Quantum field theory and Einstein’s theory of general relativity are extremely successful in predicting the outcome of particle physics and gravitational experiments. For instance, precision experiments carried out in the realm of quantum electrodynamics show an agreement between measurements and theoretical predictions to a 10 digit precision. Moreover, the recent direct detection of gravitational waves by the LIGO-VIRGO collaboration as well as the first image of a black-hole shadow by the Event-Horizon-Telescope collaboration marked striking confirmations of general relativity. Based on theoretical considerations, it is generically expected that key concepts from quantum field theory must be brought together with the fundamental insights about the nature of gravity from general relativity, in order to provide a consistent description of our universe at all scales. At microscopic scales, matter exhibits quantum properties which are successfully accounted for by quantum field theory. As with classical matter, these quantum fluctuations gravitate and spacetime has to respond to these effects. This suggests that a proper description of spacetime and matter at small scales requires a quantum theory of gravity or, more ambitiously, a quantum model of spacetime.

A primary challenge in this line of research is that it is currently not clear which structures are the relevant ones for extending the present theoretical frameworks to and possibly beyond the Planck scale $E_{\text{Planck}} \approx 10^{18}$ GeV. As a consequence, the research area is subject to a constant inflow of exciting ideas which could be relevant for describing physics at these scales. This Special Issue reflects this broad range of concepts and ideas. It collects contributions from leading experts on topics related to

- renormalization group techniques in quantum gravity,
- quantum field theory in curved and fractal spacetimes,
- geometrical aspects of spacetime and quantum field theory.

The pivotal theme are ideas related to “Asymptotic Safety”, first proposed by Nobel Laureate Steven Weinberg [1,2]. In essence, this idea states that gravity could be formulated as a consistent and predictive quantum field theory by generalizing the idea of Asymptotic Freedom beyond the realm of perturbation theory. In this case, the dynamical principle underlying the predictivity for a quantum theory of geometry is quantum scale symmetry [3]. Intriguingly, progress along this research frontier may be the key to answer fundamental questions about the structure of space, time, and matter. Following the idea of “renormalizing the non-renormalizable”, this route may even allow for a unified
description of all fundamental forces, including gravity, within the framework of renormalizable quantum field theories.

A central tool for the exploration of this territory are functional methods-based, e.g., on path integrals or functional renormalization group equations, complemented by lattice techniques as well as combinatorial, discrete methods. An indispensable tool in these investigations came with Martin Reuter’s adaptation of the functional renormalization group equation [4] for the effective average action, first developed in the context of gauge theories [5–7], to gravity. Since then, the program has made significant progress on the conceptual understanding of the asymptotic safety mechanism [8–12] as well as its phenomenological consequences for the structure of spacetime [13,14], particle physics [15], black holes [16–18], and cosmology [19]. Moreover, renormalization group ideas related to those that Martin Reuter pioneered in the context of asymptotically safe gravity are now also used to explore the properties of spacetime in other approaches to quantum gravity, including loop quantum gravity [20], Causal Dynamical Triangulations [21,22], tensor models [23], and group field theories [24]. The renormalization group is thereby starting to play an important role in linking different quantum-gravity approaches providing a common language in which distinct ideas on quantum spacetime can be discussed and compared. This Special Issue brings together an inspiring collection of research articles and reviews which reflects the wide scope of the research field, its current ideas, and future perspectives.

The Special Issue contains nine published manuscripts; six research articles and three reviews. The research articles (listed in chronological order of publication) focus on a wide range of open questions in quantum field theory, quantum gravity, and functional renormalization group techniques:

- **The Role of Riemann’s Zeta Function in Mathematics and Physics** by Walter Dittrich [25], discusses Riemann’s impact on mathematics and physics using methods originating from number theory and quantum electrodynamics. Concretely, this is illustrated by using the Riemann zeta function to regularize the Heisenberg–Euler Lagrangian. As a by-product, the work explicitly evaluates various integrals that are useful in mathematics and physics.

- **Dimension and Dimensional Reduction in Quantum Gravity** by Steven Carlip [26], reviews the concept of “dimension” in the context of quantum gravity. In many cases, operators on a fluctuating spacetime exhibit an anomalous scaling dimension that makes the theory effectively two-dimensional in a specific sense. By now, this was observed in several independent ways and in various different approaches to quantum gravity. The article offers a discussion of potential mechanisms that could explain the universality of this phenomenon. In the absence of direct observational tests of quantum gravity such common findings that arise almost universally in distinct quantum gravity approaches could provide an important guide to the development of a full-fledged theory of quantum gravity.

- **Anti-Newtonian Expansions and the Functional Renormalization Group** by Max Niedermaier [27], introduces an Anti-Newtonian expansion scheme for scalar quantum field theories and classical gravity. This scheme employs a spatial gradient expansion in the sense that the limiting theory evolves only in time while the spatial points are dynamically decoupled. In scalar quantum field theories, the limiting system consists of copies of a self-interacting quantum mechanical system which produces an (in principle) exact solution of the functional renormalization group equation for the effective average action. In Einstein gravity, the anti-Newtonian limit has no dynamical spatial gradients, yet remains fully diffeomorphism invariant and propagates the original number of degrees of freedom. In this setting, the work constructs a canonical transformation which maps the ADM action to its anti-Newtonian limit. The prospects of further applications in the context of quantum gravity are discussed. Quite intriguingly, this setting might be linked to the idea of asymptotic silence arising close to spacetime singularities which has been proposed as a potential mechanism to generate dimensional reduction, as discussed above.

- **Holographic Formulation of 3D Metric Gravity with Finite Boundaries** by Seth K. Asante, Bianca Dittrich and Florian Hopfmüller [28], constructs holographic boundary theories for linearized
three-dimensional gravity, for a general family of finite or quasi-local boundaries. These boundary theories are directly derived from the dynamics of three-dimensional gravity by computing the effective action for a geometric boundary observable which measures the geodesic length from a given boundary point to some center in the bulk manifold. The general form for these boundary theories is shown to be Liouville-like with a coupling to the boundary Ricci scalar. The discussion of several examples offers interesting insights into the structure of holographic boundary theories. An important motivation for this line of research comes from considering renormalization group flows for spin-foam models. In this context, it is related to the identification of boundary observables which encode the bulk dynamics efficiently and lend themselves to which allow computing such renormalization group flows in practise.

- **On the Structure of the Vacuum in Quantum Gravity: A View from the Asymptotic Safety Scenario** by Alfio Bonanno [28], investigates the vacuum state underlying the gravitational asymptotic safety scenario. It is demonstrated that higher derivative operators, commonly expected in this scenario, may generate additional degrees of freedom which render the standard vacuum unstable. When this happens, translation and rotational symmetries can be spontaneously broken and the effective action may give rise to a vacuum state corresponding to a “kinetic condensate”. In this scenario, the vacuum state of gravity may then be given by the gravitational analogue to the Savvidy vacuum in Quantum Chromo-Dynamics (QCD). This line of research highlights how a change in the microscopic gravitational dynamics could address the long-standing conformal factor instability in Euclidean Einstein gravity.

- **The Inflationary Mechanism in Asymptotically Safe Gravity** by Alessia Platania [29], reviews the implications of Asymptotic Safety in the cosmological context. Specifically, the work analyzes a toy model exemplifying how the departure from quantum scale invariance could explain the approximate scale-invariance observed in the power spectrum of temperature fluctuations in the cosmic microwave background. In a broader phenomenological context, this constitutes an explicit example of how quantum gravity might provide a first-principles explanation for free parameters appearing in the effective field theory framework – both in cosmology and in particle physics. In the case of inflation, Asymptotic Safety may naturally provide inflationary potentials from the gravitational dynamics without the need to introduce a scalar field with an ad-hoc potential by hand [30].

The three review articles of the Special Issue focus on the following:

- **Vacuum Effective Actions and Mass-Dependent Renormalization in Curved Space** by Sebastián A. Franchino-Viñas, Tibério de Paula Netto and Omar Zanusso, ref. [31], reviews results on the non-local form factors entering the effective action of semiclassical gravity in two and four dimensions. The relevant contributions are computed by means of a covariant expansion of the heat kernel up to the second order in the spacetime curvature. The result highlights the importance of the form factors in determining the momentum-dependence of Newton’s constant and the other gravitational couplings in Wilsonian renormalization group computations.

- **Geometric Operators in the Einstein–Hilbert Truncation** by Maximilian Becker and Carlo Pagani, ref. [32], reviews the composite operator formalism for the effective average action. Subsequently, the method is used to estimate the anomalous scaling properties of geometric operators, such as the geodesic length and the volume of hypersurfaces, in the context of the gravitational Asymptotic Safety program. The results provide several extensions of previous studies. As a future perspective, this line of research provides additional characterizations of the effective spacetime geometry beyond the dimensional estimators reviewed, e.g., in [14]. In particular, it provides tools to further test the conjecture of dynamical dimensional reduction in quantum gravity.

- **Multi-Critical Multi-Field Models: A CFT Approach to the Leading Order** by Gian Paolo Vacca, Alessandro Codello, Mahmoud Safari and Omar Zanusso, ref. [33], reviews some general results for the multi-critical multi-field models in $d > 2$, recently obtained using conformal field theory
and Schwinger–Dyson methods at the perturbative level [34,35]. Results in the leading non-trivial order are derived consistently for several conformal field theories and found to be in agreement with functional renormalization group methods. The prospects for investigating mechanisms like emergent (possibly approximate) symmetries in this framework are outlined. The characterization of the corresponding scale-invariant theories provides blueprints for fixed-point mechanisms that could also be active in a gravitational setting. In particular, the approach taken in this work may lead to a better understanding of the Reuter universality class underlying the gravitational asymptotic safety scenario in the future.

In its broad scope, the present collection of articles highlights several attractive points about the gravitational asymptotic safety scenario, which owes much to the pioneering work by Martin Reuter [4,36–40]. Currently, the approach has become a focal point of quantum gravity research, that it has enriched with new questions, concepts and techniques. From a phenomenological viewpoint, the fact that Asymptotic Safety may explain assumptions made within an effective-field-theory setting makes the approach very attractive for developing (beyond) the Standard Model particle physics, astrophysics, and cosmological scenarios. The methodological link to conformal field theory implies that insights and methods can be transferred between the physically rather distinct realms of quantum gravity and condensed matter/statistical physics, providing a fruitful cross-fertilization between these research areas.

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