Noise-induced vortex reversal of self-propelled particles

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We report an interesting phenomenon of noise-induced vortex reversal in a two-dimensional system of self-propelled particles (SPP) with soft-core interactions. With the aid of forward flux sampling, we analyze the configurations along the reversal pathway and thus identify the mechanism of vortex reversal. We find that statistically the reversal exhibits a hierarchical process: those particles at the periphery first change their motion directions, and then more inner layers of particles reverse later on. Furthermore, we calculate the dependence of the average reversal rate on noise intensity $D$ and the number $N$ of SPP. We find that the rate decreases exponentially with the reciprocal of $D$. Interestingly, the rate varies nonmonotonically with $N$ and a minimal rate exists for an intermediate value of $N$.

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In recent years, the collective dynamics of self-propelled particles (SPP) has been a subject of intense research due to the potential implications in biology, physics, and engineering (see [1] for a recent review). Examples of SPP are abundant including traffic flow [2], birds flocks [3, 4], insects swarms [5, 6], bacteria colonies [7, 8], and active granular media [9, 10], just to list a few. Especially, inspired by the seminal work of Vicsek et al. [11], many theoretical and experimental studies reported that various systems of SPP can exhibit a wealth of emergent nonequilibrium patterns like swarming, clustering and vortex [12–18].

A fascinating phenomenon about SPP is that they can abruptly change their collective motion pattern, which may be induced by either intrinsic stochasticity such as error in communication among SPP or a response to an external influence such as a predator. For example, a recent experiment showed that marching locusts can suddenly switch their direction without any change in the external environment [13]. Later, the experimental results are further explained theoretically by a mathematical modeling, highlighting the nontrivial role of randomness or noise on this transition [19]. Also, noise-induced transitions between translational motion and rotational motion of SPP have been observed [20, 21]. However, the research on this topic is still in its infancy and deserves more investigations. In particular, investigation of the mechanisms about these transitions is lacking at present. As we know, identifying the underlying mechanisms is key to understanding and controlling the collective motion of SPP.

In this paper, we report an interesting phenomenon of noise-induced vortex reversal between two different rotational directions in a two-dimensional model of SPP interacting via Morse potential [17, 18]. Vortex pattern has been commonly observed in nature, such as fish, ants [24, 25], Bacillus subtilis [26], and Dictyostelium cells [27]. By virtue of a recently developed simulation method of rare event, forward flux sampling (FFS) [28], we analyze the intermediate configurations along the reversal path and compute average reversal rate. We find that an important statistical property of the vortex reversal, that is, the reversal first starts from peripheral particles and then gradually to inner particles, so that almost all particles change their rational directions. Furthermore, we show that the reversal rate decreases exponentially with the inverse of noise intensity. Interestingly, the rate varies nonmonotonically with the number of particles and a minimal rate exists.

We consider $N$ identical SPP in two-dimensional space with positions $\vec{x}_i$, velocities $\vec{v}_i$ ($i = 1, \ldots, N$) and unit mass. The equations of motion read

$$\dot{\vec{x}}_i = \vec{v}_i, \quad \dot{\vec{v}}_i = \left(\alpha - \beta |\vec{v}_i|^2\right)\vec{v}_i - \sum_{j \neq i} \nabla U(|\vec{x}_i - \vec{x}_j|) + \sqrt{2D}\xi_i,$$

where the first and second terms on right hand side (rhs) of Eq.(2) represent self-propelled force and friction force, respectively, and the third term is pair interaction among SPP given by the generalized Morse potential

$$U(|\vec{x}_i - \vec{x}_j|) = C_e e^{-|\vec{x}_i - \vec{x}_j|/l_r} - C_a e^{-|\vec{x}_i - \vec{x}_j|/l_a}.$$  

Here, $l_a$ and $l_r$ represent the attractive and repulsive potential ranges, $C_a$ and $C_r$ represent their respective amplitudes. The last term on rhs of Eq.(2) is a stochastic force of intensity $D$ that are independent of particle index and satisfy $\langle \xi_{i,m}(t) \rangle = 0$ and $\langle \xi_{i,m}(t)\xi_{j,n}(t') \rangle = \delta_{ij}\delta_{mn}\delta(t - t')$ with $i,j \in 1, \ldots, N; m,n \in x,y$.

The model exhibits diverse dynamic patterns, such as clumps, rings and vortex, for different model parameters. Here we set $\alpha = 1.0$, $\beta = 0.5$, $l_a = 2.0$, $l_r = 0.5$, $C_a = 0.6$ and $C_r = 1.0$, which corresponds to the case of vortex. To distinguish two different rotational directions of vortex, we define average angular momentum of particles.
In Fig. 2, we show the velocity field of SPP at six different interfaces. Fig. 2(a) and Fig. 2(f) show the velocity field before and after the reversal, indicating that the system is in stable vortex with CW and CCW rotation, respectively. While Figs. 2(b-e) show the intermediate processes of the reversal. One can clearly observe that along the pathway of vortex reversal the velocity field gradually changes its sign from the periphery to center of the vortex. That is to say, vortex reversal first starts from peripheral particles and then gradually to inner particles, and finally almost all particles change original rational directions.

Further information on pathway of vortex reversal is provided by the radial distributions of average angular momentum \( \vec{L}(r) \) and average angular velocity \( \omega(r) \), where \( r \) is the distance to the center of mass. In Fig. 3 we plot \( \vec{L}(r) \) and \( \omega(r) \) for five different interfaces along the pathway of vortex reversal. From the variations of \( \vec{L}(r) \) and \( \omega(r) \) with interfaces one can observe that the whole process of the vortex reversal. Before the reversal \( \vec{L} = -1.95 \) the values of \( \vec{L}(r) \) and \( \omega(r) \) are always negative, irrespectively of \( r \). That is, the system is in stable vortex with CW rotation before the reversal. When the reversal happens, for example, for \( \vec{L} = 0 \) the values of \( \vec{L}(r) \) and \( \omega(r) \) become positive for \( r > 1.8 \), while for \( r < 1.8 \) they are always negative. With increasing \( \vec{L} \) this situation further goes till any values of \( \vec{L}(r) \) and \( \omega(r) \) become positive. Finally, the vortex rotates with CCW direction. Therefore, this further validate that the statistical property of vortex reversal is that the reversal process starts from the periphery of the vortex. Also, we calculate the radial distributions of density of particles and the absolute value of velocity of particles, and find that they do not have significant difference with the interfaces \( \vec{L} \).

It is informative to analyze the statistical properties
of critical configurations of SPP. Similar to definition in previous studies [30], the critical nucleus is determined by the committor probability $P_B(i) = 0.5$, where $P_B(i)$ is the probability of reaching $B$ state before returning to $A$ state starting from the FFS interface $i$, which can be computed by FFS sampling. This shows that when the system initially locates at the critical configurations there is equal probability of returning to CW vortex or CCW vortex. We find that $\bar{L} \simeq 0.08$ at the critical con-

![FIG. 3: (Color online) Radial distributions of average angular momentum $\bar{L}(r)$ and average angular velocity $\omega(r)$, for five different interfaces along the pathway of vortex reversal.](image)

![FIG. 4: (Color online) The distributions of angular momentum $\bar{L}_i$ near the critical configurations for these particles with $r \in [0.6, 1.0]$ (squares), $r \in [1.6, 2.0]$ (circles), and $r \in [2.6, 3.0]$ (triangles).](image)

![FIG. 5: (Color online) The natural logarithm of rate of vortex reversal $\ln R$ as a function of the inverse of noise intensity $1/D$ for different SPP number $N$ (a) and as a function of $N$ for different $D$ (b). The dashed lines in (a) are plotted by linear fitting.](image)
destroyed, and thus vertex reversal makes no sense. If $D$ is too small, $R$ is very low so that simulation will be time-consuming. In Fig.\ref{fig:5}a we show that ln $R$ as a function of $N$ for different $D$. Interestingly, we find that $R$ varies nonmonotonically with $N$. There exists a minimal value of $R$ at $N = 48$ for low $D$ and at $N = 50$ for high $D$. From Fig.\ref{fig:5}a), one may speculate that there seems to be an effective nonequilibrium potential for describing the collective transition in rotational directions. The potential barrier between CW vortex and CCW vortex is fixed if $N$ is unchanged, such that the transition rate follow classical Kramers’ law. On the other hand, the nonmonotonic dependence on $R \sim N$ implies dependence of the effective nonequilibrium potential on $N$ is nontrivial if exists. However, understanding these results from the view of theoretical analysis, if not infeasible, is at least a complex task at present.

In summary, using a two-dimensional model of SPP interacting via a soft-core potential, we have investigated the mechanism of noise-induced the changes of vortex pattern in rational direction. By virtue of FFS method we analyze the statistical property and compute the rate of the reversal. We find that the reversal process is hierarchical: the process initially inspired by the peripheral particles, and those particles gradually drive more inner layers of particles into reverse motion directions. On the other hand, we show that the rate of the reversal decreases exponentially with the inverse of noise intensity. Interestingly, the reversal rate depends nonmonotonically on the number of SPP and a minimal rate exists at a moderate number of particles. Our findings may provide us some new understanding on the transitions of collective patterns of SPP.

Acknowledgments

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