Study of Comparing Several Nonlinear Filtering Algorithms in Carrier-based Aircraft Positioning

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Abstract. In order to improve the positioning accuracy of the landing guidance radar to the carrier aircraft, the nonlinear filtering algorithm is used to estimate the positioning of the carrier aircraft. The main algorithms of nonlinear filtering, such as extended Kalman filter, unscented Kalman filter and particle filter, are analyzed and compared. A typical nonlinear model is simulated to verify the performance of these algorithms. The simulation results show that the estimation accuracy of the particle filter algorithm is better than the other two filter algorithms.

1. Introduction
When landing on the ship, the positioning requirements of the carrier aircraft are very high, and the positioning accuracy of the carrier aircraft is very important for the safety of landing. Because the mathematical model of carrier motion and landing is nonlinear, the problem of carrier positioning can be transformed into the problem of nonlinear filtering. Traditional Kalman filter is not suitable for nonlinear dynamic system. In recent years, nonlinear filtering has been widely used in target tracking, signal processing, ship navigation and other fields, and nonlinear filtering algorithm has been more and more in-depth research [1].

This paper mainly studies three nonlinear filtering algorithms. The first is the extended Kalman filter (EKF) [2]. This algorithm uses the Taylor series expansion method to transform the nonlinear model into a linear filter, and then uses the Kalman filter method to solve it. The algorithm is simple and convenient, but it is not suitable for the filtering problems under the strong nonlinear and non-Gaussian conditions. The second is unscented Kalman filter (UKF) [3, 4]. This algorithm is based on UT transformation, and does not need to linearize the nonlinear system, but uses a group of scatter points to represent the distribution of state variables. Through these points, the mean and variance of Gaussian random variables in the nonlinear system are calculated. UKF has a good estimation for the first or higher order nonlinear system accuracy. The third kind of particle filter (PF) [5, 6] is a nonlinear filtering algorithm, which is based on the optimal regression Bayesian filtering method of Monte Carlo simulation. Because it uses random samples to estimate the state, the method is not limited by the linearization error and Gaussian noise, which is superior to the first two filtering algorithms.

2. Analysis of nonlinear filtering algorithm
The state equation and observation equation of dynamic nonlinear system can be described as follows:
\[
\begin{align*}
    x_k &= f(x_{k-1}) + w_{k-1} \\
    z_k &= h(x_k) + v_k
\end{align*}
\]

Where: \( x_k \in \mathbb{R}^n \) and \( z_k \in \mathbb{R}^n \) are the state vector and observation vector of the system at \( k \) time; \( f(\cdot) \) and \( h(\cdot) \) are the nonlinear state transfer function and observation function of the system; \( w_k \in \mathbb{R}^n N(q_k, Q_k) \) and \( v_k \in \mathbb{R}^n N(r_k, R_k) \) are the process noise and observation noise of the system respectively, which are not related to each other.

### 2.1. Extended Kalman filter

The extended Kalman filter algorithm is a commonly used nonlinear filtering method. Its core idea is to expand nonlinear functions \( f(\cdot) \) and \( h(\cdot) \) into Taylor series around the filtering value, and ignore the items above the second order to get an approximate linear model. Then the extended Kalman filter equation is recursively deduced according to the Kalman filter equation.

Because the covariance matrix of the state noise and the covariance matrix of the observation noise remain unchanged all the time, the quality of the filtering result has a great relationship with the initial estimation of the process noise and the measurement noise. In addition, the algorithm has a good filtering precision only when the model linearization error is small.

The model and recurrence of the first-order extended Kalman filter algorithm are as follows.

Make

\[
\begin{align*}
    &\frac{\partial f}{\partial \hat{x}_{k-1}} = \frac{\partial f(\hat{x}_{k-1})}{\partial \hat{x}_{k-1}} \bigg|_{\hat{x}_{k-1}=x_{k-1}} = F_{k|k-1} \\
    &\frac{\partial h}{\partial \hat{x}_{k-1}} \bigg|_{\hat{x}_{k-1}=x_{k-1}} = H_{k-1}
\end{align*}
\]

One step state prediction:

\[
\hat{x}_{k|k-1} = f(\hat{x}_{k-1|k-1})
\]

One step covariance prediction:

\[
P_{k|k-1} = F_{k|k-1} P_{k|k-1} F_{k|k-1}^T + Q_{k-1}
\]

Filter gain:

\[
K_k = P_{k|k-1} H_k^T \left( H_k P_{k|k-1} H_k^T + R_k \right)^{-1}
\]

Status update:

\[
\hat{x}_{k|k} = \hat{x}_{k|k-1} + K_k \left[ z_k - h(\hat{x}_{k|k-1}) \right]
\]

Covariance matrix update:
\[
P_k = (I - K_k H_k) P_{k|k-1}
\]

Process noise:

\[
E_k = \hat{x}_{k|k} - z_k
\]

Process noise variance:

\[
Q_k = \text{var}(E_k)
\]

2.2. Unscented Kalman filter

When the nonlinear characteristic of the system is strong, the effect of the extended Kalman filter algorithm is often unsatisfactory [7]. In order to get a better filtering effect, Julier et al [8-10] proposed unscented Kalman filtering algorithm (UKF) based on UT transform. This algorithm uses a group of sample points of approximate Gauss distribution to get the approximation of state distribution function after UT transformation. This algorithm avoids the error of EKF algorithm when it approximates the nonlinear function and gets better filtering effect.

Firstly, the basic idea of UT transformation is introduced:

1. Calculate \( (2n_x + 1) \) sampling points \( \xi_i \) and their corresponding weights \( \omega_i \).

\[
\begin{align*}
\xi_0 &= \bar{x} \\
\xi_i &= \bar{x} + \left( \sqrt{n_x + \kappa} P \right), \quad i = 1, 2, \ldots, n_x \\
\xi_i + n_x &= \bar{x} - \left( \sqrt{n_x + \kappa} P \right), \quad i = 1, 2, \ldots, n_x
\end{align*}
\]

\[
\begin{align*}
\omega_0 &= \frac{\kappa}{n_x + \kappa} \\
\omega_i &= \frac{1}{2(n_x + \kappa)}, \quad i = 1, 2, \ldots, n_x \\
\omega_{i + n_x} &= \frac{1}{2(n_x + \kappa)}, \quad i = 1, 2, \ldots, n_x
\end{align*}
\]

Where, \( \kappa \) is the scale parameter, satisfying \( \kappa + n_x \neq 0 \). \( n_x \) is the state vector dimension.

2. New sample points obtained by nonlinear transfer

\[
y_i = g(\xi_i), \quad i = 1, 2, \ldots, n_x
\]

3. The estimated mean and covariance are as follows:
The algorithm flow of unscented Kalman filter is as follows:
(1) Calculate the sample point \( \chi^i_{k|k} \).
(2) According to UT transformation, predict \( \hat{x}_{k+1|k} \) and \( P_{k+1|k} \).
(3) Update observations.

\[
\tilde{z}_{k+1|k} = h_{k+1}(\chi^i_{k+1|k})
\]

\[
\hat{z}_{k+1|k} = \sum_{i=0}^{2n_x} \omega_i z^i_{k+1|k}
\]

\[
S_{k+1} = \sum_{i=0}^{2n_x} \omega_i \left( z^i_{k+1|k} - \hat{x}_{k+1|k} \right) \left( z^i_{k+1|k} - \hat{x}_{k+1|k} \right)^T + R_k
\]

\[
P_{k+1|k} = \sum_{i=0}^{2n_x} \omega_i \left( \chi^i_{k+1|k} - \hat{x}_{k+1|k} \right) \left( \chi^i_{k+1|k} - \hat{x}_{k+1|k} \right)^T
\]

\[
K_{k+1} = P_{k+1|k} S_{k+1}^{-1}
\]

\[
\hat{x}_{k+1|k+1} = \hat{x}_{k+1|k} + K_{k+1} \left( z_{k+1} - \tilde{z}_{k+1|k} \right)
\]

\[
P_{k+1|k+1} = P_{k+1|k} - K_{k+1} S_{k+1} K_{k+1}^T
\]

From the process of UKF algorithm, it can be seen that the algorithm does not need to calculate the Jacob matrix of nonlinear function, thus saving a lot of computation, so the speed is faster than EKF algorithm, and UT transformation can estimate the mean and variance of nonlinear function more accurately, so the filtering accuracy is higher than EKF algorithm. When the nonlinear function is transferred, UKF is a Gaussian filter whose posterior mean and covariance can be accurate to the second order for any nonlinear function, and EKF can only obtain the first order precision, which makes UKF can be used in any dynamic model. In addition, UKF algorithm still obeys the Gaussian distribution for the probability density function after the state vector approximation, so it will bring great error to the filtering of non-Gaussian system.

2.3. Particle filter
Particle filter is an approximate estimation algorithm which combines Monte Carlo and Bayesian theory. The basic idea is to select a group of random samples in the state space, estimate the posterior probability density function approximately according to the updated particle weight and position information, and use the mean value of random samples instead of integral operation to obtain the minimum variance estimation of state. [11]
If the initial probability density is \( p(x_0 | y_0) = p(x_0) \), the prediction and update process is as follows:

\[
p(x_k | z_{1:k-1}) = \int p(x_k, x_{k-1}) p(x_{k-1} | z_{1:k-1}) \, dx_{k-1}
\]

\[
p(x_k | z_{1:k}) = \frac{p(z_k, x_k) p(x_k | z_{1:k-1})}{\int p(z_k, x_k) p(x_k | z_{1:k-1}) \, dx_k}
\]

The specific implementation steps are summarized as follows:

1) Initialization

N particles are sampled from the distribution \( p(x_0) \), and the weight of each particle is set to \( 1/N \).

2) Sequential importance sampling

1. Important density function:

\[
q(x'_k | x'_{k-1}, z^k) = p(x'_k | x'_{k-1})
\]

Extract N prediction particles from important density function \( \{x'_i, i = 1, 2, \cdots, N\} \).

2. Calculate particle weight:

\[
\omega'_k = \omega'_{k-1} \frac{p(z_k | x'_k) p(x'_k | x'_{k-1})}{q(x'_k | x'_{k-1}, Z^k)}
\]

3. Normalized weight

\[
\hat{\omega}'_k = \omega'_k / \sqrt{\sum_{i=1}^{N} \omega'_i}
\]

3) Resampling

1. calculate the effective particle capacity

\[
N_{\text{eff}} \approx \frac{1}{\sum_{i=1}^{N} (\hat{\omega}'_i)^2}
\]

2. If \( N_{\text{eff}} < N_{\text{threshold}} \), resample and map the weighted particles \( \{x'_0, \hat{\omega}'_k\}_{i=1}^{N} \) to equal weight particles \( \{x'_{0:k}, 1/N\}_{i=1}^{N} \).

4) State and variance estimates

\[
\hat{x}_k = \sum_{i=1}^{N} \hat{\omega}'_i x'_k
\]

\[
P_k = \sum_{i=1}^{N} \hat{\omega}'_i (\hat{x}_k - x'_k)(\hat{x}_k - x'_k)^T
\]
Particle filter has the phenomenon of particle degradation in practical operation, that is, with the increase of the number of samples, the weight of many particles will become very small, and the variance of samples will increase with time. There are two methods to avoid particle degradation: resampling technique and selecting a good importance density function. Resampling technology may dry up the samples, thus losing the diversity of particles. The problem of sample drying up needs further study. There are two principles to choose a good importance density function: making the importance density function easy to sample and minimizing the variance of weight coefficient.

3. Simulation experiment and analysis
The representative model in the literature is selected as the experimental simulation model, and its process equation and measurement equation are as follows:

Equation of state:

\[
x_k = 0.5x_{k-1} + \frac{25x_{k-1}}{1 + x_{k-1}^2} + 8\cos[1.2 \cdot (k - 1)] + w_k
\]

Observation equation:

\[
z_k = \frac{x_k^2}{20} + v_k
\]

Where, \(w_k \sim N(0,1)\), \(v_k \sim N(0,1)\) are the Gaussian white noise with zero mean independent of each other, the simulation time is 50s, and the particle number of particle filter is 100. The three algorithms are tested 100 times by Monte Carlo simulation.

The simulation results of state estimation and estimation error of the three filtering algorithms are shown in Figure 1 and Figure 2 respectively.

![Figure 1. State estimation of three filters](image-url)
Figure 2. Absolute value of state estimation error of three filters

The evaluation standard of filtering accuracy adopts root mean square error, and its calculation formula is

\[ \text{RMSE} = \sqrt{\frac{1}{M} \sum_{i=1}^{M} (x_i^m - \hat{x}_i^m)^2} \]

Where: \( m \) is the number of Monte Carlo simulations, set to 100.

The mean square values of the state estimation errors of the three filters are shown in Table 1.

From the above simulation results, it can be seen that the rounding error of EKF algorithm’s first-order Taylor series expansion is large, resulting in obvious filtering divergence. UKF algorithm also produces large filtering deviation, while PF algorithm is better than EKF and UKF algorithm, with higher filtering accuracy and smaller estimation error.

### Table 1 Mean square value of state estimation error of three filters

| Algorithm | Mean square error (RMSE) |
|-----------|--------------------------|
| EKF       | 27.2675                  |
| UKF       | 14.7343                  |
| PF        | 4.7969                   |

4. Conclusion

In this paper, three kinds of nonlinear filtering algorithms are simulated. Using strong nonlinear model, EKF algorithm and UKF algorithm produce obvious divergence, while PF algorithm still has good performance. At present, PF has been applied to many fields, such as target tracking and navigation guidance system, fault detection, parameter estimation and system identification. However, the application of particle filter in ocean motion control is rare. PF can identify and locate the target and estimate the environmental parameters in complex ocean conditions. Finally, it can provide high-precision target estimation and environmental parameter estimation.

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