Dynamics of soliton self-injection locking in a photonic chip-based microresonator

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Soliton microcombs constitute chip-scale optical frequency combs, and have the potential to impact a myriad of applications from frequency synthesis and telecommunications to astronomy. The requirement on external driving lasers has been significantly relaxed with the demonstration of soliton formation via self-injection locking of the pump laser to the microresonator. Yet to date, the dynamics of this process has not been fully understood. Prior models of self-injection locking were not able to explain sufficiently large detunings, crucial for soliton formation. Here we develop a theoretical model of self-injection locking to a nonlinear microresonator (nonlinear self-injection locking) for the first time and show that self- and cross-phase modulation of the clockwise and counter-clockwise light enables soliton formation. Using an integrated soliton microcomb of directly detectable 30 GHz repetition rate, consisting of a DFB laser self-injection-locked to a Si3N4 microresonator chip, we study the soliton formation dynamics via self-injection locking, as well as the repetition rate evolution, experimentally. We reveal that Kerr nonlinearity in microresonator significantly modifies locking dynamics, making laser emission frequency red detuned. We propose and implement a novel technique for measurements of the nonlinear frequency tuning curve and concurrent observation of microcomb states switching in real time.

I. INTRODUCTION

Recent advances in bridging integrated photonics and optical microresonators1–3 have highlighted the technological potential of soliton-based microresonator frequency combs (“soliton microcombs”)4–7 in a wide domain of applications, which spans coherent communication8,9, ultrafast optical ranging10,11, dual-comb spectroscopy12, astrophysical spectrometer calibration13,14, microwave synthesis15 or chip-scale frequency synthesizers16 and atomic clocks17. Likewise, soliton microcombs also are a testbed for studying the rich nonlinear dynamics, arising from a non-equilibrium driven dissipative nonlinear system, governed by the Lugiato Lefever equation or extensions thereof, that leads to the formation of ‘dissipative structure’. To generate soliton microcombs, commonly, the cavity is pumped with a frequency agile, high power narrow-linewidth, continuous-wave laser with an optical isolator to avoid back reflections. The fast tuning of the laser frequency18 is applied to access the soliton states, which are affected by thermal resonator heating. Locking the laser frequency to the microresonator resonance by an offset Pound–Drever–Hall (PDH) or amplitude locking19 can significantly enhance the soliton stability.

Previously, laser self-injection locking (SIL) to high-Q crystalline microresonators has been used to demonstrate ultranarrow-linewidth lasers20,21, ultralow-noise photonic microwave oscillators21, and soliton microcomb generation22, i.e. soliton self-injection locking. Microresonators provide high level of integration with semiconductor devices, integrated InP-Si3N4 hybrid lasers have rapidly become the point of interest for narrow linewidth on-chip lasers23–25. Moreover, 100 mW multi-frequency Fabry-Perot lasers have recently been employed to demonstrate an electrically-driven microcomb26. Another approach was based on a Si3N4 microresonator butt-coupled to a semiconductor optical amplifier (SOA) with on-chip Vernier filters and heaters for soliton initiation and control27. The integrated soliton microcomb based on the direct pumping of a Si3N4 microresonator by a III-V DFB laser has been reported28–30. Recent demonstrations of integrated packaging of DFB lasers and Si3N4 microresonators with low repetition rates31 made turn-key operation of such devices possible. However, the principles of soliton self-injection locking (SIL) are nontrivial and have never been thoroughly studied. In particular prior theories could not account for the detuning necessary for soliton formation.

Here we demonstrate a hybrid-integrated soliton microcomb device with 30 GHz repetition rate, amenable to direct electronic detection, based on an InGaAsP distributed feedback (DFB) laser self-injection locked to high quality factor ($Q_0 > 10^5$) integrated Si3N4 microresonator. We reveal that solitons can, surprisingly, be initiated using both forward and backward laser current scanning in the SIL state. We have experimentally observed single soliton generation and characterized the phase noise of the soliton repetition, detected by conventional RF electronics32–36.

The locking of a DFB laser benefits from the absence of mode competition effects in the laser medium in comparison to multi-frequency Fabry-Perot laser pumping, thus enabling deterministic soliton generation, as well as a wider range of diode currents for stable self-injection locking37.
We further develop a novel technique to characterize the SIL dynamics experimentally. First, the direct measurement of a laser tuning curve allows us to corroborate with the theoretical model. Secondly, we observe changes in soliton repetition rate and slow switching of soliton states.

We take into account for self- and cross-phase modulation in the microresonator and consider nonlinear self-injection locking (i.e. SIL to a nonlinear microresonator), and develop a theoretical model to describe such dynamics. Principles of soliton generation in self-injection locked devices differ considerably from the conventional soliton generation techniques and provides much richer dynamics.

II. PRINCIPLE OF LASER SELF-INJECTION LOCKED SOLITON MICROCOMB SOURCE

The integrated soliton microcomb device consists of a semiconductor laser diode and a high-Q Si$_3$N$_4$ microresonator chip, as shown in Fig. 2(a). In our experiment, a commercial DFB laser diode of 120 kHz linewidth and 120 mW output power is directly butt-coupled to the Si$_3$N$_4$ chip without using an optical isolator (see Methods). The single-frequency operation of the DFB laser diode allows to control the laser dynamics via Rayleigh backscattering from the Si$_3$N$_4$ microresonator (“laser self-injection locking”), which is fabricated using the photonic Damascene reflow process$^{38,39}$ and features intrinsic quality factor $Q_0$ exceeds $10 \times 10^6$. Such backscattering is commonly undesired$^{40}$, but is critical here for laser self-injection locking whose dynamics is affected by the amplitude and phase of the backscattered light.
Figure 2. **Spectral characterization of self-injection locked soliton microcombs.** (a) Experimental setup, photos of the DFB laser diode and the Si$_3$N$_4$ photonic chip containing 9 microresonators of 30.6 GHz FSR. The RF mixer is utilized to study RF signal above 26 GHz in combination with a local oscillator (LO). TC: temperature controller. CC: current controller. AFG: arbitrary function generator. OSA: optical spectral analyzer. ESA: electrical spectral analyzer. FPD: fast photodetector. (b) The optical spectrum of laser self-injection locked single soliton microcomb. Inset: optical spectrum of the free-running DFB. (c) Comparison of laser phase noise in the free-running and SIL regimes. Inset: beat signal with the reference laser for free-running DFB and the SIL DFB (See Supplementary Information for details). (d) Phase noise of the soliton repetition rate signal in different SIL regimes and of LO. Inset: soliton repetition rate signal.

The schematic of laser self-injection locking to a linear microresonator is shown in Fig. 1(c). The frequency of the free-running DFB diode is determined by its laser cavity (LC) frequency $\omega_{LC}$ and can be tuned by varying the injection current to the diode. When $\omega_{LC}$ is tuned into a high-Q resonance of the Si$_3$N$_4$ microresonator of frequency $\omega_0$, laser self-injection locking can happen. First, we clarify the definition of laser frequency: $\omega_{LC}$ is the laser cavity frequency and defines the laser frequency of the diode in the absence of optical feedback, which exhibits a linear dependence on the injected current. $\omega_{\text{eff}}$ is the actual or effective laser emission frequency when the optical feedback from the microresonator affects the laser dynamics, i.e. self-injection locking is present. Increasing the injection current increases the normalized laser cavity to microresonator detuning $\xi = 2(\omega_0 - \omega_{LC})/\kappa$, where $\kappa$ is the loaded linewidth of the resonance. Note that $\xi$ linearly depends on the injection current $I_{\text{inj}}$. In the absence of SIL, $\omega_{\text{eff}} = \omega_{LC}$ and the actual effective detuning $\zeta = 2(\omega_0 - \omega_{\text{eff}})/\kappa$ linearly depends on the injection current $I_{\text{inj}}$. Note that, $\zeta$ is the actual detuning parameters that determines the soliton formation dynamics. Here we define the “tuning curve” as the dependence of $\omega_{\text{eff}}$ on the injection current $I_{\text{inj}}$, or equivalently, $\zeta$ dependence.
When the laser frequency $\omega_{LC}$ is far detuned from the resonance $\omega_0$, the tuning curve first follows line $\zeta = \xi$ (Fig. 1(c)) when the injection current $I_{\text{inj}}$ increases. When $\omega_{LC}$ is tuned into $\omega_0$, i.e. $\xi \to 0$, the laser frequency becomes locked to the resonance due to Rayleigh-backscattering-induced self-injection locking, such that $\zeta \approx 0$. The self-injection coefficient $K$ and the locking range $\Delta \omega_{\text{lock}}$ are determined by the amplitude and phase of the backscattered light.\(^{42}\) When $\xi$ increases further and finally moves out of the locking range, the laser becomes free-running again, such that $\zeta = \xi$. The tuning curve in the nonlinear case, when Kerr nonlinearity is present, differs drastically (see red line in Fig. 1(d)), as we illustrate below. We studied a set of parameters and found that the effective detuning $\zeta$ predominantly locks to the red detuned region, where soliton microcomb formation is possible (Fig. 1(e) shows the union of SIL regions for different locking phases).

The experimental setup is shown in Fig. 2(a) (see “Experimental setup” in Methods). Figure 2(c) shows the laser linewidth reduction at 50 times due to SIL (see “Stabilization of laser diode” in Methods). Figure 2(b) shows the single soliton spectrum with 30.6 GHz FSR (see “Soliton generation in the SIL regime” in Methods). The beatnote of the soliton repetition rate and its phase noise, measured with 100 Hz resolution bandwidth, are shown in Fig. 2(d).

### III. NONLINEAR SOLITON SELF-INJECTION LOCKING MODEL

Previous works\(^{41,43}\) have shown that soliton generation occurs in a certain range of normalized pump detunings $\zeta$, with the lower boundary being the bistability criterion and the upper boundary being the soliton existence criterion:

$$\zeta \in \left[ \left( \frac{f}{2} \right)^{2/3} + \sqrt{4 \left( \frac{f}{2} \right)^{4/3} - \frac{\pi^2 f^2}{8}}, \right]$$  \hspace{1cm} (1)

where $f = \sqrt{8 \omega_0 c n_2 \eta P_{\text{in}} / (\kappa^2 n^2 V_{\text{eff}})}$ is the normalized pump amplitude, $\omega_0$ is the resonance frequency, $c$ is the speed of light, $n_2$ is the microresonator nonlinear index, $P_{\text{in}}$ is the input pump power, $n$ is the refractive index of the microresonator mode, $V_{\text{eff}}$ is the effective mode volume, $\kappa$ is the loaded resonance linewidth and $\eta$ is the coupling efficiency ($\eta = 1/2$ for critical coupling).

The linear SIL model\(^{42}\) predicts that, in the SIL regime, the attainable detuning range of $\zeta \in [-0.7:0.7]$ in locked state does not overlap with the soliton existence range despite sufficient pump power. However, this prediction is contradictory to our experimental observation of single soliton formation, which is achieved by decreasing the diode injection current. We attribute this contradiction to the absence of the microresonator Kerr nonlinearity in the linear SIL model. The modified nonlinear SIL model including the Kerr nonlinearity\(^{44}\) and is presented as following.

Consider the microresonator coupled mode equations\(^{41}\) with backscattering\(^{15,46}\) for forward and backward travelling mode amplitudes $a_\mu$ and $b_\mu$, which is analogous to the linear SIL model\(^{42}\) with added Kerr nonlinearity:

$$\dot{a}_\mu = -(1 + i \zeta_\mu)a_\mu + i \Gamma b_\mu + i \sum_{\nu = \pm} \eta_\nu a_\nu a_\nu^* + 2i \alpha_x a_\mu \sum_{\nu = \pm} |b_\nu|^2 + f \delta_\mu 0,$$  \hspace{1cm} (2)

$$\dot{b}_\mu = -(1 + i \zeta_\mu)b_\mu + i \Gamma a_\mu + i \sum_{\nu = \pm} \eta_\nu b_\nu b_\nu^* + 2i \alpha_x b_\mu \sum_{\nu = \pm} |a_\nu|^2,$$  \hspace{1cm} (3)

where $\Gamma$ is the normalized coupling rate between forward and backward modes, $\alpha_x$ is a coefficient derived from mode overlap integrals and $\zeta_\mu = 2(\omega_\mu - \mu D_1 - \omega_{\text{eff}})/\kappa$ is the normalized detuning between the laser emission frequency $\omega_{\text{eff}}$ and the $\mu$-th cold microresonator resonance $\omega_\mu$, with $\mu = 0$ being the pumped mode and $D_1/2\pi$ is the microresonator FSR. In numerical estimations we use $\alpha_x = 1$ for modes with the same polarization. Equation 2 provides a nonlinear resonance curve and the soliton solution\(^{41,45,46}\). For SIL, the pumped mode $\mu = 0$ is of main interest, and we study the stationary case:

$$- (1 + i \zeta) a + i \Gamma b + ia(|a|^2 + 2 \alpha_x |b|^2) + i f = 0,$$  \hspace{1cm} (4)

$$- (1 + i \zeta) b + i \Gamma a + ib(|b|^2 + 2 \alpha_x |a|^2) = 0,$$  \hspace{1cm} (5)

where we define $a = a_0$, $b = b_0$ and $\zeta = \zeta_0$ for simplicity. This is often referred as the "Continuous Wave" (CW) solution, because Eq. 3 describe a constant amplitude in the entire microresonator. To solve Eq. 3 and make resemblance to the linear case, we introduce the nonlinear detuning shift $\delta_{n1}$ and nonlinear coupling shift $\delta_{\Gamma1}$:

$$\delta_{\zeta n1} = \frac{2 \alpha_x}{\Gamma} \left( |a|^2 + |b|^2 \right),$$  \hspace{1cm} (6)

$$\delta_{\Gamma n1} = \frac{2 \alpha_x}{\Gamma} \left( |a|^2 - |b|^2 \right).$$  \hspace{1cm} (7)

We further transform $\zeta = \zeta - \delta_{n1}$, $\Gamma^2 = \delta_{\Gamma n1}^2 + \Gamma^2$, in order to achieve Eq. (3) in the same form as in the linear SIL model\(^{42}\). After redefinition $\xi = \zeta - \delta_{n1}$, the nonlinear tuning curve in the new coordinates $\xi - \zeta$ becomes:

$$\xi = \zeta + \frac{K}{2} \frac{2 \zeta \cos \psi + (1 + \Gamma^2 - \zeta^2)^2 \sin \psi}{(1 + \Gamma^2 - \zeta^2)^2 + 4 \zeta^2},$$  \hspace{1cm} (8)

where $K = 8 \eta \kappa \kappa_{do} \sqrt{1 + \alpha_x^2 / (\kappa R_s)}$ is the self-injection coefficient and $\psi = \psi_0 - \kappa \zeta / 2$ is the self-injection phase, $\kappa_{do}$ is the laser diode output mirror coupling.
Figure 3. **Theoretical model of nonlinear self-injection locking.** Model parameters for (a)-(f): the normalized pump amplitude \( f = 1.6 \), the normalized mode-coupling parameter \( \Gamma = 0.11 \), the self-injection coefficient \( K = 44 \), the locking phase \( \psi_0 = 0.1 \pi \). (a, b) Model for the conventional case where the microresonator is pumped by the laser with an optical isolator, and \( \zeta = \xi \) (dashed dark green line). (c-f) Model for the self-injection locked regime. The solution of Eq. (6)-(8) (dashed orange curve) is compared with the linear tuning curve \( \zeta = \xi \) (black dashed line). While tuning the laser, the actual effective detuning \( \zeta \) and the intracavity power \( |a(\xi)|^2 \) will follow the solid orange or blue line with several jumps due to multistability of the tuning curve. The nonlinear resonance curve (purple in (a)) is deformed when translated from \( \zeta \) frame to the detuning \( \xi \) frame (d,f) with the tuning curve \( \zeta(\xi) \) (c,e). Note that the upper and lower soliton existence boundaries switched places. The locking range is larger for forward scan, but the backward scan can provide larger detuning \( \zeta \), which is crucial for soliton generation.

The rate, \( \alpha_g \) is the Henry factor\(^{47}\) and \( R_o \) is the amplitude reflection coefficient. The \( \kappa \tau_o/2 \) is usually considered to be small, i.e. \( \kappa \tau_o/2 \ll 1 \), so the locking phase \( \psi \approx \psi_0 = \omega_0 \tau_s - \arctan \alpha_g + 3 \pi/2 \) depends on both the resonance frequency \( \omega_0 \) and the round-trip time \( \tau_s \) from the laser output facet to the microresonator and back. The nonlinear detuning and coupling can be expressed as:

\[
\delta \zeta_{nl} = \frac{2 \alpha_x + 1}{2} f^2 \frac{1 + (\zeta - \delta \Gamma_{nl})^2 + \Gamma^2}{(1 + \Gamma^2 - \zeta^2)^2 + 4 \zeta^2},
\]

\[
\delta \Gamma_{nl} = \frac{2 \alpha_x - 1}{2} f^2 \frac{1 + (\zeta - \delta \Gamma_{nl})^2 - \Gamma^2}{(1 + \Gamma^2 - \zeta^2)^2 + 4 \zeta^2}.
\]

Equations (6)-(8) can be solved numerically and plotted in \( \zeta = \bar{\zeta} + \delta \zeta_{nl} \), \( \xi = \bar{\zeta} + \delta \zeta_{nl} \) coordinates\(^{44}\). We can see from Eq. (7) that the nonlinear detuning shift is positive, and allows for larger detuning \( \zeta \) (proportional to the pump power) compared to the linear case, advantageous for soliton generation. Consequently, SIL laser always locks to the red-detuned region of the microresonator resonance. Note that the cross-modulation term is important here as for \( \alpha_x < 1/2 \) (such as for cross-polarization case, when \( \alpha_x = 1/3 \)) the locking plateau do not form. Also, our nonlinear SIL model is valid for both anomalous group velocity dispersion (GVD) and normal GVD.

The principle of laser SIL to the Kerr nonlinear microresonator, as derived above, is explained as follow-
ing. Figure 3(a, b) shows the conventional case where the laser pumps the microresonator with an optical isolator. In Fig. 3(a), the solid black line corresponds to the linear tuning curve ($\zeta = \xi$) of a free-running laser. In Fig. 3(b), the thick solid black curve corresponds to the solution of Eq. (2) in the $\zeta$ frame, which provides soliton solutions\textsuperscript{45,46}. Horizontal dashed green lines are the boundaries of soliton existence range in the $\zeta$ frame, i.e. Eq. (1), highlighted also with the green area.

Next, we take into account Rayleigh backscattering. The nonlinear tuning curve, i.e. Eq. (6), is plotted in Fig. 3(c, e) in the $\zeta$-$\xi$ frame with dashed orange lines. Due to multi-stability of the curve, forward and backward laser tuning with the same diode current range results in different experimental effective detunings $\zeta$ and tuning curves (thick solid lines). Figure 3(c, e) shows the attainable values of detunings $\zeta$, and Fig. 3(d, f) shows the intracavity power $|\alpha(\zeta)|^2$ for forward and backward tuning (thick dashed). The key conclusions are the following. First, obtaining larger detuning $\zeta$ using backward tuning is possible (cf. Fig. 3(c, e)). At the same time, the locking range can be shorter in the backward tuning than that in the forward tuning. Second, while decreasing the diode current (i.e. backward tuning, the free-running laser frequency rising), the detuning $\zeta$ can grow (Fig. 3(e)) which is counter-intuitive. More study on the dependence of nonlinear tuning curves on locking phase $\psi_0$, pump power, and the mode-coupling parameter $\Gamma$ is found in Supplementary Information.

IV. EXPERIMENTAL CHARACTERIZATION OF NONLINEAR TUNING CURVES

To corroborate our findings from the theoretical model of nonlinear SIL, we develop a novel technique to experimentally investigate the soliton dynamics via laser self-injection locking, based on a spectrogram measurement of the beatnote signal between the SIL laser and a reference laser (IDPhotronics CoBrite). This approach allows experimental characterization of the nonlinear tuning curve $\zeta(\xi)$, which can be compared to the theoretical model.

The microresonator transmission trace is measured by applying 30 Hz triangle diode current modulation, such that the laser scans over a nonlinear microresonator resonance. As shown in Fig. 4(a), the resonance shape is prominently different from the typical triangle shape with soliton steps using the conventional method, which uses a laser with an isolator, and tunes the laser from the blue-detuning to the red-detuning of the resonance (i.e. forward tuning)\textsuperscript{41}. However, in the case of nonlinear SIL, soliton states can be accessed in both tuning directions, as illustrated below.

The reference laser’s frequency is set higher than the free-running DFB laser frequency, such that the heterodyne beatnote signal is observed near 15 GHz. The laser diode current is swept at 10 mHz rate in the range of 372 to 392 mA, such that the laser scans across the resonance in both directions. The spectrogram data in the range of 0 to 25 GHz is collected by an electric signal analyzer (ESA R&S FSW26), and the soliton beatnote signal is measured at 30.696 GHz (Fig. 4(c)). Based on the spectrogram data shown in Fig. 4(b), the nonlinear tuning curve $\zeta(\xi)$ is extracted, as shown in Fig. 4(d, e), given the fact that the $\zeta = \xi$ linear slope can be identified from the DFB free-running regime. These experimentally measured nonlinear tuning curves are fitted using Eq. (6)-(8). The fitted optical phase is $\psi_0 = -0.27\pi$, and the fitted normalized pump is $f = 13.1$. In our experiments DFB diode provides $7 < f < 15$.

Initially, the DFB laser is free-running (Frame I). The diode current is decreasing, such that $\xi$ is decreasing, and the laser is tuned into a nonlinear microresonator resonance (Frame II). Further decreasing the diode current locks the $\zeta$ to the microresonator in the soliton existence range (Frame III). In this case, two effects can be observed: The appearance of the 20 GHz beat signal (Frame IV) between the reference laser and the first comb line, and the signal of the 30.6 GHz soliton repetition rate, as shown in Fig. 4(c) (Frame X). Further reducing the diode current can lead to the switching of (multi-)soliton states. Then, the diode current scan changes direction (Frame V). In our experiment, the DFB laser remains locked when the current scan direction reverses from backward to forward. In Fig. 4(b), region V which represents the multi-frequency regime of the DFB (see “Linewidth of SIL DFB” in Supplementary Information), is truncated for better visual representation. The spectrogram regions IV and VII are also enhanced for this purpose.

The DFB laser is then switched to a single frequency locked state again (Frame VI), and generates a chaotic comb (Frame XI). As the effective detuning $\zeta$ changes with increasing $\xi$, switching between different soliton states is observed, as well as the appearance of breather soliton states (Frame VIII). The soliton repetition rate reduces with increasing $\zeta$, and vice versa (Frames X and XII). Further diode current tuning causes the DFB laser to jump out of the SIL regime. Such a transition, shown on the theoretical curves in Fig. 4(d, e), agrees with that observed in Fig. 4(b) (Frame II). Finally, the laser frequency returns to the free-running regime, and thus $\zeta = \xi$ again (Frame IX).

V. CONCLUSION AND DISCUSSION

In summary, We have demonstrated single-soliton generation enabled by a DFB laser self-injection locked to an integrated Si$_3$N$_4$ microresonator of 30 GHz FSR. The soliton formation dynamics is controlled by tuning the injection current of the laser diode. We have developed a theoretical model to describe self-injection locking to a nonlinear microresonator (‘nonlinear SIL’), which agrees with the experimental observations. We further develop
Figure 4. Soliton formation dynamics with nonlinear laser self-injection locking. (a) Microresonator transmission trace in backward and forward scan, at 30 Hz scan rate. (b) Spectrogram of the beat signal between the DFB and the reference laser. (c) Evolution of the soliton repetition rate around 30.6 GHz. (d,e) Measured nonlinear $\zeta(\xi)$ tuning curve and the theoretical fit. The model parameters: normalized pump amplitude $f = 13.1$, the locking phase $\psi_0 = -0.27\pi$ and the locking coefficient $K = 1464$. 

Eff. detuning $z$ (in $k/2$ units)

Transmit. power

Current sweep

Spectrogram R&S FSW26

Time (10 ms/div.)

Microwave (GHz)

RF frequency (GHz)

Noisy comb

RF frequency (GHz)

Repr. rate 30.69635 GHz (kHz)

Laser cavity-microresonator detuning $\xi$ (in $k/2$ units)

Laser cavity-microresonator detuning $\xi$ (in $k/2$ units)
a novel spectrogram technique that allows to measure accessible soliton detuning range, and to observe features of nonlinear self-injection locking. The main conclusion is that the effective emission frequency of the self-injection locked laser is red-detuned relative to the microresonator resonance and is located inside the soliton existence region for most combinations of parameters. The dependence of emission frequency on the injection current is complicated and leads to non-trivial dynamics.

Some deviations from the predicted behavior can be attributed to the nonlinear generation of sidebands not included in our theory, which depletes the power from pumped mode and changes the nonlinear detuning shift \( \frac{d\zeta}{d\xi} = 0 \). Where noise characteristics of the stabilized laser may be significantly improved.

Our results provide insight into the soliton formation dynamics via laser self-injection locking, which has received wide interest recently from the fields of integrated photonics, microresonators, and frequency metrology. Our findings, in combination with recent demonstrations of industrial packaging of DFB lasers and Si\(_3\)N\(_4\) waveguides, pave the way for highly compact microcomb devices operated at microwave repetition rates, but on commercially available CMOS-compatible components and amenable to integration. This device is a promising candidate for high-volume applications in data-centers, as scientific instrumentation, and even as wearable technology in healthcare.

**Methods**

**Silicon nitride chip information.** The Si\(_3\)N\(_4\) integrated microresonator chips were fabricated using the photonic Damascene process\(^{38,48}\). The pumped microresonator resonance is measured and fitted including backscattering\(^{49}\) to obtain the intrinsic loss \( N_0/2\pi = 20.7 \) MHz, the external coupling rate \( \kappa_\text{ex}/2\pi = 48.6 \) MHz, and the backward-coupling rate \( \gamma/2\pi = 11.8 \) MHz. These correspond to the full resonance linewidth \( \kappa/2\pi = \kappa_0/2\pi + \kappa_\text{ex}/2\pi = 69.3 \) MHz, the pump coupling efficiency \( \eta = \kappa_\text{ex}/\kappa = 0.70 \), the normalized mode-coupling parameter \( \Gamma = \gamma/\kappa = 0.17 \), and the amplitude resonant reflection coefficient from the passive microresonator \( r \approx 2\eta/(1 + \Gamma^2) = 0.23 \).

**Butt-coupling:** We do not use any optical wire bonding techniques in our work. The DFB laser diode and the Si\(_3\)N\(_4\) chip are directly butt-coupled and are mounted on precise optomechanical stages. The distance between the diode and the chip can be varied with an accuracy better than \( 100 \) nm, thus enabling the control of the accumulated optical phase from the Si\(_3\)N\(_4\) microresonator to the diode (i.e. the locking phase). The output light from the Si\(_3\)N\(_4\) chip is collected using a lensed fiber. The total insertion loss, i.e. the output power of the free-running laser diode divided by the collected power in the lensed fiber, is \( 5 \) dB. Note that matching polarization of DFB laser radiation and polarization of microcavity high-Q modes is critically important to achieve maximum pump power either via rotation of laser diode or by ‘polarization engineering’ of high-Q modes.

**Experimental setup:** The DFB laser diode’ temperature and injection current are controlled by an SRS LD501 controller and an external function generator (Tektronix AFG3102C). The output optical signal of the soliton microcomb is divided by optical fiber couplers and sent to an optical spectrum analyzer (Yokogawa AQ6370D), a fast photodetector (NewFocus 1014), an oscilloscope, and an electrical signal analyzer (Rhode&Schwarz FSW26). The heterodyne beatnote measurement of various comb lines is implemented with a narrow-linewidth reference laser (ID Photonics DX-2). A passive double balanced MMIC radio-frequency (RF) mixer (Marki MM1-1140H) is utilized to down-convert and study the RF signal above \( 26 \) GHz in combination with a local oscillator (Keysight N5183B).

**Comb generation in SIL regime:** Besides meeting the soliton power budget, a key requirement for soliton generation in the SIL regime is to reach sufficient detuning \( \zeta = 2(\omega_0 - \omega_\text{eff})/\kappa \). When the laser is tuned into resonance from the red-detuned side, SIL can occur so that the laser frequency \( \omega_0 \) becomes different from \( \omega_\text{eff} \) and is close to \( \omega_0 \). In the conventional case where soliton initiation and switching are achieved using frequency tunable lasers with isolators, the soliton switching, e.g. from a multi-soliton state to a single-soliton state, can lead to the intracavity power drop which causes the resonance frequency \( \omega_0 \) shift due to the thermal effect, and ultimately the annihilation of solitons. However, in the case of laser SIL, the optical feedback via Rayleigh backscattering is much faster (instantaneous) than the thermal relaxation time (at millisecond order), therefore the laser frequency can follow the resonance shift instantaneously such that the soliton state is maintained. The slope of the tuning curve \( d\zeta/d\xi \) in the SIL regime allows control the effective detuning \( \zeta \) by varying \( \xi \), realized by increasing the laser injection current for forward tuning\(^{41}\), or decreasing the current for backward tuning\(^{50}\). However, note that the entire soliton existence range may not be fully accessible at certain locking phases \( \psi_0 \) and locking coefficient \( K \). Therefore, a single soliton state with a large detuning and broad bandwidth may not be attained in the SIL regime. In our experiments, a variety of optical spectra of microcombs generated in the Si\(_3\)N\(_4\) microresonator is observed (see Supplementary Information), as well as the subsequent switching from chaotic combs to breather solitons and multi-solitons in forward and backward scan.

**Funding Information:** This work was supported by Contract HR0011-15-C-0055 (DODOS) from the Defense Advanced Research Projects Agency (DARPA), Microsystems Technology Office (MTO), and by the Air Force Office of Scientific Research, Air Force Material Command, USAF under Award No. FA9550-15-1-0099, and by Swiss National Science Foundation under grant agreement No. 176563 (BRIDGE). N.M.K. acknowledges the support of the Russian Science Foundation (grant 19-72-00173).

**Acknowledgments:** The authors thank Miles H. Anderson, Joseph Briggs and Vitaly V. Vasiliev for the fruitful discussion. The Si\(_3\)N\(_4\) microresonator samples were fabricated in the EPFL center of MicroNanoTechnology (CMI).

**Author contribution:** A.S.V., G.V.L. conducted the experiment. N.M.K. developed a theoretical model and performed numerical simulations. J.L. designed and fabricated the Si\(_3\)N\(_4\) chip devices. All authors analysed the data and prepared the manuscript. I.A.B. and T.J.K initiated the collaboration and supervised the project.

**Data Availability Statement:** The code and data used to produce the plots within this work will be released on the repository Zenodo upon publication of this preprint.

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LINEWIDTH OF SIL DFB

The employed DFB laser diode has the wavelength of 1547 nm, the maximum optical output power of \( \sim 120 \) mW at the injection current of \( \sim 472 \) mA and the temperature of 20 °C. Such high-power DFB laser diodes with similar characteristics have been reported previously\(^1\)–\(^4\), and are commercially available. The spacing between suppressed modes is 24.5 GHz, defined by the cavity length of the laser diode. Heterodyne measurements determine the instantaneous linewidth in the free-running regime with a reference laser (IDPhotronics CoBrite, instantaneous linewidth \( \sim 20 \) kHz), and is estimated as 120 kHz using a Voigt profile. The laser diode was installed in a custom-made mount, providing good heat sinking, but it may also be placed on a submount in a conventional butterfly package.

Compared with the 120 kHz Lorentzian linewidth of the free-running DFB diode, the fitted Lorentzian linewidth (using a Voigt profile) of the SIL DFB is reduced down to 3.8 kHz, as shown in Fig. 2(c) inset in the main text. Using the parameters such as the linewidth of the free-running DFB laser diode \( \delta \omega_{\text{free}}/2\pi = 120 \) kHz, the quality factor of the DFB laser cavity \( Q_{LC} = 10^4 \), and the linewidth enhancement Henry factor \( \alpha_g = 2.5 \)\(^1\),\(^3\), the theoretical linewidth of the SIL DFB is calculated as \( \delta \omega_{\text{locked}}/2\pi = 0.23 \) Hz. Note that, this value \( \delta \omega_{\text{locked}}/2\pi \) does not take into account other noises, and is thus much smaller than the measured value of 3.8 kHz. We attribute the linewidth broadening to the resonance frequency fluctuations, estimated to be in the Hz-level\(^6\),\(^7\), and mechanical instability of our setup, estimated to be in the kHz-level\(^8\)). The experimental SIL DFB linewidth was also estimated in special multi-frequency regime of operation.

We were not able to estimate the linewidth of self-injection locked (SIL) laser diode by the heterodyne measurement because the measured Lorentzian profile width 20 kHz equals to the Ref. laser linewidth. Therefore, the instantaneous linewidth of the SIL DFB is below 20 kHz. The phase noise of the beat between the self-injection locked DFB, and the reference laser is equal to the phase noise of the reference laser. The phase noise of the SIL DFB at offset frequency \( > 10 \) kHz is not worse than of the reference laser, as the SIL DFB linewidth is narrower than 20 kHz.

The self-injection locking range defined as the maximum difference between the laser frequency and the microresonator resonance frequency when laser SIL happens, can be estimated\(^8\) as \( \Delta \omega_{\text{lock}}/2\pi \approx r \sqrt{1 + \alpha_g^2 \omega_0/Q_{LC}}/2\pi = 12.4 \) GHz, and the experimentally measured value is more than 7 GHz. Note that increasing the locking range \( \Delta \omega_{\text{lock}}/2\pi \), e.g. by increasing the microresonator \( Q \), may lead to the overlap of the locking ranges of two neighboring resonances. The locking range overlap can lead to mode competition and an unstable SIL regime.

The mutual coherence of the SIL DFB was estimated in a special multi-frequency regime of lasing\(^9\). When the back-scattered laser radiation from the microresonator is strong enough, it may lead to a spatial hole burning (SHB) effect. The spatial structure created in the active media of the semiconductor diode may play a role of spectral selective element, so-called, self-induced grating. Therefore, the spectral selectivity of the DFB laser diode is determined not only by the DFB structure but also by the SHB structure. The influence of SHB may be stronger than the DFB structure, and the latter ceases to be a major spectral selecting element. In this case, the DFB laser diode may operate in multi-frequency regime (Fig. 1). In other words, the side mode suppression ratio of the DFB laser diode structure is significantly decreased. Such a multi-frequency regime was observed in our experiment. It should be noted that the spectral profile strongly depends on the optical feedback level.

Figure 1. Multifrequency regime of DFB diode operation with different feedback levels (regime I and regime II).
Figure 2. Different regimes of laser self-injection locking to nonlinear cavity. (a)-(b): Dependence on backscattering $\Gamma$. Higher mode-coupling parameter $\Gamma$ provides wider locking range and twists the resonance curve. (c)-(d): Increasing of pump power $f$ leads to shifting of the tuning curve to the red detuned region, necessary for soliton formation, but reduces the locking range. (e)-(f): Influence of the optical locking phase $\psi_0$ on the tuning curve. Changing the phase by $\pi$ effectively mirrors the curve over $\xi = \zeta$ in $\xi$ coordinate.
We measured SIL DFB optical line’s mutual stability in such a multi-frequency regime. The estimated beatnote Lorentzian width is 3.8 kHz.

The phase noise of the free-running DFB is 
\(-20 \text{ dBc/Hz} \) at 10 kHz offset frequency and 
\(-75 \text{ dBc/Hz} \) at 1 MHz offset frequency.

Phase noise for the free running DFB, SIL DFB and multi-frequency DFB coincide and equals 6.7 dBc/Hz at 810 kHz. At higher offsets (1 kHz - 2 MHz) the phase noise of SIL DFB is decreased by 15 dB and at offsets higher 2 MHz it is decreased more than by 20 dB which corresponds to the narrowing of the linewidth by 30 times (15 dB) from \(\delta \omega_{\text{free}}/2\pi = 120 \text{ kHz} \) to \(\delta \omega_{\text{free}}/2\pi = 3.8 \text{ kHz} \).

**DIFFERENT REGIMES OF LASER SELF-INJECTION LOCKING TO NONLINEAR CAVITY.**

We study how the nonlinear self-injection locking depends on significant parameters: locking phase (phase delay between the laser cavity output and the feedback), laser diode power, and the normalized mode-coupling parameter \(\Gamma \). The nonlinear tuning curve \(\zeta(\xi)\) and the intracavity power \(|a(\xi)|^2\) are presented in Fig. 2 left and right columns correspondingly. The first row in Fig. 2 shows how the tuning curve changes with increasing of the normalized mode-coupling parameter \(\Gamma\) from 0 to 1. First, \(\Gamma \approx 0\), and there is no optical feedback and no self-injection locking. The tuning curve is linear (\(\zeta = \xi\)), and the resonance curve is Lorentzian. Then while \(\Gamma\) increases, the locking range increases too. Laser tuning curves are asymmetrical because the pump power \(f = 1.5\) is high enough for nonlinearity manifestation.

The second row shows the dependence on the normalized pump \(f\). First, \(f\) is small, and we observe linear self-injection locking. The tuning curve becomes asymmetric with higher \(f\) and the horizontal part of it shifts to higher detunings \(\zeta\), making soliton generation possible. Note that the intracavity power \(|a(\xi)|^2/f^2\) looks very different from conventional nonlinear resonance in the presence of the optical feedback and both curves become asymmetric already at \(f = 1\). None that for high input power the intracavity power is similar to nonlinear resonance with backward wave\(^{10}\).

The third row shows the dependence on the locking phase \(\psi_0\). The tuning curve drastically changes with this parameter, the most sensitive in our experimental setup. It depends on the distance between the laser cavity and the microresonator, its change for 1.5 \(\mu\text{m}\) corresponds to change of phase at \(2\pi\).

**OPTIMAL LOCKING PHASE AND ITS CONTROL**

We observed the dependence of the microresonator transmission on the optical alignment (the distance between the laser facet and the chip with microresonator) experimentally and thus the locking phase \(\psi_0\) (see Fig. 2). The transmission trace changes drastically with different locking phases and is very different in forward and backward scans.

Note that the trace “Optical phase V” in Fig. 3 does not contain the high-Q resonance. There are some resonances with low Q but with strong back-reflection. Such resonances appear due to a Fabry-Perot cavity formed by Si\(_3\)N\(_4\) photonic chip edges. Such edges are comfortable for butt-coupling, but they cause a strong influence on the laser diode. In some cases, the high-Q resonances are hindered because the laser diode is locked to the Fabry-Perot resonances. This undesired back-reflection might be improved by introducing angled waveguide tapers.

**DIFFERENT MICROCOMB STATES IN LASER SELF-INJECTION LOCKED REGIME.**

We scanned the laser diode frequency with injection current over the nonlinear resonance and observed formation, first, of a modulation instability (characteristic primary combs) in (Fig. 4(a)), then a chaotic comb (Fig. 4(b)) with a flat top symmetric envelope\(^{11}\). Tuning further we observe soliton crystals (Fig. 4(c)) and multi-soliton state (Fig. 4(d)). Other observed comb states are shown in Fig. 4(e)-(h).
Figure 4. Different microcomb states in laser self-injection locked regime.

PHASE NOISE CHARACTERIZATION

Previously, the phase noise of soliton repetition rates has been characterized only in bulk crystalline microresonators\textsuperscript{12,13} and silica microdisks\textsuperscript{14,15}, because the repetition rates are electronically detectable. This has not been possible previously in any integrated platform, including Si\textsubscript{3}N\textsubscript{4} as integrated solitons of microwave repetition rates have not been demonstrated\textsuperscript{16–18} due to the limited microresonator Q factors. Recent advancement in the fabrication of ultralow-loss Si\textsubscript{3}N\textsubscript{4} waveguides\textsuperscript{19} has allowed the generation of single solitons with repetition rates in the widely employed microwave K- and X-band\textsuperscript{20}. Equally important, the phase noise of integrated soliton microcombs has been characterized and compared to the fundamental limit imposed by Si\textsubscript{3}N\textsubscript{4} thermo-refractive noise\textsuperscript{7}.

First, we performed a measurement of the phase noise of laser diode in different regimes using heterodyne technique and IQ-data measurements. In RF measurements, heterodyne signal represented by a sinusoid with angle modulation can be decomposed into, or synthesized from, two amplitude-modulated sinusoids that are offset in phase by one-quarter cycle ($\pi/2$ radians): $A \cos(2 \pi f t + \phi) = I \cos(2 \pi f t) - Q \sin(2 \pi f t)$. All three functions have the same center frequency. The amplitude modulated sinusoids are known as the in-phase (I) and quadrature (Q) components. This approach is very convenient for RF signal processing. Frequency and phase noises may be extracted from IQ-data as periodogram power spectral density or be found using Welch’s overlapped segment averaging estimator. Due to the fact that the 30 GHz carrier frequency was out of the detection bandwidth of our ESA (26 GHz), the signal was down-converted with an RF mixer and a local oscillator (Keysight N5183B) $f_{\text{LO}} = 20$ GHz, in order to facilitate the detection.

The free running DFB laser diode has phase noise of 6.7 dBc/Hz at 810 kHz and $-93$ dBc/Hz at 10 MHz. Some features at offsets 99 kHz with $-37$ dBc/Hz and 1.7 MHz with $-70$ dBc/Hz are clearly observed. The phase noise in the locked state could not have been measured because they are hindered by the phase noise of reference laser. The phase noise in the locked state were measured in multi-frequency DFB regime, providing $-53$ dBc/Hz at 99 MHz, $-84$ dBc/Hz at 1.7 MHz and $-115$ dBc/Hz at 10 MHz. Following feature should be noted: the self-injection locking does not lead to suppressing of phase noise at offsets below 1 kHz. Phase noise for the free running DFB, SIL DFB and multi-frequency DFB coincide and equals 6.7 dBc/Hz at 810 kHz. At higher offsets (1 kHz - 2 MHz) the phase noise of DFB is decreased by 15 dB and at offsets higher 2 MHz it is decreased more than by 20 dB which corresponds to the narrowing of the linewidth by 30 times (15 dB) from $\delta \omega_{\text{free}}/2\pi = 119$ kHz to $\delta \omega_{\text{free}}/2\pi = 3.8$ kHz.

We measured the phase fluctuations of the 30 GHz soliton repetition rate in the SIL regime. The measured phase noise of the single soliton repetition rate, shown as the red curve in Fig. 2(d) of the main text, has single-sideband phase noise of $-65$ dBc/Hz at 1 kHz Fourier offset frequency, and $-88$ dBc/Hz at 10 kHz.

For the red curve, the influence of the laser diode controller is clearly observed, one can notice the harmonics of 50 Hz caused by the power supply. The detector shot noise for the studied signal was $-144$ dBc/Hz. Thus, SIL soliton microcomb provides competitive phase noise characteristics at 1 kHz - 10 kHz offsets in comparison with breadboard implementation. The ability to switch
between soliton states and different phase noise for different soliton states indicates that, in our experiment, laser the phase noise characteristic might be further improved.

Different soliton states demonstrate different phase fluctuations, multi-soliton state (purple curve) provide the same phase noise $-65 \, \text{dBc/Hz}$ at $1 \, \text{kHz}$, but much higher phase fluctuations $-77 \, \text{dBc/Hz}$ at $10 \, \text{kHz}$ and at higher offsets. Moreover narrow peak at $32 \, \text{kHz}$, $64 \, \text{kHz}$, $226 \, \text{kHz}$ and $250 \, \text{kHz}$ have additional $20 \, \text{dB}$ in comparison with the single soliton.

Thus SIL microcomb provide competitive phase noise characteristics within $1 \, \text{kHz} - 10 \, \text{kHz}$ in comparison with breadboard implementation\textsuperscript{20}. As future investigation, we need to understand why self-injection locking does not suppress phase noise at low frequencies (below $1 \, \text{kHz}$). Also, robust packaging of integrated microcomb sources may decrease the phase noise at these frequency offsets. Then we need to ensure that "quiet point"\textsuperscript{14} is accessible in the self-injection locking regime. Furthermore, we need to find the reason why SIL soliton demonstrates a phase noise plateau within $10 \, \text{kHz} - 1 \, \text{MHz}$.

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