Unified Performance Assessment of Optical Wireless Communication over Multi-Layer Underwater Channels

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Abstract—In this paper, we model the multi-layer vertical underwater link as a cascaded channel and unify the performance analysis for the underwater optical communication (UWOC) system using generalized Gamma (GG), exponential GG (EGG), exponentiated Weibull (EW), and Gamma-Gamma (ΓΓ) oceanic turbulence models. We derive unified analytical expressions for probability density function (PDF) and cumulative distribution function (CDF) for the signal-to-noise ratios (SNR) considering independent and non-identical (i.n.i.d.) turbulent models and zero bore-sight model for pointing errors. We develop performance metrics of the considered UWOC system using outage probability, average bit error rate (BER), and ergodic capacity with asymptotic expressions for outage probability and average BER. We develop the diversity order of the proposed system to provide a better insight into the system performance at a high SNR. We also integrate a terrestrial OWC (TOWC) subjected to the combined effect of generalized Malaga atmospheric turbulence, fog-induced random path gain, and pointing errors to communicate with the UWOC link using the fixed-gain amplify-and-forward (AF) relaying. We analyze the performance of the mixed TWOC and multi-layer UWOC system by deriving PDF, CDF, outage probability, and average BER using the bivariate Fox H-function. We use Monte-Carlo simulation results to validate our exact and asymptotic expressions and demonstrate the performance of the considered underwater UWOC system using measurement-based parametric data available for turbulent oceanic channels.

Index Terms—Cascaded channels, multi-layer channels, Mellin's transform, performance analysis, oceanic turbulence, UWOC, vertical link.

I. INTRODUCTION

Underwater optical communication (UWOC) is a potential solution for broadband connectivity in oceans and seas for underwater applications providing high data rate transmission with low latency and high reliability [2]–[4]. It is a promising technology for underwater data transmission providing higher throughput with low latency and high reliability than radio frequency (RF) and acoustic wave communication systems. The underwater optical communication (UWOC) system transmits data in an unguided water environment using the wireless optical carrier for military, economic and scientific applications [4]. Despite several advantages of the UWOC, the underwater link suffers from signal attenuation, oceanic turbulence, and pointing errors. The signal attenuation occurs due to the molecular absorption and scattering effect of each photon propagating through water, generally modeled by the extinction coefficient. Oceanic turbulence is the effect of random variations in the refractive index of the UWOC channel caused by random variations of water temperature, salinity, and air bubbles. Pointing errors can also be detrimental to UWOC transmissions due to misalignment between the transmitter and detector apertures. Therefore, it is desirable to analyze the UWOC systems over various underwater channel impairments for an effective system design.

As is for any communication system, recent works developed theoretical and experimental characterization of turbulence-induced fading under various underwater conditions [5]–[11]. Research outcomes in [5]–[7] demonstrate that the log-normal distribution efficiently models weak oceanic turbulence similar to the modeling of weak atmospheric turbulence for terrestrial OWC links. The authors in [8] demonstrated higher oceanic turbulence since the scintillation index for an optical wave is very high over several meters of underwater propagation. In [9], the authors presented a holistic experimental view on the statistical characterization of oceanic turbulence in UWOC systems, considering the effect of the temperature gradient, salinity, and air bubbles. They used various statistical distributions such as log-normal, Gamma, Weibull, Exponentiated Weibull (EW), Gamma-Gamma (ΓΓ), and generalized Gamma (GG) to model underwater turbulence channels. Further, experimental investigations projected the GG distribution and EW as more generic models and valid for various underwater channel conditions [9], [10]. Recently, [11] used experimental data to propose the mixture exponential-generalized Gamma (EGG) distribution for oceanic turbulence caused by air bubbles and temperature gradient for UWOC channels, which perfectly matches the measured data, collected under different channel conditions ranging from weak to strong turbulence conditions.

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A part of this paper on the generalized gamma distribution for the multi-layer oceanic turbulence is under review in the 2022 IEEE 95th Vehicular Technology Conference: VTC2022-Spring to be held in Helsinki, Finland 19-22 June 2022 [12].
There has been tremendous research on the performance assessment of UWOC systems [11]–[21] and the mixed system consisting of UWOC and terrestrial networks [22]–[27]. In those mentioned above and related research, a single layer of oceanic turbulence channel over the entire transmission range has been considered. However, experimental results reveal ocean stratification, i.e., the temperature gradient and salinity are depth-dependent (typically varying between a few meters to tens of meters), resulting in many non-mixing layers with different oceanic turbulence [28]. Thus, considering multiple oceanic layers for vertical transmissions may provide a more realistic performance assessment for UWOC systems. In [28]–[31], the author analyzed the performance vertical UWOC links by cascading the end-to-end link as the concatenation of multiple layers considering both log-normal and IT oceanic turbulent channels for each layer. They used the method of induction to analyze the cascaded IT channel, which may not be readily applicable to other channel models. To the best of the authors’ knowledge, no analyses available for the outage probability, average BER, and ergodic capacity of a multi-layer UWOC system considering GG, EGG, and EW oceanic turbulence models. Further, it is desirable to consider a more generalized model for the terrestrial OWC (TOWC) that includes the combined effect of Malaga atmospheric turbulence, fog-induced random path gain, and pointing errors to study the mixed UWOC-TOWC transmission. It should be mentioned that the terrestrial OWC link might be affected by foggy conditions near the ocean/sea, and consideration of deterministic path loss may underestimate/overestimate the performance of the considered system [32].

This paper presents a unified performance analysis of a vertical UWOC system under the combined effect of multilayer underwater turbulence channels and pointing errors. The major contributions of the proposed work are summarized as follows:

- We apply the Mellin inverse transform to develop analytical expressions for probability density function (PDF) and cumulative distribution function (CDF) for the signal-to-noise ratios (SNR) of UWOC system unifying GG, EGG, and IT oceanic turbulent models and zero bore-sight model for pointing errors.
- We use the derived statistical results to develop performance metrics of the considered UWOC system using outage probability, average bit error rate (BER), and ergodic capacity with asymptotic expressions for the outage probability and average BER to determine the diversity order of the proposed system for a better insight into the system performance.
- We integrate a generalized terrestrial OWC (TOWC) link subjected to the combined effect of generalized Malaga atmospheric turbulence, fog-induced random path gain, and pointing errors to communicate with the UWOC link using the fixed-gain amplify-and-forward (AF) relaying. We analyze the performance of the mixed TOWC-UWOC by deriving PDF, CDF, outage probability, and average BER using bivariate Fox H-function.
- We use numerical and simulation analysis to validate our derived expressions and demonstrate the performance of the considered UWOC system for various parameters of interest.

A. Related Work

There is rich literature considering the performance analysis for the single UWOC link under various oceanic turbulence conditions [11]–[20]. In [11], the authors analyzed the outage probability, average BER, and ergodic capacity for UWOC by modeling the underwater optical turbulence channel using the EGG distribution. The authors in [12] provided an overview of various challenges associated with UWOC and proposed positioning, acquisition, and tracking scheme to mitigate the effect of pointing errors under turbulent channels. The average bit-error-rate (BER) performance under weak log-normal distributed turbulence channels was presented in [13]. The authors in [14] characterized a relay-assisted UWOC with optical code division multiple access (OCDMA) over log-normal turbulent channels. An analytic expression for the channel capacity of an orbital angular momentum (OAM) based free-space optical (FSO) communication in weak oceanic turbulence was developed in [15]. The authors in [16], [17] analyzed the performance of multi-input and multi-output (MIMO) UWOC systems over log-normal turbulent channels. Further, a multihop UWOC system was investigated in [18]. The outage probability of a multiple decode-and-forward (DF) relaying-assisted UWOC system with an on-off keying (OOK) modulation was studied in [19]. The various optical turbulence models like log-normal, Gamma, K, Weibull, and exponentiated Weibull distributions have been used to analyze the performance of underwater wireless optical communication (UWOC) systems [20]. It should be emphasized that the related work on the UWOC system consider a single channel and that there is limited research on the vertical cascaded using log-normal and IT turbulence models [28]–[31].

Since the advent of UWOC, there is an increased interest to offload the underwater data to a terrestrial network using RF technology [22], [25], [33] and terrestrial OWC [26], [27]. The authors in [22], [23] considered Nakagami-m fading for the radio frequency (RF) and EGG turbulence for the UWOC and analyzed the outage probability and average BER for the mixed RF-UWOC system. In [24], the fixed-gain AF relaying was used to mix the RF link over generalized-K distributed fading and the EGG distributed UWOC link. In [25], the authors analyzed the performance of dual-hop RF-UWOC system assisted by an unmanned aerial vehicle (UAV) using both fixed-gain AF relaying and decode-and-forward (DF) relaying schemes. The use of multiple input multiple output (MIMO) for RF transmission under mixed RF-UWOC was studied in [33]. Terrestrial transmission using optical wireless was recently investigated in [26], [27]. The authors in [26] studied the outage probability of a mixed terrestrial OWC link with multi-sensor UWOC considering weak oceanic and atmospheric turbulence conditions. Recently, the authors in [27] used the fixed-gain AF relaying to mix TOWC-UWOC communication system by modeling the TOWC channel using
Gamma-Gamma atmospheric turbulence with pointing errors and the EGG distributed UWOC channel with pointing errors.

B. Notations and Organization of the Paper

We list the main notations in Table I.

The paper is organized as follows. Section II discusses the channel models for both terrestrial and underwater optical communications. In Section III presents statistical results for the multi-layer UWOC system. In Section IV, the performance of the mixed TOWC-UWOC system in terms of outage probability and average BER is analyzed. In Section V, we present the numerical and simulation analysis of the proposed system. Finally, important conclusions are stated in Section VI.

II. SYSTEM MODEL

We consider a mixed terrestrial and underwater optical communication system integrated through a fixed-gain AF relaying protocol, as shown in Fig 1. Assume that a source on the land intends to communicate a signal $s$ with an underwater submarine. We use the non-coherent intensity modulation/direct detection (IM/DD) scheme, where the photodetector detects changes in the light intensity without employing a local oscillator. It is known that the heterodyne detection (HD) requires complex processing of mixing the received signal with a coherent signal produced by the local oscillator [34]. In the following two subsections, we describe channel and system models for terrestrial and underwater OWC systems.

A. Terrestrial OWC

In the first hop, we assume that the transmitted signal undergoes three types of fading: atmospheric turbulence-induced, pointing errors, and random fog. Thus, the received signal $y_T$ at the relay is given by

$$y = h Ts + w_T$$

where $h_T$ is the channel coefficient (including fog-induced path gain, atmospheric turbulence, and pointing errors) for the terrestrial link and $w_T$ is the additive white gaussian noise (AWGN) with variance $\sigma^2_{w_T}$. Assuming generalized Malaga distribution for atmospheric turbulence, fog-induced random path loss, and zero bore-sight pointing errors, then the PDF of the SNR $\gamma_T$ for the terrestrial link is given by

$$f_{\gamma_T}(x) = \frac{x^{\beta-1}e^{-\frac{x^\alpha}{\beta}}}{\Gamma(\alpha)}$$

where $\gamma_T$ is the average SNR, $\{\alpha,\beta, A^{mg}, b^{M}, \gamma^{M}, \Omega^{M}\}$ are Malaga parameters [36], and $\{z, k\}$ specifies the effect of fog on the signal transmission [38].

B. Underwater OWC

In the second hop, we employ a fixed-gain AF relay with gain parameter $G_R$ to forward the received to the destination over underwater channel. The gain selection $G_R$ can be entirely blind for a duration or using a semi-blind approach where it can be obtained using statistics of received signal power of the first hop (i.e., TOWC link). We consider the UWOC system by splitting the entire transmission channel in $N$ distinct layers in succession, resulting in $N$ vertical links, as depicted in Fig 1. Thus, the received electrical signal $y_U$ at

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**TABLE I**

| Notation | Description |
|----------|-------------|
| $(\cdot)_T$ | Notation for the TOWC link |
| $(\cdot)_U$ | Notation for the UWOC link |
| $\hat{I}_i$ | Notation for the $i$-th element |
| $l_T$ | TOWC link distance |
| $l_U$ | UWOC link distance |
| $\bar{\gamma}$ | Average SNR |
| $\gamma$ | Instantaneous SNR |
| $\alpha, \beta$ | Parameters for foggy channel |
| $\alpha^M, \beta^M, A^{mg}, b^{M}, \gamma^{M}, \Omega^{M}$ | Malaga distribution parameters |
| $\rho, A$ | Pointing errors parameters |
| $\omega, \lambda, a, d, p$ | EGG and GG distribution parameters |
| $\alpha^{p, p}, \beta^{p, p}, \eta^{p, p}$ | EW distribution parameters |
| $\alpha^{G, G}, \beta^{G, G}$ | IT distribution parameters |
| $\mathbb{E}[\cdot]$ | Expectation operator |
| $\Gamma(a)$ | $\int_0^\infty t^{a-1}e^{-t}dt$ |
| $C_{m,n}^{p,q}(a_k)_{k=1}^{z}$ | Meijer’s G-function |
| $H_{m,n}^{p,q}(a_k, A_k)_{k=1}^{z}$ | Fox’s H-function |

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Fig. 1. Mixed TWOC-UWOC system with multi-layer UWOC channel.
the destination can be expressed as:

\[ y_U = h_l h_p \left[ \prod_{i=1}^{N} h_i \right] G_{R|Y|} + w_U \]  

where \( h_l = e^{-\alpha_l} \) is the path gain with link distance \( l \) (in m) and extinction attenuation coefficient \( \alpha \), \( h_p \) models pointing errors, \( h_i = \prod_{j=1}^{N} h_i \) is the cascaded channel with \( h_i \) as the \( i \)-th layer of vertical link, and \( w_U \) is the AWGN with variance \( \sigma_w^2 \).

The PDF of zero-boresight pointing errors fading \( h_p \) is given as [37]:

\[ f_{h_p}(x) = \frac{\rho^2}{A_0^2} h_p^{2-1}, 0 \leq h_p \leq A_0 \]  

where \( A_0 = \text{erf}(v)^2 \) with \( v = \sqrt{\pi/2} r/\omega_z \) is the aperture radius and \( \omega_z \) is the beam width, and \( \rho = \frac{\omega_{z\max}}{\omega_{z\min}} \) as the equivalent beam width at the receiver and \( \sigma_x^2 \) as the variance of pointing errors displacement characterized by the horizontal sway and elevation [37].

Fading coefficients \( h_i, i = 1, 2, \ldots, N \) associated with different layers are modeled using various statistical distributions such as generalized Gamma, EGG, GG, and EW among others for different underwater conditions [9], [11].

The PDF of the channel coefficient for the generalized Gamma is given as

\[ f_{h_q}(x) = \frac{p_i}{a_i d_{i-1}} x^{d_{i-1}} \exp \left( -\left( \frac{x}{a_i} \right)^{d_{i}} \right) \]  

where \( a_i, d_i, \) and \( p_i \) are distribution parameters for the \( i \)-th layer to model different oceanic turbulence scenarios, as given in [9] (see Table-I, Table-II, and Table-III). As such, \( p_i = 1 \) in (5) denotes a Gamma distribution representing a thermally uniform UWOC channel.

Recently, [11] proposed EGG distribution (i.e., the combined exponential and generalized Gamma) for the oceanic turbulence with the PDF:

\[ f_{h_q}(x) = \frac{\omega_i}{\lambda_i} \exp \left( -\frac{x}{\lambda_i} \right) + (1 - \omega_i) \frac{p_i}{a_i d_{i-1}} x^{d_{i-1}} \exp \left( -\left( \frac{x}{a_i} \right)^{d_{i}} \right) \]  

where \( \omega_i \) is the mixture coefficient of the distributions (i.e., \( 0 < \omega_i > 1 \)), \( \lambda_i \) is the exponential distribution parameter. Note that experimental data is available for \( a_i, p_i, \) and \( d_i/p_i \).

Further, the PDF of the channel coefficient using the three-parameter EW distribution to model the oceanic turbulence is given by:

\[ f_{h_q}(x) = \frac{\beta_E \rho^2}{\eta^2} \left( \frac{x}{\eta} \right)^{\beta_E - 1} \exp \left( -\left( \frac{x}{\eta} \right)^{\beta_E} \right) \left[ 1 - \exp \left( -\left( \frac{x}{\eta} \right)^{\beta_E} \right) \right]^{\beta_E - 1} \]  

where \( \beta_E > 0 \) denotes the shape parameter of the scintillation index (SI), \( \eta^E > 0 \) is a scale parameter, and \( \alpha^E > 0 \) is an extra shape parameter dependent on the receiver aperture size.
TABLE II

| Oceanic Models | Unified Parameter Description |
|----------------|-------------------------------|
| EGG            | $\mathcal{P} = 1, A_i = 1, B_{i,0} = \omega_i, B_{i,1} = \frac{1}{\Gamma\left(\frac{d_{i,1}}{\rho_{i,1}}\right)}, m = N, q = N, D_{i,0} = (1, 1), D_{i,1} = \left(\frac{d_{i,1}}{\rho_{i,1}}, \frac{1}{\rho_{i,1}}\right)$, $\mathcal{E}_{i,0} = \frac{1}{\xi_{i,2}}, \mathcal{E}_{i,1} = \frac{1}{\xi_{i,2}}$ |
| GG             | $\mathcal{P} = 0, A_i = 1, \frac{1}{\Gamma\left(\frac{d_{i,1}}{\rho_{i,1}}\right)}, B_{i,j} = 1, m = N, q = N, D_{i,j} = \left(\frac{d_{i,j}}{\rho_{i,j}}, \frac{1}{\rho_{i,j}}\right), \mathcal{E}_{i,j} = \frac{1}{\xi_{i,j}}$ |
| EW             | $\mathcal{P} = \infty, A_i = \Gamma(\alpha_i^{e_p} + 1), B_{i,j} = \left(\frac{1}{\alpha_i^{e_p} + 1}, \frac{1}{\alpha_i^{e_p} + 1}\right), m = N, q = N, D_{i,j} = \left(\frac{1}{\alpha_i^{e_p} + 1}, \frac{1}{\alpha_i^{e_p} + 1}\right), \mathcal{E}_{i,j} = \frac{1}{\xi_{i,j}}$ |
| ITT            | $\mathcal{P} = 0, A_i = 1, \frac{1}{\Gamma(\alpha_i^{e_p} + 1)}, B_{i,j} = \left(\frac{1}{\alpha_i^{e_p} + 1}, \frac{1}{\alpha_i^{e_p} + 1}\right), m = 2N, q = 2N, D_{i,j} = \left(\frac{1}{\alpha_i^{e_p} + 1}, \frac{1}{\alpha_i^{e_p} + 1}\right)$ |

where $\gamma_{th}$ is the SNR threshold. Substituting (14) in (13) yields an exact expression for the outage probability. The asymptotic expression for the outage probability in the high SNR regime $\gamma \to \infty$ can be derived by applying [44] eq. 1.8.4.

\[ P_{\text{out}}^\infty = \Pi_{i=1}^N \frac{\tilde{\rho}_{i^2} A_i}{\xi_{i,2}} \sum_{j=0}^P \text{B}_{i,j} \sum_{k=1}^{m+1} \left(1 + \gamma_{\text{th}}\right)^{-1} \Gamma(1 + \frac{m+1}{\gamma_{\text{th}}}) \left(1 + \frac{b_j}{\gamma_{\text{th}}}ight) \gamma^\left(\frac{1}{\gamma_{\text{th}}}\right) \left(1 + \frac{b_k}{\gamma_{\text{th}}}ight) \Gamma(1 + \frac{b_k}{\gamma_{\text{th}}}) \right) \]

C. Average BER

In this subsection, we derive the average BER for the proposed UWOC system. Considering IM/DD, the average BER can be obtained as [45]:

\[ \overline{\text{BER}} = \frac{\delta}{21(\phi)} \sum_{n=1}^{M'} q_n^\phi \int_0^\infty \gamma^\phi \exp(-q_n \gamma) F_{\gamma}(\gamma) d\gamma \]

where the set $\{M', \delta, \phi, q_n\}$ can specify a variety of modulation schemes.

Using (14) and substituting $\exp(-q_n \gamma) = G_{0,1}^{1,0} \left[0 \left| q_n \gamma\right.\right]$ in (17) and reduce the Meijer’s G-function into Fox-H function, we get

\[ \overline{\text{BER}} = \frac{\delta \tilde{\rho}_{i^2}^\phi}{21(\phi)} \sum_{n=1}^{M'} q_n^\phi \Pi_{i=1}^N A_i \sum_{j=0}^P B_{i,j} \int_0^\infty \gamma^\phi \exp(-q_n \gamma) F_{\gamma}(\gamma) d\gamma \]

\[ \overline{\text{BER}} = \frac{\delta \tilde{\rho}_{i^2}^\phi}{21(\phi)} \sum_{n=1}^{M'} q_n^\phi \Pi_{i=1}^N A_i \sum_{j=0}^P B_{i,j} H^{m+1,1}_{2,0+2} \left[ \left(1, 1\right), \left(1 + \rho_{i,j}^\phi\right), \left(0, 1\right) \right] \left(\gamma_{\text{th}}, 0\right) \left(1 + \rho_{i,j}^\phi\right) \left(0, 1\right) \left(\gamma_{\text{th}}, 0\right) \]
for average BER at high SNR $\gamma_U \to \infty$ can be derived as

$$BER_{U}^{\infty} = \frac{\delta \gamma_{T}^{2}}{21(\phi)} \sum_{n=1}^{M} \prod_{i=1}^{N} A_{i} \sum_{j=0}^{P} B_{i,j} \sum_{k=1}^{m+1} \left[ \left( \frac{\epsilon_{i,j}}{A_{i}^{2}} \right)^{\frac{1}{4}} \Pi_{i=1}^{j} \Gamma_{i,j}^{(y_{i,j})+1} \right]$$

where $b_j = b_k = [D_{i,j}, 1]$ and $\beta_{j} = \beta_{k} = [D_{i,j}, 2]$.

Thus, the dominant SNR terms of (20) provides the diversity of proposed system as $\min\{\sum_{j=1}^{N} b_{j}^{2}/b_{j}^{2}\}$, which is exactly same as obtained using the outage probability.

D. Ergodic Capacity

The ergodic capacity for the underwater link $EC_U$ is an important performance metric for the design of communication systems and it can be defined as [46]:

$$EC_U = \int_{0}^{\infty} \log_{2}(1 + \kappa \gamma) f_{\gamma_U}(\gamma) d\gamma$$

(21)

where $\kappa = \frac{\epsilon_{U}}{\epsilon_{T}}$ for IM/DD and $\kappa = 1$ for HD (heterodyne detection).

Using (13) and substituting $\log_{2}(1 + \kappa \gamma) = 1.44G_{1,2,1}^{1,1,1}(1, 0, \kappa \gamma)$ in (21) and reduce the Meijer’s G-function into Fox-H function, we get

$$EC_U = 0.72 \rho_{U}^{2} \prod_{i=1}^{N} A_{i} \sum_{j=0}^{P} B_{i,j} \left[ \left( 1 + \rho_{U}^{2} \right) \prod_{i=2}^{N} \frac{1}{A_{i}^{2}} \sqrt{\frac{\gamma_{T}}{\gamma_{U}}} \right]$$

$$H_{p+1, q+1}^{m+1, n+1} \left[ \{D_{i,j}\}_{i=1}^{N} \right]_{j=1}^{1} , (\rho_{U}^{2}, 1) \left| \prod_{i=2}^{N} \frac{1}{A_{i}^{2}} \sqrt{\frac{\gamma_{T}}{\gamma_{U}}} \right]$$

(22)

Finally, applying the identity [44, eq. 2.8.4] to get the closed-form expression for the ergodic capacity over the cascaded channel

$$EC = 0.72 \rho_{U}^{2} \prod_{i=1}^{N} A_{i} \sum_{j=0}^{P} B_{i,j} H_{p+1, q+1}^{m+1, n+1} \left[ \left( 0, \frac{1}{2} \right), (1, \frac{1}{2}), (1, 0, 1) \right]$$

$$\left( \{D_{i,j}\}_{i=1}^{N} \right)_{j=1}^{1} , (\rho_{U}^{2}, 1) \left| \prod_{i=2}^{N} \frac{1}{A_{i}^{2}} \sqrt{\frac{\gamma_{T}}{\gamma_{U}}} \right|$$

(23)

In what follows, we employ a terrestrial optical link to communicate with the underwater transmission.

IV. PERFORMANCE OF MIXED TOWC-UWOC SYSTEM

In this section, we analyze the performance of a mixed TOWC-UWOC system when the fixed-gain AF relaying is applied. We can use (3) to express the end-to-end SNR of the dual-hop system consisting of TOWC and UWOC links [47]:

$$\gamma = \frac{\gamma_T \gamma_U}{\gamma_U + C}$$

(24)

where $C$ a constant for the fixed-gain AF relaying protocol. Standard transformation of random variables in (24) leads to the PDF of SNR for the fixed-gain AF relayed system as

$$f_{\gamma}(\gamma) = \int_{0}^{\infty} f_{\gamma_T} \left( \frac{2(x + C)}{x} \right) f_{\gamma_U}(x) \frac{x + C}{x} dx$$

(25)

where $f_{\gamma_T}(\gamma)$ and $f_{\gamma_U}(\gamma)$ are the PDF of SNR for TOWC link and UWOC link, respectively.

We use (2) and (13) in (25) to develop the PDF of the SNR for the mixed link:

Lemma 2: The PDF and CDF of the end-to-end SNR for the fixed-gain AF relay-assisted mixed TOWC-UWOC system are given as

$$f_{\gamma_T}(\gamma) = \frac{z^{k} \rho_{U}^{2} A_{mg}}{4 \gamma} \sum_{m=1}^{N} \frac{b_{m}^{2}}{\gamma_{T}} \prod_{i=1}^{N} \rho_{U}^{2} A_{i} \sum_{j=0}^{P} B_{i,j}$$

$$H_{1,0,0,0}^{1,0,1} \left[ \frac{1}{A_{i}^{2}} \sqrt{\frac{\gamma_{T}}{\gamma_{U}}} \right]$$

$$U_{1} \left| V_{1} \right. \left| U_{2} \right. \left| V_{2} \right.$$

(26)

where $U_{1} = \{ (1, \frac{1}{2}, \frac{1}{2}) : (1 - \rho_{T}^{2}, 1), (1 - \gamma_{T}^{M}, 1); (1 - m_{1}, 1); ((1 - z, 1))_{1}^{N}; (1 + \rho_{U}^{2}, 1) \}$ and $V_{1} = \{ : (-\rho_{T}^{2}, 1); (-z, 1); (1, \frac{1}{2}) ; \}$.

$$F_{\gamma_T}(\gamma) = \frac{z^{k} \rho_{U}^{2} A_{mg}}{4} \sum_{m=1}^{N} \frac{b_{m}^{2}}{\gamma_{T}} \prod_{i=1}^{N} \rho_{U}^{2} A_{i} \sum_{j=0}^{P} B_{i,j}$$

$$H_{1,0,0,0}^{1,0,1} \frac{1}{A_{i}^{2}} \sqrt{\frac{\gamma_{T}}{\gamma_{U}}} \right.$$
TABLE III
SIMULATION PARAMETERS

| Parameter                      | Value                          |
|--------------------------------|--------------------------------|
| Transmitted optical power      | $P_t$                          |
| AWGN variance                  | $\sigma_w^2$                   |
| Total link distance            | $l = (l_l + l_t) = (200 + 50)$ m |
| Extinction coefficient         | $\alpha$                       |
| Shape parameter of foggy channel | $k$                           |
| Scale parameter of foggy channel | $\beta_f$                  |
| Pointing errors parameters    | $A$, $\rho^2$                  |
| Malaga distribution parameters | $\alpha^M$, $\beta^M$          |
| GG distribution parameters    | $\omega_i$, $\lambda_i$        |
| EGG distribution parameters   | $\omega_i$, $\lambda_i$        |
| EGG distribution parameters (For Fig. 5(a) and Fig. 5(b)) | $\omega_i$, $\lambda_i$        |
| EW distribution parameters    | $\alpha^E$, $\beta^E$, $\eta^E$ |
| TT distribution parameters    | $\alpha^T$, $\beta^T$          |
| Modulation parameters         | $M^i$, $\delta$, $\phi$, $q_n$ |

$\Gamma (\phi - \frac{m+1}{2}) q_n^{-\phi+\frac{m+1}{2}}$ and applying (48), we get average BER of the mixed TWOC-UWOC system involving the bivariate Fox–H function:

$$\begin{align*}
\text{BER} &= \frac{\delta}{2\Gamma(\phi)} \sum_{n=1}^{N} \sum_{m=1}^{M} b_{m}^{M} \sum_{j=1}^{P} B_{i,j} \\
&= \frac{(\gamma^M \beta^M + \Omega^M)}{\rho^2} \left\langle \frac{U_3}{V_3} \right\rangle
\end{align*}$$

where $U_3 = \{(1 - \frac{1}{2}, \frac{1}{2}) : (1 - \rho^2, 1); (1 - \alpha^M, 1); (1 - m_1, 1); \{(1 - z, 1); (1 - \frac{1}{2}); (1 - \phi, \frac{1}{2}) ; (1 + \rho^2, 1)\}$ and $V_3 = \{(1 - \rho^2, 1); (z, 1); \{(1, \frac{1}{2}) ; (0, \frac{1}{2}) ; \{D_{i,j}^N \} : (\gamma^M, 1); (0, \frac{1}{2})\}$

It should be mentioned that bivariate Fox–H function has been extensively analyzing the fixed-gain AF relaying over complicated fading models [27].

V. SIMULATION AND NUMERICAL ANALYSIS

In this section, we demonstrate the performance of multi-layer vertical UWOC system over various oceanic turbulence conditions and the mixed TOWC-UWOC transmission. We
also compare the performance of the multi-layer UWOC with the single-layer ($N = 1$) approximation. We use Monte-Carlo (MC) simulation (averaged over $10^7$ channel realizations) to validate the derived analytical expressions. Further, the asymptotic expression of outage probability and average BER converges with analysis and simulation results in the high SNR regime. We use standard inbuilt MATLAB and Mathematica libraries to calculate Meijer’s G and Fox’s H-function, respectively. Since there is no measurement data to confirm the variation of distribution parameters with distance, we illustrate the performance by considering vertical underwater link length $l_U = 50$ m with $N = 5$ layers, and the thickness of each layer is assumed to be 10 m. We use standard simulation parameters and measurement-based parametric data for EGG, GG, EW, and $\Gamma \Gamma$, as given in Table III.

First, we demonstrate the outage probability performance of the considered UWOC system in Fig. 2. It can be seen from Fig. 2(a) that the outage probability for a specific oceanic turbulence condition using different statistical models is similar for the single-layer ($N = 1$) case validating the equivalence of different models using PDF plots, as demonstrated comprehensively in [9]. We used two different EGG turbulence since $\lambda$ and $\omega$ parameters is not available for the same experimental scenario. The figure shows that an acceptable operating outage probability of $10^{-3}$ can be achieved with an average SNR of 80 dB. To demonstrate the multi-layer performance, we consider the GG model (as depicted in Fig. 2(b)), which excellently fits the experimental data for a wide range of oceanic turbulence, as observed in [9]. Comparing Fig. 2(a) and Fig. 2(b) with $\rho_U^2 = 1$ plots, it can be seen that the single-layer model underestimates the oceanic turbulence concerning the $N = 5$ layers case justifying the use of multi-layer modeling for UWOC transmissions. Further, Fig. 2(b) shows that the outage performance of the system improves with an increase in the values of GG distribution parameters ($a$, $d$, and $p$) and a decrease in pointing errors (i.e., higher $\rho_U^2$). In the first plot of Fig. 2(b), we consider the pointing errors parameter ($\rho_U^2 = 1$) and the GG distri-
The average BER performance of the mixed TWOC and multi-layer UWOC system in Fig. 5(a) and Fig. 5(b), respectively. We consider terrestrial link distance $l_T = 400$ m, underwater link length $l_U = 50$ m with $N = 5$ layers each with EGG oceanic turbulence. In Fig. 5(a), we plot the outage probability considering light fog with weak turbulence and moderate fog with moderate turbulence for two different pointing errors parameters $\rho_T^2 = \rho_U^2 = \{1, 6\}$. The effect of fog density, the intensity of atmospheric turbulence, and pointing errors are clearly visible. It can be seen from Fig. 5(a), that 20 dBm more transmit power is required to achieve the same outage probability with moderate fog as compared with light fog. Further, the penalty for strong pointing errors is 10dBm for the same foggy and atmospheric conditions. In Fig. 5(b), we demonstrate the effect of different (weak, moderate, and strong) atmospheric turbulence on the average BER performance for both light and moderate foggy conditions. The figure shows that the effect of atmospheric turbulence on the average BER performance is less as compared with performance degradation due to the fog.

VI. CONCLUSIONS AND FUTURE WORK

We presented unified performance analysis of the UWOC system considering the vertical underwater link as a multi-layer cascaded channel considering i.i.d. GG, EGG, EW, and IT oceanic turbulence channels. We analyzed the system performance by deriving analytical expressions of the PDF and CDF of the end-to-end SNR, and developed outage probability, average BER, and ergodic capacity under the combined effect of cascaded oceanic turbulence and pointing errors in terms of Meijer-G and Fox-H functions. We provided the asymptotic expressions using Gamma functions for the outage probability and average BER to determine the diversity order of the considered system. We also employed the fixed-gain AF relaying to integrate the terrestrial OWC transmission subjected to the combined effect of generalized Malaga atmospheric turbulence, fog-induced random path gain, and pointing errors to communicate with the UWOC link. We analyzed the...
performance of the mixed link using outage probability and average BER involving bivariate Fox H-function. Simulation results showed a performance gap when the single-layer approximation was compared with the multi-layer model. The proposed analysis would be helpful for an efficient deployment for UWOC under various oceanic conditions. The existing measurement data and statistical model do not consider depth dependency for oceanic turbulence. It would be interesting to investigate the applicability of the proposed analysis using the channel measurement data considering ocean stratification for the UWOC system.

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APPENDIX A: PROOF OF THEOREM 1

First, we find the PDF of \( h_{GG} = \prod_{i=1}^{N} h_i \), where \( h_i \), \( i = 1, 2, \ldots, N \) denote i.i.d GG random variables. Substituting (5) in (10) and applying the identity \( \int_{0}^{\infty} t^a \exp(-b t^c) dt = \frac{1}{b^{c} c!} \) [49, pp. 347, eq. 3.381.10], we get the \( n \)-th order moment of \( h_c \) as:

\[
\mathbb{E}[h_{GG}^n] = \prod_{i=1}^{N} \frac{\Gamma(n + d_i)}{a_i^{-n} \Gamma(d_i)}
\]

(30)

We use (30) in (9) and apply the definition of Fox H-function to get the PDF of \( h_c \) for the generalized Gamma turbulent channel as

\[
f_{h_{GG}}(x) = \prod_{i=1}^{N} \frac{1}{\Gamma\left(\frac{d_i}{a_i}\right)}
\]

\[
\frac{1}{x} H_{0,N}^{N,0} \left[ \left\{ \left( \frac{d_i}{a_i}, \frac{1}{a_i} \right) \right\}_{i=1}^{N} \right] \left[ \prod_{i=2}^{N} \left( \frac{x}{a_{i2}} \right) \right]
\]

(31)

Next, we find the PDF of \( h_{EGG} = \prod_{i=1}^{N} h_i \), where \( h_i \), \( i = 1, 2, \ldots, N \) denote i.i.d EGG random variables. Substituting (6) in (10), we get the \( n \)-th order moment of \( h_{EGG} \) as:

\[
\mathbb{E}[h_{EGG}^n] = \prod_{i=1}^{N} \left( \omega_i \lambda_i \Gamma(1 + n) + \frac{1}{\omega_i} \Gamma \left( \frac{n + d_i}{a_i} \right) \right)
\]

(32)

Using (32) in (9) and applying the definition of Fox H-function to get the PDF of \( h_{EGG} \) for the EGG oceanic turbulent channel:

\[
f_{h_{EGG}}(x) = \prod_{i=1}^{N} \frac{\omega_i}{\Gamma\left(\frac{d_i}{a_i}\right)} H_{0,N}^{N,0} \left[ \left\{ \left( \frac{d_i}{a_i}, \frac{1}{a_i} \right) \right\}_{i=1}^{N} \right] \left[ \prod_{i=2}^{N} \left( \frac{x}{a_{i2}} \right) \right] + \frac{(1 - \omega_i)}{\Gamma\left(\frac{d_i}{a_i}\right)} H_{0,N}^{N,0} \left[ \left\{ \left( \frac{d_i}{a_i}, \frac{1}{a_i} \right) \right\}_{i=1}^{N} \right] \left[ \prod_{i=2}^{N} \left( \frac{x}{a_{i2}} \right) \right]
\]

(33)

To develop the PDF for the EW channel, we use the Newton’s generalized binomial theorem \((1 + z)^p = \sum_{j=0}^{\infty} \frac{\Gamma(p+1)z^j}{\Gamma(j+1)j!} \) for the term \([1 - (\exp(-x/\eta))^j]^{a-1} \) in (7) to get

\[
f_{h_{EW}}(x) = \frac{a\beta}{\eta} \Gamma(a) \sum_{j=0}^{\infty} \frac{(-1)^j}{j! \Gamma(a-j)j!} \left( \frac{x}{\eta} \right)^j \exp\left(-\frac{(j+1)\beta x}{\eta}\right)
\]

(34)

Substituting (34) in (10) and applying the identity \( \int_{0}^{\infty} t^a \exp(-b t^c) dt = \Gamma\left(\frac{a+1}{c}\right) \) [49, pp. 347, eq. 3.381.10], we get the \( n \)-th order moment of \( h_{EW} \) as:

\[
\mathbb{E}[h_{EW}^n] = \prod_{i=1}^{N} \Gamma(\alpha_i + 1) \sum_{j=0}^{\infty} \frac{(-1)^j}{j! \Gamma(\alpha_i-j)(j+1)\Gamma(\alpha_i+j+1)} \left( \frac{(j+1)\beta_i x}{\eta_i}\right)
\]

(35)

Using (35) in (9) and applying the definition of Fox H-function to get the PDF of \( h_{EW} \) for the EW turbulent:

\[
f_{h_{EW}}(x) = \prod_{i=1}^{N} \Gamma(\alpha_i + 1) \sum_{j=0}^{\infty} \frac{(-1)^j}{j! \Gamma(\alpha_i-j)(j+1)\Gamma(\alpha_i+j+1)} \left( \frac{(j+1)\beta_i x}{\eta_i}\right)
\]

(36)

Finally, we find the PDF of \( N \) cascaded IT channel \( h_{GG} = \prod_{i=1}^{N} h_i \), where \( h_i \), \( i = 1, 2, \ldots, N \) denote i.i.d GG random variables. Note that [9] used the method of induction to derive the PDF of cascaded channel for GG oceanic turbulence. Converting the \( v \)-th order modified Bessel’s function \( K_v(\cdot) \) into Meijer G function equivalent, we represent (8) as

\[
f_{h_{EGG}}(x) = \frac{(\alpha G \beta G)(\alpha G + \beta G)^2}{\Gamma(\alpha G) \Gamma(\beta G)} x \frac{\alpha G - \beta G}{2} - \frac{1}{\Gamma(\alpha G) \Gamma(\beta G)}
\]

(37)

Substituting (37) in (10) and applying the identity [50, eq. 07.34.21.0009.01], we get the \( n \)-th order moment of \( h_{IT} \) as:

\[
\mathbb{E}[h_{IT}^n] = \prod_{i=1}^{N} \frac{(\alpha_i \beta_i)^n}{\Gamma(\alpha_i) \Gamma(\beta_i) \Gamma(\alpha_i + n) \Gamma(\beta_i + n)}
\]

(38)

We use (38) in (9) and apply the definition of Fox H-function to get the PDF of \( h_{IT} \) as

\[
f_{h_{IT}}(x) = \prod_{i=1}^{N} \frac{1}{\Gamma(\alpha_i) \Gamma(\beta_i)} \left( \frac{\alpha_i \beta_i}{x} \right)
\]

(39)

\[
H_{0,2N}^{2N,0} \left[ \left\{ (\alpha_i, 1) \right\}_{i=1}^{N}, \left\{ (\beta_i, 1) \right\}_{i=1}^{N} \right] \prod_{i=2}^{N} \left( \frac{\alpha_i \beta_i x}{\eta_i} \right)
\]

Capitalizing (31), (33), (36), (39), we get an expression for the unified PDF in (11) with parameters as depicted Table II which concludes the proof for Theorem 1.
APPENDIX B: PROOF OF LEMMA 1

Using the product distribution [51], the PDF of the combined channel \( h = h_i h_p \) can be expressed as

\[
f_h(h) = \int_0^{h_0} \frac{1}{h_p} f_{h_p}(h_p) f_{h_i}(h) \, dh_p
\]

(40)

We substitute (6) and (11) in (40), solve the inner integral

\[\int h_p^\gamma \, dh_p = \left( \frac{\rho_1^\gamma}{\rho_1^\gamma + \nu} \right) \Gamma(1 + \frac{\nu}{\rho_1^\gamma}) \]

and apply the definition of Fox H-function [52] to get

\[
f_h(h) = \prod_{i=1}^{N} \rho_i^\gamma A_i \sum_{j=0}^{P} B_{i,j} \frac{1}{\pi} H^{n+1,0}_{1,q+1} \left[ \left( \frac{1 + \rho_1^\gamma}{\rho_1^\gamma + \nu} \right) \Gamma(N+1, \rho_1^\gamma, 1) \right] \prod_{i=1}^{N} E_{i,j}^\gamma h
\]

(41)

Thus, we use the transformation of random variable \( \gamma = \gamma_U h^2 \) to get the PDF of SNR in (13). To find the CDF of SNR under the combined channel, we use (13) in \( F_s(\gamma) = \int_0^\gamma f(\gamma') d\gamma' \) and apply the definition of Fox H-function with inner integral

\[
\int_0^\gamma \gamma^{-\frac{2}{\nu} - 1} d\gamma = \frac{2^{2/\nu} \Gamma(-n)}{\Gamma(1-n)}
\]

to get the CDF of the SNR in (14), which concludes the proof of Lemma 1.

APPENDIX C: PROOF OF LEMMA 2

Using (2) and (13) in (45) results the PDF of the mixed link as

\[
f_3(\gamma) = \frac{1}{4\gamma} \sum_{m=1}^M \left( \frac{1}{2\pi} \right)^2 \int L_i \int L_P \prod_{i=1}^P \rho_i^m A_i \frac{1}{2} \left[ \frac{1}{\pi} H^{n+1,0}_{1,q+1} \left[ \left( \frac{1 + \rho_1^\gamma}{\rho_1^\gamma + \nu} \right) \Gamma(N+1, \rho_1^\gamma, 1) \right] \prod_{i=1}^{N} E_{i,j}^\gamma h \right]
\]

(42)

Substitute (40) in (42), we get

\[
f_3(\gamma) = \frac{1}{4\gamma} \sum_{m=1}^M \left( \frac{1}{2\pi} \right)^2 \int L_i \int L_P \prod_{i=1}^P \rho_i^m A_i \frac{1}{2} \left[ \frac{1}{\pi} H^{n+1,0}_{1,q+1} \left[ \left( \frac{1 + \rho_1^\gamma}{\rho_1^\gamma + \nu} \right) \Gamma(N+1, \rho_1^\gamma, 1) \right] \prod_{i=1}^{N} E_{i,j}^\gamma h \right]
\]

(43)

The inner integral can be solved as

\[
\int_0^\gamma \gamma^{-\frac{2}{\nu} - 1} d\gamma = \frac{\gamma^{\frac{2}{\nu}}}{\frac{2}{\nu}} = \gamma^{\frac{2}{\nu} - \frac{2}{\nu}}
\]

(46)

Using (46) in (45), we get (27), which completes the proof of Lemma 2.

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