Implementation of cross-phase analysis for study of MHD instabilities arising on TUMAN-3M and Globus-M tokamaks

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Abstract. Following paper represents a method to resolve spatial mode structure of plasma MHD instabilities employing cross-phase correlation analysis applied to signals of magnetic probes, mounted on Globus-M and Tuman-3M tokamaks. The data on observations of toroidal Alfvén eigenmodes (TAEs), which were formerly identified in Globus-M tokamak discharges, has been analyzed. Possible interpretation of the phase plot, acquired from analysis of NBI-stimulated ion-cyclotron emission (ICE) on Tuman-3M is given as well. The method described below turns out to be useful instrument for resolution of spatial structure of plasma instabilities, especially in D-shaped tokamaks with low aspect ratio.

1. Introduction

Magnetic probes (Mirnov coils — MC), intended to detect a local variation of magnetic flux, are today’s most common and widespread, as well as the most practical in use diagnostic to observe plasma MHD instabilities in tokamaks. Despite some particular limitations of this diagnostic (e.g. the absence of spatial localization), an array of the probes, allocated along poloidal or toroidal turn of a tokamak, allows one to resolve spatial structure of plasma instability (mode numbers), analyzing phase shift between the probes.

Determination of spatial structure of magnetic field perturbations, registered with magnetic probes, by means of direct plotting of phase diagrams (polar plots) may cause some difficulties quite often, especially in case of tokamaks with D-shaped plasma and low aspect ratio. As an alternative technique to resolve mode structure of MHD perturbations, employment of correlation analysis is possible as well [1]. Having a pair of signals $x(t)$ and $y(t)$, with the same finite duration, one could acquire a cross-correlation function $R_{xy}(t)$:

$$R_{xy}(t) = \int_{-T}^{T} x^*(\tau)y(t-\tau)d\tau$$

Since in real experiment signals of a pair of magnetic probes are meant by $x(t)$ and $y(t)$, the value of cross-correlation (1) at frequency detected by magnetic probes is of the greatest
interest. Thus, cross-power spectral density (CPSD) or cross-spectral density (CSD), which is a Fourier transform of cross-correlation (1), is calculated.

\[
S_{xy}(\omega) = \int_{-T}^{T} R_{xy}(t)e^{-i\omega t}dt = \int_{-T}^{T} \int_{-T}^{T} e^{-i\omega t}x^*(\tau)y(t-\tau)d\tau dt
\]  

For obtaining an information on mode structure of plasma instability, one could calculate cross-phase (3) as an argument of complex value (2) for each pair of probes. Cross-phase (3) is calculated on the frequency of the detected signals, which automatically makes frequency resolution possible, however, only in case of the same frequency registered on each probe. On practice, selection of time interval \(T\) (2),(3) is mostly specified by a condition of small mode frequency deviation \(\Delta f\) during the interval \((T << 1/\Delta f)\).

\[
\theta_{xy} = \text{Arg}(S_{xy}(\omega)) \pm 2\pi n
\]

Cross-phase (3) contains information on phase shift between two probes. The values of cross-phase form a square matrix, which dimension is determined by a number of magnetic coils in array. Then, the \(\theta_{xy}\) matrix could be presented graphically, where spatial structure of observed perturbations could be estimated by a number of stripes, along which the value of phase shift between signals is quasi constant (equal phase stripes). According to cross-correlation function definition (1), matrix \(\theta_{xy}\) should be antisymmetric with zero main diagonal.

2. Spatial structure resolution of instabilities on the Globus-M tokamak

Globus-M is a small spherical tokamak \((R = 0.36\ \text{m}, \ a = 0.24\ \text{m}, \ B_0 \approx 0.5\ \text{T})\). Magnetic perturbation detection system on Globus-M tokamak consists of 28 coils, placed at one poloidal cross-section (poloidal belt) and of 4 coils, mounted along the toroidal turn, shifted at 90° to each other.

![Spectrogram of toroidal coil T1 signal, shot #33171. White rectangle corresponds to processed time interval 140.18 — 140.26 ms.](image)

Data acquisition system on Globus-M allows us to detect signals with frequencies up to 500 kHz. In Globus-M discharges signals of slow MHD instabilities (about ones of kHz) are being registered, as well as signals of high-frequency (hundreds of kHz) Alfvén waves — fig. 1.

As an illustration of possible application on provided method we consider MHD perturbations, stimulated by neutral beam injection (NBI), identified\[2\] as toroidicity-induced Alfvén eigenmodes (TAEs) — fig. 1.
2.1. Toroidal mode structure

The burst presented in fig. 1 consists of 3 pronounced frequency branches. Highest branch magnitude corresponds to the lower harmonic of frequency $\sim 130$ kHz. Cross-phase matrix was constructed for signals of four toroidal probes on frequency of lower harmonic in processed time interval 140.18 — 140.26 ms. Resulting phase plot is presented in fig. 2.

Figure 2: Left block — cross-phase matrix for signals of four toroidal probes T1 — T4, shot #33171, processed interval: 140.18 — 140.26 ms. Signal frequency — about 130 kHz, Sampling frequency – 1 MHz. Values on axis correspond to probe numbers. Cross-phase range spreads from $-\pi$ (dark blue) to $\pi$ (brown). Right block — Same phase plot, doubled in both axis directions for more visibility. Equal phase stripes are highlighted in white.

Estimating equal phase stripes in fig. 2 (two stripes on doubled plot or one on initial plot) and taking into account initial signals evolution, one could infer that acquired phase plot corresponds to spatial toroidal mode with $n = 1$.

2.2. Poloidal mode structure

An implemented method turns out to be most useful in resolution of poloidal structure of perturbations, especially, applied to data on devices with not round plasma cross-sections, since it allows us not to take into account flux surface geometry, e.g. as it is for spatial Fourier transform.

Figure 3: Cross-phase matrix for poloidal array. Shot # 36980, 140.31 — 140.42 ms. Signal frequency $\sim 130$ kHz, Sampling rate – 1 MHz. Equal phase stripes are highlighted in white.
The matrix in fig. 3 demonstrates a predominance of poloidal mode with \( m = 3 \). Certain blurriness of the phase plot could be explained by presence of spatial modes of other poloidal numbers and lower magnitude, as well as by influence of reflected signals.

3. Spatial structure resolution of instabilities on the TUMAN-3M tokamak

TUMAN-3M represents itself as a compact tokamak with round cross-section and high aspect ratio \( (R = 0.53 \text{ m}, a = 0.22 \text{ m}, A = 2.4, B < 1 \text{ T}) \). Detection of poloidal perturbations on TUMAN-3M is implemented by an array of 16 coils uniformly spread along the poloidal turn. Amplification and sampling system allows to detect signals of frequencies up to 100 MHz. There is ion-cyclotron emission being registered in tokamak shots, which is stimulated by a neutral beam (about 40% of hydrogen and 60% deuterium) injected in deuterium plasma [3].

The cross-phase method has been also applied to signals of the TUMAN-3M magnetic diagnostic. For approbation of discussed method time interval of about 5 \( \mu s \) has been selected in one of the discharges, where NBI-stimulated ICE is presented.

![Figure 4: Cross-phase matrix of poloidal array (doubled). TUMAN-3M, shot #18052810, 62.000 — 62.005 ms. Signal frequency \( \sim 15 \text{ MHz} \), that corresponds to 1st hydrogen ICE harmonic, sampling rate — 64MHz.](image)

The acquired phase plot (fig. 4) is apparently formed by simultaneous propagation of two waves in opposite directions, both having \( m = 2 \). This is evidenced by the result of processing of manually set signals of magnetic diagnostic in circle geometry with analogous spatial structure.

![Figure 5: Left block — initial phase plot (see fig. 4). Right block — result of modeling of the phase plot, formed by two waves with \( m = 2 \), propagating in opposite directions by the method discussed in section 1.](image)
On the right block of fig. 5 the result of processing of manually set ideal signals using the discussed method is presented. Two waves of the same frequency $\omega$ and wavelength (or poloidal mode number $m$), propagating towards each other form standing wave, which could be provided as $\xi = 2\sin(\omega t) \cdot \cos(m\theta)$ in the most simple case. While the factor $\cos(m\theta)$ does not change its sign, signals of two probes are in-phase, although they have different magnitude. When sign changes to opposite, phase shifts instantly to $\pi$, which explains sharp leaps on the cross-phase (see fig. 5 — right block). Since extremums and zeros on cross-phase plot of real signals are placed the same way as on the modeling one, it makes us possible to assume, that during selected time interval of the discharge an instability with poloidal structure of $m = 2$ and $m = -2$ was propagating. Blurriness of the plot is caused by imperfection of initial signals.

4. Conclusion

The cross-phase method has shown its practical efficiency for determination of spatial mode structure of plasma MHD perturbations on the base of Mirnov coil signals. It could be successfully applied on various tokamaks, in spite of their geometry, that makes it a universal instrument, giving quite visual results.

References

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[2] Petrov Yu V et al 2011 Physics Reports 37 1001 — 5
[3] Askinazi L G, Abdulina G I et al 2018 Tech. Phys. Lett. 44 48 — 56