LEARNING TO SELECT EXAMPLES FOR PROGRAM SYNTHESIS

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ABSTRACT

Program synthesis is a class of regression problems where one seeks a solution, in the form of a source-code program, mapping the inputs to their corresponding outputs exactly. Due to its precise and combinatorial nature, it is commonly formulated as a constraint satisfaction problem, where input-output examples are encoded as constraints and solved with a constraint solver. A key challenge of this formulation is scalability: while constraint solvers work well with few well-chosen examples, a large set of examples can incur significant overhead in both time and memory. We address this challenge by constructing a representative subset of examples that is both small and able to constrain the solver sufficiently. We build the subset one example at a time, using a neural network to predict the probability of unchosen input-output examples conditioned on the chosen input-output examples, and adding the least probable example to the subset. Experiment on a diagram drawing domain shows our approach produces subsets of examples that are small and representative for the constraint solver.

1 INTRODUCTION

Program synthesis (or synthesis for short) is a special class of regression problems where rather than minimizing the error on an example dataset, one seeks an exact fit of the examples in the form of a program. Applications include synthesizing database relations (Singh et al., 2017), inferring excel-formulas (Gulwani et al., 2012), and compilation (Phothilimthana et al., 2016). The synthesized programs are complex, consisting of branches, loops, and other programming constructs. Recent efforts (Ellis et al., 2015; Singh et al., 2017) show an interest in applying the synthesis technique to large sets of examples, but scalability remains an open problem. In this paper we present a technique that selects a small representative subset of examples from a large dataset of examples, such that it is sufficient to synthesize a correct program, yet small enough to encode efficiently.

There are two key ingredients to a synthesis problem: a domain specific language (DSL for short) and a specification. The DSL defines a space of candidate programs which serve as the model class. The specification is commonly expressed as a set of example input-output pairs which the candidate program needs to fit exactly. The DSL restricts the structure of the programs in such a way that it is difficult to fit the input-output examples in an ad-hoc fashion: This structure aids generalization to an unseen input despite “over” fitting the input-output examples during training.

Given the precise and combinatorial nature of synthesis, gradient-descent based approaches perform poorly and an explicit search over the solution space is required (Gaunt et al., 2016). For this reason, synthesis is commonly casted as a constraint satisfaction problem (CSP) (Solar-Lezama, 2013; Jha et al., 2010). In such a setting, the DSL and its execution can be thought of as a parametrized function \( F \); which is encoded as a logical formula. Its free variables \( s \in S \) correspond to different parametrization within the DSL, and the input-output examples \( D \) are expressed as constraints which the instantiated program needs to satisfy, namely, producing the correct output on a given input.

\[
\exists s \in S. \bigwedge_{(x_i, y_i) \in D} F(x_i; s) = y_i.
\]
The encoded formula is then given to a constraint solver such as Z3 (de Moura & Bjørner, 2008), which solves the constraint problem, producing a set of valid parameter values for $s$. These values are then used to instantiate the DSL into a concrete, executable program.

A key challenge of framing a synthesis problem as a CSP is that of scalability. While solvers have powerful heuristics to efficiently prune and search the constrained search space, constructing and maintaining the symbolic formula over a large number of constraints constitute a significant overhead. Significant efforts were put into simplifying and re-writing the constraint formula for a compact representation (Singh & Solar-Lezama, 2016; Cadar et al., 2008). To apply program synthesis to a large dataset, one needs to limit the number of examples expressed as constraints. The standard procedure to limit the number of examples is counter example guided inductive synthesis, or CEGIS for short (Solar-Lezama et al., 2006). CEGIS employs two adversarial sub-routines, a synthesizer and a checker: The synthesizer solves the CSP on a subset of examples rather than the whole set, producing a candidate program; The checker takes the candidate program and produces an adversarial counter example that invalidates the candidate program. This adversarial example is then added to the subset of examples, prompting the synthesizer to improve. CEGIS successfully terminates when the checker fails to produce an adversarial example. By iteratively adding counter examples to the subset, CEGIS can drastically reduces the size of the constraint constructed by the synthesizer, making it scalable to large domains. The subset of examples are representative in a sense that, once a candidate program is found over this subset, it is also correct over all the examples. However, CEGIS has to repeatedly invoke the constraint solver in the synthesis sub-routine to construct the subset, solving a sequence of challenging CSP problems. Moreover, due to the phase transition (Gent & Walsh, 1994) property of SAT formulas, there may be instances in the sequence of CSPs with enough constraints to make the problem non-trivial, yet not enough constraints for the solver to prune the search space, making the performance of CEGIS extremely volatile.

In this paper, we construct the representative subset in a different way. Rather than using the constraint solver as in CEGIS, we learn the relationships between the input-output examples with a neural network. Given a (potentially empty) subset of examples, the neural network computes the probability for other examples not in the subset, and grow the subset with the most “surprising” example (one with the smallest probability). The reason being if an example has a low probability conditioned on the given subset, then it is the most constraining example that can maximally prune the search space once added. We greedily add examples, stopping when all the input-output examples have a sufficiently high probability (no longer surprising). The resulting subset of examples is then given to the constraint solver. Experiments show that the trained neural network is capable of representing domain-specific relationships between the examples, and, while lacking the combinatorial precision of a constraint solver, can nonetheless finds subset of representative examples. In conclusion, our approach constructs the sufficient subset at a much cheaper computational cost and shows improvement over CEGIS in both solution time and stability.

2 An Example Synthesis Problem

To best illustrate the synthesis problem and the salient features of our approach, consider a diagram drawing DSL (Ellis et al., 2017) that allows a user to draw squares and lines. The DSL defines a draw(row, col) function, which maps a (row, col) pixel-coordinate to a boolean value indicating whether the specified pixel coordinate is contained within one of the shapes. By calling the draw function across a canvas, one obtains a rendering of the image where a pixel coordinate is colored white if it is contained in one of the shapes, and black otherwise. Figure 1 shows an example of a draw function and its generated rendering on a 32 by 32 pixel grid. The drawing DSL defines a set of parameters that allows the draw function to express different diagrams, some of which are underlined in Figure 1(left). The synthesis problem is: Given a diagram rendered in pixels, discover the hidden parameter values in the draw function so that it can reproduce the same rendering.

1Imagine a mostly empty Sudoku puzzle, the first few numbers and the last few numbers are easy to fill, whereas the intermediate set of numbers are the most challenging
The synthesized drawing program is correct when its rendered image matches the target rendering exactly. Let $S_{\text{draw}}$ be the synthesized draw function and $\text{Target}$ be the target rendering:

$$\text{correct}(S_{\text{draw}}) := \bigwedge_{(\text{row}, \text{col})} S_{\text{draw}}(\text{row}, \text{col}) = \text{Target}[\text{row}][\text{col}]$$

Because of the many possible combinations of parameters for the program, this is a difficult combinatorial problem that requires the use of a constraint solver. Each of the pixel in the target render is encoded as an input-output pair $((\text{row}, \text{col}), \text{bool})$, which can be used to generate a distinct constraint on all of the parameters. For the 32 by 32 pixel image, a total of 1024 distinct constraints are generated, which impose a significant encoding overhead for the constraint solver.

In this paper, we propose an algorithm that outputs a representative subset of input-output examples. This subset is small, which alleviates the expensive encoding overhead, yet remains representative of all the examples so that it is sufficient to constrain the parameter only on the subset. Figure 2 (left) shows the selected subset of examples: white and black pixels indicate chosen examples, grey pixels indicate unchosen ones. As we can see, from a total of 1024 examples, only 15% are selected for the representative subset. The representative subset is then given to the constraint solver, recovering the hidden parameter values in Figure 2 (right).

The algorithm constructs the representative subset iteratively. Starting with an empty subset, the algorithm uses a neural network model to compute the probability of all the examples conditioned on the chosen examples in the subset. It then adds to the subset the least probable example, the intuition being the example with the lowest probability would best restrict the space of possible solutions. The process stops when all the examples in the dataset are given a sufficiently high probability. In the context of the drawing DSL, the sampling process stops when the neural network is sufficiently confident in its reconstruction of the target rendering given the chosen subset of pixels Figure 2 (middle). The rest of the paper elaborates the specifics of our approach.
3 Examples Selection

The crux of our algorithm is an example selection scheme, which takes in a set of examples and outputs a small subset of representative examples. Let $D' \subseteq D$ be a subset of examples. Abusing notation, let us define the consistency constraint $D'(s) := \bigwedge_{(x_i, y_i) \in D'} F(x_i; s) = y_i$, that is to say, the parameter $s$ is consistent with all examples in $D'$. We define the smallest sufficient subset as:

$$D^* = \arg\min_{D' \subseteq D} |D'| \text{ s.t. } \forall s \in S. D'(s) \Rightarrow D(s).$$

$D^*$ is sufficient in a sense any parameter $s$ satisfying the subset $D^*$ must also satisfy $D$. Finding the exact minimum sized $D^*$ is intractable in practice, thus we focus on finding a sufficient subset that is as close in size to $D^*$ as possible.

3.1 Examples Selection with a Count Oracle

We describe an approximate algorithm with a count oracle $c$, which counts the number of valid solutions with respect to a subset of examples:

$$c(D') := |\{(s) \in S | D'(s)\}|.$$

This algorithm constructs the subset $D'$ greedily, choosing the example that maximally restricts the solution space.

$$D' = \emptyset$$

while True do

$$(x, y) \leftarrow \text{argmin}_{x_j, y_j} c(D' \cup \{(x_j, y_j)\}) \text{ # selection criteria}$$

if $c(D') = c(D' \cup \{(x, y)\})$ then

| return: $D'$

else

| $D' \leftarrow D' \cup \{(x, y)\}$

end

end

Algorithm 1: An example selection algorithm with a count oracle

Claim 1: Algorithm 1 produces a subset $D'$ that is sufficient, i.e. $\forall s D'(s) \Rightarrow D(s)$.

Proof 1: As $D'(s)$ is defined as a conjunction of satisfying each example, $c$ can only be monotonically decreasing with each additional example/constraint: $c(D') \geq c(D' \cup \{(x, y)\})$. At termination, the counts remain unchanged $c(D') = c(D' \cup \{(x, y)\}) \forall (x, y) \in D$, meaning no more solutions can be invalidated. Thus we obtain the sufficiency condition $\forall s. D'(s) \Rightarrow D(s)$.

Claim 2: Algorithm 1 produces a subset $D'$ that is $1 - \frac{1}{e}$ optimal

Proof Gist: We need to show the function $c(D')$ is both monotonic and sub-modular (Nemhauser et al. 1978). Proof 1 has shown monotonicity, see appendix for the sub-modularity proof.

The selection criteria for Algorithm 1 amounts to model counting (Gomes et al. 2008), which is impractical in practice. We now aim to resolve this issue by adopting an alternative selection criteria.

3.2 Example Selection without the Count Oracle

We describe an alternative selection criteria that can be approximated efficiently with a neural network. Let’s write the selected subset $D'$ as $\{(x^{(1)}, y^{(1)}), \ldots, (x^{(r)}, y^{(r)})\}$ where $(x^{(j)}, y^{(j)})$ denotes the $j^{th}$ input-output example to be added to $D'$. We define the anticipated probability:

$$Pr((x, y) \mid D') := Pr(F(x; s) = y \mid D'(s))$$

$$= Pr(F(x; s) = y | F(x^{(1)}; s) = y^{(1)}, \ldots, F(x^{(r)}; s) = y^{(r)})$$
Note that $Pr((x, y)|D')$ is not a joint distribution on the input-output pair $(x, y)$, but rather the probability for the event where the parameterized function $F(\cdot ; s)$ maps the input $x$ to $y$, conditioned on the event where $F(\cdot ; s)$ is consistent with all the input-output examples in $D'$. We claim that one can use $Pr((x, y)|D')$ as an alternative the selection criteria.

**Claim:** Under a uniform distribution of parameters $s \sim \text{unif}(S)$,

$$\arg\min_{(x, y)} c(D' \cup \{(x, y)\}) = \arg\min_{(x, y)} Pr((x, y)|D')$$

**Proof:** See appendix.

To use $\arg\min_{(x, y)} Pr((x, y)|D')$ as a selection criteria, one needs a corresponding termination condition. It is easy to see the right termination condition should be $\min_{(x, y)} Pr((x, y)|D') = 1$: when all the input-output examples are completely anticipated given $D'$, the subset is sufficient.

### 3.3 Approximating Anticipation with a Neural Network

We now describe how to model $Pr((x, y)|D')$ with a neural network. For the scope of this work, we assume there exists an uniform sampler $s \sim \text{unif}(S)$ for the possible parameters, and that the space of possible input and output values are finite and enumerable $\text{dom}(x) = \hat{x}_1 \ldots \hat{x}_N$, $\text{dom}(y) = \hat{y}_1 \ldots \hat{y}_M$. We will first describe an empirical count based approach to approximate $Pr((x, y)|D')$, then describe how to model it with a neural network to achieve generalization.

For the count-based approximation, we sample a subset of input values $X' = \{x^{(1)}, \ldots, x^{(r)}\}$, and a particular input value $x \notin X'$. We sample a parameter $s \sim \text{unif}(S)$ and evaluate the parameterized function, $F(x; ; s)$, on each of the input in $X'$, obtaining output values $F(x^{(1)}; s) = y^{(1)}$, $F(x^{(r)}; s) = y^{(r)}$, we also evaluate the function on $x$, obtaining $F(x; s) = y$. Let $\hat{c}$ denote the empirical count, we have, after sufficient number of samples:

$$Pr((x, y)|D') \approx \frac{\hat{c}(F(x^{(1)}; s) = y^{(1)}, \ldots, F(x^{(r)}; s) = y^{(r)}, F(x; s) = y)}{\hat{c}(F(x^{(1)}; s) = y^{(1)}, \ldots, F(x^{(r)}; s) = y^{(r)})}$$

The empirical count method is intractable as there are a total number of $2^N$ subsets of inputs that need to be sampled. Therefore, we approximate $Pr((x, y)|D')$ with a neural network.

![Diagram](image.png)

**Figure 3:** Our neural network architecture resembles a feed-forward auto-encoder with explicitly enumerated input and output neurons. In this figure, $|\text{dom}(x)| = 6$.

The neural network is similar to a feed-forward auto-encoder with $N$ input neurons $\hat{Y}_1 \ldots \hat{Y}_N$ and $N$ output neurons $\hat{Y}_1' \ldots \hat{Y}_N'$. That is to say, we enumerate over (the finite set of) distinct input values, $\hat{x}_1 \ldots \hat{x}_N$, creating a corresponding input and output neuron for each value. Each input neuron $\hat{Y}_i$ can take on $1 + M$ different values where $M = |\text{dom}(y)|$, and each output neuron $\hat{Y}_i'$ can take on $M$ different values. In this encoding, each input neuron $\hat{Y}_i$, and output neuron $\hat{Y}_i'$, can represent the value of running function $F(\cdot ; s)$ on the corresponding input value $\hat{x}_i$, i.e. $F(\hat{x}_i; s)$.
The input neuron \( Y_i \) can also represent the unknown value with the additional \( M + 1 \) class. Figure 3.3 shows our neural network architecture, note that we do not suggest a specific architecture for the middle layers, as one should select whichever architecture that is appropriate for the domain.

During training time, given a sampled parameter \( s \) and a subset of inputs \( X' = \{x^{(1)}, \ldots, x^{(r)}\} \), we set the input and output neurons values as follows:

\[
Y_i = \begin{cases} 
F(x_i, s) & \text{if } x_i \in X' \\
M + 1 & \text{otherwise}
\end{cases} \quad Y'_i = F(x_i, s)
\]

That is to say, the training task of the neural network is to predict the output values for all the possible input values \( x \in \text{dom}(x) \) while given only a subset of input-output values in \( D' \). This is similar to a data completion task in Boltzmann machines (Ackley et al., 1985), with the difference that we directly compute the completion rather than searching for the most probable completion.

During use time, given a subset of input-output examples \( D' \), we set input neuron values the same as in training. The neural network then computes the softmax values for all the \( M \) classes in all the output neurons, obtaining \( Pr((x, y) | D') \) for every possible input-output examples simultaneously.

3.4 Tying up the Loose Ends with CEGIS

The neural network cannot perfectly model the probability \( Pr((x, y) | D') \), therefore, one cannot guarantee that the subset produced by our selection algorithm is sufficient: There may be solutions \( s \) which satisfies the subset \( D' \) yet fails to satisfy the entire set of examples \( D \). We remedy this problem by using CEGIS (Solar-Lezama et al., 2006), which guarantees correctness on \( D \).

\[
D' = \{\} \\
\text{while } \text{True do} \\
| \quad s = \text{synthesize}(S, D') \\
| \quad (x_{\text{counter}}, y_{\text{counter}}) = \text{check}(s, D) \\
| \quad \text{if } (x_{\text{counter}}, y_{\text{counter}}) == \text{None then} \\
| \quad \quad \text{return: } s \\
| \quad \text{else} \\
| \quad \quad D' = D' \cup \{(x_{\text{counter}}, y_{\text{counter}})\} \\
\text{end} \\
\text{end}
\]

Algorithm 2: CEGIS

Like Algorithm 1, CEGIS also maintains a subset of examples \( D' \) and grows it one at a time. In CEGIS, two subroutines, synthesize and check, interacts in an adversarial manner to select the next example to add to the subset: synthesize uses a solver to produce a candidate parameter \( s \) that satisfies the current subset \( D' \); check finds a counter example \( (x_{\text{counter}}, y_{\text{counter}}) \in D \) that invalidates the candidate \( s \). This counter example is added to \( D' \), prompting the synthesizer to improve its solution. CEGIS terminates when no counter example can be found. Clearly, when CEGIS terminates, the resulting solution \( s \) is correct on all the examples in \( D \). The main drawback of CEGIS is that it requires repeated calls to the constraint solver, which is expensive.

Our synthesis algorithm Our algorithm combines example selections and CEGIS in a straightforward way. First, example selection is run until the mean anticipation probability reaches a threshold \( \beta \). The sampled examples are then used to initialize the subset \( D' \) in CEGIS. By initializing CEGIS with a set of representative examples, CEGIS will be able to find the correct solution with fewer calls to the constraint solver, saving both overhead time and solving time.

4 Experiments

We perform a set of experiments measuring the overall speed and stability of our synthesis algorithm, and the representativeness of the subset of examples produced by the selection process. We
# phase 1: examples selection
\[
D' = \emptyset
\]
while mean \((x, y) \in D \Pr((x, y) | D') \leq \beta\) do
\[
(x, y) \leftarrow \text{argmin}_{x', y'} \Pr((x', y') | D') \quad \# \text{selection criteria}
\]
\[
D' \leftarrow D' \cup \{(x_i, y_i)\}
\]
end

# phase 2: CEGIS
while True do
\[
s = \text{synthesize}(S, D')
\]
\[
(x_{\text{counter}}, y_{\text{counter}}) = \text{check}(s, D)
\]
if \((x_{\text{counter}}, y_{\text{counter}}) = None\) then
\[
\text{return}: s
\]
else
\[
D' = D' \cup \{(x_{\text{counter}}, y_{\text{counter}})\}
\]
end
end

Algorithm 3: Synthesis with example selections

evaluate our algorithm on 500 randomly sampled 32×32 renderings. For the experiment, the drawing function has a parameters space that correspond to \(1.31 \times 10^{23}\) possible programs. For each sampled rendering, the following synthesis algorithms are run:

- **full**: all 1024 examples are added to the subset, solved once
- **cegis**: CEGIS with counterexamples picked in canonical order, top-left most pixel first.
- **rcegis**: CEGIS with counterexamples picked in random order.
- **acegis**: CEGIS with counterexample picked in a fixed, arbitrary order.
- **rand+cegis**: initialize CEGIS with a random subset of 20% examples
- **ours**: initialize CEGIS with the subset produced by the example selection algorithm

All listed algorithms are guaranteed to synthesize a program that can perfectly reproduce the target render. For all details of the experiment see appendix.

For the average time plot in Figure 4 (upper left), we measure the breakdown for the different kinds of times: slanted stripes denotes the overhead time in constructing the constraints, grey denotes the solving time by the solver, and black denotes the time taken by the neural network for example selections. On average, our algorithm finishes the fastest, with **cegis** second. We remark that we achieve a similar **solve time** as the full algorithm (column 1 vs column 6, gray blocks), indicating the subset returned by our algorithm constrained the solver to a similar degree as constraining all the examples at once. In comparison, all CEGIS variants and **rand+cegis** have significantly longer solving times, indicating that these algorithms tend to under-constrain the synthesis problem, making it more difficult to solve.

Figure 4 (bottom) shows the distributions of overall times by algorithms. Our algorithm achieves the best overall median time of 7 seconds, and best maximum time of 15 seconds. **cegis** achieves a similar median time, but with significant higher variance. The different CEGIS variants, **cegis**, **rcegis**, and **acegis**, while differs only by which counter-example is added to the subset (top-left most, random, and arbitrary), results in a huge difference in the over-all time performance. We postulate that the top-left-most counter-examples chosen by **cegis** happens to be representative as they tend to lay on boundaries of the shapes, which is well suited for the drawing DSL domain. However, such coincidence is not to be expected in general: By making the counter example be given at random, or given at a fixed but arbitrary ordering, **rcegis** and **acegis** were unable to pick a representative set of examples and suffer in overall time.

Figure 4 (upper right) shows sizes of selected subset of examples: Light grey indicates the size of the initial subset of examples, chosen at random for **rand+cegis** and chosen by the example selection algorithm for **ours**; Stripped indicates the number of additional examples chosen by CEGIS. **rcegis** was able to solve the synthesis problems with the least number of examples but also performs worst in term of overall time. This suggests that while it is possible to generate a valid solution from a
small subset of examples, such subset is not sufficiently constraining for the solver to efficiently prune the search space. By comparison, our approach was able quickly select a larger number examples for the representative subset, which would have been expensive if these examples were chosen by CEGIS (requires solving a sequence of challenging CSPs). Although rand+cegis selects an initial random subset 1.5 times the size of the subset produced by the example selection algorithm, this subset is less representative: On average rand+cegis requires 5 more counter-examples to fully solve the synthesis problem while our approach require only 1 more.

Overall, our algorithm provides a quick and stable solution over existing algorithms by selecting a small and representative subset of examples.

5 RELATED WORKS

In recent years there have been an increased interest in program induction. [Graves et al., 2014], [Reed & De Freitas, 2015], [Neelakantan et al., 2015] assume a differentiable programming model and learn the operations of the program end-to-end using gradient descent. In contrast, in our work we assume a non-differentiable programming model, allowing us to use expressive program constructs without having to define their differentiable counter parts. Works such as [Reed & De Freitas, 2015] and [Cai et al., 2017] assume strong supervision in the form of complete execution traces, specifying a sequence of exact instructions to execute, while in our work we only assume labeled input-output pairs to the program, without any trace information.

[Parisotto et al., 2016] and [Balog et al., 2016] learn relationships between the input-output examples and the syntactic structures of the program that generated these examples. When given a set of input-outputs, these approach use the learned relationships to prune the search space by restricting the syntactic forms of the candidate programs. In these approaches, the learned relationship is across the semantic domain (input-output) and the syntactic domain. In contrast, in our approach we learn a relationship between the input-output examples, a relationship entirely in the semantic domain. In this sense, our approaches are complimentary.
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APPENDIX

PROOFS

Claim: Algorithm 1 produces a subset $D'$ that is $1 - \frac{1}{e}$ optimal

Proof: To show this, we need to show the count function $c(D')$ is both monotonic and sub-modular (Nemhauser et al. 1978). We have already shown monotonicity. For sub-modularity, we need to show for subsets $A \subseteq B \subseteq D$:

$$A \subseteq B \Rightarrow \forall (x, y) \in D. c(A) - c(A \cup \{(x, y)\}) \geq c(B) - c(B \cup \{(x, y)\})$$

To show this, we need to show the number of parameters $s$ invalidated by $(x, y)$ is greater in $A$ than that in $B$. Let $A'(s) := A(s) \land \neg \{(x, y)\}(s)$, the constraint stating that a parameter $s$ should satisfy $A$, but fails to satisfy $(x, y)$, similarly, let $B(s)' := B(s) \land \neg \{(x, y)\}(s)$. The count $c(A')$ indicates how many parameter $s$ becomes invalidated by introducing $(x, y)$ to $A$, i.e. $c(A') = c(A) - c(A \cup \{(x, y)\})$, similarly, $c(B') = c(B) - c(B \cup \{(x, y)\})$. Note that $A'$ and $B'$ are strictly conjunctive constraints, with $B'$ strictly more constrained than $A'$ due to $A \subseteq B$. Thus, there are more solutions to $A'$ than there are to $B'$, i.e. $c(A') \geq c(B')$, showing sub-modularity.

Claim: Under a uniform distribution of parameters $s \sim \text{unif}(S),$

$$\arg\min_{(x, y)} c(D' \cup \{(x, y)\}) = \arg\min_{(x, y)} Pr((x, y) \mid D')$$

Proof: The probability $Pr((x, y) \mid D')$ can be written as a summation over all the possible parameter values for $s$:

$$Pr((x, y) \mid D') := Pr(F(x; s) = y \mid D'(s)) = \sum_{s \in S} Pr(s \mid D'(s))Pr(F(x; s) = y|s) .$$

Note that under $s \sim \text{unif}(S)$, we have:

$$Pr(s \mid D'(s)) = \begin{cases} \frac{1}{c(D')} & \text{if } D'(s) \\ 0 & \text{otherwise} \end{cases} .$$

And since $F(\cdot ; s)$ is a function we have:

$$Pr(F(x; s) = y|s) = \begin{cases} 1 & \text{if } F(x; s) = y \\ 0 & \text{otherwise} \end{cases} .$$

Thus the summation over all $s$ results in:

$$\sum_{s \in S} Pr(s \mid D'(s))Pr(F(x; s) = y|s) = \frac{c(D') \cup \{(x, y)\}}{c(D')} .$$

As $c(D')$ is a constant given $D'$ and is invariant under $\arg\min_{(x, y)}$, we have $\arg\min_{(x, y)} c(D' \cup \{(x, y)\}) = \arg\min_{(x, y)} Pr((x, y) \mid D')$ as claimed.

EXPERIMENT DETAILS

Parameter space for the timed experiments: loop iterations in 0, 1, 2; transformations parameters integers from 0 to 10; offset parameters integers from −10 to 10, up to 2 squares and lines (per transformation). The randomly sampled renderings are filtered to have more than 100 filled pixels so the image is sufficiently complex. The neural network is a single-layer convnet with a 7x7 sliding window and 20 hidden ReLU units, trained over batches of 20 randomly sampled renderings 20000 times. Trained and tested on a laptop with core i7 and nvidia GTX 980M.
SYNTHESIZED DRAWING PROGRAMS

The following images are some synthesized drawing programs. Each row consists of: The target rendering, the subset of selected examples, the neural-network estimation of the rendering, and the synthesized parameters for the \textit{draw} function.

Sequence of Prediction Estimates

A sequence of neural-network’s estimation of the target rendering given its current subset of examples. Each column consists of the chosen subset of examples and its corresponding estimations.