Possible Revelation of Seesaw Mass Pattern in Solar and Atmospheric Neutrino Data

Ernest Ma
Department of Physics, University of California, Riverside, California 92521

J. Pantaleone
Department of Physics, University of Alaska, Anchorage, Alaska 99508

Abstract

Assuming the solar and atmospheric neutrino deficits to be due to neutrino oscillations, it is shown that the $3 \times 3$ mass matrix spanning $\nu_e, \nu_\mu,$ and $\nu_\tau$ may have already revealed a seesaw mass pattern. Also, this matrix is the natural reduction of a simple $5 \times 5$ seesaw mass matrix with one large scale, the $4 \times 4$ reduction of which predicts that a fourth neutrino would mix with $\nu_e$ and $\nu_\mu$ in such a way that $\nu_\mu \to \nu_e$ oscillations may occur just within the detection capability of the LSND (Liquid Scintillator Neutrino Detector) experiment.
Whether or not neutrinos have mass is clearly a fundamental issue in particle physics and astrophysics. There is now a good deal of indirect evidence from measurements of the solar $\nu_e$ flux\cite{1, 2, 3, 4} and the atmospheric $\nu_\mu/\nu_e$ ratio\cite{3, 5} that neutrinos oscillate from one to another. This means that $\nu_e$ and $\nu_\mu$ are not mass eigenstates; each is rather an admixture of two or more mass eigenstates. Under the simplifying assumption that only two neutrinos are involved in each case, probable central values for the mass-squared difference and the mixing angle are

$$\Delta m^2 \simeq 4 \times 10^{-6} \text{ eV}^2, \quad \sin^2 2\theta \simeq 0.01,$$

from analyzing\cite{7} solar data, assuming the small-angle, nonadiabatic solution, and

$$\Delta m^2 \simeq 10^{-2} \text{ eV}^2, \quad \sin^2 2\theta \simeq 0.5,$$

from analyzing\cite{8} atmospheric data.

Given only the above information, it is clearly not possible to reconstruct unambiguously the underlying $3 \times 3$ neutrino mass matrix. However, if a certain empirical relationship can be found among the four parameters listed above, an important insight may be gained as to the form of this mass matrix. Such an example is already very well-known in the case of the quark mass matrix. It was pointed out many years ago\cite{9} that the empirical relationship

$$\sin^2 \theta_C \simeq m_d/m_s$$

may be obtained with a $2 \times 2$ mass matrix of the form

$$\mathcal{M} = \begin{bmatrix} 0 & a \\ a & b \end{bmatrix}.$$

This simple observation has generated over the years an enormous literature on quark mass matrices. It is an especially active field of research in the past two or three years. In the jargon of neutrino physics, the form of $\mathcal{M}$ in Eq. (4) is called seesaw\cite{10}.
We propose here that Eq. (4) is also applicable to the $2 \times 2$ matrix spanning $\nu_\tau$ and the linear combination $\nu_\mu \cos \theta + \nu_e \sin \theta$, where $\theta$ is the angle measured in atmospheric neutrino oscillations. The orthogonal linear combination $\nu_e \cos \theta - \nu_\mu \sin \theta$ is assumed to be massless.

The $3 \times 3$ neutrino mass matrix spanning $\nu_e, \nu_\mu, \nu_\tau$ is then given by

$$
\mathcal{M}_\nu = \begin{bmatrix}
b \sin^2 \theta & b \sin \theta \cos \theta & a \sin \theta \\
b \sin \theta \cos \theta & b \cos^2 \theta & a \cos \theta \\
a \sin \theta & a \cos \theta & 0
\end{bmatrix}. 
$$

(5)

The eigenvalues of $\mathcal{M}_\nu$ are 0, $-a^2/b$, and $b$ for $a << b$. Let the corresponding mass eigenstates be $\nu_1, \nu_2, \nu_3$. Then the mixing matrix $U$ is easily obtained:

$$
\begin{pmatrix}
\nu_e \\
\nu_\mu \\
\nu_\tau
\end{pmatrix} = \begin{pmatrix}
cos \theta & -(a/b) \sin \theta & \sin \theta \\
-sin \theta & -(a/b) \cos \theta & \cos \theta \\
0 & 1 & a/b
\end{pmatrix} \begin{pmatrix}
\nu_1 \\
\nu_2 \\
\nu_3
\end{pmatrix}. 
$$

(6)

Following the results of the two-neutrino analyses given in Eqs. (1) and (2), let $b = 0.1$ eV and $a = 0.014$ eV, then $a^2/b \simeq 2 \times 10^{-3}$ eV, hence $\Delta m_{12}^2 \simeq 4 \times 10^{-6}$ eV$^2$ and $\Delta m_{13}^2 = 10^{-2}$ eV$^2$. Now choose $\sin^2 2\theta = 0.5$ as given in Eq. (2), then the effective $\sin^2 2\theta_{12} \equiv 4|U_{e2}|^2|U_{e1}|^2/[|U_{e1}|^2 + |U_{e2}|^2]^2$ for solar neutrino oscillations is predicted to be

$$
\sin^2 2\theta_{12} \simeq (a/b)^2 \sin^2 2\theta / \cos^4 \theta \simeq 0.014,
$$

(7)

which roughly matches the value given in Eq. (1). This marks the first time that a possible seesaw mass pattern in neutrino physics has been identified.

Since $\Delta m_{12}^2 << \Delta m_{13}^2$ and $a/b << 1$, the atmospheric neutrino deficit\[3, 4\] is explained by a simple two-neutrino oscillation between $\nu_\mu$ and $\nu_e$. However the solar neutrino deficit comes from the oscillations of all three neutrinos. If matter effects \[11\] could be neglected, the disappearance probability of solar neutrinos would be given by

$$
1 - P(\nu_e \rightarrow \nu_e) = 2|U_{e3}|^2|U_{e1}|^2 + 2|U_{e3}|^2|U_{e2}|^2 + 2|U_{e2}|^2|U_{e1}|^2 \left(1 - \cos \frac{t \Delta m_{12}^2}{2E}\right), 
$$

(8)
where $U_{\alpha i}$ are the elements of the mixing matrix in Eq. (6). We see that the effects of the heaviest mass parameter average out, but a dependence on $|U_{e3}|^2$ remains. Only in the limit of vanishing $|U_{e3}|^2 = \sin^2 \theta$ is the two-neutrino approximation valid for solar neutrinos. This general observation is also true when matter effects are included\cite{12}. Since a large $\sin^2 \theta$ is required by the atmospheric data, the naive comparison of Eq. (7) with Eq. (1) will have to be modified.

A three-neutrino analysis of all the solar neutrino data\cite{1, 2, 3, 4} is shown in Figs. (1a), (1b), and (1c). The method is the same as that of an earlier analysis\cite{13} where a continuous range of $|U_{e3}|^2$ values is considered. Three values of $\Delta m^2_{13}$ and $\sin^2 2\theta$ which fit the atmospheric neutrino data have been chosen, and their predictions for $\Delta m^2_{12}$ and $\sin^2 2\theta_e \equiv 4|U_{e2}|^2(1 - |U_{e2}|^2)$ (dashed line) have been compared to the solar data constraints (solid contours). For Fig. (1a), a small value of $\sin^2 2\theta$ is chosen. There the allowed solar neutrino solutions are close to those of the two-neutrino analysis, and the predictions of Eq. (5) match the small-angle, nonadiabatic solution. For Figs. (1b) and (1c), two larger values of $\sin^2 2\theta$ are chosen and the allowed solar neutrino solutions are very different from those found in the two-neutrino analysis. Here the predictions of Eq. (5) match an adiabatic solution which is allowed only in the three-neutrino analysis. As shown in Fig. (2), Eqs. (5) and (6) are satisfied in different ways across the parameter region indicated by the atmospheric data. Although we do not include the region allowed by the new Kamiokande multi-GeV data\cite{6} which is in conflict with Frejus\cite{14}, solutions exist there as well.

Consider now the possible origin of the $3 \times 3$ neutrino mass matrix $\mathcal{M}_\nu$ of Eq. (5). The most natural way is to find it as the seesaw\cite{10} reduction of a larger matrix which includes
a higher mass scale. A very simple solution is the following $5 \times 5$ mass matrix

$$
M_5 = \begin{bmatrix}
0 & 0 & 0 & m_1 & 0 \\
0 & 0 & 0 & m_2 & 0 \\
0 & 0 & 0 & 0 & m_3 \\
m_1 & m_2 & 0 & 0 & m_4 \\
0 & 0 & m_3 & m_4 & m_5
\end{bmatrix}
$$

(9)

spanning $\nu_e, \nu_\mu, \nu_\tau, \nu_S$, and $N$, where the last two are singlet Majorana neutrinos. In the limit of very large $m_5$, the seesaw reduction of $M_5$ is

$$
M_4 = \begin{bmatrix}
0 & 0 & 0 & m_1 \\
0 & 0 & 0 & m_2 \\
0 & 0 & -m_3^2/m_5 & -m_3 m_4/m_5 \\
m_1 & m_2 & -m_3 m_4/m_5 & -m_4^2/m_5
\end{bmatrix}.
$$

(10)

Assume now that $m_2^2/m_5$ is the dominant term in $M_4$, then a second seesaw reduction gives the following $3 \times 3$ matrix:

$$
M_3 = \begin{bmatrix}
m_1^2 m_5/m_4 & m_1 m_2 m_5/m_4 & -m_1 m_3/m_4 \\
m_1 m_2 m_5/m_4 & m_2^2 m_5/m_4 & -m_2 m_3/m_4 \\
-m_1 m_3/m_4 & -m_2 m_3/m_4 & 0
\end{bmatrix},
$$

(11)

which matches exactly $M_\nu$ of Eq. (5) with the identification

$$
b = \frac{(m_1^2 + m_2^2)m_5}{m_4^2}, \quad \tan \theta = \frac{m_1}{m_2}, \quad a = -\frac{m_3 \sqrt{m_1^2 + m_2^2}}{m_4}.
\quad (12)
$$

With the further requirement that $m_3 m_4 << m_5 \sqrt{m_1^2 + m_2^2}$ so that $a << b$, we then obtain Eq. (6). Since $M_3(= M_\nu)$ reduces further to $M$ of Eq. (4), one might even speculate that the present neutrino data are indicative of three successive seesaw reductions.

If we take $M_5$ (and thus $M_4$) seriously, then we have one more very interesting prediction. There should be a fourth neutrino ($\nu_S$) with mass given by $-m_2^2/m_5$ and mixing to $\nu_e$ and $\nu_\mu$ given by $-m_1 m_5/m_4^2$ and $-m_2 m_5/m_4^2$ respectively. Since

$$
\left( \frac{m_1^2}{m_5} \right) \left( \frac{m_1 m_5}{m_4^2} \right) \left( \frac{m_2 m_5}{m_4^2} \right) = \left( \frac{(m_1^2 + m_2^2)m_5}{m_4^2} \right) \left( \frac{m_1}{\sqrt{m_1^2 + m_2^2}} \right) \left( \frac{m_2}{\sqrt{m_1^2 + m_2^2}} \right).
\quad (13)
$$
we predict that there should be additional oscillations between $\nu_e$ and $\nu_\mu$ through this fourth neutrino mass eigenstate with restricted values of $\Delta m^2$ and $\sin^2 2\theta$ such that their product is the same as that for atmospheric oscillations. Using the numerical results obtained earlier in this paper, we would then predict

$$3 \times 10^{-3} \text{ eV}^2 < \Delta m^2 \sin^2 2\theta < 10^{-2} \text{ eV}^2.$$  \hfill (14)

Part of this region is excluded by the E776 neutrino experiment at Brookhaven National Laboratory[16], but another part ($\Delta m^2 \sim 1 \text{ eV}^2, \sin^2 2\theta \sim 5 \times 10^{-3}$) lies within the detection capability of the LSND (Liquid Scintillator Neutrino Detector) experiment at Los Alamos National Laboratory. We also note that the upper bound in the above comes from reactor experiments[15]. Relaxing it would allow a larger $\Delta m^2 (\sim 6 \text{ eV}^2)$ which is preferred by the preliminary LSND results, but in conflict with the published E776 data.

To obtain $\mathcal{M}_5$, the following softly broken discrete $Z_3$ symmetry may be considered. Let $\nu_e, \nu_\mu, \nu_\tau, \nu_S$, and $N$ transform as $\omega, \omega, 1, \omega^2$, and 1 respectively, where $\omega^3 = 1$. Then $m_1, m_2$, and $m_3$ come from the vacuum expectation value of the standard Higgs doublet, and $m_5$ is allowed by $Z_3$. However, $m_4$ breaks $Z_3$ softly and so does the diagonal mass term for $\nu_S$, but as long as the latter is much smaller than $m_4^2/m_5$, the reduction to $\mathcal{M}_\nu$ proceeds as before. With four light neutrinos, the nucleosynthesis bound[17] of $N_\nu < 3.3$ is an important constraint. Although $\nu_S$ is a singlet neutrino, it mixes with other neutrinos and may contribute significantly to $N_\nu$ through oscillations[18]. In the context of $\mathcal{M}_4$, this constraint implies that $\sqrt{m_1^2 + m_2^2}$ has to be very much smaller than $m_4^2/m_5$, hence the $\sin^2 2\theta$ parameter in Eq. (14) is too small to be in the range of the LSND detection capability. In other words, the simplest extension of our basic proposal to four neutrinos cannot accommodate simultaneously the LSND results and nucleosynthesis. The latter constraint can be avoided if we switch $\nu_\tau$ and $\nu_S$ in $\mathcal{M}_5$, but then the underlying theory becomes much more involved. For example, the $\nu_e\nu_\tau$ and $\nu_\mu\nu_\tau$ mass terms must now come from either a Higgs triplet or a
radiative mechanism as recently proposed[19]. Details will be given elsewhere.

In conclusion, we have shown in the above how present data on solar and atmospheric neutrinos may have revealed a seesaw mass pattern for the three known neutrinos. This seesaw scenario requires the solar neutrino deficit to involve the oscillations of all three neutrinos. Hence a much more extensive numerical analysis is needed[13] beyond that of the usual two-neutrino assumption[7]. It is interesting to note that with only two neutrinos, the adiabatic solution is not allowed by the combined solar data, but with three neutrinos, it becomes allowed if the mixing of the third neutrino with $\nu_e$ is large enough.

We have also shown that a simple $5 \times 5$ mass matrix with one large scale reduces naturally to the desired $3 \times 3 \mathcal{M}_\nu$ of Eq. (5). Furthermore, the intermediate $4 \times 4$ reduction predicts a fourth neutrino which mixes with $\nu_e$ and $\nu_\mu$ in such a way that $\nu_\mu \rightarrow \nu_e$ oscillations may occur just within the detection capability of the LSND experiment.

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References

[1] R. Davis, Jr. et al., Ann. Rev. Nucl. and Part. Sci. 39, 467 (1989).

[2] K.S. Hirata et al., Phys. Rev. Lett. 63, 16 (1989); 65, 1297 (1990); 66, 9 (1991).

[3] A.I. Abazov et al., Phys. Rev. Lett. 67, 3332 (1991).

[4] P. Anselmann et al., Phys. Lett. B314, 445 (1993); B327, 377 (1994).

[5] R. Becker-Szendy et al., Phys. Rev. D 46, 3720 (1992).

[6] K.S. Hirata et al., Phys. Lett. B280, 146 (1992); Y. Fukuda et al., ibid. B335, 237 (1994).

[7] See for example S.A. Bludman et al., Phys. Rev. D 47, 2220 (1993).

[8] See for example W. Frati et al., Phys. Rev. D 48, 1140 (1993).

[9] S. Weinberg, Ann. N.Y. Acad. Sci. 38, 185 (1977); see also E. Ma, Phys. Rev. D 43, R2761 (1991).

[10] M. Gell-Mann, P. Ramond, and R. Slansky, in Supergravity, Proceedings of the Workshop, Stony Brook, New York, 1979, edited by P. van Nieuwenhuizen and D.Z. Freedman (North-Holland, Amsterdam, 1979), p. 315; T. Yanagida, in Proceedings of the Workshop on the Unified Theory and the Baryon Number in the Universe, Tsukuba, Ibaraki, Japan, 1979, edited by O. Sawada and A. Sugamoto (KEK Report No. 79-18, Tsukuba, Japan, 1979).

[11] S.P. Mikheyev and A.Yu. Smirnov, Yad. Fiz. 42, 1441 (1985) [Sov. J. Nucl. Phys. 42, 913 (1985)]; Nuovo Cimento 9C, 17 (1986); L. Wolfenstein, Phys. Rev. D 17, 2369 (1978).
[12] T.K. Kuo and J. Pantaleone, Rev. Mod. Phys. 61, 937 (1989).

[13] D. Harley, T.K. Kuo and J. Pantaleone, Phys. Rev. D 47, 4059 (1993).

[14] Ch. Berger et al., Phys. Lett. B227, 489 (1989); B245, 305 (1990).

[15] G.S. Vidyakin et al., Sov. Phys. JETP 71 424 (1990); G. Zacek et al., Phys. Rev. D 34, 2621 (1986).

[16] L. Borodovsky et al., Phys. Rev. Lett. 68, 274 (1992).

[17] See for example T. Walker et al., Astrophys. J. 376, 51 (1991).

[18] See for example X. Shi et al., Phys. Rev. D 48, 2563 (1993).

[19] E. Ma, Phys. Rev. D 51 (Rapid Communication), in press.
Figure Captions

Figs. (1) Plots of $\Delta m^2_{12}$ versus $\sin^2 2\theta_{e2} \equiv 4|U_{e2}|^2(1 - |U_{e2}|^2)$ for various values of $\sin^2 2\theta$.

The solid contours surround the parameter region allowed by the solar neutrino data \[1, 4, 5, 6\] at 90% confidence level. The dashed line gives the prediction of Eq. (5) for a specific value of $\Delta m^2_{13}$ which explains the atmospheric neutrino deficit.

(a) $\sin^2 2\theta = 0.35$ and $\Delta m^2_{13} = 3.0 \times 10^{-2}$ eV$^2$.

(b) $\sin^2 2\theta = 0.50$ and $\Delta m^2_{13} = 1.0 \times 10^{-2}$ eV$^2$.

(c) $\sin^2 2\theta = 0.75$ and $\Delta m^2_{13} = 4.0 \times 10^{-3}$ eV$^2$.

Fig. (2) Plot of $\Delta m^2_{13}$ versus $\sin^2 2\theta$. The dashed contours surround the parameter region allowed by sub-GeV atmospheric neutrino measurements\[3\] but excluded by the Frejus data\[14\] and below constraints from reactor measurements\[15\]. The shaded regions are where Eq. (5) can satisfy the solar neutrino data; the left shaded region corresponds to the small-angle nonadiabatic solution and the right shaded region corresponds to the adiabatic solution. In between these two regions is a small area where it is not possible to satisfy the solar neutrino data.