We develop the Lagrangian perturbation theory in the general relativistic cosmology, which enables us to take into account the vortical effect of the dust matter. Under the Lagrangian representation of the fluid flow, the propagation equation for the vorticity as well as the density is exactly solved. Based on this, the coupling between the density and vorticity is clarified in a non-perturbative way. The relativistic correspondence to the Lagrangian perturbation theory in the Newtonian cosmology is also emphasized.

I. INTRODUCTION

The dynamics of fluids in the expanding universe is of great importance in the cosmology. For investigating such a dynamics, the Newtonian treatment is often used as a good approximation for the region $l/L \ll 1$, where $l$ is the scale of fluctuations of fluids and $L$ corresponds to the Hubble radius \[.\] In the Newtonian cosmology, the Lagrangian perturbation theory in the dust universe has been developed for non-linear density fluctuations, up to the caustic formation \[.\] In order to take into account the relativistic correction to the Newtonian dynamics, the cosmological post-Newtonian approximation has been formulated \[.\] Furthermore, in the cosmological post-Newtonian approximation, the Lagrangian perturbation theory has been also discussed \[.\] Such a treatment beyond the Newtonian approximation may become important because of not only the theoretical interest but also

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the recent progress in the observational cosmology. For example, the Sloan Digital Sky Survey (SDSS) aims at the deep survey over several hundred Mpc [10], in which it is not clear whether the Newtonian treatment is sufficient for such a large area.

Relativistic theories of linear perturbations have been developed [1,11,12]. The second-order extension of relativistic perturbation theory has been also developed [13]. However, they still depend on the assumption that the density fluctuation is small. In order to overcome the drawback, several perturbative approaches for the nonlinear dynamics have been proposed, which are based on the fluid flow approach [14], the gradient expansion method [15–17], or the relativistic Lagrangian approach [18,19]. It is shown that the relativistic post-Zel’dovich approximation, a second-order extension of the relativistic Lagrangian approach, successfully express the non-linear evolution of the density contrast with higher accuracy than the conventional second-order theory [20].

So far, however, these relativistic approaches are all restricted within the irrotational dust. There has been no established way of calculating non-linear evolution of the density fluctuation without such limitation. Our paper is aimed to add to knowledge in literature by presenting a relativistic framework of such calculation.

This paper is organized as follows. In section II, we show the integrals of the density and the vorticity in the general relativity, in the simple manner using the Lagrangian condition. This fact suggests strongly that the Lagrangian condition allows us to formulate the Lagrangian description in the relativistic cosmology. Under this condition, section III presents a perturbative Lagrangian approach. Summary and Discussion are given in section VI. For comparison, the vorticity in the Newtonian cosmology is discussed in the appendix A. Residual gauge freedoms in the Lagrangian condition are clarified in the appendix B. Greek indices run from 0 to 3, and Latin indices from 1 to 3. We use the unit, $c = 1$. 
II. LAGRANGIAN DESCRIPTION OF DUST FLUID

Let us consider a simple dust universe, in which the energy momentum tensor is written as

\[ T^{\mu\nu} = \rho u^\mu u^\nu. \]  \hspace{1cm} (2.1)

The conservation law \( T^\mu_{\mu\nu} = 0 \) gives

\[ \rho_{\mu\nu} u^\mu + \rho u^\mu_{\mu\nu} = 0, \]  \hspace{1cm} (2.2)

\[ u^\mu_{\nu\nu} u^\nu = 0, \]  \hspace{1cm} (2.3)

which are called as the continuity equation and the geodesic equation, respectively.

The vorticity \( \omega^\mu \) of the fluid flow is defined by \[ 21 \]

\[ \omega^\alpha \equiv \frac{1}{2} \epsilon^{\alpha\mu\nu\rho} u^\mu u^\nu_{\rho}, \]  \hspace{1cm} (2.4)

where \( \epsilon^{\alpha\mu\nu\rho} \) denotes the complete anti-symmetric tensor with \( \epsilon^{0123} = 1/\sqrt{-g} \) and \( g \equiv \det(g_{\mu\nu}) \). From the geodesic equation Eq. (2.3), we obtain the propagation equation for the vorticity \[ 21 \]:

\[ \omega^\mu_{\nu\nu} u^\nu + u^\nu_{\nu\nu} \omega^\mu = u^\mu_{\nu\nu} \omega^\nu. \]  \hspace{1cm} (2.5)

Using Eq. (2.2), we have

\[ \left( \frac{\omega^\mu}{\rho} \right) u^\nu = u^\nu_{\nu\nu} \left( \frac{\omega^\nu}{\rho} \right), \]  \hspace{1cm} (2.6)

which may be called as the relativistic Beltrami equation (cf. Eq. (A10) in Appendix A).

The Einstein equations are decomposed with respect to the fluid flow as follows:

\[ G_{\mu\nu} u^\mu u^\nu = 8\pi G \rho, \]  \hspace{1cm} (2.7)

\[ G_{\mu\nu} u^\mu P^\nu_\alpha = 0, \]  \hspace{1cm} (2.8)

\[ G_{\mu\nu} P^\mu_\alpha P^\nu_\beta = 0, \]  \hspace{1cm} (2.9)

where \( P^\mu_\nu \) is the projection tensor.
\[ P^\mu_\nu \equiv \delta^\mu_\nu + u^\mu u_\nu. \] (2.10)

So far the treatment is fully covariant. In the following, we adopt the Lagrangian condition (e.g. \[22\]), in which the components of the matter 4-velocity take the values of

\[ u^\mu = (1, 0, 0, 0). \] (2.11)

Under this condition, we immediately have \( g_{00} = -1 \) and \( u_\mu = (-1, g_{0i}) \). Furthermore, since the geodesic equation Eq. (2.3) under the Lagrangian condition simply tells \( u_{\mu,0} = 0 \), \( u_i(= g_{0i}) \) are functions of spatial coordinates only:

\[ u_i = u_i(x), \quad \text{and} \quad g_{0i} = g_{0i}(x). \] (2.12)

In the Lagrangian description, the continuity equation (2.2) is simply

\[ (\rho \sqrt{-g})_{,0} = 0. \] (2.13)

Therefore,

\[ \rho(x, t) = \left[ \frac{g(x, t_0)}{g(x, t_0)} \right] \rho(x, t_0). \] (2.14)

Under the Lagrangian condition, the determinant of the metric tensor can be expressed as

\[ g = -(1 + \gamma^{ij} g_{0i} g_{0j}) \det(g_{ij}), \] (2.15)

where \( \gamma^{ij} \) is the inverse of the spatial metric \( g_{ij} \). The relativistic Beltrami equation (2.6) also becomes simply

\[ \left( \frac{\omega^i}{\rho} \right)_{,0} = 0, \] (2.16)

which is integrated to give

\[ \frac{\omega^i}{\rho} = \left. \frac{\omega^i}{\rho} \right|_{t_0}. \] (2.17)

This is also expressed as
\( \omega^i(x, t) = \sqrt{g(x, t)} \omega^i(x, t_0). \) \hfill (2.18)

The \( \omega^0 \) component is not independent of \( \omega^i \). Using the relation \( \omega^\mu u_\mu = 0 \), we obtain

\[ \omega^0 = g_{0i} \omega^i, \] \hfill (2.19)

The result Eq. (2.17) tells us that the vorticity is coupled to the density enhancement and vice versa. In particular, if the vorticity does not vanish exactly at an initial time, the vorticity will blow up as the density grows larger and larger (i.e. in the collapsing region), even if it has only the decaying mode in the linear perturbation theory. It should also be emphasized that our results Eqs. (2.14) and (2.17) in the fully general relativistic treatment precisely correspond to those in the Newtonian case (see Eqs. (A7) and (A13) in the Appendix).

### III. PERTURBATIVE LAGRANGIAN APPROACH

In the previous section, we solved exactly the equations for the density and the vorticity. The results Eqs. (2.14) and (2.18) show that \( \rho \) and \( \omega^i \) are completely written in terms of the determinant of the metric tensor and their initial values. In this section, we solve the metric perturbatively. We assume that the background is spatially flat Friedmann-Lemaître-Robertson-Walker (FLRW) universe. The extension to the spatially non-flat case must be a straightforward task. The perturbed metric is decomposed into

\[ g_{0i} = B_{,i}(x) + b_i(x), \] \hfill (3.1)

\[ g_{ij} = a^2 (\delta_{ij} + 2H_L \delta_{ij} + 2H_T,ij + (h_{i,j} + h_{j,i}) + 2H_{ij}), \]

where \( B, H_L, \) and \( H_T \) are scalar mode quantities, \( b_i \) and \( h_i \) are the vector (transverse) mode, and \( H_{ij} \) is the tensor (transverse-traceless) mode satisfying

\[ b^i_{,i} = 0, \] \hfill (3.2)

\[ h^i_{,i} = 0, \] \hfill (3.3)
\( H^i_i = 0, \quad (3.4) \)
\( H^{ij} = 0. \quad (3.5) \)

Raising and lowering indices of the perturbed quantities are done by \( \delta^{ij} \) and \( \delta_{ij} \). Using the perturbed metric Eq. (3.1), the general expression for the perturbed Einstein tensor up to the linear order is as follows:

\[
G_{\mu\nu}u^\mu u^\nu = 3 \left( \frac{\dot{a}}{a} \right)^2 + 2 \frac{\dot{a}}{a} (3 \dot{H}_L + \nabla^2 \dot{H}_T) - \frac{2}{a^2} \left( \frac{\dot{a}}{a} \nabla^2 B + \nabla^2 H_L \right), \quad (3.6)
\]

\[
G_{\mu\nu}u^\mu P^\nu_i = -2 \left( \dot{H}_L + \frac{\dot{a}}{a} B \right) \delta_{ij} \quad (3.7)
\]

where an overdot (\( \dot{\cdot} \)) denotes \( \partial/\partial t \), \( \nabla^2 = \delta^{ij} \partial_i \partial_j \), and we have used the fact \( a(t) = t^{2/3} \) so that \( G_{\mu\nu}P^\mu_i P^\nu_j \) does not have the background quantity.

**A. Scalar perturbations**

Using Eqs. (3.7) and (3.8), the Einstein equations for the scalar perturbations are

\[
\left( \dot{H}_L + \frac{\dot{a}}{a} B \right) = 0, \quad (3.9)
\]
\[
\ddot{H}_L + 3 \frac{\dot{a}}{a} \dot{H}_L = 0, \quad (3.10)
\]
\[
\ddot{H}_T + 3 \frac{\dot{a}}{a} \dot{H}_T = \frac{1}{a^2} \left( H_L + \frac{\dot{a}}{a} B \right). \quad (3.11)
\]

In order to solve the above equations, we use the residual gauge freedom (see Eq. (B8) in Appendix B) to set \( B = 0 \). Then, from Eq. (3.9) we have

\[
H_L = H_L(x) \equiv \frac{10}{9} \Psi(x). \quad (3.12)
\]
Next, from Eq. (3.11), we obtain two independent solutions for $H_T$: one is proportional to $t^{2/3}$ and the other to $t^{-1}$. According to Eq. (B10), we may also use the residual gauge freedom for $H_T$ to add any function which does not depend on $t$. One may choose it as

$$H_T = \left(t^{2/3} - t_0^{2/3}\right)\Psi(x) + \left(t^{-1} - t_0^{-1}\right)\Phi(x),$$  \hspace{1cm} (3.13)

so that $H_T$ vanishes at the initial time $t = t_0$. The initial density field $\rho(x, t_0)$ is also expressed by the metric. If the density contrast $\delta \equiv (\rho - \rho_b)/\rho_b$ is sufficiently small, we obtain from Eqs. (2.7) and (3.6)

$$\delta = -\nabla^2 \left(t^{2/3}\Psi + t^{-1}\Phi\right) \hspace{1cm} (3.14)$$

Therefore, the initial density can be related with the initial metric perturbation as

$$\rho(x, t_0) = \rho_b(t_0) \left[1 - \nabla^2 \left(t_0^{2/3}\Psi + t_0^{-1}\Phi\right)\right]. \hspace{1cm} (3.15)$$

The above expression is used only to relate the initial density fluctuation to the initial metric perturbations in linear regime. Once the initial seed is given, the later non-linear evolution is evaluated by the non-perturbative expression Eq. (2.14).

**B. Vector perturbations**

The Einstein equations for the vector perturbations are

$$\nabla^2 (\dot{h}_i - \frac{1}{a^2} b_i) = 4\left(\frac{\dot{a}}{a}\right) b_i, \hspace{1cm} (3.16)$$

$$\ddot{h}_i + 3\frac{\dot{a}}{a}\dot{h}_i - \frac{\ddot{a}}{a^3} b_i = 0. \hspace{1cm} (3.17)$$

Introducing $\beta_i$ as

$$b_i(x) = \nabla^2 \beta_i(x), \hspace{1cm} (3.18)$$

Eq. (3.16) is solved to give

$$\dot{h}_i = -3 \left(t^{-1/3} - t_0^{-1/3}\right) \nabla^2 \beta_i(x) + \frac{8}{3} \left(t^{-1} - t_0^{-1}\right) \beta_i(x), \hspace{1cm} (3.19)$$
where we again used the residual gauge freedom (cf. Eq. (B11) in the Appendix B) to set $h_i(x, t_0) = 0$. The $\beta_i(x)$ is directly related to the initial value of the vorticity field. From Eq. (2.17), we have

$$\omega^i(x, t_0) = \frac{1}{2\sqrt{-g(x, t_0)}}\varepsilon^{ijk}\nabla^2\beta_{j,k},$$

(3.20)

where $\varepsilon^{ijk}$ is the 3-dimensional Levi-Civita symbol with $\varepsilon^{123} = 1$. Particularly when the deviation from the background is sufficiently small, we obtain the first-order expression

$$\omega^i(x, t_0) \simeq \frac{1}{2a^3(t_0)}\varepsilon^{ijk}\nabla^2\beta_{j,k}.$$  

(3.21)

C. Tensor perturbations

The equation for the tensor perturbations is

$$\ddot{H}_{ij} + 3\frac{\dot{a}}{a}\dot{H}_{ij} - \frac{1}{a^2}\nabla^2H_{ij} = 0.$$  

(3.22)

This is a homogeneous wave equation in the expanding universe. The solutions are well known and we will not discuss the detail here. See, e.g., [23].

Before closing this section, we emphasize the following point: As for the metric, our result is identical to that of the linear perturbation theory (e.g. [11]). However, there is an important difference. In Bardeen’s paper, it is essential to linearize the density contrast. On the other hand, our Lagrangian approach does not rely on the assumption that the density contrast should be small. It is actually an important advantage of the Lagrangian approach that it uses (or extrapolates) the well-known solutions of the linear theory to express the non-linear density contrast.

IV. SUMMARY AND DISCUSSION

Motivated by the fact that the Lagrangian condition enables us to obtain simply the integrals of the density and the vorticity along the fluid flow, we have developed the Lagrangian perturbation theory in the general relativistic cosmology, fully using this condition.
The main advantage of the present Lagrangian theory is to be amenable to the vorticity, while previous works are limited within the irrotational fluid. In this approach, only the metric is expanded and its behaviors are determined perturbatively through the evolution equations, while the density and the vorticity are calculated non-perturbatively from the integrals along the flow. Seeds of the density and vorticity fluctuations can be related with the metric perturbation at an initial time, only when the constraint equation is solved as the Poisson-like equation. Hence, this simple approach is the first that can describe the dynamics of the dust fluid with the rotational motion as well as the non-linear density fluctuations, up to the caustic formation.

As an illustration, the first order Lagrangian perturbation has been solved explicitly. It is natural that at the first order of the metric perturbation, our result agrees with that of the usual linear theory [11] when we assume the small density contrast. Contrary to the standard linear perturbation theory, it is explicitly shown in our approach that the vorticity is coupled to the density contrast and is amplified in high density (i.e., collapsing) region.

In this paper, we have presented a relativistic framework to describe non-linear evolution of the density fluctuation which is not restricted to the irrotational case. Once the framework is given, it should be straightforward to apply it to higher-order computations. Actually, an extension to the second order is being investigated by Morita [24]. More comprehensive study of non-linear couplings will be of the subject of future investigation.

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In this Appendix, we summarize the treatment of vorticity in the Newtonian cosmology. Using expanding coordinates $r = a(t)x$, the continuity equation and the equation of motion are

$$\frac{\partial \rho}{\partial t} + 3\frac{\dot{a}}{a}\rho + \frac{1}{a} \nabla \cdot (\rho \mathbf{v}) = 0,$$  \hspace{1cm} (A1)

$$\frac{\partial \mathbf{v}}{\partial t} + \frac{\dot{a}}{a} \mathbf{v} + \frac{1}{a}(\mathbf{v} \cdot \nabla) \mathbf{v} = -\frac{1}{a} \nabla \phi,$$  \hspace{1cm} (A2)

where $\rho$ is the density of dust matter, $\mathbf{v}$ is the peculiar velocity defined by

$$\mathbf{v} \equiv \frac{d\mathbf{r}}{dt} - \frac{\dot{a}}{a} \mathbf{r} = \frac{a}{dt} \mathbf{x},$$  \hspace{1cm} (A3)

and $\phi$ is the gravitational potential which satisfies the Poisson equation

$$\nabla^2 \phi = 4\pi G a^2 (\rho - \rho_b).$$  \hspace{1cm} (A4)

Let us introduce the Lagrangian time derivative

$$\frac{d}{dt} \equiv \frac{\partial}{\partial t} + \frac{1}{a} \mathbf{v} \cdot \nabla,$$  \hspace{1cm} (A5)

and the Lagrangian coordinates $\mathbf{q} \equiv \mathbf{x}(t_0)$. In the Lagrangian description, the continuity equation (A1) is

$$\frac{d\rho}{dt} + 3\frac{\dot{a}}{a}\rho + \frac{\rho}{a} \nabla \cdot \mathbf{v} = 0,$$  \hspace{1cm} (A6)

which is solved to give

$$\rho = \left(\frac{a_0}{a}\right)^3 \frac{\rho_0}{\det (\partial x^i/\partial q^j)}.$$  \hspace{1cm} (A7)

We now discuss the vortical part of the fluid flow. The vorticity is defined by

$$\mathbf{\omega} \equiv \frac{1}{a} \nabla \times \mathbf{v}.$$  \hspace{1cm} (A8)

From Eq. (A2), we obtain the propagation equation for the vorticity:
\[
\frac{d\omega}{dt} + \frac{\dot{a}}{a} \omega + \frac{\omega}{a} \nabla \cdot v = \frac{1}{a}(\omega \cdot \nabla)v. \tag{A9}
\]

Using Eq. (A9), we accordingly have the diffusion equation of Beltrami:

\[
\frac{d}{dt} \left( \frac{\omega}{a\rho} \right) = \left( \frac{\omega}{a\rho} \cdot \nabla \right) v. \tag{A10}
\]

It is instructive to define the vector components of the vorticity in the Lagrangian frame as follows:

\[
\tilde{\omega}^i \equiv \frac{\partial q^i}{\partial x^j} \omega^j. \tag{A11}
\]

Then, Beltrami’s equation (A10) simply reads

\[
\frac{d}{dt} \left( \frac{\tilde{\omega}}{a\rho} \right) = 0, \tag{A12}
\]

which is manifestly solved to give

\[
\frac{\tilde{\omega}}{a\rho} = \frac{\omega}{a\rho} \bigg|_{t_0}, \tag{A13}
\]

or

\[
\omega^i(q, t) = \frac{a\rho}{a_0 \rho_0} \frac{\partial x^i}{\partial q^j} \omega^j(q, t_0). \tag{A14}
\]

The result Eq. (A14) is known as Cauchy’s integral [3].

**APPENDIX B: RESIDUAL GAUGE FREEDOM IN THE LAGRANGIAN CONDITION**

The general gauge transformation to the first order is induced by the infinitesimal coordinate transformation

\[
\tilde{x}^\mu = x^\mu + \xi^\mu. \tag{B1}
\]

The changes due to the gauge transformation are
\[
\delta \xi g_{\mu \nu} = -g_{\mu \nu, \alpha} \xi^\alpha - g_{\mu \alpha} \xi^\alpha_{, \nu} - g_{\nu \alpha} \xi^\alpha_{, \mu}, \tag{B2}
\]
\[
\delta \xi u^\mu = \xi^\mu_{, \nu} u^\nu - u^\mu_{, \nu} \xi^\nu. \tag{B3}
\]

For the spatially flat background, \( \xi^\mu \) are decomposed into each mode,
\[
\xi^\mu = (T, \delta_i^j L_{j} + \ell^i), \tag{B4}
\]
where the vector mode quantity \( \ell^i \) satisfies \( \ell^i_{, i} = 0 \). In order to maintain the Lagrangian condition \( u^\mu = (1, 0, 0, 0) \), we have \( \delta \xi u^\mu = 0 \), which leads to \( \xi^\mu_{, 0} = 0 \). Therefore,
\[
T = T(x), \quad L_{i} = L_{i}(x), \quad \ell^i = \ell^i(x). \tag{B5}
\]
The changes of the metric tensor due to the residual gauge freedom are
\[
\delta \xi g_{0i} = T_{, i}, \tag{B6}
\]
\[
\delta \xi g_{ij} = -a^2 \left( \frac{\dot{a}}{a} T \delta_{ij} + 2L_{,ij} + \ell_{i,j} + \ell_{j,i} \right). \tag{B7}
\]

Hence we obtain
\[
\tilde{B} = B + T(x), \tag{B8}
\]
\[
\tilde{H}_L = H_L - \frac{\dot{a}}{a} T(x), \tag{B9}
\]
\[
\tilde{H}_T = H_T - L(x), \tag{B10}
\]
\[
\tilde{h}^i = h^i - \ell^i(x). \tag{B11}
\]
The vector mode quantity \( b^i \) and the tensor mode quantity \( H_{ij} \) are gauge-invariant.

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