences the EP transition involving the abrupt net loss suppression of the dominant mode in the broken phase. The filled markers represent the data with the saturated detections of the counts because of a long integration time of 60 s for the measurement. They hence do not indicate lasing; note that their corresponding linewidths are still longer than 0.2 nm.

We also show some experimental spectra representative of the EP transition in Fig. S4(b). When $I_1$ is large ($I_1 = 7.0 \, \mu A$), there are two separate peaks corresponding to the two coupled modes in the symmetric phase. The difference in their peak counts stems from the interplay of the cavity detuning $\delta$ and asymmetric pumping. As $I_1$ decreases, the peaks get closer to each other ($I_1 = 4.0 \, \mu A$) and eventually coalesce at their middle ($I_1 = 2.4 \, \mu A, \gamma_1 = 93.4 \, \text{GHz}$). Remarkably, the peak count grows by 30% from this apparent coalescence to the EP (very near $I_1 = 1.4 \, \mu A, \gamma_1 = 115.6 \, \text{GHz}$), despite that the net loss $\approx \gamma_1/2$ of the eigenmodes in the symmetric phase continues to be magnified. This indicates the peculiar peak LDOS enhancement by the EP degeneracy, i.e. the constructive interference of the two original spectral peaks, based on the non-orthogonality of non-Hermitian eigenmodes. The localization of the dominant mode and decline of its net loss in the broken phase result in a further rise in peak power and further linewidth narrowing ($I_1 = 0.2 \, \mu A$).
Figure S4. (a) Dependence of the spectral peak count on current $I_1$ for channel 1 in the spontaneous emission measurement ($I_2 = 100 \, \mu A$). (b) Transition of the spontaneous emission spectra for selected $I_1$. The peak level elevation from $I_1 = 2.4 \, \mu A$ ($\gamma_1 = 93.4 \, \text{GHz}$) to $I_1 = 1.4 \, \mu A$ ($\gamma_1 = 115.6 \, \text{GHz}$) signifies the LDOS enhancement by the EP degeneracy (constructive interference of non-Hermitian eigenstates in the spectral domain). We emphasize that our comprehensive theoretical analysis of the experimental data elucidates that $\kappa \approx 58 \, \text{GHz}$, and the EP is hence around $I_1 = 1.4 \, \mu A$.

V. Measured spectra in spontaneous emission regime

In Fig. S5, we present experimental photon count spectra for some other $I_1$ and $I_2 = 100 \, \mu A$ (blue dots) with their CME fitting (red curves), to show that our model consistently reproduces the measured data well. Here, there is small difference between them, in terms of the ridge lines and lower peaks, because we ignore $c_1(\omega)$. However, the fitting including it leads to significant complexity of the task, since the injection current to the channels does not necessarily reflect the ratio of the excitation amplitudes, as inferred in Sec. III. Nonetheless, the flat-top [Fig. S5(a)] and pointed [Fig. S5(b), (c)] peak structures in the experimental curves can simply be attributed to the fluctuation of the detection.

The emission spectra around the EP condition, together with the fitting using different trial functions, are depicted in Fig. S6. Because the transition of the system spectral response is continuous, we can also observe its particular EP-based property nearby. For both $I_1 = 1.6 \, \mu A$ [Fig. S6(a)] and $I_1 = 1.2 \, \mu A$ [Fig. S6(b)], the Lorentzian curves obviously diverge from the experimental data in peak counts and decaying tails. In contrast, our CME and squared Lorentzian
functions explain them significantly better and hence demonstrate the enhanced LDOS. In our measurement condition where both \(a_1(\omega)\) and \(a_2(\omega)\) are collected, the radiation at the EP has slightly broader spectral skirts than the squared Lorentzian function [see Eq. (2) and Methods]. Thus, Fig. S6(a) signifies the system (\(\gamma_1\)) below the EP. The narrower peak shown in Fig. S6(b) is slightly more compliant with the Lorentzian fitting compared with that for \(I_1 = 1.4 \mu\text{A}\) (Fig. 4), suggesting that the system with \(I_1 = 1.2 \mu\text{A}\) (\(\gamma_1 = 119.7 \text{GHz}\)) is just above the EP.

VI. Correction of background counts in the spontaneous emission spectra

The background emission spectrum is not completely flat, because the electroluminescence of the sample has a global spectral peak located around 1440 nm. We can correct this non-essential factor
within the first order in the following way, for clearer comparison between the theoretical and experimental results.

We first pick a raw spectral curve from 195.0 to 197.0 THz and take its moving average for denoising. Next, we draw a smooth lower envelope of the resultant data, which avoids the effect of the resonant peaks. Linear fitting of the envelope gives the background gradient that should be compensated. The linear function with this slope and reference to the least point is subtracted from the entire raw data, before the additional offsetting with the minimum count.

Figure S7 shows the experimental spontaneous emission spectra with the linear background correction for (a) $I_1 = 1.4 \mu A$ and (b) $I_1 = 0.2 \mu A$ and their theoretical fitting. Compared with Fig. 4, the corrected data here agree better with our CMT fitting in both (a) and (b), especially on the right skirts of the peaks. The discrepancy between them is just within the order of hundreds of
counts, and it is mostly attributed to the small oscillations superposed over the data, which have an amplitude of about 100 counts and correspond to the Fabry-Perot resonances by the line defects. We emphasize that the difference between the CMT and squared Lorentzian function around the EP condition [Fig. S7(a)] comes from the radiation from the excited cavity 2 that is subject to the reaction of cavity 1 and hence broadens the spectral tails via the quadratic term $(\Delta \omega - \delta)^2$ in the response function. It is also noteworthy that the CMT also explains the data for $I_1 = 0.2 \mu A$ best of the considered trial functions [Fig. S7(b)], and it becomes closer to the Lorentzian function as the system gets away from the EP.

Figure S7. Device spontaneous emission spectra with the linear background correction, to be compared with Fig. 4 in the main text. (a) $I_1 = 1.4 \mu A$ and (b) $I_1 = 0.2 \mu A$. Linear functions with slopes of 240 and 200 counts/THz are subtracted from the raw data for (a) and (b), respectively.
VII. Estimated parameters

We discuss the rest of the parameters estimated in our theoretical analysis and shown in Fig. S8. The characteristics of the cavity-mode wavelengths $\lambda_1(I_1)$ and $\lambda_2(I_1)$ [Fig. S8(a)] include the information about the average frequency $\omega_0$ and cavity detuning $\delta$. While the resonance of cavity 2 under the constant pumping ($I_2 = 100 \, \mu\text{A}$) remains near $\lambda_2 \approx 1529.9 \, \text{nm}$ in the symmetric phase, the other mode ($\lambda_1$) undergoes a continuous red shift with decreasing $I_1$ due to the suppression of the carrier plasma effect. The detuning almost vanishes around $I_1 = 1.4 \, \mu\text{A}$, i.e. the EP condition, and its sign is flipped beyond the degeneracy. The small but consistent red shift of $\lambda_2$ for $I_1 \lesssim 1.6 \, \mu\text{A}$ is present possibly because of carrier depletion due to amplified spontaneous emission. Note that the absorption loss ($Q$ factor) of the major localized eigenmode falls (scales) rapidly in the broken phase.

As mentioned in the main text, the values of the coupling rate $\kappa$ are mostly distributed between 58 and 59 GHz over our analysis [Fig. S8(b)]. As the fitting for the coalesced peaks with narrowing linewidths becomes difficult, we approximate the coupling as $\kappa = 58 \, \text{GHz}$ considering the average and analyze the data by estimating $\gamma_1$ and $\delta$ for $I_1 \leq 0.8 \, \mu\text{A}$.

Figure S8. Estimated dependence of the (a) resonance wavelengths of the cavity modes ($\lambda_1, \lambda_2$) and (b) cavity coupling rate ($\kappa$) on $I_1$ in the spontaneous emission experiment. In the shaded area of (b), $\kappa$ is approximated as 58 GHz to obtain a reliable fitting result.
VIII. Some important properties of the system’s spectral response

The photonic LDOS is directly relevant to how much a point-source dipole excitation couples with possible harmonic modes in the system. In the spectral coupled-mode model here, the field solutions \{\text{\(a_1(\omega)\), \(a_2(\omega)\)}\} denote the response of the coupled eigenmodes to the net excitations \{\text{\(c_1(\omega)\), \(c_2(\omega)\)}\} for the tiny cavities. Because the cavity coupling \(\kappa\) is approximated as a constant for the considered frequency range, it is reasonable that the intensity of cavity 1 \(|a_1(\omega)|^2\) under the excitation only for transparent cavity 2 \([c_1(\omega) = 0, \gamma_2 = 0]\) exactly involves the peculiar squared Lorentzian LDOS at the EP condition \((\delta = 0, \kappa = \gamma_1/2)\),

\[
|a_1(\omega)|_{\text{EP}}^2 = \left(\frac{\gamma_1}{2}\right)^2 \frac{|c_2(\omega)|^2}{\left(\frac{\gamma_2}{2}\right)^2 + \Delta\omega^2}, \tag{S4}
\]

and vice versa for \(c_2(\omega) = 0, \gamma_1 = 0, \delta = 0\) and \(\kappa = \gamma_2/2\),

\[
|a_2(\omega)|_{\text{EP,}\ c_2=0}^2 = \left(\frac{\gamma_2}{2}\right)^2 \frac{|c_1(\omega)|^2}{\left(\frac{\gamma_1}{2}\right)^2 + \Delta\omega^2}, \tag{S5}
\]
as mentioned in Ref. 6. Remarkably, the full width at half maximum (FWHM) of Eq. (S4) [(S5)] is \(\sqrt{\pi} - 1 \gamma_1(\gamma_2) \approx 0.644 \gamma_1(\gamma_2)\), where \(\gamma_1(\gamma_2)\) is that for the Lorentzian function with the same loss factor. This shows the linewidth narrowing of the EP resonance.

Without loss of generality, we focus on the former case as we have done in the experiment. Here, the excited cavity (#2) has an intensity additionally affected by \(\Delta\omega^2\), as the back action from the cavity 1’s resonance,

\[
|a_2(\omega)|_{\text{EP}}^2 = \left(\frac{\gamma_1}{2}\right)^2 \frac{|c_2(\omega)|^2}{\left(\frac{\gamma_1}{2}\right)^2 + \Delta\omega^2} + \Delta\omega^2, \tag{S6}
\]

Around the EP frequency \((\omega \approx \omega_0\) or \(\Delta\omega \approx 0\)), however, Eq. (S6) also features a squared Lorentzian shape, holding consistency with the result of the singular perturbation analysis. This is also the case for our fitting function, \(P(\omega) = \eta\gamma_{\text{cav}}|a_1(\omega)| + ia_2(\omega)|^2\).

The device emission in the large loss limit \((\gamma_1 \gg \kappa)\) is another important property. Cavity 1’s intensity, with \(c_1(\omega) = 0, \gamma_2 = 0, \) and \(\delta = 0\) for simplicity, can reduce to

\[
|a_1(\omega)|^2 = \frac{\kappa^2}{\gamma_1^2} \frac{|c_2(\omega)|^2}{\Delta\omega^2 + \kappa^4 \left(\frac{\Delta\omega^2}{\kappa^2}\right)^2} \approx \frac{\kappa^2}{\gamma_1^2} \frac{|c_2(\omega)|^2}{\Delta\omega^2 + \left(\frac{\kappa^2}{\gamma_1}\right)^2}, \tag{S7}
\]
when the main peak structure ($|\Delta \omega| \leq \kappa^2/\gamma_1$) is sufficiently narrower than the width of the quadratic factor $[1 - (\Delta \omega^2/\kappa^2)]$, i.e. $\gamma_1 \gg \kappa$. Because the same derivation is applicable for $P(\omega)$, a Lorentzian emission spectrum with a FWHM of $2\kappa^2/\gamma_1$ is expected in the case of a high imaginary potential contrast.

We also point out that the position of the emission peaks is not identical to the eigenfrequencies of the effective Hamiltonian. Approximate peak frequencies of $P(\omega)$ are those giving extremal values of its denominator and found as real solutions for the following cubic equation,

$$2\Delta \omega^3 + (\gamma_1^2 - 2\kappa^2 - 2\delta^2)\Delta \omega^2 + \gamma_1^2\delta = 0. \quad (S8)$$

They successfully take into consideration the effect of the overlap between the two spectral peaks with their level difference and finite linewidths, and provide the exact result for $|a_1(\omega)|^2$ under a flat (constant) excitation spectrum. We have numerically assured that Eq. (S8) and the estimated parameters reproduce the peak position of the measured spectra well. In contrast, the eigen-wavelengths of the lower peaks in Fig. 3(b) somewhat diverge from the peak locations of the experimental data, due to the skirts of the other peaks with larger peak counts.

A simpler example is the case where $\delta = 0$. Here, we obtain the analytic solutions $\Delta \omega = \pm \sqrt{\kappa^2 - (\gamma_1^2/2)}$, which are certainly different from the corresponding eigenfrequencies, $\Delta \omega_i = \pm \sqrt{\kappa^2 - (\gamma_1^2/4)}$.

**IX. Eigenvalue analysis for lasing states**

The time-domain CMEs [Eq. (1) in the main text] are solved under the condition that cavity 2 has a variable saturated gain, $\gamma_2 = -g_2$, and all the parameters are real. If we assume the eigenfrequencies $\omega_e$ are also real (i.e. the eigenmodes are lasing), the characteristic equation for $\omega_e$ can be divided into its real and imaginary parts as,

\[
\begin{align*}
\text{Re:} & \quad (\omega_e - \omega_1)(\omega_e - \omega_2) + \gamma_1 g_2 - \kappa^2 = 0, \quad (S9) \\
\text{Im:} & \quad (\omega_e - \omega_1)g_2 - (\omega_e - \omega_2)\gamma_1 = 0, \quad (S10)
\end{align*}
\]

where $\omega_1 = \omega_0 + \delta$ and $\omega_2 = \omega_0 - \delta$. Here, the latter equation reads

$$g_2 = \frac{\omega_e - \omega_2}{\omega_e - \omega_1} \gamma_1, \quad (S11)$$

and hence enables us to eliminate $g_2$ in the former\(^7\). The resultant cubic equation for $\omega_e$ is given by,

$$(\omega_e - \omega_1)^2(\omega_e - \omega_2) + \gamma_1^2(\omega_e - \omega_2) - \kappa^2(\omega_e - \omega_1) = 0. \quad (S12)$$
For our analysis here, we take into consideration the following parameter-dependent properties. First, we assume the saturable loss $\gamma_1(I_1)$ in cavity 1 as a two-step linear model comprising straight lines connecting $(I_1, \gamma_1) = (-5 \mu A, 160 \text{ GHz})$, $(8 \mu A, 33 \text{ GHz})$ and $(50 \mu A, 0.2 \text{ GHz})$, which is close to that estimated by our spontaneous emission measurement and fitting [Fig. 3(c)].

Second, we isolate the red shift $\tau_T(I_1)$ caused by the sample’s thermal expansion. This effect can be safely regarded as the proportional variation of all the resonances on the pumping current $I_1$, and the contribution of $I_1$ is given by,

$$\tau_T(I_1) = C_T I_1 = -8.547 \times 10^{-2} I_1 \text{ (GHz).} \quad (S13)$$

Here, $I_1$ is in microamperes and the coefficient $C_T \text{ [GHz/μA]}$ is calculated with the data points with the largest $I_1$, where $\gamma_1$ is well saturated.

Finally, the reduction of the carrier plasma effect $\tau_{C,L}(I_1)$ for the restored lasing state in cavity 2 is approximated as a proportional dependence on the peak power $n_P(I_1)$ in pW [Fig. 4(b)], since $I_2$ is constant and the steady active carriers in cavity 2 are consumed one by one by the stimulated photons localizing there. This is only applied for $|\lambda_-\rangle$ with $I_1 \leq 5.4 \mu A$ and shown as

$$\tau_{C,L}(I_1; I_1 \leq 5.4 \mu A) = C_C n_P(I_1) = -2.726 \times 10^{-1} n_P(I_1) \text{ (GHz)}, \quad (S14)$$

where $C_C \text{ [GHz/pW]}$ is estimated with the lasing frequencies with minimum $I_1$, where the eigenstates are deeply in the broken phase.

With all of them, we eventually reach our trial function,

$$\omega(I_1) = \omega_e[\gamma_1(I_1)] + \tau_T(I_1) + \tau_{C,L}(I_1; I_1 \leq 5.4 \mu A), \quad (S14)$$

and we use $\kappa = 71 \text{ GHz}$, $\omega_1 = 196.1143 \text{ THz (1529.72 nm)}$ and $\omega_2 = 196.1284 \text{ THz (1529.61 nm)}$ in our fitting. A somewhat large value of $\kappa$ is adopted here, since we have to average the effect of the cavity detuning, which is hard to be precisely identified. Because the equation for $\omega_e \in \mathbb{R}$ is cubic, the solution can include the eigenmodes in both the exact and broken phases.

In Fig. S9, we compare the transition of the theoretical system’s eigen-wavelengths in the lasing regime for the case of (a) no and (b) finite cavity resonance detuning. Here, we exclude the heat and carrier contributions to the resonances and just measure the impact of gain and loss, by solving the algebraic equation (S12) for the eigenfrequencies $\omega_e \in \mathbb{R}$. We consider their dependence on the loss rate of cavity 1, $\gamma_1$, without converting it into the injection current $I_1$. The cavity coupling here is $\kappa = 71 \text{ GHz}$, as adopted for fitting Fig. 2(d). Note that the saturated gain in cavity 2 is given by $g_2 = \gamma_1(\omega_e - \omega_2)/(\omega_e - \omega_1)$ for each $\omega_e$. Complex solutions $(\text{Im } \omega_e \neq 0)$ are ruled out in the analysis, because they indicate unstable or decaying modes. The data here are presented as
When the local cavity-mode wavelengths are identical, $\lambda_1 = \lambda_2 = 1529.665$ nm, the system holds the singular EP degeneracy of the coupled modes $\{|\lambda_+\rangle, |\lambda_-\rangle\}$ even under the existence of the adaptive gain due to heavy pumping [Fig. S9(a)]. The red-side $|\lambda_+\rangle$ and blue-side $|\lambda_-\rangle$ modes in the symmetric phase have the same gain $g_2 = \gamma_1$, until they reach the EP at $\gamma_1 = \kappa = 71$ GHz. Thus, both are ideally stable, and the device hence does not exhibit selective mode excitation or mode competition. On the other hand, the additional branch $|\lambda_B\rangle$ in the middle has the clamped wavelength $\lambda_B = \lambda_1 = \lambda_2$. When $\gamma_1 < \kappa$, it actually corresponds to the lossier eigenstate in the broken phase and requires the highest gain $g_2 = \kappa^2/\gamma_1$ for oscillation. Thus, it never dominates over the coupled modes $\{|\lambda_+\rangle, |\lambda_-\rangle\}$ in the symmetric phase. Beyond the EP ($\gamma_1 > \kappa$), however, $|\lambda_B\rangle$ turns into the state that localizes at the cavity with gain in the broken phase and becomes the only stable branch with the lowest saturation gain, $g_2 = \kappa^2/\gamma_1 < \gamma_1$.

Finite cavity detuning drastically alters the system response, as shown in Fig. S9(b) for $\lambda_1 = 1529.72$ nm and $\lambda_2 = 1529.61$ nm, which are also used in explaining Fig. 2 (d). In this case, the intense excitation to cavity 2 always couples better with $|\lambda_-\rangle$, because its wavelength $\lambda_-$ is closer to $\lambda_2$ than that of $|\lambda_+\rangle$. Consequently, $|\lambda_-\rangle$ always needs the smallest $g_2$ and undergoes the continuous but detuned EP transition without the degeneracy, which is confirmed by Fig. 2(d) and (e). Here, we emphasize that the analysis is performed for tracing the revived lasing resonance of $|\lambda_-\rangle$, and the effect of the detuning is averaged and constant, $2\delta = 0.11$ nm. The result is hence
approximate and does not explain detailed experimental behavior such as the switching of the dominant mode around the suppression of oscillation, which is attributed to the carrier-based inversion of $\lambda_1$ and $\lambda_2$.

The red-side mode $|\lambda_+\rangle$ exhibits peculiar properties in the process. It is stable but inferior compared to $|\lambda_-\rangle$ when $\gamma_1$ is small and thus demands a slightly larger $g_2$. Remarkably, there is an apparent singular coalescence between $|\lambda_+\rangle$ and $|\lambda_B\rangle$. However, they actually denote the single eigenstate with different variable gain $g_2$. While $|\lambda_+\rangle$ is one of the coupled modes in the symmetric phase, $|\lambda_B\rangle$ is the same but unobservable eigenstate in the broken phase with the largest $g_2$ that localizes at the lossy cavity. Thus, their coalescence is not an EP degeneracy. The combined solutions for larger $\gamma_1$ come to be a single non-steady mode with $\text{Im} \omega_c \neq 0$, which is hence omitted from Fig. S9(b). This mode destabilization would result in the unexpected experimental response with the multiple peaks split from $|\lambda_+\rangle$ just before the lasing of $|\lambda_-\rangle$. Further details are shown in the next section.

X. Selected spectra in lasing regime

Here, we present and discuss the transition of the emission spectra in the lasing regime ($I_2 = 800 \mu A$). When $I_1$ is large [Fig. S10(a)], there are two spectral peaks corresponding to the coupled ground modes with a splitting of about 1.1 nm, which is reasonable for a system with a coupling of $\kappa \approx 60$ GHz and a cavity detuning $\delta$ on an order of 10 GHz. The mode with the shorter wavelength $|\lambda_-\rangle$ decays faster [Fig. S10(b)] with decreasing $I_1$ and disappears for $I_1 < 30 \mu A$. The significant peak power difference between the two modes here is attributed to a finite and non-negligible $\delta$.

The spectral property changes drastically at $I_1 = 5.4 \mu A$ [Fig. S10(c)]. The peak power of the red-side mode $|\lambda_+\rangle$ falls by more than 10 dB from the preceding point ($I_1 = 5.6 \mu A$), and the device comes to exhibit a weak and broad spectrum spanning from 1529.1 to 1530.2 nm, which is attributed to spontaneous emission. Remarkably, $|\lambda_+\rangle$ is split into two peaks, and the minor one at 1529.9 nm shifts toward the middle of its higher counterpart and the revived $|\lambda_-\rangle$. Note that because cavity 2 is heavily pumped, the amplified spontaneous emission at resonance can have a linewidth below 0.1 nm (or the cold linewidth of each single cavity). Although we also find the first-order mode with an increased intensity (not shown), it never becomes dominant over the ground modes in the entire measurement.
Figure S10. Coherent emission spectra of the sample under high pumping for cavity 2 ($I_2 = 800 \, \mu\text{A}$), for different $I_1$. (a) $I_1 = 100 \, \mu\text{A}$, (b) $I_1 = 30 \, \mu\text{A}$, (c) $I_1 = 5.4 \, \mu\text{A}$, (d) $I_1 = 2.0 \, \mu\text{A}$, (e) $I_1 = 0.0 \, \mu\text{A}$ and (f) $I_1 = -5.0 \, \mu\text{A}$. 
The peak power of the whole spectrum becomes minimum at $I_1 = 2.0 \mu A$ [Fig. S10(d)]. Here, $|\lambda_-\rangle$ (left) exceeds the remaining peak around 1530.15 nm (right), which we denote as $|\lambda'_+\rangle$. $|\lambda'_+\rangle$ has an unconventionally spiky structure and hence is not considered as a normal cavity resonance but as a non-steady mode. The weaker itinerant bump coming off of $|\lambda_+\rangle$ reaches the right verge of $|\lambda_-\rangle$ and forms a visible plateau. $|\lambda_-\rangle$ is further enhanced and $|\lambda'_+\rangle$ is damped by reducing $I_1$, as shown in Fig. S10(e).

$|\lambda_-\rangle$ is significantly red-shifted and eventually overlaps the shoulder on its recovery [Fig. S10(f)]. Interestingly, while the spectral resolution of the measurement is limited to 0.05 nm, the emission for $I_1 = -5.0 \mu A$ (near zero bias of channel 1) fits much better with the squared Lorentzian function than with the Lorentzian. As seen by the localized emission pattern [Fig. 2(e)], the state here is in the broken phase after the detuned EP transition. However, its peculiar spectral shape and position indicate the cancelling of the cavity detuning $\delta$ by reducing $I_1$. It is noteworthy that the measurement equipment used here (OSA) is different from that for the spontaneous emission experiment (a spectrometer).

**XI. Loss-induced lasing in detuned EP transition**

The spectral variation with $I_1$ for $I_2 = 800 \mu A$ is shown in terms of the peak powers and linewidths of the two major branches in Fig. S11(a) and (b), respectively. The abrupt suppression of $|\lambda_+\rangle$ at $I_1 = 5.4 \mu A$ results in the broad spectrum plotted in Fig. S10(c), which includes the re-emergent blue-side peak $|\lambda_-\rangle$ [Fig. S11 (a)]. The power of $|\lambda_-\rangle$ then grows with a kink with decreasing $I_1$, and its linewidth is correspondingly narrowed from $\approx 0.2$ nm to below the minimum measurement resolution here, 0.05 nm [Fig. S11 (b)]. This indicates that the system undergoes the revival of lasing with $|\lambda_-\rangle$ after the switching of the dominant eigenmode.

It is notable that $|\lambda'_+\rangle$ for $I_1 \leq 5.4 \mu A$ denotes the immobile spiky peak remaining around 1530.15 nm, which is again considered unstable [Fig. S10(c)-(e)]. Although it seems to hold somewhat narrow linewidths, it soon decays as $|\lambda_-\rangle$ oscillates. Its detailed properties are beyond the scope of this work.

Finally, the spectra here do not have any hysteresis behavior. They are identical regardless of whether $I_1$ is swept upward or downward, except for a small fluctuation coming from the long-term change in the electrical contact between the device and probes.
Figure S11. (a) Peak powers and (b) linewidths of the $|\lambda_+\rangle$, $|\lambda'_\pm\rangle$ (red) and $|\lambda_-\rangle$ (blue) under small $I_1$. Filled blue markers: approximate linewidths estimated with the moving-average data. The plots for $|\lambda'_\pm\rangle$ after the drop of the power correspond to the immobile peak around 1530.15 nm in Fig. S10(c)-(e), which is attributed to the non-steady damping state. Another weaker and broader bump moving toward the middle is omitted. The peak power rising and linewidth narrowing of $|\lambda_-\rangle$ indicates the revival of lasing. Here, the minimum linewidth resolution is 0.05 nm.

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