A FRACTIONAL ORDER COVID-19 EPIDEMIC MODEL WITH MITTAG–LEFFLER KERNEL

H. Khan¹, M. Ibrahim², A. Khan³, O. Tunç⁴,⁵, and Th. Abdeljawad⁶

UDC 517.9

We consider a nonlinear fractional-order Covid-19 model in a sense of the Atagana–Baleanu fractional derivative used for the analytic and computational studies. The model consists of six classes of persons, including susceptible, protected susceptible, asymptomatic infected, symptomatic infected, quarantined, and recovered individuals. The model is studied for the existence of solution with the help of a successive iterative technique with limit point as the solution of the model. The Hyers–Ulam stability is also studied. A numerical scheme is proposed and tested on the basis of the available literature. The graphical results predict the curtail of spread within the next 5000 days. Moreover, there is a gradual increase in the population of protected susceptible individuals.

1. Introduction

The coronavirus infection 2019 (Covid-19) is a communicable respiratory disease. SARS-CoV-2 (Severe Acute Respiratory Syndrome Coronavirus) is a disease caused by a newly discovered virus strain [1]. In Wuhan, China, Covid-19 was first identified in December, 2019, and quickly spread over the next four months. Within a short period, more than 2.9 million inhabitants in 185 nations throughout the world were infected and 206 thousand persons passed away [2]. On March 11, 2020, “The World Health Organization” announced that this coronavirus infection is a pandemic [3]. This disease can spread primarily by small droplets via coughing, sneezing, or person-to-person conversations. By contacting polluted surfaces, prone individuals can also be compromised. The most prevalent signs of this disease are fever, nausea, dry coughing, fatigue, breath shortages. All these signs are parts of Covid-19 [4]. Some patients may also have joint pain, nasal stuffiness, runny nose, sore throat, or diarrhea. The symptoms are typically mild but can slowly occur. In order to prevent infection, hand washing, nose covering, and/or mouth covering are advised, while sneezing or coughing, as well as avoiding nose or mouth touching plus some preventive steps for the eyes, and keeping social distances.

Due to the seriousness of the Covid-19 pandemic, many states made drastic decisions in order to curb the distribution of Covid-19 infection. In addition, they checked and covered their healthcare systems. Hence, they ruled the cancellation of public events, closing of public events, schools, public places, borders, restrictions on travel, and lockouts, etc. While these measures were helpful, the indicated lockdown led to the socioeconomic
damage, such as bankruptcy of numerous workplaces, loss of the respective positions of a part of the staff, and so on. Further, this shutdown also disrupted supply chains and decreased productivity. The shutdown of China’s drug-producing plants, i.e., the shutdown of the second largest pharmaceutical products exporter delayed the deliveries of generic drug processing factories [5]. The sectors of tourism, air transport, and oil were visibly influenced. It is also clear that invisible impacts are expected irrespective of the duration of pandemic. According to “The International Monetary Fund,” the worldwide economy is expected to shrink by 3% in 2020 [6].

Governments try to prevent the failures of economy, thinking about security measures in order to relax the lockdown. Some advanced countries intend to grant immunity passports, which show immunity to the illness. However, this technique was disapproved by “The World Health Organization”, since there is a lack of adequate scientific proof that reinfection is not possible in this case. A risk balancing strategy was adopted by the South African government to lift the lockout restrictions progressively.

We refer the readers to some scientific works done on infectious diseases [7–9] and, in particular, to several fractional mathematical models [10, 13–34].

In the present paper, we consider the following Covid-19 model for the existence, stability, and numerical simulations based on the use of the Atanga–Baleanu fractional derivative in Caputo’s sense; for details, we refer the readers to [29, 35]:

\[
\begin{align*}
\frac{ABC}{0} D_t^{\alpha_1} S &= \Lambda_1 + \gamma Q - \alpha_1 S - \frac{(1-\alpha_1)(\alpha_2 \eta_1 I_A + \alpha_3 \eta_2 I_S + \eta_3 Q)S}{S + S_P + I_A + I_S + Q + R} - \mu S, \\
\frac{ABC}{0} D_t^{\alpha_2} S_P &= \alpha_1 S - \mu S_P, \\
\frac{ABC}{0} D_t^{\alpha_3} I_A &= \frac{(1-\alpha_1)(\alpha_2 \eta_1 I_A + \alpha_3 \eta_2 I_S + \eta_3 Q)S}{S + S_P + I_A + I_S + Q + R} - (\alpha_2 \rho + \alpha_2 r_1 + 1 - \alpha_2 + \mu) I_A, \\
\frac{ABC}{0} D_t^{\alpha_4} I_S &= \alpha_2 \rho I_A - (1 - \alpha_3 + \alpha_3 r_2 + \mu + \delta) I_S, \\
\frac{ABC}{0} D_t^{\alpha_5} Q &= (1 - \alpha_2) I_A + (1 - \alpha_3) I_S - (\gamma + r_3 + \mu + \delta) Q, \\
\frac{ABC}{0} D_t^{\alpha_6} R &= \alpha_2 r_1 I_A + \alpha_3 r_2 I_S + R_3 Q - \mu R.
\end{align*}
\]

The population is divided into six compartments. These are: susceptible class \( S \), protected susceptible class \( S_P \), asymptomatic infected but not quarantined class \( I_A \), symptomatic infected not quarantined class \( I_S \), quarantined class \( Q \), and recovered class \( R \). The fractional orders are denoted by \( \varphi^*_1 \in (0, 1] \). The parameters are as follows: \( \alpha_1 \) is the fraction of protected susceptible class, \( \alpha_2 \) is the fraction of unidentified asymptomatic infected, \( \alpha_3 \) is the fraction of unidentified symptomatic infected, \( \eta_1 \) is the contact rate between \( S \) and \( I_A \), \( \eta_2 \) is the contact rate between \( S \) and \( I_S \), \( \eta_3 \) is the contact rate between \( S \) and \( Q \), \( \rho \) is the disease progression rate from \( I_A \) to \( I_S \), \( r_1 \) is the recovery rate of \( I_A \), \( r_2 \) is the recovery rate of \( I_S \), \( r_3 \) is the recovery rate of \( Q \), \( \delta \) is the death rate caused by the Covid-19 disease, \( \gamma \) is the proportion of nonaffected quarantine class, and \( \mu \) is the natural mortality rate. As far as the ABC-fractional calculus is concerned, we highlight the following useful literature.

**Definition 1.** The ABC-fractional differential operator on \( \psi \in H^*(a, b), b > a, \) for \( \varphi^*_1 \in [0, 1] \) is defined as follows:
\[ ABC_a^\varphi \psi (\tau) = \frac{B(\varphi^\ast)}{1 - \varphi_1^\ast} \int_a^\tau \psi'(s) E_{\varphi^\ast} \left[ \frac{-\varphi^\ast(\tau-s)\varphi^\ast}{1 - \varphi^\ast} \right] ds, \] (1)

where \( B(\varphi^\ast) \) satisfies the property \( B(0) = B(1) = 1. \)

**Definition 2.** For \( \psi \in H^*(a,b), b > a, \) and \( \varphi^\ast \in [0,1], \) the ABR-fractional derivative is defined as follows:

\[ ABR_a^\varphi \psi (\tau) = \frac{B(\varphi^\ast)}{1 - \varphi^\ast} \frac{d}{d\tau} \int_a^\tau \psi(s) E_{\varphi^\ast} \left[ \frac{-\varphi^\ast(\tau-s)\varphi^\ast}{1 - \varphi^\ast} \right] ds. \]

**Definition 3.** The AB-integral of \( \psi \in H^*(a,b), b > a, \) and \( 0 < \varphi_1^\ast < 1 \) is given by

\[ AB_a^\varphi \psi (\tau) = \frac{1 - \varphi_1^\ast}{B(\varphi_1^\ast)} \psi (\tau) + \frac{\varphi_1^\ast}{B(\varphi_1^\ast) \Gamma(\varphi_1^\ast)} \int_a^\tau \psi(s)(\tau-s)^{\varphi_1^\ast-1} ds. \]

**Lemma 1.** The AB fractional derivative and the AB fractional integral of the function \( \psi \) satisfy the Newton–Leibniz formula

\[ AB_a^\varphi \left( ABC_a^\varphi \psi (\tau) \right) = \psi (\tau) - \psi (a). \]

2. Existence Criteria

By the AB-fractional integral and the Covid-19 model (1), we have

\[
S(t) - S(0) = \frac{1 - \varphi_1^\ast}{\beta(\varphi_1^\ast)} \left[ \Lambda_1 + \gamma Q - \alpha_1 S - \frac{(1 - \alpha_1)(\alpha_2 \eta_1 I_A + \alpha_3 \eta_2 I_S + \eta_3 Q)S}{S + S_P + I_A + I_S + Q + R} - \mu S \right] \\
+ \frac{\varphi_1^\ast}{\beta(\varphi_1^\ast) \Gamma(\varphi_1^\ast)} \int_0^t (t-s)^{\varphi_1^\ast-1} \left[ \Lambda_1 + \gamma Q - \alpha_1 S \right. \\
- \frac{(1 - \alpha_1)(\alpha_2 \eta_1 I_A + \alpha_3 \eta_2 I_S + \eta_3 Q)S}{S + S_P + I_A + I_S + Q + R} - \mu S \left. \right] ds, \\
S_P(t) - S_P(0) = \frac{1 - \varphi_2^\ast}{\beta(\varphi_2^\ast)} [\alpha_1 S - \mu S_P] + \frac{\varphi_1^\ast}{\beta(\varphi_2^\ast) \Gamma(\varphi_2^\ast)} \int_0^t (t-s)^{\varphi_2^\ast-1} [\alpha_1 S - \mu S_P] ds.
\]
\[ I_A(t) - I_A(0) = \frac{1 - \varphi^*_3}{\beta(\varphi^*_3)} \left[ \frac{(1 - \alpha_1)(\alpha_2 \eta_1 I_A + \alpha_3 \eta_2 I_S + \eta_3 Q)S}{S + S_P + I_A + I_S + Q + R} \right. \\
- (\alpha_2 \rho + \alpha_2 r_1 + 1 - \alpha_2 + \mu)I_A \left. \right] + \frac{\varphi^*_1}{\beta(\varphi^*_3) \Gamma \varphi^*_3} \int_0^t (t - s)\varphi^*_3 - 1 \right] ds, \\
\]

\[ I_S(t) - I_S(0) = \frac{1 - \varphi^*_4}{\beta(\varphi^*_4)} \left[ \alpha_2 \rho I_A - (1 - \alpha_3 + \alpha_3 r_2 + \mu + \delta)I_S \alpha_2 \rho I_A \right. \\
- (1 - \alpha_3 + \alpha_3 r_2 + \mu + \delta)I_S \left. \right] + \frac{\varphi^*_4}{\beta(\varphi^*_4) \Gamma \varphi^*_4} \int_0^t (t - s)\varphi^*_4 - 1 \left[ (1 - \alpha_2)I_A + (1 - \alpha_3)I_S - (\gamma + r_3 + \mu + \delta)Q \right] ds, \\
\]

\[ Q(t) - Q(0) = \frac{1 - \varphi^*_5}{\beta(\varphi^*_5)} \left[ (1 - \alpha_2)I_A + (1 - \alpha_3)I_S - (\gamma + r_3 + \mu + \delta)Q \right. \\
+ \frac{\varphi^*_5}{\beta(\varphi^*_5) \Gamma \varphi^*_5} \int_0^t (t - s)\varphi^*_5 - 1 \left[ (1 - \alpha_2)I_A + (1 - \alpha_3)I_S - (\gamma + r_3 + \mu + \delta)Q \right] ds, \\
\]

\[ R(t) - R(0) = \frac{1 - \varphi^*_6}{\beta(\varphi^*_6)} \left[ \alpha_2 r_1 I_A + \alpha_3 r_2 I_S + R_3 Q - \mu R \right. \\
+ \frac{\varphi^*_6}{\beta(\varphi^*_6) \Gamma \varphi^*_6} \int_0^t (t - s)\varphi^*_6 - 1 \left[ \alpha_2 r_1 I_A + \alpha_3 r_2 I_S + R_3 Q - \mu R \right] ds. \\
\]

Assume the functions \( Y_i, i = 1, \ldots, 6, \) are given below:

\[ Y_1(t, S) = \Lambda_1 + \gamma Q - \alpha_1 S - \frac{(1 - \alpha_1)(\alpha_2 \eta_1 I_A + \alpha_3 \eta_2 I_S + \eta_3 Q)S}{S + S_P + I_A + I_S + Q + R} - \mu S, \]

\[ Y_2(t, S_P) = \alpha_1 S - \mu S_P, \]

\[ Y_3(t, I_A) = \frac{(1 - \alpha_1)(\alpha_2 \eta_1 I_A + \alpha_3 \eta_2 I_S + \eta_3 Q)S}{S + S_P + I_A + I_S + Q + R} - (\alpha_2 \rho + \alpha_2 r_1 + 1 - \alpha_2 + \mu)I_A. \]

\[ Y_4(t, I_S) = \alpha_2 \rho I_A - (1 - \alpha_3 + \alpha_3 r_2 + \mu + \delta)I_S, \]
\[ Y_5(t, Q) = (1 - \alpha_2) I_A + (1 - \alpha_3) I_S - (\gamma + r_3 + \mu + \delta) Q. \]
\[ Y_6(t, R) = \alpha_2 r_1 I_A + \alpha_3 r_2 I_S + R_3 Q - \mu R. \]

\[ \begin{align*}
    \psi_1 &= \alpha_1 + k_1 + \mu, \\
    \psi_2 &= \mu, \\
    \psi_3 &= k_2 + (\alpha_2 \rho + \alpha_2 r_1 + 1 - \alpha_2 + \mu), \\
    \psi_4 &= \mu_c, \\
    \psi_5 &= 1 - \alpha_3 + \alpha_3 r_2 + \mu + \delta, \\
    \psi_6 &= \mu.
\end{align*} \]

**Assumption (B).** Assume that, for \( S(t), S^*(t), S_p(t), S^*_p(t), I_A(t), I^*_A(t), I_s(t), I^*_s(t), Q(t), Q^*(t), R(t), \) \( R^*(t) \in L[0, 1], \) there exists constants \( \kappa_i > 0, i = 1, \ldots, 6, \) such that

\[ \|S(t)\| \leq \kappa_1, \quad \|S_p(t)\| \leq \kappa_2, \quad \|I_A(t)\| \leq \kappa_3, \quad \|I_s(t)\| \leq \kappa_4, \quad \|Q(t)\| \leq \kappa_5, \quad \|R(t)\| \leq \kappa_6, \quad \xi_1, \xi_2 > 0, \]

and

\[ \|S(t) + I_A(t) + Q(t)\| \leq \xi_1, \]

\[ \|I_s(t) + R(t)\| \leq \xi_2. \]

**Theorem 1.** The functions \( Y_i, i \in N^6, \) satisfy the Lipschitz condition provided that Assumption (B) is obeyed.

**Proof.** For \( Y_1, \) we obtain

\[
\|Y_1(t, S) - Y_1(t, S^*)\| = \left\| \Lambda_1 + \gamma Q - \alpha_1 S - \frac{(1 - \alpha_1) (\alpha_2 \eta_1 I_A + \alpha_3 \eta_2 I_S + \eta_3 Q) S}{S + S_p + I_A + I_S + Q + R} - \mu S - \left( \Lambda_1 + \gamma Q - \alpha_1 S^* - \frac{(1 - \alpha_1) (\alpha_2 \eta_1 I_A + \alpha_3 \eta_2 I_S + \eta_3 Q) S^*}{S^* + S_p + I_A + I_S + Q + R} - \mu S^* \right) \right\|
\]

\[
\leq \left\| \alpha_1 + \frac{(1 - \alpha_1) (\alpha_2 \eta_1 I_A + \alpha_3 \eta_2 I_S + \eta_3 Q) S^*}{S^* + S_p + I_A + I_S + Q + R} + \mu \right\| \|S - S^*\|
\]

\[
\leq [\alpha_1 + k_1 + \mu]\|S_c - S^*_c\| = \psi_1\|S - S^*\|. \tag{2}
\]

For \( Y_2(t, E_c), \) we get
\[ \| Y_2(t, S_P) - Y_2(t, S_P^*) \| = \| (\alpha_1 S - \mu S_P) - (\alpha_1 S - \mu S_P^*) \| \]
\[ \leq [\mu] \| S_c - E_c^* \| \leq \psi_2 \| E_c - E_c^* \|. \] (3)

Further, for \( Y_3(t, I_A^*) \), we find
\[ \| Y_3(t, I_A) - Y_3(t, I_A^*) \| = \left\| \left( \frac{(1 - \alpha_1)(\alpha_2 \eta_1 I_A + \alpha_3 \eta_2 I_S + \eta_3 Q)S}{S + S_P + I_A + I_S + Q + R} \right) \right. \]
\[ - (\alpha_2 \rho + \alpha_2 r_1 + 1 - \alpha_2 + \mu) I_A \]
\[ \left. - \left( \frac{(1 - \alpha_1)(\alpha_2 \eta_1 I_A^* + \alpha_3 \eta_2 I_S + \eta_3 Q)S}{S + S_P + I_A^* + I_S + Q + R} \right) \right. \]
\[ - (\alpha_2 \rho + \alpha_2 r_1 + 1 - \alpha_2 + \mu) I_A^* \right\| \]
\[ \leq \left\| \left( \frac{(1 - \alpha_1)(\alpha_2 \eta_1 I_A + \alpha_3 \eta_2 I_S + \eta_3 Q)S}{S + S_P + I_A + I_S + Q + R} \right) \right. \]
\[ + (\alpha_2 \rho + \alpha_2 r_1 + 1 - \alpha_2 + \mu) \right\| I_A - I_A^* \right\| \]
\[ \leq k_2 + (\alpha_2 \rho + \alpha_2 r_1 + 1 - \alpha_2 + \mu) \| I_c - I_c^* \| \]
\[ = \psi_3 \| I_A - I_A^* \| \] (4)

For \( Y_4(t, I) \), we obtain
\[ \| Y_4(t, I_s) - Y_4(t, I_s^*) \| = \left\| (\alpha_2 \rho I_A - (1 - \alpha_3 + \alpha_3 r_2 + \mu + \delta) I_S) \right. \]
\[ - (\alpha_2 \rho I_A - (1 - \alpha_3 + \alpha_3 r_2 + \mu + \delta) I_S^*) \right\| \]
\[ \leq \| 1 - \alpha_3 + \alpha_3 r_2 + \mu + \delta \| I_s - I_s^* \|
\[ \leq \| 1 - \alpha_3 + \alpha_3 r_2 + \mu + \delta \| I_s - I_s^* \| \leq \psi_4 \| I_s - I_s^* \| \] (5)

For \( Y_5(t, Q) \), we get
\[ \| Y_5(t, Q) - Y_5(t, Q^*) \| = \left\| (1 - \alpha_2) I_A + (1 - \alpha_3) I_S - (\gamma + r_3 + \mu + \delta) Q \right. \]
\[ - (1 - \alpha_2) I_A^* - (1 - \alpha_3) I_S^* - (\gamma + r_3 + \mu + \delta) Q^* \right\| \]
Further, for $Y_6(t, R)$ we have

$$
\|Y_6(t, R) - Y_6(t, R^*)\| = \left\| (\alpha_2 r_1 I_A + \alpha_3 r_2 I_S + r_3 Q - \mu R.)
\right\|
- (\alpha_2 r_1 I_A + \alpha_3 r_2 I_S + r_3 Q - \mu R^*)
\right\|
\leq \|\mu\| \| R - R^* \| = \psi_6 \| R - R^* \|.
$$

Thus, it follows from (2)–(7) that $Y_i, i = 1, \ldots, 6,$ satisfy the Lipschitz condition.

This completes the proof.

Assume that

$$
S(0) = S_p(0) = I_A(0) = I_S(0) = Q(0) = R(0) = 0.
$$

This yields

$$
S(t) = \frac{1 - \varphi_1^*}{\beta(\varphi_1^*)} \gamma_1(t, S(t)) + \frac{\varphi_1^*}{\beta(\varphi_1^*) \Gamma(\varphi_1^*)} \int_0^t (t - s)^{\varphi_1^* - 1} \gamma_1(s, S(s)) ds,
$$

$$
S_p(t) = \frac{1 - \varphi_2^*}{\beta(\varphi_2^*)} \gamma_2(t, S_p(t)) + \frac{\varphi_2^*}{\beta(\varphi_2^*) \Gamma(\varphi_2^*)} \int_0^t (t - s)^{\varphi_2^* - 1} \gamma_2(s, S_p(s)) ds,
$$

$$
I_A(t) = \frac{1 - \varphi_3^*}{\beta(\varphi_3^*)} \gamma_3(t, I_A(t)) + \frac{\varphi_3^*}{\beta(\varphi_3^*) \Gamma(\varphi_3^*)} \int_0^t (t - s)^{\varphi_3^* - 1} \gamma_3(s, I_A(s)) ds,
$$

$$
I_S(t) = \frac{1 - \varphi_4^*}{\beta(\varphi_4^*)} \gamma_4(t, I_S(t)) + \frac{\varphi_4^*}{\beta(\varphi_4^*) \Gamma(\varphi_4^*)} \int_0^t (t - s)^{\varphi_4^* - 1} \gamma_4(s, I_S(s)) ds,
$$

$$
Q(t) = \frac{1 - \varphi_5^*}{\beta(\varphi_5^*)} \gamma_5(t, Q(t)) + \frac{\varphi_5^*}{\beta(\varphi_5^*) \Gamma(\varphi_5^*)} \int_0^t (t - s)^{\varphi_5^* - 1} \gamma_5(s, Q(s)) ds,
$$

$$
R(t) = \frac{1 - \varphi_6^*}{\beta(\varphi_6^*)} \gamma_6(t, R(t)) + \frac{\varphi_6^*}{\beta(\varphi_6^*) \Gamma(\varphi_6^*)} \int_0^t (t - s)^{\varphi_6^* - 1} \gamma_6(s, R(s)) ds.
$$
For the iterative scheme of the model (1), we define

\[
S_n(t) = \frac{1 - \varphi_1^*}{\beta(\varphi_1^*)} \gamma_1(t, S_{n-1}(t)) + \frac{\varphi_1^*}{\beta(\varphi_1^*) \Gamma(\varphi_1^*)} \int_0^t (t - s)^{\varphi_1^*-1} \gamma_1(s, S_{n-1}(s)) ds,
\]

\[
S_{p_n}(t) = \frac{1 - \varphi_2^*}{\beta(\varphi_2^*)} \gamma_2(t, S_{p_{n-1}}(t)) + \frac{\varphi_2^*}{\beta(\varphi_2^*) \Gamma(\varphi_2^*)} \int_0^t (t - s)^{\varphi_2^*-1} \gamma_2(s, S_{p_{n-1}}(s)) ds,
\]

\[
I_{A_n}(t) = \frac{1 - \varphi_3^*}{\beta(\varphi_3^*)} \gamma_3(t, I_{A_{n-1}}(t)) + \frac{\varphi_3^*}{\beta(\varphi_3^*) \Gamma(\varphi_3^*)} \int_0^t (t - s)^{\varphi_3^*-1} \gamma_3(s, I_{A_{n-1}}(s)) ds,
\]

\[
I_{S_n}(t) = \frac{1 - \varphi_4^*}{\beta(\varphi_4^*)} \gamma_4(t, I_{S_{n-1}}(t)) + \frac{\varphi_4^*}{\beta(\varphi_4^*) \Gamma(\varphi_4^*)} \int_0^t (t - s)^{\varphi_4^*-1} \gamma_4(s, I_{S_{n-1}}(s)) ds,
\]

\[
Q_n(t) = \frac{1 - \varphi_5^*}{\beta(\varphi_5^*)} \gamma_5(t, Q_{n-1}(t)) + \frac{\varphi_5^*}{\beta(\varphi_5^*) \Gamma(\varphi_5^*)} \int_0^t (t - s)^{\varphi_5^*-1} \gamma_5(s, Q_{n-1}(s)) ds,
\]

\[
R_n(t) = \frac{1 - \varphi_6^*}{\beta(\varphi_6^*)} \gamma_6(t, R_{n-1}(t)) + \frac{\varphi_6^*}{\beta(\varphi_6^*) \Gamma(\varphi_6^*)} \int_0^t (t - s)^{\varphi_6^*-1} \gamma_6(s, R_{n-1}(s)) ds.
\]

**Theorem 2.** The fractional-order Covid-19 model (1) has a solution provided that

\[
\Delta = \max \{ \Psi_i \} < 1, \quad i \in N^6.
\]

**Proof.** We define the functions

\[
K_1(t) = S_{n+1}(t) - S(t), \quad K_2(t) = S_{p_{n+1}}(t) - S_p(t), \quad K_3(t) = I_{A_{n+1}}(t) - I_A(t),
\]

\[
K_4(t) = I_{S_{n+1}}(t) - I_S(t), \quad K_5(t) = Q_{n+1}(t) - Q(t), \quad K_6(t) = R_{n+1}(t) - R(t).
\]

Thus, by using the above equations, we conclude that

\[
\| K_1 \| \leq \frac{1 - \varphi_1^*}{\beta(\varphi_1^*)} \| \gamma_1(t, S_n(t)) - \gamma_1(t, S_{n-1}(t)) \|
\]

\[
+ \frac{\varphi_1^*}{\beta(\varphi_1^*) \Gamma(\varphi_1^*)} \int_0^t (t - s)^{\varphi_1^*-1} \| \gamma_1(s, S_n(s)) - \gamma_1(t, S_{n-1}(t)) \| ds.
\]
\[
\|\mathcal{K}_{2n}\| \leq \frac{1 - \varphi_2^*}{\beta(\varphi_2^*)} \|\mathcal{Y}_2(t, S_{p_n}(t)) - \mathcal{Y}_2(t, S_{p_{n-1}}(t))\| \\
+ \frac{\varphi_2^*}{\beta(\varphi_2^*)} \Gamma(\varphi_2^*) \int_0^t (t-s)^{\varphi_2^*-1} \|\mathcal{Y}_2(s, S_{p_n}(s)) - \mathcal{Y}_2(t, S_{p_{n-1}}(t))\| ds
\]

and
\[
\|\mathcal{K}_{3n}\| \leq \frac{1 - \varphi_3^*}{\beta(\varphi_3^*)} \|\mathcal{Y}_3(t, I_{A_n}(t)) - \mathcal{Y}_3(t, I_{A_{n-1}}(t))\| \\
+ \frac{\varphi_3^*}{\beta(\varphi_3^*)} \Gamma(\varphi_3^*) \int_0^t (t-s)^{\varphi_3^*-1} \|\mathcal{Y}_3(s, I_{A_n}(s)) - \mathcal{Y}_3(t, I_{A_{n-1}}(t))\| ds
\]

Similarly,
\[
\|\mathcal{K}_{4n}\| \leq \frac{1 - \varphi_4^*}{\beta(\varphi_4^*)} \|\mathcal{Y}_4(t, I_{s_n}(t)) - \mathcal{Y}_4(t, I_{s_{n-1}}(t))\| \\
+ \frac{\varphi_4^*}{\beta(\varphi_4^*)} \Gamma(\varphi_4^*) \int_0^t (t-s)^{\varphi_4^*-1} \|\mathcal{Y}_4(s, I_{s_n}(s)) - \mathcal{Y}_4(t, I_{s_{n-1}}(t))\| ds
\]

and
\[
\|\mathcal{K}_{5n}\| \leq \frac{1 - \varphi_5^*}{\beta(\varphi_5^*)} \|\mathcal{Y}_5(t, I_{s_{n+1}}(t)) - \mathcal{Y}_5(t, I_{s_{n-1}}(t))\| \\
+ \frac{\varphi_5^*}{\beta(\varphi_5^*)} \Gamma(\varphi_5^*) \int_0^t (t-s)^{\varphi_5^*-1} \|\mathcal{Y}_5(s, I_{s_{n+1}}(s)) - \mathcal{Y}_5(t, I_{s_{n-1}}(t))\| ds
\]
\[
\frac{1 - \varphi_i^*}{\beta(\varphi_i^*)} \leq \left[ \frac{1 - \phi_i}{\beta(\phi_i)} + \frac{1}{\beta(\phi_i) \Gamma(\phi_i)} \right]^n \Delta^n \| I_i - I_s \|, \\
\| K_5 \| \leq \frac{1 - \phi_5^*}{\beta(\phi_5^*)} \| Y_5(t, Q_n(t)) - Y_5(t, Q_{n-1}(t)) \|
\]

\[
+ \frac{\phi_5^*}{\beta(\phi_5^*) \Gamma(\phi_5^*)} \int_0^t (t - s)^{\phi_5^* - 1} \| Y_5(s, Q_n(s)) - Y_5(t, Q_{n-1}(t)) \| ds
\]

\[
\leq \left[ \frac{1 - \phi_5^*}{\beta(\phi_5^*)} + \frac{1}{\beta(\phi_5^*) \Gamma(\phi_5^*)} \right]^n \Delta^n \| I_1 - I_s \|, \\
\| K_6 \| \leq \frac{1 - \phi_6^*}{\beta(\phi_6^*)} \| Y_6(t, R_n(t)) - Y_6(t, R_{n-1}(t)) \|
\]

\[
+ \frac{\phi_6^*}{\beta(\phi_6^*) \Gamma(\phi_6^*)} \int_0^t (t - s)^{\phi_6^* - 1} \| Y_6(s, R_n(s)) - Y_6(t, R_{n-1}(t)) \| ds
\]

\[
\leq \left[ \frac{1 - \phi_6^*}{\beta(\phi_6^*)} + \frac{1}{\beta(\phi_6^*) \Gamma(\phi_6^*)} \right]^n \Delta^n \| R_1 - R_s \|.
\]

Thus, we get \( K_i \| \rightarrow 0, i = 1, \ldots, 6 \), as \( n \rightarrow \infty \) for \( \Delta < 1 \), which is the required proof.

3. Uniqueness of Solution

For our suggested model (1), we now analyze the problem of uniqueness of the solution.

**Theorem 3.** The Covid-19 model (1) has a unique solution if

\[
\left[ \frac{1 - \phi_i}{\beta(\phi_i)} + \frac{1}{\beta(\phi_i) \Gamma(\phi_i)} \right] \psi_i \leq 1, \quad i \in \mathcal{N}_6.
\]

**Proof.** Assume that there exists another solution \( \overline{S}(t), \overline{S}_c(t), \overline{I}_A(t), \overline{I}_s(t), \overline{Q}(t), \overline{R}(t) \) such that

\[
\overline{S}(t) = \frac{1 - \phi_1^*}{\beta(\phi_1^*)} Y_1(t, \overline{S}(t)) + \frac{\phi_1^*}{\beta(\phi_1^*) \Gamma(\phi_1^*)} \int_0^t (t - s)^{\phi_1^* - 1} Y_1(s, \overline{S}(s)) ds,
\]

...
\[
S_p(t) = \frac{1 - \varphi_2^*}{\beta(\varphi_2^*)} \mathcal{Y}_2(t, S_p(t)) + \frac{\varphi_2^*}{\beta(\varphi_2^*) \Gamma(\varphi_2^*)} \int_0^t (t-s)^{\varphi_2^*-1} \mathcal{Y}_2(s, S_p(s)) ds.
\]

\[
T_A(t) = \frac{1 - \varphi_3^*}{\beta(\varphi_3^*)} \mathcal{Y}_3(t, T_A(t)) + \frac{\varphi_3^*}{\beta(\varphi_3^*) \Gamma(\varphi_3^*)} \int_0^t (t-s)^{\varphi_3^*-1} \mathcal{Y}_3(s, T_A(s)) ds,
\]

\[
T_s(t) = \frac{1 - \varphi_4^*}{\beta(\varphi_4^*)} \mathcal{Y}_4(t, T_s(t)) + \frac{\varphi_4^*}{\beta(\varphi_4^*) \Gamma(\varphi_4^*)} \int_0^t (t-s)^{\varphi_4^*-1} \mathcal{Y}_4(s, T_s(s)) ds.
\]

\[
\mathcal{Q}(t) = \frac{1 - \varphi_5^*}{\beta(\varphi_5^*)} \mathcal{Y}_5(t, \mathcal{Q}(t)) + \frac{\varphi_5^*}{\beta(\varphi_5^*) \Gamma(\varphi_5^*)} \int_0^t (t-s)^{\varphi_5^*-1} \mathcal{Y}_5(s, \mathcal{Q}(s)) ds,
\]

\[
\mathcal{R}(t) = \frac{1 - \varphi_6^*}{\beta(\varphi_6^*)} \mathcal{Y}_6(t, \mathcal{R}(t)) + \frac{\varphi_6^*}{\beta(\varphi_6^*) \Gamma(\varphi_6^*)} \int_0^t (t-s)^{\varphi_6^*-1} \mathcal{Y}_6(s, \mathcal{R}(s)) ds.
\]

Thus,

\[
\|S(t) - \mathcal{S}(t)\| \leq \frac{1 - \varphi_1^*}{\beta(\varphi_1^*)} \|\mathcal{Y}_1(t, S(t)) - Y_1(t, \mathcal{S}(t))\| + \frac{\varphi_1^*}{\beta(\varphi_1^*) \Gamma(\varphi_1^*)} \int_0^t (t-s)^{\varphi_1^*-1} \|Y_1(s, S(s)) - Y_1(t, \mathcal{S}(t))\| ds
\]

\[
= \left[ \frac{1 - \varphi_1^*}{\beta(\varphi_1^*)} + \frac{1}{\beta(\varphi_1^*) \Gamma(\varphi_1^*)} \right] \psi_1 \|S - \mathcal{S}\|.
\]

whence it follows that

\[
\left[ \frac{1 - \varphi_1^*}{\beta(\varphi_1^*)} \psi_1 + \frac{\psi_1}{\beta(\varphi_1^*) \Gamma(\varphi_1^*)} - 1 \right] \|S - \mathcal{S}\| \geq 0.
\]

(15)

By virtue of (14), relation (15) is true if

\[
\|S - \mathcal{S}\| = 0,
\]

which implies that \(S(t) = \mathcal{S}(t)\). Similarly, we have
\[
\|S_p(t) - \overline{S}_p(t)\| \leq \frac{1 - \phi_2^*}{\beta(\phi_2^*)} \| \mathcal{V}_2(t) - Y_2(t, \overline{S}_p(t)) \| \\
+ \frac{\phi_2^*}{\beta(\phi_2^*)\Gamma(\phi_2^*)} \int_0^t (t - s)^{\phi_2^* - 1} \| Y_2(s, S_p(s)) - Y_2(t, \overline{S}_p(t)) \| \ ds \\
\leq \left[ \frac{1 - \phi_2^*}{\beta(\phi_2^*)} + \frac{1}{\beta(\phi_2^*)\Gamma(\phi_2^*)} \right] \psi_2 \|S_p - \overline{S}_p\| .
\]

which follows. By virtue of (14), inequality (16) is true for
\[
\|S_p - \overline{S}_p\| = 0,
\]
which implies that \(S_p(t) = \overline{S}_p(t)\).

Further, for \(I_A\), we obtain
\[
\|I_A(t) - \overline{I}_A(t)\| \leq \frac{1 - \phi_3^*}{\beta(\phi_3^*)} \| \mathcal{V}_3(t) - Y_3(t, \overline{I}_A(t)) \|

+ \frac{\phi_3^*}{\beta(\phi_3^*)\Gamma(\phi_3^*)} \int_0^t (t - s)^{\phi_3^* - 1} \| Y_3(s, I_A(s)) - Y_3(t, \overline{I}_A(t)) \| \ ds \\
\leq \left[ \frac{1 - \phi_3^*}{\beta(\phi_3^*)} + \frac{1}{\beta(\phi_3^*)\Gamma(\phi_3^*)} \right] \psi_3 \|I_A - \overline{I}_A\| .
\]

This gives
\[
\left[ \frac{1 - \phi_3^*}{\beta(\phi_3^*)} \psi_3 + \frac{\psi_3}{\beta(\phi_3^*)\Gamma(\phi_3^*)} - 1 \right] \|I_A - \overline{I}_A\| \geq 0. \tag{17}
\]

Thus, in view of (14), inequality (17) is true if \(\|I_A - \overline{I}_A\| = 0\), which means that \(I_A(t) = \overline{I}_A(t)\) and, therefore,
\[
\|I_s(t) - \overline{I}_s(t)\| \leq \frac{1 - \phi_4^*}{\beta(\phi_4^*)} \| \mathcal{V}_4(t) - Y_4(t, \overline{I}_s(t)) \|

+ \frac{\phi_4^*}{\beta(\phi_4^*)\Gamma(\phi_4^*)} \int_0^t (t - s)^{\phi_4^* - 1} \| Y_4(s, I_s(s)) - Y_4(t, \overline{I}_s(t)) \| \ ds \\
\leq \left[ \frac{1 - \phi_4^*}{\beta(\phi_4^*)} + \frac{1}{\beta(\phi_4^*)\Gamma(\phi_4^*)} \right] \psi_4 \|I_s - \overline{I}_s\| .
\]

Hence, we get
\[
\left[ \frac{1 - \phi_4^*}{\beta(\phi_4^*)} \psi_4 + \frac{\psi_4}{\beta(\phi_4^*)\Gamma(\phi_4^*)} - 1 \right] \|I_s - \overline{I}_s\| \geq 0. \tag{18}
\]
By virtue of (14), inequality (18) is true if \( \|I_{\delta} - \overline{I}_{\delta}\| = 0 \). This yields \( I_{\delta}(t) = \overline{I}_{\delta}(t) \). Further, for \( Q \), we obtain

\[
\|Q(t) - \overline{Q}(t)\| \leq \frac{1 - \varphi_{5}^*}{\beta(\varphi_{5}^*)} \|Y_{5}(t, Q(t)) - Y_{5}(t, \overline{Q}(t))\| + \frac{\varphi_{5}^*}{\beta(\varphi_{5}^*) \Gamma(\varphi_{5}^*)} \int_{0}^{t} (t - s)^{\varphi_{5}^* - 1} \|Y_{5}(s, Q(s)) - Y_{5}(s, \overline{Q}(s))\| \, ds
\]

This yields

\[
\|Q(t) - \overline{Q}(t)\| \leq \left[ \frac{1 - \varphi_{5}^*}{\beta(\varphi_{5}^*)} + \frac{1}{\psi_{5}} \right] \psi_{5} \| Q - \overline{Q} \|.
\]

This yields

\[
\|R(t) - \overline{R}(t)\| \leq \frac{1 - \varphi_{6}^*}{\beta(\varphi_{6}^*)} \|Y_{6}(t, R(t)) - Y_{6}(t, \overline{R}(t))\| + \frac{\varphi_{6}^*}{\beta(\varphi_{6}^*) \Gamma(\varphi_{6}^*)} \int_{0}^{t} (t - s)^{\varphi_{6}^* - 1} \|Y_{6}(s, R(s)) - Y_{6}(s, \overline{R}(s))\| \, ds
\]

This yields

\[
\|R(t) - \overline{R}(t)\| \leq \left[ \frac{1 - \varphi_{6}^*}{\beta(\varphi_{6}^*)} + \frac{1}{\psi_{6}} \psi_{6} \right] \| R - \overline{R} \|.
\]

Therefore,

\[
\left[ \frac{1 - \varphi_{6}^*}{\beta(\varphi_{6}^*)} + \frac{\psi_{6}}{\beta(\varphi_{6}^*) \Gamma(\varphi_{6}^*)} \right] \psi_{6} \| R - \overline{R} \| \geq 0.
\]

By virtue of (14), this implies that relation (19) is true if \( \| R - \overline{R} \| = 0 \), which means that \( R(t) = \overline{R}(t) \). Thus, model (1) has a unique solution.

4. Hyers–Ulam Stability

**Definition 4.** The integral system (8)–(13) is Hyers–Ulam stable if, for \( \Delta_i > 0 \), \( i \in \mathbb{N}_1^\delta \), and \( \gamma_i > 0 \), \( i \in \mathbb{N}_1^\delta \), we have

\[
\left| S(t) - \frac{1 - \varphi_{1}^*}{\beta(\varphi_{1}^*)} Y_{1}(t, S(t)) - \frac{\varphi_{1}^*}{\beta(\varphi_{1}^*) \Gamma(\varphi_{1}^*)} \int_{0}^{t} (t - s)^{\varphi_{1}^* - 1} Y_{1}(s, S(s)) \, ds \right| \leq \gamma_{1}.
\]

\[
\left| S_{p}(t) - \frac{1 - \varphi_{2}^*}{\beta(\varphi_{2}^*)} Y_{2}(t, S_{p}(t)) - \frac{\varphi_{2}^*}{\beta(\varphi_{2}^*) \Gamma(\varphi_{2}^*)} \int_{0}^{t} (t - s)^{\varphi_{2}^* - 1} Y_{2}(s, S_{p}(s)) \, ds \right| \leq \gamma_{2}.
\]
Further, for $\dot{S}(t)$, $\dot{S}_p(t)$, $\dot{I}_A(t)$, $\dot{I}_s(t)$, $\dot{Q}(t)$, $\dot{R}(t)$, we get

\[
\dot{S}(t) = \frac{1 - x_1^*}{\beta(x_1^*)} \mathcal{Y}_1(t, S(t)) + \frac{x_1^*}{\beta(x_1^*) \Gamma(x_1^*)} \int_0^t (t-s)^{x_1^*-1} \mathcal{Y}_1(s, \dot{S}(s)) \, ds,
\]

\[
\dot{S}_p(t) = \frac{1 - x_2^*}{\beta(x_2^*)} \mathcal{Y}_2(t, S_p(t)) + \frac{x_2^*}{\beta(x_2^*) \Gamma(x_2^*)} \int_0^t (t-s)^{x_2^*-1} \mathcal{Y}_2(s, \dot{S}_p(s)) \, ds,
\]

\[
\dot{I}_A(t) = \frac{1 - x_3^*}{\beta(x_3^*)} \mathcal{Y}_3(t, I_A(t)) + \frac{x_3^*}{\beta(x_3^*) \Gamma(x_3^*)} \int_0^t (t-s)^{x_3^*-1} \mathcal{Y}_3(s, \dot{I}_A(s)) \, ds,
\]

\[
\dot{I}_s(t) = \frac{1 - x_4^*}{\beta(x_4^*)} \mathcal{Y}_4(t, I_s(t)) + \frac{x_4^*}{\beta(x_4^*) \Gamma(x_4^*)} \int_0^t (t-s)^{x_4^*-1} \mathcal{Y}_4(s, \dot{I}_s(s)) \, ds,
\]

\[
\dot{Q}(t) = \frac{1 - x_5^*}{\beta(x_5^*)} \mathcal{Y}_5(t, Q(t)) + \frac{x_5^*}{\beta(x_5^*) \Gamma(x_5^*)} \int_0^t (t-s)^{x_5^*-1} \mathcal{Y}_5(s, \dot{Q}(s)) \, ds,
\]

\[
\dot{R}(t) = \frac{1 - x_6^*}{\beta(x_6^*)} \mathcal{Y}_6(t, R(t)) + \frac{x_6^*}{\beta(x_6^*) \Gamma(x_6^*)} \int_0^t (t-s)^{x_6^*-1} \mathcal{Y}_6(s, \dot{R}(s)) \, ds
\]

such that

\[|S(t) - \dot{S}(t)| \leq \delta_1 \gamma_1,\]

\[|S_p(t) - \dot{S}_p(t)| \leq \delta_2 \gamma_2,\]

\[|I_A(t) - \dot{I}_A(t)| \leq \delta_3 \gamma_3.\]
\[ |I_s(t) - \dot{I}_s(t)| \leq \delta_4 \gamma_4, \]
\[ |Q(t) - \dot{Q}(t)| \leq \delta_5 \gamma_5, \]
\[ |R(t) - \dot{R}(t)| \leq \delta_6 \gamma_6. \]

**Theorem 4.** If Assumption (B) is satisfied, then model (1) is Hyers–Ulam-stable.

**Proof.** By Theorem 3, the Covid-19 model (1) has a unique solution, say, \( S(t), S_p(t), I_A(t), I_s(t), Q(t), R(t) \). Let \((\dot{S}(t), \dot{S}_p(t), \dot{I}_A(t), \dot{I}_s(t), \dot{Q}(t), \dot{R}(t))\) be an approximate solution of (1) satisfying (8)–(13). Thus, we get
\[
\| S(t) - \dot{S}(t) \| \leq \frac{1 - \varphi_1^*}{\beta(\varphi_1^*)} \| Y_1(t, S(t)) - Y_1(t, \dot{S}(t)) \|
\]
\[
+ \frac{\varphi_1^*}{\beta(\varphi_1^*) \Gamma(\varphi_1^*)} \int_0^t (t - s)^{\varphi_1^*-1} \| Y_1(s, S(s)) - Y_1(t, \dot{S}(t)) \| \, ds
\]
\[
\leq \left[ \frac{1 - \varphi_1^*}{\beta(\varphi_1^*)} + \frac{1}{\beta(\varphi_1^*) \Gamma(\varphi_1^*)} \right] \psi_1 \| S - \dot{S} \|.
\]
If we take \( \gamma_1 = \psi_1 \) and
\[
\Delta = \frac{1 - \varphi_1^*}{\beta(\varphi_1^*)} + \frac{\varphi_1^*}{\beta(\varphi_1^*) \Gamma(\varphi_1^*)},
\]
then we get
\[
\| S(t) - \dot{S}(t) \| \leq \gamma_1 \Delta_1.
\]
Similarly, for \( S_p(t), \dot{S}_p(t), I_A(t), \dot{I}_A(t), I_s(t), \dot{I}_s(t), Q(t), \dot{Q}(t), R(t), \dot{R}(t) \), we get
\[
\| S_p(t) - \dot{S}_p(t) \| \leq \gamma_2 \Delta_2,
\]
\[
\| I_A(t) - \dot{I}_A(t) \| \leq \gamma_3 \Delta_3,
\]
\[
\| I_s(t) - \dot{I}_s(t) \| \leq \gamma_4 \Delta_4,
\]
\[
\| Q(t) - \dot{Q}(t) \| \leq \gamma_5 \Delta_5,
\]
\[
\| R(t) - \dot{R}(t) \| \leq \gamma_6 \Delta_6.
\]
This implies that system (1) is Hyers–Ulam stable, which ultimately ensures the stability of (1).

This completes the proof.
5. Numerical Scheme

By using (2)–(7), we produce the following numerical scheme:

\[
\begin{align*}
ABC D_t^{p_1} S(t) &= Y_1(t, S), \\
ABC D_t^{p_2} S_p(t) &= Y_2(t, S_p), \\
ABC D_t^{p_3} I_A(t) &= Y_3(t, I_A), \\
ABC D_t^{p_4} I_s(t) &= Y_4(t, I_s), \\
ABC D_t^{p_5} Q(t) &= Y_5(t, Q), \\
ABC D_t^{p_6} R(t) &= Y_6(t, R).
\end{align*}
\]

With the help of the fractional AB-integral operator, relations (20) take the following form:

\[
\begin{align*}
S(t) - S(0) &= \frac{1 - \varphi_1^*}{\beta(\varphi_1^*)} \mathcal{Y}_1(t, S) + \frac{\varphi_1^*}{\beta(\varphi_1^*) \Gamma(\varphi_1^*)} \int_0^t (t-s)^{\varphi_1^*-1} \mathcal{Y}_1(s, S) ds, \\
S_p(t) - S_p(0) &= \frac{1 - \varphi_2^*}{\beta(\varphi_2^*)} \mathcal{Y}_2(t, I_s) + \frac{\varphi_2^*}{\beta(\varphi_2^*) \Gamma(\varphi_2^*)} \int_0^t (t-s)^{\varphi_2^*-1} \mathcal{Y}_2(s, S_p) ds, \\
I_A(t) - I_A(0) &= \frac{1 - \varphi_3^*}{\beta(\varphi_3^*)} \mathcal{Y}_3(t, I_A) + \frac{\varphi_3^*}{\beta(\varphi_3^*) \Gamma(\varphi_3^*)} \int_0^t (t-s)^{\varphi_3^*-1} \mathcal{Y}_3(s, I_A) ds, \\
I_s(t) - I_s(0) &= \frac{1 - \varphi_4^*}{\beta(\varphi_4^*)} \mathcal{Y}_4(t, I_s) + \frac{\varphi_4^*}{\beta(\varphi_4^*) \Gamma(\varphi_4^*)} \int_0^t (t-s)^{\varphi_4^*-1} \mathcal{Y}_4(s, I_s) ds, \\
Q(t) - Q(0) &= \frac{1 - \varphi_5^*}{\beta(\varphi_5^*)} \mathcal{Y}_5(t, Q) + \frac{\varphi_5^*}{\beta(\varphi_5^*) \Gamma(\varphi_5^*)} \int_0^t (t-s)^{\varphi_5^*-1} \mathcal{Y}_5(s, Q) ds, \\
R(t) - R(0) &= \frac{1 - \varphi_6^*}{\beta(\varphi_6^*)} \mathcal{Y}_6(t, R) + \frac{\varphi_6^*}{\beta(\varphi_6^*) \Gamma(\varphi_6^*)} \int_0^t (t-s)^{\varphi_6^*-1} \mathcal{Y}_6(s, R) ds.
\end{align*}
\]

Dividing the assumed interval \([0, t]\) into subintervals with the help of points \(t_{m+1}\), for \(m = 0, 1, 2, \ldots\), we obtain

\[
S(t_{m+1}) - S(0) = \frac{1 - \varphi_1^*}{\beta(\varphi_1^*)} \mathcal{Y}_1(t_m, S) + \frac{\varphi_1^*}{\beta(\varphi_1^*) \Gamma(\varphi_1^*)} \sum_{k=0}^n \int_{t_k}^{t_{k+1}} (t_{k+1} - s)^{\varphi_1^*-1} \mathcal{Y}_1(s, S) ds.
\]
Fig. 1. Comparative analysis for the $S(t)$ and $S_P(t)$ and the following orders: 1.0, 0.99, 0.98, and 0.97.

\[
S_P(t_{m+1}) - S_P(0) = \frac{1 - \varphi^*_2}{\beta(\varphi^*_2)} \mathcal{Y}_2(t_m, S_P) + \frac{\varphi^*_2}{\beta(\varphi^*_2) \Gamma(\varphi^*_2)} \sum_{k=0}^{t_{k+1}} \int_{t_k}^{t_{k+1}} (t_{m+1} - s)^{\varphi^*_2 - 1} \mathcal{Y}_2(s, S_P) ds,
\]

\[
I_A(t_{m+1}) - I_A(0) = \frac{1 - \varphi^*_3}{\beta(\varphi^*_3)} \mathcal{Y}_3(t_m, I_A) + \frac{\varphi^*_3}{\beta(\varphi^*_3) \Gamma(\varphi^*_3)} \sum_{k=0}^{t_{k+1}} \int_{t_k}^{t_{k+1}} (t_{m+1} - s)^{\varphi^*_3 - 1} \mathcal{Y}_3(s, I_A) ds,
\]

\[
I_S(t_{m+1}) - I_S(0) = \frac{1 - \varphi^*_4}{\beta(\varphi^*_4)} \mathcal{Y}_4(t_m, I_S) + \frac{\varphi^*_4}{\beta(\varphi^*_4) \Gamma(\varphi^*_4)} \sum_{k=0}^{t_{k+1}} \int_{t_k}^{t_{k+1}} (t_{m+1} - s)^{\varphi^*_4 - 1} \mathcal{Y}_4(s, I_S) ds,
\]

\[
Q(t_{m+1}) - Q(0) = \frac{1 - \varphi^*_5}{\beta(\varphi^*_5)} \mathcal{Y}_5(t_m, Q) + \frac{\varphi^*_5}{\beta(\varphi^*_5) \Gamma(\varphi^*_5)} \sum_{k=0}^{t_{k+1}} \int_{t_k}^{t_{k+1}} (t_{m+1} - s)^{\varphi^*_5 - 1} \mathcal{Y}_5(s, Q) ds,
\]

\[
R(t_{m+1}) - R(0) = \frac{1 - \varphi^*_6}{\beta(\varphi^*_6)} \mathcal{Y}_6(t_m, R) + \frac{\varphi^*_6}{\beta(\varphi^*_6) \Gamma(\varphi^*_6)} \sum_{k=0}^{t_{k+1}} \int_{t_k}^{t_{k+1}} (t_{m+1} - s)^{\varphi^*_6 - 1} \mathcal{Y}_6(s, R) ds.
\]

Now, by using Lagrange’s interpolation, we get
Comparison of $S(t)$ for different orders:

- $S_{\alpha = 1.0}$
- $S_{\alpha = 0.99}$
- $S_{\alpha = 0.98}$
- $S_{\alpha = 0.97}$

**Fig. 2.** Comparative analysis of the $S(t)$ for the orders 1.0, 0.99, 0.98, and 0.97.

\[
S(t_{m+1}) = S(0) + \frac{1 - \varphi_1^*}{\beta(\varphi_1^*)} \mathcal{Y}_1(t_k, S) \sum_{k=0}^{n} \frac{h^{\varphi_1^*} \mathcal{Y}_1(t_k, S)}{\Gamma(\varphi_1^* + 2)}
\]

\[
\times \left( (m + 1 - k)^{\varphi_1^*} (m - k + 2 + \varphi_1^*) - (m - k)^{\varphi_1^*} (m - k + 2 + 2 \varphi_1^*) \right)
\]

\[
- \frac{h^{\varphi_1^*} \mathcal{Y}_1(t_{k-1}, S)}{\Gamma(\varphi_1^* + 2)} \left( (m + 1 - k)^{\varphi_1^*} - (m - k)^{\varphi_1^*} (m + 1 - k + \varphi_1^*) \right).
\]

\[
S_p(t_{m+1}) = S_p(0) + \frac{1 - \varphi_2^*}{\beta(\varphi_2^*)} \mathcal{Y}_2(t_k, S_p) \sum_{k=0}^{n} \frac{h^{\varphi_2^*} \mathcal{Y}_2(t_k, S_p)}{\Gamma(\varphi_2^* + 2)}
\]

\[
\times \left( (m + 1 - k)^{\varphi_2^*} (m - k + 2 + \varphi_2^*) - (m - k)^{\varphi_2^*} (m - k + 2 + 2 \varphi_2^*) \right)
\]

\[
- \frac{h^{\varphi_2^*} \mathcal{Y}_2(t_{k-1}, S_p)}{\Gamma(\varphi_2^* + 2)} \left( (m + 1 - k)^{\varphi_2^*} - (m - k)^{\varphi_2^*} (m + 1 - k + \varphi_2^*) \right).
\]

\[
I_A(t_{m+1}) = I_A(0) + \frac{1 - \varphi_3^*}{\beta(\varphi_3^*)} \mathcal{Y}_3(t_k, I_A) \sum_{k=0}^{n} \frac{h^{\varphi_3^*} \mathcal{Y}_3(t_k, I_A)}{\Gamma(\varphi_3^* + 2)}
\]

\[
\times \left( (m + 1 - k)^{\varphi_3^*} (m - k + 2 + \varphi_3^*) - (m - k)^{\varphi_3^*} (m - k + 2 + 2 \varphi_3^*) \right)
\]
Comparison of $S_P(t)$ for different orders

Fig. 3. Comparative analysis of the $S_P(t)$ for the orders 1.0, 0.99, 0.98, and 0.97.

$S_P(t)$ at $\alpha = 0.99$

$S_P(t)$ at $\alpha = 0.98$

$S_P(t)$ at $\alpha = 0.97$
This numerical scheme helps us to predict the role of protected susceptible persons, which was practically exercised in various nations as a control strategy. Although this strategy has the worst effect on the economy of a nation, it is essential to curtail the process of spread of the infection of lethal Covid-19. The sensitivity analysis was given in [36]. It shows that the role of this strategy is very much effective in the curtail of the spread process.

5.1. Numerical Results. In this section, we provide a detail of numerical results related to the model with the data available from the literature. The parameters and initial data were taken from the available literature. The initial values are as follows: \( S(0) = 59300000, S_P(0) = 0, I_S(0) = 0, Q(0) = 0, I_A(0) = 2079, R(0) = 903, \) and the values of parameters are as follows: \( \alpha_1 = 0.0008, \alpha_2 = 0.1, \eta a_1 = 0.25, \rho = 0.0001, \eta_2 = 0, \eta_3 = 0.385, r_1 = 0.2976, r_2 = 0, \gamma = 0, r_3 = 0.2976, \mu = 0.00236/90, \delta = 0.017/90, \alpha_3 = 1, \) and \( \Lambda_1 = 296425.875/90 \) [36].

In Fig. 1, we present a joint comparative simulation for the two classes \( S(t) \) and \( S_P(t) \) for the following orders: 1.0, 0.99, 0.98, 0.97. In Fig. 2, we present a graphical study of the \( S(t) \) class for various orders 1.0, 0.99, 0.98, and 0.97 for a long period of 5000 days. There is a decrease in the population of the class. Moreover, as the order decreases, a relatively large decrease is observed in the population while the behavior of the class remains similar. Figure 3 shows the comparative analysis of the \( S_P(t) \) for the indicated orders and a gradual increase can be seen in the graph.
In Fig. 4, we present the plots for the infected population, which show an increase detected up to 300 days and a decrease observed after 300–600 days. In Fig. 5, we show a numerical representation of the class $I_S(t)$ for various orders, whereas Fig. 6 displays the plots for the $R(t)$ class.

6. Conclusions

In the present article, we focus on the theoretical and computational studies of the fractional-order Covid-19 model in the ABC-sense of derivative. The existence and uniqueness results were carried out with the help of an iterative sequential approach with limit point as the solution of the suggested model (1). We also estimated the Hyers–Ulam stability and a numerical scheme was obtained on the basis of Lagrange’s interpolation. The numerical scheme was then tested and very similar results, like the integer order, were obtained. The numerical results were interpreted with the help of six graphs. The details are as follows: In Fig. 1, we present a joint comparative simulation for the two classes $S(t)$ and $S_P(t)$ and the orders 1.0, 0.99, 0.98, and 0.97. In Fig. 2, we present a graphical study of the $S(t)$ class for various orders 1.0, 0.99, 0.98, and 0.97 for a long time of 5000 days. There is a decrease in the population of the analyzed class. Moreover, as the order decreases, we observe a relatively large decrease in the population, while the behavior of the class remains similar. In Fig. 3, we show the comparative analysis of the class $S_P(t)$ for the mentioned orders and a gradual increase can be seen in the graph. Figure 4 is for the infected population and shows an increase for up to 300 days followed by a decrease observed after 300–600 days. Figure 5 shows a numerical representation of the class $I_S(t)$ for the various orders. Finally, Fig. 6 shows the behavior of the $R(t)$ class. The reader of the paper can work on the comparative analysis of different fractional operators for higher accuracy and better results.
Acknowledgements

A. Khan and T. Abdeljawad would like to thank Prince Sultan University for the support through the TAS research lab. In addition, the authors would like to thank Prof. T. Abdeljawad for his support in improving the paper.

REFERENCES

1. A. E. Gorbalenya, S. C. Baker, R. S. Baric, R. J. de Groot, C. Drosten, A. A. Gulyaeva, B. L. Haagmans, C. Lauber, A. M. Leontovich, B. W. Neuman, and D. Penzar, “Coronaviridae study group of the international committee on taxonomy of viruses. The species severe acute respiratory syndrome-related coronavirus: classifying 2019-nCoV and naming it SARS-CoV-2,” Nat. Microbiol., 5, No. 4, 536–544 (2020).

2. S. A. Morrison, I. Gregor, and S. Gregor, “Responding to a global pandemic: Republic of Slovenia on maintaining physical activity during self-isolation,” Scand. J. Med. Sci. Sports, 30, No. 8 (2020).

3. N. Kokudo and H. Sugiyama, “Call for international cooperation and collaboration to effectively tackle the Covid-19 pandemic,” Global Health Med., 30, 2(2), 2–60 (2020).

4. S. Omer and S. Ali, “Preventive measures and management of Covid-19 in pregnancy,” Drugs Therapy Perspect., 36, No. 6 (2020).

5. K. F. Owusu, E. F. Goufo, and S. Mugisha, “Modelling intracellular delay and therapy interruptions within Ghanaian HIV population,” Adv. Difference Equat., 1, 1–9 (2020).

6. M. Anderson, M. Mckee, and E. Mossialos, “Developing a sustainable exit strategy for Covid-19: health, economic and public policy implications,” J. R. Soc. Med., 113(5), 8–176 (2020).

7. E. D. Goufo and R. Maritz, “A note on ebolas outbreak and human migration dynamic,” J. Human Ecol., 51, No. 3, 257–263 (2015).

8. P. T. Djomegni, A. Tekle, and M. Y. Dawed, “Pre-exposure prophylaxis HIV/AIDS mathematical model with non classical isolation,” Jap. J. Ind. Appl. Math., 37, 781–801 (2020).

9. K. F. Owusu, E. F. Goufo, and S. Mugisha, “Modelling intracellular delay and therapy interruptions within Ghanaian HIV population,” Adv. Difference Equat., 1, 1–9 (2020).
10. Z. A. Khan, F. Jarad, A. Khan, and H. Khan, “Nonlinear discrete fractional sum inequalities related to the theory of discrete fractional calculus with applications,” *Adv. Difference Equat.*, 1, 1–3 (2021).

11. A. Khan, H. M. Alshehri, T. Abdeljawad, Q. M. Al-Mدائل, and H. Khan, “Stability analysis of fractional nabla difference Covid-19 model,” *Results Phys.*, 4, 103–888 (2021).

12. A. Shah, R. A. Khan, A. Khan, H. Khan, and J. F. Gomez-Aguilar, “Investigation of a system of nonlinear fractional order hybrid differential equations under usual boundary conditions for existence of solution,” *Math. Meth. Appl. Sci.*, 30, No. 2, 1628–38 (2021).

13. T. Abdeljawad, “A Lyapunov type inequality for fractional operators with non singular Mittag-Leffler kernel,” *J. Inequal Appl.*, 130, (2017); DOI: 10.1186/s13660-017-1400-5.

14. T. Abdeljawad, “Fractional operators with generalized Mittag-Leffler kernels and their iterated differentials,” *Chaos*, 29, No. 2, 023102 (2019).

15. T. Abdeljawad and D. Baleanu, “Discrete fractional differences with nonsingular discrete Mittag-Leffler kernels,” *Adv. Difference Equat.*, Paper No. 232 (2016).

16. T. Abdeljawad, Q. M. Al-Mدائل, and F. Jarad, “Fractional logistic models in the frame of fractional operators generated by conformable derivatives,” *Chaos Solitons Fractals*, 119, 94–101 (2019).

17. T. Abdeljawad, M. A. Hajji, Q. M. Al-Mدائل, and F. Jarad, “Analysis of some generalized ABC-fractional logistic models,” *Alexandria Eng. J.*, 59, No. 4, 8–2141 (2020).

18. B. Acay, E. Bas, and T. Abdeljawad, “Fractional economic models based on market equilibrium in the frame of different type kernels,” *Chaos Solitons Fractals*, 130, 109438 (2020).

19. A. Atangana and S. I. Araz, “Mathematical model of Covid-19 spread in Turkey and South Africa: theory, methods and applications,” *Adv. Difference Equat.*, Paper No. 659 (2020).

20. A. Atangana and S. I. Araz, “Nonlinear equations with global differential and integral operators: existence, uniqueness with application to epidemiology,” *Results Phys.*, 20, 103593 (2021).

21. A. Atangana and D. Baleanu, “New fractional derivatives with nonlocal and non-singular kernel: theory and application to heat transfer model,” *Thermal Sci.*, 20(00), (2016); DOI: 10.2298/TSCI160111018A.

22. I. V. Atamas’ and V. I. Slinko, “Stability of the fixed points of a class of quasilinear cascades in the space conv R^n,” (Russian. English, *Ukrainian Summary*) *Ukr. Mat. Zh.*, 69, No. 8, 1166–1179 (2017).

23. M. Arfan, K. Shah, T. Abdeljawad, N. Mlaiki, and A. Ullah, “A Caputo power law model predicting the spread of the Covid-19 outbreak in Pakistan,” *Alexandria Eng. J.*, 1, 60(1), 56–447 (2021).

24. R. Begum, O. Tunç, H. Khan, H. Gulzar, and A. Khan, “A fractional order Zika virus model with Mittag-Leffler kernel,” *Chaos Solitons Fractals*, 146, No. 110 898 (2021).

25. M. Bohner, O. Tunç and C. Tunç, “Qualitative analysis of Caputo fractional integro-differential equations with constant delays,” *Comput. Appl. Math.*, 40, No. 6, Paper No. 214 (2021).

26. A. I. Dvirnyi and V. I. Slyn’ko, “Stability of solutions of pseudolinear differential equations with impulse action,” *Math. Notes*, 93, No. 5-6, 691–703 (2013).

27. J. F. Gomez, L. Torres, and R. F. Escobar, “Fractional derivatives with Mittag-Leffler kernel. Trends and applications in science and engineering,” *Studies in Systems, Decision and Control*, 194, Springer, Cham (2019).

28. F. Jarad, T. Abdeljawad, and Z. Hammouch, “On a class of ordinary differential equations in the frame of Atangana–Baleanu fractional derivative,” *Chaos Solitons Fractals*, 1, No. 117, 16–20 (2018).

29. K. M. Owolabi and A. Atangana, “On the formulation of Adams–Bashforth scheme with Atangana–Baleanu–Caputo fractional derivative to model chaotic problems,” *Chaos*, 29, No. 2, 023111 (2019).

30. K. Shah, M. A. Alqudah, F. Jarad, and T. Abdeljawad, “Semi-analytical study of pine wilt disease model with convex rate under Caputo–Febrizio fractional order derivative,” *Chaos Solitons Fractals*, 135, 109754 (2020).

31. O. Tunç, O. Atan, C. Tunç, and J. C. Yao, “Qualitative analyses of integro-fractional differential equations with Caputo derivatives and retardations via the Lyapunov–Razumikhin method,” *Axioms*, 10, No. 2 (2021); DOI: https://doi.org/10.3390/axioms10020058.

32. E. Tunç and O. Tunç, “On the oscillation of a class of damped fractional differential equations,” *Miskolc Math. Notes*, 17, No. 1, 647–656 (2016).

33. M. Yavuz and T. Abdeljawad, “Nonlinear regularized long-wave models with a new integral transformation applied to the fractional derivative with power and Mittag-Leffler kernel,” *Adv. Difference Equat.*, Paper No. 367 (2020).

34. F. B. Yousef, A. Yousef, T. Abdeljawad, and A. Kalinli, “Mathematical modeling of breast cancer in a mixed immune-chemotherapy treatment considering the effect of ketogenic diet,” *Eur. Phys. J. Plus.*, 135, No. 12, 1–23 (2020).

35. M. A. Khan, A. Atangana, and E. Alzahrani, “The dynamics of Covid-19 with quarantined and isolation,” *Adv. Difference Equat.*, 2020, No. 1, 1–22 (2020).

36. P. M. Djomegni, M. D. Haggar, W. T. Adigo, “Mathematical model for Covid-19 with “protected susceptible” in the post-lockdown era,” *Alexandria Eng. J.*, 1, 60(1), 35–527 (2021).