A framework to predict the load-settlement behavior of shallow foundations in a range of soils from silty clays to sands using CPT records

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Abstract
Using a set of cone penetration test (CPT) records, the current paper develops a general framework based on regression analyses to model the load-settlement (q-s) behavior of shallow foundations resting on a variety of soils ranging from silty clays to sands. A three-parameter hyperbolic function is employed to rigorously examine the obtained q-s curves and to determine the model parameters. Also, the results of some CPT soundings, including the corrected cone tip resistance (qt) and the skin friction (Rf), are adopted to predict the results of plate load tests (PLT). The findings corroborate the high accuracy of the proposed model, the reasonable performance of the hyperbolic function and the use of the Volterra series to predict the q-s curves. Moreover, the obtained curves from the newly developed model are compared to those from other methods in the literature which cross-confirms the efficacy of the current model. A sensitivity analysis is also conducted, and the exclusive effects of all the contributing parameters are assessed among which Rf is shown to be the most influential. Ultimately, simple solutions are adopted to determine various key geotechnical parameters, like the ultimate bearing capacity (qult), the allowable bearing capacity (qa) and the modulus of subgrade reaction (ks).

Keywords Load-settlement curve · CPT · Ultimate bearing capacity · Volterra series · Sensitivity analysis

List of symbols

| Symbol | Description |
|--------|-------------|
| a      | Net area ratio of cone tip |
| ai     | Constant coefficient of Volterra series |
| Bf     | Foundation width |
| Bp     | Plate diameter |
| CPT    | Cone penetration test |
| De     | Effective depth |
| Df     | Embedment depth |
| fs     | Shaft resistance |
| FS     | Factor of safety |
| hs     | Empirical fitting term that depends on the soil type |
| Ic     | Soil material index |
| q      | Load |
| qa     | Allowable bearing capacity |
| qc     | Cone tip resistance |
| qc, ave, qc, mean | Averaged CPT cone resistance |
| qcapacity | Foundation bearing capacity of the ground |
| qult   | Ultimate bearing capacity |
| qult, f | Ultimate bearing capacity of the foundation |
| qult, p | Ultimate bearing capacity of the plate |
| Rf     | Skin friction |
| ksf    | Modulus of subgrade reaction of full-sized footing |
| ksp    | Modulus of subgrade reaction from PLT |
| kφ     | Coefficient for ultimate bearing capacity calculation |
| MAD    | Mean absolute deviation |
| qmax   | Maximum value of applied pressure |
| qnet   | Net cone resistance |
| qt     | Corrected qc |
| qult   | Ultimate bearing capacity |

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\( R_{k/t} \) Coefficient for ultimate bearing capacity calculation
\( RMSE \) Root mean square error
\( R^2 \) Absolute fraction of variance
\( s \) Settlement
\( s_a \) Allowable settlement
\( s_f \) Settlement of the foundation
\( s_{f, all} \) Allowable settlement of the foundation
\( s_i \) Settlement of an arbitrary point
\( s_p \) Settlement of the plate
\( s_{p, all} \) Allowable settlement of the plate
\( u_2 \) Pore water pressure
\( \alpha, \beta, \gamma \) Model parameters
\( \sigma_{vo} \) Effective overburden stress at bearing elevation

1 Introduction

Load-settlement \((q-s)\) behavior of soils is commonly investigated by the plate load test \((PLT)\) through which the ultimate bearing capacity \((q_{ult})\) and allowable bearing capacity \((q_a)\) of shallow foundations are determined. As a result of \(PLT\), the settlement \((s)\) value is measured against the specified applied pressure \((q)\), which consequently provides the basic geotechnical properties required for the design of shallow foundations. Based on the \(PLT\) results, different criteria have been suggested to compute \(q_{ult}\) from the \(q-s\) curves. In the study performed by Kulhawy (2004), the loading corresponding to the settlement/width ratio of 0.1 \((\frac{s}{B} = 0.1)\) was proposed to be \(q_{ult}\) for all soil types except the sensitive clays, where the loading associated with \(\frac{s}{B} = 0.04\) introduces \(q_{ult}\). The mentioned criterion is well-known and distinguished in European standard (Amar et al. 1998).

One of the consequential benefits from \(q-s\) curves is the determination of the modulus of subgrade reaction \((k_s)\), which is simply acquired from dividing the applied pressure by the corresponding settlement \((k_s = \frac{q}{s})\). \(k_s\) can be suggested based on the allowable settlement associated with different types of foundations, including shallow footings and mat foundations, and according to various standards in different countries. It can also be suggested as the slope of the initial tangent line to the load-settlement curve at zero settlement; yet, this definition is not widely used in geotechnical engineering practice. Generally, the calculation of \(k_s = \frac{q}{s}\) yields more conservative designs in the former method, where it is dependent on the settlement level and is effectively estimated for each specific allowable settlement value. In many previously proposed correlations, like the one presented by Bowles (1996), \(k_s\) is calculated for the settlement of 1 inch which is simplified as the following relation:

\[
k_s = \frac{q_{ult}}{s = 1\text{inch}(25\text{mm})} = 40q_{ult} = 40 \times FS \times q_a\]

(1)

where \(FS\) is the safety factor, and \(q_a\) is the allowable bearing capacity.

It is worth noting that \(k_s\) depends on some parameters like the shape and width of foundation as well as the spatial variability of the stiffness/strength parameters of the soil underneath (Mohseni et al. 2018). Vesic (1975) showed different values of \(k_s\) for foundations having the same sizes and applied pressures. Also, Farouk and Farouk (2014) indicated that the rigidity of the soil-footing system should not be neglected for the determination of \(k_s\). In terms of linear elastic models, soil plasticity is ignored, which is the reason why the distribution of \(k_s\) is anomalous at the edges of the foundation. To allow for such an anomaly, non-linear models should be employed. Terzaghi (1955) proposed Eq. (2) in order to evaluate the modulus of subgrade reaction of full-sized footings resting on sandy subgrades.

\[
k_{sf} = k_{sp} \left( \frac{B_p + B_f}{2B_f} \right)^2
\]

(2)

where \(k_{sp}\) is the modulus of subgrade reaction from \(PLT\), \(k_{sf}\) is the corresponding value for the actual foundation, \(B_p\) is the plate diameter, and \(B_f\) is the foundation width.

Given the very fact that \(PLTs\) are costly and time-consuming, researchers have been recently more inclined to invoke empirical relationships to yield \(k_s\), the majority of which are endorsed by in situ tests (Naeini et al. 2018). A variety of field tests, among which the \(SPT\) and cone penetration test \((CPT)\) are the most recent common ones, have garnered the attention of researchers to provide experimental correlations for \(PLT\) parameters. So far, less attention has been paid to the correlation between the results of \(PLT\) and \(CPT\).

\(CPT\) is one of the most useful common in situ tests, which can lend itself to the prediction of non-linear soil constitutive parameters, time-dependent stress–strain characteristics and profiling the soil elasto-plastic behavior (Holz et al. 2011). On the other hand, given the very fact that repeatability is easily achieved in \(CPT\) soundings, and also that the accurate stratification and soil layering interpretations are viable with \(CPT\), even in small thicknesses, it is therefore preferred among other common in situ tests (Eslami et al. 2017, 2019).

Of late, the link between the \(PLT\) parameters and those of \(CPTs\) has been investigated in two different categories, the first of which is to directly obtain the bearing capacity based on \(CPT\) results and the second one is to predict the \(q\-
s curves through which $q_{ult}$ can be estimated. In this article, the second category is adopted. The proposed methods and relationships for the determination of the ultimate bearing capacity from CPT results are enlisted in Table 1 in chronological order (Saftner and Dagger 2018).

Deploying CPT results, many researchers have tried to correlate $q$ to the dimensionless parameter, $\frac{s}{B}$, which consequently has resulted in the following general formula (Fellenius and Altaee 1994; Decourt 1999; Briaud 2007).

$$q = a_f \left( \frac{s}{B} \right)^{b_f}$$

(3)

where $q$ is the footing bearing pressure, $s/B$ is the footing settlement normalized against the footing width $B$, and $a_f$ and $b_f$ are the model parameters. Based on Mayne et al. (2012), the value of $b_f$ was suggested to be 0.5, and the value of $a_f$ for sands was reported to be dependent on the particle size distribution, relative density, geological condition and the percentage of fine grains. Regarding the ultimate bearing capacity criterion for sandy soils, when $\frac{s}{B} = 0.1$, $q_{ult}$ equates to 0.316$a_f$. Based on the theoretical studies conducted by Briaud (2007) and Mayne et al. (2012), Eq. (4) was proposed using 32 $q$-$s$ data sets on 13 different soil types (note that $q_c$ is the cone tip resistance).

$$q = 0.585q_c \sqrt{\frac{s}{B}}$$

(4)

Stuedlein and Holtz (2010) assigned 1.77 MPa for $a_f$ of the soil known as delta soil, yet it should be noted that the number of the studied data was quite limited. Mayne and Woeller (2014) used $q_{net}$ instead of $q_c$ in Eq. (4) and rewrote it as Eq. (5).

$$q = h_vq_{net} \left( \frac{s}{B} \right)^{0.5} < q_{capacity}$$

(5)

where $q_{net}$ is the net cone resistance, $q_{capacity}$ is the foundation bearing capacity of the ground, and $h_v$ is the empirical fitting term which was considered to be 0.58, 1.12, 1.47 and 2.7, for sands, silts, crushed (fissured) soils and clays, respectively. It is worth noting that the value of $q_{capacity}$ is approximately close to the maximum value of $q$ in the load–displacement profile ($q_{max}$). Mayne (2014) incorporated the soil material index, $I_c$, as an indicative of the type of soil behavior, into the other empirical relationships and proposed a correlation which could predict the $q$-$s$ trend [Eq. (6)].

$$q = q_{net} \sqrt{\frac{s}{B}} \left[ 2.8 - \frac{2.3}{1 + \left( \frac{b}{25} \right)^{1.8}} \right]$$

(6)

Most researchers, who have previously studied the relationship between $q$-$s$ profiles and CPT results, have confirmed that $q$ is an a priori function of $\left( \frac{s}{B} \right)^{0.5}$ based on the correlations they have presented whose differences lay solely on the magnification factor. However, the mentioned relationships are open to question as $q$ grows excessively in higher settlements. Their proposed correlations were therefore more acceptable for the values of $\frac{s}{B}$ up to 0.1, beyond which they would render unrealistic $q$ values. In this study, an attempt has been made to reconcile this by establishing a novel and more appealing model, which could intrinsically capture the limiting nature of the bearing pressure.

Generally, it should be noted that the $q$-$s$ curves in this study were obtained from reliable sources including 46 PLTs and their corresponding CPT logs. In the following sections, first, the collected database and the input and

| Method | Surface footing | Remarks and embedment |
|--------|-----------------|-----------------------|
| Eslami and Gholami (2006) | $q_{ult} = R_{u1} \times q_c$ | Where $R_{u1}$ is a function of $D/B$ and $q_c/\sigma_{u1}'$<br>Note: Measured $q_c$ and $q_c/\sigma_{u1}'$ are geometric means<br>Influence zone: Depth of 2B beneath footing |
| Briaud (2007) | $q_{ult} = K_\phi \times q_c$<br>($K_\phi = 0.23$) | Based on full-scale tests at Texas A&M University<br>Influence zone: Depth of 2B beneath footing |
| Robertson and Cabal (2014) | $q_{ult} = K_\phi \times q_c$<br>($K_\phi = 0.16$) | Where $K_\phi$ is a function of $B/De$, shape, and density<br>Influence zone: Depth of 2B beneath footing |
| Mayne and Illingworth (2010) | $q_{ult} = 0.18 q_c^{\text{mean}}$ | Based on 30 footing load tests on 12 sands<br>Note: $q_c^{\text{mean}}$ is averaged CPT cone resistance<br>Influence zone: Depth of 1.5B beneath footing |
| Lehane (2012) | $q_{ult} = 0.16 q_c^{\text{ave}}$ | Based on 47 load tests<br>Note: $q_c^{\text{ave}}$ is averaged CPT cone resistance<br>Influence zone: $[B (m)]^{0.7}$ |
Table 2 A summary of the data employed in this research

| Soil type       | Case No | Footing shape | From CPT | From PLT | Reference               |
|-----------------|---------|---------------|----------|----------|-------------------------|
|                 |         |               | $q_t$ (kPa) | $R_f$ | $D_f$ (m) | $B$ (m)* | $L/B$ | $q_{ult}$ (kPa) | $s$ (mm) |             |
| Silt            | N1      | Square        | 2500     | 0.5 | 0 | 1 | 300 | 98 | Eslami and Gholami (2006) |
|                 | N2      |               | 2800     | 0.5 | 0 | 1 | 325 | 97 |
| Silty sand      | N3      | Square        | 7000     | 0.5 | 0 | 0.6 | 1 | 1260 | 55 |
|                 | N4      |               | 10,000   | 0.5 | 0 | 0.6 | 1 | 1280 | 55 |
| Silty clay      | N5      | Circular      | 1400     | 0.6 | 0 | 0.45 | 1 | 170 | 40 |
|                 | N6      |               | 1700     | 0.6 | 0 | 0.6 | 1 | 170 | 55 |
|                 | N7      |               | 2000     | 0.6 | 0 | 0.6 | 1 | 170 | 55 |
| Silty clay      | N8      | Circular      | 3100     | 0.6 | 1.5 | 0.6 | 1 | 520 | 60 |
|                 | N9      |               | 4600     | 0.6 | 1.5 | 0.6 | 1 | 310 | 55 |
|                 | N10     |               | 5400     | 0.6 | 1.5 | 0.6 | 1 | 310 | 60 |
|                 | N11     |               | 6000     | 0.6 | 1.5 | 0.6 | 1 | 690 | 60 |
| Glaciofluvial sand| N12    | Rectangular  | 10,720   | 0.51 | 0.4 | 0.6 | 1 | 1740 | 59 | Mayne and Illingworth (2010) |
|                 | N13     |               | 10,720   | 0.51 | 0.6 | 1.2 | 1 | 1740 | 119 |
|                 | N14     |               | 10,720   | 0.51 | 0.8 | 1.7 | 1.1 | 1740 | 170 |
|                 | N15     |               | 10,720   | 0.51 | 1.1 | 2.4 | 1.08 | 1740 | 245.8 |
| Silty sand      | N16     | Square        | 3440     | 0.44 | 0.45 | 0.5 | 0.5 | 1 | 480 | 51 |
|                 | N17     | Square        | 7520     | 0.65 | 0.76 | 1 | 1 | 1540 | 100 | Briaud and Gibbens (1999) |
|                 | N18     |               | 7520     | 0.65 | 0.76 | 1.5 | 1 | 1540 | 154 |
| Silt            | N19     | Square        | 1700     | 0.5 | 0 | 1 | 1 | 375 | 115 | Eslami and Gholami (2006) |
|                 | N20     |               | 2000     | 0.5 | 0 | 1 | 1 | 370 | 100 |
| Silty sand      | N21     | Square        | 3000     | 0.5 | 0 | 0.6 | 1 | 1260 | 60 |
|                 | N22     | Circular      | 500      | 0.6 | 0 | 0.3 | 1 | 170 | 33 |
|                 | N23     |               | 900      | 0.6 | 0 | 0.3 | 1 | 170 | 25 |
| Silty clay      | N24     | Circular      | 1000     | 0.6 | 1.5 | 0.6 | 1 | 600 | 72 |
|                 | N25     |               | 1700     | 0.6 | 1.5 | 0.6 | 1 | 600 | 72 |
|                 | N26     |               | 2500     | 0.6 | 1.5 | 0.6 | 1 | 600 | 60 |
| White fine sand | N27     | Square        | 3660     | 0.54 | 0.69 | 0 | 1 | 620 | 65 | Mayne and Illingworth (2010) |
| Glaciofluvial sand| N28    | Rectangular  | 4010     | 0.63 | 0 | 1 | 1.02 | 840 | 102.4 |
|                 | N29     |               | 4010     | 0.63 | 0 | 1 | 1.02 | 840 | 102.4 |
|                 | N30     |               | 3200     | 0.63 | 1 | 2.4 | 1.08 | 640 | 260 |
| Compacted fill  | N31     | Square        | 880      | 0.53 | 0 | 0.46 | 1 | 150 | 47 |
|                 | N32     |               | 3860     | 0.48 | 0 | 0.63 | 1 | 580 | 64 |
|                 | N33     |               | 2870     | 0.58 | 0 | 0.8 | 1 | 520 | 82 |
| Alluvial sand   | N34     | Circular      | 6720     | 0.6 | 2.2 | 2.2 | 1 | 1280 | 250 |
|                 | N35     |               | 6720     | 0.6 | 2.2 | 2.2 | 1 | 1280 | 250 |
|                 | N36     |               | 10,460   | 0.52 | 2.35 | 2.35 | 1 | 1730 | 245 |
|                 | N37     |               | 10,460   | 0.52 | 2.35 | 2.35 | 1 | 1730 | 245 |
| Dune sand       | N38     | Square        | 4010     | 0.66 | 0 | 0.7 | 1 | 840 | 71.7 |
|                 | N39     |               | 4010     | 0.66 | 0 | 0.7 | 1 | 840 | 71.7 |
|                 | N40     |               | 4010     | 0.66 | 0 | 1 | 1 | 840 | 102.4 |
|                 | N41     |               | 4010     | 0.66 | 0 | 1 | 1 | 840 | 102.4 |
|                 | N42     |               | 4010     | 0.66 | 0 | 1 | 1 | 840 | 102.4 |
| Silty sand      | N43     | Circular      | 1710     | 0.55 | 0.6 | 1.82 | 1 | 1710 | 186 |
| Siliceous dune sand| N44    | Square        | 480      | 0.44 | 0.5 | 0.5 | 1 | 480 | 51 |
|                 | N45     |               | 480      | 0.44 | 1 | 1 | 1 | 480 | 102 |
|                 | N46     |               | 480      | 0.44 | 1 | 1 | 1 | 480 | 102 |

*B is considered as the width and the diameter of the square and circular footings, respectively
output variables are explained, and then, the results of the modeling are provided in terms of equations and predicted versus measured trends of $q$-$s$. Also, the validation of the proposed hyperbolic function, and the comparison with previous models are discussed and the variations of $q_{ult}$, $q_s$ and $k_s$ with $CPT$ parameters are thoroughly elaborated. Finally, sensitivity analysis of the input variables is presented.

2 Method of analysis

Naturally, in $q$-$s$ curves, values of the imposed pressure tend to be bounded to two extremes. In the suggested models, two different boundary conditions are expected for the lower extreme ($s \to 0$) and the higher extreme ($s \to +\infty$), where $q$ tends to 0 and $q_{max}$, respectively.

There are some limitations associated with the previous models, including failure to incorporate the asymptotic behavior at limit settlements ($s \to +\infty$); hence, suggesting the need for simpler models with far superior applicability. It should be noted that the proposed function $q = f(s)$ to fit a PLT load–displacement profile should meet the following conditions: $f(0) = 0, f'(0)$ should be the global maximum, and $f'(\infty) \approx 0$ should be satisfied. Therefore, it is deemed that if a function meets the aforementioned criteria and is capable of precise prediction of the $q$-$s$ curve, it can accurately serve to estimate the $q_{max}$, $q_{ult}$, $q_s$ and $k_s$ parameters, as schematically illustrated in Fig. 1.

Many functions have been proposed to predict the load-settlement behavior of geo-materials among which the hyperbolic function has been admittedly used effectively in analyzing geotechnical properties, such as predicting the settlement, assessing the compressibility behavior and describing the relationship between the void ratio and effective stress (Al-Shamrani 2005; Sridharan 2006; Zhang et al. 2014; Ahmed and Siddiqua 2016; Soltani et al. 2017). Upon reviewing the literature, to the best of the authors’ knowledge, very limited efforts have been devoted to the prediction of the $q$-$s$ behavior of shallow footings on the basis of $CPT$ records using the hyperbolic function. The hyperbolic function considered in the present paper is given in Eq. (7), which has three constant coefficients.

$$q(s) = \frac{\alpha s}{\beta s + \gamma}$$

boundary conditions

$\begin{align*}
q(0) &= 0 \\
\lim_{s \to +\infty} q(s) &\approx \frac{\alpha}{\beta}
\end{align*}$

where $\alpha, \beta,$ and $\gamma$ are the model parameters, which can be obtained from PLT results using regression analysis. Schematic illustration of the proposed hyperbolic model is depicted in Fig. 1 along with the index parameters.

As shown in Fig. 1, $q_{max}$ is defined as the horizontal asymptotic of the bearing pressure function, $q_{ult}$ is the value of bearing capacity corresponding to $\frac{s}{s_i} = 0.1$, $q_s$ is the allowable bearing capacity equivalent to the allowable settlement ($s_a$) and $k_s$ is the slope of the line starting at the origin and ending at the arbitrary point with the settlement of $s_i$ and the bearing pressure of $q_i$.

$CPT$ strength measurement is based on the soil failure with respect to the cone penetration, which is driven consistently and at a slow pace. Such properties presumably make $CPT$ one of the most viable options among the in situ tests to determine the above-mentioned hyperbolic model parameters. Herein, by adopting some $CPT$ records, $\alpha, \beta,$ and $\gamma$ can be estimated as the main contribution of the current paper that can be suggested from the load–displacement profiles of PLT. In other words, these parameters are linked to $q_{max}$, $q_{ult}$, $q_s$ and $k_s$ (Eqs. (8a) to (8d)).

$$q_{max} = \frac{\alpha}{\beta}$$

$$q_{ult} = \frac{0.1 \beta B}{0.1 \beta B + \gamma}$$

$$q_s = \frac{\alpha s_a}{\beta s_a + \gamma}$$

$$k_s = \frac{\alpha}{\beta s + \gamma}$$

where $\alpha, \beta,$ and $\gamma$ can be presented as functions of the cone tip and shaft resistances, $q_i$, and $f_c$. Therefore, with the $CPT$ records at hand, model parameters $\alpha, \beta,$ and $\gamma$ can be predicted accordingly. By and large, a general model can be established to estimate the $q$-$s$ profile on the basis of a database containing $CPT$ and PLT results.

It is important to note that the foundation width ($B$) is also considered in the modeling. To this end, $\frac{\beta}{\gamma}$ has been used to consider the effect of the plate dimension. This makes the settlement dimensionless consistent with
previous studies like Mayne and Dasenbrock (2018). Ergo, Eq. (7) is rewritten as follows:

$$q_s(a) = \frac{\alpha}{\beta} + \frac{\beta}{\gamma}$$

(9)

Although $\alpha$, $\beta$, and $\gamma$ are primarily functions of $q_c$ and $f_s$, they are assumed to be functions of $q_t$ and $R_f$ herein to take the influence of pore water pressure into account. Hence, $q_t$ (corrected cone resistance) is defined as:

$$q_t = q_c + u_2(1 - a)s$$

(10)

where $u_2$ is the pore water pressure, and $a$ is the net area ratio of cone tip.

Hence, in terms of the dimensional analysis for the hyperbolic model parameters, shown in Eq. (9), the unit of $\alpha$ is $kPa$, and $\beta$ and $\gamma$ are both dimensionless. It is worth noting that the effect of embedment depth, $D_f$ is reflected in $q_t$ in such a way that the more the depth of the foundation, the more effectively it will impact $q_t$. In general, the current study seeks to validate the hyperbolic model and derive a relationship between the model and $CPT$ parameters.

### 3 Review of the database

The experimental data employed for the validation of Eq. (9) were gathered from credible sources, including 46 $CPT$ logs and 46 $q-s$ results from $PLTs$. Furthermore, other parameters like the footing width ($B$) and embedment depth ($D_f$) of the $PLTs$ were documented. For $CPT$ data, the depth of $2B$ has been regarded as the effective zone of load influence, according to the majority of the common methods for the estimation of the settlement beneath the foundation under symmetric loading (Valikah and Eslami 2019). The parameters acquired from $CPT$ and used in the analyses are the tip resistance $q_t$ ($q_c$ corrected for pore pressure) and skin friction ($R_f = \frac{f_s}{q_c}$). The mentioned $CPT$ parameters were introduced in an arithmetic average sense through a depth of $2B$ so as to reflect the mean soil shear strength (Eslami and Mohammadi 2016). Based on the soil and loading conditions and according to Schmertmann (1978), the effective depth under the foundation is equal to approximately $2B$. Table 2 lists the experimental data related to a series of tests performed on a wide range of soil types, from silty clays to sands. The parameter $q_{ult}$ is reported corresponding to $s/B \geq 0.1$.  

Fig. 2 Bar charts of the input variables
Table 3  Summary of the regression analyses for the hyperbolic model, encapsulating all the studied data

| Case No | R (MPa) | β | γ | R^2 | RMSE (kPa) | MAD (kPa) | q_max (kPa) | q_a (kPa) (s/ B = 0.05) | k_s0 (MN/m^3) | k_s (MN/m^3) (s/ B = 0.05) |
|---------|---------|---|---|-----|------------|-----------|------------|----------------|----------------|----------------|
| N1      | 59      | 168.0 | 2.65 | 0.99 | 4 | 9 | 348 | 265 | 22 | 5 |
| N2      | 62      | 162.4 | 2.62 | 0.99 | 5 | 9 | 379 | 286 | 23 | 6 |
| N3      | 247     | 167.0 | 2.67 | 0.99 | 17 | 9 | 1480 | 1121 | 154 | 22 |
| N4      | 277     | 170.3 | 2.70 | 0.98 | 34 | 15 | 1628 | 1236 | 171 | 25 |
| N5      | 34      | 168.9 | 2.69 | 0.99 | 2 | 9 | 200 | 152 | 28 | 3 |
| N6      | 32      | 162.1 | 2.62 | 0.99 | 2 | 9 | 200 | 151 | 21 | 3 |
| N7      | 33      | 170.2 | 2.70 | 0.99 | 2 | 9 | 197 | 149 | 21 | 3 |
| N8      | 101     | 166.1 | 2.66 | 0.99 | 8 | 10 | 609 | 461 | 63 | 9 |
| N9      | 62      | 169.4 | 2.69 | 0.99 | 4 | 9 | 364 | 276 | 38 | 6 |
| N10     | 60      | 165.5 | 2.66 | 0.99 | 4 | 9 | 360 | 272 | 37 | 5 |
| N11     | 135     | 169.3 | 2.69 | 0.99 | 10 | 10 | 800 | 607 | 84 | 12 |
| N12     | 307     | 163.7 | 2.64 | 0.99 | 17 | 8 | 1877 | 1419 | 194 | 28 |
| N13     | 307     | 168.9 | 2.69 | 0.99 | 20 | 9 | 1819 | 1380 | 95 | 28 |
| N14     | 307     | 165.1 | 2.65 | 0.99 | 17 | 8 | 1861 | 1409 | 68 | 28 |
| N15     | 307     | 161.3 | 2.61 | 0.997 | 17 | 8 | 1905 | 1439 | 49 | 29 |
| N16     | 87      | 166.3 | 2.66 | 0.997 | 5 | 8 | 521 | 394 | 65 | 8 |
| N17     | 267     | 163.7 | 2.64 | 0.996 | 16 | 8 | 1633 | 1235 | 101 | 25 |
| N18     | 267     | 169.8 | 2.70 | 0.994 | 20 | 9 | 1574 | 1194 | 66 | 24 |
| N19     | 73      | 170.8 | 2.71 | 0.989 | 7 | 12 | 427 | 324 | 27 | 6 |
| N20     | 58      | 163.5 | 2.63 | 0.994 | 8 | 17 | 357 | 270 | 22 | 5 |
| N21     | 236     | 161.2 | 2.61 | 0.993 | 18 | 9 | 1464 | 1106 | 151 | 22 |
| N22     | 32      | 165.6 | 2.66 | 0.991 | 3 | 10 | 195 | 147 | 40 | 3 |
| N23     | 33      | 163.4 | 2.63 | 0.995 | 2 | 11 | 203 | 153 | 42 | 3 |
| N24     | 116     | 171.0 | 2.71 | 0.988 | 11 | 13 | 678 | 515 | 71 | 10 |
| N25     | 115     | 168.6 | 2.69 | 0.988 | 11 | 13 | 680 | 515 | 71 | 10 |
| N26     | 115     | 166.3 | 2.66 | 0.994 | 8 | 9 | 692 | 524 | 72 | 10 |
| N27     | 101     | 165.5 | 2.65 | 0.985 | 13 | 17 | 612 | 464 | 55 | 9 |
| N28     | 138     | 164.7 | 2.65 | 0.990 | 14 | 12 | 837 | 633 | 52 | 13 |
| N29     | 138     | 168.5 | 2.68 | 0.987 | 16 | 14 | 818 | 620 | 51 | 12 |
| N30     | 114     | 167.0 | 2.67 | 0.997 | 6 | 7 | 680 | 515 | 18 | 10 |
| N31     | 29      | 170.7 | 2.71 | 0.995 | 2 | 9 | 170 | 129 | 23 | 3 |
| N32     | 97      | 170.9 | 2.71 | 0.988 | 11 | 13 | 570 | 432 | 57 | 9 |
| N33     | 90      | 162.6 | 2.63 | 0.996 | 5 | 8 | 552 | 417 | 43 | 8 |
| N34     | 212     | 167.4 | 2.67 | 0.993 | 18 | 8 | 1266 | 960 | 36 | 19 |
| N35     | 212     | 161.3 | 2.61 | 0.996 | 14 | 7 | 1314 | 992 | 37 | 20 |
| N36     | 303     | 166.9 | 2.67 | 0.996 | 18 | 7 | 1815 | 1375 | 48 | 28 |
| N37     | 303     | 162.4 | 2.62 | 0.997 | 16 | 8 | 1865 | 1410 | 49 | 28 |
| N38     | 150     | 170.9 | 2.71 | 0.996 | 9 | 8 | 880 | 668 | 79 | 13 |
| N39     | 150     | 167.7 | 2.68 | 0.997 | 8 | 8 | 897 | 680 | 80 | 14 |
| N40     | 150     | 162.8 | 2.63 | 0.997 | 8 | 8 | 924 | 698 | 57 | 14 |
| N41     | 150     | 161.3 | 2.61 | 0.996 | 9 | 8 | 932 | 704 | 58 | 14 |
| N42     | 150     | 169.2 | 2.69 | 0.997 | 8 | 8 | 889 | 674 | 56 | 13 |
| N43     | 327     | 165.5 | 2.66 | 0.993 | 25 | 9 | 1976 | 1495 | 68 | 30 |
| N44     | 90      | 162.5 | 2.63 | 0.993 | 7 | 9 | 554 | 418 | 68 | 8 |
| N45     | 89      | 161.2 | 2.61 | 0.994 | 7 | 9 | 552 | 417 | 34 | 8 |
| N46     | 91      | 164.5 | 2.64 | 0.993 | 7 | 9 | 553 | 419 | 34 | 8 |
4 Results

4.1 Hyperbolic model for load–displacement profiles

Based on the presented database, 46 PLTs were selected from the literature whose results were then fitted by the well-established hyperbolic model. The selected data set was compiled from all over the globe and was not restricted to a specific location so as to introduce sufficient diversity. The fitting parameters, including $a$, $b$, and $c$, were computed based on the least squares error optimization scheme. Table 3 enlists the parameters for all the 46 cases. Furthermore, absolute fraction of variance ($R^2$), root mean square error (RMSE) and mean absolute deviation (MAD) are defined in Eqs. (11), (12) and (13), respectively, and were exploited herein to assess the fitting accuracy of the relevant correlations.

$$R^2 = 1 - \frac{\sum_{i=1}^{M} (q_{mi} - q_{ci})^2}{\sum_{i=1}^{M} (q_{mi})^2}$$  \hspace{1cm} (11)

$$\text{RMSE} = \sqrt{\frac{1}{M} \sum_{i=1}^{M} (q_{mi} - q_{ci})^2}$$  \hspace{1cm} (12)

$$\text{MAD} = \frac{\sum_{i=1}^{M} |q_{mi} - q_{ci}|}{M}$$  \hspace{1cm} (13)

where $q_{mi}$ and $q_{ci}$ are the measured and calculated pressures, respectively. Lower values of RMSE and MAD (close to zero) and higher values of $R^2$ (close to one) are indicators of superior model performance. Table 3 corroborates the accuracy of Eq. (9) in replicating the load–displacement trends acquired from PLTs, substantiated in forms of high $R^2$ and low RMSE and MAD values. To be more specific, the $R^2$ values have remained higher than 99% and the RMSE and MAD values are lower than 34 and 17 kPa, respectively; hence, bearing witness to the efficacy of Eq. (9). It should be noted that $k_s$ is the maximum modulus of subgrade reaction, values of $q_a$ and $k_y$ are corresponding to $\frac{q}{p} = 0.05$, and $q_{\text{max}}$ is the asymptotic load.

Maximum soil strength means that the soil has reached the failure state or is on the verge of failure. CPT has also the same mechanism with its resistance parameters ($R_f$ and $q_{\text{t}}$) being equivalent to the soil failure state. Based on the experimental and numerical studies, a logarithmic spiral failure mode has been considered for the CPT results of homogenous soils as shown in Fig. 3. According to Eslami and Mohammadi (2016), the failure mechanism of CPT tests is approximately similar to that of shallow foundations. Therefore, it can be observed that the pressure corresponding to the ultimate bearing capacity state or the
same failure threshold can be correlated with various CPT parameters.

4.2 Prediction of model parameters from CPT data

With regard to Table 3, it is clear that $\beta$ and $\gamma$ are roughly constant and equal to approximately 166.4 and 2.66, respectively. However, $\alpha$ possesses different values. Hence, $\alpha$ in Eq. (9) can be rewritten as Eq. (14) with the objective of reaching a general correlation for the hyperbolic model parameter(s).

$$\alpha = h(q_t, R_f)$$

(14)

To date, various approaches have been adopted to derive a general correlation for $\alpha$. The aim in this study is to find a viable formulation, which correlates the model parameters in hyperbolic load–displacement model to the CPT data available in the proximity of the PLT location. To this end, the Volterra series is employed, the implication of which is on the basis of GMDH-type neural network (MolaAbasi et al. 2013; MolaAbasi and Shooshpasha 2017). Instead, the core concept of this regression scheme is to invoke genetic algorithm in combination with the multi-layer perceptron numeral networks to find an optimized configuration of hidden layers and neurons to form the polynomial correlation. Herein, the Volterra series is rewritten for the two input variables, including $q_t$ and $R_f$, and a single output, $\alpha$, as Eq. (15):

$$\alpha = h(q_t, R_f) = a_1 + a_2 R_f + a_3 q_t + a_4 R_f^2 + a_5 q_t^2 + a_6 q_t R_f$$

(15)

where the parameters $a_i$ are constant variables which are obtained from the regression analysis.

To fairly evaluate the efficacy of the proposed model and to compare it with the previous relationships, data were randomly categorized in two groups of training and validation. The statistical results of the model are given in Table 5.

| Model stage | $R^2$ | RMSE (kPa) | MAD (kPa) |
|-------------|-------|------------|-----------|
| Training    | 0.992 | 54         | 38        |
| Validation  | 0.989 | 64         | 44        |
| Total       | 0.990 | 57         | 39        |

Fig. 4 Predicted and measured pressure values for a training data set; and b validation data set

Table 5 Statistical results of the model

Fig. 5 Graphical representation of the accuracy of empirical correlations
Fig. 6 Comparison of the accuracy of the current study in estimation of the ultimate bearing capacity of shallow footings with other relationships proposed in the literature.
validation, 70% of which were used for training and the rest were kept for validation. Therefore, 14 out of the total 46 data sets, obtained from the $q$-$s$ curves, were randomly selected for validation data, and the rest were utilized for the sake of training. In order to equate the ranges of training and validation in the modeling, their statistical parameters, including minimum, maximum and average, were monitored to remain comparable (MolaAbasi et al. 2015; MolaAbasi and Eslami 2018), as enlisted in Table 4. It is evident that the statistical characteristics are in a similar level and are quite close to each other.

Aforementioned regression analysis converts Eq. (9) to Eqs. (16a) and (16b). It should be noted that these correlations are based only on the training data and the validation data have not been introduced to the regression process.

\[
q = \frac{\pi s}{2.68 + 161B}
\]

\[
\alpha(MPa) = 292.65 - 1026.95R_f - 0.008q_t + 964.37R_f^2 + 8.45 \times 10^{-7}q_t^2 + 0.05q_tR_f
\]

where $q_t$ is the corrected $q_c$ and $R_f$ is the skin friction. With regard to the proposed roughly simple equation, predicted and measured values can be compared. Cases related to the predicted and measured $q$-$s$ curves are presented in Fig. 4, which clearly proves that Eq. (16) is a powerful modelling tool, according to the statistical parameters provided in Table 5.

Fig. 7 Results of the sensitivity analysis of the input parameters
4.3 Comparison with previous models

Given the very fact that the validation data were not incorporated in the model training process, they are used hereinafter for the purpose of comparison with previous methods to elaborate on the aptitude of different models to imitate the load–displacement trends.

To compare the current method with the previous ones, two specific model sets were considered. In the first set, relationships, which predicted \( q_s \) curves, including those proposed by Mayne and Dasenbrock (2018), Mayne and Woeller (2014) and Stuedlein and Holtz (2010), were considered and in the second set, correlations proposed by Briaud (2007), Lehane et al. (2013), as well as Robertson and Cabal (2014) are employed, considering \( S/B = 0.1 \) to constitute \( q_{ult} \). In the former set, scaled cumulative frequency (SCF) is presented against the relative error defined in Eq. (17) for comparison purpose.

\[
E_r(\%) = \frac{q_{ci} - q_{mi}}{q_{mi}} \times 100
\]  

(17)

Figure 5 shows the results of the first set where the relationships suggested by Mayne and Woeller (2014) and Stuedlein and Holtz (2010) is observed to render lower estimations for \( q_{ult} \) (\( E_r \) is in the negative range). However, the prediction model proposed by Mayne and Dasenbrock (2018), which is based on the soil type, \( I_c \), seems to yield estimations at both sides of conservatism. As it is plainly visible, the proposed formula in the current study is fairly accurate as it yields SCFs close to the vertical axis (\( E_r = 0\% \)); hence, implying to provide veritable predictions.

However, it is to be noted that the provided comparisons are only valid in the deterministic (nominal) realms, and it does not bear witness to the absolute superiority of the model proposed in the current study over others. In other words, the reliability of different models and methods should be looked as if they are probabilistic in nature. To do so, different model parameters involved in the load–displacement profile predictions are considered probabilistically, and by assuming different load factors, it would be possible to yield information about the reliability index of different models. However, this issue is beyond the scope of this paper, and in this study, the model parameters have been assumed in average sense and only the resistance side of the performance function has been covered.

In the latter set, however, the calculated vs. measured ultimate bearing capacity values have been employed to indicate the accuracy of the proposed relationships.

Fig. 8 Schematic illustration of the load-settlement curves based on CPT results

4.3 Comparison with previous models
Likewise, statistical parameters, such as $R^2$, RMSE, and MAD, have been superimposed on the curves to make a quantitative comparison, as depicted in Fig. 6. It is evident that the method presented in the current study poses more aptitude to estimate $q_{ult}$ in comparison to the other approaches.

4.4 Sensitivity analysis

In order to examine the effect of every single input variable on the output parameter, sensitivity analyses were performed by changing each of the input parameters at a constant pace, while the rest were unavering (Liong et al. 2000). The considered rate of changes ranges from -10% to 10%. As a result of the changes in every input variable, RMSE in the estimation of output ($q$) was determined and presented in Fig. 7. In other words, the sensitivity analysis with respect to the changes of the parameter under study is drawn, while other parameters were adopted in the average sense; that is, changes of RMSE with respect to the input variables. As it is plainly demonstrated, the changes in parameters $B$ and $s$ barely affect $q$, whereas the variations of $R_f$ and $q_t$, particularly $R_f$, have substantial influences on the output parameters. Variations of the left sides of Fig. 7 in terms of $R_f$ and $q_t$ are almost similar, whereas changes in the right-hand side of $R_f$ curve are more pronounced. Such a result corroborates that $R_f$ is the most influential parameter on RMSE.

Another observation from Fig. 7 is that any perturbation in the measured soil parameters $q_t$ and $R_f$, which can be considered as epistemic uncertainty and could be presumably due to measurement error, will have a reflection on the accuracy of the load/pressure estimations for shallow foundations. In other words, one could easily deduce that the soil parameters measurements, substantiated in form of in situ tests logging, have paramount impacts on the accuracy of the relevant predictions as appears from Fig. 7. This would imply that the type of in situ test equipment and its measurement accuracy play a crucially important part in the appropriate estimation of other parameters, let’s say the ultimate bearing capacity of shallow footings, for example.

4.5 Application of the proposed model for future cases

The efficiency of the current approach for the new data set related to sands and silty sands and the calculation of the corresponding settlement are presented schematically in Fig. 8. First, a CPT log is considered, and then, parameters $q_t$ and $R_f$ are drawn against the depth (Fig. 8a). With regard to the defined $D_f$ and $B$, CPT data series are derived from $D_f$ to $D_f + 2B$ and their average values are taken into account (Fig. 8b). The arithmetic mean values of the parameters $q_t$ and $q_c$ obtained from CPT data are considered in the current study. Using this arithmetic mean, resistance peaks are filtered using the method presented by Eslam and Fellenius (1997). Finally, by introducing the mean values of $q_t$ and $R_f$, acquired from the CPT log, $z$ presented in Eq. (16b) is computed. Thereafter, the values of the applied pressures corresponding to various settlement levels could be obtained from Eq. (16a), and the graph presented in Fig. 8c could be drawn accordingly. To this end, six values for the settlement ($s_t$ to $s_6$) are considered schematically, and the respective values of the applied pressure ($q_1$ to $q_6$) are acquired so as to draw a complete load-settlement profile.

5 Discussion

The main purpose of the current study was to find a robust relationship for the load-settlement behavior of soils based on CPT results. Such end was obtained in accordance with the gathered in situ test results. The proposed formula is Eq. (16), which correlates $q$ to $s$, $B$ and other properties obtained from CPT data components, such as $R_f$ and $q_t$. The results show the superiority of the proposed relationship over previous ones.

5.1 Ultimate bearing capacity ($q_{ult}$)

Based on different criteria for $q_{ult}$ obtained from PLTs, loading up to 0.1B settlement is valid particularly for coarse-grained soils and sands. Hence, applying this criterion and putting it into Eq. (16), Eq. (18) can be proposed for the evaluation of the ultimate bearing capacity based on CPT results.

$$q_{ult} = 5.32(292.65 - 1026.95R_f - 0.008q_t + 964.37R_f^2 + 8.45 \times 10^{-7}q_t^2 + 0.05q_tR_f)$$

(18)

where $q_t$ is the corrected $q_t$ in kPa and $R_f$ is the skin friction in percent. A 3D illustration of the variation of $q_{ult}$ with respect to $q_t$ and $R_f$ is given in Fig. 9. As depicted, the increase in either CPT parameters leads to an increase in the ultimate bearing capacity. However, the rate of increase in the $q_{ult}$ value with the increase of $q_t$ is more than that of $R_f$. Moreover, for higher values of tip resistance, $R_f$ is observed to be more influential. This phenomenon shows that the cohesive property will be enhanced in soils due to the increase of $R_f$, which consequently gives rise to the augmentation in the ultimate bearing capacity.

Based on the results of a PLT, the settlement corresponding to the failure of the plate can be obtained. In order to link the settlement values of a PLT to a true scale foundation, Eq. (19) can be adopted from Terzaghi et al.
(1996), which relates the settlement of the foundation \( (s_f) \) to the settlement of the plate \( (s_p) \).

\[
\begin{align*}
    s_f &= s_p \frac{B_f}{B_p} \quad \text{clayey soil} \\
    s_f &= s_p \left( \frac{B_f (B_p + 0.3)}{B_p (B_f + 0.3)} \right)^2 \quad \text{sandy soil}
\end{align*}
\]

\[ (19) \]

where \( B_f \) and \( B_p \) are the width (diameter) of the foundation and plate, respectively.

Accordingly, the ultimate bearing capacity of the foundation \( (q_{ult,f}) \) and plate \( (q_{ult,p}) \) can be related as follows:

\[
\begin{align*}
    q_{ult,f} &= q_{ult,p} \frac{B_f}{B_p} \quad \text{clayey soil} \\
    q_{ult,f} &= q_{ult,p} \left( \frac{B_f}{B_p} \right)^{2} \quad \text{sandy soil}
\end{align*}
\]

\[ (20) \]

Based on the above-mentioned discussion, the factor of safety can be reached as follows:

1. Calculating the allowable settlement of the plate \( (s_{p,all}) \) based on the values of \( B_f, B_p \) and \( s_{f,all} \), using the relations provided in foundation manuals for the relevant soil type.

2. Based on the value of \( q_t \) from the CPT data (i.e., the average \( q_t \) values corresponding to the depths of \( D_f \) to \( D_f + 2B \)) and the value of \( s_{p,all} \), the value of \( q_a \) for the plate can be obtained.
3. Obtaining $q_{ult}$ based on the criterion of $\frac{q}{B} = 0.1$ [Eq. (18)] and calculating the factor of safety ($FS_p = q_{ult}/q_a$).

5.2 Load-settlement curve ($q$-$s$)

One of the applications of Eq. (16) is to delineate the $q$-$s$ profile, while representing the load–displacement behavior of the footing, as depicted in a three-dimensional fashion in Fig. 10 for $B = 1$ m. From the graphs, it is observed that $q$-$s$ surfaces grow with $R_f$, implying higher $q$ values in similar settlement levels. On the other hands, for the predetermined settlements of 50 and 100 mm, $q_a$ can be drawn in the $q$-$q_t$ plane, as also shown in Fig. 10. As it can be seen, the values of $q_a$ corresponding to 100 mm settlement are detectably higher than those corresponding to 50 mm settlement with both increasing with $R_f$ value.

5.3 Modulus of subgrade reaction ($k_s$)

Another application of Eq. (16) is to predict the modulus of subgrade reaction ($k_s$). As stated before (Fig. 1), $k_s$ can be predicted from the $q$-$s$ curve acquired from PLT data and can be computed for foundations employing Eq. (9). Figure 11 shows the three-dimensional representation of $k_s$ as a function of $q_t$ and $s$ for the case of $B = 1$ m. It should be noted that the considered value of $R_f$ for this graph is the average of all data; i.e., $R_f = 0.6$. As it is clear from the figure, settlement is inversely related to the modulus of subgrade reaction, whereas $q_t$, as an indicative of soil strength properties, has an augmenting influence on $k_s$. Generally, settlement is more of consequence than $q_t$.

6 Conclusions

In the current paper, a general framework was developed to predict and model the load-settlement response of shallow foundations based on CPT results. To this end, 46 $q$-$s$ curves were carefully assessed, and a sensitivity analysis was also conducted. Moreover, the proposed model was compared to previous relationships readily available from the literature. Prominent results obtained in the course of this study include:

- The proposed hyperbolic function was proven to have a high accuracy to predict the load-settlement behavior of shallow foundations.
- Constant parameters of the current hyperbolic model, including $\gamma$ and $\beta$, stem from the general form of the $q$-$s$ curves and the variable coefficient, $\alpha$, are dependent on the soil strength properties obtained from CPT results. $\alpha$ was related to the corrected cone tip resistance ($q_t$) and skin friction ($R_f$) via regression analysis and the Volterra series.
- Dimensionless parameter $\frac{s}{B}$ was used, and simple equations with high accuracy were proposed to estimate the allowable and ultimate bearing capacities of shallow foundations.
- To have a more accurate investigation, data were categorized into two groups of training and validation, and the model parameters were estimated. Validation data were employed to make a comparison between the proposed model and previous ones, the results of which showed considerably high accuracy of the developed model ($R^2 > 99\%$).
- Sensitivity analysis was performed to assess the individual impacts of the input variables, the results of which demonstrated the higher sensitivity of the model output to $R_f$.
- All the results of $q_{max}$, $q_{ult}$, $q_a$ and $k_s$ were given in the forms of example 3D graphs to be readily used by potential readers.

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Declarations

Conflict of interest The authors declare no conflict of interest.

Ethical approval This article does not contain any studies with human participants performed by any of the authors.

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