Development of mathematical literacy problems to empower students’ representation

L Lestariningsih¹, E Nurhayati¹, TAB Susilo¹, C Cicinidia¹, M Lutfianto²

¹STKIP PGRI Sidoarjo, Sidoarjo, Indonesia
²STKIP Al Hikmah Surabaya, Surabaya, Indonesia

e-mail: lestari@stkippgri-sidoarjo.ac.id

Abstract. Representation is the main skill needed by someone in the process of solving problems, especially mathematical literacy problems. Thus this paper aimed to describe the process and results of developing mathematical literacy problems that are valid, practical, and have the potential effect on students’ representation. In developing problems, researchers used design research with development study type. The problems was developed with a formative evaluation consisting of self evaluation, one to one, expert review, small group, and field test involving 2 experts from mathematical literacy experts, and 36 first-year students. The results of the study showed the problems developed to meet valid and practical criteria based on one to one results analysis, expert reviews, and small group. Furthermore, the problems developed also have potential effects based on a field test analysis that showed students were empowered in generating representations in solving problems.

1. Introduction

Representation is very notable in mathematics because representation is needed in understanding concepts and solving mathematical problems [1]. In addition, representation is also needed by students to find or develop ways of thinking or communicating mathematical ideas from abstract to concrete, and making these ideas easier to understand [2]. Furthermore [3] states that a successful problem solving process depends on the skills of representing problems such as constructing and using mathematical representations in words, graphs, tables, and equations, completion and manipulation of symbols. In general, representation is very important in improving mathematical competence for students.

Representation is an individual ability to express mathematical ideas contained in problems, statements, and definitions in various ways. Representation is the disclosure of mathematical ideas using various means such as spoken language, written language, symbols, pictures, diagrams, models, charts, or using physical members [4]. Furthermore, according to [1], representations are grouped into 3 types, namely (1) visual representations (images, graphs, or tables), (2) symbolic representations (mathematical statements or mathematical notations, numerical or algebraic symbols) and (3) verbal representations (written text or words). Students who face new or complex problems will use various representations as a form of strategy to solve them [2]. Problems in mathematics can be contextual problems that are presented in the form of meaningful narratives or problems related to real situations experienced by students in everyday life [5], [6].

Contextual problems or problems related to real situations are usually used as a basis for developing mathematical literacy problems because their structure emphasizes the need to develop a variety of contexts [7]. Mathematical literacy problems can be used to measure the ability of individual
mathematical literacy. Mathematical literacy problem in PISA is based on aspects of mathematical content and context [7]. Based on the results of the Program for International Student Assessment (PISA), the ability of Indonesian students in the field of mathematical literacy in general is still low. The results of the PISA 2015 show that mathematics achievement in Indonesia is ranked 63rd out of 72 countries with the score obtained is 386 [8]. According to [9] students use representation in mathematical activities when solving mathematical literacy problems. Therefore, it is important to develop mathematical literacy problems for students to support their ability in representation.

Previous research on the development of mathematical literacy problems was carried out in the form of developing mathematical questions about PISA models level 3, 4, 5, and 6 by using the Lampung context showing the results that these problems have potential effects on students' mathematical literacy abilities [10]. Other research on the development of mathematical literacy problems conducted by [11] shows that the problems developed have a potential effect on student involvement actively in generating indicators of the basic mathematical abilities mentioned by the PISA framework.

Regarding the previous explanations, i.e. [6], [7], [9], [10], and [11] the researcher considers it necessary to develop mathematical literacy problems based on the PISA framework for empowering students' representations. Thus, the purpose of this study is to describe the process and results of developing mathematical literacy problems that are valid, practical, and have a potential effect on students' representation.

2. Methods
In developing mathematical literacy problems, researchers used a design research with development study type. The main characteristic of this type of research is development with a repetitive cycle that uses formative evaluation [12]. The steps that we conducted were preliminary and prototyping stages (formative evaluation) which includes self-evaluation, expert reviews and one-to-one, small group, and field test (see Figure 1) [13][14].

![Figure 1. The flow of development of mathematical literacy problems](image)

Mathematical literacy problems were developed by referring to the characteristics of mathematical literacy problems used in PISA. After obtaining valid and practical mathematical literacy problems, the subject then completes the problem in 60 minutes. Data on the results of student answers and interview results were analysed descriptively to determine the extent of students' representations in solving mathematical literacy problems.
3. Results and Discussion
The process of developing problems started from the preliminary stage, which was designing mathematical literacy problems that were distributed into content, context, and level categories, then determined the validators of the problems, namely 5 lecturers who have expertise in the field of mathematical literacy. Next, we determined that the research subjects were 36 first year students. The results of designing mathematical literacy problems were left to the validators for an expert review process and 6 students to do one-to-one activity in order to obtain comments and suggestions. Figure 2 is an example of changing questions from the preliminary stage before revision.

In the Sidrap area, South Sulawesi, the government will build several wind-powered electric generators. The government wants to estimate the costs and benefits to be gained from the construction of a wind-powered electric generator. They use the following formula to estimate the acquisition of funds, $F$ rupiah, over the length of time (years), $y$.

$$F = \frac{40000000\, y}{\text{profits from electric production for each year}} - \frac{32000000}{\text{cost of building a wind–powered electric generator}}$$

Based on the formula used by the government, how many years does a wind-powered electric generator operate so that it can cover the cost of building it? Explain!

Figure 2. Problem before revision (Prototype 1)

Then based on the results of the validation of the expert reviews obtained some suggestions for improvement to the problem, namely writing a description of the formula separately so it did not enter in the sentence. The proposed editorial changes were $F = \text{funds (in rupiah)}\, y = \text{length of time (years)}$. Then add a comma to the number to indicate thousands. Meanwhile, from one-to-one activities, the results obtained that students already understand the intent or question contained in the problem. Furthermore, advice from expert reviews and the results of one-to-one activities were used as the basis for revising the problems that have been designed. Figure 3 showed problems that have been revised to become prototype 2.

The problems in Figure 3 used the science context related to the government's plan in one area in the province of South Sulawesi to build a wind-powered electric generator. In this problem, someone is asked to determine the minimum time needed so that the operating a wind-powered electric generator can cover the cost of building using the estimated formula for obtaining funds. Furthermore, this problem contains changes and relationships contents. Mathematical aspects in this content are related to function and algebra material [11]. Forms of algebra, equations, inequality, representations using tables and graphs are central in describing, modeling, and interpreting changes in a phenomenon.

The next step was to try out prototype 2 into a small group that involved 6 STKIP PGRI Sidoarjo students. In this stage, the researchers reviewed each item developed to be discarded, maintained with revision, or retained without revision. This decision was based on the results of the activity: (1) giving a questionnaire that asked students' opinions about the question, (2) examining the distribution of students' answers, and (3) interviewing the small group subjects to find out whether students couldn’t work on the problem because there was no scheme that helps or because of problems with readability. The contents of the questionnaire asked whether or not words/sentences/pictures/graphics were not understood and the students' general impression of the problem.
In the Sidrap area, South Sulawesi, the government will build several wind-powered electric generators. The government wants to estimate the costs and benefits to be gained from the construction of a wind-powered electric generator. They use the following formula to estimate the acquisition of funds.

\[
F = \frac{40\,000\,000 \, y \, \text{profits from electric production for each year}}{-320\,000\,000 \, \text{cost of building a wind–powered electric generator}}
\]

Note:
\( F = \) funds (in rupiah)
\( y = \) length of time (years)

Based on the formula used by the government, how many years does a wind-powered electric generator operate so that it can cover the cost of building it? Explain!

**Figure 3. Problem after revision (Prototype 2)**

Based on data from the questionnaire, it could be concluded that some general impressions of the small group results related to the problem developed, namely (1) problems related to daily life so that they could add insight, (2) more problem-solving activity required direct reasoning using formulas, (3) problems required the ability of representation to solve them, (4) these types of problems were rarely encountered before, and (5) there were problems that were difficult to understand because they used language that has a double meaning. Thus, it could be said that points (1), (2), and (3) supported the statement that this problem was developed based on the concept of mathematical literacy, and could empower students' representational abilities. Meanwhile, point (4) showed that the mathematical literacy problems developed need to be introduced more widely, while point (5) showed that still there were problems that need to be revise in the sentence structure used because of the possibility that students who were unable to answer that caused by sentence/picture arrangement or something unclear in the problems. Overall, based on the small group test results, it is necessary to revise the problem.

| No. | The representation used by students                                      | Percentages of student responses |
|-----|-------------------------------------------------------------------------|---------------------------------|
| 1   | Used visual representation (images, graph diagrams, or tables)          | 83%                             |
| 2   | Used symbolic representations (mathematical statements / mathematical notations, numeric / algebraic symbols) | 88%                             |
| 3   | Used verbal representations (written text / words)                       | 94%                             |

**Table 1. Used of representation to solve mathematical literacy problems**

Note: percentage of student responses = \( \frac{\text{the number of subject who gave respond}}{\text{the number of subjects}} \) × 100%

Next, the prototype 2 of problems were revised to prototype 3 which were used during the field test. This stage was carried out with the aim to see the potential effects of problems developed based on revisions in the small group stage. This stage was carried out by involving 36 first year students. After students finish working on the problem, we analysed the students' answers which one that using representations to solve the problem. Table 1 showed the distribution of data related to representations based on students' answers. Students who used visual representations such as triangle, square, rectangle, circle and beam images, bar charts, line diagrams, or tables were 83%. Next, symbolic representations
made by students by using mathematical statements/mathematical notations to declare the length, width, radius, diameter, circumference, and area of a building, using numerical/algebraic symbols to express the addition, subtraction, multiplication, division, and equal sign were 88%. Moreover, 94% of students used verbal representation by using written text or words to solve the problem of mathematical literacy.

Figure 4 showed an example of a visual representation made by a student from a mathematical literacy problem about building space that looks like a solid build with a length of 6 small cubes, width of 5 small cubes, and height of 4 small cubes. Students stated in the picture as many small cubes as they knew in the problem then gave label as surface 1, surface 2, surface 3, surface 4, surface 5, and surface 6.

![Figure 4. An example of a visual representation](image)

Based on the results of the analysis of students’ answers in the field test stage, it was known that students used visual representations such as triangle, square, rectangle, circle and beam images, bar charts, line diagrams, or tables. This was in accordance with the opinion of [1], [3], [4] that the representation made by students could be in the forms of pictures, graphs, charts or tables. Also, students made symbolic representations by using mathematical statements/mathematical notations to declare the length, width, radius, diameter, circumference, and area of a building, using numerical/algebraic symbols to express the addition, subtraction, multiplication, division, and equal sign [1][4]. Furthermore, students used verbal representation by using written text or words to solve the problem of mathematical literacy [1][4]. Overall, the mathematical literacy problems developed could empower students’ representation [11] as a form of strategy in completing it [2].

4. Conclusions
Based on the results of data analysis and discussion, it could be concluded that the mathematical literacy problems developed are valid, practical based on one-to-one results analysis, expert reviews, and small groups. Furthermore, the question of mathematical literacy developed has a potential effect in empowering students’ representation. Students solve mathematical literacy questions using visual, symbolic, and verbal representations. The strategies used by students in solving mathematical literacy problems were multiple representations, which means that in each problem, students could use more than one form of representation. Furthermore, for lecturers or teachers, it was important to pay attention and develop students' representation in solving mathematical literacy problems or contextual problems. From the representations made by students, the lecturers or teachers can find out the mindset and ideas of students.
Acknowledgment
The author presents her sincere appreciation which goes to her parents. In addition, this paper would not be done without the help and support from Prof. Dr. Siti M. Amin and Dr. Agung Lukito, M.S. The researcher thanks them for their advices, and guidance from the very early stage of this research until the accomplishment of this paper.

References
[1] Kartini 2009 Peranan Representasi dalam Pembelajaran Matematika Semin. Nas. Mat. dan Pendidik. Mat. UNY 978979.
[2] Sahendra A et al 2018 J. Phys.: Conf. Ser. 947 012059.
[3] Brenner, M E et al 1997 Learning by understanding: The role of multiple representations in learning algebra. American Educational Research J. 34 663689.
[4] Goldin G A 2002 Representational systems, learning, and problem solving in mathematics J. Math. Behav. 17 137165.
[5] Bates E T and Wiest L R 2003 Impact of Personalization of Mathematical Word Problems on Student Performance Mathematics Educator 14 1726.
[6] Boonen A J H, et al 2016 Word Problem Solving in Contemporary Math Education: A Plea for Reading Comprehension Skills Training Frontiers in psychology 7th February 191.
[7] Lestariningsih et al 2018 J. Phys.: Conf. Ser. 1108 012083.
[8] OECD 2017 PISA 2015 Results (Volume V): Collaborative Problem Solving PISA OECD Publishing Paris.
[9] Lestariningsih, et al 2018 Exploring mathematization underpinnings of prospective mathematics teachers in solving mathematics problems Beta: J. Tadris Matematika 11 167176.
[10] Putra YY, Zulkardi, and Hartono Y 2016 Pengembangan Soal Matematika Model PISA Level 4, 5, 6 menggunakan Konteks Lampung Kreano: J. Matematika Kreatif-Inovatif 7 1016.
[11] Kohar AW and Zulkardi 2009 Pengembangan Soal Berbasis Literasi Matematika dengan Menggunakan Kerangka PISA Tahun 2012. Prosiding Konferensi Nasional Matematika XVII – 2014 ITS Surabaya 379388.
[12] National Center for Education Statistics USA 2013 PISA 2012 Data Tables, Figures, and Exhibits.
[13] Plomp T and Nieveen N 2007 An introduction to educational design research In Proceedings of the Seminar Conducted at the East China Normal University [Z]. Shanghai: SLO- Netherlands Institute for Curriculum Development.
[14] Tessmer M 1993 Planning and Conducting Formative Evaluations: Improving the Quality of Education and Training London Kogan Page.