Development of a mathematical model of operation forming with discounting assumption about the flat-deformed state

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Abstract. For development and improvement, technological processes are a necessary strict analysis of the maximum numbering factors most strongly influencing the deformation process and flat-deformed strain state procurement blank. When determining the strained and deformed conditions of blank lead the admission about the flat-deformed state, which ensures the minimal thickness variation thin-walled detail by forming process. In this article, the complex method of researches including the theoretical analysis and check of the entered condition in the program ANSYS complex is used.

1. Introduction
The development of science-based ways and methods for creating resource-saving technologies [1, 2, 3] includes a wide range of theoretical, experimental, technological and computer-software tasks [4, 5]. These tasks, first of all, include the development of more complete and accurate mathematical models of operations, the steady flow conditions of which are achieved by maximizing the use of internal reserves of material deformation and in-depth analysis of the stress and strain state of the workpiece. The presence of a developed mathematical apparatus allows one to create a model, but, as a rule, with simplifications when using this apparatus. However, an adequate mathematical model allows you to predict and evaluate the performance of the process in the given conditions, checks the hypotheses about the causes of the observed phenomena and unwanted changes. For calculations of stresses and strains, the so-called engineering method is most often used, based on the joint solution of the equilibrium equations for the elementary volume of the metal produced in the deformation zone and the equations of plasticity. Solving problems in an analytical form is very difficult, so assumptions are used in practice to simplify.

2. Object of study
As is known from the theory of plastic deformations, the mathematical analysis of deformation processes is carried out by jointly solving the equilibrium equations, the plasticity condition, the coupling equations for stresses and strains, the deformation continuity equations, and the continuity equation [6]. For finding arbitrary constants, the boundary conditions determined by the specified deformation conditions are used. It should be noted that the solution of this cumbersome system, taking into account the simultaneous influence of many factors, encounters considerable mathematical difficulties, which in most cases do not allow to obtain exact closed solutions in the form of formulas that functionally reflect the influence of the main factors on the deformation process. At the same time, such formulas are of particular value, since they make it possible not only to realize the process
of deformation in one or another sheet punching operation but also to identify the conditions for optimizing and consciously controlling technological processes and the quality of the products obtained. These difficulties force us to resort to the analysis of form-forming operations of sheet metal stamping to significant simplifications and schematization of the process for obtaining analytical dependencies. In the engineering method of solution, a number of assumptions are used, such as the workpiece material is homogeneous, continuous, isotropic, or orthotropic; stress state schemes are reduced to axisymmetric or flat. In planar circuits, the dependence of normal stresses only on one of the coordinates appears, as a result of which the number of differential equations is reduced to one, containing simple derivatives; simplified plasticity conditions are used; tangential stresses from the action of contact friction forces are tangential to the generator of the workpiece and refer to its middle surface. The simplifying assumptions adopted here do not contradict the modern theory of plastic mechanics and the data of direct experience. Let us compare the results of a theoretical solution and simulation using a plane-deformed condition (when tangential deformation) in the case of obtaining a uniform thickness for parts of a convex shape. Let us determine the maximum possible error of the proposed assumption for extreme cases of molding.

3. The method, theoretical foundations

The shape of the part will be presented in the form of a spherical surface with relative dimensions (Figure 1). The angle between the axis of symmetry and the tangent to the generator of the smaller base is equal to the angle of the taper of the workpiece. This equality is determined by the condition of the seizure of the workpiece at the time of the beginning of the deformation. Under the conditions of the process, larger and smaller bases do not change their size until the end of shaping, i.e. the radius of the smaller base of the cone is equal to the radius of the smaller base of the part, and the radius of the larger base of the cone is equal to the radius of the larger base of the part. Consider the case of how the coordinates of an element with a radius change when it takes a position on the finished part, provided that the thickness of the resulting part is thinned and is a constant value. The value of this value is found from the known forms of the workpiece, the details and the conditions of the constancy of volumes by the known value of the thickness of the workpiece.

![Figure 1. A spherical shape detail.](image)

We define from the geometric relations:

$$V_{billet} = \frac{2\pi (R+r)^2}{2} \cdot \frac{(R-r)}{\sin \alpha_{billet}} \cdot S_{billet} = \frac{\pi (R^2-r^2)}{\sin \alpha_{billet}} \cdot S_{billet}$$

where:

- $h$ - is the distance from the larger base to the element under consideration after shaping;
- $h$ - is the height of the part obtained;
- $R$ - is the radius of the larger base of the part;
- $r$ - is the radius of the smaller base of the part;
- $\alpha_{det}$ - is the angle formed by the edge of the part of the larger base and the radius drawn to the point of the part obtained by displacing the smaller base of the part after shaping;
- $\alpha_{det1}$ - is the angle of the taper of the workpiece;
- $\alpha_{det1}$ - is the angle formed by the edge of the part of the larger base and the radius drawn to the point of the part obtained by displacing the smaller base of the part after shaping;
- $\rho_{billet}$ - is the angle formed by the edge of the part of the larger base and the radius drawn to a point with a radius $\rho_{det}$ obtained by displacement after shaping from a workpiece point with a radius $\rho_{billet}$.
\[ V_{\text{det}} = 2\pi RhS_{\text{det}} \]  \hspace{1cm} (2)

at \( h = R \sin a_{\text{det}} \)

\[ V_{\text{det}} = 2\pi R^2 \sin a_{\text{det}} S_{\text{det}} \]  \hspace{1cm} (3)

where \( V_{\text{det}} \) the volume of the spherical part of the part; \( V_{\text{billet}} \) - amount of billet.

Equating the expressions (1) and (3) determine the thickness of the part:

\[ S_{\text{det}} = \frac{(R^2 - r^2)}{2 \sin a_{\text{billet}} R^2 \sin a_{\text{det}}} \cdot S_{\text{billet}} \]  \hspace{1cm} (4)

Considering the limiting case of forming from a conical billet into a spherical shaped part, we have the value of the maximum thinning of the billet \( \frac{S_{\text{billet}}}{S_{\text{det}}} = 2 \) whatever relationship \( \frac{r}{R} \). Indeed, based on the geometric relationships and dependencies (4) with \( \sin a_{\text{billet}} = \sin a_{\text{det}} = \sqrt{1 - \cos^2 a_{\text{det}}} = \sqrt{1 - \left(\frac{r}{R}\right)^2} \) we have:

\[ \frac{S_{\text{billet}}}{S_{\text{det}}} = 2. \]  \hspace{1cm} (5)

The relationship between the coordinates of the elements on the workpiece and on the parts will find from the condition of the constancy of the volumes of the parts of the workpiece and parts. The volume of the workpiece, the limited section \( \text{obcd} \) equals \( V_{\text{billet}} = V_{\text{obcd}} : \)

\[ V_{\rho_{\text{billet}}} = \frac{\pi(R^2 - \rho_{\text{billet}}^2)}{\sin a_{\text{billet}}} \cdot S_{\text{billet}}, \]  \hspace{1cm} (6)

where \( \rho \) - current radius.

The volume of a part limited by section \( \text{obc1d1} \) current radius \( V_{\rho_{\text{det}}} = V_{\text{obc1d1}} : \)

\[ V_{\rho_{\text{det}}} = 2\pi R^2 S_{\rho_{\text{det}}} \sin a_{\text{det1}}. \]  \hspace{1cm} (7)

We use dependences:

\[ h_1 = \sqrt{R^2 - \rho_{\text{det1}}^2}, \]
\[ h = \sqrt{R^2 - \rho_{\text{billet}}^2}. \]

Equating volumes (6) and (7), we find the connection between \( \rho_{\text{billet}} \) and \( \rho_{\text{det}} : \)

\[ \rho_{\text{billet}} = \sqrt{R^2 - \sqrt{(R^2 - \rho_{\text{billet}}^2)(R^2 - \rho_{\text{det}}^2)}}. \]  \hspace{1cm} (8)

Next, we determine the relative difference (error) between the coordinates of the same element on the billet and detail.

\[ \xi = \frac{\rho_{\text{det}} - \rho_{\text{billet}}}{\rho_{\text{det}}} \]  \hspace{1cm} (9)

Substituting (8) into (9), we get:

\[ \xi = x - \sqrt{1 - \psi(1 - x^2)} \cdot \frac{x}{x}, \]  \hspace{1cm} (10)

where \( x = \frac{\rho_{\text{det}}}{R}, \psi = (1 - \frac{r^2}{R^2}). \)

Find the coordinate of the element at which \( \xi \) has a maximum. For this we take \( \frac{\partial \xi}{\partial x} = 0 \). After a series of transformations we have:

\[ \frac{\rho_{\text{det}}}{R} = x_{\text{min}} = \sqrt{1 - \frac{B^2}{\psi}}, \]  \hspace{1cm} (11)

where \( B = y = \frac{\psi}{1 - 2\psi} - \sqrt{\frac{\psi^2}{(1 - 2\psi)^2} + \frac{\psi^2}{1 - 2\psi}}. \)

4. Simulation results

In fairly time-consuming tasks, the use of modern automation tools allows us to eliminate a significant part of the routine work and to present in a visual form simulation results [7].
To confirm the admission entered, the process of forming parts belonging to the class of thin-walled was modeled: \( \frac{D}{t} < 0.008 \) using software ANSYS [3]. To analyze the process, an element was selected with the wording SHELL163 - shell element with 4 nodes, the possibility of bending and spring back. The element has 12 degrees of freedom in each node: displacement, acceleration, and speed in the directions along the x, y, z axes, as well as rotation around the x, y, z axes. To solve any physical problem by any numerical method, you must first build a geometric model of the part, body or region. This is usually one of the most time-consuming steps in solving applied problems.

Preprocessor ANSYS allows you to transfer the original geometry from other CAD-systems, so the geometry for the simulated process was transferred from the Compass-3D system using the IGS format. To reduce the number of elements, when solving an axisymmetric problem, we considered \( \frac{1}{4} \) volume. To account for the axial symmetry, in this case, it is necessary to set the appropriate boundary conditions. A special feature of LS-DYNA is the necessity of splitting even completely rigid bodies into finite elements. To reduce the number of finite elements, it is convenient to represent rigid bodies (tooling) in the form of surfaces (shells) directly in contact with the workpiece. Each shell is thick, and all calculations are made relative to the middle surface. In the calculation, the thickness of the shells representing solid bodies (punch, die, pressing, ejector) was taken to be 0.5 mm (in general, it can be any). The resulting geometric model is shown in Figure 2.

![Figure 2. Geometric model.](image)

For the preparation, physical properties were used that correspond to steel 12X18H10T. The behavior of the workpiece material is described by a bilinear strain curve (Figure 3), beginning at the origin of coordinates with positive values of deformations and stresses. The slope of the first section is determined on the basis of the elastic characteristics of the material. At the point corresponding to the yield point, the curve continues along the second angle defined by the tangent modulus, having the same units as the modulus of elasticity. It was established experimentally that this model satisfactorily describes the deformations of most metals. The model material of the matrix, clamp, punch, and ejector - a solid body. The finite element mesh can significantly affect the quality of the results obtained. Usually, a smaller partition gives the best accuracy results. However, an approximation of the geometry of a model by a large number of small elements leads to a system of algebraic equations of a large order, which can affect the speed of the calculation. You can evaluate the quality of the finite element model by successively solving several problems with different numbers of elements. If the solution (maximum displacements and stresses) ceases to noticeably change when using a more dense grid, then we can assume that the optimal level of discretization has been reached and a further increase in grid discretization is not rational. The image of the model with splitting into finite elements is shown in Figure 4.

One of the stages of preparation for carrying out the calculation and obtaining satisfactory results is the determination of external influences on a solid-state object, enclosed in a volume already divided into finite elements. Since \( \frac{1}{4} \) of the volume of the physical model is considered, one translational and two rotational degrees of freedom were prohibited in the nodes of the finite mesh elements of the workpiece located on the symmetry boundary to ensure the conditions of axial symmetry.

Also, two types of external loads were used in the task - kinematic (movement of the punch with a given speed) and force (impact of the clamp and ejector on the workpiece with a certain force). LS-DYNA has many options for creating contact conditions. As a rule, the task does not require a
redefinition of parameter values that are set “by default”. The calculation was carried out at different values of the friction coefficient. To construct the problem, a type of friction was used - “automatic, surface-to-surface”. This type of friction allows taking into account the thickness of the shells. The following components were chosen as the contacting surfaces: billet - punch; blank - matrix; blank - clamp; billet - ejector. The images obtained at the last calculation step were transferred to the Compass graphing drawing and graphic system (Figures 5, 6).

Figure 3. Bilinear curve.  
Figure 4. Model with splitting deformation into finite elements.

Figures 5. The distribution of thickness along the generatrix after shaping.  
Figures 6. Distribution of strain intensity along the generatrix.

5. Conclusions
Analyzing the simulation results, we can conclude that both in the area of elements close to the workpiece clamps and intermediate there is a tangential deformation close to zero, i.e. \( e_{\tau} \approx 0 \), which characterizes the shaping under the assumption of a plane-deformed state.

In a theoretical solution, the considered thinning is twice as applicable to the case of very plastic material, and \( r/R \) does not exceed 1/2, we get that the difference in radii for this case will correspond to \( \chi_{\min} = 0.92 \) and \( \xi \approx 11.6 \% \). For the majority of considered parts of convex shape, the radius of curvature of which is much greater than the radius of the part, this error will be much smaller (will be 5-6%). The convergence of the obtained results confirms the possibility of applying the assumptions and replacing the analytical solution of the problem by modeling in software products.

6. References
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