Single Point Positioning with Sequential Least-Squares Filter and Estimated Real-Time Stochastic Model

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Abstract  To obtain higher accurate position estimates, the stochastic model is estimated by using residual of observations, hence, the stochastic model describes the noise and bias in measurements more realistically. By using GPS data and broadcast ephemeris, the numerical results indicating the accurate position estimates at sub-meter level are obtainable.

Keywords  GPS; single point positioning; functional model; stochastic model; sequential least-square filter

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Introduction

Compared to relative GPS positioning, single-point positioning is efficient, fast and low-cost. Conventional single-point positioning provides the user’s coordinates using only pseudorange. The expected horizontal positioning accuracy of the classical approach has been improved from about 100 m when selective availability (SA) was on, to about 22 m or better in the absence of SA\(^1\). \(L_1\) and \(L_2\) measurements are combined to generate the so-called ionosphere-free linear combination, which removes the ionospheric error. The horizontal accuracy has been improved to 1-10 m by using a dual frequency receiver\(^2\). In this study, tropospheric and relativistic GPS satellite clock corrections are applied to the ionosphere-free linear combination of pseudorange and carrier phase. The carrier phase ambiguity is eliminated by differencing the carrier-phases epoch by epoch if no cycle slip occurs. In addition, the remaining errors, including uncorrected systematic errors, multipath and noise are absorbed by the stochastic model. Especially, to match the stochastic model with the environment, the stochastic model is estimated by using residuals in real-time, i.e., the current stochastic model is estimated by residuals at the previous epoch. Based on the functional model and stochastic model, the user’s coordinates are sequentially estimated by using least squares filter.

In the following sections pseudorange and carrier-phase processing are discussed, then the single receiver processing strategy is described. The test is discussed by using the broadcast orbit products and combining the IGS station, UNB1 observations. The result shows the true errors in topcentric coordinate system are under 1 m with a probability of more than 82%. Finally, conclusions are given.
1 Mathematical model and adjustment procedure

1.1 Functional model

The ionospheric-free combinations of dual-frequency GPS pseudorange, carrier phase observations and station clock offset from GPS time, respectively. Linearization of observation equations[3] gives the following mathematical model in simple form:

\[
P^\prime_{\Phi}(t) = \rho(t) + c \cdot d\tau(t) + \delta\Phi^\prime_{\Phi}(t)
\]

\[
\Phi^\prime_{\Phi}(t) = \rho(t) + c \cdot d\tau(t) + \delta\Phi^\prime_{\Phi}(t)
\]

where \(P^\prime_{\Phi}(t)\) and \(\Phi^\prime_{\Phi}(t)\) are the pseudorange and carrier-phase ionosphere-free combination, which is corrected by the above systemic effects. The time-differenced carrier-phase ionosphere-free combination is expressed as:

\[
\delta\Phi^\prime_{\Phi}(t) = \delta\Phi^\prime_{\Phi}(t) - \delta\Phi^\prime_{\Phi}(t-1) = \rho(t) - \rho(t-1) + c \cdot (d\tau(t) - d\tau(t-1)) + \\
\varepsilon^\prime(\Phi^\prime_{\Phi}(t), X(t), t-1)
\]

in which the ambiguity is eliminated if no cycle slips occur. \(t\) and \(t-1\) denote the current and previous epoch respectively. Linearization of observation Eqs.(3) and (5) around the a-priori parameters vector composed of station coordinates and receiver clock error, expressed by \(X^0(t)\) becomes, in matrix form:

\[
\begin{bmatrix}
P^\prime_{\Phi}(t) \\
\delta\Phi^\prime_{\Phi}(t)
\end{bmatrix} =
\begin{bmatrix}
0 & B_r \cdot \delta x(t-1) \\
0 & B_r \cdot \delta x(t)
\end{bmatrix} +
\begin{bmatrix}
\delta\rho^0(t) \\
\delta\rho^0(t, t-1)
\end{bmatrix}
\]

(6)

where \(B_r\) and \(B_{r-1}\) are partial derivatives with respect to \(X_r(t)\) and \(X_r(t-1)\), consisting of station coordinates and clock errors for epoch \(t\) and \(t-1\), respectively; \(\delta x(t)\) and \(\delta x(t-1)\) are the vectors of corrections to the a-priori parameters (approximate initial value) vector \(X^0(t)\) and \(X^0(t-1)\). \(\delta\rho^0(t)\) and \(\delta\rho^0(t, t-1)\) are expressed by:

\[
\delta\rho^0(t) = \|X^\prime(t) - X^0(t)\|
\]

\[
\delta\rho^0(t, t-1) = \|X^\prime(t) - X^0(t)\| - \|X^\prime(t-1) - X^0(t-1)\|
\]

(7)

(8)

where \(X^\prime(t)\) and \(X^\prime(t-1)\) are the orbits of the satellite. As can be seen by observing Eq.(6), ambiguity is eliminated and the carrier-phase smooths the pseudorange.

1.2 Stochastic model

In Eq.(6), \(\varepsilon^\prime\) represents noise, unmodeled systematic errors and multipath. As we know, the stochastic model as well as functional model plays an important role on results when applying adjustment procedure. Thus, this study takes a more realistic variance matrix to describe the characteristic of all remaining errors. The procedure to get the stochastic model is depicted as follows.