The research of cracks influence on frequencies of fluctuations of thin-walled core

N Morozov¹*, A Gavrilov¹ and V Litvinov²
¹Aerospace Institute, Orenburg State University, 13 Pobegy avenue, Orenburg 460018, Russian Federation
²Faculty of Geology and Geography, Orenburg State University, 13 Pobegy avenue, Orenburg 460018, Russian Federation
*¹E-mail: moroz.off.nick@yandex.ru

Abstract. In the article a thin-walled core of the closed cross-section which is rigidly jammed by one end was under consideration. The own frequencies of fluctuations of a core without cracks and with cracks of various depth and located in various places of a core were researched. Natural and numerical experiments were conducted. Depth’s and location’s effect of a crack on frequencies of own fluctuations of the core was estimated. Expressions for definition of the crack’s origin place depending on change of frequency of core’s own fluctuations are received.

1. Introduction
The emergence of cracks in material can be caused by various reasons: defects of production, emergence of fatigue cracks in the using process, occurrence of defects as a result of external influences and corrosion. In the last several decades considerable attention was paid to identification of cracks and damages in constructions by measurement of characteristics of their fluctuations. In most cases it allows to conduct nondestructive constructions' testing.

During a research of cracks influence on parameters of beams vibration various models are offered. It begins from a method of decrease in the general rigidity of beams and passes to more difficult models which consider the increase of flexibility due to a crack. A model of using more simple rigidity decreasing coefficient in the finite elemental models is still popular. However the using of the real cracks model is pretty limited. The classical nonlinear vibrations methods are using for detection of structural damages. They are often based on generation of the highest and subharmonic fluctuations. The main lack of these methods is the possibility of emergence of nonlinearity which is not related to cracks emergence factors (boundary conditions, errors of measurements). Graphics of fluctuations form are widely used for detection of structural damages.

In the work of Cannizzaro, F., De Los Rios, J., Caddemi, S., Calio, I., Ilanko, S. there was considered the possibility of using changes in frequencies due to attachment of additional mass to the structure, and there was presented analytical evidence that the natural frequencies of a cracked rod with a body having a mass and inertia of rotation, usually change dramatically when the body passes over a crack, provided that the crack allows for various bending rotations [1].

Also the authors propose a new solution to the Euler-Bernoulli equation for a rod with an additional moving body in the presence of multiple cracks. Liu J., Zhu W. D., Charalambides P. G., Shao Y. M, Fang X. M. developed a new three-segment model of a beam with a closed horizontal crack [2]. The
The proposed model of the rod is a linear elastic system and does not take into consideration the nonlinear effect resulting from the closure of the crack. It is believed that the upper and lower segments are in contact during the entire vibration of the rod. The novelty of the application of this model lies in the possibility of modeling local deformations near the crack tip. The usage of SWT method allows determining the length and location of a horizontal crack. Chuanchuan Hou, Yong Lu highlight the simple method of reducing stiffness, especially for the thick beams [3], recommend a number of models of rods with cracks. The crack identification procedure is based on a global search method using genetic algorithms, therefore it does not impose any restrictions on the location, size and number of cracks that can be detected.

Studies of Broda D., Pieczonka L., Hiwarkar V., Staszewski W. J., Silberschmidt V. V. are dedicated to the localization of damage-related non-linearities based on the generation of higher harmonics [4]. Numerical and experimental studies of the longitudinal vibration of beams with cracks are presented. The results show that nonlinearities in cracked rods are especially strong in the vicinity of damage, which allows not only to localize the damage, but also to separate the nonlinearity caused by the crack from other sources of nonlinearity. Khassetarash A., Hassannejad R. considered free vibration of a rod with a crack, taking into account both internal friction and attenuation due to the presence of a crack [5]. The increased flexibility due to the presence of a crack was modeled by a weightless spring, and the increased damping was modeled by a weightless damper at the location of the crack, the attenuation coefficient was calculated according to this model. The results showed that the fraction of the crack in the total energy dissipation depends on the depth of the crack, the location of the crack and internal friction.

To characterize the multiple cracks of a cantilever beam with variable sections, Kai Zhang, Xiaojun Yan suggested a mathematical model based on the concept of deformation energy [6]. To obtain more accurate natural frequencies of the cantilever beam, a signal processing method based on the Hubert-Huang transform (HHT) is proposed, which is superior to the fast Fourier transform (FFT) method in terms of frequency resolution and the ability to process nonlinear vibration caused by the closure of crack edges.

In the study of Fernández-Sáez J., Morassi A., Pressacco M., Rubio L. the identification of one open crack in a uniform rod is considered by measuring the first and second natural vibration frequencies [7]. Knowledge of this set of spectral data allows you to determine definitely the depth and position of damage of small cracks. In this article, the original constructive method shows that the above result is valid for any crack size. Yashar A., Neil Ferguson N., Ghandchi-Tehrani M. using the simplified method investigated the natural frequencies and vibration forms of a rotating cracked Euler-Bernoulli’s rod [8]. This approach is based on calculations of the transverse displacement of a rotating rod using the potential deformation energy. The values of the natural frequencies are determined using the Rayleigh-Ritz method.

Behzad M., Ghadami A., Maghsoodi A., Hale J. M. proposed a simple method for the detection of edge cracks in Euler-Bernoulli’s rods having two different types of cracks [9]. Method is based on the energy equations. Each crack is modeled as a weightless spring using the linear elastic fracture theory (LEFM). To detect cracks in the rod, the natural frequencies of a rod without cracks and the natural frequencies of a rod with cracks with fluctuations in two directions are used as input signals. Xu Y. F., Zhu W. D., Liu J., Shao Y. M. reviewed two new methods that use the resulting vibration shapes to identify horizontal cracks in the rods [10]. The proposed methods do not require any information about intact rods if the rods have constant stiffness. In the work of Khiem N. T., Toan L. K a new method is proposed for calculating the natural frequencies of a rod with several cracks and detecting cracks from measured natural frequencies [11].

At the same time there are many opportunities for optimization of search of localization of cracks. In this article the purpose of obtaining regression dependences on the basis of a numerical and natural experiment was set. The analysis of frequencies of own fluctuations of cores without cracks was made by authors in their previous study [12].
2. Experimental Part
The natural experiment was made using a vibration analyzer "Vibran 3.2" by the GOST 30630.1.1-99 method 100.3 "Test methods on resistance to the mechanical external influencing factors of machines, devices and other technical products. Determination of dynamic characteristics of a design". The used sensors are 352C04.

Cores from the rolling profile made in accordance with GOST 8645-68 acted as samples. Material is St3ps steel (analog to BS standard: 1449-27/23CR). Cores were rigidly jammed by one end. Thread with freight of weight of 4 kg was fixed in one of the points. Further thread was cut, and frequencies of own fluctuations were fixed. The crack was modelled as a section 1 mm wide with depths of 2.5, 5, 7.5 and 10 mm.

The general view of experimental installation is presented on a figure 1.

![Figure 1. The experimental installation.](image1)

The Autodesk Inventor system was applied for a numerical experiment. The solid-state model of a core was created (figure 2). One end of a core was fixed from movements and turns. The sizes of cracks and a core were accepted as in a natural experiment.

![Figure 2. The core model with sensors in Autodesk Inventor.](image2)
The location of the crack was chosen with a step of 1/20 of the rod length for a numerical experiment and with a step of 1/10 for a full-scale experiment. In modeling, bulk finite elements were used. The model of the crack and the grid are presented on the figure 3.

![Figure 3. The model of the crack.](image)

### 3. Results and Discussion

The processed experimental results are presented in the form of schedules (figures 4, 5, 6, 7) of dependence of a deviation of own frequency of a core with a crack from core frequency without a crack. Various schedules correspond to various depths of a crack.

![Figure 4. Values of a deviation of frequency of the first form of fluctuations.](image)

As the schedules show, when a crack is in a zone of "excess" of a form, zero values of deviations are received, that means that the existence of a crack does not influence a form of fluctuations.

The analysis showed that the divergence between results of numerical and natural experiments for a solvable task does not exceed 12 percent. At the same time, the maximum difference corresponds to the 4th form of fluctuations at a section within a distance of 0.1 length’s from the free end of a core. Other values lie in the range up to 8 percent.
Figure 5. Values of a deviation of frequency of the second form of fluctuations.

Figure 6. Values of a deviation of frequency of the third form of fluctuations.
Figure 7. Values of a deviation of frequency of the fourth form of fluctuations.

For forecasting of the core’s own frequency of fluctuations’ value ($v$) depending on the place of emergence and depth of a crack the equation was as following

$$ v = A_1 + (A_2 + A_3 x + A_4 y) \sin(A_5 x + A_6), $$

(1)

where $x$ is the location of a crack viewed as a relative value, $x=(0..1)$;
$y$ is the crack depth expressed as a relative value, $y=(0..0.5)$;
$A_i$ are various coefficients.

As a result, for each of four forms coefficients were received and later summarized in the table 1.

### Table 1. The values of coefficients for forms of fluctuations.

| The number of coefficient | The values of coefficients for forms of fluctuations |
|---------------------------|---------------------------------------------------|
|                           | Form 1    | Form 2    | Form 3    | Form 4    |
| 1                         | 19.19     | 116.0     | 327.5     | 603.9     |
| 2                         | 1.259     | 1.199     | -13.67    | 1.896     |
| 3                         | -18.22    | -0.4168   | 21.43     | -1.624    |
| 4                         | 22.49     | 5.074     | 13.56     | 21.41     |
| 5                         | 0.1991    | 9.890     | 11.79     | 21.30     |
| 6                         | 175.8     | 137.3     | 102.8     | 188.6     |

For positioning of a crack, the regression analysis was carried out. At the same time the dependence of the location of a crack on frequencies of the first lowest four forms of fluctuations in the form of a polynomial was established:

$$ x = B_{ijkl}v_i v_j v_k v_m. $$

(2)

Where $i,j,k,m = (0,1,2,3,4)$ - cardinal numbers of frequency, for the value «0» is accepted $v_0 = 1$.

It is established that for the considered console core values have the coefficients specified in table 2.
Table 2. The Value of coefficient.

| The index of coefficient | Value     |
|--------------------------|-----------|
| 0                        | 0.9566    |
| 1                        | -0.2056   |
| 2                        | -0.0447   |
| 4                        | -0.0066   |
| 11                       | 0.0221    |
| 22                       | 0.0040    |
| 12                       | 0.0098    |
| 13                       | -0.0003   |

4. Conclusion

Thus, in this research the following results were received:
1. The difference between results of numerical and natural experiments when determining frequencies of own fluctuations of the thin-walled core having the simulated crack does not exceed 12 percent that demonstrates that this modeling of cracks at a numerical experiment is acceptable;
2. At a crack existed in an "excess" zone of a certain fluctuations form, influence of a crack on frequency change of fluctuations for this form is insignificant;
3. The polynomial dependence of the location of a crack on frequencies of the first lowest four forms of fluctuations of the burst core is established

References

[1] Cannizzaro F, De Los Rios J, Caddemi S, Caliò I, Ilanko Cannizzaro F, De Los Rios J, Caddemi S, Caliò I, Ilanko S 2018 Journal of Sound and Vibration 425 275–300
[2] Liu J, Zhu W D, Charalambides P G, Shao Y M, Fang X M 2016 Journal of Sound and Vibration 382 274–290
[3] Chuanchuan Hou, Yong Lu 2016 Journal of Sound and Vibration 385 104–124
[4] Broda D, Pieczonka L, Hiwarkar V, Staszewski W J, Silberschmidt V V 2016 Journal of Sound and Vibration 381 206–219
[5] Khassetarash A, Hassannejad R 2016 Journal of Sound and Vibration 363 247–257
[6] Kai Zhang, Xiaojun Yan 2017 Journal of Sound and Vibration 387 53–65
[7] Fernández-Sáez J, Morassi A, Pressacco M, Rubio L 2017 Journal of Sound and Vibration 371 94–109
[8] Yashar A, Neil Ferguson N, Ghandchi-Tehrani M 2018 Journal of Sound and Vibration 420 129–141
[9] Behzad M, Ghadami A, Maghsoodi A, Hale J M 2013 Journal of Sound and Vibration 332 6312–6320
[10] Xu Y F, Zhu W D, Liu J, Shao Y M 2014 Journal of Sound and Vibration 333 6273–6294
[11] Khiem N T, Toan L K 2014 Journal of Sound and Vibration 333 4084–4103
[12] Gavrilov A, Grebenyuk G, Morozov N, Grehov A 2018 International Journal of Engineering & Technology 7 7–12