On the nature of quantum gravity

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Recently was advanced the argument that accelerating reference frames can be represented by a proper quantized space-time. Quantum space-time of this form was indicated by the definition of quantum fields in momentum space, called accelerated quantum fields. In this paper, the nature of Unruh-like effect for accelerated fields is analyzed with mass having the key-role instead of acceleration. It is shown that a classical relativistic particle behaves as a black body at a temperature, \( T \), which is inversely proportional to its mass, radiating quantum excitations of accelerated field, i.e. quanta of space-time. Comparing with the classical case of curved space-time by a massive object, we deduce that quantum space-time takes the place of the usual spacetime continuum, curvature is substituted by quantum fields and tidal effects of gravity is replaced by Unruh-like effect of accelerated fields. In case of black hole, we find that the temperature \( T \) is proportional to Hawking temperature, \( T = 4T_H \). Thus black holes radiates as any other gravitational object. Finally, the direct association of rest mass with temperature offers an explanation of quantum gravity in thermodynamic terms, namely as a gas of space-time quanta in thermodynamic equilibrium.

I. INTRODUCTION

Quantum field theory and general relativity are considered as the most successful theories we have, however they are based on contradictory hypotheses. General relativity teach us that space-time is curved and everything is smooth and deterministic. On the other hand, quantum field theory advocates that the world is formed by discrete quanta over a flat space-time, governed by global symmetries.

As the history of physics has shown, the comparison between apparently contradictory successful theories has led to major steps in science. Accelerated quantum fields \cite{1,2} are precisely an effort to solve the irreconcilable contradiction between quantum mechanics, as formulated in quantum field theory, and general relativity. In this approach, both the lessons of geometry and quantum are taken into account and we proceeded with a fresh look at the problem.

In accelerated quantum field theory, accelerating reference frames are represented by quantum space-time. It is usually assumed that space-time should be a continuum in order to define a quantum field theory in an accelerating frame of reference. This is because, the transformation of a quantum field to an accelerating reference frame is simply implemented by the coordinate transformation from inertial frame to an accelerating one. The main purpose of \cite{1} was to show that an appropriate modification of the ordinary concept of space-time redefines an accelerating quantum system of fields. The principle result in that work is that there exists a Lorentz invariant definition of space-time, with a natural unit of length which depends on the value of acceleration. In a subsequent paper and in an attempt to demonstrate the utility of accelerated fields, it was shown how Unruh effect can emerge by quantizing the space-time \cite{2}.

The incorporation of gravity into relativistic framework may be based on the equivalence of all accelerated systems. Einstein’s important advance was to realize that if all accelerated systems are equivalent, then Euclidean geometry cannot hold in all of them. Thus Riemannian geometry has been manifested itself as the natural mathematical framework to study gravity. Accelerated quantum fields modify the concept of accelerating reference frame and therefore our understanding of gravity. The description of gravity as a space-time curvature cannot survive in this theory. Our aim in this work is to show how we can understand gravity in the theory of accelerated quantum fields. As we will see, gravity appears as a feature of accelerated fields, necessary for their consistency, revealing its emergent nature.

On the way to quantum gravity, while it is generally accepted that space-time is quantized, there is disagreement as to how quantization manifests itself \cite{3}. In accelerated quantum field theory, the definition of quantum fields in momentum space indicates the quantization of space-time in a mathematically consistent way. Space and time are quantized in the way quantities like energy and momentum are quantized in ordinary quantum field theories.

The requirement of locality remains a strong motivation for studying field theories in the quantum world. However, the presence of a fundamental length scale in any theory of quantum gravity guaranties the entrance of non-locality. Therefore we cannot treat the quantum gravitational field simply as a quantum field in space. Physicists have sought a way to incorporate gravity into a quantum field theory by making conjectures about possible alterations that could be made to the theory. Maldecena suggested that holography might be the key to reconcile gravity with quantum mechanics, all you need is an extra dimension of space [AdS/CFT] \cite{4}. Our proposition renders extra dimensions needless. We share Born’s view \cite{5}, that physics should be equivalently formulated from the position and momentum point of views, and develop a theory where space-time and momentum space

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appear to hold equal parts. We promote momentum space by constructing a relativistic field theory in momentum space analogously to standard field theories in space-time. Thus, in our case, the smooth metric geometry of space, which is the ground needed to define a quantum field, is not the space-time but the momentum space.

Constructing a relativistic field theory in momentum space analogously to standard field theories in space-time it would be interesting to investigate whether and in which form the properties which characterize standard quantum field theories appear in the new construction of quantum fields. We find that Unruh effect [6], one of the most intriguing feature of quantum field theory, where different frames of reference correspond to different vacua, is also met in accelerated field theory, with the exception now that the key-role is on mass and not on acceleration.

Invoking equivalence principle, we apply accelerated field theory to uniform gravitational field and show that the corresponding Unruh ambiguity is the quantum analogue of tidal effects of gravity. In this way, gravity is represented by a type of black body radiation, which is emitted by any massive object. Instead of energy quanta, "gravitational" black body emits quanta of space-time. Alternatively, this can be expressed also in the following way: The central idea of general relativity, that matter cause space-time to curve, in quantum regime, is translated to that all objects emit quanta of accelerated field when they are massive.

We start in section II with an exposition of the concept of uniform gravitational quantum fields. Section III illustrates how quantum gravity arises in the framework of accelerated quantum field theory. In section IV we show how quantum gravity can be derived entirely from statistical arguments. The summary and discussion are presented in section V. We consider quantum field theories in two dimensions with metric signature +−. Furthermore, the units are chosen with c = 1 but h and G are kept explicit.

II. FREE-FALL OBSERVERS IN ACCELERATED FIELD THEORY

In Newtonian physics, free fall is any motion of a body where gravity is the only force acting upon it. In the context of general relativity, where gravitation is reduced to a space-time curvature, a body in free fall is subject to no force and is an inertial body moving along a geodesic. Due to equivalence principle, an infinitesimally extended, homogeneous gravitational field can be completely replaced by a state of acceleration of the reference system, thus in accelerated quantum field theory a free fall observer may be identified to quantum space-time.

Following the formulation of accelerated quantum field theory, let us consider a real field \( \tilde{G}(p^\mu) \) in momentum space, defined by the wave equation

\[
\left( \partial_E^2 - \partial_p^2 - \frac{1}{g^2} \right) \tilde{G}(p^\mu) = 0,
\]

with \( p^\mu = (E, \vec{p}) \) being the four momentum, while \( g \) represents the gravitational acceleration. We solve the theory by introducing an orthonormal set of mode solutions given by \( \tilde{u}_\mu(p^\mu) = e^{i(Et - \vec{p} \cdot \vec{x})/\sqrt{4\pi \xi_t}} \), with \( x_t \equiv x(t) = \sqrt{t^2 + 1}/g^2 \), and claiming that the definition of the positive- and negative-frequency solutions lies in the existence of a space-like Killing vector field, \( \tilde{\partial}_\mu \), in momentum space [1]. Working in the space spanned by the positive frequency modes \( \tilde{u}_\mu \), one can define the field operator \( \tilde{G} \) in the language of the second quantization in the usual way

\[
\tilde{G}(p^\mu) = \int dt \left( \tilde{a}_t \tilde{u}_t(p^\mu) + \tilde{a}^\dagger_t \tilde{u}^\dagger_t(p^\mu) \right).
\]

\( \tilde{a}_t \) (\( \tilde{a}^\dagger_t \)) is the annihilation (creation) operator, which acts on physical space and annihilates (creates) excitations of the field \( \tilde{G}(p^\mu) \) which "carries" time interval \( t \) and length \( x_t \). The operators \( \tilde{a}_t \) and \( \tilde{a}^\dagger_t \) should fulfill the typical algebra for creation and annihilation operators, i.e.

\[
[\tilde{a}_t, \tilde{a}_t^\dagger] = [\tilde{a}_t^\dagger, \tilde{a}_t] = 0, \quad [\tilde{a}_t, \tilde{a}_t^\dagger] = \delta(t - t')
\]

By defining the conjugate momentum as \( \tilde{p}(p^\mu) = \tilde{\partial}_\mu \tilde{G}(p^\mu) \), the commutation relations [3] in space-time are equivalent to the canonical equal-momentum commutation relations

\[
[\tilde{G}_p(E), \tilde{G}_p(E')] = [\tilde{p}_p(E), \tilde{p}_p(E')] = 0,
\]

\[
[\tilde{G}_p(E), \tilde{p}_p(E')] = i \delta(E - E'),
\]

in momentum space.

Our next task is to find the spectrum of the Hamiltonians. We do this by defining first the Hamiltonian of the theory. The Hamiltonian is defined in the following way. First we construct the Lagrange density associated with equation (1) by inverting the Euler-Lagrange equation. Then, Noether’s theorem provides us the momentum independent quantity that plays the role of the Hamiltonian. Finally, by using Eq. (2) we can write the Hamiltonian in the form

\[
\tilde{H} = \int \frac{dt}{\sqrt{4\pi \xi_t}} x_t \left( \tilde{a}_t^\dagger \tilde{a}_t + \frac{1}{2} [\tilde{a}_t, \tilde{a}_t^\dagger] \right).
\]

At this point we need to make two comments on the form in which Hamiltonian appears. First, Hamiltonian has dimensions of length. Second, a minimum length is obtained from the second term inside parenthesis. We cannot avoid the existence of this term, since our treatment resembles that of harmonic oscillator and this term is the field analogue of the harmonic oscillator zero-point energy. We will ignore this term in all of our calculations below. But, we intend to discuss the nature and the consequences of this term in a subsequent work.
Using Eq. (4) for the Hamiltonian, it is straightforward to evaluate the commutators
\[ [\hat{H}, \hat{a}^\dagger] = x_t \hat{a}^\dagger, \quad [\hat{H}, \hat{a}] = -x_t \hat{a}. \]
(5)
We can then write down the spectrum of the theory. There will be a single vacuum state \(|0\rangle\), characterized by the fact that it is annihilated by all \(\hat{a}_t\),
\[ \hat{a}_t |0\rangle = 0, \quad \forall t. \]
(6)
All other eigenstates can be built by letting \(\hat{a}^\dagger_t\) acting on the vacuum,
\[ |t\rangle = \hat{a}^\dagger_t |0\rangle. \]
(7)
This state has length
\[ \hat{H}|t\rangle = x_t |t\rangle \quad \text{with} \quad x_t^2 = t^2 + 1/g^2. \]
(8)
The second relation resembles the hyperbolic motion of a particle with constant acceleration \(g\). Even it is tempting, the idea to associate \(|t\rangle\) to a particle with acceleration \(g\) it would be misleading and served to veil essential aspects. The interpretation of quantum states in terms of particle configurations has been seen as a manifestation of the wave-particle duality. In standard quantum field theory the concept of wave-particle duality expresses the experimental facts that the spectrum of the operator \(E_p^2 = p^2 + m^2\) have a discrete part.

Our theory does not employ any energy operator. However, we do obtain a classical-quantum duality based on the relativistic relation \(x_t^2 = t^2 + 1/g^2\). The theoretical concepts that physicists have formed about space-time, the geometric theory of gravity among them, are based on the assumption that the variables \(x_t\) and \(t\) (if we restrict ourselves to \(1 + 1\) space-time) take on a continuum of values and they may take on these values simultaneously. If this constitute the classical aspect of space-time, the quantum is introduced by postulating that space-time is composed of discrete quanta rather than continuous variables. We theorize that the length in each quantum of space may be equal to an angular frequency \(\bar{\omega}\), while an interval of time may be equal to a wavenumber \(k\), both multiplied by a constant \(\hbar\),
\[ x_t = \hbar \bar{\omega} \quad \text{and} \quad t = \hbar \bar{k} \]
(9)
following the recipe of one of the starting milestones of quantum theory, the wave-particle duality. Notice that in this setting \(\hbar\) has dimensions of physical action thus it is nothing else than the reduced Planck constant. The equation (11) has been developed principally from the above hypothesis, a wave equation that would describe quantum space-time. In practice, natural units comprising \(\hbar = 1\) are used, except otherwise stated, allowing time, wavenumber, length and angular frequency to be used interchangeably.

In this end, let us try to interpret the eigenstates of the Hamiltonian, the states of the quantum field \(\mathcal{G}\). To begin with, we recognize \(x_t\) as the relativistic dispersion relation for a frame of reference in a state of acceleration \(g\). Thus, we may interpret the state \(|t\rangle\) as the time eigenstate of a reference frame with acceleration \(g\). Let us check this interpretation by studying the other quantum number of \(|t\rangle\). The expression for time operator, which follows from Noether’s theorem (see [1] for details), is given by
\[ \hat{T} = -\int dE \hat{a}_t \hat{E} \hat{G} = \int \frac{dt}{\sqrt{\Delta x_t}} \hat{a}_t^\dagger \hat{a}_t. \]
(10)
Acting on our state \(|t\rangle\) with \(\hat{T}\), we learn that it is indeed an eigenstate,
\[ \hat{T}|t\rangle = t|t\rangle. \]
(11)
telling us that the state \(|t\rangle\) has the time interval \(t\).
So the operator \(\hat{a}^\dagger_t\) creates time \(t\) and length \(x_t = \sqrt{t^2 + 1/g^2}\). In general, since our construction accommodates multi-quanta states, the state \(\hat{a}^\dagger_0 \hat{a}^\dagger_{t_2} \cdot \cdot \cdot |0\rangle\) is an eigenstate of \(\hat{H}\) with length \(x_{t_1} + x_{t_2} + \cdot \cdot \cdot\) and time \(t_1 + t_2 + \cdot \cdot \cdot\). The full Hilbert space of our theory is spanned by acting on the vacuum with all possible combinations of \(\hat{a}^\dagger_t\)’s. The number operator \(\hat{N}\) counts the number of particles in a given state in the full Hilbert space
\[ \hat{N} = \int \frac{dt}{\sqrt{\Delta x_t}} \hat{a}^\dagger_t \hat{a}_t, \]
(12)
and satisfies \(\hat{N}|t_1, \ldots \cdot t_n\rangle = n|t_1, \ldots \cdot t_n\rangle\). The number operator commutes with the Hamiltonian, \([\hat{N}, \hat{H}] = 0\), ensuring that particle number is conserved. This is a property of free theories, but will no longer be true when we consider interactions.

In this section, we presented the quantum theory of a field in momentum space, identified only by the gravitational acceleration, avoiding to mention the word particle. Our theory accommodates multi-quanta states, the state \(\hat{a}^\dagger_0 \hat{a}^\dagger_{t_2} \cdot \cdot \cdot |0\rangle\) is an eigenstate of \(\hat{H}\) with length \(x_{t_1} + x_{t_2} + \cdot \cdot \cdot\) and time \(t_1 + t_2 + \cdot \cdot \cdot\). The full Hilbert space of our theory is spanned by acting on the vacuum with all possible combinations of \(\hat{a}^\dagger_t\)’s. The number operator \(\hat{N}\) counts the number of particles in a given state in the full Hilbert space
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This suggests the physical interpretation of these states in terms of accelerated frames of reference. More precisely, the state $|\psi\rangle$ may be interpreted as the time eigenstate of an accelerating reference frame with acceleration $g$.

Even though, the introduction of a quantum reference frame, as a replacement for accelerating frame of reference, on its own implies a wide range of consequences, the physical interpretation given above is not the last word on the subject.

General relativity is an expression of the conviction that the proportionality between the inertial and the gravitational mass is a valid law of nature. The identity of the inertial and the gravitational mass manifests itself in the fact that gravity accelerates all objects equally regardless of their masses or the materials from which they are made. This suggests the view, now known as equivalence principle, that, a gravitational field may be replaced by a state of acceleration of the reference body. In fact, we have no means to distinguish an accelerating reference frame from a gravitational field.

In this perspective, our intention to interpret the quantum states as free-fall observers and the accelerated quantum field as a uniform gravitational field are justified. Thus we claim that we can call the quantum field $\tilde{G}$ as uniform gravitational field and its excitations as free-fall observers. By a free-fall observer here we mean a reference frame, of which the classical degrees of freedom, the coordinate time and position, are promoted to operators, $\tilde{T}$ and $\tilde{H}$ respectively, acting on a Hilbert space.

We will finish this section by discussing various features of accelerated quantum field theory as applied to uniform gravitational field.

The role of fields is to implement the principle of locality, and accelerated fields are not a exception. We just need to extend the classical notion of locality, derived from the concept of classical space-time, to momentum space. If we call here the classical locality simply as locality, then, the momentum locality expresses the idea that quantum processes, as described in this section, can be localized in energy and momentum. This is a reasonable consequence of treating energy and momentum as classical entities which appear in the argument of accelerated fields. (We do not claim, against all the experimental facts, that energy and momenta are classical concepts. We just restrict ourselves to a theory that matter fields are not included.) That non-locality must enter any theory of quantum gravity is guaranteed by the presence of a fundamental length scale. On the other hand, one of the properties that quantum field theory incorporates into quantum physics is locality. The framework we propose reconcile that contradiction by linking the generalization of locality to a new understanding of the momentum space. An idea which, in different settings, has already been used [10].

The identification of accelerated quantum fields as homogeneous gravitational fields provides a new description of space-time. In [2], accelerated quantum fields were introduced to show that Unruh effect can be attributed to the nature of space-time. The relation between inertial and accelerating reference frames is reduced to a correspondence between classical and quantum space-time, expressed by the absence or the presence of accelerated fields, respectively.

This picture cannot survive when gravity is taken into account. In such a system, due to Equivalence principle, inertial frames of reference are substituted by free fall ones. As we have shown, in our structure, free fall observers entails the quantization of space-time. Thus, space-time should be quantized when gravity is not disregarded. Quantum space-time arises as excitations of uniform gravitational fields, which are described by quanta of length $x_l = \hbar \omega$ and time $t = \hbar k$. Notice that because for each quantum holds $x_l^2 - t^2 = 1/g^2$, all the information that a free fall observer carries, i.e. the gravitational acceleration $g$, is encoded in the manner that space-time is quantized. Thus, the infinity family of free fall observers we meet in classical physics here they are translated into infinity group of quantum fields, one for each value of gravitational acceleration $g$.

III. UNRUH-LIKE EFFECT FOR ACCELERATED QUANTUM FIELDS: AN INTRINSIC EFFECT OF QUANTUM GRAVITY

In the previous section, based on the space-time quantization, as indicated by accelerated quantum field theory, and the equivalence principle, we presented a quantum theory of a uniform gravitational field. However, the statement of the equivalence principle, that the effects of gravity are indistinguishable from those of an acceleration, is not strictly true. Massive bodies give rise to tidal effects (caused by variations in the strength and direction of the gravitational field) which are absent from an accelerating observer. Thus, equivalence principle has only limited validity. Measurements over extended regions of space and time can show a difference between an acceleration and gravity, so equivalence principle is valid only locally. To that end, the quantization of the uniform gravitational field does not entail quantum gravity. Despite this, accelerated quantum field theory provides a basis for a direct effect of quantum gravity.

In classical physics, consider an observer near a massive body with mass $m$. A test particle released by the observer at her location hovers where it is released. This is the essence of equivalence principle. Acceleration has removed the effects of gravity. But, in general, situations of mass distribution lead to gravitational effects due to its tide-producing action. To illustrate this in our case, let the observer release two neighboring test particles. Given the required measurement precision, one discerns the gradual acceleration of the test particles away from each other, if they lie along a common radius through the center of the massive body; or toward each other, if their separation lies perpendicular to that line. For applica-
tions of the Equivalence Principle since there is only one acceleration that the frame of reference can have, it can only match a gravitational field at some point. Nearby points will have different values of \( g \) and thus will not be eliminated. Thus, we conclude that the Equivalence Principle cannot remove all the effects of gravity. Gravitation is manifested in relative acceleration of neighboring test particles and the magnitude of the relative acceleration is proportional to the mass of the mass body.

Einstein’s great advance was the realization that relative acceleration is caused by curvature. Hence, the appropriate mathematical framework for someone to study Einstein’s gravity is Riemannian geometry.

However, the description of space-time as a (pseudo-) Riemannian manifold cannot survive accelerated quantum field theory. We have to find a new language for describing gravity, a language that gravity is not associated with a property of Riemannian geometry, but to a property of quantum field theory. This is precisely the purpose of this section.

Let us add to our new quantum theory the massive body \( m \) we mentioned above, and investigate its effect on the uniform gravitational quantum field \( \tilde{G} \). A relativistic invariant way to implement mass is the energy momentum relation \( E^2 - p^2 = m^2 \). By construction, the theory of accelerated fields does not convey any information regarding mass. Mass never appears in the equation which defines \( \tilde{G} \). Since accelerated fields have been established in momentum space, the consideration of the energy momentum relation modifies the metric of the space, where \( \tilde{G} \) is defined, i.e. for any point it holds \((E,p) \rightarrow (\sqrt{p^2 + m^2}, p)\).

Below we will investigate the effect of, based on the presence of mass, modified momentum space on the quantum states of \( \tilde{G} \). We will not only show the disagreement between an uniform gravitational quantum field built in massless and in massive momentum space (labeled massless \( \tilde{G} \) and massive \( \tilde{G} \), respectively) on the definition of the ground state, but also we shall demonstrate that this difference is due to the thermal behavior of massive \( \tilde{G} \). This is reminiscent of the Unruh ambiguity inherent to the choice of vacuum for standard quantum fields in Rindler space. In Unruh effect, acceleration is responsible for promoting zero-point quantum field fluctuations to the level of thermal fluctuation. In our case, this role is played by mass.

The approach presented below is based on Ref. \[9\] where the thermal effect of acceleration was computed by means of the field correlation function.

For the field \( \tilde{G}(E, p) \) (from now on \( \hbar \) is recovered) satisfying the wave equation \[11\] we consider the correlation function \( \langle \tilde{G}(E_1, p_1) \tilde{G}(E_2, p_2) \rangle \), computed in the vacuum state. In this case it holds \( \langle \tilde{a}_e^\dagger \tilde{a}_e \rangle = \langle \tilde{a}_e^\dagger \tilde{a}_e \rangle_0 = 0 \) and \( \langle \tilde{a}_v^\dagger \tilde{a}_v \rangle_0 = \delta(t - t') \). Therefore we obtain

\[
\langle \tilde{G}(E_1, p_1) \tilde{G}(E_2, p_2) \rangle_0 = \frac{\hbar}{\pi} \frac{1}{\Delta E^2 - \Delta p^2},
\]

with \( \Delta E = E_2 - E_1 \) and \( \Delta p = p_2 - p_1 \).

In case we measure the vacuum correlation function in massive space, the right part of \[13\] can be expressed in another form. We can adopt a reparametrization, satisfying automatically the relation \( E^2 - p^2 = m^2 \). As this represents hyperbolic curves, we can use hyperbolic functions and set

\[
E = m \cosh s \quad p = m \sinh s
\]

with \( s = \sigma/m \) a variable and \( \sigma \) a parameter. The induced metric becomes then

\[
\begin{align*}
ds^2 &= dE^2 - dp^2 \quad (16) \\
&= dm^2 - m^2 ds^2 \quad (17)
\end{align*}
\]

representing the massive momentum space. There is a horizon at \( m = 0 \) so these coordinates are good for \( m > 0 \) and \(-\infty < \sigma < \infty \). As a consequence, the coordinates \((m, \sigma)\) only cover the patch of momentum space with \( E > 0 \) and \(|p| < E\). Thus, a frame of reference embodied with mass is effectively confined to a piece of momentum space and it feels a horizon at \( m = 0 \).

One can calculate the difference \( \Delta E^2 - \Delta p^2 \) by making use of Eqs. \[14\] and \[15\]

\[
\Delta E^2 - \Delta p^2 = -4m^2 \sinh^2 \left( \frac{\sigma_2 - \sigma_1}{2m} \right). \quad (18)
\]

Then the correlation function \( \langle \Psi(E_1, p_1) \Psi(E_2, p_2) \rangle \) in the vacuum of the uniform gravitational field, defined in massive momentum space, is given by

\[
\langle \tilde{G}(0, k) \tilde{G}(0, k + \sigma) \rangle = -\frac{\hbar}{4\pi mk} \cosh^2 \left( \frac{\sigma_2 - \sigma_1}{2m} \right). \quad (19)
\]

Next, we consider the field correlation function

\[
\langle \tilde{G}(0, k) \tilde{G}(0, k + \sigma) \rangle \quad (19)
\]

at a point in (massless) momentum space for a field in equilibrium at temperature \( T \). In order to compute this, we impose

\[
\langle \tilde{a}_e^\dagger \tilde{a}_e \rangle = \delta(t' - t) n(x_i), \quad n(x_i) = \left( \frac{e^{k_B T} - 1}{e^{k_B T}} \right)^{-1}, \quad (20)
\]

which simply implies that different modes of a thermal field are uncorrelated and that a mode of frequency \( x_i \) has an average number of quanta \( n(x_i) \). It is important to mention that due to the dimensions that the Hamiltonian of our system has, the Boltzmann constant has been substituted by \( k_B \), which has dimension length divided by temperature. Finally we take

\[
\langle \tilde{G}(0, k) \tilde{G}(0, k + \sigma) \rangle = -\frac{\hbar}{\pi} \left( \frac{\pi k_B T}{\hbar} \right)^2 \cosh^2 \left( \frac{\pi k_B T \sigma}{\hbar} \right), \quad (21)
\]
which, comparing with the correlation function \(10\), we find that they are equivalent for the temperature

\[
T = \frac{\hbar}{2\pi k_B m}. \quad (22)
\]

Assuming the existence of a detector which is specialized in detecting excitations of \(\mathcal{G}\), the meaning of this result is that this detector in the vacuum, and defined in massive momentum space, responds as a detector, defined in (massless) momentum space, in a thermal bath at temperature \(T = \hbar/2\pi k_B m\). In other words, a particle with rest mass \(m\) radiates quanta of \(\mathcal{G}\), each one carrying length \(x_i = \hbar \omega\) and time \(t = \hbar k\).

This result raises many issues that should be clarified. We will complete this section by discussing and attempting to elucidate some of these. First, let us try to give some physical content to the effect that the vacuum state of a massive field \(\mathcal{G}\) is full of space-time quanta.

The fact that in our construction the vacuum of accelerated fields is unstable to space-time quanta emission in the presence of mass, should be associated to the conclusion in classical relativistic physics that the Equivalence Principle cannot remove all the effects of gravity in case the system is equipped with mass.

In a uniform gravitational field, gravitation acts on each part of the body equally and this is weightlessness, a condition that also occurs when the gravitational field is zero. Furthermore, the dynamics of the gravitational field, as described in Einstein’s Equations, do not admit solutions that are uniform in space and time. Thus, gravitation becomes apparent through the non-uniformities in gravitational fields or the tidal forces, as they called. It is these forces, formulated geometrically as space-time curvature, that are regarded as the fundamental manifestation of gravity in general relativity. In our approach, the tidal effects of gravitation are expressed in quantum terms as quanta in the vacuum state of accelerated quantum fields. The absence of non-uniformities in gravitational field reduces its quantum description to that provided by the accelerated field theory in (massless) momentum space in which the concepts of vacuum and field excitations are well-defined.

Our result, in the first part of this section, does not only show that massless and massive reference frames extract distinct excitation contents from the same field. It also demonstrates that an "observer" in massive reference frame feels a thermal bath of quanta at temperature which is inversely proportional to the mass. Unavoidably this makes one think of Hawking effect \(\mathcal{G}\). Hawking found that a Schwarzschild black hole radiates quantum mechanically at a temperature, \(T_H = \hbar/8\pi Gk_B M\), where \(M\) is the mass of the black hole.

Probably someone could claim that this comparison would be pointless and unfounded, since involves two unrelated quantum systems. Hawking discovered that black holes emit particles with a thermal spectrum at a temperature \(T_H\) by combining matter quantum fields and classical black hole mechanics. To be precise, this radiation does not come directly from the black hole itself, but rather is a result of virtual particles being boosted by the black hole’s gravitation into becoming real particles. On the other hand, we attributed a temperature to a massive object just employing accelerated quantum fields and the relativistic energy-momentum relation. Classical gravity did not contribute, in any way, to the derivation of the effect. In a sense, mass promotes vacuum fluctuations of accelerated fields to the level of thermal fluctuations.

Nevertheless, assuming that Hawking radiation gives a hint on the nature of quantum gravity and since accelerated fields have the ambition to express quantum gravity, \(T\) and \(T_H\), should be proportional in some limit. This limit may be the coincidence of the two gravitational objects, namely the black hole mass to satisfy the relation \(E^2 - p^2 = m^2\). In doing so, we implicitly consider black hole as an elementary particle. As it is known this can happen only in Planck scale where quantum gravity dominates. In this scale holds \(k_B/k_B = G\), thus finally we derive

\[
T = 4T_H. \quad (23)
\]

We find that in Planck scale, black holes, considered just as massive objects, radiate space-time quanta in temperature which is proportional to \(T_H\). We should notice that for this result we have not used all the available degrees of freedom of the system of accelerated fields, since we were confined ourselves to \(1 + 1\) dimensions. Usually \(G\) is treated as a coupling constant. However, in general relativity due to the equivalence principle, \(G\) can be understood as a conversion parameter between space-time and energy-momentum space. \(G\) in the equation that gives Hawking temperature should be interpreted as conversion parameter as well, which just serves as an auxiliary variable that is needed for dimensional reasons.

But if black hole, as any other massive object, actually radiates space-time quanta, how, in Hawking’s analysis, they appeared to radiate energy quanta? This can be explained by the instability in particle production that a gauge/matter field exhibits when it is defined on quantum space-time \(\mathcal{G}\).

The argument given for Hawking and Unruh effects for the structure of the vacuum near a black hole and acceleration horizon, respectively, applies equally well to accelerated fields vacuum near a mass horizon in momentum space. As \((E^2 - p^2) \to \infty\) the temperature is red-shifted to zero. As the mass horizon, \(E = \pm p\), is approached the observer sees a diverging temperature.

We saw that the momentum vacuum is a thermal state in massive space. Can we say anything about the entropy associated with this thermal state? With calculation similar to the one which shows that, the Minkowski vacuum contains correlations between corresponding modes on either side of the Rindler horizon, one can demonstrate that momentum vacuum accommodates correlations between modes on either side of the mass horizon. This entanglement implies that, although the vacuum is
a pure quantum state, its restriction to a localized region is mixed. The corresponding entropy is dominated by the shortest wavelength modes, and scales as the area of the region boundary. The entropy is infinite on account of the arbitrarily short wavelength fluctuations close to the horizon \( E = p = 0 \), which are entangled with partners similarly close on the other side of the horizon.

Thus, the entropy can certainly be infinite in a theory of quantum gravity with no matter fields (remember that accelerated fields is an effort to describe gravity, while it is kept separate from the matter fields.) This result complements a conclusion in standard field theory, that entropy is infinite provided that no gravity is considered \[1\]. But how could the horizon entropy ever be finite? In case of standard field theory, it seems that gravity itself should somehow render the entropy finite. We suspect, by analogous arguments, that in accelerated field theory matter fields will be responsible for this. The investigation of this issue requires the integration of matter fields with accelerated fields, something we intend to do in a subsequent paper.

IV. GRAVITY AS GRAVITATIONAL BLACK BODY RADIATION

Black body is an idealized physical body that absorbs all radiant energy falling on it, regardless of frequency or angle of incidence. A black body in thermal equilibrium emits electromagnetic radiation with spectrum that is determined by the temperature alone, not by the body’s shape or composition.

Hawking generalized the concept of black body radiation by including black hole, a classically defined gravitational object. Even though classical black holes cannot emit anything, to an outside observer, they have a non-zero temperature and emit radiation with a nearly perfect black-body spectrum. The energy distribution of emission is described by Planck’s law with a temperature \( T \) associated with the mass of the black hole.

In this work we take this result one step further and argue that a massive body (without any restriction) emits black body radiation with temperature which is inversely proportional to its mass. The novelty is that the radiation consists of space-time quanta, making us to infer that this radiation represents gravity itself. We illustrate this inference in the following way. Let us consider a classical point particle with rest mass \( m \). It seems at first futile to attempt to associate a nonzero temperature with it, since in our framework no energy field (matter or gauge) is considered. In general relativity, this particle orders the space-time to curve. The presence of particle will be recognized by a free fall observer on account of tidal effects of gravity. In our theory, free fall observers are excitations of quantum fields in momentum space, like \( \tilde{G} \). The particle will manifest its presence by the fact that the vacuum of \( \tilde{G} \) will be full of excitations.

The fact that, Equivalence Principle is inadequate to remove all the effects of gravity, quantum mechanically is expressed by the ambiguity in the definition of a certain vacuum state. If a state of motion, that an observer carries out, eliminates all the effects of gravity, then, according to Equivalence Principle, the observer is a free fall observer which, in the scheme of accelerated quantum fields, is translated as a quantum field with a unambiguous vacuum state or a useful concept of excitations. Thus, we assert that the curvature of space-time caused by the presence of mass is physically equivalent to the radiation of space-time quanta due to existence of a gravitational black body, provided that space-time is described as excitations of accelerated quantum fields and temperature is given from eq. \[22\].

The gravitational black body radiation can be seen as a gas of space-time quanta in thermodynamic equilibrium. Below we will show that gravity, as a black body radiation, can be derived entirely from a statistical argument of space-time quanta.

Let the radiation be enclosed in a volume \( V \) in momentum space. Let there be different kind of quanta with the respective numbers \( n_r \) and lengths \( x_r \) \((r = 0 \text{ and } r = \infty)\). The total grand canonical partition function of the whole system is

\[
Z = \sum_{\{n_i\}} e^{-\frac{1}{k_B T} \sum_i n_i(x_i - \mu)} = \prod_i Z^G_i \tag{24}
\]

where

\[
Z^G_i = \left(1 - e^{\frac{x_i - \mu}{k_B T}}\right)^{-1} \tag{25}
\]

the grand canonical partition function for single-quantum state with length \( x_i \), \( \mu \) is the chemical potential. In the derivation of \( Z^G_i \) in \[24\] we have used the formula from geometric series \( 1 + b + b^2 + b^3 + \cdots = 1/(1 - b) \). The average number of quanta for that single-quantum state is given by

\[
\langle n_i \rangle = \left(\frac{e^{\frac{x_i - \mu}{k_B T}} - 1}{e^{\frac{x_i - \mu}{k_B T}} + 1}\right)^{-1} \tag{26}
\]

A result that applies for each single-quantum state and thus forms a distribution for the entire state of the system.

For \( \mu = 0 \), eq. \[26\] is equivalent to the thermal radiation formula of a massive object. In line with this, the numbers of space-time quanta are not conserved. Space-time quanta are created or annihilated in the right numbers and with the right lengths following Bose-Einstein statistic, adapted, of course, to space-time quanta. Just as the Planck’s distribution is the unique maximum entropy energy distribution for a gas of photons, so is length distribution \[26\] for a gas of space-time quanta.
Quantum gravity remains an outstanding problem of fundamental physics. The bottom line is we don't even know the nature of the system that should be quantized. In this work we argue that this system should be accelerated quantum fields.

Accelerated field theory provides a theoretical argument that any physical object with zero rest mass emits radiation. The radiation is produced as if emitted by a black body with a temperature inversely proportional to the mass of the object, but contrary to Planck's law, radiation is composed of space-time quanta and not photons. In this scheme, gravity appears as a feature of accelerated fields, which is necessary for its consistency. In order to come to this conclusion, we had to give up the classical notion of space-time. The quantization of space-time is imposed by the definition of accelerated fields in momentum space. Thus, gravity continues to be considered a property of space-time. Depending on the language we use to describe space-time, the definition of gravity is adjusted analogously. Classically, where space-time is described as (pseudo-) Riemannian manifold, gravity is represented by its curvature, whereas quantum mechanically, where space-time is expressed in terms of quantum fields, gravity arises as black body radiation.

The theory presented here on the quantum nature of gravity cannot be the whole story. Our main result, that massive objects emit black body radiation, was based on the classical form of the energy-momentum relation to serve as source of gravity. However, matter, along with energy, is quantized, so a fully quantized description of gravity requires the incorporation of standard field theory into accelerated quantum fields. The consideration of gravity's source in the same physical form as that of gravity, i.e. as quantum fields, will set us capable of providing a precise formulation of the relationship between space-time fields and matter/gauge fields.

However, we chose the semi-classical approach for two reasons. First, from this point of view, gravity appears to bear a striking resemblance to a well studied effect of standard quantum field theory, Unruh effect. Second, our findings led us to develop the statistics of space-time quanta, which is actually the Bose-Einstein statistics adapted in the scheme of accelerated quantum fields. It seems to us that the hypothesis of space-time quanta in connection with statistical mechanics is sufficient to define gravity. From this, we believe that the analysis of many authors [12–14] on the resemblance between equations of gravity and the laws of thermodynamics can be obtained.

Although we have considered field theories in 1 + 1 dimensions, our results can be extended to physical dimensions. In terms of a global inertial coordinate system t, x, y, z, let us consider the Killing field which generates a boost about the origin in the x direction. In this case, the hyperbolic surface $t^2 - x^2 = -1/g^2$ is invariant under translation in y and z direction. Given the fact that the definition of accelerated fields is based on this hyperbolic cylinder surface, in our theory, accelerated fields become invariant under translations in y and z direction. This clearly reproduces all the results presented above. In the general case, we will have three dimensional surface that will be projected into the planes (t, x), (t, y) and (t, z), thus defining three independent accelerated fields. This reflects the fact that, in our theory, geometry is translated into fields.

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