Update on the quantum properties of the Supermembrane.

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In this note we summarize some of the quantum properties found since the early 80’s until nowadays that characterize at quantum level the spectrum of the supermembrane. In particular we will focus on a topological sector of the 11D supermembrane that, contrary to the general case, has a purely discrete spectrum at supersymmetric level. This construction has been consistently implemented in different types of backgrounds: toroidal and orbifold-type with G2 structure able to lead to a true G2 compactification manifold. This theory has $N = 1$ supersymmetries in 4D. comment on the relevant points of this construction as well as on its spectral characteristics. We will also make some comments on the quantum properties of some effective formulation of multiple M2’s theories recently found.

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1 The 11D Supermembrane.

The 11D supermembrane, also called M2 brane, \cite{1} was discovered in 1987. A year later its hamiltonian formulation in the Light Cone Gauge (L.C.G.) and its matrix regularization by \cite{2} was obtained. The supermembrane is the natural extension of the string in 11D and it was thought to be a fundamental object in 11D. However, the spectral properties of the regularized M2 differed substantially from those of the string as it was shown in a rigorous proof by \cite{2}. Firstly, the theory of the supermembrane is a constrained system highly nonlinear and difficult to solve, in distinction with the harmonic oscillator-type hamiltonian of the string. Secondly, the supermembrane classically contains instabilities that make the scalar potential along directions in the configuration space vanish. They are called ‘valleys’ and render the classical system unstable. The third and most important difference is that its supersymmetric spectrum is continuous. Let us characterize it in more detail. The hamiltonian of the $D = 11$ Supermembrane \cite{1} may be defined in terms of maps $X^\mu, \mu = 0, \ldots, 10$, from a base manifold $\Sigma \times R$ onto a target manifold which we will assume to be $11D$ Minkowski. $\Sigma$ is a Riemann surface of genus $g$. $\sigma^a, a = 1, 2$ are local spatial coordinates over $\Sigma$ and $\tau \in R$ represents the worldvolume time. Decomposing $X^\mu$ and $\Gamma^\mu$ accordingly to the standard L.C.G ansatz and solving the constraints, the canonical reduced hamiltonian to the light-cone gauge has the expression

\begin{equation}
H = \int_\Sigma \sqrt{W} \left( \frac{1}{2} \left( \frac{P_m}{\sqrt{W}} \right)^2 + \frac{1}{4} \sqrt{W} \{X^m, X^n\}^2 + \sqrt{W} \partial_\tau \Gamma_m \{X^m, \theta\} \right)
\end{equation}

where the range of $m$ is now $m = 1, \ldots, 9$ corresponding to the transverse coordinates in the light-cone gauge, and

\begin{equation}
\{X^m, X^n\} = \frac{\epsilon^{ab}}{W(\sigma)} \partial_a X^m \partial_b X^n.
\end{equation}

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$W(\sigma)$ is a scalar density introduced in the light-cone gauge fixing procedure. $\theta$ represents the 11D Majorana spinors.

The theory has a residual symmetry which is the invariance under the area preserving diffeomorphism so it is subject to the following constraints,

$$\phi_1 := d\left(\frac{P_m}{\sqrt{W}} dX^m + \bar{\theta} \Gamma_- d\theta\right) = 0 \quad \phi_2 := \oint_{C_s} \left(\frac{P_m}{\sqrt{W}} dX^m + \bar{\theta} \Gamma_- d\theta\right) = 0,$$

where $C_s$, $s = 1, \ldots, 2g$ is a basis of 1-dimensional homology on $\Sigma$.

- **Classical Analysis: String-like spikes**

The scalar potential of the supermembrane vanishes along some configurations known as String-like spikes. They are singular configurations associated to configurations of the bosonic maps which depend on a single worldvolume variable (or a combination of the two spatial variables) $X^m(\alpha \sigma^1 + b \sigma^2, \tau)$ that render the classical potential unstable. The system fluctuates along all the allowed states, consequently not preserving neither the topology nor the number of particles. Since the membrane may split and merge without any cost of energy, the concept of particle looses its meaning, and this fact can be taken as a first indication that the 11D Supermembrane can be seen as an effective theory of interacting lower-dimensional objects.

- **Bosonic Analysis**

At quantum level, however, the situation changes completely and the bosonic hamiltonian has a purely discrete spectrum in spite of the classical instabilities. Its analysis was performed in the context of matrix model formulation. The regularized hamiltonian of the M2 brane in the LCG is the following \[3\]:

$$H = Tr \left[ \frac{1}{2} (P^A) \gamma^A \frac{1}{4} (X^{mB} X^{nC} f_{ABC})^2 - \frac{i}{2} f_{ABC} X^m A \theta^B \gamma_m \theta^C \right]$$

subject to the Gauss constraint

$$\phi_A = f_{ABC} (X^{mB} P^C_m - \frac{i}{2} \theta^B \theta^C) \approx 0.$$ 

The sufficient condition for the bosonic regularized hamiltonian was formerly found by Lüscher [4] and Simons [5]. In [6] we have been able to improve these bounds by finding the necessary and sufficient condition for the discreteness of the regularized 11D membrane. That condition is explained in detail in [6]. We showed that then, the most precise condition ensuring the discreteness of the spectrum of the membrane theories, is given in terms of an intrinsic moment of inertia of the membrane. It may be interpreted as if the membrane, or equivalently the D0 branes describing it, have a rotational energy. It is a quantum mechanical effect. The condition is obtained from the Molchanov [7], Maz’ya and Shubin [8] necessary and sufficient condition on the potential of a Schrödinger operator to have a discrete spectrum. The criteria is expressed not in terms of the behaviour of the potentials at each point, but by a mean value, on the configuration space. The mean value in the sense of Molchanov considers the integral of the potential on a finite region of configuration space. It can be naturally associated to a discretization of configuration space in the quantum theory. We found that the mean value in the direction of the valleys where the potential is zero, at large distances in the configuration space, is the same as a harmonic oscillator with frequencies given by the tensor of inertia of the membrane. The interesting feature is that all previously known bounds for the membrane and Yang-Mills potential were linear on the configuration variables, while the bound we found is quadratic on the configuration variables.

Since the dimensionally reduced action of SYM to $0 + 1D$ corresponds exactly to the regularized action of the 11D Supermembrane, as indicated in [3], then the previous result also holds for the bosonic YM in this regime (Slow-mode), both interpreted as the interaction of multiple D0 branes. The Slow-mode regime assumes that there is no momentum for the D0 branes but they still have energy. In [6] we also
obtained analytically bounds, based on the moment of inertia for the hamiltonian of $0 + 1$D Yang-Mills theories which allows to obtain interpretation about the mass gap of the theory. In particular we found that a lower bound for $SU(3)$ in $3 + 1$ dimensions is given by a hamiltonian whose spectrum and eigenvalues are known, and its eigenfunctions are expressed in terms of Bessel functions.

- **Supersymmetric Analysis**

A quantum mechanic supersymmetric hamiltonian is realized in a matrix whose diagonal contains the bosonic part of the hamiltonian while the fermionic potential lies in the non-diagonal entries.

$$H = -\Delta + V_B \mathbb{1} + V_F$$

We denote by $H$ the hamiltonian operator defined in the whole phase space although the hamiltonian operator if the 11D Supermembrane is defined only in an hypercone which is a subset of the previous space of those solutions that satisfy the first class constraints of the supermembrane. The core of the proof, which was done in [2], consisted in the construction of a wavefunction such that for any given energy $E \geq 0$ there exists a suitable wave function $\Psi$ with $\|\Psi\| = 1$ such that for any $\epsilon > 0$ is always satisfied

$$\| (H - E)\Psi \| < \epsilon. \quad (5)$$

The $SU(N)$ regularized model obtained from (1) [1] [3] was shown to have continuous spectrum from $[0, \infty)$, [2], [10], [3]. It happens that the fermionic contribution cancel this effect of raising the valleys and lead to a continuous spectrum at quantum level. The supermembrane in 11D could not be considered any longer as a fundamental object, contrarily it was interpreted as a second quantized theory, a theory of interacting $D_0$'s. In 1996, [9] conjectured that the action that should be taken as a fundamental in the context of M-theory should be the matrix model action of $D0$ branes (related with the IKKT formulation in terms of $D(-1)$ branes). This led to the fruitful field of matrix model formulation so much developed since then. Almost simultaneously, there was an study on the supermembrane with winding started by [11] in a series of papers, where they extended the study to compactified spaces and were able to see that compactification by itself is not able to avoid the string-like spikes that together with supersymmetry render the spectrum continuous. Fortunately a new avenue was explored: the discovering of topological sector inside the supermembrane that has a purely discrete spectrum: The MIM2.

## 2 Type of Supermembranes with discrete spectrum: The MIM2

In what follows we will consider a topological restriction on the configuration space of the 11D supermembrane. This topological sector is characterized by imposing an irreducibility winding condition on the compactified target space. It generates a central charge in the supersymmetric algebra of the 11D supermembrane. Geometrically it corresponds to a Supermembrane minimally immersed in the target space. For that reason from now on, we will refer to it as MIM2, [16]-[20]. Following [19] we may extend the original construction on a $M_9 \times T^2$ to $M_7 \times T^4$, $M_5 \times T^6$ target manifolds by considering genus 1, 2, 3 Riemann surfaces on the base respectively. We are interested in reducing the theory to a 4 dimensional model, we will then assume a target manifold $M_4 \times T^6 \times S^1$. The configuration maps satisfy:

$$\oint_{c_s} dX^r = 2\pi S^r_s R^r, \quad r, s = 1, \ldots, 6, \quad \oint_{c_s} dX^m = 0 \quad m = 8, 9; \quad \oint_{c_s} dX^7 = 2\pi L_s R, \quad (6)$$

where $S^r_s$, $L_s$ are integers and $R^r, r = 1, \ldots, 6$ are the radius of $T^6 = S^1 \times \cdots \times S^1$ while $R$ is the radius of the remaining $S^1$ on the target. We now impose the central charge condition

$$I^{rs} \equiv \int_\Sigma dX^r \wedge dX^s = (2\pi R^r R^s) \omega^{rs} n \quad (7)$$
where \( \omega^r \) is a symplectic matrix on the \( T^6 \) sector of the target and \( n \) denotes an integer representing the irreducible winding. The topological condition (7) does not change the field equations of the hamiltonian (1). In addition to the field equations obtained from (1), the classical configurations must satisfy the condition (7). In the quantum theory, the space of physical configurations is also restricted by the condition (7) [13,15].

We consider now a Hodge decomposition of the map satisfying condition (7):

\[
dX^r = M_s^r d\tilde{X}^s + dA^r
\]

where \( d\tilde{X}^s \), \( s = 1, \ldots, 2g \) is a basis of harmonic one-forms over \( \Sigma \), \( M_s^r \) is a matrix of integers carrying the d.o.f of harmonic forms and \( dA^r \) represents the single-valued one-forms. Now, we impose the constraints (3). It turns out that \( M_s^r \) gets fixed and can be expressed in terms of a matrix \( S \in Sp(2g, Z) \). The natural choice for \( \sqrt{W(\sigma)} \) in this geometrical setting is defined as \( \sqrt{W(\sigma)} = \frac{1}{2} \partial_a \tilde{X}^r \partial_b \tilde{X}^s \omega_{rs} \). \( \sqrt{W(\sigma)} \) is then invariant under the change \( d\tilde{X}^r \rightarrow S^r_s d\tilde{X}^s \). We thus conclude that the theory is invariant not only under the diffeomorphisms generated by \( \phi_1 \) and \( \phi_2 \) but also under the diffeomorphisms, biholomorphic maps, changing the canonical basis of homology by a modular transformation. The theory of supermembranes with central charges in the light cone gauge (LCG) we have constructed depends then on the moduli space of compact Riemannian surfaces \( M \) only. In addition, when compactified to 9D there has been proved in [21] that the hamiltonian is also invariant under a second \( SL(2, Z) \) symmetry associated to the \( T^2 \) target space that transform the Teichmüller parameter of the 2-torus \( \beta \).

This construction can be seen in detailed in [12]. There the theory is formulated in 4D and its hamiltonian is the following:

\[
H_d = \int \sqrt{\omega} d\sigma \wedge d\sigma^T \left( \frac{1}{2} \left( \frac{P^m}{\sqrt{W}} \right)^2 + \frac{1}{2} \left( \frac{\Pi^r}{\sqrt{W}} \right)^2 + \frac{1}{4} \{ X^m, X^m \}^2 + \frac{1}{2} (D_r X^m)^2 \\
+ \frac{1}{4} (F_{rs})^2 + \frac{1}{2} (F_{ab} \frac{\epsilon_{ab}}{\sqrt{W}})^2 + \frac{1}{8} \left( \frac{\Pi^c}{\sqrt{W}} \partial_c X^m \right)^2 + \frac{1}{8} \left[ \Pi^c \partial_c (\tilde{X}^r + A_r) \right]^2 + \\
\Lambda \left( \frac{P^m}{\sqrt{W}}, X^m \right) - D_r \Pi^r - \frac{1}{2} \Pi^c \partial_c (F_{ab} \frac{\epsilon_{ab}}{\sqrt{W}}) + \lambda \partial_b \Pi^c \right] + \\
+ \int \omega \left( - \Phi \Gamma_- \Gamma_r \Psi + \Gamma^- \Gamma_m \{ X^m, \Psi \} + 1/2 \Phi \Gamma_- \Pi^b \partial_b \Psi \right) + \Lambda \{ \Phi \Gamma_- \}, \Psi \}
\]

where \( D_r X^m = D_r X^m + \{ A_r, X^m \} \), \( F_{rs} = D_r A_s - D_s A_r + \{ A_r, A_s \} \), \( D_r = 2\pi R^r \frac{\epsilon_{ab}}{\sqrt{W}} \partial_a \tilde{X}^r \partial_b \) and \( P_m \) and \( \Pi_r \) are the conjugate momenta to \( X^m \) and \( A_r \) respectively. \( \Psi \) are \( SO(7) \) Majoron spinors. \( D_r \) and \( F_{rs} \) are the covariant derivative and curvature of a symplectic noncommutative theory [15,17], constructed from the symplectic structure \( \omega_{ab} \) introduced by the central charge.

The integral of the curvature we take it to be constant and the volume term corresponds to the value of the hamiltonian at its ground state. The physical degrees of the theory are the \( X^m, A_r, \Pi^c, \Psi \). They are single valued fields on \( \Sigma \). It was shown in [12] that the supermembrane minimally immerse on this \( T^7 \) has no moduli free for the isotropic tori, is \( N = 1 \) (4 target space supersymmetries) and has been proved in [12] that it has a purely discrete spectrum with finite multiplicity and compact resolvent following the lines of the theorems shown in Section 3. Moreover, it is possible to define the supermembrane with central charges induced through an irreducible winding on an orbifold with G2 structure and define precisely all of the maps of the twisted states. Since the symmetries of the orbifold are symmetries of the parent theory in the untwisted sector, then no state is projected out and it coincides with those of the 4D formulation of the MIM2 on the \( T^7 \). Due to the regularized parent hamiltonian symmetries one can guaranteed that the spectral properties are inherited on the orbifolded theory since the twisted sector only adds a finite number of states that do not change the qualitative properties of the spectrum. Once that the singularities are resolved by the action of the symplectomorphisms present in the theory one ends with the well-known example of compact G2 manifold with the supermembrane minimally immersed on it.
3 Discreteness of the spectrum

In this section I would like to illustrate the master lines followed to show the discreteness of this sector of the supermembrane theory. The analysis of the spectrum of the truncated Schrödinger operator associated to $\hat{H}$ without further requirements on the constants $f^N$:

i) The potential of the Schrödinger operator only vanishes at the origin of the configuration space:

$$V = 0 \rightarrow ||(X, A, \phi)|| = 0$$

where $|| . ||$ denotes the euclidean norm in $R^L$. We notice that the original hamiltonian as well as $\hat{H}$ are defined on fields up to constants. This condition guarantee the non-existence of singular configuration in the hamiltonian.

ii) There exists a constant $M > 0$ such that

$$V(\mathbf{x}, A, \phi) \geq M||(\mathbf{x}, A, \phi)||^2.$$  \hspace{1cm} (10)

Again, this bound arises from very general considerations. In fact, writing $(\mathbf{x}, A, \phi)$ in polar coordinates $\mathbf{X} = Rx \quad A = Ra \quad \phi = R\varphi$ where $\theta \equiv (x, a, \varphi)$ is defined on the unit sphere, $||(\mathbf{x}, A, \phi)|| = 1,

$$V(\mathbf{x}, A, \phi) \geq MR^2.$$  \hspace{1cm} (11)

The Schrödinger operator is then bounded by below by an harmonic oscillator and goes to infinity on every direction of the configuration space [5]. Consequently it has a compact resolvent. The result coincides qualitatively with the bosonic statement of discreteness for the regularized membrane without the central charge condition, however, the bound is not the same. It depends indirectly on the central charge condition which is responsible for the generation of the mass terms. The proof of discreteness for the bosonic part of the supermembrane with central charges without regularization was done in [20]. It corresponds to have infinite d.o.f., then spectral theorems valid for finite Hilbert spaces generically do not hold.

- The Supersymmetric Analysis.

The supersymmetric extension can be proved in different ways [18, 22] to be also discrete. See for example the guide lines of the proof in [18]. By using the Lemma.1 of [18] that states:

**Lemma 3.1** Let $v_k(x)$ be the eigenvalues of $V(x)$. If all $v_k(x) \rightarrow +\infty$ as $|x| \rightarrow \infty$, then the spectrum of $H$ is discrete.

To this end the authors of [18], decompose the resolvent of the following operator $\mu$ as

$$\mu = -\Delta + V_B \mathbb{I} + V_F$$

where $V_B$ and $V_F$ denote the bosonic and fermionic potentials respectively on the whole space of configurations. The space of solutions of the supermembrane is smaller since it is constrained. Then $V_F$ is the sum of a linear homogeneous part $M(X, A)$ and a constant matrix $C$ that may be reabsorbed, (see [18] for further details). The eigenvalues are determined by the solutions of the characteristic equation. By virtue of the homogeneity of $M$, $\lambda$ must satisfy

$$\det \left[ \frac{\lambda - V_B \mathbb{I}}{R} - M(\phi, \psi) \right] = 0, \quad R > 0.$$  \hspace{1cm} (12)

Therefore if $\tilde{\lambda}$ are the eigenvalues of $M(\phi, \psi)$, then

$$\lambda = V_B(R\phi, R\psi) + R\tilde{\lambda}.$$  \hspace{1cm} (13)

Consequently, $\lambda \rightarrow +\infty$ whenever $R \rightarrow \infty$. Notice that $V$ is discrete, hence it is automatically bounded from below. Since this is true for the operator $\mu$ whose domain is the whole configuration space without...
considering the constraints, then it also holds for the constrained theory. This statement obviously would not hold in the other way-round. Finally it was shown that the fermionic contribution to the susy hamiltonian do not change the qualitative properties of the spectrum of the hamiltonian. In fact, both contributions are linear on the configuration variables. In addition the supersymmetric contribution cancels the zero point energy of the bosonic oscillators even in the exact theory \cite{23,20}. The Schröedinger operator is then bounded by an harmonic oscillator. Consequently it has a compact resolvent. We now use theorem 2 \cite{22} to show that: i) The ghost and antighost contributions to the effective action assuming a gauge fixing condition linear on the configuration variables, ii) the fermionic contribution to the susy hamiltonian, do not change the qualitative properties of the spectrum of the hamiltonian.

**Generic Supersymmetric matrix potential Analysis**

Whenever no string-like configurations are present the following sufficient condition for discreteness hold. The assumptions on the bosonic potential are very mild, we only require the potential to be measurable, bounded from below and unbounded above in every direction (if the potential is continuous the unbounded assumption ensures the bounded-ness from below). For instance, for a quantum mechanical potential of the form

\[
V = V_B(x) + V_F(x) \in \mathbb{C}^{2n \times 2n},
\]

where \(x \in \mathbb{R}^L\), \(V_B(x)\) is continuous with the asymptotic behaviour

\[
V_B \geq c \|x\|^{2p}, \quad c > 0,
\]

and the fermionic matrix potential satisfies

\[
V_F \leq V_F \|x\|^{2q},
\]

for all \(\|x\| > R_0\) with \(2p > q\), the Hamiltonian of the quantum system has spectrum consisting exclusively of isolated eigenvalues of finite multiplicity.

**Comments on the multiple M2’s spectrum**

The discreteness of the spectrum of the bosonic M5 brane \cite{24} has been characterized following the necessary and sufficiency condition showed in Section 1 \cite{6}. More generally, the authors have proved that provided a suitable matrix regularization of a generic p-brane whose regularized hamiltonian is of the form

\[
H_L = -\Delta + V_L(X) = -\Delta + (X^a_M a_1 \ldots a_L f_{b_1 \ldots b_L}^a)^2
\]

where \(L\) is the degree of the brane considered, \(M_1 = 1, \ldots, K, a_i = 1, \ldots, N, K \geq L, N \geq L, X = X^a_M \in \mathcal{R}^{KN}\) and \(f_{b_1 \ldots b_L}^a\) is a non-singular constant tensor totally antisymmetric and it is not singular, then the spectrum of the bosonic hamiltonian is discrete \cite{24}. This condition also holds for the type of bosonic scalar potentials of the multiple M2’s actions based on 3-algebras (whenever a proper discretization is provided). For example: \cite{25,27,28}. In the case of \cite{26} their action is expressed in terms of an ordinary lie algebra whose structure constants can be seen as some particular choice of the 3-algebra structure constants that render it not fully antisymmetric, then the above criteria does not hold. One may consider a toy model whose bosonic potential is a polynomial product of an arbitrary number of operators:

\[
V(x) = \Pi_{k=1}^n |x_k|^{\alpha_k},
\]

where \(\alpha_k > 0\) for all \(k = 1, 2, \ldots, n\). Then the spectrum of the Schrödinger operator \(-\Delta + V\) in \(L^2(\mathbb{R}^n)\) is discrete. This toy model gives a hint that also the bosonic potential of the \(N = 6\) Chern-Simons term coupled to matter will probably also be discrete, although a rigorous study of its quantum properties is needed.
to establish it properly [29]. An interesting issue is the spectral characterization of the complete hamiltonians including their supersymmetric extension. The analysis is much more involved. All of these actions of multiple M2’s have in common the construction of a conformal supersymmetric gauge theory, with a sextic scalar potential, quadratic couplings in the fermionic variables and two-coupled Chern-Simons terms with a number of supersymmetries, $N = 8$ [25, 27, 28] or $N = 6$ susy in the case. A rigorous study of its spectral properties is currently under study [29], here I will just make some heuristic comments. Their fermionic contribution in distinction with the case of a single M2 brane depends quadratically on the bosonic variables. The sufficient condition for discreteness of supersymmetric potentials shown previously is no longer applicable and although it does not exclude completely the possibility of the spectrum be discrete, makes it much more fine tuned. On the other hand, the continuity of the spectrum for a single M2 in [2] lies on the result that along the singularities the potential behaves as a susy harmonic oscillator so there is no confining potential. The present dependence of the fermionic terms on the bosonic variables makes it much more involved and to determine this a much more exhaustive study is required [29] and lies out the scope of this work.

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