Alternative refined Gribov-Zwanziger Lagrangian

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Abstract. We consider the implications of the condensation of a general local BRST invariant dimension two operator built out of the localizing ghost fields of the Gribov-Zwanziger Lagrangian which is a localized Lagrangian incorporating the Gribov problem in the Landau gauge. For different colour tensor projections of the general operator, the properties of a frozen gluon propagator and unenhanced Faddeev-Popov ghost propagator, which are observed in lattice computations, can be reproduced. The alternative possibilities are distinguished by the infrared structure of the propagators of the spin-1 fields, other than those of the gluon and Faddeev-Popov ghost, for which there is no numerical simulation data to compare with yet.
1 Introduction.

Yang-Mills theories play an important role in our understanding of the fundamental particles of nature. For instance, the strong force is believed to be reliably described by an $SU(3)$ non-abelian gauge theory whose fundamental particles are quarks and gluons. In nature these particles cannot be isolated as separate entities. Instead at high energies they are asymptotically free, [1, 2], but at low energies they are present only within bound states of hadrons. Understanding the specific dynamics of the mechanism which confines quarks and gluons and prevents them from being observed in nature is currently a major topic of interest. Ordinarily in a quantum field theory the behaviour of a particle’s propagator plays a key role in its interpretation as an observed quanta. For instance, in Quantum Electrodynamics (QED) the electron propagator has a simple pole at real positive value of its squared momentum which corresponds to its physical mass when all quantum corrections have been computed to all orders in perturbation theory. The form of this fundamental propagator is therefore associated with a real observed particle. By contrast in Quantum Chromodynamics (QCD), which is the non-abelian generalization for the strong force, the propagators of the quark and gluon fields derived from the canonical Lagrangian emerge as being of the same fundamental form but in the case of the gluon it is massless. Clearly this cannot be correct since that would imply that the gluon should exist as a free massless observed particle. However, this analysis derives from a gauge fixed Lagrangian with a non-abelian symmetry. It transpires that contrary to what occurs in an abelian theory there are problems in fixing the gauge uniquely globally in Yang-Mills theories. This was first pointed out in detail by Gribov in his seminal work, [3]. In essence one can only fix a gauge uniquely locally in a non-abelian gauge theory but not globally. It turns out that one can construct different gauge configurations satisfying the same gauge condition in the non-abelian case. Therefore, there is an overcounting in the definition of the path integral and the region of integration in configuration space has to be restricted to a specific subspace. This is known as the first Gribov region and it contains the origin, [3]. Whilst this overcomes a significant amount of the copy issue the gauge is still not uniquely fixed. Only inside the fundamental modular region which is within this horizon is there a unique globally fixed gauge.

In analysing this problem for the Landau gauge Gribov, [3], managed to determine several interesting features. First, the path integral could be modified to incorporate the restriction to the Gribov region. The boundary is defined by the no-pole condition where the Faddeev-Popov operator vanishes. This means that the inverse is finite within the horizon of the Gribov region. Using a semi-classical analysis Gribov demonstrated that the path integral cut-off modified the gluon propagator. In particular the fundamental behaviour was replaced by a propagator which vanished at zero momentum, known as gluon suppression, but the simple massless pole property accepted at high energy was retained. Underlying this was a new mass parameter called the Gribov mass and denoted by $\gamma$, [3]. It is not an independent quantity but is a function of the coupling constant, $g$, and defined via the horizon condition cutting off the path integral leading to a gap equation. Only when the Gribov mass actually satisfies the gap equation can the theory be regarded as a gauge theory. This gap equation was responsible for another novel feature which is that the Faddeev-Popov ghost propagator behaved as a dipole in the infrared which is known as ghost enhancement. This latter property was subsequently reformulated in terms of the Kugo-Ojima confinement criterion, [4, 5], which was established by a BRST analysis of the Landau gauge. However, one of the main consequences is that the analysis showed that the gluon propagator ceased to be fundamental when the copy problem was taken into consideration and the hope was that the Gribov problem was fundamental to the problem of confinement, [3].

One of the main difficulties in extending the semi-classical analysis of [3] was that the no pole condition introduces a non-local operator into the Lagrangian. This is an obstacle to performing
canonical field theory calculations which only proceed when a Lagrangian is at least local and renormalizable. However, in a series of articles, [6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16], Zwanziger overcame this by managing to not only rewrite the Gribov Lagrangian in a local form but also in a way that it was renormalizable, [13, 17, 18]. The localization proceeded by the introduction of a set of additional ghost fields, \( \{ \phi_{ab}^{\mu}, \bar{\phi}_{ab}^{\mu}, \omega_{ab}^{\mu}, \bar{\omega}_{ab}^{\mu} \} \). The first pair are bosonic whilst the second set have Fermi statistics and are necessary to ensure the theory remains asymptotically free at high energy, for instance. The renormalization group functions are not affected by these additional fields whose effect is only apparent in the infrared region, [13, 17, 18]. Moreover, no extra renormalization constants are required since the anomalous dimensions of the localizing ghosts are the same as that of the Faddeev-Popov ghost and \( \gamma \) is rendered finite merely by a simple combination of the gluon and ghost renormalization constants. With these additions to the Lagrangian the original suppressed gluon propagator is retained. However, one can study the loop corrections using the localized Gribov version which is known as the Gribov-Zwanziger Lagrangian. For instance, the one loop gap equation can be computed, [8, 13], and reproduces that of Gribov, [3]. This was extended to two loops in the \( \overline{MS} \) scheme in [19] with the one loop propagator corrections also deduced in [20, 21]. As noted in [11, 12] by Zwanziger the gluon remains suppressed when quantum corrections are included. Also the Kugo-Ojima criterion was shown to be satisfied at two loops, [19, 20]. Subsequently this criterion was revisited but in the context of the Gribov-Zwanziger Lagrangian, [22], where the additional localizing fields have to be included in repeating the original BRST analysis of [4, 5]. It was shown that the Kugo-Ojima condition was valid in the Gribov-Zwanziger Lagrangian.

Having summarized the theoretical analysis of accommodating the gauge copy problem in the Landau gauge in QCD the next issue in this area relates to lattice data collected over the last few years on the gluon and Faddeev-Popov ghost propagators. Given the advances in computing power and algorithms to carry out the gauge fixing numerically on larger lattices, [23, 24, 25, 26, 27, 28, 29, 30, 31], there is now a reasonable amount of data on both propagators at low momenta. More specifically, the zero momentum behaviour appears to show clearly that contrary to the Gribov scenario the gluon propagator freezes to a non-zero value and the Faddeev-Popov ghost does not enhance. Instead its infrared form does not significantly deviate from that of its ultraviolet fundamental massless form. Indeed there is also similar evidence from Schwinger-Dyson computations. See, for example, [31, 32, 33]. Thus there are two distinct cases. One is the gluon suppressed but ghost enhanced propagators known as the scaling or conformal solution. The other is the non-zero frozen gluon propagator with the unenhanced ghost. Such a gluon propagator was originally derived in [33] in a Schwinger-Dyson analysis and referred to as the massive solution. More recently, [34], the term decoupling solution has been used. As both labels are recognised we will use them synonymously in the discussion. Therefore, there has been a debate as to how the numerical data fit in with the Gribov scenario, [3], and whether the absence of the Kugo-Ojima confinement was significant. One point of view was that satisfying the criterion or not was an additional condition on the gauge fixing procedure, [35], effective in the infrared. However, given the properties and elegance of the Gribov-Zwanziger Lagrangian it seemed appropriate to try and extract a gluon propagator with non-zero freezing and a non-enhanced Faddeev-Popov ghost in that approach. This was achieved in the series of articles, [36, 37, 38]. In essence the Lagrangian was refined to include a local BRST invariant dimension two operator built from the localizing ghost fields. Ordinarily such a term would just introduce masses for these fields only. However, the complicated way the localization arises means that in constructing the propagators from the quadratic sector the extra mass term affects the gluon propagator too. Specifically the extra mass parameter appears in the gluon propagator in such a way that there is non-zero freezing at zero momentum. Equally the extra mass excludes the enhancement of the Faddeev-Popov ghost and therefore the numerical data and Schwinger-Dyson
analyses can be modelled by what was termed a refinement of the original Gribov-Zwanziger Lagrangian. The justification for the inclusion of this additional mass operator is to consider the condensation of the operator via a dynamical mechanism. The analysis for this is achieved by the local composite operator (LCO) formalism, \[39, 40\], which was initially used in QCD to study the condensation of a gluon mass operator, \[41, 42\].

Whilst the computation of \[36, 37, 38\] clearly accommodates the current picture of the gluon and ghost propagators it transpires that the most general dimension two BRST operator was not considered. Given that the localizing fields carry more than one colour adjoint index there are six different colour projections. Therefore, it is the purpose of this article to perform a comprehensive analysis of the various different colour channels. It will turn out that there are other operators whose condensation will lead to a non-zero freezing of the gluon propagator and an unenhanced Faddeev-Popov ghost propagator. Therefore, the refined Gribov-Zwanziger Lagrangian of \[36, 37, 38\] is not the unique model of the lattice data which is why we will refer to the explanations here as an alternative. Moreover, in our results we will show that the conformal or scaling behaviour of the original or pure Gribov-Zwanziger analysis can persist even with the extra mass operator. Although the alternative we focus on here, which will be referred to as the \(\mathcal{R}\) channel for reasons which will be clear later, is equivalent to the earlier explanation, which will be called the \(\mathcal{Q}\) channel, the full infrared behaviour of both are not equivalent. For instance, the behaviour of the Bose ghost propagator is different in each channel. The study of this particular propagator has been of interest recently in the pure Gribov-Zwanziger Lagrangian, \[43, 21\]. It has been shown that there is enhancement in certain colour channels. This was observed at the one loop level, \[21\], but proved to be an all orders feature in \[43\]. Briefly, the enhanced Bose fields are the Goldstone bosons associated with the spontaneous breaking of the BRST symmetry in a theory where the fields are constrained by the horizon condition, \[43\]. In principle one could resolve which of the \(\mathcal{Q}\) or \(\mathcal{R}\) cases was correct when the BRST invariant operator is included if there was numerical data for this propagator. However, these localizing ghost fields only exist within the Gribov-Zwanziger Lagrangian itself and not within the lattice formulation of QCD. Therefore, it is not clear whether one could construct the equivalent correlation function in the lattice language. Whilst we will carry out the one loop analysis for the \(\mathcal{R}\) channel we will also give a general indication of the properties of the additional colour channel possibilities in order to have as a complete picture as possible. This would become important if additional numerical data became available which rules out either of the \(\mathcal{Q}\) or \(\mathcal{R}\) channel explanations. Interestingly in certain scenarios the colour tensor structure which was observed in \[43, 21\], but for the enhanced Bose localizing propagator at one loop, actually arises in the dominant term in the zero momentum limit in the original propagator prior to any loop analysis.

The article is organised as follows. The key properties of the pure Gribov-Zwanziger Lagrangian are reviewed in the next section. The most general local BRST invariant dimension two operator built from the localizing ghosts is introduced in section three and the propagators are constructed for each of the six potential single additional mass parameters. The justification for the inclusion of such additional masses is provided in the subsequent section where the focus is on the \(\mathcal{R}\) channel. There the one loop effective potential for the operator is constructed as well as the one loop \(\overline{\text{MS}}\) corrections to all the propagators. These are required to study the effect of the gap equation satisfied by \(\gamma\) has on the propagator corrections in order to see, for example, whether there is any Bose ghost enhancement. Our conclusions are given in section five. There are two appendices. The first gives the full propagators for the inclusion of all six colour channel projections of the operator for the case of \(SU(3)\) whilst the second records the propagators for the \(W\) and \(S\) channels separately for \(SU(N_c)\). Whilst the expressions recorded in both appendices for the propagators are cumbersome they are significantly compact compared with those for the general colour group case.
2 Background.

We begin by briefly recalling the essential features of the Gribov-Zwanziger Lagrangian and the construction of the propagators. Throughout the article we work in the Landau gauge. The basic Lagrangian is, \[^3\],

\[
L^\text{Grib} = L^\text{QCD} + \frac{C_A \gamma^4}{2} A^a_{\mu} \frac{1}{\partial^\nu D^\nu} A^a_{\mu} - \frac{d N_A \gamma^4}{2 g^2}
\]  

(2.1)

where the original gauge fixed Lagrangian is

\[
L^\text{QCD} = - \frac{1}{4} G^a_{\mu\nu} G^{a\mu\nu} - \frac{1}{2 \alpha} (\partial^\mu A^a_{\mu})^2 - \bar{c}^a \partial^\mu D^\mu c^a + i \bar{\psi}^i D^i \psi^i
\]

(2.2)

and \(\alpha\) is the covariant gauge fixing parameter. Although we have included \(\alpha\) we will focus purely on the Landau gauge, corresponding to \(\alpha = 0\), but it is retained here since it is crucial in obtaining the propagators. The parameter \(\gamma\) is the Gribov mass which is not an independent quantity but is the mass scale which is implicit in the horizon condition, \[^3\],

\[
\left\langle A^a_{\mu}(x) \frac{1}{\partial^\nu D^\nu} A^{a\mu}(x) \right\rangle = \frac{d N_A}{C_A g^2}
\]

(2.3)

where \(N_A\) is the dimension of the adjoint representation. The presence of the horizon condition alters the infrared behaviour of the gluon and Faddeev-Popov propagators in the original Gribov scenario, \[^3\]. As has already been well documented the non-local term renders practical computations impossible. However, Zwanziger’s series of articles on how one can localize the non-locality produces a renormalizable local Lagrangian, \[^7\] \[^8\] \[^11\] \[^12\] \[^13\] \[^14\] \[^44\], which is

\[
L^\text{GZ} = L^\text{QCD} + \frac{1}{2} \rho_{ab}^\mu \partial^\nu (D_\nu \rho_{ab}^\mu )_{ab} + i \frac{1}{2} \rho_{ab}^\mu \partial^\nu (D_\nu \xi_{ab}^\mu )_{ab} - i \frac{1}{2} \xi_{ab}^\mu \partial^\nu (D_\nu \omega_{ab}^\nu )_{ab} - \frac{1}{\sqrt{2}}  g f^{abc}_{\mu\nu} \bar{c}^c \partial^\nu (D_\nu)_{\omega_{ab}^\mu} - \frac{1}{\sqrt{2}}  g f^{abc}_{\mu\nu} \bar{c}^c \partial^\nu (D_\nu)_{\omega_{ab}^\mu} - \frac{d N_A \gamma^4}{2 g^2} .
\]

(2.4)

There are additional fields to the gluon, \(A^a_{\mu}\), Faddeev-Popov ghost, \(c^a\), and massless quark, \(\psi^i\), which are two Bose ghosts, \(\rho_{ab}^\mu\) and \(\rho_{ab}^\mu\), and the Grassmann ghosts \(\omega_{ab}^\mu\) and \(\omega_{ab}^\mu\). The indices range over the values \(1 \leq a \leq N_A\), \(1 \leq I \leq N_F\) and \(1 \leq i \leq N_f\) where \(N_F\) is the dimension of the fundamental representation and \(N_f\) is the number of massless quarks. In earlier localized versions of the Gribov Lagrangian the complexified Bose ghosts \(\phi_{ab}^\mu\) and \(\phi_{ab}^\mu\) were used. Given recent developments, \[^13\] \[^44\], in terms of the propagator structure of the real versions of the Bose ghosts we will use this formulation but note that the relation between the two versions are trivial and given by

\[
\phi_{ab}^\mu = \frac{1}{\sqrt{2}} (\rho_{ab}^\mu + i \xi_{ab}^\mu) , \quad \phi_{ab}^\mu = \frac{1}{\sqrt{2}} (\rho_{ab}^\mu - i \xi_{ab}^\mu)
\]

(2.5)

However, the renormalizability proofs of the Lagrangian, \[^13\] \[^17\] \[^18\], were performed for the complex Bose ghost field and used the BRST symmetry of the localized Lagrangian. Since we will be considering a BRST dimension two operator we note the BRST symmetry transformations for (2.4) given (2.5) are

\[
\delta A^a_{\mu} = -(D_\mu c)^a , \quad \delta c^a = \frac{1}{2} f^{abc} c^b c^c , \quad \delta \bar{c}^a = b^a , \quad \delta b^a = 0
\]

\[
\delta \phi_{ab}^\mu = \omega_{ab}^\mu , \quad \delta \omega_{ab}^\mu = 0 , \quad \delta \bar{\phi}_{ab}^\mu = 0 , \quad \delta \bar{\omega}_{ab}^\mu = \bar{\phi}_{ab}^\mu
\]

(2.6)
where \( b^a \) is the Nakanishi-Lautrup auxiliary field. Therefore, it is trivial to observe that the colour non-singlet operator

\[
O^{abcd} = \bar{\phi}^{ab} \phi^{cd} - \bar{\omega}^{ab} \omega^{cd} \tag{2.7}
\]

is BRST invariant. Rewriting in terms of the real Bose ghosts the operator is equivalent to

\[
O^{abcd} = \frac{1}{2} \left[ \rho^{ab} \rho^{cd} + i \xi^{ab} \rho^{cd} - i \rho^{ab} \xi^{cd} + \xi^{ab} \xi^{cd} \right] - \bar{\omega}^{ab} \omega^{cd}. \tag{2.8}
\]

In [36, 37, 38] it was the condensation of the operator \( \delta^{ac} \delta^{bd} O^{abcd} \) which was investigated using the local composite operator formalism in order to see what effect it had on the infrared structure of the propagators.

In this localized Lagrangian the horizon condition, (2.3), is replaced by the vacuum expectation value of local fields, [43, 44],

\[
f^{abc} \langle A^\mu_a(x) \xi^b_\mu(x) \rangle = \frac{idN_A \gamma^2}{g^2} \tag{2.9}
\]

which determines \( \gamma \) to two loops in the \( \overline{\text{MS}} \) scheme, [19], by the solution of

\[
1 = \frac{5}{8} - \frac{3}{8} \ln \left( \frac{C_A \gamma^4}{\mu^4} \right) a
+ \left[ \frac{2}{3} \ln \left( \frac{C_A \gamma^4}{\mu^4} \right) + \frac{35}{128} \left( \ln \left( \frac{C_A \gamma^4}{\mu^4} \right) \right)^2 \right]
+ \frac{25}{24} - \frac{7}{12} \ln \left( \frac{C_A \gamma^4}{\mu^4} \right) \tag{2.10}
\]

where \( \zeta(z) \) is the Riemann zeta function, \( s_2 = (2\sqrt{3}/9)\text{Cl}_2(2\pi/3) \) and \( \text{Cl}_2(x) \) is the Clausen function and \( a = g^2/(16\pi^2) \). However, it was demonstrated recently that the horizon condition could be replaced by a vacuum expectation value of an operator involving only the Bose ghost, albeit one involving an infinite number of terms, [45]. Specifically,

\[
f_4^{abcd} \left\langle \partial^\nu \xi^{ab}_\mu \left[ \partial_\nu \xi^{cd}_\mu - \frac{ig}{C_A \gamma^2} f_4^{cfrs} (\partial^\sigma \xi^{rs}_\nu) \xi^{fd}_\mu \right. \right.
- \frac{g^2}{C_A \gamma^4} f_4^{cfrs} f_4^{qmn} \partial^\sigma \left( \partial^\rho \partial_\sigma \xi^{mn}_\rho \right) \xi^{gs}_\nu \xi^{fd}_\mu \left. \right]
+ O(g^3) \right\rangle = \frac{dC_A N_A \gamma^4}{g^2} \tag{2.11}
\]

where \( f_4^{abcd} \equiv f^{abp} f^{cdp} \). This reproduces (2.10) to two loops and is constructed by application of the equation of motion

\[
A^a_\mu = - \frac{i}{C_A \gamma^2} f^{abc} (\partial^\nu D_\nu \xi_\mu)^bc \tag{2.12}
\]

to (2.9) via the intermediate vacuum expectation value

\[
f^{abp} f^{cdp} \left\langle \xi^{ab}_\mu (x) (\partial^\nu D_\nu \xi_\mu)^{cd} (x) \right\rangle = - \frac{dC_A N_A \gamma^4}{g^2} \tag{2.13}
\]
where \( d \) is the spacetime dimension.

One of the reasons for reviewing the various formulations of the gap equation is to motivate our alternative mechanism for modelling the massive or decoupling solution observed on the lattice. First, we recall that the Landau gauge propagators for the spin-1 fields of (2.4) are

\[
\langle A^a_\mu(p)A^b_\nu(-p) \rangle = -\frac{\delta^{ab}p^2}{[(p^2)^2 + C_A \gamma^4]} P_{\mu\nu}(p)
\]

\[
\langle A^a_\mu(p)\epsilon^{bc}_\nu(-p) \rangle = \frac{if^{abc}\gamma^\nu}{[(p^2)^2 + C_A \gamma^4]} P_{\mu\nu}(p)
\]

\[
\langle A^a_\mu(p)\eta^{bc}_\nu(-p) \rangle = 0
\]

\[
\langle \xi^{ab}_\mu(p)\xi^{cd}_\nu(-p) \rangle = -\frac{\delta^{ac}\delta^{bd}}{p^2} \eta_{\mu\nu} + \frac{f^{abe}f^{cde}\gamma^A}{p^2[(p^2)^2 + C_A \gamma^4]} P_{\mu\nu}(p)
\]

\[
\langle \xi^{ab}_\mu(p)\rho^{cd}_\nu(-p) \rangle = 0
\]

\[
\langle \rho^{ab}_\mu(p)\rho^{cd}_\nu(-p) \rangle = \langle \omega^{ab}_\mu(p)\omega^{cd}_\nu(-p) \rangle = -\frac{\delta^{ac}\delta^{bd}}{p^2} \eta_{\mu\nu}
\]

(2.14)

where

\[
P_{\mu\nu}(p) = \eta_{\mu\nu} - \frac{p_\mu p_\nu}{p^2}, \quad L_{\mu\nu}(p) = \frac{p_\mu p_\nu}{p^2}
\]

are the usual transverse and longitudinal projectors. Therefore to reproduce the leading order contribution to the gap equation, and hence obtain Gribov’s original expression, one simply integrates the mixed \( A^a_\mu\xi^{ab}_\nu \) propagator of (2.14). Equally the alternative formulation of the gap equation involving only the Bose ghost can be justified at leading order by noting that the second term of the \( \xi^{ab}_\mu \) propagator needs the massless pole to be absent. This is achieved by the inclusion of the wave operator, \( \partial^2\partial_\sigma \), at leading order, [3], and eventually correctly with the Faddeev-Popov operator in keeping with the ethos of Zwaniger’s extension of Gribov’s semiclassical horizon condition, [7]. Given this, one can obtain an idea of which of the other operators may condense and be dominant or relevant in the infrared at leading order by considering other propagators of (2.14). Clearly integrating the gluon propagator suggests the condensation of the gauge variant operator \( \frac{1}{2}A^a_\mu A^a_\mu \) which has been widely treated. However, if one considers the BRST invariant operator, (2.8), and integrating the relevant Bose and Grassmann propagators of (2.14) we find the colour dependence

\[
\langle O^{abcd} \rangle \propto f^{abe}f^{cde}.
\]

(2.16)

Clearly the other colour channel of the relevant propagators, \( \delta^{ac}\delta^{bd} \), gives zero upon integration in dimensional regularization. Whilst we are not saying that there could be no condensation in this colour channel, or any others not present here, we are suggesting that the more natural colour channel to condense is that of (2.16). It is natural in the sense that it does not differ too much from the original localized Lagrangian structure. Moreover, it would appear to be the dominant expectation value at leading order. Given this simple observation we will examine the consequences for this taking this colour channel choice in (2.8) and discuss alternative cases as well in the next section.

3 Propagators.

In this section we consider the construction of the spin-1 propagators for (2.4) when there are extra mass terms eminating from a BRST invariant mass. As \( O^{abcd} \) is BRST invariant then in
order to have a colour scalar it needs to be contracted with a rank 4 colour tensor. To be as flexible as possible at the outset we take the general operator
\[
\mathcal{O} = \left[ \mu_2^2 \delta^{ac} \delta^{bd} + \mu_2^2 f^{ace} f^{bde} + \frac{\mu_2^2}{C_A} f^{abe} f^{cde} + \mu_2^2 A_{\alpha}^{abcd} + \frac{\mu_2^2}{N_A} \delta^{ab} \delta^{cd} + \mu_2^2 \delta^{ad} \delta^{bc} \right] \mathcal{O}^{abcd}
\]
(3.1)
where \(\mu_2^2\) are the various mass parameters. The labelling is chosen in order to track the location of the various colour structures within the final propagators. The inclusion of \(C_A\) and \(N_A\) in several terms is in order to have a degree of uniformity in the various different final propagator forms and to make relative comparisons easy to follow as will be evident later. In addition to be complete we include an additional dimension two operator which is
\[
\mathcal{O}_{A_{\alpha}^{a}}^{a} = \frac{\mu_2^2}{2} A_{\alpha}^{a} A_{\alpha}^{a} .
\]
(3.2)
In a general linear covariant gauge for \(\mathcal{O}_{A_{\alpha}^{a}}^{a}\) the operator \(\frac{1}{2} A_{\alpha}^{a} A_{\alpha}^{a} \) is BRST invariant and was introduced as a potential gluon mass operator by Curci and Ferrari in \(\text{[46]}\). So in the Landau gauge \((3.2)\) represents a natural additional operator in the context of studying the BRST operator for the localizing ghost sector. So it seems appropriate to include it in our current construction to ascertain its effect. The subscripts, \(X\), \(Q\), \(W\), \(R\), \(S\), \(P\) and \(T\), derive from notation used in earlier papers, \([20\ 21]\). There the colour structure of the one loop corrections to the 2-point functions was examined and the inversion of the appropriate matrix of 2-point functions was performed to obtain the one loop propagator corrections. This involved understanding the multiplication of the colour tensors associated with each amplitude. As the colour structure for the BRST invariant mass term is now of the most general form, we will use the same approach as \([20\ 21]\) to construct the propagators and recall the essentials for this.

First, for the spin-1 sector we focus on the fields which mix through the term involving \(\gamma^2\) in \((3.3)\). Therefore, we take our basis of fields to be \(\{A_{\mu}^{a}, \xi^{ab}_{\mu}, \rho^{ab}_{\mu}\}\) in that order. Although there is no mixing with \(\rho^{ab}_{\mu}\) we have included it here since there is potentially an effect in the longitudinal sector. Next we define the matrix of quadratic terms in the momentum space version of the Lagrangian as, \([20]\),
\[
\Lambda_{ab|cd}^{\{\} \xi} = \begin{pmatrix}
\mathcal{X} \delta^{ac} & \mathcal{U} f^{acd} & \mathcal{U} f^{abcd} & 0 \\
\mathcal{U} f^{cab} & \mathcal{Q}_{\xi}^{abcd} & \mathcal{Q}_{\xi}^{abcd} & 0 \\
0 & 0 & \mathcal{Q}_{\rho}^{abcd} & \mathcal{Q}_{\rho}^{abcd}
\end{pmatrix},
\]
(3.3)
for the transverse sector and
\[
\Lambda_{ab|cd}^{L} = \begin{pmatrix}
\mathcal{X}^{L} \delta^{ac} & \mathcal{U}^{L} f^{acd} & \mathcal{U}^{L} f^{abcd} & \mathcal{U}^{L} f^{abcd} \\
\mathcal{U}^{L} f^{cab} & \mathcal{Q}_{\xi}^{abcd} & \mathcal{Q}_{\xi}^{abcd} & \mathcal{Q}_{\rho}^{abcd} \\
\mathcal{V}^{L} f^{cab} & \mathcal{Q}_{\rho}^{abcd} & \mathcal{Q}_{\rho}^{abcd}
\end{pmatrix},
\]
(3.4)
for the longitudinal sector where we will use the superscript \(L\) throughout to differentiate it from the transverse sector. The respective Lorentz projectors are passive and omitted, \([20\ 21]\). The colour decompositions are taken to be
\[
\mathcal{Q}_{\xi}^{abcd} = \mathcal{Q}_{\xi}^{abcd} \delta^{ac} \delta^{bd} + \mathcal{W}_{\xi} f^{ace} f^{bde} + \mathcal{R}_{\xi} f^{abe} f^{cde} + \mathcal{S}_{\xi} d_{A_{\alpha}^{abcd}} + \mathcal{P}_{\xi} \delta^{ab} \delta^{cd} + \mathcal{T}_{\xi} \delta^{ad} \delta^{bc}
\]
\[
\mathcal{Q}_{\rho}^{abcd} = \mathcal{Q}_{\rho}^{abcd} \delta^{ac} \delta^{bd} + \mathcal{W}_{\rho} f^{ace} f^{bde} + \mathcal{R}_{\rho} f^{abe} f^{cde} + \mathcal{S}_{\rho} d_{A_{\alpha}^{abcd}} + \mathcal{P}_{\rho} \delta^{ab} \delta^{cd} + \mathcal{T}_{\rho} \delta^{ad} \delta^{bc}
\]
(3.5)
which is the origin of our earlier notation and
\[
\mathcal{Q}_{\xi}^{L, abcd} = \mathcal{Q}_{\xi}^{L, abcd} \delta^{ac} \delta^{bd} + \mathcal{W}_{\xi}^{L} f^{ace} f^{bde} + \mathcal{R}_{\xi}^{L} f^{abe} f^{cde} + \mathcal{S}_{\xi}^{L} d_{A_{\alpha}^{abcd}} + \mathcal{P}_{\xi}^{L} \delta^{ab} \delta^{cd} + \mathcal{T}_{\xi}^{L} \delta^{ad} \delta^{bc}
\]
\[
\mathcal{Q}_{\rho}^{L, abcd} = \mathcal{Q}_{\rho}^{L, abcd} \delta^{ac} \delta^{bd} + \mathcal{W}_{\rho}^{L} f^{ace} f^{bde} + \mathcal{R}_{\rho}^{L} f^{abe} f^{cde} + \mathcal{S}_{\rho}^{L} d_{A_{\alpha}^{abcd}} + \mathcal{P}_{\rho}^{L} \delta^{ab} \delta^{cd} + \mathcal{T}_{\rho}^{L} \delta^{ad} \delta^{bc}
\]
(3.6)
with
\[ d^{a b c d}_A = \frac{1}{6} \text{Tr} \left( T_A^a T_A^b T_A^c T_A^d \right) \] (3.7)

being the rank 4 totally symmetric tensor in the adjoint representation, \[47\]. The propagators are then obtained by inverting the quadratic part of the momentum space Lagrangian. However, this is more involved than usual due to the colour structure and is formally given by the corresponding colour structures for the transverse sector,

\[ \Pi^{(c d | p q)} = \begin{pmatrix} A \delta^{c p} & B f^{c p q} & 0 \\ D f^{b c d} & D^{c d p q} & 0 \\ 0 & 0 & D^{c d p q} \end{pmatrix} \] (3.8)

and

\[ \Pi^{L (c d | p q)} = \begin{pmatrix} A^L \delta^{c p} & B^L f^{c p q} & C^L f^{c p q} \\ D^L f^{b c d} & D^{c d p q} & \xi^L \xi^{c d p q} \\ C^L f^{b c d} & \xi^L \xi^{c d p q} & \xi^L \xi^{c d p q} \end{pmatrix} \] (3.9)

for the longitudinal part of the propagators. The two sectors of the propagators can be split since we employ the projectors \( P_{\mu \nu} (p) \) and \( L_{\mu \nu} (p) \) which satisfy

\[ \eta_{\mu \nu} = P_{\mu \nu} (p) + L_{\mu \nu} (p) , \quad P_{\mu} (p) L_{\nu \sigma} (p) = 0 . \] (3.10)

We use a similar decomposition for the colour tensors with

\[ D^{c d p q} = D_{\xi} \delta^{c p} \delta^{d q} + \mathcal{J}_{\xi} f^{c p e} f^{d q e} + K_{\xi} f^{c de} f^{p q e} + L_{\xi} d^{c d p q} + M_{\xi} \delta^{c d} \delta^{p q} + N_{\xi} \delta^{e q} \delta^{d p} \]
\[ D^{c d p q} = D_{\rho} \delta^{c p} \delta^{d q} + \mathcal{J}_{\rho} f^{c p e} f^{d q e} + K_{\rho} f^{c de} f^{p q e} + L_{\rho} d^{c d p q} + M_{\rho} \delta^{c d} \delta^{p q} + N_{\rho} \delta^{e q} \delta^{d p} \] (3.11)

and

\[ D^{c d p q} = D_{\xi} \delta^{c p} \delta^{d q} + \mathcal{J}_{\xi} f^{c p e} f^{d q e} + K_{\xi} f^{c de} f^{p q e} + L_{\xi} d^{c d p q} + M_{\xi} \delta^{c d} \delta^{p q} + N_{\xi} \delta^{e q} \delta^{d p} \]
\[ E^{c d p q} = E_{\xi} \delta^{c p} \delta^{d q} + \mathcal{J}_{\xi} f^{c p e} f^{d q e} + K_{\xi} f^{c de} f^{p q e} + L_{\xi} d^{c d p q} + M_{\xi} \delta^{c d} \delta^{p q} + N_{\xi} \delta^{e q} \delta^{d p} \]
\[ E^{c d p q} = E_{\rho} \delta^{c p} \delta^{d q} + \mathcal{J}_{\rho} f^{c p e} f^{d q e} + K_{\rho} f^{c de} f^{p q e} + L_{\rho} d^{c d p q} + M_{\rho} \delta^{c d} \delta^{p q} + N_{\rho} \delta^{e q} \delta^{d p} \] (3.12)

The inversions in the two sectors proceed via

\[ \Lambda^{(a b | c d)} \Pi^{(c d | p q)} = \begin{pmatrix} \delta^{c p} & 0 & 0 \\ 0 & \delta^{c p} \delta^{d q} & 0 \\ 0 & 0 & \delta^{c p} \delta^{d q} \end{pmatrix} \] (3.13)

and

\[ \Lambda^{L (a b | c d)} \Pi^{L (c d | p q)} = \begin{pmatrix} \delta^{c p} & 0 & 0 \\ 0 & \delta^{c p} \delta^{d q} & 0 \\ 0 & 0 & \delta^{c p} \delta^{d q} \end{pmatrix} \] (3.14)

where the matrix on the right hand side is the unit matrix in this colour space basis. In order to handle the products of the colour tensors in this matrix multiplication we recall, \[21\], that

\[ d^{a b c d}_A d^{a b c d}_A = a_1 \delta^{a b} \delta^{c d} + a_2 \left( \delta^{a c} \delta^{b d} + \delta^{a d} \delta^{b c} \right) + a_3 \left( f^{a c e} f^{b d e} + f^{a d e} f^{b c e} \right) + a_4 d^{a b c d}_A \]
\[ f^{a c e} f^{b d e} d^{a b c d}_A = b_1 \delta^{a b} \delta^{c d} + b_2 \left( \delta^{a c} \delta^{b d} + \delta^{a d} \delta^{b c} \right) + b_3 \left( f^{a c e} f^{b d e} + f^{a d e} f^{b c e} \right) + b_4 d^{a b c d}_A \] (3.15)
where

\[
\begin{align*}
    a_1 &= - \left[ 540C_A^2 N_A(N_A - 3)d_{abcd}^d d_{abpq}^d d_{pq} + 144(2N_A + 19) d_{abcd}^d d_{abcd}^d \right]^2 - 150C_A^2 N_A(3N_A + 11)d_{abcd}^d d_{abcd}^d + 625C_A^4 N_A^2 \\
    &\times \frac{1}{54N_A(N_A - 3)[12(N_A + 2) d_{abcd}^d d_{abcd}^d - 25C_A^4 N_A]} \\
    a_2 &= \left[ 144(11N_A - 8) d_{abcd}^d d_{abcd}^d \right]^2 - 1080C_A^2 N_A(N_A - 3)d_{abcd}^d d_{abcd}^d d_{pq} + 625C_A^4 N_A^2 - 3000C_A^4 N_A d_{abcd}^d d_{abcd}^d \\
    &\times \frac{1}{108N_A(N_A - 3)[12(N_A + 2) d_{abcd}^d d_{abcd}^d - 25C_A^4 N_A]} \\
    a_3 &= \frac{[12(N_A + 2) d_{abcd}^d d_{abcd}^d - 25C_A^4 N_A]}{54C_A N_A(N_A - 3)} \\
    a_4 &= \frac{[216(N_A + 2)d_{abcd}^d d_{abcd}^d d_{pq} - 125C_A^4 N_A - 360C_A^2 d_{abcd}^d d_{abcd}^d]}{18[12(N_A + 2) d_{abcd}^d d_{abcd}^d - 25C_A^4 N_A]} \tag{3.16}
\end{align*}
\]

and

\[
\begin{align*}
    b_1 &= -2b_2 = \frac{5C_A^4 N_A - 12d_{abcd}^d d_{abcd}^d}{9C_A N_A(N_A - 3)} , \\
    b_3 &= \frac{6(N_A - 1)d_{abcd}^d d_{abcd}^d - 5C_A^4 N_A}{9C_A^2 N_A(N_A - 3)} , \\
    b_4 &= \frac{C_A}{3} \tag{3.17}
\end{align*}
\]

It is clear that the set of linear algebraic equations resulting in multiplying out the matrices of colour amplitudes will be very involved. For reference in the case of the absence of any conventional mass terms these are formally given in \[21\, 45\]. Indeed retaining all possible masses \(\mu^2\) will be very complicated and we have provided the propagators for this situation in Appendix A for the specific case of SU(3). Instead it seems more instructive to consider the effect one particular mass term has on the propagators in turn. The motivation for this is to see which masses can produce propagator behaviour akin to that observed in lattice simulations. We note that in solving the set for the longitudinal sector it is important one follows a specific algorithm. This is because we are working in the Landau gauge but in order to carry out the inversion correctly we must retain a non-zero \(\alpha\) at the outset. The Landau gauge propagators are deduced at the end by setting \(\alpha = 0\) which will produce a transverse gluon propagator in all cases. As an aid we note that in the absence of \(3.11\) and \(3.12\) the non-zero propagators are

\[
\begin{align*}
    \langle A_\mu^a(p) A_\nu^b(-p) \rangle &= - \frac{\delta^{ab} p^2}{[(p^2)^2 + C_A^4]} P_{\mu\nu}(p) - \frac{\alpha \delta^{ab} p^2}{[(p^2)^2 + \alpha C_A^4]} L_{\mu\nu}(p) \\
    \langle A_\mu^a(p) \xi_{\nu}^{bc}(-p) \rangle &= \frac{if_{abc\gamma}^2}{[(p^2)^2 + C_A^4]} P_{\mu\nu}(p) + \frac{i\alpha f_{abc\gamma}^2}{[(p^2)^2 + \alpha C_A^4]} L_{\mu\nu}(p) \\
    \langle A_\mu^a(p) \rho_{\nu}^{bc}(-p) \rangle &= 0 \\
    \langle \phi_{\mu}^{ab}(p) \xi^{cd}_{\nu}(-p) \rangle &= - \frac{\delta^{ac} \delta^{bd} p^2}{p^2} \eta_{\mu\nu} + \frac{f_{abc} f_{cde\gamma}^4}{p^2 [(p^2)^2 + C_A^4]} P_{\mu\nu}(p) + \frac{\alpha f_{abc} f_{cde\gamma}^4}{p^2 [(p^2)^2 + \alpha C_A^4]} L_{\mu\nu}(p) \\
    \langle \phi_{\mu}^{ab}(p) \rho^{cd}_{\nu}(-p) \rangle &= 0 \\
    \langle \rho_{\mu}^{ab}(p) \rho_{\nu}^{cd}(-p) \rangle &= \langle \omega_{\mu}^{ab}(p) \bar{\omega}_{\nu}^{cd}(-p) \rangle = - \frac{\delta^{ac} \delta^{bd}}{p^2} \eta_{\mu\nu} \tag{3.18}
\end{align*}
\]

which are clearly non-singular in the limit to the Landau gauge.
Given these considerations we record the propagators for each of the masses \( \mu_i^2 \) being non-zero in succession. We append a subscript \( i \) to the propagators themselves to keep a track of each channel. First, before looking at the six localizing ghost possibilities, including the gluon mass we find

\[
\langle A_\mu^a(p) A_\nu^b(-p) \rangle_{Q} = \frac{-\delta^{ab} [p^2 + \mu_Q^2]}{[(p^2)^2 + \mu_Q^2 p^2 + C_A \gamma^4]} P_{\mu \nu}(p)
\]

\[
\langle A_\mu^a(p) \phi^{bc}(-p) \rangle_{Q} = \frac{i f^{abc} \gamma^2}{[(p^2)^2 + \mu_Q^2 p^2 + C_A \gamma^4]} P_{\mu \nu}(p)
\]

\[
\langle A_\mu^a(p) \phi^{bc}(-p) \rangle_{C} = 0
\]

\[
\langle \xi_{\mu}^{ab}(p) \xi^{cd}(-p) \rangle_{Q} = \frac{-\delta^{ac} \delta^{bd}}{p^2 - \eta_{\mu \nu} + \frac{f^{abe} f^{cde} \gamma^4}{p^2[(p^2)^2 + \mu_Q^2 p^2 + C_A \gamma^4]} P_{\mu \nu}(p)
\]

\[
\langle \xi_{\mu}^{ab}(p) \phi^{cd}(-p) \rangle_{Q} = 0
\]

\[
\langle \rho_{\mu}^{ab}(p) \rho^{cd}(-p) \rangle_{Q} = \langle \omega_{\mu}^{ab}(p) \omega^{cd}(-p) \rangle_{Q} = \frac{-\delta^{ac} \delta^{bd}}{p^2 - \eta_{\mu \nu}} . \quad (3.19)
\]

This produces a Stirling propagator which has been observed before, and which will be a common feature in other sets of propagators. However, the gluon propagator is suppressed similarly to the original Gribov propagator. For the Bose ghost there are several massless poles. It is these features of suppression and location of massless poles which is the central focus of this propagator analysis. If instead we include the mass terms already proposed in we reproduce those results but record them in our current notation for completeness. We have

\[
\langle A_\mu^a(p) A_\nu^b(-p) \rangle_{Q} = \frac{-\delta^{ab} [p^2 + \mu_Q^2]}{[(p^2)^2 + \mu_Q^2 p^2 + C_A \gamma^4]} P_{\mu \nu}(p)
\]

\[
\langle A_\mu^a(p) \phi^{bc}(-p) \rangle_{Q} = \frac{i f^{abc} \gamma^2}{[(p^2)^2 + \mu_Q^2 p^2 + C_A \gamma^4]} P_{\mu \nu}(p)
\]

\[
\langle A_\mu^a(p) \phi^{bc}(-p) \rangle_{C} = 0
\]

\[
\langle \xi_{\mu}^{ab}(p) \xi^{cd}(-p) \rangle_{Q} = \frac{-\delta^{ac} \delta^{bd}}{p^2 - \eta_{\mu \nu} + \frac{f^{abe} f^{cde} \gamma^4}{p^2[(p^2)^2 + \mu_Q^2 p^2 + C_A \gamma^4]} P_{\mu \nu}(p)
\]

\[
\langle \xi_{\mu}^{ab}(p) \phi^{cd}(-p) \rangle_{Q} = 0
\]

\[
\langle \rho_{\mu}^{ab}(p) \rho^{cd}(-p) \rangle_{Q} = \langle \omega_{\mu}^{ab}(p) \omega^{cd}(-p) \rangle_{Q} = \frac{-\delta^{ac} \delta^{bd}}{p^2 - \eta_{\mu \nu}} . \quad (3.20)
\]

Here the gluon propagator does not vanish at zero momentum and there are no massless poles in any of the colour channels. It was in part this gluon propagator freezing which justified examining the condensation of the associated BRST invariant operator originally, and channel propagators are cumbersome we have recorded them in Appendix B for the case of SU\( (N_c) \). However, the case of the \( R \) channel is similar to that for \( Q \) since

\[
\langle A_\mu^a(p) A_\nu^b(-p) \rangle_{R} = \frac{-\delta^{ab} [p^2 + \mu_R^2]}{[(p^2)^2 + \mu_R^2 p^2 + C_A \gamma^4]} P_{\mu \nu}(p)
\]

\[
\langle A_\mu^a(p) \phi^{bc}(-p) \rangle_{R} = \frac{i f^{abc} \gamma^2}{[(p^2)^2 + \mu_R^2 p^2 + C_A \gamma^4]} P_{\mu \nu}(p)
\]

\[
\langle A_\mu^a(p) \phi^{bc}(-p) \rangle_{C} = 0
\]

\[
\langle \xi_{\mu}^{ab}(p) \xi^{cd}(-p) \rangle_{R} = \frac{-\delta^{ac} \delta^{bd}}{p^2 - \eta_{\mu \nu} + \frac{f^{abe} f^{cde} \gamma^4}{p^2[(p^2)^2 + \mu_R^2 p^2 + C_A \gamma^4]} P_{\mu \nu}(p)
\]

\[
\langle \xi_{\mu}^{ab}(p) \phi^{cd}(-p) \rangle_{R} = 0
\]

\[
\langle \rho_{\mu}^{ab}(p) \rho^{cd}(-p) \rangle_{R} = \langle \omega_{\mu}^{ab}(p) \omega^{cd}(-p) \rangle_{R} = \frac{-\delta^{ac} \delta^{bd}}{p^2 - \eta_{\mu \nu}} . \quad (3.20)
\]
\[ \langle \xi_{\mu}^{ab}(p) \xi_{\nu}^{cd}(-p) \rangle_{\mathcal{R}} = 0 \]
\[ \langle \rho_{\mu}^{ab}(p) \rho_{\nu}^{cd}(-p) \rangle_{\mathcal{R}} = \langle \omega_{\mu}^{ab}(p) \omega_{\nu}^{cd}(-p) \rangle_{\mathcal{R}} = -\frac{\delta^{ac}\delta^{bd}}{p^2} \eta_{\mu\nu} + \frac{f^{abe}f^{cde}\mu_{\mathcal{R}}^2}{C_{A}p^2 + \mu_{\mathcal{R}}^2} \eta_{\mu\nu} (3.21) \]

and we note that the inclusion of the factor of \( C_{A} \) in (3.1) eases comparison. Like (3.20) the gluon propagator freezes. However, the main difference is that there are massless poles in, for instance, the \( \xi_{\mu}^{ab} \) propagator. Whilst it was these massless poles which became enhanced when the gap equation was satisfied by \( \gamma \) in the pure Gribov-Zwanziger Lagrangian we will show later that there is no similar enhancement in this case. The situation for the \( \mathcal{P} \) channel has parallels with the previous set since
\[ \langle A_{\mu}^{a}(p) A_{\nu}^{b}(-p) \rangle_{\mathcal{P}} = -\frac{\delta^{ab}p^2}{[(p^2)^2 + C_{A}\gamma^2]} P_{\mu\nu}(p) \]
\[ \langle A_{\mu}^{a}(p) \xi_{\nu}^{bc}(-p) \rangle_{\mathcal{P}} = \frac{i f^{abc}\gamma^2}{[(p^2)^2 + C_{A}\gamma^2]} P_{\mu\nu}(p) \]
\[ \langle A_{\mu}^{a}(p) \rho_{\nu}^{bc}(-p) \rangle_{\mathcal{P}} = 0 \]
\[ \langle \xi_{\mu}^{ab}(p) \xi_{\nu}^{cd}(-p) \rangle_{\mathcal{P}} = -\frac{\delta^{ac}\delta^{bd}p^2}{p^2} \eta_{\mu\nu} + \frac{f^{abe}f^{cde}\gamma^4}{p^2[(p^2)^2 + C_{A}\gamma^2]} P_{\mu\nu}(p) + \frac{\delta^{ab}\delta^{cd}\mu_{\mathcal{P}}^2}{N_{AP}^2[p^2 + \mu_{\mathcal{P}}^2]} \eta_{\mu\nu} \]
\[ \langle \xi_{\mu}^{ab}(p) \rho_{\nu}^{cd}(-p) \rangle_{\mathcal{P}} = 0 \]
\[ \langle \rho_{\mu}^{ab}(p) \rho_{\nu}^{cd}(-p) \rangle_{\mathcal{P}} = \langle \omega_{\mu}^{ab}(p) \omega_{\nu}^{cd}(-p) \rangle_{\mathcal{P}} = -\frac{\delta^{ac}\delta^{bd}p^2}{p^2} \eta_{\mu\nu} + \frac{\delta^{ad}\delta^{bc}\mu_{\mathcal{P}}^2}{N_{AP}^2[p^2 + \mu_{\mathcal{P}}^2]} \eta_{\mu\nu} (3.22) \]

producing massless poles but gluon suppression instead of freezing. By contrast the \( \mathcal{T} \) channel has gluon freezing but no massless poles because
\[ \langle A_{\mu}^{a}(p) A_{\nu}^{b}(-p) \rangle_{\mathcal{T}} = -\frac{\delta^{ab}[p^2 - \mu_{\mathcal{T}}^2]}{[(p^2)^2 - \mu_{\mathcal{T}}^2 + C_{A}\gamma^4]} P_{\mu\nu}(p) \]
\[ \langle A_{\mu}^{a}(p) \xi_{\nu}^{bc}(-p) \rangle_{\mathcal{T}} = \frac{i f^{abc}\gamma^2}{[(p^2)^2 - \mu_{\mathcal{T}}^2 + C_{A}\gamma^4]} P_{\mu\nu}(p) \]
\[ \langle A_{\mu}^{a}(p) \rho_{\nu}^{bc}(-p) \rangle_{\mathcal{T}} = 0 \]
\[ \langle \xi_{\mu}^{ab}(p) \xi_{\nu}^{cd}(-p) \rangle_{\mathcal{T}} = -\frac{\delta^{ac}\delta^{bd}p^2}{[(p^2)^2 - \mu_{\mathcal{T}}^2]} \eta_{\mu\nu} + \frac{f^{abe}f^{cde}\gamma^4}{[(p^2)^2 - \mu_{\mathcal{T}}^2 + C_{A}\gamma^4]} P_{\mu\nu}(p) + \frac{\delta^{ab}\delta^{cd}\mu_{\mathcal{T}}^2}{\eta_{\mu\nu} \mu_{\mathcal{T}}^2} \eta_{\mu\nu} \]
\[ \langle \xi_{\mu}^{ab}(p) \rho_{\nu}^{cd}(-p) \rangle_{\mathcal{T}} = 0 \]
\[ \langle \rho_{\mu}^{ab}(p) \rho_{\nu}^{cd}(-p) \rangle_{\mathcal{T}} = \langle \omega_{\mu}^{ab}(p) \omega_{\nu}^{cd}(-p) \rangle_{\mathcal{T}} = -\frac{\delta^{ac}\delta^{bd}}{[(p^2)^2 - \mu_{\mathcal{T}}^2]} \eta_{\mu\nu} + \frac{\delta^{ad}\delta^{bc}\mu_{\mathcal{T}}^2}{\eta_{\mu\nu} [((p^2)^2 - \mu_{\mathcal{T}}^2)]} \eta_{\mu\nu} (3.23) \]

Though, of the sets we have recorded this appears to be the one which is least likely to be realistic since a type of tachyonic pole appears which would violate causality.

As there is interest in effective gluon masses we record the propagators for two more general situations. Those with a non-zero \( \mu_{\mathcal{X}}^2 \) and either a non-zero \( \mu_{\mathcal{Q}}^2 \) or a non-zero \( \mu_{\mathcal{R}}^2 \). The former case has been discussed in [35] but as we will concentrate on the \( \mathcal{R} \) channel we will examine the consequences of a gluon mass for this case briefly as well. The motivation for a non-zero \( \mu_{\mathcal{X}}^2 \) originates from the same argument as that leading to (2.16). If we integrate the gluon propagator
then one obtains a non-zero vacuum expectation value for $\frac{1}{2} A_{\mu}^a A_{\mu}^a$. For completeness and to allow us to contrast with other situations we note that the $XQ$ case is

$$\langle A_{\mu}^a(p) A_{\mu}^b(-p) \rangle_{XQ} = -\frac{\delta^{ab}[p^2 + \mu_{Q}^2]}{[(p^2)^2 + (\mu_X^2 + \mu_{Q}^2)p^2 + \mu_X^2\mu_{Q}^2 + C_A \gamma^4]} P_{\mu\nu}(p)$$

$$\langle A_{\mu}^a(p) \xi_{\nu}^{bc}(-p) \rangle_{XQ} = \frac{f_{abce\gamma}^2}{[(p^2)^2 + (\mu_X^2 + \mu_{Q}^2)p^2 + \mu_X^2\mu_{Q}^2 + C_A \gamma^4]} P_{\mu\nu}(p)$$

$$\langle A_{\mu}^a(p) \rho_{\nu}^{bc}(-p) \rangle_{XQ} = 0$$

$$\langle \xi_{\mu}^{ab}(p) \xi_{\nu}^{cd}(-p) \rangle_{XQ} = -\frac{\delta^{ab}\delta^{cd}}{[p^2 + \mu_{Q}^2]} \eta_{\mu\nu} + \frac{f_{abce\gamma}^2 \mu_{Q}^2}{C_A p^2 + \mu_{Q}^2 + \mu_X^2 + C_A \gamma^4} P_{\mu\nu}(p)$$

$$\langle \xi_{\mu}^{ab}(p) \rho_{\nu}^{cd}(-p) \rangle_{XQ} = 0$$

$$\langle \rho_{\mu}^{ab}(p) \rho_{\nu}^{cd}(-p) \rangle_{XQ} = \langle \omega_{\mu}^{ab}(p) \omega_{\nu}^{cd}(-p) \rangle_{XQ} = -\frac{\delta^{ab}\delta^{cd}}{[p^2 + \mu_{Q}^2]} \eta_{\mu\nu} + \frac{f_{abce\gamma}^2 \mu_{Q}^2}{C_A p^2 + \mu_{Q}^2 + \mu_X^2 + C_A \gamma^4} L_{\mu\nu}(p) \cdot (3.24)$$

The presence of the non-zero $\mu_{Q}^2$ does not alter the properties significantly from the pure $Q$ channel case. There is still gluon freezing and no massless poles in the full set. The structure of the Stingl propagator is unsurprisingly affected. Equally when $\mu_X^2$ and $\mu_R^2$ are both non-zero the propagators behave in essence in the same way as the $R$ channel ones since

$$\langle A_{\mu}^a(p) A_{\mu}^b(-p) \rangle_{XR} = -\frac{\delta^{ab}[p^2 + \mu_{R}^2]}{[(p^2)^2 + (\mu_X^2 + \mu_{R}^2)p^2 + \mu_X^2\mu_{R}^2 + C_A \gamma^4]} P_{\mu\nu}(p)$$

$$\langle A_{\mu}^a(p) \xi_{\nu}^{bc}(-p) \rangle_{XR} = \frac{f_{abce\gamma}^2}{[(p^2)^2 + (\mu_X^2 + \mu_{R}^2)p^2 + \mu_X^2\mu_{R}^2 + C_A \gamma^4]} P_{\mu\nu}(p)$$

$$\langle A_{\mu}^a(p) \rho_{\nu}^{bc}(-p) \rangle_{XR} = 0$$

$$\langle \xi_{\mu}^{ab}(p) \xi_{\nu}^{cd}(-p) \rangle_{XR} = -\frac{\delta^{ab}\delta^{cd}}{[p^2 + \mu_{R}^2]} \eta_{\mu\nu} + \frac{f_{abce\gamma}^2 \mu_{R}^2}{C_A p^2 + \mu_{R}^2 + \mu_X^2 + C_A \gamma^4} P_{\mu\nu}(p)$$

$$\langle \xi_{\mu}^{ab}(p) \rho_{\nu}^{cd}(-p) \rangle_{XR} = 0$$

$$\langle \rho_{\mu}^{ab}(p) \rho_{\nu}^{cd}(-p) \rangle_{XR} = \langle \omega_{\mu}^{ab}(p) \omega_{\nu}^{cd}(-p) \rangle_{XR} = -\frac{\delta^{ab}\delta^{cd}}{[p^2 + \mu_{R}^2]} \eta_{\mu\nu} + \frac{f_{abce\gamma}^2 \mu_{R}^2}{C_A p^2 + \mu_{R}^2 + \mu_X^2 + C_A \gamma^4} L_{\mu\nu}(p) \cdot (3.25)$$

Clearly there is gluon suppression together with massless poles. For both (3.24) and (3.25) one could have the situation where $\mu_X^2 = -\mu_{Q}^2$ or $\mu_X^2 = -\mu_{R}^2$ then one would return to a Gribov type denominator in the gluon propagator. However, the main point to appreciate from this analysis is that there is another possibility of obtaining a frozen gluon via the condensation of a BRST invariant dimension two operator which has a different colour structure to that considered in [36,37,38]. This derives from (3.24) and we will refer to it as the $R$ channel mechanism for the moment and study it in more detail in the next section.

Prior to that we need to briefly discuss the renormalization of the general operator $O^{abcd}$ since we will be performing loop computations. It has already been shown that the contraction with $\delta^{ac}\delta^{bd}$ produces a renormalizable operator [36,37,38]. However, the more general operator is clearly renormalizable multiplicatively in the Landau gauge by the same reasoning. To
partly verify this and check our internal conventions we have renormalized $O^{abcd}$ to two loops by inserting the operator in either a $\phi^{ab}_\mu$ or a $\omega^{ab}_\mu$ 2-point function where the momentum flows in through one of the legs and out through the operator itself. Our momentum configuration allows one to apply the MINCER algorithm of [48] but recoded, [49], in the symbolic manipulation language FORM, [50]. As the two external fields in the Green’s function each carry one Lorentz index then we have to contract all the Feynman integrals with the tensor $\eta^{\mu\nu}$ so that MINCER can be applied to a Lorentz scalar. The factor of $d$ in the denominator arises from the normalization. Concerning the colour indices we do no projection on the four free indices of the operators which allows us to check that the divergent part of the Green’s function is only associated with the colour structure of the original Feynman rule for the operator. In essence this is how we check that the renormalization of the general operator is multiplicative. For the Green’s function with $\phi^{ab}_\mu$ legs there are 4 one loop and 160 two loop Feynman diagrams. The respective figures for the $\omega^{ab}_\mu$ Green’s function are 1 and 25. The diagrams are generated by the QGRAF package, [51], before being converted into FORM input notation. Then the FORM version of the MINCER algorithm, [49], is applied. As we are using an automatic Feynman diagram computation procedure we use the method of rescaling of bare quantities such as the coupling constant in order to deduce the overall final renormalization constant for the Green’s functions we renormalize, [52]. For both Green’s functions we find that the operator is multiplicatively renormalized and with the same renormalization constant $Z_O$. Not unexpectedly like the earlier colour contraction the more general operator $O^{abcd}$ satisfies the Slavnov-Taylor identity

$$Z_\phi Z_O = 1 \quad (3.26)$$

to two loops in $\overline{\text{MS}}$ where $Z_\phi$ is the renormalization constant of the original localizing ghost field $\phi^{ab}_\mu$. The renormalization constants for the real and imaginary parts of this field $\phi^{ab}_\mu$ are equivalent.

4 \hspace{1cm} R channel mass.

Having considered the different forms the propagators of the spin-1 sector can take when there are a variety of single mass terms originating from the BRST invariant operator $O^{abcd}$, we now focus on the $R$ sector mass in detail. Like the $Q$ case it has a frozen gluon propagator. The aim is to give evidence that there is a dynamical origin for such a mass as the previous analysis merely assumed the existence of such additional mass terms. The approach we take is the same as that of [37] for the $Q$ channel. Indeed given the similarities between the $Q$ and $R$ colour contractions of $O^{abcd}$ a large amount of the results of [37] can immediately be transferred to the present case without detailed re-analysis. For instance, the inclusion of the general operator (3.1) does not destroy the renormalizability of the pure Gribov-Zwanziger Lagrangian, (2.4). Moreover, we use the same procedure to examine the dynamical origin of an $R$ channel mass which is the construction of an effective potential for the operator. From this it will transpire that there is a non-zero vacuum expectation value for the operator which therefore condenses to produce the mass term we analysed previously. However, it turns out that from the way we have set up the $R$ term of (3.2) that the effective potential is formally the same as that for the $Q$ case whence we can merely translate all those results to the present case. For instance, the starting point of the application of the local composite operator formalism, [39, 40, 41, 42], to the operator is the one loop energy functional $W[J]$ where $J$ is the source coupling to the $R$ channel operator. Summing up the conventional set of contributing one loop diagrams produces

$$W[J] = -\frac{dN_A\gamma^4}{2g^2} + \frac{dN_A\xi J\gamma^2}{g^2} + \frac{(d-1)N_A}{2} \int_k \ln \left[ k^2 + \left( \frac{CA\gamma^4}{k^2 + J} \right) \right] + O(g^2) \quad (4.1)$$
where $\zeta$ is the local composite operator parameter, $\{39, 40, 41, 42\}$. Clearly this is formally similar to that for the $Q$ channel and hence we merely recall the subsequent properties. Defining
\[
\sigma(x) = \frac{\delta W[J]}{\delta J(x)}
\] (4.2)
then the effective action, $\Gamma[\sigma]$, is constructed from the Legendre transformation
\[
\Gamma[\sigma] = W[J] - \int d^4 x \ J(x) \sigma(x) .
\] (4.3)
Setting
\[
\sigma(x) = \sigma_0 + \tilde{\sigma}(x)
\] (4.4)
where
\[
\sigma_0 = \frac{dN_A \zeta \gamma^2}{g^2}
\] (4.5)
then we find the same non-zero value for the condensate as before, $\{37\}$,
\[
\tilde{\sigma} \bigg|_{J=0} = - \frac{3N_A \sqrt{CA} \gamma^2}{64\pi} .
\] (4.6)

With this observation then the corresponding subsequent analysis of $\{37\}$ in respect of the gluon propagator will hold for the $R$ channel mass. This is due to the fact that the gluon propagators of (3.20) and (3.21) are formally the same. Therefore, the estimate for the value where the gluon propagator freezes, which is in qualitative agreement with the lattice, applies in this case too.

Given this complete parallel between the two cases it might appear that there is no justification in posing an alternative way of having a frozen gluon in a refinement of the Gribov-Zwanziger formalism. However, there is a key difference and it resides in the Bose ghost sector. From (3.20) and (3.21) there are massless poles in the $\xi_{\mu}^{ab}$ propagator for the latter case but not for the former. Recently, Zwanziger has argued, $\{43\}$, that in the pure case there is an enhancement of the Bose ghost in the infrared which is a non-perturbative property of the theory. The argument is based on the spontaneous breaking of the BRST symmetry in a theory where fields are constrained by the horizon condition. The structure of the propagator has been examined at one loop in the $\overline{\text{MS}}$ scheme in the zero momentum limit and an enhanced $\xi_{\mu}^{ab}$ emerges, $\{21\}$. There is also enhancement in the $\rho_{\mu}^{ab}$ case too. In essence the starting point for this is the massless poles of the original propagators, (2.14). However, one needs to compute the one loop corrections to all the 2-point functions of the fields in the zero momentum limit and apply the gap equation satisfied by $\gamma$. Then the leading momentum term vanishes to ensure that the resultant propagators have a dipole behaviour in the infrared as opposed to the canonical behaviour of a massless field. The analysis for $\xi_{\mu}^{ab}$ is hampered by the colour tensor structure of the 2-point functions and hence only certain colour channels of its propagator enhance.

Therefore, given that the structure of the $\xi_{\mu}^{ab}$ propagators in (3.20) and (3.21) are different it is worth examining the structure of the $R$ propagators in the infrared limit. However, as the gap equation is important for this we need to compute the one loop expression for $\gamma$. This is achieved by evaluating (2.9) using the mixed propagator of (3.21). As the denominator factors are of a Stingl form in order to evaluate the basic Feynman integrals we need to first rewrite the factor formally in a more conventional fashion. Therefore, we define two additional mass parameters, $\mu_{\pm}^2$, given by
\[
\mu_{\pm}^2 = \mu_+^2 + \mu_-^2 , \quad C_A \gamma^4 = \mu_+^2 \mu_-^2 .
\] (4.7)
This choice is motivated by the relation

\[(p^2)^2 + \mu_R^2 p^2 + C_A \gamma^4 = [p^2 + \mu^2_+][p^2 + \mu^2_-].\]  

(4.8)

Solving the relations produces the mapping

\[\mu^2_+ = \frac{1}{2} \left[ \mu_R^2 + \sqrt{\mu_R^4 - 4 C_A \gamma^4} \right],\]
\[\mu^2_- = \frac{1}{2} \left[ \mu_R^2 - \sqrt{\mu_R^4 - 4 C_A \gamma^4} \right].\]  

(4.9)

With this factorization it is a straightforward matter to determine the \(\overline{\text{MS}}\) gap equation and we find

\[1 = C_A \left[ \frac{5}{8} - \frac{3}{8} \ln \left( \frac{C_A \gamma^4}{\mu^4} \right) - \frac{3 \mu_R^2}{8 \sqrt{\mu_R^4 - 4 C_A \gamma^4}} \ln \left( \frac{\mu^2_+}{\mu^2_-} \right) \right] a + O(a^2).\]  

(4.10)

In the limit of \(\mu_R^2 \rightarrow 0\) one recovers the gap equation for \(\gamma\) of the pure Gribov-Zwanziger case. This expression is formally the same as that for the \(Q\) channel, \([36, 37]\), if one simply replaces \(\mu_R^2\) by \(\mu_Q^2\). This is not unexpected since the mixed propagators are formally the same at this order. At two loops one would expect the gap equations to be different merely because at that order the other spin-1 propagators will appear in the Feynman diagrams.

Before considering the infrared structure of the \(\xi^{ab}_\mu\) propagator we first examine the Faddeev-Popov ghost propagator. This is partly to highlight the technique one follows but in a case which is not complicated by the colour tensor structure as well as to verify the loss of ghost enhancement which has to be a feature of the \(R\) channel in order to be consistent with the evidence from the lattice. However, the situation for the behaviour of the Faddeev-Popov ghost propagator in the infrared is the same as that for \(Q\) since at one loop only one diagram contributes to the 2-point function. Defining the ghost propagator in terms of its form factor, \(D_c(p^2)\), by

\[\langle c^a(p)\bar{c}^b(-p) \rangle = \frac{D_c(p^2)}{p^2} \delta^{ab}\]  

(4.11)

we have

\[D_c(p^2) = - \left[ 1 - C_A \left[ \frac{5}{8} - \frac{3}{8} \ln \left( \frac{C_A \gamma^4}{\mu^4} \right) + \frac{3 \mu_R^2}{8 \sqrt{\mu_R^4 - 4 C_A \gamma^4}} \ln \left( \frac{\mu^2_+}{\mu^2_-} \right) \right] \right.\]
\[+ \left. \left[ \frac{1}{8} \ln \left( \frac{(p^2)^2}{C_A \gamma^4} \right) - \frac{11}{24} + \frac{1}{8} \sqrt{\mu_R^4 - 4 C_A \gamma^4} \ln \left( \frac{\mu^2_+}{\mu^2_-} \right) \right] \mu_R^2 \frac{\mu^2_+}{C_A \gamma^4} \right]
\[+ \frac{1}{4 \sqrt{\mu_R^4 - 4 C_A \gamma^4}} \ln \left( \frac{\mu^2_+}{\mu^2_-} \right) \right] p^2 + O\left( (p^2)^2 \right) a + O(a^2)\]  

(4.12)

in the limit as \(p^2 \rightarrow 0\) where we have included the \(O(p^2)\) contribution. Ordinarily this term would govern the infrared behaviour of the propagator in the infrared as the leading order term in momentum would vanish when \(\gamma\) satisfies the gap equation. That does not happen due to the \(O(\mu_R^2)\) term which has the opposite sign to the analogous term which appears in the gap equation, \([4,10]\). Therefore, as with the \(Q\) channel there is no Faddeev-Popov ghost enhancement for \(R\) either. Again this is in keeping with the lattice data which sees a minor variation from \(1/p^2\)
channel and record the explicit one loop form factors for all the spin-1 field 2-point functions.

As all the other 2-point functions, whose denominator has to be the absent then there would be one loop ghost enhancement even with the Stingl type propagator complicated to determine since it would require the structure of the earlier section will contain enhanced Faddeev-Popov propagators. This will happen for \( \mu \) which is responsible for the difference in the signs of the respective terms in the gap equation and the ghost 2-point function. In other words if the \( \mu_R^2 \) term in the gluon propagator numerator was absent then there would be one loop ghost enhancement even with the Stingl type propagator whose denominator has to be the same in both the gluon and mixed propagator. With this observation it is simple to determine which of the various channels we have introduced in an earlier section will contain enhanced Faddeev-Popov propagators. This will happen for \( \mathcal{X}, \mathcal{P} \) and \( \mathcal{S} \). So, interestingly, a massive gluon but with a Gribov width would be infrared suppressed whilst satisfying the Kugo-Ojima confinement criterion at one loop in this context. Whether there is enhancement in various colour channels for the \( \xi_{ab}^\mu \) fields and if so which ones, is more complicated to determine since it would require the structure of the \( \xi_{ab}^\mu \) 2-point function as well as all the other 2-point functions.

To illustrate the complexity of such an exercise even in a simple case we return to the \( \mathcal{R} \) channel and record the explicit one loop form factors for all the spin-1 field 2-point functions. We have

\[
\mathcal{X} = - \left[ p^2 - C_A \left[ \frac{27\mu_R^2 C_A \gamma^4}{32} \right] - \frac{27 C_A^2 \gamma^8 \sqrt{\mu_R^4 - 4C_A \gamma^4}}{16 \mu_R^4 - 4C_A \gamma^4} \ln \left[ \frac{\mu_+^2}{\mu_-^2} \right] \right. \\
+ \frac{3}{8} \mu_R^2 \ln \left[ \frac{\mu_+^2}{\mu_-^2} \right] + \frac{3}{8} \mu_R^2 \ln \left[ \frac{C_A \gamma^4}{\mu_R^4} \right] \\
+ \frac{5 \mu_R^2 p^2}{192 C_A \gamma^4} \left[ \sqrt{\mu_R^4 - 4C_A \gamma^4} \ln \left[ \frac{\mu_+^2}{\mu_-^2} \right] + \mu_R^2 \ln \left[ \frac{C_A \gamma^4}{\mu_R^4} \right] \right] \\
+ \frac{143 \mu_R^4 \ln \left[ \frac{\mu_+^2}{\mu_-^2} \right] + \mu_R^4 \ln \left[ \frac{C_A \gamma^4}{\mu_R^4} \right] \right] \\
+ \frac{9 \mu_R^4 \ln \left[ \frac{\mu_+^2}{\mu_-^2} \right] + \mu_R^4 \ln \left[ \frac{C_A \gamma^4}{\mu_R^4} \right] \right] \\
+ \frac{457 \mu_R^4 \ln \left[ \frac{\mu_+^2}{\mu_-^2} \right] + \mu_R^4 \ln \left[ \frac{C_A \gamma^4}{\mu_R^4} \right] \right] \\
+ \frac{25}{12} \ln \left[ \frac{\mu_+^2}{\mu_-^2} \right] + \frac{1}{12} \ln \left[ \frac{p^2}{\mu_-^2} \right] \right] \\
+ p^2 T_F N_f \left[ \frac{4}{3} \ln \left[ \frac{p^2}{\mu_-^2} \right] - \frac{20}{9} \right] a + O \left( p^2 \right)
\]

\[
\mathcal{U} = i \gamma^2 \left[ 1 + C_A \left[ \frac{5 \mu_R^2 p^2}{192 C_A \gamma^4} \sqrt{\mu_R^4 - 4C_A \gamma^4} \ln \left[ \frac{\mu_+^2}{\mu_-^2} \right] + \mu_R^2 \ln \left[ \frac{C_A \gamma^4}{\mu_R^4} \right] \right] \\
+ \frac{5 C_A \gamma^4 p^2}{4 \mu_R^4 - 4C_A \gamma^4} \ln \left[ \frac{\mu_+^2}{\mu_-^2} \right] \right]
\]
\[ \mathcal{W}_\xi = \left[ \frac{7\alpha_\gamma^4}{6\mu_\gamma^4 - 4\alpha_\gamma^4} \right] - \frac{7\alpha_\gamma^4 \sqrt{\mu_\gamma^4 - 4\alpha_\gamma^4}}{12\mu_\gamma^4 - 4\alpha_\gamma^4} \ln \left[ \frac{\mu_\gamma^4}{\mu_\gamma^2} \right] + O \left( (p^2)^2 \right) \]

\[ \mathcal{R}_\xi = \mu_\gamma^2 + \left[ \frac{11\alpha_\gamma^4}{72\mu_\gamma^4 - 4\alpha_\gamma^4} \right] - \frac{11\alpha_\gamma^4 \sqrt{\mu_\gamma^4 - 4\alpha_\gamma^4}}{144\mu_\gamma^4 - 4\alpha_\gamma^4} \ln \left[ \frac{\mu_\gamma^4}{\mu_\gamma^2} \right] + O \left( (p^2)^2 \right) \]

\[ \mathcal{S}_\xi = \left[ \frac{7\alpha_\gamma^4}{6\mu_\gamma^4 - 4\alpha_\gamma^4} \right] - \frac{7\alpha_\gamma^4 \sqrt{\mu_\gamma^4 - 4\alpha_\gamma^4}}{12\mu_\gamma^4 - 4\alpha_\gamma^4} \ln \left[ \frac{\mu_\gamma^4}{\mu_\gamma^2} \right] + O \left( (p^2)^2 \right) \]

\[ \mathcal{P}_\xi = \mathcal{T}_\xi = \mathcal{P}_\xi^L = \mathcal{T}_\xi^L = O(a^2) \]

\[ \mathcal{X}^L = -\left[ \frac{p^2}{a} \right] + \left[ C_A \left[ \frac{27\alpha_\gamma^4 \sqrt{\mu_\gamma^4 - 4\alpha_\gamma^4}}{16\alpha_\gamma^4 - 4\alpha_\gamma^4} \ln \left[ \frac{\mu_\gamma^2}{\mu_\gamma^2} \right] \right] + 3 \sqrt{\mu_\gamma^4 - 4\alpha_\gamma^4} \ln \left[ \frac{\mu_\gamma^4}{\mu_\gamma^2} \right] + \frac{3\alpha_\gamma^4}{8} \ln \left[ \frac{\mu_\gamma^4}{\mu_\gamma^2} \right] \right] + \mu_\gamma^2 \ln \left[ \frac{\mu_\gamma^4}{\mu_\gamma^2} \right] \right] + O \left( (p^2)^2 \right) \]

\[ \mathcal{U}^L = i\gamma^2 \left[ 1 + \left[ C_A \left[ \frac{p^2}{64\alpha_\gamma^4} \left[ \sqrt{\mu_\gamma^4 - 4\alpha_\gamma^4} \ln \left[ \frac{\mu_\gamma^2}{\mu_\gamma^2} \right] + \mu_\gamma^2 \ln \left[ \frac{\mu_\gamma^4}{\mu_\gamma^2} \right] \right] \right] \right] a + O \left( (p^2)^2 \right) \]
\[
- \frac{3C_A\gamma^4 p^2 \sqrt{\mu_R^4 - 4C_A\gamma^4}}{16[\mu_R^4 - 4C_A\gamma^4]^{3/2}} \ln \left[ \frac{\mu_+^2}{\mu_-^2} \right] \\
+ p^2 \left[ \frac{\sqrt{\mu_R^4 - 4C_A\gamma^4}}{8[\mu_R^4 - 4C_A\gamma^4]} \ln \left[ \frac{\mu_+^2}{\mu_-^2} \right] + \frac{3\mu_R^2}{32[\mu_R^4 - 4C_A\gamma^4]} \right] \right] a \\
+ O \left( (p^2)^2 \right)
\]

\[
\mathcal{V}_\xi^L = -i\gamma^2 \left[ C_A \left[ \frac{3p^2}{64C_A^4} \frac{\sqrt{\mu_R^4 - 4C_A\gamma^4}}{8[\mu_R^4 - 4C_A\gamma^4]} \ln \left[ \frac{\mu_+^2}{\mu_-^2} \right] + \mu_R^2 \ln \left[ \frac{C_A\gamma^4}{\mu_R^4} \right] \right] \\
+ \frac{3p^2}{8[\mu_R^4 - 4C_A\gamma^4]} \ln \left[ \frac{\mu_+^2}{\mu_-^2} \right] \right] a + O \left( (p^2)^2 \right)
\]

\[
Q_\xi = Q_\xi^L + O(a^2) = Q_\rho + O(a^2) = Q_\rho^L + O(a^2)
\]

\[
\mathcal{W}_\xi^L = \mu_R^2 \left[ \frac{5C_A\gamma^4}{24[\mu_R^4 - 4C_A\gamma^4]} - \frac{5\mu_R^2 C_A\gamma^4}{48[\mu_R^4 - 4C_A\gamma^4]^2} \ln \left[ \frac{\mu_+^2}{\mu_-^2} \right] \\
+ \frac{\mu_R^2 \sqrt{\mu_R^4 - 4C_A\gamma^4}}{8[\mu_R^4 - 4C_A\gamma^4]} \ln \left[ \frac{\mu_+^2}{\mu_-^2} \right] \right] a p^2 + O \left( (p^2)^2 \right)
\]

\[
\mathcal{R}_\xi^L = \mu_R^2 \left[ \frac{5\gamma^4}{2[\mu_R^4 - 4C_A\gamma^4]} - \frac{5\mu_R^2 \gamma^4}{4[\mu_R^4 - 4C_A\gamma^4]^2} \ln \left[ \frac{\mu_+^2}{\mu_-^2} \right] \\
- \frac{3\mu_R^2 \sqrt{\mu_R^4 - 4C_A\gamma^4}}{4C_A[\mu_R^4 - 4C_A\gamma^4]} \ln \left[ \frac{\mu_+^2}{\mu_-^2} \right] \right] a p^2 + O \left( (p^2)^2 \right)
\]

\[
\mathcal{W}_\rho = \mathcal{W}_\rho^L = -\frac{\mu_R^2 \sqrt{\mu_R^4 - 4C_A\gamma^4}}{8[\mu_R^4 - 4C_A\gamma^4]} \ln \left[ \frac{\mu_+^2}{\mu_-^2} \right] a p^2 + O \left( (p^2)^2 \right)
\]

\[
\mathcal{R}_\rho = \mathcal{R}_\rho^L = \mu_R^2 \left[ \frac{\mu_R^2 \sqrt{\mu_R^4 - 4C_A\gamma^4}}{8[\mu_R^4 - 4C_A\gamma^4]} \ln \left[ \frac{\mu_+^2}{\mu_-^2} \right] \right] a p^2 + O \left( (p^2)^2 \right)
\]

\[
\mathcal{S}_\rho = \mathcal{S}_\rho^L = -\frac{3\mu_R^2 \sqrt{\mu_R^4 - 4C_A\gamma^4}}{4C_A[\mu_R^4 - 4C_A\gamma^4]} \ln \left[ \frac{\mu_+^2}{\mu_-^2} \right] a p^2 + O \left( (p^2)^2 \right)
\]

(4.13)

where \( \tilde{\mu} \) is the mass scale introduced to ensure that the coupling constant remains dimensionless in \( d \)-dimensions as we are using dimensional regularization. The divergences have been removed by the \( \overline{\text{MS}} \) scheme prescription. As expected \( Q_\xi \) is similar to the Faddev-Popov ghost 2-point function. In the pure case this function was solely responsible for producing the overall enhancement of the \( \xi^{ab}_\mu \) propagator observed in [21] which was consistent with the general analysis of [43]. However, since the leading term of the momentum expansion does not vanish when the gap equation is satisfied this leads to the absence of enhancement for \( \xi^{ab}_\mu \) in the \( \mathcal{R} \) channel. To
be more specific we find the leading behaviour in the $p^2 \to 0$ limit is

\[
\langle \xi_{\mu}^{ab} (p) \xi_{\nu}^{cd} (-p) \rangle_R \sim \frac{1}{2Q_0 p^2 a} \left[ \delta^{ac} \delta^{bd} - \delta^{ad} \delta^{bc} - \frac{2}{C_A} f^{abe} f^{cde} \right] \eta_{\mu\nu} \\
\langle \rho_{\mu}^{ab} (p) \rho_{\nu}^{cd} (-p) \rangle_R \sim \frac{1}{2Q_0 p^2 a} \left[ \delta^{ac} \delta^{bd} - \delta^{ad} \delta^{bc} - \frac{2}{C_A} f^{abe} f^{cde} \right] \eta_{\mu\nu}
\]

where

\[
Q_0 = \frac{1}{4\sqrt{\mu^2_R - 4C_A \gamma^4}} \ln \left[ \frac{\mu^2_R}{\mu^2} \right] + \left[ \frac{1}{8} \sqrt{\mu^4_R - 4C_A \gamma^4} \ln \left[ \frac{\mu^2_R}{\mu^2} \right] - \frac{1}{8} \ln \left[ \frac{C_A \gamma^4}{(p^2)^2} \right] - \frac{11}{24} \right] \frac{\mu^2_R}{C_A \gamma^4}
\]

which is derived using the standard procedure from the coefficient of the leading term in the zero momentum limit after the gap equation has been set. It is the same as that for the Faddeev-Popov ghost. As expected there is no enhancement and unlike the pure Gribov-Zwanziger case the leading momentum behaviour involves a logarithm of the momentum. However, the Bose ghost propagators diverge in the same way as the Faddeev-Popov ghost. Interestingly the case the leading momentum behaviour involves a logarithm of the momentum. However, the Popov ghost. As expected there is no enhancement and unlike the pure Gribov-Zwanziger case discussed in [43, 21] and we recall the leading behaviour of both Bose ghosts in that case is

\[
\langle \xi_{\mu}^{ab} (p) \xi_{\nu}^{cd} (-p) \rangle \sim \frac{4\gamma^2}{\pi \sqrt{C_A (p^2)^2 a}} \left[ \delta^{ac} \delta^{bd} - \delta^{ad} \delta^{bc} \right] \eta_{\mu\nu} + \frac{8\gamma^2}{\pi C_A^{3/2} (p^2)^2 a} f^{abe} f^{cde} P_{\mu\nu}(p)
\\
\langle \rho_{\mu}^{ab} (p) \rho_{\nu}^{cd} (-p) \rangle \sim - \frac{8\gamma^2}{\pi \sqrt{C_A (p^2)^2 a}} \delta^{ac} \delta^{bd} \eta_{\mu\nu}.
\]

(4.14)

So far in our discussions we have noted that the properties of the $Q$ and $R$ channel masses are the same. However, (4.14) clearly represents the first departure from that similarity. For the $Q$ case all colour channels of the $\xi_{\mu}^{ab}$ propagator freeze to finite values in the zero momentum limit. By contrast the $R$ channel $\xi_{\mu}^{ab}$ propagator neither freezes nor enhances. Instead its behaviour is in essence the same as that of the Faddeev-Popov ghost since the coefficient of the leading term is the same but differs in the colour tensor structure. That the same tensor structure as (4.16) emerges at leading order for both cases is a consequence of the peculiarities of the inversion of the matrix of colour structures. However, the adjoint colour projection that Zwanziger focused on in [43] will not enhance nor diverge but freeze to a non-zero value.

There are several main consequences of our $R$ channel analysis. First, it appears that there are now two possibilities of modelling lattice data by refining the Gribov-Zwanziger Lagrangian with one additional mass operator. Both the $Q$ and $R$ channels reproduce gluon freezing to a non-zero value and ghost non-enhancement. However, to determine which one is correct would require a lattice computation of the $\xi_{\mu}^{ab}$ propagator in the zero momentum limit. It would seem to us that this would be a non-trivial exercise since the lattice gauge fixing procedure used pays no attention to a field, $\xi_{\mu}^{ab}$, which only appears in the Gribov-Zwanziger Lagrangian and is necessary to localize the non-local horizon operator. However, one could consider instead the correlation of a non-local projection of the gluon field itself since the equation of motion from (2.24)

\[
(\partial^\nu D_\nu \xi_{\mu})^{ab} = i \gamma^2 f^{abe} A^c_{\mu}
\]

produces the relation

\[
\xi_{\mu}^{ab} = i \gamma^2 \left( \frac{1}{\partial^\nu D_\nu} \right)^{ad} f^{dbe} A^c_{\mu}
\]

(4.17)

(4.18)

where we have been careful in including the colour indices of the inverse Faddeev-Popov operator to correct an error in earlier work. [20, 21]. Again this would appear to open up other difficulties.
since the presence of a non-locality via the Faddeev-Popov operator may be hard to define in a discrete spacetime in an unambiguous way. Aside from that it may even be restrictive both financially and computationally to produce accurate enough data in order to determine the behaviour at zero momentum definitively. Currently, the state of the art to obtain the gluon propagator behaviour at zero momentum involves a new formulation of the lattice definition of the linear covariant gauge, \[ \text{[53]} \]. Another problem relates to whether (4.18) is the proper definition of the field whose correlator is the relevant object to examine. For instance, (4.18) assumes there is no mass term initially for the \( \xi^{ab}_\mu \) field and that the mass term has a dynamical origin. However, if one has the mass term present then it could be argued that (4.17) should be replaced by something such as
\[
(\partial^\nu D_\nu \xi^a_\mu)^{ab} + \frac{\mu^2}{C_A} f^{abe} f^{cde} \xi^{cd}_\mu = i\gamma^2 f^{abc} A^c_\mu. \tag{4.19}
\]
Though in the infrared limit this would effectively be the same as an adjoint projection of the gluon. So it is not clear in this case whether this would allow one to make a clear statement on the behaviour of the propagators we have considered, \( \langle \xi^a_\mu(p)\xi^b_\nu(-p) \rangle \) and \( \langle \rho^a_\mu(p)\rho^b_\nu(-p) \rangle \).

5 Discussion.

There are several main features arising out of our analysis which was to attempt to model lattice data with a more natural perturbation of the original Gribov-Zwanziger Lagrangian with a different BRST invariant dimension two operator compared to \[ \text{[36, 37, 38]} \]. First, if one is to reconcile the lattice and Schwinger-Dyson observations that the Landau gauge gluon propagator freezes to a non-zero value and the Faddeev-Popov ghost does not enhance, by modelling with a refined Gribov-Zwanziger Lagrangian, then it turns out that there is more than one way to do this. The original analysis of \[ \text{[36, 37, 38]} \] used only one colour projection but it has been shown here that the alternative \( \mathcal{R} \) channel projection can equally accommodate a non-zero frozen gluon propagator and unenhanced ghost propagator. If one wished to determine which of these single operator extensions was consistent with the lattice results then the test resides in the infrared behaviour of the propagators of the Bose localizing ghost. In the earlier \( \mathcal{Q} \) channel solution the \( \xi^{ab}_\mu \) propagator freezes to a non-zero value. By contrast in the \( \mathcal{R} \) channel case the propagator of this field has the same momentum behaviour in the infrared as the Faddeev-Popov ghost propagator. Given this it might be better in future to refer instead to the \( \mathcal{Q} \) channel as the gluon-like massive or decoupling scenario and that of \( \mathcal{R} \) as the Faddeev-Popov ghost-like massive or decoupling solution. In some sense the resolution by numerical work may not in fact be possible given the amount of computing resources which would be required to determine the correlation of a non-local projection of the gluon field. Moreover, both sets of localizing ghosts are an inherent feature of accommodating the original non-local Gribov horizon operator and as such would have no parallel or direct concept in a lattice construction.

We have also noted that the inclusion of the dimension two gluon mass operator, which is BRST invariant as well, into the propagators for each of the \( \mathcal{Q} \) and \( \mathcal{R} \) channels does not alter their behaviour from the situation when the gluon operator was absent. This in fact opened a wider question as to whether all the possible colour projections should not be considered simultaneously. As the gluon and Faddeev-Popov ghost propagators are the only two which have been analysed numerically the fact that the gluon freezes to a non-zero value can also be accommodated by contributions from four different colour channels as can be seen in Appendix A. From a numerical point of view the freezing to a non-zero value for the gluon propagator can never resolve each of the four different mass scales which arise there. Again only data
on the remaining spin-1 propagators could ever possibly determine this. It would require, for instance, a substantial amount of numerical fitting of all the mass parameters. Though as there are more than seven different propagator form factors then this over-redundancy would provide independent consistency checks on the seven mass parameters where we include the gluon mass parameter in the counting here. However, we should recall one of our underlying assumptions in this context. In (3.1) we have taken the masses for $\rho_{ab}^{\mu}$, $\xi_{ab}^{\mu}$ and $\omega_{ab}^{\mu}$ to be the same in order to ensure our additional operator is BRST invariant. If the BRST symmetry is broken then there is no reason why, for example, any of the masses of the localizing fields should be equal. If this were the case then the propagators we have discussed throughout would be much more complicated. Moreover, this would become apparent in all the different propagator form factors analogous to those of Appendix A. Again to test this scenario would appear to be computationally impractical for reasons we have already mentioned. However, one alternative theoretical way to shed some light on the interplay of the different colour projections would be to extend the effective potential analysis of the single projection case for the $Q$ and $R$ channels to include all seven cases simultaneously. Whilst it is a non-trivial task to apply the local composite operator method for this, finding a stable absolute minimum of the effective potential would indicate which is the most energetically favoured solution or solutions. This is currently in progress. However, it seems from earlier experience, with the construction of the enhanced Bose ghost propagator in the pure Gribov-Zwanziger Lagrangian, that the subtleties of the intricate nature of the general colour tensors of the localizing ghosts have a significant effect on the infrared structure of this Lagrangian. One only has to recall the decompositions (3.15) to see how the effective potential construction will mix up colour tensors of the operator $O_{abcd}$.

Finally, in providing a complete analysis for each of the seven individual colour projections separately we have noticed several features in the interplay of a frozen gluon propagator and a non-enhanced Faddeev-Popov ghost propagator with the structure of the original propagators when the appropriate gap equation is implemented at one loop. Specifically if the propagators of the spin-1 fields have a massless pole in a colour or Lorentz channel then whether it becomes enhanced in the infrared depends on whether the gluon propagator freezes to a non-zero value or not. In the case of the original Gribov-Zwanziger Lagrangian this was the case as is now evident in the recent work of [43, 21] for the localizing Bose ghost. Here this particular feature emerges in several of the cases. For instance, with a gluon mass operator only or in the $S$ channel only then when the appropriate horizon condition for $\gamma$ is satisfied there appears to be ghost enhancement. Whilst there is currently no lattice evidence for this scaling scenario since the massive or decoupling solution appears to be favoured by many different simulations, [23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 34, 53], it is perhaps worth noting that if instead a scaling solution had been found then for this situation the pure Gribov-Zwanziger structure may not actually have been the unique explanation.

**A Full propagators for $SU(3)$**.

In this appendix we record the explicit expressions for the propagators for the specific colour group $SU(3)$ for all channels together. First, the transverse sector is

\[
A = - \left[ 2\mu_Q^2 + 2\mu_R^2 - 2\mu_T^2 + 3\mu_V^2 + 2p^2 \right] \\
\times \left[ 3\mu_T^2 + 2p^2 - 2\mu_T^2 + 2\mu_R^2 + 2\mu_Q^2 \right] [\mu_X^2 + p^2] + 6\gamma^4 \right]^{-1}
\]

\[
B = 2i\gamma^2 \left[ 3\mu_V^2 + 2p^2 + 2\mu_T^2 + 2\mu_R^2 + 2\mu_Q^2 \right] [\mu_X^2 + p^2] + 6\gamma^4 \right]^{-1}
\]
\[ \mathcal{D}_\xi = \mathcal{D}_\xi^L = \mathcal{D}_\rho = \mathcal{D}_\rho^L \]
\[ = \frac{1}{2} \left[ -16(\mu_2)^2 - 56(\mu_2)^2 \mu_S^2 - 32(\mu_2)^2 \mu_T^2 - 24(\mu_2)^2 \mu_V^2 + 48(\mu_2)^2 \mu_P^2 \right. \]
\[ - 54(\mu_2)^2 (\mu_S^2)^2 - 56(\mu_2)^2 (\mu_T^2)^2 - 40(\mu_2)^2 (\mu_S^2) \mu_T^2 - 112(\mu_2)^2 \mu_S^2 \mu_P^2 - 16(\mu_2)^2 (\mu_T^2)^2 \]
\[ - 24(\mu_2)^2 (\mu_T)^2 (\mu_V^2)^2 + 6(\mu_2)^2 (\mu_T^2)^2 - 48(\mu_2)^2 (\mu_V^2)^2 + 48(\mu_2)^2 (\mu_P^2)^2 \]
\[ - 9(\mu_2)^2 (\mu_T)^2 (\mu_T^2)^2 - 21(\mu_2)^2 (\mu_T^2)^2 \mu_V^2 - 54(\mu_2)^2 \mu_T^2 \mu_P^2 \]
\[ - 56(\mu_2)^2 (\mu_T)^2 (\mu_P^2)^2 - 56(\mu_2)^2 (\mu_P^2)^2 - 16(\mu_2)^2 (\mu_T^2)^2 + 24(\mu_2)^2 (\mu_T^2)^2 \mu_P^2 \]
\[ - 32(\mu_2)^2 (\mu_P^2)^2 + 12(\mu_2)^2 (\mu_T)^2 (\mu_T^2)^2 - 24(\mu_2)^2 (\mu_T^2)^2 (\mu_P^2)^2 - 16(\mu_2)^2 (\mu_P^2)^2 \]
\[ \times \left[ 2(\mu_2) + 3(\mu_2)^2 + 2(\mu_T)^2 + 3(\mu_P)^2 \right]^{-1} \left[ 2(\mu_2) + 3(\mu_2)^2 + 2(\mu_T)^2 + 2(\mu_P)^2 \right]^{-1} \]
\[ \times \left[ 2(\mu_2) + 3(\mu_2)^2 + 2(\mu_T)^2 + 2(\mu_P)^2 \right]^{-1} \left[ (\mu_2)^2 - (\mu_T)^2 + 2(\mu_P)^2 \right]^{-1} \]
\[ \mathcal{J}_\xi = \mathcal{J}_\xi^L = \mathcal{J}_\rho = \mathcal{J}_\rho^L \]
\[ = \frac{1}{3} \left[ 24(\mu_2)^2 (\mu_T)^2 + 72(\mu_2)^2 (\mu_T^2)^2 + 48(\mu_2)^2 (\mu_2)^2 + 48(\mu_2)^2 (\mu_P^2)^2 + 54(\mu_2)^2 (\mu_P^2)^2 \right. \]
\[ + 72(\mu_2)^2 (\mu_T)^2 + 18(\mu_2)^2 (\mu_T^2)^2 + 72(\mu_2)^2 (\mu_2)^2 + 24(\mu_2)^2 (\mu_T^2)^2 + 48(\mu_2)^2 (\mu_T^2)^2 + 48(\mu_2)^2 (\mu_T^2)^2 \]
\[ - 36(\mu_2)^2 (\mu_T)^2 - 24(\mu_2)^2 (\mu_T^2)^2 + 24(\mu_2)^2 (\mu_T^2)^2 + 8(\mu_2)^2 (\mu_T^2)^2 + 8(\mu_2)^2 (\mu_T^2)^2 \]
\[ + 24(\mu_2)^2 (\mu_T)^2 + 24(\mu_2)^2 (\mu_T^2)^2 + 16(\mu_2)^2 (\mu_T^2)^2 + 16(\mu_2)^2 (\mu_T^2)^2 \]
\[ - 8(\mu_2)^2 (\mu_T)^2 + 8(\mu_2)^2 (\mu_T^2)^2 + 16(\mu_2)^2 (\mu_T^2)^2 + 16(\mu_2)^2 (\mu_T^2)^2 \]
\[ + 36(\mu_2)^2 (\mu_T^2)^2 + 12(\mu_2)^2 (\mu_T^2)^2 \]
\[ - 12(\mu_2)^2 (\mu_T^2)^2 - 18(\mu_2)^2 (\mu_T^2)^2 + 18(\mu_2)^2 (\mu_T^2)^2 + 24(\mu_2)^2 (\mu_T^2)^2 + 24(\mu_2)^2 (\mu_T^2)^2 \]
\[ + 6(\mu_2)^2 (\mu_T^2)^2 + 6(\mu_2)^2 (\mu_T^2)^2 + 24(\mu_2)^2 (\mu_T^2)^2 + 24(\mu_2)^2 (\mu_T^2)^2 + 8(\mu_2)^2 (\mu_T^2)^2 \]
\[ + 8(\mu_2)^2 (\mu_T^2)^2 + 16(\mu_2)^2 (\mu_T^2)^2 + 16(\mu_2)^2 (\mu_T^2)^2 + 16(\mu_2)^2 (\mu_T^2)^2 \]
\[ - 12(\mu_2)^2 (\mu_T^2)^2 + 8(\mu_2)^2 (\mu_T^2)^2 - 8(\mu_2)^2 (\mu_T^2)^2 - 8(\mu_2)^2 (\mu_T^2)^2 \]
\[ + 8(\mu_2)^2 (\mu_T^2)^2 + 27(\mu_2)^2 (\mu_T^2)^2 + 27(\mu_2)^2 (\mu_T^2)^2 + 36(\mu_2)^2 (\mu_T^2)^2 + 36(\mu_2)^2 (\mu_T^2)^2 \]
\[ + 9(\mu_2)^2 (\mu_T^2)^2 + 9(\mu_2)^2 (\mu_T^2)^2 + 24(\mu_2)^2 (\mu_T^2)^2 + 24(\mu_2)^2 (\mu_T^2)^2 \]
\[ + 24(\mu_2)^2 (\mu_T^2)^2 + 24(\mu_2)^2 (\mu_T^2)^2 \]
\[ - 12(\mu_2)^2 (\mu_T^2)^2 - 12(\mu_2)^2 (\mu_T^2)^2 \]
\[ \times \left[ 2(\mu_2) + 2(\mu_T) - 2(\mu_T) + 3(\mu_T) - 2(\mu_T) + 3(\mu_T) - 2(\mu_T) + 2(\mu_T) \right]^{-1} \left[ 2(\mu_2) + 3(\mu_2)^2 + 2(\mu_T)^2 + 2(\mu_T)^2 \right]^{-1} \]
\[ \times \left[ 2(\mu_2) + 3(\mu_2)^2 + 2(\mu_T)^2 + 2(\mu_T)^2 \right]^{-1} \left[ (\mu_2)^2 - (\mu_T)^2 + 2(\mu_T)^2 \right]^{-1} \]
\[ \mathcal{L}_\xi = \mathcal{L}_\xi^L = \mathcal{L}_\rho = \mathcal{L}_\rho^L \]
\[ = 4 \left[ 2(\mu_2)^2 (\mu_T^2)^2 + 3(\mu_2)^2 + 2(\mu_T)^2 + 2(\mu_T)^2 + 3(\mu_T)^2 + 2(\mu_T)^2 \right. \]
\[ \times \left[ 2(\mu_2) + 3(\mu_2)^2 + 2(\mu_T)^2 + 2(\mu_T)^2 \right]^{-1} \left[ 2(\mu_2) + 3(\mu_2)^2 + 2(\mu_T)^2 + 2(\mu_T)^2 \right]^{-1} \]
\[ \times \left[ 2(\mu_2) + 3(\mu_2)^2 + 2(\mu_T)^2 + 2(\mu_T)^2 \right]^{-1} \left[ (\mu_2)^2 - (\mu_T)^2 + 2(\mu_T)^2 \right]^{-1} \]
\[ \mathcal{M}_\xi = \mathcal{M}_\xi^L = \mathcal{M}_\rho = \mathcal{M}_\rho^L \]
\[ = \frac{1}{2} \left[ 4(\mu_2)^2 (\mu_2)^2 - 16(\mu_2)^2 (\mu_T^2)^2 - 6(\mu_2)^2 (\mu_T^2)^2 + 8(\mu_2)^2 (\mu_T^2)^2 - 33(\mu_2)^2 (\mu_2)^2 \right. \]
\[ + 16(\mu_2)^2 (\mu_T^2)^2 + 13(\mu_2)^2 (\mu_T^2)^2 - 16(\mu_2)^2 (\mu_T^2)^2 + 4(\mu_2)^2 (\mu_T^2)^2 - 6(\mu_2)^2 (\mu_T^2)^2 + 8(\mu_2)^2 (\mu_T^2)^2 \]
\[ + 14(\mu_2)^2 (\mu_T^2)^2 - 6(\mu_2)^2 (\mu_T^2)^2 + 4(\mu_2)^2 (\mu_T^2)^2 - 168(\mu_2)^2 (\mu_2)^2 - 96(\mu_2)^2 (\mu_T^2)^2 - 252(\mu_2)^2 \]
\[
\begin{align*}
- 168(\mu_S^2)^2 \mu_T^2 - 12(\mu_S^2)^2 \mu_W^2 - 166(\mu_S^2)^2 p^2 - 96\mu_S^2\mu_T^2 \mu_W^2 \\
+ 144\mu_S^2(\mu_W^2)^2 - 96\mu_S^2\mu_W^2 p^2 + 48(\mu_W^2)^3 \\
\times \left[ 2\mu_Q^2 + 15\mu_S^2 + 2\mu_T^2 + 6\mu_W^2 + 2p^2 + 2\mu_P^2 \right]^{-1} \left[ 2\mu_Q^2 + 3\mu_S^2 + 2\mu_T^2 + 3\mu_W^2 + 2p^2 \right]^{-1} \\
\times \left[ 2\mu_Q^2 + 3\mu_S^2 + 2\mu_T^2 - 2\mu_W^2 + 2p^2 \right]^{-1} \left[ 2\mu_Q^2 + 3\mu_S^2 + 2\mu_T^2 + 2\mu_W^2 + 2p^2 \right]^{-1}
\end{align*}
\]

\[
N_\xi = N_\xi^L = N_\rho = N_\rho^L \\
= \frac{1}{2} \left[ 16(\mu_Q^2)^2 \mu_T^2 + 6\mu_Q^2(\mu_S^2)^2 + 56\mu_Q^2\mu_S^2\mu_T^2 + 24\mu_Q^2\mu_S^2\mu_W^2 + 32\mu_Q^2(\mu_S^2)^2 + 24\mu_Q^2\mu_T^2 \mu_W^2 \\
+ 32\mu_Q^2\mu_T^2 p^2 + 9(\mu_S^2)^3 + 54(\mu_S^2)^2 \mu_T^2 + 21(\mu_S^2)^2 \mu_W^2 + 6(\mu_S^2)^2 \mu_T^2 + 56\mu_S^2(\mu_T^2)^2 \\
+ 40\mu_S^2\mu_T^2 \mu_W^2 + 56\mu_S^2 \mu_T^2 p^2 + 24\mu_S^2 \mu_W^2 p^2 + 16(\mu_T^2)^3 + 24(\mu_T^2)^2 \mu_W^2 + 32(\mu_T^2)^2 p^2 \\
- 16\mu_T^2(\mu_W^2)^2 + 24\mu_T^2 \mu_W^2 p^2 + 16\mu_T^2 (p^2)^2 - 12(\mu_W^2)^3 \right] \\
\times \left[ 2\mu_Q^2 + 3\mu_S^2 + 2\mu_T^2 + 3\mu_W^2 + 2p^2 \right]^{-1} \left[ 2\mu_Q^2 + 3\mu_S^2 + 2\mu_T^2 - 2\mu_W^2 + 2p^2 \right]^{-1} \\
\times \left[ 2\mu_Q^2 + \mu_S^2 + 2\mu_T^2 + 2\mu_W^2 + 2p^2 \right]^{-1} \left[ \mu_Q^2 - \mu_T^2 + p^2 \right]^{-1}
\]

\[
A^L = B^L = C^L = 0 \\
K_\xi^L = K_\rho = K_\rho^L = 0
\]

\[
\begin{align*}
\frac{1}{3} \left[ 8(\mu_Q^2)^2 \mu_R^2 + 24\mu_Q^2 \mu_R^2 \mu_S^2 + 16\mu_Q^2 \mu_R^2 \mu_T^2 - 8\mu_Q^2 \mu_R^2 \mu_W^2 + 16\mu_Q^2 \mu_R^2 p^2 + 36\mu_Q^2 \mu_R^2 \mu_W^2 \\
+ 48\mu_Q^2 \mu_R^2 \mu_T^2 p^2 - 12\mu_Q^2(\mu_W^2)^2 + 18\mu_Q^2(\mu_S^2)^2 + 24\mu_R^2 \mu_S^2 \mu_T^2 + 6\mu_R^2 \mu_S^2 \mu_W^2 \\
+ 24\mu_R^2 \mu_S^2 p^2 + 8\mu_R^2(\mu_T^2)^2 + 16\mu_R^2 \mu_T^2 \mu_W^2 + 16\mu_R^2 \mu_T^2 p^2 - 12\mu_R^2(\mu_W^2)^2 \\
- 8\mu_R^2 \mu_T^2 + 8\mu_R^2(\mu_T^2)^2 + 27(\mu_S^2)^2 \mu_W^2 + 36\mu_S^2 \mu_T^2 \mu_W^2 + 9\mu_S^2(\mu_W^2)^2 p^2 \\
+ 36\mu_S^2 \mu_W^2 + 24\mu_T^2(\mu_W^2)^2 + 48\mu_T^2 \mu_W^2 p^2 - 18(\mu_W^2)^3 - 12(\mu_T^2)^3 \right] \\
\times \left[ 2\mu_Q^2 + 2\mu_R^2 - 2\mu_T^2 + 3\mu_W^2 + 2p^2 \right]^{-1} \left[ \mu_Q^2 - \mu_T^2 + p^2 \right]^{-1} \\
\times \left[ 2\mu_Q^2 + 3\mu_S^2 + 2\mu_T^2 + 3\mu_W^2 + 2p^2 \right]^{-1} \left[ 2\mu_Q^2 + 3\mu_S^2 + 2\mu_T^2 - 2\mu_W^2 + 2p^2 \right]^{-1}
\end{align*}
\]

(A.1)

Clearly there are no massless poles provided the combinations of the various masses in each of the denominators do not accidentally sum to zero. Further, the gluon propagator is transverse and freezes to a non-zero value in the infrared when again there are no cancellations between the parameters.

## B \ W and S channel propagators for SU(N_c).

As the explicit expressions for the propagators for each of the \( W \) and \( S \) channel cases are complicated for an arbitrary colour group we present the expressions for \( SU(N_c) \) only here. First, the \( W \) propagators are

\[
\left. \langle A^a_\mu (p) A^b_\nu (-p) \rangle_W \right|_{SU(N_c)} = - \frac{\delta^{ab} [2p^2 + N_c \mu_W^2]}{[2(p^2)^2 + N_c \mu_W^2 p^2 + 2N_c \eta_4]} P_{\mu\nu}(p)
\]

\[
\left. \langle A^a_\mu (p) \xi^{bc}_\nu (-p) \rangle_W \right|_{SU(N_c)} = \frac{2i\eta_{abc} \mu^2}{[2(p^2)^2 + N_c \mu_W^2 p^2 + 2N_c \eta_4]} P_{\mu\nu}(p)
\]

\[
\left. \langle A^a_\mu (p) \rho^{bc}_\nu (-p) \rangle_W \right|_{SU(N_c)} = 0
\]

\[
\left. \langle \xi^{ab}_\mu (p) \xi^{cd}_\nu (-p) \rangle_W \right|_{SU(N_c)} = - \frac{\delta^{ac} \delta^{bd} [4(p^2)^3 + 2N_c \mu_W^2 (p^2)^2 - 4N_c \mu_W^2 p^2 - N_c \mu_W^2]}{2p^2[(p^2)^2 - \mu_W^2][2p^2 + N_c \mu_W^2]} \eta_{\mu\nu}
\]

24
\[+ \frac{2 \eta_{\mu \nu}}{3p^2} \left[ 2[(N_c^2 + 6)\mu_W^4] \gamma^4 + \mu_W^4 \mu_W^2 + 3(2p^2)^3 - N_c \mu_W^6] \right] \times [2(p^2)^2 + N_c \mu_W^2p^2 + 2N_c \gamma^4]^{-1} \left[(p^2)^2 - \mu_W^4]^{-1} \times [2p^2 + N_c \mu_W^2]^{-1} P_{\mu \nu}(p) \]

\[= \frac{\eta_{\mu \nu}}{2p^2 + N_c \mu_W^2} \left[ (p^2)^2 - \mu_W^4 \right]^{-1} \eta_{\mu \nu} \]

Clearly the gluon propagator freezes to a non-zero value and the \(\xi^{ab}_\mu\) and \(\rho^{ab}_\mu\) propagators have massless poles in various colour channels. More specifically the dominant part of each propagator as \(p^2 \to 0\) is

\[\langle \xi^{ab}_\mu(p)\xi^{cd}_\nu(-p) \rangle_{W} \bigg|_{SU(N_c)} \sim - \frac{1}{2p^2} \left[ \delta^{ac} \delta^{bd} - \delta^{ad} \delta^{bc} - \frac{2}{N_c} f^{abc} f^{cde} \right] \eta_{\mu \nu} \]

\[\langle \rho^{ab}_\mu(p)\rho^{cd}_\nu(-p) \rangle_{W} \bigg|_{SU(N_c)} \sim - \frac{1}{2p^2} \left[ \delta^{ac} \delta^{bd} - \delta^{ad} \delta^{bc} - \frac{2}{N_c} f^{abc} f^{cde} \right] \eta_{\mu \nu} . \]
in the full set of $\mathcal{W}$ channel propagators will not enhance for similar reasons to those of the $\mathcal{R}$ case. Finally, just to be complete for this channel we note that the leading order behaviour of the adjoint colour projected $\xi_{\mu}^{ab}$ propagator as $p^2 \to 0$ is

$$
\left. \langle f^{ab}_{\mu} \xi_{\mu}^{ab}(p) f^{cd}_{\nu} \xi_{\nu}^{cd}(-p) \rangle \right|_{SU(N_c)} \sim - \frac{2 \delta^{pq} p^2}{(p^2)^2 + N_c \mu_W^2 p^2 + 2 N_c \gamma^4} P_{\mu \nu}(p) - \frac{2 \delta^{pq} p^2}{2 p^2 + N_c \mu_W^2} L_{\mu \nu}(p).
$$

Clearly the transverse part of this particular correlator vanishes in the infrared but the longitudinal part freezes.

For the $\mathcal{S}$ channel the propagators are equally as involved since

$$
\langle A^{a}_{\mu}(p) A^{b}_{\nu}(-p) \rangle_{SU(N_c)} = - \frac{\delta^{ab} p^2}{(p^2)^2 + N_c \gamma^4} P_{\mu \nu}(p)
$$

$$
\langle A^{a}_{\mu}(p) \xi_{b}^{a\mu}(p) \rangle_{SU(N_c)} = \frac{i f^{abc} \gamma_{\nu}}{(p^2)^2 + N_c \gamma^4} P_{\mu \nu}(p)
$$

$$
\langle A^{a}_{\mu}(p) \rho_{b}^{a\mu}(p) \rangle_{SU(N_c)} = 0
$$

$$
\langle \xi_{a}^{b\mu}(p) \xi_{b}^{a\mu}(p) \rangle_{SU(N_c)} = \frac{\delta^{ac} \delta^{bd}}{2 p^2} \left[ (N_c^2 - 36) N_c^2 \mu_S^6 - 432 (p^2)^3 - 72 (N_c^2 + 12) \mu_S^2 (p^2)^2 - 6(19 N_c^2 + 72) \mu_S^4 p^2 \right] - \frac{4 \delta^{ac} \delta^{bd}}{2 p^2} \left( N_c (N_c^2 - 9) N_c^2 \mu_S^6 \right) - \frac{12 d^{abcd} \gamma_{a}^{c}(p^2) + (N_c^2 + 18) \mu_S^2 \mu_S^2}{(p^2)^2 + N_c \gamma^4} P_{\mu \nu}(p)
$$

$$
+ \frac{6 \delta^{ac} \delta^{bd} \gamma_{a}^{c}(p^2) - (N_c^2 + 18) \mu_S^2 \mu_S^2}{2 p^2 [(p^2)^2 + N_c \gamma^4]} L_{\mu \nu}(p)
$$

$$
\langle \rho_{a}^{b\mu}(p) \rho_{b}^{a\mu}(p) \rangle_{SU(N_c)} = 0
$$

$$
\langle \omega_{a}^{b\mu}(p) \omega_{b}^{a\mu}(p) \rangle_{SU(N_c)} = \left. \langle f^{ab}_{\mu} \omega_{b}^{a\mu}(p) f^{cd}_{\nu} \omega_{c}^{d\nu}(p) \rangle \right|_{SU(N_c)}
$$

$$
= \frac{\delta^{ac} \delta^{bd}}{2 p^2} \left[ (N_c^2 - 36) N_c^2 \mu_S^6 - 432 (p^2)^3 - 72 (N_c^2 + 12) \mu_S^2 (p^2)^2 - 6(19 N_c^2 + 72) \mu_S^4 p^2 \right]
$$

$$
+ \frac{12 d^{abcd} \gamma_{a}^{c}(p^2) + (N_c^2 + 18) \mu_S^2 \mu_S^2}{(p^2)^2 + N_c \gamma^4} P_{\mu \nu}(p)
$$

$$
+ \frac{6 \delta^{ac} \delta^{bd} \gamma_{a}^{c}(p^2) - (N_c^2 + 18) \mu_S^2 \mu_S^2}{2 p^2 [(p^2)^2 + N_c \gamma^4]} L_{\mu \nu}(p)
$$

$$
= \frac{\delta^{ac} \delta^{bd}}{2 p^2} \left[ (N_c^2 - 36) N_c^2 \mu_S^6 - 432 (p^2)^3 - 72 (N_c^2 + 12) \mu_S^2 (p^2)^2 - 6(19 N_c^2 + 72) \mu_S^4 p^2 \right]
$$
here then we find that at leading order in the zero momentum limit

\[ p = 0 \]

the pure Gribov-Zwanziger case. Specifically as the zero momentum limit is more akin to that observed for the enhanced where we include that for \( \rho \)
to all the 2-point functions. If we examine the colour adjoint projection of the

\[ W \]

channels as

\[ \xi_{ab} \]

By contrast there is no gluon freezing but the massless poles are distributed to the same colour

suppressed.

the loop corrections are included in the 2-point functions since here the gluon propagator is

in the longitudinal part. As discussed earlier we would expect that this will be enhanced when

So the transverse part is finite and indeed vanishes as \( p^2 \to 0 \) whilst a massless pole is present

in the longitudinal part. As discussed earlier we would expect that this will be enhanced when

the loop corrections are included in the 2-point functions since here the gluon propagator is suppressed.

\[ \langle \xi_{ab}(p)\xi_{cd}(-p) \rangle_{SU(N_c)} \sim - \frac{1}{2p^2} \left[ \delta^{ac}\delta^{bd} - \delta^{ad}\delta^{bc} \right] \eta_{\mu\nu} + \frac{1}{N_c p^2} f^{abc} f^{ced} P_{\mu\nu}(p) \]

where we include that for \( \rho_{ab} \) for completeness. Whilst the same colour tensor structure emerges,
the associated enhancement can only be determined when the one loop corrections are computed to all the 2-point functions. If we examine the colour adjoint projection of the \( \xi_{ab} \) propagator here then we find that at leading order in the zero momentum limit

\[ \langle f^{ab\xi_{ab}}(p)\xi_{cd}(-p)\rangle_{SU(N_c)} \sim - \frac{\delta^{pq}p^2}{\left( p^2 + N_c \right)^4} P_{\mu\nu}(p) - \frac{\delta^{pq}}{p^2} L_{\mu\nu}(p) \]

So the transverse part is finite and indeed vanishes as \( p^2 \to 0 \) whilst a massless pole is present
in the longitudinal part. As discussed earlier we would expect that this will be enhanced when
the loop corrections are included in the 2-point functions since here the gluon propagator is suppressed.

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