Photo-induced tunable Anomalous Hall and Nernst effects in tilted Weyl Semimetals using Floquet engineering

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Weyl semimetals (WSMs) exhibit various exotic physical phenomena due to the linear band crossings leading to the existence of the Weyl points. While type-I WSMs have a point like Fermi surface, the recently proposed type-II WSMs have touching electron and hole Fermi surface pockets, which has since been experimentally realized. Here, we discuss the effect of periodically driven circularly polarised laser beam in the Floquet limit on different transport properties of type-I and type-II Weyl semimetals. In particular, we study the effects of the optical field on anomalous thermal Hall and Nernst conductivities, using the linearized model. We find that for type-I WSMs, the Hall conductivity grows linearly with the amplitude $A_0$ of the applied radiation for low tilt, and that non-linear dispersion induced corrections are suppressed by a factor of $C^2 v^2$, with $C$ as the tilt parameter, and $v$, the Fermi velocity. In this case, the Nernst conductivity remains insensitive to the optical field. For type-II WSMs, the Hall conductivity decreases non-linearly with $A_0$, due to the large contribution from the physical momentum cutoff, and the Nernst conductivity falls of logarithmically with $A_0$.

I. INTRODUCTION

Materials with nontrivial topological features have been a primary source of interest to condensed matter physicists over the past few decades. Distinct topological phases exhibit different physical properties which are characterized by phase invariants such as Berry curvature. In recent years, Weyl Semimetals (WSM) have been identified as such a material having a variety of exotic physical properties stemming from a Hamiltonian having a gapless spectrum with at least one of the time and inversion symmetries broken [1–4]. The minimal model, obtained by breaking time reversal symmetry, which realizes such a material consists of two distinct points in the first Brillouin zone, where the conduction and the valence bands touch. These Weyl points or Weyl nodes are topological charges acting as a source or a sink for Berry curvature [2, 5, 6] as is reflected by their occurrence in opposite chirality pairs. By contrast, the inversion symmetry breaking minimal model requires four Weyl points [1, 4]. The former is distinguished by the presence of a topological Chern-Simons term [1] in its low energy electromagnetic description. Such materials, classed a Type-I WSMs, exhibit a number of phenomena including chiral magnetic waves, chiral anomaly induced plasmon modes, and chirality induced negative magneto resistance.

Since in condensed matter physics, we are not restricted to Lorentz invariant systems, an addition of a SO(3,1) symmetry breaking term to the low energy Hamiltonian described by a 3-vector $C$, the tilt parameter, coupled to the momentum $k$ leads to a tilt in the conical dispersion of the original system. For sufficiently large tilts, it can be shown [1, 7] that a Lifshitz phase transition occurs, leading to a new phase called a Type-II WSM. The fundamental difference between the two types of WSMs can be understood by considering their respective Fermi surfaces. Type-I WSMs have a point like Fermi surface, whereas in Type II WSMs, the Fermi surface splits into two, one each for electrons and holes, such that the density of states at the Weyl point is finite. Reports on the experimental realizations of Weyl semimetals have been presented in [8], and it was shown in [9] that WTe$_2$ is a possible candidate for an experimental realization of such a type-II WSM phase. Type-II WSM phase have been shown to exhibit some different physical properties as compared to Type-I WSMs, while preserving the Chern-Simons term and the chiral anomaly, both of which survive the Lifshitz transition.

Recent literature shows that external time dependent perturbations are a rich and versatile resource that can
be utilized to achieve topological diversity in systems that are topologically trivial in equilibrium. For example, time-periodic perturbations give rise to new differential operators with spectra characteristic of topological insulators (Floquet topological insulators), possessing hallmark phenomena associated with such topological phases. This combined with the highly developed technology for controlling electromagnetic modes enable the design of devices with ultra-fast switching of edge mode transport. Indeed, light matter interactions allow for an effective mechanism to create steady state exotic phases of matter, and to this end, Floquet engineering can be implemented on ultrafast timescales. The essence of Floquet theory lies in simplification of a time dependent problem to an effective time independent form, and it can be used to diagnose the behavior of physical observables in different phases [10–12]. The use of Floquet theory requires an off resonant condition[11], where the frequency of the optical field is much larger than the bandwidth of the system. The large frequency irradiation field is not directly involved in any electron transitions, but is associated with the virtual photon absorption and emissions leading to a plethora of physically relevant effects in photonic crystals [13], graphene[14], silicene[15], and Topological insulators[16–23].

These reports motivate the study of irradiated WSMs in the Floquet limit [24, 25] and the effects generated on the transport coefficients with a goal of obtaining tunable handles on properties such as Hall conductivity, Anamolous thermal Hall conductivity and Nernst conductivity. In this work, we restrict ourselves to the linearized model and look at the stated physical properties in the inversion symmetric case, and obtain analytical expressions in the zero temperature limit. Subtleties regarding the applicability of the linearized minimal model close to the Lifshitz transition have been discussed here and in [1], and we have taken care to conduct our analyses in the heart of either WSM region. In [26], authors have discussed the occurrence of Weyl semimetals, which is obtained from illuminating a Dirac semimetal with circularly polarized light.

The paper is structured as follows: In Sec. II, we have discussed our system, which is a tilted Weyl semimetal with the external time periodic perturbation, and expounded on the regime of applicability of the linearized model. Sec III contains the calculation of the current-current correlation function using the Matsubara Green’s function approach. In Sec IV, we obtain analytical expressions for the thermal Hall and Nernst conductivities for type-I and type-II Weyl semimetals in the $T \rightarrow 0$ limit, and discuss the applicability of the results. Finally we conclude in Sec V.

II. THE SYSTEM AND THE FLOQUET HAMILTONIAN

Consider a time reversal symmetry breaking tilted Weyl semimetal with two Weyl nodes of opposite chirality. The linearized Hamiltonian for such a system around each Weyl node $s = \pm$ is given by [27]

$$H_s = \hbar C_s (k_z - sQ) + s\hbar v \sigma \cdot (k - sQe_z)$$  (1)

where $C_s$ is the tilt parameter, which also is associated with the type of the Weyl point. Here, $v$ denotes the Fermi velocity in the absence of the tilting term, $2Q$ is the distance between the Weyl points in momentum space along $e_z$, and $\sigma$ is the vectorized Pauli matrix. The basic difference between the two types of WSMs is as follows: Type-I WSMs are found to occur in the regime $|C| \ll v$, and as we increase the tilt, at some value of $C \geq v$ a Lifshitz transition occurs, leading to a type-II WSM in the $|C| \gg v$ regime.

Let’s consider the chemical potential $\mu \geq 0$ for concreteness in this discussion and look at the Fermi surface for different values of tilt in the type I case, as shown in Fig. 1. The blue lines indicate the linearized band structure
near the Weyl nodes, and the red lines indicate the Fermi energy, given by the chemical potential in the $T \to 0$ limit. It is clear from Fig. 1 (right), as we increase the tilt the higher order momentum terms in the Hamiltonian become relevant and while the actual electron pocket size is finite as indicated by the green zone (the dashed boundary corresponds to the actual band structure with higher order corrections), the linearized model predicts infinite electron pocket size. We care about the electron pocket size as it is an estimate of the number of free carriers (both electric and thermal) which influences the behavior of the Hall and Nernst conductivities.

In contrast, consider the case of the type-II WSM, as shown in Fig. 2. The left hand plot shows the presence of infinite electron and hole pockets near the Lifshitz transition, a well-established limitation of the linearized model. The right hand plot shows that on increasing the tilt further this problem persists, but, it can be treated by using a physical momentum cutoff since the true band structure admits only finite pocket sizes, assuming that the chemical potential is sufficiently close to the Weyl nodes.

We are now interested in the effect of irradiating the surface of the Weyl semimetal with a laser beam. We use a beam of polarized optical field as $E(t) = E_0(\cos \omega t, -\sin \omega t)$, where $E_0$ and $\omega$ are the amplitude and frequency of the optical field. The corresponding vector potential $\vec{A}(t) = A_0(\sin(\omega t), \cos(\omega t))$ with $A_0 = -E_0/\omega$ will modify the momentum as $\hbar \vec{k}_i \to \hbar \vec{k}_i + e A_i$, where $\vec{A}(t + T) = \vec{A}(t)$ with $T = 2\pi/\omega$ as the periodicity. The full time-dependent Hamiltonian then has the form

$$H_s(k, t) = H_0(k) + V_s(t) \quad (2)$$

with $H_0(k) = h C_s(k_z - sQ) + s h v \sigma \cdot (k - sQ e_z)$, and $V_s(t) = s h v A_0(\sigma_x \sin \omega t + \sigma_y \cos \omega t)$. Considering all time scales used in the system to be large compared to $T$, we simplify to a time-independent problem using Floquet theory $[10, 28–30]$.

For completeness, we provide a brief discussion on the Floquet theory. The off resonant condition$[11, 12]$ is maintained here, and the static effective Hamiltonian in terms of the evolution operator $U$ $[30]$ is obtained as,

$$H_{\text{eff}}(k) = \frac{i\hbar}{T} \log U$$ \quad (3)

where,

$$U = \hat{T}_\text{time} \exp[\frac{1}{i\hbar} \int_0^T \mathcal{H}(k, t) dt]$$ \quad (4)

with $\hat{T}_\text{time}$ as the time-ordering operator. The effective Hamiltonian $H_{\text{eff}}$ describes the dynamics of the system on the time scale much longer than a period $T$, thus the response can be well described by an average over a period of oscillation. The matrix elements of the time-dependent Floquet Hamiltonian is $[30, 32–36]$

$$\mathcal{H}_{F}^{m,m'} = \mathcal{H}_0 \delta_{m,m'} + m \hbar \omega \delta_{m,m'} + \mathcal{H}^{'}_{m,m'}$$ \quad (5)
where $H_{m,m'} = V_n = \frac{1}{T} \int_0^T \mathcal{V}(t) e^{i(m-m')}\, dt = \frac{1}{T} \int_0^T \mathcal{V}(t) e^{i\omega t}\, dt$, where $\mathcal{V}(t)$ is the time dependent periodic perturbation term. Considering terms up to order $1/\omega$, the static time independent effective Hamiltonian is as follows

$$H_{\text{eff}} = \mathcal{H}_0 + \frac{[V_{-1}, V_{+1}]}{\hbar \omega}. \quad (6)$$

Eqn. (6) helps us to reduce time-dependent Hamiltonian (2) to an effective time independent Hamiltonian for our system as,

$$H_F = h C_s (k_z - s(Q + \Delta)) + s h v \sigma \cdot (k - s(Q + \Delta)e_z) + s h C_s \Delta, \quad (7)$$

where $\Delta = \frac{e^2 \nu A^2}{2 \hbar}$, is the contribution of the radiation field, and $\delta = \frac{e A}{2 \hbar} << 1$ is the perturbation limit. It is to be noted here from eqn. (7) that the form of the effective Hamiltonian is similar to the original Hamiltonian in eqn. (1), with the Weyl nodes being further displaced by a distance $2\Delta$ in momentum space. Moreover, in the inversion symmetric case, $s C_s = C, \forall s = \pm$, and so there is an overall shift in the total energy of both nodes by an amount equal to $hC \Delta$. This is unique to the inversion symmetric case and also suggests that the non-inversion symmetric cases might be very interesting to study. Note that this last term is not a mass term as that would require the Weyl nodes to split and the spectrum to be gapped.

### III. CURRENT-CURRENT CORRELATION FUNCTION AND THE MATSUBARA GREEN’S FUNCTION APPROACH

The anomalous Hall conductivity for the tilted WSM under the action of the circularly polarized light may now be derived from the zero frequency and zero wave-vector limit of the current-current correlation function (we consider $h = 1$)[37]

$$\Pi_{ij}(\Omega, \mathbf{q}) = T \sum_{\omega_n} \sum_{s=\pm} \int \frac{d^3 k}{(2\pi)^3} J_i^{(s)} (i \omega_n, \mathbf{k}) J_j^{(s)} (i \omega_n - i \Omega_m, \mathbf{k} - \mathbf{q}) \bigg|_{i\Omega_m \rightarrow i\Omega + i\delta}, \quad (8)$$

where $i, j = \{x, y, z\}, T$ is the temperature (setting the Boltzmann constant as unity) and $\omega_n(\Omega_m)$ are the fermionic(bosonic) Matsubara frequencies. Here $G_s(i \omega_n, \mathbf{k})$ is the single particle Green’s function of the electron, and $J_i^{(s)} = e(C, \delta_{i z} + s v \sigma_i)$ is the current operators, with $\delta_{ij}$ as the Kronecker delta. One can relate the Hall conductivity to the current-current correlation function as follows,

$$\sigma_{xy} = \lim_{\Omega \rightarrow 0} \frac{\Pi_{xy}(\Omega, 0)}{i \Omega}. \quad (9)$$

The one-particle Green functions have the following form

$$G_s(i \omega_n, \mathbf{k}) = \frac{1}{2} \sum_{\nu = \pm} \frac{1}{i \omega_n + \mu - C_s(k_z - s(Q + \Delta)) + t v |k - s(Q + \Delta)e_z| - sC_s \Delta}, \quad (10)$$

where $\mu$ is the chemical potential. Summing over the Matsubara fermionic frequencies, and tracing over the Pauli $\sigma$-matrices, we obtain the following form for the correlation function,

$$\Pi_{xy}(\Omega, 0) = \Pi_{xy}^{(+)}(\Omega, 0) + \Pi_{xy}^{(-)}(\Omega, 0), \quad (11)$$

where we separated the contributions from the two Weyl cones.

$$\Pi_{xy}^{(+)}(\Omega, 0) = \Pi_{xy}^{(+)}(\Omega, 0) + \Pi_{xy}^{(s)}(\Omega, 0), \quad (12)$$

$$\Pi_{xy}^{(s)}(\Omega, 0) = -s e^2 \int_{-\Delta - s(Q + \Delta)}^{\Delta - s(Q + \Delta)} \frac{d k_z}{2 \pi} \int_0^\infty \frac{k \, dk}{2 \pi} \frac{2 \nu^2 \Omega_m}{\Omega_m^2 + 4 \nu^2 k^2} \bigg|_{i\Omega_m \rightarrow i\Omega + i\delta}, \quad (13)$$

$$\Pi_{xy}^{(s)}(\Omega, 0) = s e^2 \int_{-\Delta - s(Q + \Delta)}^{\Delta - s(Q + \Delta)} \frac{d k_z}{2 \pi} \int_0^\infty \frac{k \, dk}{2 \pi} \frac{2 \nu^2 \Omega_m}{\Omega_m^2 + 4 \nu^2 k^2} \bigg|_{i\Omega_m \rightarrow i\Omega + i\delta}. \quad (14)$$
\[ \Pi_0 \] denotes the vacuum contribution for \( \mu = 0 \), whereas \( \Pi_{\text{FS}} \) is the contribution of the states at the Fermi surface. The function \( n_F(E) = (e^{(E-\mu)/T} + 1)^{-1} \) gives the Fermi distribution function and \( k = \sqrt{k_x^2 + k_y^2} \). The cut-off \( \Lambda_0 \), which is introduced in the \( k_z \) integral, is known not to affect the vacuum contribution to the Hall conductivity. However, the other cutoff in \( \Pi_{\text{FS}} \), which is denoted as \( \Lambda \), is crucial for finite Fermi surface effects in the type-II regime.

Using eqn. (13), we have

\[
\sigma_{xy}^{(s)} = \sigma_0^{(s)} + \sigma_{\text{FS}}^{(s)},
\]

\[
\sigma_0^{(s)} = -e^2 \int_{-\Lambda_0}^{\Lambda_0} dk_z \frac{d}{dk_z} \frac{s k_z}{2} \text{sign}(k_z),
\]

\[
\sigma_{\text{FS}}^{(s)} = -\frac{e^2}{8\pi^2} \int_{-\Lambda_0}^{\Lambda_0} dk_z \left[ \text{sign}(k_z) - \frac{vk_z}{C_s k_z - \mu + sC_s \Delta} \right] \int_{-\Lambda_0}^{\Lambda_0} dk_z \frac{d}{dk_z} \frac{s k_z}{2} \text{sign}(k_z) = \frac{e^2}{2\pi^2} (Q + \Delta)
\]

where \( \Theta(x) \) is the Heaviside function.

\[ \sigma_{xy} = \sigma_0 + \sigma_{\text{FS}} \]

Noticeably, the field has lead to a positive offset to \( \sigma_0 \). In the next section, we calculate \( \sigma_{\text{FS}} \) for different limits of \( C_s \).

IV. THERMAL ANOMALOUS HALL AND NERNST CONDUCTIVITIES

In this section, we analyze different cases associated with type-I and type-II WSMs. Importantly, we avoid the region where \( C_s \approx v \). First we consider \( |C_s| < v \), i.e the type-I case, where only the inversion symmetric setup is discussed i.e \( C_+ = -C_- = C \). In this case the Fermi surface contribution to the Hall effect can be written as

\[
\sigma_{\text{FS}}^{(s)} = -\frac{s e^2}{8\pi^2} \int_{-\Lambda_0}^{\Lambda_0} dk_z \left[ \text{sign}(k_z) - \frac{vk_z}{C_s k_z - \mu + sC_s \Delta} \right] \left[ \Theta \left( v^2 k_z^2 - \left( C_s k_z + sC_s \Delta - \mu \right)^2 \right) - 1 \right]
\]  \[ \text{Eqn. (22)} \]

The eqn. (22) is nonzero only for \( v^2 k_z^2 - \left( C_s k_z + sC_s \Delta - \mu \right)^2 < 0 \). Under this condition there will be two cases:

i) \( \mu - C \Delta > 0 \)  ii) \( \mu - C \Delta < 0 \)

We report the Nernst [38] and anomalous thermal Hall [39–41] conductivities in these two regimes, in the zero temperature limit, in the following section.
A. Conductivities of type-I WSMs

As mentioned previously, type-I WSMs follow $|C| << v$ condition. The thermal Hall and Nernst conductivities can be obtained as \[38-41\],

\[
\alpha_{xy} = eLT \frac{d\sigma_{xy}}{d\mu}, \quad K_{xy} = LT \sigma_{xy},
\]

where \( L = \frac{\pi^2 k_B^2}{3e^2} \) is the Lorentz number.

Thus from (23), the final form of the Nernst and thermal conductivities are obtained as,

\[
\alpha_{xy} = -e\frac{K_{B}^2 T C}{18h^2v^2}, \quad K_{xy} \approx \frac{K_{B}^2 T}{6h}\left[\left(Q + \Delta\right) - \frac{C(\mu - C\Delta)}{3hv^2}\right].
\]

Firstly, we note that \( K_{xy} \) varies smoothly around the point \( \mu = C\Delta \), and there is no jump as one might expect due to the seemingly distinct cases [(i) & (ii)], mentioned above. This is consistent with the fact that there is no known phase transition at \( \mu = C\Delta \), and we note that setting \( \Delta = 0 \) gives us back the results in [1]. The result is understandable in terms of the points made at the section II. Since both nodes get an energy boost of \( Ch\Delta \), the chemical potential which shows up in the Fermi-Dirac distribution function is offset by it, and one can think of this as fixing the chemical potential and moving the band structure for both nodes vertically.

We rewrite the Hall conductivity in a suggestive manner which shows that the Hall conductivity grows monotonously with \( \Delta \).

\[
K_{xy} = \frac{k_B^2 T}{6h}\left[Q - \frac{\mu}{3hv^2}\right] + \frac{k_B^2 T}{6h}\left[1 + \frac{C^2}{3hv^2}\right]\Delta = K_{xy}^0 + K_{xy}^\Delta,
\]

where \( K_{xy}^0 \) is the Hall conductivity in the absence of irradiation and \( K_{xy}^\Delta \) is the positive contribution of the laser field. It’s clear that as we increase the amplitude of the radiation field, the effective chemical potential \( \mu - C\Delta \) decreases and ultimately vanishes and then becomes negative in the \( \mu - C\Delta < 0 \) regime. Since moving the effective chemical potential further down amounts to increasingly incorrect hole pocket size estimations in the linearized model (Fig. 5, with the dashed line indicating the actual band structure), one might worry about the limit of validity of the result. Note that even in this case, we encounter no divergences from any cutoff as the hole pocket sizes are finite - Fig. 5 (right). However, the effect on the chemical potential is a second order effect, supressed by \( \frac{C}{v^2} \), and the dominant contribution to Hall conductivity comes from the shift in node spacing, i.e. \( \Delta \) which is part of the vacuum contribution, known to be cutoff independent. The vacuum contribution could pick up corrections due to a Hamiltonian with non-linear momentum dependence, but as long as we are sufficiently far from the Lifshitz transition, these corrections will turn out to be sub-leading order. Thus we can conclude that the Hall conductivity grows with the amplitude of the radiation field far away from the linear regime, within the bounds prescribed by Floquet perturbation theory, i.e. \( \delta << 1 \).

Since the linearized model predicts a linear dependence of \( K_{xy} \) on \( \mu \) in the type I regime, the Nernst conductivity is predictably constant and remains unchanged by the optical field.

B. Thermal and Nernst conductivities: type-II WSMs

We state the results for the Hall and Nernst conductivities in the type-II WSM case below, where we stick strictly to the \( |C| \gg v \) regime. [31, 37]

\[
\sigma_{FS}^{\text{tot}} = \frac{e^2}{2\pi^2} \left[ - (Q + \Delta)\left(\frac{v}{C} - 1\right) - \frac{v(-\mu + C\Delta)}{C^2} \ln \left[\frac{C^2\Lambda}{v(C\Delta - \mu)}\right]\right].
\]
By looking at the expression for $K_{xy}$ it is clear that it decreases for increasing $\Delta$. We see from Fig. 2, that on moving the effective chemical potential $C\Delta - \mu$ away from the Weyl node (by increasing $A_0$), we eventually run into the problem shown on the left diagram. Thus, in order for the qualitative description of the transport coefficients to be accurate, we cannot increase $A_0$ beyond the point of validity of the linear model. The functional behavior of $K_{xy}$ is limited to the regime depicted in Fig. 2 (right).

In the case of type-II WSMs, the Hall conductivity is related non-linearly to the chemical potential, an effect produced by introducing the adhoc but physical momentum cutoff, as discussed in section IIB. This behaviour is qualitatively correct as long as the effective chemical potential is sufficiently close to the Weyl node (i.e. $\Delta$ is sufficiently small to compensate for the large value of $C$), and as such we find that changing the amplitude of the photon field affects the Nernst conductivity, which decreases logarithmically with increasing $\Delta$.

Finally, while the physical momentum cutoff is difficult to estimate without using a non-linear model, we can provide a way to experimentally verify our findings independent of the cutoff. Notice that we can eliminate the $\Lambda$ dependence from eqns. (26) and (27) to get:

$$ [ -\frac{6\hbar C}{k_B^2 T v} K_{xy} + Q + \Delta] \frac{\hbar C}{C \Delta - \mu} = \frac{6\hbar C^2}{e k_B^2 T v} \alpha_{xy} + 1 \tag{28} $$

Thus any measurement of $K$ and $\alpha$ deep in the type II phase should always satisfy eqn.(28).

V. CONCLUSION

In this paper we have considered the effects of an incident circularly polarized optical field on a two distinct classes of Weyl Semimetals in the Floquet theory limit. The corresponding changes in Thermal Hall conductivity and Nernst conductivity have been calculated for the linearized model, with closed form expressions for the $T \to 0$ case. These results and the underlying physics can be summed up as follows.

For the effective Floquet Hamiltonian, we find that the Weyl nodes separate further due to the radiation field dependent parameter $\Delta$. This also gives rise to a constant term in the Hamiltonian proportional to $\Delta$, which leads to a distinct shifts in the spectrum of each Weyl node and is shown to be equal in the inversion symmetric case. Thus, the effect of the latter is to change the effective Fermi surface leading to an array of consequences for the transport coefficients.

For the type-I WSM case, we find that the leading correction to the Hall conductivity arises from the Floquet parameter $\Delta$. There exist subleading order corrections which may not be accurately captured by the linearized
model. The Nernst conductivity remains unchanged by the optical field because the Hall conductivity in the type-I regime shows a linear dependence on the chemical potential.

In the type-II case, we find that the Hall conductivity decreases with the amplitude of the incident laser beam, holding the frequency fixed, again within the limit prescribed by $\delta << 1$. The Nernst conductivity for this type of WSM is affected by the radiation field as the Hall conductivity depends non-linearly on the chemical potential. With increasing $\Delta$, the Nernst conductivity falls off logarithmically.

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