Technological development of a low-cost wrist rehabilitation robot: Kinematic and static performance analysis

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Abstract. This work extents the project Technological Development of a Low-cost Wrist Rehabilitation Robot. The project goal is to develop a robotic system to aid physiatrists in their work of rehabilitating patients with injuries at the wrist joint. The rehabilitation protocol of the wrist often follows three stages: i) the physiatrist immobilizes the wrist for a time, ii) then, he/she treats the wrist by applying heat or cold over sore tendons and muscles, and iii) the physiatrist applies force on the wrist to give smooth movements until recovering its range of motion. In the latter stage, the physiatrist performs several exercises for long periods of time, which opens a great opportunity for being substituted for a robotic device. Earlier works, based on analyzing several rehabilitation therapies, allowed to set up the design specifications and a robot first design. Thus, the aims of this work is to present the kinematic and static analysis of a 2 degree of freedom serial robot designed for wrist rehabilitation. First, the physiatrists’ team from the Traumatology Service at the IAHULA, Mérida Venezuela recorded rehabilitation therapies normally used when treating an injured wrist. Then, the kinematic and static models to study recorded trajectories are developed. Findings allow us to improve the actual robot design, and they also show that the designed robot can tackle the therapies required for wrist rehabilitation.

1. Introduction

The growing of robotics from industrial to service robots opened new fields requiring robots developments. Rehabilitation therapies is one of such new fields. For instance, a review has reported 141 robotic and mechanical devices for upper limb rehabilitation [1]. The review concluded that some of the reported devices have been effective in the patient recovery. Ref. [2,3] reported a similar conclusion for upper limb rehabilitation. Moreover, other works have proposed robots for lower limb rehabilitation [4-6], as well as for knee rehabilitation [7]. Also, researchers have focused on devices for upper and lower limb distal joints where motors actuate directly on the joint axis. That is, ankle rehabilitation [8-10] and wrist rehabilitation robots [11,12] to name a few.

In this vein, this work extends the project Technological Development of a Low-cost Wrist Rehabilitation Robot. The purpose of the project is to develop a robotic system for helping physiatrists in rehabilitating injured wrist joint. The type of injuries this work is focusing on, and allowing to set up the design specification, are those due to a several traumas such as: motorcycle accidents, falls, blows, bad postures in workplaces, performing incorrect movements in the practice of sporting activities, or caused by tissues-degeneration. In this sense, the rehabilitation protocol usually follows three main stages: i) a first stage where the physiatrist immobilizes the wrist joint, ii) a second stage where he/she treats tendons and muscles using temperature changes between cold and warm, and iii) a third stage...
where the physiatrist performs slow movements in the joint to recover the range of motion of the wrist [13]. The goal of the research project is to tackle the third stage where the physiatrist is the one in charge of exercising muscles for long periods of time, encompassing a stressful work. Activities requiring a lot of repetitions for a long time open a great opportunity for developing robotic devices.

The first part of the project tackled the design of the mechanical subsystem which basically consisted of developing a robotic arm. The methodology for the robot design followed the concurrent design and the industrial design methodologies [14], which allowed to come up with a first prototype [15]. The prototype relies on a 2 degree-of-freedom (DoF) serial robot architecture. Moreover, anthropometric data for the Venezuelan population helped dimensioning robot's links, and the usability approach helped to develop the user interface and the shapes of the prototype. In addition, the use of Computer Aided Design enabled to set up geometric shapes and materials for the different parts of the rehabilitation device, which were later analyzed using the Finite Element Method (FEM).

The second step in the project was focused on the control system and the electronic components. Since one of the main aspects taking place in the robot development is the low-cost, the electronic subsystem relies on the low-cost Arduino board in addition to in-house built circuits. In this step, we have worked out the algorithms for processing motors angular position and through the direct kinematics model we have established the end-effector motion which corresponds to the wrist motion. The prototype enables the recording of the rehabilitation therapy through measuring motor angular positions. Then, following a teach pendant approach, after recording the trajectory, the robot can repeat the trajectory until the therapy is over [16].

In order to improve further the real prototype, the aim of this paper is to analyze robot design as it is right now through the use of the kinematic and static model of the 2-DoF serial robot. Both models enable to perform analysis on the rehabilitation therapies recorded in the Traumatology Service at the Instituto Autónomo Hospital de la Universidad de los Andes (AIHULA), Venezuela. In addition, performance indexes enable to analyze the robot dexterity. The analyses offer insights on the actual robot design and gives some hints on the way to go ahead in the robot development.

2. Material and methods
First, the angular position of the robot's actuators from three different rehabilitation routines were acquired. Then, by direct kinematics modeling, the robot kinematics: position, linear and angular velocities and accelerations, workspace, and performance index for such rehabilitation trajectories were found. The kinematic analysis provides insights on the robot capabilities as a rehabilitation device. For the sake of completeness, the following section briefly describes the robotic device used in the experiments.

2.1. Wrist rehabilitation robot
Figure 1 depicts a sketch of the robotic device which relies on a 2 DoF serial robot architecture. The device consists of a support frame for the forearm, a clamping element based on Velcro strap for attaching the hand to the device. The strap allows the patient to take out his/her hand when cases of fear or sudden pain, see Figure 2. The mobile camera frame model PT785-S serves as the base for constructing the links of the serial robot. Servo motors HITEC HS-785HB provide the robot actuation. To obtain the proper torque, the design required a spur gears system to connect motors to each joint, see Figure 1. Each motor axis has an angular positioning sensor (rotational potentiometer) installed to measure the angular position. In addition, the control system of the robot relies on the open-source electronics platform Arduino Uno R3 among other electronic elements.

2.2. Rehabilitation trajectories
The physiatrists at the IAHULA Traumatology Service usually follow three rehabilitation routines. They do the therapies on the patient wrist in the sequential order presented below.
2.2.1. **Routine 1.** The routine starts by performing oscillating and intermittent movements between two wrist motion planes, it starts performing flexo-extension dorsal-palmar movements while no motion at the radial-ulnar plane. The trajectory takes 12.5 seconds. Next, the trajectory follows at the ulnar radial plane while motion in flexo-extension plane remains stopped, this part of the trajectory takes 25 seconds.

2.2.2. **Routine 2.** The second trajectory is a combination of circular motion, it starts with limited circular arc movements, and then the range of the arc increases over time both in the ulnar-radial plane and in the flexo-extension plane.

2.2.3. **Routine 3.** The third routine is similar to the second routine, yet it starts with large circular arc movements in both the ulnar-radial plane and flexo-extension plane. The movements are performed at the end of the second rehabilitation phase which consists of the strengthening of the muscular mass of the wrist joint. Figure 3 shows the actuator angular position for each joint when following this routine.

![Figure 1](image1.png) **Figure 1.** Sketch of the 2-DoF rehabilitation robot.

![Figure 2](image2.png) **Figure 2.** Experiments on the Rehabilitation Robot [15].

![Figure 3](image3.png) **Figure 3.** Joint angular position for the third routine.

2.3. **Kinematic model**

The kinematic model development relies on the homogeneous transformation matrix approach using Denavit-Hartembert (D-H notation) as described in [17]. Table 1 lists the D-H parameters for the prototype. The following section describes the direct position equations.

| Link $i$ | $\alpha_i$ (rad) | $a_i$ (cm) | $\theta_i$ (rad) | $d_i$ (cm) |
|----------|------------------|------------|-----------------|------------|
| 1        | $-\pi/2$         | 0          | $\theta_1$     | 25         |
| 2        | 0                | 10         | $\theta_2$     | 0          |

*Table 1. D-H Parameters for the 2-DoF robot prototype.*
2.3.1. Direct position equations. Equation 1 presents the homogeneous transformation matrix [18] used for developing the position equations.

\[
A_i^{-1} = \begin{bmatrix}
C\theta_i & -C\alpha_i S\theta_i & S\alpha_i S\theta_i & aC\theta_i \\
S\theta_i & C\alpha_i C\theta_i & -S\alpha_i C\theta_i & aS\theta_i \\
0 & S\alpha_i & C\alpha_i & d_i \\
0 & 0 & 0 & 1
\end{bmatrix}
\] (1)

By substituting the D-H parameters on Equation (1), one can get the homogeneous transformation matrix for frame 0 to 1 and frame 1 to 2 as in Equation (2).

\[
A_0^1 = \begin{bmatrix}
C\theta_1 & 0 & -S\theta_1 & 0 \\
S\theta_1 & C\theta_1 & 0 & 0 \\
0 & -1 & 0 & d_i \\
0 & 0 & 0 & 1
\end{bmatrix}, \quad A_2^1 = \begin{bmatrix}
C\theta_2 & S\theta_2 & 0 & a_2 C\theta_1 \\
S\theta_2 & C\theta_2 & 0 & a_2 S\theta_2 \\
0 & 0 & 1 & d_i \\
0 & 0 & 0 & 1
\end{bmatrix}
\] (2)

By multiplying both homogeneous transformation matrix \(A_0^1 A_2^1\) and taking the first three rows of the fourth column, one can get the position equations for the end-effector x, y, z coordinates with respect to the joint angles as in Equations (3)-(5).

\[
x = a_2 C\theta_1 C\theta_2 \\
y = a_2 C\theta_2 S\theta_1 \\
z = d_1 - a_2 S\theta_2
\] (3-5)

2.3.2. Jacobian and velocity equations. The velocity equation relies on the Jacobian matrix computation. For the actual prototype, Equation (6) presents the Jacobian matrix.

\[
J = \begin{bmatrix}
e_0 \\
e_1 \times a_{0e} \\
e_1 \times a_{1e}
\end{bmatrix}
\] (6)

Equation (7) defines the terms for computing the Equation (6).

\[
e_0 = \begin{bmatrix}0 \\
1
\end{bmatrix}, \quad e_1 = \begin{bmatrix}-S\theta_1 \\
C\theta_1
\end{bmatrix}, \quad a_{0e} = \begin{bmatrix}a_2 C\theta_1 C\theta_2 \\
a_2 C\theta_2 S\theta_1
\end{bmatrix}, \quad a_{1e} = \begin{bmatrix}a_2 C\theta_1 C\theta_2 \\
a_2 C\theta_2 S\theta_1 \\
-d_1 - a_2 S\theta_2
\end{bmatrix}
\] (7)

Finally, Equation (8) defines the velocity equation for the 2-DoF robot.

\[
t_e = \begin{bmatrix}w_e \\
v_e
\end{bmatrix} = J\dot{\theta}
\] (8)

In equation (8), \(\dot{\theta} = [\dot{\theta}_1 \quad \dot{\theta}_2]\).

2.3.3. Performance index. The analysis of the robot requires the definition of the performance index, and their application for each rehabilitation routine. Equation (9) presents the first index which is based on the condition number of the Jacobian matrix [19].

\[
k(J) = \|J\| \cdot \|J^{-1}\|
\] (9)

The condition number in Equation (9) spans between zero and one. When the index equals to one, it means that the Jacobian matrix at that point of the trajectory is isotropic indicating that the robot has the...
same ability to move in any direction. A condition number is equal to zero means that the robot is at a singular configuration.

Equation (10) shows another index commonly used in industrial robotics, the Yoshikawa index for manipulability [19].

\[ w = \sqrt{\det(J^TJ)} \]  

(10)

2.3.4. Acceleration equations. Equation (11) presents the end-effector acceleration equations for the 2 DoF robot. By deriving Equation (8), one can get the acceleration with respect to the joint coordinates.

\[ t_e = |\dot{\Theta} + \ddot{\Theta} | \]  

(11)

2.3.5. Statics equations. Finally, Equation (12) shows the static equations, which are derived based on the transpose of the Jacobian matrix.

\[ \tau = J^T \cdot w_e \]  

(12)

In Equation (12), \( w_e \) stands for the wrench applied at the end-effector. In this case, the only force being considered is the mass of the hand in the vertical direction. Equation (13) defines the applied wrench.

\[ w_e = [0 \quad 0 \quad 0 \quad g \cdot m_{hand} \quad 0]^T \]  

(13)

3. Results and discussions

Figure 4 presents the end-effector trajectory for the rehabilitation routine, as expected the end-effector path corresponds to a concentric sphere with the center of rotation at the wrist joint. The values for the end-effector angular position vary in a range from 65 to 80 degree in the dorsi-palmar flexion plane, and it presents a range between 47 and 59 degree in the ulnar-radial plane. The position analyses verify that the robot accomplishes the rehabilitation routines.

![Figure 4. End-effector position for the third routine.](image)

Figures 5 and 6 present angular and linear velocity. With respect to the angular velocities, it is worth notice that the greatest angular velocity was 1.85rad/s in the dorsal-palmar plane. These values of velocity occur in the last stage of rehabilitation, i.e., in the empowerment phase [13].
Analyzing the condition number of the Jacobian matrix for the rehabilitation routine, it was found that the index mean value is about 0.5 which indicates the robot shows a good performance. Similarly, Figure 7 presents the dexterity for third routine. The trajectory highest value is below 6 which is as good performance taking into consideration that the value was set up as upper limit for dexterity optimization in [20].

Figure 5. End-effector angular velocity for the third routine.

Figure 6. End-effector linear velocity for the third routine.

Figure 8 displays the torque values, in the case of the plane dorsal-palmar plane. The values vary from 8Ncm to 50Ncm. The greatest value is due to the weight of the hand. The data sheet of motors HITEC HS-785HB establishes that a stall torque of the motor at 6V is of about 130Ncm. This means that moments due to static forces represent about 40% of the largest torque motors can offer. Take into account that the robot not only deals with static forces, but also wrist joint tension due to the type of injury, friction in the prototype joints, the robot will not succeed in performing the rehabilitation task. Thus, the analysis points out that the actual prototype requires an upgrade in design of motors. The preliminary results show insights for future works to guide the upgrade of the robot design, for instance, optimal design considering power consumption using global optimization algorithms as the one employed in [21].

Figure 7. Dexterity of the Robot for the third routine.

Figure 8. Joint torques required for the third routine.
4. Conclusions
This work has studied the kinematic and static equations for a 2-DoF wrist rehabilitation robot for three rehabilitation routines. In this sense, the position, velocity, performance index, acceleration and moment equations were developed. The equations presented in this work enabled to understand whether the robot accomplishes the real rehabilitation routines recorded at the IAHULA. From the kinematic point of view, the robot can deal with the three recorded trajectories. In addition, preliminary findings of the analysis showed that the actuation system of the prototype requires a redesign. Further experiments will guide the redesign of the robot.

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