Open strings and their symmetry groups

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The last three years have seen a large amount of progress in String Theory [1], and the subject itself has undergone a change of scope. This is well reflected in the content of the talks that have preceded this one.

Much of the recent progress in String Theory can be traced to a precise strategy: a careful study of the few models known since the beginnings of the subject, and the abstraction from them of basic properties that one would like to demand from other models. This could be termed a set of “model-building rules”. The approach corresponds to the fact, often a source of embarrassment to specialists, that String Theory, born as a set of rules rather than as a set of principles, has long resisted attempts to reduce it to a logically satisfying structure.

The first property is two-dimensional conformal invariance. This is the motive behind the machinery that has become known as Two-Dimensional Conformal Field Theory [2, 3]. Conformal invariance entered the subject long ago, as the cure to the unitarity problems of the bosonic model, by necessity defined in terms of oscillators of Minkowski, rather than Euclidean, signature. Conformal invariance is responsible for a transverse spectrum of states, all of positive norm, and embodies the projective invariance (duality) of the originally known amplitudes. Conformal invariance is reflected, in the context of the closed bosonic string, in the invariance under two mutually commuting Virasoro algebras.

Conformal Field Theory is an algebraic, and thus non-perturbative, framework for the description of all models that share this property of invariance under two distinct Virasoro algebras. The algebraic description proceeds via “primary fields” (tensors of the conformal group), and entails the encoding of the two-dimensional dynamics in the coefficient functions of the operator product algebra. The operator product coefficients are somewhat reminiscent, in their role, of the structure constants in ordinary Lie algebras. Like structure constants, they are subject to quadratic constraints. These embody the duality properties of amplitudes. In principle, these data are sufficient to reconstruct the correlators of the fields, and thus the scattering amplitudes of String Theory. The closed bosonic string in 26 dimensions is built out of a particular two-dimensional conformal field theory, one consisting of 26 copies of a free massless bosonic field. This model has a distinctive feature: the value of the central charge is 26 for each of the two independent Virasoro algebras. In modern terms, this compensates the contribution (equal to -26) of the conformal field theory of the reparametrization ghosts, to give a resulting model which is exactly conformally invariant, i.e. characterized by a total central charge equal to
Alternatively, this property is linked to the absence of local anomalies in the two-dimensional field theory. Generalizations of the bosonic model are to retain this property, if they are to describe a space time of Minkowski signature, and a suitable generalization is needed to deal with (possible) super-partners of the string coordinates, in the form of super-conformal invariance.

The second property is modular invariance. It is essentially the statement that the theory be a theory of closed strings. This property was identified long ago [5], again in the context of the bosonic model. In modern language, following Polyakov [4], one would correct the tree amplitudes of String Theory by the addition of terms where the Conformal Field Theory lives on surfaces of increasing genera. For simplicity, let us restrict our attention to the vacuum amplitude, since this is supposed to capture the essence of the matter anyway [3], and let us consider the first correction to the “tree” amplitude. In this case the parameter space has the topology of the torus. The complex structure of the torus, the one datum that a truly conformally invariant model feels, can be described via a two-dimensional lattice, of sides 1 and Ω, where one can take $IM(\Omega)$ to be positive (fig. 1). Ω is the “period matrix” of the torus. The crucial point is that doubly periodic (elliptic) functions, such as the bosonic string coordinates, have to way to distinguish between the two sides of the cell, and in general between choices of fundamental cell related by the familiar redefinition

$$\Omega \rightarrow \Omega + 1 \quad \text{and} \quad \Omega \rightarrow -1/\Omega.$$  \hspace{1cm} (1)

In this respect, the bosonic model is a bit too simple. In proceeding to the closed superstring, one is forced to consider conformal fields that are no more doubly periodic along the two homology cycles of the torus (in modern terms, they are sections of line bundles, rather than functions, on the torus). So, if one starts with proper (i.e. doubly anti-periodic, or Neveu-Schwarz) fermions, and if one insists on the symmetries of a theory of closed strings (i.e. modular invariance), one is forced to add more contributions. The way to do so is not unique, but a very suggestive possibility is to complete separately the contributions of left and right movers (i.e. the portions depending analytically and anti-analytically on Ω). The result is the GSO projection [6], which leads to the ten-dimensional closed superstrings. So, at times one has to work harder to attain modular invariance, and this occurs precisely when one is dealing with sections of nontrivial bundles.

It was the great contribution of [7] (see also [8]) to extend the consideration of sections
of nontrivial bundles also to the case of bosons. The resulting constructions, recognized as orbifolds of toroidal models, have turned into a fundamental way of exploring the structure of two-dimensional Conformal Field Theory. It is remarkable that all known constructions based on free fields can be understood in these terms. Indeed, this possibility for the closed superstrings was pointed out in \([7]\), and served as a motivation for the orbifold construction.

Armed with the principles of \textit{conformal invariance} and \textit{modular invariance}, one can proceed to explore the set of conformal field theories. Even restricting oneself to just twisting boundary conditions of free fields, one finds a huge number of possibilities, and the long-standing limitations on the dimensionality of space time in String Theory fall apart \([8]\).

The extent of the confidence on the properties of \textit{closed strings}, and the corresponding amount of progress, have to be contrasted with the situation for \textit{open strings}. Again, the content of the previous talks makes this point hardly in need to be stressed. Apart from some occasional mention, open strings have been completely left out. This is rather peculiar, because on the one hand String Theory was born in the form of the Veneziano amplitude for open strings, and on the other hand the resurgence of a wide interest in the subject was triggered by the discovery of the Green-Schwarz mechanism, originally motivated by an analysis of open-string amplitudes. The excuses that have been given over the times for the neglect of open strings can be traced to two main points. First of all, the way symmetry groups originally entered open string models is via the Chan-Paton ansatz \([10]\). This consists in multiplying amplitudes by traces of suitable matrices (including those of the fundamental representations of the $O(N)$, $U(N)$ and $USp(2N)$ Lie algebras \([11]\)). The whole thing looks rather ad hoc, to be contrasted with the neat role played by Kac-Moody algebras \([12]\) in the construction of the heterotic string \([13]\). Moreover, the open (and closed) bosonic model is rather complicated, and thus somewhat clumsy to deal with. It involves many more diagrams that the closed (extended Shapiro-Virasoro) model, and often delicate divergence cancellations between them. Furthermore, it is usually felt, not without regret, that modular invariance is lost in this case, and that to check for the consistency of open-string models all one can do is appeal to anomaly cancellations, whenever possible. This last point is made particularly dubious by the recent recognition that, in analogy with the special role played by the group $SO(32)$ in the type-I superstring in 10 dimensions, the group $SO(8192)$ selects a special bosonic model in 26 dimensions \([14, 15]\).
The rest of this talk is devoted to remediesing these inconveniences. My aim will be convincing the reader, as well as I hope to have convinced the listener, that the known open-string models in 26 and 10 dimensions have to be understood as parameter-space orbifolds of corresponding left-right symmetric closed models. The $Z_2$ twist involved mixes left and right movers. In more geometrical terms, it symmetrizes between the two sides of the parameter surfaces. So, the open Veneziano model in 26 dimensions is seen to descend from the closed (extended) Shapiro-Virasoro model. The same pattern is followed in ten dimensions, where the type-I superstring can be seen to descend from the chiral type-IIb superstring. It should be noticed that the orbifold construction requires symmetry between left and right waves. This means a chiral spectrum for the superstring, since the two Ramond vacua must have the same chirality. The same point I will try to make is that the size of the Chan-Paton group is determined by modular invariance, by which I mean that the orbifold projection, applied to the surfaces with automorphisms that admit it, fixes the weights of the diagrams, and explains the very emergence of open strings. Open strings are the “twisted” sector of the construction. Therefore, their vertex operators sit at the fixed points. This is familiar stuff. After all, we all knew for ages that open strings are emitted from boundaries! The powers of two that build up the “magic numbers”, 32 and 8192, can be related to the sizes of the fixed-point sets. Actually, for these models, Neil Marcus and I showed that the group theory can be generated by means of free fermions valued on the boundaries of the parameter surfaces [15]. This corresponds to the long-held picture of “quarks at the ends of strings”. From the point of view advocated here, one should keep in mind the analogy with the zero modes of the Ramond sector in the orbifold construction of the superstring.

Let me start by reviewing how Chan-Paton factors [10] originally entered the game. The crucial observation was that the cyclic symmetry of “tree” open-string amplitudes (what we would now call the “disk contribution”) is compatible with the multiplication by a trace of matrices. The cyclic symmetry (planar duality) is the remnant, in the open-string case, of the total symmetry (non-planar duality) of closed-string amplitudes. Demanding that these (Chan-Paton) factors respect the factorization properties of amplitudes imposes severe constraints. These are somewhat relaxed if, following Schwarz [16], one takes into account the twist symmetry, i.e. the simple behavior of open-string amplitudes under world-sheet parity. The result is an infinite number of constraints, solved long ago by Neil Marcus and myself [11], by appealing to a classic result in the theory of Algebras. This classifies the simple associative algebras over the real numbers [17]. These
arguments lead to exclude the exceptional groups from the original open-string models in 26 and 10 dimensions \([11]\).

The Chan-Paton ansatz can actually be replaced by a dynamical construction in terms of currents \([15]\), somewhat reminiscent of the corresponding construction for the heterotic string. However, open string currents are valued on the boundaries of the parameter surfaces. They are described, in the simplest of ways, in terms of one-dimensional fermions, quantized anti periodically along boundaries. This implements the old intuitive idea of “quarks at the ends of strings”, but for a few subtleties. First of all, in order to attain a degeneracy of order \(2^{D/2}\), one needs \(D\) “quarks”. Second, these “quarks” have no space-time attributes. Thus modifying the usual actions by adding

\[ S_{\text{group}} = \frac{i}{4} \int_{\partial \Sigma} ds \beta^I d\beta^I ds \]  

(2)

produces all the right multiplicities for empty boundaries. A corresponding modification of the vertex operators includes the \(\beta\) fields, and produces the trace factors. This is all good and well, but it would be nice to predict the number of \(\beta\) fields, especially since it turn out that one needs as many of them as the string coordinates. Indeed, ten fermions give \(\text{SO}(32)\), while 26 give \(\text{SO}(8192)\).

It has been known for a while that boundaries affect the conformal properties of two-dimensional models. For instance, in ref. \([18]\) it is shown that further divergences are introduced, proportional to the lengths of boundaries, and that the theory “feels” the geodesic curvature of boundaries. In ref. \([15]\) we noticed that the “smooth doubling” of surfaces forces the boundaries to be geodesics, which can be taken as a boundary condition on the intrinsic metric. Thus, for each boundary, one is left with a (non-logarithmic) divergence proportional to the length, with a coefficient proportional to the number of space-time coordinates, and of the right sign to cancel against the divergence introduced by the boundary fermions. The divergence being not logarithmic, its cancellation is fraught with ambiguities. Moreover, showing that the cancellation takes place requires adding contributions coming partly from the parameter surface and partly from its boundary. This involves standard ways of dealing with integrals of Dirac’s delta functions over half of the real axis. This was all known to Neil Marcus and myself at the time of writing \([13]\), but it is not stressed there, since it can capture in different amounts one’s interest, due to the ambiguities mentioned above. Still, if one takes the cancellation seriously, and if the boundaries are taken to be geodesics, the open-string models with proper groups (\(\text{SO}(32)\) and \(\text{SO}(8192)\)) exhibit the same divergence structure as closed string models. Then, are they really to be regarded as closed-string models all the way?
A related observation is that the order of both SO(32) and SO(8192) is a power of two. More simply, empty boundaries contribute a factor of two for each space-time coordinate. Such factors are familiar. There are at least two instances where they arise. One is the Ramond sector of the superstring. In this case one has gamma matrices and, after all, the quantization of the one-dimensional fermions of ref. [15] also gives gamma matrices. The other case is apparently quite different. It is the $\mathbb{Z}_2$ orbifold of a “square” torus, described by Jeff Harvey at this School. There the powers of two can be traced to the size of the fixed-point set of the involution that defines the orbifold. This encourages one to look for the same structure in the known open-string models in 26 and 10 dimensions.

As usual, it is simple and instructive to consider the genus-one contribution. For simplicity, I will do so for the bosonic string in 26 dimensions. There are then four diagrams, with parameter surfaces having, respectively, the topology of the torus, the Klein bottle, the annulus and the M"obius strip. They can be conveniently described in terms of lattices in the complex plane (figs. 1 and 2). In addition, the latter three surfaces are conveniently described in terms of their “doubles”, which are all tori [19]. $\Omega$ is the “period matrix” of the doubles. It is purely imaginary for both the Klein bottle and the
annulus, but not for the Möbius strip. It should be noticed that the torus contribution is the same as for the closed bosonic string, apart from a factor of two. Thus, it is modular invariant by itself. Moreover, the Klein bottle is seen by inspection of figure 2 to be invariant under $\Omega \rightarrow \Omega + 2$. The resemblance between what one has so achieved and the untwisted sector of the $Z_2$ orbifold described by Jeff Harvey at this School is striking. The tricky point is that now the twist affects the parameter surface. Thus, in looking for the analogue of the $\Omega \rightarrow 1/\Omega$ transformation, one better think of what this transformation is meant to achieve. Then it is seen from figure 2 that the annulus contribution is precisely what is needed, since the twist is rotated by 90 degrees with respect to the Klein bottle. Finally, the Möbius strip symmetrizes the twisted sector. Actually, I have gone a bit too fast here. First of all, the two involutions that lead to the Klein bottle and to the annulus are different. One results into two cross-caps, and the other into two boundaries. Moreover, the “shift” in $\Omega$ that leads from the annulus to the Möbius strip is just $\frac{1}{2}$, not 1. The clue is noticing that the involutions act in the same way on the homology basis, and this is all the string integrand “feels”. On the other hand, the zero modes result in the same contribution only if one works in terms of the modulus of the double. However, the “proper time” for the Klein bottle is half the modulus of its covering torus, which is again half the “proper time”. Expressing amplitudes in terms of “proper time” exhibits the spectrum, and gives a relative factor of $2^D$ between Klein bottle and annulus. This is the square of the multiplicity factor when the gauge group has order $2^{[d/2]}$. In the same way, symmetrization in the twisted (open) sector required adding $\frac{1}{2}$ to $\Omega$, which refers to the double, and generates precisely the modulus of the Möbius strip. The oscillator description accommodates the orbifold idea very nicely. Open strings take values over “one half” of the parameter surfaces, and are closed modulo the doubling of the parameter surfaces.

Actually, the preceding discussion has left out an important point. This is the choice of projection in the ground state of the twisted sector. In ref. [13] it was pointed out that a twist-even ground state, and thus the group $SO(8192)$ rather than $USp(8192)$, leads to the elimination of some divergences via a principal part prescription. The divergences manifest themselves in the small-$\Omega$ limit of the amplitudes corresponding to figure 2. We were inspired by a similar phenomenon discovered by Green and Schwarz in the $SO(32)$ superstring, and responsible for both finiteness and anomaly cancellation. The last result in discussed at length in ref. [1]. Actually, even for the superstring the same can be seen to occur directly at the level of the partition function, if one refrains from using the “aequatio” of Jacobi, which sets to zero the contributions of the individual diagrams, due
to supersymmetry. The cancellations found by Weinberg \cite{20} in the scattering amplitudes of the SO(8192) theory can be traced to the same phenomenon.

The picture that emerges from the foregoing discussion has several facets. One the one hand, it is particularly satisfying to see the structure of Conformal Field Theory at work again. One the other hand, one looses the need to consider open-string models as separate entities (or oddities). Everything fits into theories of closed strings, once one allows the possibility of **twists mixing left and right-movers**, which have been systematically avoided in discussions of orbifolds so far. The “magic rule” of modular invariance is recovered, and this should allow model building with open strings as well. These points clearly deserve a fuller discussion, which will be presented elsewhere.

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