Casimir Forces for Robin Scalar Field on Cylindrical Shell in de Sitter Space

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November 29, 2021

Abstract

The Casimir stress on a cylindrical shell in background of conformally flat space-time for massless scalar field is investigated. In the general case of Robin (mixed) boundary condition formulae are derived for the vacuum expectation values of the energy-momentum tensor and vacuum forces acting on boundaries. The special case of the dS bulk is considered then different cosmological constants are assumed for the space inside and outside of the shell to have general results applicable to the case of cylindrical domain wall formations in the early universe.

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1 Introduction

The Casimir effect is one of the most interesting manifestations of nontrivial properties of the vacuum state in quantum field theory [1,2]. Since its first prediction by Casimir in 1948 [3] this effect has been investigated for different fields having different boundary geometries[4-7]. The Casimir effect can be viewed as the polarization of vacuum by boundary conditions or geometry. Therefore, vacuum polarization induced by a gravitational field is also considered as Casimir effect. The types of boundary and conditions that have been most often studied are those associated to well known problems, e.g. plates, spheres, and vanishing conditions, perfectly conducting conditions, etc. The cylindrical problem with perfectly conducting conditions was first considered in [8], for recent study ref.[9, 10].

In the context of hot big bang cosmology, the unified theories of the fundamental interactions predict that the universe passes through a sequence of phase transitions. These phase transitions can give rise to domain wall structures determined by the topology of the manifold $M$ of degenerate vacuua [11, 12, 13]. If $M$ is disconnected, i.e. if $\pi(M)$ is nontrivial, then one can pass from one ordered phase to the other only by going through a domain wall. If $M$ has two connected components, e.g. if there is only a discrete reflection symmetry with $\pi_0(M) = \mathbb{Z}_2$, then there will be just two ordered phase separated by a domain wall.

The time evolution of topological defects have played an important role in many branches of physics, e.g., vortices in superconductors [14] and in superfluid [15], defects in liquid crystals [16], domain wall [17, 18], cosmic string [12, 13] and a flux tube in QCD [19].

Zeldovich et al [11] have been shown that the energy density of the domain walls is so large that they would dominate the universe completely, violating the observed approximation isotropy and homogeneity. In other words, the domain walls were assumed to somehow disappear again soon after their creation in the early universe, for instance, by collapse, evaporation, or simply by inflating away from our visible universe. Much later however Hill et al[20] introduced the so called light or soft domain walls. They considered a late-time phase transition and found that light domain walls could be produced, that were not necessarily in contradiction with observed large-scale structure of the universe.

In addition, whatever the cosmological effects are, we find it important to obtain a better understanding of the dynamics of domain walls.

Casimir effect in curved space-time has not been studied extensively. Casimir effect in the presence of a general relativistic domain wall is considered in [21] and a study of the relation between trace anomaly and the Casimir effect can be found in [22]. Casimir effect may have interesting implications for the early universe. It has been shown, e.g., in[23] that a closed Robertson-Walker space-time in which the only contribution to the stress tensor comes from Casimir energy of a scalar field is excluded. In inflationary models, where the dynamics of bubbles may play a major role, this dynamical Casimir effect has not yet been taken into account. Let us mention that in [24] we have investigated the Casimir effect of a massless scalar field with Dirichlet boundary condition in spherical shell having different vacuua inside and outside which represents a bubble in early universe with false/true vacuum inside/outside. In this reference the sphere have zero thickness. In another paper [25] we have extended the analysis to the spherical shell with nonvanishing thickness. Parallel plates immersed in different de Sitter spaces in- and out-side is calculated in [26].

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In the present paper we will investigate the vacuum expectation values of the energy–momentum tensor of the conformally coupled scalar field on background of the conformally flat space-time. We will consider a cylindrical shell and boundary conditions of the Robin type on the shell. The latter includes the Dirichlet and Neumann boundary conditions as special cases. The Casimir energy-momentum tensor for these geometries can be generated from the corresponding flat spacetime results by using the standard transformation formula. Then we consider cylindrical shell with constant comoving radius having different vacuums inside and outside, i.e. with false/true vacuum inside/outside. Our model may be used to study the effect of the Casimir force on the dynamics of the cylindrical domain wall appearing in the simplest Goldston model. In this model potential of the scalar field has two equal minima corresponding to degenerate vacua. Therefore, scalar field maps points at spatial infinity in physical space nontrivially into the vacuum manifold [27]. Domain wall structure occur at the boundary between these regions of space. One may assume that the outer regions of cylinder are in $\Lambda_{\text{out}}$ vacuum corresponding to degenerate vacua in domain wall configuration.

The Casimir effect for the general Robin boundary conditions on background of the Minkowski spacetime was investigated in Ref. [28] for flat boundaries, and in [29, 30] for spherically and cylindrically symmetric boundaries in the case of a general conformal coupling. Here we use the results of Ref. [30] to generate vacuum energy–momentum tensor for the cylindrical shell in conformally flat backgrounds. The paper is organized as follows. In the next section the vacuum expectation values of the energy–momentum tensor and vacuum forces acting on shell are evaluated for a general case of a conformally-flat background. In section 3 we study the bulk Casimir effect for a conformal scalar when the bulk is a 4-dimensional de Sitter space. Finally, the results are re-mentioned and discussed in last section.

## 2 Vacuum expectation values for the energy-momentum tensor

In this paper we will consider a conformally coupled massless scalar field $\varphi(x)$ satisfying the equation

$$\left(\nabla_\mu \nabla^\mu + \xi R\right) \varphi(x) = 0, \quad \xi = \frac{D - 1}{4D}$$

on background of a $D + 1$–dimensional conformally flat spacetime with the metric

$$g_{\mu\nu} = e^{-2\sigma(r)} \eta_{\mu\nu}, \quad \mu, \nu = 0, 1, \ldots, D.$$  

In Eq. (1) $\nabla_\mu$ is the operator of the covariant derivative, and $R$ is the Ricci scalar for the metric $g_{\mu\nu}$. Note that for the metric tensor from Eq. (2) one has

$$R = De^{2\sigma} \left[ 2\sigma'' - (D - 1)\sigma'^2 \right],$$

where the prime corresponds to the differentiation with respect to $r$. We will assume that the field satisfies the mixed boundary condition

$$\left(A + Bn^i \nabla_i\right) \varphi(x) = 0$$

(4)
on the cylindrical shell with radius $a$. Here $n^i$ is the normal to the boundary surface, $\nabla_i$ - is the covariant derivative operator, $A$ and $B$ are constants. The results in the following will depend on the ratio of these coefficients only. However, to keep the transition to the Dirichlet and Neumann cases transparent we will use the form (4). It can be shown that for a conformally coupled scalar by using field equation (1) the expression for the energy–momentum tensor can be presented in the form

$$T_{\mu\nu} = \nabla_{\mu}\varphi\nabla_{\nu}\varphi - \xi \left[ \frac{g_{\mu\nu}}{D-1} \nabla_{\rho} \nabla^{\rho} + \nabla_{\mu} \nabla_{\nu} + R_{\mu\nu} \right] \varphi^2,$$

where $R_{\mu\nu}$ is the Ricci tensor. The quantization of a scalar filed on background of metric (2) is standard. Let $\{\varphi_\alpha(x), \varphi_\alpha^*(x)\}$ be a complete set of orthonormalized positive and negative frequency solutions to the field equation (1), obeying boundary condition (4). By expanding the field operator over these eigenfunctions, using the standard commutation rules and the definition of the vacuum state for the vacuum expectation values of the energy-momentum tensor one obtains

$$\langle 0|T_{\mu\nu}(x)|0 \rangle = \sum_\alpha T_{\mu\nu}\{\varphi_\alpha, \varphi_\alpha^*\},$$

where $|0\rangle$ is the amplitude for the corresponding vacuum state, and the bilinear form $T_{\mu\nu}\{\varphi, \psi\}$ on the right is determined by the classical energy-momentum tensor (5). In the problem under consideration we have a conformally trivial situation: conformally invariant field on background of the conformally flat spacetime. Instead of evaluating Eq. (6) directly on background of the curved metric, the vacuum expectation values can be obtained from the corresponding flat spacetime results for a scalar field $\bar{\varphi}$ by using the conformal properties of the problem under consideration. Under the conformal transformation $g_{\mu\nu} = \Omega^2 \eta_{\mu\nu}$ the $\bar{\varphi}$ field will change by the rule

$$\varphi(x) = \Omega^{(1-D)/2}\bar{\varphi}(x),$$

where for metric (2) the conformal factor is given by $\Omega = e^{-\sigma(r)}$. The boundary conditions for the field $\bar{\varphi}(x)$ we will write as following

$$\left( \bar{A} + \bar{B}\partial_r \right) \bar{\varphi} = 0,$$

with constant Robin coefficients $\bar{A}$ and $\bar{B}$. Comparing to the boundary conditions (4) and taking into account transformation rule (7) we obtain the following relations between the corresponding Robin coefficients

$$\bar{A} = A + \frac{D-1}{2} \sigma'(a) e^{\sigma(a)} B, \quad \bar{B} = B e^{\sigma(a)}.$$  

Note that as Dirichlet boundary conditions are conformally invariant the Dirichlet scalar in the curved bulk corresponds to the Dirichlet scalar in a flat spacetime. However, for the case of Neumann scalar the flat spacetime counterpart is a Robin scalar with $\bar{A} = (D-1)\sigma'(a)/2$ and $\bar{B} = 1$. The Casimir effect with boundary conditions (8) on cylindrical shell on background of the Minkowski spacetime is investigated in Ref. [28] for a scalar field with a general conformal coupling parameter. In the case of a conformally coupled scalar the corresponding regularized VEV’s for the energy-momentum tensor have the form

$$\langle 0|T_{\mu\nu}|0 \rangle = \text{diag}(\varepsilon, -p_1, -p_2, -p_3, \ldots, -p_D).$$
Here $\varepsilon$ is the vacuum energy density, $p_1, p_2, p_3 = p_4 = \cdots = p_D$ are effective pressures in the radial, azimuthal and longitudinal directions, respectively (vacuum stresses). These quantities are determined by the relations

$$ q_{SUB} = \frac{2^{1-D} \pi^{-(D+1)/2}}{a^{D+1} \Gamma(D/2 - 1/2)} \sum_{n=\infty}^{+\infty} \int_{0}^{\infty} dz \ z^{D+3} \frac{\bar{K}_n(z)}{K_n(z)} F_n^{(q)}[I_n(zr/a)], $$

where $I_n(z)$ and $K_n(z)$ are the modified Bessel functions, and

$$ F_n^{(e)}[f(z)] = \frac{1}{D-1} f^2(z) + \left(2\xi - \frac{1}{2}\right) \left[f^2(z) + \left(\frac{n^2}{z^2} + 1\right) f^2(z)\right] $$

$$ F_n^{(p_1)}[f(z)] = \frac{1}{2} \left[\left(\frac{n^2}{z^2} + 1\right) f^2(z) - f'^2(z)\right] - \frac{2\xi}{z^2} f(z) f'(z) $$

$$ F_n^{(p_2)}[f(z)] = -\left(2\xi - \frac{1}{2}\right) \left[f^2(z) + \left(\frac{n^2}{z^2} + 1\right) f^2(z)\right] + \frac{2\xi}{z^2} f(z) f'(z) - \frac{n^2}{z^2} f^2(z) $$

$$ F_n^{(p_1)}[f(z)] = -F_n^{(e)}[f(z)], \quad i = 3, \ldots, D. $$

Here and below we use the notation

$$ \tilde{f}(z) \equiv Af(z) + (B/a)zf'(z) $$

for a given function $f(z)$. Similarly the vacuum expectation values for the exterior of a single cylindrical shell can be obtained, the result is as following [28]

$$ q_{SUB} = \frac{2^{1-D} \pi^{-(D+1)/2}}{a^{D+1} \Gamma(D/2 - 1/2)} \sum_{n=\infty}^{+\infty} \int_{0}^{\infty} dz \ z^{D+3} \frac{\bar{I}_n(z)}{I_n(z)} F_n^{(q)}[K_n(zr/a)], $$

where we use notations (12)-(15). As we see, these quantities can be obtained from the ones for interior region by the replacements $I \rightarrow K$, $K \rightarrow I$. Using the expressions for the interior and exterior quantities we have

$$ F = \frac{2^{D} \pi^{-(D+1)/2}}{a^{D+1} \Gamma(D/2 - 1/2)} \sum_{n=\infty}^{+\infty} \int_{0}^{\infty} dz \ z^{D+1} \times $$

$$ \times \left[2\beta - 4\xi + (z^2 + n^2 - \beta^2 + 4\xi\beta) \frac{\bar{I}_n(z)K_n(z)}{z^2I_n(z)K_n(z)}\right] $$

for the total vacuum force acting per unit surface of the shell. In these formulae we have introduced the notation

$$ \tilde{f}(z) = z^\beta f(z), \quad \beta = A/B $$

for a given function $f(z)$.

The vacuum energy-momentum tensor on curved background (2) is obtained by the standard transformation law between conformally related problems (see, for instance, [31]) and has the form

$$ \langle T^\mu_\nu [g_{\alpha\beta}] \rangle_{\text{ren}} = \langle T^\mu_\nu [g_{\alpha\beta}] \rangle^{(0)}_{\text{ren}} + \langle T^\mu_\nu [g_{\alpha\beta}] \rangle^{(b)}_{\text{ren}}. $$

Here the first term on the right is the vacuum energy–momentum tensor for the situation without boundaries (gravitational part), and the second one is due to the presence of boundaries. As the quantum field is conformally coupled and the background spacetime is conformally flat the gravitational part of the energy–momentum tensor is completely
determined by the trace anomaly and is related to the divergent part of the corresponding effective action by the relation [31]

$$\langle T^\mu_\nu [g_{\alpha\beta}] \rangle^{(0)}_{\text{ren}} = 2g^{\mu\sigma}(x)\frac{\delta}{\delta g^{\nu\sigma}(x)} W_{\text{div}}[g_{\alpha\beta}].$$  \hspace{1cm} (21)

Note that in odd spacetime dimensions the conformal anomaly is absent and the corresponding gravitational part vanishes:

$$\langle T^\mu_\nu [g_{\alpha\beta}] \rangle^{(0)}_{\text{ren}} = 0, \quad \text{for even } D.$$  \hspace{1cm} (22)

The boundary part in Eq. (20) is related to the corresponding flat spacetime counterpart (10) by the relation [31]

$$\langle T^\mu_\nu [g_{\alpha\beta}] \rangle^{(b)}_{\text{ren}} = \frac{1}{\sqrt{|g|}} \langle T^\mu_\nu [\eta_{\alpha\beta}] \rangle_{\text{ren}}.$$  \hspace{1cm} (23)

By taking into account Eq. (10) from here we obtain

$$\langle T^\mu_\nu [g_{\alpha\beta}] \rangle^{(b)}_{\text{ren}} = e^{(D+1)\sigma(a)} \text{diag}(\sigma, -p_1, -p_2, -p_3, \ldots, -p_D),$$  \hspace{1cm} (24)

Now we see that as gravitational part (20) is a continuous function on $r$ it does not contribute to the forces acting on the boundary and the vacuum force per unit surface acting on the boundary at $r = a$ is determined by the boundary part of the vacuum pressure, $p_D = -\langle T^D_\nu [g_{\alpha\beta}] \rangle^{(b)}_{\text{ren}}$, taken at the point $r = a$:

$$p_D(a) = e^{(D+1)\sigma(a)} F,$$  \hspace{1cm} (25)

where $F$ is given by (18).

### 3 Casimir stress on cylindrical shell in dS background

We will consider one of the simplest field-theoretical model in which the domain wall type solutions appear [13]. The model involves a single, real-valued scalar field $\varphi$ with lagrangian given by

$$L = -1/2g_{\mu\nu}\partial^\mu\varphi\partial^\nu\varphi - V(\varphi),$$  \hspace{1cm} (26)

and

$$V(\varphi) = \frac{\lambda}{2}(\varphi^2 - v^2)^2,$$  \hspace{1cm} (27)

where $\lambda$ and $v$ are positive constants. The classical ground states are given by $\varphi = \pm v$. The domain wall arises if there are regions in the space where the field $\varphi$ has different vacuum values, the domain wall interpolating between such regions. In this paper we will consider a domain wall between a cylindrical region around $z$ axis in which $\varphi = \Lambda_{in}$ and the remaining part of the space where $\varphi = \Lambda_{out}$.

As an application of the general formulae from the section-2 here we consider the important special case of the dS$_{3+1}$ bulk for which

$$ds^2 = \frac{\alpha^2}{\eta^2} [d\eta^2 - \sum_{i=1}^{3} (dx^i)^2],$$  \hspace{1cm} (28)
where \( \eta \), is the conformal time
\[
-\infty < \eta < 0.
\] (29)

The constant \( \alpha \) is related to the cosmological constant as
\[
\alpha^2 = \frac{3}{\Lambda}.
\] (30)

Now we consider the pure effect of vacuum polarization due to the gravitational field without any boundary conditions (to see such problem for spherical shell and parallel plate geometry refer to [24, 25, 26]). The renormalized stress tensor for massless scalar field in de Sitter space is given by [31, 32]
\[
<T^\nu_\mu > = \frac{1}{960\pi^2\alpha^4} \delta^\nu_\mu.
\] (31)

The corresponding effective pressure is
\[
P = -<T^1_1 > = -<T^r_r > = -\frac{1}{960\pi^2\alpha^4},
\] (32)
valid for both inside and outside the cylinder. Hence the effective force on the cylinder due to the gravitational vacuum polarization is zero.

Now, assume there are different vacuum inside and outside corresponding to \( \alpha_{\text{in}} \) and \( \alpha_{\text{out}} \) for the metric Eq.(28). Now, the effective pressure created by gravitational part Eq.(32), is different for different part of space-time
\[
P_{\text{in}} = -<T^r_r >_{\text{in}} = -\frac{1}{960\pi^2\alpha_{\text{in}}^4} = \frac{-\Lambda_{\text{in}}^2}{8640\pi^2},
\] (33)
\[
P_{\text{out}} = -<T^r_r >_{\text{out}} = -\frac{1}{960\pi^2\alpha_{\text{out}}^4} = \frac{-\Lambda_{\text{out}}^2}{8640\pi^2}.
\] (34)

Therefore the gravitational pressure over shell, \( P_g \), is given by
\[
P_g = P_{\text{in}} - P_{\text{out}} = \frac{-1}{8640\pi^2}(\Lambda_{\text{in}}^2 - \Lambda_{\text{out}}^2)
\] (35)

Now we considering the effective pressure due to the boundary condition. Under the conformal transformation in four dimensions with the conformal factor given by
\[
\Omega(\eta) = \frac{\alpha}{\eta}.
\] (36)

The vacuum force acting from inside per unit surface of the cylinder can be found using the Eqs. (11), (25) for the vacuum radial pressure:
\[
F_{\text{in}} = \frac{\eta^4}{\alpha_{\text{in}}^4} p_1 \big|_{r=a^{-}} = \frac{\eta^4}{\alpha_{\text{in}}^4} \frac{1}{4\pi^2\alpha^4} \sum_{n=-\infty}^{+\infty} \int_0^\infty dz \, z^6 \frac{\bar{K}_n(z)}{I_n(z)} F_{n}(p_1)[I_n(z)],
\] (37)
with notation (13). The expression for the radial projection of the vacuum force acting per unit surface of the cylinder from the outside directly follows from Eqs. (17),(25) with \( q = p_1 \):
\[
F_{\text{ext}} = -\frac{\eta^4}{\alpha_{\text{out}}^4} p_1 \big|_{r=a^{+}} = -\frac{\eta^4}{\alpha_{\text{out}}^4} \frac{1}{4\pi^2\alpha^4} \sum_{n=-\infty}^{+\infty} \int_0^\infty dz \, z^6 \frac{\bar{I}_n(z)}{K_n(z)} F_{n}(p_1)[K_n(z)],
\] (38)
Therefore the vacuum pressure due to the boundary condition acting on the cylinder is given by

\[
P_b = F_{\text{in}} + F_{\text{ext}} = \frac{\eta^4}{\alpha_{\text{in}}^4} \frac{1}{4\pi^2a^4} \sum_{n=-\infty}^{+\infty} \int_0^\infty dz \frac{z^6 K_n(z)}{I_n(z)} \frac{F_{n}(p_1)}{I_n(z)}[I_n(z)] + \frac{\eta^4}{\alpha_{\text{out}}^4} \sum_{n=-\infty}^{+\infty} \int_0^\infty dz \frac{z^6 I_n(z)}{K_n(z)} \frac{F_{n}(p_1)}{K_n(z)}[K_n(z)].
\] (39)

The total pressure on the cylinder, \(P\), is then given by

\[
P = p_g + p_b = \frac{1}{8640\pi^2} (\Lambda_{\text{in}}^2 - \Lambda_{\text{in}}^2) + \frac{\eta^4}{36\pi^2a^4} \sum_{n=-\infty}^{+\infty} \int_0^\infty dz \frac{z^6}{K_n(z)} F_n(p_1)[I_n(z)] - \Lambda_{\text{out}}^2 \frac{I_n(z)}{K_n(z)} F_n(p_1)[K_n(z)].
\] (40)

The \(\eta\)- or time-dependence of the pressure in intuitively clear due to the time dependence of the physical radius of cylinder. This pressure corresponds to the attractive/repulsive force on the shell if \(P < / > 0\). The equilibrium state for the cylinder correspond to the zero values of Eq. (40): \(p = 0\). Total pressure, may be negative or positive, depending on the difference between the cosmological constant in the two parts of space-time. Given a false vacuum inside of the cylinder, and true vacuum outside, i.e. \(\Lambda_{\text{in}} > \Lambda_{\text{out}}\), then the gravitational part is negative, and tends to contract the cylinder, but the boundary pressure part may be positive or negative. Therefore the total effective pressure on the cylinder may be negative, leading to a contraction of the cylinder. The contraction however, ends for a minimum of radius of the cylinder, where both part of the total pressure are equal. For the case of true vacuum inside the cylinder and false vacuum outside, i.e \(\Lambda_{\text{in}} < \Lambda_{\text{out}}\), the gravitational pressure is positive. In this case, boundary part can be negative or positive depending on the difference between \(F_{\text{in}}\) and \(F_{\text{out}}\). Hence, the total pressure may be either negative or positive.

4 Conclusion

In the present paper we have investigated the Casimir effect due to the conformally coupled massless scalar field for a cylindrical shell on background of the conformally-flat space-times. The general case of the mixed boundary conditions is considered. The vacuum expectation values of the energy-momentum tensor are derived from the corresponding flat spacetime results by using the conformal properties of the problem. Then we consider cylindrical shell with constant comoving radius having different vacuums inside and outside, i.e. with false/true vacuum inside/outside. The boundary induced part for the vacuum energy-momentum tensor is given by Eq.(24), and the corresponding vacuum forces acting per unit surface of the shell have the form Eqs. (37),(38). The effective vacuum pressure due to the boundary condition acting on the cylinder is given by Eq.(37). The vacuum polarization due to the gravitational field, without any boundary conditions is given by Eq.(31), the corresponding gravitational pressure part has the form Eq.(32), which is the same from both sides of the shell, and hence leads to zero effective force. However when we consider different cosmological constants for the space between and
outside of the shell, in this case the effective pressure created by gravitational part is different for different part of the space-time and add to the boundary part pressure. The total pressure is given by Eq.(40). Our calculation shows that for the cylindrical shell with false vacuum inside and true vacuum outside $\Lambda_{in} > \Lambda_{out}$, the gravitational pressure part is negative, but the boundary pressure part may be positive or negative. In contrast for the case of true vacuum inside the cylinder and false vacuum outside, $\Lambda_{out} > \Lambda_{in}$, the gravitational pressure is positive, and boundary part can be negative or positive depending on the difference between $F_{in}$ and $F_{out}$ in Eq.(40). Therefore the detail dynamics of the cylindrical shell depends on different parameters and all cases of contraction and expansion may appear. The result may be of interest in the case of formation of the cosmic cylindrical domain walls in early universe.

References

[1] G. Plunien, B. Mueller, W. Greiner, Phys. Rep. 134, 87(1986).

[2] V. M. Mostepanenko and N. N. Trunov. The Casimir effect and its applications. (Oxford Science Publications New York, 1997).

[3] H. B. G. Casimir, proc. K. Ned. Akad. Wet. 51, 793(1948).

[4] E. Elizalde, S. D. Odintsov, A. Romeo, A. A. Bytsenko and S. Zerbini, zeta regularization techniques with applications(World Scientific, Singapore, 1994).

[5] E. Elizalde, Ten physical applications of spectral zeta functions, lecture notes in physics, (Springer-Verlage, Berlin, 1995).

[6] K.A. Milton, The Casimir Effect: Physical Manifestations of the Zero-Point Energy, hep-th/9901011; Dimensional and Dynamical Aspects of the Casimir Effect: Understanding the Reality and Significance of Vaccum Energy, hep-th/0009173.

[7] M. Bordag, U. Mohideen, V.M. Mostepanenko, Phys. Rept. 353, 1-250, (2001).

[8] L. L. De Raad Jr and K. A. Milton, Ann. Phys. 136, 2290, (1981).

[9] P. Gosdzinsky and A. Romeo, Phys. Lett. B441, 265,(1998).

[10] K. A. Milton, A. V. Nesterenko and V. V. Nesterenko, Phys. Rev. D59, 105009, (1999).

[11] Ya. B. Zel’dovich, I. Yu. Kobzarev and L. B. Okun, Sov. Phys. JETP 40, 1 (1975).

[12] T. W. B. Kibble, J. Phys. A9, 1387 (1976).

[13] A. Vilenkin, Phys. Reports 121, 263 (1985).

[14] R. P. Huebener, Magnetic Flux Structure in Superconductor (Springer-Verlag, Berlin, 1979).

[15] R. J. Donnelly, Quantized Vortices in Helium II (Cambridge University Press, Cambridge, England, 1991).
[16] S. Chandrasekhar and G. S. Ranganath, Adv. Phys. **35**, 507 (1986).

[17] H. Arodz and A. L. Larsen. Phys. Rev. **D49**, 4154, (1994).

[18] H. Arodz, Phys. Rev. **D52**, 1082, (1995).

[19] M. Baker, J. S. Ball, and F. Zachariasen, Phys. Rep. **209**, 73 (1991).

[20] C. T. Hill, D. N. Schramm, and J. N. Fry, Comments. Nucl. Part. Phys. **19**, 25 (1998).

[21] M. R. Setare and A. A. Saharian. Int. J. Mod. Phys. **A16**, 1463(2001).

[22] M. R. Setare and A. H. Rezaeian. Mod. Phys. Lett. **A15**, 2159(2000).

[23] F. Antonsen and K. Borman. Casimir Driven Evolution of the Universe. gr-qc/9802013

[24] M. R. Setare and R. Mansouri. Class. Quant. Grav. 18 (2001) 2331

[25] M. R. Setare. Class. Quant. Grav. 18 (2001) 4823-4830

[26] M. R. Setare and R. Mansouri. Class. Quant. Grav. 18 (2001) 2659-2664

[27] A. Vilenkin and E. P. S. Shellard. COSMIC STRINGS AND OTHER TOPOLOGICAL DEFFECTS,(Cambridge University Press, 1994).

[28] A. Romeo and A. A. Saharian, J. Phys. **A35**, 1297, (2002).

[29] A. A. Saharian, Phys. Rev. **D63**, 125007 (2001).

[30] A. Romeo and A. A. Saharian, Phys. Rev. **D63**, 105019 (2001).

[31] N. D. Birrell and P. C. W. Davies, Quantum fields in curved space,(Chambridge University press, 1986).

[32] J. S. Dowker, R. Critchley, Phys. Rev. **D13**, 3224 (1976).