Quantum generative adversarial learning in a superconducting quantum circuit

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Generative adversarial learning [1] is one of the most exciting recent breakthroughs in machine learning—a subfield of artificial intelligence that is currently driving a revolution in many aspects of modern society. It has shown splendid performance in a variety of challenging tasks such as image and video generations [2]. More recently, a quantum version of generative adversarial learning has been theoretically proposed [3, 4] and shown to possess the potential of exhibiting an exponential advantage over its classical counterpart [3]. Here, we report the first proof-of-principle experimental demonstration of quantum generative adversarial learning in a superconducting quantum circuit. We demonstrate that, after several rounds of adversarial learning, a quantum state generator can be trained to replicate the statistics of the quantum data output from a digital qubit channel simulator, with a high fidelity (98.8%) on average) that the discriminator cannot distinguish between the true and the generated data. Our results pave the way for experimentally exploring the intriguing long-sought-after quantum advantages in machine learning tasks with noisy intermediate-scale quantum devices [5].

Figure 1(a) shows the schematic of the QGAN scheme. The black box provides the quantum true data which is described by a density matrix $\sigma$ of a quantum system, while both the internal physical structure and the quantum process do not need to be known. G can generate arbitrary quantum states ($\rho$) by producing an ensemble of pure quantum states, i.e. a pure state from a set is randomly selected with certain probability to mimic the quantum true data. D performs quantum measurements ($M$) on the true and the generated (fake) data, and attempts to distinguish them by the statistics of the measurement outcomes $p_\rho = \text{tr} M \rho$ and $p_\sigma = \text{tr} M \sigma$. In the QGAN, the measurement outcomes are public to both G and D. According to $p_\rho, p_\sigma$, D and G compete against each other by adaptively adjusting their strategies alternatively to distinguish $\rho$ from $\sigma$ and to fool D, respectively. It is interesting to note that $\sigma$ and $\rho$ represent two distinct interpretations of mixed quan-
 quantum states: one is the output of a physical process in which an initial pure state might be entangled with some degrees of freedom of the environment; the other is an ensemble of pure states. Our QGAN scheme can also be explained as a game trying to distinguish between these two interpretations.

A visualized illustration of the general procedure of QGAN is depicted in Fig. 1(b) by presenting $\sigma$, $\rho$ and $M$ of a qubit system in the Bloch sphere (note that we use the same notation $M$ to represent both the projective measurement and its corresponding axis). D and G play the game alternatively. D always starts first, and in her turn $M$ is optimized to maximize the difference of the measurement outcome $d = p_\rho - p_\sigma$. In an ideal case, D’s turn ends up with $d = \frac{1}{2} ||\rho - \sigma||$, corresponding to the normalized trace distance $D_{tr} = \frac{1}{n} \sum_{i=0}^{n} |\rho(\theta_i, \phi_i) - \sigma(\theta_i, \phi_i)|$, with $\rho(\theta, \phi) = e^{i\phi \sigma_z/2} e^{i\theta \sigma_x/2} e^{i\theta \sigma_x/2}$ the unitary operation on the transmon qubit, with $\sigma_x$ and $\sigma_z$ being the Pauli matrices. D performs the measurements by applying a unitary pre-rotating operation with the axis angles $(\beta, \gamma)$ on the transmon and detecting the population of the ground state $|g\rangle$, which leads to $M = U^\dagger (\beta, \gamma) |g\rangle \langle g| U (\beta, \gamma)$.

The protocol of our experimental QGAN algorithm is illustrated in Fig. 2(a). The experiment starts with a randomly generated state $\rho(r_0, \theta_0, \varphi_0)$ by G, a randomly picked measurement axis $M(\beta_0, \gamma_0)$ by D, and the quantum true data $\sigma$ from a fixed channel simulator. In each round of experiment, D plays the adversarial game first with $p$ fixed, followed by G’s turn with $M$. In all experiments, $d$ is obtained by averaging $n = 5,000$ repetitive measurements on the true and the fake data respectively. The gradient $\partial d/\partial \xi$ for the control parameter $\xi \in \{r, \theta, \varphi, \beta, \gamma\}$ is critical for the QGAN. These gradients are approximately obtained by measuring $d$ with respect to $\xi$ and $\xi + \delta$ ($\delta \ll 1$) and calculating the differential numerically in a classical computer. According to the principle of gradient descent, the parameters are updated to maximize $d$ (minimize $d$ with $d > 0$) for D’s (G’s) turn, as explained in Fig. 2(b) (see Supporting Materials for the strategy). Here, the procedure to estimate each gradient for the relevant parameters is counted as one step, and the total number of steps quantifies the consumption of time and copies of data. In practical experiments, the projective detection outcomes follow a binomial statistic, and show a standard deviation of $d$ as $sd = \sqrt{p_\rho (1 - p_\rho)/n + p_\sigma (1 - p_\sigma)/n}$. When approaching the Nash equilibrium, $p_\rho \approx p_\sigma \approx \frac{1}{2}$, then $sd \approx 1/\sqrt{2n} = 0.01$. Therefore, the measurement precision of $d$ will limit the convergence of the game. In our experiments, D’s turn ends when the differences of $d$ in the last $3$
FIG. 3. Tracking of QGAN. (a-c) Experimental results for selecting \( \sigma = |g\rangle \langle g| \) as the quantum true data. (a) shows the snapshots of the system at the particular steps indicated by arrows in b (from left to right in the same order) in the Bloch sphere representation. (b) shows the tracking of \( p_x, \rho, d \) and \( F \) during the quantum adversarial learning process. The gray shadow regions are the processes of optimizing \( D \), while the rests are those for optimizing \( G \). (c) shows the measured state tomography of the experimental \( \sigma_x \) and final \( \rho_t \) with a state fidelity \( F = 0.991 \), demonstrating a successful QGAN that \( G \) can fool \( D \) by generating quantum data highly similar to the true data. (d-f) Typical experimental results for \( \sigma \) in an arbitrary mixed state with each panel being the counterpart of (a-c) respectively.

steps are less than 0.02. The G’s turn ends when \( d < R_j \) for the \( j \)th round: \( R_j = 0.055 - 0.01 j \) when \( j \leq 3 \) and \( R_j = 0.02 \) when \( j > 3 \). These two adversarial learning procedures can be repeated many rounds until either the total count of steps \( c_{\text{step}} \) reaches a preset limit \( c_B \) or the optimized \( d \) in D’s round is smaller than a preset bound \( d_B \).

Figures 3(a-c) show the typical results for the experimental QGAN with \( \sigma = |g\rangle \langle g| \) of the transmon qubit, the highest purity state, as an example for the quantum true data. Since a digital quantum channel simulator can generate an arbitrary quantum state [21], the QGAN experiments by taking an arbitrary mixed state of the transmon as the true data is also studied and the results are depicted in Figs. 3(d-f). During the QGAN, the trajectory of control parameters are recorded [Figs. 3(b) and (e)] instead of characterizing the exact experimental \( \rho \) and \( M \). As shown in Figs. 3(a) and (d), the snapshots of the quantum states and measurement axis at the particular steps indicated by the arrows in Figs. 3(b) and (e) respectively (from left to right in the same order) are plotted on the Bloch sphere. Here, \( \sigma_t, \rho_t \) and \( M_t \) are the ideal results derived based on the calibrated control parameters. As expected, D adaptively adjusts \( M \) to be parallel with \( \rho_t - \sigma_t \), while \( G \) learns from the measurement outcomes to generate quantum data to fool \( D \), and the generated quantum data gradually converges to the plane that contains the quantum true data and is perpendicular to \( M \). As a result, \( d \) oscillates in the D’s and G’s turns due to the adversarial process and eventually approaches 0, which indicates that ultimately \( D \) fails to discriminate \( \rho \) from \( \sigma \), and G achieves his goal of replicating the statistics of the quantum true data.

To characterize the adversarial learning process, the state fidelity \( F(\sigma_t, \rho_t) = \text{trace} \sqrt{\sigma_t \rho_t \sqrt{\sigma_t}} \) in the adversarial pro-
The QGAN is implemented for two different cases, with a pure and an arbitrary mixed state as the quantum true data, respectively. The average $c_{\text{step}}$ for these two cases is 500 and 300, respectively. The obtained average $c_{\text{step}}$ for these two types of adversarial learning process is 243 and 170 respectively. The cumulative probability of final state fidelity $F$. The average fidelities for the pure and the mixed state are both 98.8%. For comparison, the noiseless numerically simulations of the adversarial learning process are also performed for 100 times, and their distributions are shown as solid lines.

By taking the total steps ($c_{\text{step}}$) and the fidelity of final generated state ($F$) as the figures of merit, the statistics of our QGAN performance is finally studied with 100 random adversarial learning processes. We study both cases with the same pure and arbitrary mixed states as the quantum true data as in Fig. 3, but with different random $p$ and $M$ at the beginning, all showing similar behaviors as those in Fig. 3. Figure 4(a) plots the cumulative probability of total steps, i.e. the probability to finish the QGAN experiment within $c_{\text{step}}$ steps. The average $c_{\text{step}}$ for these two types of adversarial learning process are 243 and 170, respectively. Figure 4(b) shows the cumulative probability of final state fidelity $F$ with the average fidelities for both the pure and the mixed quantum data of 98.8%. Comparing to the noiseless numerical simulation results, the experimental average $c_{\text{step}}$ is about twice larger, and the average $F$ is about 1% lower. These differences are mainly attributed to the decoherence processes of the qubit, the finite measurement precision of $d$, and the non-ideal measured gradients. Further studies about the effects of the experimental imperfections are provided in Supporting Materials.

In conclusion, we have demonstrated the feasibility of quantum generative adversarial learning with a superconducting quantum circuit, in which the input data, the generator and the discriminator are all quantum mechanical. Our results show that the generator can indeed learn the patterns of the input quantum data and produce quantum states with high fidelity that are not distinguishable by the discriminator. Since our QGAN experiment requires neither a quantum random accessing memory, nor a universal quantum computing device or any fine-tuning parameters (thus robust to experimental imperfections), it carries over to the noisy intermediate-scale quantum (NISQ) devices widely expected to be available in the near future [5]. An experimental demonstration of QGAN with NISQ devices promises to showcase the quantum advantages over classical GAN—a possible approach to realize quantum supremacy [22, 23] with practical applications. Our results may also have far-reaching consequences in solving quantum many-body problems with QGAN, given the recent rapid progresses in related directions [24–26]. In addition, the hybrid quantum-classical architecture demonstrated in this work can be straightforwardly extended to the optimal control [27] and self-guided quantum tomography [28], and we also anticipate their applications in other quantum machine learning/artificial intelligence algorithms.

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