A DoF-Optimal Scheme for the two-user X-channel with Synergistic Alternating CSIT

Ahmed Wagdy†, Amr El-Keyi†, Tamer Khattab⋆, Mohammed Nafie†

†Wireless Intelligent Networks Center (WINC), Nile University, Smart Village, Egypt.
⋆Electrical Engineering, Qatar University, Doha, Qatar.

Abstract—In this paper, the degrees of freedom (DoF) of the two-user single input single output (SISO) X-channel are investigated. Three cases are considered for the availability of channel state information at the transmitters (CSIT); perfect, delayed, and no-CSIT. A new achievable scheme is proposed to elucidate the potency of interference creation-resurrection (IRC) when the available CSIT alternates between these three cases. For some patterns of alternating CSIT, the proposed scheme achieves 4/3 DoF, and hence, coincides with the information theoretic upper bound on the DoF of the two-user X-channel with perfect and instantaneous CSIT. The CSIT alternation patterns are investigated where the patterns that provide extraordinary synergistic gain and dissociative ones are identified.

Index Terms—Interference alignment, degrees of freedom, X-channel, alternating CSIT, interference creation-resurrection.

I. INTRODUCTION

Breaking the interference barrier is an important step towards unleashing the capacity of wireless networks to meet future demands. Several classical interference management approaches have been utilized such as successive interference cancellation, treating the interference as noise, and orthogonalizing the channel access. Interference alignment (IA) is an interference management technique that refers to creating a correlation or an overlap between the interference signals at the receiver in order to minimize dimensions of the interference subspace and maximize the desired signal space dimensions [1]–[3]. Global, perfect, and instantaneous channel state information at the transmitters (CSIT) is an essential component for interference alignment in pioneering works [2]–[5]. On the other hand, in the complete absence of CSIT, an overly pessimistic assumption, the potential of IA fades and the degrees of freedom (DoF) of many networks collapse to what is achievable by time-division multiplexing [6].

Contrary to the popular belief that in fast fading environments delayed CSIT is vain, Maddah Ali and Tse [7] proved that completely outdated CSIT can still be useful even if the channel states are completely independent. Various interesting extensions of [7] have been considered, e.g., the 3-user interference channel [8] and the two-user MISO broadcast channel with imperfect instantaneous CSIT and perfect delayed CSIT [9].

In another research direction, Tandon et al. in [10] formalized an interesting model for the availability of the CSIT, called alternating CSIT, which provides practical standpoints. In this model, the authors allow the availability of CSIT to vary over time which is a more practical assumption and convenient to the nature of wireless networks. They proved that alternating CSIT could be beneficial. For example, in the case of a two-user MISO broadcast channel with perfect CSIT for one user and no-CSIT for the other user, the authors conjectured that the optimal DoF with fixed CSIT is only one while in the alternating CSIT setting 3/2 DoF can be achieved.

Earlier research work on the DoF of the two-user X-channel have determined that the upper bound for DoF of two-user single-input single-output (SISO) X-channel is 4/3 [2], [4], [5]. This upper bound is achievable with global, perfect, and instantaneous CSIT and when the channel coefficients are time varying. The authors of [8] showed that even in a fast fading environment and for interference networks consisting of distributed transmitters and receivers, delayed CSIT could be beneficial and have a great impact on increasing the DoF. They proved that for the two-user SISO X-channel, 8/7 DoF is achievable with delayed CSIT. New results have been demonstrated in [11] where the two user SISO X-channel with delayed CSIT could achieve 6/5 DoF.

In this work, we consider the two-user SISO X-channel with alternating CSIT. The main question we ask is whether the synergistic alternation in the availability of CSIT is beneficial in this channel as it is in collocated transmitters networks. We answer this question in an affirmative way by presenting optimal scheme that exploits the synergistic alternation of the CSIT to achieve the upper bound on the DoF of the two-user SISO X-channel.

The second question we ask is whether all alteration patterns have synergistic benefits. We negatively answer this question and show that there exists some certain alteration patterns in which some channel knowledge availability states can not work together in a cooperative way but they work individually and the corresponding DoF dwindle to the sum of their individual DoF. Furthermore we find the synergistic alternation patterns and dissociative ones.

II. SYSTEM MODEL

A two-user SISO X-channel is considered in which two transmitters $T_1$ and $T_2$ transmit four independent messages
for a given transmission power. Let $h_{ij}(t)$ be the channel coefficient from $T_j$ to $R_i$ at time slot $t$. In (1), $h_{ij}(t)$ is the channel coefficient from $T_j$ to $R_i$ and all channel coefficients are independent identically distributed and drawn from continuous distribution. Let $r_{ij}(P)$ denote the achievable rate of $W_{ij}$ for a given transmission power $P$ where $r_{ij}(P) = \frac{\log_2(1 + h_{ij}(t)N(t))}{n}$ and $n$ is the number of channel uses. We are interested in the DoF region $\mathcal{D}$, defined as the set of all achievable tuples $(d_{11}, d_{12}, d_{21}, d_{22})$, where $d_{ij} = \lim_{P \to \infty} \frac{r_{ij}(P)}{\log_2(P)}$ is the DoF for message $W_{ij}$. The total DoF of the network is defined as

$$\text{DoF} = \max_{(d_{11}, d_{12}, d_{21}, d_{22}) \in \mathcal{D}} d_{11} + d_{12} + d_{21} + d_{22} \quad (2)$$

We assume that the receivers have perfect global channel state information. Furthermore, we consider three different states of the availability of CSIT identified by:

1. Perfect CSIT (P): identifies the state of CSIT in which CSIT is available to the transmitters instantaneously and without error.
2. Delayed CSIT (D): identifies the state of CSIT in which CSIT is available to the transmitters with some delay ≥ one time slot and without error.
3. No CSIT (N): identifies the state of CSIT in which CSIT is not available to the transmitters at all.

The state of CSIT availability of the channels to the $i$th receiver is denoted by $S_i$; where, $S_i \in \{P,D,N\}$. In addition, let $S_{12}$ denote the state of CSIT availability for the channels to the first and second receivers, respectively. Therefore, $S_{12} \in \{PP,PD,PN,PD,DD,DP,PN,ND,NN\}$. For example, $S_{12} = PN$ refers to the case where $T_1$ has perfect knowledge of $h_{11}$ (and no information about $h_{21}$) and $T_2$ has perfect knowledge of $h_{12}$ (and no information about $h_{22}$).

We denote the CSIT availability of the channels to the $i$th receiver over three time slots by $S_{12}^T = (x, y, z)$ where $x, y, z \in S_i$ and $x, y, z$ denote the availability of the CSIT in the first, second, and third time slots respectively. Similarly, we denote the availability of CSIT for the channels to the first and second receivers in three time slots “CSIT pattern” denoted by $S_{12}^T = (X, Y, Z)$ where $X, Y, Z \in S_{ij}$.

III. PROPOSED ACHIEVABLE SCHEME

Motivated by the fact that multiuser networks with time varying channels, the variation in the availability of CSIT or the fluctuation in state of CSIT across different links is inevitable, we extend this verity modelled for MISO broadcast channel in [10] to the two-user SISO X-channel. Form [2], [4, 5], it is known that for the two-user SISO X-channel the DoF of the network are bounded by 4/3. This upper bound is achievable over 3-symbol channel extension if the channel coefficients vary over time and each transmitter has global and constantly perfect CSIT over the 3 time slots.

In this section, we present three illustrative examples for the proposed achievable scheme in three different patterns of CSIT availability. In all these cases, we show that 4/3 DoF is achievable by sending 4 different data symbols; 2 for each receiver over three time slots. The basic idea behind the proposed achievable scheme is to resurrect the interference formerly created, hence, interference creation-resurrection (ICR) strikes and interference alignment arises. Inspired by the MAT algorithm in [7], the proposed achievable scheme is performed in two phases over three time slots.

The first phase is associated with the delayed CSIT state where the transmitters transmit their messages. As a result, the receivers get linear combinations of their desired messages in addition to interference. This phase is called “interference creation” phase. On the other hand, the second phase is associated with the perfect CSIT state and is called the “interference resurrection” phase. In this phase, the transmitters reconstruct the old interference by exploiting the delayed CSIT received in phase one and the perfect CSIT in the second phase. Hence, after three time slots, each receiver has two different linear combinations of its desired messages and only one interference term received twice. Noteworthy, in some cases the two phases can overlap over the 3 time slots.

Let $u_1$ and $u_2$ be two independent symbols intended to $R_1$ transmitted from $T_1$ and $T_2$, respectively. Also, let $v_1$ and $v_2$ be two independent symbols intended to $R_2$ from $T_1$ and $T_2$, respectively. In the next subsections, we show that we can reliably transmit these symbols to their target destinations in 3 time slots in three different cases of alternating CSIT.

A. Scheme 1: Combined delayed and distributed perfect CSIT

As an illustrative example of this case, let us consider a 2-user SISO X-channel with alternating CSIT pattern given by (DD, PN, NP) over three time slots. Here, we have combined delayed CSIT in the first time slot and distributed perfect CSIT over the last two time slots. Consequently, the proposed scheme is performed in two separate phases as follows.

**Phase one:** In this phase, the two transmitters greedily transmit all data symbols, i.e., $X_1(1) = u_1 + v_1$ and $X_2(1) = u_2 + v_2$. As a result, the received signals are given as

$$Y_1(1) = h_{11}(1)u_1 + h_{12}(1)u_2 + h_{11}(1)v_1 + h_{12}(1)v_2$$

$$\equiv L_1^1(u_1, u_2) + I_1^1(v_1, v_2) \quad (3)$$

$$Y_2(1) = h_{21}(1)u_1 + h_{22}(1)u_2 + h_{21}(1)v_1 + h_{22}(1)v_2$$

$$\equiv I_2^1(u_1, u_2) + L_2^1(v_1, v_2) \quad (4)$$

where $L_j^i(x_1, x_2)$ denotes the $j$th linear combination of the two messages $x_1$ and $x_2$ that are intended for receiver $R_i$ and $I_l(z_1, z_2)$ denotes the interference term for receiver $R_i$ which
is a function of the messages $z_1$ and $z_2$ that are not intended for this receiver.

**Phase two:** This phase consists of two time slots where in each time slot the transmitted signals are designed such that the interference is resurrected at one receiver while the second receiver receives a new linear combination of its desired messages. Note that now the transmitters are aware of the CSIT of the previous time slot, i.e., $T_1$ knows $h_{11}(1)$ and $h_{21}(1)$ while $T_2$ knows $h_{12}(1)$ and $h_{22}(1)$. Also, at $t = 2$, the channels to the first receiver are known perfectly and instantaneously at the two transmitters, i.e., $T_1$ knows $h_{11}(2)$ and $T_2$ knows $h_{12}(2)$. As a result, the first time slot in this phase is dedicated to resurrecting the interference $I_1(v_1, v_2)$ received by $R_1$ in the first time slot. The transmitted signals of $T_1$ and $T_2$ are given by

$$X_1(2) = h_{11}^{-1}(2)h_{11}(1)v_1$$

$$X_2(2) = h_{12}^{-1}(2)h_{12}(1)v_2$$

and the received signals at $R_1$ and $R_2$ are given respectively by

$$Y_1(2) = h_{11}(1)v_1 + h_{12}(1)v_2$$

$$Y_2(2) = h_{21}(2)h_{11}^{-1}(2)h_{11}(1)v_1 + h_{22}(2)h_{12}^{-1}(2)h_{12}(1)v_2$$

Hence, at the end of sub-phase one, $R_1$ has received $I_1(v_1, v_2)$, the interference received in the first time slot, and $R_2$ has received a new linear combination $L_2^2(v_1, v_2)$.

In the second sub-phase, the transmitted signals are designed to resurrect the interference received by $R_2$ in the first time slot and provide a new linear combination of the desired messages to $R_1$. The transmitted signals of $T_1$ and $T_2$ are given respectively by

$$X_1(3) = h_{21}^{-1}(3)h_{21}(1)u_1$$

$$X_2(3) = h_{22}^{-1}(3)h_{22}(1)u_2$$

where $T_1$ and $T_2$ utilize their perfect and instantaneous knowledge of their channels to $R_2$. The received signals at $R_1$ and $R_2$ are given by:

$$Y_1(3) = h_{11}(3)h_{21}^{-1}(3)h_{21}(1)u_1 + h_{12}(3)h_{22}^{-1}(3)h_{22}(1)u_2$$

$$Y_2(3) = h_{21}(1)u_1 + h_{22}(1)u_2$$

After the third time slot, the two receivers $R_1$ and $R_2$ have enough information to decode their intended messages. In particular, $R_1$ has access to two different equations in $u_1$ and $u_2$ only. The first one is obtained by subtracting $Y_1(2)$ from $Y_1(3)$ to cancel out the interference and the second equation is $Y_1(3)$ by itself as it is received without interference. Similarly, $R_2$ forms its first equation by subtracting $Y_2(3)$ from $Y_2(1)$ to cancel out the interference while the second equation is $Y_2(2)$.

Note that this scheme could be used also when the CSIT pattern given by (DD, NP, PN) but with minor modification in phase two, where sub-phase one is dedicated to resurrecting interference of $R_2$ instead of resurrecting interference of $R_1$ and sub-phase two is dedicated to resurrecting interference of $R_1$ instead of resurrecting the interference of $R_2$.

**B. Scheme 2: Distributed delayed and combined perfect CSIT**

Let us consider the 2-user SISO X-channel with alternating CSIT given by (ND, DN, PP). Unlike case 1, here we have distributed delayed CSIT over the first two time slots and combined perfect CSIT in the last time slot. Consequently, the interference creation phase extends over two time slots while the interference resurrection phase can be executed in one time slot as follows.

**Phase one:** Each time slot of this phase is dedicated to one receiver where the two transmitters transmit the desired messages for this receiver. For example, if the first time slot is dedicated to $R_1$, then $T_1$ transmits $u_1$ and $T_2$ transmits $u_2$. The received signals at $R_1$ and $R_2$ are given respectively by

$$Y_1(1) = h_{11}(1)u_1 + h_{12}(1)u_2$$

$$Y_2(1) = h_{21}(1)u_1 + h_{22}(1)u_2$$

Therefore, $R_1$ receives linear combination $L_1^1(u_1, v_2)$ of its desired signals, while $R_2$ receives only interference $I_2(u_1, u_2)$. Similarly, in the next time slot, $T_1$ transmits $v_1$ and $T_2$ transmits $v_2$ and the received signals at $R_1$ and $R_2$ are given respectively by

$$Y_1(2) = h_{11}(2)v_1 + h_{12}(2)v_2$$

$$Y_2(2) = h_{21}(2)v_1 + h_{22}(2)v_2$$

where $R_2$ receives linear combination $L_2^1(v_1, v_2)$ of its desired signals, while $R_1$ receives only interference $I_1(v_1, v_2)$.

**Phase two:** This phase includes only one time slot where the transmitters resurrect the formerly received interference terms $I_1(v_1, v_2)$ and $I_2(u_1, u_2)$, while providing new linear combinations of the desired messages to the two receivers. In order to achieve this goal, the transmitted signals from $T_1$ and $T_2$ in the third time slot is given by

$$X_1(3) = h_{21}^{-1}(3)h_{21}(1)u_1 + h_{11}(3)h_{11}(2)v_1$$

$$X_2(3) = h_{22}^{-1}(3)h_{22}(1)u_2 + h_{12}(3)h_{12}(2)v_2$$

and the corresponding received signals at $R_1$ and $R_2$ are given respectively by

$$Y_1(3) = L_1^2(u_1, u_2) + I_1(v_1, v_2)$$

$$Y_2(3) = L_2^2(v_1, v_2) + I_2(u_1, u_2)$$

where

$$L_1^2(u_1, u_2) = h_{11}(3)h_{21}^{-1}(3)h_{21}(1)u_1 + h_{12}(3)h_{22}^{-1}(3)h_{22}(1)u_2$$

$$L_2^2(v_1, v_2) = h_{21}(3)h_{11}^{-1}(3)h_{11}(2)v_1 + h_{22}(3)h_{12}^{-1}(3)h_{12}(2)v_2$$

At the end of the third time slot, each receiver can decode its intended messages by solving two equations. For example, $R_1$ subtracts $Y_1(2)$ from $Y_1(3)$ to cancel out the interference and obtain the first equation in $u_1$ and $v_2$ while the second equation is $Y_1(1)$ by itself as it received without interference.

Noteworthy, this scheme could be used when the CSIT pattern given by (DN, ND, PP) but with minor modification in phase one where the two sub-phases swap their dedications from $R_1$ to $R_2$ and vise versa.
C. Scheme 3: Distributed delayed and distributed perfect CSIT

As an illustrative example, let us consider a 2-user SISO X channel with CSIT pattern given by (DN, PD, NP). Unlike the above two examples, we have distributed delayed CSIT over the first two time slots and distributed perfect CSIT over the last two consecutive time slots. Consequently, the proposed scheme is performed in two overlapping phases as follows.

Time slot 1: The first sub phase of phase one begins at \( t = 1 \), and is dedicated to transmitting the desired messages of \( R_2 \), i.e., \( T_1 \) transmits \( v_1 \) while \( T_2 \) transmits \( v_2 \). The received signals are given by

\[
Y_1(1) = h_{11}(1)v_1 + h_{12}(1)v_2 \equiv I_1(v_1, v_2) \quad (23)
\]
\[
Y_2(1) = h_{21}(1)v_1 + h_{22}(1)v_2 \equiv L_2^1(v_1, v_2) \quad (24)
\]

Therefore, \( R_2 \) receives the first linear combination \( L_2^1(v_1, v_2) \) of its desired signals, while \( R_1 \) receives only interference \( I_1(v_1, v_2) \).

Time slot 2: At \( t = 2 \) the overlap occurs between the two phases. In particular, sub-phase two of phase one and sub-phase one of phase two begin simultaneously. In this time slot, sub-phase two of phase one creates interference at \( R_2 \) with while sub-phase one of phase two is designed to resurrect the interference term \( I_1(v_1, v_2) \). The transmitted signals are given by:

\[
X_1(2) = u_1 + h_{11}^{-1}(2)h_{11}(1)v_1 \quad (25)
\]
\[
X_2(2) = u_2 + h_{12}^{-1}(2)h_{12}(1)v_2 \quad (26)
\]

and the corresponding received signals are given by:

\[
Y_1(2) = h_{11}(2)u_1 + h_{12}(2)u_2 + h_{11}(1)v_1 + h_{12}(1)v_2 \nonumber\]
\[
\quad \equiv L_1^1(u_1, u_2) + I_1(v_1, v_2) \quad (27)
\]
\[
Y_2(2) = h_{21}(2)h_{11}^{-1}(2)h_{11}(1)v_1 + h_{22}(2)h_{12}^{-1}(2)h_{12}(1)v_2 + h_{21}(2)u_1 + h_{22}(2)u_2 \nonumber\]
\[
\quad \equiv L_2^2(v_1, v_2) + I_2(u_1, u_2) \quad (28)
\]

Therefore, \( R_2 \) receives a new linear combination \( L_2^2(v_1, v_2) \) of its desired signals and an interference term \( I_2(u_1, u_2) \) as a by-product of the overlap, while \( R_1 \) receives the old interference \( I_1(v_1, v_2) \) and the first linear combination \( L_1^1(u_1, u_2) \) of its desired signals.

Time slot 3: In this time slot the transmitters send linear combination from \( u_1 \) and \( u_2 \) aiming to resurrect the interference terms \( I_2(u_1, u_2) \) formerly received at \( t = 2 \), while providing a new linear combinations to \( R_1 \) of its desired messages. The transmitted signals are given by:

\[
X_1(3) = h_{21}^{-1}(3)h_{21}(2)u_1 \quad (29)
\]
\[
X_2(3) = h_{22}^{-1}(3)h_{22}(2)u_2 \quad (30)
\]

and the corresponding received signals are

\[
Y_1(3) = h_{11}(3)h_{21}^{-1}(3)h_{21}(2)u_1 + h_{12}(3)h_{22}^{-1}(3)h_{22}(2)u_2 \nonumber\]
\[
\quad \equiv L_1^3(u_1, u_2) \quad (31)
\]
\[
Y_2(3) = h_{21}(2)u_1 + h_{22}(2)u_2 \equiv I_2(u_1, u_2) \quad (32)
\]

Finally, the two receivers \( R_1 \) and \( R_2 \) have enough information to decode their intended messages. In particular, \( R_1 \) has access to two different equations in \( u_1 \) and \( u_2 \) only. The first one is obtained by subtracting \( Y_1(1) \) from \( Y_1(2) \) to cancel out the interference and the second equation is \( Y_1(3) \) by itself as it is received without interference. Similarly, \( R_2 \) its first equation is \( Y_2(1) \) while forming its second equation by subtracting \( Y_2(3) \) from \( Y_2(2) \) to cancel out the interference.

Note that this scheme could be used when the CSIT pattern is given by (ND, DP, PN) but with minor modification in the two phases where the two sub-phases in each phase swap their dedications from \( R_1 \) to \( R_2 \) and vise versa.

IV. Synergistic CSIT Alteration Patterns

In this section, we discuss CSIT alternation patterns that can provide synergistic gain in the DoF of the two-user SISO X channel. We focus on a three-symbol extension of the channel. Since the possible CSIT states for the two users are given by \( S_{12} \in \{PP, PD, PN, DP, DD, DN, NP, ND, NN\} \), there are \( 9^3 \) possible alternation patterns over the three time slots.

First, we note that the aforementioned three examples in Section III present the CSIT patterns with the lowest CSIT sufficient to achieve 4/3 DoF. Definitely, any alteration pattern with channel knowledge higher than these patterns can achieve the same DoF, i.e., if we have \( S_{12} = ND \), its higher CSIT state that could provide the same synergistic DoF gain are \{NP, DD, DP, PD, PP\}. Theorem 1 presents sufficient conditions on the lowest CSIT alternation pattern among tree-symbol channel extension patterns for achieving the upper bound on the DoF of the two-user SISO X-channel.

Theorem 1: For the two-user SISO X-channel in time varying or frequency selective settings, the upper bound on the total DoF of the channel is achievable if the following requirements on the CSIT alternation pattern are satisfied.

1) Each transmitter has a delayed CSIT followed by a perfect CSIT over three time slots.
2) At each time slot, at least one transmitter should have some CSIT (perfect or delayed), i.e., the two transmitters should not be simultaneously without CSIT.
3) In the third time slot, at least one transmitter should have perfect CSIT.

Proof: We show that the three requirements of Theorem 1 limit the CSIT alternation patterns in a 3-symbol channel extension to the minimum CSIT synergistic patterns considered in the three examples of Section III and its higher CSIT patterns. Hence, the achievability of 4/3 DoF follows from the results of Section III. The first requirement in Theorem 1 yields three possible minimum states for the CSIT of the channel to the \( i \)th receiver over three time slots \( S_{12}^i \in \{(D,P,N), (D,N,P), (N,D,P)\} \). As a result we have 9 possible combinations for the CSIT of the two-user channel. Six of these 9 combinations, satisfy the second and third requirements.
in Theorem 1 and are listed as the first 6 entries in Table 1. The remaining three combinations are those which have $S_{12} = NN$ in any of the three time slots, i.e., (DD,PP,NN), (DD,NN,PP), and (NN,DD,PP). For the first combination, the minimum CSIT states that satisfy the three requirements are (DD,PP,PN), which is higher than (DD,NP,PN), and (DD,P,NP), which is higher than (DD,PN,NP). From Table 1, we can achieve 4/3 DoF using achievable scheme 1 in both cases. Similarly, for the CSIT state (DD,NN,PP), the minimum CSIT that satisfy the requirements of Theorem 1 are (DD,ND,PP) and (DD,DN,PP) for which 4/3 DoF can be achieved using scheme 2. Finally, for the CSIT state (NN,DD,PP), the minimum CSIT that satisfy the requirements of Theorem 1 are (ND,DD,PP) and (DN,DD,PP) for which 4/3 DoF can be achieved using scheme 2 too.

| CSIT state         | Scheme 1 | CSIT state         | Scheme 2 | CSIT state         | Scheme 3 |
|--------------------|----------|--------------------|----------|--------------------|----------|
| (DD,PN,NP)         | Scheme 1 | (DN,PD,PN)         | Scheme 3 | (ND,DP,PN)         | Scheme 2 |
| (DD,NPPN)          | Scheme 1 | (DN,NPP)           | Scheme 2 | (ND,DP,PN)         | Scheme 2 |
| (ND,DP,PN)         | Scheme 3 |                    |          |                    |          |

**Remark 1:** [Synergy benefits] Note that the DoF for two-user X-channel with perfect CSIT is 4/3 [21, 5], with delayed CSIT is upper bounded by 6/5 [12] and with No-CSIT is unity due to statistical symmetry of channel outputs [6]. Synergy is the interaction of multiple elements in a system to produce an effect greater than the sum of their individual effects. In particular, the alteration of CSIT states $S_{ij}$ over three time slots works cooperatively to provide a DoF greater than the DoF of the weighted average of their individual DoF for the same network.

As an example, let us consider the CSIT alternation pattern given by (DD,DD,PP). If there is no interaction between the three time slots, the DoF that can be obtained are given by

$$\frac{2}{3} + \frac{6}{5} + \frac{4}{3} = \frac{56}{35}$$

which is strictly lower than the upperbound on the DoF of the channel. However, using achievable scheme 2, we can get 4/3 DoF for this case as this CSIT pattern is higher than (DN,ND,PP) in Table 1. This illustrates the synergistic benefit that can be obtained from the alternation of CSIT over the three time slots.

Note that not all combinations of CSIT states could provide synergistic benefits or work together in a cooperative way. For example, when perfect CSIT comes before delayed and no CSIT, it loses its synergetic DoF gain; where, the DoF degrades to the sum of the individual DoF each case. on the other hand, when perfect CSIT comes after delayed and no CSIT, the synergy of the alternation appears.

**Remark 2:** [Potential of delayed followed by perfect CSIT] The extraordinary synergistic gain of delayed CSIT followed by perfect CSIT lies in the ability to upgrade the X channel to a broadcast channel with delayed CSIT. In particular, when the delayed CSIT comes first it provides the transmitters with delayed channel knowledge which combats the distributed nature of the X-channel and can be exploited in addition to the perfect CSIT to provide one message to each receiver.

**Remark 3:** [Combined No-CSIT] We note that the synergy of alternating CSIT is lost when the network has combined No-CSIT in any time slot. Intuitively, the uncertainty of distributed channel unawareness is better than blindness of combined complete ignorance. As Martin Luther King said before “Darkness cannot drive out darkness; only light can do that.”

**V. Conclusion**

We have investigated the synergistic benefits of alternating CSIT in the context of two-user X-channel. Unlike what is commonly thought that the synergistic benefits of alternating CSIT could be more sensitive to or may be lost depending on whether the transmitter of the network are distributed or collocated, we end up with the synergistic alternation of CSIT is still very beneficial in distributed transmitters network. The surprising finding of the synergistic alternating CSIT is that it is capable of achieving the information theoretic upper bound upper bound on the DoF of the two-user SISO X-channel. Hence, constantly perfect CSIT has much redundant and unnecessary channel information.

**References**

[1] S. A. Jafar, *Interference alignment: A new look at signal dimensions in a communication network*. Now Publishers Inc, 2011.
[2] M. A. Maddah-Ali, A. S. Motahari, and A. K. Khandani, “Communication over MIMO X channels: Interference alignment, decomposition, and performance analysis,” *IEEE Transactions on Information Theory*, vol. 54, no. 8, pp. 3457–3470, 2008.
[3] V. Cadambe and S. Jafar, “Interference alignment and degrees of freedom of the K-user interference channel,” *IEEE Transactions on Information Theory*, vol. 54, no. 8, pp. 3425–3441, 2008.
[4] ——, “Interference alignment and the degrees of freedom of wireless X networks,” *IEEE Transactions on Information Theory*, vol. 55, no. 9, pp. 3893–3908, 2009.
[5] S. Jafar and S. Shamai, “Degrees of freedom region of the MIMO X channel,” *IEEE Transactions on Information Theory*, vol. 54, no. 1, pp. 151–170, 2008.
[6] C. Vaze and M. Varanasi, “The degree-of-freedom regions of MIMO broadcast, interference, and cognitive radio channels with no CSIT,” *IEEE Transactions on Information Theory*, vol. 58, no. 8, pp. 5354–5374, 2012.
[7] M. A. Maddah-Ali and D. Tse, “Completely stale transmitter channel state information is still very useful,” *IEEE Transactions on Information Theory*, vol. 58, no. 7, pp. 4418–4431, 2012.
[8] H. Maleki, S. A. Jafar, and S. Shamai, “Retrospective interference alignment over interference networks,” *IEEE Journal of Selected Topics in Signal Processing*, vol. 6, no. 3, pp. 228–240, 2012.
[9] S. Yang, M. Kobayashi, D. Gesbert, and X. Yi, “Degrees of freedom of MISO broadcast channel with perfect delayed and imperfect current CSIT,” in *Information Theory Workshop (ITW)* 2012.
[10] R. Tandon, S. Jafar, S. Shamai Shitz, and H. Poor, “On the synergistic benefits of alternating csit for the miso broadcast channel,” *IEEE Transactions on Information Theory*, vol. 59, no. 7, pp. 4106–4128, 2013.
[11] A. Ghasemi, A. S. Motahari, and A. K. Khandani, “On the degrees of freedom of X channel with delayed CSIT,” in *proceedings IEEE International Symposium on Information Theory (ISIT) 2011*.
[12] S. Lashgari, A. S. Avestimehr, and C. Suh, “Linear degrees of freedom of the x-channel with delayed csit,” *CoRR*, vol. abs/1309.0799, 2013.