Hypoelastic-plastic-damage analysis of a tunnel in swelling rock mass

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Abstract. In this study, we proposed a multi-stage analysis process to simulate the mechanical behaviour of a tunnel constructed in swelling rock mass, which including self-weight analysis, excavation analysis and swelling analysis. Especially, in the swelling analysis, a hypoelastic-plastic-damage model was constructed which can express the stiffness deterioration due to the damage that occurs with the process of rock swelling, and the plastic softening and hardening behaviour based on the Modified Cam-Clay (MCC) model. Furthermore, an implicit stress-update algorithm was formulated for the swelling model to obtain an accurate and robust numerical computation. A series of numerical examples of a tunnel in swelling rocks is shown in this study to discuss the mechanical behaviour of the tunnel due to the swelling of the bedrock, aimed at evaluating the stability of a tunnel constructed in swelling rocks after some years.

1. Introduction

In recent years, a lot of tunnels in Japan have been destroyed after the completion of construction. The cause of such a serious problem is that the swelling clay minerals, called smectite, contained in the rock mass, which gives an excessive load to the structure of tunnel in the process of water absorption and swelling. The deformation of tunnel, such as uplift of the invert and extrusion of the side wall, caused by swelling minerals will affect the normal operation and endanger the structural safety seriously. In order to take measures against the swelling deformation rationally, it is an urgent issue to establish a rational prediction evaluation method for this phenomenon.

There are many model experiments and numerical analysis studies conducted to deal with the fracture of tunnel inverts, such as Kobayashi H (2019) et al. and Okui Y (2009) at al., however, no methods have been established that can evaluate stress and deformation quantitatively and accurately due to swelling of smectite by the process of water absorption. As so far, most of the analysis for tunnels where deformation has occurred in Japan are conducted using the "ground deterioration model" proposed by Matsunaga T et al. (2005, 2009) and Yashiro K et al. (2009). The "ground deterioration model" expresses the deterioration of the ground over time by reducing the strength of the ground from the peak strength to the residual strength with a curve of strength reduction assumed in advance. That is to say it expresses the deformation of tunnel only by plasticization effect, not considering the swelling effect itself.

In order to evaluate the mechanical behaviour of tunnel in swelling rock mass, we propose a hypoelastic-plastic-damage model for the swelling behaviour of rocks and formulate an implicit stress-update algorithm. The constitutive model is able to express softening and hardening behaviour, based on the Modified Cam Clay (MCC) model, and deterioration of stiffness (damage) along with the process of swelling. And the swelling characteristics of the smectite, based on laboratory experiment data, are introduced into the constitutive model. This study contributes to the prediction of the...
nonlinear mechanical response of the tunnel constructed in swelling rock mass, which contains the stress redistribution of the tunnel after excavation and the soft/hard/damage response caused by the swelling process.

2. Numerical analysis method

2.1. Analysis process

In this study, in order to evaluate the mechanical behaviour of tunnels excavated in swelling rock mass accurately, multi-stage analysis consisting of self-weight analysis, excavation analysis, and swelling analysis is conducted. And in the self-weight analysis and excavation analysis, the MCC model is applied.

2.2. Construction of swelling model

The total strain tensor $\varepsilon$ is decomposed into three parts, as the elastic strain $\varepsilon^e$, plastic strain $\varepsilon^p$ and the swelling strain $\varepsilon^s$, respectively.

$$\varepsilon = \varepsilon^e + \varepsilon^p + \varepsilon^s$$  \hspace{1cm} (1)

From the swelling experiment data of montmorillonite sample, a typical swelling clay mineral of smectite group, the typical swelling characteristic is shown in the left part of figure 1. It can be seen that the important swelling characteristic of smectite minerals is that the lower the effective stress state is ($p_s < p < p_c$), the larger the volumetric swelling strain will be ($\varepsilon_{s_{\text{max}}} > \varepsilon^s > \varepsilon^s_{\text{max}}$). $p_s$ and $p_c$ are the reference mean stress and the critical maximum mean stress for the occurrence of swelling, respectively. $\varepsilon_{s_{\text{max}}}$ and $\varepsilon^s_{\text{max}}$ are the maximum volumetric swelling strain that swelling minerals can generate under the reference mean stress state $p_s$ or mean stress state $p_c$, respectively. As a lot of relationships between the mean stress and the maximum volumetric swelling strain ($\varepsilon^s_{\text{max}}$) is plotted in $p - \varepsilon^s_{\text{max}}$ space, we can model the relationship of $p - \varepsilon^s_{\text{max}}$ as shown in the right of figure 1. That is to say that the maximum volumetric swelling strain $\varepsilon^s_{\text{max}}$ is dependent on mean stress state.

Although the details of our experiment will not be shown in this paper, there are many previous researches shows the similar relationship, such as Grob H (1972) and Thomas M (2013) et al.

![Figure 1. The relationship of the maximum volumetric strain and mean stress](image)

Therefore, the maximum volumetric swelling strain $\varepsilon^s_{\text{max}}$ can be expressed as follows.

$$\varepsilon^s_{\text{max}}(p) = \begin{cases} \frac{\ln p_s - \ln p}{\ln p_s - \ln p_{c_s}} \varepsilon^s_{\text{max}}(p_s) & (p \leq p_s) \\ \frac{\ln p - \ln p_{c_s}}{\ln p_s - \ln p_{c_s}} \varepsilon^s_{\text{max}}(p_s) & (p > p_s) \end{cases}$$  \hspace{1cm} (2)
In this study, the volumetric swelling strain $\varepsilon_s^v$ is determined by the following relationship.

$$\varepsilon^v_{s(p)} = m_s S_w \varepsilon^v_{saw(p)}$$

(3)

$m_s$ and $S_w$ are the content of swelling minerals (montmorillonite, etc.) contained in unit volumetric rocks and swelling saturation, respectively. In the swelling analysis conducted in this study, $S_w$ is treated as a swelling control parameter, changed incrementally from 0 to 1.

The hypo-elastic constitutive relation can be written as:

$$\mathbf{\varepsilon} = \mathbf{C}(\mathbf{e}, \mathbf{x}, \varepsilon_s^v) : \dot{\mathbf{\varepsilon}}$$

(4)

The elastic modulus tensor considered damage can be expressed as follows.

$$\overline{\mathbf{C}}_{(\mathbf{e}, \mathbf{x}, \varepsilon_s^v)} = \left(1 - D_{(\mathbf{e}, \mathbf{x}, \varepsilon_s^v)}\right)\overline{\mathbf{C}}(\mathbf{e})$$

(5)

Where

$$\overline{\mathbf{C}}(\mathbf{e}) = \mathbf{K} \mathbf{I} \otimes \mathbf{I} + 2 \overline{\mathbf{G}} \begin{pmatrix} (\mathbf{I}_{4\text{sym}}^{\text{sym}}) & -1/3 \mathbf{I} \otimes \mathbf{I} \end{pmatrix} p$$

$$\mathbf{K} = \frac{(1 + \epsilon_0)}{\kappa}; \quad \overline{\mathbf{G}} = \frac{3(1 - 2\nu)}{2(1 + \nu)} \mathbf{K}$$

(6)

$\overline{\mathbf{C}}_{(\mathbf{e})}$ is the hypo-elastic modulus tensor in original MCC model. $\mathbf{I}$ is the second order identity tensor and $\mathbf{I}_{4\text{sym}}^{\text{sym}}$ is a fourth order symmetric identity tensor. $\epsilon_0$, $\nu$ and $p$ are initial void ratio, Poisson’s ratio and mean stress, respectively. As the phenomenon of rocks stiffness degradation occurs with the process of swelling, the damage is taken into consideration. And the damage variable $D$ is given by the following exponential function.

$$D_{(\mathbf{e}, \mathbf{x}, \varepsilon_s^v)} = D_\infty \left[1 - \exp(-\chi)\right]$$

(7)

$D_\infty$ is the maximum value of the damage variable. It is assumed that the damage parameter $\chi$ depends on the deviatoric plastic strain and volumetric swelling strain, based on the swelling phenomenology. That is to say the damage will progress when swelling and plastic deformation occurs. Specifically, it can be expressed as follows.

$$\chi = a \sqrt{\frac{2}{3} \| \mathbf{e}^p \| - c \varepsilon_s^v}$$

(8)

$\mathbf{e}^p$ is deviatoric plastic strain, $\varepsilon_s^v$ is volumetric swelling strain, $a$ and $c$ are the coefficients representing the influence degree of damage progress relating deviatoric plastic strain and volumetric swelling strain, respectively.

The yield function of Modified Cam-Clay model is applied in this study, which is shown as follows.

$$f = MD \left[ \ln \left( \frac{p}{p_c} \right) + \ln \left( \frac{M^2 + \eta^2}{M^2} \right) \right] - \alpha$$

(9)

$M$ is the critical state parameter, $D$ is the dilatancy coefficient and $\alpha$ is the hardening parameter defined by plastic volumetric strain $\varepsilon_s^p$. The associated flow law is adopted in this study, so the rate of plastic strain is expressed as follows.

$$\dot{\mathbf{e}}^p = \gamma \frac{\partial f}{\partial \mathbf{\sigma}}$$

(10)

2.3. Implicit stress-update algorithm

In this section, an implicit stress update algorithm is formulated for the swelling model constructed above, according to the return-mapping framework proposed by Simo et al., e.g., (Simo, J.C. and Hughes, T.J.R., 1998). The variables of incremental format of the above model are $[\Delta \mathbf{\sigma}, \mathbf{e}^p, \mathbf{e}^v, \Delta \gamma]$, and as both the elastic modulus tensor and the swelling strain are dependent on the stress, the main variables are treated as $[\Delta \mathbf{\sigma}, \Delta \gamma]$ and the dependent variables are treated as $[\mathbf{e}^p, \mathbf{e}^v]$. Therefore, the residual equations to be solved by the Newton-Raphson method can be summarized as follows.
\[
\begin{align*}
R_{\nu+1}(\sigma, \Delta \gamma) &= \sigma_{\nu+1} - \sigma_{\nu} - C_{(\sigma : \varepsilon)} : \Delta \varepsilon \\
R_{f+1}(\sigma, \Delta \gamma) &= MD \left[ \ln \left( \frac{p}{p_i} \right) + \ln \left( \frac{M^2 + \eta^2}{M^2} \right) \right] - \left( \varepsilon_{\nu+1}^p + \Delta \gamma I : \frac{\partial f_{\nu+1}}{\partial \sigma_{\nu+1}} \right)
\end{align*}
\]  
(11)

Linearize the above equation and then solve the variable corrections \( \delta \sigma \) and \( \delta (\Delta \gamma) \), shown as follows.

\[
\begin{align*}
\delta (\Delta \gamma) = & \frac{\partial R_{f+1}}{\partial \sigma_{\nu+1}} : \frac{\partial [R_{\nu+1}]^{-1}}{\partial \sigma_{\nu+1}} : R_{\nu+1} \\
\delta \sigma = & - \left[ \frac{\partial R_{\nu+1}}{\partial \sigma_{\nu+1}} : [\frac{\partial [R_{\nu+1}]^{-1}}{\partial \sigma_{\nu+1}}]^{-1} \right] : R_{\nu+1} + \frac{\partial R_{\nu+1}}{\partial (\Delta \gamma_{\nu+1})} : \delta (\Delta \gamma)
\end{align*}
\]  
(12)

It should pay attention to that the stress correction amount \( \delta \sigma \) in the case of the elastic state can be expressed simply as follows.

\[
\delta \sigma = - \left[ \frac{\partial R_{\nu+1}}{\partial \sigma_{\nu+1}} : [\frac{\partial [R_{\nu+1}]^{-1}}{\partial \sigma_{\nu+1}}]^{-1} \right] : R_{\nu+1}
\]  
(13)

That is because the increment of the plastic multiplier \( \Delta \gamma \) is 0 in the elastic state.

When solving the nonlinear equilibrium equation by the Newton-Raphson method in FEM, the consistent tangent modulus \( C^{op} \) is required for obtaining the quadratic convergence rate (Simo, J.C., and Taylor, R.L., 1985). It is derived as follows.

\[
C^{op} = \Xi_{\nu+1} : C_{\nu+1} - \Xi_{\nu+1} : C_{\nu+1} \otimes \zeta_{\nu+1} : \Xi_{\nu+1} : C_{\nu+1} \otimes \Xi_{\nu+1} : C_{\nu+1}
\]  
(14)

With

\[
\begin{align*}
\Xi_{\nu+1} &= \left[ \begin{array}{l}
\frac{m_i e^e}{w} S_{w+1} I + 1 \\
\ln p_s - \ln p_s \quad p_{w+1}
\end{array} \right] \\
\beta_{\nu+1} &= \left[ \begin{array}{l}
\frac{\partial f_{\nu+1}}{\partial \sigma_{\nu+1}} - C_{\nu+1}^{-1} : \frac{\partial C_{\nu+1} \ast \Delta \varepsilon}{\partial \sigma_{\nu+1}} : \frac{\partial f_{\nu+1}}{\partial \sigma_{\nu+1}} \\
\frac{\partial f_{\nu+1}}{\partial \sigma_{\nu+1}} - \Delta \gamma_{\nu+1} I : \frac{\partial f_{\nu+1}}{\partial \sigma_{\nu+1}} \otimes \sigma_{\nu+1}
\end{array} \right]^{-1}
\]  
(15)

The operator \( \ast \) in the formula above means \( (\mathbb{I} \otimes \mathbb{I})_{ijpq} = \mathbb{I}_{ijpq} \cdot \delta_{ij} \).

3. Numerical analysis results

In this section, the numerical analysis method constructed in section 2 is used to analyze the swelling deformed tunnel called Sakazukiyama Tunnel in Japan, which belongs to Yamagata Expressway.

3.1. Self-weight analysis and excavation analysis
The dimensions and boundary conditions of the FEM model for the Sakazukiyama Tunnel is shown in figure 2. The displacement boundary condition was set as plane strain state. The dimension of the overall analysis model was set as 100m × 100m × 1m. And a tunnel with a cross section of 11m and 9.2m in the width and height, respectively, was set in the center of the model. Since the overburden at the location where the swelling deformation occurred in the tunnel is about 100 m, the tunnel surface load $\sigma_v$ was set as 1000 kN/m$^2$, which corresponds to the earth pressure of 50 m. The parameters used for self-weight analysis and excavation analysis are shown in table 1. Since the self-weight analysis and excavation analysis are analyzed as elasticity, the initial yield stress $\sigma_y$ was set as large as $3.0 \times 10^9$ kPa. And the other parameters, such as the density $\rho$, the initial porosity $e_0$, the limit state constant M, the compression index $\lambda$, and the Poisson’s ratio $\nu$ are the average value of the laboratory tests using the boring core of Sakazukiyama Tunnel[10].

![Figure 2. Boundary condition and size of cross section.](image)

![Figure 3. Mean stress distribution](image)

**Table 1. Parameters for self-weight analysis and excavation analysis**

| $\rho$ (t/m$^3$) | $e_0$ | $p_i$ (kPa) | $p_c$ (kPa) | M  | $\lambda$ | $\kappa$ | $\nu$ |
|-----------------|-------|-------------|-------------|----|-----------|----------|-------|
| 1.73            | 1.26  | 200         | $3.0 \times 10^9$ | 2.0 | 0.43      | 0.05     | 0.3   |

The mean stress distribution of self-weight analysis and excavation analysis are shown in figure 3 (a) and (b), respectively. It can be confirmed that the stress redistributed after excavation as the mean stress decrease in the vertical direction, especially the domain below the invert, but increase on both sides of the tunnel.
3.2. Swelling analysis
The swelling domain is shown in figure 2 (b), which radius is of 20 m from the center of the tunnel. Since the radius of the tunnel is about 5 m, the thickness of swelling domain is about 15 m. As it is reported that the swelling phenomenon often occurs within 10 m below the invert of tunnel, the 15 m set here is enough. And in the swelling analysis, the lining structure of tunnel is not taken in consideration as we want to know what the final deformation will be as the lining structure is in a complete destruction state. The parameters used for swelling analysis in swelling domain are shown in table 2 and table 3. As the initial mean stress \( p_0 \) of swelling analysis is used the results of excavation analysis, it is not written in the table 2. And the parameters in other domains are the same as shown in table 1.

| Table 2. Parameters for swelling analysis (1) |
|---------------------------------------------|
| Swelling domain | \( p_0 \) (kPa) | \( p_{c0} \) (kPa) | \( D_\infty \) | \( a \) | \( c \) | \( p_s \) (kPa) | \( p_c \) (kPa) |
|------------------|------------------|------------------|-------------|-------|-------|---------------|---------------|
| 1,2,2’            | /                | \( 3.5 \times 10^3 \) | 0.95        | 20.0  | 20.0  | 1.0           | 4.0 \times 10^4 |

| Table 3. Parameters for swelling analysis (2) |
|---------------------------------------------|
| Swelling domain | \( \lambda \) | \( e_0 \) | \( m_i \) | \( e_{swc} \) |
|------------------|-------------|-------|-------|-------------|
| 1                | 0.57        | 1.4   | 0.50  | -1.20       |
| 2                | 0.37        | 1.1   | 0.90  | -0.33       |
| 2’               | 0.34        | 1.3   | 0.40  | -0.30       |

The equivalent stress distribution of the swelling analysis is shown in figure 4’s (a). The equivalent stress increased as the swelling occurred, and it can be seen that the stress increased remarkably on the both sides of the tunnel, especially. In addition, the distribution of the volumetric swelling strain, damage variable, volumetric plastic strain, and deviator plastic strain occurred in swelling domain are shown in figure 4’s (b), (c), (d) and (e) respectively. Viewing the distribution of the volumetric swelling strain firstly, the largest volumetric strain occurred in the zone below the tunnel invert, moreover, the amount of volumetric swelling strain is larger in the shallow area than in the deep areas. This is because the swelling strain is dependent on the stress. Furthermore, the amount of volumetric swelling strain on the left side is larger than that on the right side. It can be explained as the content of swelling minerals in the left side of the Sakazukiyama Tunnel is about two times higher than that in the right side (see table 3). In response to a such swelling deformation, it can be confirmed in the (c) that the damage progresses remarkably below the invert, especially in the shallow area. And the amount of damage is larger in the left side than the right side of the tunnel. In the (d) and (e) of figure 4, it can be confirmed that plasticity happened at the legs on both sides of the tunnel, and the deviatoric plastic strain is about two times larger than the volumetric plastic strain. This analysis result fits well with the appearance of the destroyed state reported from the deformed tunnel.

The mean stress distribution after swelling analysis is shown in figure 5, and the ground heave (vertical displacement) and horizontal extrusions (horizontal displacement) in both sides of the tunnel is shown in (a), (b), (c), respectively. It can be seen that the stress increases significantly in the swelling domain compared with the result of excavation analysis. The maximum of vertical displacement and horizontal displacement as shown in (a), (b), (c), reaches about 375 mm, 57 mm and 23 mm, respectively. It is very close to the values reported in Deformation Countermeasure Construction Report of Sakazukiyama Tunnel, which is 477 mm, 55 mm and 28 mm. It needs to pay attention to that since the measurement range of the underground displacement measurement device is
0 to 10 m, the computation results are modified by setting the point at depth of 10 m as a fixed point, in order to compare the computation results with the field measured data.

![Figure 4. Results of swelling analysis](image)

### 4. Conclusions

In this study, a hypoelastic-plastic-damage constitutive law taking the pressure dependence property of swelling into consideration was constructed for the phenomenon of rock swelling. In addition, an implicit stress update algorithm was formulated for a good convergence and robustness in the numerical analysis. Furthermore, the simulation results of Sakazukiyma Tunnel, where the swelling deformation occurred, showed a good consistency with the actual deformation and failure mode of the tunnel. Therefore, the numerical analysis method developed in this study is valid, and it can evaluate the mechanical behavior of tunnels in swelling mountains quantitatively and accurately.
Figure 5. Mean stress distribution and displacement path

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