The generalized correlating function of capillary curves and the relationship of the filtration-capacitive parameters of reservoirs in Western Siberia with the size distribution of pore channels

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Abstract. It was shown that the Leverett J-function and the Brooks-Corey correlating function approximate the capillary curves with insufficient accuracy and therefore do not allow us to obtain an analytical expression for the density distribution of pore channels by size. The polynomial approximation proposed makes it possible to sharply increase the reliability and accuracy of the capillary curves approximation. We obtained analytical expressions that allow to get a density distribution function of pore channels by size for known values of filtration-capacitive parameters (porosity, permeability, residual water saturation). It should be noted that the correlation between the distribution of the size of the pore channels and the porometric characteristics of the reservoir can be obtained according to the geophysical methods of well research.

1. Introduction
The method of capillarimetry is based on the idea that when a certain pressure \( p \) is applied to a water-saturated sample, water is squeezed out of the pore channels of a certain size in accordance with the Laplace formula \([1, 2]\):

\[
p = \frac{2\sigma \cos \theta}{r},
\]

Where \( \sigma \) is the value of surface tension; \( \theta \) is the wetting angle; \( r \) is the radius of the pore channel (capillary).

It is assumed that the pore channels of the reservoir are a combination of capillaries of various sizes.

In the process of displacement at the lowest pressures acting on the sample, water is squeezed out of the pore channels of maximum cross section, and subsequently, as the capillary pressure increases, a wider coverage of the capillary family takes place up to the smallest ones filled with residual water.
2. Materials and methods
Under the conditions of Western Siberia, either the Brooks – Corey function [3–6] are used as a correlating function of capillary curves. Many researchers state that the correlation between water saturation and capillary pressure is power-law [7–10].

However, our studies show that power-law dependence occurs only in the middle section of the interval of variation in the coefficient of water saturation. At low values of water saturation, as well as at values close to unity, the nature of the bond differs significantly from the power-law [11–12].

In some cases, the Leverett J-function is used as a correlating function of capillary curves. This applies to clean as well as weakly clayey sandstones, in which water is mainly capillary-retained.

3. Results and Discussion
As for clay collectors in Western Siberia, they contain a rather large amount of water adsorbed by the clay component, which cannot be displaced at the existing capillary pressure values [13]. Therefore, the most common approximation method in Western Siberia is a method based on the use of normalized water saturation (Brooks-Corey method):

\[ K_w^* = \left( \frac{p}{p_0} \right)^\alpha, \]

where \( K_w^* \) – normalized water saturation, and \( K_w^* = \frac{K_w - K_{wo}}{1 - K_{wo}} \); \( K_w \) – total water saturation; \( K_{wo} \) – residual water saturation; \( p \) is capillary pressure; \( p_0 \) — initial capillary pressure; \( \alpha \) is a parameter characterizing the steepness of capillary curves.

We use the Laplace formula and in the formula (1) from capillary pressures we turn to the corresponding radii of the pore channels:

\[ K_w^* = \left( \frac{r}{r_m} \right)^\alpha, \]

where \( r \) is the current radius of the pore channel; \( r_m \) is the maximum radius of the pore channels.

Next, we determine the radius derivative and obtain a formula for the distribution of pore channels \( g(r) \) in size:

\[ g(r) = \frac{dK_w^*}{dr} = \frac{\alpha}{r_m} \left( \frac{r}{r_m} \right)^{\alpha-1}. \]

For \( \alpha > 1 \), this function monotonically increases, while for \( \alpha < 1 \) it decreases monotonically. Thus, in the interval of existence of pore channel sizes, this function does not have an extremum. However, the curves of the density of distribution of pore channels, obtained on the basis of laboratory studies of core samples, first increase with increasing radius of the pore channels, reach a maximum, and then decrease monotonically and tend to zero.

The absence of an extremum of function (2) suggests that the approximation of capillary curves by the Brooks-Corey function is not sufficiently accurate, rather rough, and needs to be modified.

The analysis shows that the Brooks-Corey function gives distorted values of capillary pressure at medium and especially at low water saturation values. Thus, real capillary curves with accuracy sufficient for practical purposes cannot be described by a simple power function.

The capillary curve is usually graphically depicted as a function of capillary pressure versus water saturation.

However, in a real reservoir, capillary pressure, depending on the size of the pore channels, determines the water saturation of the reservoir.

Because of this, in the analytical description of capillary curves, it would be more correct to consider water saturation as a function of capillary pressure.

Let us consider the correlation between the logarithm of the reduced value of the coefficient of water saturation \( K_w^* \), the logarithm of the product of capillary pressure and the parameter characterizing the average radius of the pore channels \( (r_0) \) (Figures 1 and 2). With a power-law correlation between these parameters in a logarithmic coordinate system, we should get a straight line. As shown in Figure 1, in the logarithmic coordinate system, the regression line is significantly
different from the straight line.

Thus, the correlating functions of capillary curves proposed by Leverett and Brooks-Corey are only a first approximation, and the behavior of real capillary curves is much more complicated.

It is easy to notice that the relationship between water saturation and capillary pressure in a logarithmic coordinate system has a parabolic character. Indeed, when using an algebraic polynomial of the second degree as an approximation function in the conditions of the $\text{AB}_1^3$ layer of the Las-Egan deposit in Western Siberia, the correlation coefficient exceeds 0.97 (see Figure 1).

![Figure 1](image1.png)  
**Figure 1.** Dependence of normalized water saturation on dimensionless capillary pressure in a logarithmic coordinate system. Las Yegan field, reservoir $\text{AB}_1^3$

The graph of comparing water saturation and capillary pressure in the logarithmic coordinate system for the $\text{IOB}_1^1$ formation of the Povkhovskoye field in Western Siberia has a similar character (Figure 2).

![Figure 2](image2.png)  
**Figure 2.** Dependence of normalized water saturation on dimensionless capillary pressure in a logarithmic coordinate system. Povkhovskoye field, reservoir $\text{IOB}_1^1$

Thus, the regression line for these graphs is described by the following formula:

$$\ln K_w^* = a + b \ln (pr_0) + c \ln^2 (pr_0),$$  \hspace{1cm} (3)

where $p$ is the capillary pressure; $r_0 = \sqrt{\frac{K_m}{K_n}}$ is a parameter having the dimension of radius.

Formula (3) can be reduced to the following form:

$$K_w^* = e^a \cdot (pr_0)^b \cdot \ln(pr_0).$$  \hspace{1cm} (4)

Analysis of the last expression shows that for $c = 0$ we get the Brooks-Corey formula, and the exponent $c \ln (pr_0)$ allows us to correct the correlation function in the range of medium and large capillary pressure values. Thus, expression (4) is a modified Brooks-Corey function.

We use the proposed formula (3) to identify the relationships between the filtration-capacitive parameters of the productive formations of Western Siberia: the size of the pore channels, porosity, residual water saturation, and absolute permeability.

For this purpose, on the basis of the mathematical model (3) we will get the expression for the density of distribution of pore channels of reservoir collectors by sizes $g(r)$.

Obviously, the distribution of pore channels in size is described by the following formula:
where $r$ is the radius of the pore channel.

In accordance with the formula (3)

$$K^*_B = e^{a+b \ln(p r_0)+c \ln^2(p r_0)}.$$

we get:

$$\frac{dK^*_B}{dp} = \frac{b+2c \ln(p r_0)}{p} \cdot e^{a+b \ln(p r_0)+c \ln^2(p r_0)}.$$  \hspace{1cm} (6)

The second factor on the right side of formula (5) is the derivative with respect to the radius of the pore channel of the Laplace function:

$$\frac{d}{dr} \left( \frac{\beta_0 r}{r} \right) = -\frac{\beta_0}{r^2} = -\frac{p^2}{\beta_0},$$

where $\beta_0 = 2 \sigma \cos \theta$.

Finally, for the density distribution of pore channels by size, we get the following formula:

$$g(r) = -\frac{p}{\beta_0} \left[ b + 2c \ln(p r_0) \right] \cdot K^*_B,$$ \hspace{1cm} (7)

where $p = \frac{\beta_0}{r}$.

Figure 3. Pore channel size distribution density plots for a number of reservoir samples AB of the Las Yeganskoye field, obtained by the proposed method.
Figure 4. Pore channel size distribution density plots for a number of reservoir samples IOB₁ of the Povkhovskoye field, obtained by the proposed method

Figure 3 shows graphs of the size distribution functions of pore channels for five core samples from the AB₁ layer of the Las Yegansky field in Western Siberia, which have different values of porosity and absolute permeability.

As follows from the graphs, with an increase in the permeability coefficient, the curves shift to the right in the region of large radii, and the proportion of large pore channels increases.

Similar conclusions can be drawn from the analysis of the behavior of the density distribution curves of the size of the pore channels of the reservoir of the Povkhovskoye field in Western Siberia (Figure 4).

4. Conclusion

1. Using algebraic polynomials as a correlating function of capillary curves on a logarithmic scale can dramatically increase the accuracy of the approximation over the entire range of pressure and water saturation.

2. The proposed approximation is a modification of the Brooks-Corey function. It gives more accurate results in the field of medium and large values of capillary pressure.

3. Using the modified function allows getting analytical expressions for the density distribution of the size of the pore channels for any rock sample from a given reservoir.

4. The resulting expressions are analytical relationships between the size of the pore channels and the filtration-capacitive parameters of the reservoir.

5. Analytical relationships between the distribution of the size of the pore channels and the porometric characteristics of the formation can also be obtained from the data of wells’ geophysical studies.
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