Black holes in extra dimensions can decay on the bulk

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In the extra dimensional theories, with TeV scale Plank constant, black holes may be produced in the Large Hadron Collider experiments. We have argued that in the d-dimensional black hole, the intrinsically 4-dimensional brane fields do not see the same geometry at the horizon, as in a 4-dimensional space-time. Kaluza-Klein modes invades the brane and surroundings and the brane fields can be considered as a thermal system at the temperature of the black hole. From energy and entropy consideration, we show that whether or not a six-dimensional black hole will decay by emitting Kaluza-Klein modes or the standard model particles, will depend on the length scale of the extra dimensions as well as on the mass of the black hole. For higher dimensional black holes, Kaluza-Klein modes will dominate the decay.

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Recently several authors have proposed that in addition to the usual 3+1 dimensions, our universe may contain extra spatial dimensions, as large as a millimeter [1]. The proposal gain popularity as the models contain extra spatial dimensions, as large as a millimeter. If such theories are correct, then it may be possible to produce black holes in the laboratory conditions at the Large Hadron Collider (LHC). The idea is simple. The effective 4-dimensional Plank’s constant $M_4$ is related to the fundamental d-dimensional Plank constant $M_d$ as, $M_d^{d-2} = M_4 L^{-(d-4)}$, $L$ being the compactification scale of the extra dimensions. For millimeter scale $L$, fundamental Plank constant $M_d$ could be as low as $\sim$ TeV, much smaller than the LHC cm energy ($\sqrt{s} = 14$ TeV). Banks and Fischler [2] argued that for $\sqrt{s} \gg M_d$, scattering cross sections are dominated by the black hole formation, at the impact parameters less than the Schwarzschild radius. Simple calculations [3,4] indicate that at the LHC, black hole production cross sections can be large.

Several authors have considered formation and decay of black holes in extra dimensional theories [2-10]. While LHC can produce black holes in abundant, its detection may face problems. Black holes will decay through Hawking radiation. Some authors [2,6] have argued that the d-dimensional black holes will radiate mainly into Kaluza-Klein (KK) modes. Temperature ($T_{BH}$) of extra dimensional black holes are large and all the KK modes below $T_{BH}$ will be produced. Emission of standard model (SM) particles will be limited, due to phase space reason. Emparan, Horowitz and Myers [9] on the other hand, argued that the d-dimensional black holes will radiate mainly on the brane. They argued that the phase space consideration could not be applied to SM fields, as they are intrinsically 4-dimensional. The 4-dimensional black hole law governs their emission. For black hole radius $r_d << L$, the emission of KK modes will be suppressed by a factor of $(r_d/L)$, compared to SM particles. They compared emission rates of black holes in d-dimension and in 4-dimension and showed that the ratio of emission rates gets closer to one as dimensionality increases. Thus, $(dE_4/dt)/(dE_6/dt) \sim 3.66$ and $(dE_A/dt)/(dE_W/dt) \sim 1.54$ [9].

Whether or not d-dimensional black holes decay on the bulk or on the brane is an important issue as the detection of laboratory produced black holes will depend on it. If they decay on the bulk, they will remain undetected. In the d-dimensional black hole, the intrinsically 4-dimensional brane fields do not see the same geometry at the horizon as in a 4-dimensional space-time. Matter fields (i.e. Kaluza-Klein modes) invades the brane and surroundings. In the present paper, we assume that the brane fields can be considered as a thermal system, in equilibrium with the matter fields at the black hole temperature. We then study the energy and entropy of the brane fields (i.e. SM particles) vis-a-vis Kaluza-Klein modes. It is seen that for $d > 6$, energy and entropy in the Kaluza-Klein modes greatly exceeds the energy and entropy of the SM particles. It is then expected that $d > 6$ black holes will decay mainly in to the bulk, by emitting KK modes. For $d = 6$ black holes, KK modes dominate the energy and entropy above a critical compactification scale, which depend on the black hole mass. Below that critical compactification scale, SM particles dominate the energy and entropy of the black holes.

Let us assume a mass $M$ is collapsed in to a black hole of size $r_d << L$, in $d > 4$ dimension. The extra-dimensions are of the scale $L$. The d-dimensional metric (extension of Schwarzschild solution in d-dimension) can be written as [5],

$$ds^2 = -f(r)dt^2 + f^{-1}(r)dr^2 + r^2 d\Omega_{d-2}^2,$$

with $f(r) = 1 - (r_d/r)^{d-3}$. $d\Omega_{d-2}^2$ is the line element on the unit d-sphere. The horizon lies at $r = r_d$, the black hole radius [5],

$$r_d = \left[ \frac{16\pi G_d M}{(d-2) A_{d-2}} \right]^{\frac{2}{d-3}}$$

where $G_d$ is the d-dimensional Newton’s constant and $A_{d-2}$ is the area of the unit $(d-2)$-dimensional sphere.
The induced metric in 4-dimension [9],

\[ ds^2 = -f(r)dt^2 + f^{-1}(r)dr^2 + r^2d\Omega_2^2, \]

is of the same form as the d-dimension metric, with \( f(r) \) same as before. The event horizon \( r = r_d \) is certainly not of 4-dimensional Schwarzschild geometry. Ricci tensor of this four dimensional metric is non-zero near the horizon. It can be thought upon as a black hole with matter fields (Kaluza-Klein modes) around it. This observation led us to assume that the brane fields can be considered as a thermal system of SM particles. The temperature of the thermal system is the same as the black hole temperature. The gravity being the only interaction common to the brane and the bulk, the brane will be populated by the KK modes, at the temperature of the d-dimensional black hole. Interaction of the KK modes with the SM particles will interact much more strongly than the usual gluons. Lifetime of d-dimensional black holes being large, the assumption of thermal equilibrium between the KK modes and the SM particles is reasonable.

If we approximate \((dE_d/dt)/dE_4/dt \sim E_d/E_4\), then whether the black hole will decay in to the brane or the bulk, will depend on the relative energy \(E_d \) and \(E_4\). As will be shown below, apart from the black hole mass, it depends on the number of extra dimension as well as on the compacitification scale of the extra-dimensions.

Energy density \((\varepsilon)\), pressure \((p)\) and entropy density \((\sigma)\) of the SM particles at a temperature of \(T\) can be easily calculated for a non-interacting system. For massive particles,

\[ \varepsilon_{\text{mass}}(T) = \sum_i \frac{g_i}{(2\pi)^3} \int \frac{\sqrt{k^2 + m_i^2}}{\exp(\sqrt{k^2 + m_i^2}/T) \pm 1} d^3k \]

\[ p_{\text{mass}}(T) = \sum_i \frac{g_i}{(2\pi)^3} \int \frac{k^2}{3\sqrt{k^2 + m_i^2}} \exp(\sqrt{k^2 + m_i^2}/T) \pm 1 d^3k \]

\[ \sigma_{\text{mass}}(T) = \frac{\varepsilon_{\text{mass}} + p_{\text{mass}}}{T} \]

where \( g_i \) is the degeneracy of the particle of mass \( m_i \) and the \( \pm \) is for boson/fermion. The energy density of mass less particles is calculated using the \( T^4 \) law, \( (\varepsilon_{\text{massless}} = \pi^2 N T^4/30, \) \( N \) being the effective degrees of freedom). We also assume the ideal gas equation of state, \( p = 1/3 \varepsilon \) for them. In the SM particle lists we have included six quarks \((u,d,s,c,b \) and \( t)\) and their anti particle, three electrons \((e, \mu, \tau)\) and their anti particles and \( W^\pm, Z\) bosons. The color degrees of freedom as well as the spin degeneracy is taken into account. In the mass less particles lists, we include \( 8 \times 2 \) gluons and 2 photons. In Fig.1, we have shown the energy density, pressure and the entropy density of the SM particles as a function of the temperature. For the SM particles listed above, for temperatures above 50 GeV, the energy density and pressure are well described by the \( T^4 \) law. The entropy density is then described by the \( T^3 \) law.

The energy and entropy of the SM particles, in a d-dimensional black hole of radius \( r_d \) is then obtained as,

\[ E_{SM} = [\varepsilon_{mass} + \varepsilon_{massless}] \frac{4\pi}{3} r_d^3 \]

\[ S_{SM} = [\sigma_{mass} + \sigma_{massless}] \frac{4\pi}{3} r_d^3 \]

Black hole temperatures being \( T_d = (d-3)/4\pi r_d \), in six dimension, total entropy of the SM particles is independent of the black hole mass or radius. The energy and entropy due to KK modes is then obtained as,

\[ E_{KK} = M_{BH} - E_{SM} \]

\[ S_{KK} = S_d - S_{SM} \]

where \( M_{BH} \) is the black hole mass and \( S_d \) is the entropy of the d-dimensional black hole,

\[ S_d = \frac{\pi r_d^d - 2 A_{d-2}}{4 G_d} \]

In Fig.2, we have shown the ratio of the entropies, \( S_{KK}/S_{SM} \) (panel a) and the ratio of energies \( E_{KK}/E_{SM} \) (panel b) as a function of the compactification radius \( L \) for a black hole formed in d=6 dimension. We have considered black hole masses of 1,3,5,7, and 9 TeV, accessible at the LHC. The ratios indicate that whether or not SM particles will dominate the emission spectrum depend on the compactification radius as well as on the black hole mass. For \( M_{BH}=1 \) TeV, energy available to KK modes exceeds the SM energy for \( L > 0.2 \) mm. Nearly similar value is obtained from the entropy consideration. Entropy due to KK modes exceeds the entropy due to SM particles for \( L > 0.4 \) mm. For higher black hole masses, the cross over from KK mode dominance to SM particle dominance will occur at still lower \( L \). We note that the critical compactification radius, above which the KK modes dominate the SM particles, are indicative only. We only intend to show that the emission from d=6 dimensional black holes can be dominated by the KK modes or by the SM particles. Which one will dominate will depend on the compactification radius as well as on the mass (radius) of the black holes.

Situation is completely different for higher dimensional black holes. In Fig.3, we have shown the results obtained for d=8 dimensional black holes. It can be seen that if the compactification scale lies between \( 10^{-3}\)mm-10 mm, KK modes will dominate the black hole energy and entropy. Consequently emission will be dominated by the KK modes.

To conclude, we have approximated the brane fields as a thermal system of standard model particles, in contact with the Kaluza-Klein modes. We have calculated the
energy density, pressure and entropy density of SM particles. It was seen that for six dimensional black holes, whether the decay will be dominated by the KK modes or by the SM particles depends on the compactification scale of the extra dimensions, as well as on the mass of the black holes. For 1 TeV black holes, if the compactification scale is below 0.2-0.4 mm, SM particles will dominate the decay. For higher mass black holes, SM particles will dominate, if the compactification scale is still lower. Other wise, a six dimensional black hole will decay mainly through emission of KK modes. For \( d > 6 \) dimensional black holes, the decay will be dominated by the KK modes.

\[ S_{KK}/S_{SM} \]

\[ E_{KK}/E_{SM} \]

\( L(\text{mm}) \)

\( d=6 \)

\( d=8 \)

\[ E/T \]

\[ P/T \]

\[ S/T^3 \]

\[ T \]

\[ (G_{eV}) \]

\[ \sigma/T^3 \]

\[ \sigma \]

\[ \sigma_{SM} \]

\[ \sigma_{KK} \]

\[ d \]

\( T \)

\( L(\text{mm}) \)

\( (a) \)

\( (b) \)

FIG. 1. In three panels, energy density, pressure and entropy density due to SM particles as a function of temperature are shown.

FIG. 2. (a) Ratio of entropy due to KK modes and SM particles for six-dimensional black holes of mass \( M_{BH}=1,3,5,7 \) and 9 TeV (from bottom to top) are shown, as a function of the compactification radius. (b) Ratio of energy available to KK modes and SM particles.

FIG. 3. Same as Fig.2, for 8-dimensional black holes.

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