Spinor particle creation in near extremal Reissner–Nordström black holes

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Received 25 April 2015, revised 9 July 2015
Accepted for publication 28 July 2015
Published 4 September 2015

Abstract

The pair production of spinor particles, which can be captured by the solution of the Dirac equation with an appropriate boundary condition, in charged black holes is investigated. We obtain the closed form of the production rate in the near extremal limit of Reissner–Nordström black holes. The cosmic censorship conjecture is seemingly guaranteed by the pair production process. Moreover, the absorption cross section ratio and retarded Green’s functions of the spinor fields calculated from the gravity side match well with those of spinor operators in the dual conformal field theory (CFT) side.

Keywords: black holes, spinor pair production, AdS/CFT correspondence

1. Introduction

Spontaneous pair production in which virtual particles and antiparticles from vacuum fluctuations are separated, by various mechanisms, to become real pairs is essentially a significant quantum phenomenon. In particular, the pair production occurring in charged black holes mixes two independent processes, namely the Schwinger mechanism by an electromagnetic force [1]

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and Hawking radiation by tunneling through the horizon [2]. There are numerous studies of pair production in the Reissner–Nordström (RN) black holes in the literature [3–13].

In our previous study [14], we considered the pair production of scalar particles created from a particular background—the near horizon region of near extremal RN black holes. The consideration for only taking into account the effect at the near horizon is inspired by an intuitive expectation that pair production should mainly occur in this region, which contains the causal boundary for the Hawking radiation and the dominated electric field for the Schwinger mechanism. The final results indeed confirm such anticipation. In addition, the primary motivation for our study is to understand the holographic dual interpretation for the pair production process in black holes in the context of the anti-de Sitter/conformal field theory (AdS/CFT) correspondence. The holographic description of black holes has been intensively studied after the leading work on Kerr/CFT correspondence [15], see, e.g., the recent review [16] and the references therein. The preliminary information is usually derived from the study on near extremal black holes. For example, in near extremal RN black holes the near horizon geometry has a particular AdS \times S^2 structure which allows one to verify the holographic duality with the knowledge of AdS/CFT correspondence [17–19].

In the present paper, we extend the previous study on scalar particles to explore spinor pair production. We first solve the Dirac equation for a probe massive charged spinor field in the near horizon region of the near extremal RN black holes. Due to the spherical symmetry, the spinor field can be expanded by the spherical spinors [20], then the Dirac equation can be separated and reduces to two first order coupled ordinary differential equations for two radial functions. Furthermore, these two equations can be transformed into the well-known hypergeometric equations. Consequently, the exact solutions are obtained in terms of the hypergeometric functions. From the behavior of the spinor field at the asymptotic (outer boundary) and horizon (inner boundary) we can identify the corresponding ingoing and outgoing modes on the boundaries. There are two equivalent boundary conditions for capturing the pair production process [14]. Here we will enforce the 'particle viewpoint' boundary condition by imposing a vanishing ingoing flux at the outer boundary. The other three fluxes have an intuitive physical interpretation from the particle viewpoint: the outgoing flux at the outer boundary representing the pair-produced particles, and the outgoing and ingoing fluxes at the inner boundary describing the virtual and re-annihilated particles respectively. Finally the physical quantities of pair production, i.e. the Bogoliubov coefficients (vacuum persistence and mean number of pairs) and the absorption cross section ratio, can be derived from the ratios of these three fluxes [21, 22]. Moreover, pair production can occur only when the spinor field generated at the horizon can propagate to the infinity. The existence condition, in both scalar and spinor fields, is identical to the violation of the Breitenlohner–Freedman (BF) bound [23, 24] in AdS_2. Thus, the pair production indeed corresponds to the instability of the probe field (or perturbations) in AdS_2 (or effectively AdS_2 spacetime based on the RN/CFT correspondence [25–30]). Consequently, the conformal weights of the spinor operator dual to the charged spinor field are complex, indicating an instability in the dual CFT_2. It is worth noting that the existence of pair production strongly requires the charge of the produced spinor particle to be bigger than its mass. By the charge and energy conservations\(^7\), the black holes in the pair production process should lose charge more than mass, and this property ensures the cosmic censorship conjecture. In addition, using the RN/CFT correspondence, we show that the absorption cross section ratio and the

\(^7\) This argument is not completely rigorous since we only consider the near horizon region and the gravitational backreaction is neglected in the probe limit. In addition, it was claimed in [31] that the cosmic censorship conjecture for the near extremal RN black holes can be violated by quantum tunneling of spin \(\frac{1}{2}\) charged particles.
retarded Green’s function of the bulk spinor field computed from the gravity side match well with those of the spinor operators in the dual CFT$_2$, and thus the pair production ratio also can be understood holographically. For other previous studies on Schwinger pair creation in AdS or dS spacetime backgrounds, see for example [32, 33], and see [34–36] for other studies on the dual CFT description of the Schwinger effect.

The rest of the paper is organized as follows: we analytically solve the Dirac equation for charged spinor fields in the near horizon near extreme RN black hole background in section 2; then the pair production rate and absorption cross section ratio of the charged spinor are obtained by choosing appropriate boundary conditions in section 3; in 4, the holographic description of the spinor absorption cross section ratio, the retarded Green’s function and the pair production ratio are analyzed based on the RN/CFT correspondence; we draw the conclusions and discussions in section 5. Some useful properties of spherical spinors and hypergeometric functions are listed in appendices A and B.

2. Spinor field in the RN black holes

2.1. Dirac equation

The Dirac equation for a massive and charged spinor field, $\Psi$, in a curved spacetime coupled with an electromagnetic field, $A_\mu$, is given by

$$\left[\gamma^a e^a_\mu \left(\partial_\mu + \Gamma_\mu - iqA_\mu\right) + m\right] \Psi = 0,$$

where $m$ and $q$ denote the mass and charge of the spinor field. The tetrad $e^a_\mu$ and $\Gamma$ one-form represent the curved spacetime effects in the light of the metric $g_{\mu\nu}$ and connection one-form $\omega_{\lambda\beta}$ which are defined as

$$\eta_{ab} = e^a_\mu e^b_\nu g_{\mu\nu}, \quad \Gamma = \Gamma_\mu dx^\mu = \frac{1}{8} \left[\gamma^a, \gamma^b\right] \omega_{ab} = \frac{1}{4} \gamma^a \gamma^b \omega_{ab}.\tag{2}$$

In this paper, the Greek and Latin letters symbolize the coordinate and frame indices respectively, and the convention for the gamma matrices is given in appendix A.

For a spherically symmetric spacetime coupled with an electric field, the metric and gauge potential are generally characterized by three radial dependent functions, $f(r)$, $\rho(r)$ and $\phi(r)$, as

$$ds^2 = -f(r)dr^2 + \frac{dr^2}{f(r)} + \rho^2(\text{d}^2 \Omega^2), \quad A = \phi(r)dr,\tag{3}$$

where $r$ is the usual radial coordinate and the explicit expressions of these functions depend on the exact solution. In this paper, we will consider the near horizon solution of near extremal RN black holes and the corresponding functions are

$$f = \frac{r^2 - B^2}{Q^2}, \quad \rho = Q, \quad \phi = -\frac{r}{Q},\tag{4}$$

in which $Q$ is the electric charge of the RN black hole and the parameter $B$ describes the deviation from the extreme geometry and it plays the role of the horizon radius of the near extreme geometry [14]. Besides, the near horizon near extremal geometry in equation (4) is diffeomorphic to its extremal counterpart. The Dirac equation can be formulated in different choices of frame in association with different representations of the spinor field. In the following we are going to solve the Dirac equation in the rotation frame, $\hat{e}^a := e^a_\mu dx^\mu$, defined by the tetrad
Straightforwardly, we can compute the following essential $\Gamma$-term
\[ \gamma^a e_a \rho^\nu \gamma_\nu = \gamma^1 \left( \frac{\sqrt{f} \rho'}{\rho} + \frac{f'}{4\sqrt{f}} \right) + \gamma^2 \cot \frac{\theta}{2}, \]
and therefore the Dirac equation in spherically symmetric spacetimes explicitly reduces to
\begin{equation}
\begin{aligned}
\left[ \gamma^0 \frac{1}{\sqrt{f}} \left( \partial_t - i q \phi \right) + \gamma^1 \sqrt{f} \left( \partial_r + \frac{\rho'}{\rho} + \frac{f'}{4f} \right) \\
+ \gamma^2 \frac{1}{\rho} \left( \partial_\theta + \frac{\cot \theta}{2} \right) + \gamma^3 \frac{1}{\rho \sin \theta} \partial_\phi + m \right] \Psi = 0.
\end{aligned}
\end{equation}
Moreover, the above equation can be further simplified, basically absorbing those connection terms as a rescaling of the spinor field, according to the following definition
\[ \Psi = \left( -\det g g^{rr} \right)^{-\frac{1}{2}} \Phi = \rho^{-1} \sin \frac{\theta}{2} f^{-\frac{1}{2}} \Phi. \]
Finally the considerable version of the Dirac equation is
\begin{equation}
\begin{aligned}
\left[ \gamma^0 \frac{1}{\sqrt{f}} \left( \partial_t - i q \phi \right) + \gamma^1 \sqrt{f} \partial_r + \gamma^2 \frac{1}{\rho} \partial_\theta + \gamma^3 \frac{1}{\rho \sin \theta} \partial_\phi + m \right] \Phi = 0.
\end{aligned}
\end{equation}

2.2. Exact solution

The spinor field in spherically symmetric spacetimes can be expanded in terms of the orthonormal spherical spinors $[20]$, $\Phi^\pm_{n,n}$, whose components are defined by spherical harmonics. The parameter $\kappa = \mp (j + 1/2)$ is specified by the angular momentum quantum number $j$ (half-integer) and $n$ is its projection. The spherical spinors have desirable properties with respect to gamma matrices $\gamma^0, \gamma^1$ and the operator $\mathcal{K} = \gamma^2 \partial_\theta + \gamma^3 \sin^{-1} \partial_\phi$, which allows one to separate the Dirac equation. The useful properties of the spherical spinors are summarized in appendix A. Hence, we impose the following ansatz of the spinor field
\[ \Phi(t, r, \theta, \phi) = e^{-i\omega t} [R_+(r) \Phi^+_{n,n}(\theta, \phi) + R_-(r) \Phi^-_{n,n}(\theta, \phi)]. \]
and then the Dirac equation, by using the properties (A23), reduces to two first order coupled equations for two radial functions $R_\pm(r)$ as
\begin{equation}
\begin{aligned}
\left( \sqrt{f} \partial_r \mp \frac{\kappa}{\rho} \right) R_\pm - i \left( \frac{\omega + q \phi}{\sqrt{f}} \pm m \right) R_\mp = 0.
\end{aligned}
\end{equation}
Technically, it is more proper to rewrite the equations such that the terms including function $f$ be combined together. This can be achieved by introducing new functions $\mathcal{R}_\pm = R_+ \pm R_-$, and then the first order equations are transformed to
\begin{equation}
\begin{aligned}
\left( \sqrt{f} \partial_r \mp \frac{i \omega + q \phi}{\sqrt{f}} \right) \mathcal{R}_\pm + \left( \frac{\kappa}{\rho} \pm im \right) \mathcal{R}_\mp = 0.
\end{aligned}
\end{equation}
For our considered background (4) of the near extremal RN black holes, the equations to be solved are
\[
\sqrt{r^2 - B^2} \frac{\partial}{\partial r} R_{\pm} + i \omega Q - q r \frac{\kappa}{\sqrt{r^2 - B^2}} R_{\pm} + \left( \frac{\kappa}{Q} \pm im \right) R_{\mp} = 0,
\]
(13)
in which the last coefficient is simply a constant. These two equations can be further expressed in a more desirable form in terms of the new radial coordinate \( z \),
\[
z = \frac{r + B}{2B},
\]
(14)
and two rescaled radial functions \( \tilde{R}_{\pm} \),
\[
\tilde{R}_{\pm} = \Sigma^{1/2} R_{\pm}, \quad \Sigma = (2B)^{i\omega} e^{\frac{\alpha}{2}}(z - 1)^{-i\frac{\alpha}{2}},
\]
(15)
where the parameters \( a, \tilde{a} \) and \( b \) (\( b \) is defined for later convenience) are
\[
a = qQ, \quad \tilde{a} = \frac{\omega Q^2}{B}, \quad b = \sqrt{(q^2 - m^2)Q^2 - \kappa^2}.
\]
(16)
Interestingly these three parameters are almost identical to the analogous parameters appearing in the scalar field pair production [14] except the term \( \kappa^2 = (j + 1/2)^2 \) with half-integer \( j \) in parameter \( b \) being replaced by \( (l + 1/2)^2 \) with integer \( l \). Finally, the first order coupled equations reduce to
\[
\Sigma^{1/2} \partial_z \tilde{R}_{\pm} + \frac{1}{\sqrt{z(z - 1)}} (\kappa \pm i mQ) \tilde{R}_{\mp} = 0,
\]
(17)
and they can be simply decoupled as two second order equations
\[
\left[ z(1 - z) \partial_z^2 + \frac{1}{2} \left( qQ + \frac{\omega Q^2}{B} \right) - (1 \mp i2qQ)z \right] \partial_z \tilde{R}_{\pm} + \left( \kappa^2 + m^2 Q^2 \right) \tilde{R}_{\pm} = 0.
\]
(18)
These two decoupled equations are just the hypergeometric equation (B1) with parameters
\[
\alpha_{\pm} = i(b \mp a), \quad \beta_{\pm} = -i(b \pm a), \quad \gamma_{\pm} = \frac{1}{2} \mp i(\tilde{a} + a).
\]
(19)
Thus the corresponding solutions are generically two independent hypergeometric functions, see appendix B. However, the actual solutions for functions \( \tilde{R}_{\pm} \) must satisfy the first order coupled equations (17) which should give constraints on integration constants. Therefore, there is only one free integration constant for each radial function.

Let us first show how one obtains the solutions around the point \( z = 0 \) in detail. A simple way of constructing the general solutions, suppose \( \tilde{R}_{\pm} = \tilde{R}_{\pm}^{(1)} + \tilde{R}_{\pm}^{(2)} \), consistent with the first coupled equations is to choose the first 'half' part of the solution in (B3)
\[
\tilde{R}_{\pm}^{(1)} = C_{\pm} F \left( i(b \mp a), -i(b \pm a); \frac{1}{2} \mp i(\tilde{a} + a); z \right),
\]
(20)
and then use the equations (17) to determine the complementary part, \( \tilde{R}_{\pm}^{(2)} \), as
\[
\tilde{R}_{\pm}^{(2)} = \tilde{C}_{\pm} e^{z \mp \frac{i}{2} (a + b)} F \left( \frac{1}{2} - i(b \mp \tilde{a}), \frac{1}{2} + i(b \pm \tilde{a}); 3 \mp i(\tilde{a} + a); z \right),
\]
(21)
where the coefficients $\tilde{C}_\pm$ are determined by $C_\pm$

\[
\tilde{C}_\pm = -(-1)^{\frac{1}{2} \pm (\tilde{a} - a)} C_\pm \frac{\kappa \pm i m Q}{2} (2B)^{\pm i 2 \alpha}.
\]

One can straightforwardly check that $\tilde{R}_\pm^{(2)}$ will reproduce the complementary part $\tilde{R}_\pm^{(1)}$ via the equations (17).

### 2.3. Asymptotic and near horizon behaviors

In order to obtain the ingoing and outgoing fluxes at both the inner and outer boundaries, we need the asymptotic and near horizon behaviors of the spinor field solution. For analyzing the asymptotic behaviors at $z = \infty$ ($r = \infty$), it is convenient to express the solution around the point $z = \infty$. Following a similar approach, firstly we chose a ‘half’ solution, a convenient choice is the first part in (B4) for $\tilde{R}_\pm^{(1)}$ and second part in (B4) for $\tilde{R}_\pm^{(1)}$, as

\[
\tilde{R}_\pm^{(1)} = C_\pm^\infty z^{\mp i (b - a)} F\left( \pm i(b - a), \frac{1}{2} \pm i(b + a); 1 \pm i2b; \frac{1}{z} \right).
\]

and the complementary half is determined by the first order equations (17)

\[
\tilde{R}_\pm^{(2)} = \tilde{C}_\pm^\infty z^{\pm i (b + a)} F\left( \mp i(b + a), \frac{1}{2} \mp i(b - a); 1 \mp i2b; \frac{1}{z} \right),
\]

where

\[
\tilde{C}_\pm^\infty = \mp C_\pm^\infty \frac{i(b - a)}{\kappa \mp i m Q} (2B)^{\pm i 2 \alpha}.
\]

Therefore, the asymptotic behavior of functions $\tilde{R}_\pm^\infty = \sum_{j = 1}^{\infty} \tilde{R}_\pm^{(1)} \simeq (2B)^{\pm i a} z^{\mp i b} \tilde{R}_\pm^\infty$ is

\[
\tilde{R}_\pm^\infty \simeq C_\pm^\infty (2B)^{\mp i a} z^{\mp i b} + \tilde{C}_\pm^\infty (2B)^{\pm i a} z^{\pm i b},
\]

and the corresponding ingoing and outgoing modes are

\[
\begin{align*}
\tilde{R}_+^{\infty (\text{in})} &= C_+^\infty (2B)^{-i a} z^{-i b}, & \tilde{R}_+^{\infty (\text{out})} &= \tilde{C}_+^\infty (2B)^{-i a} z^{-i b} = -\frac{b - a}{\kappa \mp i m Q} C_+^\infty (2B)^{i a} z^{+i b}, \\
\tilde{R}_-^{\infty (\text{out})} &= C_-^\infty (2B)^{i a} z^{+i b}, & \tilde{R}_-^{\infty (\text{in})} &= \tilde{C}_-^\infty (2B)^{i a} z^{+i b} = \frac{b - a}{\kappa \mp i m Q} C_-^\infty (2B)^{-i a} z^{-i b}.
\end{align*}
\]

It is obvious that the condition for the existence of propagating modes requires that the parameter $b$ has a real value, namely,

\[
(q^2 - m^2)Q^2 - \left( j + \frac{1}{2} \right)^2 > 0.
\]

with half-integer $j$. This condition directly implies $q^2 > m^2$ which is satisfied by known physical spin-$1/2$ particles, such as electrons and positrons. Moreover, according to the charge and energy conservations, the black holes in the process of pair production will lose their charge more than mass. This consequence ensures the cosmic censorship conjecture.

Similarly, for analyzing the behavior at the horizon, $z = 1$ ($r = B$), the solutions can be transformed, via (B6), to the following expressions
\[ \hat{R}_\pm^H = C_\pm^H F \left( \pm i(b - a), \mp i(b + a); \frac{1}{2} \pm i(\tilde{a} - a); 1 - z \right) \]
\[ + \tilde{C}_\pm^H (z - 1)^{\frac{1}{2} \pm i(\tilde{a} - a)} \frac{1}{2} \pm i(\tilde{a} + a) \]
\[ \times F \left( 1 \pm i(b + a), 1 \mp i(b - a); \frac{3}{2} \mp i(\tilde{a} - a); 1 - z \right). \] (29)

and the coefficients are

\[ C_\pm^H = \frac{\Gamma(1 \pm i2b) \Gamma \left( \frac{1}{2} \mp i(\tilde{a} - a) \right)}{\Gamma(1 \pm i(b + a)) \Gamma \left( \frac{1}{2} \mp i(b - \tilde{a}) \right) C_\pm^\infty} \]
\[ + \frac{\Gamma(1 \mp i2b) \Gamma \left( \frac{1}{2} \pm i(\tilde{a} - a) \right)}{\Gamma(1 \mp i(b - a)) \Gamma \left( \frac{1}{2} \pm i(b + \tilde{a}) \right) \tilde{C}_\pm^\infty}, \]
\[ \tilde{C}_\pm^H = \frac{\Gamma(1 \pm i2b) \Gamma \left( -\frac{1}{2} \pm i(\tilde{a} - a) \right)}{\Gamma(\pm i(b - a)) \Gamma \left( \frac{1}{2} \pm i(b + \tilde{a}) \right) C_\pm^\infty} \]
\[ + \frac{\Gamma(1 \mp i2b) \Gamma \left( -\frac{1}{2} \mp i(\tilde{a} - a) \right)}{\Gamma(\mp i(b + a)) \Gamma \left( \frac{1}{2} \mp i(b - \tilde{a}) \right) \tilde{C}_\pm^\infty}. \] (30)

Thus, near horizon the solutions \( \mathcal{R}_\pm^H = \Sigma^\pm \hat{R}_\pm^H \approx (2B)^{\mp i\alpha} (z - 1)^{\frac{1}{2} \pm i\frac{\alpha}{2} \pm \alpha} \hat{R}_\pm^H \) behave like

\[ \mathcal{R}_\pm^H \approx C_\pm^H (2B)^{\mp i\alpha} (z - 1)^{\frac{1}{2} \pm i\frac{\alpha}{2} \pm \alpha} + \tilde{C}_\pm^H (2B)^{\mp i\alpha} (z - 1)^{\frac{1}{2} \pm i\frac{\alpha}{2} \pm \alpha}. \] (31)

The ingoing and outgoing modes, for case \( \tilde{a} = a > 0 \) which covers the extremal limit \( B \to 0 \) (i.e. \( \tilde{a} \to \infty \)), are

\[ \mathcal{R}_+^H(\text{out}) = C_+^H (2B)^{-i\alpha} (z - 1)^{-\frac{1}{2} \mp i\frac{\alpha}{2}} \]
\[ \mathcal{R}_+^H(\text{in}) = \tilde{C}_+^H (2B)^{-i\alpha} (z - 1)^{-\frac{1}{2} \pm i\frac{\alpha}{2}} \to 0, \]
\[ \mathcal{R}_-^H(\text{in}) = C_-^H (2B)^{i\alpha} (z - 1)^{-\frac{1}{2} \mp i\frac{\alpha}{2}} \]
\[ \mathcal{R}_-^H(\text{out}) = \tilde{C}_-^H (2B)^{i\alpha} (z - 1)^{-\frac{1}{2} \pm i\frac{\alpha}{2}} \to 0. \] (32)

Due to the term \((z - 1)^{1/2}\), both modes \( \mathcal{R}_+^H(\text{in}) \) and \( \mathcal{R}_-^H(\text{out}) \) approach zero at the horizon.

### 3. Spinor particle creation

The pair production process, as discussed in the previous work for the scalar field case \([14]\), can be captured by the ratios of the ingoing/outgoing fluxes at the horizon and the asymptotic with appropriate boundary conditions. Actually, there are two equivalent boundary conditions associated with the pair production depending on two complementary particle or antiparticle viewpoints. In this paper, we adopt the particle viewpoint boundary condition which imposes no ingoing flux at the infinity. In this case, the other three non-vanishing fluxes have the following intuitive physical interpretation. The outgoing flux at infinity, \( D_\text{out}(\text{in}) \), represents the pair produced particles. The outgoing and ingoing flux at horizon, \( D_\text{out}(\text{in}) \), correspond to the virtual and re-annihilated particles respectively. The Bogoliubov coefficients, \(|A|^2\) (vacuum persistence amplitude) and \(|B|^2\) (mean number of pairs) can be obtained from the following flux ratios \([21, 22]\).
and the absorption cross section ratio is given by

$$\sigma_{\text{abs}} = \frac{|D_{H}^{(\text{out})}|}{|D_{H}^{(\text{in})}|}.$$  (34)

However, unlike the scalar field case, the flux conservation for the spinor particle production is indeed $|D_{H}^{(\text{out})}| + |D_{\infty}^{(\text{out})}| = |D_{H}^{(\text{in})}|$, see the discussion for QED in [37], which leads to the Bogoliubov relation $|A|^2 + |B|^2 = 1$. Accordingly, there is only one independent information in these three physical quantities.

The vector current density of a spinor field is given by

$$J_{\mu} = \sqrt{-g} \, \bar{\Psi} e_{\mu} \gamma^{\mu} \Psi,$$  (35)
in which the Dirac adjoint is defined as

$$\bar{\Psi} = \Psi^{\dagger} \gamma^{0}.$$  (36)

For the general spherically symmetric background it reduces to

$$J_{\mu} = f^{-1/2} \bar{\Psi} e_{\mu} \gamma^{0} \gamma^{\mu} \Psi.$$  (37)

The relevant radial flux, after integrating over whole solid angle $d\Omega = \sin \theta d\theta d\phi$, is

$$D := \int J d\Omega = - \int \left( R_{+}^{+} \Phi_{+}^{+} + R_{-}^{+} \Phi_{-}^{+} \right) \left( R_{+}^{0} \Phi_{+}^{0} + R_{-}^{0} \Phi_{-}^{0} \right) d\Omega$$

$$= - \frac{1}{2} \left( R_{+}^{+} R_{+}^{0} - R_{-}^{+} R_{-}^{0} \right).$$  (38)

The overall sign comes from the properties (A23).

By using the flux formula (38), one can straightforwardly compute the ingoing and outgoing fluxes at the horizon and asymptotic. The fluxes at the asymptotic region $r \to \infty$ ($z \to \infty$), according to (27), are

$$D_{\infty}^{(\text{in})} = - \frac{1}{2} \left( |C_{+}^{\infty}|^2 - |C_{-}^{\infty}|^2 \right) = - \frac{b}{a+b} |C_{+}^{\infty}|^2,$$

$$D_{\infty}^{(\text{out})} = - \frac{1}{2} \left( |C_{+}^{\infty}|^2 - |C_{-}^{\infty}|^2 \right) = \frac{b}{a+b} |C_{-}^{\infty}|^2.$$  (39)

Here the relations $|C_{+}^{\infty}|^2 = \frac{a-b}{a+b} |C_{-}^{\infty}|^2$ from equation (25) are used. The particle viewpoint boundary condition $D_{\infty}^{(\text{in})} = 0$ implies $C_{+}^{\infty} = 0$, and consequently by (25) implying $C_{-}^{\infty} = 0$.

The ingoing and outgoing fluxes at the near horizon region, $r \to B$ ($z \to 1$), according to (32) and the boundary condition $C_{+}^{\infty} = C_{-}^{\infty} = 0$, are

$$D_{H}^{(\text{in})} = \frac{1}{2} |C_{-}^{H}|^2 = \frac{1}{2} \left( \frac{\Gamma(1-i2b)\Gamma\left(\frac{1}{2} + i(\tilde{a} - a)\right)}{\Gamma(1 - i(b + a))\Gamma\left(\frac{1}{2} - i(b - a)\right)} \right)^2 |C_{-}^{\infty}|^2,$$

$$= \frac{\sinh(\pi a + \pi b) \cosh(\pi \tilde{a} - \pi b)}{\sinh(2\pi b) \cosh(\pi \tilde{a} - \pi a)} \frac{b}{b + a} |C_{-}^{\infty}|^2,$$  (40)

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Finally, we obtain the closed form for the Bogoliubov coefficients

\[
|A|^2 = \left|\frac{D^{\text{out}}}{D^{\text{in}}}_{H} \right|^2 = \frac{\sinh(\pi a - \pi b)\cosh(\pi a + \pi b)}{\sinh(\pi a + \pi b)\cosh(\pi a - \pi b)},
\]

\[
|B|^2 = \left|\frac{D^{\text{out}}}{D^{\text{in}}}_{H} \right|^2 = \frac{\sinh(2\pi b)\cosh(\pi a - \pi a)}{\sinh(\pi a + \pi b)\cosh(\pi a - \pi b)},
\]

and the absorption cross section ratio

\[
\sigma_{\text{abs}} = \left|\frac{D^{\text{out}}}{D^{\text{in}}}_{H} \right|^2 = \frac{\sinh(2\pi b)\cosh(\pi a - \pi a)}{\sinh(\pi a + \pi b)\cosh(\pi a + \pi b)}.
\]

It is worth noting an interesting relation between the absorption cross section ratio and the mean number of pairs: \(\sigma_{\text{abs}}(b \rightarrow -b) = -|B|^2\). The vacuum persistence amplitude, mean number of pairs and absorption cross section ratio for spinor and scalar particle productions are greatly analogous. The difference of three parameters \(a, \tilde{a}\) and \(b\) is explained after the definition in equation (16). Moreover, the expressions in equations (42), (43) for spinor particle production can be easily transformed to the results of scalar particle case by 

\[
\sinh(\pi a \pm \pi b) \rightarrow \cosh(\pi a \pm \pi b) \quad \text{and} \quad \cosh(\pi a \pm \pi a) \rightarrow \sinh(\pi a - \pi a).
\]

4. Dual CFT description

Recall that the spacetime background is the near horizon region of a near extreme RN black hole which is dual to a two-dimensional CFT with left- and right-hand central charges and temperatures

\[
c_L = c_R = \frac{6Q^3}{\ell}, \quad T_L = \frac{\ell}{2\pi Q}, \quad T_R = \frac{4B}{\pi Q^2},
\]

where \(\ell\) is a free parameter which can be interpreted as a measure of the U(1) bundle of the background spacetime\(^8\) [25–30]. In addition, from the asymptotic solution (27), one can determine the right-moving conformal dimension of the spinor operator dual to the charged spinor field via \(SL(2, R)\) isometry [38], which is complex

\[
h_R = \frac{1}{2} \pm ib,
\]

and the parameter \(b\) is defined in (16). Note that \(h_R\) is of the same form as the conformal dimension of the scalar operator dual to the charged scalar field in the same spacetime

\(^8\) The U(1) bundle is formed by the background U(1) Maxwell field. The parameter \(\ell\) has a geometric interpretation when uplifting the four-dimensional RN black hole equation (3) into its five-dimensional counterpart, in which \(\ell\) describes the radius of the extra circle in the fifth dimension [25–30].
background although $b$ possesses a different value to that in [14]. The condition for the existence of the propagating mode requires that the value of parameter $b$ should be real, i.e., 

$$(q^2 - m^2)Q^2 - \kappa^2 > 0.$$ 

From the field/operator duality in the AdS/CFT correspondence, the conformal dimension of the spinor operator in the boundary $d$-dimensional CFT is

$$\Delta = \frac{d}{2} + |m_{\text{eff}}|,$$

where $m_{\text{eff}}$ is the effective mass of the bulk spinor field [39]. The Breitenlohner–Freedman (BF) bound is violated when $m_{\text{eff}}$ becomes imaginary. Therefore, the inequality $(q^2 - m^2)Q^2 - \kappa^2 > 0$ can just be rewritten as the violation of the BF bound for the spinor field in AdS$_2$ (or effectively AdS$_3$) spacetime, i.e. the effective mass square of the spinor field satisfies

$$m_{\text{eff}}^2 \equiv m^2 - q^2 + \frac{\kappa^2}{Q^2} \leq 0,$$

resulting in a tachyonic ‘instability’$^9$ for the bulk spinor fields.

The absorption cross section ratio of the spinor field in equation (43) can be rewritten in a more explicit form as

$$\sigma_{\text{abs}} = \frac{\sinh(2\pi b)}{\pi^2(a - b)} \cosh(\pi a - \pi a)|\Gamma(1 + i(b - a))(1 - \Gamma\left(\frac{1}{2} + i(b - a)\right)|^2.$$ 

This version is more convenient for comparing with the standard absorption cross section ratio of spinor operators of the dual CFT

$$\sigma_{\text{abs}} \sim \frac{(2\pi T_L)^{2h_L - 1}}{\Gamma(2h_L)} \frac{(2\pi T_R)^{2h_R - 1}}{\Gamma(2h_R)} \cosh\left(\frac{\omega_L - q_L \Omega_L}{2T_L} + \frac{\omega_R - q_R \Omega_R}{2T_R}\right) \times \left|\Gamma\left(h_L + i\frac{\omega_L - q_L \Omega_L}{2\pi T_L}\right)\right|^2 \left|\Gamma\left(h_R + i\frac{\omega_R - q_R \Omega_R}{2\pi T_R}\right)\right|^2.$$ 

where $(q_L, q_R)$ and $(\Omega_L, \Omega_R)$ are the charges and chemical potentials of the left- and right-hand operators, respectively. In addition to the right-moving conformal weight, the left-moving conformal dimension of the spinor operator dual to the spinor field can be identified as

$$h_L = 1 \pm ib,$$ 

which satisfies the natural relation $|h_L - h_R| = \frac{1}{2} = \pm s$, giving the spins of the fermions, e.g. the electron and positron. Similar arguments can be found in the holographic study of (non)-fermion liquids in the near extreme RN–AdS black brane [40–43].

To further compare the results of the absorption cross section ratios in equations (47), (48), recall that there is an identification between the first law of thermodynamics of the black hole and that of the dual CFT, i.e. $\delta S_{\text{BH}} = \delta S_{\text{CFT}}$, we have

$$\frac{\delta M}{T_H} - \Omega_H \frac{\delta Q}{T_H} = \frac{\omega_L}{T_L} + \frac{\omega_R}{T_R}$$

where the black hole Hawking temperature and chemical potential are $T_H = \frac{B}{2\pi Q}$, $\Omega_H = A_+(B) = -B/Q$ and the ‘total’ energies are $\omega_L = \omega_L - q_L \Omega_L$, $\omega_R = \omega_R - q_R \Omega_R$. Together with the identifications $\delta M = \omega$ and $\delta Q = -q$ (the minus sign corresponds to the conventions ‘$-q$’ in the operator $D_\mu \equiv \partial_\mu + \gamma_\mu - i q A_\mu$ for the equation of motion of the charged spinor field), so we can determine that

$^9$ Note that the tachyonic ‘instability’ for fermionic fields ensures that the bulk fermionic modes propagate to the boundary of the AdS spacetime, see equation (27), it does not mean that the amplitude of the fermionic fields will grow exponentially.
\[ \tilde{\omega}_L = -q \ell \quad \text{and} \quad \tilde{\omega}_R = 2 \omega \ell. \] (51)

Then one can see that the absorption cross section ratio in equation (47) matches with the CFT’s result equation (48) only up to some numerical factors. In addition, by the property \( \sigma_{\text{abs}}(b \rightarrow -b) = -|B|^2 \), the mean number of pairs \( |B|^2 \) indeed also matches with the CFT two-point function for fermion (48).

In addition, the retarded Green’s functions can be obtained from the asymptotical behavior of equation (26) by adopting the ingoing boundary condition \( D_0^L H_{out}(w) = 0 \) at the black hole horizon implying, according to equation (32), \( C_L^+ = 0 \). There are two retarded Green’s functions depending on two possible choices of sources, either \( G_R^+ \sim \frac{C^-}{C^+} \) or \( \tilde{G}_R^+ = \frac{1}{G_R^+} \). According to the relations in equation (30), the condition \( C_L^+ = 0 \) gives

\[
G_R^+ \sim \frac{C^-}{C^+} = \frac{\Gamma(2ib)\Gamma(1 - ib + ia)\Gamma\left(\frac{1}{2} - ib - ia\right)}{\Gamma(-2ib)\Gamma(1 + ib + ia)\Gamma\left(\frac{1}{2} + ib - ia\right)}, \quad G_R^- \sim \frac{C^-}{C^+} = \frac{a + b}{a - b} G_R^+,
\] (52)

which indicates that the conformal dimensions of the left- and right-hand spin-\( \frac{1}{2} \) spinor operators are \( h_L = 1 - ib \) and \( h_R = \frac{1}{2} - ib \). While the other type of retarded Green’s functions is

\[
G_R^+ = \frac{1}{G_R^-} \sim \frac{\Gamma(-2ib)\Gamma(1 + ib + ia)\Gamma\left(\frac{1}{2} + ib - ia\right)}{\Gamma(2ib)\Gamma(1 - ib + ia)\Gamma\left(\frac{1}{2} - ib + ia\right)}, \quad G_R^- = \frac{1}{G_R^+} = \frac{a - b}{a + b} G_R^+.
\] (53)

which shows that \( h_L = 1 + ib \) and \( h_R = \frac{1}{2} + ib \). These results are also consistent with the result from the dual CFT2 side in which the retarded Green function \( G_R(\omega_L, \omega_R) \) is obtained via analytic continuation from the Euclidean correlator (in terms of the Euclidean frequencies \( \omega_{EL} = 1 \omega_L \), and \( \omega_{ER} = 1 \omega_R \))

\[
G_E(\omega_{EL}, \omega_{ER}) \sim T_L^{-2h_L - 1} T_R^{-2h_R - 1} e^{i \omega_{EL} \pi} e^{i \omega_{ER} \pi} \frac{1}{\Gamma\left(h_L - \frac{\omega_{EL}}{2\pi T_L}\right) \Gamma\left(h_R - \frac{\omega_{ER}}{2\pi T_R}\right)} \Gamma\left(h_L + \frac{\omega_{EL}}{2\pi T_L}\right) \Gamma\left(h_R + \frac{\omega_{ER}}{2\pi T_R}\right)
\] (54)

on the upper half complex \( \omega_{L,R} \)-plane

\[
G_R(\omega_L, i\omega_R) = G_E(\omega_{EL}, \omega_{ER}), \quad \omega_{EL}, \omega_{ER} > 0,
\] (55)

where \( \tilde{\omega}_{EL} = \omega_{EL} - i h_L \mu_L \) and \( \tilde{\omega}_{ER} = \omega_{ER} - i h_R \mu_R \), and the Euclidean frequencies \( \omega_{EL} \) and \( \omega_{ER} \) take discrete values of the Matsubara frequencies

\[
\omega_{EL} = 2\pi m_L T_L, \quad \omega_{ER} = 2\pi m_R T_R,
\] (56)

and \( m_L, m_R \) are half integers here.

Furthermore, the quasinormal modes are determined by the boundary conditions \( D_{out}^R = 0 \) (i.e. \( C_L^+ = 0 \) or \( C_R^+ = 0 \)) and \( D_{in}^R = 0 \) (i.e. \( C_L^+ = 0 = C_R^- \)). From equation (30), the nontrivial solution satisfying these conditions is
\[
\frac{1}{\Gamma(1 - ib + ia)\Gamma\left(\frac{1}{2} - ib - ia\right)} = 0, \tag{57}
\]

which gives
\[
\omega_N = -\frac{bB}{Q^2} - i\left(\frac{1}{2} + N\right)\frac{B}{Q^2}, \quad (N = 0, 1, \cdots), \tag{58}
\]

and they match with the poles of the retarded Green’s functions of the dual CFT$_2$, too.

5. Conclusions and discussions

In this paper, we studied the spinor particle pair production for near extremal RN black holes without backreaction. The Dirac equation was solved in the near horizon region where the spacetime structure is \(S^d \times S^2\) and the background electric field is constant in the radial direction which is effectively a warped AdS$_d$ \((=AdS_2 \times S^1)\times S^2\) [26]. The near horizon region contains the causal horizon and dominated electric field which capture both essential contributions: the Hawking radiation and the Schwinger mechanism. Exact spinor solutions were obtained in terms of spherical spinors and hypergeometric functions. Thus one can explicitly compute the ingoing and outgoing fluxes on both inner and outer boundaries. By imposing the particle viewpoint boundary condition, the physical quantities associated to the pair production can be derived by the ratios of boundary fluxes. In particular, the explicit expressions of the vacuum persistence amplitude, the mean number of pairs and the absorption cross section ratio were obtained. Similar to the charged scalar field case, the existence condition of the spinor pair production is actually corresponding to the instability of probe fields in the AdS$_2$, i.e. violating the BF bound. Moreover, the condition leads to the black holes losing their charge more than their mass in the pair production process. This consequence is in agreement with the cosmic censorship conjecture that a naked singularity cannot be evolved from the complete gravitational collapse when the matter fields satisfy appropriate energy conditions. The holographic dual CFT description of the spinor particle pair production was also studied in the light of the RN/CFT correspondence. It was showed that both the left- and right-moving conformal dimensions of the spinor operators dual to the bulk spinor field are complex, indicating that the spinor operators are unstable. In addition, the CFT fermionic absorption cross section ratio and the retarded Green’s function notably agreed with the results computed from the gravity side, and thus the holographic description of the Schwinger pair creation ratio could be understood via its relation with the absorption cross section ratio. Our results revealed further information about the dual CFT$_2$ picture for the near extreme RN black hole. It would be interesting to further study the Schwinger effect in charged black holes in asymptotically AdS$_{d+1}$ spacetime, in which the CFT$_d$ dual to near horizon near extreme geometry is the infrared (IR) one while the dual CFT$_d$ on the asymptotic boundary is the ultraviolet (UV) one. By studying the RG flow between the IR and UV CFTs we can then see how the spontaneous pair creation near the black hole horizon evolves to the boundary and affects the dual CFT$_d$, such as the condensed matter system [44].

Acknowledgments

The authors thank Rong-Gen Cai, Sang Pyo Kim, Miao Li, Chun-Yen Lin, Jian-Xin Lu for helpful discussions. C M C is grateful to the ITP, CAS for hospitality while the paper was in
Appendix A. Spherical spinors

The Dirac gamma matrices should satisfy the Clifford algebra, especially in the Minkowski spacetime
\[
\{ \gamma^a, \gamma^b \} = 2\eta^{ab}.
\]  
(A1)

There are many representations for gamma matrices and the version used in this paper is
\[
\gamma^0 = \begin{pmatrix} -i & 0 \\ 0 & i \end{pmatrix}, \quad \gamma^i = \begin{pmatrix} 0 & \sigma^i \\ \sigma^i & 0 \end{pmatrix}, \quad i = 1, 2, 3,
\]  
(A2)
in which the Pauli matrices are defined as
\[
\sigma^1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \quad \sigma^2 = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}, \quad \sigma^3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}.
\]  
(A3)
The Pauli matrices have the following relations
\[
\sigma^i \sigma^j = \delta^{ij} I + i e^{ijk} \sigma^k,
\]  
(A4)
and the operator
\[
\pi = \frac{1}{2} (I + i \sigma^1 + i \sigma^2 + i \sigma^3) = \frac{1}{2} \begin{pmatrix} 1 & i & 1 + i \\ -1 & i & 1 - i \end{pmatrix}. 
\]  
(A5)
permutes the Pauli matrices as
\[
\pi^{-1} \sigma^1 \pi = \sigma^2, \quad \pi^{-1} \sigma^2 \pi = \sigma^3, \quad \pi^{-1} \sigma^3 \pi = \sigma^1.
\]  
(A6)

In the curved spacetime, the metric can be encoded in an orthonormal frame via tetrad. However, the frame is not unique. In the following we will firstly introduce the spherical spinors in the Cartesian frame and then transform them into the rotation frame which is used in this paper.

A.1. Cartesian frame

In the Cartesian frame, \((\hat{e}_x, \hat{e}_y, \hat{e}_z)\), the momentum and angular momentum operators, \(\vec{P} = -i \nabla, \vec{L} = \hat{r} \times \vec{P}\), in spherical coordinates have the following components
\[
\vec{P}_1 = -i \left( \sin \theta \cos \varphi \partial_r + \frac{\cos \theta \cos \varphi}{r} \partial_\theta - \frac{\sin \varphi}{r \sin \theta} \partial_\varphi \right),
\]  
(A7)
\[
\vec{P}_2 = -i \left( \sin \theta \sin \varphi \partial_r + \frac{\cos \theta \sin \varphi}{r} \partial_\theta + \frac{\cos \varphi}{r \sin \theta} \partial_\varphi \right),
\]  
\[
\vec{P}_3 = -i \left( \cos \theta \partial_\theta - \frac{\sin \theta}{r} \partial_\varphi \right).
\]  
(A8)
\[ \mathbf{L}_1 = i(\sin \varphi \partial_\theta + \cot \theta \cos \varphi \partial_\varphi), \quad \mathbf{L}_2 = i(-\cos \varphi \partial_\theta + \cot \theta \sin \varphi \partial_\varphi), \quad \mathbf{L}_3 = -i\partial_\varphi, \]

(A9)

The orthogonal spherical spinors \( \Phi^\pm_{\kappa}(\theta, \varphi) \) with the value of \( \kappa = \mp(j + 1/2) \) are defined as [20]

\[ \Phi^+_{\pm(j+1/2),n} = \begin{pmatrix} i\Psi^+_{j+1/2} \\ 0 \end{pmatrix}, \quad \Phi^-_{\pm(j+1/2),n} = \begin{pmatrix} 0 \\ \Psi^-_{j+1/2} \end{pmatrix}, \]

(A10)

where \( \Psi^\pm_{j+1/2}(\theta, \varphi) \) are two-component spherical spinors which are defined in terms of the spherical harmonics \( Y^\pm_l(\theta, \varphi) \)

\[ \Psi^n_{j-1/2} = \begin{pmatrix} \sqrt{\frac{j+n}{2j}} Y^{n-1/2}_{j-1/2} \\ \sqrt{\frac{j-n}{2j}} Y^{n+1/2}_{j-1/2} \end{pmatrix}, \quad \Psi^n_{j+1/2} = \begin{pmatrix} \sqrt{\frac{j-n+1}{2j+2}} Y^{n-1/2}_{j+1/2} \\ -\sqrt{\frac{j+n+1}{2j+2}} Y^{n+1/2}_{j+1/2} \end{pmatrix}. \]

(A11)

These spherical spinors are completely specified by the quantum numbers of the angular momentum \( j (j = l \pm 1/2) \) and its projection \( n (-l \leq n \leq l) \) where \( l \) is an integer. Moreover, they are eigen-spinors of the operator \( \mathbf{I}_L = \sigma \cdot \mathbf{L} \)

\[ (\mathbf{I}_L \cdot \sigma) \Psi^n_{j+1/2} = \pm (j + 1/2)\Psi^n_{j+1/2}, \]

(A12)

where

\[ \sigma \cdot \mathbf{L} = \begin{pmatrix} \mathbf{L}_3 & -i\mathbf{L}_2 \\ i\mathbf{L}_2 & -\mathbf{L}_3 \end{pmatrix} = \begin{pmatrix} -i\partial_\varphi & e^{-i\varphi}(-\partial_\theta + i \cot \theta \partial_\varphi) \\ e^{i\varphi}(\partial_\theta + i \cot \theta \partial_\varphi) & i\partial_\varphi \end{pmatrix}. \]

(A13)

Moreover, the spherical spinors also satisfy the relation

\[ \sigma \cdot \tilde{\sigma} \Psi^n_{j+1/2} = \Psi^n_{j+1/2}, \]

(A14)

where

\[ \sigma \cdot \tilde{\sigma} = \sin \theta \cos \varphi \sigma^1 + \sin \theta \sin \varphi \sigma^2 + \cos \theta \sigma^3 = \begin{pmatrix} \cos \theta & \sin \theta e^{-i\varphi} \\ -\sin \theta e^{i\varphi} & -\cos \theta \end{pmatrix}. \]

(A15)

In the Dirac equation in flat spacetime, the spatial derivative is basically \( \sigma \cdot \tilde{\sigma} \) which can be expressed in terms of \( \sigma \cdot \tilde{\sigma} \) and \( \sigma \cdot \mathbf{L} \) via the identity

\[ \sigma \cdot \tilde{\sigma} = -i(\sigma \cdot \tilde{\sigma})\partial_\theta + \frac{1}{\sigma}(\sigma \cdot \tilde{\sigma})(\sigma \cdot \mathbf{L}). \]

(A16)

Therefore, the spherical spinors are a suitable basis to expand the general solution.

**A.2. Rotation frame**

The other convenient frame for solving the Dirac equation is the rotation frame (\( \hat{e}_r, \hat{e}_\theta, \hat{e}_\varphi \)).

For separating the Dirac equation, the analog two-component spherical spinors in rotation frame, \( \Psi^n_{j+1/2} \), must have a nice property with respect to the associated angular operator.
K = \sigma^2 \partial_\theta + \sigma^\lambda \frac{\partial_\varphi}{\sin \theta} = \begin{pmatrix} \frac{\partial_\varphi}{\sin \theta} - i \partial_\theta \\ i \partial_\theta - \frac{\partial_\varphi}{\sin \theta} \end{pmatrix}. \tag{A17}

Basically the two-component spherical spinors in the rotation and Cartesian frames should be related by a similarity transformation \( \bar{\Psi}_{j=\pm 1/2} = S^{-1} \Psi_{j=\pm 1/2} \). The similarity transformation [45] turns out to be

\[
S = \frac{1}{\sqrt{\sin \vartheta}} e^{-i \frac{\varphi}{\vartheta}} e^{-i \frac{j}{\vartheta} \varphi}, \tag{A18}
\]

which transforms the operator \( K \) as

\[
SKS^{-1} = - (\sigma \cdot \hat{r})(I + \sigma \cdot \hat{L}). \tag{A19}
\]

One can straightforwardly verify that the spherical spinors satisfy

\[
K \Psi_{j=\pm 1/2} = \mp (j + 1/2) \Psi_{j=\pm 1/2}, \tag{A20}
\]

and

\[
\sigma^4 \Psi_{j=\pm 1/2} = \Psi_{j=\mp 1/2}. \tag{A21}
\]

Therefore, the spherical Dirac spinor in the rotation frame is

\[
\Phi_{\pm n, n} = S^{-1} \Phi_{\pm n, n}, \quad S = \begin{pmatrix} S & 0 \\ 0 & S \end{pmatrix}. \tag{A22}
\]

which satisfy the following relations

\[
\gamma^0 \Phi_{n, n} = \mp i \Phi_{\mp n, n}, \quad \gamma^1 \Phi_{n, n} = \pm i \Phi_{\mp n, n}, \quad K \Phi_{n, n} = \mp i \Phi_{\mp n, n}, \tag{A23}
\]

where

\[
K = \gamma^2 \partial_\theta + \gamma^\lambda \frac{\partial_\varphi}{\sin \theta} = \begin{pmatrix} 0 & K \\ K & 0 \end{pmatrix}. \tag{A24}
\]

**Appendix B. Hypergeometric functions**

The hypergeometric equation

\[
z (1-z) \partial_z^2 w + [\gamma - (\alpha + \beta + 1)z] \partial_z w - \alpha \beta w = 0, \tag{B1}
\]

has two independent solutions which can be expressed around three regular singular points, i.e. \( z = 0, z = 1 \) and \( z = \infty \), as

\[
w = a_1 F(\alpha, \beta; \gamma; z) + a_2 z^{1-\gamma} F(\alpha - \gamma + 1, \beta - \gamma + 1; 2 - \gamma; z), \tag{B2}
\]

\[
= b_1 F(\alpha, \beta; \alpha + \beta - \gamma + 1; 1-z) + b_2 (1-z)^{\gamma - \alpha - \beta} \times F(\gamma - \alpha, \gamma - \beta; \gamma - \alpha - \beta + 1; 1-z), \tag{B3}
\]
\[= c_1 z^{-a}F\left(\alpha, \alpha - \gamma + 1; \alpha - \beta + 1; \frac{1}{z}\right)\]
\[+ c_2 z^{-\beta}F\left(\beta, \beta - \gamma + 1; \beta - \alpha + 1; \frac{1}{z}\right). \quad (B4)\]

There are a number of mathematical properties for \(F(\alpha, \beta; \gamma; z)\), in particular the following ones are useful for our analysis: (i) transformation formula
\[
F(\alpha, \beta; \gamma; z) = \frac{\Gamma(\gamma)\Gamma(\gamma - \alpha - \beta)}{\Gamma(\gamma - \alpha)\Gamma(\gamma - \beta)} z^{-\alpha}F\left(\alpha, \alpha - \gamma + 1; \alpha + \beta - \gamma + 1; 1 - \frac{1}{z}\right)
+ \frac{\Gamma(\gamma)\Gamma(\alpha + \beta - \gamma)}{\Gamma(\alpha)\Gamma(\beta)}(1 - z)^{\gamma - \alpha - \beta}z^{\alpha - \gamma}
\times F\left(\gamma - \alpha, 1 - \alpha; \gamma - \alpha - \beta + 1; 1 - \frac{1}{z}\right). \quad (B5)\]

(ii) special values
\[
F(\alpha, \beta; \gamma; 0) = 1, \quad F(\alpha, \beta; \gamma; 1) = \frac{\Gamma(\gamma)\Gamma(\gamma - \alpha - \beta)}{\Gamma(\gamma - \alpha)\Gamma(\gamma - \beta)}, \quad (B6)\]

and (iii) differential formula
\[
\partial_zF(\alpha, \beta; \gamma; z) = \frac{\alpha\beta}{\gamma}F(\alpha + 1, \beta + 1; \gamma + 1; z), \quad (B7)\]
\[
\partial_z\left[z^\alpha F(\alpha, \beta; \gamma; z)\right] = \alpha z^{\alpha - 1}F(\alpha + 1, \beta; \gamma; z). \quad (B8)\]
\[
\partial_z\left[z^{\gamma - 1}F(\alpha, \beta; \gamma; z)\right] = (\gamma - 1)z^{\gamma - 2}F(\alpha, \beta; \gamma - 1; z). \quad (B9)\]

In addition, the following properties of gamma function are also needed in our computation
\[
\Gamma(\alpha + 1) = \alpha\Gamma(\alpha), \quad \Gamma(\alpha)\Gamma(1 - \alpha) = \frac{\pi}{\sin(\alpha\pi)}, \quad (B10)\]
\[
\left|\Gamma\left(\frac{1}{2} + iy\right)\right|^2 = \frac{\pi}{\cosh(y)}, \quad \left|\Gamma(1 + iy)\right|^2 = \frac{\pi y}{\sinh(y)}, \quad \left|\Gamma(iy)\right|^2 = \frac{\pi}{y \sinh(y)}. \quad (B11)\]

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