Strong interaction of a quantum dot with the photon reservoir in one-dimensional photonic crystals

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Abstract. Quantum fluctuations as a process of the self-interaction of a system lead to a shift of the energy levels and are usually described by the self-energy function. Here quantum fluctuations caused by the interaction of a quantum dot coupled with an one-dimensional photonic crystal’s cavity with the photon reservoir are discussed. It is shown that this interaction can lead to a significant increase in quantum fluctuations. The self-interaction is of interest for solving problems of decreasing decoherence and dephasing of qubit for quantum computation.

1. Introduction
In the past decade, there has been tremendous progress in the experimental development of a quantum computer: a machine that would exploit the full complexity of a many-particle quantum wavefunction to solve a computational problem [1]. Quantum computers have many ambitious applications, e.g., Feynman’s 1980s proposal of using quantum computers for the efficient simulation of quantum systems [2], in artificial nanotechnology, in understanding of the processes in biology.

There are many different physical implementations of quantum bits (qubits) for quantum computation these days. Some of them are represented by single atoms, ions, or spins. The long history of study such systems makes it possible to manipulate their quantum states. Several proposals for scalable quantum computation rely on direct probing of the cavity-quantum dot (QD) coupling by means of resonant light scattering from strongly or weakly coupled dots [3–7]. QDs in photonic crystals (PCs) are interesting both as a testbed for fundamental cavity quantum electrodynamics (QED) experiments and as a platform for quantum and classical information processing [8]. These artificial media like PCs made from two or more dielectrics with different refractive indices periodically arranged in spatial directions have great attention in quantum technologies for observing QED effects such as the control of spontaneous emission of atoms [9], amplification of quantum interference [10,11], control of the electron mass [12,13].

The restriction of the motion of electrons and holes leads to the fact that the continuous energy bands, which would be in a bulk material, break up into separate bands, up to the formation of discrete levels. This phenomenon is called quantum confinement. The discrete structure of energy states leads to a discrete absorption spectrum, in contrast to the continuous
absorption spectrum of a bulk semiconductor. Semiconductor QDs are a physical realization of a spatial three-dimensional quantum well. Due to their nanometer size, such islands ensure the restriction of the movement of charge carriers in all three dimensions, and the electron mean free path becomes comparable to the de Broglie wavelength, providing discretization of the energy levels of the system and, accordingly, a change in its properties. This makes it possible to create new optoelectronic devices operating according to the laws of atomic physics. Examples of such devices include single photon sources, entangled photon sources, single-photon optical switches, and potential qubit registers in quantum digital technology.

The interaction of an exciton (electron-hole pair) of a QD with environment is one of the factors in the decreasing of the coherence (decoherence) of the state of light, which in turn is an obstacle to the implementation of QD’s qubit in quantum computation. This is the reason for studying the exciton-photon interaction in QDs in order to reduce decoherence. In particular, we will be interested in the case when, as a result of a series of transitions, the system goes into a state identical to the initial one. Such processes are called quantum fluctuations. By their nature, they are similar to the interaction of charged particles with their own radiation field, which are also sometimes called quantum fluctuations. In QED, such processes lead to the appearance of a shift in the energies of bound states, which is called the Lamb shift. In QDs there will also be a shift in the energy of states caused by quantum fluctuations. The intermediate state in such processes will be virtual, therefore, for the duration of such fluctuations, the energy conservation law will be violated according to Heisenberg’s uncertainty.

The energy dependence of the self-energy function having a significant effect on the emission spectra of a QD, indicates a time delay in the QD-reservoir interaction. The quantum fluctuations during which the reservoir degrees of freedom manifest themselves in virtual states influence on the state of a quantum dot. As a consequence, the state of a quantum dot becomes dressed. In this paper, the object of research is quantum fluctuations caused by the interaction of a quantum dot coupled with one-dimensional PC’s cavity (air-void) with the photon reservoir. The effect of quantum fluctuations is found in corrections to the energy levels of QDs [14].

2. Method
To describe the processes associated with quantum fluctuations as nonlocal in time interaction, we use the generalized quantum dynamics (GQD) formalism [15], which allows solving the problem nonperturbatively. The generalized dynamic equation (GDE) is written in terms of an operator $\tilde{S}(t_2, t_1)$ that determines the contribution to the evolution operator $U(t, t_0)$ from the process in which the interaction begins at time $t_1$ and ends at time $t_2$.

$$\langle \psi_2 | U(t, t_0) | \psi_1 \rangle = \langle \psi_2 | \psi_1 \rangle + \int_{t_0}^{t} dt_2 \int_{t_0}^{t_2} dt_1 \langle \psi_2 | \tilde{S}(t_2, t_1) | \psi_1 \rangle,$$  \hspace{0.5cm} (1)

$$(t_2 - t_1) \tilde{S}(t_2, t_1) = \int_{t_1}^{t_2} dt_4 \int_{t_1}^{t_4} dt_3 (t_4 - t_3) \tilde{S}(t_2, t_4) \tilde{S}(t_3, t_1).$$  \hspace{0.5cm} (2)

The boundary condition for equation (2) corresponds to the interaction on an infinitely small time interval $t_2 \to t_1$ determined by the interaction Hamiltonian $\tilde{S}(t_2, t_1) \to H_{\text{int}}(t_2, t_1)$. Taking into account the fact that in the Schrödinger picture the evolution operator can be written in terms of the Green operator $G(z)$, we have

$$U_S(t, 0) = \frac{1}{2\pi} \int_{0}^{+\infty} e^{-izt} G(z) \, dx, \hspace{0.5cm} z = x + iy.$$  \hspace{0.5cm} (3)
Then, in the energy representation, the GDE takes the form

$$\frac{dT(z)}{dz} = -T(z) (G_0(z))^2 T(z),$$

(4)

$$T(z) = i \int_0^\infty d\tau \exp (i(z - H_0)t_2) \tilde{S} (t_2, t_1) \exp (-i(z - H_0)t_1).$$

(5)

This formalism allows one to take into account from the beginning that the contribution to the Green’s operator $G(z)$, which comes from processes associated with the self-interaction of particles, has the same structure as the free Green’s operator $G_0(z)$. For this reason, it is natural to replace $G_0(z)$ by the operator $\tilde{G}_0(z)$ describing the evolution of the system, when the particles propagate freely

$$\langle m'| \tilde{G}_0(z) |m \rangle = \frac{\langle m'|m \rangle}{z - E_m - C_m(z)},$$

(6)

where $|m\rangle$ are the eigenvectors of the free Hamiltonian $(H_0 |m\rangle = E_m |m\rangle)$. In this case, the complete Green operator takes the form

$$G(z) = \tilde{G}_0(z) + \tilde{G}_0(z) M(z) \tilde{G}_0(z),$$

(7)

where the operator $M(z)$ describes the processes of the interaction of particles with each other. The equation for the self-energy function $C_m(z)$ has the form

$$\frac{dC_m(z)}{dz} = - \langle m| M(z) \big( \tilde{G}_0(z) \big)^2 M(z) |m \rangle, \quad \langle m'|m \rangle = 1,$$

(8)

and the condition $z - E_m^{(0)} - C_m(z) = 0$ determines the physical masses of the particles. In fact, since the significant contribution is made by the processes associated with the fundamental interaction in the system, in the leading order the equation for $C_m(z)$ is reduced to the equation

$$\frac{dC_m^{(0)}(z)}{dz} = - \langle m| H_I \big( \tilde{G}_0(z) \big)^2 H_I |m \rangle, \quad \langle m'|m \rangle = 1.$$

(9)

3. Quantum fluctuations in quantum dots interacted with the photon reservoir in one-dimensional photonic crystals

We consider a system where a quantum dot is placed in one-dimensional PC’s cavity and coupled to a boson reservoir. Boson reservoir consists of phonons and photons. We will take into account the interaction of excitons only with photons of the reservoir. In the natural system of units, for $\hbar = c = k_B = 1$, the complete Hamiltonian of the “QD’s exciton + photons” system can be represented as

$$H = |X\rangle \langle X| \left[ \omega_x + \sum_k g_k^2 \left( a_k + a_k^\dagger \right) \right] + \sum_k \omega_k a_k^\dagger a_k.$$

(10)

This expression describes a particle – exciton by the state $|X\rangle$ with energy $\omega_x$ interacting with photons with frequencies $\omega_k$ and momentum $k$ under coupling strength $g_k^2$. Here $a_k$ and $a_k^\dagger$ are the annihilation and creation operators for photons, respectively. Using this Hamiltonian in the one-loop approximation, the following solution for the self-energy function of equation (9) at the $z \to z - \omega_x$, where the energy of the exciton state $\omega_x = 830$ meV [16], was obtained:

$$C(z) = \sum_{kn} \left\{ \frac{|g_k^2|^2 [1 + n(k)]}{z + \frac{i\gamma}{2} - \omega_0 - \omega_{kn}} + \frac{|g_k^2|^2 n(k)}{z + \frac{i\gamma}{2} - \omega_0 + \omega_{kn}} \right\},$$

(11)
Figure 1. (a) Real part and (b) imaginary part of the self-energy interaction of a CdSe QD [19] with the virtual Bloch photons of one-dimensional PC’s cavity with the length 500 nm. The average refractive index of GaAs layers of the one-dimensional photonic structure is 3.5 [20], and its period is 1000 nm. We use $\omega_{\text{max}} = 10.65$ eV in dispersion relations.

where the summation over $n$ is performed over the number of bands in dispersion relations $\omega_{kn}$, $n(k) = \frac{1}{e^{\frac{\omega_{kn}}{k_BT}} - 1}$ is the Bose-Einstein distribution, $\gamma$ denotes the QD’s decay rate, $\omega_0$ is the resonance energy of interaction of a QD with one-dimensional PC’s cavity. Replacing the discrete sums by integrals $\sum_{kn} \rightarrow \frac{V}{(2\pi)^3} \sum_n \int d^3k$ and taking the integral $\int d^3k$ over the azimuthal angle $\varphi_k$ in the cylindrical coordinate system, we get

$$C(z) = \frac{V}{4\pi^2} \sum_{n\lambda} \int_{k_{\rho}} dk_{\rho} \int d_{k_z} \left\{ \frac{|g_{x}^{e}|^{2}}{z + \frac{\omega_{0} + \omega_{kn\lambda}}{2}} \frac{1 + n(k)}{2} + \frac{|g_{x}^{e}|^{2}n(k)}{z + \frac{\omega_{0} + \omega_{kn\lambda}}{2}} \right\}, \quad (12)$$

where $V$ is the unit volume of the PC medium, $\lambda_1$ and $\lambda_2$ denote indexes of the TE (transverse-electric) and TM (transverse-magnetic) polarization in dispersion relations $\omega_{kn\lambda}$, $g_{x}^{e} = 3.9$ meV is the constant of the exciton-photon coupling [17], $\gamma = 77$ $\mu$eV [16, 18], $\omega_0 = 2.4$ eV [19], $L = 500$ nm is the cavity-resonator length. The self-energy function of the system, which reflects the change in the energy of the state of a CdSe QD upon interaction with photons, is shown in figure 1 (a, b). Sharp peaks occur in the self-energy function $C(z)$, its correspond to the periodical behavior of dispersion relations of the PC medium, and the resonance interaction of a QD with this artificial optical medium.

4. Conclusion

The energy dependence of the self-energy function indicates a time delay in the QD-photon reservoir interaction. The quantum fluctuations during which the reservoir degrees of freedom manifest themselves in virtual states influence on the state of a quantum dot. We have estimated the value of quantum fluctuations caused by the interaction of a quantum dot with a photons reservoir of one-dimensional photonic crystal. The effect of quantum fluctuations is found in corrections to the energy levels of quantum dots. This investigation could be important for creation solid state qubits on QDs in PC’s cavity and single photon sources.
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References
[1] Ladd T D, Jelezko F, Laflamme R, Nakamura Y, Monroe C and OBrien J L 2010 Nature 464 45–53
[2] Nielsen M A and Chuang I 2002 Quantum Computation and Quantum Information (American Association of Physics Teachers)
[3] Cirac J I, Zoller P, Kimble H J and Mabuchi H 1997 Phys. Rev. Lett. 78 3221
[4] Duan L M and Kimble H 2004 Phys. Rev. Lett. 92 127902
[5] Childress L, Taylor J, Sørensen A S and Lukin M D 2005 Phys. Rev. A 72 052330
[6] Ladd T D, van Loock P, Nemoto K, Munro W J and Yamamoto Y 2006 New J. Phys. 8 184
[7] Gainutdinov R K, Nabiieva L, Garifullin A and Mutyygullina A 2020 J. Phys. Conf. Ser. 1628 012005
[8] Englund D, Fushman I, Farao A and Vučković J 2009 Photonics Nanostructures: Fundam. Appl. 7 56–62
[9] Wang W, Yang X, Luk T S and Gao J 2019 Appl. Phys. Lett. 114 021103
[10] Song G, Xu J and Yang Y 2014 Phys. Rev. A 89 053830
[11] Gainutdinov R K, Garifullin A, Khamadeev M and Salakhov M K 2020 J. Phys. Conf. Ser. 1628 012006
[12] Gainutdinov R K, Khamadeev M A and Salakhov M K 2012 Phys. Rev. A. 85 053836
[13] Gainutdinov R, Khamadeev M, Akhmadeev A and Salakhov M 2018 Modification of the electromagnetic field in the photonic crystal medium and new ways of applying the photonic band gap materials Theoretical Foundations and Application of Photonic Crystals ed Vakhrushev A (Rijeka: InTech) chap 1
[14] Calic M, Gallo P, Felici M, Atlasov K, Dwir B, Rudra A, Biasiol G, Sorba L, Tarel G, Savona V et al. 2011 Phys. Rev. Lett. 106 227402
[15] Gainutdinov R K 1999 J. Phys. A: Math. Gen. 32 5657
[16] Tarel G and Savona V 2010 Phys. Rev. B 81 075305
[17] Vamivakas A N, Zhao Y, Lu C Y and Atatürk U 2009 Nature Physics 5 198–202
[18] Hughes S, Yao P, Milde F, Knorr A, Dalacu D, Mnaymneh V, Sazonova V, Poole P, Aers G, Lapointe J et al. 2011 Phys. Rev. B 83 165313
[19] Wang T, Zhang S, Mao C, Song J, Niu H, Jin B and Tian Y 2012 Biosens. Bioelectron. 31 369–375
[20] Aspnes D, Kelso S, Logan R and Bhat R 1986 J. Appl. Phys. 60 754–767