INFLUENCE OF RESCATTERING ON THE SPECTRA OF STRANGE PARTICLES

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Abstract

Applying a new method of rescattering which is based on the neural network technique we study the influence of rescattering on the spectra of strange particles produced in heavy ion reactions. In contradistinction to former approaches the rescattering is done explicitly and not in a perturbative fashion. We present a comparison of our calculations for the system Ni(1.93AGeV)+Ni with recent data of the FOPI collaboration. We find that even for this small system rescattering changes the observables considerably but does not invalidate the role of the kaons as a messenger from the high density zone. We cannot confirm the conjecture that the kaon flow can be of use for the determination of the optical potential of the kaon.

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1 Introduction

The production of kaons in heavy ion collisions is presently one of the most challenging topics in nuclear physics. At beam energies below or close to the threshold (in NN collisions) of $E_{\text{beam}} = 1.583$ GeV we observe a strong enhancement of the kaon production as compared to the extrapolation of pp collisions. Detailed investigations have shown that there most of the kaons are created in two step processes via an intermediate $\Delta$ or $\pi$ and are produced at a density well above the nuclear matter density [1, 2]. This triggered the conjecture that kaons may be of use as a messenger of the high density zone. In this high density zone the compressional energy may be high (depending on the nuclear equation of state) and therefore, due to energy conservation, the available kinetic energy may be reduced. Hence we have to expect a lowering of the production rate. However, at this high density also the properties of the kaon may have changed. Due to the presence of strong scalar and vector fields the threshold for kaon production may be lowered leading to an enhancement of the kaon yield. Calculations have shown that the fields cause only a small correction for the $K^+$ but for the $K^-$ the effect may be large due to g-parity [3].

In infinite matter these phenomena can be studied using chiral Lagrangians. The higher the density and the temperature the more difficult it is, however, to solve the perturbation expansion of this Lagrangian. In addition a verification of the results in experiments is only possible with heavy ions. These heavy ion reactions are unfortunately far away from thermal equilibrium and finite size effects are dominant. The presently only possibility to face this problem are sophisticated numerical programs which simulate the time evolution of the reaction. These programs may reveal the influence of different conjectures on the meson production and meson properties on the observable spectra.

Due to the smallness of the cross section the kaon production has been calculated up to now only perturbatively: for every collision $i$ with $\sqrt{s} > \sqrt{s_{\text{threshold}}}$ one has registered the production probability to produce a kaon $P_i = \sigma(\text{NN} \rightarrow K)/\sigma(\text{NN} \rightarrow X)$. The total kaon cross section is then given by $\sigma_K = \sigma_{\text{reac}} \sum_i P_i$. This perturbative approach does allow to predict the cross section for $K^+$ which due to its $\bar{s}$ quark cannot be reabsorbed. The small change on the effective $K^+$ yield in isospin asymmetric systems due to $K^+ + n \rightarrow K^0 + p$ has been neglected. For generating the spectra one assumes that the $N\Lambda(\Sigma)K$ system decays according to phase space in its center of mass system. This assumption is a first order approximation to existing data. About deviations conflicting results are reported[4, 5]. Applying Monte Carlo procedures one can perform the disintegration of a single $N\Lambda(\Sigma)K$ system many times where each disintegration yields a precise value of the momenta of all
particles. This allows a very effective calculation of differential kaon cross section.

In some of these simulation programs an approximative rescattering procedure has been added which goes back to Randrup [6]. There, after the determination of the momentum of the kaon, one checks the path the kaon has to travel through nuclear matter if the nucleus rest in the configuration given at the moment of the creation of the kaon. The length of the path is then divided by the mean free path of the kaon in nuclear matter to obtain the average collision number. The number of collisions the kaon is supposed to suffer is given by a Poisson distribution around this mean value. The momentum distribution of the nucleons is taken from the measured proton spectrum. Furthermore, it is assumed that the $KN$ cross section is purely elastic and the angular distribution of the cross section is isotropic. Any feedback of the kaon production on the system is neglected, i.e. all non strange particles move as if there has been no kaon produced. An improved version of this approximative rescattering procedure has been recently proposed by Fang et al. [7]. Instead of calculating the path of the kaons in matter using the density distribution as given at the instant of the creation of the kaon Fang et al. follow the trajectory of the kaon, assuming that in between collisions the kaons move on straight lines. Collisions with the nucleons change only the momentum of the kaon whereas the nucleons move as if no collisions had appeared. The cross section of $KN$ scattering is taken as isotropic. Thus as in the aforementioned approaches there is no feedback of the kaon creation on the nuclear system.

In the last years the experiments on meson production in heavy ion collisions reached a precision which makes it necessary to minimize the systematic errors due to approximations in the simulation programs. One of the major sources of the systematic error is the approximate treatment of rescattering. In addition the use of an unrealistic isotropic cross section does not allow to address a conjecture which has been advanced recently: Does the kaon flow depend crucially on the magnitude of the vector and the scalar potential and how compares the kaon flow with that of the baryons? If kaons rescatter with baryons which possess directed flow, this directed flow may be communicated to then and shows up in the final kaon spectra.

The production of a kaon is a rare event. Even at central collisions of Ni+Ni at 1.93 A GeV, i.e. well above the threshold, a kaon is produced only in 1 event out of one thousand. Therefore it is not easy to gain sufficient statistics if one would like to follow the time evolution of the kaons in the system. It is the purpose of this article to introduce a new method to achieve this goal. In the next chapter we introduce this new method, in chapter 3 we study the influence of rescattering on the observables taking as an example the recent data obtained by the FOPI detector.
2 Neural Network for the Rescattering of Kaons

Since the kaon production is a rare event in heavy ion collisions below or close to the threshold we have to enhance artificially its production rate if we want to avoid simulations which are useless for studying kaon properties. This can be achieved by introducing an artificial kaon production enhancement factor which assures that on the average in about 30% of the heavy ion collisions 1 kaon is produced. Due to the fluctuation around the mean value we see in 7% of the reactions the production of two kaons, a quantity which we consider as tolerable.

This enhancement factor is a necessary but not a sufficient condition to obtain a reasonable kaon yield in a short amount of time. A second condition is that the treatment of the rescattering is done fast and effectively despite of the rather complicated structure of the elementary $K N$ cross section. This second condition is achieved with a modern mathematical tool, called neural network.

The basic problem in simulating a cross section by a Monte Carlo procedure is to assure that after many simulations the particles are scattered with the desired differential cross section, i.e. that the distribution $P(x)$ of scattering angles corresponds to the desired one. In one dimension and in the case that the integral over the distribution can be inverted this can be achieved with elementary methods.

$$y_i = \int_{\text{lower bound}}^{x_i} P(x) dx$$

where $y_i$ is a random number uniformly distributed in $[0,1]$. The $x_i$’s are distributed like $P(x)$ if $P(x)$ is normalized. Usually the cross section is not given in an analytical form but by experimental data points and the angular distribution varies as the energy increases. Then one approaches usually the problem by multi dimensional fits. Multiparameter fits are normally difficult to invert and in addition the inversion poses problems due to the finite accuracy of the computer, a problem one suffers already for $p p$ collisions.

The neural network takes a different approach. First one has to fit the differential cross sections according to the data by polynomials as follows

$$\frac{d\sigma}{d\Omega} = \sum_{i=0}^{n} a_i(\sqrt{s})(\cos\theta)^i$$

where $a_i(\sqrt{s})$ are the polynomial coefficients and $n$ the degree of the polynomial. Then we calculate $X$ defined as

$$X(\sqrt{s}, \cos\theta') = \frac{\int_{-1}^{\cos\theta'} \frac{d\sigma}{d\Omega}(\sqrt{s}, \cos\theta') d(\cos\theta')}{\int_{-1}^{1} \frac{d\sigma}{d\Omega}(\sqrt{s}, \cos\theta') d(\cos\theta')}$$

$X$ is calculated for many values of $\cos \theta^i$ chosen between -1 et 1 at the points of $\sqrt{s}$ where data exist. We see that $X$ is distributed between 0 and 1.

In the second step $X(\sqrt{s}, \cos \theta^i)$ and $\sqrt{s}$ are presented as input to a neural network as displayed in fig.1. As an output variable $x_{out}^i$ we would like to have the scattering angle $\cos \theta^i$. With help of the many values of $X$ prepared as above we train the network, i.e. we minimize the standard deviation

$$\sum (\cos \theta^i - x_{out}^i)^2 \quad (4)$$

by varying the free parameters of the network. For the detailed description of the free parameters of the neural network and the method to achieve this goal we refer to reference [8]. After this procedure one calls the network ”trained”. The quality of the training can be viewed from fig. 2 where we display the response of the trained network. For three different momenta we display $\cos \theta^i$ given as input into the network in form of eq. 3 and the $\cos \theta^i$ estimated by the neural network, i.e. the output value of the network. For a perfect reproduction we expect a straight line. As seen, the deviations are of minor importance.

Now the network is prepared to serve the purpose. For each collision we present to the network the center of mass energy $\sqrt{s}$ and a random number and the network responds with a scattering angle. The scattering angles are distributed like $d\sigma(\sqrt{s}) d\Omega$. This procedure is fast, precise and easy to control and will replace in near future all the standard cross section routines.

In the present case we implemented the measured $KN$ cross sections for the elastic channels [9, 10, 11, 12, 13] and the charge exchange channel with an angular distribution [14] fitted to the experimental data by use of a neural network as described above. We also implemented the $\Lambda N$ elastic scattering as isotropic with a total cross section of 16.4 mb [15].

3 Comparison with FOPI data

These neural networks are embedded in the Quantum Molecular Dynamics (QMD) approach to simulate heavy ion reactions on a event by event basis. This approach simulates the time evolution of all projectile and target nucleons from their initial separation in projectile and target up to their final fate being protons, neutrons or part of a cluster. The nucleons are represented by coherent state wave functions which depend on two parameters, the position $r_{i0}$ and the momentum $p_{i0}$. The Wigner transforms of these coherent states are gaussians in momentum and coordinate space, respectively. The time evolution of the centroids of the gaussians $r_{i0}, p_{i0}$ is determined by a generalized Ritz vari-
ational principle. The nucleons interact by mutual two and three-body potentials which reduce to a Skyrme potential in nuclear matter. For details we refer to ref [16].

These simulations have been frequently employed to understand the physical origin of the observed spectra or to predict the observable consequences of the variation of yet unknown properties of hadronic matter like the equation of state or in medium properties of particles.

The kaon are produced with the cross section of Randrup and Ko [17] who parametrized the little known cross sections $NN \rightarrow NK\Lambda$ and $NN \rightarrow NK\Sigma$. Because the rescattering cross section of $\Sigma$'s at the energy of interest is not known, we apply for the $\Sigma N$ collisions the same cross section as for the $\Lambda N$ collisions. Finally we let decay the $\Sigma^0$'s into $\Lambda$'s in order to obtain the $\Lambda$ spectrum. Due to the very similar mass of $\Lambda$'s and $\Sigma$'s the error of this approximation is small as far as the kaons are concerned and in any case smaller than the systematic error caused by the ignorance of the $np \rightarrow \Lambda$ cross section. This cross section depends on the particle which is exchanged which may be a kaon or a pion. For a pion exchange the cross section would be 2.5 time larger than that of the $pp$ channel, for a kaon exchange both are the same.

From the moment of their production the kaons move on straight line trajectories as free particles with their on shell mass. If kaons come closer to a nucleon than $r = \sqrt{\sigma_{tot}/\pi}$ they rescatter with an angular distribution of the free scattering. Charge exchange reactions and elastic scattering have a relative fraction $\sigma_{ch\ ex}/\sigma_{tot}$ and $\sigma_{el}/\sigma_{tot}$, respectively with $\sigma_{tot} = \sigma_{ch\ ex} + \sigma_{el}$. $\Lambda$'s rescatter with nucleons with a cross section given by [15] and feel the same potential as the nucleons.

For the comparison we have chosen the reaction Ni(1.93AGeV)+Ni measured by the FOPI collaboration at GSI [18]. First, because in this experiments the rapidity and transverse energy distribution of the kaons has been measured, second, because the low observed kaon flow brought up the conjecture that it is caused due to the balance of the scalar and the vector potential [19].

We start our comparison with the rapidity distribution of kaons which is represented in fig.3. We compare the rapidity distribution of kaons as created and after rescattering with the experimental results. The experimental points are supplemented with a preliminary experimental data point at midrapidity obtained by the KAOS collaboration at 1.8 GeV/N for the same projectile target combination [21]. According to our calculations this point has to be multiplied by 1.1 in order to be comparable with the results at 1.93 GeV. Without rescattering the results correspond to the distribution obtained by the perturbative treatment. We observe, first of all, a surprisingly large influence of rescattering even for systems as small as Ni + Ni. This confirms calculations of Fang et al [7]. That rescattering enlarges the width in rapidity is expected: Due to the limited available energy the creation
of strange particles is centered around midrapidity and the width of the created kaons, given by phase
space, is very limited. Rescattering tries to "thermalize" the kaons, hence the rapidity approaches
that of the nucleons which is displayed in fig. 4. We see that the kaons have finally still a smaller
variance than the nucleons which are far from representing a thermal system. Despite we are well
above the threshold about 50 % of the kaons come from ∆N and only the rest from NN collisions.

In fig. 3 we compare as well our calculation with that of G.Q.Li et al. [21, 22]. They have used
a relativistic RBUU program and an isotropic cross section of 10 mb for KN rescattering [7]. Both
programs differ in their nucleon and kaon potentials. Whereas in QMD we use for the nuclear potential
a nonrelativistic Skyrme type potential with a soft equation of state as already in ref.[2], Li et al. use
a relativistic vector and scalar potential. At energies above 1 GeV/N the optical potential becomes
energy independent and the energy dependence at lower energies becomes only important after the
kaons have been produced. Hence we have neglected the energy dependence. This constancy of the
optical potential cannot be reproduced in relativistic models which are bound to give (for energy
independent coupling constants) a linear dependence on the kinetic energy. Beside this difference the
Schrödinger equivalent potential of the relativistic potential used by Li et al. is not far away from the
Skyrme potential applied here. In addition, above threshold the influence of the potential becomes
less and less important. The kinematics is relativistic in both cases. The major difference between
both approaches is the use of a kaon optical potential by Li et al. (consisting of a scalar and a vector
part). In our calculation the kaons move in between collisions on straight lines.

It is surprising how little these differences influence the kaon rapidity distribution. Especially the
use of a kaon optical potential seems not to have any observable influence on this variable.

The similar broadening of the distribution by rescattering is observed for the transverse energy
spectra \( \frac{1}{M_\perp^2} \frac{d^2N}{dY dM_\perp} = Ae^{-M_\perp/T_B} \), fig.5, where we compare the different slopes with those measured
by the FOPI collaboration. In both cases this slope is fitted to the high energy tail of the spectra.
Before rescattering the slope is determined by the available phase space in the production collision.
Subsequent collisions increase the slope \( T_B \) of the transverse energy spectra by about 70%, almost
independent of the rapidity. We would like to mention that the increase of the slope is much larger
than the increase of the average transverse root mean square momentum, which is about 25%.

The influence of the rescattering on the observed spectra is displayed in fig.6 where we show for
3 laboratory angles ( \( \theta_{lab} = 0^\circ \), \( \theta_{lab} = 44^\circ \), the standard angle for the kaons observed by the KAOS
collaboration, and \( \theta_{lab} = 85^\circ \)) the spectra with (WR) and without (NR) rescattering. We see an
influence of the rescattering which is not at all negligible, however less dramatic as in ref.[4], where
kaon rescattering at lower energies in a smaller system has been investigated.

Fig. 7 shows the flow of kaons and Λ’s. We display the average in plan transverse momentum as a function of the rapidity. On the left hand side we have included all the particles on the right hand side those with $p_t/m > 0.5$ which corresponds to the acceptance of the FOPI detector. We see that for kaons before and after rescattering the $< p_x >$ is compatible with 0 whereas the Λ’s seem to show a finite flow even without rescattering.

Kaons and Λ’s are produced according to phase space and hence isotropically in the NN center of mass system. Hence only a finite average transverse velocity of the NN center of mass system can be the origin of a finite average transverse momentum of the particles when they get produced. This transverse velocity $< v_x(y) > = < \frac{p_x(y)}{E} >$ as a function of rapidity is displayed in fig. 8. However, a common center of mass velocity has not the same consequences for Λ’s and kaon’s. The root mean square rapidity of kaons in the NN center of mass system is 70% larger as that for the Λ’s. Hence the kaons average a finite transverse source velocity over a larger rapidity range (the variance is about $0.28 y_0$). This averaging leads - besides close to midrapidity - to a smaller transverse velocity as compared to that of the Λ’s. This is seen in fig. 8 as well.

The KN cross section is not only much smaller as the $NN$ cross section but as well forward peaked in the momentum range of interest. Thus if the kaon, produced around midrapidity, hits a nucleon of the in streaming matter it changes its direction only little as compared to the Λ which is assumed to have an isotropic cross section. As a consequence, a collective transverse velocity is much less communicated to the kaons. Due to the geometry the forward peaked cross section makes it easier for the kaons to escape from the system. This, as well as the larger velocity of the kaons results in a small number of KN collisions. Indeed the kaons suffer fewer collisions as compared to the Λ’s as expected from the cross section ratios. We see in fig. 7 that rescattering does not give the kaons a directed transverse momentum whereas for the Λ’s this is observed. Another consequence of this dynamics is the small increase of the variance in rapidity of the kaons (12%) due to rescattering as compared to that of the Λ’s (39%) which will be discussed later. We see that the velocity of the final Λ’s follows that of the nucleons except for the large rapidity were the spectators contribute. Hence rescattering increases the transverse flow of the Λ’s to that extend that it agrees with that of the protons as seen in fig. 7.

Hence we cannot confirm the conjecture that the kaon flow may be of use to measure the scalar and vector part of the kaon potential [22]. No kaon potential as in the present investigation yields already the observed flow. Of course there is a kaon potential but the only information one may extract from
the observable flow is that the combination of vector and scalar potential has to have no influence on that observable.

In view of former studies this is a very expected result. Kaons are predominately produced in central collisions where the collective flow is small \[2\]. In addition they are produced in the high density zone. Nucleons from the high density zone show a smaller flow than the average over all nucleons \[23\] because the flow is caused by the potential gradient which diverts the nucleons from the high density zone.

Fig. 9 displays the distribution of nuclear densities at the positions were the kaons are created and at the point of their last rescattering. We see that the average density at the point of creation is well above normal nuclear matter density. 79\% of the kaons are produced at \[\rho > \rho_0\]. Even further interactions do not spoil the ability of the kaons to transfer informations of the high density zone to the detectors. 60.5\% of the kaons did not suffer rescattering or had their last collision at a density larger then normal nuclear matter density.

Finally, in fig. 10 we display the rapidity distribution of the \(\Lambda\)'s before (dashed) and after (full line) rescattering. The absolute value here is even more plagued by the little knowledge of the exclusive cross section \(NN \rightarrow NK\Sigma\) resp. \(NN \rightarrow NK\Lambda\) than that for the kaons, especially in cases where a neutron is in the entrance channel. Measurements do not exist there and theoretical calculations \[1, 21\] predict quite different results depending on the type of the exchanged meson (kaon or pion). Preliminary experimental results of the FOPI collaboration come close to our rapidity distribution, for the absolute values one has, however, await the final analysis.

4 Conclusion

In conclusion we have shown, that despite of the small cross section the rescattering influences the strange particle observables even for small systems. It widens the energy distribution in transverse and in longitudinal direction. A realistic treatment with takes into account all presently known differential cross sections reproduces the kaon observables for the case investigated despite of the fact that no kaon potential has been employed. This leaves little room for the conjecture that \(K^+\) observables may yield information on the kaon potential in nuclear matter in a unique way. The experimental results are compatible with calculations which show that the \(K^+\) properties change only little in nuclear matter. The situation is probably quite different for \(K^-\). This is presently under investigation.

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FIGURE CAPTIONS

Fig.1: Schematic Neural Network used for $KN$ scattering.

Fig.2: Output of the Neural Network for $KN$ charge exchange scattering with $p_{lab} = 1.06, 1.13, 1.21 GeV/c$ as compared to the input.

Fig.3: Rapidity distribution of kaons with and without rescattering as compared to FOPI data [18] and the calculation of Li et al [19].

Fig.4: Rapidity distribution of baryons.

Fig.5: Slopes $T_B$ of the spectra $\frac{1}{M_\perp} \frac{d^2N}{dY(0)dM_\perp} = Ae^{-M_\perp/T_B}$ as a function of the rapidity of kaons with and without rescattering as compared to FOPI data [18].

Fig.6: $\frac{d^2\sigma}{dp_{lab}d\Omega}$ of kaons as a function of $p_{lab}$ for $\theta_{lab} = 0^\circ, 44^\circ, 85^\circ$.

Fig.7: $< P_x(y) >$ for kaons and $\Lambda$’s with and without rescattering. On the left hand side we include all particles, on the right hand side we applied the acceptance filter of FOPI.

Fig.8: Velocity on $x$ direction as a function of the rapidity for kaons, $\Lambda$’s and nucleons.

Fig.9: Density distribution of kaons at the production points and at the points of the last collision.

Fig.10: Rapidity distribution for $\Lambda$’s with and without rescattering.
Fig. 1
KN charge exchange scattering

Fig. 2
Fig. 3
Fig. 4

$^{58}\text{Ni}(1.93\text{AGeV}) + ^{58}\text{Ni}$, $b<4\text{ fm}$

$dN/dY(0)$ vs. $Y(0)$ (Rapidity)

Baryons
Fig. 5
WR = with rescattering
NR = no rescattering

Fig. 6
* with rescattering

◇ without rescattering

Fig. 7
Fig. 8
Fig. 10