Nuclear and particle physics aspects of the $2\nu\beta\beta$-decay of $^{150}Nd$

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A discussion is given on possible realization of the Single State Dominance (SSD) hypothesis in the case of the two-neutrino double beta decay ($2\nu\beta\beta$-decay) of $^{150}Nd$ with $1^−$ ground state of the intermediate nucleus. We conclude that the SSD hypothesis is expected to be ruled out by precision measurement of differential characteristics of this process in running NEMO 3 or planned SuperNEMO experiments unlike some unknown low-lying $1^+$ state of $^{150}Pm$ does exist. This problem can be solved via ($d,^3He$) charge-exchange experiment on $^{150}Sm$. Further, we address the question about possible violation of the Pauli exclusion principle for neutrinos and its consequences for the energy distributions of the $2\nu\beta\beta$-decay of $^{150}Nd$. This phenomenon might be a subject of interest of NEMO 3 and SuperNEMO experiments as well.

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I. INTRODUCTION

The main interest in the double beta decay is connected with the neutrinoless mode ($0\nu\beta\beta$-decay) as a probe for physics beyond the Standard Model (SM) of electroweak interactions. The detection of double beta decay with emission of two neutrinos ($2\nu\beta\beta$-decay), which is an allowed process of second order in the SM, provides the possibility for experimental determination of the corresponding nuclear matrix elements (NME’s). At present, $2\nu\beta\beta$-decay has been measured for ten nuclei $^{150}Nd$.

A subject of interest is the single state dominance (SSD) hypothesis proposed by Abad et al. some times ago. It was suggested that $2\nu\beta\beta$-decays with $1^−$ ground state of the intermediate nucleus (e.g., $A=100, 116$ and $128$ nuclear systems) are solely determined by the two virtual $\beta$-transitions: (i) the first one connecting the ground state of the initial nucleus with $1^+_2$ intermediate state; (ii) the second one proceeding from the $1^+_2$ state to the final ground state. An alternative higher state dominance (HSD) approach assumes that high-lying energy states of the intermediate nucleus give the main contribution to the $2\nu\beta\beta$-decay NMEs. In this case the sum of the two lepton energies in the denominators of the NMEs can be replaced with their average value [3].

Recently, it has been pointed out that the $2\nu\beta\beta$-decay allows to investigate also particle properties, in particular whether the Pauli exclusion principle is violated for neutrinos, and thus, neutrinos obey at least partly the Bose-Einstein statistics [4, 5]. For the $2\nu\beta\beta$-decay of $^{76}Ge$ and $^{100}Mo$ results of numerical calculations of the total rates and various distributions were presented in [6].

Currently, much attention is paid to the double beta decay of $^{150}Nd$ due to a large Q-value and low background from natural radioactivity. This isotope is considered to be a proper candidate for planned SuperNEMO, DCBA(Drift Chamber Beta-ray Analyzer) and SNO+ experiments. In this contribution two problems are addressed: Can the SSD hypothesis be confirmed or ruled out for the $2\nu\beta\beta$-decay of $^{150}Nd$? Is it possible to study also the violation of the Pauli exclusion principle for neutrinos in this process?

II. THE SSD VERSUS HSD STUDY

Till now the issue of the SSD hypothesis has been not addressed in the case of the $2\nu\beta\beta$-decay of $^{150}Nd$ as the ground state of the intermediate nucleus is $1^−$ state. It looks simply unlikely that the SSD is realized through forbidden EC and $\beta^−$-decay, nevertheless, this issue has not been checked yet. However, a negative energy difference between the initial $^{150}Nd$ and the intermediate $^{150}Pm$ ground states favors this nuclear system for the SSD/HSD analysis via differential characteristics.

It is interesting to compare the $2\nu\beta\beta$-decays of $^{100}Mo$ and $^{150}Nd$ in respect to the SSD/HSD hypothesis. The basic characteristics of these two nuclear systems are given in Table 1. The presented nuclear matrix elements for EC of $^{100}Tc$ and $\beta^−$-decays of $^{100}Tc$ and $^{150}Pm$ were calculated from experimental half-lives (see Table 1). We have

\[
\frac{m_e}{6\hbar^2 \ln 2} (G_{\beta^−}m_e^2)^2 \left|\mathcal{M}(J^+e^- \rightarrow 0^0)\right|^2 f_{\beta,EC}(Z, E_i - E_f).
\]

The phase space integrals are given by

\[T_{1/2}^{\beta,EC}(J^+e^- \rightarrow 0^0)\]

\[= \frac{m_e}{6\hbar^2 \ln 2} (G_{\beta^−}m_e^2)^2 |\mathcal{M}(J^+e^- \rightarrow 0^0)|^2 f_{\beta,EC}(Z, E_i - E_f).\]
TABLE I: Basic characteristics of the electron capture (EC) and $\beta^-$-decay of the ground state of the intermediate nucleus for $A = 100$ and 150 systems \(^{[7]}\). The half-life was estimated by assuming that values of the EC and $\beta^-$ matrix elements are comparable for a given isotope.

| Type   | Transition | $Q$ [MeV] | $J^\pi$ | $T_{1/2}^{\nu\beta\beta}$ | $|\mathcal{M}(J^\pi)|$ log $t_\nu$ |
|--------|------------|-----------|---------|--------------------------|---------------------------------|
| $\beta^-$ | $^{100}$Te($1^+_{g.s.}$) $\rightarrow$ $^{100}$Ru($0^+_{g.s.}$) | 3.202 | $1^+$ | 15.8 [s] | 0.69 | 4.6 |
| EC     | $^{100}$Te($1^+_{g.s.}$) $\rightarrow$ $^{100}$Mo($0^+_{g.s.}$) | 0.168 | $1^+$ | 8.77 $10^5$ [s] | 0.82 | 4.5 |
| $\beta^-$ | $^{150}$Pm($1^+_{g.s.}$) $\rightarrow$ $^{150}$Nd($0^+_{g.s.}$) | 0.086 | $1^-$ | $\approx 6 \times 10^5$ [y] | $\approx 0.016$ | 8.0 |
| EC     | $^{150}$Pm($1^+_{g.s.}$) $\rightarrow$ $^{150}$Sm($0^+_{g.s.}$) | 3.454 | $1^-$ | 9.64 $10^3$ [s] | 0.016 | 7.9 |

Nuclear matrix elements can be written as

$$f_{EC}(Z, E_i - E_f) = \frac{1}{m_e^2} \int E_i - E_f \ F_0(Z, p_0) p_0 (E_i - E_f - p_0)^2 dp_0, \quad (2)$$

$$f_{EC}(Z, E_i - E_f) = 2\pi^2 \left( \frac{1}{m_e^2} \frac{Z^3}{\pi a_F^3} \right) \frac{(E_i - E_f + \varepsilon_b)^2}{m_e^2}. \quad (3)$$

Nuclear matrix elements can be written as

$$\mathcal{M}(J^\pi) = < 0_f ||\mathcal{O}(J^\pi)|| 1^+_i > \quad (4)$$

with

$$\mathcal{O}_k(1^+) = ig_A \sum_m \tau^+_m (\vec{\sigma}_m)_k, \quad (5)$$

$$\mathcal{O}_k(1^-) = \left( \frac{\alpha Z}{2} \right) \sum_m \tau^-_m \frac{1}{R} (x^-_m - ig_A x^-_m \times \vec{\sigma}_m)_k. \quad (6)$$

Here, $J^\pi = 1^+, 1^-$. $E_i, E_f$ are energies of the initial and final nuclei, respectively. $g_A$ is the axial-vector coupling constant ($g_A = 1.254$). $R$ is nuclear radius.

Further we assume that the nuclear matrix element for EC of the ground state $^{150}$Pm is comparable with that for $\beta^-$-decay of this isotope. Both matrix elements are suppressed by the same coulombic factor ($\alpha Z/2$) and a similar situation occurs also for $A=100$ system (see Table I \(^{[8]}\)). Under this assumption we find the half-life of the EC of $^{150}$Pm to be about $6 \times 10^{16}$ years, i.e., not measurable with help of current technologies. The SSD prediction for the $2\nu/3\beta$-decay half-life is $4 \times 10^{24}$ years in complete disagreement with the experimental value 9.7 $10^{18}$ years \(^{[1]}\). We recall that for the $2\nu/3\beta$-decay of $^{100}$Mo there is a rather good agreement between the SSD calculated value 6.8 $10^{18}$ years and the measured value 7.1 $10^{18}$ years.

In Fig. \(^{[1]}\) we present the single electron differential decay rate for the $2\nu/3\beta$-decay of $^{100}$Mo and $^{150}$Nd to $0^+$ ground state and $2^+_1$ excited state. The SSD results were obtained by assuming the dominance of transition through $1^+$ ground state of $^{100}$Te and $1^-$ ground state of $^{150}$Pm for $2\nu/3\beta$-decay of $^{100}$Mo and $^{150}$Nd, respectively. We see that differences between the SSD and HSD predictions are even larger for $A=150$ as for $A=100$. Unlike there is an unknown $1^+$ low lying state of $^{150}$Pm the experimental measurement should confirm the HSD results for the $2\nu/3\beta$-decay of $^{150}$Nd. This spectroscopic problem can be also addressed by measuring $(d,^2\text{He})$ charge exchange reaction on $^{150}$Sm \(^{[6]}\). The recent progress in the field of charge exchange reactions is encouraging and provide new and sometimes unexpected insight into nuclear structure phenomena.

### III. THE VIOLATION OF PAULI EXCLUSION PRINCIPLE FOR NEUTRINOS

Neutrinos may possibly violate the spin-statistics theorem, and hence obey Bose statistics or mixed statistics despite having spin half \(^{[9]}\). A violation of the spin-statistics relation for neutrinos would lead to a number of observable effects in cosmology and astrophysics. In particular, bosonic neutrinos might compose all or a part of the cold cosmological dark matter (through bosonic condensate of neutrinos) and simultaneously provide some hot dark matter \(^{[1]}\). A change of neutrino statistics would have an impact on the evolution of supernovae and on the spectra of supernova neutrinos. The Pauli principle violation for neutrinos can be tested in the $2\nu/3\beta$-decay \(^{[1], [5]}\).

Qualitative features of the $2\nu/3\beta$-decay in the presence of bosonic neutrinos can be understood by the fact that two contributions to the amplitude of the decay from diagram with permuted neutrino momenta have relative plus sign instead of minus in the Fermi-Dirac case. The decay probability is proportional to the bilinear combinations
Here, \( \Delta \) is the average energy of the leptonic pair.

\[
K^b_m L_n^b, \quad K^f_m L_n^f, \quad L_m^f L_n^b, \quad \text{where}
\]

\[
K^f_m^b \equiv \frac{1}{E_m - E_i + p_{10} + k_{10}} \pm \frac{1}{E_m - E_i + p_{20} + k_{20}},
\]

\[
L^f_m^b \equiv \frac{1}{E_m - E_i + p_{20} + k_{10}} \pm \frac{1}{E_m - E_i + p_{10} + k_{20}}.
\]  

(7)

Here, \( E_i, \ E_m, \ p_{i0} \) and \( k_{i0} \) (\( i = 1, 2 \)) are the energies of the initial nucleus, intermediate nucleus, electrons and neutrinos, respectively. We notice a sign difference between the two energy denominators in (7) distinguishing the cases of fermionic (f) and bosonic (b) neutrinos.

The effect of bosonic neutrinos is different for transitions to \( 0^+ \) ground states and \( 2^+ \) excited states. It is because the decay rate to \( 0^+ \) state is governed by the combinations \( (K_m + L_m)(K_n + L_n) \) and decay rate to \( 2^+ \) state is proportional to combinations \( (K_m - L_m)(K_n - L_n) \). By approximating these combinations one finds significantly different expressions for bosonic and fermionic neutrinos:

\[
(K_m^b + L_n^b) \approx \frac{2}{(E_m - E_i + \Delta)^2} (k_{20} - k_{10}),
\]

\[
(K_m^b - L_n^b) \approx \frac{4}{(E_m - E_i + \Delta)^2} (p_{20} - p_{10}),
\]

\[
(K_m^f + L_n^f) \approx \frac{4}{E_m - E_i + \Delta},
\]

\[
(K_m^f - L_n^f) \approx \frac{2}{(E_m - E_i + \Delta)^3} (p_{20} - p_{10})(k_{20} - k_{10}).
\]  

(8)

Here, \( \Delta \) is the average energy of the leptonic pair.

In Fig. 2 we show the energy distributions of outgoing electrons calculated for the \( 2\nu\beta\beta \)-decay of \( ^{150}Nd \) to the \( 0^+ \) ground state and \( 2^+ \) excited state with the HSD assumption. Both for the single electron differential decay rate (left panel) and for differential decay rate as function of the sum of kinetic energy of outgoing electrons (right panel) the results are significantly different for bosonic and fermionic neutrinos. In particular, maxima of the distributions for transitions to \( 0^+ \) and \( 2^+ \) states (right panel) are about at the same position for fermionic neutrinos and are shifted each to other by about 0.5 MeV for bosonic neutrinos.

IV. CONCLUSIONS

We offered some arguments that the SSD hypothesis is not expected to be realized for \( 2\nu\beta\beta \)-decay of \( ^{150}Nd \). This might be confirmed within the NEMO 3 and SuperNEMO experiments. However, if there is an unknown low-lying \( 1^+ \) state of \( ^{150}Pm \) the conclusion might be opposite. This issue can be addressed by the \((d,^2He)\) charge-exchange experiment on \( ^{150}Sm \) target [6].

In addition, we showed that a study of the \( 2\nu\beta\beta \)-decay of \( ^{150}Nd \) can provide a sensitive test of the Pauli exclusion principle and statistics of neutrinos. For that purpose an experiment (e.g., SuperNEMO) is needed with a precision measurement of differential characteristics for \( 2\nu\beta\beta \)-decay transitions to \( 0^+ \) ground and \( 2^+ \) excited states.

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FIG. 2: The single electron differential rate normalized to the total decay rate vs. the electron energy (left panel) and the differential decay rate normalized to the total decay rate vs. the sum of kinetic energy for 2νββ-decay of $^{150}$Nd to the ground $0^{+}_{gs}$ and excited $2^{+}$ states of final nucleus. The results are presented for the cases of pure fermionic and pure bosonic neutrinos. The calculations have been performed with the HSD assumption.

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