Abstract

Within the framework of modified teleparallel gravity, we reconstruct a \( f(T) \) model corresponding to the QCD ghost dark energy scenario. For a spatially flat FRW universe containing only the pressureless matter, we obtain the time evolution of the torsion scalar \( T \) (or the Hubble parameter). Then, we calculate the effective torsion equation of state parameter of the QCD ghost \( f(T) \)-gravity model as well as the deceleration parameter of the universe. Furthermore, we fit the model parameters by using the latest observational data including SNeIa, CMB and BAO data. We also check the viability of our model using a cosmographic analysis approach. Moreover, we investigate the validity of the generalized second law (GSL) of gravitational thermodynamics for our model. Finally, we point out the growth rate of matter density perturbation. We conclude that in QCD ghost \( f(T) \)-gravity model, the universe begins a matter dominated phase and approaches a de Sitter regime at late times, as expected. Also this model is consistent with current data, passes the cosmographic test, satisfies the GSL and fits the data of the growth factor well as the \( \Lambda \)CDM model.

PACS numbers: 95.36.+x, 04.50.Kd

Keywords: Dark energy, Modified theories of gravity
1 Introduction

Astronomical data from the type Ia supernovae (SNeIa), cosmic microwave background (CMB) and baryon acoustic oscillation (BAO), etc., have revealed that the universe is undergoing an accelerating expansion [1]. This unexpected observed phenomenon poses one of the most puzzling problems in cosmology today. There are two representative approaches to explain this behavior. One is to introduce some unknown matters with negative pressure called “dark energy” (DE) in the framework of Einstein’s general relativity (for reviews on DE, see e.g. [2]). The other approach for describing the accelerated expansion of the universe is to modify the gravitational theory called “dark gravity” (see e.g. [3, 4] for a review on modified gravity).

More recently, a new interesting DE model called ghost DE (GDE) has been motivated from the Veneziano ghost of chromodynamics (QCD) [5]. In this proposal, it is claimed that the vacuum energy arises from the contribution of the ghost fields which are supposed to be present in the low energy effective theory of QCD. The Veneziano ghost is required to exist for the resolution of the $U(1)_A$ problem, but are completely decoupled from the physical sector. The above claim is that the ghosts are decoupled from the physical states and make no contribution in the usual Minkowski space-time, but make a small energy density contribution to the vacuum energy due to the off-set of the cancellation of their contribution in the space-time with nontrivial topology or time-dependent background such as our Friedmann-Robertson-Walker (FRW) universe. This ghost gives the vacuum energy density proportional to $\sim \Lambda_{QCD}^3 H$, where $H$ is the Hubble parameter and $\Lambda_{QCD} \sim 100$ MeV is the QCD mass scale [6]. This small contribution can play an important role in the evolutionary behavior of the universe. For instance, taking $H \sim 10^{-33}$ eV at the present, $\Lambda_{QCD}^3 H$ gives the right order of observed magnitude of the DE density. This coincidence is remarkable and implies that the GDE model gets rid of fine tuning problem [5]. In addition, the appearance of the QCD scale could be relevant for a solution to the cosmic coincidence problem, as it may be the scale at which dark matter (DM) forms [7]. It is worth to note that the GDE model does not violate unitarity, causality, gauge invariance and other important features of renormalizable quantum field theory, as advocated in [8]. This new kind of DE model has got a lot of enthusiasm recently in the literature [9, 10, 11, 12].

In the framework of modified gravity, the underlying philosophy of extended theories of gravity is that general relativity (GR) should be seen as a particular case of a more general effective theory coming from fundamental principles. The common property of all these approaches is that the DE effects can be associated to an evolving equation of state (EoS). The basic idea lies on the fact that the standard Einstein-Hilbert action is modified by additional degrees of freedom, spanning from further curvature invariant corrections, to scalar fields and Lorentz violating terms. The first need of “corrections” to GR emerges when quantum field theory is formulated on curved space-time. Normalization and regularization processes lead to non-minimal couplings and higher-order corrections in curvature invariants [4]. Here, we limit our attention to investigate the so-called $f(T)$ theories [13, 14], which represent a class of models that take into account the effects due to the torsion. Indeed, $f(T)$ theory is based on the old idea of the “teleparallel” equivalent of GR (TEGR) [15], which, instead of using the curvature defined via the Levi-Civita connection, uses the Weitzenböck connection that has no curvature but only torsion. In fact, this approach was taken by Einstein [15] in an attempt of unifying gravity and electromagnetism. TEGR is closely related to standard GR, differing only in terms involving total derivatives in the action, i.e. boundary terms [16]. $f(T)$-gravity is a modification of the teleparallel gravity in which the teleparallel Lagrangian density described by the torsion scalar $T$ has been promoted to a function of $T$. This concept is similar to the idea of $f(R)$-gravity. However, in comparison with $f(R)$-gravity, whose fourth-order equations may
lead to pathologies, $f(T)$-gravity has the significant advantage of possessing second-order field equations [17]. This feature has led to a rapidly increasing interest in the literature. Models based on $f(T)$-gravity can provide an alternative to inflation [13]. It was also found that $f(T)$ theory can explain the observed acceleration of the universe [14]. Some viable phenomenological $f(T)$ models were proposed by [18]. Observational constraints were considered in [19]. A reconstruction of the $f(T)$ theory from the background expansion history and the $f(T)$ theory driven by scalar fields were studied in [20]. It was shown that $f(T)$ theories are not dynamically equivalent to teleparallel action plus a scalar field via conformal transformation [21]. Cosmological perturbations and growth factor of matter perturbations in $f(T)$-gravity were investigated in [22]. In [23], Birkhoff’s theorem in $f(T)$-gravity was studied. Static solutions with spherical symmetry in $f(T)$ theories were discussed in [24]. In [25, 26], the cosmic expansion was studied by using cosmography. Thermodynamical description of $f(T)$-gravity was studied in [27, 28].

In the present work, our aim is to reconstruct a $f(T)$-gravity model without resorting to any additional DE, that is, considering that the ghost DE is effectively described by the modification of the gravity with respect to the teleparallel gravity. To do so, in section 2, we briefly review the $f(T)$-gravity in a spatially flat FRW universe filled only with the pressureless matter. In section 3, we reconstruct a $f(T)$ model according to the evolution of GDE density. In section 4, we fit this model and give the constraints on model parameters, with current observational data including SNeIa, CMB and BAO data. In section 5, we check the viability of our model using the cosmographic analysis method. In section 6, the validity of the generalized second law of gravitational thermodynamics for our $f(T)$ model is examined. In section 7, we study the growth of structure formation in our model. Section 8 is devoted to conclusions.

2 $f(T)$-gravity

The action of $f(T)$-gravity is given by [13, 14]

$$I = \frac{1}{2k^2} \int d^4x \ e \left[ f(T) + L_m \right],$$

(1)

where $k^2 = 8\pi G$, $e = \det(e^i_\mu)$ and $e^i_\mu$ is the vierbein field which is used as a dynamical object in the teleparallel gravity. Also $T$ and $L_m$ are the torsion scalar and the Lagrangian density of the matter inside the universe, respectively.

Taking the variation of the action (1) with respect to the vierbein $e^i_\mu$, the modified Friedmann equations in the spatially flat FRW universe can be obtained in the standard forms

$$\frac{3}{k^2} \dot{H}^2 = \rho_m + \rho_T,$$

(2)

$$\frac{1}{k^2}(2\dot{H} + 3H^2) = -(p_m + p_T),$$

(3)

where

$$\rho_T = \frac{1}{2k^2}(2Tf_T - f - T),$$

(4)

$$p_T = -\frac{1}{2k^2}[-8\dot{H}Tf_{TT} + (2T - 4\dot{H})f_T - f + 4\dot{H} - T],$$

(5)

and

$$T = -6H^2.$$  

(6)

Here, $\dot{H} = \dot{a}/a$ is the Hubble parameter and the subscript $T$ denotes a derivative with respect to the torsion scalar $T$. Also $\rho_m$ and $p_m$ are the energy density and pressure of the matter
inside the universe, respectively. Furthermore, \( \rho_T \) and \( p_T \) are the torsion contributions to the energy density and pressure, respectively. Note that in the case of \( f(T) = T \), Eqs. (4) and (5) give \( \rho_T = 0 \) and \( p_T = 0 \). Then Eqs. (2) and (3) recover the usual Friedmann equations in the teleparallel gravity.

The energy conservation equations are still given by

\[
\dot{\rho}_m + 3H(\rho_m + p_m) = 0,
\]

\[
\dot{\rho}_T + 3H(\rho_T + p_T) = 0.
\]

The effective torsion EoS parameter is defined as [17, 28]

\[
\omega_T = \frac{p_T}{\rho_T} = -1 - \frac{\dot{T}}{3H} \left( \frac{2Tf_{TT} + f_T - 1}{2Tf_T - f - T} \right).
\]

In the de Sitter universe, i.e. \( \dot{H} = 0 = \dot{T} \), Eq. (9) yields \( \omega_T = -1 \) which behaves like the \( \Lambda \)CDM model.

With the help of Eqs. (2), (4) and (6) one can get

\[
\rho_m = \frac{1}{16\pi G} (f - 2T f_T).
\]

For the pressureless matter, i.e. \( p_m = 0 \), from Eqs. (2) to (5) one can obtain

\[
\dot{H} = -\frac{4\pi G p_m}{f_T + 2T f_{TT}}.
\]

Inserting Eq. (10) into (11) and using \( \dot{T} = -12H\dot{H} \) gives

\[
\dot{T} = 3H \left( \frac{f - 2T f_T}{f_T + 2T f_{TT}} \right).
\]

Using the above relation, the effective EoS parameter (9) yields

\[
\omega_T = -\frac{f/T - f_T + 2T f_{TT}}{(f/T + 2T f_{TT})(f/T - 2f_T + 1)}.
\]

Here, we also calculate the deceleration parameter

\[
q = -1 - \frac{\dot{H}}{H^2},
\]

which can be compared with the observations. Using Eqs. (6) and (12) the deceleration parameter (14) leads to

\[
q = 2 \left( \frac{f_T - T f_{TT} - 3f}{f_T + 2T f_{TT}} \right).
\]

For \( f(T) = T \), from Eq. (15) we have \( q = 0.5 \) which corresponds to the matter dominated universe.
3 Ghost $f(T)$-gravity model

The dark torsion contribution in $f(T)$-gravity can justify the observed acceleration of the universe without resorting to the DE. This motivates us to reconstruct a $f(T)$-gravity model according to the GDE model. The GDE density, which comes from the Veneziano ghost of QCD, is proportional to the Hubble parameter [5, 9]

$$\rho_D = \alpha H,$$  \hspace{1cm} (16)

where $\alpha$ is a constant with dimension [energy]$^3$, and roughly of order of $\Lambda_Q^{3}$ where $\Lambda_{QCD} \sim 100$ MeV is the QCD mass scale.

With the help of Eq. (6) one can rewrite (16) as

$$\rho_D = \alpha \left( -\frac{T}{6} \right)^{1/2}.$$  \hspace{1cm} (17)

Equating (4) and (17), i.e. $\rho_T = \rho_D$, we obtain the following differential equation

$$2T f_T - f - T - \beta \sqrt{-T} = 0,$$  \hspace{1cm} (18)

where

$$\beta = \frac{2k^2\alpha}{\sqrt{6}}.$$  \hspace{1cm} (19)

Solving Eq. (18) yields the $f(T)$-gravity corresponding to the QCD ghost DE model as

$$f(T) = T + \sqrt{-T} \left( \epsilon + \frac{\beta}{2} \ln(-T) \right),$$  \hspace{1cm} (20)

where $\epsilon$ is an integration constant that can be determined from a boundary condition. Following [25] to recover the present day value of Newtonian gravitational constant we need to have

$$f_T(T_0) = 1,$$  \hspace{1cm} (21)

where $T_0 = -6H_0^2$ is the torsion scalar at the present time. Applying the above boundary condition to the solution (20) one can obtain

$$\epsilon = -\beta \left( 1 + \frac{1}{2} \ln(-T_0) \right).$$  \hspace{1cm} (22)

Substituting this into Eq. (20) gives

$$f(T) = T + \beta \sqrt{-T} \left[ \frac{1}{2} \ln \left( \frac{T}{T_0} \right) - 1 \right].$$  \hspace{1cm} (23)

Note that the parameter $\beta$ can be obtained by inserting Eq. (23) into the modified Friedmann equation (2). Solving the resulting equation for the present time gives

$$\beta = \sqrt{6}H_0(1 - \Omega_{m_0}),$$  \hspace{1cm} (24)

where $\Omega_{m_0} = \frac{k^2\rho_{m_0}}{3H_0^2}$ is the dimensionless matter energy density and the index 0 denotes the value of a quantity at present.
The evolution of the ghost $f(T)$-gravity model, Eq. (23), versus $T/T_0$ is shown in Fig. 1, where we also plot $f(T) = T$ corresponding to the case of teleparallel gravity for comparison. Figure 1 shows that the QCD ghost $f(T)$-gravity model (23) satisfies the condition
\[
\lim_{|T| \to \infty} \frac{f}{T} \to 1,
\]
at high redshift which is compatible with the primordial nucleosynthesis and CMB constraints [17, 18].

Inserting Eq. (23) into (12) and using $H = (-T/6)^{1/2}$ yields
\[
t - t_i = \frac{1}{\sqrt{6}} \int_{T_i}^{T} \left( \frac{2\sqrt{-T} - \beta}{\beta \sqrt{-T} + T} \right) \frac{dT}{T}.
\]

Integrating the above relation analytically gives
\[
t = \sqrt{\frac{2}{3}} \left[ \frac{1}{\sqrt{-T}} + \frac{1}{\beta} \ln \left( \frac{\sqrt{-T}}{\sqrt{-T} - \beta} \right) \right], \quad \sqrt{-T} \geq \beta,
\]
where at $T_i = -6H_i^2 = -\infty$ we have $t_i = 0$. Note that the condition $\sqrt{-T} \geq \beta$ is necessary due to having a real time. Using Eq. (24) the condition $\sqrt{-T} \geq \beta$ can be rewritten as $T/T_0 \geq (1 - \Omega_{m0})^2$. Figure 2 shows time evolution of the fractional torsion scalar $T/T_0 = (H/H_0)^2$ (or the fractional squared Hubble parameter). It clears that $T/T_0$ (or $H^2/H_0^2$) decreases with increasing the time. Figure 2 illustrates that at early ($t/t_0 \to 0$) and late ($t/t_0 \to \infty$) times we have $T/T_0 \to \infty$ and $T/T_0 \to (1 - \Omega_{m0})^2 = 0.545$, respectively, where we take $\Omega_{m0} = 0.262$ from the cosmological constraints (see section 4).

It is worth to mention that in [20], to reconstruct a $f(T)$-gravity according to a specific DE model, usually an ansatz for the scale factor $a(t)$ or the Hubble parameter $H(t)$ is assumed. Although this selection may justify the asymptotically behavior of the universe, the obtained $f(T)$ model doesn’t satisfy the full set of Eqs. (2), (3), (7) and (8). Because one cannot assume both a relation $a(t)$ (or $H(t)$) and a relation $\rho_T = \rho_D(t)$, independently. Choosing one determines the other through the Friedmann equations (2)-(3) and so it is inconsistent to choose both. Whereas in the present work, one can determine the time evolution of the Hubble parameter $H = (-T/6)^{1/2}$ with the help of Eq. (27), as plotted in Fig. 2. Besides, our $f(T)$ model (23) satisfies the full set of field equations in $f(T)$-gravity.

Inserting Eq. (23) into (13) gives the effective torsion EoS parameter of the ghost $f(T)$-gravity model as
\[
\omega_T = -\frac{T}{2T + \beta \sqrt{-T}}.
\]
The time evolution of the EoS parameter (28) is plotted in Fig. 3. It shows that at early time ($t/t_0 \to 0$) we have $\omega_T \to -0.5$ and at late time ($t/t_0 \to \infty$) we get $\omega_T \to -1$ which acts like the $\Lambda$CDM model. Also at present time we have $\omega_0 = -0.79$. Figure 3 clears that the torsion EoS parameter of the ghost $f(T)$-gravity model behaves like freezing quintessence DE [29]. This result is in complete agreement with that obtained for the GDE model in the Einstein gravity [9].

Inserting Eq. (23) into (15) one can obtain the deceleration parameter
\[
q = \frac{\sqrt{-T} - 2\beta}{2\sqrt{-T} - \beta}.
\]
Figure 4 shows the time evolution of the deceleration parameter \( q \). It clears that at early time \( (t/t_0 \to 0) \) we have \( q \to 0.5 \) which corresponds to the matter dominated universe. Also at late time \( (t/t_0 \to \infty) \) we get \( q \to -1 \) which behaves like the de Sitter universe. Figure 4 illustrates that at the near past \( t/t_0 = 0.57 \) we have a cosmic deceleration \( q > 0 \) to acceleration \( q < 0 \) transition which is compatible with the observations \[30\]. Also at present time we get \( q_0 \approx -0.4 \) which is in good agreement with the recent observational results \(-1.4 \leq q_0 \leq -0.3 \) \[30\].

### 4 Cosmological constraints

Here, we fit the free parameter \( \Omega_{m0} \) of the QCD ghost \( f(T) \)-gravity model \(23\) by using the recent observational data including SNeIa, CMB shift and BAO data.

Since SNeIa behave as excellent standard candles, they can be used to directly measure the expansion rate of the universe up to high redshifts \( (z \geq 1) \) for comparison with the present rate. Therefore, they provide direct information on the accelerating universe and constrain the model. Recently, the Supernova Cosmology Project (SCP) collaboration released the updated Union2.1 compilation which consists of 580 SNeIa \[31\]. The Union2.1 compilation is the largest published and spectroscopically confirmed SNeIa sample to date. The data points of the 580 Union2.1 SNeIa compiled in \[31\] are given in terms of the distance modulus \( \mu_{\text{obs}}(z_i) \). From the SNeIa constraint, the best fit value of the model parameters can be obtained by minimizing \[32, 33\]

\[
\chi^2_{\text{SN}} = A - \frac{B^2}{C},
\]

where

\[
A = \sum_{i=1}^{580} \left[ \mu_{\text{obs}}(z_i) - \mu_{\text{th}}(z_i) \right]^2 / \sigma_i^2,
\]

\[
B = \sum_{i=1}^{580} \left[ \mu_{\text{obs}}(z_i) - \mu_{\text{th}}(z_i) \right] / \sigma_i^2,
\]

\[
C = \sum_{i=1}^{580} 1 / \sigma_i^2,
\]

and \( \sigma_i \) stands for the 1σ uncertainty associated to the \( i \)th data point. Here, \( \mu_{\text{th}}(z) \) is the theoretical distance module defined as \[32, 33\]

\[
\mu_{\text{th}}(z) = 5 \log_{10} D_L(z) + \mu_0,
\]

where \( \mu_0 = 42.38 - 5 \log_{10} h \) and \( h \) is the Hubble constant \( H_0 \) in units of 100 km s\(^{-1}\) Mpc\(^{-1}\). Also the Hubble-free luminosity distance \( D_L(z) \) for the flat universe is given by

\[
D_L(z) = (1 + z) \int_0^z \frac{dz'}{E(z'; \mathbf{p})},
\]

with \( \mathbf{p} \) the model parameters and \( E(z; \mathbf{p}) = H(z; \mathbf{p})/H_0 \).

Next, we add the data from the observation of the CMB. The CMB peak from WMAP observations arises from acoustic oscillations of the primeval plasma just before the universe becomes transparent. The structure of the anisotropies of the CMB radiation depends on two
eras in cosmology including last scattering and today [34, 35]. They can also be applied to limit the model parameters. The $\chi^2$ from the CMB constraint is given by

$$\chi^2_{\text{CMB}} = \frac{(R_{\text{obs}} - R_{\text{th}})^2}{\sigma_R^2}. \tag{36}$$

Here, the theoretical value of CMB shift parameter, $R_{\text{th}}$, is defined as [34, 35]

$$R_{\text{th}} = \sqrt{\Omega_m} \int_0^{z_{\text{rec}}} \frac{dz'}{E(z'; \mathbf{p})}, \tag{37}$$

where $z_{\text{rec}} \simeq 1091.3$ is the redshift at the recombination epoch, which has been updated in the 7-year WMAP (WMAP7) data [36]. Also the observational value of $R_{\text{obs}}$ has been updated to $1.725 \pm 0.018$ from the WMAP7 data [36]. The shift parameter $R$ relates the angular diameter distance to the last scattering surface, the comoving size of the sound horizon at $z_{\text{rec}}$ and the angular scale of the first acoustic peak in CMB power spectrum of temperature fluctuations [34, 35].

Finally, we further add the data from the observation of the large scale structure (LSS). Here, we use the distance parameter $A$ of the measurement of the BAO peak in the distribution of Sloan Digital Sky Survey (SDSS) luminous red galaxies [37, 38], which contains the main information of the observations of LSS. Using the BAO data, one can minimize the $\chi^2_{\text{BAO}}$ defined as [37, 38],

$$\chi^2_{\text{BAO}} = \frac{(A_{\text{obs}} - A_{\text{th}})^2}{\sigma_A^2}, \tag{38}$$

where

$$A_{\text{th}} = \sqrt{\Omega_m} E(z_b; \mathbf{p})^{-1/3} \left( \frac{1}{z_b} \int_0^{z_b} \frac{dz'}{E(z'; \mathbf{p})} \right)^2, \tag{39}$$

and $z_b = 0.35$ is the redshift of luminous red galaxies sample of the SDSS. Also $A_{\text{obs}} = 0.469 (n_s/0.98)^{-0.35} \pm 0.017$ is measured from the SDSS data [38] and the scalar spectral index $n_s$ is taken to be 0.968, which has been updated from the WMAP7 data [36].

Note that to fit the model parameters one needs to determine the dimensionless Hubble parameter $E(z; \mathbf{p}) = H(z; \mathbf{p})/H_0$ appeared in Eqs. (35), (37) and (39). To do so, using Eqs. (2), (4), (23) and (24) one can obtain the dimensionless Hubble parameter as

$$E(z; \mathbf{p}) = \frac{H(z; \mathbf{p})}{H_0} = \frac{1}{2} (1 - \Omega_{m0}) + \frac{3}{4} (1 - \Omega_{m0})^2 + \Omega_{m0} (1 + z)^3 \right]^{1/2}. \tag{40}$$

As the normalized likelihood function is defined by $\mathcal{L} = e^{-(\chi^2_{\text{total}} - \chi^2_{\text{min}})/2}$ [39], the best fit value of the model parameters follows from minimizing the sum

$$\chi^2_{\text{total}} = \chi^2_{\text{SN}} + \chi^2_{\text{CMB}} + \chi^2_{\text{BAO}}. \tag{41}$$

The results are summarized in Table 1, where we also list the best fit value of the corresponding parameter of the $\Lambda$CDM model for comparison. At 68.3% and 95.4% confidence levels, we obtain the best fit value $\Omega_{m0} = 0.262^{+0.013}_{-0.011}(1\sigma)_{+0.025}^{0.027}(2\sigma)$ for the full data sets including SNeIa+CMB+BAO. The best fit value obtained for $\Omega_{m0}$ is slightly smaller than the corresponding one in the $\Lambda$CDM model. The total $\chi^2$ of the best fit value of the QCD ghost $f(T)$-gravity model is $\chi^2_{\text{min}} = 606.567$ for the full data sets with degrees of freedom (dof) = 582. The reduced $\chi^2$ is 1.042, which is acceptable, but $\chi^2_{\text{min}}$ is larger than the one for the $\Lambda$CDM model, $\chi^2_{\Lambda\text{CDM}} = 562.531$, for the same data sets. The relative (normalized) likelihood function $\mathcal{L}$ versus $\Omega_{m0}$ is also shown in Figure 5.
5 Cosmographic analysis

Here, using a cosmographic analysis approach introduced by Capozziello et al. [25] we check the viability of our model without the need of explicitly solving the field equations and fitting the data. In this approach, the parameters of a given \( f(T) \) model must be chosen in such a way that the model-independent constraints on the cosmographic parameters \( (h, q_0, j_0, s_0, l_0) \), listed in Table I in [25], are satisfied. Here, \( h, q_0, j_0, s_0 \) and \( l_0 \) are usually referred to as the Hubble constant, deceleration, jerk, snap, and lerk parameters, respectively. As we already obtained, imposing the condition (21) on our model (20) yields Eq. (22). This yields the \( f_i = f(i)(T_0)/(6H_0^2)^{(i-1)} \) values, given by Eqs. (4.23)-(4.26) in Capozziello et al. [25], for \( i = (2, 3, 4, 5) \) where \( f(i)(T) = d^i f/dT^i \) to be expressed as function of \( \beta \) only when we fix \( \Omega_{m0} = 0.1329/h^2 \) from the WMAP7 data. Following [25] for each \( f_2 \) value of the sample obtained above from the cosmographic parameters analysis, we solve \( \hat{f}_2(\beta) = f_2 \). Then, we estimate the theoretically expected values for the other derivatives \( (f_3, f_4, f_5) \). The median and 68% and 95% confidence ranges are obtained as

\[
\begin{align*}
  f_3 &= 0.208^{+0.150}_{-0.336} - 0.197^{+0.197}_{-0.789} \\
  f_4 &= 0.857^{+0.433}_{-0.966} + 0.567^{+0.567}_{-2.269} \\
  f_5 &= 3.281^{+1.658}_{-3.696} - 2.172^{+2.172}_{-8.682}.
\end{align*}
\]

Now we compare the above results with the model-independent constraints on the \( f_i \) values given in Table II in [25]. Following [25] we use only the 68% confidence level which we compare the above constraints to. Our comparison shows that the values of \( (f_3, f_4, f_5) \) take place in the 68% confidence level in Table II in [25]. Therefore, we conclude that QCD ghost \( f(T) \)-gravity model (23) is favored by the observational data.

6 Generalized second law of thermodynamics

Here, we examine the validity of the generalized second law (GSL) of gravitational thermodynamics for our model. According to the GSL, entropy of the matter inside the horizon beside the entropy associated with the surface of horizon should not decrease during the time [40]. Karami and Abdolmaleki [28] showed that within the framework of \( f(T) \)-gravity, the GSL for a spatially flat FRW universe enclosed by the dynamical apparent (Hubble) horizon \( \tilde{r}_A = H^{-1} \) and containing only the pressureless matter is given by

\[
T_A \dot{S}_{tot} = \frac{9}{8G} \left( \frac{f - 2T f_T}{f_T + 2T f_{TT}} \right) \left[ 4f_{TT} + \left( \frac{f - 2T f_T}{f_T + 2T f_{TT}} \right) \left( \frac{f_T + 5T f_{TT}}{T^2} \right) \right],
\]

where \( S_{tot} = S_m + S_A \) is the total entropy due to different contributions of the matter and the horizon. Here, \( T_A \) is the Hawking temperature on the apparent horizon \( \tilde{r}_A \). Note that the horizon entropy in \( f(T) \)-gravity given by

\[
S_A = \frac{A f_T}{4G},
\]

where \( A = 4\pi \tilde{r}_A^2 \), is valid only when \( f_{TT} \) is small [27]. To check this, we plot \( f_{TT} \) versus \( T/T_0 \) for our model (23) in Fig. 6. Figure shows that the \( f_{TT} \) is very small throughout history of the universe. This confirms the validity of Eq. (44) for our model.
Now we examine the validity of the GSL, Eq. (43), for the QCD ghost \( f(T) \)-gravity model (23). The GSL reads

\[
GTA \dot{S}_{\text{tot}} = \frac{9}{16} \left[ \frac{2 \left( 4(-T)^{3/2} + 9\beta T + (8\sqrt{-T} - 3\beta)\beta^2 \right) + \beta (T + \beta^2) \ln(T/T_0)}{\sqrt{-T} (\beta - 2\sqrt{-T})^2} \right].
\tag{45}
\]

The variation of the GSL (45) versus \( t/t_0 \) is plotted in Fig. 7. Figure shows that the GSL for our model is satisfied throughout history of the universe.

### 7 Growth rate of matter density perturbation

Here, we study the growth rate of matter density perturbation in QCD ghost \( f(T) \)-gravity model. The origin of structure in the universe is seeded by the small quantum fluctuations generated at the inflationary epoch. These small perturbations over time grew to become all of the structure we observe. Once the universe becomes matter dominated primordial density inhomogeneities \((\delta \rho_m/\rho_m \sim 10^{-5})\) are amplified by gravity and grow into the structure we see today [41]. The evolution equation for the matter density contrast \( \delta_m = \delta \rho_m/\rho_m \) in \( f(T) \)-gravity is given by [41]

\[
\ddot{\delta}_m + 2H \dot{\delta}_m - 4\pi G_{\text{eff}} \rho_m \delta_m = 0,
\tag{46}
\]

where \( G_{\text{eff}} \) is the effective Newton’s constant which is related to \( G \) by \( G_{\text{eff}} = \frac{G}{T} \). In the case of \( f(T) = T \), we have \( G_{\text{eff}} = G \) and Eq. (46) recovers the corresponding linear matter perturbation equation in TEGR.

Note that for the matter dominated universe, i.e. \( H^2 = k^2 \rho_m/3 \), solution of Eq. (46) yields \( \delta_m = a \). Hence, we introduce a new variable \( g(a) \), namely \( g(a) = \delta_m/a \) which does not depend on \( a \) during the matter dominated era. Thus, the natural choice for the initial conditions are \( g(a_i) = 1 \) and \( \frac{dg}{d\ln a} \bigg|_{a=a_i} = 0 \) [41]. The initial moment should be taken during the matter era, e.g., \( a_i = 1/31 \) (i.e., \( z_i = 30 \)).

In terms of new variable \( g(a) \), Eq. (46) becomes

\[
\frac{d^2 g}{d\ln a^2} + \left( 4 + \frac{\dot{H}}{H^2} \right) \frac{dg}{d\ln a} + \left( 3 + \frac{\dot{H}}{H^2} - \frac{4\pi G_{\text{eff}} \rho_m}{H^2} \right) g = 0.
\tag{47}
\]

From Eqs. (14), (15), (23) and (24) one can obtain

\[
\frac{\dot{H}}{H^2} = -3 \left( \frac{2f_T - f/T}{f_T + 2Tf_{TT}} \right) = -3 \left[ \frac{E + \Omega_{m_0} - 1}{2E + \Omega_{m_0} - 1} \right],
\tag{48}
\]

where \( E(z) = H(z)/H_0 \) is given by Eq. (40). Also with the help of Eqs. (11) and (48) one can get

\[
\frac{4\pi G_{\text{eff}} \rho_m}{H^2} = \frac{3}{2} \left( 2 - \frac{f}{Tf_T} \right) = \frac{3}{2} \left[ \frac{E + \Omega_{m_0} - 1}{2E + (\Omega_{m_0} - 1) \ln E} \right].
\tag{49}
\]

In general, there is no analytical solution to Eq. (47), and we need to solve it numerically. In Fig. 8, we plot the evolutionary behavior of \( g(a) \) normalized to today’s value, with the best fitting values of the QCD ghost \( f(T) \)-gravity model and the ΛCDM model. Figure 8 shows that \( g(a) \) for the ghost \( f(T) \)-gravity model like the ΛCDM decreases during history of the universe.

In Fig. 9, we also plot the evolution of the growth factor \( f(z) \) defined as [42]

\[
f(z) = \frac{d \ln \delta_m}{d \ln a} = -(1+z) \frac{d \ln \delta_m}{dz},
\tag{50}
\]
with the best fitting values of the ghost $f(T)$-gravity model and the $\Lambda$CDM model. The 11 data of the growth factor are summarized in Table 2. Figure 9 shows that the ghost $f(T)$-gravity model can not be discriminated by the data, and both this model and the $\Lambda$CDM model fit the data very well.

8 Conclusions

Here, we reconstructed a $f(T)$ model according to the QCD ghost DE paradigm. In the framework of $f(T)$ modified teleparallel theory, we considered a spatially flat FRW universe filled only with the pressureless matter. Then, we obtained the time evolution of the dark torsion scalar $T = -6H^2$ (or the Hubble parameter). We also calculated the effective torsion EoS parameter of the ghost $f(T)$-gravity model as well as the deceleration parameter of the universe. Furthermore, we fitted the model with current observational data, including SNeIa, CMB and BAO data. Using a cosmographic analysis approach, we also checked the viability of our model without the need of explicitly solving the field equations and fitting the data. We further examined the validity of the GSL of gravitational thermodynamics for the QCD ghost $f(T)$-gravity model. Finally, we pointed out the growth of structure formation in our model. Our results show the following.

(i) The condition $f/T \rightarrow 1$ is satisfied for our model at high redshift ($|T| \rightarrow \infty$) which is compatible with the primordial nucleosynthesis and CMB constraints.

(ii) The effective torsion EoS parameter $\omega_T$ varies from $-0.5$ at early time to $-1$ at late time, which is similar to freezing quintessence DE. For the present time we obtain $\omega_{T0} = -0.79$.

(iii) The variation of the deceleration parameter $q$ shows that the universe transits from an early matter dominant epoch ($q = 0.5$) to the de Sitter era ($q = -1$) in the future, as expected. Also at the near past $t/t_0 = 0.57$ we have a cosmic deceleration ($q > 0$) to acceleration ($q < 0$) transition. The deceleration parameter $q_0 \simeq -0.4$ obtained at the present is compatible with the recent observations.

(iv) The best fit value of the model parameter is $\Omega_{mb} = 0.262^{+0.013}_{-0.013}(1\sigma)^{+0.027}_{-0.025}(2\sigma)$ for the full data sets including SNeIa+CMB+BAO. The minimal $\chi^2$ gives $\chi^2_{\text{min}} = 606.567$ with dof = 582. The reduced $\chi^2$ equals to 1.042 which is acceptable. But $\chi^2_{\text{min}}$ is larger than the one for the $\Lambda$CDM model, $\chi^2_{\Lambda\text{CDM}} = 562.531$, for the same data sets.

(v) Cosmographic analysis shows that the QCD ghost $f(T)$-gravity model is favored by the observational data.

(vi) The GSL of gravitational thermodynamics holds for our $f(T)$ model throughout history of the universe.

(vii) The evolutionary behavior of the growth factor of matter perturbation shows that the ghost $f(T)$-gravity model can not be discriminated by the data, and both this model and the $\Lambda$CDM model fit the data very well.

Acknowledgements

The authors thank the reviewers for their valuable comments. The works of K. Karami and Z. Safari have been supported financially by Research Institute for Astronomy and Astrophysics of Maragha (RIAAM) under research project No. 1/2782-41.
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Figure 1: The evolution of QCD ghost $f(T)$-gravity model, Eq. (23), versus $T/T_0$. The dashed line denotes the model $f(T) = T$ corresponding to the case of teleparallel gravity for comparison. Auxiliary parameters are: $H_0 = 70.6$ km s$^{-1}$ Mpc$^{-1}$ [25] and $\Omega_{\text{m0}} = 0.262$. For these values one finds $\beta = \sqrt{6} H_0 (1 - \Omega_{\text{m0}}) = 127.625$ and $f(T_0) = -51976.9$.

Figure 2: The evolution of the torsion scalar (or the squared Hubble parameter), Eq. (27), versus $t/t_0$. Auxiliary parameters as in Fig. 1.
Figure 3: The effective torsion EoS parameter of the QCD ghost $f(T)$-gravity model, Eq. (28), versus $t/t_0$. Auxiliary parameters as in Fig. 1.

Figure 4: The deceleration parameter of the QCD ghost $f(T)$-gravity model, Eq. (29), versus $t/t_0$. Auxiliary parameters as in Fig. 1.
Table 1: The best fit value of \( \Omega_{m0} \) within the 68.3% and 95.4% confidence intervals for each observational data set for the QCD ghost \( f(T) \)-gravity model. The last column shows the best fit result of the \( \Lambda \)CDM model using the full data sets for comparison.

| Parameter | SN | SN+CMB | SN+CMB+BAO | \( \Lambda \)CDM |
|-----------|----|--------|------------|-----------------|
| \( \Omega_{m0} \) | 0.179\( ^{+0.016}_{-0.014-0.028} \) | 0.247\( ^{+0.014+0.030}_{-0.014-0.028} \) | 0.262\( ^{+0.013+0.027}_{-0.013-0.025} \) | 0.273\( ^{+0.014+0.028}_{-0.013-0.026} \) |

Figure 5: 1D Likelihood for \( \Omega_{m0} \). Horizontal lines give the bounds with 1\( \sigma \) and 2\( \sigma \) level of confidence.
Figure 6: $f_{T^T}$ versus $T/T_0$ for the QCD ghost $f(T)$-gravity model (23). Auxiliary parameters as in Fig. 1.

Figure 7: The variation of the GSL, Eq. (45), versus $t/t_0$ for model (23). Auxiliary parameters as in Fig. 1.
Table 2: The observational data for the linear growth rate $f_{\text{obs}}(z)$.

| $z$  | 0.15 | 0.22 | 0.32 | 0.35 | 0.41 | 0.55 | 0.60 | 0.77 | 0.78 | 1.4 | 3.0 |
|------|------|------|------|------|------|------|------|------|------|-----|-----|
| $f_{\text{obs}}$ | 0.51 | 0.60 | 0.654 | 0.70 | 0.50 | 0.73 | 0.91 | 0.70 | 0.90 | 1.46 |
| $1\sigma$ | 0.11 | 0.10 | 0.18 | 0.18 | 0.07 | 0.18 | 0.07 | 0.36 | 0.08 | 0.24 | 0.29 |

Ref. [43] [44] [45] [46] [44] [47] [44] [48] [44] [49] [50]

Figure 8: Linear growth function $D = \frac{\delta_m}{\delta_{m0}}$, normalized to today’s value, relative to its value in a pure-matter model ($D = a$), for the ghost $f(T)$-gravity model and the $\Lambda$CDM model using the full data sets.

Figure 9: Evolution behaviors of the growth factor for the ghost $f(T)$-gravity model and the $\Lambda$CDM model using the full data sets.