SIMILARITY AND NONSIMILARITY SOLUTIONS ON FLOW AND HEAT TRANSFER OVER A WEDGE WITH POWER LAW STREAM CONDITION

M. Ismoen1*, M. F. Karim1, R. Mohamad2 and R. Kandasamy2

1Engineering Mathematics Unit, Faculty of Engineering, Institut Teknologi Brunei, Jalan Tungku Link, Gadong BE 1410, Bandar Seri Begawan, NEGARA BRUNEI DARUSSALAM
2Computational Fluid Dynamics Group, Faculty of Science, Technology and Human Development Universiti Tun Hussein Onn Malaysia, Parit Raja 86400 Batu Pahat, Johor Bahru, MALAYSIA

Abstract
The similarity and non-similarity analysis are presented to investigate the effect of buoyancy force on the steady flow and heat transfer of fluid past a heated wedge. The fluid is assumed to be a Newtonian, viscous and incompressible. The wall of the wedge is an impermeable with power law free stream velocity and a wall temperature. Due to the effect of a buoyancy force, a power law of free stream velocity and wall temperature, then the flow field is similar when

\[ n = 2m - 1 \]

otherwise is non-similar when

\[ n \neq 2m - 1 \].

The governing boundary layer equations are written into dimensionless forms of ordinary differential equations by means of Falkner-Skan transformation. The resulting ordinary differential equations are solved by Runge-Kutta Gill with shooting method for finding a skin friction and a rate of heat transfer. The effects of buoyancy force and non-uniform wall temperature parameters on the dimensionless velocity and temperature profiles are shown graphically. Comparisons with previously published works are performed and excellent agreement between the results is obtained. The conclusion is drawn that the flow field and temperature profiles are significantly influenced by these parameters.

Keywords: similarity, non-similarity, buoyancy force, power law, stream condition

*E-mail address: muhaimin_ismoen@itb.edu.bn

1. Introduction
The problem dealing with a convective heat transfer on laminar boundary layer flow resulting from over a heated wedge plate is of considerable theoretical and practical interest. Many practical applications of convection exist, for example in chemical factories, in the heaters and coolers of electrical and mechanical devices and in lubrication of machine parts. In a wedge geometry, the buoyancy force effects may become significant when the flow velocities are relatively low and the temperature difference between the surface and the ambient fluid is relative large. It depends strongly not only on the wedge angle but also on the wedge configuration. This is because the resulting buoyancy force from the temperature difference modifies the flow field and the surface heat transfer rate.

Excellent reviews of convection flows over a wedge under various effects or conditions have been presented by many authors. The effect of thermal radiation on MHD forced convection flow adjacent to a non-isothermal wedge in the presence of a heat source or sink was reported by Chamkha et al. [1]. While, convective thermal boundary layer flow past a wedge with suction/injection were studied by [2–6]. The Falkner-Skan flow with constant wall temperature and variable viscosity was investigated by Pantokratoras [7]. Unfortunately, many contemporary problems of convective heat transfer do not admit similarity solutions [8–10]. The nonsimilarity of boundary layers may results from a variety of causes, such as nonuniform wall temperature and free stream velocity. This method was developed by Minkowycz and Sparrow [11] and the results obtained were found to be in excellent agreement [12].

The present study addresses the buoyancy force effects on the boundary-layer flow over an impermeable wedge subject to a power law of stream velocity and wall temperature. The boundary layer equations are reduced to ordinary differential equations for similar and local non-similar flows by the well-known Falkner-Skan transformation. The aim of this work is to investigate a similarity and local non-similarity solutions by applying the Runge–Kutta–Gill integration scheme [13] in conjunction with the shooting method.
2. Mathematical Analysis

Let us consider the steady convective heat transfer of an electrically conducting fluid with the magnetic field $B_0$ applied transversally to the direction of fluid flow. The fluid is assumed to be a Newtonian, a viscous, and incompressible and its property variations due to temperature are limited to density and viscosity. The density variation and the effects of the buoyancy are taken into account in the momentum equation (Boussinesq's approximation).

\[
2.\text{ Mathmatical Analysis}
\]

Let the $x$-axis be taken along the direction of the wedge and $y$-axis normal to it. A constant suction or injection is imposed at the wedge surface, as shown in Fig. 1. Under these assumptions, the governing equations describing the problem are

\[
y = \infty : u_x = u_0 x^m, \quad T = T_\infty
\]

In the above equations $u, v$ are the corresponding velocity component along and perpendicular to the wall, $u_x$ is the flow velocity at outer edge of the boundary layer, $v$ is kinematics viscosity and $\alpha$ is thermal diffusivity, $\sigma$ the electric conductivity of the fluid, $\rho$ the fluid density, $B_0$ the constant magnetic field strength, $K$ the permeability of the wedge wall, $g$ the gravitational force, $\beta$ the coefficient of thermal expansion, $T_\infty$ the constant free stream temperature, $\nu_0$ the suction/injection velocity (constant). The third term on the right hand side of Eq.2 signifies the buoyancy force acting on the fluid elements, respectively.

3. Falkner-Skan Transformation

As first step in the development of the solution method, the following dimensionless stream function $\psi$ and similarity variable for wedge flow $\eta$ will be introduced by Falkner-Skan [7]

\[
\psi(x, \eta) = \sqrt{\frac{2u_x v_x}{1 + m}} f(x, \eta)
\]

\[
\eta = y \sqrt{\frac{(1 + m)u_x}{2v_x}}
\]

\[
\theta(x, \eta) = \frac{T - T_\infty}{T_w - T_\infty}
\]

where $u_x = u_0 x^m, \quad 0 < m < 1$ where $m = \beta_1/(2 - \beta_1)$ and $\beta_1$ is the Hartree pressure gradient parameter which corresponds to $\beta_1 = \Omega / \pi$ for a total angle $\Omega$ of the wedge.

The continuity equation (1) is satisfied by defining a stream function $\psi(x, y)$ such that

\[
u = -\frac{\partial \psi}{\partial x} \quad \text{and} \quad v = -\frac{\partial \psi}{\partial x}
\]

The velocity components can be expressed as

\[
u = U \frac{\partial f}{\partial \eta}
\]

and

\[
v = -\sqrt{\frac{2u_0 v_x}{(1 + m)x}} \left( f + \frac{x}{2} \frac{\partial f}{\partial \eta} + \frac{1}{2} \frac{\partial^2 f}{\partial \eta^2} + \frac{x}{2} \frac{\partial f}{\partial \xi} \right)
\]
Introducing a buoyancy force $\gamma$ and a wedge parameter $\xi$ defined, respectively, by $\gamma = Gr_x / Re_x^2$ and $\xi = kx^{1-m}$, the governing partial differential equations of the problem can be written in dimensionless equation as

$$f'' + \beta f' + \frac{2m}{1+m} \left(1 - f'^2\right) + \frac{2\gamma_1}{1+m} \frac{2(n+1-2m)}{m} \frac{\xi}{\eta} = 0$$  \hspace{1cm} (11)$$

$$\theta'' - \frac{2n}{1+m} Pr f' \theta' + Pr f' \theta'' = 0$$  \hspace{1cm} (12)

The boundary conditions can be written as

$$\eta = 0 : \frac{\partial f}{\partial \eta} = 0, \ f(0) = 0, \ \theta(0) = 1$$
$$\eta \to \infty : \frac{\partial f}{\partial \eta} = 1, \ \theta(\infty) = 0$$  \hspace{1cm} (13)

where

$$Gr_x = \left[ g\beta(T_w - T_x)x^2 \right] \nu^2$$ \hspace{1cm} (14a)$$Re_x = u_\infty x / \nu$$ \hspace{1cm} (14b)$$Pr = \nu / \alpha$$ \hspace{1cm} (14c)$$\gamma_1 = \frac{g\beta h}{u_0^2} \frac{2(n+1-2m)}{m-1}$$ \hspace{1cm} (14d)

The convective heat transfer on boundary layer flow past an impermeable wedge with a non-uniform stream velocity and a non-uniform wall temperature in the presence of buoyancy force is described by the system of partial differential equations (11) and (12), and its boundary conditions (13). In this system of equations, it is obvious that the non-similarity aspects of the problem are embodied in the terms containing $\xi$ and its partial derivatives with respect to $\xi$. A similarity solution exists when $n = 2m - 1$, whereas when $n \neq 2m - 1$ the system only has a non-similarity solution. Thus, with $\xi$-derivatives terms retained in the system of equations (11)-(13), it is necessary to employ a numerical scheme suitable for partial differential equations for the solution.

4. Solution of the Problem

4.1 Similarity Solution

By deleting term containing partial derivative with respect to $\xi$ and choosing $n = 2m - 1$, the governing equations (11) and (12) with boundary condition (13) have a similarity solution and reduce to the following system of ordinary differential equations

$$f'' + \beta f' + \frac{2m}{1+m} \left(1 - f'^2\right) + \frac{2\gamma_1}{1+m} \frac{2(n+1-2m)}{m} \theta \sin \Omega = 0$$  \hspace{1cm} (15)$$

$$\theta'' - \frac{2n}{1+m} Pr f' \theta' + Pr f' \theta'' = 0$$  \hspace{1cm} (16)

The boundary conditions can be written as

$$\eta = 0 : \frac{\partial f}{\partial \eta} = 0, \ f(0) = 0, \ \theta(0) = 1$$
$$\eta \to \infty : \frac{\partial f}{\partial \eta} = 1, \ \theta(\infty) = 0$$  \hspace{1cm} (17)

Thus, the velocity and temperature profiles are not affected by location parameter $\xi$, where $\xi$ represents a distance from stagnation point along $x$-axis.

4.2 Non-Similarity Solution

To derive equations for first level truncation and higher levels truncation, it is convenient to define the following new functions, Minkowycz, et al. [13]:

$$g = \frac{\partial f}{\partial \xi}, \ h = \frac{\partial g}{\partial \xi} = \frac{\partial^2 f}{\partial \xi^2}, \ \phi = \frac{\partial \theta}{\partial \xi}, \ \chi = \frac{\partial \phi}{\partial \xi} = \frac{\partial^2 \theta}{\partial \xi^2}$$  \hspace{1cm} (18)

At the first level of truncation, the terms $(f'g - f' \phi)$ and $(\phi'g - f' \chi)$, which appear on the respective right-hand sides of Eqs. (11) and (12), are deleted. The resulting differential equations are written in the form of

$$f'' + \beta f' + \frac{2m}{1+m} \left(1 - f'^2\right) + \frac{2\gamma_1}{1+m} \frac{2(n+1-2m)}{m} \theta \sin \Omega = 0$$  \hspace{1cm} (19)$$

$$\phi'' - \frac{2n}{1+m} Pr f' \theta' + Pr f' \phi'' = 0$$  \hspace{1cm} (20)
with the boundary conditions

\[ \eta = 0 : \quad \frac{\partial f}{\partial \eta} = 0, \quad f(0) = 0, \quad \theta(0) = 1 \]

\[ \eta \to \infty: \quad \frac{\partial f}{\partial \eta} = 0, \quad \theta(\infty) = 0 \]  \hspace{1cm} (21)

where \( \gamma = \gamma \frac{\pi}{2} \). The equations (19) – (21) can be regarded as ordinary differential equations with \( \xi \) as a parameter.

At the second level truncation, the governing equations for \( f \) and \( \theta \), Eqs. (11) and (12), respectively, are retained without approximation.

\[ f'' + f' + \frac{2m}{1+m} (1 - f^2) + \frac{2}{1+m} \gamma \xi \frac{2(n+1-2m)}{1-m} \]
\[ \times \Omega \sin \frac{\Omega}{2} \frac{1-m}{1+m} \xi (f' g' - g f'') = 0 \]  \hspace{1cm} (22)

\[ \phi'' + \frac{\partial f}{\partial \phi} - \frac{2n}{1+m} \text{Pr} f' \phi' = 0 \]  \hspace{1cm} (23)

with the boundary conditions

\[ g'(\xi,0) = 0, \quad g'('\xi,\infty) = 0, \]

\[ \phi(\xi,0) = 0, \quad \phi(\xi,\infty) = 0 \]  \hspace{1cm} (24)

Auxiliary equations for \( g, \phi \) and their boundary conditions are derived by taking the derivatives of Eqs. (22) and Eqs. (23) with respect to \( \xi \), and terms involving \( \xi \partial g / \partial \xi = \xi h, \quad \xi \partial g' / \partial \xi = \xi h', \) and \( \xi \partial \phi / \partial \xi = \xi \chi \) are deleted, the system of equations for the second level of truncation consists of the following equations (25) – (28) with the boundary conditions given by equation (29).

\[ f'' + f' + \frac{2m}{1+m} (1 - f^2) + \frac{2}{1+m} \gamma \xi \frac{2(n+1-2m)}{1-m} \]
\[ \times \Omega \sin \frac{\Omega}{2} \frac{1-m}{1+m} \xi (f' g' - g f'') = 0 \]  \hspace{1cm} (25)

\[ \phi'' + \frac{\partial f}{\partial \phi} - \frac{2n}{1+m} \text{Pr} f' \phi' = 0 \]  \hspace{1cm} (26)

\[ g'' + g' + \frac{2m}{1+m} (1 - g^2) + \frac{2}{1+m} \gamma \xi \frac{2(n+1-2m)}{1-m} \]
\[ \times \Omega \sin \frac{\Omega}{2} \frac{1-m}{1+m} \xi (g' f' - f g'') = 0 \]  \hspace{1cm} (27)

\[ \phi'' + \frac{\partial f}{\partial \phi} - \frac{2n}{1+m} \text{Pr} f' \phi' = 0 \]  \hspace{1cm} (28)

with boundary conditions

\[ f'(\xi,0) = g'(\xi,0) = f'(\xi,\infty) = 1, \]

\[ g'(\xi,\infty) = 0, \quad \theta(\xi,0) = 1, \]

\[ \phi(\xi,0) = 0 \]  \hspace{1cm} (29)

Equations (25) - (28) with boundary conditions (29) were solved numerically using Runge-Kutta Gill [14] with shooting methods. The computations have been carried out for various values of a wedge parameter \( \xi \), buoyancy force \( \gamma \) and a parameter of wall temperature \( n \). The comparison with previous published work has been done for similar case to that of buoyancy effect on forced convection along vertical plate investigated by Minkowycz [13]. Comparison result for the values of skin friction \( f''(0) \) and rate of heat transfer \( -\theta'(0) \) for various values of \( \gamma \) (Table 1) are found in excellent agreement.

| \( \gamma \) | Minkowycz et al. [13] | Present works |
|-------------|---------------------|---------------|
| \( f''(\xi,0) \) | \( -\theta'(\xi,0) \) | \( f''(\xi,0) \) | \( -\theta'(\xi,0) \) |
| 0           | 0.33206             | 0.29268       | 0.33206             | 0.29268       |
| 0.2         | 0.55713             | 0.33213       | 0.55707             | 0.33225       |
| 0.4         | 0.75041             | 0.35879       | 0.75007             | 0.35910       |
| 0.6         | 0.92525             | 0.37937       | 0.92449             | 0.37986       |
| 0.8         | 1.08792             | 0.39640       | 1.08700             | 0.39685       |
| 1.0         | 1.24170             | 0.41106       | 1.24062             | 0.41149       |
| 2.0         | 1.92815             | 0.46524       | 1.92689             | 0.46551       |
| 10          | 5.93727             | 0.64956       | 5.93665             | 0.64959       |

5. Results and Discussion

The problem of a convective heat transfer on boundary layer flow over an impermeable wedge with
a non-uniform wall temperature and buoyancy force is analyzed. The problem under consideration has both a similarity and non-similarity solution. The similarity solution exits when the power law of the velocity stream function \( m \) and the power law of the temperature difference (between the wall temperature and the ambient temperature) \( n \) is in the form of 
\[
1 - \frac{n}{m} = 0
\]
whereas the non-similarity solutions exist when 
\[
1 - \frac{n}{m} \neq 0
\].

Fig. 2(a). Similarity solutions on dimensionless velocity and temperature distribution with various values of buoyancy force \( \gamma_1 \) for \( n = 0.2 \)

The similarity solution is shown in Fig. 2(a) for the values of \( m = 0.6 \), \( n = 0.2 \) and the non-similarity solution is shown in Fig. 2(b) and 2(c) for the values of \( m = 0.6 \), \( n = 0.1 \) and \( m = 0.6 \), \( n = 0.3 \). Fig. 2(a) – 2(c) shows the dimensionless velocity profile \( f'(\eta) \) and the dimensionless temperature are affected by buoyancy force for parameters values \( Pr = 0.70 \), \( \Omega = 720^\circ \), and \( \xi = 1 \). The positive values of buoyancy force correspond to the assisting flow past a wedge and negative value of buoyancy force correspond to the opposing flow past a wedge.

Fig. 2(b). Local non-similarity solutions on dimensionless velocity and temperature distribution with various values of buoyancy force \( \gamma_1 \) for \( n = 0.1 \)

Table 2. Comparison of the values of \( f'(0) \) for various values of buoyancy force \( \gamma_1 \)

| \( \gamma_1 \) | \( n = 0.1 \) | \( n = 0.2 \) | \( n = 0.3 \) |
|---|---|---|---|
| -1.0 | 0.27102 | 0.29009 | 0.31428 |
| -0.5 | 0.71834 | 0.72554 | 0.73560 |
| 0 | 1.09044 | 1.09044 | 1.09044 |
| 1 | 1.73713 | 1.72493 | 1.70722 |
| 5 | 3.81476 | 3.76651 | 3.67810 |
| 10 | 5.95090 | 5.87287 | 5.68620 |
| 20 | 9.62585 | 9.47750 | 9.14890 |

Table 3. Comparison the values of \( -\theta'(0) \) for various values of buoyancy force \( \gamma_1 \)

| \( \gamma_1 \) | \( n = 0.1 \) | \( n = 0.2 \) | \( n = 0.3 \) |
|---|---|---|---|
| -1.0 | 0.41890 | 0.44403 | 0.46878 |
| -0.5 | 0.47858 | 0.50566 | 0.53200 |
| 0 | 0.51732 | 0.54719 | 0.57552 |
| 1 | 0.57362 | 0.60676 | 0.63774 |
| 5 | 0.70372 | 0.74466 | 0.78446 |
| 10 | 0.80100 | 0.84672 | 0.89400 |
| 20 | 0.92480 | 0.97878 | 1.03300 |

It is observed that an increase in the values of buoyancy force \( \gamma \) increases the fluid velocity inside the boundary layer but decreases the fluid temperature inside the boundary layer, whereas a decrease in the values of buoyancy force decreases the fluid velocity inside boundary layer but increases the fluid temperature inside the boundary layer as shown in Fig. 2(a) – 2(c). It is also noted that the free convection mode dominates to the mixed and forced convection mode for velocity profile.

Fig. 3(a) and 3(b) show the local non-similarity solutions on the dimensionless velocity and temperature profiles \( f'(\eta) \) and \( \theta'(\eta) \) for different
values of a wedge parameter $\xi$ for $n > 2m - 1$ and $n < 2m - 1$ with the presence of buoyancy force. In the case of $n > 2m - 1$, if $\xi$ goes to infinity then $\xi^{2(n-2m)/(1-m)}$ also goes to infinity and if $\xi$ approaches to zero then $\xi^{2(n-2m)/(1-m)}$ also approaches to zero. However, in the case of $n < 2m - 1$, when $\xi$ goes to infinity then $\xi^{2(n+1-2m)/(1-m)}$ goes to zero and if $\xi$ approaches to zero then $\xi^{2(n+1-2m)/(1-m)}$ approaches to infinity.

The values of $\xi$ correspond to the distance from the leading edge of the wedge along $x$-axis. The variation of the value of $\xi$ actually means variation of the distance along the wedge. In the case of a negative power of a wedge parameter $n < 2m - 1$, for example, $m = 0.6$ and $n = 0.1$, it is observed that an increase in the values of $\xi$ decreases the fluid velocity but increases the temperature distribution inside the boundary layer as shown in Fig. 3(a). However, in this case of a positive power of a wedge parameter $n > 2m - 1$, for example $m = 0.6$ and $n = 0.3$, an increase of the value of $\xi$ increases the fluid velocity but decreases the temperature profile inside the boundary layer as seen in Fig. 3(b).

Fig. 3(a). Local non-similarity solution on dimensionless velocity and temperature distribution with various value of $\xi$ with $m = 0.6$, $n = 0.1$

Fig. 3(b). Local non-similarity solution on dimensionless velocity and temperature distribution with various value of $\xi$ with $m = 0.6$, $n = 0.3$

Fig. 4(a). Effect of non-uniform wall temperature on dimensionless velocity and temperature distribution for $\xi = 0.1$

Fig. 4(b). Effect of non-uniform wall temperature on dimensionless velocity and temperature distribution for $\xi = 3$

Fig. 4(a) and 4(b) describe the effects of the parameter wall temperature on the dimensionless velocity and temperature distribution on a
convective boundary layer flow over impermeable wedge in the presence of a non-uniform wall temperature for $\xi \neq 1$ and $n > 2m - 1$. It is observed that an increase in the values of the wall temperature parameter $n$ decreases the fluid velocity and the fluid temperature distribution inside the boundary layer as shown in Fig. 4(a). However, for $\xi > 1$ and $n > 2m - 1$, an increase in the values of wall temperature parameter $n$ increases the fluid velocity inside the boundary layer but decreases the fluid temperature distribution inside the boundary layer as depicted in Fig. 4(b).

6. Conclusions
This paper studied the effects of buoyancy force on convective heat transfer flow over a impermeable wedge with a power law of wall temperature and free stream velocity. The problems under considerations have a similarity and non-similarity solutions. The results are presented graphically and the conclusion is drawn that the flow field and other quantities of physical interest are significantly influenced by these parameters. Comparisons with previously published works are performed and the excellent agreements between the results are obtained.

We conclude the following from the results and discussions:
- The velocity of fluid flow inside the boundary layer increases whereas the temperature inside the boundary layer decreases with increasing buoyancy force. Positive buoyancy force means assisting flow and negative buoyancy force means opposing flow.
- The velocity of the fluid flow inside the boundary layer decreases but the temperature inside the boundary layer increase when a wedge parameter goes to infinity, in the case of a negative power of a wedge parameter. However, the velocity of the fluid flow inside the boundary layer increase but the temperature inside the boundary layer decreases when a wedge parameter goes to infinity, in the case of a positive power of a wedge parameter.
- When a wedge parameter values approaches to zero, the velocity and the temperature distribution of fluid flow inside the boundary layer decreases with increasing the wall temperature parameter. However, when a wedge parameter value goes to infinity, the velocity of the fluid inside the boundary layer increases whereas the temperature of fluid flow inside the boundary layer decreases.
- It is hoped that the present investigation may be useful to design the heaters and coolers of electrical and mechanical devices and in lubrication of machine parts.

7. Nomenclature
$u, v = $ velocity in $x$ and $y$ direction
$u_\infty = $ flow velocity away from the wedge
$g = $ acceleration due to gravity
$T = $ temperature of the fluid
$T_w = $ temperature of the wall
$T_\infty = $ temperature far away from the wall
$\beta = $ coefficient of thermal expansion
$\alpha = $ thermal diffusivity
$\rho = $ density of the fluid
$\Omega = $ total angle of the wedge
$c_p = $ specific heat at constant pressure
$\nu = $ kinematic viscosity
$Gr = $ Grashof number
$Re = $ Reynolds number
$Pr = $ Prandtl number
$\gamma_1 = $ Buoyancy parameter

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