Theory overview of \((g - 2)_\mu\)

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Abstract. For more than a decade the SM determination of the anomalous magnetic moment of the muon, \((g - 2)_\mu\), has shown a deviation from its experimental value. There is currently a more than 3 standard deviations difference. However, the SM determination could be greatly improved, mainly by the reduction of the uncertainties associated to the calculation of the hadronic contributions to \((g - 2)_\mu\). Supersymmetric models are capable to explain the difference between experimental and SM values of \((g - 2)_\mu\). In these models, the study of the correlations to other observables, for example BR(\(\mu \to e\gamma\)), can help to select models where constraints from the LHC are compatible and can give definite predictions.

1. SM overview

Current status. A lepton has a magnetic moment, \(g_\ell\), aligned along its spin: \(\vec{\mu}_\ell = g_\ell \frac{Q_\ell}{m_\ell} \vec{s}\), where \(Q_\ell\) is the charge of the lepton, \(m_\ell\) and \(s\) its spin. Without quantum corrections \(g_\ell = 2\), but with quantum corrections the anomalous moment, \(a_\ell \equiv (g - 2)_\ell/2\), has a non zero value. In the Standard Model (SM), the largest contribution to \(a_\mu\) comes from the one loop, 1\(\ell\), QED correction, Fig. 1, but EW and Hadronic corrections are crucial to determine the departure of the SM with the experimental value [1]. The up to date value of \(a_\mu^{\text{QED}}\) has been computed at 5\(\ell\) [2] and the main uncertainty comes from the fine structure constant \(\alpha\). The other uncertainties come, respectively, from the lepton mass ratios, the 4\(\ell\) and the 5\(\ell\) corrections. This value is presented in Tab. 1. The \(a_\mu^{\text{EW}}\) contribution has been calculated up to 2\(\ell\) [3], and is presented in the second row of Tab. 1. Its main uncertainty comes from hadronic considerations at 2\(\ell\), and unknown 3\(\ell\) effects. The quoted value includes a reevaluation using the latest constraints from the LHC Higgs data, based on previous calculations from [4, 5]. The hadronic contributions are divided in \(a_\mu^{\text{HLO,HNLO}}\) and Hadronic light by light contributions: \(a_\mu^{\text{HLbL}}\). The LO Hadronic

\[ \gamma \]

\[ \mu \]

\[ \gamma \]

Figure 1. 1\(\ell\) correction to \(a_\mu\) in QED.

\[ \kappa \]

\[ \gamma + \text{permutations} \]

Figure 2. \(a_\mu^{\text{HLbL}}\): Hadronic Light by Light contributions to \(a_\mu\).
contribution is calculated from the dispersion relation

\[ a_{\text{HLO}}^\mu = \left( \frac{a_{\text{HLO}}}{3\pi} \right)^2 \int_{m^2}^{\infty} \frac{ds}{s^2} K(s) R(s), \quad R \equiv \frac{\sigma_{\text{tot}}(e^+e^- \rightarrow \text{hadrons})}{\sigma(e^+e^- \rightarrow \mu^+\mu^-)} \]  

(1)

where the kinematic form factor varies from \( K(s = m_e^2) = 0.4 \) to \( K(s \rightarrow \infty) = 0 \). The dispersion relation connects the bare cross section for \( e^+e^- \) annihilation into hadrons to the hadronic vacuum polarization contribution to \( a_{\mu} \). There are presently different analyses, each with different models to calculate the dispersion relation, Eq. (1), based on existing experimental data sets. However, there has not been a consensus on how these values should be put together.

The most recent calculations can be found in [6, 7, 8, 9]. Their values are respectively data sets. However, there has not been a consensus on how these values should be put together.

If we compare the average value \( a_{\mu} \) of Tab. 1 to the experimental value from E821, \( a_{\mu}^{E821} = (116592089 \pm 63) \times 10^{-11} \), and the revised value of \( \lambda = \mu_{\mu}/\mu_p \), we obtain \( \Delta a_{\mu}(E821 - SM) = 311 \pm 81 \). Since there are different analyses of \( a_{\mu}^{HLO} \), we can say in general is that there is currently more than 3\( \sigma \) difference between SM and experimental results.

**Table 1.** Summary of SM results. The results marked with * has been computed here, taking into account references [6, 7, 8, 9]. There is not an agreed official value of the LO contribution, but taking as an average the four references quoted, we obtain \( \Delta a_{\mu}(E821 - SM) = 311 \pm 81 \).

| Contribution (\( \times 10^{-11} \)) units | QED [2] | 116 584 718,951 \( \pm 0.009 \pm 0.019 \pm 0.007 \pm 0.007 \) units | \( E\text{W} \) [3] | 154 \( \pm 1 \) |
|-----------------------------------------------|----------|-------------------------------------------------|-----------------|-----------------|
| HVP(LO) \([\star]\)                         | 6 898 \( \pm 44 \) |
| HVP(NLO) [9]                                | \(-84 \pm 0.7\) |
| HLBL [12]                                    | 105 \( \pm 26 \) |
| Total SM \([\star]\)                        | 116 591 778 \( \pm 44\) \text{HLO} \pm 26\text{HNLO} \pm 2_{\text{other}} \( \pm 51_{\text{other}} \) |

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**Future improvements in the theoretical error estimation.** The evaluation of \( a_{\mu}^{HLO} \) from \( e^+e^- \rightarrow \text{hadrons} \) requires a better combination from different experimental data sets used to determine the dispersion relations, Eq. (1), in particular some of the old sets should be revisited. The expected error reduction from the HLO evaluation of [8] is to bring \( 42 \times 10^{-11} \) down to \( 26 \times 10^{-11} \) [13]. Current lattice estimates for the error are at the \( \sim 5\% \) and the plan is to reduce them to \( \sim 2\% \), within the next five years and within a decade to be competitive to the theoretical estimates taking into account the experimental information of \( e^+e^- \rightarrow \text{hadrons} \) [14]. Since \( a_{\mu}^{HLBL} \) cannot be evaluated by dispersions relations, a big decrease is not expected but some experimental data can help to pin down related amplitudes. The ultimate goal is to reduce the
uncertainty from 25% to 10%. A calculation on the lattice even with an error \( \sim 30\% \) would have an impact since it could pin down important difference in the models used to determine short and long distance limits of \( a_\mu^{HLbL} \) [13, 14].

2. Supersymmetry overview

In the MSSM, at 1\( \ell \)\( \ell \), the relevant parameters to determine the supersymmetric (SUSY) contribution to \( a_\mu \) are: \( m_{\tilde{g}}, \tan \beta = (H_u/H_d) \mu \) (\( \mu H_u H_d \)), where \( m_{\tilde{g}} \) is the scale of the particles involved in the 1\( \ell \) correction to \( a_\mu \), Fig. 3. When all the relevant SUSY masses \( m_{\tilde{g}} \) are of the same order, the contribution to \( a_\mu \) has the form \( a_\mu^{\text{SUSY}} = 120 \times 10^{-11} \tan \beta \text{sign}(\mu) \left(100 \text{ GeV}/m_{\tilde{g}}\right)^2 \).

![Figure 3](image.png)

**Figure 3.** Top: example of a SUSY diagram mediating \( a_\mu \). Bottom: The flavor-violating analogous diagram, mediating the transition \( \mu \to e\gamma \).

![Figure 4](image.png)

**Figure 4.** SUSY contribution to \( a_\mu \) vs BR(\( \mu \to e\gamma \)) for similar SUSY masses (light blue), large \( \mu \) (orange), and heavy left-handed sleptons (\( \tilde{\chi}^0 - \tilde{\mu}_R \), violet). In each case, \( \delta_{LL} = \delta_{RR} = 2 \times 10^{-5} \), \( \tan \beta = 50 \).

The LHC searches for SUSY particles, indicate that \( m_{\tilde{q}}, m_{\tilde{g}} \gtrsim 1 \text{ TeV} \) (the masses of the squarks -in general- and the gluino, respectively). Since \( a_\mu \) requires \( m_{\tilde{g}} = O(100) \text{ GeV} \), we should consider that the supersymmetric particles involved in \( a_\mu \) should be smaller than those for which the LHC has put constraints above the 1 TeV range, hence we need \( m_{\tilde{q}}, m_{\tilde{g}} \gg m_{\tilde{g}} = m_{\tilde{g}}, m_{\tilde{\chi}^\pm}, m_{\tilde{\chi}^0} \). These last three masses are respectively the masses of the sleptons, the charginos and the neutralinos. With these considerations, a solution to the discrepancy between experimental and SM determinations to \( a_\mu \) is still possible. Furthermore, when there are some big hierarchies between some sfermions and the sleptons relevant to \( a_\mu \), 2\( \ell \) effects could be important [15]. In [16] we have studied cases where there is a hierarchy among the particles entering into the 1\( \ell \) calculation of \( a_\mu^{\text{SUSY}} \) and hence, the typical behavior of the general 1\( \ell \) corrections does not longer hold. A nice example is the case where the lightest neutralino contribution dominates and we can see an increase of the value of \( a_\mu^{\text{SUSY}} \) as the value of \( \mu \) grows. This result holds when the hierarchy is such that \( \mu > M_2 > M_1 \), where \( M_1\approx M_2 \) are the masses of the gauginos coupled to the bino and the \( SU(2)_L \) gauginos, respectively. In this case, \( a_\mu \approx g_4^2 m_{\tilde{g}}^2 \frac{\mu \tan \beta}{M_1} F \left[ \frac{m_{\tilde{\chi}^0}^2}{m_{\ell\mu}^2} \right] \), where in the range of \( a_\mu^{\text{SUSY}} \) that fits the difference \( \Delta a_\mu(E821 - SM) \), the function \( F \left[ \frac{m_{\tilde{\chi}^0}^2}{m_{\ell\mu}^2} \right] \) is \( O(1) \) and the value of \( \mu \) can be up to 5 TeV. This example is a clear departure from the typical behavior at 1\( \ell \) and is relevant for constraints at the LHC, and as we will see next, for correlations to BR(\( \mu \to e\gamma \)). We call this scenario “Large-\( \mu \”).

3. Correlation to \( \mu \to e\gamma \)

The SUSY contributions to \( a_\mu \) and to the decay \( \mu \to e\gamma \) are given by very similar Feynman diagrams. There has been already more than a decade (please see references in [16] ) of works...
reporting correlations in specific scenarios, in particular if $a_{\mu}$ is dominated by a single diagram. In [16] we disentangled at which degree the correlations depended on the specific relations on the masses of the MSSM. We have shown how the correlations are weakened by significant cancellations between diagrams in large parts of the MSSM parameter space. Nevertheless, the order of magnitude of $\text{BR}(\mu \to e \gamma)$ for a fixed flavor-violating parameter can often be predicted. We can define flavor violating parameters $\delta_{12}^{\text{HLO}}$ with $\mu |L|^2/|\mu |R|^2$, for $X = L, R$.

The Large-$\mu$ region, which we have described above, is shown in orange (dark) and finally a region labeled by “$\bar{\mu}_R$” is shown towards the left-bottom corner. The vertical green-shaded area corresponds to the upper limit of $\text{BR}(\mu \to e \gamma)$ [17].

The ultimate application of this kind of analyses would be attained if the behavior were we can identify a correlation. The use of this correlation is as follows. We can write $\text{BR}(\mu \to e \gamma) = \frac{3\alpha^2}{16\pi m_{\mu}^2} (|a_{\mu e \gamma L}|^2 + |a_{\mu e \gamma R}|^2)$, where $a_{\mu e \gamma X}$, $X = L, R$ are the contributions to the amplitude of the process $\mu \to e \gamma$, with Left and Right chiralities, respectively (see. eq. 5 of [16]). By calculating the values of $a_{\mu e \gamma X}$ and $a_{\mu}$ for the different scenarios, we are able to write $|a_{\mu e \gamma X}| = |a_{\mu}| f m_{\mu}^2$, where $f = O(1)$ number and now $m_{\mu}^2$ is the value of the SUSY mass involved in the specific scenario, for example in the Large-$\mu$ scenario, the only particles involved will be the particles in the loop of the top figure of Fig. 3.

Now, the requirement of $a_{\mu}$ to lie within the difference of the experimental and SM values fixes $m_{\mu}^2$, the coefficient $f$ changes with the scenario, but the point is that then $|a_{\mu e \gamma X}| \propto (\delta_{12}^{\text{HLO}})^2$ is constrained if we want to satisfy the bound on $\text{BR}(\mu \to e \gamma)$ and this cannot be evaded by simply raising the value of the parameter $m_{\mu}^2$. The ultimate application of this kind of analyses would be attained if $\text{BR}(\mu \to e \gamma)$ could eventually be measured: that would give definite indications for the required values of all relevant mass parameters.

Acknowledgments

I thank D. Stöckinger for collaboration, discussions and encouragement. I also would like to thank the organizers of BEACH 2014, specially C. Lazzeroni.

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