ENHANCEMENT of PERSISTENT CURRENT ON MULTICHANNEL RING*

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We describe a "train" effect which may exist in small metallic and semiconductor rings and might be associated with the long-standing problem of the persistent current enhancement. The current is associated with the cooperative motion of \( N \) electrons constituting the \( N \)-electron "train". The train arises via an excitation or a backflow of spin waves or of interchannel charge density sound modes. The impurities and defects have a little effect on the "train" current. The reason is that the "train" current is associated with a small momentum transfer \( \Delta k = k_{\text{free}}/N \), which is much smaller than the momentum transfer of the free electron current equal to \( k_{\text{free}} = 2\pi/L \). For an illustration of the "train" effect we have calculated the persistent current in the framework of the Bethe ansatz solutions for Hubbard model. The fractional \( M/N \) periodicities of the persistent current serve as an indication of the electron "train".
There is a long-standing problem of the strong persistent current observed in metallic [1,2] and semiconductor rings [3]. The present existing theories describe reasonably well the observed periodicities but fail to explain the strength of the current [4–11]. In real rings used in experiments [1–3] there are always impurities and defects. However the value of the current is similar to the one in systems without defects. The calculation of the current in cases when these defects have been taken into account by the perturbation theory gives the current value which is about two or (for metallic rings) four order of magnitude smaller than the observed one.

Thus, the experiments indicate that impurities and defects have no much influence on the electrons. The persistent current is not suppressed by defects as it is used to be in 1D or 2D metals [5]. If we take into account the impurities on a single channel ring with free electrons, the disorder induces a repulsion between energy levels in the regions of the level-crossing [6]. The level repulsion has a simple explanation based on the picture of single electron motion in a periodic potential. The motion of an electron on the ring at some configuration of the disorder may be thought as the motion of an electron in the periodic potential with the period equal to the circumference of the ring \( L \). But for the motion of a particle in a periodic potential, according to the Bloch theorem, there are bands and gaps occurring. The values of the gaps are determined by the values of the barriers, created by disorder, which are proportional to the average amplitude of the disorder \( W \). The widths of the new bands are determined by the new period \( L \) and, therefore it is proportional to \( t_1 = ta^2/L^2 - W \), where \( t \) and \( a \) are the original bandwidth and interatomic distance, respectively. One can see that the value of \( t_1 \) is nothing but the half distance between the levels created by the size quantization at zero magnetic field.

When the magnetic field changes each energy level oscillates with a single flux quantum period \( \phi_0 \) and the amplitude equals to the new bandwidth \( t_1 \). Thus, the amplitude of the oscillations of the energy flux dependence associated with this single level becomes smaller as the strength of disorder increases [6]. For an infinite system the unit flux quantum period implies a localization. In contrast, without disorder a single energy level oscillates with
the period equal $N\phi_0$ and the amplitude equal to the original bandwidth $t$ \cite{12}. The total persistent current is maximal when the magnetic flux corresponds to the level crossing. With disorder there appears a finite curvature associated with the level repulsion (the point of the level-crossing corresponds to an infinite curvature). This rounding of the intersection points gives mostly the current decrease. That is if the disorder increases, the curvature of the intersection point decreases and, therefore, the suppression of the persistent current increases (see, for comparison, Refs. \cite{3,11}). The total flux dependent energy is a sum of the single electron energies. The partial current associated with a single energy level is determined by the first derivative of its energy with respect to the flux. The total persistent current consists of the sum of these partial currents. The dependence on the flux for each single electron energy becomes smoother with disorder, which means that the persistent current is strongly suppressed \cite{10}. These arguments are very general and from the first view, it seems, that in systems with strong electron-electron interaction we would expect the same picture \cite{8}.

However, at a strong electron-electron interaction the pattern of level‘ intersection is much more dense (see, for example, Refs \cite{13,17}). Small systems with small number of particles can be investigated numerically (by exact diagonalization) \cite{16,12,18}, while big systems – with the aid of the Bethe ansatz. In the limit of the strong electron-electron interaction ($U/V_F \to \infty$) in the framework of a Hubbard model the spectrum \cite{19} has an analytic solution of the universal form \cite{13,17}:

$$K_n = \frac{2\pi n}{L} + \frac{2\pi}{L}(f + \sum_\alpha J_\alpha/N),$$

which is different from that for spinless fermions: $K_n = \frac{2\pi n}{L} + \frac{2\pi f}{L}$, by an additional statistical (or gauge) flux $\phi = \sum_\alpha J_\alpha/N$, where the $J_\alpha$ are the quantum numbers of spinons. The total energy of the system in both cases is determined in the same way by the relation $E(f) = -2t \sum_n \cos K_n$ and it is equal to the holon energy $E(f, \phi) = E_h = V_F(f - \phi)^2/L$. The gauge flux arises due to the interaction and takes the fractions $\phi = l/N$, where $l = \ldots, -2, -1, 0, 1, 2, \ldots$ is any integer number. It is related to the local $SU(2)$ symmetry of
individual spins and arises due to exclusion of doubly-occupied states, i.e. due to the decoupling of the spins on the different sites and the possibility of individual spins to rotate freely in space. Therefore, free local spin rotations may be described by the gauge field. The flux of the gauge field corresponds to the different spin configurations. Since the eq.(1), looks like single particle spectrum of free spinless fermions with the disorder one could expect the same level repulsion and the narrowing of bands as for free electrons.

At a fixed flux value each new band is associated with a single level of a size quantization. With the change of magnetic flux the level moves inside the "band" within the interval $t_1$. Let the electron be located at the bottom of the band [20]. Then with the change of the magnetic flux by $2\pi/N$ one may tune the gauge flux by the value $-2\pi/N$. This results in the location of an electron at the bottom of the band and $E_h = 0$. That is, the energy-flux dependence consists of equidistant parabolic-like curves (see, for example, Refs [13–17]) and there occurs the fractional $1/N$-periodic Aharonov-Bohm (AB) effect. The current $I$ is equal to the derivative of $E(f)$ with respect to the flux $f$: $I(f) = -\partial E(f)/\partial f$ and, therefore, the current-flux dependence is also a $1/N$ periodic function of the flux $f$ consisting of sawtooth peaks. Note, however, that the disorder has a little effect on the curvature of the bottom of the energy-flux band and is acting mostly on the region of the level intersection. Therefore, each of the current peaks has, approximately, the slope of a free electron current. The amazing fact, however, is that the weak disorder has practically no influence on this current, which, because of a small period, has a very small amplitude $V_F/(LN) = t/UL^2$. The current will be suppressed only at very strong disorder such as $W \sim t$, when the energy-flux "band" becomes flat.

The fractional $1/N$ period has a simple explanation [13] in a picture of the simultaneous, collective, cooperative motion of all $N$ electrons on the ring which we name as "$N$-electron train". After one lap of the train motion the many body wave function will get the phase factor equal to $2\pi fN$. The gauge invariance dictates $fN = 1$, which means that the minimal periodicity of the AB - effect will be equal to $f = 1/N$. Thus, the creation of this $N$-electron train gives rise to a very small $1/N$ periodicity and to a nonsensitivity of the persistent
current to the weak disorder. Thus, we offer a new mechanism of the current based on a formation of \(N\) electron trains, which are not sensitive to the disorder. Although this \(N\)-electron-train current is only slightly influenced by disorder, its value is very small due to the big charge of the train \(Q = Ne\), i.e. due to the small AB period.

The first correction in the parameter \(\alpha = V_F/U << 1\) gives rise to an antiferromagnetic interaction between the electron spins. Therefore, different spin configurations will have different magnetic energies associated with the energy of the spin waves. At a finite value \(\alpha\) the spin waves velocity is nonzero. Then the total energy of the Hubbard ring \(E(f, \phi)\) (let us assume, for simplicity, \(N_\uparrow = M = N/2\)) is the sum of the holon energy \(E_h = V_F(f - \phi)^2/L\) and the energy of the spin waves, i.e.

\[
E(f, \phi) = \frac{V_F}{L} (f - \phi)^2 + \frac{V_F}{L} \alpha |\sin 2\pi \phi| \tag{2}
\]

At zero gauge flux \(\phi\) the holon energy \(E_h\) increases rapidly \(\sim f^2\). However, if together with \(f\) we also change \(\phi\), and excite the spin-waves, then the contribution of the first term may vanish and the total energy decreases. Although the energy in the new state with the external flux equal \(2\pi/N\) is different from the energy at zero flux by the spinon energy, we still have a pronounced energy-flux oscillation with the quasi-period equal to \(2\pi/N\). The value \(\phi\) is always chosen to minimize the total energy. One general conclusion is that the structure of the spin-wave excitation spectrum, i.e. the energy dependence on the spin wave momentum determines the AB periodicity as well as the parity effect.

For the calculation of the energy-flux dependence at different ratios \(M/N\) in the framework of the Bethe ansatz solution we use the method, recently, presented in Ref. \([17]\). Since the \(E(f)\) dependence consists of many segments of parabolic like curves associated with the \(1/N\) oscillations (see, Figs in Ref. \([17]\)), the current-flux dependence is calculated at each segment independently as \(I(f) = -\partial E(f)/\partial f\) and is a straight line. The length of these straight lines is determined by points of intersection of neighboring parabolic curves \([17]\). The total current-flux dependence consists of many parallel lines associated mostly with single \(1/N\) oscillations. We have calculated the persistent current at different ratios of \(M/N\).
The most striking result, however, is that the persistent current is perfectly $M/N$ periodic \[21\]. On the Fig.1A we present the results of our calculations when the number of down spins $M = 101$ and $N = 303$. The current is a perfect $1/3$– periodic function of the flux, which was not obvious from the analysis of the energy-flux dependence \[17\]. We would expect that this new periodicity will be very approximate. It seems that the $M/N$-fractional AB effect is much more pronounced for the current than for the energy-flux dependence (see, for comparison, Figs in Ref. \[17\]).

There are two different regions, where the current effectively increases with the flux and monotonously decreases. In the flux region where the current effectively increases there occurs the perfect $1/N$– or, here, $1/303$- periodic oscillations, which are practically not resolved on the scale of the Figure. If the number of particles $N$ decreases such oscillations may be already seen. On the Fig.1B we present the case when $M = 33$ and $N = 99$. Again one can see perfectly $1/3$-flux quantum periodic oscillations, where, however, $1/99$ periodic current oscillations are already resolved and may be clearly seen. The current decreases linearly within each $1/N$ flux region. The slope of this decrease is equal to the slope of the free electron current decrease. The large amplitude of the current is determined by a large number of small $1/N$– oscillations, related to the $N$– particle bound coherent state \[13\] or to the formation of the $N$– electron ”train”. One sees from Fig.1, that it can be very large $I_{\text{max}} \sim 10V_F/L \sim 0.3I_{\text{free}}$, where the maximum amplitude of the free electron current is $I_{\text{free}} = 4\pi^2V_F/L$. Since the weak disorder has a little effect on the motion of this ”electron” train, we expect that at the disordered ring the current will be big as well. Loosely speaking, since the current is associated with this $N$– particle coherent state, which spreads over the perimeter of the ring, its localization length is in $N$– times larger than for free electrons. Therefore, on the Hubbard ring with the disorder the localization effects are suppressed by the strong interaction, and the current is big. With the interaction also the temperature suppression of the current decreases \[22\].

When the value of $\alpha$ increases the number of the $N$– electron train excitations (or the number of the $1/N$ oscillations) in the single AB half period decreases. Then there the
regimes with $N - 2$, with $N - 6$, with $N - 8$, ..., with $N - 2K$ oscillations will occur, $K$ being any integer (see, for comparison, the numerical simulations in Ref. [18]). At $\alpha > \alpha_c \sim 0.02$ the $N-$ electron trains disappear and a complete decoupling of spin and charge degrees of freedom (Luttinger Liquid) occurs [22]. However, the fractional $M/N$ oscillations do still survive, since they are related to the $2k_F s$ excitations of the spinon Fermi see. On Fig. 2, we show the $1/3$ periodic oscillations of the ground state energy-flux dependence which consists of low energy parabolic curves associated only with charge degrees of freedom. The neighboring parabolic curves differ by $2k_F s$ excitations of the spinon Fermi see. The total spin wave excitation spectrum is presented by short horizontal lines on this Figure (instead of the flux $f$ the energy here depends on the spin wave momentum $k_s$, which, in our notations, is equal to $2\pi f$). The parabolic curves create the cusps at the flux values $f_c \sim 0.35$ and $f_c = 0$. For other half-flux regions this dependence must be continued with the aid of symmetrical reflections. One sees that in this case the persistent current is determined by lowest parabolas or, practically, by charge degrees of freedom. Except $2k_F s$ excitations the spinon energy is always bigger and, therefore, switches off and does not participate in the AB effect. One sees on this Figure that the $M/N$ oscillations of the current survive even if $\alpha$ increases, which is due to the existence of the $2k_F s$ excitations.

However, when $\alpha$ decreases, the amplitude of the spin wave spectrum ($\sim \alpha$) decreases. If $\alpha \leq \alpha_c$ at some flux values the spin wave energy becomes lower than the cusp energy associated with the intersection of parabolic curves describing charge degrees of freedom (see, on Fig. 3A, where the notations are the same as for Fig. 2). Then, in these regions (for example, one of these regions is at $0.31 < f < 0.35$, see, Fig. 3A) it is energetically favorable to have a spin wave excitations, i.e. here the charge degrees of freedom are coupled with the spin ones. This gives rise to the N-electron trains and to the $1/N$ oscillations of the energy-flux dependence, which envelope function is determined by the spin wave spectrum.

At $U = \infty$ ($\alpha = 0$) the spin waves have zero velocity, and, therefore, the spin currents $I_s$ do not exist. If $\alpha \neq 0$ the spin-current does occur ($I_s \sim \alpha$), and it depends on the flux of the magnetic field as a nonanalytic function which is also $M/N$ periodic. The spin current
remains constant during each of the $1/N$ oscillations of the electric current and decreases, step by step, with every next $1/N$ oscillation of the electric current. The magnetic flux-dependence of the spin current and its periodicity with the magnetic flux are absolutely new effects, which are different from Aharonov-Bohm effect or from Aharonov-Casher (AC) effect. Here we have the dependence of the spin current on the flux of the magnetic field, whereas in AB effect an electric current depends on a magnetic flux, while in the AC effect a spin current depends on the flux of electric field.

The maximum amplitude of the spin current $I_{s,max} \sim V_{F_s}N_s/L \sim V_F \alpha N_s/NL$ is determined by the spinon Fermi velocity $V_{F_s}$ and it also depends on the number of steps $N_s$ or the number of $1/N$-- periodic oscillations of the electric current equal to one of the following numbers: $N, N - 2, N - 4, \ldots$. The number of steps $N_s$ depends on the ratio of the spinon and the holon Fermi velocities and decreases when this ratio increases. The amplitude of the $1/N$ oscillations of the electric current is proportional to $\sim V_F/LN = t/L^2$, which is a very small value when the ring is very large. The surprising fact is that the amplitude of $M/N$ periodic oscillations of the electric current is proportional to $\sim V_F(1 - N_s/N)/L$. If $N_s << N$, it is of the order of a free electron current, which is much larger than the value of the spin current.

The results obtained in the framework of the Hubbard model are very general. We have checked this picture for the Kondo ring (a single Kondo impurity on the thick metallic ring) where the results are identical to those described above and for the $SU(N) \ast SU(M)$ invariant models [23,24]. At strong coupling there occurs a strong degeneracy (or a local symmetry) associated with the channel or spin index. Strong local Coulomb interaction gives a freedom to local spin rotations or to interchannel transitions and, therefore, generates a local gauge field. The value of the gauge flux $\phi$, which may be not single, are related to the total momentum of "spinons" or to the total momentum of the relative charge density waves created between channels, which may take the small fractions $\sim 1/N$. This small fraction of gauge flux $\sim 1/N$, in which the "holons" are moved, generates the "N-electron trains" giving rise the fine structure of the AB effect (the fractional $1/N$ and $M/N$ periodic oscillations).
and making the current insensitive to the disorder. Of course, for the real interaction, which is not infinite, this degeneracy is lifted. This means that the creation of the gauge flux costs some energy, which is, however, small if the parameter $V_F/E_c$ is small. The picture is valid for any multichannel or 2D ring, where the interchannel interaction is so strong that the gauge field arises due to the interchannel transitions (see, for comparison, Ref. [12]).

Whether or not this "train" effect can explain the discrepancy between experiments [1–3] and existing theories is still unclear. The detection of any described fractional periodicities may resolve this question and prove the existence of the "train" excitations on the quantum rings. Since the distribution of particles over the channels is, probably, well controllable, it should be a realizable experiment to see the predicted fractional periods. One may also expect the described persistent current with fractional periods $p/q$ in higher magnetic field, where electrons are partially polarized.

I thank Markus Buttiker, C.A.Stafford, D.E. Khmel’nitzkii, D.V. Khveshchenko, Alan Luther, E. Mucciolo for nice conversations.

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[20] For simplicity we assume that there is odd number of electrons on the ring. In this case at small values of the flux the current is determined by electron located on the bottom of the band. The other situations with different parity are considered analogously.

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tude is, however, very small when $M$ of the order $N/2$ and which amplitude increases when
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**Figure Captions**

**Fig.1** The behavior of the persistent current as a function of flux $f$ within the half of
fundamental flux quantum: A) for 303 electrons at the values $L = 20000$ and $U = 10$ and
there are $M = 101$ particles with up-spin; B) for 99 electrons at the values $L = 10000,$
$U = 10$ and $M = 33$. On the slope where the current increases, there are perfect $1/303$
(for A) and $1/99$ (for B) flux quantum periodic oscillations. In both cases the maximal
current amplitude is equal to $\sim 10V_F/L$ and its main periodicity is equal to $1/3$ in units of
elementary flux quantum.

**Fig.2** The behavior of the ground state energy as a function of flux $f$ (the lower parabolic-
cusp curve) and the spinon energy (indicated by the short horizontal lines) for 303 electrons
with the 101 up spin electrons at the values $L = 1000$ and $U = 10$ in the region within
the half of fundamental flux quantum. The energy is expressed in the units $t10^2$. The zero
energy corresponds to $-520.0t$.

**Fig.3** A) The behavior of the holon energy (the parabolic-cusp curve) and the spinon
energy, indicated by short horizontal lines as a function of flux $f$ ($\Phi$) for 303 electrons
($M = 101$) at the values $L = 20000$ and $U = 10$ in the region within the half of fundamental
flux quantum. The zero energy corresponds to $-605.771443t$. The energy is expressed in
the units $t10^7$. B) The behavior of the ground state energy as a function of flux $f$. 

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