Third Quantization and Black Holes

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ABSTRACT

Recent results on the quantum mechanics of black holes indicate that the space-time singularity can be avoided if the space of fields is extended in its domain as well as the vanishing of beta functions required as an equation in target space. Such an approach seems to inevitably lead to the quantization in this space that is the so-called third quantization. We discuss some of the implications of third quantization for the physics of blackholes in two dimensions. By discretizing the transverse dimensions describing the horizon it may also be possible to describe regulated four dimensional gravity in this manner and perhaps shed some light on the meaning of black hole entropy.

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An interesting proposal for recovering the information lost in blackhole evaporation involves the nucleation of a baby universe\cite{1}. In this picture the quantization of gravity permits topology change to take place during gravitational collapse and the new universe created carries away all the information about the initial quantum state (For a nice discussion of this and other issues involving the information loss paradox see \cite{2}). Also systems such as the axionic blackhole\cite{3} which are problematic in the sense that the axionic quantum hair would be present after blackhole evaporation also admit a wormhole solution which can carry off the axionic quantum number\cite{4}. In any case the non-renormalizability of gravity inhibits the inclusion of such processes in usual four dimensional general relativity. Clearly it would useful to study a simplified model where the impact of topology change on blackhole dynamics can be adequately discussed.

Recently much attention has been focused on two dimensional dilaton gravity as it is a renormalizable theory containing black hole solutions. However results of these studies indicate that a semiclassical approximation breaks down just before a naked singularity appears\cite{5,6}. Thus in order to address questions about the last stages of Hawking evaporation as well as to incorporate the effects of topology change on black hole radiance it is necessary to go beyond the semiclassical approximation and quantize the 2d dilaton gravity system. A useful approach to quantization was taken by\cite{7} who chose the conformal gauge and obtained as consistency conditions the vanishing of the beta functions in target space\cite{8,9,10}. Such an approach seems to inevitably lead to the quantization in this space that is the so-called third quantization. Indeed a closely related model is the $c = 1$ matrix model and there the target space was indeed quantized with the ground state of the system being promoted to the tachyon field\cite{11}.

In this paper we discuss the 2d dilaton gravity arising from the reduction of 3+1 dimensions to 1+1 dimensions upon the assumption of spherical symmetry. This leads to a description of the s-wave sector of general relativity as well as blackhole solutions with the usual values of blackhole temperature and entropy. Further description of the beta function technique applied to this and other reductions of
general relativity to 1+1 dimensions as well as additional references are contained in\cite{12}.

We start in four dimensions and make a spherical symmetric ansatz for the metric which gives

\[
[g_{\mu\nu}] = \begin{pmatrix}
g_{00} & g_{01} & 0 & 0 \\
g_{10} & g_{11} & 0 & 0 \\
0 & 0 & L^2 e^{-2\phi} & 0 \\
0 & 0 & 0 & L^2 e^{-2\phi} \sin^2 \theta
\end{pmatrix}
\]

The Einstein Hilbert action then takes the form

\[
S = \frac{L^2}{4G} \int d^2x \sqrt{-g} e^{-2\phi} \left( R + 2(\nabla \phi)^2 + 2L^{-2}e^{2\phi} \right).
\]

A useful gauge choice for models of this type is the conformal gauge \( g_{ab} = \hat{e}^{2\rho} \hat{g}_{ab} \). In this gauge the action becomes:

\[
S = \frac{L^2}{4G} \int d^2x \sqrt{-\hat{g}} e^{-2\phi} \left( 2\hat{g}^{ab} \partial_a \phi \partial_b \phi - 4\hat{g}^{ab} \partial_a \phi \partial_b \rho + \hat{R} + 2L^{-2}e^{2\phi+2\rho} \right).
\]

This form of the action can be easily compared with the sigma model action

\[
S = -\frac{1}{4\pi \alpha'} \int d^2x \sqrt{-\hat{g}} \left( \hat{g}^{ab} \partial_a X^I \partial_b X^J G_{IJ}(X) + \alpha' \hat{R} \Phi(X) + T(X) \right).
\]

with \( G_{IJ}(X), \Phi(X) \) and \( T(X) \) given by

\[
G_{IJ}(X) dX^I dX^J = \frac{\pi L^2 \alpha'}{G} (-2d\phi d\phi + 4d\phi d\rho) e^{-2\phi}
\]

\[
\Phi(X) = -\frac{\pi L^2}{G} e^{-2\phi}
\]

\[
T(X) = -\frac{2\pi \alpha'}{G} e^{2\rho}
\]

The metric is indeed flat as can be seen by the coordinate choice \( X^0 = \frac{\ell}{2}(e^{-2\phi} + 2\rho - \phi) \) and \( X^1 = \frac{\ell}{2}(e^{-2\phi} - 2\rho + \phi) \) where \( \ell = \sqrt{\alpha' \pi \frac{L}{\sqrt{G}}} \). In these variables the the
backgrounds take the form:

\[ G_{IJ}(X)dX^IdX^J = dX^+dX^- \]

\[ \Phi(X) = -\frac{\ell}{\alpha'}X^+ \]

\[ T(X) = -\frac{2\ell^2}{L^2}\sqrt{\frac{\ell}{X^+}}e^{-\frac{X^-}{2}} \] (2)

The theory must contain the symmetry \( \hat{g}_{ab} \rightarrow e^{2\omega}\hat{g}_{ab} \) while \( \rho \rightarrow \rho - \omega \). At the quantum level this amounts to the vanishing of the beta functions in target space. These beta functions are given by\(^{[13,14]}\)

\[ \beta^G_{IJ} = R_{IJ} + 2\nabla_I\nabla_J\Phi - \frac{\sigma}{2}\partial_I\partial_JT \]

\[ \beta^\Phi = -\frac{16}{\alpha'} + 4(\nabla\Phi)^2 - 4\nabla^2\Phi - R - \frac{2\sigma}{\alpha'}T^2 + \frac{\sigma}{2}(\nabla T)^2 \]

\[ \beta^T = \nabla^2 T - 2G^{IJ}\partial_I\partial_J\Phi + \frac{4}{\alpha'}T \]

The coefficient \( \sigma \) was determined by Das and Sathiapalan\(^{[13]}\) to be \( \sigma = \frac{1}{2}\alpha'^4 \) where \( a \) is a lattice spacing on the world sheet used as a regulator. The factor \( \sigma \) can be absorbed into the definition of \( T \). These beta functions do not vanish for the background given by (1). If one shifts the dilaton field to \( \Phi(X) = -\frac{2}{\sqrt{\alpha'}}X^1 \) and fixes \( G = 2L^2 = 2\alpha' \) and the beta functions do vanish. So we arrive at the following action describing 3+1 to 1+1 dimensional reduction on \( S^2 \).

\[ S = \int d^2x \sqrt{-\hat{g}}\left( \frac{1}{4}e^{-2\phi}(\hat{g}^{ab}\partial_a\phi\partial_b\phi-2\hat{g}^{ab}\partial_a\phi\partial_b\rho)+R\frac{1}{4}\sqrt{\frac{1}{2\pi}}(e^{-2\phi+\phi-2\rho}) + \frac{1}{2G}e^{2\rho} \right) \] (3)

For a general sigma model the \( \hat{T}_{00} \) and \( \hat{T}_{01} \) components of the stress energy
tensor are given by

\[ 4\pi\alpha' \hat{T}_{00} = G_{IJ}(\dot{X}^I \dot{X}^J + X'^I X'^J) - \alpha'^2 \Phi'' + T \]

\[ 4\pi\alpha' \hat{T}_{01} = G_{IJ}(\dot{X}^I X'^J + X'^I \dot{X}^J) - \alpha'^2 \dot{\Phi} \]

In terms of the action (3) these constraints become

\[ \hat{T}_{00} = \frac{L^2}{2G}(\dot{\phi}^2 - \phi'^2 + 2\dot{\rho} \dot{\phi} + 2\phi' \dot{\rho})e^{-2\phi} + \frac{1}{2} \sqrt{\frac{1}{2\pi}}(e^{-2\phi} + \phi - 2\rho)'' - \frac{1}{2G}e^{2\rho} \]

\[ \hat{T}_{01} = \frac{L^2}{G}(\dot{\phi}\phi' + \phi' \dot{\rho} + \phi' \dot{\rho})e^{-2\phi} + \frac{1}{2} \sqrt{\frac{1}{2\pi}}(e^{-2\phi} + \phi - 2\rho)' \]

In the back hole interior \( r < 2MG \) the component \( g_{tt} \) becomes positive and \( \partial_t \) becomes a spacelike killing vector of the schwarzschild solution\(^{[15]}\). The minisuperspace approximation takes the independence of \( t \) as a property of the quantum solution and the \( \hat{T}_{00} \) becomes (where dot refers to the derivative with respect to \( r^* \) satisfying \( \frac{dr}{dr^*} = \frac{2MG}{r} - 1 \), which is a timelike coordinate in the blackhole interior):

\[ \frac{1}{4}(-q^2 + \dot{\rho}^2)e^{2(q-\rho)} - \frac{1}{2G}e^{2\rho} = 0 \]

Where \( q = \rho - \phi \). In terms of canonical momentum \( \pi_q = -\frac{\dot{q}}{2}e^{2(q-\rho)}v \) \( \pi_\rho = \frac{\dot{\rho}}{2}e^{2(q-\rho)}v \)

and taken as a quantum equation we obtain the Wheeler-DeWitt equation

\[ (\pi_q^2 - \pi_\rho^2 + \frac{v^2}{2G}e^{2q})\psi(q, \rho) = 0 \quad (4) \]

This Wheeler-DeWitt equation has two basic solutions

\[ \psi_1(x, \rho) = e^{ik\rho}K_{i|k|}(\sqrt{\frac{x}{2G}}) \]

\[ \psi_2(x, \rho) = e^{ik\rho}I_{i|k|}(\sqrt{\frac{x}{2G}}) \]

where \( x = \frac{2v}{\sqrt{2G}}Le^q \) and \( K_\nu(z), I_\nu(z) \) are modified Bessel functions. The former decays exponentially for large \( x \) and is real. It yields a superposition of plane
waves for small $x$. The latter exponentially grows for large $x$ and is complex. It yields a single plane wave for small $x$.

The universal Green function is the path integral from one spacelike slice to another, and has dependence on the initial and final values of the two dimensional fields.

$$
G(x', \rho'; x, \rho) = \int_{q=\log x'/v, \rho=\rho'} DqD\rho e^{\frac{i}{2}(-q^2+\rho^2)e^{2(\rho-\rho')}+\frac{1}{2\pi}e^{2\rho}}
$$

It can be evaluated by the method of chitre and Hartle and yields the expression

$$
G(x', \rho'; x, \rho) = -\frac{1}{2\pi} \int_{-\infty}^{\infty} dk e^{ik(\phi'-\phi)} K_{i|k|}\left(\frac{x_>}\sqrt{2G}\right) I_{i|k|}\left(\frac{x_<}\sqrt{2G}\right)
$$

where $x_>$ or $x_<$ is the greater or lesser of $x, x'$. The product structure is consistent with a general result of Brown and Martinez\cite{17}. Note that this is identical with the loop-loop transition amplitude of $c = 1$ string theory\cite{18}.

If one introduces a quantum field

$$
\Psi(x, \rho) = \sum_k (A_{\psi}^1 k_{\psi}^1 + A_{\psi}^1 k_{\psi}^{1*}) = \sum_k (A_{\psi}^2 k_{\psi}^2 + A_{\psi}^2 k_{\psi}^{2*})
$$

then the creation operators $A_{\psi}^{1\dagger}, A_{\psi}^{2\dagger}$ act in an extended Hilbert space. It can be verified, using the classical solutions’ that the parameter $k$ which is the eigenvalue of the canonical momentum $\pi_{\rho}$ is related to the blackhole mass by $k = -2vM$.

Thus the above expansion can be interpreted a sum over all black holes of variable mass. The universal Greens function can then be represented as:

$$
G(x', \rho'; x, \rho) = <0^1|T\Psi(x', \rho')\Psi(x, \rho)|0^2>
$$

Where $T$ represents ordering with respect to $x$. As the $\psi_k$ themselves are ground states of a second quantized theory the introduction of $|0^1>$ and $|0^2>$ as states
annihilated by $A_1^k$ and $A_2^k$ respectively this represents in effect a third quantized
description of the path integral. This third quantized description of the path
integral at first sight does not appear to offer anything new but as we shall see
the non-uniqueness of the third-quantized vacuum has important implications for
the nonconservation of universe number and topology change for this 2$d$ dilaton
gravity.

It is actually necessary to inspect the universal Greens function to decide
whether third quantization is appropriate. For example Chamseddine’s description
of the 2$d$ dilaton gravity$^{[19]} S = \int d^2x\Omega(x)\sqrt{-g}R$ resulting from the dimensional
reduction of $2 + 1$ dimensional gravity, yields an entirely different type of universal
Greens function. Essentially the $\rho$ field is entirely nailed down to its classical
value inside the path integral. Similar statements also apply to the Chern-Simon
description of $2 + 1$ dimensional gravity$^{[20]}$. There the path integral for the action
$\int e \wedge R$ reduces to a sum over flat classical solutions$^{[21]}$.

Now we can apply this formalism of 2$d$ dilaton gravity to the dynamics of
a black hole interior. It has been known since the work of DeWitt$^{[22]}$ that the
classical solutions to the Einstein equations are given by geodesics in the space
of metrics. For the solution corresponding to the interior of the black hole the
geodesic through target space is given by:

$$e^{2\rho} = \frac{2GM}{r} - 1$$

$$e^{-2\phi} = \frac{2r^2}{G}$$

So that upon solving the above equation for $r$ in terms of $\rho$ we find that $q = \rho - \phi$
can be expressed as:

$$e^q = \sqrt{2M^2G} \frac{1}{\cosh \rho}$$

Note that this is the same trajectory of a particle in Rindler space$^{[23]}$ (see figure
Indeed one can define new regions of target space through

\[ I: \begin{align*}
X &= e^q \cosh \rho \\
T &= e^q \sinh \rho
\end{align*} \]

Then one sees that as one goes from the horizon to the singularity the original target space variables \( q \) and \( \rho \) only parametrize the region of target space \( X < |T| \).

One can use new target space coordinates to describe the trajectory beyond the singularity. If we denote the region \( |X| > T \) by region II one can parametrize this region by

\[ II: \begin{align*}
X &= e^{q'} \sinh \rho' \\
T &= e^{q'} \cosh \rho
\end{align*} \]

The easiest way to continue the blackhole solution beyond the singularity is to continue the solution \( X = \sqrt{2M^2G} = \text{const} \) into region II. That is to continue the upward vertical line in figure 2 into this region.

It is intriguing that the equation (4) describing the blackhole interior is identical to a particle bouncing off an exponential barrier i.e. the Liouville equation. It is possible upon hitting the barrier for a single universe to split and for a multiple universe state to bounce back, this has verified explicitly in two dimensional models of quantum gravity. Similar phenomena occur for the two dimensional dilaton gravity considered here. In terms of the variables \( X \) and \( T \) the canonical momentum are defined by \( P_X = -\frac{1}{2}e^{-2\rho} \dot{X}v \) and \( P_T = \frac{1}{2}e^{-2\rho} \dot{T}v \). The Wheeler-DeWitt equation becomes:

\[ (-P_T^2 + P_X^2 + \frac{v^2}{2G})\Psi(X, T) = 0 \]

The solution is simply

\[ \psi_{out}^K = e^{iKX-i\sqrt{K^2 + \frac{v^2}{2g}}T} \]

Promoting this solution to a field one has the expansion

\[ \Psi(x, \rho) = \sum_K (A_K \psi_{out}^K + A_K^\dagger \psi_{out}^K) \]

Third quantization interprets \( A_K \) and \( A_K^\dagger \) as operators and allows one to define a
no-universe state or third quantized vacuum by $A_K|\text{out}>=0$ and $A_K^1|\text{in}>=0$.

The interesting fact here is that in expanding one solution in terms of the other one finds that the (out) vacuum is not annihilated by the $A_k^1$ in particular

$$<\text{out}|A_k^{1\dagger}A_k^1|\text{out}>=\frac{1}{e^{2\pi|k|-1}}$$

The computation is essentially the same as counting the number of Rindler particles in Minkowski space$^{[24]}$. Here the interpretation is that the extended Hilbert space of third quantization has allowed the universe number to fluctuate from the initial one universe state of the black hole interior emerging at the horizon.

So far all of our discussion has been based the two dimensional dilaton gravity described by (1). What possible connection does this have with the real four dimensional world? Perhaps the hint of such a connection lies in the work of Verlinde and Verlinde$^{[25]}$. In this work they studied 4d gravity in the gauge

$$[g_{\mu\nu}] = \begin{pmatrix} g_{ab} & 0 \\ 0 & h_{ij} \end{pmatrix}$$

where the action can be written$^{[25]}$:

$$S = \int \sqrt{-g}(\sqrt{-h}R_h + \frac{1}{4}\sqrt{h}h^{ij}\partial_i g_{ab} \partial_j g_{cd}(g^{ab}g^{cd} - g^{ad}g^{bc}))$$

$$+ \sqrt{h}(\sqrt{-g}R_g + \frac{1}{4}\sqrt{-gg^{ab}}\partial_a h_{ij} \partial_b h_{kl}(h^{ij} h^{kl} - h^{il} h^{jk}))$$

This action still contains the residual gauge invariance $g_{ab} \rightarrow g_{ab} + \nabla_a \epsilon_b + \nabla_b \epsilon_a$ if $\epsilon_a$ is independent of $x_2, x_3$. We can fix this symmetry by choosing the conformal gauge $g_{ab} = e^{2\phi} \hat{g}_{ab}$ and if we parametrize

$$[h_{ij}] = L^2 e^{-2\phi} \begin{pmatrix} \frac{\tau_1^2 + \tau_2^2}{\tau_2} & \frac{\tau_1}{\tau_2} \\ \frac{\tau_1}{\tau_2} & \frac{1}{\tau_2} \end{pmatrix}$$
the action becomes:

\[ S = \frac{L^2}{4G} \sum_{\ell_2 \ell_3} \int d^2 x \sqrt{-g} (e^{-2\phi} (2\hat{g}^{ab} \partial_a \phi \partial_b \phi - 4\hat{g}^{ab} \partial_a \phi \partial_{\bar{b}} \rho + \hat{R} + \chi L^{-2} e^{2\phi + 2\rho}) \\
- \frac{e^{-2\phi}}{2\tau_2^2} \hat{g}^{ab} (\partial_a \tau_1 \partial_b \tau_1 + \partial_a \tau_2 \partial_b \tau_2) + \frac{2e^{2\rho}}{L^2 \tau_2} ((\Delta_2 \rho)^2 - 2\tau_1 \Delta_2 \rho \Delta_3 \rho + (\tau_1^2 + \tau_2^2) \Delta_3 \rho \Delta_3 \rho)) \]

Where \( \chi = \frac{1}{4\pi} \int \sqrt{h} R_h \) is the Euler characteristic of the transverse space, which if taken to be topologically a sphere equals 2. The variables \( \tau_1, \tau_2 \) encode the two remaining degrees of freedom of the graviton. \( \Delta_i \rho = a^{-1}(\rho(x_i + a) - \rho(x_i)) \) is the finite difference defined on a lattice with lattice spacing \( a \). Indeed a similar 1 + 1 dimensional transverse lattice approach has yielded a tractible description of 4d QCD\(^{[26,27]}\).

It is possible that a Lagrangian in this form can be applied to the blackhole entropy problem. A microcanonical description of blackhole entropy is embodied in the formula:

\[ tr(\delta (M - \hat{M})) = e^{4\pi GM^2} \]

Here the trace is over the Hilbert space of the blackhole interior and \( \hat{M} \) is a quantum mass operator which reduces to the usual ADM value for the mass in the classical limit. However the right states to be included in the trace as well a suitable choice of the mass operator remains an open problem.

In paper we have seen paper we have seen that the canonical momentum of the \( \rho \) field has eigenvalues which classically reduce to the mass. Also the Bondi mass \( M_{\text{Bondi}} = \frac{L^3}{2\pi} e^{-2\rho - 3\phi} (\dot{\phi}^2 - \phi'^2) + \frac{L^3}{2\pi} e^{-\phi} \), taken as a quantum operator has eigenvalues proportional to the classical ADM mass. This gives a spacetime dependent mass, crucial for the phenomena of mass inflation\(^{[28]}\). However it is not clear either is the correct expression to insert in the entropy formula.

The introduction of a transverse lattice and working in the gauge of Verlinde and Verlinde has reduced the Einstein-Hilbert action to type of string theory.
albeit one of a rather complicated form, so it worth noting that the entropy of string theory is determined by:

$$\text{tr}(\delta(E - \hat{H})) = E^{-a}e^{bE}$$

In that case the Hilbert space was over multi-string states with the dominant contribution comprising many strings in their ground state and one highly excited string with the bulk of the energy\(^{[29]}\). The analog for the blackhole entropy is clearly a sum over multi-universe states with many of the universes nearly empty and one containing nearly all the mass \(M\), but the role of these multiverse states in the trace could be more subtle. Specific models of quantum cosmology have been investigated in which the average number of universes in the final state is given by \(e^{\frac{4\pi}{\Lambda}}\) where \(\Lambda\) is the cosmological constant. This number is so huge that that it was given the name googolplexus\(^{[31]}\). In the Schwarzschild-DeSitter universe with blackhole mass \(GM = \frac{1}{\sqrt{\Lambda}}\) this number is \(e^{\frac{2\pi}{\Lambda}}\)\(^{[32]}\) and is equally large. Significantly this coincides with the Euclidean saddle point computation of blackhole entropy of the Schwarzschild-DeSitter solution. Even if each universe carried off one bit of information they huge number of them could contain the entropy present in the initial state before blackhole evaporation and topology change.
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**Figure Captions**
figure 1. Representation of blackhole interior metric as a geodesic in target space. Here $\rho$ going to plus or minus infinity corresponds to the horizon and singularity respectively.

figure 2. Extended target space describing the blackhole interior. Points $a$ and $b$ denote the horizon and singularity. The geodesic can be continued beyond the singularity by extending the vertical line into region II.