SU(3) Flavor Breaking in Hadronic Matrix Elements for $B - \bar{B}$ Oscillations

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We present an analysis, using quenched configurations at $6/g^2 = 5.7, 5.85, 6.0, \text{ and } 6.3$ of the matrix element $M_{hl} \equiv \langle P_{hl}|\bar{h}\gamma_\mu(1 - \gamma_5)|\bar{l}\gamma_\mu(1 - \gamma_5)|P_{hl}\rangle$ for heavy-light pseudoscalar mesons. The results are extrapolated to the physical $B$ meson states, $B^0$ and $B^0_d$. We directly compute the ratio $M_{bs}/M_{bd}$, and obtain the preliminary result $M_{bs}/M_{bd} = 1.54(13)(32)$. A precise value of this SU(3) breaking ratio is important for determining $V_{td}$ once the mixing parameter $x_d$, for $B^0 - B^0$ is measured experimentally. We also determine values for the corresponding B parameters, $B_{bs}(2\text{GeV}) = B_{bd}(2\text{GeV}) = 1.02(13)$, which we cannot distinguish in the present analysis.

1. Introduction

We present a direct calculation of the $\Delta F = 2$ heavy-light mixing matrix element,

$$M_{hl}(\mu) \equiv \langle P_{hl}|\bar{h}\gamma_\mu(1 - \gamma_5)|\bar{l}\gamma_\mu(1 - \gamma_5)|P_{hl}\rangle,$$  

(1)

where we have suppressed the scale dependence of the local four quark operator. As is well known, these matrix elements govern $B^0 - \bar{B}^0$ and $B^0_d - \bar{B}^0_d$ oscillations \cite{2}. In the above $h$ and $l$ denote heavy and light quark fields, $P_{hl}$ the corresponding pseudoscalar meson, and $\mu$ is the energy scale appropriate to the calculation. In particular, we present a first direct calculation of the SU(3) flavor breaking ratio,

$$r_{sd} = M_{bs}(\mu)/M_{bd}(\mu)$$

(2)

Our preliminary result is $r_{sd} = 1.54(13)(32)$ where the first error is statistical and the second is from uncertainty in extrapolating to the $B$ mass and to $a \to 0$. The importance of this ratio is that, in conjunction with the experimental measurement of $B^0 - \bar{B}^0$ oscillation (when that becomes available), it should allow the cleanest extraction of the crucial CKM parameter $V_{td}$. We note that the above value of $r_{sd}$ is lower than the result reported at the conference (1.81) which is due primarily to a change in how we extrapolate to the $B$; the difference gives a systematic error.

Presently, $V_{td}$ is deduced from $B^0 - \bar{B}^0$ oscillation via \cite{3}

$$x_{bd} = \frac{(\Delta M)_{bd}}{\Gamma_{bd}} \propto m^2_{bd}B_{bd}(\mu)f^2_{bd}|V_{td}|^2$$

(3)

where $m_{bd}$, $\Gamma_{bd}^{-1} \equiv \tau_{bd}$, and $f_{bd}$ are the mass, life time, and decay constant of the $B^0$ meson, and $(\Delta M)_{bd}$ is the mass difference of the two mass eigenstates of the $B^0 - \bar{B}^0$ system. $x_{bd}$ is the mixing parameter characterizing the oscillation and has been determined experimentally, $x_{bd} = 0.71(6)$ \cite{4}. $B_{bd}$ is the so called bag, or B, parameter. To extract $V_{td}$ from Eq. (3) requires knowledge of two hadronic matrix elements, $f_{bd}$ and $B_{bd}$. These are being calculated by using lattice and other methods. $f_{bd}$ may eventually even be measured experimentally through, for example, the decay $B \to \tau \nu_\tau$. However, $B_{bd}$ is a purely theoretical construct which is inaccessible to experiment. Thus determination of $V_{td}$ from experiment through use of Eq. (3) will ultimately be limited by the precision of the nonperturbative quantity $f^2_{bd}B_{bd}$. These parameters are related to the matrix element, Eq. (4), via

$$M_{bd}(\mu) = \frac{8}{3} f^2_{bd}m^2_{bd}B_{bd}.$$  

(4)

Now making the replacement $d \to s$ in Eq. (3) and taking the ratio with Eq. (3), we arrive at an
alternate way to extract \( V_{td} \),

\[
\frac{|V_{td}|^2}{|V_{ts}|^2} = r_{td} \frac{\tau_{ls} x_{td}}{\tau_{ls} x_{bs}} \quad (5)
\]

Note that \( V_{ts} \) in Eq. (3) is related by three generation unitarity to \( V_{cb} \), and is therefore already quite well determined, \( |V_{ts}| \approx |V_{cb}| = 0.041 \pm 0.003 \) [4]. The important distinction between using Eq. (3) instead of Eq. (5) is that the former requires only knowledge of corrections to SU(3) flavor symmetry while the latter requires the \textit{absolute} value of the matrix element \( M_{bd} \). It is also important to realize that since \( r_{sd} \) is a ratio of two very similar hadronic matrix elements, it is less susceptible to common systematic errors in lattice calculations, including scale dependence, matching of continuum and lattice operators, and heavy quark mass dependence. Indeed, the ratio \( r_{sd} \) is, to an excellent approximation, renormalization group invariant, even though the individual matrix elements \( M_{bs} \) and \( M_{bd} \) are scale dependent.

2. Simulations and Results

Table 1 summarizes our quenched lattices and the valence Wilson quark hopping parameters, \( \kappa_l \) and \( \kappa_h \), used to construct quark propagators. For each \( \kappa_l \) and \( \kappa_h \) in Table 1 we calculate a quark propagator using a single point source at the center of the lattice and a point sink.

In Fig. 1 we show example results at \( 6/g^2 = 6.3 \) for \( M_{hl} \) versus \( \kappa_h \) for each value of \( \kappa_l \). The lattice matrix elements are found through simultaneous fits to the three point function corresponding to Eq. 1 and the two point function of the corresponding heavy-light interpolating operator. In Fig. 1 and the following, we have already matched the lattice operator to the continuum one by using the one loop perturbative result from Refs. [7].

The scale dependent renormalization \( Z \) factor is calculated in the NDR scheme at scale \( \mu = 2.0 \) GeV. The coupling is tadpole improved and evaluated at scale \( 1/a \), and we include the KLM normalization for heavy quarks.

Results for the physical \( B \) and \( B_s \) meson systems follow from a series of fits to the lattice data which we use to extrapolate in the two parameters \( \kappa_h \) and \( \kappa_l \). Since the data are highly correlated, we use covariant fits and a jackknife procedure at each step to account for the correlations. We take the form of our fits from chiral perturbation theory and expectations based on heavy quark effective theory (HQET).

\begin{figure}[h]
\centering
\includegraphics[width=0.5\textwidth]{fig1.png}
\caption{The matrix element \( M_{hl} \) at \( 6/g^2 = 6.3 \).}
\end{figure}

\( \kappa_c(6/g^2) \) and \( \kappa_s(6/g^2) \) are determined from either linear or quadratic fits to \( m_{ll}^2 \), as a function of \( \kappa_l^{-1} \) and \( \kappa_l'^{-1} \) (\( \kappa_l' \) refers to the strange quark). The values for \( \kappa_c \) and \( \kappa_s \) are summarized in Table 2.

Finding \( \kappa_s \) requires the scale \( a \), which we set from \( a f_s \), to determine the lattice value of the kaon mass \( m_K \) \( (a^{-1} \) is tabulated in Table 2). We note that at \( 6/g^2 = 5.7 \) the choice of the coupling constant scale for \( Z_A \), the axial current renormalization, has a significant effect on the lattice spacing determination; \( Z_A \) differs by \( \sim 7\% \) when the scale changes from \( 1/a \) to \( \pi/a \).

Next, using chiral perturbation theory for heavy-light mesons [8], we extrapolate \( M_{hl} \) to \( \kappa_l = \kappa_c \). We do not include chiral logarithms in our fits since the light quark masses used in the extrapolations are relatively heavy, \( \kappa_l \approx \kappa_c \). The results for \( M_{hl} \) at \( 6/g^2 = 6.3 \) (see Fig. 1) show a smooth linear behavior. The confidence levels for the extrapolations are much lower for \( 6/g^2 = 6.0(243) \) than for the other points; in addition, the effective values of \( M_{hl} \) show poor
Table 1
Summary of simulation parameters.

| $6/g^2$ | conf. size | $\kappa_l$ | $\kappa_h$ |
|---------|------------|------------|------------|
| 5.7     | 83 $16^3 \times 33$ | 0.160 0.164 0.166 | 0.115 0.125 0.135 0.145 |
| 5.85    | 100 $20^3 \times 61$ | 0.158 0.159 0.160 | 0.107 0.122 0.130 0.138 0.143 |
| 6.0     | 60 $16^3 \times 39$ | 0.152 0.154 0.155 | 0.103 0.118 0.130 0.135 0.142 |
| 6.0     | 100 $24^3 \times 39$ | 0.152 0.154 0.155 | 0.103 0.118 0.130 0.135 0.142 |
| 6.3     | 60 $24^3 \times 61$ | 0.148 0.149 0.150 0.1507 | 0.100 0.110 0.125 0.133 0.140 |

Table 2
Inverse lattice spacing and critical and strange hopping parameters.

| $6/g^2$ | $a^{-1}$(GeV) | $\kappa_c$ | $\kappa_s$ |
|---------|---------------|------------|------------|
| 5.7     | 1.45(10)      | 0.16969(10) | 0.1642(7)  |
| 5.85    | 1.64(20)      | 0.16163(9)  | 0.1577(9)  |
| 6.0     | 2.06(17)      | 0.15711(7)  | 0.1548(4)  |
| 6.0     | 2.08(13)      | 0.15714(4)  | 0.1543(4)  |
| 6.3     | 3.37(47)      | 0.15226(16) | 0.1506(4)  |

plateaus, so we exclude this point in our final determination of $r_{sd}$.

Inspired by HQET, we continue by fitting $M_{hc}$ to a polynomial in the inverse heavy meson mass, $m_{hc}^{-1}$, and then extrapolating to the $B$ meson mass (see Fig. 3). Again, we use $f_\pi$ to set the scale. Of course, the same procedure can be carried through for the heavy-strange mesons by first extrapolating the data to $\kappa_s$ instead of $\kappa_c$. The resulting curve is also shown in Fig. 3. Fits which include all mass values (dashed line) generally have low confidence levels. Moreover, the lighter points have smaller statistical errors and dominate the fits, yet they are far from the $B$ mass. We therefore omit the lightest two points at each value of $6/g^2$ and use a completely constrained fit to extrapolate to the $B$. Compared with fitting all points, this results in systematically lower values of $r_{sd}$, changing the central value from 1.81 (which was reported at the conference) to 1.54. We take the 0.27 difference as the systematic error of the extrapolation. The ratio $M_{bs}/M_{bd}$ is shown as a function of $a(6/g^2)$ in Fig. 4. At $6/g^2 = 5.7$, our heaviest mass is still quite far from the $B$ mass, so we also ignore this point in the extrapolation to the continuum limit.

Generally, we expect the Wilson quark action to introduce discretization errors of order $a$ in all observables. However, in a ratio of two similar quantities, we might expect a large cancellation of the lowest order discretization errors. A constant fit gives $M_{bs}/M_{bd} = 1.54(13)$ while a linear fit gives $1.72(67)$. Since the coefficient of the linear term is only 0.3 sigma from 0, we quote the constant fit result as our central value and use the difference as an estimate of the systematic error of the continuum extrapolation. Adding that error in quadrature with the systematic error from the extrapolation to the $B$ mass, we get $r_{sd} = 1.54(13)(32)$.

The extraction of the individual values of $M_{bd}$ and $M_{bs}$ is clearly expected to have larger errors. Conventionally these matrix elements are given in terms of the corresponding $B$ parameter.
defined in Eq. (4). Carrying out the above fitting procedure for $B_{bd}(\mu)$, we find a constant fit yields $B_{bd}(2 \text{ GeV}) = 0.97(3)$ while linear extrapolation gives $1.02(13)$. We cannot however distinguish $B_{bs}(2 \text{ GeV})$ from $B_{bd}(2 \text{ GeV})$ since our data for $B_{hl}$ versus $\kappa_l$ are fit equally well to constant or linear forms. This was not true for $M_{hl}$ as is evident from Fig. 2. Taking the linear result, we quote $B_{bd}(2 \text{ GeV}) = B_{bs}(2 \text{ GeV}) = 1.02(13)$. We recall that until now [1,2], lattice results for the SU(3) breaking ratio $r_{sd}$ have been obtained by using Eqs. (2) and (4) and the lattice measurements of $f_{bd(s)}$ and $B_{bd(s)}$. A reasonable estimate is $f_{bs}/f_{bd} \approx 1.13 \pm .10$ (we are presently calculating this ratio on our lattices). As indicated above, the ratio of B parameters is consistent with unity, and the ratio of masses is $1.017 \pm .03$. Therefore, the conventional method leads to $r_{sd} \approx 1.32 \pm .23$ compared to $1.54 \pm .13 \pm .32$ obtained with our direct method. Thus the two methods are quite consistent. However, as we have emphasized, the direct method offers many distinct advantages, and future lattice computations should be able to improve the precision in our determination of the ratio $r_{sd}$.

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