Modeling the non-destructive control of road surfaces compaction

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Abstract. In this paper, based on the state space method, a mathematical model of the “stacker-mix” dynamic system as an object of continuous non-destructive testing of compaction of asphalt-concrete mix is obtained. The system of differential equations takes into account the structural elements of the working body of the stacker and the medium to be sealed. The compacted medium is described by a rheological model of an elastic-viscous body. A simulation model of a dynamic system is obtained in the MATLAB/Simulink program. The results of numerical simulation are presented.

1. Introduction
Increasing the degree of compaction of asphalt concrete mix by working bodies of pavers makes it possible to increase the efficiency of road construction [1].

Based on studies of the asphalt paver carried out under real operating conditions, experimental dependences have been obtained that make it possible to realize the idea of a continuous non-destructive testing of compaction for pavers [1].

The conducted experimental studies allow us to conclude that there is a dependence of the compaction coefficient on the maximum force in the tappets of the tamper at the final stage of the compaction cycle with increasing resistance to deformation of the mixture. The increase in mixture resistance to compaction occurs gradually and reaches its maximum value after 4-5 impacts with a tamper.

The work [2] is well-known, the results of which are the dependences of the acceleration of the tamper of the paver from the operating modes. Rollers equipped with a Continuous Compaction Control (CCC) system utilize vibration drum acceleration data, which is used to determine the compaction index. To calculate the compaction index, the spectral Fourier transform is used [3, 4].

This work is devoted to a theoretical description of the process of compaction of a road-building mixture by the working body of the paver as an object of control. Several varieties of working bodies are known including a smoothing plate (static or vibrational), a tamper beam, and pressing strips.

Theoretical studies of the workflow of asphalt pavers were carried out by many foreign and Russian scientists. Experimental studies of pavers in real conditions were carried out by a small number of researchers, which is associated with significant difficulties in organizing work when laying hot asphalt mixtures. Therefore, methods of mathematical modeling are an effective means of research and design of control systems (control) [5].

The disadvantages of previously developed mathematical models are computational difficulties in solving the design problems of automated control systems. To increase the efficiency of a theoretical
study of the control object using modern software, it is recommended to use state space methods that allow for clear formalization and automation of computational procedures [6-8].

The description of systems in the state space allows one to detect and investigate properties that, using classical methods of frequency analysis and description in terms of “input-output”, would remain hidden. The matrix form of recording used in the state space method has an undeniable advantage in the numerical solution [7].

The purpose of the work is to develop a mathematical model of the process of compaction of the road-building mixture by the working body of the paver as an object of quality control of compaction using the state space method. The object of the study is the process of compaction of the road-building mixture by the working body of the paver as a part of: ramming beam + vibration smoothing plate. The subject of the study is the dependence of the dynamic parameters of the working process on the paver operating modes and the characteristics of the compacted medium.

The working process is characterized by deformation of the mixture with a tamper with a kinematic drive (main seal in 4-6 movements) and a vibrating plate. Vibration plate improves the structure of the pavement and fixation of the achieved degree of compaction of the pavement. The process of compaction of the mixture is performed with constant contact of the vibrating plate with the mixture.

The main dynamic parameters of the vibrations of the beam, plate and particles of the medium being compacted are frequency, amplitude, speed, acceleration. The oscillation amplitude of any sealant depends on the physicomechanical characteristics of the material being compacted and changes during its compaction.

The process of compaction of road building materials is the accumulation of residual deformations by the material. For effective sealing, it is necessary to comply with the condition under which the contact pressures arising under the working body must not exceed the tensile strength of the material.

2. A model of the compaction process in the state space

In compiling the mathematical model of the compaction process, the following assumptions are made:

1) the structural elements of the machine have absolute rigidity;
2) the compacted layer has elastic-viscous properties;
3) only the vertical component of vibration is considered;
4) the inertial properties of the medium to be sealed are taken into account.

An elastic-viscous Kelvin-Voigt model is used for a theoretical description of the medium being compacted.

The design model of the dynamic system model of the compacting working body of the stacker is shown in figure 1. In the diagram of figure 1, the following notation is used: $m$ – mass of the unbalanced shaft; $m_1$ – mass of vibration plate, kg; $m_2$ – mass of the vibration module, kg; $m_3$ – mass of tamper beam, kg; $m_4$ – the mass of the medium under the tamper, kg; $m_5$ – the mass of the medium under the vibrating plate, kg; $k_1$ – coefficient of elastic resistance of the medium being compacted under the stove, N/m; $c_1$ – damping coefficient of the sealed medium under the stove, N s/m; $k_2$ – coefficient of elastic resistance of the shock absorbers of the vibrator, N/m; $c_2$ – damping coefficient of shock absorbers of the vibrator, N s/m; $k_3$ – coefficient of elastic resistance of the medium being compacted under the tamper, N/m; $c_3$ – damping coefficient of the compacted medium under the tamper, N s/m; $y_1, y_2, y_3$ – moving elements of the system, respectively, m.
Figure 1. Design scheme of the dynamic system.

The differential equations of the vibrating system “tamper-vibrating plate-mixture” are obtained.

Differential equation of vibrations of a vibrator

\[ m_2 \cdot \ddot{y}_2 - c_2 \cdot \dot{y}_1 + c_2 \cdot \ddot{y}_2 - k_2 \cdot y_1 + k_2 \cdot y_2 = m \cdot r \cdot \omega_2^2 \cdot \sin(\omega_2 \cdot t), \tag{1} \]

where \( r \) – radius of the eccentricity of the unbalanced shaft, m; \( \omega_2 \) – angular frequency of rotation of the unbalanced shaft of the vibrator, rad/s; \( t \) – time, s.

The differential equation of motion of the tamper

\[ (m_3 + m_4) \cdot \ddot{y}_3 + c_3 \cdot \dot{y}_3 + k_3 \cdot y_3 = F_3 + m_4 \cdot g, \tag{2} \]

where \( F_3 \) – ram pusher force, N.

Taking into account the principle of relative motion, we obtain an additional equation

\[ y_3 = y_1 + e \cdot \sin(\omega_3 \cdot t), \tag{3} \]

where \( e \) – radius of the eccentricity of the tamper drive, m; \( \omega_3 \) – angular speed of rotation of the drive shaft of the tamper, rad/s.

Substituting equation (3) in (2) with transformations, we obtain the following equation

\[ F_3 = \left( m_3 + m_4 \right) \cdot \ddot{y}_3 + c_3 \cdot \dot{y}_3 + k_3 \cdot y_3 - \left( m_3 + m_4 \right) \cdot e \cdot \omega_3^2 \cdot \sin(\omega_3 \cdot t) + \\
+ k_3 \cdot e \cdot \sin(\omega_3 \cdot t) + c_3 \cdot e \cdot \omega_3 \cdot \sin \left( \omega_3 \cdot t + \pi/2 \right) - m_4 \cdot g. \tag{4} \]

Differential equation of motion of a screed

\[ (m_1 + m_2) \cdot \ddot{y}_1 + (c_1 + c_2) \cdot \dot{y}_1 - c_2 \cdot \ddot{y}_2 + (k_1 + k_2) \cdot y_1 - k_1 \cdot y_1 = -F_3 + m_5 \cdot g. \tag{5} \]

Moving equation (4) to (5), we obtain the following expression

\[ (m_1 + m_2 + m_3 + m_4) \cdot \ddot{y}_1 + (c_1 + c_2 + c_3) \cdot \dot{y}_1 - c_2 \cdot \ddot{y}_2 + (k_1 + k_2 + k_3) \cdot y_1 - k_1 \cdot y_1 = \\
= \left( m_1 + m_2 \right) \cdot e \cdot \omega_3^2 \cdot \sin(\omega_3 \cdot t) - c_1 \cdot e \cdot \omega_3 \cdot \sin \left( \omega_3 \cdot t + \pi/2 \right) + (m_3 + m_4) \cdot g. \tag{6} \]

The state space method can be represented as a system of equations \([1, 7]\):

\[ \dot{x}(t) = A(t) \cdot x(t) + B(t) \cdot u(t); \tag{7} \]

\[ y(t) = C(t) \cdot x(t) + D(t) \cdot u(t), \tag{8} \]

where \( x(t) \) – state vector whose components are state variables of the \( n \)-th order system; \( y(t) \) – output vector the components of which are the output variables of the system; \( A(t) \) – system coefficient matrix \((n \times n)\); \( B(t) \) – input matrix \((r \times n)\), \( r \) – number of impacts; \( u(t) \) – input vector, the
components of which are the input variables of the system; $C(t)$ – output matrix $(n \times p)$, $p$ – number of output quantities; $D(t)$ – bypass matrix that determines the direct dependence of the output on the input.

System state variables are defined: $z_1$ – vertical movement of the screed, $z_1 = y_1$; $z_2$ – speed of vertical movement of the screed, $z_2 = \dot{y}_1$; $z_3$ – vibrator vertical movement, $z_3 = y_2$; $z_4$ – vibrator vertical speed, $z_4 = \dot{y}_2$.

The system of equations (1) – (6) after transformations, taking into account the accepted parameters of the state space in the form of Cauchy

$$
\dot{z}_1 = z_2; \\
\dot{z}_2 = \frac{1}{m_1 + m_3 + m_4 + m_5} \left[ -(c_1 + c_2 + c_3) \cdot z_2 + c_2 \cdot z_4 + (k_1 + k_2 + k_3) \cdot z_1 + k_2 \cdot z_3 + \\
+((m_3 + m_4) \cdot e \cdot \omega_3^2 - k_1 \cdot e) \cdot \sin(\omega_3 \cdot t) + \\
+c_3 \cdot e \cdot \omega_3 \cdot \sin(\omega_3 \cdot t + \pi/2) + (m_4 + m_5) \cdot g \right]; \\
\dot{z}_3 = z_4; \\
\dot{z}_4 = \frac{1}{m_2} \left( c_2 \cdot z_2 - c_2 \cdot z_4 + k_2 \cdot z_1 - k_2 \cdot z_3 + m \cdot r \cdot \omega_2^3 \cdot \sin(\omega_2 \cdot t) \right).
$$

(9)

The following parts of the model are obtained in the state space

$$
\dot{Z} = \begin{bmatrix} \dot{z}_1 \\ \dot{z}_2 \\ \dot{z}_3 \\ \dot{z}_4 \end{bmatrix}^T; \\
Z = \begin{bmatrix} z_1 \\ z_2 \\ z_3 \\ z_4 \end{bmatrix}^T;
$$

system coefficient matrix

$$
A = \begin{bmatrix}
0 & -c_1 & c_2 & 0 \\
-k_1 & -c_1 & -c_2 & 0 \\
k_2 & c_2 & -k_2 & 0 \\
0 & 0 & 0 & 1 \\
\end{bmatrix}.
$$

input matrix $B$, output matrix $C$, bypass matrix $D$, output vector $Y$

$$
B = \begin{bmatrix}
0 & 0 \\
1 & 0 \\
0 & 0 \\
0 & \frac{1}{m_2} \\
\end{bmatrix}; \\
C = \begin{bmatrix}
1 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 \\
0 & 0 & 1 & 0 \\
0 & 0 & 0 & 1 \\
\end{bmatrix}; \\
D = \begin{bmatrix}
0 & 0 \\
0 & 0 \\
\end{bmatrix}; \\
Y = \begin{bmatrix}
y_1 \\
y_2 \\
\end{bmatrix}^T.
$$

input vector $U$

$$
U = \begin{bmatrix}
\left((m_3 + m_4) \cdot e \cdot \omega_3^2 - k_1 \cdot e\right) \cdot \sin(\omega_3 \cdot t) - c_3 \cdot e \cdot \omega_3 \cdot \sin(\omega_3 \cdot t + \pi/2) + (m_4 + m_5) \cdot g \\
m \cdot r \cdot \omega_2^3 \cdot \sin(\omega_2 \cdot t) \\
\end{bmatrix}.
$$

3. The study of the mathematical model

To check the adequacy of the model, calculations were performed in the MATLAB program. A simulation model of the studied process is presented in Figure 2.

To simulate the process the initial data from [9] are used
\[ k_1 = 4 \cdot 10^6 \text{ N/m}; \quad k_2 = 1.1 \cdot 10^7 \text{ N/m}; \quad k_3 = 8.5 \cdot 10^5 \text{ N/m}; \quad c_1 = 3200 \text{ N} \cdot \text{s/m}; \quad c_2 = 1800 \text{ N} \cdot \text{s/m}; \quad c_3 = 1200 \text{ N} \cdot \text{s/m}; \quad m = 21.6 \text{ kg}; \quad m_1 = 682 \text{ kg}; \quad m_2 = 80 \text{ kg}; \quad m_3 = 71.3 \text{ kg}; \quad m_4 = 0.1 \cdot m_3; \quad m_5 = 0.2 \cdot m_3; \quad r = 0.03 \text{ m}; \quad e = 0.006 \text{ m}; \quad f_2 = 20 \text{ Hz}; \quad f_3 = 15 \text{ Hz}. \]

**Figure 2.** Simulation model of the process under study in MATLAB/Simulink.

As a result of computer simulation, the parameters of the process of compaction of the mixture by the working body of the paver were obtained: acceleration (figure 3); effort of a pusher of a ram (figure 4).

**Figure 3.** Time dependences of the acceleration

**Figure 4.** Time dependence of tamper force
The simulation results showed good reproducibility of the process.

4. Conclusions
The idea of using maximum force in the pusher of the tamper beam of the stacker for continuous non-destructive testing of the compaction of the mixture is proposed.

A mathematical model of the dynamic system “stacker - mixture” as an object of compaction control in the state space is obtained. The results of the work are a stage of scientific research in the field of automation of the processes of compaction by pavers.

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References
[1] Prokopyev A P and Emelyanov R T 2004 Automation of the process of compaction of the paver Izv. of higher institutions, construction 7(547) 82-4
[2] Sun J and Xu G 2013 Dynamics Modeling and Analysis of Paver Screed Based on Computer Simulation J. Applied Sciences 13 1059-65
[3] Pistrol J and Adam D 2018 Fundamentals of roller integrated compaction control for oscillatory rollers and comparison with conventional testing methods Transportation Geotechnics DOI: https://doi.org/10.1016/j.targeo.2018.09.010
[4] Adam D and Pistrol J 2016 Dynamic roller compaction for earthworks and roller-integrated continuous compaction control: State of the art overview and recent developments Institute of Geotechnics, Vienna University of Technology Austria 1-41
[5] Nosov S V 2017 Generalized dynamic model of the interaction of compactors with road construction materials Russian J. Building Construction and Architecture 2(34) 35-44
[6] Derusso P M, Roy R J and Close C M 1998 State Variables for Engineers (New York: Wiley)
[7] Strejc V 1981 State space theory of discrete linear control (ACADEMIA/ Prague) p 426
[8] Phillips C L and Harbor R D 2000 Feedback Control Systems (Prentice Hall) p 658
[9] Jian S and Guiyun X 2013 Dynamics Modeling and Analysis of Paver Screed Based on Computer Simulation J. of Applied Sciences 13(7) 1059-65