Exact closed equation for reduced equilibrium distribution functions of the many-particle system

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May 11, 2014

Abstract
An exact closed equation for $s$-particle equilibrium distribution function ($s < N$) of the system of $N \gg 1$ interacting particles is obtained. This integra-differential $\beta$-convolution equation ($\beta = 1/k_BT$) follows from the Bloch equation for the canonical distribution function by applying the projection operator integrating off the coordinates of $N-s$ irrelevant particles. The method of expansion of the obtained equation kernel in the particle density $n$ is suggested. The solution to this equation in the linear in $n$ approximation for the kernel is found.

1 Introduction

In equilibrium statistical mechanics, calculation of thermodynamic functions can be performed either with the help of the corresponding partition function or by means of the reduced distribution functions. For many-body nonideal systems, both approaches are rather involved and based on the Ursell-Mayer cluster expansion of the partition function and distribution functions (see, e.g. [1] and references herein). The particle density expansions of the equation of state and of a pair distribution function for a nonideal gas can then be obtained. However, for dense gases and liquids these density expansions poorly converge or even diverge.

This problem is conventionally cured with the help of the equations for the reduced distribution functions. For example, one may use the BBGKY hierarchy of coupled integra-differential (in coordinate variables) equations for the reduced distribution functions and close it by employing some approximation procedure, e.g., the Kirkwood superposition approximation (see, e.g. [2]). This approximation cannot however be strictly justified. There are other approaches leading to the approximate closed equations for a pair distribution function such as the Percus-Yevick (PY) nonlinear integral equation and the Hypernetted Chain (HNC) equation (see, e.g. [3]). The approximations used for obtaining these
equations can be analyzed in terms of the Mayer diagrams (the PY and HNC approximations are correct to the first order diagram in the particle density) but, again, these approximations are rather guessed than substantiated.

The principal and interesting question then arises: Is it possible to derive an **exact** closed equation for \( s \)-particle distribution function \((s < N)\) for the system of \( N \) \((N \gg 1)\) interacting particles? The goal of this paper is to demonstrate that, surprisingly, this is possible. We will derive such closed equation by applying the projection operator technique to the Bloch differential equation (with respect to \( \beta = 1/k_B T \)) for the \( N \)-particle canonical classical Gibbs distribution function. The obtained exact equation is the integra-differential equation for the \( s \)-particle distribution function with a complicated integral (\( \beta \)-convolution) kernel. This equation can be approximately solved by, e.g., the expansion of the kernel in the particle density series. The expansion of the kernel, however, is much more effective than the expansion of the distribution function itself. For example, the linear in the particle density term of the kernel expansion corresponds to the summing up of the infinite series of the distribution function density expansion.

2 Projection operator derivation of the exact closed equation for a reduced distribution function

We start with the classical equilibrium Gibbs distribution function for the system of \( N \gg 1 \) interacting classical particles with coordinates \( x_i = \{x_i^\alpha\} \) \((i = 1, \ldots, N, \alpha = 1, 2, 3)\) occupying a volume \( V \) at temperature \( T \)

\[
D_N(\beta) = D_N(\beta; \{x_i\}_N) = \frac{1}{Z_N(\beta)} \exp(-\beta U_N), \quad \beta = 1/k_B T,
\]

\[
Z_N(\beta) = \int dx_1 \ldots \int dx_N \exp(-\beta U_N),
\]

\[\{x_i\}_N = x_1, \ldots, x_N\] (1)

where \( U_N \) is the system’s potential energy depending on the whole set of spatial variables \( \{x_i\}_N \)

\[
U_N = \sum_{1 \leq i < j \leq N} \Phi_{ij}, \quad \Phi_{ij} = \Phi(|x_i - x_j|)
\] (2)

and integration is performed over the whole system’s volume \( V \).

Let us consider the quantity

\[
f_N(\beta) = f_N(\beta; \{x_i\}_N) = Z_N(\beta) D_N(\beta) = \exp(-\beta U_N).
\] (3)

The Bloch equation for function \( f_N \) reads

\[
\frac{\partial f_N(\beta)}{\partial \beta} = -U_N f_N(\beta).
\] (4)
The formal solution to this equation is
\[ f_N(\beta) = \exp[-(\beta - \beta_0)U_N]f_N(\beta_0). \] (5)

If we choose \( \beta_0 = 0 \), the initial value of the distribution function \( f_N(\beta_0) \) is
\[ f_N(0) = 1. \] (6)

Let us introduce the operator
\[ P_s = \frac{1}{V_{N-s}} \int dx_{s+1} \ldots \int dx_N, s < N \] (7)
acting on any function in the coordinate space, where integrations are supposed to be performed over the whole system’s volume \( V \). Evidently, this operator is a projection one satisfying the condition \( P_s P_s = P_s \). The complementary operator \( Q_s = 1 - P_s \) is also a projection operator.

Now we apply the operators \( P_s \) and \( Q_s \) to Eq. (4). This procedure is completely equivalent to the well known approach leading to the generalized master equations for the relevant distribution functions in the kinetic theory (see, \[4\], \[5\], \[6\]). The only difference is that our formalism is related to the temperature domain instead of the time domain in the kinetic theory. Thus we have
\[ \frac{\partial f_s^r(\beta)}{\partial \beta} = -P_s U_N[f_s^r(\beta) + f_s^i(\beta)], \]
\[ \frac{\partial f_s^i(\beta)}{\partial \beta} = -Q_s U_N[f_s^i(\beta) + f_s^r(\beta)]. \] (8)

The relevant distribution \( f_s^r(\beta) \) in our case is
\[ f_s^r(\beta) = f_s^r(\beta; \{x_i\}_s) = P_s f_N(\beta) = \frac{Z_N(\beta)}{V_N} F_s(\beta), \]
\[ \{x_i\}_s = x_1, \ldots, x_s, \] (9)
where \( F_s(\beta) \) is the \( s \)-particle distribution function depending on the set \( \{x_i\}_s \) of the coordinates of \( s < N \) particles (see \[2\])
\[ F_s(\beta) = F_s(\beta; \{x_i\}_s) = V^s \int dx_{s+1} \ldots \int dx_N D_N(\beta), \] (10)
which is normalized in such a way that
\[ \frac{1}{V^s} F_s(\beta) dx_1 \ldots dx_s \] (11)
is the probability for finding the particles of the given group of \( s \) particles in the infinitesimal volumes \( dx_1, \ldots, dx_s \) near the points with the coordinates \( x_1, \ldots, x_s \). The irrelevant distribution function is then given by
\[ f_N^i(\beta) = f_N^i(\beta; \{x_i\}_N) = Q_s f_N(\beta) \]
\[ = f_N(\beta) - \frac{1}{V_{N-s}} \int dx_{s+1} \ldots \int dx_N f_N(\beta). \] (12)
Note, that the irrelevant distribution function (12) depends on the complete set of variables \( \{x_i\}_N \), while the relevant function (9) is the reduced distribution function of interest depending on the coordinates \( \{x_i\}_s \) of the cluster of \( s < N \) particles. The reduced distribution functions (10) are sufficient for calculations of the thermodynamic functions as the averages of the corresponding dynamical functions which depend on the much smaller number of variables than the original distribution function \( D_N(\beta) \). Especially important is the pair distribution function \( F_2(\beta; x_1, x_2) \) which enables obtaining the system’s equation of state.

Now we need to solve the system of two equations (8). The solution of the second equation (8) for the irrelevant distribution function reads

\[
f_i^N(\beta) = \frac{\beta}{\beta_0} \int_0^{\beta_0} d\beta_1 \exp[-Q_s U_N(\beta - \beta_1)](-Q_s U_N)f_r^s(\beta_1) \]

\[
+ \exp[-Q_s U_N(\beta - \beta_0)]f_i^N(\beta_0),
\]

(13)

Substituting (13) into the first equation (8), we arrive at the integra-differential equation for the relevant distribution function \( f_r^s(\beta) \), with a source

\[
- P_s U_N \exp[-Q_s U_N(\beta - \beta_0)]f_i^N(\beta_0)
\]

containing the irrelevant part of the distribution function \( f_i^N(\beta_0) \) depending on the whole set of variables \( x_1, \ldots, x_N \). In the kinetic theory this source represents the initial (at the initial moment of time \( t_0 \)) correlations in the many-particle system and poses some problem to deal with (see [7]). Fortunately, in our case, if we choose the initial value of the temperature parameter \( \beta_0 = 0 \) in (13), then, as it is seen from (6) and (12),

\[
f_i^N(0) = 0.
\]

(14)

Thus, we obtain the following exact closed equation for the \( s \)-particle distribution function \( f_r^s(\beta) \)

\[
\frac{\partial f_r^s(\beta)}{\partial \beta} = -P_s U_N f_r^s(\beta)
\]

\[
+ P_s U_N \int_0^{\beta} d\beta_1 \exp[-Q_s U_N(\beta - \beta_1)]Q_s U_N f_r^s(\beta_1)\}
\]

(15)

Equation (15) can be further specialized. It is useful to split the energy \( U_N \) (2) into two parts

\[
U_N = U_s + U_{s,N-s},
\]

\[
U_s = \sum_{1 \leq i < j \leq s} \Phi_{ij}, U_{s,N-s} = \sum_{i=1}^{s} \sum_{j=s+1}^{N} \Phi_{ij} + \sum_{s+1 \leq i < j \leq N} \Phi_{ij}.
\]

(16)

Here \( U_s \) is the energy of the cluster of \( s \) interacting particles with coordinates \( x_1, \ldots, x_s \) (\( s \)-cluster), the first term in \( U_{s,N-s} \) is the potential energy of interaction of \( s \)-cluster with remaining \( N-s \) particles, and the second term in \( U_{s,N-s} \) is the interaction energy of the of \( N-s \) particles not belonging to the
Note, that the projector \( P_s \) commutes with \( U_s Q_s = U_s Q_s \) (the same is of course true for \( P_s \)), but it is not the case for \( Q_s(P_s) \) and \( U_{s,N-s} \). Thus, \( \exp[-Q_s U_N(\beta - \beta_1)] = \exp[-(U_s Q_s + Q_s U_{s,N-s})(\beta - \beta_1)] \).

For what follows, the expansion of the operator exponent

\[
e^{(A+B)\beta} = e^{A\beta} + \int_0^\beta d\beta_1 e^{A(\beta-\beta_1)} Be^{(A+B)\beta_1} = e^{A\beta} + \int_0^\beta d\beta_1 e^{A(\beta-\beta_1)} Be^{A\beta_1} + \int_0^\beta \int_0^{\beta_1} d\beta_2 e^{A(\beta-\beta_1)} Be^{A(\beta_1-\beta_2)} Be^{A\beta_2} + \ldots
\]

(17)

is useful, where \( A \) and \( B \) are generally noncommuting operators.

Then, accounting for (17) and the properties of projection operators \( (P_s Q_s = 0, P_s^2 = P_s, Q_s^2 = Q_s) \), Eq. (15) can be represented as

\[
\frac{\partial f^r_s(\beta)}{\partial \beta} = -(U_s + P_s U_{s,N-s}) f^r_s(\beta)
+ P_s U_{s,N-s} \int_0^\beta d\beta_1 e^{-Q_s U_{s,N-s} \beta_1} Q_s U_{s,N-s} e^{-U_s \beta_1} f^r_s(\beta - \beta_1).
\]

(18)

This exact closed equation for the \( s \)-particle function \( f^r_s(\beta) \) can also be rewritten as

\[
\frac{\partial f^r_s(\beta)}{\partial \beta} = -(U_s + P_s U_{s,N-s}) f^r_s(\beta)
- P_s U_{s,N-s} \int_0^\beta d\beta_1 \frac{\partial}{\partial \beta_1} (e^{-Q_s U_{s,N-s} \beta_1} e^{-U_s \beta_1} f^r_s(\beta - \beta_1)).
\]

(19)

Note, that the operator \( Q_s U_{s,N-s} \beta_1 \) commutes with \( U_s \beta_1 \).

Equation (15), or rather its more explicit versions (18) and (19), is our main result which demonstrates the existence of the exact closed equation for the reduced \( s \)-particle distribution function. The known closed equations, which follow from the decoupling of the BBGKY chain or such as the Percus-Yevick (PY) and the Hypernetted Chain (HNC) nonlinear integral equations for the pair distribution function, are approximate equations in the coordinate space. The obtained closed equation is the exact one but in the space-temperature domain. The process, described by this equation, may be regarded as the evolution of the distribution function for \( s \)-cluster with particles coordinates \( \{x_i\}_s \) at infinite temperature \( (\beta = 0) \) to the same configuration at the heat bath temperature \( T = 1/k_B \beta \). This equation for the \( s \)-particle function \( f^r_s(\beta) \) (9), differing
from the conventional $s$ - particle distribution function only by the temperature dependent factor $Z_N(\beta)/V^N$, is the complicated integro-differential non-Markovian ($\beta$ - convolution) equation. However, the existence of such an equation is of principal importance and rather surprising.

3 Solution of the equation for a reduced distribution function in the Markovian approximation

We should offer some recipe for resolving the obtained equation. One of the possibilities is the expansion of the kernel of equation (18) or (19), e.g. in the particle density $n = N/V$. Such an expansion is, as a rule, much more effective than the direct expansion of the distribution function itself. Each term of such an expansion generally accounts for the partial infinite series of terms (diagrams) in the expansion of the distribution function. Thus, the sums of these infinite series of terms may obey the closed equation which approximates the exact equation (18). This fact is important for finding the reduced distribution function (e.g. the pair distribution function) in the dense liquids and gases.

The operator exponent $e^{-Q_sU_{s,N-s}\beta_1}$ in Eqs. (18) and (19) can be expanded in series either directly in the powers of $Q_sU_{s,N-s}\beta_1$ or (by using (17)) in the powers of $P_sU_{s,N-s}$ as

$$e^{-Q_sU_{s,N-s}\beta_1} = e^{-U_{s,N-s}\beta_1} + \int_0^{\beta_1} d\beta_2 e^{-U_{s,N-s}(\beta_1-\beta_2)} P_sU_{s,N-s} e^{-U_{s,N-s}\beta_2}$$

$$+ \int_0^{\beta_2} d\beta_3 e^{-U_{s,N-s}(\beta_1-\beta_2)} P_sU_{s,N-s} e^{-U_{s,N-s}\beta_3} + \ldots.$$ (20)

It is seen from (19) and (20) that the Eq. (19) can be expanded in the series in $P_sU_{s,N-s}$. In the zero approximation in $P_sU_{s,N-s}$, Eq. (19) (or (18)) can be easily solved and the solution is

$$f^r_s(\beta) = \exp(-U_s\beta),$$ (21)

which represents the distribution function for the isolated $s$ - cluster. Equation (21) coincides with the known result for the correlation function $F_s(\beta;x_1,\ldots,x_s)$ (9) (see, e.g. 24) in the zero approximation in $n$ (an isolated $s$-cluster), because
\[
\frac{Z_N(\beta)}{V^N} = V^{-s} \int dx_1 \ldots \int dx_s \prod_{1 \leq i < j \leq s} (1 + f_{ij}) \\
= 1 + V^{-1} \frac{s(s-1)}{2} \int dx_2 f_{12} \\
+ V^{-2} \frac{s(s-1)(s-2)}{3!} \int dx_2 \int dx_3 f_{12} f_{13} + \ldots,
\]
\[
f_{ij} = \exp(-\Phi_{ij}\beta) - 1,
\]
where \( f_{ij} \) is the Mayer function, and we take into account that \( f_{ij} \) depends on the difference of coordinates \(|x_i - x_j|\). Now, taking the limit \( V \to \infty \) (while \( s \) remains finite), we see that in this limit the normalized partition function \((22)\)
\[
Z_N(\beta)/V^N = 1 \text{ and, therefore, } F_s(\beta) = f_s(\beta) = \exp(-U_s\beta) \text{ in the zero approximation in } n (P_s U_{s,N}). \text{ The terms in Eqs. (18), (19), proportional to } P_s U_{s,N} - s, \text{ describe the interaction of } s \text{- cluster with the "environment" of the remaining } N - s \text{ particles. Now, we will restrict ourselves to the first approximation in } P_s U_{s,N} - s \text{ of the Eq. (19) kernel. We will show now, that each term containing } P_s U_{s,N} - s \text{ is proportional to at least the first power of } n. \text{ Using (7) and (16), we have }
\]
\[
P_s U_{s,N} - s = P_s \sum_{i=1}^{s} \sum_{j=s+1}^{N} \Phi_{ij} + P_s \sum_{s+1 \leq j < k \leq N} \Phi_{jk} \\
= \frac{V^{N-s-1}}{V^{N-s}} (N-s) \sum_{i=1}^{s} \int dx_{s+1} \Phi(|x_i - x_{s+1}|) \\
+ \frac{1}{2} \frac{V^{N-s-2}}{V^{N-s}} (N-s)(N-s-1) \int dx_{s+1} \int dx_{s+2} \Phi(|x_{s+1} - x_{s+2}|).
\]
\[
(23)
\]
For a many-particle system with \( N \gg 1 \) (and \( N \gg s \)), Eq. \((23)\) takes the form
\[
P_s U_{s,N} - s = n \sum_{i=1}^{s} \int dx_{s+1} \Phi_{is+1} + \frac{n^2}{2} \int dx_{s+1} \int dx_{s+2} \Phi_{s+1,s+2}.
\]
\[
(24)
\]
Although, the second term in the right-hand side of \((24)\) is formally of the second order in the density \( n \), it is additionally proportional to the system volume \( V \) due to the fact that \( \Phi_{s+1,s+2} \) depends on the difference \(|x_{s+1} - x_{s+2}|\) of coordinates. Thus, strictly speaking, the terms of all orders in \( \Phi_{ij} \) \((s+1 \leq i < j \leq N)\) should be accounted for. On the other hand, these terms do not depend on the particles’ coordinates and, therefore, contribute only to the temperature dependent factor.

Because the integrand of Eq. \((19)\) is at least of the first order in \( P_s U_{s,N} - s \), we can try introducing the zero approximation \((24)\) for \( f_s(\beta - \beta_i) \) into the second right-hand term of \((19)\). Such a substitution means that we should further remain within the first approximation in \( n \) for the kernel of Eq. \((19)\). It
follows, that in this approximation equation (19) becomes Markovian and can be formally integrated. The result is

\[ f_s^\prime(\beta) = \exp[-U_s\beta - P_s U_{s,N-s} \int_0^\beta d\beta' e^{-Q_s U_{s,N-s}\beta'}], \] (25)

where we have used that \( f_s^\prime(0) = 1 \).

In order to calculate the integral in (25), we consider the expansion of \( e^{-Q_s U_{s,N-s}\beta'} \) in \( P_s U_{s,N-s} \). Remaining in the first order in \( P_s U_{s,N-s} \), we substitute \( e^{-Q_s U_{s,N-s}\beta'} \) in (25) with the first term in the right-hand side of (20). Thus, in the considered approximation we obtain

\[ f_s^\prime(\beta) = \exp[-U_s\beta + P_s e^{-U_s,N-s}\beta - 1]. \] (26)

In the adopted first in the density approximation for the kernel \(-U_s\beta + P_s e^{-U_s,N-s}\beta - 1\), the projected exponential \( P_s e^{-U_s,N-s}\beta \) in (20) should be calculated in the first order in \( n \) as to the terms containing energy of interaction of the \( s \)-cluster particles with remaining \( N-s \) particles, i.e. \( \Phi_{ij} \) \( (1 \leq i \leq s, s+1 \leq j \leq N) \), but all terms related to the mutual interactions of the irrelevant \( N-s \) particles should be calculated in all orders in the corresponding \( \Phi_{ij} \) (see [24] and the subsequent comment). It can be easily done, if we take into account that the quantity \( P_s e^{-U_s,N-s}\beta \) can be expressed in terms of the canonical \( s \)-particle distribution function \( F_s(\beta) \) (10), related to the relevant distribution function \( f_s^\prime(\beta) \) by Eq. (9), as

\[ P_s e^{-U_s,N-s}\beta = \frac{Z_N(\beta)}{V^N} e^{U_s\beta} F_s(\beta). \] (27)

We can now use the known density expansion of \( F_s(\beta) \)

\[ F_s(\beta; \{x_i\}) = e^{-U_s\beta}[1 + \sum_{k=1}^\infty \beta_k^\prime(\beta; \{x_i\})], \] (28)

where the factors \( \beta_k^\prime(\beta; x_1, \ldots, x_s) \) are the cluster integrals corresponding to cluster diagrams with the fixed \( s \) points \( x_i \) \( (1 \leq i \leq s) \) of the Ursell-Mayer theory (for details see, e.g., [5]). In particular, for a pair distribution function \( F_2(\beta; x_1, x_2) \) we should make in (28) the substitutions \( U_s \to \Phi_{12}, \beta_k^\prime(x_1, \ldots, x_s) \to \beta_k^\prime(1, x_2) \), where

\[ \beta_k^\prime(1,x_1,x_2) = \int dx_3 f_{13} f_{23}. \] (29)

According to the adopted approximation for the kernel of Eq. (19), we should insert the linear in \( n \) term of (28) into (27). Thus, in the considered approximation we obtain from (26) - (28) and (9)

\[ F_s(\beta; \{x_i\}) = \frac{V^N}{Z_N(\beta)} e^{-U_s\beta} \exp \left[ \frac{Z_N(\beta)}{V^N} - 1 \right] e^{-U_s\beta} \exp \left[ \frac{Z_N(\beta)}{V^N} n \beta_1^\prime(\beta; \{x_i\}) \right]. \] (30)
where the first two factors depend only on the temperature, and the subsequent exponentials define the dependence of the distribution function on the particles coordinates.

Now we can use the following known density expansion for the partition function $Z_N(\beta)$ (see, e.g., [1])

$$
\frac{Z_N(\beta)}{V^N} = \exp\left[N \sum_{k=1}^{\infty} n^k \frac{\beta_k(\beta)}{k + 1}\right],
$$

(31)

where $\beta_k(\beta)$ are Mayer’s irreducible cluster integrals. For example the first irreducible cluster integral is defined as

$$
\beta_1(\beta) = \int \left[e^{-\beta \Phi(r)} - 1\right] dr.
$$

(32)

Note, that the formal expansion of the partition function $Z_N(\beta)$ (31) in the density $n = N/V$

$$
Z_N(\beta) = V^N \left[1 + N \sum_{k=1}^{\infty} n^k \frac{\beta_k(\beta)}{k + 1} + \ldots\right]
$$

(33)

leads to the correct equation of state virial expansion, i.e. $Nn^k\beta_k/(k + 1)$ can formally be regarded as a small parameter $\sim n^k$ [9].

In the spirit of the adopted approximation we retain only the first term of $Z_N(\beta)/V^N - 1$ expansion (33) (containing linear in $n$ term) into the first exponential of (30), and take $Z_N(\beta)/V^N = 1$ in the third exponential. Thus, we finally obtain from (30) and (31)

$$
F_s(\beta; x_1, \ldots, x_s) = e^{-U_s\beta} \exp[n\beta'_1(\beta; x_1, \ldots, x_s)].
$$

(34)

Expansion of (34) in the particle density agrees, naturally, with (28).

The solution for the distribution function (34) going beyond the result (28) (in the linear in $n$ approximation, which can turn insufficient) and takes into account the infinite in the density series of terms (diagrams). This demonstrates the advantage of the expansion of the kernel (of the equation for a function) as compared to the corresponding expansion of the function itself. We can expect that the result (34) will provide a reasonable approximation for not very large densities.

One can try to improve Eq. (34) by adding the higher order in $n$ terms of the expansion (28) to the kernel (27). For example, by taking into account the quadratic in $n$ term of (28) we obtain

$$
F_s(\beta; \{x_i\}_s) = e^{-U_s\beta} \exp[n\beta'_1(\beta; \{x_i\}_s) + n^2\beta'_2(\beta; \{x_i\}_s)].
$$

(35)

The approximation (35) (although not quite consistent with the accepted approach to the kernel expansion) accounts for much more Mayer’s diagrams than the linear approximation for the kernel (34).
Next terms of expansion (20) contribute to the higher powers in the projector $P_s$ terms of Eq. (19) kernel expansion and may change the obtained results.

Generally, the non-Markovian equation (18) or (19) is to be solved with some approximation for the kernel. This may be done, e.g., with the help of the Laplace transform of Eq. (18).

4 Conclusion

We have obtained the exact closed equation (18) (or (19)) for the $s$-particle distribution function which is (from our point of view) surprising and of principal importance, because the known closed equations, such as PY and HNC equations or those which follow from disentangling the BBGKY chain, are approximate equations. It is not also quite clear how to improve the mentioned approximate equations in the regular way by taking into account the additional terms of, e.g., the expansion in the particle density. Our equation is an integro-differential equation with respect to the inverse temperature $\beta$ variable (the mentioned known equations are the nonlinear equations with respect to the coordinate variables). The projection operator approach to the Bloch equation for the classical distribution function of the system of $N \gg 1$ particles, which enabled us to reach a goal, is, as to our knowledge, a new one. To some extent this method is reminiscent of the approach used in the paper [10] (see also [1]), where the solution of the Bloch equation and the partition function for the system of quantum particles are represented as the series in the particles’ interaction in the $\{x_i, \beta\}$ space. For obtained rather complicated non-Markovian ($\beta$-convolution) equation the methods of its solution should be developed. One of the possibilities is to expand the kernel of this equation into the particle density series. The expansion of the kernel of equation for the distribution function is generally much more effective than that of this distribution function which is characteristic of the Ursell - Mayer theory. That is why, the finding of the closed equations for reduced distribution functions is important for the theory of dense gases and liquids. We have suggested the expansion of the obtained equation kernel in the particle density and found the solution for the $s$-particle distribution function in the linear in $n$ approximation for the kernel.

5 References

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