The goal of this message is to calculate radiative corrections to the Sommerfeld fine structure constant in the framework of a new QED in which particles are described by bilocal fields. The bare constant is $1/136$ where $136$ is a dimension of the dynamical group of the bihamiltonian system underlying the suggested elementary particle theory. Our calculations in the second order of perturbation theory give the renormalized Sommerfeld constant $1/137.0345$. We believe the difference $(137.0359 - 137.0345)$ between corresponding experimental and theoretical values may be understood as corrections of the fourth order.

11.10.Gh, 11.10.Lm

I. INTRODUCTION

The aim of this paper is to show how to calculate the main radiative corrections in quantum electrodynamics improved on the bases of the general elementary particle theory suggested in [1]. The keystone of the theory is the assumption that the true mechanism of production of elementary particles is not interactions between them (or between their hypothetical constituents), but is a certain quantum-dynamical system determining the special physics at supersmall distances where the space-time is discontinuum, i.e. it is the quite non-connected manifold there. The transition in such a system lead to the creation of fundamental particle fields which are bilocal wave functions $\psi(X,Y)$ in our theory (the Heisenberg-Schrödinger-Dirac theory postulates the existence of local fields $\psi(X)$, but in that theory there are ultraviolet divergences). The initial principles of our approach to the elementary particle problem have been stated in the Russian periodicals [1–5].

The dynamical system mentioned above has been described in [2] and named as a relativistic bi-Hamiltonian system. Owing to the discontinuity of space in small, the quantum theory of the system is non-unitary; the non-standard (non-Fock) representations of the Heisenberg algebra $h^{(+)}_16$ described in [3] (extraction of square root of Grassmann spinors) leads to such algebras [3] and non-unitary (infinite-dimensional) representations of the rotation group $SO(3)$ and Lorentz group $SO(3,1)$, induced by them and characterize by the arbitrary complex spin found in [5] earlier, form the mathematical foundation of this theory. Thus these representations stand for a new physical reality. The elementary particle theory based on them is more like the atomic spectrum theory rather than any composite model.

In the framework of this theory (quantum electrodynamics with bilocal fields) we consider here only one question — the charge renormalization which was not solved up till now.

II. BILOCAL FIELDS AND THEIR INTERACTIONS

The field bilocality $\psi(X,Y)$ is the direct consequence of the semispinor structure of the particle fields $\psi^\Sigma \sim \langle \hat{f}, O^\Sigma f \rangle (O^\Sigma)$ is elements of the Heisenberg algebra $h^{(+)}_16$) discovered by means of extracting the square root of Grassmann spinors, see [3] (this structure is quite analogous to the spinor structure of current $j_\mu \sim \psi \gamma_\mu \psi$ where $\gamma_\mu$ are elements of the Clifford algebra discovered by means of extracting the Dirac square root of vectors).

The bilocal field $\psi^\Sigma(X,Y)$ defined by the transition amplitude $\langle \hat{f}(X - Y), O^\Sigma f(X + Y) \rangle$ where $O^\Sigma f(x)$ and $\hat{f}(x)$ are the initial (excited) and final (ground) states of the relativistic bi-Hamiltonian system respectively (the explicit form of these states found in [3]) is written down as
\[
\psi(X,Y) = \frac{1}{(2\pi)^{3/2}} \int e^{ipX + iqY} \theta(p_0 + q_0) \theta(p_0 - q_0) \delta(p^2 + q^2) \delta(pq) \delta(p^2 - m^2) \psi(p,q) d^4p \frac{d^4q}{2\pi}.
\] (1)

Here \(X_\mu\) are coordinates in Minkowsky space (\(p_\mu\) is a 4-momentum of a particle) and \(Y_\mu\) are internal coordinates (which are not fixed in the experiment and therefore we call them hidden) describing the space-time structure of particles (\(q_\mu\) is a 4-momentum of such a particle). Interactions between bilocal fields are described by differential equations in Minkowsky space. We are interested in the Dirac field in the form of \(\psi\) (an interaction Lagrangian). In the perturbation theory the interaction picture may be described by the interaction \(\delta\) function: \(\delta(pq)\).

It follows from (1) that if \(|X| \gg |Y|\) then the bilocal field \(\psi(X,Y)\) transforms into the usual local field \(\psi(X) = \psi(X,0)\) (hence, in the suggested scheme the local fields appear as asymptotic fields; it is a principal point of a new correspondence principle). It also follows from (1) that \(\psi(X,Y)\) may be represented in the form of \(\psi(X,Y) = F(Y, -i\frac{\partial}{\partial X})\psi(X)\) where \(\psi(X)\) is a local field and \(F\) is the so-called smearing operator which has the form in the case of massive particles (\(p_\mu\) is a 4-momentum of such a particle)

\[
F(Y, p) = \frac{1}{2\pi} \int e^{iqY} \delta(p^2 + q^2) \delta(pq) d^4q.
\] (2)

Another form of the smearing operator takes place in the case of massless particles (it follows from the explicit form of the leptonic transition amplitude; it is necessary to note that operator (4) does not transform into (3) when \(p^2 = 0\); in the case we have the stochastic integral

\[
\frac{1}{2} \int_{-1}^{1} e^{i\alpha pY} d\alpha,
\]

namely:

\[
F_0(Y, k) = e^{iYk}.
\] (3)

Proceeding from (4) we may construct the \(S\)-matrix: \(S = T \exp(i \int L_i(X) d^4X)\) where \(L_i(X) = \frac{1}{2} [\bar{\phi}(X) J(X) + \bar{J}(X) \psi(X)]\) is an interaction Lagrangian. In the perturbation theory the interaction picture may be described by the well-known Feynman diagrams in vertices of which the electron-photon formfactor arises

\[
\rho(p, k) = \int F(Y, p) F_0(Y', k) d\mu(Y, Y') = \frac{1}{2\pi} \int e^{ik\theta} \delta(p^2 + q^2) \delta(pq) d^4q.
\] (5)

### III. THE MAIN FORMULA

First of all we state the result of our calculations of radiative corrections to the Sommerfeld fine structure constant \(\alpha = e^2/4\pi\). In the suggested theory the renormalized constant \(\tilde{\alpha}\) connects with the “bare” constant \(\alpha\) by the formula

\[
\tilde{\alpha} = \left(\frac{Z_1}{Z_2}\right)^2 \frac{Z_4}{Z_3} \alpha
\] (6)
where $Z_1, Z_2, Z_3, Z_4$ are the renormalization constants of the fermion Green function, vertex function, Lagrangian of classical electromagnetic field and three-tail, respectively. Here all these quantities are calculated in the second order of perturbation theory.

In the suggested theory the “bare” constant $\alpha$ is equal to $1/136$ (Eddington formula) where 136 is the dimension of the dynamical group (the group of automorphisms $Sp^{(*)}(8, \mathbb{C})$ for the Heisenberg algebra $h_1^{(*)}$) in our relativistic bi-Hamiltonian system, see [3].

We see formula (3) essentially differs from the local theory formula $\tilde{\alpha} = Z_1^{-1} \alpha$ being a consequence of the Ward identity $Z_1 = Z_2$ (in this theorem for regularized constants, see [7], the regularized fermion self-energy operator $\Sigma(p)$ is assumed to be an analytic function at point $p^2 = 0$; but it does not take place in the suggested theory: it follows from [8] that $\Sigma(p) \sim \ln p^2$ when $p^2 \to 0$) and also the Furry theorem (which does not take place in the suggested theory too due to the presence of hidden parameters $Y_\mu$ for bare particles and the absence of them for bare antiparticles, see further).

**IV. CALCULATION OF $Z_1/Z_2$**

In our theory the Ward identity

$$\frac{\partial \Sigma(p)}{\partial p_\mu} + \Lambda_\mu(p, 0) = 0$$

($\Lambda_\mu$ is the vertex function) is replaced by a more general identity

$$\frac{\partial \Sigma(p)}{\partial p_\mu} + \Lambda_\mu(p, 0) = \Sigma_\mu(p) \quad (7)$$

where $\Sigma_\mu(p)$ is the following operator

$$\Sigma_\mu(p) = \frac{e^2}{i(2\pi)^4} \int \frac{d^4k}{(p-k)^2 - m^2} \left[ \frac{\partial}{\partial p_\mu} \rho(p, k) \right]$$

$$= \frac{ie^2}{(2\pi)^4} \int_0^1 dz \int_0^\infty \frac{d\sigma}{\sigma^2} \exp \left[ \frac{p^2}{2\sigma} - i\sigma(m^2z - p^2z(1-z)) \right] \left[ p_\mu(2m - \hat{p}(1-z)) + \frac{z}{3}(p_\mu\hat{p} - \gamma_\mu p^2) \right].$$

To express the quantity $(Z_1/Z_2 - 1)$ of interest to us in terms of $\Sigma_\mu(p)$, it is necessary to take the operator on the mass shell $\bar{p} = m$ by means of the formula

$$\left( \frac{Z_1}{Z_2} \right) \gamma_\mu = \Sigma_\mu(m) = -\gamma_\mu \frac{e^2}{4\pi^2} m^2 \int_0^1 z (1 + z) K_1(m^2z) dz$$

where $K_1$ is the MacDonald function. From here we get

$$\frac{Z_1}{Z_2} = \begin{cases} 1 - \frac{3\alpha}{2m^2}, & m \ll 1; \\ 1 - \frac{3\alpha}{2m}, & m \gg 1 \end{cases} \quad (8)$$

**V. CALCULATION OF $Z_1/Z_3$**

Similarly, another Ward identity

$$\frac{\partial \Pi_{\mu\nu}(k)}{\partial k_\sigma} + \Delta_{\mu\nu\sigma}(k, 0) = 0$$

($\Pi_{\mu\nu}$ is the polarization tensor), see [3], is replaced by a more general identity

$$\frac{\partial \Pi_{\mu\nu}(k)}{\partial k_\sigma} + \Delta_{\mu\nu\sigma}(k, 0) = \Pi_{\mu\nu\sigma}(k) \quad (9)$$
where $\Pi_{\mu\nu\sigma}(k)$ is the following expression

$$
\Pi_{\mu\nu\sigma}^{(1/2)}(k) = \frac{ie^2}{(2\pi)^4} \int \frac{2p_\mu p_\nu + 2p_\mu k_\nu - \delta_{\mu\nu}(p^2 + pk)}{p^2 (p + k)^2} \left[ \frac{\partial}{\partial k_\sigma} \rho((p + k)^2) \right] d^4p
$$

in the case of Weyl’s dissociation, and

$$
\Pi_{\mu\nu\sigma}^{(0)}(k) = \frac{ie^2}{(2\pi)^4} \int \frac{p_\mu p_\nu + p_\mu k_\nu}{p^2 (p + k)^2} \left[ \frac{\partial}{\partial k_\sigma} \rho((p + k)^2) \right] d^4p
$$

in the case of Klein-Gordon’s dissociation.

Speaking about the electromagnetic wave dissociation we should explain two points. Firstly calculating $\Pi_{\mu\nu}$ we use quite another form factor not \[8\], but

$$
\rho(p^2) = \int F(Y, p) F(Y', p) d\mu(Y, Y') = \frac{\sin p^2}{p^2}
$$

(10)

because the Lagrangian $\hat{A}_\mu \hat{\psi} \gamma_\mu \hat{\psi}$ all fields of which are quantized does not give any contribution to the charge renormalization, see \[9\]. Another Lagrangian, namely $A_\mu \hat{\psi} \gamma_\mu \hat{\psi}$ ($A_\mu$ is a classical field), gives such a contribution. If the wave function of photons $A_\mu(X, Y)$ has the internal variables $Y_\mu$, then the classical one (Maxwell field $A_\mu(X)$), as an essential alloy of indefinite number of photons (light molecule), does not have such variables. Therefore in the case only internal variables of intermediate particles (not antiparticles) are paired. This operation leads to the form factor \[10\].

It is important to note that the bare particles as objects being created in the transition $f \rightarrow \bar{f}$ have the additional variables $Y$. The bare antiparticles arised in consequence of interactions do not have such variables (T-asymmetry of 100 per cent or complete fermion-antifermion asymmetry of the theory, see \[1\]). Under these circumstances the well-known Furry theorem is invalid.

Secondly, the polarization tensor $\Pi_{\mu\nu}$ having a finite value $\Pi_{\mu\nu}(k) = (k\mu k_\nu - \delta_{\mu\nu}k^2)P(k^2) + \delta_{\mu\nu}d(k^2)$ where

$$
d(k, m) = -\frac{e^2}{4\pi^2} \int_{-1}^{1} \frac{d\alpha}{2} \int_{0}^{\infty} dz \int_{0}^{\infty} \frac{\sigma d\sigma}{(\sigma + \alpha)^2} \left[ m^2 - \frac{i}{\sigma + \alpha} - k^2 \frac{\sigma z}{\sigma + \alpha} \left( 1 - \frac{\sigma z}{\sigma + \alpha} \right) \right] \times

\times \exp \left[ -i\sigma m^2 + ik^2 \alpha \right]
$$

for both the Dirac and Kemmer-Duffin (or Klein-Gordon) polarizations (the expression for $\Pi$ is not given here) must be a gauge-invariant quantity. Therefore we require $d(k) = 0$ at least in the region $k^2 = 0$. The last condition leads to the equation

$$
\int_{0}^{\infty} \frac{\sin x}{x + m^2} dx + m^2 \int_{0}^{\infty} \frac{\cos x}{x + m^2} dx = \frac{\pi}{2}
$$

which has the only solution $m = 0$.

Hence a classical electromagnetic wave may dissociate on massless particles only. Essentially, in the suggested theory there are two and only two charged particles with zero bare mass: positron (in our scheme it is the fundamental fermion with spin 1/2; electron is antifermion) and $\pi$-meson (quantum of degeneration fields with spin 0). Therefore we consider only two these cases.

Since $\Pi_{\mu\nu\sigma}$ leads the Lagrangian to the form of $\Pi_{\mu\nu\sigma}(k) A_\mu(k) A_\nu(k) A_\sigma(0)$ and in consequence of the Lorentz-gauge $k_\mu A_\mu(k) = k_\nu A_\nu(k) = 0$ we should hold only the term $\delta_{\mu\nu}k_\sigma$ in $\Pi_{\mu\nu\sigma}(k)$. Therefore we write $\Pi_{\mu\nu\sigma}(k) = \delta_{\mu\nu}k_\sigma I(k)$. Our calculations give

$$
I^{(1/2)}(k) = \frac{e^2}{4\pi^2} \int_{-1}^{1} \frac{\alpha d\alpha}{2} \int_{0}^{1} dz \int_{0}^{\infty} \frac{\sigma d\sigma}{(\sigma + \alpha)^3} \left( \frac{1}{2} - \frac{2\sigma z}{\sigma + \alpha} \right) \exp \left[ ik^2 \sigma z \left( 1 - \frac{\sigma z}{\sigma + \alpha} \right) \right],
$$

$$
I^{(0)}(k) = -\frac{e^2}{4\pi^2} \int_{-1}^{1} \frac{\alpha d\alpha}{2} \int_{0}^{1} z d\sigma \int_{0}^{\infty} \frac{\sigma^2 d\sigma}{(\sigma + \alpha)^4} \exp \left[ ik^2 \sigma z \left( 1 - \frac{\sigma z}{\sigma + \alpha} \right) \right].
$$
On the mass shell \( k^2 = 0 \) we get

\[
I^{(1/2)}(0) = -\frac{e^2}{48\pi^2}, \quad I^{(0)}(0) = -\frac{e^2}{24\pi^2}.
\]

The quantity \((Z_4/Z_3 - 1)\) of interest to us is determined by the sum \(I^{(1/2)}(0) + I^{(0)}(0)\) and we have

\[
\frac{Z_4}{Z_3} = 1 - \frac{\alpha}{12\pi} - \frac{\alpha}{6\pi} = 1 - \frac{\alpha}{4\pi}.
\]  \(\text{(11)}\)

**VI. THE PRINCIPAL RESULT**

Expressions (8) and (11) together give

\[
\left(\frac{Z_2}{Z_1}\right)^2 \frac{Z_3}{Z_4} = \left(1 + \frac{3\alpha}{\pi}\right) \left(1 + \frac{\alpha}{4\pi}\right) = 1 + \frac{13\alpha}{4\pi}.
\]

\[(Z_2/Z_1)^2 \frac{Z_3}{Z_4} = \left(1 + \frac{3\alpha}{\pi}\right) \left(1 + \frac{\alpha}{4\pi}\right) = 1 + \frac{13\alpha}{4\pi}.
\]

From (11) it follows now

\[
\tilde{\alpha}^{-1} = \alpha^{-1} + \frac{13}{4\pi}.
\]  \(\text{(12)}\)

Since in the suggested theory \(\alpha^{-1} = 136\), the renormalized constant \(\tilde{\alpha}^{-1}\) is \(\tilde{\alpha}^{-1} = 136 + 1.0345 = 137.0345\). We believe the difference 0.0014 (indeed, 0.00085 only) may be explained by the fourth order radiative corrections.

**VII. FERMION ANOMALOUS MAGNETIC MOMENT**

According to the suggested theory, calculations of the vertex operator in the third order of the perturbation theory lead to the following formula of the fermion anomalous magnetic moment

\[
\Delta \mu = \frac{\alpha}{\pi} m^2 \int_0^1 z (1-z) K_1(m^2 z) \, dz.
\]

\[(\Delta \mu) = \frac{\alpha}{\pi} m^2 \int_0^1 z (1-z) K_1(m^2 z) \, dz.
\]

a) In the case \(m \ll 1\) the formula (13) gives Schwinger’s result \(\frac{\alpha}{2\pi}\) with a correction

\[
\Delta \mu \simeq \frac{\alpha}{2\pi} \left[1 + \frac{m^4}{12} \left(C - \frac{13}{12} - \ln 2 + \ln m^2\right)\right].
\]

\[(\Delta \mu) \simeq \frac{\alpha}{2\pi} \left[1 + \frac{m^4}{12} \left(C - \frac{13}{12} - \ln 2 + \ln m^2\right)\right].
\]

The electron has \(m = \frac{m_e}{\kappa h} = 5 \cdot 10^{-4}\) and the correction \(\frac{\alpha}{2\pi} \frac{m^4}{12} \left(C - \frac{13}{12} - \ln 2 + \ln m^2\right) \approx -9.8 \cdot 10^{-17}\) is far beyond the experimental possibilities of today. The \(\mu\)-meson has \(m_\mu = 0.1\) and the correction is equal \(-5.6 \cdot 10^{-8}\) within the bounds of possibility. The correction should be added to the factor \((\frac{g-2}{2})_{\text{theory}} = \frac{\alpha}{2\pi} + 0.76 (\frac{\alpha}{2\pi})^2 = 0.0011655102\) calculated by means of the local theory. Its experimental value is \((\frac{g-2}{2})_{\text{exper}} = 0.001165923\). The difference \((\frac{g-2}{2})_{\text{exper}} - (\frac{g-2}{2})_{\text{theory}} = 0.0000000413\) (together with our correction the value is equal 0.000000493) is usually accounted for by influence of the strong interaction the correct theory of which is known to be wanting as yet (and all the calculations are not strictly defined). However there is a correction close to it in magnitude because of nonlocality (of both electromagnetic and strong interactions), i. e. owing to the finite third fundamental constant \(\kappa\).

b) For \(m \gg 1\) the formula (13) gives

\[
\frac{g - 2}{2} \simeq \frac{\alpha}{2m^2}.
\]

\[(\frac{g - 2}{2}) \simeq \frac{\alpha}{2m^2}.
\]

Let us apply it to the \(\tau\)-meson having \(m_\tau = 1.78\) and obtain \(\frac{g-2}{2} = 0.001151584\).
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