Rare decays of $\eta$-meson: the background of $\rho$- and $\omega$-mesons

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Abstract

The problem of $\rho$- and $\omega$-mesons contributions to the background for $\eta \to \pi^0 e^+ e^-$ decay and to the asymmetries in the decays $\eta \to \pi^+ \pi^- \pi^0$ and $\eta \to \pi^+ \pi^- \gamma$ is considered.

1 Introduction

The reaction $pp \to pp\eta$ at the proton energy $E_p = 2.3 GeV$ ($\sqrt{s} = 2.47 GeV$) has the cross section about $5 \mu b$ [1, 2]. Then for the luminosity of WASA at CELSIUS about $10^{32} cm^2/sec$ [1, 2] 500 $\eta$-mesons will be produced every second. This means that the reaction $\eta \to \pi^0 e^+ e^-$ with the expected branching ratio

$$Br(\eta \to \pi^0 e^+ e^-) \sim 10^{-9}$$ [3, 4]
and the asymmetries in the reactions $\eta \to \pi^+\pi^-\pi^0$ and $\eta \to \pi^+\pi^-\gamma$ with the expected level

$$A(\eta \to \pi^+\pi^-\pi^0) \sim 10^{-5}$$

$$A(\eta \to \pi^+\pi^-\gamma) \sim 10^{-5}$$

will become accessible for investigations after a year of running (approximately $10^{10}$ $\eta$'s).

Taking into account the expected level of the effects there should be the strong restrictions on the background processes. In this note we are considering the background decays of $\rho$- and $\omega$- mesons that can be produced in the $pp$-collisions at $E_p = 2.3\text{GeV}$ under the threshold. The relevant parameters of $\rho$- and $\omega$- mesons decays are the following

$$\text{Br}(\rho \to \pi^0 e^+ e^-) \approx 4 \times 10^{-6}$$

$$\text{Br}(\omega \to \pi^0 e^+ e^-) \approx 6 \times 10^{-4}$$

$$\text{Br}(\omega \to \pi^+\pi^-\pi^0) \approx 0.9$$

$$\text{Br}(\rho \to \pi^+\pi^-\gamma) \approx 10^{-2}$$

In the next section we will evaluate the cross sections of $\rho$- and $\omega$- mesons production and the corresponding background in the considered decay channels of $\eta$- meson.

## 2 Under the threshold production of $\rho$- and $\omega$- mesons

We will evaluate the cross sections of $\rho$- and $\omega$- mesons production in the region of invariant masses ($\sqrt{q^2}$) near the mass of $\eta$- meson ($m_\eta \pm \Delta m$, $\Delta m = 10\text{MeV}$). We will describe the production of $\rho$- and $\omega$- mesons by the diagrams Fig.1 where the vertices of $pp$- interaction and vector meson emission are considered as constants, i.e. independent on the energy of the incident proton ($E_p$) and the invariant mass of emitted meson ($\sqrt{q^2}$).

Then the production cross section is defined by the accessible phase space factors, the resonance form of $\rho$- and $\omega$- mesons propagators and the $q^2$- dependence of $\rho$- and $\omega$-mesons decay widths. Let us therefore write the differential production cross section in the following form

$$\frac{d\sigma_V}{dq^2} = \sigma_V(E_p, q^2) \frac{1}{((q^2 - m_V^2)^2 + m_V^4 \Gamma_V(q^2)^2)} \frac{m_V \Gamma_V(q^2)}{\pi},$$

where $\sigma_V(E_p, q^2)$ is the production cross section of a particle with invariant mass $\sqrt{q^2}$ that depends on $E_p$ and $q^2$ only due to accessible phase space (approximation), $\Gamma_V(q^2)$ is the $q^2$- dependent width of the resonance.
In order to get the desirable cross sections of $\rho$- and $\omega$-mesons production the differential cross section (2.1) should be integrated over $q^2$ in the region $(m_\eta - 5\text{MeV})^2 < q^2 < (m_\eta + 5\text{MeV})^2$. The unknown constant in $\sigma_V(E_p, q^2)$ can be found using experimental data on $\rho$- and $\omega$-mesons production at energies $E_p$ above the threshold (see later).

Let us now describe the $q^2$-dependence of $\rho$- and $\omega$-mesons decay widths $\Gamma_V(q^2)$. For $\rho$-meson we will take into account the two particle phase space ($\rho \to \pi\pi$ is the main decay channel of $\rho$-meson) and the $p$-wave character of $\rho \to \pi\pi$ decay

$$\Gamma_\rho(q^2) \sim \sqrt{q^2 - 4m_\rho^2} \pi(q^2 - 4m_\rho^2).$$

(2.2)

For $\omega$-meson the main decay channel is $\omega \to \pi\pi\pi$. The decay width can be calculated numerically using the following form of matrix element (uniquely determined)

$$M(\omega \to \pi\pi\pi) \sim \epsilon_{\mu\alpha\beta\gamma} p_1^\mu p_2^\nu p_3^\alpha p_3^\beta \frac{1}{q^3},$$

(2.3)

where $\epsilon_\mu$ is the polarization vector of $\omega$-meson and $p_i$ are the momenta of pions.

The $\rho$-meson production cross section is known at $\sqrt{s} = 3.5\text{GeV}$ \cite{7} and is equal to $\approx 80\mu b$. Recalculating it at the $\sqrt{s} = 2.47\text{GeV}$ and $(m_\eta - 5\text{MeV})^2 < q^2 < (m_\eta + 5\text{MeV})^2$ we get

$$\sigma_\rho \approx \frac{80}{2 \times 10^9} = 4 \times 10^{-4}\mu b.$$

The $\omega$-meson production cross section is known at $\sqrt{s} = 3.08\text{GeV}$ \cite{8} and is equal to $\approx 80\mu b$. Recalculating it at the $\sqrt{s} = 2.47\text{GeV}$ and $(m_\eta - 5\text{MeV})^2 < q^2 < (m_\eta + 5\text{MeV})^2$ we get

$$\sigma_\omega \approx \frac{80}{5 \times 10^6} = 1.6 \times 10^{-5}\mu b.$$

Using the calculated cross sections and the branching ratios of $\rho$- and $\omega$-decays mentioned in the Introduction we get $\rho$- and $\omega$- backgrounds for $\eta \to \pi^0 e^+ e^-$ decay

$$\frac{\sigma_\rho Br(\rho \to \pi^0 e^+ e^-)}{\sigma_\eta Br(\eta \to \pi^0 e^+ e^-)} = 0.32$$

and the following background in the asymmetries \cite{9} of $\eta \to \pi^+ \pi^- \pi^0$ and $\eta \to \pi^+ \pi^- \gamma$-decays

$$A_\omega(\eta \to \pi^+ \pi^- \pi^0) = \frac{2\pi \Gamma_\eta \sigma_\omega Br(\omega \to \pi^+ \pi^- \pi^0)}{\Delta m \sigma_\eta Br(\eta \to \pi^+ \pi^- \pi^0)} \approx 10^{-4}$$

$$A_\rho(\eta \to \pi^+ \pi^- \gamma) = \frac{2\pi \Gamma_\eta \sigma_\rho Br(\rho \to \pi^+ \pi^- \gamma)}{\Delta m \sigma_\eta Br(\eta \to \pi^+ \pi^- \gamma)} \approx 10^{-4}.$$
Here $\Delta m$ is equal to 10$MeV$. The asymmetries in both cases are due to the charged pions in the background amplitudes have the opposite symmetry under the interchange of momenta comparing to the charged pions from the $\eta$-decays amplitudes and the amplitudes of the process and of the background interfere.

3 Conclusion

So, we have considered the problem of $\rho$- and $\omega$- mesons contributions to the background for $\eta \rightarrow \pi^0 e^+ e^-$ decay and to the asymmetries in the decays $\eta \rightarrow \pi^+ \pi^- \pi^0$ and $\eta \rightarrow \pi^+ \pi^- \gamma$. We have found that the background due to $\rho$- and $\omega$- mesons is comparable with the expected signal from $\eta$- meson decay. For the case of $\eta \rightarrow \pi^0 e^+ e^-$ decay the background can be lowered by lowering $\Delta m$ (better energy resolution) or by selecting the events with the invariant mass of $e^+ e^-$ pair in the region where the decays of vector mesons do not substantially contribute. In the case of asymmetries the first way (lowering of $\Delta m$) will not change the situation because of the effect (interference with the background) does not depend on $\Delta m$. So, in this case there should be looked for the regions on the Dalitz plot where the ratio of the signal to background is higher. Now this work is in progress.

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Figure caption Fig.1 Diagrams of $\rho$ and $\omega$ production