Implications of scaling violations of $F_2$ at HERA for perturbative QCD

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Abstract

We critically examine the QCD predictions for the $Q^2$ dependence of the electron-proton deep-inelastic structure function $F_2(x, Q^2)$ in the small $x$ region, which is being probed at HERA. The standard results based on next-to-leading order Altarelli-Parisi evolution are compared with those that follow from the BFKL equation, which corresponds to the resummation of the leading log($1/x$) terms. The effects of parton screening are also quantified. The theoretical predictions are confronted with each other, and with existing data from HERA.

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The first measurements of the proton structure function \( F_2(x, Q^2) \) at small \( x \) have been made by the H1 and ZEUS collaborations at HERA. A striking increase of \( F_2 \) with decreasing \( x \) is observed which is consistent with the expectations of perturbative QCD at small \( x \) as embodied in the BFKL equation. This equation effectively performs a leading \( \alpha_s \log(1/x) \) resummation of soft gluon emissions, which results in a small \( x \) behaviour \( F_2 \sim x^{-\lambda} \) with \( \lambda \sim 0.5 \).

The data at \( Q^2 = 15 \) and 30 GeV\(^2\) are shown in Fig.1, together with a representative set of predictions and extrapolations, whose distinguishing features we elucidate below. These curves fall into two general categories. The first, category (A), is phenomenological and is based on parametric forms extrapolated to small \( x \) with \( Q^2 \) behaviour governed by the next-to-leading order Altarelli-Parisi equations. The parameters are determined by global fits to data at larger \( x \) (examples are the curves in Fig. 1 labelled MRS(D\(_\prime\)) [3], MRS(H) [5] and, to some extent, also GRV [1], but see below). The second approach, denoted (B), is, in principle, more fundamental. Here perturbative QCD is used in the form of the BFKL equation to evolve to small \( x \) from known behaviour at larger \( x \) (e.g. AKMS [7]). In other words in approach (A) the small \( x \) behaviour is input in the parametric forms used for the parton distributions at some scale \( Q^2 = Q_0^2 \), whereas in (B) an \( x^{-\lambda} \) behaviour at small \( x \) is generated dynamically with a determined value of \( \lambda \). Of course in the phenomenological approach, (A), it is possible to input a BFKL-motivated small \( x \) behaviour into the starting distributions (e.g. MRS(D\(_\prime\)) and MRS(H) have \( xg, xq \sim x^{-\lambda} \) with \( \lambda = 0.5 \) and 0.3 respectively). Since the \( x^{-\lambda} \) behaviour, for these values of \( \lambda \), is stable to evolution in \( Q^2 \) we may anticipate that it will be difficult to distinguish approaches (A) and (B). However the \( Q^2 \) behaviour (or scaling violations) of \( F_2 \) is, in principle, different in the two approaches.

The Altarelli-Parisi \( Q^2 \) evolution is controlled by the anomalous dimensions of the splitting functions (and by the coefficient functions) which have been computed perturbatively up to next-to-leading order. On the other hand the BFKL approach, at small \( x \), corresponds to an infinite order resummation of these quantities, keeping only leading \( \log(1/x) \) terms. Summing the leading \( \log(1/x) \) terms, besides generating an \( x^{-\lambda} \) behaviour, gives its own characteristic \( Q^2 \) dependence. One of our main purposes is to study whether or not the BFKL behaviour, which is more theoretically valid at small \( x \), can be distinguished from the approximate Altarelli-Parisi parametric forms which neglect the \( \log(1/x) \) resummation.

If we were to assume that Altarelli-Parisi evolution is valid at small \( x \) then

\[
\frac{\partial F_2(x, Q^2)}{\partial \log Q^2} \approx 2 \sum_q e_q^2 \frac{\alpha_s(Q^2)}{2\pi} \int_x^1 dy \frac{x}{y} P_{qg} \left( \frac{x}{y} \right) y g(y, Q^2) + \ldots ,
\]

and hence the \( Q^2 \) behaviour of \( F_2 \) can be varied by simply exploiting the freedom in the gluon distribution at small \( x \). However the situation is much more constrained when the BFKL equation is used to determine the (unintegrated) gluon distribution \( f(x, k_T^2) \). Then \( F_2 \) may be calculated using the \( k_T \)-factorization theorem

\[
F_2(x, Q^2) = \int_{x'}^1 dx' \frac{dx'}{x'} \int \frac{dk_T^2}{k_T^2} f \left( \frac{x}{x'}, k_T^2 \right) F_2^{(0)}(x', k_T^2, Q^2)
\]

where \( x/x' \) and \( k_T \) are the longitudinal momentum fraction and transverse momentum that are carried by the gluon which dissociates into the \( q\bar{q} \) pair, see Fig. 2. \( F_2^{(0)} \) is the quark box (and crossed box) amplitude for gluon-virtual photon fusion.
In order to gain insight into the different possible $Q^2$ dependences of $F_2$ it is useful to introduce the moment function of the (unintegrated) gluon distribution

$$f(n, k_T^2) = \int_0^1 dx x^{n-2} f(x, k_T^2). \quad (3)$$

The evolution of the moment function is given by the renormalization group equation

$$f(n, k_T^2) = f(n, k_0^2) \exp \left[ \int_{k_0^2}^{k_T^2} \frac{dk^2}{k^2} \gamma(n, \alpha_s(k_T^2)) \right] \quad (4)$$

where the anomalous dimension $\gamma(n, \alpha_s)$ is known. From eq.(3) we see that the behaviour at small $x$ is controlled by the leading singularity of $f(n, k_T^2)$ in the $n$ plane. In the leading log$(1/x)$ approximation $\gamma(n, \alpha_s)$ is just a function of the single variable $\alpha_s(k_T^2)/(n-1)$ and is determined by the BFKL kernel. Its value is such that

$$1 - \frac{3\alpha_s(k_T^2)}{\pi(n-1)} \tilde{K}(\gamma) = 0 \quad (5)$$

is satisfied, with

$$\tilde{K}(\gamma) = 2\Psi(1) - \Psi(\gamma) - \Psi(1-\gamma), \quad (6)$$

where $\Psi$ is the logarithmic derivative of the Euler gamma function.

For fixed $\alpha_s$ the leading singularity of $f(n, k_T^2)$ is a square root branch point at $n = 1 + \lambda_L$ where $\lambda_L = 3\alpha_s \tilde{K}(\frac{1}{2})/\pi = 12\alpha_s \log 2 / \pi$. Comparing with eq.(5) we find that $\gamma(1 + \lambda_L, \alpha_s) = \frac{1}{2}$. Thus, from eq.(4), it directly follows that

$$f(x, k_T^2) \sim (k_T^2)^{\frac{1}{2}} x^{-\lambda_L}. \quad (7)$$

Since $F_2^{(0)}/k_T^2$ in eq.(2) is simply a function of $k_T^2/Q^2$, this leading behaviour feeds through into $F_2$ to give

$$F_2(x, Q^2) \sim (Q^2)^{\frac{1}{2}} x^{-\lambda_L}, \quad (8)$$

where in (7) and (8) we have omitted slowly varying logarithmic factors.

Formula (4) is valid for running $\alpha_s$, provided $n$ remains to the right of the branch point throughout the region of integration, that is provided $n > 1 + 12\alpha_s(k_0^2)\log 2 / \pi$. (For smaller values of $n$ the $k_T^2$ dependence of $f(n, k_T^2)$ is more involved.) For running $\alpha_s$ the small $x$ behaviour of $f(x, k_T^2)$ is controlled by the leading pole singularity of $f(n, k_T^2)$ which occurs at $n = 1 + \bar{\lambda}$, where now $\bar{\lambda}$ has to be calculated numerically. A value of $\bar{\lambda} \approx 0.5$ is found, with rather little sensitivity to the treatment of the infrared region of the BFKL equation. The $k_T^2$ dependence of $f$ (and hence the $Q^2$ dependence of $F_2$) is determined by the residue $\beta$ of this pole. Using eq.(4) we have

$$f \sim \beta(k_T^2) x^{-\bar{\lambda}} \quad (9)$$

where

$$\beta(k_T^2) \sim \exp \left[ \int_{k_0^2}^{k_T^2} \frac{dk^2}{k^2} \gamma(1 + \bar{\lambda}, \alpha_s(k_T^2)) \right]. \quad (10)$$
From the above discussion we see that this form is valid provided \( k_T^2 \geq k_0^2 \geq \kappa^2(\bar{\lambda}) \), where \( \kappa^2(\bar{\lambda}) \) satisfies the implicit equation \( \bar{\lambda} = 12\alpha_s(\kappa^2(\bar{\lambda}))\log 2/\pi \). Similarly, provided that \( Q^2 \geq \kappa^2(\bar{\lambda}) \), we have

\[
F_2(x, Q^2) \sim \beta(Q^2)x^{-\bar{\lambda}},
\]

up to slight modifications which result from known \( Q^2 \) effects embedded in \( F_2^{(0)} \). Note that for \( Q^2 \gtrsim \kappa^2(\bar{\lambda}) \) we should again get an approximate \( (Q^2)^{1/2} \) behaviour of \( F_2(x, Q^2) \), although it may (at moderately small values of \( x \)) be modified by the non-leading contributions. Here we are also interested in \( Q^2 < \kappa^2(\bar{\lambda}) \) and then the form of \( \beta \) is more involved [10].

In the leading \( \log(1/x) \) approximation the anomalous dimension, \( \gamma(n, \alpha_s) \), is a power series in \( \alpha_s/(n-1) \). For the BFKL approach \( \gamma(n, \alpha_s) \) contains the sum of all these terms. If only the first term were retained then the \( Q^2 \) behaviour would correspond to Altarelli-Parisi evolution from a singular \( x^{-\lambda} \) gluon starting distribution with only \( g \to gg \) transitions included and with the splitting function \( P_{gg}(z) \) approximated by its singular \( 1/z \) term.

It is useful to compare the \( Q^2 \) dependence of \( F_2 \) which results from the theoretically motivated BFKL approach, (B), with that of the Altarelli-Parisi \( Q^2 \) evolution of approach (A). For Altarelli-Parisi evolution the \( Q^2 \) behaviour of \( F_2 \) depends on the small \( x \) behaviour of the parton starting distributions. If we assume that the starting distributions are non-singular at small \( x \) (i.e. \( xg(x, Q^2_0) \) and \( xq_{\text{sea}}(x, Q^2_0) \) approach a constant limit for \( x \to 0 \)), then the leading term, which drives both the \( Q^2 \) and \( x \) dependence at small \( x \), is of the double logarithmic form

\[
F_2(x, Q^2) \sim \exp \left[ 2\{\xi(Q^2_0, Q^2)\log(1/x)\}^{1/2} \right],
\]

where

\[
\xi(Q^2_0, Q^2) = \int_{Q^2_0}^{Q^2} dq^2 \frac{3\alpha_s(q^2)}{q^2} \frac{1}{\pi}.
\]

From (12) we see that, as \( x \) decreases, \( F_2 \) increases faster than any power of \( \log(1/x) \) but slower than any power of \( x \).

If, on the other hand, the starting gluon and sea quark distributions are assumed to have singular behaviour in the small \( x \) limit i.e.

\[
xg(x, Q^2_0), xq_{\text{sea}}(x, Q^2_0) \sim x^{-\lambda}
\]

with \( \lambda > 0 \), then the structure function \( F_2(x, Q^2) \) behaves as

\[
F_2(x, Q^2) \sim x^{-\lambda}h(Q^2)
\]

where the function \( h(Q^2) \) is determined by the corresponding anomalous dimensions of the moments of the (singlet) parton distributions at \( n = 1+\lambda \), as well as by the coefficient functions.

We emphasize again that, in contrast to the BFKL approach, for (next-to-leading order) Altarelli-Parisi evolution the relevant quantities which determine \( h(Q^2) \) are computed from the first (two) terms in the perturbative expansion in \( \alpha_s \). Thus terms are neglected, which may in principle be important at small \( x \), corresponding to the infinite sum of powers of \( \alpha_s/(n-1) \) in \( \gamma \) (and in the coefficient function).
Note that in both cases (i.e. eqs.(12) and (15)) Altarelli-Parisi evolution gives a slope of the structure function, \( \partial F_2(x,Q^2)/\partial \log(Q^2) \), which increases with decreasing \( x \). The MRS(D′)_L\cite{4} and MRS(H)\cite{5} extrapolations are examples of (12), with \( \lambda = 0.5 \) and 0.3 respectively. On the other hand, the behaviour of \( F_2 \) obtained from the GRV\cite{6} partons is an example of (12). In the GRV model the partons are generated from a valence-like input at a very low scale, \( Q_0^2 = 0.3 \text{GeV}^2 \) (and then the valence is matched to MRS at much higher \( Q^2 \)). Due to the long evolution length, \( \xi(Q_0^2, Q^2) \), in reaching the \( Q^2 \) values corresponding to the small \( x \) HERA data the GRV prediction tends to the double logarithmic form of (12). The GRV model is probably best regarded as a phenomenological way of obtaining steep distributions at a conventional input scale, say \( 4 \text{GeV}^2 \), since the steepness is mainly generated in the very low \( Q^2 \) region where perturbative QCD is unreliable\cite{13}. Note, however, that the steepness is specified by the evolution and is not a free parameter. In fact, in the region of the HERA data, the GRV form mimics an \( x^{-\lambda} \) behaviour with \( \lambda \sim 0.4 \), although for smaller \( x \) it is less steep.

To summarize, we have discussed four different ways of generating a steep \( x \) behaviour of \( F_2(x,Q^2) \) at small \( x \), each with its own characteristic \( Q^2 \) dependence: the BFKL fixed and running \( \alpha_s \) forms, (8) and (11), the Altarelli-Parisi double leading logarithmic form with a long \( Q^2 \) evolution, (12), and finally Altarelli-Parisi evolution from a steep \( x^{-\lambda} \) input, (15). Examples of such forms are, respectively, the fixed and running \( \alpha_s \) AKMS predictions\cite{7,12}, and the GRV\cite{6} and MRS(H)\cite{5} extrapolations. Their \( Q^2 \) dependences are compared with each other in Fig. 3 at given values of small \( x \) in the HERA regime. For reference the MRS(D′)_L\cite{4} extrapolation is also shown. The theoretical curves are calculated either from eq.(2) (where \( f \) is the complete numerical solution of the BFKL equation obtained as described in ref.\cite{12}) or from the full next-to-leading order Altarelli-Parisi evolution. We also show, in Fig. 3, H1\cite{14} and ZEUS\cite{2} measurements of \( F_2 \) made during the 1992 HERA run, corresponding to an integrated luminosity of \( 25 \text{nb}^{-1} \). Only the statistical errors of the data are shown. Measurements will be made with much higher luminosity, and at smaller \( x \) values, in the future.

Several features of this plot are noteworthy. First, if we compare the data with the “\( x^{-\lambda} \)” dependences” of the Altarelli-Parisi forms of MRS(D′)_L, GRV and MRS(H) (which have respectively \( \lambda = 0.5, \approx 0.4 \), and 0.3), then we see that MRS(D′)_L and GRV are disfavoured. So we are left with MRS(H), which, in fact, was devised simply to reproduce\cite{6} the HERA data of refs.\cite{1,2}.

Second, we see that the AKMS prediction (which pre-dated the HERA data) is, like MRS(H), in good agreement with the \( x \) and \( Q^2 \) dependence of the data. In principle, it is an absolute perturbative QCD prediction of \( F_2(x,Q^2) \) at small \( x \) in terms of the known behaviour at larger \( x \), but, in practice, the overall normalization depends on the treatment of the infrared region of the BFKL equation\cite{4,13}. We can therefore normalise the BFKL-based predictions so as to approximately describe the data at \( x = 0.0027 \) by adjusting a parameter which is introduced\cite{12} in the description of the infrared region. For the running \( \alpha_s \) AKMS calculation, this is achieved if the infrared parameter \( k_0^2 \approx 2 \text{GeV}^2 \) (with \( k_c^2 = 1 \text{GeV}^2 \)), in the notation of ref.\cite{12}. Strictly speaking, within the genuine leading \( \log(1/x) \) approximation the coupling \( \alpha_s \) should be kept fixed\cite{12}. We therefore also solved the BFKL equation with fixed \( \alpha_s \),

\(^1\)See also the partons of the CTEQ collaboration which have \( \lambda = 0.27 \)\cite{3}.

\(^2\)The use of running \( \alpha_s \) has the advantage that then the BFKL equation reduces to the Altarelli-Parisi equation in the double leading logarithm approximation when the transverse momenta of the gluons become strongly ordered.
choosing a value $\alpha_s = 0.25$ so as to have a satisfactory normalization. The resulting $Q^2$ dependence of $F_2(x, Q^2)$ turned out to be almost identical to that calculated from the solution of the BFKL equation with running $\alpha_s$. For clarity, we therefore have omitted the fixed $\alpha_s$ curve from Fig. 3. Also a background (or non-BFKL) contribution to $F_2$ has to be included in the AKMS calculation\[9\]; this explains why MRS(H), with $\lambda = 0.3$, and AKMS, with $\lambda \approx 0.5$, both give equally good descriptions of the HERA data. However, by the smallest $x$ value shown we see that the BFKL-based AKMS predictions for $F_2$ begin to lie significantly above those for MRS(H), due to this difference in $\lambda$.

A third feature of Fig. 3 is the stronger $Q^2$ dependence of the AKMS predictions as compared with the MRS and GRV extrapolations which are based on Altarelli-Parisi evolution. This we had anticipated, with a growth approaching $(Q^2)^{3/2}$ for BFKL as compared with the approximately linear $\log Q^2$ behaviour characteristic of Altarelli-Parisi evolution. In reality, at the smallest $x$ value shown we find that the AKMS growth is reduced to about $(Q^2)^{1/4}$, due to the fact that $F_2$(background) is still significant. Although we see that the BFKL and Altarelli-Parisi $Q^2$ behaviours are quite distinctive, to actually distinguish between them will clearly be an experimental challenge, particularly since $Q^2 \lesssim 15$GeV$^2$ is the kinematic reach of HERA at the lowest $x$ value shown. Recall that the BFKL and Altarelli-Parisi equations effectively resum the leading $\log(1/x)$ and $\log(Q^2)$ contributions respectively. Thus the BFKL equation is appropriate in the small $x$ region where $\alpha_s \log(1/x) \sim 1$ yet $\alpha_s \log(Q^2/Q_0^2) \ll 1$, where $Q_0^2$ is some (sufficiently large) reference scale. If the latter were also $\sim 1$ then both $\log(1/x)$ and $\log(Q^2/Q_0^2)$ have to be treated on an equal footing\[17\], as is done, for instance, in the unified equation proposed by Marchesini et al.\[18\]. For this reason we restrict our study of small $x$ via the BFKL equation to the region $5 \lesssim Q^2 \lesssim 50$GeV$^2$. As it happens, the very small $x$ HERA data lie well within this limited $Q^2$ interval.

So far we have neglected the effects of parton shadowing. If, as is conventionally expected, the gluons are spread reasonably uniformly across the proton then we anticipate that the effects will be small in the HERA regime\[12\]. For illustration we have therefore shown the effects of (speculative) “hot-spot” shadowing, corresponding to concentrations of gluons in small hot-spots of transverse area $\pi R^2$ inside the proton with, say, $R = 2$GeV$^{-1}$. In this case, to normalise the predictions at $x=0.0027$, we need to take the infrared parameter $k_0^2 \approx 1.5$GeV$^2$. With decreasing $x$, we see from Fig. 3, that this shadowed AKMS prediction increases more slowly than the unshadowed one, but that it keeps the characteristic “BFKL $Q^2$ curvature”.

To conclude, we have performed a detailed analysis of the $Q^2$ dependence of the structure function $F_2(x, Q^2)$ in the small $x$ region which is being probed at HERA. We have found that the theoretically-motivated BFKL-based predictions do indeed lead, in the HERA small $x$ regime, to a more pronounced curvature of $F_2(x, Q^2)$ than those based on next-to-leading order Altarelli-Parisi evolution. The difference is illustrated in Fig. 3 by the comparison of the AKMS curve with that for MRS(H). From the figure we see that data at the smallest possible $x$ values will be the most revealing. The measurements shown are from the 1992 run, but data with much higher luminosity, and at smaller $x$, will become available in the near future. Clearly the experimental identification of the characteristic BFKL $Q^2$ behaviour will pose a difficult, though hopefully not an impossible, task.

\[3\]To be precise, we take $F_2$(background) = $F_2(x_0 = 0.1, Q^2)(x/x_0)^{-0.08}$\[12\]; a form which is motivated by “soft” Pomeron Regge behaviour\[10\]. Other reasonable choices of the background do not change our conclusions.
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Figure Captions

Fig. 1: The measurements of $F_2(x,Q^2)$ at $Q^2 = 15$ and 30 GeV$^2$ by the H1$^1$ and ZEUS$^2$ collaborations shown by closed and open data points respectively, with the statistical and systematic errors added in quadrature; the H1 and ZEUS data have a global normalization uncertainty of ±8% and ±7% respectively. The continuous, dotted and dashed curves respectively correspond to the values of $F_2$ obtained from MRS(H)$^3$, GRV$^6$ and MRS(D$^4$) partons. The curves that are shown as a sequence of small squares (triangles) correspond to the unshadowed (strong or “hot-spot” shadowing) AKMS predictions obtained by computing $F_2 = f \otimes F_2^{(0)} + F_2^{(\text{background})}$ as in ref.$^{12}$ and as described in the text.

Fig. 2: Diagrammatic display of the $k_T$-factorization formula (2), which is symbolically of the form $F_2 = f \otimes F_2^{(0)}$, where $f$ denotes the gluon ladder and $F_2^{(0)}$ the quark box (and crossed box) amplitude.

Fig. 3: The $Q^2$ dependence of $F_2(x,Q^2)$ at small $x$ (note the shifts of scale between the plots at the different $x$ values, which have been introduced for clarity). The curves are as in Fig. 1. Also shown are the measurements of the 1992 HERA run obtained by the ZEUS collaboration$^2$ (open points) and, by the H1 collaboration$^{14}$ using their “electron” analysis (closed points). Only statistical errors of the data are shown. The ZEUS points shown on the $x=0.00098$ curves are measured at an average $x=0.00085$. A challenge for future experiments is to distinguish between curves like AKMS and MRS(H), both of which give a satisfactory description of the existing data.