Chirality dependence of thermoelectric response in a thermal QCD medium

Debarshi Dey and Binoy Krishna Patra

Indian Institute of Technology Roorkee, Roorkee 247667, India

(Dated: )

The lifting of the degeneracy between $L$- and $R$-modes of massless flavors in a weakly magnetized thermal QCD medium leads to a novel phenomenon of chirality dependence of the thermoelectric tensor, whose diagonal and non-diagonal elements are the Seebeck and Hall-type Nernst coefficient, respectively. Both coefficients in $L$-mode have been found to be greater than their counterparts in $R$-mode, however the disparity is more pronounced in the Nernst coefficient. Another noteworthy observation is the impact of the dimensionality of temperature ($T$) profile on the Seebeck coefficient, wherein we find that the coefficient magnitude is significantly enhanced ($\sim$ one order of magnitude) in the 2-D setup, compared to a 1-D $T$ profile. Further, the chiral dependent quasifermion masses constrain the range of magnetic field ($B$) and $T$ in a manner so as to enforce the weak magnetic field ($eB \ll T^2$) condition.

Introduction- Nuclear matter at a very high temperature and/or baryon density is conceived in terms of deconfined quarks and gluons (dubbed as QGP). Transport coefficients serve as input for modelling the flow of such matter created in relativistic heavy ion collision experiments[1–3]. These experiments indicate that the created matter is very nearly an ideal fluid with the viscosity being close to the conjectured lower bound[4] arrived at from AdS/CFT correspondence. The QGP may be exposed to magnetic fields ($B$) arising from non-central nucleus-nucleus collisions[5, 6]; its strength depending on the time scale of evolution. A strong $B$ provides the ground for probing the topological properties of QCD vacuum[7, 8], whereas weak $B$ yields some novel phenomenological consequences through the lifting of degeneracy between left and right handed quarks. Our aim is to explore the consequence of this splitting on the thermoelectric response of the medium.

In weak $B$ regime ($m_0^2 \ll eB \ll T^2$), the thermoelectric response of the thermal QCD medium assumes a matrix structure:

$$
\begin{pmatrix}
E_x \\
E_y
\end{pmatrix}
= \begin{pmatrix}
S & N|B| \\
N|B| & S
\end{pmatrix}
\begin{pmatrix}
\nabla T|_x \\
\nabla T|_y
\end{pmatrix}.
$$

(1)

The diagonal and nondiagonal elements are Seebeck ($S$) and Nernst ($N|B|$) coefficients, respectively, which are the measures of ‘longitudinal’ and ‘transverse’ (analogous to Hall effect) induced electric fields. Large fluctuations in the initial energy density in the heavy-ion collisions[9] translate to significant temperature gradients between the central and peripheral regions of the produced fireball, providing the ideal ground to study thermoelectric phenomena.

We also look at the impact of dimensionality of temperature profile (1-D/2-D) on the Seebeck coefficient. At strong $B$, fermions are constrained to move only in one dimension, i.e. only the lowest Landau level (LLL) is populated. When the strength of $B$ decreases, higher Landau levels start getting occupied, consequently, the constraint on motion of fermions is relaxed and the study of thermoelectric response with a 1-D or 2-D temperature profile becomes plausible. However, the Nernst effect is only manifested in the weak $B$ regime, since the transverse current vanishes at strong $B$.

Dispersion Relations in weak $B$: Chiral modes- The dispersion relation of quarks is obtained from the zeros of the inverse resummed quark propagator:

$$
S^{-1}(K) = S_0^{-1}(K) - \Sigma(K),
$$

(2)

where the quark self-energy, $\Sigma(K)$ is to be calculated up to one-loop from thermal QCD in weak $B$ and the bare quark propagator, $S_0(K)$ in weak $B$, up to power $|q_f B|$, is given by

$$
iS_0(K) = \frac{i}{K^2 - m_f^2} - \frac{\gamma_1 \gamma_2 (\gamma \cdot K)_f + m_f}{(K^2 - m_f^2)^2}(q_f B),
$$

(3)

which can be written in terms of the fluid four-velocity $u^\mu = (1, 0, 0, 0)$ and $b^\mu = \vec{b}$.
\[ \frac{1}{\mathcal{B}} \varepsilon^{\mu \nu \lambda \rho} u^\nu F_{\mu \lambda} = (0, 0, 0, 1), \] as,

\[ iS_0(K) = \frac{i \langle \vec{K} \rangle}{K^2 - m_J^2} - \frac{i \gamma_5 [(K.b) \gamma_5 - (K.u) \gamma_5]}{(K^2 - m_J^2)^2} \langle q_f B \rangle. \]

The one loop quark self energy is then given by

\[ \Sigma(P) = g^2 C_F T \sum_n \int \frac{d^3 k}{(2\pi)^3} \gamma_\mu \left( \frac{\vec{K}}{K^2 - m_J^2} - \frac{\gamma_5 [(K.b) \gamma_5 - (K.u) \gamma_5]}{(K^2 - m_J^2)^2} \langle q_f B \rangle \right) \gamma_\mu \frac{1}{(P - K)^2}. \]

In a covariant tensor basis, the above self energy can be expressed in terms of structure constants \( A, B, C, D \) as

\[ \Sigma(P) = -A \bar{P} - B \gamma_5 \gamma_5 - C \gamma_5 \gamma_5 - D \gamma_5 \gamma_5, \]

The self energy and the full propagator can then be rewritten in terms of projection operators \( P_L = (\gamma_5)/2 \) and \( P_R = (\gamma_5)/2 \) as

\[ \Sigma(p) = -P_R \bar{A}' P_L - P_L \bar{B}' P_R, \]

\[ S^{-1}(P) = P_L \bar{P} L + P_R \bar{R} R, \]

\[ S(P) = \frac{1}{2} \left[ P_L \frac{L}{L^2/2} P_R + \frac{1}{2} P_R \frac{R}{R^2/2} P_L \right], \]

where,

\[ L^2 = (1 + A)^2 P^2 + 2(1 + A)(B + C) p_0 - 2D(1 + a)p_z + (B + C)^2 - D^2, \]

\[ R^2 = (1 + A)^2 P^2 + 2(1 + A)(B - C) p_0 + 2D(1 + A)p_z + (B - C)^2 - D^2. \]

The \( p_0 = 0, \ p \rightarrow 0 \) limit of the denominator of the effective propagator yields the quasiparticle masses as[10, 11]

\[ m_L^2 = \frac{L^2}{2} \bigg|_{p_0=0,|p|\rightarrow 0} = m_{th}^2 + 4g^2 C_F M^2, \]

\[ m_R^2 = \frac{R^2}{2} \bigg|_{p_0=0,|p|\rightarrow 0} = m_{th}^2 - 4g^2 C_F M^2, \]

thus lifting the degeneracy. Here,

\[ M^2 = \frac{|q_f B|}{16\pi^4} \left( \frac{\pi T}{2m_f} - \ln 2 + \frac{7\mu^2 \zeta(3)}{8\pi^2 T^2} \right), \]

\[ m_{th}^2 = \frac{1}{8} g^2 C_F \left( T^2 + \frac{\mu^2}{\pi^2} \right). \]

The coupling constant, \( g \) is used as in[12]. In the ultra relativistic limit, the chirality of a particle is the same as it’s helicity, so that the right and left chiral modes can be thought of as the up/down spin projections in the direction of momentum \((s_z = \pm 1/2)\) of the particle, suggesting that the medium generated mass of a collective fermion excitation at very high temperatures \((T \gg m_0)\) is \( s_z \) (helicity)-dependent. Such \( s_z \) dependent quasiparticle masses are already known in condensed matter systems[13, 14].

The thermoelectric coefficients - We make use of the Boltzmann transport equation to calculate the infinitesimal deviation from equilibrium of the system caused by the temperature gradient.

\[ p^\mu \frac{\partial f_i(x, p)}{\partial x^\mu} + q_i F_{\mu \nu} p_\nu \frac{\partial f_i(x, p)}{\partial p^\mu} = \left( \frac{\partial f_i}{\partial t} \right)_{\text{coll}} \]

The highly non linear collision integral on the R.H.S. can be linearized using the relaxation time approximation which reads (suppressing the flavor index \( i \))

\[ \left( \frac{\partial f}{\partial t} \right)_{\text{coll}} \sim - p^\mu u_\mu \delta f = - \frac{f - f_0}{\tau}, \]

where, \( \tau \) is the relaxation time[15] and \( f_0 \) is the Fermi-Dirac distribution function. \( \delta f \) is then used to calculate the induced current which is set to zero (enforcing the equilibrium condition) to evaluate the relevant response functions[16] (Seebeck and Nernst coefficients). A non-zero \( \mu \) leads to a non-zero net thermocurrent and induces an electric field which grows until the thermocurrent is neutralised. In condensed matter systems, an electric field is applied externally to achieve the condition of zero thermocurrent and the coefficient (Seebeck/Nernst) is read off therefrom.
We use the following Ansatz\cite{17} to solve for $\delta f$ from Eq.\eqref{16}

$$f^{L/R} = f_0^{L/R} - \tau q E \cdot \frac{\partial f_0^{L/R}}{\partial p} - \chi \frac{\partial f_0^{L/R}}{\partial p},$$

where, the effect of $B$ is encoded in $\chi$ ($L/R$ denotes the handedness). We begin with a single flavor system. The components of $\chi$, after some algebra, can be expressed conveniently in a matrix form

$$\begin{bmatrix}
\chi_x \\
\chi_y
\end{bmatrix} = \begin{bmatrix}
\frac{-\omega_\tau^2 q}{1 + \omega_\tau^2} & \frac{-\omega_\tau^2 q}{1 + \omega_\tau^2} \\
\frac{-\omega_\tau^2 q}{1 + \omega_\tau^2} & \frac{-\omega_\tau^2 q}{1 + \omega_\tau^2}
\end{bmatrix} \begin{bmatrix}
E_x \\
E_y
\end{bmatrix} + \begin{bmatrix}
\frac{\omega_\tau^2}{1 + \omega_\tau^2} \left( \frac{\tau}{\bar{T}} - \frac{\tau}{\bar{T}} \right) - \frac{\omega_\tau^2}{1 + \omega_\tau^2} \left( \frac{\tau}{\bar{T}} - \frac{\tau}{\bar{T}} \right)
\end{bmatrix} \begin{bmatrix}
\frac{\partial \tau}{\partial x} \\
\frac{\partial \tau}{\partial y}
\end{bmatrix}.
$$

The induced 4-current is given by (We drop the $L/R$ notation for brevity):

$$J^\mu = q g \int \frac{d^3p}{(2\pi)^3} \frac{\partial^3}{\partial p^3} p^\mu \left[ \delta f - \bar{f} \right],$$

where, $\bar{f}$ denotes the contribution from antiparticles. For $\nabla T = \frac{\partial T}{\partial x} \hat{x} + \frac{\partial T}{\partial y} \hat{y}$ the equilibrium condition is $J_x = J_y = 0$, which yields the following equations

$$\begin{bmatrix}
(C_1)E_x + (C_2)E_y + (C_3)\frac{\partial T}{\partial x} + (C_4)\frac{\partial T}{\partial y} \\
-(C_2)E_x + (C_1)E_y + (C_4)\frac{\partial T}{\partial x} + (C_3)\frac{\partial T}{\partial y}
\end{bmatrix} = 0$$

Solving for $E$ in terms of $\nabla T$ leads to the structure

$$\begin{bmatrix}
E_x \\
E_y
\end{bmatrix} = \begin{bmatrix}
S & N|B| \\
-N|B| & S
\end{bmatrix} \begin{bmatrix}
\frac{\partial \tau}{\partial x} \\
\frac{\partial \tau}{\partial y}
\end{bmatrix},$$

with

$$S = -\frac{C_1 C_3 + C_2 C_4}{C_1^2 + C_2^2},$$

$$N|B| = \frac{C_2 C_3 - C_1 C_4}{C_1^2 + C_2^2}.$$
with

\[ I_1 = q \int dp p^4 \frac{\tau}{\epsilon^2(1 + \omega_c^2 \tau^2)} \left\{ f_0(1 - f_0) + \bar{f}_0(1 - \bar{f}_0) \right\}; \]
\[ I_2 = q \int dp p^4 \frac{\tau}{\epsilon^2(1 + \omega_c^2 \tau^2)} \left\{ (\epsilon - \mu) f_0(1 - f_0) - (\epsilon + \mu) \bar{f}_0(1 - \bar{f}_0) \right\}. \]

Thus, the total Seebeck coefficient is expressed as a weighted average of the single component coefficients. Such a mathematical structure is absent for the 2D $T$ profile. Since the Nernst coefficient relates the induced electric field and the temperature gradient in mutually transverse directions, it emerges along with the Seebeck coefficient naturally in the 2D setup.

Figures (1) and (2) show the variation of Seebeck and Nernst coefficients of the medium with temperature. It can be seen that the magnitude of the induced electric field in the longitudinal (along $\nabla T$) and transverse (perpendicular to $\nabla T$) directions shows similar trends as far as variation with temperature is considered; for both the modes, the magnitudes decrease with temperature. Also, for both the coefficients, the $L$ mode elicits a larger comparative response. This can be understood from a numerical perspective. Each of the integrals $C_1, C_2, C_3, C_4$ are decreasing functions of the effective mass. However, the extent of decrease follows the hierarchy $\Delta C_2 > \Delta C_1 > \Delta C_4 > \Delta C_3$, where $\Delta C$ denotes change in the value of the integral due to a given change in mass. The mathematical expressions of $S$ and $N|B|$ then imply that whereas both the numerator and denominator of the expressions decrease with increasing mass, the denominator decreases by a larger amount (because of the presence of $C_1$ and $C_2$) than the numerator. The value of the fraction, therefore, increases with increasing mass. For comparison, the $B = 0$ case is also shown for the Seebeck coefficient. The Nernst coefficient is, however zero for $B = 0$, as should be the case.

Fig.(1) shows the impact of the dimension of $T$ profile on the magnitude and temperature behaviour of the Seebeck coefficient. For the 1D setup, Both the $I_1$ and $I_2$ integrals are decreasing functions of mass, i.e. their values increase in going to the $R$ mode from the $L$ mode. The
increase is however greater for $I_1$, compared to $I_2$ and hence, Eq. (27) dictates the hierarchy that is observed in Fig. (1). As can be seen, there is almost an order magnitude increase in the medium Seebeck coefficient values in the 2-D case. The hierarchy with respect to magnitudes remains the same in the 1-D case with the $L$ mode lying above the $R$ mode for the entire temperature range considered. Compared to the 2-D case, there is stark contrast regarding the extent of splitting (in the Seebeck coefficient magnitude) witnessed in the 1-D setup with a maximum relative difference of 24.5% between the $L$ and $R$ modes (57.1% in the 2-D case). Thus, the magnitude as well as sensitivities to mass and temperature of the thermoelectric response are heightened in the 2-D temperature profile.

We found that the mass (squared) of the $R$ mode fermion, (Eq. (12)) comes out to be negative for a certain range of combinations of $T$ and $B$ values, which is brought out by Fig. (3). As the magnetic field is increased, the temperature (above $T_c \sim 155$ MeV) upto which $m_R^2$ is negative, also increases. This suggests that the perturbative framework used by us to study the chirality dependence of the thermoelectric response is valid only at regions sufficiently far ($\sim 200$ MeV) from the crossover region in the QCD phase diagram for $|eB| > 0.2m_{\pi}^2$. Another way to look at it is that the condition $eB \ll T^2$ is strictly enforced. For $T > T_c$, we find from Fig. (3) that $|\frac{eB}{T^2}|_{\text{max}} \sim 0.07$ for $eB = 0.1m_{\pi}^2$ and $\sim 0.03$ for $eB = 0.35m_{\pi}^2$. For values of $T$ and $B$ leading to higher values of $|\frac{eB}{T^2}|$, $m_R^2$ is negative and thus unphysical.

The Seebeck coefficient does not have an explicit $B$ dependence; its $B$ dependence stems from that of the quasiparticle mass. The Nernst coefficient additionally carries an explicit $B$ dependence. As such, its sensitivity to changes in magnetic field strength is comparatively more pronounced. The maximum percentage difference in magnitudes between the $L$ and $R$ modes for the Seebeck and Nernst coefficients is 57.1% and 118.6%, respectively.
[1] F. Karsch, D. Kharzeev, and K. Tuchin, Physics Letters B 663, 217 (2008).
[2] C. Sasaki and K. Redlich, Nuclear Physics A 832, 62 (2010).
[3] H. M. S. I. Finazzo, R. Rougemont and J. Noronha, Journal of High Energy Physics 051, 10.1007/JHEP02(2015)051 (2015).
[4] P. K. Kovtun, D. T. Son, and A. O. Starinets, Phys. Rev. Lett. 94, 111601 (2005).
[5] V. V. Sokov, A. Y. Illarionov, and V. D. Toneev, International Journal of Modern Physics A 24, 5925 (2009).
[6] K. Tuchin, Phys. Rev. C 82, 034904 (2010).
[7] D. E. Kharzeev, L. D. McLerran, and H. J. Warringa, Nuclear Physics A 803, 227 (2008).
[8] K. Fukushima and Y. Hidaka, Phys. Rev. Lett. 120, 162301 (2018).
[9] B. Schenke, P. Tribedy, and R. Venugopalan, Phys. Rev. Lett. 108, 252301 (2012).
[10] A. Das, A. Bandyopadhyay, P. K. Roy, and M. G. Mustafa, Phys. Rev. D 97, 034024 (2018).
[11] Pushpa and B. K. Patra, (2022), arXiv:2112.12950 [nucl-th].
[12] A. Ayala, C. A. Dominguez, S. Hernandez-Ortiz, L. A. Hernandez, M. Loewe, D. M. Paret, and R. Zamora, Phys. Rev. D 98, 031501 (2018).
[13] J. Spaek and P. Gopalan, Phys. Rev. Lett. 64, 2823 (1990).
[14] P. S. Riseborough, Philosophical Magazine 86, 2581 (2006).
[15] A. Hosoya and K. Kajantie, Nuclear Physics B 250, 666 (1985).
[16] H. B. Callen, Thermodynamics and an Introduction to Thermostatistics, 2nd ed. (Wiley, New York, 1991).