Weighing a galaxy bar in the lens Q2237+0305

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1 INTRODUCTION

Gravitational lensing provides a unique way to weigh objects at cosmological distances without any assumption about the connection between light and dark matter. Since the discovery of the first gravitational lens (Walsh, Carswell & Weymann 1979) several gravitational lenses have been found and this method has been used many times to explore the mass distribution of galaxies. In this paper, we model the lens system Q2237+0305 in order to weigh the bar in the lensing galaxy.

The quasar Q2237+0305 (z = 1.695) was found by Huchra et al. (1985) at the centre of an SBB spiral galaxy (z = 0.0394) that is situated in the outskirts of the Pegasus II cluster. The quasar was later resolved into four images that are situated around the core of the galaxy within a radius of one arcsecond [Yee 1988; Schneider et al. 1988].

Two fundamentally different approaches have been used to model the lensing galaxy. One was to fit a parametric mass profile with several free parameters to the observed quasar image configuration [Kent & Falco 1988]; the other to use a model of the light distribution of the galaxy and to fit for the mass-to-light ratio as the single free parameter (Schneider et al. 1988; Rix et al. 1993). The former approach was naturally much more precise in the reproduction of the observed image geometry due to the greater number of free parameters.

The length scale over which the lensing galaxy influences a light bundle from the quasar is small compared to the cosmological distances between observer, lens and source. The lens can therefore be treated as a mass sheet at the position of the galaxy. Since the galaxy disk of 2237+0305 is inclined with respect to the sky, elliptical surface mass distributions must be used in the models of this system.

Interestingly, the position angle, counted counterclockwise from north, of the major axis of the elliptical lens models found by Kent & Falco (1988) was about 67°. This is almost parallel to the axis through images C and D and just between the angle of the inclination axis of the galaxy (77°, Yee 1988) and the angle of the bar (39°, also Yee 1988). This situation is shown in figure 1. This was also found by Kochanek (1991) and Wambsganss & Paczyński (1994) who used simple circular mass distributions with an additional quadrupole perturbation. When fitted to the observed image geometry, the direction of the perturbation turned out to be close to the one Kent & Falco (1988) found for their model major axis. More recent investigations by Witt, Mao & Schechter (1992), Kassiola & Kovner (1993) and Witt (1996) obtained the same position angle for the perturbation or major model axis.

On the other hand, the bar shows up prominently in CCD images of the galaxy. It has been noted (Tyson & Gorenstein 1983, Yee 1988, Foltz et al. 1992) that it might contribute significantly to the lensing in the system – initially this was actually an aid to explain the lensing effect when only the images A, B and C were known (Tyson & Gorenstein 1983).

The motivation for this paper is the idea that the apparent misalignment of the predicted model major axis and observed galaxy inclination axis as shown in figure 1 is due to the lensing influence of the bar. In section 2 we construct...
and analyse a lensing model that includes the bar component and takes the observed position angle for the inclina-
tion axis of the galaxy into account. Section 3 deals with the implications of this model for the bar. In section 4 we
finally discuss our results. We use a cosmological model with

$$H_0 = 75 \text{ km s}^{-1} \text{ Mpc}^{-1}, \Omega = 1 \text{ and } \Lambda = 0.$$  

2 THEORETICAL MODEL

The lensing model we construct has two components, each with several free parameters. In this section, we introduce the components and determine the values for these that provide the best description of the observations.

2.1 The bulge

Yee (1988) identified three components in the inner part of the galaxy: bulge, disk and bar. In the lensing models for this system, bulge and disk have been represented by just one effective component since there is only limited observational data from the quasar image and galaxy positions to constrain free parameters of the model. For simplicity, we call this composite component ‘bulge’.

Moreover, Kochanek (1993) and Wambsganss & Paczyński (1994) found that the quasar image positions and the galaxy position in this system do not constrain the parameters even of simple lens models. In particular, Wambsganss & Paczyński (1994) showed that for a circular power-law mass distribution with an external shear there is a whole family of models that fit the observations; they discovered that for this family there is a linear relation between the magnitude of the external shear and the exponent of the mass profile for a vast range of exponents. The shear and the mass exponent are degenerate and one needs more information than only the positions of the quasar images and the galaxy to break this degeneracy. If one uses a two-component galaxy model the shear contributions from the two components will also be degenerate since the resulting shear is degenerate.

One way to get around the shear degeneracy is to use an elliptical mass distribution instead of a circular profile. In this case, the ellipticity–parameter of the mass distribution replaces the shear–parameter as a free parameter of the model. An analogous degeneracy in the ellipticity can then be broken by using the observed ellipticity of the isophotes of the galaxy.

Generalising the approach by Wambsganss & Paczyński (1994), we accordingly modelled the bulge with an elliptical power-law mass distribution with a major axis position angle of 77°. There is some disagreement in the literature on the value of this position angle (for example Rix, Schneider & Bahcall 1992). Fitte & Adam (1994) showed that this is because the position angle of elliptical isophotes is increasingly twisted towards the bar with increasing distance from the galaxy centre. The value we adopted from Yee (1988) is identical with the position angle determined from the galaxy continuum map within a radius of one arcsecond from the galaxy centre (Fitte & Adam 1994) where most of the lensing mass is situated. Let $\epsilon$ be the elliptical parameter, so that the ratio $b/a$ of minor and major axis of concentric elliptical shells is

$$\frac{b}{a} = \frac{1 - \epsilon}{1 + \epsilon}. \quad (1)$$

It is useful to express surface mass densities in units of the critical lensing density (Schneider et al. 1992) $\Sigma_{\text{crit}} = c^2 D_s/(4\pi G D_a D_\text{ls})$, where $D_s$, $D_a$ and $D_\text{ls}$ are the angular size distances between observer and source, observer and deflect (lens), as well as deflector and source. The surface mass density $\kappa$ of a power-law elliptical mass distribution in units of $\Sigma_{\text{crit}}$ is given by

$$\kappa(\theta_1, \theta_2) = \frac{E_0}{2 \theta_1^\nu}. \quad (2)$$

$\theta_1$ and $\theta_2$ are the coordinates on the sky as measured in a coordinate system oriented with the observed major and minor axis of the bulge. $\theta_e$ is the elliptical radius

$$\theta_e = \sqrt{\frac{\theta_1^2}{(1 + \epsilon)^2} + \frac{\theta_2^2}{(1 - \epsilon)^2}}. \quad (3)$$

$\nu$ is the power-law exponent of the elliptical mass distribution and $E_0$ is a constant. In the analysis of our results we use the ellipticity $\epsilon = 1 - \frac{b}{a} = \frac{2 \alpha}{1 + \alpha}$ since this is the value that is usually used in the observations. In Appendix A, deflection potential and angles for this mass distribution are described.

2.2 The bar

The light distribution of bars has a well-defined elongated shape, and is non-singular and centrally condensed (Sellwood & Wilkinson 1993). It is not straightforward to determine the true form of bar mass distributions from this,
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so that we have to assume a model. Very simple models with these properties of the bar light are the Ferrers profiles (Ferrers 1877). They were used in dynamical studies of bars (Ferrers 1866a,b,c). Since they can be treated analytically. To model the surface mass distribution of the bar of 2237+0305 we used two-dimensional Ferrers profiles of the form

$$\kappa(\theta_1, \theta_2) = \begin{cases} \kappa_c (1 - \frac{\theta_1^2}{a^2} - \frac{\theta_2^2}{b^2})^\lambda, & \text{if } \frac{\theta_1^2}{a^2} + \frac{\theta_2^2}{b^2} \leq 1, \\ 0, & \text{otherwise} \end{cases}$$

(4)

In this equation, $\theta_1$ and $\theta_2$ are the coordinates on the sky as measured in a coordinate system oriented with the observed major and minor axis of the bar. $\lambda$ is a real number, $\kappa_c$ is the central surface density in units of the critical lensing density $\Sigma_{\text{crit}}$ defined in section 2.1 and $a$, $b$ are the semi-major respectively semi-minor axis of the bar.

In the analysis we restricted ourselves to moderate exponents $\lambda = 0.5$, 1 and 2. The deflection potential and angles for integer values of $\lambda$ can be calculated analytically as described in Appendix 2. For $\lambda = 0.5$ the profiles were constructed through the numerical superposition of many elliptical slices of constant density ($\lambda = 0$) and different size (Schramm 1994).

### 2.3 The effect of shear

The lensing influence of the bar can be understood by considering a system with two shear tensors with shear directions as shown in figure 2 for galaxy inclination axis and bar (see Schneider, Ehlers & Falco (1992) for the definition of the shear tensor). The resulting shear tensor can be found by adding up the single tensors and the resulting shear direction is determined by the shear ratio of the two components.

A source almost directly behind the core of the lensing galaxy appears lensed with four of the five images in a cross formation aligned with the axes parallel and perpendicular to the resulting shear direction, while the fifth is seen to the resulting shear direction, which almost coincides with the galaxy inclination axis (Schneider et al. 1992). In the case of 2237+0305, the major axis position angle as found by the one-component lens models (670) can be interpreted to be this resulting shear direction, which almost coincides with the axis through images C and D. The axis through images C and D has effectively been twisted away from the galaxy inclination axis by the bar.

In figure 2 this twisting is illustrated by plotting the critical lines, caustics and image positions of two barred lenses with identical source positions. The caustics are the lines in the source plane that separate regions of different image multiplicity. The critical lines are the corresponding lines in the lens plane where pairs of images are created or destroyed (Schneider et al. 1992).

In this figure, the first lens has a weak bar that barely changes the elliptical shape of the bulge’s critical line or the corresponding diamond shape of the caustic. The other bar is significantly more massive; it warps the shape of these structures and shifts the image positions.

### 2.4 Detailed modelling

Our barred galaxy model has ten adjustable parameters. These are the positions of the galaxy and the source plus the constant $E_0$, the ellipticity $e$ and the exponent $\nu$ for the bulge as well as the bar mass normalisation $\kappa_c$, the semi-minor axis $b$ and the exponent $\lambda$ for the bar. This number can be reduced by two if the observed ellipticity of the bulge and only fixed values for $\lambda$ are used. For the length of the semi-major axis of the bar we used the observed value of $a \approx 9$ arcsec (taken from the figures in Yee 1988 or Irwin et al. 1989). The lens effect is insensitive to the precise value of $a$ because $a$ is much larger than the radius of the ring of images ($\approx 1$ arcsec). The length of the semi-minor axis must, however, remain a free parameter of the model since it is comparable to this radius, but not known well enough ($b \approx 1 - 2$ arcsec, see section 2.1).

There are ten observational constraints the system imposes upon theoretical models. These are the coordinates of the four observed quasar images and the galaxy centre. The positions for the images and galaxy centre were taken from Crane et al. (1991). These positions have been determined from Hubble Space Telescope (HST) observations and have quoted measurement errors of $0.005$. In general, the ratios of the fluxes of the different images of a gravitational lens also provide good constraints for a model. Unfortunately, in

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Figure 2. Illustration of the effect of a strong bar. Assuming identical source positions, the diamond-shaped caustics, the corresponding critical curves and the image positions are plotted for two different lenses. The smaller caustic and the almost elliptical critical curve belong to a model with a $\nu = 1$ power–law bulge and a $\lambda = 2$ Ferrers bar that was fitted to the observed parameters of Q2237+0305. The positions of the images created by this model are labelled with unprimed letters as in figure 1. In this model, the bulge mass inside the ring of images is about 20 times larger than the bar mass (see table 1). The caustic and the critical curve transform into the two elongated curves if the mass of the bar inside the ring of images is increased to half the mass of the bulge, while the bulge mass is kept fixed. The images are shifted by the more massive bar to the positions labelled with the primed letters.
the case of Q2237+0305 the lightcurves from Corrigan et al. (1996) or Østensen et al. (1996) clearly show that all optical image fluxes are subject to flux variations due to microlensing of the quasar light from the stars in the lensing galaxy. In addition, the light from the quasar is non-uniformly dust reddened during the passage through the galaxy. The fluxes were, therefore, neglected in the modelling procedure. We will, however, compare the model predictions with the recently measured radio flux densities by Falco et al. (1998).

Let \( \theta_k, \theta_\nu \) be the positions on the sky the model predicts for quasar images and the galaxy centre and \( \theta_{k0}, \theta_{\nu0} \) the observed positions with their positional uncertainties \( \sigma_k, \sigma_\nu \).

To find the best fit model, the expression

\[
\chi^2 = \sum_{k=1}^{4} \frac{(\theta_k - \theta_{k0})^2}{\sigma_k^2} + \frac{(\theta_\nu - \theta_{\nu0})^2}{\sigma_\nu^2}
\]

(Wambsganss & Paczyński 1994) was minimised through variation of the model parameters using a multidimensional minimisation routine (direction set or downhill simplex methods according to Press et al. 1992).

In order to find an estimator for the separations \( \theta_k - \theta_{k0} \) between modelled and observed images for the first term on the right hand side of Equation (3), we used the method by Kochanek (1991); the separations between an optimally weighted source position and the positions in the source plane where the observed image positions are mapped to by a given lens model are propagated back into the lens plane.

### 2.5 Analysis

For a nearly circularly symmetric lens with a source almost in the origin, it follows from Newton’s theorem in two dimensions (Foltz et al. 1992; Schramm 1994) that the mass inside the circle of images is approximately given by the separation \( \Delta \theta \) of the images at opposite ends of the cross via

\[
M = \frac{c^2}{16G} \frac{D_{\Delta \theta}}{D_{\Delta \theta}} (\Delta \theta)^2
\]

(see for example Narayan & Bartelmann 1996). The galaxy 2237+0305 is situated relatively close to us at an angular size distance \( D_\Delta = 0.15 h^{-1}_{75} \) Gpc, so that \( D_{\Delta \theta}/D_{\Delta} \approx 1 \). The separations of the quasar images are \( \Delta \theta \approx 1.8 \) arcsec, so that we get \( M \approx 1.5 \times 10^{10} h^{-1}_{75} M_\odot \). This value was also found in previous models for this system (Rix et al. 1992: 1.44 ± 0.08 \( \times 10^{10} h^{-1}_{75} M_\odot \), Wambsganss & Paczyński 1994: 1.48 ± 0.01 \( \times 10^{10} h^{-1}_{75} M_\odot \)). The other value theoretical models for 2237+0305 agreed on was the resulting shear direction of \( \approx 67^\circ \).

In our barred lens model, the bulge acts as the main lensing mass and the bar as a perturbation; for a given ellipticity \( e \) or exponent \( \nu \), the bulge parameter \( E_0 \) and hence the bulge mass do not change very much for different \( \lambda \)-bar models. In fact, experiments with different bar masses as in figure 3 showed that for similar masses of bulge and bar inside the quasar images the cruciform image symmetry gets skewed. Models with a strong bar thus cannot reproduce a symmetric image geometry as in Q2237+0305. In order to explore the effect of \( \lambda \) on the models, we used fixed values \( \lambda = 0.5, 1 \) and 2.

\[ \quad \]

Figure 3. Contour plot of confidence regions in the parameter space of bulge parameter \( e \), bulge exponent \( \nu \), bar exponent \( \kappa \), and bar axis ratio \( \lambda \). The other value theoretical models for 2237+0305 are very similar. The parameters were determined through minimisation and a \( \chi^2 \)-value was computed. The best fit models lie in a long valley that extends up to \( \nu \approx 1.25 \). The unclear structure at the lower end of the valley for \( \nu \lesssim 0.5 \) and \( e \lesssim 0.1 \) is due to numerical effects. Detailed investigation shows that the valley continues towards smaller \( \nu \), becoming shallower. The models in this region, with low \( e \) and \( \nu \), are similar to circular disks with constant surface mass density and are not examined here because they do not represent realistic galaxy models.

Besides \( E_0 \) and \( \nu \), the parameter space of \( e \), \( \nu \), \( \kappa \), and \( \lambda \) had to be examined. We first scanned the parameter space of \( e \) and \( \nu \) while leaving \( \kappa \) and \( \lambda \) as free parameters. In figure 3 a contour plot of the confidence regions that contain 68.3%, 95.4% and 99.7% of normally distributed models around the minimum of \( \chi^2 \) (Press et al. 1992) in the parameter space of \( \nu \) and \( e \) is shown. Only the plot for \( \nu = 2 \) is presented; the cases with \( \nu = 0.5 \) and \( \nu = 1 \) are very similar. The parameter space was scanned with a stepsize of 0.02 for \( \nu \) and 0.01 for \( e \). For every point the best parameters were determined through minimisation and a \( \chi^2 \)-value was computed. The best fit models lie in a long valley that extends up to \( \nu \approx 1.25 \). The unclear structure at the lower end of the valley for \( \nu \lesssim 0.5 \) and \( e \lesssim 0.1 \) is due to numerical effects. Detailed investigation shows that the valley continues towards smaller \( \nu \), becoming shallower. The models in this region, with low \( e \) and \( \nu \), are similar to circular disks with constant surface mass density and are not examined here because they do not represent realistic galaxy models.

The external-shear models by Wambsganss & Paczyński (1994) yielded no constraints on their circular mass distributions for the whole range of parameters from \( e = 0.07 \) to \( e = 0.2 \). The smaller allowed range for \( e \) in figure 3 shows that the ellipticity does not allow the same kind of freedom as the shear. For high ellipticities of the power–law bulges, the quasar image geometry cannot be reproduced anymore.

In figure 4 the total mass, the bulge mass, and the bar mass inside a circle of 0.9 arcsec as well as the semi-minor axis \( b \) and the total magnification \( \mu \) along the valley of best fits around the \( \nu = 1 \) profile are shown for 0.75 \( \leq \nu \leq 1.3 \) and the three values of \( \lambda \). The plots for different \( \lambda \) only differ noticeably in their semi-minor axis predictions. Beginning with \( \nu \approx 1.2 \) the minimisation produces numerical noise at the upper end of the valley from figure 3 since the fits get worse.
Figure 4. Model parameters along the valley of best fits. Plotted against \( \nu \) are the total mass and the masses of bulge and bar inside a circle of 0.9 arcsec in \( 10^{10} h^{-1}_7 M_\odot \) as well as the semi-minor bar axis \( b \) in arcsec and the total magnification \( \mu \). Different line styles have been used for different values of \( \lambda \) as indicated in the \( b-\nu \) panel.

It can be seen that the total mass of the model inside a circle of 0.9 arcsec is almost constant. The constituent masses of the bar and the bulge exhibit a dependence on \( \nu \). It can be interpreted that the bar mass increases with \( \nu \) in order to counter the increased contribution of the bulge to the resulting shear due to the increase of \( \epsilon \), and hence the bulge mass decreases in order to conserve the mass inside 0.9 arcsec given by equation (3). The magnification drops strongly with increasing \( \nu \), which was also observed by Wambsganss & Paczyński (1994).

As motivated in the introduction, we chose the model from the family of best models in figure 3 that exhibits the observed ellipticity of the bulge. The ellipticity has been observed elliptical or nearly elliptical profiles. For MG 1654+134, \( M_\text{bar} \) obtained \( \nu = 1.05, \epsilon = 0.31 \) and bar exponent \( \lambda = 2 \). The indicated contours contain 68.3%, 95.4% and 99.7% of normally distributed models around the minimum of \( \chi^2 \).

Kochanek (1995) obtained \( \nu = 1.0 \pm 0.1 \), and for QSO 0957+561, Grogin & Narayan (1996) obtained \( \nu = 1.1 \pm 0.1 \). The predictions of our model for the bulge are in good agreement with these values.

In table 3, the model parameters for the different values of \( \lambda \) and the uncertainties due to the uncertainty in \( \nu \) are shown. The \( \chi^2 \)-values from equation (5) have been divided by the number of degrees of freedom, giving a measure of the quality of the fit. The number of degrees of freedom is the number of constraints minus the number of free parameters; here we have three degrees of freedom since \( \epsilon, \nu \) and \( \lambda \) are fixed.

The value for the total magnification predicted from the \( \nu = 1.05 \) model is about half the value found by Wambsganss & Paczyński (1994) for their \( \nu = 1 \) circular power law profile with an external shear. This illustrates that the use of an elliptical mass distribution drastically changes the predictions of the model; a similar discrepancy is apparent for the time-delays. Note that \( \mu_{\text{total}} \approx \frac{1}{2} = 16.7 \) with \( \epsilon = 0.18 \) (corresponds to \( \nu = 0.31 \)), and derived from a \( \nu = 1 \) power-law lens with small \( \epsilon \) and a source in the origin by Kassiola & Kovner (1993). In contrast to the non-singular mass models by Kent & Falco (1988) our lens models do not produce a fifth image in the centre due to the central singularity of the mass distribution. The predicted source positions are very similar to the ones by Kent & Falco.

The remaining parameter space of the bar models is illustrated in figure 4. Similar to figure 3, the contours of the confidence regions of normally distributed models around the minimum of \( \chi^2 \) in the parameter space of \( \kappa_c \) and \( b \) are shown for the model with \( \nu = 1.05, \epsilon = 0.31 \) and \( \lambda = 2 \). The parameter space was scanned with a stepsize of 0.003 in \( \kappa_c \) and 0.025 arcsec in \( b \). The bar parameters are well constrained at the bottom of a steep boomerang-shaped valley.
Table 1. Model Parameters for Q2237+0305 for three values of the bar exponent $\lambda$ and the bulge exponent $\nu$. The columns for $\lambda = 0.5, 1.0$ and $2.0$ are indicated. Each table entry contains the value for $\nu = 1.05$ and the differences to the corresponding values for $\nu = 1.15$ (upper index) and 0.95 (lower index). Only one value is given for a line if the differences between the bar models are below the rounding precision. $\chi^2 / \beta$ is the total magnification, $\beta$ are the relative magnification ratios between the images. $\Delta t_{ij}$ are the relative time-delays between the images in h$^{-1}_{75}$ hours. $M_{\text{bulge}}(< 0.9''')$ and $M_{\text{bar}}(< 0.9''')$ are the masses of bulge and bar inside a circle of 0.9 arcsec and $M_{\text{bar, total}}$ is the total mass of the bar. Masses are given in $10^{10} h_{75}^{-1} M_\odot$, the semi-major bar axis was assumed as $a = 9$ arcsec. The VLA flux ratios are the 3.6cm radio flux ratios measured by Falco et al. [1999].

| $\lambda$ | 0.5 | 1.0 | 2.0 | VLA flux ratios |
|-----------|-----|-----|-----|----------------|
| $\chi^2 / \beta$ | $2.1^{+1.3}_{-0.8}$ | 
| $E_0$ | $0.81^{+0.09}_{-0.09}$ | $0.85^{+0.023}_{-0.017}$ |
| $e$ | $0.31^{+0.06}_{-0.06}$ | $0.85^{+0.023}_{-0.017}$ |
| $\kappa$ | $0.074^{+0.022}_{-0.014}$ | $0.079^{+0.021}_{-0.016}$ | $0.085^{+0.023}_{-0.017}$ |
| $b$ | $0.58^{+0.04}_{-0.06}$ | $0.67^{+0.07}_{-0.06}$ | $0.85^{+0.023}_{-0.017}$ |
| $\beta_1$ | $-0.063^{+0.009}_{-0.010}$ | $-0.014^{+0.003}_{-0.001}$ |
| $\beta_2$ | $-0.014^{+0.003}_{-0.001}$ | $0.55^{+0.022}_{-0.021}$ |
| $\mu_{\text{total}}$ | $15.6^{+3.5}_{-4.8}$ | $16.0^{+3.6}_{-4.8}$ | $16.2^{+3.6}_{-4.8}$ |
| $\mu_{\text{BA}}$ | $1.12^{+0.02}_{-0.01}$ | $1.08^{+0.03}_{-0.02}$ | $1.04^{+0.03}_{-0.02}$ |
| $\mu_{\text{CA}}$ | $0.62^{+0.04}_{-0.04}$ | $0.61^{+0.03}_{-0.02}$ | $0.60^{+0.03}_{-0.02}$ |
| $\mu_{\text{DA}}$ | $1.26^{+0.04}_{-0.06}$ | $1.26^{+0.04}_{-0.06}$ | $1.24^{+0.04}_{-0.06}$ |
| $\Delta t_{\text{BA}}$ | $2.0^{+0.4}_{-0.3}$ | $16.2^{+2.7}_{-4.4}$ | $4.9^{+1.0}_{-0.8}$ |
| $\Delta t_{\text{CA}}$ | $4.9^{+1.0}_{-0.8}$ | $16.2^{+2.7}_{-4.4}$ | $4.9^{+1.0}_{-0.8}$ |
| $\Delta t_{\text{DA}}$ | $4.9^{+1.0}_{-0.8}$ | $16.2^{+2.7}_{-4.4}$ | $4.9^{+1.0}_{-0.8}$ |
| $M_{\text{bulge}}(< 0.9''')$ | $1.42^{+0.02}_{-0.01}$ | $1.42^{+0.02}_{-0.01}$ | $1.41^{+0.01}_{-0.01}$ |
| $M_{\text{bar}}(< 0.9''')$ | $0.07^{+0.01}_{-0.01}$ | $0.07^{+0.01}_{-0.01}$ | $0.06^{+0.01}_{-0.01}$ |
| $M_{\text{bar, total}}$ | $0.47^{+0.04}_{-0.06}$ | $0.43^{+0.03}_{-0.06}$ | $0.39^{+0.05}_{-0.05}$ |

Table 2. Local lensing parameters at the positions of the quasar images. The values have been determined with a $\lambda = 1$ bar, but they are identical for $\lambda = 0.5$ and 2 within $\pm 0.01$. Each table entry contains the value for $\nu = 1.05$ and the differences to the corresponding values for $\nu = 1.15$ (upper index) and 0.95 (lower index).

| Image | $\kappa$ | $\gamma$ |
|-------|---------|---------|
| A     | $0.36^{+0.07}_{-0.06}$ | $0.40^{+0.03}_{-0.03}$ |
| B     | $0.36^{+0.06}_{-0.06}$ | $0.42^{+0.03}_{-0.03}$ |
| C     | $0.69^{+0.03}_{-0.03}$ | $0.71^{+0.03}_{-0.03}$ |
| D     | $0.59^{+0.02}_{-0.02}$ | $0.61^{+0.03}_{-0.03}$ |

The surface mass density $\kappa$ in units of the critical density and the local shear $\gamma$ at the positions of the images are given in table 2. The values for $\nu = 0.95$ and 1.05 are similar to what one gets for a circular $\nu = 1$ power-law mass distribution (singular isothermal sphere) with a mixture of internal and external shear. The values can be taken from tables 1 and 2 that a change of $\lambda$ does not cause a measurable change of observable quantities except for the semi-minor axis of the bar; the uncertainty of the model parameters is dominated by the uncertainty of the bulge ellipticity $e / \nu$ exponent $\nu$.

3 PROPERTIES OF THE BAR

The width of the bar has not been measured previously. In figure 1 of their paper, Irwin et al. [1988] present a contour plot of the galaxy where the bar has been separated from the disk. In this plot, the bar appears about 18 arcsec long, but only $\approx 2 - 4$ arcsec wide. It is just the less well-known minor axis that enters the lensing model since the quasar images are situated in the centre of the galaxy. In their image analysis Irwin et al. subtracted structure from their image that is smooth on scales of $\approx 5 - 10$ arcsec. This procedure removed the galaxy disk very efficiently, but, being long as well as thin, the bar shown in their figure 1 could also be affected by this procedure.

A different approach to obtain the light distribution of the bar was pursued by Schmidt [1996]. Using a Hubble Space Telescope (HST) I-band image taken with the Planetary Camera prior to the first servicing mission [Westphal 1992], the galaxy was decomposed into bulge, disk and bar with analytical profiles that have been convolved with the...
point-spread function of the telescope. Exponential profiles
(Andredakis & Sanders 1993) were fitted to bulge and disk.
After these components were removed from the image, the
bar and the spiral arms remained. The bar light distribution
could be fit with a $\lambda = 2$ Ferrers profile with $a = 9.5 \pm 1.0$
arsec and $b = 1.0 \pm 0.3$ arcsec. This light model can be com-
bined with the mass model for $\lambda = 2$ from table 2 due to the
similar value for $b$; the I-band mass-to-light ratios of these
model components inside a circle of 0.9 arcsec are given by
$M/L_\lambda \approx 4.8\,h_{75}$ for bulge plus disk and $M/L_\lambda \approx 5.0\,h_{75}$
for the bar. The bar mass detected with gravitational lensing
and the bar light in the I-band in this model both consti-
tute a 5% fraction of the total mass respectively light inside
0.9 arcsec.

This result is, however, dependent on the bulge model.
For a de Vaucouleurs–bulge (de Vaucouleurs 1948), a $\lambda = 0.5$
Ferrers profile fitted the bar light much better with a similar
value for $a$, but a much larger value $b = 3.1 \pm 0.9$ arcsec which
cannot be combined with the lensing model from table 2 since
$b$ is very different. The unrefurbished HST point spread
function inhibited a clear distinction between the quality of
fit of these different bulge models, so that the question about
the minor bar axis is not decided yet.

A value of $b > 1$ arcsec could in connection with the values
for $b$ in table 2 be taken as evidence in favour of steeper
bar models, $\lambda \geq 2$. There is observational evidence from the
light distributions of real bars for more boxy mass distribu-
tions (Freeman 1996), so that it has to be ascertained that
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4 DISCUSSION

In this paper we have presented a barred galaxy model for
the gravitational lens 2237+0305. We used a power–law el-
liptical mass distribution for the bulge and chose the model
for which the observed ellipticity is predicted. It turned out
that this model has an exponent close to unity, which is
compatible with other determinations of lens mass profiles
through gravitational lensing (Kochanek 1995; Grogin &
Narayan 1996). The bar represents a small perturbation of
the deflection field of the bulge of the galaxy, amounting to
$7.5 \pm 1.5 \times 10^8\,h_{75}^{-1}\,M_\odot$ or about 5% of the bulge mass in the
critical region inside the quasar images.

The relative magnifications our model predicts for the
quasar images can be compared with the 3.6cm radio flux
ratios published by Falco et al. (1996). Their results are also
given in table 2. Falco et al. argue that it is unlikely that
the radio flux densities are variable from microlensing due
to the larger size of the radio emitting region as compared to
the optical continuum emitting region, although they cannot
completely rule out microlensing as an important effect in
the radio. If microlensing is not important, the model magni-
fication ratios should be identical to these measured flux
ratios. It can be seen that only the ratio $\mu_{DA}$ between imag-
es D and A is not compatible with their results although no
effort was made to fit the flux ratios.

In order to find out more about the discrepancy of the ratio $\mu_{DA}$ between observation and model one has to make the relatively large error bars from Falco et al. smaller
through longer radio observation of the object. Unfortunately,
2237+0305 has a radio flux density of only $\approx 1\,mJy$
(Falco et al. 1996), so that radio observations of this object
are very time-consuming; Falco et al. observed for 11 hours
of which only five could be used eventually due to weather
conditions.

To get additional, independent arbiters for the model,
it would be very helpful to measure the time–delays in this
system. Since the time–delays are of the order of several
hours, this has to be done in a wavelength domain where
the necessary intra–day variability is likely to occur for a
radio–quiet quasar, for example in the x–ray regime as pro-
posed by Wambsganss & Paczyński (1992). Also, monitoring
in the radio would show if the quasar image flux densities
vary at these frequencies. Unless we learn more about the
radio flux densities and the time delays, it is not possible
to decide whether or not it is microlensing that causes the
low magnification of image D. In the optical, image D has
always been the faintest quasar image. In fact, in the first
resolved image of the quasar, image D was not visible at
all (Tyson & Gorenstein 1985). There is also spectroscopic
evidence from optical data that image D is undergoing de-
magnification (Lewis et al. 1996).

If image D is in fact microlensed in the radio, the con-
sequences are interesting. The scale size of the radio region
could be less than the characteristic scale of the caustic net-
work. Alternatively, the radio source could have an asym-
metric structure like a jet that would have differing mi-
crolensing properties for different paths of the microlenses
across the source.

Yet another way to significantly change the radio flux
density of image D would be a globular cluster or black hole
(Lacey & Ostriker 1983) with a mass of about $10^6\,M_\odot$
in the halo of the lensing galaxy that is situated close to image
D. An object of this mass would magnify or demagnify the
radio image of the quasar, depending on its location with
respect to the direction of the local shear. This effect, the
perturbation of lens models by $10^6\,M_\odot$ objects, has recently
been treated by Mao & Schneider (1993). With this, we can
estimate that a surface mass density of globular clusters or
black holes of approximately $0.04\,\Sigma_{crit}$ ($\Sigma_{crit}$ is defined in
section 2) or $470\,h_{75}\,M_\odot/pc^2$ is needed to observe a de-
magnification of image D by 40% or more with a probability
of 20%. Higher surface mass densities would make it more
likely. This is much more than the globular cluster surface
mass density of about $1\,M_\odot/pc^2$ seen in our Galaxy within
5 kpc of the Galactic centre (Mao & Schneider 1993). It thus
seems unlikely that the demagnification is due to a globular
cluster.

The question of the existence of such a massive object
near image D could be solved with a method that was pro-
posed by Wambsganss & Paczyński (1992). They showed that
these objects would bend or even create holes in the
radio maps of milliarcsecond jets of gravitationally lensed
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APPENDIX A: POWER–LAW ELLIPTICAL MASS DISTRIBUTIONS

The lensing properties of the elliptical power–law mass distributions from Equation (2) with $\nu = 1$ have been described by Kassiola & Kovner (1993) and Kormann, Schneider, & Bartelmann (1994). Kormann et al. determined these by solving the Poisson equation for the deflection potential $\psi$ (see Schneider et al. 1992 for the definition of the deflection potential) in polar coordinates $\theta = \sqrt{\theta_1^2 + \theta_2^2}$ and $\varphi$

$$\frac{1}{\theta} \frac{\partial}{\partial \theta} \left( \theta \frac{\partial \psi}{\partial \theta} \right) + \frac{1}{\varphi} \frac{\partial^2 \psi}{\partial \varphi^2} = 2\kappa (\theta, \varphi). \quad (A1)$$

Their approach can be generalised for arbitrary real numbers $\nu$ with $0 \leq \nu < 2$. Define $n = 2 - \nu$, $s_n = \sin \frac{n\pi}{2}$, $c_n = \cos \frac{n\pi}{2}$ and

$$\Delta (\varphi) = \sqrt{\cos^2 \varphi + \sin^2 \varphi} \left( 1 + \frac{n^2}{4} \right) \left( 1 - \frac{n^2}{4} \right). \quad (A2)$$

Using the integrals

$$N_1 = \int_0^\pi \cos n \varphi' \Delta (\varphi') d\varphi'$$

$$N_2 = \int_0^\pi \sin n \varphi' \Delta (\varphi') d\varphi'$$
\[ N_3 = N_1 + \int_0^\pi \frac{\cos n \varphi'}{\Delta'(\varphi')} \, d\varphi', \] (A3)

defining the deflection potential for \( 0 \leq \varphi \leq \frac{\pi}{2} \) is given by
\[ \psi(\theta, \varphi) = E_0 \theta^n \left[ \frac{1}{n} \int_0^\alpha \frac{\cos n \varphi}{\Delta(\varphi)} \, d\varphi + \left( N_2 + \frac{c_n}{s_n} N_3 \right) \cos n \varphi \right]. \] (A4)

The potential for the other quadrants can be taken from this result because of the elliptical symmetry of the mass distribution. The deflection angles \( \vec{\alpha} = \vec{\nabla} \psi \) for this quadrant can be calculated from Equation (A4). The cartesian components are
\[ \alpha_1(\theta, \varphi) = E_0 \theta^{n-1} \left[ N_1 \left( \sin n \varphi \cos \varphi - \cos n \varphi \sin \varphi \right) \right. \]
\[ + \left( N_2 + \frac{c_n}{s_n} N_3 \right) \cos n \varphi \cos \varphi + \sin n \varphi \sin \varphi \right] \]
\[ \alpha_2(\theta, \varphi) = E_0 \theta^{n-1} \left[ N_1 \left( \sin n \varphi \sin \varphi + \cos n \varphi \cos \varphi \right) \right. \]
\[ + \left( N_2 + \frac{c_n}{s_n} N_3 \right) \cos n \varphi \sin \varphi - \sin n \varphi \cos \varphi \]. (A5)

These expressions can be evaluated numerically. For \( n = 1 \) the integrals are analytically solvable and the formulas become identical to the results by Kassiola & Kovner (1993) and Kormann et al. (1994). After this work was completed, we discovered that Grogin & Narayan (1996) also used power-law elliptical mass profiles in their model of the lens of the double quasar 0957+561. In their paper they present the deflection angles of power-law elliptical mass distributions in a complex-valued lensing formalism in terms of the complex hypergeometric function.

**APPENDIX B: FERRERS PROFILES**

The deflection potential \( \psi_0 \) and the deflection angles \( \vec{\alpha}_0 \) of an elliptical slice of constant surface density, a Ferrers profile from Equation (B1) with \( \lambda = 0 \), are given by
\[ \psi_0(\theta_1, \theta_2) = \frac{1}{2} ab \kappa c \left( Q_{00} - \theta_1^2 Q_{10} - \theta_2^2 Q_{01} \right) \] (B1)

and the cartesian components
\[ \alpha_{01}(\theta_1, \theta_2) = ab \kappa c \theta_1 Q_{10}, \]
\[ \alpha_{02}(\theta_1, \theta_2) = ab \kappa c \theta_2 Q_{01}, \] (B2)

where
\[ Q_{00} = 2 \ln \left( \sqrt{a^2 + \rho} + \sqrt{b^2 + \rho} \right)^{-1} \]
\[ Q_{01} = \frac{2}{a^2 - b^2} \left( \sqrt{\frac{a^2 + \rho}{b^2 + \rho}} - 1 \right) \]
\[ Q_{10} = 2 / \Delta(\rho) - Q_{01}. \] (B3)

\( \rho \) is the positive solution of
\[ \frac{\theta_1^2}{a^2 + \rho} + \frac{\theta_2^2}{b^2 + \rho} = 1 \] (B4)

outside the bar, and \( \rho = 0 \) inside the bar. \( \Delta(\rho) \) is given by
\[ \Delta(\rho) = \sqrt{(a^2 + \rho)(b^2 + \rho)}. \] (B5)

\( \vec{\alpha}_0 \) has first been derived by Schramm (1990) by using the known force field for a homogenous ellipsoid and letting the largest axis go to infinity in order to get the corresponding 2-dimensional equations. To derive the potential \( \psi_0 \) in terms of real-numbered coordinates \( \theta_1, \theta_2 \), we applied Schramm’s method to the results by Pfenniger (1984) for the potential and force fields of 3-dimensional Ferrers ellipsoids. This enables us to calculate deflection potential and deflection angles for a whole family of Ferrers profiles. The deflection potential for Ferrers surface-density profiles with integer exponents \( \lambda \) is given by
\[ \psi_\lambda(\theta_1, \theta_2) = \frac{ab \kappa c}{2(\lambda + 1)} \int_0^{\infty} \frac{du}{\Delta(u)} \left( 1 - \frac{\theta_1^2}{a^2 + u} - \frac{\theta_2^2}{b^2 + u} \right)^{\lambda + 1}. \] (B6)

The corresponding 3-dimensional expression was first derived by Ferrers (1877). For integers \( j, k \neq 0 \) or \( k \neq 0 \), this integral can be split up into a sum containing the coefficients
\[ Q_{jk} = \int_0^{\infty} \frac{du}{\Delta(u)} \left( a^2 + u \right)^j \left( b^2 + u \right)^k. \] (B7)

For \( Q_{00} \) use Equation (B3); a remaining contribution from the infinite axis had to be subtracted here. Pfenniger (1984) solved his corresponding integrals using recurrence relations. Translated into two dimensions, the \( Q_{jk} \) obey the relation
\[ Q_{j,k} = (Q_{j-1,k} - Q_{j,k-1}) / (a^2 - b^2), \] (B8)

as well as for \( n > 0 \)
\[ Q_{nn} = \frac{1}{2n - 1} \left[ \frac{2}{\Delta(\rho)} \left( a^2 + \rho \right)^{n-1} - Q_{n-1,1} \right]. \] (B9)
\[ Q_{0n} = \frac{1}{2n - 1} \left[ \frac{2}{\Delta(\rho)} \left( b^2 + \rho \right)^{n-1} - Q_{1,1-n} \right]. \] (B10)

With these equations, the deflection potential \( \psi_\lambda \) as well as the deflection angles \( \vec{\alpha}_\lambda = \vec{\nabla} \psi_\lambda \) can be calculated from Equation (B4). The coefficients \( Q_{jk} \) can be treated as constant for the derivation with respect to \( \theta_1 \) and \( \theta_2 \) since the definition of \( \rho \) in Equation (B3) implies that \( \frac{\partial \rho}{\partial \rho} = 0 \).

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