Profile of Metacognition of Mathematics Pre-Service Teachers in Understanding the Concept of Integral Calculus with Regard Gender Differences

L Misu\textsuperscript{1,2}, I K Budayasa\textsuperscript{2,3}, and A Lukito\textsuperscript{2,5}

\textsuperscript{1}Department of Mathematics Education Universitas Halu Oleo, Kendari-Indonesia
\textsuperscript{2}Department of Mathematics Universitas Negeri Surabaya-Indonesia

Email: \textsuperscript{3)lamisuhamid@yahoo.co.id}, \textsuperscript{4)ketutbudayasa@unesa.ac.id}, \textsuperscript{5)agunglukito@unesa.ac.id}

Abstract. This research is to describe metacognition profile of female and male mathematics’ pre-service teachers in understanding the concept of integral calculus. The subjects of this study are one female and 1 male mathematics’ pre-service teachers who have studied integral calculus. This research type is an explorative study with the qualitative approach. The main data collection of this research was obtained by using Interview technique. In addition, there are supporting data which is the result of the written work of research subjects (SP) in understanding the question of integral calculus. The results of this study are as follows: There is a difference in metacognition profiles between male and female mathematics’ pre-service teachers in the understanding concept of integral calculus in the interpreting category, especially the definite integral concept. While in the category of exemplifying, there is no difference in metacognition profile between male and female mathematics’ pre-service teachers either the definite integral concept and the indefinite integral concept.

1. Introduction
The metacognition profile is a natural and an intact description of a person's cognition that involves his own thinking in terms of using his knowledge, and the ability to plan and monitor his thinking process, and evaluate the process and outcome of one's thinking when understands a concept. According to Koriat, (in Goh [7] that Metacognition refers to what people know about cognition in general and about their own cognitive processes, in particular, as well as how they use this knowledge to adjust their informational processes and behavior. While the cognitive notion as described by Gagne [6] is a process that occurs internally within the central nervous system at a time when people are thinking. Then Neisser [12] says that cognition is the acquisition, arrangement, and use of knowledge.

The concept examined in this paper is the concept of integral calculus, consisting of the definite integral concept and indefinite integral concept. To understand the concept of integral calculus using the Bloom theory developed by Anderson et al. [1], and Mayer [11]. There are 7 categories to understanding the concept of integrals based on the theory, namely: Interpreting, Exemplifying, Classifying, Summarizing, Inferring, and Explaining. This paper discusses only two categories of conceptual understanding, i.e: Interpreting and Exemplifying. The interpreting category is defined as changing from
one form of representation to another, while the exemplifying category is defined as finding a specific example or illustration of a concept or principle [11].

The metacognition profile in understanding the concept of integral calculus, especially the category of interpreting and exemplifying can be seen from 2 components of metacognition, i.e. cognitive knowledge consists of declarative knowledge, procedural knowledge, and conditional knowledge (Flavell [5]), and metacognition skills consisting of planning, monitoring and evaluating (Dawson [4] and Livingston [9]).

The students’ metacognition profile review distinguishes between male and female mathematics’ pre-service teachers. Thus, the problems in this research are (1) how the metacognition profile of female mathematics’ pre-service teachers in understanding the concept of integral calculus, and (2) how the metacognition profile of male mathematics’ pre-service teachers in understanding the concept of integral calculus.

2. Method
This type of research is an explorative study with the qualitative approach. The subjects of this study are mathematics’ pre-service teachers who are studying integral calculus, consisting of 1 female and 1 male. Selection of the subject of this study based on the highest score on the Mathematics Ability Test (a score of at least 70), and consider the value of the Student Achievement Index. The main data collection of this research was obtained by using interview technique. In addition, there are supporting data which are the result of written work of research subjects in understanding the question of integral calculus task.

3. Results and Discussion

3.1. The Results of the male mathematics’ pre-service teachers interviews about understanding the concept of integral calculus in the category of interpreting

3.1.1. The Indefinite Integral Concepts
The result of male mathematics’ pre-service teachers interview in understanding the concept of indefinite integral by interpreting the form of narration to the form of indefinite integral notation as follows: (1) The male mathematics’ pre-service teachers can interpret the form “The first derivative function $F(x)$ to $x$ is $f(x)$” into the indefinite integral notation “$\int f(x)\,dx = F(x) + C$”, (2) The male mathematics’ pre-service teachers can argue that “an indefinite integral is the opposite of a derivative, so it is called anti-derivative, (3) The male mathematics’ pre-service teachers may use the procedure to obtain indefinite integral notation based on the definition of anti-derivative (the indefinite integral), (4) The male mathematics’ pre-service teachers can determine the hook element of the indefinite integral concept of the derivative, since the indefinite integral is the opposite of a derivative, (5) The male mathematics’ pre-service teachers can explain easily in interpreting the narrative form to an indefinite integral notation, and (6) The male mathematics’ pre-service teachers can confirm the interpretation of the narrative form to the indefinite integral notation “$\int f(x)\,dx = F(x) + C$” is true.

Based on the interview result of male mathematics’ pre-service teachers, the male mathematics’ pre-service teachers have used metacognition knowledge and metacognition skills in understanding the concept of the indefinite integral in the category of interpreting.

3.1.2. The Definite Integral Concepts
Based on the results of male mathematics’ pre-service teachers interviews in understanding the concept of the definite integral by interpreting the image form to the form of the definite integral notation as follows: (1) The male mathematics’ pre-service teachers can interpret the image form “The region bounded by the curve $y=f(x)$, the lines $x=a$, $x=b$ and the $x$-axis” to the definite integral notation
L = \int_a^b f(x)dx, \quad (2)\text{ The male mathematics’ pre-service teachers can not give a reason that the definite integral notation from “The region bounded by the curve } y=f(x)\text{, the lines } x = a, x = b \text{ and the } x\text{-axis” is } L = \int_a^b f(x)dx, \quad (3)\text{ The male mathematics’ pre-service teachers were not able to use the procedure to obtain an the definite integral notation based on the definition of Riemann sums, (4) The male mathematics’ pre-service teachers can not determine the element of the hook of the definite integral concept of } \lim_{\Delta x \to 0} \sum f(x_i)\Delta x, \quad (5)\text{ The male mathematics’ pre-service teachers find it difficult to explain the steps to calculate the region bounded by the curve } y=f(x), \text{ the lines } x = a, x = b \text{ and the } x\text{-axis” becomes an the definite integral notation } L = \int_a^b f(x)dx, \text{ and (6) The male mathematics’ pre-service teachers do not have the procedures to obtain an the definite integral notation based on the definition of Riemann sums.}

Based on the interview result of male mathematics’ pre-service teachers, then the male mathematics’ pre-service teachers can only use knowledge of declarative metacognition to understand the definite integral concept of the category of interpreting.

3.2. The Results of the female mathematics’ pre-service teachers interview about understanding the concept of integral calculus in the category of interpreting

3.2.1. The Indefinite Integral Concepts
The results of female mathematics’ pre-service teachers interviews in understanding the concept of indefinite integrals by interpreting the form of narration to the form of the indefinite integral notation as follows: (1) The female mathematics’ pre-service teachers can interpret the form ”The first derivative function } F(x) \text{ to } x \text{ is } f(x)” into the indefinite integral notation”\int f(x)dx = F(x)+C”, (2) The female mathematics’ pre-service teachers can give the reason that “Dx[F(x)+C] = f(x)means an indefinite integral is the opposite of a derivative, (3) The female mathematics’ pre-service teachers may use the procedure for obtaining indefinite integral notation based on the definition of anti-derivative (the indefinite integral), (4) The female mathematics’ pre-service teachers can define the relationship between the concept of indefinite integral and the derivative of the indefinite integral as the opposite of the derivative, (5) The female mathematics’ pre-service teachers can explain easily in interpreting the narrative form to an indefinite integral notation, and (6) The female mathematics’ pre-service teachers can confirm that the interpretation of the narrative form to the indefinite integral notation is “\int f(x)dx = F(x)+C” is true.

Based on the interview result of female mathematics’ pre-service teachers, the female mathematics’ pre-service teachers have used metacognition knowledge and metacognition skills in understand the indefinite integral concepts in the interpreting category.

3.2.2. The Definite Integral Concepts
Based on the results of female mathematics’ pre-service teachers interviews in understanding the concept of the definite integral by interpreting the image form to the form of the definite integral notation as follows: (1) The female mathematics’ pre-service teachers can interpret the image form “the region bounded by the curve } y=f(x)\text{, the lines } x = a, x = b \text{ and the } x\text{-axis” to the definite integral notation } L = \int_a^b f(x)dx = \lim_{\Delta x \to 0} \sum f(x_i)\Delta x, \quad (2)\text{ The female mathematics’ pre-service teachers can give the reason that the definite integral notation from } ”\text{ the region bounded by the curve } y=f(x), \text{ the lines } x = a, x = b \text{ and the } x\text{-axis” is } L = \int_a^b f(x)dx. (3)\text{ The female mathematics’ pre-service teachers are not perfect using procedures to obtain the definite integral notation based on the definition of Riemann sums, (4) The female mathematics’ pre-service teachers can determine the hook element of the definite integral}
concept that is \( \lim_{n \to \infty} \sum_{i=1}^{n} f(\xi_i) \Delta x \), (5) The female mathematics’ pre-service teachers explains the steps of calculating the area of “the region bounded by the curve \( y=f(x) \), the lines \( x=a, x=b \) and the \( x \)-axis” being the definite integral notation \( L=\int_{a}^{b} f(x) \, dx \), and (6) The female mathematics’ pre-service teachers master the procedure for obtaining definite integral notation based on the definition of Riemann sums.

Based on the interview result of female mathematics’ pre-service teachers, then the female mathematics’ pre-service teachers can use metacognition knowledge and metacognition skills in understanding the definite integral concept of the category of interpreting, except the knowledge of Conditional.

3.3. The Results of the male mathematics’ pre-service teachers interview about understanding the concept of integral calculus in the category of exemplifying

3.3.1. The Indefinite Integral Concepts
The result of interviews of male mathematics’ pre-service teachers in giving examples of the indefinite integral as follows: (1) The male mathematics’ pre-service teachers can exemplify the indefinite integrals based on indefinite integral definitions, is \( \int (2x+3) \, dx = x^2 + 3x + C \), (2) The male mathematics’ pre-service teachers may argue that "the indefinite integral is the inverse of the derivative” so that the derivative \( x^2 + 3x + C \) is \( 2x + 3 \), (3) The male mathematics’ pre-service teachers can use the procedure to obtain \( x^2 + 3x + C \) ie \( \int (2x+3) \, dx = \frac{2}{1+1} x^{1+1} + \frac{3}{0+1} x^{0+1} + C \), (4) The male mathematics’ pre-service teachers can determine the characteristics of the indefinite integral example ie if it is derived to produce the original function in the added number of constants, (5) The male mathematics’ pre-service teachers can provide other examples in addition to polynomial functions namely trigonometric functions, exponential functions, and logarithmic functions, and (6) The male mathematics’ pre-service teachers are only able to give an example of a simple indefinite integral. While the examples are complicated, for example: \( \int \ln x \, dx \) and \( \int \log x \, dx \) cannot be completed.

Based on the interviews of male mathematics’ pre-service teachers, then the male mathematics’ pre-service teachers can only use metacognition knowledge and metacognition skills in giving an example of a simple indefinite integral.

3.3.2. The Definite Integral Concepts
The result of interviews of male mathematics’ pre-service teachers in giving examples of the definite integral as follows: (1) The male mathematics’ pre-service teachers can provide an definite integral example based on the fundamental theorem of calculus, i.e. \( \int_{0}^{4} (2x-1) \, dx = 12 \), (2) The male mathematics’ pre-service teachers may argue that "the definite integral is necessarily the region bounded by the function \( f(x) = 2x-1 \), the lines \( x = 0, x = 4 \) and the \( x \)-axis” So the breadth is 12 unit area, (3) The male mathematics’ pre-service teachers can use the procedure to obtain the region bounded by the functions of \( f(x) = 2x - 1 \), the lines \( x = 0, x = 4 \) and the \( x \)-axis, ie \( \int_{0}^{4} (2x-1) \, dx = (4^2 - 4) - (0^2 - 0) = 12 \), (4) The male mathematics’ pre-service teachers have not been able to determine the sample characteristics of a definite integral, (5) The male mathematics’ pre-service teachers can provide other examples in addition to polynomial functions namely trigonometric functions, exponential functions, and logarithmic functions, and (6) The male mathematics’ pre-service teachers are only able to give an example of a simple definite integration. While the examples are complicated, for example: \( \int_{0}^{4} \ln x \, dx \) and \( \int_{0}^{4} \log x \, dx \) cannot be completed.
Based on the interviews of male mathematics’ pre-service teachers, then the male mathematics’ pre-service teachers can only use metacognition knowledge and metacognition skills in giving an example of a simple definite integral.

3.4. The Results of the female mathematics’ pre-service teachers interview about understanding the concept of integral calculus in the category of exemplifying

3.4.1. The Indefinite Integral Concepts
The result of interviews of female mathematics’ pre-service teachers in giving examples of the indefinite integral as follows: (1) The female mathematics’ pre-service teachers can instantiate indefinite integrals based on the definition of indefinite integrals, i.e. \( \int (x^{4/3})dx = \frac{3}{7}x^{7/3} + C \), (2) The female mathematics’ pre-service teachers may argue that “the indefinite integral is the inverse of the derivative” so that the derivative \( \frac{3}{7}x^{7/3} + C \) is \( x^{4/3} \), (3) The female mathematics’ pre-service teachers can use the procedure to obtain \( \frac{3}{7}x^{7/3} + C \) i.e. \( \int (x^{4/3})dx = \frac{1}{3/4 + 1}x^{4/3 + 1} + C \), (4) The female mathematics’ pre-service teachers can determine the characteristics of the indefinite integral example i.e if it is derived to produce the original function in the added number of constants, (5) The female mathematics’ pre-service teachers can only give examples of indefinite integrals of polynomial functions, root functions, and fractional functions, and (6) The female mathematics’ pre-service teachers are only able to give an example of a simple indefinite integral, while a complex example can not yet.

Based on the interview result of female mathematics’ pre-service teachers, the female mathematics’ pre-service teachers can only use the knowledge and skills of metacognition in giving an example of the simple indefinite integral.

3.4.2. The Definite Integral Concepts
The result of interviews of female mathematics’ pre-service teachers in giving examples of the definite integral as follows: (1) The female mathematics’ pre-service teachers can provide an definite integral example of course based on the fundamental theorem of calculus, i.e. \( \int_2^5 x^2dx = 39 \), (2) The female mathematics’ pre-service teachers may argue that “the definite integral is necessarily the region bounded by the function \( f(x) = x^2 \), the lines \( x = 2, x = 5 \) and the \( x \)-axis” So the breadth is \( 39 \) unit area, (3) The female mathematics’ pre-service teachers can use the procedure to obtain the region bounded by the functions of \( f(x) = x^2 \), the lines \( x = 2, x = 5 \) and the \( x \)-axis, i.e. \( \int_2^5 x^2dx = \frac{1}{3}(5)^3 - \frac{1}{3}(2)^3 = 39 \), (4) The female mathematics’ pre-service teachers have not been able to determine the sample characteristics of an definite integral, (5) The female mathematics’ pre-service teachers have not been able to provide other examples other than polynomial functions, and (6) The female mathematics’ pre-service teachers are only able to give an integral example of a simple one.

Based on the interview result of female mathematics’ pre-service teachers, the female mathematics’ pre-service teachers can only use the knowledge and skills of metacognition in giving an example of simple definite integral.

The results above show that there are differences in metacognition profiles between male and female mathematics’ pre-service teachers in understanding the concept of integral calculus, especially the integral concept of the category of interpreting. The female mathematics’ pre-service teachers are able to explain the reason or procedure for obtaining the definite integral notation through the Riemann sum approach. While the male mathematics’ pre-service teachers are only able to write the definite integral notation, but cannot explain the reason or procedure to obtain of about the definite integral notation. Means the female mathematics’ pre-service teachers are able to analyze an image of the curve region so as to obtain an integral notation. This is not in line with the opinion of Casey [2], Maccoby & Jacklyn
that boys as a group rely on spatial strategies when completing mental-rotation tasks, while girls as groups tend to use verbal strategies to accomplish this task. However, this difference is influenced by motivation and craft factors in learning mathematics. This is as explained by Mitsos and Browne (in Haralambos and Holborn [8]) that female have better learning achievement rates than male, as females are more motivated and work more diligently than male in doing school work.

Then, in the exemplifying category, there is no difference in metacognition profile among male and female mathematics’ pre-service teachers. However, the examples given for both the concept of indefinite integral and definite integrals are of course only polynomial functions. While other functions, such as trigonometric functions, root functions, and exponential functions give a simple example. This shows that the knowledge of mathematics’ pre-service teachers about integral calculus, especially the Exemplifying category is still low.

4. Conclusion
Based on the results of research and discussion above, it can be concluded that:

In the category of interpreting, the male mathematics’ pre-service teachers can use metacognition knowledge and metacognition skills in understanding the concept of indefinite integrals, while the concept of the definite integral only uses declarative knowledge. While in the exemplifying category, the male mathematics’ pre-service teachers can use metacognition knowledge and metacognition skills to give examples of the indefinite integral and the definite integrals, but only a simple example;

In the category of interpreting, the female mathematics’ pre-service teachers have used metacognition knowledge and metacognition skills in understanding the concept of the indefinite integral and the definite integral, except the conditional knowledge of the concept of the definite integral. While in the exemplifying category, the female mathematics’ pre-service teachers can use metacognition knowledge and metacognition skills to give examples of the indefinite integral and the definite integral, but only a simple example;

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