Probing the Supersymmetric Grand Unified Theories at the Future Proton-Proton Colliders and Hyper-Kamiokande Experiment

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Gauge coupling unification in the Supersymmetric Standard Models strongly implies the Grand Unified Theories (GUTs). With the grand desert hypothesis, we show that the supersymmetric GUTs with gravity mediated supersymmetry breaking can be probed at the future proton-proton (pp) colliders and Hyper-Kamiokande experiment. For the GUTs with the GUT scale $M_{GUT} \lesssim 1.0 \times 10^{16}$ GeV, we can probe the dimension-six proton decay via heavy gauge boson exchange at the Hyper-Kamiokande experiment. Moreover, for the GUTs with $M_{GUT} \gtrsim 1.0 \times 10^{16}$ GeV, we for the first time study the upper bounds on the gaugino and sneutrino masses. We show that the GUTs with anomaly and gauge mediated supersymmetry breakings are well within the reaches of the future 100 TeV pp colliders such as the FCC$_{hh}$ and SpPC, and the supersymmetric GUTs with gravity mediated supersymmetry breaking can be probed at the future 160 TeV pp collider.

Introduction.—Supersymmetry (SUSY) provides a natural solution to the gauge hierarchy problem in the Standard Model (SM). In the supersymmetric SMs (SSMs) with R-parity, gauge coupling unification can be achieved [1], the Lightest Supersymmetric Particle (LSP) such as the lightest neutralino can be a dark matter (DM) candidate [2], and the electroweak (EW) gauge symmetry can be broken radiatively due to the large top quark Yukawa coupling, etc. In particular, gauge coupling unification strongly suggests Grand Unified Theories (GUTs) [3–7], which may be constructed from superstring theory. Therefore, supersymmetry is a bridge between the low energy phenomenology and high-energy fundamental physics, and thus is the promising new physics beyond the SM.

However, after the LHC Run 2, the null results of the SUSY searches have given strong constraints on the SSMs. For example, the low mass bounds on the gluino, first-two generation squarks, stop, and sbottom are about 2.3 TeV, 1.9 TeV, 1.25 TeV, and 1.5 TeV, respectively [8–12]. Thus, there might exist SUSY EW fine-tuning (EWFT) problem. And there are some promising and successful solutions available in literatures, for example, Refs. [13–27]. In particular, in the Super-Natural SUSY [28,30], it was shown that the fine-tuning measure defined by Ellis-Enqvist-Nanopoulos-Zwirner [31] and Barbieri-Giudice [32] is at the order of one naturally, despite having relatively heavy supersymmetric particle (sparticle) spectra. The previous natural SSMs generically predict some relatively light sparticles, for instance, Higgsino, stop, gluino, and sleptons, which can be tested at the future proton-proton (pp) colliders such as the FCC$_{hh}$ [33] and SpPC [34].

Because the gauge coupling unification in the SSMs strongly suggests GUTs, the interesting and challenging question is: can we probe the supersymmetric GUTs at the future pp colliders and other experiments even if there does exist the SUSY EWFT problem? If yes, what is the center-of-mass energy of the future pp collider needed? We shall study it in this paper. In the GUTs, the well-know prediction is the dimension-six proton decay $p \rightarrow e^+\pi^0$ via heavy gauge boson exchange, and the proton lifetime is given by

$$\tau_p(e^+\pi^0) \simeq 1.0 \times 10^{34} \times \left( \frac{2.5}{\alpha_{GUT}} \right)^2 \times \left( \frac{M_{GUT}}{1.0 \times 10^{16} \text{ GeV}} \right)^4 \times \left( \frac{0.04}{\alpha_{GUT}} \right)^2 \times \left( \frac{A_R}{1.0 \times 10^{16} \text{ GeV}} \right)^4 \times \left( \frac{1}{10^{35} \text{ years}} \right) \times \left( \frac{1}{10^{34} \text{ years}} \right) \times \left( \frac{1}{10^{34} \text{ years}} \right) \times \left( \frac{1}{10^{34} \text{ years}} \right),$$

where $A_R$ is the dimensionless one-loop renormalization factor associated with anomalous dimension of the relevant baryon-number violating operators, $\alpha_{GUT}$ is the unified gauge coupling, and $M_{GUT}$ is the GUT scale. The current lower limit on the proton lifetime from the Super-Kamiokande experiment is $\tau_p > 1.6 \times 10^{34}$ years [36]. Thus, we obtain $M_{GUT} \gtrsim 1.0 \times 10^{16}$ GeV. At the future Hyper-Kamiokande experiment, we can probe the proton lifetime at least above $1.0 \times 10^{35}$ years [37]. Therefore, the GUTs with $M_{GUT} \lesssim 1.0 \times 10^{16}$ GeV is within the reach of the future Hyper-Kamiokande experiment.

In the following, with the grand desert hypothesis from the EW scale to the GUT scale, we shall show that the supersymmetric GUTs with $M_{GUT} \gtrsim 1.0 \times 10^{16}$ GeV can be probed at the future pp colliders. The supersymmetry searches at the 100 TeV pp colliders have been studied previously [33] [38,41]. For the integrated luminosity...
30 ab$^{-1}$, Wino via Bino decay, gluino $\tilde{g}$ via heavy flavor decay, gluino via light flavor decay, first-two generation squarks $\tilde{q}$, and stop can be discovered for their masses up to about 6.5 TeV, 11 TeV, 17 TeV, 14 TeV, and 11 TeV, respectively. Moreover, if the gluino and first-two generation squark masses are similar, they can be probed up to 20 TeV.

Moreover, in the SSMs, supersymmetry is broken in the hidden sector, and then supersymmetry breaking is mediated to the SM observable sector via gravity mediation or anomaly mediation. For the supersymmetric GUTs with $M_{GUT} \geq 1.0 \times 10^{16}$ GeV, we for the first time study the upper bounds on the gaugino and sfermion masses. We show that the GUTs with anomaly and gauge mediated supersymmetry breakings are well within the reaches of the future 100 TeV pp colliders such as the FCC$\text{hh}$, SppC, and the supersymmetric GUTs with gravity mediated supersymmetry breaking can be probed at the future 160 TeV pp collider. The interesting viable parameter spaces for gravity mediation, which can be probed at the FCC$\text{hh}$ and SppC, have been discussed as well.

**Scanning Codes and Constraints.**–We use the ISAJET 7.85 package to perform random scans over the parameter space of gravity mediated SUSY breaking via the minimal supergravity (mSUGRA) or Constrained MSSM (CMSSM), as well as the anomaly mediated SUSY breaking. To study the gauge mediated SUSY breaking, we also employ the SPheno 4.0.4 package generated with SARAH 4.14.3.

The collected data points all satisfy the requirement of the Radiative Electroweak Symmetry Breaking (REWSB), has the lightest neutralino being the LSP for gravity and anomaly mediations, SM-like Higgs boson mass $m_h \subset [123, 127]$ GeV, and gluino mass $m_{\tilde{g}} > 2.2$ TeV. After collecting the data, we impose the constraints from rare decay processes $B_s \to \mu^+\mu^-$, $b \to s\gamma$, and $B_s \to \tau\nu\tau$. To be general, we do not require the relic abundance of the LSP neutralino to satisfy the Planck bound within $5\sigma$ $0.114 \leq \Omega_{\text{CDM}}h^2(\text{Planck}) \leq 0.126$.

**Gravity Mediated Supersymmetry Breaking: mSUGRA/CMSSM.**–The mSUGRA/CMSSM is based on the GUTs and $N = 1$ supergravity where supersymmetry breaking is communicated through the supergravity interaction. It is one of the most widely studied SUSY scenarios, and has three supersymmetry breaking soft terms at the GUT scale: the universal gaugino mass $M_{1/2}$, universal scalar mass $M_0$, and universal trilinear coupling $A_0$. The other free parameter $\tan \beta$ is the ratio of vacuum expectation values (VEVs) of two Higgs-doublets, and a discrete parameter $\text{sign}(\mu) = \pm 1$. We perform the random scans for the following mSUGRA/CMSSM parameter space

$$0 \leq M_0 \leq 90 \text{ TeV},$$
$$0 \leq M_{1/2} \leq 30 \text{ TeV},$$
$$-3 \leq A_0/M_0 \leq 3,$$
$$2 \leq \tan \beta \leq 60$$

with $\mu > 0$ and $m_t = 173.2$ GeV. The results are not too sensitive to one or two sigma variations in the value of $m_t$. We use $m_0^{\text{Planck}}(M_Z) = 2.83$ GeV as well which is hard-coded into the ISAJET.

Because the sfermions in the SSMs form the complete GUT multiplets while gauginos do not, the universal gaugino mass $M_{1/2}$ has big effects on gauge coupling unification. We present the plot $M_{GUT}$ vs $M_{1/2}$ in Fig. 1. In our figures, gray points are consistent with the REWSB and LSP neutralino. Orange points satisfy the mass bounds and the constraints from rare $B$-meson decays. Green points are a subset of orange points and satisfy $M_{GUT} \geq 1.0 \times 10^{16}$ GeV. Thus, we obtain that the upper bound on $M_{1/2}$ is about 7 TeV. This bound can be translated into the upper bound 15 TeV on gluino mass, as shown below.

In the left panel of Fig. 2, we show results of our scans in $M_{1/2} - m_{\tilde{g}}$ plane. We first find that the upper bound on the gluino mass is 15 TeV. In addition, the red points ($\tan \beta > 7.5$) and blue points ($\tan \beta < 7.5$) are the subsets of green points and satisfy the Planck 2018 $5\sigma$ bounds on dark matter relic density. Interestingly, gluino masses for the red points are lighter than 11 TeV, and thus the gluino for the red points is within the reach of the FCC$\text{hh}$ and SppC. In the right panel of Fig. 2, we present the scan results in the first-two generation squark mass $m_{\tilde{q}}$ and $M_0$ plane. In particular, $M_0$ can be very heavy up to 65 TeV. Similarly, the red points ($\tan \beta > 9$) and blue points ($\tan \beta < 9$) are also the subsets of green points and satisfy the Planck 2018 $5\sigma$ bounds on dark mat-
FIG. 2. The color coding for gray, orange, and green points is the same as the Fig. [4] Left: plot in the $m_{\tilde{g}}$ and $M_{1/2}$ plane. Red ($\tan \beta > 7.5$) and blue ($\tan \beta < 7.5$) points are subset of green points and represent solutions which satisfy the Planck $5\sigma$ bound. Right: plot in the first two generation squark mass $m_{\tilde{q}}$ and $M_0$ plane. Red ($\tan \beta > 9$) and blue ($\tan \beta < 9$) points are subset of green points and represent solutions which satisfy the Planck $5\sigma$ bound.

...ter relic density. We see that the maximum value of $M_0$ for most of red points is about 20 TeV. Because $m_{\tilde{q}} \simeq M_0^2 + (5 - 6)M_{1/2}$ \cite{58} and the maximum value of $M_{1/2} \sim 7$ TeV, we obtain that the maximum value of the first-two generation squark masses for most of red points is about $m_{\tilde{q}} \simeq 20$ TeV, as shown clearly in $m_{\tilde{q}} - M_0$ plot. Thus, most of the red points can be probed at the FCC$\,\!hh$ and SppC.\cite{38, 41} Because $M_0$ can be very large up to 65 TeV, it will be difficult to search for the squarks and sleptons at the FCC$\,\!hh$ and SppC in general. Thus, we can look for the gauginos at the future pp colliders. For the integrated luminosity 30 ab$^{-1}$ at the FCC$\,\!hh$ and SppC, gluino via heavy and light flavor decays can be discovered for the masses up to about 11 TeV and 17 TeV, respectively. Thus, if gluino decays via light flavor squarks, it can be discovered at the FCC$\,\!hh$ and SppC. However, in our viable parameter space, the lightest squark is generically to be light stop, and thus we do have gluino via heavy flavor decay. To probe such gluino with mass up to 15 TeV, we find that the center-of-mass energy of the future pp collider needs to be about 160 TeV. And we can discover Wino at this energy as well.

**Anomaly Mediated Supersymmetry Breaking.**—Anomaly mediated supersymmetry breaking (AMSB) is a special type of gravity mediated SUSY breaking. In this case, SUSY breaking is communicated to the visible sector from the hidden sector via a super-Weyl anomaly \cite{48, 49}. In the minimal AMSB, there are three basic parameters in addition to $\text{sign}(\mu)$: $\tan \beta$, the universal scalar mass $M_0$ at the GUT scale which is introduced to solve the tachyonic slepton mass problem, and gravitino mass $M_{3/2}$. We have performed the random scans over the following parameter space of the minimal AMSB

$$\begin{align*}
1 \text{ TeV} \leq M_0 &\leq 75 \text{ TeV}, \\
100 \text{ TeV} \leq M_{3/2} &\leq 30 \text{ TeV}, \\
2 \leq \tan \beta &\leq 60
\end{align*}$$

with $\mu > 0$ and $m_t = 173.2$ GeV \cite{57}. In the left panel of Fig. 3 we present the results of our scan in the $m_{\tilde{q}} - m_{\tilde{g}}$ plane. All the points, which satisfy the current experimental constraints and have $M_U > 1 \times 10^{16}$ GeV, are shown in green color. We obtain that the upper bounds on the masses of both the first-two generation squarks and gluino are around 5 TeV, and thus they are well within the reaches of the FCC$\,\!hh$ and SppC.\cite{38, 41} Moreover, the neutralinos, charginos, and sleptons can be discovered at the FCC$\,\!hh$ and SppC as well.

**Gauge Mediated Supersymmetry Breaking.**—Finally, we study the Gauge Mediated Supersymmetry Breaking (GMSB) \cite{45–47}. The GMSB is a method of communicating SUSY breaking to the SSMs from the hidden sector through the SM gauge interactions. The basic parameters of the minimal GMSB are: $\tan \beta$, $\text{sign}(\mu)$, the messenger field mass scale $M_{\text{mess}}$, the number of $SU(5)$ representations of the messenger fields $N_{\text{mess}}$, and the SUSY breaking scale in the visible sector $\Lambda$. The messenger fields induce the gaugino masses at one loop and then they are transmitted on to the squark and slepton masses at two loops. To preserve the gauge coupling unification, we consider the messenger fields which form the complete GUT multiplets. For simplicity, we introduce one pair of the messenger fields in the $\mathbf{5}$ and $\mathbf{\bar{5}}$ representations of $SU(5)$, i.e., $N_{\text{mess}} = 1$. Also, we take
M current experimental constraints and have respectively. Similarly, all the points, which satisfy the ation squarks and gluino are well within the reaches of and 6 TeV, respectively. Therefore, the first-two generation squarks and gluino are 8 TeV, Fig. 3. We see that the upper bounds on the masses M and α M compute and output the SM gauge couplings at the use Spheno to do these calculations since it can com-

\[ 5 \times 10^5 \, \text{GeV} \leq \Lambda \leq 10^7 \, \text{TeV}, \]
\[ 2 \times \Lambda \leq M_{\text{mess}} \leq 10^{15} \, \text{GeV}, \]
\[ 2 \leq \tan \beta \leq 60 \]  

with $\mu > 0$ and $m_l = 173.2 \, \text{GeV}$ [57]. Because $M_{\text{GUT}}$ is not calculated in all the current codes, we esti-

\[ M_{\text{GUT}} \geq \alpha_{12}(Q) \equiv \alpha^{-1}_1(Q) - \alpha^{-1}_2(Q), \]  

and make a plot of $\alpha^{-1}_{12}(Q)$ from the Renormalization Group Equation (RGE) run-

\[ \frac{M}{M_{\text{GUT}}} \text{ to the weak scale } M_W \text{ as functions of renormalization scale } Q. \]  

We then fit the $\alpha^{-1}_{12}(Q)$ curve by a polynomial function $f(Q)$ via Mathematia. For any point at the messenger scale $M_{\text{mess}}$, we calculate $\alpha^{-1}_1(M_{\text{mess}})$ and $\alpha^{-1}_2(M_{\text{mess}})$ via the codes SARAH 4.14.3 and Sphenov4.0.4. We use Spheno to do these calculations since it can com-

\[ M_{\text{GUT}} > 1 \times 10^{16} \, \text{GeV} \text{ and } M_{\text{GUT}} < 1 \times 10^{16} \, \text{GeV}, \]  

respectively. Similarly, all the points, which satisfy the current experimental constraints and have $M_{\text{GUT}} > 1 \times 10^{16} \, \text{GeV}$, are shown in green color in the right panel of Fig. 3. We see that the upper bounds on the masses of the first-two generation squarks and gluino are 8 TeV, and 6 TeV, respectively. Therefore, the first-two generation squarks and gluino are well within the reaches of the FCC$_{hh}$ and SppC [38, 41]. Moreover, the neutrali-

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