Access via laboratory experiments to the level mixing effect induced by blackbody radiation and its influence on the cosmological hydrogen recombination problem

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Two different effects of the blackbody radiation (BBR)-induced atomic line broadening are compared. The first one (stimulated Raman scattering) was discussed by many authors, the second one (quadratic level mixing) was predicted earlier in our publication. It is shown that the mixing effect gives the most significant contribution to the line broadening and it is indicated how to distinguish these two effects in laboratory experiments. The influence of the level mixing on the recombination history of primordial plasma is also discussed.

The influence of external fields on atomic characteristics is still one of the interesting subjects for investigations in modern atomic physics. In particular a question about the blackbody radiation (BBR) influence on atoms is widely discussed. First the BBR induced effects were observed experimentally and then the theoretical description was given in [1,2] within the frameworks of quantum mechanical (QM) approach. In particular, it was shown that the blackbody radiation induces the ac-Stark shift of energy levels and an additional line broadening in atoms. Theoretical calculations of the dynamic Stark shifts and depopulation rates of Rydberg energy levels caused by the BBR and the corresponding experimental measurements were widely discussed in literature [3-8]. The most important consequences of these investigations corresponds to the improvement of atomic clocks and the development of optical standards of frequency measurements [9].

Finally, in [10] the effect of level mixing induced by the blackbody radiation was firstly described theoretically within the rigorous quantum electromagnetic (QED) theory. The mixing effect for the states of opposite parity arising in the presence of an external electric field leads to a significant changes in the decay channel which was forbidden by the selection rules in the absence of an external field. As a consequence the mixing effect gives the most significant contribution to the level mixing of 2s and 2p states in hydrogen atom can mimick the parity non-conservation phenomenon.

In conjunction with the expression (2) the mixing effect induced by the thermal radiation can be introduced. The level mixing effect in an external electric field was considered in connection with the Lamb shift measurements in hydrogen and hydrogen-like ions [13,14] and the corresponding theoretical analysis of the electric field influence on atomic levels can be found in [15,16]. An accurate description of level mixing effect in hydrogen atom was given in [17]. In particular, the authors of [17] have shown that the mixing of 2s and 2p states in hydrogen atom can mimick the parity non-conservation phenomenon.

As a result of level mixing effect in presence of an external electric field the essential modification of the 2s state decay in hydrogen atom arises. This is due to the appearance of the one-photon electric dipole decay channel which was forbidden by the selection rules in absence of an external field. As a consequence the 2s state level in hydrogen atom does not remain a metastable one in presence of an external field.

We should note that in what concerns level mixing we include only 2p₁/₂ and 2s₁/₂ mixing and neglect 2p₃/₂ and 2s₃/₂ mixing. The reason is that the fine structure interval E₂p₅/₂ - E₂s₁/₂ is much larger than the Lamb shift E₂s₁/₂ - E₂p₁/₂ and consequently the mixing effect for 2p₅/₂ state is much smaller.

The one-photon decay rate of the mixed 2s state can be expressed as

\[ W^{(1γ)}_{2s₁}(k) = W^{(1γ)}_{2s₁γ}(k) \left[ 1 + e^2 \alpha E_0 |\eta|^2 \Gamma_{2p} \right] \frac{w}{w^2} \]

(3)

where E represents the electric field, \( n_k \) is the unit vector corresponding to the wave vector k of photon, w = \( \sqrt{W^{M1}_{2s₁γ}/W^{E1}_{2p₁γ}} \), the electron charge e and the Bohr’s radius \( a_0 \) are written explicitly for clarity. \( \Gamma_{2p} \) is the 2p level width and \( \eta = (\Delta E_{2p₁/₂}^L - 1/2 \Gamma_{2p})^{-1} \). \( \Delta E_{2p₁/₂}^L \) represents the Lamb shift between 2s₁/₂ and 2p₁/₂ levels, the one-photon transition probabilities W^{M1}_{2s₁γ} and W^{E1}_{2p₁γ} correspond to the emission of the magnetic dipole and electric dipole photons, respectively. Integration over photon emission direction \( n_k \) and frequency of the emitted photon \( \omega = |k| \) yields the expression

\[ \langle E^2(\omega) \rangle = \frac{8}{15} \alpha^3 (k_B T)^4 \]

(1)

where \( \langle E^2(\omega) \rangle \) is rms electric field strength, \( \omega \) is the radiation frequency. The Planck’s distribution function is presented by \( n_\beta(\omega) = 1/k_B T \) with \( k_B \) is the Boltzmann’s constant, \( T \) is the temperature in Kelvin and \( \alpha \) is the fine structure constant. Then the integral rms value of electric field is

\[ \langle E^2 \rangle = \frac{1}{2} \int_0^\infty \langle E^2(\omega) \rangle d\omega = \frac{4}{15} \alpha^3 (k_B T)^4 \]

(2)

= (8.319 V/cm)^2 [T(K)/300]^4.
\[ W_{2p1s}^{(1s)} = W_{2s1s}^{M1} + \frac{e^2 a_0^2 E_0^2}{4 \Gamma_1^2} W_{2p1s}^{E1}, \] (4)

where \( E_0 \) is the field amplitude. This expression shows that the additional one-photon electric dipole emission channel is allowed for the hydrogen-like atom in the metastable \( 2s \) state in presence of an external electric field. The term linear in the field in Eq. (3) vanishes after the integration over photon emission directions. In contrast, the term quadratic in the field does not depend on the photon emission or field directions. This contribution represents the quadratic mixing effect.

Since the decay rate of the E1 transition, \( W_{2p1s}^{E1} = 6.265 \times 10^8 \text{ s}^{-1} \), exceeds strongly the one-photon magnetic decay channel, \( W_{2s1s}^{M1} = 2.496 \times 10^{-6} \text{ s}^{-1} \), the second term in Eq. (4) may become the dominant decay channel of the mixed \( 2s \) state with increasing strength of the external electric field. The contribution of second term in (4) at the field strength 475 V/cm (easily achievable in laboratory experiments) becomes equal to the decay rate of the \( 2p \) level in hydrogen atom (the case of complete mixing) and is much larger than the main two-photon E1E1 decay rate of the \( 2s \) state in absence of the electric field, \( W_{2s1s}^{E1E1} = 8.299 \text{ s}^{-1} \). In turn, the same scenario can be considered for the mixing of \( 2p \) state in hydrogen atom. In this case there is no essential difference in the decay rate in external field because of the small additional contribution of the transition rates of the \( 2s \) level. Note, that this effect arises in presence of static electric field. According to the description above the rms value \( \langle \Delta E^2 \rangle \) of electric field caused by the BBR can be estimated by Eq. (2). Thus, the effect of level mixing should arise in presence of thermal radiation as it was demonstrated in [10].

However, the thermal radiation can not be described completely as a static electric field. The significant dynamical character of BBR modifies the form of the transition rate.

The full description of the dynamical effects as well as mixing effects was given in [10]. The QED expression for the BBR broadening is

\[ \Gamma_a^{BBR-QED} = \frac{2e^2}{3\pi} \sum_n \langle \langle a|f|n \rangle \rangle^2 \int_0^\infty d\omega \omega_3 \times \] (5)

\[ \left[ \frac{\Gamma_{na}}{(\omega_{na} + \omega)^2 + \frac{4}{4} \Gamma_{na}^2} + \frac{\Gamma_{na}}{(\omega_{na} - \omega)^2 + \frac{4}{4} \Gamma_{na}^2} \right], \]

where \( a \) and \( n \) denote the set of quantum numbers of corresponding atomic state, \( \omega_{na} \equiv E_n - E_a + \Delta E_{na}^E \), \( \Delta E_{na}^E \) is the corresponding Lamb shift and \( \Gamma_{na} \equiv \Gamma_n + \Gamma_a \). Expression (5) is the width of resonant emission line profile in presence of BBR.

For the dynamical effects in Eq. (5) the frequency-dependent energy denominators are responsible. The mixing effect is incorporated when the summation over \( n \) in Eq. (5) extends over the states with the space parity opposite to the parity of the state \( a \). In this case the main contribution comes from the state \( n \) close to \( a \). In hydrogen atom such states (\( 2s_{1/2} \) and \( 2p_{1/2} \) for example) are degenerate and \( \omega_{an} = \Delta E_{an}^E \) is the Lamb shift. For such levels as \( 2s \) in hydrogen which will be of our interest below, the mixing effect becomes dominant.

To clarify the physical situation we have to compare the result Eq. (5) with the well-known effect of the level broadening by the multiple photon scattering (Raman scattering in general case) on atomic levels: \( i + \gamma \rightarrow a \rightarrow f + \gamma' \), where \( i, f \) denote the initial and final states, respectively, \( a \) is the excited intermediate state and \( \gamma \) represents the emitted/absorbed photon. In case of BBR this effect was described in [2] and has application in cosmological recombination [18]. Below we will show that 1) the mixing broadening can not be reduced to the Raman scattering (RS); it is independent effect 2) mixing effect dominates over RS in line broadening 3) mixing effect leads to the emission lines that occur at frequencies Lamb shifted from frequencies corresponding to the emission lines broadened by the RS process 4) This frequency difference in principle is possible to observe in laboratory experiments what may give an access via these experiments to the study of primordial plasma.

For this purpose below we consider the simple RS process. The multiple photon scattering will be taken into account by introducing the Einstein coefficients, or the number of photons. In case of the BBR the number of photons is defined by the Planck’s distribution function.

The S-matrix element of the RS process can be written in the form [19], [21]:

\[ \tilde{S}^{(2)}_{fi} = (-ie)^2 \int dx_1 dx_2 \tilde{\psi}_f(x_1) \gamma_{\mu f} A_{\mu i}^{(k,e)_l}(x_1) \]

\[ \times S(x_1x_2) \gamma_{\mu i} A_{\mu i}^{(k,e)}(x_2) \tilde{\psi}_i(x_2), \]

where \( \psi_i(x) \) and Dirac conjugated \( \tilde{\psi}_f(x) \) represent the wave functions of the initial and final states, respectively, \( \gamma_{\mu i} \) are the Dirac matrices with \( \mu = 0, 1, 2, 3 \). The photon wave function (electromagnetic field potential) is described by

\[ A_{\mu}^{(k,e)}(x) = \sqrt{2} \frac{e}{\omega} e^{i(kx - \omega t)} = A_{\mu}^{(k,e)}(r) e^{-i \omega t}, \] (7)

where \( k \equiv (k, \omega) \) is the photon momentum 4-vector, \( k \) is the photon wave vector, \( \omega = |k| \) is the photon frequency, \( e_{\lambda}^{(k)} \) are the components of the photon polarization 4-vector. \( A_{\mu}^{(k,e)} \) and \( A_{\mu}^{(k,e)} \) in (6) correspond to the absorbed and emitted photon, respectively. In the Furry picture the eigenmode decomposition of Feynman electron propagator \( S(x_1, x_2) \) reads

\[ S(x_1x_2) = \frac{1}{2\pi} \int_{-\infty}^{\infty} d\omega \ e^{i\Omega(t_1-t_2)} \sum_n \psi_n(r_1) \tilde{\psi}_n(r_2) E_n(1 - i\Omega) + \omega, \] (8)

where the summation over \( n \) in Eq. (8) extends over the entire Dirac spectrum.

The differential absolute probability of emission process resulting from RS cross-section in case of resonant scattering is

\[ dw_{af}(\omega) = \frac{1}{2\pi} \frac{dW_{af}(\omega)}{(E_a - E_f - \omega)^2 + \frac{4}{4} \Gamma_{af}^2} \] (9)
where \( dW_{af} \) is the differential partial transition rate \( a \rightarrow f \).

Expression (9) follows from separation of absorption and emission processes, which become independent within the resonant approximation. The result (9) represents the emission line profile, i.e. the photon emission occurs at the resonant frequency \( \omega_{af} = E_a - E_f \). Integrating Eq. (9) over frequency \( \omega \) we find

\[
\omega_{af} = \frac{W_{af}^{E1}(\omega_{af})}{\Gamma_a}, \tag{10}
\]

where \( \omega_{af} \) is the absolute transition probability \( a \rightarrow f \) and \( W_{af}^{E1}(\omega_{af}) \) denotes the electric dipole one-photon spontaneous emission rate. Expression (10) is given for the case of one-photon emission process that arises as a result of Raman scattering in the resonant approximation. However, the presence of a photon field (in particular BBR) induces additional emission with the same frequency \([24]\). According to \([24]\) an induced photon emission probability is expressed via the number of photons. In our case this number is defined by the Planck’s distribution function, \( n_\beta \):

\[
W_{af}^{ind} = n_\beta(\omega_{af}) W_{af}^{E1}(\omega_{af}). \tag{11}
\]

Thus, the emission probability corresponding to the stimulated RS process is given by

\[
W_{af}(T) = (1 + n_\beta(\omega_{af})) W_{af}^{E1}(\omega_{af}). \tag{12}
\]

Then the total BBR-induced level broadening via RS process for an arbitrary level \( a \) in the nonrelativistic limit (neglecting all the types of photons except E1) looks like

\[
\Gamma_{a}^{BBR-RS} = \frac{4e^2}{3} \sum_f |\langle a|f\rangle|^2 \frac{\omega_{af}^3}{e^{3\omega_{af}} - 1}, \tag{13}
\]

and coincides precisely with the result obtained in \([2]\) within the QM approach. Here we have used an explicit expression for \( W_{af}^{E1} \). Expression (13) is the sum of all the partial transition probabilities including the higher excited states. The one-photon emission occurs at the corresponding resonant frequency \( \omega_{af} \).

Thus, we have two effects induced by the BBR: Raman scattering and level mixing. They have similar structure. The former one is described by the expression (13). The expression (5) is more general, it incorporates both effects. The result (13) can be obtained from Eq. (5) if we fully neglect the widths of the states \( n, a \). The limit \( \Gamma_{na} \rightarrow 0 \) in square brackets in Eq. (5) gives the sum of two delta functions \( \delta(\omega_{na} + \omega) + \delta(\omega_{na} - \omega) \) and, therefore, the RS result (13) arises immediately.

The QM mixing effect follow from Eq. (4) by substituting the rms value Eq. (2) instead of \( E_2 \). This result, also can be obtained from Eq. (5) but using different approximations than for RS effect. To do this we have to set \( \omega = 0 \) in square brackets in Eq. (5). It can be justified if we remember that function \( n_\beta(\omega) \) is concentrated at small \( \omega \) values for not too high temperatures. Moreover Eq. (4) follows from Eq. (5) when we take the contribution \( \Gamma_n \) from the sum \( \Gamma_n = \Gamma_n + \Gamma_a \) in the numerators in square brackets in Eq. (5). The contribution of \( \Gamma_a \) also appears to be important, but it corresponds not to the mixing but to the general ‘dynamic’ effect.

According to \([15, 17]\) the mixing of atomic levels with opposite parity occurs in a static electric field. Such a field can not induce any electron transitions in atoms. In laboratory experiments an electric field leading to the complete mixing of \( 2s, 2p \) states in hydrogen atom produces the Stark shift, which is much smaller than the Lamb shift. This circumstance allows the measurements of the Lamb shift with a high accuracy \([15, 16]\). The one-photon dipole emission (3) occurs at the resonant frequency \( \omega_{2s1s} = \omega_{2s1s} + \Delta E_{2s}^{Stark} \approx \omega_{2s1s} \), which is Lamb shifted from Ly\(_a \), transition frequency \( \omega_{2p1s} \approx \omega_{2p1s} \).

With the growth of strength of the electric field the intensity of emitted photons at the frequency \( \omega_{2s1s} \) increases and reaches the Ly\(_a \) value at the field strength corresponding to the complete mixing of \( 2s, 2p \) states. The difference of emission frequencies can serve as a tool for distinguishing of mixing and RS processes in laboratory experiments, see Fig. 1.

To make this physical picture clearer, the RS process should be considered in the case when the excited state \( a \) is mixed (we denote it by \( a' \)). Formally, it can be obtained by the substitutions \( a \rightarrow a' \) and \( E_a \rightarrow E_{a'} = E_a + \Delta E_{a}^{Stark} \approx E_a \), \( \Gamma_{a} = \Gamma_{a} + \Gamma_{a}^{BBR-QED} \), \( W_{af} \rightarrow W_{af}(\omega) \) into Eq. (9). Then in the resonant approximation we find

\[
d\omega_{af}(\omega) = \frac{1}{2\pi} \frac{dW_{af}(\omega_{af})}{(E_{af} - E_2)^2 + \frac{4\Gamma_{a}^{2}}{\omega_{af}}}. \tag{14}
\]

In this expression the resonance frequency \( \omega_{af} \) is concentrated at small \( \omega \) values for not too high temperatures. Moreover Eq. (4) follows from Eq. (5) when we take the contribution \( \Gamma_n \) from the sum \( \Gamma_n = \Gamma_n + \Gamma_a \) in the numerators in square brackets in Eq. (5). The contribution of the Ly\(_a \) frequency corresponds to \( \omega = 0.5 \). The corresponding values of transition rates and level widths are presented in Table I.
emission lines differ by the Lamb shift:

\[
\omega_{2p1s} - \omega_{2p1s} = E_{2p} - E_{2s} + \delta E_{2p}^L - \delta E_{2s}^L - \Delta E_{\text{Stark}}^L = \delta E_{2p}^L - \delta E_{2s}^L - \Delta E_{\text{Stark}}^L \approx \Delta E_{2s2p}^L, \tag{15}
\]

where \(\delta E_{a}^L\) denotes the Lamb shift of the state \(a\). In total, the situation with \(2p\) and \(2s\) levels in the BBR field looks as follows. Both levels are broadened by RS and mixing effects. The broadening of \(2p\) level modifies \(Ly_{\alpha}\) spectral emission line but does not change the emission frequency. The broadening of \(2s\) level leads to the arrival of the new spectral line (one-photon \(E1\) transition) with the frequency Lamb-shifted from \(Ly_{\alpha}\). This happens exclusively due to the mixing effect, not by RS. The broadening effects produced by the mixing are much stronger than the broadening effects produced by RS.

The numerical calculations of \(\Gamma_{a}^{BBR-QED}\) \([10]\) show that the mixing effect is dominant in comparison to the RS process, see Table I.

### Table I

Numerical values \(\Gamma_{a}^{BBR-QED}\) for \(2p\) and \(2s\) states in hydrogen atom in \(s^{-1}\) (the last two columns) for different values of radiation temperature \(T\) in Kelvin (first column). The corresponding \(\Gamma_{2p}^{BBR-RS}\) and \(\Gamma_{2s}^{BBR-RS}\) in \(s^{-1}\) are listed in second and third columns. The number in parentheses indicates the power of ten.

| \(T, K\) | \(\Gamma_{2p}^{BBR-RS}\) | \(\Gamma_{2s}^{BBR-RS}\) | \(\Gamma_{2p}^{BBR-QED}\) | \(\Gamma_{2s}^{BBR-QED}\) |
|---|---|---|---|---|
| 3 | 4.782(−8) | 1.434(−7) | 0.475 | 1.42 |
| 300 | 4.743(−6) | 1.422(−5) | 3.572(3) | 1.070(4) |
| 1000 | 0.033 | 2.023(−2) | 5.265(4) | 1.208(5) |
| 2000 | 1.916(3) | 11.783(2) | 7.134(8) | 1.207(8) |
| 3000 | 7.583(4) | 470.062(2) | 2.759(10) | 4.651(9) |
| 5000 | 1.522(6) | 967.009(3) | 5.198(11) | 8.760(10) |

In following the BBR-induced level mixing effect \([5]\) is discussed in application to the astrophysical investigation of the cosmological recombination epoch of the early universe (in SI units). The corresponding contribution can be evaluated similarly for the level mixing in helium atom caused by the spin-orbit interaction \([25, 26]\). Within the ‘three-level’ approach \([18]\) only the emission line corresponding to the one-photon decay in RS \(2p \rightarrow 1s + \gamma(E1)\) together with the two-photon decay of the \(2s\) state in hydrogen \(2s \rightarrow 1s + 2\gamma(E1)\) are taken into account. According to the discussion above the additional electric dipole decay channel \(2s \rightarrow 1s + \gamma(E1)\) should be included into the rate equations.

The latter can be transformed to the differential equation for the ionization fraction \(x_e = n_e/n_H\), where \(n_e\) is the free electron number density and \(n_H\) is the total number density of hydrogen atoms and ions. The time evolution of the density number of free electrons in a homogeneous, isotropic expanding universe can be described by the following differential equation

\[
\frac{dn_e}{dt} = - \sum_{nl} (\alpha_{H,nl}n_en_p - \beta_{H,nl}n_{nl}) - 3n_e, \tag{16}
\]

where \(n_{nl}\) is the number density of neutral hydrogen in the state with principal quantum number \(n\) and orbital momentum \(l\), \(n_p \approx n_e\) is a number density of protons, \(\alpha_{H,nl}\) is the recombination coefficient for the level \(nl\) and \(\beta_{H,nl}\) is the corresponding ionization coefficient. The last term in Eq. \(16\) describes the decreasing of number density \(n_e\) due to the cosmological expansion. The redshift \(z\) is related to time by the expression \(dz/dt = -(1+z)H(z)\), where \(H(z)\) is the Hubble factor \([18]\). The radiation temperature \(T_R\) is related to redshift as \(T_R = T_0(1+z)\), where \(T_0 = 2.725\) K is the present Cosmic Microwave Background (CMB) temperature. Then Eq. \(16\) can be rewritten in terms of ionization fraction \(x_e\) with the notations \(x_e = n_p/n_H\) and \(x_{2s} = n_{2s}/n_H\):

\[
\frac{dx_e}{dt} = - (\alpha_H x_e x_p - \beta_H x_{2s}) \equiv J_H, \tag{17}
\]

where \(\alpha_H\) and \(\beta_H\) are the total coefficients of recombination and ionization, respectively. Assuming that all the uncompensated transitions to the ground state \(J_H\) proceed via the two-photon decay \(2s \rightarrow 1s + 2\gamma(E1)\) and escape of \(Ly_{\alpha}\) photons \(2p \rightarrow 1s + \gamma(E1)\) due to the cosmological expansion \([27, 28]\) we arrive at the balance condition

\[
J_H = J_{2s}^{E1} + J_{2p}^{E1}, \tag{18}
\]

where \(J_{2s}^{E1}\) and \(J_{2p}^{E1}\) are the corresponding uncompensated transition rates.

Following to derivation of differential equation for the ionization fraction \(x_e\) \([18]\) we can introduce the contribution \(J_{2s}^{E1}\) for the one-photon transition rate \(2s \rightarrow 1s + \gamma(E1)\):

\[
\dot{J}_H = J_{2s}^{E1} + J_{2p}^{E1} + J_{2s}^{E1}. \tag{19}
\]

The contribution of \(J_{2s}^{E1}\) can be written in the same form as \(J_{2p}^{E1}\) \([18]\):

\[
J_{2s}^{E1} = P_{2s1s} A_{2s1s} \left( x_{2s} - \exp \left( -\frac{E_{2s} - E_{1s}}{k_BT} \right) x_{1s} \right). \tag{20}
\]

The two terms in right-hand side of Eq. \(20\) represent the difference between forward and backward one-photon transitions \(2s \leftrightarrow 1s\). The Einstein coefficient \(A_{2s1s}\) is defined as the partial transition rate in Eq. \(5\), i.e. \(A_{2s1s} = \Gamma_{a0}^{BBR-QED}\), where only one term from the sum over \(n\) is retained with \(n = a_0 = 2p\) and \(a = 2s\).
and differential equation for the variable \( x_e \). Insertion of (19)-(22) into Eq. (17) gives the differential equation for the variable \( x_e \).

The Sobolev escape probability \( P_{2s1s} \) and optical depth \( \tau_{2s} \) can be written as [18]

\[
P_{2s1s} = \frac{1 - e^{-\tau_{2s}}}{\tau_{2s}},
\]

(21)

\[
\tau_{2s} = \frac{A_{2s1s} n_{1s} c^3}{8 \pi H(z) \nu_{2s1s} g_{1s}}.
\]

(22)

Here \( g_{2s} \) and \( g_{1s} \) are the statistical weights of the states \( 2s \) and \( 1s \), respectively, \( \nu_{2s1s} \) is the corresponding transition frequency. Insertion of (19)-(22) into Eq. (17) gives the differential equation for the variable \( x_e \).

Then the modified equation for the ionization fraction \( x_e \) with respect to the redshift \( z \) is

\[
\frac{d\tilde{x_e}}{dz} = C_H \left( \alpha_H n_e \tilde{x_e} - \beta_H \exp \left( -\frac{\Delta E_e}{k_B T(z)} \right) (1 - \tilde{x_e}) \right),
\]

(23)

\[
C_H = \frac{g_{2s} A_{r_{1s}}^r}{g_{1s}} + \frac{g_{2s} A_{r_{2s}}^r}{g_{1s} + g_{1s}} \frac{A_{1s}^e}{A_{2s1s}} + A_{2s1s}.
\]

(24)

Here we have used the short notation for the effective coefficient \( A_{r_{2s}}^r A_{1s}^e = P_{2s(2s)1s} A_{2s1s} \). Thus, an additional decay channel of the \( 2s \) level has arisen in \( C_H \), that gives the difference with the standard case Eq. (18).

The ionization fraction \( x_e \) was evaluated with the use of Mathematica code. All necessary cosmological parameters are taken from [29]. The corresponding graph is presented in Fig. 2, where the function \( x_e(z) \) for the case of local thermodynamic equilibrium (LTE), i.e. evaluated with the use of Saha equation, is depicted as a dotted-green line. Evaluation of ‘ordinary’ rate equation [18] is shown by the dashed-yellow line and solid-blue line represents ionization fraction with the account for BBR-induced level mixing.

We find a significant influence of the BBR-induced mixing effect on the ionization fraction in the cosmological recombination epoch of the early universe. However, the period of recombination is almost the same. Thus, the possible modification of the CMB temperature fluctuations map can be expected in the far tail of multipole expansion. The relative difference between ionization fraction from Eq. (23) and calculated within the ‘ordinary’ approach \( \Delta x_e/x_e \approx (\tilde{x_e} - x_e)/x_e \) is presented in Fig. 3. It is shown that the mixing effect is important during the period of cosmological recombination and reaches 20% at \( z \approx 1000 \). Therefore the contribution of level mixing effect should be taken into account in detailed investigation of cosmological recombination epoch. It is important that the existence of the level mixing effect can be tested in laboratory experiments as it was discussed above. Hence the laboratory studies may give an access to the details of cosmological recombination.

FIG. 2. Ionization fraction \( x_e \) as a function of redshift \( z \). The dotted-green line corresponds to the LTE case (Saha equation), the dashed-yellow line is given by the evaluation of ‘ordinary’ rate equation and the solid-blue line represents ionization fraction with the account for BBR-induced level mixing.

FIG. 3. Relative difference \( \Delta x_e/x_e \) as a function of redshift \( z \).

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