Forward-backward multiplicity fluctuations in heavy nuclei collisions in the wounded nucleon model

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Abstract

We use the wounded nucleon model to study the forward-backward multiplicity fluctuations measured by the PHOBOS Collaboration in $Au + Au$ collisions at $\sqrt{s_{NN}} = 200$ GeV. The enhancement of forward-backward fluctuations in $Au + Au$ collisions with respect to the elementary $p+p$ interactions is in this model explained by the asymmetric shape of the pseudorapidity density of produced particles from a single wounded nucleon and the fluctuations of the number of wounded nucleons in the colliding nuclei. The wounded nucleon model describes these experimental data better than the HIJING, AMPT or UrQMD models do.

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1 Introduction

The studies of the fluctuations of the charged particles multiplicity in heavy nuclei collisions provide information on the particle production and the properties of the

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matter created in these collisions \cite{1}. The multiplicity measured in the symmetric pseudorapidity intervals in the forward and backward hemisphere appears to be a suitable observable for such analysis. Recently, the direct forward-backward correlations were studied by the STAR Collaboration \cite{2}, while the difference between forward-backward multiplicities was used by the PHOBOS Collaboration \cite{3}.

The PHOBOS Collaboration measured the variance $\sigma^2_C$ of the forward-backward asymmetry variable $C = (N_F - N_B)/\sqrt{N_F + N_B}$

$$\sigma^2_C = \left\langle \frac{(N_B - N_F)^2}{N_B + N_F} \right\rangle,$$

where $N_B$ and $N_F$ are the event-by-event multiplicities in the backward $B$ and forward $F$ pseudorapidity intervals, respectively. The measurement was performed for many different symmetric (with respect to $\eta = 0$) pseudorapidity bins in the range $|\eta| < 3$. It was found that:

(i) for $\eta$ bins with fixed width but changing position in pseudorapidity $\sigma^2_C$ increases with increasing distance between bin centers

(ii) for $\eta$ bins centered at the same position ($\eta = \pm 2$) $\sigma^2_C$ increases with increasing width of the bins

(iii) $\sigma^2_C$ is significantly larger for peripheral (centrality 40 – 60%) collisions than for central (centrality 0 – 20%) collisions.

In this paper we use the wounded nucleon model \cite{4, 5} to calculate the values of $\sigma^2_C$ for the the same pseudorapidity bins as in the PHOBOS experiment. First we perform calculations for the simple case of elementary $p + p$ interactions and then we extend them to $Au + Au$ collisions and different centralities. We show that the wounded nucleon model reproduces reasonably well the experimental data, especially the centrality dependence of the forward-backward fluctuations.

In the next section we define the model in detail. In Section 3 we describe our Monte Carlo calculations and present the approximate analytical formula for $\sigma^2_C$ derived in the framework of the wounded nucleon model in the Appendix. In Section 4 our results are compared with the PHOBOS data and also predictions of other models of nucleus-nucleus collisions are discussed. We end our paper with conclusions.

2 Wounded nucleon model

The wounded nucleon model \cite{4, 5} assumes that after the collision of two nuclei, the secondary particles are produced in the process of independent fragmentation\cite{1} of

\footnote{\text{It is similar to the assumption of independent hadronization of strings in the dual parton model \cite{6}.}}
these nucleons (called "wounded nucleons"), which underwent at least one inelastic collision. The contribution from a wounded nucleon does not depend on the number of such collisions, thus the secondary particles are produced according to a universal fragmentation function, $\rho(\eta)$. Number of produced particles may vary following a multiplicity distribution.

The shape of the fragmentation function was extracted from the PHOBOS data on $d + Au$ collisions at $\sqrt{s} = 200$ GeV and is presented in Fig. 1 for a wounded nucleon from the forward moving nucleus. The fragmentation function for nucleons from the opposite nuclei is a reflection with respect to $\eta = 0$.

In this model the single particle density in elementary $p + p$ collisions is described as $\rho(\eta) + \rho(-\eta)$. In the case of nucleus-nucleus collision it becomes a sum of these functions multiplied by the number of wounded nucleons in the appropriate nuclei: $\langle w_F \rangle \rho(\eta) + \langle w_B \rangle \rho(-\eta)$, where $\langle w_F \rangle$ and $\langle w_B \rangle$ are the mean values of wounded nucleons in the forward and backward moving nucleus, respectively. In consequence at midrapidity the particle density in $A + A$ equals $\langle w \rangle \rho(0)$ (where $\langle w \rangle = \langle w_F \rangle + \langle w_B \rangle$) and is thus $\langle w \rangle / 2$ times larger than in elementary collisions. This prediction does not agree with the experimental data, however, as the particle density $dN/d\eta|_{\eta=0}$ is larger than $\langle w \rangle \rho(0)$ [10]. This can be easily corrected by introduction of a centrality dependent factor $\gamma$ which appropriately increases the multiplicity of particles generated in the fragmentation of a wounded nucleon that

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2In fact in our calculations we will modify this assumption. We will come back to this point later.

3An analogous wounded nucleon fragmentation function was constructed in Ref. [9].
underwent many inelastic collisions\footnote{Contrary to the wounded nucleon model assumption multiplicity from a wounded nucleon slightly depends on the number of collisions it underwent.}

The necessity of the above correction results from the composite structure of the nucleons. It is accounted for in the wounded quark-diquark model \cite{7}, where the fragmenting objects are quarks and diquarks, and which reproduces the particle density measured in $A + A$ collisions. The fragmentation of a wounded nucleon is equal in this approach to a sum of fragmentation functions of the quark and diquark weighted by the probability of "wounding" each of them. The scaling factor $\gamma$ introduced by us reflects the changes of these probabilities for different centralities of the collisions. The wounded nucleon model with our correction is equivalent to the wounded quark-diquark model in the kinematical area, where the fragmentation of spectators (unwounded constituents) is negligible \cite{7}. This is fully justified in our case, as we are analyzing data restricted to $|\eta| < 3$.

The fragmentation function $\rho(\eta)$ represents the mean yield from a wounded nucleon, whereas for calculation of fluctuations the changes occurring event-by-event have to be included. We assume that the shape of $\rho(\eta)$ is unchanged, even if the integral of $\rho(\eta)$ follows a multiplicity distribution. Further we also assume that the multiplicity distributions in $p + p$ collisions in the combined pseudorapidity bin $B + F$ can be parameterized by negative binomial (NB) distribution \cite{11}

$$P(n, \bar{n}, k) = \frac{\Gamma(n + k)}{\Gamma(\bar{n} + 1)\Gamma(k)} \left(\frac{\bar{n}}{k}\right)^n \left(1 + \frac{\bar{n}}{k}\right)^{-n-k},$$

where $\bar{n}$ is the average multiplicity in the combined interval $B + F$, $1/k$ measures deviation from Poisson distribution and $\Gamma(n)$ is the gamma function. In our calculations we need to know the multiplicity distribution from a single wounded nucleon. Assuming that soft particle production in $p + p$ collisions can be described by independent contributions from each nucleon \cite{12}, it is straightforward to show that the multiplicity distribution from a single wounded nucleon follows also NB distribution \cite{2} but with $\bar{n}$ and $k$ substituted by $\bar{n}/2$ and $k/2$, respectively\footnote{In general the multiplicity distribution from $w$ independent wounded nucleons is given by NB distribution \cite{2} with $\bar{n} \to w\bar{n}/2$ and $k \to wk/2$, where $\bar{n}$ and $k$ are measured in $p + p$ collisions.}

To perform the necessary calculations we also need to know the probability that a particle originating from the forward moving wounded nucleon goes to $F$ interval, under the condition it is found either in $B$ or $F$. It can be easily calculated as

$$p = \frac{\int_F \rho(\eta) \, d\eta}{\int_B \rho(\eta) \, d\eta + \int_F \rho(\eta) \, d\eta},$$

where $\rho(\eta)$ is the pseudorapidity density of produced particles shown in Fig. \ref{fig:1}.
wounded nucleon goes to $B$ interval equals $1 - p$. For the backward moving wounded nucleon $p$ has the same meaning when the intervals $F$ and $B$ are exchanged. The $\bar{n}$ parameter from NB distribution can be also calculated using the fragmentation function

$$\bar{n} = 2 \int_{F} \rho(\eta) \, d\eta + 2 \int_{B} \rho(\eta) \, d\eta.$$  \hspace{1cm} (4)

For the nucleus-nucleus collisions in our modified version of the wounded nucleon model the multiplicity of particles from a single wounded nucleon is rescaled by $\gamma$ factor. It reflects the fact that a wounded nucleon which underwent many inelastic collisions produces more particles than a wounded proton in $p + p$ collisions. Accordingly, in Eq. (2) we substitute $\bar{n}$ not by $\bar{n}/2$ but by $\gamma \bar{n}/2$. Less obvious is the necessity of modification of $k$ parameter, which we decided to multiply also by the factor $\gamma$. This is justified if we assume that each nucleon is composed of two constituents, a quark and a diquark, which populate particles independently. However, we show also results for unmodified $k$ and we consider the difference between them as an additional systematic uncertainty of the wounded nucleon model.

3 Calculations

In the present section we describe the MC calculation of $\sigma_{C}^{2}$ defined in Eq. (1) in the modified wounded nucleon model. In addition we present also approximate analytical formulae (with $\bar{n}$ and $k$ rescaled by the factor $\gamma$) derived in the Appendix.

We performed our calculations in three steps described below.

(A) In order to study effects of simple superposition of nucleon-nucleon collisions we assume that in both nuclei we have fixed number of wounded nucleons $w_{F} = w_{B}$. Each wounded nucleon adds its contribution generated independently according to the NB distribution (Eq. 2) with parameters $\bar{n}$ and $k$ substituted by $\gamma \bar{n}/2$ and $\gamma k/2$ respectively, where $\bar{n}$ and $k$ are measured in $p + p$ collisions. The generated particles are then assigned randomly, with probability $p$ and $1 - p$, to the intervals $F$ and $B$ (for the forward moving wounded nucleon; to $B$ and $F$ for the backward moving nucleon).

Our main observation is that this contribution very quickly saturates with increasing $w_{F}$. For $w_{F} > 10$ it practically does not depend on $w_{F}$. In the Appendix we present the approximate analytical derivation of the asymptotic formula for large $w_{F}$, which is:

$$\sigma_{C}^{2} \big|_{A} \approx 1 + \frac{\bar{n}}{k} (2p - 1)^{2}.$$  \hspace{1cm} (5)

This term depends only on the ratio $\bar{n}/k$, thus it is insensitive to our modification of the wounded nucleon model, i.e. the simultaneous substitution $\bar{n} \mapsto \gamma \bar{n}$ and
k \mapsto \gamma k$. Except the most peripheral events $\sigma^2_C|_A$ is centrality independent.

(B) Contribution solely from the fluctuations in the number of wounded nucleons can be obtained when we assume that each wounded nucleon fragments always into the same, mean number of particles. Then each forward moving wounded nucleon populates $\gamma \bar{n}p/2$ particles into the forward $F$ interval and $\gamma \bar{n}(1-p)/2$ particles into the backward $B$ interval (and analogously for the backward moving nucleons). In consequence we obtain

$$
N_F = \frac{1}{2} \gamma \bar{n} \left[ w_F p + w_B (1-p) \right],
$$

$$
N_B = \frac{1}{2} \gamma \bar{n} \left[ w_B p + w_F (1-p) \right],
$$

where $w_F$ and $w_B$ are the numbers of wounded nucleons in the forward and backward moving nuclei, respectively. The above formulae allow to substitute $N_F$ and $N_B$ in the definition of $\sigma^2_C$ (Eq. 1):

$$
\sigma^2_C|_B = \frac{1}{2} \gamma \bar{n} (2p-1)^2 \left< \frac{(w_F - w_B)^2}{w_F + w_B} \right>,
$$

This term explicitly depends on $w_F$ and $w_B$ and thus is sensitive to the centrality of the events sample. Moreover this term is sensitive to the $\gamma$ factor which was introduced to correct the wounded nucleon model. The distribution of $w_F$ and $w_B$ cannot be obtained in a model independent way. In order to match the experimental centrality definition we are using the events from the HIJING generator, for which cuts compatible with these for the PHOBOS experimental data were performed \cite{13}. However, similar results can be obtained if $w_F$ and $w_B$ are obtained from the Glauber type MC generator using centrality cuts on impact parameter or on $w_F + w_B$.

(C) The full calculations combine (A) and (B) together and include both the multiplicity distribution and the fluctuations of the number of wounded nucleons. The $w_F$ and $w_B$ are taken from generated HIJING events. Next each wounded nucleon populates particles according to the distribution (2) in a procedure similar to that from (A).

Results of full simulations are practically identical with the sum of two previously described terms (A) and (B). This is confirmed by the analytical calculations in the Appendix as in the first approximation (neglecting terms of the order $1/\langle w_F \rangle$) we obtain

$$
\sigma^2_C \approx \sigma^2_C|_A + \sigma^2_C|_B
$$

$$
\approx 1 + \bar{n} (2p-1)^2 \left[ \frac{1}{k} + \frac{1}{2} \gamma \left< \frac{(w_F - w_B)^2}{w_F + w_B} \right> \right].
$$

\footnote{We checked it with our own MC generator with the Gaussian nucleon-nucleon interaction profile.}
According to this formula, the values of $\sigma_C^2 > 1$ can be obtained in the wounded nucleon model only when the wounded nucleon fragmentation function is asymmetric with respect to $\eta = 0$ (leading to $p \neq 0.5$). The effects increasing $\sigma_C^2$, already contained in the multiplicity distribution measured in elementary $p + p$ collisions are enhanced by superposition of nucleon-nucleon collisions. The fluctuations of the number of wounded nucleons, that cause the difference $w_B - w_F$ to be non-zero, further increase the $\sigma_C^2$.

This closes theoretical discussion of the problem.

## 4 Results and discussion

In the present section we compare results for the variance $\sigma_C^2$ of the forward-backward asymmetry variable $C = (N_F - N_B)/\sqrt{N_F + N_B}$ obtained in our wounded nucleon model MC simulations with the recently published PHOBOS data \[3\]. The measurement was performed for $Au + Au$ collisions at $\sqrt{s} = 200$ GeV in the pseudorapidity range of $|\eta| < 3$ for various symmetric forward and backward intervals.

The model calculations were performed for the same intervals as in the experiment and for the experimentally studied centrality classes. The parameters $p$ and $\bar{n}$ were obtained from the wounded nucleon fragmentation function shown in Fig. 1 using Eqs. (3) and (4) and are listed in Tab. 1. The $\gamma$ factor responsible for the correction of the wounded nucleon model was estimated by comparing $\langle w \rangle \rho(0)$ and the PHOBOS data on $dN/d\eta$ at midrapidity \[10\], and equals $\gamma = 1.35 \pm 0.15$ and $1.6 \pm 0.1$ for peripheral and central collisions, respectively.

The dependence of $\sigma_C^2$ on the bin width $\Delta \eta$ presented in Fig. 2 includes points with the largest values of $\sigma_C^2$, measured with the smallest relative errors, and is thus more challenging for the models. Here the center of each bin is fixed at $\eta_F = 2$ and

| $\eta$ | $\bar{n}$ | $k$ | $p$ |
|-------|-------|-----|-----|
| 0.25  | 2.35  | 2.0 | 0.52|
| 0.75  | 2.41  | 2.3 | 0.57|
| 1.25  | 2.47  | 2.6 | 0.62|
| 1.75  | 2.47  | 2.9 | 0.67|
| 2.25  | 2.31  | 3.2 | 0.73|
| 2.75  | 5.10  | 3.5 | 0.8 |

| $\Delta \eta$ | $\bar{n}$ | $k$ | $p$ |
|---------|-------|-----|-----|
| 0.25    | 1.21  | 3.0 | 0.7 |
| 0.50    | 2.41  | 3.0 | 0.7 |
| 1.00    | 4.78  | 3.0 | 0.7 |
| 1.50    | 7.10  | 3.0 | 0.7 |
| 2.00    | 9.35  | 3.0 | 0.7 |

Table 1: The parameters used in the calculations of wounded nucleon model predictions presented in Figs. 2 and 3. We estimate the uncertainty of the values of the parameters to be 7%, 20% and 10% for $\Delta \bar{n}/\bar{n}$, $\Delta k/k$ and $\Delta p/p$, respectively.
Figure 2: The variance of the forward-backward asymmetry variable $C = (N_F - N_B)/\sqrt{N_F + N_B}$ as a function of the width of the forward and backward intervals for most central collisions (left) and the peripheral collisions (right). The position of the center of each bin is fixed at $\eta_F = 2$ and $\eta_B = -2$. The PHOBOS data points (dots) are compared with the prediction of the modified wounded nucleon model (continuous and dashed lines, for the scaled and unmodified $k$ parameter respectively). The grey bands reflect the systematic error of our calculations, due to the uncertainty of the parameters (see the text near the end of Section 4).

Our modified wounded nucleon model correctly reproduces the general trends observed in the data, especially the centrality dependence. The predictions obtained for the case with scaling of $k$ are slightly below the experimental points, but in most cases are consistent within the systematic errors. Interestingly, in the case of unmodified $k$, which we consider less probable, the agreement is much better.

In Fig. 3 the values of $\sigma_C^2$ obtained for fixed width of each interval ($\Delta \eta = 0.5$) and changing the distance between bin centers are shown. The wounded nucleon model is in this case in worse agreement with the data, there are discrepancies for $0.5 < \eta < 2$. However, for narrow $\eta$ bins the values of $\sigma_C^2$ are closer to 1, the systematic errors are relatively larger and also the influence of effects not included in the model may be thus stronger. The NB distribution which is supposed to contain effectively all correlations present in the elementary collisions may be not sufficient to fully represent the short range correlations (resonance decays, clusters).

In Fig. 4 we show different components of $\sigma_C^2$ present in the wounded nucleons model (with scaled $k$ parameter), as discussed in the previous section. For elementary $p + p$ collisions the value of $\sigma_C^2$ for the widest interval ($\Delta \eta = 2$) reaches 1.34, the superposition of nucleon-nucleon collisions increases it to about 1.5. There is almost no difference between $w_F = 30$ and $w_F = 140$, which correspond to the mean values for the peripheral and central collision analysed, as the superposition effects
Figure 3: The same as in Fig. 2 but now the width of each bin is fixed and equals $\Delta \eta = 0.5$, changing $\eta$ is the position of the center of the forward bin (the backward one is symmetric around $\eta = 0$).

fast saturate. Much stronger increase is due to contribution from the fluctuations of the number of wounded nucleons, they add to the previous numbers directly and give $1.94$ and $2.41$ for central and peripheral collisions respectively.

Let us notice that all presented results\textsuperscript{7} can be obtained from our analytical approximation (8), which agrees with the presented MC results with the accuracy better than $3\%$. In the case of analytical calculations we are using the values of $\langle (w_F-w_B)^2 \rangle$ extracted from samples of peripheral and central HIJING events selected according to the experimental centrality cuts, which are $0.38 \pm 0.05$ and $0.90 \pm 0.09$ for centrality $0−20\%$ and $40−60\%$, respectively \cite{13}.

Finally, let us discuss the uncertainty of our approach, represented by the grey bands in Figs. 2 and 3. The systematic errors of model calculations can be estimated using the analytical formula (8). We calculate the error of $\sigma^2_C$ caused by the uncertainty of each parameter separately and add them in quadrature. In the case of $k$ parameter the systematic error contains only the error of $k$ (20%), the uncertainty of scaling $k$ for $A+A$ collisions is represented by two extreme cases, for which the systematic errors are calculated separately. All contributions due to errors of parameters are of the same order, but the uncertainty of the probability $p$ has always the largest impact. The systematic error of predicted $\sigma^2_C$ calculated this way is approximately equal to $0.17 (\sigma^2_C - 1)$.

The data analyzed in this paper were previously compared with expectations from several models of nucleus-nucleus collisions: HIJING \cite{15}, AMPT \cite{16} in Ref. \cite{3} and UrQMD in Ref. \cite{14}. As we can see in Fig. 5 the differences between them

\textsuperscript{7}Except the predicted values of $\sigma^2_C$ for elementary $p+p$ collisions presented in Fig. 3
Figure 4: Wounded nucleon model predictions (with scaled $k$ parameter) for $\sigma_C^2$ values obtained for the pseudorapidity bins centered at $\eta = 2$ with changing width $\Delta \eta$. The model calculations for central 0–20% ($\langle w_F \rangle = 140$) and peripheral 40–60% ($\langle w_F \rangle = 30$) $Au + Au$ collisions with all effects included (full and open circles) are compared with results for collisions with exactly 140 and 30 wounded nucleons in each nuclei (full and open triangles), the fluctuations in the number of wounded nucleons is responsible for the difference between corresponding sets of points. In addition the predictions for $p+p$ collisions are also shown (stars).

are large, but none of these models correctly describes all features of the data listed in the Introduction. HIJING gives approximately correct values of $\sigma_C^2$ for central collisions, but they do not grow for peripheral collisions. AMPT features expected centrality dependence, but always significantly underestimates $\sigma_C^2$ values. UrQMD calculations agree with the experimental results for peripheral events, but this model predicts a slight increase of $\sigma_C^2$ for central collisions in place of an observed significant drop. The predictions of the wounded nucleon model are closest to the experimental data - especially for the version with $k$ parameter not scaled.

5 Conclusions

Our conclusions can be formulated as follows.

(i) We show that the PHOBOS Collaboration data [3] on the variance $\sigma_C^2$ of the forward-backward asymmetry variable $C = (N_F - N_B)/\sqrt{N_F + N_B}$ measured in $Au + Au$ collisions at $\sqrt{s} = 200$ GeV can be reasonably described in the framework
Figure 5: Comparison of the experimental data with predictions from several models for $\sigma_C^2$ dependence on the width $\Delta \eta$ of the pseudorapidity bin. Results for wounded nucleon model are shown for two cases, $k$ parameter unscaled and scaled, as dashed and continuous lines respectively. UrQMD, HIJING and AMPT predictions are represented by open symbols described in the plot.

of the wounded nucleon model. The wounded nucleon model predictions in most cases are consistent within the errors with the data and slightly lower values of $\sigma_C^2$ may be due to short range correlations, which are only partially accounted for in the model. Anyway, we reproduce both the pseudorapidity and centrality dependence of $\sigma_C^2$ in contrast to much more advanced HIJING, AMPT or UrQMD models.

(ii) The key ingredients of our approach are: the assumption of independent hadronization of each wounded nucleon; the wounded nucleon multiplicity distribution described by the negative binomial distribution (2) and; the wounded nucleon pseudorapidity fragmentation function, shown in Fig. 1. Moreover, we correct the wounded nucleon model by taking into account the dependence of the multiplicity from a single wounded nucleon on the number of collisions.

(iii) We also derive approximate analytical formula for $\sigma_C^2$ (5). It can be observed that $\sigma_C^2 - 1$ is proportional to $(1 - 2p)^2$ where $p$ determines the partition of each particle between forward and backward intervals. Thus the PHOBOS data can be reasonably described only if the wounded nucleon fragmentation function is indeed asymmetric in pseudorapidity, i.e., $p \neq 0.5$. It is also interesting to note that the centrality dependence of $\sigma_C^2$ is fully determined by the simple factor $\left\langle \frac{(w_B-w_F)^2}{w_B+w_F} \right\rangle$.

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A Appendix: Calculation of $\sigma_C^2$

In this Appendix we present the approximate analytical calculation of $\sigma_C^2$. Let us introduce the following notation

$$
N_B = \langle N_B \rangle (1 + \varepsilon_B), \\
N_F = \langle N_F \rangle (1 + \varepsilon_F).
$$

Although generally $\varepsilon_B$ and $\varepsilon_F$ may have any value from $-1$ to infinity, if we exclude the most peripheral events, the values of $|\varepsilon_B|$ and $|\varepsilon_F|$ are much smaller than 1. In such a case it is possible to expand the denominator of $\sigma_C^2$ in the powers of $\varepsilon_B$ and $\varepsilon_F$ (here we consider collisions of identical nucleus and symmetric $B$ and $F$ bins, i.e. $\langle N_B \rangle = \langle N_F \rangle$)

$$
\frac{1}{N_B + N_F} = \frac{1}{\langle N_B \rangle} \left( \frac{1}{2 + (\varepsilon_B + \varepsilon_F)} \right) \\
= \frac{1}{\langle N_B \rangle} \left( \frac{1}{2} - \frac{\varepsilon_B + \varepsilon_F}{4} + \frac{(\varepsilon_B + \varepsilon_F)^2}{8} + ... \right).
$$

Taking the first term of this expansion into account we obtain much simpler formula to handle

$$
\left\langle \frac{(N_B - N_F)^2}{N_B + N_F} \right\rangle \approx 1 + \frac{\langle N_B(N_B - 1) \rangle - \langle N_B N_F \rangle}{\langle N_B \rangle}.
$$

Indeed, now we can apply exactly the same method as in Ref. [17], where the forward-backward correlation coefficient in the wounded nucleon model was calculated. Let us construct the generating function

$$
H(z_B, z_F) = \sum_{N_B, N_F} P(N_B, N_F) z_B^{N_B} z_F^{N_F},
$$

where $P(N_B, N_F)$ is the probability to find $N_B$ particles in the backward $B$ interval and $N_F$ particles in the forward $F$ one. Of course $P(N_B, N_F)$ depends on the...
centrality of the collision. In Ref. [17] we have shown that in the framework of the wounded nucleon model, defined in Section 2, we obtain

\[
H(z_B, z_F) = \sum_{w_B, w_F} W(w_B, w_F) \left\{ 1 + \frac{\bar{n}}{k} \left[ p \left( 1 - z_B \right) + p' \left( 1 - z_F \right) \right] \right\}^{-kw_B/2} \times \\
\times \left\{ 1 + \frac{\bar{n}}{k} \left[ p' \left( 1 - z_B \right) + p \left( 1 - z_F \right) \right] \right\}^{-kw_F/2},
\]

(13)

where \( W(w_B, w_F) \) is the probability distribution of the numbers of wounded nucleons in the backward and forward moving nucleus, respectively. \( p' = 1 - p \) (\( B \) and \( F \) are symmetric around \( \eta = 0 \)) where \( p \) is defined in Eq. (3). \( \bar{n} \) and \( k \) come from the NB fits [2] to the \( p + p \) multiplicity data in the combined bin \( B + F \). Now the averages present in (11) can be directly calculated using the generating function (13)

\[
\langle N_B \rangle = \frac{\partial H(z_B, z_F)}{\partial z_B} \bigg|_{z_B=z_F=1}, \\
\langle N_B(N_B - 1) \rangle = \frac{\partial^2 H(z_B, z_F)}{\partial z_B^2} \bigg|_{z_B=z_F=1}, \\
\langle N_B N_F \rangle = \frac{\partial^2 H(z_B, z_F)}{\partial z_B \partial z_F} \bigg|_{z_B=z_F=1}. 
\]

(14)

Performing appropriate differentiations, multiplying \( \bar{n} \) and \( k \) by the \( \gamma \) factor and taking the following relation into account

\[
\sum_{w_B, w_F} W(w_B, w_F) \frac{(w_B - w_F)^2}{2 \langle w_B \rangle} = \frac{\langle (w_B - w_F)^2 \rangle}{2 \langle w_B \rangle} \approx \frac{\langle (w_B - w_F)^2 \rangle}{\langle w_B + w_F \rangle},
\]

(15)

we obtain our final result (8).

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