EXACTLY SOLVABLE PAIRING MODELS

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Abstract

Some results for two distinct but complementary exactly solvable algebraic models for pairing in atomic nuclei are presented: 1) binding energy predictions for isotopic chains of nuclei based on an extended pairing model that includes multi-pair excitations; and 2) fine structure effects among excited $0^+$ states in $N \approx Z$ nuclei that track with the proton-neutron ($pn$) and like-particle isovector pairing interactions as realized within an algebraic $sp(4)$ shell model. The results show that these models can be used to reproduce significant ranges of known experimental data, and in so doing, confirm their power to predict pairing-dominated phenomena in domains where data is unavailable.

1. Introduction

Pairing is an important interaction that is widely used in nuclear and other branches of physics. In this contribution we present some results that follow from exact algebraic solutions of an extended pairing model that includes multi-pair excitations and that is designed to reproduce binding energies of deformed nuclei, and the $sp(4)$ pairing model that can be used to track...
fine structure effects in excited $0^+$ states in medium mass nuclei.\textsuperscript{2} The results show that these models can be used to reproduce significant ranges of known experimental data, and in so doing, confirm their power to predict pairing-like phenomena in domains where data is unavailable or simply not well understood, such as binding energies for proton or neutron rich nuclei far off the line of stability and the fine structure of proton-neutron systems that are critical to understanding the $rp$-process in nucleosynthesis.

The Bardeen-Cooper-Schrieffer (BCS)\textsuperscript{3} and Hartree-Fock-Bogolyubov (HFB)\textsuperscript{4} methods for finding approximate solutions when pairing plays an important role are well known. However, the limitations of BCS methods, when applied in nuclear physics, are also well understood. First of all, the number of valence particles ($n \sim 10$) that dominate the behavior of low-lying states is too few to support the underlying assumptions of the approximations, that is, particle number fluctuations are non-negligible. As a result, particle number-nonconservation effects can lead to serious problems such as spurious states, nonorthogonal solutions, and so on. In addition, an essential feature of pairing correlations are differences between neighboring even and odd mass nuclei, which are driven mainly by Pauli blocking effects. It is difficult to treat these even-odd differences with either the BCS or HFB theories because different quasi-particle bases must be introduced for different blocked levels. Another difficulty with approximate treatments of the pairing interaction is related to the fact that both the BCS and the HFB approximations break down for an important class of physical situations. A remedy that uses particle number projection techniques complicates these methods and does not help achieve a better description of higher-lying states.

2. Mean-field plus Extended Pairing Model

The importance of having exact solutions of the pairing Hamiltonian has driven a great deal of work in recent years. In particular, building on Richardson’s early work\textsuperscript{5} and extensions to it based on the Bethe ansatz, several authors have introduced novel approaches.\textsuperscript{5,7} For the algebraic approaches based on the Bethe ansatz, the solutions are provided by a set of highly non-linear Bethe Ansatz Equations (BAE). Although these applications demonstrate that the pairing problem is exactly solvable, solutions are not easily obtained and normally require extensive numerical work, especially when the number of levels and valence pairs are large. This limits the applicability of the methodology to relatively small systems; in particular,
it cannot be applied to large systems such as well-deformed nuclei.

2.1. Algebraic Underpinnings of the Theory

The standard pairing Hamiltonian for well-deformed nuclei is given by

\[ \hat{H} = \sum_{j=1}^{p} \epsilon_j n_j - G \sum_{i,j=1}^{p} a_i^+ a_j, \quad (1) \]

where \( p \) is the total number of single-particle levels, \( G > 0 \) is the pairing strength, \( \epsilon_j \) is single-particle energies taken for example from a Nilsson model, \( n_j = c_j^\dagger c_j \) is the fermion number operator for the \( j \)-th single particle level, and \( a_i^+ = c_i^\dagger c_i^\dagger \) are pair creation (annihilation) operators. The up and down arrows in these expressions denote time-reversed states. Since each level can only be occupied by one pair due to the Pauli Exclusion Principle, the Hamiltonian (1) is also equivalent to a finite site hard-core Bose-Hubbard model with infinite range one-pair hopping and infinite on-site repulsion. Specifically, the operators \( a_i^+, a_i \), and \( n_i^a = n_i/2 \) satisfy the following hard-core boson algebra:

\[ (a_i^+)^2 = 0, \quad [a_i, a_j^+] = \delta_{ij}(1 - 2n_i^a), \quad [a_i^+, a_j^+] = [a_i, a_j] = 0. \quad (2) \]

The extended pairing Hamiltonian adds multiple-pair excitations to the standard pairing interaction (1):

\[ \hat{H} = \sum_{j=1}^{p} \epsilon_j n_j - G \sum_{i,j=1}^{p} a_i^+ a_j - G \sum_{\mu=2}^{\infty} \frac{1}{(\mu!)^2} \sum_{i_1 \neq \cdots \neq i_{2\mu}} a_{i_1}^+ \cdots a_{i_{2\mu}}^+ a_{i_{2\mu+1}} \cdots a_{i_{2\mu}}, \quad (3) \]

where \( \mu \) is a quantum number. With this extension, the model is exactly solvable.1 In particular, the \( k \)-pair excitation energies of (3) are given by the expression:

\[ E_k^{(G)} = \frac{2}{x^{(G)}} - G(k - 1), \quad (4) \]

where the undetermined variable \( x^{(G)} \) satisfies

\[ \frac{2}{x^{(G)}} + \sum_{1 \leq i_1 < i_2 < \cdots < i_k \leq p} \frac{G}{(1 - x^{(G)} \sum_{\mu=1}^{k} \epsilon_{i_{2\mu}})} = 0. \quad (5) \]

The additional quantum number \( \zeta \) can be understood as the \( \zeta \)-th solution of (5). Similar results can be shown to hold for even-odd systems except that the index \( j \) of the level occupied by the single nucleon should be excluded.
from the summation and the single-particle energy term $\epsilon_j$ contributing to the eigenenergy from the first term of (3) should be included. Extensions to many broken-pair cases are straightforward. If (5) is rewritten in terms of a new variable $\zeta^{(G)} = \frac{2}{[G \epsilon^{(G)}]}$ and the dimensionless energy of a ‘grand’ boson $\tilde{E}_{i_1 i_2 \ldots i_k} = \sum_{\mu=1}^{k} \frac{2\epsilon_{i\mu}}{G}$, (5) reduces to:

$$1 = \sum_{1 \leq i_1 < i_2 < \ldots < i_k \leq P} \frac{1}{(\tilde{E}_{i_1 i_2 \ldots i_k} - \zeta^{(G)})}. \quad (6)$$

Since there is only a single variable $\zeta^{(G)}$ in (6), the zero points of the function can be determined graphically in a manner that is similar to the one-pair solution of the TDA and RPA approximations with separable potentials. \(^4\)

### 2.2. Application to the \(^{154-181}Yb\) Isotopes

A study of the binding energies of well-deformed nuclei within the framework of the extended pairing model is currently in progress. Typically, the single-particle energies of each nucleus are calculated within the deformed Nilsson shell model with deformation parameters taken from Moller and Nix; \(^10\) experimental binding energies are taken from Audi, et al; \(^8\) and, theoretical binding energies are calculated relative to a particular core. For an even number of neutrons, only pairs of particles (bosons-like structures) are considered. For an odd number of neutrons, Pauli blocking of the Fermi level of the last unpaired fermion is invoked with the remaining fermions considered to be an even $A$ fermion system. Using (4) and (5), values of $G$ are calculated so that the experimental and theoretical binding energy match exactly. Note that for a given set of single-particle energies there is an upper limit to the binding energy for which a physically meaningful exact solution can be constructed. This upper value on the binding energy is given by the energy of the lowest ‘grand’ boson, with energy given by $\sum_{\mu=1}^{k} 2\epsilon_{i\mu}$.

As a first application of the theory, we calculated the binding energies for the \(^{154-181}Yb\) isotopes and extracted the corresponding $\log(G)$ values for the extended pairing model. The binding energy of the closed neutron shell nucleus \(^{132}Yb\) was taken to be the zero-energy reference point. Its odd-$A$ \(^{153}Yb\) neighbor was assumed to be well described by the independent particle model with Nilsson single-particle energies; this means that the pairing interaction terms have no affect on \(^{153}Yb\). The energy scale applied to the Nilsson single-particle energies, which is $3/4$ for pure harmonic oscillator interaction, was set so that the binding energy of \(^{153}Yb\)
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Figure 1. The solid line gives the theoretical binding energies of the Yb isotopes relative to that of the $^{152}\text{Yb}$ core. The single-particle energy scale is set from the binding energy of $^{153}\text{Yb}$. The inset shows the fit to values of $G$ that reproduce exactly the experimental data. The two fitting functions are: $\log(G(A)) = 662.2247 - 7.7912A + 0.0226A^2$ for even values of $A$ and $\log(G(A)) = 716.3279 - 8.4049A + 0.0244A^2$ for odd values of $A$. The Nilsson BE energy is the lowest configuration energy of the non-interacting system.

is reproduced by the independent particle model. For all the other nuclei we solved for the pairing strength $G(A)$ that reproduces the experimental binding energies exactly within the selected model space, the latter consisting of the neutron single-particle levels between the closed shells with magic numbers 50 and 82. The structure of the model space is reflected in the values of $G(A)$. In particular, $\log(G(A))$ has a smooth quadratic behavior for even- and odd-$A$ values with a minimum in the middle of the model space where the size of the space is a maximal. As shown in Figure 1, although the even- and odd-$A$ curves are very similar, they are shifted from one another due to the even-odd mass difference.

To summarize, in this section we reviewed the extended pairing model and tested its predictive power using the $^{172-177}\text{Yb}$ isotopic chain as an example. In particular, calculations of the pairing strength $G$ were carried out for the $^{154-171}\text{Yb}$ and $^{178-181}\text{Yb}$ isotopes but not for the $^{172-177}\text{Yb}$ isotopes that are in the middle of the model space where the computations are
more involved. The even- and odd-\(A\) \(\log(G)\) curves, which were assumed to have a quadratic polynomial form and therefore determined by three parameters, were fit to the two data sets which consist of 11 data points each, one for the even-\(A\) isotopes and another for odd \(A\). From the quadratic polynomial fit to the \(\log(G)\) values, we then calculate the theoretical values of the binding energy for all the nuclei shown in Figure 1. The prediction is very good when compared to the experimental numbers. Thus, based on experimental data of the nuclei in the upper and lower parts of the shell and an assumed quadratic from for \(\log(G)\) that was fit to this data, we were able to make reasonable estimates for the binding energies of mid-shell nuclei. Based on this simple exercise, we conclude that the extended pairing model has good predictive power for binding energies. Indeed, this early success suggests that the extended pairing model may have broader applicability to other well-deformed nuclei as well as other physical systems where pairing plays an important role.

3. Algebraic \(sp(4)\) Pairing Model

The recent renaissance of studies on pairing is related to the search of a reliable microscopic theory for a description of medium nuclei around the \(N = Z\) line, where like-particle pairing comprises only a part of the complicated nuclear interaction in this region. This is because for such nuclei protons and neutrons occupy the same major shells and their mutual interactions are expected to influence significantly the structure and decay of these nuclei. Such a microscopic framework is as well essential for astrophysical applications, for example the description of the \(rp\)-process in nucleosynthesis, which runs close to the proton-rich side of the valley of stability through reaction sequences of proton captures and competing \(\beta\) decays.\(^{11}\) The revival of interest in pairing correlations is also prompted by the initiation of radioactive beam experiments, which advance towards exploration of ‘exotic’ nuclei, such as neutron-deficient or \(N \approx Z\) nuclei far off the valley of stability.

In our search for a microscopic description of pairing in the broad range of nuclei with mass numbers \(32 \leq A \leq 100\) with protons and neutrons filling the same major shell, we employ an \(sp(4)\) algebraic model that accounts for proton-neutron and like-particle pairing correlations and higher-\(J\) proton-neutron interactions, including the so-called symmetry and Wigner energies.\(^{2}\) The nuclei classified within a major shell possess a clear \(Sp(4)\) dynamical symmetry. The basis operators of the \(sp(4)\) al-
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Algebra (\(\sim so(5)^{12,13}\)) have a distinct physical meaning: \(N_{\pm 1}\) counts the total number of protons (neutrons) (and hence \(\hat{N} = N_{+1} + N_{-1}\) is the total number operator), the operators \(T_{0,\pm}\) are related to isospin (where \(T_0 = (N_{+1} - N_{-1})/2\) is the third projection of isospin), while the six operators \(A_{-1,0,1}^\dagger (A_{-1,0,1})\) create (annihilate) a pair of total angular momentum \(J^\pi = 0^+\) and isospin \(T = 1\). The model Hamiltonian with an \(Sp(4)\) dynamical symmetry,

\[
H = -G \sum_{i=-1}^{-1} A_i^\dagger A_i - FA_0 A_0 - \frac{E}{2\Omega} (T^2 - \frac{3\hat{N}}{4}) - D(T_0^2 - \frac{\hat{N}}{4}) - C\hat{N}(\hat{N} - 1) - \epsilon\hat{N},
\]

includes a two-body isovector \((T = 1)\) pairing interaction and a diagonal isoscalar \((T = 0)\) force, which is proportional to a symmetry and Wigner term \((T(T+1))-like dependence). In addition, the \(D\)-term introduces isospin symmetry breaking and the \(F\)-term accounts for a plausible, still extremely weak, isospin mixing. This Hamiltonian conserves the number of particles \((N)\), the third projection of isospin \((T_0)\) and angular momentum, and changes the like-particle seniority quantum number by zero or \(\pm 2\), the latter implies scattering of a \(pp\) pair and a \(nn\) pair into two \(pn\) pairs and vice versa. The interaction strength parameters in (7) are estimated in optimum fits to the lowest isobaric analog \(0^+\) state experimental energies of total of 149 nuclei\(^2\) and are found to have a smooth dependence on the nuclear mass \(A\),

\[
\frac{G}{\Omega} = \frac{23.9 \pm 1.1}{A}, \quad \frac{E}{2\Omega} = \frac{-52 \pm 5}{A},
\]

\[
D = \frac{-37 \pm 5}{A} + (-0.24 \pm 0.09), \quad C = \left(\frac{32 \pm 1}{A}\right)^{1.7 \pm 0.2},
\]

where \(2\Omega = \Sigma_{2j}(2j + 1)\) is the shell dimension.

The basis states are constructed as \((T = 1)\)-paired fermions, \(|n_1,n_0,n_{-1}\rangle = (A_1^\dagger)^{n_1} (A_0^\dagger)^{n_0} (A_{-1}^\dagger)^{n_{-1}} |0\rangle\), and model the \(0^+\) ground state for even-even and some odd-odd nuclei and the corresponding isobaric analog excited \(0^+\) state for even-\(A\) nuclei in a significant range of nuclei, \(32 \leq A \leq 100\). The properties of these states are described well by the \(Sp(4)\) dynamical symmetry model, including quite good agreement of the isobaric analog \(0^+\) state energy spectra with experiment, and in addition the remarkable reproduction of their detailed structure properties.

3.1. Energy Spectra of Isobaric Analog \(0^+\) States

The \(Sp(4)\) model leads to a very good reproduction of the experimental energies of the lowest isobaric analog \(0^+\) state for even-\(A\) nuclei (that is, bind-
ing energies for even-even and some odd-odd nuclei) with nuclear masses $32 \leq A \leq 100$. This result follows from the very small deviation (estimated by the $\chi$-statistics) between experimental energies and the corresponding theoretical energies predicted in optimization procedures, namely $\chi = 0.496$ in the $1d_{3/2}$ shell, $\chi = 0.732$ in the $1f_{7/2}$ shell and $\chi = 1.787$ in the $1f_{5/2}2p_{1/2}2p_{3/2}1g_{9/2}$ major shell. Without varying the values of the interaction strength parameters, the energy of the higher-lying isobaric analog $0^+$ states can be theoretically calculated and they agree remarkably well with the available experimental values for the single-$j$ $1d_{3/2}$ and $1f_{7/2}$ orbits (Figure 2). However, such a comparison to experiment is impossible for the nuclei in the region with nuclear masses $56 < A < 100$, since their energy spectra are not yet completely measured, especially the higher-lying $0^+$ states.

The agreement, which is observed throughout both single-$j$ shells, represents an important result. This is because the higher-lying isobaric analog $0^+$ states constitute an experimental set independent of the data that determines the interaction strength parameters in (7). Therefore, such a result is, first, an independent test of the physical validity of the strength parameters, and, second, an indication that the interactions interpreted by the model Hamiltonian are the main driving force that defines the properties of these states. In this way, the simple $Sp(4)$ model provides for a reasonable prediction of the isobaric analog (ground and/or excited) $0^+$ states in proton-rich nuclei with energy spectra not yet experimentally fully explored. For example, in the case of the $1f_{7/2}$ level the binding energy of the proton-rich $^{48}\text{Ni}$ nucleus is estimated to be $E_0 = 348.19$ MeV, which is 0.07% greater than the sophisticated semi-empirical estimate of Moller and Nix. Likewise, for the odd-odd nuclei that do not have measured energy spectra the theory can predict the energy of their lowest $0^+$ isobaric analog state: $358.62$ MeV ($^{44}\text{V}$), $359.34$ MeV ($^{46}\text{Mn}$), $357.49$ MeV ($^{48}\text{Co}$), $394.20$ MeV ($^{50}\text{Co}$). The $Sp(4)$ model predicts the relevant $0^+$ state energies for additional 165 even-$A$ nuclei in the medium mass region of the $1f_{5/2}2p_{1/2}2p_{3/2}1g_{9/2}$ major shell. The binding energies for 25 of them are also calculated in Moller and Nix. For these even-even nuclei, we predict binding energies that on average are by 0.05% less than the semi-empirical approximation.
3.2. \( N = Z \) Irregularities, Staggering and the Pairing Gap

The theoretical \( Sp(4) \) model can be further tested through second- and higher-order discrete derivatives of the energies of the lowest isobaric analog \( 0^+ \) states in the \( Sp(4) \) systematics, without any parameter variation. The theoretical discrete derivatives under investigation not only follow the experimental patterns but their magnitude was found to be in a remarkable agreement with the data. The proposed model has been used to successfully interpret: the two-proton (two-neutron) separation energy \( S_{2p(2n)} \) for even-even nuclei (hence determined the two-proton drip line), the \( S_{pn} \) energy difference when a \( pn \) \( T = 1 \) pair is added, the observed\(^{14}\) irregularities around \( N = Z \) (Figure 3), the like-particle and \( pn \) isovector pairing gaps, and the prominent “ee-oo” staggering between even-even and odd-odd nuclides. We suggest that the oscillating “ee-oo” effects correlate with the alternating of the seniority numbers related to the \( pn \) and like-particle isovector pairing, which is in addition to the larger contribution due to the discontinuous change in isospin values associated with the symmetry.

Figure 2. Theoretical (‘th’) and experimental (‘exp’) energy spectra of the higher-lying isobaric analog \( 0^+ \) states for isotopes in \( 1f_{7/2} \) (in \( 1d_{3/2} \) (insert)).
energy.\textsuperscript{15}

![Diagram](image-url)

Figure 3. Second discrete derivatives of the energy function $E_0$: (a) $\delta I_{pp(nn)}(N\pm1) = E_0(N\pm1+2) - 2E_0(N\pm1) + E_0(N\pm1-2)$ versus $N\pm1$, as an estimate for the non-pairing like-particle nuclear interaction in MeV for the $N(Z) = 34, 36, 38$-multiplets; (b) $\delta V_{pn}(N_{+1}, N_{-1}) = E_0(N_{+1}+2, N_{-1}+2) - E_0(N_{+1}+2, N_{-1}) - E_0(N_{+1}, N_{-1}+2) + E_0(N_{+1}, N_{-1})$ versus $N_{+1}$ and $N_{-1}$, as an estimate for the residual interaction between the last proton and the last neutron in MeV for Zn, Ge, Sr isotopes.

The present study brings forward a very useful result. We find a finite energy difference of the energy function $E_0$,

$$E_0(N, T_0 + 1) - 2E_0(N, T_0) + E_0(N, T_0 - 1) =$$

$$= E_0(N_{+1} + 1, N_{-1} - 1) - 2E_0(N_{+1}, N_{-1}) + E_0(N_{+1} - 1, N_{-1} + 1),$$

(9)

that, for the specific case $T_0 = 0$ (or $N = Z$), can be interpreted as an isovector pairing gap, $\tilde{\Delta} = \Delta_{pp} + \Delta_{nn} - 2\Delta_{pn}$, which is related to the like-particle and $pn$ isovector pairing gaps. Indeed, they correspond to the $T = 1$ pairing mode because we do not consider the binding energies for all the nuclei but the respective isobaric analog $0^+$ states for the odd-odd nuclei with a $J \neq 0^+$ ground state. This investigation is the first of its kind. Moreover, the relevant energies are corrected for the Coulomb interaction and therefore the isolated effects reflect solely the nature of the nuclear interaction. In addition, the discrete derivative filter (9) can be used to estimate the pairing gaps for all the nuclei within a major shell.
when only the contribution of the pairing energy is considered in the $E_0$ energy function. In this way, the like-particle pairing gap is found to be in a very good agreement with the $12/\sqrt{A}$ experimental approximation. Small deviations from the experimental data are attributed to other two-body interactions or higher-order correlations that are not included in the theoretical model.

In summary, the symplectic $Sp(4)$ scheme allows not only for an extensive systematic study of various experimental patterns of the even-$A$ nuclei, it also offers a simple $sp(4)$ algebraic model for interpreting the results and predicting properties of nuclei that are not yet experimentally explored. The outcome of the present investigation shows that, in comparison to experiment, the $sp(4)$ algebraic approach reproduces not only overall trends of the relevant energies but as well the smaller fine features driven by isovector pairing correlations and higher-$J$ $pn$ and like-particle nuclear interactions.

4. Conclusion

Results for two distinct but complementary exactly solvable algebraic models for pairing in atomic nuclei have been presented: 1) binding energy predictions for isotopic chains of nuclei based on an extended pairing model that includes multi-pair excitations; and 2) fine structure effects among excited $0^+$ states in $N \approx Z$ nuclei that track with the proton-neutron ($pn$) and like-particle isovector pairing interactions as realized within an algebraic $sp(4)$ shell model. The results show that both models can be used to reproduce significant ranges of known experimental data, and in so doing confirm their power to predict pairing-dominated phenomena in domains where data is either not, or only partially available or simply not well understood in terms of applicable models.

In addition, it is important to reiterate that both approaches, the extended pairing model and the algebraic $sp(4)$ model, yield exact analytic solutions to their respective pairing problems. As the examples show, this is important for applications, but it is also important for theory as having exact solutions available gives one an opportunity to test approximate and perhaps simpler to apply approaches, such as the BCS scheme. Other limits as well as extensions of these theories are under investigation.

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References

1. Feng Pan, V. G. Gueorguiev and J. P. Draayer, Phys. Rev. Lett. 92, 112503 (2004).
2. K. D. Sviratcheva, A. I. Georgieva and J. P. Draayer, J. Phys. G: Nucl. Part. Phys. 29, 1281 (2003).
3. J. Bardeen, L. N. Cooper and J. R. Schrieffer, Phys. Rev. 108, 1175 (1957).
4. P. Ring and P. Schuck, *The Nuclear Many-Body Problem* (Springer Verlag, Berlin, 1980).
5. R. W. Richardson, Phys. Lett. 3, 277 (1963); Phys. Lett. 5, 82 (1963); R. W. Richardson and N. Sherman, Nucl. Phys. 52, 221 (1964).
6. Feng Pan, J. P. Draayer and W. E. Ormand, Phys. Lett. B422, 1 (1998); Feng Pan and J. P. Draayer, Phys. Lett. B442, 7 (1998); Feng Pan, J. P. Draayer and Lu Guo, J. Phys. A: Math. Gen. 33, 1597 (2000); J. Dukelsky, C. Esebbag and P. Schuck, Phys. Rev. Lett. 87, 066403 (2001); J. Dukelsky, C. Esebbag and S. Pittel, Phys. Rev. Lett. 88, 062501 (2002); H.-Q. Zhou, J. Links, R. H. McKenzie and M. D. Gould, Phys. Rev. B 65, 060502(R) (2002).
7. Feng Pan and J. P. Draayer, Ann. Phys. (NY) 271, 120 (1999).
8. G. Audi and A. H. Wapstra Nucl. Phys. A595, 409 (1995); G. Audi, O. Bersillon, J. Blachot and A. H. Wapstra, Nucl. Phys. A624, 1 (1997); (http://csnwww.in2p3.fr/AMDC/web/amdcw_en.html).
9. V. G. Gueorguiev, Feng Pan and J. P. Draayer, nucl-th/0403055.
10. P. Möller, J. R. Nix and K. L. Kratz, Atomic Data Nucl. Data Tables 66, 131 (1997); P. Möller, J. R. Nix, W. D. Myers and W. J. Swiatecki, Atomic Data Nucl. Data Tables 59, 185-381 (1995); (http://t2.lanl.gov/data/astro/molnx96/massd.html).
11. K. Langanke, Nucl. Phys. A630, 368c (1998); H. Schatz et. al, Phys. Rep. 294, 167 (1998).
12. K. Helmers, Nucl. Phys. 23, 594 (1961); K. T. Hecht, Nucl. Phys. 63, 177(1965); Phys. Rev. 139, B794 (1965); Nucl. Phys. A102, 11 (1967); J. N. Ginocchio, Nucl. Phys. 74, 321 (1965).
13. J. Engel, K. Langanke and P. Vogel, Phys. Lett.B389, 211 (1996).
14. D. S. Brenner, C. Wesselborg, R. F. Casten, D. D. Warner and J.-Y. Zhang, Phys. Lett. B243, 1 (1990); N. V. Zamfir and R. F. Casten, Phys. Rev. C43, 2879 (1991).
15. K. D. Sviratcheva, A. I. Georgieva and J. P. Draayer, Phys. Rev. C69 (2004) 024313.
16. A. Bohr and B. R. Mottelson, *Nuclear Structure* (Benjamin, New York), Vol. I (1969); Vol. II (1975).