STATISTICAL TESTS FOR CHDM AND ΛCDM COSMOLOGIES

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ABSTRACT

We apply several statistical estimators to high-resolution N-body simulations of two currently viable cosmological models: a mixed dark matter model, having Ω_0 = 0.2 contributed by two massive neutrinos (C + 2νDM), and a cold dark matter model with cosmological constant (ΛCDM) with Ω_0 = 0.3 and h = 0.7. Our aim is to compare simulated galaxy samples with the Perseus-Pisces redshift survey (PPS). We consider the n-point correlation functions (n = 2–4); the N-count probability functions P_N, including the void probability function P_0; and the underdensity probability function U_ε (where ε fixes the under-density threshold in percentage of the average). We find that P_0 (for which PPS and CfA2 data agree) and P_1 distinguish efficiently between the models, while U_ε is only marginally discriminatory. On the contrary, the reduced skewness and kurtosis are, respectively, S_3 ≈ 2.2 and S_4 ≈ 6–7 in all cases, quite independent of the scale, in agreement with hierarchical scaling predictions and estimates based on redshift surveys. Among our results, we emphasize the remarkable agreement between PPS data and C + 2νDM in all the tests performed. In contrast, the above ΛCDM model has serious difficulties in reproducing observational data if galaxies and matter overdensities are related in a simple way.

Subject headings: cosmology: theory — dark matter — galaxies: clusters: general — large-scale structure of universe

1. INTRODUCTION

Among the various cosmological models considered in the literature, much effort has been concentrated on models inspired by the hypotheses of inflation. In such a context, the “natural” choice is to consider COBE normalized models with negligible spatial curvature, Gaussian and adiabatic primordial fluctuations, and a spectrum close to the Zeldovich one.

Among them, models that agree with the available data on large scales, where fluctuations are still in the linear regime, deserve further inspection at smaller scales. Critical differences can be expected to exist for fluctuations still in a weakly nonlinear regime. Here we need statistical tests, which are simultaneously robust and discriminatory, to compare real data with N-body simulations. On the contrary, when much smaller scales are inspected, it is not clear whether some residual signal coming from the shape of the postrecombination spectrum can still be appreciated, and in any case it will be necessary to include astrophysics that is still poorly understood, such as star formation and feedback effects.

In a previous paper (Ghigna et al. 1994, hereafter Paper G) we showed that the void probability function (VPF), P_0, can be a robust and discriminatory test on the distribution of matter in underdense regions of the universe, which are, however, in a weakly nonlinear regime (|δρ| ≤ 0). In Paper G, voids in a volume-limited sample of the Perseus-Pisces survey (PPS; Giovanelli & Haynes 1991) were analyzed and the function P_0(r) was computed. This VPF was then compared with simulations based on the Ω_0 = 1 cold dark matter (CDM) model and on the cold and hot dark matter (CHDM) model with Ω_0 = 0.3 and one massive neutrino (m_ν ≈ 7 eV). We found that this CHDM model produces too many intermediate-size voids.

In this paper we extend the comparison between PPS data and simulations by considering new models and new tests. We study a ΛCDM model with Ω_0 = 0.3 and h = 0.7 (hereafter this model will be called ΛCDM_0.3), and a mixed dark matter model with Ω_0 = 0.2, h = 0.5, and two massive neutrinos, each having a mass of m_ν = 2.3 eV (hereafter this mix will be called C + 2νDM). The ΛCDM_0.3 model has been found to reproduce the power-spectrum shape on intermediate (∼ 20 h^{-1} Mpc) scales (e.g., Peacock & Dodds 1994; Borgani et al. 1996), as well as the abundance of galaxy clusters (e.g., Eke, Cole, & Frenk 1996 and references therein), Klypin, Primack, & Holtzman (1996) showed, however, that it predicts too strong a galaxy clustering on scales ≤ 10 h^{-1} Mpc. As for the C + 2νDM model, it was first considered by Primack et al. (1995). Sharing Ω_0, between two massive neutrino species decreases the fluctuation amplitude on ∼ 10 h^{-1} Mpc scales with respect to the standard CHDM model (thus alleviating cluster overproduction), without reducing the small-scale (∼ 1 h^{-1} Mpc) power to an unacceptable level for early galaxy formation.

Among previous works on the VPF, it should be mentioned that Fry et al. (1989) estimated it for a preliminary version of PPS and compared the results with CDM N-body simulations. Weinberg & Cole (1992) showed that the VPF can also discriminate between Gaussian and non-Gaussian initial conditions. Finally, Ghigna et al. (1996a) compared the void statistics in the PPS sample analyzed here to simulations of the broken scale invariance (BSI) model (CDM with a characteristic scale in the postinflationary spectrum; see, e.g., Gottlöber, Mücke, & Starobinski 1994), which was found to agree with observations.
The $n$-point correlation functions $\xi_n$ for the PPS, evaluated through the counts-of-neighbors technique, were also considered in previous papers (Bonometto et al. 1993, 1995; Ghigna et al. 1996b), but their comparison with $N$-body simulations did not show a strong discriminatory power. Here we work them out through a different technique (counts-in-cells), which, however, confirms previous results.

In addition to the $n$-point correlation functions and the VPF, we address the $N$-count probability functions $P_n(r)$ and the underdensity function $U_n(r)$, defined as the probability that a randomly placed sphere has a galaxy density below $\Delta$% of the average. The functions $P_n(r)$ ($N \geq 1$) and $U_n(r)$ were never estimated before in samples with depth comparable to PPS.

PPS results are given here in the cosmic microwave background (CMB) reference frame (at variance with Paper G, where the volume-limited subsample was worked out in the Local Group [LG] rest frame) and are therefore more suitable for comparison with our simulations. As a matter of fact, however, there is hardly any difference between the analyses realized in the two frames (Fig. 1: LG, filled circles; CMB, triangles).

Void analyses were also performed on the CfA2 survey (Vogeley, Geller, & Huchra 1991; Vogeley et al. 1994), the SSRS (Maurogordato, Schaeffer, & da Costa 1992; Fig. 1: open circles) and the 1.2 Jy IRAS redshift survey (Bouchet et al. 1993). It is remarkable that these results are globally consistent with the PPS ones, despite the fact that they refer to samples defined in different ways and differently located in the sky.

A crucial point, when testing cosmological models through the VPF statistics, concerns the identification of galaxies. Indeed, a change in the efficiency of galaxy formation in underdense regions has an immediate impact on galaxies. Indeed, a change in the efficiency of galaxy formation through the VPF statistics, concerns the identification of galaxies in the sky.

There are, however, some concerns about whether the linear biasing approach yields the seeds where nonlinear structures later form (e.g., Katz, Quinn, & Gelb 1993). Furthermore, although physically motivated, the Cen & Ostriker results were from CDM simulations performed within a limited dynamical range. For these reasons, we decided to identify galaxies as corresponding to high peaks of the evolved density field. However, even within this choice, different criteria to fragment overmerged structures into individual objects can be proposed (e.g., Gelb & Bertschinger 1994). As a general criterion, galaxy identification should be required to produce the basic observed properties of the galaxy distribution, i.e., their average separation, two-point correlation function, and, possibly, the observed luminosity function. In the next section we will discuss the simple technique used here to identify galaxies, and compare it with the approach adopted in Paper G.

Based on the simulation outputs, we generate artificial samples in redshift space having the same geometry and number of galaxies as the volume-limited sample extracted from PPS. We extract several samples from each simulation box corresponding to different viewpoints, so as to obtain an estimate of the sky variance within a given real-space volume.

2. REAL AND SIMULATED DATA SETS

2.1. Real Data

The PPS database (Giovanelli & Haynes 1989, 1991) is limited to the region bound by $22^h \leq \alpha \leq 3^h 10^m$, $0^\circ \leq \delta \leq 42^\circ 30'$ to avoid areas of high Galactic extinction.

Zwicky magnitudes of all galaxies brighter than $m_{25_\text{w}} = 15.5$ are, however, corrected for extinction, by using the absorption maps of Burstein & Heiles (1978). The resulting sample includes 3395 galaxies and is virtually 100% complete for all morphological types up to $m_{25_\text{w}} = 15.5$. Observed velocities are then corrected by subtracting the component of our velocity relative to the CMB, therefore putting the observer...
at rest in the CMB frame. A volume-limited subsample (VLS) is then extracted, whose limiting magnitude $M_{\text{lim}} = -19 + 5 \log h$ corresponds to a limiting depth of 79 $h^{-1}$ Mpc. This sample contains 902 galaxies with mean galaxy separation $d = 5.5$ $h^{-1}$ Mpc. This sample differs from the one used in Paper G, which was obtained by setting the observer at rest with respect to the centroid of the Local Group. The presence of a large volume, moving coherently with the Local Group, allowed us to include several middle-distance faint galaxies in the old volume-limited sample. Hence the total number of galaxies it contained was 1032, and their average separation was 5.2 $h^{-1}$ Mpc. The decrease of the number of galaxies, with consequent slightly worse statistics, is the price to be paid to have full coherence between observed and simulated data.

2.2. Simulated Samples

We used four different particle-mesh (PM) simulations obtained evolving 256$^3$ cold particles on a 800$^3$ cell grid. More in detail: (1) One realization of C + 2νDM, with an additional 2 $\times$ 256$^3$ hot particles, in a box with side $l = 50$ $h^{-1}$ Mpc ($h = 0.5$), normalized to a COBE quadrupole $Q_{\text{COBE}} = 17$ μK and yielding $\sigma_8 = 0.67$ (Primack et al. 1995). (2 and 3) Two realizations of ΛCDM$_{0.3}$ ($\Lambda$CDM$_1$, and ΛCDM$_{3}$), in a box with side $l = 50$ $h^{-1}$ Mpc ($h = 0.7$), normalized to a COBE quadrupole $Q_{\text{COBE}} = 21.6$ μK and yielding $\sigma_8 = 1.10$. The first one started from the same random numbers as C + 2νDM. (4) A further realization of ΛCDM$_{0.3}$ in a box with side $l = 80$ $h^{-1}$ Mpc ($\Lambda$CDM-80), with the same normalization as above. (The ΛCDM$_{0.3}$ simulations are from Klypin et al. 1996). All these models assume a primordial spectral index $n = 1$.

Let us now discuss the criteria followed to identify galaxies. As in Paper G, galaxies are set in overdensities exceeding a given threshold, but here the simulation output was preliminarily treated in such a way as to provide a direct individuation of overdensity regions. In Paper G we had first found the number $n_i$ of particles in each cell to single out local density maxima. However, single cells are below the resolution allowed by PM codes. So now, the density of each cell has been gauged by considering the sum $\sum_{p=1}^{27} n_i$ of the particles contained in a 3 $\times$ 3 $\times$ 3 cell box centered on each cell. Here the simulation output, in addition to listing coordinates and velocities for DM particles, also gives us directly the density contrast $\delta$ in a 27 cell volume centered on each of them (actually we use a large random subsample of particles with uniform probability, amounting to a 20% fraction of the total).

Therefore, we can simply select a priori a threshold density contrast $\delta_{\text{th}}$, and consider only particles with $\delta \geq \delta_{\text{th}}$. We considered three values: $\delta_{\text{th}} = 100, 150, 400$. They are large enough to ensure that peaks above threshold correspond to virialized structures. The two lowest values are (more and less) conservative estimates of the typical density contrast associated with structures becoming virialized at the present epoch, while the highest value allows us to significantly perturb this basic distribution of objects.

The total numbers of particles above the $\delta_{\text{th}}$ selected are still quite large, as expected (about 5%–10% of the total). Among them we select a small subset at random (751 particles for the box of side $l = 50$ $h^{-1}$ Mpc and 3077 for the box of side $l = 80$ $h^{-1}$ Mpc), in order that the average interparticle separation is $d_{\text{gal}} = 5.5$ $h^{-1}$ Mpc, i.e., the average galaxy separation in the real volume-limited sample.

By construction, the surviving particles are located in regions whose overdensities are above the thresholds selected and the distributions of those particles inside the parent overdensities automatically fit the different density profiles of such regions within the “noise interval” introduced by the randomization process, which anyway should be expected to occur in the real world as well.

In principle, passing from $\sim 10^{-1}$ of dark matter particles down to $\sim 10^{-3}$ could introduce a bias, namely, when small overdensity regions are considered. The volume-limited sample extracted from PPS contains galaxies with luminosity exceeding $L_{\text{a}} \simeq 10^{10}$ $h^{-2}$ $L_{\odot}$. Accordingly, overdensity regions whose mass is $\sim 10^{12}$ $h^{-1}$ $M_{\odot}$, and that typically yield one or two galaxies, can be casually included or excluded from the artificial samples. This point is potentially delicate, especially for measures like VPF, whose output could be affected by the inclusion or exclusion of a few isolated galaxies.

We addressed much care to this point, by building artificial samples from different random choices and comparing the outputs of our statistical measures for them, although most results reported in this paper, for the sake of homogeneity, come from a single realization. In the next section we will debate this point further. We only anticipate here that the effect of changing the random subset of particles is always quite modest, apart from a few cases whose anomaly is apparent. Moreover, the scatter induced by such an effect is smaller than that associated with the change of the observer setting within a given realization. The results reported were, however, checked and found to be typical, by comparison of 10 different realizations.

As a further check of the robustness of the results based on the above galaxy identification method, we implemented in the C + 2νDM simulation two further prescriptions, both starting from the identification of DM halos and, therefore, free of this possible source of bias. Such prescriptions were also meant to approach the procedure followed in Paper G, in spite of the different characteristics of the simulations used here. The method of this countercheck is described in the next paragraph, and the results are reported in the next section (cf. Fig. 7); they confirm the validity of our standard procedure.

As a starting point to identify halos, we select local density maxima on the grid, whose overdensity is greater than 200. Afterward, we center a sphere on this point, with radius equal to that at which the overdensity drops to 200. The center of mass of the cold particles falling within the sphere is then computed and used as the starting point for the next iteration. We always find that this procedure converges after a few iterations. At the end, the mass of the halo is defined as the sum of the masses of all the member DM particles. The resulting sample of DM halos is then used to identify galaxies. The two prescriptions correspond then to two extreme cases:

1. No fragmentation.—$N_{\text{gal}} = (l/d_{\text{gal}})^3 = 750$ galaxies are identified as the $N_{\text{gal}}$ most massive halos. Each halo is then identified with a single galaxy. The resulting halo mass threshold is $M_{\text{th}} \simeq 1.5 \times 10^{11}$ $h^{-1}$ $M_{\odot}$.

2. Fragmentation.—In order to break up halos, we follow the same simple prescription described by Bonometto et al. (1995). After a halo mass threshold is chosen, the number $N_i$ of galaxies belonging to the ith halo of mass $M_i$ is assumed to be $N_i = [M_i/M_{\text{th}}]$, where $[x]$ denotes the largest integer
that does not exceed \( x \). Therefore, the resulting mass threshold, \( M_{\text{th}} \approx 2.4 \times 10^{12} h^{-1} M_\odot \), is fixed by requiring that the total number of galaxies matches \( N_{\text{gal}} \). Fragments are assigned random positions within the radius of the parent halo, and velocities drawn from a Gaussian distribution having mean equal to the halo peculiar velocity and dispersion equal to the rms velocity of the member cold particles.

As already outlined and discussed in Paper G, these two prescriptions represent extreme cases within a class of fragmentation methods not relying on local antibiasing. Therefore, although we do not attach to them any strong physical motivation, they can be reliably used for bracketing results based on more refined approaches.

At variance with Paper G, both the standard procedure used in the present work and the two latter prescriptions do not have recourse to the galaxy luminosity function. In the present work and the two latter prescriptions do not have recourse to the galaxy luminosity function. Therefore, although we do not attach to them any strong physical motivation, they can be reliably used for bracketing results based on more refined approaches.

At variance with Paper G, both the standard procedure used in the present work and the two latter prescriptions do not have recourse to the galaxy luminosity function. In the simplest way, this would require one additional parameter, the mass-to-light ratio \( M/L \) of overdensity regions, which cannot be easily related to the physical \( M/L \) of well-defined objects and generally would depend on the resolutions of the simulations (see also Ghigna et al. 1996b). The outputs are, however, strictly analogous. In conclusion, what we work out are galaxies, located in overdensity regions, with suitable individual velocities, which are essential to set them in redshift space. Overdensities were verified to be essentially in virial equilibrium. Henceforth, the velocity distribution for each region above threshold is quite similar to the one considered in Paper G, where each cell above threshold was given a total galaxy mass proportional to \( \Sigma_{n=1}^{N} n_g \) and virial equilibrium was explicitly imposed to obtain individual galaxy velocities. As a final consideration, let us notice that these procedures, as well as the one adopted in Paper G, do not leave room for any form of velocity bias. It is, however, important to notice that, in turn, we were able to keep the number of parameters fixing the distribution down to one.

### 2.3. Data-Simulation Comparison

The comparison between real and simulated data is performed in redshift space, by extracting from the periodic simulation box (with replication) a volume with the same location of the observer and the direction of the axis of the volume observed. However, for each random setting, we first verified that the galaxy density in the artificial PPS sample differed by less than 2\% from the expected one \((= 902/V_{\text{VLS}})\). In this way five different observer settings were selected for each case. As we shall see below, the scatter among observers, which is a measure of the sky variance, is always small and approximately of the same order as bootstrap errors.

### 3. STATISTICAL ANALYSES

We estimate the statistical distribution of galaxies in each sample through the counts-in-cells technique. We work out the probabilities \( P_N \) that a randomly placed cell contains \( N \) galaxies. From this we compute the moments of counts and obtain the volume-averaged correlation functions \( \xi_n \), after subtracting shot-noise contributions (see Bonometto et al. 1995 for more details). As in Paper G, we use spherical cells completely contained in the sample boundaries whose radii \( R \) are in the range 1–13 \( h^{-1} \) Mpc, and at each \( R \) we take \( N_R = 2V_{\text{VLS}}/V_R \) spheres distributed in the sample volume. Here \( V_{\text{VLS}} \approx 1.5 \times 10^7 h^{-3} \) Mpc\(^3\) is the volume of the sample, and \( V_R = 4\pi R^3/3 \). As suggested by Fry & Gaztañaga (1994), \( N_R \) should give a sensible estimate of the number of independent cells that can be allocated in the volume \( V_{\text{VLS}} \) in the presence of clustering (therefore the factor of 2). This argument works well, at least at relatively large scales for which \( \xi_4(R) \approx 1 \). At smaller \( R \), underestimating \( N_R \) can in principle make the outcome of a measure excessively dependent on the set of \( N_R \) spheres chosen. We verified that this is not the case, by analyzing the PPS sample for 20 different realizations of the positions of the spheres. The small shifts occurring at the smallest radii are anyway accounted for by our estimates of errors, which we obtain through the bootstrap resampling technique (e.g., Ling, Frenk, & Barrow 1986). We consider up to 50 resamplings, even though we find rapid convergence and a value of 20 would already provide satisfactory estimates. In the following figures, for reasons of clarity, bootstrap errors will be reported only for the observational data, but they also affect the results on simulated data, with similar magnitudes. (For a careful analysis of the uncertainties in counts-in-cells statistics see Colombi, Bouchet, & Schaeffer 1994, 1995 and Szapudi & Colombi 1996.)

For each galaxy sample, we worked out \( \xi_n(R) \) for \( n = 2, 3 \), and 4, i.e., variance, skewness, and kurtosis, respectively. As far as \( \xi_2 \) and \( \xi_4 \) are concerned, we will refer to the reduced cumulants \( S_k \equiv \xi_k/\xi_2^k \) and \( S_4 \equiv \xi_4/\xi_2^2 \).

As for the \( P_N \), we examined them up to \( N = 5 \) and \( U_\epsilon \), for \( \epsilon \) in the range 30\%–70\%, but for \( N \geq 2 \) and \( \epsilon > 30\% \) the discriminatory power is virtually absent. Values of \( \epsilon \) less than 30\% are hardly distinguishable from \( P_0 \) over most of the range of scales considered. For these reasons, we will report results only for \( P_0, P_1, \) and \( U_{30} \).

As mentioned in the Introduction, an important point concerning the general significance of our analysis is whether the PPS catalog provides us with a fair sample of the universe. Although we cannot give an answer to this question, we can at least check its reliability against similar data available in the literature for other galaxy surveys.

In Figure 1 we compare the results of our PPS analysis on \( \xi_4(R) \) and \( \text{VPF} \) both in the CMB (triangles) and in the LG (filled circles) frame with that by Vogele et al. (1994) for a volume-limited subsample of the CfA2 survey, having the same limiting magnitude of the PPS VLS that we consider (open circles; the average between northern and southern CfA2 samples is plotted here). In order to be consistent with the analysis by Vogele et al., their results must be compared with those of PPS in the LG frame. Therefore, only for the purpose of this comparison, we resort to the same version of the PPS sample as that considered in Paper G. In any case, it turns out that results are very weakly dependent on the frame in which redshifts are measured. Error bars for PPS correspond to 3 \( \sigma \) bootstrap uncertainties. It is remarkable how close the results for the two surveys are over the whole explored scale range, thus indi-
cating that PPS and CfA2 are essentially equivalent for the statistical analyses we are considering here.

In Figure 2 we plot $\xi_2(R)$ for the four simulations considered at the three overdensity thresholds, $\delta_{th} = 100, 150,$ and 400 (note that the finite volume of the simulation affects scales $\gtrsim 10 \, h^{-1} \, \text{Mpc}$). The curves are obtained by averaging over five observer locations. The points (filled circles) refer to the PPS sample, and their error bars are $3 \sigma$ from 20 bootstrap resamplings of PPS data. The $\Lambda CDM_{0.3}$ model clearly provides a good fit to the observational data, especially for $\delta_{th} = 150$, and an even better fit could be obtained by setting $\delta_{th} = 180$. Let us recall that this is roughly the density contrast expected for a virialized system in the approximation of spherical collapse. On the contrary, the artificial galaxy samples that we extract from the simulations do not reproduce PPS data. Both the amplitude and the slope of $\xi_2$ are not satisfactory. Moreover, the dependence on $\delta_{th}$ seems weaker here than for $C + 2\nu DM$. The effect of sky variance can be seen in Figure 3, which shows again how hard it is to find an observer setting in $\Lambda CDM_{0.3}$ whose sky has a $\xi_2$ consistent with the PPS one. These difficulties of $\Lambda CDM_{0.3}$ were, however, already known (Klypin et al. 1996).

The dependence of $S_3$ on $R$ is shown in Figure 4, where, as usual, error bars are $3 \sigma$ for PPS data and curves refer to simulations. The figure reveals a fair agreement of models with observational data and predictions from the hierarchical scaling model (HS; see, e.g., Bonometto et al. 1995 and references therein), which requires a constant $S_3$. In all cases a satisfactory fit is obtained with $S_3 \approx 2.2$ (values are slightly higher for the $\Lambda CDM_{0.3}$ simulations than for $C + 2\nu DM$, but the difference is within the error bars). For $R > 6 \, h^{-1} \, \text{Mpc}$ the values of $S_3$ decrease, rather abruptly for $C + 2\nu DM$ and $\Lambda CDM_{1}$, and gently in the other cases. The significance of this trend is questionable anyway in view of the large uncertainties at these (relatively) large scales where the number of sampling spheres is small and there may be effects due to the finite size of the simulation box. Let us also recall that $C + 2\nu DM$ and $\Lambda CDM_{1}$ have the same initial random numbers. Also, $S_4$, though rather noisy, is compatible with HS by allowing a fit with a constant value in the range 6–7. This rather good agreement of

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**Figure 2.** Variance $\xi_2$ vs. scale $R$ for the set of $800^3$ mesh simulations (continuous curves; curves correspond to three values of the overdensity threshold $\delta_{th}$ as shown in the first panel) and for the PPS sample (filled circles; error bars are $3 \sigma$ bootstrap errors). For each simulation and each $\delta_{th}$, the curves are averages over five artificial samples differing in the observer location. Of the three $\Lambda CDM_{0.3}$ simulations, $\Lambda CDM_{1}$ has the same initial random numbers as $C + 2\nu DM$ and a $50 \, h^{-1} \, \text{Mpc}$ box; $\Lambda CDM_{2}$ has the same box size but an independent set of random numbers, while $\Lambda CDM-80$ has a $80 \, h^{-1} \, \text{Mpc}$ box.
"galaxies" in the simulations with redshift survey results and HS confirms and extends the results of Bonometto et al. (1995) and, in turn, can be taken as an indication that our galaxy identification procedure is a sensible one.

It should be mentioned that the values of the reduced cumulants, $S_n = \frac{\bar{\xi}_n}{\xi_2^{n-1}}$, obtained from angular samples exceed those obtained from redshift surveys by a factor of ~3 (Fry & Gaztanaga 1994; Gaztanaga 1994; see also Peebles 1980). The origin of this discrepancy is still unclear. Since the galaxies included in angular catalogs span much larger volumes of space than redshift surveys, it could be ascribed to sampling effects, i.e., that our local neighborhood is not a fair sample or finite statistics effects (Colombi et al. 1994, 1995; Szapudi & Colombi 1996). Indeed, the last authors point out that the volumes of current redshift surveys and the number of galaxies they contain appear to be too small for a meaningful estimate of the n-point functions. On the other hand, in our previous analysis of CDM and CHDM $N$-body simulations (Bonometto et al. 1995), we found that the $S_n$ values are decreasing functions of the halo mass cutoff for galaxy identification. Therefore, since projected samples include fainter galaxies, which could be less biased tracers of the density field, this can partly account for the discrepancy between angular and redshift-space analyses. In any case, if observational data and simulations are compared on strictly similar grounds, as we do, finite statistics should affect results for real and artificial galaxies by the same amount. However, it is worth stressing here that the limits of our analysis should not be forgotten, especially when comparing our results with data from large angular samples.

Figures 5 and 6 give the results for the void probability function $P_0(R)$ for simulated and PPS data. Error bars for observational data are again $3\sigma$, and the solid line represents the expected behavior for a Poisson sample with the same number of objects as the real one. In Figure 5 the VPFs for different $\delta_{th}$ are plotted. In Figure 6, which also shows the effect of the sky variance, the expected contribution to $P_0$ coming from Poisson noise is subtracted, and the resulting difference is divided by the volume $V(R) = (4\pi/3)R^3$. Plotting $(P_0 - P_{0,\text{Poisson}})/V$ magnifies the detailed behavior of the VPF at small $R$.

From Figures 5 and 6, it is clear that the $C+2\nu DM$ model agrees with PPS data at all scales, independently of
the choice of \( \delta_0 \). In contrast, as before for \( \xi_3 \), it is difficult for \( \Lambda CDM_{0.3} \) to yield an observer setting whose sky is also marginally consistent with PPS data, also when the overdensity threshold is pushed down to its lowest value \( \delta_{th} = 100 \).

Figure 7 shows the results obtained for \( C + 2\nu DM \) from artificial galaxy samples built starting from halo identification. The 2-point function and VPF are shown for the two galaxy identification methods described in the previous section, applied to \( C + 2\nu DM \) halos. Note that the effect of halo fragmentation is rather limited, especially for \( P_0(R) \). This is essentially due to the presence of two effects, which act in opposite directions in determining the strength of the "galaxy" clustering. On the one hand, breaking up halos increases the mass threshold. Therefore, galaxies are identified to correspond to higher peaks of the DM density field, which in turn leads to an increase of their clustering. On the other hand, since fragments generated by the same halo are assigned different peculiar velocities, redshift-space distortions cause a suppression of the clustering. The resulting stability of \( P_0(R) \) results can be also appreciated by comparing them with Figure 5. This confirms that VPF results are connected with DM composition or model, while the method of galaxy identification, within the class we considered here, which is based on local and positive biasing, has only a modest relevance.

In Figure 8 we report the behavior of \( P_1(R) \) for data and models. Observational bootstrap errors are fairly wide here, especially at large \( R \). In spite of that, at \( R < 4 \, h^{-1} \) Mpc, \( \Lambda CDM_{0.3} \) samples miss PPS data, while \( C + 2\nu DM \) is once more in good agreement with them. Similar considerations hold for the underdensity probability function \( U_v \), which is illustrated in Figure 9 for a 30% underdensity threshold. Notice that, because of the pointlike nature of the distribution, \( U_v \) carries new information with respect to \( P_0 \) when \( R \) approaches the average interparticle separation, precisely when \( R \geq \frac{[300/(4\pi e)]^{1/3} d_{\text{gal}}}{h} = 3.42(e/100)^{-1/3} \) h\(^{-1}\) Mpc. This is the reason why we plot results on \( U_{30} \) only for \( R \geq 4.5 \, h^{-1} \) Mpc.

Finally, Figure 10 is aimed at illustrating the effects of changing the sampling of galaxies in overdensities (we take \( \delta_0 = 150 \) in all panels). Here we report the results for \( P_0 \) and \( P_1 \), whose measures are potentially most sensitive to the sampling choice. In each panel the dashed curve shows the scatter between different observer settings in the "usual" realization, i.e., the one used to draw the previous
Fig. 5.—Void probability function $P_0$ vs. $R$ for simulated samples and PPS. Symbols are the same as in Fig. 2. In each panel, the solid curve is what is expected for a Poissonian distribution of points with average separation $d_{\text{gal}}$.

We also plot $1\sigma$ error bars that we obtain from such results. Superimposed on them, solid lines give the results for two typical observer settings “observing” a different subset of particles, thus showing the limited effects of changing realization. In fact, the difference between the averages over five observers in two different realizations is smaller than the difference among observer settings in a single realization by a factor of $\sim 3-10$.

As a general remark, it can be said that cosmic variance does not appear to play an important role (an idea of its effect can be obtained by comparing $\Lambda\text{CDM}_1$ and $\Lambda\text{CDM}_2$). In contrast, comparing $\Lambda\text{CDM}-80$ with its smaller box companions, there are nonnegligible differences. Since effects of finite box size are expected to play a role on large scales ($\gtrsim 10\, h^{-1}\, \text{Mpc}$), differences on scales of a few Mpc are unlikely to be directly related to the size of the simulation box. For instance, Kauffmann & Melott (1993) pointed out that the scaling of the VPF starts feeling the box limits at about $L/4$. Therefore, any difference on scales $\approx 2-5\, h^{-1}\, \text{Mpc}$, where we are mostly able to discriminate between models, seems more likely to be an effect of different resolutions: in $\Lambda\text{CDM}-80$ the linear cell size is a factor of 1.6 larger, and the mass of each DM particle is increased by a factor of $1.6^3 \approx 4.1$.

4. CONCLUSIONS

In this work we tested the statistical properties of artificial galaxy samples extracted from high-resolution simulations of $\mathcal{C} + 2\nu\text{DM}$ with $\Omega_0 = 1$, $\Omega_r = 0.2$, $h = 0.5$ and $\Lambda\text{CDM}$ with $\Omega_0 = 0.3$, $\Omega_\Lambda = 0.7$, $h = 0.7$ ($\Lambda\text{CDM}_{0.3}$) against a similar volume-limited sample of the PPS. Artificial galaxies reside in overdensity regions of the evolved density field whose density contrasts are above a suitable threshold $\delta_{\text{th}}$. We showed that, while the reduced skewness $S_3$ yields almost identical results for the two models (and so does kurtosis $S_4$, but with larger uncertainties), variance, $P_0$, and $P_1$ are able to discriminate efficiently between them (also $U_s$, though marginally). In particular, the $\mathcal{C} + 2\nu\text{DM}$ model agrees with our observational data, while it is quite difficult to find an observer setting from which $\Lambda\text{CDM}_{0.3}$ is consistent with PPS data. The latter results confirm the analysis of Klypin et al. (1996) and show that the excessive small-scale clustering of $\Lambda\text{CDM}_{0.3}$ is apparent in redshift space as well and makes this model hardly viable, at least as
long as galaxies follow DM overdensities. In contrast, the analysis of $S_3$ (and $S_4$) does not distinguish between the models, as said before, and agrees with PPS data and HS predictions with constant values of $\approx 2.2$ (and 6–7). This extends the results of Bonometto et al. (1995), which also found good agreement of "galaxies" with HS in CDM and CHDM N-body simulations at variance with DM particles.

The values of $S_3$ and $S_4$ that we found agree with those derived from other redshift surveys, which, as is known, are markedly smaller than those obtained from angular samples (see, e.g., Fry & Gaztanaga 1994). As already stressed before, to address the origin of this discrepancy is beyond the scope of this paper. However, we would like to notice here that the remarkable stability of our results seems to indicate that sampling effects do not play an important role. If redshift distortions, projection effects, and the mixing of galaxies of largely different luminosities do not contribute either (see, e.g., Fry & Gaztañaga 1994 and Gaztanaga 1994, who, however, used the shallow CfA1 sample), the reason could very likely be finite statistics effects, which indeed tend to decrease the estimates of the hierarchical coefficients (Colombi et al. 1994; see also Szapudi & Colombi 1996). This should not be a cause of concern for our analysis, since we compare observational data and simulations through "galaxy" samples of equal geometry, volume, and interparticle separation. Even smaller effects are expected for the VPF, which has been found to be less sensitive to finite-volume effects (Colombi et al. 1995).

Our analysis shows that $\Lambda$CDM$_{0.3}$ tends to overproduce low-density regions. This is shown both by $P_0$ and by $U_{30}$. Also, the probability of finding a single galaxy in volumes smaller than $\sim 3–4 \, h^{-1}$ Mpc is smaller than in the PPS data. These inconsistencies are more or less relevant in

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**Fig. 6.** Here the VPF is plotted after subtracting the Poissonian $P_0$, and then dividing by the volume $V(R)$ of a sphere of radius $R$, which allows us to magnify the small-scale behavior. Each plot also shows $P_d(R)$ for five typical different settings in the simulations (dotted lines) and gives an indication of the sky variance. We have chosen the $\delta_0$ values for which the $P_0$ values of the models best approach the observational curve. In the top left panel, the heavy "T" at the bottom sets the boundary of the region where the signal is indistinguishable from Poissonian. They are obtained from the 3 $\sigma$ scatters among measures for 50 different realizations of the Poissonian distribution in the same volume as our samples.
Fig. 7.—$\xi_{c}(R)$ and $P_{0}(R)$ from artificial samples based on halo identification. Dashed and dotted lines refer to no fragmentation and full fragmentation of the DM halos, respectively, and are obtained as averages over 10 realizations of artificial samples. For the sake of comparison, the result for PPS (filled circles) is also given.

Fig. 8.—Results for $P_{1}(R)$, the probability of counting a galaxy in a sphere of radius $R$. Symbols as in Fig. 2.
Fig. 9.—Results for $U_{30}$, the probability that the number density $n$ in a sphere of radius $R$ is less than 30% of the average $n$

various realizations, and depend on the threshold selected, but are present everywhere. It seems clear that the galaxy number distribution in random spheres is significantly different in $\Lambda$CDM$_{0.3}$ and in the real world. However, let us add a word of caution about our conclusions in view of the limitations of our analysis, especially those related to the uncertainties on how galaxies actually form and on the way their real distribution relates to that of DM particles.

As a concluding remark, it is worth pointing out what we have learned here about the ultimate goal of picking up the “final” cosmological model. As for the $\Lambda$CDM models, the one we considered here appears to have serious troubles in reproducing the galaxy clustering below $10 h^{-1}$ Mpc. It is, however, clear that, by suitably changing the model parameters, one may get substantial improvements (we reserve to a forthcoming paper the study of larger simulations of a larger suite of models). As for the class of CHDM models, while the model with one massive neutrino providing $\Omega_X = 0.3$ fails to pass the VPF test (Ghigna et al. 1994), $C+2\nu$DM with $\Omega_X = 0.2$ is in good agreement with all data considered here. Therefore, having one single massive neutrino flavor with $m_\nu = 7$ eV instead of two massive neutrino flavors with $m_\nu = 2.3$ eV seems completely sufficient to alter the void distribution in a detectable way. The remarkable performance of the $C+2\nu$DM model in this small-scale redshift-space analysis adds to previous favorable results from numerical and linear theory calculations (see Primack et al. 1995; Primack 1996). This makes it a good candidate to interpret the large-scale structure of the universe.

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FIG. 10.—Effects of changing galaxy sampling in overdensities, for $P_0$ and $P_1$. Dashed curves are results for different settings in the realization used for the previous figures; 1σ error bars are worked out from their variance. Continuous lines give results for two observer settings within a different realization. Results for two models with $\delta_n = 150$ only are plotted.

REFERENCES

Betancort-Rijo, J. 1990, MNRAS, 246, 608
Bonometto, S. A., Borgani, S., Ghigna, S., Klypin, A., & Primack, J. P. 1995, MNRAS, 273, 101
Bonometto, S. A., Iovino, A., Guzzo, L., Giovaneli, R., & Haynes, M. 1993, ApJ, 419, 451
Borgani, S., et al. 1996, New Astron., in press
Bouchet, F. R., Strauss, M. A., Davis, M., Fisher, K. B., Yahil, A., & Huchra, J. P. 1993, ApJ, 417, 36
Burstein, D., & Heiles, C. 1978, ApJ, 225, 40
Cen, R., & Ostriker, J. P. 1993, ApJ, 417, 415
Colombi, S., Bouchet, F. R., & Schaeffer, R. 1994, A&A, 281, 301
Davies, R., Elfstathiou, G., Frenk, C. S., & White, S. D. M. 1985, ApJ, 292, 371
Einasto, J., Einasto, M., Gramann, M., & Saar, E. 1991, MNRAS, 248, 593
Eke, V. R., Cole, S., & Frenk, C. S. 1996, MNRAS, in press
Fry, J. N., & Gaztañaga, E. 1994, ApJ, 425, 1
Fry, J. N., Giovanelli, R., Haynes, M. P., Melott, A. L., & Scherrera, R. J. 1989, ApJ, 340, 11
Gaztañaga, E. 1994, MNRAS, 268, 913
Gelb, J. M., & Bertshinger, E. 1994, ApJ, 436, 467
Ghigna, S., Bonometto, S. A., Guzzo, L., Giovanelli, R., Haynes, M. P., Klypin, A., & Primack, J. R. 1996a, ApJ, 463, 395
Ghigna, S., Bonometto, S. A., Retzlaff, J., Gottloeber, S., & Murante, G. 1996b, ApJ, 469, 40
Ghigna, S., Borgani, S., Bonometto, S. A., Guzzo, L., Klypin, A., Primack, J. R., Giovaneli, R., & Haynes, M. P. 1994, ApJ, 437, L71 (Paper G)
Giovanelli, R., & Haynes, M. P. 1989, AJ, 97, 633
Gottlöber, S., Müller, V., & Starobinski, A. A. 1994, ApJ, 434, 417
Katz, N., Quinn, T., & Gelb, J. M. 1993, 265, 689
Kaufmann, G., & Melott, A. L. 1993, ApJ, 393, 415
Klypin, A., Primack, J. R., & Holtzman, J. 1996, ApJ, 466, 13
Ling, E. N., Frenk, C. S., & Barrow, J. D. 1986, MNRAS, 223, 21P
Little, B., & Weinberg, D. H. 1994, MNRAS, 267, 605
Maurogordato, S., Schaeffer, R., & da Costa, L. N. 1992, ApJ, 434, 745
Peacock, J. A., & Dodds, S. J. 1994, MNRAS, 267, 1020
Peebles, P. J. E. 1980, The Large-Scale Structure of the Universe (Princeton: Princeton Univ. Press)
Primack, J. R. 1996, in Critical Dialogues in Cosmology, ed. N. Turok (Singapore: World Scientific), in press
Primack, J. R., Holtzman, J., Klypin, A., & Caldwell, D. O. 1995, Phys. Rev. Lett., 74, 2160
Szapudi, I., & Colomb, S. 1996, ApJ, 470, 131
Vogeley, M. S., Geller, M. J., & Huchra, J. P. 1991, ApJ, 382, 44
Vogeley, M. S., Geller, M. J., Park, C., & Huchra, J. P. 1994, AJ, 108, 745
Weinberg, D. H., & Cole, S. 1992, MNRAS, 259, 652