Sensitivity Analysis of Factors Influencing Accelerometer Hysteresis Based on Morris Screening Method

Kun Ye*, Lin Teng, Pengfei Li, Liang Chen
Xi’an Flight Automatic Control Research Institute of AviC, Xi’an, Shanxi, China
*Corresponding author: email: yekun84@aliyun.com.cn

Abstract: Aiming at the fuzzy cognition of the key process parameters that affect the accelerometer hysteresis performance in the assembly and manufacturing process, a sensitivity analysis method for the influencing factors of accelerometer hysteresis based on Morris screening method was proposed in this paper. Firstly, the assembly and manufacturing process parameters and their range of values of core structure pendulum components are analyzed statistically. Secondly, the mean value and variance of the basic factors of each parameter are obtained based on finite element numerical simulation method. The results show that the bonding Angle between the pad I and the pendulum plate and the bonding Angle between the counterweight and the pendulum plate have the greatest influence on the hysteresis performance of the accelerometer. The interaction between the counterweight and the bonding Angle and other parameters is the most nonlinear.

1. Introduction
As the core inertial device of the navigation system, the accelerometer has been widely used in key fields, such as aerospace, electronic communication, and engineering machinery [1-3]. With the upgrading of special airborne equipment, modern high-precision inertial navigation equipment has put forward higher and higher requirements for the performance of accelerometers. Therefore, many scholars have carried out a series of investigations on the performance analysis of accelerometers. Liu et al. [4] used sensitivity analysis technology to obtain the key parameters of accelerometer structures, and proposed a robust design method using the multi-island genetic algorithm for the accelerometer structures. Cheng et al. [5] established an accelerometer performance analysis platform and constructed an accelerometer performance parameter model to improve the calibration and test rate for the accelerometer. Levy et al. [6] studied the temperature stability of the bias value and noise level of the vibrating beam micro-inertial accelerometer, and designed a new structure with a second resonator, which improved the sensitivity and linearity of bias value with respect to change of the temperature.

The above studies mainly focused on the output bias and stability of accelerometer. However, in the process of accelerometer service, hysteresis is also one of the key indicators to evaluate its performance. At present, it seriously restricts the improvement of the overall performance of the accelerometer, but few scholars have conducted in-depth research on it. Since the component structures and assembly process of the accelerometer are complex, which makes it difficult to improve the hysteresis performance. Hence, it is urgent to carry out parameter sensitivity analysis of the accelerometer during the assembly and manufacturing process and identify key sensitive parameters to improve the overall performance of the accelerometer.

The objective of parameter sensitivity analysis is to qualitatively or quantitatively evaluate the influence of model input parameters on output, and to screen out factors that have an important
influence on model output. At present, a series of sensitivity analysis methods have been established, including multiple regression method [7], Fourier sensitivity test method [8], Morris screening method [9], variance decomposition method [10], etc. Among these methods, Morris screening method has been widely used in mechanical engineering, physics, chemistry and other fields [11-13] since it has the advantages in less calculation and considering the interaction between parameters.

Hence, this paper introduces the Morris screening method into the field of inertial navigation, and proposes a sensitivity analysis method for the parameters affecting the hysteresis of the accelerometer. Firstly, the value ranges of the parameters are determined through the statistical analysis of the assembly manufacturing process parameters. Secondly, the parameters that have an important influence on the hysteresis performance of the accelerometer during the assembly and manufacturing process are obtained by combining the Morris screening method and the finite element numerical analysis technology.

2. Morris screening method [9]

The Morris screening method can be summarized as follows. Firstly, the initial value of the parameter should be determined. Secondly, the system output can be calculated by adding a perturbation $\Delta x_i$ to a certain parameter and keeping other parameters unchanged. Finally, the ratio of the system output can be calculated by changing to the perturbation $\Delta x_i$. This ratio is called the elementary effect, which is an important evaluation index in Morris screening method. The elementary effect for the $i$-th parameter is defined as:

$$EE_i(x^*) = \left[ \frac{f(x_1^*, \ldots, x_{i-1}^*, x_i^* + \Delta x_i, x_{i+1}^*, \ldots, x_n^*) - f(x_1^*, \ldots, x_{i-1}^*, x_i^*, x_{i+1}^*, \ldots, x_n^*)}{\Delta x_i} \right]$$

(1)

$$\Delta x_i = \frac{1}{p-1}$$

(2)

where $x_i^*$ is the initial value for the $i$-th parameter, $\Delta x_i$ is perturbation of $x_i^*$, $f(x_1^*, \ldots, x_{i-1}^*, x_i^*, x_{i+1}^*, \ldots, x_n^*)$ is the system output, $p$ is the number of parameter sampling points.

Assuming that $EE(x)$ for the basic factors obey a certain distribution, after multiple rounds of calculation, compare the mean and variance of each variable and conduct corresponding sensitivity evaluations. The mean $\mu_i$ and variance $\sigma_i$ of basic factor for the $i$-the parameter can be obtained by:

$$\mu_i = \frac{\sum_{j=1}^{r'} EE_j(x^*)}{r}$$

(3)

$$\sigma_i = \sqrt{\frac{\sum_{j=1}^{r'} (EE_j(x^*) - \mu_i)^2}{r}}$$

(4)

where $r$ is number of samples. The greater the mean for $EE(x)$ is, the greater the sensitivity of the parameter is; the larger the variance is, the stronger the nonlinearity of the interaction between the variable and other variables is.

3. Parameter sensitivity analysis of accelerometer hysteresis performance

3.1 Accelerometer hysteresis phenomenon

Accelerometer consists of shell, pendulum assembly, fixed plate, magnetic steel and signal processing circuit. As the most core component of the accelerometer, the swing component is composed of
counterweight, pad, swing plate and torque coil, as shown in Fig. 1. The components of the swing component are connected by adhesives, and the coating position of adhesives is shown in Fig. 2. A differential capacitance sensor is composed of a pendulum and a fixed plate. The accelerometer senses the acceleration through the variation of the capacitance plate spacing.

As a common phenomenon in the accelerometer's service process, the hysteresis is a key indicator to evaluate the performance of the accelerometer. Apply the cyclic temperature load shown in Fig. 3 to the accelerometer without acceleration input, and collect the output data of the accelerometer in the segments $ab$ and $cd$, as shown in Fig. 4. Calculate the difference between two accelerometer output at the same temperature point in the temperature interval $-40 \sim 80 \degree C$, the largest of all differences is the hysteresis performance.
3.2 Sensitivity analysis of assembly and manufacturing process parameters

This subsection will study the influence of assembly and manufacturing process parameters of accelerometer on the hysteresis performance of accelerometer. Accelerometer assembly manufacturing process parameters are treated as input parameters, accelerometer hysteresis performance is treated as output response. Since it is difficult to construct the mathematical expression between input parameters and output response, this paper obtains the output response based on the CAE model of accelerometer hysteresis performance. For a given set of input parameters (pendulum assembly structure and adhesive process parameters), the corresponding output response \( f(X) \) is solved through the CAE model of accelerometer hysteresis performance. According to the relationship between the input parameters and the output response, the sensitivity analysis is carried out by using the Morris screening method to obtain the sensitivity of each parameter in the assembly manufacturing process to the hysteresis performance and select the parameters with significant influence.

According to the relevant process specifications of accelerometer core structure pendulum assembly manufacturing and engineering experience, 10 assembly manufacturing process parameters and their value range are determined, as listed in Table 1.

| No. | Physical parameters                                | Initial values | Parameter value intervals |
|-----|-----------------------------------------------------|----------------|--------------------------|
| 1   | Adhesion angle of cushion block I and pendulum piece | 10°            | [10°, 20°]               |
| 2   | Bonding angle of spacer I and coil                  | 10°            | [10°, 20°]               |
| 3   | Adhesion angle of spacer II and swing piece         | 10°            | [10°, 30°]               |
| 4   | Bonding angle of spacer II and coil                 | 10°            | [10°, 30°]               |
| 5   | Bonding angle of counterweight and swing piece      | 10°            | [10°, 30°]               |
| 6   | Spacer I assembly error                             | -30'           | [-30’, 30’]              |
| 7   | Assembly error of spacer II                         | -30’           | [-30’, 30’]              |
| 8   | Coil processing error (coil roundness)              | 0              | [0, 0.05]                |
| 9   | Coil assembly error (coaxiality of coil and pendulum)| 0              | [0, 0.05]                |
| 10  | Weight assembly error                               | -30’           | [-30’, 30’]              |

The value range of each parameter is mapped to an interval \( \eta = [0, 1] \) by the following equation:

\[
x_i = \bar{x}_i + (\bar{x}_i - \underline{x}_i) \eta
\]

(5)

where \( \underline{x}_i \) and \( \bar{x}_i \) are the lower and upper bounds of parameter \( x_i \), respectively. This paper set the perturbation of parameters a fixed step \( \frac{1}{10} \) and consider a subdivision for the interval \( \eta \) as follows:

\[0, \frac{1}{10}, \frac{2}{10}, ..., \frac{9}{10}, 1\]. Firstly, establish the accelerometer hysteresis performance CAE model
corresponding to the initial values listed in Table 1, and obtain $f(X^*)$ through simulation calculation. Then, select the values from $0, \frac{1}{10}, \frac{2}{10}, ..., \frac{9}{10}, 1$ in order and calculate the elementary effect of the parameters using the Morris screening method. In the process of calculating the elementary effect, the input parameters and output response are normalized. The system output rate of change and the slope of the perturbation are calculated

$$EE_i(x^*) = \frac{f(x^*_1, ..., x_{i-1}^*, x_i^* + \Delta x_i, ..., x_{10}^*) - f(X^*)}{\Delta x_i}$$

This step is repeated 10 times; the mean and variance can be solved through Eqs. (3) and (4). After multiple rounds of simulation analysis and solution, the sensitivity analysis results of assembly manufacturing process parameters are shown in Table 2.

| No. | $\mu_i$  | $\sigma_i$ |
|-----|----------|------------|
| $x_1$ | 0.651    | 0.014      |
| $x_2$ | 0.027    | 0.008      |
| $x_3$ | -0.139   | 0.022      |
| $x_4$ | 0.005    | 0.002      |
| $x_5$ | 0.367    | 0.365      |
| $x_6$ | 0.013    | 0.000081   |
| $x_7$ | 0.016    | 0.000018   |
| $x_8$ | 0.013    | 0.000013   |
| $x_9$ | 0.004    | 0.000023   |
| $x_{10}$ | 0.022  | 0.000074   |

As can be seen from Table 2, the mean value of the basic factors of the parameters $x_1$ and $x_5$ is larger, while the mean value of the basic factors of the parameters $x_4$ and $x_9$ is smaller. It shows that the parameter $x_1$ and $x_5$ has a great influence on the accelerometer hysteresis performance, but the parameter $x_4$ and $x_9$ has a small influence. In the actual manufacturing of accelerometer, the parameters $x_1$ and $x_5$ should be strictly controlled. That is to strictly control the bonding Angle between the adhesion angle of cushion block I and pendulum piece and Bonding angle of counterweight and swing piece. At the same time, it can be seen from the right column of Table 2 that the variance of the basic factor of the parameter $x_5$ is the largest, that is, the interaction between the parameter and other parameters has the strongest non-linearity.
4. Conclusions
Morris screening method is introduced into the field of inertial navigation in this paper, and a sensitivity analysis method for accelerometer hysteresis factors is proposed. Through calculation and analysis, it can be seen that the bonding angle between the adhesion angle of cushion block I and pendulum piece and bonding angle of counterweight and swing piece have the most obvious influence on the hysteresis performance of the accelerometer. The interaction between the counterweight and the bonding angle of the swing plate and other parameters is the most nonlinear. In the actual production process, the above two parameters should be strictly controlled to improve the comprehensive performance and performance consistency of the product.

References
[1] Dong, Y., Zwahlen, P., Nguyen, A.-M, (2010) Rudolf, F. Stauffer, J-M. High performance inertial navigation grade sigma-delta MEMS accelerometer. IEEE Symposium on Position Location and Navigation, pp:32-36.
[2] Ullah, P., Ragot, V., Zwahlen, P., Rudolf, F. (2015) A new high performance sigma-delta MEMS accelerometer for inertial navigation. DGON Inertial Sens. Syst. Symp, pp:2-13.
[3] Traon, O. L., D Janiaud, Pernice, M., Masson, S., Tridera, J. Y. (2006) A new quartz monolithic differential vibrating beam accelerometer. Navigation Symposium, IEEE/ION, pp:6-15.
[4] Liu, G. J., Jiang, T., Wang, A. L. (2009) Robust optimization of an accelerometer considering fabrication errors. Materials Science Forum, 628-629: 353-356.
[5] Cheng, H., Yuan, Z., Qian, Z., Li, B. (2009) Research on performance analyzing and modeling of quartzose flexible accelerometer. International Conference on Electronic Measurement & Instruments. IEEE.
[6] Levy, R., Traon, O. L., Masson, S., Ducloux, O., Chartier, C. (2012) An integrated resonator-based thermal compensation for Vibrating Beam Accelerometers. Sensors,IEEE, pp:1-5.
[7] M.D. Mckay, Conover R. J. B. J. (1979) A comparison of three methods for selecting values of input variables in the analysis of output from a computer code. Technometrics, 21(2): 239-245.
[8] Robert I., Cukier, C. M., Fortuin, K., et al. (1973) Study of the sensitivity of coupled reaction systems to uncertainties in rate coefficients. I theory. Journal of Chemical Physics, 59.
[9] MD Morris. (1991) Factorial sampling plans for preliminary computational experiments. Technometrics, 33(2): 161-174.
[10] Sobol, I. M. (1993) Sensitivity estimates for nonlinear mathematical models. Math model computer expriment, 1(1): 112–118.
[11] Qin, Y., Wang, Z., Xiang, C., Dong, M., Hu, C., Wang, R. (2018) A novel global sensitivity analysis on the observation accuracy of the coupled vehicle model. Vehicle System Dynamics, pp:1-22.
[12] Idrisi, K., Klimke, A., Wohlmuth, B. (2014) Global sensitivity analysis of expensive models using sparse grids with applications to laminated composite structure. Noise Control Engineering Journal, 62(6): 521-534.
[13] Tao, C., Yang, Y. (2011) Interpretation of non-linear empirical data-based process models using global sensitivity analysis. Chemometrics and Intelligent Laboratory Systems, 107(1): 116-123.