Wormhole solutions to Hořava gravity

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Abstract

We present wormhole solutions to Hořava non-relativistic gravity theory in vacuum. We show that, if the parameter \( \lambda \) is set to one, transversable wormholes connecting two asymptotically de Sitter or anti-de Sitter regions exist. In the case of arbitrary \( \lambda \), the asymptotic regions have a more complicated metric with constant curvature. We also show that, when the detailed balance condition is violated softly, transversable and asymptotically Minkowski, de Sitter or anti-de Sitter wormholes exist.

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I. INTRODUCTION

A. Wormholes

In general relativity, a wormhole solution is often defined as a solution of Einstein equations with non-trivial topology that interpolates between two asymptotically flat spacetimes. A slightly more general definition replaces the asymptotically flat regions by other vacua as AdS or dS spacetimes, modelling gravitational solitons. A wormhole is said to be traversable whenever matter can travel from one asymptotic region to the other by passing through the throat. The classical example of a wormhole is the Einstein-Rosen bridge [1] which combines models of a Schwarzschild black hole and a white hole, and it is not traversable. There are no other traversable wormhole solutions in standard gravity in vacuum, nor in presence of physically acceptable matter sources (see e.g. [2]-[4]).

It is a widely accepted point of view that a correct theory of quantum gravity will modify our understanding of spacetime structure at small scales. In particular, it is generally assumed that General Relativity is a theory that describes large scales, and that at the UV it should be modified by quantum effects. In this context, it seems natural to wonder whether small wormholes in vacuum could be supported by such quantum corrections, without requiring the addition of any exotic matter. Indeed, in theories that correct General Relativity at short distances, like Einstein-Gauss-Bonnet, such wormholes have been shown to exist [5]-[8]. They would be created at small scales, and would be invisible at the scales at which Einstein gravity is recovered.

B. Hořava-Lifshitz gravity

Recently, a power counting renormalizable non-relativistic theory of gravity was proposed by Hořava [9]. Since then, a lot of work was made on the subject. Formal developments were presented in [10]-[17], some spherically symmetric solutions were presented in [18]-[23], toroidal solutions were found in [24], gravitational waves were studied in [25],[26], cosmological implications were investigated in [27]-[34], and interesting features of field theory in curved space and black hole physics were presented in [35]-[39].

A state of the theory is defined by a four-dimensional manifold $\mathcal{M}$ equipped with a
three dimensional foliation $\mathcal{F}$, with a Riemannian structure defined by an Euclidean three dimensional metric in each slice of the foliation $g_{ij}(\vec{x}, t)$, a shift vector $N^i(\vec{x}, t)$ and a lapse function $N(\vec{x}, t)$. This structure can be encoded in the ADM-decomposed metric

$$ds^2 = -N^2(\vec{x}, t)dt^2 + g_{ij}(\vec{x}, t) \left( dx^i + N^i(\vec{x}, t)dt \right) \left( dx^j + N^j(\vec{x}, t)dt \right).$$

(1)

The dynamics for the set $(\mathcal{M}, \mathcal{F}, g_{ij}, N_i, N)$ is defined as being gauge invariant with respect to foliation-preserving diffeomorphisms, and having a UV fixed point at $z = 3$, where $z$ is defined as the scaling dimension of time as compared to that of space directions $[\vec{x}] = -1, [t] = -z$. This choice leads to power counting renormalizability of the theory in the UV. To the resulting action one may add relevant deformations by operators of lower dimensions, that lead the theory to a IR fixed point with $z = 1$, in which symmetry between space and time is restored, and thus a general relativistic theory may emerge. This is close in spirit to the idea of recovering four-dimensional Einstein gravity as a sort of macroscopic limit ($z = 1$) of a collection of three dimensional theories whose quantization is better understood.

In order to have a control on the number of terms arising as possible potential terms, one may impose the so-called detailed balance condition: the potential term in the action for 3+1 dimensional non-relativistic gravity is built from the square of the functional derivative of a suitable action for Euclidean three-dimensional gravity (here three-dimensional indices are contracted with the inverse De Witt metric). Condensed matter experience on this kind of construction tells us that the higher dimensional theory satisfying the detailed balance condition inherits the quantum properties of the lower dimensional one. It has to be noted that the construction still works even when detailed balance condition is broken softly, in the sense of adding relevant operators of dimension lower than that of the operators appearing at the short distance fixed point $z = 3$. In the UV, the theory still satisfies detailed balance, while in the IR, the theory still flows to a $z = 1$ fixed point.

We won’t go through the above described steps in more detail, but state the resulting action together with the deformations that will be relevant to our purposes. The interested reader can refer to the original paper. The action for non-relativistic gravity satisfying the detailed balance condition can be written as

$$S = \int \sqrt{g} N (\mathcal{L}_0 + \mathcal{L}_1),$$

(2)
where we have defined the Lagrangians

\[
\mathcal{L}_0 = \frac{2}{\kappa^2} (K_{ij} K^{ij} - \lambda K^2) + \frac{\kappa^2 \mu^2 (\Lambda_w R - 3 \Lambda_W^2)}{8(1 - 3\lambda)},
\]
\[
\mathcal{L}_1 = \frac{\kappa^2 \mu^2 (1 - 4\lambda)}{32(1 - 3\lambda)} R^2 - \frac{\kappa^2}{2 w^4} \left( C_{ij} - \frac{\mu w^2}{2} R_{ij} \right) \left( C^{ij} - \frac{\mu w^2}{2} R^{ij} \right).
\]

(3)

Here \(\kappa, \lambda, \mu\) and \(w\) are arbitrary couplings. The dynamics in the infrared is controlled by \(\mathcal{L}_0\) and then, if \(\lambda = 1\), general relativity is recovered. On the other hand, in the UV the terms in \(\mathcal{L}_1\) become dominant, and the anisotropy between space and time is explicit.

To define soft violations of the detailed balance condition, we may add to the above Lagrangians a new term with the form

\[
\mathcal{L}_2 = \frac{\kappa^2 \mu^6}{8(1 - 3\lambda)} R,
\]

(4)

or we can distort the relative weight of \(\mathcal{L}_0\) and \(\mathcal{L}_1\). For our purposes, it will be enough to write the deformed action as

\[
S = \int dt d^3x \left( \mathcal{L}_0 + (1 - \epsilon_1^2) \mathcal{L}_1 \right) + \epsilon_2^2 \mathcal{L}_2,
\]

the detailed balance condition is recovered for \(\epsilon_1 = \epsilon_2 = 0\).

Since Ho\'řava-Lifshitz theory realizes a field theory model for quantum gravity that is power counting renormalizable in the UV, one might expect that its classical solutions differ from those of General Relativity at small scales, providing a mechanism to avoid singularities. For example it may have vacuum wormholes of the kind described above, only perceivable at microscopic scales and hidden at large scale. The purpose of this paper is to show that exact solutions with this behavior exist in the Ho\'řava model.

II. REFLECTING A SOLUTION AROUND A FIXED RADIUS

We are going to focus our work on the simplest case of static spherically symmetric wormholes. Under this assumption, the Riemannian structure of Ho\'řava gravity can be written in terms of the metric

\[
ds^2 = -\tilde{N}^2(\rho) dt^2 + \frac{1}{f(\rho)} dr^2 + (\rho^2 + r_o^2) d\Omega_2^2, \quad -\infty < \rho < \infty.
\]

(6)
This metric has two asymptotic regions \( \rho \to \pm \infty \) connected at \( \rho = 0 \) by a \( S_2 \) throat of minimal radius \( r_o \). We will also require the additional condition of \( Z_2 \) reflection symmetry with respect to the throat of the wormhole, namely

\[
\tilde{f}(\rho) = \tilde{f}(-\rho) \quad \text{and} \quad \tilde{N}^2(\rho) = \tilde{N}^2(-\rho).
\]

As we will see, this last condition simplifies the construction of smooth wormhole solutions in vacuum, free of discontinuities of the metric or its spatial derivatives. The presence of such discontinuities would require using surgery methods to glue two solutions, where the corresponding junction conditions, analog to the Israel’s conditions of standard General Relativity, must be solved [45].

To start the wormhole construction, we will take a static spherically symmetric solution written in the standard form

\[
ds^2 = -N^2(r)dt^2 + \frac{1}{f(r)}dr^2 + r^2d\Omega_2^2, \quad 0 < r < \infty.\]

This may or may not represent a wormhole, and it may have singularities and/or event horizons. In the present paper we will concentrate in the solutions of this kind presented in [18, 19]. Such solutions are defined by the set

\[
S \equiv (\mathcal{M}, f(r), N(r)), \quad 0 < r < \infty.\]

Since \( N_i(r) = 0 \) we have omitted explicit mention to the shift vector. The foliation \( \mathcal{F} \) is implicit in the form of the metric as being defined by constant \( t \) surfaces. We will take the restriction of \( \mathcal{M} \) to \( \mathcal{M}^+ \) whose boundary is a sphere of radius \( r_o \), namely

\[
S^+ \equiv (\mathcal{M}^+, f^+(r), N^+(r)), \quad r_o \leq r < \infty.\]

By defining the new variable \( \rho^2 \equiv r - r_0, \quad \rho \in \mathbb{R}^+_0 \) one may write

\[
S^+ = (\mathcal{M}^+, \tilde{f}^+(\rho), \tilde{N}^+(\rho)), \quad 0 \leq \rho < \infty,\]

where \( \tilde{N}^+(\rho) = N^+(\rho^2 + r_o) \) and \( \tilde{f}^+(\rho) = f^+(\rho^2 + r_o) \). Now one may define the reflected set coordenatized with \( \rho \in \mathbb{R}^-_0 \) as

\[
S^- \equiv (\mathcal{M}^-, \tilde{f}^-(\rho), \tilde{N}^-(\rho)), \quad -\infty < \rho \leq 0,\]

where \( \tilde{N}^-(\rho) = N^+(\rho^2 + r_o) = \tilde{N}^+(\rho) \) and \( \tilde{f}^-(\rho) = f^+(\rho^2 + r_o) = \tilde{f}^+(\rho) \). Here \( \mathcal{M}^- \) is a copy of \( \mathcal{M}^+ \).
So the extended manifold may be finally defined by joining $\mathcal{M}^+$ and $\mathcal{M}^-$ at their boundaries at $\rho = 0$, with the foliation extended accordingly.

$$\tilde{S} \equiv (\tilde{\mathcal{M}}, \tilde{f}(\rho), \tilde{N}(\rho)), \quad -\infty < \rho < \infty.$$ (13)

where $\tilde{N}(\rho) = N^+(\rho^2 + r_o)$, $\tilde{f}(\rho) = f^+(\rho^2 + r_o)$. Note that these functions satisfy (7) at $r_o$.

That the resulting set $\tilde{S}$ is a solution of the theory at any point $\rho \neq 0$ is made evident by changing variables back to $r > r_o$, what results in the original solution $S^+$. On the other hand, the change of variables is singular at $\rho = 0$ ($r = r_o$), which implies that special attention should be paid to the equations of motion there. In order to get a smooth solution when completing the space with the point $r = r_o$, we need to impose the condition

$$f_{,r}(r_o) = 0.$$ (14)

This implies that, in order for the extended set to be a solution, the reflection point must be chosen as a stationary point of the function $f(r)$. This condition ensures that both sides of the solution can be unique and smoothly completed at this point. On the other hand, one might directly verify that the ansatz constructed in this way is a solution of the vacuum equations of motion.

This extended solution has two asymptotic regions $\rho \to \pm \infty$, that share the asymptotic behavior of the original solution at $r \to \infty$, connected by an $S_2$ throat of radius $r_o$ at $\rho = 0$, and $Z_2$ symmetric with respect to it.

Since the starting solution may have event horizons, namely points $r_h$ at which

$$N(r_h) = 0,$$ (15)

then in such case to get a transversable wormhole, a sufficient condition is

$$r_o > r_{h^+},$$ (16)

where $r_{h^+}$ is the position of the outmost horizon.

### III. WORMHOLE SOLUTIONS TO HOŘAVA GRAVITY

#### A. Asymptotically flat wormholes

A spherically symmetric solution with a flat asymptotic region was found in \[19\] by setting $\epsilon_1 = 0$, $\epsilon_2 = 1$ and $\lambda = 1$ in \[5\]. This theory has a Minkowski vacuum, and it is natural to
look for spherically symmetric solutions that approaches that vacuum at infinity. As shown there, such solution exist and takes the form

\[ N^2 = f = 1 + \omega r^2 - \sqrt{r(\omega^2 r^3 + 4\omega M)} , \]  

(17)

here \( M \) is an integration constant and \( \omega = 16\mu^2/\kappa^2 \). This solution has two event horizons at radius

\[ r_{h\pm} = M \left( 1 \pm \sqrt{1 - \frac{1}{2\omega M^2}} \right) , \]  

(18)

which disappear leaving a naked singularity whenever

\[ 2\omega M^2 < 1 . \]  

(19)

To find a wormhole solution we will apply to (17) the above described reflection technique. As discussed above, this entails to look for an stationary point of the spherically symmetric solution

\[ f, r_o(r_o) = 2\omega r_o - 2 \frac{\omega^2 r_o^3 + \omega M}{\sqrt{r_o(\omega^2 r_o^3 + 4\omega M)}} = 0 . \]  

(20)

By multiplying by \( \sqrt{r_o(\omega^2 r_o^3 + 4\omega M)} \), taking the square and rewriting in terms of the variable \( x = \omega^2 r^3 \), this equation can be solved, resulting in

\[ r_o = \left( \frac{M}{2\omega} \right)^{\frac{1}{3}} , \]  

(21)

thus we can build a wormhole by taking a solution of the form (17) with positive \( M \), and reflecting it at the value of \( r_o \) given by (21).

In terms of this, the transversability condition (16) may be written as

\[ \left( \frac{M}{2\omega} \right)^{\frac{1}{3}} > M \left( 1 + \sqrt{1 - \frac{1}{2\omega M^2}} \right) , \]  

(22)

and can be solved by writing \( x = (2\omega M^2)^{-1/3} \) and solving for \( x \), getting

\[ 2\omega M^2 < 1 , \]  

(23)

which is exactly (19). This implies that, in order to build a transversable wormhole, we should start with the solution that has naked singularity. Note that after reflection the singularity disappears, leaving us with a completely regular wormhole.
B. Asymptotically (A)dS wormholes

If the detailed balance condition is satisfied $\epsilon_1 = \epsilon_2 = 0$, a spherically symmetric solution with AdS asymptotic behavior was found in [18], for the particular case in which $\lambda = 1$. They have the form:

$$N^2 = f = 1 + (\beta r)^2 - \alpha \sqrt{\beta r}, \quad (24)$$

where $\beta = \sqrt{-\Lambda}$ and $\alpha$ is a constant of integration.

A horizon will be present at the value of $r$ satisfying $N^2(r_h) = 0$, which can be rewritten as ($x = \beta r_h$)

$$x^4 + 2x^2 - \alpha^2 x + 1 = 0. \quad (25)$$

This polynomial has at most two real roots, as can be seen from the fact that its second derivative is always positive. Its discriminant reads

$$\Delta = \alpha^4(256 - 27\alpha^4), \quad (26)$$

and it vanishes whenever there is a double root. This implies that the two horizons will join into a single one when $\alpha = \pm 4/3^{3/4}$. For smaller $|\alpha|$'s there is no event horizon but a naked singularity. This can also be seen in Figure 1.

To find the reflection point to build our wormhole, we write the derivative as

$$f(r_o)_{,r} = 2\beta^2 r_o - \beta \frac{\alpha}{2\sqrt{\beta r_o}} = 0. \quad (27)$$

This condition uniquely defines the point at which the solution must be reflected as

$$r_o = \frac{1}{\beta} \left(\frac{\alpha}{4}\right)^{\frac{2}{3}}. \quad (28)$$

As mentioned before, in order to ensure transversability, the reflection point must be at the right of the outer horizon $r_o > r_{h^+}$. This cannot be satisfied for any value of $r_o$, as can be seen in Figure 1. In consequence, as it happened for the asymptotically flat case, we should build our wormhole from the solution without horizons $|\alpha| < 4/3^{3/4}$, which in turn implies $r_o < 1/\sqrt{3}\beta$. The wormhole obtained in this way is transversable and asymptotically (A)dS by construction.
FIG. 1: In this plot the relation between the horizons and the reflection point is shown, for the case $\lambda = 1$, $\epsilon_1 = \epsilon_2 = 0$. As can be seen in the plot, the radius of the outer horizon, when it is present, is always larger than that of the reflection point, implying that a transversable wormhole must be constructed from a solution with a naked singularity.

If the detailed balance condition is relaxed to $\epsilon_1 \neq 0$, $\epsilon_2 = 0$, a more general spherically symmetric solution with an (A)dS asymptotic behavior has also been found with $\lambda = 1$ [18]. It takes the form

$$N^2 = f = 1 + \beta^2 \frac{r^2}{1 - \epsilon_1^2} - \frac{\sqrt{r(\epsilon_1^2\beta^4r^3 + 4M(1 - \epsilon_1^2)\beta)}}{1 - \epsilon_1^2},$$

where $\beta = \sqrt{-\Lambda_W}$ and $M$ a constant of integration. Despite of its similarity with (17), this solution takes an (A)dS form asymptotically, as can be easily verified. If $\epsilon_1^2 < 1$ the solution has a curvature singularity at the origin. On the other hand if $\epsilon_1^2 > 1$, the square root becomes complex below a finite radius $r_c^3 = 4M(\epsilon_1^2 - 1)/\epsilon_1^2\beta^3$.

To find the event horizons, we rewrite the condition $f(r_h) = 0$ as $(x = \beta r_h)$

$$x^4 + 2x^2 - 4Mx + (1 - \epsilon_1^2) = 0.$$
a single double root, namely when the discriminant vanishes

\[-256 \left( 27M^4 + 2 \left( 9\epsilon_1^2 - 8 \right) M^2 + \epsilon_1^4 \left( \epsilon_1^2 - 1 \right) \right) = 0, \tag{31}\]

or in other words when

\[M_{\pm}^2 = \frac{1}{27} \left( 8 - 9\epsilon_1^2 \pm (4 - 3\epsilon_1^2)\sqrt{4 - 3\epsilon_1^2} \right). \tag{32}\]

Here we see that in order to have a horizon we need \(\epsilon_1^2 < 4/3\). Under this assumption, the value of \(M_{\pm}^2\) is always negative, implying an imaginary value of \(M_{\mp}\) that should be discarded. Regarding \(M_{\pm}^2\), it is always positive, determining a bound that \(M^2\) should satisfy in order to have a horizon. Such bound read \(M^2 > M_{\pm}^2\), or more explicitly

\[M^2 > \frac{1}{27} \left( 8 - 9\epsilon_1^2 \pm (4 - 3\epsilon_1^2)\sqrt{4 - 3\epsilon_1^2} \right). \tag{33}\]

For values of \(M\) satisfying this inequality, the discriminant is negative and two horizons exists. On the other hand for \(M^2 < M_{\pm}^2\), there is no event horizon.

To build an asymptotically (A)dS wormhole, we reflect the above solution at the stationary point \(r = r_o\) defined by

\[f_{,r}(r_o) = 2\beta^2 \frac{r_o}{1 - \epsilon_1^2} - 2\frac{\epsilon_1^2\beta^4 r_o^3 + M(1 - \epsilon_1^2)\beta}{(1 - \epsilon_1^2)\sqrt{r_o(\epsilon_1^2\beta^4 r_o^3 + 4M(1 - \epsilon_1^2)\beta)}} = 0, \tag{34}\]

this is solved by

\[r_{o\pm} = M_{\pm}^4 \frac{1}{\beta} \left( 1 - \frac{2}{\epsilon_1^2} \pm \frac{1}{\epsilon_1} \sqrt{\frac{4}{\epsilon_1^2} - 3} \right)^{1/4}. \tag{35}\]

Again the condition \(\epsilon_1^2 < 4/3\) shows up, now to ensure that a stationary point exist at which the solution can be reflected. For \(1 < \epsilon_1^2 < 4/3\), the value \(r_{o+}\) is always smaller than the value \(r_c\) at which the solution become complex. This implies that no wormhole can be used to cure this problem, suggesting that only small violations of the detailed balance condition \(\epsilon_1^2 < 1\) should be accepted.

In the case \(\epsilon^2 < 1\), it can be checked that \(r_{o+}\) is never larger than \(r_{h+}\). In other words, to build a transversable wormhole one should start with a solution without horizon, \(i.e.\) one satisfying \(M^2 < M_{\pm}^2\). Such solutions contain a naked singularity at the origin that disappears after reflection. This is analogous to what happened for cases studied in the previous sections.
C. Exotic asymptotic behavior

Spherically symmetric solutions of the action satisfying the detailed balance condition \( \epsilon_1 = \epsilon_2 = 0 \) were found for arbitrary \( \lambda \) in [18]. They take the form

\[
f = 1 + (\beta r)^2 - \alpha (\beta r)^{\frac{2\lambda + \sqrt{6\lambda - 2}}{\lambda - 1}}, \tag{36}
\]

\[
N^2 = (\beta r)^{\frac{2\lambda - \sqrt{6\lambda - 2}}{\lambda - 1} } f(r), \tag{37}
\]

where \( \beta = \sqrt{-\Lambda_W} \) and \( \alpha \) is a constant of integration. There are two solutions for each value of \( \lambda \), defined by the choice of the sign in front of the square root.

To build a \( Z_2 \) symmetric solution (that we will loosely call a wormhole, even if, as we will see, its asymptotic behavior is nontrivial) we take the derivative and look for a stationary point

\[
f_{,r}(r_o) = 2\beta^2 r_o - \alpha \beta \frac{2\lambda \pm \sqrt{6\lambda - 2}}{\lambda - 1} (\beta r_o)^{\frac{2\lambda \pm \sqrt{6\lambda - 2}}{\lambda - 1} - 1} = 0, \tag{38}
\]

\[
N^2_{,r}(r_o) = -2\beta \frac{1 + 3\lambda \pm 2\sqrt{6\lambda - 2}}{\lambda - 1} (\beta r_o)^{\frac{2\lambda + \sqrt{6\lambda - 2}}{\lambda - 1} - 1} f(r_o) = 0. \tag{39}
\]

This equations can be simplified to \((x = \beta r_o)\)

\[
2x^2 = \alpha \frac{2\lambda \pm \sqrt{6\lambda - 2}}{\lambda - 1} x^{\frac{2\lambda \pm \sqrt{6\lambda - 2}}{\lambda - 1}}, \tag{40}
\]

\[
\left(1 + 3\lambda \pm 2\sqrt{6\lambda - 2}\right) f(r_o) = 0, \tag{41}
\]

we see from the second equation that either the value of \( \lambda \) satisfies

\[
1 + 3\lambda \pm 2\sqrt{6\lambda - 2} = 0, \tag{42}
\]

that is solved only by \( \lambda = 1 \) when the minus sign is chosen, corresponding to the AdS case already analyzed in the previous section; or the solutions must be reflected at the horizon \( f(r_o) = 0 \). Even if this may result on a non-transversable wormhole, we will study the possibility of a solution of this form. In this last case equations read

\[
\alpha \frac{2\lambda \pm \sqrt{6\lambda - 2}}{\lambda - 1} x^{\frac{2\lambda \pm \sqrt{6\lambda - 2}}{\lambda - 1}} = 2x^2, \tag{43}
\]

\[
1 + x^2 - \alpha x^{\frac{2\lambda \pm \sqrt{6\lambda - 2}}{\lambda - 1}} = 0, \tag{44}
\]

and can be simplified to

\[
\alpha \frac{2\lambda \pm \sqrt{6\lambda - 2}}{\lambda - 1} x^{\frac{2\lambda \pm \sqrt{6\lambda - 2}}{\lambda - 1}} = 2x^2, \tag{45}\]

\[
\frac{2\lambda \pm \sqrt{6\lambda - 2}}{-2 \mp \sqrt{6\lambda - 2}} = x^2.
\]
The second equation has no solution if we choose the plus sign. On the other hand, for the minus sign, two possibilities arise from the second line, either

$$2\lambda - \sqrt{6\lambda - 2} > 0 \quad \text{and} \quad -2 + \sqrt{6\lambda - 2} > 0,$$

(46)
or

$$2\lambda - \sqrt{6\lambda - 2} < 0 \quad \text{and} \quad -2 + \sqrt{6\lambda - 2} < 0.$$

(47)
The first is solved by $\lambda > 1$, the second instead is solved by $1/2 < \lambda < 1$.

We conclude that, when $\lambda > 1/2$, the second equation in (45) provides a value of $r_o$ at which the solution can be reflected. The first equation on the other hand, determines the value of $\alpha$ that is compatible with that $r_o$. In other words wormhole solutions can be built from the solution with the minus sign whenever $\lambda > 1/2$, by reflecting the exterior region at the horizon.

To explore the asymptotic behavior of the resulting wormhole, we take the $r \to \infty$ limit. We see that the second term in $f(r)$ in (36) will dominate whenever

$$\frac{2\lambda - \sqrt{6\lambda - 2}}{\lambda - 1} < 2,$$

(48)
which is always satisfied for $\lambda > 1/2$. Regarding $N^2(r)$ in (37), we see that for large $r$ it behaves as a power law with exponent

$$n = -2 \left( 1 + 3\lambda - 2\sqrt{6\lambda - 2} \right) \frac{1}{\lambda - 1} + 2,$$

(49)
which is positive and smaller than 4 for $1/2 < \lambda < 3$ and negative for $\lambda > 3$. The (A)dS case $n = 2$ corresponds to $\lambda = 1$ and has been studied in the previous sections.

This results in an asymptotic metric with the form

$$ds^2 = -(\beta r)^n dt^2 + \frac{dr^2}{(\beta r)^2} + r^2 d\Omega^2_2.$$ 

(50)
The scalar curvature is asymptotically constant and finite

$$R = \frac{\Lambda_W}{2} (12 + 4n + n^2) + \frac{2}{r^2},$$

(51)
and it never vanishes as a function of $n$. As can be easily seen, the asymptotic isometry group is $SO(3) \times \mathbb{R}$, the first factor representing the rotations on the sphere, while the second refers to time translations.
IV. CONCLUDING REMARKS

In this work we have constructed wormhole solutions of Hořava gravity theory in vacuum by reflection of known spherically symmetric metrics. For both the asymptotically Minkowski or (A)dS cases, the condition to have purely exterior solutions, namely wormholes constructed with regions exterior to the horizon of the starting solution, that we imposed in order to ensure transversability, forces us to choose the parameters in the starting spherically symmetric solutions that imply a naked singularity. After reflection the singularity disappears, leaving us with a $Z_2$ and spherically symmetric wormhole, where two asymptotic regions are connected through a topologically $S^2$ neck.

Under the detailed balance condition, the $\lambda = 1$ case provides a wormhole with dS or AdS asymptotic regions. A similar solution is obtained when the detailed balance condition is softly broken by a deformation of the kind parameterized by $\epsilon_1$, whenever $\epsilon_1^2 < 1$. In the case $\epsilon_1^2 > 1$ no wormhole exist and the solution become complex at finite distance to the origin, what suggest that only small violations of the detailed balance condition $\epsilon_1^2 < 1$ should be allowed. On the other hand, if $\epsilon_2$ is turned on, the resulting asymptotic regions correspond to flat Minkowkian spacetime. On the general $\lambda > 1/2$ case, the asymptotic regions have constant curvature, and isometry group $SO(3) \times \mathbb{R}$.

The fact that a transversable wormhole can be constructed when the starting solution has no horizon suggest the following interpretation: if we start with value of the integration constant $\alpha$ (or $M$) for which a horizon exists, an outside observer cannot say whether the solution represents a microscopic wormhole hidden inside the horizon, or a black-hole with a censored singularity at the center. When varying $\alpha$ the horizon shrinks and at some point it disappears. From the macroscopic point of view, described by a relativistic theory of gravity, the singularity would become naked violating the cosmic censorship principle. But from the microscopic non-relativistic point of view, the wormhole provides a mechanism to avoid it. Therefore, we may observe a sort of complementarity between these two mechanisms to censure the singularities.

As another related point, we may see that for instance in the solution (24) the condition to have a well defined Hawking temperature [18] is exactly the opposite to the one that allows to form a transversable wormhole. This is consistent with the fact that in the last case, the degrees of freedom in both sides of the throat are causally connected, and therefore,
one should’t take traces to evaluate the asymptotic quantum amplitudes.

A remark should be made about our definition of tranversability. In this paper we have constructed wormholes that are tranversable for particles that move according to the causal structure of the metric, namely inside the light cones. Since in the present context symmetry between space and time is broken, one may imagine particles with a more complicated causal behavior. For example a Lifshitz scalar has corrections to its dispersion relation that are quartic in momenta. Depending on the sign of the correction, the field quanta may eventually follow worldlines that travels outside the metric light cones. This may indeed enlarge the set of solutions which are microscopically transversable by such particles.

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[43] Although in principle, the model could be generalized to foliations where the hypersurfaces are not spacelike.

[44] While this paper was in preparation, important objections were raised to the possibility of Hořava gravity would indeed flow to General Relativity, due to the presence of an additional degree of freedom associated to foliation violating diffeomorphisms, that is not gauged in Hořava theory as it is in General Relativity.

[45] Junction conditions in Hořava gravity will be studied elsewhere.