The decay $\phi \rightarrow f_0(980)\gamma$ and the process $e^+e^- \rightarrow \phi f_0(980)$

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Abstract

The decay $\phi \rightarrow f_0(980)\gamma$ and process $e^+e^- \rightarrow \phi f_0(980)$ are considered within the local Nambu-Jona-Lasinio model. In the amplitudes of these processes contributions of $s$-quark and kaon loops are taken into account. The kaon loop gives a dominant contribution. Our estimation for the decay width of $\phi \rightarrow f_0\gamma$ is in satisfactory agreement with recent experimental data. This allows us to make some predictions for cross sections of the process $e^+e^- \rightarrow \gamma^* \rightarrow \phi f_0$ which can be tested in the C-τ factory. The total and differential cross sections of this process are calculated and presented in the figures.
I. INTRODUCTION

In the last years a lot of experimental [1, 2] and theoretical [3, 4, 5, 6, 7, 8, 9, 10] papers have been devoted to the description of $\phi$-meson decays with the production of scalar isoscalar $f_0$ mesons.

There are different theoretical interpretations of the $f_0(980)$ meson structure. In papers [3, 11, 12, 13], for example, this meson is considered as a kaon molecule. In other papers, this meson is described as a four quark state [3, 4] or as an admixture of quark-antiquark and diquark-antidiquark states [5, 6, 7]. Recently, the decays of $\phi$-mesons were considered within the ChPT [8].

In this paper, the local Nambu-Jona-Lasinio (NJL) model will be used. All mesons are treated as quark-antiquark states in this model. In particular, the $f_0(980)$ meson is the admixture of light $u\bar{u}$ and $d\bar{d}$ and strange $s\bar{s}$ quarks [9]. In the framework of this model we describe the decay $\phi \rightarrow f_0\gamma$.

This decay channel was successfully described in terms of one-loop Feynman amplitudes with intermediate state of $K^+K^-$ mesons [3].

The amplitude of this process in our approach we express in terms of $s$-quark and kaon loops. The obtained result is in satisfactory agreement with recent experimental data [14].

Using the same approximations we calculate the total and differential cross sections for the $e^+e^- \rightarrow \gamma^* \rightarrow \phi f_0$ process. A comparison with the results obtained in ChPT approach [15] and the recent experimental data [16] are discussed in Conclusion.

II. PROCESS $\phi \rightarrow f_0(980)\gamma$

The inner parameters of the NJL model are the constituent quark masses $m_u = m_d = 263$ MeV, $m_s = 407$ MeV and the ultraviolet cut-off parameter $\Lambda = 1.27$ GeV [9, 17]. These parameters are fixed by a value of the weak pion decay $\pi \rightarrow \mu\nu$ constant $f_\pi = 92.4$ MeV and by the strong decay $\rho \rightarrow \pi\pi$, $g_\rho = 5.94$ (that correspond to the width $\Gamma_{\rho \rightarrow \pi\pi} = 149.4$ MeV) [18].

\footnote{Let us note that in [17] some other values of these parameters were used which corresponded to $f_\pi = 93$ MeV, $g_\rho = 6.14$ (in that case the width $\Gamma_{\rho \rightarrow \pi\pi} = 155$ MeV). Here we use modern experimental data [18] for fixing our model parameters.}
Besides, we use the angle $\alpha$ that describes the deviation from the angle of ideal mixing of scalar mesons in the singlet-octet sector. In the case of ideal mixing we have two states: the state $\sigma_u$ consists of light $u$ and $d$ quarks, and the state $\sigma_s$ consists of $s$ quarks only. The angle $\alpha$ allows us to express physical states $f_0(980)$ and $\sigma$ through the states $\sigma_u$ and $\sigma_s$:

$$\sigma = \sigma_u \cos \alpha - \sigma_s \sin \alpha,$$

$$f_0 = \sigma_u \sin \alpha + \sigma_s \cos \alpha.$$  

The value of $\alpha = 11.85^\circ$ was obtained by using 't Hooft interaction and mass difference of $\eta$ and $\eta'$ mesons [19, 20].

Part of Lagrangian corresponding to a quark-meson interaction has the form

$$\Delta L_{int} = \bar{q} \left\{ g_{\sigma_u} \lambda_u \sigma_u + g_{\sigma_s} \lambda_s \sigma_s + i \gamma_5 g_K \left( \lambda_{K^+} K^+ + \lambda_{K^-} K^- \right) + \frac{g_\phi}{2} \gamma_{5} \lambda_s \phi^+ \right\} q. \quad (1)$$

Using the parameters of the model it is possible to calculate all meson-quark coupling constants and the constant corresponding to additional renormalization of the pseudoscalar fields $Z_\pi$ and $Z_K$ which takes into account the transition of pseudoscalar mesons to axial-vector mesons [17]

$$g_{\sigma_u} = (4I(m_u, m_u))^{-1/2} = \frac{g_\rho}{\sqrt{6}} = 2.42, \quad g_{\sigma_s} = (4I(m_s, m_s))^{-1/2} = 2.98, \quad (2)$$

$$g_{K_0^*} = (4I(m_s, m_u))^{-1/2} = 2.71, \quad g_\rho = \sqrt{6} g_{\sigma_s} = 7.32,$$

$$Z_K = \left( 1 - \frac{3(m_u + m_s)^2}{2M_{K_1}^2} \right)^{-1} = 1.52, \quad g_K = g_{K_0^*} Z_K^{1/2} = 3.34,$$

where $M_{K_1} = 1403$ MeV is the mass of the strange axial-vector meson $K_1$ and

$$I(m, m) = \frac{3}{(2\pi)^4} \int \frac{d^4 k}{(k^2 + m^2)^2} \frac{\theta(A^2 - k^2)}{(k^2 + m^2)^2} = \frac{3}{(4\pi)^2} \left( \ln \left( \frac{A^2}{m^2} + 1 \right) - \frac{A^2}{A^2 + m^2} \right),$$

$$I(m_1, m_2) = \frac{3}{(2\pi)^4} \int \frac{d^4 k}{(k^2 + m_1^2)(k^2 + m_2^2)} \frac{\theta(A^2 - k^2)}{(k^2 + m_1^2)(k^2 + m_2^2)} =$$

$$= \frac{3}{(4\pi)^2 (m_2^2 - m_1^2)} \left( m_2^2 \ln \left( \frac{A^2}{m_2^2} + 1 \right) - m_1^2 \ln \left( \frac{A^2}{m_1^2} + 1 \right) \right).$$

We used in (1) the following combinations of the Gell-Mann matrices:

$$\lambda_u = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{pmatrix} = \frac{(\lambda_s + \sqrt{2}\lambda_0)}{\sqrt{3}}, \quad \lambda_s = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & -\sqrt{2} \end{pmatrix} = \frac{(-\lambda_0 + \sqrt{2}\lambda_s)}{\sqrt{3}}.$$
The decay $\phi \to f_0 \gamma$ in the local NJL model is described by the diagrams shown in Fig. 1, where the first diagram contains the $s$-quark loop and presents the contribution of the first order of $1/N_c$ expansion (where $N_c = 3$ is the number of quark colors), and the last two diagrams describe the contribution of kaon loops (next order of $1/N_c$ expansion). All vertices of these diagrams were calculated in terms of the quark loops. Only the divergent part of this quark loop integrals with the appropriate ultraviolet regularization with the cut-off parameter $\Lambda$ was taken into account. As a result, all coupling constants in formula (2) was calculated. Let us consider, for example, the calculation of the vertex $f_0 K^+ K^-$. The integral corresponding to this vertex has the form

$$
g_{f_0 K^+ K^-} = i \frac{3}{(2\pi)^4} g_K^2 \int dk \times$$

\[ \times \left\{ \text{Sp} \left[ (\lambda_s g_{\sigma_u} \cos \alpha + \lambda_u g_{\sigma_u} \sin \alpha) S (k - p_+) \lambda_{K^+} \gamma_5 S (k) \lambda_{K^-} \gamma_5 S (k + p_-) \right] \right\} + \]

\[ + \text{Sp} \left[ (\lambda_s g_{\sigma_u} \cos \alpha + \lambda_u g_{\sigma_u} \sin \alpha) S (k - p_-) \lambda_{K^-} \gamma_5 S (k) \lambda_{K^+} \gamma_5 S (k + p_+) \right] \right\} \right\} \right\} \right\} (3)\]

where $S (k)$ is the matrix of quark propagators:

$$S (k) = \text{diag} \left( \frac{\hat{k} + m_u}{k^2 - m_u^2}, \frac{\hat{k} + m_d}{k^2 - m_d^2}, \frac{\hat{k} + m_s}{k^2 - m_s^2} \right), \quad (4)$$

and Sp[..]$ is the trace over flavour indices. Calculation of this trace leads to the following expression:

$$g_{f_0 K^+ K^-} = i \frac{3}{(2\pi)^4} g_K^2 \int dk \times$$
\[
\times \left\{ (-2\sqrt{2}) g_{\sigma} \cos \alpha \frac{\text{Sp}\left[\left(\hat{k} - \hat{p}_+ + m_s\right) \gamma_5 \left(\hat{k} + m_u\right) \gamma_5 \left(\hat{k} + \hat{p}_- + m_s\right)\right]}{(k - p_+)^2 - m_u^2) (k^2 - m_u^2) (k + p_-)^2 - m_s^2)} + \\
+ 2 g_{\sigma} \sin \alpha \frac{\text{Sp}\left[\left(\hat{k} - \hat{p}_+ + m_u\right) \gamma_5 \left(\hat{k} + m_u\right) \gamma_5 \left(\hat{k} + \hat{p}_- + m_u\right)\right]}{(k - p_+)^2 - m_u^2) (k^2 - m_u^2) (k + p_-)^2 - m_s^2}) \right\}.
\]

After calculation of the trace over the Dirac matrices we separate out the divergent terms of the quark loop and calculate them in Euclidean metric:

\[
g_{f_0K^+K^-} = i \frac{3}{(2\pi)^2} g_K^2 \int \frac{dk}{i\pi^2} \left\{ (-2\sqrt{2}) g_{\sigma} \cos \alpha \frac{4 \left((k^2 - m_u^2) (m_u - 2m_s) + \text{finite terms}\right)}{(k^2 - m_u^2) (k^2 - m_s^2)^2} + \\
+ 2 g_{\sigma} \sin \alpha \frac{4 \left((k^2 - m_u^2) (m_s - 2m_u) + \text{finite terms}\right)}{(k^2 - m_s^2) (k^2 - m_u^2)^2} \right\} = \\
g_K^2 \left\{ (-2\sqrt{2}) g_{\sigma} \cos \alpha (m_u - 2m_s) \left(\frac{4}{(2\pi)^4} \int \frac{dk}{(k^2 + m_u^2) (k^2 + m_s^2)}\right) + \\
+ 2 g_{\sigma} \sin \alpha (m_s - 2m_u) \left(\frac{4}{(2\pi)^4} \int \frac{dk}{(k^2 + m_s^2) (k^2 + m_u^2)}\right) \right\}. \tag{5}
\]

Recalling (2) the expression in the round brackets can be rewritten as \((\ldots)_1 = 4I (m_s, m_s) = (g_{\sigma_s})^{-2}\) and \((\ldots)_2 = 4I (m_u, m_s) = (g_{K_0^*})^{-2}\) and the vertex obtains the form

\[
g_{f_0K^+K^-} = 2 \left\{ \sqrt{2} g_{\sigma} \cos \alpha (2m_u - m_u) \left(\frac{g_K}{g_{\sigma_s}}\right)^2 - g_{\sigma} \sin \alpha (2m_u - m_u) \left(\frac{g_K}{g_{K_0^*}}\right)^2 \right\} = \\
= 5.51 \text{ GeV}. \tag{6}
\]

Similar calculations give us the following expressions for other constants:

\[
g_{\phi^0K^+K^-} = \frac{g_{\phi}}{\sqrt{2}} \left(\frac{g_{K_0^*}}{g_{\sigma_s}}\right)^2 Z_K \left(p^+ + p^-\right)^\mu, \\
g_{A^0K^+K^-} = e \left(p^+ + p^-\right)^\mu, \tag{7}
\]

where \(p^\pm\) are the \(K^\pm\) momenta and \(e\) is the electric charge \((e^2/4\pi = 1/137)\). Let us note that in the vertices \(g_{\phi^0K^+K^-}\) and \(g_{\phi^0\gamma\gamma K^+K^-}\) the factor \(Z_K\) disappears after taking into account \(K^+ \to K^+\) transitions on the kaon line. A similar situation takes place in the decay of \(\rho \to \pi\pi\) [17]. The vertex \(g_{\phi^0K^+K^-}\) was derived in [21, 22] and leads to satisfactory agreement with the experiment – we get the decay width \(\Gamma_{\phi^0 \to KK} = 1.88\) MeV while the experimental value is \(\Gamma_{\phi^0 \to KK}^{\text{exp}} = 2.1\) MeV [18].

Now we can calculate the contributions of the quark and kaon loops (see Fig. II to the process \(\phi \to f_0\gamma\). The quark loop gives the amplitude

\[
M_{\phi^0 \to f_0\gamma}^{(s)} = C_{\phi^0 \to f_0\gamma}^{(s)} \left( g_{\mu\nu} (p_1 p_2) - p_1^\mu p_2^\nu \right) e_\mu (p_1) e_\nu (p_2), \tag{8}
\]
\[ C^{(s)} = \frac{e}{(4\pi)^2} g_\rho g_{\sigma_s} \cos \alpha, \]
\[ A^{(s)}_{\phi \rightarrow f_0 \gamma} = \int_0^1 dx \int_0^{1-x} dy \frac{8m_s (4xy - 1)}{m_s^2 - y(1 - y)M_\phi^2 + xy(M_\phi^2 - M_{f_0}^2) + i\epsilon}. \]

Following the quark confinement condition we take into account only the real part of this amplitude. Then the amplitude square is
\[
|\text{Re} \left( M^{(s)}_{\phi \rightarrow f_0 \gamma} \right) |^2 = \frac{1}{2} \left( M_\phi^2 - M_{f_0}^2 \right)^2 |C^{(s)} \text{Re} \left( A^{(s)}_{\phi \rightarrow f_0 \gamma} \right) |^2. \tag{9} \]

It gives the following contribution to the decay width:
\[
\Gamma^{(s)}_{\phi \rightarrow f_0 \gamma} = \frac{1}{2^5 3\pi} \frac{\left( M_\phi^2 - M_{f_0}^2 \right)^3}{M_\phi^3} |C^{(s)} \text{Re} \left( A^{(s)}_{\phi \rightarrow f_0 \gamma} \right) |^2 = 6.75 \text{ eV}. \tag{10} \]

The kaon loop gives the amplitude
\[
M^{(K)}_{\phi \rightarrow f_0 \gamma} = C^{(K)} A^{(K)}_{\phi \rightarrow f_0 \gamma} (g_{\mu\nu} (p_1p_2) - p_1^\mu p_2^\nu) e_\mu(p_1) e_\nu(p_2), \tag{11} \]
\[
C^{(K)} = \frac{e}{(4\pi)^2} g_\rho \sqrt{2} g_{f_0 K^+ K^-}, \]
\[
A^{(K)}_{\phi \rightarrow f_0 \gamma} = \int_0^1 dx \int_0^{1-x} dy \frac{8(4xy)}{M_K^2 - y(1 - y)M_\phi^2 + xy(M_\phi^2 - M_{f_0}^2) + i\epsilon}. \]

Its contribution to the decay width \( \phi \rightarrow f_0 \gamma \) is dominant
\[
\Gamma^{(K)}_{\phi \rightarrow f_0 \gamma} = \frac{1}{2^5 3\pi} \frac{\left( M_\phi^2 - M_{f_0}^2 \right)^3}{M_\phi^3} |C^{(K)} A^{(K)}_{\phi \rightarrow f_0 \gamma} |^2. \tag{12} \]

It is worth noticing that a theoretical prediction has a strong dependence on the mass of the \( f_0 \)-meson value. Experimental value is \( M_{f_0} = 980 \pm 10 \text{ MeV} \) and changing \( M_{f_0} \) in the interval \( 970 \text{ MeV} \leq M_{f_0} \leq 990 \text{ MeV} \) we obtain the following interval for a theoretical prediction of decay width \( 2.39 \text{ KeV} \geq \Gamma^{(K)}_{\phi \rightarrow f_0 \gamma} \geq 0.66 \text{ KeV} \). With the contribution of the quark loop taken into account these values slightly change
\[
\Gamma^{(K+s)}_{\phi \rightarrow f_0 \gamma} \approx 0.65 \text{ KeV}. \tag{13} \]

The experimental value is \( \Gamma^{(exp)}_{\phi \rightarrow f_0 \gamma} = 0.47 \pm 0.03 \text{ KeV} \) [14]. So we can see that our prediction is in qualitative agreement with experiment at \( M_{f_0} = 990 \text{ MeV} \).
III. SUBPROCESS $\gamma^* \rightarrow \phi f_0(980)$

Let us now consider the cross process for the $\phi \rightarrow f_0\gamma$ decay, namely, the $\gamma^* \rightarrow \phi f_0$. Due to the off-mass-shell photon here the additional gauge invariant Lorentz structure appears and the amplitude can be written in the form

$$M (\gamma^*(p_2, \nu) \rightarrow \phi(p_1, \mu)f_0(p_3)) =$$

$$= \sum_{i=s,K} \frac{C_{(i)}}{24\pi^2} \epsilon_\mu(p_1)\epsilon_\nu(p_2) \left( A_{(i)} R^{\mu\nu}_{(1)} + B_{(i)} R^{\mu\nu}_{(2)} \right),$$

where $i$ denotes the type of a contribution ($i = s$ corresponds to the $s$-quark loop contribution and $i = K$ to the kaon loop contribution). Two gauge invariant structures $R^{\mu\nu}_{(1,2)}$ are

$$R^{\mu\nu}_{(1)} = \frac{g^{\mu\nu}(p_1p_2)}{p_1^2p_2^2} - \frac{p_1^\mu p_2^\nu}{p_1^2p_2^2},$$

$$R^{\mu\nu}_{(2)} = \left( p_1 - p_2 \frac{p_1^2}{p_1^2p_2^2} \right)^\mu \left( p_2 - p_1 \frac{p_2^2}{p_1^2p_2^2} \right)^\nu,$$

$$p_1^\mu R_{\mu\nu}^{(i)} = p_2^\nu R_{\mu\nu}^{(i)} = 0, \quad i = 1, 2.$$  \(15\)

The quantities $A_{(i)}, B_{(i)}$ in (14) depend only on momentum squares $(p_1^2, p_2^2, p_3^2)$ and $C_{(i)}$ are the product of the coupling constants.

Quark-loop contribution takes the form (which differs from the case of $\phi \rightarrow f_0\gamma$ by nonzero virtuality of photon)

$$C_{(q)} = e g_\rho g_{\sigma_\gamma} \cos \alpha,$$

$$A_{(q)} = -\alpha_{(q)} + \beta_{(q)} \frac{p_1^2p_2^2}{(p_1p_2)^2},$$

$$B_{(q)} = \beta_{(q)},$$

$$\alpha_{(q)} = \frac{1}{1-x} \int_0^{1-x} \int_0^{1-x} \frac{8m_s(4xy-1)}{m_s^2 - x z p_1^2 - y z p_2^2 - x y p_3^2 - i\epsilon},$$

$$\beta_{(q)} = \frac{1}{1-x} \int_0^{1-x} \int_0^{1-x} \frac{8m_s(2(x+y) - 4xy - 1)}{m_s^2 - x z p_1^2 - y z p_2^2 - x y p_3^2 - i\epsilon},$$

where $z = 1 - x - y$. The kaon-loop contribution reads as:

$$C_{(K)} = e g_{\phi K^+ K^-} g_{f_0 K^+ K^-},$$

$$A_{(K)} = -\alpha_{(K)} + \beta_{(K)} \frac{p_1^2p_2^2}{(p_1p_2)^2},$$

$$B_{(K)} = \beta_{(K)},$$
\[ e^{-}(p_{-}) \rightarrow \gamma^{*} \rightarrow \phi f_{0}(980) \]

\[ \begin{align*}
\alpha(K) &= \frac{1}{s} \int_{0}^{1-x} dx \int_{0}^{1-x} dy \frac{8 \left(4xy\right)}{M_{K}^{2} - xz p_{1}^{2} - yz p_{2}^{2} - xy p_{3}^{2} - i\epsilon}, \\
\beta(K) &= \frac{1}{s} \int_{0}^{1-x} dx \int_{0}^{1-x} dy \frac{8 \left(2(x + y) - 4xy - 1\right)}{M_{K}^{2} - xz p_{1}^{2} - yz p_{2}^{2} - xy p_{3}^{2} - i\epsilon}.
\end{align*} \]

**IV. PROCESS** \( e^{+}e^{-} \rightarrow \gamma^{*} \rightarrow \phi f_{0}(980) \)

Using the amplitude (14) we can write the amplitude for the process \( e^{+}e^{-} \rightarrow \gamma^{*} \rightarrow \phi f_{0}(980) \) (see Fig. 2)

\[ M \left( e^{+}(p_{+})e^{-}(p_{-}) \rightarrow \gamma^{*}(p_{2}) \rightarrow \phi(p_{1})f_{0}(p_{3}) \right) = \]

\[ = \frac{4\pi\alpha}{s} J_{\mu}^{QED} e_{\nu}(p_{2}) \sum_{i=s,K} C_{(i)} \frac{24\pi^{2}}{s^{2}} \left( A_{(i)} R_{(1)}^{\mu\nu} + B_{(i)} R_{(2)}^{\mu\nu} \right), \]

(16)

where \( J_{\mu}^{QED} = \bar{v}(p_{+})\gamma_{\mu}u(p_{-}) \) is the electromagnetic current of electron and positron annihilation \( (J_{\mu}p_{\mu}^{2} = 0) \), and \( e_{\nu}(p_{1}) \) is the polarization 4-vector of the \( \phi \)-meson \( (p_{1}^{\nu}e_{\nu}(p_{1}) = 0) \).

The square modulus of the amplitude (16) after summation over polarization states has the form

\[ \sum_{pol} |M|^{2} = \frac{8\pi\alpha}{s} \left\{ \frac{s_{1}^{2}}{4} |A|^{2} - \frac{1}{2} \left( |A - \bar{B}|^{2}s - |\bar{B}|^{2}\frac{s_{1}^{2}}{4M_{\phi}^{2}} \right) \left( E_{\phi}^{2} \left( 1 - \beta_{\phi}^{2}c^{2} \right) - M_{\phi}^{2} \right) \right\}, \]

(17)

where \( s_{1} = 2(p_{1}p_{2}) = s + M_{\phi}^{2} - M_{f_{0}}^{2} \), \( \bar{B} = B(4sM_{\phi}^{2}/s_{1}^{2}) \), \( E_{\phi} = \left( s + M_{\phi}^{2} - M_{f_{0}}^{2} \right) / (2\sqrt{s}) \) is the \( \phi \)-meson energy in the center-of-mass system \( c = \cos\theta = \cos(\bar{p}_{-}, \bar{p}_{1}) \) is the cosine of the emission angle of the \( \phi \)-meson, and \( \beta_{\phi} = \sqrt{\lambda(s, M_{\phi}^{2}, M_{f_{0}}^{2}) / (s + M_{\phi}^{2} - M_{f_{0}}^{2})} \) is the velocity.
of the φ-meson \((\lambda(x, y, z) = x^2 + y^2 + z^2 - 2xy - 2xz - 2yz)\) is the well-known triangle function). The quantities \(A\) and \(B\) in (17) are the sums of quark and kaon contributions

\[
A = C_{(q)}A_{(q)} + C_{(K)}A_{(K)},
\]

\[
B = C_{(q)}B_{(q)} + C_{(K)}B_{(K)}.
\]

The phase volume is

\[
d\Gamma_2 = \frac{d^3p_1 d^3p_3 (2\pi)^4}{2E_\phi 2E_{f_0} (2\pi)^6} \delta^4(p_+ + p_ - - p_1 - p_3) = \frac{\sqrt{\lambda(s, M^2_\phi, M^2_{f_0})}}{16\pi s} dc. \tag{18}
\]

The differential cross section can be written in the form

\[
\frac{d\sigma^{e^+e^-\rightarrow f_0}}{d\cos \theta} = \frac{\pi\alpha^2}{s} \left(D(s) + E(s) \cos^2 \theta \right), \tag{19}
\]

where

\[
D(s) = \frac{4\pi \lambda(s, M^2_\phi, M^2_{f_0})}{2^7 \pi s^2 \alpha} \left\{ s_1^2 |A|^2 - 2 \left( |A - \bar{B}|^2 s - |\bar{B}|^2 \frac{s_1^2}{4M^2_\phi} \right) \left( E^2_\phi - M^2_\phi \right) \right\}, \tag{20}
\]

\[
E(s) = \frac{4\pi \lambda(s, M^2_\phi, M^2_{f_0})}{2^7 \pi s^2 \alpha} 2\beta^2_\phi E^2_\phi \left( |A - \bar{B}|^2 s - |\bar{B}|^2 \frac{s_1^2}{4M^2_\phi} \right). \tag{21}
\]

The total cross section then reads as:

\[
\sigma(s) = \frac{2\alpha^2}{s} \left( D(s) + \frac{1}{3} E(s) \right). \tag{22}
\]

Unlike the \(\phi \rightarrow f_0\gamma\) decay, where the contribution of the quark loop was negligible, in the process \(e^+e^- \rightarrow \gamma^* \rightarrow \phi f_0(980)\) both contributions are of the same order. In Figs. 3 and 4 we show the contributions of quarks and kaons separately for the values of \(D(s)\) and \(E(s)\). In Fig. 5 the same contributions are shown for the total cross section.

V. CONCLUSION

The decay \(\phi \rightarrow f_0(980)\gamma\) was calculated in the framework of local NJL model. We suppose that all mesons are the quark-antiquark states. It turns out that the lowest order of \(1/N_c\) expansion (where \(N_c = 3\) is the number of colors, Hartree-Fock approximation) where only quark loops are taken into account does not give satisfactory agreement with the experimental data (see (10)). In the next order of \(1/N_c\) expansion we have to consider
FIG. 3: Coefficient function $D(s)$ from differential cross section (see (20)). The dotted line is the quark loop contribution, dashed line is the kaon loop contribution and the solid line is the total value of $D(s)$.

The meson loops and they give the dominant contribution to the amplitude of the process $\phi \to f_0\gamma$, which leads to satisfactory agreement with experimental data. By the way, a similar approximation was also used in other models for description of this process (see [3, 8, 13]).

In the same approximation of the local NJL model the total probability and the differential cross section of the process $e^+e^- \to \phi f_0(980)$ were calculated.

The recent experiment [16] of production $K^+K^-\pi^+\pi^-$ in annihilation channel at high energy of $e^+e^-$ collision show some structure of 0.7 nb size in the region $\sqrt{s} \approx 2.175$ GeV, which was treated as a resonance state. The value of the cross section exceeds the theoretical (non-resonant) cross section calculated in frames of ChPT which is equal 0.15 nb. Our result exceeds the experimental value by a factor 1.2 in this energy range. The difference can be associated with the background from the channel $e^+e^- \to \phi \pi^+\pi^-$ with the effective mass of $\pi^+\pi^-$ outside the $f_0$ meson width.

The investigation of this process and the set of similar ones with production of heavy
FIG. 4: Coefficient function $E(s)$ from differential cross section (see (21)). The dotted line is the quark loop contribution, dashed line is the kaon loop contribution and the solid line is the total value of $E(s)$.

and radially excited mesons could be part of the physical program of the BABAR and the BES-III experiment.
FIG. 5: Total cross section of the $e^+e^- \rightarrow \gamma^* \rightarrow \phi f_0(980)$ process (see (22)). The dotted line is the quark loop contribution, dashed line is the kaon loop contribution and the solid line is the total value of cross section.

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