CONSTITUENT QUARK REST ENERGY AND WAVE FUNCTION ACROSS THE LIGHT CONE

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Abstract

It is shown that for a constituent quark in the intra-nucleon self-consistent field the spin-orbit interaction lowers the quark rest energy to values $\sim 100$ MeV, which agrees with the DIS momentum sum rule. The possibility of violation of the spectral condition for the light-cone momentum component of a bound quark is discussed.

Given the information from the DIS momentum sum rule [1] that half of the nucleon mass is carried by gluons, one may lower the expectations for the constituent quark mass from $M_N/3$ to $\lesssim M_N/6$. This may also provide grounds for formation of a gluonic mean-field in the nucleon, permitting single-quark description of the wave-function, and (by analogy with the situation in atomic mean fields), making LS-interaction dominant over the SS. Here, adopting the mentioned assumptions for the nucleon wave function, we will show that the constituent quark mass $\sim 50 \div 100$ MeV, in fact, results from the empirical values for the nucleon mean charge radius and the magnetic moment. We will also address the issue of valence quark wave function continuity and sizeable value at zero light-front momentum, which owes to rather low a value of the constituent quark mass, and discuss phenomenological implications thereof.

Wave function ansatz  For a spherically symmetric ground state, by parity reasons, the upper Dirac component has orbital momentum $l = 0$, whereas the lower one has $l = 1$ and flipped spin. This is accounted for by writing the single-quark wave function

$$\psi_q(\mathbf{r},t) = e^{-i\kappa_0 t} \left( \begin{array}{c} \varphi(r)w \\ -i\sigma \cdot \xi \chi(r)w \end{array} \right) \quad (\kappa_0 > 0)$$

(1)

with $w$ being an $\mathbf{r}$-independent Pauli spinor.

The relation between $\varphi$ and $\chi$ to a first rough approximation is determined by the quark rest energy. For instance, for a Dirac particle of mass $m$ moving in a static potential $V(r)$ (being either a 4th component of vector, or a scalar – cf. [2]), in the ground state

$$\chi(r) = \frac{1}{m + E - V(r)} \varphi'(r).$$

(2)

Thereat, a reasonable model may result from replacing the denominator of (2) by its average $\langle \kappa_0^0 \rangle$ (which may somewhat differ from $\kappa_0^0$ in (1)):

$$\chi(r) = \frac{1}{2 \langle \kappa_0^0 \rangle} \varphi'(r).$$

Neglecting the SS-interaction and employing the single-quark model for the nucleon wave function, one must be ready that the accuracy is not better than $\frac{M_\Delta - M_N}{M_N} \sim 30 \div 40\%$. 

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For bag models [1], the proportionality between $\chi$ and $\varphi'$ holds exactly, but the particular shape of bag model wave functions, especially $\chi$, may be not phenomenologically reliable due to the influence of the bag sharp boundary. Instead, we prefer to take $\varphi$ a smooth function for all $r$ from 0 to $\infty$:

$$\psi_q(\kappa) = \left(1 + \frac{1}{2} \langle \kappa^0 \rangle \gamma^0 \gamma \cdot \kappa \right) \varphi(\kappa) \left( \begin{array}{c} w \\ 0 \end{array} \right) \equiv \varphi(\kappa) \left( \begin{array}{c} w \\ \frac{\sigma \cdot \kappa}{2 \langle \kappa^0 \rangle} \end{array} \right),$$  

(3)

$$\varphi(\kappa) \propto e^{-a^2 \kappa^2/2}. \quad (4)$$

Thereby, the model involves only 2 parameters: $\langle \kappa^0 \rangle$ and the Gaussian radius $a$. It may describe the valence quark component in the nucleon, and is suitable for calculation of matrix elements of vector (conserved) currents, whereas axial vector currents require the account of sea quarks, which is beyond our scope here.

Proton mean charge radius and magnetic moment  With recoilless spectators, our model can describe only formfactors at $Q \ll M_N$. Equating $Q/M_N$ to our accuracy $30 \div 40\%$, one finds $Q < 0.4$ GeV. In this region the decrease of the form-factors is approximately linear [1], whereby they are characterized basically by the proton’s magnetic moment and the mean charge (or magnetic) radius [2].

The form-factors are defined as overlaps of $\psi_q$ with $\psi'_q$, of the same ground state, but with a shifted momentum and a different polarization $w'$:

$$\int \frac{d^3\kappa}{(2\pi)^3} \bar{\psi}^I_q \left( \kappa + \frac{Q}{2} \right) \gamma^0 \psi_q \left( \kappa - \frac{Q}{2} \right) = F_1(Q^2) w' + w, \quad (F_1(0) = 1) \quad (5)$$

$$\int \frac{d^3\kappa}{(2\pi)^3} \bar{\psi}^I_q \left( \kappa + \frac{Q}{2} \right) \gamma \psi_q \left( \kappa - \frac{Q}{2} \right) = F_2(Q^2) w' + \frac{i}{2} [Q \times \sigma] w \quad (6)$$

(the symmetric shift of momenta preserves the parity of the integrand with respect to the origin, simplifying the calculations). Since we neglect the recoil, $F_1$, $F_2$ may be associated equally well with Dirac or with Sachs form-factors. That will be non-contradictory provided in the exact identities relating those types of form-factors

$$F_e = F_1 - \frac{Q^2}{4M_N^2} F_2, \quad F_m = F_2 + F_1 \quad (7)$$

the last terms are small. For the proton, $F_1(Q^2)/F_2(Q^2) \approx F_1(0)/F_2(0) = 1/2.79$, and within our accuracy $30 \div 40\%$ this contribution is indeed negligible. Also, the Foldy term for the proton’s mean square charge radius $\frac{3F_2(0)}{2M_N^2} = (0.4\text{fm})^2$ is small compared to

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[2] By the isospin symmetry, neutron’s magnetic moment $\mu_n = -\frac{2}{3}\mu_p$, and its magnetic radius is about the same as proton’s [1].
\( \langle r_{ch}^2 \rangle \approx 1 \text{fm}^2 \). Now with Eqs. (5) \( \text{[3]} \), let us actually evaluate the mean square charge radius:

\[
\langle r_{ch}^2 \rangle = -\frac{6}{F_1(0)} \frac{dF_1(Q^2)}{dQ^2} \bigg|_{Q^2=0} \\
= -\frac{6}{F_1(0)} \frac{\partial}{\partial Q^2} \int d^3 \kappa \left[ \frac{\kappa^2 - \langle Q^2 \rangle}{4 \langle \kappa^2 \rangle^2} \right] \varphi \left( \kappa + \frac{Q}{2} \right) \varphi \left( \kappa - \frac{Q}{2} \right) \bigg|_{Q=0} \\
= a^2 \left( \frac{3}{2} + \frac{1}{1 + \frac{8 \langle \kappa^2 \rangle a^2}{3}} \right). \quad (8)
\]

Another constraint on the model parameters comes from the proton magnetic moment, which in our additive quark model with charges \( e_u = \frac{2}{3} e_p, e_d = -\frac{1}{3} e_p \) appears to equal

\[
\frac{\mu_p}{e_p} = \frac{\mu_q}{e_q} = \frac{1}{2 \langle \kappa \rangle} F_2(0) = \frac{1}{2 \langle \kappa \rangle} \int d^3 \kappa \left[ \frac{\kappa^2}{2 \langle \kappa \rangle} \right] \varphi^2 \left( \kappa \right) = \frac{1}{2 \langle \kappa \rangle} + \frac{1}{8 \langle \kappa \rangle^2 a^2}. \quad (9)
\]

Together, constraints (8,9) may determine both free parameters of the wave function. With the experimental values \( \mu_p/e_p \approx \frac{\mu_q}{e_q} \approx (0.9 \text{fm})^2 \) \text{[1]}, Eqs. (3, 4) are, strictly speaking, mutually inconsistent (see Fig. 1). But within our accuracy \( 30 \div 40\% \) they are sufficiently consistent, yielding \( a = 0.55 \div 0.85 \text{ fm} \) and \( \langle \kappa \rangle = 50 \div 250 \text{ MeV} \). Similar values for \( a \) were found by other authors \text{[3]}.\(^3\) As for \( \langle \kappa \rangle \), it is about twice smaller than typical quark mass in non-relativistic constituent models \text{[1]}.

Valence quark distribution function \  The constructed model should also be able to describe valence quark distribution at Bjorken \( x < 0.4 \), i. e., including the region of the quark distribution maximum. The definition of a quark parton distribution (cf., e.g., \text{[4]}) leads to

\[
q_v(x) = \int \frac{d^3 \kappa}{(2\pi)^3} \delta \left( x - \frac{\kappa^0 + \kappa^3}{M_N} \right) \bar{\psi}_q(\kappa) \left( \gamma^0 + \gamma^3 \right) \psi_q(\kappa) = M_N \int d^2 \kappa_\perp \left[ \left( 1 - \frac{\kappa^0}{2 \langle \kappa^0 \rangle} + \frac{x M_N}{2 \langle \kappa^0 \rangle} \right)^2 + \frac{\kappa_\perp^2}{4 \langle \kappa^0 \rangle^2} \right] \varphi^2(\kappa) \bigg|_{\kappa^3 = x M_N - \kappa^0} \\
= \frac{M_N a}{\sqrt{\pi} (4 \langle \kappa^0 \rangle^2 a^2 + \frac{3}{2})} \left[ (x M_N - \kappa^0 + 2 \langle \kappa^0 \rangle)^2 a^2 + 1 \right] e^{- (x M_N - \kappa^0)^2 a^2}. \quad (10)
\]

The behavior of function (10) at different \( \langle \kappa^0 \rangle \) is shown in Fig. 2. Values \( \langle \kappa^0 \rangle > 150 \text{ MeV} \) look rather unrealistic. At \( \langle \kappa^0 \rangle \approx 50 \text{ MeV} \) our \( x q_v(x) \) holds within the declared \( 30 \div 40\% \) accuracy with the MRST, CTEQ pdf fits. It seems interesting that the constituent quark “mass” \( \langle \kappa^0 \rangle = 50 \div 100 \text{ MeV} \) gets already commensurable with the pion mass scale \( m_\pi \approx 140 \text{ MeV} \).

\(^3\)However, if instead of \text{[4]} one took a model \( \varphi \propto \left( 1 + \frac{e^2 \kappa^2}{2\alpha} \right)^{-\alpha} \) with \( \alpha < 4 \), the value for \( a \) might be larger.
The features of Fig. 2a are \( \int_{0}^{1} dx q_v(x) < 1 \) and high value of \( q_v(0) \). That is expectable: since the quark rest energy is smaller than its typical momentum \( \sim (0.7 \text{ fm})^{-1} \approx 300 \text{ MeV} \), the wave function must reach well across the light cone. Note that \( q_v(x) \) at negative values of \( x \) should not be associated with antiquarks [4], but still they may describe quarks, only ones inaccessible to leading twist DIS. In fact, for strongly bound relativistic states the positivity condition does not apply to light-cone components of momenta. Even considering the DIS handbag diagram in terms of 2-particle intermediate states with Bethe-Salpeter wave functions in nucleon vertices, the \( \kappa^- \)-integration will give a non-zero result for \( \kappa^+ \) beyond the interval \( [0, P^+] \), because due the BS vertex functions \( G(\kappa) \) depending on \( \kappa^+ \) through \( \kappa^3 \) one can not completely withdraw the integration contour to \( \infty \).

In conclusion, let us point out that the continuity of a valence quark wave function around \( x = 0 \) permits its use for perturbative description of hadron-hadron reactions with single quark exchange. Thereat, non-zero \( q_v(0) \) opens a specific possibility of transverse polarization generation in reaction \( np \to pn \) [6].

References

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\( q_v(0) \) is finite also in bag models [5], whereas in light-cone models often \( q_v(x) \to 0 \). The latter property seems phenomenologically unlikely, even assuming further Regge enhancements.

Figure 2. (a) Valence quark distribution for \( a = 0.7 \text{ fm} \) and \( m = 50 \text{ MeV} \) (solid line), \( m = 100 \text{ MeV} \) (dashed), \( m = 200 \text{ MeV} \) (dotted). In thin red the transverse spin asymmetry \( \Delta_T q(x) \) is shown. (b) Same as (a) but for the valence quark distribution weighted with \( x \). The momentum sums are, correspondingly, \( 3 \int_{0}^{1} dx x q_v(x) = 0.5, 0.7, 1.0 \).