Systematic study of cluster radioactivity half-lives based on a modified Gamow-like model

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Abstract

In this study, taking into account of the contribution of the centrifugal potential on half-life and the effect of electrostatic shielding, we modify the Gamow-like model proposed by Zdeb et al [2013 Phys. Rev. C87, 024308] to systematically investigate the cluster radioactivity half-lives of nuclei ranging from $^{221}$Fr to $^{238}$U. The calculated results can reproduce the experimental data well. Additionally, this modified Gamow-like model is applied to predict the half-lives of cluster radioactivity nuclei whose experimental half-lives have the lower limit. It is found that the predicted results are in good agreement with the ones obtained by using the Gamow-like model and a Viola-Seaborg type formula.

1. Introduction

Cluster radioactivity, the spontaneous emission of particle lighter than fission fragments but heavier than $^4$He particle from radioactive nuclei, was predicted by Sandulescu, Poenaru and Greiner for the first time in 1980 [1]. Since then, masses of works began to focus on the exploration of this radioactive decay mode because it can provide abundant information on nuclear structure such as ground state lifetime, nuclear spin and parity, shell effects, nuclear deformations and so on and also play a key role in identification and synthesis of new isotopes or elements [2–11]. Experimentally, the cluster radioactivity phenomenon was first discovered with the observation of $^{22}$Ra $\rightarrow$ $^{20}$Pb $+$ $^{14}$C in experiment by Rose et al in 1984 [12]. Soon afterwards, the emissions of heavier clusters than $^{14}$C from superheavy nuclei e.g. $^{20}$O, $^{23}$F, $^{22,24}$Ne, $^{28,30}$Mg, $^{32,34}$Si and so on were also observed experimentally [13–15].

From the theoretical point of view, the pictures of cluster radioactivity process can be described as a quantum tunneling process or a very mass asymmetric spontaneous fission process [16–20]. Based on the above theories, extensive theoretical models have been put forward to deal with this type radioactivity including the preformed cluster model (PCM) [21, 22], Coulomb and proximity potential model (CPPM) [23, 24], the generalized liquid drop model (GLDM) [2, 25], among others [8, 26–31]. Meanwhile, a lot of empirical and semi-empirical formulas are also considered as effective tools to describe this process [32–41]. The calculations of cluster radioactivity half-lives using all these theoretical models and formulas are in satisfactory agreement with the experimental data.

Gamow-like model was firstly proposed by Zdeb et al in 2013 based on the Gamow theory [42] for calculating the half-lives of $\alpha$ decay and cluster radioactivity of nuclei [13, 44]. In this model, the inner potential associated with the nuclear interaction is represented by a square potential well and the outer one is composed by electrostatic repulsion which defaults to the Coloumb potential under the hypothesis of spherical uniform charge distribution. However, the effect of electrostatic shielding should be considered for the inhomogeneous charge distribution of the nucleus, the charge superposition of the emitted particle, etc. Recently, R. Budaca et al
introduced an exponential-type electrostatic potential called as Hulthén potential to account for the electrostatic repulsion between the emitted cluster and daughter nucleus and proposed an analytical model to compute proton radioactivity half-lives [45]. In this model, the only one parameter $a$ defines the electrostatic screening effect on Coloumb potential which can be understood as short-range effects such as charge diffuseness, inhomogeneous charge distribution, charge superposition and so on [46]. Their calculated results are in good agreement with the experimental data. The same successes were also achieved with respect to $\alpha$ decay and two-proton (2p) radioactivity [47, 48]. In 2019, considering the effect of electrostatic shielding, we modified Gamow-like model and used this model to systematically calculate the half-lives of $\alpha$, proton and 2p radioactive emitters [49–51]. Our calculated results can reproduce the experimental data well. From this perspective, since cluster radioactivity may be described as a quantum tunnelling process the same as $\alpha$ decay and proton radioactivity [52–58]. Whether this modified Gamow-like model can be used to uniformly describe $\alpha$, proton, cluster and 2p decays or not is of particular interest. For this purpose, in the present work, based on this model, we systematically investigate the half-lives of cluster radioactive nuclei with $87 \leq Z \leq 95$.

This article is organized as follows. In section 2, the theoretical framework of the modified Gamow-like model is briefly described. In section 3 the results and discussion are presented. Finally, a summary is given in section 4.

2. Theoretical framework

The cluster radioactivity half-life $T_{1/2}$ of a decaying nucleus is defined as [43]

$$T_{1/2} = \frac{\ln 2}{\lambda} 10^h,$$

(1)

where $h$ represents the hindrance factor associated with unpaired nucleons. $\lambda$ is decay constant, it can be given by

$$\lambda = S_c \nu P,$$

(2)

where $S_c$ is the preformation probability of the cluster in decaying parent nucleus. In this work, we choose $S_c = 1$, which is obtained by fitting the experimental data [43]. $\nu$ represents the collision frequency of cluster in the potential barrier, which is given by an approximate equation for the ground state of the spherical square well potential with the radius $R_{in}$ [43]. It can be expressed as

$$\nu = \frac{\pi h}{2\mu R_{in}^2},$$

(3)

where $\mu$ denotes the reduced mass of the emitted cluster and daughter nucleus in center of mass system, $h$ is the reduced Planck constant. $R_{in} = r_0 (A_1^{1/3} + A_2^{1/3})$, where $r_0$ is effective nuclear radius parameter, $A_1$ and $A_2$ are mass numbers of the emitted cluster and daughter nucleus, respectively.

$P$ is the Gamow penetrability factor through the barrier, which can be calculated by means of the semi-classical Wentzel–Kramers–Brillouin (WKB) approximation:

$$P = \exp \left[ -\frac{2}{h} \int_{R_{in}}^{R_{out}} \sqrt{2\mu[V(r) - E_k]} \, dr \right],$$

(4)

where $V(r)$ is the total emitted cluster–daughter nucleus interaction potential. $R_{out}$ represents the outer turning point from the potential barrier, which satisfies the condition $V(R_{out}) = E_k$ with $E_k$ being the kinetic energy of emitted cluster.

In the framework of Gamow-like model, $V(r)$ is devided into two parts i.e. the inner potential and the outer one. The inner potential associated with the nuclear interaction is represented by a square potential well, while the outer one associated with electrostatic potential is represented by default of the Coulomb potential $V_C(r) = Z_1Z_2e^2/r$ with $Z_1$ and $Z_2$ being the proton numbers of emitted cluster and daughter nucleus, respectively. $r$ denotes the center-of-mass distance between the emitted cluster and daughter nucleus. However, for the superposition of the involved charges, movement of the emitted cluster which generates a magnetic field and the inhomogeneous charge distribution of the nucleus, in the process of cluster radioactivity process, the emitted cluster–daughter nucleus electrostatic potential behaves as a $V_C(r)$ at short distance ($r \rightarrow 0$) and drops exponentially at large distance ($r \gg 0$) i.e. the screened electrostatic effect [46]. This behaviour of electrostatic potential can be described as the Hulthén type potential which is widely applied in the fields of nuclear, atomic, molecular and solid state physics [59–63]. It is defined as

$$V_H(r) = \frac{aZ_1Z_2e^2}{e^{ar} - 1},$$

(5)

where $a$ represents the screening parameter. As we can see from figure 1, when $r \gg 0$, relative to $V_C(r)$, $V_H(r)$ decays exponentially more rapidly.
In addition, the contribution of the centrifugal potential \( V_l (r) \) on cluster radioactivity half-lives was considered in this modified Gamow-like model. We adopt the Langer modified centrifugal barrier \( V_l (r) \), which can be expressed as

\[
V_l(r) = \frac{(l + 1/2)^2 \hbar^2}{2mr^2}.
\]

Here \( l \) is the orbital angular momentum taken away by the emitted cluster, which satisfies the angular momentum and parity conservation laws. In this framework, \( V_l(r) \) can be expressed as

\[
V(r) = \begin{cases} 
-V_0, & 0 \leq r \leq R_{in}, \\
V_{H}(r) + V_l(r), & r > R_{in},
\end{cases}
\]

where \( V_0 \) is the depth of the potential well.

3. Results and discussion

In 2015, considering the electrons in different external environments, Wan et al calculated the half-lives of screened \( \alpha \) decay for the proton number \( Z = 52 - 105 \) nuclei [65]. Their results indicated that the electrostatic shielding effect is positively correlated with the proton number of daughter nucleus and negatively related to \( \alpha \) decay energy. In our previous works [49, 50], we modified the Gamow-like model and used this model to systematically calculate the half-lives of 41 proton radioactive nuclei and 461 \( \alpha \) decay nuclei, respectively. Our calculated results indicated that for the nuclei having the larger proton number, the experimental data can be better reproduced. Recently, WA Yahya computed the \( \alpha \) decay half-lives for \( ^{171-189}\text{Hg} \) isotopes using the Gamow-like model, modified Gamow-like model, temperature-independent Coulomb and proximity potential model and temperature-dependent Coulomb and proximity potential model, it is found that the calculated \( \alpha \) decay half-lives obtained by using the modified Gamow-like model, temperature-independent Coulomb and proximity potential model are in better agreement with the experimental data [66].

In order to investigate the influence of electrostatic screening effect on the cluster radioactivity half-lives of trans-lead nuclei, in the present work, we systematically study the half-lives of 19 cluster radioactive nuclei using our modified Gamow-like model, which contains two adjusted parameters i.e. the screening parameter \( \alpha \) and effective nuclear radius parameter \( r_0 \). Considering the kinds of emitted clusters and structural characteristics of these nuclei are not exactly same, the corresponding screened electrostatic effect may be different with respect to 19 cluster radioactive nuclei. According to the odd–even property of the parent nucleus, these 19 cluster radioactive nuclei can be divided into two kinds i.e. 10 even–even nuclei and 9 odd–\( A \) nuclei. Firstly, by fitting the experimental data [67], we obtain the values of \( \alpha \) and \( r_0 \) for 10 even–even nuclei that emit \( ^{14}\text{C} \) (for \( ^{222,224,226}\text{Ra} \), \( ^{224}\text{Ne} \) (for \( ^{230}\text{Th} \) and \( ^{232,234}\text{U} \)) and \( ^{28}\text{Mg} \) (for \( ^{234,236}\text{U} \) and \( ^{236,238}\text{Pu} \)) and 9 odd–\( A \) nuclei that emit \( ^{14}\text{C} \) (for \( ^{221}\text{Fr} \), \( ^{221}\text{Ra} \) and \( ^{225}\text{Ac} \), \( ^{24}\text{Ne} \) (for \( ^{231}\text{Pa} \) and \( ^{233,235}\text{U} \)) and \( ^{25}\text{Ne} \) (for \( ^{233,235}\text{U} \)), respectively. All the numerical results are presented in table 1. In this table, the first two columns represent the cluster radioactive parent nucleus and the emitted clusters, the last two columns represent the fitted values of \( r_0 \) and \( \alpha \), respectively. From this table, we can clearly see that for 10 even–even nuclei, the fitted values of \( r_0 \) and \( \alpha \) are quite close, although the emitted clusters are not entirely same. The values of \( r_0 \) and \( \alpha \) are basically same for 9 odd–\( A \) nuclei which emit \( ^{14}\text{C} \), \( ^{24}\text{Ne} \) or \( ^{25}\text{Ne} \). Then we can obtain a unified \( \alpha \) and \( r_0 \) values for
and effective nuclear radius parameter $r_0$ for 10 even–even and 9 odd-$A$ nuclei.

| Nucleus | Cluster | $r_0$/fm | $a$/fm$^{-1}$ | Nuclei | Clusters | $r_0$/fm | $a$/fm$^{-1}$ |
|---------|---------|----------|-------------|--------|----------|----------|-------------|
| $^{222}$Ra | $^{14}$C | 1.16 | $1.223 \times 10^{-3}$ | $^{222}$Fr | $^{14}$C | 1.05 | $9.598 \times 10^{-3}$ |
| $^{224}$Ra | $^{14}$C | 1.16 | $1.223 \times 10^{-3}$ | $^{222}$Ra | $^{14}$C | 1.05 | $9.598 \times 10^{-3}$ |
| $^{226}$Ra | $^{14}$C | 1.16 | $1.223 \times 10^{-3}$ | $^{222}$Ra | $^{14}$C | 1.05 | $9.598 \times 10^{-3}$ |
| $^{230}$U | $^{24}$Ne | 1.07 | $7.449 \times 10^{-3}$ | $^{226}$Ac | $^{14}$C | 1.05 | $9.598 \times 10^{-3}$ |
| $^{232}$U | $^{24}$Ne | 1.07 | $7.449 \times 10^{-3}$ | $^{231}$Pa | $^{28}$Ne | 1.06 | $9.695 \times 10^{-3}$ |
| $^{234}$U | $^{24}$Ne | 1.07 | $7.449 \times 10^{-3}$ | $^{233}$U | $^{28}$Ne | 1.06 | $9.695 \times 10^{-3}$ |
| $^{236}$U | $^{28}$Mg | 1.06 | $9.576 \times 10^{-3}$ | $^{235}$U | $^{28}$Ne | 1.04 | $9.634 \times 10^{-3}$ |
| $^{238}$Pu | $^{28}$Mg | 1.06 | $9.576 \times 10^{-3}$ | $^{235}$U | $^{28}$Ne | 1.04 | $9.634 \times 10^{-3}$ |

Table 2. The comparison between the experimental data and calculated half-lives obtained by using our modified Gamow-like model, Gamow-like model and Viola-Seaborg type formula for 19 cluster radioactive nuclei. The $Q_{exp}$ and $T_{1/2}^{exp}$ are taken from the evaluated atomic mass table AME2016 [68,69] and evaluated nuclear properties table NUBASE2016 [67], respectively.

| Nucleus | Cluster | $Q_{exp}$/MeV | $\log_{10} T_{1/2}^{exp}$/s | $\log_{10} T_{1/2}^{GGLM}$/s | $\log_{10} T_{1/2}^{MGLM}$/s | $\log_{10} T_{1/2}^{V-S}$/s |
|---------|---------|--------------|----------------|----------------|----------------|----------------|
| $^{222}$Ra | $^{14}$C | 33.05 | 11.00 | 10.96 | 10.45 | 10.73 |
| $^{224}$Ra | $^{14}$C | 30.54 | 15.92 | 15.52 | 15.11 | 15.97 |
| $^{226}$Ra | $^{14}$C | 28.20 | 21.34 | 20.40 | 20.10 | 21.46 |
| $^{230}$U | $^{24}$Ne | 57.57 | 24.64 | 24.90 | 23.99 | 24.43 |
| $^{232}$U | $^{24}$Ne | 62.31 | 20.40 | 20.35 | 19.34 | 21.00 |
| $^{234}$U | $^{24}$Ne | 58.84 | 25.92 | 25.29 | 24.36 | 25.58 |
| $^{236}$U | $^{28}$Mg | 74.11 | 25.74 | 25.81 | 24.64 | 25.12 |
| $^{238}$Pu | $^{28}$Mg | 71.69 | 27.58 | 28.96 | 27.84 | 27.95 |
| $^{232}$Fr | $^{14}$C | 31.32 | 14.52 | 14.21 | 14.76 | 14.37 |
| $^{234}$Ra | $^{14}$C | 32.40 | 13.39 | 13.27 | 13.66 | 13.43 |
| $^{236}$Ra | $^{14}$C | 31.83 | 15.20 | 14.17 | 14.60 | 14.60 |
| $^{238}$Ac | $^{14}$C | 30.48 | 17.34 | 17.26 | 18.12 | 18.83 |
| $^{231}$Pa | $^{28}$Ne | 60.42 | 23.38 | 23.09 | 22.84 | 23.43 |
| $^{233}$U | $^{28}$Ne | 60.49 | 24.82 | 24.06 | 23.88 | 24.76 |
| $^{235}$U | $^{28}$Ne | 57.36 | 27.42 | 28.25 | 28.63 | 29.07 |
| $^{237}$U | $^{28}$Ne | 60.78 | 24.83 | 24.61 | 24.41 | 24.38 |
| $^{239}$U | $^{28}$Ne | 57.76 | 27.42 | 28.74 | 29.03 | 28.50 |

Table 1. The values of screening parameter $a$ and effective nuclear radius parameter $r_0$ for 10 even–even and 9 odd-$A$ nuclei.

these 10 cluster radioactive even–even nuclei and 9 odd-$A$ nuclei by fitting the experimental data as $r_0(e^{-e^-}) = 1.17$ fm, $a(e^{-e^-}) = 1.39 \times 10^{-3}$ fm$^{-1}$ and $r_0(o-A) = 1.08$ fm, $a(o-A) = 8.30 \times 10^{-3}$ fm$^{-1}$, which is convenient to investigate the half-lives for different kinds of nuclei. Using these unified parameters values, we calculate the cluster radioactivity half-lives of these 19 nuclei. The logarithmic forms of calculated half-lives are listed in the sixth column of table 2, denoted as $\log_{10} T_{1/2}^{GGLM}$. For comparing, the logarithmic forms of half-lives calculated by using Gamow-like model proposed by Zdeb et al [43] and a Viola-Seaborg type formula proposed by Ren et al [11] are also listed in the last two columns of this table, denoted as $\log_{10} T_{1/2}^{GGLM}$ and $\log_{10} T_{1/2}^{V-S}$, respectively. In this table, the first two columns denote the parent nuclei and the emitted clusters, the third and forth columns denote the experimental cluster radioactivity released energy Q, and logarithmic form of experimental half-life $\log_{10} T_{1/2}^{exp}$, which are taken from the evaluated atomic mass table AME2016 [68,69] and evaluated nuclear properties table NUBASE2016 [67], respectively. As we can see from this table, the calculated half-lives using our modified Gamow-like model are in good agreement with experimental data as well as the calculated results given by using Gamow-like model and Viola-Seaborg type formula. For more on this, intuitively, we calculate the standard deviation $\sigma = \sqrt{\frac{1}{N} \sum (\log_{10} T_{1/2}^{exp} - \log_{10} T_{1/2}^{GGLM})^2}$ between the experimental data and calculated ones obtained by using these three models i.e., the modified Gamow-like model, Gamow-like model and V-S for 10 even–even nuclei and 9 odd-$A$ nuclei. The results are listed in table 3. As we can see from this table, the $\sigma$ value for all 19 nuclei within modified Gamow-like model is smaller than the ones within Gamow-like model and Viola-Seaborg type formula.
In particular, the $\sigma$ value for even–even nuclei (odd-$A$ nuclei) within modified Gamow-like model drops $1.061 - 0.601 = 43.36\%$ relative to the one within Gamow-like model, it indicates that the calculated half-lives given by our modified Gamow-like model can better reproduce the experimental data.

Encouraged by the good agreement between the experimental cluster radioactivity half-lives and calculated ones obtained by our modified Gamow-like model, in the following, using this model, we predict the half-lives of 13 cluster radioactive nuclei (including 9 even–even nuclei and 4 odd-$A$ nuclei) whose only the lower limit of the half-lives are measured and the predicted results are presented in the fifth column of table 4. In this table, the meaning of first four columns are exactly the same as in table 2. In addition, the predicted results of other theoretical studies within the Gamow-like model and Viola-Seaborg type formula are shown in the following two columns for comparison. As can be seen from table 2, all the predicted results can basically reproduce the experimental lower limit of cluster radioactivity half-lives of these nuclei. For more intuitively, figure 2 shows the predicted results within the modified Gamow-like model, Gamow-like model and Viola-Seaborg type formula as well as the corresponding experimental data. In this figure, the black open hexagons represents the experimental data of these 13 nuclei, the red, blue and green spheres represent corresponding predicted half-lives within our modified Gamow-like model, Gamow-like model and Viola-Seaborg type formula, respectively. From figure 2, we can clearly see that the predicted half-lives within our modified Gamow-like model, Gamow-like model and Viola-Seaborg type formula are larger than the lower limit of the experimental data except for the half-lives of $^{232}$Th $\rightarrow ^{208}$Hg + $^{24}$Ne and $^{233}$U $\rightarrow ^{205}$Hg + $^{28}$Mg. As for this two decays, it is obvious that the half-lives obtained by our modified Gamow-like model are closer to the lower limit of the experimental data comparing to the ones obtained by Gamow-like model and Viola-Seaborg type formula. Furthermore, for the decay of $^{232}$Th $\rightarrow ^{208}$Hg + $^{26}$Ne, only the half-life obtained by our modified Gamow-like model is larger than the lower limit of the experimental data.

### Table 3. The standard deviation $\sigma$ between the experimental data and calculated half-lives obtained by using our modified Gamow-like model, Gamow-like model and Viola-Seaborg type formula for 19 cluster radioactive nuclei.

| Model | Even–even (n = 10) | odd-$A$ (n = 9) | total (n = 19) |
|-------|-------------------|----------------|----------------|
| MGLM  | 0.601             | 0.690          | 0.645          |
| GLM   | 1.061             | 0.849          | 0.966          |
| V-S   | 0.357             | 0.862          | 0.647          |

### Table 4. The predicted half-lives obtained by using our modified Gamow-like model, Gamow-like model and Viola-Seaborg type formula for the 13 radioactive nuclei, whose only the lower limit of the half-lives are measured. $Q_{\text{exp}}$ and $T_{1/2}^{\text{exp}}$ are taken from the evaluated atomic mass table AME2016 [68, 69] and evaluated nuclear properties table NUBASE2016 [67], respectively.

| Nucleus | Cluster | $Q_{\text{exp}}$/MeV | $\log_{10} T_{1/2}^{\text{exp}}$/s | $\log_{10} T_{1/2}^{\text{MGLM}}$/s | $\log_{10} T_{1/2}^{\text{GLM}}$/s | $\log_{10} T_{1/2}^{\text{V-S}}$/s |
|---------|---------|----------------------|-----------------------------------|-----------------------------------|-----------------------------------|----------------------------------|
| $^{226}$Th | $^{14}$C | 30.67 | $>15.30$ | 17.06 | 16.67 | 18.33 |
| $^{226}$Th | $^{18}$O | 45.88 | $>15.30$ | 19.51 | 18.83 | 21.81 |
| $^{232}$Th | $^{24}$Ne | 55.62 | $>29.20$ | 27.86 | 26.99 | 27.21 |
| $^{230}$U | $^{24}$Ne | 61.55 | $>18.20$ | 21.65 | 20.66 | 21.97 |
| $^{230}$U | $^{25}$Ne | 55.95 | $>29.50$ | 31.60 | 30.72 | 29.72 |
| $^{232}$Th | $^{27}$Ne | 55.97 | $>29.20$ | 29.44 | 28.54 | 26.70 |
| $^{230}$U | $^{27}$Ne | 56.75 | $>29.50$ | 30.64 | 29.75 | 28.55 |
| $^{232}$U | $^{28}$Mg | 74.32 | $>22.26$ | 25.79 | 24.61 | 24.88 |
| $^{240}$Pu | $^{34}$Si | 90.95 | $>25.52$ | 28.41 | 27.38 | 25.57 |
| $^{233}$U | $^{28}$Mg | 74.24 | $>27.59$ | 27.16 | 26.56 | 26.37 |
| $^{233}$U | $^{28}$Mg | 72.2 | $>28.10$ | 29.46 | 29.19 | 28.74 |
| $^{237}$Np | $^{30}$Mg | 75.02 | $>26.93$ | 28.95 | 28.42 | 26.95 |
| $^{241}$Am | $^{34}$Si | 93.84 | $>24.41$ | 28.60 | 27.29 | 25.65 |
4. Summary

In summary, based on Gamow-like model with a screened electrostatic barrier and take into consideration the contribution of the centrifugal potential on cluster radioactivity half-life, we systematically study the half-lives of 19 cluster radioactive nuclei having experimental data. The values of screening parameter $a$ and effective nuclear radius parameter $r_0$ equal to $1.17 \times 10^{-3}$ fm and $1.39 \times 10^{-1}$ for even–even nuclei and $1.08$ fm and $8.30 \times 10^{-3}$ fm $^{-1}$ for odd-$A$ ones, respectively. The calculated results obtained by using our modified Gamow-like model are in satisfactory agreement with the experimental data. Moreover, using this model, we predict the half-lives of 13 cluster radioactive nuclei whose only the lower limit of the half-lives are measured. It is found that the predicted results have a good consistency with the ones obtained by using Gamow-like model and Viola-Seaborg type formula. This work is useful for the future theoretical and experimental research for cluster radioactivity.

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Data availability statement

The data that support the findings of this study are available upon reasonable request from the authors.

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