Determination of vector meson properties by matching resonance saturation to a constituent quark model

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Abstract. The properties of mesonic resonances can be calculated in terms of the low-energy coefficients of chiral perturbation theory ($\chi$PT) by extending unitarized $\chi$PT to higher energies. On the other hand these low-energy coefficients can be calculated in two different models, namely (i) by assuming resonance saturation and (ii) within a constituent quark model. By matching the expressions of the two models combined with the results of unitarized $\chi$PT and the Weinberg sum rules the properties of vector and axial-vector mesons can be calculated in the combined large-$N_c$ and chiral limit.

PACS. 12.39.Fe Phenomenological quark models; chiral Lagrangians – 14.40.Cs Properties of specific particles; other mesons with $S=C=0$, mass $< 2.5$ GeV

1 Introduction

What determines the properties of hadrons made from light quarks, chiral symmetry breaking ($\chi$SB) and/or confinement? It is nowadays common wisdom that the mechanism of $\chi$SB causes large constituent quark masses of the order of 300 – 400 MeV. Hence even without confinement the creation of a quark-antiquark pair is rather expensive. Therefore the role of confinement for the description of light hadrons is at least diminished by the appearance of $\chi$SB. This suggests that the properties of light hadrons are quantitatively determined by the effect of $\chi$SB. In such a scenario confinement enters only qualitatively by excluding non-white states and quark-antiquark thresholds. It is well known that such a picture works very well for pions (e.g. [2] and references therein). It is the purpose of the present work to apply that picture to $\rho$- and $a_1$-mesons. One reason why one does not need confinement to describe the properties of pions can be found in the fact that the mass of these quasi-Goldstone bosons is much below the (constituent!) quark-antiquark threshold. This is of course different for other types of mesons. At first glance it seems that this messes up the line of reasoning given above. The point however is that e.g. $\rho$-mesons leave a trace also in the low-energy region much below their pole mass by mediating e.g. pion-pion interactions [3]. Hence the key idea is that on the one hand side ($\chi$SB aspect) one can describe the low-energy region reliably by a (chiral!) quark model (without confinement) — as this region is far away from the quark-antiquark production threshold. On the other hand side (confinement aspect) the mesonic resonances are supposed to mediate the interactions in this low-energy regime. By matching corresponding expressions it should be possible to determine masses and coupling constants of mesonic resonances in terms of quark model expressions. This procedure is depicted schematically in Fig. 1. For simplicity I work in the following in the combined large-$N_c$ and chiral limit.\footnote{To be specific I take the large-$N_c$ limit first, i.e. neglect the chiral log’s which are suppressed by $1/N_c$.}

2 Chiral perturbation theory and unitarization

At low energies QCD reduces to an effective theory where only the lightest mesons — the pseudoscalar Goldstone bosons — appear which interact with each other and with external sources. $\chi$SB demands that the meson interaction vanishes with vanishing energy. Therefore a systematic expansion in terms of the derivatives of the meson fields is possible. These considerations lead to the effective lagrangian of chiral perturbation theory ($\chi$PT) [4]:

$$L_{\chi\text{PT}} = L_1 + L_2 + \text{higher order derivatives} \quad (1)$$

Fig. 1. Schematic view of the resonance saturation model (a) and the constituent quark model (c) and their respective low-energy reduction to $\chi$PT (b). The dashed lines denote Goldstone bosons, the double line mesonic resonances and the full lines quarks.
with
\[ L_1 = \frac{1}{4} F_\pi^2 \text{tr}(\nabla_\mu U^\dagger \nabla^\mu U) + \ldots , \]
\[ L_2 = L_1 (\nabla_\mu U^\dagger \nabla^\mu U)^2 + L_2 (\nabla_\mu U^\dagger \nabla_\nu U) (\nabla^\mu U^\dagger \nabla^\nu U) \]
\[ + L_3 (\nabla_\mu U^\dagger \nabla^\nu U \nabla_\nu U^\dagger \nabla^\mu U) - i L_9 \left[ F_{\mu\nu}^\pi \nabla^\mu U^\dagger \nabla^\nu U^\dagger + F_{\mu\nu}^1 \nabla_\mu U \nabla_\nu U^\dagger \nabla^\mu U^\dagger \nabla^\nu U \right] + \ldots \quad (3) \]
where I have only displayed the terms which are relevant for later use. In \( U \) the pseudoscalar meson fields are encoded. \( F_{\mu\nu}^\pi \) denotes the field strength which corresponds to (chirally covariant combinations of) external vector fields \( v_\mu \) and axial-vector fields \( a_\mu \). \( F_\pi \) denotes the pion decay constant (in the chiral limit). I refer to \[ \text{Fig. 1b) for further details. The four-point meson interaction induced by (3) is depicted schematically in Fig. 1b.} \]
As it stands the effective theory \[ \text{is valid at low energies only. Especially unitarity is not fulfilled. In Sec. 5 the inverse amplitude method (IAM) is used to unitarize the effective theory and extend its applicability to the mesonic resonance region. The IAM is very well suited to recover a resonance from its trace left at low energies} \[ \text{and chiral limit of the results presented in Sec. 5. It is easy to show that in this limit the mass of the} \]
\[ M^2_V = -\frac{F_\pi^2}{4 L_3} . \quad (4) \]

3 Chiral constituent quark model

As already pointed out it should be reasonable to calculate the coefficients of the effective theory \[ \text{from a chiral constituent quark model as the low-energy region is}\] (much) below the quark-antiquark production threshold. In the following I use the quark-Goldstone boson lagrangian (in euclidean space)
\[ \mathcal{L}_\text{quark} = \bar{q} \left( \gamma_\mu \partial_\mu - M U^{\gamma_5} + \gamma_\mu v_\mu + \gamma_\mu \gamma_5 a_\mu + \ldots \right) q \quad (5) \]
This lagrangian can be motivated in several ways (e.g. \[ 8 \text{ and 11). I would like to stress that it is also the simplest model which one can write down which couples quarks to the Goldstone bosons of } \chi_\text{SB}. The latter are encoded in
\[ U^{\gamma_5} = \frac{1 - \gamma_5}{2} U + \frac{1 + \gamma_5}{2} U^\dagger . \quad (6) \]
\( M \) denotes the mass of the constituent quark. The dots in \[ 8 \text{ denote further couplings to external sources besides the displayed ones for vector and axial-vector fields. By integrating out the quarks, expanding the obtained effective action in terms of meson field derivatives and finally transforming the result to Minkowski space one arrives at the } \chi_\text{PT lagrangian} \[ \text{with predictions for the low-energy constants. This procedure is shown in Fig. 1c as the} \]

4 Resonance saturation model

Assuming that (only) resonances mediate the low-energy interactions of \[ \text{one can write down a lagrangian which couples resonances to Goldstone bosons} \[ \text{Here I concentrate on the } \rho \text{-meson and its interaction lagrangian}\]
\[ \mathcal{L}_\text{int} = \frac{F_V}{2 \sqrt{2}} \text{tr}(V_{\mu\nu} f_\mu^\nu) + \frac{g_V}{2 \sqrt{2}} \text{tr}(V_{\mu\nu} u^\mu u^\nu) \quad (8) \]
where basically \( u_\mu \) is obtained from \( U \), i.e. contains the pseudoscalar fields, while \( f_\mu^\nu \) contains the external vector fields (see 10 for details). \( V_{\mu\nu} \) denotes the meson resonance in the tensor representation. Note that one does not assume here that the vector meson couples with the same strength to the external vector field (photon) as it couples to the pseudoscalars (e.g. pions). To phrase it differently, universality of the \( \rho \)-meson coupling is not an input of the resonance saturation model. As I will show below, however, one gains the universality as an output of my approach. Integrating out the resonance fields one obtains \[ 10 \text{ with predictions for the low-energy coefficients} \]
\[ L_2^\text{res} = \frac{G_\pi^2}{4 M_V^2} , \quad L_9^\text{res} = \frac{F_V G_V}{2 M_V^2} . \quad (9) \]
Note that all the other low-energy constants are additionally influenced by the exchange of mesons with different quantum numbers.

5 Results from matching

In the last sections I have basically collected results from the literature. The new aspect is now that the results from the approaches with hadronic degrees of freedom (Secs. 2 and 11) are matched to the quark model calculations of Sec. 3. As already pointed out in the introduction the idea behind that matching is that on the one hand side the chiral quark model is supposed to give reliable results in the low-energy regime. On the other hand side confinement enforces the formation of resonances (instead of the production of free quarks and antiquarks). These resonances
however are also visible at low energies, i.e. determine the low-energy structure of the strong interaction. From the matching procedure one obtains information about the resonances in terms of quark degrees of freedom.

Even without the results of Sec. 4 one obtains from (4) and (7) for the $\rho$-meson mass:

$$M^2 = \frac{24\pi^2 F^2_\pi}{N_c}. \quad (10)$$

Using the physical value for the pion decay constant $F_\pi \approx 93$ MeV the $\rho$-meson mass turns out to be $M_\rho \approx 786$ MeV, already close to the physical $\rho$-meson mass of 770 MeV. Note that this result was obtained in the chiral and large-$N_c$ limit. Hence pion loops are absent in this framework, i.e. I have determined the mass of a bare $\rho$-meson without its pion cloud. Typically the $\rho$-pion interaction reduces the bare $\rho$-mass by approximately 5 – 10\% [11,12].

Using in addition (10) yields the coupling constants

$$F^2_V = 2F^2_\pi \quad \text{and} \quad G^2_V = \frac{F^2_\pi}{2}. \quad (11)$$

In particular the relation $F_V = 2G_V$ states the universality of the vector meson coupling as can be most easily seen by inspecting $\Phi$. This automatically implies that the KSFR relation is fulfilled [13].

The connection of $G_V$ to the usual $\rho\pi\pi$ coupling is provided by

$$g = \frac{G_V M_V}{F^2_\pi} \quad \text{(12)}$$

with $g$ defined via the lagrangian [12]

$$\mathcal{L}_{\text{int}} = \frac{i g}{4} \text{tr}(V^\mu [\partial_\mu \Phi, \Phi]) - \frac{g^2}{16} \text{tr}([V^\mu, \Phi]^2) \quad (13)$$

where $V^\mu$ is the vector meson resonance in the vector representation and $\Phi$ is connected to $U$ via $U = \exp(i\Phi/F_\pi)$. Relation (12) can be obtained by calculating the decay width $\Gamma(\rho \to \pi\pi)$ in both approaches [3] and [13]. From (10), (11) one gets

$$g = \sqrt{\frac{3}{N_c}} \frac{2\pi}{N_c} \quad (14)$$

as compared to the experimental value of 6.05 extracted from the decay width $\Gamma(\rho \to \pi\pi)$ [12].

Finally properties of the axial-vector meson $a_1$ can be deduced from the Weinberg sum rules [13,14]:

$$F^2_a = F^2_A + F^2_\pi, \quad M^2_A F^2_a = M^2_A F^2_A. \quad (15)$$

In combination with the previous results this yields the $a_1$-mass

$$M_A = \sqrt{\frac{3}{N_c}} 4\pi F_\pi \approx 1169 \text{ MeV} \quad (16)$$

and the coupling of the $a_1$ to an external axial-vector current

$$F_A = F_\pi \approx 93 \text{ MeV} \quad (17)$$

to be compared to the experimental values $M_{a_1} = 1230 \pm 40$ MeV and $F_{a_1} = 124 \pm 27$ MeV [10].

### 6 Summary and outlook

I have presented a somewhat indirect way to determine the properties of vector and axial-vector mesons in terms of quark degrees of freedom. The success of the presented approach suggests that it is indeed the phenomenon of $\chi$SB which quantitatively determines the properties of the studied mesonic resonances. Confinement enters the framework only qualitatively by demanding that color-white resonances are formed instead of quark-antiquark pairs.

There are things which still have to be clarified: First, I have utilized two different versions of resonance saturation. In the first one (Sec. 2) resonances are created from a combination of the two lagrangians [2] and [3] (cf. [3] for details) while in the second version (Sec. 4) the resonances only influence the fourth order lagrangian [3]. The connection of these two versions has to be studied in more detail. Second, there is a low-energy constant, namely $L_{10}$, which can be calculated both from the quark model and from the resonance saturation model using vector and axial-vector mesons. It turns out that one needs more than one meson per channel to achieve an agreement between the two calculations [13].

I expect that the presented framework can be extended from the vacuum case studied here to the case of a medium with finite temperature and/or quark density. This should provide interesting insight in the in-medium changes e.g. of the $\rho$-meson mass, its coupling to pions and photons and possible differences between longitudinal and transverse $\rho$-mesons.

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