Intermediate and Power-law Inflation in the Tachyon Model with Constant Sound Speed

Narges Rashidi\textsuperscript{1,2}\thanks{Department of Theoretical Physics, Faculty of Science, University of Mazandaran, P. O. Box 47416-95447, Babolsar, Iran; n.rashidi@umz.ac.ir}
\textsuperscript{2}ICRANet-Mazandaran, University of Mazandaran, P. O. Box 47416-95447, Babolsar, Iran

Received 2022 April 6; revised 2022 May 13; accepted 2022 May 14; published 2022 July 1

Abstract

By adopting the intermediate and power-law scale factors, we study the tachyon inflation with constant sound speed. We perform some numerical analysis on the perturbation and non-Gaussianity parameters in this model and compare the results with observational data. By using the constraints on the scalar spectral index and tensor-to-scalar ratio obtained from Planck2018 TT, TE, and EE+lowE+lensing+BAO+BK14 data; the constraints on the running of the scalar spectral index obtained from Planck2018 TT, TE, and EE+lowEB+lensing data; and the constraints on tensor spectral index obtained from Planck2018 TT, TE, and EE+lowE+lensing+BK14+BAO +LIGO and Virgo2016 data, we find the observationally viable ranges of the model’s parameters at both 68% CL and 95% CL. We also analyze the non-Gaussian features of the model in the equilateral and orthogonal configurations. Based on Planck2018 TTT, EEE, TTE, and EET data, we find constraints on the sound speed of 0.276 \(\leq c_s \leq 1\) at 68% CL, 0.213 \(\leq c_s \leq 1\) at 95% CL, and 0.186 \(\leq c_s \leq 1\) at 97% CL.

Unified Astronomy Thesaurus concepts: Inflationary universe (784); Observational cosmology (1146); Early universe (435)

1. Introduction

Considering that the early-time inflation in the history of the universe seems to have occurred around Planck’s scale, the M/string theory inspired fields can be a possible candidate for the field responsible for inflation. Tachyon inflation is one of these fields leading to interesting cosmological results. The tachyon field, which has attained a lot of attention, is associated with the D-branes in string theory. One interesting implication of this field is that its slow rolling down to the potential leads to the smooth evolution of the universe from the accelerating phase of the expansion to the era dominated by the nonrelativistic fluid (Sen 1999, 2002a, 2002b; Gibbons 2002).

On the other hand, we know that in the simple inflation model with canonical scalar fields we find the primordial perturbation to be scale invariant and Gaussian (Guth 1981; Albrecht & Steinhardt 1982; Linde 1982, 1990; Lidsey et al. 1997; Liddle & Lyth 2000; Riotto 2002; Maldacena 2003; Lyth & Liddle 2009). When we consider an inflation model with a canonical scalar field, we find a Gaussian perturbation that the values of its tensor-to-scalar ratio for $\phi$, $\phi^2$, and $\phi^{4/3}$ are not consistent with observational data (Akrami et al. 2020a). Therefore, we have to seek some other specific potential, like the hilltop potential, to get an observationally viable single canonical field inflation model (Akrami et al. 2020a). However, with every potential, the amplitude of the primordial perturbation in the single canonical field inflation is almost Gaussian. Or, we should consider a nonminimal inflation model to give a better explanation of the early-time cosmological inflation. However, with a noncanonical scalar field like tachyon, it is possible to get the scale-dependent and non-Gaussian distribution for the amplitude of the primordial perturbations. Another interesting point about the tachyon field’s cosmology is its equation of state parameter. This parameter in the tachyon field can be $-1$, which describes both very early time and very late time accelerating expansion of the universe. In addition, its value can be 0, describing the matter/dark-matter-dominated era. Therefore, with the tachyon field it is possible to explain the thermal history of the universe in a reasonable way (Gibbons 2002). These are interesting issues that motivate cosmologists to consider and study the inflation models driven by a tachyon field as a noncanonical scalar field. In this regard, a lot of interesting works based on tachyon inflation have been done. For instance, Kamali et al. (2018) have considered a tachyon field in the context of quantum loop gravity. In this way, they have obtained the inflation and perturbation parameters and tested the observational viability of some inflation models. In Bilic et al. (2019), by considering the holographic cosmology, the tachyon inflation has been studied and the results have been compared to the observational data. Mohammadi et al. (2020) have studied an inflation model where a tachyon field interacts with photon gas. They have shown that under some assumptions and conditions this model shows some agreement with observational data. In Rashidi (2021), the tachyon model with a superpotential, a potential based on supersymmetry, has been studied. It has been shown that tachyon inflation with a superpotential, at least in some ranges of the model’s parameter space, is consistent with observational data. There are other works on tachyon inflation leading to interesting cosmological results (Nojiri & Odintsov 2003; Nozari & Rashidi 2013; Bouaballaoui et al. 2016; Rezaazadeh et al. 2017; Rashidi & Nozari 2018, 2020), and all of these works show that the tachyon field can be considered as a field running the inflation.

One of the important parameters in the inflation models is the sound speed of the primordial perturbation, shown by $c_s$. The sound speed corresponds to the Lorentz factor $\gamma$ as $c_s = \frac{1}{\gamma}$. For the canonical scalar field, we get the sound speed equal to unity. However, with the noncanonical scalar fields, we have $c_s^2 \neq 1$. The values of parameter $\gamma$ and correspondingly sound speed determine the deformation of the field’s kinetic energy from the canonical one. The constant sound speed is an interesting idea...
motivated by Spalinski (2007) and Tsujikawa et al. (2013). Considering that in the noncanonical scalar field the sound speed is related to the time variation of the field, the constant sound speed corresponds to the constant field’s variation in times. This can lead to interesting results. Now, the question is what constant values of $c_s$ we should adopt, to study the model numerically. Considering that the sound speed is related to the amplitude of the non-Gaussianity, by using the observational constraints on the amplitude of the non-Gaussianity, we can find suitable ranges of the sound speed. In addition, there are some other parameters, such as the scalar spectral index, tensor spectral index, and tensor-to-scalar ratio, that help us to find some constraints on the sound speed values.

To perform a numerical analysis, we use the Planck2018 constraints on the inflation parameters. Based on the $\Lambda$CDM + $r + \frac{dn_s}{d\ln k}$ model, the Planck2018 TT, TE, and EE +lowE+lensing+BAO+BK14 data give the value of the scalar spectral index as $n_s = 0.9658 \pm 0.0038$ and imply a constraint on the tensor-to-scalar ratio of $r < 0.072$ (Aghanim et al. 2020; Akrami et al. 2020a). The constraint on the tensor spectral index, released by Planck2018 TT, TE, and EE +lowE+lensing+BAO+LIGO and Virgo2016 data, is $-0.62 < n_T < 0.53$ (Aghanim et al. 2020; Akrami et al. 2020a). In addition, from Planck2018 TT, TE, and EE+lowEB+lensing data, we have the value of the running of the scalar spectral index as $\alpha_s = -0.0085 \pm 0.0073$ (Aghanim et al. 2020; Akrami et al. 2020a). Other useful constraints are related to the amplitude of the non-Gaussianity. The Planck2018 combined temperature and polarization analysis gives the constraints on the amplitude of the non-Gaussianity in the equilateral configuration as

$$\eta_{eq} = -26 \pm 47$$

and in the orthogonal configuration as

$$\eta_{orth} = -38 \pm 24$$

(Akrami et al. 2020b). These several constraints determine the observational viability of every inflation model.

With these preliminaries, this paper is organized as follows. In Section 2, we review the inflation in a tachyon model with constant sound speed. In this section, we present the main equations governing the dynamics of the model in terms of the sound speed. In Section 3, we consider an intermediate scale factor and study the perturbation and non-Gaussianity parameters in this model numerically. We compare the results with several data sets to find some constraints on the model’s parameter space. In Section 4, we study the power-law tachyon inflation with constant sound speed and compare the results with observational data. In Section 5, we provide some discussion on unifying the initial inflation with late-time dark energy in the tachyon model with constant sound speed. In Section 6, we present a summary of this work.

2. Review on the Tachyon Inflation with Constant Sound Speed

For the tachyon field, we have the following Dirac–Born–Infeld–type effective four-dimensional action:

$$S = \int d^4x \sqrt{-g} \left[ \frac{1}{2\kappa^2} R - V(\phi) \sqrt{1 - 2X} \right],$$

(1)

where $\kappa$ is the gravitational constant, $R$ is the Ricci scalar, and $X = -\frac{1}{2} \partial_\mu \phi \partial^\mu \phi$. In addition, the tachyon field $\phi$ has the potential $V(\phi)$. Cosmologists believe that the physics of the tachyon condensation can be described by such an action. Einstein’s field equations, corresponding to action (1), are given by

$$G_{\mu\nu} = \kappa^2 \left[ -g_{\mu\nu} V(\phi) \sqrt{1 - 2X} + \frac{V(\phi) \partial_\mu \phi \partial_\nu \phi}{\sqrt{1 - 2X}} \right],$$

(2)

which have been obtained by varying action (1) with respect to the metric. Considering the FRW metric as

$$ds^2 = -dt^2 + a^2(t) \delta_{ij} dx^i dx^j,$$

(3)

we find the following Friedmann equation:

$$3H^2 = \frac{k^2 V(\phi)}{\sqrt{1 - \dot{\phi}^2}},$$

(4)

corresponding to the $(0, 0)$ component of Einstein’s field equations. Note that a dot on the parameter shows the derivative of the parameter with respect to the time. In addition, the $(i, i)$ component of Einstein’s field equations gives the following second Friedmann equation:

$$2\dot{H} + 3H^2 = k^2 [V(\phi) \sqrt{1 - \dot{\phi}^2}].$$

(5)

We find the equation of motion of the tachyon field by varying action (1) with respect to the field as

$$\ddot{\phi} + 3H \dot{\phi} + \frac{V(\phi)}{V(\phi)} \dot{V}(\phi) = 0,$$

(6)

where a prime shows the derivative with respect to the tachyon field. Since our purpose in this paper is to study the tachyon model with constant sound speed, we should rewrite the above main equations in terms of the sound speed. To this end, we need to define the sound speed of the tachyon field. In general, the square of the sound speed is given by $c_s^2 = \frac{p_x}{\rho_x}$, which in the tachyon model takes the following form:

$$c_s = \sqrt{1 - \dot{\phi}^2}.$$

(7)

In this regard, Equations (4)–(6) take the following forms:

$$3H^2 = \frac{k^2 V(\phi)}{c_s^2},$$

(8)

$$2\dot{H} + 3H^2 = k^2 V(\phi) c_s,$$

(9)

and

$$3H \sqrt{1 - c_s^2} + \frac{V(\phi)}{V(\phi)} \dot{V}(\phi) = 0,$$

(10)

where we have considered the sound speed as a constant parameter. To study cosmological inflation, we need the following slow-roll parameters:

$$\epsilon = -\frac{\dot{H}}{H^2},$$

(11)

$$\eta = -\frac{1}{3} \frac{\ddot{H}}{H^2}.$$

(12)

Note that, in the case of the constant sound speed, the third slow-roll parameter $s = \frac{1}{3} \epsilon$ is zero. Another needed parameter to study inflation is the number of $e$-folds, defined as

$$N = \int H dt.$$
These parameters are useful to study the primordial perturbations in the model and compare the results with observational data. The observational data that we use in our work are the data released by the Planck2018 team (Aghanim et al. 2020; Akrami et al. 2020a, 2020b). Based on the fact that, under the assumption of statistical isotropy, the two-point correlations of the cosmic microwave background (CMB) anisotropies are described by the angular power spectra $C_l^{TT}, C_l^{TE}, C_l^{EE},$ and $C_l^{BB}$ (where the subscript $l$ shows the multipole moment number; Hu & White 1997; Kamionkowski et al. 1997; Seljak & Zaldarriaga 1997; Zaldarriaga & Seljak 1997; Hu et al. 1998), the Planck team has obtained some constraints on the important perturbation parameters. To include the contributions from the scalar and tensor perturbations in the CMB angular power spectra, the Planck team has used the following expressions (Ade et al. 2016):

$$C_{l}^{ab,T} = \int_0^{\infty} \frac{dk}{k} \Delta_{l,a}^s(k) \Delta_{l,b}^s(k) \mathcal{A}_i(k),$$

$$C_{l}^{ab,T} = \int_0^{\infty} \frac{dk}{k} \Delta_{l,a}^T(k) \Delta_{l,b}^T(k) \mathcal{A}_T(k).$$

In Equations (14) and (15), we have shown the transfer functions by $\Delta_{l,a}^s(k)$ and $\Delta_{l,a}^T(k)$. The parameters $a$ and $b$ are given as $a, b = T, E, B$. In addition, the primordial power spectrum, the parameter identified by the physics of the primordial universe (Ade et al. 2016), is shown by $\mathcal{A}_i(k)$ (where $i = s, T$). Model-independent forms of the scalar and tensor power spectra are given by

$$\mathcal{A}_i(k) = \mathcal{A}_i \left( \frac{k}{k_H} \right)^{n_i - 1 + \frac{d n_i}{d \ln k}} + \frac{1}{6} \frac{d n_i}{d \ln k} \left( \frac{k}{k_H} \right)^{n_i - 1 + \frac{d n_i}{d \ln k}} + \ldots,$$

$$\mathcal{A}_T(k) = \mathcal{A}_T \left( \frac{k}{k_H} \right)^{n_T + \frac{d n_T}{d \ln k}} + \frac{1}{2} \frac{d n_T}{d \ln k} \left( \frac{k}{k_H} \right)^{n_T + \frac{d n_T}{d \ln k}} + \ldots.$$  

These forms of the scalar and tensor power spectra have been used by the Planck team to compare the perturbation parameters with data. Note that, in Equations (16) and (17), $\mathcal{A}_j$ shows the scalar (corresponding to $j = s$) and tensor (corresponding to $j = T$) perturbations. In addition, the running of the scalar or tensor spectral index and the running of the scalar spectral index are given by $\frac{d n_i}{d \ln k}$ and $\frac{d n_T}{d \ln k}$, respectively. One can also find the tensor-to-scalar ratio by using the power spectra as

$$r = \frac{\mathcal{A}_T(k_H)}{\mathcal{A}_s(k_H)},$$

which is a very important parameter in studying the inflation models. The parameters $n_s$ and $n_T$ are the scalar and tensor spectral indices showing the scale dependence of the primordial perturbations. The scalar spectral index, at the time of sound horizon exit of the physical scales, is defined as

$$n_s - 1 = \frac{d \ln A_s}{d \ln k} \bigg|_{k=k_H},$$

where the parameter $A_s$, the amplitude of the scalar spectral index, is given by

$$A_s = \frac{H^2}{8 \pi^2 \mathcal{W}_s \mathcal{C}_s^s},$$

with

$$\mathcal{W}_s = \frac{V(1 - c_s^2)}{2 H^2 c_s^2}.$$  

The scalar spectral index, in terms of the slow-roll parameters, is written as

$$n_s = 1 - 6 \epsilon + 2 \eta.$$  

This is one of the parameters that can be compared with observational data. Another important parameter in studying the inflation models is the running of the scalar spectral index, defined as

$$\zeta = \frac{H}{H^2}.$$  

The tensor spectral index, obtained from Equation (17), is defined as follows:

$$n_T = \frac{d \ln A_T}{d \ln k} \bigg|_{k=k_H},$$

with

$$A_T = \frac{2 \kappa^2 H^2}{\pi^2},$$

being the amplitude of the tensor perturbations. In terms of the slow-roll parameters, the tensor spectral index is expressed as

$$n_T = -2 \epsilon.$$  

Finally, we have the following expression for the tensor-to-scalar ratio:

$$r = 16 \epsilon c_s.$$  

As we see in Equation (28), the tensor-to-scalar ratio depends explicitly on the sound speed of the perturbations. Therefore, from the observationally viable value of $r$, we can constrain the values of the sound speed.

Another important parameter that depends on the sound speed parameter is the nonlinearity parameter. The nonlinearity parameter demonstrates the non-Gaussian property of the amplitude of the primordial perturbation. The Gaussian distributed primordial perturbations are characterized by a two-point correlation. To seek the additional statistical information, corresponding to the non-Gaussian distributed perturbation, it is necessary to consider the higher-order correlations. The three-point correlation function for the spatial curvature perturbation in the interaction picture is given by (Maldacena 2003)

$$\langle \Phi(k_1) \Phi(k_2) \Phi(k_3) \rangle = (2 \pi)^3 \delta^3(k_1 + k_2 + k_3) B_0(k_1, k_2, k_3).$$
with the following definition for the parameter $B_k$:

$$B_k(k_1, k_2, k_3) = \frac{(2\pi)^4 A^2}{\prod_{i=1}^3 k_i^3} D_b(k_1, k_2, k_3).$$  \hfill (30)

The parameter $D_b$ in Equation (40) is expressed as follows:

$$D_b = \left(1 - \frac{1}{c_s^2}\right) \left[\frac{3}{4} J_1 - \frac{3}{2} J_2 - \frac{1}{4} J_3\right],$$  \hfill (31)

where

$$J_1 = \frac{2 \sum_{i>j} k_i^2 k_j^2}{k_1 + k_2 + k_3} - \frac{\sum_{i>j} k_i^2 k_j^3}{(k_1 + k_2 + k_3)^2},$$  \hfill (32)

and

$$J_3 = \frac{2 \sum_{i>j} k_i^2 k_j^3}{k_1 + k_2 + k_3} - \frac{\sum_{i>j} k_i^2 k_j^3}{(k_1 + k_2 + k_3)^2} + \frac{1}{2} \sum_i k_i^3.$$  \hfill (34)

By using the parameter $D_b$, one can find the nonlinear parameter as

$$f_{NL} = \frac{10}{3} \frac{D_b}{\sum_{i=1}^3 k_i^3}.$$  \hfill (35)

Note that, in Equation (35), there is a parameter $k_i$, which is the momentum. Depending on different values of the momenta ($k_1$, $k_2$, and $k_3$), we get different shapes of the non-Gaussianity. In every shape, there is a maximal peak in the amplitude of the perturbations that defines the kind of the corresponding shape. We are interested in the case with a peak at the $k_1 = k_2 = k_3$ limit, leading to an equilateral shape, and the case orthogonal to it (Babich et al. 2004; Chen et al. 2007; De Felice & Tsujikawa 2013; Baumann 2009). It is possible to write bispectrum (31) in terms of the equilateral and orthogonal shapes basis as follows (De Felice & Tsujikawa 2013):

$$D_b = M_1 \hat{J}^{\text{equil}} + M_2 \hat{J}^{\text{ortho}},$$  \hfill (36)

where

$$\hat{J}^{\text{equil}} = -\frac{12}{13} (3J_1 - J_2),$$  \hfill (37)

$$\hat{J}^{\text{ortho}} = \frac{12}{14 - 13C} (C(3J_1 - J_2) + 3J_1 - J_2),$$  \hfill (38)

$$M_1 = \frac{13}{12} \left[\frac{1}{24} (1 - \frac{1}{c_s^2}) (2 + 3C)\right],$$  \hfill (39)

and

$$M_2 = \frac{14 - 13C}{12} \left[\frac{1}{8} (1 - \frac{1}{c_s^2})\right].$$  \hfill (40)

with $C \approx 1.1967996$. For more details on obtaining the nonlinear parameter, see Fergusson & Shellard (2009), De Felice & Tsujikawa (2013), and Byrne (2014). Now, we find the amplitudes of the non-Gaussianity in the equilateral and orthogonal configurations as

$$f_{\text{NL}}^{\text{equil}} = \frac{130}{36 \sum_{i=1}^3 k_i^3} \left[\frac{1}{24} (1 - \frac{1}{c_s^2}) (2 + 3C)\right] \xi_{\text{equil}},$$  \hfill (41)

and

$$f_{\text{NL}}^{\text{ortho}} = \frac{140 - 13C}{36 \sum_{i=1}^3 k_i^3} \left[\frac{1}{8} (1 - \frac{1}{c_s^2})\right] \xi_{\text{ortho}}.$$  \hfill (42)

Equations (41) and (42) in the $k_1 = k_2 = k_3$ limit become

$$f_{\text{NL}}^{\text{equil}} = \frac{325}{18} \left[\frac{1}{24} (1 - \frac{1}{c_s^2}) - 1\right] (2 + 3C)$$  \hfill (43)

and

$$f_{\text{NL}}^{\text{ortho}} = \frac{10}{9} \left[\frac{65}{4} C + \frac{71}{6}\right] \left[\frac{1}{8} (1 - \frac{1}{c_s^2})\right].$$  \hfill (44)

Up to this point, we have presented the main equations describing the cosmological dynamics of the tachyon model. In the following, we adopt two types of scale factor (power law and intermediate) for the tachyon model with constant sound speed and study its observational viability.

### 3. Intermediate Tachyon Inflation with Constant Sound Speed

In this section, we study the intermediate inflation in the tachyon model with constant sound speed. In the intermediate inflation, we have the following scale factor (Barrow 1990; Barrow & Liddle 1993; Barrow & Nunes 2007):

$$a = a_0 \exp(b t^\beta),$$  \hfill (45)

where $0 < \beta < 1$ and $b$ is a constant. This scale factor demonstrates that in this case the expansion of the universe is slower than an exponential expansion and faster than a power-law expansion. From the intermediate scale factor (41) we find the Hubble parameter as

$$H(N) = N \left(\frac{N}{b}\right)^{-\frac{1}{\beta}}.$$  \hfill (46)

To use this Hubble parameter and study the observational viability of the model, we need to reconstruct the potential and the inflation parameters in terms of the Hubble parameter and its derivatives (Bamba et al. 2014; Odintsov & Oikonomou 2015). From Equation (8) we obtain the potential as

$$V = 3 \frac{H^2(N)c_s}{\kappa^2}.$$  \hfill (47)

In addition, the slow-roll parameters take the following form:

$$\epsilon = -\frac{d}{dN} \frac{H(N)}{H(N)},$$  \hfill (48)

and
derivatives. After that, by using Equation Planck2018 TT, TE, and EE scalar spectral index in this model, in the background of the intermediate tachyon model with constant sound speed versus the results with observational data. The results have been shown to obtain some constraints on the model sound speed. Then, we perform some numerical analysis, and to to-scalar ratio in terms of the intermediate parameters and also the sound speed. Then, we perform some numerical analysis, and to obtain some constraints on the model’s parameters, we compare the results with observational data. The results have been shown in several figures. Figure 1 shows the tensor-to-scalar ratio of the intermediate tachyon model with constant sound speed versus the scalar spectral index in this model, in the background of the Planck2018 TT, TE, and EE+lowE+lensing+BAO+BK14 data at 68% CL and 95% CL. As we can see from Figure 1, this model in some ranges of the model’s parameter space is consistent with observational data. Our numerical analysis shows that the intermediate tachyon model with constant sound speed is consistent with Planck2018 TT, TE, and EE+lowE+lensing+BAO+BK14 data at 68% CL at 68% CL and 95% CL. We have also performed some numerical analysis to find the domain of β and c*, leading to observationally viable values of scalar spectral index, tensor spectral index, and tensor-to-scalar ratio. The results are shown in Figure 4, where we have used both Planck2018 TT, TE, EE+lowE+lensing+BAO+BK14 and Planck2018 TT, TE, EE+lowE+lensing+BAO+BK14+BAO +LIGO and Virgo2016 data sets at 68% CL and 95% CL. Both panels confirm that, depending on the value of sound speed, the observationally viable values of β start from almost 0.75. The interesting point is that for these ranges of parameter space the amplitude of the scalar perturbation is consistent with observational data too. The constraint on the amplitude of the scalar perturbation, obtained from Planck2018 TT, TE, and EE+lowE+lensing, is \( \ln(10^{0.05}) = 3.044 \pm 0.014 \). By this observational constraint, we have obtained the observational viable domain of \( c* \) and \( \beta \), shown in Figure 5. Another way to check the observational viability of the model is to study the amplitudes of the non-Gaussianity numerically and compare the results with observational data. To this end, we consider the amplitudes of the non-Gaussianity in equilateral and orthogonal configurations, given by Equations (43) and (44). In this way, we plot the behavior of the orthogonal amplitude of the non-Gaussianity versus the equilateral amplitude of the non-Gaussianity, in the background of Planck2018 TT, EEE, and BAO+

\[
\eta = \frac{\left( \frac{d}{dN} H(N) \right)^2 \sqrt{1 - c_*^2} + (c_* - 1)(c_* + 1) \left( H(N) \frac{d^2}{dN^2} H(N) + 2 \left( \frac{d}{dN} H(N) \right)^2 \right)}{\sqrt{1 - c_*^2} H(N) \frac{d}{dN} H(N)}.
\]

Figure 1. Tensor-to-scalar ratio vs. the scalar spectral index in the intermediate tachyon model with constant sound speed.
TTE, and EET data. The result is shown in the left panel of Figure 6. Our numerical analysis shows that the orthogonal and equilateral amplitudes of the non-Gaussianity in the intermediate tachyon model with constant sound speed are consistent with Planck2018 TTT, EEE, TTE, and EET data at 68% CL if \( 0.276 < c_s \), at 95% CL if \( 0.213 < c_s \), and at 97% CL if \( 0.186 < c_s \). These ranges of the sound speed lead to the observationally viable values of the tensor-to-scalar ratio too. In the right panel of Figure 6, we have plotted the phase space of the tensor-to-scalar ratio and the sound speed squared, based on the domain of the sound speed leading to the observationally viable values of the amplitudes of the non-Gaussianity. As the figure shows, the values of the tensor-to-scalar ratio are in the domain consistent with Planck2018 TT, TE, and EE+lowE+lensing+BAO+BK14 data. We have summarized the prediction of the model for some sample values of the sound speed in Table 1. According to our analysis, it seems that the intermediate tachyon model with constant sound speed is an observationally viable inflation model.

4. Power-Law Tachyon Inflation with Constant Sound Speed

Now, we study the power-law inflation in the tachyon model with constant sound speed. In the power-law inflation, the scale factor is given by

\[ a = a_0 t^n, \]

leading to the following Hubble parameter:

\[ H(N) = N e^{-\frac{N}{2}}. \]

We use this Hubble parameter and substitute it in Equations (47)-(49) to find the inflation parameter in the power-law case. In this way, we can obtain Equations (22), (23), (27), and (28) in terms of the model’s parameters and perform some numerical analysis on the power-law tachyon inflation with constant sound speed. To obtain some constraints on the model’s parameters, we compare the numerical results with observational data. According to our numerical analysis,
the power-law tachyon model with constant sound speed is consistent with Planck2018 TT, TE, and EE+lowE+lensing+BAO+BK14 data at 68% CL if $0 < c_s < 1$ and $222 < n < 565$. In addition, this model is consistent with Planck2018 TT, TE, and EE+lowE+lensing+BAO+BK14 data at 68% CL if $0 < c_s < 0.990$ and $293 < n < 485$. The behavior of the tensor-to-scalar ratio has been shown in Figure 7. The running of the scalar spectral index of the power-law tachyon model with constant sound speed versus the scalar spectral index in this model is shown in Figure 8. By studying these parameters, we find that, in this case, the model is consistent with Planck2018 TT, TE, and EE+lowE+lensing data at 68% CL if $0 < c_s < 1$ and $299 < n < 473$ and at 95% CL if $0 < c_s < 1$ and $231 < n < 551$. Figure 9 shows the evolution of the tensor-to-scalar ratio of the power-law tachyon model with constant sound speed versus the tensor spectral index in this model, in the background of the Planck2018 TT, TE, and EE+lowE+lensing+BK14+BAO+LIGO and Virgo2016 data. In this case, the model has observational viability if $0 < c_s < 1$ and $27.3 < n < 3200$ at 68% CL and $0 < c_s < 1$ and $1.67 < n < 95$% CL.

The domain of $n$ and $c_s$, leading to observationally viable values of scalar spectral index, tensor spectral index, and the tensor-to-scalar ratio is shown in Figure 10, where we have used both Planck2018 TT, TE, EE+lowE+lensing+BAO+BK14 and Planck2018 TT, TE, EE+lowE+lensing+BK14+BAO+LIGO and Virgo2016 data sets at 68% CL and 95% CL. It is clear that, although from the observationally viable values of the tensor spectral index we cannot find an upper bound on the parameter $n$, the observational values of the scalar spectral index imply an upper limit on it. These observationally viable ranges of $c_s$ and $n$ lead to the value of the amplitude of the scalar perturbations released by Planck2018 data (ln$(10^{10} A_s) = 3.044 \pm 0.014$). Figure 11 shows this issue clearly. Considering that the sound speed is constant, the behavior of the orthogonal amplitude of the non-Gaussianity versus the equilateral amplitude of the non-Gaussianity in the power-law tachyon case is the same as the one in the intermediate tachyon case. This means that in this case also we have consistency with Planck2018 TTT, EEE, TTE, and EET data at 68% CL if $0.276 < c_s < 1$ at 95% CL if $0.213 < c_s$ and at 97% CL if $0.186 < c_s$. These ranges are compatible with
the ranges obtained from $n_s$, $\alpha_s$, $n_p$, and $r$. Therefore, the power-law tachyon inflation with constant sound speed seems to be a fine model that in some ranges of its parameter space is consistent with observational data. In Table 2, we have summarized some constraints obtained in studying the power-law tachyon inflation with constant sound speed.

| $c_s$  | 68% CL     | 95% CL     | 68% CL     | 95% CL     |
|-------|------------|------------|------------|------------|
| 0.1   | $0.981 < \beta < 1$ | $0.965 < \beta < 1$ | $0.312 < \beta < 0.859$ | $0.250 < \beta < 1$ |
| 0.4   | $0.968 < \beta < 1$ | $0.953 < \beta < 1$ | $0.630 < \beta < 0.960$ | $0.554 < \beta < 1$ |
| 0.7   | $0.935 < \beta < 0.970$ | $0.920 < \beta < 0.995$ | $0.748 < \beta < 0.976$ | $0.684 < \beta < 1$ |
| 0.9   | $0.885 < \beta < 0.931$ | $0.871 < \beta < 0.931$ | $0.792 < \beta < 0.982$ | $0.731 < \beta < 1$ |

5. A Discussion on Unifying the Inflation with Dark Energy

Currently, cosmologists have been interested in the models covering a larger domain in the thermal history of the universe. In this way, the models that can describe both very early time and very late time accelerating expansion of the universe have attracted a lot of attention. In this respect, in some viable models of $f(R)$ gravity it is possible to describe both early-time inflation and late-time acceleration of the universe (Nojiri & Odintsov 2007; Cognola et al. 2008; Nojiri & Odintsov 2008, 2011). For instance, Nojiri & Odintsov (2008) have considered a model of $f(R)$ gravity where the universe effectively starts with a large cosmological constant at the early universe (leading to inflation) and, after passing the radiation/matter-dominated era, reaches the small values of the cosmological constant (leading to late-time accelerating expansion). Nojiri & Odintsov (2011) consider another model of $f(R)$ gravity that can explain the initial inflation and at late times can reproduce the behavior of the $\Lambda$CDM model. As we have mentioned in Section 1, the equation of state of the tachyon field can be $-1$, corresponding to late-time dark energy and early-time inflationary phases of the universe. Therefore, in our model also, it seems possible to unify the inflation with dark energy. As we know, the tachyon inflation with constant sound speed is an observationally viable inflation model, at least in some domains of its parameter space. After inflation ends and the tachyon field reaches the nonzero minimum of the potential, the universe becomes radiation dominated. During the radiation- and matter-dominated universe, the minimum value of the potential does not disturb the thermal history of the universe. With more expansion of the universe, the energy densities of the radiation and matter become small, and eventually at late times the potential becomes the dominant component in the energy density of the universe. In fact, this dominant component of the energy density can be considered as an effective cosmological constant, leading to late-time accelerating expansion of the universe. It is also possible to consider some extension of the tachyon field so we can get both inflation and late-time acceleration in the model. For instance, by considering the nonminimal coupling or nonminimal derivative coupling between the tachyon field and the gravity, depending on the
Figure 7. Tensor-to-scalar ratio vs. the scalar spectral index in the power-law tachyon model with constant sound speed.

Figure 8. Running of the scalar spectral index vs. the scalar spectral index in the power-law tachyon model with constant sound speed.

Figure 9. Tensor-to-scalar ratio vs. the tensor spectral index in the power-law tachyon model with constant sound speed.
values of the nonminimal parameter, it may possible to have a model covering both initial inflation and late-time acceleration.

6. Conclusion

In this paper, we have considered tachyon inflation where the sound speed is constant. We have obtained the Friedmann equations and equation of motion in terms of sound speed. After that, we have presented the main perturbation parameters, like the scalar spectral index, its running, tensor spectral index, and tensor-to-scalar ratio. We have also, by considering the three-point correlations, presented the amplitude of the non-Gaussianity in both equilateral and orthogonal configurations. Then, we have adopted two types of scale factor: intermediate and power law. We have studied both cases separately. To study these cases, we have obtained the potential and inflation parameters in terms of the Hubble parameter, its derivative, and also the sound speed. In this way, we were able to study the model based on two types of scale factor. By considering the intermediate scale factor, we have studied $r - n_s$ behavior in the background of Planck2018 TT, TE, and EE+lowE+lensing+BAO+BK14 data at both 68% CL and 95% CL and found some constraints on the model’s parameters. The behavior of $\alpha_s - n_s$ in the background of Planck2018 TT, TE, and EE+lowE+lensing data has been studied too. Another important parameter is the tensor scalar spectral index corresponding to the primordial perturbations. In this way, we have studied $r - n_T$ behavior in the background of Planck2018 TT, TE, and EE+lowE+lensing+BAO+BK14 data to find some constraints on the model’s parameter space. We have also analyzed $c_s$ and $\beta$ phase space in both 68% CL and 95% CL. By studying these parameters numerically, we have found that intermediate tachyon inflation with constant sound speed is observationally viable if $0 < c_s < 0.997$ and $0.833 < \beta < 0.984$ at 68% CL and $0 < c_s < 1$ and $0.787 < \beta < 1$ at 95% CL. To check the viability of the model more precisely, we have studied the non-Gaussian feature of the primordial perturbations. In this regard, we have considered the Planck2018 TTT, EEE, TTE, and EET constraints on the equilateral and orthogonal configurations of the non-Gaussianity. In this way, we have obtained the constraints on the sound speed as $0.276 < c_s < 1$ at 68% CL, $0.213 < c_s < 1$ at 95% CL, and $0.186 < c_s < 1$ at 97% CL. We have also studied $c_s^2 - r$ phase space and found that the ranges of the sound speed, obtained from the observational constraints on the non-Gaussianity, lead to the observationally viable values of the tensor-to-scalar ratio.
The power-law tachyon inflation with constant sound speed is another model that has been studied in this paper. Similar to the intermediate tachyon inflation case, we have studied the perturbation parameters in this model, but here based on the power-law scale factor. In this case, by using different data sets, we have studied $r - n_a$, $r - n_s$, and $r - n_l$ behavior. The phase space of the parameters $c_s$ and $n_s$ based on the observationally viable ranges of the scalar and tensor spectral indices, has been studied too. Our numerical analysis has shown that the power-law tachyon inflation with constant sound speed is observationally viable for $0 < c_s \leq 0.990$ and $299 < n_s < 473$ at 68% CL and $0 < c_s \leq 1$ and $231 < n_s < 551$ at 95% CL. For these obtained ranges, the amplitude of the scalar spectral index is obtained ranges, the amplitude of the scalar spectral index is consistent with observational data. We have also discussed the issue of unifying the inflation with dark energy in the tachyon model with constant sound speed. We have argued that the nonzero minimum value of the tachyon potential becomes important at late times, after the energy density of the radiation/matter decreases. Based on our analysis, it seems that both intermediate and power-law tachyon models in some ranges of their parameter space are consistent with observational data.

I thank the referee for the very insightful comments that have improved the quality of the paper considerably.

ORCID iDs

Narges Rashidi https://orcid.org/0000-0002-7413-1500

References

Ade, P. A. R., Aghanim, N., Arnaud, M., et al. 2016, A&A, 594, A20
Aghanim, N., Akrami, Y., Ashdown, M., et al. 2020, A&A, 641, A6
Akrami, Y., Arroja, F., Ashdown, M., et al. 2020a, A&A, 641, A10
Akrami, Y., Arroja, F., Ashdown, M., et al. 2020b, A&A, 641, A9
Albrecht, A., & Steinhardt, P. 1982, PhRvD, 26, 1220
Babich, D., Creminelli, P., & Zaldarriaga, M. 2004, JCAP, 2004, 009
Bamba, K., Nojiri, S., Odintsov, S. D., & Sáez-Gómez, D. 2014, PhRvD, 90, 124061
Barrow, J. D. 1990, PhLB, 235, 40
Barrow, J. D., & Liddle, A. R. 1993, PhRvD, 47, 5219

Table 2

Ranges of the Parameter $n$ in Which the Tensor-to-scalar Ratio, the Scalar Spectral Index, and the Tensor Spectral Index of the Power-law Tachyon Model with Constant Sound Speed Are Consistent with Different Data Sets

| $c_s$ | Planck2018 TT,TE,EE+lowE lensing+BK14+BAO | Planck2018 TT,TE,EE+lowE lensing+BK14+BAO | Planck2018 TT,TE,EE+lowE lensing+BK14+BAO | Planck2018 TT,TE,EE+lowE lensing+BK14+BAO |
|-------|------------------------------------------|------------------------------------------|------------------------------------------|------------------------------------------|
|       | 68% CL                                   | 95% CL                                   | 68% CL                                   | 95% CL                                   |
| 0.1   | $368 < n < 492$                          | $336 < n < 569$                          | $27.3 < n < 300$                        | $20.1 < n$                               |
| 0.4   | $354 < n < 488$                          | $324 < n < 559$                          | $102 < n < 1600$                       | $76.0 < n$                               |
| 0.7   | $330 < n < 447$                          | $301 < n < 511$                          | $177 < n < 2930$                       | $131 < n$                                |
| 0.9   | $305 < n < 373$                          | $270 < n < 483$                          | $228 < n < 3130$                       | $167 < n$                                |

Barrow, J. D., & Nunes, N. J. 2007, PhRvD, 76, 043501
Baumann, D. 2009, arXiv:0907.5424
Bilic, N., Dimitrijevic, D. D., Djordjevic, G. S., Milošević, M., & Stojanović, M. 2019, JCAP, 2019, 034
BouabdaHousni, Z., Errahmani, A., Bouhmadi-López, M., & Ouati, T. 2016, PhRvD, 94, 123508
Byrnes, C. T. 2014, arXiv:1411.7002
Chen, X., Huang, M.-X., Kachru, S., & Shiu, G. 2007, JCAP, 2007, 002
Cognola, G., Elizalde, E., Nojiri, S., et al. 2008, PhRvD, 77, 046009
De Felice, A., & Tsujikawa, S. 2013, JCAP, 2013, 030
Fergusson, J. R., & Shellard, E. P. S. 2009, PhRvD, 80, 043510
Gibbons, G. W. 2002, PhLB, 537, 1
Guth, A. H. 1981, PhRvD, 23, 347
Hu, W., Seljak, U., White, M., & Zaldarriaga, M. 1998, PhRvD, 57, 3290
Hu, W., & White, M. J. 1997, PhRvD, 56, 596
Kamali, V., Basilakos, S., Mehrabi, A., Motaharfar, M., & Massaelli, E. 2018, JMPD, 27, 1850056
Kamionkowski, M., Kosowsky, A., Stebbins, A., et al. 1997, PhRvD, 55, 7368
Liddle, A., & Lyth, D. 2000, Cosmological Inflation and Large-Scale Structure (Cambridge: Cambridge Univ. Press)
Lidsey, J. E., Liddle, A. R., Kolb, E. W., et al. 1997, RvMP, 69, 373
Linde, A. D. 1982, PhLB, 108, 389
Linde, A. D. 1990, Particle Physics and Inflationary Cosmology (Switzerland: Harwood Academic)
Lyth, D. H., & Liddle, A. R. 2009, The Primordial Density Perturbation (Cambridge: Cambridge Univ. Press)
Maldacena, J. M. 2003, JHEP, 0305, 013
Mohammadi, A., Golanbari, T., Sheikhahmadi, H., et al. 2020, ChinC, 44, 095101
Nojiri, S., & Odintsov, S. D. 2003, PhLB, 571, 1
Nojiri, S., & Odintsov, S. D. 2007, PhLB, 657, 238
Nojiri, S., & Odintsov, S. D. 2008, PhRvD, 77, 026007
Nojiri, S., & Odintsov, S. D. 2011, PhR, 505, 59
Nozari, K., & Rashidi, N. 2013, PhRvD, 88, 023519
Odintsov, S. D., & Oikonomou, V. K. 2015, AnPhy, 363, 503
Rashidi, N., & Nozari, K. 2018, IJMPD, 27, 1850076
Rashidi, N., & Nozari, K. 2020, ApJ, 890, 58
Rezzadaleh, K., Karami, K., & Hashemi, S. 2017, PhRvD, 95, 103506
Riotto, A. 2002, arXiv:hep-ph/0210162
Seljak, U., & Zaldarriaga, M. 1997, PhRl, 78, 2054
Sen, A. 1999, JHEP, 10, 008
Sen, A. 2002a, JHEP, 07, 065
Sen, A. 2002b, MPLA, 17, 1797
Spalinski, M. 2007, JCAP, 2007, 017
Tsujikawa, S., Ohashi, J., Kuroyanagi, S., & De Felice, A. 2013, PhRvD, 88, 023529
Zaldarriaga, M., & Seljak, U. 1997, PhRvD, 55, 1830