Opportunistic Relaying in Wireless Networks

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Abstract

Relay networks having $n$ ad hoc nodes and $m$ half-duplex relays, all operating in the same frequency band in the presence of fading, are analyzed. This setup has attracted significant attention and several relaying protocols have been reported in the literature. However, most of the proposed solutions require either centrally coordinated scheduling or detailed channel state information (CSI) at the source nodes. Here, an opportunistic relaying scheme is proposed, which alleviates these limitations without sacrificing the system throughput scaling in the regime of large $n$. The scheme entails a two-hop communication protocol, in which sources communicate with destinations only through half-duplex relays. The key idea is to schedule at each hop only a subset of nodes that can benefit from multiuser diversity. To select the source and destination nodes for each hop, it requires only CSI at receivers (relays for the first hop, and destination nodes for the second hop) and an integer-value CSI feedback to the transmitters. Moreover, the relays operate in a completely distributed fashion, with no cooperation. For the case when $n$ is large and $m$ is fixed, it is shown that the proposed scheme achieves a system throughput of $m/2$ bits/s/Hz. In contrast, the information-theoretic upper bound of $(m/2) \log \log n$ bits/s/Hz is achievable only with more demanding CSI assumptions and full cooperation between the relays. Furthermore, it is shown that the system throughput of the proposed scheme scales as $\Theta(\log n)$.
Index Terms

Opportunistic communication, channel state information (CSI), multiuser diversity, ad hoc networks, throughput, scaling law.

I. INTRODUCTION

The demand for ever larger and more efficient wireless communication networks necessitates new network architectures, such as ad hoc networks and relay networks. As such, there has been significant activity in the past decade toward understanding the fundamental system throughput limits and developing communication schemes that seek to approach these limits.

Among other notable recent results on the throughput scaling of wireless networks, Gowaikar et al. [1] proposed a new wireless ad hoc network model, whereby the strengths of the connections between nodes are drawn independently from a common distribution, and analyzed the maximum system throughput under different fading distributions. Such a model is appropriate for environments with rich scattering but small physical size, so that the connections are governed by random fading instead of deterministic path loss attenuations. When the random channel strengths follow a Rayleigh fading model, the system throughput scales as $\log n$. This result is achievable through a multihop scheme that requires central coordination of the routing of nodes. Moreover, full CSI is needed to enable the central coordination.

Along with the work on multihop schemes, such as [1] and [2], there is another line of work characterizing the system throughput for wireless networks operating with two-hop relaying. The listen-and-transmit protocol, studied by Dana and Hassibi [3] from the power-efficiency perspective, turns out to have interesting results from the system throughput’s standpoint as well. This is in fact a two-hop amplify-and-forward scheme, where relays are allowed to adjust the phase and amplitude of the received signals. A throughput of $\Theta(n)$ bits/s/Hz throughput is achieved by allowing $n$ source–destination (S–D) pairs to communicate, while $m \geq n^2$ nodes in the network act as relays. It is assumed that each relay node has full knowledge of its local channels (backward channels from all source nodes, and forward channels to all destination nodes), so that the relays can perform distributed beamforming. Morgenshtern and Bölcskei worked in [4] with a similar distributed beamforming setup, and their results unveil trade-offs between the level of available channel state information (CSI) and the system throughput. In particular, utilizing a scheme with relays partitioned into groups, and where relays in each
group have CSI knowledge of only one backward and one forward channel, the number of relays required to support a $\Theta(n)$ throughput is $m \geq n^3$. In other words, with less CSI, the number of required relays increases from $n^2$ to $n^3$. An equivalent point of view is to state the throughput in terms of the total number of transmitting nodes in the system, $p = n + m$. Then the system throughput is $\Theta(p^{1/3})$, when the relays in each group know the channel for only one source-destination pair. When relays know the channels for all source and destination nodes, the throughput scales as $\Theta(p^{1/2})$.

Although these works have made great strides toward understanding wireless ad hoc network capacity, implementations of the schemes require either central coordination among nodes [1], [2] or some level of CSI (channel amplitude and/or phase) at the transmitter side [3], [4]. In a large system, obtaining this level of CSI, especially at the transmit side, may not be feasible. Likewise, the cooperation between wireless relays does not come for free, since the overhead to set up the cooperation may drastically reduce the useful throughput [5]. This paper addresses the need to alleviate these limitations by proposing an opportunistic relaying scheme that works in a completely decentralized fashion and imposes less stringent CSI requirements.

A. Main Contributions and Related Work

The main contributions of this work can be summarized as follows.

- A two-hop opportunistic relaying scheme for operating over fading channels is proposed and analyzed. The scheme’s salient features are:
  - It operates in a decentralized fashion. No cooperation among relays is assumed or required.
  - Only modest CSI requirements are imposed. At each hop, receivers are assumed to have knowledge of incoming channel realizations, while transmitters have access to only an index-valued CSI via low-rate feedback from the receivers.
- The throughput of the proposed scheme is characterized by:
  - A closed-form lower bound on the throughput is derived for the first hop (also referred to as Phase 1), and an exact throughput expression is derived for the second hop (also referred to as Phase 2). Both results are valid for a finite number of nodes $n$ and of relays $m$. This distinguishes the proposed scheme from other works [1]–[6], in which only asymptotic results are provided.
— In the regime of a large number of nodes $n$ and fixed number of relays $m$, the proposed scheme is shown to achieve a system throughput of $m/2$ bits/s/Hz. This can be contrasted with the information-theoretic upper bound on the scaling of the throughput $(m/2) \log \log n$, achievable only with full cooperation among the relays and full CSI (backward and forward) at the relays.

— The throughput analysis reveals an interesting feature of multiuser diversity: whereas when full cooperation between relays is allowed multiuser diversity boosts the throughput from $m/2$ to $(m/2) \log \log n$ bits/s/Hz, when cooperation is not possible multiuser diversity succeeds in restoring the parallel channels, but must forswake the power gain.

— We show that $m$ can grow (as a function of $n$) as rapidly as $\Theta(\log n)$, while still guaranteeing the linear throughput scaling in $m$. The linearity breaks down if $m$ grows faster than $\Theta(\log n)$. Therefore, the throughput scaling of the proposed opportunistic relaying scheme is given by $\Theta(\log n)$.

— The proof of the aforementioned optimal $m$ scaling implies a stronger result: $\Theta(\log n)$ is in fact the throughput scaling for any two-hop relaying scheme, given the assumption of decentralized operation of relays and CSI at the receivers.

The key idea behind the proposed scheme is to schedule at each hop only the subset of nodes that can benefit from multiuser diversity gain. The concept of multiuser diversity gain was originally studied in the context of cellular systems [7], [8]. At any given time, in a system with a large number of independently fading users, with a high probability there is a strong user to whom resources (bandwidth and power) can be allocated. By doing so, the system performance can be significantly boosted. The concept is by now well understood in the context of infrastructure wireless networks and has been adopted in 3G cellular systems and other emerging wireless standards. However, to the best of our knowledge, it has received less attention for wireless access networks, with some exceptions [9], where the potential of opportunistic relaying is reported in the framework of a diversity-multiplexing trade-off. In this work, we highlight another aspect of multiuser diversity: its application to simplify network operations and its effect on throughput scaling. The opportunistic scheme proposed here is in the spirit of [6], where distance-dependent, random channel gains were exploited in scheduling.
B. Organization of the Paper

The rest of the paper is organized as follows. The system model and the proposed two-phase relay protocol are introduced in Section II. Section III characterizes the system throughput in the regime where $n$ is large and $m$ is fixed. The throughput scaling of the proposed scheme is evaluated in Section IV. Finally, Section VI, contains a discussion and conclusion.

Notation: The symbol $|\mathcal{X}|$ denotes for the cardinality of the set $\mathcal{X}$, and $\log(\cdot)$ indicates the natural logarithm. We write $X \sim \text{Exp}(1)$ to indicate that the random variable $X$ follows the standard exponential distribution with probability density function (pdf) given by $f_X(x) = \exp(-x), x > 0$. The indicator function is denoted by $1(\cdot)$, and we use “$\chi^2(2p)$” to denote a chi-square random variable with $2p$ degrees of freedom.

II. System Model

Consider a wireless network with $n$ ad hoc nodes and $m$ relay nodes. Ad hoc nodes that generate data traffic are referred as source nodes, nodes that receive data traffic are referred to as destination nodes. Relay nodes have no intrinsic traffic demands. Since we are concerned with asymptotic behavior, and without loss of generality, we do not distinguish between source and destination nodes. We consider a two-hop, decode-and-forward communication protocol, in which sources can communicate with their destinations only through half-duplex relays. Specifically, in Phase 1 of the protocol, a subset of sources is scheduled for transmission to relays. The relays decode and buffer the packets received in Phase 1. During Phase 2 of the protocol, the relays forward packets to a subset of destinations (not necessarily the set of destinations associated with the source set in Phase 1). These two phases are interleaved: in the even-indexed time-slots, Phase 1 is run; in the odd-indexed time-slots, Phase 2 is run. The selection process for source/destination sets is of an opportunistic nature and will be elaborated upon in the sequel. Once scheduled for transmission, the transmission rate is fixed at 1 bit/s/Hz, and therefore the communication can be supported in an information theoretic sense if the corresponding signal to interference plus noise ratio (SINR) is greater than or equal to 1.\footnote{Generalizing to higher SINR thresholds and transmission rates is straightforward, but it encumbers notation without adding insight.} An example of the two-hop relay protocol is depicted in Fig. 1.
We now describe the channel model. It is assumed that the wireless network has independent and identically distributed (i.i.d.) Rayleigh connections $h_{i,r}$ from source nodes $i$, $1 \leq i \leq n$, to relay nodes $r$, $1 \leq r \leq m$. Thus, the channel gains follow an exponential distribution, i.e., $\gamma_{i,r} = |h_{i,r}|^2 \sim \text{Exp}(1)$. Likewise, we assume that the channel gains $\xi_{k,j}$ from relays $k$, $1 \leq k \leq m$, to destination nodes $j$, $1 \leq j \leq n$, be i.i.d. Exp(1), and that channel gains $\gamma_{i,r}$ and $\xi_{k,j}$ are independent for all $i$, $r$, $k$, and $j$. Quasi-static fading is assumed, with channel gains fixed during the transmission of each hop, and taking on independent values at different transmission times. In this model, channel gains are dominated by the effects of small-scale fading, making it possible to neglect path loss effects. Regarding CSI, we assume that at both hops, the receivers have detailed CSI of all senders, while the transmitters have access only to an index-value used to indicate a source chosen for transmission. This simple transmit CSI is obtained via receiver feedback. This CSI assumption is reasonable in practice as most wireless access network standards incorporate some form of pilot signals, and the type of feedback specified has low overhead.

We now describe the scheduling at each hop. We start with the first hop (Phase 1). All relays operate independently. Thus, without loss of generality, let us focus on any specific relay, say $r$. Relay $r$ measures the channels $\gamma_{i',r}$, $1 \leq i' \leq n$, and schedules the transmission of the strongest source node, say $i = \arg \max_{i'} |\gamma_{i',r}|^2$, by feeding back the index $i$. The overhead of this phase of the protocol is a single integer per relay node. Suppose the scheduled nodes constitute a set.
$\mathcal{K} \subset \{1, \ldots, n\}$; then since there are $m$ relays, up to $m$ source nodes can be scheduled in this fashion, i.e., $|\mathcal{K}| \leq m$. It is worth mentioning that it is possible for multiple relays to schedule the same source. In such cases, $|\mathcal{K}| < m$. The scheduled source nodes transmit simultaneously, each at a rate of 1 bit/s/Hz. The communication from source $i$ to relay $r$ is successful if the corresponding SINR

$$\text{SINR}_{i,r}^\text{P1} = \frac{\gamma_{i,r}}{1/\rho + \sum_{t \in \mathcal{K}, t \neq i} \gamma_{t,r}} \geq 1,$$

where $\rho$ is the average signal to noise ratio (SNR) of the source-relay link.

The scheduling at the second hop (Phase 2) works as follows: Each destination node $j$, $1 \leq j \leq n$, measures the forward channel strengths, $\xi_{k,j}$, $1 \leq k \leq m$, and computes $m$ SINRs by assuming that relay $k$ is the desired sender and the other relays are interference as follows:

$$\text{SINR}_{k,j}^\text{P2} = \frac{\xi_{k,j}}{1/\rho_R + \sum_{1 \leq t < m, t \neq k} \xi_{t,j}},$$

where $\rho_R$ denotes the average SNR of a relay–destination link. If the destination node $j$ captures one good SINR, say, $\text{SINR}_{k,j}^\text{P2} \geq 1$ for some $k$, it instructs relay $k$ to send data by feeding back the relay index $k$. Otherwise, the node $j$ does not provide feedback. It follows that the overhead of the second hop is also at most an index value per destination node. When scheduled by a feedback message, relay $k$ relays the data to the destination node at rate 1 bit/s/Hz. In case a relay receives multiple feedback messages, it randomly chooses one destination for transmission.

It is noted that in steady-state operation of the system, the relays have the ability to buffer the data received from source nodes, such that it is available when the opportunity arises to transmit it to the intended destination nodes over the second hop of the protocol. This ensures that relays always have packets destined to the nodes that are scheduled. In addition, due to the opportunistic nature of scheduling, the received packets at the destinations are possibly out of order and therefore each destination is assumed to be able to buffer them before decoding.

\textbf{Remark 1:} It is noteworthy to draw a comparison between Phase 1 and Phase 2. From the relays’ perspective, both hops of the communication protocol rely on scheduling a subset of “good” source/destination nodes for transmission. However, these two phases of the protocol differ in one key aspect: transmission over the second hop can be guaranteed to be successful since relays have access to the SINRs, but this is not the case for the first hop. This is because in the first hop, each relay selects a source node without knowledge of what the other relays
select. As a consequence, each relay has no control over the interferences stemming from all other concurrent transmitting sources, and therefore has no a priori knowledge of its own SINR. For example, in (1) relay $r$ knows the desired link strength $\gamma_{i,r}$, but does not know $\mathcal{K}$ and the corresponding interference term $\sum_{t \in \mathcal{K}, t \neq i} \gamma_{j,r}$. For the second hop, the senders (now the relays) are known a priori, and therefore the destination nodes have direct access to SINRs. This implies that once the destination node captures an SINR $P_2 \geq 1$, and accordingly requests a transmission, this transmission will be successful at a data rate of 1 bit/s/Hz. This key difference between the two phases is mirrored in the analysis in Section III.

### III. THROUGHPUT: LARGE $n$ AND FIXED $m$

Motivated by the observation that as communication devices (source and destination nodes in our system) become more and more pervasive, the number of infrastructure nodes (relays) is not likely to keep pace, the throughput analysis in this paper pays special attention to a regime in which the number of source and destination nodes, $n$, is large, while the number of relay nodes, $m$, is relatively small. We will show that both Phase 1 and Phase 2 achieve throughput of $m$ bits/s/Hz, yielding a $m/2$ bits/s/Hz throughput for the complete two-hop scheme. We also show that for any two-hop protocol, the throughput is upper-bounded by $m^2 \log \log n$ bits/s/Hz. This information-theoretic upper bound holds even if we allow full cooperation between relays and assume full CSI is available at the relays. Thus, the proposed scheme, with much simplified assumptions of decentralized relay operations and CSI at the receiver, succeeds in capturing the pre-log factor of the throughput.

#### A. Phase 1: Source Nodes to Relays

In Phase 1, $m$ relays operate independently and each schedules one source node for transmission. Hence, the total number of scheduled source nodes can be any integer between 1 and $m$, i.e., $|\mathcal{K}| \leq m$. In cases when $|\mathcal{K}| < m$, multiple relays schedule the same source node, and the analysis of the probability of successful transmission should consider explicitly those links with multiple receivers. Due to the multiplicity of possible combinations, the exact characterization of the throughput of Phase 1, $R_1$, is mathematically involved. Fortunately, in order to show the achievability of $m$ successful concurrent transmissions, it suffices to lower-bound $R_1$ by
considering only cases in which the \( m \) scheduled source nodes are distinct (thereby discarding the contributions to the throughput of the other combinations).

By symmetry, each source node has a probability of \( 1/n \) to be the best node with respect to a relay. Thus, the probability that the scheduled users are distinct, i.e., no source node is scheduled by more than one relay, is given by \( \Pr[N_m] = n(n-1) \cdots (n-m+1)/n^m \), where \( N_m \) denotes the event “\( m \) distinct source nodes are scheduled.” Now, a lower bound on \( R_1 \) is

\[
R_1 \geq m \cdot \Pr[N_m] \cdot \Pr[S_m],
\]

where \( S_m \) denotes the event of a successful transmission, i.e., \( \text{SINR} \geq 1 \). For notational brevity, in the following, we drop the source node and relay indices from the description of the channel gains \( \gamma_{i,r} \). From (1), it follows that the probability of a successful transmission between a source and a relay can be written

\[
\Pr[S_m] = \Pr[\text{SINR}^{P_1} \geq 1] = \Pr[\frac{X}{1/\rho + Y} \geq 1],
\]

where \( X \) is the highest channel gain between source nodes and the respective relay, and \( Y \) is the aggregate interference from all other \((m-1)\) concurrent transmitting source nodes. Since random variables \( X \) and \( Y \) are not independent, there is no apparent way to proceed with the computation of \( \Pr[S_m] \). Instead, we further lower-bound (4) by introducing a real variable \( s \) \((s > 0)\). By total probability, we have

\[
\Pr[S_m] = \Pr[\frac{X}{1/\rho + Y} \geq 1] = \Pr[X \geq s] \cdot \Pr[\frac{X}{1/\rho + Y} \geq 1 | X \geq s] + \Pr[X \leq s] \cdot \Pr[\frac{X}{1/\rho + Y} \geq 1 | X \leq s]
\geq \Pr[X \geq s] \cdot \Pr[\frac{X}{1/\rho + Y} \geq 1] \geq \Pr[X \geq s] \cdot \Pr[\frac{s}{1/\rho + Y} \geq 1]
= (1 - F_X(s)) F_Y(s - 1/\rho).
\]

The operational meaning of (5) can be interpreted as follows: Each relay sets a threshold \( s \), and it schedules the transmission of the strongest source only when the power gain of the
link exceeds the threshold. The probability of such event is given by $1 - F_X(s)$. Upon being scheduled and transmitting at 1 bit/s/Hz, the probability of successful communication with the relay is at least $F_Y(s - 1/\rho)$.

The result in (5) is general and holds for any channel fading model. Particularizing the analysis to the Rayleigh fading case, in which the link strengths are i.i.d. Exp(1) random variables, the cumulative distribution function (cdf) of $X$ (largest of $n$ i.i.d. Exp(1) random variables) can be written explicitly as

$$F_X(x) = (1 - e^{-x})^n. \quad (6)$$

The characterization of the interference term $Y$ in (4) needs more care. This is due to the fact that, conditioned on not being the maximum among $n$ channel strengths, each interference term in $Y$ is no longer exponentially distributed, and the interference terms are not necessarily independent. However, as shown in Appendix I, asymptotically with $n$, these properties still hold even after removing the largest channel gain. Numerical results show that these asymptotic trends are achieved for relatively small values of $n$, enabling the approximation of $Y$ as chi-square random variable with $2(m - 1)$ degree of freedom, whose cdf is thus given by

$$F_Y(y) = 1 - e^{-y} \sum_{k=0}^{m-2} \frac{1}{k!} y^k. \quad (7)$$

Substituting (5), (6), and (7) into (3) yields the following lower bound on the throughput of Phase 1.

**Lemma 1**: For any $\rho$, $n \gg m$ and $s > 0$, the achievable throughput of the opportunistic relay scheme in Phase 1 is lower-bounded by

$$R_1 \geq m \frac{n(n-1)\cdots(n-m+1)}{n^m} \left(1 - (1 - e^{-s})^n\right) F_Y\left(s - \frac{1}{\rho}\right). \quad (8)$$

A tighter lower bound can be found by maximizing (5) over $s$, but we find that little insight can be gained from this exercise. The tightness of the lower bound (8) is substantiated by numerical results in Fig. 2 of Section V.

For the regime of interest, where $n$ is large and $m$ fixed, it can be trivially shown, e.g., by setting $s = \log n - \log \log n$, that the above lower bound approaches $m$. Note that this is also the best we can hope for in Phase 1, since the SINR constraint implies that no more than $m$ sources can be successful.

The following corollary to Lemma 1 follows immediately.
**Corollary 1:** For the opportunistic relaying scheme with a fixed number of relays \( m \), the throughput for the first hop satisfies \( R_1 \to m \) as the number of source nodes \( n \to \infty \).

To summarize Phase 1, the core idea is that multiuser diversity enables a link to overcome the interference stemming from the other links.

**Remark 2:** Inspecting (8), we note that the lower bound on \( R_1 \) exhibits a tradeoff between quantity and quality of scheduled links. By increasing the number of relays \( m \), one can schedule more simultaneous transmissions, which is beneficial from the throughput perspective. However, more transmissions generate more interference, degrading the SINR and lowering the probability of successful transmissions. Not only the lower bound discussed here, but also the actual throughput \( R_1 \) demonstrates this tradeoff. This phenomenon will be discussed in Section V in connection with Fig. 2.

**B. Phase 2: Relays to Destination Nodes**

We now develop an expression for the sum-rate of the relay-destination links. This is done by first showing that only a single relay per destination can produce a required SINR larger than one, and then computing the probability of the event that a relay is scheduled and consequently delivers throughput.

Note that in Phase 2, unlike in Phase 1, the scheduling is based directly on SINR. Given the i.i.d. channel model introduced in the previous section, the SINRs measured at each destination (cf. (2)) are of the generic form \( \text{SINR}^{\text{P2}}_{k,j} = \frac{x^2(2)}{1 + x^{2(m-2)}} \). With the help of (7), the pdf of SINR can be shown as [10]:

\[
\begin{align*}
  f(x) &= \int_0^\infty f(x|y)f_Y(y)dy \\
       &= \frac{e^{-x/\rho_R}}{(1 + x)^m} \left( \frac{1}{\rho_R} (1 + x) + m - 1 \right).
\end{align*}
\]

The corresponding cdf is

\[
F(x) = 1 - \frac{e^{-x/\rho_R}}{(1 + x)^{m-1}}, \quad x \geq 0.
\]

Note that the SINR\(^{\text{P2}}_{k,j}\)s are i.i.d. over \( j = 1, \ldots, n \) (but are not independent over \( k = 1, \ldots, m \)).

In contrast to Phase 1, where we had to be satisfied with a lower bound on the throughput, for Phase 2 of the communication protocol we are able to write an exact expression for the system throughput.
Lemma 2: For any $\rho_R$, $m$ and $n$, the achievable throughput of the opportunistic relay scheme at Phase 2 is given by

$$R_2 = m \left(1 - \left(1 - \frac{e^{-1/\rho_R}}{2^{m-1}}\right)^n\right).$$

(11)

Proof: First we observe that each destination node $j$ has at most one $\text{SINR}_{\ell,j}^{P_2} \geq 1$ for all relays $1 \leq \ell \leq m$. To see this, assume $\text{SINR}_{k,j}^{P_2} \geq 1$ for some relay $k$, and consider another index $k' \neq k$. From (2), we have

$$\xi_{k,j} \geq 1/\rho_R + \sum_{1 \leq \ell \leq m, \ell \neq k} \xi_{\ell,j},$$

from which it follows that

$$\xi_{k,j} > \xi_{k',j}, \quad \forall k' \neq k.$$

Therefore

$$\text{SINR}_{k',j}^{P_2} = \frac{\xi_{k',j}}{1/\rho_R + \sum_{1 \leq \ell \leq m, \ell \neq k'} \xi_{\ell,j}} < \frac{\xi_{k',j}}{\xi_{k,j}} < 1.$$

Thus, each destination node can have at most one good relay as its sender.

Now the sum-rate for Phase 2 depends on how many relays are scheduled by destinations. The probability that a relay finds no destination satisfying $\text{SINR} \geq 1$ is\(^2\)

$$\Pr[\text{relay } k \text{ finds no destination satisfying } \text{SINR} \geq 1]$$

$$= \Pr[\forall j, \text{SINR}_{k,j}^{P_2} \leq 1]$$

$$= \left((F(1))^n\right)$$

$$= \left(1 - \frac{e^{-1/\rho_R}}{2^{m-1}}\right)^n.$$

The throughput of the relay-destination links is given by summing the probabilities of the relays engaged in transmission. Accounting for the 1 bit/s/Hz rate per relay, we have that the

\(^2\)While it may seem logical to turn off a relay for which the highest SINR is still $\leq 1$, we still allow such relays to transmit (say, control information). This is because, as shown by numerical results, the performance is limited by the source-relay link.
average throughput of the second hop is given by

\[ R_2 = \sum_{k=1}^{m} \Pr[\text{relay } k \text{ transmits data to a destination}] \cdot 1 \]

\[ = m \left( 1 - (F(1))^n \right) \]  
\[ = m \left( 1 - \left( 1 - \frac{e^{-1/\rho_R}}{2^{m-1}} \right)^n \right). \]  

The following corollary ensues:

**Corollary 2:** For fixed \( m \), \( R_2 \to m \) as \( n \to \infty \).

**Remark 3:** It is interesting at this point to draw a connection between the scheduling of Phase 2 of the opportunistic scheme proposed here with the random beamforming scheme due to Sharif and Hassibi in the context of multiple-input multiple-output broadcast channels (MIMO-BCs) [10]. Seemingly unrelated, the SINRs of both setups turn out to have the same distribution (cf. (2)). To explain this subtlety, note that in the random beamforming scheme of [10], a random unitary matrix \( \Phi \) is applied to the data streams before sending them over the channel \( H \) (hence the terminology “random beamforming”). With the assumption of i.i.d. Rayleigh fading, entries of \( H \) follow i.i.d. circularly symmetric complex Gaussian random variables \( \mathcal{CN}(0, 1) \). By the isotropic property of the i.i.d. complex Gaussian random matrix \( H \), \( \Phi H \) has the same distribution as \( H \) [11]. It follows that the channel statistics of the random beamforming scheme in the beam domain are the same as in the original antenna domain. In other words, the SINR in the beam domain is still of the generic form

\[ \text{SINR} = \frac{\chi^2(2)}{1/\rho_R + \chi^2(2m-2)}, \]  

which is the same as in our Phase 2 (cf. (2)).

Despite the mathematical equivalence, our proposed scheme for Phase 2 simplifies the random beamforming scheme in several respects:

- While random beamforming requires cooperation among the transmitters to form a beam, opportunistic relaying operates in a completely decentralized fashion.
- While random beamforming requires the feedback of an integer (the beam index) as well as a real number (the instantaneous SINR), the proposed opportunistic relaying scheme requires the feedback of only an index number. This simplification is justified by [10, Th. 2] which
implies that when the system operates in the limit as $n \to \infty$ with $m = \Theta(\log n)$, the aggregate interference from concurrent transmissions eventually hardens the instantaneous SINR near the value 1. Thus, there is no need to feed back the SINR value anymore. Furthermore, in terms of the throughput scaling law (as discussed later in Section IV), this simplification incurs no loss.

C. Two-Hop Communication

With the help of Corollaries 1 and 2, and by taking into account the $1/2$ penalty due to the two hops, the overall system throughput, defined as $\frac{1}{2} \min(R_1, R_2)$, can be readily shown to be given as follows.

**Theorem 1:** For fixed $m$, the two-hop opportunistic relaying scheme achieves a system throughput of $m/2$ bits/s/Hz as $n \to \infty$.

Since the proposed scheme works in a decentralized fashion and with low rate CSI feedback, it is natural to expect some throughput degradation compared to more intensive schemes. We will show that the opportunistic relaying scheme exhibits the pre-log factor of the scaling law of the throughput of more intensive schemes. To see this, we find an information-theoretic upper bound on the achievable scaling law for the aggregate throughput of any two-hop relaying scheme.

**Lemma 3:** For any two-hop relaying architecture, with fixed $m$ and SNR, and large $n$, the sum rate capacity scales at most as $\frac{1}{2} m \log \log n$.

**Proof:** In two-hop relay schemes, all data traffic passes through relays. Therefore, the best scheme would be one in which all $m$ relay nodes can cooperate and the relays have full CSI (i.e., backward as well as forward channel realizations). In such case, the two-hop communication can be interpreted as MIMO (multiple-input multiple-output) multiple access channels (MACs) followed by a MIMO-BC. The capacity region of the MIMO-BC, and the optimality of dirty-paper-coding (DPC) in achieving the capacity region have been shown in [12]. Furthermore, the capacity scaling of the DPC scheme is shown in [10] to be $m \log \log n$, which is also the capacity scaling for MIMO-MAC due to the MAC–BC duality [13]. Now, Lemma 3 follows by taking the two-hop penalty $1/2$ into account. 

Contrasting Theorem 1 to Lemma 3, we see that the proposed scheme simplifies network operation (central coordination and CSI assumption) at the cost of losing the $\log \log n$ term. However, it succeeds in preserving the pre-log factor of any two-hop relaying scheme.
**Remark 4:** Contrasting Theorem 1 to Lemma 3 reveals two different facets of multiuser diversity gain. Fundamentally, multiuser diversity gain is a power gain, e.g., in the Rayleigh fading case, multiuser diversity schedules the best user for transmission, and boosts the average power by a factor of $\log n$ [14]. With the assumption of relay cooperation, as in Lemma 3, a spatial multiplexing gain equal to the number of relays $m$ can be readily achieved (e.g., even by a suboptimal zero-forcing receiver [11]). Then, multiuser diversity can further boost the rate of each parallel channel by $\log \log n$, as shown by Lemma 3. In contrast, with the proposed opportunistic scheme, where relays operate independently, there is no guarantee of achieving the multiple parallel channels. Here, multiuser diversity is used as a mechanism that compensates for the interference plus noise so that the scheduled link can support 1 bit/s/Hz. Ultimately, one achieves the linear scaling in $m$. Note that only with multiuser diversity gain does the SINR of each noncooperative link have the chance to meet the threshold.\(^3\)

**IV. HOW FAST CAN m GROW?**

As discussed in Remark 2, in Phase 1 there is a tradeoff between the number of relays $m$ that serve as conduits between the source and destination nodes and the mutual interference caused by the transmissions. The same is true for Phase 2. This brings up the question: *What is the optimal $m$?* In this section, we show that the best $m$, in terms of order, is $\log n$. In light of Theorem 1, finding the optimal order of $m = \Theta(\log n)$ is equivalent to concluding that the throughput scaling of the proposed two-hop opportunistic relaying scheme is $\Theta(\log n)$.

**A. Phase 1**

Earlier, in Section III-A, the lower bound (8) of the system throughput of Phase 1 was found. This lower bound was adequate for the discussion in that section which assumed a large $n$ and fixed $m$ regime. However, in seeking to determine how the throughput scales with $m$, the lower bound (8) might considerably underestimate the true throughput. In light of this, in order to address the question of optimal $m$, we reason as follows: First, we consider a genie-aided scheme by relaxing the assumptions of decentralized relay scheduling and local CSI knowledge. Thus, the throughput scaling with the number of relays $m$ of the genie-aided network serves as

\[^3\text{If one randomly schedules the transmission, the average receiver SINR can be shown to be } \frac{1}{m - 1}.\]
an upper bound on the proposed decentralized scheme. We show that the throughput scaling law of this genie-aided scheme is $\Theta(\log n)$. Next, we show that the lower bound (8) of the proposed decentralized scheme does achieve the $\Theta(\log n)$ scaling. Thus, we are able to conclude that the throughput scaling of the original scheme of Phase 1 is given by $\Theta(\log n)$ and is optimal in a scaling law sense.

1) Upper Bound on Throughput Scaling of Phase 1: Consider a genie-aided network, in which the genie has access to the full CSI of the network, and can coordinate the operation of the entire network. Nevertheless, we still assume that each relay decodes data independently and that SINR $\geq 1$ is required for successful decoding. Since the genie has full CSI of the network, it can enumerate all $m$-subsets of sources and test, for each subset, all $m$-to-$m$ mappings between sources and relays. Consequently, the genie schedules the one subset of sources for which all the SINRs are $\geq 1$, that is, all $m$ scheduled sources are successful. Note that, given a set of channel realizations, the successful source–relay pairs in the proposed decentralized scheme (cf. Section II) must also be successful in the genie-aided scheduling scheme. Thus, the throughput of the genie-aided scheduling scheme upper-bounds the proposed decentralized scheme.

We address the question of how fast $m$ can grow using a probabilistic argument [15]. Letting $X(m)$ be the number of groups of $m$ source nodes that can be scheduled simultaneously such that all $m$ nodes in the group transmissions are successful (i.e., $\text{SINR}^{P1} \geq 1$), we will show that, for the genie-aided scheme, $\Pr[X(m) \geq 1] \to 0$, when $m = \frac{\log n}{\log 2} + 2$, while $\Pr[X(m) = 0] \to 0$ when $m = (1 - \epsilon) \frac{\log n}{2 \log 2} + 2$.

**Theorem 2:** Under the two-hop opportunistic relaying scheme, and assuming genie-aided scheduling, then with probability approaching 1, one cannot find a set of $\frac{\log n}{\log 2} + 2$ nodes whose simultaneous transmissions to relays are all successful. The converse is also true: with high probability, there exists a set of $(1 - \epsilon) \frac{\log n}{2 \log 2} + 2$ nodes whose simultaneous transmissions to relays are all successful.

**Proof:** See Appendix II. \qed

In terms of scaling law, Theorem 2 shows that the throughput of the genie-aided scheme is given by $\Theta(\log n)$. This establishes the throughput upper bound of the proposed decentralized scheme of Section II.
2) Achievability of the Upper Bound: Theorem 2 states that, with high probability, there exists a valid group with \( m = (1 - \epsilon) \frac{\log n}{2 \log 2} + 2 \) sources such that all transmissions are successful. However, the proof is nonconstructive: it does not afford insight into how to find such a set in practice. The proof also assumes that there is a genie with global channel information that can enumerate all \( \left( 1 - \epsilon \right) \frac{n}{2 \log 2} + 2 \) possibilities and select a good one for scheduling. In contrast to the genie-aided scheme, the opportunistic relaying scheme seeks to operate in a decentralized manner, and it is not clear whether this operational simplification incurs a loss in the scaling order of the throughput. Serendipitously, it can be shown that the \( \log n \) scaling is also met by the lower bound in (8). To see this, we examine the asymptotic behavior of (8).

Consider the case of \( m = \log n \) and \( s = \log n - \log \log n \). With \( n \rightarrow \infty \), the term \( \frac{n(n-1)\cdots(n-m+1)}{n^m} \rightarrow 1 \). The term \( (1 - (1 - e^{-s})^n) \) is independent of \( m \), and approaches 1 for \( s = \log n - \log \log n \) and asymptotically in \( n \). Therefore, a throughput of \( \Theta (\log n) \) can be achieved as long as \( F_Y(s - \frac{1}{\rho}) = \Theta (1) \). Indeed, for \( m = \log n \), the interference term \( Y \) in (7) is (asymptotically) a chi-square distributed random variable with \( 2(\log n - 1) \) degrees of freedom. By the central limit theorem, \( Y \) can be further approximated as being Gaussian with mean and variance both equal to \( \log n \). Now, we have

\[
F_Y(\log n - \log \log n - 1/\rho) \approx F_Y(\log n) = \frac{1}{2},
\]

(15)
due to the symmetry of the Gaussian distribution. Consequently \( R_1 \approx \frac{1}{2} \log n \). This result implies that for \( m = \log n \) relays, each running the two-hop opportunistic relaying protocol, it is possible to schedule up to \( \log n \) source nodes to transmit simultaneously, but half of them will fail to satisfy the SINR requirement due to the multiple access interference. In terms of throughput, this example yields \( \frac{1}{2} \log n \), which confirms that the scheme is in fact order-optimal in achieving a throughput of \( \Theta (\log n) \) at Phase 1.

Remark 5: Theorem 2 for the genie-aided scheme implies that for any two-hop relaying scheme without cooperation among relays and without full CSI, the throughput scaling is upper-bounded by \( \Theta (\log n) \). The proposed opportunistic relaying scheme meets an optimality criterion in the sense that the scaling law of its throughput achieves this upper bound.
B. Phase 2

We will show in this subsection that the optimal value of $m$ in Phase 2 exhibits a sharp phase transition phenomenon. That is, $m = \frac{\log n - \log \log n - 1}{\log 2} + 1$ succeeds in retaining the linearity of $R_2$ in $m$, but $m = \frac{\log n + \log \log n - 1}{\log 2} + 1$ does not. As far as the scaling law is concerned, this implies that the throughput of Phase 2 scales as $\Theta(\log n)$.

**Theorem 3:** For Phase 2 of the two-hop opportunistic relaying scheme, if the number of relays $m = \frac{\log n - \log \log n - 1}{\log 2} + 1$, then $R_2 = \Theta (m) = \Theta (\log n)$. Conversely, if $m = \frac{\log n + \log \log n - 1}{\log 2} + 1$, then $R_2 = o(m)$.

**Proof:** For convenience, we repeat $R_2$ of (12) and (13):

$$R_2 = m \left( 1 - \left( F(1) \right)^n \right) = m \left( 1 - \left( 1 - \frac{e^{-1/\rho R}}{2m-1} \right)^n \right).$$

With $m = \frac{\log n - \log \log n - 1}{\log 2} + 1$, we have

$$1 - F(1) = \frac{e^{-1/\rho R}}{2m-1} = e^{-(m-1) \log 2 - 1/\rho R} = \frac{\log n}{n}.$$

Then,

$$\left( (F(1))^n \right) = \left( 1 - \frac{\log n}{n} \right)^n = e^{n \log (1 - \frac{\log n}{n})} = e^{-\log n + O(\log^2 n)} = e^{-\log n + o(\log n)} = O \left( \frac{1}{n} \right),$$

where we have used the fact that, for small $x$, $\log(1 - x) = -x + O(x^2)$ and $e^x = 1 + O(x)$. Thus, most of the transmissions meet the SINR threshold (with probability $1 - O(1/n)$) and consequently the throughput $R_2$ is given by $m(1 - O(\frac{1}{n})$, $R_2 = \Theta(m) = \Theta(\log n)$ follows readily.

Similarly, when $m = \frac{\log n + \log \log n - 1}{\log 2} + 1$ we have $1 - F(1) = \frac{1}{n \log n}$ and

$$\left( (F(1))^n \right) = e^{-\frac{1}{n \log n} + O(\frac{1}{n \log^2 n})} = e^{-\frac{1}{n \log n} + o(\log n)} = 1 - O(1/ \log n).$$
Now, in contrast to the case of \( m = \frac{\log n - \log \log n - 1/\rho_R}{\log 2} + 1 \), when we increase \( m \) to \( \frac{\log n + \log \log n - 1/\rho_R}{\log 2} + 1 \), Phase 2 of the two-hop scheme cannot support a throughput that scales with \( m \). With probability one, the SINRs cannot meet the threshold. In this case, the throughput does not scale linearity with \( m \) anymore, i.e., \( R_2 = o(m) \).

This \( \Theta(\log n) \) scaling result is consistent with the random beamforming scheme of [10], an outcome that is not surprising in light of the connection discussed in Remark 3.

C. Two-Hop Communications

From Theorems 2 and 3, it follows that there is a solution to the opportunistic relaying scheme as long as \( m \) is of the order of \( \log n \). We conclude that the achievable throughput scaling of the scheme is given by \( \log n \).

*Theorem 4:* Under the setup of Section II, the proposed two-hop opportunistic relaying scheme yields a maximum achievable throughput of \( \Theta(\log n) \).

*Remark 6:* In the case when \( m = \Theta(\log n) \), the scheme is interference limited, i.e., one cannot further increase the throughput by increasing the transmitted power. As discussed later in Section VI-A, to break the barrier of this limitation, one needs both cooperation between relays and CSI at the transmitter side.

V. Numerical Results

In this section, we provide some numerical examples by simulations of the proposed opportunistic relaying scheme under Rayleigh fading. Throughout these examples, the SNR for both hops is set at 10 dB (\( \rho = \rho_R = 10 \) dB).

We first examine in Fig. 2 the throughput \( R_1 \) of the first hop of the protocol and its various lower bounds. The figure contains four curves. The two simulation curves were obtained by averaging throughput over 2,000 channel realizations. The “simulated \( R_1 \)” curve was obtained using all assignments of source nodes, while the curve marked “simulated \( R_1 \) with distinct nodes” represents only assignments of distinct source nodes. The other two lower bounds shown are computed with (8): one is obtained by optimizing (8) over \( s \) (numerically); the other lower bound is for \( s = \log n - \log \log n \). Three observations are noteworthy relative to Fig. 2. First, both the simulated throughput and the analytical lower bound (8) exhibit linearity with respect to \( m \), consistent with the analysis of Section III-A. Second, it is observed that when \( m \) exceeds a
Fig. 2. First hop throughput $R_1$ as a function of the number of relays for $n = 1200$ nodes. From the top: simulation results utilizing all source node assignments, simulated results with distinct scheduled nodes, lower bound with optimized threshold $s$, and lower bound with threshold $s = \log n - \log \log n$.

Fig. 3. First hop throughput $R_1$, second hop throughput $R_2$, and the system throughput $R$ as a function of the number of relays $m$ for $n = 1200$ nodes.
certain value (in this case, 6), the throughput $R_1$ starts to fall off. Noting that $\log 1200 = 7$, this effect is consistent with the analysis in Section IV-A which established that the linear increase in throughput with the number of relays holds only as long as $m$ is of the order $\log n$. Third, the lower bound of $R_1$ of (8) becomes loose when $m$ grows. The development leading to (5) suggests two possible reasons for this behavior. The first is that the computation of $\Pr [N_m]$ is based on only distinct source nodes. However, the close match between the two simulation curves in Fig. 2 eliminates this possibility. It follows then that the bound is loosened due to the series of lower-boundings of $\Pr [S_m]$ leading to (5) being too conservative.

In Fig. 3, we illustrate throughputs $R_1$ and $R_2$, as well as the corresponding system throughput of the full scheme given by $R = \frac{1}{2} \min\{R_1, R_2\}$. As discussed in Section II, the transmissions over the second hop are destined to be successful since they are scheduled based on SINR measurements at the destination nodes, whereas the transmissions over the first hop are not guaranteed to be successful since they are based only on SNR measurements. As a consequence, we observe from Fig. 3 that $R_1$ is lower than $R_2$, and is the bottleneck to the system throughput, i.e., $R = \frac{1}{2} R_1$. In addition, we observe that the optimal number of relays for Phase 2 is consistent with the analysis of Theorem 3 in Section III-B. Nevertheless, both $R_1$ and $R_2$ display the linearity in $m$ as predicted by Corollaries 1 and 2 in Section III.

The total throughput of the two-hop opportunistic relaying scheme is shown in Fig. 4 as a function of the number of nodes $n$. We observe that the throughput exhibits the $\log n$ trend, as predicted by Theorem 4. In fact, the system throughput curve can be perfectly approximated by $0.36 \log n$. Since the system throughput is always limited by Phase 1, i.e., $R = \frac{1}{2} \min\{R_1, R_2\} = \frac{1}{2} R_1$, we also plot two bounds of $\frac{1}{2} R_1$ for references. More specifically, the genie bound $\frac{\log n}{4 \log 2} + 1$ (cf. Theorem 2) serves as an upper bound and the $\frac{1}{4} \log n$ curve from (15) serves as a lower bound for the system throughput. In Fig. 5, the optimal value of the number of relays $m$, was obtained numerically versus the number of nodes, $n$. Comparing the values of $m$ from the curve, with the value $\frac{\log n}{2 \log 2} + 2$, which is the bound on the number of relays for the genie scheme in Theorem 2, we observe that the optimal $m$ is very close to that of the genie-bound. This explains why the scheme can harness large portions of throughput as promised by Theorem 4.
Fig. 4. Simulated system throughput of the proposed scheme as a function of the number of nodes $n$ and for optimized number of relays $m$. Also shown are a genie upper bound and the lower bound $\frac{1}{4} \log n$.

Fig. 5. Optimal value of $m$ that maximizes the throughput, and $\frac{\log n}{2 \log 2} + 2$ curve (cf. Theorem 2).
VI. DISCUSSION AND CONCLUSION

One of the key contributions of this work is to propose an opportunistic relaying scheme that features decentralized relay operations and practical CSI assumptions (backward channels and integer-valued feedback). In this section, we discuss the case in which relays are allowed to cooperate with each other. In order to isolate the impact of the relay cooperation on the two-hop scheme, we leave unchanged the CSI assumptions. This discussion will help identify the fundamental limits of the opportunistic relaying scheme. Finally, we briefly address the issue of network delay.

A. Cooperative Relays

In the proposed opportunistic relaying scheme, we assume no cooperation among the relays. In particular, the relays can treat the received interference only as noise. In both phases of the two-hop protocol, the relays cannot perform joint processing in order to cancel the mutual interference caused by concurrent transmissions. As a consequence, the system is interference limited. In this subsection, we address the question: How will cooperation between relays change the scheduling operation and the throughput scaling? For example, it is conceivable that the relays could be implemented as infrastructure nodes that are connected to a wired backbone. This setup has been referred to as a hybrid network (see for example [16] and references therein).

When the relays are allowed to fully cooperate, they can be considered to be a distributed MIMO system. Accordingly, the first and second hops are equivalent to a MIMO MAC with receiver CSI, and a MIMO BC without transmitter CSI, respectively. Now, the scheduling in Phase 1 can be simplified. It is well-known that for the MIMO MAC, the sum-capacity can be achieved by allowing all users to transmit. The receiver can retrieve the data via some sophisticated signal processing algorithm, e.g., MMSE-SIC (minimum-mean square estimator with successive interference cancelation) [17]. The optimal scaling in the large $n$ and fixed $m$ regime is given by $m \log \log n$. However, if we seek to achieve only linear scaling in $m$, it suffices to schedule any $m$ source nodes for transmission. With high probability, the resulting $m \times m$ channel is well-conditioned, and a spatial multiplexing gain of $m$ is achieved [11]. In contrast, Phase 2 does not benefit from the cooperation of relays. This is because, without transmitter CSI, and since destination nodes are not allowed to collaborate, arbitrarily selecting $m$ destination
nodes cannot yield a throughput linear in \( m \). In this case, we still need a feedback mechanism as introduced in Section II.

The impact of relay cooperation on throughput scaling exhibits similar behavior to that demonstrated for scheduling. Phase 1 benefits from relay cooperation, and in principle \( R_1 = \Theta(n) \) is possible (note that the capacity of a \( n \times n \) MIMO channel scales linearly with \( n \) [18]); Phase 2 is still bounded by \( \Theta(\log n) \) and becomes the bottleneck of the two-hop scheme. The reason for this is that, due to the lack of CSI at the relays, there is no way to generate more than \( \Theta(\log n) \) parallel channels. The reader is referred to [10] for a discussion of the impact of CSI knowledge on MIMO downlink channels.

We summarize the discussion of relay cooperation in Table I, where the case of full relay cooperation together with the full CSI assumption is also considered. From the table, one can readily identify that, for the two-hop opportunistic relaying scheme, both relay cooperation and CSI at the transmitters are essential for achieving throughput scaling beyond \( \Theta(\log n) \). Alternatively, when full CSI is available, but cooperation of relays is not possible, one can operate the two-hop amplify-and-forward scheme [3] to achieve \( \Theta(n^{1/2}) \).

| Assumption of | throughput scaling | throughput scaling | throughput scaling |
| cooperation/CSI | of Phase 1 | of Phase 2 | of the scheme |
|----------------|-------------|-------------|---------------|
| no relay cooperation / CSIR with feedback | \( \Theta(\log n) \) | \( \Theta(\log n) \) | \( \Theta(\log n) \) |
| relay cooperation / CSIR with feedback | \( \Theta(n) \) | \( \Theta(\log n) \) | \( \Theta(\log n) \) |
| relay cooperation / full CSI | \( \Theta(n) \) | \( \Theta(n) \) | \( \Theta(n) \) |

It is important to point out that the discussion applies only to the underlying Rayleigh fading model. For other fading models, the opportunistic relaying scheme may exhibit a different scaling law.

**B. Delay Considerations**

There is a tension between the simplification of network operations and delay considerations. The delay issue is more salient in the two-hop scheme than in the cellular setup [14], because
packets transmitted by one particular source in Phase 1 must be buffered at a relay until that relay schedules the original destination during Phase 2. While one can partially relieve the problem by, say, prioritizing the destination in cases when a relay receives multiple requests from multiple destinations (including the destination of interest, of course), the delay may still be large. The detailed study of end-to-end delay is currently underway.

C. Conclusion

In this work, we have proposed an opportunistic relaying scheme that alleviates the demanding assumptions of central scheduling and CSI at transmitters. The scheme entails a two-hop communication protocol, in which sources can communicate with destinations only through half-duplex relays. The key idea is to schedule at each hop only a subset of nodes that can benefit from multiuser diversity. To select the source and destination nodes for each hop, relays operate independently with receiver CSI only, and with an integer-valued feedback to the transmitter. The system throughput has been characterized for the operating regime, where \( n \) is large and \( m \) is relatively small. In this case, the proposed scheme achieves a system throughput of \( \frac{m}{2} \) bits/s/Hz, while the upper bound with full cooperation among relays and full CSI is \( (\frac{m}{2}) \log \log n \). Moreover, we have further characterized the throughput scaling to be \( \Theta(\log n) \).

The delay behavior of the proposed opportunistic relaying scheme is left for future work.

APPENDIX I

CHARACTERIZATION OF INTERFERENCE \( Y \) OF (4)

In this appendix, we characterize the statistical properties of the interference term \( Y \) in (4). More specifically, it is shown that, asymptotically in \( n \), each individual term that comprises \( Y \) has an exponential distribution, and all interferers are asymptotically independent. It is also illustrated by numerical results that these asymptotic trends are achieved quickly, enabling the approximation of \( Y \) as a chi-square random variable with \( 2(m-1) \) degrees of freedom.

For notional convenience, we denote the channel connections from \( n \) sources to the relay as \( X_1, \ldots, X_n \). According to the scheduling of Phase 1, for each time-slot, i.e., each realization of \( X_1, \ldots, X_n \), the desired signal strength is the maximum among all connections. The interference term \( Y \) is the summation of \( (m-1) \) out of the remaining \( (n-1) \) channel connections.
We first show that each interferer is asymptotically exponentially distributed. By the law of total probability, for events $B$ and $A$, we have

$$\Pr[B] = \Pr[B|A] \Pr[A] + \Pr[B|\overline{A}] \Pr[\overline{A}],$$

(20)

where $\overline{A}$ denotes the complement of the event $A$. Now define the event $B$ as “$X_i \leq x_i$,” and $A$ as “$X_i$ is not the maximum.” Then we have for the cdfs

$$F_{X_i}(x_i) = F_{X_i}(x_i|A) \Pr[A] + F_{X_i}(x_i|\overline{A}) \Pr[\overline{A}].$$

(21)

In our i.i.d. model, by symmetry, each node has probability of $1/n$ to be the maximum, i.e., $\Pr[A] = 1/n$. Thus, the above equation can be written as

$$F_{X_i}(x_i) = F_{X_i}(x_i|A) \left(1 - \frac{1}{n}\right) + F_{X_i}(x_i|\overline{A}) \frac{1}{n}. \quad (22)$$

Therefore, we have

$$F_{X_i}(x_i|A) \rightarrow F_{X_i}(x_i) \quad \text{as } n \rightarrow \infty,$$

(23)

and thus, asymptotically, each interferer is still exponentially distributed.

While the above results are of an asymptotic nature, numerical result shows that they hold for practical values of $n$ as well. For example, in Fig. 6, the empirical pdf $f(x_i|X_i$ is not the maximum) is plotted together with the pdf of $\text{Exp}(1)$, i.e., $f(x) = e^{-x}, x \geq 0$, for various values of $n$. It is seen that the empirical pdf is well approximated by the standard exponential distribution.

Next, we show that the interferers are asymptotically independent. Define $A$ as “none of $X_1, \ldots, X_{m-1}$ is the maximum.” The event $\overline{A}$ is then “at least one of $X_1, \ldots, X_{m-1}$ is the maximum.” Again, by the law of total probability,

$$F(x_1, \ldots, x_{m-1}) = F(x_1, \ldots, x_{m-1}|A) \Pr[A]$$

$$+ F(x_1, \ldots, x_{m-1}|\overline{A}) \Pr[\overline{A}].$$

(24)

Due to the underlying i.i.d. assumption, $\Pr[A] = (1 - \frac{1}{n})^{m-1} \rightarrow 1$ and $\Pr[\overline{A}] = 1 - (1 - \frac{1}{n})^{m-1} \rightarrow 0$ when $n$ is large and $m$ small relative to $n$. Then it follows readily from (24) that in the regime of interest

$$f(x_1, \ldots, x_{m-1}|\overline{A}) \rightarrow f(x_1, \ldots, x_{m-1}) = \prod_{i=1}^{m-1} f(x_i).$$

(25)
Theorem 2: Under the two-hop opportunistic relaying scheme, and assuming genie-aided scheduling, then with probability approaching 1, there is no set of $\frac{\log n}{\log 2} + 2$ nodes whose simultaneous transmissions to relays are all successful. The converse is also true: there is a set of $(1 - \epsilon)\frac{\log n}{2\log 2} + 2$ nodes whose simultaneous transmissions to relays are all successful.
Proof: The proof relies on the probabilistic method [15]. The basic idea of the probabilistic method is that in order to prove the existence of a structure with certain properties, one defines an appropriate probability space of structures and then shows that the desired properties hold in this space with positive probability. This method of proof has been seen in various subjects of information theory, for instance, see [19, Ch. 8] which studies the bandwidth scaling problem in the context of spectrum sharing. The line of our proof follows [19].

To proceed, we need the following definitions. Define a probability space \( \Omega = \{ (\mathcal{A}, \pi) : \mathcal{A} \subseteq \{1, \ldots, n\}, |\mathcal{A}| = m, \pi \text{ is any permutation on } \{1, \ldots, m\} \} \) where \( \mathcal{A} \) denotes a random \( m \)-set of all \( n \) source nodes and \( \pi \) denotes any possible \( m \)-to-\( m \) mappings from \( m \) source nodes in \( \mathcal{A} \) to \( m \) relays. Let \( B_{\mathcal{A}}^\pi \) be the event “all nodes in \( \mathcal{A} \) can transmit simultaneously and successfully under mapping rule \( \pi \)” and \( X_{\mathcal{A}}^\pi \) the corresponding indicator random variable, i.e.,

\[
X_{\mathcal{A}}^\pi = 1 \left( \text{SINR}_{i,R(i)}^{P1} \geq 1, \ \forall i \in \mathcal{A} \right) \tag{26}
\]

\[
= 1 \left( \frac{\gamma_{i,R(i)}}{1/\rho + \sum_{i \neq i} \gamma_{i,R(i)}} \geq 1, \ \forall i \in \mathcal{A} \right), \tag{27}
\]

where the subscript \( R(i) \) in (26) denotes the corresponding relay for source \( i \) under mapping rule \( \pi \). In other words, for all \( i \in \mathcal{A} \), the \( R(i) \)'s are uniquely determined by the given \( \pi \).

Then

\[
\mathbb{E}[X_{\mathcal{A}}^\pi] = \Pr[B_{\mathcal{A}}^\pi] \\
= \Pr[\text{SINR}_{i,R(i)}^{P1} \geq 1, \ \forall i \in \mathcal{A}] \\
= \left( \Pr[\text{SINR}_{i,R(i)}^{P1} \geq 1] \right)^m \\
= (p_m)^m, \tag{28}
\]

where (28) follows from the fact that for \( r \neq r' \ (1 \leq r, r' \leq m) \) and any \( i \) and \( i' \), \( \text{SINR}_{i,r}^{P1} \) and \( \text{SINR}_{i',r'}^{P1} \) are i.i.d. The term \( p_m = 1 - F(1) \) in (29) is the probability that a transmission is successful when there are \( m \) concurrent transmissions, and \( F(\cdot) \) is the cdf of the SINR computed in (10).

The number of valid sets that satisfy the SINR threshold is given by

\[
X(m) = \sum_{\mathcal{A}} \sum_{\pi} X_{\mathcal{A}}^\pi. \tag{30}
\]
The linearity of the expectation yields
\[ \mathbb{E}[X(m)] = \binom{n}{m} m! (p_m)^m. \] (31)

Now, we first prove the nonexistence part, i.e., \( \Pr[X(m) \geq 1] \to 0 \) when \( m = \frac{\log n}{\log 2} + 2 \). This can be seen from Markov’s inequality:
\[
\Pr[X(m) \geq 1] \leq \frac{n!}{(n-m)!} \left( \frac{p_m}{m} \right)^m \\
\leq (np_m)^m \leq \left( \frac{ne^{-1/\rho}}{2^m} \right)^m \\
= e^{m(\log n - (m-1)\log 2 - 1/\rho)} \\
\leq e^{m(\log n - (m-1)\log 2)}. \] (32)

Now substituting \( m = \frac{\log n}{\log 2} + 2 \) into (32), we have
\[
\Pr[X(m) \geq 1] \leq e^{-\log n + o(\log n)} \\
= O\left(\frac{1}{n}\right). \] (33)

What (33) tells us is that when \( m = \frac{\log n}{\log 2} + 2 \), the probability of finding a set of \( m \) nodes for concurrent successful transmissions decreases to zero as \( n \) increases.

Next we look at the existence of a good set. We find an upper bound on \( \Pr[X(m) = 0] \) and show that the upper bound approaches zero when \( m = (1 - \epsilon) \frac{\log n}{\log 2} + 1 \). We need the following probabilistic tool from [15, Th. 8.1.2] (we shall also adopt the conventional notations of the probabilistic method of [15]).

**Theorem 5 (The Extended Janson Inequality):** Let \( B^a_{\mathcal{A}} \) and \( \mu = \mathbb{E}[X(m)] \) be as above. Furthermore, we write \( (\mathcal{A}, \pi) \sim (\mathcal{A}', \pi') \) if \( (\mathcal{A}, \pi) \neq (\mathcal{A}', \pi') \) and \( B^a_{\mathcal{A}} \) and \( B^{\pi'}_{\mathcal{A}'} \) are not independent. Define
\[
\Delta = \sum_{(\mathcal{A}, \pi) \sim (\mathcal{A}', \pi')} \mathbb{E}[X^a_{\mathcal{A}} X^{\pi'}_{\mathcal{A}'}]. \] (34)

If \( \Delta \geq \mu \), then
\[
\Pr[X(m) = 0] \leq e^{-\frac{\mu^2}{2\Delta}}. \] (35)

Given disjoint \( \mathcal{A} \) and \( \mathcal{A}' \), i.e., \( |\mathcal{A} \cap \mathcal{A}'| = 0 \), then for any \( \pi \) and \( \pi' \), \( B^a_{\mathcal{A}} \) and \( B^{\pi'}_{\mathcal{A}'} \) involve independent random variables, and therefore are independent. Thus, we need to consider only
the case in which \( A \) and \( A' \) have common elements, i.e., \( |A \cap A'| = q \), where \( 1 \leq q \leq m \). Conditional on \( |A| = |A'| = m \), \( |A \cap A'| = q \), and for all \( \pi, \pi' \), we have

\[
\mathbb{E} \left[ X_{A}^{\pi} X_{A'}^{\pi'} \right|_{|A'\cap A|=q} = \mathbb{E} \left[ \prod_{k \in A} 1 \left( \frac{\gamma_{k,R}(k)}{\sigma^2/P + \sum_{t \in A} \gamma_{t,R}(k)} \geq 1 \right) \prod_{\ell \in A'} 1 \left( \frac{\gamma_{\ell,R}(\ell)}{\sigma^2/P + \sum_{t \in A'} \gamma_{t,R}(\ell)} \geq 1 \right) \right]
\]

\[
\leq \mathbb{E} \left[ \prod_{k \in A} 1 \left( \frac{\gamma_{k,R}(k)}{\sigma^2/P + \sum_{t \in A} \gamma_{t,R}(k)} \geq 1 \right) \prod_{\ell \in A' \setminus A} 1 \left( \frac{\gamma_{\ell,R}(\ell)}{\sigma^2/P + \sum_{t \in A'} \gamma_{t,R}(\ell)} \geq 1 \right) \right] \tag{36}
\]

\[
= \left( p_{m} \right)^{m} \left( p_{m-q} \right)^{m-q} \tag{37}
\]

where (37) upper-bounds (36) by neglecting the interference coming from the sources belonging to \( A' \cap A \) in the latter product. In so doing, the two products \( \prod_{k \in A} 1(\cdot) \) and \( \prod_{\ell \in A' \setminus A} 1(\cdot) \) in (37) involve independent random variables now and therefore are independent (note that this is not true in (36)). A minimal example is illustrated in Fig. 7. The upper-bounding in (37) can be thought of as reducing the number of concurrent transmissions from \( m \) to \( m - q \) by keeping the elements \( \{ t : t \in A' \cap A \} \) silent. The probability of successful transmission when there are \( m - q \) concurrent transmissions, denoted as \( p_{m-q} \), can be shown to be \( p_{m-q} = \frac{e^{-1/\rho}}{2^{m-q-1}} \).

![Fig. 7. Example of \( A = \{1, 2\} \) and \( A' = \{1, 3\} \). We can upper-bound the SINR of source 3 in \( A' \) as \( 1/\rho_{3} + 2 \leq 1/\rho' \), which now is independent of the SINRs of source nodes in \( A \).](image-url)
Now, we are able to proceed with $\Delta$ in (34). In particular, we have

$$\Delta = \sum_{(A, \pi) \sim (A', \pi') \in \mathcal{X}} \mathbb{E}[X^\pi_A X^{\pi'}_{A'}]$$

$$= \sum_{q=1} \sum_{|A|=m} \sum_{|A'|=m} \sum_{\pi} \sum_{\pi'} \mathbb{E}[X^\pi_A X^{\pi'}_{A'} \mid |A|=|A'|=m]$$

$$= (m!)^2 \sum_{q=1} \sum_{|A|=m} \sum_{|A'|=m} \mathbb{E}[X^\pi_A X^{\pi'}_{A'} \mid |A|=|A'|=m]$$

$$= (m!)^2 \left( \sum_{q=1}^m \binom{n}{m} \binom{m}{q} \binom{n-m}{m-q} \binom{p_m}{m}^m (p_{m-q})^{m-q} \right).$$

In order to apply Theorem 5, we check $\frac{\Delta}{\mu^2}$:

$$\frac{\Delta}{\mu^2} = \sum_{q=1}^m \frac{\binom{m}{q} \binom{n-m}{m-q} \binom{p_m}{m-q}^{m-q}}{\binom{n}{m} \binom{p_m}{m}^m} = \sum_{q=1}^m a_q,$$

where $a_q = \frac{(m-q)\binom{n-m}{m-q} \binom{p_{m-q}}{m-q}^{m-q}}{(n-m) \binom{p_m}{m}^m}$. Now on defining $b_q = a_{q+1}/a_q$, we have that $b_q = \frac{(m-q)^2 e^{1/\rho}}{(q+1)(n-2m+q+1)} e^{2(m-q-1) \log 2}$ decreases with $q$, $b_q \leq b_1$. Therefore, $a_q \leq b_1^{q-1} a_1$.

On setting $m = (1 - \epsilon) \frac{\log n}{2 \log 2} + 2$ we have

$$b_1 = \frac{(m-1)^2}{2(n-2m+2)} e^{2(m-2) \log 2 + 1/\rho}$$

$$= e^{2 \log(m-1) - \log 2 - \log(n-2m+2) + 2(m-2) \log 2 + 1/\rho}$$

$$= e^{(1-\epsilon) \log n - \log \left(n-2(1-\epsilon) \frac{\log n}{2 \log 2} - 2\right) + o(\log n)}$$

$$= e^{-\epsilon \log n + o(\log n)}$$

$$= O\left( \frac{1}{n^\epsilon} \right).$$
Furthermore, with \( m = (1 - \epsilon) \frac{\log n}{2 \log 2} + 2 \), we have

\[
\frac{\Delta}{\mu^2} = \sum_{q=1}^{m} a_q \leq a_1 \sum_{q=1}^{m} b_1^{q-1} \leq \frac{a_1}{1 - b_1}
\]

\[
= \frac{\binom{m}{1} \binom{n-m}{m-1}}{\binom{n}{m}} \frac{1}{1 - b_1} = \frac{\binom{m}{1} \binom{n-m}{m-1}}{\binom{n}{m}} e^{2(m-1) \log 2 + 1/\rho}
\]

\[
= m^2 \prod_{i=0}^{m-2} \frac{(n - m - i)}{1 - b_1} \frac{e^{2(m-1) \log 2 + 1/\rho}}{1 - b_1}
\]

\[
< \frac{m^2}{n - m} \frac{e^{2(m-1) \log 2 + 1/\rho}}{1 - b_1}
\]

\[
= e^{2 \log m - \log(n-m) + 2(m-1) \log 2 - \log(1-b_1) + 1/\rho}
\]

\[
= e^{-\epsilon \log n + o(\log n)}
\]

\[
= O\left( \frac{1}{n^\epsilon} \right).
\]

Finally, Theorem 5 yields

\[
\Pr[X(m) = 0] < e^{-n^\epsilon}.
\]  \hspace{1cm} (39)

This completes the proof.

\[\square\]

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