Design and research of two-point contact type stacked roller support two-dimensional piston pump

Chuantan Ruan, Chenchen Zhang, Heyuan Wang, Jian Ruan and Sheng Li

Abstract
To address the effect of clearance on the efficiency of two-dimensional piston pumps, this paper proposes a novel two-point contact type stacked roller support two-dimensional piston pump that eliminates the effect of guide rail and cone roller clearance on mechanical efficiency and volumetric efficiency in the inertia force-balance two-dimensions piston pump. It achieves two-way force balance support for the hydraulic static pressure and inertia force of the cone rollers through the overlapping mutual support and friction transmission of the double-layered cone rollers, which can effectively compensate for the gap between the rollers and the guide rail. The structure and operating principle of a two-dimensional piston pump are described first in this study, establish the mechanical efficiency model and volumetric efficiency model. Building an experimental bench for testing, the outcome is consistent with the theoretical analysis, and the difference is within 6%, demonstrating that the theoretical analysis is correct.

Keywords
Two-dimensional piston pump, mechanical efficiency, volumetric efficiency, design for manufacture, experimental research

Date received: 22 August 2022; accepted: 24 October 2022
Handling Editor: Chenhui Liang

Introduction
Compared with other transmission methods such as mechanical transmission and electric transmission, hydraulic transmission has a series of advantages such as high power density, compact structure, easy automation, and good dynamic performance. It is widely used in construction machinery, robots fields. The hydraulic pump, located at the heart of the hydraulic system, is in charge of transforming mechanical energy into hydraulic energy by supplying the hydraulic system with exact pressure and flow of a transmission medium. The axial piston pump is the most common of these, with three critical friction pairs: cylinder-port plate, piston-cylinder block, and slipper-swash plate. These three friction pairings limit the axial piston pump’s speed and power density growth. Additionally, the axial piston pump’s cylinder body will experience the piston’s overturning moment while it is operating, which could result in failure and fracture in extreme circumstances. Scholars at home and abroad have worked tirelessly to analyze the sliding friction pair in the axial piston pumps, striving to free it from its restrictions on speed and power density.
Xing et al.’s team has been engaged in the research of two-dimensional hydraulic components for a long time, applied the two-dimensional concept to hydraulic pumps, and proposed a two-dimensional piston pump. Because the two-dimensional piston pump has an axisymmetric structure, the piston is constantly in a condition of balance between radial and axial forces. The two-dimensional piston pump’s guide-roller mechanism uses rolling friction as its primary kind of friction to reduce the effects of conventional friction and circumvent the performance limitations imposed by conventional friction pairs. The two-dimensional piston pump is a positive displacement pump essentially, so there are some challenges such as cavitation and overturning moment. The working chamber of the two-dimensional piston pump’s pressure impact product and the instantaneous displacement change are what create flow pulsation. Therefore, Jin et al. presented a two-dimensional piston dual pump that uses the insertion and misalignment of the two pistons to lessen the hydraulic pump’s flow pulsation. In order to minimize the negative effects brought on by the axial force overturning moment, Huang et al. showed a two-dimensional piston pump that was force-balanced. This pump balances the cylinder body and reduces vibration and noise by using an inventive construction of inner and outer pistons. Huang et al. analyze of the force-balanced two-dimensional transmission mechanism’s churning loss using CFD and experimental research to the development of an empirical equation for churning loss torque. However, during the experiment, it was found that there is a gap which product by machining errors and assembly errors between the two guides rail sets and the cone roller. At high speeds, the existence of this gap will make the guide rail continue to hit the stacked rollers, aggravate vibration.

In order to reduce the gap’s impact on the two-dimensional pump’s efficacy, the author suggests a brand-new two-point contact type stacked roller support two-dimensional piston pump (two-point stacked roller two-dimensional piston pump). The double-layer cone rollers’ overlapping mutual support and friction transmission allow for two-way force balancing of the double-piston hydraulic static pressure and inertia force. When the number of rollers is increased, the distance between the guide rail and the cone roller is effectively reduced while the number of strokes and displacement are increased. The cross-sectional area and working stroke of the piston are significantly reduced due to the 18 times that oil is sucked in and discharged in one spin, and the improvement in power density. The piston’s cross-sectional area is reduced, which lessens the pressure exerted on it by the high-pressure chamber, lowers the input torque needed to operate, and improves mechanical efficiency. The structure and operation of the two-point stacked-roller two-dimensional piston pump are thoroughly described in this paper. Following that, a mechanical and volumetric efficiency analytical model was developed. First the force analysis of a single piston, and a mechanical efficiency model is built after modeling the guide rail’s space surface. The regularity of the flow and pressure changes in a single oil chamber is then used to create a volumetric efficiency model. Based on this, an experimental prototype was created, processed, and an expert experimental bench was constructed. Test the experimental prototype’s mechanical and volumetric efficiency at varied load pressures and speed. The sources of the inconsistencies in the results are then determined by comparing the theoretical analysis and experimental outcomes.

Structure

Figure 1 depicts the two-point stacked-roll two-dimensional piston pump’s structural layout. The two-point stacked-roll two-dimensional piston pump consists of a drive rail group, a balance rail group, a stacked roller set, and a distribution cylinder group. The stacked roller set of the 2D piston pump consists of six cone rollers stacked in pairs. During operation, the stacked cone roller sets adhere closely to the surfaces of the two guide sets by load pressure to ensure that the 2D piston and piston ring work with the required stroke. The drive rail group, as depicted in Figure 1(b), is made up of pistons and drive guide rails. A pair of conjugated end-face cams with equal acceleration and deceleration surfaces make up the left drive guide rail and the right drive guide rail. The right drive rail is mounted on the piston in a peak-to-valley fashion. As shown in Figure 1(c), the balance rail group is composed of pistons and balance guide rails. The driving guide rail group and the balancing rail group always have opposite axial movement directions, but their structure is the same. The installation phase is different by half of the operating cycle. The distribution cylinder assembly is made up of the main shaft, the distribution cylinder block, the left flange baffle plate, the right flange baffle plate, and the bushings, as seen in Figure 1(d). The distribution cylinder has six distribution windows evenly distributed in the last week, and there are three left distribution grooves and three right distribution grooves on the pump casing. To achieve oil suction and discharge, they are positioned in a staggered axial orientation and communicate alternately with the distribution window on the distribution cylinder.

Figure 2 depicts the two-point stacked-roller two-dimensional piston pump’s flow distribution method: A, B, and C are used to represent the left distribution grooves in the figure, while D, E, and F are used to
represent the right distribution grooves. M stands for
the piston, and G, H, I, J, K, and N are used to represen-
t the distribution windows. Low-pressure oil is shown in
the blue area and high-pressure oil is shown in
the red area. The main shaft’s rotational angle is 0°,
the distribution grooves A, B, C, D, E, and F do not
connect with the distribution windows G, H, I, J, K,
and N, the piston M is at the right-most end, the left
chamber has the highest volume, and the right chamber
has the smallest volume, as shown in Figure 2(a). The
distribution grooves A, B, C, D, E, F and the distribu-
tion window G, H, I, J, K, N start to communicate
when the main shaft rotates from 0° to 30°, which also
causes the piston M to accelerate and travel to the left.
When the main shaft rotates at a 30° angle, as shown
in Figure 2(b), the communications area between the
distribution grooves A, B, C, D, E, and F and the distribu-
tion windows G, H, I, J, K, N is at its greatest. The piston M slows down and moves to the left as
the main shaft rotates from 30° to 60°. Additionally,
when the right chamber R expands and the left
chamber L contracts, the communication space
between the distribution grooves A, B, C, D, E, F and
the distribution windows G, H, I, J, K, N is confined. The distribution windows G, H, I, J, K, and N are in a
state of non-communication when the main shaft
rotates at 60°, as shown in Figure 2(c), with the left
chamber L having the smallest volume and the right
chamber R having the largest volume. As the main
shaft rotates from 60° to 90°, the volume of the left
chamber L increases, the volume of the right chamber
R decreases, the piston M accelerates and travels to the
right, and the distribution grooves A, B, C, D, E, F
and the distribution window G, H, I, J, K, N begin to
communicate. When the main shaft rotates at 90°,
as shown in Figure 2(d), the volumes of the left L and
right chambers R are equal, and the communication
area between distribution grooves A, B, C, D, E, F and
distribution windows G, H, I, J, K, N is maximized.
The piston M slows down and slides to the right while
the main shaft rotates from 90° to 120°. At the same
time, the right chamber R continues to get smaller and

Figure 1. The structure schematic diagram of the two-point stacked-roller two-dimensional piston pump: (a) structure
diagram, (b) drive rail set, (c) balance rail set, and (d) distribution cylinder assembly.
Figure 2. The flow distribution diagram of two-point stacked-roll two-dimensional piston pump: (a) the rotating 0° state, (b) the rotating 30° state, (c) the rotating 60° state, and (d) the rotating 90° state.
the left chamber L continues to get bigger. Until the spindle spins at 120° and returns to the condition as shown in Figure 2(a), the communication area between the distribution grooves A, B, C, D, E, and F and the distribution windows G, H, I, J, K, and N decreases. During this operation, the right distribution grooves D, E, and F continue to feed low-pressure oil while the left distribution grooves A, B, and C always discharge high pressure oil. The left and right chambers alternately operate as oil suction and discharge units. For each motor spin, the left and right chambers complete three cycles of oil suction and discharge.

Analytical modeling

Mechanical efficiency

The rail Surface Equation. The two-point stacked-roll twodimensional piston pump relies on the stacked roller set to force the guide rail and the piston to reciprocate to ensure that the two-dimensional piston pump can work normally. As a result, the space surface model of the stacked roller set and the guide rail must be created in order to develop the mechanical efficiency model of the two-point stacked-roll two-dimensional piston pump.

Figure 3 depicts the cone roller’s angled relationship. Among them, γ is the angle formed by the cone roller’s axis and the contact plane, and β is the cone roller’s half-cone angle.

The space surface equation of the guide rail can be established using information from the cone roller’s shape, motion law, and guide rail. Through homogeneous coordinate transformation, the process of transforming the motion coordinate system $O_3 - X_3Y_3Z_3$ of the cone roller to the global coordinate system $O_0 - X_0Y_0Z_0$ is shown in Figure 4.

Rotate the motion coordinate system $O_3 - X_3Y_3Z_3$ around $X_3$ by $\pi / 2 - \gamma$ degrees, rotate the $Z_3$ axis to the $Z_2$ axis, which is the central axis of the guide rail, and obtain the intermediate coordinate system $O_2 - X_2Y_2Z_2$. Rotate the intermediate coordinate system $O_2 - X_2Y_2Z_2$ around the $Z_2$ axis by $\phi$ degrees to obtain the intermediate coordinate system $O_1 - X_1Y_1Z_1$. Move the intermediate coordinate system $O_1 - X_1Y_1Z_1$ along the $Z_1$ axis by $s$ to obtain the global coordinate system $O_0 - X_0Y_0Z_0$. As a result of the aforementioned procedure, the cone roller’s outer surface equation $R$ is expressed in the global coordinate system $O_0 - X_0Y_0Z_0$ as indicated in equation (1).

$$
R = \frac{L \cos \phi \tan \beta \cos \theta + L \cos \gamma \sin \phi - L \tan \beta \sin \gamma \sin \phi \sin \theta}{L \tan \beta \cos \phi + L \cos \tan \beta \sin \gamma \sin \phi - L \cos \gamma \cos \phi} \\
\frac{L \cos \gamma \tan \beta \sin \theta \sin \gamma + s}{L \cos \gamma \tan \beta \sin \theta}
$$

Among them, $s$ is the guide rail’s axial displacement, $\phi$ is the guide rail’s angle of rotation, $L$ is the separation between any exterior cone roller point and the plane $X_3O_3Z_3$, $\theta$ is the cone roller’s rotational angle.

Only one of the surfaces needs to be studied because the guide rail’s surface is made up of three equal acceleration and deceleration surfaces. The expression of the guide rail’s axial displacement is shown in equation (2).

$$
s = \begin{cases} 
\frac{18h}{\pi^2} \phi^2 & \phi \in \left[0, \frac{\pi}{6}\right) \\
-h - \frac{16h}{\pi^2} \phi^2 + \frac{12h}{\pi} \phi & \phi \in \left[\frac{\pi}{6}, \frac{\pi}{2}\right) \\
8h + \frac{16h}{\pi^2} \phi^2 - \frac{24h}{\pi} \phi & \phi \in \left[\frac{\pi}{2}, \frac{2\pi}{3}\right)
\end{cases}
$$
Where \( h \) is the guide rail’s axial movement stroke located.

According to the single-parameter surface family envelope theory, equation (1) can be written as a function with three variables. \( f = f(\theta, L, \varphi) \). The motion relationship between the guide rail and the cone roller can be used to generate the constraint equation, as shown in equation (3).

\[
\frac{\partial f}{\partial \theta} \times \frac{\partial f}{\partial L} \cdot \frac{\partial f}{\partial \varphi} = 0
\]  

(3)

Substituting equation (3) into \( f = f(\theta, L, \varphi) \), the guide rail surface equation can be obtained.

\[
f = f(L, \varphi)
\]  

(4)

**Spatial angle function.** To solve the tangent vector, normal vector at the contact point, and their projection on the bottom surface of the roller body, the parameterization of the guide rail surface equation is used. This makes it possible to calculate the relationship between each vector’s space angles, the rotation angle \( \varphi \), the distance \( L \), and the angle of the active force in the global coordinate system \( O_0 - X_0Y_0Z_0 \).

The friction force \( f_\theta \) and sliding friction force \( f_L \) along the cone roller’s bus can be calculated using \( f = f(\theta, L, \varphi) \) for the parameters \( \theta \) and \( L \). Concurrently, the normal vector \( n_e \) of the cone roller’s outer surface can be obtained, as shown in equation (5).

\[
n_e = \frac{f_\theta \times f_L}{|f_\theta \times f_L|}
\]  

(5)

Consider the direction of the positive pressure component as being the projection \( \vec{n}_e \) it makes on the cone roller’s bottom.

Referring to Figure 4, the unit vectors \( i \) in the \( Y_3 \) direction, \( k \) in the \( Z_0 \) direction, and the unit vector in the circumferential direction of the guide rail are established at the point of contact to reflect the force’s direction. The resulting spatial angle from the unit vector mentioned before is as follows:

Where \( \alpha_N \) is the pressure angle, \( \alpha_f \) is the angle between the friction force and the guide’s axial motion, \( \alpha_{NC} \) is the angle between the positive pressure and the contact point’s circumferential direction, \( \alpha_{FC} \) is the angle between the friction force and the contact point’s circumferential direction, and \( \theta_N \) is the angle between the friction force and the axial direction.

\[
\alpha_N = \alpha_N(L, \varphi) = \arccos \left( \frac{n_e \cdot k}{|n_e|} \right)
\]  

(6)

\[
\alpha_f = \alpha_f(L, \varphi) = \arccos \left( \frac{|r_\theta \cdot k|}{|r_\theta|} \right)
\]  

(7)

**Mathematical model**

The guideway of the two-point stacked-roller two-dimensional piston pump is made up of six parts of equal acceleration and equal deceleration surfaces, thus one of these sections is chosen for investigation.

The inner and outer guide rail groups of the cam guide rail group are constantly in touch with the stacked roller set due to the gap compensating capabilities of the pump, which puts them under stress. The drive rail group and the balance rail group are in comparable situations, so the study will use the balance rail group as an example.

Taking the right limit position of the balance guide rail group as the starting point, when the pump spins from \( 0^\circ \) to \( 30^\circ \), the rails accelerate and travel straight to the left; when the pump rotates from \( 30^\circ \) to \( 60^\circ \), the rails decelerate and go straight to the left. The expression for the acceleration \( a \) as show in equation (11).

\[
a = \begin{cases} 
\frac{h}{23} n^2 & \varphi \in [0, \frac{\pi}{6}) \\
-\frac{h}{23} n^2 & \varphi \in [\frac{\pi}{6}, \frac{5\pi}{6})
\end{cases}
\]  

(11)

Where \( n \) is the cam guide rail group’s rotational speed.

Figure 5 depicts the force analysis of the balanced guide set. Newton’s second theorem can be used to list
the balancing equation in the axial direction of the balance guide rail group, as illustrated in equation (12).

\[3F_{N1} \cos \alpha_N - 3F_{N2} \cos \alpha_N' - 3F_{f1} \cos \alpha_f - 3F_{f2} \cos \alpha_f' - 3F_p - 3F_{sh} = m_1 \alpha\]

(12)

Where \(F_p\) is the piston force on the high pressure chamber, \(F_{N1}\) and \(F_{f1}\) stand for the support force and friction force of the roller group acting on the left guide, \(F_{N2}\) and \(F_{f2}\) stand for the support force and friction force of the roller group acting on the right guide, \(T_c\) is the torque lost by the pump churning the oil, and \(T_{sh}\) is the oil shear torque brought up by the circumferential rotation of the flow distribution cylinder. \(F_{sh}\) represents the oil shear force brought by the axial movement of the piston, \(T_d\) is the motor torque, and because only the force analysis on the balanced guide group is performed, \(T_c\), \(T_{sh}\), and \(T_d\) are all half of their values.

Because the large guide rail is in the return section, the applied force is at the same angle as mentioned in the previous section. The small guide rail, on the other hand, is in the push section and its angular phase differs from the angle mentioned in the previous section by half a working cycle. When the number of cone rollers is taken into account, the force of the cone rollers on the rail must be multiplied by half.

Where \(m_1\) is the weight of the balance rail group, and \(\alpha_N' = \alpha_N(L, \varphi + \pi/3)\), \(\alpha_f' = \alpha_f(L, \varphi + \pi/3)\)

The high-pressure force \(F_p\) on the piston, the oil shear force \(F_{sh}\) brought by the axial movement of the piston, the friction force \(F_{f1}\) of the stacked roller set to the left guide rail, and the friction force \(F_{f2}\) of the stacked roller set to the right guide rail, as shown in the following equation.

\[F_p = \Delta p A = \frac{p \pi(D_1^2 - D_2^2)}{4}\]  

(13)

\[F_{sh} = \mu_L \pi D_1 L_1 \frac{v_1}{\delta} + 2\mu_L \pi D_2 L_2 \frac{v_1}{\delta}\]  

(14)

\[F_{f1} = \mu_g F_{N1}\]  

(15)

\[F_{f2} = \mu_g F_{N2}\]  

(16)

In equation (13), \(\Delta p\) denotes the pressure differential between high and low pressure oil, \(A\) is the force area of the left chamber, \(D_1\) is the piston’s diameter, while \(D_2\) is the piston rod’s diameter.

In equation (14), \(L_1\) is the distance between the piston’s contact with the flow distribution cylinder, and \(L_2\) is the distance between the piston’s contact with the bushing, \(v_1\) is the speed of the balance guide rail group in relation to the distribution cylinder, \(\mu_L\) is the oil dynamic viscosity, \(\delta\) is the gap width.

Where \(\mu_g\) in equations (15) and (16) are the friction coefficient of the guide rail.

According to the torque balance, the circumferential balance equation of the balance guide rail group can be listed, as shown in equation (17)

\[\frac{1}{2}T_d - \frac{1}{2}T_c - \frac{1}{2}T_{sh} - 3F_{N1} \cos \alpha_N r_1 + 3F_{N2} \cos \alpha_N' r_2 - 3F_{f1} \cos \alpha_f r_1 - 3F_{f2} \cos \alpha_f' r_2 = 0\]

(17)

In the equation, \(r_1\) is the force arm of the pressure \(F_{N1}\) and the friction force \(F_{f1}\), which is the distance between the large guide rail’s axis and its midpoint, and \(r_2\) is the force arm of the pressure \(F_{N2}\) and the friction force \(F_{f2}\), which is the distance between the small guide rail’s axis and its midpoint. Where \(\alpha_N' = \alpha_{NC}(L, \varphi + \pi/3)\), \(\alpha_f' = \alpha_{FC}(L, \varphi + \pi/3)\)

The following equation shows the oil shear torque \(T_{sh}\) and the oil churning loss torque \(T_c\).

\[T_{sh} = \mu_L \pi D_2 (L_3 + L_4) \frac{\pi D_1}{120\delta}\]  

(18)

\[T_c = 8.019 \times 10^{-9} \cdot n^2 + 8.085 \times 10^{-6} \cdot n - 2.185 \times 10^{-3}\]  

(19)

In equation (18), \(L_3\) is the smallest distance from the distribution window to the left side, \(L_4\) is the shortest distance from the distribution window to the right side, and \(D_2\) is the diameter of the distribution cylinder.

The churning loss \(T_c\) is calculated by fitting equation (19) to simulation and experimental data.

The stacked roller set force analysis is used to calculate the force exerted by the stacked roller set on the guide rail. Because the bearing pressure and operating conditions of the six cone rollers are the same, only one of them is studied, as shown in Figure 6.

Where \(F_u\) and \(F_b\) are the forces exerted by the adjacent roller on this roller, respectively. \(F_{N1}', F_{f1}\) are the right guide of the drive guide set’s friction and support force on the roller set, \(F_{N2}', F_{f2}\) are the right guide of

Figure 6. The schematic diagram of the force of the cone roller.
the balance guide set’s friction and support force on the roller set, and \( F_s \) is the compression force provided by the oil.

Newton’s second theorem states that the equilibrium equations in the \( Y_3 \) and \( Z_3 \) directions can be listed, as shown in equations (20) and (21).

\[
F'_{N_i} \sin \beta + F'_{N_i} \sin \beta + F_a \sin \beta - F_s = 0
\] (20)

\[
F_a \cos \beta - F'_{N_i} \cos \beta \cos \theta_s' + F'_{N_i} \sin \theta_s' - F'_{N_i} \cos \beta \cos \theta_s = 0
\]

\[
F_s = \Delta P A_s
\] (22)

Where \( \theta_s' = \theta_s (L, \phi + \pi/3) \), \( A_s \) is the inner chamber area of the roller.

To make the mathematical model simpler, convert all frictional forces to the friction coefficient normal pressure product. And consider that \( F_a \) and \( F_s \) are equal in size, \( F'_{N_i} \) and \( F'_{N_i} \) are equal in size, and \( F'_{N_i} \) and \( F'_{N_i} \) are equal in size.

According to the above premise, the expression of \( T_d \) can be obtained by combining equations (12), (17), (20), and (21).

\[
T_d = T_d(\phi, \Delta \rho, n)
\] (23)

The mechanical efficiency of the two-point stacked-roll two-dimensional piston pump can be calculated by equation (24).

\[
\eta_m = \frac{\Delta P V_D t_{00, deg}}{2 \pi \int_{0}^{t_{00, deg}} T_d dt}
\] (24)

Where \( t_{00, deg} \) is the amount of time it takes for the pump to revolve from 0° to 60° and \( V_D \) is the pump displacement.

**Volumetric efficiency**

The driving guide rail group and the balance guide rail group of the two-point stacked roller two-dimensional piston pump function on the same principles, and the volumetric efficiency is modeled using the balancing guide rail group as an example. The left chamber has the most capacity in its initial position.

When the compressibility of the oil is considered, equation (25) describes the instantaneous pressure shift in the left chamber.

\[
\frac{dp}{dt} = \frac{\beta_s}{V_L} (q_o + q_i + q_L + \frac{dV_L}{dt})
\] (25)

where \( p \) is the left chamber’s instantaneous pressure, \( \beta_s \) is the oil’s elastic modulus, \( q_o \) is the oil discharge flow of the left chamber, \( q_i \) is the left chamber’s oil suction flow, \( q_L \) is the left chamber’s leakage flow, and \( V_L \) is the left chamber’s immediate volume.

The interaction of the piston and bushing, which are both fixed to the guide rail, determines the instantaneous volume of the left chamber. The piston’s movement speed \( v_1 \), which affects how quickly the balanced guide rail group moves, can be expressed as follows:

\[
v_1 = \begin{cases} 
    at & 0 < t \leq \frac{5}{n} \\
    a(t - \frac{10}{n}) & \frac{5}{n} < t \leq \frac{15}{n} \\
    a(t - \frac{20}{n}) & \frac{15}{n} < t \leq \frac{20}{n}
\end{cases}
\] (26)

Where \( \omega \) is the rotation’s angular velocity of the guide rail. The \( t \) in formula (26) represents the rotation of the spindle from 0° to 30°, from 30° to 60°, from 60° to 90°, and from 90° to 120°, respectively.

The speed \( v_1 \) and acceleration \( a \) of the piston can be used to compute the instantaneous volume change \( V_L \) of the left chamber volume, which can be represented as follows:

\[
V_L = \begin{cases} 
    V_M - \frac{1}{2} Aa t^2 & 0 < t \leq \frac{5}{n} \\
    V_M - Aa \frac{25}{n} + A \cdot \frac{1}{2} (a(t - \frac{10}{n}) - t)^2 & \frac{5}{n} < t \leq \frac{10}{n} \\
    V_M - Aa \frac{25}{n} + A \cdot \frac{1}{2} (t - \frac{10}{n})^2 & \frac{10}{n} < t \leq \frac{15}{n} \\
    V_M - \frac{1}{2} Aa (t - \frac{20}{n})^2 & \frac{15}{n} < t \leq \frac{20}{n}
\end{cases}
\] (27)

Where \( V_M \) denotes the left chamber’s maximal volume. The \( t \) in formula (27) represents the rotation of the spindle from 0° to 30°, from 30° to 60°, from 60° to 90°, and from 90° to 120°, respectively.

\[
\frac{dV_L}{dt} \text{ could derivation from equation (27) gives:}
\]

\[
\frac{dV_L}{dt} = \begin{cases} 
    -Aa t & 0 < t \leq \frac{5}{n} \\
    -Aa (t - \frac{10}{n}) & \frac{5}{n} < t \leq \frac{15}{n} \\
    -Aa (t - \frac{20}{n}) & \frac{15}{n} < t \leq \frac{20}{n}
\end{cases}
\] (28)

The \( t \) in formula (28) represents the rotation of the spindle from 0° to 30°, from 30° to 90°, and from 90° to 120°, respectively.

According to standard orifice can be used to express inlet flow and output flow, as show in equations (29) and (30).

\[
q_i = C_d A_{in} \sqrt{\frac{2[p_T - p]}{\rho} } \sign(p_T - p)
\] (29)

\[
q_o = C_d A_{out} \sqrt{\frac{2[p - p_{load}]}{\rho} } \sign(p - p_{load})
\] (30)

Where \( C_d \) is the flow coefficient, \( p_T \) is the tank pressure, \( p_{load} \) is the load pressure, \( \rho \) is the oil density, \( A_{in} \) is the outlet area and \( A_{in} \) is the oil inlet area, which can be expressed as:
and differential pressure flow (Couette-Poiseuille flow) can be described by equation (35):

\[ q_{Li1} = \frac{\Delta p \pi D_3}{12 \mu L_1} \delta^3 - \frac{\pi D_3 \delta}{2} v_1 \]  

Internal leakage \( q_{Li2} \) passes via the area between the distribution cylinder and the casing to reach the low-pressure chamber from the high-pressure chamber. The internal leakage \( q_{Li2} \) may be described by the small hole leakage model when the distribution cylinder spins at \( 0 \)°, as the distribution cylinder rotates, the distribution grooves and windows begin to interact, and the internal leakage \( q_{Li2} \) may be represented by the gap leakage model. It can be expresses by equation (36).

\[ q_{Li2} = \begin{cases} L_g \cdot \frac{1}{12 \mu_2} \delta^3 (p - p_T) & h < k \\ \delta C_d L_g \frac{[p - p_T]}{2} \text{sign}(p - p_T) & h \geq k \end{cases} \]

\[ h = \left| L_g \cdot \frac{1}{12 \mu_2} \delta^3 (p - p_T) \right| k = \left[ \delta C_d L_g \frac{[p - p_T]}{2} \right] \]  

\[ L_r \] is the distance between the distribution cylinder’s outer and inner walls, and its variation law is shown in Figure 8, which may be explained by equation (37).

\[ L_r = \begin{cases} \frac{\pi D_3}{60} & 0 < t \leq \frac{5}{n} \\ \frac{\pi D_3}{60} \left( \frac{10}{n} - t \right) & \frac{5}{n} < t \leq \frac{10}{n} \\ \frac{\pi D_3}{60} \left( \frac{15}{n} - t \right) & \frac{10}{n} < t \leq \frac{15}{n} \\ \frac{\pi D_3}{60} \left( \frac{20}{n} - t \right) & \frac{15}{n} < t \leq \frac{20}{n} \end{cases} \]  

To sum up, the leakage \( q_L \) of the two-point stacked-roll two-dimensional piston pump can be expressed by equation (38).

\[ q_L = q_{Lo1} + q_{Lo2} + q_{Li1} + q_{Li2} \]
Table 1. Simulation parameter table.

| Parameter                                      | Value  |
|------------------------------------------------|--------|
| Oil Bulk Elastic Modulus $\beta$ (MPa)         | 1400   |
| Oil density $\rho$ (kg/m$^3$)                  | 850    |
| Oil viscosity $\mu$ (Pa s)                     | $3.91 \times 10^{-2}$ |
| Friction coefficient $\mu_f$                   | 0.070  |
| Flow Coefficient $C_f$                         | 0.62   |
| Inlet pressure $p_{in}$ (MPa)                  | 0.4    |
| Piston diameter $D_1$ (m)                      | $8 \times 10^{-3}$ |
| Piston rod diameter $D_2$ (m)                  | $5 \times 10^{-3}$ |
| The diameter of the distribution cylinder $D_3$ (m) | $33 \times 10^{-3}$ |
| The distance between the piston’s contact with the flow distribution cylinder $L_1$ (m) | $6 \times 10^{-3}$ |
| The distance between the piston’s contact with the bushing $L_2$ (m) | $10 \times 10^{-3}$ |
| The smallest distance from the distribution window to the left side $L_3$ (m) | $8 \times 10^{-3}$ |
| The shortest distance from the distribution window to the right side $L_4$ (m) | $8 \times 10^{-3}$ |
| Suction and discharge port width $L_c$ (m)      | $12 \times 10^{-3}$ |
| The stroke of the balance rail set $h$ (m)      | $1.5 \times 10^{-3}$ |
| Maximum left chamber volume $V_m$ (m$^3$)       | $1.1 \times 10^{-7}$ |
| Gap width $d$ (m)                               | $5 \times 10^{-6}$ |
| The angle formed by the cone roller’s axis and the contact plane $\gamma$ (deg) | 17.5 |
| The cone roller’s half-cone angle $\beta$ (deg) | 17.5  |
| The distance from the axis of the large guide rail to the curved surface of the large guide rail $r_1$ (m) | $24.5 \times 10^{-3}$ |
| The distance from the axis of the small guide rail to the surface of the small guide rail $r_2$ (m) | $21.5 \times 10^{-3}$ |
| Pump’s displacement $V_o$ (cc/rad)              | 5.3    |
| The mass of the balancing guide rail $m_1$ (kg)  | $90 \times 10^{-3}$ |

Finally, throughout the process from 0° to 120°, by integrating the output flow $q_o$ and comparing it to the theoretical flow rate, the discharge flow rate is calculated. As shown in equation (39).

$$\eta_v = \frac{\int_{0}^{120\text{deg}} q_o}{2Ah}$$ (39)

The time $t_{120\text{deg}}$ takes for the pump to revolve from 0° to 120°.

Theoretical analysis

The mathematical model described above is used to investigate the effect of rotational speed and load pressure on mechanical and volumetric efficiency. No. 15 hydraulic oil is selected as the working medium. Table 1 displays the key parameters.

Influence of rotational speed and load pressure on mechanical efficiency

Take the balanced guide rail group as an example, Figure 9 is the torque change curve. The graphic shows that the torque grows with increasing load pressure and speed, and that it changes quickly when the guide rail reaches the neutral point. This is due to the change in acceleration direction and the sudden change in the support force applied by the cone roller to the group of balance guide rails.

As illustrated in Figure 9(a), as the load pressure increases, so does the torque due to a variety of factors, including an increase in the support force $F_{N_i}$ of the cone roller to the balance guide rail group and an increase in the force $F_p$ of the high-pressure chamber on the piston. Figure 9(b) depicts how the torque increases with increasing rotational speed.

As the rotating speed increases, both the oil shear torque $T_{sh}$ and the churning loss torque $T_c$ increase. As the speed increases, so does the acceleration, which boosts the cone roller’s support force $F_{N_i}$ to the balance guide rail group, causing the torque to increase, as shown in Figure 10.

The link between mechanical efficiency, speed, and pressure is depicted in Figure 11. The two-point stacked-roll two-dimensional piston pump eventually requires more torque as rotational speed rises, and mechanical efficiency declines; The mechanical efficiency rises as the load pressure steadily rises because the pressure rise speed is greater than the torque rise speed.

Influence of rotational speed and load pressure on volumetric efficiency

Figures 12 and 13 show the output flow curves of the left chamber for various load pressures and speed conditions. The graphic shows that the output flow rises as the rotational speed rises and is less influenced by the load pressure. Because the left chamber’s immediate pressure is lower than the load pressure when oil discharge begins and the left chamber stops sucking in oil, oil will backflow.

While the rotating speed remains constant, the peak and duration of the backflow flow increase as the load pressure increases, as shown in Figure 12(b). As the load pressure rises, so does the pressure difference between the left chamber’s instantaneous pressure and the load pressure. This allows the backfill flow to assist the left chamber’s instantaneous pressure to rise to system pressure and start oil discharge.

As shown in Figure 13, increasing the rotating speed increases the output flow and peak value of the oil backflow, but decreases the duration. This is because, when a particular load pressure is assumed, the rotation speed increases, the rotation length decreases, and the peak value of oil backflow rises while the demand for oil backflow remains constant.
Figure 9. Torque $T_d$ curve: (a) different load pressure at speed at 4000 r/min and (b) different speed at load pressure at 8 MPa.

Figure 10. Pressure $F_{Ni}$ curve: (a) different load pressure at speed at 4000 r/min and (b) different speed at load pressure at 8 MPa.

Figure 11. The connection between rotational speed, load pressure, and mechanical efficiency: (a) different load pressure at speed at 4000 r/min and (b) different speed at load pressure at 8 MPa.
The leakage flow fluctuation curve for varied load pressures and rotational speeds is shown in Figure 14. When can be seen in Figure 14(a), as the pressure rises, the leakage flow rate increases, and as the rotating speed rises, the minimum value lowers. This is because leakage flow is estimated by subtracting shear flow from differential pressure flow; the greater the load pressure, the greater the differential pressure flow. When the rotational speed is fixed, the piston moves at the same rate as before, conserving shear flow and increasing the leakage flow rate. The differential pressure flow does not vary as the rotational speed increases while the load pressure remains constant; nevertheless, as the piston moves faster, the minimum value of the leakage flow drops. When the oil is released from the left chamber, the piston’s movement speed rises and then falls as the rotation angle increases, causing the leakage flow to climb and then reduce.

The relationship between volumetric efficiency, load pressure, and rotating speed is depicted in Figure 15. The volumetric efficiency curve, as depicted in the diagram, is consistent with the previous investigation’s findings. Backflow and oil loss increase as load pressure increases when the rotation speed remains constant, while volumetric efficiency decreases. While the load pressure remains constant and the rotating speed increases, oil backflow and leakage are reduced, and volumetric efficiency improves.

**Experiment analysis**

The two-point stacked-roll two-dimensional piston pump is displayed in Figure 16 and its important parameters are presented in Table 2 to confirm the aforementioned mathematical model and simulation findings.
An experimental bench, as shown in Figure 17, was built to test the volumetric and mechanical performance of the experimental prototype at various rotational speeds and load pressures. A gasoline supply pump, motor, torque-speed sensor, flow meter, and overflow valve,
and pressure sensor comprise the test bench. The test bench uses the data acquisition card to send data from the flowmeter and torque-speed sensor to the computer. The critical sensor parameters are listed in Table 3.

The temperature of the experimental environment is controlled at 25°C. The pump will be cooled to room temperature before the experiment, and during the test, the speed and pressure of the pump under test are rapidly ramped up to the target speed and pressure to minimize the effect of temperature rise.

**Experimental analysis of load pressure and rotational speed on mechanical efficiency**

Set the motor speed to 4000 rpm before adjusting the relief valve to control the pump station’s output pressure. When the torque stable, record the information to determine the link between mechanical efficiency and

| Description       | Parameter                                      |
|-------------------|-----------------------------------------------|
| Torque sensor     | Torque range: 0–20 Nm, accuracy ± 0.1%         |
|                   | rotational speed range: 0–18,000 r/min         |
| Pressure sensor   | Range: 0.05–80 L/min, accuracy ± 0.3%         |
| Flowmeter         | Range: 0–10 MPa, accuracy ± 0.3%              |

The temperature of the experimental environment is controlled at 25°C. The pump will be cooled to room temperature before the experiment, and during the test, the speed and pressure of the pump under test are rapidly ramped up to the target speed and pressure to minimize the effect of temperature rise.

**Experimental analysis of load pressure and rotational speed on mechanical efficiency**

Set the motor speed to 4000 rpm before adjusting the relief valve to control the pump station’s output pressure. When the torque stable, record the information to determine the link between mechanical efficiency and
load pressure, and compare it to the simulation’s outcomes, as illustrated in Figure 18(a). When the torque display was stable, the pump station’s output pressure was adjusted to stabilize it at 8 MPa, and the speed was increased in units of 1000 r/min. The simulation results were then compared to the data to determine the link between mechanical efficiency and rotational speed, as illustrated in Figure 18(b).

Although the modeling findings and experimental data appear to be in good agreement, the gap grows as load pressure and speed do as well. One reason for this is that as the experiment goes on, the oil temperature rises and the viscosity drops, reducing the lubricating effect and affecting mechanical efficiency. On the other hand, the cone roll’s condition changes as a result of the change in rotational speed and load pressure.

**Experimental analysis of volumetric efficiency of load pressure and rotational speed**

Repeat the aforementioned experimental processes and examine the flowmeter findings to determine the volumetric efficiency at various load pressures and rotating speeds.

The experimental findings, which are depicted in Figure 19, are in line with the shifting trend of the modeling curve. The experimental and simulated results differ just slightly.

On the one hand, this is because after a time of operation, the temperature of the oil rises and the viscosity lowers, increasing leakage. When the cone roller is running, the cone roller will move in the direction of the central axis due to the constant change in pressing conditions.
force, causing the eccentricity of the piston and the flow distribution.

**In conclusion**

In this paper, a novel two-point contact type stacked roller support two-dimensional piston pump is proposed. The mathematical modeling of the pump’s mechanical and volumetric efficiency was followed by an analysis of the impact of load pressure and speed on efficiency and the construction of an experimental bench for testing the two-dimensional piston pump. Following the validation of the mathematical model, the following results were drawn:

1. It is clear from the experimental and modeling results that mechanical efficiency will improve along with load pressure. The two effects of increasing the load pressure are as follows: To begin with, it increases the force between the cone roller and the guide rail, lowering mechanical efficiency. Second, increasing the load pressure allows more force from the high-pressure chamber to be exerted on the piston, boosting mechanical efficiency. However, because the former’s magnitude is worse than the latter’s, mechanical efficiency increases as load pressure increases. Furthermore, increasing the rotation speed improves mechanical efficiency by reducing losses caused by oil churning and shear flow. Although there is only a minor difference between the modeling and experimental results, the inaccuracy grows with speed and load pressure. This is because the oil’s temperature rises and viscosity decreases as the experiment goes, reducing lubrication and mechanical efficiency. The type of rolling friction between the guide rail and the cone roller is altered by increases in rotational speed and load pressure, on the other hand, which lowers mechanical efficiency.

2. The experimental and modeling results reveal that when load pressure increases, volumetric efficiency decreases due to increased oil loss and backflow. Increasing rotational speed also increases shear flow, which decreases leakage and improves volumetric efficiency. Despite the minor gap between the modeling and experimental results, the inaccuracy grows with speed and load pressure. This occurs as the temperature of the oil rises and the viscosity lowers with time, resulting in increased leakage. In contrast, as the pressing force changes, the cone roller shifts in the direction of the central axis, causing the piston and the flow distribution cylinder to be eccentric and increasing leakage.

**Author contributions**

Data curation, C.R.; writing – original draft preparation, C.R.; writing – review and editing, C.Z.; funding acquisition, J.R. and S.L.; formal analysis, H.W.

**Declaration of conflicting interests**

The author(s) declared no potential conflicts of interest with respect to the research, authorship, and/or publication of this article.

**Funding**

The author(s) disclosed receipt of the following financial support for the research, authorship, and/or publication of this article: This research was funded by the National Key Research and Development Program of China, grant number 2019YFB2005202.

**ORCID iD**

Chuantan Ruan https://orcid.org/0000-0002-4681-0215

**References**

1. Zhang J, Chao Q, Wang Q, et al. Experimental investigations of the slipper spin in an axial piston pump. *Measurement* 2017; 102: 112–120.
2. Chao Q, Zhang J, Xu B, et al. Test rigs and experimental studies of the slipper bearing in axial piston pumps: A review. *Measurement* 2019; 132: 135–149.
3. Yang H, Zhang B and Xu B. Development of axial piston pump/motor technology. *Chin J Mech Eng* 2008; 44: 1–8.
4. Chao Q, Zhang J, Xu B, et al. A review of high-speed electro-hydrostatic actuator pumps in aerospace applications: challenges and solutions. *J Mech Des* 2019; 141: 050801.
5. Zhang C, Huang S, Du J, et al. A new dynamic seven-stage model for thickness prediction of the film between valve plate and cylinder block in axial piston pumps. *Adv Mech Eng* Epub ahead of print 26 September 2016. DOI: 10.1177/1687814016671446.
6. Ouyang X, Wang T and Fang X. Research status of the high speed aircraft piston pump. *Chin Hydraul Pneum* 2018; 2: 1–8.
7. Zhang J, Chao Q and Xu B. Analysis of the cylinder block tilting inertia moment and its effect on the performance of high-speed electro-hydrostatic actuator pumps of aircraft. *Chin J Aeronaut* 2018; 31: 169–177.
8. Zhang J, Li Y, Xu B, et al. Experimental study on the influence of the rotating cylinder block and pistons on churning losses in axial piston pumps. *Energies* 2017; 10: 662.
9. Manring ND. Tipping the cylinder block of an axial-piston swash-plate type hydrostatic machine. *J Dyn Syst Meas Control* 2000; 122: 216–221.
10. Ivantysyn R. Computational design of swash plate type axial piston pumps a framework for computational design. MS Thesis, Purdue University, Indiana, 2011.
11. Wegner S, Gels S and Jang DS. Experimental investigation of the cylinder block movement in an axial piston
machine. In: ASME/BATH Symposium on fluid power and motion control, Chicago, IL, 12–14 October 2015. New York: ASME.

12. Chao Q, Tao J, Lei J, et al. Fast scaling approach based on cavitation conditions to estimate the speed limitation for axial piston pump design. *Front Mech Eng* 2021; 16: 176–185.

13. Ye S, Zhang J, Xu B, et al. A theoretical dynamic model to study the vibration response characteristics of an axial piston pump. *Mech Syst Signal Process* 2021; 150: 107237.

14. Ye S, Zhang J, Xu B, et al. Theoretical investigation of the contributions of the excitation forces to the vibration of an axial piston pump. *Mech Syst Signal Process* 2019; 129: 201–217.

15. Wegner DS, Lôschnier F and Gels S. Validation of the physical effect implementation in a simulation model for the cylinder block/valve plate contact supported by experimental investigations. In: 10th International fluid power conference, 2016.

16. Wu S, Yu B, Jiao Z, et al. Preliminary design and multi-objective optimization of electro-hydrostatic actuator. *Proc IMechE, Part G: J Aerospace Engineering* 2017; 231: 1258–1268.

17. Tsukiji T, Nakayama K, Saito K, et al. Study on the cavitating flow in an oil hydraulic pump. In: Proceedings of 2011 International conference on fluid power and mechatronics, Beijing, China, 17–20 August 2011, pp.253–258. New York: IEEE.

18. Zhao K, He T, Wang C, et al. Lubrication characteristics analysis of slipper pair of digital valve distribution axial piston pump. *Adv Mech Eng*, Epub ahead of print 16 March 2022. DOI: 10.1177/1687812221085442.

19. Wang T, Yin Y, Wang H, et al. Numerical study on tribological performance of the floating valve-plate pair in axial piston pump. *Adv Mech Eng*. Epub ahead of print 1 November 2014. DOI: 10.1177/1687814020983820.

20. Murrenhoff H and Scharf S. Wear and friction of ZRCG-coated pistons of axial piston pumps. *Int J Fluid Power* 2006; 7: 13–20.

21. Tao R, Xiao R, Yang W, et al. Investigation of the hydrodynamics of sweep blade in hi-speed axial fuel pump impeller. *Adv Mech Eng*. Epub ahead of print 1 January 2013. DOI: 10.1155/2013/174017.

22. Li WY, Zhang XY, Shuai ZJ, et al. CFD numerical simulation of the complex turbulent flow field in an axial-flow water pump. *Adv Mech Eng*. Epub ahead of print 1 January 2014. DOI: 10.1155/2014/521706.

23. Lin C, Wei Y and Zhao L. Design and analysis of new type of piston pump. *J Southwest Jiaotong Univ* 2018; 53: 602–609.

24. Zuti Z, Shuping C, Xiaohui L, et al. Design and research on the new type water hydraulic axis piston pump. *J Press Vessel Technol* 2016; 138: 031203.

25. Ericson L and Forssell J. A novel axial piston pump/motor principle with floating pistons: design and testing. In: *Proceedings of the fluid power systems technology, Bath, UK*, 12–14 September 2018.

26. Zhao J, Fu Y, Ma J, et al. Review of cylinder block/valve plate interface in axial piston pumps: theoretical models, experimental investigations, and optimal design. *Chin J Aeronaut* 2021; 34: 111–134.

27. Xing T, Xu Y and Ruan J. Two-dimensional piston pump: principle, design, and testing for aviation fuel pumps. *Chin J Aeronaut* 2020; 33: 1349–1360.

28. Jin D and Ruan J. Design and research of two-dimensional fuel pump. *Acta Aeronaut Astronaut Sin* 2019; 40: 318–327.

29. Qian J, Shentu S and Ruan J. Volumetric efficiency analysis of two-dimensional piston aviation fuel pump. *Aeronaut Astronaut Sin* 2020; 41: 271–283.

30. Shentu S, Ruan J and Qian J. Optimization analysis of flow characteristic and distribution window of 2D pump. *Trans Chin Soc Agric Mach* 2019; 50(12): 403–410.

31. Jin DC, Ruan J, Li S, et al. Modelling and validation of a roller-cam rail mechanism used in a 2D piston pump. *Journal of Zhejiang University-Science A* 2019; 20: 201–217.

32. Ruan J, Jin D and Shentu S. Research and feasibility verification of two-dimensional (2D) tandem pump. *Chin Hydraul Pneum* 2017; 11: 1–5.

33. Shentu S, Ruan J, Qian J, et al. Study of flow ripple characteristics in an innovative two-dimensional fuel piston pump. *JBraz Soc Mech Sci Eng* 2019; 41: 1–15.

34. Huang Y, Ruan J, Zhang C, et al. Research on the mechanical efficiency of high-speed 2D piston pumps. *Processes* 2020; 8: 853.

35. Huang Y, Ruan J, Chen Y, et al. Research on the volumetric efficiency of 2D piston pumps with a balanced force. *Energies* 2020; 13: 4796.

36. Huang Y, Ding C, Wang H, et al. Numerical and experimental study on the churning losses of 2D high-speed piston pumps. *Eng Appl Comput Fluid Mech* 2020; 14: 764–777.