Flavor “Conservation” and Hierarchy in TeV-scale Supersymmetric Standard Model

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(February 11, 1999)

Abstract

Recently a TeV-scale Supersymmetric Standard Model (TSSM) was proposed in which the gauge coupling unification is as precise (at one loop) as in the MSSM, and occurs in the TeV range. Proton stability in the TSSM is due to an anomaly free $Z_3 \otimes Z_3$ discrete gauge symmetry, which is also essential for successfully generating neutrino masses in the desirable range. In this paper we show that the TSSM admits anomaly free non-Abelian discrete flavor gauge symmetries (based on a left-right product tetrahedral group) which together with a “vector-like” Abelian (discrete) flavor gauge symmetry suppresses dangerous higher dimensional operators corresponding to flavor changing neutral currents (FCNCs) to an acceptable level. Discrete flavor gauge symmetries are more advantageous compared with continuous flavor gauge symmetries as the latter must be broken, which generically results in unacceptably large gauge mediated flavor violation. In contrast, in the case of discrete flavor gauge symmetries the only possibly dangerous sources of flavor violation either come from the corresponding “bulk” flavon (that is, flavor symmetry breaking Higgs) exchanges, or are induced by flavon VEVs. These sources of flavor violation, however, are adequately suppressed by the above flavor gauge symmetries for the string scale $\sim 10^{100}$ TeV.

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I. INTRODUCTION

Recently it has become clear that the discovery of D-branes [1] may have profound phenomenological implications. In particular, the Standard Model gauge fields (as well as the corresponding charged matter) may reside inside of \( p \leq 9 \) spatial dimensional \( p \)-branes (or a set of overlapping branes), while gravity lives in a larger (10 or 11) dimensional bulk of space-time. This “Brane World” scenario appears to be flexible enough so that satisfying various requirements such as gauge and gravitational coupling unification, dilaton stabilization and weakness of the Standard Model gauge couplings seems to be possible [11] within this framework (provided that the Standard Model fields live on branes with \( 3 < p < 9 \)). This suggests that the brane world scenario might be a coherent picture for describing our universe [11].

In string theory, which is the only known theory that consistently incorporates quantum gravity, the gauge and gravitational couplings are expected to unify (up to an order one factor due to various thresholds [14,15]) at the string scale \( M_s = 1/\sqrt{\alpha'} \). In the brane world scenario the string scale is \textit{a priori} undetermined, and can be anywhere between the electroweak scale \( M_{ew} \) and the Planck scale \( M_P = 1/\sqrt{G_N} \) (where \( G_N \) is the Newton’s constant). Thus, if we assume that the bulk is ten dimensional, then the four dimensional gauge and gravitational couplings scale as \( \alpha \sim g_s/V_{p-3}M_{p}^{-3} \) respectively \( G_N \sim g_s^2/V_{p-3}V_{9-p}M_p^8 \), where \( g_s \) is the string coupling, and \( V_{p-3} \) and \( V_{9-p} \) are the compactification volumes inside and transverse to the \( p \)-branes, respectively. For \( 3 < p < 9 \) there are two \textit{a priori} independent volume factors, and, for the fixed gauge coupling \( \alpha \) (at the unification, that is, string scale) and four dimensional Planck scale \( M_P \), the string scale is not determined. Based on this fact, in [2] it was proposed that the gauge and gravitational coupling unification problem can be ameliorated in this context by lowering the string scale \( M_s \) down to the GUT scale \( M_{GUT} \approx 2 \times 10^{16} \) GeV [18]. In [3] it was noticed that \( M_s \) can be lowered all the way down to TeV.

More recently it was proposed in [4] that \( M_s \) as well as the fundamental (10 or 11 dimensional) Planck scale can be around TeV. The observed weakness of the four dimensional

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1For recent developments, see, \textit{e.g.}, [2–11].

2The brane world picture in the effective field theory context was discussed in [12,13].

3For illustrative purposes here we are using the corresponding tree-level relations in Type I (or Type I’) theory.

4For a review of the gauge and gravitational coupling unification problem in the perturbative heterotic string context, see, \textit{e.g.}, [11], and references therein. In the Type I context the discussions on this issue can be found in [17,11].

5By the GUT scale here we mean the usual scale of gauge coupling unification in the MSSM obtained by extrapolating the LEP data in the assumption of the standard “desert” scenario.

6Note that the string scale \( M_s \) cannot be too much lower than the fundamental Planck scale or
gravitational coupling then requires the presence of at least two large \((\gg 1/M_s)\) compact directions transverse to the \(p\)-branes on which the Standard Model fields are localized. A general discussion of possible brane world embeddings of such a scenario was given in \([7,8,11]\).

In \([10]\) various non-trivial phenomenological issues were discussed in the context of the TeV-scale brane world scenario, and it was argued that this possibility does not appear to be automatically ruled out\(^7\).

In such a scenario, however, as well as in any scenario with \(M_s \ll M_{\text{GUT}}\), the gauge coupling unification at \(M_s\) would have to arise via a mechanism rather different from the usual MSSM unification which occurs with a remarkable precision \([13]\). In thebrane world picture there appears to exist such a mechanism \([1]\) for lowering the unification scale. Thus, let the “size” \(R\) of the compact dimensions inside of the \(p\)-brane (where \(p > 3\)) be somewhat large compared with \(1/M_s\). Then the evolution of the gauge couplings above the Kaluza-Klein (KK) threshold \(1/R\) is no longer logarithmic but power-like \([30]\). This observation was used in \([5]\) to argue that the gauge coupling unification might occur at a scale (which in the brane world context would be identified with the string scale) much lower than \(M_{\text{GUT}}\). For successfully implementing this mechanism, however, it is also necessary to find a concrete extension of the MSSM such that the unification prediction is just as precise as in the MSSM (at least at one loop). In fact, one could also require that such an extension explain why couplings unify in the MSSM at all, that is, why the unification in the MSSM is not just an “accident” (assuming that the TeV-scale brane world scenario has the pretense of replacing the old framework).

In \([22]\) a TeV-scale Supersymmetric Standard Model (TSSM) was proposed in which the gauge coupling unification indeed occurs via such a higher dimensional mechanism. Moreover, the unification in the TSSM is as precise (at one loop) as in the MSSM, and occurs in the TeV range\(^8\). In particular, the key ingredient of the TSSM is the presence of new (compared with the MSSM) light states neutral under \(SU(3)_c \otimes SU(2)_w\) but charged under \(U(1)_Y\) whose mass scale is around that of the electroweak Higgs doublets. It is the heavy Kaluza-Klein tower corresponding to these new states which makes it possible to satisfy the requirement that the unification in the TSSM be as precise (at one loop) as in the MSSM. In fact, as was pointed out in \([22]\), after a rather systematic search the TSSM was the only (simple) solution found for this constraint. The TSSM also explains why the unification in the MSSM is not an accident - if the TSSM is indeed (a part of) the correct description of nature above the electroweak scale, then the gauge coupling unification in the

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else the string coupling \(g_s\) as well as all the gauge couplings would come out too small contradicting the experimental data.

\(^7\)For other recent works on TeV-scale string/gravity scenarios, see, e.g., \([3,19–27]\). For other scenarios with lowered string scale and related works, see, e.g., \([23]\). TeV-scale compactifications were studied in \([29]\) in the context of supersymmetry breaking.

\(^8\)By the TeV range we do not necessarily mean that \(M_s \sim 1\text{ TeV}\). In fact, as was argued in \([22]\), the gauge coupling unification constraints seem to imply that \(M_s\) cannot really be lower than \(10–100\) TeV.
MSSM is explained by the present lack of experimental data which leads to the standard “desert” assumption.

One of the most obvious worries with the TeV-scale brane world scenario in general is the proton stability problem - higher dimensional baryon and lepton number violating operators are generically suppressed only by powers of $1/M_\text{s}$ with $M_\text{s}$ in the TeV range, which is inadequate to ensure the observed proton longevity. In [24] it was shown that introduction of the new states responsible for the gauge coupling unification in the TSSM also allows to gauge anomaly free discrete symmetries which suppress dangerous higher dimensional operators and stabilize proton. In particular, in [24] an anomaly free $\mathbb{Z}_3 \otimes \mathbb{Z}_3$ discrete gauge symmetry which makes proton completely stable was explicitly constructed. In [24] it was also argued that this discrete gauge symmetry is essential for successfully generating neutrino masses in the desirable range via a higher dimensional mechanism recently proposed in [21]. In particular, in [24] it was pointed out that certain dimension 5 lepton number violating operators must be absent or else unacceptably large Majorana neutrino masses would be generated upon the electroweak symmetry breaking. More concretely, from this viewpoint unsuppressed dimension 5 operators of the form $L L H \pm H_\pm$ would be disastrous, where $L$ is the $SU(2)_\text{w}$ doublet containing a left-handed neutrino and the corresponding charged lepton (we are suppressing the flavor indices), and $H_\pm$ is the electroweak Higgs doublet with the hypercharge $\pm 1$. The $\mathbb{Z}_3 \otimes \mathbb{Z}_3$ discrete gauge symmetry mentioned above stabilizes proton and forbids these dimension 5 operators in one shot.

The fact that the gauge coupling unification, proton stability and neutrino mass problems can be solved within the TSSM suggests that it is reasonable to take the TSSM as a starting point for addressing many other open questions that the TeV-scale brane world scenario faces. In this paper we will focus on another obvious worry with the TeV-scale brane world scenario: flavor changing neutral currents (FCNCs). Here we will take the approach where we will try to find possible solutions to this problem in ways consistent with other features described above such as proton stability and neutrino masses. This appears to imply that the number of viable possibilities is rather limited which allows to explore them in a systematic fashion.

Certain generic aspects of the FCNC problem in the context of the TeV-scale brane world scenario were discussed in [19,20]. Thus, in [19] global flavor symmetries were considered in the context of generating the desired flavor hierarchy. It was pointed out in [19] that hierarchical breaking of such flavor symmetries on “distant” branes could account for the observed fermion mass hierarchy in the Standard Model. Breaking global flavor symmetries on “distant” branes would be compatible with the current experimental bounds on FCNCs as the only possibly dangerous sources of flavor violation induced by such breaking come from the corresponding “bulk” flavon (that is, flavor symmetry breaking Higgs) exchanges. The latter are adequately suppressed by the volume of the corresponding large dimensions (required to be present in the TeV-scale brane world scenario) transverse to the $p$-branes on which the Standard Model fields are localized. However, global continuous symmetries or non-gauge discrete symmetries may not be completely adequate in this context. First,

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\footnote{Supersymmetry breaking in the TSSM was recently discussed in [23] in the context of the Scherk-Schwarz mechanism [24].}
generically quantum gravity effects (wormholes, etc.) are expected to violate such global symmetries and induce effective flavor violating higher dimensional operators which would only be suppressed by the corresponding powers of $1/M_s$ [32]. Second, it is believed that there are no global symmetries in string theory [33]. This then implies that we should consider either continuous or discrete flavor gauge symmetries. Discrete gauge symmetries are believed to be stable under quantum gravity effects [34] (also see, e.g., [35]), and they do arise in string theory. At first it might appear that continuous flavor gauge symmetries would suppress FCNCs much more “efficiently” than discrete flavor gauge symmetries. However, it was pointed out in [20] that gauging continuous flavor symmetries immediately runs into the following problem. First, it was shown in [20] that Abelian continuous flavor gauge symmetries cannot do the job of adequately suppressing all dangerous FCNCs (for instance, those relevant for the Kaon system). This implies that we would have to consider non-Abelian continuous flavor gauge symmetries. However, these gauge symmetries must be broken at some scale (below $M_s$) or else the corresponding massless gauge bosons would give rise to experimentally excluded long range forces. Also, the observed fermion hierarchy in the Standard Model is incompatible with such unbroken flavor symmetries. In [20] it was shown that even if these flavor symmetries are “bulk" gauge symmetries, the gauge mediated flavor violation triggered by the flavor symmetry breaking generically is not so small after all, and can lead to unacceptably large FCNCs. More precisely, it was shown in [20] that the flavor mediated FCNCs are not at all suppressed in the case of 2 large dimensions transverse to the $p$-branes, and an adequate suppression can only be achieved if the number of large transverse dimensions is $\geq 4$ (provided that $M_s$ is not lower than $10 - 100$ TeV). A priori this fact may not look like a big deal, albeit it would rule out (provided that there are no solutions to the problem other than gauging continuous flavor symmetries in the bulk) the cases with less than 4 large transverse dimensions. Here it is reasonable to ask whether one can find a much more “efficient” way of solving the FCNC problem in the TeV-scale brane world context. This becomes especially desirable taking into account that, as was pointed out in [24], in the brane world (that is, string theory) framework it might be necessary to have 2 and only 2 large transverse directions (at least in the TSSM context) if one would like the higher dimensional mechanism of [21] for generating the correct neutrino masses to work (and, at the same time, be compatible with $\mathcal{N} = 1$ supersymmetry which seems to be essential for the gauge coupling unification [22]). Moreover, as was pointed out in [24], the discussion of dilaton stabilization in [11], which takes into account various observations of [36] as well as the explicit mechanisms of dilaton stabilization [37], suggests that to achieve

10 These two observations may not be completely unrelated.

11 Note that such flavor gauge symmetries would actually have to be “bulk" gauge symmetries. Thus, if the corresponding gauge bosons are localized on the same $p$-branes as the Standard Model fields, the tree-level exchanges of the horizontal gauge boson(s) would ultimately reintroduce the exact same types of FCNCs (with unacceptable strengths) as those which the flavor gauge symmetry was supposed to suppress in the first place.
the latter we do not seem to be allowed to have more than 2 large transverse dimensions in the TeV-scale brane world context. The recent discussion in [23] also suggests that radius stabilization at large values in the brane world framework seems to favor having 2 large transverse directions. These considerations all indicate that the case of 2 large transverse directions could be the most interesting one, so letting it be ruled out so easily may not be desirable.

Motivated by the above considerations, in this paper we consider anomaly free non-Abelian discrete flavor gauge symmetries for suppressing FCNCs in the TSSM. The advantage of gauging discrete rather than continuous flavor symmetries is that, unlike in the latter case, upon breaking the former no gauge mediated flavor violation occurs whatsoever - there are no horizontal gauge bosons to start with. Thus, just as in the case of global flavor symmetries, the only possibly dangerous sources of flavor violation come from the corresponding “bulk” flavon exchanges which are adequately suppressed. On the other hand, as we have already pointed out, considering gauge rather than global symmetries appears to be necessary in this context since we are dealing with a theory where quantum gravity becomes strongly coupled at energies around $M_s$ which sets the “cut-off” for the induced effective flavor violating higher dimensional operators. Gauging non-Abelian discrete symmetries in the TSSM is not completely trivial - we must ensure that these gauge symmetries are anomaly free. This puts tight constraints on possible flavor symmetries we can gauge, especially that they must be compatible with other anomaly free gauge symmetries (such as the $Z_3 \otimes Z_3$ discrete gauge symmetry responsible for proton longevity in the TSSM). In fact, the number of possibilities we find is rather limited, and the conclusive solution to the FCNC problem we present in this paper is based on a left-right product (non-Abelian) tetrahedral group $T_L \otimes T_R$ (which can be viewed as a discrete subgroup of the $SU(2)_L \otimes SU(2)_R$ flavor symmetry group) accompanied by a “vector-like” $U(1)_V$ flavor gauge symmetry (or its appropriate discrete subgroup). The non-Abelian part of the flavor group together with the $U(1)_V$ subgroup adequately suppresses FCNCs (for any number $\geq 2$ of large transverse dimensions, and the string scale $M_s \sim 10 - 100 \text{ TeV}$) provided that the flavor symmetry breaking Higgses (flavons) are “bulk” fields.

The rest of this paper is organized as follows. In section II we briefly review the TSSM proposed in [22]. We mainly focus on the light spectrum as the heavy KK modes are not going to be relevant for the subsequent discussions. We also briefly review the discrete gauge symmetries proposed in [24] which ensure proton stability and successful generation of small neutrino masses. These discrete gauge symmetries are relevant in the following as the flavor symmetries we are going to gauge must be anomaly free without spoiling the anomaly freedom condition for the former discrete gauge symmetries. In section III we briefly review the discussion in [20], which we tailor to our purposes in this paper. In section IV we explicitly construct an anomaly free discrete flavor gauge symmetry which, as we show, adequately suppresses FCNCs in the TSSM. There we also discuss various phenomenological implications (including possible collider signatures) of these symmetries together with other symmetries in the TSSM. In section V we briefly summarize our results.

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\[12\] Note that having only one large transverse dimension is experimentally excluded [4] for otherwise there would be deviations from the Newtonian gravity over solar system distances.
We have been recently informed that some issues related to flavor violation in the TeV-scale brane world context are also going to be discussed in [38,39].

II. THE TSSM

In this section we briefly review the TSSM proposed in [22]. The gauge group of this model is the same as in the MSSM, that is, $SU(3)_c \otimes SU(2)_w \otimes U(1)_Y$. The light spectrum of the model is $N = 1$ supersymmetric, and along with the vector superfields $V$ transforming in the adjoint of $SU(3)_c \otimes SU(2)_w \otimes U(1)_Y$ we also have the following chiral superfields (corresponding to the matter and Higgs particles):

$Q_i = 3 \times (\mathbf{3}, \mathbf{2})(+1/3)$,  $D_i = 3 \times (\mathbf{3}, \mathbf{1})(+2/3)$,  $U_i = 3 \times (\mathbf{3}, \mathbf{1})(-4/3)$,

$L_i = 3 \times (\mathbf{1}, \mathbf{2})(-1)$,  $E_i = 3 \times (\mathbf{1}, \mathbf{1})(+2)$,  $N_i = 3 \times (\mathbf{1}, \mathbf{1})(0)$,

$H_+ = (\mathbf{1}, \mathbf{2})(+1)$,  $H_- = (\mathbf{1}, \mathbf{2})(-1)$,

$F_+ = (\mathbf{1}, \mathbf{1})(+2)$,  $F_- = (\mathbf{1}, \mathbf{1})(-2)$.

Here the $SU(3)_c \otimes SU(2)_w$ quantum numbers are given in bold font, whereas the $U(1)_Y$ hypercharge is given in parentheses. The three generations ($i = 1, 2, 3$) of quarks and leptons are given by $Q_i, D_i, U_i$ respectively $L_i, E_i, N_i$ (the chiral superfields $N_i$ correspond to the right-handed neutrinos), whereas $H_\pm$ correspond to the electroweak Higgs doublets. Note that the chiral superfields $F_\pm$ are new; they were not present in the MSSM.

The massive spectrum of the TSSM contains Kaluza-Klein (KK) states. These states correspond to compact $p - 3$ dimensions inside of the $p$-branes ($p = 4$ or 5) on which the gauge fields are localized. The heavy KK levels are populated by $N = 2$ supermultiplets with the quantum numbers given by $(\bar{V}, \bar{H}_+, \bar{H}_-, \bar{F}_+, \bar{F}_-)$, where $\bar{V}$ stands for the $N = 2$ vector superfield transforming in the adjoint of $SU(3)_c \otimes SU(2)_w \otimes U(1)_Y$, whereas $\bar{H}_\pm, \bar{F}_\pm$ are the $N = 2$ hypermultiplets with the gauge quantum numbers of $H_\pm, F_\pm$. (The exact massive KK spectrum, which is not going to be important in the subsequent discussions, can be found in [22,24].) Here we would like to point out some of the features of the model which are going to be relevant for discussions in this section as well as sections III and IV. Note that the massless superfields $V, H_\pm, F_\pm$ have heavy KK counterparts. We can think about these states together with the corresponding heavy KK modes as arising upon compactification of a $p + 1$ dimensional theory on a $p - 3$ dimensional compact space with the volume $V_{p-3}$. On the other hand, the massless superfields $Q_i, D_i, U_i, L_i, E_i, N_i$ do not possess heavy KK counterparts corresponding to these $p - 3$ dimensions. (A concrete mechanism for localizing these fields in the brane world context was discussed in detail in [22,24].)

The gauge coupling unification in the TSSM is just as precise (at one loop) as in the MSSM [22], and the unification scale $M_u$ is in the TeV range (provided that the mass scale of the superfields $F_\pm$ is around that of the electroweak Higgs doublets). The lowering of the unification scale here occurs along the lines of [3] due to the power-like running of the

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13By the light spectrum we mean the states which are massless before the supersymmetry/electroweak symmetry breaking.
gauge couplings above the KK threshold scale \([30]\). On the other hand, the fact that the one-loop unification in the TSSM is as precise as in the MSSM crucially depends on the KK tower of the new states \(F_\pm\). (The actual value of \(M_s\) \([22,24]\) depends on the volume \(V_{p-3}\) which is assumed to be (relatively) large, that is, \(V_{p-3}/(2\pi)^{p-3} \gg 1/M_{p-3}^2\).) The unified gauge coupling \(\alpha\) in the TSSM is small\(^{14}\).

Thus, for instance, for \(M_s \approx 10\) TeV we have \(\alpha \approx 1/37.5\).

### A. Proton Stability and Neutrino Masses

To stabilize proton, in \([24]\) a \(Z_3 \otimes Z_3\) discrete gauge symmetry was introduced. Following \([24]\) we will refer to this discrete gauge symmetry as \(\tilde{L}_3 \otimes \tilde{R}_3\). The corresponding discrete charges (which are conserved modulo 3) are given by:

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\begin{align*}
Q & : (0,0), & D & : (0,1), & U & : (0,-1), \\
L & : (+1,0), & E & : (-1,1), & N & : (-1,-1), \\
H_+ & : (0,1), & H_- & : (0,-1), \\
F_+ & : (0,-1), & F_- & : (0,1).
\end{align*}
\]

In \([24]\) it was shown that this discrete gauge symmetry is anomaly free. In particular, it satisfies the anomaly freedom conditions discussed in \([1]\). In fact, this gauge symmetry satisfies even stronger conditions as it can be embedded into an anomaly free continuous gauge symmetry which we refer to \(U(1)_L \otimes U(1)_R\) \([24]\). This fact will be useful in section IV.

In \([24]\) it was shown that the \(\tilde{L}_3 \otimes \tilde{R}_3\) discrete gauge symmetry stabilizes proton. Thus, it forbids all dangerous baryon and lepton number violating operators potentially leading to proton decay. In particular, dimension 4 \([12]\), dimension 5 (such as \(QQQL\)) as well as all other higher dimensional operators of this type are forbidden by this discrete gauge symmetry.

The \(\tilde{L}_3 \otimes \tilde{R}_3\) discrete gauge symmetry also forbids the dangerous lepton number violating dimension 5 operator \(LLH_+H_+\) which would be disastrous\(^{15}\) for neutrino masses \([24]\) - this operator is suppressed only by \(1/M_s\) (with \(M_s\) in the TeV range), and would result in unacceptably large Majorana neutrino masses \(m_\nu \sim M_{\text{ew}}^2/M_s\) upon the electroweak symmetry breaking. The desirable Dirac neutrino masses in the TSSM can then be generated via the higher dimensional mechanism of \([21]\).

\(^{14}\)Note, however, that the true loop expansion parameter is of order one \([22]\) for it is enhanced by a factor proportional to the number of the heavy KK states which is large. (This enhancement is analogous to that in large \(N\) gauge theories \([10]\).) Nonetheless, as explained in \([22]\), one-loop corrections to the gauge couplings are still dominant due to supersymmetry.

\(^{15}\)Note that this operator is precisely the one responsible for generating the correct neutrino masses in the old “see-saw” mechanism \([43]\) in scenarios with high \(M_s \sim M_{\text{GUT}}\).
III. FLAVOR VIOLATION IN THE TEV-SCALE BRANE WORLD SCENARIO

In this section we would like to review some of the discussions of [20] on flavor violation in the TeV-scale brane world scenario. In particular, one of the important points in [20] is the estimate for the expected flavor violation coming from the “bulk” flavon as well as horizontal gauge boson (in the cases with continuous flavor gauge symmetries) exchanges. The latter are dominant, and lead to unacceptably large flavor violation unless the number of large transverse dimensions is $\geq 4$.

A. Why Do We Need Flavor Symmetries?

There are tight experimental bounds on the flavor changing neutral currents [12], the most constraining being the Kaon system. Thus, let us consider the constraints from the mass splitting between $K^0_S$ and $K^0_L$. The lowest dimensional operators relevant in this context are the dimension 6 four-fermion operators of the form (for simplicity our notations here are symbolic, and we suppress the corresponding color and Lorentz indices as well as the chiral projection operators for the left- and right-handed fields):

$$\xi (\bar{s}d)^2. \quad (1)$$

The experimental bounds on the dimensionful coupling $\xi$ imply that we must have $\text{Re}(\xi) \lesssim 10^{-8} - 10^{-7}$ TeV$^{-2}$ [13]. Without any flavor symmetries, we generically expect the above operator to be suppressed at most by $1/M_s^2$. This would imply the following lower bound: $M_s \gtrsim 10^3 - 10^4$ TeV. To lower this bound on $M_s$ we would need to impose some symmetry which acts on $s$ and $d$ states differently, that is, we would have to impose a flavor symmetry [20]. On the other hand, to be compatible with, say, the observed fermion hierarchy in the Standard Model, this flavor symmetry would have to be broken. So suppressing FCNCs is a non-trivial task: the suppression should arise in a subtle way due to a broken flavor symmetry.

Next, we can ask what kind of flavor symmetries should be imposed to possibly suppress the above operator. First, we should consider (continuous or discrete) gauge flavor symmetries (see Introduction). Second, this gauge symmetry must be anomaly free. Before we plunge into more technical issues such as anomaly freedom, we can ask a more basic question: Would Abelian flavor symmetries suffice, or should we impose non-Abelian flavor symmetries? As pointed out in [20], Abelian symmetries alone cannot do the job. Here we would like to briefly review the arguments of [20].

$^{16}$A rough estimate for the $K^0_L - K^0_S$ mass splitting due to [13] is given by $\Delta m_K/m_K \sim \text{Re}(\xi)m_K^2$, where $m_K$ is the Kaon mass. Note that $\text{Im}(\xi)$ is constrained by the CP violation parameters, and this constraint is about 100 times stronger than that for $\text{Re}(\xi)$. We will discuss this issue in section IV.
B. Possible Flavor Symmetries

To understand why Abelian flavor symmetries are inadequate for suppressing FCNCs in the present context, let us consider dimension 6 four-fermion operators containing the fields $Q_i$ and $\overline{Q}^i$, where the latter fields are just the conjugates of the former, and we are using superscript for the corresponding flavor indices as $\overline{Q}^i$ transform in the representation of the flavor group which is complex conjugate of the representation in which $Q_i$ transform. Here the indices $i, j, \ldots$ refer to flavor indices in the flavor basis, that is, the basis in which the charged electroweak currents are diagonal. Ultimately we will be interested in understanding FCNCs in the physical basis in which the up and down quark mass matrices are diagonal. These two bases are related by the usual unitary rotations (see below).

Thus, let us consider the most general four fermion operators of the form:

$$C_{kl}^{ij}(\overline{Q}^i Q_k)(\overline{Q}^j Q_l).$$

(2)

Note that no Abelian (continuous or discrete) symmetry can forbid the terms with $i = k$ and $j = l$. Moreover, the corresponding coefficients a priori are completely unconstrained. These terms do not contain the dangerous operator $(\overline{s}d)^2$ in the flavor basis. However, in the physical basis this term will be generated unless the flavor and physical quark states are identical, at least for the first two generations. This could a priori be the case in, say, the down quark sector. However, this cannot be the case in both up and down quark sectors: indeed, if both up and down quark mass matrices were diagonal in both bases (or, more precisely, if this was the case for the corresponding $1 - 2$ blocks, $1, 2, 3$ referring to the three generations), there would be no Cabibbo mixing between the first and the second generations, which would contradict the experimental data. This implies that either $(\overline{s}d)^2$ or $(\overline{u}c)^2$ (or both at the same time) operators would be present in the physical basis with the coupling $\xi \sim \sin^2(\theta_C)/M_s^2$ ($\theta_C$ is the Cabibbo angle) if the flavor symmetry is Abelian. This is unacceptable as the corresponding strengths of flavor violation in the $K^0 - \overline{K}^0$ and/or $D^0 - \overline{D}^0$ transitions would be above the present experimental bounds (note that $\sin \theta_C \simeq 0.22$). The only way around this problem is to impose a non-Abelian flavor symmetry which is more constraining and can a priori result in a non-trivial “conspiracy” between the coefficients $C_{kl}^{ij}$ such that the disastrous flavor violating operators are not induced in the physical basis.

We are therefore led to the conclusion that some type of non-Abelian flavor symmetry must be invoked. If all the Yukawa couplings in the Standard Model are set to zero, then we have the following flavor symmetry group: $G_F = \bigotimes_A U(3)_A$, where $A = Q, D, U, L, E, N$. This is the largest flavor group we can attempt to introduce. However, it is clear that at least in the quark sector the $U(3)_Q$ and $U(3)_U$ must be broken at the string scale to have large top mass. In fact, this is also the case for $U(3)_D$ as the top-bottom splitting is generally considered to be due to a “vertical” hierarchy: we can either have large $\tan \beta$, or the $QDH_-$ coupling could arise as an effective Yukawa coupling descending from a higher dimensional

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\[17\text{Some early works on non-Abelian flavor symmetries include [46]. Some of the more recent works can be found in [47].}\]
operator \( SQDH \) upon the additional singlet field \( S \) acquiring a VEV \( \langle S \rangle /M_s \sim m_b/m_t \) (here \( m_b \) and \( m_t \) are the bottom and top quark masses, respectively, and \( \tan \beta \) is assumed to be \( \sim 1 \)). As was pointed out in [2], the latter possibility can naturally arise in the TSSM. The same is also expected to be the case in the lepton sector. Thus, we should really consider the \( U(2)_A \) subgroup of the \( U(3)_A \) flavor group as our starting point.

Thus, let us consider gauging as large a non-Abelian flavor group as possible. In the next section we will argue that the anomaly cancellation conditions in the TSSM require that we identify \( SU(2)_Q \) with \( SU(2)_L \) - we will refer to this flavor subgroup as the left-handed flavor group \( SU(2)_L \). Similarly, we must identify the other four subgroups \( SU(2)_D \), \( SU(2)_U \), \( SU(2)_E \) and \( SU(2)_N \) with each other - we will refer to this flavor subgroup as the right-handed flavor group \( SU(2)_R \). We can also gauge additional flavor \( U(1) \)'s (albeit the number of relevant anomaly free possibilities is rather limited). We will consider such an Abelian flavor group (as well as its discrete subgroups) in the next section in more detail. Here, however, we would like to concentrate on the non-Abelian part of the flavor group, the largest possibility being \( SU(2)_L \otimes SU(2)_R \).

Let us see if the \( SU(2)_L \otimes SU(2)_R \) flavor symmetry can suppress FCNCs adequately. Let us concentrate on the first and the second generations for the moment. In particular, in the expressions we are about to write down the flavor indices \( i, j \ldots \) take values 1, 2. Thus, consider four-fermion operators in the presence of the \( SU(2)_L \otimes SU(2)_R \) flavor symmetry. The \( SU(2)_L \otimes SU(2)_R \) invariant operators involving only the fields \( Q_i \) and \( \bar{Q} \) are given by

\[
C(\overline{Q}Q_i)(\overline{Q}^jQ_j) + C''\epsilon_{ij}\epsilon^{kl}(\overline{Q}Q_k)(\overline{Q}^lQ_i) .
\]

These operators do not contain the dangerous four-fermion interactions in the flavor basis. However, we really need to go to the physical basis. This is accomplished by means of the rotation \( Q_I = Q^IQ_I \), which is unitary if we consider all three generations. Here \( Q_I \) contains the left-handed up and down quarks \( (Q_U)_I \) respectively \( (Q_D)_I \) in the physical basis, and by \( Q^IQ_I \) we really mean two \emph{a priori} independent matrices \((Q_U)_i^I\) and \((Q_D)_i^I \) acting on \((Q_U)_I \) respectively \((Q_D)_I \). The condition that ensures that the dangerous operator \( \bar{s}d^2 \) does not appear in the physical basis is then given by the requirement that the \( 2 \times 2 \) matrix \( (Q_D)_i^I \) be almost unitary. A similar constraint arises if we consider the four-fermion operators involving \( Q \)'s and/or \( D \)'s (as well as their conjugates): the \( 2 \times 2 \) matrix \( D^I_i \) must be almost unitary. Here we are using the bar notation to distinguish the \( SU(2)_R \) flavor indices \( \bar{i}, \bar{I}, \ldots \) from the \( SU(2)_L \) flavor indices \( i, I, \ldots \), and the left-handed down anti-quarks in the physical basis are given by \( D_I \), where \( D_I = D^{I\bar{i}}D^I_{\bar{i}} \). Finally, the \( 2 \times 2 \) matrix \( U^I_i \), which relates the left-handed up anti-quarks in the flavor and physical bases via \( U_i = U_IU^I_i \), must also be almost unitary. (Note that \((Q_D)_i^I\) and \(D^I_i\) together diagonalize the down quark mass matrix \( (M_D)^{\bar{i}i} \) via \( D_I(M_D)^{\bar{i}i}(Q_D)_i = D_I(M_D)^{\bar{i}i}(Q_D)_i \), where the diagonal mass matrix \( (M_D)^{\bar{i}i} \equiv D^{I\bar{i}}_I(M_D)^{\bar{i}i}(Q_D)_i^I \). Similarly, \((Q_U)_i^I\) and \(U^I_i\) together diagonalize the up quark mass matrix \( (M_U)^{i\bar{i}} \) via \( U_I(M_U)^{i\bar{i}}(Q_U)_i^I = U_I(M_U)^{i\bar{i}}(Q_U)_i^I \), where the diagonal mass matrix \( (M_U)^{i\bar{i}} \equiv U^I_i(M_U)^{i\bar{i}}(Q_U)_i^I \). Here we are considering all three generations so that all matrices are \( 3 \times 3 \) matrices.)

The above “unitarity” constraints imply that the 1–3 and 2–3 mixing in the down as well as the up quark sectors cannot be large. This requirement, however, is not so constraining. Thus, if we take the 1–3 and 2–3 mixings in the up and down quark sectors to be of order of the corresponding mixings in the Cabibbo-Kobayashi-Maskawa

\[
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\]
(CKM) matrix $V_{CKM} \equiv Q_D^T Q_U$, then the FCNCs induced by the “non-unitarity” of the $1-2$ diagonalizations in the up and down quark sectors will be adequately suppressed for $M_s \sim 10 - 100$ TeV. In particular, the $1-3$ and $2-3$ CKM mixings are given by $|V_{ub}| \sim 3 \times 10^{-3}$ and $|V_{cb}| \sim 3 \times 10^{-2}$. This implies that the corresponding flavor violating couplings are going to be given by:

$$\xi_{(u3)}^2 \sim |V_{ub}|^2 / M_s^2 \sim 10^{-8} / M_s^2,$$

$$\xi_{(d3)}^2 \sim |V_{cd}|^2 / M_s^2 \sim 10^{-5} / M_s^2,$$

$$\xi_{(s3)}^2 \sim |V_{cb}|^2 / M_s^2 \sim 10^{-3} / M_s^2.$$

These couplings are within the experimental bounds for $M_s \sim 10 - 100$ TeV.

C. Gauge and Flavon Mediated Flavor Violation

Imposing a flavor symmetry is not the end of the story, however, as it has to be broken. As pointed out in [20], it cannot be broken on the $p$-branes as this would be in contradiction with various cosmological as well as other constraints [19]. This implies that it has to be broken in the bulk, and if we gauge a continuous flavor symmetry, it must be a “bulk” gauge symmetry. Note that the flavor symmetry breaking source could be on a “distant” brane [19], but it would have to be transmitted to the $p$-branes via some “bulk” fields anyways, so for our purposes here we can treat the flavor symmetry breaking as arising due to the corresponding “bulk” flavons acquiring non-zero VEVs.

The flavor symmetry breaking due to the “bulk” flavons is felt by the $p$-brane fields via the couplings of the latter to the former. The non-trivial flavor hierarchy in the Standard Model then would have to be due to the “bulk” flavons. It is then clear that once the “bulk” flavons acquire non-zero VEVs, there are going to be induced effective flavor violating four-fermion operators. We will discuss the flavon VEV-induced flavor violation in detail in section IV. Here, however, we will focus on the other two sources of flavor violation which we must take into account.

First, we must take into account the flavon mediated flavor violation due to the flavon exchanges. Let $n$ be the number of large transverse dimensions in which the flavons can propagate. Then it is straightforward to estimate the corresponding flavor non-universal contributions due to the light ($\lesssim m_M$) and heavy ($\gtrsim m_M$) KK flavon modes in these directions ($m_M$ is the corresponding meson mass, $M = K, D, B$) [20]:

$$\xi_{\text{light}} \sim (\lambda M_s)^{-2} (m_M / M_s)^n,$$

$$\xi_{\text{heavy}} \sim \lambda^{n-4} (m_M / M_s)^2.$$

Here $\lambda \sim m_c / m_t \sim 10^{-2}$ ($m_c$ is the charm quark mass) is a measure of the flavor symmetry breaking. Thus, in the worst case where $n = 2$ we have flavor violation which is in the acceptable range for $M_s \gtrsim 30$ TeV or so.

Suppose, however, we have a continuous non-Abelian flavor gauge symmetry in the bulk. Then we must also consider gauge mediated flavor violation due to the exchanges of horizontal gauge bosons and their KK counterparts all of which have flavor non-universal contributions ($\sim (\lambda M_s)^2$) to their squared masses. The induced flavor violation is given by [20]:

$$\xi_{\text{light}} \sim (\lambda M_s)^{-2} (m_M / M_s)^n,$$

$$\xi_{\text{heavy}} \sim \lambda^{n-4} (m_M / M_s)^2.$$
\[ \xi_{\text{gauge}} \sim \lambda^{n-2} M_s^{-2}, \quad n < 4 , \]
\[ \xi_{\text{gauge}} \sim \lambda^2 M_s^{-2}, \quad n \geq 4 . \]

Note that for \( n < 4 \) the gauge mediated flavor violation is unacceptably large. In fact, for \( n = 2 \) there is no suppression at all (which is due to the fact that in this case the flavor non-universal contribution to the gauge boson mass squared scales the same way with \( \lambda \) as the number of the “light” KK modes for which this contribution is dominant\(^1\)). We are therefore led to the conclusion that the FCNC constraints rule out gauging continuous flavor symmetries in the bulk for \( n < 4 \). In the next section we will circumvent this difficulty by explicitly constructing a discrete flavor gauge symmetry that suppresses the FCNCs adequately, yet has the advantage of not reintroducing gauge mediated flavor violation as there are no gauge bosons in this case to begin with.

**IV. DISCRETE FLAVOR GAUGE SYMMETRIES AND FCNC SUPPRESSION**

In this section we will explicitly construct an anomaly free non-Abelian discrete flavor gauge symmetry which suppresses FCNCs just as well as the \( SU(2)_L \otimes SU(2)_R \) continuous flavor gauge symmetry. In fact, the discrete flavor group we are going to discuss is a subgroup of \( SU(2)_L \otimes SU(2)_R \). Since we are going to gauge it, we must make sure that all anomalies cancel. The discrete anomaly cancellation conditions are similar to those for continuous gauge symmetries\(^1\). To illustrate some of the non-trivial issues arising when discussing discrete anomaly cancellation conditions, let us first consider these conditions in the case of an Abelian \( \mathbb{Z}_N \) discrete gauge symmetry.

A \( \mathbb{Z}_N \) discrete gauge symmetry can be thought of as follows. Consider a theory with some matter charged under an anomaly free \( U(1) \) gauge symmetry. Let all the \( U(1) \) charge assignments be integer. Consider now adding a pair of chiral superfields which are neutral under all the other gauge subgroups but carry \(+N\) and \( -N \) charges under the above \( U(1) \). Suppose there is a flat direction along which these chiral superfields acquire non-zero VEVs. Then the \( U(1) \) gauge symmetry is broken down to its \( \mathbb{Z}_N \) subgroup. This \( \mathbb{Z}_N \) is then an anomaly free discrete gauge symmetry. In the above approach the anomalies for the \( \mathbb{Z}_N \) gauge symmetry mimic those for the original \( U(1) \) gauge symmetry (except that the former anomalies are only defined “modulo \( N \)”). Thus, we have the \( \text{Tr}(\mathbb{Z}_N^3) \) anomaly as well as the mixed \( \text{Tr}(\mathbb{Z}_N) \) gravitational anomaly. If there are non-Abelian gauge subgroups \( \otimes_i G_i \) in the theory, one also needs to consider the mixed \( \text{Tr}(G_i^2 \mathbb{Z}_N) \) non-Abelian gauge anomalies. Finally, if there are additional Abelian subgroups \( \otimes_a U(1)_a \), then we must also consider the mixed \( \text{Tr}(U(1)_a U(1)_b \mathbb{Z}_N) \) and \( \text{Tr}(U(1)_a \mathbb{Z}_N^3) \) gauge anomalies. The last two anomalies are somewhat tricky compared with the rest of the anomalies. The reason is that to compute them one is required to know the massive spectrum of the theory. More concretely, these anomalies depend on details of the parent \( U(1) \) breaking\(^1\). Since we are going to attempt to gauge discrete symmetries in the TSSM which contains an Abelian

\(^{18}\)By such “light” KK modes we mean those for which the flavor universal mass squared contributions due to the non-zero KK momenta are \( \lesssim (\lambda M_s)^2 \).
gauge subgroup (namely, $U(1)_Y$), we cannot ignore these anomalies. There is however, a way out of this difficulty. We can explicitly gauge a continuous (that is, $U(1)$) symmetry and make sure that all the anomalies cancel before we break it to the corresponding discrete subgroup. This way we are guaranteed to have an anomaly free discrete gauge symmetry at the end of the day.

Similar considerations apply to any discrete flavor gauge symmetry $\Gamma_F$. To make sure that it is anomaly free, we can first consistently gauge a continuous flavor symmetry $G_F$ which contains $\Gamma_F$ as a subgroup (note that $G_F$ as well as $\Gamma_F$ can contain Abelian as well as non-Abelian factors). Then the discrete flavor symmetry $\Gamma_F$ is guaranteed to be anomaly free. This is precisely the strategy we will follow in this section.

A. The $SU(2)_L \otimes SU(2)_R$ Flavor Gauge Symmetry

As our starting point, we would like to consistently gauge the $SU(2)_L \otimes SU(2)_R$ flavor symmetry. A priori we could start from the full flavor group $G_F = \bigotimes_A U(3)_A$, where $A = Q, D, U, L, E, N$, where the 3 generations transform in the corresponding fundamental representations of the subgroups $U(3)_A$. However, as we already mentioned in the previous section, we need not bother trying to gauge the full $U(3)_A$ subgroups as they would have to be broken at the string scale $M_s$ to account for the large quark and lepton masses in the third generation. We can therefore restrict our attention to the flavor group $G_F = \bigotimes_A U(2)_A$.

Here we must address the issue of the mixed $\text{Tr}(G_F^2 \otimes U(1)_Y)$ gauge anomalies. These could a priori be canceled by introducing additional states charged under $G_F \otimes U(1)_Y$ (but neutral under $SU(3)_c \otimes SU(2)_w$). The generic problem with this is that we would have to add many such states, and they would have to be chiral (to be able to cancel anomalies). They then are generically massless unless $U(1)_Y$ is broken, which only happens upon the electroweak symmetry breaking\(^{19}\). This then implies that there would be many additional light states on top of those we already have in the TSSM. Note that the situation is even worse once we consider the mixed $\text{Tr}(G_F^2 \otimes (\tilde{L}_3 \otimes \tilde{R}_3))$ gauge anomalies, where $\tilde{L}_3 \otimes \tilde{R}_3$ is the Abelian discrete gauge symmetry discussed in section II which is responsible for proton stabilization.

The above difficulties can be ameliorated by considering the flavor group $G_F = SU(2)_L \otimes SU(2)_R$, where we identify $SU(2)_A$, $A = Q, L$, with the left-handed flavor group $SU(2)_L$, and $SU(2)_A$, $A = D, U, E, N$, with the right-handed flavor group $SU(2)_R$. A priori we could also consider additional $U(1)$ factors, but the anomaly cancellation conditions are very tight, so the number of consistent possibilities is actually very limited. We will consider adding such a $U(1)$ factor in a moment. First, however, let us discuss gauging $G_F = SU(2)_L \otimes SU(2)_R$.

Next, we give the quantum numbers of the TSSM fields under $[SU(2)_L \otimes SU(2)_R] \otimes [U(1)_L \otimes U(1)_R]$ (here $U(1)_L$ and $U(1)_R$ are the “parent” $U(1)$ symmetries\(^{20}\) for the discrete subgroups $\tilde{L}_3$ respectively $\tilde{R}_3$):

\(^{19}\)In fact, some of these additional states would actually have to be $SU(2)_w$ doublets to have appropriate couplings with the electroweak Higgs doublets $H_{\pm}$, which makes the anomaly cancellation (as well as other issues) even more problematic.

\(^{20}\)Here we choose to work with these parent gauge symmetries as the mixed anomalies are more
\[ Q_i : (2, 1)(0, 0), \quad Q_3 : (1, 1)(0, 0), \]

\[ D_i : (1, 2)(0, +1), \quad D_3 : (1, 1)(0, +1), \]

\[ U_i : (1, 2)(0, -1), \quad U_3 : (1, 1)(0, -1), \]

\[ L_i : (2, 1)(+1, 0), \quad L_3 : (1, 1)(+1, 0), \]

\[ E_i : (1, 2)(-1, +1), \quad E_3 : (1, 1)(-1, +1), \]

\[ N_i : (1, 2)(-1, -1), \quad N_3 : (1, 1)(-1, -1), \]

\[ \chi_{i\alpha} : (2, 1)(-1, 0), \quad \chi'_{i\alpha'} : (1, 2)(+1, 0). \]

Here the \( SU(2)_L \otimes SU(2)_R \) quantum numbers are given in bold font, whereas the \( U(1)_{\mathcal{L}} \otimes U(1)_{\mathcal{R}} \) charges are given in parentheses. The TSSM states \( H_{\pm} \) and \( F_{\pm} \) are neutral under \( SU(2)_L \otimes SU(2)_R \), and are not shown. The flavor indices \( i \) and \( \tilde{i} \) take values 1, 2 and 1, 2, respectively. The new states \( \chi_\alpha \) and \( \chi'_{\alpha'} \) (\( \alpha, \alpha' = 1, 2 \)) are new indices not related to the flavor indices for the quarks and leptons) are neutral under the Standard Model gauge group \( SU(3)_c \otimes SU(2)_w \otimes U(1)_{Y} \), and they are required by the \( \text{Tr}(SU(2)^2_{\mathcal{L}}U(1)_{\mathcal{L}}) \) and \( \text{Tr}(SU(2)^2_{\mathcal{R}}U(1)_{\mathcal{R}}) \) mixed gauge anomaly cancellation, respectively. Note that these new states carry non-zero \( U(1)_{\mathcal{L}} \) charges. This could be dangerous for proton stability which is based on a non-trivial selection rule (due to the \( \tilde{\mathcal{L}}_3 \otimes \mathcal{R}_3 \) discrete gauge symmetry) discussed in detail in \cite{24}. To avoid jeopardizing proton stability, we can simply require that the new states \( \chi_\alpha, \chi'_{\alpha'} \) be heavier than proton (and their VEVs are zero). This can be achieved by introducing \( n_{\Sigma} \) additional “flavon” fields (which are neutral under \( SU(3)_c \otimes SU(2)_w \otimes U(1)_{Y} \))

\[ \Sigma^i_a (2, 2)(0, 0), \quad a = 1, \ldots, n_{\Sigma}, \quad (4) \]

with the following Yukawa couplings to \( \chi_\alpha, \chi'_{\alpha'} \):

\[ y_{aa'a'} \Sigma^i_a \chi_{i\alpha} \chi'_{i\alpha'}. \quad (5) \]

Upon the fields \( \Sigma_a \) (which we assume to be “bulk” flavon fields) acquiring VEVs (which break the \( SU(2)_L \otimes SU(2)_R \) flavor symmetry), the fields \( \chi_\alpha, \chi'_{\alpha'} \) pick up large masses (of order of the flavor symmetry breaking scale which is \( \sim \lambda M_{\chi} \sim 100 \text{ GeV} - 1 \text{ TeV} \), where \( \lambda \sim 10^{-2} \) was introduced in the previous section), so that proton decay via channels involving \( \chi_\alpha \) and/or \( \chi'_{\alpha'} \) in the final state is forbidden due to the kinematics. Note that such heavy fields could \textit{a priori} be dangerous - they could potentially mediate unacceptably large FCNCs through a tree-level exchange. However, precisely due to their non-zero \( U(1)_{\mathcal{L}} \) (or, more precisely, \( \tilde{\mathcal{L}}_3 \)) charges, the \( \chi_\alpha, \chi'_{\alpha'} \) states do not have the required couplings (such as, say, \( \chi_{QDH} \)) to the quarks (or leptons) to mediate flavor violation in this way, and are therefore safe.

The above spectrum (together with the states \( H_{\pm}, F_{\pm} \)) is completely anomaly free. This implies that if we considered the same spectrum with the discrete flavor gauge symmetry \( \Gamma_L \otimes \Gamma_R \) (instead of the continuous flavor gauge symmetry \( SU(2)_L \otimes SU(2)_R \)), where \( \Gamma \subset \)

transparent in this language.

\footnote{At the end of the day we are going to consider the discrete subgroup \( \tilde{\mathcal{L}}_3 \otimes \mathcal{R}_3 \) of the \( U(1)_{\mathcal{L}} \otimes U(1)_{\mathcal{R}} \) gauge group as in \cite{24}.}
SU(2), the resulting spectrum would also be completely anomaly free. In the next subsection we will make a particular choice of $\Gamma$ such that in the unbroken flavor symmetry limit the FCNCs are suppressed just as well as in the case of the full $SU(2)_L \otimes SU(2)_R$ flavor gauge symmetry.

B. A Conclusive Solution: The $T_L \otimes T_R$ Discrete Flavor Gauge Symmetry

Next, we need to identify a discrete subgroup $\Gamma$ of $SU(2)$ such that the $\Gamma_L \otimes \Gamma_R$ discrete flavor gauge symmetry suppresses FCNCs just as well as the $SU(2)_L \otimes SU(2)_R$ flavor gauge symmetry itself. It is not difficult to identify such a subgroup - the discrete subgroups of $SU(2)$ are classified in terms of the A-D-E series. The infinite $A_N$ series corresponds to the Abelian $\mathbb{Z}_N$ subgroups of $SU(2)$. The second infinite $D_N$ series contains non-Abelian dihedral groups (with $D_3$ isomorphic to (the double cover of) the permutation group $S_3$). Finally, the finite E-series contains three “exceptional” non-Abelian subgroups (analogous to $E_6$, $E_7$, $E_8$): the tetrahedral group $T$, the octahedral group $O$, and the icosahedral group $I$. Here we are interested in the non-Abelian groups only. It is not difficult to show that the dihedral groups $D_N$ cannot do the job. The tetrahedral group $T$, however, is perfectly adequate for our purposes. Thus, it only allows the $SU(2)$ invariant four-fermion operators. For instance, the only operators containing $Q_i$’s and $\overline{Q}_i$’s are those given in (3). This follows from the action of the generators of the tetrahedral group $T$ on the fields $Q_i, Q_3 (i = 1, 2)$. Let these generators be $\theta, R, R'$. Then their action on $Q_3$ is trivial. The fields $Q_i$, however, transform in one, namely, $2_0$, of the three non-trivial two dimensional representations $2_k, k = 0, 1, 2$, of $T$. These two dimensional representations are given by

$$\theta = \frac{\omega^k}{2} (1 - i\sigma_1 + i\sigma_2 - i\sigma_3),$$

$$R = i\sigma_1, \quad R' = i\sigma_3,$$

where $1$ stands for the $2 \times 2$ identity matrix, and $\sigma_1, \sigma_2, \sigma_3$ are the Pauli matrices. Also, $\omega \equiv \exp(2\pi i/3)$. Note that $\theta^3 = R^2 = R'^2 = -1$, and $R\theta = \theta R'R, R'\theta = \theta R, R'R\theta = \theta R'$, and $R'R'R' = R$. (These are the defining commutation relations for the tetrahedral group.)

Note that we have just described the action of $T_L$ (on $Q$’s and $L$’s). The action of $T_R$ (on $D$’s, $U$’s, $E$’s and $N$’s) is similar.

Thus, the $T_L \otimes T_R$ discrete flavor gauge symmetry is just as efficient in constraining four-fermion operators as the continuous $SU(2)_L \otimes SU(2)_R$ flavor gauge symmetry whose subgroup $T_L \otimes T_R$ is. This discrete symmetry, however, does not guarantee that all dangerous flavor violating operators are suppressed after the flavor symmetry breaking (neither does $SU(2)_L \otimes SU(2)_R$). In the following we will consider an additional Abelian flavor symmetry which is required to ensure that the flavon VEV-induced flavor violation is also adequately suppressed.

C. Suppressing Flavon VEV-induced FCNCs: The $U(1)_V$ Flavor Gauge Symmetry

The $SU(2)_L \otimes SU(2)_R$ flavor gauge symmetry (or its discrete subgroup $T_L \otimes T_R$) suppresses dangerous flavor violating four-fermion operators before the flavor symmetry breaking takes
place. Once the flavor symmetry is broken, there are going to be induced such four-fermion operators, but the corresponding couplings will be suppressed by the flavon VEVs. Nonetheless, we must make sure that this suppression is adequate, that is, that these couplings are within the experimental bounds. In this respect it does not make any difference whether we are breaking the $SU(2)_L \otimes SU(2)_R$ flavor gauge symmetry or its discrete subgroup $T_L \otimes T_R$, so we will use the language of the full $SU(2)_L \otimes SU(2)_R$ flavor gauge symmetry in this subsection.

Let us first consider the operators involving only the first two generation quarks and anti-quarks in the flavor basis. The possible VEV-induced operators of this type are always suppressed by two powers of $\lambda \sim m_c/m_t$. To see this, note that, say, in the operator $(D_i Q_i)(\bar{Q}^j \bar{D}^j)$ we must contract two pairs of the $SU(2)$ indices. Let us first consider the flavon VEV-induced operators of this type involving only flavons in the bifundamental representation of $SU(2)_L \otimes SU(2)_R$, say, the fields $\Sigma_a^{ij}$ introduced above. The corresponding four-fermion operators then read:

$$
(D_i Q_i)(\bar{Q}^j \bar{D}^j)\epsilon_{ijk}\epsilon_{j\bar{k}} \left( C_{a_1a_2}\Sigma_{a_1}^{ik} \Sigma_{a_2}^{k\bar{k}} + C'_{a_1a_2}\Sigma_{a_1}^{i\bar{k}} \Sigma_{a_2}^{k\bar{j}} \right),
$$

where $C_{a_1a_2}$ and $C'_{a_1a_2}$ are some dimensionful couplings of order $\sim 1/M_s^4$. The strength of the induced FCNC operator of the type $(\pi d)^2$ is then going to be suppressed at least as $\xi^{(\pi d)^2} \sim \lambda^2/M_s^2$. Here we are taking into account that the $\Sigma_a^{ij}$ VEVs are expected to be such that $\Sigma_a^{22} \sim \lambda M_s$ to explain the flavor hierarchy between the charm mass $m_c$ and the top mass $m_t$. The above coupling then follows if we assume the maximal mixing between the first and the second generations. However, in the case of bifundamental flavons there is going to be an additional suppression factor as the first and the second generation mixing is expected to be somewhat smaller, and it is reasonable to assume that it is (roughly) given by the Cabibbo angle $\theta_C$. Then the corresponding coupling is going to be even more suppressed: $\xi^{(\pi d)^2} \sim \lambda^2 \sin^2(\theta_C)/M_s^2$. In fact, to avoid fine-tuning in the determinants of the $1-2$ blocks in the up and/or down quark mass matrices, we can assume that $\Sigma_{a_1}^{12}, \Sigma_{a_1}^{21} \sim \lambda \sin(\theta_C)M_s$, which would lead to the same conclusion for the coupling $\xi^{(\pi d)^2}$. (We will discuss the up and down quark mass matrices in more detail in the next subsection.) The above suppression is completely adequate for $M_s \sim 10 - 100 \text{ TeV}$. A similar conclusion also holds for flavons transforming in the fundamental representations.

There are, however, other operators we must worry about which generically are not as suppressed. These are the operators involving the third generation quarks or anti-quarks. Thus, consider the following operator:

$$
C_a(\bar{Q}^3 Q_i)(D_i \bar{D}^j)\Sigma_a^{ij} \epsilon_{ij},
$$

where the dimensionful coupling $C_a \sim 1/M_s^3$. It is then not difficult to see that the VEV-induced four-fermion operators of the type $(\bar{b}d)^2$ (and $(\bar{b}s)^2$) will be suppressed only by the suppression factor $\sim (\langle H^- \rangle/M_s)^2 \sim (M_{ew}/M_s)^2$ as they carry two units of the weak isospin. In the following we will therefore not discuss such operators, but rather focus on “self-conjugate” operators of the type mentioned above.

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22Note that operators of, say, the form $(D_i Q_i)(D_j Q_j)$ are further suppressed by a factor $\sim (\langle H^- \rangle/M_s)^2 \sim (M_{ew}/M_s)^2$ as they carry two units of the weak isospin. In the following we will therefore not discuss such operators, but rather focus on “self-conjugate” operators of the type mentioned above.
couplings of order $\xi \sim \lambda \sin(\theta_C)/M_s^2$. However, the above operator can be further suppressed by imposing an additional “vector-like” flavor symmetry that acts non-trivially, say, on the third generation quarks and anti-quarks only. In fact, even an Abelian symmetry would suffice for this purpose. We can therefore augment the $SU(2)_L \otimes SU(2)_R$ flavor gauge symmetry by such a $U(1)_V$ flavor gauge symmetry. Thus, let the $U(1)_V$ charge assignments be the following: $+1$ for $Q_3$, and $-1$ for $D_3$ and $U_3$. Then it is clear that the above coupling is going to be suppressed. (Note that the flavon $\Sigma_i^{a\bar{a}}$ must be neutral under $U(1)_V$ to have the required couplings with the first and the second generations.) In fact, all the other flavon VEV-induced FCNCs can be suppressed this way. Thus, for instance, the operators of the type
\[
(Q_3 Q_i)(Q_3 Q_j),
\]
where the $SU(2)_L$ indices must be contracted with flavons transforming in the fundamental representation, are also suppressed provided that we can find an appropriate $U(1)_V$ charge assignment for the flavons.

It is, however, non-trivial to construct an anomaly free $U(1)_V$ flavor gauge symmetry. The anomaly cancellation constraints are very tight as we have to consider mixed anomalies involving $SU(2)_L \otimes SU(2)_R$, $SU(3)_c \otimes SU(2)_w \otimes U(1)_Y$, and $U(1)_L \otimes U(1)_R$ (or, more precisely, the discrete $\tilde{L}_3 \otimes \tilde{R}_3$ subgroup of the latter). Fortunately, there exists a simple $U(1)_V$ flavor gauge symmetry which is anomaly free. Here we will first write down the $U(1)_V$ charge assignments for the quarks and leptons, and then explain why this choice turns out to be anomaly free.

Thus, consider the following $U(1)_V$ charge assignments (the $SU(2)_L \otimes SU(2)_R$ quantum numbers are given in bold font, whereas the $U(1)_V$ charges are given in parenthesis):

\[
\begin{align*}
Q_i : & \ (2,1)(0), \quad Q_3 : \ (1,1)(+1), \\
D_i : & \ (1,2)(0), \quad D_3 : \ (1,1)(-1), \\
U_i : & \ (1,2)(0), \quad U_3 : \ (1,1)(-1), \\
L_i : & \ (2,1)(0), \quad L_3 : \ (1,1)(-3), \\
E_i : & \ (1,2)(0), \quad E_3 : \ (1,1)(+3), \\
N_i : & \ (1,2)(0), \quad N_3 : \ (1,1)(+3), \\
\chi_{\alpha} : & \ (2,1)(0), \quad \chi'_{\alpha'} : \ (1,2)(0), \\
\Sigma_i^{a\bar{a}} : & \ (2,2)(0),
\end{align*}
\]

Here we should point out that the flavon VEV-induced FCNCs due to the flavons transforming in the fundamental representations can be suppressed even more if we utilize the vertical (that is, up-down) hierarchy. Thus, consider a scenario where the $1-3$ and $2-3$ mixing arises only in the down quark sector, and the flavons transforming in the fundamental representations responsible for this mixing carry non-zero (namely, $+3$) $U(1)_L$ charges (the corresponding electroweak Higgs doublet $H_-$ has $U(1)_L$ charge $-3$ - see 24 for details). This implies that the corresponding flavon VEV-induced FCNCs are suppressed by extra factors of $(m_b/m_t)^2$ (note that $\Lambda/M_s \sim m_b/m_t$, where $\Lambda$ is the $U(1)_L \supset \tilde{L}_3$ breaking scale [24]).
\[\rho_\beta^i : (2,1)(+1) , \quad (\rho_\beta')^\dagger : (1,2)(-1) ,
\]
\[\eta^i : (2,1)(-3) , \quad (\eta')^\dagger : (1,2)(+3) .\]

All the other fields (such as \(H_\pm, F_\pm\)) are neutral under the \(U(1)_V\) flavor gauge symmetry. On the other hand, the fields \((\beta, \beta' = 1, 2, 3) \rho_\beta, \rho'_\beta, \eta, \eta'\) (as well as \(\Sigma_a\)) are all neutral under \(SU(3)_c \otimes SU(2)_w \otimes U(1)_Y \otimes \tilde{\mathcal{L}}_3 \otimes \tilde{\mathcal{R}}_3\). The above spectrum is completely anomaly free. This can be seen as follows. Thus, the \(U(1)_V\) symmetry acts as 3 times \(B-L\) (baryon minus lepton number) for the third generation only (in the flavor basis). This is precisely the reason why it is anomaly free and compatible with all the other gauge symmetries we have introduced so far in the TSSM. Thus, as was explained at length in \([24]\), we can consistently gauge any linear combination of the three \(U(1)'s\), namely, \(U(1)_Y, U(1)_L, U(1)_R\). In fact, any linear combination of \(U(1)_V\) and \(U(1)_R\) can be gauged generation-by-generation. The linear combination \(V \equiv 3Y - 3R\) is precisely the generator corresponding to 3 times \(B-L\), which we choose to act on the third generation only. In particular, the anomaly cancellation requires that the \(U(1)_V\) charge assignments be different in the quark and lepton sectors. Note that the fields \(\rho_\beta, \rho'_\beta, \eta, \eta'\) are chosen so that the corresponding \(U(1)_V\) anomalies cancel.

It is not difficult to check that with the above \(U(1)_V\) charge assignments all the flavon VEV-induced FCNCs are adequately suppressed. In fact, the corresponding couplings are at least as suppressed as those due to the “non-unitarity” of the \(1-2\) diagonalization discussed in subsection B of section III. Here we would like to point out that suppressing flavon VEV-induced FCNCs in the up and down quark sectors does not require the full \(U(1)_V\) symmetry. More concretely, any discrete \(\mathbb{Z}_N\) subgroup of \(U(1)_V\) with \(N \neq 2, 4\) would do the job. The \(N = 4\) case is also acceptable if the \(U(1)_V \supset \mathbb{Z}_4\) breaking scale is \(10^{-2} M_s\) or lower\(^{24}\).

Next, we would like to briefly discuss the flavor hierarchy in the above model. Note that the couplings \(Q_3 U_3 H_+\) and \(Q_3 D_3 H_-\) are allowed by the \(G_F = SU(2)_L \otimes SU(2)_R \otimes U(1)_V\) symmetry. (This, in particular, is one of the reasons why we have chosen the \(U(1)_V\) symmetry to be “vector-like”.) This implies that we have large top and bottom quark masses with the “vertical” hierarchy generated as in \([24]\). On the other hand, the first two generations can only couple via
\[
\begin{align*}
z_a H_+ Q_i U_i \Sigma_a^i , \\
\bar{z}_a H_- Q_i D_i \Sigma_a^i ,
\end{align*}
\]
where the dimensionful couplings \(z_a \sim 1/M_s\) and \(\bar{z}_a \sim (m_b/m_t) z_a\) (here we are taking into account the “vertical” hierarchy mentioned above). Upon the fields \(\Sigma_a\) acquiring non-zero VEVs, we can generate non-zero masses for the second and first generations\(^{25}\). In particular,

\(^{24}\)One of the interesting phenomenological implications of the \(U(1)_V\) breaking would be the existence of new sub-millimeter forces which would compete with gravity \([14]\), and could be accessible at the upcoming sub-millimeter experiments \([13]\). Thus, \(U(1)_V\) must be a “bulk” gauge symmetry since “bulk” flavons are charged under it. This implies that upon breaking \(U(1)_V\) on a brane, the corresponding gauge \(U(1)_V\) boson acquires the mass around several inverse millimeters or so \([10]\).

\(^{25}\)Here we are not going to consider the detailed mechanism of how the VEVs for \(\Sigma_a\) are generated.
we can consider scenarios where the $1 - 2$ mixing occurs in both the up and down quark sectors, or mostly in, say, the down quark sector.

The $1 - 3$ and $2 - 3$ mixing arises due to the corresponding couplings to the flavons $\rho_3^\beta$ and $(\rho_{3'}^\beta)^*$:

$$v_\beta H U_i Q_3^\beta, \quad w_\beta H D_i Q_3^\beta,$$

$$v_{3'} H U_i Q_3^3(\rho_{3'}^\beta)^*,$$

$$w_{3'} H D_i Q_3^3(\rho_{3'}^\beta)^*,$$

where $v_\beta, v_{3'} \sim 1/M_s$, and $w_\beta, w_{3'} \sim (m_b/m_t)/M_s$. As we discussed in section III, we will assume that the $\rho_3^\beta, \rho_{3'}^\beta$ VEVs give rise to the $1 - 3$ and $2 - 3$ mixings comparable with those in the CKM matrix. This is compatible with the observed flavor hierarchy and the CKM matrix (for a recent discussion, see, e.g., [50], and references therein).

Before we end this subsection, we would like to comment on CP violation. Up until now we have been implicitly discussing the real parts of the dimensionful couplings $\xi$ corresponding to flavor violating four-fermion operators. The experimental bounds on the imaginary part of, say, $\xi_{(sd)^2}$ are about 100 times stronger than those on the real part. Here we would like to address the issue of whether these imaginary flavor violating couplings are adequately suppressed as well.

Let us first discuss the $\xi_{(sd)^2}$ and $\xi_{(uc)^2}$ couplings. Note that in the absence of the $1 - 3$ and $2 - 3$ mixing the up and down quark mass matrices are block-diagonal, and the CP violation is absent. Thus, in this limit we expect the imaginary parts of the corresponding flavor violating couplings to be vanishing. This implies that the CP violating imaginary couplings are suppressed by the $1 - 3$ and $2 - 3$ mixing angles. In fact, for these particular couplings this suppression can be seen to be adequate for $M_s \sim 10 - 100$ TeV.

Another coupling we must consider is $\text{Im}(\xi_{(bd)^2})$. A priori this coupling is not suppressed any more than the corresponding real part. However, suppose that the CP violating phases in the up and down quark mass matrices are exactly equal (or very close to) $\pi/2$ (that is, we have the maximal CP violation). When translated into the corresponding four-fermion couplings, these phases are squared, so that the imaginary parts of these couplings are vanishing. Thus, the maximal CP violation may provide a framework for suppressing the corresponding flavor violating couplings.

D. The Lepton Sector

So far we have discussed the quark sector of the model. The lepton sector deserves a separate consideration as the $U(1)_V$ charge assignments here are different from those in the to give a desirable flavor hierarchy. This is a very model dependent question, which needs to be addressed in any extension of the Standard Model. Let us, however, mention that there are more than one possibilities for generating such a hierarchy. Thus, for instance, the flavor symmetry could be broken via some dynamical mechanism on “distant” branes [19]. Then the flavor hierarchy in the “observable” sector is determined by a model dependent flavor symmetry breaking dynamics on these branes as well as their locations relative to the $p$-branes on which the quarks and leptons are localized.
quark sector. In particular, relatively large mixing angles between the first two and the third lepton generations might be desirable in the light of [51] (for a recent discussion, see, e.g. [52], and references therein). In principle there appears to be no difficulty in obtaining such large mixing angles as the corresponding flavons \( \eta, \eta' \) can acquire VEVs independent from those in the quark sector.

Next, we would like to discuss the following possibility. As was pointed out in [24], the \( \mathcal{L}_3 \otimes \mathcal{R}_3 \) discrete gauge symmetry is too strong for just proton stabilization purposes. Thus, its subgroup \( \mathcal{Y}_3 \) is just as efficient. This subgroup can be viewed as the \( \mathbb{Z}_3 \) subgroup of the \( U(1)_Y \) gauge symmetry, where the generator \( Y \) is given by \( Y = L - R \). The advantage of having the full \( \mathcal{L}_3 \otimes \mathcal{R}_3 \) discrete symmetry is that it automatically forbids the dangerous dimension 5 operator \( LLH^+H_+ \) which generically would result in too large Majorana neutrino masses (see section II for details). A priori there is a possibility, however, that this operator is suppressed due to the non-Abelian flavor gauge symmetries we have been considering in this paper. This possibility was originally pointed out in [21], and also briefly discussed in [24]. Now we are in the position to see whether this operator is indeed sufficiently suppressed in the context of a particular model we are considering in this paper.

It is convenient to discuss this issue using the language of the \( SU(2)_L \otimes SU(2)_R \otimes U(1)_V \) flavor gauge symmetry (as the conclusions are the same for the \( T_L \otimes T_R \) discrete flavor gauge symmetry). Let us first consider the unbroken flavor symmetry limit. Note that the \( SU(2)_L \otimes SU(2)_R \) invariant operator \( e^{ij}L_iL_jH_+H_+ \) vanishes due to antisymmetry. On the other hand, the \( L_3L_3H_+H_+ \) operator is forbidden by the \( U(1)_V \) flavor symmetry. In fact, even if we confine our attention to its \( \mathbb{Z}_N \) subgroup with \( N \geq 3 \), this latter operator is still absent.

Let us now consider the \( SU(2)_L \otimes SU(2)_R \) breaking by the flavon VEVs. Then a priori we are going to have VEV-induced operators of the form \( C^{ij}L_iL_jH_+H_+ \). Thus, for instance, we can have such operators with \( C^{ij} = \eta^i\eta^j \). (Here we could also consider the \( \rho_\beta \) flavons instead of the \( \eta \) flavons.) To suppress such operators we can introduce a \( U(1)_{V'} \) flavor gauge symmetry (or its appropriate discrete \( \mathbb{Z}_N \) subgroup), where the vector-like \( U(1)_{V'} \) symmetry acts as 3 times \( B - L \) on the first two generations only. However, then we have a “generation-blind” subgroup \( U(1)_Z \) of \( U(1)_{V'} \otimes U(1)_{V'} \) that acts as 3 times \( B - L \) on all three generations. As was explained in detail in [24], gauging \( U(1)_{V'} \) together with such a \( U(1)_Z \) gauge symmetry is equivalent (for our purposes here) to gauging the full \( U(1)_L \otimes U(1)_R \) symmetry. In particular, note that \( \mathcal{Z} = 3Y - 3\mathcal{R} \), where \( \mathcal{Z} \) acts on all three generations. (Here the generalization to the corresponding discrete subgroups should be evident.) In fact, it appears to be the case that we need two generation-blind discrete gauge symmetries to both ensure proton longevity and suppress the \( LLH_+H_+ \) operator.

The above discussion might have interesting phenomenological implications. Note that the \( \mathcal{Y}_3 \) discrete gauge symmetry allows all lepton number violating dimension 3 and 4 couplings in the TSSM [24], whereas the full \( \mathcal{L}_3 \otimes \mathcal{R}_3 \) discrete gauge symmetry forbids all such couplings. On the other hand, in the MSSM with high \( M_\star \) (of order \( M_{GUT} \)) the only Abelian generation-blind discrete gauge symmetry that forbids the dimension 5 operator \( QQQL \) (which would otherwise be disastrous for proton stability even in the case \( M_\star \sim M_{GUT} \)) and at the same time allows the \( LLH_+H_+ \) operator (which in this case is needed to generate the correct neutrino masses via the old “see-saw” mechanism [43]) is the \( \mathcal{Y}_3 \) discrete gauge symmetry [53,24]. This would lead to an interesting experimental “prediction” that if \( M_\star \)
is high (that is, $\sim M_{\text{GUT}}$ or so), the upcoming collider experiments should detect lepton number violating dimension 4 operators via channels involving sleptons. On the other hand, if $M_s$ is low (that is, in the TeV range), such operators should be absent. This implies that it might indeed be possible to indirectly deduce whether $M_s$ is high or low by examining the corresponding processes at the upcoming collider experiments without directly producing the heavy Kaluza-Klein or string states. This might be important as the nearest future collider experiments may not be able to directly probe such states if they are heavier than a few TeV.

V. SUMMARY AND OPEN QUESTIONS

Let us briefly summarize the discussions in the previous sections. We have considered the issue of FCNC suppression in the TSSM. We have shown that the $T_L \otimes T_R$ non-Abelian discrete flavor gauge symmetry is just as efficient in suppressing the corresponding four-fermion operators as $SU(2)_L \otimes SU(2)_R$ (whose subgroup the former is). On the other hand, the flavon VEV-induced operators can be adequately suppressed by introducing an additional $U(1)_V$ flavor gauge symmetry (or its appropriate discrete subgroup). Thus, flavor violation in the TSSM appears to be in the acceptable range for $M_s \sim 10 - 100$ TeV.

One of the many remaining open questions is how to explicitly embed the TSSM (or its variations) in the brane world framework. That is, it would be nice to have an explicit string construction of such a model. One of the promising directions in this regard appears to be a possible embedding into the Type I (Type I') framework. The recent progress in understanding four dimensional Type I compactifications [54,6,9] raises hope that this might not be out of reach. However, as was pointed out in [22,24], if there exists an embedding of the TSSM in the Type I framework, it appears to be within non-perturbative Type I compactifications. A better understanding of such Type I (as well as four dimensional F-theory [53]) compactifications is therefore more than desirable.

ACKNOWLEDGMENTS

I would like to thank Nima Arkani-Hamed, Tom Banks and especially Gia Dvali for useful discussions. This work was supported in part by the grant NSF PHY-96-02074, and the DOE 1994 OJI award. I would also like to thank Albert and Ribena Yu for financial support.
REFERENCES

[1] J. Polchinski, Phys. Rev. Lett. 75 (1995) 4724.
[2] E. Witten, Nucl. Phys. B471 (1996) 135.
[3] J. Lykken, Phys. Rev. D54 (1996) 3693.
[4] N. Arkani-Hamed, S. Dimopoulos and G. Dvali, Phys. Lett. B429 (1998) 263.
[5] K.R. Dienes, E. Dudas and T. Gherghetta, Phys. Lett. B436 (1998) 55; hep-ph/9806292, hep-ph/9807522.
[6] Z. Kakushadze, Phys. Lett. B434 (1998) 269; Nucl. Phys. B535 (1998) 311; Phys. Rev. D58 (1998) 101901.
[7] I. Antoniadis, N. Arkani-Hamed, S. Dimopoulos and G. Dvali, Phys. Lett. B436 (1998) 257.
[8] G. Shiu and S.-H.H. Tye, Phys. Rev. D58 (1998) 106007.
[9] Z. Kakushadze and S.-H.H. Tye, Phys. Rev. D58 (1998) 126001.
[10] N. Arkani-Hamed, S. Dimopoulos and G. Dvali, hep-ph/9807344.
[11] Z. Kakushadze and S.-H.H. Tye, hep-th/9809147.
[12] V. Rubakov and M. Shaposhnikov, Phys. Lett. B125 (1983) 136; A. Barnaveli and O. Kancheli, Sov. J. Nucl. Phys. 52 (1990) 576.
[13] G. Dvali and M. Shifman, Nucl. Phys. B504 (1997) 127; Phys. Lett. B396 (1997) 64; Erratum, ibid B407 (1997) 452.
[14] V. Kaplunovsky, Nucl. Phys. B307 (1988) 145; (E) Nucl. Phys. B382 (1992) 436.
[15] C. Bachas and C. Fabre, Nucl. Phys. B476 (1996) 418; I. Antoniadis, H. Partouche and T.R. Taylor, Nucl. Phys. B499 (1997) 29.
[16] K.R. Dienes, Phys. Rept. 287 (1997) 447.
[17] E. Cáceres, V.S. Kaplunovsky and I.M. Mandelberg, Nucl. Phys. B493 (1997) 73.
[18] S. Dimopoulos and H. Georgi, Nucl. Phys. B150 (1981) 193; W.J. Marciano and G. Senjanović, Phys. Rev. D25 (1982) 3092; C. Giunti, C.W. Kim and U.W. Lee, Mod. Phys. Lett. A6 (1991) 1745; J. Ellis, S. Kelley and D.V. Nanopoulos, Phys Lett. B249 (1990) 441; U. Amaldi, W. de Boer and H. Furstenau, Phys. Lett. B260 (1991) 447; P. Langacker and M. Luo, Phys. Rev. D44 (1991) 817.
[19] N. Arkani-Hamed and S. Dimopoulos, hep-ph/9811353.
[20] Z. Berezhiani and G. Dvali, hep-ph/9811375.
[21] N. Arkani-Hamed, S. Dimopoulos, G. Dvali and J. March-Russell, talk presented by S. Dimopoulos at SUSY’98, hep-ph/9811448.
[22] Z. Kakushadze, hep-th/9811193.
[23] I. Antoniadis and C. Bachas, hep-th/9812093.
[24] Z. Kakushadze, hep-th/9812163.
[25] G. Dvali and S.-H.H. Tye, hep-ph/9812483.
[26] A. Delgado, A. Pomarol and M. Quirós, hep-th/9812489.
[27] A. Pomarol and M. Quirós, Phys. Lett. B438 (1998) 255; C.P. Bachas, hep-ph/9807413; P.C. Argyres, S. Dimopoulos and J. March-Russell, hep-th/9808133; N. Arkani-Hamed, S. Dimopoulos and J. March-Russell, hep-th/9809124; D. Ghiilencea and G.G. Ross, hep-ph/9809217; K.R. Dienes, E. Dudas, T. Gherghetta and A. Riotto, hep-ph/9809406.
S. Abel and S. King, hep-ph/9809467; 
D. Lyth, hep-ph/9810320; 
I. Antoniadis, S. Dimopoulos, A. Pomarol and M. Quirós, hep-ph/9810410; 
G.F. Giudice, R. Rattazzi and J.D. Wells, hep-ph/9811291; 
S. Nussinov and R. Shrock, hep-ph/9811323; 
E.A. Mirabelli, M. Perelstein and M.E. Peskin, hep-ph/9811337; 
T. Han, J.D. Lykken and R.-J. Zhang, hep-ph/9811350; 
N. Kaloper and A. Linde, hep-th/9811141; 
J.L. Hewett, hep-ph/9811356; 
K.R. Dienes, E. Dudas and T. Gherghetta, hep-ph/9811428; 
P. Mathews, S. Raychaudhuri and K. Sridhar, hep-ph/9811501, hep-ph/9812486; 
T. Kobayashi, J. Kubo, M. Mondragon and G. Zoupanos, hep-ph/9812221; 
B.A. Dobrescu, hep-ph/9812349; 
T.G. Rizzo, hep-ph/9901209, hep-ph/9902273; 
A.E. Faraggi and M. Pospelov, hep-ph/9901299; 
K. Agashe and N.G. Deshpande, hep-ph/9902263; 
M.L. Graesser, hep-ph/9902310. 

[28] H. Hatanaka, T. Inami and C.S. Lim, Mod. Phys. Lett. A13 (1998) 2601; 
R. Sundrum, hep-ph/9805471, hep-ph/9807348; 
K. Benakli, hep-ph/9809582; 
L. Randall and R. Sundrum, hep-th/9810155; 
K. Benakli and S. Davidson, hep-ph/9810280; 
G.F. de Téramond, hep-ph/9810438; 
C.P. Burgess, L.E. Ibáñez and F. Quevedo, hep-ph/9810535; 
M. Maggiore and A. Riotto, hep-th/9811089; 
M. Drees, O.J.P. Éboli and J.K. Mizukoshi, hep-ph/9811343; 
M. Gogberashvili, hep-ph/9812296; 
L.E. Ibáñez, C. Muñoz and S. Rigolin, hep-ph/9812397; 
A. Donini and S. Rigolin, hep-ph/9901443; 
I. Antoniadis and B. Pioline, hep-th/9902055; 
M. Sakamoto, M. Tachibana and K. Takenaga, hep-th/9902070.

[29] I. Antoniadis, Phys. Lett. B246 (1990) 377; 
I. Antoniadis, C. Muñoz and M. Quirós, Nucl. Phys. B397 (1993) 515; 
I. Antoniadis, K. Benakli and M. Quirós, Phys. Lett. B331 (1994) 313; 
I. Antoniadis, S. Dimopoulos and G. Dvali, Nucl. Phys. B516 (1998) 70.

[30] T.R. Taylor and G. Veneziano, Phys. Lett. B212 (1988) 147.

[31] J. Scherk and J.H. Schwarz, Nucl. Phys. B153 (1979) 61; Phys. Lett. B82 (1979) 60; 
E. Cremmer, J. Scherk and J.H. Schwarz, Phys. Lett. B84 (1979) 83; 
P. Fayet, Phys. Lett. B159 (1985) 121; Nucl. Phys. B263 (1986) 649.

[32] See, e.g., 
S. Giddings and A. Strominger, Nucl. Phys. B307 (1988) 854; 
S. Coleman, Nucl. Phys. B310 (1988) 643; 
G. Gilbert, Nucl. Phys. B328 (1989) 159.

[33] M. Dine and N. Seiberg, Nucl. Phys. B306 (1988) 137; 
T. Banks and L. Dixon, Nucl. Phys. B307 (1988) 93.
[34] L. Krauss and F. Wilczek, Phys. Rev. Lett. **62** (1989) 1221.
[35] T. Banks, Nucl. Phys. **B323** (1990) 90;
   L. Krauss, Gen. Rel. Grav. **22** (1990) 50;
   M. Alford, J. March-Russell and F. Wilczek, Nucl. Phys. **B337** (1990) 695;
   J. Preskill and L. Krauss, Nucl. Phys. **B341** (1990) 50;
   M. Alford, S. Coleman and J. March-Russell, Nucl. Phys. **B351** (1991) 735.
[36] M. Dine and N. Seiberg, Phys. Rev. Lett. **55** (1985) 366; Phys. Lett. **B162** (1985) 299;
   V.S. Kaplunovsky, Phys. Rev. Lett. **55** (1985) 1036;
   T. Banks and M. Dine, Phys. Rev. **D50** (1994) 7454.
[37] See, *e.g.*, N.V. Krasnikov, Phys. Lett. **B193** (1987) 37;
   L. Dixon, V. Kaplunovsky, J. Louis and M. Peskin, SLAC-PUB-5229 (1990);
   J.A. Casas, Z. Lalak, C. Muñoz and G.G. Ross, Nucl. Phys. **B347** (1990) 243;
   T.R. Taylor, Phys. Lett. **B252** (1990) 59;
   V.S. Kaplunovsky and J. Louis, Phys. Lett. **B417** (1998) 45;
   G. Dvali and Z. Kakushadze, Phys. Lett. **B417** (1998) 50;
   M. Klein and J. Louis, Nucl. Phys. **B533** (1998) 163;
   C.P. Burgess, A. de la Macorra, I. Maksymyk and F. Quevedo, J. High Energy Phys. **9809** (1998) 007;
   K.-I. Izawa and T. Yanagida, [hep-ph/9809366](http://arxiv.org/abs/hep-ph/9809366).
[38] N. Arkani-Hamed and M. Schmaltz, to appear.
[39] T. Banks, M. Dine and A. Nelson, to appear.
[40] G. ’t Hooft, Nucl. Phys. **B72** (1974) 461.
[41] L.E. Ibáñez and G.G. Ross, Phys. Lett. **B260** (1991) 291; Nucl. Phys. **B368** (1992) 3;
   J. Preskill, S.P. Trivedi, F. Wilczek and M.B. Wise, Nucl. Phys. **363** (1991) 207;
   T. Banks and M. Dine, Phys. Rev. **D45** (1992) 1424.
[42] S. Weinberg, Phys. Rev. **D26** (1982) 287;
   N. Sakai and T. Yanagida, Nucl. Phys. **B197** (1982) 533.
[43] M. Gell-Mann, P. Ramond and R. Slansky, in: “Supergravity”, eds. P. van Nieuwenhuizen and D. Freedman (Amsterdam, North-Holland, 1979) p. 315;
   T. Yanagida, in: “Workshop on Unified Theory and Baryon Number in the Universe”,
   eds. O. Sawada and A. Sugamoto (KEK, Tsukuba, 1979) p. 95;
   R.N. Mohapatra and G. Senjanović, Phys. Rev. Lett. **44** (1980) 912.
[44] C. Caso *et al.*, Review of Particle Physics, Eur. Phys. J. **C3** (1998) 1.
[45] See, *e.g.*, N. Arkani-Hamed, C.D. Carone, L.J. Hall and H. Murayama, Phys. Rev. **D54** (1996) 7032, and references therein.
[46] For some early works, see, *e.g.*, F. Wilczek and A. Zee, Phys. Rev. Lett. **42** (1979) 421;
   J. Chkareuli, JETP Lett. **32** (1980) 671;
   Z. Berezhiani and J. Chkareuli, Sov. J. Nucl. Phys. **37** (1983) 618.
[47] See, *e.g.*, M. Dine, R. Leigh and A. Kagan, Phys. Rev. **D48** (1993) 4269;
   Y. Nir and N. Seiberg, Phys. Lett. **B309** (1993) 337;
   P. Pouliot and N. Seiberg, Phys. Lett. **B318** (1993) 169;
D. Kaplan and M. Schmaltz, Phys. Rev. D48 (1993) 4269;
R. Barbieri, G. Dvali and A. Strumia, Nucl. Phys. B435 (1995) 102;
A. Pomarol and D. Tommasini, Nucl. Phys. B466 (1996) 3;
L.J. Hall and H. Murayama, Phys. Rev. Lett. 75 (1995) 3985;
P.H. Frampton and O.C.W. Kong, Phys. Rev. D53 (1995) 2293; Phys. Rev. Lett. 77 (1996) 1699;
P.H. Frampton and T.W. Kephart, Phys. Rev. D51 (1995) 1;
R. Barbieri, G. Dvali and L.J. Hall, Phys. Lett. B337 (1996) 76;
N. Arkani-Hamed, H.C. Cheng and L.J. Hall, Nucl. Phys. B472 (1996) 95; Phys. Rev. D54 (1996) 2242;
K.S. Babu and S.M. Barr, Phys. Lett. B387 (1996) 87;
R. Barbieri and L.J. Hall, Nuovo Cim. 110A (1997) 1;
Z. Berezhiani, Nucl. Phys. Proc. Suppl. 52A (1997) 153; hep-ph/9609342;
A. Rašin, Phys. Rev. D57 (1998) 3977;
C.D. Carone, L.J. Hall and T. Moroi, Phys. Rev. D56 (1997) 7183;
G. Dvali and Z. Kakushadze, Phys. Lett. B426 (1998) 78.

[48] See, e.g.,
F. Wilczek, Phys. Rev. Lett. 49 (1982) 1549;
D. Reiss, Phys. Lett. B115 (1982) 217;
G. Gelmini, S. Nussinov and T. Yanagida, Nucl. Phys. B219 (1983) 311;
A. Anselm and N. Uraltsev, Sov. Phys. JETP 57 (1983) 1142;
A. Anselm and Z. Berezhiani, Phys. Lett. B162 (1985) 349;
A. Anselm, N. Uraltsev and M. Khlopov, Sov. J. Nucl. Phys. 41 (1985) 1056;
J.E. Kim, Phys. Rept. 150 (1987) 1;
M. Khlopov and R. Homeriki, Sov. J. Nucl. Phys. 51 (1990) 935;
J.L. Feng, T. Moroi, H. Murayama and E. Schnapka, Phys. Rev. D57 (1998) 5875.

[49] J.C. Price, in: Proceedings of the International Symposium on Experimental Gravita-
tional Physics, ed. P.F. Michelson (World Scientific, Singapore, 1988) 436-439;
J.C. Price et al., NSF proposal (1996);
A. Kapitulnik and T. Kenny, NSF proposal (1997);
J.C. Long, H.W. Chan and J.C. Price, hep-ph/9805217.

[50] J.L. Chkareuli and C.D. Froggatt, hep-ph/9812499.

[51] Super-Kamiokande Collaboration, Y. Fukuda et al., Phys. Lett. B433 (1998) 9; ibid. B436 (1998) 33;
Kamiokande Collaboration, S. Hatakeyama et al., Phys. Rev. Lett. 81 (1998) 2016.

[52] R. Barbieri, L.J. Hall, G.L. Kane and G.G. Ross, hep-ph/9901228.

[53] L.E. Ibáñez and G.G. Ross, Nucl. Phys. B368 (1992) 3.

[54] M. Berkooz and R.G. Leigh, Nucl. Phys. B483 (1997) 187;
C. Angelantonj, M. Bianchi, G. Pradisi, A. Sagnotti and Ya.S. Stanev, Phys. Lett. B385 (1996) 96;
Z. Kakushadze, Nucl. Phys. B512 (1998) 221;
Z. Kakushadze and G. Shiu, Phys. Rev. D56 (1997) 3686; Nucl. Phys. B520 (1998) 75;
G. Aldazabal, A. Font, L.E. Ibáñez, A.M. Uranga and G. Violero, Nucl. Phys. B519 (1998) 239;
G. Zwart, Nucl. Phys. B526 (1998) 378;
G. Aldazabal, A. Font, L.E. Ibáñez and G. Violero, Nucl. Phys. B536 (1999) 29;
Z. Kakushadze, G. Shiu and S.-H.H. Tye, Nucl. Phys. B533 (1998) 25;
R. Blumenhagen and A. Wisskirchen, Phys. Lett. B438 (1998) 52;
J.D. Lykken, E. Poppitz and S. Trivedi, hep-th/9806080;
L.E. Ibáñez, R. Rabadan and A.M. Uranga, hep-th/9808133;
C. Angelantonj, hep-th/9810214;
I. Antoniadis, G. D’Appollonio, E. Dudas and A. Sagnotti, hep-th/9812118.
[55] C. Vafa, Nucl. Phys. B469 (1996) 403.