Interpolating gauge fixing for Chern-Simons theory

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Abstract

Chern-Simons theory is analyzed with a gauge-fixing which allows to discuss the Landau gauge and the light-cone gauge at the same time.

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1 Introduction

The Chern-Simons theory in three space-time dimensions \([1]\) has been the object of continuous investigations during the last years. In particular, its topological nature has led to the beautiful and powerful relation between topological invariants of three dimensional manifolds and the vacuum expectation value of Wilson lines \([1, 2, 3]\).

The Chern-Simons theory has been extensively studied also as a pure three-dimensional gauge field model. It turns out to be an example of a fully ultraviolet finite theory \([1, 2]\). This is due to the existence, besides the usual gauge invariance, of an additional symmetry, usually called topological vector-supersymmetry, whose generators carry a Lorentz index. This supersymmetry, originally found in the covariant Landau gauge \([3, 4]\), has been recently observed also in the noncovariant algebraic gauges \([5, 6]\) and imposes quite strong constraints on the theory. Indeed, as it has been shown in the case of the axial gauge \([10]\), it allows to compute exactly all the Green functions without the need of an action principle and of the usual Feynman graphs expansion derived from it.

Let us mention also that, actually, the topological supersymmetry seems to be a common feature of a very large set of topological field theories \([11]\) including the bosonic string \([12]\). Moreover, as proven in \([13]\), it provides an elegant and simple way for solving the descent equations associated to the integrated BRST cohomology, yielding then an algebraic characterization of anomalies and invariant counterterms.

The aim of this letter is to examine the symmetry content of the Chern-Simons theory in a more general gauge known as the interpolating gauge. This gauge, introduced by \([14]\) in the case of four-dimensional Yang-Mills theories, interpolates between the Landau gauge and the light-cone gauge with the Leibbrandt-Mandelstam prescription \([15]\). This is done by means of the introduction of a gauge parameter \(\zeta\) running from 0 to \(\infty\), the two values correspond respectively to the Landau gauge \((\zeta = 0)\) and to the the light-cone gauge \((\zeta = \infty)\).

The main advantage of the use of the interpolating gauge in four dimensional gauge theories relies on the possibility of regularizing the nonlocal divergences which arise when pure noncovariant gauges are adopted \([15]\). Indeed, as discussed in details in \([14]\), for any finite value of the gauge parameter \(\zeta\) the ultraviolet behaviour of the propagators turns out to be compatible with the requirement of the power-counting theorem. This implies that only local counterterms are needed in order to renormalize the theory. The limit \(\zeta \to \infty\) is, of course, singular and one recovers the usual nonlocal divergences. However, being \(\zeta\) a gauge parameter, it follows that the matrix element of gauge invariant operators does not depend on \(\zeta\), i.e. the limit \(\zeta \to \infty\) exists. One sees then that the use of the interpolating gauge allows to prove in an algebraic way the important result that in a four-dimensional gauge theory the nonlocal divergences drop out in the matrix element of physical quantities.
Let us turn now to the case of the three-dimensional Chern-Simons theory. Antici-
pating the conclusions, we may say that the use of the interpolating gauge gives
quite interesting results also in the case of the topological field theories. In par-
ticular, as we shall see in the following, the topological supersymmetry holds also
in the interpolating gauge. We will be able then to prove that, as in the case of a
pure Landau gauge \([4, 5]\), the model is ultraviolet finite. Therefore, contrary to the
four-dimensional case, the limit \(\zeta \to \infty\) is not singular.

This feature has an important consequence: it implies that the transition between
a covariant Landau gauge and a noncovariant algebraic gauge can be done in a soft
way, i.e. in the case of topological field theories the use of the interpolating gauge
allows to reach smoothly an algebraic noncovariant gauge starting from a covariant
one. In addition it suggests also that the matrix element of Wilson loops, being a
gauge invariant quantity, does not depend on the gauge parameter \(\zeta\) and then its
value remains unchanged when one moves from \(\zeta = 0\) to \(\zeta = \infty\). This result has
been confirmed by \([16]\) with a different technique.

The work is organized as follows. In Sect.2 we present the quantization of the
model in the interpolating gauge, in Sect.3 we establish the topological vector super-
symmetry and, finally, in Sect.4 we discuss the ultraviolet finiteness of the model.

## 2 The classical action in the interpolating gauge

The classical gauge-fixed action we start with is given by

\[ S = S_{\text{inv}} + S_{\text{gf}}, \]  

(2.1)

where \(S_{\text{inv}}\) and \(S_{\text{gf}}\) are respectively the gauge invariant Chern-Simons action and
the interpolating gauge-fixing term \([14]\). They read

\[ S_{\text{inv}} = -\frac{1}{2k}\int d^3x \text{Tr} \epsilon^{\mu\nu\rho}\left( A_\mu \partial_\nu A_\rho + \frac{2}{3} A_\mu A_\nu A_\rho \right), \]  

(2.2)

and

\[ S_{\text{gf}} = s \int d^3x \text{Tr} \left( \bar{K} (\zeta (n^* \partial) D - (\partial A)) + \bar{c} (D + nA) \right), \]  

(2.3)

where \(s\) denotes the BRST-transformations

\[ sA_\mu = -D_\mu c, \quad sc = c^2, \]

\[ s\bar{c} = b, \quad sb = 0, \]

\[ s\bar{K} = K, \quad sK = 0, \]  

(2.4)

\[ sD = \dot{D}, \quad s\dot{D} = 0, \]

2
with
\[ D_\mu c = \partial_\mu c + [A_\mu, c] . \] (2.5)

All the fields \( \phi \) are Lie algebra-valued \( \phi = \phi^a T^a \), \( T^a \) being the generators of a simple gauge-group. We use a Minkowskian metric \( \eta_{\mu\nu} = \text{diag}(+,-,-) \) and we assume that \( n^\mu \) is a light-like vector \( (|\vec{n}|, \vec{n}) \) with \( n^\mu \) the dual vector \( (|\vec{n}|, -\vec{n}) \). The gauge-fixing (2.3) interpolates between the Landau-gauge \( (\zeta = 0) \) and the light-cone gauge \( (\zeta \to \infty) \) as one can see from the expressions of the propagators. The latters are computed to be

\[
\begin{pmatrix}
\varepsilon_{\alpha\mu\nu} \frac{\partial^n}{\partial^\zeta} (\partial^\alpha + n^\alpha \zeta (n^* \partial)) & -\zeta (n^* \partial) \partial_\nu & \varepsilon_{\alpha\beta\nu} \frac{n^\beta \partial^\alpha}{\partial^\zeta} & \frac{\partial_\nu}{\partial^\zeta} \\
-\frac{\zeta (n^* \partial)}{\partial^\zeta} & 0 & -\frac{\partial^2}{\partial^\zeta} & 0 \\
\varepsilon_{\alpha\beta\mu} \frac{n^\alpha \partial^\beta}{\partial^\zeta} & -\frac{\partial^2}{\partial^\zeta} & 0 & \frac{(n\partial)}{\partial^\zeta} \\
\frac{\partial_\mu}{\partial^\zeta} & 0 & \frac{(n\partial)}{\partial^\zeta} & 0
\end{pmatrix}
\delta(x - y) \delta^{ab} , \quad (2.6)
\]

for the \( (A_\mu, b, D, K) \) sector and

\[
\begin{pmatrix}
0 & -\frac{\zeta (n^* \partial)}{\partial^\zeta} & 0 & \frac{1}{\partial^\zeta} \\
-\frac{\zeta (n^* \partial)}{\partial^\zeta} & 0 & \frac{\partial^2}{\partial^\zeta} & 0 \\
0 & -\frac{\partial^2}{\partial^\zeta} & 0 & \frac{(n\partial)}{\partial^\zeta} \\
-\frac{1}{\partial^\zeta} & 0 & \frac{(n\partial)}{\partial^\zeta} & 0
\end{pmatrix}
\delta(x - y) \delta^{ab} . \quad (2.7)
\]

for the ghost sector \( (c, \bar{c}, \hat{D}, \hat{K}) \) with

\[ \partial^2 = \partial^2 + \zeta (n^* \partial) (n\partial) . \quad (2.8) \]

It is easy now to recover the propagators of the Landau-gauge for \( \zeta = 0 \) and of the light-cone gauge with the Leibbrandt-Mandelstam prescription for \( \zeta \to \infty \).

Notice that, as already discussed in [14], for any finite value of \( \zeta \) all the relevant propagators are compatible with the requirement of the power-counting theorem. Therefore we are allowed to study the ultraviolet behaviour of the model by making use of the Quantum Action Principle [17] which, in turn, guarantees that all possible counterterms and anomalies are local functionals of the fields. Let us remark also that the parameter \( \zeta \), due to the fact that it appears only in the pure gauge-fixing part of the action (2.1), is a gauge parameter.
3 Symmetries

It is well known that the Chern-Simons theory, when quantized in the pure Landau gauge \[6, 7\] or in a noncovariant algebraic gauge \[8, 9\], possesses a vector-supersymmetry which turns out to be very powerful for studying its perturbative behaviour. Let us show that this supersymmetric structure is also present in the interpolating gauge (2.3). In order to derive it let us consider, following \[18\], the energy-momentum tensor \(T_{\mu\nu}\) which, due to the topological character of the action (2.1), turns out to be a BRST-variation

\[
T_{\mu\nu} = s\Lambda_{\mu\nu} ,
\]

with

\[
\Lambda_{\mu\nu} = +\eta_{\mu\nu}\left(\zeta\bar{K}(n^*\partial)D + (A\partial)\bar{K} + \bar{c}(nA) + \bar{c}D\right)
+ \left(\zeta\bar{n}^*\partial_\nu D + \partial_\mu \bar{K}A_\nu + \bar{c}n_\mu A_\nu + \mu \leftrightarrow \nu\right) .
\]

In particular the divergence of \(\Lambda_{\mu\nu}\) reads

\[
\partial^\mu \Lambda_{\mu\nu} = \text{Tr} \left( \partial_\nu \frac{\delta S}{\delta b} + \partial_\nu \bar{K} \frac{\delta S}{\delta K} + \partial_\nu D \frac{\delta S}{\delta D} + \varepsilon_{\mu\nu\rho}(\partial^\rho \bar{K} + n^\rho \bar{c}) \frac{\delta S}{\delta A_\rho} - A_\nu \frac{\delta S}{\delta c} \right)
+ s\text{Tr} \left( \varepsilon_{\mu\nu\rho}n^\rho \bar{c}\partial^\rho \bar{K} \right) + \text{tot. deriv}.
\]

so that, integrating on space-time one gets the equation

\[
\int d^3x \text{Tr} \left( \partial_\nu \frac{\delta S}{\delta b} + \partial_\nu \bar{K} \frac{\delta S}{\delta K} + \partial_\nu D \frac{\delta S}{\delta D} + \varepsilon_{\mu\nu\rho}(\partial^\rho \bar{K} + n^\rho \bar{c}) \frac{\delta S}{\delta A_\rho} - A_\nu \frac{\delta S}{\delta c} \right) =
- \int d^3x \text{Tr} \left( \varepsilon_{\mu\nu\rho}n^\rho \bar{c}\partial^\rho \bar{K} \right) .
\]

This identity, due to the fact that the quadratic breaking in the right hand side is a BRST variation, can be converted into an exact symmetry of the classical action. This is easily done by introducing a BRST doublet of external sources \((\lambda^\nu, \bar{\lambda}^\nu)\) which are singlet with respect to the gauge group

\[
s\lambda^\mu = \bar{\lambda}^\mu , \quad s\bar{\lambda}^\mu = 0 .
\]

Adding now to the classical action \(S\) of (2.1) the external coupling

\[
S^{(\lambda)} = \text{Tr} \int d^3x s \left( \varepsilon_{\mu\nu\rho}\lambda^\mu n^\nu \bar{c}\partial^\rho \bar{K} \right) .
\]

one easily checks that the modified action

\[
\hat{S} = S + S^{(\lambda)} ,
\]
is left invariant by the following transformations

\[ \delta_\mu A_\nu = \varepsilon_{\mu\nu\rho}(\partial^\rho \bar{K} + n^\rho \bar{c}), \quad \delta_\mu b = \partial_\mu \bar{c}, \]
\[ \delta_\mu K = \partial_\mu \bar{K}, \quad \delta_\mu \bar{D} = \partial_\mu D, \]
\[ \delta_\mu \bar{\lambda}^\nu = \partial_\mu \lambda^\nu, \quad \delta_\mu c = -A_\mu, \]
\[ \delta_\mu \lambda^\nu = \delta^\nu_\mu, \quad \delta_\mu \bar{c} = 0, \]
\[ \delta_\mu D = 0, \quad \delta_\mu \bar{K} = 0, \]

and

\[ \delta_\mu \hat{S} = 0, \]

The operator \( \delta_\mu \) gives rise, together with the BRST operator, to the following on-shell supersymmetric structure

\[ \{s, \delta_\mu\} \phi = \partial_\mu \phi + \text{eq. of motion}, \]
\[ \{\delta_\mu, \delta_\nu\} = s^2 = 0, \]

where \( \phi \) collects all the fields.

Finally, coupling the non-linear BRST-variations of \( A_\mu \) and \( c \) to the external sources \( \gamma^\mu \) and \( \sigma \),

\[ \Sigma = \hat{S} + \text{Tr} \int d^3x \left( -\gamma^\mu \left( \partial_\mu c + [A_\mu, c] \right) + \sigma c^2 \right). \]

one gets the classical Slavnov identity

\[ \mathcal{B}(\Sigma) = \text{Tr} \int d^3x \left( \frac{\delta \Sigma}{\delta \gamma^\nu} \frac{\delta \Sigma}{\delta A_\mu} + \frac{\delta \Sigma}{\delta \sigma} \frac{\delta \Sigma}{\delta \bar{c}} + b \frac{\delta \Sigma}{\delta \bar{c}} + \bar{D} \frac{\delta \Sigma}{\delta D} + K \frac{\delta \Sigma}{\delta \bar{K}} + \bar{\lambda}^\nu \frac{\delta \Sigma}{\delta \lambda^\mu} \right) = 0, \]

and the supersymmetric Ward identity

\[ \mathcal{W}_\mu \Sigma = \Delta^c_\mu \]

with

\[ \mathcal{W}_\mu = \text{Tr} \int d^3x \left( \varepsilon_{\mu\nu\rho}(\partial^\rho \bar{K} + n^\rho \bar{c} - \gamma^\rho) \frac{\delta}{\delta A_\nu} - A_\mu \frac{\delta}{\delta c} + \partial_\mu \bar{c} \frac{\delta}{\delta b} + \partial_\mu K \frac{\delta}{\delta \bar{K}} \right) \]
\[ + \partial_\mu D \frac{\delta}{\delta D} + \partial_\mu \lambda^\nu \frac{\delta}{\delta \lambda^\nu} + \frac{\delta}{\delta \lambda^\mu} - \sigma \frac{\delta}{\delta \gamma^\mu} \), \]

and

\[ \Delta^c_\mu = \text{Tr} \int d^3x \left( \sigma \partial_\mu c - \gamma^\nu \partial_\mu A_\nu - \varepsilon_{\alpha\mu\beta} \gamma^\alpha (\partial^\beta \bar{K} + n^\beta b) \right). \]

Notice that the breaking \( \Delta^c_\mu \), being linear in the quantum fields, is a classical breaking.
Introducing now the nilpotent linearized Slavnov operator

\[ B_\Sigma = \text{Tr} \int d^3x \left( \frac{\delta \Sigma}{\delta A_\mu} \frac{\delta}{\delta A_\mu} + \frac{\delta \Sigma}{\delta \gamma_\mu} \frac{\delta}{\delta \gamma_\mu} + \frac{\delta \Sigma}{\delta \bar{c}} \frac{\delta}{\delta \bar{c}} + \frac{\delta \Sigma}{\delta c} \frac{\delta}{\delta c} \right) + b \frac{\delta}{\delta \bar{c}} + \hat{D} \frac{\delta}{\delta D} + K \frac{\delta}{\delta K} + \bar{\lambda}^\mu \frac{\delta}{\delta \lambda^\mu} \), \tag{3.16} \]

we get the off-shell Wess-Zumino type algebra

\[ B_\Sigma B_\Sigma = 0, \]
\[ \{W_\mu, W_\nu\} = 0, \]
\[ \{W_\mu, B_\Sigma\} = P_\mu, \tag{3.17} \]

where \( P_\mu \) is the translation Ward operator

\[ P_\mu = \sum_{\text{all fields } \phi} \text{Tr} \int d^3x \partial_\mu \phi \frac{\delta}{\delta \phi}. \tag{3.18} \]

We remark that the supersymmetric Ward operator \( W_\mu \) doesn’t depend explicitly on the interpolating gauge parameter \( \zeta \), implying that the supersymmetric structure (3.17) remains unchanged when one performs the limits \( \zeta = 0 \) and \( \zeta = \infty \). This is in agreement with the results obtained in \([7, 8]\).

Besides the Slavnov identity (3.12) and the supersymmetric Ward identity (3.13), the classical action (3.11) turns out to be constrained also by:

(i) four gauge conditions

\[ \frac{\delta \Sigma}{\delta b} = D + (nA) + \varepsilon_{\mu \rho \nu} \partial^\nu \bar{K} \lambda^\rho; \]
\[ \frac{\delta \Sigma}{\delta D} = -\zeta (n^\ast \partial) K + b; \]
\[ \frac{\delta \Sigma}{\delta \bar{c}} = -\zeta (n^\ast \partial) \bar{c}; \]
\[ \frac{\delta \Sigma}{\delta c} = \zeta (n^\ast \partial) D - (\partial A) - \varepsilon_{\mu \rho \nu} \bar{\partial}^\mu (\lambda^\nu n^\rho \bar{c}); \tag{3.19} \]

(ii) two anti-ghost equations

\[ \frac{\delta \Sigma}{\delta \bar{K}} - \partial^\mu \frac{\delta \Sigma}{\delta \gamma_\mu} = -\zeta (n^\ast \partial) \hat{D} + \varepsilon_{\mu \rho \nu} (\bar{\lambda}^\nu n^\rho \bar{c} - \lambda^\nu n^\rho b); \]
\[ \frac{\delta \Sigma}{\delta \bar{c}} + n^\mu \frac{\delta \Sigma}{\delta \gamma_\mu} = -\hat{D} + \varepsilon_{\mu \rho \nu} (\bar{\lambda}^\nu \partial^\rho \bar{K} - \lambda^\nu \partial^\rho K); \tag{3.20} \]

(iii) the ghost equation, usually valid in the Landau gauge \([19]\), in this case extends to

\[ G_\Sigma = \int d^3x \left( \frac{\delta \Sigma}{\delta c} + \left[ \bar{c}, \frac{\delta \Sigma}{\delta \bar{K}} \right] + \left[ \bar{c}, \frac{\delta \Sigma}{\delta \bar{c}} \right] + \left[ D, \frac{\delta \Sigma}{\delta \bar{K}} \right] \right) \]
\[ = \int d^3x \left( [c, \sigma] - [A_\mu, \gamma_\mu] \right), \tag{3.21} \]
(iv) two transversality conditions

\[ \mathcal{U}_\Sigma = \int d^3x \, n^\mu \frac{\delta \Sigma}{\delta \lambda^\mu} = 0, \]

\[ \mathcal{V}_\Sigma = \int d^3x \, n^\mu \frac{\delta \Sigma}{\delta \bar{\lambda}^\mu} = 0, \] (3.22)

which express the fact that the composite operators defined in (3.6) are orthogonal to the vector \( n^\mu \),

(v) the rigid gauge-invariance

\[ \mathcal{H}^a_\Sigma = \int d^3x \sum_{\text{all fields } \phi} f^{abc}_\phi \phi^b \frac{\delta \Sigma}{\delta \phi^c}, \] (3.23)

In addition, one has the following algebraic relations

\[ \{ \mathcal{G}^a, \mathcal{G}^b \} = 0, \quad \{ \mathcal{G}^a, \mathcal{B}_\Sigma \} = \mathcal{H}^a, \quad [\mathcal{H}^a, \mathcal{G}^b] = -f^{abc} \mathcal{G}^c, \]

\[ [\mathcal{H}^a, \mathcal{B}_\Sigma] = 0, \quad [\mathcal{B}_\Sigma, \mathcal{V}] = -\mathcal{U}, \quad [\mathcal{H}^a, \mathcal{H}^b] = -f^{abc} \mathcal{H}^c, \] (3.24)

which, taken together with (3.17), show that the operators \((\mathcal{B}_\Sigma, \mathcal{W}_\mu, \mathcal{P}_\mu, \mathcal{G}^a, \mathcal{H}^a, \mathcal{V}, \mathcal{U})\) form a closed algebra.

Let us conclude this section by displaying in Table 1 the dimension and the ghost number of all the fields and sources.

| \( A_\mu \) | \( b \) | \( \bar{c} \) | \( c \) | \( D \) | \( D \) | \( \bar{K} \) | \( K \) | \( \lambda^\mu \) | \( \bar{\lambda}^\mu \) | \( \gamma^\mu \) | \( \sigma \) |
|---|---|---|---|---|---|---|---|---|---|---|---|
| dim | 1 | 2 | 2 | 0 | 1 | 1 | 1 | -1 | -1 | 2 | 3 |
| ghost# | 0 | 0 | -1 | 1 | 0 | 1 | 0 | -1 | 1 | 2 | -1 | -2 |

Table 1: Dimensions and ghost numbers

4 Renormalization and finiteness

In order to study the ultraviolet behaviour of the Chern-Simons theory in the interpolating gauge let us begin by showing that the Slavnov and the supersymmetric Ward identities (3.12), (3.13) as well as the symmetries (iii),(iv) and (v) are not anomalous. Following [11], this is easily done by collecting all the operators into a unique nilpotent operator by means of additional global ghost parameters \((\theta, \varepsilon, \alpha, \beta, x, y)\) whose dimensions and ghost number are given in Table 2.

Let us introduce then the nilpotent operator \( Q \) defined as

\[ Q = \mathcal{B}_\Sigma + \theta^\lambda \mathcal{W}_\lambda + \varepsilon^\lambda \mathcal{P}_\lambda + x^a \mathcal{G}^a + y^a \mathcal{H}^a + \alpha \mathcal{U} + \beta \mathcal{V} \]

\[ - (x^a - \frac{1}{2} f^{abc} y^b y^c) \frac{\partial}{\partial y^a} - f^{abc} x^b y^c \frac{\partial}{\partial x^a} - \theta^\lambda \frac{\partial}{\partial \varepsilon^\lambda} - \beta \frac{\partial}{\partial \alpha}, \] (4.1)
We have shown in [11] that the operator $Q$ collects all the symmetries of the action $\Sigma$. Moreover, as shown in [11], the absence of anomalies for the operator $Q$ is equivalent to the absence of anomalies for each single operator entering the expression (4.1).

Let us consider then the consistency condition

$$QA = 0 ,$$

(4.3)

where the possible anomaly $A$ is an integrated local polynomial of dimension three and ghost number one. To study the cohomology of $Q$ we introduce the filtering operator $N$

$$N = y^a \frac{\partial}{\partial y^a} + x^a \frac{\partial}{\partial x^a} + \theta^\lambda \frac{\partial}{\partial \theta^\lambda} + \varepsilon^\lambda \frac{\partial}{\partial \varepsilon^\lambda} + \alpha \frac{\partial}{\partial \alpha} + \beta \frac{\partial}{\partial \beta}$$

(4.4)

according to which the operator (4.1) decomposes as

$$Q = Q^{(0)} + Q^{(R)} ,$$

(4.5)

with

$$[Q^{(0)}, N] = 0 ,$$

$$Q^{(0)} = B_\Sigma - x^a \frac{\partial}{\partial y^a} - \theta^\lambda \frac{\partial}{\partial \theta^\lambda} - \beta \frac{\partial}{\partial \alpha} .$$

(4.6)

Since the cohomology of $Q$ is isomorphic to a subspace of the cohomology of $Q^{(0)}$ we focus on this last operator. From (4.6) one immediately sees that the ghosts $(x, y, \theta, \varepsilon, \beta, \alpha)$ are grouped in BRST-doublets which are known to yield a vanishing cohomology. Therefore the characterization of the cohomology of the operator $Q^{(0)}$ reduces to that of the Slavnov-Taylor operator $B_\Sigma$. However, due to fact that in three dimensions there are no gauge anomalies, it follows that $Q^{(0)}$ has vanishing cohomology in the sector of field polynomials of dimension three and ghost number one, implying then that the consistency condition (4.3) has only trivial solutions, i.e.

$$A = Q \hat{A}$$

(4.7)

We have proven thus the absence of anomalies for all the operators entering in (4.1).

Let us proceed now to the analysis of the invariant local counterterms $\Gamma_{\text{count}}$ corresponding to possible coupling constant and field amplitudes renormalization. The gauge conditions (3.19) and the anti-ghost equations (3.20) imply that $\Gamma_{\text{count}}$
does not depend on \((b, D, \hat{D}, K)\) and that the fields \(\bar{K}, \gamma\) and \(\bar{c}\) enter only in the combination
\[
\Omega^\mu = \gamma^\mu - n^\mu \bar{c} - \partial^\mu \bar{K}.
\] (4.8)

Taking into account also the Slavnov identity (3.12) and the ghost equation (3.21), it follows that the most general local invariant counterterm can be parametrized as
\[
\Gamma_{\text{count}} = \text{Tr} \int d^3 x \left( -\frac{a}{2} \varepsilon^{\mu \nu \rho} \left( A_\mu \partial_\nu A_\rho + \frac{2}{3} A_\mu A_\nu A_\rho \right) \right) + B \Sigma \text{Tr} \int d^3 x \left( \alpha^\mu A_\mu \Omega^\nu + \beta_{\mu \nu \rho} \Omega^\mu \Omega^\nu \lambda^\rho + \tau^\mu A_\mu \sigma \lambda^\nu 
\right.
\[
\left. + \delta \varepsilon_{\mu \nu \rho} \sigma^2 \lambda^\mu \lambda^\nu \lambda^\rho + \omega_{\mu \nu \rho} \Omega^\mu \sigma \lambda^\nu \lambda^\rho \right),
\] (4.9)

where \((a, \alpha^\mu, \beta_{\mu \nu \rho}, \tau^\mu, \delta, \omega_{\mu \nu \rho})\) are arbitrary quantities which depend on the vectors \((n, n^*)\).

Applying now the supersymmetric Ward identity (3.13) we get the homogeneous condition
\[
\mathcal{W}_\mu \Gamma_{\text{count}} = 0,
\] (4.10)

from which it follows
\[
\begin{align*}
\alpha^\mu &= -\tau^\mu = -\delta^\mu \left( \frac{a}{3} + 2\delta \right) \\
\omega_{\mu \nu} &= 3\delta \varepsilon_{\mu \nu}
\end{align*}
\] (4.11)

so that \(\Gamma_{\text{count}}\) takes the following restricted form
\[
\Gamma_{\text{count}} = \text{Tr} \int d^3 x \bar{a} \left( \frac{1}{2} \varepsilon^{\mu \nu \rho} A_\mu \partial_\nu A_\rho + \Omega^\mu \partial_\mu c - \partial_\mu \sigma \lambda^\mu + A_\mu \sigma \bar{\lambda}^\mu - D_\rho \Omega^\rho A_\mu \lambda^\mu 
\right.
\[
\left. + (\partial_\mu A_\rho - \partial_\rho A_\mu + [A_\mu, A_\rho]) \Omega^\rho \lambda^\mu + \frac{1}{2} \varepsilon_{\mu \nu \rho} \Omega^\rho \lambda^\mu \right),
\] (4.12)

with \(\bar{a}\) defined as \(\bar{a} \equiv -\frac{a}{3} - 2\delta\).

It remains now to impose the two transversality conditions (3.22). It is very easy to see that these conditions enforce the vanishing of all the terms of (4.12) containing the external sources \(\lambda^\mu\) or \(\bar{\lambda}^\mu\). Therefore
\[
a = 0, \quad \delta = 0,
\] (4.13)

and
\[
\Gamma_{\text{count}} = 0,
\] (4.14)
meaning that there is no possible local counterterm compatible with the symmetries and constraints of the classical action. This proves the perturbative finiteness of the Chern-Simons theory in the interpolating gauge (2.3).

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