Topological defects in 1D elastic waves

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It has been recently shown theoretically that a topological defect in a 1D periodic potential may give rise to two localized states within the energy gaps. In this work we present an experimental realization of this effect for the case of torsional waves in elastic rods. We also show numerically that three, or even more, localized states can be present if the parameters characterizing the topological defect are suitably varied.

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I. INTRODUCTION

Gumen et al [1] have considered recently the consequences of introducing a topological defect, that is, a defect that breaks the long range order in an otherwise 1D periodic structure. They analyze the case of two semi-infinite equal lattices that match at a point which is not at the center of the unit cell. As an example, they consider the potential

\[ V(x) = V_0 \cos \left( \frac{2\pi x}{d} - \frac{\Delta}{2} \text{sgn}(x) \right), \]

where \( \Delta \) is related to the strength of the defect. The authors then solve the stationary Schrödinger equation, which is Mathieu's equation at both sides of the defect, and find that for special values of the defect strength \( \Delta \), two levels appear in the forbidden band. The situation is different from what occurs with a point defect, where typically only one level lies in the energy gaps. In this paper we shall demonstrate this effect for the normal-mode frequencies of torsional waves in rods with notches, both numerically and experimentally.

II. TORSIONAL WAVES FOR RODS WITH TOPOLOGICAL DEFECTS: THEORY AND EXPERIMENT

In this note we deal with torsional vibrations of the elastic rod shown in Fig. 1. The rods consist of \( N \) unit cells plus a defective cell, which is the topological defect. With the exception of the defect, each cell is formed by three cylinders, one of length \( l - \epsilon \) with cross section area \( S \) and two cylinders of length \( \epsilon/2 \) with cross section area \( s \). In the defect the central cylinder has length \( (l - \epsilon) + \chi \). The wavelength is much larger than the radius of the rod, so the system is indeed one dimensional.

Using the electromagnetic acoustic transducer (EMAT) for low frequencies that we have recently developed [2], we can excite and measure normal-mode frequencies and wave amplitudes for torsional waves in metallic rods. The experimental apparatus has been described in detail elsewhere [3]. For locally periodic systems a band structure emerges. As shown in Ref. [3],

FIG. 1: Geometry of the aluminum rod. The length of the unit cell is \( l \) and \( \epsilon \) is the width of the notch. Here \( S \) and \( s \) are the cross section areas of the rod and notches, respectively. The defective cell has length \( (l - \epsilon) + \chi \).

FIG. 2: Experimental (crosses) and numerical (circles) frequency spectra of torsional waves for a defective rod with \( N = 10, l = 10.00 \) cm, \( \epsilon = 1.00 \) cm, and \( \chi = 6.00 \) cm. In the case of the fourth band only four frequencies are shown.
FIG. 3: Experimental (dashed lines) and theoretical (continuous lines) wave amplitudes for a) \( n=22 \) and b) \( n=23 \) nodes. Since the vertical scale is logarithmic, it can be seen that theory and experiment agree, except for very small amplitudes. Experimental values at the right-hand side were not measured, since in this portion of the rod the exciter interferes with the detector.

these normal-mode properties can also be computed using the transfer matrix method. The theoretical results agree very well with our experimental measurements; we should emphasize that this is a parameter free fit.

In Fig. 2 we present the band spectrum obtained for \( N = 10 \), both from the theoretical and experimental points of view. We see that in the gap between the first and second bands only one frequency appears, but in the second forbidden gap two levels lie. In some of the higher gaps two frequencies are also found. This is an experimental realization of the theoretical findings of Gumen et al. In Fig. 3 we show the wave amplitudes of these two states; they are localized around the topological defect with an exponential decay. The theoretical values, both for the frequencies and wave amplitudes coincide well with our measurements, as these two figures show. The theoretical results are easily extended for wider ranges of \( \chi \). The band spectrum as a function of \( \chi \) is given in Fig. 4. Since the frequencies of the localized states are proportional to \( m/(l - \epsilon + \chi) \), where \( m \) is an integer number, more than one level can lie in the forbidden band. For example, as will be seen in Fig. 4 for \( \chi = 18 \) cm two levels appear, whereas for higher values of \( \chi \) three levels are located in the second forbidden band.

FIG. 4: Theoretical frequency spectrum as a function of \( \chi \). For \( \chi = 18 \) cm two levels lie between the second and third bands. For larger values of \( \chi \) three levels are in the forbidden band.

III. CONCLUSION

We have measured and calculated numerically normal-mode frequencies and amplitudes of torsional waves in a rod with a topological defect. The theoretical prediction of Gumen et al. that for certain topological defects two levels instead of one become localized is shown to be true, and an experimental example is provided. We also give, via a numerical calculation, a generalization of the results found in Ref. 1, since for other special values of the topological defect more than two frequencies can lie within the forbidden bands.

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