Research Article
Dynamic Analysis of a 5-DOF Flexure-Based Nanopositioning Stage

Yiping Shen 1, Xin Luo 2, Songlai Wang 1, and Xuejun Li 1

1 Hunan Provincial Key Laboratory of Health Maintenance for Mechanical Equipment, Hunan University of Science and Technology, Xiangtan, Hunan 411201, China
2 State Key Laboratory of Digital Manufacturing Equipment and Technology, Huazhong University of Science and Technology, Wuhan, Hubei 430074, China

Correspondence should be addressed to Yiping Shen; yiping1011@163.com

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A multibody dynamic model is developed for dynamic analysis of a 5-DOF flexure-based nanopositioning stage in the projection optical system of the semiconductor lithography in this paper. The 5-DOF stage is considered as an assembly of rigid bodies interconnected by elastic flexure hinges. Considering the length effects of flexure hinges, multibody dynamic equations are established according to spatial motions of rigid bodies by using Lagrangian method. The shear effects and the torsional compliances of the commonly used circular flexure hinges are considered to enhance the modeling accuracy. The accuracies of various out-of-plane compliance formulas are also discussed. To verify the developed dynamic model, the finite element analyses (FEA) by using ANSYS and modal hammer experimental tests of the primary flexure-based composition structures and the integral 5-DOF stage are performed. The analytical modal frequencies are well in agreement with FEA and experimental test. The results are significant to analyze and optimize the 5-DOF flexure-based nanopositioning stage.

1. Introduction

To achieve high imaging resolution in the projection optical system of the semiconductor lithography, flexure-based nanopositioning stages are commonly used to adjust the respective lens during actual use [1, 2]. Such stage is required to provide five degrees of freedom (DOF) of motions except the rotational motion along the optical axis due to the symmetry of the circular lens. The desired ultraprecision motions are in the micrometer range with nanometer-scale precision. The extreme precision motions are achieved through the elastic deformations of flexure hinges, which can eliminate friction, wear, backlash, and lubrication commonly seen in traditional engineered artifacts [3–5]. Challenges in design of the 5-DOF flexure-based stage arise from the low stiffness in the allowable motion directions and high dynamic performance under external vibration. For better dynamic imaging resolution of the projection optical system, it is necessary to perform dynamic analysis for performance prediction and parameter optimization of the 5-DOF flexure-based nanopositioning stage.

In the past decades, numerous flexure-based stages were designed to achieve one-three DOF of motions. Ryu [6] developed a parallel 3-DOF in-plane wafer stage with three double compound mechanical levers. Awtar [7] presented a decoupled parallel in-plane XY stage composed of two double parallelogram flexure-based mechanisms. Kim [8] developed a 3-DOF out-of-plane positioning stage for optical instrument alignment. As for 6-DOF stage, parallel mechanisms with six legs were extensively applied [9, 10]. Recently, the hybrid serial-parallel flexure-based mechanisms are increasingly applied due to their special merits. Such stages combine the advantages of the parallel and serial mechanisms, for example, compact structure, stability, and manipulability [11]. Liang [12] and Brouwer [13] presented a 6-DOF flexure-based mechanism by mounting the out-of-plane mechanism on the in-plane stage. Considering the carrying capacity of the stage, Cai [11] proposed a novel 6-DOF stage placing the in-plane
stage on the top of the out-of-plane stage. The 5-DOF stage investigated in this paper has a similar design.

Cross-axis couplings impose difficulties on the dynamic performance evaluation of the 5-DOF flexure-based nanopositioning stage [5, 14–16]. Cross-axis couplings are impossible to be eliminated due to the inherent elasticities of flexure hinges. Especially for the hybrid 5-DOF nanopositioning stage, cross-axis couplings between the 5-DOF of motions are critical. The finite element method (FEM) is the most accurate computational method to estimate its dynamic performance. However, FEM requires an elaborate and time-consuming analysis procedure. The meshes of the thinnest region of a flexure hinge should be far finer than other parts of the hinge, which results in massive 3D/2D elements for a single flexure hinge [17]. Although improved FEM is proposed for model reduction, such as using spatial nonprismatic beam element to model circular flexure hinge [18] and the Krylov subspace reduction scheme [19], it is still not convenient to provide the intrinsic relation between the dynamic performance and design parameters of the multiple DOF flexure-based stage.

Many theoretical methods are developed to perform dynamic analysis of the flexure-based stage; pseudo-rigid-body model (PRBM) is the most extensively utilized [3]. In PRBM, a flexure hinge is considered as a single DOF rotation joint; the cross-axis coupling is naturally neglected, which may cause noticeable errors of modal frequencies between the analytical and experimental results [20–25]. Ling [26] presented three criteria to perform dynamic modeling with high accuracy and efficiency and proposed a semianalytical modeling method considering the compliances of the flexure hinges as well as the flexible beams. Considering the 5-DOF nanopositioning stage as an assembly of rigid bodies interconnected by elastic flexure hinges, multibody system approach is suitable to model such system [6, 16, 27]. In the conventional dynamic equation, the length of flexure hinge is not considered. Choi [28] demonstrated that the conventional equation can cause incorrect modal analysis results because the length of flexure hinge influences the off-diagonal stiffness elements.

The parametric modeling of the compliance matrix for flexure hinge is the pioneer work for dynamic modeling of the 5-DOF stage. The in-plane compliances of flexure hinges are generally concerned in many applications by using the analytical formulas developed by Paros and Weisbord [29], Lobontiu [4], and other scholars [30–32]. As for the 5-DOF stage, the out-of-plane compliances should be modeled so as to explore the cross-axis coupling effects between the 5 DOFs of motion directions. The shear effects and the torsional compliance should be treated carefully to obtain accurate compliance matrix. The shear effect is important for short flexure hinge [4], such as the commonly used circular flexure hinges. However, the shear effect is commonly not taken into account when performing kinematic or dynamic analyses in most applications. The torsional compliance is only a few less than the bending compliance and should be concerned especially in spatial flexure-based stage. The torsional compliance depends on the thickness-to-width ratio. As for the varied thickness of the circular flexure hinge, the torsional compliance is difficult to formulate an exact calculation equation [33]. Koseki [30] presented a simplified torsional compliance equation for the wider flexure hinge. Lobontiu [34] formulated torsional compliance equation for the flexure hinge when the thickness-to-width ratio is equal to or larger than 1. Chen and Howell [33] derived two new precise and effective equations independent of thickness-to-width ratio.

This paper attempts to establish an accurate multibody dynamic model for a 5-DOF flexure-based nanopositioning stage in the projection optical system of the semiconductor lithography. The stage is considered as an assembly of rigid bodies interconnected by elastic flexure hinges; multibody dynamic model is developed to include the length effects of flexure hinges. The shear effects and the torsional compliances of circular flexure hinges are investigated to obtain accurate compliance matrix. FEA and experimental test of the 5-DOF stage are performed to validate the developed multibody dynamic model.

This paper is organized as follows. Section 2 introduces the design of the 5-DOF flexure-based nanopositioning stage. In Section 3, considering the length effects of flexure hinges, multibody dynamic equation is established according to spatial motions of rigid bodies by using Lagrangian method, and the accuracies of various out-of-plane compliance formulas are discussed. In Section 4, dynamic model analytic results are validated by FEA and experimental test. A summary and some concluding remarks are given in Section 5.

2. Design of the 5-DOF Flexure-Based Nanopositioning Stage

The 5-DOF flexure-based nanopositioning stage is designed to provide translational motion range of 25 μm in X and Y directions, 100 μm in Z direction, and rotational motion range of 30° about the x-axis and y-axis. The stage is constructed as a 5-DOF serial-parallel stage by bolting a 2-DOF in-plane XY stage and another 3-DOF out-of-plane \( \theta_x, \theta_y, Z \) stage together, as shown in Figure 1. The lens is arranged in the in-plane stage.
The in-plane stage is composed of two double parallelogram mechanisms in series connection, and each one is driven by a piezoelectric actuator. The out-of-plane stage is a parallel flexure mechanism with three limbs placed 120° apart from each other. Each limb is driven by a piezoelectric actuator. The in-plane stage is bolted on the top of the out-of-plane stage. As discussed in [11], such design will improve the stability of the stage and ensure good dynamic performance and high-accuracy positioning. The 5-DOF flexure-based nanopositioning stage is considered as an assembly of ideal rigid bodies interconnected by elastic flexure hinges. The total number of rigid bodies is 28, and that of flexure hinges is 43. The flexure-based mechanisms of the 5-DOF stage are illustrated in Figure 2.

As shown in Figure 2, double parallelogram mechanism is applied to eliminate the parasitic motion in the X and Y directional motion. Eight circular flexure hinges are applied in each double parallelogram mechanism. Two mechanical levers are employed to amplify the forces provided by the piezoelectric actuators in the X and Y directions, respectively. Three circular flexure hinges are used in each mechanical lever: one is used for amplification and the other two are arranged to minimize parasitic motions. Mechanical lever 1 connects plate 2 and motion plate 1 by 4 M4 bolts; its output amplified force drives motion plate 1 along the X direction. Mechanical lever 2 connects plate 2 and plate 3; its output amplified force drives motion plate 1 and plate 2 along the Y direction together. The in-plane stage has 22 identical circular flexure hinges; their minimum thickness is 0.6mm, the radius is 3mm, and the width is 15mm. Both the XY stage and its mechanical levers are manufactured monolithically using wire electrical discharge machining (WEDM) technology. The in-plane stage is made of alloy steel with a Young's modulus of 200 GPa, Poisson's ratio of 0.288, and density of 7820 kg/m³.

As shown in Figure 3, three parallel limbs are placed with axial symmetry around the circumference of Base 16. The upper end of each limb is bolted to plate 3 by 2 M5 bolts; the lower end is bolted to Base 16 by 4 M5 bolts. Each limb is composed of two half bridge-type mechanisms and spring leafs to amplify the forces provided by the piezoelectric actuators. Two bridge-type mechanisms are symmetrically arranged to avoid the bending moments and improve the stability and the positioning accuracy of the stage [35, 36].
The spring leafs (numbered as (23) and (24) in limb 1, seen in Figure 3(b)) are used to enlarge the amplification ratio for the bridge-type mechanism. The output amplified forces of three limbs are used to realize the 3-DOF out-of-plane motions of motion plate 1. Each limb has seven flexure hinges. The dimensions of the spring leafs are 12mm×4mm×0.6mm. Four circular flexure hinges of the bridge-type mechanism (numbered as (25) to (28) in limb 1, seen in Figure 3(b)) are identical, their minimum thickness is 0.6mm, the radius is 1.5mm, and the width is 4mm. The circular flexure hinge (29) has the same parameters except that the width is 13mm. Three limbs are manufactured monolithically using WEDM technology. Their material is beryllium bronze with Young’s modulus of 126 GPa, Poisson’s ratio of 0.35, and density of 8300 kg/m³.

3. Dynamic Modeling of the 5-DOF Flexure-Based Nanopositioning Stage

3.1. Multibody Dynamic Equations of the 5-DOF Stage considering the Length of the Flexure Hinge. Based on Lagrange equations, the derivation of the potential energies of elastic flexure hinges is elaborated according to spatial motions of rigid bodies, as shown in Figure 4. Considering the length of the flexure hinge, the translation and rotation motions of rigid bodies are described by three sets of reference frames [37], including the global frame XYZ, the rigid body frame $x_iy_iz_i$ of the rigid body $B_i$, and the spring frame of the spring $S_i$, between the rigid bodies $B_i$ and $B_{i+1}$, $x_{j,i+1}y_{j,i+1}z_{j,i+1}$. The global frame is located at the center of motion plate 1 of the in-plane XY stage. The origin of the spring frame $x_{j,i+1}y_{j,i+1}z_{j,i+1}$ is fixed at the end $s_i$ of the flexure hinge on the rigid body $B_i$, and the positive direction of its abscissa is oriented towards the end $s_{i+1}$ on the rigid body $B_{i+1}$.

In the deformed state, the position vector of the point $s_i$ with respect to the rigid body frame $x_iy_z$ is defined as [38]

$$\mathbf{x}_{i,s} = \begin{bmatrix} r_{i,s} \\ \theta_{i,s} \end{bmatrix}$$

(1)

where $r_{i} = [x_i \ y_i \ z_i]^T$ and $\theta_{i} = [\theta_{x_i} \ \theta_{y_i} \ \theta_{z_i}]^T$ represent the translational and rotational motion of the rigid body $B_i$, respectively. $\mathbf{u}_{i,s} = [x_{is} \ y_{is} \ z_{is}]^T$ is the position vector of the point $s_i$ with respect to the rigid body frame $x_iy_z$ in the undeformed state. $A$ is the transformation matrix due to rotational motion of rigid body $B_i$, and $I$ is an $3 \times 3$ identity matrix. Because the rotational motions of rigid bodies in the 5-DOF stage are generally small, $A$ is simplistically defined as

$$A = \begin{bmatrix} 1 & \theta_{zi} & -\theta_{yi} \\ -\theta_{zi} & 1 & \theta_{xi} \\ \theta_{yi} & -\theta_{xi} & 1 \end{bmatrix}$$

(2)

Substitute (2) into (1), and yield

$$\mathbf{x}_{i,s} = \begin{bmatrix} 1 \\ U_{is} \\ 0 \end{bmatrix} \mathbf{x}_i = T_{i,s} \mathbf{x}_i$$

(3)

where $0$ is a $3 \times 3$ zero matrix and $U_{is}$ is a skew symmetric matrix determined by $u_{zi}$:

$$U_{is} = \begin{bmatrix} 0 & -z_{is} & y_{is} \\ z_{is} & 0 & -x_{is} \\ -y_{is} & x_{is} & 0 \end{bmatrix}$$

(4)

The position vector of the point $s_{i+1}$ can be similarly derived as

$$\mathbf{x}_{i+1,s} = \begin{bmatrix} 1 \\ U_{i+1,s} \\ 0 \end{bmatrix} \mathbf{x}_{i+1} = T_{i+1,s} \mathbf{x}_{i+1}$$

(5)

where $U_{i+1,s}$ is determined by position vector $u_{i+1,s}$ of the point $s_{i+1}$ with respect to the rigid body frame $x_{i+1}s_{i+1}$ in the undeformed state, which is similar to (4).

The potential energy of the 5-DOF stage is determined by the small deformations of flexure hinges, which can be directly calculated from the spatial motions of two rigid bodies. Suppose that the flexure hinge was free of deformation by cutting from rigid body $B_{i+1}$; the virtual point $\ddot{s}_{i+1}$ can be expressed as

$$\ddot{s}_{i+1} = \begin{bmatrix} \dot{r}_{i+1,s} \\ \dot{\theta}_{i+1,s} \end{bmatrix} = \begin{bmatrix} r_{is} + (A-1)h_{i,s} \\ \dot{\theta}_{i,s} \end{bmatrix}$$

(6)
where $h_{ij+1}$ is the position vector of the point $s_{i+1}$ with respect to the spring frame $x_{ij}, y_{ij}, z_{ij}$ in the undeformed state.

Substitute (2) into (6), and yield

$$
\ddot{\mathbf{x}}_{i+1} = \begin{bmatrix}
1 & U_{i,i+1} \\
0 & 1
\end{bmatrix} \mathbf{x}_{i+1} = H_{ij+1} T_{is} \mathbf{x}_i
$$

where $U_{ij+1}$ is determined by position vector $h_{ij+1}$:

$$
U_{ij+1} = \begin{bmatrix}
0 & z_{ij+1} & y_{ij+1} \\
-z_{ij+1} & 0 & x_{ij+1} \\
y_{ij+1} & -x_{ij+1} & 0
\end{bmatrix}
$$

Consequently, the small deformation of the flexure hinge can be measured from the virtual point $s'_{i+1}$ to the point $s_{i+1}$ and is expressed as

$$
\Delta \mathbf{x}_{ij+1} = R_{ij+1} \left( \mathbf{x}'_{i+1} - \mathbf{x}_{i+1} \right)
$$

where $R_{ij+1}$ is the rotation matrix from the spring frame $x_{ij+1}, y_{ij+1}, z_{ij+1}$ to the global frame $XYZ$. $H_{ij+1}$ is the vector about the length of the flexure hinge in the undeformed state, and $T_{is}$ and $T_{is+1}$ are the position vectors of the points $s_i$ and $s_{i+1}$ with respect to their rigid body frame in the undeformed state, respectively.

As a result, the potential energy in the flexure hinge is yielded as

$$
U_{ij+1} = \frac{1}{2} (\Delta \mathbf{x}_{ij+1})^T K_{ij+1} \Delta \mathbf{x}_{ij+1}
$$

where $K_{ij+1}$ is the stiffness matrix of the flexure hinge $S_i$ with respect to the global frame.

Rigid bodies are generally connected together through multiple flexure hinges. Let $n$ denote the number of rigid bodies and let $s_{ij}$ denote the number of flexure hinges between $B_i$ and $B_j$ ($i, j = 1, 2, \ldots , n, i \neq j$). According to (10), the total potential energy of flexure hinges in the stage is written as

$$
U = \sum_{i=0}^{n-1} \sum_{j=i+1}^{n} \sum_{s=1}^{s_{ij}} \frac{1}{2} \left( H_{ij} T_{js} \mathbf{x}_j - T_{ji} \mathbf{x}_i \right)^T K_{ij} \left( H_{ij} T_{js} \mathbf{x}_j - T_{ji} \mathbf{x}_i \right)
$$

Differentiation in the potential energy with respect to the rigid body coordinate leads to

$$
K_{ii} = \sum_{j=0}^{n} \sum_{s_{ij}} \sum_{s=1}^{s_{ij}} T_{ij}^T H_{ij} \left( R_{ij} \right)^T K_{ij} R_{ij} H_{ij} T_{js}
$$

$$
K_{ij} = \sum_{s=1}^{s_{ij}} T_{ij}^T H_{ij} \left( R_{ij} \right)^T K_{ij} R_{ij} T_{js}
$$

where $K_{ii}$ and $K_{ij}$ are the equivalent stiffness matrix of the 5-DOF stage.

In the conventional dynamic equations of the flexure-based mechanism, the length of the flexure hinges is ignored. Considering the length of the flexure hinge, Choi [28] derived
the equivalent stiffness matrices based on force and moment equilibrium of the rigid body, as yielded as

\[ K_{ij} = \sum_{s=1}^{n} T_{ij}^T H_{ij} S_{ij} K_{ij} S_{ij} T_{ij} \]  

\[ K_{ij} = \sum_{s=1}^{n} T_{ij}^T H_{ij} S_{ij} K_{ij} S_{ij} T_{ij} \]  

Compared with the proposed equations (12) and (13) in this study, there are two major differences among these equivalent stiffness equations. Firstly, the term \( H_{ij} \) is omitted in Choi's equation (14), which would cause incorrect mathematical results especially when long flexure hinges are employed. Secondly, the transformation matrix is varied in Choi's equations, which is \( R_{ij} \) in (14) but \( R_{ij} \) in (15). It can be found that the derivation in this study, based on the spatial motions of rigid bodies, is more rigorous and concise than Choi's derivation.

The mass matrix of rigid body \( B_i \) with respect to the global frame is given by

\[ M_i = \text{diag}(m_i \mathbf{1} J_i) \]  

where \( m_i \) is the mass of \( B_i \) and \( J_i \) is the inertia tensor with respect to the global frame. The mass matrices of rigid bodies can be arranged as

\[ M = \text{diag}(M_1 M_2 \cdots M_n) \]  

According to the Lagrange equations, the dynamic equation can be written as

\[
\begin{bmatrix}
M_1 & & & & \\
& M_2 & & & \\
& & \ddots & & \\
& & & M_n & \\
\end{bmatrix}
\begin{bmatrix}
\ddot{x}_1 \\
\ddot{x}_2 \\
\vdots \\
\ddot{x}_n \\
\end{bmatrix}
= \begin{bmatrix}
K_{11} & -K_{12} & \cdots & -K_{1n} \\
-K_{21} & K_{22} & \cdots & \vdots \\
\vdots & \ddots & \ddots & \vdots \\
-K_{n1} & \cdots & \cdots & K_{nn} \\
\end{bmatrix}
\begin{bmatrix}
x_1 \\
x_2 \\
\vdots \\
x_n \\
\end{bmatrix}
\]  

\[ \begin{bmatrix}
f_1 \\
f_2 \\
\vdots \\
f_n \\
\end{bmatrix}
\]

Modal frequencies of the 5-DOF stage can be obtained by solving the eigenvalue equation of (18).

3.2. Compliance Matrix of Circular Flexure Hinge. The accuracy of the dynamic model primarily depends on the accuracy of the compliance matrices of the flexure hinges. There are two types of flexure hinges used in the 5-DOF stage, circular flexure hinge and spring leaf. The stiffness matrix of spring leaf is easy to be calculated using compliance equations of prismatic beam [28]. The geometry of the circular flexure hinge is described by the minimum thickness \( t \), the radius \( r \), the width \( w \), and the length \( L \) as shown in Figure 5. The thickness of circular flexure hinge is varied along the length direction, as defined as

\[ t(x) = t + 2 \left[ r - \sqrt{r^2 - (r - x)^2} \right] \]  

The compliance matrix of the circular flexure hinge is non-diagonal because of the coupled translational and rotational displacements in the Y and Z directions, as expressed as

\[
\begin{bmatrix}
C_{x-F_x} & 0 & 0 & 0 & 0 \\
0 & C_{y-F_y} & 0 & 0 & 0 \\
0 & 0 & C_{z-F_z} & 0 & 0 \\
0 & 0 & 0 & C_{\theta_{y-M_y}} & 0 \\
0 & 0 & 0 & 0 & C_{\theta_{z-M_z}} \\
\end{bmatrix}
\]

\[ K_{ij} = (C_{ij})^{-1} \]  

The accuracies of various in-plane compliance equations have been reviewed by Yong [32]. In this paper, the accuracies of the out-of-plane compliance equations are mainly discussed. The out-of-plane compliance equations are expressed as

\[
C_{z-F_z} = \frac{12}{Ew^3} \int_0^L \frac{x^2}{t(x)} dx + \frac{5}{6Gw} \int_0^L \frac{1}{t(x)} dx \]  

\[ C_{z-M_z} = \frac{12}{Ew^3} \int_0^L \frac{x}{t(x)} dx \]  

\[ C_{\theta_{z-M_z}} = \frac{12}{Ew^3} \int_0^L \frac{1}{t(x)} dx \]  

\[ C_{\theta_{x-M_x}} = \frac{1}{G} \int_0^L \frac{1}{I_x(x)} dx \]
The torsional compliance depends on the ratio between the thickness and the width of the circular flexure hinge. Young's simplified equation [39] is used to calculate the torsional compliance in this paper.

\[
I_x (x) = w^3 t (x) \left[ \frac{1}{3} - 0.21 \frac{w}{t(x)} \left( 1 - \frac{w^5}{t^5(x)} \right) \right]
\]

where the subscript \( \delta \) represents that the shear effects are considered.

From Table 1, the accuracies of various out-of-plane compliance equations can be obtained. Wu and Zhou's compliance results are almost the same as the mathematical results, but the shear constant 5/6 for the rectangle cross-section of the circular flexure hinge is ignored in \( \mathcal{C}_{(\Delta x/F_x)} \). Koseki's compliance results have larger difference than Wu and Zhou's compliance results, and the shear effects are not considered. Lobontiu's compliance results have larger differences than the others' results and are not concerned much about the directions of the force and the displacement. Koseki's torsional compliance is double the mathematical torsional compliance. Chen's torsional compliance has smaller differences compared with the mathematical torsional compliance result.

The out-of-plane compliance matrices of circular flexure hinges are calculated by using Mathematica software, as discussed in Section 3.2. Symbolic formulations are applied and implemented in MATLAB [38]. By using the developed multibody dynamic model, modal analyses of two primary composition structures, the in-plane XY stage and limb 1 of the out-of-plane \( \theta_x, \theta_y, Z \) stage, and the integral 5-DOF stage are performed. The analytical modal analysis results are compared with FEA and modal experimental test results to validate the developed multibody dynamic model.

### Table 1: Compliance results with various out-of-plane compliance equations.

|       | \( \mathcal{C}_{(\Delta x/F_x)} \) (m/N) | \( \mathcal{C}_{(\Delta x/F_y)} \) (m/N) | \( \mathcal{C}_{(\Delta y/M_y)} \) (rad/Nm) | \( \mathcal{C}_{(\Delta x/M_y)} \) (rad/Nm) | \( \mathcal{C}_{(\Delta y/M_x)} \) (rad/Nm) |
|-------|------------------------------------------|------------------------------------------|------------------------------------------|------------------------------------------|------------------------------------------|
| Math.  | \( 9.26 \times 10^{-10} \)              | \( 6.01 \times 10^{-9} \)              | \( 8.78 \times 10^{-5} \)              | \( -2.63 \times 10^{-7} \)              | \( 1.90 \times 10^{-2} \)              |
| Wu and Zhou | \( 9.26 \times 10^{-10} \)        | \( 5.17 \times 10^{-9} \)        | \( 8.78 \times 10^{-5} \)        | \( -2.63 \times 10^{-7} \)        | \( / \)                           |
| Koseki | \( 9.98 \times 10^{-10} \)              | \( / \)                              | \( 7.92 \times 10^{-5} \)              | \( -3.8 \times 10^{-7} \)              | \( 3.77 \times 10^{-2} \)              |
| Lobontiu | \( -7.67 \times 10^{-10} \)        | \( 4.30 \times 10^{-9} \)        | \( -9.59 \times 10^{-6} \)        | \( -1.28 \times 10^{-7} \)        | \( / \)                           |
| Chen   | /                                        | /                                    | /                                        | /                                        | \( 2.15 \times 10^{-2} \)              |

### 4. Modal Analysis of the 5-DOF Flexure-Based Nanopositioning Stage

As described in Section 2, the 5-DOF flexure-based nanopositioning stage is consisted of 28 rigid bodies and 43 flexure hinges. According to topologies of rigid bodies and flexure hinges of the 5-DOF stage, multibody dynamic model can be established with the proposed dynamic equation in Section 3. The mathematical compliance matrices of circular flexure hinges are calculated by using Mathematica software, as discussed in Section 3.2. Symbolic formulations are applied and implemented in MATLAB [38]. By using the developed multibody dynamic model, modal analyses of two primary composition structures, the in-plane XY stage and limb 1 of the out-of-plane \( \theta_x, \theta_y, Z \) stage, and the integral 5-DOF stage are performed. The analytical modal analysis results are compared with FEA and modal experimental test results to validate the developed multibody dynamic model.

#### 4.1. Finite Element (FE) Model of the 5-DOF Stage

As described in Section 2, the 5-DOF flexure-based nanopositioning stage is consisted of 28 rigid bodies and 43 flexure hinges. According to topologies of rigid bodies and flexure hinges of the 5-DOF stage, multibody dynamic model can be established with the proposed dynamic equation in Section 3. The mathematical compliance matrices of circular flexure hinges are calculated by using Mathematica software, as discussed in Section 3.2. Symbolic formulations are applied and implemented in MATLAB [38]. By using the developed multibody dynamic model, modal analyses of two primary composition structures, the in-plane XY stage and limb 1 of the out-of-plane \( \theta_x, \theta_y, Z \) stage, and the integral 5-DOF stage are performed. The analytical modal analysis results are compared with FEA and modal experimental test results to validate the developed multibody dynamic model.
Table 2: The first two order modal frequencies results of the XY stage and a limb of the $\theta_x\theta_yZ$ stage.

| Order                        | Analytic /Hz | %E  | FEA /Hz | %E  | Test /Hz | Mode shapes                                      |
|------------------------------|--------------|-----|---------|-----|----------|--------------------------------------------------|
| 1st order of the XY stage    | 46.9         | 5.6 | 51      | 2.6 | 49.7     | Translation of motion plate 1 along the y-axis    |
| 2nd order of the XY stage    | 55.6         | 6.1 | 58.6    | 11.8| 52.4     | Translation of motion plate 1 along the x-axis    |
| 1st order of a limb of the $\theta_x\theta_yZ$ stage | 43.0   | 10.2| 44.7    | 6.7 | 47.9     | Bending about the leaf springs                    |
| 2nd order of a limb of the $\theta_x\theta_yZ$ stage | 123.7          | 9.5 | 128.2   | 6.2 | 136.7    | Out-of-plane rotation about the leaf springs      |

4.2. Experimental Test Setup. Modal experimental test is carried out by means of hammer impact testing. The schematic diagram of experimental setup is based on the LMS Test.Lab Vibration testing and analysis system, as shown in Figure 7.

In the process of modal experimental test, the 5-DOF stage and the primary composition structures are hung up with rubber thread. Free modal analyses of two primary composition structures, that is, the in-plane XY stage and limb 1 of the out-of-plane $\theta_x\theta_yZ$ stage, and the integral 5-DOF stage are performed by ANSYS.

4.3. Modal Analysis Results Discussion. Considering the complexity of the 5-DOF stage, dynamic characteristics of two primary composition structures, that is, the in-plane XY stage and limb 1 of the out-of-plane $\theta_x\theta_yZ$ stage, are firstly analyzed to ensure the analysis accuracy of the integral 5-DOF stage. Their first two order modal shapes are shown in Figure 9 and the modal frequencies are listed in Table 2. The analytical modal frequencies of the XY stage and the limb of the $\theta_x\theta_yZ$ stage are well in agreement with experimental test results; the errors are mostly less than 10%. FEA results are also well in agreement with experimental test, which validate the precision of the FE model with local fine meshes in flexure hinges. The errors are probably from the manufacture error because of the small size of the flexure hinge. The
comparisons of two primary composition structures can ensure the accuracy of the integral stage model.

As for the assembled 5-DOF stage, the first eight modal results of the multibody dynamic model, FEA, and experimental test are listed in Table 3. The first six mode shapes are induced by the low stiffness of three limbs of the $\theta_1\theta_2 Z$ stage, and the seventh and eighth modes are related with the XY stage. Compared with the above modal results of two composition structures, the integral 5-DOF stage has higher modal frequencies. This is because the hybrid serial-parallel hybrid construction enhances its dynamic performance. It can be found from Table 3 that FEA results are well in agreement with experimental test results, but the analytical modal results have larger difference. The reason is that the proposed multibody dynamic model only takes into account the flexibilities of flexure hinges [26]. Although the analytical modal results are mostly well in agreement with FEA and experimental test results, the errors of lower order modal frequencies are less than the higher order modal frequencies, up to about 20%. The developed multibody dynamic model is convenient to establish the mathematical relationship between the dynamic performance and the design parameters of the 5-DOF stage.

5. Conclusions

This paper focuses on the accurate multibody dynamic modeling of a 5-DOF flexure-based nanopositioning stage in the projection optical system of the semiconductor lithography. The stage is a serial-parallel stage by bolting a 2-DOF in-plane and a 3-DOF out-of-plane stage together. The 5-DOF stage is considered as an assembly of rigid bodies interconnected by elastic flexure hinges. Considering the length effects of flexure hinges, multibody dynamic equations are established according to spatial motions of rigid bodies by using Lagrangian method. It can be found that the developed dynamic equations are more rigorous and concise than Choi’s derivation. The accuracies of out-of-plane compliance matrices of the circular flexure hinge are investigated to enhance the modeling accuracy. FEA and modal hammer experimental tests of the primary flexure-based composition structures, that is, the in-plane XY stage and limb 1 of
Table 3: Modal results of the dynamic model of the 5-DOF nanopositioning stage.

| Order | Analytic /Hz | %E | FEA /Hz | %E | Test /Hz | Mode shapes |
|-------|--------------|----|---------|----|----------|-------------|
| 1     | 71.1         | 5.8| 72.2    | 7.4| 67.2     | Translation of the base along the z-axis |
| 2     | 78.4         | 6.4| 75.9    | 3.0| 73.7     | Rotation of the base about the x-axis |
| 3     | 79.0         | 7.2| 76.2    | 3.4| 73.7     | Rotation of the base about the y-axis |
| 4     | 106.9        | 15.3| 98.3    | 6.0| 92.7     | Translation of plate 2 along the y-axis |
| 5     | 116.1        | 12.7| 104.6   | 1.6| 103      | Translation of the base along the x-axis |
| 6     | 169.8        | 18.1| 144.1   | 0.2| 143.8    | Rotation of the XY stage about the z-axis |
| 7     | 184.2        | 20.3| 161.3   | 5.4| 153.1    | Translation of plate 3 along the y-axis |
| 8     | 202.8        | 19.4| 176.1   | 3.7| 169.8    | Translation of motion plate 1 along the x-axis |

the out-of-plane $\theta_1, \theta_2, Z$ stage, and the integral 5-DOF stage are performed to validate the accuracy of the developed multibody dynamic model. The analytical modal frequencies are well in agreement with FEA and experimental test results. The results show that the established multibody dynamic model is effective to establish a mathematical relationship between the design parameters and the dynamic performance of the 5-DOF stage. It is significant to provide a mathematical model to analyze and optimize the 5-DOF flexure-based nanopositioning stage.

Data Availability

All mass, position, and stiffness data of the 5-DOF flexure-based nanopositioning stage is published in the Figshare database (https://figshare.com/articles/mass_and_stiffness_data_of_nano-motion_stage/6115484).

Conflicts of Interest

The authors declare that they have no conflicts of interest.

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