Soft-gluon resummation for gluon-induced Higgs-Strahlung

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Importance of $gg \rightarrow HZ$

- A Higgs boson found with a mass of 125 GeV
- Precision study needed to determine if it is SM Higgs
- One process is Higgs-strahlung (H+Z final state)
- At LO $pp \rightarrow HZ$ is described by $q\bar{q} \rightarrow HZ$
- Drell-Yan corrections up to NNLO \cite{Hamberg:1991dc, Harlander:2002wh, Brein:2004nt}
- $gg \rightarrow HZ$ at NLO \cite{Altenkamp:2012uh}
  - Large corrections (factor of 2)
  - Still has significant scale dependence
## Results NLO $gg \rightarrow HZ$

*Altenkamp, Dittmaier, Harlander, Rzehak, Zirke, ’12*

| $\sqrt{s}$ [TeV] | $m_H$ [GeV] | $\sigma_{gg}^{LO}$ [fb] | $\sigma_{gg}^{NLO}$ [fb] |
|------------------|--------------|-------------------------|------------------------|
| 8                | 115          | $19.8^{+61\%}_{-34\%}$ | $39.3^{+32\%}_{-24\%}$ |
| 8                | 120          | $18.7^{+61\%}_{-34\%}$ | $37.2^{+32\%}_{-24\%}$ |
| 8                | 125          | $17.7^{+61\%}_{-34\%}$ | $35.1^{+32\%}_{-24\%}$ |
| 8                | 130          | $16.7^{+61\%}_{-34\%}$ | $33.1^{+32\%}_{-24\%}$ |
| 14               | 115          | $79.1^{+51\%}_{-31\%}$ | $152^{+27\%}_{-21\%}$  |
| 14               | 120          | $75.1^{+51\%}_{-31\%}$ | $144^{+27\%}_{-21\%}$  |
| 14               | 125          | $71.1^{+51\%}_{-31\%}$ | $136^{+27\%}_{-21\%}$  |
| 14               | 130          | $67.2^{+51\%}_{-31\%}$ | $129^{+27\%}_{-21\%}$  |
Importance of Resummation

- Resummation up to NNLL already improved Higgs production results [Catani, de Florian, Grazzini, Nason, '03] [de Florian, Grazzini, '09] [de Florian, Grazzini, '12]

- $gg \rightarrow HZ$ similar loop induced process $\Rightarrow$ threshold resummation could help further improve results

Agrees with [Catani, de Florian, Grazzini, Nason, '03]
Definition of Threshold

Q-approach

Threshold variable $\hat{\tau}_Q = \frac{Q^2}{\sqrt{s}}$

$Q^2$: the invariant mass final state particles

$1 - \hat{\tau}_Q = 1 - \frac{Q^2}{\sqrt{s}}$

$\sim \frac{\text{energy of the emitted gluons}}{\text{total available energy}}$

M-approach (absolute threshold)

Threshold variable $\hat{\tau}_M = \frac{M^2}{\sqrt{s}}$

$M = m_H + m_Z$

$1 - \hat{\tau}_M = 1 - \frac{M^2}{\sqrt{s}}$

$\sim \frac{\text{maximum energy of the emitted gluons}}{\text{total available energy}}$

$\sqrt{s}$: the partonic center of mass energy
Logarithms

Q-approach

The IR divergences lead to logarithms:

\[ \alpha_s^n \left( \frac{\log^m (1 - \hat{\tau}_Q)}{1 - \hat{\tau}_Q} \right) \equiv \alpha_s^n D_{Q,m}(\hat{\tau}_Q), \; m \leq 2n - 1 \]

In general logarithms of \( 1 - \hat{\tau}_Q \)

M-approach

For \( 2 \rightarrow 2 \) process: logarithms of \( 1 - \hat{\tau}_M \):

\[ \alpha_s^n \log^m (1 - \hat{\tau}_M) \equiv \alpha_s^n D_{M,m-1}(\hat{\tau}_M), \; m \leq 2n \]

Logarithms become large in threshold: \( \hat{\tau} \rightarrow 1 \)
Mellin Transform

Mellin transform is used with respect to $\tau$ (needed for factorization of phase space):

$$
\tilde{\Sigma}_{pp \to HZ}(N) \equiv \int_{0}^{1} d\tau \, \tau^{N-1} \Sigma_{pp \to HZ}(\tau, m_Z, m_H, \mu_R, \mu_F)
$$

$$
= \sum_{i,j} \tilde{f}_{i/p}(N+1, \mu_F) \tilde{f}_{j/p}(N+1, \mu_F) \tilde{\Sigma}_{ij \to HZ}(N, \mu_R, \mu_F)
$$

- $\tilde{f}_{i/p}(N+1, \mu_F)$: Mellin transform with respect to $x$
- $\tilde{\Sigma}_{ij \to HZ}(N, \mu_R, \mu_F)$: Mellin transform with respect to $\hat{\tau}$

$$
D_n(\hat{\tau}) \Rightarrow \log^{n+1} N \text{ and threshold } \hat{\tau} \to 1 \sim N \to \infty
$$

Threshold resummation for $gg \to HZ$
Orders of Resummation

Large logarithms $\log N \equiv L$ for $N \to \infty$

Perturbation needs to be reordered in $\alpha_s$ and $L$:

\[ \tilde{\sigma} \sim \tilde{\sigma}_{LO} \times C(\alpha_s) \exp \left[ L g_1(\alpha_s L) + g_2(\alpha_s L) + \alpha_s g_3(\alpha_s L) + \cdots \right] \]

With orders of precision: LL NLL NNLL

\[ \alpha_s^n \log^{n+1}(N) \quad \alpha_s^n \log^n(N) \quad \alpha_s^{n+1} \log^n(N) \]

Exponential functions are well known and the same as for $gg \to H$

[Catani, de Florian, Grazzini, Nason, '03]
Hard Matching Coefficient (Schematically)

\[ C(\alpha_s) = 1 + \frac{\alpha_s}{\pi} C^{(1)} + \cdots \]

Originates from NLO calculation. Using terms proportional to:

\[ \Rightarrow \sigma_{LO}, \quad \sigma_{LO} D_{M,0}, \quad \sigma_{LO} D_{M,1} \]

OR

\[ \Rightarrow \sigma_{LO} \delta(Q^2 - \hat{s}), \quad \sigma_{LO} D_{Q,0}, \quad \sigma_{LO} D_{Q,1} \]

Mellin transform leads to:

\[ \frac{\alpha_s}{\pi} [C^{(1)} \tilde{\Sigma}_{LO} + \mathcal{O}(\tilde{\Sigma}_{LO} \log(N), \tilde{\Sigma}_{LO} \log^2(N)) + \cdots] \]

\[ \downarrow \]

Expansion of exponential
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Mellin transform leads to:

\[ \frac{\alpha_s}{\pi} [C^{(1)} \tilde{\Sigma}_{LO} + O(\tilde{\Sigma}_{LO} \log(N), \tilde{\Sigma}_{LO} \log^2(N)) + \cdots] \]

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Mellin transform leads to:

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\[ \Downarrow \]

Expansion of exponential
\[ \hat{\Sigma}^{NLO} = \hat{\Sigma}^R + \hat{\Sigma}^V + \hat{\Sigma}^C \]

\[ = \int_3 \left[ d\hat{\Sigma}^R|_{\epsilon=0} - d\hat{\Sigma}^A|_{\epsilon=0} \right] + \int_2 \left[ d\hat{\Sigma}^V + \int_1 d\hat{\Sigma}^A \right]_{\epsilon=0} + \hat{\Sigma}^C \]
Hard Matching Coefficient

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↓

Supressed in threshold limit
Hard Matching Coefficient

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\[ \Downarrow \]

Supressed in threshold limit

\[ \Rightarrow C^{(1)} \text{ calculated by: } \hat{\Sigma}^V + \hat{\Sigma}^A + \hat{\Sigma}^C \]

In agreement with: [Catani, Cieri, de Florian, Ferrera, Grazzini, '13]
**Motivation**

Threshold Resummation Numerical Results

**Hard Matching Coefficient (Result)**

\[
C^{(1)} = \frac{\hat{\sigma}_{\text{virt}}}{\hat{\sigma}_{\text{LO}}} \frac{\pi}{\alpha_s} + \left[ \frac{2}{3} T_R n_l - \left( \frac{11}{6} - 2\gamma_E \right) C_A \right] \log \left( \frac{\mu^2}{W^2} \right) \\
- \left( \frac{50}{9} - \frac{2\pi^2}{3} - 2\gamma_E^2 \right) C_A + \frac{16}{9} T_R n_l
\]

- **Q-approach:** Absolute threshold expansion \( \hat{\sigma}_{\text{virt}} \) and \( \hat{\sigma}_{\text{LO}} \), \( W^2 = Q^2 \)

- **W-approach:** \( W^2 = M^2 \)
Hard Matching Coefficient (Result)

\[ C^{(1)} = \frac{\hat{\sigma}_{\text{virt}}}{\hat{\sigma}_{\text{LO}}} \frac{\pi}{\alpha_s} + \left[ \frac{2}{3} T_R \ n_l - \left( \frac{11}{6} - 2\gamma_E \right) C_A \right] \log \left( \frac{\mu^2}{W^2} \right) \]

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- **Q-approach:** Absolute threshold expansion \( \hat{\sigma}_{\text{virt}} \) and \( \hat{\sigma}_{\text{LO}} \), \( W^2 = Q^2 \)

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Motivation

Threshold Resummation

Numerical Results

Matching to Fixed Order

Resummed Cross Section

\[ \Sigma_{gg \rightarrow HZ}^{(NLO+NLL)}(\tau) = \Sigma_{gg \rightarrow HZ}^{(NLO)}(\tau) \]

\[ + \int_{CT} \frac{dN}{2\pi i} \tau^{-N} \tilde{f}_{g/p}(N+1) \tilde{f}_{g/p}(N+1) \]

\[ \times \left[ \tilde{\Sigma}_{gg \rightarrow HZ}^{(NLL)}(N) - \tilde{\Sigma}_{gg \rightarrow HZ}^{(NLL)}(N) \right]_{(NLO)} \]

Matching to fixed order required to avoid double counting.
Threshold resummation for $gg \rightarrow HZ$
Q-approach (NLL resummation)

PDFs used: MSTW2008NNLO

$\sigma(gg \rightarrow HZ + X)$ [fb]

$\sqrt{S} = 8$ TeV

LO
NLO
NLO+NLL

$m_H = 125$ GeV
$\mu_0^2 = (p_H + p_Z)^2$

$\mu/\mu_0$

$\sigma(gg \rightarrow HZ + X)$ [fb]

$\sqrt{S} = 14$ TeV

LO
NLO
NLO+NLL

$m_H = 125$ GeV
$\mu_0^2 = (p_H + p_Z)^2$

$\mu/\mu_0$
Conclusions

- Improvement in scale dependence:
  \[ \sigma^{NLO} = 32.7^{+31\%}_{-24\%} \text{ fb and } \sigma^{NLO+NLL}_Q = 38.8^{+8.3\%}_{-6.9\%} \text{ fb for 8 TeV} \]
  \[ \sigma^{NLO} = 124^{+26\%}_{-21\%} \text{ fb and } \sigma^{NLO+NLL}_Q = 143^{+6.9\%}_{-5.1\%} \text{ fb for 14 TeV} \]
  Error determined at \( Q^2/3 \) and \( 3Q^2 \)
- Sizable correction: \( \frac{\sigma^{NLO+NLL}}{\sigma^{NLO}} = 1.18 \) (1.15) for 8 (14) TeV

Outlook

- NNLL resummation
- Combine into \( pp \to HZ \) results
Conclusions

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  \[ \sigma_{\text{NLO}} = 32.7^{+31\%}_{-24\%} \text{ fb and } \sigma_{Q}^{\text{NLO+NLL}} = 38.8^{+8.3\%}_{-6.9\%} \text{ fb for 8 TeV} \]
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Thank you for your attention