INTRODUCTION

In recent years, the rapid development of wind power industry in China poses a great challenge for the utilization of wind energy. By the end of 2017, the national average curtailment rate of wind power was up to 12%, which was much higher than that of other countries, such as Denmark (about 1%) and America (about 2%). Most of the wind power curtailment occurs in the power system of Northern China, where the power supply is mainly dependent on the coal-fired combined heat and power (CHP) units. The CHP units are operated at a “heat-led” mode; namely, the output of electric power is dominated by the thermal power. It is very difficult to regulate the electric power flexibly because the heating demand has a priority to be met, especially during heating periods. Hence, it is in urgent need to promote the flexibility of the power system for better utilization of wind energy.

Optimal operation of the combined heat and power system equipped with power-to-heat devices for the improvement of wind energy utilization

Yanjuan Yu | Hongkun Chen | Lei Chen | Cheng Chen | Jun Wu

Abstract

The combined heat and power system (CHPS) is one important type of the integrated energy system in Northern China. Compared with the independently operated power system, the combined heat and power system (CHPS) has a great potential to improve the utilization of wind energy in virtue of the energy storage of the heating system and power-to-heat (P2H) devices. This paper studies the optimal operation of the CHPS equipped with the electric boiler (EB) and heat pump (HP). Comprehensive factors are considered, including P2H conversion, the thermal inertia of district heating network (DHN) and buildings, and adjustable heat loads. An equivalent model of the heating system is developed to express the direct mathematical relations between the heat source and heat loads. Redundant details of the DHN are simplified to reduce the complexity of the optimization problem. Furthermore, a self-repair method incorporated with the real-coded genetic algorithm (RCGA) is proposed to efficiently handle the nonlinear constraints. The effectiveness of the solving method is verified in case studies. The impacts of the EB and the HP on the CHPS are compared in details. The results show that the HP offers better performance than the EB on coal saving and improving wind power integration.

KEYWORDS
combined heat and power system, power-to-heat device, real-coded genetic algorithm, wind power integration
At present, the combined operation of the heating system and electric power system (well known as combined heat and power system, CHPS) has been recognized as a promising approach to breaking through the limited flexibility of the independent electric power system.\(^4_5\) By utilizing the energy storage of the heating system\(^6\) or auxiliary heating devices, such as the heat storage (HS)\(^7\) and power-to-heat (P2H) devices,\(^8_9\) the joint production of electric and heat power of the CHP units can be decoupled, contributing to the ability enhancement of the electric power system for better wind power integration.

In respect to the application of the energy storage of heating system, a few works of literature can be found. Interaction mechanisms of the electric and heating systems are introduced in Reference,\(^10\) and a quasi-steady multi-energy flow model is proposed to elucidate operation characteristics of the main components in the integrated energy systems by dividing the interaction process into four quasi-steady stages. Zheng et al\(^11\) demonstrate that the district heating network (DHN) can be simulated as a huge HS to provide extra flexibility for wind power accommodation. The buildings also play a role of thermal energy storage owing to their thermal inertia, and they are compared with the HS in reference.\(^12\) The results indicate that both the buildings and the HS benefit the operation of the district heating system. Li et al propose a detailed district heating system model for the optimal scheduling of the CHPS, including heat stations, heat-exchanger stations, and the DHN.\(^13_14\) However, the thermal inertia of buildings is not mentioned. In contrast, the works in ref.\(^15\) focus on the application of building’s short-term storage capacity in the scheduling of CHPS, and the characteristics of the DHN are not embodied. In studies,\(^16_19\) both of the DHN and buildings are considered in the modeling of the CHPS. The authors in refs.\(^16\) and \(^17\) establish the models of mass flow and dynamic temperatures at each node of the DHN and buildings. Dan et al\(^18\) investigate the energy flow in the DHN with multiple heat sources under the quality regulation mode and exploit the impact of different heating regulation modes on the thermal capacity of buildings. Similarly, Jin et al\(^19\) contribute to applying different regulation modes into the dispatch model of the CHPS in consideration of the hydraulic and dynamic thermal characteristics of the heating system.

The works of literature mentioned above have verified that the energy storage of heating system is beneficial for accepting more wind power in the CHPS. However, utilizing the thermal energy storage of heating system solely is insufficient for eliminating waste of wind energy when the wind power penetration reaches a high level. Thus, many scholars devote to the application of auxiliary heating devices in the CHPS. The HS is proved to play an essential role in heat load shifting.\(^20\) Since part of the heat load can be served by the thermal power from the HS, the CHP units are allowed to reduce power production when the wind energy is abundant.\(^21\)

A latent HS facility with phase-change material is applied in reference\(^22\) to increase the flexibility of the CHP unit, and it is verified to reduce wind energy loss effectively. As popular as the HS is employed in the CHPS,\(^23\) the EB and the HP are usually utilized to consume surplus wind power by converting it into thermal power.\(^24_25\) The authors in reference\(^26\) explore the effectiveness of the EB incorporated with heat storages on the improvement of wind power integration and propose a start-stop strategy for the control of the EB and heat storages. The performance of the EB and the HP in the CHPS of West Denmark is compared in reference.\(^27\) The results show that both the EB and the HP are helpful for increasing the penetration of renewable energy, but the HP is more cost-effective than the EB in future markets. Economic valuation of the EBs and HPs with different capacities and energy efficiencies is studied based on a two-stage stochastic model in reference.\(^28\) Hao et al\(^29\) claim that both the heat transfer unit and the HS are beneficial to improve renewable energy accommodation, and the authors propose a novel heat current method for analyzing the energy flow in the electric and heating systems. Rong et al\(^30\) provide several indexes to evaluate the effects of the HS and EB on decoupling the production of electric and thermal power. In addition, some researchers focus on the combination of P2H devices and demand-side management technologies.\(^31\) Demand-side management technology can be applied to the adjustable electric power loads\(^32\) or heat loads.\(^33\) It is also suitable for the P2H devices installed at the load side,\(^34\) which provides new opportunities for increasing the flexibility of the CHPS.

Researches mentioned above have paid much attention to the accommodation of renewable energy in the CHPS, but few of them have simultaneously utilized the P2H devices and the thermal inertia of the DHN and buildings. Moreover, the established models of the DHN usually contain many intermediate variables, and thus, the models are too complex to be applied to the optimal operation of the CHPS, such as studies.\(^13_14_16_17\) Therefore, based on the research achievements in the previous studies, this paper devotes to further research on the optimal operation of the CHPS regarding modeling and solution strategy. The main contributions of the present work are as follows:

1. Comprehensive factors are considered in the optimal operation of the CHPS, including thermal inertia of the DHN and buildings, adjustable heat loads, and P2H devices (the EB and the HP). Different constraints are established for the EB and the HP, respectively, in the optimization model.

2. Given the actual situation of central heating in Northern China, an equivalent model of the heating system is formulated to express the direct mathematical relations between the heat source and heat load. Key factors of the modeling, including the transmission delay, transmission loss, and
dynamic temperatures of the hot water in pipelines, are concerned. Complicated details are simplified reasonably, which is beneficial for introducing the heating system model into the scheduling model of the electric power system.

3. A self-repair method incorporated with the real-coded genetic algorithm (RCGA) is proposed for solving the optimal scheduling problem of the CHPS. All the constraints can be satisfied effectively by the proposed method.

4. Three cases are carried out to verify the effectiveness of the solving algorithm, as well as comparing the impacts of the P2H devices (the EB and the HP) on the operation of the CHPS.

The remainder of this paper is organized as follows: In Section 2, an equivalent model of the heating system is formulated. In Section 3, the optimization model is established for the dispatch of the CHPS, and the proposed solving method is developed in Section 4. Section 5 carries out numerical case studies. Finally, in Section 6, the conclusions are given out.

2 | EQUIVALENT MODEL OF THE DISTRICT CENTRAL HEATING SYSTEM

A typical district central heating system is composed of six parts: heat sources, the DHN, heat stations, heat exchangers, water pumps, and heat consumers, as shown in Figure 1. For the sake of brevity, heat exchangers and water pumps are not displayed. The thermal power output of heat sources is transported to heat stations by the primary pipelines of the DHN and distributed to heat consumers by the secondary pipelines. It is notable that modeling of the secondary pipelines is out of the scope here for the following two factors: (I) Thermal inertia of the DHN is embodied in transmission delay which mitigates the tight relation between supply and demand of thermal power, and the transmission delay is mainly determined by the length of the primary pipelines because the primary pipelines are much longer than the secondary pipelines.16 (II) The topology of the secondary pipelines is too intricate to be modeled owing to the widely distributed buildings.35 Hence, the equivalent model of the district central heating system consists of heat sources, the primary pipelines, and heat stations.

2.1 | Heat sources

2.1.1 | Coal-fired CHP units

The coal-fired CHP units are usually classified into two types: extraction-condensing units and back-pressure units. Since most of the CHP units belong to the former type in Northern China, this kind of CHP is the dominant heat source in the heating system. The CHP unit is operated within the region as shown in Figure 2, and the constraints of the generation are expressed as:

\[
\begin{align*}
0 & \leq P_{i,t}^{CHP} \leq P_{\max,i}^{CHP} \\
0 & \leq Q_{i,t}^{CHP} \leq Q_{\max,i}^{CHP} \\
C_{\min,i} & = \max \left\{ c_i - k_{DC}^{CHP} \cdot Q_{i,t}^{CHP}, k_{BC}^{CHP} \cdot Q_{i,t}^{CHP} + b_i \right\} \\
C_{\max,i} & = a_i - k_{AB}^{CHP} \cdot Q_{i,t}^{CHP}
\end{align*}
\]

where \( t \) is the index of scheduling time interval; \( P_{i,t}^{CHP} \) and \( Q_{i,t}^{CHP} \) are the electric power and thermal power produced by the \( i \)th CHP unit at time \( t \), respectively; \( P_{\max,i}^{CHP} \) and \( P_{\min,i}^{CHP} \) are the maximum and minimum electric power outputs; \( Q_{\max,i}^{CHP} \) is the maximum thermal power output; \( a_i, k_{AB}^{CHP}, b_i, k_{BC}^{CHP}, c_i, \) and \( k_{DC}^{CHP} \) are the expression coefficients of the segments AB, BC, and DC in Figure 2.

2.1.2 | EB

In the CHPS, the EBs can be utilized to consume the surplus wind power by converting electric energy into thermal
energy. The thermal power produced by the EB is proportional to the consumed electric power:

$$Q_{tEB}^E = \beta_{EB} \cdot P_{tEB}$$

$$0 \leq P_{tEB} \leq P_{tEB}^{max}$$ (2)

### 2.1.3 | HP

Except for the EB, the HP is another promising candidate to enhance the wind power integration. When there is a great demand for central heating, the HP can be activated by electric power to produce thermal power. The model of the HP is expressed as:

$$Q_{tHP}^E = \beta_{HP} \cdot P_{tHP}$$

$$0 \leq P_{tHP} \leq P_{tHP}^{max}$$ (3)

where $P_{tEB}$ and $P_{tHP}$ are the electric power consumed by the EB and the HP, respectively; $Q_{tEB}^E$ and $Q_{tHP}^E$ are the produced thermal power; $P_{tEB}^{max}$ and $P_{tHP}^{max}$ are the maximum power capacities of the EB and the HP, respectively; $\beta_{EB}$ and $\beta_{HP}$ are the average annual coefficients of performance for the EB and HP, respectively.

### 2.2 | District heating network

The DHN in this work is operated at the quality regulation mode which is the most widely used mode in China. It means that the quantity of the mass flow keeps constant with the decreasing the water temperature is regulated for following the change of thermal demand. Under the quality regulation mode, the hydraulic conditions always remain stable, and the electric power consumed by the water pump is almost constant.

A schematic diagram of the DHN is shown in Figure 3, where each heat station represents a heat load. The heat station is denoted as $H_m (m = 1, \ldots, M)$. According to the engineering design of the DHN, supply pipelines and return pipelines are buried underground symmetrically. Sections of the supply and the return pipeline are denoted as $L^S_n$ and $L^R_n (n = 1, \ldots, 2M)$, respectively, and the nodes of the pipes are denoted as $S$ and $R$.

#### 2.2.1 | Transmission time

The thermal energy from the heat source is distributed to heat users by hot water in the supply pipelines. It takes a long time for the water flowing from the heat source to the consumers and returning back. The transmission time spent in a segment of the supply pipeline is expressed as:

$$\Delta t^S_n = \frac{\rho^w l^S_n A^S_n}{G^S_n} \quad n = 1, \ldots, 2M$$ (4)

where $\rho^S$ is the density of the hot water; $l^S_n$ and $A^S_n$ are the length and sectional area of the pipe $L^S_n$, respectively; $G^S_n$ is the mass flow rate in the pipe $L^S_n$; $\Delta t^S_n$ is the transmission time.

The supply and return pipelines, which are placed in parallel, have the same characteristics, such as the length, the sectional area, and the material. Therefore, the time spent in the return pipelines equals to that in the supply pipelines:

$$G^R_n = G^S_n$$

$$\Delta t^R_n = \Delta t^S_n$$ (5)

where $G^R_n$ and $\Delta t^R_n$ are the mass flow rate and the transmission time in the pipe $L^R_n$.

#### 2.2.2 | Dynamic temperature

The thermal energy from the heat source is consumed by heat users and wasted in transmission, leading to the variation of the water temperature in the pipelines. The variation of the water temperature is expressed as:

$$\Delta T^S_n = \frac{\Delta Q^S_n}{G^S_n \cdot c_w^S}$$ (6)

$$\Delta T^R_n = \frac{\Delta Q^R_n}{G^R_n \cdot c_w^R}$$ (7)
\[
\Delta T^H_m = \frac{h_m}{C^w_{2m} c^w} \tag{8}
\]

where \(\Delta T^S_n\), \(\Delta T^R_n\), and \(\Delta T^H_m\) are the temperature differences between the inlet and the outlet sides of the supply pipe \(L^S_n\), the return pipe \(L^R_n\), and the \(m\)th heat station, respectively; \(G^S_{2m}\) is the mass flow rate in the supply pipe connected with the \(m\)th heat station; \(c^w\) is the specific heat capacity of hot water; \(h_m\) is the heat load; \(\Delta Q^S_n\) and \(\Delta Q^R_n\) are the transmission losses in the \(n\)th supply pipe and return pipe. The transmission loss in the \(n\)th pipeline is assumed to be the product of the average loss coefficient and the length of the pipeline:

\[
\Delta Q^S_n = \lambda^S L^S_n \tag{9}
\]
\[
\Delta Q^R_n = \lambda^R L^R_n
\]

where \(\lambda^S\) and \(\lambda^R\) are the average loss coefficients of the supply and return pipelines, respectively.

In the preliminary studies, the scholars generally formulate the dynamic temperatures of each node. This method introduces significant intermediate variables into the dispatch model, increasing the difficulty in solving process. In fact, only the temperature difference and time difference between the supply and return water at the source side are necessary for the operation of the CHPS. In accordance with the energy conservation law, the relationship between the supply and return water temperatures can be formulated as a nonlinear function of the heat loads. Detailed derivation of the Equation (10) is given in Appendix A.

\[
\left( \sum_{m=1}^{M} G^S_{2m} T^S_{1_{m},T-\Delta t_n} - G^S_{1_{m}} T^S_{1_{m},T} \right) \cdot c^w = \sum_{m=1}^{M} h_{m,T-\Delta t_n} \\
+ \sum_{n=1}^{2M} \left( \Delta Q^S_n + \Delta Q^R_n \right) T^S_{1_{m},T} \leq T^S_{\text{max}}, \quad T^S_{1_{m},T} \geq T^R_{1_{m},T} \tag{10}
\]

\[
\alpha_m = \frac{\Delta T^S_{1_{m},T} + \Delta T^R_{1_{m},T} + \sum_{n=1}^{m-1} \Delta T^S_{2m+1} + \sum_{n=1}^{m+1} \Delta T^R_{2m+1}}{\Delta t} \tag{11}
\]

where \(T^S_{1_{m},T}\) and \(T^R_{1_{m},T}\) are temperatures of the supply and return water at the source side. \(\alpha_m\) is the total time spent by the supply water moving from the heat source to the \(n\)th heat load, and it is expressed as the integral multiple of the scheduling time interval \(\Delta t\); \(T^S_{\text{max}}\) is the maximum water temperature allowed by the heating pipeline.

### 2.3 Heat load

In practice, the heat transfer process of building envelopes is very complex in terms of air convection, heat conduction, and radiation. The intact model of the building is too complicated to be applied to the optimal operation of the CHPS. In this paper, we only focus on the changes of indoor temperatures at the beginning and end of each discrete dispatch period. Then, the modeling of the heat load can be simplified.

The transient heat balance equation of a room is expressed as:

\[
\frac{\partial T^\text{in}}{\partial t} = \frac{h - (T^\text{in} - T^\text{out}) \cdot K \cdot A}{c^\text{air} \rho^\text{air} V} \tag{12}
\]

where \(T^\text{in}\) and \(T^\text{out}\) are indoor and outdoor temperatures, respectively; \(K, A, \text{ and } V\) are the average heat conductivity, external surface area, and total volume of a room, respectively; \(c^\text{air}\) and \(\rho^\text{air}\) are the specific heat capacity and density of air; \(h\) is the thermal power supplied by the heating system.

The indoor temperature changes very slowly because of the thermal inertia of building’s maintenance structure. We assumed that during a scheduling time interval, the indoor and out temperatures remain unchanged. Thus, the Equation (12) can be converted to a discrete equation\(^{(38,39)}:\)

\[
T^\text{in}_{t+1} = \frac{h_t}{K \cdot A} + \left( T^\text{in}_{t-1} - \frac{h_t}{K \cdot A} \right) \chi \\
\chi = \exp \left( -\frac{K\cdot A}{c^\text{air}\rho^\text{air}V} \cdot \Delta t \right) \tag{13}
\]

All the buildings served by the same heat station are abstracted into a large room by assuming that indoor temperatures of buildings in the same area have the same variation. Then, from the Equation (13), the thermal power supplied by the \(m\)th heat station can be expressed as the function of the indoor and outdoor temperatures:

\[
h_{m,T} = \frac{T^\text{in}_{m,T} - T^\text{out}}{1 - \chi_m} \cdot K_m A_m \\
\chi_m = \exp \left( -\frac{K_m A_m}{c^\text{air}\rho^\text{air}V_m} \cdot \Delta t \right) \\
T^\text{in}_{\text{min}} \leq T^\text{in}_{m,T} \leq T^\text{in}_{\text{max}}
\]

where \(T^\text{in}_{\text{min}}\) and \(T^\text{in}_{\text{max}}\) are the minimum and maximum indoor temperatures.

### 3 Optimal Operation Model of the CHPS

A simplified structure of the CHPS is presented in Figure 4. The interaction between the electric power system and the heating system is mainly dependent on the CHP units and
the P2H devices. In this work, only one kind of the P2H devices (the EB and the HP) is employed at a time in view of their similar acts. In general, the HP is usually installed at the demand side due to its limited supply temperature (about only up to 85°C), whereas the EB does not have such a limit of location. For the convenience of centralized control, the EB is usually located at the supply side. In addition to the above members, wind farms (WF), coal-fired condensing (CON) units, and other (OTH) peak-load regulation units, such as hydropower units, are included in the CHPS for power supply.

### 3.1 | Objective function

The objective of the dispatch is to minimize the coal cost of the CHP units and CON units. In order to improve the wind power integration, penalty cost of the curtailed wind energy is added to the objective function.

\[
\min C_{\text{coal}} = C_{\text{CHP}}^{\text{coal}} + C_{\text{CON}}^{\text{coal}} + C_{\text{pen}}^{\text{WF}}
\]

\[
C_{\text{CHP}}^{\text{coal}} = \sum_{i=1}^{N_t} \sum_{j=1}^{N_1} \left( k_{1,j}^{\text{CHP}} P_{i,j}^{\text{CHP}} + k_{2,j}^{\text{CHP}} P_{i,j}^{\text{TH}} + k_{3,j}^{\text{CHP}} P_{i,j}^{\text{TH}} \right) + C_{\text{pen}}^{\text{WF}}
\]

\[
C_{\text{CON}}^{\text{coal}} = \sum_{i=1}^{N_t} \sum_{j=1}^{N_2} \left( k_{1,j}^{\text{CON}} P_{i,j}^{\text{CON}} + k_{2,j}^{\text{CON}} P_{i,j}^{\text{TH}} + k_{3,j}^{\text{CON}} P_{i,j}^{\text{TH}} \right) \Delta t
\]

\[
C_{\text{pen}}^{\text{WF}} = \sum_{i=1}^{N_t} \left( P_{\text{gen},i}^{\text{WF}} - P_{\text{int},i}^{\text{WF}} \right) \Delta t
\]

where \( C_{\text{coal}} \) is the total coal cost; \( C_{\text{CHP}}^{\text{coal}} \) and \( C_{\text{CON}}^{\text{coal}} \) are the coal costs of CHP units and CON units during a scheduling period; \( C_{\text{pen}}^{\text{WF}} \) is the penalty cost of wind power curtailment; \( P_{i,j}^{\text{CON}} \) is the power output of the \( j \)th CON unit; \( k_{1,j}^{\text{CHP}} \), \( k_{2,j}^{\text{CHP}} \), \( k_{3,j}^{\text{CHP}} \), \( k_{1,j}^{\text{CON}} \), \( k_{2,j}^{\text{CON}} \), and \( k_{3,j}^{\text{CON}} \) are the generation cost coefficients of the \( j \)th CHP unit; \( k_{1,i}^{\text{CON}} \), \( k_{2,i}^{\text{CON}} \), and \( k_{3,i}^{\text{CON}} \) are the coefficients of the \( j \)th CON unit \( i \); \( P_{\text{gen},i}^{\text{WF}} \) and \( P_{\text{int},i}^{\text{WF}} \) are the generated and integrated wind power; \( \Delta t \) is the penalty cost coefficient; \( N_t \) is the last scheduling time interval.

### 3.2 | Constraints

The objective function should subject to the constraints of the electric power system and the heating system.

1. **Application of the EB**: The EB is installed at the heat source side for centralized control, and the electric power balance is:

\[
\sum_{i=1}^{N_1} p_{i,j}^{\text{CHP}} + \sum_{j=1}^{N_2} p_{j}^{\text{CON}} + p_{\text{OTH}} + p_{\text{int},i}^{\text{WF}} = P_{\text{e},i} + P_{\text{EB}}
\]

The thermal power balance is:

\[
\sum_{i=1}^{N_1} Q_{i,j}^{\text{CHP}} + Q_{i,j}^{\text{EB}} = G_i^s e^{\beta} \left( T_{1,i}^S - T_{1,i}^R \right) = Ph_i
\]

\[
Ph_i = \sum_{m=1}^{M} h_{m,t} + \sum_{n=1}^{2M} \left( \Delta Q_{n}^S + \Delta Q_{n}^R \right)
\]

where \( N_1 \) and \( N_2 \) are the amounts of CHP units and CON units, respectively; \( P_{\text{e},i} \) is the power produced by other generators; \( P_{\text{e},i} \) and \( Ph_i \) are the total electric power load and thermal power load.
2. Application of the HP: Each heat station is equipped with an HP for auxiliary heating. The electric power balance is:

$$\sum_{j=1}^{N_1} P_{CHP,ij} + \sum_{j=1}^{N_2} P_{CON,ij} + P_{OTH, t} + P_{WF, t} = P_e + \sum_{m=1}^{M} P_{HP, mt}$$

(18)

The thermal power balance is:

$$\sum_{j=1}^{N_1} Q_{CHP,ij}^{cw} = G_{i}^{w} (T_{s,ij}^{S} - T_{r,ij}^{R}) = P_{h, t}$$

(19)

Generation output constraints are:

$$P_{CON, min, j} \leq P_{CON, j, t} - P_{CON, j, t+1} \leq P_{CON, max, j}$$

$$P_{OTH, min} \leq P_{OTH, t} - P_{OTH, t+1} \leq P_{OTH, max}$$

$$0 \leq P_{WF, min} \leq P_{WF, t} \leq P_{WF, max}$$

(20)

Ramp rate constraints are:

$$-U_{CHP} \leq P_{CHP, j, t} - P_{CHP, j, t+1} \leq D_{CHP}$$

$$-U_{CON} \leq P_{CON, j, t} - P_{CON, j, t+1} \leq D_{CON}$$

$$-U_{OTH} \leq P_{OTH, t} - P_{OTH, t+1} \leq D_{OTH}$$

(21)

where $P_{CON, min}$, $P_{CON, max}$, $P_{OTH, min}$, and $P_{OTH, max}$ are the output bounds of the CON units and OTH units; $U$ and $D$ represent the upward and downward ramping limits of the units.

Other constraints of the electric power production are defined in Equations (1-3), and the constraints of the DHN and the heat load are defined in Equations (10), (11), and (14).

4 | Solution strategy

The RCGA is applied to solve the nonlinear optimization problem in this section. Traditionally, penalty approach is used for handling of the complex constraints. However, the constraints, especially the equality constraints, are not satisfied strictly by this approach. Aiming at this problem, we propose a self-repair method incorporated with the RCGA.

4.1 | Solving process

The solving process is comprised of population initialization, fitness calculation, roulette wheel selection, arithmetic crossover, uniform mutation, and self-repair program. In a population, the individual represents a dispatch plan of the generators during a scheduling period, and it is coded by a real matrix:

$$P = [P_{1, 1}; \ldots; P_{1, i}; \ldots; P_{N, 1}; \ldots; P_{N, t}; \ldots; P_{N, N}]$$

(22)

The fitness value of the individual is derived by:

$$Fit(P) = A_{fit} - C_{coal}(P)$$

(23)

where $A_{fit}$ is the scaling factor of the fitness value.

The flow chart of the solving process is displayed in Figure 5. The initialized power outputs of the generators are adjusted by the self-repair method to satisfy all the constraints. After finishing the steps of fitness calculation, crossover, mutation, and selection, the parent generation is prepared. Thereupon, an iteration, is completed. The solving process will not be terminated until the number of iterations, $g$, reaches the threshold value, $N_g$. More information about the critical steps of the RCGA can be found in reference. Details of the self-repair method are explained in the following section.

4.2 | Self-repair method

The initialized power outputs of the generators are modified to satisfy the electric and thermal power constraints. It is notable that the thermal power has the priority to be settled as the CHP units are operated at the “heat-led” mode. The flow chart of the self-repair method is shown in Figure 6, and the steps are described as follows.

Initialization: Let $t = 0$. Calculate the initial value of the total heat load (denoted as $P_{h, 0}$) by substituting the indoor temperature ($T_{m}^{in}$ = 20°C) into Equations (14) and (17). The total heat load is dispatched to the CHP units according to their capacities:

$$Q_{0, ij}^{CHP} - 0 = \frac{Q_{CHP, max, j}}{\sum_{j=1}^{N_1} Q_{CHP, max, j}} \cdot P_{h, 0}$$

(24)

where $Q_{ij}^{CHP, 0}$ is the initial thermal power of the $i$th CHP unit.

Step 1: Let $t=t+1$. The upper and lower limits of the generators at time $t$ are determined by the generation output constraints and ramp rate limits:

$$P_{min, CHP, i, t} = \max \left( P_{CHP, min, i, t} - P_{CHP, max, i, t} - D_{CHP} \right)$$

$$P_{min, CON, j, t} = \max \left( P_{CON, min, j, t} - P_{CON, max, j, t} - D_{CON} \right)$$

$$P_{max, CHP, i, t} = \min \left( P_{CHP, max, i, t} + U_{CHP} \right)$$

$$P_{max, CON, j, t} = \min \left( P_{CON, max, j, t} + U_{CON} \right)$$

(25)

where $P_{max, CHP, i, t}$, $P_{max, CON, j, t}$, $P_{min, CHP, i, t}$, and $P_{min, CON, j, t}$ are the real-time upper and lower limits of the CHP and CON units.
Then, the original electric power of each unit is randomly generated with the limits in Equation (25), and they are denoted as $P_{CHP_i,t-0}$, $P_{CON_j,t-0}$, and $P_{OTH_{t-0}}$.

Step 2: A criterion, denoted as $delta_{WF}$, is derived by Equation (26) to judge whether the generated wind power is surplus or not.

$$delta_{WF} = \sum_{i=1}^{N1} P_{minCHP_i,t} + \sum_{j=1}^{N2} P_{minCON_j,t} + P_{CON_{min}} + P_{WF_{gen,t}} - P_{e_t}$$  \hspace{1cm} (26)$$

If $delta_{WF} > 0$, wind power is surplus, then go to Step 3_1. If $delta_{WF} < 0$, no wind power is wasted, then go to Step 3_2.

Step 3_1: At this step, the power outputs of the generators are regulated and the EB/HP is activated to consume the surplus wind power. In order to control the operating power of the EB/HP, three factors have to be considered:

1. A part of the thermal power from the CHP units is replaced by that from the EB/HP. As regulation of the CHP units should subject to the operating limits in Equations (1), (20), and (21), the allowed reduction of the CHP’s thermal power is constrained by:

$$allow_{CHP_{i,t}} = \min \left( \frac{D_i^{CHP}}{k_i^{BC}}, \frac{P_{minCHP_{i,t}} - P_{CHP_{min,i,t}}}{k_i^{BC}} \right)$$  \hspace{1cm} (27)$$

2. The required thermal power from the EB/HP is constrained by the demand of wind power integration. If all the wind energy is expected to be consumed, the required thermal power from the EB/HP is given by:

$$\sum_{i=1}^{N1} \left( P_{minCHP_{i,t}} - \frac{1}{Q_{max,i}} \frac{Q_{CHP_{max,i}} \cdot req_{EB}}{Q_{CHP_{max,i}} \cdot req_{EB}} \right) + \sum_{j=1}^{N2} P_{CON_{min,j,t}} + P_{OTH_{min,t}} = P_{e_t} + \frac{req_{EB}}{P_{gen,j}} - P_{WF_{gen,j}}$$  \hspace{1cm} (28)$$

where $req_{EB}$ is the required thermal power from the EB, and the expression is the same for the HP.

3. In view of the installed capacity of the EB/HP, the available thermal power from the EB/HP is determined by:

$$\text{for the EB: } avail_{EB}^{t,EB} = \beta_{EB} \cdot P_{max}^{EB}$$

$$\text{for the HP: } avail_{HP}^{t,HP} = \beta_{HP} \cdot \sum_{m=1}^{M} P_{m,HP,\text{max}}$$  \hspace{1cm} (29)$$

where $P_{m,HP,\text{max}}$ is the maximum capacity of the HP installed at the $m$th heat station.

At last, the actual thermal power supplied by the EB/HP is derived by:
\[ Q_{\text{EB}}^{\text{sup}, j} = \min \left( \sum_{i=1}^{N_1} \text{allow}_{t_{i,t}, \text{EB}}^{\text{CHP}}, \text{req}_{t_{i,t}, \text{EB}}^{\text{CHP}}, \text{avail}_{t_{i,t}, \text{EB}}^{\text{CHP}} \right) \]
\[ Q_{\text{HP}}^{\text{sup}, j} = \min \left( \sum_{i=1}^{N_1} \text{allow}_{t_{i,t}, \text{HP}}^{\text{CHP}}, \text{req}_{t_{i,t}, \text{HP}}^{\text{CHP}}, \text{avail}_{t_{i,t}, \text{HP}}^{\text{CHP}} \right) \] (30)

The electric power consumed by the EB/HP is given by:
\[ p_{EB}^t = \frac{Q_{EB}^{\text{sup}, j}}{\beta_{EB}} \] (31)
\[ p_{HP}^t = \frac{Q_{HP}^{\text{sup}, j}}{\beta_{HP}} \]

Finally, at step 3.1, the value of the total heat load \( P_{h,0} \) is updated to the \( P_{h,1} \) by Equations (17) and (19), and the output power of the generators is determined by:
\[ Q_{CHP}^{\text{sup}, t} = \frac{Q_{CHP}^{\text{max}, t}}{\sum_{i=1}^{N_1} Q_{CHP}^{\text{max}, i}} \cdot P_{h,1} \]
\[ p_{CHP}^{\text{sup}, t} = P_{\text{min}}^{\text{CHP}, t} - \epsilon_i \left( Q_{CHP}^{\text{max}, t} - Q_{CHP}^{\text{sup}, t} \right) \] (32)
\[ p_{CON}^{\text{sup}, t} = P_{\text{min}}^{\text{CON}, t} \]
\[ p_{OTH}^{\text{sup}, t} = P_{\text{min}}^{\text{OTH}, t} \]

Step 3.2: Let \( PWF_{int,t} = PWF_{gen,t} \), the randomly generated power of the units has to be modified to meet the power balance constraint. The difference between the power load and the generated power is defined as \( \Delta P_{e}^t \):
\[ \Delta P_{e}^t = P_{\text{gen},t} - \sum_{i=1}^{N_1} p_{CHP}^{\text{sup}, t} + \sum_{j=1}^{N_2} p_{CON}^{\text{sup}, t} - \sum_{j=1}^{N_2} p_{OTH}^{\text{sup}, t} - P_{e}^t \] (33)

The upward and downward regulating spaces of the CHP units are defined as \( spU_{t_{i,t}}^{\text{CHP}} \) and \( spD_{t_{i,t}}^{\text{CHP}} \):
\[ spU_{t_{i,t}}^{\text{CHP}} = P_{\text{max}}^{\text{CHP}, t} - p_{CHP}^{\text{sup}, t} - 0 \]
\[ spD_{t_{i,t}}^{\text{CHP}} = p_{CHP}^{\text{sup}, t} - 0 - P_{\text{min}}^{\text{CHP}, t} \] (34)

For the CON units and the OTH units, the expression of the regulating spaces is similar to Equation (34). If \( \Delta P_{e}^t > 0 \), the generators should reduce the electric power in the range of the regulating spaces until \( \Delta P_{e}^t = 0 \), and vice versa. The thermal power of the CHP unit remains the same. Finally, the value of the generated power is updated.

Step 4: Since the total heat load has been determined, the supply water temperature is given by:
\[ T_{1,t}^{S} = \frac{P_{h,1}}{G_{i}^{c,w}} - T_{1,t}^{R} \] (35)

Repeating step 1 to step 4 until \( t = N_t \), then the self-repair process is finished.

5  |  NUMERICAL SIMULATIONS

5.1  |  System description

A modified configuration of the CHPS is shown in Figure 7, consisting of a heating system with four heat stations and an electric system with six buses. There are two CHP units (300 MW and 200 MW, respectively), one CON unit (300 MW), one wind farm (190MW), and other units are equivalent to one generator (denoted as the OTH unit, 260MW). The installed capacities of the sources are set concerning the actual system data of the Jilin province in China. Specific parameters of the generators and the heating system are given in Tables B1-B4 in Appendix B. The optimal scheduling is carried out during a typical day of the heating season in four scenarios: the CHPS only considering the thermal inertia of the heating system, the CHPS with adjustable indoor temperature, the CHPS with the EB, and the CHPS with the HP. The scheduling time interval is set as 15 minutes.

Three cases are carried out in the following. In case I, the convergence of the proposed solving method is verified, and the optimal dispatch results of the CHPS are discussed. In case II, dispatch results of the CHPS are compared in terms of the power outputs, the integrated wind power, and the consumed coal. In case III, the performance of the EB and the HP with different capacities is investigated. The solving strategy is coded with the Matlab, and the parameters of the RCGA used in the program are listed in Table B5 in Appendix B.

5.2  |  Case I

The solving method is applied to the optimization problem in four different scenarios. The scenarios are described as:

Scenario (a): The CHPS is operated only considering the thermal inertia of the heating system, and the indoor temperature is maintained at 20°C.

Scenario (b): In consideration of the thermal comfort, the supplied thermal power is allowed to change on condition that the indoor temperature can be maintained within the range of 18-22°C.

Scenario (c): An EB with the capacity of 30 MW and the energy efficiency of 0.98 is installed at the CHP plant side.

Scenario (d): Each heat station is equipped with an HP. The total capacity of the HPs is 30MW, and the capacities of the distributed HPs are allocated according to the proportion of the heating area. The capacities are 7 MW, 8 MW, 6 MW, and 9 MW, respectively. All the HPs are operated with the energy efficiency of 3.
5.2.1 | Algorithm convergence

For better reliability, iteration curves of ten times repeated computations are shown in Figure 8. It can be seen that the solving algorithm tends to be convergent after 100 iterations. In each scenario, the iteration curves of the consumed coal are almost overlapped. Other results, such as the power dispatch, the wind power integration, are almost the same in the repeated computations, and they are not displayed on account of limited space. It can be convinced that the proposed solving strategy is robust for the optimization problem.

5.2.2 | Optimal dispatch of the original CHPS

The curve of the supplied heat power seems to move left relative to the heat demand curve in Figure 9A because the CHP units are planned to produce heat power earlier in consideration of the transmission delay. During the scheduling period, the electric power produced by the CHP units fluctuates slightly as the units are operated at the “heat-led” mode. Hence, only the CON and the OTH units can be regulated for accommodating wind power as shown in Figure 9B. In Figure 10, the supply water temperature at the source side changes along with the variation of the supplied heat power. The return water temperature at the source side changes in a small range as the supply water temperature plays a role in following the change of the heat demand. Curves of the integrated and the generated wind power are shown in Figure 11. In comparison with the consumed wind power without considering thermal inertia (2966.69 MWh), the consumed wind power (2977.13 MWh) is increased by 10.44 MWh when the thermal inertia of the heating system is utilized in the scheduling. However, during the time intervals of 1-19 and 87-96, much wind power has to be curtailed because the flexibility of the CHPS is still limited only relying on the energy storage of the heating system. Thus, the approaches introduced in scenarios (b)–(d) are exploited to further promote the integration of wind power.
5.3 | Case II

In the scenarios (b)–(d), operating flexibility of the CHP units is promoted by decoupling the joint production of electric power and thermal power. As shown in Figure 12, when the auxiliary approach is introduced into the CHPS, the electric and thermal power outputs of the CHP units in the scenarios (b)–(d) are possible to be reduced obviously during time intervals of 1-19 and 87-96.

Curves of the integrated wind power in different scenarios are shown in Figure 13. Ratios of the curtailed wind power to the generated wind power are listed in Table 1.
For the scenario (b), discrete changes of the indoor temperatures at the heat stations are displayed in Figure 14. As the transmission delay is up to five scheduling intervals from the heat source to the fourth heat station, 101 time intervals are shown in Figure 14. The maximum temperature is maintained at 20°C to reduce the coal consumption required by heat production when the wind power can be integrated totally. The indoor temperature at each station is decreased to 18°C with a delay of several time intervals for releasing the flexibility of the CHP units when the wind energy is abundant. Thus, in this scenario, the curtailed wind power is slightly less than that in the scenario (a).

For the scenarios (c)–(d), the operating power of the EB and the HPs are displayed in Figures 15 and 16. Owing to the transmission delay expressed in Equation (19), the distributed HPs are activated later than the EB. Thus, the integrated wind power in the scenario (d) is less than that in the scenario (c) during the time intervals of 1-3, which is marked in Figure 13. However, the HPs still have the best performance on reducing the total surplus wind power (the curtailed wind power ratio is only 2.46%) by virtue of the high efficiency.

Since the integrated wind power is increased, the output power of the coal-fired units is decreased, leading to the reduction of consumed coal. Results of the consumed coal gotten from the repeated computations are displayed in Figure 17. One can see that the consumed coal in the scenarios (b)–(d) is reduced in comparison with the scenario (a). Notably, although the wind power integration in the scenario (c) is much better than the scenario (b), the consumed coal is similar in these two scenarios. It is because that a considerable amount of electric energy is consumed by the EB (180.90 MWh) which is higher than that consumed by the HPs (146.68 MWh), leading to the extra coal consumption. In this respect of coal saving, the HP has the best performance.

### Table 1: Ratios of the curtailed wind power to the generated wind power

| Scenarios | a     | b     | c     | d     |
|-----------|-------|-------|-------|-------|
| Ratios    | 11.53%| 8.35% | 3.73% | 2.46% |

FIGURE 12 Power outputs of the CHP units

FIGURE 13 Curves of the integrated wind power in different scenarios

FIGURE 14 Discrete changes of indoor temperatures at the heat stations

FIGURE 15 Operating power of the EB
In this case, the capacities of the EB and the HP are set from 10 to 80 MW. The utilized wind energy increases along with the growth of installed capacity as presented in Figure 18A, and the consumed coal tends to decrease as drawn in Figure 18B. The required capacity of the HP is smaller than that of the EB owing to the high efficiency if the same amount of wind power is integrated. Furthermore, the operating power of the EB/HP is constrained by the allowable reduction of the CHP’s thermal power as expressed in Equation (30). For the HP, the capacity of 40 MW is big enough for auxiliary heating when the thermal power from the CHP units is reduced to the boundary. However, for the EB, the capacity larger than 40 MW is required.

In addition, the HP has a better performance on enhancing the utilization of wind energy when the installed capacity is smaller than the critical point (about 40 MW in this case) as shown in Figure 18A. After that, the EB is superior to the HP. The reason is that the electric energy consumed by the EB is much more than that by the HP if the capacity exceeds 40 MW, as shown in Figure 19. In other words, the total electric power load under the action of the EB is much bigger, contributing to more wind power integration. However, in respect to reducing the coal consumption, the HP is a better choice whatever the capacity is.

6 | CONCLUSIONS

This paper focuses on the optimal operation of the CHPS equipped with the P2H devices for improving the utilization of wind energy. The optimization model is constructed in terms of the coupling relationship between the electric and heating systems. In consideration of transmission delay, transmission loss, and dynamic temperatures, an equivalent model of the heating system is developed. The dynamic temperatures of the supply and return water are expressed as a function of heat loads for the convenience of integrating the heating system model into the scheduling model of the electric power system. A self-repair method incorporated with the RCGA is proposed to address the nonlinear optimization problem. The effectiveness of the solving method is verified in case studies, and several conclusions are drawn as follows:
1. Compared with the application of adjustable heat load which is constrained by the thermal comfort, employing the P2H devices has a better performance on enhancing the wind power integration and reducing the coal consumption.

2. The integrated wind power tends to increase along with the growth of the installed capacity of the EB/HP. However, the functional capacity of the EB/HP is constrained by the operating limits of the CHP units. Hence, it is necessary to design the optimal capacity of the P2H devices according to the configuration of the CHPS.

3. For a given system, there exists a critical capacity derived from the comparison between the EB and the HP. The HP is more effective in utilizing surplus wind power than the EB if the configured capacities of them are smaller than the critical point. Nevertheless, the EB with a capacity larger than the critical point is promising to surpass the HP in the aspect of more integrated wind power. For the saving of coal consumption, the HP is always superior to the EB whatever the capacity is.

Although the scale of the CHPS used in the case study is smaller than the actual system, the conclusions are still valid. Investigation of the optimal operation method applied to the actual system will be conducted in our further work.

ACKNOWLEDGMENTS

This work is supported by the National Key Technologies R&D Program of China under Grant No. 2018YFB0904800 and the Science and Technology Project of State Grid Corporation of China under Grant “Coordinating Development Model and Reliability Assessment of the Combination of Generation, Grid, Load and Storage Considering the Security Requirement of Power Grid.”

CONFLICT OF INTEREST

The authors declare no conflict of interest.

ORCID

Yanjuan Yu https://orcid.org/0000-0002-0018-2006

REFERENCES

1. Global Wind Energy Council (GWEC). Global Wind Reports. 2017; http://gwecc.net/publications/global-wind-report-2/
2. Zhang N, Lu XI, McElroy MB, et al. Reducing curtailment of wind electricity in China by employing electric boilers for heat and pumped hydro for energy storage. Appl Energy. 2016;184:987-994.
3. Yu Y, Chen H, Chen L. Comparative study of electric energy storages and thermal energy auxiliaries for improving wind power integration in the cogeneration system. Energies. 2018;11(2):263-278.
4. Ko W, Kim J. Generation expansion planning model for integrated energy system considering feasible operation region and generation efficiency of combined heat and power. Energies. 2019;12(2):226.
5. Yao S, Gu W, Zhou S, Lu S, Wu C, Pan G. Hybrid timescale dispatch hierarchy for combined heat and power system considering the thermal inertia of heat sector. IEEE Access. 2018;6:63033-63044.
6. Michal T, Sekret R. Buildings and a district heating network as thermal energy storages in the district heating system. Energy Build 2018;179:49-56.
7. Nuyten T, Claessens B, Paredis K, Van Bael J, Six D. Flexibility of a combined heat and power system with thermal energy storage for district heating. Appl Energy. 2013;104:583-591.
8. Zhao S, Ge Z, Sun J, Ding Y, Yang Y. Comparative study of flexibility enhancement technologies for the coal-fired combined heat and power plant. Energy Convers Manage. 2019;184:15-23.
9. Mollenhauer E, Christidis A, Tsatsaronis G. Increasing the flexibility of combined heat and power plants with heat pumps and thermal energy storage. J Energy Res Technol. 2018;140(2);020907.
10. Pan Z, Guo Q, Sun H. Interactions of district electricity and heating systems considering time-scale characteristics based on quasi-steady multi-energy flow. Appl Energy. 2016;167:230-243.
11. Zheng J, Zhou Z, Zhao J, Wang J. Integrated heat and power dispatch truly utilizing thermal inertia of district heating network for wind power integration. Appl Energy. 2018;211:865-874.
12. Romanchenko D, Kensby J, Odenberger M, Johnsson F. Thermal energy storage in district heating: centralised storage vs. storage in thermal inertia of buildings. Energy Convers Manage. 2018;162:26-38.
13. Li Z, Wu W, Wang J, Zhang B, Zheng T. Transmission-constrained unit commitment considering combined electricity and district heating networks. IEEE Trans Sustain Energy. 2016;7(2):480-492.
14. Li Z, Wu W, Shahidehpour M, Wang J, Zhang B. Combined heat and power dispatch considering pipeline energy storage of district heating network. IEEE Trans Sustain Energy. 2016;7(1):12-22.
15. Zhang L, Luo Y. Combined heat and power scheduling: Utilizing building-level thermal inertia for short-term thermal energy storage in district heat system. IEEE Trans Elect Electron Eng. 2018;13(6):804-814.
16. Gu W, Wang J, Lu S, Luo Z, Wu C. Optimal operation for integrated energy system considering thermal inertia of district heating network and buildings. Appl Energy. 2017;199;234-246.
17. Wu C, Gu W, Jiang P, et al. Combined economic dispatch considering the time-delay of a district heating network and multi-regional indoor temperature control. IEEE Trans Sustain Energy. 2018;9(1):118-127.
18. Wang D, Zhi Y-Q, Jia H-J, et al. Optimal scheduling strategy of district integrated heat and power system with wind power and multiple energy stations considering thermal inertia of buildings under different heating regulation modes. Appl Energy. 2019;240:341-358.
19. Zheng J, Zhou Z, Zhao J, Wang J. Effects of the operation regulation modes of district heating system on an integrated heat and power dispatch system for wind power integration. Appl Energy. 2018;230:1126-1139.
20. Chen X, Kang C, O’Malley M, et al. Increasing the flexibility of combined heat and power for wind power integration in China: Modeling and implications. IEEE Trans Power Syst. 2015;30(4):1848-1857.
21. Rinne S, Syri S. The possibilities of combined heat and power production balancing large amounts of wind power in Finland. Energy. 2015;82:1034-1046.
22. Hu K, Chen L, Chen Q, et al. Phase-change heat storage installation in combined heat and power plants for integration of renewable energy sources into power system. Energy. 2017;124:640–651.

23. Dai Y, Chen L, Min Y, et al. Integrated dispatch model for combined heat and power plant with phase-change thermal energy storage considering heat transfer process. *IEEE Trans Sustain Energy*. 2018;9(3):1234–1243.

24. Han X, Wang F, Tian C, Xue K, Zhang J. Economic evaluation of actively consuming wind power for an integrated energy system based on game theory. *Energies*. 2018;11(6):1476–1500.

25. Bloess A, Schill WP, Zerrahn A. Power-to-heat for renewable energy integration: A review of technologies, modeling approaches, and flexibility potentials. *Appl Energy*. 2018;212:1611–1626.

26. Yang L, Zhang X, Gao P. Research on heat and electricity coordinated dispatch model for better integration of wind power based on electric boiler with thermal storage. *IET Trans Sustain Energy*. 2018;12(15):3736–3743.

27. Blake MB. Towards an intermittency-energy friendly system: comparing electric boilers and heat pumps in distributed cogeneration. *Appl Energy*. 2012;91(1):349–365.

28. Nielsen MG, Morales JM, Zunino M, Pedersen TE, Madsen H. Economic valuation of heat pumps and electric boilers in the Danish energy system. *Appl Energy*. 2016;167:189–200.

29. Hao J, Chen Q, He K, et al. A heat current model for heat transfer/storage systems and its application in integrated analysis and optimization with power systems. *IEEE Trans Sustain Energy*. 2018;1:1–10.

30. Rong S, Li Z, Li W. Investigation of the promotion of wind power consumption using the thermal-electric decoupling techniques. *Energies*. 2015;8:8613–8629.

31. Yang Y, Yan G, Mu G. Integrated heating load and storage control in combined heat and power plant with phase-change thermal energy storage. *Int J Energy Res*. 2017;42(11):3477–3495.

32. Salpakari J, Mikkola J, Lund PD. Improved flexibility with large-scale variable renewable power in cities through optimal demand side management and power-to-heat conversion. *Energy Convers Manage*. 2016;126:649–661.

APPENDIX A

When the water from different pipes flows into the same node, the water temperatures are mixed, and this kind of node is defined as a mixed-flow node in this work. Under the energy conservation law, the thermal energy flows into the same mixed-flow node equals to the energy flows out. Taking the mixed-flow node $R_{2M-2}$ at the end of the DHN in Figure 3 as an example:

\[
G^R_{2M-1} \left( TH - \frac{\Delta Q^S_{2M-1}}{G^S_{2M-1}c_v} - \frac{\Delta Q^R_{2M-1}}{G^R_{2M-1}c_v} \right) + G^R_{2M-2} \left( TH - \frac{\Delta Q^S_{2M-2}}{G^S_{2M-2}c_v} - \frac{\Delta Q^R_{2M-2}}{G^R_{2M-2}c_v} \right) = G^R_{2M-3} \Delta Q^R_{2M-3} (A1)
\]

where $TH$, $TM–1$, and $TRmix$ are the temperatures of the water flowing out from the $M$th and $M–1$th heat stations, and the mixed-flow node $R_{2M-2}$.

Similarly, for other mixed-flow nodes $R_{2m}$ ($m = 1, ..., M–2$), we get the equations as following:

\[
G^R_{2m} \left( TH - \frac{\Delta Q^S_{2m}}{G^S_{2m}c_v} - \frac{\Delta Q^R_{2m}}{G^R_{2m}c_v} \right) + G^R_{2m+1} \left( TH - \frac{\Delta Q^S_{2m+1}}{G^S_{2m+1}c_v} - \frac{\Delta Q^R_{2m+1}}{G^R_{2m+1}c_v} \right) = G^R_{2m+2} \Delta Q^R_{2m+2} (A2)
\]

Then, the temperature of the mixed water out from the node $R_{2m}$ is expressed as:

\[
TRmix = \frac{G^R_{2m+1} \Delta Q^R_{2m+1}}{G^R_{2m+1}} (A3)
\]

Considering the total energy loss occurs in the pipelines from the heat source to the $m$th heat load and the energy consumed by heat users, the $TH$ is given by:

\[
TH = T^S_1 - \frac{\Delta Q^S_1}{G^S_1c_v} - \frac{\Delta Q^S_{2m}}{G^S_{2m}c_v} - \frac{\Delta Q^R_{2m+1}}{G^R_{2m+1}c_v} \sum_{n=1}^{m-1} \frac{\Delta Q^S_{2n+1}}{G^S_{2n+1}c_v} (A4)
\]
On the basis of Equations (A1-A4), the TRmix 2 can be derived as:

\[
T_{\text{mix}}^2 = \frac{\sum_{m=1}^{M} G_s^{2m} \cdot T_s^1}{G_1^*} - \frac{\sum_{m=1}^{M} h_m - \sum_{m=1}^{2M} \Delta Q_{n}^s + \sum_{m=1}^{2M} \Delta Q_{n}^r}{G_1^* \cdot c_w}
\]  

(A5)

Considering the heat loss in the last return pipeline, LR 1, the value of the return water temperature is given by:

\[
T_{r}^1 = \frac{\sum_{m=1}^{M} G_s^{2m} \cdot T_s^{1-t_2 \alpha m} - \sum_{m=1}^{M} h_m - \sum_{m=1}^{2M} \Delta Q_{n}^s + \sum_{m=1}^{2M} \Delta Q_{n}^r}{G_1^* \cdot c_w}
\]  

(A6)

Owing to the transmission delay, the time spent by the hot water flowing from the different nodes (R_{2m+1}, m = 1,2,...M) at the heat station to the node (R_{1}) at the end of the return pipeline is different. It means that at time \( t \), the water at the node R_{1} is mixed by a part of the water flows from the heat source at \( t-2 \alpha_m \) time intervals ago, a part of the water flows from the heat source at \( t-2 \alpha_2 \) time intervals ago,..., a part of the water flows from the heat source at \( t-2 \alpha_m \) time intervals ago, and the water flows from the heat source at \( t-2 \alpha_m \) time intervals ago. Thus, the water temperature at node R_{1} is influenced by the supply water temperatures \( T_{1,t_1-t_2 a_m} \),... \( T_{1,t_1-t_2 a_m} \), and the heat loads \( h_{1,t_1-t_2 a_m} \),... \( h_{1,t_1-t_2 a_m} \). Thus, by integrating the time delay into Equation (A6), the return water temperature at the source side is expressed as a function of the supply water temperature and heat loads with the consideration transmission delay:

\[
T_{1,t}^r = \frac{\sum_{m=1}^{M} G_s^{2m} \cdot T_s^{1-t_2 a_m} - \sum_{m=1}^{M} h_m - \sum_{m=1}^{2M} \Delta Q_{n}^s + \sum_{m=1}^{2M} \Delta Q_{n}^r}{G_1^* \cdot c_w}
\]  

(A7)

Besides, since the temperature of the mass flow declines gradually from the first node to the end node in the pipelines, the temperature of the first node should be limited by the maximum for the guarantee that the temperatures of all the nodes are within the bound.

\[
T_{1,t}^s \leq T_{\text{max}}^s, T_{1,t}^s > T_{1,t}^r
\]  

(A8)

### APPENDIX B

#### TABLE B1 Parameters of the generators

| Type  | \( k_1 \) | \( k_2 \) | \( k_3 \) | \( k_4 \) | \( k_5 \) | \( k_6 \) |
|-------|---------|---------|---------|---------|---------|---------|
| CHP1  | 7.6e-5  | 0.2716  | 18.822  | 4.0e-6  | 0.0625  | 3.5e-5  |
| CHP2  | 1.7e-4  | 0.2705  | 11.537  | 7.5e-6  | 0.0568  | 7.2e-5  |
| CON   | 7.6e-5  | 0.2716  | 18.823  | —       | —       | —       |

#### TABLE B2 Operating limits of the generators

| Type  | \( P_{\text{max}}/\text{MW} \) | \( P_{\text{min}}/\text{MW} \) | \( U/\text{MW/h} \) | \( D/\text{MW/h} \) |
|-------|----------------|----------------|----------------|----------------|
| CHP1  | 323            | 150            | 100            | 100            |
| CHP2  | 210            | 100            | 80             | 80             |
| CON   | 300            | 150            | 100            | 100            |
| OTH   | 260            | 130            | 260            | 260            |

#### TABLE B3 Parameters of the DHN

| Node   | \( G^s/\text{kg/s} \) | \( \ell/\text{m} \) | \( A^y/\text{m}^2 \) | \( \Delta x^y/\text{s} \) |
|--------|----------------|----------------|----------------|----------------|
| S_1→S_2| 2904           | 3600           | 1.13           | 1402.04        |
| S_2→S_4| 673            | 800            | 0.38           | 457.47         |
| S_2→S_3| 2231           | 2000           | 1.13           | 1013.87        |
| S_1→S_3| 673            | 500            | 0.38           | 285.92         |
| S_1→S_6| 158            | 2000           | 0.79           | 1008.22        |
| S_1→S_7| 673            | 500            | 0.38           | 285.92         |
| S_6→S_9| 885            | 1500           | 0.64           | 1078.26        |

#### TABLE B4 Parameters of the heat load

| \( A_x/10^6 \text{ m}^2 \) | \( A_y/10^6 \text{ m}^2 \) | \( A_x/10^6 \text{ m}^2 \) | \( A_y/10^6 \text{ m}^2 \) |
|----------------|----------------|----------------|----------------|
| 3.10           | 3.77           | 2.79           | 4.35           |

\[ c_{\text{air}}/\text{kJ/kg}^\circ\text{C} \]

#### TABLE B5 Parameters of the MCGA

| Population size | Number of iterations | Crossover probability | Mutation probability | \( A_{\text{fit}} \) |
|-----------------|----------------------|-----------------------|----------------------|------------------|
| 100             | 100                  | 0.65                  | 0.05                 | 11 000           |