Abstract

We discuss the underlying connections among the thermodynamic properties of short-ranged spin glasses, their behavior in large finite volumes, and the interfaces that separate different pure states, and also ground states and low-lying excitations.

KEY WORDS: spin glass; Edwards-Anderson model; replica symmetry breaking; mean-field theory; pure states; ground states; domain walls; interfaces; incongruence

1 Introduction

In this talk, we focus on the following question: assuming that there exists in finite dimensions a low-temperature spin glass phase with broken spin-inversion symmetry, what are the relationships among the possible types of ground states that might be present, their low-energy excitations, and the interfaces that separate those states? Answering this question provides a handle on the broader nature of the low-temperature spin glass phase, if it exists.

While our results apply to a wide class of short-ranged models, we focus here for specificity on the Edwards-Anderson (EA) Ising model [1], with Hamiltonian:

$$\mathcal{H}_J = - \sum_{<x,y>} J_{xy} \sigma_x \sigma_y - h \sum_x \sigma_x ,$$

(1)
where $x$ is a site in a $d$-dimensional cubic lattice, $\sigma_x = \pm 1$ is the Ising spin at site $x$, $h$ is an external magnetic field, and the first sum is over nearest neighbor pairs of sites. To keep things simple, we take $h = 0$ and the spin couplings $J_{xy}$ to be independent Gaussian random variables whose common distribution has mean zero and variance one. The absence of a field and the symmetry of the coupling distribution results in an overall global spin inversion symmetry. We denote by $J$ a particular realization of the couplings, corresponding physically to a specific spin glass sample.

It is important to consider both equilibrium and nonequilibrium properties arising from (1), but we consider here only the former. We remain far from a comprehensive theory of the statistical mechanics of the EA Hamiltonian, but nonetheless there has in recent years been substantial analytical — and even some rigorous — progress in understanding what can and cannot occur in a putative low-temperature spin glass phase. In this talk we examine the relationships among three properties of this phase:

- **Interfaces between ground states.** A ground state is an infinite-volume spin configuration whose energy cannot be lowered by the flip of any finite subset of spins. It can be constructed as the limit of a sequence of finite-volume ground states, i.e., the lowest-energy spin configuration(s) consistent with some boundary condition in a finite volume. The *interface* between two ground states is the set of all couplings that are satisfied in one and not the other.

- **Thermodynamic volume ($V \to \infty$) behavior.**

- **Behavior in typical large finite volumes.**

  Our emphasis will be on the interconnectedness of these three: conclusions about any one can provide important information about the others. However, these relationships may not be at all straightforward; one theme emerging from our work is that they are far more complicated than in homogeneous systems.

## 2 ‘Observable’ vs. ‘Invisible’ States

How might one “see” the structure of the spin glass phase? There are several possible procedures. As in many other statistical mechanical problems, one can study the effects of a change in boundary conditions. Consider a cube $\Lambda_L$ of side $L$ centered at the origin with periodic boundary conditions. There will a spin-flip-related pair of spin configurations within $\Lambda_L$ of lowest energy. This finite-volume ground state pair will change in general with
the boundary condition; for example, if one switches to antiperiodic boundary conditions, then there will be a new ground state pair with some interface relative to the old one.

One important question is whether such interfaces are pinned or deflect to infinity: if for any fixed $L_0$, the relative interface eventually moves (and stays) outside of $\Lambda_{L_0}$ as $L \to \infty$, the interface has ‘deflected to infinity’; if it remains inside $\Lambda_{L_0}$, it is ‘pinned’ (for an illustration, see Fig. 2 of [2]). Pinned interfaces imply the existence of infinite-volume multiple ground state pairs.

More recent techniques are useful for studying excitations whose energies above a finite-volume ground state are of $O(1)$ independent of the volume size. Consider again a cube $\Lambda_L$ with periodic boundary conditions. The method of Krzakala and Martin (KM) [3] is to choose an arbitrary pair of spins, and force them to have an orientation opposite to that in the ground state pair. The method of Palassini and Young (PY) [4] is to add a perturbation that lowers the energy of a spin configuration by an amount proportional to the fraction of bonds in an interface relative to the ground state pair. Either way, there will be a new spin configuration pair that minimizes the energy.

What these and most other widely-used procedures have in common is that they are not explicitly coupling-dependent; e.g., the boundary conditions used are independent of the coupling realization $J$. We call ‘observable’ any interfaces (and resulting states) that can be constructed in this way; interfaces or states that can only be seen using coupling-dependent boundary conditions we call ‘invisible’. A detailed discussion of the reasons behind these designations is given in [5] (see also the discussion in Sec. 3 of [6]).

3 Interfaces

We now focus on interfaces separating spin configurations. The analysis is at $T = 0$ but can be extended to nonzero temperature. That is, we confine our attention here to interfaces separating either different ground states or ground states and excitations. For now, we won’t worry how the interfaces arise: they might have arisen between ground states in a single volume under a switch from periodic to antiperiodic boundary conditions, or from using the KM or PY procedures, or through some other coupling-independent method. Our discussion below applies to all.

The main features we study include:

• **Spatial Structure.** In other words, is the interface ‘space-filling’ or ‘zero-density’? By space-filling we mean the following. Consider in $d$ dimensions a sequence of cubes $\Lambda_L$, each
containing an interface generated through a common procedure (e.g., switching from periodic to antiperiodic boundary conditions). If, for each $L$, the number of bonds in the interface scales as $L^{d_s}$ with $d_s = d$, then the interface is space-filling. If $d_s < d$, it is zero-density.

One of the interesting possibilities arising in spin glasses is the possibility of space-filling interfaces separating ground or pure states; such a possibility cannot arise, e.g., in ferromagnets. Space-filling interfaces (suitably redefined) are believed [7] to separate different pure states in the low-temperature phase of the infinite-ranged Sherrington-Kirkpatrick (SK) model [8].

- **Energetics.** Spin glass interfaces can also have unusual energetic properties. In a ferromagnet, whether homogeneous or disordered, the energy of an interface scales linearly with the number of bonds comprising it. This is not necessarily so in a spin glass. One intriguing possibility is that — as happens in the SK model — an interface might have $O(1)$ energy independently of its size. The other possibility is that the energy of an interface in the volume $\Lambda_L$ is $O(L^\theta)$, with $\theta > 0$. (We ignore unlikely special cases such as logarithmic and other non-power-law dependences.)

- **Pinning.** A property crucial to the nature of states separated by an interface is whether the interface is pinned or deflects to infinity, as discussed in Sec. 2. As shown in [2, 9], which of these occurs is not independent of the spatial structure, for interfaces separating observable states: a space-filling interface must be pinned, and a zero-density one (generated by a coupling-independent procedure) must deflect to infinity.

Each of these interface properties corresponds to a major scenario proposed for the low-temperature spin glass phase of the EA model in finite dimensions. This is summarized in Fig. 1, and each will be discussed in turn.

### 3.1 Mean-Field Picture

The picture arising from Parisi’s solution [7, 10, 11, 12, 13] of the infinite-ranged SK model is known as the replica symmetry breaking (RSB) theory. There have been many papers written in support of the notion that the RSB theory should apply as well to short-ranged spin glasses. We will not discuss all of RSB’s features here; given the subject of this talk, we focus on the prediction [14, 15] of RSB theory that interfaces separating ground states are both space-filling and can have energies of $O(1)$, i.e., that don’t scale with the size of the system. Hence the designation ‘RSB interfaces’ in the upper left-hand corner of the table in Fig. 1.
Figure 1: Table illustrating the correspondence between a type of interface and a scenario for the structure of the low-temperature spin glass phase in finite dimensions.

A central result of [2] was a proof that the existence of space-filling interfaces (regardless of how their energies scale, as long as the interfaces are generated by coupling-independent methods) is a sufficient condition for the existence of multiple thermodynamic pure state pairs. In order to have a nontrivial overlap function (at nonzero temperature) of the sort characteristic of RSB theory [7], we have shown that the set of all states generated through sequences of finite-volume Gibbs states must partition into a union of thermodynamic mixed states $\Gamma$, with each $\Gamma$ being a nontrivial collection of infinitely many pure states with certain weights. Different $\Gamma$’s would appear in different finite volumes; for details, see [16, 17, 18, 19]. One can also prove that there must be an uncountable infinity of pure states in the union of all $\Gamma$’s [20].

So in this picture the connections among the three properties listed in Sec. 1 is that RSB interfaces imply (and are implied by) this complex thermodynamic structure, and by nontrivial overlap structure in large finite volumes. However, thermodynamic arguments show [2, 16, 17, 18, 19, 21] that this thermodynamic structure cannot be supported in finite-dimensional, short-ranged spin glasses, in turn ruling out the possibility that space-filling, $O(1)$ energy interfaces can arise in these systems.
3.2 Chaotic Pairs Picture

The upper right-hand corner of Fig. 1 raises the possibility of space-filling interfaces whose energy scales as $L^\theta$, with $\theta > 0$. (For reasons that won’t be discussed here, there is an upper bound [22, 23] of $\theta \leq (d - 1)/2$ for observable states.) This leads directly to the chaotic pairs picture [16, 17, 18, 19, 23] in which many pure state pairs exist, but only a single spin-reversed pair appears in any large finite volume, leading to a trivial overlap structure and no ultrametricity or related features of the RSB scenario.

3.3 Droplet/Scaling Picture

Continuing clockwise around the table in Fig. 1, we come to interfaces that are zero-density and with energy scaling as $L^\theta$, with $\theta > 0$. These interfaces characterize droplet excitations in a two-state picture developed by Macmillan [24], Bray and Moore [25], and Fisher and Huse [22]. This picture is well-known and won’t be discussed in detail here.

3.4 ‘TNT’ Picture

We come finally to the last entry, which conjectures interfaces that are both zero-density and whose energies remain $O(1)$ independently of lengthscale. This picture was proposed by Krzakala and Martin [3] (who denoted the picture ‘TNT’ for trivial link overlap and nontrivial spin overlap) and Palassini and Young [4]. It can be shown that zero-density interfaces cannot separate observable pure or ground states [9], so these states, should they exist, would necessarily be excitations, as in the droplet-scaling picture.

4 Conclusion

The purpose of this talk has been to demonstrate that there are deep connections among the thermodynamic structure of spin glass states, their behavior in large finite volumes, and the interfaces that separate these states (and/or their excitations). Tools developed for any one can lead to strong conclusions about the other two. This interconnection has allowed substantial analytical, and even rigorous, progress on short-ranged spin glasses.

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