Observation of $e^+e^- \rightarrow \gamma \chi_{c1}$ and search for $e^+e^- \rightarrow \gamma \chi_{c0}, \gamma \chi_{c2}$, and $\gamma \eta_c$ at $\sqrt{s}$ near 10.6 GeV at Belle

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The production of heavy quark pairs in high-energy lepton collisions is described well by perturbative Quantum Chromodynamics (pQCD). Yet a description of these pairs forming quarkonium—charmonium or bottomonium—is theoretically challenging. Quarkonium formation is governed by nonperturbative long-distance effects [1]. Nonrelativistic Quantum Chromodynamics (NRQCD) factorization was used to compute the cross section for several processes, including the double-charmonium production cross section [2, 3], \( e^+e^- \to \gamma \chi_{cJ} \) \( (J = 0, 1, 2) \) [4-8] and \( e^+e^- \to \gamma \eta_c \) [5, 6, 9, 10] at \( B \) factories with relativistic and higher-order corrections included.

Electromagnetic quarkonium production is relatively simpler than other production mechanisms, and therefore it serves as a good testing ground for such NRQCD predictions. The BESIII experiment measured \( e^+e^- \to \gamma \chi_{cJ} \) cross sections at \( \sqrt{s} = 4.01 \text{ GeV}, 4.23 \text{ GeV}, 4.26 \text{ GeV}, \) and \( 4.36 \text{ GeV} \) and \( e^+e^- \to \gamma \eta_c \) cross section at the same energies and additionally at \( 4.42 \text{ GeV} \) and \( 4.60 \text{ GeV} \) [11, 12]. At none of the individual energy points does the statistical significance for production of \( \chi_{cJ} \) or \( \eta_c \) exceed \( 3\sigma \), and when the data from all energy points are combined, the statistical significances for \( \chi_{c1}, \chi_{c2}, \) and \( \eta_c \) are \( 3.0\sigma, 3.4\sigma, \) and greater than \( 3.6\sigma \), respectively.

In addition, the BESIII experiment reported evidence for \( X(3872) \) production via \( e^+e^- \to \gamma X(3872) \) [13]. Precise and unambiguous measurement of \( e^+e^- \to \gamma \chi_{cJ} \) and \( \gamma \eta_c \) is useful for understanding C-even quarkonia and exotic \( XYZ \) particles [14-16], e.g., \( X(3872) \).

In this paper, we report cross-section measurements for the exclusive reactions \( e^+e^- \to \gamma \chi_{cJ} \) and \( \gamma \eta_c \) with data recorded at \( \sqrt{s} \sim 10.6 \text{ GeV} \) by the Belle experiment at the KEKB asymmetric-energy \( e^+e^- \) collider [17, 18]. The data used in this analysis corresponds to 89.5 fb\(^{-1} \) of integrated luminosity at 10.52 GeV, referred to as the continuum sample; 711 fb\(^{-1} \) at 10.58 GeV, referred to as the \( Y(4S) \) sample; and 121.4 fb\(^{-1} \) at 10.867 GeV, referred to as the \( \Upsilon(5S) \) sample.

The Belle detector [19, 20] is a large solid-angle magnetic spectrometer that consists of a silicon vertex detector, a 50-layer central drift chamber (CDC), an array of aerogel threshold Cherenkov counters (ACC), a barrel-like arrangement of time-of-flight scintillation counters (TOF), and an electromagnetic calorimeter comprised of CsI(Tl) crystals (ECL) located inside a superconducting solenoid coil that provides a 1.5 T magnetic field. An iron flux-return yoke instrumented with resistive plate chambers (KLM) located outside the coil is used to detect \( K^0_L \) mesons and to identify muons. The Belle detector is described in detail elsewhere [19, 20].

We determined event-selection criteria using a large sample of Monte Carlo (MC) signal events (100k) for \( e^+e^- \to \gamma \chi_{cJ} \) and \( \gamma \eta_c \) at \( \sqrt{s} = 10.52, 10.58, \) and \( 10.867 \text{ GeV} \) generated with EvtGen [21]. In the generator, the polar angle of the transition photon in the \( e^+e^- \) C.M. system (\( \theta_\gamma \)) is distributed according to \( (1 + \cos^2 \theta_\gamma) \) for \( \gamma \chi_{c0} \) and \( \gamma \eta_c \) production, and \( (1 + 0.63 \cos^2 \theta_\gamma) \) for \( \gamma \chi_{c1} \) production [22]. No definite model exists for the distribution of \( \theta_\gamma \) in \( \gamma \chi_{c2} \) production because the combination of tensor-meson production and \( \gamma \) emission is theoretically complicated and requires experimental input. So we model the production of this channel as evenly distributed in phase space and account for differences from \( (1 \pm \cos^2 \theta_\gamma) \) distributions as systematic uncertainties.

Corrections due to initial-state radiation (ISR) are taken into account in all studied channels, where we assume \( \sigma(e^+e^- \to \gamma \chi_{cJ}/\gamma \eta_c) \sim 1/s^n \) in the calculation of the radiative-correction factor. The values of \( n \), determined from Refs. [8, 10], are 1.4 for \( \chi_{c0} \), 2.1 for \( \chi_{c1} \), 2.4 for \( \chi_{c2} \), and 1.3 for \( \eta_c \) in the predictions of next-to-leading order (NLO) QCD, and 1.4 for \( \eta_c \) in leading order (LO) QCD. Possible sources of background events from \( \Upsilon(nS) \to B\bar{B} \) \( (n = 4, 5) \), \( \Upsilon(5S) \to B_s^+\bar{B}_s^- \), and \( e^+e^- \to q\bar{q} \) \( (q = u, d, s, c) \) are checked with a MC sample four times larger than the data sample, and are also gen-
rerated with EvtGen [21], GEANT3 [23] is used to simulate the detector response to all MC events. The \( \chi_{cJ} \) candidates are reconstructed from their decays to \( \gamma J/\psi \) with \( J/\psi \to \mu^+ \mu^- \), and the \( \eta_c \) candidates are reconstructed from five hadronic decays into \( K_S^0 K^+\pi^- \), \( \pi^+\pi^- K^+K^- \), \( 2(\pi^+\pi^-) \), \( 2(K^+K^-) \), and \( 3(\pi^+\pi^-) \) [24].

We define a well-reconstructed charged track as having impact parameters with respect to the nominal interaction point of less than 0.5 cm and 4 cm perpendicular to and along the beam direction, respectively. For an \( e^+e^- \to \gamma \chi_{cJ} \) candidate event, we require the number of well-reconstructed charged tracks, \( N_{\text{trk}} \), to be two, and the net charge be zero. For \( e^+e^- \to \gamma \eta_c \), we require \( N_{\text{trk}} = 6 \) for the \( 3(\pi^+\pi^-) \) final state and \( N_{\text{trk}} = 4 \) for the other final states, also with a zero net charge. For the particle identification (PID) of a well-reconstructed charged track, information from different detector sub-systems, including specific ionization in the CDC, time measurement in the TOF, and the response of the ACC, is combined to form a likelihood \( L_i \) [25] for particle species \( i \). Tracks with \( R_K = L_K/(L_K + L_{\pi}) < 0.4 \) are identified as pions with an efficiency of 96%, while 9% of kaons are misidentified as pions; tracks with \( R_K > 0.6 \) are identified as kaons with an efficiency of 98%, while 8% of pions are misidentified as kaons.

For muons from \( J/\psi \to \mu^+ \mu^- \), we require at least one of the paired tracks to have \( R_\mu = L_\mu/(L_\mu + L_K + L_\pi) > 0.95 \); if one track has \( R_\mu < 0.95 \), it must have associated hits in the KLM agreeing with the extrapolated trajectory provided by the CDC [26]. The efficiency of muon-pair identification is 94%.

Using a multivariate analysis with a neural network [27] based on two sets of input variables [28], a \( K_S^0 \) candidate is reconstructed from a pair of oppositely-charged tracks that are treated as pions. An ECL cluster with energy higher than 50 MeV is treated as a photon candidate if it does not match the extrapolation of any charged track. The photon with the maximum energy in the \( e^+e^- \) C.M. system is taken as the transition photon. Since there are two photons in the \( e^+e^- \to \gamma \chi_{cJ} \) channel, the transition photon is denoted as \( \gamma_{h} \), and the one with the second highest energy is denoted as \( \gamma_{l} \) and is taken as the photon from the \( \chi_{cJ} \to \gamma J/\psi \) decay. We require \( E(\gamma_{l}) > 300 \) MeV to suppress the backgrounds from fake photons.

If there are more than two photons, to suppress the background from the ISR process \( e^+e^- \to \gamma_{\text{ISR}} \psi(2S) \to \gamma_{\text{ISR}} \gamma \chi_{cJ} \), an extra photon \( (\gamma_{\text{ext}}) \) besides \( \gamma_{h} \) and \( \gamma_{l} \) is selected, and \( M(\gamma_{\text{ext}}\gamma_{l}\mu^+\mu^-) < 3.60 \) GeV/c\(^2\) or \( M(\gamma_{\text{ext}}\gamma_{h}\mu^+\mu^-) > 3.78 \) GeV/c\(^2\) is required. This requirement removes 92.2% and 91.5% of the ISR \( \psi(2S) \to \gamma \chi_{c1} \) and \( \gamma \chi_{c2} \) background events, respectively. The residual yields of \( \chi_{c1} \) and \( \chi_{c2} \) events from ISR \( \psi(2S) \) decays are expected to be \( 0.84 \pm 0.15 \) and \( 0.43 \pm 0.05 \), respectively, where the uncertainties from intermediate branching fractions and \( \psi(2S) \) production cross-section via ISR are considered. The selection efficiency of this requirement is 85.5% for the \( \chi_{c1} \) signal and 80.9% for the \( \chi_{c2} \) one.

A four-constraint (4C) kinematic fit constraining the four-momenta of the final-state particles to the initial \( e^+e^- \) collision system is performed. In \( e^+e^- \to \gamma \chi_{cJ} \), an additional constraint is used, constraining the mass of the \( \mu^+\mu^- \) pair to the \( J/\psi \) nominal mass, giving a five-constraint (5C) kinematic fit. Kinematic fits with \( \chi_{3C}^2 < 25 \) for \( e^+e^- \to \gamma \chi_{cJ} \) and \( \chi_{4C}^2 < 30 \) for \( e^+e^- \to \gamma \eta_c \) are required to improve the resolutions of the momenta of charged tracks and the energies of photons, and to suppress backgrounds with more than two photons, such as ISR processes.

The invariant mass distribution of the \( \mu^+\mu^- \) pair from the continuum, \( \Upsilon(4S) \), and \( \Upsilon(5S) \) data samples, prior to the application of the 5C fit, is shown in Fig. 1, together with the result of fitting the data to the sum of a Gaussian function for the \( J/\psi \) and a first order polynomial for the background. In the plot, a clear \( J/\psi \) signal is observed. We define the \( J/\psi \) signal region as \( |M(\mu^+\mu^-) - M_{J/\psi}| < 48 \) MeV/c\(^2\) corresponding to three times the detector resolution, where \( m_{J/\psi} \) is the \( J/\psi \) mass [29].

![FIG. 1: The invariant mass distribution of \( \mu^+\mu^- \) pairs from all the data samples before the application of the 5C kinematic fit. The solid curve is the fit, and the dotted line is the fitted background. The arrows show the boundaries of the defined \( J/\psi \) signal region.](image-url)

After all of the above requirements, some non-peaking background events are observed in the processes \( e^+e^- \to \gamma \chi_{cJ} \) and \( \gamma \eta_c \) at the studied C.M. energy points.

Figure 2 shows the \( \gamma_{J/\psi} \) invariant mass distributions for the data. A clear \( \chi_{c1} \) signal is observed in the \( \Upsilon(4S) \) data sample, but is not evident in the other two data samples. Unbinned extended maximum-likelihood fits to the \( M_{J/\psi} \) distributions are performed to extract the \( \chi_{cJ} \) signal yields. The \( \chi_{cJ} \) signal shapes in the fits are a Breit-Wigner (BW) function convolved with a Log-normal [30] function with all the values of the \( \chi_{cJ} \) resonance parameters fixed from the fits to MC simulations; second-order polynomial functions are used to describe the background distributions. The MC-simulated \( \chi_{cJ} \) signals have mass
resolutions around 6 MeV/c² with small low-mass tails due to the measurement of $E(\gamma)$. The results from the fits are listed in Table I. The statistical significances of the $\chi_{cJ}$ signals are calculated using $\sqrt{-2 \ln \mathcal{L}_0/\mathcal{L}_{\text{max}}}$, where $\mathcal{L}_0$ and $\mathcal{L}_{\text{max}}$ are the maximized likelihoods of the fits without and with the $\chi_{cJ}$ signal, respectively. The statistical significance of the $\chi_{c1}$ signal in the $\Upsilon(4S)$ sample is 5.2$\sigma$. The signal significance remains at 5.1$\sigma$ when convolving the likelihood profile with a Gaussian function of width equal to the total systematic uncertainty discussed below. The $\chi_{c3}$ signals in the continuum sample and the $\Upsilon(5S)$ sample are not significant, as indicated in Table I.

Figure 2: The $\gamma J/\psi$ invariant mass spectra at $\sqrt{s} = 10.52$ (bottom), 10.58 (middle), and 10.867 GeV (top) together with fit results. The points with error bars show the data and the solid curves are the fit functions; the dashed curves show the fitted backgrounds contributions. The arrows show the expected peak positions for the $\chi_{c0}$, $\chi_{c1}$, and $\chi_{c2}$ states.

Figure 3 shows the $\eta_c$ invariant mass distributions for the five hadronic final states combined. Clear signals resulting from the production of $J/\psi$ by ISR are present, while no significant $\eta_c$ signal is evident.

We perform a simultaneous fit to the five $\eta_c$ final states, in which the ratio of the yields in each channel is fixed to the ratio of $B_i\varepsilon_i$, where $i$ is the $\eta_c$ decay-mode index, $B_i$ is the branching fraction taken from the Particle Data Group (PDG) [29], and $\varepsilon_i$ is the reconstruction efficiency determined from MC simulation. In the fit, we use a BW function convolved with a Gaussian resolution function to describe the $\eta_c$ signal; the values of all parameters are fixed from the fits to MC simulations. A Gaussian function with free parameters is used to describe the $J/\psi$ signal, and a second-order Chebyshev polynomial function is used for the backgrounds. The fit results are shown in Fig. 3 and summarized in Table I.

The Born cross section for $e^+e^- \rightarrow \gamma X$ is given by the formula

$$\sigma_B(e^+e^- \rightarrow \gamma X) = \frac{N_{\text{obs}}}{\mathcal{L}} \frac{|1 - \prod B_i \varepsilon_i| \times (1 + \delta)_{\text{ISR}}}{\sum_i B_i \varepsilon_i \times (1 + \delta)_{\text{ISR}}}.$$  

where $N_{\text{obs}}$ is the number of signal events obtained from the fit, $\mathcal{L}$ is the integrated luminosity of the data sample, $B_i$ and $\varepsilon_i$ are the branching fraction and the detection efficiency of the $i$-th $X$ decay mode ($\chi_{cJ}$ is reconstructed in one decay mode and $\eta_c$ in five decay modes), $(1 + \delta)_{\text{ISR}}$ is the radiative-correction factor, calculated using the formula given in Ref. [31], and $|1 - \prod|^2$ is the vacuum polarization factor, calculated according to Ref. [32]. The obtained Born cross sections for $e^+e^- \rightarrow \gamma \chi_{cJ}$ and $\gamma \eta_c$ are listed in Table I together with all the parameter results needed for the cross section calculation.

For all processes but $e^+e^- \rightarrow \gamma \chi_{c1}$ at $\sqrt{s} = 10.58$ GeV, upper limits at 90% credibility level (C.L.) [33] on the numbers of signal events ($N_{\text{UL}}$) and the Born cross sections ($\sigma_B^{UL}$) are determined by solving the equation

$$\frac{\int_0^{x_{\text{UL}}} \mathcal{F}_{\text{likelihood}}(x) dx}{\int_0^{\infty} \mathcal{F}_{\text{likelihood}}(x) dx} = 90\%,$$

where $x$ is the assumed signal yield or Born cross section, and $\mathcal{F}_{\text{likelihood}}(x)$ is the corresponding maximized likelihood from a fit to the data. To take into account the systematic uncertainties discussed below, the likelihood is convolved with a Gaussian function whose width equals the corresponding systematic uncertainty.

Combining the measurement of $\sigma_B(e^+e^- \rightarrow \gamma \chi_{c1})$ from BESIII [11] and this analysis, we show the cross section as a function of $\sqrt{s}$ in Fig. 4. We fit these data points with a function proportional to $1/s^n$ assuming that the reaction $e^+e^- \rightarrow \gamma \chi_{c1}$ proceeds through the continuum process only: from a fit to the seven points for $e^+e^- \rightarrow \gamma \chi_{c1}$, we find $n = 2.1^{+0.3}_{-0.4}$. The significance of the fitted $n$ is 2.2$\sigma$, calculated using $\sqrt{\chi^2_0 - \chi^2_{\text{min}}} = 2.2$, where $\chi^2_0$ is the $\chi^2$ with $n$ fixed at 0, and $\chi^2_{\text{min}}$ is the mini-

![Figure 2](image1.png)

![Figure 3](image2.png)
TABLE I: Measurements of $e^+ e^- \to \gamma \chi_{cJ}$ and $e^+ e^- \to \gamma \eta_c$ at $\sqrt{s} = 10.52$ GeV, 10.58 GeV, and 10.867 GeV. $\varepsilon(\%)$ represents efficiency for the $e^+ e^- \to \gamma \chi_{cJ}$, and value of $\Sigma_i B_{\ell i}$ for the $e^+ e^- \to \gamma \eta_c$. $\Sigma(\sigma)$ is the statistical signal significance; $\sigma_{\text{syst}}(\%)$ is the systematic uncertainty on $\sigma_B$. The Born cross sections are given with statistical (first) and systematic (second) uncertainties.

| Channel                              | $\sqrt{s}$ (GeV) | $\Lambda^{\text{obs}}$ | $\Lambda^{\text{UL}}$ | $\Sigma(\sigma)$ | $\varepsilon(\%)$ | $|1 - \prod_i^2|$ | $(1 + \delta)_{\text{ISR}}$ | $\sigma_{\text{syst}}(\%)$ | $\sigma_B$ (fb) | $\sigma_B^{\text{UL}}$ (fb) |
|--------------------------------------|-------------------|-------------------------|------------------------|-------------------|------------------|--------------------|--------------------------|---------------------|------------------------|---------------------|
| $e^+ e^- \to \gamma \chi_{c0}$      | 10.52             | 2.9$^{+4.0}_{-3.3}$     | 9.6                    | 0.9               | 19.0             | 0.73268            | 10.7                     | 286.2$^{+394.7}_{-325.6}$ | 30.7                  | 957.2                 |
| $e^+ e^- \to \gamma \chi_{c1}$      |                   | 4.8$^{+3.3}_{-2.9}$     | 10.4                   | 1.9               | 20.8             | 0.733432           | 8.9                      | 16.2$^{+12.1}_{-9.8}$    | ± 1.4                  | 34.9                  |
| $e^+ e^- \to \gamma \chi_{c2}$      |                   | 0.8$^{+2.6}_{-1.0}$     | 4.5                    | -                 | 19.9             | 0.733675           | 12.8                     | $-5.0^{+14.3}_{-10.0}$   | ± 0.6                  | 28.9                  |
| $e^+ e^- \to \gamma \eta_c$         |                   | 6.8$^{+14.8}_{-14.3}$   | 30.8                   | 0.5               | 0.79             | 0.732788           | 11.3                     | 9.0$^{+19.5}_{-18.1}$    | ± 1.0                  | 40.6                  |
| $e^+ e^- \to \gamma \chi_{c0}$      | 10.58             | -1.6$^{+9.9}_{-8.9}$    | 16.5                   | -                 | 18.9             | 0.732725           | 13.1                     | $-20.0^{+122.3}_{-111.0}$| ± 2.6                  | 205.9                 |
| $e^+ e^- \to \gamma \chi_{c1}$      |                   | 39.0$^{+9.5}_{-8.8}$    | -                      | 5.2               | 19.9             | 0.73329            | 10.0                     | 17.3$^{+12.2}_{-9.8}$    | ± 1.7                  | -                     |
| $e^+ e^- \to \gamma \chi_{c2}$      |                   | -8.7$^{+5.7}_{-5.0}$    | 7.2                    | -                 | 19.8             | 0.733532           | 20.9                     | $-6.8^{+4.5}_{-3.8}$     | ± 1.4                  | 5.7                   |
| $e^+ e^- \to \gamma \eta_c$         |                   | 67.2$^{+42.0}_{-39.2}$  | 125.9                  | 1.8               | 0.78             | 0.732645           | 13.0                     | 11.3$^{+7.0}_{-6.6}$     | ± 1.5                  | 21.1                  |
| $e^+ e^- \to \gamma \chi_{c0}$      | 10.867            | -1.3$^{+4.0}_{-3.2}$    | 7.0                    | -                 | 17.7             | 0.732054           | 9.4                      | $-101.4^{+312.0}_{-240.6}$| ± 9.5                  | 543.7                 |
| $e^+ e^- \to \gamma \chi_{c1}$      |                   | 1.9$^{+3.4}_{-2.6}$     | 7.9                    | 0.7               | 16.8             | 0.732623           | 13.4                     | 5.8$^{+10.9}_{-8.0}$     | ± 0.8                  | 24.3                  |
| $e^+ e^- \to \gamma \chi_{c2}$      |                   | -2.8$^{+3.2}_{-2.4}$    | 5.3                    | -                 | 16.3             | 0.732863           | 14.4                     | $-15.7^{+17.9}_{-13.4}$  | ± 2.3                  | 30.3                  |
| $e^+ e^- \to \gamma \eta_c$         |                   | 12.3$^{+18.2}_{-17.4}$  | 42.3                   | 0.9               | 0.76             | 0.731974           | 9.1                      | 12.3$^{+17.3}_{-18.1}$   | ± 1.1                  | 42.2                  |

minimum $\chi^2$ with the value of $n$ free, respectively. Adding an additional possible resonance, such as $\psi(4040)$, $\psi(4160)$, $Y(4260)$, or $\Upsilon(4S)$, the largest change in the fitted value of $n$ is 0.3. The result is consistent with the prediction by NRQCD with all leading relativistic corrections included in Ref. [8]. Due to the large uncertainties, we do not fit the $\sqrt{s}$ dependence of $e^+ e^- \to \gamma \chi_{c0}$, $e^+ e^- \to \gamma \chi_{c2}$, or $e^+ e^- \to \gamma \eta_c$.

![FIG. 4: Measured cross sections for $e^+ e^- \to \gamma \chi_{c1}$ as a function of C.M. energy. Error bars contain both the statistical and systematic uncertainties. The curve shows the result of fit with a function proportional to $1/s^n$.](image)

There are several sources of systematic uncertainty in the Born cross section measurements, including detection efficiency, the statistical error of the MC efficiency, trigger simulation, intermediate state branching fractions, resonance parameters, the distribution of $\theta_1$ for $e^+ e^- \to \gamma \chi_{c2}$ fit uncertainty, the $s$ dependence of the cross sections, and the integrated luminosity. The systematic uncertainty for detection efficiency is a final-state-dependent combined uncertainty for all the different types of particles detected, including tracking efficiency, PID, $K^0_S$ selection, and photon reconstruction.

Based on a study of $D^{+} \to D^{0}(\to K^0_S \pi^+ \pi^-)\pi^+$, the uncertainty in tracking efficiency is taken to be 0.35% per track. The uncertainties in PID are studied via $\gamma \gamma \to \ell^+ \ell^-$ for leptons and a low-background sample of $D^*$ decay for charged kaons and pions. The studies show uncertainties of 2.2% for each muon, 1.0% for each charged kaon, and 1.2% for each charged pion.

Comparison of the $K^0_S$ selection efficiencies determined from data and MC results in $1 - \delta_{\text{ISR}}^{\text{MC}} \approx (1.4 \pm 0.3)$%; 1.7% is taken as a conservative systematic uncertainty. The uncertainty in the photon reconstruction is 2.0% per photon, according to a study of radiative Bhabha events. For each final state, the final detection efficiency uncertainty is obtained by adding all sources in quadrature.

The statistical uncertainty in the determination of efficiency from MC is less than 1.0%. We include uncertainties of 4.8% and 0.6% from trigger simulations for $e^+ e^- \to \gamma \chi_{cJ}$ and $\gamma \eta_c$, respectively. The uncertainties from the intermediate decay branching fractions are taken from Ref. [29]. For $e^+ e^- \to \gamma \chi_{cJ}$, the total uncertainties from the branching fractions are obtained by adding all relative uncertainties in quadrature. For $e^+ e^- \to \gamma \eta_c$, the total uncertainty from the branching fraction is obtained by summing in quadrature over the five decay modes with weight factors equal to the corresponding efficiency. The uncertainties from the resonance
parameters are estimated by changing the values of mass and width of a resonance by 1σ in the fits [29]. Additionally, for the mode $e^+e^- \rightarrow \gamma \chi_{c1}$, the uncertainty from simulating the $\theta_s$ dependence is estimated to be 8.2% by comparing the difference between a phase space distribution and the angular distributions of $(1 \pm \cos^2 \theta_s)$.

In determining the number of signal events from the fits to data, the fit range and the choice of the function to describe the backgrounds are the main sources of systematic uncertainty. For the latter, the background shapes are replaced by an exponential form or a higher-order Chebyshev polynomial, and the largest difference compared to the nominal fit result is taken as the related systematic uncertainty. Changing the $s$ dependence of the cross sections from fitted values of $n$ to a large number, e.g., $n = 4$, gives very small differences in the radiative-correction factor ($< 1\%$). The total luminosity is determined to 1.4% precision using wide-angle Bhabha scattering events. All the uncertainties are summarized in Table II and, assuming all the sources are independent, summed in quadrature for the total systematic uncertainties.

In summary, we perform measurements of $e^+e^- \rightarrow \gamma \chi_{cJ}$ ($J = 0, 1, 2$) and $\gamma \eta_c$ at $\sqrt{s} = 10.52$ GeV, 10.58 GeV, and 10.867 GeV using a 921.9 fb$^{-1}$ data sample taken by the Belle detector. A clear $\chi_{c1}$ signal is observed at 10.58 GeV with a statistical significance of 5.2$\sigma$, and the Born cross section is measured to be $(17.3^{+4.2}_{-3.9(\text{stat.})} \pm 1.7(\text{syst.}))$ fb. For the cases where a $\chi_{cJ}$ or $\eta_c$ signal is not evident, upper limits on the Born cross sections are determined at 90% C.L. Using the cross sections measured at three different $\sqrt{s}$ in this analysis and from BESIII at much lower $\sqrt{s}$ and assuming the reaction $e^+e^- \rightarrow \gamma \chi_{c1}$ proceeds through the continuum process only, we determine the cross section $s$-dependence to be $1/s^{2.1^{+0.3}_{-1.4}}$ for $e^+e^- \rightarrow \gamma \chi_{c1}$.

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TABLE II: Relative systematic uncertainties (%) in the cross-section measurements for \(e^+e^- \rightarrow \gamma\chi_cJ\) and \(\gamma\eta_c\) at \(\sqrt{s} = 10.52\) GeV, 10.58 GeV, and 10.867 GeV. When three values are given in a cell, they apply to \(\chi_{c0}\), \(\chi_{c1}\), and \(\chi_{c2}\), respectively; otherwise a single number applies to all states.

| Final state                  | \(\gamma\chi_{c0}/\chi_{c1}/\chi_{c2}\) | \(\gamma\eta_c\) |
|-----------------------------|-------------------------------------------|-------------------|
| C.M. energy (GeV)           |                                           |                   |
| Detection efficiency        | 6.0                                       | 2.8               |
| MC sample size              | 1.0                                       | 1.0               |
| Trigger                     | 4.8                                       | 0.6               |
| Branching fractions         | 4.8/3.7/3.8                               | 7.5               |
| Resonance parameters        | 1.7/0.3/0.7                               | 0.6               |
| \(\theta_\gamma\) distribution | -/-/8.2                              | -                  |
| Fit uncertainty             | 5.3/1.9/4.5                               | 7.7               |
| Integrated luminosity       | 1.4                                       | 1.4               |
| Sum in quadrature           | 10.7/8.9/12.8                             | 11.3              |

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