Structure, formation, and decay of $\bar{K}NN$ system by Faddeev-AGS calculations

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Abstract: The Faddeev AGS equations for the coupled-channels $\bar{K}NN - \pi\Sigma N$ system with quantum numbers $I = 1/2$ and $S = 0$ are solved. Using separable potentials for the $\bar{K}N - \pi\Sigma$ interaction, we calculate the transition probability for the $(Y_K)_{I=0} + N \rightarrow \pi\Sigma N$ reaction. The possibility to observe the trace of the $K^- pp$ quasi-bound state in $\pi\Sigma N$ mass spectra was studied. Various types of chiral-based and phenomenological potentials are used to describe the $\bar{K}N - \pi\Sigma$ interaction. Finally, we show that we can observe the signature of the $K^- pp$ quasi-bound state in the mass spectra, as well as the trace of branch points in the observables.

Keywords: Faddeev method, $\bar{K}N$ interaction, $K^- pp$ system, quasi-bound state

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1 Introduction

Defining a precise interaction model for the $\bar{K}N$ interaction is a basic aim in strangeness nuclear physics. For the past two decades, an enormous amount of effort was put into the study of the structure of dense kaonic nuclear clusters [1-4]. The $K^- pp$ system is an important kaonic cluster, which is a highly controversial issue in the study of kaonic systems. Many theoretical calculations were performed with focus on the $K^- pp$ system [5-15]. All few-body calculations show that the $K^- pp$ is bound, with some variation in the values of the extracted pole energy.

If the $K^- pp$ system is indeed bound, then the question remains whether this state is sufficiently narrow to allow observation and identification. Due to strong absorption of antikaon by the nucleus, the quasi-bound state in the $\bar{K}NN$ system can exhibit a large width. This may provide difficulties in the direct experimental observation of kaonic bound states in nuclei [16]. Numerous experimental efforts have been made to explore the pole structure of the $K^- pp$ system. An exclusive analysis of the $p + p \rightarrow X + K^*, X \rightarrow p + \Lambda$ reaction at 2.85 GeV [17] indicated a large peak both in the $\Lambda p$ invariant-mass and $K^*$ missing-mass spectra, which had been predicted in theoretical studies [18, 19]. The observed peak corresponds to the binding energy of about 103 MeV, and the width is given as $\Gamma = 118$ MeV. The $K^- pp$ quasi-bound state can occur as a result of kaon-induced reactions on light nuclei, such as the $^3$He and deuteron, and resonance may be observed in the mass spectra of the final particles. The investigation of the $K^- pp$ quasi-bound state was further explored through the $\pi^+ \bar{K}^0 \rightarrow \pi^- p$ reaction in the $E27$ experiment at J-PARC [20]. The $d(\pi^+, K^+)$ experiment also revealed a distinct peak in the $K^+$ missing-mass spectrum, nearly at the same mass and width as the DISTO peak $X$. The investigation for the $K^- pp$ quasi-bound state can be reached through the $K^- + ^3$He reaction (see Fig. 1). This reaction was performed as an E15 experiment at J-PARC [21]. The E15 group suggests a broad $K^- pp$ quasi-bound state structure at 15 MeV just below the $\bar{K} NN$ threshold [21].

![Fig. 1. (color online) Schematic diagram for the $^3$He($K^-,N$) $K^- pp$ reactions.](image-url)

The kaon-induced reactions have been studied by Koike-Harada [22] and Yamagata et al. [23] using the op-
tical potential method. In Ref. [24], the \((Y_K)_{i=0}+N \to \pi \Sigma N\) reaction has been studied by the Faddeev approach. In this calculation, they employed chiral-based potentials for the s-wave \(KN\) interaction. Within their model, the authors found a clear signal corresponding to the strange-dibaryon resonances in the Faddeev scattering amplitudes and the \((Y_K)_{i=0}+N \to \pi \Sigma N\) transition probabilities. The \(K^+\) He 

\(\to \Lambda pn\) reaction was also studied by Sekihara et al., [25-27] to investigate the origin of the observed peak close to the \(K^- pp\) threshold in the first run of the E15 experiment at J-PARC [21]. Two scenarios were considered in the generation of the peak. In the first scenario, the \(\Lambda(1405)\) resonance can occur, however this does not correlate with \(p\), and the uncorrelated \(\Lambda(1405)p\) system subsequently decays into \(\Lambda p\). In the other scenario, the \(KNN\) quasi-bound state should be generated and decays into \(\Lambda p\). The calculation of the \(\Lambda p\) invariant mass spectrum reproduced the experimental signal in the second scenario, and the authors discarded the scenario where the \(\Lambda(1405)\) does not correlate with \(p\).

The present study is devoted to study the pole structure of the \(K^- pp\) three-body system. We study the observation of the \(K^- pp\) quasi-bound state in the \(\pi \Sigma N\) mass spectra resulting from the considered reaction. We performed few-body calculations for the \(KNN - \pi \Sigma N\) system by coupled-channels Faddeev AGS equations. Transition probabilities for the \((Y_K)_{i=0}+N \to \pi \Sigma N\) reaction are calculated. With this behavior, we investigated the behavior of the transition probability for the \((Y_K)_{i=0}+N \to \pi \Sigma N\) reaction. Several chiral-based and phenomenological \(KN - \pi \Sigma\) potentials are used [24, 28, 29] to investigate the sensitivity of the three-body observables on two-body inputs.

The paper is organized as follows: in Section 2, we explain the formalism used for the coupled channel three-body \(KNN - \pi \Sigma N\) system and give a brief description of the transition probability formula for break-up reactions. Section 3 is devoted to the two-body inputs in the calculations. The computed transition probabilities are presented in Section 4, and we provide conclusions in Section 5.

## 2 Three particle system \(KNN - \pi \Sigma N\)

The calculation of the \(KNN - \pi \Sigma N\) three body system is based on the Faddeev treatment [30]. Separable potentials were used to describe the two-body interactions

\[
V_{i,j}^{\alpha \beta}(k_i^q, k_j^q; z) = g_{i,j}^{\alpha \beta}(k_i^q) \chi_{i,j}^{\alpha \beta}(k_j^q),
\]

where \(g_{i,j}^{\alpha \beta}(k_i^q)\) is the form factor of the interacting two-body subsystem \((ikj)\), with relative momentum \(k_i^q\) and isospin \(I_i\). Here, \(\chi_{i,j}^{\alpha \beta}\) is the strength parameter of the interaction. To directly consider \(KN - \pi \Sigma\) coupling, the potentials are further labeled with the \(\alpha\) values. The two-body \(T\)-matrices in separable form are given by

\[
T_{i,j}^{\alpha \beta}(k_i^q, k_j^q; z; \omega) = R_{i,j}^{\alpha \beta}(k_i^q) \chi_{i,j}^{\alpha \beta}(k_j^q)\chi_{j,i}^{\alpha \beta}(k_i^q),
\]

where \(z\) and \(\omega\) are the total energy of the system and the energy of the spectator particle in the \(\alpha\) channel, respectively.

\[
E^\alpha_i(p_i^\alpha) = \frac{(p_i^\alpha)^2}{2m^\alpha_i},
\]

where the quantity \(v_i = m_i^\alpha(m_i^\gamma + m_i^\gamma + m_i^\gamma)\) is the reduced mass with particle \(i\) in channel \(\alpha\) as the spectator. The operator \(x_{i,j}^{\alpha \beta}(z - E_i(p_i^\alpha))\) also depicts the usual two-body propagator. Using separable potential for two-body interactions, the three-body Faddeev equations [6] in the AGS take the form

\[
\mathcal{K}_{i,j,l}^{\alpha \beta}(\vec{p}_i^\alpha, \vec{p}_j^\beta; z) = \delta_{\alpha \beta}\mathcal{M}_{i,j,l}^{\alpha \beta}(\vec{p}_i^\alpha, \vec{p}_j^\beta; z) + \sum_{k,l} \int d^3 p_k \mathcal{M}_{i,k,l}^{\alpha \beta}(\vec{p}_i^\alpha, \vec{p}_k^\beta; z) \times \tau_{k,l}^{\gamma \gamma}(z - E_k(p_k^\gamma)) \times \mathcal{K}_{k,j,l}^{\gamma \gamma}(\vec{p}_k^\gamma, \vec{p}_j^\beta; z).
\]

After the partial wave decomposition, assuming that the s-wave as the sole significant contribution in our calculations, we obtain the following equations

\[
\mathcal{K}_{i,j,l}^{\alpha \beta}(p_i^\alpha, p_j^\beta; z) = \delta_{\alpha \beta} \mathcal{M}_{i,j,l}^{\alpha \beta}(p_i^\alpha, p_j^\beta; z) + \sum_{k,l} \int d^3 p_k \mathcal{M}_{i,k,l}^{\alpha \beta}(p_i^\alpha, p_k^\beta; z) \times \tau_{k,l}^{\gamma \gamma}(z - E_k(p_k^\gamma)) \times \mathcal{K}_{k,j,l}^{\gamma \gamma}(p_k^\gamma, p_j^\beta; z).
\]

Here, the operators \(\mathcal{K}_{i,j,l}^{\alpha \beta}\) denote the transition amplitudes between Faddeev and particle channels [6]. The operators \(\mathcal{M}_{i,j,l}^{\alpha \beta}\) are the corresponding Born terms. The inputs for the AGS system of Eq. (5) are two-body \(T\)-matrices, embedded in the three-body Hilbert space. Faddeev partition indices \(i, j, k = 1, 2, 3\) are used to define the interacting pair and the spectator particle. The Faddeev equations are modified [6, 7] to take the \(KN - \pi \Sigma\) coupling into account directly. Thus, in addition to the Faddeev indices, the particle indices \((\alpha, \beta, \gamma = 1, 2, 3)\) are also added for each state \((i)\) [31].

\[
\alpha = (1, 2, 3) = (KNN, n\Sigma N_2, \pi N_1 \Sigma).
\]

Since the total isospin of the system is \(I = 1/2\), depending on the spin of the two baryons, we should treat the \(K^- pp\) or \(K^- d\) system. The total spin of the system remains unchanged. Therefore, the baryon spins do not enter explicitly in this formalism, and the operators are labeled by isospin indices. In the \(K^- pp\) system, the spin component is antisymmetric, such that all operators in the isospin base should be symmetric.

In the present calculations, we employed the quasi-particle approach to solve the Faddeev equations for the
bound state problem. The most important aspect of the quasi-particle method is the separable expansion of the scattering amplitudes in the two- and three-body systems [32-34]. To find the \( K^-pp \) pole position, the separable representation must be defined for the three-body amplitudes and driving terms. For this purpose, we used the Hilbert-Schmidt expansion method [33, 35].

\[
\mathcal{M}_{i,j,l}(\vec{p}_i, \vec{p}_j; z) = -\sum_{n=1}^{\infty} \alpha_n(z) u_{n,j,l}^a(\vec{p}_i^a; z) \times u_{n,i,l}^a(\vec{p}_j^a; z),
\]

where the form factors \( u_{n,j,l}^a(\vec{p}_i^a; z) \) are taken as the eigenfunctions of the kernel of Eq. (5). The separable form of the Faddeev transition amplitudes is given by

\[
\mathcal{K}_{i,j,l}(\vec{p}_i^a, \vec{p}_j^a; z) = \sum_n^\infty \zeta_n(z) u_{n,j,l}^a(\vec{p}_i^a; z),
\]

where the functions \( \zeta_n(z) \) obey the equation

\[
\zeta_n(z) = \alpha_n(z)/(\alpha_n(z) - 1).
\]

Applying the Hilbert-Schmidt expansion method [35] to the Faddeev equations of \( KNN - \pi\Sigma N \), the following homogeneous integral equations for \( u_{n,j,l}^a(\vec{p}_i^a; z) \) are obtained

\[
u_{n,j,l}^a(\vec{p}_i^a; z) = \frac{1}{\alpha_n} \sum_{\gamma=1}^{\infty} \sum_{l} \int d^3 p_i^\gamma [\mathcal{M}_{\gamma,l,l}(\vec{p}_i^\gamma, \vec{p}_j^a; z) \\
\times \alpha_{\gamma} T_{n,l,l}^a(z - E_i^\gamma(p_i^\gamma)) u_{n,i,l}^a(\vec{p}_j^a; z)].
\]

To solve the homogeneous system, we search for a complex energy at which one of the eigenvalues \( \lambda_n(z) \) of the kernel matrix becomes equal to one. Hence, we work on the physical and non-physical energy sheet of the \( KNN \) and \( \pi\Sigma N \) channels, respectively.

Another purpose of this work is to study the possible signature of the \( K^-pp \) quasi-bound state in the \( \pi\Sigma N \) mass spectra from the reaction \( (K^-)_{l=0} + N \rightarrow \pi\Sigma N \). The breakup amplitudes for this reaction in terms of the Faddeev transition amplitudes is given by [36].

\[
T_{\pi\Sigma N \rightarrow (K^-)_{l=0} + N}(\vec{k}_N, \vec{p}_N, \vec{p}_N^\prime; z) = \sum_{l} \mathcal{g}_{\pi\Sigma}(\vec{k}_N) \mathcal{r}_{(\Sigma \Xi), N}(z - E_N(\vec{p}_N)) \\
\times \mathcal{K}_{(\Sigma \Xi), N}(p_N, p_N^\prime; z) + \sum_{l} \mathcal{g}_{\pi\Sigma}(\vec{k}_N) \mathcal{r}_{(\Sigma \Xi), N}(z - E_N(\vec{p}_N)) \\
\times \mathcal{K}_{(\Sigma \Xi), N}(p_N, p_N^\prime; z) + \sum_{l} \langle |\pi \otimes \Sigma| p_N^\prime, N | \pi \otimes [\Sigma \Xi]| \rangle \mathcal{g}_{\Sigma\Xi N}(\vec{k}_N) \\
\times \mathcal{T}_{\pi\Sigma N \rightarrow (\Sigma \Xi), N}(z - E_N(\vec{p}_N)) \\
\times \mathcal{K}_{(\Sigma \Xi), N}(p_N, p_N^\prime; z),
\]

where \( z \) is the three-body energy. To find the two-body energy, we reduce it by the spectator particle energy \( E_i(\vec{p}_i) \). Here, \( k_i \) is the relative momentum between the interacting pair \( (jk) \). The quantities \( \mathcal{K}_{i,j} \) denote the Faddeev amplitudes that are derived from Faddeev Eq. (5). Since the \( \pi N \) interaction is neglected, the Faddeev transition amplitudes corresponding to \( \pi N \) system are missing in this equation. Using Eq. (10), we define the transition probability of \( (K^-)_{l=0} + N \rightarrow \pi\Sigma N \) as follows,

\[
w(p_{N^3}, z) = \int d^3 p_N \int d^3 k_N \delta(z - Q(p_N, k_N)) \\
\times |\mathcal{T}_{\pi\Sigma N \rightarrow (\Sigma \Xi), N}(\vec{k}_N, \vec{p}_N, p_N^\prime; z)|^2,
\]

where \( Q(p_N, k_N) \) is given by

\[
Q(p_N, k_N) = \frac{p_{N^3}^2(m_N + m_\pi + m_\Sigma)}{2m_N(m_\pi + m_\Sigma)} - \frac{k_N^2(m_\pi + m_\Sigma)}{2m_\pi m_\Sigma}.
\]

3. Two-body input

In this section, we provide a brief survey on two-body interactions, which are employed as the central inputs for the present few-body calculations. The \( KN - \pi\Sigma \) interaction is dominated by the \( s \)-wave \( \Lambda(1405) \) resonance. Therefore, the orbital angular momentum for the \( KN \) interaction is assumed to be zero. The \( NN \) and \( \Sigma \Sigma \) interactions were assumed to be in the \( l = 0 \) state. Since the interaction in the \( \pi N \) subsystem is dominated by the \( p \)-wave component, the \( \pi \Sigma \) interaction is neglected in our few-body calculations. All separable potentials in the momentum representation assume the form of Eq. (1).

3.1 \( \bar{K}N - \pi\Sigma \) coupled-channel system

During the past two decades, different phenomenological [1, 28, 37] and chiral-based [38-44] potentials are constructed to describe the \( KN \) interaction. The phenomenological models of the interaction consider the \( \Lambda(1405) \) resonance as a quasi-bound state in the \( KN \) system embedded in the \( \pi\Sigma \) continuum. The chiral \( SU(3) \) dynamics was also considered a successful approach to describe the \( KN \) interaction and the \( \Lambda(1405) \) resonance [38-44]. At and above the \( KN \) threshold, the phenomenological and the chiral \( SU(3) \) \( KN \) models of interaction generate comparable results, while for subthreshold energies, their results are different. The phenomenological \( KN \) interactions are constructed to reproduce a single pole nature for the \( \Lambda(1405) \) resonance as a quasi-bound state of the \( KN \) system around 1405 MeV. The \( KN - \pi\Sigma \) coupled-channels amplitude resulting from chiral \( SU(3) \)-based potentials has two poles, one of which is located around 1420 MeV [39], while the other pole with large width is located above the \( \pi\Sigma \) threshold. Therefore, the chiral-based potentials generate a binding energy of about 15 MeV for the \( KN \) system, which is about half the binding energy produced with the purely phenomenological \( \bar{K}N \) models of interaction.
We used different models to describe the $s$-wave $KN-\pi\Sigma$ interaction, which is the most important interaction in the present three-body calculations with $\bar{K}NN$ and $\pi\Sigma N$ coupled-channels. We considered four types of phenomenological potentials. They reproduce the one- and two-pole structure for the $\Lambda(1405)$ resonance, and their parameters are given in Refs. [28, 29]. The parameters are adjusted to reproduce all existing data on the low-energy $\bar{K}N$ interaction. The $\bar{K}N-\pi\Sigma$ potentials in Ref. [29] are adjusted to reproduce the experimental results of the SIDDHARTA experiment [45]. Depending on a pole structure of the $\Lambda(1405)$, we refer to these potentials as the "SIDD, one-pole" and "SIDD, two-pole" potential. The parameters of the potentials in Ref. [28] are adjusted to reproduce the experimental results of the KEK experiment [46, 47]. Depending on the pole structure of the $\Lambda(1405)$, we refer to these potentials as the "KEK, one-pole" and "KEK, two-pole" potential.

In addition to the phenomenological potentials, we also used two chiral-based $\bar{K}N-\pi\Sigma$ potentials, which are provided in Ref. [24]. These chiral-based potentials reproduce the elastic and inelastic cross-sections for the $K^-p$ reaction as well as the $\pi\Sigma$ mass spectra.

3.2 $NN$ interaction

In order to investigate the dependence of the $\pi\Sigma N$ invariant mass on nucleon-nucleon interaction models, we used two different potentials for the $NN$ interaction. The first one is a two-term separable potential [48]

$$V_{NN}^I = \sum_{m=1}^{2} g_{NN}^{I m} I_{NN}^{m} s_{NN}^{m} (k),$$

where $I_{NN}^{m}$ is negative to take into account the short range repulsion in the interaction. The parameters of this potential are given in Ref. [48].

We also used the one-term PEST potential from Ref. [49]. The strength parameter of the PEST potential is equal to one, and the form-factor is defined by

$$g_{NN}^{I}(k) = \frac{1}{2}\sqrt{\sum_{i=1}^{6} \frac{c_{NN}^{I}}{(b_{NN}^{I})^2 + k^2}},$$

where the parameters of the potential are given in Ref. [49]. The PEST potential is not repulsive at short distances, however at low energies its phase shifts are close to the rank-two potential.

For the $s$-wave $\Sigma N$ interaction, we assume the form given in Ref. [50],

$$V_{\sigma\sigma}^I (k', k^2) = C_{\sigma\sigma}^{I} (\Lambda_r \Lambda_p) \frac{1}{(\mu_r \mu_p)^{1/2}} (\mu_r \mu_p)^{-1/2} g_{\sigma}^{I} (k') g_{\sigma}^{I} (k'),$$

where the form factor is defined by $g_{\sigma}^{I} (k') = 1/(k'^2 + \Lambda_r^2)$. The coupling constants, $C_{\sigma\sigma}^{I}$, and the range parameters $\Lambda_r$ are given in Ref. [50].

4 Results and discussion

Solutions of the Faddeev equations corresponding to bound states and resonance poles in the $(I, J^P) = \left( \frac{1}{2}^-, 0^- \right)$ channel of the $\bar{K}NN-\pi\Sigma N$ three-body system were found by applying search procedures described in Section 2. In Tables 1 and 2, the results for the three-body $\bar{K}NN-\pi\Sigma N$ quasi-bound state of the present study are presented. The sensitivity of the $\bar{K}NN$ pole position to the $\bar{K}N-\pi\Sigma$ interaction is investigated using different potential models. In Table 1, the pole position of the quasi-bound states in $\bar{K}NN$ systems is calculated for the phenomenological models of the $\bar{K}N-\pi\Sigma$ interaction, and in Table 2, we calculated the pole energies for energy-dependent and energy-independent chiral potentials.

| Table 1. Sensitivity of the pole position(s) (in MeV), $z_{\Lambda(1405)}^{pole}$, of the $\bar{K}N$ and $\bar{K}NN$ systems to the different phenomenological models of the $\bar{K}N-\pi\Sigma$ interaction. $V_{NN-\Sigma\pi}$ and $V_{NN-\Sigma\pi}$ depict a one- and a two-pole structure of the $\Lambda(1405)$ resonance, respectively. |
|-------------------------------------------------|
| SIDD pot. [28]: |
| $z_{\Lambda(1405)}^{pole}$ | 1428.1 − i46.6 | 1418.1 156.9 |
| $z_{\Lambda(1405)}^{pole}$ | 1382.0 1104.2 |
| KEK pot. [29]: |
| $z_{\Lambda(1405)}^{pole}$ | 1411.3 − i35.8 | 1410.8 135.9 |
| $z_{\Lambda(1405)}^{pole}$ | 1380.8 1104.8 |

| Table 2. Pole position(s) (in MeV), $z_{\Lambda(1405)}^{pole}$, of the quasi-bound states in the $\bar{K}N$ and $\bar{K}NN$ systems to the different phenomenological models of the $\bar{K}N-\pi\Sigma$ interaction. $V_{NN-\Sigma\pi}$ and $V_{NN-\Sigma\pi}$ depict a one- and a two-pole structure of the $\Lambda(1405)$ resonance, respectively. |
|-------------------------------------------------|
|-------------------------------------------------|
| $z_{\Lambda(1405)}^{pole}$ | 1407.2−i18.5 | 1420.6−i20.3 |
| $z_{\Lambda(1405)}^{pole}$ | 1343.0−i72.5 |

The position of a quasi-bound state in the three-body problem is usually defined by solving the homogeneous integral Eq. (9), which is obtained from the separable expansion of the Faddeev amplitudes. To obtain the resonance energy of the system using these equations, one should search for a complex energy where one of the eigenvalues of the kernel matrix is equal to one. Therefore, as deduced from Eqs. (7) and (8), the Faddeev amplitudes have a pole at this energy.

To examine the efficiency of the separable expansion
method, we employed another method to find the $K^-pp$ pole position(s) without integration in the complex momentum plane. The signal of the $K^-pp$ bound state is reflected in the Faddeev transition amplitudes. In the present study, we investigate how the signature of the $K^-pp$ quasi-bound state arises in the three-body scattering amplitudes using coupled-channel Faddeev AGS equations. To achieve this goal, we must solve the inhomogeneous integral equations for the amplitudes defined in Eq. (5).

Figure 2 shows the calculated three-body scattering amplitude $\mathcal{M}_{(\bar{K}N)I=0 + N}^{KN-\pi\Sigma}\!(\bar{K}N_{I=0} + N)$, whose initial and final states are $(\bar{K}N)_{I=0} + N$. The off-shell momenta $p$ and $p'$ are equal to 150 MeV/c, and the real and imaginary parts of the three-body energy, $z$, change from $-100$ MeV to 0 MeV. We used the one-pole (left) and two-pole (right) version of the KEK potential to describe the $\bar{K}N-\pi\Sigma$ interaction. Since the input energy of the AGS equations is complex, the moving singularities that are caused by the open channel $\pi\Sigma N$, do not appear in the three-body amplitudes. The resonance energies of the $KNN$ system calculated by this method are presented in Table 3. By comparison of these results with those in Table 1, one can see that both are in good agreement with each other.

When at least one of the intermediate particles is unstable, plus the signal of the resonance states, one can see a branch point in the complex plane. In Fig. 2, plus the signature of $K^-pp$ pole position, we can see the branch points i.e., a threshold opening associated with the $\Lambda(1405)$ pole, situated at $z = M_N + M_{\Lambda(1405)}$, sum of nucleon mass and $\Lambda(1405)$ pole position. The second row of Fig. 2 illustrates the branch points for the $\Lambda(1405)N$ intermediate state. The branch point is clearly visible, along with the cut chosen in the positive $\Re z$ direction. In the second row, to make the branch points more visible the imaginary part of the three-body energy, $z$ was chosen to be in the range between $-50$ MeV and $-12$ MeV, while the real part changed from 2347 MeV to 2375 MeV. We used the one-pole (left) and two-pole (right) version of the KEK potential to extract the branch points.

We investigated the dependence of the two- and three-body pole energy trajectories on the magnitude $\lambda_{KN-\bar{K}N}^{I=0}$, when the $KN$ strength parameter is increased from its physical value. Let $\kappa$ stand for a strength enhancement factor for the $I=0 \bar{K}N$ interaction:

$$\lambda_{KN-\bar{K}N}^{I=0} = \kappa \lambda_{KN-\bar{K}N}^{I=0}. \quad (16)$$

We calculated the pole trajectory for the one- and two-pole versions of the KEK potential. The behavior of the two-body $K^-p$ and three-body $K^-pp$ pole energy trajectories are considerably different at $\pi\Sigma$ threshold. The pole energies obtained for the three-body system are shown in Fig. 3 (b). The blue dashed and black solid curves in Fig. 3 (a) correspond to the two-body results.

Table 3. Pole position (in MeV) of the Faddeev amplitude for $(\bar{K}N)_{I=0} + N \rightarrow (\bar{K}N)_{I=0} + N$ reaction. The pole positions are calculated for one- and two-pole models of the KEK potential.

| $z^\text{pole}_{KN-\pi\Sigma}$ | $\nu_{\text{KEK},\text{One-pole}}^{\bar{K}N-\pi\Sigma}$ | $\nu_{\text{KEK},\text{Two-pole}}^{\bar{K}N-\pi\Sigma}$ |
|-------------------------------|---------------------------------|---------------------------------|
| $\nu_{\text{KEK},\text{One-pole}}^{\bar{K}N-\pi\Sigma}$ | 2331.0 − i27.2 | 2328.5 − i19.8 |

Fig. 2. (color online) Faddeev amplitude for $(\bar{K}N)_{I=0} + N \rightarrow (\bar{K}N)_{I=0} + N$ reaction. The transition amplitudes are calculated using one-pole (left) and two-pole (right) models of the KEK potential. In the second row, the imaginary part of $z$ is chosen to be between $-50$ MeV and $-12$ MeV, while the real part changes from 2347 MeV to 2375 MeV to make the branch points more visible. The three-body calculations are performed using the PEST potential model for $NN$ interaction.
The numbers attached to the circles and squares depict the corresponding values of the enhancement factor $\kappa$. As $\kappa$ increases, the binding energy of the system increases for both two- and three-body systems. In the two-body calculations of the two-pole potential, the imaginary part of the resonance energy becomes smaller as the binding energy increases, and for $\kappa \approx 1.33$ (at $\pi\Sigma$ threshold) the resonance almost becomes a bound state in the $\pi\Sigma$ channel. In contrast, in the three-body system, the resonance energy will have a non-zero imaginary part at the $\pi\Sigma$ threshold with the increase in $\kappa$, since the $\pi\Lambda$ channel is effectively included.

4.1 Calculation of the transition probability

The calculated resonance energies presented in Table 1 and 2, only provide the pole positions of the $K^-pp$ system. However, these results are not a quantity that can be directly measured in any experiment. To experimentally examine the existence of the quasi-bound state in the $K^-pp$ system, the cross-sections of $K^-pp$ production reactions have to be calculated. We can use the calculated results in Table 1 and 2 and also in Fig. 3 as guideline to study these reactions. As previously mentioned in Section 1, the $K^-pp$ quasi-bound state can occur as a result of kaon-induced reactions on light nuclei such as the $^3$He and deuteron. The resonance traces are observable in the mass spectra of the final particles. In the present calculations, we studied to what extent the signature of the $K^-pp$ system arises in the observables of the three-body reactions by using coupled-channel Faddeev equations in the AGS form. To achieve this goal, we must solve the coupled integral equations for the amplitudes defined in Eq. (5), and then construct the breakup amplitudes $T_{\pi\Sigma N \rightarrow \{Y_K\}_{I=0} + N}$ defined in Eq. (10). Since the kernel of AGS equations has standard moving singularities resulting from the opened channel $\pi\Sigma N$, which are encountered in any three-body breakup problem, we followed the same procedure implemented in Refs. [51, 52].

Using the so called “point-method”, we computed the cross section of $(Y_K)_{I=0} + N \rightarrow \pi\Sigma N$ reaction and studied the behavior of $\pi\Sigma N$ mass spectra. The transition probabilities for phenomenological potentials are depicted in Figs. 4 and 5. In Fig. 6, the three-body calculations are performed by chiral-based potentials for the $KN$ interaction and the PEST potential for the $NN$ interaction.

In Fig. 4, we calculated the $\pi\Sigma N$ mass spectra using one-pole (a) and two-pole (b) versions of KEK potentials for the $KN - \pi\Sigma$ interaction given in Ref. [28]. To investigate the energy dependence of the transition probability, we calculated $w(p_{N,N})$ for $p_N = 100 - 250$ MeV/c. We investigated the dependence of $\pi\Sigma N$ mass spectra on two-body $KN - \pi\Sigma$ interactions, necessary for describing the $KN - \pi\Sigma$ system. Therefore, in Fig. 5, we calculated the $\pi\Sigma N$ mass spectra using the one- and two-pole version of the SIDDHARTA potential for the $KN - \pi\Sigma$ interaction given in Ref. [28]. The results suggest that a distinct peak of bound kaonic states should be observed, regardless of the momentum value and the class of the $KN - \pi\Sigma$ interaction. In the calculated mass spectra for the two-pole model of the KEK and SIDDHARTA potentials, the second pole of $\Lambda(1405)$ resonance, with its large width, does not affect the $\pi\Sigma N$ invariant mass. As observed in Figs. 4 and 5, all potential models generate the mass spectra with similar behavior, and two distinct bump structures can be seen in the $\pi\Sigma N$ invariant mass.

The results obtained from the full coupled-channel calculations of $(Y_K)_{I=0} + N \rightarrow \pi\Sigma N$ scattering using two versions of the energy-dependent and energy-independ-
Fig. 4. (color online) $\pi\Sigma N$ mass spectra for $(Y_N)_{K^+} + N \rightarrow \pi\Sigma N$ reaction. The $M_{\pi\Sigma N}$ spectra calculated using KEK potentials for the $\tilde{K}N - \pi\Sigma$ interaction given in Ref. [28]. The calculations are performed by the PEST potential for the $NN$ interaction. To investigate the energy dependence of the transition probability, we calculated $w(p_N,z)$ for $p_N = 100 - 250$ MeV/c.

Fig. 5. (color online) As in Fig. 4, but using SIDDHARTA potentials [29] for the $\tilde{K}N - \pi\Sigma$ interaction, which reproduce the one- and two-pole structure of $\Lambda(1405)$ resonance.

Fig. 6. (color online) As in Fig. 4, but for the energy-dependent (b) and energy-independent (a) potentials given in Ref. [24].
ent $\bar{K}N-\pi\Sigma$ potentials derived based on chiral $SU(3)$ dynamic and non relativistic kinematics are shown in Fig. 6. The three-body results corresponding to each version of the $\bar{K}N$ interaction exhibit sufficient differences. Therefore, in principle, it is possible to favor one version of the $\bar{K}N-\pi\Sigma$ potential by comparison with experimental results. Within this model, two bump structures appear in the $(Y_K)_{l=0} + N \rightarrow \pi\Sigma N$ transition probabilities in the energy region around the $\bar{K}NN$ pole position and $z = M_{\pi \Sigma} + M_{\Lambda(1405)}$. As mentioned above, the second bump, situated at $z = M_{\pi \Sigma} + M_{\Lambda(1405)}$, actually originates from a branch point in the complex plane (see Fig. 2), i.e., a threshold opening associated with the $\Lambda(1405)$ pole.

To demonstrate this, we show that these bumps truly correspond to the quasi-bound state in the $\bar{K}NN$ system and $\Lambda(1405)$ pole and are not due to threshold effects. We investigate these bump structures in the $\pi\Sigma N$ mass spectra and clarify the origin of these bumps. In Fig. 7, we calculated the $\pi\Sigma N$ mass spectra using the one- and two-pole version of the KEK as well as energy-dependent potentials for the $\bar{K}N-\pi\Sigma$ interaction when the magnitude of the $(I = 0)$ $\bar{K}N$ strength parameter is increased above its physical value. We calculated the $\pi\Sigma N$ mass spectra for three values of the enhancement factor $\kappa = 1.0, 1.05, \text{and } 1.10$. As is shown in Fig. 3 and Table 4, when the $\kappa$ parameter is increased, the binding energy of the system increases for both the two- and three-body systems, and the pole energies proceed toward the $\pi\Sigma(+/N)$ and $\pi\Sigma N$ threshold, respectively. Comparing the results of the mass spectra with those presented in Table 4 and Fig. 3, one can see that the bump structures in the mass spectra and the quasi-bound states in Table 4 are located at the same energy and have the same shift. Therefore, the first bump structure should correspond to a quasi-bound state in the $\bar{K}NN$ system, and the second bump structure is derived from a branch point in the complex plane.

To compare the present results with those in Ref. [24], we calculate $w(p_N,z)$ for $p_N = 100$ MeV/c using the same $\bar{K}N-\pi\Sigma$ and a two-term type potential [48] with a

![Fig. 7. (color online) Our results for $\pi\Sigma N$ mass spectra are presented using “KEK, one-pole”, “KEK, two-pole”, and energy-dependent potentials for the $\bar{K}N$ interaction and PEST potential for $NN$ interaction. We calculated the $\pi\Sigma N$ invariant mass for three different values of the $\kappa$ coefficient. As one can see, when we increase the $\kappa$ parameter, the shift in the bumps' location is very similar to the shift in the pole positions in the $\bar{K}N(+N)$ and $\bar{K}NN$ systems, which are presented in Table 4 and Fig. 3.](image-url)
repulsive core. An intermediate-range attraction is used to describe the nucleon-nucleon interaction. The $\pi\Sigma N$ invariant mass obtained with the two-terms $V_{NN}^{\prime}$ is shown in Fig. 8. The energy-dependent set of the $KN-\pi\Sigma$ potential was used together.

In contrast to the results of Ref. [24], our results show that, in addition to the bump related to the quasi-bound state in $KNN$ systems, a typical bump structure manifests itself in the $\pi\Sigma N$ invariant mass at the energy related to the quasi-bound state of the $KNN(\pi N)$ system. However, this bump structure in the observables is not derived from a resonance pole, and the origin of this structure is the branch points. The behavior of the mass spectra is similar to the extracted results for the phenomenological potentials. The difference between the present results and those obtained by Ohnishi et al. is apparent. Our results exhibit two bump structures that are close to each other in the mass spectra, one of which is related to the quasi-bound state in the $K^-pp$ system, while the other corresponds to branch points that originate from the intermediate $\Lambda(1405)$. Therefore, this effect should be taken into account in the theoretical interpretation of the experimental results by E15.

![Fig. 8.](image)

**Fig. 8.** (color online) Calculated $\pi\Sigma N$ mass spectra for energy-dependent potential is compared with other theoretical results. The $\pi\Sigma N$ mass spectra for incident momentum $p_N = 100\text{ MeV/c}$ was calculated. Our result is depicted by red solid curve, and the result by Ohnishi et al.[24] is shown by the blue dashed curve.

### 4.2 Averaged transition probability

In Subsection 4.1, we calculated the transition probabilities for four discrete values of momenta, however, in an actual scenario the momentum $p_N^\prime$ can occupy any value over a continuous range. To take all these momenta into consideration, in this subsection, we calculated the averaged transition probability $\tilde{w}$, which is given by

$$
\tilde{w}(z) = \int d^3p_N \int d^3p_N^\prime \delta(z - Q(p_N, k_N))
\times \left| \frac{p_N^\prime}{p_N} \right|^2 
\times \left| \frac{d^4p_N}{d^4p_N^\prime} \right|^2
\times |\langle \phi|\psi \rangle|^2
$$

where the function $\rho(p_N)$ can be defined by

$$
\rho(p_N) = \int_0^{p_N^\prime} d^3p_N^\prime \left| \langle \phi|\psi \rangle \right|^2.
$$

To define the wave function of the $K^-pp$ three-body system, we used the so-called exact optical potential [28]. Therefore, the $\pi\Sigma N$ channel is not directly included, and we can omit the particle channel indices. Using Faddeev equations in 9, we can calculate three Faddeev components, $\phi_{i,L}(p_i,k_i;z)$, given by

$$
\phi_{i,L}(p_i,k_i;z) = \frac{1}{z - k_i^2/2\mu_i - p_i^2/2\nu_i}
\times g_{i,L}(k_i) r_{i,L}(z - E_i(p_i)) u_{i,L}(p_i,z),
$$

where $\mu_i = m_j m_k / m_j + m_k$ is the reduced mass of the interacting pairs $jk$, while $p_i$ and $k_i$ depict the Jacobi momenta of the spectator and interacting particles, respectively. The $K^-pp$ three-body wave function can be defined by

$$
|\psi\rangle = \sum_{i,L} |\phi_{i,L}\rangle,
$$

where $\psi$ is the normalized wave function of the $K^-pp$ system, which is defined as a sum of the above components. Figure 9 (left) shows the momentum distribution of the spectator nucleon, $\rho_S(p)$, in the $K^-pp$ system for various models of the $KNN-\pi\Sigma$ potential, and Fig. 9 (right) shows the same distributions multiplied by $p_N^4$. Figure 10 shows the calculated $\pi\Sigma N$ mass spectra using the one- and two-pole version of KEK (black curves) and SIDD potential (blue curves) for the $KNN-\pi\Sigma$ interaction. The mass spectra around the $KNN$ threshold are affected by the kinematical effects and the peaks corresponding to the $\Lambda(1405) + N$ branch point and $K^-pp$ quasi-bound states are not as clear as in Figs. 4 and 5. According to the Figs. 4, 5, and 9, these changes were expected, because the threshold effects are stronger for low values of the $p_N^\prime$ momentum, and the weight of these in the momentum distribution are larger than at high momentum.

In general, Faddeev equations require, as input, a potential that describes the interaction between two indi-

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**Table 4.** Dependence of the $K\bar{N}$ (first pole) and $K\bar{N}N$ quasi-bound state positions (in MeV) on the $\lambda_{K\bar{N}N}$ strength parameter in the $I = 0$ state. We calculated the pole positions for three values of parameter $x$.

| $x$   | $\epsilon^\prime_{K\bar{N}}$ | $\epsilon^\prime_{KN}$ | $\epsilon^\prime_{KNN}$ |
|-------|-----------------------------|------------------------|------------------------|
| 1.00  | 1420.6 − i20.25             | 1414.4 − i21.8         | 1407.7 − i24.0         |
| 1.05  | 2359.5 − i20.25             | 2353.3 − i21.8         | 2346.6 − i24.0         |
| 1.10  | 2346.5 − i22.0              | 2339.0 − i22.1         | 2331.6 − i21.5         |

---

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Fig. 9. (color online) Momentum distribution of spectator nucleon, (left) $\rho(p_N)$ and (right) $p^2\rho(p_N)$, in $K^-pp$ system. We used different kinds of potentials to study the model dependence of the momentum distributions.

Fig. 10. (color online) Energy dependence of averaged transition probability, $\bar{\sigma}(\omega)$. To study the model dependence of the results, we used KEK potential (black curves) and SIDD potentials (blue curves) in our calculations. The solid curves represent the results of the one-pole models and the dashed curves belong to the two-pole models.

An individual particle. A term can also be introduced into the equation in order to take into account three-body forces. Although, we assume, that while our information about the two-body $KN$ interaction is not complete, the inclusion of three-body forces is not necessary. One can also investigate the dependence of mass spectra on the two-body local potentials [44]. The three-body reaction theory can also be formulated for local potentials based on Faddeev equations. This work is progress and will be reported in a future study.

5 Conclusion

In summary, this study performs exact Faddeev-type calculations of the $KNN-\pi\Sigma N$ system to define the binding energy and width of the $K^-pp$ system. The efficiency of the so-called HSE method was investigated. We calculated the transition probability (11) for the $(Y_K)_{I=0} + N \rightarrow \pi\Sigma N$ reaction in the energy region between the $KNN$ and $\pi\Sigma N$ thresholds. Moreover, we examined how the signature of the $K^-pp$ quasi-bound state in the three-body $KNN-\pi\Sigma N$ system manifests itself in the transition probabilities on the real energy axis. To investigate the dependence of the resulting transition probabilities on models of the $KNN-\pi\Sigma$ interaction, several versions of $KNN-\pi\Sigma$ potentials that generate different structures for the $\Lambda(1405)$ resonance were employed. Within this model, we discovered a bump produced by the $KNN-\pi\Sigma N$ system in the $(Y_K)_{I=0} + N \rightarrow \pi\Sigma N$ transition probabilities within the energy region around the $KNN-\pi\Sigma N$ pole position. Furthermore, we found a distinct peak in the mass spectrum for the momenta range of $p_N = 100 – 250$ MeV/c. In the calculations, we also found that the shape and position of the peaks in the transition probability are independent of the momentum $p_N$ of the initial $(Y_K) + N$ channel. This implies that the bumps correspond to the $\Lambda(1405)$ and $K^-pp$ quasi-bound states. Since the nucleon in the initial state covers a continuous range, the effect of all momenta in the transition amplitude should be included. We calculated the averaged transition probability $(Y_K)_{I=0} + N \rightarrow \pi\Sigma N$. Furthermore, we showed that the signature of the $K^-pp$ quasi-bound state can be observed, along with the effect of the branch points in the Faddeev amplitudes (complex plane) and transition probabilities, which are a consequence of the $\Lambda(1405)$ pole. Moreover, we showed that the bump structures related to the branch points can affect the peak corresponding to the $K^-pp$ quasi-bound state. Thus, in the mesonic decay channel, the effect of the branch points should be considered, and this reaction would also be helpful to reveal the dynamic origins of the $\Lambda(1405)$ resonance.
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