Analysis of measurement errors for Thomson diagnostics of non-Maxwellian plasmas in tokamak reactors

P A Sdvizhenskii1,5, A B Kukushkin1,2, G S Kurskiev3, E E Mukhin3 and M Bassan4
1 National Research Center “Kurchatov Institute”, Kurchaton square 1, Moscow, 123182, Russian Federation
2 National Research Nuclear University MEPhI, Kashirskoe shosse 31, Moscow, 115409, Russian Federation
3 Ioffe Physical Technical Institute, Politekhnicheskaya 26, St. Petersburg, 194021, Russian Federation
4 ITER Organization, Route de Vinon sur Verdon, 13115 St Paul Lez Durance, France

E-mail: sdvinpt@gmail.com

Abstract. The study is stimulated by the expected noticeable deviation of the electron velocity distribution function (eVDF) from a Maxwellian under condition of a strong auxiliary heating of electron plasmas in tokamak-reactors. The key principles of accuracy estimation of the Thomson scattering diagnostic of non-Maxwellian plasmas in tokamak-reactors are presented. The algorithm extends the conventional approach to the assessment of non-Maxwellian plasmas measurements errors for a broad class of deviations of the eVDF from a Maxwellian. The algorithm is based on solving the inverse problem many times to determine main parameters of the eVDF with allowance for all possible sources of error and statistical variation of the input parameters of the problem. The method is applied to a preliminary analysis of the advantages of the formerly suggested use of various wavelengths of probing laser radiation in the Thomson diagnostics of non-Maxwellian plasma on the example of the core plasma Thomson scattering diagnostic system which is under design for ITER tokamak. The results obtained confirm the relevance of the diversification of the probing laser radiation wavelength.

1. Introduction
The Thomson Scattering (TS) diagnostics of high electron temperature $T_e$ in the core plasma in tokamak reactors (DEMO and ITER) has to operate in a limited spectral range of a strongly broadened TS spectrum and allow for possible deviation of the electron velocity distribution function (eVDF) from a Maxwellian under condition of a strong auxiliary heating. The analysis [1] of opportunities to enhance the accuracy of the core plasma TS diagnostics suggested the use of several probing wavelengths to increase the number of signals from different spectral channels and to be able to interpret the TS data with allowance for possible deviation from a Maxwellian eVDF in the range of weakly/moderate superthermal energy, with account of data for higher energy from other diagnostics (e.g., from that suggested in [2]).

In this paper we present the results of further development of the method [3-5] for detailed accuracy analysis of the Thomson scattering diagnostics of non-Maxwellian plasmas in tokamak-
The method is based on the correct error assessment to determine main parameters of the eVDF from the TS spectrum measurements with allowance for all possible sources of error. The method is applied for analyzing the advantages of using several probing wavelengths for the core plasma TS diagnostics for Maxwellian and non-Maxwellian plasmas via solving an inverse problem for error assessment. This includes evaluation of the possibility to recover, from visible and infrared light spectral range ~400—1000 nm, (i) the electron temperature $T_e$ of the bulk (Maxwellian) plasma up to $\sim 40$ keV and (ii) a moderate anisotropy of the eVDF in electron pitch angles in the weakly/moderate superthermal energy range. A particular example of the ITER core plasma TS system design is considered for a test of suggested approach, and a comparative analysis of conservative and advanced approaches is given.

### 2. Algorithm of error assessment

The algorithm to assess the measurement error should be formulated in the frame of a synthetic diagnostics. Such a diagnostic generates “phantom” experimental data and allows direct comparison of the pristine (i.e. taken as known, “assumed”) and the recovered values of diagnosed parameters. The number of photoelectrons in a given spectral channel due to Thomson scattering is as follows:

$$
[ N_{\text{ph-el}} ]_{\text{Laser}} = \frac{E_{\text{L}}}{h\nu_0} \Delta x_{\text{L}} \Delta \Omega_{\text{pupil}} n_e \int_{\Delta x_{\text{L}}} \eta_e(X) \eta_{\text{Laser}}(X) \sigma_{\text{TS}} \mathrm{d}X,
$$

where $X = \lambda/\lambda_0 - 1$, $\Delta X_j$ is the spectral width of the $j$-th channel; $\lambda$ and $\lambda_0$, the wavelength of scattered and probing radiation; $E_{\text{L}}$, laser pulse energy; $\Delta x_{\text{L}}$, the length of a radiating cylinder viewed by the detector (“scattering length”); $n_e$, electron density; $\Delta \Omega_{\text{pupil}} = \Delta S_{\text{pupil}} (r_{\text{emis}} - r_{\text{pupil}})$, where $\Delta S_{\text{pupil}}$ is the area of the pupil, and the vectors stand for the coordinate of the radiating cylinder and the pupil; $r_{\text{emis}}$ electron classical radius; $\eta_e$ and $\eta_0$, directions of incident and scattered light; $\mathbf{e}_i$ and $\mathbf{e}_s$, polarization vectors of incident and scattered light; the factors $\eta$ allow for the properties of optical system. The normalized cross-section $\sigma_{\text{TS}}$ of the Thomson scattering (i.e. incoherent, non-collective scattering by relativistic electrons with neglect of the electron’s recoil in the elementary act of scattering the photon) can be calculated with formulas [6-8].

In the present paper we consider the following model eVDF, which accords with results of the eVDF numerical modelling studies:

$$
f(p) = (1 - \delta_{\text{Hot}}) \cdot f_{\text{Maxw}}(p) + \delta_{\text{Hot}} \cdot f_{\text{Hot}}(p_{||}, p_{\perp}),
$$

$$
f_{\text{Hot}}(p_{||}, p_{\perp}) = C_{\text{Hot}} \exp \left[-mc^2 \frac{\sqrt{1 + (p/lmc)^2} - 1}{p^2 T_{e||} + p_{\perp}^2 T_{e\perp}} \right]
$$

Here $f_{\text{Maxw}}(p)$ is the relativistic Maxwellian distribution; $C_{\text{Hot}}$, normalization factor (in momentum space); $p_{||}$ and $p_{\perp}$, the components of momentum $p$, parallel and perpendicular to local magnetic field; $\delta_{\text{Hot}}$, total fraction of superthermal electrons. The background signal is assumed to be measured in a separate time window of similar duration. Different lasers are assumed to act in different but very close time instants.

The TS-based eVDF diagnostic algorithm [3-5] includes the calculation of the number of photoelectrons, $[ N_{\text{ph-el}} ]_{\text{Laser}}$, in a given spectral channel of the detector (where $j$ numerates spectral channels) in terms of the normalized cross-section $\sigma_{\text{TS}}$ of the Thomson scattering, which is averaged over the assumed model eVDF (2), (3). For each spectral channel, the difference between total, $[ N_{\text{ph-el}} ]_{\text{Total}}$, and background, $[ N_{\text{ph-el}} ]_{\text{Background}}$, signals is calculated. Randomization of input parameters is applied to estimate the diagnostic’s accuracy (for $[ N_{\text{ph-el}} ]_{\text{Laser}} >> 1$, one can take a Gaussian with the average value $[ N_{\text{ph-el}} ]_{\text{Laser}}$ and the mean square deviation

$$
\sigma_j = \left\{ [ N_{\text{ph-el}} ]_{\text{Laser}} + 2 [ N_{\text{ph-el}} ]_{\text{Background}} + 2 [ \sigma^2 ]_{\text{Ampl}} \right\}^{1/2},
$$

where $[ \sigma_{\text{Ampl}} ]$ describes the noise of
The factor 2.5 in (5) corresponds to 98% probability to find the value of \( B \) in the range \( \{B_{\text{assumed}} - 2.5\delta B, B_{\text{assumed}} + 2.5\delta B\} \), where \( \delta B = \langle(B - B_{\text{assumed}})^2\rangle^{1/2} \).

For a non-Maxwellian eVDF, one has to analyze the accuracy of measuring the mean kinetic energy:

\[
\langle E_{\text{kin}} \rangle = T_\parallel - K_2 (1/T_\parallel) / K_2 (1/T_\parallel); \quad E_{\text{kin}} = \sqrt{(p/mc)^2 + 1} - 1,
\]

where \( K_2 \) is the MacDonald function of the 2nd order, the prime denotes the derivative. In the non-relativistic case, this yields the well-known relation \( \langle E_{\text{kin}} \rangle = (3/2)T \).

Figure 1. The comparison of errors of recovering the plasma parameters. (a) Calculation of the phantom spectrum for the non-Maxwellian eVDF and interpretation assuming the non-Maxwellian eVDF of Equations (2), (3): conventional approach with one color probing radiation 1064 nm (blue); multicolor (946; 1064; 1320 nm) probing radiation (red). (b) Calculation of the phantom spectrum for the non-Maxwellian eVDF and interpretation assuming a Maxwellian eVDF: one color probing radiation 1064 nm (blue); the same but for multicolor (1064; 1320 nm) probing radiation (green). Calculation of the phantom spectrum for the Maxwellian eVDF and interpretation assuming the non-Maxwellian eVDF of Equations (2), (3) (yellow).

Figure 1 shows the results for 200 runs of solving the inverse problem for the following input parameters which are considered in the ongoing analysis of the core plasma TS system in ITER. Figure 1(a) shows the results in the case of calculating the phantom spectrum for a non-Maxwellian eVDF and interpreting it assuming the non-Maxwellian eVDF of Equations (2), (3): the case of three lasers with \( \lambda_0 = 1064 \) nm and the case of three lasers with different probing wavelengths \( \lambda_0 = 946; 1064; 1320 \) nm; \( n_e = 3 \times 10^{19} \) m\(^{-3} \); \( E_{\text{Las}} = J \) (the same for all lasers); \( \eta_t = 0.168 \) is the total transmission factor of optical system (a flat efficiency of 30% for the collection optics, a 70% packing efficiency for the fiber bundles, a 80% optical efficiency of the spectrometers); the quantum detector sensitivity, \( \eta_q \), is taken to have the APDs spectral sensitivity like the Excelitas C30956EH in the range above 650 nm and like the Hamamatsu S8664 in the range below 650 nm; \( [\sigma_j]_{\text{Ampl}} = 50 \); \( \Delta x_{\text{Las}} = 0.07 \) m;
\[ \Delta \Omega_{\text{pupil}} = 1.38 \times 10^{-3} \text{ sr}. \] The angle between \( \mathbf{n}_0 \) and \( \mathbf{n} \) is taken 159.5º that corresponds to observation of the plasma column center; linear polarization vectors \( \mathbf{e}_0 \) and \( \mathbf{e} \) coincide and are perpendicular to scattering plane. The background signal is formed by the direct signal from the plasma Bremsstrahlung along the line of sight and the reflected-from-the-wall light which is determined by the line radiation emitted in the divertor (the background signal data are provided in [9]). Errors for the parameter \( \delta_{\text{hot}} \) are about several hundred percent (not shown in the figure 1(a)). Note that the high error of recovering the parameters, which describe particular type of the deviation from a Maxwellian (especially, parameter \( \delta_{\text{hot}} \)), does not influence the accuracy of recovering the mean energy of the non-Maxwellian eVDF. Thus, the inverse problem solution is stable with respect to recovery of the mean energy regardless of particular form of the deviation from a Maxwellian in the thermal and weakly superthermal range of electron energy.

Figure 1(b) shows the results of similar error estimations for the case where the phantom spectrum is taken for a non-Maxwellian VDF whereas the interpretation of this spectrum assumes a Maxwellian VDF, and the opposite case of a small deviation from a Maxwellian VDF and interpretation of data assuming a non-Maxwellian VDF (2), (3). Note that the increase of the errors of multicolor probing wavelength compared to the one color probing wavelength is attributed to a partial loss of accuracy in solving the inverse problem with an increased number of unknown, sought-for parameters.

3. Conclusions

The results of analysing the opportunity to enhance the accuracy of the core plasma Thomson scattering diagnostics in tokamak-reactors -- under conditions close to those in the diagnostics under design for ITER - illustrate the trend towards increasing the accuracy of measuring the parameters of non-Maxwellian electron velocity distribution function (eVDF) with increasing range of \( T_e \) measurements that confirms the relevance of the diversification of the probing laser radiation wavelength.

The method also enables one to evaluate the accuracy of measuring the mean energy of electrons in non-Maxwellian plasmas under assumption of a Maxwellian eVDF. The results obtained give quantitative illustration of the well-known high sensitivity of Thomson diagnostics to the main Maxwellian bulk and appreciably less sensitivity to a small superthermal fraction of the eVDF.

The results obtained show the necessity of a detailed analysis of the Thomson scattering diagnostics accuracy in tokamak-reactors.

The views and opinions expressed herein do not necessarily reflect those of the ITER Organization.

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