Volume effects and pressure effects on ferromagnetic superconductors

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Abstract. Thermal expansion of ferromagnetic superconductors below the superconducting transition temperature $T_{sc}$ of a majority spin conduction band is re-examined. In the previous study [J. Phys. Conf. Ser. 200 012056 (2010)] the volume differential of the kinetic energy of conduction electrons is constant. In this study, the volume differential of the kinetic energy of conduction electrons is inconstant, however. The superconducting gaps used in this study are like those of the thin film of A2 phase in liquid $^3$He. We find that the thermal expansion coefficient has the divergence at the superconducting transition temperatures. Thermodynamic Grüneisen’s relation is satisfied. The pressure effects on the superconductors is also investigated. As an example, the pressure differential of the superconducting transition temperatures and that of the Curie temperature are studied. We find the analytical expression of the pressure differential of the superconducting transition temperatures and that of the Curie temperature.

1. Introduction

The ferromagnetic superconductors [1, 2] has attracted many researchers again since the ferromagnetic superconductors in UGe$_2$ [3], UCoGe [4] and URhGe [5] was discovered. Recently, Hatayama and Konno [6] investigated the volume effects on ferromagnetic superconductors based on the free energy derived by Linder and Sudbo [7] by applying Takahashi’s method [8].

The first purpose of this study is stated. It is assumed that the volume differential of the kinetic energy of conduction electrons is constant in the previous study [6]. The volume differential of the kinetic energy of conduction electrons is usually inconstant. In this study, thermal expansion behaviour is re-examined by making the volume differential of the kinetic energy of conduction electrons inconstant. It is assumed that the Curie temperature $T_C$ is much larger than the superconducting transition temperatures $T_{sc}$ of a majority spin band and that the Fermi temperature $T_F$ is much higher than the Curie temperature $T_C$.

Next, we mention the antecedent studies of the pressure effects on magnetism and superconductivity chronologically. For example, Pfleiderer et al. studied critical behaviour at the magnetic transition as a function of hydrostatic pressure phenomenologically [9]. Aso et al. obtained the pressure dependence of Stoner gap based on the Stoner model from the neutron intensities [10]. Recently, Shopova and Uzunov investigated the pressure dependence of the superconducting transition temperature and that of the Curie temperature in ferromagnetic superconductors phenomenologically [11]. Shopova and Uzunov used the free energy of the Landau expansion about the superconducting order parameters and the magnetisation. Huang et al. reported the pressure differential of the superconducting transition temperature in FeSe$_{1-x}$Te$_x$ experimentally [12].

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The second purpose of this study is expressed. The pressure effects on ferromagnetic superconductors based on the microscopic model have been unresolved yet. The pressure differential of the superconducting transition temperatures and that of the Curie temperature derived from the single band model by Linder and Subo [7] is analysed in the context of the mean-field theory. We will obtain the analytical expression of the pressure differential of the superconducting transition temperatures and that of the Curie temperature.

This paper is organised as follows. In the next section thermal expansion of ferromagnetic superconductors will be derived. In section 3, the numerical results will be provided. In section 4, the pressure differential of the superconducting transition temperatures and that of the Curie temperature will be derived. Section 5 will be devoted to conclusions.

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2. The derivation of thermal expansion of ferromagnetic superconductors

Thermal expansion of ferromagnetic superconductors is derived in this section. We begin with the following free energy [6, 7]:

\[ F/N = F_0/N + F_T/N \]  

with

\[ F_0/N = \frac{IM^2}{2} + \sum_{\sigma} \frac{\Delta_{\sigma,0}^2}{2g} - \sum_{\sigma} \int_0^{E_F} d\epsilon N(\epsilon) \sqrt{(\epsilon - \sigma IM - E_F)^2 + \Delta_{\sigma,0}^2}/2, \]  

\[ F_T/N = T \sum_{\sigma} \int_0^{\infty} d\epsilon N(\epsilon) \ln(1 + e^{-\sqrt{(\epsilon - \sigma IM - E_F)^2 + \Delta_{\sigma,0}^2}/T}), \]  

\[ \sigma = 1(\uparrow) \text{or} -1(\downarrow), \]  

\[ \Delta_{k\sigma\sigma} = \frac{\Delta_{\sigma,0}}{\sqrt{3/8\pi}} Y_{l=1}^{\sigma}(\theta, \phi) \]  

where \( N(\epsilon) \) is the density of states. \( g, M, IM \) are the effective attractive pairing coupling, the magnetisation and the magnetic exchange energy, respectively. \( Y_{l=1}^{\sigma}(\theta, \phi) \) is the spherical harmonics. \( \Delta_{k\sigma\sigma} \) is the superconducting gap. We shall consider \( \sin \phi = 1 \) similar to the A2 phase in liquid \( ^3\)He. The superconducting order parameters at \( T=0K \) are obtained

\[ \Delta_{\sigma,0}(0) = 2E_0e^{-1/c\sqrt{1+\sigma M}} \]  

where \( \tilde{M} = IM/E_F \) and \( E_F \) is the Fermi energy. \( E_0 \) is the cutoff energy. \( E_0/E_F \) is set to 0.01. The weak-coupling constant \( c = gN(0)/2 \) is set to 0.2. The temperature dependence of the superconducting order parameters is as follows:

\[ \Delta_{\sigma,0}(T) = \Delta_{\sigma,0}(0) \tanh(1.74\sqrt{T_{sc\sigma}/T - 1}) \]  

where \( T_{sc\sigma} \) is the superconducting transition temperature of the spin \( \sigma \) band. Thermal expansion \( \omega \) is given by \( \omega = -K \frac{\partial \ln \epsilon}{\partial T} \). \( K \) is the compressibility. By our paying attention to \( \frac{\partial \ln \epsilon}{\partial T} \) where \( t \) is a transfer integral of electrons, from Eq.(1), thermal expansion of ferromagnetic superconductors is derived as follows:

\[ \frac{\omega}{(NE_F)} = \frac{\omega_0}{(NE_F)} + \frac{\omega_T}{(NE_F)} \]
with

\[ \omega_0/(NE_F) = -K \left[ \frac{1}{2} E_F \left( \frac{\partial \ln I}{\partial V} \right) \bar{M}^2 + \frac{1}{2} E_F \sum_{\sigma} \frac{\partial}{\partial V} \left( \frac{1}{g} \tilde{\Delta}^2_{\sigma,0} \right) \right. \]

\[ - N(0) E_F \left( \sum_{\sigma} \int_0^1 dx \sum_{\sigma} \frac{\sqrt{(x - \sigma \bar{M} - 1)^2 + \tilde{\Delta}^2_{\sigma,0}}}{2} \right) \]

\[ + \frac{1}{2} \sum_{\sigma} A_\sigma \left( \sqrt{(-\sigma \bar{M})^2 - \sqrt{(-\sigma \bar{M} - 1)^2 + \tilde{\Delta}^2_{\sigma,0}}} \right) \]

\[ + \sum_{\sigma} \frac{\partial \ln t}{\partial V} \int_0^1 dxx \frac{x - 1 - \sigma \bar{M}}{2\sqrt{(x - \sigma \bar{M} - 1)^2 + \tilde{\Delta}^2_{\sigma,0}}}, \]

\[ (9) \]

\[ A_\sigma = -\sigma \frac{\partial \ln I}{\partial V} \bar{M} - \frac{\partial \ln E_F}{\partial V}, \]

\[ (10) \]

where \( x = \epsilon/E_F \) and \( \tilde{\Delta}_{\sigma,0} = \Delta_{\sigma,0}/E_F \). \( N(0) \) is the density of states at the Fermi energy. \( N \) is the number of the magnetic atoms. The derivatives of \( \Delta_{\sigma,0} \) and \( \bar{M} \) do not appear in Eqs. (8), (9), and (10) because of the minimal conditions \( \frac{\partial F}{\partial E_F} = 0 \) and \( \frac{\partial F}{\partial M} = 0 \). The thermal expansion \( \omega_T \) originated from the explicitly thermal part of the free energy \( F_T \) is negligible because the temperature is very low compared with the Fermi temperature \( T_F \). The corresponding thermal expansion coefficient is given by

\[ \alpha = \frac{\partial \omega}{\partial T}. \]

(11)

In the next section, the temperature dependence of the thermal expansion coefficient of ferromagnetic superconductors will be provided numerically.

3. Results

Numerical results of the temperature dependence of the thermal expansion coefficient of ferromagnetic superconductors is given in this section. The temperature dependence of the thermal expansion coefficient of ferromagnetic superconductors is investigated with Eqs. (6), (7), (8), (9), (10), and (11). Fig. 1 shows the temperature dependence of the thermal expansion coefficient. Fig. 2 shows the temperature dependence of the thermal expansion coefficient in the vicinity of \( T_{sc} \). The divergence of the thermal expansion coefficient appears at \( T_{sc} \). The temperature dependence originates from the superconducting order parameters \( \Delta_{\sigma,0}(T) \) because the magnetisation at low temperatures is constant and because \( T_{sc} \) is much lower than the Curie temperature. These results show the similar behaviour to the previous study [6]. Moreover, the thermodynamic Grüneisen’s relation between the temperature dependence of the thermal expansion and that of the magnetic specific heat is satisfied because we use the same free energy as the free energy when the expression of the specific heat is derived. We will derive the pressure differential of the superconducting transition temperatures \( T_{sc} \) and that of the Curie temperature \( T_C \) in the next section.

4. The pressure differential of the superconducting transition temperatures and that of the Curie temperature

In this section, the pressure differential of the superconducting transition temperatures \( T_{sc} \) and that of the Curie temperature \( T_C \). The superconducting transition temperatures are given by [7]

\[ T_{sc} = 1.13E_0 \exp(-1/c\sqrt{1 + \sigma \bar{M}(T_{sc})}). \]

(12)
Figure 1. The reduced temperature $T/T_F$ dependence of the thermal expansion coefficient of ferromagnetic superconductors when $E_F/I = 0.1$, $\frac{\partial \ln I}{\partial T} = 0.1$, $E_F \frac{\partial}{\partial T} \left( \frac{T}{T_F} \right) = 0.1$, $N(0)E_F \frac{\partial T}{\partial T} = 0.1$, $N(0)E_F = 0.1$, and $N(0)E_F \frac{\partial \ln N(0)}{\partial T} = 0.1$.

Figure 2. The reduced temperature dependence of the thermal expansion coefficient at very low temperatures when $E_F/I = 0.1$, $\frac{\partial \ln I}{\partial T} = 0.1$, $E_F \frac{\partial}{\partial T} \left( \frac{T}{T_F} \right) = 0.1$, $N(0)E_F \frac{\partial T}{\partial T} = 0.1$, $N(0)E_F = 0.1$, and $N(0)E_F \frac{\partial \ln N(0)}{\partial T} = 0.1$. 
This expression is differentiated about the pressure $P$. We get the pressure differential of the superconducting transition temperature as follows:

$$\frac{\partial T_{sc\sigma}}{\partial P} = T_{sc\sigma}\left[\frac{\partial}{\partial P}(\ln 1.13E_0) + \frac{1}{e^2\sqrt{1+\sigma M(T_{sc\sigma})}} \frac{\partial c}{\partial P} \right] + \frac{1}{2e(1+\sigma M(T_{sc\sigma}))^{3/2}} \frac{\partial(\sigma M(T_{sc\sigma}))}{\partial P}.$$  \hspace{1cm} (13)

The pressure differential of the cutoff energy $E_0$, that of the magnetisation, and that of both of the weak coupling constant and density of states will be able to be estimated if this pressure differential of the superconducting transition temperatures and the magnetisation is compared with the experimental data.

We proceed to the pressure differential of the Curie temperature $T_C$. The magnetisation is determined by the following equation [7]

$$M = -\frac{1}{2} \sum\sigma \int_0^\infty d\epsilon N(0) \tanh\left[\frac{\epsilon - E_F - \sigma IM}{2T}\right].$$  \hspace{1cm} (14)

In the right hand-side, we perform the integration about $\epsilon$. We get the equation of the magnetisation

$$M = \frac{1}{2}N(0) + TN(0) \ln \frac{\cosh\left(E_F + 1\right)}{\cosh\left(E_F - 1\right)}.$$  \hspace{1cm} (15)

In $T \to T_C$, this equation is expanded around the small $M$. After that, $M$ is equal to zero at $T = T_C$. We obtain the Curie temperature

$$T_C = \frac{T_F}{\ln(2IN(0) - 1)}.$$  \hspace{1cm} (16)

If $1 < IN(0) < \frac{e+1}{2}$, the Curie temperature $T_C$ is higher than the Fermi temperature $T_F$. If $\frac{e+1}{2} < IN(0)$, the Curie temperature $T_C$ is lower than the Fermi temperature $T_F$. From Eq. (16) we differentiate the Curie temperature $T_C$ about the pressure $P$. We obtain the pressure differential of the Curie temperature $T_C$

$$\frac{\partial T_C}{\partial P} = \frac{\partial T_F}{\partial P} \left[\ln(2IN(0) - 1) - \left(2IN(0) - 1\right)\ln(2IN(0) - 1)\right]^2.$$  \hspace{1cm} (17)

If we analyse the pressure differential of $T_C$ in experimental data by this theory, the pressure differential of $T_F$ and that of $IN(0)$ will be able to be estimated.

5. Conclusions

The thermal expansion on ferromagnetic superconductors by making the volume differential of the kinetic energy of electrons inconstant have been investigated. The superconducting gaps similar to those of the thin film of the A2 phase in liquid $^3$He are assumed. We find that the divergence of the thermal expansion coefficient exists at the superconducting transition temperatures like the previous study [6]. The thermodynamic Grüneisen’s relation between the temperature dependence of the thermal expansion coefficient and that of the magnetic specific heat is satisfied.

The expression of the Curie temperature is derived. The Curie temperature depends on the inverse of logarithm of the electron-electron interaction.
The pressure effects on ferromagnetic superconductors have been analyzed. Both analytical expression of the pressure differential of the superconducting transition temperatures and that of the Curie temperature have obtained. If the expression of the pressure differential of the superconducting transition temperatures and that of the Curie temperature are compared with the experimental data, the pressure differential of the Fermi temperature, that of the weak coupling constant and that of the density of states will be determined.

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