Research on rapid planning of intelligent aircraft trajectory under multiple constraints

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Abstract. Intelligent aircraft trajectory planning is an important support to ensure the successful completion of the mission of the aircraft, and it is also a key component of the intelligent aircraft mission planning system. Due to system performance constraints and environmental constraints, the positioning system of the intelligent aircraft cannot be accurately positioned during flight, which will cause flight errors. Excessive flight errors will lead to the failure of the mission. Therefore, error correction plays an increasingly important role in trajectory planning. This paper studies the rapid trajectory planning of intelligent aircraft under the limitation of system positioning accuracy. Based on the optimization theory, with the error generation rules and correction rules of the intelligent aircraft in flight as constraints, establishes the optimization model, and uses the fast non-dominated multi-objective optimization algorithm (NSGA-II) with elite retention strategy. This paper corrects the vertical and horizontal errors during the flight, and finally designs a reasonable intelligent aircraft trajectory that meets the minimum trajectory length and the minimum number of corrections during the flight.

1. Introduction
Intelligent aircraft trajectory planning is to determine a reasonable and optimal route according to the mission objectives. The trajectory meets the aircraft performance constraints, environmental constraints and mission requirements, and optimizes the intelligent aircraft mission effectiveness. The traditional idea is to combine multiple indexes into a single index by weighting. However, this method has inherent defects such as inconsistent dimension between indexes, excessive dependence on weight, strong subjectivity, and uncontrollable excellent progress [1]. The three-dimensional path planning problem is a typical NP-hard problem. Multi-objective optimization has been widely concerned by scholars in recent years because it can achieve joint optimization of multiple indicators [2, 3]. Among the multi-objective optimization algorithms, the genetic algorithm (NSGA-II) with elitist non-dominated solution ranking strategy has a milestone significance, and has become the benchmark for performance comparison of other multi-objective optimization algorithms [4]. Deb K [5] improved the non-dominated sorting genetic algorithm (NSGA), proposed a lower complexity, more able to maintain population diversity, with Pareto dominant NSGA-II algorithm, has been more and more widely used to solve goal optimization problem [6, 7]. Xu Xiaojun [8] studied the multi-satellite navigation system star selection problem as a constrained multi-objective optimization problem. Guo Lake [9] adopted the Pareto optimal solution idea, a fast non-dominated sorting algorithm, and the track length, concealment, and safety as specific evaluation indicators, and used NSGA-II algorithm to optimize the UAV multi-target mission.
This paper studies the rapid flight trajectory planning problem of intelligent aircraft under the limitation of system positioning accuracy. The vertical and horizontal errors generated by the aircraft during flight, the type of correction point, the correction rules, and the minimum turning radius when the aircraft turns are considered, taking into account the shortest track and the least number of correction points, to find the optimal or satisfactory flight trajectory between the departure point and the destination, and gives the model solution method based on NSGA-II. It provides new ideas and methods for intelligent aircraft trajectory planning research.

2. Problem description and preliminaries
The intelligent aircraft needs real-time positioning during space flight, and its positioning errors include vertical errors and horizontal errors. For every 1m of flight, the vertical error and horizontal error will increase by δ units. When reaching the end point, the vertical error and the horizontal error should be less than θ units. There are some safe positions (called correction points) in the flight area that can be used for error correction. When the aircraft reaches the correction point, it can perform error correction according to the type of error correction at that position. If the vertical error and horizontal error can be corrected in time, the aircraft can fly according to the predetermined route. Considering the error generation rules and error correction rules during the flight, at point A, the vertical and horizontal errors of the aircraft are both zero. Only vertical error correction can be performed at the vertical error correction point. During the correction, the vertical error must not be greater than α₁ units and the horizontal error should not be greater than α₂ units. After correction, the vertical error will become 0 and the horizontal error will remain unchanged. Similarly, only horizontal error correction can be performed at the horizontal error correction point. During the correction, the vertical error must not exceed β₁ units, and the horizontal error should not exceed β₂ units. After correction, the horizontal error will become 0, and the vertical error will remain unchanged. Under the above constraint conditions, the length of the trajectory between the departure point A and the destination point B is made as small as possible, and the number of corrections when passing through the correction area is as small as possible. The two goals are mutually constrained, looking for an optimal or satisfactory route that satisfies the multiple constraints of the two goals.

The variables in this research are defined as follows:
- $X_k$ is the error correction point type.
- $X_k^*$ is whether the correction node is selected.
- $L_{kk+1}$ is the Euclidean distance between the correction point k and k+1.
- $l_k$ is the horizontal error cumulative at point k.
- $v_k$ is the vertical error accumulation at point k.
- $α_1$ is the vertical error threshold to be met at the vertical correction type node.
- $α_2$ is the horizontal error threshold to be met at the vertical correction type node.
- $β_1$ is the vertical error threshold to be met at the horizontal correction type node.
- $β_2$ is the horizontal error threshold to be met at the horizontal correction type node.
- $θ$ is the maximum vertical error and horizontal error that the intelligent aircraft can still fly during the flight.

For simplicity, the author put forward the following assumptions.
Assumption 1, the intelligent aircraft will not malfunction during the flight.
Assumption 2, the intelligent aircraft can navigate according to any planned feasible trajectory path.
Assumption 3, the intelligent aircraft ignores the influence of factors such as terrain and obstacles during the flight.
Assumption 4, In the problem, the correction nodes can successfully correct the errors, that is, the error is zero after the correction of the corresponding correction type node.
3. Model construction and solution

This paper studies the establishment of the trajectory of the intelligent aircraft from the starting point A to the destination point B in three-dimensional space. According to the existing information, there are N waypoints including start point A and target point B, the vertical and horizontal errors generated by the aircraft during the flight, the type of error correction point, the threshold for correction of the horizontal error correction point and the vertical error correction point, the maximum threshold that the aircraft can still fly from point A to point B, so that the track length is as small as possible, and the number of corrections through the correction area is as small as possible. Therefore, an optimal or satisfactory trajectory plan is obtained, which provides an effective reference basis for the trajectory plan in the aviation field.

Step 1: Determine the decision variables of the model:

(1) Distinguish the types of error correction point k.

\[ X_k = \begin{cases} 
0, & k \text{ is horizontal type} \\
1, & k \text{ is vertical type}
\end{cases} \]  

\(X_k\) is the set of error correction points, k is one of the correction points, \(k = 1...n\), \(X_k\) is the type of error correction points, and distinguishes whether the correction points are vertical error correction points or horizontal error correction points.

(2) Distinguish the decision variable of error correction node status \(X^s_k\), that is, whether the correction node is added to the path.

\[ X^s_k = \begin{cases} 
0, & k \text{ is unselected} \\
1, & k \text{ is selected}
\end{cases} \]  

\(X^s_k\) is the state of the correction point, \(X^s_k = 1\) indicates the current correction point has been selected. According to the method in this article, the times of corrections is the same as the number of correction points, that is, each selected correction point needs to be corrected.

(3) Decision variable \(L_{kk+1}\) to determine the distance between the correction points k and k+1.

\[ L_{kk+1} = \sqrt{(a_{k+1}-a_k)^2 + (b_{k+1}-b_k)^2 + (c_{k+1}-c_k)^2} \]  

\((a_k, b_k, c_k)\) is the coordinate of the point k, \((a_{k+1}, b_{k+1}, c_{k+1})\) is the coordinate of the point k+1.

Step 2: Determine the objective function of the model.

According to the requirements of the problem, the track length needs to be as small as possible. The objective function can be set to minimize the sum of the length of each track. The objective function is as follows:

\[ \min Z_1 \sum_{k \in N} L_{kk+1} X^s_k \]  

According to the requirements of the problem, it is necessary to make the number of corrections through the correction area as few as possible, which can be converted into solving the sum of the states of each point, and the following objective function two is established:

\[ \min Z_2 = \sum_{k=1}^{N} X^s_k \]  

Among them, \(X^s_k\) is a decision variable, used to judge whether the current correction point is selected. The requirement of this question is to reduce the number of correction points as much as possible on the basis of satisfying the shortest track in each segment. And the method used in this paper is that correction must be done as long as the correction point is selected, so the objective function \(Z_2\) established in this paper is to minimize the number of selected correction points.

Step 3: Determine the constraints of the model. The constraints of this problem are as follows:
(1) At point A, the vertical and horizontal errors of the aircraft are both 0.

\[ l_0 = 0, v_0 = 0 \]  \hspace{1cm} (6)

(2) Ensure that the vertical and horizontal errors during flight are less than \( \theta \).

\[ l_k < \theta, v_k < \theta, k \in \{1, 2, \ldots, N\} \]  \hspace{1cm} (7)

(3) Ensure that when the aircraft reaches the vertical error correction point, the vertical error of the aircraft is not greater than \( \alpha_1 \) and the horizontal error is not greater than \( \alpha_2 \).

\[ l_k < \alpha_1, v_k < \alpha_2, k \in \{1, 2, \ldots, N\} \]  \hspace{1cm} (8)

(4) Ensure that the vertical error of the aircraft is not greater than \( \beta_1 \) and the horizontal error is not greater than \( \beta_2 \) when the aircraft reaches the horizontal error correction point.

\[ l_k < \beta_1, v_k < \beta_2, k \in \{1, 2, \ldots, N\} \]  \hspace{1cm} (9)

\( l_k \) represents the accumulation of horizontal errors at point k, and \( v_k \) represents the accumulation of vertical errors at point k. The formula is directional. When the aircraft flies from k to k+1, the recursive formula is as follows.

\[
\begin{align*}
l_{k+1} &= l_k + \delta l_{kk+1} \\
v_{k+1} &= v_k + \delta v_{kk+1}
\end{align*}
\]  \hspace{1cm} (10)

In summary, this paper establishes the following dual-objective multi-constrained mathematical programming model:

\[
\begin{align*}
\min Z_1 &= \sum_{k \in N} L_{kk+1} X_k^s \\
\min Z_2 &= \sum_{k=1}^{N} X_k^s \\
l_0 &= 0, v_0 = 0 \\
l_k &< \theta, v_k < \theta \\
l_k &< \alpha_1, v_k < \alpha_2 \\
l_k &< \beta_1, v_k < \beta_2
\end{align*}
\]  \hspace{1cm} (11)

4. NSGA-II algorithm for aircraft multi-target track planning

The NSGA-II algorithm has the advantages of fast running speed, good convergence, uniform solution set distribution, and strong robustness. It is currently the most widely used multi-objective evolutionary algorithm (MOEA). In this paper, the algorithm is applied to the solution of trajectory planning under multi-object and multi-constraint conditions. The specific steps are as follows:

Step 1: Coding design, after all the correction nodes are arranged in order, each correction node is set as a gene, which is marked as 1 when selected and 0 when not selected. All the corrected nodes after encoding form a binary string of length \( n \), called a chromosome.

Step 2: Determine the initial parameters, the number of population iterations is \( M \), the population size is \( P \), the crossover probability is \( P_c \), the mutation probability is \( P_m \), input the coordinates of each point in the planned three-dimensional space, and set the starting point A and end point B of the planned trajectory.

Step 3: Initialize population 1, population 2, using three-dimensional coordinate coding, the initialization results of the two populations are diverse.
Step 4: Apply a little crossover and a little variation to the population to generate a new population and record it as the offspring $Q_t$.

Step 5: Combine the population $P_t$ with the progeny population $Q_t$, the new population is recorded as $R_t$, the non-dominant sorting of the population $R_t$, the crowding degree calculation, and a new population $P_{t+1}$.

Step 6: Repeat Step 3 and Step 4 continuously to determine whether the number of iterations has been reached, and if so, output the current optimal solution.

5. Numerical example
From point A to point B, there are 613 nodes containing the position and type of the correction point. For every 1m of flight, the unit of vertical error and horizontal error increase is $\delta = 0.001$. When performing vertical error correction, the maximum vertical error is $\alpha_1 = 25$ and the maximum horizontal error value is $\alpha_2 = 15$. When performing horizontal error correction, the maximum vertical error is $\beta_1 = 20$, the maximum horizontal error is $\beta_2 = 25$. When the intelligent aircraft reaches the end point, the unit of which both the vertical error and the horizontal error should be less than is $\theta = 30$. Set the chromosome length is 50, the amplification scale is 60, and the maximum number of iterations is 100. The convergence curve of the current population track length with the number of iterations is shown in the Figure 1. The aircraft needs to fly through 9 calibration points in the calibration area. The night correction point types are 1, 0, 1, 0, 1, 0, 1, 0, 1; the total length of the track is 108,104.5 m. The vertical and horizontal errors of the aircraft to the end point are 8.49 m and 20.2296 m, respectively. The trajectory is shown in the Figure 2 and the error before correction of each point is shown in the following Table 1:

| Calibration point number | Vertical error before correction | Horizontal error before correction | Calibration point type |
|--------------------------|---------------------------------|-----------------------------------|------------------------|
| 0                        | 0                               | 0                                 | Starting point A       |
| 347                      | 11.2575                         | 11.2575                           | 1                      |
| 70                       | 11.8149                         | 23.0724                           | 0                      |
| 238                      | 24.3143                         | 12.4994                           | 1                      |
| 156                      | 11.2008                         | 23.7002                           | 0                      |
| 143                      | 21.4584                         | 10.2576                           | 1                      |
| 251                      | 8.3167                          | 18.5743                           | 0                      |
| 404                      | 19.8243                         | 11.5076                           | 1                      |
| 249                      | 11.0198                         | 22.5274                           | 0                      |
| 502                      | 22.7594                         | 11.7396                           | 1                      |
| 613                      | 8.49                            | 20.2296                           | End point B            |
Since the coordinates of the original data correction points may change due to the influence of the actual environment, we need to check the stability of the model. After randomly changing the coordinates of the 20% points of the original data by 3%, we use our model to solve again. It is obtained that the aircraft needs to fly through 9 correction points in the area. The night correction point types are 0, 1, 0, 1, 0, 1, 0, 1, 0; the total length of the track is 109244.1471 meters. The vertical and horizontal errors at the end point are 10.0812m and 21.3234m, respectively. It can be obtained from the results that when the original data changes slightly, the results will not change significantly. Therefore, it shows that our model has better stability.

6. Conclusions
In this paper, the intelligent aircraft trajectory planning problem is studied as a multi-objective optimization problem. The article establishes a multi-objective optimization model for intelligent aircraft trajectory planning, and gives a solution method based on NSGA-II. In this way, the accuracy and real-time requirements of the aircraft's positioning solution can be met at the same time. Compared with the traditional weighted single-objective optimization, this method can take into account the requirements of multiple different indicators of intelligent aircraft trajectory planning, and has strong flexibility, stability and adaptability. Next, we will improve the NSGA-II algorithm to study the trajectory planning of the aircraft.

Figure 1. The track length convergence curve.

Figure 2. The track map of data set.
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