Study of the dynamics of a sheet cavitation

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Abstract. A compressible, two-phase, one-fluid solver has been developed to investigate the behaviour of cavitation models. The model is based on a transport equation for the volume fraction of gas. Numerical simulations are performed on a cavitating Venturi flow where a self-sustained cavitation pocket develops. The sheet dynamics involving traveling pressure waves is investigated.

1. Introduction
The cavitation simulation is a challenging problem both in modelling of the physics and in developing robust numerical methodologies. Several physical and numerical models have been developed to investigate cavitating flows within the framework of the averaged approach. In its implementation, there are different approaches according to the assumptions made on the thermodynamic path during the phase transition and the slip condition between phases. A hierarchy of models exists, with the numbers of equations ranging from seven to three only. In the present paper, we study an homogeneous, one-pressure and one-temperature model, composed by four equations: three mixture conservation laws and a void ratio transport equation. The mass transfer between phases is closed assuming its proportionality with the divergence of velocity [3, 4, 6]. This model is implemented in a compressible preconditioned Reynolds averaged solver with transport equation turbulence models. A self-sustained oscillating cavitation pocket developing on a Venturi geometry is investigated. The importance of the propagation of pressure waves in the physical mechanism is exhibited.

2. A four-equation model
The homogeneous mixture approach is used with the assumption of thermal and mechanical local equilibrium between pure phases, liquid and vapour. The model consists in three conservation laws for mixture quantities (mass, momentum and total energy) and an additional equation for the void ratio. The expression for the void ratio equation is:

\[
\frac{\partial \alpha}{\partial t} + \text{div} (\alpha \vec{V}) = \left( \frac{\rho_l c_l^2 - \rho_v c_v^2}{\rho_l - \frac{\rho_l c_l^2}{1 - \alpha} + \rho_v c_v^2} + \alpha \right) \text{div} \vec{V} + \left( \frac{c_l^2}{\rho_l} + \frac{c_v^2}{\rho_v} \right) \frac{\dot{m}}{\frac{\rho_l c_l^2}{1 - \alpha} + \rho_v c_v^2} \tag{1}
\]

where \( \dot{m} \) is the mass transfer between phases, \( \vec{V} \) is the velocity vector and \( \rho_k, c_k \) are the pure phase density and speed of sound, respectively. Pure phases follow the stiffened gas equation of
state. By assuming that the mass transfer is proportional to the divergence of the velocity, it is possible to build a family of models in which the mass transfer $\dot{m}$ is expressed as [3]

$$\dot{m} = \frac{\rho_l \rho_u}{\rho_l - \rho_u} \left( 1 - \frac{c^2}{c_{wallis}^2} \right) \text{div} \vec{V}$$

where $c_{wallis}$ is the propagation of acoustic waves without mass transfer. The system is closed by an equation of state for the mixture. Two formulations are considered to determine the pressure in the mixture. A sinusoidal law deduces the pressure from the void ratio:

$$p_m(\alpha) = P_{vap}(T_{ref}) + \left( \frac{\rho_l^{sat} - \rho_u^{sat}}{2} \right) c_{min}^2 \text{Arcsin} (1 - 2\alpha)$$

while the stiffened gas EOS, written with mixture quantities, proposes the following formulation:

$$p_m(\rho_m, \epsilon_m, \alpha) = (\gamma_m(\alpha) - 1)\rho_m(\epsilon_m - q_m(\alpha)) - \gamma_m(\alpha)p_m(\infty)(\alpha)$$

The parameter $c_{min}$ refers to the minimal speed of sound in the mixture. Its influence and the EOS properties are detailed in [1]. Each closure expression has been studied on an expansion tube [3] and a Venturi flow [6].

3. The numerical tool

The numerical simulations are carried out using an implicit RANS solver for multi-domain structured meshes. This solver is based on a cell-centered finite-volume discretization. The Smith $k - \ell$ turbulence model is used. Unsteady simulations are performed using the dual time stepping method. Numerical schemes are second-order accurate in both time and space. All computations are performed on a H-type grid containing 174 nodes in the flow direction and 56 in the orthogonal direction. More details are given in [4, 6].

4. Computations and comparisons

4.1. Experimental conditions

The test case is Venturi geometry constituted by an angle of the divergent part of $8^\circ$. In experiments, the pressure $P_{inlet}$ was lowered until the desired cavitation number. The lower wall was equipped with probe holes to evaluate the local pressure, void ratio and velocity. The selected operation point is characterized by the following parameters [2]:

- $V_{inlet} = 7.04 \text{ m/s}$, the inlet velocity;
- $T_{ref} = 293 \text{ K}$, the reference temperature;
- $\sigma_{inlet} \approx 2.15$, the cavitation parameter in the inlet section.

A typical self-oscillation behavior was observed with quasi-periodic vapour clouds shedding which frequency is measured closed to 45 Hz. Figure 1 shows an instantaneous photograph of the cavity and a scheme of the venturi profile.
4.2. 4-equation models comparison

The objective of the study is to investigate the ability for the 4-equation model to correctly reproduce the cloud cavitation shedding. The influence of the tunable parameter $c_{\text{min}}$ of the sinusoidal EOS (3) is evaluated. Two values are tested: 0.47 m/s and 1.36 m/s. Cavity pocket motions are illustrated on Figure 2 by representing the void ratio on a spatio-temporal graph $(x, t)$. The Venturi throat is located at the abscissa $x = 0$ m. The cavitating flow dynamic mainly depends on the considered equation of state. Moreover, variations of $c_{\text{min}}$ modify the pressure computation and influence the cyclic phenomenon frequency.

Figure 2. Sheet cavitation dynamic and pressure oscillation frequency using (from left to right): sinusoidal law $c_{\text{min}} = 0.47$ m/s (freq = 24.13 Hz); sinusoidal law $c_{\text{min}} = 1.36$ m/s (freq = 42.24 Hz) and stiffened gas law (freq = 40.94 Hz).

The physical mechanism of break-off cycles and cloud shedding are not clearly understood. The motion of a re-entrant liquid jet [2] is known as a main responsible of cavitation instability but recent experimental works [5] also report the presence of travelling pressure waves created by cloud collapses. In our simulations, the re-entrant jet is clearly observed by negative time-averaged velocity profiles near the wall. As presented on Figure 3, it remains identical for all computations and argue that the pressure and the speed of sound determine the periodic shedding frequency without changing the velocity field.

Figure 3. From left to right: comparison of void ratio and velocity profiles and pressure oscillation frequency for sinusoidal and stiffened gas law (probe number 2 : $x = 0.0315$ m).

In order to identify pressure waves, a numerical captor is located in the region of cloud condensation and the pressure signal is plotted on Figure 4. It reveals the presence of high pressure values between consecutive clouds cavitation which could disturb their advection.

4.3. Saturation versus metastable value for $\rho_v$

The added transport equation allows to relax the local thermodynamic equilibrium and to introduce a metastable state for the vapour phase. Comparisons have been done for the sinusoidal EOS (3) between the two cases: equilibrium case with the saturation value $\rho_v = \rho_v^{\text{sat}}$ and metastable case according to the stiffened gas EOS. Figure 4 compares the pressure evolutions for both cases in the collapse region and demonstrates that a non-equilibrium state
for $\rho_v$ increases the pressure magnitude. Similar results have been observed for speed of sound variations and consequently the Mach number computation in the shedding region. Thus, the pressure wave propagation caused by the vapour structure collapse is modified when the pure vapour density is no more constant.

![Figure 4. Comparison of pressure evolution in the collapse region ($x = 0.0415$ m; $y = 0.0025$ m) for metastable vapour density (top) and saturation value (bottom).](image)

5. Conclusion
One-fluid RANS simulations based on a void ratio transport-equation model were performed to study an oscillating cavitation pocket developing along a Venturi geometry. The ability of simulation to reproduce the cloud shedding phenomenon is investigated by comparison with experimental data. Two equations of state are proposed (sinusoidal and stiffened gas). Results show that these relations and their dedicated parameters pilot the cavitating flow dynamic. Moreover, the introduction of a metastable state for the vapour phase modifies the speed of sound and increases pressure oscillations in the collapse region. These observations question the need of considering a non-constant vapour density for such cavitating flows.

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References
[1] E. Goncalves and R. Fortes Patella, “Numerical simulation of cavitating flows with homogeneous models”, *Computers & Fluids*, **38**, pp. 1682–1696, (2009).
[2] V. Aeuchlimann, S. Barre and H. Djeridi, “Unsteady cavitation analysis using phase averaging and conditional approaches in a 2D Venturi”. *Open J. of Fluids Dynamics*, **3**, pp.171–183, (2013).
[3] E. Goncalves, ”Numerical study of expansion tube problems: Toward the simulation of cavitation”, *Computers & Fluids*, **72**, pp.1–19, (2013).
[4] E. Goncalves and B. charriere, ”Modelling for isothermal cavitation with a four-equation model”, *Int. J. Multiphase Flow*, **59**, pp.54–72, (2014).
[5] C. Stanley, T. Barber and G. Rosengarten, ”Re-entrant jet mechanism for periodic cavitation shedding in a cylindrical orifice”. *Int. J. Heat and Fluid Flow*, **50**, pp.169–176, (2014).
[6] B. Charriere, J. Decaix and E. Goncalves, ”A comparative study of cavitation models in Venturi flow”, *Eur. J. of Mech. B/Fluids*, **49**, pp.287–297, (2015).