Detailed Study of the Decay
$\Lambda_b \rightarrow \Lambda_c \tau \bar{\nu}_\tau$

E. Di Salvo$^{a,b}$, F. Fontanelli$^b$ and Z. J. Ajaltouni$^{a,3}$

$^a$ Laboratoire de Physique Corpusculaire de Clermont-Ferrand, IN2P3/CNRS Université Blaise Pascal, F-63177 Aubière Cedex, France
$^b$ Dipartimento di Fisica Università di Genova and I.N.F.N. - Sez. Genova, Via Dodecaneso, 33, 16146 Genova, Italy

Abstract
We examine in detail the semi-leptonic decay $\Lambda_b \rightarrow \Lambda_c \tau \bar{\nu}_\tau$, which may confirm previous hints, from the analogous $B$ decay, of a new physics beyond the standard model. First of all, starting from rather general assumptions, we predict the partial width of the decay. Then we analyze the effects of five possible new physics interactions, adopting five different form factors. In particular, for each term beyond the standard model, we find some constraints on the strength and phase of the coupling. On the one hand, some dimensionless quantities, on which our analysis is based, are substantially independent of the form factor. On the other hand, our study, when combined with those of the $B$ semi-leptonic decay, privileges two of the new physics interactions; moreover, these may be distinguished by a suitable observable involving the differential decay width.

PACS numbers: 13.30.Ce, 12.15.-y, 12.60.-i

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$^1$disalvo@ge.infn.it
$^2$fontanelli@ge.infn.it
$^3$ziad@clermont.in2p3.fr
1 Introduction

The high energy physicists have been looking for physics beyond the standard model (SM) for some decades. This research has recently received a new impulse from the Higgs discovery\cite{1, 2} and from the data of the semi-leptonic decays $B \rightarrow D^{(*)}\ell\nu_\ell[3-8]$ and $B \rightarrow K^*\ell^+\ell^-$\cite{9, 10}, which have exhibited strong tensions with the SM predictions\cite{11-13}. Indeed, the SM entails lepton flavor universality (LFU), which seems to be contradicted by the measurements of the observables

$$R_{D^{(*)}} = \frac{\mathcal{B}(B \rightarrow D^{(*)}\tau\bar{\nu}_\tau)}{\mathcal{B}(B \rightarrow D^{(*)}\ell\bar{\nu}_\ell)} \quad \text{and} \quad R_{K^*} = \frac{\mathcal{B}(B \rightarrow K^*\mu^+\mu^-)}{\mathcal{B}(B \rightarrow K^*e^+e^-)},$$  

(1)

$\ell$ denoting, in this case, a light lepton. These quantities attenuate the biases related to the experimental efficiency and to the value of $V_{cb}$ and are affected by quite small theoretical uncertainties, due to the form factors (FF).

We are mainly concerned with the experimental results of $B \rightarrow D^{(*)}\ell\nu_\ell$ decays, about which some authors have performed model independent analyses\cite{14-23}, while other people have interpreted them in terms of specific new physics (NP) models, like two-higgs-doublet\cite{24-27} (2HDM), leptoquark\cite{14,28-34} (LQ), left-right symmetric\cite{35, 36} (LR) or extra-dimension\cite{37} model. The anomaly has been connected to the leptonic $B$ and $B_c$ decays to $\tau\bar{\nu}_\tau$\cite{24-27, 35} and a new light has been cast on the muon anomalous magnetic moment\cite{27, 32} (see also ref. 38).

All that is a goad to further searches for confirmations of NP. In this sense, the $\Lambda_b$ decays to $\Lambda\ell^+\ell^-$\cite{39, 40} and to $\Lambda_c\ell^-\bar{\nu}_\ell$\cite{41-47}, as well as the decay $B_c \rightarrow J/\psi(\eta_c)\ell^-\bar{\nu}_\ell$\cite{48}, could give definitive confirmations of NP, in particular of LFU violation (LFUV); indeed, these presumably share the same basic processes as the two above mentioned $B$ decays.

In the present paper, we consider the baryonic decay

$$\Lambda_b \rightarrow \Lambda_c\tau^-\bar{\nu}_\tau,$$  

(2)

to which a previous letter\cite{45} has been dedicated. Here we give a more in-depth analysis of this decay. Precisely, we limit ourselves to the spin-independent observables and analyze the NP dependence of suitable dimensionless ratios, among which, analogously to (1),

$$R_{\Lambda_c} = \frac{\mathcal{B}(\Lambda_b \rightarrow \Lambda_c\tau\bar{\nu}_\tau)}{\mathcal{B}(\Lambda_b \rightarrow \Lambda_c\ell\bar{\nu}_\ell)}.$$  

(3)

To this end, we propose for the NP interaction five different dimension 6 operators, chosen according to the most popular models - typically, the above mentioned 2HDM, LQ or LR - and similarly to other model independent analyses\cite{41, 43}. Moreover, in order to probe the FF dependence of our predictions, we compare the results obtained by means of five different kinds of such factors. We find that, while the partial decay width depends rather strongly on the FF, the above mentioned ratios depend much more mildly on them. We infer important constraints on the NP couplings, also thanks to previous analyses on the
semileptonic $B$ decays\cite{15,19,17}; moreover we single out an observable which allows to discriminate between two of the NP interactions.

Sect. 2 resumes the assumptions commonly shared by the authors. In sect. 3, we give the general formulae for the matrix element, introducing the various FF. In sect. 4, we sketch the expressions of the differential and partial widths of the decays of interest. In sect. 5, we show predictions of the partial decay widths, both according to the SM and to our assumptions about NP. Sect. 6 is devoted to illustrating and discussing the constraints on the various NP couplings. In sect. 7, we show the predictions of the differential decay widths according to two different NP interactions. Lastly, a summary and some conclusions are presented in sect. 8.

2 Assumptions

We recall here the assumptions shared by the authors that have interpreted the anomalies of the semileptonic $B$ decays mentioned above.

1) The NP process entails LFUV, therefore it does not act on $\tau$ in the same way as on the light leptons. In a simplifying assumption, the NP does not concern at all the electron and the muon.

2) The basic process that gives rise to the NP in the semileptonic decays $B \rightarrow D^{(*)}\tau \nu_\tau$ consists uniquely of $b \rightarrow c \tau \nu_\tau$ and does not involve any spectator partons.

3) Only one type of interaction — scalar, vector, \textit{etc.} — is present in the effective lagrangian.

4) The double ratio $R_D^{\text{exp}}/R_D^{\text{SM}}$ depends only mildly on the FF.

This last assumption is supported by the analyses relative to the semileptonic $\Lambda_b$\cite{11,40} and $B$\cite{3-8,28,29} decays. In particular, according to refs. 28 and 49, it results

$$ R_D^{\text{exp}}/R_D^{\text{SM}} = 1.30 \pm 0.17, \quad R_{D^{(*)}}^{\text{exp}}/R_{D^{(*)}}^{\text{SM}} = 1.25 \pm 0.08, $$

quite compatible with each other. Further arguments will be exposed below.

Taking into account the more restrictive of the two results (4), our assumptions imply immediately that

$$ R_{\Lambda_c} = \xi \frac{\Gamma_{\tau}^{\text{SM}}}{\Gamma_{\mu}^{\text{SM}}}, \quad \xi = 1.25 \pm 0.08 $$

and that the predicted value of the partial decay width with the $\tau$ in the final state amounts to

$$ \Gamma_\tau = \xi \Gamma_{\tau}^{\text{SM}}. $$
3 Matrix Element of the Decay

3.1 SM and NP Amplitudes

We consider the matrix element for the decay $\Lambda_b \rightarrow \Lambda_c \ell \bar{\nu}_\ell$, with $\ell$ denoting either $\tau$ or a light lepton. To this end, we set, in quite a general way,

$$\mathcal{M} = V_{cb} \frac{G}{\sqrt{2}} (J^L_\mu j^\mu + g_r \mathcal{I}).$$

(7)

Here $V_{cb}$ is the CKM matrix element of the quark transition considered, $\mathcal{I}$ is the NP interaction and

$$g_r = xe^{i\varphi}$$

the corresponding relative coupling[III], with $x$ and $\varphi$ real, $x > 0$. We consider five types of effective dimension 6 operators, according to the most frequently used models:

$$\mathcal{I} = J^L_\mu j^\mu, \quad J^R_\mu j^\mu, \quad J^S j, \quad J^P j, \quad J^H j.$$  

(9)

Here

$$j_\mu = \bar{u}_\ell \gamma_\mu (1 - \gamma_5) v, \quad j = \bar{u}_\ell (1 - \gamma_5) v,$$

(10)

$$J^L_\mu = \langle \Lambda_c | \bar{c} \gamma_\mu (1 - \gamma_5) b | \Lambda_b \rangle, \quad J^S = \langle \Lambda_c | \bar{c} b | \Lambda_b \rangle,$$

(11)

$$J^P = \langle \Lambda_c | \bar{c} \gamma_5 b | \Lambda_b \rangle, \quad J^H = J^S - J^P.$$  

(12)

and $u_\ell$ and $v$ are the four-spinors of the charged lepton and of the anti-neutrino respectively; lastly, $L, R, S, P$ and $H$ denote, respectively, left-handed vector, right-handed vector, scalar, pseudo-scalar and $S - P$-interaction.

3.2 Form Factors

The most general expressions of the vector and axial hadronic currents are

$$\langle \Lambda_c | \bar{c} \gamma_\mu b | \Lambda_b \rangle = \bar{u}_f V_\mu u_i = \bar{u}_f (f_1 \gamma_\mu + f_2 i\sigma_{\mu\nu} q^\nu + f_3 q_\mu) u_i,$$

$$\langle \Lambda_c | \bar{c} \gamma_5 b | \Lambda_b \rangle = \bar{u}_f A_\mu \gamma_5 u_i = \bar{u}_f (g_1 \gamma_\mu + g_2 i\sigma_{\mu\nu} q^\nu + g_3 q_\mu) \gamma_5 u_i.$$  

(13)

(14)

Here the $f_i$ and the $g_i$ ($i = 1,2,3$) are functions of $q^2$, $u_{i(f)}$ the four-spinor of the initial (final) baryon,

$$q = p_i - p_f = p_\ell + p$$

(15)

and $p_{i(f)}, p_\ell$ and $p$ are, respectively, the four-momenta of the baryons, of the charged lepton and of the anti-neutrino.

Using the equations of motion (eom), the operators $V_\mu$ and $A_\mu$, which appear in Eqs. (13) and (14), can be re-written as (see Appendix A)

$$V_\mu = X_0 \gamma_\mu + f_2 P_\mu + f_3 q_\mu, \quad A_\mu = Y_0 \gamma_\mu + g_2 P_\mu + g_3 q_\mu,$$

(16)
Table 1: The four different FF inferred from sum rules: $f_1$ is dimensionless, $f_2$ is expressed in $\text{GeV}^{-1}$.

|       | SR1            | SR2            | SR3            | SR4            |
|-------|----------------|----------------|----------------|----------------|
| $f_1(q^2)$ | 6.66/(20.27 - $q^2$) | 8.13/(22.50 - $q^2$) | 13.74/(26.68 - $q^2$) | 16.17/(29.12 - $q^2$) |
| $f_2(q^2)$ | -0.21/(15.15 - $q^2$) | -0.22/(13.63 - $q^2$) | -0.41/(18.65 - $q^2$) | -0.45/(19.04 - $q^2$) |

where

$$X_0 = f_1 - (m_i + m_f)f_2, \quad Y_0 = g_1 + (m_i - m_f)g_2, \quad P = p_i + p_f$$  \hspace{1cm} (17)

and $m_{i(f)}$ is the mass of the initial (final) baryon: $m_i = 5.619 \text{ GeV}$, $m_f = 2.286 \text{ GeV}$.

For the heavy quark transition $b \to c$, some approximations are generally assumed\[51\]:

$$f_1 = g_1, \quad f_2 = g_2 = A, \quad f_3 = g_3 = 0.$$  \hspace{1cm} (18)

In order to test the FF dependence, we perform the calculations using five different FF, the Isgur-Wise one\[50\] (IW) and four of them, derived from sum rules\[51, 41, 46\] (SR). The IW FF reads as

$$f_1(q^2) = \zeta_0[\omega(q^2)] = 1 - 1.47[\omega(q^2) - 1] + 0.95[\omega(q^2) - 1]^2, \quad \omega(q^2) = \frac{m_i^2 + m_f^2 - q^2}{2m_im_f};$$  \hspace{1cm} (19)

$$f_2(q^2) = 0.$$  \hspace{1cm} (20)

The parametrizations of the SR FF are reported in Table 1.

Still, the eom imply\[45\]

$$J^S = \frac{q^a}{\delta m_Q} \bar{u}_f V_{\alpha} u_i, \quad J^P = -\rho \frac{q^a}{\delta m_Q} \bar{u}_f A_{\alpha} u_i,$$  \hspace{1cm} (21)

with

$$\delta m_Q = m_b - m_c, \quad \rho = \frac{m_b - m_c}{m_b + m_c} \sim 0.53,$$  \hspace{1cm} (22)

$m_b = 4.18 \text{ GeV}$ and $m_c = 1.28 \text{ GeV}$ being the masses of the $b$- and $c$-quark respectively.

4 Decay Width

4.1 Derivation of Basic Formulae

The observables that we study in this paper are derived from

$$d\Gamma = \frac{1}{2m_i} \sum |\mathcal{M}|^2 d\Phi.$$  \hspace{1cm} (23)
Here $d\Phi$ is the phase space and the symbol $\sum$ denotes the average over the polarization of the initial baryon and the sum over the polarizations of the final particles. We have

$$\sum |M|^2 = |V_{cb}|^2 \frac{G^2}{2} [T_{SM} + 2x3R(T_i e^{-i\phi}) + x^2 T_N].$$

(24)

Here $T_{SM}$ is the SM contribution,

$$T_{SM} = \sum H_{\mu\nu}\ell^{\mu\nu}, \quad H_{\mu\nu} = J_{\mu}^{L}J_{\nu}^{L*}, \quad \ell^{\mu\nu} = j_{\mu}j_{\nu}^*.$$

(25)

As to the terms $T_i$ and $T_N$, they correspond, respectively, to the interference between the SM and the NP amplitude and to the modulus square of the NP amplitude. Specifically, we have

$$T_i^L = T_i^N = T_{SM}, \quad T_i^R = \sum J_{\mu}^{L}J_{\nu}^{R*}\ell^{\mu\nu}, \quad T_N^R = \sum J_{\mu}^{R}J_{\nu}^{R*}\ell^{\mu\nu},$$

(26)

$$T_i^{S(P)} = \sum J_{\mu}^{L}J_{\nu}^{S(P)*}\ell^{\mu\nu}, \quad T_N^{S(P)} = \sum J_{\mu}^{S(P)}J_{\nu}^{S(P)*}\ell^{\mu\nu},$$

(27)

$$T_i^H = \sum J_{\mu}^{L}J_{\nu}^{H*}\ell^{\mu\nu}, \quad T_N^H = \sum J_{\mu}^{H}J_{\nu}^{H*}\ell^{\mu\nu},$$

(28)

the upper indices denoting the various NP interactions.

In the present paper we are not concerned with spin, therefore we consider an unpolarized initial baryon. A standard calculation leads to

$$T_{SM} = 2^5\{(X_0 + Y_0)^2h_1 + (X_0 - Y_0)^2h_2 + (Y_0^2 - X_0^2)h_3$$

$$+ A[m_f(X_0 + Y_0)\mathcal{L}_i + m_i(X_0 - Y_0)\mathcal{L}_f] + A^2\mathcal{P}_j \cdot \mathcal{P}_i \mathcal{L}_P\},$$

(29)

where $A$ is defined by the second Eq. (18) and

$$h_1 = p_f \cdot p_{\ell} p_i \cdot p, \quad h_2 = p_f \cdot p p_i \cdot p_\ell, \quad h_3 = m_i m_f p \cdot p_\ell, \quad \mathcal{L}_{i(f)} = p_{i(f)} \cdot p_{\ell} P \cdot p_i \cdot p_\ell - p_{i(f)} \cdot P p \cdot p_\ell,$$

(30)

$$\mathcal{L}_P = 2p_\ell \cdot P p \cdot P - P^2 p \cdot p_\ell.$$

(31)

As regards the remaining tensors, one has

$$T_i^R = 2^5\{(X_0^2 - Y_0^2)(k_1 + k_2) - (X_0^2 + Y_0^2)k_3$$

$$+ A[m_f(X_0 + Y_0)\mathcal{L}_i + m_i(X_0 - Y_0)\mathcal{L}_f] + A^2m_f m_i \mathcal{L}_P\},$$

(33)

$$T_N^R = 2^5\{(X_0 - Y_0)^2k_1 + (X_0 + Y_0)^2k_2 + (Y_0^2 - X_0^2)k_3$$

$$+ A[m_f(X_0 + Y_0)\mathcal{L}_i + m_i(X_0 - Y_0)\mathcal{L}_f] + A^2m_f m_i \mathcal{L}_P\},$$

(34)

$$T_i^S = 2^5\frac{m_i}{\delta m_Q}[X_0^2(k_1 + k_2) + AX_0(k_3 + k_4) + A^2q P k_+],$$

(35)

$$T_N^S = 2^5\frac{m_i}{\delta m_Q}[X_0^2(k_5 + k_6) + AX_0(k_7 + k_8) + A^2(q P)^2 k_+],$$

(36)

$$T_i^P = 2^5\frac{m_i}{\delta m_Q}[Y_0^2( - k_1 + k_2) + AY_0(k_3 - k_4) - A^2q P k_+],$$

(37)

$$T_N^P = 2^5\frac{m_i}{\delta m_Q}[Y_0^2(k_5 - k_6) + AY_0(k_7 - k_8) + A^2(q P)^2 k_+],$$

(38)

$$T_i^H = T_i^S + \rho T_i^P, \quad T_N^H = T_N^S + \rho^2 T_N^P.$$
Here

\begin{align*}
  k_1 &= p \cdot p_f q \cdot p_i + p \cdot p_i q \cdot p_f - p \cdot q p_f \cdot p_i, \\
  k_2 &= m_i m_f p \cdot q, \\
  k_3 &= m_i (p \cdot p_f q \cdot P + p \cdot P q \cdot p_f), \\
  k_4 &= m_f (p \cdot p_i q \cdot P + q \cdot p_i p \cdot P), \\
  k_5 &= 2q^2 p_f \cdot q p_i \cdot q p_i \cdot p_f, \\
  k_6 &= m_i m_f q^2, \\
  k_7 &= m_i p_f \cdot P q, \\
  k_8 &= m_f p_i \cdot P q, \\
  k_+ &= p_i \cdot p_f + m_i m_f, \\
  k_- &= p_i \cdot p_f - m_i m_f.
\end{align*}

(39)

4.2 Differential and Partial Decay Width

The integration over the phase space is suitably performed by fixing a reference frame at rest with respect to $\Lambda_b$; to this end, it is also worth recalling the relation of $q^2$ to the energy $E_f$ of the final baryon in that frame:

\[ q^2 = m_i^2 + m_f^2 - 2m_iE_f. \]

(44)

After integrating Eq. (23) over the angular variables, the differential decay width reads as:

\[ d\Gamma = \frac{1}{2\pi^3 m_i^2} \int_{E_i^+}^{E_i^-} dE \sum |M|^2. \]

(45)

Here $\ell$ denotes the charged lepton, which may be either a light one (typically a $\mu$) or a $\tau$, for which one has, respectively, $m_{\mu} = 0.106$ GeV and $m_{\tau} = 1.777$ GeV. Moreover, $E_\ell$ is the energy of the charged lepton in the above mentioned frame and

\begin{align*}
  E_\ell^\pm &= \frac{b \pm \sqrt{\Delta}}{2q^2}, \\
  \Delta &= b^2 + 4q^2c, \\
  b &= 2m_iE_f^2 - (2m_i^2 + M^2)E_f + M^2m_i, \\
  c &= -(m_i^2 + m_\ell^2)E_f^2 + m_i M^2 E_f + m_\ell^2 m_i^2 - \frac{1}{4}M^4, \\
  M^2 &= m_i^2 + m_f^2 + m_\ell^2.
\end{align*}

(46) (47) (48) (49)

The partial decay width is obtained by integrating Eq. (45) over $q^2$:

\[ \Gamma_\ell = \int_{q^2_-}^{q^2_+} dq^2 \frac{d\Gamma}{dq^2}. \]

(50)

Here the limits $q^2_\pm$ are related, through Eq. (44), respectively to $E_f = m_f$ and $E_f = E_f^m$, with

\[ E_f^m = \sqrt{m_f^2 + p_m^2}, \quad p_m = \frac{1}{2}(m_i - m_\ell - \frac{m_\ell^2}{m_i - m_\ell}). \]

(51)

For later convenience, we re-write Eq. (50) as

\[ \Gamma_\ell = \Gamma_\ell^{SM} + 2x \cos \varphi \Gamma_\ell^I + x^2 \Gamma_\ell^N. \]

(52)
Table 2: $\Gamma_{\mu}^{SM}$, $\Gamma_{\tau}^{SM}$ (in $\mu$eV) and the ratio $R_{\Lambda_c}^{SM} = \Gamma_{\tau}^{SM}/\Gamma_{\mu}^{SM}$, for the five different FF considered.

| FF  | $\Gamma_{\mu}^{SM}$ | $\Gamma_{\tau}^{SM}$ | $R_{\Lambda_c}^{SM}$ |
|-----|----------------------|----------------------|----------------------|
| IW  | 31.6                 | 5.63                 | 0.18                 |
| SR1 | 10.8                 | 1.95                 | 0.18                 |
| SR2 | 11.5                 | 1.80                 | 0.16                 |
| SR3 | 22.1                 | 3.40                 | 0.15                 |
| SR4 | 24.5                 | 3.61                 | 0.15                 |

Here, taking account of Eqs. (45) and (24), we have

$$\Gamma_{\ell}^{SM} = \frac{|V_{cb}|^2}{2\pi^3 m_i^2} \int_{q^2}^{q^2_{\ell}} dq^2 \int_{E_\ell^+}^{E_\ell^-} dE_{\ell} T_{SM},$$

(53)

similar expressions holding for $\Gamma_{I}^{SM}$ and $\Gamma_{N}^{SM}$, with $T_I$ and $T_N$ in place of $T_{SM}$. A check of the formulae used is given in Appendix B.

5 Predictions of Partial Decay Widths

Table 2 shows the values of $\Gamma_{\mu}^{SM}$, $\Gamma_{\tau}^{SM}$, calculated by means of Eq. (53), and the ratio $R_{\Lambda_c}^{SM} = \Gamma_{\tau}^{SM}/\Gamma_{\mu}^{SM}$, for the five different FF considered in the article. It can be seen that the SM results of the partial widths depend strongly on the FF. In particular, as regards $\Gamma_{\mu}^{SM}$, the IW FF gives the best approximation of the experimental value, i.e. 

$$\Gamma_{\ell}^{exp} = (29.5^{+14.5}_{-11.4}) \mu eV.$$  

(54)

Instead, two of the SR FF differ from this value by more than one standard deviation and probably they need an overall normalization factor. However, we consider in the present article mainly ratios between dimensional quantities, which appear to be barely dependent on the FF. A first example is offered by the ratio $R_{\Lambda_c}^{SM}$, listed in the last column of Table 2.

This table and Eq. (5) entail a prediction for $R_{\Lambda_c}$. Indeed, averaging over the five values yields

$$\bar{R}_{\Lambda_c}^{SM} = 0.164 \pm 0.006, \quad \bar{R}_{\Lambda_c} = 0.205 \pm 0.013 \pm 0.008.$$  

(55)

Here the former ratio is only affected by the systematic error caused by the FF uncertainty, while for the latter also the statistical one (0.013) has to be accounted for. The smallness of the theoretical error confirms our assumption 4).
Table 3: Values of $\Gamma_I^\tau$ and $\Gamma_N^\tau$ (in $\mu$eV) for $S$, $P$ and $R$-interactions and for the five different FF

| FF  | $\Gamma_I^{I,S}$ | $\Gamma_I^{I,P}$ | $\Gamma_I^{I,R}$ | $\Gamma_N^{N,S}$ | $\Gamma_N^{N,P}$ | $\Gamma_N^{N,R}$ |
|-----|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|
| IW  | 1.28            | 0.26            | -3.32           | 2.26            | 0.44            | 5.63            |
| SR1 | 0.58            | 0.12            | -0.67           | 1.03            | 0.19            | 1.95            |
| SR2 | 0.60            | 0.12            | -0.39           | 1.06            | 0.20            | 1.80            |
| SR3 | 0.99            | 0.20            | -1.21           | 1.75            | 0.34            | 3.40            |
| SR4 | 1.05            | 0.22            | -1.25           | 1.86            | 0.36            | 3.61            |

6 Couplings of the Various NP Interactions

6.1 Argand Diagrams for the NP Couplings

Table 3 provides the values of $\Gamma_I^\tau$ and $\Gamma_N^\tau$ for the $S$, $P$ and $R$-interaction, calculated by Eq. (52) together with the equations analogous to (53). The parameters corresponding to the $H$-interaction can be deduced from the following linear combinations:

$$\Gamma_I^{I,H}^\tau = \Gamma_I^{I,S}^\tau - \rho \Gamma_I^{I,P}^\tau, \quad \Gamma_N^{N,H}^\tau = \Gamma_N^{N,S}^\tau + \rho^2 \Gamma_N^{N,P}^\tau. \quad (56)$$

As regards the $L$-interaction, we have

$$|1 + x_L e^{i\varphi}|^2 = \xi, \quad (57)$$

independent of the FF.

Eq. (52) yields, together with Eq. (6), a relation between $x$ and $\varphi$. Taking account of the statistical and systematic errors, the allowed region consists of a circular crown in the Argand plane of the coupling $g_r$, centered at

$$g_c \equiv (\chi, 0), \quad \chi = -\frac{\Gamma_I^\tau}{\Gamma_N^\tau}, \quad (58)$$

and with radii

$$r_\pm = \sqrt{\frac{\Delta_\tau}{\Gamma_N^\tau}}, \quad \Delta_\tau = (\Gamma_I^\tau)^2 + (\Gamma_{\tau\pm} - \Gamma_{SM}^\tau)\Gamma_N^\tau, \quad (59)$$

here $\Gamma_{\tau\pm}$ takes into account the statistical error of $\xi_\pm$, Eq. (5), and the systematic one, related to the FF. Exceptionally, the latter is absent for $L$-interaction, as Eq. (57) entails, independent of the FF,

$$g_c \equiv (-1, 0), \quad r_\pm = \sqrt{\xi_\pm}. \quad (60)$$

The mean values and the statistical and systematic errors of the radii and the coordinates of centers of the Argand diagrams are listed in Table 4. Again, we remark the small theoretical errors of the parameters, which reflect the mild FF dependence.
Table 4: The mean values of the radii and of the centers of the Argand diagrams for the relative couplings $g_r$. $\bar{r}$ is affected both by a statistical and a systematic error, the former and the latter one respectively.

|       | $S$       | $P$        | $H$        | $L$        | $R$        |
|-------|-----------|------------|------------|------------|------------|
| $\bar{r}$ | 0.90±0.04±0.02 | 3.21±0.20±0.08 | 0.83±0.04±0.02 | 1.12±0.02 | 0.65±0.03±0.04 |
| $g_c$   | (-0.56, 0) | (-1.12, 0) | (-0.48, 0) | (-1.0, 0) | (0.37±0.10, 0) |

6.2 Remarks

Two remarks are in order for the case of $\phi = \pi/2$, where the interference between the SM amplitude and the NP one vanishes.

- Firstly, note that comparing the results of Table 2 and of Table 3 yields

$$\Gamma^{N,R}_\tau = \Gamma^{SM}_\tau; \quad (61)$$

this is a consequence of the integration of Eq. (23) over the phase space, which washes out the interference term between the vector and the axial current. Therefore we have, again independent of the FF,

$$x_R(\pi/2) = x_L(\pi/2) = 0.50 \pm 0.04. \quad (62)$$

- Secondly, if one considers the possibility of decays $\Lambda_b \to \Lambda_c \tau^- \bar{\nu}_\ell$, with $\ell = e, \mu, \tau$, the coupling strength for $\ell = \mu$ and $e$ can be inferred just for $\phi = \pi/2$.

6.3 Minimal Values of the Couplings

In order to compare the strengths of the various NP interactions, we may, for example, calculate their minimal values. These occur at $\phi = 0$, except for the $R$-interaction, for which one has to set $\phi = \pi$, owing to the negative value of the real part of $g_c$. Indeed,

$$x_{min} = r - |\chi|. \quad (63)$$

The values of $x_{min}$ — once more barely FF dependent — are listed in Table 5.

6.4 Comparison with Previous Analyses

Now we compare our results with those of previous analyses, both for the semi-leptonic $B$ decay\[15, 16, 17, 19, 53\] and for the $\Lambda_b$ one\[41, 47\]. In particular, we examine the consequences of those contributions, especially of the most recent ones\[19\], on each NP interaction.

- As regards the $L$-interaction, the agreement with all of the previous papers is trivial, because the NP term just re-scales the SM interaction.
Figure 1: The Argand diagram for the relative coupling $g_r$, $H$-interaction. The thinner
of the two circular crowns is inferred from our analysis, the other one from the second
ref. 19, where also the bounds on the phase have been established. The shaded regions
correspond to the allowed intervals of $g_r$.

- As shown before, the minimum value of $x$ for the $R$-interaction occurs in
correspondence of $\varphi = \pi$. This property is shared by the $B \to D^*\tau\nu_\tau$ decay, whereas the
$B \to D\tau\nu_\tau$ decay requires $\varphi = 0$, which strongly restricts the phase of the $R$-interaction
to a narrow nearby of $\varphi = \pi/2$ [15, 17, 19]. But we have seen, Eq. (62), that for $\varphi = \pi/2$
one has $x_R = x_L$, again independent of the FF and of the specific semi-leptonic decay.

- The $P$-interaction demands a quite large coupling ($x > 2$), in order to compensate
the smallness of the matrix element of the corresponding operator between the initial and
final state. This appears unrealistic, also in view of the considerations by Datta et al.[47],
who discard this interaction when compared with the data of the decay $B_c \to \tau\bar{\nu}_\tau$.

- As a consequence, for $x \leq 1$, the $H$-interaction ($H = S - P$) has a behavior which
is quite similar to the $S$ one, as can be seen in Tables 4 and 5. Also this interaction
exhibits strong limitations on its phase. Indeed, we have to take into account the results
of Table 4, together with those by refs. 19, that is, $g_c = (-0.76, 0.0)$, $r = 1.03 \pm 0.25$.
As illustrated in Fig. 1, only two very small intervals around $\pm 2.18$ rad are allowed. In
order to determine the corresponding minimal values of $x$, we set, at the left-hand side of
Eq. (52), $\Gamma_\ell = \Gamma_\tau = \xi_7^{SM}$, according to Eq. (6). Solving this equation yields

$$
x_0 = 1.18 \pm 0.12, \quad x_1 = 1.09 \pm 0.10, \quad x_2 = 1.05 \pm 0.09,
\quad x_3 = 1.09 \pm 0.10, \quad x_4 = 1.09 \pm 0.10, \quad (64)
$$

where $x_0$ corresponds to the IW FF, the remaining $x_i$ to the SR FF.

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Ivanov et al.[19], private communication.
Table 5: Minimal values of the relative strength $x$ for the various interactions and for the different FF: one has to set $\varphi = \pi$ for the $R$-interaction, 0 for the remaining ones.

| $FF$ | $x_S$ | $x_P$ | $x_H$ | $x_L$ | $x_R$ |
|------|------|------|------|------|------|
| IW   | 0.41±0.10 | 2.44±0.51 | 0.43±0.10 | 0.12±0.04 | 0.18±0.05 |
| SR1  | 0.33±0.09 | 2.08±0.45 | 0.35±0.09 | 0.12±0.04 | 0.26±0.07 |
| SR2  | 0.30±0.08 | 1.91±0.42 | 0.32±0.08 | 0.12±0.04 | 0.33±0.07 |
| SR3  | 0.33±0.09 | 2.08±0.45 | 0.35±0.09 | 0.12±0.04 | 0.26±0.07 |
| SR4  | 0.33±0.09 | 2.06±0.45 | 0.35±0.09 | 0.12±0.04 | 0.26±0.07 |

Lastly, the $S+P$-interaction is excluded by analyses of the semi-leptonic $B$ decays\cite{19}; similarly, the tensor interaction does not find an appreciable room\cite{17, 19}.

7 Predictions of the Differential Decay Width

We define the following differential observable $vs$ $q^2$:

$$\Delta r(q^2) = \frac{d\Gamma}{dq^2}/(d\Gamma/dq^2)_{SM} - 1.$$  \hspace{1cm} (65)

Fig. 2 shows the behavior of this quantity in the case of the $H$-interaction, assuming, as found before, $\varphi = \pm2.18$ rad and the strengths (64) for the different FF. The band represented there depends only mildly on the phase; it is not so different than the one corresponding to $\varphi = 0$, since, owing to condition (52), the increasing strength is compensated by a decreasing interference. Also this observable does not depend so dramatically on the FF.

As regards the $L$-interaction, one has, independent of the FF,

$$\Delta r(q^2) = 0.25 \pm 0.04$$  \hspace{1cm} (66)

for any $\varphi$, which is equal to $\Delta r(q^2)$ for the $R$-interaction if $\varphi = \pi/2$, the only allowed value in this case, as shown above.

8 Summary and Conclusions

Let us stress a few important points of our paper.

A) As already observed in sect. 5, the results concerning the dimensional quantities listed in Table 2 and 3 depend rather strongly on the FF. On the contrary, the dimensionless $r$, $\chi$ and $x$, defined as ratios of those quantities, exhibit, similarly to ref. 41, a mild FF dependence, contained within $\sim 2 - 3\%$. Actually, such uncertainties vanish at all if the $L$-interaction is assumed.
B) The $L$, $H$- and $R$-interaction recur in the most popular models used to explain NP effects of the semi-leptonic decay. The first two interactions might be compatible with the anomaly seen in the $B \to K^* \ell^+ \ell^-$ decay \cite{22, 31, 32, 54, 55}.

C) The $H$-interaction includes the 2HDM, a simple extension of the SM, whose coupling depends on the flavor, as required by LFUV. However, such a model presents difficulties in explaining the anomaly \cite{24, 25, 26, 27}. Moreover, the data and the analysis of the $B$ semi-leptonic decays, together with our calculation about the $\Lambda_b$ decay, constrain severely the relative phase. As a consequence, $x$ is rather large and, since the mass of the intermediate boson for the NP is estimated to be $\simeq 1 \text{ TeV}$ \cite{23, 29, 34, 54}, a value of $x \sim 1$ implies a $g_{NP} \sim 10$ times greater than the electroweak coupling constant.

D) A narrow interval is allowed also for the phase of the $R$-interaction.

E) On the contrary, as regards the $L$-interaction, any value of $\varphi$ is admitted by the analyses. This entails the possibility of a small ($\sim 0.12$) value of the relative strength, then the NP coupling may be not much greater ($\sim 3$ times) than the electroweak one. Moreover, in the optics of MFV \cite{29}, the $L$-interaction is favored, as it does not imply a CP violation phase out of the CKM scheme.

To conclude, if we assume for NP a shorter range than for weak interactions, so that the observed anomalies depend just on the basic process $b \to c \tau \nu_\tau$ and not on the specific hadronic decay (see assumption 2), the simplest and most natural explanation of the tensions appears the $L$-interaction \cite{34, 54}, which re-scales the SM one. It is also in qualitative agreement with a possible solution to the anomaly observed in the $B \to K^* \ell^+ \ell^-$ decay \cite{56, 54}, for which a very small relative strength is required. However, alternative criteria of analyzing data \cite{21, 57} lead to a different conclusion. Therefore, in order to discriminate among the various NP interactions, we suggest to measure the differential observable \cite{65}, or, alternatively, some asymmetry \cite{38, 53} and the polarization of one of the final products \cite{12, 14, 17, 19, 27, 58}, especially the T-odd component \cite{19}.

Acknowledgments

The authors are thankful to their colleagues Fajfer et al. \cite{12, 15} and Ivanov et al. \cite{19} for helpful communications and suggestions.

Appendix A

Here we show that the operators

\[ V_\mu = f_1 \gamma_\mu + f_2 i \sigma_\mu \nu q^\nu + f_3 q_\mu, \]  

(A. 1)

and

\[ A_\mu = (g_1 \gamma_\mu + g_2 i \sigma_\mu \nu q^\nu + g_3 q_\mu) \gamma_5, \]  

(A. 2)

when inserted between the initial and the final baryon state, can be re-written as

\[ V_\mu = [f_1 - (m_i + m_f)f_2] \gamma_\mu + f_2 P_\mu + f_3 q_\mu, \]  

(A. 3)

\[ A_\mu = \{(g_1 + (m_i - m_f)g_2) \gamma_\mu + g_2 P_\mu + g_3 q_\mu\} \gamma_5, \]  

(A. 4)
Figure 2: The observable $\Delta r$, Eq. (65), as a function of $q^2$, with $\varphi = \pm 2.18 \text{ rad}$; see Eq. (64) for the corresponding minimal values of $x$. The upper and lower curve delimit the allowed band.

thanks to the equations of motion (eom). Here

$$q = p_i - p_f, \quad P = p_i + p_f$$  \hspace{1cm} (A. 5)

and $p_{i(f)}$ is the four-momentum of the initial (final) baryon.

To this end, we consider the matrix element

$$i\bar{u}_f\sigma^{\mu\nu}u_iq_\nu = -\frac{1}{2}\bar{u}_f(\gamma_\mu\gamma_\nu - \gamma_\nu\gamma_\mu)u_i(p_i^\nu - p_f^\nu).$$  \hspace{1cm} (A. 6)

By using the eom and the relationship

$$(\gamma_\mu\gamma_\nu + \gamma_\nu\gamma_\mu) = 2g_{\mu\nu},$$  \hspace{1cm} (A. 7)

we get

$$\bar{u}_f(\gamma_\mu\gamma_\nu - \gamma_\nu\gamma_\mu)u_iP_\mu = 2(m_i\bar{u}_f\gamma_\mu u_i - \bar{u}_f u_i p_\mu),$$  \hspace{1cm} (A. 8)

$$\bar{u}_f(\gamma_\mu\gamma_\nu - \gamma_\nu\gamma_\mu)u_iP_\mu' = 2(\bar{u}_f u_i p_{f\mu} - m_f\bar{u}_f\gamma_\mu u_i).$$  \hspace{1cm} (A. 9)

Inserting these two expressions into Eq. (A. 6) yields

$$i\bar{u}_f\sigma^{\mu\nu}u_iq_\nu = \bar{u}_f u_i P_\mu - (m_i + m_f)\bar{u}_f\gamma_\mu u_i.$$  \hspace{1cm} (A. 10)

By considering the matrix element $\bar{u}_fV_\mu u_i$ and taking account of Eq. (A. 10), we get Eq. (A. 3). As far as $A_\mu$ is concerned, a quite analogous procedure for leads to Eq. (A. 4).
Appendix B

Here we check the formula used for the differential decay width, by verifying that it amounts, under suitable substitutions, to the muon decay, $\mu^- \to e^- \nu_\mu \bar{\nu}_e$. Eqs. (44), (45) and (24) yield

$$d\Gamma_{SM}^{SM} = 2m_i \frac{d\Gamma_{SM}^{SM}}{dq^2} = \frac{|V_{cb}|^2 G^2}{2^{6\pi^3} m_i} \frac{1}{2} \int_{E^-}^{E^+} dE_\ell T_{SM}.$$

(B. 1)

Here $E_\ell$ and $E_f$ are respectively the energies of the final baryon and of the charged lepton in the $\Lambda_\bar{b}$ rest frame, with $E_\ell^\pm$ given by Eqs. (46). For the sake of simplicity, we assume the Isgur-Wise form factor; then Eq. (29) entails

$$T_{SM} = 2 \frac{p_f \cdot p_\ell \cdot p_i \cdot p}{p \cdot p} \zeta_0^2[\omega(E_f)], \quad \omega(E_f) = \frac{E_f}{m_f},$$

(B. 2)

with $\zeta_0[\omega(E_f)]$ given by Eq. (20). Then

$$d\Gamma_{SM}^{SM} = \frac{|V_{cb}|^2 G^2}{\pi^3} \int_{E^-}^{E^+} dE_\ell [-m_i E_\ell^2 + A_0(E_\ell) E_\ell + B_0(E_\ell)],$$

(B. 3)

with

$$A_0(E_\ell) = -2m_i E_\ell + \frac{1}{2} M^2 + m_i^2,$$

(B. 4)

$$B_0(E_\ell) = -m_i E_\ell^2 + (m_i M_0 + \frac{1}{2} M^2) E_\ell + \frac{1}{2} (m_i^2 m_f - M^2 M_0),$$

(B. 5)

$$M^2 = m_i^2 + m_\ell^2 + m_f^2, \quad M_0 = m_i + \frac{1}{2} m_f.$$

(B. 6)

In order to recover the differential width of the muon decay, we substitute

$$m_i \to m_\mu, \quad m_f, m_\ell \to 0, \quad \zeta_0, V_{cb} \to 1.$$

(B. 7)

Therefore Eq. (B. 3) yields

$$d\Gamma_{SM}^{SM} = \frac{|V_{cb}|^2 G^2}{\pi^3} m_\mu \left[ -\frac{1}{3} \delta_3(E_e) + \frac{1}{2} A(E_e) \delta_2(E_e) - B(E_e) \delta_1(E_e) \right].$$

(B. 8)

Here

$$A(E_e) = \frac{1}{2} (3m_\mu - 4E_e), \quad B(E_e) = \frac{1}{4} (2E_e^2 - 3m_\mu E_e + m_\mu^2),$$

(B. 9)

$$\delta_1(E_e) = \frac{\sqrt{\Delta}}{q^2}, \quad \delta_2(E_e) = \delta_1(E_e) \frac{b}{q^4}, \quad \delta_3(E_e) = \delta_1(E_e) \frac{b^2 + q^2 c}{(q^2)^2};$$

(B. 10)

moreover, $\Delta, b$ and $c$ are given by Eqs. (46) to (49), taking into account the substitutions (B. 7). Substituting Eqs. (B. 9) and (B. 10) into Eq. (B. 3), we get the energy spectrum of the electron emerging from the muon decay:

$$d\Gamma_{SM}^{SM} = \frac{m_\mu G^2}{12\pi^3} E_e^2 (3m_\mu - 4E_e).$$

(B. 11)
The calculation of the SM differential and decay width, Eq. (B. 3), has been performed analytically. The partial decay width has been obtained by integrating numerically the differential one between \( m_f \) and \( E_m^m \), according to Eqs. (51). An analogous procedure has been employed for the contributions due to new physics. To this end, the tool Mathematica\textsuperscript{60} has been used. The same results, exposed in Tables 2 and 3, have been obtained also by means of Matlab\textsuperscript{61}.

References

[1] S. Chatrchyan et al., CMS Coll.: Phys. Lett. B 716 (2012) 30
[2] G. Aad et al., ATLAS Coll.: Phys. Lett. B 716 (2012) 1
[3] J.P. Lees et al., BaBar Coll.: Phys. Rev. Lett. 109 (2012) 101802
[4] J.P. Lees et al., BaBar Coll.: Phys. Rev. D 88 (2013) 072012
[5] M. Huschle et al., Belle Coll.: Phys. Rev. D 92 (2015) 072014
[6] Y. Sato et al., Belle Coll.: Phys. Rev. D 94 (2016) 072007
[7] S. Hirose et al., Belle Coll.: Phys. Rev. Lett. 118 (2017) 211801
[8] R. Aaij et al., LHCb Coll.: Phys. Rev. Lett. 115 (2015) 111803
[9] R. Aaij et al., LHCb Coll.: Phys. Rev. Lett. 111 (2013) 191801
[10] R. Aaij et al., LHCb Coll.: Phys. Rev. Lett. 113 (2014) 151601
[11] J.F. Kamenik and F. Mescia: Phys. Rev. D 78 (2008) 014003
[12] S. Fajfer et al.: Phys. Rev. D 85 (2012) 094025
[13] J.A. Bailey et al., Fermilab Lattice and Milc Coll.: Phys. Rev. Lett. 109 (2012) 071802; Phys. Rev. D 92 (2015) 034506
[14] J.P. Lee: Phys. Lett. B 526 (2002) 61
[15] S. Fajfer et al.: Phys. Rev. Lett. 109 (2012) 161801
[16] A. Datta et al.: Phys. Rev. D 86 (2012) 034027
[17] M. Tanaka and R. Watanabe: Phys. Rev. D 87 (2013) 034028
[18] P. Biancofiore et al.: Phys. Rev. D 87 (2013) 074010
[19] M.A. Ivanov et al.: Phys. Rev. D 94 (2016) 094028; Phys. Rev. D 95 (2017) 036021
[20] D. Choudhury et al.: Phys.Rev. D 95 (2017) 035021
[21] S. Bhattacharya et al.: Phys. Rev. D 93 (2016)034011; Phys. Rev. D 95 (2017) 075012
[22] L. Di Luzio and M. Nardecchia: Eur. Phys. Jou. C 77 (2017) 536
[23] F.U. Bernlochner et al.: Phys. Rev. D 95 (2017) 115008
[24] L. Dhargyal: Phys. Rev. D 93 (2016) 115009
[25] J.P. Lee: Phys. Rev. D 96 (2017) 055005
[26] S. Iguro and K. Tobe: Nucl. Phys. B 925 (2017) 560
[27] C.-H. Chen and T. Nomura: Eur. Phys. Jou. C 77 (2017) 631
[28] Y. Sakaki et al.: Phys. Rev. D 88 (2013) 094012
[29] M. Freytsis et al.: Phys. Rev. D 92 (2015) 054018
[30] R. Barbieri et al.: Eur. Phys. Jou. C 77 (2017) 8
[31] A. Crivellin et al.: JHEP 1709 (2017) 040
[32] Y. Cai et al.: JHEP 1710 (2017) 047
[33] D. Buttazzo et al.: JHEP 1711 (2017) 044
[34] M. Bordone et al.: Phys. Rev. D 96 (2017) 015038
[35] X.-G. He and G. Valencia: Phys. Rev. D 87 (2013) 014014; Phys. Lett. B 779 (2018) 52
[36] W. Altmannshofer et al.: Phys. Rev. D 96 (2017) 095010
[37] A. Biswas et al.: Phys.Rev. D 97 (2018) 035019
[38] C.-H. Chen and C.-Q. Geng: Phys. Rev. D 71 (2005) 077501
[39] W. Detmold and S. Meinel: Phys. Rev. D 93 (2016) 074501
[40] T. Gutsche et al.: Phys. Rev. D 87 (2013) 074031
[41] S. Shivashankara et al.: Phys. Rev. D 91 (2015) 115003
[42] T. Gutsche et al.: Phys. Rev. D 91 (2015) 074001
[43] R. Dutta: Phys. Rev. D 93 (2016) 054003
[44] N. Haby et al.: Int. Jou. Mod. Phys. Conf. Ser. 39 (2015) 1560112
[45] E. Di Salvo and Z.J. Ajaltouni: Mod. Phys. Lett. A 32 (2017) 1750043
[46] X.Q. Li et al.: JHEP 1702 (2017) 068
[47] A. Datta et al.: JHEP 1708 (2017) 131
[48] R. Dutta and A. Bohl: Phys. Rev. D 96 (2017) 076001
[49] Y. Amhis et al., HFLAV coll.: Eur. Phys. Jou. C 77 (2017) 895
[50] H.-W. Ke et al.: Phys. Rev. D 77 (2008) 014020
[51] R.S.M. De Carvalho et al.: Phys. Rev. D 60 (1999) 034009
[52] C. Patrignani et al.: Chin. Phys. C 40 (2016) 100001
[53] M. Duraisamy and A. Datta: JHEP 1309 (2013) 059
[54] D. Choudhury et al.: Phys. Rev. Lett. 119 (2017) 151801
[55] B. Bhattacharya et al.: Phys. Lett. B 742 (2015) 370
[56] S. Glashow et al.: Phys. Rev. Lett. 114 (2015) 091801
[57] A. Celis et al.: Phys. Lett. B 771 (2017) 168
[58] A.K. Alok et al.: Phys. Rev. D 95 (2017) 115038
[59] L.B. Okun: Leptons and Quarks, Elsevier Science Publishers B.V., 1982, 1984
[60] Wolfram Research, Inc., Mathematica, Version 11.3, Champaign, IL (2018)
[61] MATLAB Release 2016b, The MathWorks, Inc., Natick, Massachusetts, United States