Theoretical uncertainties of muon transport calculations for very large volume neutrino telescopes

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Abstract. Underground cosmic-ray experiments, including very large volume neutrino telescopes, depend on a precise description of the interaction cross sections of muons, which can travel large distances before reaching the detector. High-energy muons lose their energy almost exclusively via four processes: ionization, electron-positron pair production, bremsstrahlung and inelastic interaction with nuclei. At low energies, ionization is the dominant process, while above energies of about a TeV, the three other processes dominate the energy loss. We discuss the uncertainties of the cross sections of the energy loss processes used in the simulation chain of current very large volume neutrino telescopes and review recent theoretical improvements.

1. Muons in very large volume neutrino telescopes

Most events recorded in very large volume neutrino telescopes (VLVνT) are muon events. These muons are mainly either products of muon neutrino interactions in or around the detector, or they originate in extensive air showers (EAS) from hadron decays. A small part is also due to muon pair production.

Muons are comparatively heavy particles and thus loose their energy slowly, while they are comparatively long-lived. This gives high-energy muons a large range which can reach several kilometers. Therefore muons from EAS can reach cosmic ray and neutrino detectors even deep underground.

Propagating through matter, muons loose their energy by ionization, pair production, bremsstrahlung and inelastic interaction with nuclei. Ionization and pair production mainly lead to small quasi-continuous losses, while bremsstrahlung and inelastic interaction lead on average to larger losses of some tenths of the muon energy per interaction, leading to large secondary cascades along the muon track. Because the energy loss is a stochastic process, the problem lends itself to the application of Monte Carlo techniques.

The theoretical uncertainties of muon propagation have been treated several times in the past (e.g. [1,2,3]). We first discuss the cross sections used in current VLVνT muon simulation chains; then we discuss recent theoretical developments and their effect on the average energy loss.

2. Muon propagation software used in current VLVνT

The currently operating VLVνT are the neutrino observatory IceCube at the South Pole, which uses the lepton propagator PROPOSAL [4,5], the Gigaton volume detector in Lake Baikal using MUM [6,7], and ANTARES in the Mediterranean Sea, which uses MUSIC [8] and MUM.
PROPOSAL, MUSIC and MUM offer a selection of possible cross sections for muon interactions. The standard selection in the IceCube simulations are the Bethe-Bloch equation [9] with density correction [10] and radiative corrections [11] for ionization; for pair production the parametrization of Kokoulin and Petrukhin [12,13] with the atomic electron contribution as calculated by Kelner [14]; for bremsstrahlung the parametrization of Kelner, Kokoulin and Petrukhin [15] with the atomic electron contribution from [11]; and for inelastic interaction with nuclei the ALLM97 parametrization [16] with shadowing calculated as in [17].

The standard recommended for MUSIC [8] is the Bethe-Bloch equation with density correction for ionization; the pair production parametrization according to [12–14] and the bremsstrahlung parametrization according to [11,15], as in the IceCube simulation chain; and the two-component parametrization of [18] for the inelastic interaction.

The standard cross sections in MUM 1.5 [6] are the Bethe-Bloch equation with density correction and radiative corrections for ionization; the pair production parametrization according to [12–14] as in the IceCube simulation chain; the bremsstrahlung parametrization according to [19]; and the parametrization of the inelastic interaction with nuclei according to [18].

The ionization parametrization differs by the omission of the radiative corrections in MUSIC. The pair production cross section parametrization is the same in all three cases. The bremsstrahlung parametrization is either [11,15] or [19] for MUM. The parametrization of the inelastic interaction with nuclei is either [16] or [18].

3. Recent theoretical developments and differences between currently used cross sections

3.1. Bremsstrahlung

The bremsstrahlung parametrizations [15,19] differ in the high- and low-energy regions for a number of reasons. At high-energies, the main difference is due to the different radiation logarithms [20,21] and the different treatment of the contribution of atomic electrons, while at low energies the difference is due to the different treatment of the screening functions $\Phi_{1,2}$ introduced by [22,23], which differ by 1/6 in the no-screening region due to the nuclear formfactor and are approximated by $\Phi = \Phi_1 \approx \Phi_2$ in [15]. Between the regions dominated by complete screening at high energies and absence of screening at low energies, the different atomic formfactor shows a small influence; [19] chose a dipole formfactor from [20], which allows to calculate the cross section analytically, [15] use an analytical interpolation from [24] which approximates numerical results for the Thomas-Fermi formfactor.

In [25], the analytical interpolation of [24] for the screening functions was carried out for $\Phi_{1,2}$ separately; in addition, radiative corrections were calculated and parametrized, leading in total to an increase of 2% of the average bremsstrahlung energy loss at high energies (see also [26]).

An additional correction to the bremsstrahlung energy loss are diffractive corrections, which were first considered in [27]. These correspond to the diffractive scattering of virtual photons by the nucleus $\gamma^* A \rightarrow \gamma A$, where the virtual photons are produced by the muon. Since the initial and final states coincide with the respective states of bremsstrahlung, the amplitudes interfere. Recently, these corrections were considered again [28]. It turned out, that the approach to calculations of [27] are essentially correct, but due to a mistake in the calculation of the real part of the nuclear scattering amplitude the effect of diffractive corrections was overestimated.

The effect of these different parametrizations on the average energy loss is shown in figure 1(a).

3.2. Pair production

The pair production cross section of [12,14] is used as a default in all cases. Similar improvements with regard to the screening functions as in the bremsstrahlung case were carried out for the pair production cross section [25]. The effect on the average energy loss is about $-0.5\%$. 
Complete higher-order corrections to the pair production have not yet been calculated. Coulomb corrections, i.e., higher-order corrections in $Z\alpha$ have been calculated in [29]. The effect depends on energy, in standard rock or lighter media the energy loss is decreased by no more than 0.5%. The effect of other higher-order corrections has by now only been estimated with the equivalent photon method. The dominant contribution to the energy loss comes from the $\gamma$-diagrams, where two photons couple to the electron-positron pair. The effect of radiative corrections to the pair production cross section can be estimated using the equivalent photon approximation; the conditions for the applicability of this method are not satisfied over the whole phase space essential for the energy loss, however, so this can serve only as an estimate of the order of magnitude of radiative corrections. The corrections to the $\gamma$-diagrams can be subdivided into the production of two electron-positron pairs, loop corrections and emission of a bremsstrahlung photon from the electron-positron pair or the muon, and exchange of an additional photon between the muon and the electron-positron pair.

The ratio of the energy loss due to production of one and two pairs is given in the equivalent photon approximation by

$$\frac{\langle -dE/dx \rangle |_{2\gamma} + e^-}{\langle -dE/dx \rangle |_{\gamma} + e^-} = \frac{\alpha^2 (175\zeta(3) - 38)}{56\pi^2} \ln^2 \frac{E_{\mu}}{m_{\mu}},$$

which reaches half a percent for muon energies in the PeV range.

The loop corrections and emission of a bremsstrahlung photon in pair production by a real photon was treated by [31], who found a relative increase of about 0.93% with respect to the tree-level cross section, leading to a corresponding increase of about a percent for corrections in the electron-positron pair.

The loop corrections and emission of a bremsstrahlung photon by the muon can be calculated similar to radiative corrections to inelastic interaction with nuclei, which was considered in [32]. Preliminary numerical results show an increase of about 0.5%. In addition, vacuum polarization corrections to the photon coupling the muon and the electron-positron pair increase the cross section by about 0.2%.

The exchange of additional photons between the muon and the electron is similar to Coulomb corrections on a nucleus with charge number $Z_{\mu} = 1$, which are very small, so we neglect them for this estimate, leading to a total increase of the energy loss on account of NLO effects of 1.6%.

The effect of the discussed corrections on the average energy loss is shown in figure 1b.

3.3. Inelastic interaction with nuclei

To describe the energy loss of high-energy muons due to inelastic interaction with nuclei, one needs a description of the nucleon structure functions at low $x, Q^2$ and a description of collective nuclear effects such as shadowing. These phenomena are not strictly independent of each other. At high $Q^2$, perturbative quantum chromodynamics is a well-understood and successful theory for the description of quark and gluon interactions. However, quarks and gluons are never observed as free particles due to confinement. Moreover, the evolution equations for the parton distribution functions are not applicable in the photoabsorption limit $Q^2 = 0$. This problem was discussed in [33, 34].

The two parametrizations currently used are based on very different descriptions. The two-component model of [18] is based on generalized vector meson dominance for the soft component and the color dipole model for the hard component, while the model of [16] is based on Regge theory.

The two-component model has been updated in [37, 38]. New models which have appeared in recent years and address also the low-$Q^2$ region including the photoproduction limit which

1 In [30], the total cross section for this process was considered.
Figure 1. Average muon energy loss in water due to (a) bremsstrahlung (b) pair production.

Figure 2. (a) Average inelastic nuclear energy loss of muons on protons. (b) Effect of shadowing prescriptions of \([17, 35, 36]\) for the \(\gamma p\) structure function parametrization of \([16]\) in water.

Concerns us in cosmic ray calculations, are the Froissart-bounded parametrization of \([39]\), the generalized vector meson dominance color dipole model parametrization of \([40, 41]\), and the tensor pomeron model of \([42]\).

The problem of the description of the proton structure functions at low \(x, Q^2\) has improved, as can be seen by the small scatter between conceptually completely different parametrizations in figure 2. This is also a consequence of the high-quality \(ep\) scattering data collected at the HERA collider, the combined data of which have become available in recent years \([43, 44]\). To assess the effect of the precise combined HERA on the energy loss, we have repeated the fit of \([39]\) on these data with the parametrization of \([45]\) as photonuclear boundary condition; with the uncertainties and the correlation between the fit parameters thus determined, we have determined the uncertainty on the energy loss of muons on protons, which turns out to be about 3–5%, slowly rising with increasing energy (cf. figure 3 (a)). Different nuclear shadowing prescriptions give predictions differing by about 5–10% (cf. figure 3 (b)).

4. Effect on muon spectra

The resulting spectrum underground at a depth \(h\) from a powerlaw distribution at the surface with spectral index \(\gamma\) is approximately determined by the product \(b\gamma h\), where \(\langle -dE/dx \rangle \approx\)
Figure 3. (a) Uncertainty on inelastic nuclear energy loss, based on a fit to combined HERA data [43] with the parametrization of [39]. (b) Total uncertainty on the energy loss per energy.

\[ a + bE, \]

such that uncertainties on the energy losses translate to uncertainties of the spectral index estimate.

In the estimation of the energy of muon events inside the detector, use is made of the fact, that small energy losses, mainly due to ionization and pair production, are well-correlated to the energy of the muon, while large stochastic losses are ill-correlated and may be considered separately in the experiment. Therefore, the uncertainties on pair production are most significant for this application.

For propagation outside the detector, where the losses are not observable, all energy loss processes contribute. For atmospheric muons, the depth traversed from the surface to the detector is given by geometry, and increased energy losses translate to larger energies at the surface.

5. Conclusion
The total uncertainty of the average energy loss of high-energy muons is dominated by different processes at different energies. The dominant uncertainties are now radiative corrections to the pair production loss and the nuclear structure function parametrization and shadowing effect. For the energy estimation inside VLVνT the dominant uncertainty are the radiative corrections to pair production, whose magnitude has been estimated in this work. Uncertainties have decreased in recent years due to improved parametrizations and calculation of radiative corrections.

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