1. Introduction

Intersections are the road network nodes that frequently cause traffic disruption, severe delays, and accidents in a city [1]. Therefore, corridor progression has always been an effective way to ensure traffic safety and efficiency at intersections [2, 3]. Generally speaking, the solution methods of the corridor progression design scheme can be roughly divided into three types: model method, algebraic method, and graphical method. In comparison, the model method can obtain multiple different ideal coordination design schemes. However, its modeling process is complex, requiring a long solution time for the sizeable calculated amount. The algebraic method, which can meet the requirements of corridor progression in various situations, has the advantages of good operability and robust reproducibility. However, practitioners often need to have a relatively complete theoretical knowledge of coordination planning. Meanwhile, the graphical method obtains coordination planning using time-space diagrams, illustrating the relationship between intersection spacing, signal timing, and vehicle movement. Although the graphical method is challenging to ensure the optimal corridor progression effect and measure the solution efficiency by manual drawing, it is suitable for engineering applications due to its slight computational complexity, strong operability, and intuitive reflection of the coordinated optimization process.

The model method constructs a linear or nonlinear programming optimization model based on the relationships between the progression bandwidth and signal timing parameters, travel time, and progression boundary trajectory [4]. Moreover, the mixed-integer linear programming method is usually used to realize the optimal solution of signal timing parameters. The most classic models are the MAXBAND model proposed by Morgan and Little [5] and the MULTIBAND model constructed by Gartner et al. [6]. The MAXBAND model optimized common signal cycle,
offsets, vehicle speed, and the order of left-turn phases to maximize the bandwidths, ensuring that most vehicles can drive through the downstream signal intersections without stopping [7]. The MULTIBAND model designed an individually weighted bandwidth for each directional road section, considering the traffic volumes and flow capacities. Extensive research has been constantly provided based on the MAXBAND and MULTIBAND models [8–10]. Optimizing the left-turn phase sequence of the MAXBAND model, Chang et al. [11] further established the MAXBAND-86 model. Lu et al. [12] introduced a bandwidth proration model, solving the OD-based traffic signal coordination problem. Defining the signal phase sequence as a decision variable, Yang et al. [13] constructed three types of multipath progression models based on the MAXBAND model. Zhang et al. [14] established an asymmetrical multi-band (AM-BAND) model and successfully achieved an excellent coordination effect. Zhang et al. [15] constructed the MAXBANDLA model, and numerical tests proved that the model could significantly reduce the number of stops and the average delay of vehicles on long corridors. Zhou et al. [16] built two multiobjectives uneven double cycles. By doing this, they allowed minor intersections to adopt uneven double cycling models to ensure the coordinated efficiency of minor intersections. Aiming to provide progression bands for all major O-D flows, Arsava et al. [17] constructed the OD-BAND model, which considered the significant turning traffic flows from and to cross streets. Arsava et al. [18] expanded the OD-BAND model to the OD-NETBAND model, solving the OD-based traffic signal coordination problem in multiarterial grid networks.

The algebraic method utilizes the numerical calculation method to obtain the optimal corridor progression design schemes, realizing the comprehensive optimization of signal phases. Xu [19] expounded on the classical algebraic method, that is, to determine the optimal common signal cycle and offsets according to the ideal intersection distance that best matches the actual one. Moreover, this method is only applicable under symmetrical phasing. On this basis, the related research of the algebraic method has gradually gained traction. Lu et al. [20] first addressed the theoretical limitations of the classical algebraic method and improved the algebraic method for symmetrical corridor progression. Analyzing the advantages and disadvantages of the classical algebraic method, Wang et al. [21] further improved the calculation method of the bandwidths and the offsets and the location matching method of actual intersection and ideal intersection. As research progressed, the research focus of the corridor progression has been gradually expanded from the algebraic method under the symmetrical phasing to split phasing and asymmetrical phasing. Lu et al. [22] established a bidirectional algebraic method under the split phasing, which can adapt to different corridor conditions, such as asymmetric geometry, large left-turn traffic volume, uneven traffic flow. Ji and Song [23] proposed an asymmetrical bidirectional algebraic method and analyzed the effects of vehicle speed and queued vehicles during the red light. Using the speed transformation and phase combination method, Lu et al. [24] also constructed a bidirectional algebraic method under asymmetrical phasing, effectively solving each road section’s asymmetric bidirectional distance and unequal speed.

The graphical method establishes a set of easy-to-understand drawing rules and utilizes the transformation of basic elements on the time-space diagram to obtain ideal progression bandwidths. However, scholars have done little research on the graphical method of corridor progression until now. China Highway Association [25] defined the intuitive and straightforward graphical method as one of the earliest timing design methods for corridor progression. Xu [19] provided the basic solution idea of the graphical method, but there are still many deficiencies during the solution process. It lacked a specific design principle and well-defined rules for optimizing the common signal cycle and offsets. Lu and Cheng [26] introduced the concept of the NEMA phase based on the graphical method and optimized the phase sequence and offsets by the graphical way. Although an ideal bidirectional bandwidth was obtained, there was no amendment to the shortcomings of the existing graphical method.

In this study, we define the trajectory lines that reflect the unrestrained movement of characteristic vehicles in the progression as progression trajectory characteristic lines (PTC lines). Then, based on the rotation transformation of the PTC line, a graphical method for symmetrical bidirectional corridor progression is proposed to improve further and optimize the overall process of the graphical method. The corresponding progression coordination design process will also be presented in detail. The remainder of the article is organized as follows: Section 3 introduces the design principle of the proposed method. Section 4 presents a case study, which helps to compare the coordinated optimization effect between the proposed method, the improved algebraic method, and the MAXBAND model. Concluding remarks are provided in the last section.

2. Materials and Methods

2.1. Design Process. The graphical method proposed can be solved by a normal step-by-step graphical process and programming. We define the horizontal scaling of the time-space diagram to convert the progression design speed of the road section to the corridor progression design speed $V$. Meanwhile, we also define the vertical scaling of the time-space diagram to realize the transformation of the optimization of the common signal cycle into the optimization of the corridor progression speed. The design process of the proposed graphical method is refined and organized as shown in Figure 1, which is mainly divided into four parts: initialization, the first round of rotation transformation, the second round of rotation transformation, and scheme generation.

2.2. Initialization

2.2.1. Initial Common Signal Cycle. Let us suppose that there are $n$ signalized intersections along the corridor and number the intersections in the ascending order in the outbound
direction. Then, the $p$th ($1 \leq p \leq n$) intersection is defined as $I_p$ in sequence along the outbound direction.

The common signal cycle range will be determined by the signal cycle range of each intersection, denoted as $[C_{\text{min}}, C_{\text{max}}]$. The initial common signal cycle $C_1$ can be valued as the midpoint between the minimum common signal cycle $C_{\text{min}}$ and the maximum common signal cycle $C_{\text{max}}$.

2.2.2. Adjusted Distance and Speed. Let $d_p$ and $v_p$ be the actual distance and the progression design speed between intersection $I_p$ and $I_{p+1}$ ($1 \leq p \leq n-1$), respectively. Then, we find that the optimal effect of progression coordination does not change in the time-space diagram as long as the travel time of the road section remains unchanged. For example, as shown in Figure 2, when we simultaneously change the progression design speed and distance of the road section highlighted by the blue and purple arrows in Figure 2(a), but keep the vehicle travel time unchanged, the progression bandwidth of the corridor remains unchanged in Figure 2(b). Therefore, it can be said that the proposed method can be applied to coordinate corridors with inconsistent progression design speeds across road segments. We define it as the horizontal scaling of the time-space diagram.

Therefore, to facilitate the diagrammatic design process, when the design speed of the road section is inconsistent with the corridor progression design speed $V$, we will convert the progression design speed of the road section to $V$ by adjusting the intersection distance. The adjusted distance

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**Figure 1:** The design flow chart of the proposed method.
2.2.3. Possible Coordinated Mode. To balance the effect of bidirectional corridor progression, the coordinated mode of each intersection must adopt synchronous coordination or backstepping coordination. Synchronous coordination means that the green center point of the coordinated phase is consistent with intersection \( I_1 \), whereas backstepping coordination means that the red center point is consistent with the green center point of the coordinated phase at \( I_1 \). The progressions in the outbound direction and the inbound direction have the characteristic of time symmetry, and then, only the outbound direction needs to be considered. Then, we define a Boolean variable \( F_p \) as the coordinated mode judgment factor. When \( F_p \) is 0, the optimal coordinated mode of the intersection \( I_p \) is synchronous coordination. When \( F_p \) is 1, the optimal coordinated mode of \( I_p \) is backstepping coordination.

2.3. The First Round of Rotation Transformation. The optimization of the intersection coordinated mode can be realized by defining the rules of the first round of PTC line rotation transformation. First, we define the green center point of the coordinated phase at \( I_1 \) as the reference point \( O_{1,1} \) of the time-space diagram coordinate system. A horizontal line can be drawn from \( O_{1,1} \) and rotated until the cotangent of its angle with the \( x \)-axis equals the corridor progression design speed \( V \). This process is defined as the first rotation transformation of the PTC line in the first round. We define the rotated ray as the initial PTC line \( L_1 \) and then assign \( V \) to adjusted speed \( V_{(i,1)} \).

The following steps have to be executed cyclically until the coordinated modes of all intersections have been determined.

2.3.1. Primary Judgment Factor Calculation. The intersection \( I_p \) is selected as the current coordinated intersection during the first round’s \( pth (2 \leq p \leq n) \) rotation transformation. According to PTC line \( L_{p-1} \) obtained by the last rotation transformation, the crossing point of \( L_{p-1} \) and the timeline of \( I_p \) is marked as \( O_{1p} \) as shown in Figure 4(a). The coordinated phase horizontal red center lines at \( I_1 \) have crossing points with the timeline of \( I_p \), and so we define the point closest to \( O_{1p} \) as \( O_{R_p} \). The coordinated phase horizontal green center lines at \( I_1 \) have crossing points with the timeline of \( I_p \) and so we define the point closest to \( O_{1p} \) as \( O_{G_p} \).
Our first step is to define and calculate some basic parameters, and the initialization routine can be presented below.
Initialization procedure

**Step 0.** Determine the range of the common signal cycle \([C_{\text{min}}, C_{\text{max}}]\) and calculate the initial common signal cycle \(C_1 = 0.5 (C_{\text{min}} + C_{\text{max}})\).

**Step 1.** Make a judgment of whether the progression design speeds of all road sections equal the corridor progression design speed or not. If there is an inequality, calculate the intersection adjusted distance \(D_p\) for the corresponding road section as shown in Equation (1).

**Step 2.** Determine the range of adjusted speed \(V_{(i,j)}\) obtained after the \(j\)th rotation transformation in the \(i\)th round, as shown in Equation (2).

\[
\begin{align*}
D_p &= \frac{b}{C_1} \\
V_{(i,j)} &= \frac{D_p}{y_{(i,j)}}
\end{align*}
\]

**Algorithm 1:** Initialization procedure.

Starting at the point \(O_{i1}\), we can make a ray \(l_{Rp} (l_{Gp})\) passing through the point \(O_{Rp} (O_{Gp})\), and the corresponding adjusted speed \(V_{Rp} (V_{Gp})\) satisfies the following:

\[
V_{Rp} (V_{Gp}) = \frac{D_p}{y_{Rp}} \left( \frac{D_p}{y_{Gp}} \right)
\]
The time difference between points \( O_{LP} \) and \( O_{RP} (O_{GP}) \), which corresponds to the distance between the ordinate of points \( y_{LP} \) and \( y_{RP} (y_{GP}) \), is recorded as \( T_{RP} (T_{GP}) \). Then, the primary judgment factor \( f_p \) can be constructed according to \( T_{RP} \) and \( T_{GP} \):

\[
f_p = T_{RP} - T_{GP} = \left| y_{LP} - y_{RP} \right| - \left| y_{LP} - y_{GP} \right|.
\]

### 2.3.2. Optimal Coordinated Mode Calculation

We define \( f_{\Gamma R_p} (f_{\Gamma G_p}) \) \( (1 < q < p) \) as a Boolean variable. When ray \( I_{p+} (I_{GP}) \) does not pass through the red interval of the intersection \( I_{q} (1 < q < p) \), \( f_{\Gamma R_p} (f_{\Gamma G_p}) \) is 0. When ray \( I_{p+} (I_{GP}) \) passes through the red interval of \( I_{p+} (I_{GP}) \) \((f_{\Gamma R_p} (f_{\Gamma G_p}) = 1 \). Also, the red interval crossing amount of \( I_{p+} (I_{GP}) \) in \( I_{q} \) can be defined as \( T_{R} (p, q) \) \( (T_{GP} (p, q)) \).

When \( f_p \geq 0 \), the situation is shown in Figure 4. If ray \( I_{GP} \) does not pass through any red interval of the coordinated phase, all \( f_{\Gamma (p,q)} \) equals 0. The current coordinated intersection can form a synchronous coordinated mode with \( I_{1} \), where \( F_p = 0 \). If ray \( I_{GP} \) passes through any red interval of the coordinated phase while ray \( I_{gp} \), \( I_{RP} \) does not pass through any red interval, as shown in Figure 4(b), the backstepping coordinated mode can be chosen, where \( F_p = 1 \). Both \( I_{GP} \) and \( I_{RP} \) pass through the red interval of the coordinated phase, and if the maximum red interval crossing amount of \( I_{GP} \) is less than or equal to \( I_{LP} \), as shown in Figure 4(c), these two intersections can form a synchronous coordinated mode, where \( F_p = 0 \). Otherwise, the backstepping coordinated mode will be chosen as \( F_p = 1 \).

Similarly, when \( f_p < 0 \) if ray \( I_{RP} \) does not pass through any red interval of the coordinated phase, all \( f_{\Gamma R_p} (p, q) \) equals 0, \( F_p = 1 \). If ray \( I_{RP} \) passes through any red interval of the coordinated phase while ray \( I_{GP} \), \( I_{RP} \) does not pass through any red interval, then \( F_p = 0 \). If both \( I_{RP} \) and \( I_{GP} \) pass through the red interval of the coordinated phase, and if the maximum red interval crossing amount of \( I_{LP} \) is less than or equal to \( I_{GP} \), \( F_p = 1 \). Otherwise, \( F_p = 0 \).

### 2.3.3. PTC Line Optimization

If the current coordinated intersection has formed a synchronous coordinated mode with \( I_{1} \) and \( V_{GP} \) is within the rotation range of the PTC line, we can define \( I_{GP} \) as PTC line \( L_p \) and assign the value of \( V_{GP} \) to the adjusted speed \( V_{(t, p)} \) as shown in Figure 5(a). When \( V_{GP} \) is out of the rotation range, we can keep the PTC line unchanged, that is, as shown in Figure 5(b), \( L_p \) is the same as \( L_{p-1} \), and \( V_{(t, p)} \) equals \( V_{(t, p-1)} \).

Similarly, if the current coordinated intersection has formed a backstepping coordinated mode with \( I_{1} \) and \( V_{GP} \) is within the rotation range of the PTC line, we can define \( I_{GP} \) as PTC line \( L_p \) and assign the value of \( V_{GP} \) to \( V_{(t, p)} \). When \( V_{GP} \) is out of the rotation range, we can keep the PTC line unchanged. \( L_p \) is the same as \( L_{p-1} \), and \( V_{(t, p)} \) equals \( V_{(t, p-1)} \).

### 2.4. The Second Round of Rotation Transformation

The final optimization of the common signal cycle and the offsets will be determined upon completing second round of PTC line rotation transformation.

After the first round of rotation transformation, the coordinated modes of all signalized intersections have been determined. We assign adjusted speed \( V_{(1, p)} \) to \( V_{(2, 0)} \). Then, the beginning PTC line \( L_{B0} \) and the end PTC line \( L_{E0} \) of the outbound progression can be obtained. Then, the initial bandwidth \( B_0 \) and ratio \( R_0 \) can also be calculated. Meanwhile, the bottleneck intersections of the beginning PTC line are put into the intersection set \( S_{B0} \) and the bottleneck intersections of the end PTC line are put into the intersection set \( S_{E0} \). The calculation of related parameters will be introduced in the next section.

We define a termination decision parameter \( F_A \), which is a Boolean variable. The following steps have to be executed cyclically until the termination decision parameter \( F_A \) equals 1.

#### 2.4.1. Bottleneck Intersection Recognition

We assume that the obtained beginning PTC line, end PTC line, adjusted speed, bandwidth, and bandwidth ratio after the \( m \)th rotation transformation in the second round are defined as \( L_{BM}, L_{EM}, V_{(2, m)}, b_{m} \), and \( R_m \), respectively. The parameters obtained from the \( m \)-1th rotation transformation need to be used during the \( m \)th rotation transformation in the second round.

According to \( F_p \), \( V_{(2, m-1)}, C_1 \) and \( \lambda_p \), we can draw the beginning and end PTC lines in the time-space diagram. We define the Boolean variables \( K_B (m-1,p) \) and \( K_E (m-1,p) \) as the judgment factors of the bottleneck intersection of the beginning and end PTC lines. When \( K_B (m-1,p) \) equals 1, intersection \( I_p \) is the bottleneck intersection of the beginning PTC line after the \( m \)-1th rotation transformation, \( I_p \in S_{BM-1} \), and the corresponding bottleneck point is defined as \( P_{BP} \). When \( K_E (m-1,p) \) equals 1, \( I_p \) is the bottleneck intersection of the end PTC line after the \( m \)-1th rotation transformation, \( I_p \in S_{EM-1} \), and the corresponding bottleneck point is recorded as \( P_{EP} \).

Before the other calculation steps of the \( m \)th rotation transformation in the second round, it is necessary to judge whether the sets \( S_{BM-1} \) and \( S_{EM-1} \) meet the conditions for stopping rotation. If the conditions for stopping rotation are unsatisfied, then it must be continued to complete the \( m \)th rotation transformation until the bottleneck intersections meet the conditions of stopping rotation, where \( F_A = 1 \).

#### 2.4.2. Conditions for Stopping the Rotation

**Condition 1.** \( \exists I_k \in (S_{BM-1}\cap S_{EM-1}) \), that is, there are both bottleneck points of the beginning and end PTC lines at intersection \( I_k \). At this time, the ratio of the bandwidth \( R_m \) equals the split of the coordinated phase at \( I_k \), which means that it has already reached the maximum value of the bandwidth ratio. Therefore, there is no need to continue the rotation transformation of the PTC line.

**Condition 2.** \( \exists I_j \in S_{BM-1} \), \( I_j \in S_{BM-1} \), \( I_k \in S_{EM-1} \), and \( i < j < k \), that is, there are two bottleneck intersections of the end PTC line located upstream and downstream of a bottleneck intersection of the beginning PTC line, respectively.
The corresponding bandwidth will decrease when we increase the progression speed and rotate the PTC line clockwise, as shown in Figure 7(a). Furthermore, when we reduce the progression speed and rotate the PTC line counter-clockwise the corresponding bandwidth will decrease, as shown in Figure 6(b). Therefore, the further rotation transformation of the PTC line has to be finished.

**Algorithm 2: First round rotation transformation procedure.**

In Figure 5: The PTC line updating under synchronous coordinated mode.

**Condition 3.** \( \exists I_i \in S_{Bm-1}, I_j \in S_{Bm-1}, I_k \in S_{Bm-1} \), and \( i < j < k \), that is, there are two bottleneck intersections of the beginning PTC line located upstream and downstream of a bottleneck intersection of the end PTC line, respectively. The corresponding bandwidth will decrease when we increase the progression speed and rotate the PTC line clockwise, as shown in Figure 7(a). Furthermore, when we reduce the progression speed and rotate the PTC line counter-clockwise the corresponding bandwidth will decrease, as shown in Figure 7(b). Therefore, the further rotation transformation of the PTC line has to be finished.

2.4.3. Rotation Points and Direction Recognition. It is necessary to determine the rotation points and direction according to the relationship between the rotation
transformation and the progression bandwidth during the $m$th rotation transformation in the second round.

When increasing the progression speed, that is, $V_{(2,m)} > V_{(2,m-1)}$, the rotation point of $L_{Bm}$ should fall at the most downstream bottleneck intersection in $S_{E_{m-1}}$, and the rotation point of $L_{Em}$ should fall at the most upstream bottleneck intersection in $S_{E_{m-1}}$, as shown in Figures 8(a) and 8(b). Therefore, when rotating the PTC line clockwise, we should select intersections $I_i \in S_{E_{m-1}}$, $I_j \in S_{B_{m-1}}$ to ensure that $\forall I_k \in S_{E_{m-1}}$ satisfies $i \leq k$ and $\forall I_j \in S_{B_{m-1}}$ satisfies $j \geq l$. Then, the bottleneck point $P_{Ei}$ and $P_{Bj}$ should be taken as the rotation points.
When reducing the progression speed, that is, \(V(2,m) < V(2,m-1)\), the rotation point of \(L_{Em}\) should fall at the most downstream bottleneck intersection in \(S_{Em-1}\), and the rotation point of \(L_{Bm}\) should fall at the most upstream bottleneck intersection in \(S_{Bm-1}\), as shown in Figures 8(c) and 8(d). Therefore, when rotating the PTC line counter-clockwise, we should select intersections \(I_i \in S_{Em-1}, I_j \in S_{Bm-1}\) to ensure that \(\forall I_k \in S_{Em-1}\) satisfies \(i \geq k\) and \(\forall I_l \in S_{Bm-1}\) satisfies \(j \leq l\). Then, the bottleneck point \(P_{Ei}\) and \(P_{Bj}\) should be taken as the rotation points.
\( \bar{D}_m \) is defined as the abscissa difference between the bottleneck rotation points \( P_{Bj} \) and \( P_{Ei} \) in the \( m \)th rotation transformation in the second round, that is, \( \bar{D}_m = x_j - x_i \). Then, the bandwidth \( b_m \) can be calculated as follows:

\[
b_m^* = b_m - \Delta b_m = b_m^* + \bar{D}_m \cdot \left( \frac{1}{V_{(2,m)}} - \frac{1}{V_{(2,m-1)}} \right),
\]

where \( \Delta b_m \) is the bandwidth increment resulting from the transformation of the progression speed from \( V_{(2,m-1)} \) to \( V_{(2,m)} \).

We define a Boolean variable \( K_{Dm} \) as the judgment factor of the rotation direction. When \( K_{Dm} \) equals 0, the rotation direction is the counter-clockwise rotation, whereas the rotation direction is the clockwise rotation when \( K_{Dm} \) equals 1.

**Scenario 1.** If \( \forall I_k \in S_{E_{m-1}} \) and \( \forall I_l \in S_{B_{m-1}} \), satisfy \( k < l \), that is, all the bottleneck intersections of the end PTC line are located upstream of any bottleneck intersection of the beginning PTC line, then \( \bar{D}_m > 0 \). At this time, if the PTC line is rotated clockwise, \( V_{(2,m)} > V_{(2,m-1)} \), the obtained bandwidth after the rotation will decrease according to Equation (5). If the PTC line is rotated counterclockwise, \( V_{(2,m)} < V_{(2,m-1)} \), the obtained bandwidth after the rotation will increase according to Equation (5). Therefore, the rotation direction in Scenario 1 should be determined as the counter-clockwise rotation, \( K_{Dm} = 0 \).

**Scenario 2.** If \( \forall I_k \in S_{E_{m-1}} \) and \( \forall I_l \in S_{B_{m-1}} \), satisfy \( k > l \), that is, all the bottleneck intersections of the end PTC line are located downstream of any bottleneck intersection of the beginning PTC line, then \( \bar{D}_m < 0 \). At this time, if the PTC line is rotated clockwise, \( V_{(2,m)} > V_{(2,m-1)} \), the obtained bandwidth after the rotation will increase according to Equation (5). If the PTC line is rotated counterclockwise, \( V_{(2,m)} < V_{(2,m-1)} \), the obtained bandwidth after the rotation will decrease according to Equation (5). Therefore, the rotation direction in Scenario 2 should be determined as the clockwise rotation, \( K_{Dm} = 1 \).

### 2.4.4. Rotation Angle Calculation

Taking the bottleneck point of the end PTC line \( P_{Ei} \) and the bottleneck point of the beginning PTC line \( P_{Bj} \) as extreme points, we can calculate the rotation angle formed with other designated crossing points and determine the adjusted speed comprehensively after the rotation transformation.

The green endpoint of the coordinated phase crossed by the end PTC line \( L_{E_{m-1}} \) at intersection \( I_k (1 \leq k \leq n) \) is defined as \( P_{Fk} \), and the green start point of the coordinated phase crossed by the beginning PTC line \( L_{B_{m-1}} \) at \( I_k (1 \leq k \leq n) \) is defined as \( P_{Sk} \).

The adjusted speed \( V_{Fk} (V_{Sk}) \) corresponding to the rotation line \( L_{Fk} (L_{Sk}) \) formed by connecting points \( P_{Bj} (P_{Bi}) \) and \( P_{Fk} (P_{Sk}) \) can be calculated by the following equation:

\[
V_{Fk} (V_{Sk}) = \frac{x_k - x_i}{y_{Fk} - y_{Ei}} \cdot \left( \frac{x_k - x_i}{y_{Sk} - y_{Bj}} \right).
\]

Here, \( y_{Fk} \) and \( y_{Sk} \) represent the ordinate of points \( P_{Fk} \) and \( P_{Sk} \), respectively; \( y_{Ei} \) and \( y_{Bj} \) represent the ordinate of points \( P_{Ei} \) and \( P_{Bi} \), respectively.

If \( K_{Dm} \) equals 0, we should connect points \( P_{Fg} \) and \( P_{Sk} \) in turn to form the rotation line \( L_{Fg} \). When the corresponding adjusted speed \( V_{Fg} \) is within the adjusted speed range, the eligible \( V_{Fg} \) is incorporated into the optional speed set, \( S_{V_{m-1}} \), obtained after the \( m \)th rotation transformation. Meanwhile, \( P_{Bj} \) and \( P_{Sk} \) \( (1 \leq h \leq j-1) \) are connected to form the rotation line \( L_{Sh} \). When the corresponding adjusted speed \( V_{Sh} \) is within the adjusted speed range, the eligible \( V_{Sh} \) is incorporated into \( S_{V_{m}} \). Then, we have to select the maximum value in \( S_{V_{m}} \) as the adjusted speed \( V_{(2,m)} \) determined by the \( m \)th rotation transformation of the vehicle PTC line in the second round, as shown in Figure 9(a).

If \( K_{Dm} \) equals 1, we should connect points \( P_{Bi} \) and \( P_{Fg} \) \( (1 \leq g \leq i-1) \) in turn to form the rotation line \( L_{Bg} \). When \( V_{Bg} \) is within the adjusted speed range, the eligible \( V_{Bg} \) is incorporated into \( S_{V_{m}} \). Meanwhile, points \( P_{Bj} \) and \( P_{Sh} \) \( (j + 1 \leq h \leq n) \) are connected to form the rotation line \( L_{Bh} \). When \( V_{Bh} \) is within the adjusted speed range, the eligible \( V_{Bh} \) is incorporated into \( S_{V_{m}} \). Then, we have to select the minimum value in \( S_{V_{m}} \) as \( V_{(2,m)} \), as shown in Figure 9(b).

According to \( V_{(2,m)} \), the end PTC line \( L_{Em} \) and the beginning PTC line \( L_{Bm} \) in the outbound direction can be obtained after the rotation transformation. Before entering the next rotation transformation in the second round, the sets of the bottleneck intersections can be updated as \( S_{Em} \) and \( S_{Bm} \) according to \( L_{Em} \) and \( L_{Bm} \).

### 2.5. Scheme Generation

#### 2.5.1. Optimal Common Signal Cycle Calculation

The optimal adjusted speed \( V_B \) can be calculated according to the final PTC line obtained after the rotation transformations in the second round. According to the vertical scaling of the time-space diagram, the optimal common signal cycle \( C_B \) can be calculated by the following equation:

\[
C_B = \frac{C_1 \cdot V_B}{V}.
\]

#### 2.5.2. Offset Calculation

When the coordinated mode of each intersection and the optimal common signal cycle have been determined, combined with the known split distribution scheme, the green interval of the coordinated phase and absolute offset of each intersection can be calculated. The absolute offset of the intersection \( I_p \) \( (1 \leq p \leq n) \) is the beginning of the coordinated phase at \( I_p \) defined as \( O_p \). While the green center point of the coordinated phase at intersection \( I_i \) is defined as the offset reference point with a value
Our third step is to optimize the common signal cycle and offsets, and the second round of the rotation transformation routine can be presented below.

**Second Round Rotation Transformation Procedure**

**Step0.** Determine the bottleneck intersections and the set $S_{Bbm-1}$ and $S_{Em-1}$.

According to $E_p$, $V_{(2, m-1)}$, $C_i$, and $\lambda_p$, draw the beginning and end PTC lines in the time-space diagram. If $I_p$ is the bottleneck intersection of the beginning PTC line, let $K_{E_1(m-1, p)} = 1$ and add $I_p$ in set $S_{Bbm-1}$, else $K_{E_1(m-1, p)} = 0$. And if $I_p$ is the bottleneck intersection of the end PTC line, let $K_{E_1(m-1, p)} = 1$ and add $I_p$ in set $S_{Em-1}$, else $K_{E_1(m-1, p)} = 0$.

**Step1.** Make a judgment of whether the sets $S_{Bbm-1}$ and $S_{Em-1}$ meet any condition for stopping rotation. If $\exists I_k \in (S_{Bbm-1} \cap S_{Em-1})$, or $\exists I_l \in S_{Bbm-1}, I_j \in S_{Em-1}, I_k \in S_{Bbm-1}$ and $i < j < k$, or $\exists I_m \in S_{Bbm-1}, I_n \in S_{Em-1}, I_k \in S_{Bbm-1}$ and $i < j < k$, then $F_A = 1$, finish the second round of rotation transformation. Otherwise, $F_A = 0$, proceed to Step 2.

**Step2.** Determine the rotation direction.

If $\forall I_k \in S_{Em-1}, \forall I_l \in S_{Bbm-1}$, and $k < l$, let $K_{Dm} = 0$, else if $\forall I_k \in S_{Em-1}, \forall I_l \in S_{Bbm-1}$, and $k > l$, let $K_{Dm} = 1$.

**Step3.** Determine the rotation angle.

If $K_{Dm} = 0$, connect points $P_{E1}$ and $P_{Eg}(i + 1 \leq g \leq n)$, points $P_{Eg}$ and $P_{sh}(1 \leq h \leq j - 1)$ in turn, calculate the corresponding $V_{Eg}$ and $V_{sh}$ as shown in Equation (6), add the eligible $V_{Eg}$ and $V_{sh}$ in set $S_{vm}$, then select the maximum value in $S_{vm}$ as the updated adjusted speed $V_{(2, m)}$.

If $K_{Dm} = 1$, connect points $P_{E1}$ and $P_{Eg}(1 \leq g \leq i - 1)$, points $P_{Eg}$ and $P_{sh}(j + 1 \leq h \leq n)$ in turn, calculate the corresponding $V_{Eg}$ and $V_{sh}$ as shown in Equation (6), add the eligible $V_{Eg}$ and $V_{sh}$ in set $S_{vm}$, then select the minimum value in $S_{vm}$ as the updated adjusted speed $V_{(2, m)}$.

**Step4.** Let $m = m + 1$, and return to Step 0.

**Algorithm 3:** Second round rotation transformation procedure.

of zero, the absolute offset of each intersection can be calculated by the following equation:

$$O_p = \begin{cases} C_B - 0.5 \cdot C_B \cdot \lambda_p, & 1 \leq p \leq n \text{ and } F_p = 0, \\ 0.5 \cdot C_B - 0.5 \cdot C_B \cdot \lambda_p, & 1 \leq p \leq n \text{ and } F_p = 1. \end{cases}$$

(8)

**2.6. Case Study.** To facilitate the comparative analysis of the optimization effect of the graphical method proposed in this paper, we cite a case study that has been used in many books and papers [19, 20, 25]. The corridor being analyzed is an east-west roadway, and there are eight signalized intersections along the corridor. The direction from west to east is defined as the outbound direction, and the western-most intersection is selected as intersection $I_1$. The schematic diagram of the selected corridor is shown in Figure 10. It has already known that each intersection adopts symmetrical phasing. The split of each intersection is 55%, 60%, 65%, 65%, 60%, 65%, 70%, and 50%, respectively. The optimization range of the common signal cycle is [60, 100] s, and
the progression design speed is 40 km/h, which approximately equals 11 m/s.

2.7. Scheme Generation. The proposed method can obtain the scheme through a limited number of rule-guided drawings, and it is suitable for engineering applications. The related process can be shown as follows:

2.7.1. Initialization. The initial common signal cycle $C_1$ is valued as the midpoint between the minimum common signal cycle $C_{\text{min}}$ and the maximum common signal cycle $C_{\text{max}}$, which is 80 s. Because the progression design speeds of all the road sections are 11 m/s, there is no need to adjust the intersection distance. When the initial common signal cycle $C_1$ is unchanged, the adjusted speed range can be determined as $[8.25, 13.75]$ m/s.

2.7.2. The First Round of Rotation Transformation. The green center point of the coordinated phase at intersection $I_1$ is defined as the reference point $O_{I_1}$ of the coordinate system of the time-space diagram. A horizontal line from the reference point $O_{I_1}$ can be drawn, and then, it is rotated until the cotangent of its angle with the x-axis equals the corridor progression design speed $V$. We can define the rotated ray as the initial PTC line $L_1$ and assign the value of $V$ to adjusted speed $V_{(1,1)}$. Then, the first rotation transformation in the first round is completed.

The intersection $I_2$ is selected as the current coordinated intersection. According to PTC line $L_1$ obtained by the first rotation transformation, the crossing point of the PTC line $L_1$ and the timeline of $I_2$ is marked as $O_{I_2}$. The nearest crossing point of the coordinated phase horizontal red (green) center line at $I_1$, and the timeline of $I_2$ is identified and recorded as $O_{R2} (O_{G2})$, as shown in Figure 11(a). The time difference $T_{R2}$ between points $O_{I_2}$ and $O_{R2}$ is 8 s, and the time difference $T_{G2}$ between points $O_{I_2}$ and $O_{G2}$ is 32 s. Then, $f_2 = -24$ s. Ray $l_{R2}$ does not pass through the red interval of the coordinated phase, and $f_2 < 0$, and then, $I_2$ can form a backstepping coordinated mode with $I_1$, where $F_2 = 1$.

The corresponding adjusted speed $V_{R2}$ of $l_{R2}$ is $8.75$ m/s, within the adjusted speed range $[8.25, 13.75]$ m/s. Therefore, $l_{R2}$ is defined as PTC line $L_2$, and the value of $V_{R2}$ is assigned to the adjusted speed $V_{(1,2)}$. Up to now, the second rotation transformation in the first round is finished.

Repeat the above steps to complete the 3rd to 8th rotation transformation in the first round in turn. Then, the coordinated mode between $I_1$ and $I_3$, $I_4$, $I_5$, $I_6$, $I_7$, and $I_8$ can be determined, respectively. The coordination process is shown in Figures 11(b)–11(g).

After the first round of rotation transformation, $I_2$, $I_5$, and $I_8$ form a backstepping coordinated mode with $I_1$, $F_2 = F_5 = F_8 = 1$, whereas the intersections $I_3$, $I_4$, $I_6$, and $I_7$ form a synchronous coordinated mode with $I_1$, $F_3 = F_4 = F_6 = F_7 = 0$. The adjusted speed is $11.40$ m/s, and the time-space diagram obtained after the first round of rotation transformation is shown in Figure 11(h). The corresponding bandwidth is 24 s, and the bandwidth ratio is 30.0%.

2.7.3. The Second Round of Rotation Transformation. According to the adjusted speed $V_{(1,8)}$ finally obtained after the first round of rotation transformation, the end PTC line of vehicle band $L_{EH0}$ and the beginning PTC line of vehicle band $L_{BH0}$ of the outbound progression can be drawn, as shown in Figure 12. Meanwhile, the bottleneck intersections of the end PTC line are put into the set $S_{EH0} = \{I_8\}$. Also, the bottleneck intersections of the beginning PTC line are put into the set $S_{BH0} = \{I_1\}$.

The set $S_{BH0}$ and $S_{EH0}$ do not meet any conditions for stopping rotation. Then, we have to complete the first rotation transformation in the second round.

Because the only bottleneck intersection of the end PTC line $I_8$ is located downstream of the only bottleneck intersection of the beginning PTC line $I_1$, it corresponds to the rotation direction judgment Scenario 2. Therefore, the PTC
Figure 11: Continued.
Figure 11: The coordination process of the first round of rotation transformation. (a) The 2nd rotation transformation. (b) The 3rd rotation transformation. (c) The 4th rotation transformation. (d) The 5th rotation transformation. (e) The 6th rotation transformation. (f) The 7th rotation transformation. (g) The 8th rotation transformation. (h) The obtained time-space diagram.

Figure 12: The second round of rotation transformation.

line can be rotated clockwise. Furthermore, the most upstream bottleneck point $P_{E7}$ is selected as the rotation point of the end PTC line, whereas the most downstream bottleneck point $P_{B3}$ is selected as the rotation point of the beginning PTC line.

As shown in Figure 12, we can connect points $P_{E7}$ ($P_{B3}$) and $P_{Eg}$ ($P_{Bg}$) (1 $\leq g \leq 6$ ($P_{Sh}$ (4 $\leq h \leq 8$)) to form the rotation line $L_{Eg}$ ($L_{Bg}$) and determine whether the corresponding adjusted speed $V_{Eg}$ ($V_{Bg}$) is within the adjusted speed range. The calculation results are shown in Table 1.

It can be seen from Table 1 that $S_{V1} = \{V_{F1}, V_{F2}, V_{F4}, V_{F5}, V_{S6}, V_{S8}\}$. Then, select the minimum value $V_{F1}$ in $S_{V1}$ as the adjusted speed $V_{(2, 1)}$. Then, $V_{(2, 1)}$ is 12.11 m/s.

According to $V_{(2, 1)}$, the end PTC line $L_{E1}$ and the beginning PTC line $L_{B1}$ of the outbound progression can be drawn. Determine the bottleneck intersections set of the end PTC line $S_{E1} = \{I_1, I_2\}$ and the beginning PTC line bottleneck intersections set $S_{B1} = \{I_3\}$, according to $L_{E1}$ and $L_{B1}$. Then, the updating of the sets of bottleneck intersections is completed, as shown in Figure 13.

The sets $S_{E1}$ and $S_{B1}$ meet one of the conditions for stopping rotation. At this time, $\exists I_1 \in S_{E1}, I_3 \in S_{B1}, I_7 \in S_{E1}$, that is, there are two bottleneck intersections of the end PTC line located upstream and downstream of a bottleneck intersection of the beginning PTC line, which satisfies Condition 2 for stopping the rotation. Therefore, we can stop the further rotation transformation of the PTC line, and the second round of rotation transformation is finished.

2.7.4. Scheme Generation. The optimal adjusted speed $V_B$ is 12.11 m/s, according to the final PTC line obtained after the second round of rotation transformation. Combined with the progression design speed $V$, it can be calculated that the optimal common signal cycle $C_B$ is 88 s.
According to the coordinated mode of each intersection and the optimal common signal cycle obtained after the two rounds of rotation transformation, combined with the green signal ratio distribution scheme, the green interval of the coordinated phase and absolute offset of each intersection can be calculated. The time-space diagram finally obtained is shown in Figure 14. The corresponding progression bandwidth is 32.85 s, and the bandwidth ratio reaches 37.3%.

2.7.5. Programmatic Solution. The proposed solution logic steps can be solved graphically and programmed to quickly obtain an optimal signal coordination control scheme. The above case can be solved by python programming to obtain the optimal signal coordination control scheme. The experimental environment of the model solver is the Windows10 64-bit operating system, and the CPU is Intel i5-6600 3.30GHz. The optimal signal coordination control scheme obtained by programming is the same as the graphical way, with a solution time of only $2.3 \times 10^{-3}$ s.

2.8. Model Comparison. The MAXBAND model [7] and the improved algebraic method [20] can also be used to design the corridor progression schemes for the same case, and the calculation results are shown in Table 2. As for the coordinated mode between intersection $I_1$ and other intersections, the graphical method proposed in this study is consistent with the improved algebraic method and the MAXBAND model. As for the bandwidth ratio, the proposed graphical method can obtain 37.3% in both coordination directions, which is higher than the improved
algebraic method. Moreover, it also achieves the global optimal coordination effect, the same as the MAXBAND model.

Using the LINGO programming solution, the MAXBAND model can be solved in 3 s. Meanwhile, the proposed graphical method obtains the global optimal scheme in $2.3 \times 10^{-3}$ s, which means the solving efficiency is more than a thousand times higher than the MAXBAND model.

It can be seen from Table 2 that for the symmetrical corridor progression design, the method proposed in this study has the advantages of solid operability, solving simplicity, and a small amount of calculation. It can obtain an ideal coordination effect and be solved by a rule-guided graphical method with a limited number of steps.

### 3. Conclusions

In summary, a symmetrical bidirectional corridor progression method based on graphical optimization theory is established in this study. According to the internal relationship between the PTC lines rotation transformation and the progression bandwidth, a step-by-step optimization process of symmetrical corridor progression based on the rotation transformation of the PTC lines is proposed. After a two-round rotation transformation of the PTC line, the comprehensive optimization of the intersection coordinated mode, common signal cycle, and offsets are realized.

In the first round of PTC line rotation transformation, based on the existing graphical method designing flow, we have innovatively added a process for judging whether the PTC line passes through the red interval of the intersections with a determined coordinated mode or not, achieving a more optimized choice of coordinated mode at each intersection. Meanwhile, our innovative proposed the second round of rotation transformation of the PTC line, the regular bandwidth pattern during the rotation transformation of the PTC line is explored. Innovative rules for rotation transformation stop conditions and selection of the rotation points and direction are given to achieve the reoptimization of the common signal cycle.

The proposed graphical method can be used to obtain a signal coordination control scheme by a rule-guided graphical method and be programmed. It can be seen from the case study that the bandwidth ratio obtained by the first round of rotation transformation of the proposed graphical method is 30.0%, with the optimization of the coordinated mode and the initial offsets. Then, through the second round of rotation transformation, the optimization of the common signal cycle and the offsets increases the bandwidth ratio to 37.3%. Finally, the effect of the corridor progression control scheme, which is better than the improved algebraic method, is ultimately the same as the global optimal result of the MAXBAND model with a more than a thousand times higher solving efficiency.

The graphical method proposed in this study has the advantages of intuitive readability, strong operability, and is particularly suitable for engineering applications. The horizontal and vertical scaling of the time-space diagram is one of the few attempts to standardize a graphic solution to corridor traffic coordination control. However, extending the graphical method to the corridor progression under the asymmetrical phasing and constructing a more applicable graphical method for corridor progression will be important for future follow-up research.

### Notations

- $a_p$: The ideal distance between intersection $I_p$ and $I_i$
- $b_m$: The bandwidth obtained after the $m$th rotation transformation in the second round
- $C$: The common signal cycle
- $C_i$ ($C_0$): The initial (optimal) common signal cycle
- $C_{\max}$ ($C_{\min}$): The maximum (minimum) common signal cycle
- $d_p$: The actual distance between intersection $I_p$ and $I_{p+1}$ ($1 \leq p \leq n-1$)
- $d'_p$: The adjusted distance between intersection $I_p$ and $I_{p+1}$ ($1 \leq p \leq n-1$)
- $D_p$: The adjusted distance between the nonbenchmark intersection $I_p$ and the benchmark intersection $I_1$
- $D_m$: The difference between the abscissa of the bottleneck rotation point of the beginning and end PTC line in the $m$th rotation transformation in the second round
- $f_p$: The primary judgment factor for the coordinated mode of intersection $I_p$

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**Table 2**: The comparison table of the coordination effects of various models.

| Model                          | Intersection | $I_1$ | $I_2$ | $I_3$ | $I_4$ | $I_5$ | $I_6$ | $I_7$ | $I_8$ |
|-------------------------------|--------------|-------|-------|-------|-------|-------|-------|-------|-------|
| Proposed graphical method     | Coordinated mode | $S^*$ | $B^*$ | $S^*$ | $B^*$ | $S^*$ | $B^*$ | $S^*$ | $B^*$ |
| Proposed graphical method     | Absolute offset (s) | 63.8 | 17.6  | 59.4  | 59.4  | 17.6  | 59.4  | 57.2  | 22.0  |
| Proposed graphical method     | Common signal cycle (s) | 88    |       |       |       |       |       |       |       |
| Proposed graphical method     | Bandwidth ratio (%) | 37.3  |       |       |       |       |       |       |       |
| Improved algebraic method     | Coordinated mode | $S^*$ | $B^*$ | $S^*$ | $B^*$ | $S^*$ | $B^*$ | $S^*$ | $B^*$ |
| Improved algebraic method     | Absolute offset (s) | 66.0  | 18.2  | 61.4  | 61.4  | 18.2  | 61.4  | 59.2  | 22.8  |
| Improved algebraic method     | Common signal cycle (s) | 91    |       |       |       |       |       |       |       |
| Improved algebraic method     | Bandwidth ratio (%) | 30.5  |       |       |       |       |       |       |       |

$S^*$ = “synchronous coordinated mode” and $B^*$ = “backstepping coordinated mode”. 
\( f_{G(p,q)}(f_{R(p,q)}) \): The judgment factor of whether ray \( l_{Gp} (l_{Rp}) \) passes through the red interval of the intersection \( I_p \) \((1 < q < p)\) or not

\( F_p^c \): The coordinated mode judgment factor of intersection \( I_p \)

\( F_A^c \): The termination decision parameter of the second round of rotation transformation

\( I_p^c \): The \( q \)th signalized intersection along the corridor in the outbound direction

\( K_{Dm}^c \): The rotation direction judgment factor in the \( m \)th rotation transformation in the second round

\( K_{E(m-1,p)} \) \((K_{R(m-1,p)})\): The judgment factors of the bottleneck intersection of the end and beginning PTC lines

\( l_{Gp} (l_{Rp}) \): A ray passes through point \( O_{Gp} (O_{Rp}) \) started at point \( O_{I_1} \)

\( l_{p} \): The PTC line obtained after the \( q \)th rotation transformation in the first round

\( L_{Bm} \) \((L_{Em})\): The beginning (end) PTC line obtained after the \( m \)th rotation transformation in the second round

\( L_{Fk} \) \((L_{Sk})\): The rotation line formed by connecting the bottleneck rotation point of the end and beginning PTC line and the coordinated phase endpoint \( P_{Fk} \) (start point \( P_{Sk} \))

\( n \): The number of signalized intersections on the corridor

\( O_{Ip}^c \): The absolute offset of the intersection \( I_p \) \((1 \leq p \leq n)\)

\( O_{Gp} \) \((O_{Rp})\): The nearest crossing point of the coordinated phase horizontal green (red) center line at the intersection \( I_1 \) and the timeline of \( I_p \)

\( O_{Ip}p \): The crossing point of PTC line \( L_{p-1} \) and the timeline of intersection \( I_p \)

\( P_{Ej} \) \((P_{Bj})\): The bottleneck point of the end (beginning) PTC line at the bottleneck intersection \( I_j \)

\( P_{Fk} \) \((P_{Sk})\): The green interval end (start) point of the coordinated phase that is crossed by the PTC line at intersection \( I_k \)

\( R_{mi} \): The ratio of the bandwidth obtained after the \( m \)th rotation transformation in the second round

\( S_{Em} \) \((S_{Bm})\): The set of bottleneck intersections of the end (beginning) PTC line obtained after the \( m \)th rotation transformation in the second round

\( S_{Vm} \): The set of optional vehicle speed obtained after the \( m \)th rotation transformation in the second round

\( T_{Gp} \) \((T_{Rp})\): The time difference between points \( O_{Ip} \) and \( O_{Gp} \) \((O_{Rp})\)

\( T_{G(p,q)} \) \((T_{R(p,q)})\): The red interval crossing amount of \( l_{Rp} \) \((l_{Gp})\) in \( I_j \) \((1 < q < p)\)

\( v_p^c \): The progression design speed between intersection \( I_p \) and \( I_{p+1} \) \((1 \leq p \leq n-1)\)

\( V \): The corridor progression design speed

\( V_{(i,j)} \): The adjusted speed obtained after the \( j \)th rotation transformation in the \( i \)th round

\( V_{p}^c \): The optimal adjusted speed

\( V_{Fk} \) \((V_{Sk})\): The adjusted speed corresponding to the rotation line \( L_{Fk} \) \((L_{Sk})\)

\( V_{Gp} \) \((V_{Rp})\): The adjusted speed corresponding to the line \( l_{Gp} \) \((l_{Rp})\)

\( x_{p}^c \): The abscissa of intersection \( I_p \) in the time-space diagram

\( y_{Gp} \) \((y_{Rp})\): The ordinate of point \( O_{Gp} \) \((O_{Rp})\) in the time-space diagram

\( y_{G(p,q)} \) \((y_{R(p,q)})\): The ordinate of the green center point of \( I_p \) which is closest to the intersection of ray \( l_{Gp} \) \((l_{Rp})\) and the timeline of \( I_p \) \((1 < q < p)\)

\( y_{Ip} \): The ordinate of point \( O_{Ip} \) in the time-space diagram

\( \lambda_p \): The split of the coordinated phase of the intersection \( I_p \)

**Data Availability**

The data used to support the findings of this study are available from the corresponding author upon request.

**Conflicts of Interest**

The authors declare that there are no conflicts of interest regarding the publication of this paper.

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