Abstract

We review the status of theoretical predictions for W-pair production at high energies. We discuss a systematic scenario towards a Monte-Carlo generator for $e^+e^- \rightarrow 4f(\gamma)$, which meets the experimental requirements. In particular we summarize the recent developments in this field.
1. Introduction

The gauge-boson production processes allow an accurate direct study of triple and quartic gauge-boson couplings. Owing to the presence of unitarity cancellations for longitudinal gauge bosons at high energies, the sensitivity to anomalous gauge couplings, which in general spoil these cancellations, grows with energy.

For $e^+e^-\rightarrow W^+W^-$, the most prominent gauge-boson production process, the sensitivity to anomalous couplings is roughly given by $\beta^2 t^2 / s^2 \times s/M_W^2$, with $\beta = \sqrt{1 - 4M_W^2/s}$. Here the first factor originates from the suppression of the $s$-channel diagrams, containing the triple gauge couplings, compared to the dominant $t$-channel diagram. The second factor is due to the enhancement in the absence of unitarity cancellations. Accordingly one should go to energies as high as possible* and consider observables that are not dominated by the $t$-channel pole. In view of the latter, one needs to consider angular distributions of the $W$ bosons and their decay products, which in particular allow to derive information on the polarization of the $W$ bosons.

As the $W$ bosons decay mainly into fermion–antifermion pairs, one has to consider the process $e^+e^-\rightarrow 4f(\gamma)$. For this one needs a (fast) Monte-Carlo generator which includes anomalous couplings and has an accuracy of better than 1%, in order to allow theoretical predictions with an uncertainty below the experimental precision.

The lowest-order process $e^+e^-\rightarrow 4f$ involves diagrams with internal $W$- and $Z$-boson propagators, which may become resonant and yield the dominant contributions. In order to define gauge-invariant resonant contributions and to introduce finite width effects in a gauge-invariant way, we adopt the pole scheme†. This is based on a split-up of the matrix elements according to the poles of the $W$- and $Z$-boson propagators with corresponding constant residues‡. As there can be two resonant gauge

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*For LEP200 the highest energy possible should be aimed at in order to overcome the $\beta^2$ suppression.
bosons in $e^+e^- \rightarrow 4f$, there are double-pole, single-pole, and non-resonant contributions. The dominant contributions are given by the double-pole terms. Since the $Z$-boson double-pole terms are suppressed by roughly a factor of 10 relative to the $W$-boson ones, we focus on the latter. In that case the doubly-resonant diagrams can be related to on-shell $W$-pair production in the following way:

$$
\mathcal{M}_{e^+e^-\rightarrow 4f}^{\text{pole}} = \sum_{\lambda_+,\lambda_-} \mathcal{M}_{+\rightarrow W^+}^{\lambda_+} \times \frac{\mathcal{M}_{W^+\rightarrow f_1f_2}^{\lambda_+}}{k_+^2 - M_W^2 + iM_W\Gamma_W} \times \frac{\mathcal{M}_{W^-\rightarrow f_3f_4}^{\lambda_-}}{k_-^2 - M_W^2 + iM_W\Gamma_W}.
$$

(1)

where $\mathcal{M}_{e^+e^-\rightarrow W^+W^-}$, $\mathcal{M}_{W^+\rightarrow f_1f_2}$, $\mathcal{M}_{W^-\rightarrow f_3f_4}$ denote the matrix elements for the production of two on-shell $W$ bosons and their subsequent decay into fermion–antifermion pairs, and $\lambda_+$ the helicities of the $W$ bosons. The finite width effects are included by introducing the physical width $\Gamma_W$ into the resonant propagators.

At the cross-section level, all other (non-doubly-resonant) contributions are typically suppressed by a factor $\Gamma_W/M_W \approx 2.5\%$ for each non-resonant $W$ propagator). The non-resonant contributions can be further reduced by a cut on the invariant masses of the decay products $M_W - \Delta < \sqrt{k^2} < M_W + \Delta$, which typically suppresses a flat background by an additional factor $\Delta/M_W$. For small enough $\Delta$ this also applies to the background from $Z$-pair production contributions.

2. Radiative Corrections

As far as the $O(\alpha)$ corrections are concerned, it is most likely sufficient to take into account only the doubly-resonant contributions, as the others are suppressed by an additional factor $\Gamma_W/M_W$. The only exceptions might be enhanced corrections, which can usually be treated by renormalization group methods (leading collinear QED logarithms, running $\alpha$, . . . ) and which can thus simply be combined with the lowest-order cross-section.

There are two different sources of double-pole terms in the virtual corrections, factorizable corrections and non-factorizable photonic corrections. The factorizable corrections consist of all those diagrams where the $W$-production and decay parts can be separated by cutting the two resonant $W$ propagators, i.e. the ones that are reducible at both $W$ lines. Consequently they can be related to the on-shell matrix elements according to Eq. (1). At $O(\alpha)$ they comprise the complete resonant non-photonic corrections and the most important resonant photonic corrections, in particular those involving leading logarithms. The on-shell matrix elements are known, including the complete $O(\alpha)$ corrections. Thus regarding the factorizable corrections the only task left is their implementation into a Monte-Carlo generator. For the resonant diagrams contributing to the $O(\alpha)$ real photonic corrections, such a generator already exists.

†It should be noted that below the $W$-pair production threshold the doubly-resonant contributions vanish and the singly-resonant ones become dominant. Moreover there are problems when defining the pole-scheme split-up in the vicinity of the threshold. This is, however, not relevant at high energies.
Feynman diagrams that are not reducible at both W lines do in general not yield doubly-resonant contributions. This does not hold for non-factorizable contributions resulting from diagrams where a virtual photon is exchanged between the two decay parts or between the production and a decay part of the diagram (see Fig. 1). These diagrams contain resonant contributions related to the IR limit, i.e. to those parts of the integration region where both the energy and the momentum of the massless photon are zero.

3. Recent Developments

In this section we summarize the developments in the field of radiative corrections to $e^+e^- \rightarrow 4f$ achieved by the European working group on electroweak gauge bosons since the 1991 workshop.

By a systematic expansion of the existing complete virtual and soft-photonic $\mathcal{O}(\alpha)$ corrections, a high-energy approximation has been constructed for the process $e^+e^- \rightarrow W^+W^-$ in the limit $s, |t|, |u| \gg M_{W,Z}^2 \gg m_{e,\mu,\tau,d,u,s,c,b}^2$, keeping $m_t$ and $M_H$ arbitrary. For intermediate energies (500 GeV – 2 TeV) the high-energy approximation has been improved by exactly taking into account the leading low-energy universal corrections. In the angular range $-0.9 \leq \cos \theta_{W} \leq 0.9$ and for energies above 500 GeV, the approximation reproduces the complete results for all relevant polarizations essentially within 1%, which roughly matches the expected experimental accuracy.

A semi-Monte-Carlo program, GENTLE, has been developed for $e^+e^- \rightarrow 4f(\gamma)$. It includes the lowest-order resonant diagrams and the complete initial-state $\mathcal{O}(\alpha)$ QED corrections, defined in a gauge-invariant way through the current-splitting technique. In addition soft-photon exponentiation and the universal s-channel $\mathcal{O}(\alpha^2)$ QED corrections to the initial state, known from LEP1, have been taken into account. The program generates distributions based on a compact analytical formula for $d^3\sigma/(dk^2 dk^2 ds')$, with $s' = s - 2\sqrt{s}E_{\gamma}$, and hence does not yet allow studies of angular distributions. A generalization to angular distributions is in progress.
A Monte-Carlo event-generator, WOPPER, has been created for $e^+e^- \rightarrow 4f(n\gamma)$. Using techniques known from parton-shower algorithms, it calculates initial-state QED corrections in the leading-log approximation, with resummation to all orders and soft-photon exponentiation. It is based on the resonant diagrams and provides full spin transmission to the final state. Moreover it yields exclusive photons with full kinematics.

4. Outlook

Finally we would like to summarize what, in our opinion, still has to be completed to arrive at a Monte-Carlo generator that meets all requirements. The full lowest-order process $e^+e^- \rightarrow 4f$ should be calculated, including all non-resonant contributions and also anomalous couplings. Already at this level the leading corrections should be implemented. For the remaining $\mathcal{O}(\alpha)$ corrections it is sufficient to evaluate the doubly-resonant contributions. These involve electroweak as well as QCD corrections. Of course all relevant leading higher-order effects should be taken into account.

References

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