Strong magnetic coupling of an ultracold gas to a superconducting waveguide cavity

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Placing an ensemble of $10^6$ ultracold atoms in the near field of a superconducting coplanar waveguide resonator (CPWR) with $Q \sim 10^6$ one can achieve strong coupling between a single microwave photon in the CPWR and a collective hyperfine qubit state in the ensemble with $g_{eff}/2\pi \sim 40$ kHz larger than the cavity line width of $\kappa/2\pi \sim 7$ kHz. Integrated on an atomchip such a system constitutes a hybrid quantum device, which also can be used to interconnect solid-state and atomic qubits, to study and control atomic motion via the microwave field, observe microwave super-radiance, build an integrated micro maser or even cool the resonator field via the atoms.

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In the past decade important breakthroughs in implementing quantum information processing were reached in different physical implementations [1], each showing advantages and shortcomings. For quantum information to emerge as a valuable technology, it is mandatory to pool their strengths. Solid-state systems allow fast processing and dense integration; atom or ion based systems are slower but exhibit long qubit coherence times. Ensembles of atoms constitute a quantum memory, it can be read out onto photons [2] which can then be transmitted over long distances [3]. Here we analyze a device to quantum interconnect superconducting solid-state qubits to an atomic quantum memory.

The challenge in transferring the state of a solid-state qubit to atoms is bridging the tremendous gap in time scales that govern solid-state and atomic physics devices. This difference can be overcome using a coplanar waveguide resonator (CPWR) [4,5,6], which can be electrically coupled to single superconducting qubits [7,8,9,10,11]. Various ways were proposed to couple to atomic and molecular systems [12,13,14,15,16,17,18]. The small effective mode volume together with the long photon lifetime allow a strong coupling.

The superconducting qubit to CPWR coupling has been implemented and studied by several groups [7,8,9,10,11]. In this letter we concentrate on the magnetic coupling of a microwave photon in a CPWR to a collective hyperfine qubit in an ensemble of ultracold atoms. We show below that even though the magnetic coupling strength is much weaker than the optical dipole coupling, one can achieve strong coupling with currently available technology of circuit cavity quantum electro dynamics and ultracold atomic ensembles on an atomchip.

As particular qubit example we consider a hyperfine transition in $^87$Rb between $|F = 2, m_F\rangle$ and $|F = 1, m_F\rangle$ states which frequency of 6.83 GHz being ideally suited for a CPWR. In principle both systems can be integrated in a hybrid device on an single superconducting atomchip [19,20]. Besides the transfer of a single photon to the atomic ensemble as a quantum memory and back, such a hybrid quantum system opens up many different other possibilities. For example nondestructive microwave detection of the atomic density will allow to continuously monitor BEC formation or changing operating parameters one can achieve a superradiant microwave source, a micro maser. Optically pumped atoms are a heat bath close to $T = 0$ and will strongly suppress thermal photons in the coupled resonator mode. Adiabatic microwave potentials will allow to couple the quantum properties of the resonator mode to the mechanical motion of the atoms.

The coplanar wave guide resonators developed for
circuit cavity QED consist of three conducting stripes: the central conductor plus two ground planes (Fig. 1). Their electromagnetic field is strongly confined near the gap between conductor and the ground planes. Using atomchip technology [21, 22] large ensembles of ultracold atoms exceeding \( N > 10^3 \) can be positioned only a few \( \mu m \) above this gap [23, 24], where they experience the very strong localized magnetic field of the CPW-mode. The high concentration of field energy near the surface results in a dramatic reduction of the effective volume \( V_{\text{eff}} \sim \frac{\lambda}{2} \lambda^2 \) of the resonator mode. For the \(^{87}\text{Rb} \) microwave transition at 6.83 GHz (wavelength \( \lambda \sim 3 \text{ cm} \)) and a typical decay length \( l \sim 3 \mu m \) of the field of the order of the gap size \( W \), one expects an enhancement of the atom-photon coupling strength of \( \gamma_\lambda \sim 10000 \) for \( (W = 3 \mu m ) \). A full calculation of the local electromagnetic field [23, 26] as shown in Fig. 1 confirms these estimates [27], and we obtain at the location of the atoms a fields of \( > 40 \mu G \) for a single photon.

For an detailed treatment of the coupling we write the single mode electromagnetic field operators as:

\[
\vec{E}^\gamma(\vec{r}, t) = \frac{\epsilon^{\gamma}_{\text{tr}}(x, y)}{\sqrt{2}} \left( a_\gamma e^{i(\gamma z - \omega t)} + a_\gamma^\dagger e^{-i(\gamma z - \omega t)} \right),
\]

\[
\vec{B}^\gamma(\vec{r}, t) = i \frac{\hat{b}^{\gamma}_{\text{tr}}(x, y)}{\sqrt{2}} \left( a_\gamma e^{i(\gamma z - \omega t)} - a_\gamma^\dagger e^{-i(\gamma z - \omega t)} \right),
\]

where \( a_\gamma^\dagger \) and \( a_\gamma \) represent the boson creation and destruction operators for the microwave photons. \( \omega = 2\pi \nu \) is the angular frequency of the microwave and \( k_\gamma = 2\pi/\lambda \) is the propagation constant with wavelength \( \lambda = c/\sqrt{\epsilon_{\text{eff}}} \). The effective relative dielectric constant \( \epsilon_{\text{eff}} \) has a value between the substrate value and 1 (vacuum) and depends on the actual dimensions of the CPW [28].

The corresponding mode functions \( \epsilon^{\gamma}_{\text{tr}}(x, y) \) and \( \hat{b}^{\gamma}_{\text{tr}}(x, y) \) are strongly varying in space depending on the CPW geometry and have to be determined numerically. To satisfy the proper field commutators they have to be normalized to:

\[
\frac{1}{2} \int dV \epsilon(\vec{r}) |\epsilon^{\gamma}_{\text{tr}}|^2 = \frac{1}{2\mu_0} \int dV |\hat{b}^{\gamma}_{\text{tr}}|^2 = \frac{1}{2} \hbar \omega_\gamma. \quad (1)
\]

They represent the field amplitude per photon. Note that the permittivity has to be included in the integral. In the following we assumed the substrate to be non magnetic. The field Hamiltonian then reads:

\[
\mathcal{H}_\epsilon = \hbar \omega_\gamma (a_\gamma^\dagger a_\gamma + \frac{1}{2}),
\]

where \( \omega_\gamma \) is the cavity resonance frequency.

For a ground state Rb atom the dominant interaction with a microwave field are the M1-dipole transitions between the atomic hyperfine states \( |F = 2, m_F \rangle \leftrightarrow |F = 1, m_F' \rangle \). This leads to the interaction Hamiltonian:

\[
\mathcal{H}_{\text{int}} = \vec{\mu} \cdot \vec{B}^\gamma = \frac{\mu_B}{\hbar} \left( g_S S - \frac{\mu_N}{\mu_B} g_I I \right) \cdot \vec{B}^\gamma \quad (2)
\]

Assuming an external bias field \( \vec{B}_0 \) as quantization axis for the atomic magnetic moment, transitions driven by a transverse field \( \vec{B}^\gamma \parallel \vec{B}_0 \) follow the selection rules \( \Delta m_F = m_F - m_F' = \pm 1 \). Longitudinal fields \( \vec{B}^\gamma \parallel \vec{B}_0 \) induce \( \Delta m_F = 0 \) transitions.

The transverse fields generated by the quasi TEM mode of the CPWR cavity couples therefore to the two transitions \( \Delta m_F = 1 \) and \( \Delta m_F = -1 \). By adjusting the Zeeman splitting of the hyperfine states via a typical longitudinal Ioffe bias field of \( B_0 \sim 1 \text{ Gauss} \) we can ensure that the CPW-mode is only resonant with one of those two transitions. Thus the atom can be modelled effectively by a two level system and we simply denote the two coupled atomic states by \( |2 \rangle = |F = 2, m_F \rangle \) and \( |1 \rangle = |F = 1, m_F' \rangle \) with energies \( E_2 \) and \( E_1 \).

For the internal dynamics of an ensemble of \( N \) atoms, we thus get an effective Hamiltonian in a standard Jaynes-Cummings form [29]:

\[
\mathcal{H}_{\text{atom}} = \sum_i \hbar \omega_\gamma \hat{n}_i \hat{a}_i - \sum_i \left\{ \hat{\mu}_i \cdot \vec{B}(\vec{r}_i) + \hat{\mu}_i^\dagger \cdot \vec{B}^\gamma(\vec{r}_i) \right\}.
\]

Here \( \omega_\gamma \) is the atomic transition frequency and \( \hat{n}_i = \hat{a}_i^\dagger \hat{a}_i \) is the excitation operator of the \( i^\text{th} \) atom and \( \hat{\mu}_i = \vec{\mu}_i \cdot \vec{\hat{R}} \) with \( \vec{\mu}_i \) is the transition matrix element of the magnetic dipole moment as defined in Eq. (2).

Typically an ensemble of ultracold Rb atoms is confined in an elongated potential on the atomchip and has a transverse extension of \( d < 1 \mu m \) and a length of up to a few mm, much smaller then the transverse variation of the magnitude, respectively the wavelength of the microwave field. Positioning the atomic ensemble longitudinally in vicinity of the field maximum we can, to a first approximation, neglect in the change of the magnetic field along the cloud in magnitude and direction and fix \( \hat{b}^{\gamma}_{\text{trans}}(\vec{r}_i) \approx \hat{b}^{\gamma}_{\text{trans}}(\vec{R}) \) for all atoms. Here \( \vec{R} \) is the mean transverse position. This allows to define a simple collective atomic excitation operator of the form \( \hat{n} = \frac{1}{\sqrt{N}} \sum_i \hat{n}_i \) to rewrite the interaction Hamiltonian in the Tavis Cummings form [30]:

\[
H = \hbar \omega_\gamma \hat{a}_i^\dagger \hat{a}_i + \hbar \omega_\gamma \hat{\pi}^\dagger \hat{\pi} + \hbar g_{\text{eff}} \hat{\pi}^\dagger \hat{a}_i + \hbar g_{\text{eff}}^* \hat{a}_i^\dagger \hat{\pi}, \quad (3)
\]

where \( g_{\text{eff}} = \sqrt{N} g \) and \( \hbar g = \frac{1}{\sqrt{2}} (\hat{b}^{\gamma}_{\text{trans}}(\vec{R}) \cdot \vec{\mu}) \). From this we can immediately read off the effective coupling strength \( g_{\text{eff}} \) for the first symmetric atomic excitation, which is enhanced by a factor of \( \sqrt{N} \).

As a concrete example we obtain a matrix element of 0.86\( \mu_B \) for a \( m_F = 2 \) to \( m_F = 1 \) transition in the \(^{87}\text{Rb} \) ground state. Taking into account the calculated field of a CPWR (see Fig. 1) we obtain a single photon - single atom Rabi frequency of typically \( g/2\pi \sim 40 \text{ Hz} \) at a height of a few \( \mu m \). For an atomic ensemble of \( N \sim 10^6 \) \(^{87}\text{Rb} \) atoms coherent collective coupling \( g/2\pi = \sqrt{N}g/2\pi \sim 40 \text{ kHz} \) dominates over cavity decay \( \kappa/2\pi = \nu/Q \sim 7 \text{ kHz} \) and one would get several exchanges between a microwave photon in he cavity...
and a collective atomic excitation before the photon decays.

The Hamiltonian (Eq. 3) can be diagonalized by eigenstates forming a weighted coherent superposition of collective atomic excitations and a photon depending on system parameters. Controlling the relative weights of the superposition via atomic or cavity tuning adiabatically switches excitation between the microwave and the atomic qubit. The strong coupling regime allows to perform this transfer fast enough to avoid decoherence.

The upper collective atomic qubit state is a delocalized symmetric superposition of all possible single atom excitations and one can in principle achieve long coherence times for the collective atomic hyperfine qubit. Decay rates of $\gamma/2\pi \sim 0.3$ Hz were recently demonstrated for hyperfine excitations [31]. As demonstrated in atom ensemble experiments [2] such an excitation can be efficiently read out by forward coherent Raman scattering into a travelling wave optical photon. This will allow to complete the transfer from solid-state qubits via an atomic quantum memory to photons as flying qubits.

Besides implementing a quantum interconnect between solid-state qubits, atoms and even photons, our system offers many further interesting possibilities, which we will discuss in short examples below.

Whenever the effective Rabi splitting $g_{\text{eff}}$ is larger then the cavity line width $\kappa$, it manifests itself in the spectrum of the transmitted and reflected fields, exhibiting resonances at these collective excitations (see Fig. 2). Hence one can use the microwave field as non-destructive probe for the integrated atomic density in the mode.

Let us note that, as compared to standard cavity QED here the line width of the atomic excitation is not limited by spontaneous decay as for all practical purposes both hyperfine states can be regarded as stable. Hence the decay rate it will effectively be given by non radiative losses such as the lifetime of the atoms in the trap, which can be in the order of seconds. In view of the large difference of atomic and cavity decay the single atom cooperativity parameter $C = g^2/(\kappa\gamma)$, which can reach $C \sim 1$, is only partly meaningful. While a single excited atom will still emit one photon predominantly into the cavity mode, the large cavity line width prevents direct single atom detection via resonator transmission.

In the microwave regime the transmitted field amplitude and phase are directly accessible. The corresponding phase shift of the transmitted field is plotted in Fig. 2, for an empty cavity and one filled with $10^6$ atoms. Note that measuring the phase shift will not only be a sensitive probe of the atom number but, as the phase will change sign when the atoms are transferred to the upper hyperfine state, it can also be used for preparation and readout of spin states.

One important aspect neglected so far are thermal photons in the resonator. The number of photons in the cavity is given by $n_T = \exp(-\hbar\omega_T/k_B T)$. With 6.83 GHz corresponding to a temperature $T \sim 350$ mK cooling to below 100 mK is required to have an empty cavity ($n_T < 0.1$). However a perfectly polarized BEC with all atoms in the lower hyperfine state has a very low effective internal temperature. A relative purity in the polarization of $10^{-5}$ corresponds to $\hbar\omega_a/k_B T = 10^{-5} = 11.5$ or a temperature of $T \sim 30$ mK. Hence the coupling of the two systems can lead to an energy flow towards the ensemble of ultracold atoms. We estimate the photon absorption rate from the cavity into the atomic ensemble to $\gamma_c/2\pi \sim g^2 N/(\gamma\alpha 2\pi) \sim 8.6$ MHz, assuming an upper state with a lifetime of $\gamma_a^{-1} \sim 1$ ms (for this experiment a state that is very short lived is used) which has to be compared to the heating rate $R \sim \kappa n_T$. The suppression of the thermal photons is then given by $\kappa/(\gamma_a + \kappa)$. We can remove thermal photons from the mode as long as $\gamma_c \gg \kappa$, which can be for several 10 seconds.

Superradiance from a completely inverted atomic ensemble has been first discussed in the microwave context [32] and lead to extensive theoretical and experimental studies [33]. Here we could study its magnetic analogue in a very clean form by preparing an almost perfectly inverted atomic system with all atoms in $F = 2$. This situation is very close to the original model for a superradiant system proposed by Dicke [32], where a highly excited atomic system is supposed to spontaneously emits coherent multi-photon pulses. Here, with the spontaneous life time of the excited state practically infinite, we can assume the dominant decay to happen solely via the cavity mode and we get an emission rate of $N g^2/k \sim 5 \cdot 10^7 s^{-1}$ at which about $10^6$ photons are collectively emitted into the microwave mode in a coherent pulse.

The field in the cavity mode can exert forces onto the cloud of ultracold atoms through microwave induced dressed state potentials (microwave ac-Stark
A coupling strength of $g/2\pi \approx 40$ Hz results in small modifications (dressed state shifts) of the trapping potential and forces are small on the few photon level. Potential energies in the order of a typical chemical potential of a trapped 1d cloud $V \sim 1kH\Omega$ appear for microwave fields of 1000 photons in the mode, which still correspond to a minute powers of only $P \sim 5 \times 10^{-16} \omega_0 \sim 10^{-16}$ W. In the quantum noise of the cavity field, and its time evolution will have a strong influence on the atomic motion in the trap. In addition a microwave power in the nano Watt regime will create very large forces and could help for atomic positioning towards optimum coupling.

Comparing with other related CQED systems, the cooperativity for the collective qubit state is very large and even comes close to the values for a BEC coupled to an resonator on a strong optical transition [36, 37, 38]. Note that the involved transition frequency is much lower in our case. Hence one could even envisage reaching the regime of the quantum phase transition to a collective superradiant phase, predicted for $gN \approx \omega_c$ in a classic paper by Hepp [39].

In conclusion we found that coherent strong coupling between a collective spin Dicke state, as it is used for ensemble qubits, and a microwave photon from a CPW resonator is feasible by combining current state of the art technology of atomchips and superconducting microwave resonators. Such a quantum interconnect will allow to transfer a quantum state of a solid-state qubit into the atomic ensemble and store it there as interface for long distance quantum communication. In addition we get a microwave realization of the Tavis Cummings model, where super radiance can be studied and maybe even used for cooling and generating strong coherent light forces with microwaves.

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[1] D. Bouwmeester, A. K. Ekert, and A. Zeilinger, *The Physics of Quantum Information*, 2000.
[2] M. D. Lukin, Rev. Mod. Phys. **75**, 457 (2003).
[3] N. Gisin, G. Ribordy, W. Tittel, and H. Zbinden, Rev. Mod. Phys. **74**, 145 (2002).
[4] P. K. Day, et al. Nature **425**, 817 (2003).
[5] B. A. Mazin, Ph.D. thesis, California Institute of Technology (2004).
[6] L. Frunzio, et al. Applied Superconductivity, IEEE Transactions on **15**, 860 (2005).
[7] A. Blais, et al. Phys. Rev. A **69**, 062320 (2004).
[8] A. Wallraff, et al. Nature **431**, 162 (2004).
[9] M. A. Sillanpaa, J. I. Park, and R. W. Simmonds, Nature **449**, 438 (2007).
[10] M. Hofheinz, et al. Nature **454**, 310 (2008).
[11] J. M. Fink, et al. Nature **454**, 315 (2008).
[12] A. S. Sorensen, C. H. van der Wal, L. I. Childress, and M. D. Lukin, Phys. Rev. Lett. **92**, 063601 (2004).
[13] L. Tian, P. Rabl, R. Blatt, and P. Zoller, Phys. Rev. Lett. **92**, 247902 (2004).
[14] A. Andre, et al. Nature Physics **2**, 636 (2006).
[15] P. Rabl, et al. Phys. Rev. Lett. **97**, 033003 (2006).
[16] R. J. Schoelkopf and S. M. Girvin Nature, **451**, 664 (2008).
[17] D. Petrosyan and M. Fleischhauer, Phys. Rev. Lett. **100**, 170501 (2008).
[18] K. Tordrup and K. Molmer, Phys. Rev. A **77**, 020301(R) (2008).
[19] T. Nirrengarten, et al. Phys. Rev. Lett. **97**, 200405 (2006).
[20] T. Mukai, et al. Phys. Rev. Lett. **98**, 260407 (2007).
[21] R. Folman, P. Krüger, J. Schmiedmayer, J. Denschlag, and C. Henkel, Adv. At. Mol. Opt. Physics **48** (2002).
[22] J. Fortagh and C. Zimmermann, Rev. Mod. Phys. **79**, 235 (2007).
[23] Y. Lin, I. Teper, C. Chin, and V. Vuletic, Phys. Rev. Lett. **92**, 050404 (2004).
[24] S. Aigner, et al. Science **319**, 1226 (2008).
[25] R. N. Simons, IEEE Transactions on Microwave Theory Techniques **32**, 116 (1984).
[26] R. E. Collin, *Foundations for Microwave Engineering* (Wiley-IEEE Press, 2001), 2nd ed.
[27] For full details of the calculations see: J. Verdu, et al. (to be published).
[28] B. C. Wadell, *Transmission Line Design Handbook* (Artech House Publishers, 1991), 1st ed.
[29] P. L. Knight and B. W. Shore, Phys. Rev. A **48**, 642 (1993).
[30] M. Tavis and F. W. Cummings, Phys. Rev. **170**, 379 (1968).
[31] P. Treutlein, et al. Phys. Rev. Lett. **92**, 203005 (2004).
[32] R. H. Dicke, Phys. Rev. **93**, 99 (1954).
[33] M. Gross, S. Haroche, Physics Letters (reports section) **93**, 301 (1982).
[34] E. Muskat, D. Dubbers and O. Scharpf, Phys. Rev. **58**, 2047 (1987).
[35] C. C. Agosta, I. F. Silvera, . H. T. C. Stoof and B. J. Verhaar Phys. Rev. Lett. **62**, 2361 (1989).
[36] S. Slama, et al. Phys. Rev. Lett. **98**, 053603 (2007).
[37] F. Brennecke, et al. Nature **450**, 268 (2007).
[38] Y. Colombe, et al. Nature **450**, 272 (2007).
[39] K. Hepp and E. H. Lieb, Annals of Physics **76**, 360 (1973).