Communication skill and mathematics conceptual understanding of senior high school students

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Abstract. The aims of this research are to describe about mathematical communication ability either written or oral form and find the relation between mathematical communication and mathematics conceptual understanding of senior high school students. Mathematical communication is process of social interaction involve process of exchange and reflection of information, ideas, and understanding the mathematics from someone to others. Conceptual understanding of mathematics involve understanding of the basic concepts behind algorithms in mathematics. This study used mixed method, quantitative and qualitative method. Mathematical communication ability is measured by written test, and non-test with specific indicators to measure mathematical communication. In other hand, mathematics conceptual understanding is measured by written test. The number of subject in qualitative phase is four subjects categorized based on their mathematical communication ability, namely very low (VL), low (L), high (H), and very high (VH). The result of this research revealed the students in doing mathematical communication of every ability level. In addition, we find that there is no relation between mathematical communication and mathematics conceptual understanding of students.

1. Introduction
Communication is an important skill in learning mathematics. Qohar [1] clarify that the communication is needed to understand the mathematical ideas correctly. Weak communication skills will result in a lack of other mathematical abilities. Students who have high mathematical communication skills will be able to create a diverse representation, it will be easier in finding alternatives to solving problems that resulted in the increased ability to solve mathematical problems.

Mathematical communication as the foundation of the teaching and learning process is based on the recognition of social interactions in the classroom between the teacher, the students and mathematical knowledge. It is relevant with the statement of NCTM in Kadir [2] that communication is an essential part of mathematics and mathematics education. This argument clarify more and more the importance of communication in the learning of mathematics. The communication enables students to express their ideas to teachers and other students. Tinungki [3] also states that the mathematics communication ability is very important for the students so that they can solve mathematics problems by using high reasoning. Moreover, Marthino and Ponte [4] state that communication in the mathematics classroom is a special element in most recent curriculum reform movements.

In addition to communication, one aspect of critical concern in mathematics education is mathematics conceptual understanding of students. In mathematics education, understanding of mathematical concepts is an important basis for thinking in solving mathematical problems and real life problems.

Diverse students produce different abilities to understand concepts. Some possible causes of the diversity of students' ability of understanding mathematical concepts is the ability for different
mathematical communication. Ontario Ministry of Education [5] states that in order to observe the mathematical concepts and solve the problem mathematically, students need to read and interpret the information, stating his thoughts orally and in writing, listening to others, and thinking critically about ideas. Many students have low mathematical communication skills and also followed by the low in understanding of mathematical concepts.

These two aspects, communication and conceptual understanding of mathematics are the focus of this study. This study analysed the mathematical communication skills and students' understanding of mathematical concepts. In addition, it is also look at whether there is a relationship between mathematical communication skills and students' understanding of mathematical concepts. If this is true then efforts to increase students 'understanding of mathematical concepts can be done by increasing students' mathematical communication skill.

2. Method
This study is a mixed method using exploratory sequential design. Exploratory sequential design begins with the collection of qualitative data to explore a phenomenon, then proceed with the collection of quantitative data to explain the relationship between the variables in the study [6]. Subjects of this study is students in grade 10. They were given test to classify students’ mathematical communication ability and mathematics conceptual understanding with using QUASAR Cognitive Assessment Instrument (QCAI). The students then classified into four levels: very low (VL), low (L), High (H), and Very High (VH). In quantitative phase, the variables were students’ mathematical communication ability and students’ conceptual understanding of mathematics. The data were gathered through observation with field notes, interview, mathematical communication test, and conceptual understanding of mathematics test. In qualitative phase, the data analyzed using six steps: (1) preparing and organizing the data, (2) transcribing the interview, (3) coding the transcript, (4) interpreting the data based on the coding and themes, (5) reflecting the interpretation based on the literature, and (6) validating the result. In quantitative phase, the data of communication and conceptual understanding were analyzed descriptively and inferentially to look at whether there is relationship between communication and conceptual understanding of mathematics. The analysis used software R commander.

3. Result and Discussion

3.1. Qualitative Phase

Subject with very low in mathematical communication (Subject VL)
It can be revealed that the subject VL did not write the known information and the objectives of the problem. Students also did not write down the information specified in each matter. Students did not write it down because student was not accustomed to write down the answers or solution along with the known information and the objectives whereas one of the steps that students must do in solving mathematical problems is to make observations about the problem. It is relevant to ideas presented by Sedig in Yang, et al. [7] that in mathematical problem solving student can learn by observing, interacting and manipulating objects and represents the objects and concepts. Students who are not able or wrong in identifying the problem will have any errors in solving problem so they will obtain the wrong answer.

In addition, based on the results of students' work, student frequently described the problem and stated the solutions of problems with the wrong presentation of algebra. Students also did not use picture or charts that could help in solving mathematics problems whereas use of pictures and chart is one of the important things in the process of mathematical communication. This is relevant with the statement of Mooney et al. in Yang et al. [7] that students can perform mathematical communication in writing using descriptions, pictures, tables, diagrams, or mathematical symbols to provide important evidence of the ideas and concepts in math. In this case, student did not use the help of picture so that students was wrong in answering problem involving the picture.
Student are already able to use half complete representation to express mathematics concepts and his solutions. It can be seen from the student were able to answer almost right on several problems. For example, in problem number 1, student had been able to make a correct equation to solve the problem is by forming equation \[256,000 = 4000 \times 2^4\]. But on the next line student writes down the wrong process. In problem 2, students had been able to write comprehensive concept of the rectangle that the area of a rectangle can be obtained by multiplying the length and width of the rectangle. But after students multiplied the two fractions whose irrational denominator subject VL experienced errors by dividing the square root in the denominator in length and the width of the rectangle. Student was still not familiar with the rules of the division. But in another problems, subject erred in determining the strength of algebraic operations. This happened on the number 3. Students had been able to apply some of the concepts of logarithms. But subject VL was wrong in determining the algebraic operations which did before. The students preferred to work on subtraction operation and then proceed to the multiplication operation whereas algebraically, when an addition or subtraction operation and multiplication operation in an equation, then the first which is done is multiplication then operations of addition or subtraction.

Subject VL in some problems did not give a reason in the process of problem solving whereas reason or explanation was very important to be included in the problem solving. It is relevant to Pugalee statement in Risnanosnti [8] in learning process student should give arguments on each answer and provided feedback in the answers given, so that the material learned was more meaningful for students.

Subject with low in mathematical communication (Subject L)

Subject L did not write down the information that was known from the problem completely. Students just wrote this in problem 3. As for the other problems, the students did not write information known and the objective.

In this problem, subject L was also error in presenting solution algebraically. In determining the area of rectangle, subject L wrote the results of \[21 \times 1 = 3\] whereas should be \[21^2\]. In addition to the problem number 1, subject L actually had been able to write the correct form of the initial equation. However, because students was not confident with the equation that was made, then subject L deleted his answer and replaced with the equation that has the same values. Next in problem 6 Students was less precise in writing algebraic form of solution. In this problem, \[8 - 2 \cdot 2 \log A\] was the formula function which maps \[A\] to \[B\]. So he should write in form \[B = 8 - 2 \cdot 2 \log A\]. But students only wrote \[8 - 2 \cdot 2 \log A\] in his solution, so it did not explain to the reader the meaning of \[8 - 2 \cdot 2 \log A\]. In addition, in the 5th row, he wrote the number 2 not as numerous of logarithm but as a multiplier of the logarithm. In the solution of problem 7, subject L used equal sign directly in the first step without any numbers or variables in front of the equal sign. Besides writing errors algebraic form of an equation, he was also able to write some equations correctly, for example, equation in problem 3 was \[k = 2p + 2l\].

From problem 1 to problem 7, he barely able to write down the answers in accordance with the problem objective, although in some problems students got wrong calculation results. For example, in problem number 1. In the other hand, the problem number 5, students misunderstood the problem objective, so that his answer was only partially correct. Students equated form \[0.15 \times 10^7\] with \[150000000\]. Whereas actually the objective of this problem was subject able to write the simplest scientific notation form and then compared the value of \[0.15 \times 10^7\] but he directly compared 150000000 with \[0.15 \times 10^9\] .

Students had already able to use half comprehensive representation to express mathematics concepts and solutions. It can be seen from the subject L was able to answer almost right in some problems using the correct concept. For example, in problem 3, students had been able to write the concept of perimeter correctly. Moreover, the same thing happened in problem 6. Students had been able to apply some of the concepts of logarithms. But the he was wrong in determining the algebraic operations must be do fist. Students preferred working on subtraction operation and then proceed to
the multiplication operation algebraically, whereas when an addition operation or subtraction and multiplication in an equation, then multiplication done before operations of addition or subtraction.

Subject L did not give reasons in his solution. For example, the solution in problem 5. Students would not give or explain an idea used in solving the problem whether in the form of information or in any other form.

Subject with high in mathematical communication (Subject H)
Subject H was able to write the known information from the problems completely in several problems. Students wrote the known information and the objective completely in problem 1 and 2. However, the students did not write the information and the objective of problem 3.

Subject H was able to describe the problem and the solution correctly in some problem. In problem 1, subject H was able to write the equation of the number of viruses at a certain time correctly. Subject H also was correct in writing the concept of the perimeter as the sum of the four sides. But the mistake occurred in the form of presentation algebraic form was done in problem 4 and 7. In problem 4, subject H used equal sign directly in comparing the value between $(-2^8)^4$ and $-(2^8)^4$. It also happened in problem 7, which directly wrote the equal sign when comparing the value of $\sqrt{777^2 + 777^2}$ and $\sqrt{(777)^2}$.

From problem 1 to 7, subject H barely able to write down the answers in accordance with the problem objective, although in some problems subject H got wrong calculation results. For example in problem number 1. In the other hand, the problem number 5, students misunderstood the problem objective, so that his answer was only partially correct. Students equated form $150000000$ with $0.15 \times 10^8$. Whereas actually the objective of this problem was subject able to write the simplest scientific notation form and then compared the value of $150000000$ with $0.15 \times 10^8$.

Subject H are already able to use half representation to express the mathematics concepts and solutions. It can be seen from subject H were able to answer almost right on some problems using the correct concept. For example, in problem 3, subject H had been able to write perimeter concept correctly was $K = 2(p + l)$. Moreover, the same thing happened to number 6. Subject H had been able to apply some of the concepts of logarithm. Subject H was also able to express multiplication of exponential into other simpler form. Subject H used the concept of the exponential correctly. But subject H had gone wrong in determining the algebraic operations. Subject H preferred to work subtraction operation first and then proceed to the multiplication operation algebraically, when an addition operation or subtraction and multiplication in an equation, then multiplication is done before the operations of addition or subtraction.

Subject H wrote down the reasons in determining the solution. But subject H was wrong so that the final answers were also wrong. For example, in problem number 4, subject H used equal sign in the first step and immediately assume that these two values were equal. Though both of these values to be compared. This is the reason given by subject H in determining the difference or not depending on the value. In addition, subject H did not include a reason in the selection of a final answer. For example, the completion of Problem 1. Subject H did not give or explain the reasons for selecting a value to obtain the amount of the viruses as much as 256000.

Subject with very high in mathematical communication (Subject VH)
Subject VH was able to write known information from the problems in some problems. Students wrote completely the known information and the objective of problem 2 and 3. Thus, students could explain the problem to be solved. This is relevant to Ministry Education of Ontario [5] which states that the identifying information provided and the information needed will help students explain the problem in order to be better understood.

Subject VH was able to describe a problem and the solution correctly in all problems. Students had been able to write every equation of any settlement of problems correctly. Students are also able to determine the order of algebraic operation. This indicator has been in line with any mathematical
communication abilities indicators proposed by Yang, et al. [7] that the student is able to express the concept of mathematics and mathematical symbols in solving problems.

From problem 1 to 7, subject VH were able to write down the answers according to the problem objective. For example, in problem 3. Most of the students in her solution only equated with \( 0.15 \times 10^9 \) without wrote the simplest scientific notation of \( 1.5 \times 10^9 \) then compared its value with \( 150000000 \) but subject VH had wrote the simplest form of scientific notation and compared them.

Students was already able to use a comprehensive representation to express mathematical concepts and solutions. It could be seen from the students were able to answer correctly at all using the right concepts. Subject VH also had been able to connect between one concept to another. For example, in solution of problem 3. Students used the Pythagorean concept and radical form.

In addition, students wrote down the reasons in determining solution. Every reason given by students was logical and understandable. Thus if someone asked her about the her way, she understood and going feedback. This is relevant to the statement of Pugalee in Risnanosanti [8] which states that student learning should give arguments on each answer and provide feedback on the answers given, so that what is learned is more meaningful for students.

### 3.2. Quantitative Phase

As result of mathematical communication ability, the average value is 63.85, median is 65.00, minimum is 25.00 and maximum value is 95.00. Meanwhile, the average score of conceptual understanding ability is 55.38, median is 54.00, minimum score is 25.00 and maximum score is 86.00. The data of communication and conceptual understanding of mathematics are shown in Table 1.

| Interval | Category | Communication | Conceptual understanding |
|----------|----------|---------------|--------------------------|
| 0 – 25   | Very Low | 3             | 1                        |
| 26 – 50  | Low      | 10            | 13                       |
| 51 – 75  | High     | 12            | 22                       |
| 76 – 100 | Very High| 14            | 3                        |

In table 1, the data shows that the number of students who already have a category level very high in communication skills are more than the number of students in the other categories. Meanwhile the number of students who already have a category level very low in conceptual understanding ability is more than the number of students in the other categories.

Inferential analysis is used to test the hypotheses of quantitative research phase. The test used is Chi-square test \( (\chi^2) \). Chi-Square test is used to find whether there is any relationship between the levels of mathematical communication skills of students with the levels of mathematics conceptual understanding students'. This testing used statistics software, R Commander. The result shown in Figure 1.

Figure 1. Chi-Squared test with R Commander

\[ \chi^2 = 8.7727, \text{ df } = 9, \text{ p-value } = 0.4585 \]

The value of Chi-Square \( (\chi^2) \) count obtained is 8.7727 with a degree of freedom 9. The significance value was \( \alpha = 0.05 \). Chi-Square \( (\chi^2) \) Value in table test with 9 degrees of freedom and significance was 16.919. Because \( (\chi^2) \) count less than \( (\chi^2) \) table, which is 8.7727 < 16.919, then \( H_0 \) was accepted. In other words, the data showed that there was no relation between the levels of students
'mathematical communication ability with the levels of students' mathematics conceptual understanding.

Lack of relation between mathematical communication ability and mathematics conceptual understanding is also seen from the result of their test. Students whose various ability in mathematical communication and mathematics conceptual understanding. Some of students whose high mathematical communication ability also have various mathematics conceptual understanding. It means that for every level of mathematical communication, there are students whose very low, low, high, or very high mathematics conceptual understanding. Some of students who able to use concept well, don’t have high written mathematical communication. Students in low mathematical communication ability but have high mathematics conceptual understanding just solve the problems with use concept they know without note the aspect of communication. For example students didn’t write known information and the objective of problems and also didn’t give reasonable explanation in problem solving. In other side, students whose low mathematics conceptual understanding ability but have high mathematical communication ability wrote the known information and the objectives of problem although in their solution, they use wrong concept. Also, students provided information in sentences form in their solution. Therefore, this confirms the conclusion that there is no relation between mathematical communication ability and mathematics conceptual understanding of students.

4. Conclusion

This study describes students’ mathematical communication and conceptual understanding for four level of communication. It also shows that there is no relation between the mathematical communication and conceptual understanding. However, it is need further study on relation between writing and oral communication. It is interesting to reveal the connection between these two kinds of communication in mathematics.

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