Neutron star natal kicks: Collisions, $\mu$TDEs, faint SNe, GRBs and GW sources with preceding electromagnetic counterparts

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Abstract

Based on the observed high velocity of pulsars it is thought that neutron stars (NSs) receive a significant velocity kick at birth. Such natal kicks are considered to play an important role in the the evolution of binary-NS systems. The kick given to the NS (together with the effect of mass loss due to the supernova explosion of the NS progenitor) may result in the binary disruption or lead to a significant change of the binary orbital properties. Here we explore in detail the dynamical aftermath of natal kicks in binary systems, determine their possible outcomes and characterize their relative frequency, making use of analytic arguments and detailed population synthesis models. In a fraction of the cases the kick may cast the NS in such a trajectory as to collide with the binary companion, or pass sufficiently close to it as to disrupt it (micro tidal disruption event; $\mu$TDE), or alternatively it could be tidally-captured into a close orbit, eventually forming an X-ray binary. We calculate the rates of direct post-kick physical collisions and the possible potential production of Thorne-Zytkow objects or long-GRBs through this process, estimate the rates X-ray binaries formation and determine the rates of $\mu$TDEs and faint supernovae from white dwarf disruptions by NSs. Finally we suggest that natal kicks can produce BH-NS binaries with very short gravitational-wave merger time, possibly giving rise to a new type of promptly appearing eLISA gravitational wave (GW) sources, as well as producing aLIGO binary-merger GW sources with a unique (likely type Ibyc) supernova electromagnetic counterpart which precedes the GW merger.

Key words: binaries: general – gravitational waves – stars: neutron – gamma-ray burst: general

1 Introduction

Neutron stars (NSs) are born in supernova (SN) explosions of massive stars with initial mass of $\gtrsim 8 M_{\odot}$ (Smartt et al. 2009; Smartt 2015). Some of these are pulsars which can be observed through their frequent radio pulses. Proper motion measurements of pulsar velocities have indicated that newly born NSs receive a large natal kick reaching up to $\sim 1,500 \text{ km s}^{-1}$ (Hobbs et al. 2005). The origin of the natal kick is still debated, but it’s typically thought to be the results of an asymmetric explosion of the SN.

Several studies focused on identifying the natal kick velocity distribution, by measuring the proper motion of isolated pulsars. Arzoumanian et al. (2002) modeled the kick velocity distribution as two overlapping Gaussians, the first with low characteristic velocity of $\sigma_1 \approx 90 \text{ km s}^{-1}$, consisting of 40% of the pulsars, and the second with high characteristic velocity of $\sigma_2 \approx 500 \text{ km s}^{-1}$. Later, Hobbs et al. (2005), using a sample of 73 young pulsars, modeled the velocity distribution as a Maxwellian with a velocity disper-

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Over 80% of the massive O and B stars reside in binaries or higher multiplicity systems, and when accounting for observational biases the multiplicity fraction is consistent with 100% (Sana et al. 2012, 2014; Moe & Di Stefano 2016). Moe & Di Stefano (2016) showed that up to a semi major axis (sma) of $\sim 50 \text{ AU}$ all massive stars are in binaries. Therefore a non-negligible fraction of all NSs will host a companion in the binary systems, and go through a post SN interaction between the newly born NS and its companion. This can result in several possible outcomes: a direct physical collision, a disruption of the companion due to a very close passage near the NS (a micro tidal disruption event; $\mu$TDE; Perets et al. (2016)), a tidal capture into a close orbit, or even significant gravitational wave emission through a close approach of the NS to another compact object. Previous studies have discussed some aspects of this prob-
lem (e.g. Leonard & Davies 1994; Troja et al. 2010; Hills 1983), as we discuss below; here we focus on characterizing and mapping all of these possible outcomes.

In this work we calculate the possible interaction between the newly born NS and its binary companion. After the formation of the NS it may pass sufficiently close to its stellar or Black Hole (BH) companion to strongly interact with it and produce an electromagnetic transient, a gravitational wave source or evolve to become a short-period binary to eventually form an X-ray binary system. The exact dynamical outcome of the SN explosion is determined by the natal kick velocity vector, the mass loss in the SN explosion, the initial Keplerian orbit and the companion radius, which are determined by the long term stellar evolution of the binary prior to the explosion. All of these issues play an important role in the final outcomes of NS kicks and are addressed in the following.

We note that the large body of literature of NS binaries and their outcomes prohibits us from a detailed review of all previous works, and we therefore refer the reader to various review papers (e.g. Kalogera et al. 2007; Abadie et al. 2010; Postnov & Yungelson 2014). These studies typically aimed to calculate the production rate of specific types of objects and/or transients; we emphasize that our aim is not to reproduce these extensive studies, but rather to focus on the novel aspects of the close encounters following NS natal kicks. It’s nevertheless important to point out at least briefly some of these works. In particular over the last two decades Kalogera, Belczyski, Fryer and collaborators have begun and continued a long-term study of stellar binary evolution (e.g. Fryer et al. 1999; Kalogera et al. 2007), mostly using the STARTRACK population synthesis module they developed to explore the rate of properties of a wide range of transients and objects, and in particular GW sources and short-GRBs. Similar efforts have been done by other groups, using other models such as the Scenario Machine (e.g. Lipunov & Puzhinskaya 2014 and references therein) and various other models (e.g. Duchêne & Kraus 2013 and references therein).

This paper is organized as follows: in Section 2 we present an analytical treatment to calculate the periapsis distance of the binary post the SN accounting for a random position in the initial Keplerian orbit, mass loss and a random kick velocity and direction. In Section 3 we describe the population synthesis calculation. In Section 4 we present the simulation results for 587,019 binary systems. In the following Section 5 we explain the results analytically. The implication of the results are described in Section 6, and the discussion and summary of the paper is presented in Section 7.

2 CALCULATING THE MINIMAL DISTANCE

We begin by deriving the closest approach distance between the kicked NS and the secondary. The closest approach is determined by the state of the binary system right before the SN explosion, the prompt mass lost during the SN, and the natal kick velocity vector. The binary state before the SN is defined by the binary semimajor axis (sma), $a$, the eccentricity $e$, the secondary mass, $m_2$, the primary mass before the SN, $m_1$, and the specific orbital phase of the stars in their orbit, given by the true anomaly, $\nu$.

The known solution to the two body Kepler problem is

$$ E = \frac{1}{2} \mu \mathbf{v}^2 - \frac{G m_1 m_2}{r} $$

where $r$ is the separation between $m_1$ and $m_2$, $G$ is the Newton’s constant, $\mu = m_1 m_2 / (m_1 + m_2)$ is the reduced mass of the binary, $E$ is the total energy, $r$ is the center of mass reference frame, $\nu = \mu |\mathbf{v} \times \mathbf{r}|$ is the system total angular momentum and $\nu$ is the true anomaly.

The timescale of the SN explosion and the mass-loss is short compared with the dynamical time of the system, and we can therefore assume that during the SN the primary object is not changing its position, namely $r$ is not changed. After the SN the primary star with mass $m_p$ undergoes prompt mass loss, and the NS remnant, with mass $m_{NS}$, is given a natal kick, $\Delta \mathbf{v}_{kick}$. In order to calculate the relative velocities between the binary components just after the kick one needs to add the random natal kick velocity vector to the relative velocity between the objects, i.e.

$$ \mathbf{r}_{SN} = \mathbf{r} + \Delta \mathbf{v}_{kick} $$

The solution for the new two body problem is then

$$ \frac{1}{r} = \frac{\mu_{SN} G m_1 m_2}{E_{SN}} \left( 1 + \sqrt{1 + \frac{2E_{SN}^2}{\mu_{SN} G^2 m_1^2 m_2^2} \cos \nu} \right) $$

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From the last equation we can easily compute the closest approach of the NS to its companion, $r_{min}$. If the binary is disrupted i.e. $E_{SN} > 0$ and $r > 0$, the closest separation is the separation at the moment of the SN, otherwise it is given by

$$ r_{min} = \frac{\mu_{SN} G m_1 m_2}{E_{SN}} \left( 1 + \sqrt{1 + \frac{2E_{SN}^2}{\mu_{SN} G^2 m_1^2 m_2^2} \cos \nu} \right)^{-1} $$

3 POPULATION SYNTHESIS AND NUMERICAL SETUP

In order to obtain the statistics of a large population we used the publicly available open code - BSE (Hurley et al. 2002). The parameters used are presented in Table 1. The initial conditions for the binary population were constructed as to follow the observed binary parameters as reviewed by Duchêne & Kraus (2013). We modeled $n = 587,019$ binaries with primary mass $m_1 > 8 M_\odot$ from a Salpeter initial mass function, the sma, $a$, was randomly chosen from a peak-power law distribution (see Figure 2 in Duchêne & Kraus 2013). The mass ratio $q$ was randomly generated from a uniform distribution for $a < 0.45 AU$ (corresponding to orbital periods of < 20 days for a $30 M_\odot$ binaries); for wider separations ($0.45 AU \leq a \leq 50 AU$) a power law distribution $f(q) \propto q^{-3}$ with $\gamma = -2$ was used (Moe & Di Stefano 2016). The eccentricities of the binaries were chosen from a thermal distribution $f(e) \propto e^\eta$ with $\eta = 1$. The metallicity was chosen to be solar, namely $z = 0.02$.

We evolved the system until one of the companions becomes a NS through a SN or accretion induced collapse (AIC). The population synthesis code provides the orbital and stellar parameters, i.e.
the sma $a$, the eccentricity $e$, the mass and radius of the companion and the mass of the NS. Next, we generated a random position in the binary orbit by randomizing the mean motion and calculating the eccentric anomaly, $E$ from it. The eccentric anomaly translates to separation, $r$, by $r = a(1 - e \cos E)$, and hence specifies the velocity $v$. In order to produce the natal kick velocity vector we randomized the kick velocities chosen from a Maxwellian distribution with a velocity dispersion of $\sigma = 270 \text{km s}^{-1}$ (Hobbs et al. 2005), and isotropic unit vector around the NS. We used eq. (7) to determine the minimal separation between the newly born NS and its companion. Accounting for the closest approach together with the radius of the companion at that point in the evolution we can determine the outcome, and identify cases that produce direct collisions, $\mu$TDEs or tidal captures.

We flag a direct collision if

$$r_{\text{min}} < R_s + R_{\text{NS}} \quad (8)$$

where $R_s$ is the secondary radius and $R_{\text{NS}}$ is the NS radius given from the population synthesis code. The criteria for $\mu$TDE is given by the following condition

$$R_s + R_{\text{NS}} < r_{\text{min}} < R_{\text{Tidal}} \quad (9)$$

where $R_{\text{Tidal}} = R_s (2m_{\text{NS}}/m_s)^{1/3}$. The criteria for tidal capture is

$$r_{\text{min}} < R_s \left( \frac{GM_{\text{NS}} (m_{\text{NS}} + m_s)}{R_s v_{\text{kick}}^2} \right)^{1/b} \quad (10)$$

is taken from Fabian et al. (1975).

Note that when $R_{\text{Tidal}} < R_s$, the tidal radius is inside the companion radius. In such cases we flag the outcome as a collision.

4 RESULTS

4.1 Stellar/dynamical evolution before the SN

In this subsection we present the evolution of a population binaries from the initial conditions at zero age main sequence (ZAMS) until the formation of the first NS in the system, either through a core-collapse SN or an AIC, if such occurs, as determined by the BSE code (Hurley et al. 2002). These set the conditions of the binaries just before the SN kick.

In the left panel of Figure 1 we present the initial sma, $a$, distribution at ZAMS, the beginning of the integration, for the 587,019 systems. On the right panel we present the sma distribution just before the SN, $a_{\text{SN}}$. The number of systems that survive as binaries up to the stage where a SN explodes or an AIC event takes place leaving behind a NS is 144,915. About three quarters of the systems the binary components merged or the binary was disrupted before any NS was formed. The remaining systems have a wider sma distribution compared with the initial population due binary interaction or stellar evolution, e.g. common envelope, tidal friction or mass loss from one of the companions.

In Figure 2 we present the eccentricity distribution of the population just before the SN. One can see the effects of binary evolution, mainly the circularization of the binaries due to tidal interaction during the giant phase.

The post-SN outcome strongly depends on the binary mass and separation. Figure 7 shows the average mass of binaries just prior to the SN as a function of their orbital separation. Binaries with separation wider than $\sim 10$ AU have a typical mass of $m_b \approx 15 M_\odot$ while binaries with separation of $\sim 1$ AU have a wider mass range between $m_b \approx 20 - 28 M_\odot$. Closer binaries have lower total masses ranging between $m_b \approx 2 - 10 M_\odot$. In the right panel of Figure 7 we plot the averaged normalized binary mass loss, $\langle \Delta m/m_b \rangle$, namely the ratio between the mass loss due to the SN and the binary total mass prior the SN. Hills (1983) showed that the normalized mass loss is a measure for dissociation. All binaries dissociate if the following condition is met

$$\Delta m/m_b \approx \frac{r}{2a_{\text{SN}}} \left( \frac{1}{v_{\text{kick}}} \right)^2 - 2 \left( \frac{1}{v_c} \right) \left( \frac{\Delta v_{\text{kick}}}{v_c} \right) \cos \theta \quad (11)$$

where $v_c$ is the relative orbital velocity between the stars when $r = a_{\text{SN}}$, $v$ is the relative orbital velocity at the moment of the explosion and $\theta$ is the angle between $v$ and $\Delta v_{\text{kick}}$. Hence for sufficiently large mass loss any binary would dissociate.

4.2 Post-SN dynamical evolution

Figure 3 shows the average eccentricity as a function of separation before the SN (left panel) and immediately after the SN (right panel). In the left panel we explicitly see the circularization of the binaries with separation smaller than $\sim 1$ AU and a monotonically increasing trend with increasing separation. Disrupted binaries have a hyperbolic trajectory that can result in two outcomes: (i) the NS is kicked away from its companion never to interact with it (ii) the NS is kicked towards its companion and may interact with it as discussed in the previous subsection. In Figure 4 we present the fraction of post-kick bound systems as a function of $a_{\text{SN}}$. The data shows an inverse trend of the fraction of bound systems with sma, as expected, given the ratio of the kick velocity $v_{\text{kick}}$ and the orbital velocity, $v_{\text{o}}$, at the moment of the SN. The data indicates that at separation of $\sim 1$ AU most binaries are disrupted following the natal kick imparted by the SN explosion.

4.2.1 The Distribution of closest approaches after the natal kick

In this subsection we present the main numerical results; the distribution of the closest approaches of the newly born NSs to their companions. In Figure 5 we compare the distribution of the closest approaches with the separation (and not the sma) between the objects just before the SN.

Most of the systems will not interact after the SN kick. As discussed above, the systems which do interact can be divided between three non-trivial outcomes: (i) Collision (eq. (8)) (ii) $\mu$TDE (eq. (9)). (iii) Tidal capture (eq. (10)). We note that the majority of systems that result in one of these three outcomes are considered bound in the moment following the SN, $E_{\text{NS}} < 0$. Note that the tidal capture case includes binaries that were bound following the SN, and therefore tidal capture is somewhat of a misnomer. However, the strong tidal interaction dissipates a significant fraction of the orbital energy as to dynamically capture the binary into a much closer binary compared with its face value properties post-kick in the absence of tidal interactions.

In Table 2 we present a summary of the results. The number of systems resulting in a direct collision is 2,176 which is $\approx 0.9$% of the surviving binaries and $\approx 0.37$% of the total sample. In order to provide rate estimate per galaxy, we consider fiducial number for the Milky-Way galaxy, taking $2 \times 10^{11}$ stars, of which $4 \times 10^8$ are stars with mass greater than $8M_\odot$. Taking these values, and assuming the stars in the Galaxy were formed continuously over 10Gyr, our results translate to a post-SN close encounters rate of $\approx 1 \times 10^{-4}$ yr$^{-1}$. Among these systems 1,809 ($\approx 83.1$%) were bound the moment following the SN and 367 ($\approx 26.9$%) were unbound.
presents the distribution of the elapsed times since the SN. we present the pre-explosion sma distribution for each distribution of the surviving binaries, 144 systems. Approximately 55% of the binaries are circular due to tidal interactions. The number of systems that satisfied the TDE criteria is 309 (giving a TDE rate of $\sim 2 \times 10^3$ yr$^{-1}$) of which 236 ($\sim 76.3\%$) were bound and 73 ($\sim 23.7\%$) were unbound. The number of tidally captured system is 156 (production rate of $\sim 1 \times 10^2$ yr$^{-1}$), comprised out of 151 ($\sim 96.7\%$) bound and 5 ($\sim 3.3\%$) unbound systems.

The close encounters occur when the stars in the binary system are sufficiently close, namely at periapsis of the trajectory (either elliptical or hyperbolic). The elapsed time since the SN down to the periapsis approach is given by the known formula from Kepler equations. For a bound binary,

$$\Delta t = \sqrt{\frac{a_{\text{post}}}{G M_{\text{binary}}}} \left( \frac{\pi}{2} - \arcsin \left( \frac{1 - \hat{a}}{e} \right) \right)$$

if $r \cdot \hat{r} < 0$

$$\Delta t = \sqrt{\frac{a_{\text{post}}}{G M_{\text{binary}}}} \left( 2\pi - \left( \frac{\pi}{2} - \arcsin \left( \frac{1 - \hat{a}}{e} \right) \right) \right)$$

if $r \cdot \hat{r} > 0$ (12)

and for an unbound binary,

$$\Delta t = \sqrt{\frac{-a^3}{G M_{\text{binary}}}} \log \left( 1 - \hat{a} + \sqrt{\hat{a}^2 - 2 \hat{a} - (e^2 - 1)} \right)$$

(13)

where $G$ is Newton’s constant, $\hat{a} \equiv r/a$ is the ratio between the separation and the sma immediately after the SN; and $M_{\text{binary}}$ and $e_{\text{post}}$ are the post-SN binary mass and eccentricity, respectively. Figure 6 presents the distribution of the elapsed times since the SN (“delay times” hereafter). The distributions are shown for each type of interaction. The delay time distribution for collisions and tidal-captures follow similar behavior, with $\langle \Delta t \rangle_{\text{collision}} \approx 4.7 \times 10^6$ sec and $\langle \Delta t \rangle_{\text{capture}} \approx 9.3 \times 10^5$ sec. The $\mu$TDEs follow a different distribution with much shorter delay times, with an average time of $\langle \Delta t \rangle_{\text{TDE}} \approx 3 \times 10^3$ sec $\approx 8$ hours.

4.2.2 Dependence of post-SN outcomes on pre-SN binary configurations and kicks

In Figure 8 we present the pre-explosion sma distribution for each type of interactions. Direct collisions and tidal captures follow similar distributions with an sma range of $\sim 0.05 - 1$AU with $\langle a_{\text{pre}} \rangle_{\text{collision}} \approx 0.44$ AU and $\langle a_{\text{pre}} \rangle_{\text{capture}} \approx 0.41$AU, while $\mu$TDEs follow a much tighter sma of $10^{-4} - 10^{-1}$ AU with $\langle a_{\text{pre}} \rangle_{\text{TDE}} \approx 0.012$ AU. These follow the same trends as seen for the delay-time
Close encounters in NS binaries

Figure 3. **Left panel:** The average eccentricity of the binary prior to the SN. All systems with separation of < 1 AU circularized due to tidal interactions during the course of the binary stellar evolution. **Right panel:** The average eccentricity of bound binaries after the SN and natal kick.

| Outcome       | Systems   | Rates (yr⁻¹) | Bound systems | Unbound systems |
|---------------|-----------|--------------|---------------|-----------------|
| Collisions    | 2176 (0.37%) | 1 × 10⁻⁴    | 1809 (83.1%)  | 367 (16.9%)     |
| µTDE          | 309 (0.05%)  | 2 × 10⁻⁵    | 236 (76.3%)   | 73 (23.7%)      |
| Tidal capture | 156 (0.02%)  | 1 × 10⁻⁵    | 151 (96.7%)   | 5 (3.3%)        |

Table 2. Summary of the possible close encounter outcomes from NS natal kick (and the rates per Milky-Way galaxy). The numbers are out of a total of 587,019 modeled systems.

Figure 4. The fraction of bound systems as a function of semi major axis. An inverse trend is visible. Roughly all systems are disrupted at ~ 1 AU.

distributions (Figure 6), as expected from the relation between delay times and pre-SN separations.

Figure 9 shows the kick velocity distribution for all types of systems found to be strongly interacting. The averaged velocity distribution for all types of systems is \( \sim 349\text{km} \cdot \text{s}^{-1} \) (compared with \( \sim 430\text{km} \cdot \text{s}^{-1} \) for the primordial averaged kick, taken from the Maxwellian distribution; not shown) and the standard deviation (std) is \( \sim 151\text{km} \cdot \text{s}^{-1} \) (compared with \( \sim 180\text{km} \cdot \text{s}^{-1} \) for the Maxwellian distribution). The systems undergoing collisions have a kick distribution with an average value of \( v_k \approx 345\text{km} \cdot \text{s}^{-1} \) with std of \( \sim 140\text{km} \cdot \text{s}^{-1} \). Tidal captured systems experience an average kick velocity of \( v_k \approx 223\text{km} \cdot \text{s}^{-1} \) with std of \( \sim 88\text{km} \cdot \text{s}^{-1} \). The µTDEs have an average kick velocity of \( v_k \approx 439\text{km} \cdot \text{s}^{-1} \) with std of \( \sim 188\text{km} \cdot \text{s}^{-1} \). These result are also consistent with Figure 8 indicating that shorter period binaries (higher orbital velocities) are the progenitors for µTDEs.

4.2.3 Distribution of stellar components

The final outcome of the post-SN close encounters is strongly dependent on the radius of the stellar companion of the NS, which itself is related to the mass and stellar evolutionary stage of the star. These dependencies are well reflected in the distributions of these properties, as we show below.

The distribution of companion radii is shown in Figure 10. The average values of the companion radius in the collision, capture and
In the previous subsection we described the results for $\sigma_{\text{kick}} = 270 \text{ km/s}$, in this subsection we will compare the results to the same simulation for a different natal kick velocity dispersion, $\sigma_{\text{kick}} = 190 \text{ km/s}$ (Hurley et al. 2002), in order to explore the dependence of the outcomes on the assumed distribution of natal kicks. The number of events in all three outcomes is 2,560 compared with 2,641 events with the higher velocity dispersion, this change is insignificant (smaller than 0.02%). We found 2,108 direct collisions, 271 $\mu$TDEs and 181 tidal captures into a close binary. The slow distribution (not shown) is somewhat altered (compared with Figure 8), and gives rise to larger separations, on average, as expected from a smaller natal kick. The average values for the different type of interaction are $\langle a_{\text{pre}} \rangle_{\text{collision}} = 0.52 \mu$A, $\langle a_{\text{pre}} \rangle_{\text{capture}} = 0.47 \mu$A and $\langle a_{\text{pre}} \rangle_{\mu\text{TDE}} = 0.017 \mu$A, the overall average for all interactions is $\langle a_{\text{pre}} \rangle = 0.445 \mu$A.

4.4 Gravitational wave sources and short-GRBs

In the case of a Black Hole (BH) companion an extremely close periastron can result in the formation of a binary which could merge through GW emission in a Hubble-time and produce a GW source detectable by LIGO. The merger itself might give rise to the accretion of the disrupted NS on the BH and possible production of a short-GRB. The equation that governs the dynamics in a GW emitting systems is given by Peters (1964)

$$\frac{da}{dt} = -\frac{64}{5} G^2 m_1 m_2 (m_1 + m_2) \left(1 + \frac{73}{24} e^2 + \frac{37}{96} e^4 + \frac{121}{304} e^6\right)$$

$$\frac{de}{dt} = -e \frac{304}{15} G^2 m_1 m_2 (m_1 + m_2) \left(1 + \frac{121}{304} e^6\right)$$

where $c$ is the speed of light. Given the mass of the binary components (for a BH companion) $m_1$, the sma $a_{\text{pre}}$, and the binary eccentricity $e_{\text{pre}}$, we can compute the merger time for each binary. Our simulations produced 46, 155 BH-NS binaries, out of which 484 bound systems and the rest are unbound. In Figure 14 we present the number of systems that merge as a function of time since the SN (merger time).

5 ANALYTIC UNDERSTANDING

In this subsection we present our analytic treatment of the natal-kick scenario. The binary system is comprised by a NS progenitor with mass $m_p$ that in turn becomes a NS with mass $m_{\text{NS}}$, the prompt mass loss is denoted by $\Delta m$, and a companion with mass $m_c$ and radius of $R_c$, with a separation $r$, sma $a_0$ and eccentricity $e$; for $e = 0$ we get $r = a_0$. We denote the combined binary mass prior the SN by $m_{\text{SN}} \equiv m_p + m_c$, the combined mass post-SN is denoted by $m_{\text{SN}} \equiv m_p + m_{\text{NS}}$. The companion can be of any stellar type, and therefore its size is important for the interaction cross section with the kicked NS.

The SN explosion governs the binary dynamical outcome through the prompt mass loss, $\Delta m/m_c$ and through the natal kick velocity vector $\mathbf{v}_{\text{kick}}$. For a detailed treatment of sudden mass loss and natal kicks see Hills (1983). The natal kick, $\mathbf{v}_{\text{kick}}$, changes both the energy and the angular momentum of the binary, hence it correspondingly changes both the sma and the eccentricity. Hills (1983) finds the changes in the sma due to the natal kick and the mass loss (for initially circular orbit) to be

$$\frac{a_{\text{SN}}}{a} = \frac{1 - 2 \Delta m/m_c - \mathbf{v}_{\text{kick}}^2}{2 - 2 \Delta m/m_c - \mathbf{v}_{\text{kick}}^2},$$

where $\mathbf{v}_{\text{kick}}$ is the relative velocity of the binary components after the SN, namely $\mathbf{v}_{\text{kick}} = \mathbf{v}_0 + \mathbf{v}_{\text{kick}}$. From the cosine theorem we get

\[ a_{\text{SN}}/a \approx 1 - \frac{2 \Delta m/m_c}{2 - 2 \Delta m/m_c - \mathbf{v}_{\text{kick}}^2}. \]
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Figure 7. Left panel: The average binary mass, \( \langle m_b \rangle \) just prior to the SN, as a function of the orbital separation. Right panel: The average of the normalized mass loss \( \langle \Delta m/m_b \rangle \), as a function of orbital separation. The higher the value \( \Delta m/m_b \), the higher is the probability to disrupt the binary.

Table 3. Summary of the possible close encounter outcomes from NS natal kick for AIC-formed NSs (and the rates per Milky-Way galaxy). Natal kicks drawn from a Maxwellian distribution with \( \sigma_{\text{AIC}} = 20 \text{ km s}^{-1} \).

| Outcome       | Systems (from AIC events) | Rates (yr\(^{-1}\)) | Bound systems | Unbound systems |
|---------------|---------------------------|----------------------|---------------|-----------------|
| Collisions    | 82 (3.8\%)                | 5 \times 10^{-6}    | 80 (97.5\%)  | 2 (2.5\%)       |
| \( \mu \text{TDE} \) | 185 (8.6\%)               | 1 \times 10^{-5}    | 185 (100\%)  | 0               |
| Tidal capture | 5 (0.2\%)                 | 3 \times 10^{-7}    | 5 (100\%)    | 0               |

Table 4. Same as Table 2 but with \( \sigma_{\text{kick}} = 190 \text{ km s}^{-1} \). Total number of interacting systems after NS formation is 2,560. The rates are calculated per a Milky-Way galaxy.

| Outcome       | Systems (from AIC events) | Rates (yr\(^{-1}\)) | Bound systems | Unbound systems |
|---------------|---------------------------|----------------------|---------------|-----------------|
| Collisions    | 2108 (0.35\%)             | 1 \times 10^{-4}    | 1727 (82\%)  | 381 (18\%)      |
| \( \mu \text{TDE} \) | 271 (0.03\%)              | 2 \times 10^{-5}    | 205 (75.6\%) | 66 (24.4\%)     |
| Tidal capture | 181 (0.03\%)              | 1 \times 10^{-5}    | 173 (95.5\%) | 5 (4.5\%)       |

Figure 8. The pre-SN sma distribution of all (post-SN) interacting binaries. Colors and line types the same as in figure 6.

The following geometric relation

\[
\frac{v_{\text{SN}}^2}{v_0^2} = \left( \frac{v_{\text{kick}}}{v_0} \right)^2 + 2 \frac{v_{\text{kick}}}{v_0} \cos \theta + 1,  \tag{17}
\]

where \( \theta \) is the angle of the natal kick velocity vector and the orbital velocity vector. From equation (16) and (17) we find a minimum value of \( v_{\text{SN}} \) that dissociates the binary, namely for any value of \( v_{\text{SN}} \) that satisfies the following condition

\[
v_{\text{SN}} \geq v_0 \left( 1 + \sqrt{2m_l/m_b} \right),  \tag{18}
\]

the binary is disrupted. This explains the result presented in Figure 4, that states that all binaries with initial sma larger than \( \sim 1 \text{ AU} \) are disrupted by the natal kick received from the SN explosion.
Collisions and tidal captures typically occur with large, \( \sim 10 R_{\odot} \) stars while \( \mu \)TDEs cases are typically disruptions of WD companions.

For simplicity we consider all binaries prior the SN to be circular, namely \( e \approx 0 \). Hence there are two relevant velocity scales in the system that corresponds to three regimes. First, the relative orbital velocity at the pre-SN, \( v_0 = \sqrt{Gm_{\text{NS}}/a_0} \). Second, the natal kick velocity, \( v_{\text{kick}} \), drawn randomly from a kick velocity distribution \( f(v_{\text{kick}}) \). These velocities together with the mass loss \( \Delta m \) govern the dynamics of the system. We identify three regimes (i) \( \tilde{v} \equiv v_{\text{kick}}/v_0 \gg 1 \) where the kick velocity is much bigger than the orbital velocity, this implies large separation for a given kick velocity distribution, (ii) \( \tilde{v} \ll 1 \), where the kick velocity is much smaller than the orbital velocity, this implies extremely close binary at the moment of SN, (iii) \( \tilde{v} \approx 1 \), where the kick velocity comparable to the orbital velocity.

In the first regime \( v_{\text{kick}} \gg v_0 \) all systems are disrupted following the SN explosion. Further, the new relative velocity is \( v_{\text{post}} \approx v_{\text{kick}} \) and in approximately half the cases the NSs will be kicked in the direction of the companion, namely the closest approach will be closer than the separation of the binary in the instant of the SN. For these systems we can estimate the cross-section for different close-interactions with the companion. For peri-apsis passage with a distant \( R \) of the companion we can estimate the cross section, \( \sigma_{\text{ps}} \) to be

\[
\sigma_{\text{ps}} = \sigma_{\text{geom}} \left( 1 + \left( \frac{v_{\text{esc}}}{v_{\text{ps}}(R)} \right)^2 \right),
\]

where \( \sigma_{\text{geom}} = \pi R^2 \) is the geometric cross section, and \( R \) is the relevant radius, and the escape velocity is calculated at \( R \), namely \( v_{\text{esc}} = \sqrt{Gm_{\text{NS}}/m_{\text{ps}}(R)} \). We are interested in very close encounters: collisions, \( \mu \)TDEs or tidal-captures. These correspond to \( v_{\text{esc}}/v_{\text{ps}}(R) \geq 1 \). With such parameters the systems are well inside the gravitational focusing regime, in which case the cross-section for close-interactions scales linearly with \( R \),

\[
\sigma_{\text{ps}} \propto \sigma_{\text{geom}} \left( \frac{v_{\text{esc}}}{v_{\text{ps}}(R)} \right)^2 = \pi R^2 \left( \frac{G(m_{\text{NS}} + m_{\text{ps}})}{R} \right) \propto R^2,
\]

The fraction of the systems in which the NS encounters the companion at a distance closer than \( R \) is

\[
F_1 = \int \frac{\sigma_{\text{ps}}}{4\pi R^2} f(r) \, dr,
\]

where \( f(r) \) is the separation distribution within the relevant integration boundaries, i.e. including the separations of all binaries. It is clear from the structure of eq. (21) that \( F_1 \propto R_1/r \), i.e. the probability for a kick in a binary with initial sma of 10AU with a MS companion (taking \( R_1 = 1R_\odot \)) to result in a direct collision scales like \( \sim 1R_\odot/10AU \approx 4 \times 10^4 \).

In the second regime, \( v_{\text{kick}} \ll v_0 \), and third regime, \( v_{\text{kick}} \approx v_0 \) most of the binaries survive the SN, this implies close pre-SN binaries are involved. The most compact pre-SN binaries are on circular orbits due to tidal interaction that lead to their small sma, post-common-envelope. In this case we can simply estimate the periapsis, \( q \) of the new Keplerian orbit due to the kick and mass loss. The periapsis of the new orbit with the sma taken from (16)

\[
q = a_{\text{PSN}} \left( 1 - e_{\text{PSN}} \right).
\]

The angular momentum and the energy of the binary after the SN determines the value of \( q \). We can calculate \( a_{\text{PSN}} \) from eq. (16) and can find \( e_{\text{PSN}} \) from the specific angular momentum equation,

\[
l = r x v_{\text{PSN}} = \sqrt{Gm_{\text{NS}} \left( 1 - e_{\text{PSN}}^2 \right)},
\]

We use the angles defined by Troja et al. (2010), where \( \theta \) is the angle between the \( v_0 \) and \( v_{\text{kick}} \) and \( \phi \) is the angle between the initial orbital plane and the plane span by \( (v_0, v_{\text{kick}}) \). Following the calculation done by Troja et al. (2010) and from eq. (23) we get the following conditions for a close encounter within \( q \leq R \),

\[
\sin^2 \phi \leq \frac{\xi^2 - (1 + \tilde{v} \cos \theta)^2}{\tilde{v}^2 \sin^2 \theta} \leq \frac{-\left( \xi + 1 \right)}{\tilde{v}} \leq \cos \theta \leq \xi - 1 \tilde{v}
\]

We note that Troja et al. (2010) approximated eq. (26) by \( \xi = (2m_{\text{NS}}/m_{\text{MS}})^{1/2} \) while we keep this term because we find several very close binaries prior the SN. In Figure (14) we present the probability for interaction between a NS and a MS star companion with mass of 1 Ms and radius of 1 R_\odot at a separation of 1 R_\odot (collision) as a function of the \( \tilde{v} \). We find that the probability distribution peaks around unity, namely when the kick velocity approximately equals the orbital velocity, but with the direction opposite direction in respect to the orbital velocity; hence, the resulting angular momentum is minimized. Consequently, we expect that in the second regime no interacting binaries will be proceeded, while we expect most of the interacting binaries to originate from the third regime, where \( \tilde{v} \approx 1 \). Indeed, these conclusions are consistent with our numerical results shown in Figure 8. We note that the difference in comparison with the \( \mu \)TDE is due to the difference in the typical companion radius, which is smaller by an order of magnitude lower, on average, in the \( \mu \)TDE case when comparing with the other type of interactions (see Figure 10).

Following the analytic treatment we reach the conclusion that the third regime is the most conducive to form a nontrivial interaction between the components. We present the distribution of \( \tilde{v} \) in Figure 15: \( \sim 91.5% \) of the interacting systems have a value of \( \log \tilde{v} \) between the values of \( -0.5 \) and \( 0.5 \).

For a given binary mass in the pre-SN system and given the natal kick velocity distribution we can define the separation scale \( r_i \) by the following relation:

\[
\langle v_{\text{kick}} \rangle = v = \frac{Gm_{\text{NS}}}{R_i},
\]

Figure 10. Distribution of the companion radii for each type of interaction. Colors and line types the same as in figure 6. Collisions and tidal captures typically occur with large, \( \sim 10 R_{\odot} \) stars while \( \mu \)TDEs cases are typically disruptions of WD companions. The fraction of the systems in which the NS encounters the companion at a distance closer than \( R \) is

\[
F_1 = \int \frac{\sigma_{\text{ps}}}{4\pi R^2} f(r) \, dr,
\]

where \( f(r) \) is the separation distribution within the relevant integration boundaries, i.e. including the separations of all binaries. It is clear from the structure of eq. (21) that \( F_1 \propto R_1/r \), i.e. the probability for a kick in a binary with initial sma of 10AU with a MS companion (taking \( R_1 = 1R_\odot \)) to result in a direct collision scales like \( \sim 1R_\odot/10AU \approx 4 \times 10^4 \).
where \((\langle v_{\text{kick}} \rangle)\) is the mean kick velocity from distribution \(f(v_{\text{kick}})\).

We note an inverse relation between the kick velocity and the separation scale \(r_s\), and therefore smaller \((\langle v_{\text{kick}} \rangle)\), corresponds to larger separation scales, which naturally explains the results in section 4.3.

6 IMPLICATIONS

Many studies explored binary stellar evolution in NS binary systems and its outcomes and implications (see e.g. Yoon 2015), and in particular accounted for the effects of NS natal kicks. These works explored the long term evolution of binaries and their various transient outcomes (GRBs, GW sources) or the interacting X-ray binaries or exotic objects they could produce. Nevertheless, most of the studies did little study of the direct implications of close encounters occurring on dynamical timescales following the SN, and in particular did not consider binaries that formally became unbound even if they approached sufficiently close to strongly interact immediately after the kick. Our work focuses on these latter issues, namely the implications of close encounters immediately after the supernova and the natal kick, which were not, or scarcely studied before. In addition we do consider some aspects of the longer timescale evo-

Figure 11. Distribution of NS-companion and progenitor stellar types. Upper left panel: Collisions, ~ 86.8% of all collisions have a MS companion. Middle left panel: Tidal capture, ~ 94.9% of all tidal capture NSs in a new orbit have a MS companion. Bottom left panel: \(\mu\)TDE, ~ 44% of all \(\mu\)TDE have a Hertzsprung gap He star companions. Upper right panel: Collisions, ~ 77% of all collisions are from a Hertzsprung gap He star progenitors. Middle right panel: Tidal capture, ~ 86.5% of progenitors are from a Hertzsprung gap He star. Bottom right panel: \(\mu\)TDE, ~ 97% of progenitors are of O-He WD stellar type.
Figure 12. The pre-SN sma distribution of all (post-SN) interacting binaries with AIC-formed NSs, with kick velocity dispersion of $\sigma_{\text{AIC}} = 20\text{km s}^{-1}$: Colors and line types the same as in figure 6. This distribution resembles the distribution in figure 8 but with a shift for larger values of sma due to the smaller kick.

Figure 13. Cumulative number of BH-NS mergers, GW sources, as a function of elapsed time from the SN. The post-SN merger with the shortest delay time found in our sample occurs after only 51.6 yr after the SN explosion.

...solution, and in particular novel implications for eLISA GWs, as well as novel electromagnetic counterparts for GWs.

...Generally we find that in a fraction of $\sim 1 - 3 \times 10^{-3}$ of the massive systems we modeled the NSs either collided with companions; disrupt their companions via $\mu$TDE; or formed a close binary that in turn will eventually form an X-ray source. We note that the systems we consider also include cases of NS-BH binaries which will merge through GW waves inspiral in less than a Hubble time, and serve as GW sources observable by current (aLIGO) and next generation GW detectors (eLISA). These are extensively studied in the literature and here we only point out novel aspects of the formation of such system and their electromagnetic counterparts.

$\mu$TDEs and faint peculiar SNe: $\mu$TDEs are tidal disruption events by stellar compact objects Perets et al. (2016) that may explain ultra long Gamma ray bursts (GRBs). We find that $2 - 4 \times 10^{-4}$ of the NS binary systems undergo tidal disruption of the stellar companion. The majority of the companion are found to be either He stars or WDs. While the former may produce $\mu$TDEs as envisioned by Perets et al. (2016), the latter disruptions of WDs by NSs might produce faint transients, possibly resembling some types of peculiar faint SNe (Metzger 2012; Fernández & Metzger 2013) such as Calcium-rich SNe (Perets et al. 2010), though further studies of such events may shed more light on their actual observational properties. Note that the relatively short delay times between the SN and the $\mu$TDE would give rise to an ultra-long GRB accompanied by a SN (likely a type Ib/c given the stripped companion), or in the case of disruption of a WD, the initial SN might be accompanied by the faint peculiar transient, in which case the former, regular SN might mask the appearance of the latter.

Short GRBs: Our models only consider the case of a single NS in a binary, and generally did not follow the binary later evolution after the formation of the first NS (besides the case of a BH companion to the newly formed NS, in which case only dynamical evolution, but not stellar evolution is considered, as we discuss below). In principle, if the binary survives the kick, the companion to the NS star could itself become a NS later on, for a sufficiently massive progenitor. In this case NS-NS interactions should be con-
sidered. Troja et al. (2010) calculated the possibility for NS-NS collisions following natal kicks, which may result in GW sources accompanied by short GRBs. They studied the likelihood of such events as a function of the initial orbital parameters, and the kick velocity (but did not consider any previous stellar evolution leading to the pre-kick configurations). The authors found that in order to get a direct NS-NS collision from an initially circular orbit the kick velocity $v_{\text{kick}} \geq v_{\text{i}}$, where $v_{\text{i}}$ is the relative velocity between the NS progenitor and its NS companion in the circular per-expiration orbit; consistent with our analytic results.

We do consider other possible short-GRB progenitors such as BH-NS mergers. Most interestingly, the earliest merger (due to GW inspiral) we identify occurs only 50 yr post-SN. This suggests that short merger times are possible. Such short merger-time events can give rise to detectable SNe preceding short-GRBs, where only long-GRBs were suggested (and observed) to be connected to SNe.

GW sources: The possibility of short merger times for the BH-NS case gives rise to potentially important implications for GW sources and their electromagnetic counterparts. Sufficiently short merger-time can result in a unique novel type of eLISA GW sources; since eLISA is sensitive to GW sources from binaries at large separations, it can generally track a GW binary as it inspirals from larger separations to smaller ones for months or years (but before they enter the aLIGO band; Sesana 2016). In the case of NS-kick - formed GW source, the source could promptly appear at small separations, without showing the prior typically expected long term inspiral from larger separations, thus providing a smoking gun signature for its natal-kick formation process. Moreover, cases of short (<yr) merger times, if such exist, can provide novel type of electromagnetic counterparts to aLIGO GW sources, i.e. a SN which accompanies a long-GRB. In this case one might expect a long-GRB which is preceded by the SN, rather than the typical case discussed in the literature where the initial collision produced the long GRB.

X-ray binaries: X-ray binaries with a NS primary are binary systems with short period orbits allowing for mass transfer from the primary, usually evolved star, to an accretion disk around the NS. We find that $\sim 1 - 3 \times 10^{-4}$ of the NSs end up in sufficiently close eccentric orbits as to become X-ray binaries either immediately after the SN or later on when the companion evolves into its giant phase and Roche-lobe overflow commences. Out of 156 NSs that are captured into a close orbit (181 systems for $\sigma_{\text{kick}} = 190 \text{ km s}^{-1}$) 11 host low MS ($< 2 M_{\odot}$) companions; progenitors for a low mass X-ray binaries systems (LMXBs), while the rest are more massive progenitors producing high Mass X-ray binaries (HMXBs), usually on a wider orbit than the LMXBx.

**7 SUMMARY**

In this work we have explored the end result of natal-kicks given to NS in binary systems. We identified three regimes corresponding to the value of the kick velocity, and the importance of the mass loss, $\Delta m$ and $\text{sm}a$. We preformed population synthesis of 587,019 binary systems and evolved them until NS is formed. Next, we randomized the position in the binary orbit and drawn a natal kick from a Maxwellian velocity distribution with $\sigma_{\text{kick}} = 270 \text{ km s}^{-1}$ ($\sigma_{\text{kick}} = 190 \text{ km s}^{-1}$). Using eq. (7) we determined the closest approach $r_{\text{min}}$ of any NS to its companion, whether the binary survived the SN or whether it was disrupted. From the value of $r_{\text{min}}$ and the companion radius, $R$, and mass we identified the systems that underwent a strong encounter: (i) direct collision, namely $r_{\text{min}} < R + R_{\text{NS}}$ (ii) tidal disruption event, namely $R_{\text{NS}} \leq R_{\text{max}}(< R_{\text{NS}} < R_{\text{max}} < R_{\text{NS}})$ (iii) tidal capture, satisfying the condition in eq. (10) and not flagged as collision or TDE.

Using a simple analytic treatment we identified the important parameters that determine the natal-kick outcomes, and find them to explain well the results qualitatively.

Our calculations provide us for estimate of the rate of various type of transient events and possible production of exotic stars induced by the natal kicks, which are discussed in details. In particular we calculate the rates of TDEs as well as faint SNe from accretion of WDs on NSs; and we find the production rates of Thor-Henskag objects (if such exist; or the possible production of long-GRBs if the NS rapidly accrete the star with which they collide to collapse into a BH) and NS X-ray binaries (the latter were also studied by others e.g. Brandt & Podsadlowski 1995; Podsadlowski et al. 1995; Belczynski et al. 2006). Finally we consider the possible production of short GRBs as well GW sources

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from BH-NS mergers and point out the possible existence of short merger times since the SN; such cases could give rise to novel types of eLISA GW sources, as well as provide SN electromagnetic counterparts preceding aLIGO GW sources from BH-NS mergers.

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REFERENCES

Abadie J., et al., 2010, Classical and Quantum Gravity, 27, 173001
Arzoumanian Z., Chernoff D. F., Cordes J. M., 2002, ApJ, 568, 289
Belczynski K., Perna R., Bulik T., Kalogera V., Ivanova N., Lamb D. Q., 2006, ApJ, 648, 1110
Benamini P., Piran T., 2016, MNRAS, 456, 4089
Brandt N., Podsiadlowski P., 1995, MNRAS, 274, 461
Duchêne G., Kraus A., 2013, ARA&A, 51, 269
Fabian A. C., Pringle J. E., Rees M. J., 1975, MNRAS, 172, 15P
Fernández R., Metzger B. D., 2013, ApJ, 763, 108
Fryer C. L., Woosley S. E., Hartmann D. H., 1999, ApJ, 526, 152
Hills J. G., 1983, ApJ, 267, 322
Hobbs G., Lorimer D. R., Lyne A. G., Kramer M., 2005, MNRAS, 360, 974
Hurley J. R., Tout C. A., Pols O. R., 2002, MNRAS, 329, 897
Kalogera V., Belczynski K., Kim C., O'Shaughnessy R., Willems B., 2007, Phys. Rep., 442, 75
Leonard P. J. T., Davies M. B., 1994, in Holt S., Day C. S., eds, American Institute of Physics Conference Series Vol. 308, The Evolution of X-ray Binaries. p. 315
Moe M., Di Stefano R., 2016, preprint, (arXiv:1606.05347)
Nakar E., 2007, Phys. Rep., 442, 166
Perets H. B., et al., 2010, Nature, 465, 322
Perets H. B., Li Z., Lombardi Jr. J. C., Milcarek Jr. S. R., 2016, ApJ, 823, 113
Peters P. C., 1964, Physical Review, 136, B1224
Podsiadlowski P., Cannon R. C., Rees M. J., 1995, MNRAS, 274, 485
Podsiadlowski P., Langer N., Poelarends A. J. T., Rappaport S., Heger A., Pfahl E., 2004, ApJ, 612, 1044
Postnov K. A., Yungelson L. R., 2014, Living Reviews in Relativity, 17
Sana H., et al., 2012, Science, 337, 444
Sana H., et al., 2014, ApJS, 215, 15
Sesana A., 2016, Physical Review Letters, 116, 231102
Smartt S. J., 2015, Publ. Astron. Soc. Australia, 32, e016
Smartt S. J., Eldridge J. J., Crockett R. M., Maund J. R., 2009, MNRAS, 395, 1409
Soderberg A. M., et al., 2007, ApJ, 661, 982
Thorne K. S., Zykow A. N., 1977, ApJ, 212, 832
Troja E., Wynn G. A., O'Brien P. T., Rosswoog S., 2010, MNRAS, 401, 1381
Yoon S.-C., 2015, Publ. Astron. Soc. Australia, 32, e015
de Mink S. E., Belczynski K., 2015, ApJ, 814, 58