Abstract
A lepton flavor violating process $\mu \to e\gamma$ is investigated in the supersymmetric extra $U(1)$ models, which often appear as the low energy effective models of superstring and can potentially solve the $\mu$-problem. The branching ratio of this process is calculated. It is numerically estimated and compared with that of the MSSM. In this study we take account of an abelian gaugino kinetic term mixing and discuss its influence on this process. The possibility to find the extra gauge structure through this process is discussed.
The standard model (SM) has shown its incredible accuracy to describe the electroweak interaction through the precise measurement at LEP. Nevertheless, physics beyond the SM is eagerly explored because of its unsatisfactory feature for the explanation of the origin of weak scale and its stability. Supersymmetrization of the SM is now considered as the most promising extension to solve this problem\cite{1}. However, even in this minimal supersymmetric standard model (MSSM) there still remains a theoretically unsatisfactory feature which is known as the $\mu$-problem\cite{2}. To cause an appropriate radiative symmetry breaking at the weak scale, we need a Higgs mixing term $\mu H_1 H_2$, where $\mu \sim O(G_F^{-1/2})$ and $G_F$ is a Fermi constant. However, there is no reason why $\mu$ should be such a scale because it is usually considered to be irrelevant to the supersymmetry breaking. A solution for this problem is to consider $\mu$ as a dynamical variable\cite{3}. The introduction of a singlet field $S$ with a Yukawa type coupling $\lambda SH_1 H_2$ can realize this scenario in the simplest way\cite{4}. That is, if $S$ gets a vacuum expectation value (VEV) of order 1 TeV as a result of radiative corrections to the soft supersymmetry breaking parameters\cite{5}, $\mu \sim O(G_F^{-1/2})$ will be realized dynamically through the relation $\mu = \lambda \langle S \rangle$.

The extra $U(1)$ models are the typical extensions of gauge structure of the SM. It is very interesting to note that many extra $U(1)$ models have the above mentioned feature inevitably\cite{6,7}. Low energy models derived from superstring often have accompanied with extra $U(1)$ factors in their gauge structure\cite{8}. It seems to be natural that these aspects motivate us to investigate extra $U(1)$ models and try to look for a clue of such a gauge structure. Recent precise measurements at LEP and also the Tevatron experiments show us that the lower bound for its gauge boson mass is rather large and then it may not be so easy to find it directly\cite{9}. Even in that case if nature is supersymmetric and the gauge bosons have their superpartners, there may be other possibilities to investigate the gauge structure through examining the processes to which their superpartners contribute.

In this letter we study the lepton flavor violating $\mu \to e\gamma$ process. The gauginos of extra $U(1)$s can affect this process. Our purpose here is to estimate their effect and discuss the possibility to find the extra gauge structure through this process. Its comparison with the results in the MSSM will also be useful for the future experimental analysis. We consider the minimal models which have only one extra $U(1)_X$ and a singlet Higgs $S$ with a $U(1)_X$ charge besides the MSSM contents\cite{10}. These fields are assumed to remain light\footnote{In order to induce the symmetry breaking radiatively, it is necessary to introduce the vector like}.
around the TeV region. The neutralino sector in this model is extended by an extra $U(1)_X$ gaugino and a fermionic partner of the singlet Higgs $S$ in addition to the ingredients of the MSSM.

Before proceeding to the detailed study we should note an additional feature of the neutralino sector of these extra $U(1)$ models. It has been well known that in principle there can be kinetic term mixings among abelian gauge fields because these field strengths are gauge invariant. Supersymmetrization of the models introduces kinetic term mixings among abelian gauginos. In the analysis of multi $U(1)$ models we generally need to take account of these effects. Thus at first we briefly summarize the mixing effects in the gaugino sector for the usage in the later study.

In supersymmetric models gauge fields are extended to vector superfields

$$V_{WZ}(x, \theta, \bar{\theta}) = -\theta \sigma_\mu \bar{\theta} V^\mu + i \theta \bar{\theta} \lambda - i \theta \bar{\theta} \lambda + \frac{1}{2} \theta \bar{\theta} \bar{\theta} D, \quad (1)$$

where we used the Wess-Zumino gauge. A gauge field strength is included in the chiral superfield constructed from $V_{WZ}$ in the well known procedure,

$$W_\alpha(x, \theta) = (\bar{D} D) D_\alpha V_{WZ}$$

$$= 4i \lambda_\alpha - 4 \theta_\alpha D + 4i \theta^\beta \sigma_{\nu \alpha \beta} \bar{\sigma}^\beta \partial^\nu V^\mu - 4 \theta \sigma_{\mu \alpha \beta} \bar{\partial}^\nu \lambda^\beta. \quad (2)$$

In terms of these superfields the supersymmetric Lagrangian can be written as

$$\mathcal{L} = \frac{1}{32} (W^\alpha W_\alpha)_F + (\Phi^\dagger \exp(2g^0 Q V_{WZ}) \Phi)_D, \quad (3)$$

where $\Phi = (\phi, \psi, F)$ is the chiral superfield representing matter fields. This Lagrangian is easily extended to multi $U(1)$ models. In the models with two $U(1)$ factor groups, the supersymmetric gauge invariant kinetic terms are most generally written by using chiral superfields $\hat{W}^a_\alpha$ and $\hat{W}^b_\alpha$ for $U(1)_a \times U(1)_b$ as

$$\frac{1}{32} (\hat{W}^{a\alpha} \hat{W}^{\alpha}_a)_F + \frac{1}{32} (\hat{W}^{b\alpha} \hat{W}^{\alpha}_b)_F + \sin \chi \frac{1}{16} (\hat{W}^{a\alpha} \hat{W}^{b}_a)_F \quad (4)$$

extra color triplets $(g, \bar{g})$ which have the coupling to the singlet $S$ as $\kappa S g \bar{g}$. But in the present study they play no role and then we will ignore them.

In fact there are some works in which it is discussed in what case kinetic term mixings can occur.
where we introduced the mixing terms. These can be canonically diagonalized by performing the transformation,

\[
\begin{pmatrix}
\hat{W}^a \\
\hat{W}^b
\end{pmatrix} = \begin{pmatrix}
1 & -\tan \chi \\
0 & 1/\cos \chi
\end{pmatrix}
\begin{pmatrix}
W^a \\
W^b
\end{pmatrix}.
\]

This transformation affects not only the gauge vector fields but also the sector of gauginos \(\lambda_{a,b}\) and auxiliary fields \(D_{a,b}\). The modification due to this transformation in the gaugino sector can be summarized as

\[
g_0^a Q_a \hat{\lambda}^a + g_0^b Q_b \hat{\lambda}^b = g_a Q_a \lambda^a + (g_{ab} Q_a + g_b Q_b) \lambda^b,
\]

where \(\lambda_{a,b}\) are canonically normalized gauginos. \(Q_a\) and \(Q_b\) stand for the charges of matter fields for \(U(1)_a\) and \(U(1)_b\). Gauge coupling constants \(g_a, g_{ab}\) and \(g_b\) are related to the original ones as,

\[
g_a = g_0^a, \quad g_{ab} = g_0^a \tan \chi, \quad g_b = \frac{g_0^b}{\cos \chi}.
\]

These low energy values are determined by using the renormalization group equations. However, in the present study we will treat them as parameters.

For the study of the \(\mu \rightarrow e\gamma\) process in the supersymmetric models, it is necessary to clarify both of the neutralino and chargino sector. The relevant part of the superpotential and soft supersymmetry breaking terms are

\[
W = \lambda S H_1 H_2 + \cdots,
\]

\[
\mathcal{L}_{\text{soft}} = -\sum_i m_i^2 |\phi_i|^2 + M_W \lambda_W \lambda_W + M_Y \lambda_Y \lambda_Y + M_X \lambda_X \lambda_X + M_{YX} \lambda_Y \lambda_X + \text{h.c.} + \cdots,
\]

where \(\phi_i\) represents the scalar components contained in the models. \(M_W, M_Y\) and \(M_X\) are soft supersymmetry breaking gaugino masses\(^3\) for \(SU(2)_L, U(1)_Y\) and \(U(1)_X\). These parameters and a Yukawa coupling \(\lambda\) are assumed to be real and it should not be confused with the gaugino fields \(\lambda_a\). Using the canonically normalized basis, we can write down the relevant quantities in the neutralino sector modified by the kinetic

\(^3\) We introduce the effect caused from the abelian gaugino mass mixing as \(M_{YX}\), which may exist at the Planck and may also be yielded through the loop effects. We need to estimate its low energy value by using the renormalization group equations. In the later numerical study we put \(M_{YX} = 0\), for simplicity.
term mixing. They are the neutralino mass matrix and the vertex factors of gaugino-fermion-sfermion interactions. If we take the canonically normalized gaugino basis as $N^T = (-i\lambda_{W_3}, -i\lambda_Y, -i\lambda_X, \tilde{H}_1, \tilde{H}_2, \tilde{S})$ and define the mass terms as

$$L_{\text{mass}}^n = -\frac{1}{2} N^T M N + \text{h.c.},$$

the $6 \times 6$ neutralino mass matrix $M$ can be expressed as

$$
\begin{pmatrix}
M_W & 0 & 0 & m_Z c_W \cos \beta & -m_Z c_W \sin \beta & 0 \\
0 & M_Y & C_1 & -m_Z s_W \cos \beta & m_Z s_W \sin \beta & 0 \\
0 & C_1 & C_2 & C_3 & C_4 & C_5 \\
m_Z c_W \cos \beta & -m_Z s_W \cos \beta & C_3 & 0 & \lambda u & \lambda v \sin \beta \\
-m_Z c_W \sin \beta & m_Z s_W \sin \beta & C_4 & \lambda u & 0 & \lambda v \cos \beta \\
0 & 0 & C_5 & \lambda v \sin \beta & \lambda v \cos \beta & 0
\end{pmatrix}.
$$

Matrix elements $C_1 \sim C_5$ are components which are affected by the kinetic term mixing. They are represented as

$$
C_1 = -M_Y \tan \chi + \frac{M_Y X}{\cos \chi}, \quad C_2 = M_Y \tan^2 \chi + \frac{M_X}{\cos^2 \chi} - \frac{2M_Y X \sin \chi}{\cos^2 \chi}, \\
C_3 = \frac{1}{\sqrt{2}} \left( g_Y \tan \chi + \frac{g_X Q_1}{\cos \chi} \right) v \cos \beta, \quad C_4 = \frac{1}{\sqrt{2}} \left( -g_Y \tan \chi + \frac{g_X Q_2}{\cos \chi} \right) v \sin \beta, \\
C_5 = \frac{1}{\sqrt{2}} \frac{g_X Q_S}{\cos \chi} u,
$$

where $Q_1, Q_2$ and $Q_S$ are the extra $U(1)_X$ charges of Higgs chiral superfields $H_1, H_2$ and $S$.

Neutralino mass eigenstates $\tilde{\chi}_i^0 (i = 1 \sim 6)$ are related to $N_j$ through the mixing matrix $U$ as

$$
\tilde{\chi}_i^0 = \sum_{j=1}^{6} U_{ij}^T N_j.
$$

The change induced by the kinetic term mixing in the gaugino interactions can be confined into the extra $U(1)_X$ gaugino sector and by using eq.(6) new interaction terms can be expressed as,

$$
\begin{split}
\frac{i}{\sqrt{2}} & \left[ \bar{\psi}^* \left( -g_Y \tan \chi + \frac{g_X Q_X}{\cos \chi} \right) \lambda_X \psi - \left( -g_Y \tan \chi + \frac{g_X Q_X}{\cos \chi} \right) \bar{\lambda}_X \bar{\psi} \bar{\psi} \\
&+ H^* \left( -g_Y \tan \chi + \frac{g_X Q_X}{\cos \chi} \right) \lambda_X \tilde{H} - \left( -g_Y \tan \chi + \frac{g_X Q_X}{\cos \chi} \right) \bar{\lambda}_X \bar{\tilde{H}} \right] \quad (12)
\end{split}
$$
where $\psi$ and $\tilde{\psi}$ represent the quarks/leptons and the squarks/sleptons, respectively. Higgs fields ($H_1, H_2, S$) are summarized as $H$ and the corresponding Higgsinos $\tilde{H}_1, \tilde{H}_2$ and $\tilde{S}$ are denoted as $\tilde{H}$. Taking account of this, gaugino-fermion-sfermion vertices in the basis of mass eigenstates are assigned by the following factors,

\[
Z_i^L(Y, Q_X) = -\frac{1}{\sqrt{2}} \left[ g_W \tau_3 U_{1i} + g_Y Y U_{2i} + \left( -g_Y Y \tan \chi + \frac{g_X Q_X}{\cos \chi} \right) U_{3i} \right],
\]

\[
Z_i^R(Y, Q_X) = \frac{1}{\sqrt{2}} \left[ g_Y Y U_{2i} + \left( -g_Y Y \tan \chi + \frac{g_X Q_X}{\cos \chi} \right) U_{3i} \right],
\]

where we used the left handed basis for the chiral superfields. It is also necessary to define the chargino mass eigenstates for the following calculation. The chargino mass term is given in the matrix form as

\[
\mathcal{L}_{\text{mass}}^c = -\left( H^+_1, -i\lambda^+_Y \right) \begin{pmatrix} -\lambda u & \sqrt{2}m_Z c_W \cos \beta \\ \sqrt{2}m_Z c_W \sin \beta & M_W \end{pmatrix} \begin{pmatrix} H^+_2 \\ -i\lambda^+_Y \end{pmatrix} + \text{h.c.}
\]

The mass eigenstates are defined in terms of the weak interaction eigenstates through unitary transformations,

\[
\begin{pmatrix} \tilde{\chi}^+_1 \\ \tilde{\chi}^+_2 \end{pmatrix} = W^{(+)}\dagger \begin{pmatrix} H^+_2 \\ -i\lambda^+_Y \end{pmatrix}, \quad \begin{pmatrix} \tilde{\chi}^{-}_1 \\ \tilde{\chi}^{-}_2 \end{pmatrix} = W^{(-)}\dagger \begin{pmatrix} H^-_1 \\ -i\lambda^-_Y \end{pmatrix}.
\]

Based on these preparations, we proceed to the estimation of the $\mu \rightarrow e\gamma$ process in the present models. The flavor changing processes are strongly suppressed through the experimental results\cite{12}. In the supersymmetric models, however, there are generally many sources for these processes in the superpartner sector besides the ones of the SM\cite{13-16}. Colored superpartners cause the dominant contributions in many hadronic flavor changing neutral processes. Thus the contribution from the neutralino sector may not be clearly seen through such processes. In order to see the structure of the neutralino sector, we need non-hadronic process and $\mu \rightarrow e\gamma$ seems to be particurally interesting in the relation to our present purpose as far as R-parity violating terms are absent. At one-loop level this process can occur because of the existence of nontrivial flavor structure of soft supersymmetry breaking terms in the slepton sector. Various studies of $\mu \rightarrow e\gamma$ in the MSSM framework and some extended models have been done by now\cite{14, 16, 17}. We extend these analyses to the multi $U(1)$s case. One-loop diagrams contributing to this process in the present models are shown in Fig.1.
The effective interaction describing this decay is given as

\[ \mathcal{L}_{\text{eff}} = \mathcal{G}_L\bar{\psi}_eR\sigma_{\mu\nu}\psi_{\mu L}F^{\mu\nu} + \mathcal{G}_R\bar{\psi}_eL\sigma_{\mu\nu}\psi_{\mu R}F^{\mu\nu}. \] (16)

By carrying out the calculation of diagrams in Fig.1, we can obtain the effective couplings \(\mathcal{G}_L\) and \(\mathcal{G}_R\). In the diagonalizing basis of the lepton mass matrix \(m_l\), the origin of flavor changings is the off-diagonal elements of Kobayashi-Maskawa matrix in the lepton sector\(^4\) and also the slepton mass matrix. The slepton mass matrix is written as

\[
\begin{pmatrix}
M^2_{LL} & M^2_{LR} \\
M^2_{RL} & M^2_{RR}
\end{pmatrix}
\]

(17)

where \(M^2_{LR} = m_l(A + \lambda u \tan \beta)\) for charged sleptons. In the neutrino sector right-handed sneutrinos are assumed to have the large supersymmetric masses and their relevant part to the present analysis is only the part of \(M^2_{LL}\). To reduce the number of free parameters we make the following assumptions for components of the slepton mass matrix,

\[
(M^e_{\alpha\alpha})^2 = (M^e_{RR})^2 \equiv M^2, \quad (M^e_{\alpha\beta})^2 = (M^e_{RR})_{\alpha\beta} \equiv \Delta_{\alpha\beta}^2.
\] (18)

As shown in Fig.1, there are two types of diagrams which are distinguished by the place of the chirality flip. For our present purpose, it will be enough to concentrate our attention on the diagrams with the chirality flip on the internal line (Figs (a) and (c)).\(^5\) Under these assumptions the effective couplings \(\mathcal{G}_L\) and \(\mathcal{G}_R\) can be summarized as

\[
\mathcal{G}_L = -\frac{e}{32\pi^2} \sum_{i=1}^{6} m_i^n \left\{ \left(\frac{M^e_{LR}}{M^4}\right) Z^L_{2i}(-1, Q_{eL}) Z^R_i(2, Q_{eR}) + \frac{g^2 m_i m_e}{2m_W^2 \cos^2 \beta} F_i \right\},
\]

\[
-\frac{g_W}{\sqrt{2}m_W \cos \beta} \frac{\Delta_{\mu e}^2}{M^4} \left( m_e Z^L_{2i}(-1, Q_{eL}) + m_{\mu} Z^R_i(2, Q_{eR}) \right) U_{4i},
\]

\[
-\frac{g^2 W m_e}{\sqrt{2}m_W \cos \beta} \sum_{i=1}^{2} \left( K_{\nu e\mu} \frac{m_i^c}{M^2} J \left( \frac{m_i^{\nu U}}{M^2} \right) + \sum_{\alpha=\mu,\tau} K_{\nu \alpha \mu} \frac{\Delta_{\alpha e}^2}{m_i^3} F_i \right) W^{(+)}_{2i} W^{(-)}_{1i},
\]

\[
\mathcal{G}_R = -\frac{e}{32\pi^2} \sum_{i=1}^{6} m_i^n \left\{ \left(\frac{M^e_{RL}}{M^4}\right) Z^L_{2i}(-1, Q_{eL}) Z^R_i(2, Q_{eR}) + \frac{g^2 m_i m_e}{2m_W^2 \cos^2 \beta} F_i \right\},
\]

\(\Delta_{\alpha\beta}^2 \equiv (M^e_{\alpha\beta})^2 - (M^e_{RR})_{\alpha\beta} - (M^e_{\alpha\alpha})^2 \equiv (M^e_{RR})_{\alpha\beta} \equiv \Delta_{\alpha\beta}^2\).

\(^4\)We assume the non-zero Majorana neutrino masses induced from the seesaw mechanism in view of the solar neutrino problem.

\(^5\)The origin of flavor violating off-diagonal elements of \(M^2_{LL}\), \(M^2_{RR}\) and \(M^2_{LR}\) are discussed from various viewpoints\(^6\), \(^7\). In this analysis we donot refer to it and only treat them as parameters.

\(^6\)This treatment may not be bad even in the quantitative view point since the neutralino masses are expected to be much larger than charged lepton masses. For the completeness of our formuluses, however, we will present the contribution to the effective couplings from Figs. (b) and (d) in the appendix.
\[ -\frac{g_W}{\sqrt{2}m_W} \frac{\Delta_{\mu e}^2}{M^4} \left( m_\mu Z_{2i}^L(-1, Q_{e_L}) + m_e Z_{3i}^R(2, Q_{e_R}) \right) U_{4i} \right\} F_1 \left( \frac{m_\mu^2}{M^2} \right) \\
- \frac{g_W^2 m_\mu}{\sqrt{2}m_W \cos \beta} \sum_{i=1}^{2} \left( K_{\nu e}^i \frac{m_\tau^i}{M^2} J \left( \frac{m_\tau^2}{M^2} \right) + \sum_{\alpha=e,\tau} K_{\nu e} \frac{\Delta_{\nu e}^2}{m_\tau^3} F_1 \left( \frac{M^2}{m_\tau^2} \right) \right) W_{2i}^{(+)} W_{1i}^{(-)} \] (19)

where \( m_\mu^i \) and \( m_\tau^i \) represent the i-th mass eigenvalues of neutralinos and charginos, respectively. \( \Delta_{\mu e}^2 \) stands for an off-diagonal element between the e- and \( \mu \)-generation of slepton mass matrices as defined by eq.(18). Its allowed range may be estimated at a few GeV\(^2\) or less depending on other soft supersymmetry breaking parameters\[15\]. \( K_{\alpha\beta} \) is the Kobayashi-Maskawa matrix element in the lepton sector. Kinematical functions \( F_1(r) \) and \( J(r) \) appearing from the loop integrals are defined by

\[ F_1(r) = \frac{1}{2(1-r)^2} \left[ 1 + 4r - 5r^2 + 2r(r+2) \ln r \right], \]

\[ J(r) = \frac{1}{2(1-r)^2} \left[ -3 + r - \frac{2}{(1-r)} \ln r \right]. \] (20)

Using these results, we can represent the branching ratio of this decay process as

\[ B(\mu \rightarrow e\gamma) = \frac{48\pi^2}{G_F^2 m_\mu^2} \left( |G_L|^2 + |G_R|^2 \right). \] (21)

In order to compare this result with the MSSM one, it is useful to list up the extra parameters added to the ones contained in the MSSM formulism:

\[ \tan \beta, \ (M_{LR}^e)_{\alpha\beta}^2, \ M^2, \ \Delta_{\alpha\beta}^2, \ K_{\alpha\beta}, \ M_W, \ M_Y. \]

Additional parameters to these are new gaugino masses (\( M_X, M_{YX} \)), the kinetic term mixing parameter \( \sin \chi \), the extra \( U(1) \) coupling \( g_X \) and charges\[1](\( Q_1, Q_2 \)) and also the \( \mu \)-term relevant parameters (\( \lambda, \langle S \rangle (\equiv u) \)). We can easily check that the neutralino contribution to eq.(21) results in the expression given in refs.\[14, 15\] in the case of the photino dominated neutralino, if we put these additional parameters zero instead of keeping \( \mu (= \lambda u) \) constant.

Before choosing a parameter set for the numerical analysis, we should note some features of our models. In these models the vacuum expectation value \( u \) of the singlet Higgs \( S \) is relevant to the extra \( U(1)_X \) gauge boson mass besides determining the \( \mu \)-scale. The mixing between the ordinary \( Z^0 \) and the \( U(1)_X \) boson is severely constrained by

\[ \sum Q_1 + Q_2 + Q_S = 0 \] is satisfied because of the form of superpotential.\[7\]
the precise measurement at LEP and the direct search at Tevatron. This constraint requires that the mass of the $U(1)_X$ boson is large enough and in that case its mass eigenvalue is given by

$$m_{Z'}^2 \simeq \frac{1}{2 \cos^2 \chi} g_X^2 (Q_1^2 v_1^2 + Q_2^2 v_2^2 + Q_3^2 u^2).$$

(22)

The experimental bound on $m_{Z'}^2$ determines the lower bound on $u$. On the other hand, $u$ determines the $\mu$-scale as $\mu = \lambda u$. Thus to keep $\mu$ in the suitable range we need to put the upper bound on $\lambda$. For its rough estimation, we take $m_{Z'} > \sim 400$ GeV and also assume $g_X = g_Y$, which is satisfied, for example, in the abelian subgroup of $E_6$ if the full components of 27 of $E_6$ contribute to $\beta$-functions. In this case if we require $\mu < \sim 1$ TeV, we obtain the upper bound on $\lambda$ as $\lambda < \sim 0.6 Q_S$. This bound seems to be reasonable from the view point of the analysis of the radiative symmetry breaking.

Another feature which we should note is the dependence of the effective couplings $G_L$ and $G_R$ on the neutralino and chargino mass eigenvalues $m_\nu$ and $m_\chi$. These dependence can be factorized as $r^{1/2} F_1(r)$ where $r = (m_\nu/M)^2$ for neutralinos and also $r^{-3/2} F_1(r^{-1})$, $r^{1/2} J(r)$ where $r = (m_\chi/M)^2$ for charginos. They vary in the range $0 < r^{1/2} F_1(r) \lesssim 0.1$, $0 < r^{-3/2} F_1(r^{-1}) \lesssim 0.25$ and $0 < r^{1/2} J(r) \lesssim 0.44$. Each maximum value is realized at $r \sim 0.27$, $r \sim 0.025$ and $r \sim 0.12$, respectively. Thus all of these factors can be considered as the same order at least except for $r \sim 0$.

Taking account of this, we can roughly estimate the condition for the neutralino contribution dominance by comparing the neutralino contribution to the branching ratio with the chargino contribution. The couplings of the neutralinos to leptons come from gauge couplings and Yukawa couplings. Because Yukawa couplings are small enough, the dominant contribution will be yielded by the neutralinos which are dominantly composed of the gauginos $\lambda_W, \lambda_Y$ and $\lambda_X$. As seen from eq.(19), it is naively required

$$\frac{\Delta_{\mu e}^2}{(M^2_{LR})_{\mu e}} > \frac{m_W \cos \beta}{m_\mu g_W} \sim 10^3$$

(23)

in order to guarantee the similar order contribution from all neutralinos. This requirement seems to be difficult to be satisfied. So we focus our study to the case where main neutralino contribution comes from the term associated with $(M^2_{LR})_{\mu e}$ in eq.(19). Next
we compare it with the chargino contribution. If we pay attention on the factors in each terms of eq.(19) except for the mixing matrix elements, the condition for the neutralino contribution becoming larger than the chargino one can be roughly estimated as

\[ (M_{LR}^e)^2 > \frac{m_\mu}{m_W \cos \beta} K_{\nu\mu} M^2 \sim 20 K_{\nu\mu}. \]  

In this estimation we assumed \( \tan \beta \sim 1 \) and \( M \sim 100 \text{ GeV} \). If we note that \( (M_{LR}^e)^2 = m_\mu A_{\mu e} \) and \( m_{t\mu} \tan \beta \) does not contribute to it, the above condition for \( (M_{LR}^e)^2 \) corresponds to \( A_{\mu e} \gtrsim 200 K_{\nu\mu} \text{ GeV} \). Thus if we take \( K_{\nu\mu} \sim 5 \times 10^{-4} \) as the KM-matrix element in the lepton sector\(^9\) the neutralino contribution assumed above can be expected to be dominant under the condition\(^9\)

\[ \Delta_{\mu e} < 1 \text{ GeV}, \quad A_{\mu e} \gtrsim 10^{-1} \text{ GeV}. \]  

This argument suggests that the gaugino components of the neutralino contribution can be dominant one in the rather general situation. Moreover, under this condition \( B(\mu \to e\gamma) \) takes the value just below the present experimental bound. Thus in such a parameter range we may have a chance to get a hint for an additional abelian gauge structure through the \( \mu \to e\gamma \) process. Our main interest is the effect coming from new ingredients so that in the following numerical study we assume the measure of flavor mixing \( (M_{LR}^e)^2 \) so as \( B(\mu \to e\gamma) \) to be the same order as the present experimental bound in the MSSM case.

Now we give our result of the numerical analysis. As the typical values of free parameters, we take

\[ \tan \beta = 1.5, \quad A_{\mu e} = 0.2 \text{ GeV}, \quad M = 100 \text{ GeV}, \]
\[ M_Y = M_X = \frac{5}{3} \tan^2 \theta_W M_W, \quad M_{YX} = 0, \quad \lambda = 0.5, \]  

where we assumed the unification relation for the gaugino masses. As a typical example of the low energy extra \( U(1) \), we take the \( \eta \)-model induced from \( E_6 \). Their charge assignment for the relevant fields is listed in Table 1. Under this parameter setting, in Fig.2 \( B(\mu \to e\gamma) \) in this model is drawn as a function of \( u \) in the case of \( M_W = 80, 180 \text{ GeV} \) and \( \sin \chi = 0, 0.3 \). The horizontal axis should be converted to \( u \) by \( u = 50(u' + 2) \). Thus the

\(^9\)This small value does not contradict the neutrino oscillation solution for the solar neutrino problem, if we assume the existence of a sterile neutrino as ref.\(^{18}\).

\(^{10}\)These values are very similar to ones given in ref.\(^{15}\).
Table 1  The charge assignments of extra $U(1)$s induced from $E_6$. These charges are normalized as $\sum_{i\in 27} Q_i = 20$. Only relevant fields to our study are listed from 27 of $E_6$.

| fields | $Q$ | $U^c$ | $D^c$ | $L$ | $E^c$ | $H_1$ | $H_2$ | $S$ |
|--------|-----|-------|-------|-----|-------|-------|-------|-----|
| $Y$    | $1/3$ | $-4/3$ | $2/3$  | $-1$ | $2$   | $-1$  | $1$    | $0$ |
| $Q_\eta$ | $-2/3$ | $-2/3$ | $1/3$  | $1/3$ | $-2/3$ | $1/3$  | $4/3$  | $-5/3$ |

The charge assignments of extra $U(1)$s induced from $E_6$. These charges are normalized as $\sum_{i\in 27} Q_i = 20$. Only relevant fields to our study are listed from 27 of $E_6$.

range of $u$ is $100 \text{ GeV} \leq u \leq 2000 \text{ GeV}$. Here it is useful to recall again that $m_{Z'}$ is given by eq.(22) and also the relation $\mu = \lambda u$. From the recent chargino and neutralino search at LEP[19], the allowed region of $(\mu, M_{W})$ plane should satisfy $\mu, M_{W} > \sim 100 \text{ GeV}$ as far as $\mu > 0$. This corresponds to $u' > 2$ for $\lambda = 0.5$. Moreover, taking account of the small mixing constraint on $Z^0$ and the extra $U(1)$, $u \geq 800 \text{ GeV}$ and then $u' \geq 14$ should be satisfied if we require $m_{Z'} \geq 400 \text{ GeV}$.

The ratio $R$ of this $B(\mu \to e\gamma)$ against the one of the MSSM is also presented in Fig.3 in the case of $\sin \chi = 0$ and 0.3 for each value of $M_{W}$. As easily seen from Figs.2 and 3, the influence appearing in $B(\mu \to e\gamma)$ due to the extra $U(1)$ gaugino is not so large but non-negligible. These figures also show that the gaugino kinetic term mixing has an enhancement effect on $B(\mu \to e\gamma)$ in the $\eta$-model. This effect is considered to be mainly caused from the change of the effective couplings $Z^L_i(Y, Q_X)$ and $Z^R_i(Y, Q_X)$. The smaller $M_{W}$ and $u$ give the larger $B(\mu \to e\gamma)$. The effect of $\sin \chi \neq 0$ can be easily seen for the smaller $M_W$ and $u$.

It should be noted that there can remain the deviation of $O(10^{-12})$ from the MSSM value at $u' \sim 14$. As $u'$ becomes larger, $B(\mu \to e\gamma)$ monotonically decreases but the $O(10^{-13})$ deviation can still remain. For the smaller $u(u' < 14)$, the suitable condition should be satisfied for the consistency with the bound of the mixing between $Z^0$ and the extra $U(1)_X$, as already remarked in the footnote. if such a condition is satisfied and then the small value of $u$ is allowed from the precise measurement at LEP, we may have an important clue to study the extra gauge structure because of the large deviation from the MSSM prediction. Anyway, these results seem to be interesting. Particularly, the fact that $B(\mu \to e\gamma)$ can deviate by $O(10^{-12\sim-13})$ from the MSSM value even at the rather large $u$ region seems to be encouraging. It may be possible to find some clues of the extra
$U(1)$ gauge structure through $B(\mu \rightarrow e\gamma)$ if its experimental bound is improved by an order from the present value.

Some comments should be ordered on the various parameter dependences of $B(\mu \rightarrow e\gamma)$. In particular, $\lambda$ and $\tan \beta$ dependence seems to be important.

1. In the present models $\lambda$ can be an independent parameter and it affects $B(\mu \rightarrow e\gamma)$ through the neutralino mass matrix directly besides through the combination with $u$. As a result, $B(\mu \rightarrow e\gamma)$ shows the substantial $\lambda$ dependence, although no significant $\lambda$ dependence can be seen in the ratio $R$ at the large $u$ region.

2. The $\tan \beta$ dependence seems to be very crucial for the absolute value of $B(\mu \rightarrow e\gamma)$ in the whole range of $u$. The large $\tan \beta$ makes $B(\mu \rightarrow e\gamma)$ small for all $u$ region and also increases the sensitivity to $u$ at the small $u$ region. No significant $\tan \beta$ dependence can be seen in $R$ at the large $u$ region.

We may also be interested in the $M_{YX}$ dependence in the case of $\sin \chi \neq 0$. As far as $M_{YX}$ is induced by the loop effects, $M_{YX}$ may be roughly estimated as $M_{YX} \sim g_Y g_{YX} M_Y / 8 \pi^2$, which is completely negligible. Unless $M_{YX}$ is produced at $M_{pl}$ as a rather large value, it can be safely neglected in this type of study.

In summary we investigated the $\mu \rightarrow e\gamma$ process in the extra $U(1)$ models taking account of the gaugino kinetic term mixing. After driving the formulas for the branching ratio of $\mu \rightarrow e\gamma$ in the general framework, we practiced the numerical study and showed that in the $\eta$-model the deviation from the MSSM can be seen at the level of $O(10^{-12-13})$ through this process. The abelian gaugino kinetic term mixing has some effects on this process and we may find the suitable clue of the extra gauge structure by investigating this process. It will be useful to note that the usage of this process may open the alternative window to search the extra $U(1)$ gauge structure for an appropriate parameter region. It will be necessary to investigate this process in more general parameter region and other extra $U(1)$ models. The combined study with other rare process related to the neutralino sector like the electron electric dipole moment is also interesting[20].

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Appendix

The contribution to the effective couplings $G_L$ and $G_R$ coming from Figs.(b) and (d) are summarized as,

\[
G_L = -\frac{e m_\mu}{32\pi^2} \left[ \sum_{i=1}^{6} F_3 \left( \frac{m_{\mu}^2}{M^2} \right) \left\{ \frac{(M_{LR})_\mu e g_W (m_\mu + m_e)}{M^4} \sqrt{2 m_W \cos \beta} Z_{2i} (-1, Q_{eL}) U_{4i} \right. \right.
\]
\[
+ \frac{\Delta_{\mu e}^2}{M^4} \left. \left( Z_{2i}^L (-1, Q_{eL}) Z_{2i}^L (-1, Q_{eL}) + \frac{g_W^2 m_\mu m_e}{2 m_W^2 \cos^2 \beta} U_{4i}^2 \right) \right\}
\]
\[
+ \frac{2}{\sum_{i=1}^{2} g_W F_4 \left( \frac{m_{\mu}^2}{M^2} \right) \left( \sum_{\alpha, \beta (\alpha \neq \beta)} e_{\alpha, \beta} \frac{\Delta_{\alpha \beta}^2}{M^4} K_{\nu, \alpha \mu} K_{\nu, \beta e} \right) W_{2i}^{(+)} W_{2i}^{(+)} \right],
\]
\[
G_R = -\frac{e m_\mu}{32\pi^2} \left[ \sum_{i=1}^{6} F_3 \left( \frac{m_{\mu}^2}{M^2} \right) \left\{ \frac{(M_{LR})_\mu e g_W (m_\mu + m_e)}{M^4} \sqrt{2 m_W \cos \beta} Z_i^R (2, Q_{eR}) U_{4i} \right. \right.
\]
\[
+ \frac{\Delta_{\mu e}^2}{M^4} \left. \left( Z_i^R (2, Q_{eR}) Z_i^R (2, Q_{eR}) + \frac{g_W^2 m_\mu m_e}{2 m_W^2 \cos^2 \beta} U_{4i}^2 \right) \right\}
\]
\[
+ \frac{2}{\sum_{i=1}^{2} g_W F_4 \left( \frac{m_{\mu}^2}{M^2} \right) \frac{\Delta_{\mu e}^2}{M^4} \frac{m_\mu m_e}{2 m_W^2 \cos^2 \beta} W_{1i}^{(-)} W_{1i}^{(-)} \right],
\]

(27)

where we used the fact $m_\mu \gg m_e$ and the kinematical functions $F_3(r)$ and $F_4(r)$ are defined by

\[
F_3(r) = \frac{1}{12(1-r)^5} \left[ -1 + 9r + 9r^2 - 17r^3 + 6r^2(r+3) \ln r \right],
\]
\[
F_4(r) = \frac{1}{6(1-r)^5} \left[ 1 + 9r - 9r^2 - r^3 + 6r(r+1) \ln r \right].
\]

(28)

In the limit of $m_{\mu} \to 0$, $B(\mu \to e\gamma)$ calculated from the photino contribution in $G_L$ can be easily checked to be reduced to the result given in ref. [14].
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Figure Captions

Fig. 1
One-loop diagram contributing to the effective coupling $G_L$ of $\mu_L \to e_R \gamma$. Figs.(a) and (b) represent the neutralino contribution and Figs.(c) and (d) represent the chargino contribution. There are similar diagrams in which the chirality of the external lines are exchanged. Flavor mixings are induced by the off-diagonal elements of slepton mass matrices which are expressed by $\bullet$. It should be noted that the chirality flip occurs on the internal line in (a) and (c) and on the external line in (b) and (d).

Fig. 2
$B(\mu \to e\gamma)$ as a function of $u$ in the $\eta$-model. The vertical axis $Br$ stands for $10^{11} \times B(\mu \to e\gamma)$ and the horizontal axis $u'$ should be understood as $u = 50(u' + 2)$. Each line corresponds to the various parameter settings for $(M_W, \sin \chi)$ and their values are taken as $A(80, 0), B(180, 0), C(80, 0.3)$ and $D(180, 0.3)$.

Fig. 3
The ratio $R$ of $B(\mu \to e\gamma)$ against the MSSM as a function of $u$ in $\eta$-model. $R$ is defined as $R = B_\eta(M_W, \sin \chi)/B_{\text{MSSM}}(M_W)$. Each line corresponds to the various parameter settings for $(M_W, \sin \chi)$ and their values are taken as $A(80, 0), B(180, 0), C(80, 0.3)$ and $D(180, 0.3)$. 
Fig. 1
Fig. 2
