NONTHERMAL EMISSION FROM A SUPERNOVA REMNANT IN A MOLECULAR CLOUD

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ABSTRACT

In evolved supernova remnants (SNRs) interacting with molecular clouds, a highly inhomogeneous structure consisting of a forward shock of moderate Mach number, a cooling layer, a dense radiative shell and an interior region filled with hot tenuous plasma is expected. We present a model of nonthermal electron injection, acceleration and propagation in that environment and find that these SNRs are efficient electron accelerators and sources of hard X- and γ-ray emission. A forward shock of velocity \(v_s \gtrsim 100 \text{ km s}^{-1}\) with an ionized precursor propagating into the molecular cloud accompanied by magnetohydrodynamic turbulence provides a spatially inhomogeneous distribution of nonthermal electrons. The energy spectrum of the nonthermal electrons is shaped by the joint action of first and second order Fermi acceleration in a turbulent plasma with substantial Coulomb losses. Bremsstrahlung, synchrotron, and inverse Compton radiation of the nonthermal electrons produce multiwavelength photon spectra in quantitative agreement with the radio and the hard emission observed by ASCA and EGRET from IC 443. We distinguish interclump shock wave emission from molecular clump shock wave emission; particles reach higher energies in the interclump shock and that is the likely source of γ-ray emission and radio synchrotron emission. Spatially resolved X- and γ-ray spectra from the supernova remnants IC 443, W44, and 3C391 as might be observed with BeppoSAX, Chandra XRO, XMM, INTEGRAL and GLAST would distinguish the contribution of the energetic lepton component to the γ-rays observed by EGRET, constraining the cosmic ray nuclear component spectra in these SNRs. These data would provide a valuable tool for studying the complex structure of molecular clouds where SNR radiative shocks interact with dense molecular clumps.

Subject headings: acceleration of particles — cosmic rays — ISM: Supernova Remnants — radiation mechanisms: nonthermal

1. INTRODUCTION

Supernovae and their remnants have long been identified as likely sites of galactic cosmic ray acceleration. Evidence for electron acceleration first came from observations of radio synchrotron radiation. The observation of nonthermal X-ray emission from SN 1006 by ASCA has provided evidence for electron acceleration up to \(\sim 100 \text{ TeV}\) (Koyama et al. 1995; Reynolds 1998). The proton cosmic ray component is more difficult to observe, but it has long been recognized that pion decays from collisions with interstellar gas could give an observable flux of \(0.1-1 \text{ GeV}\) photons. Initial estimates of γ-ray emission from supernova remnants (SNRs), concentrating on the component resulting from pion decays, were made by Chevalier (1977), Blandford & Cowie (1982), and Drury, Aharonian, & Völk (1994). When γ-ray emission was apparently detected from SNRs by CGRO, the γ-ray spectrum could not be fitted by a pure pion decay spectrum and some other component was needed (Esposito et al. 1996; Sturter et al. 1997; Gaissier, Protheroe, & Stanev 1998). In addition to the pion decays, the relevant processes are bremsstrahlung emission of relativistic electrons and inverse Compton emission. Gaissier et al. (1998) modeled these processes in detail in order to fit the observed γ-ray spectra of the remnants IC 443 and γ Cygni. They assumed acceleration to a power law spectrum in the shock front and determined the spectral index, electron to proton ratio, and the upper energy cutoff.

The evolution of the relativistic component in a supernova remnant has been modeled by a number of groups. Sturter et al. (1997) assumed particle acceleration in the shock front with an \(E^{-2}\) energy spectrum and followed the evolution of the particle spectrum. Their SNR model was homogeneous: a factor 4 density jump at the shock front, constant density in the interior, and a constant magnetic field in the interior. Their shock acceleration model had a diffusion length parameter that was important for the maximum energy reached by the particles. They modeled the nonthermal emission from IC 443 as synchrotron emission in the radio and bremsstrahlung in γ-rays. de Jager & Mastichiadis (1997) dealt with the same processes in W44, as well as inverse Compton emission. They noted that the observed radio spectrum is flatter than would be expected from shock acceleration of newly injected particles and suggested that the particles originated from a pulsar in the supernova remnant. Recently, Ostrowski (1999) showed that second-order electron acceleration by the turbulent medium just after the shock could flatten the spectra to account for the radio observations of IC 443. Baring et al. (1999) presented calculations of the broadband emission from non-linear shock models of shell-type SNRs. They used Sedov adiabatic shock dynamics in a homogeneous medium and a Monte Carlo simulation of the particle acceleration taking into account the nonlinear shock structure. The set of models considered by Baring et al. (1999) covers the range of shock speeds \(490 \leq v_s \leq 4000 \text{ km s}^{-1}\) and ambient medium number densities \(10^{-3} \leq n \leq 1 \text{ cm}^{-3}\).

Massive stars that are the likely progenitors of core collapse supernovae are expected to be spatially correlated with molecular clouds. The remnants that are likely to be γ-ray sources in CGRO observations (Esposito et al. 1996) also show evidence for interaction with molecular gas. Chevalier (1999) re-

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ently studied the evolution of supernova remnants in molecular clouds and concluded that many aspects of the multiwavelength observations could be understood in a model where the remnants evolve in the interclump medium of a molecular cloud, which has density of $5 \sim 25$ H atoms cm$^{-3}$, and become radiative at radii $\sim 6$ pc. The compression in the radiative shell is limited by the magnetic field. Molecular emission occurs when the radiative shell collides with molecular clumps. IC 443 is the remnant with the best evidence for high energy emission and also shows clear evidence for interaction of the shock with a molecular cloud (e.g., Burton et al. 1990; van Dishoeck et al. 1993; Cesarsky et al. 1999).

Most of the current models of particle acceleration in SNRs have dealt with adiabatic remnants in a homogeneous medium (see, however, the model of Boulares & Cox 1988 for the Cygnus Loop and the model of Jones & Kang 1993). In contrast, our aims here are to examine in detail the nonthermal emission of radiative supernova remnants in molecular clouds using a kinetic model of electron injection, acceleration and propagation, including the inhomogeneous structure deduced from multiwavelength SNR observations. We use global MHD (magnetohydrodynamic) models of supernova remnants interacting with molecular clouds as described by Chevalier (1999), including the theory of radiative shock structure by Shull & McKee (1979) and the kinetic model of electron injection and acceleration by Bykov & Uvarov (1999). The model described here is relevant to the mixed-morphology SNRs (Rho & Petre 1998), if they are interacting with molecular clouds. These remnants comprise a sizable fraction of the Galactic SNR population.

The plan of our paper is as follows. In § 2, we treat the energy spectrum of energetic electrons as they pass through the regions associated with a radiative shock wave. We include the shock acceleration of particles from the thermal pool. In § 3, we present calculations of the nonthermal emission from the relativistic particles, including emission from shocks in clumps as well as emission from the radiative shock wave in the interclump region. Our model is specifically applied to the well-observed remnant IC 443, although we expect it to more generally apply to other supernova remnants in molecular clouds. We also examine the ionization by energetic particles and the energy involved in the relativistic particles. A discussion of results and future prospects is in § 4. A detailed discussion of expected range of densities, shock velocities and magnetic fields determined from multiwavelength observations of IC 443, W44 and 3C391 was given in the paper by Chevalier (1999, and references therein).

2. EVOLUTION OF NONTHERMAL ELECTRONS

The structure of the flow expected in a radiative shock can be divided into the following regions (see, e.g., Draine & McKee 1993):

(I) preshock gas ionized and heated by UV (ultraviolet) emission and fast particles from the shock;

(II) shock transition region;

(III) postshock cooling layer;

(IV) cold shell of swept-up gas;

(V) hot low density gas interior to the shock.

Radiative shocks are subject to thermal instabilities (Chevalier & Imamura 1982; Bertchinger 1986) as well as dynamical instabilities (Vishniac 1983; Blondin et al. 1998). Although the effect of inhomogeneous magnetic fields on the stability of radiative shock flow is yet to be studied, MHD turbulence is expected in such a system. The transport and acceleration of nonthermal particles in the violent SNR environment are governed by the MHD flow and depend on the ionization structure and on the spectrum of MHD turbulence. Shocks with velocity $v_{77} \geq 1.1$ generate sufficient ultraviolet UV radiation for full pre-ionization of hydrogen and helium (He$^+$) in the preshock region, where the gas temperature is typically $\sim 10^4$ K (Shull & McKee 1979). Here, $v_{77}$ is the shock velocity measured in units of 100 km s$^{-1}$, and $n_1$ is the ion number density measured in units of 10 cm$^{-3}$. Shocks with $v_{77} < 1.5$ propagating in a medium with $n_1 \sim 1$ photoionize gas ahead of the viscous jump to a depth $N_1 \geq 3 \times 10^{17}$ cm$^{-2}$. The shock transition in such a case is collisionless and supercritical unless the preshock magnetic field is above $\sim 4 \times 10^{-3}$ G.

The geometry of the magnetic field is important for the efficiency of electron injection and acceleration. It has been argued (e.g., Levinson 1996; McClements et al. 1997) that quasi-perpendicular shocks are an efficient source of freshly injected electrons if their speed $v_e \geq (m_e/m_p)^{0.5} c$; the injection efficiency drops sharply below that threshold. On the other hand, Bykov & Uvarov (1999) have shown that quasi-parallel MHD shocks of moderate Alfvén Mach number $M_A \leq (m_e/m_p)^{0.5}$ are suitable sites of electron injection and acceleration. The SNR shock waves in molecular clouds are expected to be in this regime of Mach number. Thus we shall discuss below the quasi-parallel portion of the shock (magnetic field inclinations $\leq \pi/4$) propagating into a molecular cloud with $v_{77} \sim 1.5$.

Nonresonant interactions of the electrons with fluctuations, generated by kinetic instabilities of the ions in the transition region of a quasi-parallel supercritical shock, play the main role in heating and preacceleration of the electrons. The electron heating and preacceleration (injection) occur in the collisionless shock front transition region on a scale $\Delta$ of several hundred inertial lengths of the ions, $l_i$ (Bykov & Uvarov 1999). Here $l_i = c/\omega_{pi} \sim 9 \times 10^6 n_1^{-0.5}$ cm and $\omega_{pi}$ is the ion plasma frequency. For a SNR shock in a molecular cloud, the scale $\Delta \sim 10^9$ cm is much shorter than the scales of all the other regions.

To accelerate electrons injected in the shock transition region to relativistic energy $E(p)$, MHD turbulence should fill the acceleration region, which has a scale $l_a \sim k_0(p)v_e$. Here, $k_0 = \max\{k_i\}$, where $k_i(p)$ is the diffusion coefficient of an electron of momentum $p$ and $i = (I)$, (III) refers to the regions defined above. To provide the conditions for efficient transformation of the MHD flow power to the accelerated electrons, the MHD turbulent fluctuation spectrum in the vicinity of the shock should extend to scales $l \geq r_s(p)$ (resonant scattering), where $r_s(p)$ is the gyroradius of an electron with momentum $p$. The MHD fluctuations responsible for the electron scattering are collisionless for particles with energies up to a GeV because the ion-neutral collision length is above $10^{14}$ cm for typical parameters of the radiative shock described below (a keV electron has $r_s \approx 10^6 [B/10^{-6} G]^{-1}$ cm).

To calculate the spectra of nonthermal electrons in the regions $i = I - IV$, we use a kinetic equation for the nearly-isotropic distribution function $N(z, p, t)$:

$$\frac{\partial}{\partial t} N_i + u_i(z) \frac{\partial}{\partial z} N_i + \frac{p}{3} \frac{\partial}{\partial p} N_i \left( \frac{\partial}{\partial u_i} N_i \right) =$$

$$k_i(p) \frac{\partial^2 N_i(z, p, t)}{\partial z^2} + \frac{1}{p^2} \frac{\partial}{\partial p} \frac{\partial}{\partial p} D_i(p) \frac{\partial^2 N_i}{\partial p^2} + \frac{1}{p^2} \frac{\partial}{\partial p} \left\{ p^2 L_i(p) N_i \right\}.$$  

The Fokker-Planck type equation (1) takes into account diffu-
sion and advection (with bulk velocity $u(z)$) of an electron in phase space due to particle interaction with MHD waves and the large scale MHD flow (Bykov & Toptygin 1993). Here, $L_i(p)$ is the momentum loss rate of an electron due to Coulomb collisions in a partially ionized plasma (e.g., Ginzburg 1979). The momentum diffusion coefficient $D(p)$ is responsible for second order Fermi acceleration, and $k_i(p)$ is the fast particle spatial diffusion coefficient. For low energy electrons (i.e., $E \leq E_c$), Coulomb and ionization losses are important in the regions ($I - V$), except for the narrow shock transition region ($II$) where acceleration is fast enough to overcome losses and nonthermal electron injection occurs. The characteristic energy $E_c$ depends on the plasma density and ionization state, magnetic field, and fast particle diffusion coefficients $k_i(p)$ and $D_i(p)$. The corresponding momentum $p_m$ can be estimated as the point where the last two terms in equation (1) are equal. We used the energy $E_c$ calculated simultaneously with the electron distribution function as a convenient parameter to distinguish between different possible models of MHD turbulence in the postshock cooling layer of the radiative shock structure (see below in this section).

The diffusion coefficients $k_i(p)$ and $D_i(p)$ depend on the spectrum of the collisionless MHD turbulence, which is poorly understood. A plausible approximation for the diffusion coefficients $k_i(p)$ was assumed here. For all of the regions we used the following parameterization:

$$k_i(p) = k_{i0} \begin{cases} 
1, & p_T \leq p \leq p_s, \\
{v_p^3/v_e p_s^2}, & p_s \leq p \leq p_{ss}, \\
{c p^2/(p_{ss}^3 + p_e^3 v_e)}, & p_{ss} \leq p \leq p_m.
\end{cases} \tag{2}$$

Here, $p_T$ is the momentum of the upstream thermal electrons, $p_m$ is the momentum corresponding to the upper cut-off energy $E_m$ (see equation [3]), and $p_s$ and $p_{ss}$ are defined in the next paragraph. The standard relation for the momentum diffusion coefficient $D_i(p) = p^2 w_i^3 / 9 k_i(p)$ (e.g., Bereczinszky et al. 1990) was used. In the low energy regime $p \leq p_s$, the particle transport is dominated by large scale turbulent advection (Bykov & Toptygin 1993). The large scale turbulence is due to stochastic MHD plasma motions on scales $\lambda$ longer than the particle mean free path due to resonant scatterings. The sources of MHD turbulent motions are the shock wave instabilities mentioned at the beginning of this section. The large scale vortex rms-velocity $w_i$ typically is a fraction of bulk speed $u_i$. The spatial diffusion is energy independent below $p_s$ and $k_{i0} \approx w_i \Lambda$. For $p_s \leq p \leq p_{ss}$, electrons are scattered by resonant MHD waves presumably generated by the streaming instability of shock accelerated particles (e.g., Blandford & Eichler 1987; Jones & Ellison 1991). In this case $w_i$ is close to the Alfvén velocity $v_a$ and $k_i(p)$ is the electron diffusion coefficient due to resonant wave-particle interactions. If the power spectrum of magnetic field fluctuations of Alfvénic turbulence for the resonant fluctuations is approximated as $dB_i^2/dk \sim k^{-\delta}$ then the power-law index $a$ (equation [2]) is related to the index $\theta$ as $a = 2 - \theta$. An important particular case corresponding to $a \sim 1$ can be written as $k_i(p) = (1/3) \eta \nu r_i(p)$, where $\eta$ determines the scattering “strength” and strong scattering (Bohm limit) corresponds to $\eta \sim 1$. The diffusion model with $a = 1$ was successfully used for modeling the observed anomalous cosmic ray proton fluxes in the interplanetary medium (Ellison et al. 1999) and in the Monte Carlo simulations of nonlinear shocks in the shell type SNRs (e.g., Baring et al. 1999). The momentum $p_m$ in our model can be estimated from the equation (1/3) $\eta \nu r_i(p_m) = w_i \Lambda$.

MHD waves in the partially ionized plasma of the radiative shock flow are subject to ion-neutral damping (e.g., Völk et al. 1981). We define the momentum $p_{ss}$ from the condition $r_i(p_{ss}) = \lambda_{in}$, where $\lambda_{in}$ is determined from the relation $\lambda_{in} = 2 \nu r_i / \Gamma_{in}$ and $\Gamma_{in}$ is the Alfvén wave damping rate due to ion-neutral collisions (Kulsrud & Cesarsky 1971). For $p \geq p_{ss}$, electrons are scattered by nonresonant small scale waves.

The limits imposed on the diffusive shock acceleration of particles due to ion-neutral MHD wave damping have been discussed by Draine & McKee (1993) and Drury et al. (1996). They considered the spectra of Alfvén waves driven by the instabilities of shock accelerated ions in a partially ionized plasma. Drury et al. (1996) obtained the upper cut-off energy of a proton due to ion-neutral wave damping

$$E_m \leq v_{A}^3 T_4^{-0.4} n_i^{-1} \lambda_{ii}^{0.5} \mathcal{P}^{CR}_{-1} \text{GeV}, \tag{3}$$

where $n_i$ is the neutral particle density, $\lambda_{ii}$ is the ion density (both are measured in $\text{cm}^{-3}$), and $\mathcal{P}^{CR}$ is the total particle pressure normalized to 10% of the shock ram pressure. The ionization structure of the preshock region is inhomogeneous so the ion number density, temperature and other numbers in equation (3) must be taken at an upstream distance $\sim k_i(E_m)/v_a$ from the collisionless shock transition. Application to our model yields $E_m \sim 10 \text{ GeV}$ if $v_{A} \sim (1.1 - 1.5)$ and $k_i(E_m) \sim 10^{24} \text{ cm}^2 \text{s}^{-1}$ because of the high ionization in the preshock region ($I$) and the postshock cooling layer ($III$) provided by the UV radiation. One can justify the same value of $E_m$ for the electrons because the synchrotron and inverse Compton losses are relatively unimportant for $E_m \sim 10 \text{ GeV}$.

**Region I.** In the preshock region of the shock with $v_{A} \sim 1.1$, we assume a highly ionized plasma of temperature $T_i \sim 1$ up to depths $\sim$ a few times $10^{16} \text{ cm}^{-3}$ upstream. For the IC 443 preshock region, we adopted a density $\sim 25 \text{ cm}^{-3}$ and $v_{A} \sim 1.5$ (Fesen & Kirshner 1980). The magnetic field $B_{1} \approx 10^{-5} \text{ G}$ (Chevalier 1999), while $B_1 \approx 5 \times 10^{-6} \text{ G}$ (the inclination is assumed to be $\sim \pi/6$). The Alfvén velocity here is about 5 km s$^{-1}$. We assume a conservative diffusion model in the shock upstream region where $k_i(p)$ has $a = 1$ and the moderate scattering “strength” $10 \leq \eta \leq 100$ (Ellison et al. 1999).

**Region II.** In the shock transition region, injection and heating of the electrons occur due to nonresonant interactions with strong MHD fluctuations generated by the ions. Following the model developed by Bykov & Uvarov (1999) we take

$$D_{II}(p) \approx p^2 \mathcal{C} \left( \frac{\delta B}{B_0} \right)^2 \left( \frac{\nu_r}{v} \right)^2 \left( \frac{v}{v_A} \right), \tag{4}$$

where \( \mathcal{C} \sim 1 \) and \( \delta B \gtrsim B_0 \). The Alfvénic Mach number of the shock, $M_{\perp} \gtrsim 10$, is moderate for our situation. Nevertheless, the effect of the accelerated particle pressure on the shock structure could be important (see, e.g., Jones & Ellison 1991; Baring et al. 1999). An exact treatment of the effect is not feasible in our model because it would require a kinetic description of the ion injection. We accounted for the effect by correction of the total shock compression ratio $\delta_i$ and by introducing an extended modified shock structure with a precursor and a subshock with compression ratio $\delta_i \leq \delta_i$ (e.g., Jones & Ellison 1991). In our case with Mach numbers $M_{\perp} \gtrsim 15$, the estimated extra compression was about 10% and thus $\delta_i \approx 4.4$. A subshock compression ratio $\delta_i \approx 3.3$ is consistent with the assumption of substantial heating of the precursor gas due to wave dissipation (Berezhko & Ellison 1999). These numbers are in agreement...
with those from a hybrid simulation of nonlinear shock structure (Giacalone et al. 1997). We found that the resulting high energy emission spectra are sensitive to the particular choice of precursor flow structure in the MeV regime. The keV and GeV emission is not sensitive to the structure if the compression ratios \( \delta_1 \) and \( \delta_2 \) are fixed.

Region III. The postshock cooling layer has about a column density \( \sim 3 \times 10^{17} \text{ cm}^{-2} \) of highly ionized plasma with an initial density about \( \delta_1 \) times that of the preshock one. The Alfvén velocity in the highly ionized portion of the region (III) is similar to that at the preshock region, \( \sim 5 \text{ km s}^{-1} \). Large scale vorticity with an amplitude \( \lesssim 20 \text{ km s}^{-1} \) on scales \( \Lambda \lesssim 10^{14} \text{ cm} \) may occur here because of shell instabilities of the radiative shock with \( v_{\text{SH}} \sim 1.5 \) (see, e.g., section 5 in the review by Draine & McKee 1993). This would dominate the low energy electron propagation, providing a spatial diffusion coefficient of \( \sim 10^{20} \text{ cm}^2 \text{ s}^{-1} \) for the radiative shock parameters described above.

Coulomb losses are important here for electrons with energies below \( E_C \approx 20 \text{ keV} \) if the large scale turbulent velocity has a substantial longitudinal component with \( w_{\text{III}} \approx 20 \text{ km s}^{-1} \). \( E_C \gtrsim 8 \text{ MeV} \) is expected in the absence of a longitudinal component of large scale turbulence in the postshock cooling layer because the Coulomb losses are overcome by resonant interaction with Alfvén waves. Since the Alfvén velocity is relatively low (\( \sim 5 \text{ km s}^{-1} \)), the value of \( E_C \) is much higher in that case. An accurate description of MHD turbulence in the postshock cooling layer is not available now. Thus we considered both limiting cases described above and show the uncertainty introduced by the lack of data concerning the large scale turbulence properties in Fig. 1. To avoid overestimating the nonthermal emission, we used a conservative minimum value of \( E_C = 120 \text{ keV} \) for the radiative shock structure described above, although one could expect \( E_C \sim 20 \text{ keV} \) in the most favorable case.

Region IV. The density and temperature in the radiative shell depend on the preshock density, magnetic field and abundances (Shull & McKee 1979; Chevalier 1999). From the model of Chevalier (1999) for the above preshock parameters, we adopted \( n \sim 275 \text{ cm}^{-3} \) (\( N_{\text{III}} \approx 1.6 \times 10^{20} \text{ cm}^{-2} \)) and \( T \sim 10^4 \text{ K} \) in the dense radiative shell. The mass of the dense radiative shell is \( \sim 1000 M_\odot \) and its magnetic field is \( \sim 6 \times 10^{-5} \text{ G} \). The ionization in the dense radiative shell is supported at the level of about a percent by shock accelerated particles penetrating into the shell (see section 3.5 for details). The phase velocity of the high frequency collisionless Alfvén waves which are responsible for electron resonant scattering is about \( 50 \text{ km s}^{-1} \); it is determined by the number density of the ions. The Coulomb loss rate of the electrons is then dominated by the neutral component.

We numerically calculated the electron distribution function in regions (I-IV). The results were then used in calculations of the nonthermal emission.

3. NONTHERMAL EMISSION FROM IC 443

The SNR IC 443 (G189.1+3.0) is a very good candidate for testing our model because the interaction of the SNR with a molecular cloud was established from radio and infrared line observations (e.g., DeNoyer 1979; Mufson et al. 1986; Burton et al. 1990; van Dishoeck et al. 1993; Richter et al. 1995; Claussen et al. 1997). The high energy fluxes are somewhat better determined than in other cases. Although we concentrate on IC 443, we briefly mention the case of W44, where observations of OH masers and CO clumps again provide convincing evidence of molecular cloud interaction (Claussen et al. 1997; Frail & Mitchell 1998).

To model the nonthermal emission from the shell of IC 443 we integrated the local emissivities over the radiative shock structure (regions I-IV). As a parcel of gas containing accelerated nonthermal electrons evolves through the radiative shock structure including the dense shell, the local emissivity from bremsstrahlung and inverse Compton was calculated using standard theory (see Appendix A for details). The electron - ion bremsstrahlung emission differential cross sections (Bethe-Heitler formulae) were taken from Akhiezer & Berestetskii (1957) taking into account the Elwert factor, which is important for modeling the keV emission. The electron - electron bremsstrahlung contribution to the \( \gamma \)-ray emissivity was calculated using cross sections derived by Haug (1975). For the inverse Compton emissivity of IC 443, we used the same description of the background photon field as Gaisser et al. (1998). The hard emission production cross sections used in our calculations are similar to those used recently by other authors (Asvarov et al. 1990; Sturmer et al. 1997; Gaisser et al. 1998; Baring et al. 1999). We also estimated the contribution of pion decays to the \( \gamma \)-ray emission using a model for the pion component and the cross sections from Dermer (1986). The resulting emission in the \textit{EGRET} regime was roughly 7 times below that from electron bremsstrahlung, which is consistent with the results of Sturmer et al. (1997). We do not discuss the pion component further.
the scheme described by Ginzburg (1979) can be compared to the radio observations and to the upper limits at higher frequencies. We also included free-free absorption of low frequency radio waves (dotted line in Fig. 1). The synchrotron losses of a relativistic electron were included following Ginzburg (1979); bremsstrahlung energy losses are relatively unimportant in our case.

3.1. Energetic Nonthermal Emission from the Shell

The emission was integrated over the regions of the radiative shock in order to model the spectrum of the entire shell. Region IV is the dominant contributor to the synchrotron and bremsstrahlung emission. The results in Fig. 1 can be compared to the ASCA and EGRET observations of SNRs, as well as to the significant upper limits that have been set at TeV energies by Whipple observations (Buckley et al. 1998) and from the preliminary analysis of CGRO OSSE observations presented by Sturmer et al. (1997). We tried different parameter sets to fit the observations.

In Fig. 1, $\nu F_\nu$ fits to the observations of IC 443 are presented for the model described in the previous section. The preshock density is taken to be $n = 25$ cm$^{-3}$ and the assumed distance to IC 443 is 1.5 kpc (e.g., Fesen & Kirshner 1980). Interstellar photoelectric absorption of the X-ray spectrum is accounted for with a line-of-sight column density $N_H = 2 \times 10^{21}$ cm$^{-2}$ using the cross sections from Morrison & McCammon (1983).

For $\nu \approx 1.5$ in the diffusion model described by equation (2), we used $k_{\text{HII}} = 1.1 \times 10^{20}$ cm$^2$ s$^{-1}$, $a = 1.0$, $E(p_\ast) = 1$ MeV, and $E(p_{\ast\ast}) = 20$ GeV, which are compatible with theoretical estimates of resonant wave-particle interactions. The corresponding value $\eta \approx 30$. While in principle $\eta$ can be determined from observations of heliospheric shocks, single spacecraft observations do not provide direct values and are subject to ambiguous interpretation. Values of $\eta \approx 30$ are consistent with modeling of the highly oblique solar wind termination shock (Ellison et al. 1999), but smaller values ($\eta \approx 3 - 10$) have been inferred for interplanetary traveling shocks (Baring et al. 1997). On the other hand, very weak scattering ($\eta \approx 100$) has been inferred from observations near corotating interaction regions (e.g., Fisk, Schwadron, & Gloeckler 1997) and in the solar wind when the interplanetary magnetic field is nearly radial (e.g., Mőbius et al. 1998). We use $\eta \approx 30$ as a conservative value. If strong MHD turbulence is present in the upstream shock region, the $\eta$ values could be closer to 1. This might allow an electron acceleration model similar to that described earlier, but for slower shock velocities ($\lesssim 100$ km s$^{-1}$).

The diffusion propagation model is not unique. We obtained a similar fit to that presented in Fig. 1 for a diffusion model with $k_{\text{HII}} = 10^{20}$ cm$^2$ s$^{-1}$, $a = 0.7$, $E(p_\ast) = 1$ MeV, $E(p_{\ast\ast}) = 20$ GeV. As mentioned in §2, there is an important uncertainty concerning the lack of a quantitative model for the generation of large scale MHD turbulence in the postshock cooling layer. The uncertainty affects the hard X-ray spectral calculations. In view of this, we calculated the expected bremsstrahlung emission for two limiting cases covering the range of uncertainties. Curve 1 in Fig. 1 corresponds to fully developed Alfvénic large scale MHD turbulence in the postshock cooling layer ($E_C = 120$ keV). Curve 2 corresponds to the case of a lack of large scale MHD turbulence ($E_C = 2$ GeV). While the GeV bremsstrahlung emission (as well as synchrotron and inverse Compton emission) is similar in the 2 cases, the hard X-ray emission is sensitive to the uncertainty. We also obtained a similar fit to that presented in Fig. 1 for a model with $\nu \approx 1.1$ and a diffusion model described by $k_{\text{HII}} = 5 \times 10^{19}$ cm$^2$ s$^{-1}$, $a = 1.0$, $E(p_\ast) = 1$ MeV, and $E(p_{\ast\ast}) = 10$ GeV. The model with $\nu \approx 1.1$ requires a somewhat higher power conversion efficiency than that with $\nu \approx 1.5$ (see §3.6), but this value is preferred for the IC 443 radiative shock velocity (Chevalier 1999).

The model of electron injection from the cloud thermal pool described above is based on the ionization structure of a radiative shock, which is very sensitive to the shock velocity if $\nu \approx 1$ (Shull & McKee 1979; Hollenbach & McKee 1989). In this respect, it is instructive to consider a model in which high energy electrons are not related to freshly injected particles, but are reaccelerated cosmic ray electrons (Blanford & Cowie 1982; Chevalier 1999; Cox et al. 1999). To model that case we suppose that the far upstream electron flux is just that observed for Galactic cosmic ray electrons near Earth, but extrapolated back to the energy $E_{\text{crm}}$ with the same slope as observed above a GeV. The calculations of $\nu F_\nu$ for this case are shown in Fig. 2. We consider the radiative shock structure with the parameters described in §2 (the same as were used for Fig. 1). The Galactic cosmic ray electron flux in the far upstream region was taken to be $3 \times 10^{-5} (E/\text{GeV})^{-3}$ e$^{-}$ cm$^{-2}$ s$^{-1}$ GeV$^{-1}$ sr$^{-1}$ (e.g., Berezinsky et al. 1990) for $E \geq E_{\text{crm}}$. We assumed a flat cosmic ray electron spectrum below $E_{\text{crm}}$, corresponding to flattening due to Coulomb losses in the interstellar medium. The maximum energy of cosmic ray electrons reaccelerated by the radiative shock is $E_{\text{sh}} \sim 10$ GeV (see equation [3]), as in the case shown in Fig. 1. The energy $E_{\text{crm}}$ was considered as a free parameter here.

![Fig. 2. — Broadband $\nu F_\nu$ spectrum of the shell of IC 443 calculated for the model of Galactic cosmic ray electron reacceleration by a radiative shock. The basic shock parameters are as in Fig. 1. The low energy cut-off of the preexisting Galactic cosmic ray electron spectrum in the cloud is 3 MeV for curve 1, 8 MeV for curve 2, and 50 MeV for curve 3. The inverse Compton emission from the whole remnant is shown as dashed lines.](image-url)
emission if $E_{\text{crm}} \leq 10$ MeV for the radiative shell parameters described above, but fall short for $E_{\text{crm}} \approx 100$ MeV.

The relatively low energies, $E_{\text{crm}} \sim 10$ MeV, required for the IC443 shell are a potential problem for this model. The radio synchrotron radiation from the Galaxy indicates that the cosmic ray electron spectrum flattens to $N(E) \propto E^{-2}$ below energies $\sim 1$ GeV (Webber 1983). If we use such a spectrum in our model, the nonthermal emission is significantly below that observed. However, there are uncertainties about betatron-type acceleration in the shock compression, about the magnetic field strength, and about the volume of the emitting region.

Duin & van der Laan (1975) did successfully model the IC 443 radio flux with a cosmic ray compression model. We found that models with a higher magnetic field and/or larger volume could reproduce the radio flux, but that the $\gamma$-ray fluxes observed by EGRET could not be achieved. Cox et al. (1999) have recently concluded that cosmic ray compression by radiative shocks could provide a good fit to radio and gamma-ray emission from W44 if the cosmic ray electron population in the vicinity of the SNR is higher than that of measured in solar neighborhood. They used a somewhat higher magnetic field ($B \sim 200 \mu G$) in the radiative shell of W44 than the $60 \mu G$ field we assumed for the IC 443 shell.

The acceleration time is a constraining parameter in this model because of the relatively low shock speed $v_{57} \sim 1.5$ in the dense medium. The test particle shock acceleration time $t_{\text{a}}(E)$ can be estimated from the equation

$$t_{\text{a}} = \frac{3}{v_{i} - v_{II}} \int_{p_{0}}^{p} \left( \frac{k_{IP}(p)}{v_{I}} + \frac{k_{IP}(p)}{v_{II}} \right) \frac{dp}{p}$$

where the velocities $v_{I}$ are measured in the shock rest frame (e.g., Axford 1981). For the radiative shock parameters described above, the acceleration time for $E \gtrsim 10$ GeV is about 3,000 years, for $\eta \sim 30$ (note that $t_{\text{a}} \propto \eta$), consistent with the age of IC 443 estimated from the hydrodynamical model by Chevalier (1999). On the basis of Ginga observations of IC 443, Wang et al. (1992) suggested an age of about one thousand years and an association with the supernova of AD 837. The reasons for the low age are related to a thermal interpretation of the observed X-ray emission above $10$ keV. We shall argue below for the nonthermal origin of the hard emission in a radiative shock model. A distinctive feature of the radiative shock SNR shell model with direct injection of the electrons from the thermal pool is a hard spectrum of nonthermal emission extending from keV to GeV energies (Fig. 1). There is also a hard emission component due to inverse Compton emission of relativistic electrons having a scale comparable to the scale of the whole radio image of the remnant.

### 3.2. Radio Emission from the Shell

Observations of the radio emission from IC 443 were performed with different instruments over the last 35 years (see, e.g., Erickson & Mahoney 1985; Green 1986; Claassen et al. 1997 and references therein). The 151 and 1419 MHz radio maps (Green 1986) reveal that the most intense flux comes from the shell of the remnant. We suggest that the eastern part of the shell is a radiative shock which has a high magnetic field and electron flux, dominating the synchrotron radiation from the remnant.

We calculated the synchrotron radiation from the remnant using the local emissivity given by equation (A17). The integrated spectrum from the remnant is dominated by the emission from the radiative shell (having magnetic field $B \sim 6 \times 10^{-5}$ G) if the magnetic field in the central parts of the remnant (region $V$) is moderate, $B \leq 4 \times 10^{-6}$ G. For modeling the shell emission, we took into account the interstellar free-free absorption at low radio frequencies, assuming a column density to IC 443 of $N_{H} \approx 2 \times 10^{21} \text{cm}^{-2}$ and an average interstellar medium temperature $T \approx 10^{3}$ K. This column density is similar to that adopted for the recent analysis of ASCA data by Keohane et al. (1997) and is about four times less than the value used earlier by Petre et al. (1988) and Wang et al. (1992). We assumed that the ionization fraction is $\sim 15\%$. The free-free absorption inside the cool radiative shell is not very important above 50 MHz because the emission measure is about $1 \text{ cm}^{-6} \text{ pc}$. The solid and dot-dashed lines on Fig. 1 show the total synchrotron radiation from IC 443 (dominated by shell emission in our model) expected at the Earth compared to the observational data (Erickson & Mahoney 1985). Both of our models are illustrated: direct injection of thermal electrons (Fig. 1) and reacceleration of preexisting cosmic ray electrons by a radiative shock (Fig. 2). A radiative shock with total compression ratio $\delta = 4.4$, including a shock precursor and subshock ($\delta_{s} = 3.3$), was considered. An important feature of the radio observations of IC 443 and W44 is that their spectral indices are relatively flat, implying electron energy spectra flatter than those produced by standard strong shock acceleration in the test particle limit. In our model, the calculated radio spectrum fits that observed from the IC 443 shell because of the large shock compression ratio $\delta_{s} = 4.4$ and to a lesser extent because of the effect of second order Fermi acceleration discussed recently by Ostrowski (1999).

### 3.3. Nonthermal Emission from a Shocked Clump

Molecular clouds show complex structure with an ensemble of dense massive clumps embedded in the interclump medium (Blitz 1993). We considered the possibility of radiative shock interaction with dense molecular clumps, resulting in high energy emission. In IC 443 and W44, those regions do not appear to be significant sites of GHz radio continuum emission, but the clumps may contain most of the cloud mass and are possible sites of hard X-rays and MeV $\gamma$-ray emission. The interaction of a radiative shock with a molecular clump generates a dense slab bounded by two shocks (Chevalier 1999). The forward shock propagating into the clump is strong (see also the simulations of Jones & Kang 1993). Since the clump size is typically smaller or comparable to that of the radiative shock layer, it is embedded in a pool of nonthermal particles created by the radiative shock according to the scenario described above. Accelerated particles as well as ionizing radiation from the hot interior regions of the SNR should provide ionization of the external portions of a dense clump at the level $\gtrsim 1\%$. This is sufficient for a shock propagating into the dense clump with $n \sim 10^{4}$ cm$^{-3}$ and $B \sim 0.1$ mG (e.g., Claassen et al. 1997) to be collisionless. The percent ionization level provides Alfvén Mach numbers $M_{a} \gtrsim 1$ and weak damping of collisionless Alfvén waves up to wavelengths $\sim 10^{10}$ cm for a flow with velocity $\gtrsim 25$ km s$^{-1}$ in the clump. Then the injection and acceleration processes considered above would be efficient and the ionization in the preshock region would be maintained self-consistently at least at a percent level even for a shock propagating inside the dense clump.

The gas ionization by UV radiation in the radiative shock precursor is sensitive to the shock speed if $v_{57} \lesssim 1.1$ (Shull & McKee 1979; Hollenbach & McKee 1989). This implies that a
shock having \( v_{S7} \leq 1 \) propagating in a dense clump would be able to accelerate electrons only below \( E_m < 1 \) GeV (see equation [3]). We shall estimate below also the gas ionization rate due to accelerated nonthermal particles. The lifetime of a pre-existing electron of MeV energy against ionization losses in the interclump medium (\( n \sim 10 \text{ cm}^{-3} \)) is about \( 10^4 \) years. Unless a nearby source of fresh accelerated electrons is present, Galactic cosmic ray MeV electrons are unlikely to be present in the cloud. Direct injection of electrons from the thermal pool of the weakly ionized precursor would be a dominant source of shock ionization [3]). We shall estimate below also the gas ionization rate in the interclump medium (\( \gamma \)).

CGRO few hundreds of parsecs) to be detected with Keohane et al. (1997). The hard X-ray nonthermal emission feature. Nevertheless, these shocks might manifest themselves as hard X-ray and MeV \( \gamma \)-ray emission sources with \( \nu F_{\nu} \propto \nu^a \) \((a \gtrsim 0.5)\) up to a few MeV and with a soft spectrum at higher energies. A comparison of Fig. 3 with Fig. 1 shows that the hard X-ray flux at 7 keV and above could be dominated by the emission from the shocked clumps, which is consistent with ASCA observations of IC 443 presented by Keohane et al. (1997). The hard X-ray nonthermal emission flux may be variable on a timescale of years. A relatively high MeV emissivity is a distinctive feature of shocked clumps, but they are difficult to observe. They must be nearby (within a few hundreds of parsecs) to be detected with COMPTEL/OSS. Cgro.

For a forward shock of velocity \( v_{S7} \leq 1 \) propagating into a dense clump of number density \( n \sim 10^3 \text{ cm}^{-3} \) with upstream ionization level of a few percent \((\sim k_i [E_m] / v_S \text{ distance})\), the upper cut-off energy \( E_m \lesssim 0.5 \) GeV. This is about 40 times less than that for a radiative shock in the interclump medium. The spatial diffusion \( k_0 \) below \( p_s \) scales as \( w_1 / \Lambda \). Since the scale of a postshock radiative cooling layer \( \propto n^{-1} \Lambda \) and \( \Lambda \) is a fraction of that, we have \( k_{10} \sim 3 \times 10^{16} \text{ cm}^2 \text{ s}^{-1} \) in the clump case.

The effective compression of the nonthermal electrons as well as the magnetic field by the system of two shocks bounding the slab could be high. The magnetic field in the clump is expected to be high. Claussen et al. (1997) derived line-of-sight magnetic field strengths of \( \sim 0.2 \) mG, remarkably uniform on the scale of several parsecs. Relativistic electrons accelerated by a shock propagating into a clump should produce radio emission with a flat spectrum up to some hundreds of MHz, which is lower than the maximum radio frequency from the shell because of the lower \( E_m \) in the clumps. The emission measure of a clump interacting with a radiative shock could be \( EM \lesssim 10^4 \text{ cm}^{-6} \text{ pc} \), which is much higher than that of the shell. Thus, the internal free-free absorption could be important here providing a flatter radio spectrum in the 100s of MHz regime. This is possibly a reason for the observed spatial variations of the radio spectral index in IC 443 (e.g., Green 1986).

We present in Fig. 3 the calculated \( \nu F_{\nu} \) spectrum of nonthermal emission from a radiative shock - molecular clump interaction region. The forward shock velocity was \( 100 \text{ km s}^{-1} \), the initial number density in the clump \( 1,000 \text{ cm}^{-3} \), and the shocked clump radius \( 0.5 \text{ pc} \). The shock compression ratio was the same as that in the radiative shell model: \( \delta_i \approx 3 \) and \( \delta_i \approx 4.4 \). The ionization of gas in the shock upstream was about 5% (at the distances \( \sim k_i [E_m] / v_S \)). To account for some uncertainty in the gas ionization, we present the radio spectra for two possible values of the emission measure \( EM \) of the shocked clump: \( EM = 110 \text{ cm}^{-6} \text{ pc} \) (solid line) and \( EM = 900 \text{ cm}^{-6} \text{ pc} \) (dashed line). The diffusion coefficient was \( k_{10} \sim 3 \times 10^{16} \text{ cm}^2 \text{ s}^{-1} \), \( E(p_s) = 1 \) MeV. We illustrate the results for two possible maximal energies of the electrons (depending on the time of shock - clump interaction). Curve 1 in Fig. 3 corresponds to \( E_m = 0.5 \) GeV and \( E_m = 0.05 \) GeV for curve 2. The synchrotron radio spectrum for curve 2 is below the scale of the plot. The radio spectrum corresponding to curve 1 is very hard because of internal absorption in the shocked clump medium.

![Fig. 3.—Broadband \( \nu F_{\nu} \) spectrum for a model of nonthermal electron production by a radiative shock interacting with a molecular clump (distance 1.5 kpc). The model spectra are calculated for the case of a shock velocity in the clump of 100 km s\(^{-1}\), a number density of \( 10^3 \) cm\(^{-3}\), and a magnetic field strength of \( 2 \times 10^{-4} \) G. The electron diffusion coefficient (see equation [2]) is \( k_{10} = 3 \times 10^{18} \text{ cm}^2 \text{ s}^{-1}\), \( a = 1.0\), and \( E(p_s) = 1 \) MeV. The maximum energy of accelerated electrons is \( E_m = 0.5 \) GeV (curve 1) and \( E_m = 0.05 \) GeV (curve 2). The model spectrum for the case of a 30 km s\(^{-1}\) shock in a clump with number density \( 10^4 \) cm\(^{-3}\) is shown by curve 3 (see the text). Interstellar absorption of the radio emission is as in Fig. 1. The internal absorption at the shocked clump is shown for \( EM \approx 110 \text{ cm}^{-6} \text{ pc} \) (solid line radio spectrum) and \( EM \approx 900 \text{ cm}^{-6} \text{ pc} \) (dashed line radio spectrum). The dot-dashed curves are the shell emission spectra from Fig. 1 given for comparison.

We presented above the results for nonthermal emission of a strong radiative shock of \( v_{S7} \sim 1 \) propagating into a molecular clump. For the case of radiative shock interaction with the molecular clumps in IC 443, lower shock velocities, \( v_{S7} \sim 0.3 \), are more realistic (e.g., Cesarsky et al. 1999; Chevalier 1999). The preshock ionization may be dominated mostly by nonthermal particles in such a shock because UV radiation is inefficient. A self-consistent model of MHD collisionless shock propagation into a dense clump requires simultaneous simulations of the MHD turbulence spectral properties and the nonthermal particle kinetics. Instead, we suppose that a collisionless shock transition exists for \( v_{S7} \sim 0.3 \) and \( n \sim 10^4 \) cm\(^{-3}\) and assume the same diffusion model as described in §2. We calculated the upstream gas ionization, finding that the ionized fraction \( x \gtrsim 0.9 \) may hold up to depths about \( 10^{15} \) cm\(^{-2}\) due to shock accelerated electrons. The Mach number of the shock with \( v_{S7} \sim 0.3 \) is typically below 5 because of heating of upstream gas up to \( 10^6 \) K by accelerated particles and the compression ratio is lower than 4. The maximum energy of accelerated electrons in that case is typically below an MeV. We illustrate the calculated \( \nu F_{\nu} \) spectrum of nonthermal emission from a 30 km s\(^{-1}\) velocity shock (\( \delta_i = 3 \)) interacting with a molecular clump of density \( 10^3 \) cm\(^{-3}\) by curve 3 in Fig. 3; this
model corresponds to $k_{110} \sim 7 \times 10^{16} \text{ cm}^2 \text{s}^{-1}$ and $E(p_e) = 10 \text{ keV}$. Such collisionless shocks, if they exist, would provide a softer spectrum of nonthermal radiation than that for high velocity shocks with $v_{\text{SH}} \gtrsim 1$. A more comprehensive study is required to model the nonthermal emission from the low velocity shocks in detail. A search for hard X-ray emission correlated with molecular emission predicted by the simplified model described above is possible with high resolution instruments like Chandra and XMM.

3.4. Hard X-ray Emission

IC 443 was a target of X-ray observations with HEAO 1 (Petre et al. 1988), Ginga (Wang et al. 1992), ROSAT (Asaoka & Aschenbach 1994) and ASCA (Keohane et al. 1997). ASCA GIS observations discovered the localized character of the hard X-ray emission (Keohane et al. 1997). Most of the 2-10 keV GIS photons came from an isolated emitting feature and from the southeast elongated ridge of hard emission. The ridge is also coincident with the 95% confidence error circle of the EGRET CGRO source (Esposito et al. 1996), while the isolated feature is outside the circle (Keohane et al. 1997). The integrated 2-10 keV flux observed by ASCA GIS was $(5\pm1) \times 10^{-11} \text{ erg cm}^{-2} \text{s}^{-1}$, representing more than 90% of the total flux from the SNR (Keohane et al. 1997). The hard X-ray ridge, as defined by Keohane et al. (1997), is only a part of the more extended radio shell discussed in the previous section. The flux density at 7 keV from the isolated emitting feature was about $4 \times 10^{-2} \text{ photons cm}^{-2} \text{s}^{-1} \text{ keV}^{-1}$ and that from the ridge was $2 \times 10^{-2} \text{ photons cm}^{-2} \text{s}^{-1} \text{ keV}^{-1}$. HEAO A-2 measured a flux at 2-10 keV of $(7\pm1) \times 10^{-11} \text{ erg cm}^{-2} \text{s}^{-1}$ (Petre et al. 1988), while Ginga measured a 2-20 keV flux of about $9 \times 10^{-11} \text{ erg cm}^{-2} \text{s}^{-1}$ (Wang et al. 1992). These numbers are consistent with the assumption that the integrated flux is dominated by the extended soft component coming from the central parts of the remnant.

The soft X-ray 0.2–3.1 keV surface brightness map of IC 443 from the Einstein Observatory (Petre et al. 1988) shows bright features in the northeastern part of the remnant. The presence of nearly uniform X-ray emission from the central part of the remnant is a characteristic feature of mixed-morphology SNRs (Rho & Petre 1998). It might be due to the effect of thermal conduction (Cox et al. 1999; Shelton et al. 1999), although Harrus et al. (1997) argued for an alternative radiative shock model for W44, another mixed-morphology SNR. That feature in our model corresponds to the emission from hot ($T \lesssim 10^7 \text{ K}$), low density gas interior to the shock (region V).

In the radiative shock model with direct injection from the thermal pool and second order Fermi acceleration in postshock cooling layer, most of the hard X-ray emission ($\gtrsim 7 \text{ keV}$) comes from the shell - radiative shock structure with a rising $\nu F_{\nu}$ X-ray spectrum (curve 1 in Fig. 1 for the shell and curves 1 and 2 in Fig. 3 for a shocked molecular clump). In addition to the shell, there is a more extended source of hard X-rays from inverse Compton emission. It is the dominant source of hard X-rays in the scenario with cosmic ray electron reacceleration (Fig. 2) and in the case of a lack of second order Fermi acceleration in the radiative shell (see curve 2 in Fig. 1). In the scenario with reacceleration of cosmic ray electrons, it is impossible to obtain the 7 keV flux observed by ASCA from the IC 443 remnant. Thus, a resolved hard X-ray image of the IC 443 remnant may be very informative in determining the electron acceleration scenario.

The isolated emitting feature in the southern part of IC 443 detected by ASCA and brightest around 7 keV, has a flat spectrum low frequency radio continuum source (Green 1986) and is close to a region of excess H$^2$ emission. High spatial resolution observations of this isolated emitting feature are required to decide between a low-luminosity pulsar nebula and shock interaction models (e.g., radiative shock interaction with a dense molecular clump, Fig. 3). Observations of the hard emission with the sensitivity and resolution available with BeppoSAX, Chandra XRO, XMM and GLAST would be required to constrain the extension and spectral properties of the hard X-ray emission from IC 443.

3.5. Gas Ionization by Nonthermal Electrons

Nonthermal particles accelerated by a MHD collisionless shock wave provide an efficient ionizing agent. We calculated above the nonthermal emission generated by accelerated energetic electrons; one can also calculate the ionization produced by the same electrons in the molecular cloud. This might connect the high energy observations with radio, IR and optical emission from shocked atomic and molecular gas. To estimate the gas ionization due to accelerated electrons in the radiative shock structure we used the electron spectra calculated with the kinetic model of electron acceleration and propagation described in §2 and electron ionization cross sections from Appendix B. In Fig. 4, we present the calculated ionization rate $\zeta_e$ and estimated ionized fraction $x$ in the shell of a radiative shock due to accelerated electrons for the case of $v_{\text{SH}} = 1.5$ for both models of large-scale turbulence in the postshock cooling layer considered in §3.1.
cleons and from UV radiation which are not included in Fig. 4. We accounted for only radiative recombination, assuming a small molecular fraction in the radiative shell at column densities below $2 \times 10^{20} \text{cm}^{-2}$.

The solid curves in both panels of Fig. 4 correspond to the case of fully developed Alfvenic turbulence in the postshock cooling layer, which provides efficient second order Fermi acceleration making Coulomb losses relatively unimportant above $E_C = 120 \text{ keV}$. This case corresponds to curve 1 in Fig. 1 for emission from the shell with substantial hard X-ray emission. The dotted lines in Fig. 4 correspond to the case of a lack of large scale MHD turbulence where the Coulomb losses overcome the second order Fermi acceleration up to $E_C = 2 \text{ GeV}$. The nonthermal emission expected in this case is illustrated by curve 2 in Fig. 1. The radiative shell ionization by nonthermal electrons is sensitive to the turbulent structure of the postshock cooling layer and there is a correlation between the hard X-ray spectrum and the ionization structure of the radiative shell. We shall consider the effect of nonthermal particles on the ionization structure of a shocked clump elsewhere.

3.6. Energy in Nonthermal Electrons

The energy of the electron component for the parameter set used to compute the spectra in Fig. 1 is substantial. The power required for electron acceleration and maintenance for curve 1 in Fig. 1, $\sim 1.5 \times 10^{37} \text{ erg s}^{-1}$, is high because of the strong Coulomb losses of keV electrons in the dense ionized medium, but it is consistent with the estimate of the total radiative losses from IC 443, dominated by infrared emission (Mufson et al. 1986; Burton et al. 1990). The implied power is required for both models 1 and 2 in Fig. 3, where radiative shock - molecular clump interaction is illustrated. For curve 2, the power requirements are less, $\sim 6 \times 10^{36} \text{ erg s}^{-1}$. The implied efficiency of power conversion from the MHD shock flow (which can be estimated as $3 \times 10^{38} \text{ erg s}^{-1}$) to the nonthermal electrons is about 5% in these cases. The model with $\gamma_{\gamma} \approx 1$ would provide emission similar to that shown in Fig. 1, but requires about three times higher efficiency. This is higher than the electron acceleration efficiency estimated from GeV regime cosmic rays observed near the Earth, but evolved SNRs interacting with dense molecular gas probably cannot be considered as the main source of the observed Galactic cosmic rays because of the relatively low maximum energies of accelerated particles.

The pressure of the nonthermal electron component downstream of the shock (region $IV$) is $P_e \approx 7 \times 10^{-11} \text{ erg cm}^{-3}$. The magnetic pressure in the dense shell is about $1.5 \times 10^{-10} \text{ erg cm}^{-3}$, which is higher than the thermal gas pressure. The uniform ($\sim 60 \mu \text{G}$) and stochastic magnetic field components dominate the total pressure in the shell.

Let us consider the energy requirements for the scenario with electron injection from preexisting cosmic rays (Fig. 2). The nonthermal electron pressure in this case is $P_e \approx 5.2 \times 10^{-11} \text{ erg cm}^{-3}$ (for $E_{\gamma} \leq 8 \text{ MeV}$), similar to that for the model with injection from the thermal pool, because it is dominated by GeV particles. An advantage of this scenario is that much less power is required. It is about $4 \times 10^{35} \text{ erg s}^{-1}$, which is less than 10% of that for the scenario with injection from the thermal pool. The difference is because Coulomb losses are unimportant for the high energy electrons involved in that scenario. However, a potential problem is the relatively low $E_{\gamma}$ required. The lifetime of a 20 MeV electron against ionization losses in a neutral medium of number density $\sim 1 \text{ cm}^{-3}$ is about 10$^6$ years. This implies that the source of MeV cosmic ray electrons should exist within about 100 pc of the molecular cloud if the diffusion coefficient is $\sim 3 \times 10^{26} \text{ cm}^2 \text{s}^{-1}$ at these energies.

4. DISCUSSION AND FUTURE PROSPECTS

For individual supernova remnants, the ambient density is an important parameter that can be estimated from multiwavelength observations (Chevalier 1999). Another important parameter for modeling the nonthermal particles in a SNR is the molecular cloud is the collisionless MHD turbulence spectrum, particularly in the radiative shock cooling layer. A substantial level of MHD collisionless turbulence could overcome Coulomb losses in the dense plasma downstream from the radiative shock. An accurate model of MHD turbulence in the postshock cooling layer is not available now. Thus we considered both limiting cases described in the § 2 and show in Fig. 1 the uncertainty introduced by the lack of data concerning the MHD turbulence properties. Coulomb losses are important in the postshock cooling region only for electrons with energies below $E_C \approx 20 \text{ keV}$ if the large scale turbulent velocity has a substantial longitudinal component of $\sim 20 \text{ km s}^{-1}$. A higher value ($E_C \approx 8 \text{ MeV}$) is expected in the absence of a longitudinal component of large scale turbulence in the postshock cooling layer because the Coulomb losses are overcome by resonant interaction with Alfven waves. Since the Alfven velocity is relatively low ($\sim 5 \text{ km s}^{-1}$), the value of $E_C$ is much higher in that case. We used a conservative minimum value of $E_C = 120 \text{ keV}$ for the radiative shock structure described above, although one could expect $E_C \sim 20 \text{ keV}$ in the most favorable case. The nonthermal emission of SNR in a molecular cloud (especially in the hard X-ray regime) and ionization structure of the radiative shock are sensitive to the MHD turbulence model. Spatially resolved observations of the nonthermal emission from SNRs may be able to constrain models of MHD turbulence.

All of the SNR candidates from the $EGRET$ list: IC 443, $\gamma$ Cyg, W44, and Monoceros (Esposito et al. 1996) are old remnants interacting with molecular clouds. The $\gamma$-ray emission from electrons accelerated by the radiative shock calculated for the IC 443 parameters presented in Figs. 1 and 2 is in good agreement with that observed by $EGRET$ (Esposito et al. 1996), as well as with the upper limits established by Whipple (Buckley et al. 1998). The high energy $\gamma$-rays ($\gtrsim 50 \text{ MeV}$) from the radiative shell should be spatially correlated with the radio emission. The $EGRET$ telescope detected an extended excess ($95\%$ confidence circle $\sim 42'$ in radius) from the Monoceros SNR correlated with the 1.42 GHz radio emission (Esposito et al. 1996). This is in accordance with the model of a leptonic origin of GeV emission from extended SNR interactions with clouds (de Jager & Mastichiadis 1997). $GLAST$ (Gamma-ray Large Area Space Telescope) will be an excellent instrument for future $\gamma$-ray observations, because it will have the capability of spatially resolving the $\gamma$-ray emission from SNRs.

Spatially resolved observations of W44, IC 443, 3C391 and some other mixed-morphology SNRs from the list given by Rho & Petre (1998) with $BeppoSAX$, $Chandra$ $XRO$, $XMM$ would be very valuable tools to test our model. The hard X-ray detector (HXD) aboard the forthcoming $ASTRO-E$ mission could be used for observations of the hard 10–700 keV continuum with a field of view of 0.8° FWHM at 60 keV and 2.8° at 500 keV from the extended hard X-ray emission from IC 443, W44 and the Monoceros SNR predicted by the radiative shock model (see Fig. 1 and Fig. 3). Spatially resolved spectra are needed to distinguish the shell emission from the shocked clump emis-
Molecular clumps interacting with moderately fast radiative shocks are also expected to be sources of hard X-rays and MeV γ-rays (up to 100s of MeV). Nonthermal continuum radio emission (100s of MHz) with a sharply rising νFν spectrum (Fig. 3) and a time dependent cut-off frequency is expected in the fast shock model. Due to potentially substantial internal free-free absorption, the spectrum of radio emission from a localized clump might constrain the ionized gas density in the clump. The MeV γ-ray spectrum of a localized clump may be resolved with the forthcoming γ-ray missions INTEGRAL (e.g., Schönfelder 1999; Winkler 1999) and GLAST. With an expected angular resolution about 12′ FWHM, imager IBIS aboard INTEGRAL would allow detection of hard X-ray (∼ 50 keV) emission from molecular clouds within ∼ 1 kpc. A comparison of hard emission spectra with the radio spectrum can provide valuable information about the density and magnetic field in a clump. A possible variability on a timescale of a few years for clump hard emission and radio emission (on a longer time scale) could further constrain the model.

For low velocity (below ∼ 30 km s⁻¹) shocks interacting with a dense (∼ 10⁴ cm⁻³), magnetized (B ∼ 0.5 mG) molecular clump, one may expect hard X-ray emission below the MeV regime correlated with the regions of molecular emission of shocked gas. Nonthermal radio continuum is not expected in the case of low velocity shocks because of a lack of accelerated relativistic electrons at sufficiently high energies.

Finally, we note that the high density of energetic particles in the vicinity of a shock wave in a molecular cloud can affect the ionization and thermal properties of the gas. Rich molecular spectra have been observed from IC 443 and 3C391 (Reach & Rho 1999). It will be interesting to see whether there is a signature of the presence of nonthermal particles that can be discerned from the molecular spectra.

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APPENDIX

APPENDIX A: RADIATIVE PROCESSES

Given the electron distribution function, the high energy emission flux J(Eγ) at a distance R from the source can be calculated from the equation:

\[ J(E_\gamma) = \frac{1}{4\pi R^2} \int dV \frac{dn_\gamma(E_\gamma,\vec{r})}{dt} \]

where \( dn_\gamma(E_\gamma,\vec{r})/dt \) is the emissivity, which here includes bremsstrahlung, synchrotron radiation and the inverse Compton effect. We generally assumed a pure hydrogen composition in our calculations.

High energy particles penetrating through a partially ionized medium produce photons due to interactions with atoms, electrons and ions. We assumed a Maxwellian distribution for the ambient matter and in most cases took the target ambient particles to be at rest. Then, the emissivity can be calculated from

\[ \frac{dn_\gamma(E_\gamma)}{dt} = 4\pi \int_{E_\gamma}^{\infty} dE_e N_e \sigma_{ep}(E_\gamma, E_e) J(E_e) + \int_{E_{\text{min}}}^{E_\gamma} dE_e N_e \sigma_{ee}(E_\gamma, E_e) J(E_e), \]

where \( J(E_e) \) is the electron flux measured in units s⁻¹ cm⁻² keV⁻¹ sr⁻¹, \( \sigma_{ep} \) and \( \sigma_{ee} \) are differential cross sections of one photon emission due to electron-proton and electron-electron interactions integrated over angles, and \( E_{\text{min}} \) is the minimum energy of incoming electron that could generate a photon with energy \( E_\gamma \). For e-p interactions, \( E_{\text{min}} = E_\gamma \) is a good approximation. The correction due to finite proton mass is less than the uncertainty in the cross section. In the case of e-e interactions, we used the following equation for \( E_{\text{min}} \):

\[ E_{\gamma} = E_{\text{min}} m_e c^2 / \left[ 2m_e c^2 + E_{\text{min}} - \sqrt{E_{\text{min}}(E_{\text{min}} + 2m_e c^2)} \right]. \]

The inverse bremsstrahlung contribution is negligible unless there is a very high (far in excess of 100) ratio of protons to electrons.

We used the following cross section approximations in our calculations:

**e-e bremsstrahlung.**

Haug (1975) has obtained a general formula for the e-e bremsstrahlung cross section, but it is more convenient to use simplified approximations. We used relativistic and non-relativistic asymptotic expressions, which were matched in the intermediate energy region (see Baring et al. 1999).

In the relativistic limit, we used approximate formulae given by Baier et al. (1967) with a correction factor from Baring et al. (1999):

\[ \sigma_{ee}(E_e, E_\gamma) = (\sigma_1 + \sigma_2)A(\epsilon_\gamma, \gamma_e), \]

where \( \sigma_1 \) and \( \sigma_2 \) are from Baier et al. (1967) and the factor \( A \) was determined by Baring et al. (1999).

Equations (A4) and (A6) are somewhat different from that given by Baring et al. (1999). M.G. Baring (private communication) made available the corrections to their original formulae.

\[ \text{Equation (A4)} \text{ and (A6) are somewhat different from that given by Baring et al. (1999). M.G. Baring (private communication) made available the corrections to their original formulae.} \]
where $\gamma_e = (E_e + mc^2)/mc^2$, $\epsilon_e = E_e/mc^2$, 

$$
\sigma_1(\gamma_e, \epsilon_e) = \frac{4r_0^2\alpha}{\epsilon_e} \left[ 1 + \left( \frac{3 - \epsilon_e}{\gamma_e - 1} \right) \left( 1 - \frac{\epsilon_e}{\gamma_e - 1} \right) \right] \left[ \ln \left( \frac{2(\gamma_e - 1)\gamma_e - 1 - \epsilon_e}{\epsilon_e} \right) - \frac{1}{2} \right], \quad (A4)
$$

$$
\sigma_2 = \frac{r_0^2\alpha}{3\epsilon_e} \left\{ \frac{16(1-\epsilon_e + \epsilon_e^2)\ln \left( \frac{2c}{\epsilon_e} \right)}{\epsilon_e^2 - \gamma_e - 1} - \frac{3}{2} \epsilon_e + 4 + 4\epsilon_e - 8\epsilon_e^2 \right\} - (A5)
$$

$$
\frac{2}{\epsilon_e} \left[ \left( \frac{4}{\epsilon_e} + \frac{2}{\epsilon_e^2} \right) \ln (2\gamma_e) - 2 + \frac{1}{\epsilon_e} - \frac{1}{2\epsilon_e^2} \right], \quad \epsilon_e \leq \frac{1}{2},
$$

$$
\frac{2}{\epsilon_e} \left[ \left( \frac{4}{\epsilon_e} + \frac{2}{\epsilon_e^2} \right) \ln (2\gamma_e) - 2 + \frac{1}{\epsilon_e} - \frac{1}{2\epsilon_e^2} \right], \quad \epsilon_e > \frac{1}{2},
$$

\(r_o = e^2/m_e c^2\) is the classical electron radius, $\alpha = e^2/\hbar c$, and

$$
A(\epsilon_e, \gamma_e) = 1 - \frac{10(\gamma_e - 1)^{\frac{1}{2}}}{3\gamma_e + 1} \left( \frac{\epsilon_e}{\gamma_e} \right)^{\frac{1}{2}}, \quad (A6)
$$

According to Baring et al. (1999), these formulae agree with those from Haug (1975) with an accuracy $\sim 10\%$ for electrons of energy $\sim 5$ MeV.

In the non-relativistic limit, we used equations from Garibyan (1952) and Fedynushin (1952) with correction coefficients from Baring et al. (1999):

$$
\sigma_{ee} = \frac{4r_0^2\alpha}{15\epsilon_e} F \left( \frac{4\epsilon_e}{\gamma_e^2 - 1} \right), \quad (A7)
$$

where the photon energy is in the range $0 < \epsilon_e < \frac{1}{4}(\gamma_e^2 - 1)$ and $F(x) \; (0 < x < 1)$ has the form:

$$
F(x) = B(\gamma_e) \left[ 17 - \frac{2}{(2x)^x} - C(\gamma_e, x) \right] \sqrt{1-x} + \left[ 12(2-x) - \frac{2}{2(2x)^x} \right] \ln \left( \frac{1+\sqrt{1-x}}{\sqrt{1-x}} \right). \quad (A8)
$$

The correction coefficients are:

$$
B(\gamma_e) = 1 + \frac{1}{2}(\gamma_e^2 - 1); \quad C(\gamma_e, x) = \frac{10x\gamma_e^2(2+\gamma_e^2)}{1+x^2(\gamma_e^2 - 1)}, \quad (A9)
$$

Baring et al. (1999) found less than a $10\%$ difference between equation (A7) and the expressions of Haug (1975) for electron energies $E_e \lesssim 500$ keV.

**e-p bremsstrahlung.**

In the non-relativistic limit, we took the e-p bremsstrahlung cross section from Akhiezer & Berestetsky (1957):

$$
\sigma_{ep} = \frac{16r_0^2Z^2\alpha m_e c^2}{3p_1^2} \left( \epsilon_e^2 - 1 \right) \left( 1 - e^{2\pi\zeta_1} \right) \frac{1}{1 + x^2(\gamma_e^2 - 1)} \ln \left( \frac{p_1 + p_2}{p_1 - p_2} \right), \quad (A10)
$$

which is valid if $\zeta_1 \ll 1$; here $\zeta_1 = \alpha Z/\beta_1$.

For the relativistic limit, we used Born approximation formulae from Akhiezer & Berestetsky (1957):

$$
\sigma_{ep} = \frac{r_0^2Z^2\alpha p_2}{3p_1^2} \left( \frac{4 - 2\epsilon_1\epsilon_2}{p_1 p_2} \right) + \frac{\ln \left( \frac{1 + \epsilon_1}{1 - \epsilon_1} + \frac{1 + \epsilon_2}{1 - \epsilon_2} \right)}{\ln p_1 p_2} + \frac{L}{\ln p_1 p_2} \left( \frac{\epsilon_1 \epsilon_2}{p_1 p_2} + \frac{\epsilon_2 \epsilon_2}{p_1 p_2} + \frac{\epsilon_1 \epsilon_2}{p_1 p_2} \right), \quad (A11)
$$

where $L = \ln \left( \frac{p_1 p_2 + 1 - 2 \epsilon_1 \epsilon_2}{m_e c^2} \right)$, $\eta_1 = \ln \left( \frac{1 + \epsilon_1}{1 - \epsilon_1} \right)$, $\eta_2 = \ln \left( \frac{1 + \epsilon_2}{1 - \epsilon_2} \right)$. In the velocity range $Z\alpha(\beta_2^{-1} - \beta_1^{-1})$, we included the Elwert factor from Koch & Motz (1959): $f_E = \frac{\beta_1(1 - \exp(-2\pi \alpha Z \alpha/\beta_1))}{\beta_1(1 - \exp(-2\pi \alpha Z \alpha/\beta_2))}$.

In equation (A11), $c = 1$ is assumed. In the intermediate energy range, we matched equation (A10) and equation (A11).

**Synchrotron radiation.**

We used standard formulae for synchrotron radiation in our calculations (e.g., Ginzburg 1979). The emissivity can be written as:

$$
E_\gamma \frac{d\epsilon}{dt} = \frac{\sqrt{\alpha} \alpha}{m_e c} \int d\chi dEN(E, \chi) H \sin^2(\chi) F \left( \frac{\chi}{\mu} \right),
$$

$$
F(x) = x \int K_{5/3}(\eta)d\eta, \quad (A12)
$$
where $\chi$ is the angle between the electron velocity and the magnetic field, $N(E, \chi)$ is the electron distribution function over energy and angle, $\nu_c = \frac{3eH\sin(\chi)}{4\pi m_e c} \left( \frac{E}{m_e c^2} \right)^2$, and $K_{3/2}(x)$ is the McDonald function. Integrating equation (12) over angle under the assumption of a chaotic magnetic field orientation, we obtain the equation

$$E_\gamma \frac{dn_\gamma(E_\gamma, \vec{\theta})}{dt} = \frac{\sqrt{3} \,\alpha e}{2\pi m_e c} \int dE \, N(E, \vec{\theta}) H_{\perp} F \left( \frac{\nu_\gamma}{\nu_c} \right),$$

(A13)

where $N(E, \vec{\theta})$ is the electron distribution function as a function of energy and position and $H_{\perp} = \sqrt{2/3} H(\vec{\theta})$ is the averaged magnetic field.

**Inverse Compton emission**

High energy electrons colliding with photons result in high energy photon production (e.g., Jones 1968; Gaisser et al. 1998; Sturmer et al. 1997; Baring et al. 1999). If the electron energy $E_\gamma \ll \frac{m_e^2 c^4}{eH}$ $\approx \frac{6 \times 10^{10}}{E_\gamma (eV)}$ eV ($E_\gamma$ is the energy of the emitted photon), then the formulae in the Thompson limit are (e.g., Ginzburg 1979):

$$\frac{dn_\gamma(E_\gamma)}{dt} = \sqrt{\frac{3\pi}{\alpha} \sigma_T m_e c^2} \int dE_{ph} \frac{E_\gamma}{E_{ph}} N_{ph}(E_{ph}, \vec{R}) J_\epsilon \left( \frac{m_e c^2}{4E_{ph}} \frac{3E_\gamma}{E_{ph}} \frac{\vec{R}}{R} \right),$$

(A14)

where $\sigma_T = \frac{8\pi}{\alpha} \left( \frac{e}{m_e c} \right)^2$ is the Thompson cross section, $N_{ph}(E_{ph})$ is the background photon energy spectrum, and $J_\epsilon(E)$ is the incoming electron flux.

If the electron energy $E \geq \frac{m_e^2 c^4}{4eH} \approx \frac{6 \times 10^{10}}{E_\gamma (eV)}$ eV, we must use the Klein-Nishina formulae (integrated over angles):

$$\frac{dn_\gamma(E_\gamma)}{dt} = 4\pi \int dE_{ph} dE_{ph} N_{ph}(E_{ph}) \sigma_{KN}(E_\gamma, E_c, E_{ph}) J_\epsilon,$$

(A15)

$$\sigma_{KN}(E_\gamma, E_c, E_{ph}) = \frac{2\pi r_e^2}{c^2 \epsilon_{ph} m_e c^2} \left[ 2q \ln(q) + 1 + q - 2q^2 + \frac{q^2 (1-q)^2}{2(1+q)} \right],$$

(A16)

where $\Gamma = 4\epsilon_{ph}\gamma_e$, $q = \frac{\epsilon_\gamma}{(\gamma_\gamma - \epsilon_e)\Gamma}$, $0 \leq q \leq 1$.

In the above, $\epsilon_\gamma = E_\gamma / m_e c^2$, $\epsilon_{ph} = E_{ph} / m_e c^2$, $\gamma_e$ is the electron Lorentz factor, and $r_e = e^2 / m_e c^2$ is the classical electron radius. The interstellar background photon spectrum, $N_{ph}(E_{ph}, R)$, was adopted from Mezger et al. (1982) and Mathis et al. (1983) taking into account infrared data from Saken et al. (1992) for the case of IC443 (see also Gaisser et al. 1998).

**Free-free absorption**

Radio waves propagating in the partially ionized thermal plasma are subject to thermal free-free absorption. For $h\nu \ll kT$ (the Rayleigh – Jeans regime), the free-free absorption coefficient $\alpha_{ff}(\nu)$ (cm$^{-1}$) can be calculated from

$$\alpha_{ff} = \frac{4e^6}{3m_e k c} \left( \frac{2\pi}{3km_e} \right)^{1/2} T^{-3/2} Z^2 n_e n_i \nu^{-2} g_{ff}.$$  

(A17)

Using an approximate formula for the Gaunt factor $g_{ff}(\nu, T)$ from Rybicki & Lightman (1979) appropriate to our parameter range, we obtained a simplified expression for the optical depth $\tau(\nu, T)$

$$\tau \approx 0.022 \nu_{100}^{-2.1} T_{100}^{-1.34} EM,$$

(A18)

where the radio wave frequency $\nu_{100}$ is measured in units of 100 MHz, the plasma temperature in units of 100 K is $T_{100}$, and the emission measure $EM$ is in units of cm$^{-6}$ pc.

**APPENDIX B: ELECTRON IONIZATION**

The primary ionization rate $\zeta_e$ due to electron impact was calculated from the equation:

$$\zeta_e = 4\pi \int_{E_{min}}^{\infty} dE \sigma_{eion}(E) J_e(E),$$

(B1)

where $J_e(E)$ is the electron flux measured in units s$^{-1}$ cm$^{-2}$ keV$^{-1}$ sr$^{-1}$, $\sigma_{eion}$ is the differential cross section for hydrogen atom primary ionization by electron-atom interactions integrated over atomic electron states, and $E_{min}$ is the minimum energy of the ionizing incoming electron.
In the regime below 1 keV, we obtained the following fit to the ionization cross section \( \sigma_{\text{ion}} \) using experimental data from Fite & Brackman (1958):

\[
\sigma_{\text{ion}}(E) = \pi a_0^2 A_1 \exp(-bA_1 - b^2 A_2 - b^2 A_0) / E(\text{eV})^3,
\]

where \( b = A_2 / E(\text{eV}) \) and \( E(\text{eV}) \) is the incoming electron energy measured in eV. The fitting coefficients are \( A_1 = 76.11 \), \( A_2 = 14.34 \), \( A_3 = 0.89 \), \( A_4 = 3.82 \), \( A_5 = -2.55 \), and \( A_6 = 4.5 \). \( a_0 = h^2 / m_e e^2 \) is the Bohr radius.

At high energies ( \( \gtrsim 1 \) keV), the Born approximation provides good accuracy (e.g., Mott & Massey 1965):

\[
\sigma_{\text{ion}} = \int_0^{k_{\text{max}}} I_{0n}(K) dK d\kappa, \quad (B3)
\]

where

\[
I_{0n}(K) = \frac{2\pi Z^{10} \kappa}{a_0^2 k^2 K} \left[ K^2 + (\mu^2 + \kappa^2)^2 / 3 \right]^{\frac{1}{2}} \exp \left[ -\frac{2\mu}{\kappa} \text{arctg} \left( \frac{2\mu K}{K^2 - \kappa^2 + \mu^2} \right) \right] \frac{1}{1 - \exp(-2\mu / \kappa)}, \quad (B4)
\]

\( \mu = Z / a_0 \), \( k(E) \) is the incoming electron wave number, \( k_\kappa \) is the outgoing electron wave number, and \( \kappa \) is the emitted atomic electron wave number. In addition, \( K = |K - K_\kappa| \), \( K_{\text{max}} = k + k_\kappa \), and \( K_{\text{min}} = k - k_\kappa \).

We found a convenient approximation providing a reasonably accurate fit (better than 10% above 1 keV and better than 5% above 3 keV) to equations (B3) and (B4):

\[
\sigma_{\text{ion}}(E) = \pi a_0^2 P_1 E(\text{eV})^{-p_2}, \quad (B5)
\]

where \( P_1 = 97.72 \) and \( P_2 = 0.93 \).

The gas ionization fraction \( x = n_i / (n_i + n_n) \) (we consider here a hydrogen plasma) can be expressed through the ionization rate \( \zeta \) and recombination rate \( \alpha(T) \) as:

\[
x = \left( s^2 / 4 + s \right)^{0.5} - s / 2 \quad (B6)
\]

where \( s = \zeta / (\alpha n) \). We used the radiative recombination rates given by Spitzer (1978) and dissociative recombination rates from Schneider et al. (1994).

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