The comparison of predictive scheduling algorithms for
different sizes of job shop scheduling problems

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Abstract. In the paper a survey of predictive and reactive scheduling methods is done in order
to evaluate how the ability of prediction of reliability characteristics influences over robustness
criteria. The most important reliability characteristics are: Mean Time to Failure, Mean Time of
Repair. Survey analysis is done for a job shop scheduling problem. The paper answers the
question: what method generates robust schedules in the case of a bottleneck failure occurrence
before, at the beginning of planned maintenance actions or after planned maintenance actions?
Efficiency of predictive schedules is evaluated using criteria: makespan, total tardiness, flow
time, idle time. Efficiency of reactive schedules is evaluated using: solution robustness
criterion and quality robustness criterion. This paper is the continuation of the research
conducted in the paper [1], where the survey of predictive and reactive scheduling methods is
done only for small size scheduling problems.

1. Introduction

In the paper a survey of predictive and reactive scheduling methods is done in order to evaluate how
the ability of prediction of reliability characteristics influences over robustness criteria. The most
important reliability characteristics are: Mean Time to Failure, Mean Time of Repair. Survey analysis
is done for job shop (JS) scheduling problem. The scheduling problem i.e. 15 jobs have to be
performed on 10 machines (15x10) is investigated. The paper answers the question: what method
generates robust schedules in the case of a bottleneck failure occurrence before, at the beginning of
planned maintenance actions or after planned maintenance actions? Efficiency of predictive schedules
is evaluated using criteria: makespan ($C_{max}$), total tardiness ($T$), flow time ($F$), idle time ($I$). Efficiency
of reactive schedules is evaluated using: solution robustness criterion (SR) and quality robustness
criterion (QR). This paper is the continuation of the research conducted in the paper [1], where the survey of predictive and reactive scheduling methods is done only for small size scheduling problems.
It is important to elaborate a method which generates robust schedules in case of disturbance
occurrence e.g. a machine failure for various production systems [5, 8, 9, 11]. However, predictive
scheduling methods should be aided with methods of monitoring [12], diagnosis [6, 7, 10] and
mistakes proofing [13] to increase failure-free time of the bottleneck.
The Hybrid - Multi Objective Immune Algorithm (H-MOIA) is aided with heuristics: Minimal Impact of Disturbed Operation on the Schedule (MIDOS) for predictive scheduling and Minimal Impact of Rescheduled Operation on the Schedule (MIROS) for reactive scheduling. Sensitivity analysis is done for predictive scheduling methods 1) H-MOIA +MIDOS, 2) algorithm based on priority rules: the Least Flexible Job First (LFJ) and the Longest Processing Time (LPT) [3] and 3) the Average Slack Method [2]. Reactive schedules are generated for various scenarios of the bottleneck occurrence in order to evaluate the efficiency of predictive scheduling methods. Reactive schedules are generated using 1) H-MOIA +MIROS, 2) the algorithm based on priority rules: the LFJ and LPT and 3) Shifted Gap-Reduction [4]. The paper presents the research results and computer simulations. This paper is the continuation of the research conducted in the paper [1], where reactive schedules are generated using different methods: 1) Right Shifting, 2) rescheduling of disturbed operation to the parallel machine first available.

2. Job shop scheduling problem

This Section presents job shop (JS) scheduling problem with interruptions for experimental study as well as a schedule of surveys.

The objective is to obtain a solution for four objective functions: $C_{max}$, $F$, $T$, $I$ with priorities: $w_1=0.3$ for $C_{max}$, $w_2=0.2$ for $F$, $w_3=0.3$ for $T$, $w_4=0.2$ for $I$ for (15x10) scheduling problem. Efficiency of predictive schedules is evaluated using: SR and QR [1].

There are three stages of the survey, i.e. first – the generation of the basic schedule, second - the generation of PS and third - the generation of RS for JS scheduling problems and given criteria. The basic schedule is generated in order to define which machine constitutes the bottleneck. The PS is generated in order to meet the deadline. The RS is generated if an unpredicted disturbance occurs. Three different scenarios of the bottleneck failure are considered. The method which generates robust schedules is searched for, for the three cases of a bottleneck failure occurrence: before planned maintenance actions, at the beginning of planned maintenance actions or after planned maintenance actions (Fig. 1).

![Figure 1](image-url)  
Figure 1. The time period of increased probability of a disturbance occurrence, $[a, b + MTTR]$  

3. Results of computer simulations

This section is concerned with the job shop scheduling problem (10×15). Machine $w = 1$ is the most loaded. The assumption is: $a=60$ and $b=72$ and MTTF = 66, MTTR = 6 for machine $w = 1$.

In order to achieve the basic schedule for JS problem (10×15) using the H-MOIA, three simulations are generated. Each simulation consists of 30 iterations. In the first simulation, the best basic schedule is generated according to the rule of { 2 5 14 1 3 0 4 6 7 10 8 9 11 12 13 }. The quality of the schedule is $C_{max} = 117$, $F = 511$, $I = 638$ and $T = 0$. Remaining solutions are described in Table 1.
Table 1. The best basic schedules achieved by the MOIA

| Method   | No simulation | Job shop scheduling problem (15x10) |
|----------|---------------|-------------------------------------|
|          |               | The priority rule of the basic schedule | The quality of the schedule |
|          |               |                                      | $C_{max}$ | $F$ | $I$ | $T$ | $F_{xy}$ |
| H-MOIA   | 1             | 2 5 14 1 3 0 4 6 7 10 8 9 11 12 13 | 117 | 511 | 638 | 0 | 264.9 |
|          | 2             | 10 14 2 5 3 1 0 4 7 8 9 6 11 12 13 | 117 | 497 | 638 | 0 | 262.1 |
|          | 3             | 2 5 8 0 14 1 4 3 6 7 9 10 11 12 13 | 117 | 534 | 638 | 0 | 269.5 |

The best basic schedule is generated in the second simulation. Thus, the issue will be analysed using the example of the second rule.

PSs are generated using the methods of H-MOIA+MIDOS, ASM and LFJ/LPT.

3.1. **H-MOIA+MIDOS**

First, the PS is generated using the second stage of the H-MOIA. Input data to the second stage of H-MOIA+MIDOS constitutes the basic schedule generated according to the rule of \{10 14 2 5 3 1 0 4 7 8 9 6 11 12 13\}. It is predicted that in the time period \([a,b]+MTTR\) operations performed on machine $w = 1$ can be disturbed. Disturbed batches $\hat{s}_{j}$ are deleted from the basic schedule. First, in the PS, in the time period \([MTTF, MTTF+MTTR]= [66, 66+6]\), the technical inspection of the bottleneck is scheduled. Next, the most flexible operation of each deleted job in the time period \([60, 72+6]\) is scheduled. For the remaining operations, backward and forward scheduling algorithms are applied. The quality of the PS obtained at the second stage of the H-MOIA is $C_{max} = 110$, $F = 611$, $I = 562$ and $T = 0$ (Tab. 2).

3.2. **ASM**

In order to achieve a PS using the ASM, three simulations are generated (the number of iteration = 30 and is the same as in the first stage of the MOIA). Afterwards, the efficiency of two algorithms, i.e. the MOIA and the ASM in searching a solution space is compared. Therefore, the input data to the neighbourhood searching heuristic (ASM) is the permutation of jobs obtained at the first stage of the H-MOIA. After running the experiment, the PS generated using the ASM is the flow according to the rule of \{10 14 2 5 3 1 0 4 7 8 9 6 11 12 13\}. Although, the ASM improve the average quality value of population, the ASM did not achieve a better quality solution than that generated at the first stage of the H-MOIA.

3.3. **LFJ/LPT**

The PS generated using the LFJ/LPT is the flow according to rule of \{11 10 6 5 0 14 13 8 4 3 9 1 12 7 2\}. The quality of the PS obtained by the LFJ/LPT is $C_{max} = 139$, $F = 576$, $I = 858$ and $T = 145$.

Table 2. Evaluation of predictive scheduling methods for a job shop problem

| Method   | $C_{max}$ | $F$ | $T$ | $I$ | $F_{xy}$ |
|----------|-----------|-----|-----|-----|----------|
| H-MOIA   | 110       | 611 | 0   | 562 | 267.6    |
| LFJ, LPT | 139       | 576 | 145 | 858 | 372      |
| ASM      | 110       | 611 | 0   | 562 | 267.6    |

The quality of the PSs generated using various methods is presented in Table 2. Taking into account the criteria of $C_{max}$, $F$, $I$, and $T$, the solutions generated using the MOIA+MIDOS are better than those generated using the LFJ/LPT. It should be noted that the algorithms of ASM and LFJ/LPT do not insert the maintenance task into a schedule. The MOIA enables the achievement of the best quality predictive schedule including the technical inspection of the bottleneck at time 66 (Fig. 4 and 5).
Undeleted jobs are generated in the PS generated using the MOIA+MIDOS (Tab. 3, 4 and 5). The main advantage of the MOIA+MIDOS is the minimization of the probability of the bottleneck breaking down due to the insertion of the additional task, i.e. planned technical inspection, into the basic schedule.

The question which arises in such a situation is what happens if the bottleneck fails before the maintenance actions are performed?. In order to answer the question, for each PS obtained using three methods, i.e. the H-MOIA+MIDOS, ASM and LFJ/LPT RSs are generated using the methods H-MOIA+MIROS, LFJ/LPT and SGR (Tab. 3). RSs are evaluated using the solution robustness criterion $SR$ and the quality robustness $QR$ [1]. For the assumption that the real $MTTF$ of the bottleneck equals 63 the detailed results generated for JS scheduling problem (15x10) using different algorithms are presented in Table 3. Taking into account criteria of $C_{max}$, $F$, $T$ and $I$, the best PS seems to be generated by the ASM. However, further analysis indicated that the PS generated by the H-MOIA+MIDOS+MIROS absorbs the effect of the bottleneck failure more efficiently (Tab. 3, Fig. 4 and 6). The PS generated by the H-MOIA+MIDOS is robust and the most stable for three RS.

| Predictive method | Reactive scheduling | $C_{max}$ | $F$ | $T$ | $I$ | $FF_{vs}$ | $SR$ | $QR$ |
|-------------------|---------------------|-----------|-----|-----|-----|----------|------|------|
| H-MOIA +MIDOS     | LFJ, LPT            | 110       | 630 | 0   | 559 | 270.8    | 34   | 3    |
|                   | MIROS               | 110       | 601 | 0   | 559 | 265      | 12   | 2    |
|                   | SGR                 | 110       | 630 | 0   | 559 | 270.8    | 34   | 3    |
| LFJ, LPT          | LFJ, LPT            | 135       | 627 | 150 | 821 | 375.1    | 1567 | 3    |
|                   | MIROS               | 130       | 547 | 128 | 762 | 339.2    | 294  | 32   |
|                   | SGR                 | 135       | 544 | 153 | 812 | 357.6    | 1156 | 14   |
| ASM               | LFJ, LPT            | 89        | 212 | 0   | 235 | 116.1    | 326  | 8.1  |
|                   | MIROS               | 83        | 204 | 0   | 203 | 106.3    | 93   | 17.9 |
|                   | SGR                 | 87        | 189 | 0   | 223 | 108.5    | 175  | 15.7 |

The second question which also arises in such a situation is what happens if the bottleneck fails at the time of the planned maintenance actions?. For the assumption that the real $MTTF$ of the bottleneck equals 66 the detailed results generated for JS scheduling problem (15x10) using different algorithms are presented in Table 4. Taking into account criteria of $C_{max}$, $F$, $T$ and $I$, the best PS is generated by the H-MOIA+MIDOS+MIROS. The PS generated by the H-MOIA+MIDOS+MIROS absorbs the effect of the bottleneck failure.

| Predictive method | Reactive scheduling | $C_{max}$ | $F$ | $T$ | $I$ | $FF_{vs}$ | $SR$ | $QR$ |
|-------------------|---------------------|-----------|-----|-----|-----|----------|------|------|
| H-MOIA +MIDOS     | LFJ, LPT            | 110       | 611 | 0   | 562 | 267.6    | 0    | 0    |
|                   | MIROS               | 110       | 611 | 0   | 562 | 267.6    | 0    | 0    |
|                   | SGR                 | 110       | 611 | 0   | 562 | 267.6    | 0    | 0    |
| LFJ, LPT          | LFJ, LPT            | 134       | 667 | 165 | 803 | 383.7    | 1238 | 11   |
|                   | MIROS               | 134       | 585 | 169 | 802 | 368.3    | 1020 | 3    |
|                   | SGR                 | 135       | 559 | 165 | 812 | 364.2    | 698  | 7    |
| ASM               | LFJ, LPT            | 139       | 576 | 28  | 582 | 335.7    | 919  | 74   |
|                   | MIROS               | 117       | 502 | 0   | 632 | 261.9    | 122  | 0    |
|                   | SGR                 | 136       | 579 | 31  | 822 | 330.3    | 927  | 69   |
The third question which also arises in such a situation is what happens if the bottleneck fails even though the maintenance actions have been performed?. For the assumption that the real MTTF of the bottleneck equals 75 the detailed results generated for JS scheduling problem (15x10) using different algorithms are presented in Table 5. The PS generated by the H-MOIA+MIDOS and ASM absorbs the effect of the bottleneck failure more efficiently (Tab. 5). The PS generated by the ASM is robust under the constraint that the MIROS is used for rescheduling.

Table 5. Evaluation of reactive scheduling methods for JS problem (15x10) and for the scenario of the real MTTF of the bottleneck = 75

| Predictive method | Reactive scheduling | $C_{max}$ | $F$ | $T$ | $FF_y$ | SR | QR |
|-------------------|---------------------|-----------|-----|-----|-------|----|----|
| H-MOIA +MIDOS     | LFJ, LPT            | 122       | 596 | 0   | 685   | 292.8 | 699 | 25 |
|                   | MIROS               | 110       | 604 | 0   | 556   | 265  | 68  | 2  |
|                   | SGR                 | 126       | 627 | 0   | 716   | 306.4 | 309 | 38 |
| LFJ, LPT          | LFJ, LPT            | 130       | 639 | 132 | 771   | 360.6 | 1269 | 11 |
|                   | MIROS               | 136       | 580 | 140 | 822   | 363.2 | 141 | 9  |
|                   | SGR                 | 140       | 611 | 169 | 862   | 387.3 | 564 | 14 |
| ASM               | LFJ, LPT            | 135       | 529 | 33  | 815   | 319.2 | 732 | 57 |
|                   | MIROS               | 117       | 504 | 0   | 632   | 262.3 | 71  | 1  |
|                   | SGR                 | 130       | 529 | 9   | 770   | 301.5 | 588 | 40 |

In order to answer the question which method generates the most robust schedules for three different scenarios (the real MTTF of the bottleneck equals 63, 66 and 75) following criteria $QR(y)$ and $SR(y)$ are considered. In Figure 2 and 3, the vertical axis represents values of $QR(y)$ or $SR(y)$ achieved by methods described in the horizontal axis. The order of researched methods is according to the order presented in Tables 3 - 5.

![Solution Robustness](image)

Figure 2. Solution robustness criterion of reactive schedules for JS problem (10x15) achieved using following methods 1) H-MOIA+MIDOS+LFJ,LPT, 2) H-MOIA+MIDOS+MIROS, 3) H-MOIA+MIDOS+SGR, 4) LFJ, LPT, 5) LFJ, LPT +MIROS, 6) LFJ, LPT +SGR, 7) ASM+LFJ,LPT, 8) ASM+MIROS, 9) ASM+SGR.
Taking into account criterion of $SR$ the best PSs are generated by the second set of methods e.g. H-MOIA+MIDOS+MIROS (Fig. 2). Taking into account criterion of $QR$, the best PSs are also generated by the second set of methods e.g. H-MOIA+MIDOS+MIROS (Fig. 3). The H-MOIA+MIDOS+MIROS generate the best schedules for three scenarios: the real $MTTF$ of the bottleneck equals 63, 66 and 75.

![Quality Robustness](image)

**Figure 3.** Quality robustness criterion of reactive schedules for JS problem (10×15) achieved using following methods 1) H-MOIA+MIDOS+LFJ,LPT, 2) H-MOIA+MIDOS+MIROS, 3) H-MOIA+MIDOS+SGR, 4) LFJ, LPT, 5) LFJ, LPT +MIROS, 6) LFJ, LPT +SGR, 7) ASM+LFJ,LPT, 8) ASM+MIROS, 9) ASM+SGR.

Analysing the schedules separately and taking into account the criterion of $SR$, the best RSs are generated using the MIROS under the constraint that the MIDOS is used for predictive scheduling. The regardless of the real failure occurrence scenario the LFJ, LPT achieves the worst quality schedules. The regardless of the method for predictive scheduling the MIROS achieves the best quality schedules for two scenarios, namely: real failure time equals 63 and real failure time equals 75.

Analysing the schedules separately and taking into account the criterion of $QR$, the best RSs are generated using the MIROS under the constraint that the MIDOS is used for predictive scheduling. The LFJ, LPT achieves the worst quality schedules for the scenario of the bottleneck failure equals 63. The regardless of the method for predictive scheduling the MIROS achieves the best quality schedules for all three scenarios of the bottleneck failure occurrence.

4. Conclusions
The LFJ/LPT is less effective than the H-MOIA comparing results achieved for the the job shop scheduling problem. This is because the LFJ/LPT is based on heuristics and no solution space is searched. In the ASM the uncertainty is handled by proposing the initial schedule with the best performance in the case of a disruption occurrence. The ASM provides better predictive schedules than the schedules achieved by the LFJ/LPT taking into account the criterion of solution robustness. The LFJ/LPT provides better predictive schedules than the schedules achieved by the ASM taking into account the criterion of quality robustness for two scenarios e.g. the real $MTTF = 66$ and 75. Comparing the performance of schedules generated by two algorithms, i.e. the H-MOIA and the ASM
applied for the job shop scheduling problem it is noticed that the H-MOIA performs better regardless of the method applied as the rescheduling procedure.

**Figure 4.** The first part of PS obtained at the second stage of the H-MOIA+MIDOS and the first part of RS obtained by the H-MOIA+MOIA+MIROS. The schedule is presented from start time = 0 till time = 62, \(j = \{1,2,...,15\} - \text{no. of job.}\)

**Figure 5.** PS obtained at the second stage of the H-MOIA+MIDOS from time = 63 till end time = 110, \((j = \{1,2,...,15\} - \text{no. of job, 99- predicted technical inspection at time 66.}\)

**Figure 6.** PS obtained at the third stage of H-MOIA+MIDOS+MIROS from time = 63 till end time = 110, \((j = \{1,2,...,15\} - \text{no. of job, 99- repair and predicted technical inspection at time 63.}\)

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