Scalable Planning with Tensorflow for Hybrid Nonlinear Domains

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Abstract
Given recent deep learning results that demonstrate the ability to effectively optimize high-dimensional non-convex functions with gradient descent optimization on GPUs, we ask in this paper whether symbolic gradient optimization tools such as Tensorflow can be effective for planning in hybrid (mixed discrete and continuous) nonlinear domains with high dimensional state and action spaces? To this end, we demonstrate that hybrid planning with Tensorflow and RMSProp gradient descent is competitive with mixed integer linear program (MILP) based optimization on piecewise linear planning domains (where we can compute optimal solutions) and substantially outperforms state-of-the-art interior point methods for nonlinear planning domains. Furthermore, we remark that Tensorflow is highly scalable, converging to a strong policy on a large-scale concurrent domain with a total of 576,000 continuous actions over a horizon of 96 time steps in only 4 minutes. We provide a number of insights that clarify such strong performance including observations that despite long horizons, RMSProp avoids both the vanishing and exploding gradients problem. Together these results suggest a new frontier for highly scalable planning in nonlinear hybrid domains by leveraging GPUs and the power of recent advances in gradient descent with highly optimized toolkits like Tensorflow.

Introduction
Many real-world hybrid (mixed discrete continuous) planning problems such as Reservoir Control [Yeh, 1985], Heating, Ventilation and Air Conditioning (HVAC) [Erickson et al., 2009; Agarwal et al., 2010], and Navigation [Paulwasser and Findeisen, 2009] have highly nonlinear transition and (possibly nonlinear) reward functions to optimize. Unfortunately, existing state-of-the-art hybrid planners [Ivankovic et al., 2014; Lohr et al., 2012; Coles et al., 2013; Piotrowski et al., 2016] are not compatible with arbitrary nonlinear transition and reward models. Monte Carlo Tree Search (MCTS) methods [Coulom, 2006; Kocsis and Szepesvári, 2006; Keller and Helmert, 2013] including AlphaGo [Silver et al., 2016] that can use any (nonlinear) black box model of transition dynamics do not inherently work with continuous action spaces due to the infinite branching factor. While MCTS with continuous action extensions such as HOOT [Weinstein and Littman, 2012] have been proposed, their continuous partitioning methods do not scale to high-dimensional continuous action spaces (e.g., 100’s or 1,000’s of dimensions as used in this paper). Finally, offline model-free reinforcement learning (e.g., Q-learning) with function approximation [Sutton and Barto, 1998; Szepesvári, 2010] and deep extensions [Mnih et al., 2013] do not require any knowledge of the (nonlinear) transition model or reward, but they also do not directly apply to domains with high-dimensional continuous action spaces. I.e., offline learning methods like Q-learning require action maximization for every update, but in high-dimensional continuous action spaces such nonlinear function maximization is non-convex and computationally intractable at the scale of millions or billions of updates.

To address the above scalability and expressivity limitations of existing methods, we turn to Tensorflow [Abadi et al., 2015], which is a symbolic computation platform used in the machine learning community for deep...
learning due to its compilation of complex layered symbolic functions into a representation amenable to fast GPU-based reverse-mode automatic differentiation [Linnainmaa, 1970] for gradient-based optimization. Given recent results in gradient descent optimization with deep learning that demonstrate the ability to effectively optimize high-dimensional non-convex functions, we ask whether Tensorflow can be effective for planning in hybrid (mixed discrete and continuous) nonlinear domains with high dimensional state and action spaces?

Our results answer this question affirmatively, where we demonstrate that hybrid planning with Tensorflow and RMSProp gradient descent [Tieleman and Hinton, 2012] is surprisingly effective at planning in complex hybrid nonlinear domains. As evidence, we reference figure I where we show Tensorflow with RMSProp optimizing a path in a 2d nonlinear Navigation domain. In general, Tensorflow with RMSProp planning results are competitive with optimal MILP-based planning on piecewise linear planning domains and demonstrate the ability to effectively optimize high-dimensional (deep) neural networks in a single linear time pass in the size of the network using what is now simply known as reverse-mode automatic differentiation [Linnainmaa, 1970]. Despite its relative efficiency, backpropagation in large-scale (deep) neural networks is still computationally expensive and it is only with the advent of recent GPU-based symbolic toolkits like Tensorflow [Abadi et al., 2015] that recent advances in training very large deep neural networks have become possible.

In this paper, we reverse the idea of training parameters of the network given fixed inputs to instead optimizing the inputs (i.e., actions) subject to fixed parameters (effectively the transition and reward parameterization assumed a priori known in planning). That is, given transition $T(s_t, a_t)$ and reward function $R(s_{t+1})$ whose parameters are fixed, we want to optimize the input $a_t$ for all $t$ to minimize the overall cost-based reward. Specifically, we want to optimize all actions

$$a = a - \eta \frac{\partial L}{\partial a},$$

where $\eta$ is the optimization rate and the partial derivatives comprising the gradient based optimization are computed as

$$\frac{\partial L}{\partial a_{t+1}} = \frac{\partial L}{\partial a_{t+1}} \frac{\partial s_{t+1}}{\partial a_{t+1}} \frac{\partial r_{t+1}}{\partial s_{t+1}} \prod_{\kappa=t}^{t+2} \frac{\partial s_{t+1}}{\partial s_{t+1}}.$$ (2)

Tools such as Tensorflow typically assume that a loss function is being minimized. To connect our planning objective to a standard Tensorflow loss function, we choose Mean Squared Error (MSE), which given two continuous vectors $Y$ and $Y^*$ is defined as

$$MSE(Y, Y^*) = \frac{1}{2} \|Y^* - Y\|^2.$$ (3)

We adopt $MSE(0, V)$ to minimize our cost-based cumulative reward objective; here MSE takes as its input a constant vector $0$ and objective value vector $V = (\ldots, V_i, \ldots)$ where $V_i$ is the value of the $i$th problem instance. We will further explain the use of MSE in a moment, but first we digress to explain why we need to solve multiple problem instances $i$.

Since both transition and reward functions are not assumed to be convex, optimization on such a domain could result in a local minimum. To mitigate this problem, we use randomly

**Hybrid Nonlinear Planning via Tensorflow**

**Hybrid Planning**

A hybrid planning problem is a tuple $\langle S, A, T, R, C \rangle$ with $S$ denoting the set of states, $A$ the set of actions bounded by action constraints $C$, $R: S \times A \to \mathbb{R}$ the reward function and $T: S \times A \to S$ the transition function. There is also an initial state $s_0$ and the planning objective is to maximize the cumulative reward over a decision horizon of $H$ time steps. Before proceeding, we outline necessary notation:

- $s_t$: mixed discrete, continuous state vector at time $t$.
- $a_t$: mixed discrete, continuous action vector at time $t$.
- $s_{t+1}$: the $j$th dimension of state vector of problem $i$ at time $t$.
- $a_{t+1}$: the $j$th dimension of action vector of problem $i$ at time $t$.
- $R(s_t, a_t)$: a non-positive reward function — higher absolute values indicate higher cost.
- $T(s_t, a_t)$: a (nonlinear) transition function.

- $V$: cumulative value of reward to maximize:

$$V = \sum_{t=1}^{H} r_t = \sum_{t=0}^{H-1} R(s_t, a_t).$$

**Planning through Backpropagation**

Backpropagation [Rumelhart et al., ] is a standard method for optimizing parameters of large multilayer neural networks via gradient descent. Via the chain rule of derivatives, backpropagation passes the derivative of the output error of a neural network back to each of its parameters in a single linear time pass in the size of the network using what is now simply known as reverse-mode automatic differentiation [Linnainmaa, 1970]. Despite its relative efficiency, backpropagation in large-scale (deep) neural networks is still computationally expensive and it is only with the advent of recent GPU-based symbolic toolkits like Tensorflow [Abadi et al., 2015] that recent advances in training very large deep neural networks have become possible.

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- $V$: cumulative value of reward to maximize:

$$V = \sum_{t=1}^{H} r_t = \sum_{t=0}^{H-1} R(s_t, a_t).$$
initialized actions in a batch optimization: we optimize multiple mutually independent planning problems \(i\) simultaneously since the GPU can exploit their parallel computation, and then select the best-performing action sequence among the independent simultaneously solved problems. MSE then has dual effects of optimizing each problem instance \(i\) independently and providing fast convergence. We remark that simply defining the objective \(V\) and the definition of all state variables in terms of predecessor state and action variables via the transition dynamics (back to the known initial state constants) is enough for Tensorflow to build the symbolic directed acyclic graph (DAG) representing the objective and take its gradient w.r.t. to all free action parameters as shown in (2) using reverse-mode automatic differentiation.

**Long Horizon Planning**

Given that the transition dynamics of planning problems are typically non-stationary, the Tensorflow compilation of a nonlinear planning problem reflects the same structure as recurrent neural network (RNN) deep nets commonly used in deep learning. The connection here is not superficial since a long-standing difficulty with training RNNs lies in the vanishing gradient problem, i.e., multiplying long sequences of gradients in the chain rule usually renders them extremely small and irrelevant for weight updates, especially when using nonlinear transfer functions such as a sigmoid. However in hybrid planning problems, continuous state updates often take the form \(s_{i(t+1)} = s_{i(t)} + \Delta\) for some \(\Delta\) function of the state and action at time \(t\). Critically we note that the transfer function here is linear in \(s_{i(t)}\), which is the largest determiner of \(s_{i(t+1)}\), hence avoiding vanishing gradients.

In addition, a gradient can explode with the chain rule through backpropagation if the elements of the Jacobian matrix of state transitions are too large:

\[
\frac{\partial s'}{\partial s} = \left[ \begin{array}{cccc}
\frac{\partial s'_1}{\partial s_1} & \frac{\partial s'_1}{\partial s_2} & \cdots & \frac{\partial s'_1}{\partial s_n} \\
\frac{\partial s'_2}{\partial s_1} & \frac{\partial s'_2}{\partial s_2} & \cdots & \frac{\partial s'_2}{\partial s_n} \\
\vdots & \vdots & \ddots & \vdots \\
\frac{\partial s'_n}{\partial s_1} & \frac{\partial s'_n}{\partial s_2} & \cdots & \frac{\partial s'_n}{\partial s_n}
\end{array} \right]
\]  
(4)

In this case, if the planning horizon is large enough, a simple Stochastic Gradient Descent (SGD) optimizer would suffer from overshooting the optimum and never converge. RMSProp optimization algorithm has a significant advantage on preventing from overshooting the optimum and never converge. RMSProp maintains a decaying root mean squared gradients value \(G\) for each variable, which averages over squared gradients of previous epochs

\[
G'_{a_{ij}} = 0.9G_{a_{ij}} + 0.1(\frac{\partial L}{\partial a_{ij}})^2,
\]  
(6)

and updates each action variable through

\[
a_{ij} = a_{ij} - \eta \frac{\partial L}{\sqrt{G_{a_{ij}} + \epsilon}},
\]  
(7)

Here, the gradient is relatively small and consistent over iterations. Although Adagrad and Adadelta optimization algorithm have similar mechanisms, their learning rate could quickly reduce to an extremely small value when encountering large gradients. We compare all methods in experiments.

**Experiments**

In this section, we introduce our three benchmark domains and then validate Tensorflow planning performance in the following steps. (1) We evaluate the optimality of the Tensorflow backpropagation planning on linear and bilinear domains through comparison with the optimal solution given by Mixture Integer Linear Programming (MILP). (2) We evaluate the performance of Tensorflow backpropagation planning on nonlinear domains (that MILPs cannot handle) through comparison with the Matlab-based interior point nonlinear solver FMINCON. (4) We investigate the impact of several popular gradient descent optimizers on planning performance. (5) We evaluate optimization of the learning rate.

**Domain Descriptions**

**Navigation:** The Navigation domain is designed to test the ability of optimization of Tensorflow in a relatively small environment that supports different complexity transitions. Navigation has a 2D state of the agent location \(s\) and a 2D action \(a\). Both of the states and action spaces are continuous and constrained by their maximum and minimum boundaries separately.

The goal of the problem is for an agent move to the target state as soon as possible (cf. figure 1). Therefore, we compute the reward based on the Manhattan distance from the agent to the target state at each time step as

\[
R(s_t, a_t) = -||s_t - g||_1,
\]  
(8)

where \(g\) is the goal state.

We designed three different transition functions: nonlinear, bilinear and linear. The nonlinear transition has a radius deceleration zone in the center of the field. The agent’s movement distance is reduced based on its Euclidean distance to the center of deceleration zone. The following equation shows the transition function:

\[
d_t = ||s_t - z|| \\
\lambda = \frac{2}{1 + \exp(-2d_t)} - 0.99
\]  
(9)

\[T(s_t, a_t) = \max(\mathbf{u}, \min(1, \mathbf{p})),\]

where \(d_t\) is the distance from the deceleration zone \(z\), \(\mathbf{p}\) is the proposed next state and \(\mathbf{u}1\) are upper and lower boundaries of domain respectively.

The bilinear domain is designed to compare with MILP where domain discretization is possible. In this setting, we evaluate the efficacy of approximately discretizing bilinear planning problems into MILPs. The following equation
Reservoir Control: Reservoir Control \cite{Yeh1985} is a system to control multiple connected reservoirs. Each of the reservoirs in the system has a single state \( s_j \in \mathbb{R} \) that denotes the water level of the reservoir \( j \) and a corresponding action to permit a flow \( a_j \in [0, s_j] \) from the reservoir to the next downstream reservoir.

The goal of this problem is to maintain the target water level of each reservoir in a safe range and as close to half of its capacity as possible. Therefore, we compute the reward based on the following equation:

\[
\begin{align*}
c_j &= \begin{cases} 
0, & L_j \leq s_j \leq U_j \\
-5, & s_j < L_j \\
-100, & s_j > U_j 
\end{cases} \\
R(s_t, a_t) &= -\|c - 0.1 \cdot \left(\frac{u - 1}{2}\right) - s_t\|_1, 
\end{align*}
\]

where \( c_j \) is the cost value of Reservoir \( j \) that penalizes water levels outside a safe range.

We introduce two settings: Nonlinear and Linear. For the nonlinear domain, nonlinearity due to the water loss \( c_j \) for each reservoir \( j \) includes water usage and evaporation. The transition function is

\[
T(s_t, a_t) = s_t + r_t - e_t - a_t + a_t \Sigma, 
\]

where \( \circ \) represents an elementwise product, \( r \) is a rain quantity parameter, \( m \) is the maximum capacity of the largest tank, and \( \Sigma \) is a lower triangular adjacency matrix that indicates connections to upstream reservoirs.

For the linear domain, we only replace the nonlinear function of water loss by a linear function:

\[
T(s_t, a_t) = s_t + r_t - e_t - a_t + a_t \Sigma, 
\]

Unlike Navigation, We do not limit the state dimension of the whole system into 2D. In the experiments, we use domain setting of a network with 20 reservoirs.

HVAC: Heating, Ventilation, and Air Conditioning \cite{Erickson2009, Agarwal2010} is a centralized control problem, with concurrent controls of multiple rooms and even multiple connected buildings. For each room \( j \) there is a state variable \( s_j \) denoting the temperature and an action \( a_j \) for sending the specified volume of heated air to each room \( j \) via vent actuation.

The goal of this problem is to maintain the temperature of each room in a comfortable range and consume as little energy as possible in doing so. Therefore, we compute the reward based on the following equation:

\[
\begin{align*}
d_t &= \|u - 1\|_2 - s_t \\
e_t &= a_t \cdot C \\
R(s_t, a_t) &= -\|e_t + d_t\|_1, 
\end{align*}
\]
vent, outside and hallway, respectively, of outside areas, and the adjacency vector of hallways.

\[ F(t) = \text{function} \]

We only present one domain version with a nonlinear transition function:

\[ \theta_t = a_t \odot (F_{\text{vent}} - s_t) \]

\[ \phi_t = (s_t Q - s_t \odot \sum_{j=1}^{J} q_j) / w_q \]

\[ \varphi_t = (F_{\text{out}}^t - s_t) \odot \alpha / w_o \]

\[ \varphi_t = (F_{\text{hall}}^t - s_t) \odot h / w_h \]

\[ T(s_t, a_t) = s_t + \alpha * (\theta_t + \phi_t + \varphi_t + \phi_t) \]

where \( F_{\text{vent}}, F_{\text{out}}^t \) and \( F_{\text{hall}}^t \) are temperatures of the room vent, outside and hallway, respectively, \( Q, \alpha \) and \( h \) are respectively the adjacency matrix of rooms, adjacency vector of outside areas, and the adjacency vector of hallways. \( w_q, w_o \) and \( w_h \) are thermal resistances with a room and the hallway and outside walls, respectively.

In the experiments, we work with a building layout with five floors and 12 rooms on each floor for a total of 60 rooms. For scalability testing, we apply batched backpropagation on 100 instances of such domain simultaneously, of which, there are 576,000 actions needed to plan concurrently over all time steps.

**Planning Performance**

In this section, we investigate the performance of Tensorflow optimization through comparison with the MILP on linear domains and with Matlab’s fmincon nonlinear interior point solver on nonlinear domains. We ran our experiments on Ubuntu Linux system with one E5-1620 v4 CPU, 16GB RAM, and one GTX1080 GPU. The Tensorflow version is beta 0.12.1, the Matlab version is R2016b, and the MILP version is IBM ILOG CPLEX 12.6.3.

**Performance in Linear Domains**

In figure 2 we show Tensorflow backpropagation planning results in lower cost plans than domain-specific heuristic policies, and the overall cost is relatively close to the optimal reward given by the MILP.

While Tensorflow backpropagation planning always shows good performance, when comparing the performance of Tensorflow on bilinear and linear domains of Navigation to the MILP solution (recall that the linear domain was discretized from the bilinear case), we notice that Tensorflow does much better relative to the MILP on the bilinear domain than the discretized linear domain. The reason for this is quite simple: gradient optimization of smooth bilinear functions is actually much easier for Tensorflow than the piecewise linear discretized version which has large piecewise steps that make it hard for RMSProp to get a consistent and smooth gradient signal.

**Performance in Nonlinear Domains**

In figure 3 we show Tensorflow backpropagation planning always achieves the best performance among all of the three methods. For relatively simple domains like Navigation, we see the fmincon nonlinear solver provides a very competitive
Figure 5: Comparison of Tensorflow gradient methods in the HVAC domain. The vanilla stochastic gradient descent (SGD) never converges and the curve becomes a wide green bar. Adagrad and Adadelta show extremely slow optimization progress. Adam optimizes the reward in a relatively smooth curve. RMSProp shows fast convergence in only 500 epochs. All of these optimizers use the same learning rate of 0.001.

Table 1: Timing evaluation of the largest instances of the three domains we tested. All of these tests were performed on the nonlinear versions of the respectively named domains.

| Domain | Dim | Horizon | Batch | Action | Time |
|--------|-----|---------|-------|--------|------|
| Nav.   | 2   | 120     | 100   | 24000  | <1mins|
| Res.   | 20  | 120     | 100   | 240000 | 4mins |
| HVAC   | 60  | 96      | 100   | 576000 | 4mins |

solution, while, for the complex domain HVAC with a large concurrent action space, the fmincon solver shows a complete failure at solving the problem in the given time period.

In figure 4(a), Tensorflow backpropagation planning shows 16 times faster optimization in the first 15s, which is close to the result given by fmincon at 4mins. In figure 4(b), the optimization speed of it shows it to be hundreds of times faster than fmincon nonlinear solver to achieve the same value (if fmincon does ever reach it). These remarkable results demonstrate the power of fast parallel GPU computation of the Tensorflow framework.

Scalability
In table 1, we show the scalability of Tensorflow backpropagation planning via the running times required to converge to for different domains. The results demonstrate the extreme efficiency with which Tensorflow can converge on exceptionally large nonlinear hybrid planning domains.

Optimization Methods
In this experiment, we investigate the effects of different backpropagation optimizers. In figure 5 we show that the RMSProp optimizer provides exceptionally fast convergence among the five standard optimizers of Tensorflow. This observation reflects the previous analysis and discussion concerning equation (7) that RMSProp manages to avoid exploding gradients. As mentioned, although Adagrad and Adadelta have similar mechanisms, their normalization methods may cause vanishing gradients after several epochs, which corresponds to our observation of nearly flat curves for these methods. This is a strong indicator that exploding gradients are a significant concern for hybrid planning with gradient descent and that RMSProp performs well despite this well-known potential problem for long horizon gradients.

Optimization Rate
In figure 6, we show the best learning optimization rate for the HVAC domain is 0.01 since this rate converges to near-optimal extremely fast. The overall trend is smaller optimization rates have a better opportunity to reach a better final optimization solution, but can be extremely slow as shown for optimization rate 0.001. Hence, while larger optimization rates may cause overshooting, rates that are too small may simply converge too slowly for practical use. This suggests a critical need to tune the optimization rate per planning domain.

Conclusion
We investigated the practical feasibility of using the Tensorflow toolbox to do fast, large-scale planning in hybrid nonlinear domains. We worked with a direct symbolic (nonlinear) planning domain compilation to Tensorflow for which we optimized planning actions directly through gradient-based backpropagation. We then investigated long horizon planning and suggested that RMSProp avoids both the vanishing and
expanding gradient problems and showed experiments to corroborate this finding. Our key empirical results demonstrated that Tensorflow with RMSProp is competitive with MILPs on linear domains (where the optimal solution is known — indicating near optimality of Tensorflow and RMSProp for these non-convex functions) and strongly outperforms Matlab’s state-of-the-art interior point optimizer on nonlinear domains, optimizing up to 576,000 actions in under 4 minutes. These results suggest a new frontier for highly scalable planning in nonlinear hybrid domains by leveraging GPUs and the power of recent advances in gradient descent such as RMSProp with highly optimized toolkits like Tensorflow.

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Appendix

Network Structure Clarification

Figure 7: Planning with RNN Network Structure. Reward function and Transition function are embedded in an RNN cell. Inputs for each time step corresponds to actions in planning problem. Hidden recurrent outputs corresponds to intermediate states.

Forward propagation represents execution process of a plan given actions are filled into the network sequentially. A intermediate state of the planning problem is computed through the RNN like network structure as a hidden states. Cumulative reward function is

\[ L(r_1, r_2, r_3 \cdots) = \sum_{i}^{H} \gamma^i r_i \]

Back-propagation of the negative cumulative reward with respect to action sequence means updating actions to optimize cumulative reward, which is formally defined in equation 1 and 2.

Additional Visualization on HVAC problem

Figure 8: Temperature and Energy evolution comparison between Tensorflow back-propagation with current state-of-the-art heuristic method.

Figure 8 shows that even though our model and Heuristic method show same temperature control performance, our model saves more energy.

\[1\text{ Github Repository: } https://github.com/wuga214/TOOLBOX-Learning-and-Planning-through-Backpropagation\]