Schrödinger-Milne Big Bang—Creating a ‘Universe of Threeness’

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Summary
A Schrödinger-evolving forward-lightcone-interior ‘Milne’ universe (‘SMU’) is governed by ‘centered-Lorentz’ (CL) symmetry—that of a 9-parameter Lie group with a 6-parameter SL(2,c) ‘exterior’ and a 3-parameter ‘quality-space’ center. ‘Reality’ resides in current densities of electric charge and energy-momentum—the Dalemberian of an SMU-ray-specified classical retarded Lorentz-tensor field with $2^2$ electromagnetic and $3^2$ gravitational components.

Nine conserved Dirac momenta comprise the CL algebra. We here propose a Dirac self-adjoint CL-invariant Hamiltonian—kinetic energy plus electromagnetic and gravitational potential energy—to evolve the SMU ray from a featureless beginning. Illuminated here via discreteness of electric charge are baryon number (‘nuclear forces’ arising from ‘almost-screened’ electric charge), Bohm’s ‘hidden variables’—dark matter and dark energy—and three generations of ‘elementary’ fermions.
Introduction

Dirac’s nonrelativistic quantum theory (1) was based on Hilbert-space self-adjoint operators—Dirac coordinates and Dirac momenta plus a Hamiltonian which commuted with the Dirac-momentum-represented algebra of a Euclidean Lie symmetry group. Dirac coordinates represent the symmetry-group’s manifold. Ray evolution was according to a Schrödinger first-order differential equation. Such quantum theory has for ‘relativistic’ physics been obstructed by absence of unitary finite-dimensional Lorentz-group representations.

The Gelfand-Naimark (GN) unitary Hilbert-space infinite-dimensional representations, (2) although inapplicable to a physics focused on ‘objective reality’—i.e., on particles—apply quantum-cosmologically to a Schrödinger-Milne Planck-scale-originated--‘big-bang’—a continuously-evolving universe (SMU) whose galaxies presently contain more ‘dark matter’ than particulate. Non-galactic non-particulate unphysical energy continues at present to exceed that of galaxies.

Milne’s universe (3) situates inside a Lorentz-Minkowski forward lightcone—a 4-dimensional manifold whose boundary locates outside Milne’s universe-occupied submanifold. A positive Lorentz-invariant ‘age’, $\tau \geq \tau_0 > 0$, of any location within Milne’s universe is defined to be its ‘Minkowski distance’ (not a ‘Riemann geometrical distance’) from the lightcone vertex. Universe age at big bang was $\tau_0$.

Milne’s geometrical 3-space at any fixed age is hyperbolic—negatively curved in Riemann sense as suggested by the surprising astronomically-observed Nobel-acknowledged correlation between luminosity distance and redshift. (4) The present paper will ‘embellish’ GN’s unitary Hilbert-space Lorentz-group representation with discrete electric charge. A 9-parameter SMU Lie symmetry group, here denoted ‘CL’, then enriches Milne’s 6-dimensional SL(2,c) ‘universe-exterior’ by a 3-dimensional group center that, in spanning the universe’s ‘quality space’, recognizes discreteness both of electric charge and of energy.

Through an approximate relation between redshift and the (geometrical) distance light travels through Milne’s negatively-curved noncompact 3-space, age is astronomically estimate-able. Present age is approximated by the reciprocal of astronomy’s ‘Hubble constant’. (4)

‘Cosmological photons’ ($\gamma, \chi$), first created at extremely early SMU ages, enjoy a ‘semi-foundational’ status in SMU’s unification of electromagnetism and gravity—a GN-dependent unification without ‘gravitons’ that attributes the strength and ‘short range’ of (present-age) ‘nuclear forces’ to any nucleon (of unit baryon number) being a composite of nine electrically-charged ‘quantum-universe constituents’—qucs. One of many important outcomes of discrete-charge-screening is ‘short-range nuclear forces’. A principal task of the present paper is to define ‘quc’. An electron comprises three electrically-charged qucs, and a photon or neutrino two. Dark-matter-composing qucs are chargeless.

Decades of thought about alternative possibilities, plus attention to Occam’s principle, have led the author here to propose SMU birth without any ‘particles’ at a Planck scale age. But there was, at SMU’s start, a huge (although finite) energy which has diminished subsequently by inverse age-proportionality of each quc’s energy. The here-proposed set of SMU-composing qucs is finite and fixed, although ginormous.

The SMU ray at age $\tau \geq \tau_0 > 0$—a sum of (‘tensor’) products of single-quc Hilbert vectors—is an indefinitely-differentiable function of $\tau$—without singularities at any age greater than or equal to the positive starting-age $\tau_0$ whose value, together with the (permanently-fixed) number of (different) qucs and an electric-charge unit, provides SMU’s foundation.
At any age (above \(\tau_0\)), ray expectations of certain here-defined self-adjoint Hilbert-space operators specify a ‘reality’ consistent at present and recent ages with Karl Popper’s humanistic classical meaning for ‘physical measurement’. But SMU’s age-varying 3-space curvature requires that an inherently-approximate Euclidean meaning for ‘quantum physics’ be distinguished from the meaning of ‘quantum-cosmology’.

Specified here is a (Dirac-sense) self-adjoint (even though ‘cosmological’) Hilbert-space Hamiltonian operator whose potential energy generates ‘creative’ ray dynamics (crd). Kinetic energy perpetuates the creativity of evolution. Appendix A proposes a creation-bereft (particle-absent) initial ray all of whose qucs carried positive energy.

SMU’s attribution, through mathematical language, of cosmological meaning to Hamiltonian kinetic and potential energy as well as to ‘momentum’, ‘angular momentum’, ‘electric charge’ and ‘energy’—all notions uncovered by human physics—has amazed the author. Is it conceivable that the human-uncovered mathematical notions of ‘Riemannian geometry’, ‘Lie-group algebra’, ‘Hilbert space’, ‘fiber bundle’, ‘differential equation’ and ‘Mersenne prime’ play ‘cosmological’ roles that transcend humanity? This paper supposes such to be the case.

Milne’s hyperbolic three-dimensional space at fixed universe age is CL-invariantly metricized and thereby, although (negatively) curved, endowed with an unambiguous (positive, Lorentz-frame-independent) shortest path (geodesic) between any pair of spatial locations.\(^{(4)}\) Completely age-determined (without dependence on 3-space location), Milne’s spatial curvature ignores energy distribution—curvature being initially at Planck scale and diminishing thereafter—paralleling diminishment with advancing age of any and all qucs energies. Hubble’s ‘constant’ approximates both the inverse of present-universe age and the current magnitude of (negative) 3-space curvature.\(^{(4)}\)

Above emphasized is SMU governance by the symmetry of a 9-parameter CL ‘centered-Lorentz’ Lie group whose (below-compactified) center is 3-parameter U(1) \(\times\) SL(2,\(c\),\(D\)).\(^{(4)}\) Here the symbol D denotes 2-parameter left-acting diagonal complex unimodular 2\(\times\)2 matrices. The (non-compact) CL exterior is 6-parameter right-acting SL(2,\(c\)). Formula (8) below, within our main text, shows how ‘left-right’ distinction is achieved by GN’s remarkable Hilbert space (never associated by its discoverers to cosmology).

CL extends SL(2,\(c\)) by a trio of single-parameter CL-commuting subgroups. Algebra extension from 6 to 9 elements defines, firstly, additively-conserved (ac) discrete and superselected electric charge with units proportional to \((ke)^\frac{1}{2}\), secondly, a discrete ‘semi-superselected’ attribute dubbed ‘chirality’ with \#\(\sqrt{2}\) units and, finally, discrete ac energy with \#\(2\pi\) units. As later elaborated, the universe’s 3-dimensional ‘quality space’ is spanned by CL’s center.

Energy discreteness does not mean Hamiltonian diagonalization. (Spacing between successive possible single-quc energies is \#\(2\pi\).) The quantum-physics notion of ‘stationary state’ enjoys no cosmological meaning. SMU displays ‘onflow’—never-ending continuous development of ‘newness’.

The 6-element (‘acting from the right’) exterior CL algebra defines angular momentum and momentum. Although the former (when Stone-Dirac represented) is discrete, the latter is not. Non-compactness of CL exterior associates in SMU to continuous spectra for a (3-vector) trio of Hilbert-space self-adjoint (Dirac) quc-momentum operators, whose non-Euclidean (cosmological) failure to mutually commute lacks physics precedent. Physics meaning for the term ‘boost’ is absent from SMU’s cosmological vocabulary. An SMU Stone-Dirac momentum 3-vector operator generates displacements in Milne’s curved 3-space.
Conservation of electric charge and chirality, as well as of angular momentum, is SMU (Noether) sustained by unitary Hilbert-space CL representation and CL-invariance of Hamiltonian. Despite inverse age-proportionality of all quc energies and momenta (not of ‘particle masses’, whose significance is physical—not cosmological), the CL-central algebra adjoins a positive ac energy-integer to charge and chirality integers—thereby defining a (permanent) ‘complete set’ of different SMU constituents.

Wigner’s flat 3-space (10-parameter) ‘Poincaré group’, foundation for quantum field theory (QFT) and the S matrix, fails to admit (Dirac-required) unitary Hilbert-space representation. Having been apprised by D. Finkelstein (private communication) that the Lorentz group ‘Inonu-contracts’ to the Euclidean group in a \( \tau \to \infty \) limit, we have come to regard QFT as a ‘3-space-flattened’ micro-macro-scale approximation which, for human-history values of \( \tau/\tau_0 \sim 10^{30} \), is adequate for human physics purposes (FHPF, imitating John Bell’s acronym) although not for all purposes of Schrödinger-Milne cosmology. (The author believes Bell, philosopher as well as physicist but not cosmologist, to have regarded humanity’s 3-space as FAPP Euclidean.)

Physics, able to ‘Popper-define’ particles but not qucs, is unable to describe ‘dark matter’. Bewilderingly (to the author), Maxwell classical-electromagnetic theoretical physics, through 4-vector electric-charge current density with discrete electric charge, plus symmetric-tensor energy-momentum current density, seems capable of classical Popper-physics (discrete) ‘particle’ definition, regardless of 3-space curvature. But QFT requires flatness for its 3-space. There is no SMU meaning for ‘quantum radiation field’.

QFT had become an accurate ‘local’ approximation for micro to macro spatial scales by the macro-scale ages, \( \tau/\tau_0 \sim 10^{38} \), when galaxy and star-building commenced—well after the micro-scale ages, \( \tau/\tau_0 \sim 10^{19} \), when SMU’s massive-elementary-particle-building got underway. The Poincaré group and the associated QFT identical-elementary-particle micro-macro physics approximation will be addressed by other papers. Appendices here make a start.

QFT’s ‘parity-reflection’ (not a Lie-group generator) parallels sign-reversal of SMU’s self-adjoint ‘chirality’—one of the 3 central CL generators. (All 9 generators are ‘Noether conserved’). Dirac’s writings never mention ‘parity’ but he proposed a ‘doubling’ of electron spin through a velocity direction (not momentum direction) that was either parallel or antiparallel to spin direction.

Dirac’s ‘relativistic-electron (quantum-physics) wave function’ (frustratingly for Dirac, not a ‘Hilbert vector’) satisfied a first-order (Schrödinger) equation of motion via his doubling of 2-valued electron spin. Later this doubling became represented by the notation \((0, \frac{1}{2})\) and \((\frac{1}{2}, 0)\) for a pair of inequivalent nonunitary finite-dimensional SL(2,c) representations. The three eigenvalues, \(0, \pm 1\), of SMU’s self-adjoint quc ‘chirality’ amount to a ‘Dirac tripling’ relevant to all particles, not only spin-½ fermions.

A pair of main-text 4-vector fiber-bundle quc Dirac-coordinates, here employed to achieve a retarded Lorentz-tensor-field classical-cosmological definition of ‘reality’, represent the ‘Maxwell-Lorentz’ group SO(3,1) while also providing \((\frac{1}{2}, \frac{1}{2})\) representation of SL(2,c)_R. Classical electromagnetic fields provide \((0, 1)\) and \((1, 0)\) representations of SL(2,c)_R, but strikingly-absent from the definition of these fields is any reference to chirality—a term here accorded cosmological ‘Dirac-tripling’ meaning.

Although no finite-dimensional representation of any ‘Lorentz’ group is unitary, a classical bridge between ‘particulate Popper physics’ and cosmology is provided by a below-specified Lorentz-tensor ‘reality’ that does not require 3-space to be Euclidean and makes no reference to chirality. Cosmological absence of QFT’s quantum-theoretic (S-matrix) meaning for ‘particle’ is alleviated by a
classical cosmological ‘Popper’ meaning that is based on charge discreteness together with Maxwell’s equations and an energy-momentum tensor.

Formula (8) here shows how, with GN’s unitary representation, Dirac might, in the spirit of later-appreciated ‘supersymmetry’, have extended the Hilbert-space-representable U(1) group generated by a self-adjoint operator representing electric charge. The 1-dimensional compact manifold spanned by chirality-generated GN Hilbert-vector argument displacements covers an interval twice that spanned by charge-generated displacements of a (complex) Hilbert vector’s phase.

Formula (8) formalizes the foregoing. Unitary SL(2,c) representations remained undiscovered for more than a decade after Dirac’s attempt to ‘relativize’ the electron. Dirac’s opinion, either about Milne cosmology or about GN’s unitary SL(2,c) representations, is unknown to the author.

The ‘exterior’ 6-element nonabelian SL(2,c)_R algebra, a subalgebra of the CL group, \(^{(4)}\) defines SMU conserved momentum and angular momentum—the generators, respectively, of infinitesimal displacements of location and orientation within Milne’s negatively-curved metricized 3-space. Milne seems not to have represented either energy or electric charge; he almost certainly did not represent chirality.

The 9-member CL algebra provides a complete set of conservation laws to govern SMU evolution of a reality that encompasses Hubble redshift together with non-particle extra-galactic dark energy, galactic ‘dark matter’ and QFT’s set of macro-scale-observable micro-scale ‘identical’ elementary particles within our galaxy. (Certain other galaxies may be found to contain an alternative set of elementary particles—with QFT’s ‘left-handed weak-vector bosons’ replaced by ‘right-handed’ counterparts.)

The terms ‘particle’ and ‘identical particles’ are FHP-meaningful at present and recent SMU ages. But Milne-Lorentz cosmological symmetry transcends physics by attending to dark matter and dark energy as well as to ‘early’ SMU history when 3-space curvature was huge on macro scale. [Note: Our adjective ‘macro’ applies both to the (kilometer) ‘lab’ spatial scale at which ‘measurements’ are performed by ‘conscious beings’ and to the temporal scale of those SMU ages when galaxy-clumping began.]

SMU recognizes, while not depending on, approximate scale-dependent human-language meaning for ‘free-will measurement by an observer’—a macro-scale Galilean notion on which human (Popper) physics is founded. The author believes human language incapable of exactly true sentences’. Any human-language ‘truth’ we believe a scale-dependent approximation, which sometimes may enjoy high accuracy because of ginormous differences between five different spatial scales currently recognizable in SMU—GUT scale, micro scale, macro scale, galactic scale and Hubble scale. Human language is macro. Far above macro while still far below the (present) scale of Hubble is the galactic scale of dark matter. All five scales play roles in the present paper’s content, which to the author appears consistent with the belief, by an increasing number of philosophers, that ‘free will is a (macro-scale) human illusion’.

Both universe age (approximately measurable by redshift) and the proposed SMU Hamiltonian—Formula (21) below—which analytically generates universe-ray evolution with increasing age, are CL invariant, with the CL algebra (Stone) representable by self-adjoint GN-Hilbert-space (Dirac-momentum) operators. Once again: CL algebra comprises (Noether-conserved) continuous-momentum times age, angular momentum, electric-charge, chirality and energy times age—the latter sextet all Dirac-discrete.
SMU Elemental Constituents

The unitary CL representation by Formula (8) below has led the author to recognize a set of SMU elemental constituents, each here bearing the (pronounceable) name ‘quc’ (quantum-universe constituent), which we here suggest compose the entire universe in a sense evocative of that accorded by nuclear physicists to Gell-Mann’s acronym ‘quark’. No single quc, individuated by 3 central integers specifying its electric charge, its chirality and its energy, is a 'particle'. SMU, nevertheless, we propose to be completely ‘quc-composed’.

A subscript q, equivalent to a trio of integers, $Q_q, N_q$ and $M_q$, here distinguishes any quc from all other qucs. It will below be seen that the total number of different qucs is $21 M_{\text{max}}$, with $M_{\text{max}}$, a ginormous but finite (age-independent) positive integer. Dependence of any single-quc wave function on one of its 6 ‘Dirac’ quc coordinates—that which is ‘canonically-conjugate’ to $M_q$—‘spreads’ this quc over the entire Milne 3-space. Any individual particle ‘location’ similarly is spread. All ‘meaning’ is relative. Application of any element of the CL symmetry group to a (multi-quc) SMU ray ‘changes nothing’.

An SMU ray, at some fixed age, is a sum of products with 21 $M_{\text{max}}$ factors—each of these a wave function of a different quc. An ‘elementary particle’ is Newton-Maxwell-theoretically (Popper) a classical ‘clump’ of conserved energy, momentum and angular momentum, with an (approximate) ‘FHPP mass’, a ‘spin’ and an electric charge equal to some integral multiple of a universal charge unit.

Such ‘fixed and settled’ Popper reality is later prescribed by (mathematically-defined) expectations of certain self-adjoint operators for the SMU ray at the age in question. Any particle, whether or not considered ‘elementary’, associates to electro-dynamically-correlated wave functions of many different charged qucs.

Two-Quc ‘Cores’ of Massive Elementary Particles

Within the individual terms of an SMU ray’s tensor-product summation, a zero-chirality zero-charge 2-charged-quc ‘core’ factor associates to any elementary particle except photons and neutrinos. A core factor depends on the quc-pair’s ‘relative’ coordinate, which later in this paper will be seen essential in a more general context to the SMU Hamiltonian’s potential energy. A particle’s mass reflects its core.

Not only does the net charge and chirality of a ‘particle-core’ quc-pair vanish but so does its below-specified baryon number. (A photon or neutrino is a neutral zero-baryon-number but chirality-bearing quc pair.) When the elementary particle in question is charged, the charge is provided by a ‘valence’ quc. Also established by valence is particle chirality and baryon number.

What about elementary-particle energy, momentum and angular momentum? A valence quc adds its contribution of the foregoing conserved attributes to that of particle core. A Hamiltonian-prescribed superposition of products with variable values of total energy, momentum and angular momentum, as well as different distributions thereof between the 3 qucs, represents the particle. Different energy distributions require different qucs to appear within the 2-quc or 3-quc sets that quantum-theoretically represent a single elementary particle.

Each quc carries a positive energy integer with one of a ginormous although finite set of possible values, an electric-charge integer with one of the 7 values 0, $\pm 1$, $\pm 2$, $\pm 3$ and a ‘chirality’ integer with one of the 3 values, 0, $\pm 1$. (All particle-building qucs have nonvanishing electric charge!) The foregoing options became author-appreciated after decades of Occam-guided contemplation that included many discussions with colleagues. Also additively-carried by a particle’s constituent-qucs are (‘familiar’) momentum and angular momentum.
Baryon Number and Chirality

Temporal microscale stability of a particle requires any particle-composing quc to have non-vanishing electric charge. The electric-charge integer \( Q_q \) defines quc-\( q \)'s baryon number, \( B_q \), which vanishes when \( Q_q \) takes any one of the three values 0, ±3. If \( Q_q \) is either +2 or −1, \( B_q \) is +1/2. If \( Q_q \) is either −2 or +1, \( B_q \) is −1/2. Individual qucs thereby may be categorized as either ‘baryonic’ or ‘nonbaryonic’; total-universe baryon number vanishes. (See Table I in Appendix E.)

Despite absence of ‘particle’ status for single qucs, the 9 conserved Dirac-momentum quc attributes as well as baryon number are each additively manifested by particles ‘built’ from qucs. ‘Particles’—(spatial) ‘clumps of reality’ each with integer net charge and one-third-integer baryon number, odd or even chirality and an ‘FHPP mass’—are, in a flat 3-space (QFT) approximation, fermions (bosons). Not addressed by this paper is the error in the S-matrix concept of ‘identical’ particles.

Particles of common FHPP mass, spin, charge, baryon number and momentum but with differing energy-distribution among constituent qucs, are ‘physically identical’—FHPP identical—despite not being ‘cosmologically identical’. (Appendix C.) Pedagogically-useful quc segregation, both by angular momentum and by chirality, into ‘fermionic’ and ‘bosonic’ categories is unaccompanied by any meaning for ‘quc statistics’.

The physics word ‘particle’ lacks precise SMU meaning! SMU classical cosmology is a ‘continuous onflow of electro-gravitational reality’, with discretely-conserved (because of quantum superselection) electric charge and baryon number and continuously-conserved energy and momentum times age. The author believes the term ‘plasma’ to be useful, both physically and cosmologically, but there is no cosmological usefulness for the term ‘vacuum’.

[Energy integer, whose definition in Reference (4) is repeated below in this paper’s main text, was suggested to the author not by QFT but by Fermi momentum notions useful in condensed-matter physics. At present-universe age the difference in energy associating to successive SMU energy integers—a difference determined by SMU age—is ‘ginormously tiny.’]

The SMU Hamiltonian, specified by Formula (21) to be a CL invariant (not a 4-vector component), comprises a sum of single-quc kinetic-energy and quc-pair electro-gravitational potential-energy self-adjoint Dirac operators. The finite although ginormous set of SMU qucs is age-independent.

Cosmological vs. Physical Photons

Other papers, through ‘recent-age’ flat 3-space (Euclidean) micro-macro approximations, will depict as Schrödinger-Dirac charged-quc composites not only QFT quarks, charged leptons and W bosons but neutrinos, \( Z_0 \)'s, and Higgs bosons. Transcending Euclidean 3-space QFT (whose accuracy derives from recent macro-scale hugeness of SMU age) are net electrically-neutral but chirality-carrying zero-mass cosmological photons (\( \gamma_c \))—each a pair of nonbaryonic qucs oppositely charged but of same nonzero chirality (total \( \gamma_c \), chirality being ±2).

Equality, between the value of its chirality and that of twice its helicity, accompanies physical-photon unique masslessness—equality of energy and kinetic energy. The quc structure of a propagating \( \gamma_c \) might spatio-temporally be described as an ‘electro-gravitationally-stabilized double helix’—a ‘perfect quc marriage’—remarkably enjoying the same number (6) of ‘Dirac degrees of freedom’ (Ddof) as a single chargeless quc. (Appendix C) All other quc marriages are ‘imperfect’.

Two of a \( \gamma_c \)'s 6 Ddof are ‘relative’ (internal and fluctuating) coordinates on which a unique \( \gamma_c \) ‘ground-state’ internal wave function depends. Four of the 6 Ddof are external ‘Dirac momenta’ (3-vector momentum plus helicity). Future investigation we expect to expose not only the ‘shape’ of the \( \gamma_c \)'s
internal ground state but absence of other stable internal states. (The double-helix characterization involves both external and internal DoF of a γc.)

Pedagogically, the term ‘mass’ helps to distinguish the ‘particle’ concept from that of ‘quc’. There is no meaning for ‘quc mass’—in contrast to the long-appreciated physics meaning for zero photon mass, which Appendix C, via the 2-quc γc, accords to FHPP ‘identical photons’ obeying Bose statistics.

Miscellany, Perhaps Helpful to Reader Thinking

Throughout this paper reference to a single quc may for convenience of reader thinking be understood as in an ‘SMU local frame’ whose definition relates to later-defined quc ‘Dirac coordinates’. FHPP meaning for local frame relates to Milne’s celebrated ‘cosmological principle’. In local frame, present-universe cosmic background radiation (CMB) is isotropic. (QFT’s meaning for ‘lab frame’ roughly matches—with error ~10⁻³—that of SMU local frame!) In local frame, time change and age change are equal—‘time interval since big bang’ being equal to τ_0. ‘Local frame’ associates to the CL-invariant meaning of ‘quc energy’. Wherever the latter term here is used it refers to the quc’s energy in a space where a (3-dimensional) spherically-symmetric big bang occurred’—all clumping of spatial energy being a consequence of cd after universe birth. (Appendix A) Because of clumpings developed before (‘recent’) CMB decoupling from atoms (at τ/τ₀ ~10³), the local frame is only approximately establishable by CMB astronomy.

The quc-composed Schrödinger-Milne universe may not be described as ‘composed of elementary particles’. Additivity of quc ‘Dirac momenta’ nevertheless allows an ‘elementary-particle set’ of 2 or 3 (different) charged qucs to represent the CL symmetry group through a Dirac-momentum unirrep csco—a complete set of 7 commuting self-adjoint operators that adjoins two CL Casimirs to the direction (2 operators) of (exterior) conserved momentum and the 3 central momenta. SMU’s multi-quc ‘external-momentum csco’ parallels the ‘asymptotic Hilbert space’ of the particle-physics S matrix. Appendix B addresses the single-quc ‘Dirac-momentum’ csco.

The SMU Hamiltonian—to which this paper’s main text leads—-is expressed through Dirac quc coordinates and Dirac quc momenta, but not through particles. The author nevertheless expects SMU’s Hamiltonian (eventually) to explain (approximately) the observed values of elementary-particle masses and other arbitrary QFT parameters. (Appendix E)

Each of 9 conserved elementary-particle attributes is the sum of (corresponding) constituent-quc attributes. QFT elementary-particle ‘asymptotic Hilbert space’ enjoys useful approximate (flat 3-space) S-matrix meaning because the macro scale of a human laboratory, although huge on micro scale, is tiny on Hubble scale. Sums of quc (Dirac) momenta approximate particle (S-matrix) momenta. No such feature attaches to (non-conserved) quc (Dirac) coordinates.

Even with quantum fluctuation of the energy integer, M_q (essential to single-quc identification), it is useful to think of any QFT elementary particle as a macro-stable (‘Popper-measurement-accessible’) potential-energy-induced marriage (after universe birth and of GUT or micro internal spatial scale) between 2 or 3 electrically-charged chirality-carrying qucs whose wave functions at SMU birth were uncorrelated. (Appendix A)

An ‘early’ such marriage—at GUT or micro-scale ages—might dynamically be described as Hamiltonian-generated ‘collapse’ of a product of uncorrelated charged single-quc wave functions into a macro-temporally-stable micro or GUT-sized ‘object’ wave function. (The photon is the only ‘elementary’ particle where, because of GUT-scale internal extension, gravity significantly contributes to
stability.) In any present-day S-matrix ‘connected part’, it is electromagnetic potential energy that causes charged married qucs to ‘change partners’.

Dark matter comprises gravitationally-sustained galactic-scale ‘colonies’ of individually-chargeless bachelor qucs. (A dark-matter bachelor-quc wave function spatially spans its galaxy.) Inter-galactic universe-spanning bachelor qucs, both charged and uncharged but without previous ‘marriage history’ because of energy too high for electromagnetic macro-clumping, constitute ‘dark energy’.

Particle mass, a physics word without (onflow) cosmological meaning, reflects Hilbert-space collaboration between the discrete ca trio (electric charge, chirality, energy) and continuous zitterbewegung (zbw). The latter (Schrödinger-coined) term refers to fluctuation of lightlike velocity direction—a non-conserved Dirac coordinate—at fixed conserved Dirac momenta. As above noted, the CL unirrep csco provides a Hilbert-space FHPP elementary-particle-momentum basis that, although without particle mass among its labels, resembles ‘in’→‘out’ S-matrix ‘asymptotic Hilbert space’.

Physics-fundamental, however, are conserved (and with commuting components) energy-momentum 4-vector Poincaré-group generators. Essential SMU roles are played in this paper’s main text by two non-conserved positive-4-vector quc Dirac coordinates, specifying the quc’s spatio-temporal location and its lightlike-velocity direction, but there are no SMU quc-energy-momentum 4-vectors. SMU momentum-basis portrayal of a photon’s external Ddof extends to massive elementary particles—via CL-invariant particle energy and a smaller invariant, although not conserved, momentum magnitude (‘relativistic kinetic energy’). But neither portrayal is 4-vector!

For pedagogical reasons the CL unirrep (Dirac momentum basis) is not addressed by this paper’s main text. Appendix B exposes the quc Dirac-momentum csco that complements the main-text-explored quc Dirac-coordinate csco.

QFT is a spatially-restricted scale-dependent approximation that accords meaning to ‘vacuum’, ignores (photon-redshifting) universe expansion and relies on an S matrix with a priori elementary-particle masses and electric-charge screening. Electron mass is treated by QFT as non-fluctuating and age independent.

SMU (approximate) age-independence of electron mass we associate to ongoing interchange between electron-attached qucs and charged intergalactic bachelor (un-clumped ‘dark-energy’) qucs of, on average, slightly higher $M_e$, than those of the electron qucs they replace. The author counts on eventual verifcation of such interchange by Hamiltonian-based computation. The ‘vacuum’ concept is absent from SMU, where the set of qucs is fixed. Incompleteness of electric-charge screening is of major SMU importance.

Any quc subset represents CL. The total set of universe-comprising qucs, although ginormous, is finite and constant (ε-independent). Qucs are never created or annihilated. We repeat: Despite each of 8 CL generators—all except chirality—representing a conserved quc attribute to which a physics-familiar name may be attached, while all 9 ca are particle-carried, no quc may ever individually be called ‘particle’.

Six conserved ‘exterior’ attributes of a quc—its momentum and its angular momentum—may change from crd interaction with other qucs as universe-age increases. Three integer-specified conserved ‘central’ attributes—electric charge, chirality and energy times age—remain unchanged. Continuous momentum-basis variability of a quc’s momentum-magnitude times age, while its energy integer remains fixed, renders impossible any definition of ‘single-quc mass’.

Objective reality—spatially-localized and temporally-stable current density—involves at the very least two qucs. A single quc cannot represent an ‘object’—the definition of which requires a self-
sustaining relationship between different qucs. Dark matter, despite its gravitationally-sustained localization being at galactic scale, is ‘objectively real’. A galaxy, although not built entirely from ‘particles’, may astronomically be described (approximately) as a ‘spatially-localized object’.

Any future SMU (of age greater than present age) is determined by the quantum state of SMU now (as the reader is seeing this sentence). SMU’s quc foundation transcends measurement-based physics. A self-adjoint Hamiltonian operator is a Schrödinger-Dirac-imitating sum of single-quc kinetic-energy and quc-pair potential-energy operators.

Each quc carries a discrete energy that permanently remains positive despite perpetual decrease by its inverse proportionality to positive-increasing universe age. Accompanying perpetual photon redshift is ongoing (never complete) flattening of hyperbolic 3-space. \(^{(4)}\) These SMU features (in profound contrast to general relativity) involve no reference to ‘local’ energy density!

Current densities of (classical) energy-momentum and electric charge—specified by expectations of E-G field-operators—define at every age an SMU ‘reality’. Only a portion of this reality, nevertheless, is ‘objective’—expressible through the ‘stable object’ notion (which includes a ‘galaxy of stars with attached dark matter’). Most of the present universe’s energy remains non-objective. Bohm hidden-reality comprises both dark energy and, wrt ‘Copenhagen (S-matrix) quantum physics’, dark matter. Disregard of SMU’s hidden reality has required probabilistic interpretation for ‘Copenhagen’ quantum theory.

At any universe age, \(\tau \geq \tau_0\), an SMU Hilbert-space ray, ‘regularly’ representing CL, is a complex normed multiply-differentiable function of the 6 Dirac coordinates of each member of a ginormous but finite age-independent set of qucs. We shall see how the six coordinates of any quc, equivalent to 3 complex coordinates, specify via a complex unimodular \(2 \times 2\) coordinate matrix the quc’s location within a 6-dimensional manifold. A unit such matrix locates the quc at an SMU ‘oriented center’. Any ‘exterior quc location’—later shown to include a 5-dimensional fiber bundle—is rendered ‘central’ by that right-SL\(2(c)\) transformation (an exterior element of the CL group) which transforms this quc’s Dirac coordinate to a unit matrix.

Fixed Popper reality—current densities of energy, momentum, angular momentum and electric charge (not chirality)—is prescribed by the Dalembertian of a 13-component \((2^2+3^2)\) retarded classical electro-gravitational (E-G) Lorentz-tensor field. The present paper’s main text specifies this field by SMU-ray expectations of self-adjoint Lorentz-tensor field (not radiation-field) operators.

At SMU birth all reality was hidden and full reality fails (at any age) to specify the SMU ray that is Schrödinger-determined by the ‘immediately-preceding’ ray. Present reality fails to determine future reality! Hilbert space with Schrödinger Hamiltonian dynamics is SMU essential.

**Unification**

This paper ‘unifies’ gravity and electromagnetism by ‘bundling’ classical Newton-Maxwell \((G-c)\), quantum Planck-Schrödinger-Dirac \((\hbar)\) and classical Hubble-Milne \((\tau)\). The foregoing brackets associate to natural philosophers symbols for 4 positive dimension-ful real parameters--3 constant \((G, c, \hbar)\) and one perpetually increasing \((\tau)\)—that underpin the present paper.

Any objective reality, such as a photon, a proton, a molecule, a planet, a star or a galaxy, associates to exceptional temporally-stable spatially-localized multi-quc wave functions where the ‘expansion’ tendency of positive quc kinetic energy—to increase spatial separation between different qucs—is opposable by negative gravitational and (or) electromagnetic potential energy that tends to decrease separation.
‘Strong interactions’ (‘nuclear forces’) arise from the Formula (21) Hamiltonian’s kinetic plus electromagnetic potential energy, applied to systems with baryon-number-carrying valence qucs. Gravitational potential energy, together with electromagnetic, we believe essential at GUT scale to the photon double helix as well as, at ginormously larger scales, to planets, stars and galaxies.

‘Quality’ 3-Space; Natural Units
SMU displays a 3-dimensional ‘quality space’. The central SMU ‘momentum dimensionalities’—those of energy, electric charge and chirality—span all universe dimensionality. SMU chirality shares the dimensionality of angular momentum. SMU associates G (Newton) to energy, c (Maxwell) to electric charge and \( \hbar \) (Planck) to chirality.

Dimensionless non-conserved quc attributes prominently include (as will later be seen) 2-dimensional positive-lightlike velocity direction—an SMU ‘Dirac quc coordinate’. Mathematics distinction between number-theoretic ‘topology’ and analysis-theoretic ‘geometry’ provides some analog to the distinction between ‘dimensionless’ and ‘dimension-ful’ quc attributes, but neither mathematics nor theoretical physics has so far dignified through a symbol the 3-dimensional ‘quality space’ displayed by the universe. Such a symbol would ‘legitimatz’ SMU’s Lie symmetry-group center—currently puzzling to mathematicians as well as physicists.

Quality-space 3-dimensionality dovetails with the trio of independent universal dimension-ful constants with independent dimensionalities. To these constants the symbols G, c and \( \hbar \) have become attached. Occam-inclined natural philosophers expect no universal dimension-ful constant (udc) beyond this trio.

Notational economy has long been appreciated for units that assign the value 1 to each member of the udc trio. This paper will henceforth employ such units, thereby attaching unambiguous numerical value to the dimension-ful SMU age symbol \( \tau \) (present-universe age in ‘natural’ units being \( \sim 10^{10} \)), to the ‘big-bang’ birth age \( \tau_0 \) (1 or ten to power zero), and to dimension-ful particle-physics parameters such as Higgs mass \( \sim 10^{17} \) and electron mass \( \sim 10^{-33} \).

Values for all the many arbitrary QFT parameters we expect to be shown SMU-Hamiltonian-determined by the (macro) temporal-stability spatial-localizability requirement implicit in objectivity. [Our guess for ‘diameter of photon double helix’—a notion unrecognized by QFT—locates in the neighborhood of QFT’s ‘GUT’ spacetime scale—\( \sim 10^4 \) in natural units (Appendix C).]

Mysterious Physics-Enabling Macro-Scale; Avogadro Number; Feynman’s Perturbative S Matrix
This paper assigns ‘foundational’ status neither to the micro spatial scale, \( \sim 10^{-19} \), set (inversely) by elementary-particle masses nor to the much larger \( \sim 10^{39} \) ‘macro’ spatial scale that characterizes at once the (kilometer) scale of human ‘laboratories’ and the Schwartzschild radius of stellar mass. These scales we portray here as \( \text{crd outcomes} \). Far below micro spatial scale is SMU-foundational GUT scale—that we associate to the electric-charge unit \( g \). Far above macro are galactic and (present) Hubble scales. ‘Macro’ locates in the ‘logarithmic middle’. Two sections below we mention a huge Mersenne prime that, via \( M_{\text{max}} \), might relate to ‘macro’.

QFT’s spatial parameters are ‘micro’—in a neighborhood below macro by a factor larger than the cube root of Avogadro’s mysteriously-huge number. Long appreciated is dependence of physical chemistry, statistical mechanics and condensed-matter physics on the latter’s largeness.

Macro scale understood as ‘lab scale’—the ‘scale of measurement’—is recognized by the QFT S-matrix as well as by Avogadro-number-based atomic and molecular physics, statistical mechanics and condensed-matter physics. How does QFT recognize macro scale? Within each denominator of

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Feynman’s perturbative series for an S-matrix element there appears the symbol $\varepsilon$—representing an energy which, although ‘vanishingly-small’ for S-matrix purposes, cannot be ignored. S-matrix definition regards ‘macro time’ as ‘ginormously larger’ than the times associating to elementary-particle masses.

The inverse of Feynman’s $\varepsilon$ may be regarded QFT’s definition of ‘macro’ time scale—huge for particle-physics purposes, as well as for those of any science relying on Avogadro-number hugeness, while ‘tiny’ compared to galactic scale and yet tinier when compared to that of Hubble. ‘Macro scale’ includes that extremely narrow scale range where ‘free-will measurement by conscious life’ enjoys meaning. Alfred North Whitehead acknowledged mysterious macro scale not by a number but through use of the term, ‘God’. Schwartzschild was thinking ‘stellar black-hole radius’ when recognizing macro scale.

**Age-Independent Huge Finite Set of Different Qucs**

SMU is populated by a finite set of different qucs—distinguished by an electric-charge integer, $Q_q$, a chirality integer, $N_q$, and a positive energy integer, $M_q$. Each of the corresponding attributes enjoys a separate dimensionful unit. QMU has exactly one quc for any $Q_q$, $N_q$, $M_q$ integer-trio, with $Q_q$ allowed the 7 values 0, ±1, ±2, ±3 and $N_q$ allowed the 3 values 0, ±1 while $M_q$ is allowed $M_{\text{max}}$ possible values, 1, 2, ... $M_{\text{max}}$. The total number of SMU qucs is thus 21 $M_{\text{max}}$. Our choice of allowed values for $Q_q$ and $N_q$ has been influenced by Occam, by Mersenne and by QFT’s set of elementary particles. (Appendix E.) For present-paper purposes we elect to leave still unspecified the ginormous value of $M_{\text{max}}$.

Any DMU ray is a sum of products of 21 $M_{\text{max}}$ single-quc normed functions, any quc appearing exactly once in each such ‘tensor’ product. It will below be seen that GN unitary Hilbert-space representation of SL(2,c) requires each single-quc Dirac-coordinate-basis wave function to depend on that quc’s location within a noncompact 4-dimensional manifold—not a spacetime manifold but a product of two complex-variable-coordinated manifolds.

An approximately temporally-stable multi-quc although single-particle wave function, such as that of an electron, correlates constituent-quc energy-integers with constituent-quc chiralities and charges. In ‘reactions’ that annihilate this particle while creating other particles, quc ‘creation’ or ‘annihilation’ never occurs. Instead there is quc ‘relocalization’.

Sub-product clusters of 2 or 3 qucs represent elementary particles of sharply (integer)-specified electric charge and baryon number. Chirality evenness or oddness of any particle is unambiguous. Always, within any elementary-particle wave function there is superposition of different energies whose spacing, $(2\pi)^{-1}$, is presently smaller than any ‘soft-photon’ energy by a factor of order $10^{22}$.

The author anticipates eventual number-theoretic specification of $M_{\text{max}}$, the maximum quc local-frame energy in units of $1/2\pi$. Perhaps $M_{\text{max}}$ is a ginormous Mersenne prime. $M_{\text{max}}$ is one of several SMU ‘foundational numbers’. Another is the dimensionless coefficient $g$, related to the ‘fine-structure constant’ of particle physics, that determines GUT scale and appears below in our formulas for electric-current density and Hamiltonian electromagnetic potential energy. The ‘large’ Mersenne prime $2^{127} - 1$ has often been conjectured to set the value of $g^{-2}$. Our number-theoretic Occam-choice for a third foundational parameter—the SMU birth age $\tau_0$ in ‘natural’ units—is 1. The huge Mersenne prime, 2$^{127} - 1$, may relate to $M_{\text{max}}$ and to mysterious macro scale.

As age $\tau$ ($>\tau_0$) continuously increases, the value of every quc’s energy—some fixed-integer multiple of $1/2\pi$—decreases in inverse proportion to age: Milne redshift. The value of (continuous, positive, dimensionful and global) age establishes an SMU scale (presently ‘Hubble’) not only for quc total and kinetic energies [see Formula (21)], but for quc momentum (main-text section on EL Casimir)
and for distance (at same age) between different quc locations in the hyperbolic Milne 3-space \(^{(3)}\) whose Riemann (negative) curvature is \(r\)-determined. \(^{(4)}\)

Classical (although quc-sourced) retarded E-G tensor fields whose Dalambertians prescribe SMU reality, will be seen below to have 13 components. The electromagnetic and gravitational fields sourced by \(\text{quc} q\) are proportional, respectively, to \(gQ_q\) and to \(M_q/2r\). The former is the quc’s electric charge while the latter is its energy. Total universe energy at age \(r\) is, by a simple computation, \((21M_{\text{max}}/2)^2 \cdot r^{-1}\), while total charge and total chirality vanish, together with total baryon number.

**Unitary Hilbert-Space Dirac-Coordinate Representation of 9-Parameter CL**

We now reproduce, with minor notation adjustment, certain formalism from Reference (4). An SMU ray at Age \(r\) is a sum of (‘tensor’) products, each with \(21M_{\text{max}}\) factors, of single-quc Hilbert vectors. In the ‘Dirac-coordinate’ Hilbert-space basis, each of the latter is a normed complex differentiable function, \(\psi_q^r(a_q)\), of \(\text{quc} q\)’s location \(a_q\) in a 6-dimensional manifold.

The symbol \(a_q\) denotes the spectra of a complete set of 6 commuting self-adjoint operators—a ‘Dirac-coordinate’ cso that complements a 6-element, Casimir-based, ‘Dirac-momentum’ cso which our main text ignores, apart from references to CL Casimirs. Appendix B attends to the momentum basis.

Reference (4) has shown how \(a_q\) comprises a 3-dimensional metricized base-space location and a (non-metricized) fiber-space location whose dimensionality is \(2 + 1 = 3\). The coordinate of the 1-dimensional fiber subspace will be seen Dirac-conjugate to chirality. The complete 6-dimensional ‘quc-locating’ coordinate amounts to a \(2\times2\) complex unimodular matrix. Henceforth in this paper’s main text any boldface symbol is to be understood as denoting a \(2\times2\) matrix.

The single-quc Dirac-coordinate-basis wave function, \(\psi_q^r(a_q)\), unitarily representing CL at each age \(r \geq \tau_0\) [see Formula (8)] , is a Hilbert vector with the invariant (finite) norm,

\[
\int |da_q| |\psi_q^r(a_q)|^2.
\]

The CL-invariant 6-dimensional volume element (Haar measure) \(da_q\) we below express through a trio of complex Dirac coordinates equivalent to the matrix \(a_q\).

Because the present section and that following refer to a single quc and a single age, we shall in these sections omit both the superscript \(r\) and the subscript \(q\). Also ignored, except in Eq. (8), is the charge integer \(Q_q\). U(1) transformation does not affect the wave function’s (Dirac-coordinate) argument--merely shifting complex-wave-function phase (in any basis) by an increment proportional to \(Q_q\).

The unimodular \(2\times2\) complex (Dirac) quc-coordinate matrix \(a\) is equivalent to three complex variables: \(s, y, z\) (six real variables), according to the following product of three unimodular \(2\times2\) matrices, each of which coordinates the manifold of a 2-parameter abelian CL subgroup:

\[
a(s, y, z) = \exp(-\sigma_z s) \times \exp(\sigma_y y) \times \exp(\sigma_z z).
\]

The \(s\) subgroup lies within the CL (diagonal-matrix) center. The complex variables \(s\) and \(y\) together coordinate the manifold of a 4-parameter nonabelian CL subgroup. The latter feature is essential both to GN’ s unitary transformation between Dirac-coordinate and Dirac-momentum bases \(^{(2)}\) and to coordination of base and fiber spaces.

The \(2\times2\) real-matrix pair \(\sigma_3\) is defined as \((1/2)(\sigma_1 \pm i\sigma_2)\). The (familiar to physicists) Pauli-matrix symbols \(\sigma_3\) and \(\sigma_1\) represent hermitian real self-inverse traceless \(2\times2\) matrices with determinant \(-1\), \(\sigma_3\) being diagonal and \(\sigma_1\) off-diagonal, while the symbol \(\sigma_2\) represents an imaginary such matrix equal to \(-i\sigma_3\sigma_1\). The 6-dimensional Haar measure,
\[ da = ds \, dy \, dz, \quad (3) \]

is invariant under \( a \to a' = a \Gamma^{-1} \), with \( \Gamma \) a 2×2 unimodular complex matrix representing a \textit{right} \( SL(2,c) \) transformation of the coordinate \( a \). The measure (3) is also invariant under analogous \textit{left} transformation. Any ‘volume-element’ symbol \( d\zeta \) in (3), with \( \zeta \) complex, means \( d \, Re \, \xi \times d \, Im \, \xi \).

The Hilbert-vector norm (and inner-product)-defining integration (1) is, \( w.r.t \, Im \, s \), over any continuous \( 2\pi \) interval of \( Im \, s \). Interpreting \( 2\pi Re \, s \) as \textit{periodic ‘quc’ time} we shall below shrink the Hilbert space so that \( Re \, s \) and \( Im \, s \) enjoy similar status in vector-norm (and inner-product) regular-basis definition. \textit{Full} real lines for \( Re \, y \), \( Im \, y \), \( Re \, z \) and \( Im \, z \) remain spanned by the shrunken space’s vector-norm definition.

A transformation specified by the 2×2 complex unimodular right-acting matrix \( \Gamma \) is \textit{unitarily} Hilbert-space represented by

\[ \Psi(a) \to \Psi(a \Gamma^{-1}). \quad (4) \]

Straightforward calculation shows \( a \Gamma^{-1} \) to be equivalent to

\[ \xi^\prime = (\Gamma_{22} \xi - \Gamma_{21}) / (\Gamma_{11} - \Gamma_{12} \xi), \quad (5) \]
\[ y^\prime = (\Gamma_{11} - \Gamma_{12}) [ (\Gamma_{11} - \Gamma_{12} \xi) y - \Gamma_{12} ], \quad (6) \]
\[ s^\prime = s + \ln (\Gamma_{11} - \Gamma_{12} \xi). \quad (7) \]

Under the 9-parameter CL symmetry group the 2-dimensional volume element \( ds \) within the Haar measure (3) is seen from (7) to be invariant. Also invariant is the 4-dimensional volume element \( dy \, dz \).

We now explicitly display CL representation by the single-\textit{quc} Dirac-coordinate Hilbert-space basis. A CL element (a location within the 9-dimensional CL manifold) is specified by a \( U(1) \)-representing angle, \( \omega \), with \( 0 \leq \omega < 2\pi \) (1 parameter), by a \textit{left-acting} \( SL(2,c,D) \) complex \textit{argument} displacement \( s \to s + \Delta \) (2-parameter) and, finally, by the Formula (4) \( SL(2,c) \)-representing argument displacement (6-parameter). Under a so-specified (9-parameter) CL, element,

\[ \Psi_{\omega}(s,y,z) \to e^{i\omega} \Psi(s^\prime + \Delta, y^\prime, z^\prime). \quad (8) \]

\textit{Essential} is commutativity of the \textit{two} \( s \)-displacements. The 1-dimensional Haar-measure volume elements \( d(Re \, s) \) and \( d(Im \, s) \) are \textit{separately} CL invariant, together with the (4-dimensional) \( dy \, dz \) volume element.

**Periodicity in ‘Quc-Time’--a Hilbert-Space Reduction**

Displacement in the coordinate \( Re \, s \), at fixed \( Im \, s \), \( y \), \( z \) and \( \tau \), displaces what we choose to call ‘\textit{quc} local time’ at fixed values of global age and the \textit{quc}’s 5 other coordinates. \textit{Quc} energy--a self-adjoint Hilbert-space operator representing a member of the CL \textit{center} subalgebra and the source of SMU gravity—is \textit{canonically-conjugate} in Dirac sense to \( 2\pi Re \, s \).

Although positive lightlikeness of a \textit{quc} velocity 4-vector (defined in the following section) invites confusion between ‘temporal’ and ‘spatial’ \textit{quc} displacement, the group algebra unambiguously distinguishes CL-invariant \textit{quc} energy from any non-invariant \textit{quc-momentum} 3-vector component of a right 6-vector—an algebra member that generates infinitesimal \textit{quc} spatial displacement (at fixed age) in some (arbitrarily-specifiable) direction through curved metricized 3-dimensional base-space. \( (4) \)

[Because the infinitesimal-displacement direction is specified in some \textit{fixed} right-Lorentz frame, whereas a geodesic follows a curved path requiring \textit{parallel transport} of direction-defining axes, the]
later-defined invariant self-adjoint quc kinetic energy—a function of Casimir geodesic-associated second
derivatives—is not proportional to the 3-vector inner product with itself of quc-momentum.]

Already noted has been the explicit indication by Formula (7) that (fixed-r,y,z) displacements in s
are right-Lorentz invariant (both real and imaginary parts). They further are invariant under the 3-
parameter symmetry (central) subgroup (with energy, chirality and electric charge as generators) that
defines quc type, despite failure to be invariant under the full (6-parameter) left-Lorentz group. A quc
Hilbert-space shrinkage that requires ray periodicity in regular-basis dependence on Re s (location of a
‘quc-timepiece hand’) maintains quc capacity to represent 9-parameter CL.

We therefore diminish quc Hilbert space by the Dirac-coordinate-basis periodicity constraint,

$$\Psi(s, y, z) = \Psi(s + 2\pi, y, z), \quad (9)$$

modifying the Hilbert-vector norm to integration in (1) over any (single, continuous) 2\pi interval of Re s,
as well as of Im s. The constraint (9) specifies integer eigenvalues for the self-adjoint operator that is
canonically-conjugate to Re s. (The quc-energy operator has eigenvalues M/2\pi.) We further reduce the
Hilbert space by requiring energy to be positive and upper bounded (M a finite positive integer).

For each quc there are 9 ‘conserved-momentum’ self-adjoint operators, although they do not all
commute with each other. A 6-element Dirac-momentum cSCO that includes the operators M and N is
identified in Appendix B with an ‘irreducible’ CL representation. The operators M and N are, respectively,
canonically-conjugate to the operators Re s and Im s.

‘Dirac-Coordinate’ 4-Vector Operators that Locate a Quc in a 5-Dimensional Fiber Bundle

Reference (4) defines a (classical, ‘exterior’) positive 4-vector as a (4-parameter) positive-
hermitian 2x2 matrix that transforms under an exterior Lorentz transformation \( \Gamma \) through right
multiplication by \( \Gamma^{-1} \) and left multiplication by the hermitian conjugate of \( \Gamma^{-1} \). A 4-vector’s invariant
‘squared magnitude’ is the hermitian-matrix’s determinant. (The timelike component is half the
hermitian-matrix trace.) Timelike, lightlike and spacelike 4-vector matrices have, respectively, positive,
zero and negative determinants.

A pair of commuting (exterior) 4-vector Dirac-coordinate self-adjoint operators, one positive-
timelike and one positive lightlike, are equivalent to a quintet of real quc coordinates that specifies the
following Quc-q 2x2 unimodular matrix, \((4)\)

$$b_\gamma \equiv \exp (\sigma_3 s_\gamma) \times a_\gamma \quad (10)$$

$$= \exp(\sigma_+ y_\gamma) \times \exp(\sigma_- z_\gamma). \quad (11)$$

Henceforth any symbol with a \( q \) subscript, whether attached to a unit 3-vector (see below), a 2x2
matrix (boldface indicated, as in the foregoing formulas), to a Lorentz tensor with 4-valued indices or to
an EL Casimir, is to be understood as a self-adjoint (Dirac) Quc-q operator on the SMU Hilbert space.

The left multiplication (10) of the quc-coordinate matrix \( a_\gamma \) by the diagonal unitary unimodular
matrix, \( \exp (\sigma_3 s_\gamma) \), has deleted the coordinates \( s_\gamma \) from the coordinate-sixt set \( s_q, y_q, z_q \) (a cSCO). The
eliminated coordinate pair (understood as self-adjoint operators), are canonically-conjugate in Dirac sense
to those conserved integer-eigenvalued Quc-q momenta (elements of the CL algebra) which we have
called ‘energy’ and ‘chirality’.

Quc-q’s ‘fiber-bundle location’, either in 3-dimensional metricized base space or in a 2-
dimensional (unmetricized) ‘velocity-direction’ fiber space, fails to depend on \( s_q \). We now show that base
3-space location and velocity-direction fiber 2-space location are equivalent to the 4 coordinates, \( y_q, z_q \)—
the latter symbols representing a set of 4 commuting self-adjoint Dirac-coordinate operators collectively
representable by the (single) 2x2 unimodular-matrix symbol \( b_\gamma \).

Either coordination associates in Dirac sense to a quartet of commuting self-adjoint coordinate
operators on a single-quc Hilbert space. Throughout the remainder of this paper’s main text, all quc-
coordinate symbols are to be understood as referring to self-adjoint Dirac operators, either on single-quc or multi-quc Hilbert spaces. Such operators commute neither with CL’s momentum algebra nor with Casimir quadratic functions of that algebra.

Formulas (2) and (9) together expose as positive-hermitian the unimodular (single-quc) 2×2 coordinate matrix,

\[ B_q \equiv b_q^* b_q = \exp(-\beta_q \sigma n_q), \]

the (non-matrix) symbol \( \beta_q \) in (13) denoting a rotationally-invariant non-negative continuous-spectrum self-adjoint operator while the (non-matrix) symbol \( n_q \) denotes a unit 3-vector self-adjoint operator that commutes with \( \beta_q \). The symbol \( \sigma n_q \) denotes the inner product of two 3-vectors—one a Hilbert-space operator and the other a 2×2 hermitian matrix. (4) The hermitian unimodular 2×2 matrix (and self-adjoint Dirac-operator) symbol \( B_q \) denotes a positive-timelike dimensionless (exterior) 4-vector of unit ‘Minkowski magnitude’.

Through the dimension-ful positive-real factor \( \tau \) (age) the positive-timelike 4-vector (operator) symbol \( x_q = \tau B_q \) locates the quc within (non-Riemannian) Milne (Minkowski) spacetime—prescribing (in Dirac-operator sense) its displacement from the vertex of the universal forward lightcone. In a (to physicists) more familiar notation, the 4 (operator) components of \( x_q \) are \( \tau \cosh \beta_q, \tau n_q \sinh \beta_q \).

[Warning to physicists familiar with the Copenhagen statistical interpretation of Dirac quantum theory: SMU reality does not associate to the expectation of single-quc self-adjoint operators, such as \( x_q \)-attaching rather to the expectations of self-adjoint E-G potential-field operators on the multi-quc Hilbert space—operators defined in the section which follows. The foregoing operators do not include zero-Dalambertian ‘radiation’ fields of the kind that QFT associates to its elementary particles.]

Complementing dimensionless \( B_q \), which coordinates a quc-fiber-bundle’s metricized base space, is a second dimensionless positive 4-vector—this one lightlike—-to be denoted by the symbol \( v_q \) and coordinating a 2-dimensional unmetricized fiber space. The pair of 4-vectors, \( B_q, v_q \), is equivalent to \( b_q \) and thereby to \( y_q, z_q \).

The latter equivalence is below exhibited via invariant 4-vector inner products. The inner product of two 4-vectors will be denoted by the symbol \( \cdot \). The inner product of two positive 4-vectors is non-negative. [Because the inner product of any two right (exterior) 4-vector operators may be shown equal to the inner product of a unitarily-equivalent left 4-vector pair, (4) either product is invariant under the 12-parameter group \( \text{SL}(2c) \times \text{SL}(2c) \) and thereby is CL invariant.]

The quc-velocity-direction positive-lightlike 4-vector self-adjoint Dirac-coordinate operator, \( v_q \), is defined to be the dimensionless zero-determinant positive-hermitian matrix

\[ v_q \equiv b_q^* (\sigma_0 - \sigma_3) b_q, \]

the symbol \( \sigma_0 \) here denoting the unit 2×2 matrix. Equivalence of the coordinate matrix \( b_q \) to the dimensionless positive 4-vector pair \( B_q, v_q \), follows from the inner-product trio, \( B_q \cdot v_q = 1, B_q \cdot B_q = 1 \) and \( v_q \cdot v_q = 0 \), deducible by going to the special (‘local’) frame where \( B_q = \sigma_0 \) (i.e., where \( \beta_q = 0 \)).

In the physicist-familiar 4-component notation, the 4-vector self-adjoint operator \( v_q \) is equal to

\[ (1, u_q) / (\cosh \beta_q - u_q \cdot n_q \sinh \beta_q), \]

\( u_q \) a \( z_q \)-equivalent (2-dimensional, see Appendix B) unit 3-vector (a self-adjoint Dirac-coordinate operator accompanying the unit 3-vector \( n_q \) but independent thereof) that admits the name, ‘direction of quc lightlike-velocity’. A quc fiber bundle is thus coordinated by the (3-dimensional) base-space coordinates,
\( \beta_q, n_q, \) together with the (2-dimensional) velocity-direction fiber coordinate \( u_q. \) An equivalent Dirac-coordinate set is \( y_q, z_q. \)

The positive-lightlike \( quc \) 4-velocity \( v_q \) will be seen in the following section, together with \( Q_q \) and \( M_q, \) ‘almost completely’ to specify ‘electromagnetism-gravity from \( quc-q \) source’.

**Classical Retarded E-G (Non-Radiation) Fields**

As emphasized in the preceding section, throughout this paper’s remaining main text any symbol with a \( quc \)-designating subscript is to be understood as representing a self-adjoint age-independent Dirac operator. The only age-dependent self-adjoint SMU operator is the Hamiltonian. (SMU does not admit the ‘Heisenberg picture’. No operator represents ‘\( quc \) acceleration’.)

The present section deals with classical retarded electromagnetic and gravitational Lorentz-tensor fields (not zero-Dalembertian ‘radiation fields’) whose Dalembertian prescribes SMU’s ‘reality’. At any age \( \tau \geq \tau_0, \) such fields are prescribed by expectations, with respect to the SMU ray at that age, of certain self-adjoint retarded Hilbert-space operators which sum over all \( quc \) sources of the field in question. These retarded-field tensor operators we now define through the previous section’s pair of 4-vector single-\( quc \) Dirac-coordinate operators.

Four electromagnetic SMU field components are complemented by nine gravitational-field components. The preceding section’s \( quc \)-fiber-bundle coordinating 2-parameter positive lightlike \( quc \)-velocity 4-vector and 3-parameter \( quc \) spacetime-location 4-vector, together with \( quc \) electric charge and energy, ‘co-variantly’ prescribe a ‘\( quc \)-source’ for 13 E-G retarded-field operators. Positivity of \( quc \)-velocity 4-vector associates to the source’s retarded nature; this 4-vector also prescribes the ‘direction’ of the generated tensor field. The source-location 4-vector, through the denominator of the Lienard-Wiechert (\( LW \)) formula, joins \( quc \) charge and energy to determine magnitude of the \( quc \)-generated field.

We define retarded electro-gravitational field operators--not ‘radiation’ quantum fields--by applying the \( LW \) formula to \( Quc-q \) source of gravity and electromagnetism. We begin with the latter, for which the 4-vector retarded potential at some ‘field point’ \( x, (x \cdot x = r^2), \) generated by Source \( q, \) is

\[
\mathcal{J}_q^\mu(x) \equiv gQ_q \theta_{\text{ret}}(b_q, x) v_q^\mu / (v_q \cdot x - \tau),
\]  

(16)

the retardation step function, \( \theta_{\text{ret}}(b_q, x), \) being defined below via the lightcone whose vertex locates at \( x. \)

The symbol \( \theta_{\text{ret}}(b_q, x) \) in (16) denotes an operator function equal to 1 iff the spacetime straight line of direction \( v_q, \) that passes at age \( \tau \) through the ‘source’ spacetime location \( x_q, \) intersects the \( x \) backward lightcone (which does not include this lightcone’s vertex). Otherwise \( \theta_{\text{ret}}(b_q, x) \) vanishes. (Any lightlike straight line not passing through \( x \) intersects the \( x \) lightcone exactly once.)

The Age-\( \tau \) expectation, of the self-adjoint operator \( \mathcal{J}(x) \) that sums (16) over all \( qucs, \) prescribes the reality-defining classical electromagnetic 4-vector field \( A^\mu(x). \) The Dalembertian of \( A^\mu(x) \) is the electric-charge current density—an aspect of reality that, despite ‘classical’ status, manifests electric-charge discreteness.

Discretization of \( quc \) energy renders almost straightforward an extension of the foregoing to gravity. In place of (16) the 9-component ‘traceless’ symmetric Lorentz-tensor retarded self-adjoint gravitational potential operator, that Source-\( q \) generates, is defined by

\[
P_{\mu q}^{\alpha q}(x) \equiv -M/2\tau \theta_{\text{ret}}(b_q, x) v_q^\alpha v_q^\rho / (v_q \cdot x - \tau).
\]  

(17)

The Age-\( \tau \) expectation of the self-adjoint operator \( P_{\mu q}^{\alpha q}(x) \) that sums (17) over all \( qucs, \) prescribes the classical gravitational traceless-symmetric-tensor field \( \Phi_{\mu q}^{\alpha q}(x). \) The Dalembertian of \(- \Phi_{\mu q}^{\alpha q}(x) \) is the energy-momentum tensor.

There is no retarded SMU-\( LW \) ‘acceleration field’. Photons are represented, within SMU reality, not by a zero-Dalembertian electromagnetic field but via the energy-momentum tensor—which comprises current densities of all energy, momentum and angular momentum.
Photon annihilations or creations—aspects of ‘objective reality’—become (Popper) physically inferrable from positive-energy-momentum current density together with classically-discrete electric-charge current density—the Dalambertian of the classical but discretely-sourced electromagnetic vector field. Charge discreteness together with energy positivity allows experimenters to infer discrete photons from observed current densities by application of (classical) Newton-Maxwell theory to current densities of electric charge and energy-momentum.

**Self-Adjoint Single-Quc Kinetic-Energy Operator—a Positive Function of CL Casimirs**

The algebra of the 6-parameter semi-simple non-abelian exterior-SL(2,c) CL subgroup, comprises the conserved components of a 6-vector—a second-rank antisymmetric exterior-Lorentz tensor.

Three algebra members associate to quc angular momentum, \( J_q \) (a 3-vector), and three to \( qq \) momentum, \( K_q \) (also a 3-vector). Each sixet-algebra member is represented by a self-adjoint operator on the quc Hilbert space. In the Dirac-coordinate \((s, q, \tau)\) basis each of these momentum operators linearly and homogeneously superposes first (partial) derivatives. \(^2\) As emphasized by Appendix B, each algebra member is a Dirac momentum—not a Dirac coordinate. The two (invariant) CL-group Casimirs (commuting with all 9 of the conserved CL generators) are the 3-vector operator inner products \( K_q J_q \) and \( K_q K_q - J_q J_q \)—homogeneous in regular-basis (partial) second derivatives. \(^2\) Neither of the foregoing forms is positive, but Reference (2) displays algebraic equivalence to another pair of invariant self-adjoint operators, one of which has (positive-negative) integral eigenvalues while its companion enjoys a continuous positive spectrum. Denoting the former by the symbol \( m_q \) and the latter by the symbol \( b_q \), the algebraic relation is

\[
K_q J_q = (\rho_q/2)(m_q/2), \quad K_q K_q - J_q J_q = (\rho_q/2)^2 - (m_q/2)^2 + 1. \quad (18)
\]

The positive continuous-spectrum SMU quc-kinetic-energy operator is \( \rho_q/2\tau \), joining in the 6-element unirrep cso (see Appendix B) the positive discrete-spectrum quc-energy operator, \( M_q/2\tau \).

**Self-Adjoint Quc-Pair Potential-Energy Operators**

The SMU Hamiltonian potential-energy operator is a sum over \(21M_{max}(21M_{max} -1)/2\) quc pairs of CL-invariant electromagnetic-gravitational potential energies, \( V_{qq}(\tau) = V_{\varepsilon q}(\tau) \), whose individual status parallels that of the Euclidean-group-invariant ‘Coulomb-gravity’ potential energy in a Hamiltonian for two (slowly-charged) massive particles. We postulate an SMU Hamiltonian potential-energy operator, for the quc pair, \( qq' \), that depends on the exterior-invariant ‘relative Dirac coordinate’ \( b_{qq'} \equiv b_q b_{q'}^{-1} \). Notice that \( b_{qq'}^{-1} = b_{qq} \).

Guided by Reference (4) and the LW denominator in Formulas (16) and (17), we further postulate inverse proportionality to \( e^{\beta_{qq}} - 1 \). Here the positive symbol \( \beta_{qq} = \beta_{qq'} \) stands for (EL-invariant) shortest distance in (curved) relative base-space between the locations of Quc \( q \) and Quc \( q' \). This distance equals \( \cosh^{-1}[\sqrt{2}tr(b_{qq'} b_{qq'})] \)—the same function of \( b_{qq} \) as that which in Formula (13) above specified the single-quc coordinate \( \beta_q \) in terms of \( b_q \).

Beginning with electromagnetism, as we did above when defining classical E-G fields via the LW formulas (16) and (17) for field operators, we postulate

\[
V_{qq}(\tau) = g^2 \tau^{-1} Q_q Q_q' \left( e^{\beta_{qq}} - 1 \right)^{-1}. \quad (19)
\]

The corresponding CL-invariant gravitationnal potential-energy operator is

\[
V_{qq'}(\tau) = - \tau^{-1} (M_q/2\tau) (M_q/2\tau) \left( e^{\beta_{qq'}} - 1 \right)^{-1}, \quad (20)
\]

the complete quc-pair potential-energy operator being the sum, \( V_{qq}(\tau) \equiv V_{qq}(\tau) + V_{qq'}(\tau) \). (The absence, anticipated earlier, of a separate ‘nuclear-force’ potential-energy Hamiltonian component will be
SMU Hamiltonian and Schrödinger Equation

As was the case for Schrödinger, our Hamiltonian sums symmetry-group-invariant self-adjoint kinetic-energy and potential-energy operators that do not commute. SMU dynamics proceeds through a multi-quc Schrödinger (first-order) differential equation where, at each post-big-bang age, a CL-invariant although age-dependent self-adjoint Hamiltonian operator (not a CL-algebra member) generates an infinitesimal wave-function change that prescribes the ‘immediately-subsequent’ universe wave function. Schrödinger’s 1927 equation was similar although based on a 7-parameter extended-Euclidean group with flat 3-space translations (instead of the 9-parameter CL group with curved 3-space translations).

The invariant age-dependent self-adjoint Hamiltonian operator is

$$ H(\tau) = \sum_q \rho_q/2\tau + \sum_q \epsilon_q q^2 \Psi_q(q), $$

while the evolution equation for the universe ray is

$$ i\partial \Psi(t)/\partial \tau = H(t) \Psi(t). $$

An initial ray of uncorrelated (bachelor) qucs, at $\tau = \tau_0$, is proposed in Appendix A. Notice that our Schrödinger equation, in absence of gravitational potential energy, is conformally (‘scale’) invariant—dependent only on age ratios and thereby paralleling a QFT feature important to renormalization. Related is our conjecture that Maxwell’s equations are satisfied by SMU classical electromagnetic fields.

Conclusion

A 9-parameter ‘centered-Lorentz’ (CL) Lie symmetry group collaborates with a Schrödinger equation that prescribes Schrödinger-Milne quantum-universe electro-gravitational evolution with increasing universe age. SMU resides inside a forward lightcone, the age of any location its ‘Minkowski distance’ from lightcone vertex. Age is a CL-invariant nongeometrical perpetually-increasing parameter approximately equal at present to the reciprocal of Hubble’s ‘constant’.

At each age greater than or equal to a starting (big-bang) age (1, in units where $G = \hbar = c = 1$; in seconds, big bang age is $\sim 10^{43}$), the Dirac-coordinate-basis universe ray is a sum of (tensor) products of single-quc wave functions. A ‘quc’ is an SMU constituent. The argument of a quc’s wave function in the latter’s ‘Dirac-coordinate’ basis specifies the quc’s location within a CL-dictated 6-dimensional manifold.

The number of qucs is ginormous but finite and unchanging. Each quc represents CL [Formula (8)] through displacements of its 6 Dirac coordinates and of its wave-function phase. The total number of qucs and the unit of electric-charge remain to be specified. An estimate of the latter is provided by the (incomplete) ‘Standard Model of particle-physics’.

Appendix A proposes an initial ray of ‘bachelor’ qucs--devoid, at the beginning, of mutual correlations. We suppose the first ‘marriages’ of electrically-charged qucs to have emerged at GUT-scale ages ($\sim 10^{39}$ sec) with creation of 2-quc ‘double-helix cosmological photons’. Later, at micro-scale universe ages ($\sim 10^{24}$ sec), there emerged ‘massive elementary particles—electrically-neutral 2-quc neutrinos, Higgs bosons and Z’s, together with charged 3-quc quarks, leptons and W bosons. (Charged-quc composition of elementary particles, broached in Appendices C and E, will be addressed in separate papers.) At macro-scale universe ages ($\sim 10^{5}$ sec), we believe stellar construction began. All such conjectures are in principle verifiable by computation.

Present-age ($\sim 10^{17}$ sec) ‘dark matter’ comprises galactic-scale colonies of electrically-neutral ‘bachelor’ qucs that gravitationally attach to entire stellar galaxies. A (still larger) universe component, remaining today galaxy-unassociated, comprises quc bachelors that so far have maintained their independence.
Quc chirality (conjugate to one of six quc Dirac coordinates) we have (‘Occam’) limited to the values $0, \pm 1$—a ‘Dirac-tripling’ that accompanies limitation on any quc’s electric-charge integer to the 7 values $0, \pm 1, \pm 2, \pm 3$. In early-universe dynamics we believe the combination of chirality and electric charge at particle-physics micro scale to have (dynamically) distinguished baryon-number-carrying quarks and associated ‘strong interactions’ from elementary bosons and leptons with zero baryon number (Appendix, Table I).

We expect the SMU Schrödinger equation (22) to reveal ‘nuclear forces’, along with other particle-physics, as an approximate notion—useful at micro scale but not at all SMU scales and not a foundational feature of a quantum universe—all of whose ‘forces’ our Hamiltonian proposes to be electro-gravitational. The ‘short range’ of nuclear forces manifests electric-charge screening—important whenever the number of qucs in some charged-quc set exceeds the sum of this set’s charge integers.

All physics measurements ‘Popper-rely’ for interpretation of observed objective reality, on classical-physics electro-gravitational theory. The SMU ray specifies, through expectations of self-adjoint electromagnetic and gravitational field operators, a physics-enabling ‘fixed and settled reality’ that includes locally-unobservable energy (Bohm hidden reality) together with macro-scale ‘observable objectivity’.

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Appendix A. Initial-Universe (Planck-Scale) Wave Function
The initial ($\tau = \tau_0 = 1$) SMU wave function we propose to have been a single product of $21 M_{\text{max}}$ single-quc wave functions that represent uncorrelated ‘initially-bachelor’ qucs. No (GUT-scale or micro-scale) marriages between 2 or 3 qucs—electro-gravitationally-stabilized elementary particles—were a priori. All particles and dark matter—all objective reality—we presume to have resulted from Schrödinger-equation Hamiltonian-generated evolution. Present-universe Hubble scale—the spacetime scale set by our universe’s age—is larger than the scale of $\tau_0$ by a factor of order $10^{60}$.

Von Neumann ideas have led us to propose, as initial ray in the regular CL-representation (‘Dirac-coordinate’) basis, the following product of $21 M_{\text{max}}$ ‘gaussian’ factors, with no dependence on the arguments of the complex coordinates $y_q$ and $z_q$—dependence only on their magnitudes,

$$\Psi(\tau_0) = \Pi_q \exp \left(-i N_q \text{Im } s_q\right) \exp \left(-i M_q \text{Re } s_q\right) |y_q|^2 |z_q|^2 \exp \left(-\frac{1}{2} \ln |y_q|^2 - \frac{1}{2} \ln |z_q|^2\right). \quad (A.1)$$

With the starting ray (A.1), which recognizes electric charge by the subscript $q$ on any GN-Dirac coordinate being defined as synonymous with the integer trio $Q_q, N_q, M_q$, the 3-vector momentum operator of each quc has vanishing expectation. This ray, further, is an eigenvector of total 3-vector angular momentum with zero eigenvalues for all components thereof—thereby satisfying classical Mach-
Milne principles perpetuated by (main-text) Eq. (22). Total electric charge and chirality have zero eigenvalues in the initial ray (A.1) as well as in all subsequent rays. Total starting energy has the value, \(21(M_{\text{max}}/2)^2 \tau_0^{-1}\). Subsequent total SMU energies replace \(\tau_0 (=1)\) in the foregoing formula by \(\tau\).

**Appendix B. CL Unirrep-Csco as ‘Dirac Momentum’ Basis**

GN’s unitary \(\text{SL}(2,c)\) Hilbert-space representation via normed complex differentiable functions, \(\mathcal{H}(a)\), of (single-\(q\)-c) location within the 6-dimensional group manifold, was called by these authors the ‘regular’ Lorentz-group representation. \(^{(2)}\) Our main text has characterized the corresponding complete set of 6 commuting self-adjoint operators (‘csco’) as a ‘Dirac-coordinate’ Hilbert-space basis. Not GN-emphasized is representability, by this same csco, of the 12-parameter group \(\text{SL}(2,c)_L \times \text{SL}(2,c)_R\). An 8-parameter subgroup of the latter—keeping only the diagonal left—multiplying 2x2 complex-unimodular matrices—is the electric-charge-ignoring subgroup of the 9-parameter SMU-foundational CL group. \(^{(4)}\) [The CL center augments \(\text{SL}(2,c,D)_L\) with \(U(1)\).]

Exposed in detail by GN, beyond their regular \(\text{SL}(2,c)\) representation, was the latter group’s unitary irreducible representation by a ‘unirrep-csco’ that also \[\text{GN-} \]
represented 12-parameter \(\text{SL}(2,c)_L \times \text{SL}(2,c)_R\). GN’s ‘Lorentz unirrep’ becomes a ‘Dirac-momentum-basis’ when 2 of its 6 csco members are ‘Fourier-replaced’ by GN-center generators. (The 4-member remainder of GN’s 6-member unirrep-csco commutes with the foregoing pair of Dirac-momentum csco members.)

*Both* the GN unirrep-csco and its ‘Dirac-momentum counterpart’ include the two Lorentz-group Casimirs appearing in the main-text paragraph preceding Formula (18). These Casimirs commute with all 12 generators of the ‘left-right Lorentz group’ and with all 9 generators of the CL group. The pair of self-adjoint operators appearing in the Dirac-momentum csco (but *not* in GN’s unirrep csco) are the main-text energy and chirality integer-eigenvalued operators. CL unirrep then follows from 6-dimensional unirrep-csco augmentation by electric charge.

The complete set of 7 commuting self adjoint ‘Dirac-momentum’ single-\(q\)-c operators comprises \(Q_q, N_q, M_q, m_q, \rho_q, \text{and } z_{iq}\), where \(Q_q\) and \(N_q\) take (the by now familiar) possible values \(Q_q = 0, \pm 1, \pm 2, \pm 3\) and \(N_q = 0, \pm 1\). The integer \(m_q\) takes *all* positive-negative-integer values while \(\rho_q\) has a continuous spectrum spanning the *positive* real line and \(M_q\) takes (main-text, positive-integer) values \(1, 2... M_{\text{max}}\). Evenness (oddness) of \(m_q\) is accompanied by evenness (oddness) of \(N_q\). Earlier we have described \(\rho_q/2\pi\) as the ‘local-frame kinetic energy of Quc \(q\)’; in the present context, ‘local-frame magnitude of Quc-\(q\) momentum’ is a more appropriate appellation. (Remember that ‘quc mass’ is devoid of meaning!)

The continuous spectra of both \(\text{Re } z_{iq}\) and \(\text{Im } z_{iq}\) span (full) real lines; we choose to describe the ‘meaning’ of \(z_{iq}\) as ‘local-frame-direction of Quc-\(q\) 3-vector momentum’. Reminder of trickiness in terminology choice is the Haar measure for the 3-dimensional (noncompact) \(\rho_q, z_{iq}\) subspace: 
\[
(m_q^2 + \rho_q^2) \, d\rho_q \, dz_{iq}. \quad (2)
\]
[In terms of directional (real) polar angles, \(\pi \geq \theta_{iq} \geq 0\) and \(2\pi \geq \varphi_{iq} \geq 0\), the complex \(z_{iq}\) may be written \(i \tan(\theta_{iq}/2) \exp(\varphi_{iq})\). The absolute value of \(z_{iq}\) is then \(\tan(\theta_{iq}/2)\).]

We perceive the set of 4 commuting self-adjoint operators, \(m_q, \rho_q, \text{and } z_{iq}\), as ‘Dirac-momenta’ even though *not* ‘canonically conjugate’ to the \(y_q, z_q\) quartet of Dirac coordinates—the complex commuting operators that, despite lack of ‘ordinary-language’ names, have appeared prominently in this paper’s main text. We think of \(m_q\) as ‘quc helicity’—the ‘component of quc angular momentum in the direction of its momentum’. The author is comfortable in calling the GN-Dirac coordinate \(z_q\ ‘quc\ velocity direction’ (related to velocity polar angles in the manner above used for ‘momentum direction’) but has yet to achieve comfort with any (physics-familiar) name for the GN-Dirac coordinate \(y_q\).

**Appendix C. Photons of Differing Diameters Although Same Momentum and Helicity**

Main-text noted has been SMU’s (2-\(q\)-c) 4-dimensional (3-momentum plus helicity) ‘Dirac-momentum’ basis for the ‘external’ properties of a single photon. But \(\gamma_t\) also has an ‘internal’ Hilbert vector—a function of the 2-\(q\) relative coordinate, whose spatial extension transverse to momentum direction might be called ‘photon diameter’.
The internal $\gamma$, Hilbert vector is a complex normalizable function of location within a Dirac-relative-coordinate manifold. Among elementary particles, photons are ‘special’ by important dependence of their internal quc dynamics on gravitational attraction between qucs, as well as on electromagnetic inter-quc attraction or repulsion.

The foregoing we have main-text summarized by attaching to the photon the acronym, ‘double helix’. A physics-unappreciated photon attribute is GUT-scale double-helix diameter.

With (see Table I below) $Q_\gamma = \pm 3$ and $Q_\phi = -Q_\gamma$, (local-frame) $qq$-composed photon energy at age $t$ is $E_r = (2\pi)^{-1}(M_\gamma + M_\phi)$. This ‘external’ energy remains unchanged if $M_\gamma \rightarrow M_\gamma + \sigma, \ M_\phi \rightarrow M_\phi - \sigma$, with $\sigma$ an integer whose absolute value is smaller than either $M_\gamma$ or $M_\phi$. Gravitational potential energy--$V_{q\phi}^{\gamma\phi}$--by Formula (20), however, is changed, with an associated change in double-helix diameter. (Plausibly the smallest diameter associates to $M_\gamma = M_\phi$.)

The total number of different double-helix quc pairs, with same momentum and helicity (and same chirality) but differing helix diameters, is $tE_r$. Even for ‘soft’ photons in the present universe (those of wavelength ~ km or greater), the number of different SMU ($\gamma$,) photons sharing the same momentum and helicity is of order $10^{22}$. (A huge number have ‘almost the same’ diameter.)

It follows that, despite SMU finiteness of photon total number, high accuracy may attach to physics coherent-state QFT representation of classical electromagnetic radiation—a $t\rightarrow\infty$ approximation that recognizes indefinitely many FHPP-identical photons. Present-universe FHPP accuracy of Bose-Einstein identical-photon statistics is understandable even though any $\gamma$ is different from any other. Identity of all ‘photons with common momentum and helicity’ is one of many physics approximations that accompany 3-space flattening.

Already at spatial micro-scales (far above GUT scale although far below macro scale) physics notions such as Bose-Einstein and Fermi-Dirac statistics become accurate. Lack of meaning for ‘quc statistics’ accompanies higher dimensionality of ‘quc space’ compared to that of ‘particle space’.

**Appendix D. Dark Matter as Non-Particulate (Bohm) ‘Hidden Reality’**

Via a self-adjoint Hamiltonian with kinetic- and potential-energy components, the universe’s evolving ‘ray’ specifies ‘evolving reality’ through expectations of self-adjoint operators that represent current densities of energy, momentum, angular momentum and electric charge. A ‘particle’ is a micro-scale clump of energy-momentum with some integral electric charge, some baryon number, some integral or half-integral angular momentum in units of $\hbar$ and some approximately-determined mass. All qucs in particle clumps are electrically charged.

Any ‘observer’ is a macro-scale clump of particles with approximately-zero total charge (charge screening). Distinction between ‘particle’ and ‘observer’ resides not in the SMU Hamiltonian but in ray aspects that emerge as distinct scales develop with universe expansion. ‘Dark matter’ resides in non-particulate galactic-scale clumps of chargeless but energy-carrying qucs. Neither particle nor observer ‘contains’ dark matter.

Schrödinger’s equation determines reality evolution without requiring either that all energy density be ‘particulate’ or that all reality be micro-macro-scale. Galactic-scale ‘dark matter’ is the non-particulate electrically-neutral source of gravitational potential energy which helps determine, via the SMU Schrödinger equation, the age at which a radioactive particle decays. Dark matter constitutes reality that Bohm characterized as ‘hidden’. All SMU history is ‘deterministic’—with ‘observations’ merely one among many ‘onflow’ (Ralph Pred’s term) aspects.
Appendix E. 3 Elementary-Fermion Generations

The famously-mysterious 3 generations of QFT elementary fermions associate to the three possible absolute values for nonvanishing quc electric charge. (All QFT elementary particles, whether charged or neutral, are composed exclusively of charged qucs.) Generation mass magnitudes we believe associate inversely to the quc-charge integers 1, 2, 3. Elementary-fermion mass ratios are, to an accurate approximation, electrodynamically determined. Approximate mass-ratio gravity-ignoring computation should be possible.

Each of the three qucs (approximately) building any charged elementary fermion carries a charge integer, ±1, ±2, ±3. Two of the three individually carry zero chirality while opposite charges—the net chirality and charge of this “core” pair vanishing. The remaining ‘valence’-quc carries the fermion’s chirality, charge and spin as well as baryon number. For a charged lepton the charge, is −3q while the (fluctuating) chirality is ±h/2. For a quark the valence quc carries either the charge 2q or the charge −q, together with baryon number ⅓ and ±h/2 (fluctuating) chirality.

Three generations of charged elementary fermions associate to 3 possibilities for the electrically-neutral ‘core’-pair of individually-zero-chirality charged qucs. We expect the lowest-mass generation to be that with the $Q = ±3$ pair because here the negative electromagnetic potential energy is greatest. The highest-mass generation we expect to be that with the $Q = ±1$ quc pair.

Three ‘types’ of neutrino, each a $Q = ±1, ±2$ or $±3$, quc pair, differ from the foregoing 3 charged ‘generations’ by one member of any neutrino quc-pair having $N = ±1$, with a sign that in our galaxy agrees with that of this quc’s electric charge; the other quc has zero chirality.

|   | Dark | Baryonic | Bright |
|---|------|----------|--------|
| $Q_q$ | 0    | ±1       | ±2     | ±3     |
| $B_q$ | 0    | ±⅓      | ±½     | 0      |
