Kinematics Analysis and Trajectory Planning of the Working Device for Hydraulic Excavators

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Abstract. Robotization has become a research hotspot in the field of excavation machinery. In this paper, the excavator working device is regarded as the open-chain 4 DOF manipulator. The link coordinate system is established by D-H method, and forward kinematics from the configuration space to the joint space is analysed. Moreover, inverse kinematics and the mapping relationship between the joint space and the driving space are solved by geometric approach. The manipulator kinematics model is developed based on MATLAB Robotics Toolbox. Meanwhile, the point cloud image of the workspace for the excavator working device is plotted by the Latin hypercube sampling, and the complete working sequence for the excavator is planned and simulated. To verify the effect of trajectory planning, a multi-physics domain fusion model including the mechanical kinematics model, the hydraulic subsystem and the trajectory tracking control strategy was implemented by hybrid modeling method in MATLAB. At the same time, this work also presents a theoretical reference for the research of the motion trajectory for the excavator working device.

1. Introduction

Among all kinds of mechatronics products, hydraulic excavators as the core force of infrastructure can realize one machine multipurpose, they are widely used in foundation excavation, material loading, obstacles dredging and so on. With the development of hydraulic excavators, its robotization has become the focus of current research. The hydraulic excavator working device can be considered as an open-chain 4 DOF manipulator driven by hydraulic cylinder [1]. As an important basic technology, the trajectory planning of the working device helps the excavator to realize the automatic operation. At present, many scholars have studied the trajectory planning of hydraulic excavator working device. A novel trajectory planning and control method for autonomous mining was proposed by Jud et al, it can be independent of the soil composition and go beyond a single dig [2]. In order to achieve a well-matched smooth trajectory with the desired trajectory, the trajectory generation scheme based on ANFIS was presented by Vu et al [3]. With the aim of continuous motion and small vibration impact, Wenwen studied the trajectory planning method in the operation space and joint space [4]. In addition, a kinematics simulation model of the excavator working device is established using Robotics Toolbox and the point-to-point trajectory scheduling and simulation are carried out by Hailang [5].

In this paper, the mainstream medium hydraulic excavator is selected as the research object, and its working device is mainly composed of the swing, the boom, the arm, the bucket and their corresponding hydraulic actuator. Firstly, the forward kinematics, the inverse kinematics and the mutual mapping of the workspaces of the excavator working device are analyzed and given. Secondly, the trajectory planning for the excavator working device are studied using MATLAB Robotics Toolbox.
Toolbox. Finally, the multi-physics domain fusion simulation model is developed using hybrid modeling method and the trajectory planning of the working device is verified based on this model.

2. Kinematic analysis

2.1. Establishment of a reference frame
For the excavator working device, let the base coordinate system, the boom joint coordinate system, the arm joint coordinate system and the bucket joint coordinate system be \( \{O_0\} \), \( \{O_1\} \), \( \{O_2\} \), \( \{O_3\} \) and \( \{O_4\} \) respectively. It should be noted that all joints except the rotary device are hinged in the same longitudinal plane. They are presented in Figure 1. And Table 1 presents the D-H parameters of the excavator working device.

![Figure 1. Structure and coordinate space.](image1)

![Figure 2. Coordinate transformation.](image2)

**Table 1. D-H parameters of the excavator working device**

| Joint Number | \( \theta_i (\degree) \) | \( d_i (\text{m}) \) | \( a_i (\text{m}) \) | \( \alpha_i (\degree) \) | \( \sigma_i \) |
|--------------|-----------------|-----------------|-----------------|-----------------|-----------------|
| 0            | \( \theta_0 \)   | 1.480           | 0.071           | 90              | 0               |
| 1            | \( \theta_1 \)   | 0               | 5.669           | 0               | 0               |
| 2            | \( \theta_2 \)   | 0               | 2.756           | 0               | 0               |
| 3            | \( \theta_3 \)   | 0               | 1.570           | 0               | 0               |

2.2. Analysis of the forward kinematics and inverse kinematics
By the forward kinematics analysis from the configuration space to the joint space, the position of the excavator bucket can be controlled by each joint. In the coordinate frame of link, the transformation method between two adjacent coordinate systems is 
\[
^{i-i-1}A_i (\theta_i, d_i, a_i, \alpha_i) = R_z (\theta_i) T_x (d_i) T_x (a_i) R_x (\alpha_i). 
\]

And the coordinate transformation process is verified using Matlab Robotics Toolbox, as shown in Figure 2. Meanwhile, the transformation matrix can be expressed as:

\[
^{i-i-1}T_i = \begin{bmatrix}
\cos \theta_i & -\cos \alpha_i \sin \theta_i & \sin \alpha_i \sin \theta_i & a_i \cos \theta_i & 0 \\
\sin \theta_i & \cos \alpha_i \cos \theta_i & -\sin \alpha_i \cos \theta_i & a_i \sin \theta_i & 0 \\
0 & \sin \alpha_i & \cos \alpha_i & d_i & 0 \\
0 & 0 & 0 & 1 & 1
\end{bmatrix}
\]

(1)

where \( \theta_i \) is the joint angle, \( \alpha_i \) is the torsion angle, \( d_i \) is the rod migration, and \( a_i \) is the rod length.

According to formula (1), the transformation matrix of the coordinate system \( \{O_4\} \) relative to the base coordinate system \( \{O_0\} \) is further solved:
\[
{i}T_i = \begin{bmatrix}
    c_0 c_{123} & -c_0 s_{123} & s_0 & c_0 (a_3 c_{123} + a_2 c_{12} + a_1 c + a_0) \\
    s_0 c_{123} & -s_0 s_{123} & -c_0 & s_0 (a_3 c_{123} + a_2 c_{12} + a_1 c + a_0) \\
    s_{123} & c_{123} & 0 & a_3 s_{123} + a_2 s_{12} + a_1 s + a_0 \\
    0 & 0 & 0 & 1
\end{bmatrix}
\]

where \(c_i = \cos \theta_i\); \(s_i = \sin(\theta_i)\); \(c_{ijk} = \cos(\theta_i + \theta_j + \theta_k)\).

In the coordinate system \(\{O_4\}\), let the bucket tip coordinates be \(4P = [0,0,0,1]^T\). According to the formula (2), the coordinates in the base coordinate system \(\{O_0\}\) can be calculated as:

\[
^0P = ^0T_i {^iP} = \begin{bmatrix}
    c_0 (a_3 c_{123} + a_2 c_{12} + a_1 c + a_0) \\
    s_0 (a_3 c_{123} + a_2 c_{12} + a_1 c + a_0) \\
    a_3 s_{123} + a_2 s_{12} + a_1 s + a_0 \\
    \theta_1 + \theta_2 + \theta_3
\end{bmatrix}
\]

During the excavation process, let the \(\theta_0\) keep invariant. Therefore, the bucket direction can be uniquely determined by \(\theta_\omega = \theta_1 + \theta_2 + \theta_3\), and the position vector of the bucket is calculated as:

\[
\begin{bmatrix}
    x \\
    y \\
    z \\
    \theta_\omega
\end{bmatrix} = \begin{bmatrix}
    c_0 (a_3 c_{123} + a_2 c_{12} + a_1 c + a_0) \\
    s_0 (a_3 c_{123} + a_2 c_{12} + a_1 c + a_0) \\
    a_3 s_{123} + a_2 s_{12} + a_1 s + a_0 \\
    \theta_1 + \theta_2 + \theta_3
\end{bmatrix}
\]

Each joint angle of the excavator working device can be determined by the position of the bucket according to the inverse kinematics analysis. Considering that the excavator working device belongs to the four degrees manipulator and its structure is simple, the inverse kinematics is solved by geometric method in this section. Let the new base coordinate system be \(O_\alpha\{X_o,Y_o,Z_o\}\), which is derived from the translation of the original base coordinate system \(\{O_0\}\) to \(\{O_\alpha\}\). The coordinates of the bucket tip relative to the new base coordinate system \(\{O_\alpha\}\) can be obtained by geometric relation:

\[
\begin{bmatrix}
    \theta_0 \\
    \theta_1 \\
    \theta_2 \\
    \theta_3
\end{bmatrix} = \begin{bmatrix}
    \arctan(y/x) \\
    \alpha + \beta - \gamma \\
    \arccos([a_1^2 + a_2^2 - a_0^2] / 2a_1 a_2) - \pi \\
    \theta_\omega - \theta_1 - \theta_2
\end{bmatrix}
\]

In Figure 1, the mutual mapping between the joint space and the actuating space can be acquired according to the geometric relation.

\[
\begin{bmatrix}
    \lambda_2 \\
    \lambda_3 \\
    \lambda_4
\end{bmatrix} = \begin{bmatrix}
    \sqrt{A_{02}^2 + B_{02}^2 - 2A_{02} \cdot B_{02} \cos \angle AO_B} \\
    \sqrt{C_{02}^2 + D_{02}^2 - 2C_{02} \cdot D_{02} \cos \angle CO_D} \\
    \sqrt{G_{03}^2 + E_{03}^2 - 2G_{03} \cdot E_{03} \cos \angle EFG}
\end{bmatrix}
\]

\[
\begin{bmatrix}
    \theta_1 \\
    \theta_2 \\
    \theta_3
\end{bmatrix} = \begin{bmatrix}
    \angle AO_B - \angle BO_C - \angle AO_I \\
    180^\circ - \angle O_2 O_1 C - \angle O_2 O_0 D - \angle CO_D \\
    180^\circ - \angle O_2 O_3 F - \angle FO G - \angle GO_H - \angle O_4 O_3 H
\end{bmatrix}
\]
where $\lambda_1$, $\lambda_2$, and $\lambda_4$ denote the lengths of the hydraulic actuators for the boom, arm and bucket, respectively.

3. Trajectory planning

According to the standard D-H parameters of the hydraulic excavator working device, the manipulator simulation model was established using MATLAB Robotics Toolbox. The workspace envelope diagram for the mining robot is solved by Latin hypercube sampling. All the angles of the swing joint, the boom joint, the arm joint and the bucket joint are randomly discretized in their corresponding range of variation. The coordinate value of the Cartesian coordinate space can be calculated according to the forward kinematics analysis, and the program is written in MATLAB to draw the simulation results, as shown in Figure 3 and Figure 4.

![Figure 3. Workspace in 3-D space.](image1.png)

![Figure 4. Workspace in X-Z plane.](image2.png)

In this section, the jtraj function in Robotics Toolbox is used to simulate the trajectory planning for the joint space. In the configuration space, eleven target points are selected as follows: (6.18, 0.00, -0.39), (6.14, 0.00, -1.93), (5.97, 0.00, -2.23), (5.45, 0.00, -3.04), (4.28, 0.00, -2.99), (3.59, 0.00, -2.45), (3.49, 0.00, -0.74), (4.28, 0.00, 4.12), (1.11, 4.13, 4.12), (1.48, 5.52, 3.41), (6.08, 1.11, -0.39). And, the angle values of each joint are solved by inverse kinematics. The swing angle is \([0, 0, 0, 0, 0, 0, 0, 0, 75.00, 75.00, 10.30]\); The boom angle is \([24.20, 8.74, 5.71, -3.15, -5.53, -7.91, 5.17, 51.60, 51.60, 62.30, 24.20]\); The arm angle is \([-96.90, -87.20, -86.30, -81.10, -93.20, -94.50, -109.00, -87.20, -87.20, -104.00, -96.90]\); The boom bucket angle is \([-12.90, -14.60, -16.40, -26.50, -31.60, -60.50, -74.10, -137.00, -137.00, -11.20, -12.90]\). The trajectory planning simulation model is schematically illustrated in Figure 5. Other simulation results are shown in Figure 6, in Figure 7, and in Figure 8.

![Figure 5. Trajectory planning.](image3.png)

![Figure 6. Joint angle curves.](image4.png)
4. Validation of Trajectory Planning

For the verification of the trajectory planning results, the multi-physics domain model of the working device for a medium hydraulic excavator is established by using multisystem hybrid modeling method in MATLAB [6]. Firstly, the 3D model of the real excavator is drawn by SolidWorks. After the complete assembly body is set up with actual properties and reasonable constraints, it is exported as a description file by means of Simscape Multibody plug-in. And the mechanical system is developed when the description file is imported into Simscape Multibody using the smimport function. Then, the hydraulic driving system is developed by calling various hydraulic components in the Simscape Fluids library. Finally, the controller applied to the signal regulation of each main valve is built in Simulink, and the sensors are reasonably added at each joint for data measurement. Each module is selected into different separate subsystems according to the function, and the multi-physics domain fusion simulation model is obtained by reasonable connection, as shown in Figure 9.

The action of the excavator working device can be observed by the simulation based on the multi-physics domain model. Comparing the planned motion trajectory, the measured data proves that the
designed parameters are feasible and can meet the intended objectives. Figure 10 and Figure 11 show the verification results.

5. Conclusion
In this paper, the forward kinematics and inverse kinematics for the hydraulic excavator working device are analyzed by the standard D-H approach and the geometric approach, respectively. The manipulator simulation model is developed using MATLAB Robotics Toolbox. It can be seen that the joint angle curve is smooth, the angular velocity and the angular acceleration are continuous. And it is shown that the bucket does not produce vibration, which ensures the smoothness of the motion for the excavator working device. In addition, the multi-physics domain fusion model driven by the trajectory signal shows that the trajectory planning is reasonable.

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