Chapter 9:

A New Optimal Orthogonal Additive Randomized Response Model Based on Moments Ratios of Scrambling Variable

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Additional information is available at the end of the chapter

Introduction

The Randomized response (RR) technique was first presented by Warner (1965) mainly to cut down the probability of (i) reduced response rate and (ii) inflated response bias experienced in direct or open survey relating to sensitive issues. Some recent involvement to randomized response sampling is given by Fox and Tracy (1986), Singh and Mathur (2004, 2005), Gjestvang and Singh (2006), Singh and Tarray (2013, 2014, 2015, 2016) and Tarray and Singh (2016, 2017, 2018). We below give the description of the model due to Singh (2010):

Singh (2010) Additive Model

Let there be k scrambling variables denoted by $S_j, j = 1, 2, \ldots, k$ whose mean $\theta_j$ (i.e. $E(S_j) = \theta_j$) and variance $\gamma_j^2$ (i.e. $V(S_j) = \gamma_j^2$) are known. In Singh’s (2010) proposed optimal new orthogonal additive model named as (POONAM), each respondent selected in the sample is requested to rotate a spinner, as shown in Fig. 9.1, in which the proportion of the k shaded areas, say $P_1, P_2, \ldots, P_k$ are orthogonal to the means of the k scrambling variables, say $\theta_1, \theta_2, \ldots, \theta_k$ such that:

$$\sum_{j=1}^{k} P_j \theta_j = 0 \quad (9.1)$$

and

$$\sum_{j=1}^{k} P_j = 1 \quad (9.2)$$
Figure 9.1: Spinner for POONAM (Singh (2010))

Now if the pointer stops in the $j$th shaded area, then the $i$th respondent with real value of the sensitive variable, say $Y_i$, is requested report the scrambled response $Z_i$ as:

$$Z_i = Y_i + S_j$$  \hspace{1cm} (9.3)

Assuming that the sample of size $n$ is drawn from the population using simple random sampling with replacement (SRSWR). Singh (2010) suggested an unbiased estimator of the population mean $\mu_Y$ as

$$\hat{\mu}_Y = \frac{1}{n} \sum_{j=1}^{n} Z_j$$  \hspace{1cm} (9.4)

The variance of $\hat{\mu}_Y$ is given by

$$V(\hat{\mu}_Y) = \frac{1}{n} \left[ \sigma^2_y + \sum_{j=1}^{k} P_j (\theta^2_j + \gamma^2_j) \right]$$  \hspace{1cm} (9.5)

The proposed procedure

It is to be noted that the mean $\theta_j$ and variance $\gamma^2_j$ of the $j$th scrambling variable $S_j$ ($j=1,2,\ldots,k$) are known. Author has to propose a new additive model based on standardized scrambling variable $S^*_j = \left( \frac{S^2_j}{\theta_j(1+C^2_j)} \right)$, $j = 1,2,\ldots,k$. 
As demonstrated in Fig. 9.2, in which the proportion of the k shaded areas, say $P_1, P_2, \ldots P_k$ are orthogonal to the means of the k scrambling variables ($S_j^*, \forall j = 1,2,\ldots,k$), say $\theta_1, \theta_2,\ldots, \theta_k$ such that:

$$\sum_{j=1}^{k} P_j \theta_j = 0 \quad (9.6)$$

and

$$\sum_{j=1}^{k} P_j = 1 \quad (9.7)$$

Now if the pointer stops in the $j^{th}$ shaded area, then the $i^{th}$ respondent with real value of the sensitive variable, say $Y_i$, is requested report the scrambled response $Z_i^*$ as:

$$Z_i^* = Y_i + S_j^* \quad (9.8)$$

we prove the following theorems.

Figure 9.2: Spinner for proposed procedure.
Theorem 9.1

\[ \hat{\mu}_{ST} = \frac{1}{n} \sum_{i=1}^{n} Z_i^* \]  

(9.9)

Proof

Let \( E_1 \) and \( E_2 \) denote the expectations, then we have

\[ E(\hat{\mu}_{ST}) = E_1 E_2 \left[ \frac{1}{n} \sum_{i=1}^{n} Z_i^* \right] \]

\[ = E_1 \left[ \frac{1}{n} \sum_{i=1}^{n} E_2(Z_i^*) \right] \]

\[ = E_1 \left[ \frac{1}{n} \sum_{i=1}^{n} \left\{ Y_i \sum_{j=1}^{k} P_j + \sum_{j=1}^{k} P_j E_2(S_j^*) \right\} \right] \]

\[ = E_1 \left[ \frac{1}{n} \sum_{i=1}^{n} Y_i \right] = \mu_Y \]

, since \( \sum_{j=1}^{k} P_j = 1 \) and \( E_2(S_j^*) = \theta_j \),

which proves the theorem.

Theorem 9.2

\[ V(\hat{\mu}_{ST}) = \frac{1}{n} \left[ \sigma_Y^2 + \frac{k}{\sum_{j=1}^{k} (1 + C_j^2)^2} \right] \]  

(9.10)

where

\[ A_j = \left[ \beta_2(S_j) C_j^4 + 4 C_j^2 G_1(S_j) + 6 C_j^2 + 1 \right] \]

\[ \beta_2(S_j) = \frac{\mu_4(S_j)}{\gamma_j^4} \]

\[ G_1(S_j) = \frac{\mu_3(S_j)}{\gamma_j^3} \]

is the Fisher’s measure of skewness , \( \mu_3(S_j) \) and \( \mu_4(S_j) \) are third and fourth central moments of the scrambling variable \( S_j \).

Proof

\[ V(\hat{\mu}_Y) = E_1 V_2(\hat{\mu}_Y) + V_1 E_2(\hat{\mu}_Y) \]

\[ = E_1 \left[ V_2 \left( \frac{1}{n} \sum_{i=1}^{n} Z_i^* \right) \right] + V_1 \left[ E_2 \left( \frac{1}{n} \sum_{i=1}^{n} Z_i^* \right) \right] \]

\[ = E_1 \left[ \frac{1}{n^2} \sum_{i=1}^{n} V_2(Z_i^*) \right] + V_1 \left[ \frac{1}{n} \sum_{i=1}^{n} E_2(Z_i^*) \right] \]
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\[
= \frac{1}{n} \sum_{j=1}^{k} \frac{P_j A_j}{(1 + C_j^2)^2} + \frac{\sigma_y^2}{n}
\]

\[
= \frac{1}{n} \left[ \sigma_y^2 + \sum_{j=1}^{k} \frac{P_j A_j}{(1 + C_j^2)^2} \right]
\]

Note that:

\[
V_2(Z_i^*) = E_2(Z_i^{*2}) - (E_2(Z_i^*))^2
\]

\[
E_2(Z_i^{*2}) = E_2(Y_i + S_j^*)^2 = E_2[Y_i^2 + S_j^{*2} + 2Y_i S_j^*]
\]

\[
= Y_i^2 \sum_{j=1}^{k} P_j + \sum_{j=1}^{k} P_j E_2(S_j^{*2}) + 2Y_i \sum_{j=1}^{k} P_j E_2(S_j^*)
\]

\[
= Y_i^2 + \sum_{j=1}^{k} P_j E_2(S_j^{*2}) + 2Y_i \sum_{j=1}^{k} P_j \theta_j
\]

\[
= Y_i^2 + \sum_{j=1}^{k} P_j E_2(S_j^{*2}),
\]

Since \( \sum_{j=1}^{k} P_j \theta_j = 0, \)

and \( E_2(S_j^{*2}) = E_2 \left\{ \frac{S_j^4}{\theta_j^2 (1 + C_j^2)^2} \right\} \)

\[
= \frac{1}{\theta_j^2 (1 + C_j^2)^2} E_2(S_j^4)
\]

\[
= \frac{1}{\theta_j^2 (1 + C_j^2)^2} \left\{ (S_j - \theta_j)^4 + 6\theta_j^2 (S_j - \theta_j)^2 + 4\theta_j (S_j - \theta_j)^3 + 4 \theta_j^2 (S_j - \theta_j) \right\}
\]

\[
= \frac{1}{\theta_j^2 (1 + C_j^2)^2} \left\{ \mu_4(S_j) + 4 \theta_j \mu_3(S_j) + 6 \theta_j^2 \gamma_j^2 + \theta_j^4 \right\}
\]
\[\mu_4(S_j) + 4\theta_j \mu_3(S_j) + 6\theta_j^2 r_j^2 + \theta_j^4\]

\[= \frac{\theta_j^4 A_j}{\theta_j^2 (1 + C_j^2)^2} = \frac{\theta_j^2 A_j}{(1 + C_j^2)^2}\]

Thus \(E_2(Z_i^2) = Y_i^2 + \sum_{j=1}^{k} \frac{P_j A_j}{(1 + C_j^2)^2}\)

Therefore \(V_2(Z_i^2) = Y_i^2 + \sum_{j=1}^{k} \frac{P_j A_j}{(1 + C_j^2)^2} - Y_i^2\)

\[= \sum_{j=1}^{k} \frac{P_j A_j}{(1 + C_j^2)^2}\]

**Efficiency Comparison**

From (9.5) and (9.4), we have

\[
\text{V}(\hat{\mu}_{ST}) < \text{V}(\hat{\mu}_Y) \text{ if } \frac{1}{n} \left[ \sigma_y^2 + \sum_{j=1}^{k} \frac{P_j A_j}{(1 + C_j^2)^2} \right] < \frac{1}{n} \left[ \sigma_y^2 + \sum_{j=1}^{k} P_j \theta_j^2 (1 + C_j^2) \right] \]

i.e. if \(\sum_{j=1}^{k} \frac{P_j A_j}{(1 + C_j^2)^2} < \sum_{j=1}^{k} P_j \theta_j^2 (1 + C_j^2)\)

i.e. if \(\sum_{j=1}^{k} P_j \left[ \frac{A_j}{(1 + C_j^2)^2} - \theta_j^2 (1 + C_j^2) \right] < 0\)

i.e. if \(\frac{A_j}{(1 + C_j^2)^2} < \theta_j^2 (1 + C_j^2) \quad \forall \ j = 1, 2, \ldots, k,\)

i.e. if \(A_j < \theta_j^2 (1 + C_j^2)^3 \quad \forall \ j = 1, 2, \ldots, k,\)

i.e. if \(\theta_j^2 > \frac{A_j^2}{(1 + C_j^2)^3} \quad \forall \ j = 1, 2, \ldots, k.\) (9.11)
In case the scrambling variable $S_j$ follows a normal distribution (i.e. $S_j \sim N(\theta_j, \gamma_j^2)$, \( j = 1,2,\ldots,k \)), then $A_i$ reduces to:

$$A_i^* = \left\{ 1 + 3C_j^2 (2 + C_j^2) \right\}$$  \hspace{1cm} (9.12)

Thus the condition (9.1) reduces to:

$$\theta_j^2 > \frac{(1 + 6C_j^2 + 3C_j^4)}{(1 + C_j^2)^3}$$  \hspace{1cm} (9.13)

The condition (9.3) clearly indicates that \( \left\{ \theta_j^2 > \frac{(1 + 6C_j^2 + 3C_j^4)}{(1 + C_j^2)^3}, \forall j = 1,2,\ldots,k \right\} \), then the proposed model is always better.

Further, suppose $S_j \sim N(0, \gamma_j^2)$, \( \forall j = 1,2,\ldots,k \), \( \theta_j = 0 \) and \( \theta_j = 0 \) \( \forall j = 1,2,\ldots,k \), then the variance expression in (9.5) and (9.4) respectively reduce to:

$$V(\hat{\mu}_Y) = \frac{1}{n} \left[ \sigma_y^2 + \sum_{j=1}^{k} P_j \gamma_j^2 \right]$$  \hspace{1cm} (9.14)

and

$$V(\hat{\mu}_{ST}) = \frac{1}{n} \left[ \sigma_y^2 + 3 \right]$$  \hspace{1cm} (9.15)

From (9.4) and (9.5) we have

$$V(\hat{\mu}_{ST}) - V(\hat{\mu}_Y) = \frac{1}{n} \left( \sum_{j=1}^{k} P_j \gamma_j^2 - 3 \right)$$

$$= \frac{1}{n} \sum_{j=1}^{k} P_j (\gamma_j^2 - 3)$$  \hspace{1cm} (9.16)

which is always positive if

$$ (\gamma_j^2 - 3) > 0 \hspace{1cm} \forall j = 1,2,\ldots,k$$

i.e. if \( \gamma_j^2 > 3 \hspace{1cm} \forall j = 1,2,\ldots,k \) \hspace{1cm} (9.17)

Thus when $S_i \sim N(0, \gamma_i^2)$, \( \forall j = 1,2,\ldots,k \), $\hat{\mu}_{ST}$ is more efficient as long as the condition (9.7) is satisfied.
In case $S_j$ follows a normal distribution (i.e. $S_j \sim N(\theta_j, \gamma_j^2)$, $\forall j = 1, 2, \ldots, k$), PRE of $\hat{\mu}_{ST}$ with $\hat{\mu}_Y$ by using the formula:

$$\text{PRE}(\hat{\mu}_{ST}, \hat{\mu}_Y) = \frac{\sigma_Y^2 + \sum_{j=1}^{k} P_j (\theta_j^2 + \gamma_j^2)}{\sigma_Y^2 + \sum_{j=1}^{k} \frac{P_j A_j^*}{(1 + C_j^2)^2}} \times 100$$

(9.18)

where $A_j^*$ is given in (9.2).

Suppose $\gamma=40$, $\gamma_1=30$, $\gamma_2=40$, $\gamma_3=20$, $\gamma_4=10$, $P_1=0.02$, $P_2=0.05$, $P_3=0.06$, $P_4=0.87$ with $k = 4$. $\sigma_Y^2, \theta_1, \theta_2, \theta_3$ and $\theta_4$ as listed in Table 9.1.

**Table 9.1: $\text{PRE}(\hat{\mu}_{ST}, \hat{\mu}_Y)$**

| $\sigma_Y^2$ | $\theta_1$ | $\theta_2$ | $\theta_3$ | $\theta_4$ | PRE   |
|--------------|-------------|-------------|-------------|-------------|-------|
| 25           | 300         | 200         | 100         | -25.20      | 18523.16 |
|              | 800         | 700         | 600         | -100.00     | 242264.29 |
|              | 1300        | 1200        | 1100        | -174.70     | 732808.85 |
|              | 1800        | 1700        | 1600        | -249.40     | 1490172.55 |
| 125          | 300         | 200         | 100         | -25.20      | 4130.07  |
|              | 800         | 700         | 600         | -100.00     | 53073.44 |
|              | 1300        | 1200        | 1100        | -174.70     | 160380.06 |
|              | 1800        | 1700        | 1600        | -249.40     | 326053.37 |
| 225          | 300         | 200         | 100         | -25.20      | 2362.49  |
|              | 800         | 700         | 600         | -100.00     | 29839.47 |
|              | 1300        | 1200        | 1100        | -174.70     | 90081.79 |
|              | 1800        | 1700        | 1600        | -249.40     | 183091.37 |
| 325          | 300         | 200         | 100         | -25.20      | 1672.71  |
|              | 800         | 700         | 600         | -100.00     | 20772.56 |
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| 425 | 1300 | 1200 | 1100 | -174.70 | 62648.32 |
|-----|------|------|------|---------|----------|
|     | 1800 | 1700 | 1600 | -249.40 | 127301.32 |

| 525 | 300  | 200  | 100  | -25.20  | 1305.25  |
|-----|------|------|------|---------|----------|
|     | 800  | 700  | 600  | -100.00 | 15942.52 |
|     | 1300 | 1200 | 1100 | -174.70 | 48034.22 |
|     | 1800 | 1700 | 1600 | -249.40 | 97581.38 |

| 625 | 300  | 200  | 100  | -25.20  | 1076.99  |
|-----|------|------|------|---------|----------|
|     | 800  | 700  | 600  | -100.00 | 12942.05 |
|     | 1300 | 1200 | 1100 | -174.70 | 38955.77 |
|     | 1800 | 1700 | 1600 | -249.40 | 79119.00 |

| 725 | 300  | 200  | 100  | -25.20  | 921.41   |
|-----|------|------|------|---------|----------|
|     | 800  | 700  | 600  | -100.00 | 10897.13 |
|     | 1300 | 1200 | 1100 | -174.70 | 32768.55 |
|     | 1800 | 1700 | 1600 | -249.40 | 66536.36 |

| 825 | 300  | 200  | 100  | -25.20  | 808.58   |
|-----|------|------|------|---------|----------|
|     | 800  | 700  | 600  | -100.00 | 9414.01  |
|     | 1300 | 1200 | 1100 | -174.70 | 28281.11 |
|     | 1800 | 1700 | 1600 | -249.40 | 57410.48 |

| 925 | 300  | 200  | 100  | -25.20  | 723.01   |
|-----|------|------|------|---------|----------|
|     | 800  | 700  | 600  | -100.00 | 8289.13  |
|     | 1300 | 1200 | 1100 | -174.70 | 24877.59 |
|     | 1800 | 1700 | 1600 | -249.40 | 50488.93 |
From Table 9.1 \( \text{PRE}(\hat{\mu}_ST, \hat{\mu}_Y) \) are greater than 100. It shows \( \hat{\mu}_ST \) is more efficient than \( \hat{\mu}_Y \) with substantial gain. Thus, the estimator \( \hat{\mu}_ST \) over \( \hat{\mu}_Y \) is recommended.

**Table 9.2: PRE of \( \hat{\mu}_ST \) over \( \hat{\mu}_Y \).**

| \( \sigma^2_Y \) | 25  | 125 | 225 | 325 | 425 | 525 | 625 | 725 | 825 |
|-------------------|-----|-----|-----|-----|-----|-----|-----|-----|-----|
| PRE               | 835.71 | 260.94 | 190.35 | 162.80 | 148.13 | 139.02 | 132.80 | 128.30 | 124.88 |

The minimum values from 9.2 is observed as 124.88 and maximum 835.71 with a median of 148.13.

Table 9.2 PRE remains higher if the value of \( \sigma^2_Y \) is small. In that case the value of \( \sigma^2_Y \) will be around 0.5 to 5.0 (see Singh (2010), p. 67). It is observed that the PRE value decreases from 5985.71 to 2675.00 as the value of \( \sigma^2_Y \) increases from 0.5 to 5.0.

Case \( k = 2 \) and the \( \text{PRE}(\hat{\mu}_ST, \hat{\mu}_Y) \) for different parameters. Results are shown in Table 9.3.

Thus, the estimator \( \hat{\mu}_ST \) over the estimator \( \hat{\mu}_Y \) is recommended.

**Table 9.3: PRE of the estimator \( \hat{\mu}_ST \) over the estimator \( \hat{\mu}_Y \) with \( k = 2 \).**

| \( P_1 \) | \( \theta_1 \) | \( \theta_2 \) | \( \sigma^2_Y \) | PRE          |
|----------|---------------|---------------|----------------|--------------|
| 0.2      | 1300          | -325.0        | 25             | 1514232.14   |
|          |               |               | 125            | 331316.41    |
|          |               |               | 225            | 186046.05    |
|          |               |               | 325            | 129355.18    |
|          |               |               | 425            | 99155.37     |
|          |               |               | 525            | 80394.89     |
|          |               |               | 625            | 67609.08     |
|          |               |               | 725            | 58335.85     |
|          |               |               | 825            | 51302.54     |
| 0.4      | 300           | -200.0        | 25             | 219089.29    |
|          |               |               | 125            | 48003.91     |
|          |               |               | 225            | 26993.42     |
|          |               |               | 325            | 18794.21     |
|          |               |               | 425            | 14426.40     |
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|       |       |       |       |
|-------|-------|-------|-------|
| 0.4   | 800   | -533.3|       |
| 525   | 11713.07 |       |
| 625   | 9863.85 |       |
| 725   | 8522.66 |       |
| 825   | 7505.43 |       |
| 25    | 1528536.91|       |
| 125   | 334445.57 |       |
| 225   | 187802.78 |       |
| 325   | 130576.32 |       |
| 425   | 100091.20 |       |
| 525   | 81153.47 |       |
| 625   | 68246.87 |       |
| 725   | 58886.03 |       |
| 825   | 51786.27 |       |
| 0.8   | 300   | -1200.0|       |
| 25    | 1289517.86 |   |
| 125   | 282160.16 |   |
| 225   | 158449.56 |   |
| 325   | 110172.26 |   |
| 425   | 84454.44  |   |
| 525   | 68478.22  |   |
| 625   | 57589.97  |   |
| 725   | 49692.99  |   |
| 825   | 43703.50  |   |

Conclusion

This paper elucidates amelioration over the Singh’s (2010) randomized response model. We have advocated the optimal orthogonal additive randomized response model. The proposed model is found to be more resourceful both theoretically as well as numerically than the additive randomized response model studied by Singh (2010). Thus, the suggested RR procedure is therefore indorsed for its use in practice as an alternative to Singh’s (2010) model.

Author’s Detail

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