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Research on the Economic Impact of Environmental Tax under the Background of State-Owned Enterprise Shareholding Reform in an Open Economic System

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Abstract
In this paper, we construct a Stackelberg-Cournot tripartite game model and discuss the impact of tariff policy on the privatization of the state-owned enterprise, environmental tax, pollutant emission and social welfare in an open economic system. We find that with the increase in tariff, the proportion of privatization of the state-owned enterprise increases, environmental tax falls, and environmental pollution alleviates. The relationship between social welfare and tariff is inverted U-shaped. A closed trade environment is most conducive to environmental protection, but the environmental tax at this time is very detrimental to social welfare. When social welfare is optimal, environmental damage is not the smallest.

Keywords: Tariff, Privatization, Environmental tax, Nash Equilibrium

1. Introduction
The rapid economic development has led to aggravation of environmental pollution. China pays more and more attention to environmental protection. In terms of policies and regulations, China is gradually in line with international standards. In 2018, the "Environmental Protection Tax Law of the People's Republic of China" was promulgated to reduce pollutant emissions by imposing environmental tax on enterprises. Chinese capita PM2.5 emissions of provinces in 2012, 2014, 2016, and 2018 are shown in Figure 1. We can see that with the attention of the government, pollutant emissions have reduced.

Figure 1: Regional pollution of PM2.5.

In recent years, in order to promote further economic development, many state-owned enterprises try to increase the proportion of privatization. Because of the special social responsibilities of state-owned enterprises, social welfare is included in enterprises’ goals. Improving the environment is essential to improving social welfare. With the development of globalization, foreign companies enter the domestic market. Domestic companies may increase production to protect domestic market. However, pollutant emissions will also increase. Levying environmental taxes can help reduce pollutant emissions and protect the environment. Considering the above issues, we
aim to discuss the role of tariffs on the reform of state-owned enterprises and environmental taxes in an open economy.

In the early stage, Matsumura (1998) proposed that proper privatization of state-owned enterprises would help to optimize social welfare. On the basis of the mixed oligarchs’ theory, a group of scholars have carried out researches on the effects of privatization on pollutant emissions. These researches can be roughly divided into two categories. In the first type of studies, scholars assumed that companies produce similar products (Beladi & Chao 2006, Ohori 2006, Wang et al. 2009, Xie et al. 2012). In the second type of studies, scholars brought product differences into the hybrid oligopoly model (Wang & Wang 2009, Rupayan & Bibhas 2015). Both types of studies have verified partial privatization is conducive to reducing pollutant emissions and achieving optimal social welfare. Some scholars have brought foreign private enterprises into the hybrid oligarchic model, studying the relationship between the privatization of state-owned enterprises and tariff in an open economy. Pal & White (1998), Xiang et al. (2008), Ye & Deng (2010) all thought that partial privatization is conducive to the increase of social welfare. Some scholars have included the privatization of state-owned enterprises, environmental tax and tariff into the mixed oligarchic model for discussion. Xie et al. (2015) found that the time sequence of companies entering the market and the size of the foreign capital shares will have different effects on the optimal trade policy and environmental pollution. Ohori (2004) found that imposing tariff is a good choice when the environmental tax is optimal, and privatization can improve the environment when the environmental tax reaches the optimal level. Ohori (2006) attributed environmental pollution to consumption and believed that trade liberalization would not have any impact on the environment. Wang et al. (2007) also considered the environmental pollution caused by consumption. They found that the environmental tax decreases with the increase of tariff, and the environmental impact under the mixed oligarchs is not monotonous.

In addition to constructing game models, scholars used to make dynamic analysis. The application of dynamic theory and chaos theory to the analysis of game models is very common. Most scholars assumed that the market is an oligopoly market and established a dynamic oligopoly game model basing on bounded rationality, then dynamic analysis was carried out (Puu 1998). They used different equations to construct oligopoly game models such as price competition, output competition, and cost competition to analyze the stability of the Nash equilibrium point of the system.
The dynamic game model was mainly manifested in the form of duopoly (Dai et al. 2008, Agiza & Elsadany 2004, Bischi et al. 1999), triple oligarch (Elsadany 2012, Peng et al. 2011) and quadruple oligopoly (Ma & Sun 2018). But these game models always used in supply chain and stability analysis (Xie et al. 2020). Scholars have applied this type of method in different fields, just like advertising market (Mu et al. 2010), electricity market (Zhu et al. 2022, Zhang et al. 2009), household appliances market (Lou & Ma 2018), electric vehicle field (Bao et al. 2020, Ma & Xu 2020), solar photovoltaic (Xu & Ma 2021), agricultural field (Liu et al. 2020, Liu & Zhan 2019, Su et al. 2014, Su et al. 2018). There were even recycling system (Zhu et al. 2021, Ma & Ren 2018), and trade friction (Wang et al. 2019) in the research field. Some scholars have explored the improvement of the model, such as Ma and Ren (2016) and Yan and Sun (2017).

Most scholars only cared about the effects of a single policy, ignoring the interaction between multiple policies in an open environment. In this article, we comprehensively consider the tariff policy, environmental tax policy and the privatization reform policy of state-owned enterprises. By constructing a Stackelberg-Cournot three-stage game model, we investigate the impact of tariff policy on the reform of state-owned enterprise’s shareholding system and environmental tax policy. Using this model helps us understand the interaction between variables.

The structure of this paper is arranged as follows. In the second part, we construct a Stackelberg-Cournot tripartite game model basing on some assumptions. In the third part, we solve the Nash equilibrium of the game model and analyze the impact of tariff policies on the privatization of state-owned enterprise, environmental tax, pollutant emissions and social welfare. In the last part, we summarize the conclusions of the research and propose policy suggestions.

2. Modelling

Oligopoly competition is very common in some industries. In reality, various elements are intertwined with each other, and it is difficult to use a single model to describe every detail. Therefore, we construct a simplified model to describe oligopoly competition through some assumptions.

An industry is considered to be oligopoly competition, and the companies are the participants in the game. State-owned enterprises, private enterprises and foreign enterprises are representative enterprises in the market. We select these three types of
Assumption 1. There are three enterprises in the domestic market, namely enterprise 1, enterprise 2 and enterprise 3. Among them, enterprise 1 is a state-owned enterprise, enterprise 2 is a domestic private enterprise, and enterprise 3 is a foreign private enterprise. These three companies produce homogeneous products, $q_i(t)$ represents the output of the enterprise $i(i=1,2,3)$ at time $t$.

In the simple model, all the products produced by the company are purchased by consumers. Therefore the utility function of representative consumers and consumer surplus may be related to the output quantity, we refer to the research of Singh and Vives (1984), Ma et al. (2021), Fanti and Gori (2012) and make the following assumptions.

Assumption 2. We assume that all consumers are the same and the representative consumer’s utility function of $q_1,q_2,q_3$ is

$$U(q_1,q_2,q_3) = a_1q_1 + a_2q_2 + a_3q_3$$

$$- \frac{1}{2}(b_1q_1^2 + b_2q_2^2 + b_3q_3^2 + 2d_1q_1q_2 + 2d_2q_2q_3 + 2d_3q_3q_3),$$

where $a_i > 0, a_2 > 0, a_3 > 0, b_1 > 0, b_2 > 0, b_3 > 0$ , $d_i \in (-1,1)$ ($i=1,2,3$) represents the horizontal product differentiation. When $d_i \in (0,1)$, these products are imperfect substitutable. As $d_i$ approaches 0, the difference between products becomes larger. When $d_i =1$, the products of the three firms are homogeneous. When $d_i \in (-1,0)$, these products are complements. The value $d_i = -1$ reflects the existence of complete complementarity. Note that we abbreviate the output as $q_i(i=1,2,3)$.

Since three companies produce homogeneous products, we set $d_i = 1$. Then the utility function is

$$U(q_1,q_2,q_3) = a_1q_1 + a_2q_2 + a_3q_3$$

$$- \frac{1}{2}(b_1q_1^2 + b_2q_2^2 + b_3q_3^2 + 2q_1q_2 + 2q_2q_3 + 2q_3q_3).$$

Assumption 3. We assume that the inverse demand functions of products at time $t$ are

$$p_1(t) = a_1 - b_1q_1(t) - d_1q_2(t) - d_2q_3(t),$$

$$p_2(t) = a_2 - b_2q_2(t) - d_2q_1(t) - d_3q_3(t),$$

$$p_3(t) = a_3 - b_3q_3(t) - d_3q_1(t) - d_1q_2(t).$$

In this article, since we assume that the products of the three oligarchs are homogeneous, $d_1 = d_2 = d_3 = 1$. Hence we have the inverse demand functions
From (1) and (2), consumer surplus can be expressed as

$$CS(t) = U - \sum_{i=1}^{3} p_i q_i(t).$$

The cost of an enterprise usually includes fixed costs and variable costs, and variable costs can be approximated in a linear form. So we made the following assumption.

**Assumption 4.** The three companies have no technical differences in product production and their marginal costs are same. Then the form of the cost functions are same

$$C(q_i(t)) = f + c q_i(t).$$

Enterprises 1, 2, and 3 all produce pollutant emissions during the production process. Enterprises adopt emission reduction measures to reduce pollution, and certain costs are usually incurred in the process. In this article, we focus on the pollutant emissions of state-owned enterprises and domestic private enterprises, the pollutant emissions of foreign private enterprises have negligible impact on domestic social welfare. So the pollutant discharge and pollution cost of enterprise 3 are not considered here. Regarding corporates’ pollution emissions, we have made the following assumptions.

**Assumption 5.** The total emissions of enterprise \( i(i = 1,2) \) is \( e_i(t)(i = 1,2) \). For the convenience of analysis, we refer to the research of Rupayan and Bibhas (2015), assuming that every unit product produced by a company will emit \( \theta_i > 0 (i = 1,2) \) pollutant. Therefore, the company’s pollutant emission is \( \theta_i q_i(t) \). We set the company’s emission reduction to \( h_i(t) \), and the company’s actual pollutant emission is

$$e_i(t) = \theta_i q_i(t) - h_i(t)$$

for \( i = 1,2 \). The setting of corporate abatement costs draws on the method of Ulph (1996), and the corporate abatement cost is expressed as \( \frac{v_i(h_i(t))^2}{2} \), where abatement cost parameter \( v_i > 0 (i = 1,2) \). The environmental damage caused by pollutant
discharge is $D(e(t))$. Its relationship with pollutant emissions is

$$D(e(t)) = \frac{\varepsilon(e_1(t) + e_2(t))^2}{2},$$

where the pollution parameter $\varepsilon > 0$.

In order to protect the environment and control the discharge of pollutants, the government will levy environmental tax on enterprises. Hence we make the following assumption.

**Assumption 6.** Government imposes $x(t)$ as environmental tax on each unit of pollutants discharged.

In an open economic system, in order to protect domestic industries, the domestic government often imposes tariff on imported products. Therefore we make the following assumption.

**Assumption 7.** The government imposes a specific tariff on the foreign firm (enterprise 3) and the tariff rate is $r(t)$.

Using assumption 1-7, profit of enterprises 1, 2, and 3 can be written as the following forms.

\[
\pi_1(t) = p_1(t)q_1(t) - (f + cq_1(t)) - x(t)e_1(t) - \frac{v_1(h_1(t))^2}{2},
\]

\[
\pi_2(t) = p_2(t)q_2(t) - (f + cq_2(t)) - x(t)e_2(t) - \frac{v_2(h_2(t))^2}{2},
\]

\[
\pi_3(t) = (p_3(t) - r(t))q_3(t) - (f + cq_3(t)),
\]

where $\pi_i(t)$ represents the profit of enterprise $i (i = 1, 2, 3)$.

Private enterprises usually aim at maximizing profits, while state-owned enterprises have a different objective function from private enterprises due to their special social responsibilities. Hence we make the following assumption.

**Assumption 8.** The objective function of the state-owned enterprise is

$$O_i(t) = k(t)\pi_i(t) + (1 - k(t))W_p(t).$$

The objective function of a state-owned enterprise is composed of the profit of the state-owned enterprise and $W_p(t)$. We continue to make the following assumptions.

**Assumption 9.** Suppose that the state-owned enterprise does not care about environmental damage and the government’s tax revenue, and then $W_p(t) = \pi_1(t) + \pi_2(t) + CS(t)$. The parameter $k(t) (0 \leq k(t) \leq 1)$ represents the
privatization ratio of state-owned enterprise. If \( k(t) = 0 \), enterprise 1 is a completely state-owned enterprise, and if \( k(t) = 1 \), enterprise 1 is completely privatized.

However, in order to facilitate the distinction, no matter how privatized enterprise 1 is, enterprise 1 will still be referred to as a state-owned enterprise later.

The government takes the maximization of social welfare as the objective function. So we make the following assumptions.

**Assumption 10.** The country’s social welfare expression is

\[
W(t) = \pi_1(t) + \pi_2(t) + CS(t) + x(t)(e_1(t) + e_2(t)) + r(t)q_3(t) - \frac{\varepsilon(e_1(t) + e_2(t))^2}{2}.
\]

(5)

The game in this article is divided into three stages in order ¹: In the first stage, according to the optimization of social welfare, state-owned enterprises determine the privatization ratio. According to assumption 1-10, the privatization reform of state-owned enterprise in the first stage is

\[
\max_{k \in [0,1]} W(t) = \left[ p_1(t)q_1(t) - \left( f + cq_1(t) \right) - x(t)e_1(t) - \frac{v_1(h_1(t))^2}{2} + p_2(t)q_2(t) - \left( f + cq_2(t) \right) - x(t)e_2(t) - \frac{v_2(h_2(t))^2}{2} + a_1q_1(t) + a_2q_2(t) + a_3q_3(t) - \frac{1}{2}(b_1(q_1(t)))^2 + b_2(q_2(t))^2 + b_3(q_3(t))^2 + 2q_1(t)q_2(t) + 2q_1(t)q_3(t) + 2q_2(t)q_3(t) - (p_1(t)q_1(t) + p_2(t)q_2(t) + p_3(t)q_3(t)) + x(t)(e_1(t) + e_2(t)) + r(t)q_3(t) - \frac{\varepsilon(e_1(t) + e_2(t))^2}{2} \right].
\]

(\(U_1\))

In the second stage, the government formulates an environmental tax to maximize social welfare. According to assumption 1-10, in the second stage, the government formulated an environmental tax plan, which is

¹In accordance with the timing of the mixed reform of state-owned enterprises and the environmental tax policy, we regard the reform of the state-owned enterprise shareholding system as the first stage of the game, and the formulation of the environmental tax as the second stage. If the shareholding reform of state-owned enterprises and the formulation of environmental taxes are regarded as occurring at the same time, a two-stage game is constructed, and the final result is exactly the same as the three-stage game.
\[
\max_{t \in (0, +\infty)} W(t) = \left[ p_1(t)q_1(t) - \left( f + cq_1(t) \right) - x(t)e_1(t) - \frac{v_1(h_1(t))^2}{2} \right. \\
+ p_2(t)q_2(t) - \left( f + cq_2(t) \right) - x(t)e_2(t) - \frac{v_2(h_2(t))^2}{2} \\
+ a_1q_1(t) + a_2q_2(t) + a_3q_3(t) - \frac{1}{2}(b_1(q_1(t))^2 + b_2(q_2(t))^2) \\
+ b_3(q_3(t))^2 + 2q_1(t)q_2(t) + 2q_1(t)q_3(t) + 2q_2(t)q_3(t) \\
- (p_1(t)q_1(t) + p_2(t)q_2(t) + p_3(t)q_3(t))] + x(t)(e_1(t) + e_2(t)) \\
+ r(t)q_3(t) - \frac{\varepsilon(e_1(t) + e_2(t))^2}{2}.
\]

In the third stage, under the constraints of the privatization ratio and environmental tax, enterprise 1, enterprise 2, enterprise 3 determine the outputs and pollutant emission reduction, and then achieve the maximum goal of the enterprises. According to assumption 1-10 and equations (3) and (4), the plan of the business objectives of the third stage is

\[
\begin{align*}
\max_{q_1, q_2 \in (0, +\infty)} O_1(t) &= k(t)[p_1(t)q_1(t) - \left( f + cq_1(t) \right) - x(t)e_1(t)] \\
&- \frac{v_1(h_1(t))^2}{2} + (1 - k(t))[p_1(t)q_1(t) - \left( f + cq_1(t) \right)] \\
&- x(t)e_1(t) - \frac{v_1(h_1(t))^2}{2} + p_2(t)q_2(t) - \left( f + cq_2(t) \right) \\
&- x(t)e_2(t) - \frac{v_2(h_2(t))^2}{2} + a_1q_1(t) + a_2q_2(t) + a_3q_3(t) \\
&- \frac{1}{2}(b_1(q_1(t))^2 + b_2(q_2(t))^2 + b_3(q_3(t))^2 + 2q_1(t)q_2(t) \\
&+ 2q_1(t)q_3(t) + 2q_2(t)q_3(t)) - (p_1(t)q_1(t) + p_2(t)q_2(t) \\
&+ p_3(t)q_3(t))], \\
\max_{q_2, q_3 \in (0, +\infty)} \pi_2(t) &= p_2(t)q_2(t) - \left( f + cq_2(t) \right) - x(t)e_2(t) - \frac{v_2(h_2(t))^2}{2}, \\
\max_{q_3 \in (0, +\infty)} \pi_3(t) &= (p_3(t) - r(t))q_3(t) - \left( f + cq_3(t) \right).
\end{align*}
\]

Note that

\[
\begin{align*}
p_1(t) &= a_1 - b_1q_1(t) - q_2(t) - q_3(t), \\
p_2(t) &= a_2 - b_2q_2(t) - q_1(t) - q_3(t), \\
p_3(t) &= a_3 - b_3q_3(t) - q_1(t) - q_2(t), \\
e_1(t) &= \theta q_1(t) - h_1(t), \\
e_2(t) &= \theta q_2(t) - h_2(t).
\end{align*}
\]
3. Nash Equilibrium Analysis

According to the general method of solving this kind of game problem, we adopt the reverse induction method to solve it.

Since the main purpose of this article is to examine the impact of tariff on the degree of privatization of state-owned enterprises, environmental pollution and social welfare, it is assumed that tariff $r(t)$ is given exogenously. In order to simplify the analysis below, we might as well set $\theta_i = 1, v_i = 1, e = 1, a_i = a, b_i = 1$. In order to visualize the relationship between variables with images, we will assume $f = 1, c = 0.01, a = 1, b = 1$ when drawing figures. First, we analyze the third stage. At this stage, each enterprise determines the emission reduction and output according to the objective function.

**Proposition 1.** Under the conditions of mixed tri-oligarchic competition, the privatization of state-owned enterprises increases with the increase in tariff.

**Proof.** Since $e_i(t)$ are functions of $h_i(t)$ and $\theta_i$, while $p_i(t)$ are functions of $q_1, q_2, q_3$ for $i = 1, 2, 3$, according to the first-order condition of the objective function of state-owned enterprise, we have

\[
\frac{\partial O_i(t)}{\partial q_i(t)} = a - c - (1 + k(t))q_i(t) - q_2(t) - k(t)q_3(t) - x(t) = 0, \quad (6)
\]

\[
\frac{\partial O_i(t)}{\partial h_i(t)} = x(t) - h_i(t) = 0.
\]

Since $\frac{\partial^2 O_i(t)}{\partial (q_i(t))^2} < 0$, $\frac{\partial^2 O_i(t)}{\partial (h_i(t))^2} < 0$, the optimal condition of the objective function of enterprise 1 is satisfied. From the first-order condition of the objective function of the domestic private enterprise, we have

\[
\frac{\partial \pi_2(t)}{\partial q_2(t)} = a - c - q_1(t) - 2q_2(t) - q_3(t) - x(t) = 0, \quad (7)
\]

\[
\frac{\partial \pi_2(t)}{\partial h_2(t)} = x(t) - h_2(t) = 0.
\]

Similarly, $\frac{\partial^2 \pi_2(t)}{\partial (q_2(t))^2} < 0$, $\frac{\partial^2 \pi_2(t)}{\partial (h_2(t))^2} < 0$, enterprise 2 also has the maximum value of the objective function. From the first-order condition of the objective function of the foreign enterprise, we have

\[
\frac{\partial \pi_3(t)}{\partial q_3(t)} = a - c - q_1(t) - q_2(t) - 2q_3(t) - r(t) = 0, \quad (8)
\]
Using (6)(7)(8), the output and emission reduction of enterprise 1, enterprise 2, and enterprise 3 at this stage are obtained as follows

\[ q_1(t) = \frac{(a-c)(2-k(t)) + r(t)(2k(t)-1) - x(t)(1+k(t))}{2(1+k(t))}, \]
\[ q_2(t) = \frac{r(t) + (a-c)k(t) - (1+k)x(t)}{2(1+k(t))}, \]
\[ q_3(t) = \frac{(a-c)k(t) - (1+2k(t))r(t) + (1+k(t))x(t)}{2(1+k(t))}, \]
\[ h_1(t) = x(t), h_2(t) = x(t). \]

Next, let us consider the second stage of the game. The government sets the environmental tax rate according to the principle of maximizing social welfare.

According to (5) and (9), we get

\[ W(t) = \frac{\alpha_1 + \alpha_2 + \alpha_3 + \alpha_4}{8(1+k(t))^2}, \] (10)

where
\[ \alpha_1 = c^2k(t)(8+k(t)) - 16f(1+k(t))^2 + a^2k(t)(8+k(t)), \]
\[ \alpha_2 = -(3+8k(t) + 12k(t)^2)(r(t))^2 - 2c[-3k(t)(r(t) - 3x(t)) + (k(t))^2(4r(t) - 3x(t)) + 12x(t)], \]
\[ \alpha_3 = x(t)(1+k(t))[2 + 32k(t)r(t) - 47(1+k(t))x(t)], \]
\[ \alpha_4 = -2ak(t)[c(8+k(t)) + (3-4k(t))r(t)] - 6ax(t)(k(t)-4)(k(t)+1). \]

From the first-order condition of (10), the environmental tax formulated by the government is

\[ x(t) = \frac{-3(a-c)(k(t)-4) + r(t) + 16k(t)r(t)}{47(1+k(t))}. \] (11)

In the first stage of the game, the government determines the degree of privatization of state-owned enterprises from the perspective of maximizing social welfare. Using (11) and (10), we get

\[ W(t) = \frac{\beta_1 + \beta_2 + \beta_3}{94(1+k(t))^2}, \] (12)

where
\[ \beta_1 = 188f(1+k(t))^2 - 2(a^2 + c^2)(18 + 38k(t) + 7(k(t))^2), \]
\[ \beta_2 = r(t)[c(6 + 24k(t) + 70(k(t))^2) + r(t)(35 + 86k(t) + 77k(t))^2)], \]
\[ \beta_3 = 4ac[18 + k(t)(38 + 7k(t))] - 2ar(t)[3 + k(t)(12 + 35k(t))]. \]

From the first-order condition of (12), the optimal privatization ratio expression is
Substituting (13) into (9), we obtain the output of enterprise 1, enterprise 2 and enterprise 3 as follows

\[
q_1^*(t) = \frac{10(a-c)-12r(t)}{13},
\]
\[
q_2^*(t) = \frac{c-a+9r(t)}{13},
\]
\[
q_3^*(t) = \frac{2(a-c)-5r(t)}{13}.
\]

Equation (14) shows equilibrium outputs of three firms.

Furthermore, we obtain the expression of profit and consumer surplus as (15)

\[
\pi_1(t) = \frac{-11(a-c)^2 + 198r(t)(a-c) - 338f - 215(r(t))^2}{338},
\]
\[
\pi_2(t) = \frac{11(a-c)^2 - 42r(t)(a-c) - 338f + 163(r(t))^2}{338},
\]
\[
\pi_3(t) = \frac{-169f + (2c - 2a + 5r(t))^2}{169},
\]
\[
CS(t) = \frac{(11c - 11a + 8r(t))^2}{338}.
\]

where

\[
O_{11}(t) = 33a^2(40a - 120c - 79r(t)),
\]
\[
O_{12}(t) = a(3960c^2 - 7774f + 5214cr(t) + 1129(r(t))^2),
\]
\[
O_{13}(t) = 33c^2(79r - 40c) + 338f(23c + 38r),
\]
\[
O_{14}(t) = -(r(t))^2(1129c + 1112r(t)).
\]

Since there is a foreign enterprise included in the mixed oligarchic model, the export volume \(q_3(t)\) of enterprise 3 to the country should be greater than or equal to 0.

According (14), we can know that \(\frac{2(a-c)-5r(t)}{13} \geq 0\), because \(r(t) > 0\), then

\(a - c \geq 0\). Taking first-order derivative of (13) with respect to \(r(t)\), we can get that

\[
\frac{\partial k(t)}{\partial r(t)} = \frac{65(a-c)}{(12c-12a+17r(t))^2} \geq 0.
\]
In reality, the number of imported products cannot be less than 0, so
\[
q_3(t) = \frac{2(a-c) - 5r(t)}{13} \geq 0.
\]
We have assumed \( f = 1, c = 0.01, a = 1, b = 1 \), then
\[
0 \leq r \leq 0.396.
\]
Once \( r \) exceeds the range, other variables stop changing. So in Figure 2, there is a constant branch. The relationship between \( r \) and \( k \) is shown in Figure 2, and the relationship between \( r, k \) and \( O_i \) is shown in Figure 3.

**Proposition 2.** Under the conditions of mixed three-oligarchic competition, the government's optimal environmental tax will increase with the increase of tariff.

**Proof.** Substituting (13) into (11), the reaction function of environmental tax rate \( x(t) \) with tariff \( r(t) \) is obtained.
Taking first-order derivative of (16) with respect to \( r(t) \), we can get that \[ \frac{\partial x(t)}{\partial r(t)} = \frac{1}{13} < 0. \] This completes the proof.

We have known that \( 0 \leq r \leq 0.396 \). So in Figure 4, there still exists a constant branch. The relationship between \( r \) and \( x \) is shown in Figure 4, and the relationship between \( r \), \( k \) and \( x \) is shown in Figure 5.

**Proposition 3.** Under the conditions of mixed tri-oligopoly competition, society's pollutant emissions decrease with the increase of tariff.

**Proof.** According to (13) and (16), we have the total pollutant emissions of the society \( e(t) = e_1(t) + e_2(t) = \frac{3(a-c) - r(t)}{13} \). Doing the first-order derivation on this basis, we get \[ \frac{\partial e(t)}{\partial r(t)} = \frac{1}{13} < 0. \] This completes the proof.

Because \( 0 \leq r \leq 0.396 \), there is also a constant branch in Figure 6. The relationship between \( r \) and \( e \) is shown in Figure 6, and the relationship between \( r \), \( x \) and \( e \) are shown in Figure 7.

**Proposition 4.** Under the conditions of mixed tri-oligarchic competition, the country's social welfare and tariff are in an inverted U-shaped relationship.
Proof. Substituting (13) into (12), we can further obtain the social welfare of the country as

\[
W(t) = \frac{10(a-c)^2 + 2r(a-c) - 52f - 9(r(t))^2}{26}.
\] (17)

Calculate the first derivative of (17) with respect to \( r(t) \), we get

\[
\frac{dW(t)}{dr(t)} = \frac{a-c-9r(t)}{13}.
\]

When \( a-c \geq 9r(t) \), \( \frac{dW(t)}{dr(t)} \geq 0 \); when \( a-c < 9r(t) \), \( \frac{dW(t)}{dr(t)} < 0 \). This completes the proof.
There is a constant branch in Figure 8 because \(0 \leq r \leq 0.396\). The relationship between \(r\) and \(W\) is shown in Figure 8, and the relationship between \(r, e\) and \(W\) are shown in Figure 9, the relationship between \(r, k\) and \(W\) is shown in Figure 10, the relationship between \(r, x\) and \(W\) is shown in Figure 11.

4. Conclusion and Policy Implications

In this article, by constructing a three-stage game model of mixed oligarchy, we analyze the impact of import tariff on the state-owned enterprise shareholding reform, environmental pollution and social welfare in an open economic environment. After research in this article, we get some decisions.

(1) Under the conditions of mixed three-oligarchic competition, the privatization of the state-owned enterprise increases with the increase in tariff. With the increase in tariff, the export of foreign company is suppressed, and domestic consumer surplus is lost. Although tariff increases government revenue, the loss of consumer surplus may worsen social welfare. In this case, the government prefers to improve the profitability of the state-owned enterprise by increasing its proportion of privatization, so as to maintain the stability of social welfare.

(2) The government's optimal environmental tax increases with the increase of tariff. The export of foreign company is suppressed because of higher tax. In order to meet market demand, the overall production volume of domestic enterprises has increased, and there are more and more pollutant emissions. For the purpose of environmental protection, the government will increase environmental tax.

(3) Society's pollutant emissions decrease with the increase of tariff. The increase in tariff causes the government to levy more environmental tax. The environmental tax helps to reduce domestic companies’ pollutant emissions, thereby protecting the environment.

(4) The relationship between social welfare and tariff is in an inverted U shape. From the view of real economics, although the highest tariff is conducive to protecting the environment and increasing the level of privatization, it is not conducive to social welfare. When the tariff is lower than the prohibitive tariff, the privatization of the state-owned enterprise becomes higher with the increase of tariff, outputs are reduced, pollutant emissions are reduced, and the environment is improved. So the environmental tax and pollutant emissions will decrease with a relatively small growth of the tariff. However, too high tariff will hinder import and
the competitiveness of the domestic market will decline, which raises the price of
product, reduces the output and profit of the state-owned enterprise, adversely affects
consumers and leads to consumer surplus losses. Since social welfare includes the
interests of enterprises, consumers, the government and environment, although the
government's tariff revenue increases and the environment is improved, the losses
from consumers and enterprises are larger, which leads to a decrease in social welfare.

Based on the research and analysis of this article, we have the following policy
suggestions. The environmental tax rate reaches the highest without trade, and the
total pollutant emissions of domestic enterprises are the lowest at this time. When
social welfare is maximized, the environmental tax corresponding to the tariff is not
the level that is most conducive to environmental protection. Too low tariff or too
high tariff does not benefit for maximizing social welfare. This shows that there is a
conflict in the formulation of environmental policy and tariff policy. This requires
improvement to environmental tax. Changes of tariff cause changes in environmental
tax and privatization. Therefore, when the government sets tariff, it must take into
account the impact of tariff on other factors in order to help optimize social welfare.
Considering interest of only one aspect is not beneficial to social welfare. The
government should improve the environmental taxation system, so that other related
taxes and environmental tax can cooperate with each other, so as to more effectively
play the role of environmental protection.

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