Research article

Design and analysis of a multiple collaborative beamforming scheme in the realm of Wireless Sensor Networks featuring 3-dimension node configuration

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ABSTRACT

Collaborative Beamforming (CB) is an essential tool towards achieving long-range transmission in Wireless Sensor Networks (WSNs). In some instances, there may be multiple intended data destinations (sinks) in a WSN. This calls for multi-CB. In comparison to sink-by-sink CB, multi-CB implies improved data rates and decreased co-channel interference; and consequently increased network capacity. In current literature, there is no research in multi-CB particularly in 3-dimension WSNs. In this paper, a novel multi-CB mechanism is brought to the fore. This is from the point of view of a random arrangement of sensor nodes in a 3-dimension manner. It is assumed that all sinks' directions are known at the CB cluster head. Node transmit amplitude and phase are optimized using a Particle Swarm Optimization (PSO) algorithm variant to concurrently achieve balanced multiple narrow beams and minimal radiation in undesired directions. The performance of the proposed scheme is checked against that of a pure multiple beam steering approach (without beam power balancing and minimization of radiation in undesired directions). Moreover, an analysis of beam power, width and steering accuracy is done upon varying the number of collaborating nodes and the collaborating cluster radius. Increasing the count of collaborating nodes yields improved beam precision/accuracy, lower radiation in undesired directions and appreciable stability in beam power performance increasing the collaborating nodes’ cluster radius yields narrow beams, improved beam precision and appreciably lower radiation in undesired directions. The contributions of this work to current literature include: (i) formulation and analysis of a multiple beamforming scheme in the realm of 3-dimension WSNs; (ii) design of a multi-CB scheme taking into account minimization of radiation in undesired directions; (iii) a statistical multi-CB performance analysis upon varying collaborating nodes’ cluster radius and collaborating node count.

1. Introduction

Traditionally, multi-hop transmission has been utilized in solving the “long distance” propagation problem in Wireless Sensor Networks (WSNs). However, multi-hop transmission in WSNs has a number of drawbacks, which are listed below [1].

• Lack of energy consumption equity in multi-hop routing: In multi-hop routing, some sensor nodes are overused, resulting in rapid energy depletion/low node life-time, and, as a result, connection failure.

• Multi-hop transmission is ineffective when a long-distance hop is required along a routing path.

• Transmission quality is dependant on individual sensors.

• Multi-hop routing causes increased intra-network interference due to the presence of multiple radiations.

• There are high communication overheads, with concomitant transmission delays.

Recently, Collaborative Beamforming (CB) has been proposed as an alternative valuable technology for constructing reliable and energy-efficient WSNs. CB is an emerging technique that involves the utilization of multiple sensor nodes to provide a high-quality beam direction and communication path. CB is an attractive approach for WSNs because it can simultaneously improve the transmission quality and reduce energy consumption. CB can be achieved through various methods, including beamforming, beam steering, and beam combination. In this paper, we propose a novel multi-CB mechanism that utilizes Particle Swarm Optimization (PSO) algorithm variant to concurrently achieve balanced multiple narrow beams and minimal radiation in undesired directions. The performance of the proposed scheme is checked against that of a pure multiple beam steering approach (without beam power balancing and minimization of radiation in undesired directions). Moreover, an analysis of beam power, width and steering accuracy is done upon varying the number of collaborating nodes and the collaborating cluster radius. Increasing the count of collaborating nodes yields improved beam precision/accuracy, lower radiation in undesired directions and appreciable stability in beam power performance increasing the collaborating nodes’ cluster radius yields narrow beams, improved beam precision and appreciably lower radiation in undesired directions. The contributions of this work to current literature include: (i) formulation and analysis of a multiple beamforming scheme in the realm of 3-dimension WSNs; (ii) design of a multi-CB scheme taking into account minimization of radiation in undesired directions; (iii) a statistical multi-CB performance analysis upon varying collaborating nodes’ cluster radius and collaborating node count.
efficient communication routes between WSN nodes and distant sinks [2, 3, 4].

CB entails optimal transmit amplitude/phase weighting at collaborating nodes with the goal of enhancing radiation in the sink’s direction. The nodes’ radiation energy constructively combines in the direction of the sink, resulting in radiation enhancement. Collaborating nodes typically create a virtual antenna array.

A typical feature of CB radiation is the presence of high sidelobes. High levelled sidelobes are disadvantageous in the below listed ways.

- Sidelobes imply wasted radiation energy.
- Sidelobes are bound to cause interference at other co-channel networks/devices.
- Sidelobes may lead to interception of confidential information by unintended receivers: This is highly disadvantageous in networks with stringent information security requirements.

Research entailing sidelobe minimization in CB can be found in [2, 3, 5, 6, 7]. In [2], the authors give a review of recent trends in collaborative beamforming. Research entailing sidelobe minimization is captured by the authors. Noteworthy, sidelobe minimization is commonly performed from the perspective of reducing the peak sidelobe of the CB beam pattern. Commonly used sidelobe minimization schemes cut across node selection and node transmit coefficient perturbation. The authors in [3] present a node selection based beam pattern optimization algorithm. Simulation results show lower peak sidelobe level of the radiation beam pattern compared to the outcomes of other beam pattern optimization schemes. In [5], a hybrid discrete and continuous optimization scheme aimed at peak sidelobe level reduction is presented. The scheme entails aspects of node selection and optimization of the excitation current weights of the selected nodes. Simulation results bring to the fore the effectiveness of the proposed sidelobe minimization strategies. In [6], the effect of peak sidelobe minimization in collaborative beamforming on the communication capacity of an unintended receiver terminal is examined. Select metaheuristic optimization algorithms are used in the peak sidelobe minimization process. In the simulation study, a reduction of 20 dB in peak sidelobe level and a capacity improvement of 162% is obtained in the worst-case scenario. The authors in [7] present a sidelobe suppression strategy aimed at reducing the peak sidelobe level through jointly optimizing node locations and amplitude weights. worthwhile sidelobe level improvements are noted. A common observation cutting across the research work in [2, 3, 5, 6, 7] is a focus on minimizing the peak sidelobe only. It would be worthwhile to extend sidelobe minimization to a generalized minimization of undesired radiation. Use of node selection based schemes tends to overuse some nodes, inevitably leading to premature node failure.

In networks featuring multiple data sinks/receivers/base stations, multiple beamforming is of essence. Research in multiple beamforming entailing centralized antenna arrays can be found in [8, 9, 10, 11, 12, 13, 14]. In [8], a multiple beamforming approach based on Butler matrices is presented. The intended area of application is wideband beamforming in 4G Long-Term Evolution (LTE) networks. A prototype based on a linear antenna array has been developed. It has been established that the developed prototype can yield three simultaneous beams. In [9], a Millimeter Wave (mmWave) technology based multiple beamforming scheme for use in High Speed Railway (HSR) communication systems is proposed. This is in line with enhancing the capacity in HSR communication systems. The outcomes of a theoretical analysis process and simulations affirm that the developed multiple beamforming can improve throughput and decrease outage probability in HSR communication systems. In [10] an improved Least Mean Square (LMS) algorithm is formulated in a bid to speed up antenna array elements’ phase evaluation in a multiple beamforming process. A 61-element hexagonal array is utilized as the multiple beamforming platform in a high-throughput satellite scenario. The intention is to form a seven-point beam in an environment featuring substantial rainfall attenuation. Improvement in beamforming convergence speed is realized. As per the reviewed literature, research in beamforming in centralized antenna arrays is mostly based on uniform array geometries (particularly linear array element arrangements). WSN nodes, on the other hand, are typically positioned randomly.

Research in CB in WSNs is an active area particularly in matters to do with sidelobe minimization and application of Artificial Intelligence (AI) in solving CB problems. The authors in [15] propose a node selection mechanism aimed at minimizing sidelobes in a CB process. A novel algorithm utilizing Cross-Entropy Optimization (CEO) is applied in selecting an optimal combination of CB nodes from a random planar arrangement of nodes. In [16], a collaborative beamforming scheme (Stochastic Collaborative Beamforming (SCB)) that takes advantage of collaborating nodes’ clock synchronization errors is proposed. The proposed scheme yields sufficient gain in the beamsteering direction with limited node power consumption. In [17], a peak sidelobe minimization scheme in CB is proposed. The scheme utilizes a hybrid meta-heuristic optimization algorithm: Particle Swarm Optimization and Gravitational Search Algorithm-Explore (PSOGSA-E) to optimize node transmit weights. In [18], sidelobe minimization in CB through node position perturbation is proposed. The authors propose a multi-objective optimization approach aimed at concurrently optimizing peak sidelobe level, node transmit power and node motion energy. An improved version of the Genetic Algorithm (GA) is utilized in solving the proposed multi-objective problem. In [19] an improved meta-heuristic optimization algorithm (bat-chicken swarm optimization algorithm) is utilized in minimizing the peak sidelobe level in a distributed beamforming problem.

CB in the presence of phase and frequency synchronization errors is a current subject of research [20, 21, 22, 23]. Notably, transmit phase at collaborating nodes ought to be synchronized perfectly to obtain ideal CB outcomes. Phase synchronization errors during a CB process are bound to negatively affect beamforming outcomes. The authors in [24] propose use of a master-slave synchronization protocol to appropriately adjust phase and carrier frequency at every collaborating node in a distributed manner. In [25], a variety of schemes focused on phase synchronization between collaborating nodes in a CB process are presented. The presented schemes cut across open-loop and closed-loop approaches. In closed-loop schemes, the WSN sink (base station) gives appropriate feedback towards compensation of the overall phase offset. In the open-loop case, the WSN sink broadcasts an unmodulated sinusoidal beacon to the nodes. Collaborating nodes make use of the beacon towards appropriate phase compensation in the CB process. This approach befits the research work carried out in this paper.

Other research work pertaining CB in WSNs can be found in [26, 27, 28, 29, 30, 31].

To the authors’ best knowledge, the multiple beamforming concept has not been applied in WSNs. In general, observations made in current CB research (in the context of WSNs) are:

- Multiple data sink(s) have not been considered.
- 3-dimension WSN node arrangements have not been considered.

The contributions brought forward in this paper include:

1. Design of a balanced multi-CB scheme with the implementation platform being a WSN in which nodes are arranged in a 3-dimension layout.
2. Design of a scheme entailing concurrent balanced multi-CB and minimization of radiation in undesired directions (sidelobes included).
3. Multi-CB performance analysis upon varying the number of collaborating nodes and CB cluster radius.

Performance measures used in the analysis process include:
Fig. 1. 3-dimension WSN, both in terms of node distribution and sink placement. Cluster head: green star symbol; Other sensor nodes: yellow star symbol; Sinks: red circular symbol.

- Relative beam power.
- Beam width.
- Beamsteering accuracy.
- Average radiation in undesired directions.

The rest of the paper is organized as follows. The utilized WSN/CB model is presented in Section 2. Section 3 presents the proposed multi-CB scheme. The performance of the proposed multi-CB approach is analyzed in Section 6. Section 7 gives the overall conclusion.

2. 3-dimension WSN/CB model

Fig. 1 shows a random arrangement of WSN nodes/sinks (3-dimension arrangement). The model benefits a wide range of WSNs.

In general, considerations/assumptions made in the model are:

- The geometric locations of the sinks and collaborating nodes are mapped out with reference to the cluster head (selected from the collaborating nodes).
- The sinks are located in the far-field region of the nodes.
- Collaborating nodes-to-sink paths bear equivalent and time-invariant path loss.
- The collaborating nodes and sinks are immobile over the entire CB procedure.
- Ideal and equivalent isotropic antennas are utilized by the collaborating nodes.
- Mutual coupling between collaborating nodes is insignificant.
- The collaborating nodes are frequency and phase synchronized.
- The collaborating nodes are fully location aware (for instance, through the Global Positioning System (GPS)).

In subsequent expressions, in relation to Fig. 1:

- The symbol \( w \) is used to denote angle \( P'OQ \).
- The position vector of a select sink (in the far field region of the radiators (nodes)) is denoted as \( r \).
- The position vector of a select node (radiator) is denoted as \( r' \).

When formulating electromagnetic radiation problems, it is usually convenient to work with electric and magnetic potential parameters rather than the electric (E) and magnetic (H) fields themselves. Basically, Maxwell’s equations give room towards introduction/formulation of the electric and magnetic potentials. Outside of magnetic and/or electric current density source regions, electric and magnetic fields can be derived from the electric and magnetic potentials. The 4 Maxwell’s equations are as per Eq. (1), (2), (3) and (4).

\[
\begin{align*}
\nabla \times E &= -\frac{\partial B}{\partial t} \\
\nabla \times H &= J + \frac{\partial D}{\partial t} \\
\n\nabla \cdot D &= \rho \\
\n\nabla \cdot B &= 0
\end{align*}
\]

(1) (2) (3) (4)

Two of Maxwell’s equations allow the introduction of electric scalar potential (\( \Phi \)) and magnetic vector potential (\( \mathbf{A} \)). The other two Maxwell’s equations, written in terms of these potentials (\( \Phi \) and \( \mathbf{A} \)), take a wave-equation format. The Maxwell’s equations given in Eq. (1) and (4) imply the existence of the magnetic and electric potentials \( \mathbf{A}(r,t) \) and \( \Phi(r,t) \), in a manner that the Electric (E) and Magnetic (H) fields are obtainable as per Eq. (5) and (6) where \( \mathbf{H} = \mathbf{B}/\mu \) [32]. \( \mathbf{B} \) is representative of magnetic field density and \( \mu \) is the permeability constant.

\[
\begin{align*}
\mathbf{E} &= -\nabla \Phi - \frac{\partial \mathbf{A}}{\partial t} \\
\mathbf{B} &= \nabla \times \mathbf{A}
\end{align*}
\]

(5) (6)

Consider a sensor node antenna fed with some current whose density is \( \mathbf{J} \). The resultant magnetic potential at a far-field observation point (Fig. 1) is as per Eq. (7) [32]. The integration operation in Eq. (7) is over the volume \( v \) wherein current density \( \mathbf{J} \) is non-zero.

\[
\mathbf{A}(r,t) = \frac{\int \mu \mathbf{J}(r',t) \cdot \hat{r}}{4\pi R} d^3r'
\]

(7)

where \( d \) represents the radiating antenna dimensions and \( C \) represents wave velocity.

Taking into consideration a single frequency wave, \( \mathbf{A}(r,t) \) can be expressed as per Eq. (8).

\[
\mathbf{A}(r) e^{j\omega t} = \frac{\int \mu \mathbf{J}(r') e^{j\omega t - jk R}}{4\pi R} d^3r'
\]

(8)

Cancelling a common factor \( e^{j\omega t} \) from both sides of Eq. (8), \( \mathbf{A}(r,t) \) takes the form given in Eq. (9).

\[
\mathbf{A}(r) = \frac{\int \mu \mathbf{J}(r') e^{-jk R}}{4\pi R} d^3r'
\]

(9)

where \( k = \frac{\omega}{c} = \frac{2\pi}{\lambda} \) represents the free space wave-number.

Taking into consideration far placement of the observation point \( P \) in relation to the origin \( O \) in Fig. 1, \( PP' \approx OP - OP' \). \( OP = r \) and \( OP' = r' \cos(\psi) \). Consequently, the distance \( R (PP') \) can be approximated expressed as per Eq. (10).

\[
R \approx r - \hat{r} \cdot r' = r - r' \cos(\psi)
\]

(10)

where \( \hat{r} \) is the radial unit vector.

The approximation \( R \approx r \) can be applied in the denominator part of Eq. (9). The approximation in Eq. (10) can be applied in the exponential part of Eq. (9). The approximations give rise to Eq. (11).

\[
\mathbf{A}(r) = \frac{\int \mu \mathbf{J}(r') e^{-jk (r - r')} d^3r'}{4\pi R}
\]

(11)

Upon factoring out the constants in Eq. (11), the resultant form of \( \mathbf{A}(r) \) is as per Eq. (12).
\[
A(r) = \frac{\mu_0 e^{-jkr}}{4\pi r} \int J(r') e^{jkr'} d^3r'
\] (12)

In Eq. (12), the integral factor contributes to the radiated field’s directional properties. The integral factor can be termed as the radiation vector and is explicitly given in Eq. (13).

\[
F(k) = \int J(r') e^{jk\cdot r'} d^3r'
\] (13)

where \( k = kr \) is the wave-number vector.

The collaborating nodes in Fig. 1 more or less form a virtual antenna array. Consider a three-dimension array of equivalent antennas positioned at \([R_0, R_1, R_2, \ldots, R_J]\). Given relative feed current coefficients \([a_0, a_1, a_2, \ldots, a_J]\), the current density at the \(i^{th}\) antenna is as given in Eq. (14).

\[
J_i(r') = a_i J(r - R_i)
\] (14)

The radiation vector is the three-dimensional Fourier transform of the current density \( J \) (Eq. (13)). Consequently, the radiation vector corresponding to the translated current density in Eq. (14) is as given in Eq. (15).

\[
F_i(k) = a_i e^{jk\cdot R_i} F(k)
\] (15)

Given \( T \) radiators, the resultant radiation vector is as given in Eq. (16). In Eq. (16), \( \sum_{i=0}^{J} a_i e^{jk\cdot R_i} \) is the array factor.

\[
F_{\text{tot}}(k) = \sum_{i=0}^{T} a_i e^{jk\cdot R_i} F(k)
\]

Going by Eq. (16), given a set of \( T \) nodes featuring 3-dimension placement, the magnitude of the array factor in the direction \((\phi, \theta)\) is as given in Eq. (17).

\[
AF_{\phi, \theta} \approx \sum_{i=0}^{T} |w_i|^2 \sum_{l=0}^{J} |R_i, l|
\] (17)

where:

- \( w_i \) represents the node transmit phase and amplitude weighting.
- \( R_i \) represents the position vector of the \(i^{th}\) node.
- \( \cdot \) represents the scalar product operator.
- \( \hat{r} = \sin(\theta) \cos(\phi) a_u + \sin(\theta) \sin(\phi) a_v + \cos(\theta) a_w \) represents radial unit vector.

Upon wavelength normalization, Eq. (17) takes the form of Eq. (18).

\[
AF_{\phi, \theta} \approx \sum_{i=0}^{T} |w_i|^2 \sum_{l=0}^{J} |R_i, l|
\] (18)

where \( \tilde{R}_i = \frac{R_i}{\lambda} \) is the wavelength-normalized position vector of the \(i^{th}\) node.

3. Proposed multi-CB scheme

The proposed multiple beamforming scheme entails maximizing radiation towards a multiple number of intended sinks whilst minimizing radiation towards undesired directions.

Let:

- \((\phi_s, \theta_s), s = 1, \ldots, S\) be the directions of the intended sinks from the point of view of the cluster head.
- \((\phi_u, \theta_u), u = 1, \ldots, U\) be a set of select directions apart from those of the intended sinks from the point of view of the cluster head.
- \((HPBW)_s, s = 1, \ldots, S\) be the half-power beamwidth values corresponding to the multi-beamformed radiation.

The objectives to be optimized are as per Eq. (19)–(21).

Eq. (19) focuses on maximizing radiation towards the intended multiple directions. \( |AF_{\phi_s, \theta_s}(\mathbf{w}_s)|^2 \) represents the radiation power in the desired direction \((\phi_s, \theta_s)\), where \( |AF_{\phi_s, \theta_s}(\mathbf{w}_s)| \) is the magnitude of the array factor in the desired direction \((\phi, \theta)\). In essence, Eq. (19) gives the average radiation power in a set of \( S \) desired directions.

\[
\text{maximize } \frac{1}{S} \sum_{s=1}^{S} |AF_{\phi_s, \theta_s}(\mathbf{w}_s)|^2
\] (19)

Eq. (20) focuses on minimizing radiation towards select unintended directions. The unintended directions are selected as per the grid in Fig. 2. \( |AF_{\phi_u, \theta_u}(\mathbf{w}_u)|^2 \) represents the radiation power in the selected unintended directions \((\phi_u, \theta_u)\), where \( |AF_{\phi_u, \theta_u}(\mathbf{w}_u)| \) is the magnitude of the array factor in the undesired direction \((\phi_u, \theta_u)\). In essence, Eq. (20) gives the average radiation power in a set of \( U \) unintended directions.

\[
\text{minimize } \frac{1}{U} \sum_{u=1}^{U} |AF_{\phi_u, \theta_u}(\mathbf{w}_u)|^2
\] (20)

Eq. (21) focuses on minimizing the half-power beam width corresponding to the multiple beams. \( HPBW_s(\mathbf{w}_s) \) represents the half-power beamwidth in the beam oriented towards direction \((\phi_s, \theta_s)\). In essence, Eq. (21) gives the average half-power beam width corresponding to \( S \) multiple beams.

\[
\text{minimize } \frac{1}{S} \sum_{s=1}^{S} HPBW_s(\mathbf{w}_s)
\] (21)

The overall multi-objective function to be optimized is a weighted combination of the singular objectives given in Eq. (19)–(21) as per Eq. (22). The weight values \((g_1, g_2, g_3)\) have been carefully chosen in order to generate optimal results. The chosen values are: 0.5, 0.3 and 0.2 for \( g_1, g_2, g_3 \) respectively. The collaborating nodes’ transmit amplitude and phase values (encapsulated in \( w_i \) in Eq. (22)) have been optimized using a PSO algorithm variant (Culled-Fuzzy-Adaptive PSO algorithm [33]).

\[
\text{minimize } f(\mathbf{w}_s) = -g_1 \frac{1}{S} \sum_{s=1}^{S} |AF_{\phi_s, \theta_s}(\mathbf{w}_s)|^2 + g_2 |AF_{\phi_u, \theta_u}(\mathbf{w}_u)|^2 + g_3 \frac{1}{S} \sum_{s=1}^{S} HPBW_s(\mathbf{w}_s)
\] (22)

\( z_i \) is a weight factor chosen over each optimization iteration in relation to the value of beam power levels \((|AF_{\phi, \theta}|^2)\). This ensures that all beam power values are nearly equivalent. The chosen values of \( z_i \) are 0.2, 0.3 and 0.5 (for a 3-beam case) for reducing values of \(|AF_{\phi, \theta}|^2 \).

4. Culled-Fuzzy-Adaptive PSO algorithm

As previously stated, a PSO algorithm variant (Culled-Fuzzy-Adaptive PSO (CFAPSO)) has been utilized in solving/optimizing the proposed multi-CB scheme as per Section 3. PSO algorithm is a controlled “flow” of a swarm of “particles” (holding potential solutions to an
optimization problem) in some bounded search space \([34, 35, 36]\). Eqs. (23) and (24) capture the standard PSO algorithm.

\[
v_i(t + 1) = (w v_i(t)) + (c_1 r_1 (p_i - x_i)) + (c_2 r_2 (l_i - x_i)) \tag{23}
\]

\[
x_i(t + 1) = x_i(t) + v_i(t + 1) \tag{24}
\]

Eq. (23) is the velocity update equation and Eq. (24) is the position update equation. In the equations, \(i\) denotes a swarm particle, \(t\) denotes iteration, \(x_i\) is particle’s \(i\) position, \(v_i\) is particle’s \(i\) velocity, \(w\) is the inertia weight, \(c_p\) is the self/personal confidence parameter, \(c_s\) is the social confidence parameter, \(r_1\) and \(r_2\) are random values within \([0 - 1]\), \(p_i\) is the personal best position and \(l_i\) is the local/global best position.

In the velocity update equation, the inertia weight controls the influence of the immediate previous velocity. In the standard PSO algorithm, it is decreased linearly from 0.9 in the initial algorithm iteration to 0.4 in the final iteration \([37]\). In the velocity update equation, the personal confidence parameter controls the influence of the personal best position. In the standard PSO algorithm, the personal confidence parameter is fixed at 2 \([38]\). The social confidence parameter controls the influence of the local best position in the velocity update equation. In the standard PSO algorithm, the social confidence parameter is fixed at 2 \([38]\). The random values \(r_1\) and \(r_2\) enhance the exploration capabilities of the personal and social influences.

PSO confidence parameter values have a profound effect on the algorithm performance. The values ought to be carefully selected in line with the problem at hand. Large values of \(c_p\) in comparison to \(c_s\) yield enhanced exploration. Large values of \(c_s\) in comparison to \(c_p\) yield enhanced exploitation and convergence to a local (or global) solution. Therefore, it is necessary to have adaptive confidence parameters rather than the static values typical of the standard PSO algorithm. A large inertia weight value is preferable in the early iterations (exploration stages) and a smaller value in later iterations (exploitation stages).

In the CFWPSO algorithm utilized in this paper, the value of parameter \(c_s\) is mapped onto the range \([2.2 - 2.4]\) using a particle performance index alongside iteration count \([33]\). The value of parameter \(c_i\) is mapped onto the range \([2.2 - 2.6]\). The inertia weight is mapped onto the range \([0.4 - 0.9]\). The mapping process is done using a fuzzy logic system.

In general, the procedure followed is as per the following steps.

1. Step 1: Initialize swarm particles randomly.
2. Step 2:
   (a) Evaluate the objective function at all swarm particles.
   (b) Pick the best swarm particle in accordance with the obtained objective function values.
3. Step 3: Check if the maximum number of iterations allowed has been covered. If so, terminate the algorithm, else proceed to step 4.
4. Step 4: Check if the number of iterations is at half the maximum value. If so, proceed to step 5, else proceed to step 7.
5. Step 5: Sort and rank swarm particles in order of performance.
6. Step 6: Cull and randomly re-initialize poorly performing swarm particles.
7. Step 7: Update inertia and personal/social confidence parameter values (using a fuzzy logic based look-up table).
8. Step 8: Update swarm particle positions. Revert to Step 2.

5. Simulation procedure

The research work presented in this paper has been analyzed and validated through simulations in Matlab software. The Matlab platform offers a convenient, “self-contained”, and accurate programming/numeric computing platform in regard to beamforming schemes modelling, simulation, and analysis.

In every optimization procedure in the research work, 50 independent trials have been used. This fulfils the condition that sample sizes \(\geq 30\) are generally sufficient for a majority of data distributions for the central limit theorem to hold \([39]\). The central limit theorem is critical in statistical data analysis for 2 major reasons \([40]\):

1. Accurate analysis estimates: Sampling distributions of the mean bunch more tightly about the population mean upon increasing the sample size.
2. Normality assumption: That sampling distributions can approximate a Gaussian distribution is vital towards application of parametric hypothesis tests of the mean.

The statistical tools utilized in the performance analysis process are:

- Analysis of Variance (ANOVA) test: a statistical test as to whether 2 or more population means are equivalent \([41]\).
- Tukey-Kramer post-hoc analysis test: a statistical test to find the exact population means that bear significant differences \([42]\).

6. Performance analysis of the proposed multi-CB scheme

In this section, the performance of the proposed multi-CB scheme (as captured in Eq. (22)) is compared to that of a pure multi-CB scheme. Aspects of beam power fine-tuning, beam width control and generalized minimization of radiation in unwanted directions as captured in Eq. (22) are not considered in the pure multi-CB method. Multi-CB performance is also examined against variation in the collaborating nodes’ cluster radius and number of collaborating nodes. Achieved beam power, beam width and beamforming accuracy are among the performance metrics used.

6.1. Performance comparison against a pure multi-CB scheme

The performance of the proposed multi-CB scheme (as given in Eq. (22)) is weighed against that of a pure multi-CB scheme given in Eq. (25).

\[
\text{maximize } f(w) = \frac{1}{S} \sum_{s=1}^{S} |AF_{\psi, \theta}(w)|^2 \tag{25}
\]

where \(|AF_{\psi, \theta}(w)|\) is the array factor magnitude in the beam steering direction \((\psi, \theta)\).

The used beam steering directions are: a common elevation angle 50 degrees, azimuth angles -130, -20, and 130 degrees for beams 1, 2, and 3 respectively.

Figs. 3 and 4 show typical radiation power patterns (in mesh plot form) obtained after multi-CB employing the two techniques under consideration.
The exact normalized beam power values obtained upon using the proposed and pure multi-CB scheme are given in Table 1. The proposed multi-CB scheme approach yields higher and more balanced (low variance) beam power values in comparison to the pure multi-CB scheme.

Table 1. Normalized beam power values.

|                | Proposed multi-CB | Basic multi-CB |
|----------------|-------------------|----------------|
| Beam 1         | 0.9594            | 0.4973         |
| Beam 2         | 0.9641            | 0.6977         |
| Beam 3         | 0.9690            | 0.9820         |
| Average        | 0.9642            | 0.7256         |
| Variance       | 0.0000            | 0.0395         |

Upon qualitatively analyzing the radiation patterns presented in Figs. 3 and 4 the following outcomes are observed:

- The proposed multi-CB scheme leads to balanced beam power levels in comparison to the pure multi-CB scheme.
- The proposed multi-CB scheme in general leads to lower average radiation in undesired directions in comparison to the pure multi-CB scheme.

Azimuth cut radiation power patterns corresponding to the two multi-CB schemes under study are compared in Fig. 5. Upon qualitatively analyzing the azimuth cut radiation patterns, the following outcomes are observed:

- The proposed multi-CB scheme yields slightly more accurate beams (in particular with reference to Beam 3).
- The proposed multi-CB scheme yields slightly narrower beam width (particularly with reference to Beam 3).

6.2. Performance analysis with variation in the count of collaborating nodes

In this part, analysis of 3-beam placement with varying node count is considered. The used beam steering directions are: a common elevation angle 45 degrees, azimuth angles -120, 5, and 120 degrees for beams 1, 2, and 3 respectively.

The number of collaborating nodes used in the multi-CB comparative process are five, ten, fifteen and twenty. The respective node arrangements are captured in Table 2. The presented curves are average outcomes of fifty independent runs for each of the four cases under investigation. The performance of the twenty-node arrangement is clearly superior to that of the other arrangements.

6.2.1. Radiation power pattern

Figs. 7–10 illustrate the normalized power patterns obtained upon multi-CB using five, ten, fifteen and twenty collaborating nodes. The
Table 2. Collaborating nodes’ positions: 5, 10, 15, and 20 node configurations. R: Normalised radial distance (m); El.: Elevation angle in degrees; Az.: Azimuth angle in degrees.

| S. No. | 20 nodes | 15 nodes | 10 nodes | 5 nodes |
|--------|----------|----------|----------|---------|
|        | R El. Az. | R El. Az. | R El. Az. | R El. Az. |
| 1      | 0.4 61 -27 | 0.4 61 -27 | 0.4 61 -27 | 0.7 14 14 |
| 2      | 0.7 14 14 | 0.7 14 14 | 0.7 14 14 | 0.8 25 144 |
| 3      | 0.5 10 -59 | 0.5 10 -59 | 0.5 10 -59 | 0.6 26 99 |
| 4      | 0.5 58 108 | 0.5 58 108 | 0.8 25 144 | 0.5 34 -117 |
| 5      | 0.8 25 144 | 0.8 25 144 | 0.6 -9 72 | 0.7 -53 -63 |
| 6      | 0.8 43 -39 | 0.8 43 -39 | 0.8 12 49 | 0.8 12 49 |
| 7      | 0.3 56 180 | 0.3 56 180 | 0.6 26 99 | 0.6 26 99 |
| 8      | 0.6 -9 72 | 0.6 -9 72 | 0.5 34 -117 | 0.7 -53 -63 |
| 9      | 0.8 -12 49 | 0.8 -12 49 | 0.7 -53 -63 | 0.7 -53 -63 |
| 10     | 0.7 54 -143 | 0.7 54 -143 | 0.6 16 -135 | 0.6 16 -135 |
| 11     | 0.4 24 117 | 0.6 26 99 | 0.6 26 99 | 0.6 26 99 |
| 12     | 0.6 26 99 | 0.5 34 -117 | 0.7 -53 -63 | 0.7 -53 -63 |
| 13     | 0.5 34 -117 | 0.7 -53 -63 | 0.6 16 -135 | 0.6 16 -135 |
| 14     | 0.8 13 -36 | 0.6 16 -135 | 0.8 40 -34 | 0.8 40 -34 |
| 15     | 0.8 40 -34 | 0.8 40 -34 | 0.8 -42 -153 | 0.8 -42 -153 |
| 16     | 0.5 -22 37 | 0.5 -22 37 | 0.5 -22 37 | 0.5 -22 37 |
| 17     | 0.7 -22 -164 | 0.7 -22 -164 | 0.7 -22 -164 | 0.7 -22 -164 |
| 18     | 0.7 -53 -63 | 0.7 -53 -63 | 0.7 -53 -63 | 0.7 -53 -63 |
| 19     | 0.6 16 -135 | 0.6 16 -135 | 0.6 16 -135 | 0.6 16 -135 |
| 20     | 0.8 -42 -153 | 0.8 -42 -153 | 0.8 -42 -153 | 0.8 -42 -153 |

Fig. 9. Normalized power pattern in the form of a mesh plot: 15 nodes.

Fig. 10. Normalized power pattern in the form of a mesh plot: 20 nodes.

patterns shown are the average of fifty independent results for each of the four cases under investigation. Higher collaborating node counts are associated with better outcomes in terms of beam precision and overall radiation power spread.

Fig. 11 shows the azimuth cut (Angle of elevation: 45 degrees) of the normalized radiation power pattern. It can be seen that as the number of collaborating nodes grows, the beams become more precise.

6.2.2. Beam power values

Table 3 shows the obtained beam power values. The values are mean results of fifty independent evaluations for each of the 4 cases under study. P-values obtained upon an ANOVA test are 4.6E-07, 6.7E-32 and 1.1E-32 for Beams one, two and three respectively. The beam power levels in Table 3 have statistically significant differences, based on the low P-values. Following a Tukey-Kramer comparison test, the differences are laid out in Table 4.

Table 5 shows the ranking of beam power performance (based on the Tukey-Kramer comparison test findings) when using twenty, fifteen, ten, and five nodes. In the table, a rank tie indicates statistically equal beam power values. Overall, the 5-node multi-CB procedure yields the
Table 4. Tukey-Kramer comparison test results. VnD stands for “Values Different” and VnD stands for “Values not Different”.

| Comparison     | Beam 1 | Beam 2 | Beam 3 |
|----------------|--------|--------|--------|
| 20 vs 15       | VnD    | VnD    | VnD    |
| 20 vs 10       | VnD    | VnD    | VnD    |
| 20 vs 5        | VD     | VD     | VD     |
| 15 vs 10       | VnD    | VnD    | VnD    |
| 15 vs 5        | VD     | VD     | VD     |
| 10 vs 5        | VD     | VD     | VD     |

Table 5. Ranking of beam power performance for multi-CB with twenty, fifteen, ten, and five nodes.

| Nodes | Beam 1 | Beam 2 | Beam 3 | Total |
|-------|--------|--------|--------|-------|
| 20 nodes | 1      | 1      | 1      | 3     |
| 15 nodes  | 1      | 1      | 1      | 3     |
| 10 nodes  | 1      | 1      | 1      | 3     |
| 5 nodes   | 4      | 4      | 4      | 12    |

Table 6. Half-power beam width (in degrees) corresponding to multi-CB using twenty, fifteen, ten, and five nodes.

| Nodes | Beam 1 | Beam 2 | Beam 3 | Average |
|-------|--------|--------|--------|---------|
| 20     | 50     | 55     | 55     | 50.67   |
| 15     | 55     | 53     | 53     | 51.67   |
| 10     | 47     | 47     | 49     | 52.33   |
| 5      | 47     | 47     | 49     | 53.00   |

Table 7. Beamsteering accuracy (in degrees) corresponding to multi-CB using twenty, fifteen, ten, and five nodes.

| Nodes | Avg. degrees off. |
|-------|--------------------|
| 20    | 3.33               |
| 15    | 4.00               |
| 10    | 4.67               |
| 5     | 8.00               |

Table 8. Average power in all undesired directions corresponding to multi-CB using twenty, fifteen, ten, and five nodes.

| No. of nodes | Avg. power | SD |
|--------------|------------|----|
| 20           | 0.1979     | 0.0213 |
| 15           | 0.1982     | 0.0287 |
| 10           | 0.2486     | 0.0302 |
| 5            | 0.2700     | 0.0375 |

least favourable beam power performance. The 10, 15, and 20 node procedures yield statistically equivalent beam power performance.

6.2.3. Beam width values

Tabulated in Table 6 are half-power beam width values as per the azimuth-cut radiation pattern given in Fig. 11. For the 4 cases under study, there is no discernible trend in beam width values.

6.2.4. Beamsteering accuracy

Beamsteering accuracy values are given in Table 7. Higher node count yields better beamsteering precision.

6.2.5. Power in the undesired directions

Table 8 shows the average normalized power values for all undesired directions. The values are the mean results of fifty independent runs. An ANOVA test yields a P-value 5.3E-32 implying statistically significant differences in the data. Following a Tukey-Kramer comparison test, the only pair found to bear statistically similar values is 20/15.

The overall performance rank from the best to the worst is: 1. A tie between 20 and 15 nodes; 2. 10 nodes; 3. 5 nodes. Overall, an increase in node count is associated with a decrease in power radiated towards undesired directions.

6.3. Performance analysis with change in the beamforming cluster radius

In this part, analysis of 3-beam placement with varying cluster radius is considered. The used beam steering directions are: a common elevation angle 50 degrees, azimuth angles -150, -30, and 105 degrees for beams 1, 2, and 3 respectively.

The CB cluster radius values (wavelength-normalized) used in the multi-CB comparative process are one, two, three and four. The respective node arrangements are captured in Table 9.

Fig. 12 depicts the evolution of the multi-CB cost function using cluster radius values one, two, three and four. The presented curves are average outcomes of fifty independent runs for each of the four cases under investigation. An increase in Cluster radius is generally associated with better performance.

6.3.1. Radiation power pattern

Figs. 13–16 illustrate the normalized power patterns obtained upon multi-CB using cluster radius values one, two, three and four respectively. The patterns shown are the average of fifty independent results for each of the four cases under investigation. Higher cluster radius values are associated with:
Fig. 13. Normalized power pattern in the form of a mesh plot: cluster radius 1.

Fig. 14. Normalized power pattern in the form of a mesh plot: cluster radius 2.

Fig. 15. Normalized power pattern in the form of a mesh plot: cluster radius 3.

Fig. 16. Normalized power pattern in the form of a mesh plot: cluster radius 4.

- Narrower beam width.
- Higher sidelobe levels.
- Presence of grating lobes.

Fig. 17. Normalized power pattern azimuth cut (Angle of elevation: 50 degrees).

Table 10. Beam power corresponding to multi-CB using cluster radius values one, two, three and four.

| Rad 1 | Rad 2 | Rad 3 | Rad 4 |
|-------|-------|-------|-------|
| Power | SD    | Power | SD    |
| Beam 1| 0.911 | 0.068 | 0.859 | 0.077 |
| Beam 2| 0.901 | 0.074 | 0.792 | 0.065 |
| Beam 3| 0.928 | 0.057 | 0.737 | 0.071 |
| Average| 0.913 | 0.053 | 0.859 | 0.077 |

Table 11. Tukey-Kramer comparison test results.
VD stands for “Values Different” and VnD stands for “Values not Different”.

| Comparison     | Beam 1 | Beam 2 | Beam 3 |
|----------------|--------|--------|--------|
| Radii1 vs Radii2| VD     | VD     | VD     |
| Radii1 vs Radii3| VnD    | VnD    | VD     |
| Radii1 vs Radii4| VnD    | VnD    | VD     |
| Radii2 vs Radii3| VD     | VD     | VD     |
| Radii2 vs Radii4| VD     | VD     | VD     |
| Radii3 vs Radii4| VnD    | VD     | VD     |

Fig. 17 shows the azimuth cut (Angle of elevation: 50 degrees) of the normalized radiation power pattern. It can be seen that as the cluster radius increases, the beams become narrow.

6.3.2. Beam power values

Table 10 shows the obtained beam power values. The values are mean results of fifty independent evaluations for each of the 4 cases under study. P-values obtained upon an ANOVA test are 7.5E-10, 4.3E-26 and 2.9E-41 for beams one, two and three respectively. The beam power levels in Table 10 have statistically significant differences, based on the low P-values. Following a Tukey-Kramer comparison test, the differences are laid out in Table 11.

Table 12 shows the ranking of beam power performance (based on the Tukey-Kramer comparison test findings) when using cluster radius values one, two, three and four. In the table, a rank tie indicates statistically equal beam power values. Generally, cluster radius four and one multi-CB procedures yield the best beam power performance.

6.3.3. Beam width values

Tabulated in Table 13 are half-power beam width values as per the azimuth-cut radiation pattern given in Fig. 17. Generally, beam width decreases with increase in cluster radius.
Further analysis has been carried out on the basis of varying the number of collaborating nodes and CB cluster radius. Increasing the number of collaborating nodes leads to increase in beam precision/accuracy, decrease in power radiated towards undesired directions and appreciable stability in beam power performance. Over and above the afore-listed advantages, a large number of collaborating nodes implies increased distribution of radiation power requirements resulting in WSN energy conservation. Negatively, a large number of collaborating nodes imply increased intra-cluster communication (in the information sharing stages of the CB process). Increasing the CB cluster radius leads to decrease in beam width, increase in beam precision/accuracy, appreciable decrease in power radiated towards undesired directions (due to reduction in beamwidth) and higher sidelobe levels/formation of grating lobes.

As per the observed outcomes, collaborating nodes’ count and CB cluster radius should be selected carefully in order to obtain an optimal multi-CB outcome (narrow precise beams with minimal radiation in undesired directions).

**Declarations**

**Author contribution statement**

Robert Macharia Maina: Conceived and designed the experiments; Performed the experiments; Analyzed and interpreted the data; Wrote the paper.

Philip Kibet Lang’at, Peter Kamita Kihato: Conceived and designed the experiments; Analyzed and interpreted the data; Wrote the paper.

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**Data availability statement**

Data will be made available on request.

**Declaration of interests statement**

The authors declare no conflict of interest.

**Additional information**

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