Damping of neutron star shear modes by superfluid friction

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ABSTRACT
The forced motion of superfluid vortices in shear oscillations of rotating solid neutron star matter produces damping of the mode. A simple model of the unpinning and repinning processes is described, with numerical calculations of the consequent energy decay times. These are of the order of 1 s for typical anomalous X-ray pulsars with rotational angular velocity $\Omega \approx 1$ rad s$^{-1}$ but become very short for radio pulsars with $\Omega \approx 10 - 100$ rad s$^{-1}$. The superfluid friction processes considered here may also be significant for the damping of r-modes in rapidly rotating neutron stars.

Key words: stars: neutron-pulsars: general.

1 INTRODUCTION
Although there has been much recent interest in shear modes of neutron star oscillation (Blaes et al 1989; Thompson & Duncan 1995; Duncan 1998), relatively little attention has been paid to the associated damping mechanisms apart from coupling with magnetospheric Alfvén waves. Blaes et al consider that stress-induced motion of dislocations may be an important process, as is believed to be the case for shear dislocation movement and which might be capable of damping shear waves enough to invalidate this assumption. Shear waves are reflected many times from the boundary with the magnetosphere and possibly from the inner boundary of the crust with the liquid core of the star where the matter density is $\rho_c \approx 2 \times 10^{14}$ g cm$^{-3}$. The number of reflections must be at least of the order of $c_s \tau_s/R \approx 10^7$, where $R$ is the neutron star radius and $c_s \approx 10^3$ cm s$^{-1}$ is the shear-wave propagation velocity. Reflection from each boundary is complicated because it couples shear waves with longitudinal sound (see, for example, Landau & Lifschitz 1970, p.103) and at $\rho_c$, may even depend on the dimensionality of the solid.

It is known that longitudinal sound is strongly absorbed in the core of a neutron star owing to bulk viscosity produced by the strangeness-changing four-baryon weak interaction. The magnitude of the coefficient is very dependent on the assumptions which must be made, principally about the equation of state at core densities, but the absorption is always strong enough, within certain temperature intervals, to be significant for shear-wave damping.

The shear-wave velocity depends on nuclear parameters according to $c_s \propto Z A^{-2/3} \rho_N^{-1/6}$, where $Z$ and $A$ are the nuclear charge and mass number, and $\rho_N$ is the spatially-averaged matter density derived from the nuclei alone and excluding the superfluid neutron continuum external to them. This elementary expression assumes a nuclear effective mass equal to the free mass and, in conjunction with the nuclear parameters listed by Negele & Vautherin (1973), gives a rapid increase of $c_s$ with depth at $\rho < \rho_{nd}$ but no...
more than a very slow increase at $\rho > \rho_{nd}$. (The latter variation seems to be in disagreement with Blaes et al who state that $\rho_{nd}$ is a decreasing function of depth at $\rho > \rho_{nd}$. As a consequence of this variation, shear waves will be refracted radially outward in all regions, as are longitudinal sound waves, but only weakly so at $\rho > \rho_{nd}$. But in a typical 1.4$M_\odot$ neutron star, the layers of strong refraction at $\rho < \rho_{nd}$ have depth only of the same order as the wavelength at the relevant frequency of $10^4$ Hz. Even assuming, with Blaes et al, that the Maxwell-stress generated events producing shear waves occur predominantly within this latter region, it follows from this small depth and from the multiplicity of reflections that shear-wave propagation in the neutron drip region at $\rho > \rho_{nd}$ must be considered as a significant source of damping. Thus most of the present paper will describe an investigation in this context of phenomena involving the rapid forced motion of vortices which, for want of any better name, we shall refer to as superfluid friction. The relevant results for longitudinal sound absorption in the liquid core are considered in Section 3.

2 DESCRIPTION OF SUPERFLUID FRICTION

An elementary description of superfluid friction is given by considering a region of solid in which the neutron superfluid has rotational angular velocity $\Omega_n$. The superfluid velocity $v_s$ is defined (see Sonin 1987) as the average over the unit cell of the local vortex lattice. All velocities are referred to cylindrical polar coordinates rotating with constant angular velocity $\Omega$, coincident at some fiducial time with the angular velocity of the charged components of the star. The difference $\Omega_n - \Omega$ is always small compared with $\Omega$. To the extent that steady spin-down of the superfluid occurs, it is maintained by radial outward expansion of its vortex lattice; the Magnus force $f_M$ acting on an individual vortex is balanced by the force per unit length $f$ generated through its motion with velocity $-u$, perpendicular to its local axis and relative to the solid. The quantities $f$ and $u$ have to be treated as averages over macroscopic lengths and times, as a force $f(u)$ per unit length of vortex, and that fluctuations from it are not so large as to make its use meaningless. The component of this force parallel with $u$ is the dissipative force $f_d(u)$. Its properties were considered briefly in a previous paper (Jones 2002), but it seems appropriate to give here a more detailed account of what can be deduced about it from the interaction of a vortex, conoving with superfluid of velocity $v_s$, and a single point defect in the solid structure. This leads to the excitation of phonons in the solid and, at finite temperatures, of vortex-core quasiparticles (Jones 1990a,b), but Kelvin wave excitation is easily the most important process (Epstein & Baym 1992, Jones 1998). In the case that the interaction potential is attractive, the coupling always becomes strong at sufficiently small $v_s$ so that the energy transfer to Kelvin waves must be calculated non-perturbatively. In the strong-coupling region, the vortex is trapped in the potential for a time $t_m \propto v_s^{-2}$ and the energy transfer is $E_n \propto v_s^{-1}$. We refer to Jones (1998) for details and for a diagram showing a typical vortex orbit within the potential. (For weak coupling, realized only at values of $v_s$ larger than those considered here, the energy transfer changes to $E_n \propto v_s^{-1/2}$ and the vortex orbit is little perturbed by the potential.) Fig. 1 shows schematically the effect of a sequence of such interactions, with a dilute distribution of point defects, on the vortex path. Each interaction produces a lateral displacement and introduces a trapping time $t_m$. It is found that, averaged over impact parameter, the components of the computed impulse on the vortex are approximately equal,

$$\int_{-\infty}^{\infty} F_1(t)dt \approx \int_{-\infty}^{\infty} F_2(t)dt,$$  

where the axes are as labelled in the Fig. 1. The connection between $F$, defined for interaction with an isolated point defect, and the force per unit length $f$, which is an average over macroscopic lengths and times for motion through a dense system of point defects, is a complex problem not considered here. On microscopic length and time-scales, the vortex has fluctuating motion and, as shown in Fig. 1, the macroscopically-averaged vortex velocity $-u = -\hat{u}$ is smaller in magnitude than the superfluid velocity $v_s$ with which the vortex would comove in the absence of interaction, and has a component perpendicular to it. For general values of $v_s$ and for a dense system of point defects, it must be presumed that as a consequence of equation (1) the force $f$ has, in addition to the dissipative component $f_d$, a component $f_\perp$ parallel with $\kappa \times \hat{u}$, where the circulation is $\kappa = \kappa_0$ and $\kappa_0$ is its quantum. In other superfluid problems, the existence of this non-dissipative component has been viewed as controversial. Thouless, Ao & Nin (1996) have claimed that it does not exist for neutral superfluids in a uniform normal background (see, however, Hall & Hook 1998, Sonin 1998, and the replies therein), but a system of point defects is not of this class. However, in the special case of an infinite linear array of point defects, parallel with $\kappa$, the Green function used by Jones (1998) allows one to see that the condition

$$\int_{-\infty}^{t} F_2(t')dt' \rightarrow 0,$$  

which is necessary for the onset of pinning, can always be satisfied at sufficiently small $v_s$.

The qualitative form inferred for the dissipative force

$$f_d(u) = f \propto u \times \hat{u} \times \kappa_0,$$  

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per unit length of vortex, \( f_\parallel(u) \), is shown in Fig. 2. Finite temperatures and quantum-mechanical tunnelling lead to vortex creep and therefore a large and rapidly varying gradient close to \( u = 0 \). However, those properties of \( f_\parallel \) which are important here can be expressed in the simple function

\[
f_\parallel(u) = f_m \left(1 - e^{-u/a}\right) \left(1 - e^{-b/u}\right)
\]  

where \( a, b \) and \( f_m \) are constants. We assume that \( b \) is of the order of \( v_m = f_m/\rho_n \kappa \), where \( \rho_n \) is the density of the superfluid continuum outside the nuclei. It is the only naturally occurring temperature-independent scale of velocity in the problem. The precise value of the parameter \( a \) is not important here except in connection with rotational noise generation as mentioned in Section 3; it is very many orders of magnitude smaller than \( b \).

The linearized equations for the macroscopic hydrodynamics of an incompressible superfluid rotating with angular velocity \( \Omega \) are:

\[
\rho_n \kappa \times (\nabla v - \nu_n) + f_\parallel(u) \hat{u} + f_\perp(u) \hat{k} \times \hat{u} = 0,
\]

\[
\frac{\partial v_n}{\partial t} + 2 \Omega \times v_n = -\frac{1}{\rho_n} \nabla P,
\]

where \( P \) is the pressure. Comparison with the equations given by Sonin (1987; equations 4.36 - 4.38) shows that the Tkachenko force, derived from the vortex lattice structure, and the force dependent on vortex curvature have been excluded from equation (4). Both are quite negligible in the present context. These equations are macroscopic in that equation (5) contains the vortex density \( 2\Omega/\kappa \), although equation (4), with no Tkachenko force, is valid for an isolated vortex. At the low vortex densities (\( 10^{3-5} \) cm\(^{-2} \)) of rotating neutron stars, \( \nu_n \) does not change rapidly with time and, for the purpose of understanding vortex unpinning and repinning at a qualitative level, can be treated as a constant in equation (4). Remembering that \( u = v - v_L \), and that all the velocity components with which we are concerned are perpendicular to \( \hat{k} \), the components of equation (4) parallel with \( \hat{u} \) and \( \hat{k} \times \hat{u} \) give an equation for \( u \),

\[
(v - \nu_n)^2 = \left(u - \frac{f_\parallel}{\rho_n \kappa}\right)^2 + \left(\frac{f_\perp}{\rho_n \kappa}\right)^2.
\]

The roots of this equation are determined by the \( u \)-dependence of the functions \( f_\parallel \) and \( f_\perp \). For consistency with equation (2), the latter must satisfy the limit \( f_\perp(u) \to 0 \) as \( u \to 0 \). In general, there can be either one or three roots in each of the sectors \( |v - \nu_n| < v_m \) and \( |v - \nu_n| > v_m \). A simple example is the instance in which the solid is stationary in the rotating coordinates so that \( u = -v_L \), with force components \( f_\parallel = 0 \), and \( f_\perp \) given by equation (3). The solutions are shown in Fig. 2 for a set of four values of \( \nu_n \).

In case (1), the single root corresponds with an unpinned vortex moving with a velocity \( v_L \approx \nu_n \). If \( \nu_n \) were decreased to case (2), there would be three roots including a pinned solution at very small \( u \). Further reduction to case (3) leads to a critical value \( u = u_c \), \( u_c = 0.3v_m \) for \( b = 0.3v_m \) and to a transition to the pinned solution. At the smaller \( \nu_n \) of case (4), only the pinned solution exists. If equation (6) has only one root, there is, of course, no discontinuity and \( u \) varies continuously with \( \nu_n \). Obviously, this discussion is based on our definition and use of macroscopic averaging, but gives some idea of the unpinning and repinning processes for an isolated vortex. The very low vortex densities in neutron matter suggest that these, rather than multiple-vortex processes involving vortex-vortex interactions, are the significant consideration.

2.2 The hydrodynamic equations

Although the non-dissipative force component \( f_\perp \) is expected to be finite for general values of \( u \), any attempt to give it an explicit form would be much less well-based than equation (3). Thus the assumption \( f_\perp = 0 \) adopted here is the only practical basis for obtaining numerical solutions of the hydrodynamic equations. Resolved into components parallel with the unit vectors \( \hat{r} \) and \( \hat{\theta} \) in cylindrical polar coordinates, equations (4) and (5) are,

\[
f_\parallel(\hat{r} - \rho_n \kappa (v_L \hat{\theta} - \nu_n \hat{\theta})) = 0,
\]
shear displacement is negligibly small and it is therefore appropriate to adopt the initial conditions \( \nabla r P = \nabla \theta P = 0 \). It will be convenient to eliminate \( \nu_L \) from equations (7)-(10) in favour of \( \mathbf{u} \). We also neglect any small differences between the local \( \mathbf{k} \) and the \( \mathbf{z} \) axis. The solid velocity \( \mathbf{v} = \nu_s \sin \omega t \) is resolved into components parallel and perpendicular to the \( \mathbf{z} \) axis, only the latter, \( \nu_L \), being present in equations (7)-(10). The velocity variables are then \( u_r, u_{\theta}, v_{nr} \) and \( v_{n\theta} \), with initial conditions of \( v_{nr}(0) = 0 \) and a specified value of \( v_{n\theta}(0) \). The initial values, \( u_r(0), u_{\theta}, v_{nr}, v_{n\theta} \), are given by the intersection of this function with the circles of radius \( v_n/v_m \). For \( v_n > v_m \) (case 1), the vortex approximately comoves with the superfluid. With decreasing \( v_n \) (cases 2 and 3), the critical value \( u_c \) is approached, leading to vortex repinning (case 4)

\[
\begin{align*}
\dot{v}_{n\theta} &= \rho_n \kappa (v_{nr} - v_{n\theta}) = 0, \\
v_{nr} &= 2\Omega v_{L\theta} - \frac{1}{\rho_n} \nabla_r P, \\
v_{n\theta} &= -2\Omega v_{Lr} - \frac{1}{\rho_n} \nabla_{\theta} P.
\end{align*}
\]

With regard to boundary conditions, the only practical way of obtaining numerical solutions is to assume an infinite system. Thus the boundary condition that \( \mathbf{v}_B \) should have no perpendicular component on the spherical surface of the superfluid at \( \rho_B \mathbf{d} \) is neglected. Solutions are therefore valid only for wavenumbers \( q \approx 10^{-4} \text{ cm}^{-1} \) large enough to be consistent with the infinite system assumption. Equations (7)-(10) also neglect fluctuations in pressure and gravitational potential, a reasonable approximation at the wavenumbers considered here. Any pressure fluctuation caused by the local

For any physically reasonable values of the parameters concerned, this is many orders of magnitude too small to be of interest in the present context.

The rate of energy dissipation per unit length of vortex is \( \mathbf{f} \cdot \mathbf{u} \), which satisfies

\[
\mathbf{f} \cdot \mathbf{u} = \mathbf{f} \cdot \mathbf{v} - \mathbf{f} \cdot \mathbf{v}_L.
\]

Because \( \mathbf{f} + \mathbf{f}_M = 0 \) and \( \mathbf{f}_M \cdot (\mathbf{v}_L - \mathbf{v}_n) = 0 \), this can be rewritten as

\[
\mathbf{f} \cdot \mathbf{u} = \mathbf{f} \cdot \mathbf{v} - \mathbf{f} \cdot \mathbf{v}_n.
\]

This merely states that the dissipation rate is the sum of the energy loss rates for the shear mode (\( \mathbf{f} \cdot \mathbf{v} \)) and for the superfluid. For the chosen initial conditions, equations (7)-(10) have been solved numerically, with attention to the instabilities discussed in Section 2.1, for several complete cycles of the shear mode in order to obtain values of its time-averaged energy loss rate. These rates have been calculated for the interval \( 10^4 < v_{nr} < 10^5 \text{ cm}^{-1} \), where \( v_{nr} \) is the component of the displacement velocity \( \mathbf{v} \) perpendicular to \( \mathbf{k} \), and for a distribution of values of its \( r, \theta \) components. This allows us to obtain, for a given value of \( v_0 \) and for each of the shear mode polarization states, the energy loss rate averaged over an isotropic distribution of its wave vector \( \mathbf{q} \). (The multiple reflection of shear waves makes this average the most appropriate way of presenting the data.) The energy-loss rates per unit length of vortex, divided by \( \rho_n \kappa \) and averaged over both polarization states, are defined as \( D_{\alpha \nu} \) and shown in Fig. 3 as functions of \( v_0 \) for pairs of values of \( f_M/\rho_n \kappa \) and of \( b \). They are quite insensitive to \( \omega, \Omega \) and \( a \). Therefore, we have assumed \( \omega = 10^4 \text{ rad s}^{-1}, \Omega = 10 \text{ rad s}^{-1} \) and \( a = 10^{-4} \text{ cm s}^{-1} \) throughout. Except for velocities close to the unpinning threshold, loss-rates do not depend strongly on the initial pinned-vortex velocity. This is demonstrated by the broken curve in Fig. 3 which is for \( u_r(0) = 0 \), an initial state of an exactly corotating superfluid, \( v_{nr} = v_{n\theta} = 0 \). Away from the threshold, it is almost indistinguishable from the curve for \( u_r(0) = a \) and so we have assumed this latter initial condition for all other curves shown. In general, energy loss-rates become large at values of \( v_0 \) sufficient to produce vortex unpinning and increase to asymptotic values \( \rho_n \kappa D_{\alpha \nu} \approx f_M b \). The mode decay time \( \tau \) at a given amplitude \( v_0 \) can be found from its energy density \( \frac{1}{2} \rho_n v_0^2 \) and the

**Figure 2.** The force per unit length \( f_\parallel \) (in units of \( f_m \)) is shown versus \( u \) (in units of \( v_m \)). The function is that of equation (3) except that the value of the parameter \( a \) has been greatly exaggerated for ease of representation. For the same reason, the second parameter is \( b = 0.3v_m \), a value intermediate between those considered in Figs. 3 and 4. Possible solutions of equation (6), for the case \( v = f_\parallel = 0 \), are given by the intersection of this function with the circles of radius \( v_n/v_m \). For \( v_n > v_m \) (case 1), the vortex approximately comoves with the superfluid. With decreasing \( v_n \) (cases 2 and 3), the critical value \( u_c \) is approached, leading to vortex repinning (case 4)
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The shear-mode energy loss-rate per unit length of vortex, averaged over polarization and for an isotropic distribution of wavevectors, is shown as a function of velocity amplitude \( v_0 \) for given \( v_m \) and \( b \). The broken curve, for \( b = v_m = 10^5 \text{ cm s}^{-1} \), is for the initial condition \( v_{nr} = v_{n\theta} = 0 \).

\[
\frac{v_0^2}{D_{av}} = 4\Omega\tau \frac{\rho_n}{\rho_N} \tag{12}
\]

is shown in Fig. 4. Assuming the lattice parameters calculated by Negele & Vautherin (1973), the density ratio \( \rho_n/\rho_N \) appearing in our, perhaps naive, model of the shear wave varies from zero at the threshold \( \rho_{nd} \) to approximately 8 in the interval \( 10^{13} < \rho < 10^{14} \text{ g cm}^{-3} \) which includes most of the neutron-drip volume. The product \( \Omega\tau \) is less than unity for a substantial interval of shear-wave amplitude.

\[
\tau = \frac{\rho_N v_0^2}{4\rho_n \Omega v_m^2 \theta(v_0 - v_m)} \tag{13}
\]

Figure 3. The shear-mode energy loss-rate per unit length of vortex, averaged over polarization and for an isotropic distribution of wavevectors, is shown as a function of velocity amplitude \( v_0 \) for given \( v_m \) and \( b \). The broken curve, for \( b = v_m = 10^5 \text{ cm s}^{-1} \), is for the initial condition \( v_{nr} = v_{n\theta} = 0 \).

Figure 4. The vertical axis gives the right-hand side of equation (12), which is proportional to \( \Omega\tau \) averaged over polarization and for an isotropic distribution of wavevectors. The parameter sets are those of Fig. 3. Damping is strongest at values of \( v_0 \) in the vicinity of \( v_m \).

3 DISCUSSION AND CONCLUSIONS

The motivation for expressing the force parameter \( f_m \) as in Figs. 3 and 4 is that \( v_m = f_m/\rho_n\kappa \) is an upper limit for the superfluid velocity \( v_{n\theta} \) relative to coordinates rotating with angular velocity \( \Omega \). Thus values of the order of \( 10^3 - 10^6 \text{ cm s}^{-1} \) must exist in some intervals of matter density if angular momentum transfer from crust superfluid is the origin of, or a substantial contributor to, pulsar glitches. Much smaller values may exist in other regions, and the limit \( v_m \rightarrow 0 \) must be approached near \( \rho_{nd} \) and \( \rho_c \). In Section 2.1, it was argued that \( b \) should be of the order of \( \rho_m \), but in Figs. 3 and 4, results are also given for values an order of magnitude smaller. From Fig. 3, we can see that the asymptotic value \( D_{av} \approx v_m b \) gives a very approximate estimate, \( 2\Omega\rho_n v_m b \theta(v_0 - v_m) \), for the energy-loss rate per unit volume, where \( \theta \) is here the unit step function. The order of magnitude of the energy decay time is then

\[
\tau = \frac{\rho_N v_0^2}{4\rho_n \Omega v_m^2 \theta(v_0 - v_m)}.
\]
greater than a. The conventionally defined shear viscosity $\eta$ for the solid (see Landau & Lifshitz 1970, p. 153) is related with the decay time by $\eta = \mu/\tau\omega^2$, where $\mu$ is the shear modulus.

This process has some similarity with stress-induced motion of dislocations, considered briefly by Blaes et al. Both involve dissipative motion of a topological defect, either in the superfluid order parameter or in the lattice, but it is not possible to obtain even an order of magnitude estimate of the importance of the latter process. The force per unit length on a line dislocation is of the order of $b\mu v_{\text{rel}}/c_s$, where $b$ is the Burgers vector (see Landau & Lifshitz 1970, p. 133); it is typically some orders of magnitude smaller than $\rho_s\kappa D_{\text{vis}}$. The dislocation density is not known, but could be many orders of magnitude greater than the vortex density, $2\Omega/\kappa \approx 10^{-3.5}$ cm$^{-2}$. But the time-averaged velocity of an individual dislocation under stress is probably very many orders of magnitude smaller than $v_m$.

The coupling with longitudinal sound referred to in Section 1 introduces additional damping. The most significant source is the bulk viscosity produced by the strangeness-changing four-baryon weak interaction. The amplitude decay time is

$$\tau_{\text{nl}} = \frac{2\rho c_l^2(0)}{\zeta \omega^2}, \quad (14)$$

at matter density $\rho$ and angular frequency $\omega$, where $c_l(0)$ is the longitudinal sound velocity in the zero-frequency limit. The bulk viscosity coefficient $\zeta$ is a function both of temperature and angular frequency. It also depends on the equation of state at core densities. For recent evaluations, based on differing assumptions, we refer to Jones (2001), Haensel et al (2002) and Lindblom & Owen (2002). For $\omega = 10^9$ rad s$^{-1}$, a temperature of $10^9\text{K}$ and $\zeta = 3 \times 10^{28}$ g cm$^{-1}$ s$^{-1}$ (see Jones 2001; equation 45), the decay time is $\tau_{\text{nl}} = 3 \times 10^{-2}$ s, which would increase by several orders of magnitude if the temperature were to decrease to $10^8\text{K}$. The evaluation of Lindblom & Owen gives a larger $\zeta$ and thus an even shorter decay time.

Fig. 3 of Blaes et al shows the computed Alfvén wave luminosity as a function of time following generation of shear waves by a starquake, assuming a magnetic flux density equal to their reference value of $10^{31}$ G. Energy begins to enter the magnetosphere about $10^{-3}$ s after the event and the luminosity persists for times just exceeding $6 \times 10^{-1}$ s. Anomalous X-ray pulsars (AXP) and soft gamma repeaters (SGR) are associated with long rotation periods, $\Omega \approx 1$ rad s$^{-1}$ and, in the magnetar model, with surface magnetic fields exceeding $10^{14}$ G. These very large fields indicate, assuming the $B$-dependence of $\tau_\nu$ given by Blaes et al and referred to in Section 1, that superfluid friction damping times are probably rather longer than the time $\tau_\nu$ for energy transfer by coupling of the shear modes with magnetospheric Alfvén waves. In magnetars, superfluid friction is unlikely to limit the efficiency of this process. A similar conclusion, though depending on temperature, holds for the bulk viscosity damping.

Duncan (1998) has examined the excitation and detectability of global seismic modes, particularly low-order toroidal modes, by starquake events. He assumes that these modes are damped through their interaction with fault lines in the solid and the consequent generation of high-frequency shear waves. But the superfluid friction calculated in the present paper damps such modes, based on shear deformations, directly. The damping times found here are much shorter than those considered by Duncan and would rule out the existence of other than small mode-ampitudes.

The presence of $\Omega$ in equations (12) and (13) represents no more than the mean vortex density of the superfluid. Therefore, superfluid friction would produce extremely strong damping in more rapidly rotating neutron stars, such as the typical radio pulsar with $\Omega \approx 10^{11.2}$ rad s$^{-1}$. This may also be of relevance to the AXP and SGR sources, of which all examples so far observed have long periods. Energy transfer through coupling between shear waves and magnetospheric Alfvén waves, as in the magnetar model of Thompson & Duncan (1995), is unlikely to be efficient at short periods owing to the very strong damping.

Superfluid friction is also likely to be relevant to other current problems such as the possibility of vortex unpinning in precessing neutron stars (Link & Cutler 2002), and the damping of r-modes in rapidly rotating neutron stars (Lindblom, Owen & Morsink 1998; Andersson, Kokkotas & Schutz 1999). Many papers on the latter problem have assumed that the solid crust does not participate in the displacement amplitude of the mode, is stationary in rotating coordinates, and therefore acts only as a boundary condition on a thin viscous or turbulent dissipative boundary layer. But more recent work (Levin & Ushomirsky 2001; Yoshida & Lee 2001) has recognized that the mode amplitude must penetrate the crust significantly at high values of $\Omega$. (The radial component of the r-mode displacement, though small, must penetrate the crust. Also, an element of solid crust moving with velocity $v$ in the rotating coordinates experiences a Coriolis force which, for $\Omega \approx 10^9$ rad s$^{-1}$, is of the same order of magnitude as the shear stress integrated over its surface.) A mode with amplitude large enough for its velocity of displacement to be of the order of $v_m$ will be subject to strong dissipation in the crust, the energy loss-rate there being of the order of $2\Omega p_s v_m b(\nu = v_m)$ per unit volume, as for shear waves. Unlike kinetic theory or weak-interaction mechanisms of viscous dissipation (Jones 2001; Haensel et al 2002; Lindblom & Owen 2002), this is not cut off by exponential factors involving superfluid energy gaps but is approximately temperature-independent below the critical temperatures. However, the detailed calculation required to obtain the r-mode lifetime is beyond the scope of the present paper.

The second term on the right-hand side of equation (11) is the energy-loss rate for the neutron superfluid. For all the cases in Fig. 3, except for that with the initial condition $u_n(0) = 0$, the superfluid loses kinetic energy in the rotating frame of reference and is gradually brought into corotation with the charged components of the star. The initial rate is of the same order as for the shear mode.

Superfluid spin-down in the presence of shear waves is governed by the form of $f_1(u)$ at $u < u_m$, the position $u_m$ of its maximum being $u_m \approx \sqrt{ab}$ for the specific form given by equation (3). In the absence of shear waves, the solid matter velocity is $v = 0$ and steady-state superfluid spin-down is possible, in principle, if the required vortex-creep velocities, $v_L = -r\dot{\Omega}/2\Omega$ at radius $r$, are smaller than $u_m$. The actual form of $f_1$ in this region is unknown, because it is dependent on thermal activation and quantum-mechanical
tunnelling, but it would not be unreasonable to assume that it allows this condition to be satisfied given the very small values, \( v_L < 10^{-5} \text{ cm s}^{-1} \) required. Under this assumption, some perturbing movement of the solid is needed to induce vortex unpinning. Its magnitude depends on the actual form of \( f_R(u) \) near its maximum at \( u_m \), but it is likely that quite small values would be sufficient to satisfy the condition \( |v - v_n| \geq v_m \) in one or more, possibly small, regions of superfluid. Following our discussion at the end of Section 2.1, we can see that vortices in these regions would remain unpinned with rapid superfluid spin-down, even if the initial disturbance were to decay to negligible amplitudes, until the critical value \( u = u_c \) is reached. Thus a significant fraction, \( 1 - \sqrt{(u_c/v_m)^2 + (f_R(u_c)/f_m)^2} \), of the available superfluid angular momentum in these regions is transferred to the charged components of the star in discontinuous glitch or noise events.

It is worth comparing the crust movements considered here with those assumed in the glitch model of Ruderman, Zhu & Chen (1998). The poloidal velocities of the latter model are not necessarily large enough to generate a significant shear-wave energy density; it is possible that they are no more than episodes of plastic flow occurring in times smaller than the observed upper limit (\( \approx 10^2 \text{ s} \)) for pulsar spin-up, thus with velocities many orders of magnitude smaller than the shear-wave velocity \( c_s \). In this case, superfluid is spun up or down by the inward or outward radial displacement of the solid with pinned vortices (see Jones 2002). Vortices are not necessarily unpinned although this could occur in any region where the superfluid velocity \( v_n \) reaches \( v_m \). But crust movement with poloidal velocities of the order of \( c_s \) is not excluded, in which case glitches would be initiated by the \( (v \neq 0) \) velocity perturbations considered here. It is possible that both kinds of movement contribute to the observed glitch population.

This would not be inconsistent with the value of the glitch activity parameter defined by Lyne et al (2000), which is based on the observation that for all except very young or old pulsars, the summed glitch amplitudes \( \delta \Omega_{gi} \) divided by the total observation time is \( T^{-1} \sum_i \delta \Omega_{gi} \approx -1.7 \times 10^{-2} \Omega \). For this group of pulsars, angular momentum of the same order as that of the total crust superfluid is transferred by glitches rather than by the steady-state superfluid spin-down described earlier in this paragraph. Thus all glitches in radio pulsars may be initiated by crust movements, with poloidal velocities of the order of \( c_s \) or much smaller. Such movements occur less frequently, if at all, in very young or old neutron stars for which steady-state superfluid spin-down is more usual. These modes of glitch formation may well extend to very small events, so giving a model for the observed timing noise in the residual phase and its time derivatives.

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