Abstract
Understanding the vibrational characteristics of power transformers is significantly important in their design and monitoring. In this contribution, a model with a multi-physics coupling simulation of the electrical circuit, magnetic field, and solid mechanics is developed to investigate the characteristics of the transformer vibration. After describing the model, the harmonic contents of the vibration signals and their variation in the case of mechanical faults are studied. It is shown that under normal operating conditions, the fundamental vibration frequency of 100 Hz has the maximum amplitude, while in the case of mechanical faults, the amplitudes of 200 Hz and 300 Hz harmonics increase dramatically compared to the fundamental harmonic. The influence of vibration sensor position is investigated too, which indicates that the area near the tank bottom is the best position to gather vibration signals. Moreover, the mechanical resonance frequencies of the transformer, along with their mode shapes, are addressed in this paper. Finally, the influence of mechanical changes on the vibration energy distribution in the tank surface is explored. The results of the paper suggest possible diagnosis methods for condition monitoring of transformers, such as using the vibration energy distribution on the tank surface or analyzing the vibration harmonics.

Keywords: Condition monitoring, finite element method (FEM), transformer winding, vibration analysis, vibration modes, winding deformation;

1. Introduction

Power transformers are important components in the power system that have a major impact on the reliability of the power delivery. Accordingly, various diagnostic and monitoring tools have been developed for them to reduce their unexpected failures while these diagnostic methods are still being developed. Referring to recent research, transformer failures can be classified as electrical, mechanical, or thermal defects [1], [2]. Several diagnostic approaches for power transformers, such as dissolved gas analysis [3], partial discharge measurements [4], [5], leakage reactance measurements [6], and frequency response analysis [7], have been proposed in the literature to prevent unexpected failures. The use of vibration signals to evaluate transformer health is a relatively new approach compared to other transformer condition monitoring approaches, and its research is still in its early stages. Because the vibration-based method is non-intrusive, it is an appropriate online approach.

In a large electric power system, the currents in electrical apparatuses such as transformers increase as a result of higher loading in the power transformers, which increases the electromagnetic forces produced [8], [9]. Furthermore, because electromagnetic forces are proportional to the square of the current flowing through the transformer windings, mechanical stress and mechanical oscillations on the windings are significantly increased in the event of a short circuit or inrush current [10], [11]. The corresponding stresses can cause the failure of the transformer internal structure, which is one of the most difficult problems to be distinguished. The winding displacement or deformation are examples of serious problems among various mechanical failures [12]. For detecting such mechanical problems in a transformer, vibration-based monitoring systems are proposed in the
Vibration-based condition monitoring refers to the use of non-destructive sensing and analysis of system characteristics in the time, frequency, or modal domains for the purpose of detecting changes, which may indicate damage or degradation [13]–[18]. In recent years, researchers have also conducted research on the recognition of transformer vibrational properties. In [19], Fahnoe et al. conducted a study of the vibrational harmonics generated by the transformer core under magnetostriction. In this study, a large number of modes in which core various structures may vibrate were determined experimentally and analytically. In [20], a numerical study of membrane vibration properties in transformer oil was performed. This study showed that mechanical energy is transformed into thermal energy during vibration and that its amplitude continuously decreases with time. In [21], Foster et al. employed simulation techniques to investigate the impact of material and processing on the vibration properties and vibration frequency spectrum of a power transformer. Researchers in [22]–[25] developed mathematical models of the electromagnetic vibration of the power transformers core by coupling the electromagnetic-field theory with the structural mechanics theory. The magnetostriction of silicon steel sheets was simulated using these models. In [26], Ji et al. explored in detail the connection between the vibration of the winding and the iron core developing a technique for obtaining the features of the transformer's vibration signal using wavelet analysis. In [27], Yu et al. investigated the connection between the tightening force and the natural frequency of the windings using modeling and testing. This study determined that winding pretension can influence the natural frequencies and vibration modes.

In recent years, researchers have also paid more attention to vibration-based techniques for detecting the condition of transformer windings, which can be divided into signal-based and model-based methods. The signal-based methods typically extract information from vibration signals. Researchers in [28]–[30] have developed a set of indicators to evaluate the condition of transformers using vibration signals. In these studies, faulty transformers show an increase in vibrational harmonics compared with healthy transformers. The use of model-based methods is the second approach to vibration diagnostic methods. For example, mathematical models were used to detect the mechanical condition of the winding in [31]–[33]. These studies use transformer parameters such as electric current, voltage, and temperature as model inputs.

In most studied research, the used methods were based on analytical models and experimental data. The most significant gap in these studies is that they emphasize only certain vibrational features and are unable to present comprehensive information about the transformer's vibration mechanisms. To obtain a precision plan and reach the proper results, a robust method is needed, which considers the linear and non-linear characteristics of the transformer core and winding in terms of electromagnetic and mechanical features. Accordingly, the finite element method (FEM) is an alternative technique for performing structural dynamic and electromagnetic field analysis because it provides precise insight into how the various elements of a design interact [34], [35]. According to the aforementioned points, the motivation behind this paper is to provide a vibration-based method with regard to experimental tests limitations. Because experimental tests are destructive to transformer windings, this paper uses a multi-physics coupled simulation to find a solution for diagnosing three types of winding defects named LV winding free buckling, winding elongation and winding twisting. Accordingly, the main contributions of the paper can be summarized as follows:

- Developing a numerical model for vibration analysis of a transformer, which can involve accurate modeling complex structures such as core and winding assemblies. In past papers, these modeling considerations are not applied to models simultaneously;
- Employing the FEM to analyze and study the problem of the paper, which uses a multi-physics coupled simulation of the electrical circuit, magnetic field, and solid mechanics to simulate the characteristics of the transformer vibration;
- Investigating the frequency response function (FRF) of the transformer structure vibration by means of vibration modal simulation, which extends transformer modal analysis to a new level;
• Presenting a useful framework to analyze the response of transformer vibration under various operating conditions and adopt them for vibration behaviors extraction, which can be directly applied to transformer condition monitoring;

• Providing an analysis framework of the natural frequencies of the winding-related modes in the presence of winding damages as well as extracting the pattern of vibration energy distribution on the transformer tank, which has the potential to be used for the mechanical monitoring of transformers. In past researches, the energy distribution patterns have not been analyzed.

The rest of the paper is structured as follows. Section 2 studies the calculation of electromagnetic forces of winding. Section 3 presents transformer modeling considerations in modeling the active part and the tank. Section 4 describes a multi-physics coupling simulation of the circuit, magnetic field, and solid mechanics. Section 5 discusses vibration spectrum analysis, including the influence of sensor positions on the tank, and compares vibration harmonics under winding mechanical faults. Afterward, Section 6 explores the mechanical resonance frequency and their corresponding mode shapes. Moreover, the effect of a possible mechanical change on the vibration energy distribution is addressed in this section. Finally, Section 7 concludes this paper.

2. Theoretical Background

The transformer vibration has two main constituents: (i) winding vibration caused by the electromagnetic force generated by the interaction of the current in a winding with leakage flux, and (ii) iron core vibration caused by the magnetostriction forces in the silicon steel sheets [36], [37]. In addition to the vibrations generated by the core and the windings, other components also vibrate during the operation of the transformer. Each tap changer operation produces a specific vibration wave that propagates through the oil and the structure of the transformer. Vibration analysis is usually based on periodic oscillations and does not account for tap changer vibrations, which appear only after a tap operation and produce temporary vibrations. Furthermore, experimental results show that the main components of tap changer vibrations have a frequency of a few kHz and that the large difference in frequency amplitude between the vibrations generated by the core and windings justifies analyzing them separately. The cooling system (oil pumps and fans) also generates vibrations that are added to the main vibrations produced by the core and the windings. These components do not work continuously, because depending on the operating temperature of the transformer, they turn on or off, but a periodic vibration signal appears during operation. In general, vibrations in the core/winding occur at a variety of frequencies up to several hundred Hz. These frequencies appear as even, odd, and rational multiplications of the fundamental frequency, with 100 Hz and its integer multiples being the most dominant components. Vibrations also are highly correlated with transformer loads, with the 100Hz, 200Hz, and 300Hz components being the most sensitive [38], [39]. Vibrations with a frequency less than 100 Hz are then usually the production of cooling systems and the operation of oil pumps [1], [40].

By putting aside the vibration of the cooling system and the tap-changer, two vibration sources remain, including the core and the winding. Moreover, the vibration of the winding is significantly larger than the vibration of the core during both the normal operating and short circuit conditions. As a result, the observed vibration signals can be estimated as comprising mainly the winding vibration signals. The current flowing through the transformer winding can be expressed as follows:

\[ i(t) = I \cos \omega t \]  

(1)

In Equation (1), \( \omega \) is the angular frequency, and \( I \) denotes the current amplitude. The leakage magnetic flux around the transformer winding is a function of current and changes with time. When the winding is dislocated, the distribution of the leakage magnetic flux around the winding also changes. Magnetic flux density (B) can be described as [41], [42]:

3
\[
B(t) = \frac{u_0}{4\pi} i(t) \int_{l}^{r} \frac{dl \times \hat{r}}{r^2}
\]

(2)

where \( r \) is the radius and \( dl \) is infinitesimal elements of winding. Except for \( i(t) \), all values at a given place in space are constant. As a result, \( B(t) \) can be simplified as follows:

\[
B(t) = kI \cos \omega t
\]

(3)

where \( k \) is the proportionality constant between \( I \) and \( B \) the, and \( I \) is the current amplitude. The axial leakage flux density \( B_a \) interacts with the current passing from the windings to produce a radial force \( F_r \). As well, the interaction between the current and radial component of the leakage flux \( B_r \), generates the axial forces \( F_a \). Then using Equation (4), the electromagnetic forces in the radial and axial directions can be defined as follows [41]:

\[
F_r = i(t)B_a(t)2\pi R, \quad F_a = i(t)B_r(t)2\pi R
\]

(4)

\[
F = \sqrt{F_r^2 + F_a^2} = 2\pi RkI^2\left(\frac{1}{2} + \frac{1}{2}\cos \omega t\right)
\]

(5)

These calculated forces are then utilized as an excitation source for calculating the vibrations in the transformer structure. After describing the model in section 3, the rest of the relevant equations are given in sections 4.1 and 4.2.

3. Model description

In this study, a single-phase transformer with the disc-type winding configuration similar to a large power transformer is considered for the modeling to simplify the electromagnetic loading conditions applied to the model. It is expected that simulating a three-phase transformer does not discredit the validity of the vibration model presented in this study, provided that the model loading conditions are appropriately determined.

3.1. Modeling of the active part

Because the transformer construction is a complicated system with many different components, only those that affect the vibration response are modeled in this study. Figure 1 describes the transformer’s three-dimensional (3D) FEM model. Tables 1 and 2 also introduce the model’s parameters, such as the geometric, electrical, and mechanical properties of the transformer components.

In practical transformers, windings are frequently made up of continuously transposed copper strands to transmit current with low eddy current losses. This design satisfies the electrical requirements, but it makes vibration modeling more difficult. To reduce the time it takes to solve the FEM model, the winding structures must be replaced with a simpler homogenized 3D model with the same form and dimension. The next challenge in the simulation is the layered core of the transformer. The meshing becomes very dense in the core due to its layered nature and, therefore, the model needs a high computational power to be solved. In transformer vibration modeling, how to deal with this challenge is crucial. An analogous methodology based on Young’s modulus of the transformer core structure is offered to overcome this problem. Young’s modulus is a mechanical property that measures the tensile stiffness of a solid material. To clarify this concept, a test specimen is made up of SiFe sheets, as shown in Figure 2. The specimen is made up of 100 layers of SiFe sheets with a thickness of 0.3 mm. A bolt hole is bored at each end of the SiFe sheet to clamp the laminations once assembled. According to the Euler–Bernoulli beam theory, a simulation is carried out to determine the natural frequencies and, hence, Young’s modulus. The natural frequency can be calculated analytically as follows [43]:
where $\omega_n$ is the natural frequency, $E$ is the elastic modulus, $\beta_n L$ is a constant relating to each natural frequency, $A_0$ is the area of cross-section, $\rho$ is the material density, $L$ is the length of core laminate, and $I$ is the area moment of inertia.

The anisotropic Young's modulus of the test specimen is determined by measuring the vibration in both the in-plane and out-of-plane directions independently. The out-of-plane direction is along the Z-axis in Figure 2. The results of the input mobility test for in-plane and out-of-plane vibrations are reported in Table 3. Two resonance frequencies of 2562 Hz and 5388 Hz are detected in the in-plane testing findings. The calculated mode shapes at the first resonance frequency always agree well with the characteristics of SiFe sheets. Equation (6) calculates the test specimen modulus of elasticity to be $E = 174.5$ GPa, while SiFe has an elastic modulus of around 180 GPa, which is close to the estimated elastic modulus in the in-plane direction. This means that the SiFe sheet's lamination has little effect on material characteristics in the in-plane direction, and the core lamination can be considered as a solid body. Compared to the in-plane direction, the natural frequencies in the out-of-plane direction are significantly lower. This means that the vibrational properties of the core are very anisotropic. However, in practice, the core stiffness is controlled not only by the elastic modulus of the SiFe core but also by the clamping forces of the core holding structure. By considering the clamping structure, the solid body shows a similar behavior to the layered core too. Based on the aforementioned points, the core is then modeled as a solid body, which has different Young’s Modulus along various axes.

### 3.2. Modeling of the tank

There are various reasons for modeling the transformer tank. The most important reason is the requirement to detect instances in which vibrations propagating from the active part excite a natural frequency of the tank construction. There are two ways in which the mechanical vibrations of the transformer winding can be transmitted to the tank wall [44], [45]:

- **Direct transfer through mechanical connections:** A direct coupling between the tank and the active part is achieved through mechanical connections, with transmission taking place via the tank base, where the active part is located.
- **Indirect transfer through the insulating fluid:** The fluid-structure interaction provides an indirect link between the insulating fluid and the tank structure, allowing vibration waves generated by the active part to reach the tank structure.

The vibration analysis technique can diagnose internal defects in transformers as the mechanical features of the windings affect the tank vibration response. Therefore, by measuring the vibration signals from the surface of the transformer tank, the mechanical health condition of the windings can be analyzed. The tank structure model is illustrated in Figure 3. Also, a summary of the steps taken to utilize the FEM model to predict the vibration characteristics of a transformer is presented in Figure 4.

### 4. Multi-physics model for transformer vibration

#### 4.1. Electromagnetic field model

To calculate electric and magnetic fields, the primary winding is connected to a 50 Hz alternating source, while the secondary winding is short-circuited. The electric field equation can be defined as follows [42], [46]:

\[
\omega_n = (\beta_n L)^2 \sqrt{\frac{E I}{\rho A_0 L^3}}
\]
\[-\nabla \cdot (\epsilon_0 \varepsilon_r \nabla V) - \nabla \cdot (\sigma \nabla V - J) = 0 \quad (7)\]

where \(\epsilon_0\) is the dielectric constant in free space and equals \(8.85 \times 10^{-12}\) F/m, \(\varepsilon_r\) is the relative dielectric constant, \(\sigma\) is conductivity, \(J\) is external current density, and \(V\) is the potential. The current density \(J\) is then incorporated into the magnetic-field differential equation as follows [46]:

\[\sigma \frac{\partial A}{\partial t} + \nabla \times (\mu_0^{-1} \mu_r^{-1} \nabla \times A) = J \quad (8)\]

where \(\mu_0\) is permeability in free space and its value is \(4\pi \times 10^{-7}\) H/m, \(\mu_r\) is the relative permeability, and \(A\) is magnetic vector potential. In addition, the magnetic field's governing equation is as follows [47]:

\[B = \mu_0 \mu_r H = \nabla \times A \quad (9)\]

\[J = \sigma E + J_e \quad (10)\]

where \(B\), \(H\), and \(E\) represent the magnetic flux density, the magnetic field, and the electric field, respectively. The meshes were built using adaptive refinement methods, thus they are substantially finer in areas of high variation and strength of the magnetic field, as shown in Figure 5. Figure 6 also shows mesh quality, and the closer the value is to 1, the higher the mesh quality. Figure 7(a, b) shows the results of the magnetic model at the point where the current going through the winding reaches 70% of the peak value. At the time, the highest \(B\) in the winding area is 0.06 T, whereas the maximum \(B\) in the core is 1.6 T. The simulation findings are consistent with the single-phase transformer specifications, which indicates the model correctness implicitly. Now, the model can be developed to simulate the vibration analysis as will be discussed in the next subsection.

4.2. Mechanical vibration model

Winding vibration is influenced by the mass inertia, elasticity, and damping of the winding. The differential equations of motion for solid mechanics are given in Equation (11) [46], [48]:

\[M_i \frac{d^2 z}{dt^2} + C_i \frac{dz}{dt} + K_i z = f(t) \quad (11)\]

where \(M_i\) denotes the mass matrix, \(C_i\) denotes the damping coefficient matrix, \(K_i\) denotes the stiffness coefficient matrix, \(z\) denotes the winding’s deformation (displacement), the first derivative of \(z\) indicates the winding’s deformation velocity, the second derivative of \(z\) denotes the winding’s deformation acceleration, and \(f(t)\) denotes the force magnitude. The displacement equation of the transformer winding can be calculated by adding Equation (5) into Equation (11) [41], [46]:

\[z(t) = Ye^{C_i t / 2M_i} \sin(\omega_0 t + \theta) + G \cos(2\omega t + \varphi) \quad (12)\]

in which:

\[G = \frac{BI^2}{\sqrt{(K_i - 4M_i \omega^2)^2 + 4C_i \omega^2}} \quad , \quad \tan \varphi = \frac{-2C_i \omega}{K_i - 4M_i \omega^2} \]
In Equation (12), \( Y \) and \( \theta \) are set by the initial conditions, \( \omega \) and \( \omega_0 \) are the current angular frequency and the natural oscillation frequency of the transformer winding, \( I \) is the current amplitude, and \( B \) is the magnetic flux density. Afterward, the acceleration of the winding vibrations according to Equation (13) can now be calculated using the second derivative of Equation (12) [41]:

\[
a = -\omega_0 Y e^{\frac{C_1 t}{2M}} \sin(\omega_0 t + \theta) - 4\omega^2 G \sin(2\omega t + \varphi)
\]  

(13)

Furthermore, for the solution to converge, the fixed boundary condition is applied at both ends of the winding. It is noteworthy that both winding ends are clamped by press rings and, therefore, the fixed boundary condition is a suitable choice.

Figure 8 shows an output result of the multi-physics model where the radial forces on the transformer. The radial forces of the windings, as shown in Figure 8, cause mechanical stress on the high voltage winding in the outward direction, while the internal low voltage winding is compressed inwards. These results are in accordance with the expected behavior of the winding. This indicates the model functionality in investigating the mechanical structure of the transformer with electrical inputs.

5. Analysis of transformer vibration signals

In this section, the vibration signals of the winding are measured from the aforementioned model and analyzed. To guarantee that the location of the tank where the vibrations are measured appropriately represents the transformer internal behavior, the first step is to evaluate a correlation between several measurement points on the tank wall and the resulting acceleration signals. Then the spectrum of vibration signals in different operating conditions is studied as a criterion for determining the mechanical condition of the winding.

5.1. Influence of sensor positions

First, the correlation between various measurement locations on the tank wall and the resulting vibration signals is investigated. Figure 9(a) depicts the transformer tank with 5 points for measurements. These test points are located in the mid-height of the tank. To capture the vibration signals, the acceleration is measured at each test point, similar to most practical vibration sensors. Figure 9(b) depicts the observed vibration harmonics at the tank center point (test point 3). As can be observed, higher vibration harmonics have a smaller amplitude than the 100 Hz component. Because of the high signal power of 100 Hz, this vibration component is considered the vibration basic frequency. Therefore, it is used in other subsequent analyses of the transformer vibration signals.

Figure 10 depicts the vibrations observed at 5 test locations at the vibration fundamental frequency (100 Hz). The results show that the signal amplitude at the edges of the transformer tank (at test points 1 and 5) is higher. This indicates that the use of vibration to assess transformer condition is highly dependent on sensor positions. In other words, the mechanical changes are measurable at any measurement position, but they are not comparable to each other. These results are consistent with the results obtained from laboratory experiments performed on a real power transformer in [49].

Because the amplitude of the vibration signals measured from the transformer tank varies at different measuring locations, an additional test is carried out to find the optimal zone for vibration signals measurement. Figure 11(a) depicts the position of the measuring locations on the tank surface. The vibration measured at point 4 near the tank bottom shows the highest power of the detected vibration signals, as illustrated in Figure 11(b). Based on the results of these tests, it can be concluded that the best location to measure the mechanical oscillations on a transformer is
near the tank bottom. These results are consistent with ones based on laboratory studies on large real power transformers in [32], [41], [50], and [51]. This can also indicate the accuracy of the proposed finite element model in monitoring transformer vibration signals, which can be used for further vibration signals analysis.

5.2. Transformer vibration analysis under winding mechanical faults

Analyzing the vibration features of transformer windings reveals that as the mechanical condition of the windings varies, so does the corresponding mechanical vibration. According to [28], [30], and [33], the fundamental frequency component consists of vibration signals of the windings, and the high-frequency vibrations of the tank surface are mainly caused by magnetostriction of the core and independent of windings vibration. Therefore, the low-frequency vibrations measured on the tank surface can be used to diagnose the condition of winding directly. Moreover, according to [52], when the relative value of the transformer vibration harmonics magnitude is changed significantly, the windings were deemed to have a serious fault, and the transformer must be out of running. This point is demonstrated in this section using the proposed model too. In this section, the multi-physics model is used to detect three types of winding defects: LV winding free buckling, winding elongation, and winding twisting, as shown in Figure 12.

Figure 13 shows a spectrum-analysis diagram of the vibration signals at a measurement point near the tank bottom when the transformer is working under healthy winding conditions and when winding mechanical changes occur. As shown in Figure 13(a), under normal operating conditions, the maximum amplitude of vibration signals is obtained at the basic vibration frequency of 100 Hz (0.95 m/s²). Other vibrating harmonics with frequencies of 50 Hz (0.33 m/s²), 200 Hz (0.4 m/s²), 300 Hz (0.59 m/s²), and 400 Hz (0.19 m/s²) with amplitudes less than 100 Hz are also seen in this diagram, which can indicate the healthy mechanical condition of the winding. In the case of mechanical faults in the winding, as shown in Figure 13(b, c, d), the amplitudes of the 200 Hz and 300 Hz harmonics increase dramatically compared to 100 Hz. When a buckling fault occurs for the winding, the maximum harmonic amplitude occurs at 200 Hz (1.08 m/s²), and the 300 Hz (0.64 m/s²) harmonic exhibits higher amplitudes than the fundamental frequency (0.37 m/s²). In the case of the winding elongation fault, the 300 Hz (1.09 m/s²) harmonic has a maximum amplitude, and the 200 Hz (0.77 m/s²) harmonic has an amplitude greater than the fundamental frequency (0.43 m/s²). In the case of the winding twisting fault, other vibrating harmonics, including 500 Hz (0.27 m/s²) and 600 Hz (0.16 m/s²), also appear in the vibration spectrum. As shown in Figure 13(d), in this case, the 300 Hz (1.05 m/s²) and 200 Hz (1.01 m/s²) harmonics have also amplitudes greater than the fundamental frequency (0.65 m/s²). Table 4 addresses the amplitudes of all harmonics in four cases. According to these results, it is reasonable to conclude that in the vibration frequency spectrum, when the amplitude of other harmonics is greater than the amplitude of the main vibration frequency (100 Hz), the transformer winding can be considered to have a serious mechanical fault. Furthermore, the presence of higher-order vibration harmonics in the vibration frequency spectrum, such as 500 Hz and 600 Hz, can be an indication of mechanical twisting in the winding. This study shows that analyzing the vibration harmonics and their amplitudes (ratios) during the transformer operation has the potential to detect a mechanical change in the winding.

6. Analysis of transformer vibration modes

The introduced FEM model can also serve for further studies on transformer vibration. In this section, the model is used to investigate the vibration frequency response of the transformer. By the frequency response, different modes and resonances can be distinguished, and it is also possible to relate each vibration mode to special sections
of the transformer. Moreover, utilizing the provided model, it is shown in this section that the mechanical changes within the transformer affect these modes and, therefore, can be used as a diagnosis tool.

6.1. Dynamic analysis of the transformer structure

Figure 14 demonstrates the average FRF of the transformer vibration. The resonant behavior of the transformer structure is visible in the frequency ranges so that the response amplitude gradually increases with increasing frequency. This means that at high frequencies, vibrations are excited simpler by an input force. Because of the sensitivity of the human ear, restrictions on noise output at higher frequencies are critical. As a result, transformer designers need to optimize their design to reduce high-frequency components. Furthermore, the number of resonant peaks in the low-frequency range, i.e., in the operating frequency range (50 Hz and its harmonics), is greater than in the high-frequency range. This necessitates the optimal design of transformers to weaken resonant modes in the low-frequency range.

The suggested model also can define which elements are involved in each resonance mode. For this purpose, each resonance should be excited to realize what elements are active in that mode. This feature has been implemented for all resonance points, and the results of the first thirteen natural frequencies are summarized in Table 5. In this table, mode shapes are divided into three types based on their participation in the vibration of the transformer structure. These three types of transformer modes are core-related modes, windings-related modes, and coupled modes. If the highest contribution to the vibrations of the transformer structure is related to the core vibrations, the vibration mode is called the core-related. Winding-related modes have a similar definition. If both the winding and the core have a significant role, the mode distribution is said to be coupled. As shown in Table 5, the vibration modes of the core and the coupled modes are in the lower frequency range, while the vibration modes of the winding are often in the higher frequency range. In addition, in the frequency range above 1000 Hz, most of the vibrational resonance frequencies are related to single disks and winding assembly.

Since a quantitative understanding of the resonance of the transformer structure can benefit from mode shape analysis, representative mode shapes are provided for vibration resonances of the winding-related modes in Figure 15. This figure shows the vibration resonances of the first four mode shapes of the winding-related cases. Since most windings of practical transformers are cylindrical, the results obtained from the shape of the winding modes can also be useful for the winding-related modes of other power transformers. The first winding-related mode in Figure 15(a) is an axial and radial bending mode with both ends constrained, while the second winding-related mode is a cylindrical mode. Most of the stress is applied to the upper part of the winding in these two modes. Based on these mode shapes, it can be concluded that the vibrations of the windings are most likely similar to a cylindrical shell. Excessive vibration in this situation produces buckling and deformations in the windings. As a result, the winding assembly cannot be regarded as a ring stack or a collection of lumped masses. In the case of other modes, as can be seen, the third mode is a combination of winding axial displacement and cylindrical modes. The fourth mode is a twisting mode that deforms the transformer winding in two directions simultaneously.

6.2. Transformer vibration modes under winding mechanical faults

The properties of transformer winding vibration modes in the presence of winding abnormalities are studied in this section. Based on the FEM analysis, the differences in natural frequencies and the associated vibration energy distribution are demonstrated and discussed. In the presence of winding damage, the natural frequencies of the winding-related modes shift, as illustrated in Table 6. In all of the examined modes, the maximum deviations of the natural frequency are negative. The free buckling of the LV winding, on the other hand, rises the 3rd natural frequency by 9 Hz, suggesting that when the winding is damaged, the natural frequency shift is not necessarily
negative or positive. In theory, using the modal displacement distribution as a monitoring indicator is straightforward. However, even when the transformer is off-line, measuring winding modes is difficult. To use mode-shape-based monitoring techniques, the experimental approach and feasible online implementation must be improved.

As mentioned before, the vibrations are transmitted through mechanical connections to the transformer tank. Therefore, when the windings are damaged, if the winding buckling occurs, for example, the vibration energy distribution of the winding changes, and the vibration distribution of the transformer tank becomes different as a result of these connections. In other words, if several sensors are attached in different positions of the transformer tank, the amplitude ratio of their captured signals varies when a mechanical change is introduced in the winding. The vibration energy distribution of the tank is compared in Figure 16 for the healthy condition of the winding with the mechanically damaged conditions on one of the tank faces. By comparing the vibration energy distribution of the tank in different states, some conclusions are drawn, including the fact that the amplitude of the vibration energy in the mechanical fault condition is obviously greater than that in the normal condition. It can also be seen from the energy distribution diagrams that the maximum energy density in mechanical faults modes, including the winding buckling with an amplitude of 16 J/m$^3$, the HV elongation with an amplitude of 9 J/m$^3$, and the winding twisting with an amplitude of 12 J/m$^3$, is much larger than the amplitude of vibration energy in the normal state with an amplitude of 1.5 J/m$^3$. These characteristics can be used to characterize the mechanical condition of the winding. In addition, as can be seen, under normal conditions of the winding, the vibration energy distribution of the tank is symmetrical. In buckling and HV elongation faults condition, the vibration energy is moved by specific patterns to the middle of the tank. In the winding twisting condition, the vibration energy is also distributed asymmetrically on the surface of the tank. Therefore, by monitoring the vibration energy distribution on the transformer tank, for example by multiple sensors, it is possible to detect a mechanical variation in the transformer. In fact, the energy distribution patterns can effectively compare the energy distribution of vibration signals under different mechanical conditions.

7. Conclusions

Based on the vibration mechanism, a finite element model of transformer vibration stimulated by electromagnetic forces was proposed in this paper. The proposed model has been used to simulate the coupled field in transformers. This paper showed that mechanical changes in the active part influence mechanical oscillations on both the winding itself and on the transformer’s tank wall. From the results of the vibrations measured from the tank surface, it is found that the signal amplitude recorded by sensors in different positions is not the same. Moreover, it has been shown that in case of a mechanical change in the winding, the harmonic contents of the captured vibration signal change, and this can be used as an indicator for transformer diagnosis. In the next section, the vibration modes of the winding have been studied. Using the FEM technique, the sections of the transformer involved in each resonance frequency and the related mode shapes can be determined. Furthermore, the mechanical changes shift the resonance frequencies and vary the energy distribution of the vibration on the tank surface. Therefore, these two features can also be used for further assessment of the transformer mechanical situation. In summary, the finite element model could show possible methods for the vibration monitoring of a transformer. As the next step in future work, these observations have to be implemented in an experimental setup to prove their practicality in the field.

References

[1] Secic, A., Krpan, M., and Kuzle, I. “Vibro-acoustic methods in the condition assessment of power transformers: A Survey”, IEEE Access, 7, pp. 83915-83931 (2019).
[2] Tran, Q.T., Davies, K., Roose, L. “A review of health assessment techniques for distribution transformers”, Applied Sciences, 10(22), 8115 (2020).
[3] Emara, M.M., Peppas, G.D., and Gonos I.F. “Two graphical shapes based on DGA for power transformer fault types discrimination”, *IEEE Transactions on Dielectrics and Electrical Insulation*, 28(3), pp. 981-987 (2021).

[4] Gao, C., Yu, L., Xu, Y., et al. “Partial discharge localization inside transformer windings via fiber-optic acoustic sensor array”, *IEEE Transactions on Power Delivery*, 34(4), pp. 1251-1260 (2019).

[5] Tenbohlen, S., Beltle, M., and Siegel, M., “PD monitoring of power transformers by UHF sensors”, *International Symposium on Electrical Insulating Materials (ISEIM)*, Toyohashi, Japan, pp. 303-306 (2017).

[6] Wang, H. and Butler, K.L. “Modeling transformer with internal winding faults by calculating leakage factors”, *Proc. 31st North American Power Symposium (NAPS)* Web-based Computing for Power System Applications, San Luis Obispo, USA, pp. 176-182 (1999).

[7] Samimi, M.H., Tenbohlen, S., Akmal, A.A.S., et al. “Dismissing uncertainties in the FRA interpretation”, *IEEE Transactions on Power Delivery*, 33(4), pp. 2041–2043 (2018).

[8] Samimi, M.H. and Dadashi Ilkhechi, H. “Survey of different sensors employed for the power transformer monitoring”, *IET Science, Measurement & Technology*, 14, pp.1–8 (2020).

[9] Wang, Y., Zhang, J., Zhou, B., et al. “Magnetic shunt design and their effects on transformer winding electromagnetic forces”, *Iranian Journal of Science and Technology, Transactions of Electrical Engineering*, 43, pp. 97-105 (2019).

[10] Geißler, D. and Leibfried, T. “Short-circuit strength of power transformer windings-verification of tests by a finite element analysis-based model”, *IEEE Transactions on Power Delivery*, 32(4), pp. 1705-1712 (2017).

[11] Wang, S., Wang, S., and Zhang, N. “Calculation and analysis of mechanical characteristics of transformer windings under short-circuit condition”, *IEEE Transactions on Magnetics*, 55(7), pp.1-4 (2019).

[12] Murugan, R. and Ramasamy, R. “Understanding the power transformer component failures for health index-based maintenance planning in electric utilities”, *Engineering Failure Analysis*, 96, pp. 274-288 (2019).

[13] Seyedshenava, S. and Ahmadpour, A. “Finite element method for optimal transformer connection based on induction motor characteristics analysis”, *Ain Shams Engineering Journal*, 12(2), pp. 1943-1957 (2021).

[14] Guo, J., Ma, X., and Ahmadpour, A. “Electrical-mechanical evaluation of the multi–cascaded induction motors under different conditions”. *Energy*, 229, p. 120664 (2021).

[15] Kim, H., Chung, Y., Jin, J., et al. “Manifestation of flexural vibration modes of rails by the phase-based magnification method”, *IEEE Access*, 9, pp. 98121-98131 (2021).

[16] Sarrafi, A., Mao, Z., Nizeirecki, C., et al. “Vibration-based damage detection in wind turbine blades using Phase-based Motion Estimation and motion magnification”, *Journal of Sound and Vibration*, 421, pp. 300-318 (2018).

[17] Civera, M., Zanotti Fragonara, L., and Surace, C. “An experimental study of the feasibility of phase-based video magnification for damage detection and localization in operational deflection shapes”, *Strain*, 56(2), (2020).

[18] Report, A.C. “Bibliography on transformer noise”, *IEEE Transactions on Power Apparatus and Systems*, PAS-87(2), pp. 372-387 (1968).

[19] Fahnoe, H. “A study of sound levels of transformers”, *Electrical Engineering*, 60(6), pp. 277-282 (1941).

[20] Si, W., Yao, W., Guan, H., et al. “Numerical study of vibration characteristics for sensor membrane in transformer oil”, *Energies*, 14(6), 1662 (2021).

[21] Foster, S.L. and Reiplinger, E. “Characteristics and control of transformer sound”, *IEEE Transactions on Power Apparatus and Systems*, PAS-100, pp. 1072–1077 (1981).

[22] Xu, L. and Liu, X. “Study on the three dimensions attenuated model and the algorithm of environmental noise in substations”, *Proc. CSEE, Mathematics*, Dodoma, Tanzania, 32, 10024 (2012).

[23] Wu, G., Cheng, S.G., Huang, L., et al. “Prediction on the noise of 220 kV outdoor substation to environmental infection”, *Noise and Vibration Control*, 27, pp. 135–137 (2007).

[24] Zhu, L., Yang, Q., Yan, R., et al. “Research on vibration and noise of power transformer cores including magnetostriction effects”, *Transactions of China Electrotechnical Society*, 28, pp. 1–6 (2013).

[25] Hu, J., Liu, D., Liao, Q., et al. “Analysis of transformer electromagnetic vibration noise based on finite element method”, *Transactions of China Electrotechnical Society*, 31, pp. 81–88 (2016).

[26] Ji, S., Li, Y., and Fu, C. “Application of on-load current method in monitoring the condition of transformer’s core based on the vibration analysis method”, *Proc. CSEE, Madrid*, Madrid, Spain, 2, pp. 154–158 (2003).

[27] Yu, X., Li, Y., Jing, Y., et al. “Calculation and analysis of natural frequency of winding model of transformer”, *Transformer*, 47, 5–8 (2010).

[28] Bartoletti, C., Desiderio, M., Carlo, D.D., et al. “Vibro-acoustic techniques to diagnose power transformers”, *IEEE Transactions on Power Delivery*, 19(1), pp. 221-229 (2004).

[29] Hong, K., Huang, H., and Zhou, J. “A method of real-time fault diagnosis for power transformers based on vibration analysis”, *IET Measurement Science and Technology*, 26, p. 115011 (2015).

[30] Berler, Z., Golubev, A., Rusov, V., et al. “Vibro-acoustic method of transformer clamping pressure monitoring”, *Conference Record of the 2000 IEEE International Symposium on Electrical Insulation*, pp. 263-266 (2000).

[31] García, B., Burgos, J.C., and Alonso, Á.M. “Transformer tank vibration modeling as a method of detecting winding deformations-part I:
former tank vibration characteristics in the field and its application

[32] Garcia, B., Burgos, J.C., and Alonso, Â.M. “Transformer tank vibration modeling as a method of detecting winding deformations-part II: experimental verification”, IEEE Transactions on Power Delivery, 21(1), pp. 157-163 (2005).

[33] Ji, S., Luo, Y., and Li, Y. “Research on extraction technique of transformer core fundamental frequency vibration based on OLCM”, IEEE Transactions on Power Delivery, 21(1), pp. 164-169 (2005).

[34] Naranpanawe, L. and Ekanayake, C. “Finite element modelling of a transformer winding for vibration analysis”, 2016 Australasian Universities Power Engineering Conference (AUPEC), Brisbane, QLD, Australia, pp. 1-6 (2016).

[35] Qian, G., Lu, Y., Wang, F., et al. “Vibration response analysis of transformer winding by finite element method”, 2016 IEEE/PES Transmission and Distribution Conference and Exposition (T&D), Dallas, TX, USA, pp. 1-5 (2016).

[36] Jin, M. and Pan, J. “Vibration transmission from internal structures to the tank of an Applied Acoustics oil-filled power transformer”, Applied Acoustics, 113, pp. 1-6, (2016).

[37] Liu, M., Hubert, O., Mininger, X., et al. “Homogenized magnetoelastic behavior model for the computation of strain due to magnetostriction in transformers”, IEEE Transactions on Magnetics, 52(2), pp. 1-12 (2016).

[38] Beltle, M. and Tenbohlen, S. “Vibration analysis of power transformers”, 18th International Symposium on High Voltage Engineering, Seoul, Korea, pp. 1816–1821 (2013).

[39] Rivas, E., Burgos, J.C., and Garcia-Prada, J.C. “Condition assessment of power OLTC by vibration analysis using wavelet transform”, IEEE Transactions on Power Delivery, 24(2), pp. 687–694 (2009).

[40] Wang, Z., Zhang, Y., Zhang, D., et al. “Modeling of magnetostrictive property of electrical steel sheet under vectorial excitation”, IEEE Transactions on Magnetics, 55(6), pp. 1-4 (2019).

[41] Xiaomu, D., Tong, Z., and Jinxin, L. “Analysis of winding vibration characteristics of power transformers based on the finite-element method”, Energies, 11(9), 2404 (2018).

[42] Wan, D.F. “Magnetic theory and its application”, pp. 56–62, Huazhong University of Science and Technology Press, Wuhan, China (1996).

[43] Togun, N. and Bağdattalı, S.M. “Nonlinear vibration of a nanobeam on a pasternak elastic foundation based on non-local euler-bernoulli beam theory”, Mathematical and Computational Applications, 21(1), 3 (2016).

[44] Hong, K., Huang, H., and Zhou, J. “Winding condition assessment of power transformers based on vibration correlation”, IEEE Transactions on Power Delivery, 30(4), pp. 1735-1742 (2015).

[45] Zhang, F., Ji, S., Shi, Y., et al. “Investigation on the action of eddy current on tank vibration characteristics in dry-type transformer”, IEEE Transactions on Magnetics, 55(2), pp. 1-8, (2019).

[46] Wang, J., Gao, C., Duan, X., et al. “Multi-field coupling simulation and experimental study on transformer vibration caused by DC bias”, Journal of Electrical Engineering and Technology, 10(1), pp. 176–187 (2015).

[47] Behjat, V., Shams, A. Tamjidi, V. “Characterization of power transformer electromagnetic forces affected by winding faults”, Journal of Operation and Automation in Power Engineering, 6(1), pp. 40-49 (2018).

[48] Yan, T., Ren, C., Zhou, J., et al. “The study on vibration reduction of nonlinear time-delay dynamic absorber under external excitation”, Mathematical Problems in Engineering, 2020, pp. 1–11 (2020).

[49] Shengchang, J., Lingyu, Z., and Yanning, L. “Study on transformer tank vibration characteristics in the field and its application”, Przegląd Elektrotechniczny, 87(2), pp. 205-211 (2011).

[50] Beltle, M. and Tenbohlen, S. “Power transformer diagnosis based on mechanical oscillations due to AC and DC currents”, IEEE Transactions on Dielectrics and Electrical Insulation, 23(3), pp. 1515-1522 (2016).

[51] Tavakoli, A., De Maria, L., Bartalesi, D., et al. “Diagnosis of transformers based on vibration data”, 2019 IEEE 20th International Conference on Dielectric Liquids (ICDL), Roma, Italy, pp. 1-4 (2019).

[52] Leibfried, T. and Feser, K. “Monitoring of power transformers using the transfer function method”, IEEE Transactions on Power Delivery, 14(4), pp. 1333–1339 (1999).
Biographies

Amir Esmaeili Nezhad has professional education with a Master's degree (GPA= 4/4) from the Faculty of Electrical and Computer Engineering, University of Tehran, Iran. His research interests include Invasive and noninvasive methods for condition assessment of power system equipment, with the focus on transformers and on-load tap changers. Additionally, he has worked on power system optimization, power system flexibility, and integration of renewable energy systems in power systems.

Mohammad Hamed Samimi received the Ph.D. degree (Hons.) in electrical engineering from the University of Tehran, Tehran, Iran, in 2017. He also carried out part of his Ph.D. research at the University of Stuttgart, Stuttgart, Germany, by receiving the DAAD scholarship award for the “bi-nationally supervised doctoral degree” program in 2015 and 2016. He has been with the High Voltage Institute of the University of Tehran as a Researcher and Assistant since 2009. He cooperated with the R&D Department of Nirou Trans Company, Shiraz, working as a Consultant. He was also with the Iran Grid Management Company (IGMC), Tehran, for two years as a researcher in the power system protection office. In 2018, he joined the School of Electrical and Computer Engineering, University of Tehran, where he is currently working as an Assistant Professor of electrical engineering. His research interests include the modeling, testing, condition monitoring, and diagnosis of high voltage apparatuses.
Figures and Tables’ Caption:

Figure 1. Transformer model for vibration analysis.
Figure 2. Test specimen laminated by SiFe sheets.
Figure 3. Finite element model of (a) the transformer tank and (b) the transformer tank with auxiliary equipment.
Figure 4. Steps in utilizing the finite element tool to predict the vibration response of the transformer.
Figure 5. Mesh-generation results for (a) transformer 3D geometric model and (b) transformer tank.
Figure 6. Quality diagram of mesh grid division for (a) transformer model and, (b) transformer tank.
Figure 7. Distribution of magnetic flux density (a) in the winding and (b) in the core.
Figure 8. Distribution diagram of the radial stress (N/m3) in the transformer winding.
Figure 9. (a) Locations of measurement points on the transformer tank wall, (b) Vibration harmonic spectrum measured at test point 3.
Figure 10. Fundamental frequency (100 Hz) amplitude of the vibration at different test point locations, compare positions to Figure 9.
Figure 11. (a) Measurement points on the test transformer tank, (b) fundamental frequency (100 Hz) amplitude of the vibration at different test point locations.
Figure 12. Schematics of types of winding damage.
Figure 13. Vibration spectrum measured at pint 4 under different operating conditions: (a) normal condition; (b) LV free buckling; (c) HV elongation; and (d) winding twisting.
Figure 14. Vibration frequency response of the modeled transformer.
Figure 15. The first four winding-related modes at (a) 370 Hz, (b) 398 Hz, (c) 450 Hz, and (d) 478 Hz. The figures show the displacement in mm.
Figure 16. The vibration energy distribution (J/m3) of the transformer tank in the (a) healthy condition; (b) LV free buckling; (c) HV elongation; and (d) winding twisting.

Table 1. Specifications of the test transformer.
Table 2. Mechanical specifications of transformer components.
Table 3. Test results for the in-plane and out-of-plane direction vibrations.
Table 4. The amplitudes of vibration harmonics in different cases of Figure 13 in m/s² (NS declares not significant).
Table 5. Classification of the transformer modes based on mode shape type.
Table 6. Changes in the natural frequencies of the winding-related modes under fault conditions.
Figures and Tables

Figure 1. Transformer model for vibration analysis.

Figure 2. Test specimen laminated by SiFe sheets.
Figure 3. Finite element model of (a) the transformer tank and (b) the transformer tank with auxiliary equipment.

Figure 4. Steps in utilizing the finite element tool to predict the vibration response of the transformer.

Figure 5. Mesh-generation results for (a) transformer 3D geometric model and (b) transformer tank.

Figure 6. Quality diagram of mesh grid division for (a) transformer model and, (b) transformer tank.
Figure 7. Distribution of magnetic flux density (a) in the winding and (b) in the core.

Figure 8. Distribution diagram of the radial stress (N/m$^3$) in the transformer winding.

Figure 9. (a) Locations of measurement points on the transformer tank wall, (b) Vibration harmonic spectrum measured at test point 3.
Figure 10. Fundamental frequency (100 Hz) amplitude of the vibration at different test point locations, compare positions to Figure 9.

Figure 11. (a) Measurement points on the test transformer tank, (b) fundamental frequency (100 Hz) amplitude of the vibration at different test point locations.
Figure 12. Schematics of types of winding damage.

Figure 13. Vibration spectrum measured at pint 4 under different operating conditions: (a) normal condition; (b) LV free buckling; (c) HV elongation; and (d) winding twisting.
Figure 14. Vibration frequency response of the modeled transformer.

Figure 15. The first four winding-related modes at (a) 370 Hz, (b) 398 Hz, (c) 450 Hz, and (d) 478 Hz. The figures show the displacement in mm.
Figure 16. The vibration energy distribution (J/m$^3$) of the transformer tank in the (a) healthy condition; (b) LV free buckling; (c) HV elongation; and (d) winding twisting.

Table 1. Specifications of the test transformer.

| Main Technical Indicators | Parameter |
|---------------------------|-----------|
| Phase number              | single-phase |
| Frequency                 | 50 Hz     |
| Rated power               | 6.3 MVA   |
| HV rated voltage          | 10.5 kV   |
| LV rated voltage          | 710 V     |
| Model height (cm)         | 100.4     |
| Outer/Inner diameter of LV winding (mm) | 200/146 |
| Outer/Inner diameter of HV winding (mm) | 285/215 |

Table 2. Mechanical specifications of transformer components.

| Component     | Material | Density (kg/m$^3$) | Young’s Modulus (GPa) | Poisson ratio |
|---------------|----------|--------------------|-----------------------|---------------|
| Windings      | Copper   | 8900               | 115                   | 0.35          |
| Core          | SiFe     | 7650               | 180                   | 0.25          |
| Clamping plate| PBT      | 900                | 7.69                  | 0.48          |
| Bolts         | Steel    | 7850               | 210                   | 0.3           |
| Brace         | Steel    | 7850               | 210                   | 0.3           |
Table 3. Test results for the in-plane and out-of-plane direction vibrations.

| ID                        | Natural frequency (Hz) | Young’s Modulus (GPa) |
|---------------------------|------------------------|-----------------------|
| In-plane vibration        | 2562                   | 174.5                 |
| Out-of-plane vibration    | 135                    | 1.9                   |

Table 4. The amplitudes of vibration harmonics in different cases of Figure 13 in m/s² (NS declares not significant).

| Cases            | Frequency of the Harmonic |
|------------------|---------------------------|
|                  | 50 Hz | 100 Hz | 200 Hz | 300 Hz | 400 Hz | 500 Hz | 600 Hz |
| Normal condition | 0.33  | 0.95   | 0.40   | 0.59   | 0.19   | NS     | NS     |
| LV free buckling | 0.20  | 0.37   | 1.08   | 0.64   | 0.33   | NS     | NS     |
| HV elongation    | 0.17  | 0.43   | 0.77   | 1.09   | 0.36   | NS     | NS     |
| Winding twisting | 0.24  | 0.65   | 1.01   | 1.05   | 0.61   | 0.27   | 0.16   |

Table 5. Classification of the transformer modes based on mode shape type.

| ID                        | Natural frequency (Hz) | Mode shape summary           |
|---------------------------|------------------------|------------------------------|
| Modes related to the      | 34                     | symmetrical twist            |
| transformer core (first   | 54                     | asymmetrical deformation     |
| five)                     | 78                     | asymmetrical deformation     |
|                           | 108                    | asymmetrical bending         |
|                           | 205                    | symmetrical bending          |
| Modes related to the      | 370                    | axial and radial bending      |
| transformer windings      | 398                    | cylindrical mode             |
| (first five)              | 450                    | axial displacement           |
|                           | 478                    | twisting mode                |
|                           | 520                    | radial bending               |
| Modes related to the      | 4                      | out-of-plane deformation      |
| core and windings together| 18                     | out-of-plane deformation      |
| together (first three)    | 36                     | out-of-plane bending         |

Table 6. Changes in the natural frequencies of the winding-related modes under fault conditions.

| Winding-related modes | Normal condition | LV free buckling | HV elongation | Winding twisting | $\Delta f_{\text{MAX}}$ |
|-----------------------|------------------|------------------|--------------|------------------|------------------------|
| 1                     | 370              | 368.2            | 369.5        | 369              | -1.8                   |
| 2                     | 398              | 391.6            | 396          | 398              | -6.4                   |
| 3                     | 450              | 459              | 439          | 452.5            | -11                    |
| 4                     | 478              | 469              | 463          | 480              | -15                    |