TWIST–3 EFFECTS IN DEEPLY VIRTUAL COMPTON SCATTERING MADE SIMPLE

C. WEISS
Institut für Theoretische Physik
Universität Regensburg, D–93053 Regensburg, Germany
E-mail: christian.weiss@physik.uni-regensburg.de

We show that electromagnetic gauge invariance requires a “spin rotation” of the quarks in the usual twist–2 contribution to the amplitude for Deeply Virtual Compton Scattering. This rotation is equivalent to the inclusion of certain kinematical twist–3 (“Wandzura–Wilczek type”) terms, which have been derived previously using other methods. The new representation of the twist–3 terms is very compact and allows for a simple physical interpretation.

Deeply Virtual Compton Scattering (DVCS), $\gamma^*(q) + N(p) \to \gamma(q') + N(p')$ at large $q^2$ and finite $t = (p'-p)^2$, is the simplest process which could probe the generalized parton distributions (GPD’s) in the nucleon. New experimental results for spin and charge asymmetries of the cross section have been reported at this meeting, allowing for a first comparison of GPD models with data.

The crucial property of DVCS (and a number of other hard electroproduction processes) is that the amplitude can be factorized in a hard photon–quark amplitude, and a soft matrix element containing the relevant information about the structure of the nucleon. Technically, this factorization can be accomplished using QCD expansion techniques familiar from the theory of deep–inelastic scattering. Originally only the contribution from twist–2 operators was included. It was realized that in this approximation the amplitude is not transverse (electromagnetically gauge invariant); the violation is proportional to the transverse component of the momentum transfer, which is not suppressed at large $q^2$. A gauge invariant amplitude up to terms $O(t/q^2)$ is obtained by including certain “kinematical” twist–3 contributions. These have been derived in various approaches: Momentum–space collinear expansion, coordinate–space light cone expansion, and a parton–model based approach. In the usual formulation the twist–3 terms are parametrized by auxiliary GPD’s given by certain integrals over the basic twist–2 GPD’s, much like the Wandzura–Wilczek part of the spin structure function $g_2(x)$ in inclusive DIS. In addition to restoring gauge invariance of the twist–2 contribution, the twist–3 terms give rise to new helicity amplitudes and strongly influence the predictions for the spin and charge asymmetries of the DVCS cross section.
In this talk I would like to point out that the kinematical twist–3 terms in the DVCS amplitude have a simple physical interpretation as being due to a “spin rotation” applied to the twist–2 quark density matrix in the nucleon. This allows for a very compact representation of the twist–3 effects. Most important, it shows that, in spite of the apparent complexity of the amplitude at twist–3 level, DVCS is still a “simple” process. The results reported here have been obtained in collaboration with A. V. Radyushkin (Jefferson Lab and Old Dominion U.).

Consider virtual Compton scattering off an electron in QED at tree level, see Fig. 1a. It is well–known that transversality of the amplitude, $q_{\mu} T_{\mu\nu} = 0$ and $T_{\mu\nu} q_{\nu} = 0$, requires not only the Ward identities relating the electromagnetic vertex and the free–field propagator, but also the on–shell conditions for the external particles, i.e., the Dirac equations for the electron spinors.

Turning now to DVCS off a hadron, the twist–2 contribution to the amplitude in QCD is given by exactly the same diagrams as Fig. 1a, describing virtual Compton scattering off a free quark, only the wave functions of the initial and final particle have been replaced by the transition matrix element of the appropriate non-local quark/antiquark density matrix between the hadronic states, see Fig. 1b. The twist–2 part of the latter is defined as

$$M_{ij}(z|X)^{\text{twist–2}} = \int_0^1 d\lambda (\gamma_\sigma)_{ij} \frac{\partial}{\partial z_\sigma} \langle p'| \bar{\psi}(X - \lambda z/2) \gamma_5 \hat{z} \psi(X + \lambda z/2) | p \rangle$$

plus a similar contribution with $\gamma_\sigma \rightarrow \gamma_5 \gamma_\sigma$ and $\hat{z} \rightarrow \hat{\gamma}_5$. The density matrix is presented here in perhaps somewhat unusual form, in coordinate space, with the quark/antiquark “ends” located at $X \pm z/2$ ($X$ is the center coordinate, $z$ the separation); $i$ and $j$ are the Dirac spinor indices. Here $\bar{\psi}$ and $\psi$ are

![Figure 1.](image-url)
the quark fields (we omit the flavor labels), and the bilinear operator is really a traceless QCD string operator, see Ref. for details. What is important is that this twist–2 density matrix does not satisfy the free–field Dirac equations with respect to the quark/antiquark “ends”; the violation is proportional to the momentum transfer $\Delta = p' - p$. The reason is, simply put, that in the twist–2 operator in Eq.(1) the quark spin is projected on a fixed direction, determined by the vector $z$, while the Dirac equations require that the spin projection changes between the two ends in accordance with the momentum transfer between the quark lines. As a consequence, the twist–2 part of the DVCS amplitude alone is not electromagnetically gauge invariant; the amplitude violates transversality by terms proportional to $\Delta$.

It is not difficult to see what must be done in order to fix this problem. We must rotate the spin projection of the quarks in the density matrix (1) such as to align it with the momenta of the incoming and outgoing quark ends. This is achieved by a position–dependent rotation with a matrix

$$\Sigma (z/2) \equiv \exp \left[ -\frac{i}{4} z_\alpha \sigma_{\alpha\beta} \Delta_{\beta} \right].$$

The modified density matrix is ($\bar{\lambda} \equiv 1 - \lambda$)

$$M_{ij}(z|X)^{\text{rot}} = \int_0^1 d\lambda \left[ \Sigma (\bar{\lambda} z/2) \gamma_\sigma \Sigma (\bar{\lambda} z/2) \right]_{ij}$$

$$\times \frac{\partial}{\partial z_\sigma} \langle p' | \bar{\psi}(X - \lambda z/2) \hat{\sigma} \psi(X + \lambda z/2) | p \rangle \quad (3)$$

\[a\] In the usual collinear expansion around a fixed light–like direction, the vector operator in Eq.(1) would have a large “plus” component, while the quark/antiquark ends have transverse momenta because of $\Delta_\perp \neq 0$. 

Figure 2.
plus the same with \( \gamma_\sigma \to \gamma_5 \gamma_\sigma \) and \( \hat{z} \to \hat{z} \gamma_5 \). This “rotated” form satisfies the Dirac equations with respect to the external ends, up to terms proportional to \( t \), see Ref.\(^9\) for details. As a result, the DVCS amplitude obtained with Eq.(3) is gauge invariant up to terms of order \( O(t/q^2) \). Schematically, our modification of the twist–2 contribution to the DVCS amplitude can be represented as in Fig.\(^3\), with the spin rotation as an “intermediate step” between the twist–2 density matrix and the free quark Compton amplitude.

In the terminology of the light cone expansion, the spin rotation of Eq.(3) amounts to the inclusion of certain twist–3 operators, which, however, are completely given in terms of total derivatives of twist–2 operators (“kinematical twist–3”). When substituting parametrizations for the basic twist–2 matrix elements, Eq.(3) reproduces the Wandzura–Wilczek type relations for the twist–3 GPD’s, which were derived previously using other techniques.\(^6\) Thus, all the complexity of the kinematical twist–3 effects in DVCS can be reduced to the simple spin rotation of Eq.(3).

The effect of kinematical twist–3 terms on DVCS observables have been discussed in the literature.\(^6\) The twist–3 terms affect in particular the spin and charge asymmetries of the cross section. The spin rotation representation could be helpful in developing a more intuitive understanding of the twist–3 effects in DVCS observables. This problem certainly deserves further study.

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References

1. G. van der Steenhoven (HERMES Collaboration), this meeting.
2. L. Elouadrhiri (CLAS Collaboration), this meeting.
3. X. Ji, Phys. Rev. D55 (1997) 7114; J. C. Collins, L. Frankfurt and M. Strikman, Phys. Rev. D56 (1997) 2982; A. V. Radyushkin, Phys. Rev. D56 (1997) 5524.
4. I. V. Anikin, B. Pire and O. V. Teryaev, Phys. Rev. D62, 071501 (2000).
5. A. V. Belitsky and D. Muller, Nucl. Phys. B589 (2000) 611.
6. A. V. Radyushkin and C. Weiss, Phys. Lett. B493 (2000) 332; Phys. Rev. D 63 (2001) 114012.
7. M. Penttinen et al., Phys. Lett. B491 (2000) 96.
8. N. Kivel, M. V. Polyakov and M. Vanderhaeghen, Phys. Rev. D 63, 114014 (2001), A. V. Belitsky, A. Kirchner, D. Müller and A. Schäfer, Phys. Rev. D 64, 116002 (2001); Phys. Lett. B 510, 117 (2001).
9. A. V. Radyushkin and C. Weiss, Phys. Rev. D 63 (2001) 114012.