1. Introduction

In practice many components and systems exhibit more than two output performances, these systems are called multi-state systems (MSSs) [6, 9, 10, 22]. Since the mid-1970s, numerous researches have been conducted which focus on MSS reliability [2]. Four commonly used approaches about MSS reliability have been formed gradually: the extension of Boolean model [26], stochastic process theory [18], universal generating function (UGF) technology [13,14] and Monte-Carlo simulation [23].

As to the stochastic process theory used in reliability analysis of MSSs, when the numbers of failures between arbitrary time intervals can be described as a Poisson process, Markov processes are often introduced to solve these questions [16, 20, 21]. When the operating time and repair time are non-exponentially distributed, a Semi-Markov process is often considered [7]. Besides Markov processes and Semi-Markov processes, the Wiener process [15, 19], the Gamma process [27] and the cumulative exposure process [10] are also considered in MSS reliability modelling. Research about MSS reliability has been a highlight topic in recent years and many new achievements are constantly emerging [17].

Studying on Markov repairable systems has always been an active branch in reliability theory. Jinhua Cao [11] studied the general model of Markov repairable systems, concluded the reliability analysis steps and deduced reliability indexes of voting systems, cold standby systems and warm standard systems. Cui et al. [5] proposed the definition of aggregated stochastic processes and applied into reliability analysis of repairable systems. Lisnianski [17] constructed a Markov reward model for reliability assessment of a multi-state system with variable demand. In his study, the process was assumed to be a homogenous Continuous Time Markov Chain (CTMC) with different possible states and corresponding transition possibility intensities. Other studies of reliability of multi-state systems using stochastic processes can be found in [25] and [29].

For a Markov repairable component containing \( N \) different output states, i.e., whose output performance is 
\[
G_1(t) > \cdots > G_w(t) \geq w > G_{w+1}(t) > \cdots > G_N(t) = 0,
\]
then \( G_N(t) = 0 \), so when output performance rate enters the \( N-th \) state, fault occurs.
maintenance is arranged immediately and after each repair, the component can return to the best state 1. When the component transfers from working states 1, 2, …, s to N and the repair time Y is less than a critical value ρ, we consider the fault doesn’t affect the output performance during the very short interval. For example, a daily water supply system exhibits plenty of output performances. If failure occurs and the system is repaired perfectly in a very short time interval, fault effect is neglected because the water reserved in the pipes is sufficient for the urban residents. That phenomenon is firstly noticed in [30] in 2006. Zheng et al [30] Studies a single-unit Markov repairable system with repair time omission and introduced a new stochastic process. By means of Ion-Channel theory [3, 4], she modeled the repairable system with repair time omission. Based on Zheng’s research, other scholars expand her model to several components [1, 12, 18, 24, 28]. This paper tries to expand Zheng’s conclusions to multiple states and will be more useful for actual multi-state repairable systems.

In this paper, we deduce reliability indexes according to the relationship between output performance and demand. Assuming that system performance keeps stationary when fault time is less than the threshold value, this paper builds a new stochastic process considering the neglected fault effect based on the original Markov process. Image in a power supply system, though output power capacity is lower than the demand, the system may not fail immediately due to some accumulators or external power sources. So we can think the system is still operating during the very short time. That is similar to the time-interval omission problem in Ion-Channel theory [3, 4].

In general, the organization of this paper is as follows. Section 2 deduces reliability indexes such as the instantaneous availability, steady-state availability, reliability and mean time to first failure (MTTFF) of the multi-state Markov repairable component. Section 3 builds a new stochastic process considering neglected fault effect and compares the change of reliability indexes. Both constants and non-negative random variables of the fault threshold are modeled. In section 4, numerical examples are given to clarify the comparisons of two different stochastic processes. Finally, we get conclusions in this paper in section 5.

2. Multi-state repairable components

In a multi-state component containing \( N \) ( 0 < N < +∞ ) states, output performance rate at time \( t \) is \( G(t) \) ∈ \( \{ G_1(t), G_2(t), \ldots, G_N(t) \} \) and its corresponding state possibility is \( P(t) \) ∈ \( \{ P_1(t), P_2(t), \ldots, P_N(t) \} \). When system demand \( W(t) \) is a constant, i.e., \( W(t) = w \) [13]. Suppose \( G_1(t) \geq w \geq G_k(t), N > s \geq 1 \). Fig. 1 shows a possible behavior of MSS performance and demand as the realizations of a stochastic process. In this paper, the original system is referred as the old stochastic process while the new system considers neglected fault effect.

![Fig. 1. A MSS behavior described with a stochastic process](image)

2.1. Assumptions

Before we construct a Markov process for the multi-state repairable component, some proper assumptions should be given.

1. Suppose \( G_1(t) > \cdots > G_s(t) > \cdots > G_N(t) = 0 \), the component has two styles of failures including major failure and minor failure. When fault comes, repair is arranged immediately. The system can reach the best state 1 after each maintenance, which is shown in Fig. 2.

![Fig. 2. State transition process of the multi-state component](image)

2. The residence time at each state and the repair time have independent exponential distributions, i.e., \( \lambda_1, \lambda_2, \ldots, \lambda_{N-1}, N \) and \( \mu \) in Fig. 2 are constants and they are independent.

2.2. Reliability indexes

The possibility when the component is in state \( j ( j = 1, 2, \ldots, N ) \) at time \( t \) is \( P_j(t) \), let \( P_j(t) = P(X(t) = j) \), then \( \{ X(t), t \geq 0 \} \) is a homogenous Markov Chain based on the assumptions above and \( P_j(t) = P(X(t + \Delta t) = j | X(t) = j) \) is independent of \( t \). Suppose the component is at its best output at \( t = 0 \), that is, \( P_1(0) = 1, P_2(0) = 0, \ldots, P_N(0) = 0 \). According Chapman-Kolmogorov (C-K) equations [8], Eq.(1) is obtained:

\[
\begin{aligned}
&P_1(t), P_2(t), \ldots, P_N(t) = P(t)Q \\
&P_1(0) = 1, P_2(0) = 0, \ldots, P_N(0) = 0
\end{aligned}
\]

In Eq. (1), \( Q \) is called the transition possibility intensity matrix and \( Q = (q_{ij}), i, j = 1, 2, \ldots, N \), where \( q_{ij} = \lim_{\Delta t \to 0} \frac{P_j(t) - P_i(t)}{\Delta t}, i \neq j \). From Fig. 2, \( Q \) can be easily obtained:

\[
Q = \begin{pmatrix}
-\lambda_1 & \lambda_2 & \cdots & \lambda_{N-1} \\
0 & -\lambda_2 & \cdots & \lambda_{N-2} \\
\vdots & \vdots & \ddots & \vdots \\
0 & 0 & \cdots & -\lambda_N \\
\mu & 0 & \cdots & -\mu
\end{pmatrix}
\]

where \( \lambda_j = \sum_{j=2}^{N} \lambda_j, \lambda_2 = \sum_{j=3}^{N} \lambda_j, \ldots, \lambda_{N-1} = \sum_{j=2}^{N} \lambda_{N-1,j} = \lambda_{N-1,N} \).

Therefore, Eq.(1) becomes:


\[
\begin{align*}
P_1(t) &= -\sum_{j=2}^{N} \lambda_j P_j(t) + \mu P_N(t) \\
P_2(t) &= \lambda_2 P_1(t) - \sum_{j=3}^{N} \lambda_j P_j(t) \\
&\vdots \\
P_N(t) &= \lambda_N P_{N-1}(t) + \cdots + \lambda_{N-s} P_{N-s}(t) - \mu P_N(t) \\
\sum_{j=1}^{N} P_j(t) &= 1 \\
P_1(0) &= 1, P_2(0) = 0, \ldots, P_N(0) = 0
\end{align*}
\]

After Laplace transform and inverse Laplace transform, \( P_1(t), P_2(t), \ldots, P_N(t) \) are obtained.

We use \( A(t) \) to denote the instantaneous availability of the component at time \( t \), then:

\[
A(t) = \sum_{j=1}^{s} P_j(t) = \sum_{G(0)\leq w} P_j(t)
\]

Steady-state availability \( A \) represents the ratio whether output performance \( G(t) \) satisfies demand \( w \) after a long service time:

\[
A = \lim_{t \to \infty} A(t)
\]

Also, for the Markov process, we can get \( A \) from the following equations:

\[
(\pi_1, \pi_2, \ldots, \pi_N)Q = (0, 0, \ldots, 0)
\]

\[
\sum_{j=1}^{N} \pi_j = 1
\]

In which \( \pi_1, \pi_2, \ldots, \pi_N \) is the limiting distribution (stationary distribution) of \( P_1(t), P_2(t), \ldots, P_N(t) \). Thus:

\[
A = \sum_{j=1}^{s} \pi_j
\]

When reliability \( R(t) \) of the multi-state component is required, we can introduce a new Markov process \( \{\hat{X}(t), t \geq 0\} \). The state space \( S = \{1, 2, \ldots, N\} \) can be divided into two parts: \( S_a = \{1, 2, \ldots, s\} \), \( S_b = \{s+1, s+2, \ldots, N\} \) and \( S = S_a \cup S_b \). \( S_a \) is called an acceptable state subset while \( S_b \) an unacceptable state subset. Let \( Q_j(t) = P(\hat{X}(t) = j), j = 1, 2, \ldots, s \), then we get a new C-K equation:

\[
\left\{ \begin{array}{c}
(\hat{Q}_1(t), \hat{Q}_2(t), \ldots, \hat{Q}_N(t)) = (Q_1(t), Q_2(t), \ldots, Q_N(t))Q \\
\hat{Q}_1(0) = 1, \hat{Q}_2(0) = 0, \ldots, \hat{Q}_N(0) = 0
\end{array} \right.
\]

\( \hat{Q} \) comes from \( Q \) and:

\[
Q = \begin{bmatrix}
1 & -\lambda_1 & \cdots & -\lambda_s & \lambda_{s+1} & \cdots & \lambda_{N-s}
\vdots & \vdots & \ddots & \vdots & \vdots & \ddots & \vdots \\
0 & \cdots & \cdots & -\lambda_s & -\lambda_{s+1} & \cdots & \lambda_{N-s}
\end{bmatrix}
\]

It means states \( s+1, s+2, \ldots, N \) are regarded as absorbing states, then we can get \( Q_1(t), \ldots, Q_s(t) \) as:

\[
Q_j(t) = \exp(-\sum_{j=2}^{N} \lambda_j t)Q_j(0)
\]

\[
Q_2(t) = \frac{\lambda_2 Q_1(t)}{\lambda_2 + \frac{1}{N-1} [1 - \exp(-\sum_{j=3}^{N} \lambda_j t)]}
\]

\[
Q_j(t) = \frac{\lambda_j Q_{j-1}(t) + \cdots + \lambda_{N-s} Q_{N-s}(t) + \cdots + \lambda_{N-1} Q_{N-1}(t)}{\sum_{j=s+1}^{N} \lambda_j} [1 - \exp(-\sum_{j=s+1}^{N} \lambda_j t)]
\]

Mean time to first failure (MTTFF) is:

\[
MTTFF = \int_{0}^{\infty} R(t)dt = \sum_{j=1}^{s} \int_{0}^{\infty} Q_j(t)dt
\]

3. Reliability with neglected fault effect

In this paper, two situations are considered for the threshold \( \tau \) is a constant and \( \tau \) is a non-negative random variable with its distribution function \( H(\tau) \).

3.1. The threshold is a constant

Here we introduce a new stochastic process \( \{\tilde{X}(t), t \geq 0\} \) when considering neglected fault effect and:

\[
\tilde{X}(t) = \begin{cases}
1, & \text{when the system is up} \\
0, & \text{when the system is down}
\end{cases}
\]

Obviously, the new stochastic process \( \{\tilde{X}(t), t \geq 0\} \) has a tight connection with \( \{X(t), t \geq 0\} \) but it is not a Markov process any more. Fig. 3 shows the relationship between the two stochastic processes \( \{X(t), t \geq 0\} \) and \( \{\tilde{X}(t), t \geq 0\} \).

For \( \{\tilde{X}(t), t \geq 0\} \), the instantaneous availability becomes:
At the system is operating at time \( t \)

\[
\tilde{A}(t) = P(\text{the system is operating at time } t)
\]

\[
= P(X(t) = 1)
\]

\[
= P(\tilde{X}(t) = 1, X(t) = 1) + \ldots + P(\tilde{X}(t) = 1, X(t) = N)
\]

\[
= \sum_{i=1}^{N} P(\tilde{X}(t) = 1, X(t) = i)
\]

(12)

From Fig. 3, when the original system is at state \( i (i_1 = 1, 2, \ldots, s) \), the new system is always operating. When the original system is at state \( i_2 (i_2 = s + 1, s + 2, \ldots, N - 1) \), the new system is always down. When the system transfers from state \( i (i = 1, 2, \ldots, N - 1) \) to state \( N \), the system can work as long as the repair time is less than fault threshold \( \tau \). Therefore, Eq.(12) becomes:

\[
\tilde{A}(t) = \sum_{i=1}^{N} P(X(t) = 1, X(t) = i)
\]

\[
= \sum_{i=1}^{s} P(\tilde{X}(t) = 1, X(t) = i_1) + \sum_{i_2=s+1}^{N-1} P(\tilde{X}(t) = 1, X(t) = i_2) + P(\tilde{X}(t) = 1, X(t) = N)
\]

\[
= \sum_{i=1}^{s} [P(X(t) = i_1)P(\tilde{X}(t) = i_1)] + \sum_{i_2=s+1}^{N-1} [P(X(t) = i_2)P(\tilde{X}(t) = i_2)] + P(X(t) = 1, X(t) = N)
\]

\[
= \sum_{i=1}^{s} P(X(t) = i_1) + P(\tilde{X}(t) = 1, X(t) = N)
\]

(13)

The 1st part of the last equation in Eq.(13) \( \sum_{i=1}^{s} P(X(t) = i_1) \) is \( \tilde{A}(t) \) in Eq.(2). So Eq.(13) becomes:

\[
\tilde{A}(t) = \tilde{A}(t) + P(\tilde{X}(t) = 1, X(t) = N)
\]

(14)

As for \( P(\tilde{X}(t) = 1, X(t) = N) \), it represents that the original system is under repair while the repair time is less than the threshold \( \tau \), as shown in Fig.4. For the original system, \( P(X(t) = 1, X(t) = N) \) is associated tightly with the nearest state before \( N \). Here we use \( P_{j \rightarrow N} \) to represent the transmission possibility of state \( j (j = 1, 2, \ldots, N - 1) \) to state \( N \).

\[
P(\tilde{X}(t) = 1, X(t) = N)
\]

\[
= p_{t \rightarrow N} \int_{0}^{\text{min}(\tau, \tau)} P(X(t - v) = 1)P(v < Y < \tau)dv + \ldots
\]

\[
+ p_{N-1 \rightarrow N} \int_{0}^{\text{min}(\tau, \tau)} P(X(t - v) = N - 1)P(v < Y < \tau)dv
\]

\[
= \int_{0}^{\text{min}(\tau, \tau)} \left[ \sum_{j=1}^{\text{min}(\tau, \tau)} P(t - v)\lambda_{Nj}dv \right] P(v < Y < \tau)dv
\]

\[
= \frac{1}{\text{min}(\tau, \tau)} \int_{0}^{\text{min}(\tau, \tau)} P(t - v)\lambda_{Nj}dv
\]

(15)

In which \( Y \) is the repair time and it’s exponentially distributed with \( Y \sim G(t) = 1 - e^{-\mu t} \), so \( P(v < Y < \tau) = e^{-\mu \tau} - e^{-\mu v} \). The instantaneous availability \( A(t) \) becomes:

\[
\tilde{A}(t) = \tilde{A}(t) + \frac{1}{\text{min}(\tau, \tau)} \int_{0}^{\text{min}(\tau, \tau)} P(t - v)\lambda_{Nj}dv
\]

(16)

The steady-state availability \( \tilde{A} \) can be deduced when \( t \rightarrow \infty \), and obviously \( \text{min}(\tau, \tau) = \tau \) at this time.
3.2. The threshold is a random variable

When \( \tau \) is a nonnegative random variable with its distribution function is \( H(\tau) \), Eq.(15) becomes:

\[
P(X(t)=1|X(t)=N) = p_{N-1} \sum_{j=0}^{\min(1,N)} P(X(t-v)=j|X(t)=N)P(X(t-v)=j|P(X(t-v)=j\tau_{X(t-v)=j}dvH(\tau) + \cdots
\]

\[
= \frac{\lambda_{N}}{\lambda_{N} + \cdots + \lambda_{N-L,N}} \sum_{j=0}^{\min(1,N)} P(X(t-v)=j|P(X(t-v)=j|\lambda_{N}dvH(\tau) + \cdots
\]

\[
= \frac{1}{N-1} \sum_{j=0}^{\min(1,N)} \frac{1}{\sum_{j'=0}^{\min(1,N)} P(t-v)P(X(t)=j|\lambda_{N}dvH(\tau)}\quad(17)
\]

And \( \tilde{A}(t) \) is:

\[
\tilde{A}(t) = A(t) + \frac{1}{N-1} \sum_{j=0}^{\min(1,N)} \frac{1}{\sum_{j'=0}^{\min(1,N)} P(t-v)P(X(t)=j|\lambda_{N}dvH(\tau)}\quad(18)
\]

The steady-state availability becomes:

\[
\hat{A} = \lim_{t \to \infty} \tilde{A}(t) \quad(19)
\]

From above equations, when neglected fault effect is considered for a multi-state repairable component, availabilities (instantaneous availability and steady-state availability) are obviously higher than the ones in the original system. It is rational for certain conditions. Consider a flow transmission system which transfers liquid through pipes. For such a multi-state system, even when the system reaches a complete failure state, as long as the failure time is short enough to affect the output performance, the flow reserved in the pipes can satisfy the demand and at this very moment we regard the repair time ignorable or the fault effect neglected. So the system can work more hours than before when fault interval omission is considered.

3.3. Optimum maintenance cost rate

When fault interval omission of multi-state components is considered, different maintenance thresholds \( \tau_{1}, \tau_{2}, \cdots, \tau_{n} \) are introduced. Under various \( \tau_{1}, \tau_{2}, \cdots, \tau_{n} \), the lifetime of multi-state components is \( \tau_{1}, \tau_{2}, \cdots, \tau_{n} \) respectively. As \( \tau \) grows, though the lifetime \( \tau \) prolongs, the maintenance cost increases accordingly when fault incurs. Therefore, how to optimize the maintenance cost under different lifetime \( \tau_{1}, \tau_{2}, \cdots, \tau_{n} \) with respect to thresholds \( \tau_{1}, \tau_{2}, \cdots, \tau_{n} \) becomes significant in reliability engineering.

Suppose the total maintenance cost \( C(t) \) of a multi-state component contains replacement cost \( c_{f}(t) \) and preventive cost \( c_{p}(t) \), depreciation rate of the component is \( \alpha(t > 0) \). At the initial moment \( c_{f}(0) = c_{f} \), then total maintenance cost \( C(t) \) becomes:

\[
C(t) = c_{f}e^{-\alpha t} + c_{p}(e^{\alpha t} - 1) \quad(20)
\]

During the whole life cycle \((0,T)\) of the multi-state component, the tendency of maintenance cost \( C(t) \) changes with time \( t \in (0,T) \), as shown in Fig. 5.

The approach to get the best maintenance moment \( T^{*} \) is to differential \( C(t) \) with respect to \( t \), i.e.,

\[
T^{*} = (1/2\alpha)\ln(c_{f}/c_{p}) \quad(21)
\]

Then according to the best \( T^{*} \), the best maintenance threshold \( \tau^{*} \) is obtained.

4. Illustrative examples

To illustrate the results obtained above in this paper, we consider a power generation system which contains four different output performance levels. Its output performance \( G(t) \) (s\(^{-1}\)) can be denoted as the generating capacity and \( G_{1}(t) = 100, G_{2}(t) = 80, G_{3}(t) = 60, G_{4}(t) = 0 \). Obviously, the power generator is a multi-state unit and state 1 shows the perfect output while state 4 is the complete failure state. After statistical analysis, the failure rates (year\(^{-1}\)) are:

\[
\lambda_{12} = 1, \lambda_{13} = 2, \lambda_{14} = 1, \lambda_{23} = 1, \lambda_{24} = 2, \lambda_{34} = 1
\]

Assume that repair is implemented immediately when the system reaches state 4 and the repair rate \( \mu = 6 \text{year}^{-1} \). Demand of the system is \( w = 50 \text{MW} \) and the system is in the best state 1 at beginning. Let \( P_{j}(t) = P_{j}(X(t) = j), j = 1,2,3,4 \), and obviously \( \{X(t), t \geq 0\} \) is a homogenous CTMC. Probability intensity matrix of \( \{X(t), t \geq 0\} \) is:

\[
Q = \begin{bmatrix}
-4 & 1 & 2 & 1 \\
0 & -3 & 1 & 2 \\
0 & 0 & -1 & 1 \\
6 & 0 & 0 & -6
\end{bmatrix}
\]

According to Eq.(1), \( P_{1}(t) = 0.231 + 0.651e^{-4.732t} + 0.118e^{-2.268t} \), \( P_{2}(t) = 0.077 - 0.238e^{-4.732t} + 0.161e^{-2.268t} \), \( P_{3}(t) = 0.538 - 0.225e^{-4.732t} - 0.314e^{-2.268t} \), \( P_{4}(t) = 0.154 - 0.188e^{-4.732t} + 0.034e^{-2.268t} \). For the original Markov process, according to Eq. (3), instantaneous availability \( A(t) \) is:
Also we can get the steady-state availability $A$ when $t \to \infty$, i.e.,

$$A = \lim_{t \to \infty} A(t) = 0.846.$$ As for the reliability $R(t)$ of the original system, according to Eqs. (7) and (8), a new stochastic process \( \{X(t), t \geq 0\} \) can be defined and:

$$R(t) = 0.833e^{-t} + 0.5e^{-3t} - 0.333e^{-4t}$$

which is shown as Fig. 6.

From Eq. (10), MTTFF of the multi-state component is:

$$MTTFF = \int_0^\infty (0.833e^{-t} + 0.5e^{-3t} - 0.333e^{-4t})dt = 0.916 \text{ year} = 330 \text{ days}$$

When the failure time is too short to be detected or the fault will not affect the component’s output performance, we can set a threshold value $\tau$ for the maintenance. In fact, this phenomenon is rather common in fault-tolerant design. When we use a program or software in a personal computer, we may endure “program nonresponse” for several seconds (similar to $\tau$). As long as the program or software works well during $\tau$, the short fault time can be ignored and does not affect the PC’s performance. Here a new stochastic process \( \{\tilde{X}(t), t \geq 0\} \) which contains only two states is introduced and first we consider $\tau$ a constant 0.6.

From Eqs.(15) and (16), the instantaneous availability of \( \{\tilde{X}(t), t \geq 0\} \) is:

$$A(t) = A(t) + P(\tilde{X}(t)=1, X(t) = 4) = 0.9805 + 0.1716e^{-0.732t} - 0.0082e^{-2.268t}$$

Obviously, when $t \to \infty$, $t$ is always larger than $\tau$, so

$$A = \lim_{t \to \infty} A(t) = 0.9805.$$ Fig. 7 shows the change of instantaneous availabilities of two conditions when $\tau = 0.6$.

In addition, when we have different variable $\tau$, availability of the new system will definitely not be the same. Fig. 8 shows the change of availabilities with different maintenance threshold values with respect to $\tau = 0, 0.2, 0.4, 0.6.$

From Fig.8, the higher $\tau$ is, the larger the steady-state availability $\tilde{A}$ will be. When $\tau = 0$, the new system is equal to the original system and the new stochastic process is the old Markov process itself. When $\tau \to \infty$, the system will never fail as long as its output performance is larger than demand.

Then we consider the failure threshold $\tau$ is a random variable with a Gamma distribution. Suppose the density function of $\tau$ is

$$f(\tau) = \frac{\lambda^\alpha}{\Gamma(\alpha)} \tau^{\alpha-1}e^{-\lambda\tau} \quad \text{and} \quad \alpha = 1, \lambda = 2, \ dH(\tau) = 2e^{-2\tau}d\tau.$$ According to Eqs.(17) and (18), $\tilde{A}(t)$ becomes:

\[
\tilde{A}(t) = A(t) + P(\tilde{X}(t)=1, X(t) = 4) = 0.9325 - 0.6088e^{-16.732t} + 0.065e^{-12t} - 0.097e^{-10.268t} + 0.0038e^{-8.732t} - 1.0722e^{-8t} + 1.1057e^{-6.732t} + 0.1727e^{-4.732t} + 0.1683e^{-4.426t} - 0.0175e^{-2.268t}
\]
Fig.9 shows the change of availability when $\tau$ obeys a Gamma distribution $Ga(1,2)$. The steady-state availability is $A = 0.9325$.

From Fig. 9, we can see that the steady-state availability becomes 0.9325 compared with the original steady-state availability 0.846. The rational explanation is that we ignore some fault effect when the repair time is less than $\tau$. As a matter of fact, steady-state availability reflects the working proportion of the power generator. Fig. 10 shows the change of steady-state availabilities when fault interval omission is considered or not.

Next we’ll consider the effect on maintenance cost $C(t)$ of fault interval $\tau$, including constants 0,0.2,0.4,0.6 and random variable with distribution function $H(\tau)$. Suppose the initial replacement cost of the power generation system $c_f = £100$ and the initial preventive cost $c_p = £60$, according to Eq.(18), $C(t) = 100e^{-\mu t} + 60(e^{\mu t} - 1)$.

Depreciation rate $\alpha (\alpha > 0)$ is set by reliability engineers and after a serious evaluation of the power generator, $\alpha = 8 \times 10^{-4}$ is suggested. According to Eq.(19), the best maintenance moment $T^* = (1 / 2\alpha)ln(c_f / c_p) = 319.27$ days. From Fig. 10, when the fault interval $\tau = 0$, $T_0 = 304.56$ days. As $\tau$ grows from $\tau_1 = 0.2$ to $\tau_2 = 0.4$ and $\tau_3 = 0.6$, lifetime of the power generator increases from $T_1 = 323.24$ days to $T_2 = 342.87$ days and $T_3 = 352.98$ days. Similarly, maintenance cost rises from $C(T_1) = £94.92$ to $C(T_2) = £94.947$ and $C(T_3) = £94.976$.

On one aspect, the lifetime of the power generator can prolong as $\tau$ grows, while the maintenance cost rises correspondingly. Choosing an appropriate threshold value $\tau$ for the fault interval can not only extend the equipment’s lifetime but also manage the maintenance cost. In this illustrative example, the best threshold interval $\tau^*$ should be $0 < \tau^* < 0.2$ and that parameter is of vital importance to make proper maintenance policies.

5. Conclusions

In this paper, we build a Markov process for the multi-state repairable component which contains $N$ output performances based on a homogenous CTMC. Under the assumption that residence time and repair time are exponentially distributed, Kolmogorov equations are built. Based on the possibility of each state of the multi-state component, availability, reliability and mean time to first failure are deduced.

When the fault time is too short to be detected, a new stochastic process considering neglected fault effect is determined. Though it is associated with the original system tightly, it is not a Markov process any more. When the threshold failure time is a constant or a random variable, we compare the change of instantaneous availability. Numerical examples show that the availability will be larger when repair time omission is considered. At the same time, when maintenance cost is introduced, the best policy of choosing an appropriate threshold is to balance the maintenance cost and the lifetime.

Relevant results can also be used in queueing theory and management science. For example, in a queueing theory problem, whether a customer leaves or not depends on the tolerant interval one can accept. If the endurable time is extremely long, no matter how many people are queuing before him or her, one will always wait for his or her service. At the same time, with the increasing of state numbers, state exploration will definitely come up. Markov method may have a problem in solving differential functions and universal generating function (UGF) technology [13] can be considered. Therefore, the future emphasis is on the mixture of Markov process and UGF with neglected fault effect or delayed failures.

Acknowledgements

The authors would like to thank the anonymous reviewers for their constructive suggestions to improve the original manuscript. In addition, this work is partially supported by the National Natural Science Foundation of China (grant number: 71671091, 71801121) and China Postdoctoral Science Foundation (grant number: 2018M630561).
References

1. Bao X, Cui L. An Analysis of Availability for Series Markov Repairable System With Neglected or Delayed Failures. IEEE Transactions on Reliability 2010; 59(4):734-743, https://doi.org/10.1109/TR.2010.2055915.
2. Calderod L. Coherent systems with multistate components. Nuclear Engineering & Design 1980; 58(1):127-139, https://doi.org/10.1016/0029-5493(80)90102-8.
3. Calderhead B, Epstein M, Sivilotti L, et al. Bayesian Approaches for Mechanistic Ion Channel Modeling. Methods Mol Biol. 2013; 1021: 247-272, https://doi.org/10.1007/978-1-62703-450-0_13.
4. Calimet N, Simoes M, Changeux J P, et al. A gating mechanism of pentameric ligand-gated ion channels. Proceedings of the National Academy of Sciences of the United States of America 2013; 110 (42): 3987-3996, https://doi.org/10.1073/pnas.1313785110.
5. Cui L, Zhang Q, Kong D. Some New Concepts and Their Computational Formulae in Aggregated Stochastic Processes with Classifications Based on Sojourn Times. Methodology & Computing in Applied Probability 2015; 18(4): 1-21.
6. Dong W, Liu S, Fang Z, et al. A model based on hidden graphic evaluation and review technique network to evaluate reliability and lifetime of multi-state systems. Proceedings of the Institution of Mechanical Engineers, Part O: Journal of Risk and Reliability 2018, https://doi.org/10.1177/1748006X18788414.
7. Chen R. Semi-Markov Processes: Applications in System Reliability and Maintenance. Graphical models and image processing. Academic Press 2014: 161-176.
8. He Q M. Fundamentals of Matrix-Analytic Methods. Springer New York, 2014, https://doi.org/10.1007/978-1-4614-7330-5.
9. He Q, Zha Y, Zhang R, et al. Reliability analysis for multi-state system based on triangular fuzzy variety subset bayesian networks. Eksplotacja i Niezawodność - Maintenance and Reliability 2017; 19(2): 152-165, https://doi.org/10.17531/ein.2017.2.2.
10. Hirose H, Sakumura T. The Extended Cumulative Exposure Model (ECEM) and Its Application to Oil Insulation Tests. IEEE Transactions on Reliability 2012; 61(3): 625-633, https://doi.org/10.1109/TR.2012.2207575.
11. Cao J H, Cheng K. Introduction to Reliability Mathematics. Beijing: Science Press, 1986, (In Chinese).
12. Jia X, Shen J, Xing R. Reliability Analysis for Repairable Multistate Two-Unit Series Systems When Repair Time Can Be Neglected. IEEE Transactions on Reliability 2016; 65(1): 208-216, https://doi.org/10.1109/TR.2015.2461218.
13. Levitin G. The Universal Generating Function in Reliability Analysis and Optimization. Springer, 2005.
14. Levitin G, Amari S V. Multistate systems with multi-fault coverage. Reliability Engineering & System Safety 2017; 93 (11): 1730-1739, https://doi.org/10.1016/j.ress.2017.12.004.
15. Li M, Ma X, Zhang X, et al. Reliability analysis of non-repairable cold-standby system based on the Wiener process. International Conference on System Reliability and Safety 2017: 151-155.
16. Lisnianski A, Elmakias D, Laredo D, et al. A multi-state Markov model for a short-term reliability analysis of a power generating unit. Reliability Engineering & System Safety 2017; 98(1): 1-6, https://doi.org/10.1016/j.ress.2017.10.008.
17. Lisnianski A, Frenkel I, Karagrigorious A. Recent Advances in Multi-state Systems Reliability. Springer 2018, https://doi.org/10.1007/978-3-319-63423-4.
18. Liu B, Cui L, Wen Y. Interval reliability for aggregated Markov repairable system with repair time omission. Annals of Operations Research 2014; 212(1): 169-183, https://doi.org/10.1007/s10479-013-1402-8.
19. Pan D, Liu J B, Huang F, et al. An Wiener Process Model With Truncated Normal Distribution for Reliability Analysis. Applied Mathematical Modelling 2017; 50: 333-346.
20. Pham T, Pham H. A generalized software reliability model with stochastic fault-detection rate. Annals of Operations Research 2017; 4(4): 1-11, https://doi.org/10.1007/s10479-017-2486-3.
21. Pirious P Y, Faure J M, Lesage J J. Generalized Boolean logic Driven Markov Processes: A powerful modeling framework for Model-Based Safety Analysis of dynamic repairable and reconfigurable systems. Reliability Engineering & System Safety 2017; 163: 57-68, https://doi.org/10.1016/j.ress.2017.02.001.
22. Qin J, Niu Y, Li Z. A combined method for reliability analysis of multi-state system of minor-repairable components. Eksplotacja i Niezawodność - Maintenance and Reliability 2016; 18 (1): 80-88, https://doi.org/10.17531/ein.2016.1.11.
23. Wang W, Maio F D, Zio E. Three-Loop Monte Carlo Simulation Approach to Multi-State Physics Modeling for System Reliability Assessment. Reliability Engineering & System Safety 2017; 167: 276-289, https://doi.org/10.1016/j.ress.2017.06.003.
24. Bao X Z, Cui L. A Study on Reliability for A Two-Item Cold Standby Markov Repairable System with Neglected Failures. Communications in Statistics 2012; 41(21): 3988-3999, https://doi.org/10.1080/03610926.2012.700736.
25. Yu H, Yang J, Mo H. Reliability analysis of repairable multi-state system with common bus performance sharing. Reliability Engineering & System Safety 2014; 132: 90-96, https://doi.org/10.1016/j.ress.2014.07.017.
26. Zaitseva E, Levashenko V. Multiple-Valued Logic mathematical approaches for multi-state system reliability analysis. Journal of Applied Logic 2013; 11(3): 350-362, https://doi.org/10.1016/j.jal.2013.05.005.
27. Zhang C, Lu X, Tan Y, et al. Reliability demonstration methodology for products with Gamma Process by optimal accelerated degradation testing. Reliability Engineering & System Safety 2015; 142:369-377, https://doi.org/10.1016/j.ress.2015.05.011.
28. Zhang Q, Cui L, Yi H. A study on a single-unit repairable system with working and repair time omission under an alternative renewal process. Journal of Risk & Reliability 2017; 233(1): 1748006X1769351.
29. Zhao X, Qian C, Nakagawa T. Comparisons of Replacement Policies with Periodic Times and Repair Numbers. Reliability Engineering & System Safety 2017; 168: 161-170, https://doi.org/10.1016/j.ress.2017.05.015.
30. Zheng Z, Cui L, Hawkes A G. A study on a single-unit Markov repairable system with repair time omission. IEEE Transactions on Reliability 2006; 55(2): 182-188, https://doi.org/10.1109/TR.2006.874933.
Wenjie DONG
College of Economics and Management
Nanjing University of Aeronautics and Astronautics
Nanjing 211106, Jiangsu, PR China

Sifeng LIU
College of Economics and Management
Nanjing University of Aeronautics and Astronautics
Nanjing 211106, Jiangsu, PR China

Xiaoyu YANG
College of Economics and Management
Nanjing University of Aeronautics and Astronautics
Nanjing 211106, Jiangsu, PR China

Huan WANG
College of Economics and Management
Nanjing University of Aeronautics and Astronautics
Nanjing 211106, Jiangsu, PR China

Zhigeng FANG
College of Economics and Management
Nanjing University of Aeronautics and Astronautics
Nanjing 211106, Jiangsu, PR China

E-mails: dongwenjie@nuaa.edu.cn, sfliu@nuaa.edu.cn,
516276255@qq.com, 535107751@qq.com, zhigengfang@163.com