Research Article

New Stability Analysis Results for Linear System with Two Additive Time-Varying Delay Components

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This paper focuses on the stability problem of continuous linear systems with two additive time-varying delay components. Firstly, an effective and simple Lyapunov–Krasovskii functional (LKF) is established, which permits unrestricted use, distribution, and reproduction in any medium, provided the original work is properly cited.

1. Introduction

It is a phenomenon that time delay often occurs in various dynamic systems such as economics, network transportation, circuit signal system, engineering systems, and so on [1–3]. The existence of the time-varying delay has a negative influence on system performance, which will have an adverse effect on the dynamic performance of the system and even make it unstable. Moreover, for the remote control systems or networked control systems, signals transmitted between different points may pass different segments of networks, in which there may exist time delays of different characteristics. For instance, the signal transmitted from the sensor to the controller and the other from the controller to the actuator have different properties for the reason of variable transmission conditions [4]. Therefore, it is necessary to investigate the stability problem of systems with additive time delays and assess the effects from different delays with various properties, which motivates the research about linear system with two additive time delay components in this paper.

Since the distribution characteristic of delays mentioned above has been modeled firstly as a system with two additive time-varying delay components in [4], many significant results have been devoted to this issue. As a most general form, linear systems with two additive delay components have drawn lots of attention [5–15]. Moreover, neural networks [16–21], T-S fuzzy systems [22, 23], and Lur’e systems [24], considering two additive delay components in the state, have been also investigated recently. Among these works, two main techniques that contain constructing an appropriate LKF [25, 26] and estimating its derivative [27–32] are usually used to reduce conservatism of the derived criteria. Additionally, reciprocally convex combination lemma [33] and its extensive form [34] also contribute to analyzing the time-delay system stability [15]. Some criteria with less conservatism are obtained in [8] by taking the delay, its upper bound, and the relationship between them into account. A novel LKF [9] is constructed, for which positive definiteness is ensured by the whole LKF rather than each term. In [10], a delay-partitioning-based LKF was established, which greatly reduces the
conservatism. In [11], a novel LKF containing delay-product type terms [12, 13] is constructed; meanwhile, both the Wirtinger-based inequality [28] and reciprocally convex combination technique [33] are utilized to estimate the derivative of LKF. Recently, a delay interconnection LKF and its improved form are constructed in [14, 15], respectively; meanwhile, the free-matrix-based inequality [30] is utilized to calculate the single integral terms in their derivatives, which is demonstrated to be effective in reducing the conservatism.

For the sake of further study, the relevant effort is to decrease the conservatism and computational complexity of the derived result from the viewpoint of constructing an appropriate LKF and estimating its derivative. Although great development has been devoted to the issue, there still exists room for further study to take both the conservatism and the computational complexity into account. For example, in [14, 15], delay interconnection LKF and its improved form are verified for their effectiveness for reducing the conservatism of the derived results; however, the reduction in conservatism is based on the increase in computational complexity. Therefore, it is a challenging and meaningful problem to ensure the low conservativeness of the stability criteria and reduce the computational complexity of the result as possible. This paper will study this issue from two aspects, one is to construct an effective and simple LKF which not only considers adequate information of delay components and their upper bounds but also establishes a simpler form in comparison with the existing works to decrease the computational complexity. In addition, the GFWM-based inequality [35] reduces the estimation error of the single integral terms appearing in the derivative of the proposed LKF, which has not been utilized for analyzing stability of the system with additive delay components. Hence, improved results can be derived of less conservatism and computational complexity while using GFWM-based inequality to estimate the single integral terms, which will further reduce the computational complexity.

(iii) With the techniques mentioned above employed in this paper, improved results with less conservatism and lower computational complexity can be presented, whose validity and superiority can be demonstrated with a representative numerical example in comparison with the works in [4, 6–9, 14, 15].

This paper is organized as follows. Problem formulation and preliminary are given in Section 2. In Section 3, an improved stability criterion and its corollary are developed. A numerical example is used to demonstrate the validity and superiority of the proposed criteria in Section 4. At last, Section 5 presents the conclusion.

Notations: the notation \( \mathbb{R}^n \) denotes the n-dimensional Euclidean space; \( P > 0 (\geq 0) \) means that \( P \) is a real symmetric and positive definite (semipositive definite) matrix; \( I \) and \( 0 \) represent an appropriately dimensioned identity matrix and zero matrix, respectively; \( * \) stands for the symmetric term in the symmetric matrix; the transpose and the inverse of a matrix are denoted by the superscripts \( T \) and \( -1 \), respectively; and Sym\( \{X\} = X + X^T \).

### 2. Problem Formulation and Preliminary

Consider the following continuous linear system with two additive time-varying delay components:

\[
\begin{aligned}
\dot{x}(t) &= Ax(t) + A_d x(t - d_1(t) - d_2(t)), \\
\forall t &\in [-h, 0], \\
x(t) &= \phi(t),
\end{aligned}
\]

where \( x(t) = [x_1(t) \ x_2(t) \ \cdots \ x_n(t)]^T \in \mathbb{R}^n \) is the state vector; \( A, A_d \in \mathbb{R}^{n \times n} \) are known constant matrices; \( \phi(t) \) is the initial condition; \( d_1(t) \) and \( d_2(t) \) are continuous and differential functions, which stand for two time delays and satisfy the following conditions:

\[
\begin{aligned}
0 &\leq d_1(t) \leq h_1, \quad d_1(t) \leq \mu_1, \\
0 &\leq d_2(t) \leq h_2, \quad d_2(t) \leq \mu_2,
\end{aligned}
\]

where \( h_i, i = 1, 2 \) and \( \mu_i, i = 1, 2 \) are constants. Let \( h = h_1 + h_2, \mu = \mu_1 + \mu_2, d(t) = d_1(t) + d_2(t) \).

The necessary lemmas are introduced as follows to develop the results in Section 3.

**Lemma 1** (GFWM-based inequality [35]). For symmetric positive definite matrix \( R \in \mathbb{R}^{m \times n} \), any matrices \( L, M, \) and an arbitrary vector \( \omega : [a, b] \rightarrow \mathbb{R}^n \). Then, the following inequality holds:
Lemma 2 (Jensen’s inequality [27]). For any symmetric positive definite matrix \( R \in \mathbb{R}^{n \times n} \), scalars \( a \) and \( b \), and vector \( x : [a, b] \to \mathbb{R}^n \), the following inequality holds:

\[
\frac{(b-a)^2}{2} \int_a^b \int_\theta^b x^T(s)Rx(s)ds d\theta \geq \left( \int_a^b x(s)ds \right)^T R \left( \int_a^b x(s)ds \right),
\]

where \( \bar{x}_1 \) is an arbitrary vector and \( \bar{x}_2 = \frac{(2/a-b) \int_a^b \omega(u)du ds}{(a+1/2)M} \).

Lemma 3 (Wirtinger-based inequality [28]). For any symmetric positive definite matrix \( R \in \mathbb{R}^{n \times n} \), scalars \( a \) and \( b \), and vector \( x : [a, b] \to \mathbb{R}^n \), the following inequality holds:

\[
\int_a^b x^T(s)Rx(s)ds \geq \frac{1}{b-a} \begin{bmatrix} \bar{x}_1^T \cr 0 \end{bmatrix} \begin{bmatrix} R & 0 \\ 0 & 3R \end{bmatrix} \begin{bmatrix} \bar{x}_1 \\ \bar{x}_2 \end{bmatrix},
\]

where \( \bar{x}_1 \) and \( \bar{x}_2 \) are defined in Lemma 1.

Lemma 4 (extended reciprocally convex inequality [34]). For a real scalar \( a \in (0, 1) \), symmetric matrices \( X_1 > 0 \) and \( X_2 > 0 \), and any matrices \( S_1 \) and \( S_2 \), the following matrix inequality holds:

\[
\begin{bmatrix} \frac{1}{X_1} & 0 \\ 0 & \frac{1}{1-a}X_2 \end{bmatrix} \succeq \begin{bmatrix} X_1 + (1-a)T_1 & (1-a)S_1 + aS_2 \\ (1-a)S_1^T + aS_2^T & X_2 + aT_2 \end{bmatrix},
\]

where \( T_1 = X_1 - S_2X_1^{-1}S_2^T \) and \( T_2 = X_2 - S_1X_1^{-1}S_1^T \).

3. Results and Discussion

In this section, by constructing a novel LKF and using GFWM-based inequality together with some other advanced techniques to calculate its derivative, a stability criterion and its corollary are derived for system (1). Meanwhile, some discussions are provided to illustrate the superiority of the methods used in this paper.

3.1. A Stability Criterion with Less Conservatism and Lower Computational Complexity

A novel LKF together with the aforementioned lemmas is used to derive the following criterion.

**Theorem 1.** For given scalars \( h_1, h_2, \mu_1, \) and \( \mu_2 \), system (1) with two additive time-varying delay components satisfying (2) is asymptotically stable if there exist symmetric positive definite matrices \( P \in \mathbb{R}^{2m \times 2n} \), \( Q_i \in \mathbb{R}^{2n \times 2n} \), \( i = 1, 2, 3, 4 \), \( R_i \in \mathbb{R}^{n \times n} \), \( \alpha \in [a, b] \), and \( L_k \in \mathbb{R}^{3n \times 3n} \), \( M_k \in \mathbb{R}^{3n \times 3n} \), \( k = 1, 2, 3, 4, 5, 6 \), \( S_1, S_2 \in \mathbb{R}^{2m \times 2n} \) such that the following matrix inequalities hold:

\[
\Sigma(0, 0) + \Omega(0, 0) < 0,
\]

\[
\Sigma(h_1, 0) + \Omega(h_1, 0) < 0,
\]

\[
\Sigma(h_1, h_2) + \Omega(h_1, h_2) < 0,
\]

where
\[ \Sigma(d_1(t), d_2(t)) = \Xi_1(d_1(t), d_2(t)) + \Xi_2 + \Xi_3 + \Xi_4(d_1(t), d_2(t)). \]
\[ \Omega(d_1(t), d_2(t)) = \Xi_5(d_1(t), d_2(t)) + \Xi_6(d_1(t), d_2(t)). \]
\[ \Xi_1(d_1(t), d_2(t)) = \text{Sym}[\Pi_1^T \Pi_1]. \]
\[ \Xi_2 = \varepsilon_1^T (Q_1 + Q_3 + Q_2) \varepsilon_1 - \varepsilon_1^T Q_1 \varepsilon_1 - (1 - \mu) \varepsilon_1^T Q_2 \varepsilon_1 - (1 - \mu) \varepsilon_1^T Q_3 \varepsilon_1. \]
\[ \Xi_3 = \Pi_1^T (h \mathcal{R}_1 + h \mathcal{R}_2 + h \mathcal{R}_3) \Pi_1 + \text{Sym}[\varepsilon_1^T \mathcal{L} \Pi_1 + \varepsilon_2^T \mathcal{M} \Pi_1] + \text{Sym}[\varepsilon_1^T \mathcal{M} \Pi_1 + \varepsilon_2^T \mathcal{L} \Pi_1] + \text{Sym}[\varepsilon_1^T \mathcal{M} \Pi_1 + \varepsilon_2^T \mathcal{M} \Pi_1]. \]
\[ \Xi_4(d_1(t), d_2(t)) = (h_1 - d_1(t)) \varepsilon_1^T \left( L_1 R_1^T L_1^T + \frac{1}{3} M_1 R_1^T M_1^T \right) \varepsilon_1 + d_1(t) \varepsilon_1^T \left( L_1 R_1^T L_1^T + \frac{1}{3} M_1 R_1^T M_1^T \right) \varepsilon_1 + \left( h_2 - d_2(t) \right) \varepsilon_2^T \left( L_1 R_1^T L_1^T + \frac{1}{3} M_1 R_1^T M_1^T \right) \varepsilon_2 + d_2(t) \varepsilon_2^T \left( L_1 R_1^T L_1^T + \frac{1}{3} M_1 R_1^T M_1^T \right) \varepsilon_2. \]
\[ \Xi_5(d_1(t), d_2(t)) = \frac{h - d(t)}{h} \Pi_1^T \Pi_1 S_{\nu} S_{\nu}^T \Pi_1 - \frac{h - d(t)}{h} \text{Sym}[\Pi_1^T S_{\nu} \Pi_1] - \frac{d(t)}{h} \Pi_1^T S_{\nu} \Pi_1 - \frac{d(t)}{h} \text{Sym}[\Pi_1^T S_{\nu} \Pi_1]. \]
\[ \Pi_1 = [e_1^T d(t) e_1^T + (h - d(t)) e_1^T] \right]. \]
Proof. A vector $\xi(t) \in \mathbb{R}^{13n}$ is defined as follows:

$$\xi(t) = \begin{bmatrix} x^T(t) & x^T(t-h_1) & x^T(t-h_2) & x^T(t-h) & x^T(t-d_1(t)) & x^T(t-d_2(t)) \end{bmatrix}^T$$

$$x^T(t-d(t)) \frac{1}{d_1(t)} \int_{t-d_1(t)}^t x^T(s)ds + \frac{1}{h_1-d_1(t)} \int_{t-h_1}^{t-d_1(t)} x^T(s)ds + \frac{1}{d_2(t)} \int_{t-d_2(t)}^t x^T(s)ds \tag{9}$$

A novel LKF containing adequate information about delay components and their derivatives with a simper form is constructed as follows:

$$V_1(x_t) = c^T(t)Pc(t)$$
$$V_2(x_t) = \int_{t-d_1(t)}^t x^T(s)Q_1x(s)ds + \int_{t-d_2(t)}^t x^T(s)Q_2x(s)ds + \int_{t-h_1}^{t-d_1(t)} x^T(s)Q_3x(s)ds + \int_{t-h_2}^{t-d_2(t)} x^T(s)Q_4x(s)ds,$$

$$V_3(x_t) = \int_{t-h}^{t} x^T(s)R_1\dot{x}(s)ds \frac{d\theta}{h} + \int_{-h}^{0} \dot{x}^T(s)R_2\dot{x}(s)ds \frac{d\theta}{h} + \int_{0}^{t} \dot{x}^T(s)R_3\dot{x}(s)ds \frac{d\theta}{h},$$

$$V_4(x_t) = \int_{t-h}^{t} \mu x^T(s)Z\dot{x}(s)ds \frac{d\mu}{h} + \int_{-h}^{0} \dot{x}^T(s)Z_2\dot{x}(s)ds \frac{d\mu}{h},$$

with

$$c(t) = \begin{bmatrix} x^T(t) & \int_{t-h}^{t} x^T(s)ds \end{bmatrix}^T.$$  \tag{12}

Firstly, the matrices declared in Theorem 1 including $P, Q_i (i = 1, 2, 3, 4), R_i (i = 1, 2, 3), Z, Z_\mu$ are positive definite, which insures the positive definitiveness of the proposed LKF. Therefore, $V(x_t) \geq \epsilon_1 \|x(t)\|$ holds for a sufficiently small scalar $\epsilon_1 > 0$.

Secondly, the derivative of the proposed LKF with respect to time along the trajectory of system (1) is calculated as follows:

$$\dot{V}(x_t) = \dot{V}_1(x_t) + \dot{V}_2(x_t) + \dot{V}_3(x_t) + \dot{V}_4(x_t).$$ \tag{13}

Calculating the derivatives of $V_1(x_t), V_2(x_t), V_3(x_t), V_4(x_t)$ yields

$$\dot{V}_1(x_t) = 2c^T(t)Pc(t) = \xi^T(t)(\Pi_1^TP\Pi_1 + \Pi_2^TP\Pi_1)\xi(t) = \xi^T(t)\text{Sym}[^TP\Pi_1]\xi(t) = \xi^T(t)\Xi_1(d_1(t), d_2(t))\xi(t),$$ \tag{14}

$$\dot{V}_2(x_t) = \xi^T(t)\left[c_1^T(Q_1 + Q_2 + Q_3 + Q_4)e_1 - c_2^TQ_4e_4 - (1 - \dot{d}_1(t))c_5^TQ_1e_5 - (1 - \dot{d}_2(t))c_7^TQ_3e_7 - (1 - \dot{d}(t))c_9^TQ_5e_9 - (1 - \dot{d}(t))c_3^TQ_3e_5 \right] \xi(t)$$

$$\leq \xi^T(t)\left[c_1^T(Q_1 + Q_2 + Q_3 + Q_4)e_1 - c_2^TQ_4e_4 - (1 - \mu_1)c_5^TQ_1e_5 - (1 - \mu_2)c_7^TQ_3e_7 \right] \xi(t),$$

$$= \xi^T(t)\Xi_2\xi(t),$$ \tag{15}

$$\dot{V}_3(x_t) = \xi^T(t)\left[^TP_1^T(h_1R_1 + h_2R_2 + hR_3)\Pi_1\right]\xi(t)$$

$$- \int_{t-h_1}^{t} \dot{x}^T(s)R_1\dot{x}(s)ds - \int_{t-h_2}^{t} \dot{x}^T(s)R_2\dot{x}(s)ds - \int_{t-h}^{t} \dot{x}^T(s)R_3\dot{x}(s)ds.$$ \tag{16}
Applying Lemma 1 to estimate the $R_1$-dependent single integral term in equation (16) yields

$$- \int_{t-h_1}^{t} \mathbf{x}^T(s) R_1 \dot{\mathbf{x}}(s) ds = - \int_{t-h_1}^{t-d_1(t)} \mathbf{x}^T(s) R_1 \dot{\mathbf{x}}(s) ds - \int_{t-d_1(t)}^{t} \mathbf{x}^T(s) R_1 \dot{\mathbf{x}}(s) ds$$

\[ \leq \text{Sym} \left\{ \eta_1^T(t) L_1 \chi_1 + \eta_1^T(t) M_1 \chi_2 \right\} + (h_1 - d_1(t)) \eta_1^T(t) \left( L_1 R_1^{-1} L_1^T + \frac{1}{3} M_1 R_1^{-1} M_1^T \right) \eta_1(t) \]

\[ + \text{Sym} \left\{ \eta_1^T(t) L_2 \chi_3 + \eta_1^T(t) M_2 \chi_4 \right\} + d_1(t) \eta_1^T(t) \left( L_2 R_1^{-1} L_2^T + \frac{1}{3} M_2 R_1^{-1} M_2^T \right) \eta_1(t), \]

where $\eta_1(t)$ is selected as

$$\eta_1(t) = \left[ x^T(t) \quad x^T(t - h_1) \quad x^T(t - d_1(t)) \right] \frac{1}{d_1(t)} \int_{t-d_1(t)}^{t} x^T(s) ds \quad \frac{1}{h_1 - d_1(t)} \int_{t-h_1}^{t-d_1(t)} x^T(s) ds \right]^T \xi(t) = e_{g1} \xi(t),$$

and the other vectors are derived based on Lemma 1:

$$\chi_1(t) = x(t - d_1(t)) - x(t - h_1) = \Pi_4 \xi(t),$$

$$\chi_2(t) = x(t - h_1) + x(t - d_1(t)) - \frac{2}{h_1 - d_1(t)} \int_{t-h_1}^{t-d_1(t)} x(s) ds = \Pi_5 \xi(t),$$

$$\chi_3(t) = x(t - d_1(t)) = \Pi_6 \xi(t),$$

$$\chi_4(t) = x(t) + x(t - d_1(t)) - \frac{2}{d_1(t)} \int_{t-d_1(t)}^{t} x(s) ds = \Pi_7 \xi(t),$$

which implies

$$- \int_{t-h_1}^{t} \mathbf{x}^T(s) R_1 \dot{\mathbf{x}}(s) ds \leq \xi^T(t) \left\{ \text{Sym} \left\{ e_{g1}^T L_1 \Pi_4 + e_{g1}^T M_1 \Pi_5 \right\} + \text{Sym} \left\{ e_{g1}^T L_2 \Pi_6 + e_{g1}^T M_2 \Pi_7 \right\} \right\} \xi(t),$$

\[ + (h_1 - d_1(t)) e_{g1}^T \left( L_1 R_1^{-1} L_1^T + \frac{1}{3} M_1 R_1^{-1} M_1^T \right) e_{g1} + d_1(t) e_{g1}^T \left( L_2 R_1^{-1} L_2^T + \frac{1}{3} M_2 R_1^{-1} M_2^T \right) e_{g1} \right\} \xi(t). \]

Similarly, the $R_2$-dependent single integral term in equation (16) can be estimated via Lemma 1:

$$- \int_{t-h_2}^{t} \mathbf{x}^T(s) R_2 \dot{\mathbf{x}}(s) ds = - \int_{t-h_2}^{t-d_2(t)} \mathbf{x}^T(s) R_2 \dot{\mathbf{x}}(s) ds - \int_{t-d_2(t)}^{t} \mathbf{x}^T(s) R_2 \dot{\mathbf{x}}(s) ds \leq \text{Sym} \left\{ \eta_2^T(t) L_3 \chi_5 + \eta_2^T(t) M_3 \chi_6 \right\}$$

\[ + (h_2 - d_2(t)) \eta_2^T(t) \left( L_3 R_2^{-1} L_3^T + \frac{1}{3} M_3 R_2^{-1} M_3^T \right) \eta_2(t) \]

\[ + \text{Sym} \left\{ \eta_2^T(t) L_4 \chi_7 + \eta_2^T(t) M_4 \chi_8 \right\} + d_2(t) \eta_2^T(t) \left( L_4 R_2^{-1} L_4^T + \frac{1}{3} M_4 R_2^{-1} M_4^T \right) \eta_2(t), \]
where \( \eta_2(t) \) is selected as

\[
\eta_2(t) = \begin{bmatrix} x^T(t) & x^T(t - h_2) & x^T(t - d_2(t)) \end{bmatrix} \begin{bmatrix} \frac{1}{d_2(t)} \int_{t-d_2(t)}^{t} x^T(s)ds & \frac{1}{h_2 - d_2(t)} \int_{t-h_2}^{t-d_2(t)} x^T(s)ds \end{bmatrix}^T
\]

where

\[
\xi(t) = e_{g3} \xi(t),
\]

\[
\eta_5 = x(t-d_2(t)) - x(t) = \Pi_{g5}(t),
\]

\[
\eta_6 = x(t-h_2) + x(t-d_2(t)) - \frac{2}{h_2-d_2(t)} \int_{t-h_2}^{t-d_2(t)} x(s)ds = \Pi_{g6}(t),
\]

\[
\eta_7 = x(t) - x(t-d_2(t)) = \Pi_{g7}(t),
\]

\[
\eta_8 = x(t) + x(t-d_2(t)) - \frac{2}{d_2(t)} \int_{t-d_2(t)}^{t} x(s)ds = \Pi_{g8}(t).
\]

Thus, inequality (21) can be rewritten as

\[
- \int_{t-h_2}^{t} \chi^T(s) R_2 \dot{x}(s) ds \leq \xi(t) \left[ \text{Sym} \left\{ e_{g2} L_3 \Pi_5 + e_{g3} M_5 \Pi_9 \right\} + \text{Sym} \left\{ e_{g2} L_4 \Pi_{10} + e_{g3} M_4 \Pi_{11} \right\} + \frac{(h_2 - d_2(t))e_{g2}^T \left( L_3 R_2^{-1} L_4 + \frac{1}{3} M_5 R_2^{-1} M_4 \right) e_{g2} + d_2(t) e_{g2}^T \left( L_4 R_2^{-1} L_4 + \frac{1}{3} M_4 R_2^{-1} M_4 \right) e_{g2}}{d_2(t)} \right] \xi(t).
\]

The \( R_1 \)-dependent single integral term in equation (16) is also calculated with Lemma 1, which yields

\[
- \int_{t-h}^{t} \chi^T(s) R_3 \dot{x}(s) ds = - \int_{t-h}^{t-d(t)} \chi^T(s) R_3 \dot{x}(s) ds - \int_{t-d(t)}^{t} \chi^T(s) R_3 \dot{x}(s) ds
\]

\[
\leq \text{Sym} \left\{ \eta_5^T(t) L_5 \chi_5 + \eta_3^T(t) M_5 \right\} + (h - d(t)) \eta_5^T(t) \left( L_3 R_3^{-1} L_5 + \frac{1}{3} M_5 R_3^{-1} M_5 \right) \eta_5(t)
\]

\[
+ \text{Sym} \left\{ \eta_3^T(t) L_6 \chi_11 + \eta_5^T(t) M_6 \chi_{12} \right\} + d(t) \eta_5^T(t) \left( L_6 R_6^{-1} L_6 + \frac{1}{3} M_6 R_6^{-1} M_6 \right) \eta_5(t),
\]

where \( \eta_3(t) \) is selected as

\[
\eta_3(t) = \begin{bmatrix} x^T(t) & x^T(t - h) & x^T(t - d(t)) \end{bmatrix} \begin{bmatrix} \frac{1}{d(t)} \int_{t-d}^{t} x^T(s)ds & \frac{1}{h-d(t)} \int_{t-h}^{t-d(t)} x^T(s)ds \end{bmatrix}^T
\]

where

\[
\xi(t) = e_{g3} \xi(t),
\]

\[
\eta_9 = x(t-d(t)) - x(t) = \Pi_{g9}(t),
\]

\[
\eta_{10} = x(t-h) + x(t-d(t)) - \frac{2}{h-d(t)} \int_{t-h}^{t-d(t)} x(s)ds = \Pi_{g10}(t),
\]

\[
\eta_{11} = x(t) - x(t-d(t)) = \Pi_{g11}(t),
\]

\[
\eta_{12} = x(t) + x(t-d(t)) - \frac{2}{d(t)} \int_{t-d(t)}^{t} x(s)ds = \Pi_{g12}(t).
\]
Therefore, inequality (25) can be rewritten as

\[
\int_{t-h}^{t} x^T(s) R_3 x(s) \, ds \leq \xi^T(t) \left\{ \text{Sym} \left[ e_{g3}^T L_5 \Pi_{12} + e_{g3}^T M_5 \Pi_{13} \right] + \text{Sym} \left[ e_{g3}^T L_6 \Pi_{14} + e_{g3}^T M_6 \Pi_{15} \right] \right\} + (h-d(t)) e_{g3}^T \left( L_2 R_4^T L_5 + \frac{1}{3} M_2 R_5^T M_5 \right) e_{g3} + d(t) e_{g3}^T \left( L_4 R_4^T L_6 + \frac{1}{3} M_4 R_5^T M_6 \right) e_{g3} \right\} \xi(t).
\]

Combining inequalities (16), (20), (24), and (28), the estimation of $V_5(x_t)$ can be derived as follows:

\[
\dot{V}_5(x_t) \leq \xi^T(t) \left( \Xi_{31} + \Xi_{32}(d_1(t), d_2(t)) \right),
\]

where $\Xi_{31}$ and $\Xi_{32}(d_1(t), d_2(t))$ are defined in equation (8).

The double integral terms in equation (30) can be estimated via Lemma 2, which yields

\[
\int_{t-h}^{t} \chi^T(s) Z \chi(s) \, ds \, d\theta \int_{\theta}^{t} \chi^T(s) Z \chi(s) \, ds \, d\theta \leq -\frac{2}{(h-d(t))} \left( \int_{t-h}^{t} \chi(s) \, ds \right)^T \left( \int_{t-h}^{t} \chi(s) \, ds \right) \int_{\theta}^{t} \chi^T(s) Z \chi(s) \, ds \, d\theta \leq \frac{2}{d(t)} \left( \int_{t-h}^{t} \chi(s) \, ds \right)^T \left( \int_{t-h}^{t} \chi(s) \, ds \right) \int_{\theta}^{t} \chi^T(s) Z \chi(s) \, ds \, d\theta
\]

\[
= -2\Pi_{16}^T Z \Pi_{16} - 2\Pi_{17}^T Z \Pi_{17}.
\]

The single integral terms in equation (30) is estimated via Lemma 3, which yields

\[
\int_{t-h}^{t} \chi^T(s) Z \chi(s) \, ds - h \int_{t-h}^{t} \chi^T(s) Z \chi(s) \, ds 
\]

\[
= -(h \, d(t)) \int_{t-h}^{t} \chi^T(s) Z \chi(s) \, ds - h \int_{t-h}^{t} \chi^T(s) Z \chi(s) \, ds \leq \left( \frac{h}{d(t)} - 1 \right) \xi^T(t) \Pi_{18}^T Z \Pi_{18} \xi(t) - \frac{h}{h-d(t)} \xi^T(t) \Pi_{19}^T Z_a \Pi_{19} \xi(t) - \frac{h}{d(t)} \xi^T(t) \Pi_{18}^T Z_a \Pi_{18} \xi(t)
\]

\[
= \xi^T(t) \Pi_{18}^T Z \Pi_{18} \xi(t) - \frac{h}{h-d(t)} \xi^T(t) \Pi_{19}^T Z_a \Pi_{19} \xi(t) - \frac{h}{d(t)} \xi^T(t) \Pi_{18}^T (Z + Z_a) \Pi_{18} \xi(t).
\]
where \( \bar{Z} \) and \( \bar{Z}_a \) are defined in equation (8), and the \( d(t) \)-dependent terms in equation (32) can be calculated with Lemma 4.

\[
-\frac{h}{d(t)}\Pi_{18}^T(\bar{Z} + \bar{Z}_a)\Pi_{18} - \frac{h}{h - d(t)}\Pi_{19}^T\bar{Z}_a\Pi_{19} = \begin{bmatrix}
\Pi_{18}^T & 0 \\
0 & \Pi_{19}
\end{bmatrix}
\begin{bmatrix}
\frac{h}{d(t)}(\bar{Z} + \bar{Z}_a) \\
\frac{h}{h - d(t)}\bar{Z}_a
\end{bmatrix} \leq -\Pi_{18}^T(\bar{Z} + \bar{Z}_a)\Pi_{18}
- \frac{h - d(t)}{h}\Pi_{18}(\bar{Z} + \bar{Z}_a)\Pi_{18} + \frac{h - d(t)}{h}\Pi_{18}\bar{Z}_a\Pi_{19},
\]

\[
- \frac{h - d(t)}{h}\Pi_{18}(\bar{Z} + \bar{Z}_a)\Pi_{18} + \frac{h - d(t)}{h}\Pi_{18}\bar{Z}_a\Pi_{19} = \frac{d(t)}{h}\text{Sym}\{\Pi_{18}^TS_1\Pi_{19}\}
- \frac{d(t)}{h}\text{Sym}\{\Pi_{18}^TS_1\Pi_{19}\} - \Pi_{18}\bar{Z}_a\Pi_{19} - \frac{d(t)}{h}\Pi_{19}\bar{Z}_a\Pi_{19} + \frac{d(t)}{h}\Pi_{18}^TS_1(\bar{Z} + \bar{Z}_a)^{-1}S_1\Pi_{19}.
\]

\[
(33)
\]

Taking equations (30)–(33) into consideration comprehensively, the upper bound of \( \bar{V}_4(x_t) \) can be obtained as follows:

\[
\bar{V}_4(x_t) \leq \xi^T(t)[\Omega(\xi_3(x_t))]\xi(t).
\]

\[
(34)
\]

Taking equations (13)–(15), (29), and (34) into account, the upper bound of \( \bar{V}(x_t) \) can be obtained as follows:

\[
\bar{V}(x_t) \leq \xi^T(t)[\Sigma(\eta_1(t),\eta_2(t)) + \Omega(\eta_1(t),\eta_2(t))]\xi(t).
\]

\[
(35)
\]

Therefore, the derivative of the proposed LKF is negative definite if inequality \( \Sigma(\eta_1(t),\eta_2(t)) + \Omega(\eta_1(t),\eta_2(t)) < 0 \) holds for all \( \eta_1(t) \in [0,h_1] \) and \( \eta_2(t) \in [0,h_2] \), which is equivalent to the inequalities in equation (7) based on the convex combination method [3].

From what has been mentioned above, the positive definiteness of the matrices declared in Theorem 1 and the inequalities in (7) ensure \( \bar{V}(x_t) \geq \varepsilon_1\|x(t)\| \) and \( \bar{V}(x_t) \leq -\varepsilon_2\|x(t)\|, \) respectively, for any existing sufficient small scalars \( \varepsilon_1, \varepsilon_2 > 0 \), which implies system (1) with two time-varying delay components satisfying (2) is asymptotically stable. This completes the proof. \( \square \)

3.2. Discussion. By constructing a novel LKF and using GFWM-based inequality to estimate its derivative, Theorem 1 is derived with less conservatism and lower computational complexity. Moreover, its superiority can be reflected in the following discussion.

Firstly, among the conditions of Theorem 1, four matrix inequalities included in equation (7) are not linear matrix inequalities (LMIs); hence, LMI Toolbox in MATLAB cannot solve these inequalities directly on account of the existence of the inverse matrices in \( \Omega(d_1(t),d_2(t)) \). Nevertheless, based on Schur complement, the matrix inequalities in equation (7) can be transformed into LMIs. Therefore, the stability problem of system (1) is equivalent to the feasibility checking problem of LMIs.

Secondly, some advanced techniques are applied to derive Theorem 1, which will be conducive to reducing its conservativeness and computational complexity.

(i) The LKF constructed in this paper plays an important role to decrease the computational complexity. It not only contains adequate information about delay components and their upper bounds but also has a simpler form in comparison with the existing works. It is obvious that the nonlinear terms, quadratic single integral terms, and quadratic double integral terms in the LKF constructed in this paper have been simplified compared with the latest works [14, 15], and the useless terms that contribute nothing to reducing the introduced conservatism are not included. The simplification of LKF will undoubtedly greatly reduce the computational complexity of Theorem 1; moreover, it ensures the low conservatism of Theorem 1.

(ii) GFWM-based inequality is used to estimate the single integral term in \( \bar{V}_3(x_t) \), which further decreases the conservatism and the computational complexity of Theorem 1. The superiority of GFWM-based inequality in comparison with FMBII utilized in [14, 15] is verified theoretically in [35]. Moreover, in the proof process of Theorem 1 in this paper, three arbitrary vectors \( \eta_1(t), \eta_2(t), \) and \( \eta_3(t) \) defined in equations (18), (22), and (26) are chosen to estimate the \( R_1 \gamma, R_2 \gamma, \) and \( \gamma \)-dependent single integral terms in \( \bar{V}_3(x_t) \), respectively, which are verified to be the simplest form to minimize the conservatism introduced in equations (20), (24), and (28). In other words, adding additional terms to \( \eta_1(t), \eta_2(t), \) and \( \eta_3(t) \) will not contribute to the reduction of conservatism. Hence, both the conservatism and the computational complexity will be further reduced with the techniques used in estimating the derivative of the LKF.

Last but not the least, both the conservatism and the computational complexity are considered in this paper, while the only objective is to decrease the conservatism in most existing works. The effect of LKF components is fully investigated; thus, an effective and simple LKF constructed in this paper will greatly reduce the decision variables without adversely affecting the conservatism introduced. Moreover, GFWM-based inequality is utilized to deal with three important single integral terms appearing in \( \bar{V}_3(x_t) \), which reduces both the introduced conservatism and decision variables. Therefore, the method proposed in this
where \( \Sigma \) hold:

\[
\begin{align*}
\Sigma_c \left( h_1, h_2 \right) + \Xi_{32} \left( h_1, h_2 \right) &< 0, \\
\Sigma_\varepsilon \left( h_1, 0 \right) + \Xi_{32} \left( h_1, 0 \right) &< 0, \\
\Sigma_\varepsilon \left( h_1, h_2 \right) + \Xi_{32} \left( h_1, h_2 \right) &< 0,
\end{align*}
\]

where \( \Sigma_c (d_1(t), d_2(t)) = \Xi_1 (d_1(t), d_2(t)) + \Xi_2 + \Xi_{31} \), and the related notations are given in equation (8).

4. A Numerical Example

In this section, a typical numerical example is provided to illustrate both the less conservativeness and the lower computational complexity of the derived criteria in comparison with the existing works.

**Example 1.** Consider system (1) with the following parameters:

\[
A = \begin{bmatrix}
-2 & 0 \\
0 & -0.9
\end{bmatrix}, \\
A_d = \begin{bmatrix}
-1 & 0 \\
-1 & -1
\end{bmatrix}.
\]

(37)

Suppose that \( \hat{d}_1(t) \leq 0.1 \) and \( \hat{d}_2(t) \leq 0.8 \), which means \( \mu_1 = 0.1 \) and \( \mu_2 = 0.8 \).

This is a representative number example to analyze the stability of system (1) satisfying condition (2), which is also used in [4, 6–9, 14, 15] to illustrate the conservatism of the derived criteria. As known, the stability criterion obtained with Lyapunov stability theory is usually a sufficient condition. Hence, the calculated upper bounds of delays based on different criteria are less than their analytical value, which represent the conservatism of criteria. In other words, the criterion with bigger calculated values of delays is less conservative.

To verify the validity of the method presented in this paper, Tables 1 and 2 are listed on behalf of two cases. For one case, while the upper bound of \( d_1(t) \) is known, i.e., \( h_1 = 0.3, 0.4, 0.5 \), the calculated upper bound of \( d_1(t) \) obtained in this paper and other existing works is listed in Table 1. For another, while the upper bound of \( d_1(t) \) is given, i.e., \( h_1 = 1, 1.1, 1.2, 1.5 \), the calculated upper bound of \( d_1(t) \)

### Table 1: The calculated upper bounds of \( d_1(t) \) for different \( h_2 \).  

| Methods         | \( h_2 \) | \( h_2 \) | \( h_2 \) | NoVs          |
|-----------------|-----------|-----------|-----------|---------------|
| Theorem 1 [4]   | 1.324     | 1.039     | 0.806     | 12.5\( n^2 \) + 4.5\( n \) |
| Theorem 1 [6]   | 1.453     | 1.214     | 1.021     | 19.5\( n^2 \) + 3.5\( n \) |
| Theorem 1 [7]   | 1.572     | 1.472     | 1.372     | 32.5\( n^2 \) + 8.5\( n \) |
| Proposition 2 [8] | 1.573   | 1.473     | 1.373     | 7\( n^2 \) + 5\( n \) |
| Theorem 1 [9]   | 1.707     | 1.637     | 1.557     | 8.5\( n^2 \) + 3.5\( n \) |
| Theorem 1 [14]  | 1.8804    | 1.7798    | 1.6759    | 189\( n^2 \) + 30\( n \) |
| Theorem 1 [15]  | 1.9137    | 1.8137    | 1.7136    | 195.5\( n^2 \) + 30.5\( n \) |
| Theorem 1       | 1.9511    | 1.8511    | 1.7511    | 74.5\( n^2 \) + 5.5\( n \) |
| Corollary 1     | 1.9432    | 1.8432    | 1.7432    | 65.5\( n^2 \) + 4.5\( n \) |

### Table 2: The calculated upper bounds of \( d_2(t) \) for different \( h_1 \).  

| Methods         | \( h_1 \) | \( h_1 \) | \( h_1 \) | NoVs          |
|-----------------|-----------|-----------|-----------|---------------|
| Theorem 1 [4]   | 0.415     | 0.376     | 0.340     | 0.248         | 12.5\( n^2 \) + 4.5\( n \) |
| Theorem 1 [6]   | 0.512     | 0.457     | 0.406     | 0.283         | 19.5\( n^2 \) + 3.5\( n \) |
| Theorem 1 [7]   | 0.872     | 0.722     | 0.672     | 0.371         | 32.5\( n^2 \) + 8.5\( n \) |
| Theorem 1 [9]   | 0.989     | 0.914     | 0.836     | 0.564         | 8.5\( n^2 \) + 3.5\( n \) |
| Theorem 1 [14]  | 0.9999    | 1.0770    | 0.9725    | 0.6807        | 189\( n^2 \) + 30\( n \) |
| Theorem 1 [15]  | 1.2136    | 1.1136    | 1.0137    | 0.7173        | 195.5\( n^2 \) + 30.5\( n \) |
| Theorem 1       | 1.2511    | 1.1511    | 1.0511    | 0.7511        | 74.5\( n^2 \) + 5.5\( n \) |
| Corollary 1     | 1.2432    | 1.1432    | 1.0432    | 0.7432        | 65.5\( n^2 \) + 4.5\( n \) |
obtained by Theorem 1 and Corollary 1 in this paper and other existing works is listed in Table 2. Moreover, the number of decision variables (NoVs) of the stability criteria is also presented in tables to demonstrate the superiority of the proposed method.

Based on the comparison of the calculated upper bounds and NoVs for different cases, it is obvious that the method used in this paper contributes to the stability analysis of system (1) on account of not only the further decrease of the conservatism but also the significant reduction of the decision variables. Compared with the existing works, for instance, Theorem 1 in [15], the derived criteria in this paper have greatly reduced the computational complexity while being less conservative, which is also the main contribution of this paper.

Moreover, based on the comparison of the calculated upper bounds and NoVs from Theorem 1 and Corollary 1, the triple integral term constructed in \( V_3(x_i) \) is verified beneficial to decrease the conservativeness. Compared with the existing works, Corollary 1 is less conservative without the triple integral terms introduced in LKF, which further illustrate the effect of the method used in this paper.

5. Conclusions

This paper investigates the stability of linear system with two additive time-varying delays. The main contribution of this paper is that the derived results have greatly reduced the computational complexity while being less conservative with comparison with the existing works. To achieve the goal, an effective and simple LKF is established in this paper, which is verified to be a great contribution to reduce the computational complexity. Additionally, GFWM-based inequality together with some other advanced techniques is utilized in this paper to calculate the derivative of the proposed LKF, which is verified to be significant to further decrease the conservatism of the derived criteria. A representative numerical example is presented to illustrate the validity and superiority of the method used in this paper.

Data Availability

The data used to support the findings of the study are included within the article.

Conflicts of Interest

The authors declare that there are no conflicts of interest regarding the publication of this paper.

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