The rare top decay $t \rightarrow cg$ in topcolor assisted technicolor models

Chongxing Yue$^{(a,b)}$, Gongru Lu$^{(a,b)}$, Guoli Liu$^{b}$, Qingjun Xu$^{b}$

a: CCAST (World Laboratory) P.O. BOX 8730. B.J. 100080 P.R. China
b: College of Physics and Information Engineering,
Henan Normal University, Xinxiang 453002. P.R.China ∗†

March 25, 2022

Abstract

In the framework of topcolor-assisted technicolor(TC2) models, we calculate the contributions of flavor changing scalar couplings involved the neutral top-pion $\pi_t^0$ and top-Higgs $h_t^0$ to the branching ratio $B_r(t \rightarrow cg)$. We find that the value of $B_r(t \rightarrow cg)$ can reach $1 \times 10^{-3}$ with reasonable values of the parameters in TC2 models, which may be testable in the future experiments.

PACS number: 14.65Ha, 12.60.Nz, 13.30Eg

∗This work is supported by the National Natural Science Foundation of China(19905004), the Excellent Youth Foundation of Henan Scientific Committee(9911); and Foundation of Henan Educational Committee.
†E-mail:cxyue@public.xxptt.ha.cn
It is widely believed that the top quark, with a mass of the order of the electroweak scale, will be a sensitive probe into physics beyond the standard (SM). Indeed, the properties of the top quark could reveal information of flavor physics, electroweak symmetry breaking (EWSB) as well as new physics beyond the SM [4]. One of these consequences is that the rare top decays can be used to detect new physics. A lot of the theoretical activity involving the rare top decays have been given within some specific models beyond the SM [2].

At the tree-level there are no flavor changing neutral currents processes in the SM, and at one-loop they are induced by charged current interactions, which are GIM-suppressed. In particular, the rates for the flavor changing rare decays of the top quark $t \rightarrow cv (v = g, \gamma, Z)$ are very small. With the current experimental value of the top quark mass, the branching ratios are $B_r(t \rightarrow cg) \sim 4 \times 10^{-13}$, $B_r(t \rightarrow c\gamma) \sim 5 \times 10^{-13}$ and $B_r(t \rightarrow cZ) \sim 1 \times 10^{-13}$ in the SM [3], which are far below the feasible experimental possibilities at the future colliders (LHC or LC) [4]. The values of the branching ratios $B_r(t \rightarrow cvv) (v = W, g, \gamma, or Z)$ predicted by the SM are more small. These have been studied in Ref.[5]. It is widely believed that, in many simple SM extensions, the branching ratios $B_r(t \rightarrow cv)$ can be enhanced by several orders of magnitude. Detection of the rare top decays at visible levels by any of the future colliders would be instant evidence of new physics. This fact has lead to a lot of studies involving the rare top decays within some specific models beyond the SM. For instance, studies of the rare top decays $B_r(t \rightarrow cv)$ in the multi Higgs doublets models (MHDM) [3, 6], in supersymmetry with R-Parity conservation [7] and with R-Parity violation [8], and in technicolor models [9].

There are many ways in which the extensions of the SM lead to flavor changing scalar (FCS) couplings at the tree-level. FCS couplings can normally enhance the values of branching ratios $B_r(t \rightarrow cv)$. This has been shown that the FCS couplings predicted by the general 2HDM type III can significantly enhance the branching ratios of the rare top decays $t \rightarrow cv$ [10]. Recently, Ref.[11] has calculated the branching ratio for $t \rightarrow c\gamma$ in the framework of the most general CP-conserving 2HDM type III. They have shown that, with reasonable values for the parameter $\tan \beta$, the branching ratio $B_r(t \rightarrow cv)$ could be
within the observable threshold of near future experiments. The aim of this paper is to point out that the FCS couplings predicted by topcolor assisted technicolor (TC2) models also can significantly enhance the values of the branching ratios $B_r(t \rightarrow cv)$, which may approach the detectability threshold of near future experiments.

An important issue in high-energy physics is to understand the mechanism of the mass generation. There may be a common origin for EWSB and top quark mass generation. Much theoretical work has been carried out in connection to the top quark and EWSB. TC2 models and the top see-saw models are two of such examples. Such type of models generally predict a number of scalars with large Yukawa coupling to the third generation fermions. For example, TC2 models predict the existence of scalars including the technipions ($\pi^0, \pi^\pm, \pi^0_a, \pi^\pm_a$) in the technicolor sector and the top-pions ($\pi^0_t, \pi^\pm_t$) and top-Higgs $h^0_t$ in the topcolor sector. Ref.[9] has considered the contributions of these new particles to the rare top decays $t \rightarrow cv$, which gave $B_r(t \rightarrow cg) \sim 10^{-6}$, $B_r(t \rightarrow c\gamma) \sim 10^{-7}$ and $B_r(t \rightarrow cZ) \sim 10^{-8}$. However, Ref.[9] only considered the contributions of charged scalars ($\pi^\pm, \pi^\pm_a$ and $\pi^\pm_t$) via the couplings $s^\pm u_i u_j$ and did not consider the contributions of the neutral top-pion $\pi^0_t$ and top-Higgs $h^0_t$ via the FCS coupling $s^t c$, in which, $s$ is behalf of $\pi^0_t$ or $h^0_t$. In fact, it is an important feature of TC2 models that the neutral scalars can induce the tree-level FCS couplings. It has been shown that the FCS couplings can give distinct new flavor mixing phenomena which may be tested at both low and high energy experiments. Thus, in this paper, we will calculate the contributions of the FCS coupling $s^t c$ to the rare top decays $t \rightarrow cv$ in the framework of TC2 models and see whether $t \rightarrow cv$ can be used to test TC2 models.

Ref.[9] has shown that the branching ratios $B_r(t \rightarrow cv)$ can be enhanced several orders of magnitude by charged scalars. The rare top decays $t \rightarrow cv(v = \gamma, Z)$ are far below the observable threshold of near future experiments. However, the branching ratio $B_r(t \rightarrow cg)$ can reach to $10^{-5}$ for the favorable parameter values of TC2 models, which is just below the expected experimental sensibility. Thus $t \rightarrow cg$ is the most promising one among the rare top decays into gauge bosons $t \rightarrow cv(v = g, \gamma, Z)$. Certainly, we must separate the signals from the large backgrounds before observation of the rare decay
t \rightarrow cg at the future LHC experiments. However, a future high-energy linear e^+e^- collider (LC) could, in principle, be the ideal place to study t \rightarrow cg, as top quark events can be clearly separated by tagging the isolated lepton from the top semileptonic decay. Perhaps, with high integrated luminosity, the largest decay channel t \rightarrow cg can be detected at LC. Thus, our attention will mainly focus on the rare top decay t \rightarrow cg.

Our results show that the branching ratio \( B_r(t \rightarrow cg) \) may approach the observable threshold of near future experiments. For \( \epsilon = 0.08, m_{\pi_t} = 200GeV \), the branching ratio \( B_r(t \rightarrow cg) \) contributed by the neutral top-pion \( \pi^0_t \) can reach \( 6 \times 10^{-4} \) and for \( \epsilon = 0.08, m_{h_t} = 200GeV \), we have \( B_r(t \rightarrow cg) \approx 1 \times 10^{-3} \) arised from the top-Higgs \( h^0_t \), which may be the order of sensitivity of the future experiments LHC or LC.

In TC2 models, the TC interactions play a main role in breaking the electroweak gauge symmetry. The ETC interactions give rise to the masses of the ordinary fermions including a very small portion of the top quark mass, namely \( \epsilon m_t \) with a model dependent parameter \( \epsilon \ll 1 \). The topcolor interactions also make small contributions to the EWSB, and give rise to the main part of the top quark mass, \( (1 - \epsilon)m_t \), similar to the constituent masses of the light quarks in QCD. So that the heaviness of the top-quark emerges naturally in TC2 models. This kind of models predict three top-pions with large Yukawa couplings to the third generation. This induces the new FCS couplings. The relevant couplings including the t-c transition for the neutral top-pion \( \pi^0_t \) can be written as [12, 14]:

\[
\frac{m_t}{\sqrt{2}F_t} \left[ K^{tt}_{UL}K^{tt*}_{LR}t_R\pi^0_t + K^{tc}_{UL}K^{tt*}_{LR}c_R\pi^0_t + h.c. \right],
\]

where the factor \( \sqrt{\nu_w^2 - F_t^2}/\nu_w \) reflects the effects of the mixing between the neutral top-pion \( \pi^0_t \) and the would be Goldstone boson with \( \nu_w = v/\sqrt{2} = 174GeV \) and \( F_t = 50GeV \) which is the top-pion decay constant. \( k^{ij}_{UL} \) is the matrix element of unitary matrix \( k^{ij}_{ UR} \) is the matrix element of the right-handed relation matrix \( k_{UR} \). Ref.[14] has shown that their values can be taken as:

\[
K^{tt}_{UL} = 1, \quad K^{tt}_{UR} = 1 - \epsilon, \quad K^{tc}_{UR} \leq \sqrt{2\epsilon - \epsilon^2}.
\]

In the following calculation, we will take \( K^{tc}_{UR} = \sqrt{2\epsilon - \epsilon^2} \) and take \( \epsilon \) as a free parameter.
The relevant Feynman diagrams for the contributions of the neutral top-pion $\pi^0_t$ to the rare top decay $t \to cg$ via the FCS coupling are shown in Fig.1. Using Eq.[1], we can give the decay width $\Gamma(t \to cg)$:

$$\Gamma(t \to cg) = \frac{\alpha_s m_t^5}{512\pi^4 F_t^4} \frac{v_w^2 - F_t^2}{v_w^2} (K^{tc})^2 [m_t^2 A_1 (C_{12} - 2C_{21} + 3C_{23} - C_{22}) + 2A_1^2$$

$$+ 2m_t^4 (-C_{12} C_{21} - C_{12} C_{22} + 2C_{12} C_{23} + C_{21}^2 + C_{21} C_{22} - 3C_{21} C_{23}$$

$$- C_{22} C_{23} + 2C_{23}^2)]$$

(3)

where

$$A_1 = m_t^2 (C_{11} - C_{12}) - 2C_{24} + B_0 + m_t^2 C_0 + \frac{m_c}{m_t - m_c} B_1 - B_0^* - \frac{m_t}{m_t - m_c} B_1^*,$$

(4)

with

$$B_0 = B_0(p_g, m_t, m_t, m_\mu), \quad B_0^* = B_0(-p_c, m_s, m_t, m_\mu),$$

(5)

$$B_1 = B_1(-p_c, m_s, m_t, m_\mu), \quad B_1^* = B_1(-p_t, m_\pi, m_t),$$

(6)

$$C_0 = C_0(-p_t, m_s, m_t, m_\mu), \quad C_{24} = C_{24}(-p_t, p_g, m_s, m_t, m_\mu),$$

(7)

$$C_{ij} = C_{ij}(-p_t, m_s, m_t, m_\mu), \quad i, j = 1, 2, 3.$$  

(8)

$B_0$, $B_0^*$, $B_1$, $B_1^*$, $C_0$, $C_{24}$ and $C_{ij}$ are standard feynman integrals, in which variable $p_c$ is the monument of charm quarks, $p_t$ is the monument of top quarks, $p_g$ is the monument of the gluons and $m_\mu$ is the scale of the TC2 models. Since the $\bar{c}Lt_R$ coupling is very small [14], we have assumed $K_{UR}^{tc} \approx K^{tc} = \sqrt{|K_{UL}^{tc}|^2 + |K_{UR}^{tc}|^2}$ in the above equations.

Ref.[12] has estimated the mass of the top-pion in the fermion loop approximation and given $180\, GeV \leq m_{\pi_t} \leq 250\, GeV$ for $m_t = 180\, GeV$ and $0.03 \leq \epsilon \leq 0.1$. Since the negative top-pion corrections to the $Z \to b\bar{b}$ branching ratio $R_b$ becomes smaller when the top-pion is heavier, the LEP/SLD data of $R_b$ give rise to certain lower bound on the top-pion mass [16]. It was shown that the top-pion mass should not be lighter than the order of 1TeV to make the TC2 models consistent with the LEP/SLD data. However, we restudied this problem in Ref.[17]. Our results show that the top-pion mass $m_{\pi_t}$ is
allowed to be in the region of a few hundred GeV depending on the models. Thus, the top-pion mass depends on the value of the parameters in TC2 models. As estimation the contributions of the neutral top-pion $\pi^0_t$ to the rare top decay $t \rightarrow cg$, we take the mass of $\pi^0_t$ to vary in the range of $200 GeV - 400 GeV$ in this paper.

The branching ratio $B_r(t \rightarrow cg)$ contributed by the neutral top-pion $\pi^0_t$ is plotted in Fig.2 as a function of the top-pion mass $m_{\pi_t}$ for three values of the parameter $\epsilon$. In Fig.2 we have assumed that the total width is dominated by the decay channel $t \rightarrow Wb$ and taken $\Gamma(t \rightarrow Wb) = 1.56 GeV$. \[1\]. The integral was performed numerically for the parameter values $m_t = 175 GeV$, $m_c = 1.2 GeV$ and $\alpha_s = 0.118$ [18]. From Fig.2, we can see that the FCS coupling $\pi^0_t tc$ indeed could give significantly contributions to the rare top decay $t \rightarrow cg$. The value of $B_r(t \rightarrow cg)$ increases with $\epsilon$ increasing and $m_{\pi_t}$ decreasing. For $m_{\pi_t} = 300 GeV$, the branching ratio $B_r(t \rightarrow cg)$ varies between $60 \times 10^{-4}$ and $3.45 \times 10^{-4}$ for the parameter $\epsilon$ in the range of 0.01-0.08. For $m_{\pi_t} = 200 GeV$ and $\epsilon = 0.08$, the value of $B_r(t \rightarrow cg)$ can reach $6.1 \times 10^{-4}$.

One possible set of anomalous interactions for top quark is given by the flavor-changing chromo-magnetic operators[19, 20, 21]:

\[ \frac{K_f}{\Lambda} g_s f \sigma^{\mu\nu} \frac{X^a}{2} t G^a_{\mu\nu} + h.c. \]

where $\Lambda$ is the new physics scale, $f=\text{u or c}$, the $K_f$ define the strength of the $\overline{t}ug$ or $\overline{t}cg$ anomalous couplings, and $G^a_{\mu\nu}$ is the gauge field tensor of the gluon. Ref.[19] has studied the discovery limits of $K_c/\Lambda$ and $K_u/\Lambda$ at Tevatron and LHC using direct production of an s-channel top quark. The minimum values of $K_c/\Lambda$ and $K_u/\Lambda$ observable at Tevatron and LHC are also calculated in Ref.[20] using single top quark production. Their results are summarized in Ref.[1] by M.Beneke et.al. Ref.[19] has shown that the strength of the anomalous coupling $\overline{t}cg$ may be detected down to $K_c = 0.03$ at the Tevatron with 30 $fb^{-1}$ of data at 2 TeV and $K_c = 0.0084$ at the LHC with 10 $fb^{-1}$ of data at 14 TeV, and the results of Ref.[20] are that the anomalous coupling $\overline{t}cg$ may be detected down to $K_c = 0.046$ at the Tevatron with 30 $fb^{-1}$ of data at 2 TeV and $K_c = 0.013$ at the LHC with 10 $fb^{-1}$ of data at 14 TeV.
If we assume that the anomalous coupling $7cg$ comes from the neutral top-pion $\pi^0_t$, the contributions of $\pi^0_t$ to $Br(t \rightarrow cg)$ can be transposed to that of the strength $K_c$. According to our calculation, we have that, for $m_{\pi_t} = 300 GeV$, the value of $K_c$ increases from 0.0081 to 0.019 as the parameter $\epsilon$ increases from 0.01 to 0.08. For $m_{\pi_t} = 200 GeV$ and $\epsilon = 0.08$, the value of $K_c$ contributed by the neutral top-pion $\pi^0_t$ can reach 0.026. Thus, the values of the branching ratio $Br(t \rightarrow cg)$ are in the range of the sensitivity of the future LHC experiments for the favorable parameter values of TC2 models. The effects of the FCS coupling $\pi^0_t7c$ predicted by TC2 models on the rare decay $t \rightarrow cg$ can not be probed at the Tevatron experiments, but may be probed at the future LHC experiments.

TC2 models also predict the existence of the neutral CP-even state, called top Higgs $h^0_t$. The main difference between the neutral top-pion $\pi^0_t$ and top-Higgs $h^0_t$ is that $h^0_t$ can couple to gauge boson pairs WW and ZZ at tree level, which is similarly to that of the SM Higgs $H^0$ (The couplings $h^0_t WW$ and $h^0_t ZZ$ are suppressed by the factor $F_t/\nu_w$ with respect to that of $H^0$[14]). Thus, the contributions of $h^0_t$ to the rare decay $t \rightarrow cg$ are similar to that of $\pi^0_t$. The relevant Feynman diagrams are also shown in Fig.1. Our results are plotted in Fig.3. To compare our results with that of Refs[19,20] and see whether the effects of $h^0_t$ on the rare decay $t \rightarrow cg$ can be detected, we plot the anomalous coupling strength $K_c$ as a function of the top-Higgs mass $m_{h_t}$ for the three values of parameter $\epsilon$ in Fig.3. The other parameter values are the same as those of Fig.2. From Fig.3, we can see that the contributions of the top-Higgs $h^0_t$ to the rare top decay $t \rightarrow cg$ also decrease with increasing $m_{h_t}$, which is similar to that of the neutral top-pion $\pi^0_t$. However, the contributions of $h^0_t$ to the branching ratio $Br(t \rightarrow cg)$ is larger than that of $\pi^0_t$. For $m_{h_t} = 300 GeV$, the strength $K_c$ varies between 0.011 and 0.026 for the parameter $\epsilon$ in the range of 0.01-0.08. For $m_{h_t} = 200 GeV$ and $\epsilon = 0.08$, the value of the strength $K_c$ can reach 0.033.

If the effects of the new particles such as $\pi^0_t$ and $h^0_t$ on the rare decay $t \rightarrow cg$ could be measured in the future colliders, then one should consider to search directly for these new particles. In fact, these has been done by many authors. For example, G. Burdman[14]...
has studied the observability of the neutral top-pion and top-Higgs at Tevatron or LHC, and H.-J. He and C. P. Yuan [14] have studied the observability of the charged top-pion in the future colliders. We have shown that the neutral top-pion may be detected via the processes \( e^+e^- \rightarrow \bar{t}c, e^+e^- \rightarrow \bar{t}e\bar{e}\nu_e \), and \( e^+e^- \rightarrow \gamma\pi^0_t \rightarrow \gamma\bar{t}c \) in the future LC experiments [15]. Certainly, the neutral scalars \( h^0_t, \pi^0_t \) can also be produced via the process \( q\bar{q} \rightarrow g \rightarrow \pi^0_t(h^0_t)g \). In the future, we will discuss the possibility of detecting \( \pi^0_t, h^0_t \) via this process in hadron colliders.

In conclusion, we have conformed that the FCS couplings predicted by TC2 models can give significantly contributions to the rare top decay \( t \rightarrow cg \). The branching ratio \( B_r(t \rightarrow cg) \) can be highly enhanced, which is in the range of the sensitivity of future experiments. Further, it is worth to remark that this enhancement in TC2 models also appear in other rare top decays. For instance, the value of the branching ratio \( B_r(t \rightarrow c\gamma) \) varies between \( 7.95 \times 10^{-7} \) and \( 4.58 \times 10^{-6} \) for \( m_{h_t} = 300 GeV \) and the parameter \( \epsilon \) in the range of 0.01-0.08, which approach the corresponding experimental threshold.

Acknowledgments

Chongxing Yue would like to thank C. P. Yuan for many useful discussions.
Figure Captions

**Fig.1:** Feynman diagrams for the contributions of the neutral top-pion $\pi_t^0$ or the top-Higgs $h_t^0$ via the FCS couplings($\pi_t^0\overline{tc}$ or $h_t^0\overline{tc}$) to the rare decay $t \rightarrow cg$.

**Fig.2:** The branching ratio $B_r(t \rightarrow cg)$ as a function of the neutral top-pion mass $m_{\pi_t}$ for the parameter $\epsilon = 0.08$(solid line), 0.05(dotted line) and 0.01(dashed line).

**Fig.3:** The strength $K_c$ of the anomalous coupling as a function of the neutral top-Higgs mass $m_{h_t}$ for the parameter $\epsilon = 0.08$(solid line), 0.05(dotted line) and 0.01(dashed line).
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Fig. 1

Fig. 2
Fig. 3

$\kappa$ vs. $m_n$ (GeV)

- $\varepsilon = 0.08$
- $\varepsilon = 0.05$
- $\varepsilon = 0.01$