Cluster Formation and the ISM

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Abstract. We review the physics of star formation, and its links with the state of the ISM in galaxies. Current observations indicate that the preferred mode of star formation is clustered. Given that OB associations provide the dominant energy input into the ISM, deep links exist between the ISM and star formation. We present a multi-scale discussion of star formation, and attempt to create an integrated vision of these processes.

1. Introduction

Star formation and the state of the interstellar medium involve strongly coupled physical processes. This is because the primary energy inputs into the ISM derive from the products of massive star formation in OB associations, namely, correlated supernova explosions (Type II), stellar winds, and photoionizing flux. The resulting pressure, shock waves, FUV, and cosmic ray content of the diffuse medium determine the surface pressures and ionization of molecular clouds. These mechanisms are the means by which the ISM regulates the surface density of molecular clouds, as well as the coupling of their internal magnetic fields; and thereby the process of star formation.

This review highlights some of the central processes in star formation and their links to ISM physics. We first give an overview of the basic physical processes in star formation (§1). In §2, we review the basic physics of molecular clouds and their cluster forming, more massive cores. We then highlight some current work on the physics of filamentary molecular clouds (§3). Finally, we review some of the basic ideas of star formation theory with an eye to understanding cluster formation (§4). The reader may consult recent related reviews by McKee et. al. (1993), McKee (1995), and Elmegreen et. al. (1999).

2. Overview: From the ISM to Protostars

2.1. Diffuse ISM

The physical scales of the diffuse ISM span the range $10^2 - 10^3$ pc. The diffuse medium consists of cooler clouds that are in pressure balance with a hot surrounding medium, following the ideas of Field (1965), and McKee and Ostriker (1977, henceforth MO). The existence of a multiphase medium hinges upon the structure of the cooling function of atomic gas. The diffuse ISM has three phases in its inventory (see review; McKee 1995):
• **cold neutral medium** of clouds (CNM) with a temperature $T \simeq 50^\circ K$ and density $n \simeq 40 \text{cm}^{-3}$;

• **warm neutral medium** (WNM) of temperature $T \leq 8,000^\circ K$ which surrounds the CNM and pervades much of space and, a **warm ionized medium** (WIM) at $T \simeq 8,000^\circ K$ and $n \simeq 0.26 \text{cm}^{-3}$ which consists of highly ionized warm hydrogen;

• **hot ionized medium** (HIM) with $T \simeq 5 \times 10^5 K$ and $n \simeq 10^{-2.5} \text{cm}^{-3}$.

The ISM contains other important components however. Of greater significance, energetically, is the pressure associated with the magnetic field that pervades the ISM. It has a strength of $B \simeq \text{several } \mu \text{G}$. Cosmic rays have comparable energy densities and are thought to be accelerated in supernova remnants. Thus, the bulk of the cosmic ray component of the ISM can be viewed as a consequence of star formation (see Duric, this volume). Finally, the ISM has non-thermal gas motions whose energy density is comparable to the magnetic and cosmic ray contributions as well. The fact that the energy density in the non-thermal component dominates the thermal one is reminiscent of the physics of molecular gas, which we address below.

The total pressure in the ISM can be ascertained by applying the condition of hydrostatic balance: the pressure in the galactic plane must be sufficient to support the weight of a hot galactic corona, detected through X-ray emission. The result (see Boulares and Cox, 1990) is that $P_{\text{ISM}} \simeq (3.9 \pm 0.6) \times 10^{-12} \text{dyne cm}^{-2}$. The constituent pressures that add up to this value are, in units of $10^{-12} \text{dyne cm}^{-2}$ (from Boulares and Cox):

- thermal pressure, $\simeq 0.3$
- cosmic rays, $\simeq 1.0$
- magnetic fields, $\simeq 1.0$ (field strengths locally 2-3 $\mu \text{G}$)
- kinetic pressure, $\simeq 1.8$.

The thermal energy contributes little to the overall budget because the gas cools so efficiently, the primary coolant being the CII line. Thus the total energy budget for the diffuse ISM may be written as,

$$E_{\text{therm}} << E_{\text{nontherm}} \simeq E_{\text{CR}} \simeq E_{\text{mag}} >> E_{\text{grav}} \quad (1)$$

The MO theory predicts that the hot phase of the ISM is heated by supernova explosions. The SN are assumed to occur at random throughout the Galactic disk. Uncorrelated Type I supernovae as well as a distributed Type II population are the drivers of ISM energetics in this picture. The HI survey of the Milky Way by Colomb, Poppel, and Heiles (1980) revealed, however, that vast atomic clouds are organized in filamentary structures. Here and there, large cavities and shells are obviously discerned. Heiles (1984) catalogued a large number of large HI shells in the ISM which he suggested are produced by the combined energy input from winds and supernova explosions that accompany OB associations. Note that OB stars formed in associations account for 90% of the supernovae in the galaxy (van den Bergh and Tammann 1991).

Theoretical models of the energy input from massive stars in the HI of the galactic disk show that the correlated massive stellar winds and supernovae in OB associations create large superbubbles in the galactic HI (Bruhweiler et al. 1980). The swept up gas forms dense, compressed supershells. Filaments could be produced as these shells cool and undergo thermal instabilities. Sufficiently
intense wind and supernova activity can also lead to break-out into the galactic halo. The plumes of hot gas flowing vertically out of the disk are akin to "chimneys" located above the OB associations (eg. Norman and Ikeuchi 1989). A beautiful example of this process in action is the CGPS 21 cm map of the chimney in the W3 region (Normandeau et. al. 1996).

Large scale interstellar magnetic fields can strongly affect the evolution of superbubbles. Recent numerical calculations by Tomisaka (1998) as an example, show that blow-out depends upon the structure of the magnetic field in the ISM. For a uniform field parallel to the galactic plane, expansion perpendicular to the plane is suppressed and a strongly magnetized shell is swept up. On the other hand, if the field strength scales with the gas density as $B \propto \rho^{1/2}$, superbubble expansion and blow-out occur more readily, the results being similar to the unmagnetized case. These calculations also show that the field is stretched in the walls of the expanding shell and appears to be well ordered.

We now turn to the question of how the ISM affects the physical properties of molecular clouds. There are at least two important influences:

- ISM Pressure $\rightarrow$ Molecular Cloud Properties
- Cosmic Ray production $\rightarrow$ Molecular Cloud Heating/Ionization

The ISM pressure upon the surfaces of molecular clouds truncates them and in so doing, determines their size. Thus, the ISM pressure essentially determines the surface density of a cloud. The second point is that the ionization of molecular clouds is determined by two types of radiation; FUV photons and high energy cosmic rays. The cosmic rays penetrate into the heart of high column density clumps in the cloud and partially ionize the gas, while FUV photoionization dominates in gas with visual extinctions $A_V \leq 4$ mag (McKee 1989). By contributing to the ionization rate in a cloud in this way, the ISM activity helps to control the star formation rate.

### 2.2. Molecular Clouds

Stars form in self-gravitating, molecular clouds. These are highly inhomogenous structures that are filamentary (eg. Johnstone and Bally, 1999), and have a rich sub-structure. Molecular clouds are host to a spectrum of dense, high pressure sub-regions known as molecular cloud clumps and smaller cores. These are the actual sites of star formation within molecular clouds.

Most of the self-gravitating gas in the Milky Way is gathered in the distribution of Giant Molecular Clouds (GMCs). Surveys show (see eg. Scoville and Sanders, 1987) that GMCs range in mass from $10^5 - 10^6 M_\odot$ although smaller molecular clouds may be as low as several $10^2 M_\odot$. GMCs have a mass spectrum, $dN/dM_{GMC} \propto M_{GMC}^{-1.6}$. The range in physical scales is about $10 - 100$ pc, with the median cloud having a mass of $3.3 \times 10^5 M_\odot$ and a median radius of 20 pc. The median cloud density is then $180$ cm$^{-3}$, and column density $1.4 \times 10^{22}$ cm$^{-2}$ or equivalently, $260 M_\odot$ pc$^{-2}$ (see Harris and Pudritz 1994). The pressures within molecular clouds are much higher than the ISM pressure surrounding them. Typically $P_{GMC}/P_{ISM} \approx 20 - 30$. Thus, molecular clouds are not another phase of the interstellar medium because self-gravity, not pressure, dominates their internal physics. Direct Zeeman measurements of molecular clouds also show that they are pervaded by strong magnetic fields $B \simeq 30 \mu$G.
The corresponding magnetic pressure is comparable with the gravitational energy density of the a cloud.

The line-profiles of molecular clouds are dominated by non-thermal motions. Larson (1981) was able to establish two important empirical cloud properties. If \( r \) represents the physical scale of a CO map, and one measures the average column density \( \Sigma \) of gas within this scale, Larson found that \( \Sigma \propto \rho r \propto \text{const} \). Moreover, if \( \sigma \) is the velocity dispersion of the gas on this scale, he also determined that \( \sigma \propto r^{1/2} \). The physical explanation of these relations came with the work of Chieze (1987) and Elmegreen (1989). The theory of self-gravitating spheres that are embedded within an external medium of pressure \( P_s \) was worked out in classic papers by Ebert (1955), and Bonner (1956). They showed that an isothermal cloud of mass \( M \) and average velocity dispersion \( \sigma_{\text{ave}} \) has a critically stable state, described by

\[
M_{\text{crit}} = 1.18 \frac{\sigma_{\text{ave}}^4}{(G^3 P_s)^{1/2}} \tag{2}
\]

\[
\Sigma_{\text{crit}} = 1.60 (P_s/G)^{1/2} \tag{3}
\]

We note that these relations may also be determined from the virial theorem (eg. McCrae 1957). They give a natural explanation for Larson’s empirical laws. The surface density of a molecular cloud, so interpreted, depends only upon the ISM pressure \( \Sigma \propto P_{\text{ISM}}^{1/2} \). It is the relative constancy of the ISM surface pressure over significant portions of the disk then, that accounts for the near constancy of molecular cloud surface densities. Similarly, by noting that \( M = \pi r^2 \Sigma \), one sees that the first equation yields Larson’s size-line width relation. Exactly the same type of scalings, as well as coefficients of the same order of magnitude pertain to the description of a filamentary cloud, as Fiege and Pudritz (1999, FP) show.

The energetics of a molecular cloud may be summarized as follows;

\[
E_{\text{therm}} \ll E_{\text{nontherm}} \approx E_{\text{mag}} \geq (1/2) E_{\text{grav}} \tag{4}
\]

where the last inequality follows from the fact that ordered fields, and non-thermal (MHD) motions contribute equally to GMC cloud support (eg. review McKee et. al., 1993). The explanation of the insignificant contribution of thermal support to this balance is again related (as in the ISM) to the efficiency of cooling; the predominant coolant in molecular gas now being the CO rotational lines \( J = 1 \rightarrow 0 \) etc. By balancing the heating rate of molecular clouds due to cosmic ray bombardment, with the molecular cooling rates one can show that clouds are maintained at a temperature of \( 10^{-20} K \) over a wide range of densities \( 10^2 \rightarrow 10^4 \) cm\(^{-3}\) (eg. Goldsmith and Langer 1978). It turns out that the column density at which HI clouds attain sufficiently high column density, ( several \( 10^{21} \) cm\(^{-3}\) ) to become self-gravitating, is also the column at which self-shielding from the galactic FUV field takes place allowing the gas to become molecular.

The onset of molecular cooling in sufficiently high-column gas drops the cloud to low temperatures where it is thermostatically controlled. Molecular clouds also have low rotational energies compared with gravity. Thus, it is left to magnetic pressure as well as the non-thermal motions (MHD waves or turbulence) to provide support of the cloud against gravitational collapse. Molecular
clouds are exotic structures in which magnetism plays out a slow, and ultimately losing battle against self-gravity.

The literature contains four basic mechanisms for cloud formation that may all be active in the galaxy:

- Cloud-cloud agglomeration in spiral waves (eg. Kwan & Valdes 1983)
- Supershell fragmentation (eg. McCray and Kafatos 1987)
- Large-scale gravitational instability: \( Q \) criterion (Kennicutt, 1989)
  \[
  Q = \frac{c_s \kappa}{\pi G \Sigma} < 1 \quad \text{gas}
  \]
- Parker Instability (eg. Blitz & Shu 1980, Elmegreen 1982)

The evidence for the first comes from CO observations of spiral galaxies. One invariably finds that GMCs inhabit the spiral arms of galaxies (eg. Rand 1993). The idea is that the passage of an arm leads to focussing of orbits of more diffuse clouds. These collide and agglomerate in the potential well of the arm. The form of the power law mass spectrum of clouds, given above, is in good support of this mechanism as Kwan and Valdes showed. The second mechanism also appears to operate; within the walls of supershells. The fragmentation mass is of order \( 5 \times 10^4 M_\odot \) by the time that \( 3 \times 10^7 \) years have elapsed (see McCray and Kafatos, 1987). The third mechanism, that of gravitational instability in a self-gravitating gas layer, employs the well-known Toomre 'Q' criterion where \( c_s \) is the effective sound speed of the medium and \( \kappa \) is the epicyclic frequency. Kennicutt (1989) showed that significant star formation in spiral galaxies only occurs in gas whose column density exceeds the limit given by the Toomre criterion, and is related, therefore, to gravitational instability of the gas. Finally, the Parker instability arises from the intrinsic buoyancy that a magnetic field, oriented parallel to the galactic plane, has within a self-gravitating gas layer. As the field rises and bubbles out of a gas layer, the gas flows back to the disk along magnetic field lines and gathers in magnetic "valleys". The predicted scale length of this instability is about 1 kpc along spiral arms (eg. Elmegreen 1982), which is precisely the kind of distance one sees between GMCs and giant HII regions in galaxies. The complete picture of GMC formation probably involves all of these processes. Gas clouds agglomerate within spiral arms and finally reach the critical column density at which gravitational instability, and magnetic buoyancy become important. The subsequent formation of OB associations and supershells leads to further gas compression, and also the possible formation of chimneys that channels some of the energy release into the galactic halo.

### 2.3. Molecular Cloud Clumps: Cluster Formation

The clump mass spectrum within molecular clouds is, interestingly enough, similar to the GMC mass spectrum. Thus, Blitz (review, 1993) reported that the spectrum \( dN/dM \propto M^{-1.6} \), with a 10% error in the index, fit the data for a number of different clouds rather well. The masses of such clumps range from \( 1 - 1000 M_\odot \), so that one sees that the median clump mass is only a thousandth of the median GMC cloud mass. The fact that only a few percent of the mass of a molecular cloud is tied up in these star forming clumps is, empirically, the reason why star formation is a rather inefficient process.
The overall energetics of clumps are more dominated by gravity than the diffuse inter-clump regions of the cloud. These clumps have strong fields and their internal kinematics are dominated by non-thermal motions (e.g., Casselli and Myers, 1995). Maps of the magnetic structures associated with clumps are now becoming available through the technique of submillimetre polarimetry. Schleuning’s (1998) map of a clump in the Orion cloud as an example, shows that there is a well ordered, hour-glass shaped magnetic field structure present on these scales. Not all clumps are expected to lie near the critical threshold for gravitational collapse. As an example, low mass clumps in the logatropic model for cloud cores developed by McLaughlin and Pudritz (1996) are pressure dominated structures. In that model, a critical magnetized clump has a mass of $\approx 250M_\odot$.

Infrared camera observations of clumps in molecular clouds such as Orion reveal that the most massive clumps are forming star clusters. The most massive clump in the Orion cloud as an example, has a mass of $\approx 500M_\odot$ and has a star formation efficiency approaching 40% (E. Lada 1992; also reviews by Zinnecker et. al. 1993, Elmegreen et. al. 1999). Stars typically form as a member of a group or cluster, and not in isolation. This may be explained by noting that clump mass spectra are rather different than the Salpeter IMF for the stars. Beyond 0.3$M_\odot$ or so, the IMF has a much steeper power-law than that describing molecular clumps; $dN_*/dM_* \propto M_*^{-2.35}$. By weighting the clump mass spectrum with the mass, and integrating over mass, one finds that the total mass of the clumps scales as $M_{\text{clump,tot}} \propto M_{\text{max}}^{0.4}$. Thus, most of the mass resides in the massive end of the spectrum of clumps. For stars however, this same procedure shows that it is the lower mass end of the IMF that contains most of the stellar mass. Therefore, cluster formation must be the preferred mode of star formation (Patel and Pudritz 1994).

How do the clumps form? Processes that have been proposed include:

- Clump-clump agglomeration (Carlberg and Pudritz 1990, McLaughlin and Pudritz 1996)
- Structures in turbulent flow (see Pouquet’s contribution).

The fact that clumps and their parental clouds share similar mass spectral forms suggests a common formation mechanism. Thus clumps could be built up by the same type of agglomerative and gravitational instability process that produced molecular clouds. Turbulence has also been proposed as the origin of a wide range of structure and substructure (Elmegreen and Efremov 1997).

The finite amplitude Alfvén waves within molecular clouds and their clumps contribute not only to the support of the gas, but also to the process of clump and structure formation within them. Dewar (1970) showed that a collection of Alfvén waves exerts a nearly isotropic gas pressure that can support gas in a direction parallel to the magnetic field lines. Arons and Max (1975) later championed the idea that such waves might account for non-thermal motions in clouds. Alfvén waves of finite amplitude are different than linear waves however, in that they produce density fluctuations. Carlberg and Pudritz (1990) suggested that such non-linear waves could produce the small density fluctuations within clouds that subsequently agglomerate to produce the observed clump mass spectrum.

This behaviour of non-linear Alfvén waves is akin to pressure (magneto-acoustic) waves, which damp out quickly. Numerical simulations of the be-
haviour of such waves find that they damp very quickly; with $\tau_{\text{damp}} \leq 2t_{\text{ff}}$ (see Vasquez-Semadeni, Passot and Pouquet 1995, Maclow et al 1998, Ostriker et al 1998). Thus, unless MHD waves and turbulence are replenished very quickly, the turbulent support of clumps would quickly vanish. It has often been suggested that bipolar outflows, examined below, can solve this problem. One caveat here is that turbulence is also observed in starless clumps and cores.

2.4. Molecular Cloud Cores: Individual Star Formation

On scales of 0.01-0.1 pc one encounters dense cores ($n_{\text{core}} \geq 10^4 \text{ cm}^{-3}$) of several solar masses that may be associated with the formation of individual protostars (Benson and Myers, 1989). Their internal velocity dispersions are less dominated by non-thermal motion (see Casselli and Myers 1995).

We have arrived at perhaps the most intensively studied level of star formation. Individual star formation is an incredibly rich process involving the simultaneous presence of:

- Accretion disks
- Bipolar outflows and jets
- Gravitational collapse

Jet activity and accretion disks are strongly correlated because whenever one sees a jet and an outflow, there is good evidence that an accretion disk is also present. Bipolar molecular outflows are the most obvious and ubiquitous sign-post of star formation. There are now more than 200 molecular outflows known (eg. reviews; Bachiller 1996, Padman, Bence and Richer 1997). They persist for at least several $10^5$ years, which is a good fraction of the pre-main-sequence evolution timescale. The outflows consist of material in the natal molecular cloud core of a protostar that is swept up and put into motion by a faster, underlying jet from the central source. Outflows are associated with stars of all masses; from the common low-mass T-Tauri stars, to high mass stars in the process of formation. A model for the underlying jets that appears to fit all of the observed facts predicts that jets are centrifugally accelerated winds driven from the surfaces of magnetized protostellar accretion disks (eg. review, Königl and Pudritz 1999). Another suggestion is that jets originate at the interface between the magnetosphere surrounding an active young protostar, and the surrounding disk (eg. review Shu et. al. 1999).

**How do cores form?** Several ideas have been suggested in the literature, including:

- Ambipolar diffusion (eg. Mouschovias 1993)
- MHD wave damping (Langer 1978, Mouschovias 1987, Pudritz 1990)

The first mechanism arises from the fact that the magnetic field within a partially ionized gas (typically one ion in $10^7$ neutrals in a molecular cloud) is fairly diffusive. A field will slowly slip out of overdense regions because in a magnetic field gradient, the Lorentz force pushes outwards on the ions to which the field is directly coupled. Collisions of the slowly outward moving ions with the neutrals communicate the magnetic force to the overwhelmingly neutral gas. This magnetic pressure support cannot be maintained forever because the ions are gradually pushed out of the dense region, dragging the field with them. The neutral density, on the other hand, slowly increases. The drift time scale of the field $\nu_B$ is thus proportional to the gravitational acceleration $g$ in the clump times the time-
scale for a neutral to collide with any ion, $\nu_{ni}^{-1}$. The ambipolar diffusion time-scale is therefore (eg. McKee et al 1993), $\tau_D = (R/v_D) = (R/g\nu_{ni}^{-1}) \propto (\zeta/\rho)^{1/2}$ where $\zeta$ is the ionization rate per molecule. Compare this with the free-fall time; $t_{ff} = (3\pi/32G\rho)^{1/2} = 1.4 \times 10^6 (n/10^3 cm^{-3})^{1/2}$ yr. Both these time scales have the same power-law dependence upon the cloud density. Thus, their ratio, $\tau_D/t_{ff} \approx 10$ for standard ionizing flux rates. This result is independent of the cloud density. We see that magnetic pressure can stave off gravitational collapse for a significant number of dynamical time-scales, but that the battle is finally lost. The time-scale required to form a core from the background molecular cloud (of density $10^2 cm^{-3}$) has been calculated by taking into account a host of complicating factors such as the effect of grains on the ionization state of the gas. The results (Ciolek and Mouschovias 1995) suggest that it would take nearly 20 million years to form a core. This may be too long in comparison with the lifetime of a molecular cloud, because it suggests that most cores should appear to be starless.

The process of ambipolar diffusion and the gradual loss of magnetic support leading to gravitational collapse is the current paradigm for star formation in a molecular core (eg. review Shu et al 1987). By taking into account the angular momentum of the parent core, the collapse gives rise to the formation of a magnetized disk. While attractive, one may well wonder whether or not this vision of individual star formation is able to account for the formation of star clusters? Of particular concern is the question of what determines the mass of a star in this scenario. We address this concern in §4.

The second mechanism for core formation begins with the fact that MHD waves in a partially ionized medium are damped on a small enough scale. The physical idea is that a wave in the combined ion-neutral fluid can only be sustained if a neutral particle collides with any ion within the period of an Alfvén wave. It is easily shown that on scales of approximately the size of cores, this condition breaks down and that waves must damp (eg. Mouschovias 1987, Pudritz, 1990). Without this wave support, gas may be more prone to increase its density, and to eventually collapse.

### 3. Molecular Cloud Structure: Topology and Helical Fields

Let us now consider models for filamentary clouds, namely upon magnetized, pressure-truncated cylinders. An analytic solution for the radial structure of a self-gravitating, isothermal filament (without pressure truncation) was found by Ostriker (1964);

$$\rho = \frac{\rho_c}{(1 + (r^2/r_0^2))^{1/2}}.$$  \hspace{1cm} (5)

The point to note about this solution is the steep fall-off of the gas density at larger scales; $\rho \propto r^{-4}$. Such a steep density gradient has never been seen in any study of molecular clouds. Molecular cloud core density profiles are never steeper than $r^{-2}$. Recent infrared-studies of the molecular filaments (eg. Lada et al., 1998) find radial profiles that fall off as $r^{-2}$. These results raise the basic question; what is a self-consistent model for filamentary clouds?
The MHD models in the literature posit predominantly poloidal magnetic fields (e.g., Nagasawa 1987, Gehman et al. 1996). Such a field contributes its pressure to the support against gravitational collapse. What is often ignored in such treatments is the effect of external pressure, as well as the possibility that filaments have a toroidal component of the field as well. The possibility that filaments are associated with helical magnetic fields was raised in the observations by Heiles (1987), and discussed by Bally (1987).

How would helical fields arise? All that is required is to twist one end of a filament containing a poloidal field, with respect to the other end. This could easily occur in the interstellar medium. We (Fiege and Pudritz 1999, FP; see Fiege and Pudritz, this volume) have developed a rather general model in which we idealize molecular filaments as infinitely long cylinders of self-gravitating gas, that are truncated at a finite radius by the pressure $P_s$ of the external medium. The general internal field consists of both poloidal $B_z$ and toroidal $B_\phi$ field components. We also include an internal, non-thermal gas pressure. One then applies the tensor virial theorem (FP) to derive a new form of the scalar virial theorem that is appropriate to the radial equilibrium of an infinite cylinder.

In the absence of an ordered magnetic field; the virial theorem for pressure truncated cylinders takes the simple form:

$$\frac{m}{m_{\text{vir}}} = 1 - \frac{P_s}{\langle P \rangle},$$

where $m$ is the mass per unit length of a self-gravitating cylinder whose average internal pressure is $\langle P \rangle$, and $m_{\text{vir}} = 2 \langle \sigma^2 \rangle / G$ is a critical mass per unit length (see e.g. McCrae 1957). This solution demonstrates that the stability of cylinders is different than for spheres. In the latter, a cloud of fixed mass, when squeezed with ever higher external pressure, reaches a critical radius below which the solution is unstable and spherical collapse ensues. For the cylinder on the other hand, as long as its mass per unit length is small enough that, $m < m_{\text{vir}}$, it doesn’t matter how much you squeeze; the filament will remain stable against radial collapse for any external pressure. Solutions to this virial equation depend upon observationally measurable quantities such as the ratio, $P_s/\langle P \rangle$ and $m/m_{\text{vir}}$. We compare our models with the available data by plotting these two virial parameters for a number of observed filaments.

Let us now turn to magnetic field effects (FP). A poloidal field adds magnetic pressure support to the filament, so that its critical mass per unit length increases. For a purely toroidal field however, the opposite trend occurs because the hoop stress associated with this component squeezes the filament. A toroidal field also produces a much shallower density gradient. By including these two field components in the virial theorem, one can draw curves through the two dimensional observational plane discussed above. We plot, in Figure 1, the predictions of the magnetic version of the virial theorem. The observer is not required to have any field strengths available, but must have enough data on hand to ascribe to any filament, the two ratios (involving the pressure and mass per unit length) discussed above. Each curve represents the solution for a different relative contribution of poloidal and toroidal field components to the virial equation, as measured by a magnetic virial parameter. For the limited amount of data available, it appears that filaments lie in a magnetic field regime...
wherein the pinch of the toroidal field dominates the support of the poloidal field component (see FP). Helical fields, it appears, may explain the data. A very important vindication of this result is that the radial density profiles for filaments with helical fields follow $\rho(r) \propto r^{-2}$, or shallower, in agreement with the current data.

4. The Clustered Mode of Star Formation

We have seen that OB associations provide the dominant input into heating and sculpting the properties of the ISM. It also appears that star formation is dominated by the process of cluster formation. Thus, from both the point of view of star formation, and ISM theory, it is imperative that we understand how stellar clusters are formed. The crucial question is, what processes determine the mass of stars and hence, the form of the IMF?

As is well known, there are two physical pictures of individual star formation that have dominated the discussion in the last decades (e.g. review; Pudritz et. al. 1996). The first view, based on the gravitational stability of B-E spheres,
posits that the mass of a star is essentially a Jeans mass. At a temperature of $10^6$ K, and for reasonable core pressures, the critical B-E mass is indeed around $1 M_\odot$ (e.g., Larson 1992). The basic problem with this approach, however, is that in a highly inhomogenous medium the Jeans mass becomes an ambiguous quantity. In order to explain the IMF therefore, one would have to posit that it is "laid-in" in the mass spectrum of clumps. We have noted that this is unlikely for larger mass scales. However, recent submm observations by Motte et. al. (1998) of clumps in the $\rho$ Oph cloud claim to have found a core spectrum much like the Salpeter IMF.

The second basic model for star formation is the idea that it is not the mass, but rather the mass accretion rate $\dot{M}$ that is determined by a molecular cloud. The self-similar collapse picture, (eg. Shu 1977), require that some process turn-off the infall. The observed outflows have been hypothesized to play this role. Note however, that jets in Class 0 objects are highly collimated and will not intercept most of the infalling envelope. In addition, there is a strong positive relation between disk accretion, and jet outflow which seems to indicate that plenty of matter can collapse onto a disk, and accrete through it onto the central protostar.

Both of the previous pictures appear to be inadequate to provide a theory for the IMF. A promising new approach however, posits that stellar mass is acquired through a process of competitive accretion for the gas that is available in a clump. In the specific numerical simulation of Bonnell et. al. (1997) as an example, the accretion rate is modelled as a Bondi-Hoyle process onto distributed set of initial objects of mass $M_o$. The stellar velocity is $v_\infty$, and the gas density of the clump is $\rho_\infty$. The simulations showed that seed objects in the higher, central density region became more massive than the outliers. This can be understood analytically since the time dependence of accretion at the Bondi accretion rate, given by $\dot{M}_{BH} = \gamma M^2$ where $\gamma \equiv 4\pi \rho_\infty G^2 / (v_\infty^2 + c_s^2)^{3/2}$ is readily found. It is easy to show that the accretion time scale in this picture is $t_{\text{accr}} = (1/\gamma M_o) \propto (1/\rho_\infty M_o)$ Shorter accretion times occur in higher density regions of a clump, which is where one might expect more massive stars to be formed. These results appear to strongly dependent on mass of input objects which is not determined by the theory. Nevertheless, this type of approach holds considerable promise for a theory of cluster formation.

Acknowledgments. We are grateful to the organizers of this meeting for the invitation to participate in this most stimulating work shop. We thank Chris McKee and Dean McLaughlin for interesting discussions about these topics. REP’s research is supported through a grant from the Natural Sciences and Engineering Research Council of Canada.

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