Radiative Corrections to $W$ Pair Production at High Energies

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1 Introduction

The spectacular success of the high energy electron positron colliders SLC at SLAC and LEP at CERN has confirmed the predictions of the Standard Model (SM) for the interactions between the gauge bosons and the fermions event at the level of electroweak radiative corrections. However, the non-abelian structure of the gauge sector of the SM with its couplings between the electroweak gauge bosons have not been tested directly. In addition, the origin of electroweak symmetry breaking giving longitudinal components to the electroweak gauge bosons, is still obscure.

Anomalous couplings will disturb the extensive gauge cancellations taking place in the standard model, and possible new physics will show up in particular in the cross section for the production of longitudinally polarized $W$ bosons. On the other hand, possible new physics is already severely constrained by LEP100 data, and the effects to be expected at a 500 GeV linear collider are small.

In order to extract the small effects of physics beyond the SM one has to have a precise knowledge of the radiative corrections within the SM. The electroweak radiative corrections to the production of on-shell $W$ bosons to one-loop order are by now well established. The influence of the finite width of the $W$'s has been investigated. Also, the higher order QED corrections have been calculated in the leading log approximation (LLA).

However, the experimental reconstruction of the $W$'s is complicated by the fact that they may decay either into leptons with an escaping neutrino, or into hadrons, where the jet energies may be poorly known due to undetected particles. In addition, the radiative corrections due to emission of photons produce a systematic shift of the effective center of mass energy towards smaller values. Such effects may best be studied with the help of a Monte Carlo event generator.

In the present contribution we present first results obtained with Monte Carlo event generator WOPPER which allows to simulate $W$ pair production including radiative corrections and effects from finite $W$ width. We shall consider two applications, the reconstruction of the $W$ boson helicities from the semileptonic final states and the effects of the radiative corrections and the finite width of the $W$ bosons on the total cross section.

2 The Monte Carlo WOPPER

The Monte Carlo Event generator WOPPER is capable of a full simulation of the cross section for $e^+e^- \rightarrow 4$ fermions + $n\gamma$ via the resonant channel containing two $W$ bosons. The finite width of the $W$ bosons is included as well as QED radiative corrections in all orders of the leading logarithmic approximation (LLA). At very high energies these corrections are indeed the ones which are numerically most important, since

$$\frac{\alpha}{\pi} \log \left( \frac{s}{m_W^2} \right) \approx 6\% \quad \text{(at LEP200 and EE500 energies)} \quad (1)$$

*Presented by Thomas Mannel
The LLA is conveniently incorporated using the so called structure function formalism \[4\]. In this formalism the expression for the radiatively corrected cross section reads

$$\sigma(s) = \int_0^1 dx_+ dx_- D(x_+, Q^2) D(x_-, Q^2) \, \hat{\sigma}(x_+ x_- s) \, ,$$

(2)

where \( \hat{\sigma} \) is the Born level cross section of the hard process, \( D(x, Q^2) \) are the structure functions for initial state radiation, and \( Q^2 \sim s \) is the factorization scale. The structure function satisfies the evolution equation

$$Q^2 \frac{\partial}{\partial Q^2} D(x, Q^2) = \frac{\alpha_s}{2\pi} \int_x^1 \frac{dz}{z} [P_{ee}(z)]_+ D\left(\frac{x}{z}, Q^2\right)$$

with \( P_{ee}(z) = 1 + \frac{z^2}{1-z} \)

(3)

with the initial condition \( D(x, m^2_e) = \delta(1-x) \). The solution to eq. (3) automatically includes the exponentiation of the soft photon contributions as well as a resummation of the large logarithms of the form \( \ln(s/m^2_e) \) from multiple hard photon emission.

The radiatively corrected cross section (2) is implemented in a Monte Carlo event generator by solving (3) by iteration. This procedure is well known from the corresponding QCD applications \[5\] and, as a by-product of the algorithm used, the four-momenta of the radiated photons may be generated explicitly. For more details we refer the reader to \[6\].

The Monte Carlo WOPPER also includes the effects of the finite width of the W bosons. To introduce finite width for the decaying W bosons one has various possibilities \[7\]. The one chosen in the Monte Carlo WOPPER is to start from an off-shell cross section \( \sigma_{\text{os}} \) for the process \( e^+ e^- \rightarrow W^+ W^- \) where the momenta \( k_\pm \) of the W bosons are not on shell

$$\sigma_{\text{os}} = \sigma_{\text{os}}(s; k_+^2, k_-^2)$$

(4)

which is obtained by continuing the momenta of the W to off-shell values. The cross section for the process \( e^+ e^- \rightarrow 4 \text{ fermions} \) is then obtained by convoluting \( \sigma_{\text{os}} \) with propagators for the W bosons multiplied by the decay probability for the subsequent W decay

$$\sigma = \int \frac{ds_+}{s_+} \frac{ds_-}{s_-} \frac{\sqrt{s_+ \Gamma_{W}(s_+)} \Gamma_{W}(s_-)}{(s_+ - M_{W}^2)^2 + s_+ \Gamma_{W}(s_+)^2} \frac{\sqrt{s_- \Gamma_{W}(s_-)} \Gamma_{W}(s_-)}{(s_- - M_{W}^2)^2 + s_- \Gamma_{W}(s_-)^2} \sigma_{\text{os}}(s; s_+, s_-)$$

(5)

where \( \Gamma_{W}(s) \) denotes the width of the W boson taken also off-shell. In the Monte Carlo WOPPER the four fermion final states are generated according to the distribution (3). For more details see \[6\].

3 Applications

3.1 Reconstruction of the W Helicities

The longitudinal modes of the electroweak gauge bosons play a specific role in investigating the origin of electroweak symmetry breaking and in extracting effects of physics beyond the SM. Hence a reconstruction of the helicities of the W bosons from their decay products is mandatory. There are three possibilities. Firstly, there may be a hadronic final state which is not well suited for a reconstruction of the W helicities, since it requires to measure jet charge and/or jet flavor, which is not yet feasible. The second possibility is a leptonic final state which contains two neutrinos. Aside from the fact that this channel is suppressed by a relatively small branching fraction, the reconstruction is difficult due to initial state radiation. Finally, there is the possibility of a semi-leptonic final state which is the best way of reconstructing W helicities, since the charge of the electron or muon may be determined and only one neutrino is present which hence may be reconstructed.
The lowest order results for the reconstruction of the $W$ helicities have been investigated in some detail in the past \cite{8}. In order to reconstruct the neutrino momentum in the case where photons are radiated we shall employ the fact that most of the radiated photons are collinear with the beam. Due to this, most of the radiated photons will be lost in the beam pipe and thus have to be treated inclusively. Hence most of the photons have small transverse momentum and thus one may neglect their transverse momentum. In the narrow width approximation one may solve the equations

$$p_{\perp}^\nu + p_{\perp}^l + p_{\perp}^x = 0, \quad (p_l^\nu + p_x^\nu)^2 = M_W^2 \tag{6}$$

for the longitudinal momentum of the neutrino up to a twofold ambiguity. Here $p_l$ is the momentum of the charged lepton and $p_x$ is the total hadronic momentum. The ambiguity in (6) is resolved by choosing the solution for which the invariant mass $M_{WW}^2 = (p_\nu^\nu + p_x^\nu + p_l^\nu)^2$ is closer to $\sqrt{s}$.

After having solved for the neutrino momentum we reconstruct the $W$ boson decay angle $\theta^*$ by a boost to the rest frame. The cross section corresponding to a definite helicity of the decaying $W^-$ is obtained by convoluting the decay angular distribution with the functions

$$f_\pm = \frac{1}{2}(5 \cos^2 \theta^* - 1 \mp 2 \cos \theta^*), \quad f_0 = 2 - 5 \cos^2 \theta^* \tag{7}$$

In fig.1 we compare the cross sections for the three helicities. The left column is the Born cross section, the radiatively corrected cross section is the right column. In both cases the above reconstruction of the neutrino momentum is used. This simulation is performed with 65 000 events in this channel which is an optimistic view of what may be expected at EE500. As may be seen from the figure the well known enhancement of the longitudinal polarization due to radiative corrections is clearly visible even at this level of statistics.

| WOPPER — Preliminary |
|-----------------------|
| EE 500               |
| Born formulas        |
| $\lambda = -1$       |
| $\lambda = 0$        |
| $\lambda = +1$       |

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Figure 1: Differential cross section for the $W$ helicities reconstructed as described in the text. Left column: Born cross section, Right column: cross section including QED corrections in LLA.

### 3.2 Finite $W$ Width Effects

Finally we shall discuss the effects of a finite $W$ width. In principle we may study these effects for any exclusive quantity, since WOPPER is a full Monte Carlo generator. However, for the sake of
comparison with other work we will consider here only the total cross section and study the finite width effects near threshold as well as at very high energies. In fig. 2 we show the total cross section in the vicinity of the $W$ pair production threshold and the corrections relative to the lowest order cross section due to finite width and QED corrections in the region from 200 GeV to 1 TeV. As it was pointed out earlier the effects from the finite $W$ width do not vanish at high energies but rather enhance the cross section by about 6% at 1 TeV.

![Graph showing total cross section and corrections](image)

Figure 2: Left column: cross section in the threshold region (full line: fully corrected, dashed line: finite width only, dotted line: Born formula). Right column: corrections relative to the Born cross section (stars: fully corrected, open symbols: finite width only).

References

[1] M. Böhm, A. Denner, T. Sack, W. Beenakker, F. Berends, H. Kuif, *Nucl. Phys.* B304 (1988) 463; J. Fleischer, F. Jegerlehner, M. Zralek, *Z. Phys.* C42 (1989) 409.

[2] T. Muta, R. Najima, S. Wakaizumi, *Mod. Phys. Lett.* A1 (1986) 203; D. Bardin, M. Bilenky, A. Olchevski, T. Riemann, DESY 93-053, BI-TP 93/09, March 1993

[3] M. Cacciari, A. Deandrea, G. Montagna, O. Nicosini, *Z. Phys.* C52 (1991) 421.

[4] E.A. Kuraev, V.S. Fadin, * Yad. Fiz.* 41 (1985) 733; G. Altarelli, G. Martinelli, in J. Ellis, R. Peccei (eds.), *Physics at LEP*, CERN Report 86-02 (1986); W. Beenakker, F.A. Berends, W.L. van Neerven, Contribution to the Ringberg workshop, April 1989.

[5] T. Sjöstrand, *Comp. Phys. Comp.* 39 (1986) 347.

[6] H. Anlauf et al., Darmstadt-Siegen Collaboration: WOPPER – A Monte Carlo event generator for $W$ off-shell pair production including higher order QED corrections, in preparation.

[7] A. Aeppli, F. Cuypers, G.J. van Oldenborgh, LMU-21/92, PSI-PR-93-05, TTP92-36

[8] Proc. of the Workshop – Munich, Annecy, Hamburg – Feb. 4 to Sep. 3, 1991, DESY 92-123