An Analysis of Students’ Error in Solving Abstract Algebra Tasks

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Abstract. This research aimed at determining student errors in completing abstract algebra tasks. This study can be categorized as descriptive research involving 31 college students of mathematics education program as research participant, Riau Kepulauan University, Batam. The research instrument used 4 essays of abstract algebra tests. The students’ errors were identified and categorized into 3 types of errors namely, careless errors, computational errors, and conceptual errors. The result of this study showed that most of the students’ errors was on conceptual error with the percentage of 57.94% and followed by the careless error with 33.33%, and the computational error was 8.73%. The analysis results based on the student ability level, it was found that the high competence students mostly conducting careless error and followed by conceptual error, and computational error. Meanwhile the moderate and low competence students, mostly doing conceptual error that followed by careless error, and computational error. Also, based on the proportion of errors, the high performance students made careless error and computational error more than the students with moderate and low performance. Moreover, in case of conceptual error, the pattern was the opposite where students with moderate and lower performance conducted conceptual errors more than high performance students.

1. Introduction
Abstract algebra is a developmental topic of algebra dealing with the structure of abstract algebra. It consists of axiomatic theories that provide opportunities to consider many different mathematical systems as being special cases of the same abstract structure. The theories are called axiomatic because the structures are defined by axioms, and group theory as one of the oldest (and also one of the simplest) of axiomatic theories [1]. Abstract algebra is important in the education of a mathematically trained person. Its terminology and methodology are used wider in computer science, physics, chemistry, and data communications, and of course, abstract algebra still has a central role in advanced mathematics itself [2]. In addition, abstract algebra is an important course for the undergraduate mathematics and mathematics education curriculum as said in [3], [4].

For most undergraduates, this course is one of their earliest experiences coping with the difficult notions of mathematical abstraction and formal proof. According to [1], there are several reasons why the learning of linear algebra is difficult for students that consider as the initial difficulty characterization of abstract algebra. Some problems related to these issues are: firstly, the concepts are
abstract structures that serve as categories for a broad and diverse range of examples. The objects are defined by their properties, and the properties rather than the examples are primary that making it hard for students to conceive of them. Secondly, many of the examples are unfamiliar to the students. Next, many students are not yet comfortable with proof and the axiomatic method. Empirical research studies attest to students’ difficulties in abstract algebra. These studies have shown that many students do not understand the fundamental concepts in group theory [5], [6] and have difficulty in writing proofs of a group theoretic context [4], [7], [8], [9]. In addition, other studies also state that student understanding of the concepts in abstract algebra is less than satisfactory [10], [11].

Further, [7], [12], and [13] state that students often make mistakes in solving abstract mathematical problems including group theory. Those made the student obtain low learning outcomes. Based on some previous research, the low learning outcomes of students in abstract algebra courses are also experienced by students in Indonesia [3], [14], [15], [16], [17], [18]. Meanwhile, based on the researcher’s experience during the abstract algebra course from year to year, it has not been able to get satisfactory learning outcomes, where there are still many students who get low scores on abstract algebra courses in the last two years as presented in Table 1 below.

### Table 1. The final score of abstract algebra course among the student of mathematics education study program at Riau Kepulauan University in last two years.

| Academic Year | Abstract Algebra I | Abstract Algebra II |
|---------------|--------------------|--------------------|
|               | A  | B  | C  | D  | A  | B  | C  | D  |
| 2015/201      | n %| n %| n %| n %| n %| n %| n %| n %|
| 6             | 7  | 9.7| 20  | 27.8| 2  | 28. | 1  | 23. | 18.0| 50. |
| 2016/201      | 1  | 17. | 31  | 42.5| 2  | 31. | 6  | 8.2 | 9  | 15.8|
| 7             | 3  | 8  | 3  | 5  | 6  | 8.2 | 6  | 12  | 21. | 8  |

The A-D scores are using 0-100 scales where the range is A ≥ 81, 68≤B <81, 56≤C <68, and D <56 referring to the standard score at Riau Kepulauan University. Based on table 1 above, it can be seen that there are still 40-50% of students who get C and D score. Many factors cause the difficulties of students in studying abstract algebra. In order to find out the student's learning difficulties, it is very important for a lecturer to perform an error analysis in order to know exactly which steps the student having difficulty and misconception, as said in [19], [20], and [21].

Error analysis is one form of diagnostic assessment. The collected data can help teachers align instruction with the student’s specific needs [22]. According to [23], error analysis involves reviewing the student’s independent work (e.g., seatwork, quizzes, tests, progress monitoring) to be identified. The specific error types and patterns error analysis can help to set teaching priorities. Further, [24] states that error analysis also referred to error pattern analysis, i.e. the study of errors in learners’ work to looking for possible explanations of these errors. It is a multifaceted activity involving analysis of correct, partially correct and incorrect processes and thinking about possible remedying strategies. The analysis provided some insight into more common procedural and conceptual errors in the learners’ scripts.

In addition, there are many advantages of doing students error analysis [25]. They are (1) to identify which steps the student is able to perform correctly (as opposed to simply marking answers either correct or incorrect, something that might the student is doing right), (2) to determine what types of errors made by the student, (3) to determine whether an error is a one-time miscalculation or a persistent issue that indicates an important misunderstanding of a mathematical concept or procedure, (4) to select an effective instructional approach to address the student’s misconceptions and to teach the correct concept, strategy, or procedure.

Some research on mathematical error analysis categorizes types of errors with different concepts. Like [23] divides the mathematical error type into 4 types: conceptual, factual (aka computational), procedural, and careless. Then, [26] classified the types of students’ mathematical errors into 4 types of
errors: conceptual, careless, problem-solving and value errors. Meanwhile, [9] categorized students' mathematical errors into three main categories: calculation errors, procedural errors, and symbolic errors. Furthermore, [25] divided the type of mathematical error into 3, namely factual errors, procedural errors, and conceptual errors. However, the conceptual and procedural knowledge often marked interchangeably that make it difficult to distinguish between those two errors [27], [28]. Similar to [26], [29] categorizes 3 types of math errors, namely careless errors, computational errors, and conceptual errors.

Careless error is defined as a form of human error as unintended action [30]. Careless errors occur because they are not paying attention, or working too fast [29]. Furthermore, [31] explains that computational errors can be generalized as errors in addition, subtraction, multiplication, and division of numbers. While conceptual errors are due to misconceptions of the underlying principles and ideas related to the mathematical problem (e.g., relationship between numbers, characteristics, and properties of shapes). [32] explains that misconceptions are systematic patterns of errors in interpreting, understanding or applying mathematical concepts. These conceptual errors are considered as the cause of the students' errors.

Based on findings from the previous studies, 3 types of mistakes put forward by [29] according is considered appropriate with the errors that are often done by students in abstract algebra course at Riau Kepulauan University. Therefore, this research focus on careless error, computational error and conceptual error by using abstract algebra test results on group material. In general, there are two research questions of this research: (1) what errors do the students of Riau Kepulauan University make when solving abstract algebra tasks? As well as (2) what is the relation between the types of errors and the student performance levels?

2. Method

2.1. Research design

Due to the purpose of this study is to describe the errors made by students in solving the problem of abstract algebra in group material, then this type of research can be categorized as descriptive research. It refers to [33] which states that descriptive research as a study that attempts to describe a symptom, event, event without giving special treatment to the event.

2.2. Participants

The research population was all fifth semester students of Mathematics Education Study Program, Faculty of Teacher Training and Education, Riau Kepulauan University in the academic year of 2017/2018. It consisted of two classes with the number of 53 students. The sampling technique was using purposive sampling with 31 student as research subject. The purposive sampling was done by selecting the class with big portion of errors occurrence.

2.3. Data collection

To obtain data errors made by students in abstract algebra course, this study used the test instrument. The test was in the form of essay consisting of 4 pieces of matter on group material, namely group concept, subgroup, cycle, and cyclic group. Before the question was given to the students, firstly, it was conducted the validity of the content, reliability and level of difficulty of the test. The validity of the contents of an instrument used to measure the what extent the items in the instrument represented the components in the whole area of the content of the object being measured and the what extent the items reflected the behavioral being measured [34].

The content validity of this research instrument was determined by using expert agreement. In this case, the instrument had been consulted with peer-leaucturers of abstract algebra course. The peer assessment results stated that the test instrument was valid and it can be used to identify the three types of student errors. In case of reliability test, cronbach alpha technique was employed and it obtained reliability coefficient was 0.6, where it means the reliability of the instrument included in the medium
category [35]. Furthermore, the problem level of problem analysis, from 4 questions, it was obtained 3 problems with the medium category and 1 problem (2nd question) with the difficult category. Thus, these four questions had met the standard of good instruments.

2.4. Analysis of data
After the instrument was considered valid and reliable, it was held field testing to 31 students. Since the research focus was errors investigation, it was only the students’ incorrect responses in whole or in part was coded. Missing responses that can also be categorized as wrong answer, were not coded and excluded from the analysis because students’ errors cannot be identified from a blank response. The scheme used to code the careless, computational, and conceptual errors. These category were derived from the most frequently occurring errors based on the literature review. To facilitate the analysis, the following researchers formulated the definition of the three types of errors that examined based on the results of literature studies in table 2 below.

Table 2. Summary of error categories.

| No. | Type of Errors   | Code | Definition                                                                                                                                                                                                 |
|-----|------------------|------|-----------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| 1.  | Careless error   | C1   | The student fails to correctly solve given mathematical problem despite having the necessary skills or knowledge (e.g. copying the problem wrong to begin with, not reading the problem, dropping a negative sign, sloppy handwriting, not following the directions, incorrect label, writing a wrong number) [29]. |
| 2.  | Computational error | C2   | The students made mistakes in counting such as incorrectly adding, mis-subtracting, mis-multiplying, or mis-dividing [31].                                                                                     |
| 3.  | Conceptual error | C3   | Students’ errors due to misconceptions or a faulty understanding of the underlying principles and ideas connected to the mathematical problem. It can be errors in interpreting, understanding or applying mathematical concepts [32]. |

After identifying and categorizing student errors in accordance with the formulation of table 2, it then tried to found the percentage of students who do the above three errors with the following formula:

\[ P = \frac{f}{N} \times 100\% \]

Where, \( P \) means the percentage, \( f \) was for the frequency, and \( N \) represented the sum.

3. Results

3.1. Overview of the observed types of errors
Overall, there were 31 student responses for all four given test questions. For the question number 1, there were 16 correct student answers, 3 blank responses, and 12 wrong answers. For question number 2, there was 1 correct answer, 2 blank answer, and 28 wrong answers. For question number 3, there were 11 correct answers and 20 wrong answers. For question number 4, there were 2 correct answers
and 29 wrong answers. Table 3 below was the distribution of student response results with correct, incorrect and blank answers.

**Table 3. Frequencies of student response.**

| Category of student response | Total |
|------------------------------|-------|
| Correct answer               |       |
| Wrong answer                 |       |
| Blank                        |       |
| Question 1                   | 31    |
| Question 2                   | 31    |
| Question 3                   | 31    |
| Question 4                   | 31    |
| Total                        | 124   |

Based on the total answers of students who were wrong for the problem number 1 to 4, it was 89 answers were then analyzed to determine the errors occurrence. Below was the results of the analysis.

**Table 4. The result analysis of student errors.**

| Type of Error | The error frequency for the question of | Total | % |
|---------------|----------------------------------------|-------|---|
|               | 1   | 2   | 3   | 4   |     |     |
| Careless      | 8   | 10  | 6   | 18  | 42  | 33.33 |
| Computational | 0   | 7   | 3   | 1   | 11  | 8.73  |
| Conceptual    | 11  | 23  | 16  | 23  | 73  | 57.94 |
| Total of errors | 19 | 40  | 25  | 42  | 126 | 100  |

Based on the results in table 4, it can be seen that a total of 89 students' answers were wrong, there were 126 errors. From 126 errors, 42 times error occurred or 33.33% were careless errors, with the most errors in the question number 4. The computational error occurred as much as 11 times or 8.73% with the most frequencies in the question number 2. In case of the question number 1, it cannot be categorized as computational error because it did not contain any computation element. Furthermore, the most common mistake was the conceptual error as much as 73 errors or 57.94% from the total of 126 errors.

3.2. **Observed careless errors**

According to [29] careless errors occur simply because the students are not paying attention, or working too fast. Meanwhile, [25] states this might happen because the student is tired or distracted by other activity in the classroom. Based on the analysis results in table 4, the careless errors found on each question where mostly, on the fourth question. Figure 1 shows the frequency of careless error in each question.

![Careless Error (C1)](image)

**Figure 1. Frequencies of careless error.**
It can be seen, from Figure 1, that careless error was most common in question number 4, followed by second, first, and third questions. The mistakes made by the student was wrong in copying the problem, mis-copying the number, wrong in determining the original number of prime with 11, did not read the problem thoroughly that make them answering the test not based on the instruction, forgetting to write the terms $L \subseteq K$ and $L \neq \emptyset$ in subgroup verification and less accurate in placing cycle as the instruction. Figure 2 show One of the responses of a student who made a careless error.

\[ K = \begin{bmatrix} a & b \\ c & d \end{bmatrix} \mid a, b, c, d \in \mathbb{Q} \text{ and } ad \neq bc \] under multiplication was group. Prove that $L = \begin{bmatrix} x & 0 \\ y & z \end{bmatrix} \mid x, y, z \in \mathbb{Q} \text{ dan } xz \neq 0$ is subgroup of $K$.” Instead of doing the task correctly, the student work contained an error in copying the problem information. In the question, the second element of the matrix $L$ was zero, but their completion was zero as the third element of the $L$ matrix that made the wrong answer. Though the student was correct in copying the questions in the answer sheet.

3.3. Observed computational error
Mathematical computational errors correspond to students’ failure in calculating, such as incorrectly added, incorrectly subtracted, incorrectly multiplied, or incorrectly divided. In the calculation, here, was not limited to arithmetic calculations only, but also in the calculation of matrix form, cycle form, addition and multiplication modulo form. Below was a graph of distribution results of computational errors done by students.
It can be seen, from Figure 3, that in question 1 there was no computational error because the computational element that must be done did not exist. Meanwhile, from 11 computational errors, computational errors occurred most often in the question number 2 and at least on item number 4. The occurred computational error was a mistake in matrix multiplication operation, wrong in calculating the product of cycle, and wrong in counting result of modulo powers. An example of calculation error in the matrix multiplication operation can also be seen in the student's answer in Figure 2 marked with the symbol C2.

3.4. **Observed Conceptual Error**

Conceptual errors are errors due to misconceptions or a faulty understanding of the underlying principles and ideas connected to the mathematical problem. According to [32], misconceptions are systematic patterns in errors in interpreting, understanding or applying mathematical concepts. These conceptual errors are responsible for students' mistakes. The results of the conceptual errors analysis done by the students presented in Figure 4 below.

Based on Figure 4, the most frequencies error occurred in questions 2 and 4 followed by the numbers 3 and 1. The error made by students for the question number 1 was the wrong question interpretation “where the inverse of each element group was the element itself” and wrong in proving the commutative nature of the group. For question number 2, the error occurred in the student's uncertainty in using the theorem subgroup verification. For question number 3, the student did not
understand the concept of permutation in the form of cycle, while for question number 4, the student had not understood the concept of determining the non trivial subgroup and its relation with the cyclic group concept so as to make a mistake in determining the non trivial subgroup of the group. Figure 5 presented one example of student concept mistakes.

Figure 5. Example of conceptual error.

The question was “If the inverse of each element in a group is its own element, then proof that the group is commutative.” In Figure 5, the concept \((ab)^{-1} = ab\) written by the students was in accordance with the purpose of the question, but the work showed that the students did not understand the concept of commutative group proving.

3.5. The relation between the types of error and the students’ performance level
To determine whether there was a difference in the percentage of frequency of errors committed by high, moderate, and low group of students, the researcher grouped the students into the three categories based on theory [36]. Next was calculated percentage of error frequency that the student committed on the three types of error categories with the following results.

| Type of Error   | High Level | Moderate Level | Low Level |
|----------------|------------|----------------|-----------|
|                | n   | %        | n   | %        | N   | %        |
| Careless       | 8  | 42.11    | 18  | 27.27    | 16  | 39.02    |
| Computational  | 4  | 21.05    | 4   | 6.06     | 3   | 7.32     |
| Conceptual     | 7  | 36.84    | 44  | 66.67    | 22  | 53.66    |
| Total          | 19 | 100      | 66  | 100      | 41  | 100      |

Based on the results of student response that have been classified based on high, moderate, and low performance levels, it was found that the students with high performance mostly conducted careless errors, followed by conceptual error, and computational error. Meanwhile, the students with moderate and low performance level, made errors mostly in conceptual error, followed by careless error, and computational error. Furthermore, from the proportion of errors made, it was found, that the low and moderate competence students were made more conceptual errors than the high competence students, but, for the computational errors, the pattern was opposite. It was found that computational errors occurred more often for the low and moderate level students. As for careless error, it was revealed that the students with high ability committed careless error more than the students with moderate and low ability.

4. Discussion
This study aimed at gaining better understanding of the errors made by students when solving abstract algebra problems. From the three types of errors according to [29], it was found that the most dominant error in solving abstract algebra problems was conceptual and careless errors while the least
mistake was computational error. To clarify our findings about the types of mistakes made by the students in each performance level, it was presented in Figure 6 below.

![Figure 6. Types of Errors for Different Performance Level.](image)

From the three types of errors made by students, it was found that conceptual errors were the most commonly types of errors occured among high, medium, or low-competence students (see Figure 6), it was followed by careless and computational errors. Based on our findings, the student misconceptions was varied. In question number 1, for example, in addition to the errors as explained in Figure 2, another misconception that also happened was that students still used an arbitrary example in the evidentiary issues, as in proving question items. The concept errors in other questions related to students' lack of understanding on the use of the theorems in the subgroup proof of matrix form, the permutation in the form of cyce, the cyclic group proof, and their relation to finding non trivial subgroups of the $\mathbb{Z}_n$ group. From these findings, students were, in fact, still found difficulties in understanding the concepts of abstract algebra, whereas this kind of problem had often been practiced in lecturing class. The concept errors were experienced by many students who were in a transitional position from arithmetic thinking to abstract thinking as revealed by [37]. Further [37] urged that this misunderstanding could hamper their learning and performance in the course as experienced by students in this study.

Furthermore, in the category of careless error, the analysis results revealed that most students tend to be less thoroughly in copying the problems and were not following the directions of the question. For example, in problem 2, the matrix elements written by the students in the solution were not in accordance with the question (see Figure 2). Also, on question number 3 where the student was asked to determine the outcome of $\alpha^{-1} \beta \alpha$ with $\alpha = (1 \ 3 \ 5)(1 \ 2)(1 \ 3); \beta = (1 \ 5 \ 7 \ 9)$, student compiled the cycle in the wrong order, and on item 4 when the student asked to determine all the generators of the cyclic group $\mathbb{Z}_{11}$, the student completed for one generator only. The error was classified in careless error because it was done by students with high performance leverl and the result of the written completion was true. So, the indication of it, the students were less thoroughly that make them failed to follow the correct instruction of test item. Another careless mistake that also happened was in writing down the negative sign of the inverse result of the matrix and writing down the number as shown in Figure 7 below.
Figure 7. The students errors in writing the numbers.

Figure 7 show that the students solved the problem correctly but made mistake copying numbers. He should had written 29 instead of 28 as a generator. According to [38], [39], and [40], careless mistakes in mathematics occured when the student was basically knowing to perform the correct computation but due to distractions, boredom, or a lapse of attention, they made careless errors. The results of research on the analysis of computational errors in the use of division algorithms found that the most common error type was careless error [39].

Then, the least error type in solving abstract algebra problem in this research was error counting. [41] mentioned that in mathematics, the ability to count was not the main focus because the primary concern in mathematics learning was to train students to think systematically. However, calculations in various forms still had an important role in the learning of mathematics and in solving problems in this life. Similarly, in this abstract algebra course, counting was not the main focus, but the ability to calculate was required as a prerequisite in proofing the main group in the matrix group, the $\mathbb{Z}_n$ group, or the permutation group. The intended calculation was not limited to arithmetic calculations but also the calculation of matrix form, cycle form, and calculation with module rules in group $\mathbb{Z}_n$. Our findings, the student errors in the most calculations occured in the process of matrix multiplication and modulo form. This became the an important information that the student had not mastered the prerequisite material in the abstract algebra course related to the theory of number and linear algebra.

The analysis results, as presented in Figure 6, obtained information that high-competence students tend to make careless and computational errors more than students of medium and low ability. Conversely, conceptual errors were mostly done by moderate and low-ability students compared to high-level students. Meanwhile, overall, conceptual errors were the most common mistakes at all levels of ability. The findings in this study indicated that the learning activities in abstract algebra courses should be really focused on understanding the basic concept first, especially in the proof method that the students had no longer using an arbitrary examples and can use their reasoning in the proofing materials. In addition, as was suggested by [42] it was very important to pay special attention to students who had not mastered prerequisite materials and often performed carelessness action.

5. Conclusion

Based on the three types of errors made by students in completing the task of abstract algebra, it was found that the most dominant errors made by the students were conceptual error, followed by careless error, and computational error with the percentage of 57.94%, 33.33 %, and 8.73% respectively. The results of the analysis based on the level of student ability, it was found that the students with high competence mostly conducted careless error, then conceptual error, and computational error. The moderate and low-ability students, errors occured most often in conceptual error, followed by careless error, and computational error. Furthermore, based on the proportion of errors, the students with high competence performed careless error and computational error more than the moderate and low-ability students. Meanwhile, for conceptual error, the pattern was the opposite where the student with
moderate and lower ability performed conceptual errors more than the high-competence students. The concept of errors related to the students’ lack of understanding so they could not solve the problems properly. Meanwhile, careless errors occurred in copying the test item, un-following the correct directions, as well as putting wrong number. Then, the calculation errors mostly occurred in the matrix multiplication process.

The results of our study provide basic information for further research into the possible factors causes of students’ difficulties in solving abstract algebra tasks. To find out the causes of students’ difficulties in solving abstract algebra tasks, it is also important to check the facilities as well as the opportunity that have been given to the students to dealing with these type of tasks. Investigating this case, will be our next step in the PKPT research.

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