Determination of the orthotropic parameters of a representative sample by computed tomography

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Abstract. The simulation of the stress-strain state of porous media is actual problem nowadays. The application of the mathematical apparatus of continuum mechanics to such media will make it possible to extend the scope of the problems to be solved. In this paper we study method of determination of mechanical properties in representative element bases on computer tomography data. To this task algorithm to create finite element grid connected with the data of the computed tomography was developed. Orthotropic properties can be determined by using target function. Unknown components of the target function are orthotropic directions. These directions can be determined by minimization of the target function. Rotating the matrix of anisotropic constants leads to components of the orthotropic tensor in orthotropic axes. These tensors are compared using of stress invariant.

1. Introduction

The simulation of the stress-strain state of porous media is actual problem nowadays. The application of the mathematical apparatus of continuum mechanics to such media will make it possible to extend the scope of the problems to be solved. But how continuum material can be compared with the whole porous material? Nowadays methods of non-destructive analyses of materials are developing at a rapid pace, and it’s becomes possible to analyze the structure of various heterogeneous materials. Actual task is to determine relationship between structural and mechanical properties of material [1-3]. This task is popular in clinical medicine [4-6], special case it is accounting for changes in bone tissue under external force [7-9]. Information about the structure of bone tissue can play a decisive role in planning treatment, and this data can improve the quality biomechanical modeling of joints and organs of musculoskeletal system [4-6]. In this paper, a technique is proposed for determining the mechanical properties of a porous material based on computed tomography (CT) data.

2. Materials and Methods

The main hypothesis of the method is that material is isotropic, all anisotropy appears because distribution of the pores. The dimensions and distribution of pores through the volume quantitatively characterize the anisotropic properties [10 - 12]. As previously mentioned, information about material structure is received from CT. In this case, the input data for constructing the geometry of the object is a three-dimensional array of X-ray attenuation coefficient data [13]. The physical size of the array element corresponds to the size of the voxel, and the size of the entire array corresponds to the dimension of the working field of the CT. Usually, the attenuation coefficient is normalized...
(Hounsfield scale) [13]. To differentiate the material from air, the binarization threshold is determined. Then the data array can be transformed into a binary one, where 1 means the presence of material and 0 - its absence (pore). For optimal determination of the binarization threshold the Otsu’s method was used [14]. To construct a finite-element model, a hexagonal uniform grid is constructed, in which the size of each element is determined by the dimensions of the voxel. The grid area corresponds to the CT data area. To construct a representative model the grid is modified according to the binary data, in this case the elements corresponding to the pores were deleted.

In numerical experiments, the kinematic loading of the sample was used. In this case, on one face (passive) the conditions of zero displacements were imposed, and on the opposite (active) - displacement, corresponding to the specified deformations were used. After solving the stress-strain state (SST) problem, the average for all components of the stress is determined on the passive side. The deformations on active face should provide elastic behavior of the material. In the general case, the tensor of elastic constants contains 21 independent components. Numerical experiments were carried out to determine these constants. Based on the finite element method, 6 tests were performed for the representative model: 3 uniaxial compression tests and 3 for pure shear [15, 16].

Thus, for all calculated cases, 36 equations are obtained, while unknowns - 21. In Voigt's notation, this can be represented by the expressions given below, in which the index $j$ corresponds to the number of the numerical experiment (in the described method $j$ changes from 1 to 6):

$$\frac{1}{A} \int \sigma_j dA = \bar{C} \cdot \bar{\varepsilon}^o$$

To solve an overdetermined system, the least squares method (LSM) was used. Thus, it is possible to obtain the averaged anisotropic properties of the representative model.

To determine the orthotropic axes, the tensor of elastic constants in a rotated coordinate system was considered. Using the fact that in orthotropic axes some of the elastic components equivalent to zero, the target function can be defined [2, 17]:

$$f = \frac{2\sum_{j=1}^{3} \sum_{i=4}^{6} (C^i_{oj})^2 + 2\sum_{i=4}^{6} \sum_{j=1}^{3} (C^i_{oj})^2 + 4\sum_{i,j=4}^{6} (C^i_{oj})^2}{\sum_{i,j=4}^{6} (C^i_{oj})^2 + 4\sum_{i=4}^{6} (C^i_{oj})^2}$$

This function equal to zero in the orthotropic axes, and so we can formulate the problem of minimizing the objective function with respect to the desired angles [2, 17]:

$$f (\varphi, \psi, \theta) \rightarrow \min$$

In practice, such an idealization is rarely realized, due to errors associated with the noise, binarization and discretization of CT data, as well as errors in solving an overdetermined task. Therefore, after turning the tensor by the angles of orthotropy, compulsory recalculation of components with the condition of zeroing of the corresponding components. Thus, it is possible to obtain the averaged orthotropic properties of the representative model and the direction of the orthotropic axes.

3. Results and Discussion
For tests we use a cube of dimensions 50x50x50 mm. The size of the voxel is 0.2 mm. Material is isotropic, but there are some pores in the object, so it will be anisotropic (see figure 1). Using described methods we get results.
Figure 1. Representative sample.

Anisotropic tensor is obtained by using the technique described in earlier in this page looks like:

\[
C_{\text{anis}} = \begin{pmatrix}
4593.97 & 28.46 & 94.96 & 6.91 & -11.39 & 13.88 \\
3870.69 & 17.11 & 5.65 & 3.87 & 1.42 \\
3923.84 & 2.50 & 15.45 & 12.74 \\
1080.78 & -3.70 & 1.45 \\
1085.37 & 3.74 \\
1260.15
\end{pmatrix}
\]

In orthotropic cases measurement error does not give desired results. The complementary least squares method helps us to nullify the necessary elements. New tensor of form is:

\[
C_{\text{ort}} = \begin{pmatrix}
2467.25 & 100.33 & 1952.45 & 0 & 0 & 0 \\
3754.05 & 68.25 & 0 & 0 & 0 \\
2548.94 & 0 & 0 & 0 \\
1299.36 & 0 & 0 \\
9149.18 & 0 \\
1221.33
\end{pmatrix}
\]

To assess quality of model, on the one hand, conditions are created for rigid embedding and on the other hand a force on 400 N. In this work we have 3 models: full models, it has 15625 finite elements; anisotropic and orthotropic model, they have 1 finite element. Three models are compared using Mises stress. We obtained the following results: 16.59 MPa, 15.98 MPa, 30.82 MPa for isotropic, anisotropic and orthotropic. In ortotropic model stress has twice the value. This phenomenon can be explained by stress concentrators. Increasing number of finite elements, reduce value of the concentrators and average stress became 16.94 MPa. These result have good match with experimental data [18].

4. Conclusion

In this paper methodology of producing the representative element by computed tomography data is presented. To obtain components of elasticity tensor, we used technique of numerical experiments, to solve an overdetermined system the least squares method was used. To find orthotropic axes and
elasticity components minimizing the target function was used. The methods was used on real object and results was analyzed.

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