Casimir-Polder interaction between two atoms in electromagnetic fields

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Abstract. We reexamine the dispersion interaction between two atoms in external electromagnetic fields, provided the atoms are not in the equilibrium with the field. We show that the processes of absorption and spontaneous/stimulated emission should be taken into account in the case of non-equilibrium dynamics.

1. Introduction
Recent progress in experimental methods stimulated a new wave of research in Casimir, Casimir-Polder (CP), and van der Waals dispersion interaction under real experimental conditions, e.g. at finite temperature and at the presence of electromagnetic (EM) fields [1, 2, 3, 4, 5]. Although the theoretical base is quite robust, there are some problems to be resolved. One of these problems is the Casimir-Lifshitz interaction between real metals. Although the interaction is described by the famous Lifshitz theory, the theoretical predictions dramatically depend on the model describing the real metal (see for example [6, 7, 8] and references therein). Another problem to be clarified is the non-equilibrium CP interaction between a single atom and a metal (or dielectric) plate if the temperature of the plate differs from the temperature of the atomic gas. This force was recently measured with the help of reflection spectroscopy of the atom interacting with the dielectric slab [9], but the agreement with the theoretical predictions has not been achieved [9].

The theoretical treatment of the CP interaction under non-equilibrium conditions is based either on the linear-responce (or Lifshitz-type) theory [3, 2, 10], or macroscopic quantum electrodynamics [11]. As it has been demonstrated by Buhmann and Scheel [12] one should distinguish two different problems. If the atom is thermalized and coupled to its thermal bath, the both approaches lead to the same result. If the atom is not thermalized, the results of the linear-responce theory could underestimate the the CP force [12]. The macroscopic quantum-electrodynamical (QED) approach takes into account the possible absorption and spontaneous/stimulated emission of a thermal photon by the interacting atom. These contributions result in the enhancement of the CP force even if the atom is in its ground state. For the case of thermalized atom, this terms cancel out and one arrives at the predictions of Lifshitz theory [13].

The dispersion interaction between two atoms embedded in an EM field was considered by Milonni and Smith [14]. They showed that the interaction potential between them can be obtained from the vacuum Casimir-Polder one by a simple substitution $\hbar \omega_k/2 \rightarrow$
where $\omega_k$ is the energy of the photon of the state \{k, \lambda\}, with k and $\lambda$ the wave vector and the polarisation index of EM field, $N_{k\lambda}$ is the number of photons in the state \{k, \lambda\}. But what happens at the initial stage of interaction, when the atoms are not in the equilibrium with the EM field? Will the Milonni-Smith result still hold, or should it be modified as in the case of atom-slab interaction considered in Ref.[12]? The aim of this paper is to answer these questions.

2. Interaction between two atoms

We consider dispersion interaction between two dissimilar atoms A and B embedded in an EM field. Let atom A be situated at a point with the radius-vector $R_A$, and B in its arbitrary state (either excited or ground-state) and situated at a point $R_B$. The distance between the atoms is large enough to neglect the exchange interaction.

The Hamiltonian of the system atom A-EM field -atom B is ($\hbar = 1, c = 1$)

$$H = H_A + H_B + H_{ph} + H_{int},$$

where $H_A = \sum_i \epsilon_A b_i^\dagger b_i$, $H_B = \sum_i \epsilon_B b_i^\dagger b_i$ are the Hamiltonians of non-interacting atoms A and B, $\epsilon_{A(B)i}$ is the energy of i-th state of atom A (B). The annihilation operators of the i-th state of atom A (B) is $b_i(\beta_i)$. $H_{ph} = \sum k\lambda \epsilon_{k\lambda} c_{k\lambda}^\dagger c_{k\lambda} + 1/2$ is the Hamiltonian of free photons. The interaction Hamiltonian is

$$H_{int} = -\int \phi_A^\dagger(X) dE(X) \phi_A(X) d\mathbf{r} - \int \phi_B^\dagger(X) dE(X) \phi_B(X) d\mathbf{r},$$

where

$$\phi_A(X) = \sum_j \phi_j(\mathbf{r} - R_A) e^{-i\epsilon_A t} b_j, \phi_B(X) = \sum_j \phi_j(\mathbf{r} - R_B) e^{-i\epsilon_B t} \beta_j,$$

with $\phi_j(\mathbf{r} - R_{A(B)})$ the wave function of state j of the corresponding atom. $d\mathbf{r}$ is the operator of dipole moment,

$$E(X) = i \sum_{k\lambda} (2\pi k/V)^{1/2} e_{k\lambda} \left( \alpha_{k\lambda} e^{ikr} e^{-ikt} - \alpha_{k\lambda}^* e^{-ikr} e^{ikt} \right)$$

is the operator of free EM field, V is the volume, $e_{k\lambda}$ is the polarisation unit vector of EM field, $X = \{\mathbf{r}, t\}$.

To calculate the interaction potential we will first assume that atom A is in its ground state and the probability of excitation is negligible. In other words the number of photons at the transition frequency $\omega_A$ is negligible, although the number of photons at the transition frequency $\omega_B$ of atom B is sufficiently large $N(\omega_A) \ll N(\omega_B)$. This assumption enables us to calculate the interaction potential as the energy shift of atom A. Then, we will obtain a general result for arbitrary initial state of atom A and arbitrary $N(\omega_A)$ (but it still should not be too large to neglect the ionisation of the atoms) using symmetry properties of the potential.

If the atom does not change its initial state, the interaction potential, as one could easily show with the help of Hartree-Fock method, can be represented as the energy shift of the atomic level.

$$U = \Delta \epsilon_A$$

To calculate the energy shift, we will use the Keldysh-Green function approach adopted to the microscopic systems [15, 16]. The Keldysh Green function of atom A is [17]

$$G_{\alpha\beta}^R(X, X') = -i \langle T_c \psi_{\alpha i}(X) \psi_{\beta i}^\dagger(X') S_c \rangle$$
where $T_c$ is the time-ordering operator given on the Keldysh contour $c$, which goes from $-\infty$ to $+\infty$ for $l = 1$ and from $+\infty$ to $-\infty$ for $l = 2$. So, for $l = 1$ $T_c$ is a conventional ordering operator, while for $l = 2$ it is an antiordering operator. Further we omit index $A$. The operators are given in the interaction representation. $\langle \ldots \rangle$ means the averaging over the initial state of the system. The scattering matrix $S_c$ is

$$S_c = T_c \exp \left( \sum_{l=1,2} \frac{(-1)^l}{l!} \int_{-\infty}^{\infty} H_{\text{int}}(t) dt \right)$$

(7)

Using the Green function (6), one can readily find the density matrix of atom $A$

$$\rho(X, X') = iG_{12}(X, X')$$

(8)

To calculate the density matrix $\rho$ we expand Ex. (6) as a perturbation series

$$\rho(X, X') = \rho_0(X, X') - \frac{1}{2} \langle T_c \int dX_1 dX_2 \psi_l(X) \psi^\dagger_l(X') (-1)^{l_1+l_2} \psi^\dagger_{l_1}(X_1) E_{l_1}(X_1) d\psi_{l_1}(X_1) \times \psi^\dagger_{l_2}(X_2) E_{l_2}(X_2) d\psi_{l_2}(X_2) \rangle + \ldots$$

(9)

Index "0" means non-interacting atoms. To tackle the EM field we will use the standard Keldysh formalism [17]. We use the exact Wick’s theorem to average the atomic operators [18]:

$$\langle T_c \psi_l(X) \psi^\dagger_l(X') \rangle = \langle \psi_l(X) \psi^\dagger_l(X') \rangle : + ig_{ll'}(X, X')$$

(10)

where $:\ldots :$ means normal ordering,

$$g_{ll'}(X, X') = -i \langle T_c \psi_l(X) \psi^\dagger_l(X') \rangle_{\text{vacuum}}$$

(11)

is the electron propagator, $\langle \ldots \rangle_{\text{vacuum}}$ means the averaging over vacuum state. The average of normal products over the initial state occupied by a single atom $\langle \psi_l(X) \psi^\dagger_l(X') \psi_{l_1}(X_1) \psi^\dagger_{l_1}(X_2) \ldots : \rangle = 0$ for all the orders but the second one. For the second order it is just the density matrix of non-interacting atom $A$, $\rho_0(X, X') = \langle \psi_l(X) \psi^\dagger_l(X') \rangle$. Taking into account the processes, which do not change the initial state of atom $A$, we find

$$\rho(X, X') = \rho_0(X, X') + \int dX_1 dX_2 \left[ g_{11}(X, X_1) M_{11}(X_1, X_2) \rho_0(X_2, X') + \rho_0(X, X_1) M_{22}(X_1, X_2) g_{22}(X_2, X') \right]$$

$$+ \int dX_1 dX_2 dX_3 dX_4 g_{11}(X, X_1) M_{11}(X_1, X_2) \rho_0(X_2, X_3) M_{22}(X_3, X_4) g_{22}(X_4, X')$$

(12)

where the electron propagator $g_{ll'}(X, X') = -i \langle T_c \psi_l(X) \psi^\dagger_l(X') \rangle_{\text{vacuum}}$ obeys the Dyson equation

$$g_{ll'}(X, X') = g_{ll'}(X, X') + \int dX_1 dX_2 \sum_{l_1 l_2} g_{l_1 l_2}(X, X_1) M_{l_1 l_2}(X_1, X_2) g_{ll'}(X_2, X')$$

(13)

The mass operator $M$ is

$$M_{ll'}(X, X') = i (-1)^{l+l'} g_{ll'}(X, X') d\nu d\nu' D^{\nu\nu'}_{ll'}(X', X)$$

(14)

$D$ is the photonic Green tensor, $\nu$ is the tensor index.

$$D^{\nu\nu'}_{ll'}(X, X') = -i \langle T_c E_{l'}(X) E_{l'}^{\nu'}(X') S_c \rangle$$
Thus, the energy shift of the atomic level along with the interaction potential is

\[
g_{11}^0(E, r, r') = \sum_i \phi_i(r - R_A) \phi_i^*(r' - R_A) \frac{\delta_{12}^0}{E - \epsilon_i + i0},
\]

(16)

\[
g_{22}^0(E, r, r') = - \sum_i \phi_i(r - R_A) \phi_i^*(r' - R_A) \frac{\delta_{12}^0}{E - \epsilon_i - i0},
\]

(17)

\[
\text{Im} M = \langle \phi | M_{11} | \phi \rangle
\]

(18)

we can solve the Dyson equation (13) in the pole approximation.

\[
g_{11}(E, r, r') = \sum_i \frac{\phi_i(r - R_A) \phi_i^*(r' - R_A)}{E - \epsilon_i - \langle \phi_i | M_{11}(\epsilon_i) | \phi_i \rangle},
\]

(19)

\[
g_{22}(E, r, r') = - \sum_i \frac{\phi_i(r - R_A) \phi_i^*(r' - R_A)}{E - \epsilon_i - \langle \phi_i | M_{22}(\epsilon_i) | \phi_i \rangle}
\]

(20)

We suppose that the interaction between the atom A and the EM field was switched on at a moment \( t_0 \). Thus, the density matrix of the free atom A in its initial state \( p \) is

\[
\rho_0(X, X') = \phi_p(r - R_A) \phi_p^*(r' - R_A) e^{-i\epsilon_p(t-t')\theta(t-t_0)\theta(t'-t_0)}
\]

(21)

where \( \theta \) is the Heavyside function. Substituting the Exs.(18),(20), and (21) into Ex.(12), we obtain

\[
\langle \phi_p | \rho(X, X') | \phi_p \rangle = \exp[-i(\epsilon_p - \langle \phi_p | M_{11}(\epsilon_p) | \phi_p \rangle)(t-t_0)] \exp[i(\epsilon_p - \langle \phi_p | M_{22}(\epsilon_p) | \phi_p \rangle)(t'-t_0)]
\]

(22)

Thus, the energy shift of the atomic level along with the interaction potential is

\[
U = \Delta \epsilon_p = \text{Re} \langle \phi_p | M_{11}(\epsilon_p) | \phi_p \rangle
\]

(23)

The imaginary part \( \text{Im} M_{11} \) describes the decay of the initial state as a result of excitation. Substituting Ex. (14) into (23), we get

\[
U = \text{Re} \left[ \frac{i}{2\pi} \int_{-\infty}^{\infty} d\omega \int d\mathbf{r} d\mathbf{r}' \phi_p^*(\mathbf{r}') g_{11}(\omega + \epsilon_p, r, r') d\omega' D_{11}^{\nu'}(\omega, \mathbf{r}', \mathbf{r}) d\omega \phi_p(\mathbf{r}) \right]
\]

(24)

After integration over spatial coordinates and making use of the Ex.(18), we obtain:

\[
U = \text{Re} \left[ \frac{i}{2\pi} \sum_j \int_{-\infty}^{\infty} d\omega \frac{d\omega'}{d\omega'} D_{11}^{\nu'}(\omega, \mathbf{R}_A, \mathbf{R}_A) d\omega \phi_p^*(\mathbf{r}) \right]
\]

(25)

where \( \omega_{jp} = \epsilon_j - \epsilon_p \). If atom A is in its ground state

\[
U = -\text{Re} \left[ \frac{i}{2\pi} \sum_j \int_{-\infty}^{\infty} d\omega \alpha_A^{\nu'}(\omega) D_{11}^{\nu'}(\omega, \mathbf{R}_A, \mathbf{R}_A) \right]
\]

(26)

where

\[
\alpha_A^{\nu'}(\omega) = \sum_j \left( \frac{d_{pj}^\nu d_{jp}^\nu}{\omega_{jp} - \omega - i0} + \frac{d_{jp}^\nu d_{pj}^\nu}{\omega_{jp} + \omega + i0} \right)
\]

(27)

is the polarisability of atom A.
Figure 1. Feynman’s diagrams for the interaction of atom A with EM field. The solid line corresponds to $g$, the dashed line corresponds to $D$, and the dashed-dotted line to $\rho_0$

Ex. (26) generalises the vacuum interaction potential obtained in Ref.[16] to the case of external EM field.

To calculate the Green tensor $D_{11}$ we use the standard Keldysh technique [17, 19]. First we notice that the casual Green tensor $D_{11}$ can be written as a sum of the retarded Green tensor $D_r$ and the Green tensor $D_{12}$ [17, 19]

$$D = D_r + D_{12}$$  \hspace{1cm} (28)

where

$$D_{12}^{\nu\nu'}(X, X') = -i \langle \hat{E}^{\nu'}(X') \hat{E}^{\nu}(X) \rangle$$  \hspace{1cm} (29)

Note, that here $\hat{E}$ is in Heisenberg representation. Substituting (29) and (28) into (26) we find

$$U = -Re \frac{i}{2\pi} \int_0^\infty \alpha^{\nu\nu'}_A(\omega) D_r^{\nu\nu'}(\omega, R_A, R_A) d\omega$$

$$-Re \frac{i}{2\pi} \int_0^\infty \alpha^{\nu\nu'}_A(\omega) D_{12}^{\nu\nu'}(\omega, R_A, R_A) d\omega$$  \hspace{1cm} (30)

The first term of the Eq.(30) describes the interaction with the vacuum EM field, modified by the presence of another atom or macroscopic body [20]. The second term describes the interaction with the photons of the external EM field.

For zero temperature the tensor $D_{12}$ describes only the spontaneous emission by the medium (if it is excited) and Ex. (30) results in Ex. (41) of [16].

Let atom $B$ at a position $R_B$. It can be either in the excited state or ground one. For simplicity we suppose that the atoms are exposed to isotropic unpolarised EM field, i.e. $N_{k\lambda}$ depends only on $|k| = \omega$, $N_{k\lambda} = N(\omega)$. We will attack the problem perturbatively. The density matrix $D_{12}$ can be calculated with the help of Keldysh technique [17, 19]. According to (26), we should integrate the Green tensor with respect to positive frequencies $\omega > 0$. Thus

$$D_{12}(X, X') = D_{12}^{00}(X, X') + \int D_{12}^{0}(X, X_1) \Pi_{a}(X_1, X_2) D_{12}^{0}(X_2, X') dX_1 dX_2$$

$$+ \int D_{12}^{a0}(X, X_1) \Pi_{a}(X_1, X_2) D_{12}^{00}(X_2, X') dX_1 dX_2$$

$$- \int D_{12}^{00}(X, X_1) \Pi_{12}(X_1, X_2) D_{12}^{00}(X_2, X') dX_1 dX_2, (\omega > 0)$$  \hspace{1cm} (31)
Here we suppressed the tensor indices $\nu$. $D^0$ means free photon Green tensor. The polarisation operators $\Pi$ describe the interaction between the EM field and atom $B$ which is in its initial state $n$ [16]

$$
\Pi_{\nu\nu}'(\omega, r, r') = -\alpha_{\nu\nu}'(\omega)\delta(r-r') = \Pi_{\nu\nu}'(\omega, r, r')
$$

$$
\Pi_{12}^{\nu\nu}(\omega, r, r') = \sum_m 2\pi i\delta_{nm} \delta(\omega - \omega_{nm})\delta(r-r')
$$

(32)

The polarisation operators $\Pi_r$ and $\Pi_a$ describe the elastic scattering of light on the atom $B$, while $\Pi_{12}$ describes the downward transition of the atom as a result of spontaneous or stimulated emission. For positive frequencies, the free field Green tensor $D_{12}^0$ is

$$
D_{12}^0(\omega, r, r') = N(\omega)(D_{12}^0(\omega, r, r') - D_{12}^0(\omega, r, r'))
$$

(33)

The formula (31) along with (32) and (33) yield

$$
D_{12}(\omega, R_A, R_A) = D_{12}^0(\omega, R_A, R_A) - 2N(\omega)D_{12}^0(\omega, R_A, R_B)\alpha_{\nu\nu}'(\omega)\alpha_{\nu\nu}'(\omega, R_B, R_A)
$$

$$
+ 2N(\omega)\text{Re}D_{r}^0(\omega, R_A, R_B)\alpha_{\nu\nu}'(\omega, R_B, R_A)
$$

$$
+ \sum_m 2\pi i\delta_{nm} \delta(\omega - \omega_{nm})N(\omega)D_{12}^0(\omega, R_A, R_B)D_{12}^0(\omega, R_B, R_A)
$$

$$
- \sum_m 2\pi i\delta_{nm} \delta(\omega - \omega_{nm})(N(\omega) + 1)D_{r}^0(\omega, R_A, R_B)D_{12}^0(\omega, R_B, R_A)
$$

(34)

The first term of the r.h.s of Eq.(34) describes the free EM field. This term results in the optical Stark shift of the energy levels of atom $A$ and should be omitted. The second and the third terms describe the elastic scattering of light on atom $B$. The fourth term is responsible for the absorption of a photon by atom $B$ and the last term is for the spontaneous and stimulated emission of a photon by atom $B$. For isotopic atom $B$ ($\alpha_{\nu\nu}' \propto \delta_{\nu,\nu'}$) prepared in a mixed state with the probability to find the atom in $n$-th state $p_n$ we obtain

$$
U = \text{Re} \frac{i}{\pi} \int_0^\infty d\omega (N(\omega) + 1/2)\alpha_A(\omega)\alpha_B(\omega)(D_{r}^0(\omega, R_A R_B))^2
$$

$$
+ \frac{1}{3} \text{Re} \sum_{mn} [\delta_{mn}]^2 \alpha_A(\omega_{mn})|D_{r}^0(\omega_{mn}, R_A, R_B)|^2 [p_nN(\omega_{mn})\theta(\omega_{mn}) - p_n (N(\omega_{mn}) + 1) \theta(\omega_{mn})]
$$

(35)

The first term of the Eq. (35) coincides with the one obtained by Milonni and Smith with the help of the source theory [14]. The second term describes the absorption of a photon by atom $B$ (first term in the square brackets) as well as spontaneous and stimulated emission by atom $B$ (second term in the square brackets). At thermal equilibrium the probability to find an atom in its $n$-th state $p_n$ is given by the Boltzmann distribution, while the number of photons obeys the Bose statistics. One can easily check that in the equilibrium, the second term is zero due to the detailed balance, and we find that the equation (35) coincides with the Lifshitz formula [13].

Thus we arrive at a conclusion that the Milonni-Smith formula describes only the interaction between two atoms at equilibrium. But for the non-equilibrium case, if one or both atoms are not equilibrated, the Milonni-Smith formula, which does not take into account the absorption and emission of photons, underestimates the interaction potential between two atoms in the EM field even if the atoms are in their ground states.

Now to obtain quantitative results, we substitute the explicite expression for the retarded Green tensor [21]

$$
D_{r}^{\nu,\nu'}(\omega, r, r') = -\omega^2 \left[ \delta_{\nu,\nu'} \left( 1 + \frac{i}{\omega R} - \frac{1}{(\omega R)^2} \right) \right]
$$

$$
+ \text{i} \frac{3}{(\omega R)^2} \left( \frac{3}{\omega R} - 1 \right) \exp\left( i\omega R \right),
$$

(36)
where \( R = |\mathbf{r} - \mathbf{r}'| \) and \( s' = (r - r')^\nu / |\mathbf{r} - \mathbf{r}'| \), into (35). The interaction potential is the sum of the equilibrium potential \( U_{eq} \), which coincides with the Milonni-Smith result, and non-equilibrium one \( U_{neq} \).

\[
U = U_{eq} + U_{neq}
\]

\[
U_{eq} = \frac{1}{\pi R^2} \int_0^\infty \alpha_A(i\omega)\alpha_B(i\omega) u^4 \left[ 1 + \frac{2}{uR} + \frac{5}{(uR)^2} + \frac{6}{(uR)^3} + \frac{3}{(uR)^4} \right] \times (2N(\omega) + 1) \exp[-2iuR] du \tag{37}
\]

\[
U_{neq} = \frac{4}{9R^2} \sum_{mnj} \frac{|d_{pj}^A|^2 |d_{mn}^B|^2 \omega_{jp}(A) \omega_{mn}(B)}{\omega_{jp}(A)^2 - \omega_{mn}(B)^2} \left( 1 + \frac{1}{(\omega_{mn}(B)R)^2} + \frac{3}{(\omega_{mn}(B)R)^4} \right) \times \left[ p_n N(\omega_{mn}(B)) \theta(\omega_{mn}(B)) - p_n N(\omega_{mn}(B)) + 1 \right] \theta(\omega_{mn}(B)). \tag{38}
\]

The interaction potential between two atoms in EM vacuum has been calculated by Power and Thirunamachandran \[22, 23\].

\[
U_{neq} = -\frac{4}{9R^2} \sum_{mnj} \frac{|d_{pj}^A|^2 |d_{mn}^B|^2 \omega_{jp}(A) \omega_{mn}(B)}{\omega_{jp}(A)^2 - \omega_{mn}(B)^2} \left( 1 + \frac{1}{(\omega_{mn}(B)R)^2} + \frac{3}{(\omega_{mn}(B)R)^4} \right) \theta(\omega_{mn}(B)). \tag{39}
\]

Eqs (38) and (39) obviously coincide if \( N(\omega) = 0 \) in Eq.(38). Thus, Eq. (38) generalizes the Power-Thirunamachandran theory to the case of atoms embedded in a EM field. For the limit \( R >> \lambda \) (\( \lambda \) is the characteristic wavelength of the atomic transition), the potential drops with the distance between the atoms as \( U_{neq} \propto R^{-2} \) (compare to the Casimir-Polder formula \( U \propto R^{-7} \)), this peculiar behaviour of the potential was discussed by Power and Thirunamachandran \[22, 23\], (see also \[16\], where the interaction potential between an excited atom and a ground-state one embedded in a dielectric medium was calculated for vacuum state of EM field). In the presence of EM field, the non-equilibrium contribution to the interaction between a ground-state atom and an excited one is enhanced by factor \( N(\omega_B) + 1 \) compared to the interaction potential in EM vacuum.

If both atoms are in their ground states, we should take into account only the first term in the square brackets of (40) For the retarded regime \( R >> \lambda \)

\[
U_{neq} = \frac{4}{9R^2} \sum_{mnj} \frac{|d_{pj}^A|^2 |d_{mn}^B|^2 \omega_{jp}(A) \omega_{mn}(B)}{\omega_{jp}(A)^2 - \omega_{mn}(B)^2} N(\omega_{mn}(B)) \theta(\omega_{mn}(B)). \tag{40}
\]

This potential drops as \( R^{-2} \) with the distance. It means that for non-equilibrium case, the contribution of the non-equilibrium term due to photon absorption could be much greater then the contribution of the equilibrium one described by the Milonni-Smith formula. We should mention, that the approach we developed in this presentation is valid for the initial stage of interaction provided the atoms and the EM field are not in the equilibrium. Obviously, after the equilibration the non-equilibrium term is zero and we obtain the Milonni-Smith result.

The results obtained above are valid only if the probe atom A is in its ground state during the interaction process. It means that the number of photons at the transition energies of atom A is negligible \( N(\omega_A) \approx 0 \), while the number of photons at the frequency of atom B is large \( N(\omega_B) \gg N(\omega_A) \). Our formalism does not allow us to calculate the interaction potential for the case when atom A can be excited by the electromagnetic field. But to extend our results
to the case free from the above mentioned assumption, we can use the symmetry properties of the potential: (i) the potential is obviously symmetric with respect to atomic indices; (ii) if both atoms are thermalized the non-equilibrium contribution to the potential should be equal to zero. Using these properties, we get the non-equilibrium potential in general case

\[ U_{neq} = \sum_{mnjp} S(|\omega_A|, |\omega_B|) \left[ p_n N(\omega_B) \theta(\omega_B) - p_n N(-\omega_B) + 1 \right] \left[ p_B \theta(\omega_A) - p_B \theta(-\omega_A) \right] \\
+ \sum_{mnjp} S(|\omega_B|, |\omega_A|) \left[ p_B N(\omega_A) \theta(\omega_A) - p_B N(-\omega_A) + 1 \right] \left[ p_A \theta(\omega_B) - p_A \theta(-\omega_B) \right] \]  \hspace{1cm} (41)

with \( \omega_A = \omega_{j\ell(A)}, \omega_B = \omega_{m\ell(B)} \) (note that the frequencies \( \omega_A \) and \( \omega_B \) are positive for upward transitions and negative otherwise). \( S \) is given by

\[ S(\omega_A, \omega_B) = \frac{4}{9R^2} \frac{|d^A_{\ell|j}|^2 |d^B_{m|\ell}|^2 \omega_A \omega_B}{\omega_A^2 - \omega_B^2} \left( 1 + \frac{1}{(\omega_B R)^2} + \frac{3}{(\omega_B R)^4} \right) \]  \hspace{1cm} (42)

For vacuum state of the EM field we recover all the corresponding results by Power and Thirunamachandran [22, 23].

As an example we consider two two-level atoms embedded in a thermal EM field at temperature \( T \).

1. Let both atoms be in their ground-states.

If the temperature is low compared to the transition frequencies \( T << \omega_{A,B} \), but high compared to the frequency difference \( T >> |\omega_A - \omega_B| \), we find

\[ U_{neq} = \frac{4}{9R^2} \exp(-\omega_A/T) |d^A| \omega_A \omega_B \left( 1 + \frac{1}{(\omega_B R)^2} + \frac{3}{(\omega_B R)^4} \right), (\omega_A > \omega_B); \]  \hspace{1cm} (43)

\[ U_{neq} = -\frac{4}{9R^2} \exp(-\omega_A/T) |d^A| \omega_A \omega_B \left( 1 + \frac{1}{(\omega_A R)^2} + \frac{3}{(\omega_A R)^4} \right), (\omega_A < \omega_B). \]  \hspace{1cm} (44)

This result is not resonant and exponentially small compared to the equilibrium contribution, but the asymptotic behaviour of Ex.(43) for large distances \( R >> \lambda \) is \( 1/R^2 \), while the asymptotics of the equilibrium contribution \( R >> 1/T \) is \( 1/R^6 \) [21].

For \( T << |\omega_A - \omega_B| \),

\[ U_{neq} = \frac{4}{9R^2} \exp(-\omega_B/T) |d^A| \omega_A \omega_B \left( 1 + \frac{1}{(\omega_B R)^2} + \frac{3}{(\omega_B R)^4} \right), (\omega_A > \omega_B); \]  \hspace{1cm} (45)

\[ U_{neq} = -\frac{4}{9R^2} \exp(-\omega_A/T) |d^A| \omega_A \omega_B \left( 1 + \frac{1}{(\omega_A R)^2} + \frac{3}{(\omega_A R)^4} \right), (\omega_A < \omega_B). \]  \hspace{1cm} (46)

The contributions of the non-equilibrium potential are resonant now.

For high temperature case \( T >> \omega_{A,B} \) we obtain a universal expression which is valid independent of the initial states of the atoms.

\[ U_{neq} = \frac{4}{9R^2} T |d^A|^2 |d^B|^2 \omega_A \omega_B \left( 1 + \frac{3}{(\omega_B \omega_A)^2 R^4} \right), \]  \hspace{1cm} (47)
The total potential is

\[ U = U_{eq} + U_{neq} = \frac{4}{9R^2} |d_A|^2 |d_B|^2 \omega_A \omega_B \]

(48)

So for \( T >> \omega_{A,B} \), \( T >> 1/R \) the total interaction potential drops like \( 1/R^2 \) with the distance \( R \), even though \( R << \lambda \).

2. Two excited atoms.

If \( T << \omega_{A,B} \),

\[ U_{neq} = \frac{4}{9R^2} \frac{|d_A|^2 |d_B|^2 \omega_A \omega_B}{\omega_A + \omega_B} \left( -\left( \omega_B^2 + \omega_A \omega_B + \omega_A^2 \right) \frac{1}{R^2} + \frac{3}{\omega_A \omega_B R^4} \right) \]

(49)

This result is no more exponentially small (although is not resonant) compared to the equilibrium potential and could be greater than the latter. For large distances we once again obtained the \( R^{-2} \) dependence of the potential. If \( T >> \omega_{A,B} \), we obtain Ex. (48).

3. Atom A is ground-state, atom B is excited.

\( T << \omega_{A,B} \),

\[ U_{neq} = \frac{4}{9R^2} \frac{|d_A|^2 |d_B|^2 \omega_A \omega_B^2}{\omega_A^2 - \omega_B^2} \left( 1 + \frac{1}{(\omega_B R)^2} + \frac{3}{(\omega_B R)^4} \right) \]

(50)

This non-equilibrium contribution is resonant and exceeds the equilibrium contribution.

For \( T >> \omega_{A,B} \) we again obtain the result (48).

3. Conclusions

We reexamined the dispersion interaction between two atom in external EM field. We showed that the Milonni-Smith approach is valid only if the atoms are in equilibrium with the EM field. To study the initial stage of the non-equilibrium dynamics, one should take into account absorption and spontaneous/stimulated emission. The latter significantly modifies the interaction potential, resulting in some cases in resonance contributions. We generalised the Power- Thirunamachandran theory for two atoms in EM vacuum and included the effects of external EM field. As en example we considered the interaction between two atoms in thermal EM field. For low temperatures, the result dramatically depends on the initial state of the atoms, while for large temperatures the interaction potential is universal, at least for the pure initial states of atoms.

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