ACTIVE LEARNING FOR REGRESSION WITH AGGREGATED OUTPUTS

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ABSTRACT

Due to the privacy protection or the difficulty of data collection, we cannot observe individual outputs for each instance, but we can observe aggregated outputs that are summed over multiple instances in a set in some real-world applications. To reduce the labeling cost for training regression models for such aggregated data, we propose an active learning method that sequentially selects sets to be labeled to improve the predictive performance with fewer labeled sets. For the selection measurement, the proposed method uses the mutual information, which quantifies the reduction of the uncertainty of the model parameters by observing the aggregated output. With Bayesian linear basis functions for modeling outputs given an input, which include approximated Gaussian processes and neural networks, we can efficiently calculate the mutual information in a closed form. With the experiments using various datasets, we demonstrate that the proposed method achieves better predictive performance with fewer labeled sets than existing methods.

1 Introduction

Data are often aggregated for privacy protection, cost reduction, or the difficulty of data collection [28, I1, I3]. For example, census data are averaged over spatial regions, IoT data are aggregated to reduce the communication overhead, the gene expression level is measured for each set of multiple cells, and brain imaging data are observed for each set of voxels. Since learning from such aggregated data is important for applications where only aggregated data are available, many machine learning methods for aggregated data have been proposed [32, 38, 4].

Although the predictive performance of the machine learning model improves as the number of training labeled data increases in general, obtaining many labeled data incurs considerable cost. Active learning has been successfully used for reducing the labeling cost, where instances to be labeled are sequentially selected to improve the predictive performance [41, 29, 48, 45]. However, there have been no active learning methods for regression with aggregated data.

In this paper, we propose an active learning method for regression with aggregated outputs. At the beginning of the active learning process, we are given unlabeled sets of instances. Then, for each active learning step, we select a set to observe its aggregated output, where we cannot observe outputs for each instance. Our aim is to improve the predictive performance of the outputs for each test instance. The proposed method selects a set that maximizes the mutual information between the aggregated output and model parameters, which corresponds to the reduction of the uncertainty of the model parameters by observing the aggregated output of the set. Mutual information-based active learning has been successfully used for non-aggregated data [30, 25, 21].

We derive the mutual information using linear basis function models as a regression model that predicts the non-aggregated output given an input vector. Various regression models can be formulated as a linear basis function model, which include polynomial regression, approximated Gaussian processes [39], and neural networks by changing basis functions. With the Bayesian inference framework of the linear basis function models, we can model the distribution of the aggregated output as a Gaussian distribution, and we can calculate the mutual information on aggregated outputs efficiently in a closed form. Figure 1 shows the framework of our active learning.

The major contributions of this paper are as follows:

1. We propose the first active learning method for regression with aggregated outputs.
Figure 1: Our framework of active learning with aggregated outputs. In the beginning, we are given unlabeled sets of instances. For each step, we iterate the following procedures: 1) Predict the distribution of the aggregated output for each of the unlabeled sets using the model. 2) Select a set from the unlabeled sets to be labeled using mutual information calculated based on the predicted distributions, and query the oracle. 3) Observe the aggregated output of the selected set, include it in the labeled sets, and exclude it from the unlabeled sets. 4) Retrain the model using the updated labeled sets.

2. The proposed method is based on entropy and mutual information, which are calculated efficiently using Bayesian linear basis function models, and considers the correlation among the instances in each set.

3. We demonstrate the effectiveness of the proposed method with various datasets compared with existing active learning methods for non-aggregated data.

The remainder of this paper is organized as follows. In Section 2, we briefly review related work. In Section 3, we describe the probability distribution of the weighted sum of Gaussian distributed random variables, which is used in the proposed method. In Section 4, we define our task, and propose our active learning method for regression with aggregated outputs based on entropy and mutual information. In Section 5, we evaluate the performance of our method by comparing existing methods. Finally, we present concluding remarks and a discussion of future work in Section 6.

2 Related work

Several frameworks of learning from aggregated data have been proposed [13, 11, 53]. Learning from label proportions [38, 35] considers classification tasks, where outputs are categorical. Multiple instance learning [31] learns classification models from labeled sets, where each set is positively labeled if at least one individual is positive, and it is otherwise negatively labeled. Collective graphical models learn from contingency tables [42, 23, 17]. Regression from aggregated data has also been considered, where outputs are continuous [4, 52, 47, 24]. Summed or averaged values are assumed to be observed in [33, 52, 47, 24] as with our setting, and histograms are assumed to be observed in [4]. Some methods assume that both inputs and outputs are aggregated [4], and others assume that only outputs are aggregated while inputs are not aggregated [32, 22]. In this paper, we consider regression with aggregated outputs by a linear weighted summation.

Many active learning methods have been proposed [41, 49, 5, 2, 29, 48], which include those for multiple instance learning [8], and those for learning from label proportions [37]. However, they are not for regression with aggregated outputs, and they are inapplicable to our task. Batch active learning [20, 14, 36] selects multiple instances to be labeled, where outputs for each instance are observed. It is different from our task, where aggregated outputs are observed, but individual outputs cannot be observed.

3 Preliminaries

Let \( y = [y_1, \cdots, y_N] \in \mathbb{R}^N \) be jointly Gaussian distributed random variables,

\[
y \sim \mathcal{N}(\mu, \Sigma),
\]

(1)
4 Proposed method

In Section 4.1 we define our task of active learning for aggregated outputs. In Section 4.2 we present our model for predicting outputs that are trained from labeled sets with aggregated outputs based on linear basis function models. In Sections 4.3 and 4.4 we propose entropy-based and mutual information-based active learning methods using our model that select a set to be observed next to improve the predictive performance, respectively. In Section 4.5 we present the procedures of the proposed method.

4.1 Problem formulation

Suppose that we are given sets of input vectors \( \{ \mathbf{X}_a \}_{a=1}^A \), where \( \mathbf{X}_a = \{ \mathbf{x}_{an} \}_{n=1}^{N_a} \) is the \( a \)th set of input vectors, \( \mathbf{x}_{an} \in \mathbb{R}^D \) is the \( r \)th input vector, \( D \) is the number of attributes, and \( N_a \) is the number of input vectors in the set. For each active learning step, we select a set from \( \{1, \ldots, A\} \), and observe the aggregated output of selected set \( a \) that is obtained by the weighted sum of the output of the input vectors in the set,

\[
\bar{y}_a = \sum_{n=1}^{N_a} \theta_{an} y_{an} = \theta_a^\top \mathbf{y}_a \in \mathbb{R},
\]

where \( \bar{y}_a \) indicates that it is an aggregated value, \( y_{an} \in \mathbb{R} \) is the unknown output of input vector \( \mathbf{x}_{an} \), \( \mathbf{y}_a = [y_{a1}, \ldots, y_{aN_a}] \in \mathbb{R}^{N_a} \), and \( \theta_a = [\theta_{a1}, \ldots, \theta_{aN_a}] \in \mathbb{R}^{N_a} \) is the weights. We assume that weights \( \theta_a \) for all sets are known. For example, \( \theta_{an} = 1 \) when the aggregated data are obtained by summation, and \( \theta_{an} = \frac{1}{N_a} \) when they are obtained by average. Our aim is to improve the test predictive performance of the outputs with as few observations of aggregated outputs as possible. Table 1 shows our notation. Although we assume that outputs are scalar, the proposed method can be straightforwardly extended to multivariate outputs. When the aggregated response value is obtained by integral \( \bar{y}_a = \int \theta_{an} y_{an} dn \), we can apply the proposed method by approximating the integral by the summation by dividing the space into a finite number of bins.

4.2 Model

Let \( \hat{y}(\mathbf{x}; \mathbf{w}) \) be a regression model to predict the non-aggregated output given input vector \( \mathbf{x} \), where \( \mathbf{w} \) is the parameters to be considered as random variables. We consider the following linear basis function model,

\[
\hat{y}(\mathbf{x}; \mathbf{w}) = \mathbf{w}^\top \phi(\mathbf{x}),
\]

where \( \phi(\mathbf{x}) \in \mathbb{R}^K \) is the nonlinear basis function that transforms a \( D \)-dimensional input vector to a \( K \)-dimensional vector, and \( \mathbf{w} \in \mathbb{R}^K \). A wide variety of regression models can be formulated by a linear basis function model, which include linear regression, polynomial regression, approximated Gaussian processes with random features-based basis functions [39], and neural networks with the last layer represented by random variables and neural network-based
basis functions. For aggregated data, linear regression [43, 45, 51], Gaussian processes [46, 10, 24, 47], and neural networks [39] have been used. The linear basis function formulation is preferable especially when training data are small, which is a situation of active learning, since the number of random variable parameters to be estimated is small, and the posterior of the model parameters are estimated analytically.

### 4.3 Entropy-based active learning

In the non-aggregated setting, the entropy-based active learning selects an instance that maximizes the entropy of the output [40]. It corresponds to the uncertainty sampling [27, 41], which selects a set whose aggregated output is least certain, where the uncertainty is quantified by the entropy. The high entropy indicates the high uncertainty. In the aggregated setting, a set that maximizes the entropy of aggregated output \( \bar{y}_a \) given input vector \( X_a \) is selected

\[
\arg \max_a H[\bar{y}_a | X_a, D], \tag{5}
\]

where \( H[x] = - \int p(x) \log p(x) dx \) represents the entropy, and \( D = \{ (X_a, \bar{y}_a) \}_{a=1}^A \) is the current set of labeled sets with the aggregated outputs.

With linear basis function models in Eq. (4), the entropy in Eq. (5) can be calculated in a closed form. We assume the following Gaussian distribution for the prior of parameters \( w \),

\[
p(w) = \mathcal{N}(w|0, \lambda^{-1}I), \tag{6}
\]

where \( \lambda \in \mathbb{R}_{>0} \) is the precision hyperparameter.

We assume the following Gaussian observation noise for the non-aggregated output given an input vector,

\[
p(y|x, w) = \mathcal{N}(y|\hat{y}(x; w), \beta^{-1}), \tag{7}
\]

where \( \beta \in \mathbb{R}_{>0} \) is the precision hyperparameter. Then, the distribution of aggregated output \( \bar{y}_a \) given set of input vectors \( X_a \) and parameters \( w \) is the following Gaussian distribution using Eqs. (2,3),

\[
p(\bar{y}_a | X_a, w) = \mathcal{N} \left( \bar{y}_a | \theta_a^\top \hat{y}_a, \beta^{-1} \| \theta_a \|^2 \right), \tag{8}
\]

where \( \bar{y}_a = [\hat{y}(x_{a1}; w), \ldots, \hat{y}(x_{aN_a}; w)] \). Note that output values \( y_{a1}, \ldots, y_{aN_a} \) to be aggregated are independent Gaussian random variables given parameters \( w \) as described in Eq. (7). The likelihood of labeled set \( D \) is given by

\[
p(D|w) = \prod_{a=1}^{N_a} p(\bar{y}_a | X_a, w). \tag{9}
\]

Figure 2 shows the graphical model representation of the proposed model.

Using the Bayes rule and Eqs. (4,6,9), the posterior distribution of parameters \( w \) given \( D \) is given by

\[
p(w|D) = \frac{p(w)p(D|w)}{\int p(w)p(D|w) dw} = \mathcal{N}(w|m, S), \tag{10}
\]

where

\[
m = \beta S \sum_{a=1}^{A} \frac{\bar{y}_a}{\| \theta_a \|^2} \Phi_a \theta_a \in \mathbb{R}^K, \tag{11}
\]

\[
S = \sum_{a=1}^{A} \frac{1}{\| \theta_a \|^2} \Phi_a \theta_a \Phi_a^\top \tag{12}
\]

Figure 2: Graphical model representation of the proposed model. The shaded and unshaded circles represent observed and unobserved variables, respectively, the squares represent hyperparameters that can be trained, the rectangles represent repetition, and the numbers at the corner represent the number of repetitions.
where

$$I = \beta \sum_{a=1}^{A} \frac{1}{\| \theta_a \|^2} (\Phi_a \theta_a)(\Phi_a \theta_a)^\top \in \mathbb{R}^{K \times K},$$

(12)

and $\Phi_a = [\phi(x_{a1}), \ldots, \phi(x_{aN_a})] \in \mathbb{R}^{K \times N_a}$. By using a conjugate Gaussian prior in Eq. (6) with a Gaussian observation noise in Eq. (7), the posterior is Gaussian, and its mean and covariance are calculated in a closed form.

The joint predictive distribution of output vectors $y_a = [y_{a1}, \ldots, y_{aN_a}] \in \mathbb{R}^{N_a}$ for input vectors $X_a$ is given by

$$p(y_a|X_a, D) = \int \prod_{n=1}^{N_a} p(y_{an}|x_{an}, w)p(w|D)dw = \mathcal{N}(y_a|m^\top \Phi_a, \beta^{-1}I + \Phi_a^\top S \Phi_a),$$

(13)

using Eqs. (4,7,10). Then, the predictive distribution of aggregated output $\bar{y}_a$ is

$$p(\bar{y}_a|X_a, D) = \mathcal{N}(\bar{y}_a|m^\top \Phi_a \theta_a, \theta^\top_a (\beta^{-1}I + \Phi_a^\top S \Phi_a) \theta_a),$$

(14)

using Eqs. (2,3,13). Since the entropy of the Gaussian distribution is

$$\mathbb{H}[\mathcal{N}(\mu, \sigma^2)] = \frac{1}{2}(\log \sigma^2 + \log(2\pi) + 1),$$

(15)

the entropy of the aggregated output in Eq. (5) is given by

$$\mathbb{H}[\bar{y}_a|X_a, D] = \frac{1}{2}(\log \theta^\top_a (\beta^{-1}I + \Phi_a^\top S \Phi_a) \theta_a + \log(2\pi) + 1),$$

(16)

using Eqs. (14,15). Intuitively speaking, when a set contains input vectors whose predicted outputs $y_a$ are highly correlated and have high variance, the entropy becomes high, and such set is likely to be selected.

### 4.4 Mutual information-based active learning

In the non-aggregated setting, the mutual information-based active learning selects an instance that maximizes the mutual information between the output and parameters $[30,28,21]$. In the aggregated setting, a set that maximizes the mutual information between aggregated output $\bar{y}_a$ and parameters $w$ are selected,

$$\arg \max_a \mathbb{I}[\bar{y}_a, w|D],$$

(17)

where $\mathbb{I}[x, x'] = \int p(x, x') \log \frac{p(x|x')}{p(x)p(x')} dx dx'$ represents the mutual information, and $D$ is the current labeled aggregated data.

The mutual information in Eq. (17) equals to the decrease of the uncertainty of the parameters by observing the aggregated output,

$$\mathbb{I}[\bar{y}_a, w|D] = \mathbb{H}[w|D] - \mathbb{E}_{p(\bar{y}_a|X_a, D)}[\mathbb{H}[w|\bar{y}_a, X_a, D]],$$

(18)

where $\mathbb{E}_{p(\bar{y}_a|X_a, D)}[.] = \int p(\bar{y}_a|X_a, D)[] d\bar{y}_a$ represents the expectation, the first term is the entropy of the parameters given the current labeled aggregated data, and the second term is the expected entropy of the parameter when aggregated output $\bar{y}_a$ is additionally observed. Since aggregated output $\bar{y}_a$ has not been observed yet, the expectation is taken in the second term using current model $p(\bar{y}_a|X_a, D)$. We can rearrange the mutual information as follows $[13, 16, 12]$,

$$\mathbb{I}[\bar{y}_a, w|D] = \mathbb{H}[\bar{y}_a|X_a, D] - \mathbb{E}_{p(w|D)}[\mathbb{H}[\bar{y}_a|X_a, w]],$$

(19)

since the mutual information is symmetric, $\mathbb{I}[\bar{y}_a, w|D] = \mathbb{I}[w, \bar{y}_a|D]$. The first term is equivalent to the objective function of the entropy-based active learning in Eq. (5). The entropy in the second term is calculated by

$$\mathbb{H}[\bar{y}_a|X_a, w] = \frac{1}{2}(\log \beta^{-1} \| \theta_a \|^2 + \log(2\pi) + 1),$$

(20)

using Eqs. (8,15). Then, the mutual information for linear basis function models is given by

$$\mathbb{I}[\bar{y}_a, w|D] = \frac{1}{2}(\log \theta^\top_a (\beta^{-1}I + \Phi_a^\top S \Phi_a) \theta_a - \log \beta^{-1} \| \theta_a \|^2),$$

(21)

using Eqs. (16,19,20).
We evaluated the proposed method using regression datasets in LIBSVM [9], which were generated. We subsampled 336 locations (instances) with the interval of three degrees of longitude and latitude. The climate data contain 26 bio-climate values, such as mean annual temperature, length of the frost-free period, precipitation as snow, summer heat-moisture index, and annual heat-moisture index, for each location defined by longitude, latitude, and elevation. One of the 26 bio-climate values was used as outputs, and the others including location information were used as attributes. In total, 26 datasets for regression with 28 attributes and a scalar output were generated. We subsampled 336 locations (instances) with the interval of three degrees of longitude and latitude. One of the 26 bio-climate values was used as outputs, and the others including location information were used as attributes. In total, 26 datasets for regression with 28 attributes and a scalar output were generated. We subsampled 336 locations (instances) with the interval of three degrees of longitude and latitude.

Algorithm 1 shows the active learning procedures by the proposed method. In Line 8, we estimate precision hyperparameters in Algorithm 1 by maximizing the log marginal likelihood in Eq. (22).

\[
\log p(D|\beta, \lambda) = \log \int p(D|w)p(w)dw \\
= \frac{A}{2} \log \beta + \frac{K}{2} \log \lambda - \frac{1}{2} \sum_{a=1}^{A} \log ||\theta_a||^2 + \frac{1}{2} \log |S| - \frac{1}{2} \beta \sum_{a=1}^{A} ||\theta_a||^2 + \frac{1}{2} m^T S^{-1} m - \frac{A}{2} \log(2\pi),
\]

which is derived using Eqs. (6,9), where we omit the precision hyperparameters in \(p(D|w, \beta) = p(D|w)\) and \(p(w|\lambda) = p(w)\) for simplicity. When we use neural networks for basis function \(\phi(x)\), we can also estimate the neural network parameters by maximizing this log marginal likelihood. The predictive distribution of the non-aggregated output given an input vector is calculated by

\[
p(y|x, D) = \int p(y|x, w)p(w|D)dw = N(y|m^T \phi(x), \beta^{-1} + \phi(x)^T S \phi(x)),
\]

using Eq. (7) and the posterior in Eq. (10).

4.5 Procedures

Algorithm 1 shows the active learning procedures by the proposed method. In Line 8, we estimate precision hyperparameters \(\beta\) and \(\lambda\) by maximizing the following log marginal likelihood,

\[
\log p(D|\beta, \lambda) = \log \int p(D|w)p(w)dw \\
= \frac{A}{2} \log \beta + \frac{K}{2} \log \lambda - \frac{1}{2} \sum_{a=1}^{A} \log ||\theta_a||^2 + \frac{1}{2} \log |S| - \frac{1}{2} \beta \sum_{a=1}^{A} ||\theta_a||^2 + \frac{1}{2} m^T S^{-1} m - \frac{A}{2} \log(2\pi),
\]

which is derived using Eqs. (6,9), where we omit the precision hyperparameters in \(p(D|w, \beta) = p(D|w)\) and \(p(w|\lambda) = p(w)\) for simplicity. When we use neural networks for basis function \(\phi(x)\), we can also estimate the neural network parameters by maximizing this log marginal likelihood. The predictive distribution of the non-aggregated output given an input vector is calculated by

\[
p(y|x, D) = \int p(y|x, w)p(w|D)dw = N(y|m^T \phi(x), \beta^{-1} + \phi(x)^T S \phi(x)),
\]

using Eq. (7) and the posterior in Eq. (10).

5 Experiments

5.1 Data

We evaluated the proposed method using regression datasets in LIBSVM [9] and the climate data in North American [2]. Table 2 shows the statistics of the LIBSVM datasets. For each of the LIBSVM datasets, we randomly selected 80% of the instances for training, and the remaining for testing. Using the training instances, we generated sets of aggregated data by randomly selecting instances without replacement, where the number of instances in a set is between one to 20.

The climate data contain 26 bio-climate values, such as mean annual temperature, length of the frost-free period, precipitation as snow, summer heat-moisture index, and annual heat-moisture index, for each location defined by longitude, latitude, and elevation. One of the 26 bio-climate values was used as outputs, and the others including location information were used as attributes. In total, 26 datasets for regression with 28 attributes and a scalar output were generated. We subsampled 336 locations (instances) with the interval of three degrees of longitude and latitude. The data were aggregated by the state, and 72 sets were obtained. The minimum, average, and maximum numbers of instances in a set were 1, 4, 67, and 51. We randomly selected 80% of the sets for training, and instances in the remaining sets were used for testing.

For each set in both the LIBSVM and climate datasets, the aggregated output was calculated by summing the individual outputs in each set. Note that when the aggregated output is the average of the outputs, we can transform it to the sum.
of the outputs by multiplying the number of instances without knowing individual outputs as preprocessing. The test data were not aggregated, where the output for each test instance was predicted. The number of queries to observe aggregated outputs was $T = 30$. We performed 50 experiments with different splits of training and test data for each dataset. We did not use validation data since we considered a situation where a limited number of labeled sets are available.

### 5.2 Comparing methods

We compared the proposed active learning methods for aggregated data based on the mutual information (AggMI) and entropy (AggEnt) with the following methods: the mutual information-based method for non-aggregated data (MI), the entropy-based method for non-aggregated data (Ent), the variance of the input vectors (Var), query by committee \([6]\) (QBC), expected model change maximization \([7]\) (EMCM), the maximum of the number of instances (MaxN), the minimum of the number of instances (MinN), and random (Rand).

MI (Ent) selects a set that maximizes the sum of the mutual informations (entropies) over the instances in the set. MI (Ent) corresponds to AggMI (AggEnt) without consideration of covariance across instances in a set, and can be seen as existing mutual information-based (entropy-based) active learning method for non-aggregated data. Var selects a set whose variance in the input vectors is maximum. QBC selects a set whose variance in the predictions of the committee members is maximum. The predictions of the committee members were obtained by the models with parameters that are sampled from the posterior. EMCM selects a set that maximizes the expected model change, which was approximated with an ensemble that was created as in QBC. MaxN (MinN) selects a set that maximizes (minimizes) the number of instances. Rand randomly selects a set.

For all methods including the proposed method, we used approximated Gaussian processes \([39]\) for modeling the non-aggregated output. With the approximated Gaussian processes, the basis function in Eq. (4) was set as follows,

$$
\phi(x) = \left[ \sqrt{\frac{2}{K-1}} \cos(-Bx + c), 1 \right],
$$

where $B \in \mathbb{R}^{(K-1) \times D}$ is generated from the standard Gaussian distribution, $b_{kd} \sim \mathcal{N}(0, 1)$, and $c \in \mathbb{R}^{K-1}$ is generated from a uniform distribution $c_k \sim \text{Uniform}(0, 2\pi)$. In Eq. (24), the last element of the basis function is set to one, $\phi_K(x) = 1$, for a bias parameter. We used $K = 128$. The precision hyperparameters were trained by maximizing the marginal likelihood in Eq. (22) using Adam \([19]\) with learning rate $10^{-3}$ and 1000 epochs. We implemented the proposed method with PyTorch \([34]\).

### 5.3 Results

Table 3 shows the mean squared error averaged over different numbers of queries. The proposed AggMI method achieved the lowest mean squared error on most datasets. The AggEnt method selects sets whose labels are the most uncertain. Therefore, although the AggEnt method can reduce the uncertainty in the selected sets, it does not necessarily reduce the uncertainty of the model parameters. On the other hand, since the AggMI method selects sets that directly maximize the decrease of the uncertainty of the parameters as in Eq. (18), it performed better than the AggEnt method. The methods that do not consider covariance in each set (MI and Ent) performed worse than those that consider covariance (AggMI and AggEnt). This result indicates that the covariance is important for active learning with aggregated outputs. The active learning methods for non-aggregated data (QBC and EMCM) did not perform well. The variance of the input vectors (Var) was worse than AggEnt, which considers the variance of the aggregated outputs. The error by the MaxN method was lower than that by the MinN method. Figure 3 shows the mean squared error for each number of queries. All methods decreased the error as the number of queries with most datasets.

Figure 4 shows the visualization of the Abalone dataset at different numbers of queries by the AggMI method, where the aggregated data were generated by randomly selected sets with sizes one to ten. The data consisted of three clusters.
Table 3: Mean squared error averaged over different numbers of queries from one to 30. Values in bold typeface are not statistically different at 5% level from the best performing method in each row by a paired t-test. The bottom row shows the number of datasets the method achieved the performance that are not statistically different from the best performing method.

| Method | AggMI | AggEnt | MI | Ent | QBC | EMCM | Var | MaxN | MinN | Rand |
|--------|-------|--------|----|-----|-----|------|-----|------|------|------|
| Abalone | 0.041 | 0.042 | 0.052 | 0.048 | 0.044 | 0.046 | 0.056 | 0.048 | 0.059 | 0.051 |
| Cadata | 0.183 | 0.185 | 0.228 | 0.206 | 0.199 | 0.218 | 0.238 | 0.227 | 0.227 | 0.353 | 0.221 |
| Cpu | 0.031 | 0.033 | 0.168 | 0.043 | 0.036 | 0.044 | 0.125 | 0.043 | 0.104 | 0.046 |
| Housing | 0.119 | 0.124 | 0.133 | 0.129 | 0.124 | 0.128 | 0.137 | 0.130 | 0.144 | 0.128 |
| Mg | 0.199 | 0.209 | 0.234 | 0.213 | 0.214 | 0.215 | 0.225 | 0.211 | 0.234 | 0.226 |
| Space | 0.011 | 0.011 | 0.015 | 0.012 | 0.011 | 0.013 | 0.014 | 0.013 | 0.017 | 0.013 |
| eFFP | 0.080 | 0.093 | 0.147 | 0.089 | 0.095 | 0.097 | 0.167 | 0.112 | 0.387 | 0.157 |
| bFFP | 0.081 | 0.089 | 0.150 | 0.088 | 0.090 | 0.090 | 0.177 | 0.119 | 0.394 | 0.137 |
| Tave-wt | 0.081 | 0.090 | 0.158 | 0.086 | 0.092 | 0.092 | 0.170 | 0.105 | 0.473 | 0.171 |
| Tave-sm | 0.094 | 0.099 | 0.119 | 0.093 | 0.092 | 0.091 | 0.136 | 0.103 | 0.207 | 0.108 |
| TD | 0.072 | 0.076 | 0.107 | 0.074 | 0.076 | 0.077 | 0.180 | 0.087 | 0.524 | 0.180 |
| SHM | 0.028 | 0.030 | 0.040 | 0.032 | 0.029 | 0.031 | 0.079 | 0.031 | 0.039 | 0.041 |
| RH | 0.106 | 0.105 | 0.143 | 0.107 | 0.108 | 0.106 | 0.132 | 0.112 | 0.130 | 0.133 |
| PPT-wt | 0.052 | 0.053 | 0.066 | 0.053 | 0.053 | 0.054 | 0.078 | 0.056 | 0.099 | 0.066 |
| PPT-sm | 0.045 | 0.053 | 0.194 | 0.052 | 0.053 | 0.055 | 0.064 | 0.062 | 0.273 | 0.084 |
| PAS | 0.024 | 0.028 | 0.029 | 0.027 | 0.027 | 0.028 | 0.029 | 0.031 | 0.036 | 0.030 |
| NFFD | 0.125 | 0.145 | 0.248 | 0.139 | 0.144 | 0.148 | 0.212 | 0.170 | 0.550 | 0.232 |
| MWMT | 0.061 | 0.063 | 0.097 | 0.063 | 0.063 | 0.064 | 0.113 | 0.071 | 0.272 | 0.115 |
| MSP | 0.053 | 0.060 | 0.171 | 0.059 | 0.061 | 0.063 | 0.080 | 0.069 | 0.285 | 0.098 |
| MCMT | 0.068 | 0.078 | 0.137 | 0.075 | 0.078 | 0.079 | 0.175 | 0.092 | 0.481 | 0.169 |
| MAT | 0.082 | 0.088 | 0.094 | 0.089 | 0.087 | 0.090 | 0.133 | 0.104 | 0.338 | 0.116 |
| MAP | 0.040 | 0.041 | 0.063 | 0.042 | 0.041 | 0.042 | 0.054 | 0.043 | 0.158 | 0.070 |
| FFP | 0.088 | 0.101 | 0.236 | 0.096 | 0.102 | 0.104 | 0.123 | 0.125 | 0.451 | 0.162 |
| Eref | 0.102 | 0.117 | 0.327 | 0.111 | 0.118 | 0.120 | 0.154 | 0.143 | 0.487 | 0.227 |
| EXT | 0.075 | 0.074 | 0.099 | 0.074 | 0.074 | 0.074 | 0.125 | 0.079 | 0.190 | 0.127 |
| EMT | 0.073 | 0.081 | 0.190 | 0.078 | 0.082 | 0.083 | 0.087 | 0.094 | 0.339 | 0.136 |
| DD-18 | 0.121 | 0.128 | 0.099 | 0.118 | 0.125 | 0.126 | 0.112 | 0.139 | 0.383 | 0.190 |
| DD-0 | 0.329 | 0.327 | 0.092 | 0.329 | 0.326 | 0.328 | 0.111 | 0.328 | 0.297 | 0.172 |
| DD5 | 0.057 | 0.068 | 0.130 | 0.065 | 0.068 | 0.071 | 0.161 | 0.084 | 0.477 | 0.194 |
| DD18 | 0.047 | 0.054 | 0.337 | 0.051 | 0.054 | 0.055 | 0.115 | 0.066 | 0.457 | 0.129 |
| CMD | 0.085 | 0.095 | 0.089 | 0.091 | 0.095 | 0.097 | 0.147 | 0.108 | 0.353 | 0.175 |
| AHM | 0.018 | 0.018 | 0.024 | 0.018 | 0.018 | 0.018 | 0.036 | 0.019 | 0.042 | 0.030 |

# best: 30 8 4 4 5 4 1 0 0 1

Table 4: Training computational time in seconds with the Abalone dataset.

| Method | AggMI | AggEnt | MI | Ent | QBC | EMCM | Var | MaxN | MinN | Rand |
|--------|-------|--------|----|-----|-----|------|-----|------|------|------|
| 2599.831 | 2544.971 | 2521.719 | 2577.427 | 3072.736 | 3111.591 | 3036.683 | 2542.546 | 2425.869 | 2523.622 |
5.4 Results with neural network models

We evaluated the proposed method with neural networks for modeling the non-aggregated output. For all methods including the proposed method, the basis function is modeled by four-layered feed-forward neural networks with 32 hidden units and 128 output units. The neural network parameters were trained by maximizing the marginal likelihood using Adam [19] with learning rate $10^{-3}$ and 1000 epochs. In addition to the comparing methods described in the paper, we compared with dropout-based active learning methods: dropout-based AggMI (DAMI) and dropout-based AggEnt (DAEnt). With the dropout-based methods, the entropy and mutual information are calculated by the Monte Carlo
method with dropout \[44\]. By approximating the predictive distribution of aggregated output \(\bar{y}_a\) with a Gaussian distribution, the variance is given by

\[
\sigma^2 = \frac{1}{L} \sum_{\ell=1}^{L} \left( \sum_{n=1}^{N_a} \theta_n \hat{y}'(x_{an}; \mathbf{w}_\ell) \right)^2 - \left( \frac{1}{L} \sum_{\ell=1}^{L} \sum_{n=1}^{N_a} \theta_n \hat{y}'(x_{an}; \mathbf{w}_\ell) \right)^2,
\]

where \(L\) is the number of dropout samples, and \(\mathbf{w}_\ell\) is the \(\ell\)th sampled parameters by dropout, and \(\hat{y}'(x_{an}; \mathbf{w}_\ell) = \hat{y}(x_{an}; \mathbf{w}_\ell) + \beta \epsilon\) is the non-aggregated output prediction by a neural network with parameter \(\mathbf{w}_\ell\) and standard Gaussian noise \(\epsilon\). The entropy of the aggregated output (DAEnt) is given by

\[
H[\bar{y}_a | X_a, \mathcal{D}] = \frac{1}{2} \left( \log \sigma^2 + \log(2\pi) + 1 \right).
\]

Intuitively speaking, sets where the aggregated output predictions vary across different dropout samples are likely to be selected. The mutual information (DAMI) is calculated by

\[
I[\bar{y}_a, \mathbf{w} | \mathcal{D}] = \frac{1}{2} \left( \log \sigma^2 - \log \beta^{-1} \| \theta_a \|^2 \right).
\]

The results are shown in Table 5. The proposed method (AggMI and AggEnt) achieved the better performance than the other methods. Since the neural network-based basis functions have many parameters that are not considered as random variables, the estimated entropy and mutual information were not accurate compared with those with the approximated Gaussian processes by random features. Therefore, the difference of the performance between the proposed method and the other methods, with the neural network-based basis functions in Table 5 were smaller than that with the approximated Gaussian processes in the main paper. For the same reason, the errors by AggMI and AggEnt were similar. Dropout-based methods (DAMI and DAEnt) were worse than the proposed method. This result indicates the effectiveness of the Bayesian linear basis function models.

### 6 Conclusion

We proposed an active learning method for aggregated data that sequentially selects sets to observe its aggregated output to improve the predictive performance with fewer observations. The proposed method selects sets using the entropy of aggregated outputs, or mutual information between aggregated outputs and model parameters. We derived the analytical solution of the entropy and mutual information with Bayesian linear basis function modeling. We experimentally demonstrated that the proposed method achieved lower test errors than the existing active learning methods.

Although our results are encouraging, we must extend our approach in several directions. First, we will apply our approach to non-Gaussian likelihoods for tasks such as classification and Poisson regression using models in [4, 53, 24].

Second, we want to extend the proposed method such that it selects a subset of input vectors to be labeled from a set of all candidate input vectors although we assume that the sets of instances are fixed and given in this paper.

### References

[1] M. P. Armstrong, G. Rushton, and D. L. Zimmerman. Geographically masking health data to preserve confidentiality. *Statistics in Medicine*, 18(5):497–525, 1999.

[2] P. Bachman, A. Sordoni, and A. Trischler. Learning algorithms for active learning. In *International Conference on Machine Learning*, pages 301–310, 2017.

[3] A. Bhowmik. *Learning from aggregated data*. PhD thesis, 2019.

[4] A. Bhowmik, J. Ghosh, and O. Koyejo. Generalized linear models for aggregated data. In *Artificial Intelligence and Statistics*, pages 93–101, 2015.

[5] A. Bordes, S. Ertekin, J. Weston, L. Botton, and N. Cristianini. Fast kernel classifiers with online and active learning. *Journal of Machine Learning Research*, 6(9), 2005.

[6] R. Burbidge, J. J. Rowland, and R. D. King. Active learning for regression based on query by committee. In *International Conference on Intelligent Data Engineering and Automated Learning*, pages 209–218. Springer, 2007.

[7] W. Cai, Y. Zhang, and J. Zhou. Maximizing expected model change for active learning in regression. In *International Conference on Data Mining*, pages 51–60. IEEE, 2013.
Table 5: Mean squared error averaged over different numbers of queries from one to 30. Values in bold typeface are not statistically different at 5% level from the best performing method in each row by a paired t-test. The bottom row shows the number of datasets the method achieved the performance that are not statistically different from the best performing method.

| Method   | Abalone | Cad-data | Cpu | Housing | Mg | Space | eEFFP | bEFFP | Tave-wt | Tave-sm | TD | SHM | RH | PPT-wt | PPT-sm | NFFD | MWMT | MSP | MAT | MAP | FFP | Eref | EXT | EMT | DD-18 | DD-0 | DD5 | DD18 | CMD | AHM | Rand |
|----------|---------|----------|-----|---------|----|-------|------|-------|--------|--------|----|-----|----|-------|-------|-----|------|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|
| AggMI    | 0.052   | 0.200    | 0.047 | 0.134   | 0.206 | 0.016 | 0.045 | 0.032 | 0.040  | 0.040  | 0.045 | 0.034 | 0.040 | 0.042  | 0.048 | 0.043 | 0.031 | 0.035 | 0.034 | 0.051 | 0.056 | 0.029 | 0.034 | 0.022 |
| DAMI     | 0.050   | 0.198    | 0.074 | 0.128   | 0.203 | 0.015 | 0.051 | 0.030 | 0.056  | 0.030  | 0.053 | 0.035 | 0.056 | 0.030  | 0.058 | 0.036 | 0.040 | 0.044 | 0.045 | 0.055 | 0.060 | 0.047 | 0.048 | 0.032 |
| AggEnt   | 0.051   | 0.207    | 0.051 | 0.136   | 0.219 | 0.017 | 0.032 | 0.030 | 0.056  | 0.030  | 0.053 | 0.035 | 0.056 | 0.030  | 0.058 | 0.036 | 0.040 | 0.044 | 0.045 | 0.055 | 0.060 | 0.047 | 0.048 | 0.032 |
| DAEnt    | 0.052   | 0.205    | 0.051 | 0.127   | 0.219 | 0.014 | 0.042 | 0.037 | 0.040  | 0.030  | 0.053 | 0.035 | 0.056 | 0.030  | 0.058 | 0.036 | 0.040 | 0.044 | 0.045 | 0.055 | 0.060 | 0.047 | 0.048 | 0.032 |
| MI       | 0.053   | 0.201    | 0.051 | 0.130   | 0.220 | 0.024 | 0.042 | 0.037 | 0.040  | 0.030  | 0.053 | 0.035 | 0.056 | 0.030  | 0.058 | 0.036 | 0.040 | 0.044 | 0.045 | 0.055 | 0.060 | 0.047 | 0.048 | 0.032 |
| Ent      | 0.052   | 0.206    | 0.051 | 0.136   | 0.220 | 0.024 | 0.042 | 0.037 | 0.040  | 0.030  | 0.053 | 0.035 | 0.056 | 0.030  | 0.058 | 0.036 | 0.040 | 0.044 | 0.045 | 0.055 | 0.060 | 0.047 | 0.048 | 0.032 |
| QBC      | 0.052   | 0.206    | 0.051 | 0.136   | 0.220 | 0.024 | 0.042 | 0.037 | 0.040  | 0.030  | 0.053 | 0.035 | 0.056 | 0.030  | 0.058 | 0.036 | 0.040 | 0.044 | 0.045 | 0.055 | 0.060 | 0.047 | 0.048 | 0.032 |
| ECMC     | 0.052   | 0.206    | 0.051 | 0.136   | 0.220 | 0.024 | 0.042 | 0.037 | 0.040  | 0.030  | 0.053 | 0.035 | 0.056 | 0.030  | 0.058 | 0.036 | 0.040 | 0.044 | 0.045 | 0.055 | 0.060 | 0.047 | 0.048 | 0.032 |
| Var      | 0.052   | 0.206    | 0.051 | 0.136   | 0.220 | 0.024 | 0.042 | 0.037 | 0.040  | 0.030  | 0.053 | 0.035 | 0.056 | 0.030  | 0.058 | 0.036 | 0.040 | 0.044 | 0.045 | 0.055 | 0.060 | 0.047 | 0.048 | 0.032 |
| MaxN     | 0.053   | 0.207    | 0.051 | 0.136   | 0.220 | 0.024 | 0.042 | 0.037 | 0.040  | 0.030  | 0.053 | 0.035 | 0.056 | 0.030  | 0.058 | 0.036 | 0.040 | 0.044 | 0.045 | 0.055 | 0.060 | 0.047 | 0.048 | 0.032 |
| MinN     | 0.052   | 0.206    | 0.051 | 0.136   | 0.220 | 0.024 | 0.042 | 0.037 | 0.040  | 0.030  | 0.053 | 0.035 | 0.056 | 0.030  | 0.058 | 0.036 | 0.040 | 0.044 | 0.045 | 0.055 | 0.060 | 0.047 | 0.048 | 0.032 |
| Rand     | 0.053   | 0.207    | 0.051 | 0.136   | 0.220 | 0.024 | 0.042 | 0.037 | 0.040  | 0.030  | 0.053 | 0.035 | 0.056 | 0.030  | 0.058 | 0.036 | 0.040 | 0.044 | 0.045 | 0.055 | 0.060 | 0.047 | 0.048 | 0.032 |

# best  17   8   16   8  11  3  4  3  6  4  1  6

[8] M.-A. Carboneau, E. Granger, and G. Gagnon. Bag-level aggregation for multiple-instance active learning in instance classification problems. *IEEE Transactions on Neural Networks and Learning Systems*, 30(5):1441–1451, 2018.

[9] C.-C. Chang and C.-J. Lin. LIBSVM: a library for support vector machines. *ACM Transactions on Intelligent Systems and Technology (TIST)*, 2(3):1–27, 2011.

[10] S. R. Flaxman, Y.-X. Wang, and A. J. Smola. Who supported obama in 2012? ecological inference through distribution regression. In *Proceedings of the 21th ACM SIGKDD International Conference on Knowledge Discovery and Data Mining*, pages 289–298, 2015.

[11] D. A. Freedman, S. P. Klein, J. Sacks, C. A. Smyth, and C. G. Everett. Ecological regression and voting rights. *Evaluation Review*, 15(6):673–711, 1991.

[12] Y. Gal, R. Islam, and Z. Ghahramani. Deep Bayesian active learning with image data. In *International Conference on Machine Learning*, pages 1183–1192, 2017.

[13] L. A. Goodman. Ecological regressions and behavior of individuals. *American Sociological Review*, 1953.

[14] Y. Guo and D. Schuurmans. Discriminative batch mode active learning. pages 593–600, 2007.

[15] N. Houlsby, F. Huszár, Z. Ghahramani, and M. Lengyel. Bayesian active learning for classification and preference learning. *arXiv preprint arXiv:1112.5745*, 2011.
[16] T. Iwata, N. Houlsby, and Z. Ghahramani. Active learning for interactive visualization. In *Artificial Intelligence and Statistics*, pages 342–350, 2013.

[17] T. Iwata and N. Marumo. Co-occurrence estimation from aggregated data with auxiliary information. In *Proceedings of the AAAI Conference on Artificial Intelligence*, volume 34, pages 4247–4254, 2020.

[18] R. A. Johnson and D. W. Wichern. *Applied multivariate statistical analysis*. Prentice hall Upper Saddle River, NJ, 2002.

[19] D. P. Kingma and J. Ba. Adam: A method for stochastic optimization. In *International Conference on Learning Representations*, 2015.

[20] A. Kirsch, J. Van Amersfoort, and Y. Gal. Batchbald: Efficient and diverse batch acquisition for deep Bayesian active learning. *Advances in Neural Information Processing Systems*, 32:7026–7037, 2019.

[21] B. Krishnapuram, D. Williams, Y. Xue, L. Carin, M. Figueiredo, and A. Hartemink. On semi-supervised classification. In *Advances in Neural Information Processing Systems*, pages 721–728, 2004.

[22] H. Kück and N. de Freitas. Learning about individuals from group statistics. In *Proceedings of the Twenty-First Conference on Uncertainty in Artificial Intelligence*, pages 332–339, 2005.

[23] A. Kumar, D. Sheldon, and B. Srivastava. Collective diffusion over networks: Models and inference. In *Uncertainty in Artificial Intelligence*, page 351, 2013.

[24] H. C. L. Law, D. Sejdinovic, E. Cameron, T. C. Lucas, S. Flaxman, K. Battle, and K. Fukumizu. Variational learning on aggregate outputs with Gaussian processes. In *Advances in Neural Information Processing Systems*, pages 6084–6094, 2018.

[25] N. Lawrence, M. Seeger, and R. Herbrich. Fast sparse Gaussian process methods: The informative vector machine. In *Advances in Neural Information Processing Systems*, pages 609–616, 2003.

[26] D. S. Lemons and P. Langevin. *An introduction to stochastic processes in physics*. JHU Press, 2002.

[27] D. D. Lewis and W. A. Gale. A sequential algorithm for training text classifiers. In *SIGIR*, pages 3–12. Springer, 1994.

[28] S. Li, L. Da Xu, and X. Wang. Compressed sensing signal and data acquisition in wireless sensor networks and internet of things. *IEEE Transactions on Industrial Informatics*, 9(4):2177–2186, 2012.

[29] C. Liang, J. Ye, S. Wang, B. Pursel, and C. L. Giles. Investigating active learning for concept prerequisite learning. In *AAAI Conference on Artificial Intelligence*, 2013.

[30] D. J. MacKay. Information-based objective functions for active data selection. *Neural Computation*, 4(4):590–604, 1992.

[31] O. Maron and T. Lozano-Pérez. A framework for multiple-instance learning. *Advances in Neural Information Processing Systems*, pages 570–576, 1998.

[32] D. R. Musicant, J. M. Christensen, and J. F. Olson. Supervised learning by training on aggregate outputs. In *Seventh IEEE International Conference on Data Mining*, pages 252–261. IEEE, 2007.

[33] Y. Park and J. Ghosh. Ludia: An aggregate-constrained low-rank reconstruction algorithm to leverage publicly released health data. In *Proceedings of the 20th ACM SIGKDD International Conference on Knowledge Discovery and Data Mining*, pages 55–64, 2014.

[34] A. Paszke, S. Gross, F. Massa, A. Lerer, J. Bradbury, G. Chanan, T. Killeen, Z. Lin, N. Gimelshein, L. Antiga, A. Desmaison, A. Kopf, E. Yang, Z. DeVito, M. Raison, A. Tejani, S. Chilamkurthy, B. Steiner, L. Fang, J. Bai, and S. Chintala. PyTorch: An imperative style, high-performance deep learning library. In *Advances in Neural Information Processing Systems*, volume 32, 2019.

[35] G. Patrini, R. Nock, P. Rivera, and T. Caetano. (almost) no label no cry. *Advances in Neural Information Processing Systems*, 27:190–198, 2014.

[36] R. Pinsler, J. Gordon, E. Nalisnick, and J. M. Hernández-Lobato. Bayesian batch active learning as sparse subset approximation. *Advances in Neural Information Processing Systems*, 32:6359–6370, 2019.

[37] R. Poyiadzis, R. Santos-Rodriguez, and N. Twomey. Active learning with label proportions. In *IEEE International Conference on Acoustics, Speech and Signal Processing (ICASSP)*, pages 3097–3101. IEEE, 2019.

[38] N. Quadrianto, A. J. Smola, T. S. Caetano, and Q. V. Le. Estimating labels from label proportions. *Journal of Machine Learning Research*, 10(10), 2009.

[39] A. Rahimi and B. Recht. Random features for large-scale kernel machines. In *Advances of Neural Information Processing Systems*, pages 1177–1184, 2008.
[40] P. Sebastiani and H. P. Wynn. Maximum entropy sampling and optimal Bayesian experimental design. *Journal of the Royal Statistical Society: Series B (Statistical Methodology)*, 62(1):145–157, 2000.

[41] B. Settles. Active learning literature survey. 2009.

[42] D. Sheldon, T. Sun, A. Kumar, and T. Dietterich. Approximate inference in collective graphical models. In *International Conference on Machine Learning*, pages 1004–1012, 2013.

[43] C. Smith-Clarke, A. Mashhadi, and L. Capra. Poverty on the cheap: Estimating poverty maps using aggregated mobile communication networks. In *Proceedings of the SIGCHI Conference on Human Factors in Computing Systems*, pages 511–520, 2014.

[44] N. Srivastava, G. Hinton, A. Krizhevsky, I. Sutskever, and R. Salakhutdinov. Dropout: a simple way to prevent neural networks from overfitting. *Journal of Machine Learning Research*, 15(1):1929–1958, 2014.

[45] M. Sugiyama and S. Nakajima. Pool-based active learning in approximate linear regression. *Machine Learning*, 75(3):249–274, 2009.

[46] Y. Tanaka, T. Iwata, T. Tanaka, T. Kurashima, M. Okawa, and H. Toda. Refining coarse-grained spatial data using auxiliary spatial data sets with various granularities. In *Proceedings of the AAAI Conference on Artificial Intelligence*, volume 33, pages 5091–5099, 2019.

[47] Y. Tanaka, T. Tanaka, T. Iwata, T. Kurashima, M. Okawa, Y. Akagi, and H. Toda. Spatially aggregated Gaussian processes with multivariate areal outputs. *Advances in Neural Information Processing Systems*, 32:3005–3015, 2019.

[48] Y.-P. Tang and S.-J. Huang. Self-paced active learning: Query the right thing at the right time. In *AAAI Conference on Artificial Intelligence*, volume 33, pages 5117–5124, 2019.

[49] S. Tong and D. Koller. Support vector machine active learning with applications to text classification. *Journal of Machine Learning Research*, 2:45–66, 2001.

[50] L. Van der Maaten and G. Hinton. Visualizing data using t-sne. *Journal of Machine Learning Research*, 9(11), 2008.

[51] H. Wang, D. Kifer, C. Graif, and Z. Li. Crime rate inference with big data. In *Proceedings of the 22nd ACM SIGKDD International Conference on Knowledge Discovery and Data Mining*, pages 635–644, 2016.

[52] F. Yousefi, M. T. Smith, and M. A. Álvarez. Multi-task learning for aggregated data using Gaussian processes. In *Advances in Neural Information Processing Systems*, 2019.

[53] Y. Zhang, N. Charoenphakdee, Z. Wu, and M. Sugiyama. Learning from aggregate observations. *Advances in Neural Information Processing Systems*, 33, 2020.