Extremal Black Attractors in 8D Maximal Supergravity

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Abstract

Motivated by the new higher D-supergravity solutions on intersecting attractors obtained by Ferrara et al. in [Phys.Rev.D79:065031-2009], we focus in this paper on 8D maximal supergravity with moduli space $\text{SL}(3,R) \times \text{SO}(3) \times \text{SL}(2,R) \times \text{SO}(2)$ and study explicitly the attractor mechanism for various configurations of extremal black p-branes (anti-branes) with the typical near horizon geometries $\text{AdS}_{p+2} \times S^m \times T^{6-p-m}$ and $p = 0, 1, 2, 3, 4; 2 \leq m \leq 6$. Interpretations in terms of wrapped M2 and M5 branes of the 11D M-theory on 3-torus are also given.

Keywords: 8D supergravity, black p-branes, attractor mechanism, M-theory.

1 Introduction

Since its discovery by D.Z Freedman, P.van Nieuwenhuizen and S.Ferrara at the mid of seventieth of the last century\textsuperscript{[1,2]}, the properties of supergravity theories, based on the gauging of Haag-Lopozansky-Sohinus (HLS) supersymmetry\textsuperscript{[3]}, have been intensively studied in four and higher dimensions; for reviews see\textsuperscript{[4,5,6]}. These studies allowed more insight into supersymmetric gauge theories in diverse dimensions and led to the observation of superstrings\textsuperscript{[7]} containing the various 4D and higher supergravities as Planck scale limits of 10D superstrings and 11D M theory compactifications\textsuperscript{[8]}. Besides usual properties, supergravity in higher dimensions have moreover specific features; in particular they need a graded Lie algebraic structure going beyond the LHS superalgebra by implementing exotic “central” charges $\mathcal{Z}_{\mu_1...\mu_p} \equiv \mathcal{Z}_p$ that transform in non trivial representations of $SO(1,D-1)$ space time symmetry\textsuperscript{[9,10]}. They also have $(p+1)$- form gauge fields $\mathcal{A}_{p+1} \equiv \mathcal{A}_{\mu_1...\mu_{p+1}}$ in addition to the graviton $\mathcal{G}_{\mu\nu}$, the usual 1-form gauge
fields $A_\mu$, scalars $\{\phi^I\}$ and their supersymmetric partners $\psi_\mu^\alpha$, $\chi^\alpha$. These central charges $Z_p$ and gauge fields $A_{p+1}$ play, like in the case of 4D black holes [11, 12, 13], a crucial role in dealing with static, asymptotically flat and spherically symmetric extremal black p-brane solutions living in higher dimensions. In this regards and following the first original works [14, 15, 16, 17, 18, 19] and subsequent ones led by S.Ferrara and collaborators [20, 21] and refs therein, growing attention has been devoted to the study of the black hole solutions in various dimensions and their attractor mechanism taking into account p-branes carrying non trivial magnetic $p^A$ and electric charges $q_A$ of the $(p+2)$-form gauge field strengths $F_{p+2}^A$ and their magnetic duals $\tilde{F}_{D-p-2\Lambda}$. The attractor equations are obtained by minimization of the effective potential $V_{\text{eff}}(\phi)$ induced by the kinetic energies of the gauge field strengths of the supergravity theory. The minima of the effective potential, solving the conditions $\partial_I V_{\text{eff}} = 0$, $\det(\partial_I \partial_J V_{\text{eff}}) > 0$, $\delta V_{\text{eff}} = 0$, determine the values of the scalars at the horizon in terms of the black brane charges $p^A$ and $q_A$.

Motivated by the new solutions on higher dimensional intersecting attractors recently obtained in [33], we focus in this paper on maximal supergravity in 8D with moduli space $[SL(3,R)/SO(3)] \times [SL(2,R)/SO(2)]$ and study explicitly the attractor mechanism for various configurations of black p-branes and anti-branes living in 8D and having the typical near horizon geometries $AdS_{p+2} \times S^m \times T^{6-p-m}$, with $0 \leq p \leq 4$, and $2 \leq p + m \leq 6$. We also complete some partial results of [33]; in particular the strand on the black dyonic membrane and the dual black attractor pairs string/(anti) 3-branes, holes/(anti) 4-branes.

The presentation is as follows: In section 2, we first study the 8D $\mathcal{N} = (2,2)$ supersymmetric algebra in presence of p-branes; then we consider the embedding of this non chiral supersymmetric field theory into 11D M theory on the 3-torus. This is useful for learning the group theoretic representations in which the gauge and scalar fields transform. In section 3, we study the attractor eqs for the black branes in 8D. We first consider an unconstrained parametrization of the moduli space, then we study the total effective potential and we derive the general form of the attractor eqs depending on the values of the Maurer Cartan 1-forms. In section 4, we study the solutions for the attractors equations. We study the explicit solutions for the dyonic membrane; actually, this completes the analysis done in [33]. Then we consider the general solutions for case of black strings and black 3-branes. This study extends directly to the case of black holes / black 4-branes; which is omitted. In section 5, we make an explicit study of the intersecting attractors in 8D by using the approach of [33]. In section 6 we give our conclusion and in in section 7, we give an appendix on useful properties on the algebras of spinors in 8D space time.
2 \textbf{Z- charges in 8D } \mathcal{N} = (2, 2) \textbf{ supergravity}

We begin by studying maximal supersymmetry in eight dimensional space time with p-branes. Then, we consider the embedding of 8D $\mathcal{N} = (2, 2)$ supergravity into the 11D M-theory compactification on the 3-torus $T^3$. Configurations based on M2 and M5 branes wrapping various cycles of $T^3$ are also considered in connection with black p-branes in 8D.

2.1 $\mathcal{N} = (2, 2)$ superalgebra with branes

In eight dimensions, non chiral $\mathcal{N} = (2, 2)$ supersymmetry has 32 conserved supersymmetric charges given by the 8D fermionic generators,

\[
\begin{align*}
generators & : \quad SO(1, 7) \times SU(2) \times U(1) \\
Q^+_\alpha A & \sim (8_s, 2)_+ \\
Q^-_{\alpha A} & \sim (8_c, 2)_-
\end{align*}
\]

In addition to the $SO(1, 7)$ space time, we also have an extra $U(2) = U(1) \times SU(2)$ invariance; this is an automorphism symmetry group with the $U(1)$ factor capturing the $\pm$ chirality charges of the Weyl spinors in 8D and the $SU(2)$ rotating the two supercharges $Q^{\pm A}$.

\[
Q^+_A \rightarrow e^{i\frac{\theta}{2}} (U^A_B Q^+_B) \quad , \quad Q^-_A \rightarrow Q^-_B (U^+_B)^A \quad e^{-i\frac{\theta}{2}},
\]

with $U(1)$ charge $q = 1$ and $U$ a unimodular $2 \times 2$ unitary matrix. This $SU(2)$ automorphism symmetry has also an interpretation as an internal symmetry in terms of embedding $\mathcal{N} = (2, 2)$ 8D supergravity in the 11D M-theory compactification on $T^3$. Under the reduction from 11D down to 8D, the $SO(1, 10)$ Lorentz group at each point of space time $\mathcal{M}_{11}$ gets broken down to $SO(1, 7) \times SO(3)$ where the internal $SO(3)$ orthogonal group is thought of in terms of the covering $SU(2)$ symmetry.

To get the general structure of the supersymmetric Lie algebra satisfied by the $Q^+_\alpha A$ and $Q^-_{\alpha A}$ operators, we use results on the tensor products of $SO(1, 7) \times SU(2) \times U(1)$ representations; in particular the $SO(1, 7)$ ones,

\[
\begin{align*}
8_i \times 8_i & = 1 + 28 + 35_i \\
8_i \times 8_j & = 8_k + 56_k
\end{align*}
\]

with $i, j, k$ cyclic and where $8_i$ stand for $8_s, 8_c, 8_v$ describing the basic eight dimensional representations of $SO(1, 7)$. For the case of spinor doublets of eqs(2.1), we have the
decompositions,
\[(8,2) \otimes (8,2) = (1,4) \oplus (28,4) \oplus (35,4)\]
\[(8,2) \otimes (8c,2) = (8,4) \oplus (56v,4)\]
\[(8c,2) \otimes (8c,2) = (1,4) \oplus (28,4) \oplus (35c,4)\]

with the complex \((1 + 35)\) associated with the \(\frac{8s9}{2}\) symmetric part of the product and the complex \(28 = \frac{8c7}{2}\) with the antisymmetric component. Using these relations, the general form of the anticommutation relations between the fermionic generators \(Q^+_\alpha\) and \(Q^-_{\dot{\alpha}}\) may be written as follows,

\[
\{Q^+_{\alpha\gamma}, Q^-_{\delta\beta}\} = \Gamma_{\gamma\delta}^{\mu} \delta_B^A P_\mu + Z^{0A}_{\gamma\delta|B} \\
\{Q^+_{\alpha}, Q^+_{\beta}\} = Z^{++AB}_{\alpha\beta} \\
\{Q^-_{\dot{\alpha}}, Q^-_{\dot{\beta}}\} = Z^{--}_{\dot{\alpha}\dot{\beta}}|AB
\]

where in addition to the usual terms \(\Gamma_{\gamma\delta}^{\mu} \delta_B^A P_\mu\), we have moreover other charge operators transforming into non trivial representations of \(SO(1,7)\). These operators have the following expansion properties

\[
Z_{\gamma\delta|B} = \varepsilon_{BC} \Gamma_{\gamma\delta}^{\mu} Z_{\mu|AC} = \varepsilon_{BC} \Gamma_{\gamma\delta}^{\mu} Z_{\mu|AC} \\
Z^{++AB}_{\alpha\beta} = \delta_{\alpha\beta} Z^{++(AB)} + \delta_{\alpha\beta} Z^{++(AB)} + \Gamma_{\alpha\beta} \delta_{\mu\rho} Z^{++(AB)} \\
Z^{--}_{\gamma\delta|AB} = \delta_{\gamma\delta} Z^{--(CD)} + \varepsilon_{AB} \Gamma_{\gamma\delta}^{\mu} Z^{--(CD)} + \varepsilon_{AC} \varepsilon_{BD} \Gamma_{\gamma\delta}^{\mu} Z^{--(CD)}
\]

where anti-symmetrization with respect to the space time indices is understood; see also appendix for more details on \(\Gamma\)- matrices. Obviously, the charge operators \(Z^{0A}_{\gamma\delta|B}\), \(Z^{++AB}_{\alpha\beta}\) and \(Z^{--}_{\gamma\delta|AB}\) are bosonic and generally take non zero values; they transform non trivially under \(SO(1,7)\) rotations and obey commutation relations \([9, 10]\), that are obtained as usual by solving the graded Jacobi identities. Let us comment much more these objects as they are crucial in studying black p-branes. The operators \(Z^{++AB}_{\alpha\beta}\) are complex and correspond to taking the symmetric part of the following tensor product relation,

\[
(8,2)_+ \otimes (8,2)_+ = (1,1)_+ \oplus (28,1)_+ \oplus (35s,1)_+ \oplus (1,3)_+ \oplus (28,3)_+ \oplus (35s,3)_+
\]

where the \(U(1)\) charges are exhibited as sub-indices. This decomposition leads to the identifications

\[
Z^{++(AB)} \sim (1,3)_+ , \quad Z^{++}_\mu \sim (28,1)_+ , \quad Z^{++(AB)}_{\mu\rho\lambda} \sim (35s,3)_+
\]

and shows that there various kinds of \(Z\)-charge operators capturing a priori different information of \(\mathcal{N} = (2,2)\) supersymmetric theory in 8D. This is in fact what happens as
we will see throughout this study. Notice that similar relations are valid for \((8c, 2)_+ \otimes (8c, 2)_-\); they are just the complex conjugates of the above ones. We also have

\[
(8s, 2)_+ \otimes (8c, 2)_- = (8v, 1)_0 \oplus (56v, 1)_0 \oplus (8v, 3)_0 \oplus (56v, 3)_0
\] (2.9)

with the correspondence

\[
Z^{(AB)}_\mu \sim (8v, 3)_0 , \quad Z^0_{\mu \nu \rho} \sim (56v, 1)_0 , \quad Z^{(AB)}_{\mu \nu \rho} \sim (56v, 3)_0
\] (2.10)

From this analysis, we learn amongst others that these bosonic \(Z\)-generators appearing in the supersymmetric algebra (2.5) exhibit a set of remarkable properties; in particular the two following:

(a) Like the other generators of the superalgebra (2.5), the \(Z\)’s are generally charged under the internal \(SU(2) \times U(1)\) automorphism symmetry and \(SO(1, 7)\) invariance since, in addition to the quantum numbers \((A, \pm q)\), they also carry space time indices in the antisymmetric representations. The last property allows to associate to each \(Z_{\mu_1...\mu_p}\) operator the space time \(p\)-form operator density

\[
Z_p = \frac{1}{p!} dx^{\mu_1} \wedge ... \wedge dx^{\mu_p} Z_{\mu_1...\mu_p},
\] (2.11)

together with the \(J(M_p) \equiv J_p\) invariant, \(J_p = \int_{M_p} Z_p\), where \(M_p\) is a \(p\)-dimensional space time submanifold which may be thought of as the world volume of a \(p\)-brane. Using eqs (2.8-2.10), it follows that in 8D maximal supergravity, we have for the complex \(SU(2)\) singlets \(Z^{\pm \pm}_{\mu \nu}\), the 2-form operators

\[
Z_2 = \frac{1}{2} dx^\mu \wedge dx^\nu Z_{\mu \nu} , \quad J_2 = \int_{M_2} Z_2 ,
\] (2.12)

and the \(p\)-forms,

\[
Z^{(AB)}_p = \frac{1}{p!} dx^{\mu_1} \wedge ... \wedge dx^{\mu_p} Z^{(AB)}_{\mu_1...\mu_p} , \quad J^{(AB)}_p = \int_{M_p} Z^{(AB)}_p,
\] (2.13)

for \(p = 0, 1, 3, 4\) for the \(SU(2)\) triplets.

(b) From eqs (2.8, 2.10), we learn as well that the \(Z_{\mu_1...\mu_p}\) operators have at most four space time indices. This property allows to give them an interpretation in terms of fluxes of gauge fields in 8D supergravity. Indeed, using the usual relations \(m = \int_{S^2} F_2\) and \(e = \int_{S^2} \tilde{F}_2\) giving the magnetic and electric charges of particles coupled to 4D Maxwell gauge fields and thinking about the \(Z_p\)’s in the same manner, we end with the following relations

\[
Z_p = \int_{S^2} F_{p+2} , \quad \tilde{Z}_{4-p} = \int_{S^2} \tilde{F}_{6-p}
\]

\[
J_p = \int_{M_{p+2}} F_{p+2} , \quad \tilde{J}_{4-p} = \int_{M_{6-p}} \tilde{F}_{6-p}
\] (2.14)
teaching us that the $Z_p$’s describe precisely charges of $p$-branes that couple to the $8D$ supergravity $(p + 1)$- form gauge fields $A_{p+1}$ with the field strengths $F_{p+2}$ and their magnetic duals $\tilde{F}_{6-p}$. In these relations, the spaces $M_{p+2} \sim M_p \times S^2$ and $\tilde{M}_{6-p} \sim \tilde{M}_{4-p} \times \tilde{S}^2$ are dual sub-manifolds of the $8D$ space time $\mathcal{M}_8$ with the typical fibration,

$$\tilde{M}_{6-p} \rightarrow \mathcal{M}_8 \quad \downarrow_{\pi_{p+2}} \quad M_{p+2}$$

(2.15)

where $M_{p+2}$ is thought as the $(p + 2)$- dimensional base sub-manifold and $\tilde{M}_{6-p}$ as its $(6 - p)$- dimensional fiber. For later use notice also that $p$-branes and their $(4 - p)$- duals extend along the dimensions of the respective $M_p$ and $\tilde{M}_{4-p}$ base sub-manifolds of $M_{p+2}$ and $\tilde{M}_{4-p}$,

$$S^2 \rightarrow M_{p+2} \quad \downarrow_{\pi_p} \quad p\text{-branes}$$

$$\tilde{S}^2 \rightarrow \tilde{M}_{6-p} \quad \downarrow_{\tilde{\pi}_{4-p}} \quad \text{4-p branes}$$

(2.16)

Below we study some aspects on gauge fields in $8D$ supergravity; in particular the gauge field content, the connection with $p$-branes and the embedding in M-theory compactification the 3-torus.

## 2.2 Embedding $8D$ $\mathcal{N} = (2, 2)$ supergravity in M-theory

The massless spectrum of the supergravity limit of M-theory has, besides the $11D$ field metric $G_{MN}^{(11D)}$, an antisymmetric gauge 3-form $C_{MNP}^{(11D)}$ that couples to M2 brane as well as fermionic partners. Under compactification of M-theory on the 3-torus, the M2 and M5 get wrapped and the fields $G_{MN}^{(11D)}$ and $C_{MNP}^{(11D)}$ get reduced to,

$$G_{\mu\nu}^{(8D)} \quad , \quad C_{\mu\nu\rho}^{(8D)}$$

(2.17)

together with the following $8D$ bosonic fields namely

$$B_{\mu\nu}^a \quad , \quad A_\mu^{ai} \quad , \quad \phi^1, ..., \phi^7$$

(2.18)

where the indices $a = 1, 2, 3$ and $i = 1, 2$. The field $C_{\mu\nu\rho}^{(8D)}$ couples to the membrane M2 living in $8D$ as in eqs (2.15-2.16), the three $B_{\mu\nu}^a$’s couple to the three kinds of strings
obtained by wrapping the M2 brane on each $S^1$ cycle of $T^3$ as given below,

\[
\begin{array}{cccccccc}
| M8 space time | T^3 |
|--------------|------|
| 0 1 2 3 4 5 6 7 | 8 9 10 |
| M2 | $\times\times\odot\odot\odot\odot\odot\odot$ |
| M5 | $\odot\odot\odot\times\times\times\times\times$ |
\end{array}
\] (2.19)

Regarding the six 1-form gauge fields

\[
A^{a_i}_\mu = (A^{a_1}_\mu, A^{a_2}_\mu), \quad A^{a_1}_\mu \equiv A^a_\mu, \quad A^{a_2}_\mu \equiv K^a_\mu
\] (2.20)

three of them; say $A^{a_1}_\mu = G^{(11D)}_{\mu a}$, are Kaluza Klein type obtained from the metric reduction; and the three $A^{a_2}_\mu$ others follow from the reduction of $C^{(11D)}_{MNP}$ as,

\[
C^{(11D)}_{\mu[ab]} = \varepsilon_{abc}K^c_\mu
\] (2.21)

where $\varepsilon_{abc}$ is the usual completely antisymmetric tensor of the real 3D space. These fields are associated with the three gauge particles given by the wrapping of M2 brane on the three 2-cycles of $T^3$ as illustrated on the following table,

\[
\begin{array}{cccccccc}
| M8 space time | T^3 |
|--------------|------|
| 0 1 2 3 4 5 6 7 | 8 9 10 |
| M2 | $\times\odot\odot\times\times\odot\odot\times$ |
| M5 | $\odot\times\times\times\times\times\times\times$ |
\end{array}
\] (2.22)

Before proceeding let us give some useful details.

8D $\mathcal{N} = 2$ supergravity fields

Under reduction to eight dimensions, the initial $128 + 128$ degrees of freedom of 11D supergravity decomposes into various $SO(1,7)$ representations. For the fermionic sector, we have

\[
128 = (2 \times [6 \times 8 - 8]) + (6 \times 8).
\] (2.23)

The first block describes the degrees of freedom of two 8D Ravita Schwinger fields $\psi^A_{\alpha\mu}$, $\bar{\psi}_{\dot{\alpha}\mu A}$ ($A = 1, 2$) and the second one captures the degrees of freedom of $(2 \times 3)$ gauginos.
$\lambda_{Aa}^a$, $\tilde{\lambda}_{Aa}^a$. For the 128 bosonic degrees of freedom; they decompose as follows

$$128 = (1 + 20) + 20 + (3 \times 15) + (2 \times 3 \times 6) + (5 + 1)$$

(2.24)

where $(1 + 20)$ stand for the dilaton $\sigma$ and the metric fields $G_{\mu\nu}$ and the second 20 for the antisymmetric $C_{\mu\nu\rho}$. The $3 \times 15$ are the degrees of freedom of the $B_{\mu\nu}^a$ triplet, the number $2 \times 3 \times 6$ describe two triplets of gauge fields $A_{\mu}^{ai}$ and finally $(5 + 1)$ stand for the scalars $\varphi^{(ab)}$, $\varphi$. The later follow respectively from the reduction of the metric and the 3-form gauge field on $T^3$ leading to the quintet $G_{ab} \equiv \varphi^{(ab)}$ and the singlet $C_{abc} \sim \varepsilon_{abc}\varphi$. In summary, the bosonic content of 8D $N = (2, 2)$ supergravity is,

$$G_{\mu\nu}, \ C_{\mu\nu\rho}, \ B_{\mu\nu}^a, \ A_{\mu}^{ai}, \ \varphi^{(ab)}, \ \sigma, \ \varphi,$$

(2.25)

with $\sum_{a=1}^{3} \varphi^{(aa)} = 0$. Notice that in addition to: (1) the seven scalar fields $\{\phi\}$ that parameterize the moduli space $\frac{SU(3,R)}{SO(3)} \times \frac{SU(2,R)}{SO(2)} \equiv \frac{SU(1,3)}{SU(2) \times U(1)}$ to be discussed in details later on; and (2) the 8D graviton $G_{\mu\nu}$ with scalar curvature $R_{8}$ and energy density,

$$L_{\text{gravity}} = -\frac{1}{16\pi G_{8}} \int_{M_{8}} d^{8}x \sqrt{-G} \ R_{8}$$

(2.26)

with $G = \det G_{\mu\nu}$, we have moreover the following:

(a) the antisymmetric field $C_{\mu\nu\rho}$ defining a real gauge 3-form $C_{3} = \frac{1}{3} dx^{\mu} \wedge dx^{\nu} \wedge dx^{\rho} C_{\mu\nu\rho}$ together with the 4-form field strength $F_{4} = dC_{3}$ and its magnetic dual $\tilde{F}_{4} = \ast F_{4}$. The lagrangian density describing the coupled dynamics of this field reads in general as follows

$$L_{3\text{-form}} = \frac{1}{32 \pi G_{8}} \int_{M_{8}} \sqrt{-G} \ N_{FF}^{(2)}(\phi) \ F_{\mu_{1}...\mu_{4}}^{i} \ F_{\mu_{1}...\mu_{4}}^{j}$$

(2.27)

where the second term is topological. Implementing the duality relation $\tilde{F}_{\mu_{5}...\mu_{8}} \sim \frac{1}{8!} \varepsilon_{\mu_{1}...\mu_{8}} F_{\mu_{1}...\mu_{4}}$ by a Lagrange multiplier $N_{FF}^{(2)}$, we end with the gauge field action

$$L_{3\text{-form}} = \frac{1}{32 \pi G_{8}} \int_{M_{8}} \sqrt{-G} \ N_{ij}^{(2)}(\phi) \ F_{\mu\nu\rho\sigma}^{i} \ F_{\mu\nu\rho\sigma}^{j}$$

(2.28)

where the field matrix $N_{ij}^{(2)}(\phi)$, which can be factorized as $K_{i}^{m}(\phi) \delta_{mn} K_{j}^{n}(\phi)$, provides the field coupling metric for the kinetic terms. In this equation, we have also included the topological term and set

$$F_{4} = \begin{pmatrix} F_{4} \\ \tilde{F}_{4} \end{pmatrix}, \quad F_{4i} = N_{ij}^{(2)} F_{ij}^{i}$$

(2.29)

transforming as an $SL(2, R)$ doublet.

(b) three antisymmetric gauge fields $B_{\mu\nu}^{a}$ defining a triplet of real gauge 2-forms $B_{2}^{a}$ with field strengths $F_{3}^{a} = dB_{2}^{a}$ and magnetic duals $\tilde{F}_{5[a]} = \ast (F_{3}^{a})$. The lagrangian density describing their coupled dynamics reads as follows

$$L_{2\text{-form}} = \frac{1}{32 \pi G_{8}} \int_{M_{8}} \sqrt{-G} \ N_{ab}^{(1)}(\phi) \ F_{\mu\nu\rho\sigma}^{a} F_{\mu\nu\rho\sigma}^{b}$$

(2.30)
with field metric $\mathcal{N}_{ab}^{(1)}(\phi)$ that can be factorized like $L_c^a(\phi)\delta_{cd}L_b^d(\phi)$.

(c) six real gauge fields $A^a_\mu$ defining six real 1-forms $A^a_i = dx^\mu A^a_\mu$ with field strengths $F^a_2 = dA^a_i$ and magnetic duals $\tilde{F}^a_6 = *(F^a_2)$. For later physical interpretation, it is interesting to set,

$$A^a_\mu = \left( \begin{array}{c} A^a_\mu \\ \kappa^a_\mu \end{array} \right), \quad F^a_{\mu\nu} = \left( \begin{array}{c} F^a_{\mu\nu} \\ \mathcal{H}^a_{\mu\nu} \end{array} \right).$$ (2.31)

The lagrangian density of these gauge fields reads as follows

$$\mathcal{L}_{1-form} = \frac{1}{32\pi G_8} \sqrt{-g} \mathcal{N}^{(0)}_{ai,bj}(\phi) F^a_i \mathcal{F}^b_{ij}. \quad (2.32)$$

Notice that because of the factorization of the moduli space, the field coupling $\mathcal{N}^{(0)}_{ai,bj}$ decomposes as well like $\mathcal{N}^{(0)}_{ab} \times \mathcal{N}^{(0)}_{ij}$. Notice also that the two internal indices $(a, i)$ carried by the above gauge field strengths refer to $SL(3,\mathbb{R}) \times SL(2,\mathbb{R})$ representations.

We have,

| fields strenghts | $F^a_4$ | $F^a_{3i}$ | $\tilde{F}^a_5$ | $F^a_2$ | $\tilde{F}^a_{6,ai}$ |
|------------------|---------|-----------|----------------|---------|---------------|
| $SL(3) \times SL(2)$ | $(1,2)$ | $(3,1)$ | $(3,2)$ | $(3',2')$ |

(2.33)

For simplicity, we will sometimes refer collectively to these field strengths as $F^I_{p+2} = dA^I_{p+1}$, $\tilde{F}_{8-p-2|I} = *(F^I_{p+2})$ with $A^I_{p+1}$ the gauge $(p+1)$-forms taking values on $SL(3) \times SL(2)$ representations designated by the $I$ index. These gauge invariant fields are associated with p-branes (anti-p-branes) having electric charges $q_I$ and magnetic ones $p_I$ given by the generic relations

$$p_I = \int_{\Sigma_{p+2}} F^I_{p+2}, \quad q_I = \int_{\Sigma_{6-p}} \tilde{F}^6_{6-p|I}. \quad (2.34)$$

where the cycles $\Sigma_{p+2}$ and the dual $\tilde{\Sigma}_{6-p}$ may be thought of as given by the spheres $S^{p+2}$ and $S^{6-p}$ respectively.

**Brane configurations**

Along with the M2 brane and the M2/$S^1$ as well as the M2/T$^2$ wrapped geometries, we also have wrapped configurations induced by the M5 brane. Since M5 is the magnetic dual of M2, the corresponding wrapped configurations are dual to the ones associated with the membrane. In the case of 8D $\mathcal{N} = (2,2)$ supergravity, the electric /magnetic duality that relates pairs of black p- and q- branes requires $p+q = 4$ from which we read the various black brane configurations in 8D:

(i) there are six black holes given by wrapping M2/T$^2$; these black holes have magnetic charges $P^{ai}$ and transform in the bi-fundamental of $SL(3,\mathbb{R}) \times SL(2,\mathbb{R})$,

$$P^{ai} = \int_{S^2} F^{ai}_2, \quad \mathcal{N}^{(1)}_{ai,bj} F^{bi}_2 = F_{2|ai}, \quad \mathcal{N}^{(1)}_{akc} \mathcal{N}^{(1)}_{ckbj} = \delta^b_k \delta^i_j \quad (2.35)$$
(ii) six black 4-branes obtained by wrapping M5/S1; these black objects are the dual of the black holes and carry the electric charges,

\[ Q_{ai} = \int_{S^6} \tilde{F}_{6|ai} \quad , \]

with \( P^{ai}Q_{bj} \sim n \delta^a_i \delta^j_b \) and where \( n \) an integer.

(iii) three black strings obtained by wrapping M2/S1; they are magnetically charged,

\[ p_a = \int_{S^3} \mathcal{F}_{3|a} \quad , \]

(iv) three black 3-branes following from the wrapping M5/T2; their electric charge reads as follows,

\[ q^a = \int_{S^5} \tilde{F}_5^a \quad , \quad q^a p_b \sim n \delta^a_b . \]

these are the dual of the black strings.

(v) a dyonic black 2-brane given by the fundamental M2 and the wrapped M5/T3. Its electric \( e \) and magnetic \( g \) charges are as follows,

\[ h^i = \int_{S^4} \mathcal{F}_4^i \quad , \quad h^i = (g, e) \quad , \quad eg \sim n . \]

From this analysis, we learn that the full abelian gauge symmetry of the 8D \( \mathcal{N} = 2 \) supergravity is given by

\[ U_{M2} (1) \otimes U^3_{M2/S1} (1) \otimes U^3_{M2/T2} (1) \otimes U^3_{KK} (1) \quad , \]

where \( U_{M2} (1) \) stands for the gauge group associated with the gauge 3-form, \( U^3_{M2/S1} (1) \) for strings and \( U^3_{M2/T2} (1) \times U^3_{KK} (1) \) for the gauge particles.

### 3 Attractor eqs of black p-branes

In this section, we study an unconstrained parametrization of the moduli space \( SL(3,R) \times SL(2,R) / SO(3) \times SO(2) \) of the 8D maximal supergravity. This parametrization is based on using field matrices in \( SL(3,R) \times SL(2,R) \) and gauging out the \( SO(3) \times SO(2) \) isometries of the moduli space. Then, we examine the total expression of the effective scalar potential \( V_{eff} \) of the black p-branes and derive the general expression of the attractor equations associated with the various black p-branes configurations living in 8D.

#### 3.1 Moduli space of 8D supergravity

##### 3.1.1 Scalar fields

In addition to the gauge fields and gauginos, the eight dimensional \( \mathcal{N} = (2,2) \) supergravity multiplet \( (2,25) \) has seven real scalar fields \( \{ \phi^1, ..., \phi^7 \} \) parameterizing a non
trivial scalar manifold. The first six scalars, to be denoted like \( \{\sigma, \varphi_{(ab)}\} \), \( Tr (\varphi) = 0 \), have a geometric interpretation in M theory compactification on the 3-torus; the seventh, denoted as \( \vartheta \), has rather a stringy origin as the value of the gauge 3-form \( C_{MNP}^{(11D)} \) on \( T^3 \). These scalars capture special features on maximal supergravity in 8D; in particular the two useful properties reported below.

First, the seven scalars \( \phi^1, ..., \phi^7 \) organize into two irreducible multiplets \( \varphi_{(ab)} \oplus \xi_{(ij)} \) with 5 + 2 field components with the property \( \sum_a \varphi_{(aa)} = 0, \sum_i \xi_{(ii)} = 0 \). The fields \( \varphi_{(ab)} \) are given by the following real symmetric and traceless 3 \( \times \) 3 matrix, 

\[
S_0 = \begin{pmatrix}
\phi^1 & \phi^3 & \phi^4 \\
\phi^3 & \phi^2 & \phi^5 \\
\phi^4 & \phi^5 & \phi^0
\end{pmatrix}, \quad S_0^T = S_0
\]

where we have set \( \phi^0 = -\phi^1 - \phi^2 \) since \( Tr S_0 = 0 \). In group theoretic language [37], this \( S_0 \) matrix is associated with a particular real group element 

\[
M_0 = \exp S_0
\]

of the \( SL (3, R) \) group manifold, \( M_0^T = M_0 \), \( det M_0 = 1 \); but moreover \( M_0^T = M_0 \) due to \( S_0^T = S_0 \). By using the general result that each generic \( SL (n, R) \) matrix \( M \) can be usually decomposed as the product \( U_0 \times M_0 \times U_0^T \) of an orthogonal \( SO (n) \) matrix \( U_0 \) and a symmetric \( M_0 \) one, it follows then that \( M_0 \) is just a representative matrix of the coset \( SL (3, R) / SO (3) \); that is a representative element of the class

\[
M \equiv U^T M U, \quad M \in SL (3, R), \quad U \in SO (3).
\]

The same analysis holds for the other two real fields \( \xi_{(ij)} \); they organize into a real symmetric and traceless 2 \( \times \) 2 matrix of the form

\[
P_0 = \begin{pmatrix}
\sigma & \vartheta \\
\vartheta & -\sigma
\end{pmatrix}, \quad Q_0 = \exp P_0,
\]

with \( \phi^6 = \sigma, \phi^7 = \vartheta \). Here also, the real 2 \( \times \) 2 matrix \( Q_0 \) is a representative matrix of the class \( Q \equiv V^T Q V \) with \( Q \in SL (2, R) \) and \( V \in SO (2) \). Therefore, the seven scalar fields \( \{\phi^1, ..., \phi^7\} \) of the 8D maximal supergravity, organized as in eqs (3.1-3.4), parameterize the real seven dimension non compact moduli space

\[
\frac{SL (3, R)}{SO (3)} \times \frac{SL (2, R)}{SO (2)} \sim \frac{SU (1, 2)}{SU (2)} \times \frac{SU (1, 1)}{U (2)}.
\]

The second property, we want to comment is that the scalars \( \{\phi^1, ..., \phi^7\} \) generate \( \phi \)-dependent couplings among the components of the supergravity multiplet (2.25). Some
of these couplings are given by the scalar functions $N_{IJ}^{(p)}(\phi)$ encountered previously [228, 230]. The other couplings are given by self interactions as well as the coupling to the gravity field $G_{\mu\nu}$ as shown on the lagrangian density

$$S_{Scalars} = \frac{1}{32\pi G_8} \int d^8 x \sqrt{-\det G} \, g_{IJ}(\phi) \partial_\mu \phi^I \partial_\nu \phi^J$$

(3.6)

where $g_{IJ}(\phi)$ is the metric of (3.5).

To deal with the various couplings of these scalar fields as well as the effective potential of the black branes $V_{eff}(\phi)$ to be considered later on, we shall develop a formalism based on the typical relations (3.3) and to which we refer to as the *unconstrained method*. This formalism relies on working with two real matrix fields; namely a $3 \times 3$ matrix field $(L_{ab})$ and a $2 \times 2$ matrix $(K_{ij})$ that are valued in the $SL(3,R) \times SL(2,R)$ Lie group,

$$L_{ab} \in SL(3,R) \quad , \quad K_{ij} \in SL(2,R)$$

(3.7)

and think about the $SO(3) \times SO(2)$ isometry of the moduli space as an auxiliary gauge symmetry captured by auxiliary gauge fields $A_{\mu}^{SO(3) \times SO(2)}$. In this set up, physical observables are expressed in terms of the $L$ and $K$ matrices; but are $SO(3) \times SO(2)$ invariant. Let us give some useful details.

### 3.1.2 More on unconstrained method

Being group elements of $SL(3,R) \times SL(2,R)$ group manifold, the real matrices $L$ and $K$ satisfy the group theoretical constraint eqs,

$$L_{ab} \tilde{L}^{cb} = \delta_a^b \quad , \quad L^{-1} = \tilde{L} \quad , \quad \det L = 1$$

$$K_{ab} \tilde{K}^{ij} = \delta_i^j \quad , \quad K^{-1} = \tilde{K} \quad , \quad \det K = 1$$

(3.8)

fixing two real degrees of freedom among the real $9 + 4$. The other $(3+1)$ undesired variables are fixed by requiring the following identifications under the $SO(3) \times SO(2)$ symmetry of the moduli space

$$K \equiv V^T K V \quad , \quad V \in SO(2)$$

$$L \equiv U^T L U \quad , \quad U \in SO(3)$$

(3.9)

where $V = \exp{\eta \tau}$ with $\tau \equiv \tau^2$ given by eq(3.15) and $U = \exp{\zeta_a T^a}$ are gauge transformations with respective gauge parameters $\eta = \eta(\phi)$ and $\zeta_a = \zeta_a(\phi)$. Notice that the three $T^a$'s are given by the following antisymmetric $3 \times 3$ matrices,

$$T_1 = \begin{pmatrix} 0 & 1 & 0 \\ -1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \quad T_2 = \begin{pmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ -1 & 0 & 0 \end{pmatrix}, \quad T_3 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & -1 & 0 \end{pmatrix}.$$  

(3.10)
We also require that the matrix gradients $\nabla_{\mu}^{SO(3)} L$ and $\nabla_{\mu}^{SO(2)} K$, with $\nabla_{\mu}^{SO} = \partial_{\mu} - A_{\mu}$, are gauge covariant under these transformations so that their space time kinetic energies
\[
\frac{1}{2} \sqrt{-\mathcal{G}} \text{Tr} (\mathcal{G}^{\mu \nu} (\nabla_{\mu} L)^{-1} (\nabla_{\nu} L)^{-1}) + \frac{1}{2} \sqrt{-\mathcal{G}} \text{Tr} (\mathcal{G}^{\mu \nu} (\nabla_{\mu} K)^{-1} (\nabla_{\nu} K)^{-1})
\]
are gauge invariant. More precisely, we have
\[
(\nabla_{\mu} L)^{-1} \equiv U^T (\nabla_{\mu} L) U^{-1},
(\nabla_{\mu} K)^{-1} \equiv V^T (\nabla_{\mu} K) V^{-1},
\]
where the 8D vector fields $(A_{\mu}^{SO(3)}, A_{\mu}^{SO(2)})$ are gauge fields associated with the $SO(3) \times SO(2)$ isometry of the moduli space. Under $SO(3) \times SO(2)$ change generated by the $(U, V)$ orthogonal matrices, these gauge fields transform respectively as $A_{\mu}^{SO(3)} = U \partial_{\mu} U^T$, $A_{\mu}^{SO(2)} = V \partial_{\mu} V^T$. Notice also that the gauge fields $A_{\mu}^{SO(3)}$ and $A_{\mu}^{SO(2)}$ are auxiliary fields in the sense that they do not have kinetic terms; the elimination of these fields through their equations of motion allows to express them as functions of the $L$ and $K$ matrices and their space time derivatives,
\[
A_{\mu}^{SO(3)} = F (L, \partial_{\mu} L), \quad A_{\mu}^{SO(2)} = F (K, \partial_{\mu} K),
\]
which, up on substitution, induce non trivial self interactions amongst the matrix fields leading to the metric of the moduli space $\frac{SL(3, R) \times SL(2, R)}{SO(3) \times SO(2)}$. The $SO(3) \times SO(2)$ identifications (3.9,3.11) can be explicitly illustrated by expressing the field matrices $L$ and $K$ as Lie group elements like,
\[
L = \exp \varphi, \quad K = \exp \xi, \quad U = \exp \zeta, \quad V = \exp \eta \tau^2,
\]
with
\[
\varphi = \sum_{a,b} (\varphi_{ab} - \frac{1}{3} \zeta \delta_{ab}) T^{ab}, \quad \xi = \sum_{m,n} (\xi_{mn} - \frac{1}{2} \eta \delta_{MN}) \tau^{mn}
\]
which read also like,
\[
\varphi = \sum_{A=1}^{8} \varphi_A T^A, \quad \xi = \sum_{a=1}^{3} \xi_a \tau^a, \quad \zeta = \sum_{a=1}^{3} \zeta_a T^a.
\]
Here the eight traceless 3×3 matrices $T^{ab}$ (or equivalently $T^A$) are the generators of $SL(3, R)$; they may be split as $(3 + 5)$ describing respectively $T^{[ab]} = \varepsilon^{abc} T^c$ generating the subgroup $SO(3)$ and $T^{(ab)}$ generating the $SL(3, R)/SO(3)$ manifold. The three 2×2 traceless matrices $\tau^{mn}$ (or equivalently $\tau^a$) are the generators of $SL(2, R)$; they split as $(1 + 2)$ describing respectively $\tau^{[mn]} = \varepsilon^{mn} \tau^2$ generating $SO(2)$ and $\tau^{(mn)}$ generating the space $SL(2, R)/SO(2)$. These generators read as follows:
\[
\tau^1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \quad \tau^2 = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}, \quad \tau^3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}
\]
with the relations
\[
\tau^1 = \tau^{12} + \tau^{21}, \quad \tau^3 = \tau^{11} - \tau^{22}, \quad \tau^2 = \tau^{12} - \tau^{21}, \quad \delta^0 = \tau^{11} + \tau^{22}
\]
where sometimes we also use the notation $\tau^3 = \tau^0$. The net field variables parameterizing the typical coset manifolds $SL(n, R)/SO(n)$ may be obtained by decomposing the adjoint representation of $SL(n, R)$ with respect to the irreducible representations of $SO(n)$. We have

$$n^2 - 1 = \frac{n(n-1)}{2} \oplus \frac{n(n+1)-2}{2} \quad (3.17)$$

where $\frac{n(n-1)}{2}$ stands for $ad_{SO(n)}$ and $\frac{n(n+1)-2}{2}$ for the traceless symmetric representation. Gauge symmetry under $SO(n)$ may be used to fix the antisymmetric part $\sum_{a,b} \varphi_{[ab]} T^{[ab]}$ in the typical expansions (3.14) leaving free the real $\frac{n(n+1)-2}{2}$ variables.

In $\mathcal{N} = (2, 2)$ supergravity where the role of $SL(n, R)$ is played by the direct product $SL(3, R) \times SL(2, R)$, the decomposition with respect to $SO(3) \times SO(2)$ reads as,

| $SL(3, R)$ | $\supset$ | $SO(3)$ | $SL(2, R)$ | $\supset$ | $SO(2)$ |
|------------|-----------|---------|------------|-----------|---------|
| 8          | =         | $3 \oplus 5$ | 3          | =         | $1 \oplus 2$ | (3.18) |

The $(5 + 2)$ physical degrees of freedom $\varphi^{(ab)}$, $\xi^{(ij)}$ parameterizing (3.5) may be explicitly exhibited by solving the above $SO(3) \times SO(2)$ identifications (3.9) to end with the gauge fixed representatives $L_0$ and $K_0$ given by,

$$L_0 = e^{\varphi_0}, \quad \varphi_0 = \sum_{a,b} \varphi_{(ab)} T^{(ab)}, \quad K_0 = e^{\xi_0}, \quad \xi_0 = \sum_{m,n} \xi_{(mn)} T^{(mn)}, \quad (3.19)$$

where $\varphi_{(ab)} = \varphi_{(ba)}$, $\xi_{(mn)} = \xi_{(nm)}$ and $\sum_a \varphi_{(aa)} = 0$, $\sum_m \xi_{(mm)} = 0$. For later use, we rewrite the matrix $K_0$ like,

$$K_0 = \exp (\vartheta \tau^1 + \sigma \tau^3) \quad (3.20)$$

These gauge fixed matrices should be compared with (3.1 3.4).

### 3.1.3 Maurer Cartan forms

In the unconstrained parametrization of the moduli space (3.5), the basic field variables are the matrices $L$ and $K$ obeying (3.9 3.11). The variations of these field matrices are captured by the Maurer Cartan 1-forms $\Omega_{SL_3} \equiv \Omega$ and $\Omega_{SL_2} \equiv \omega$ [34 37] living on the $SL(3, R) \times SL(2, R)$ group manifold. These real 1-forms,

$$\Omega = -L (dL^{-1}), \quad \omega = -K (dK^{-1}) \quad (3.21)$$

depend implicitly of the group parameters $\varphi^A$, $\xi^\alpha$ and their $d\varphi^A$, $d\xi^\alpha$ differentials and exhibit two basic kinds of expansions; the first one with respect to the field differential
basis \( \{ d\varphi^A, d\xi^\alpha \} \) and the second with respect to the generators \( \{ T_A, \tau_\alpha \} \) of the \( sl(3, R) \oplus sl(2, R) \) Lie algebra.

**Differential basis \( \{ d\varphi^A, d\xi^\alpha \} \)**

Substituting the matrices \( L \) and \( K \) by their respective expressions \( \exp \left( \sum_A \varphi_A T^A \right) \) and \( \exp \left( \sum_\alpha \xi_\alpha \tau_\alpha \right) \) into eq[3.21], we see that we can expand these Maurer Cartan forms in a series as follows

\[
\Omega = \sum_{A=1}^8 d\varphi^A \Omega_A, \quad \omega = \sum_{\alpha=1}^3 d\xi^\alpha \omega_\alpha, \quad (3.22)
\]

with sections \( \Omega_A = -LT_A L^{-1} \) and \( \omega_\alpha = -K \tau_\alpha K^{-1} \) which are nothing but the Lie group adjoint actions on the \( sl(3, R) \oplus sl(2, R) \) generators,

\[
\Omega_A = -e^{ad_\varphi} T_A, \quad \omega_\alpha = -e^{ad_\xi} \tau_\alpha. \quad (3.23)
\]

These sections are real matrices that are respectively valued in the \( sl(3, R) \) and \( sl(2, R) \) Lie algebras as shown by the traces \( Tr(\Omega_A) = 0, Tr(\omega_\alpha) = 0 \). Notice also that thinking about \( sl(3, R) \oplus sl(2, R) \) as a vector space, the Maurer Cartan fields \( \Omega \) and \( \omega \) may be split as follows

\[
\Omega = \Omega^{SO_3} + \Omega^{SL_3/\text{SO}_3}, \quad \omega = \omega^{SO_2} + \omega^{SL_2/\text{SO}_2}, \quad (3.24)
\]

where the terms \( \Omega^{SO_n} \) and \( \Omega^{SL_n/\text{SO}_n} \) are respectively the Maurer Cartan forms associated with \( SO(n) \) group and the coset space \( SL(n, R)/SO(n) \).

**Lie algebra basis \( \{ T_A, \tau_\alpha \} \)**

Using specific properties of \( SL(n, R) \) matrices; in particular the adjoint action \( e^A B e^{-A} = e^{ad_A} B \) with \( ad_A B = AB - BA \) and applying this to the field matrices \( L = e^\varphi, K = e^\xi \), we can express the sections \( \Omega_A = -e^{ad_\varphi} T_A \) and \( \omega_\alpha = -e^{ad_\xi} \tau_\alpha \) as an infinite series like,

\[
\Omega_A = -T_A - \sum_{n=1}^{\infty} \frac{1}{n!} [\varphi, [\varphi, ... [\varphi, T_A] ...]], \quad (3.25)
\]

\[
\omega_\alpha = -\tau_\alpha - \sum_{n=1}^{\infty} \frac{1}{n!} [\xi, [\xi, ... [\xi, \tau_\alpha] ...]].
\]

Now substituting \( \varphi = \sum_B \varphi_B T^B \) and \( \xi = \sum_\beta \xi_\beta \tau^\beta \) in these relations and using \( [\varphi, T_A] = F^C_{BA} \varphi^B T^C \) and \( [\xi, \tau_\alpha] = f^\gamma_{\beta\alpha} \varphi^\beta \tau^\gamma \) with \( F^C_{BA} \) and \( f^\gamma_{\beta\alpha} \) standing for the structure constants of the \( sl(3, R) \) and \( sl(2, R) \) Lie algebras, we learn that the \( \Omega_A \) and \( \omega_\alpha \) matrices may be expanded in terms of the generators \( \{ T^B, \tau^\beta \} \) as follows

\[
\Omega_A = \sum_B \Theta^B_A T_B, \quad \omega_\alpha = \sum_\beta \theta^\beta_\alpha \tau^\beta. \quad (3.26)
\]

Seen that these expansions are useful in dealing with attractor eqs of black p-branes; let us collect here below the relevant relations:
(i) The Maurer Cartan 1-forms $\Omega$ and $\omega$ \textcolor{blue}{(3.22)} may be expanded into different ways, either with respect to the differential form basis as

$$\Omega = \sum_{A=1}^{8} d\varphi^{A}\Omega_{A}, \quad \omega = \sum_{\alpha=1}^{3} d\xi^{\alpha} \omega_{\alpha}, \quad (3.27)$$

or with respect to the Lie algebra generators like

$$\Omega = \sum_{B=1}^{8} \Delta^{B}\Theta_{B}, \quad \omega = \sum_{\alpha,\beta=1}^{3} d\xi^{\alpha} \theta^{\beta}_{\alpha}, \quad (3.28)$$

In the first expansion $\Omega_{A}$ and $\omega_{\alpha}$ are matrices valued in the Lie algebras and in the second development $\Delta^{B}$ and $\lambda^{\beta}$ are real 1-forms. Combining the two expansions, we get,

$$\Omega = \sum_{A,B=1}^{8} d\varphi^{A}\Theta_{A}^{B}\Theta_{B}, \quad \omega = \sum_{\alpha,\beta=1}^{3} d\xi^{\alpha} \theta^{\beta}_{\alpha}, \quad (3.29)$$

with

$$\Delta^{B} = \sum_{A} d\varphi^{A}\Theta_{A}^{B}, \quad \lambda^{\beta} = \sum_{\alpha} d\xi^{\alpha} \theta^{\beta}_{\alpha} \quad (3.30)$$

and

$$\Theta_{A}^{B} = - (e^{ad\varphi})_{A}^{B}, \quad \theta^{\beta}_{\alpha} = - (e^{ad\xi})_{\alpha}^{\beta}. \quad (3.31)$$

(ii) In the Cartan Weyl basis $\{H_{i}, E^{\pm\eta}\} \oplus \{\tau^{0}, \tau^{\pm}\}$ of $sl_{2}(3, R) \oplus sl_{2}(2, R)$, the Maurer Cartan fields $\Omega$ and $\omega$ read as

$$\Omega = \sum_{i} \Delta^{i} H_{i} + \sum_{\eta} (\Delta^{-\eta} E^{+\eta} + \Lambda^{+\eta} E^{-\eta}), \quad \omega = \sum_{\lambda^{0}, \lambda^{\pm}} \lambda^{0} \tau^{0} + \sum_{\lambda^{+}, \lambda^{-}} \lambda^{+} \tau^{-} \quad (3.32)$$

where $\eta$ refers to the positive roots of $sl_{2}(3, R)$ and where $(\Delta^{i}, \Delta^{\pm\eta})$ and $(\lambda^{0}, \lambda^{\pm})$ are differential forms given by \textcolor{blue}{(3.30)}.

(iii) To solve the attractor eqs, we will use different representations to deal with the Maurer Cartan forms; in particular the above ones but also $\Omega = \sum d\varphi^{ab}\Omega_{ab}$, $\omega = \sum d\xi^{mn} \omega_{mn}$, where $\Omega_{ab}$ and $\omega_{mn}$ are related to $\Omega_{A}$ and $\omega_{\alpha}$ as $\Omega_{ab} = \sum_{A} \Omega_{A}^{T} \tau_{ab}$ and $\omega_{mn} = \sum_{\alpha} \omega_{\alpha} \tau_{mn}$. Similarly, we also have $d\varphi^{A} = \sum d\varphi^{ab} \tau_{ab}^{A}$ and $d\xi^{\alpha} = \sum d\xi^{mn} \tau_{mn}^{\alpha}$. In this basis, the Maurer Cartan forms associated with the $SO(n)$ and $SL_{2}(n, R) / SO(n)$ are given by the antisymmetric and symmetric parts as shown below

$$\Omega = \sum_{a,b} d\varphi^{[ab]} \Omega_{[ab]}^{SO_{3}} + \sum_{a,b} d\varphi^{[ab]} \Omega_{[ab]}^{SL_{2}/SO_{3}}, \quad (3.33)$$

where $\Omega_{[ab]}^{SO_{3}} = 0$, $\omega_{[mn]}^{SO_{2}} = 0$ leaving only the desired components $\Omega_{\{ab\}}^{SL_{2}/SO_{3}}$ and $\omega_{\{mn\}}^{SL_{2}/SO_{2}}$.

In this representation, the $SO(3) \times SO(2)$ symmetry of the moduli space may be gauged out by setting $\Omega_{[ab]}^{SO_{3}} = 0$, $\omega_{[mn]}^{SO_{2}} = 0$ leaving only the desired components $\Omega_{(ab)}^{SL_{2}/SO_{3}}$ and $\omega_{(mn)}^{SL_{2}/SO_{2}}$.
3.2 Attractor equations

Attractor equations of black objects in 8D maximal supergravity are obtained by minimizing their effective potential $V_{\text{eff}}(\phi)$; its Hessian matrix is then positive \[33\]. This scalar potential depend on the coordinates $\{\phi^I\}$ of the moduli space of the theory. So, attractor eqs follow from $\delta V_{\text{eff}} = \sum \frac{\partial V_{\text{eff}}}{\partial \phi^I} \delta \phi^I$. For arbitrary variations $\delta \phi^I$, we have the following constraint relations:

$$\frac{\partial V_{\text{eff}}}{\partial \phi^I} = 0 \quad \text{det} \left( \frac{\partial^2 V_{\text{eff}}}{\partial \phi^I \partial \phi^J} \right) > 0 \quad I, J = 1, ..., N \quad (3.34)$$

whose solutions fix the values of the field moduli at the black object near horizon geometry in terms of the charges $q$ and $p$; i.e $\phi_I = \phi_I(p, q)$.

3.2.1 Effective potential

In 8D maximal supergravity where lives several kinds of black p-branes, the total effective potential, induced from the kinetic energies of the gauge fields strengths at the horizon, is given by the sum over individual components $V_p$ associated with each black p-brane as given below,

$$V_{\text{eff}} = (V_0 + V_4) + (V_1 + V_3) + V_2. \quad (3.35)$$

The scalar components $V_p$, which are related by the electric/magnetic duality property $\tilde{V}_p = V_{4-p}$, are functions of the scalar fields \[33,1\] and the charges $\{g^I, e_I\}$ of the branes,

$$V_p = V_p(\phi^1, ..., \phi^7; g^I, e_I). \quad (3.36)$$

In the unconstrained formulation of the moduli space, the effective potential dependence in the $\phi$’s is realized through the field matrices $L_{ab} = L_{ab}(\phi)$, $K_{ij} = K_{ij}(\phi)$ so that the $V_p$ components are functionals like,

$$V_p = V_p[L(\phi), K(\phi); g^I, e_I], \quad (3.37)$$

with the symmetry property

$$V_p[L, K] = V_p[L', K'], \quad (3.38)$$

where

$$L' = U^T L U, \quad K' = V^T K V, \quad (3.39)$$

are gauge transformations expressing invariance under $SO(3) \times SO(2)$ isometry of the moduli space. The individual potentials $V_0$, $V_1$ and $\tilde{V}_0 = V_4$, $\tilde{V}_1 = V_3$ are explicitly expressed like

$$V_0 = \frac{1}{2} \sum X^{ai} \delta_{ab} \delta_{ij} X^{bj}, \quad \tilde{V}_0 = \frac{1}{2} \sum \tilde{X}_{ai} \delta^{ab} \delta^{ij} \tilde{X}_{bj}, \quad (3.40)$$

$$V_1 = \frac{1}{2} \sum Y^a \delta_{ab} Y^b, \quad \tilde{V}_1 = \frac{1}{2} \sum \tilde{Y}_a \delta^{ab} \tilde{Y}_b.$$
In these relations, $X^{ai}$ and $\tilde{X}^{ai}$ are respectively the dressed central charges of the black holes and their 4-branes dual while the fields $Y^{a}$, $\tilde{Y}^{a}$ are the dressed central charges of the black strings and their 3-branes dual. We also have

$$V_2 = \frac{1}{2}Z_{el}^2 + \frac{1}{2}Z_{mag}^2,$$

(3.41)
describing the effective potential of the black membrane. To exhibit the $SO(3) \times SO(2)$ symmetry of this potential as in (3.40), it is interesting to think about $V_2$ as given by the following symplectic form

$$V_2 = \sum Z^i \delta_{ij} Z^j,$$

(3.42)
with $Z^i = (Z_{mag}, Z_{el})$. Moreover, by using the field matrices $L$, $K$ and the bare electric and magnetic charges associated with the various fields strengths of the supergravity theory, we can express the above dressed charges as follows

$$X^{ai} = \sum P^{bj} L^a_b K^i_j, \quad \tilde{X}^{ai} = \sum (L^{-1})^b_a (K^{-1})^i_j Q_{bj},$$

(3.43)
and

$$Y^{a} = \sum p^b L^a_b, \quad \tilde{Y}^{a} = \sum (L^{-1})^b_a q_{b},$$

(3.44)
as well as

$$Z^i = \sum K^j_i h^j.$$

(3.45)
These dressed charges obey the typical $SO(3) \times SO(2)$ symmetry properties

$$X^{ai} \equiv \sum X^{bj} U^a_b V^i_j, \quad Y^{a} \equiv \sum Y^{b} U^a_b, \quad Z^i \equiv \sum Z^{j} V^i_j$$

(3.46)
that are induced by the symmetric features satisfied by the fields matrices $L$ and $K$.

### 3.2.2 Attractor equations

To get the attractor equations of the black p-branes, we extremize the above effective potential $V_{eff}$ with respect to the scalar fields $\{\phi^I\}$. Since $V_{eff}$ is a functional of these scalar fields that can be either thought of as

$$V_{eff} = V_{eff}[L_{ab}(\phi), K_{ij}(\phi)]$$

(3.47)
or in terms of the dressed central charges given by eqs (3.43, 3.45),

$$V_{eff} = V_{eff} \left[ X^{ai}(\phi), Y^{a}(\phi), Z^i(\phi), \tilde{X}^{ai}(\phi), \tilde{Y}^{a}(\phi) \right],$$

(3.48)
we can state its extremum in two different, but equivalent, ways. Below, we shall refer to these dressed central charges collectively by $\Psi^I(\phi)$ and express the attractor eqs both in terms of (3.47) and (3.48).
Using eq (3.47) By using the matrix fields \( L \) and \( K \) as well as symmetry under \( SO(3) \times SO(2) \), the extremization of the potential is given by,

\[
\sum \left( \frac{\partial \psi_{eff}}{\partial \varphi^A} \right) \frac{\partial L_{ab}}{\partial \varphi^A} d\varphi^A + \sum \left( \frac{\partial \psi_{eff}}{\partial \xi^a} \right) \frac{\partial K_{ab}}{\partial \xi^a} d\xi^a = 0 \quad (3.49)
\]

So the attractor equations read, up to \( SO(3) \times SO(2) \) transformations, as follows:

\[
d\varphi^A Tr \left[ L T_A \left( \frac{\partial \psi_{eff}}{\partial L} \right) \right] = 0 , \quad A = 1, \ldots , 8
\]
\[
d\xi^\alpha Tr \left[ K T_\alpha \left( \frac{\partial \psi_{eff}}{\partial K} \right) \right] = 0 , \quad \alpha = 0, 1, 2
\]

(3.50)

in agreement with the \( \frac{SL(3,R)}{SO(3)} \times \frac{SL(2,R)}{SO(2)} \) factorization of the moduli space.

Using eq (3.48) In this case the extremization condition \( \delta \mathcal{V}_{eff} = 0 \) may be also written as,

\[
\sum \left( \frac{\partial \psi_{eff}}{\partial \varphi^A} \right) \frac{\partial L_{ab}}{\partial \varphi^A} \delta L_{ab} + \sum \left( \frac{\partial \psi_{eff}}{\partial \xi^a} \right) \frac{\partial K_{ab}}{\partial \xi^a} \delta K_{ab} = 0
\]

(3.51)

Using eqs (3.35-3.40), we can bring this constraint relation into the form,

\[
\delta \mathcal{V}_{eff} = + \sum \delta_{ab} \delta_{ij} \left( X^{b j} \delta X^{a i} \right) + \sum \delta^{a b} \delta^{i j} \left( \tilde{X}_{b j} \delta \tilde{X}_{a i} \right) + \sum \delta_{a b} Y^b \delta Y^a + \sum \delta^{a b} \tilde{Y}_b \delta \tilde{Y}_a + \sum \delta_{i j} Z^j \delta Z^i
\]

(3.52)

with

\[
\delta X^{a i} = + \sum \left( \Omega^b_{a} \delta_{j}^{i} + \omega_j^{i} \delta_{a}^{b} \right) X^{b j}, \quad \delta Y^a = + \sum \Omega^b_{a} Y^b
\]

\[
\delta \tilde{X}_{a i} = - \sum \left( \Omega^b_{a} \delta_{j}^{i} + \omega_j^{i} \delta_{a}^{b} \right) \tilde{X}_{b j}, \quad \delta \tilde{Y}_a = - \sum \Omega^b_{a} \tilde{Y}_b
\]

\[
\delta Z^i = + \sum \omega_j^{i} Z^j
\]

(3.53)

and where the 1- forms \( \Omega = (\delta L) L^{-1} \) and \( \omega = (\delta K) K^{-1} \) are respectively the Cartan Maurer forms of the \( SL(3,R) \) and \( SL(2,R) \) that we have studied previously. Putting these relations back into (3.52), we can read the attractor equations from the following relation,

\[
\delta \mathcal{V}_{eff} = Tr \left( X \Omega X - \tilde{X} \Omega \tilde{X} \right) + Tr \left( Y \Omega Y - \tilde{Y} \Omega \tilde{Y} \right) + Tr \left( Z \omega Z \right) = 0
\]

(3.54)

Rewriting this constraint equation as a linear combination of the Maurer Cartan 1-forms on the \( SL(3) \otimes SL(2) \) group manifold like

\[
\sum_{a,b} \gamma^{a b} \Omega_{a b} + \sum_{i,j} \Gamma^{i j} \omega_{i j} = 0
\]

(3.55)

with \( \Omega_{a b} = \sum_{A} d\varphi^A \left( \Omega_{A} \right)_{a b} \), \( \omega_{i j} = \sum_{a} d\xi^a \left( \omega_{a} \right)_{i j} \) and,

\[
\gamma^{a b} = \sum \left( \delta_{i j} X^{a i} X^{b j} - \delta^{i j} \tilde{X}_{a i} \tilde{X}_{b j} \right) + \left( Y^a Y^b - \tilde{Y}_a \tilde{Y}_b \right)
\]

\[
\Gamma^{i j} = \sum \left( \delta^{i l} \delta_{a b} X^{a l} X^{b j} - \sum \delta^{a b} \delta^{i k} \delta^{j l} \tilde{X}_{a k} \tilde{X}_{b l} \right) + Z^i Z^j
\]

(3.56)
the attractor equations can be expressed as follows:

$$
\Upsilon_{ab} \Omega_{[ab]}^{SO_3} + \Upsilon_{i[j} \Omega_{i]}^{SL_3/SO_3} = 0 , \\
F_{ij} \omega_{(ij)}^{SO_2} + F_{ij} \omega_{(ij)}^{SL_2/SO_2} = 0 ,
$$

(3.57)

where the Maurer Cartan 1-forms are as before. Notice that from eqs (3.56), the tensors \( \Upsilon_{ab} \) and \( F_{ij} \) are symmetric (\( \Upsilon_{ab} = \Upsilon_{(ab)} \), \( F_{ij} = F_{(ij)} \)) and so the first terms of above relations namely \( \Upsilon_{(ab)} \Omega_{(ab)}^{SO_3} \) and \( F_{(ij)} \omega_{(ij)}^{SO_2} \) vanish identically. This property reflects just invariance under the \( SO(3) \times SO(2) \) isometry of the moduli space. So, the attractor equations of the black p-branes of maximal supergravity in 8D read as follows,

$$
\Upsilon_{(ab)} \Omega_{(ab)}^{SL_3/SO_3} = 0 , \\
F_{(ij)} \omega_{(ij)}^{SL_2/SO_2} = 0
$$

(3.58)

where now \( \Upsilon_{(ab)} \) and \( F_{(ij)} \) are precisely as in eqs (3.56).

### 4 Solving the attractor eqs

We first study the solutions of the attractor eqs for the dyonic black membrane with the near horizon geometry \( AdS_4 \times S^4 \); actually this completes the partial results given in [33]. Then, we examine the other black attractor solutions corresponding to the p-branes with near horizon geometries \( AdS_{p+2} \times S^{6-p} \).

#### 4.1 Dyonic membrane

The dyonic black membrane of 8D maximal supergravity has a near horizon geometry \( AdS_4 \times S^4 \) in which the electric \( e \) and magnetic \( g \) charges of the 4-form \( F_4 \) are switched on

$$
F_4 = e\alpha_4 + g\beta_4 , \\
\tilde{F}_4 \sim F_4
$$

(4.1)

The real 4-forms \( \alpha_4 = \alpha_{AdS_4} \) and \( \beta_4 = \beta_{S^4} \) are respectively the volume forms on the non compact \( AdS_4 \) and the compact n-sphere \( S^4 \). We have

$$
\frac{1}{V_{AdS_4}} \int_{AdS_4}^\Lambda \alpha_4 = 1 , \\
\frac{1}{V_{S^4}} \int_{S^4} \beta_4 = 1
$$

(4.2)

with \( V_{AdS_4} \) describing a regularized volume with \( \Lambda \) some UV regularization parameter. We also have

$$
\int_{AdS_4 \times S^4} F_4 \wedge \tilde{F}_4 \sim egV_{tot}
$$

(4.3)

where \( V_{tot} = V_{AdS_4} \times V_{S^4} \) and where the electric and magnetic charges of the membrane are related by the Dirac quantization relation \( eg \sim n \). Notice that p-forms with \( p \neq 4 \), which are associated with the other p-black branes of the 8D supergravity, are not
supported by the $AdS_4 \times S^4$ geometry since there is no p-cycles allowing relations type $[4.1]$. As such the charges $P^{ai}$, $Q_{ia}$, $p^a$, $q_a$ are switched off; so the attractor eqs reduce to the following conditions on the dressed central charges,

\[ Z^i Z^j \omega_{ij} = 0 \quad . \] (4.4)

A standard way to deal with relation is to suppose $\omega_{ij} \neq 0 \ \forall i, j$; and end with the constraint eqs $Z^i Z^j = 0$ leading to the trivial solution $Z^i = 0$; ie $Z_{ele} = Z_{mag} = 0$. This solution, which requires the vanishing of the bare electric and magnetic charges;

\[ g = 0 \quad , \quad e = 0, \] (4.5)

corresponds then to a trivial configuration with no black membrane charges. To get more insight into eq$(4.4)$, let us work out explicitly the minimum of the effective potential $V_{eff} = V_{eff}(\sigma, \vartheta)$ of the black membrane whose explicit expression is as in eq$(3.41)$. The corresponding attractor equations,

\[ \frac{\partial V_{eff}}{\partial \sigma} = 0 \quad , \quad \frac{\partial V_{eff}}{\partial \vartheta} = 0 \quad , \] (4.6)

lead to,

\[ 2Z^2 (Z_1 Z_1 - Z_2 Z_2) = 0 \quad , \]
\[ 2Z^2 (Z_1 Z_2 + Z_2 Z_1) = 0 \quad , \] (4.7)

where we have set $Z^2 = \sum_{ij} (Z^i \delta_{ij} Z^j)$. Moreover, the Hessian matrix is given by,

\[ \frac{\partial^2 V_{eff}}{\partial \sigma^2} = 4Z^2 (Z_1 Z_1 - Z_2 Z_2)^2 \quad , \]
\[ \frac{\partial^2 V_{eff}}{\partial \vartheta^2} = 4Z^2 (Z_1 Z_2 + Z_2 Z_1)^2 \quad , \]
\[ \frac{\partial^2 V_{eff}}{\partial \vartheta \partial \sigma} = 4Z^2 (Z_1 Z_2 + Z_2 Z_1) (Z_1 Z_1 - Z_2 Z_2) \quad . \] (4.8)

From these relations, we see that the solution of the attractor eqs $(4.6)$ is given by the trivial values $Z_1 = 0$, $Z_2 = 0$ requiring $g = 0$, $e = 0$ in agreement with eqs$(4.4)$(4.5).

### 4.2 Black pairs in $AdS_{2+p} \times S^{6-p}$ geometries

Here we study the general solutions of the attractor equations concerning the system made of black strings and their dual magnetic 3-branes in the $AdS_3 \times S^5$ geometry. Then, we consider explicit solutions for black strings recovered from the pair strings/3-branes by switching off the electric charges $q_a$. Similar analysis may be done for the pair black holes/4-branes; it is omitted.
4.2.1 black pair: strings/3-branes

We begin by recalling that the near horizon geometries of the black strings and the black 3-branes in \(8D\) maximal supergravity are respectively given by \(AdS_3 \times S^5\) and \(AdS_5 \times S^3\). But here we will mainly focus on the \(AdS_3 \times S^5\) geometry; a similar analysis is also valid for \(AdS_5 \times S^3\). Recall that, generally speaking, the metric of \(AdS_{p+2} \times S^{6-p}\) geometry of black p-branes reads as follows,

\[
 ds_8^2 = R^2_{AdS_p} ds_{AdS_n}^2 + R^2_{S^{8-n}} ds_{S^{8-n}}^2
\]

with

\[
 ds_{AdS_n}^2 = d\rho^2 - \sinh^2 \rho \, d\tau + \cosh \rho \, d\varpi_{n-2}^2
\]

and \(\rho \geq 0, \, \tau \in [0, 2\pi]\) and \(d\varpi_{n}^2\) the length element on the unit n-sphere \(S^n\) inside the non compact AdS space.

In the case of the \(AdS_3 \times S^5\) near horizon geometry, the black strings are located in the \(AdS_3\) part of the \(AdS_3 \times S^5\) space and their dual magnetic 3-branes wrap the \(S^5/S^2\) as illustrated below,

\[
 \begin{array}{cccccccc}
 AdS_3 & S^5 & T^3 \\
 0 & 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 & 10 \\
 M2 & \times & \times & \times & \circ & \circ & \circ & \circ & \times & \circ & \circ \\
 M5 & \times & \circ & \circ & \times & \times & \times & \times & \circ & \circ & \circ \\
 \end{array}
\]

Together with the two other permutations in the 3-torus directions. In this view, the field strengths \(F_3^a\) that couple the strings and their magnetic duals \(\tilde{F}_{5a}\) that couple the 3-branes are respectively given by \(F_3^a = p^a \alpha_3\) and \(\tilde{F}_{5a} = q_a \beta_5\). The magnetic charges \(p^a\) of the strings and the electric \(q_a\) of the 3-branes are given by the fluxes,

\[
 p^a \sim \int_{AdS_3} F_3^a = p^a \int_{AdS_3} \alpha_3,
 q_a \sim \int_{S^5} \tilde{F}_{5a} = q_a \int_{S^5} \beta_5,
\]

where \(\alpha_3\) and \(\beta_5\) stand for the volume forms \(\alpha_3^{AdS_3}\) and \(\beta_5^{S^5}\). We also have,

\[
 \int_{AdS_3 \times S^5} F_3^a \wedge \tilde{F}_{5b} \sim p^a q_b \mathcal{V}_{AdS_3 \times S^5},
\]

where \(\mathcal{V}_{AdS_3 \times S^5} = \mathcal{V}_{AdS_3} \times \mathcal{V}_{S^5}\).

The attractor equations describing the system of black strings/3-branes in the \(AdS_3 \times S^5\) geometry follow from the extremization of their effective potential \(\mathcal{V}_1 + \mathcal{V}_3 = \mathcal{V}_1 + \tilde{\mathcal{V}}_1\) which reads in terms of the dressed charges \(Y^a, \tilde{Y}_a\) as follows

\[
 \mathcal{V}_1 + \tilde{\mathcal{V}}_1 = \frac{1}{2} \sum Y^a \delta_{ab} Y^b + \frac{1}{2} \sum \tilde{Y}_a \delta^{ab} \tilde{Y}_b .
\]
The computation of \( \delta (V_1 + \tilde{V}_1) = 0 \) leads to the condition 
\[
(Y^a Y^b - \tilde{Y}^a \tilde{Y}^b) \Omega_{(ab)} = 0
\]
where the \( 3 \times 3 \) matrix field \( \Omega \) is the Maurer Cartan 1-form on \( SL(3, R) \). Now, seen that \( (Y^a Y^b - \tilde{Y}^a \tilde{Y}^b) \) is a symmetric matrix, the above condition reduces to,
\[
(Y^a Y^b - \tilde{Y}^a \tilde{Y}^b) \Omega^{SL3/SO3}_{ab} = 0 ,
\]
where now \( \Omega^{SL3/SO3}_{ab} \) is the Maurer Cartan 1-forms on the \( SL(3, R) \) coset manifold, in agreement with \( SO(3) \times SO(2) \) isometry of the moduli space. The attractor equations reads then as follows:
\[
\Omega^{SL3/SO3}_{ab} \neq 0 , \quad (Y^a Y^b - \tilde{Y}^a \tilde{Y}^b) = 0 .
\]
Since the dressed charges \( Y^a, \tilde{Y}^a \) depend on the matrix field \( L \) and the charges \( p^a, q_a \), the solving of the above equations turns to fixing the fields \( L_{ab} \) in terms of the electric and magnetic charges of the black objects. A particular solution with \( p \neq 0, q \neq 0 \) is given by
\[
(L^2)_{ba} \sim \frac{p_{ba}}{p^2} , \quad (L^{-2})^{ab} \sim \frac{q^{ab}}{q^2} ,
\]
with \( p^2 = \sum p_c p^c \) and \( q^2 = \sum q_c q^c \).
Notice that expanding the real 1-form matrix field \( \Omega \) along the basis of \( SL(3, R) \) as follows,
\[
\Omega = \sum_{A=1}^{8} T^A \Delta_A ,
\]
we can also put (4.15) in the equivalent form
\[
\sum_{A=1}^{8} \Delta_A \sum \left( Y^a T^A_{ab} Y^b - \tilde{Y}^a T^A_{ab} \tilde{Y}^b \right) = 0 ,
\]
In terms of these matrices, the attractor eqs read as follows,
\[
(Y^a T^A Y^b) - (\tilde{Y}^a T^A \tilde{Y}^b) = 0 , \quad \Delta_A \neq 0 .
\]
Notice the three following:
(i) the attractor eqs associated with the \( SO(3) \) generators given by values \( A = 1, 2, 3 \) vanish identically since for these values we have \( T^A_{ab} = -T^A_{ba} \).
(ii) the non-zero contributions comes from the remaining five generators \( T_4, \ldots, T_8 \). These \( 3 \times 3 \) generators are real, symmetric and traceless matrices.
(iii) Altogether with the \( SO(3) \) generators \( T_1, T_2, T_3 \) given by (3.10), the real matrices \( T_4, \ldots, T_8 \) generate the eight dimensional \( SL(3, R) \) symmetry. These \( T_4, \ldots, T_8 \) matrices can be explicitly read by help of (3.1); from which we learn that the two diagonal
generators $T_4$ and $T_5$ are given by
\begin{align}
T_4 &= \begin{pmatrix}
1 & 0 & 0 \\
0 & 0 & 0 \\
0 & 0 & -1
\end{pmatrix}, \\
T_5 &= \begin{pmatrix}
0 & 0 & 0 \\
0 & 1 & 0 \\
0 & 0 & -1
\end{pmatrix}
\end{align}
(4.21)
and the three non diagonal ones are as follows
\begin{align}
T_6 &= \begin{pmatrix}
0 & 1 & 0 \\
1 & 0 & 0 \\
0 & 0 & 0
\end{pmatrix}, \\
T_7 &= \begin{pmatrix}
0 & 0 & 0 \\
0 & 0 & 1 \\
1 & 0 & 0
\end{pmatrix}, \\
T_8 &= \begin{pmatrix}
0 & 0 & 0 \\
0 & 0 & 1 \\
0 & 1 & 0
\end{pmatrix}
\end{align}
(4.22)
Putting the decomposition (4.18) back into eq(4.15), we can bring it to the form
$$
\sum_{A=4}^{8} \mathcal{A}^A = 4 \mathcal{Y}^A - 3 \Delta_A = 0,
$$
with
$$
\mathcal{Y}^A = \sum_{a,b=1}^{3} (Y^a Y^b - \tilde{Y}^a \tilde{Y}^b) T^A_{ab}.
$$
(4.23)
By substituting $T^A_{ab}$ by their values (4.21), we can also put the components $\mathcal{Y}^A$ like,
\begin{align}
\mathcal{Y}^1 &= (Y^1 Y^1 - Y^3 Y^3) - (\tilde{Y}^1 \tilde{Y}^1 - \tilde{Y}^3 \tilde{Y}^3) \\
\mathcal{Y}^2 &= (Y^2 Y^2 - Y^3 Y^3) - (\tilde{Y}^2 \tilde{Y}^2 - \tilde{Y}^3 \tilde{Y}^3) \\
\mathcal{Y}^3 &= 2 (Y^1 Y^2 - \tilde{Y}^1 \tilde{Y}^2) \\
\mathcal{Y}^4 &= 2 (Y^1 Y^3 - \tilde{Y}^1 \tilde{Y}^3) \\
\mathcal{Y}^5 &= 2 (Y^2 Y^3 - \tilde{Y}^2 \tilde{Y}^3)
\end{align}
(4.24)
where $Y^a$ and $\tilde{Y}^a$ are as in eqs(3.44). Combining eqs(??-4.23), we learn that, depending on the values of dressed charges $Y$ and $\tilde{Y}$, there are several kinds of solutions for the attractor equations
$$
\mathcal{Y}^1 = \mathcal{Y}^2 = \mathcal{Y}^3 = \mathcal{Y}^4 = \mathcal{Y}^5 = 0,
$$
(4.25)
whose solution reads, up to $SO(3)$ transformation, as follows,
\begin{align}
Y^a &= \begin{pmatrix}
y \\
0 \\
0
\end{pmatrix}, \\
\tilde{Y}^a &= \pm \begin{pmatrix}
y \\
0 \\
0
\end{pmatrix}
\end{align}
(4.26)
with $y$ is a non zero real number. These relations describe two solutions; one with a sign (+) corresponding to a black string/3-brane pair and the second with sign (−) associated with a black string/anti 3- brane pair. These solutions fix three real scalars amongst the seven ones parameterizing $\frac{SL(3,R) \times SL(2,R)}{SO(3) \times SO(2)}$, reducing thus the moduli space down to $\frac{SL(2,R) \times SL(2,R)}{SO(2) \times SO(2)}$. 24
4.2.2 black strings

The attractor equations for the black strings may be obtained by starting from eqs(4.24) describing the attractor eqs of the strings/ (anti) 3-brane pairs; then set to zero the charges of the (anti) 3-branes

\[ q_a = \tilde{Y}_a = 0. \]  

(4.27)

This leads to the relations

\[ \begin{align*}
\theta_1 (Y^1Y^1 - Y^3Y^3) &= 0 \\
\theta_2 (Y^2Y^2 - Y^3Y^3) &= 0 \\
\theta_3 Y^1Y^2 &= 0 \\
\theta_4 Y^1Y^3 &= 0 \\
\theta_5 Y^2Y^3 &= 0
\end{align*} \]  

(4.28)

where \( Y^a \) are the dressed charges associated with the black strings and where we have set \( \theta_A = \Delta_A - 4 \). In the case where all the \( \Delta_A \)'s are non zero, it is clear that all the dressed charges should vanish

\[ Y^a = 0 \ , \ \theta_A \neq 0 \ . \]  

(4.29)

5 Intersecting attractors

From the results of [33], we learn that one should distinguish two main classes of black p-brane solutions in higher dimensional supergravity. In the 8D case we are interested in here, these are:

(1) the standard black p- brane solutions based on \( AdS_{2+p} \times S^{6-p}, p = 0, 1, 2, 3, 4 \), whose features have been explicitly analyzed in previous sections.

(2) the so called intersecting attractors with the typical near horizon geometries

\[ AdS_{2+p} \times S^m \times T^{6-p-m} \ , \ p = 0, 1, 2, 3, 4 \ , \ p + m \leq 5. \]  

(5.1)

The novelty with these geometries is that they allow the two following: (i) a variety of irreducible sub-manifolds that support various kinds of branes and so a rich spectrum of electric and magnetic charges; (ii) non trivial intersections between \( p_\tau/p_\sigma \) cycles of (5.1) leading to intersecting (BPS and non BPS) attractors. To illustrate the first point, we consider the example of the two compact manifolds \( S^{m+n} \) and \( M^{m+n} = S^m \times T^n \) with same dimension. While the sphere \( S^{m+n} \) supports only charges of \( (m + n - 2) \)-brane charges

\[ F_{n+m} = g \omega_{n+m} \ , \ g = \int_{S^{m+n}} F_{n+m} \ , \]  

(5.2)
and no \((m-1)\)-brane nor others, the manifold \(S^m \times T^n\) allows however many possibilities. It has several irreducible \(p\)-cycles that support, in addition to \((m + n - 2)\)-branes, other kinds; in particular \(n\) types of \((m-1)\)-branes with charges given by,

\[
g^a = \int_{C^{(a)}_{m+1}} F_{m+1} \quad , \quad F_{m+1} = \sum_a g^a \omega_{m+1|a} \quad , \quad \int_{C^{(a)}_{m+1}} \omega_{m+1|b} = \delta^a_b \quad , \quad a = 1, \ldots, n
\]

where the \(C^{(a)}_{m+1}\) cycles stand for \(\bigcup_{a=1}^n \left(S^1_a \times S^m\right)\) with \(\prod_{a=1}^n S^1_a = T^n\). The branes may be imagined as filling the fiber \(F^{(a)}_{m-1}\) of these cycles \(C^{(a)}_{m+1}\) thought of in terms of the fibration \(C^{(a)}_{m+1} \sim F^{(a)}_{m-1} \times S^2\) with field strength \(F_{m+1} = \beta_{S^2} \wedge \left(\sum_a g^a \beta^{(a)}_{F_{m-1}}\right)\).

Using the anzats of [33], we focus below on the study of various examples of these typical horizon geometries and work out new and explicit solutions regarding intersecting attractors in the case of 8D maximal supergravity. As the solutions are very technical, we will concentrate on drawing the crucial lines and give the results.

### 5.1 Geometries with AdS\(_4\) factor

We consider two examples: (a) \(AdS_4 \times S^3 \times S^1\) and (b) \(AdS_4 \times S^2 \times T^2\); the other possibility namely \(AdS_4 \times S^4\) has been considered in subsection 4.1. On the \(AdS_4 \times S^3 \times S^1\) near horizon geometry, the non vanishing field strength charges of the 8D maximal supergravity are: (i) the magnetic \(p^a\) of the strings, (ii) the \(q_a\) of the 3-branes and (iii) the \((e, g)\) charge of the dyonic membrane. In the case of \(AdS_4 \times S^2 \times T^2\), there are moreover non trivial black hole charges and 4-brane ones; more precisely:

**Field strengths on \(AdS_4 \times S^3 \times S^1\)**

Using the volume forms \(\alpha_{AdS^4}\) and \(\beta_{S^n}\) with \(n = 1, 3\), we have the following field strengths relations:

\[
\begin{array}{ccc}
\text{p-branes} & \text{\((4-p)\)-branes} \\
p = 0 & F_2^{(ai)} = 0 & F_6^{(ai)} = 0 \\
p = 1 & F_3 = p^a \beta_{S^3} & F_5^{(a)} = q_a \alpha_{AdS^4} \wedge \beta_{S^1} \\
p = 2 & F_4 = e^a \alpha_{AdS^4} + g(\beta_{S^3} \wedge \beta_{S^1}) & \\
\end{array}
\]

(5.4)

The vanishing of the charges of the fields strengths \(F_2^{(ai)}\) and \(F_6^{(ai)}\) are due to the fact that there is no compact 2-cycles nor 6-cycles on the \(AdS_4 \times S^3 \times S^1\) geometry that support black holes and black 4-brane charges.

**Field strengths on \(AdS_4 \times S^2 \times T^2\)**

This horizon geometry has, in addition to non compact \(AdS_4\) thought of as a regularized 4-cycle \(\int_{AdS^4}^\Lambda \alpha_{AdS^4} = V_{AdS^4} (\Lambda)\) with regularization parameter \(\Lambda\), compact n-cycles \(C^{(a)}_n \subseteq\)
$S^2 \times T^2$, with $n = 1, 2, 3, 4$ and regularized m-cycles $R_m = AdS_4 \times C_m^{(x)}$ with $m = n + 4$ supporting brane charges. Using the anzats of [33], we have the corresponding field strengths:

| p-branes         | (4 - p)-branes          |
|------------------|-------------------------|
| $p = 0$          | $F_2 = \frac{P_{ai}}{2} \beta_{S^2}$ | $\tilde{F}_{6[a} = Q_{ai} \alpha_{AdS_4} \wedge \beta_{T^2}$ |
| $p = 1$          | $F_3 = \frac{2}{3} \sum_{i=1}^{p} p_{i} \beta_{S^2} \wedge \beta_{S^1} | \tilde{F}_{5[a} = \sum_{r} q_{ar} \alpha_{AdS_4} \wedge \beta_{S^1} |
| $p = 2$          | $F_4 = e \alpha_{AdS_4} + g (\beta_{S^2} \wedge \beta_{T^2})$                |

where the black string and the black 3-brane fill respectively the $S^1$ and $\tilde{S}^1$ cycles in the 2-torus $T^2 = S^1 \times \tilde{S}^1$. With these electric and magnetic bare charges, we can deduce the dressed ones and derive the effective potentials associated with these configurations.

### 5.1.1 $AdS_4 \times S^3 \times S^1$

In the case of $AdS_4 \times S^3 \times S^1$ geometry, the corresponding effective potential $V_{eff}$ can be read from (3.35,3.40,3.41) namely

$$\frac{1}{2} Y^a \delta_{ab} Y^b + \frac{1}{2} \tilde{Y}_a \delta^{ab} \tilde{Y}_b + \frac{1}{2} Z^i \delta_{ij} Z^j;$$

its extremization $\delta V_{eff} = 0$ gives

$$\sum \Delta_A \left( Y T^A Y - \tilde{Y} T^A \tilde{Y} \right) + \sum \lambda_\alpha \left( Z \tau^\alpha Z \right) = 0 ,$$

where $\Delta_A$ and $\lambda_\alpha$ are 1-forms on the moduli space as in (3.30) and where $T^A$, $\tau^\alpha$ are respectively the generators of SL(3, $R$) and SL(2, $R$). The attractor eqs following from the above extremum namely

$$Y T^A Y - \tilde{Y} T^A \tilde{Y} = 0 ,$$

$$Z \tau^\alpha Z = 0 ,$$

are solved as $Y^a = +\tilde{Y}^a$ (or $Y^a = -\tilde{Y}^a$ ) and $Z_{elec} = Z_{mag} = 0$. This configuration describes three dual pairs of strings/3-branes (or strings/anti- 3-branes) intersecting along the time direction and has a nice interpretation in M-theory compactified on $T^3$. A typical configuration involving one string and one 3-brane is given by the following wrapped M2/M5 system

| $AdS_4$ | $S^3$ | $S^1$ | $T^3$ |
|---------|------|------|-------|
| 0       | 1    | 2    | 3     | 4    | 5    | 6    | 7    | 8    | 9    | 10   |
| M2      | $\times$ | $\times$ | $\circ$ | $\circ$ | $\circ$ | $\circ$ | $\circ$ | $\circ$ | $\times$ | $\circ$ |
| M5      | $\times$ | $\circ$ | $\circ$ | $\times$ | $\times$ | $\times$ | $\circ$ | $\circ$ | $\times$ | $\times$ |

The two other possible configurations correspond to permuting the role of the coordinates of the 3-torus.
5.1.2 \( \text{AdS}_4 \times S^2 \times T^2 \) case

On the geometry \( \text{AdS}_4 \times S^2 \times T^2 \) involving the volume forms \( \alpha_{\text{AdS}_4}, \beta_{S^2}, \beta_{S^1} \), the nonvanishing field strength charges are given by eq (5.5). Using same approach as before, we can determine the effective potential \( V_{\text{eff}} \) associated with this configuration; its extremization \( \delta V_{\text{eff}} = 0 \) leads to

\[
\sum_A \Delta_A \sum_{i=1}^{2} \left( X^{ai} T_{ab}^A X^{bi} - \bar{X}^{ai} T_{ab}^A \bar{X}^{bi} + Y^{ai} T_{ab}^A Y^{bi} - \bar{Y}^{ai} T_{ab}^A \bar{Y}^{bi} \right) + \sum_{\alpha} \lambda_{\alpha} \left( \delta_{ab} \left[ X^{ai} \tau_{ij}^\alpha X^{bj} - \bar{X}^{ai} \tau_{ij}^\alpha \bar{X}^{bj} \right] + Z^i \tau_{ij}^\alpha Z^j \right) = 0 \tag{5.9}
\]

where \( \Delta_A, \lambda_{\alpha}, T^A, \tau^\alpha \) are same as before. The attractor eqs,

\[
\sum_{i=1}^{2} \left( X^{ai} T_{ab}^A X^{bi} - \bar{X}^{ai} T_{ab}^A \bar{X}^{bi} + Y^{ai} T_{ab}^A Y^{bi} - \bar{Y}^{ai} T_{ab}^A \bar{Y}^{bi} \right) = 0 \tag{5.10}
\]

have two classes of solutions depending on whether \( Z^i = 0 \) or \( Z^i \neq 0 \). We have:

\text{Case} \ (Z_{\text{elec}}, Z_{\text{mag}}) = (0, 0)

In this case, the corresponding attractor eqs may be solved in various ways; in particular by compensating the terms of the sum like \( X^{ai} = \pm \bar{X}^{ai}, \ Y^{ai} = \pm \bar{Y}^{ai} \). These configurations describe intersecting attractors involving black holes, black strings and their duals.

\text{Case} \ (Z_{\text{elec}}, Z_{\text{mag}}) \neq (0, 0)

One of the solutions of the attractor eqs (5.10) consists to compensate the terms of the sum as follows:

First solve the first relation of (5.10) like \( X^{ai} = \pm \bar{Y}^{ai}, \ Y^{ai} = \pm \bar{X}^{ai} \),

Then, solve the second relation by taking \( X^{ai} = \pm \nu \bar{X}^{ai} \) with \( \nu \) some real number; this leads to

\[
\left[ (\nu^2 - 1) \tilde{X}^{ai} \tau_{ij}^\alpha \tilde{X}^{bj} \delta_{ab} + Z^i \tau_{ij}^\alpha Z^j \right] = 0 \tag{5.11}
\]

from which we learn

\[
Z^i = \pm \sqrt{\frac{1 - \nu^2}{3}} \sum_{a=1}^{3} \tilde{X}^{ai} \tag{5.12}
\]

where reality property of the central charges \( Z^i \) imposes to the free parameter \( \nu \) to belong to the set \([0, 1]\).

5.2 Geometries with AdS_3

We distinguish two cases: \( \text{AdS}_3 \times S^3 \times T^2 \) and \( \text{AdS}_3 \times S^2 \times T^3 \); here we focus on the first case as it allows more possibilities. The fluxes emanating from the black branes associated
with $AdS_3 \times S^3 \times T^2$ are given by,

| $p$  | $p$-branes | $(4-p)$-branes |
|------|-------------|-----------------|
| 0    | $F_{2i} = 0$ | $\tilde{F}_{6i} = 0$ |
| 1    | $F_3 = p^a \beta_{S^3}$ | $\tilde{F}_{5i} = q^{a} \alpha_{AdS_3} \wedge \beta_{T^2}$ |
| 2    | $F_4 = \sum_{r=1,2} e_r \alpha_{AdS_3} \wedge \alpha_{S^3} + \sum_{r=1,2} g_r \beta_{S^3} \wedge \beta_{S^3}$ |

(5.13)

from which we read the total effective potential,

$$V_{eff} = \frac{1}{2} \sum_{r=1}^{2} (Z_{el,r}^2 + Z_{mag,r}^2) + \frac{1}{2} \left( Y^{ai} \delta_{ab} Y^{bi} + \tilde{Y}^{aj} \delta_{ab} \tilde{Y}^{bj} \right)$$

(5.14)

and whose extremization of $V_{eff}$ gives,

$$\sum_{\alpha} \Delta_{\alpha} \left( Y T^{A} Y - \tilde{Y} T^{A} \tilde{Y} \right) + \sum_{\alpha} \lambda_{\alpha} Z_{\tau^{\alpha} Z} = 0.$$

The corresponding attractor equations are given by

$$\sum_{a,b} \left( Y^{a} T_{ab} Y^{b} - \tilde{Y}^{a} T_{ab} \tilde{Y}^{b} \right) = 0 , \quad A = 1, \ldots, 8$$

$$\sum_{i,j,r} \left( Z_{i,r}^{i} \tau_{ij}^{0} Z_{r}^{j} \right) = 0 , \quad \alpha = 1, 2, 3$$

(5.15)

The first relations are solved as usual that is $Y^{ai} = \pm \tilde{Y}^{ai}$ and the second ones like $Z_{i,r}^{i} = z_{r}^{i} \delta_{r}^{i}$; thanks to the identity $z^{2} T_{r} (\tau^{\alpha}) = 0$. This configuration describes an intersecting attractor made of black string/black 3-branes and black membrane,

| $AdS_3$ | $S^3$ | $T^2$ | $T^3$ |
|---------|-------|-------|-------|
| 0       | 1     | 2     | 3     | 4     | 5     | 6     | 7     | 8     | 9     | 10    |
| M2      | $\times$ | $\times$ | $\times$ | $\circ$ | $\circ$ | $\circ$ | $\circ$ | $\circ$ | $\circ$ | $\circ$ |
| M2      | $\times$ | $\times$ | $\circ$ | $\circ$ | $\circ$ | $\circ$ | $\circ$ | $\circ$ | $\circ$ | $\circ$ |
| M5      | $\times$ | $\times$ | $\circ$ | $\circ$ | $\circ$ | $\circ$ | $\times$ | $\circ$ | $\times$ |

(5.16)

In the case where the 1-forms $\Delta_{i} \neq 0$ and $\lambda_{\alpha} \neq 0$ and the other vanishing, the attractor eqs reduce to

$$\left( Y^{a} H_{ab} Y^{b} - \tilde{Y}^{a} H_{ab} \tilde{Y}^{b} \right) = 0 , \quad \sum_{i,j,r} \left( Z_{i,r}^{i} \tau_{ij}^{0} Z_{r}^{j} \right) = 0 ,$$

(5.17)

lead to $Y^{b} = \pm \tilde{Y}^{a}$ and $Z_{el,r} = \pm Z_{mag,r}$.
5.3 Geometries with AdS$_2$

In this subsection, we study explicitly the attractor mechanism for three examples of near horizon geometries with non compact AdS$_2$ factors; these are: (a) AdS$_2 \times S^3 \times T^3$, (b) AdS$_2 \times S^2 \times T^4$ and (c) AdS$_2 \times S^4 \times T^2$

5.3.1 AdS$_2 \times S^3 \times T^3$

On this geometry, the non vanishing field strength charges are as follows

\[
\begin{align*}
\text{p-branes} & \quad (4 - p)\text{- branes} \\
p = 0 & \quad F_2^{ai} = Q^{ai} \alpha_{AdS_2} & \quad \tilde{F}_6 |_{ai} = P_{ai} \beta_{S^3} \wedge \beta_{T^3} \\
p = 1 & \quad F_3^a = \sum_{r=1}^2 p^r \beta_{S^3} & \quad \tilde{F}_5 |_{a} = \sum_{r} q_a \alpha_{AdS_2} \wedge \beta_{T^3} \\
p = 2 & \quad F_4 = \sum \epsilon_1 \epsilon^{ijk} \alpha_{AdS_2} \wedge \beta_{S^1} \wedge \beta_{S^3} + \sum g^I \beta_{S^3} \wedge \beta_{S^1}
\end{align*}
\]

where $I, J, K = 1, 2, 3$ are associated with the three 1-cycle generators of the 3-torus. From these field strengths, we learn that this geometry supports:

(i) $3 \times 2$ electrically charge black holes with charges $Q^{ai}$ and their magnetic duals with magnetic charges $P_{ai}$,

(ii) three magnetically charged strings with charge $p^a$, and three electrically charged 3-branes with charge $q_a$,

(iii) three dyonic membranes with charges $(g^I, \epsilon_I)$.

Following the same approach we have been using, the effective potential $V_{eff}$ of these black brane configurations reads as follows,

\[
V_{eff} = \frac{1}{2} \sum \left( X^{ai} \delta_{ab} \delta_{ij} X^{bj} + \tilde{X}^{ai} \delta_{ab} \delta_{ij} \tilde{X}^{bj} \right) + \frac{1}{2} \sum \left( Y^{a} \delta_{ab} Y^{b} + \tilde{Y}^{a} \delta_{ab} \tilde{Y}^{b} \right) + \frac{1}{2} \sum Z^i \delta_{ij} Z^j
\]

where summation over the various indices is understood. The extremization of this effective potential leads to

\[
\delta V_{eff} = \sum \Delta_A \Upsilon^A + \sum \lambda_{\alpha} \Gamma^\alpha = 0,
\]

with

\[
\begin{align*}
\Upsilon^A &= \left( Y^a \mathcal{T}_{ab}^A Y^b - \tilde{Y}^a \mathcal{T}_{ab}^A \tilde{Y}^b \right) + \sum_{i=1}^2 \left( X^{ai} \mathcal{T}_{ab}^A X^{bi} - \tilde{X}^{ai} \mathcal{T}_{ab}^A \tilde{X}^{bi} \right) \\
\Gamma^\alpha &= \left[ \sum_{a=1}^3 \left( X^{ai} \mathcal{I}_{ij}^{\alpha} X^{aj} - \tilde{X}^{ai} \mathcal{I}_{ij}^{\alpha} \tilde{X}^{aj} \right) + \sum_I \mathcal{I}_I \mathcal{I}_{ij}^\alpha Z^i \right]
\end{align*}
\]
The solutions of these attractor eqs \( \Upsilon^A = 0, F^a = 0 \) may be realized in various ways; one of them is given by the following:

\[
Y^a = \pm \tilde{Y}^a, \quad X^{ai} = \pm \tilde{X}^{ai}, \quad Z^i_I = 0 \tag{5.22}
\]

The solutions with plus signs describe intersecting attractor involving three strings, three 3-branes, six black holes and six 4-branes; but no membrane; those with minus signs are associated with the corresponding anti-branes.

### 5.3.2 \( \text{AdS}_2 \times S^2 \times T^4 \)

On this geometry, the general form of the field strengths reads as follows,

| p-branes                          | \((4-p)\)-branes |
|-----------------------------------|------------------|
| \( F^{ai}_2 = P^{ia} \alpha_{S^2} \) | \( F^{6ai}_6 = Q^{ia} \alpha_{AdS_2} \wedge \beta_{T^4} \) |
| \( F^3_a = \sum p^{ak} (\beta_{S^2} \wedge \beta_{S^1}^k) \) | \( F^{5ai}_5 = \sum q_{ai} \varepsilon_{ijkl} (\alpha_{AdS_2} \wedge \beta_{S^1}^j \wedge \beta_{S^1}^k \wedge \beta_{S^1}^l) \) |
| \( F_4 = \sum e_{[kl]} \varepsilon_{klrs} (\alpha_{AdS_2} \wedge \beta_{S^1}^r \wedge \beta_{S^1}^s) + g^{[rs]} \beta_{S^2} \wedge \beta_{S^1}^r \wedge \beta_{S^1}^s \) | \( 5.23 \)

The total effective potential reads, in terms of the dressed central charges of the black holes/4-branes \( \left( X^{ia}, \tilde{X}^{ia} \right) \), the four triplets of black strings/3-branes \( \left( Y^a, \tilde{Y}^a \right) \) \( 1 \leq k \leq 4 \) and the six-uplet dressed electric charges \( Z^{elc}_{[kl]} = Z^1_{[kl]} \) and magnetic \( Z^{mag}_{[kl]} = Z^2_{[kl]} \) of the dyonic membranes, as follows:

\[
V_{\text{eff}} = \frac{1}{2} \sum_{a,b=1}^{3} \sum_{i,j=1}^{2} \left( X^{ai}_l \delta_{ab} \delta_{ij} X^{bj}_l + \tilde{X}^{ai}_l \delta_{ab} \delta_{ij} \tilde{X}^{bj}_l \right) + \frac{1}{2} \sum_{k=1}^{4} \sum_{a,b=1}^{3} \left( Y^a_k \delta_{ab} Y^b_k + \tilde{Y}^a_j \delta_{ab} \tilde{Y}^b_j \right) + \frac{1}{2} \sum_{k,l=1}^{4} \sum_{i,j=1}^{2} \left( Z^{ij}_{[kl]} \delta_{ij} Z_{[kl]}^j \right) \tag{5.24}
\]

Its gives the attractor eqs

\[
Tr \left( X T^A X \right) - Tr \left( \tilde{X} T^A \tilde{X} \right) = 0, \quad \sum_{k=1}^{4} \left[ Tr \left( Y_k T^A Y_k \right) - Tr \left( \tilde{Y}_k T^A \tilde{Y}_k \right) \right] = 0, \quad \sum_{k,l=1}^{4} Tr \left( Z_{[kl]} T^a Z_{[kl]} \right) = 0. \tag{5.25}
\]

The two first relations are solved as usual; i.e \( X^{ai} = \pm \tilde{X}^{ai}, \quad Y^a_k = \pm \tilde{Y}^a_k \) while the third has various solution based on choices that lead to \( Tr \left( \tau^a \right) \). These solutions corresponds to diverse configurations involving intersecting of black hole, black 4-brane, black string, black 3-brane and black 2-brane.
5.3.3 \( \text{AdS}_2 \times S^4 \times T^2 \)

Using the various n-cycles of \( \text{AdS}_2 \times S^4 \times T^2 \) and the corresponding n-forms that could live on, the general expressions of the field strengths on this geometry reads as follows,

| p-branes \( F_{\alpha}^{ai} \) | (4 - p)- branes \( \tilde{F}_{0|ai} \) |
|-----------------|-----------------|
| \( = Q_{\alpha}^{ia} \alpha_{\text{AdS}_2} \) | \( = P_{\alpha}^{ia} \beta_{S^4} \alpha_{T^2} \) |
| \( F_{3}^{a} = \sum_{k=1}^{q_{k}} (\alpha_{\text{AdS}_2} \wedge \alpha_{S^4}^{k}) \) | \( \tilde{F}_{5|a} = \sum_{k=1}^{p_{k}} (\beta_{S^4} \wedge \alpha_{S^4}^{k}) \) |
| \( F_{4}^{i} = e (\alpha_{\text{AdS}_2} \wedge \alpha_{T^2}) + g_{S^4} \) |

where now the strings are charged electrically and the 3-branes magnetically. The total effective potential \( V_{\text{eff}} \) associated with this system is given as usual by the sum of the contribution of each extremal black-brane. The attractor equations following from the extremization of \( V_{\text{eff}} \) are then given by:

\[
\begin{align*}
Tr \tilde{X}T^A \tilde{X} - Tr (XT^AX) + \sum_{k=1}^{2} \left[ Tr \left( \tilde{Y}_k T^A \tilde{Y}_k \right) - Tr \left( Y_k T^A Y_k \right) \right] &= 0 , \\
Tr (Z\tau^aZ) + Tr (XT^AX) - Tr \left( \tilde{X} \tau^a \tilde{X} \right) &= 0 ,
\end{align*}
\]

whose solutions are given by \( \tilde{Y}_k = \pm Y_k^a \), \( X^{ia} = \pm \tilde{X}^{ia} \), \( Z^i = 0 \).

6 Conclusion

Motivated by the new results obtained in [33], we have focused in this paper on 8D maximal supergravity embedded in 11D M-theory on \( T^3 \); and studied the attractor mechanism of black p-branes and their intersections. In particular, we have considered different configurations of black brane systems and derived various classes of solutions of their attractor eqs depending on the values of the dressed charges.

To do so, we first studied the general structure of 8D non chiral maximal supersymmetric algebra with p-branes as well its link with M- theory compactified on \( T^3 \). Then we have developed an unconstrained formalism to approach the geometry of the moduli space \([\text{SL}(3,R) \times \text{SL}(2,R)] / [\text{SO}(3) \times \text{SO}(2)]\) and the symmetries of effective potential \( V_{\text{eff}} \) of the black p-branes of the 8D maximal supergravity. In this way the scalar moduli of the supergravity are captured by two matrices \( L_{ab} \) and \( K_{ij} \) valued in the \( \text{SL}(3,R) \times \text{SL}(2,R) \) Lie group manifold; the extra degrees of freedom are suppressed by requiring gauge invariance under the \( \text{SO}(3) \times \text{SO}(2) \) isometry. The attractor eqs of the black object of 8D supergravity have the remarkable factorization,

\[
\begin{align*}
\sum_{A=1}^{8} \Delta_A Y^A &= 0 , \\
\sum_{\alpha=1}^{3} \lambda_\alpha F^\alpha &= 0 ,
\end{align*}
\]
with the two terms respectively associated with \([SL(3,R)/SO(3)]\) and \([SL(2,R)/SO(2)]\); in agreement with the factorized structure of the moduli space. Using the identities \(\Upsilon^A = \Upsilon^{(ab)} T^A_{ab} \), \(\Delta_A = \Delta_{(ab)} T^A_{ab} \) and quite similar relations for the \(SL(2,R)\) factor, these attractor equation may be also put in the equivalent forms \(\Delta_{ab} \Upsilon^{ab} = 0\), \(\lambda_{ij} \lambda_{ij} = 0\). In \([6.1]\), \(\Delta_A, \lambda_{ij}\) (or equivalently \(\Delta_{ab}, \lambda_{ij}\)) are 1-forms given by eqs \([3.30]\) and \(\Upsilon^A, F^\alpha\) (or equivalently \(\Upsilon_{ab}, F^{ij}\)) have the typical expressions,

\[
\Upsilon^A = \sum I \text{Tr}(Y_I T^A Y_I - \tilde{Y}_I T^A \tilde{Y}_I), \quad F^\alpha = \sum r \text{Tr}(Z_r \tau^\alpha Z_r) . \tag{6.2}
\]

where \(Y_I, \tilde{Y}_I, Z_r\) stand for dressed charges and \(T^A, \tau^\alpha\) for the generators of \(SL(3,R) \otimes SL(2,R)\). Similar expression can be written down for \(\Upsilon^{ab}, F^{ij}\) leading to \(\Upsilon^{ab} = Y^a_I Y^b_I - \tilde{Y}^a_I \tilde{Y}^b_I\) and \(F^{ij} = Z^i_r Z^j_r\). One of the outcome of these expression is that \(\Upsilon^{ab} = \Upsilon^{ba}\), \(\lambda_{ij} = \lambda_{ji}\); that is symmetric tensors showing that there is no contribution to the attractor eqs coming from those generators with \(T^A_{ab} = -T^A_{ba}\) and \(\tau^{\alpha}_{ij} = -\tau^{\alpha}_{ji}\). This property reflects just the decoupling of the contribution associated with the \(SO(3)\) factor inside \(SL(3,R)\). A similar conclusion is also valid for the \(SL(2,R)\) component and the \(SO(2)\) subgroup.

We end this discussion by noting that the solutions worked out in this study are real solutions since the moduli space \([SL(3,R) \times SL(2,R)] / [SO(3) \times SO(2)]\) is a real manifold. Complex solutions, such as eq \([5.12]\) with \(\nu \notin [0,1]\), can be found if instead of \([3.5]\), we use the complex manifold \([SU(1,2)/SU(2)] \times [SU(1,1)/U(1)]\).

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A Dirac matrices in 8D

Here we give some useful properties on the algebra of \(\Gamma\)- matrices in 8D space time dimensions. In a quite similar way as in 4D, there are eight \(\Gamma\)- matrices in 8D namely \(\Gamma^\mu\) with \(\mu = 0, \ldots, 7\); they generate the Clifford algebra \([38]\),

\[
\Gamma^\mu \Gamma^\nu + \Gamma^\nu \Gamma^\mu = 2\eta^{\mu\nu} \tag{A.1}
\]

with \(\eta^{\mu\nu} = \text{diag}(-,+,\ldots,+)\) standing for the metric of \(R^{1,7}\) with rotation symmetry \(SO(1,7)\). From these relations we learn amongst others that \((\Gamma^0)^2 = -I_{id}\) and \((\Gamma^i)^2 = +I_{id}\) for \(i = 1, \ldots, 7\). The simplest realization of the \(\Gamma^\mu\)'s is given by \(n \times n\) matrices with \(n = 2^4\) and so act on 16 components objects: Dirac spinors. With these matrices, one can build others carrying several space time vectors by taking the completely antisymmetric products as follows:

\[
\Gamma^{\mu_1 \cdots \mu_p} = \frac{1}{p!} (\Gamma^{\mu_1} \ldots \Gamma^{\mu_p} \pm \text{permutations}) , \tag{A.2}
\]
where (+) and (−) stand respectively for even and odd permutations. For the leading $p = 2$ case, we have just the commutators of $\Gamma$-matrices which give a realization of the $SO(1,7)$ rotation generators $M^{\mu\nu}$ in the 16 dimensional spinor representation,

$$M^{\mu\nu} = \frac{i}{2} \Gamma^{\mu\nu}. \quad (A.3)$$

Moreover and like in 4D, we distinguish three kinds of spinors in 8D; the first one is the $SO(1,7)$ Dirac spinor $\Psi^{\text{Dirac}}$ having 16 complex components. But this spinor is reducible into 8 + 8 components describing each a complex Weyl spinor defining the two chiralities of $\Psi^{\text{Dirac}}$ namely the left $\Psi_L \equiv (\psi)_a$ and the right $\Psi_R \equiv (\chi_a)$ related to the Dirac $\Psi$ as follows,

$$\Psi_L = \frac{1}{2} (1 - \Gamma_8) \Psi, \quad \Gamma_8 \Psi_L = -\Psi_L,$$

$$\Psi_R = \frac{1}{2} (1 + \Gamma_8) \Psi, \quad \Gamma_8 \Psi_R = +\Psi_R. \quad (A.4)$$

In these relations, the matrix $\Gamma_8$ is the chirality operator given by

$$\Gamma_8 = e^{-i\frac{3\pi}{2} \Gamma^0...\Gamma^7}, \quad (A.5)$$

satisfying, amongst others, the following properties

$$(\Gamma_8)^2 = I, \quad \{\Gamma_8, \Gamma^\mu\} = 0, \quad [\Gamma_8, \Gamma^{\mu\nu}] = 0. \quad (A.6)$$

The third kind of spinors in 8D is of Majorana type; that is a Dirac spinor constrained by the typical reality condition

$$\Psi^* = B \Psi, \quad (A.7)$$

where $B$ is 16 × 16 matrix. This condition which also reads as $\Psi = B^* B \Psi$ should be as well consistent with Lorentz transformation $\delta \Psi = i \omega_{\mu\nu} M^{\mu\nu} \Psi$. The solution of the constraint relations leads to

$$B^* B = I, \quad M^{*\mu\nu} = -BM^{\mu\nu}B^{-1}, \quad (A.8)$$

where $M^{*\mu\nu}$ is the Lorentz matrix generating rotation $\Psi^*$; that is $\delta \Psi^* = -i\omega_{\mu\nu} (M^{*\mu\nu}) \Psi^*$.

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