Implementation of three-qubit Grover search in cavity QED

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Abstract

Using resonant interaction of three Rydberg atoms with a single-mode microwave cavity, we consider a realization of three-qubit Grover search algorithm in the presence of weak cavity decay, based on a previous idea for three-qubit quantum gate [Phys. Rev. A 73, 064304 (2006)]. We simulate the searching process under the influence of the cavity decay and show that our scheme could be achieved efficiently to find the marked state with high fidelity. The required operations are very close to the reach with current cavity QED techniques.

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Quantum algorithms, such as Shor’s algorithm [1], Grover search [2], and Deutsch-Jozsa’s algorithm [3], have attracted much interest since they could work in quantum computers, which are in principle able to solve intractable computational problems more efficiently than present classical computers. Many efforts have been devoted to achievement of these quantum algorithms theoretically and experimentally by using trapped ions [4-6], NMR system [7-8], superconducting mesocircuits [9], cavity quantum electrodynamics (QED) [10-14], and linear optical elements [15,16].

In this Brief Report, we focus on a scheme of three-qubit Grover search with cavity QED. Cavity QED has been considering to be an efficient candidate for small-scale quantum information processing and for quantum network. The rapid development in relevant experimental technologies has enabled us to achieve entanglement between two atoms in a microwave cavity [11], based on which there have been some proposals for two-qubit Grover search with cavity QED [12,13]. We have also noticed a very recent publication for three-qubit Grover search with three four-level atoms going through a three-mode cavity [14]. Actually, the important difference of the three-qubit Grover search from the two-qubit case is the probabilistic achievement. To reach a case with high success probability, we have to implement the basic searching step (also called iteration) for several times. So implementation of a three-qubit Grover search is much more complex than that of a two-qubit case. In contrast to [14], we will design a simpler but efficient Grover search scheme by three identical Rydberg atoms sent through a single-mode microwave cavity. We will store quantum information in long-lived internal levels of the Rydberg atoms, and consider the resonant interaction between the atoms and the cavity mode, which yields a very fast implementation of the search. As the cavity decay is the main dissipative factor of our design, we will seriously consider its detrimental effect on our scheme.

Let us first briefly review the main points of a Grover search algorithm, which consists of three kinds of operations [4]. The first one is to prepare a superposition state \( |\Psi_0 \rangle = \left( \frac{1}{\sqrt{N}} \right) \sum_{i=0}^{N-1} |i \rangle \) using Hadamard gates. The second is for an iteration \( Q \), following two operations: (a) Inverting the amplitude of the marked state \( |\tau \rangle \) using a quantum phase gate \( I_\tau = I - 2|\tau \rangle \langle \tau | \), with \( \tau \) the identity matrix; (b) Inversion about average of the amplitudes of all states using the diffusion transform \( D \), with \( \hat{D}_{ij} = \frac{1}{N} - \delta_{ij} \) \((i,j = 1,2,3,\ldots,N)\) and \( N = 2^q \) (being the qubit number). This step should be carried out by at least \( \pi \sqrt{N}/4 \) times to maximize the probability for finding the marked state. Finally, a measurement of the whole system is done to get the marked state. In other word, the Grover search consists in a repetition of the transformation \( Q = H_{atom}HI_\tau \), with \( H_{atom} = I - 2 |000 \rangle \langle 000 | \) (defined later). In three-qubit case, the number of possible quantum states is \( 2^3 \), and the operation to label a marked state by conditional quantum phase gate is \( I_\tau \) with \( \tau \) read as \( 1 \). To carry out the Grover search, we need that the three atoms couple to the cavity mode by \( \Omega_1 \), \( \Omega_2 \), \( \Omega_3 \) = 1 : \( \sqrt{35} : 8 \) and the gating time be \( \frac{\pi}{\Omega_1} \). In our proposal, the qubit definitions are not the same for each atom. The logic state \( |1 \rangle \) of the qubit 1 is denoted by \( |g_1 \rangle \) of the atomic state \( 1 \); \( |g_2 \rangle \) and \( |g_3 \rangle \) of the qubit 2 encode the logic state \( |0 \rangle \) of the qubit 2; The logic state \( |0 \rangle \) of the qubit 3 is represented by \( |g_3 \rangle \) of the qubit 3. Ref. [17] has shown us the possibility to achieve an approximate three-qubit quantum phase gate \( I_{e_1 e_2 e_3} = I_{000} = \text{diag} \{ -1, \gamma_0, 1, 1, 1, 1, 1 \} \) in a computational subspace spanned by \( |e_1 \rangle |e_2 \rangle |e_3 \rangle, |e_1 \rangle |e_2 \rangle |g_3 \rangle, |e_1 \rangle |g_2 \rangle |e_3 \rangle, |e_1 \rangle |g_2 \rangle |g_3 \rangle, |g_1 \rangle |e_2 \rangle |e_3 \rangle, |g_1 \rangle |e_2 \rangle |g_3 \rangle, |g_1 \rangle |g_2 \rangle |e_3 \rangle, |g_1 \rangle |g_2 \rangle |g_3 \rangle \), where \( \gamma_0 = \frac{\Omega_1^2}{\Omega_1^2 + \Omega_3^2} \cos \sqrt{65} \pi \) and \( \Omega_3 = \frac{\Omega_1^2 + \Omega_3^2}{\Omega_1^2 + \Omega_3^2} = 0.9997 \). To carry out the Grover search, we define the three-qubit Hadamard gate,

\[
H_{3} = \prod_{i=1}^{3} H_i = \left( \frac{1}{\sqrt{2}} \right)^3 \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & -1 \\ -1 & 1 & 1 \end{bmatrix} \otimes \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & -1 \\ -1 & 1 & 1 \end{bmatrix},
\]

where \( H_i \) is the Hadamard gate acting on the \( i \)th atom, transforming states as \( |e_1 \rangle \rightarrow (1/\sqrt{2})(|e_1 \rangle + |g_1 \rangle) \), \( |g_1 \rangle \rightarrow (1/\sqrt{2})(|e_1 \rangle - |g_1 \rangle) \), \( |e_2 \rangle \rightarrow (1/\sqrt{2})(|e_2 \rangle + |g_2 \rangle) \), \( |g_2 \rangle \rightarrow (1/\sqrt{2})(|e_2 \rangle - |g_2 \rangle) \), \( |e_3 \rangle \rightarrow (1/\sqrt{2})(|e_3 \rangle + |g_3 \rangle) \), \( |g_3 \rangle \rightarrow (1/\sqrt{2})(|e_3 \rangle - |g_3 \rangle) \). These gatings could be performed by external microwave pulses.

It is easy to find that the transformation \( Q = H_{3} I_{000} H_{3} I_\tau = H_{3} I_{e_1 e_2 e_3} H_{3} I_\tau = -\hat{D} I_\tau \), which implies that the diffusion transform \( \hat{D} \) is always unchanged, no matter which state is to be searched. The only change is the phase
gate $I_t$ for different marked states. Based on the gate $I_{000}$ to mark the state $|e_1 i_2 i_3\rangle$, we could construct other seven gates for the marking job as,

\[
I_{e_1 i_2 g_3} = I_{001} = \sigma_{x,1} I_{000} \sigma_{x,3}, \quad I_{e_1 g_2 i_3} = I_{010} = \sigma_{x,2} I_{000} \sigma_{x,2},
I_{e_1 g_2 g_3} = I_{011} = \sigma_{x,3} \sigma_{x,2} I_{000} \sigma_{x,2} \sigma_{x,3},
I_{g_1 i_2 g_3} = I_{100} = \sigma_{x,1} I_{000} \sigma_{x,1}, \quad I_{g_1 i_2 i_3} = I_{101} = \sigma_{x,3} \sigma_{x,1} I_{000} \sigma_{x,1} \sigma_{x,3},
I_{g_1 g_2 i_3} = I_{110} = \sigma_{x,2} \sigma_{x,1} I_{000} \sigma_{x,1} \sigma_{x,2},
I_{g_1 g_2 g_3} = I_{111} = \sigma_{x,3} \sigma_{x,2} \sigma_{x,1} I_{000} \sigma_{x,1} \sigma_{x,2} \sigma_{x,3}.
\]

So with the state marked, and the three-qubit diffusion transform $\hat{D}$ which is generated by combining two Hadamard gates $H \otimes I$ with the quantum phase gate $I_{000}$, a full Grover search for three qubits is available.

Taking the marked state $|010\rangle$ as an example, we design a three-qubit Grover search setup in Fig. 1. The cavity is a microwave cavity sustaining a single mode with a standing-wave pattern along the $z$-axis. The atoms 1, 2 and 3 prepared in high-lying circular Rydberg states are sent through the cavity with proper speed, resonantly interacting with the cavity mode. Single-qubit rotations are made at certain times by external microwave pulses, and the state-selective field-ionization detectors $D_1$, $D_2$, $D_3$ are settled at the end of the passage for checking the states of the atoms 1, 2 and 3, respectively. One point to mention is that, in searching the state $|011\rangle$ or $|111\rangle$, inhomogeneous electric fields are needed to tune the atomic transitions through the Stark effect [12,13], which make the single-qubit operations completed individually. But these inhomogeneous electric fields are unnecessary in searching other states.

As the resonant interaction actually excites the cavity mode, although we could carry out the scheme very fast, we should consider the cavity decay seriously. Under the assumption of weak cavity decay that no photon actually leaks out of the microwave cavity during our implementation time, we employ the quantum trajectory method by the Hamiltonian,

\[
H = \sum_{j=1}^{3} \Omega_{j}\{a^{+} S_{j}^{+} + a S_{j}^{+} - \frac{i}{2} \alpha \},
\]

where $\alpha$ is the cavity decay rate. As discussed in [17], under the weak decay condition, the cavity dissipation only affects the diagonal elements of the matrix for the phase gate. For example, by choosing the interaction time $t_{\Omega} = \pi/A_{1\kappa}$ with $A_{1\kappa} = \sqrt{\Omega_{1c}^{2} - \kappa^{2}/16}$ and the condition $\Omega_{1c} : \Omega_{2c} : \Omega_{3c} = 1 : \sqrt{35} : 8$, we generate the three-qubit phase gate $I_{000}$ in the decay case,

\[
I_{e_1 i_2 i_3} = diag\{-\mu_1, \gamma_1, \mu_1, \alpha_1, 1, 1, 1\} = U_{0}(t),
\]

where $\alpha_1 = 1 - \frac{\Omega_{1c}^{2}}{\Omega_{1c}^{2} + \Omega_{2c}^{2} + \Omega_{3c}^{2}}(1 - e^{-\kappa t/4})$, $\beta_1 = 1 - \frac{\Omega_{1c}^{2}}{\Omega_{1c}^{2} + \Omega_{2c}^{2} + \Omega_{3c}^{2}}(1 - e^{-\kappa t/4})$, $\mu_1 = e^{-\kappa t/4}$, and $\gamma_1 = 1 - \frac{\Omega_{1c}^{2}}{\Omega_{1c}^{2} + \Omega_{2c}^{2} + \Omega_{3c}^{2}}[1 - e^{-\kappa t/4} \cos(\sqrt{6} \kappa \pi)]$ with the negligible term $\frac{1}{2 A_{1\kappa} \sqrt{\kappa}} \sin(A_{1\kappa})$ is omitted. So for a state $|\Psi\rangle = \frac{1}{2^{\frac{1}{2}}} (A_j |e_1 i_2 g_3\rangle + B_j |e_1 i_2 i_3\rangle + C_j |e_1 g_2 i_3\rangle + D_j |e_1 g_2 g_3\rangle + E_j |g_1 i_2 g_3\rangle + F_j |g_1 i_2 i_3\rangle + G_j |g_1 g_2 i_3\rangle + H_j |g_1 g_2 g_3\rangle)$, the success probability of the phase gate is defined as

\[
P_j = (|D_j|^{2} \sigma_{1}^{2} + |C_j|^{2} \beta_{1}^{2} + |B_j|^{2} \gamma_{1}^{2} + |A_j|^{2} \mu_{1}^{2} + |E_j|^{2} + |F_j|^{2} + |G_j|^{2} + |H_j|^{2})/8,
\]

where $j = 0, 1$ correspond to the ideal and decay cases, respectively, with $\alpha_0 = \beta_0 = \mu_0 = 1$. In our case, the atomic system is initially prepared in $|\Psi_0\rangle = \frac{1}{2^{\frac{1}{2}}} (|g_1\rangle + |e_1\rangle) (|g_2\rangle + |i_2\rangle) (|g_3\rangle + |i_3\rangle)$, which corresponds to a success probability of the three-qubit phase gate $P_j = (4 + \alpha_{j}^{2} + \beta_{j}^{2} + \gamma_{j}^{2} + \mu_{j}^{2})/8$.

As mentioned previously, the three-qubit Grover search is carried out only probabilistically. So how to obtain a high success rate of the search is the problem of much interest, particularly in the presence of weak cavity decay. We have numerically simulated the Grover search for finding different marked states in the cases of $\kappa = 0$ (the ideal case), $\kappa = \Omega_{1c}/50$, and $\kappa = \Omega_{1c}/10$. Due to the similarity, we only demonstrate the search for a marked state $|e_1\rangle |i_2\rangle |i_3\rangle$ in Fig. 2 as an example. Considering the success rates of the three-qubit phase gating (i.e., Eq. (6)) and the Grover search itself, we show in Fig. 2(a) that the success probability is smaller and smaller with the increase of the decay rate and the iteration number. This implies that, although the sixth iteration could reach the largest success rate in the ideal consideration, we prefer the second iteration in the presence of dissipation. The detrimental effect of the cavity decay is also reflected in the estimate of fidelity in Fig. 2(b).

We briefly address the experimental feasibility of our scheme with current microwave cavity technology by considering three Rydberg atoms with principal quantum numbers $49$, $50$ and $51$ to be levels $|i\rangle$, $|g\rangle$ and $|e\rangle$, respectively. Based on the experimental numbers reported in [10,11], the coupling strength at the cavity centre could be $\Omega_{c} = 2\pi \times 49 k_{Hz}$, and the Rydberg atomic lifetime is $30$ ms. Since the single-qubit operation takes negligible time in comparison with that for the three-qubit phase gating, an iteration of our proposed Grover search would take $t_{0} = 2\pi / \sqrt{\Omega_{c}^{2} - \kappa^{2}/16}$. 
Direct calculation shows that the time for one iteration is about $160\mu s$, much shorter than the cavity decay time for both cases of $\kappa = \Omega_{1c}/50$, and $\Omega_{1c}/10$. So our treatment with quantum trajectory method is physically reasonable.

With current cavity QED techniques, the design in Fig. 1 should be realized by four separate microwave cavities with each Ramsey zone located by a cavity. Since each microwave cavity is employed to carry out a three-qubit phase gate $T_{90}$, the four cavities should be identical. While for searching different states, we employ different single-qubit operations, as shown in Eq. (3). So the Ramsey zones should be long enough to finish at most two consecutive single-qubit operations, for example, to search states $|g_yg_yg_y\rangle$, we have to carry out a Hadamad gate $H$ and a gate $I_{111}$ including three simultaneous single-qubit rotations. Above requirements are due to the fact that each atom is sent by a fix velocity to fly through the design in Fig. 1, and each single-qubit operation takes a time (although it is very short so that we roughly omitted this time in above assessment of the implementation time). In principle, if each component of the design is available, our scheme would be achievable experimentally. However, we have not yet found an experimental report for three atoms simultaneously going through a microwave cavity, and the two-atom entaglement in a microwave cavity was done by using van der Waals collision between the atoms [11] under a non-resonant condition. Nevertheless, compared to [14] with four-level atoms sent through a three-mode cavity, our proposal involving a single-mode cavity is much simpler and is closer to the reach with the current cavity QED technology. Considering the intra-atom interaction occurs in the central region of the cavity, we have $\Omega_{je} \approx \Omega_0 \cos(2\pi z / \lambda_0)$. So the three atoms should be sent through the cavity with the atom 3 going along the $y$-axis ($x_3 = z_3 = 0$) and atoms 1 and 2 away from the atom 3 by $|z_1|/|z_2| = \arccos(0.125) / \arccos(\sqrt{23}/8) \approx 1.957$. With the manipulation designed in Fig. 1, a three-qubit Grover search for the marked state $|101\rangle$ would be achievable.

We have noticed that four-qubit Grover search with linear optical elements has been achieved [16]. While as photons are always flying, they are actually unsuitable for a practical quantum computing. In contrast, the atoms move much more slowly than photons, and are thereby relatively easier for manipulation. In addition, the three-qubit gating we employed simplifies the implementation and reduces the probability of error in comparison with the series of two-qubit gateings in [16]. More importantly, our scheme could be straightforwardly applied to the ion-trap-cavity combinatory setup [18] or cavity-embedded optical lattices confining atoms [19], in which the atoms are localized and the model we employ here still works. For these considerations, however, the cavities should be optical ones, for which we have to consider both the cavity decay and the atomic spontaneous emission. Based on a previous treatment [20], as long as these dissipations are weak, the three-qubit phase gating would also be available, and thereby our scheme is in principle workable in optical regime.

Besides the imperfection considered above, there are other unpredictable imperfection in an actual experiment, such as diversity in atomic velocities, deflected atomic trajectories, classical pulse imperfection, slight difference of the cavities and so on. Let us take examples to assess the influence from imperfection. First, as it is still a challenge to simultaneously send three Rydberg atoms through a cavity with precise velocities in experimental performance, we consider an imperfection in this respect. For the clarity and convenience of our discussion, we simply consider a situation that the atom 1 moves a little bit slower than the atoms 2 and 3, i.e., the times of the atoms passing through the cavity $t_1 = t_0 + \delta t$ and $t_2 = t_3 = t_0$, with $t_0$ the desired interaction time for the three-qubit phase gate $T_{90}$. Direct calculation yields the infidelity in a single three-qubit phase gate due to the imperfection in atomic velocity to be,

$$In\text{fidelity} = 1 - \frac{4 + \xi \alpha_1 + \xi \beta_1 + \xi \mu_1 + \xi \gamma_1 - \Omega_{1c}^2/(A_{1c}A_{3c}) \exp(-\kappa \delta t/4) \sin(A_{1c} \delta t) \sin(\sqrt{65\pi})/2}{8[4 + (\xi \alpha_1)^2 + (\xi \beta_1)^2 + (\xi \mu_1)^2 + (\xi \gamma_1 - \Omega_{1c}^2/(A_{1c}A_{3c}) \exp(-\kappa \delta t/4) \sin(A_{1c} \delta t) \sin(\sqrt{65\pi})/2]^2}, \quad (7)$$

where $\xi = \exp(\kappa \delta t/4) \cos(A_{1c} \delta t) + \sqrt{\kappa \delta t} \sin(A_{1c} \delta t)$, and $A_{3c} = 2\pi/\sqrt{\Omega_{1c}^2 + \Omega_{3c}^2 - \kappa^2/16}$. Due to the additional interaction regarding the atom 1, an enlarging infidelity occurs with respect to the time difference $\delta t$ and the decay rate $\kappa$, as shown in Fig. 3. Secondly, we consider the unfavorable influence from the coupling strength $\Omega'_{je}$ in some cavities with the offset $\eta \Omega_{je}$ from the ideal number. We find the infidelity due to these offsets for a Grover search implementation to be,

$$In\text{fidelity}' = 1 - \frac{(4 + \alpha'_x + \beta'_x + \gamma'_x + \mu'_x)^2}{8[4 + \alpha'^2_x + \beta'^2_x + \gamma'^2_x + \mu'^2_x]}, \quad (8)$$

where $\alpha'_x = 1 - \frac{\Omega'^2_{1c}}{\Omega_{1c}^2 + \Omega_{3c}^2} (1 - e^{-\kappa t_0/4}) \chi \xi_{1c}^4 \chi$, $\beta'_x = 1 - \frac{\Omega'^2_{3c}}{\Omega_{1c}^2 + \Omega_{3c}^2} (1 - e^{-\kappa t_0/4}) \chi \beta_{3c}^4 \chi$, $\gamma'_x = 1 - \frac{\Omega'^2_{1c}}{\Omega_{1c}^2 + \Omega_{3c}^2} (1 - e^{-\kappa t_0/4} \cos(\sqrt{65\pi})) \gamma_{1c}^4 \chi$, and $\mu'_x = e^{-\kappa t_0/4} \mu_{3c}^4 \chi$, with $\chi = 1, 2, 3, 4$ the number of the cavities with the coupling strength offsets. We plot the dependence of the infidelity on different $\eta$ and $\chi$ in the case of $\kappa = \Omega_{1c}/10$ in Fig. 4.

The error assessments in Figs. 3 and 4 are actually for the simplest consideration about imperfection. In a realistic experiment, situation would be more complicated. So to carry out our scheme efficiently and with high fidelity, we have to suppress these imperfect factors as much as we can.

In conclusion, we have proposed a potentially practical scheme for realizing a three-qubit Grover search by resonant interaction of three Rydberg atoms in a microwave cavity. We have estimated the influence from the cavity decay on
our scheme and shown that large enough success rate and fidelity could be reached for a three-qubit Grover search with current or near-future technique of cavity QED. Although we have not yet found our idea to be extendable to more than three-qubit case, our scheme could be extended to trapped ions embedded in a cavity or atoms in cavity-embedded optical lattices. So we argue that our present scheme is helpful for demonstration of Grover search algorithm by small-scale quantum information processing devices.

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Note added: After finishing this work, we became aware of a work for N-qubit Toffoli gate in a cavity by resonant interaction [21], in which the only difference from [17] is the different setting of atom-cavity coupling strength. This means that our idea for Grover search would be in principle extended to N-qubit case after slight modification.

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Captions of Figures

FIG. 1. Schematic setup for finding the marked state $|101\rangle$ in a three-qubit Grover search. Three atoms initially prepared in a superposition state $|\Psi_0\rangle$ go through the cavity with the identical velocity from the box B. We send the atom 3 through the center of the microwave cavity along the y axis and other two atoms away from the y axis. We consider twice searching steps in the setup, which yields the largest success rate in the presence of dissipation. The operations $H^\otimes 3, \sigma_x, 1, \sigma_x, 3$ and $U_0(t)$ are defined in the text. Only in the case that the marked state is $|011\rangle$ or $|111\rangle$, should additional inhomogeneous electric fields be applied on the regions for single-qubit operation to distinguish the atoms 2 and 3.

FIG. 2. Numerical results for a three-qubit Grover search for the marked state $|e_1\rangle|i_2\rangle|i_3\rangle$, where $k = \kappa$ and $g = \Omega_{1c}$. (a) Probability for finding the marked state in the case of $k = 0, \Omega_{1c}/50$ and $\Omega_{1c}/10$; (b) Fidelity of the searched state in the case of $k = \Omega_{1c}/50$ and $\Omega_{1c}/10$.

FIG. 3. Infidelity in a three-qubit phase gate versus time delay, where the solid and dashed curves represent the cases of $k = \Omega_{1c}/50$ and $\Omega_{1c}/10$, respectively.

FIG. 4. Infidelity in a Grover search versus offset constant $\eta$, where the four solid curves from bottom to top correspond to the number of imperfect cavities varying from 1 to 4 in the case of $k = \Omega_{1c}/10$. 

Fig. 1
The iteration number

Fig. 2 a
The iteration number

Fidelity

\( k = g/50 \)
\( k = g/10 \)

Fig 2 b
Fig 3
Fig 4