Hysteresis in One-Dimensional Anti-Ferromagnetic Random-Field Ising Model at Zero-Temperature

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Abstract

We analyse hysteresis in a one-dimensional anti-ferromagnetic random field Ising model at zero-temperature. The random field is taken to have a rectangular distribution of width $2\Delta$ centered about the origin. A uniform applied field is varied slowly from $-\infty$ to $+\infty$ and back. Analytic expression for the hysteresis loop is obtained in the case $\Delta \leq |J|$, where $|J|$ is the strength of the nearest neighbor interaction.

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1 Introduction

In a recent paper [1], we presented an analytic solution of the zero-temperature dynamics of the anti-ferromagnetic random field Ising model in a slowly varying uniform applied field over a truncated range of the field. The purpose of the present paper is to extend the earlier solution to cover the entire range of the applied field, and hence to obtain an analytic expression for the hysteresis loop. We refer the reader to reference [1] for a detailed discussion of the model, and the method of its solution. Here, we recapitulate the salient points of reference [1] for maintaining continuity with it, and to set up the notation.

The Hamiltonian of the system is given by

\[ H = -J \sum_i s_i s_{i+1} - \sum_i h_i s_i - h_a \sum_i s_i, \]  

(1)

where \( s_i = \pm 1 \) are Ising spins at sites \( i = 1, 2, 3 \ldots \) of a one-dimensional lattice, \( h_a \) is a uniform applied field, and \( h_i \) is a quenched random field drawn independently from a continuous bounded distribution,

\[ p(h_i) = \frac{1}{2\Delta} \quad \text{if} \quad -\Delta \leq h_i \leq \Delta \]

\[ = 0, \quad \text{otherwise.} \]  

(2)

The external field \( h_a \) is slowly increased from \(-\infty\) to \(+\infty\), and the system is allowed to relax by the single-spin-flip Glauber dynamics at zero temperature, i.e. spins are allowed to flip one at a time if it lowers their energy. This dynamics takes the system to a stable state where each spin has the same sign as the net local field \( l_i \) at its site.
\[ s_i = \text{sign} \ l_i = \text{sign} \ [J(s_{i-1} + s_{i+1}) + h_i + h_a] \]  

The magnetization \( m(h_a) \) per spin at the applied field \( h_a \) is given by

\[ m(h_a) = \frac{1}{N} \sum_i s_i \]  

Our object is to calculate \( m(h_a) \) for all values of \( h_a \) in the range \([-\infty \text{ to } +\infty]\). In reference [1], we obtained \( m(h_a) \) for \([-\infty \leq h_a \leq 2|J| - \Delta]\) in the case \([\Delta \leq |J|]\). The result is expressed conveniently in terms of variables \( h \) and \( p \) defined as follows:

\[ h = -h_a \text{ mod } 2|J|; \quad p(h) = \int_h^\Delta p(h_i)dh_i = \frac{\Delta - h}{2\Delta} \]

The field \( h \) serves as a reduced applied field whose importance is explained below. If \( \Delta \leq |J| \), the spins turn up in three categories. The first to turn up are the spins whose nearest neighbors are both down (ramp-I), then the spins with one neighbor up and one down (ramp-II), and finally the spins with both neighbors up (ramp-III). On each ramp, a spin with quenched field \( h_i \) turns up when the reduced field \( h \) crosses the threshold \( h + h_i = 0 \). Thus the reduced field acts as an effective applied field on each ramp. The reduced field covers the same range as the quenched field, i.e. \([-\Delta \leq (h, h_i) \leq \Delta]\). The parameter \( p \) is the fraction of sites in the system with \( h_i \geq h \). This parameter also serves as a convenient measure of the reduced field on each ramp. A key quantity on each ramp is the fraction of spins whose quenched field is larger than the reduced field \( h \). This quantity determines the shape of the ramp. The difficulty in calculating this quantity comes from the fact that the distribution of quenched fields...
on each ramp is modified a posteriori by the dynamics of the system. In reference [1], we obtained the following results:

Ramp-I: \([-2|J| - \Delta \leq h_a \leq -2|J| + \Delta]\)

\[m(h) = -e^{-2p(h)}\]

Plateau-I: \([-2|J| + \Delta \leq h_a \leq -\Delta]\)

\[m(h) = e^{-2}\]

Ramp-II: \([-\Delta \leq h_a \leq \Delta]\)

\[m(h) = -\left(3 + 8e^{-1} + e^{-2}\right) + 8 \left(1 + e^{-1}\right) p - 4p^2 - \frac{2}{3} \left(1 + e^{-1}\right)^2 p^3 + \frac{5}{6} \left(1 + e^{-1}\right) p^4 - \frac{4}{15} p^5 + \left\{8 \left(1 + e^{-1}\right) - 8p - 2 \left(1 + e^{-1}\right) p^2 + \frac{4}{3} p^3\right\} e^{-p} - (5 + 2p) e^{-2p}\]

Plateau-II: \([\Delta \leq h_a \leq 2|J| - \Delta]\)

\[m(h) = \begin{bmatrix} 27 & 7 & -8 \\ 30 & e^{-1} & e^{-2} \end{bmatrix}\]

Reference [1] did not contain results for ramp-III, i.e. \(m(h_a)\) in the range \([2|J| - \Delta \leq h_a \leq 2|J| + \Delta]\). This calculation is taken up in the following.
2 The Ground State

Plateau-II forms the ground state for the evolution of ramp-III in an increasing applied field. The ground state has solitary down spins dispersed in a sea of up spins. Following the nomenclature of reference [1], we call the down spins singlets. The net field on a singlet is equal to $-2|J| + h_i + h_a$. This lies in the range $[-2|J| - \Delta + h_a]$ to $[-2|J| + \Delta + h_a]$. Therefore, the singlets begin to turn up at $h_a = 2|J| - \Delta$, and are all up at $h_a = 2|J| + \Delta$. These limits mark the beginning and the end of ramp-III. The shape of ramp-III is determined by the fraction of up spins at applied fields between these limits. We take an arbitrary value of the applied field $h_a = -2|J| - h$ on ramp-III, and calculate the fraction of singlets which are up at this field. In other words, we calculate the fraction of singlets with $h_i \geq h$ or equivalently $p_i \leq p$, where

$$p_i = \frac{\Delta - h_i}{2\Delta}.$$

As discussed in reference [1], the singlets on plateau-I fall into three categories. In category (1) are the singlets which were created on ramp-I. The fraction of singlets which belong to this category, and whose quenched field is greater than $h$ is given by,

$$P_{II}^1(p) = p - \frac{1}{2} \left[1 - e^{-2p}\right] - \frac{2}{e} \left[e^{-p} - (1 - p)\right]$$

In category (2) are the singlets which were created on ramp-II by a vanishing doublet. The fraction of singlets in this category whose quenched field is greater than $h$ is equal to,
\[ P_{II}^{2}(p) = \left[ e^{-p} - (1 - p) \right]^2 \]  

(6)

Finally, category (3) singlets are those that were created on ramp-II by the central spin of an unstable up triplet flipping down. The fraction of singlets in this category whose quenched field is larger than \( h \) is given by,

\[
P_{III}^{II}(p) = - \left( 1 + 2e^{-1} \right) + \left[ \frac{1}{4} + 2e^{-1} + 4e^{-2} \right] p \\
- \frac{1}{2} \left[ 1 + 5e^{-1} + 4e^{-2} \right] p^2 + \frac{1}{3} \left[ \frac{3}{2} + 4e^{-1} + e^{-2} \right] p^3 \\
- \frac{1}{4} \left[ 1 + e^{-1} \right] p^4 + \frac{1}{20} p^5 + \left[ 1 + 2e^{-1} \right] e^{-p} \\
- 2pe^{-(1+p)} + p^2 e^{-p} + \frac{1}{2} \left[ 1 - e^{-2p} \right] \]  

(7)

The total fraction of singlets present on plateau-II whose quenched fields is larger than \( h \) is equal to,

\[ P^{II}(p) = P_{I}^{II}(p) + P_{II}^{II}(p) + P_{III}^{II}(p) \]  

(8)

The above equation gives the fraction of singlets which will turn up in an applied field \( h_a = 2|J| - h \) from among the singlets initially present on plateau-II. However, when these singlets turn up, some of their nearest neighbors turn down. This process creates new singlets \([2]\). The newly created singlets turn up later at a higher applied field on ramp-III. We need to calculate the fraction of newly created singlets, and their restoration to the original state as function of the applied field before we can determine ramp-III.
3 Creation of New Singlets

In this section, we consider the circumstances in which the destruction of a singlet on ramp-III accompanies the creation of a new singlet. Consider a singlet in the ground state, say the spin at site 3 in Figure 1. Its nearest neighbors at sites 2 and 4 are up. A next nearest neighbor at site 1 is down, and the other next nearest neighbour at site 5 is up. Suppose the singlet just flips up on ramp-III at an applied field $h$, i.e. $h_3 - 2|J| + h_a = \epsilon$, where $\epsilon \geq 0$. The applied field at this point is $h_a = 2|J| - h_3 + \epsilon$. We ask the question, could a nearest neighbor of the singlet flip down when the singlet flips up?

First, consider the nearest neighbor at site 2 in Figure 1. After site 3 has flipped up, site 2 has one nearest neighbor up and one down. Thus site 2 may flip down if $h_2 + h_a \leq 0$, or $h_2 \leq h_3 - 2|J| - \epsilon$. However, this is not possible because $h_2$ and $h_3$ lie in the range $[-\Delta]$ to $[+\Delta]$, and we are considering the case $\Delta \leq |J|$. Thus a nearest neighbor of a singlet which has both its nearest neighbors down will stay up when the singlet turns up as long as $\Delta \leq |J|$.

Next, consider the spin at site 4. After the singlet has turned up, site 4 has both its nearest neighbors up. It will turn down if $-2|J| + h_4 + h_a \leq 0$, or $h_4 \leq h_3 - \epsilon$. If $h_4 \leq h_3$, site 4 will turn down when site 3 turns up. After site 4 has turned down, site 3 has one neighbor up and one down. The net field at site 3 is then $h_3 + h_a = 2|J| + \epsilon$, which is positive. Thus site 3 will stay up after site 4 has turned down. We conclude that, when a singlet turns up, its nearest neighbor may turn down if that nearest neighbor has less quenched field than the singlet, and also if it had one nearest neighbor already up before the singlet.
turned up. Note that when site 4 turns down, it increases the upward field at site 5. There is no possibility of site 5 turning down as a result of site 4 turning down. Consequently, a spin turning up on ramp-III does not cause any change in the state of spins beyond the nearest neighbors (absence of avalanches).

Before proceeding further, we rewrite the two rules which will guide us in the following analysis.

Rule-1: When a singlet turns up on ramp-III, its nearest neighbor stays up if the adjacent next nearest neighbor is down, and $\Delta \leq |J|$.

Rule-2: When a singlet turns up on ramp-III, its nearest neighbor turns down if both of the following conditions are satisfied:

(a.) the adjacent next nearest neighbor is up, and
(b.) the nearest neighbor has less quenched field than the singlet.

The next question is, in what circumstances, the quenched field on a nearest neighbor of a singlet on plateau-II can be smaller than the quenched field on the singlet. We examine various possible cases. Suppose the singlet in question was created on ramp-II by the destruction of a doublet. For example, look at Figure 1 again and suppose sites 2 and 5 were up, and 3 and 4 were down on plateau-I. This requires $h_3 \leq h_2$ and $h_4 \leq h_5$. Let $h_3 \leq h_4$ without any loss of generality. In this case, site 4 will turn up on ramp-II. When site 3 turns up on ramp-III, no new singlet can be created because both nearest neighbors of site 3 have a larger quenched field than site 3 (Rule-2).

Next, let us assume that the singlet in question was created on ramp-II by the unstable central spin of an up triplet flipping
down. The central spin of an up triplet flips up for the first time on ramp-I. It separates two adjacent doublets on plateau-I. Therefore, the quenched field on the central spin is larger than the quenched field on each of its nearest neighbors. The nearest neighbors of the central spin flip up on ramp-II (the central spin flips down at this event). Therefore each nearest neighbor of the central spin has a larger quenched field than the next nearest neighbor of the central spin which is adjacent to it. Consequently the central spin has a larger quenched field than each of its next nearest neighbors. When the central spin flips up for the second time on ramp-III, its next nearest neighbors are still down. Therefore there is no chance (Rule-2) for the nearest neighbors of the central spin to flip down when the central spin flips up on ramp-III.

Having ruled that the destruction of a singlet created on ramp-II does not give rise to a new singlet on ramp-III, we are left with singlets created on ramp-I and present on plateau-I. When one of these singlets disappears on ramp-III, it may create a new singlet if one of its nearest neighbors has a smaller quenched field than the singlet, and if the next nearest neighbor is up. We note that only one of the two nearest neighbors of a singlet on plateau-I can have a quenched field which is smaller than the singlet. A site which has a larger quenched field than both of its nearest neighbors must be up on plateau-I. Now consider a singlet on plateau-I as shown at site 2 in Figure 2. Let its nearest neighbor on the right (at site 3) have a smaller quenched field \((h_3 \leq h_2)\). Site 4 must be down because there are no strings of up spins of length greater than unity on plateau-I. Site 5 may be up or down. The two possibilities for site 5 are depicted in Figures 2 and 3 respectively. In Figure 2, a singlet is followed
by a singlet on plateau-I. In Figure 3, a singlet is followed by a
doublet.
Consider Figure 2 first. What is the probability per site that
such an object occurs on plateau-I? A singlet on plateau-I must
be followed by a singlet or a doublet. It was shown in reference
[1], that the probability per site of the occurrence of a doublet
is equal to $e^{-2}$, and the probability that a doublet is followed
by a doublet is equal to $\frac{1}{3}e^{-2}$. Therefore, the probability that a
doublet is followed by a singlet (or vice-versa) is equal to $\frac{2}{3}e^{-2}$.
It was also shown that the probability of the occurrence of a
singlet is equal to $\frac{1}{2}[1 - 3e^{-2}]$. Therefore, the probability per
site that a singlet is followed by a singlet on plateau-I is equal
to $\frac{1}{2}[1 - \frac{13}{3}e^{-2}]$.
We have obtained above the probability of the occurrence
of objects shown in Figure 2. Our immediate interest lies in
a subset of these objects with $h_3 \leq \min (h_2, h_4)$. This subset
determines the creation of new singlets. Suppose $h_2 \leq h_4$. Then
on ramp-III, site 4 will flip up before site 2. When site 2 flips
up, site 3 will flip down because the conditions for the creation
of a new singlet are fulfilled (Rule-2). In order to calculate the
fraction of newly created singlets, we need to know the distribu-
tion of fields $h_2$, $h_3$, and $h_4$. These may be obtained on the
lines of reference [1] if one notes that sites 1 and 5 must have
flipped up on ramp-I before site 3. Site 1 must have flipped up
before site 3 because $h_1 \geq h_2 \geq h_3$. Similarly, site 5 must have
flipped up before site 3 because $h_5 \geq h_4$, $h_4 \geq h_2$, $h_2 \geq h_3$, and
therefore $h_5 \geq h_3$. This proves that just before site 3 flipped up
on ramp-I, sites 2, 3, and 4 formed a string of three down spins
bordered by up spins at sites 1 and 5. The screening property
discussed in reference [1] can be applied here to conclude that
the distributions of $h_2$, and $h_4$ are independent of each other, and each is given by

$$
\text{Prob}[p \leq p_2 \leq p + dp] = [1 - e^{-p}]dp
$$

$$
\text{Prob}[p \leq p_4 \leq p + dp] = [1 - e^{-p}]dp
$$

where

$$
p_i = \frac{\Delta - h_i}{2\Delta}
$$

Given that $h_2$ lies in the range $[h \text{ to } h + dh]$, $h_3$ must be uniformly distributed in the range $[-\Delta \text{ to } h]$. Thus,

$$
\text{Prob}[p \leq p_3 \leq p + dp] = dp \text{ (if } p_3 \geq p_2)\n$$

The contribution of Figure 2 to the fraction of newly created singlets when all sites with $h_i \geq h$ have been relaxed on ramp-III is given by,

$$
P^{III}_4(p) = 2 \int_0^p [1 - e^{-p_2}] dp_2 \int_{p_2}^1 dp_3 \int_0^p [1 - e^{-p_4}] dp_4$$

$$
= \frac{3}{2} - 2p(1 - p) - \frac{2}{3}p^3 - 2(1 - p + p^2)e^{-p} + \frac{1}{2}(1 - 2p)e^{-2p}
$$

(9)

Consider Figure 3 next. It shows a singlet at site 2 followed by a doublet at sites 4 and 5. An identical contribution will come from a situation where the doublet occurs at sites 2 and 3, and the singlet at site 5 as shown in Figure 4. We work out the contribution from Figure 3 explicitly, and then multiply it by a factor 2 to take into account the contribution from Figure 4. When site 2 flips up, site 3 will flip down if $h_3 \leq h_2$, and if site 4 is up as well. Note that site 4 would have flipped up earlier on
ramp-II if $h_4 \geq h_5$. If $h_5 \geq h_4$, site 5 would have flipped up on ramp-II. In this case there is no possibility of site 3 flipping down when site 2 flips up. The reason is as follows. The presence of a doublet at sites 4 and 5 on plateau-I means that $h_4 \leq h_3$ and $h_5 \leq h_6$. We must have $h_3 \leq h_2$ for the creation of a new singlet (Rule-2). If $h_4 \leq h_3$, and $h_3 \leq h_2$, then we have $h_4 \leq h_2$. Thus, when site 2 turns up on ramp-III, site 4 will be down, and one of the two conditions for the creation of a new singlet is not satisfied.

A similar reasoning as applied in the analysis of Figure 2 reveals that site 3 must have flipped up after sites 1 and 6. The distribution of fields at sites 2 and 5 in Figure 3 must be similar to the distribution of fields at sites 2 and 4 in Figure 2. The distribution of fields at sites 3 and 4 must be uniform over the interval $[-\Delta \text{ to } \min(h_1, h_6)]$. Thus the contributions of Figures 3 and 4 to the fraction of newly created singlets when sites with $h_i \geq h$ have been relaxed on ramp-III is given by,

$$ P_{III}^5(p) = 2 \int_0^p [1 - e^{-p_2}] dp_2 \int_{p_2}^1 dp_3 \int_{p_3}^1 dp_4 \int_{p_4}^1 [1 - e^{-p_5}] dp_5 $$

$$ = -\left(\frac{1}{3} + 3e^{-1}\right) + \left(\frac{1}{3} + 5e^{-1}\right)p - \left(\frac{1}{2} + 2e^{-1}\right)p^2 + \frac{1}{3} (1 + e^{-1})p^3 $$

$$ - \frac{1}{12}p^4 + \left\{\left(\frac{4}{3} + 3e^{-1}\right) - (1 + 2e^{-1})p + e^{-1}p^2 - \frac{1}{3}p^3\right\}e^p - e^{-2p} $$

(10)

4 **Destruction of New Singlets**

The destruction of newly created singlets on ramp-III can be analysed in a similar manner as their creation. For example,
refer to Figure 2 again, and recall that $h_2 = \min(h_2, h_4)$, and $h_3 \leq h_2$. In this figure, the creation of new singlets when all sites with $h_i \geq h$ have been relaxed is determined by $h_2 \geq h$. The destruction of new singlets when all sites with $h_i \geq h$ have been relaxed is given by $h_3 \geq h$. The result is an integral similar to the expression for the creation of singlets. Only the limits of the integration are altered. We obtain,

$$P_{III}^{6}(p) = 2 \int_0^p dp_3 \int_0^{p_3} [1 - e^{-p_2}] dp_2 \int_0^{p_3} [1 - e^{-p_4}] dp_4$$

$$= \frac{1}{2} \left[ 1 - e^{-2p} \right] - 2p e^{-p} + p(1 - p) + \frac{1}{3} p^3$$

(11)

Similarly, the contribution from Figures 3 and 4 is given by,

$$P_{III}^{7}(p) = 2 \int_0^p dp_3 \int_0^{p_3} [1 - e^{-p_2}] dp_2 \int_{p_3}^{1} dp_4 \int_{p_4}^{1} [1 - e^{-p_5}] dp_5$$

$$= 2e^{-1} - (1 + 4e^{-1}) p + \left( \frac{3}{2} + 3e^{-1} \right) p^2 - \left( 1 + \frac{2}{3} e^{-1} \right) p^3 + \frac{1}{4} p^4$$

$$- \left\{ (1 + 2e^{-1}) - 2 \left( 1 + e^{-1} \right) p + p^2 \right\} e^{-p} + e^{-2p}$$

(12)

5 Ramp-III

Putting the various terms together, the probability that a randomly chosen spin on the chain is up on ramp-III is given by

$$P_{III}^\uparrow(p) = P_{II}^\uparrow(1) + P_{1III}^\uparrow(p) + P_{2III}^\uparrow(p) + P_{3III}^\uparrow(p)$$

$$- P_{4III}^\uparrow(p) - P_{5III}^\uparrow(p) + P_{6III}^\uparrow(p) + P_{7III}^\uparrow(p)$$

(13)
The magnetization on ramp-III is given by,

\[ m^{III}(h) = 2P_{↑}^{III}(p) - 1 \]  

(14)

After some simplification, we obtain

\[
m^{III}(h) = -\left\{ \frac{13}{30} - \frac{53}{6} e^{-1} + \frac{8}{3} e^{-2} \right\} + \left\{ \frac{11}{6} - 18 e^{-1} + 8 e^{-2} \right\} p \\
- (1 - 5 e^{-1} + 4 e^{-2}) p^2 + \frac{1}{3} (1 + 2 e^{-1} + 2 e^{-2}) p^3 \\
+ \frac{1}{6} (1 - 3 e^{-1}) p^4 + \frac{1}{10} p^5 \\
- \left\{ \left( \frac{8}{3} + 10 e^{-1} \right) - (2 + 4 e^{-1}) p - (4 - 2 e^{-1}) p^2 - \frac{2}{3} p^3 \right\} e^{-p} \\
+ (4 + 2p) e^{-2p} \]  

(15)

The above expression has been superposed on the simulation data in Figure 5. The agreement is excellent. The analytic result is indistinguishable from the simulation on the scale of Figure (5).

6 Hysteresis Loop

So far, we have analysed the magnetization \( m(h_a) \) in increasing applied field. We have shown that the analytic result agrees with the simulation rather well. The magnetization \( m_R(h_a) \) in decreasing field (return loop) is related to \( m(h_a) \) by a symmetry of the model, i.e. \( m_R(h_a) = -m(-h_a) \). Thus we have implicitly determined the return loop as well. The return loop has been shown in Figure (5) by a broken line. The hysteresis in the anti-ferromagnetic RFIM is rather small, and the two halves of the hysteresis loop lie very close to each other. In order
to show them more clearly, we have plotted in Figure (6) the separation between the two halves of the hysteresis loop versus the applied field. To be precise, we have plotted \([m(h_a) - \bar{m}]\) and \([m_R(h_a) - \bar{m}]\) vs \(h_a\), where \(\bar{m} = \frac{1}{2}[m_R(h_a) + m(h_a)]\).

As we may expect, the agreement between the theoretical expression and the simulation is excellent on the scale of Figure 6 as well. In Figure 6 the simulation data was obtained from a system of \(10^3\) spins, and averaged over \(10^3\) independent realizations of the random field distribution. The set of applied fields where the spins flip on each half of the hysteresis loop is of course different for each realization of the random field distribution. Therefore, the average over different realizations requires a judgement on how to group the data. We divided the entire range of the applied field from \([-2|J| - \Delta\) to \(2|J| + \Delta\)] into \(3 \times 10^3\) sections (bins) of equal width. The data in each bin was averaged separately. The simulation data shown in Figure 6 is a much sparser set of data (in order not to crowd the figure). We have shown the simulation data at intervals of \(\delta h_a = .1\), and the theoretical expression at intervals of \(\delta h_a = .01\) (joined by a continuous line on the lower half, and a broken line on the upper half of the hysteresis loop).

7 Discussion

We have considered the zero-temperature dynamics of a one-dimensional anti-ferromagnetic random field Ising model, and obtained an analytic solution of the model if the following conditions apply:

1. All spins are down initially, and the applied field is swept
from $h_a = -\infty$ to $h_a = +\infty$ infinitely slowly. The solution is also applicable by symmetry to the case when all spins are up in the initial state, and $h_a$ is decreased from $h_a = +\infty$ to $h_a = -\infty$.

2. The random field has a uniform bounded distribution in the interval $[-\Delta \text{ to } +\Delta]$. We considered the case $\Delta \leq |J|$. The simplifying feature of this case is that the increasing applied field exhausts all strings of down spins of length three or more (ramp-I) before working on strings of down spins of length two (ramp-II). Similarly, strings of down spins of length one (singlets) are turned up (ramp-III) only after the doublets are finished.

The second condition mentioned above has been adopted essentially for simplicity. It serves to illustrate the method of solution with a minimum of algebraic detail. We do not see a conceptual difficulty in applying the same method in the case $\Delta \geq |J|$, or when the random field has an unbounded continuous distribution but we do not go into these details here.

The restriction to an initial state where spins are either all down or all up appears to be necessary so far. A similar difficulty is encountered in the ferromagnetic random field Ising model [3]. The relaxation dynamics of the ferromagnetic model is qualitatively different from that of the anti-ferromagnetic model. The relaxation process in the ferromagnetic model is abelian, while in the anti-ferromagnetic model it is not. We have made some progress in the methods of analytic solutions in both cases, but a qualitatively new idea appears to be needed in extending these methods to an arbitrary initial state.
One of us (PS) thanks D Dhar for several useful comments during the course of this work.

References

[1] P Shukla, to be published in Physica A; See also P Shukla, Physica A 233, 235 (1996).

[2] Although the new singlets make the analysis of ramp-III somewhat tedious, but they save our model from an unphysical feature. If we neglect the new singlets, the two halves of the hysteresis loop cross each other twice so as to make a series of three subloops. In this case, one could engineer a violation of the second law of thermodynamics by running an engine over two of the subloops. However, the new singlets on ramp-III push down the lower half of the hysteresis loop and prevent the upper half from crossing it.

[3] Sanjib Sabhapandit, Prabodh Shukla, and Deepak Dhar, to be published in J Stat Phys.

Figure Captions

Figure 1: A singlet (site 3) with one next nearest neighbor down (site 1), and one next nearest neighbor up (site 5). When the singlet turns up at an applied field $h_a$, the spin at site 2 stays up if $\Delta \leq |J|$, but the spin at site 4 flips down if $h_4 \leq h_3$.

Figure 2: Two adjacent singlets on Plateau-I: If $h_2 = \min(h_2, h_4)$, and $h_3 \leq h_2$, then the spin at site 3 will flip down when the spin at
site 2 flips up on ramp-III. This process creates a new singlet on ramp-III.

Figure 3:
A singlet followed by a doublet on Plateau-I: If \( h_4 \geq h_5 \), and \( h_3 \leq h_2 \), then a new singlet will be created at site 3 when the spin at site 2 turns up on ramp-III.

Figure 4:
A doublet followed by a singlet on Plateau-I: If \( h_3 \geq h_2 \), and \( h_4 \leq h_5 \), then a new singlet will be created at site 4 when the spin at site 5 turns up on ramp-III.

Figure 5:
Magnetization per spin \( m \) in an applied field \( h_a \) for an antiferromagnetic RFIM \((J = -1)\) for a rectangular distribution of the random field of width \( 2\Delta = 1 \). The analytic result in increasing applied field is shown by the solid line, and in decreasing field by a broken line. Numerical result from a simulation of \( 10^3 \) spins averaged over \( 3 \times 10^3 \) different realizations of the random field distribution has been superposed on the theoretical result by dots. The numerical result is indistinguishable from the theoretical result on the scale of this figure.

Figure 6:
Separation between the two halves of the hysteresis loop in Figure 5 has been magnified by plotting it relative to the average value of the two halves. The solid line shows \( \delta m = [m - \overline{m}] \), and the broken line \( \delta m = [m_R - \overline{m}] \) vs the applied field \( h_a \); \( \overline{m} = \frac{1}{2} [m_R + m] \); \( m \) and \( m_R \) are the magnetizations at applied field \( h_a \) in increasing and decreasing field respectively.
Figure 1

Figure 2
Figure 3

Figure 4
