EFT* for electroweak processes of light nuclei

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Recently we succeeded to make a reliable EFT prediction in a totally parameter-free manner for the $S$ factors for the solar $pp$ and $hep$ processes, $p + p \rightarrow d + e^+ + \nu_e$ and $^3\text{He} + p \rightarrow ^4\text{He} + e^+ + \nu_e$[1]. The strategy used in there is to embed a highly sophisticated standard nuclear physics approach (SNPA) exploiting realistic potentials[2,3] into an EFT framework, that we refer to as EFT*. Up to next-next-to-leading order in chiral expansion, it turns out that there is effectively only one counter-term relevant to this process, the coefficient of which – $(\hat{d}^R)$ – has been renormalized to reproduce the experimental value of the tritium-beta decay. Our study has also led to very accurate EFT calculations on two-body weak processes that also receive contributions from the $\hat{d}^R$ term, $\mu - d$ capture rate[4], and $\nu - d$ scattering cross section[5].

1. Introduction

In making theoretical predictions on electroweak processes of nuclear systems at low energy, we have basically two distinct approaches: One is the traditional model-dependent approach based on accurate nuclear potentials and the other is effective field theory (EFT) approach. The former, that we refer to as “standard nuclear physics approach (SNPA)”, has so far scored an enormous success in wide areas of nuclear physics, achieving in some cases an accuracy that defies the existing experimental precision. This suggests that all of the essential ingredients of QCD – believed to be the mother theory of strong interactions – have been correctly encoded in SNPA. It however suffers from model-dependence and lack of systematic expansion scheme. In EFT approach, processes are described consistently and systematically, and free from the mentioned problems. Respecting these nice aspects, intensive works based on EFT are recently being made with impressive successes. For complicated systems with $A = 3, 4, \cdots$, it is technically very hard to apply EFT directly and not much progress has been made up to date, although intensive efforts are being made in this direction. Now our question is, can we construct an EFT that exploits phenomenological but accurate SNPA wave functions, taking advantage of the merits of these two processes? Our answer on this question is yes, and we have recently developed such a formalism which we call “more effective effective field theory (MEEFT)” or simply EFT*[6,7,8,9]. In EFT*, it is possible to have the consistency and systematic aspect of EFT with the wave functions obtained accurately in SNPA. In this talk, I wish to report our developments on EFT*.

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To be concrete, we shall discuss the following two solar nuclear fusion processes:

\[pp : \quad p + p \rightarrow d + e^+ + \nu_e,\]
\[hep : \quad p + ^3\text{He} \rightarrow ^4\text{He} + e^+ + \nu_e.\] (1, 2)

Both figure importantly in the solar neutrino problems. Since the thermal energy of the interior of the Sun is of the order of keV, and since no experimental data is available for such low-energy regimes, one must rely on theory for determining the astrophysical $S$-factors of the solar nuclear processes. Here we concentrate on the threshold $S$-factor, $S(0)$, for the reactions (1) and (2). The necessity of a very accurate estimate of the threshold $S$-factor for the $pp$ process, $S_{pp}(0)$, comes from the fact that $pp$ fusion essentially governs the solar burning rate and the vast majority of the solar neutrinos come from this reaction. Meanwhile, the $hep$ process is important in a different context. The $hep$ reaction can produce the highest-energy solar neutrinos with their spectrum extending beyond the maximum energy of the $^8\text{B}$ neutrinos. Therefore, even though the flux of the $hep$ neutrinos is small, there can be, at some level, a significant distortion of the higher end of the $^8\text{B}$ neutrino spectrum due to the $hep$ neutrinos. This change can influence the interpretation of the results of a recent Super-Kamiokande experiment that have generated many controversies related to neutrino oscillations [10,11]. To address these issues quantitatively, a reliable estimate of $S_{hep}(0)$ is indispensable.

Before going further, I would like to explain the difficulty in making a reliable estimation of $S_{hep}(0)$. First of all, the leading one-body contribution for the $hep$ process is strongly suppressed due to the pseudo-orthogonality between initial and final wave functions. Secondly the main two-body (2B) corrections to the “leading” 1B term tend to come with the opposite sign causing a large cancellation. The 2B terms therefore need to be calculated with great precision, which is a highly non-trivial task. Indeed, an accurate evaluation of the $hep$ rate has been a long-standing challenge in nuclear physics [10]. The degree of this difficulty may be appreciated by noting that theoretical estimates of the $hep$ $S$-factor have varied by orders of magnitude in the literature.

2. Theory

The primary amplitudes for both the $pp$ and $hep$ processes are of the Gamow-Teller (GT) type. Since the single-particle GT operator is well known at low energy, a major theoretical task is the accurate estimation of the meson-exchange current (MEC) contributions. In getting the current operators, we rely on the heavy-baryon chiral perturbation theory (HB\ensuremath{\chi}\PT) with the Weinberg’s power counting rule, which is a well-studied EFT that has been proven to be quite powerful and successful in describing low-energy nuclear systems. In HB\ensuremath{\chi}\PT, we have pions and nucleons as pertinent degrees of freedom, with all other massive degrees of freedom integrated out. The expansion parameter in HB\ensuremath{\chi}\PT is $Q/\Lambda_{\chi}$, where $Q$ stands for the pion mass and/or the typical momentum scale of the system, and $\Lambda_{\chi} \approx 1$ GeV. In our studies, we have calculated the MEC of the GT operator (space part of the axial-vector currents) up to $N^3\text{LO}$, where $N^\nu\text{LO}$ stands for the order of $(Q/\Lambda_{\chi})^\nu$ compared to the leading order (LO) one-body operator. Up to this order, MEC consists of one-pion-ranged and contact (zero-ranged) two-body currents, $A_{2B} = A^{1\pi} + A^\delta$. Three-body and many-body operators appear only in higher orders.
$A^\pi$ can be determined unambiguously, thanks to available accurate $\pi N$ scattering data. $A^\delta$ however contains one unknown parameter, the overall strength of the contact-term contribution, which we denote by $\hat{d}^R$,

$$A^\delta \propto \hat{d}^R \sum_{i<j} (\tau_i - \tau_j)^{1-2}(\sigma_j \times \sigma_j)\delta_A^{(3)}(r_{ij}),$$

(3)

where $\Lambda$ is the cutoff (which we introduce in the procedure of Fourier transformation from momentum space to coordinate space), and $\delta_A^{(3)}(r)$ is the smeared delta-function with the radius $\simeq 1/\Lambda$. $\hat{d}^R$ represents all the heavy degrees of freedom integrated out, and chiral symmetry (or any other symmetry) does not tell us the value of it. We can however fix $\hat{d}^R$ by studying other processes which are sensitive on it. In other words, we adjust the value of $\hat{d}^R$ to reproduce the experimental data of those other processes, a procedure which is the renormalization condition of $\hat{d}^R$. This procedure is closely analogous to the EFT approach to effective nuclear potential backed by renormalization group equations as explained in [12]. The power of the approach is that the constant $\hat{d}^R$ appears in tritium $\beta$-decay, $\mu$-capture on a deuteron, and $\nu$–$d$ scattering, as well as in $pp$ and $hep$ processes and hence is completely fixed. Among them, accurate experimental data is available for the tritium-beta decay rate, $\Gamma_t^\beta$, which we use to fix $\hat{d}^R$.

In EFT*, we evaluate the transition matrix elements by sandwiching the obtained current operators between phenomenological but accurate SNPA wave functions. Recalling the fact that, the short-distance behaviors can be substantially different from model to model even among modern potentials, one may worry about model-dependence. To address this question, it is important to observe that, as was also recently discussed in Refs. [12], the dependence occurs only at short-distance, and thus the model-dependence is local. And we know that local (short-distance) contribution is well described (or controlled) in terms of the counter-terms, whose generic form is $\sum_\nu c_\nu \nabla^\nu \delta_A^{(3)}(r)$. The $\hat{d}^R$-term is the leading counter-term. It means that, for a fixed $\Lambda$, the matrix element of $\delta_A^{(3)}$, $\langle \delta_A^{(3)} \rangle$, may have substantial model-dependence (due to the model-dependent short-range behavior), which is, however, compensated by the model-dependence of the value of $\hat{d}^R$, to reproduce $\Gamma_t^\beta$. As a result, we can have the model-independent theory prediction for the total net amplitude. In other words, EFT* can be different from EFT only at short-distance, which is to be renormalized away order by order.

3. Results

To determine $\hat{d}^R$ from $\Gamma_t^\beta$, we calculate $\Gamma_t^\beta$ from the matrix elements of the current operators evaluated for accurate $A=3$ nuclear wave functions. We employ here the wave functions obtained in Refs. [3,13] using the correlated-hyperspherical-harmonics (CHH) method [14,15]. It is obviously important to maintain consistency between the treatments of the $A=2$, 3 and 4 systems. We shall use here the same Argonne $v_{18}$ (AV18) potential [16] for all these nuclei. For the $A \geq 3$ systems we add the Urbana-IX (AV18/UIX) three-nucleon potential [17]. Furthermore, we apply the same regularization method to all the systems in order to control short-range physics in a consistent manner.

In Table 1, we have listed the value of $\hat{d}^R$ and matrix elements of MEC, $\mathcal{M}_{2B}$. The table indicates that, although the value of $\hat{d}^R$ is sensitive to $\Lambda$, $\mathcal{M}_{2B}$ is amazingly stable against...
Table 1
The strength $\hat{d}^R$ of the contact term and the two-body GT matrix element, $\mathcal{M}_{2B}$, for the $pp$ process calculated for representative values of $\Lambda$.

| $\Lambda$ (MeV) | $\hat{d}^R$ | $\mathcal{M}_{2B}$ (fm) |
|----------------|-------------|--------------------------|
| 500           | 1.00 ± 0.07 | 0.076 − 0.035 $d^R \simeq 0.041 \pm 0.002$ |
| 600           | 1.78 ± 0.08 | 0.097 − 0.031 $d^R \simeq 0.042 \pm 0.002$ |
| 800           | 3.90 ± 0.10 | 0.129 − 0.022 $d^R \simeq 0.042 \pm 0.002$ |

the variation of $\Lambda$ within the stated range. The $\Lambda$-independence of the physical quantity $\mathcal{M}$, which is in conformity with the tenet of EFT, is a crucial feature of the present EFT result. Since the $pp$ amplitude is dominated by the well-known one-body LO contribution, the resulting $S$ factor at threshold, $S_{pp}(0)$, can be predicted with extreme accuracy,

$$S_{pp}(0) = 3.94 \times (1 \pm 0.0015 \pm 0.0010) \times 10^{-25} \text{ MeV-b.} \quad (4)$$

Here the first error is due to uncertainties in the input parameters in the one-body part, while the second error represents the uncertainties in the two-body part.

In Table 2 we have listed results for the matrix element of the GT operators for the $hep$ processes in arbitrary unit. We see from the table that the variation of the two-body GT amplitude (row labelled “2B-total”) is $\sim 10\%$ for the range of $\Lambda$ under study. It is also noteworthy that the variation of the 2B amplitude as a function of $\Lambda$ is reduced by a factor of $\sim 7$ by introducing the $\hat{d}^R$-term contributions; compare the third and fifth rows (labelled “2B (without $\hat{d}^R$)” and “2B-total”, respectively) in Table 2. As discussed, the $\Lambda$-dependence is amplified in total (1B + 2B) amplitude due to the substantial cancellation between 1B and 2B contributions. The resulting $S$-factor (adding the contributions from non-GT channels) reads

$$S_{hep}(0) = (8.6 \pm 1.3) \times 10^{-20} \text{ keV-b,} \quad (5)$$

where the “error” spans the range of the $\Lambda$-dependence for $\Lambda=500$–800 MeV. This result should be compared to that obtained by Marcucci et. al. [3], $S_{hep}(0) = 9.64 \times 10^{-20}$ keV-b.

Table 2
Values of the two-body GT amplitude (in arbitrary unit) for the $hep$ process calculated as functions of the cutoff $\Lambda$. The individual contributions from the one-body (1B) and two-body (2B) operators are also listed.

| $\Lambda$ (MeV) | 500 | 600 | 800 |
|----------------|-----|-----|-----|
| 1B            | −0.81 | −0.81 | −0.81 |
| 2B (without $\hat{d}^R$) | 0.93 | 1.22 | 1.66 |
| 2B ($\propto \hat{d}^R$) | −0.44 | −0.70 | −1.07 |
| 2B-total      | 0.49 | 0.52 | 0.59 |
4. Discussions

The \( ^3\text{He} + n \rightarrow ^4\text{He} + \gamma \) process, shares many features with the \( \text{hep} \) process, including the suppression of the leading one-body contribution due to the pseudo-orthogonality of the wave functions. Especially, the \( \text{hen} \) process also contains two counter-terms, one in isoscalar and the one in isovector channel, which can be renormalized by the magnetic moments of \( ^3\text{He} \) and \( ^3\text{H} \). We note that the MEC of the \( \text{hen} \) process starts from NLO coming from unsuppressed one-pion-exchange diagrams, while that of \( \text{hep} \) starts from N\(^3\)LO. We are applying exactly the same strategy used in the \( \text{hep} \) process (work in progress with Y.-H. Song). Accurate experimental data is available for the \( \text{hen} \) cross section, but so far no theoretical calculations have succeeded in explaining the data quantitatively. Thus applying the same EFT technique will provide a useful check of the validity of EFT* as a bona-fide EFT of QCD as discussed in Ref. \[7, 12\].

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