Chiral Anomaly at Infinite Temperature

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Abstract

Using the heat kernel regularization we show that the Abelian chiral anomaly in the limit of infinite temperature it is not a well defined quantity, contrary to what happens at any finite temperature. We show that there is an ambiguity in the ordering of the limits of infinite temperature and removal of the cut-off so that changing this ordering we find different results for the chiral anomaly. We discuss these cases and their possible interpretation.

PACS numbers: 03.70.+k, 11.15.Tk, 11.30.Rd.
1 Introduction

The chiral anomaly consists in the anomalous divergence of the axial-vector current \( \bar{\psi} \gamma_{\mu} \gamma_5 \psi \), which is classically conserved but not at the quantum level. It was originally calculated by Schwinger [1] and was later rediscovered by Adler, Bell and Jackiw [2] studying the Veltman-Shutherland paradox through the current algebra, in connection with the \( \pi^0 \to 2\gamma \) decay [3].

Since then, anomalies have played a special role in quantum field theory because chiral anomalies mean violation of standard Ward-Takahashi identities which are the guide for renormalizable theories. The presence of chiral anomalies can be circumvented by considering fermion families in which they cancel as a whole. Particularly, this is the case of the Weinberg-Salam electroweak theory. Another remarkable feature in this theory, is the fact that, to warrant renormalizability, as is well known, the gauge fields are not genuine massive but acquire mass through the spontaneous breakdown of the vacuum symmetry.

In the beginning of seventies, Kirzinite and Linde [4] proposed that the spontaneous symmetry breakdown mechanism may be associated with the cooling of the universe and this could be understood as a phase transition which had occurred at a certain critical temperature.

The behavior of chiral anomalies at finite temperature was first investigated by Dolan and Jackiw [5] who showed that in the case of massless two dimensional quantum electrodynamics the anomalous divergence of the axial current was independent of any finite temperature. Then, this result was rederived by other methods and shown to be valid even in the non-Abelian case in four and arbitrary (even) dimensions [6]. Particularly, this was done recently using the heat kernel regularization [7] with the Fujikawa’s method [8], [9]. This method was also applied successfully to models with curved background fields [10].

The relevance of the infinite temperature limit in quantum field theory is manifold. First of all, this limit is considered in the study of the very early universe as time ap-
proaches zero [11]. Secondly, it is very useful in dimensional reduction which is a requisite for compactification on string theory [12]. Finally, Polyakov has considered this limit to show the existence of a deconfined phase in QCD [13]. He had also shown that multi-nstanton configurations in $2 + 1$ dimensional QCD could be associated with negative temperatures [14], which are usually recognized as temperatures “higher” than infinity in paramagnetic systems.

In this paper we are going to discuss the infinite temperature limit for the Abelian chiral anomaly. This is done using the heat kernel regularization at finite temperature which we review in section 2. In section 3 we take the infinite temperature limit on the chiral anomaly, as calculated in the preceeding section. When we take the infinite temperature limit we find that the ordering of this limit and the removal of the cut-off is ambiguous, i.e., changing this ordering we are lead to different results for the chiral anomaly. When the cut-off is firstly removed we find the usual result for the chiral anomaly, but if we first take the infinite temperature limit we find an ill defined quantity which must be carefully treated. This is done with the help of a $\zeta$-function prescription and alternatively with a $\theta$-function transformation. A numerical approach is also sketched at the end of section 3. Discussion and conclusions are left for section 4.

2 The anomaly at non-zero temperature

Let us briefly review the paths leading to the Abelian chiral anomaly at finite temperature $T$ which are essential to the discussion of its infinite temperature limit.

In order to discuss non-zero temperature effects at equilibrium conditions we start with a theory in Euclidean time restricted to the interval $(0, \beta = 1/T)$. With this prescription and the (anti)periodicity of the (fermi) boson fields we can map generating functionals, in the absence of sources, on partition functions.

Fujikawa [8] showed that the chiral anomaly can be obtained non-perturbatively in the path integral approach, observing that the functional measures $D\psi$ and $D\bar{\psi}$ of the fermionic fields are not invariant under the infinitesimal chiral transformations
\[ \psi(x) \rightarrow \psi'(x) = e^{i\alpha(x)\gamma_5}\psi(x) \]

\[ \bar{\psi}(x) \rightarrow \bar{\psi}'(x) = \bar{\psi}(x)e^{i\alpha(x)\gamma_5} \]

where \( \gamma_5 = i\gamma_0\gamma_1\gamma_2\gamma_3 \). This noninvariance may be expressed through a Jacobian which can be calculated expanding the fields \( \psi(\bar{\psi}) \) and \( \psi'(\bar{\psi}') \) over complete orthogonal basis \( \{\phi_n^\beta\} \) \( \{\phi_n^\beta\} \), which are eigenstates of the Dirac operator \( \mathcal{D}\phi_n^\beta = \lambda_n\phi_n^\beta \) and are antiperiodic in the Euclidean time interval \((0, \beta)\). At finite temperature \( \beta^{-1} \), this Jacobian is given by [4, 7]

\[ J_\beta = \exp\{-2i \int_0^\beta d\tau \int d^3x \alpha(x)A_\beta(x)\} \]

where \( A_\beta(x) \) is an ill defined sum, usually called anomaly,

\[ A_\beta(x) = \sum_n \phi_n^{\beta\dagger}(x)\gamma_5\phi_n^\beta(x) \]

which, after regularization leads to anomalous Ward-Takahashi identities.

We are going to use the heat kernel regularization which consists in the introduction of a damping factor \( \exp\{-\epsilon\lambda_n^2\} \) in the expression of the anomaly, to cut-off the high frequencies leading to its regularized expression

\[ A_\beta^R(x) = \lim_{\epsilon \to 0} \sum_n \phi_n^{\beta\dagger}(x)\gamma_5 \exp\{-\epsilon\lambda_n^2\}\phi_n^\beta(x) \]

\[ = \lim_{\epsilon \to 0} \sum_n \phi_n^{\beta\dagger}(x)\gamma_5 \exp\{-\epsilon \mathcal{D}^2\}\phi_n^\beta(x) \]

\[ = \lim_{\epsilon \to 0} \lim_{x' \to x} tr(\gamma_5 \exp\{-\epsilon \mathcal{D}^2\}\delta^\beta(x-x')) \]

\[ = \lim_{\epsilon \to 0} \lim_{x' \to x} tr(\gamma_5 H_\beta(x, x'; \epsilon)) \] (1)

where \( H_\beta(x, x'; \epsilon) \) is the finite temperature heat kernel

\[ H_\beta(x, x'; \epsilon) = \exp\{-\epsilon \mathcal{D}^2\}\delta^\beta(x-x') \] (2)
which is related to the zero temperature one by \[7\] \[10\]

\[ H_\beta(x, x; \epsilon) = H(x, x; \epsilon)S(\beta^2/\epsilon) \]  \hspace{1cm} (3)

where \( S(\beta^2/\epsilon) \) contains all the temperature dependence of the heat kernel \( H_\beta(x, x; \epsilon) \):

\[ S(\beta^2/\epsilon) = 1 + 2 \sum_{n=1}^{\infty} (-1)^n \exp \left\{ -\frac{n^2 \beta^2}{4\epsilon} \right\} \]  \hspace{1cm} (4)

In order to calculate explicitly the anomaly one can invoke the Schwinger-DeWitt \[15\] ansatz for its diagonal part

\[ H(x, x; \epsilon) = (4\pi \epsilon)^{-d/2} \sum_{m=0}^{\infty} a_m(x) \epsilon^m \]  \hspace{1cm} (5)

where \( d \) is the space-time dimension and \( a_m(x) \) are the well known Seeley’s coefficients \[15\]. Owing to the properties of gamma matrices it is possible to define a matrix \( \gamma_{d+1} \) with the same features of \( \gamma_5 \), i.e., anticommuting with all other \( \gamma \)s, for every even dimensional space-time. This way, one can show that

\[ \mathcal{A}_\beta^R = \lim_{\epsilon \to 0} tr[\gamma_{d+1} H_\beta(x, x; \epsilon)] \]
\[ = \lim_{\epsilon \to 0} tr[\gamma_{d+1} H(x, x; \epsilon)S(\beta^2/\epsilon)] \]
\[ = (4\pi)^{-d/2} tr[\gamma_{d+1} a_{d/2}(x)] \lim_{\epsilon \to 0} S(\beta^2/\epsilon) \]  \hspace{1cm} (6)

For any finite temperature, which is equivalent to have \( \beta \neq 0 \), the above expression can easily be calculated since

\[ \lim_{\epsilon \to 0} S(\beta^2/\epsilon) = 1 \]  \hspace{1cm} (7)

This assures the final temperature independence of the chiral anomaly, leading to

\[ \mathcal{A}_{\beta \neq 0}^R(x) = (4\pi)^{-d/2} tr[\gamma_{d+1} a_{d/2}(x)] \equiv \mathcal{A}^R(x) \]  \hspace{1cm} (8)
which is the zero temperature anomaly in d (even) dimensions [5]-[7]. If d was odd we would have found a null result.

Another way of seeing this result is noting that the limit $\epsilon \rightarrow 0$ on the function $S(\beta^2/\epsilon)$ imposed on the regularized anomaly as the removal of the cut-off is equivalent to the zero temperature $\beta \rightarrow \infty$ one, for which $S(\beta^2/\epsilon)$ is again equal to unity.

3 The infinite temperature limit

Now, let us discuss the infinite temperature limit of the chiral anomaly. As we are going to show, depending on the ordering of infinite temperature and removal of cut-off limits on the calculation of the chiral anomaly we get different results. Let us study these cases separately.

3.1 Case A

Firstly we analyze the case of the ordering of limits where we first remove the regulator ($\epsilon \rightarrow 0$) and then let the temperature approach infinity ($\beta \rightarrow 0$). In this case we have:

$$\mathcal{A}_{\beta \rightarrow 0}(x) = \lim_{\beta \rightarrow 0} \left( \lim_{\epsilon \rightarrow 0} tr[\gamma_{d+1}H_{\beta}(x, x; \epsilon)] \right)$$

$$= \lim_{\beta \rightarrow 0} \left( \lim_{\epsilon \rightarrow 0} tr[\gamma_{d+1}H(x, x; \epsilon)S(\beta^2/\epsilon)] \right)$$

$$= \lim_{\beta \rightarrow 0} \left( (4\pi)^{-d/2} tr[\gamma_{d+1} a_{d/2}(x)] \lim_{\epsilon \rightarrow 0} S(\beta^2/\epsilon) \right)$$

(9)

Note that in the last line the limit $\epsilon \rightarrow 0$ was taken only on the heat kernel $H(x, x; \epsilon)$ and this limit on the function $S(\beta^2/\epsilon)$ is given by Eq.(7) above, so

$$\mathcal{A}_{\beta \rightarrow 0}(x) = \lim_{\beta \rightarrow 0} (4\pi)^{-d/2} tr[\gamma_{d+1} a_{d/2}(x)]$$

$$= (4\pi)^{-d/2} tr[\gamma_{d+1} a_{d/2}(x)]$$

$$= \mathcal{A}(x)$$

(10)
which is the same result as that for the anomaly at zero temperature, Eq.(8), since the removal of the cut-off left the anomaly temperature independent.

3.2 Case B

Now, let us examine the case of the ordering of limits where we take first the infinite temperature limit and then remove the cut-off:

\[
A_{\beta \to 0}^R (x) = \lim_{\epsilon \to 0} \left\{ \lim_{\beta \to 0} tr[\gamma_{d+1}H_\beta(x,x;\epsilon)] \right\} \\
= \lim_{\epsilon \to 0} \left\{ (4\pi)^{-d/2} tr[\gamma_{d+1}H(x,x;\epsilon)] \lim_{\beta \to 0} S(\beta^2/\epsilon) \right\} \tag{11}
\]

Note that, in this case, the series (4) seems to be not convergent, oscillating between \(\pm 1\)

\[
\lim_{\beta \to 0} S(\beta^2/\epsilon) = 1 + 2 \sum_{n=1}^{\infty} (-1)^n \to \pm 1 \tag{12}
\]

(It may be interesting to note that this oscillating series appears also in the quantization of superstrings involving an infinite tower of ghosts [16]). To remedy this situation we are going to employ two approaches: (i) relate the series \(S(\beta^2/\epsilon)\) with a \(\zeta\)-function which has a well defined limit for \(\beta \to 0\); and (ii) use a \(\theta\)-function transformation to evaluate it.

3.2.1 \(\zeta\)-function approach

To relate the series \(S(\beta^2/\epsilon)\) in the limit \(\beta \to 0\) with the \(\zeta\)-function [17]-[18]

\[
\zeta (s) = \sum_{n=1}^{\infty} n^{-s} \quad (\text{Re } s > 1) \tag{13}
\]

we recall the relation

\[
\sum_{n=1}^{\infty} (-1)^n n^{-s} = - \sum_{n=1}^{\infty} n^{-s} + 2 \sum_{n=1}^{\infty} (2n)^{-s} \\
= -(1 - 2^{1-s}) \zeta (s) \tag{14}
\]
so that comparing Eqs. (12) and (14) we choose as a prescription

$$\lim_{\beta \to 0} S(\beta^2/\epsilon) = \lim_{s \to 0} [1 - 2(1 - 2^{1-s})\zeta(s)]$$

which is still ill defined since the $\zeta$-function (13) is convergent for $Re \ s > 1$ only. This problem has a well known solution which is that of rewriting it through an analytic continuation [17] - [18]

$$\zeta_R(s) = \frac{1}{2} + \frac{1}{s - 1} + 2 \int_0^\infty \frac{(1 + t^2)^{-s/2}}{e^{2\pi t} - 1} \sin s \tan^{-1} t \ dt$$

for which the limit $s \to 0$ is well defined

$$\zeta_R(0) = -\frac{1}{2} ,$$

so that substituting this result in Eq. (15) we show that the temperature dependent sum $S(\beta^2/\epsilon)$ vanishes as $\beta$ approaches zero

$$\lim_{\beta \to 0} S(\beta^2/\epsilon) = 0 .$$

Here, the analytic continuation of the $\zeta$-function can be thought as a complementary regularization needed to handle the undefined sum $S(\beta^2/\epsilon)$ in the limit $\beta \to 0$ (or $T \to \infty$).

Now, substituting these results in Eq.(11), we can find the regularized anomaly for this case

$$\mathcal{A}_{\beta \to 0}^R(x) = \lim_{\epsilon \to 0} \left\{ (4\pi)^{-d/2} tr[\gamma_{d+1} H(x, x; \epsilon)] \lim_{\beta \to 0} S(\beta^2/\epsilon) \right\}$$

$$= \lim_{\epsilon \to 0} \left\{ (4\pi)^{-d/2} tr[\gamma_{d+1} H(x, x; \epsilon)] \lim_{s \to 0} [1 - 2(1 - 2^{1-s})\zeta_R(s)] \right\}$$

$$= 0$$

(18)

since the limit $\epsilon \to 0$ which removes the regulator was taken after the limit $\beta \to 0$ of infinite temperature.
3.2.2 $\theta$-function transformation approach

We have shown above that the series $S(\beta^2/\epsilon)$ in the limit $\beta \to 0$ can be regularized through the use of a $\zeta$-function. Now we are going to show that to evaluate this sum we can alternatively use the $\theta$-function transformation [18]

$$\sum_{n=1}^{\infty} e^{-\alpha n^2} = -\frac{1}{2} + \frac{1}{2} \sqrt{\frac{\alpha}{\pi}} + \sum_{m=1}^{\infty} e^{-\pi^2 m^2/\alpha}$$  \hspace{1cm} (19)

The idea here is to rewrite $S(\beta^2/\epsilon)$, Eq.(4), with the help of the above equation. Note that the limit $\beta \to 0$ in (4) is equivalent to $\alpha \to 0$ in Eq. (19), which diverges because of the presence of the term $1/2\sqrt{\pi/\alpha}$. As was done for the $\zeta$-function we put

$$\sum_{n=1}^{\infty} (-1)^n e^{-\alpha n^2} = -\sum_{n=1}^{\infty} e^{-\alpha n^2} + 2 \sum_{n=1}^{\infty} e^{-\alpha(2n)^2}$$

$$= - \left[ -\frac{1}{2} + \frac{1}{2} \sqrt{\frac{\alpha}{\pi}} + \sum_{m=1}^{\infty} e^{-\pi^2 m^2/\alpha} \right]$$

$$+ 2 \left[ -\frac{1}{2} + \frac{1}{2} \sqrt{\frac{\alpha}{4\pi}} + \sum_{m=1}^{\infty} e^{-\pi^2 m^2/4\alpha} \right]$$

$$= -\frac{1}{2} - \sum_{m=1}^{\infty} e^{-\pi^2 m^2/\alpha} + 2 \sum_{m=1}^{\infty} e^{-\pi^2 m^2/4\alpha}$$

$$= -\frac{1}{2} - \sum_{m=1}^{\infty} e^{-\pi^2 m^2/\alpha} \left( 1 - 2e^{3\pi^2 m^2/4\alpha} \right)$$ \hspace{1cm} (20)

Observe that the divergent terms of Eq. (19) for each $\theta$-function transformation mutually cancel in the above expression, so that identifying $\alpha = \beta^2/4\epsilon$, we can adopt as an alternative prescription for calculating the divergent series (4)

$$S^R(\beta^2/\epsilon) = 1 + 2 \sum_{n=1}^{\infty} (-1)^n e^{-\alpha n^2}$$

$$= 1 + 2 \left[ -\frac{1}{2} - \sum_{m=1}^{\infty} e^{-4\pi^2 m^2\epsilon/\beta^2} \left( 1 - 2e^{3\pi^2 m^2\epsilon/\beta^2} \right) \right]$$

$$= -2 \sum_{m=1}^{\infty} e^{-4\pi^2 m^2\epsilon/\beta^2} \left( 1 - 2e^{3\pi^2 m^2\epsilon/\beta^2} \right)$$ \hspace{1cm} (21)
so that the anomaly at infinite temperature within this prescription is

\[ \mathcal{A}_{\beta \to 0}^{R\theta}(x) = \lim_{\epsilon \to 0} \left\{ (4\pi)^{-d/2} tr[\gamma_{d+1} H(x, x; \epsilon)] \lim_{\beta \to 0} S^R(\beta^2/\epsilon) \right\}. \] (22)

As the series \( S^R(\beta^2/\epsilon) \) is well behaved in the limit \( \beta \to 0 \)

\[ \lim_{\beta \to 0} S^R(\beta^2/\epsilon) = 0, \] (23)

we find that

\[ \mathcal{A}_{\beta \to 0}^{R\theta}(x) = 0, \] (24)

reproducing the result obtained with the \( \zeta \)-function approach. Note that in the expression \( S^R(\beta^2/\epsilon) \) one can see the naive behavior of anomalies since it has the form \( 0 \times \infty \) as \( \beta \) approaches zero. However the vanishing term goes faster to zero than the divergent one to infinity.

### 3.3 Case C

We have shown above that changing the ordering of limits \( \beta \to 0 \) and \( \epsilon \to 0 \) we obtain different results for the chiral anomaly. So a natural question is that if one can accommodate these results in a single fashion. This is not an easy task since the approaches for cases A and B above were quite different.

However, it is possible to show that if one takes these limits simultaneously, this corresponds to establishing a relation between these parameters as, for example, \( \xi = \beta^2/\epsilon \). So, depending on the path in which the limits \( \beta \to 0 \) and \( \epsilon \to 0 \) are reached we have different values of \( \xi \) and also for the anomaly. So cases A and B correspond to \( \xi \to \infty \) and \( \xi \to 0 \), respectively. For any intermediate value of \( \xi \) the series \( S(\beta^2/\epsilon) \), Eq. (21) can be treated numerically, since for any finite and non-zero \( \xi \) the sum
Table 1: Numerical values for $S_N(\xi)$

| $\xi$  | $S_{N=10}(\xi)$ | $S_{N=40}(\xi)$ |
|--------|-----------------|-----------------|
| 0.001  | 0.97287133747   | 0.66361623101   |
| 0.01   | 0.75931044053   | 0.01648789112   |
| 0.1    | 0.06173907523   | 9.95794274078 * 10^{-19} |
| 1      | 3.66139425893 * 10^{-4} | 3.66139425893 * 10^{-4} |
| 4      | 0.30062579744   | 0.30062579744   |
| 10     | 0.83592080900   | 0.83592080900   |
| 100    | 0.99999999997   | 0.99999999997   |

$$S_N(\xi) = 1 + 2 \sum_{n=1}^{N} (-1)^n e^{-n^2 \xi/4}$$  \hspace{1cm} (25)

is ordinarily summable and approaches $S(\beta^2/\epsilon)$ as $N \to \infty$. For example, if $\xi = 0.5$ and $N = 40$ we have

$$S_{N=40}(\xi = 0.5) \simeq 2.68238 \times 10^{-8}$$  \hspace{1cm} (26)

which leads to an anomaly value intermediate between $A^R$ and $A^R_{\beta \to 0}$ (or $A^{R\theta}_{\beta \to 0}$). Some other values for $S_N(\xi)$ are given in Table 1. Note that the series is rapidly convergent for $\xi \geq 1$.

4 Discussion and Conclusions

We have shown above that the chiral anomaly at the infinite temperature limit is not a well defined quantity. First, because of an ambiguity in the ordering of limits of infinite temperature and removal of the cut-off. Two possible orderings were examined as cases A and B of section 3 and shown to lead to different results for the chiral anomaly. In particular, case A leads to the usual anomaly and in case B, where we first take the infinite temperature limit, we have found that the already regularized anomaly becomes again ill
defined. To handle this problem we have used \( \zeta \)- and \( \theta \)-functions prescriptions which lead to the anomaly cancellation.

Now, we are going to do some speculation about the interpretation of these results. Case A seems to represent a genuine theory at finite temperature for which we take the infinite temperature limit. Since the chiral anomaly (after the removal of the cut-off) is temperature independent the heating of the thermal bath in which the theory is immersed has no effect on it.

Case B, in which the infinite temperature limit was taken before the removal of the cut-off, led to the cancellation of the anomaly and must correspond to a different, if any, physical situation. This sequence of limits suggests that this could be interpreted as a genuine theory at infinite temperature which is not the limit of an usual theory at finite temperature as that discussed in case A. This situation may have occurred, for example, in the primordial singularity of the very early universe, where the temperature was perhaps infinite. But the connection between the singular case B and the usual one A is not clear at all. Some effort in this direction has been made as was shown in case C of section 3, where we had considered the two vanishing limits on \( \epsilon \) and \( \beta \) to be taken simultaneously. The parameter \( \xi = \beta^2/\epsilon \) approximately describes the intermediate cases between A and B. However this was not a satisfactory discussion since only a rough numerical approach was given.

These calculations were done using the heat kernel regularization (and a complementary \( \zeta \)- or \( \theta \)-function prescription) but it could be done as well with other schemes such as the Pauli-Villars one or dimensional regularization. It is worth mentioning that the cancellation of the chiral anomaly occurs in perturbation theory at finite cut-off [19]. This may be related to the present calculations if one assumes the infinite temperature limit to be taken before the removal of the cut-off.

Throughout this letter we have not mentioned background gravitational fields which are relevant for cosmological considerations. In fact the results derived above are still valid in the case where a background is present if it is at least quasistatic (and may also
have small anisotropies) in order to insure thermodynamic quasiequilibrium conditions, since the calculation of the chiral anomaly in this case is analogous to the flat space-time one even at non-zero temperatures \[10\].

From the topological point of view, we know that the chiral anomaly is an invariant, even at finite temperatures, so what happens in the infinite temperature limit? One way of answering this question is to state that it must be invariant if and only if the topology of the space-time is also unchanging. To see this let us remember that at finite temperatures the topology of \((3+1)\) space-time is just \(\mathbb{R}^3 \times S^1\) with the one-sphere having a radius \(\beta\). When we discussed case B we take the limit \(\beta \to 0\) (before the removal of the cut-off) reducing the space-time dimension to \(3 + 0\), which is equivalent to a time compactification. So we find a null result for the chiral anomaly since we have arrived at an odd dimensional space-time. However, in case A we obtain the usual anomaly taking first \(\epsilon \to 0\). Then we let the temperature approach infinity. This case corresponds to an anomaly calculated in an even dimensional space-time (since the final result was obtained before the dimensional reduction). We believe that his picture can be better understood if cases A and B could be described in an unified way. This is presently under investigation.

**ACKNOWLEDGEMENTS.** The author would like to acknowledge C. Wotzasek for calling his attention to this problem and for useful discussions. The author was also benefitted from conversations with C.P. Natividade, M.S. Alves, C. Farina, N. Braga and S. Rabello. This work was partially supported by CNPq - Brazilian agency.
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