A catalogue of double-mode high amplitude $\delta$ Scuti stars in the Galaxy and their statistical properties

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ABSTRACT

We present the first catalogue of double-mode and multi-mode high amplitude $\delta$ Scuti star (HADS) in the Galaxy. The catalogue contains source name, coordinates, radial modes such as the fundamental (F, period $P_0$), first-overtone (1O, period $P_1$), second-overtone (2O, period $P_2$), and the third-overtone (3O, period $P_3$) if available, period ratios, magnitude, and the relevant literature. Totally, 155 sources were collected until March 2021, in which 142 HADS with double-mode (132 with F and 1O, and 10 with 1O and 2O), 11 triple-mode, and 2 quadruple-mode. Statistical analysis shows clear features: $P_0$ lies in a range of 0.05 days to 0.175 days (sample: 132 double-mode HADS pulsating in F and 1O); $P_1/P_0$ lies in a range of $0.761 - 0.787$ (sample: 142 with $P_0$ and $P_1$), in which about 90% in $0.765 - 0.785$, which is wider than previous studies. The Petersen diagram was created with a much larger sample (144 HADS with $P_0$ and $P_1$) and we find that stars with periods in $0.05 - 0.1$ days scatter largely from the updated linear relation (i.e., Eq. [1]), the reason of which however needs further investigation. Particularly, we discover that the ratio $P_2/P_1$ (sample: 21 HADS with $P_1$ and $P_2$) equals 0.802$\pm$0.004, which could be viewed as a possible indicator to identify the modes 1O and 2O for multi-mode HADS. In addition, several unusual stars are pointed out, which may need more attention to their pulsations and stellar parameters in the future.

Subject headings: Stellar oscillations (1617); Delta Scuti variable stars (370); Short period variable stars (1453)

1. Introduction

$\delta$ Scuti (\$\delta$ Sct) stars are intermediate-mass pulsating stars with periods below 0.3 d and light amplitudes from mmag to 0.9 mag (Chang et al. 2013; Bowman 2017). They pulsate in radial as well as non-radial modes excited mainly in the $\kappa$ mechanism (Breger 2000; Aerts et al. 2010; Antoci et al. 2014; Murphy et al. 2020). In the Hertzsprung-Russell (HR) diagram, these pulsating stars lie in the overlapping region of the main sequence (MS) and the classical instability strip, corresponding to spectral types from A0 to F6 and luminosity
classess from III to V. They can be found at MS and post-MS, as well as pre-MS stage. The mass of $\delta$ Sct stars is usually metallicity-dependent, that’s to say, Population I with mass of about 1.5 to 2.3 $M_\odot$ is higher than Population II of about 1.0 to 1.3 $M_\odot$ (McNamara 2011). Usually, $\delta$ Sct stars are considered as the Population I stars of the flat Milk Way component in the young Galactic disk, while the Population II analogues are sometimes classified as SX Phoenicis stars (SX Phe) (McNamara 2011). The majority of $\delta$ Sct stars pulsate in many non-radial modes and their amplitude spectra appear very messy, which challenges the mode identification.

Amongst the $\delta$ Sct stars, one species that pulsates in one or several radial modes with large amplitudes, is usually called as High-Amplitude $\delta$ Scuti stars (HADS). HADS are easily recognized by their large-amplitude of $A_V \geq 0.3$ mag and non-sinusoidal light curves from observations (Breger 2000). Compared to $\delta$ Sct stars, HADS usually rotate with slow rotational velocities of $v \sin i \leq 40$ km s$^{-1}$ (Zorec & Royer 2012; Niemczura et al. 2015; Bowman et al. 2021), which might be a precondition for their high amplitudes (Breger 2000). In HR Diagram, ground-based observations indicate that HADS seem to occupy a narrow strip in the $\delta$ Sct instability region with a width of about 300 K in temperature, where the efficiency of mode excitation might be maximized (Petersen & Christensen-Dalsgaard 1996; McNamara 2000). However, photometric observations from space mission reveal that HADS could be spread beyond the narrow strip (Balona 2016). Although some studies suggest the properties of HADS might be due to their evolution stage being in post-MS state (Petersen & Christensen-Dalsgaard 1996; Bowman 2017), there is still no clear consensus on the physical difference between HADS and the lower amplitude $\delta$ Sct stars (Antoci et al. 2019). Another subgroup, which is not easy to be distinguished from HADS, called "SX Phe". Typically, they have high spatial motion, low metallicities (Breger 1980; Eggen & Iben 1989; Nemec 1989; Nemec et al. 2017; Eggen 1970, 1979; Li & Qian 2010; Yang et al. 2012; Zong et al. 2019), and usually located in globular clusters and dwarf galaxies (Rodríguez & López-González 2000; Jeon et al. 2003, 2004; Mazur et al. 2003; Garg et al. 2010). However field SX Phe stars were also found ($\sim$ 50, Fauvaud et al. 2010; Balona & Nemec 2012), which is very similar to Population I HADS, in light variations and pulsation modes (Fauvaud et al. 2010). In this work, any confirmed SX Phe star was removed from the sample.

Generally, HADS mainly pulsate in one or two dominant frequencies which are radial modes (Breger 2000; Yang et al. 2018; Xue et al. 2018). In fact, low-amplitude non-radial modes can also be found in some HADS, for example, AI Vel (Walraven et al. 1992) and V974 Oph (Poretti 2003). For double-mode HADS, which pulsate in the first two radial modes, i.e. the radial fundamental mode (F, period P0) and first-overtone mode (1O, period P1), the AAVSO International Variable Star Index (VSX$^1$, Watson et al. 2014) designate them as HADS(B). The main advantage of studying HADS(B) is the simplification of mode identification through period ratios, as the ratio of $P_1/P_0$ of most HADS(B) lies in a narrow range of $0.77 \leq P_1/P_0 \leq 0.78$ (McNamara 2000).

However for a long time, the sample of HADS is small, not to mention HADS(B),

$^1$the AAVSO International Variable Star Index: https://www.aavso.org/vsx/
which hampered the study among HADS, that’s to say, the relation between the single and
double/multi-mode HADS and their stellar parameters (Yang et al. 2021; Pietrukowicz et
al. 2013), the distribution of dominant periods and statistical properties of HADS(B), and
so on. In recent years, with more ground-based surveys and space missions, e.g. Catalina
Sky Survey (CSS, Larson et al. 2003; Drake et al. 2014, 2017), LINEAR (Palaversa et al.
2013), RATS (Ramsay et al. 2011), MACHO (Alcock et al. 1997, 2000), ZTF (Bellm et al.
2019; Graham et al. 2019), ASAS-SN (Shappee et al. 2014; Jayasinghe et al. 2020), OGLE
survey (Pietrukowicz et al. 2013, 2020) and Kepler mission (Borucki et al. 2010; Koch et
al. 2010), the number of HADS has increased dramatically (Süveges et al. 2012; Furgoni
2016). So it is a very good opportunity to investigate the overall properties and stellar
parameters of HADS(B), which may help us understand what determines the number of
radial modes in which a star pulsates and explore if there is different physics between the
single and double/multi-mode HADS.

In this paper, we collected HADS(B) and multi-mode HADS as far as possible, and
presented the first catalogue for this kind of objects. The aim of this catalogue is to provide
some basic information on HADS(B) and multi-mode HADS, including their coordinate,
pulsating modes, period ratios, and the corresponding reference. However, no attempt has
been made to compile complete reference lists and much effort has been made to avoid
errors to the best of our knowlewdge. With this catalogue, a statistical analysis between
the periods and their ratios was performed, and the relationship between them was further
investigated.

2. Data collection

The VSX is a public database, which aims to bring all the information on variable
stars in new and/or ongoing surveys together. It provides a simple web interface for public
to access, review and revise the metadata of variable stars (Watson et al. 2014).

In this work, we collected 152 HADS stars signed with HADS(B) stars from the
database. Among them, five stars (i.e. MGAB-V1192, MGAB-V1182, MGAB-V1185,
MGAB-V1163, MGAB-V1190) with only P0 (i.e., 0.05314, 0.05545, 0.06125, 0.04332,
and 0.05598 d, respectively) are provided in the database. To verify their double-mode
features, we re-analysed their pulsations using the public photometric data and found:
MGAB-V1192 and MGAB-V1163 have no other periods; MGAB-V1185 has another pe-
riod: P1=0.04233 d, but with a period ratio of 0.04233/0.06125=0.6911, not within the
typical ratio of radial overtones to fundamental mode; MGAB-V1182 has another period:
P1=0.04333 d, with a period ratio of 0.04333/0.05545=0.78142; MGAB-V1190 has an-
other period P1: 0.07205 d, if assumed it is the fundamental radial mode, then it has a
period ratio of 0.05598/0.07205=0.77696, which is within the typical value of period ra-
tio. Hence MGAB-V1182 and MGAB-V1190 are included in our sample. Another three
stars: OGLE-GD-DSCT-0018, OGLE-GD-DSCT-0024, and OGLE-GD-DSCT-0036, are
pulsators with multi-periods: (P0, P2, P3, no P1), (P0, P2, no P1) and (P1, P3, no P0),
respectively. Due to the lack of P0 or P1, they are not included in the sample.
In addition, we also collected 9 stars, i.e., AE UMa (Pócs & Szeidl 2001), GSC 02583-00504 (Wils et al. 2003), CzeV293 and CzeV581 (Skarka & Cagaš 2016), KIC 5950759 (Yang et al. 2018), KIC 10284901 (Yang & Esamdin 2019), ASASSN-V J061658.97-213318.9 (Jayasinghe et al. 2020), CSS J213934.3-050020 (Shi et al. 2021), and KIC 10975348 (Yang et al. 2021) from recent literature. Among them, two stars, i.e., AE UMa and GSC 02583-00504, are already listed in VSX but classified as SXPHE(B), i.e., double-mode SX Phe stars. However, recent studies suggest they should belong to Population I HADS stars (Niu et al. 2017, Wils et al. 2003). So they are also included in this sample.

In total, 155 stars were collected in this sample, among which each star possesses at least two radial pulsation modes. We note in the present sample, only Galactic HADS stars were taken into account, although HADS stars in other galaxies (Magellanic Clouds in particular) is large, which may need a separate study. For simplification, the radial fundamental mode and the first three radial overtones are abbreviated as F, 1O, 2O, and 3O in this work. With carefully inspection, we found there are 132 stars pulsating in F+1O, 10 in 1O+2O, 11 in triple-mode (10 stars in F+1O+2O, and 1 in 1O+2O+3O), and 2 stars in quadruple-mode (F+1O+2O+3O).
| Name                  | R.A.(J2000) | Dec.(J2000) | P0 (day) | P1 (day) | P2 (day) | P1/P0 | P2/P1 | Mag       | Reference               |
|-----------------------|-------------|-------------|----------|----------|----------|-------|-------|-----------|-------------------------|
| NSV 14800             | 00 01 16.22 | -60 36 57.1 | 0.15784  | 0.12207  | -        | 0.77339 | -     | 9.70 - 10.20 V | Poretti et al. (2005)   |
| V0823 Cas             | 00 05 42.38 | +63 24 14.2 | 0.66900  | 0.51269  | 0.41092  | 0.76635 | 0.80150 | 10.80 - 11.37 V | Jurcsik et al. (2006)   |
| FASTT 8               | 00 39 09.42 | +00 40 12.1 | 0.07302  | 0.05712  | -        | 0.78223 | -     | 13.7 - 14.1 V | Drake et al. (2009)     |
| CSS_J021145.1+371038  | 02 11 45.13 | +37 10 38.3 | 0.10529  | 0.08151  | -        | 0.77412 | -     | 15.81 - 16.38 V | Khruslov (2018)         |
| GSC 03693-01705       | 02 12 19.83 | +57 00 16.4 | 0.09109  | 0.07047  | -        | 0.77367 | -     | 13.41 - 13.80 V | Woźniak et al. (2004)   |
| RV Ari                | 02 15 07.46 | +18 04 28.0 | 0.09313  | 0.07195  | -        | 0.77256 | -     | 11.61 - 12.30 V | Pócs et al. (2002)      |
| CSS_J022438.9+262101  | 02 24 38.93 | +26 21 01.6 | 0.10158  | 0.08012  | -        | 0.78871 | -     | 13.36 - 13.83 V | Khruslov (2018)         |
| CSS_J023722.9+382847  | 02 37 22.96 | +38 28 47.2 | 0.11068  | 0.08544  | -        | 0.77199 | -     | 14.40 - 14.98 V | Khruslov (2018)         |
| CSS_J025539.8+314518  | 02 55 39.90 | +31 45 18.7 | 0.12890  | 0.09952  | -        | 0.77205 | -     | 16.20 - 16.78 V | Khruslov (2018)         |
| GSC 02860-01552       | 03 16 02.70 | +43 20 34.3 | 0.13831  | 0.10675  | -        | 0.77182 | -     | 12.52 - 13.02 V | Butters et al. (2010)   |
| DDE 107               | 03 41 53.63 | -06 53 52.7 | 0.08005  | 0.06199  | -        | 0.77445 | -     | 13.20 - 13.80 V | Drake et al. (2009)     |
| CSS_J034452.1+171634  | 03 44 52.19 | +17 16 34.6 | 0.07482  | 0.05784  | -        | 0.77306 | -     | 15.27 - 15.76 V | Khruslov (2018)         |
| V1384 Tau             | 03 54 07.27 | +07 59 15.4 | 0.13979  | 0.10739  | -        | 0.76823 | -     | 11.0 - 11.4 V  | Khruslov (2009)         |
| V1392 Tau             | 04 26 05.90 | +01 26 26.2 | 0.07443  | 0.05790  | -        | 0.77795 | -     | 12.00 - 12.55 V | Khruslov & Kusakin (2013)|
| USNO-B1.0 1332-0126848| 04 28 53.38 | +43 15 10.9 | 0.13715  | 0.10559  | -        | 0.76988 | -     | 14.4 - 15.0 V  | Khruslov (2018)         |
| USNO-B1.0 1329-0132547| 04 44 37.78 | +42 54 34.4 | 0.16190  | 0.12413  | -        | 0.76676 | -     | 15.0 - 15.3 V  | Khruslov & Kusakin (2014)|
| V542 Cam              | 04 53 46.52 | +68 28 26.5 | 0.17477  | 0.13400  | -        | 0.76663 | -     | 11.55 - 12.0 R | Khruslov (2012)         |
| CSS_J051346.4+065924  | 05 13 46.48 | +06 59 24.2 | 0.10297  | 0.07972  | -        | 0.77418 | -     | 16.09 - 16.66 V| Khruslov (2018)         |
| GSC 02900-00317       | 05 16 25.51 | +41 03 56.8 | 0.16046  | 0.13064  | -        | 0.81416 | -     | 12.65 - 12.96 V| Khruslov (2018)         |
| GSC 04757-00461       | 05 23 54.48 | +03 07 32.3 | 0.13253  | 0.10194  | -        | 0.76916 | -     | 14.70 - 15.28 V| Drake et al. (2009)     |
| CzeV581               | 05 46 27.21 | +31 11 09.9 | 0.08814  | 0.06822  | -        | 0.77389 | -     | 15.35 V      | Skarka & Cagaš (2016)   |
| V803 Aur              | 06 12 13.90 | +31 48 24.4 | 0.07106  | 0.05503  | 0.04439  | 0.77442 | 0.80665 | 12.89 - 13.20 V | Khruslov (2014)         |
| V2855 Ori             | 06 15 17.73 | +06 04 12.6 | 0.05808  | 0.04483  | -        | 0.77182 | -     | 9.93 - 10.36 V | Poretti et al. (2005)   |
| Name                  | R.A.(J2000) | Dec.(J2000) | P0 (day) | P1 (day) | P2 (day) | P1/P0 | P2/P1 | Mag       | Reference                  |
|-----------------------|-------------|-------------|----------|----------|----------|-------|-------|-----------|---------------------------|
| ASASSN-V J061658.97-213318.9 | 06 16 58.97 | -21 33 18.9 | 0.10183  | 0.07875  | -        | 0.77336 | -     | 13.27 - 13.67 V | Jayasinghe et al. (2020) |
| ASAS J062542+2206.4    | 06 25 41.61 | +22 06 19.5 | 0.15265  | 0.11731  | -        | 0.76848 | -     | 12.20 - 12.65 V | Khruslov (2015)          |
| Brh V128              | 06 44 01.06 | +22 44 31.7 | 0.15338  | 0.11770  | -        | 0.76736 | -     | 12.4 - 12.7 V  | Bernhard et al. (2004)   |
| 2MASS J06451725+4122158 | 06 45 17.25 | +41 22 15.9 | 0.05001  | 0.03869  | -        | 0.77369 | -     | 14.00 - 14.50 V | Jeon et al. (2010)       |
| GSC 762-110           | 07 12 19.41 | +09 21 06.7 | 0.19451  | 0.14862  | 0.11908  | 0.76407 | 0.80124 | 10.54 - 10.73 V | Wils et al. (2008)       |
| CSS_J072643.6+413522  | 07 26 43.58 | +41 35 23.1 | 0.09032  | 0.07023  | -        | 0.77758 | -     | 16.71 - 17.41 V | Khruslov (2018)          |
| NSVS 7293918          | 07 44 38.60 | +29 12 22.8 | 0.08854  | 0.06850  | -        | 0.77372 | -     | 12.649 (0.313) R | Wils et al. (2012)       |
| AI Vel                | 08 14 05.15 | -22 14 07.7 | 0.22871  | 0.17423  | -        | 0.76180 | -     | 12.05 - 12.8 V  | Bates & Halliwell (1985) |
| V733 Pup              | 08 18 06.98 | -22 14 07.7 | 0.11157  | 0.08621  | -        | 0.77266 | -     | 6.15 - 6.76 V   | Khruslov (2011)          |
| CSS_J082237.3+030441  | 08 22 37.42 | +03 04 41.8 | 0.09055  | 0.07030  | -        | 0.77636 | -     | 13.31 - 13.83 V | Khruslov (2018)          |
| NT Cam                | 08 24 17.52 | +74 30 25.4 | 0.08242  | 0.06505  | -        | 0.78925 | -     | 12.74 - 13.13 V | Kochanek et al. (2017)   |
| VZ Cnc                | 08 40 52.12 | +09 49 27.2 | 0.17836  | 0.14280  | -        | 0.80063 | 7.18 - 7.91 V | Mahra & Sanyal (1968)    |
| CSS_J085636.3-022534  | 08 56 36.42 | -02 25 35.2 | 0.08827  | 0.06839  | -        | 0.77471 | -     | 14.32 - 14.85 V | Khruslov (2018)          |
| LINEAR 5588339        | 08 58 54.72 | +15 22 09.7 | 0.05763  | 0.04460  | -        | 0.77385 | -     | 13.72 - 14.10 V | Drake et al. (2009)      |
| V0526 Vel             | 09 03 13.34 | -52 02 28.7 | 0.07746  | 0.06245  | -        | 0.80622 | 9.60 - 9.95 V | Khruslov (2011)          |
| GSC 04135-00504       | 09 30 23.95 | +61 20 33.9 | 0.06150  | 0.04771  | -        | 0.77581 | -     | 12.40 - 12.90 V | Kochanek et al. (2017)   |
| SSS_J093025.5-215630  | 09 30 25.52 | -21 56 31.6 | 0.09585  | 0.07424  | -        | 0.77449 | -     | 12.67 - 13.21 V | Khruslov (2018)          |
| AE UMa                | 09 36 53.16 | +44 04 00.4 | 0.08602  | 0.06653  | -        | 0.77343 | -     | 10.86 - 11.52 V | Pocs & Szeidl (2001)     |
| ASAS J094303-1707.3   | 09 43 02.81 | -17 07 15.9 | 0.09918  | 0.07652  | -        | 0.77153 | -     | 11.75 - 12.12 V | Poretti et al. (2005)    |
| VX Hya                | 09 45 46.85 | -12 00 14.3 | 0.22339  | 0.17272  | -        | 0.77318 | -     | 10.21 - 10.96 V | Fitch (1966)             |
| SSS_J095011.1-244057  | 09 50 11.12 | -24 40 58.0 | 0.06839  | 0.05302  | -        | 0.77525 | -     | 15.18 - 15.50 V | Drake et al. (2009)      |
| SSS_J095657.2-231722  | 09 56 57.19 | -23 17 22.9 | 0.05667  | 0.04425  | -        | 0.78083 | -     | 16.2 - 16.74 V  | Drake et al. (2009)      |
| OGLE-GD-DSCT-0003     | 10 36 21.58 | -62 33 14.7 | 0.07517  | 0.05827  | -        | 0.77515 | -     | 18.33 - 18.53 I | Pietrukowicz et al. (2013)|
| Name                  | R.A.(J2000) | Dec.(J2000) | P0 (day) | P1 (day) | P2 (day) | P1/P0 | P2/P1 | Mag     | Reference                  |
|-----------------------|-------------|-------------|----------|----------|----------|-------|-------|---------|-----------------------------|
| OGLE-GD-DSCT-0007     | 10 41 08.32 | -61 42 16.9 | 0.11941  | 0.09222  | -        | 0.77233 | -     | 14.01 - 14.14 I             | Pietrukowicz et al. (2013) |
| OGLE-GD-DSCT-0008     | 10 41 47.88 | -61 39 55.7 | 0.19804  | 0.15152  | -        | 0.76509 | -     | 13.27 - 13.41 I             | Pietrukowicz et al. (2013) |
| OGLE-GD-DSCT-0010     | 10 42 42.80 | -61 40 59.7 | 0.11594  | 0.08934  | -        | 0.77058 | -     | 18.14 - 18.27 I             | Pietrukowicz et al. (2013) |
| OGLE-GD-DSCT-0012     | 10 42 51.62 | -61 35 21.4 | 0.16748  | 0.13153  | -        | 0.78537 | -     | 13.62 - 13.75 I             | Pietrukowicz et al. (2013) |
| OGLE-GD-DSCT-0014     | 10 43 44.31 | -61 30 41.2 | 0.07185  | 0.05567  | -        | 0.77471 | -     | 18.55 - 19.00 I             | Pietrukowicz et al. (2013) |
| OGLE-GD-DSCT-0016     | 10 46 27.35 | -60 44 01.2 | 0.06743  | 0.05223  | -        | 0.77464 | -     | 16.09 - 16.36 I             | Pietrukowicz et al. (2013) |
| OGLE-GD-DSCT-0020     | 10 48 57.42 | -61 36 34.1 | 0.21544  | 0.16404  | -        | 0.76143 | -     | 13.88 - 14.07 I             | Pietrukowicz et al. (2013) |
| BPS BS 16553-0026     | 10 52 48.49 | +41 54 35.3 | 0.12551  | 0.09695  | -        | 0.77248 | -     | 12.75 - 12.95 R             | Butters et al. (2010)      |
| OGLE-GD-DSCT-0025     | 10 53 17.74 | -61 14 47.7 | 0.17406  | 0.13311  | 0.10646  | 0.76473 | 0.79983 | 13.16 - 13.29 I             | Pietrukowicz et al. (2013) |
| V0474 UMa            | 10 55 02.50 | +61 42 17.2 | 0.06404  | 0.04961  | -        | 0.77469 | -     | 13.43 - 13.86 V             | Wils et al. (2014)         |
| OGLE-GD-DSCT-0027     | 10 56 22.73 | -61 33 28.0 | 0.11764  | 0.09067  | -        | 0.77072 | -     | 16.04 - 16.16 I             | Pietrukowicz et al. (2013) |
| OGLE-GD-DSCT-0028     | 10 56 25.25 | -61 49 38.1 | 0.06893  | 0.05338  | -        | 0.77445 | -     | 15.54 - 15.69 I             | Pietrukowicz et al. (2013) |
| OGLE-GD-DSCT-0029     | 10 56 44.62 | -61 24 23.7 | 0.06143  | 0.04740  | -        | 0.77155 | -     | 16.52 - 16.65 I             | Pietrukowicz et al. (2013) |
| OGLE-GD-DSCT-0030     | 10 56 52.71 | -60 41 36.1 | 0.12661  | 0.09754  | -        | 0.77043 | -     | 16.46 - 16.63 I             | Pietrukowicz et al. (2013) |
| OGLE-GD-DSCT-0031     | 10 57 53.54 | -61 54 01.7 | 0.11676  | 0.09061  | -        | 0.77606 | -     | 18.55 - 18.93 I             | Pietrukowicz et al. (2013) |
| OGLE-GD-DSCT-0032     | 10 58 26.38 | -61 14 42.0 | 0.08271  | 0.06464  | -        | 0.78156 | -     | 16.47 - 16.83 I             | Pietrukowicz et al. (2013) |
| OGLE-GD-DSCT-0033     | 10 59 21.02 | -60 48 00.9 | 0.06331  | 0.04894  | 0.03949  | 0.77302 | 0.80691 | 15.88 - 15.99 I             | Pietrukowicz et al. (2013) |
| OGLE-GD-DSCT-0037     | 11 09 23.01 | -60 51 20.8 | 0.23460  | 0.17878  | -        | 0.76209 | -     | 15.80 - 15.94 I             | Pietrukowicz et al. (2013) |
| V899 Car              | 11 09 52.24 | -60 57 56.7 | 0.11080  | 0.08585  | -        | 0.77482 | -     | 11.8 - 12.4 V               | Khruslov (2009)            |
| LINEAR 1683151        | 11 32 05.40 | -03 48 27.5 | 0.06185  | 0.04821  | -        | 0.77949 | -     | 16.24 (0.51) V              | Poleski (2014)             |
| OGLE-GD-DSCT-0040     | 11 32 10.16 | -60 42 44.6 | 0.09581  | 0.07406  | -        | 0.77304 | -     | 18.22 - 18.57 I             | Pietrukowicz et al. (2013) |
| CSS_J114944.1+185128  | 11 49 44.16 | +18 51 28.1 | 0.08968  | 0.07021  | -        | 0.78289 | -     | 16.89 - 17.60 V             | Khruslov (2018)            |
| LINEAR 2653935        | 11 59 42.51 | +06 08 22.0 | 0.09521  | 0.07460  | -        | 0.78357 | -     | 16.52 (0.35) V              | Poleski (2014)             |
| Name                  | R.A.(J2000)   | Dec.(J2000)   | P0 (day) | P1 (day) | P2 (day) | P1/P0 | P2/P1 | Mag   | Reference                  |
|-----------------------|---------------|---------------|----------|----------|----------|--------|--------|-------|---------------------------|
| CSS_J120037.4+122805  | 12 00 37.39   | +12 28 05.3   | 0.09032  | 0.07086  | -        | 0.78454 | -      | 16.76 - 17.66 V | Khruslov (2018) |
| GSC 07243-00871       | 12 08 49.77   | -36 33 11.1   | 0.06003  | 0.04648  | -        | 0.77427 | -      | 13.02 - 13.15 V | Butters et al. (2010) |
| SSS_J130636.7-354622  | 13 06 36.82   | -35 46 23.0   | 0.13799  | 0.10639  | -        | 0.77099 | -      | 13.99 - 14.74 V | Khruslov (2018) |
| OGLE-GD-DSCT-0044     | 13 15 58.51   | -64 37 25.7   | -        | 0.15639  | 0.12483  | -      | 0.79820 | 14.74 - 14.88 I | Pietrukowicz et al. (2013) |
| OGLE-GD-DSCT-0045     | 13 16 00.48   | -64 58 28.1   | 0.11781  | 0.09105  | -        | 0.77281 | -      | 14.96 - 15.12 I | Pietrukowicz et al. (2013) |
| OGLE-GD-DSCT-0048     | 13 24 14.87   | -65 09 03.9   | 0.10740  | 0.08293  | 0.06655  | 0.77216 | 0.80248 | 16.22 - 16.37 I | Pietrukowicz et al. (2013) |
| OGLE-GD-DSCT-0049     | 13 24 21.24   | -64 55 55.3   | 0.36142  | 0.27315  | 0.21699  | 0.75577 | 0.79440 | 14.43 - 14.70 I | Pietrukowicz et al. (2013) |
| OGLE-GD-DSCT-0071(b)  | 13 26 16.46   | -64 55 40.0   | -        | 0.14610  | 0.11689  | -      | 0.80007 | 15.49 - 15.63 I | Pietrukowicz et al. (2013) |
| OGLE-GD-DSCT-0054     | 13 31 35.98   | -64 09 35.7   | 0.10994  | 0.08539  | -        | 0.77669 | -      | 17.05 - 17.30 I | Pietrukowicz et al. (2013) |
| OGLE-GD-DSCT-0056     | 13 33 45.66   | -64 01 17.1   | -        | 0.44410  | 0.35882  | -      | 0.80797 | 15.63 - 15.73 I | Pietrukowicz et al. (2013) |
| LINEAR 9328902        | 13 35 49.76   | +26 55 16.7   | 0.05175  | 0.04047  | -        | 0.78203 | -      | 16.24 (0.45) V | Poleski (2014) |
| GSC 03851-00240       | 13 45 21.66   | +54 11 51.2   | 0.06794  | 0.05260  | -        | 0.77418 | -      | 12.7 - 13.09 V | Wils et al. (2015) |
| V1393 Cen            | 13 57 15.60   | -52 55 22.6   | 0.11778  | 0.09083  | -        | 0.77118 | -      | 9.16 - 9.63 V   | Khruslov (2009) |
| GSC 02008-00003       | 14 22 31.21   | +24 34 57.0   | 0.05960  | 0.04614  | -        | 0.77415 | -      | 14.05 - 14.30 V | Butters et al. (2010) |
| LINEAR 13836407       | 15 12 53.99   | +23 17 48.3   | 0.06001  | 0.04674  | -        | 0.77884 | -      | 16.78 - 17.23 V | Palaversa et al. (2013) |
| V0638 Ser            | 15 13 22.01   | +18 15 58.3   | 0.05419  | 0.04191  | -        | 0.77338 | -      | 13.42 - 13.72 V | Wils et al. (2014) |
| CSS_J151435.5-145958  | 15 14 35.52   | -14 59 58.4   | 0.04932  | 0.03845  | -        | 0.77962 | -      | 14.90 - 15.45 V | Drake et al. (2009) |
| QS Dra               | 15 21 34.64   | +61 29 22.7   | 0.09442  | 0.07304  | -        | 0.77358 | -      | 12.63 - 13.16 V | Khruslov (2007) |
| ASAS J152315-5603.7   | 15 23 15.43   | -56 03 43.2   | 0.12675  | 0.09767  | -        | 0.77061 | -      | 11.22 - 11.57 V | Khruslov (2014) |
| LINEAR 15289666       | 15 45 18.61   | +16 40 51.6   | 0.08654  | 0.06870  | -        | 0.79383 | -      | 15.98 - 16.42 V | Khruslov (2018) |
| V0647 Ser            | 15 52 51.39   | +06 06 06.1   | 0.05492  | 0.04267  | -        | 0.77689 | -      | 15.8 - 16.4 V   | Palaversa et al. (2013) |
| GSC 2583-00504        | 16 13 31.71   | +32 34 42.8   | 0.05172  | 0.03999  | -        | 0.77320 | -      | 12.30 - 12.50 V | Wils et al. (2003) |
| LINEAR 16586778       | 16 13 57.55   | +28 28 57.2   | 0.07075  | 0.05570  | -        | 0.78728 | -      | 15.30 - 15.97 V | Drake et al. (2009) |
| Name       | R.A.(J2000) | Dec.(J2000) | P0 (day) | P1 (day) | P2 (day) | P1/P0     | P2/P1     | Mag        | Reference                   |
|------------|-------------|-------------|----------|----------|----------|-----------|-----------|------------|-----------------------------|
| V1553 Sco  | 16 20 21.77 | -35 41 16.0 | -        | 0.18430  | 0.14704  | -         | 0.79783   | 9.41 - 9.68 V | Khruslov (2009)              |
| CSS_J162243.6+000503 | 16 22 43.63 | +00 05 03.1 | 0.10516  | 0.08130  | -        | 0.77312   | -         | 14.73 - 15.31 V | Khruslov (2018)              |
| CSS_J162818.8+032651 | 16 28 18.88 | +03 26 51.1 | 0.12535  | 0.09859  | -        | 0.78649   | -         | 16.52 - 17.51 V | Khruslov (2018)              |
| V0552 Dra  | 16 29 40.31 | +57 20 33.3 | 0.06114  | 0.04750  | -        | 0.77693   | -         | 13.24 - 13.48 V | Khruslov (2014)              |
| NSV 7805   | 16 32 20.12 | -02 12 08.3 | 0.06460  | 0.05070  | -        | 0.78477   | -         | 16.2 - 16.8 V   | Drake et al. (2014)          |
| GSC 03887-00087 | 17 08 14.77 | +52 53 53.4 | 0.10718  | 0.08293  | -        | 0.77374   | -         | 13.45 - 13.80 V | Butters et al. (2010)        |
| V879 Her    | 17 31 12.72 | +28 03 16.8 | 0.05689  | 0.04413  | -        | 0.77564   | -         | 15.23 - 15.88 V | Wils et al. (2014)           |
| V703 Sco    | 17 42 16.81 | -32 31 23.6 | 0.14996  | 0.11522  | -        | 0.76832   | -         | 7.58 - 8.04 V   | Oosterhoff (1966)            |
| MGAB-V1182  | 17 45 32.66 | +28 45 51.7 | 0.05545  | 0.04333  | -        | 0.78142   | -         | 17.45 - 17.95 V | this work                   |
| CSS_J174643.7+285533 | 17 46 43.81 | +28 55 33.3 | 0.08153  | 0.06346  | -        | 0.77835   | -         | 14.60 - 15.17 V | Khruslov (2018)              |
| NSV 9856    | 17 56 00.20 | -30 42 46.6 | 0.11849  | 0.09127  | 0.07324  | 0.77028   | 0.80245   | 12.10 - 12.50 V | Khruslov (2014)              |
| MACHO 119.19574.1169 | 18 02 00.37 | -29 48 43.2 | 0.10685  | 0.08272  | -        | 0.77421   | -         | 16.60 - 16.88 V | Alcock et al. (2000)         |
| OGLE BW1 V207 | 18 02 14.98 | -29 54 08.8 | 0.08560  | 0.06623  | -        | 0.77375   | -         | 18.67 V      | Poretti et al. (2005)        |
| OGLE BW2 V142 | 18 02 18.04 | -30 08 11.4 | 0.06604  | 0.05140  | -        | 0.77836   | -         | 17.77 V      | Poretti et al. (2005)        |
| MACHO 114.19840.890 | 18 02 31.85 | -29 27 03.9 | 0.12557  | 0.09679  | -        | 0.77082   | -         | 17.99 V      | Alcock et al. (2000)         |
| MACHO 114.19969.980 | 18 02 52.20 | -29 30 24.5 | 0.10327  | 0.07981  | -        | 0.77282   | -         | 17.03 - 17.35 I | Alcock et al. (2000)         |
| MACHO 128.21542.753 | 18 06 35.93 | -28 39 31.3 | 0.12005  | 0.09254  | -        | 0.77083   | -         | 17.79 V      | Alcock et al. (2000)         |
| MACHO 115.22573.263 | 18 09 00.48 | -29 14 30.9 | 0.09175  | 0.07087  | -        | 0.77243   | -         | 17.95 V      | Alcock et al. (2000)         |
| MACHO 116.24384.481 | 18 13 16.45 | -29 49 27.0 | 0.08691  | 0.06716  | -        | 0.77272   | -         | 17.29 V      | Alcock et al. (2000)         |
| MACHO 162.25343.874 | 18 15 16.33 | -26 35 40.2 | 0.11128  | 0.08591  | -        | 0.77196   | -         | 17.96 V      | Alcock et al. (2000)         |
| ATO J274.5653+06.5568 | 18 18 15.68 | +06 33 24.8 | 0.11151  | 0.08609  | -        | 0.77204   | -         | 13.80 - 14.35 V | Kochanek et al. (2017)       |
| ASAS J182536-4213.6 | 18 25 36.26 | -42 13 35.8 | 0.10719  | 0.08216  | -        | 0.76648   | -         | 11.43 - 12.0 V | Khruslov (2011)              |
| CzeV293     | 18 28 54.89 | +12 21 24.4 | 0.05572  | 0.04312  | -        | 0.77387   | -         | 15.67 V      | Skarka & Cagaš (2016)        |
| Name            | R.A.(J2000)  | Dec.(J2000)  | P0 (day) | P1 (day) | P2 (day) | P1/P0 | P2/P1 | Mag       | Reference                  |
|-----------------|--------------|--------------|----------|----------|----------|-------|-------|-----------|----------------------------|
| V575 Lyr        | 18 29 43.24  | +28 09 54.6  | 0.14556  | 0.11150  | -        | 0.76602 | -     | 12.55 - 12.85 V | van Cauteren & Wils (2001) |
| V0836 Lyr       | 18 29 47.55  | +37 45 01.5  | 0.11658  | 0.09030  | 0.05084  | 0.77458 | 0.56308 | 12.7 - 13.0 V | Butters et al. (2010)      |
| MGAB-V1190      | 18 44 23.90  | +40 03 17.5  | 0.07205  | 0.05598  | -        | 0.77696 | -     | 17.40 - 17.60 g | this work                  |
| SSS_J184425.3-563648 | 18 44 25.40 | -56 36 46.2  | 0.09437  | 0.07283  | -        | 0.77169 | -     | 13.58 - 13.99 V | Khruslov (2018)            |
| KIC 5950759     | 19 15 00.54  | +41 13 55.4  | 0.07032  | 0.05453  | -        | 0.77555 | -     | 13.516 V | Yang et al. (2018)        |
| ASAS J192227-5622.5 | 19 22 27.39 | -56 22 28.1  | 0.14909  | 0.11277  | -        | 0.75639 | -     | 11.64 - 12.08 V | Khruslov (2011)            |
| KIC 10975348    | 19 26 46.11  | +48 25 30.8  | 0.09773  | 0.07408  | -        | 0.75799 | -     | 18.598 (0.7) Kp | Yang et al. (2021)         |
| KIC 2857323     | 19 29 49.16  | +38 01 21.7  | 0.07618  | 0.05897  | -        | 0.77409 | -     | 13.342 (0.27) Kp | Barceló Forteza et al. (2018) |
| V0798 Cyg       | 19 38 06.90  | +30 54 33.5  | -        | 0.19477  | 0.15602  | -     | 0.80105 | 12.28 - 12.66 V | Musazzi et al. (1998)      |
| SSS_J194149.5-533246 | 19 41 49.54 | -53 32 46.6  | 0.11106  | 0.08578  | -        | 0.77236 | -     | 15.70 - 16.14 V | Khruslov (2018)            |
| KIC 10284901    | 19 43 46.28  | +47 20 32.8  | 0.05265  | 0.04109  | -        | 0.78044 | -     | 15.459 Kp | Yang & Esamdin (2019)     |
| V829 Aql        | 19 46 57.29  | +03 30 28.5  | 0.29244  | 0.22097  | 0.17650  | 0.75561 | 0.79875 | 10.00 - 10.50 V | Handler et al. (1998)      |
| GSC 03144-595   | 19 48 02.92  | +41 46 55.8  | 0.20364  | 0.15549  | 0.12445  | 0.76355 | 0.80037 | 10.45 - 10.95 V | Mow et al. (2016)          |
| GSC 06905-01641 | 20 10 22.51  | -23 10 59.7  | 0.06880  | 0.05329  | -        | 0.77459 | -     | 14.84 - 15.29 V | Khruslov (2014)            |
| GSC 03949-00386 | 20 19 44.95  | +58 29 20.0  | 0.09578  | 0.07394  | -        | 0.77193 | -     | 11.0 - 11.2 V | Bernhard et al. (2012)     |
| GSC 03949-00811 | 20 26 01.74  | +59 30 53.5  | 0.16975  | 0.13008  | -        | 0.76629 | -     | 11.2 V | Poretti et al. (2005) |
| GSC 07460-01520 | 20 33 38.54  | -32 55 03.6  | 0.08701  | 0.06815  | -        | 0.78326 | -     | 14.30 - 14.65 V | Butters et al. (2010)      |
| CSS_J205540.0-020320 | 20 55 40.03 | -02 03 21.2  | 0.08394  | 0.06556  | -        | 0.78097 | -     | 15.67 - 16.40 V | Khruslov (2018)            |
| VY Equ          | 20 58 49.64  | +08 54 05.3  | -        | 0.17645  | 0.14078  | -     | 0.79785 | 10.21 - 10.51 V | Khruslov (2011)            |
| GSC 05183-01938 | 20 59 10.15  | -03 28 20.7  | 0.07622  | 0.05902  | -        | 0.77437 | -     | 14.45 - 15.11 V | Khruslov (2018)            |
| [SIG2010] 3269918 | 20 59 27.28 | -01 13 49.0  | 0.05238  | 0.04089  | -        | 0.78061 | -     | 17.68 (0.49) r | Süveges et al. (2012)      |
| V1719 Cyg       | 21 04 32.92  | +50 47 03.3  | -        | 0.26730  | 0.21378  | -     | 0.79978 | 7.81 - 8.21 V | Mantegazza & Poretti (1986) |
| V3124 Cyg       | 21 12 53.69  | +33 17 34.3  | -        | 0.10530  | 0.08785  | -     | 0.83428 | 10.47 - 10.69 V | Khruslov (2011)            |
Table 1—Continued

| Name                    | R.A.(J2000)   | Dec.(J2000) | P0 (day) | P1 (day) | P2 (day) | P1/P0   | P2/P1   | Mag      | Reference                      |
|-------------------------|--------------|-------------|----------|----------|----------|---------|---------|----------|--------------------------------|
| GSC 04257-00471         | 21 26 01.11  | +64 30 57.5 | 0.17380  | 0.13308  | -        | 0.76574 | -       | 11.4 V   | Poretti et al. (2005)          |
| CSS_J212609.1+145319    | 21 26 09.20  | +14 53 18.7 | 0.09042  | 0.07054  | -        | 0.78017 | -       | 13.99 - 14.74 V | Khruslov (2018) |
| V0371 Aqr               | 21 29 52.69  | -01 10 18.9 | 0.08059  | 0.06244  | -        | 0.77479 | -       | 14.22 - 14.65 V | Khruslov et al. (2013)       |
| BP Peg                  | 21 33 13.53  | +22 44 24.3 | 0.10954  | 0.08451  | -        | 0.77148 | -       | 11.69 - 12.28 V | Figer (1983); Wang et al. (2014) |
| GSC 01128-00127         | 21 35 32.99  | +12 43 41.3 | 0.06305  | 0.04878  | -        | 0.77359 | -       | 14.73 - 15.12 V | Drake et al. (2009)          |
| GSC2.3 SBA9013761       | 21 36 30.17  | -00 21 27.6 | 0.10740  | 0.08368  | -        | 0.77907 | -       | 18.19 (0.22) r | Süveges et al. (2012)        |
| CSS_J213934.3-050020    | 21 39 34.33  | -05 00 20.3 | 0.14664  | 0.11273  | -        | 0.76875 | -       | 13.90 V  | Shi et al. (2021)             |
| CSS_J214745.8+122726    | 21 47 45.78  | +12 27 26.6 | 0.07820  | 0.06062  | -        | 0.77518 | -       | 14.18 - 14.36 V | Drake et al. (2009)          |
| DDE 147                 | 21 53 05.55  | +23 53 03.2 | 0.06340  | 0.04987  | -        | 0.78656 | -       | 14.15 - 14.55 V | Drake et al. (2009)          |
| USNO-A2.0 1425-12623576 | 21 59 23.24  | +59 24 56.9 | 0.10273  | 0.07917  | -        | 0.77061 | -       | 14.33 - 14.72 R | Lapukhin et al. (2015)        |
| AG Aqr                  | 22 05 31.82  | -22 30 00.7 | 0.29174  | 0.22220  | -        | 0.76165 | -       | 14.58 - 14.96 V | Khruslov et al. (2013)        |
| DDE 148                 | 22 20 39.22  | +23 43 13.8 | 0.11261  | 0.08699  | -        | 0.77247 | -       | 13.23 - 13.73 V | Drake et al. (2009)          |
| GSC 06385-00170         | 22 24 23.25  | -15 38 05.5 | 0.05718  | 0.04453  | -        | 0.77881 | -       | 14.371 (0.487) V | Drake et al. (2009)          |
| DDE 145                 | 22 25 27.04  | +08 00 13.8 | 0.05451  | 0.04235  | -        | 0.77690 | -       | 14.20 - 14.70 V | Drake et al. (2009)          |
| ASAS J231801-4520.0     | 23 18 01.14  | -45 19 55.0 | 0.11501  | 0.08892  | -        | 0.77313 | -       | 12.79 - 13.74 V | Khruslov (2015)              |
| V0761 Peg               | 23 20 56.45  | +34 51 50.9 | 0.14194  | 0.10925  | 0.08767  | 0.76969 | 0.80247 | 13.26 - 13.50 R | Kehoe et al. (2002)          |
| CSS_J235659.3+063132    | 23 56 59.35  | +06 31 32.5 | 0.09470  | 0.07327  | -        | 0.77371 | -       | 14.91 - 15.39 V | Khruslov (2018)              |

Note: A list of 155 double- and multi-mode HADS, including 132 stars pulsating in both P0 and P1, ten stars in both P1 and P2, 11 triple-mode HADS, and 2 quadruple-mode HADS. (a) this star is a quadruple-mode HADS with P0, P1, P2 and P3 (= 0.08960 days). (b) this star is also a triple-mode pulsator, but with P1, P2, and P3 (= 0.09742 days). (c) this star is a quadruple-mode HADS with P0, P1, P2 and P3 (= 0.05829 days).
3. Statistical analysis of the sample

3.1. The distribution of the fundamental period (P0) and the period ratio P1/P0 of HADS(B)

In Figure 1, we plot a histogram of the fundamental period (P0) with 132 stars pulsating in F+1O, as well as a Petersen diagram of P0 and the ratio P1/P0. We found most of the stars are clearly grouped in a range of 0.05 days to 0.175 days. In the period range of 0.12 days to 0.175 days, the number of stars shows a clear decreasing trend, and if P0 > 0.18 days, there are only six stars known at present.

Moreover, the distribution of stars shows a bimodal structure, which is not very significant but with possible peaks at P0 ∼ 0.06 days and 0.09 days. The underlying reason is unclear, but possibly due to the currently limited sample. We suggest more such sample particularly in short periods is still needed to further verify this feature.

From Petersen diagram in Figure 1 we note six stars (i.e. LINEAR 15289666, KIC 10975348, CSS_J162818.8+032651, ASAS J192227-5622.5, OGLE-GD-DSCT-0012, VX Hya) deviate largely from the downward trend of P1/P0, compared with the corresponding P0. To help readers investigate these stars easily, a brief introduction to these six stars is listed below:

- **LINEAR 15289666**: Its value of P1/P0 is 0.7938, which is higher than that of all others. The variability was discovered by Palaversa et al. (2013). Drake et al. (2014) detected the star’s variability independently and gave the type HADS with period of 0.86539 days. This star is actually a HADS(B), but it is difficult to determine the correct periods due to the one-day alias. The currently determined period ratio (P1/P0 = 0.7938) is clearly higher than typical value of P1/P0. Khruslov (2018) noticed that if considering the one-day-alias corrected periods (P0c =0.094763 days, P1c =0.0737655 days), the corrected period ratio, P1c/P0c = 0.7784, is more typical. Continuous photometric observations from space missions and/or multi-site campaigns could provide more reliable periods.

- **KIC 10975348**: Its value of P1/P0 is 0.7580, which is a little lower than that of other stars with the same P0. It was classified as a δ Sct star with a pulsation period of 0.0979 days by Ramsay et al. (2014). Subsequently, Yang et al. (2021) analyzed the pulsating behavior of this star using the time-series data from Kepler mission and reported two new independent frequencies F1(=13.4988 c/day) and F2(=19.0002 c/day). It has a relatively low period ratio of 0.7580, revealing that it might be a metal-rich variable star. The frequency F2 may be a third overtone, making this star a radial triple-mode HADS candidate. The O–C analysis suggests this star seem to show no obvious period change, which is in contrast to the majority of HADS. To understand why it has a lower period ratio and verify the period variations, seismic modelling and regular observations from space with a longer time span are needed in the future.

- **CSS_J162818.8+032651**: the value of P1/P0 is 0.78649, which is higher than that of the stars with similar P0. It was classified as a HADS with a fundamental period of 0.125352 days by Drake et al. (2014). Subsequently, Khruslov (2018) re-analyzed the light variations of this star using thephotometric data from the Catalina Sky Surveys (CSS,
Drake et al. (2009) and identified it as a double-mode HADS. There is no more available information about this star at present. To explore the reason for its relatively higher period ratio, seismic modelling and spectroscopic observations are needed in the future.

- **ASAS J192227-5622.5**: the value of $P_1/P_0$ is 0.7639, which is the lowest value in the present sample. The variability of this star was reported by Pojmanski (2002). Subsequently, Khruslov (2011) re-analyzed this star using ASAS-3 data (Pojmanski 2002) and classified it as a double-mode HADS. There is no more available information about this star at present. To explore the reason for its lowest period ratio, seismic modelling and spectroscopic observations are needed in the future.

- **OGLE-GD-DSCT-0012**: the value of $P_1/P_0$ is 0.78537, which is higher than that of the stars with similar $P_0$. This star was discovered by Pietrukowicz et al. (2013) in OGLE-III Galactic disk area. Pietrukowicz et al. (2013) found the period ratio of this star cannot be reproduced by their theoretical models. Subsequently, Pietrukowicz et al. (2015) presented a spectroscopic follow-up observation for this star and gave the spectral type as F0. There is no more photometry available for this star at present. To explore the reason for its higher period ratio, seismic modelling and spectroscopic observations are needed in the future.

- **VX Hya**: it was discovered by Hoffmeister (1931) and then observational studies were performed by many researchers (Lause 1932; Oosterhoff 1938; Fitch 1966; Breger 1977). Templeton et al. (2009) determined its frequency of the fundamental mode as $f_0 = 4.4765$ c/day and the first overtone $f_1 = 5.7898$ c/day through a two-year, multi-site observing campaign. Recently, Xue et al. (2018) derived its mass as $2.385 \pm 0.025 \, M_\odot$ with asteroseismic models and found it is located at the post-MS stage with a helium core and a hydrogen-burning shell in H-R diagram. Its higher mass and late evolutionary stage may be responsible for its higher ratio of 0.77318, compared with other five stars.

The period ratio $P_1/P_0$ has long been taken as the critical parameter to identify a HADS(B). The typical value of $P_1/P_0$ is about 0.77, which might be slightly shifted depending on metallicity and/or mass of the star (Poretti et al. 2005). Using a much larger sample (142 HADS(B) with both $P_0$ and $P_1$), we further investigated the distribution of $P_1/P_0$ and created a histogram of $P_1/P_0$, as shown in Figure 2. We found that most stars lie in a range of 0.761 - 0.787, in which 87.8 percent of the stars concentrate in a range of 0.766 - 0.785, which is slightly wider than the previous studies (0.77 - 0.78 of Breger 2000). And in this narrow range, the ratio $P_1/P_0$ seems to have a peak at $P_1/P_0 = 0.774$, which is slightly higher than the value of 0.772 in the metal-rich HADS stars presented by McNamara (2000). We note the stars with the highest and lowest values of $P_1/P_0$ have been listed above and may need more attention to their pulsations.

### 3.2. The relationship between $P_1/P_0$ and $P_0$ of the HADS(B) stars

Petersen Diagram, showing the period ratio of $1O/F$ as a function of the fundamental mode, is a well-known tool to study radial pulsators, especially double-mode pulsators (Petersen 1973, 1978). For HADS(B), the relationship between the fundamental radial mode...
Fig. 1.— Top panel: the distribution of $P_0$ of 132 stars pulsating in F and I O. It is clear that most of the stars lie in a range of 0.05 days to 0.175 days. Bottom panel: Petersen diagram of $P_1/P_0$ and $P_0$. The red dots indicate six stars with unusual period ratios.

Fig. 2.— The distribution of period ratio $P_1/P_0$ of 144 stars with both $P_0$ and $P_1$. It clearly shows that most stars lie in a wide range of 0.761 - 0.787, compared to that in Breger (2000).
and period ratio of $1O/F$ was first studied by Petersen & Christensen-Dalsgaard (1996) using 7 HADS(B). In their work, a series of theoretical models were created and the model predicted a peak value of $P_1/P_0$ around 0.774 when $\log P_0$ equals $-0.9$. For both increasing and decreasing value of $P_0$, the value of $P_1/P_0$ shows a downward trend, i.e. $P_1/P_0$ equals 0.764 for the value of $\log P_0$ is about $-0.55$, and equals 0.770 for $\log P_0 = -1.1$. In the case of extreme simplification of the model, the relation could be treated as a downward parabola shape with vertex 0.774 with $\log P_0 = -0.9$ for the HADS(B) (Petersen & Christensen-Dalsgaard 1996).

Subsequently, with 25 HADS(B), Poretti et al. (2005) calculated a new set of theoretical models and found the period ratio $P_1/P_0$ decreases toward longer periods overall. Specifically, the decreasing trends in the theoretical sequences remain a long standstill between $P_0 = 0.05$ days and $P_0 = 0.126$ days, while for stars with longer period $P_0 > 0.126$ days, the ratio decreases rapidly, as shown in Figure 3. During the period standstill, Poretti et al. (2005) found the period ratios are more sensitive to the changes in metallicity than in mass, but at longer periods, small difference in mass can strongly influence the period ratio.

Using a larger sample of 77 HADS(B), Furgoni (2016) pointed out there seems to exist a linear relation between the ratio $P_1/P_0$ and the period $P_0$, and gave a relation as: $Y = -0.084809(\pm 0.008298)P_0 + 0.782048(\pm 0.000995)$, where $Y$ is the period ratio $P_1/P_0$, and $P_0$ is the radial fundamental period in days.

To further investigate the relationship between $P_1/P_0$ and $P_0$ of HADS(B), we plot a Petersen diagram with 144 stars with both $P_0$ and $P_1$ (i.e. stars pulsating in F+1O, F+1O+2O, and F+1O+2O+3O, as listed in Table 1). Figure 3 shows the Petersen diagram in this work. To explore the relation between $P_1/P_0$ and $P_0$, we attempt to make a linear-fit for these stars, excluding six unusual stars mentioned above. The corresponding linear equation was derived as:

$$Y = -0.0940(\pm 0.0091)P_0 + 0.7835(\pm 0.0009),$$

with a correlation coefficient of 0.6865. This possible linear relation is similar to the result in Furgoni (2016). Although Furgoni (2016) believe that more HADS(B) could improve the parameters of the possible linear relation, the derived parameters in our study are only slightly higher than that obtained by Furgoni (2016).

For the theoretical relationship between $P_0$ and $P_1/P_0$ derived by Petersen & Christensen-Dalsgaard (1996), we find that most of HADS(B) in our sample are not in agreement with the predicted values. As a comparison, we also plot the theoretical sequences calculated by Poretti et al. (2005) in Figure 3 as well as 25 HADS(B) in their work. From the figure, most of the stars are far away from the theoretical predicted values in periods of 0.05 to 0.10 days. Between periods of 0.10 and 0.20 days, most of the stars are essentially in agreement with the predicted values. But if the period is larger than 0.20 days, three stars (i.e., VX Hya, AG Aqr, and V829 Aql) are far away from the predictions.

For the scattering of $P_1/P_0$ with $P_0$ between 0.05 and 0.10 days, one of the possible reasons may be stellar rotation. For instance, Suárez et al. (2006) investigated the effect
of rotation on period ratios for double-mode pulsators and found that the differences in period ratios increase with the rotational velocities for a given metallicity. Moreover, it was also found that the differences in period ratios caused by rotation are equivalent to the differences by metallicity up to 0.30 dex, and it should be taken into account when one wants to accurately determine the mass and metallicity of a star. On the other hand, Suárez et al. (2007) also investigated the influence of near-degeneracy on period ratios and found that near-degeneracy could change the oscillation frequencies through the coupling strength and thereby modified the period ratios significantly even for the relatively slow rotators. In terms of frequency variations, the coupling strength analysis suggests that near-degeneracy may modify the frequency of the fundamental radial mode and the first overtone up to 0.3 µHz.

Therefore, for the scatters far away from the typical values of P1/P0, especially in short periods between 0.05 and 0.10 days in Figure 3, the possible reasons for their abnormality may include the differences of stellar mass, rotational velocity, and metallicity, as well as other physical parameters. On the other hand, we cannot exclude the possibility that these scatters are due to a pair of consecutive radial overtones, radial and non-radial mode or even two non-radial modes (Templeton et al. 2002; Pigulski et al. 2006). We suggest detailed seismic modelling for these stars considering these factors is needed, which is helpful to understand their peculiarity and the internal physics in the future.

3.3. The period ratios of multi-mode HADS

In this catalogue, there are 13 HADS pulsating in multi-mode, i.e. 11 star with triple-mode and 2 with quadruple-mode (F+1O+2O+3O). In addition, there are also 10 double-mode HADS pulsating in 1O+2O. These stars provide a good opportunity to explore the period ratios between the radial overtones and the fundamental mode.

Figure 4 shows a Petersen diagram for multi-mode HADS. From this figure, we found that almost all the ratios of P2/P1 lie in a narrow horizontal strip around the value of 0.80, except for two points in obviously puzzling positions from stars V3124 Cyg and V0836 Lyr. If these two outliers are excluded, we estimate the mean value of P2/P1 of the 21 stars is 0.802 with standard deviation of 0.004. In this strip, the distribution of P2/P1 do not show any up or down trend. The ratios of P1/P0 of these stars are also within the typical ratio range of 0.761 - 0.787, in consistent with that of HADS(B), suggesting there seems no difference in terms of P1/P0 between multi-mode HADS and usual HADS(B). For the values of P2/P0 and P3/P0, these stars are also nearly in the range of period ratios predicted by Stellingwerf (1979) and Netzel et al. (2021).

A brief introduction to the two stars showing unusual period ratio P2/P1 in Figure 4 is presented below:

- V3124 Cyg: Khruslov (2014) provided two pulsating periods and their modes: 0.10530 days (P1) and 0.08785 days (P2) and the P2/P1 ratio is 0.834. We re-analyzed the available light curve and found the amplitude of P2 is larger than that of P1. The pulsations might be caused by the second-overtone and third-overtone modes. Further observations
and seismic modellings are needed to verify the modes.

- V0836 Lyr: four independent frequencies are listed in the database, but other periods with lower amplitudes still may exist. Its P1/P0 ratio is 0.77458, but P2/P1 equals 0.56308, which is inconsistent with the typical value (i.e., 0.80) of P2/P1. Also, the period ratio of 2O/F (0.05084/0.11658=0.4361) is far away from the typical value of 2O/F. So we suspect the P2 is not correctly identified at present. Further analysis with seismic modellings may help to identify the modes.

4. Summary

In this work, we collected all the double-mode and multi-mode HADS as far as possible, and first created a catalogue with a sample of 155 stars, including 142 double-mode HADS (i.e., 132 stars pulsating in both the fundamental radial mode and first-overtone, 10 stars in both the first-overtone and second-overtone), 11 triple-mode HADS, and 2 quadruple-mode HADS.

Statistical analysis of the 132 double-mode HADS pulsating in both P0 and P1 indicate that most of them are pulsating with a fundamental period (P0) in the range of 0.05 days to 0.175 days, and the distribution of P0 shows a hint of bimodal structure with peaks at 0.06 days and 0.09 days, which still needs further investigation. Using 142 stars with both P0 and P1, we investigated the distribution of P1/P0 and find that the values of P1/P0 lie in a wide range of 0.761−0.787. Nearly 90 percent of the stars concentrate in a relatively narrow range of 0.765−0.785.

With 144 HADS(B) collected in this work, we created a new Petersen diagram between P1/P0 and P0, and derived an updated linear relation between P1/P0 and P0 as: P1/P0 = −0.0940(± 0.0091) P0 + 0.7835( ± 0.0009), with a correlation coefficient of 0.6865. Compared to the result obtained by Furgoni (2016), we found the new result is essentially in agreement with theirs, only with slightly higher values in both the slope and intercept of the linear relation.

We also examined the relation between P2/P1 and P1 for 23 HADS pulsating in both P1 and P2, and found the values of P2/P1 fall in a horizontal strip with mean value of 0.802 and standard deviation of 0.004, if the two obvious outliers were excluded. It seems to indicate that this narrow strip could be considered as a possible observational indicator to identify the modes 1O and 2O for the multi-mode HADS.

This research is supported by the National Natural Science Foundation of China (grant Nos. 11573021, U1938104, 12003020) and the Fundamental Research Funds for the Central Universities. We would like to thank the AAVSO International Variable Star Index for providing such data.
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Fig. 3.— Petersen diagram of 144 HADS(B) stars with both P0 and P1. The plus signs indicate 25 stars used in Poretti et al. (2005). Three dash-dotted lines are the theoretical sequences calculated by Poretti et al. (2005). The dashed line is the relationship given by Petersen & Christensen-Dalsgaard (1996) using seven stars. The solid line with shaded area is the best-fitted line in this work.
Fig. 4.— Petersen diagram for multi-mode pulsators in this work. Open circles, triangles and squares correspond to the double-mode HADS with P1 and P2, triple-mode and quadruple-mode HADS, respectively. The blue strip shows the ratios of 2O/1O, which lie in a range of $0.802 \pm 3 \sigma$, $\sigma$ is 0.004, derived in this work. The red strip shows the ratios of 1O/F determined in Sec. 3. The green and grey stand for the ratios of the radial second-overtone and third-overtone to the fundamental mode, respectively.