Comment on “The Effect of Vacuum Polarization ... in Strongly Magnetized Plasmas” by Ozel (astro-ph/0203449)

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ABSTRACT

We have recently shown that in a highly magnetized neutron star atmospheric plasma, vacuum polarization can induce resonant conversion of photon polarization modes via a mechanism analogous to MSW neutrino oscillation. In a recent paper Ozel has dismissed this mode conversion effect as “mistakes”. Here we explain why our arguments/calculations of this effect are correct.

1. Introduction

There are two polarization modes for photons (with energy $E$ much less than the electron cyclotron energy) propagating in a strongly magnetized plasma: The extraordinary mode (X-mode) and the ordinary mode (O-mode). These two modes have very different opacities in the atmosphere of a magnetized neutron star (NS). In a recent paper (Lai & Ho 2002; hereafter LH02), we discussed the effect of resonant mode conversion due to vacuum polarization: A photon propagating outward in the NS atmosphere can convert from one polarization mode into another as it traverses the resonant density, $\rho_v \simeq Y_e^{-1} \eta^{-2} (B/10^{14} G)^2 (E/1 \text{ keV})^2 \text{ g cm}^{-3}$, where $Y_e$ is the electron fraction, and $\eta \sim 1$ is a slowly varying function of the magnetic field $B$. This mode conversion is analogous to the Mikheyev-Smirnov-Wolfenstein mechanism for neutrino oscillation and is effective for $E \gtrsim$ a few keV. Because the two photon modes have vastly different opacities, the vacuum-induced mode conversion can affect radiative transport and surface emission from strongly magnetized NSs. In Ho & Lai (2002) (hereafter HL02), this effect was explored further in numerical models of magnetar atmospheres (see HL02 for references of earlier works related to strong-field vacuum polarization and its effects on radiative transport).

In a recent paper (Ozel 2002; hereafter Ozel02), Ozel has dismissed the mode conversion effect discussed our papers as “mistakes”. The purpose of this note is to clarify why our arguments/calculations of this effect are correct. Indeed, there are a number of problems in Ozel02, and many of its criticism and description of our work are incorrect or inaccurate. We restrict our comments to problems in Ozel02 that are directly related to our work.

\footnote{We have previously communicated to Ozel in private our comments described in this note, but the comments were rejected by her. We therefore feel it is necessary to put this note on astro-ph.}
2. Problems with Ozel (2002)

1. Throughout the paper (especially in the appendix), Ozel02 dismisses the MSW-like mode conversion effect first discussed in our papers. The arguments in Ozel02 are incorrect. Here is how to understand the effect. Adiabatic evolution of a state (whether it is quantum mechanical wavefunction or polarization state of a classical EM wave) means that the evolution is continuous, with no sudden change of the parameters that characterize the state (wavefunction or eigenvalue). Thus the different ways of describing the modes are of real significance. The mode eigenfunctions are characterized by the polarization ellipticity $K$: Eq. (2.27) of HL02 gives $K_j = \beta \left[ 1 + (-1)^j (1 + r/\beta^2)^{1/2} \right]$ for the X-mode and O-mode ($j = 1, 2$), and eq. (2.43) gives $K_{\pm} = \beta \pm (\beta^2 + r)^{1/2}$ for the plus-mode and minus-mode. Obviously, $K_1$ and $K_2$ are discontinuous across the vacuum resonance (note that $\beta = 0$ at vacuum resonance, and $\beta$ changes sign across the resonance), while $K_+$ and $K_-$ are continuous (see Fig.1 of LH02). Similarly, the indices of refraction (the eigenvalues) $n_1$ and $n_2$ are discontinuous, while $n_+$ and $n_-$ are continuous across the resonance (in quantum mechanics, this is called “avoid crossing”). Thus, under adiabatic condition (i.e., when the density gradient of the medium is sufficiently gentle), the polarization state will evolve along the $K_+$ or $K_-$ curve, rather than along the $K_1$ or $K_2$ curve. Since $K_+$ manifests as the O-mode at high density and as X-mode at low density (see Fig.1 of LH02), we have adiabatic mode conversion across the resonance. This conversion is not a matter of semantics, but has true physical significance: For example, without this conversion the mode opacity exhibits a spike at the vacuum resonance (see the left panel of Fig. 3 of HL02) but with conversion the opacity has a plateau-like transition (the right panel of Fig.3). As we show both analytically (see sect. 4 of LH02, where we discuss photon decoupling depth in the case of no mode conversion and the case with mode conversion) and numerically (see HL02), this will lead to a genuine difference in the radiative transfer and the emergent spectra.

Note that the adiabatic condition should not be confused with the condition of “Faraday depolarization”. The adiabatic condition requires the background density to vary slowly (the precise condition is derived in LH02), while Faraday depolarization implies that a modal description of radiative transport is possible (as first discussed by Gnedin & Pavlov 1974 and repeated in sect. 2 of Ozel02; the issue of Faraday depolarization was briefly discussed in sect. 2.4 of HL02, as well as in sect. 2.1 of Ho & Lai 2001). Adiabatic mode conversion exists even in a lossless medium (with no absorption and scattering) for which Faraday depolarization is always satisfied. The MSW analogy is: Neutrinos are essentially lossless in the sun, yet they can convert from one flavor to another.

Also, the adiabatic mode conversion should not be confused with mode switching due to scattering; the latter was treated in Ozel02, in HL02, and in many previous papers (see footnote 1 of LH02 and sect. 2.4 of HL02).

2. In addition to the conceptual problems in Ozel02 as discussed above, there are also several (less important) issues we would like to clarify in response to Ozel02:

(1) When mode conversion is neglected, the photon opacity exhibits a sharp feature at vacuum resonance, and in HL02 we adopted an approximate “equal-grid” numerical scheme to handle this
feature. Ozel02 has criticized our “equal-grid” scheme. While it is certainly fine to criticize this, it should be noted that the limitation (including the one discussed in Ozel02) and usefulness of the equal-grid method are fully discussed in HL02 (sect. 5.1; this is also mentioned at the end of sect. 6.3, after we comment on previous works). In fact, the subtlety in resolving the density-dependent, narrow vacuum resonance feature (and in particular the fact that the region very close to the resonance contributes most to the optical depth) was first discussed in our papers [see sect. 4 of LH02 (in particular footnote 2) and sect. 3, 4, 5.1 of HL02] — there was no hint that this subtlety was appreciated in previous papers (including Ozel 2001; this is why we discussed in detail these issues in our paper, including comparison with previous works in sect. 6.3).

(2) We have emphasized several times in our paper [e.g., see HL02, sect. 1 after eq. (1.2), the end of sect. 2.4, the second to last paragraph of sect. 7] that a more rigorous treatment of the problem of radiative transfer in strong magnetic fields with vacuum polarization requires one to go beyond the modal description (as used in all NS atmosphere works so far) and to solve the transport equations for the Stokes parameters (i.e., including the “off-diagonal” terms of the photon density matrix).

3. Conclusion

We stand by the main results of LH02 and HL02, although one should keep in mind the limitations (as discussed explicitly in sect. 7 of HL02). Strong-field vacuum polarization introduces several novel features in the radiative transfer whose full, rigorous solution remains an important problem. Much work remains to be done in the area of highly magnetized NS atmospheres/surfaces in order to confront current and future observational data.

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