'Galaxy Associations' as Possible Common Features of Galaxy Clusters

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Abstract. The results of the analysis of the subclustering in a sample of ESO Nearby Abell Cluster Survey (ENACS) galaxy clusters, with data complemented from other studies, are presented. The analysis is performed by using the S-tree method enabling one to study the hierarchical properties of clusters, namely by the detection of the main physical cluster and of its subgroups. The results indicate: (a) systematically lower genuine cluster velocity dispersions than known from previous studies; (b) existence of 2-3 subgroups in each cluster. Due to certain properties of specific dynamical entities, we denote these subgroups as galaxy associations, which can become an essential challenge for formation mechanisms of galaxy clusters.

1 Introduction

The study of the hierarchical subclustering in clusters of galaxies is one among the important problems of observational cosmology. Since clusters of galaxies as distinct from stellar systems (star clusters, galaxies) have characteristic time scales exceeding or comparable to the Hubble time, their physical and dynamical properties should contain direct information on the mechanisms of their formation and the early evolution of the Universe.

Though the conclusions are not absolutely unambiguous with respect to individual clusters, the studies in general do support the existence of subgroups in clusters of galaxies (e.g. West et al. 1988; Bird 1994; West 1994; Escalera et al. 1994), supported also by X-ray data on clusters (Moher et al. 1993; Fabricant et al. 1993; Bohringer et al. 1994; Grebenev et al. 1995; Zabludoff and Zaritsky 1995; West et al. 1995). Substructuring of clusters is also anticipated by theoretical studies (e.g. White 1976; Cavaliere, Colafrancesco & Menci 1992, Jing et al. 1995).
The ESO Nearby Abell Cluster Survey (ENACS) being a large homogeneous dataset of redshifts of galaxies in clusters (Mazure et al, 1996, Katgert et al 1996), provide an interesting possibility to investigate this problem. This dataset contains high reliability redshifts on 5636 galaxies in the direction of 107 clusters selected from the Abell-Corwin-Olowin (ACO) catalogue with richness \( R \geq 1 \) and mean redshifts \( z \leq 0.1 \).

The proper study of the profound properties of substructures, besides accurate observational data, requires also refined methods of statistical analysis. Among others, well known methods are the correlation functions - mainly two-point, but also occasionally using higher order ones, the minimal spanning tree, topological measures, wavelets, etc (for review of these methods, see Gurzadyan and Kocharyan 1994a). These methods are using the positional information usually completed with redshift data, e.g. in the case of wavelet technique (Slezak et al 1990; Escalera et al 1994; Grebenev et al 1995), one can combine 2D wavelets for positions, and 1D ones - for redshift information.

The present analysis is using the S-tree method (Gurzadyan et al, 1991, 1994; Gurzadyan and Kocharyan 1994a) developed for the study of the properties of the many dimensional nonlinear systems and is essentially based on the concepts of the theory of dynamical systems. In the context of galaxy clusters its advantage is in the *self-consistent* use of both, the kinematical information - the redshifts, and positional one - the 2-coordinates, as well as the data on individual observable properties of galaxies - the magnitudes. The method has been already applied to the study of the substructures of the Local Group, Virgo cluster, Coma cluster, Abell clusters (Gurzadyan et al, 1993; Petrosian et al 1997; Gurzadyan and Kocharyan 1994a; Gurzadyan and Mazure, 1996, 1997): e.g. in the case of the Local Group, aside confirmation of the general picture known from other studies, some new associations between particular galaxies have been observed.

Here we report the results of the analysis of 10 Abell clusters - A119, A151, A262, A539, A978, A1060, A2717, A3651, A3667, A3822. The redshift data used are mainly those of ENACS, complemented in some cases with other sources. Data for clusters A262, A539, A1060 are taken from literature.

The study of the substructuring properties of clusters can enable one to investigate the behavior of various important parameters, such as the velocity dispersion variation with radius (den Hartog and Katgert, 1996), or the luminosity and morphological segregation in the subgroups. The latter problem was recently considered in the case of the core of Virgo cluster (Pet-
rosian et al 1997), analyzed again via the S-tree method and morphological segregation between the subgroups has been noticed. The importance of such an information is in its direct consequences for cosmological theories.

Our aim here, however, is to concentrate only on two concrete points: the estimates of genuine velocity dispersions of clusters and the typical dynamical characteristics of substructures, since the advantage of the S-tree method is precisely in the detection of small-scale structures. More comprehensive study of the results of the dynamical substructuring for each individual cluster will be performed elsewhere.

The results of our analysis show:

First, that 1D velocity dispersions of physical clusters are typically lower than those obtained by other methods (e.g. Zabludoff, Huchra and Geller 1990; den Hartog and Katgert, 1996). If so, this fact should be considered as supporting cosmological models with $\Omega < 1$. This conclusion is close to that of (Bahcall and Oh 1996), based on the use of Tully-Fisher distances of Sc galaxies;

Second, that clusters do contain bound subgroups, with non-equal number of galaxies, but with typical velocity dispersions in the range 100-200 km $s^{-1}$. The most important is that we have found some indication concerning their truncated Gaussian, i.e. "box" like velocity histograms. Simple dynamical considerations enable us to understand the nature of this specific galaxy configurations, which, if confirmed, can have essential cosmological consequences. In view of their probable primordial nature, is some analogy with the role of stellar associations in galaxies, we call these dynamical entities galaxy associations.

We start with the description of the database in Section 2, and a brief summary of the S-tree technique in Section 3. The revealed substructures, i.e. the galaxy associations in the sample of clusters of galaxies, are presented in Section 4. The dynamical properties of galaxy associations are analyzed in Section 5. The results of the study and the main conclusions are discussed in Section 6.

2 The Data

The following data were used in our analysis. The choice of these 10 clusters was determined by the maximal completeness in galaxies in the range of the 2 brightest magnitudes (for details see, Girardi et al 1997).

A119. ENACS, (Fabricant et al 1993).
3 The S-Tree technique

In this section we outline the key points of the S-tree technique (see Appendix), referring for details to the original papers (Gurzadyan et al, 1991, 1994, and to the monograph (Gurzadyan and Kocharyan 1994a). One can also refer to the papers where these geometrical concepts have been firstly applied to N-body gravitating systems (Gurzadyan and Savvidy, 1984, 1986).

The main idea of the S-tree method is based on the property of structural stability (coarseness), well known in theory of dynamical systems, enabling to reveal and study the robust properties of nonlinear systems based on a limited amount of information. As it was known before, gravitating systems are exponentially unstable ones, and therefore, belong among systems possessing strong statistical properties.

One application of this approach is the derivation of a formula for the Hausdorff dimension depending on the dynamical properties of the clusters of galaxies (Gurzadyan and Kocharyan 1991; Monaco 1994). The S-tree method can also be used for the reconstruction of the 3D-velocities of clusters (Gurzadyan and Rauzy 1996).

The problem is formulated in a way to find out the correlation which
should exist between the parameters of an interacting N-body gravitating system, i.e. between the velocity and coordinate components of the interacting particles. Particles not satisfying the correlation found for a given subgroup are considered as uninfluenced by (i.e. unbound to) that subgroup. Thus, the particles of set \( A \subset N \) are considered to be influenced by each other if one can find out the ordinarity class

\[
\Phi_c : \mathbb{R}^6 \times \Lambda \rightarrow \mathbb{R}^3,
\]

and if there exists \( c \in \mathbb{R}^d \) such that for any \( i \in A \)

\[
\Phi_c(x_i, v_i, \lambda_i) = 0,
\]

where \( \lambda \in \Lambda \) describes the internal properties of the members of the system - the masses in our case.

The following two main concepts are basic ones for the S-tree method: the degree of boundness of the various members of the system (galaxies), and the tree-diagram – S-tree – being constructed using the boundness results and representing, therefore, the information on the hierarchical substructure of the clusters.

The S-tree method offers the possibility to use various affine parameters while constructing the tree-diagram, such as the force of interaction, momentum, potential, etc., as listed in (Gurzadyan and Kocharyan, 1994). However, the most complete information on the dynamics of the system is extracted when the transition from the correlation to the degree of boundness is realized by means of the simultaneous consideration of all particles of the system, i.e. inquiring into the properties of phase trajectories of the systems in \( 6N \)-dimensional phase space with properly defined measure. This powerful method, well developed in the theory of dynamical systems (see e.g. Arnold 1989), enables one to reduce the problem of the dynamics of the N-body system to the study of the geometrical properties of the phase space via the estimation of the two-dimensional curvature:

\[
K_{\mu\nu} = R_{\mu\nu\lambda\rho} u^\lambda u^\rho, \tag{3}
\]

where \( u \) is the velocity vector.

Then, the degree of boundness is obtained by means of the following symmetrical matrix:

\[
D_{ab} = \{-K_{\mu\nu}, 0\}; \tag{4}
\]

\[
\mu = (a, i), \nu = (b, j); a, b = 1, 2, ..., N; i, j = 1, 2, 3.
\]
Obviously, the tensor $K$ is containing self-consistently the information both on the coordinates and velocities of all the particles of the system, as well as on those individual properties of particles which are determining their mutual interaction – the masses.

The results of the analysis of the hierarchical substructure of the system, are represented via tree-diagrams – S-tree – as defined in graph theory, by means of corresponding transition matrices $D_{ab}$:

$$\Gamma_{ab} = 0 \text{when } D_{ab} < \rho, \Gamma_{ab} = 1 \text{when } D_{ab} > \rho. \quad (5)$$

Obviously, the significance of the detection of the subgrouping hierarchy of the system, as in any statistical problem, depends on the total number of particles of the system $N$; numerical experiments described in the above cited papers, show that the confidence level of the substructures is > 90 per cent if $N > 30 – 35$; note, that all substructures have the same confidence level, since they are parts of the same interacting system.

As in previous studies of real clusters (Gurzadyan et al 1993; Petrosian et al 1997)), we presently used the assumption $M = const L^n$ where $L$ is the luminosity, $M$ is the mass of galaxies, and $n = 1$ or $n = 1/2$; the results are checked to be robust with respect to these laws.

Let us briefly discuss the question of the correspondence of the S-tree method with other statistical studies of clusters of galaxies; for a review of the mathematical background of these various methods see (Gurzadyan and Kocharyan 1994a). Most thoroughly, the S-tree results have been compared (together with Eric Escalera) with those of wavelets by applying this technique to the same initial sample of Abell clusters (Girardi et al 1997). We saw that the main systems defined by both methods for most of the compared clusters are in fair agreement. The S-tree method, however, enables also to reveal with the same confidence level, substructures with smaller number of galaxies, for which wavelets have a limited sensitivity. The reason is obvious: for wavelets, a cD galaxy and its satellite have the same statistical weight in attracting the companions, while the S-tree method is using the information on the magnitudes, and hence, on the masses of galaxies in a self-consistent way, which is especially crucial while revealing small-scale subgroups. In other words, though wavelets uses also the redshift and positional information, for example via 2D-wavelets for coordinates and 1D-wavelets for redshifts, both data are complementary but not self-consistent; while the S-tree method inquiring into the dynamics of an Hamiltonian system, is considering the coordinates, velocities, and magnitude as subsets of a single correlated set. Thus, one can easily understand
the difference of the S-tree technique not only with respect to wavelets, but also to other existing methods.

Among methodical aspects of the use of the S-tree method, let us mention also the role of the magnitude completeness of the data sample in the final results. Previous applications of the S-tree technique both to toy models and to real clusters (see e.g. Gurzadyan et al 1991, 1993, 1994; Petrosian et al 1997) had shown that, the main substructure of the revealed system, as a rule, is determined by the information on the brightest 2-3 magnitudes (depending on the total number of galaxies in cluster, etc) galaxies of the system. In other words, additional information on more fainter objects does not radically change the revealed picture of substructuring. Indeed, faint objects can be either projected more distant galaxies, which in any way would have correlation with the parameters of the system, or galaxies of much lower masses situated within the system. Since the dynamics of the system is governed mainly by massive objects, the corresponding magnitude limited samples will be reflecting the genuine substructure of the clusters. The choice of the sample of Abell clusters studied in present paper was done in view of these conditions.

4 The Substructure of the Sample of Abell Clusters: Existence of Galaxy Associations

The S-tree analysis of the sample of Abell clusters defines: a) the main physical system, as a system of interacting galaxies (MS), b) the subgroups of the main system. This includes the individual membership of galaxies in the main system and in each subgroup. After that, the statistical analysis has been performed in order to reveal the main physical parameters of each subgroup and of the system.

The main results for the studied clusters are represented in Table 1. It includes the total number of galaxies in the sample (T), in the revealed main system (MS) and in the subgroups (1s, 2s, 3s), as well as their median velocity ($m$), standard deviation ($\sigma$), skewness ($s$) and curtosis ($c$) of the velocity distribution. The redshift histograms are given in Figure 1.

In addition to the parameters given in the Table 1, the analysis reveals more features for each cluster. We briefly mention some of them. For example, for A119 a fraction of the second system (12 galaxies) has been indicated with a mean redshift around 14,200 km s$^{-1}$, which probably has physical relation to the MS, so far as a weak correlation between them is
Table 1: Main parameters of the clusters: T denotes the total number of galaxies in the sample, MS is the main system, 1s, 2s, 3s are the corresponding subgroups, m is the median, $\sigma$ is the standard deviation, s is the skewness and c is the curtosis of velocity distribution

| Cluster | N | MS | 1s | 2s | 3s |
|---------|---|----|----|----|----|
| A119    | 142 | 109 | 53 | 18 | 13 |
|         | m  | 13256 | 13152 | 13702 | 12487 |
|         | $\sigma$ | 604 | 212 | 100 | 80 |
|         | s  | 0.2 | -0.2 | 0.4 | 3.1 |
|         | c  | 0.3 | -1.0 | -1.0 | -1.1 |

| Cluster | N | MS | 1s | 2s |
|---------|---|----|----|----|
| A151    | 134 | 59 | 23 | 16 |
|         | m  | 15996 | 15845 | 16340 |
|         | $\sigma$ | 615 | 381 | 134 |
|         | s  | -0.1 | 1.9 | 0.18 |
|         | c  | -0.8 | 5.1 | -1.34 |

| Cluster | N | MS | 1s | 2s |
|---------|---|----|----|----|
| A262    | 128 | 95 | 27 | 25 |
|         | m  | 4864 | 4920 | 4600 |
|         | $\sigma$ | 481 | 120 | 116 |
|         | s  | 2.84 | 0.41 | -0.5 |
|         | c  | -0.93 | -1.20 | -1.24 |

| Cluster | N | MS | 1s | 2s |
|---------|---|----|----|----|
| A539    | 107 | 82 | 31 | 15 |
|         | m  | 8737 | 8797 | 8279 |
|         | $\sigma$ | 627 | 120 | 143 |
|         | s  | 0.52 | 0.23 | -0.1 |
|         | c  | -0.35 | -0.90 | -0.94 |
|     | T | MS | 1s | 2s | 3s |
|-----|---|----|----|----|----|
| A978|   |    |    |    |    |
| N   | 73| 58 | 12 | 20 | 6  |
| m   | 16303| 16771| 16333| 16058|
| σ   | 567  | 196  | 84  | 85 |
| s   | 0.23 | 0.3  | -0.05 | -0.2|
| c   | -0.2 | -1.6 | -1.4 | -1.9|
| A1060|   |    |    |    |    |
| N   | 105| 82 | 20 | 44 | 13 |
| m   | 3609| 2957| 3612| 4409|
| σ   | 515  | 188  | 243 | 112|
| s   | -3.82| 0.1  | 0.2 | 0.24|
| c   | -1.0 | -1.57| -1.33| -1.4|
| A2717|   |    |    |    |    |
| N   | 81 | 43 | 28 | 13 |
| m   | 14814| 14687| 15125|
| σ   | 338  | 184  | 126 |
| s   | 0.32 | -0.25| -0.01|
| c   | -1.0 | -1.1 | -1.5 |
| A3651|   |    |    |    |    |
| N   | 80 | 61 | 28 | 9  |
| m   | 17943| 17962| 17544|
| σ   | 531  | 157  | 91 |
| s   | -0.05| 0.3  | -0.4 |
| c   | -0.6 | -0.7 | -1.44 |
| A3667|   |    |    |    |    |
| N   | 113| 90 | 23 | 10 | 24 |
| m   | 16589| 15847| 17350| 16792|
| σ   | 585  | 211  | 137 | 208|
| s   | -1.4 | -0.3 | -0.1 | 0.6|
| c   | -1.1 | -1.3 | -1.5 | -0.85|
observed; for details see the paper by Gurzadyan and Mazure (1996) devoted entirely to A119, containing also the comparison of S-tree results with the X-ray data, etc. The sample of A151 also contains 36 galaxies of the second system with a mean redshift around 30,000 km s$^{-1}$, and fragments of other projected clusters. In A2717, in the 1st subgroup, there is an indication of a core of 10 galaxies, with median velocity 14527 km s$^{-1}$. The initial sample of 81 galaxies, probably, contains fractions of a second system (11 galaxies). A1060 also shows a fragment of a related second system (6 galaxies) with mean redshift around 4800 km s$^{-1}$.

These and other details about the clusters revealed by the S-tree analysis will be discussed elsewhere.

What are the basic features of the substructuring paradigm indicated by our analysis?

First, the existence of 2-3 subsystems is typical, at least for the studied clusters.

Second, the magnitude of the velocity dispersions of the subsystems appears to be around 100-200 km s$^{-1}$. As follows from Table 1, the 4th momentum (curtosis) of the redshift distribution shows a clear preference of negative sign. This fact can be interpreted as a tendency to have truncated Gaussian distributions. For comparison, the 3rd momentum, as expected, shows no preference in sign.

Although the conclusion on the existence of subgroups with truncated Gaussians – galaxy associations – is strengthened by the fact that the S-tree method extracts the subgroups and their parameters at the same confidence level as the whole cluster, such a conclusion should be considered as a preliminary one. However, if confirmed, this phenomenon can have deep physical consequences, directly reflecting both – the initial conditions of the cluster, and its evolutionary paths, it makes sense to discuss at least briefly some of its dynamical aspects.

| A3822 | T | MS | 1s | 2s |
|-------|---|----|----|----|
| N     | 101| 55 | 18 | 10 |
| m     | 22883 | 22372 | 22958 |
| σ     | 537  | 273 | 64 |
| s     | -0.05 | 1.0 | 0.2 |
| c     | -1.2 | 0.4 | -1.5 |
5 Why the galaxy associations can have ”box” structures?

Let us discuss now, what can be the reason of truncated Gaussian, i.e.“box” velocity distribution of the revealed substructures of the clusters. Can it be an apparent effect, i.e. determined, say, by the data incompleteness, especially for faint galaxies or can this be a genuine feature of those configurations?

We have a N-body gravitating system, which is a subsystem of another gravitating system (main system), with larger N. What will be the velocity distributions of both systems? This problem is not new and, at first glance, is close to that of the dynamics of globular clusters within the galactic field. The situation in that case is basically understood. So far as both systems (star clusters and host galaxies) are autonomous by their dynamics, and if their corresponding relaxation time scales are less than the Hubble time, then both stellar velocity distributions should be close to Maxwellian; here we do not discuss possible distortions of that law due to various effects, such as evaporation of stars from the system, core collapse, dissolution and formation of binaries, presence of massive black holes, etc. The observations generally confirm this behaviour.

Quite different is the situation in the case of clusters of galaxies. It is well known, that due to the difference in the order of N, many stellar dynamical results cannot be directly applied to clusters of galaxies. In our study of the substructures in clusters, we have one more demonstration of this fact.

If the cluster of galaxies is a more or less isolated system, then depending on the total number of galaxies, its velocity distribution can be close to Gaussian. This is what is generally supported by observations.

However, the typical subsystems will not have Gaussian distribution, i.e. distribution with “wings”, because of the following reason. During the time scale of the order of the dynamical time of the subsystem, $\tau_{\text{dyn}}$, the galaxies situated in the wings of the velocity distribution should undergo stronger attraction by the host cluster and have to escape from the subsystem. The time scale of the cut-off of the Gaussian distribution is $\tau_{\text{dyn}}$, while the time scale to recover the “wings” should be $N^\alpha \tau_{\text{dyn}}$, where $N$ is the number of objects in the subsystem, and $1/3 < \alpha < 1$ is a positive number; its numerical value depends whether two-body or N-body interactions have the main role in that process (see Gurzadyan and Pfenniger 1994). Thus, the time scale of the cut-off will always be shorter, than that of the recovering of the
“wings”, i.e. typically one will observe only truncated Gaussian configurations - boxes. Note, that this dynamical process is somewhat different from the tidal cut-off of globular clusters in the field of the host galaxy.

Another feature, which again is indicated by our analysis of observational data, is the stability of standard deviations for the subgroups velocities, i.e. the stability of the widths of the boxes. The reason is again, understandable. The width of boxes is essentially determined by the parameters of the host cluster, which is cutting the “wings”.

Below, on a simple example we will illustrate this fact i.e. we will show in which way the properties of such subgroups can be determined by the parameters of the host cluster.

Since the number of galaxies within the subgroup is relatively small, instead of stellar dynamical technique one has to recall the methods of celestial mechanics.

Assume that the galaxies in the subgroup are moving by elliptic orbits. Then, the observed redshift interval $\Delta v = [v_{\text{min}}, v_{\text{max}}]$ should be determined by the maximal and minimal velocities of galaxies on the orbits:

$$v_{\text{max}}/v_{\text{min}} = (1 + e)/(1 - e),$$

where $e$ is the eccentricity of the orbit.

Therefore, the cut-off of the redshift “wings” will mean the absence of galaxies with eccentricities higher than some $e_{\text{max}}$; i.e. galaxies with too elongated orbits have to be captured by the host system and will not return to the system. So, the problem comes to the determination of the population of systems with eccentricity cut-off at $e_{\text{max}}$.

In this formulation we have deliberately reduced the problem to the classical one of the binary stellar systems (Ambartzumian 1937), just treated by methods of celestial mechanics.

Indeed, consider the phase density of an $N$-body system, when the distribution function of galaxies is an arbitrary function only of energy

$$dn = f(L_1, ... L_N) dL_1, ... dL_N dG_1, ... dG_N dH_1, ... dH_N$$

$$dl_1, ... dl_N dg_1, ... dg_N dh_1, ... dh_N,$$

where the Delaunay canonical elements $L_j, G_j, H_j, l_j, g_j, and h_j$ are defined as:

$$L_j = m_j (GM \left( \sum_{i=1}^{N-1} m_i / \sum_{i=1}^{N} m_i \right) a_j)^{1/2}, \quad l_j = n_j (t - T_j),$$

where $m_j$ is the mass of the $j$th galaxy, $G$ is the gravitational constant, $M$ is the mass of the central body, and $a_j$ is the semi-major axis of the $j$th galaxy's orbit.
\[ G_j = L_j (1 - e_j^2)^{1/2}, \quad g_j = \omega_j = \pi - \Omega_j, \]
\[ H_j = G_j \cos i_j, \quad h_j = \Omega_j, \]

Here \( a \) and \( e \) are the major semi-axis and the eccentricity of the orbit of a galaxy, respectively, \( i \) is the inclination of its orbital plane, \( n(t - T) \) is the mean anomaly, \( \omega \) is the angular distance of perihelion from the node, \( \Omega \) is the longitude of the ascending node, and \( M \) is the total mass of the system, \( m \) is the galactic mass.

The advantage of considering the case of elliptic orbits is in the possibility to use the Delaunay canonical formalism; note, that \( G \) and \( H \) are positive, so far as \( 0 < e < 1 \).

We are interested in the number of systems with cut-off of galactic orbits at \( e > e_{\text{max}} \):

\[
n(G_j < G_{\text{max}}) = 8\pi^N \int_0^\infty f(L_1, \ldots L_N) dL_1 \ldots dL_N \]
\[ \int_0^{L_1(1-e^2)^{1/2} \ldots L_N(1-e^2)^{1/2}} dG_1 \ldots dG_N G_1 \ldots G_N, \tag{9} \]

since the integration over \( G_j \) from 0 to \( L_j (1 - e^2)^{1/2} \) implies the variation of eccentricity from \( e = 1 \) to \( e_{\text{max}} \).

Therefore, the number of systems with cut-off of all orbits at \( e_{\text{max}} \), is

\[
n(e_j > e_{\text{max}}) = A(1 - e_{\text{max}}^2), \tag{10} \]

where

\[
A = \text{const} \int_0^\infty f(L_1, \ldots L_N) L_1^2 \ldots L_N^2 \ dL_1 \ldots dL_N. \tag{11} \]

i.e. the number of such systems crucially depends on \( e_{\text{max}} \), as, obviously, it was for the binary systems. From here one can conclude that the cut-off subsystems should be quite common within the host clusters, with the cut-off parameter \( e_{\text{max}} \), determined essentially by the parameters of the host system. The marginal stability of velocity dispersions around \( \sigma \approx 100 - 200 \) \( \text{km s}^{-1} \) is an indication of this conclusion. Some proportionality of \( \sigma \) on the number of galaxies of subsystem can also be felt for some clusters (A119, A151, A262, etc), which is again understandable.

One can even roughly estimate the range of peculiar velocities of such configurations. Indeed, assume that the maximal distance, “perigee”, of the elliptic orbit from the mass center, \( r_{\text{max}} \), does not exceed by the order of magnitude the mean distance between the galaxies of the main cluster
(otherwise the galaxy will be captured), while \( r_{\text{min}} \) should be of the order of the mean distance between the galaxies in the subsystem. Then, one can readily have: \( e_{\text{max}} = 1 - 2(R/R_0)(N_0/N)^{1/3} \), where \( N_0 \) is the number of galaxies in the main cluster, \( R_0 \) and \( R \) are the dimensions of the main system and of the subgroup, respectively. If \( e << 1 \), one has for their peculiar velocity interval: \( \Delta v \simeq 2v_0 e_{\text{max}} \simeq (0.2 - 0.4)v_0, \) (\( v_0 \) is the mean velocity of main cluster) for the parameters of the above studied clusters, i.e. in marginal agreement with the obtained velocity range. Note, the weak proportionality of \( \Delta v \) on the number of galaxies in the subsystem.

By this schematic analysis we aimed to demonstrate only the basic features of the galaxy associations, namely the typicalness of their velocity cut-off and the crucial dependence on the parameters of the host cluster. More refined theoretical studies are desirable to study their detailed properties.

6 Discussion.

Thus, we have analyzed by using the S-tree method the substructure properties of a sample of clusters of galaxies, mainly issued from ENACS data, extracting with the same statistical significance level both the main cluster and its subgroups.

As a result, we have obtained the main parameters of the revealed clusters and their subgroups. The 1D velocity dispersions obtained for the main systems are very close to those obtained by Bahcall and Oh (1996). This fact, as discussed by these authors, is supporting the low \( \Omega \) Friedmann Universe. The hierarchical structure of the clusters revealed by the present analysis, with different velocity dispersions, and hence, defining potential wells of various scales, is in agreement with the hierarchical structure of the dark matter, indicated by X-ray data (Ikebe et al 1996).

Among the basic properties of the subgroups appears to be their non-Gaussian, namely, box-like 1D velocity distribution. Though, any system with little number of gravitating bodies, will not in principle have “smooth” Gaussian distribution, the box-like truncation, if confirmed, can be a result of definite physical evolution. Indeed, simple theoretical considerations enable to conclude that box-like shape cannot be due to statistical incompleteness or other systematic errors, but most probably indicates the genuine property of these configurations – galaxy associations – namely, as subsystems, whose part of galaxies have been captured by the main cluster. De-
pending on the initial conditions and the paths of their evolution, i.e. the degree of stripping during the evolution, the core of these configurations can be close or far from the Gaussian cusp, though without “wings”.

Also note little overlapping between the 1D velocity distributions of the subgroups (Figure 1). Namely, this yields in average about 10 percent of members of other groups and 20 percent of members of the host galaxy within the redshift interval of a given subgroup which are not considered as its members.

This can be understood as follows: if the mutual bulk velocities of the subgroups are larger than their velocity dispersions, then one will typically observe redshift separation. If the galaxy associations are young mergers, one has to expect bulk velocities which could well exceed the above given values of velocity dispersions of subgroups; old mergers, on the contrary, will have more overlappings. Thus, the problem of the reconstruction of the bulk motions within the clusters (Gurzadyan and Rauzy 1997), particularly, the determination of the ratio \( \frac{\text{bulk}\text{velocity}}{\text{velocity}\text{dispersion}} \) for the galaxy associations can be important for revealing their nature. These issues also are topics of further studies with more data.

Thus, galaxy associations can have essential role in the understanding of the mechanisms of the formation of clusters of galaxies, since obviously, they should have primordial nature. By analogy with stellar associations and globular clusters which have primordial stellar population, the problem of the study of the population of galaxies of the galaxy associations, is therefore, arising.

Observational, theoretical and numerical study of galaxy associations, in particular, their morphological features, the process of their survival depending on their initial parameters and those of the main cluster, seems of particular interest.

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7 Appendix

In this Appendix we describe the main steps in introducing the concept degree of boundness used in Section 3, and, hence, on S-tree technique; for detailed account we refer to the original papers (Gurzadyan, Harutyunyan and Kocharyan 1991, 1994; Bekarian and Melkonian 1997), and particularly to (Gurzadyan, Kocharyan 1994a).

Let us start with the simplest system of point particles interacting according to a given force law in $\mathbb{R}^3$. We wish to describe the degree of boundness of those particles in various ways.

To accomplish this goal we propose the following natural approach. Let $x_1(t)$ and $x_2(t)$, $t \in (-T, T)$ be the trajectories of the two particles when the interaction is taken into account, and let $y_1(t)$ and $y_2(t)$ be the corresponding trajectories when the interaction is “switched off”, related to the former trajectories by sharing the same initial data at time zero as the corresponding interacting particles. In other words $y_1(t)$ and $y_2(t)$ are the trajectories of free particles for which $y_i(0) = x_i(0)$, $\dot{y}_i(0) = \dot{x}_i(0)$, $i = 1, 2$.

Let us measure the degree of boundness of the two interacting particles over this time period by the function

$$m = \max_{i=1,2} \mathcal{N}(x_i(\cdot) - y_i(\cdot)), \quad (12)$$

i.e., the maximum “deviation” of either of the particles from its corresponding free trajectory, where this deviation is measured with respect to the following local norm $\mathcal{N}$ on $C^\infty$ parametrized curves in $\mathbb{R}^3$ over certain interval.

$$\mathcal{N}(x(\cdot)) = \sup_{t \in (-T,T)} \{|x(t)|, |\dot{x}(t)|\}. \quad (13)$$

Now consider balls of radius $r$ at each point of trajectories of the two interacting particles $x_i$. The union of these balls will form the two spaces

$$\mathcal{C}_i(r) = \bigcup_{t \in (-T,T)} B_{x_i(t)}(r), \quad i = 1, 2.$$ 

$m$ is the minimal allowed radius such that neither of the free particle trajectories escapes from its corresponding space $\mathcal{C}_i(m)$.

Two particles are considered to be $\rho$-bound for $\rho > 0$ if $m \leq \rho$.

This is easily generalized to any finite number of particles. $N$ particles labeled by the set of integers $\mathcal{A} = \{1, \ldots, N\}$ form a $\rho$-bound cluster if the
distance between the corresponding trajectories of the system of interacting particles and free ones is less than the maximal deviation of all of the particles:

\[ m = \max_{a \in A} \mathcal{N}(x_a(\cdot) - y_a(\cdot)) \leq \rho . \tag{14} \]

Consider now a dynamical system characterized by the following equations:

\[ \ddot{x}_a = f_a(x), \ x \in \mathbb{R}^{dN}, \ x_a \in \mathbb{R}^d . \]

For the problem of interest the smooth functions \( f_a \) have the form

\[ f_a(x) = \sum_{b=1}^{N} f_{ab}(x_a, x_b) , \]

where \( f_{aa}(x_a, x_a) \) indicates the influence of an external field on the \( a \)-th particle.

We now define the boundness function \( \mathcal{P}_Z(Y) \), where \( Y \subset Z \subset \mathcal{A} \).

Consider the following two systems:

I.

\[ \ddot{z}_a = \sum_{b \in Z} f_{ab}(z_a, z_b) , \]

\[ z_a(0) = c_a, \ \dot{z}_a(0) = v_a, \ \text{where} \ a \in Z , \]

II.

\[ \ddot{y}_a = \sum_{b \in Y} f_{ab}(y_a, y_b) , \]

\[ y_a(0) = c_a, \ \dot{y}_a(0) = v_a, \ \text{where} \ a \in Y , \]

where \( c \) and \( v \) are constant vectors, the initial position and velocity, respectively, of the \( a \)-th particle. The first dynamical system represents the subsystem of particles in \( Z \) switching off the interactions with external particles, while the second is the same for \( Y \).

Then for the given local norm \( \mathcal{N} \) we take

\[ \mathcal{P}_Z(Y) = \max_{a \in Y} \mathcal{N}(z_a(\cdot) - y_a(\cdot)), \tag{15} \]

where \( z_a(t), y_a(t) \) are the solutions of the systems of equations I and II respectively for some time interval \((-T, T)\). In other words the boundness of \( Y \) in \( Z \) is the maximum deviation of the trajectories of its particles taking into account only internal interactions compared to the situation when
interactions with particles in $Z$ are also included. Our goal is to split $A$ into $\rho$–subsystems, i.e., to obtain the map $\Sigma$ for this choice of boundness function.

The description of the algorithm of the construction of tree-diagram let us begin by corresponding to the system of a matrix $D_{ab}$, $a, b \in A$, (with positive elements) which determines the degree of boundness of a pair $a$ and $b$. The concrete choice of $D[i]$ depends on the physical system and its properties. For our purposes we should inquire into the systems interacting via Newtonian gravity. For given $\rho$ it is convenient to correspond $D$ another matrix $\Gamma$ in a manner:

$$
\Gamma_{ab} = 0 \text{ if } D_{ab} < \rho \text{ and } D_{ba} < \rho,
\Gamma_{ab} = 1 \text{ if } D_{ab} \geq \rho \text{ or } D_{ba} \geq \rho;
$$

(16)

$\Gamma$ can be considered as a matrix describing a graph with $\{1, \ldots, N\}$ apexes, with two connected apexes $a$ and $b$, if $\Gamma_{ab} = 1$. The $A$-set of a $\Gamma$ would be connected if $\forall a', a'' \in A \exists a_i \in A$, $i = 1, \ldots, k$ sequence of apexes so that $a_1 = a'$, $a_k = a''$ and $\forall i \ 1 \leq i \leq k - 1 \ \Gamma_{i,i+1} = 1$.

The definition of a $\rho$–bound cluster given above can be reformulated now as a set of corresponding particles $A$ being a connected subgraph of the graph $\Gamma$ (equivalent the matrix) so that there is no other connected subgraph $B$ including $A$: $A \subset B$. If one defines $P$ as follows

$$
\mathcal{P}_{X,Y} = \max_{y \in Y \setminus X} \{D_{yx}\},
$$

(17)

the problem of the search of a $\rho$-bound cluster is reduced to that of a connected $\Gamma$-graph.

Consider now what matrices $D$ one can choose, if $d = 3$ and

$$
M_a \ddot{x}_a = - \sum_{b \in A \setminus \{a\}} f_{ab}^i
$$

1 A list of possible choices of $D$ with different informativeness is given in (Gurzadyan and Kocharyan 1994a, Chapter 4.1). For example, the use of projected distances, which is equivalent to spatial correlation functions, is evidently not enough informative in order to speak seriously about subgroupings. The use of ‘projected’ energy of gravitating particles (Serna and Gerbal 1996) is also not relevant, since it does not contain information on the degree of interaction (gradient), while arbitrary interpretation of simple numerical experiments without theoretical background can be misleading; e.g. in (Gurzadyan and Kocharyan 1994b) the non-equivalence of the physical N-body system with its computer image for certain parameters is strongly proved.
\[ f_{ab} = G M_a M_b \frac{x_a - x_b}{|x_a - x_b|^3}, \quad a \neq b, \]
\[ f_{aa} = 0, \]

where \( M_a \) is the mass of \( a \)-th particle and

\[ |x_a| = \left( \sum_{i=1}^{3} \left[ x_a^i \right]^2 \right)^{1/2}. \]

One can show that to account for the information on the dynamics of the system one can choose the matrix \( D \) using the Riemannian curvature of the phase space of the system (Arnold 1989). It is well known that the Hamiltonian system by means of Maupertuis principle can be represented as a geodesic flow in a Riemannian \( 3 \cdot N \) dimensional manifold with a defined metric. The deviation of the geodesics of the configurational space and hence the properties of the Hamiltonian system are determined by Riemannian tensor of curvature \( Riem \) via Jacobi equation:

\[ \nabla_u \nabla_u n + Riem(n, u)u = 0, \quad (18) \]

where \( u \) is the tangential vector to the geodesic, \( n \) is a deviation vector.

In some basis the Jacobi equation can be written as

\[ \ddot{n}^\mu + K^\mu_\nu u^\nu = 0, \]

where \( u^\mu = \langle e^\mu, u \rangle \), \( n^\mu = \langle e^\mu, n \rangle \), \( K^\mu_\nu = \langle e^\mu, Riem(e_\nu, e_\rho)e_\lambda \rangle \). For a system of \( N \) gravitating particles the \( K \) and Riemannian tensor has the form (Gurzadyan and Savvidy 1984, 1986)

\[ K^\mu_\nu = Riem^\mu_{\lambda\rho\nu} u^\lambda u^\rho = \]
\[ -\frac{1}{2W} \left[ \delta^\mu_\nu W_{\lambda\rho} u^\lambda u^\rho + W^\mu_\nu u^2 - u^\mu W_{\lambda\nu} u^\lambda - u_\nu W^\mu_\lambda u^\lambda \right] - \]
\[ \frac{3}{4W^2} \left[ (u^\mu W_{\nu} - \delta^\mu_\nu W_{\lambda} u^\lambda) W_\rho u^\rho + (u_\nu W_{\lambda} u^\lambda - W_{\nu} u^2 W^\mu) - \right] \]
\[ \frac{1}{4W^2} \left[ \delta^\mu_\nu u^2 - u^\mu u_\nu \right] ||dW||^2, \]

where

\[ R^i_{ab} = x^i_a - x^i_b, \quad R_{ab} = |R^i_{ab}|, \]

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\[ W = E + \sum_{a=1}^{N} \sum_{b=1}^{a-1} G \frac{M_a M_b}{R_{ab}}, \]

\[ u^\mu = u^{(a,i)} = \frac{\dot{x}^i_a}{\sqrt{2W}}, \]

\[ g_{\mu\nu} = M_a \delta_{\mu\nu} \]

\[ u_\mu = g_{\mu\nu} u^\nu, \quad u^2 = u^\nu u_\nu = 1/W. \]

The derivatives of \( W \) have the following form

\[ W_\mu = W_{(a,i)} = - \sum_{c=1}^{N} \sum_{c \neq a} G \frac{M_a M_c R_{ic}}{R_{ac}^3}, \]

\[ W_{\mu\nu} = W_{(a,i)(b,j)} = G \frac{M_a M_b}{R_{ab}^3} \left( \delta_{ij} - \frac{3R_{ia}^2 R_{jb}^2}{R_{ab}^2} \right), \quad \text{if } a \neq b, \]

\[ W_{\mu\nu} = W_{(a,i)(a,j)} = - \sum_{c=1}^{N} \sum_{c \neq a} G \frac{M_a M_c}{R_{ac}^3} \left( \delta_{ij} - \frac{3R_{ia}^2 R_{jc}^2}{R_{ac}^2} \right), \]

\[ \|dW\|^2 = \sum_{a=1}^{N} \sum_{i=1}^{3} W_{(a,i)}^2 / M_a, \]

\[ W_\mu u^\mu = \sum_{a=1}^{N} \sum_{i=1}^{3} W_{(a,i)} u^{(a,i)}, \]

\[ W_{\mu\nu} u^\mu u^\nu = \sum_{a=1}^{N} \sum_{i=1}^{3} \sum_{b=1}^{3} W_{(a,i)(b,j)} u^{(a,i)} u^{(b,j)}, \]

\[ W_{\mu\nu} u^\nu = \sum_{b=1}^{N} \sum_{j=1}^{3} W_{(a,i)(b,j)} u^{(b,j)}. \]

From here we have

\[ D_{ab} = \max \left\{ -K_{\nu}^\mu, 0 \right\}, \quad (19) \]

where \( K \) is the curvature and the system consisting of the all \( a \) and \( b \) pairs is reduced to a geodesic flow in a Riemannian 3N-dimensional manifold.
The algorithm of computer analysis is described in (Gurzadyan et al 1994). Therefore here we briefly mention the procedure of its operation and the ways of presentation of the obtained results via S-tree diagrams.

Consider a system of \( N \) particles of masses \( M_a \) with given coordinates \( x_a \) and velocities \( \dot{x}_a \). By first step we find out the multifunction \( \Sigma \) for concrete matrix \( D \). As we showed above, for given \( D \) and \( \rho \) the problem is reduced to the search of connected parts of the graph \( \Gamma(\rho) \). In general, however, for subsystems defined by \( P \) another algorithm can be used.

Thus we use an algorithm which finds out the connected parts of the graph \( \Gamma \) for given \( \rho \), but simultaneously obtains all those parts for all \( \rho \). In other words, given the parameters

\[
M_a, \ x_a(0), \ \dot{x}_a(0), \ a \in A
\]

and the matrix \( D \) the algorithm finds out the function \( \Sigma \).

The latter can be represented in a form of tree-diagram (S-tree), so that its height is determined by the value of \( \rho \). For each value of the latter the sets

\[
\{A_1(\rho), \ldots, A_d(\rho)\},
\]

are constructed in a way that each line \( L_a \) corresponds to a certain particle. As a result one has a tree-diagram indicating the set \( \{L_a : a \in A\} \).

The computer experiments on toy models using the described technique are described in (Gurzadyan et al 1991, 1994). For further development of S-tree technique see (Bekarian, Melkonian 1997).
Figure caption.

The redshift histograms of the main clusters (solid line) with indicated subgroups (dashed and dotted lines) for a sample of Abell clusters studied by S-tree method.
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