1. INTRODUCTION

The interstellar medium (ISM) is known to be composed by a multi-phase, turbulent and magnetized gas (see Brunt & Heyer 2002; Elmegreen & Scalo 2004; Crutcher 2004; McKee & Ostriker 2007). However, the relative importance of turbulence and the magnetic field in the ISM dynamics and in the formation of structures is still a matter of debate. More specifically, typical molecular clouds present densities in the range of 10^2–10^5 cm^-3, sizes L ~ 0.1–100 pc, temperature T ~ 10–20 K, and lifetimes that are larger than the Jeans gravitational collapse timescale. The role of the magnetic field in preventing the collapse is hotly debated in the literature (see Fiedge & Pudritz 2000; Falceta-Gonçalves et al. 2003; MacLow & Klessen 2004). The magnetic field can be removed from clouds in the presence of ambipolar diffusion arising from the differential drift of neutrals and ions (Mestel & Spitzer 1956; Shu 1983) and reconnection diffusion which arises from fast magnetic reconnection of the turbulent magnetic field (Lazarian 2005). Nevertheless, the role of magnetic fields in the dynamics of ISM is difficult to underestimate.

We feel that a lot of the unresolved issues in the theory of star formation are in part due to the fact that the amount of information on magnetic fields obtainable through presently used techniques is very limited. For molecular cloud the major ways of obtaining information about magnetic fields amount to Zeeman broadening of spectral lines, which provides measures of the field strength along the line of sight (LOS; see Crutcher et al. 1999) and the Chandrashekar–Fermi (CF) method, which uses the statistics of polarization vectors to provide the amplitude of the plane of the sky component of the field (see Hildebrand et al. 2000). However, since Zeeman measurements are restricted to rather strong magnetic fields (due to current observational sensitivity) and therefore the measurements are restricted to dense clouds and new measurements require a lot of observational time. At the same time, the CF method relies on all grains being perfectly aligned, which is known not to be the case in molecular clouds (see Lazarian 2007 for a review). The CR technique is also known to systematically overestimate the field intensity (Houde et al. 2009; Hildebrand et al. 2009), and to poorly map the magnetic field topology for super-Alfvénic turbulence (see Falceta-Gonçalves et al. 2008).

The difficulty of the traditional techniques calls for new approaches to measuring astrophysical magnetic fields. Recently, a number of such techniques have been proposed. For instance, Yan & Lazarian (2006, 2007, 2008) discussed using the radiative alignment of atoms and ions having fine or hyperfine split of the ground of metastable levels. The technique is based on
the successful alignment of atoms in the laboratory conditions, but it requires environments where radiative pumping dominates the collisional de-excitation of the levels.

Another new approach which we dwell upon in this paper is based on the comparison of the ion–neutral spectral lines. Houde et al. (2000a, 2000b) identified the differences of the width of the lines of neutral atoms and ions as arising from their differential interaction with magnetic fields. It was assumed that because ions are forced into gyromagnetic motions about magnetic field lines that their spectral line profiles would thus reveal the imprint of the magnetic field on their dynamics.

In particular, as observations of HCN and HCO\(^+\) in molecular clouds revealed significantly and systematically narrower ion lines, Houde et al. (2000a) proposed a simple explanation for these observations. The model was solely based on the strong Lorentz interaction between the ion and the magnetic field lines, but also required the presence of turbulent motions in the gas. More precisely, it was found that the observations of the narrower HCO\(^+\) lines when compared to that from the coexistent HCN species could potentially be explained if neutral particles stream pass magnetic field lines with the entrained ions. Such a picture could be a particular manifestation of the ambipolar diffusion phenomenon.

Although this model was successful in explaining the differences between the velocities of ions and neutrals, the quantitative description of the model of drift was oversimplified. For example, it was neither possible to infer anything about the strength of the magnetic field nor was the “amount” of ambipolar diffusion, which is at the root of the observable effect described by the model, quantifiable in any obvious manner. The main reason for these shortcomings resides in the way that turbulence and its interplay with the magnetic field were treated in the analysis of Houde et al. (2000a); a more complete and powerful model was required. The next step in the study of magnetized turbulence and ambipolar diffusion through the comparison of the coexistent ion/neutral spectral lines was taken by Li & Houde (2008) where a model of turbulence damping in partially ionized gas was employed. Assuming that the main damping mechanism is associated with ambipolar diffusion, they proposed a way for evaluating the strength of the plane-of-the-sky component of the magnetic field in molecular clouds.

The main idea is that, in a magnetically dominated scenario, cloud collapse and magnetic energy removal may be accelerated due to ambipolar diffusion of ions and neutral particles. Although as gravity becomes dominant the collapsing cloud continuously drags material to its core including the ions, which are frozen to the magnetic field lines, magnetic pressure slows down their infall, but not that of the neutrals. At this stage most of the matter, in neutral phase, continues to decouple from the ionic fluid and the field lines leading to the diffusion of magnetic energy and the collapse may develop further. This ion–neutral drift, excited by the ambipolar diffusion, is also responsible for damping the ion turbulent motions. The increase in the net viscosity of the flow provides a cutoff in the turbulent cells with turnover timescales lower than the period of collisions \(\tau_{\text{in}}\) (see Lazarian et al. 2004 for detailed review).

Since the turbulent cascade is dramatically changed by the decoupling of the ion and neutral fluids the observed velocity dispersion could reveal much of the physics of collapse length scales. The interpretation and reliability of this technique, however, still need to be corroborated with more detailed theoretical analysis, as well as numerical simulations of magnetized turbulence.

For the past decade, because of its complicated, fully non-linear and time-dependent nature, magnetized turbulence has been mostly studied by numerical simulations (see Ostriker et al. 2001; Cho & Lazarian 2005; Kowal et al. 2007). Simulations can uniquely provide the three-dimensional structure for the density, velocity, and magnetic fields, as well as two-dimensional maps that can be compared to observations (e.g., column density, line profiles, polarization maps). Therefore, direct comparison of observed and synthetic maps may help reveal the magnetic topology and velocity structure.

In this paper, we re-examine the assumptions made in this model and test some of these assumptions using the MHD numerical simulations. In particular, we provide a number of numerical simulations of MHD turbulent flows, with different sonic and Alfvénic Mach numbers. In Section 2, we describe the Neutral-to-Ion Dispersion Ratio (NIDR) technique for the determination of the damping scales and magnetic field intensity from dispersion of velocities and the main theoretical aspects of turbulence in partially ionized gases. In Section 3, we describe the numerical simulations and present the results and the statistical analysis of the data. In Section 4, we discuss the systematic errors intrinsic to the procedures involved, followed by the discussion of the results and summary, in Section 5.

2. TURBULENCE IN PARTIALLY IONIZED GAS

2.1. Challenge of Interstellar Turbulence

In 1941, Kolmogorov proposed the well-known theory for energy cascade in incompressible fluids. Under Kolmogorov’s approximation, turbulence evolves from the largest to smaller scales, up to the dissipation scales, as follows. Within the so-called inertial range, i.e., the range of scales large enough for dissipation to be negligible but still smaller than the injection scales, the energy spectrum may be well described by

\[ P(k) \sim \hat{\epsilon}^{\alpha-1} k^{-\alpha}, \]

where \(\hat{\epsilon}\) is the energy transfer rate between scales and \(\alpha \sim 5/3\). In this approximation, within the inertial range, the energy transfer rate is assumed to be constant for all scales. Therefore, integrating Equation (3) over \(k\) for \(\alpha = 5/3\), we obtain

\[ \sigma^2(k) \propto \left(3\hat{\epsilon}^{2/3}/2\right) k^{-2/3}. \]

However, reality is far more complicated. First, the ISM is threaded by magnetic fields, which may be strong enough to play a role on the dynamics of eddies and change the scaling relations. Second, observations suggest that the ISM is, at large scales, highly compressible. Third, many phases of the ISM (see Draine & Lazarian 1999 for typical parameters) are partially ionized.

Attempts to include magnetic fields in the picture of turbulence include works by Iroshnikov (1964) and Kraichnan (1965), which were done assuming that magnetized turbulence stays isotropic. Later studies proved that the magnetic field introduces anisotropy into turbulence (Shebalin et al. 1983; Higdon 1984; Zank & Matthaeus 1992; see also book by Biskamp 2003).

Goldreich & Sridhar (1995, henceforth GS95) proposed a model for magnetic incompressible turbulence\(^4\) based on

\[ \text{in the original treatment of GS95, the description of turbulence is limited to a situation of the velocity of injection at the injection scale \(V_0\) being equal to \(V_A\). The generalization of the scalings when \(V_c < V_A\) can be found in Lazarian & Vishniac (1999). The generalization for the \(V_c > V_A\) is also straightforward.} \]
the anisotropies in scaling relations, as eddies would evolve differently in directions parallel and perpendicular to the field lines: \( V_A/\Lambda_k \sim v_L/\kappa \), where \( \Lambda_k \) is the parallel scale of the eddy and \( \lambda \) is its perpendicular scale. These scales are measured in respect to the local\(^5\) magnetic field. Combining this to the assumption of self-similarity in energy transfer rate, we get a Kolmogorov-like spectrum for perpendicular motions with \( \alpha = 5/3 \) and, most importantly, the anisotropy in the eddies scales as \( \Lambda_k \propto \lambda^{2/3} \).

In spite of the intensive recent work on the incompressible turbulence (see Boldyrev 2005, 2006; Beresnyak & Lazarian 2006, 2009), we feel that the GS95 is the model that can guide us in the research in the absence of a better alternative. The generalization of the GS95 for compressible motions is available (Lithwick & Goldreich 2001; Cho & Lazarian 2002, 2003) and they consider scalings of the fast and slow MHD modes.

2.2. Turbulence Damping in Partially Ionized Gas

As stated before, the turbulent cascade is expected to develop down to scales where dissipation processes become dominant. The dissipation scales are associated with the viscous damping, which is responsible for the transfer of kinetic into thermal energy of any eddy smaller than the viscous cut-off scale.

In the ISM, e.g., in cold clouds, the gas is partially ionized and the coupling between neutrals, ions, and magnetic fields gives rise to interesting processes. As far as damping is concerned one of the most interesting is the energy dissipation as the motions of ions and neutral particles decouple. While the issue of turbulence dissipation has been discussed extensively in the literature (see Minter & Spangler 1997), a generalization of the GS95 model of turbulence for the case of the partially ionized gas was presented in Lazarian et al. (2004, hereafter LVC04). If the eddy turnover time (\( \tau \)) is of the order of the inverse of the ion–neutral collision rate (\( t_{in} \)), then the two fluids are strongly coupled. In this situation a cascade cutoff is present.

In a strongly coupled fluid, using the scaling relation for the inertial range \( v_{damp} \sim U_{inj}(L_{inj}^{-1}+\text{damp})^{1/3} \), the damping scale is given by (LVC04),

\[ l_{\text{damp}} \sim \frac{\epsilon}{\lambda_{\text{inj}}} \left( \frac{c_n}{v_L} \right)^{1/3} f_n, \]

where \( f_n \) is the neutral fraction, \( \lambda_{\text{inj}} \) and \( c_n \) are the mean free path and sound speed for the neutrals, respectively, \( V_A \) is the Alfvén speed, and subscript “inj” refers to the injection scale. Since the Alfvén speed depends on the magnetic field, \( V_A = B(4\pi \rho)^{-1/2} \), Equation (3) is rewritten as

\[ B \sim \left( \frac{l_{\text{damp}}}{\lambda_{\text{inj}}} \right)^{3} \left( \frac{v_L}{c_n} \right)^{3} \left( 4\pi \rho \right)^{1/2} \frac{f_n}{v_L^{3}}. \]

Therefore, for a given molecular cloud, if the decoupling of ions and neutrals is the main process responsible for the ion turbulence damping, Equation (4) may be a complementary estimation of \( B \). The main advantage of this method is that \( B \) is the total magnetic field and not a component, parallel or perpendicular to the LOS, as respectively obtained from the Zeeman or CF method from polarization maps.

\(^5\) The latter issue does not formally allow us to describe turbulence in the Fourier space, as the latter calls for the description with respect to the global magnetic field.

2.3. Approach by Li & Houde (2008)

From the perspective of the turbulence above we can discuss the model adopted by Li & Houde (2008) for their study. The authors considered that damping of ion motions happens earlier than those by neutrals at sufficiently small scales. At large scales, ions and neutrals are well coupled through flux freezing and their power spectra should be similar. At small scales the ion turbulence damps while the turbulence of neutral particles continues cascading to smaller scales. This difference may be detected in the velocity dispersions \( (\sigma) \) obtained from the integration of the velocity power spectrum over the wavenumber \( \kappa \),

\[ \sigma^2(k) \propto \int_k^{\infty} P(k')dk' \]
\[ \sim b k^{-n}, \]

considering \( P(k) \) a power-law spectrum function. Since the turbulence of ions is damped at the diffusion/dissipation scale \( (L_D) \), while the turbulence of neutral particles may develop up to higher wavenumbers we may consider that the ions and neutral particles present the same distribution of velocities (well coupled) for \( L > L_D \). In this sense, the dispersion of neutral particles may be written as

\[ \sigma_n^2(k) \propto \int_k^{\infty} P(k')dk' \]
\[ = \int_k^{k_0} P(k')dk' + \int_{k_0}^{\infty} P(k')dk' \]
\[ \sim \sigma_i^2(k) + b k_D^{-n}, \]

where \( k_D \approx L_D^{-1} \). Equation (5) may be directly compared to Equation (2). In this case, we would obtain \( n \sim 2/3 \) and \( b \) is related to the energy transfer rate \( \dot{\epsilon} \). Therefore, once the fitting parameters of Equation (1) are obtained from the observational data, it is possible to obtain the cascading constants \( \dot{\epsilon} \) and \( \alpha \).

With the dispersion of velocities for both ions and neutrals at different scales \( k \) it is possible to calculate the damping scale \( k_D \). From Equation (5), the dispersion of neutral particles provides \( b \) and \( n \) constants and, by combining neutral and ion dispersions, it is possible to get \( k_D \) (Equation (6)), i.e., the damping scale.

Finally, as proposed by Li & Houde (2008) in a different context, it is possible to evaluate magnetic field strength by Equation (4). We feel that the procedure of the magnetic field study requires a separate discussion, due to its complexity, but in what follows we concentrate on the interesting facts of observational determining of the characteristics of turbulence and its damping that are employed in the technique by Li & Houde (2008).

2.4. Observational Perspective

Equations (1) and (2) are based on the dispersion of a three-dimensional velocity field, i.e., subvolumes with dimensions \( k^{-3} \). Observational maps of line profiles, on the other hand, provide measurements of the velocity field integrated along the LOS within the area of the beam, i.e., a total volume of \( k^{-3}k_{\text{min}}^{-1} \) (with \( k = 1/l \) and \( k_{\text{min}} = 1/L \), as \( L \) represents the total depth of the structure observed—typically larger than \( l \)). Also, velocity dispersions are obtained from spectral line profiles, which are strongly dependent on the column density, i.e., the distribution of matter along the LOS. These factors make the comparison
between observed lines and theoretical distribution of velocity fields a hard task.

Fortunately, three-dimensional numerical simulations of MHD turbulence may be useful in providing both the volumetric properties of the plasma parameters as well as their synthetic measurements projected along given lines of sight, which may be compared directly to observations, such as the spectral line dispersion. In this sense, based on the simulations of Ostriker et al. (2001), Li & Houde (2008) stated that the actual dispersion of velocity is, approximately, the minimum value of the LOS dispersion, at each beam size \( l \), obtained in a large sample of measurements. However, Ostriker et al. (2001) presented a single simulation, exclusively for supersonic and sub-Alfvénic turbulent regime, with limited resolution (256\(^3\)). They also did not study increased viscosity, nor the correlation of minima of the synthetic dispersion and the turbulent regimes and the distribution of gas along the LOS.

In the following sections, we will describe the details regarding the estimation of line dispersions, but now comparing it with a larger set of numerical simulations with different turbulent regimes, i.e., (sub)supersonic and (sub)super-Alfvénic. Solid lines represent the non-viscous cases, and the dotted line the viscous cases. The spectra are normalized by a Kolmogorov power function \( P \propto k^{-5/3} \). The inertial range of the scales is given by the horizontal part of the spectra. For the inviscid fluid, subsonic turbulence presents approximately flat spectra for \( 2 < k \leq 50 \). Supersonic turbulence, on the other hand, shows steeper power spectra within this range. Actually, as shown from numerical simulations by Kritsuk et al. (2007) and Kowal & Lazarian (2007), shocks in supersonic flows are responsible for the filamentation of structures and the increase in the velocity flux cascade, resulting in a power spectrum slope \( \sim -2.0 \). For \( k > 50 \), the power spectra show a strong damping of the turbulence, resulting from the numerical viscosity. For the viscous fluid, the damped region is broadened (\( k_{\text{cut-off}} \sim 20 \)), due to the stronger viscosity.

3. NUMERICAL SIMULATIONS

In order to test the NIDR model, i.e., to verify if the minimum dispersion of the velocity measured along the LOS for a given beam size \( l \times l \) is approximately the actual value calculated for a volume \( l^3 \), we used a total of 12 three-dimensional MHD numerical simulations, with 512\(^3\) resolution, for six different turbulent regimes as described in Table 1, but repeated for viscous and inviscid models.

The simulations were performed solving the set of ideal MHD isothermal equations, in conservative form, as follows:

\[
\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \mathbf{v}) = 0, \\
\frac{\partial \rho \mathbf{v}}{\partial t} + \nabla \cdot \left[ \rho \mathbf{v} \mathbf{v} + \left( \frac{2}{3} \frac{\mathbf{B} \cdot \mathbf{B}}{\mathbf{B}} \right) \mathbf{I} - \frac{1}{4 \pi} \nabla \times \mathbf{B} \right] = \mathbf{f}, \\
\frac{\partial \mathbf{B}}{\partial t} - \nabla \times (\mathbf{v} \times \mathbf{B}) = 0, \\
\nabla \cdot \mathbf{B} = 0, \\
\rho = c_s^2 \rho,
\]

where \( \rho \), \( \mathbf{v} \), and \( p \) are the plasma density, velocity, and pressure, respectively, \( \mathbf{B} = \nabla \times \mathbf{A} \) is the magnetic field, \( \mathbf{A} \) is the vector potential and \( \mathbf{f} = \mathbf{f}_{\text{sub}} + \mathbf{f}_{\text{osc}} \) represents the external source terms, responsible for the turbulence injection and explicit viscosity. The code solves the set of MHD equations using a Godunov-type scheme, based on a second-order accurate and the non-oscillatory spatial reconstruction (see Del Zanna et al. (2003)). The shock-capture method is based on the Harten et al. (1983) Riemann solver. The magnetic divergence-free is assured by the use of a constrained transport method for the induction equation and the non-centered positioning of the magnetic field variables (see Londrillo & Del Zanna 2000). The code has been extensively tested and successfully used in several works (Falceta-Gonçalves et al. 2008, 2010; Leão et al. 2009; Burkhart et al. 2009; Kowal et al. 2009).

The turbulence is triggered by the injection of solenoidal perturbations in Fourier space of the velocity field. Here, we solve the explicit viscous term as \( \mathbf{f}_{\text{visc}} = -\nu \mathbf{V}^2 \mathbf{v} \), where \( \nu \) represents the viscous coefficient and is set arbitrarily to simulate the increased viscosity of the ionic flows due to the ambipolar diffusion. We run all the initial conditions given in Table 1 for both \( \nu = 0 \) and \( \nu = 10^{-3} \), representing the neutral and ion particle fluids, respectively. Each simulations is initiated with a uniform density distribution, threaded by a uniform magnetic field. The simulations were run until the power spectrum is fully developed. The simulated box boundaries were set as periodic.

In Figure 1, we show the resulting velocity power spectra of four of our models, representing the four different turbulent regimes, i.e., (sub)supersonic and (sub)super-Alfvénic. Solid lines represent the non-viscous cases, and the dotted line the viscous cases. The spectra are normalized by a Kolmogorov power function \( P \propto k^{-5/3} \). The inertial range of the scales is given by the horizontal part of the spectra. For the inviscid fluid, subsonic turbulence presents approximately flat spectra for \( 2 < k \leq 50 \). Supersonic turbulence, on the other hand, shows steeper power spectra within this range. Actually, as shown from numerical simulations by Kritsuk et al. (2007) and Kowal & Lazarian (2007), shocks in supersonic flows are responsible for the filamentation of structures and the increase in the energy flux cascade, resulting in a power spectrum slope \( \sim -2.0 \). For \( k > 50 \), the power spectra show a strong damping of the turbulence, resulting from the numerical viscosity. For the viscous fluid, the damped region is broadened (\( k_{\text{cut-off}} \sim 20 \)), due to the stronger viscosity.

4. RELATIONSHIP BETWEEN TWO-DIMENSIONAL AND THREE-DIMENSIONAL DISPERSION OF VELOCITIES

4.1. Comparing the Synthetic to the Three-dimensional Dispersion of Velocities

Theoretically, as given by Equation (6), the difference between the two spectra for each run may be obtained from the observed dispersion of velocities. The next step then is to obtain the dispersion of velocity, for different scales \( l \), from our simulations. However, as explained previously, there are two different methods to obtain this parameter. One represents the actual dispersion, calculated within subvolumes \( l^3 \) of the computational box, while the second represents the observational measurements and is the dispersion of the velocity within the subvolume \( l^2 \mathcal{L} \) (assuming the gas is optically thin), where \( \mathcal{L} \) is the total depth of the box. In order to match our calculations to observational measurements we will use the density-weighted

![Table 1 Description of the Simulations](image-url)
velocity $v^* = \rho v$ (see Esquivel & Lazarian 2005), which characterizes the line emission intensity proportional to the local density.

To obtain the actual dispersion of velocities as a function of the scale $l$, we subdivide the box in $N$ volumes of size $l^3$. Then, we calculate the dispersion of $v^*$, normalized by the sound speed $c_s$, as the mean value of the local dispersions obtained for each subvolume. For the synthetic observational dispersion, we must first choose a given LOS. Here, for the results shown in Figure 2 we adopted the $x$-direction. After, we subdivide the orthogonal plane ($y$–$z$) in squares of area $l^2$, representing the beamsize. Finally, we calculate the dispersion of $v^*$ within each of the volumes $l^2 L$, normalized by the sound speed $c_s$, for different values of $l/L$. The results of these calculations for each of the non-viscous models are shown in Figure 2.

The solid line and the triangles represent the average of the actual mean dispersion of velocities, while the crosses represent each of the synthetic observed dispersion within $l^2 L$. The general trends shown in Figure 2 are also observed, but a slight difference appears as $l \to L$, exactly as explained above. This difference is expected to be seen in sub-Alfvénic cases because of the anisotropy in the velocity field regarding the magnetic field, as the velocity components may be different along and perpendicular to $B$.

Despite this effect, the results presented in this work do not change when a different orientation for the LOS is chosen. Even though not shown in Figure 2, we have calculated the dispersion of velocity for LOS in the $y$- and $z$-directions. The general trends shown in Figure 2 are also observed, but a slight difference appears as $l \to L$, exactly as explained above. This difference is expected to be seen in sub-Alfvénic cases because of the anisotropy in the velocity distribution.

It is clear from Figure 2 that the actual dispersion of velocities and a given observational line width may be very different. Li & Houde (2008), based on Ostriker et al.’s (2001) work, assumed that if one chooses, from a large number of observational measurements along different LOSs, the minimum observational dispersion as the best estimation for the actual dispersion, the associated error is minimized. Considering the broad range of observed dispersions obtained from the simulations for a given $l$, the minimum value should correspond to the actual velocity dispersion. Actually, from Figure 2 we see that the validity of such statement depends on $l$ and on the turbulent regime, though as a general result the scaling of the minimum observed dispersion follows the actual one.
Figure 2. Dispersion of velocity for models 1–6 obtained for different beam sizes ($l^2$) integrated along the $x$-direction. The solid line (and triangles) represent the actual dispersion, defined as the averaged dispersion measured within all cubes with size $l^3$. For a given beamsize, the crosses represent each of the synthetic observed dispersion within $l^2 L$.

For the subsonic models, the actual dispersion is lower than $\sigma_v^*(l)$, with increasing difference as $l/L \to 0$. In these models, we see that for $l > 0.05L$ there is a convergence of the actual dispersion to the synthetic observational measurements. At these scales the minimum value of $\sigma_v^*(l)$ is a good estimate of the velocity dispersion of the turbulence at the given scale $l$. The difference between both values is less than a factor of 3 for all $l$'s, being of a few percent for $l > 0.03L$. In this turbulent regime, mainly at the smaller scales, $\sigma_v^*(l)$ overestimates the true dispersion. For larger scales the associated error is very small and the two quantities give similar values.

On the other hand, for the highly supersonic models ($M_s \sim 7.0$), the minimum value of $\sigma_v^*(l)$ underestimates the actual value, at most scales. Under this regime, the difference to the actual dispersion is of a factor of $\sim 2-4$. The best matching between the two measurements occurred for the marginally supersonic cases ($M_s \sim 1.5$).

As a major result, we found that the uncertainties associated with the NIDR technique depend on the sonic Mach number of the system, though the associated errors are not extreme in any case. We see no major role of the Alfvénic Mach number on this technique.

4.2. Minimum Two-dimensional Velocity Dispersion versus Three-dimensional Statistics

We have shown that the synthetic observed dispersion minima represent a fair approximation for the actual three-dimensional dispersion of velocities for the supersonic models, though it is slightly overestimated in subsonic cases, and underestimated in highly supersonic cases. What is the physical reason for that?

One of the most dramatic differences between subsonic and supersonic turbulence is the mass density distribution. Subsonic turbulence is almost incompressible, which means that density fluctuations and contrast are small. Supersonic turbulence, on the other hand, presents strong contrast and large fluctuation of density within the volume. Strong shocks play a major role on the formation of high density contrasts, and are the main cause of high density sheets and filamentary structures in simulations. Strong shocks also modify the turbulent energy cascade, opening the possibility for a more efficient transfer of energy from large to small scales, resulting in a steeper energy spectra (typically with index $\sim -2$, instead of the Kolmogorov's $\sim -5/3$).

Observationally, the determination of the velocity dispersion along the LOS is always biased by the density distribution, i.e., a
In Figure 4, we compare the average number of structures intercepting the LOSs and their average sizes as a function of the sonic Mach number. It is shown that both the number of sources and their intrinsic sizes are inversely correlated with the sonic Mach number. Subsonic models present lower contrast of densities, which corresponds to larger overdense structures. For \( M_S = 0.7 \) we obtained a range of densities \( 0.3 < \rho / \bar{\rho} < 3 \). As discussed above, large structures correspond to integrated volumes \( \sim \ell^2 L \), which deviates from the actual three-dimensional velocity dispersion. The result is that the minimum synthetic dispersion obtained from subsonic turbulence will overestimate the actual value. For supersonic models, where clumps are systematically small, the dispersion minima correspond, from Figure 3, to the single structures with volumes \( \sim \ell^3 \), very close to the actual values. On the other hand, for \( M_S = 7.0 \) we obtained \( 0.01 < \rho / \bar{\rho} < 90 \).

4.3. Turbulence Dissipation Scales

In order to obtain the dissipation scales of the ionized flows, we must apply Equations (5) and (6) to the simulated data. For the inviscid flows, we calculate \( b \) and \( n \) (Equation (5)) using both the actual and minimum observed velocity dispersions. Then, we use the data from the viscous simulation to calculate the difference \( \sigma_n^2 - \sigma_i^2 \), Finally, from Equation (6), knowing \( \sigma_n^2 - \sigma_i^2 \), \( n \) and \( b \) it is possible to obtain \( \lambda_D \). In Figure 5, we show the data used to compute Equations (5) and (6), i.e., both the actual (lines) and the minimum observed (symbols) dispersions of velocity for the viscous (stars) and inviscid fluids (squares). The fit parameters (Equation (5)) for the inviscid simulated data are shown in Table 2, where \( b_{\text{vac}} \) and \( n_{\text{vac}} \) were obtained for the actual velocity dispersion and \( b_{\text{obs}} \) and \( n_{\text{obs}} \) for the synthetic observed line widths. We see that \( n \) increases with the sonic Mach number \( (M_S) \), as explained below. In Table 2, we also present the damping scale \( L_D \), obtained from Equation (6). In the last column we show the ratio between the actual and observational scales \( L_{D,\text{obs}} / L_D \). The ratio of the obtained scales showed a maximum difference of a factor of 5 between the actual value of the dispersion and the one obtained from the synthetic observational maps. Also, compared to the expected values obtained visually from the spectra (Figure 1), there is a good correspondence with \( L_{D,\text{obs}} \) given in Table 2. This fact shows that the method indeed might be useful.

Regarding the spectral index \( \alpha \) (Equation (1)), Table 2 gives \( \alpha \sim 1.4\text{–}1.8 \) for the subsonic models and \( \alpha \sim 2.0\text{–}2.1 \) for the supersonic models. These parameters may be directly compared to the values \( \alpha \sim 1.7 \) and \( \alpha \sim 2.0 \) expected for theoretical incompressible and compressible turbulent spectra, respectively. Also, the increase of \( b \) as \( M_S \) increases shows that the energy transfer rate is larger for compressible models. From Table 2, we obtain an averaged value of \( \bar{\varepsilon} \sim 0.6 \) for subsonic and 1.4 for supersonic models, in code units.

4.4. Accuracy of the Method

Before describing a potential technique for future observations, we must stress out that this work may be divided into two independent parts. The first is related to the characterization of the energy power spectrum of turbulence, including the energy transfer rate and cut-off length, while the second is based on the use of this damping length which is used in the Li & Houde (2008) approach to determine the magnetic field.
As mentioned before, in magnetized partially ionized gases, several different mechanisms may act in order to damp turbulent motions. The determination of the magnetic field intensity from the damping scale is strictly dependent on the assumption that the turbulent cutoff is due to the ambipolar diffusion, i.e., the ion–neutral diffusion scale is larger than the scales of any other dissipation mechanism. Unfortunately, this could not be tested in the simulations since we did not include explicit two fluid equations to check the role of ambipolar diffusion and other damping mechanisms in the turbulent spectra.

In the simulations, the cutoff is obtained via an explicit and arbitrary viscous coefficient. The result is clear in the power spectra (Figure 1), where the damping length shifts to lower values of \( k \). From those, we found out that the “observational” velocity dispersions for a given beam size \( l \times l \), in most cases, do not coincide with the actual dispersion at scale \( l \). Also, the estimation from the minimum value of the observed dispersions may also be different from the expected measure. The associated errors depend on the sonic Mach number and on the scale itself, as shown in the previous section. However, the parameters obtained for the power law of the synthetic observations and actual three-dimensional distributions showed to be quite similar. The theory behind this method is very simple, but still reasonable, as it assumes that the two fluids (ions and neutrals) would present the same cascade down to the dissipation scale, when they decouple, where the ion turbulence would be sharply damped. We believe that these errors may, eventually, be originated by the short inertial range of the simulated data.
We see from Figure 1 that the turbulent damping range is broad, and not sharp compared to the inertial range, as assumed in this model. In the real ISM, the power spectrum presents a constant slope within a much broader range, and the errors with real data may be smaller.

5. DISCUSSION

In this work, we studied the relationship between the actual three-dimensional dispersion of velocities and the ones obtained from synthetic observational line profiles, i.e., the density-weighted line profile widths, along different LOSs. We study the possibility of the scaling relation of the turbulent velocity dispersion being determined from spectral line profiles. If correct, the observed line profiles could allow us to determine in details the turbulence cascade and the dissipation lengths of turbulent eddies in the ISM. However, in order to check the validity of this approach, we performed a number of higher resolution turbulent MHD simulations under different turbulent regimes, i.e., for different sonic and Alfvénic Mach numbers, and for different viscosity coefficients.

Based on the simulations, we showed that the synthetic observed line width ($\sigma_v$) is related to the number of dense structures intercepted by the LOS. Therefore, the actual dispersion at scale $l$ tends to be similar to the line width obtained by a LOS within a beam size $l \times l$ intercepting a single dense structure. It means that the minimum observed dispersion may be the best estimator for the actual dispersion of velocities, if a large number of LOS is considered, as assumed in the theoretical model.

Moreover, by adopting a power law for the spectrum function of $\sigma_v$, we were able to estimate the spectral index $n$ and constant $b$ (Equation (5)), given in Table 2. We see a good correspondence between the parameters obtained for the three-dimensional dispersions and synthetic line profiles. Furthermore, since $n$ is associated with the turbulent spectral index $\alpha$, and $b$ with the energy transfer rate between scales, line profiles may be useful in characterizing the turbulent cascade.

Also, we showed that the models under similar turbulent regime but with different viscosities will result in different dispersions of velocity, on both three-dimensional and synthetic line profile measurements. The difference of $\sigma_v$ for the inviscid and viscous models, associated with the parameters $n$ and $b$ previously obtained, gives an estimative of the damping scale $L_D$ of the viscous model (Equation (6)). The ratio between the damping scales obtained from the three-dimensional and synthetic profile dispersions vary only by a factor of $\lesssim 5$.

A good estimate of the cut-off scale of the ISM turbulence may bring light to much of the uncertainties about the mechanisms that are responsible for the damping of the turbulent eddies. As we showed, if ambipolar diffusion is the main phenomenon responsible for the dissipation of turbulence, then it is possible to provide another method for the determination of magnetic field strength in dense cores, besides Zeeman splitting.

We believe that (1) the numerical resolution used in our models and (2) the single fluid approximation, with different viscosities to simulate neutrals and ions separately, are the main limitations of this work. The ISM may present more than five decades of inertial range in its power spectrum while numerical simulations are, at best, limited to 2–3. On the other hand, since the validity of the theoretical approximation presented in this work depends on the broadness of the inertial range, we expect this model to be even more accurate for finer resolution, or for the ISM itself. Under the single fluid approximation made in this work we fail in correlating the ambipolar diffusion with increased viscosity and, therefore it is not possible to test the estimation of $B$ from the damping scales as proposed above, though the results related to the damping length and energy transfer rates remain unchanged. In a two-fluid simulation this is more likely to be fulfilled. We are currently implementing the two-fluid set of equations in the code and will test this hypothesis in the future.

6. SUMMARY

In this work, we presented an extensive analysis of the applicability of the NIDR method for the determination of the turbulence damping scales and the magnetic field intensity, if ambipolar diffusion is present, based on numerical simulations of viscous MHD turbulence. We performed simulations with different characteristic sonic and Alfvénic Mach numbers, and different explicit viscous coefficients to account for the physical damping mechanisms. As main results we showed the following.

1. The correspondence between the synthetic observational dispersion of velocities (i.e., from the two-dimensional observed maps) and the actual three-dimensional dispersion of velocities depends on the turbulent regime.
2. For subsonic turbulence, the minimum inferred dispersion tends to overestimate the actual dispersion of velocities for small scales ($l \ll L$), but presented good convergence at large scales ($l \rightarrow L$).
3. For supersonic turbulence, on the other hand, there is a convergence at small scales ($l \ll L$), but the minimum inferred dispersion tends to underestimate the actual dispersion of velocities at large scales ($l \rightarrow L$).
4. Even though not precisely matching, the actual velocity and the minimum velocity dispersion from spectral lines were well fitted by a power-law distribution. We obtained similar slopes and linear coefficients for both measurements, with $\alpha \sim -1.7$ and $-2.0$ for subsonic and supersonic cases, respectively, as expected theoretically.
5. The damping scales obtained from the fit for the both cases were similar. The difference between the scales obtained

| Actual | Synthetic Maps |
|--------|----------------|
| $M_\Sigma$ | $M_A$ | $b_{\sigma v}$ | $n_{\sigma v}$ | $L_D^{\sigma v}$ | $L_D^{\sigma v} / L_D$ |
| 0.7 | 0.7 | 0.63(16) | 0.39(2) | 0.244(21) | 1.01(9) | 0.80(5) | 0.088(13) | 2.7 |
| 0.7 | 7.0 | 1.38(12) | 0.65(3) | 0.246(21) | 1.00(10) | 0.76(4) | 0.215(17) | 1.1 |
| 7.0 | 0.7 | 1.58(21) | 1.01(4) | 0.162(17) | 1.58(13) | 1.08(5) | 0.318(23) | 0.5 |
| 7.0 | 7.0 | 1.82(18) | 1.05(4) | 0.041(6) | 2.51(18) | 0.95(6) | 0.197(25) | 0.2 |

Note. $L_D^{\sigma v}$ and $L_D^{\sigma v}$ are placed in terms of the total size of the box, i.e., $L_D / L$. 

Table 2 Parameters of Best Fit and Damping Scales
from the two fits was less than a factor of 5 for all models, indicating that the method may be robust and used for observational data.

The work presented in this paper tests some of the key assumptions important process by a technique of Li & Houde (2008). Evidently, more work is still required in order to test the full range of applicability of the method (e.g., test cases of both magnetically and neutral-driven turbulence). The aforementioned implementation of two-fluid numerical simulations to better mimic ambipolar diffusion will be an important step in that direction.

Note added in proof: We have recently learned about the two fluid simulations, provided by Tiley & Balsara (arXiv:1002.0018), which addresses some of the issues we have stressed in the present paper.

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