Multi-robot spot-welding cell design: problem formalization and proposed architecture

Stefania Pellegrinelli, Nicola Pedrocchi, Lorenzo Molinari-Tosatti, Anath Fischer, Tullio Tolio

Abstract

The multi-robot cell design for car-body spot welding is faced by industry as a sequence of tasks, where researches are focused on issues of the problem as a whole. In authors’ knowledge, none work in literature have suggested any formalization for the complete process. This paper tries to bridges the gap proposing coherent process formalization, and presenting a corresponding innovative architecture for the automatic optimal cell design. Specifically, the formalization involves the identification and allocation of the resources in terms of a set of decisional variables (e.g. robot model/positioning/number, welding gun models/allocation/number, welding point allocation etc.); then, the design optimization process minimizes the investment costs granting the cycle time. The multi-loop optimization architecture integrates both new algorithms and existent procedures from different fields. Test-bed showing its feasibility is reported.

1. Introduction

Recently, multi-robot assembly systems have been largely employed in the industrial environment. In 2011, 36% of new robot installations worldwide were required by the automotive sector and mainly destined for welding applications [1]. This demand for welding robots and multi-robot cells for car-body spot welding is well explained by two different factors. First, the recent reduction in the automobile life cycle has led to the exploitation of robot flexible and reconfigurable systems that can be used to assembly several car-body models limiting system reconfiguration costs. Second, the competitiveness of the automotive market led to the obtaining of low cycle times through the employment of several robots working simultaneously on a car body and performing high number of welding operations in parallel.

Although the wide use of multi-robot cells has well answered automotive market demand, companies have had to face several issues: the need to shorten the system design time; the demand of high accuracy in motion plans; the influence of cell and system configurations on the motion plan; the need to reduce the ramp-up and the expensive on-line modifications of the part programs through plug and produce systems and extremely realistic simulations (i.e. virtual commissioning). In this context, the design of multi-robot systems plays a relevant role. Indeed, the robots number and model, the robot fixturing support structure, the welding guns number and models, the position and orientation of the robots in the cell, and the allocation of the welding guns and welding points to the robots influence the motion planning as well as the cell cycle time. Imprecisions in the cell design could lead to high complexity or infeasibility during robot motion planning.

From the literature point of view, the problem of multi-robot cell design for spot welding applications has been widely discussed over the last decades. Two kinds of approaches can be identified: (i) approaches addressing technical issues but not
covering the complexity of the cell design process [2-5] and (ii) more general approaches able to provide several design alternatives but disregarding or partially considering relevant technical aspects specifically related to the design of multi-
robot spot-welding cells [8-10].
Nakamura et al. [2] proposes an approach for a single-robot spot welding cell design in terms of robot and gun model selection, robot installation position. They take into account the equipment versatility, while neglecting the welding-point allocation problem. Similarly, approaches [3-5] focus on the robot model selection and the robot positioning optimization without addressing the complete problem formalization.

Sarkans and Roosimölder [6,7] develop an information platform to facilitate the introduction of robot cells in small and medium enterprises. The user generates cell alternatives guided by seven predefined design steps: product analysis, system design, simulation, facilities, installation, jig and programs; and to compare the alternatives from the economic point of view (return of investments). However, the paper addressed some aspects of the cell-design process without entering technical details necessary for the formalization of the problem. Michalos et al. [8] propose the generation of alternative design solution on the basis of product specifications and in terms of possible technologies, layouts, resources and operations (investment costs, cell availability, resource reutilization, flexibility and mean annual production volume are calculated for each solution). The best solution is selected by the employment of a normalization criterion that takes into account all the listed criteria. Although the presented methodology was successfully applied to an automotive case, it does not focus on multi-robot cells for spot welding, thus not taking into account the welding point allocation problems and the influence of the system design on motion planning. The motion planning and collision problems are partially considered in [9,10]. Starting from an initial and rough solution, the approach leads to the definition of a final collision-free solution with optimized cycle time. However, robot path is investigated through the analysis of the robot tool center point instead of the whole robot structure. Therefore, only collisions between the robot end effector and the upper part of the floor and the part are analyzed. Moreover, the approach considers the robot position in the cell, thus disregarding the robot orientation.

A relevant approach for the assembly of all the lower and upper body of the car through multiple welding robots is presented in [11]. However, this paper as well as all the described papers does not provide a detailed and complete formalization of the design problem for the specific application field of multi-robot spot-welding cells while taking into account the problem of spot welding allocation as well as the influence of the design on the motion planning. Thus, the proposed paper aims at exhaustively formalizing the problem and, successively, to propose a new approach. The formalization as well as the proposed approach is the result of several interviews to the robot manufacturer COMAU.

The paper is structured as follows: in Section 2 the problem formalization is described; in Section 3 and 4 the proposed approach is depicted; Section 5 presents an ad-hoc test-bed showing the approach feasibility; finally, Section 6 drives conclusions and future works.

2. Formalization of multi-robot cell design problem

Multi-robot manufacturing cells for spot-welding are characterized by different robots working at the same time on a single car body that is handled by a transporter or a robot. The car body is generally composed by two or more metal-sheet components that are blocked during the welding process by ad-hoc fixtures. The problem of multi-robot cell design for spot welding consists of determining the main structure of the cell in terms of robots, welding guns and auxiliary devices, such as the car-body fixture and the robot support structure for the robot positioning and orientation. The goal is to find out the best compromise among productivity, costs, flexibility and reconfigurability [12]. For sake of simplicity, the following hypothesis are introduced: (i) the transportation device is not under study; (ii) the cell manages only one type of body car; (iii) welding gun changes are neglected; (iv) the methodology represents a compromise between productivity and costs.

2.1. Client and Supplier Inputs

The cell design inputs defined by the client are the car-body metal sheets (CB), the welding points (WPs), the car body fixture (BF) and the cell cycle time (RCT). The design of the fixture, that strictly influences the final quality of the body, is generally provided by the user together with the product specifications (localization of the welding points; fixture composed by different elements; clamps the metal sheets that have to be welded). Together with the client’s inputs, the following inputs have to be considered: the robot support structure models (RSMs), the robot models (RMs), the welding gun models (WGMs). These inputs can be selected among all the resources available on the market.

2.2. Decision Variables

The cell is configured when the robot support structure model (SRSM), the robot model and number (SRM, TNR), the welding gun models and number (TNWGWGM), the position and orientation of the robots in the cell (RPO), the allocation of the welding gun models (RGFWGM,RPO) and welding points (WGRPO,WP) to the robots are determined. Robot support structure models generally have a double configuration: cell or ground. Cell and ground support structure allow a different robot positioning: in cell grids, robots can be mounted in every position according to the space discretization; on the contrary, in ground grid, the welding part is generally held in the grid center allowing the robot to be placed on the structure boundaries. The employment of robots belonging to a same family or, at least, to a same producer is an utmost aspect in cell design. Indeed, synchronization among devices of different family or producers still represents a huge technical problem. According to the industrial practice, only one robot model is mounted per assembly cell. On the contrary, different welding guns can be selected considering the accessibility granted by a specific gun and the influence of the gun on the welding process.
2.3. Decision Criteria

Cell design can be led according to different criteria such as investment costs. In order to correctly evaluate the investment costs, it is necessary to consider the already available resources (NCRSM, NCRSRO, NCWGWM), i.e. the resources the company has already acquired and that can be employed in the new assembly cell. Thus, the number of resources to be acquired (NAR, NARS, NAWGWM) is evaluated as the total number of the resources (TNR, TNWGWM) minus the number of available resources taking into account the selected robot model and support structure.

However, whichever is the optimization criterion, it is necessary to cope during cell design with the required cell cycle time (RCT). When evaluating the cell cycle time (OCTBPO, MAXOCT) and the production rate, it is necessary to consider: (i) the availability of the resources in terms of welding guns and robots availability (aWGMWM, aRM); (ii) the welding time (WTWP) and the time associated to the motion plan (MPWGM, BPO, WPWP). i.e. the time necessary to the robots to reach one welding point from their current position (MTWGM, BPO, WPWP). In case the required cell cycle time is granted, it is possible to evaluate this underperformance of the system design (MCT) in terms of cost introducing a penalty cost (PC) for each lost time unit.

The problem formalization is resumed in Table A.1.

3. Method and implementation

The proposed approach exploits the problem formalization depicted in the previous section in order to optimize the spot-welding system configuration while defining the allocation of the welding points to the robots and taking into account the possible robot motion plans. The approach is based on 4 steps (Fig. 1). The approach represents an evolution of [13]. In comparison to [13], the proposed approach considers as output variables: (i) the orientations of the robots in the cell; (ii) the orientations of the welding guns around the Z axis of the welding point (sliding direction of the welding).

Step 1 selects a feasible couple <RMRM, RSM>, among all the available inputs robot models and robot support structure models. The couple is then the input for step 2 and step 3. Thus, SRM and SRSM presented in Table A.1 are initialized. The methodology does not depend on the particular shape of the transporter system, car body fixture and car body metal sheets. The 3D representations (STL files) of these elements are required for collision detection.

Step 2 defines a motion plan MPWGM, BPO, WPWP, for each possible <BPO, WGM> for the inputs <RMRM, RSM>. The motion plan describes all the couples <WP1, WP2> of reachable and admissible WPs, the collision-free trajectory for moving from WP1 to WP2 and the time associated to the trajectory. Motion plans are defined according to probabilistic roadmaps technique [14-17] where roadmap generation is based on Halton point technique to sample the joint space [18] and nearest-n technique to connect the sampled points [18], while roadmap queries employ the Dijkstra’s algorithm [19]. Collision-free trajectories are defined through the Open Realistic Library (ORL) that is the robot motion planner module of COMAU controllers [20] and the “RAPID” library [21] for collision detection.

Step 3 considers all the motion plans MPWGM, BPO, WPWP and the inputs in Table A.1 in order to identify the optimized cell design on the basis of an investment cost minimization criterion. Step 3 is widely described in Section 4.

All the solutions provided by step 1, 2 and 3 according to different RS and RSM inputs are evaluated in step 4 according the several criteria such as investment costs, return of investments, flexibility and cycle time.

Fig. 1. The approach: it is a four-step method; the input variables and the decision variable for step 1 and 2 are reported. Their meaning is in Table A.1

4. Multi-robot spot-welding cell design

Step 3 aims at defining the optimal cell design considering motion plans MPWGM, BPO, WPWP. Design stands on a mathematical model described as a MIP (Mixed Integer Programming) in terms of parameters, variables, objective functions and constraints.

4.1. Decision Variables and Criteria

In comparison to the problem formalization presented in Table A.1 three modifications are introduced as shown in Table A.2: (i) some output variables defined during Step 2 become an input parameter for Step 3; (ii) some input parameters and some output variables are added in order to introduce into the model the outputs of the stage 2; (iii) the input data WPCOV is introduced in order to reduce the complexity of the problem. The user can define the precision on the final solution: if WPCOV is equal to 1, the allocation of the welding point will be the final one; if WPCOV is greater than 1 the solutions still has some degrees of freedom in the allocation of some welding points to be exploited when coordinating the robots. The objective function of the model is presented in (1). The equation terms are: the cost of the acquired resources, i.e. COSTRM, COSTWGMWM and COSTRS, and a penalty cost PC whenever the provided solution exceeds the cycle time.

\[
\min \left\{ \text{NAR} \cdot \text{COSTRM} + \sum_{\text{WGM}} \left( \text{NAWGWM} \cdot \text{COSTWGMWM} \right) + \text{NAR} \cdot \text{COSTRS} + \text{PC} \cdot \text{MCT} \right\}
\]
Above issues on the multi-robot design approach bring to 26 constraint equations divided into 3 groups: resource constraints, motion planning constraints and cycle time constraints. For sake of brevity, the most relevant constraints are presented hereafter.

4.2. Resource constraints

9 resource constraints define the number of robots, guns and support structure to be acquired as well as the position of the robot in the cell and the allocation of the welding guns to the robots. Specifically, constraints (2)-(6) define the number and type of acquired robots, welding guns and support structures. Acquired resources are represented as the difference between the number of resources estimated by the final cell configuration minus the number of resources already owned by the company. Constraint (7) imposes that the total number of selected guns is equal to the total number of robots, so that each robot mounts one gun. Similarly, constraint (8) defines equality between the number of robots in the assembly cell and the number of selected robots. Moreover, since different gun models can be selected, constraint (9) identifies the number of selected guns for each model. Constraint (10) prevents the selection of two robots in the same position.

\[
\begin{align*}
SNR &= SNR - NCR \\
NAR &\geq SNR \\
SNWG_{WGM} &= TNWG_{WGM} - NCWG_{WGM} \quad \forall WGM \\
TNWG_{WGM} &\geq SNWG_{WGM} \quad \forall WGM \\
NARS &= 1 - NCRS \\
\sum_{WGM} TNWG_{WGM} &= TNR \\
\sum_{WGM} RGP_{WGM, RPO} &\leq TNR \\
\sum_{RPO} RGP_{WGM, RPO} &= TNWG_{WGM} \quad \forall WGM \\
\left( \sum_{WGM} RGP_{WGM, RPO} + \sum_{WGM} RGP_{WGM, ARO2} \right) RGP_{WGM, ARO2} &\leq 1 \\
&\quad \forall RPO, RPO2
\end{align*}
\]

4.3. Motion planning constraints

13 constraints grant the coherence among the variable related to the robot motion plan. 5 of the 13 constraints are described in the followings. Specifically, constraints (11) and (12) state that for each robot position/orientation to which a welding point is associated, a welding gun has to be selected. Thus, each welding point is indirectly related to a specific gun. Constraint (13) defines the maximal number of robot positions/orientations to which the same welding point can be associated. Constraint (14) defines the motion plan taking into account the information related to existing collision-free trajectory between two welding points: only collision-free trajectories can be selected in order to compose the final motion plan. Constraint (15) allows the temporization of the motion plan for each position/orientation: thus, for each robot the sequence of execution of the welding points is given.

\[
\begin{align*}
\sum_{WGM} WPA_{WGM, WP} &\leq L \sum_{WGM} RGP_{WGM, RPO} \quad \forall RPO \\
\sum_{WGM} WPA_{WGM, WP} &\geq 1 \quad \forall WGM \\
\sum_{WGM} WPA_{RPO, WP} &\leq WPCOV \quad \forall WP \neq 0 \\
MP_{WGM, RPO, WP1, WP2} &\leq MPf_{WGM, RPO, WP1, WP2} \\
&\quad \forall WGM, RPO, WP1, WP2 \\
\sum_{WGM} MPS_{WGM, RPO, WP1, WP2} &= MPS_{WGM, RPO, WP1, WP2} \\
\sum_{WGM} MPS_{WGM, RPO, WP1} &= WPA_{RPO, WP} \\
&\quad \forall RPO, WP \neq 0
\end{align*}
\]

4.4. Cycle time constraints

4 constraints evaluate the cell cycle time. Constraint (16) evaluates the cycle time for each robot, while constraint (17) defines the cycle time of the manufacturing cell. Besides, constraints (18) and (19) calculate, if existing, the lacking cycle time that is the positive difference between the obtained cycle time and the required cycle time.

\[
\begin{align*}
\alpha &\cdot \sum_{WGM} \left( MT_{WGM, RPO, WP1, WP2} + MP_{WGM, RPO, WP1, WP2} \right) + \\
\sum_{WGM} \left( MP_{WGM, RPO, WP1, WP2} + WT_{WP} - a WGM_{WGM} \right) = OCT_{RPO} \\
&\quad \forall RPO \\
OCT_{RPO} &\leq MAXOCT \quad \forall RPO \\
SCT &= MAXOCT - RCT \\
MCT &\geq SCT
\end{align*}
\]

5. Approach validation

The validation of the approach is based on a test-bed case. The goal is to show the feasibility of the proposed approach. The considered inputs are described in Table 1. For sake of brevity, only one robot model and one robot structure are considered in input selection (stage 1). During stage 2, 16 roadmaps (NRPOxNWGM=8x2) are built to define the trajectories between the reachable WPs. These roadmaps are based on 150 sampled points. Each point is connected, if possible, to the 15 nearest sampled points. The results of stage 2 are synthetized into the variables MT, MP and MPf. Collision check during robot generation is at 10 Hz. Fig. 3 depicts an example of a roadmap generation and a query. The solution generated by step 3 consists in 3 robots in RPO 1, 3 and 5 (Fig. 4). Robots in RPO 1 and 3 mount the welding gun model 2, while robot in RPO 5 mounts the welding gun model 1. The total cost of the cell is 336,000€. All the welding points are allocated to at least one robot; welding point 1, 2 and 9 are allocated to 2 robots.

Stage 2 of the problem is solved through an ad-hoc software developed in C++ and partially integrating existing open-
source library. Stage 3 exploits a commercial software for the resolution of the proposed mathematical model. Specifically, stage 3 problem is solved through a dynamic search algorithm, while root and node relaxation are solved through simplex algorithm. The resolution emphasis is balanced between optimization and feasibility. The tolerance on the final solution is 0.01%. The resolution of the problem on a 2.66GHz processor laptop requires around 2 hours for Stage 2 and a few seconds for Stage 3.

6. Conclusion and future work

The paper presents a complete formalization for the design of multi-robot spot-welding cells. The formalization can be exploited in order to develop new and exhaustive approaches for the resolution of the problem. The proposed approach optimize the cell design by minimizing the investment costs.

Table 1. Test case inputs.

| NWP | 13 |
|-----|----|
| WP  | 1500.57 -181.52 31.02 -58.26 148.37 -126.83 |
|     | 441.63 -1449.19 -770.59 +134.71 +164.12 +131.02 |
|     | -659.36 -784.45 +357.2 -156.36 +92.54 +77.26 |
|     | 601.99 -1299.56 1165.64 -139.71 108.49 -80.62 |
|     | 97.33 +1802.31 +2403.67 +68.7 +35.56 +70.39 |
|     | -689.36 -784.45 +257.2 -156.36 +92.54 +77.26 |
|     | 2698.07 +327.73 +966.75 -87.685181 +96.95 +159.24 |
|     | -1592.62 -767.12 -191.64 +149.63 +163.7 +149.38 |
|     | 1201.99 -1098.56 195.64 -59.71 108.49 -60.62 |
|     | 601.99 -1098.56 1165.64 -139.71 108.49 -80.62 |
|     | -689.36 -1284.45 +257.2 -166.36 +92.54 +77.26 |
|     | -689.36 -1284.45 +657.2 -166.36 +92.54 +77.26 |
|     | -689.36 -1184.45 +257.2 -166.36 +92.54 +77.26 |
| BF  | 3 parallelepipeds |
| SRM | 1 |
| RM  | COMAU SMART-5-NJ4-175-2.2 |
| SRSM | 1 |
| RSM | Ground support structure |
| NRPO | 8 |
| RPO | 4 possible positions with respectively 2, 1, 2 and 3 possible orientations. (Fig. 2). |
| NWGM | 2 |
| WGM | VCGCA1300001001; VCGSG0000001001 |
| NCR | [0] |
| NCWG | [0,0] |
| NCRS | [0] |
| aWGMc | [0.98 0.95] |
| aRM | [0.95] |
| WTP | [3 3 3 3 3 3 3 3 3 3 3 3 3] |
| RCT | 120 |
| COSTRMext | [100000] |
| COSTWGMext | [1000 800] |
| PC | 1000000 |
| WPCOV | 2 |
| IGP | [0 1 0 0 0 0 0; 1 0 0 0 0 0 0; 0 0 1 0 0 0 0; 0 1 0 0 0 0 0 1; 0 0 0 0 0 0 0 0; 1 0 0 0 0 0 0 0; 0 1 0 0 0 0 0 0; 0 1 1 0] |

The robot number and model, the robot support structure and the position and orientation of the robot in the cell are selected, while the welding gun models and the welding points are allocated to the robots. Thus, the influence of the motion planning is considered. Future work concerns the modification of the model in order to reduce the intersection between the working areas of the robots so that the probability of collision during robot cooperation is decreased. Moreover, the approach will be tested on real industrial cases.

Fig. 2. Examples of robot initial configuration according to different RPO and WGM: (a) RPO 1, WGM 1; (b) RPO 3, WGM 2; (c) RPO 5, WGM 1; (d) RPO 6, WGM 2

Fig. 3. Examples of (a) roadmap generation for RPO 6 and WGM 1; (b) roadmap query for RPO 6, WGM 1 from robot initial position to WP4

Fig. 4. Proposed cell design

Appendix A. Inputs and outputs

Table A.1 presents the input parameters and output variables necessary to formalize the problem of the multi-robot spot-welding cell design. Table A.2 introduces a modification to Table A.1 in order to cope with the proposed approach.
Table A.1. Problem formalization.

| INPUT PARAMETERS | OUTPUT |
|------------------|--------|
| BF | SNR < N: Support variable equal to 0 if TNR-NVR=0; otherwise equal to TNR-NVR. |
| CR | SNWGWGM < N: Support variable equal to 0 if TNGWGM-NWGGM=0; otherwise equal to TNGWGM-NWGGM. |
| WPI(s) | SCT < R: Support variable representing the difference between the obtained and the required Cycle Time. |
| RM(s) | |

Table A.2. Additional inputs and outputs. * indicates a modification in comparison to the inputs and outputs of Table 1; ° indicates an output variable of Table 1 become an input parameters.

| INPUT |
|-------|
| ° SRSM < N: Index of the selected RSM |
| ° SRM < N: Index of the selected RM |
| ° NCR < R: Motion Time according to MPWGM,RPO,WP1,WP2 | |
| ° NCR < R: Motion Time according to MPWGM,RPO,WP1,WP2 | |
| ° NCR < R: Motion Time according to MPWGM,RPO,WP1,WP2 | |
| ° NCR < R: Motion Time according to MPWGM,RPO,WP1,WP2 | |
| ° NCR < R: Motion Time according to MPWGM,RPO,WP1,WP2 | |
| ° NCR < R: Motion Time according to MPWGM,RPO,WP1,WP2 | |
| ° NCR < R: Motion Time according to MPWGM,RPO,WP1,WP2 | |
| ° NCR < R: Motion Time according to MPWGM,RPO,WP1,WP2 | |
| ° NCR < R: Motion Time according to MPWGM,RPO,WP1,WP2 | |
| ° NCR < R: Motion Time according to MPWGM,RPO,WP1,WP2 | |
| ° NCR < R: Motion Time according to MPWGM,RPO,WP1,WP2 | |
| ° NCR < R: Motion Time according to MPWGM,RPO,WP1,WP2 | |
| ° NCR < R: Motion Time according to MPWGM,RPO,WP1,WP2 | |
| ° NCR < R: Motion Time according to MPWGM,RPO,WP1,WP2 | |
| ° NCR < R: Motion Time according to MPWGM,RPO,WP1,WP2 | |

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