S- AND D-WAVE PAIRING IN SHORT COHERENCE LENGTH SUPERCONDUCTORS

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1. Introduction

Over the past three years or so the evidence has become overwhelming that the cuprate high \( T_c \) superconductors have a \( d \)-wave pairing state\[1\]. The measurements of Wollman \textit{et al.}\[2\] and of Tsuei \textit{et al.}\[3\] are especially convincing since they do not depend on the microscopic physics of the energy gap, but instead depend only on the order parameter phase. Other experiments, such as photoemission\[4, 5\] and the temperature dependence of the penetration depth\[6, 7, 8\], also strongly support the \( d \)-wave picture.

On the other hand, there is continuing controversy over whether the pairing state is a pure \( d \)-wave or an \( s-d \) mixture\[9, 10\]. There is indeed evidence for a significant \( s \)-wave component in YBa\(_2\)Cu\(_3\)O\(_7\)\[11\]. A subdominant \( s \)-wave component could be compatible with the Wollman \textit{et al.} and the Tsuei \textit{et al.} experiments provided that it was not too large. The photoemission and penetration depth measurements also cannot rule out a small \( s \)-wave component (either \( s \pm d \) or \( s \pm id \)), although they can possibly put upper bounds on the magnitude of the \( s \) component.

Any observations of an \( s \) component have important implications for the various theories of the pairing mechanism. For example, antiferromagnetic spin fluctuations lead to attraction in the \( dx^2-\!y^2 \) pairing channel, but are pair breaking in the \( s \)-wave channel\[12\]. Similarly the Hubbard model with a positive on-site interaction \( U \) may have a \( dx^2-\!y^2 \) paired ground state, but would presumably not support \( s \)-wave Cooper pairs. In the case of YBa\(_2\)Cu\(_3\)O\(_7\) the orthorhombic crystal symmetry makes some non-zero \( s \)-wave component inevitable, but a large \( s \)-wave component would be dif-
ficult to reconcile with either of these pairing mechanisms. On the other hand, pairing mechanisms based on electron-phonon interactions, polarons, or other non-magnetic excitations (e.g. excitons, acoustic plasmons) could be compatible with either s-wave or d-wave pairing states[13]. In these models, whether s-wave, d-wave or mixed pairs are more strongly favoured would depend on details of the model parameters, and could even vary from compound to compound. Indeed there is some evidence that the n-type cuprate superconductors are s-wave[14] (or at least they have no zeros in the gap $|\Delta(k)|$ on the Fermi surface). This would imply that either the pairing mechanism is different for the n- and p-type materials, or that the mechanism allows both s-wave or d-wave ground states depending on the band filling.

In this paper we examine the attractive nearest neighbour Hubbard model, which is the simplest model that allows s-wave, d-wave or mixed pairing states to occur[15]. We examine the overall phase diagram, paying particular attention to the regions near the phase transitions between the s, d and $s \pm id$ phases. We derive an appropriate Landau-Ginzburg-Wilson (LGW) effective action for the model. In the mean-field approximation this LGW functional reproduces the usual Hartree-Fock-Gor’kov equations. Beyond mean field theory this functional allows us to examine the large interaction limit, in which the superconducting phase transition becomes a Bose-Einstein condensation of preformed pairs. In this limit there will be a pseudogap in the normal state density of states. The strong coupling limit is especially interesting in this model because, unlike the case of on-site interactions, there are at least two ‘species’ of preformed pairs, $d_{x^2-y^2}$ and $s$. We show below that these become degenerate in the strong coupling Bose-Einstein limit.

2. The Nearest Neighbour Attractive Hubbard Model

Our starting point is the nearest-neighbour attractive Hubbard model in two dimensions,

$$\hat{H} = -t \sum_{\langle ij \rangle \sigma} (c_{i\sigma}^\dagger c_{j\sigma} + H.C.) - V \sum_{\langle ij \rangle} n_i n_j, \tag{1}$$

where $\sum_{\langle ij \rangle}$ denotes summation over all of the bonds between the nearest neighbour sites. For simplicity we ignore any on-site interaction terms.

The mean-field gap equations can be used straightforwardly to calculate $T_c$ for this model (Section 4). The gap equation has stable solutions for (extended) s-wave, p-wave, and d-wave pairing states, depending on the model parameters. One can see in Fig. 1(a)-(c) that, for all $V$, the d-wave
Figure 1. Mean-field transition temperature, $T_c$, versus chemical potential, $\mu$, for $s$-, $p$- and $d$-wave superconductivity at different interaction strengths; (a) $V = 1t$, (b) $V = 5t$ and (c) $V = 8t$. 
state is strongly favoured at and near to the van Hove singularity at \( \mu = 0 \), while the \( s \)-wave state is favoured in the limits of nearly full or empty bands. The \( p \)-wave solution also has a maximum at \( \mu = 0 \) but is always small compared with the \( d \)-wave. The mean-field phase diagram also includes a small region of \( s \pm id \) pairing (not shown in Fig. 1) just below the points where the \( d \)- and \( s \)-wave \( T_c \) cross. These crossing points are therefore tetracritical points, where all four phases (normal, \( s \), \( d \) and \( s \pm id \)) meet. Below we shall closely examine the nature of the phase diagram in the vicinity of these tetracritical points.

If next nearest neighbour hopping terms are also included in the Hamiltonian then there is no longer particle-hole symmetry about half-filling and it is possible to have \( d \)-wave pairing predominantly for \( p \)-type doping and \( s \)-wave pairing predominantly for \( n \)-type doping. This would be qualitatively consistent with the experimental evidence for \( s \)-wave pairing in the \( n \)-type cuprates[14]. However, for simplicity, below we shall concentrate on the case of nearest-neighbour hopping only.

3. The Landau-Ginzburg-Wilson Effective Action

In order to examine how fluctuations modify the mean-field picture of Fig. 1 it is useful to develop a Landau-Ginzburg-Wilson (LGW) effective action. This allows us to examine both the weak coupling, \( V \ll 8t \), and the strong coupling, \( V \gg 8t \), limits on an equal footing. Only in the strong or intermediate coupling regime will fluctuation effects lead to phenomena such as the normal-state pseudogap which is observed in several different high \( T_c \) materials[16].

The Landau-Ginzburg-Wilson effective action of the nearest-neighbour Hubbard model can be derived in a way similar to the usual \( s \)-wave case of on-site interaction[17]. Firstly the grand canonical partition function is written:

\[
Z = \int \mathcal{D}[c, c^\dagger] e^S, \tag{2}
\]

\[
S = \int_0^\beta d\tau \left[ \sum \sigma c^\dagger_{i\sigma}(\tau) \partial_\tau c_{i\sigma}(\tau) - \hat{H}_0 - \hat{H}_I \right], \tag{3}
\]

where \( \hat{H}_0 \) is given by the hopping term in the Hamiltonian Eq. 1 (including a chemical potential term, \(-\mu \hat{N}\)) and \( \hat{H}_I \) is the interaction term.

The interacting part may be neatly written in terms of pairing operators

\[
\]
on nearest neighbour sites:

\[ \hat{H}_I = -V \sum_{\langle ij \rangle} \text{tr} \left( F_{ij}^\dagger F_{ij} \right), \]  

(4)

where \( F_{ij} \) is defined by:

\[ F_{ij} = \begin{pmatrix} c_{j\uparrow}c_{i\uparrow} & c_{j\downarrow}c_{i\uparrow} \\ c_{j\uparrow}c_{i\downarrow} & c_{j\downarrow}c_{i\downarrow} \end{pmatrix}. \]  

(5)

The physical nature of the interaction is best illustrated by introducing a new set of operators,

\[ B_{i0}^{00} \equiv \frac{1}{\sqrt{2}}(c_{j\uparrow}c_{i\downarrow} - c_{j\downarrow}c_{i\uparrow}), \]  

(6)

\[ B_{i1}^{11} \equiv c_{j\uparrow}c_{i\uparrow}, \]  

(7)

\[ B_{i0}^{10} \equiv \frac{1}{\sqrt{2}}(c_{j\uparrow}c_{i\downarrow} + c_{j\downarrow}c_{i\uparrow}), \]  

(8)

\[ B_{i1}^{1\bar{1}} \equiv c_{j\downarrow}c_{i\downarrow}. \]  

(9)

In terms of these, \( F_{ij} \) may be decomposed as

\[ F_{ij} = \frac{i\sigma_y}{\sqrt{2}} (B_{ij} - \sigma \cdot B_{ij}), \]  

(10)

where \( \sigma \) is the vector of Pauli matrices, and

\[ B_{ij} \equiv B_{i0}^{00}, \]  

(11)

\[ B_{ij} \equiv \begin{pmatrix} B_{ij}^{11} - B_{ij}^{1\bar{1}}, B_{ij}^{11} + B_{ij}^{1\bar{1}} \\ B_{ij}^{1\bar{1}}, -i\sqrt{2} \end{pmatrix}. \]  

(12)

The scalar, \( B_{ij} \), is even under both time reversal and parity whilst the vector, \( B_{ij} \), is odd under both. We interpret \( B_{ij} \) and \( B_{ij} \) as the annihilation operators for singlet and triplet Cooper pairs on the bond \( \langle ij \rangle \)[18].

In terms of these operators the interaction Hamiltonian becomes:

\[ \hat{H}_I = -V \sum_{\langle ij \rangle} \text{tr} \left( F_{ij}^\dagger F_{ij} \right), \]

\[ = -V \sum_{\langle ij \rangle} \left( B_{ij}^\dagger B_{ij} + B_{ij}^\dagger \cdot B_{ij} \right), \]  

(13)

where we have used \( \text{tr}(\sigma_i) = 0 \) and \( \text{tr}(\sigma_i\sigma_j) = 2\delta_{ij} \).
Since this is a sum of squared bilinear Fermi operators, the Hubbard-Stratonović transformation [17] can be employed. The Gaussian identity,

$$e^{+VA^\dagger A} = \frac{V}{2\pi i} \int d\phi d\phi^* e^{-V(|\phi|^2 + A\phi + A^\dagger \phi)},$$

(14)

where $A$ is a bilinear Fermi operator and $\phi$ is a c-number, allows us to decouple the quartic interaction term in terms of new bosonic fields. In the case of Eq. 13 this is accomplished by introducing a complex scalar field, $\psi_{ij}(\tau)$, and a complex vector field, $\Psi_{ij}(\tau)$, for each bond in the lattice and every imaginary time. The partition function can now be expressed in terms of these Bose fields:

$$Z \equiv \int \mathcal{D}[\psi, \psi^*; \Psi, \Psi^*] e^{S_b}.$$  

(15)

The effective action is

$$S_b \equiv -V \int_0^\beta d\tau \sum_{<ij>} \left( |\psi_{ij}(\tau)|^2 + |\Psi_{ij}(\tau)|^2 \right) + \ln \int \mathcal{D}[c, c^\dagger] e^{S_f},$$

(16)

with the remaining fermions contained in

$$S_f \equiv \int_0^\beta d\tau \left[ \sum_{i\sigma} c_{i\sigma}^\dagger(\tau) \partial_\tau c_{i\sigma}(\tau) - \hat{H}_0 
- V \sum_{<ij>} \left( \psi_{ij}^*(\tau) B_{ij}(\tau) + \Psi_{ij}^*(\tau) \cdot B_{ij}(\tau) + H.C. \right) \right].$$

(17)

Equations 15-17 are a formally exact representation of the nearest-neighbour attractive Hubbard model.

The bosonic fields introduced above are defined separately for each bond $<ij>$ in the lattice. It is more convenient to form site-centred combinations with a definite symmetry. We define two singlet fields at site $r_i$

$$\psi_s = \frac{1}{2} (\psi_x + \psi_{-x} + \psi_y + \psi_{-y}),$$

$$\psi_d = \frac{1}{2} (\psi_x + \psi_{-x} - \psi_y - \psi_{-y}),$$

(18)

where $\psi_a(r_i, \tau) \equiv \psi_{ii+a}(\tau)$ for $a = \pm x, \pm y$ according to the direction of the bond $<ij>$. Similarly we can define the symmetrised triplet fields

$$\Psi_{p_y} = \Psi_y - \Psi_{-y},$$

$$\Psi_{p_y} = \Psi_y - \Psi_{-y}.$$  

(19)
In the limit of fields varying slowly in space and time these correspond to the Ginzburg-Landau order parameters for (extended) s-wave, \( d_{x^2-y^2} \) and \( p \)-wave pairing, respectively. The s- and d-wave pairing fields are even under inversion symmetry about lattice site \( r_i \), while the triplet fields are odd. In the notation of Ref. [19], these fields are the order parameters for superconductivity in, respectively, the \( A_{1g} \) and \( B_{1g} \) and \( E_u \) representations of the tetragonal point group \( D_{4h} \).

4. The Saddle Point Solutions

The saddle points of the effective action generate Hartree-Fock-Gor’kov mean field theory, where the Bose fields become static and spatially uniform order parameters:

\[
\psi_\alpha = -\frac{1}{\beta N} \int_0^{\beta} \sum_i \langle B_\alpha(r_i, \tau) \rangle_f, \quad \alpha = s, d
\]

\[
\Psi_\alpha = -\frac{1}{\beta N} \int_0^{\beta} \sum_i \langle B_\alpha(r_i, \tau) \rangle_f, \quad \alpha = x, y
\]

where \( N \) is the number of sites and \( \langle \ldots \rangle_f \) denotes self-consistent averaging with respect to the fermionic part of the action, \( S_f \).

The transition temperature for a given order parameter, in the absence of any of the others, is given by the solution of

\[
1 = \frac{V}{2} \sum_\epsilon \frac{N^\alpha(\epsilon)}{\epsilon - \mu} \tanh \left( \frac{\epsilon - \mu}{2T_c^\alpha} \right),
\]

in which the weighted density of states, \( N^\alpha(\epsilon) \), is

\[
N^\alpha = \frac{1}{N} \sum_k \zeta^\alpha(k)\zeta^\alpha(k)\delta(\epsilon - \epsilon_k).
\]

The form factors reflect the point group symmetries of the order parameters:

\[
\zeta^\alpha(k) = \begin{cases} 
\cos(k_x) + \cos(k_y) & \alpha = s, \\
\cos(k_x) - \cos(k_y) & \alpha = d, \\
\sin(k_x) & \alpha = p_x, \\
\sin(k_y) & \alpha = p_y.
\end{cases}
\]

The solutions of Eq. 22 are shown in Fig. 1 for three values of \( V \). The s-wave solution dominates for small and large fillings with d-wave dominant near the van Hove peak at the centre of the band. The p-wave solutions are sub-dominant everywhere. It is expected that interaction with the large
\(d\)-wave order parameter will suppress the \(p\)-wave \(T_c\) even further. It is clear from this that \(p\)-wave pairing is irrelevant in the bulk superconductor, and henceforth we ignore it and concentrate on \(s\)- and \(d\)-wave pairs only.

5. Beyond the Saddle Point: \(s\)- and \(d\)-wave Mixing

Starting with the effective action, \(S_b\), we Fourier transform in space and (imaginary) time and integrate out the fermions to give a purely bosonic action,

\[
S_b = -\frac{V}{2} \sum_{q,\alpha} |\psi_{\alpha}(q)|^2 + \text{Tr} \ln(1 - VG_0),
\]

where \(q \equiv (q, i\omega)\), and the trace is over both space-time and spinor indices. The fermions of the original theory live on in the form of the Nambu Green’s function matrix,

\[
G_0(k, k') \equiv \left( \begin{array}{cc} G_0(k) & 0 \\ 0 & -G_0^*(k) \end{array} \right) \delta_{kk'},
\]

in which \(G_0(k) = (i\omega_n - \epsilon_k + \mu)^{-1}\) is the Green’s function for non-interacting fermions. The interaction of fermions and bosons occurs through the potential matrix,

\[
V(k, k') \equiv \frac{V}{\sqrt{2\beta N}} \left( \begin{array}{cc} 0 & \psi_{\alpha}(k - k') \\ \psi_{\alpha}^*(-k + k') & 0 \end{array} \right) \zeta_{\alpha k, k'},
\]

where the Einstein summation convention has been used for repeated Greek indices and,

\[
\zeta_{\alpha k, k'} \equiv \frac{1}{2} \left( \cos(k_x) + \cos(k'_x) \right) \pm \frac{1}{2} \left( \cos(k_y) + \cos(k'_y) \right), \quad \alpha = s, d.
\]

When \(\psi_{\alpha}\) is constant in real space (i.e. \(k = k'\)), this takes a particularly simple form,

\[
\zeta^\alpha(k) \equiv \zeta_{\alpha k, k}^\alpha = \cos(k_x) \pm \cos(k_y), \quad \alpha = s, d
\]

as seen in the weighted densities of states.

Near \(T_c\), where the \(\psi_{\alpha}\) are small, we expand the logarithm as a power series up to fourth order;

\[
\text{Tr} \ln(1 - VG_0) = -\sum_{m=1}^{4} \frac{1}{m} \text{Tr} (VG_0)^m + O(\psi^5).
\]
The odd terms in the series are exactly zero but the even terms survive. The quadratic contribution is
\[
\text{Tr} (VG_0)^2 = V^2 \sum_q \psi^*_\alpha(q) \psi_\beta(q) \chi^{\alpha\beta}(q),
\]
where the susceptibility is:
\[
\chi^{\alpha\beta}(q) = -\frac{1}{\beta N} \sum_k G_0(k) G_0^*(k+q) \zeta_{k,k+q}^\alpha \zeta_{k+q,k}^\beta,
\]

(32)

The quartic term is
\[
\text{Tr} (VG_0)^4 = \frac{V^4}{2\beta N} \sum_{\{q\}} \psi^*_\alpha(q) \psi_\beta(q') \psi^*_\gamma(q'') \psi_\delta(q-q'+q'') \chi^{\alpha\beta\gamma\delta}(q,q',q'').
\]

(33)

It is sufficient to evaluate the four body susceptibility at \(q = q' = q'' = 0\) and then to treat it as a constant, \(\chi \equiv \chi(0,0,0):\)
\[
\chi^{\alpha\beta\gamma\delta} = \frac{1}{\beta N} \sum_k |G_0(k)|^4 \zeta_{k}^\alpha \zeta_{k}^\beta \zeta_{k}^\gamma \zeta_{k}^\delta,
\]

(34)

where \(\xi \equiv \epsilon_k - \mu\).

Thus, after a trivial rescaling of \(\psi\) by \(V^{1/2}\), the bosonic action to fourth order reads:
\[
\mathcal{S}_b \approx -\frac{1}{2} \sum_{q,\alpha\beta} \left( \delta^{\alpha\beta} + V \chi^{\alpha\beta}(q) \right) \psi^*_\alpha(q) \psi_\beta(q) - \frac{V^2}{8\beta N} \chi^{\alpha\beta\gamma\delta} \sum_{\{q\}} \psi^*_\alpha(q) \psi_\beta(q') \psi^*_\gamma(q'') \psi_\delta(q-q'+q'').
\]

(35)

In order to derive a Landau-Ginzburg-Wilson functional from \(\mathcal{S}_b\), we expand to lowest order in small \(i\omega_\nu\) and \(|q|\):
\[
\frac{1}{2} \left( \delta^{\alpha\beta} + V \chi^{\alpha\beta}(q) \right) \approx \alpha^{\alpha\beta} - id^{\alpha\beta} \omega_\nu + \sum_{\mu=x,y} \frac{q_\mu}{2m^{\mu}_{\alpha\beta}},
\]

(36)
where:

\[ a^{\alpha\beta} = \frac{1}{2} \left( \delta^{\alpha\beta} - \frac{V}{2N} \sum_{k} \left( \frac{\tanh (\beta \xi/2)}{\xi} \right) \zeta^{\alpha}(k) \zeta^{\beta}(k) \right), \quad (37) \]

\[ d^{\alpha\beta} = -\frac{V}{8N} \sum_{k} \left( \frac{\tanh (\beta \xi/2)}{\xi^2} \right) \zeta^{\alpha}(k) \zeta^{\beta}(k), \quad (38) \]

\[ \frac{1}{2m'_{\alpha\beta}} = -\frac{V}{16N} \sum_{k} \left\{ \left( \frac{\partial^2 \epsilon_k}{\partial k_{\mu}^2} \right) \left( \frac{d}{d\xi} \left( \frac{\tanh (\beta \xi/2)}{\xi} \right) \right) \zeta^{\alpha}(k) \zeta^{\beta}(k) \right. \]

\[ + \left( \frac{\partial \epsilon_k}{\partial k_{\mu}} \right) \left( \frac{d^2}{d\xi^2} + \frac{1}{\xi} \frac{d}{d\xi} \right) \left( \frac{\tanh (\beta \xi/2)}{\xi} \right) \zeta^{\alpha}(k) \zeta^{\beta}(k) \]

\[ + 2 \left( \frac{\partial \epsilon_k}{\partial k_{\mu}} \right) \frac{d}{d\xi} \left( \frac{\tanh (\beta \xi/2)}{\xi} \right) \frac{\partial (\zeta^{\alpha}_{k,k+q} \zeta^{\beta}_{k,k+q,k})}{\partial q_{\mu}} \right|_{q=0} \]

\[ + 2 \left( \frac{\tanh (\beta \xi/2)}{\xi} \right) \frac{\partial^2 (\zeta^{\alpha}_{k,k+q} \zeta^{\beta}_{k,k+q,k})}{\partial q_{\mu}^2} \right|_{q=0} \}

where \( \xi \equiv \epsilon_k - \mu \). The last two terms in Eq. 39 are due to the \( q \) dependence of \( \zeta^{\alpha} \).

The resulting LGW functional is shown below expressed in real space:

\[ S_{\text{LGW}} = -\int_0^\beta d\tau \sum_r [K + V], \quad (40) \]

where \( K \) is the ‘kinetic’ part and \( V \) the ‘potential’ part of the action. They are given by,

\[ K = d^{ss} \psi_s^* \partial \tau \psi_s + \frac{[\nabla \psi_s]^2}{2m_{ss}} + d^{dd} \psi_d^* \partial \tau \psi_d + \frac{[\nabla \psi_d]^2}{2m_{dd}} \]

\[ + \frac{1}{2m_{sd}} \left( \psi_s^* (\nabla_x^2 - \nabla_y^2) \psi_d + C.C. \right), \]

\[ V = a^{ss} |\psi_s|^2 + a^{dd} |\psi_d|^2 + b_s |\psi_s|^4 + b_d |\psi_d|^4 \]

\[ + \kappa |\psi_s|^2 |\psi_d|^2 + \frac{\kappa}{4} (\psi_s \psi_s \psi_d^* \psi_d^* + C.C.), \quad (42) \]

where,

\[ b_s = \frac{V^2}{8} \chi^{ssss}, \quad b_d = \frac{V^2}{8} \chi^{dddd}, \quad \kappa = \frac{V^2}{2} \chi^{ssdd}. \quad (43) \]

In the static and spatially uniform limit, \( V \) may be interpreted as the Landau free energy. We will examine the phases arising from this before considering further the effects of fluctuations. The free energy given in Eqs. 41-42 is of the form found earlier by Joynt [20] and by Feder and Kallin [21].
6. Landau Theory

The phase diagram generated by $V$ is found by simultaneously solving,
\[
\frac{\partial V}{\partial \psi_s} = \frac{\partial V}{\partial \psi_d} = 0. \tag{44}
\]

When $\kappa = 0$, i.e. there is no coupling between the $s$- and $d$-wave order parameters, solving $a^{ss} = 0$ and $a^{dd} = 0$ as functions of $\mu$ gives the critical temperature for each. This is equivalent to solving the linearised gap equation, Eq. 22.

For non-zero $\kappa$ it is possible to have mixed phases. To determine the stable phase, two parameters are needed. These are:
\[
B_s = \frac{b_s}{b_d} \left| \frac{a^{dd}}{a^{ss}} \right|^2, \tag{45}
\]
\[
K_c = \frac{\kappa(1 + \frac{1}{2} \cos(2\Delta \theta))}{b_d} \left| \frac{a^{dd}}{a^{ss}} \right|, \tag{46}
\]

where $\Delta \theta$ is the phase difference between $\psi_s$ and $\psi_d$. Fig. 2 shows the stable superconducting phase as a function of these two dimensionless parameters. Note that in Fig. 2 the definition of $K_c$ is different for $\kappa > 0$, where $(\Delta \theta = \pi/2)$, and $\kappa < 0$, where $(\Delta \theta = 0)$.

As Fig. 2 shows, depending on the parameter values $s$, $d$, $s \pm d$ or $s \pm id$ phases are possible. The transition from the pure $s$ to the pure $d$ phase can occur in a number of ways. For $K_c > 2$, a single first-order transition separates the two phases, analogous to a ‘spin flop’ transition. In this case, the $O(4)$ point (corresponding to $B_s = 1, K_c = 2$) represents the bicritical point where the line of first-order transitions $s \rightarrow d$ meets the two normal $\rightarrow$ superconducting second-order lines. When $K_c < 2$ the transition occurs via two second-order phase transitions with an intermediate mixed symmetry state, either $s \pm d$ or $s \pm id$. The $O(4)$ point here represents the meeting of the four second-order lines at a tetracritical point.

Using parameters derived from the nearest neighbour attractive Hubbard model, the dashed line in Fig. 2 shows the evolution of $(B_s, K_c)$ for $V = 2$ as $\mu$ changes in the region near the crossing of $T_{sc}^s$ and $T_{dc}^d$. As $\mu$ increases we move from the far right hand side of the figure, where $T=T_s^d <T_c^d$ and $d$-wave is dominant, to the far left, where $T=T_c^d <T_s^d$ and $s$-wave is dominant. In these extremes, the dominant $d$-wave ($s$-wave) order parameter suppresses the sub-dominant $s$-wave ($d$-wave) one even though $T<T_c^s(T_c^d)$. 
Eventually, however, a mixed $s \pm id$ phase arises. This behaviour is seen for all $V$. In particular the mixed $s \pm d$ phase is never realised for any values of $B_s$ and $K_c$ derived from the nearest neighbour attractive Hubbard model unless an orthorhombic distortion is introduced which is beyond the scope of this paper.

7. The Large $V$ Limit

In the large $V$ limit it is necessary to retain the thermal and quantum fluctuations in the Bose fields derived in section 4. The superconducting phase transition then becomes the (Kosterlitz-Thouless in 2-dimensions) Bose condensation of these quantum fields. A detailed analysis is beyond the scope of the present paper, but it is possible to draw some qualitative conclusions based on the effective action derived in sections 3 and 5.

Fig. 1 shows the evolution of the $T_c$ for $s$, $d$ and $p$-wave pairing as a function of $\mu$ for various values of $V/t$. The small $V$ limit, represented by Fig. 1(a), corresponds to the BCS weak coupling limit in which the pairing is a small perturbation on the filled Fermi sea. On the other hand as $V$ is increased (Fig 1(b)-(c)) the behaviour changes markedly. One can see that the $s$-wave
$T_c$ becomes significant even at $\mu = 0$, and that the $s$ and $d$-wave curves become increasingly similar as $V$ is increased. This has a simple physical interpretation: in the large $V$ limit the pairs become nearly localised on bonds, and the most favourable state is a singlet pair (stable compared to triplet by an energy of order $t^2/V$). As $V$ increases the effective hopping for pairs decreases (also as $t^2/V$) and so $s$- and $d$-wave pairs become nearly degenerate. This degeneracy implies that the Bose condensation of pairs in the large $V$ nearest-neighbour Hubbard model is qualitatively different from that in the on-site attractive Hubbard model.

Figures 1(b) and 1(c) also shows that for large $V$ superconductivity occurs even when the chemical potential is below the bottom of the electronic band at $-4t$. For these values of $\mu$ the fermions can only exist as bound pairs, and not as free fermions. This is analogous to the large $V$ on-site $s$-wave case discussed by Randeria, Duan and Shieh [22], in which the criterion for Bose condensation to occur in the low density limit was shown to be the same as the criterion for formation of a two particle bound state. Figure 1(c) implies that in the low density, $\mu < -4t$, limit two-particle bound states are formed. An extended $s$-wave superconducting state then occurs when these preformed pairs Bose condense at temperatures below the mean-field $T_c$ shown in Fig. 1(c).

8. Conclusions

We have derived an appropriate Landau-Ginzburg-Wilson effective action to describe the nearest neighbour attractive Hubbard model. This allows us to discuss both the small $V$ BCS and the large $V$ Bose-Einstein condensation limits on the same footing, as was done for the on-site attractive Hubbard model some years ago[23]. The nearest neighbour Hubbard model is especially interesting because the phase diagram includes regions of both (extended) $s$-wave superconductivity and $d$-wave pairing. We find that $p$-wave pairing is never stable. We have studied closely the region of the phase diagram near the cross-over from $s$- to $d$-wave pairing, and find that the two phases are always separated by two second-order phase transitions with an intermediate phase of $s \pm id$ superconductivity.

In future it will be interesting to examine more carefully the large $V$ Bose-Einstein condensation limit of this model, since the extended $s$-wave and $d$-wave pairing states become nearly degenerate. It will also be interesting to see how closely features of this model, such as the existence of $s$, $d$ and $s \pm id$ phases, correspond to the actual experimental features of the cuprates.
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