PAM-4 eye-opening monitoring techniques based on Gaussian mixture model fitting

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Abstract: Four-level pulse amplitude modulation (PAM-4) data formats are adopted to achieve next-generation high-speed data transmission standards. In this letter, a novel eye-opening monitoring technique based on machine learning is proposed to evaluate the received signal quality for the adaptive coefficients setting of a transmitter feed-forward equalizer for PAM-4 signaling. The monitoring technique employs a Gaussian mixture model (GMM) to classify the received PAM-4 symbols. Simulation and measured results of the coefficient adjustment using the GMM method are presented.

Keywords: multi-valued logic, PAM-4, eye monitoring, Gaussian mixture model

Classification: Transmission Systems and Transmission Equipment for Communications

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1 Introduction

High-speed Input/output (I/O) data rates are continually increasing to fulfil the requirement of aggregate I/O bandwidth for chip-to-chip links, backplanes, and data-center transmission. However, as the rate increases, signal distortions due to the intersymbol interference (ISI) caused by the finite bandwidth of wire channels, and reflection, significantly limit the I/O bandwidth. Therefore, advanced signaling techniques, such as the four-level pulse amplitude modulation (PAM-4) data format, are employed to improve signal bandwidth. PAM-4 signaling can transmit data at the same baud rate using a twice-slower symbol rate. Moreover, feed-forward equalizer (FFE) and pre-emphasis are used to mitigate the ISI effects at the transmitter side [1, 2, 3, 4]. Because the FFE can be implemented using simple digital circuitry, the parameters of FFE circuits can be adaptively adjusted following the transmission line characteristics. An eye-opening monitoring (EOM) technique [5, 6] is used to evaluate the quality of the received signal. However, because PAM-4 has three eyes, the EOM algorithm and circuitry for PAM-4 tend to be complicated.

To address the above problems, we propose a machine learning method for statistical symbol detection from deteriorated transmitted PAM-4 signaling to perform adaptive FFE coefficient setting. The parameter optimization problem for the unknown transmission characteristics can be considered to be an unsupervised learning problem. Thus, an EOM method is developed, using a Gaussian mixture model (GMM) to estimate the effect of waveform distortion. Each PAM-4 symbol level exhibits four Gaussian distributions at the receiver side, and the distributions are mixed together owing to ISI. GMM fitting can classify the mixed distributions into each transmitter PAM-4 symbol. Therefore, we can adjust the FFE parameters to eliminate ISI, obtaining the PAM-4 eye-opening diagram. In this letter, simulation and measured results of the coefficient adjustment using the GMM method are presented to demonstrate the feasibility of the proposed PAM-4 EOM technique.

2 Evaluation of received PAM-4 waveform distortion using Gaussian mixture model

Eye diagrams at the receiver are widely used to evaluate the signal quality of received waveforms in serial links. Figure 1(a) shows the measured examples of PAM-4 eye diagrams and the corresponding histogram of symbols distribution in a vertical region. The vertical histogram has four distinct and separate distributions. The distribution of each symbol exhibits Gaussian distribution because of ISI. As the data rate increases, the ISI also increases, which broadens the Gaussian distribution of each symbol. Subsequently, the four Gaussian distributions overlap each other,
making it difficult to detect the symbol. The conventional EOM technique cannot detect the symbol from the histogram composed of superposed distributions [6].

The estimation of the received symbols passing through an unknown transmission characteristic can be considered as unsupervised learning. Therefore, by applying GMM fitting for estimating symbol distributions against an unknown transmission line, a novel PAM-4 EOM technique relying on a statistical classification is proposed.

A linear combination of multiple Gaussian distributions can be modeled as a GMM [7], which is expressed as follows:

\[
p(x) = \sum_{k=1}^{K} \pi_k N(x|\mu_k, \sigma_k),
\]

where \(N(x|\mu, \sigma)\) has a Gaussian distribution with a mean \(\mu\) and variance \(\sigma\), \(\pi_k\) is the mixing coefficient corresponding to the weight for each Gaussian distribution, which is normalized as \(\sum_{k=1}^{K} \pi_k = 1\). A GMM is a probabilistic model that assumes that all data points are generated from a mixture of unknown Gaussian distributions. By GMM fitting from the distribution of the observed data samples, we can estimate the mean and variance of each Gaussian distribution symbol. Accordingly, the received PAM-4 symbol can be separated and estimated.

In the case of PAM-4 symbol GMM estimation, we can assume that the received symbols are a mixture of four Gaussian distributions. Typically, the expectation maximization (EM) algorithm is used for estimating the parameters, i.e., mean, variance, and weight, of each Gaussian distribution. The EM algorithm repeatedly calculates a solution that maximizes the likelihood of the observed values, and the convergence calculation is executed using the following steps: (1) Initial value setting, (2) Expectation step, (3) Maximization step, and (4) Convergence check.
tation step, (3) Maximization step, (4) Repeat step (2) until convergence is achieved (Fig. 1(b)). The initial value is set by using a $k$-means clustering method and setting appropriate initial groups from the observed values. Subsequently, steps (2)-(4) are repeated until convergence is achieved.

Figure 1(c) shows an overview of the proposed GMM-based PAM-4 EOM technique. First, the histogram of the sampling values of PAM-4 signal at the receiver is developed. Subsequently, the symbol distributions are clustered using GMM fitting. The average and variance of each PAM-4 symbol can be estimated by fitting each symbol distribution with GMM from the sampling values at the receiver. By clustering the symbol distribution with GMM fitting, we can evaluate the effect of the distortion at the receiving end even when the eye is entirely closed. In GMM, the distance between symbols can be evaluated from the average values $\mu_k$. Moreover, the effect of ISI at each symbol is evaluated from the variance values $\sigma_k$.

3 Simulation and measured results for PAM-4 eye monitoring

Figure 2(a) shows the simulation results of 2 Gb/s PAM-4 transmission over a 1 m micro-strip line (MSL) and the symbol distribution histograms at the receiving end. In the simulation, the signal response and distortion were calculated using the frequency characteristics of the measured MSL’s S-parameters. As shown in the histogram of 2 Gb/s PAM-4 received signal (Fig. 2(b)), the symbol values spread owing to the ISI effect; hence, the eye height and width decrease. However, the eye is opened enough to detect each symbol level. Figure 2(b) also shows the GMM estimation results, which are fitted as four Gaussian distributions. In this simulation, we assumed that the effect of ISI in each symbol is almost similar, and the variance of each distribution is equal. Each symbol distribution of 2 Gb/s symbol is fitted using

![Fig. 2. Simulation result of PAM-4 data transmission (a) eye-diagram at 2 Gb/s, (b) the result of GMM fitting of symbol distribution histogram at 2 Gb/s, (c) eye-diagram at 4 Gb/s, (d) the result of GMM fitting of symbol distribution histogram at 4 Gb/s](image-url)
Fig. 3. (a) Parameter adjustment transition at 4 Gb/s, (b) measurement setup, (c) measured 4 Gb/s PAM-4 eye-diagram using adjusted parameters of FFE

GMM, where each symbol value is estimated as average values ($\mu_0 = -0.5678$, $\mu_1 = -0.1939$, $\mu_2 = 0.1720$ and $\mu_3 = 0.5525$) and a variance value ($\sigma = 0.0024$).

Figure 2(c) shows the simulation results of 4 Gb/s PAM-4 data transmission; these data exhibit severe ISI. The resulting closed eye diagram has a histogram of overlapped Gaussian distributions (Fig. 2(d)). Even in such a harsh condition, GMM can classify each symbol distribution. Moreover, we obtained fitting results such that the average values and variance are $\mu_0 = -0.4366$, $\mu_1 = -0.1427$, $\mu_2 = 0.1686$ and $\mu_3 = 0.4485$ and $\sigma = 0.0108$, respectively, as shown in Fig. 2(d). The results can be used to adjust the FFE coefficients’ parameters.

As a verification experiment, parameters of the FFE were adjusted, provided that no information on transmission data and transmission characteristics is available. In this experiment, we assume an FFE with two taps and two adjustment parameters. The COBYLA algorithm in the python scipy library was used for parameter optimization. The objective function for the parameter adjustment sets the variance value $\sigma$, which is correlated with the effect of ISI. The variance value was calculated from the received symbols by using a GMM fitting assuming that each symbol variances are equal. The optimization algorithm attempts to search the parameters minimizing the variance.

Figure 3(a) shows the transition of the parameter adjustment results at 4 Gb/s PAM-4 data transmission. The optimization algorithm searched the set of parameters providing the smallest variance, which was calculated using 500 sampling symbols per each iteration. Consequently, it is possible to minimize the symbol variation owing to the effect of ISI at the receiver; therefore, we can open the eye. The sampling timing is randomly changed within the range of $-40\%$ to $20\%$ UI (unit interval) from the judgment timing, and the symbol values are acquired at the receiving end. The results confirm that the eye-opening ratio improves along with
minimizing the evaluation value by iterating the GMM fitting. In particular, we can observe that even if the initial eye is entirely closed, the parameter adjustment operates efficiently for improving the eye opening.

Figures 3(b) and (c) show the measurement setup and the measured PAM-4 eye-diagram (4 Gb/s) using the adjusted FFE parameters. The FFE modulated signal was calculated by simulation and was transferred to an arbitrary waveform generator (Tektronix AWG70001A; 50 GS/s) to emulate FFE operation. The PAM-4 eye-diagram obtained from waveforms passing through 1 m MSL shows that the optimized FFE mitigates the ISI; therefore, the 4-valued signal eye is open to 0.58 UI.

4 Conclusion

In this letter, we proposed a novel PAM-4 eye-monitoring technique based on machine learning using a GMM fitting. The classification technique could optimize the coefficients of the FFE for unknown transmission characteristics. Moreover, owing to the GMM fitting, the PAM-4 eye-monitoring technique made it possible to estimate ISI effects even when the eye was closed. Therefore, the proposed EOM method effectively adjusts the parameters of the FFE even when the channel conditions are not predicted.

Acknowledgments

This work was supported by JSPS KAKENHI Grant Numbers JP18H01488J and P18K11232.