Improved edge detection based on fractional derivatives for real-time measurement systems

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Since edges are one of the most commonly used recognition features in object detection and image segmentation, their robust and clear recognition as well as the calculation in real time is important for most applications. It could be shown that the fractional filter CRONE leads to significantly better results in noisy images than classical filters such as Sobel or Prewitt, especially under the condition of real-time capability. Unfortunately, the CRONE filter is a one-dimensional filter and only leads to good results if its filter direction is perpendicular to the edge. Therefore, this paper presents the novel approach CRONE2D, the extension of the CRONE for surface input data like images, with much better results in detection of arbitrarily oriented edges.

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1 Introduction

Most object detection and segmentation algorithms are based on the edge-feature. Unfortunately, the quality of outdoor pictures varies greatly, depending on the weather, lightning and environmental conditions like rain, fog, sun or dust. The Track Measurement System (TMS) for light rail vehicles, developed by the Chair of Dynamics and Mechanism Design of the Technische Universität Dresden, deals with all the problems above. Hence, an extension of the standard CRONE (Contour Robust d’Ordre Non Entier)-operator [1] called eCRONE [2] was developed for a more robust edge filtering-process under real-time conditions. This extension leads to much better results for noisy images compared to the established CRONE-, SOBEL- or PREWITT-operator. Unfortunately, this also leads to a heavy smearing effect for lines which are not perpendicular to the filtering direction and the loss of interpretability of the filtering direction as the direction of fractional derivation for surfaces. Therefore, this paper introduces the novel approach CRONE2D, as the extension of the standard CRONE-operator for the second dimension.

2 Edge detection based on fractional derivatives

2.1 Preliminaries

The CRONE-operator, introduced by B. Mathieu et al. [1], represents the superposition of the fractional backward and forward derivation based on GRÜNWALD-LETNIKOV [3]

\[
\mathcal{D}_{\nu}^{x} f(x) = \lim_{N \to \infty} \left\{ (\delta_N x)^{-\nu} \sum_{j=0}^{N-1} \frac{\Gamma(j-\nu)}{\Gamma(j+1)} \left[ f(x-j\delta_N x) - f(x) \right] \right\},
\]

where \( \nu \) is the fractional order, \( \delta_N x := \frac{x-a}{N} \), \( N \in \mathbb{N} \), \( \{ a \in \mathbb{R} | a < x \} \) a lower limit [4, p. 28] and \( g_j^{(\nu)} \) the GRÜNWALD-coefficients. For discrete input functions like images \( \mathbf{B}(i,j) \in \mathbb{R}^{J \times J} \), the calculation simplifies to a convolution\( \mathcal{D}_{\nu}^{x} \mathbf{B} = \mathbf{B} \ast \mathcal{D}_{\nu}^{x} \) of the image and the discrete operator

\[
\mathcal{D}_{\nu}^{x} = \begin{bmatrix} g_{mK}^{(\nu)} & \cdots & g_{m}^{(\nu)} & \cdots & 0 & \cdots & -g_{1}^{(\nu)} & \cdots & -g_{m}^{(\nu)} & \cdots & -g_{mK}^{(\nu)} \end{bmatrix} \in \mathbb{R}^{1 \times (2mK+1)},
\]

where \( \mathcal{D}_{\nu}^{y} = \mathcal{D}_{\nu}^{x \top} \) represents the operator for the y-direction. Since the gradient information of the neighboring elements remain unused, C. Telke introduced an extended version called eCRONE [2]

\[
e\mathcal{D}_{\nu}^{(n)} = \begin{bmatrix} \mathcal{D}_{\nu}^{x(nK)} \top & \cdots & \mathcal{D}_{\nu}^{x(n)} \top & \cdots & \mathcal{D}_{\nu}^{x(0)} \top & \cdots & \mathcal{D}_{\nu}^{x(-n)} \top & \cdots & \mathcal{D}_{\nu}^{x(-nK)} \top \end{bmatrix} \top
\]

where \( \nu(n) \) is an arbitrary function for a varying, fractional order of the combined CRONE-operators.

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2.2 CRONE2D

Neither the CRONE-operator nor the eCRONE-operator could be interpreted as a superposed fractional derivative of a surface, since the neighboring elements are not considered at all or are arbitrarily, which leads to the novel approach CRONE2D

\[
\begin{align*}
\mathcal{D}^{\nu}_{\theta} f(x_1, x_2) &= \lim_{\nu \to \infty} \left\{ \frac{\Gamma(\frac{r}{N})^{-\nu} \sum_{j=0}^{N-1} \Gamma(j - \nu) \Gamma(j + 1)}{\Gamma(\nu)} \cos(\varphi + \varphi_0) \right\} \left( f(x_1 - j \cos(\varphi) \frac{r}{N}, x_2 - j \sin(\varphi) \frac{r}{N}) - f(x_1 + j \cos(\varphi) \frac{r}{N}, x_2 + j \sin(\varphi) \frac{r}{N}) \right) \right\} d\varphi,
\end{align*}
\]

where \( r = \sqrt{(x_1 - a_1)^2 + (x_2 - a_2)^2} \) and \( e_\theta = e_x \cos(\varphi_0) + e_y \sin(\varphi_0) \) is the derivation-direction. The discrete operator is defined as

\[
\mathcal{D}^{\nu}_{\theta} f(x_1, x_2) = \left\{ -g^{(e)}_{\nu, n m} \cos(\varphi_{n m} + \varphi_0), \quad \forall \quad r \neq 0 \right\} \left\{ -g^{(e)}_{\nu, n m} \cos(\varphi_{n m} + \varphi_0), \quad \forall \quad r = 0 \right\}, \quad \mathcal{D}^{\nu}_{\theta} f(x_1, x_2) \in \mathbb{R}^{I \times J}
\]

where \( n = \frac{I + 1}{2} - i \), \{ \( I \mid \exists k \in \mathbb{N} : I = 2k - 1 \} \), \( m = j - \frac{J + 1}{2} \), \{ \( J \mid \exists k \in \mathbb{N} : J = 2k - 1 \} \), \( r = \sqrt{m^2 + n^2} \) and \( \varphi_{n m} \) is the angle between the derivation-direction and the matrix element \( \mathcal{D}^{\nu}_{\theta} f(x_1, x_2) \) with respect to the center of the matrix \( \mathcal{D}^{\nu}_{\theta} f(x_1, x_2) \).

2.3 Numerical example

![Fig. 1: Comparison of the CRONE2D ()] with the eCRONE () by two noisy gray-value images (step height: 20 bit; step width: 7 px; noise: normal distributed with \( \sigma = 50\% \) of the step height). Both filter matrices are normalized such that each satisfies the condition \( \sum_{i=1}^{I} \sum_{j=1}^{J} |F_{i,j}| = 1 \). a.2) and b.2) illustrate the section through the center of the picture, perpendicular to the gray-value step (dashed line).

In comparison of the CRONE2D and the eCRONE through a gray-value image with a line perpendicular to the filtering direction (Figure 1.a), both leads to a clear recognition of the edge. In contrast, the results are very different for the same filter matrices applied on a 45° rotated line (Figure 1.b). Here, the novel CRONE2D-operator still leads to a clear and sharp edge, while the peak of the eCRONE-operator is barely higher than the noise.

3 Conclusion

This paper introduced extension CRONE2D of the CRONE-operator for two-dimensional input functions like images. It was shown that the novel operator leads to much more clear and sharp edge detection for normal distributed noisy images compared to the eCRONE-operator, especially for lines or more commonly curves which are not perpendicular to the search direction. In addition, not only the discrete filter matrix but also the calculation for continuous input-functions was presented.

References

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