General Stress Fields of Orthotropic Plate with Mode II Crack

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Abstract. The plane crack problem of the orthotropic plate is discussed for the shear loading. The solution of this problem is mainly achieved by the complex variable method. Essential equations are introduced fully on the basis of linear elastic mechanics. The stress functions have been given to meet the needs of shear mode II crack states for orthotropic materials. The formulae for stress fields are derived by the complex functions in two cases. With the simplification of relative functions, the expressions of the singular stress fields in the crack-tip are determined by the real functions.

1. Introduction

Conventional fracture mechanics discusses on the homogeneous materials in great detail and has been highly successful. Nowadays the fracture mechanics of anisotropic materials may be relative to many engineering structures [1, 2]. The orthotropic plates must be for the base of composite materials in common use. The valid method to solve crack tip field problems in anisotropic materials may be in using complex analytic function theory, and the results have been reported [3–5]. But the general solutions for composite materials have not given completely or perfectly. Therefore, it is necessary to make up a new solution for the singular stresses of cracked composite materials [6–8]. Particularly, the study of the shear mode II crack problem as shown in Figure 1 must be very necessary for the plane fracture mechanics of orthotropic materials.

![Figure 1. Model of the plate with mode II crack](image-url)

2. Elastic Equations and Stress Function

To solve the plane stress problems of anisotropic materials, the body forces can not be taken into consideration, and the equilibrium equations are in the following form:
\[
\frac{\partial \sigma_x}{\partial x} + \frac{\partial \tau_{xy}}{\partial y} = 0 \quad , \quad \frac{\partial \tau_{xy}}{\partial x} + \frac{\partial \sigma_y}{\partial y} = 0
\] (1)

In order to satisfy the equilibrium equations, the plane stress components are defined by the stress function \( F = F(x,y) \) as follows:

\[
\sigma_x = \frac{\partial^2 F}{\partial y^2} , \quad \sigma_y = \frac{\partial^2 F}{\partial x^2} , \quad \tau_{xy} = -\frac{\partial^2 F}{\partial x \partial y}
\] (2)

The orthotropic coefficients are often expressed as \( E_1 \) , \( E_2 \) , \( \nu_{12} \) , \( G_{12} \). For the convenience of the below explanation, we use the characters, \( B \) and \( C \), as the material parameters, they are:

\[
B = \frac{E_1}{2G_{12}} - \nu_{12} , \quad C = \frac{E_1}{E_2}
\] (3)

The governing equation of the strain compatibility condition can be expressed by

\[
\frac{\partial^4 F}{\partial y^4} + 2B \frac{\partial^4 F}{\partial x^2 \partial y^2} + C \frac{\partial^4 F}{\partial x^4} = 0
\] (4)

To determine the solution of the partial differential equation, it is very advantageous to use complex variables. Thus, we introduce the complex variable (\( w \)) and its conjugate (\( \overline{w} \)), they are defined by: \( w = x + ihy \) , \( \overline{w} = x - ihy \), where \( h \) is a real arbitrary constant, and also we suppose that \( h > 0 \). By above definition, the real stress function \( F(x,y) \) can be expressed by the complex variables. The partial derivations of \( F \) with \( x \) or \( y \) can be transformed by following expressions:

\[
\frac{\partial F}{\partial x} = \frac{\partial F}{\partial w} \frac{\partial w}{\partial x} + \frac{\partial F}{\partial \overline{w}} \frac{\partial \overline{w}}{\partial x} , \quad \frac{\partial F}{\partial y} = \frac{\partial F}{\partial w} \frac{\partial w}{\partial y} + \frac{\partial F}{\partial \overline{w}} \frac{\partial \overline{w}}{\partial y} = ih(\frac{\partial F}{\partial w} - \frac{\partial F}{\partial \overline{w}})
\] (5)

The important relations of partial derivations are as

\[
\frac{\partial^2 F}{\partial x^2} + \frac{1}{h^2} \frac{\partial^2 F}{\partial y^2} = 4 \frac{\partial^2 F}{\partial w \partial \overline{w}} , \quad \left( \frac{\partial^2}{\partial x^2} + \frac{1}{h^2} \frac{\partial^2}{\partial y^2} \right)^2 F = 16 \frac{\partial^4 F}{\partial^2 w \partial^2 \overline{w}}
\]

Using above relations, the partial derivative equation (4) can be written by

\[
D_1 \left( \frac{\partial^4 F}{\partial w^4} + \frac{\partial^4 F}{\partial \overline{w}^4} \right) + D_2 \left( \frac{\partial^2}{\partial w^2} + \frac{\partial^2}{\partial \overline{w}^2} \right) \frac{\partial^2 F}{\partial w \partial \overline{w}} + D_3 \frac{\partial^4 F}{\partial^2 w \partial^2 \overline{w}} = 0
\] (6)

Where, \( D_1 = h^4 - 2Bh^2 + C \), \( D_2 = 4C - 4h^4 \), \( D_3 = 6h^4 + 4Bh^2 + 6C \).

In the following, we shall solve this equation (6).

Case 1: \( \frac{\partial^3 F}{\partial w^2 \partial \overline{w}^2} = 0 \) , \( D_1 = D_2 = 0 \)

In this case, two parameters must be given as follows

\[
C = h^4 , \quad B = h^2
\]

We know that, \( h > 0 \), the solution can be determined by

\[
h = \sqrt[4]{\frac{E_1}{2G_{12}} - \nu_{12}} = \sqrt[4]{\frac{E_1}{E_2}}
\] (7)
By the equation \( \frac{\partial^4 F}{\partial w^2 \partial \bar{w}^2} = 0 \), we can determine the stress function \( F \), that is:

\[
F = A_1 \Psi_1 + A_2 \overline{\Psi}_1 + A_3 \overline{w} \Psi_2 + A_4 \overline{w} \overline{\Psi}_2
\]

(8)

Where, \( \Psi_1 = \Psi_1(w) \) and \( \Psi_2 = \Psi_2(w) \) are the analytic functions.

For Mode II crack problem, the constants and complex functions can be selected as:

\[
A_1 = A_4 = -iA_0 \quad \text{and} \quad A_2 = A_3 = iA_0 \quad \Psi_1 = w \Phi \quad \Psi_2 = \Phi
\]

Then, the stress function can be determined by

\[
F = A_0 i(w - w)(\Phi + \overline{\Phi}) = 2A_0 hy(\Phi + \overline{\Phi})
\]

(9)

The partial derivations can be given as follows

\[
\frac{\partial F}{\partial x} = 2A_0 h y (\Phi' + \overline{\Phi}') \quad \frac{\partial F}{\partial y} = 2A_0 h [\Phi + \overline{\Phi} + i hy(\Phi' - \overline{\Phi}')] \]

\[
\frac{\partial^2 F}{\partial x^2} = 2A_0 h y (\Phi'' + \overline{\Phi}'') \quad \frac{\partial^2 F}{\partial y^2} = 2A_0 h^2 [2i(\Phi' - \overline{\Phi}') - hy(\Phi'' + \overline{\Phi}'')] \]

\[
\frac{\partial^2 F}{\partial x \partial y} = 2A_0 h[\Phi' + \overline{\Phi}' + i hy(\Phi'' - \overline{\Phi}'')]
\]

According to the stress expressions (2), the plane stress components can be expressed as:

\[
\begin{align*}
\sigma_x &= -4A_0 h^2 (2 \text{Im} \Phi' + hy \text{Re} \Phi'') \\
\sigma_y &= 4A_0 h^2 \text{Re} \Phi'' \\
\tau_w &= -4A_0 h(\text{Re} \Phi' - hy \text{Im} \Phi'')
\end{align*}
\]

(10)

**Case 2:** \( \frac{\partial^2 F}{\partial \bar{w} \partial w} = 0 \), \( D_1 = 0 \)

Under the case, the characteristic equation is given as

\[
h^4 - 2Bh^2 + C = 0
\]

Obviously, the solution can be determined by

\[
h^2 = B \pm \sqrt{B^2 - C} \quad (B^2 > C)
\]

Where, we suppose that: \( B^2 > C \), and also let \( h_1 > h_2 > 0 \). Then, two solutions are:

\[
\begin{align*}
h_1 &= \sqrt{E_1 - \nu_{12} + \sqrt{(E_1 - \nu_{12})^2 - E_1 E_2}} \\
&= \sqrt{\frac{E_1}{2G_{12}} - \nu_{12} + \sqrt{(\frac{E_1}{2G_{12}} - \nu_{12})^2 - \frac{E_1}{E_2}}} \\
h_2 &= \sqrt{E_1 - \nu_{12} - \sqrt{(E_1 - \nu_{12})^2 - E_1 E_2}} \\
&= \sqrt{\frac{E_1}{2G_{12}} - \nu_{12} - \sqrt{(\frac{E_1}{2G_{12}} - \nu_{12})^2 - \frac{E_1}{E_2}}}
\end{align*}
\]

(11)

By the equation \( \frac{\partial^2 F}{\partial \bar{w} \partial w} = 0 \), the stress function \( F \) may be in the following form

\[
F = A_1 \Psi_1 + A_2 \overline{\Psi}_1 + A_3 \Psi_2 + A_4 \overline{\Psi}_2
\]

(12)
Where, $\Psi_1 = \Psi_1(w_1)$ and $\Psi_2 = \Psi_2(w_2)$ are the analytic functions. The complex variables are

$$w_1 = x + ih_1 y, \quad w_2 = x + ih_2 y$$

The constants can be given by: $A_1 = A_4 = iA_0$, $A_2 = A_3 = -iA_0$. Therefore, we have:

$$F = A_0 j \cdot (\Psi_1 - \bar{\Psi}_1 - \Psi_2 + \bar{\Psi}_2)$$

(13)

The partial derivations can be given as follows

$$\frac{\partial F}{\partial x} = A_0 j \cdot (\Psi_1' - \bar{\Psi}_1' - \Psi_2' + \bar{\Psi}_2')$$

$$\frac{\partial F}{\partial y} = A_0 j \cdot [ih_1 (\Psi_1' + \bar{\Psi}_1') - ih_2 (\Psi_2' + \bar{\Psi}_2')]$$

According to the stress expressions (2), the plane stress components can be expressed as:

$$\begin{align*}
\sigma_x &= -A_0 j \cdot [h_1^2 (\Psi_1'' - \bar{\Psi}_1'') - h_2^2 (\Psi_2'' - \bar{\Psi}_2'')] = 2A_0 \text{ Im}(h_1^3 \Psi_1'' - h_2^3 \Psi_2'') \\
\sigma_y &= A_0 j \cdot (\Psi_1'' - \bar{\Psi}_1'') = -2A_0 \text{ Im}(\Psi_1'' - \Psi_2'') \\
\tau_{xy} &= A_0 [h_1 (\Psi_1'' + \bar{\Psi}_1'') - h_2 (\Psi_2'' + \bar{\Psi}_2'')] = 2A_0 \text{ Re}(h_1 \Psi_1'' - h_2 \Psi_2'')
\end{align*}$$

(14)

Evidently, the stress expressions are given with the application of complex variable method.

3. Solution of Mode II Crack

To solve the Mode II crack boundary problem as shown in Figure 1, we must consider the stress boundary conditions as follows:

At $|x| < a$, $y = 0$: $\sigma_y = \tau_{xy} = 0$. At $x^2 + y^2 \to \infty$: $\sigma_x = \sigma_y = 0$, $\tau_{xy} = \tau$.

In the following, we discuss the solutions in above section for case I and case II.

For the first case, the stress components are expressed as equation (10). We select the constant and complex function as:

$$4A_0 h = -\tau, \quad \Phi' = \sqrt{\frac{w^2}{w^2 - a^2}}, \quad \text{then:} \quad \Phi'' = -a^2 \frac{w}{w} \sqrt{\frac{w^2}{(w^2 - a^2)^3}}$$

Thus, the stresses can be determined by

$$\begin{align*}
\frac{\sigma_x}{\tau} &= 2h \text{ Im} \left[ \frac{w^2}{w^2 - a^2} - h^2 y \text{ Re} \left( \frac{a^2}{w} \sqrt{\frac{w^2}{(w^2 - a^2)^3}} \right) \right] \\
\frac{\sigma_y}{\tau} &= y \text{ Re} \left( \frac{a^2}{w} \sqrt{\frac{w^2}{(w^2 - a^2)^3}} \right) \\
\frac{\tau_{xy}}{\tau} &= \text{ Re} \left[ \frac{w^2}{w^2 - a^2} + hy \text{ Im} \left( \frac{a^2}{w} \sqrt{\frac{w^2}{(w^2 - a^2)^3}} \right) \right]
\end{align*}$$

(15)

Obviously, the stresses can meet the needs of the boundary conditions. The solution is suitable to whole plate as shown in Figure 1. Nevertheless, the local region near the crack tip is more important. So we select one small region at the crack tip on the right. By using the polar coordinate system, the functions in above equation can be simplified as follows ($r << a$):

$$x = a + r \cos \theta, \quad y = r \sin \theta, \quad w = a + r \cos \theta + ihr \sin \theta$$
\[ w \approx a \; , \; w + a \approx 2a \; , \; \frac{w^2}{w^2 - a^2} = \frac{a}{2r(\cos \theta + ih \sin \theta)} \]

Then the stresses at near crack-tip can be simplified as:

\[
\begin{align*}
\sigma_x &= \frac{K_{II}}{\sqrt{2r}} [2h \text{Im} \left( \frac{1}{\cos \theta + ih \sin \theta} - \frac{h^2 \sin \theta}{2} \text{Re} \left( \frac{1}{\cos \theta + ih \sin \theta} \right) \right)] \\
\sigma_y &= \frac{K_{II}}{\sqrt{2r}} \sin \theta \text{Re} \left( \frac{1}{\cos \theta + ih \sin \theta} \right) \\
\tau_{xy} &= \frac{K_{II}}{\sqrt{2r}} \text{Re} \left( \frac{1}{\cos \theta + ih \sin \theta} + \frac{h \sin \theta}{2} \text{Im} \left( \frac{1}{\cos \theta + ih \sin \theta} \right) \right)
\end{align*}
\] (16)

Where, \( K_{II} = \tau \sqrt{a} \) is called the stress intensity factors for mode II crack. The stress expressions show that the stress components tend to infinity at the crack tip (when, \( r \to 0 \)), this is so-called the stress singularity. As yet, the functions in above equation remain the complex form.

For the second case, the stress components are expressed by equation (14). To meet the stress boundary conditions, the complex functions can be selected as:

\[
\Psi_1' = \sqrt{\frac{w_1^2}{w_1^2 - a^2}} \; , \; \Psi_2'' = \sqrt{\frac{w_2^2}{w_2^2 - a^2}} \; . \; \text{And} \; 2A_0 = \frac{\tau}{h_1 - h_2}
\]

Then, the stresses in equation (14) can be transformed as follows

\[
\begin{align*}
\sigma_x &= \frac{\tau}{h_1 - h_2} \text{Im}(\frac{h_1^2}{w_1^2 - a^2} - \frac{h_2^2}{w_2^2 - a^2}) \\
\sigma_y &= -\frac{\tau}{h_1 - h_2} \text{Im}(\frac{w_1^2}{w_1^2 - a^2} - \frac{w_2^2}{w_2^2 - a^2}) \\
\tau_{xy} &= \frac{\tau}{h_1 - h_2} \text{Re}(\frac{w_1^2}{w_1^2 - a^2} - \frac{w_2^2}{w_2^2 - a^2})
\end{align*}
\] (17)

In the vicinity of the crack tip (\( r \ll a \)), the complex functions can be simplified by using the polar coordinate system \((x = a + r \cos \theta, \; y = r \sin \theta) : \; w_1 \approx a \; , \; w_1 + a \approx 2a \)

\[ w_2 \approx a \; , \; w_2 + a \approx 2a \; , \; w_1 - a = r(\cos \theta + ih_1 \sin \theta) \; , \; w_2 - a = r(\cos \theta + ih_2 \sin \theta) \]

Then the stresses at near crack-tip can be expressed as \( (K_{II} = \tau \sqrt{a}) : \)

\[
\begin{align*}
\sigma_x &= \frac{1}{h_1 - h_2} \frac{K_{II}}{\sqrt{2r}} \text{Im}(\frac{h_1^2}{\cos \theta + ih_1 \sin \theta} - \frac{h_2^2}{\cos \theta + ih_2 \sin \theta}) \\
\sigma_y &= -\frac{1}{h_1 - h_2} \frac{K_{II}}{\sqrt{2r}} \text{Im}(\frac{1}{\cos \theta + ih_1 \sin \theta} - \frac{1}{\cos \theta + ih_2 \sin \theta}) \\
\tau_{xy} &= \frac{1}{h_1 - h_2} \frac{K_{II}}{\sqrt{2r}} \text{Re}(\frac{h_1}{\cos \theta + ih_1 \sin \theta} - \frac{h_2}{\cos \theta + ih_2 \sin \theta})
\end{align*}
\] (18)

As yet, the functions in above equation remain the complex form.
4. Function Simplification and Stresses

In order to simplify above stress expressions, it is necessary to make some function transformation. Now, we let:

\[
\cos \theta = g \cos \beta , \quad h \sin \theta = g \sin \beta
\]

(19)

We can obtain that:

\[
h \tan \theta = \tan \beta , \quad \cos^2 \theta + h^2 \sin^2 \theta = g^2 (\cos^2 \beta + \sin^2 \beta) = g^2
\]

\[
\beta = \arctan(h \tan \theta) , \quad g = \sqrt{\cos^2 \theta + h^2 \sin^2 \theta}
\]

(20)

Then, the complex function can be simplified by:

\[
1 \frac{1}{\sqrt{\cos \theta + ih \sin \theta}} = \sqrt{g \cos \beta + ig \sin \beta} = \frac{1}{\sqrt{g}} e^{-j \beta / 2} = \frac{1}{\sqrt{g}} (\cos \beta - i \sin \beta / 2)
\]

\[
\sqrt{\frac{1}{(\cos \theta + ih \sin \theta)} = \frac{1}{\sqrt{g^3}} (\cos 3 \beta / 2 - i \sin 3 \beta / 2)}
\]

For Case 1, the stresses in equation (16) can be transformed as

\[
\sigma_x = \frac{K_u}{\sqrt{2r}} [2h \text{Im} \frac{1}{\sqrt{g}} (\cos \beta / 2 - i \sin \beta / 2) - h g \sin \beta / 2 \text{Re} \frac{1}{\sqrt{g^3}} (\cos 3 \beta / 2 - i \sin 3 \beta / 2)]
\]

\[
\sigma_y = \frac{K_u}{\sqrt{2r}} g \sin \beta / 2 \text{Re} \frac{1}{\sqrt{g^3}} (\cos 3 \beta / 2 - i \sin 3 \beta / 2)
\]

\[
\tau_{xy} = \frac{K_u}{\sqrt{2r}} [\text{Re} \frac{1}{\sqrt{g}} (\cos \beta / 2 - i \sin \beta / 2) + g \sin \beta / 2 \text{Im} \frac{1}{\sqrt{g^3}} (\cos 3 \beta / 2 - i \sin 3 \beta / 2)]
\]

Evidently, the functions can be simplified easily. And then, the singular stress fields in the crack-tip for the first case must well be determined by

\[
\sigma_x = -\frac{K_u}{\sqrt{2r}} h \frac{1}{\sqrt{g}} \sin \beta / 2 (2 + \cos \beta / 2 \cos 3 \beta / 2)
\]

\[
\sigma_y = \frac{K_u}{\sqrt{2r}} \frac{1}{h \sqrt{g}} \sin \beta / 2 \cos \beta / 2 \cos 3 \beta / 2
\]

\[
\tau_{xy} = \frac{K_u}{\sqrt{2r}} \frac{1}{\sqrt{g}} \cos \beta / 2 (1 - \sin \beta / 2 \sin 3 \beta / 2)
\]

When \( h = 1 \), then \( g = 1 , \quad \beta = \theta \).

For Case 2, it is similar to have the transformation as the first case. We let:

\[
\cos \theta = g_1 \cos \beta_1 = g_2 \cos \beta_2 , \quad h_1 \sin \theta = g_1 \sin \beta_1 , \quad h_2 \sin \theta = g_2 \sin \beta_2
\]

(22)

We can obtain that:

\[
h_1 \tan \theta = \tan \beta_1 , \quad h_2 \tan \theta = \tan \beta_2 , \quad \cos^2 \theta + h_1^2 \sin^2 \theta = g_1^2 , \quad \cos^2 \theta + h_2^2 \sin^2 \theta = g_2^2
\]

\[
\beta_1 = \arctan(h_1 \tan \theta) , \quad g_1 = \sqrt{\cos^2 \theta + h_1^2 \sin^2 \theta}
\]

\[
\beta_2 = \arctan(h_2 \tan \theta) , \quad g_2 = \sqrt{\cos^2 \theta + h_2^2 \sin^2 \theta}
\]

(23)
Then, the complex functions can be simplified by:

\[
\frac{1}{\sqrt{\cos \theta + i h_1 \sin \theta}} = \frac{1}{\sqrt{g_1 \cos \beta_1 + i g_1 \sin \beta_1}} = \frac{1}{\sqrt{g_1}} (\cos \frac{\beta_1}{2} - i \sin \frac{\beta_1}{2})
\]

\[
\frac{1}{\sqrt{\cos \theta + i h_2 \sin \theta}} = \frac{1}{\sqrt{g_2}} (\cos \frac{\beta_2}{2} - i \sin \frac{\beta_2}{2})
\]

The stresses in equation (18) can be transformed as

\[
\begin{align*}
\sigma_x &= \frac{1}{h_1 - h_2} \frac{K_{\mu}}{2r} \text{Im} \left( i \frac{h_1^2}{g_1} (\cos \frac{\beta_1}{2} - i \sin \frac{\beta_1}{2}) - i \frac{h_2^2}{g_2} (\cos \frac{\beta_2}{2} - i \sin \frac{\beta_2}{2}) \right) \\
\sigma_y &= -\frac{1}{h_1 - h_2} \frac{K_{\mu}}{2r} \text{Im} \left( \frac{1}{g_1} (\cos \frac{\beta_1}{2} - i \sin \frac{\beta_1}{2}) - \frac{1}{g_2} (\cos \frac{\beta_2}{2} - i \sin \frac{\beta_2}{2}) \right) \\
\tau_{xy} &= \frac{1}{h_1 - h_2} \frac{K_{\mu}}{2r} \text{Re} \left( \frac{h_1}{g_1} (\cos \frac{\beta_1}{2} - i \sin \frac{\beta_1}{2}) - \frac{h_2}{g_2} (\cos \frac{\beta_2}{2} - i \sin \frac{\beta_2}{2}) \right)
\end{align*}
\]

Evidently, the complex functions can be simplified easily. And at last, the singular stress fields in the crack-tip for the second case must be determined as follows:

\[
\begin{align*}
\sigma_x &= -\frac{1}{h_1 - h_2} \frac{K_{\mu}}{2r} \left( i \frac{h_1^2}{g_1} \sin \frac{\beta_1}{2} - \frac{h_2^2}{g_2} \sin \frac{\beta_2}{2} \right) \\
\sigma_y &= \frac{1}{h_1 - h_2} \frac{K_{\mu}}{2r} \left( \frac{1}{g_1} \sin \frac{\beta_1}{2} - \frac{1}{g_2} \sin \frac{\beta_2}{2} \right) \\
\tau_{xy} &= \frac{1}{h_1 - h_2} \frac{K_{\mu}}{2r} \text{Re} \left( \frac{h_1}{g_1} \cos \frac{\beta_1}{2} - \frac{h_2}{g_2} \cos \frac{\beta_2}{2} \right)
\end{align*}
\]

Above equations are the singular stress fields to be expressed as basic real functions.

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