Extracting Messages Masked by Chaotic Signals of Time-delay Systems

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Abstract

We show how to extract messages masked by a chaotic signal of a time-delay system with very high dimension and many positive Lyapunov exponents. Using a special embedding coordinate, the infinite dimensional phase space of the time-delay system is projected to a special three-dimensional space, which enables us to identify the time-delay of the system from the transmitted signal, and reconstruct the chaotic dynamics to unmask the hidden message successfully. The message extraction procedure is illustrated by simulations with Mackey-Glass time-delay system for two type of masking schemes and different kinds of messages.

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Application of chaotic synchronization systems to secure communication has been a field of great research interest [1, 2, 3, 4, 5]. However, it has been shown that in some low-dimensional chaotic systems with only one positive Lyapunov exponent, the hidden message can be unmasked by dynamical reconstruction of the chaotic signal using nonlinear dynamical forecasting (NLDF) methods [3, 7], by some simple return maps [8], or some other methods [9]. It has been suggested that one possible way to improve the security is to employ hyperchaos in communication [5, 10, 11], based on the consideration that increased randomness and unpredictability of the hyperchaotic signals will make it more difficult to extract a masked message. Lately, it has been shown that messages masked by hyperchaos of a six-dimensional systems can also be attacked using the NLDF methods [15], showing that going to higher dimensions does not produce a drastic improvement in the security of the system if the local dynamics are still quite low dimensional.

It has been known that very simple time-delay systems [12] are able to exhibit hyperchaos [13]. Therefore, it has been proposed in a recent report that time-delay systems provide alternative simple and efficient tools for chaos communication with low detectability [14]. Chaotic attractors of time-delay systems can have much higher dimension and many more positive Lyapunov exponents than the system studied in Ref. [15], and whether the communication is as secure as expected has not been examined yet. In this letter, we will show that messages masked by chaos of a time-delay system with very high dimension and many positive Lyapunov exponents can be extracted successfully, not using some well-established dynamical reconstruction methods [3, 7, 15], but using a special yet simple embedding approach proposed recently for time-series analysis of time-delay systems [16, 17, 18].

We focus our attention in this letter on scalar time-delay systems of the form

$$\dot{x} = f(x, x_{\tau_0}), \quad x_{\tau_0} = x(t - \tau_0).$$ (1)

For such systems with large time-delay $\tau_0$, some well-established nonlinear time-series analysis methods [19, 20, 21] run into severe problems [22, 23].

An observation of the Eq. (1) shows that in a special three-dimensional space $(x_{\tau_0}, x, \dot{x})$, the dynamics of the system is restricted to a smooth manifold defined by Eq. (1), namely

$$\dot{x} - f(x, x_{\tau_0}) = 0.$$ (2)

However, in a similar space $(x, x, \dot{x})$ with $\tau \neq \tau_0$, the trajectory is no longer restricted to a smooth hypersurface, but fills a great part of the space, resulting a more complicated structure. This makes it possible to detect the time-delay $\tau_0$ of the system by some measures of the complexity as a function of embedding delay $\tau$, and then recover the dynamics of the system [16, 17, 18].

This approach is applied in this letter to extract messages masked by chaotic signals of the above time-delay system. In the context of synchronization, we consider the following communication system with two masking schemes considered in Ref. [14]:

Scheme I:

$$\dot{x} = f(x, x_{\tau_0}) + kI,$$
$$s = x + I,$$ (3)

and Scheme II:

$$\dot{x} = f(x, x_{\tau_0}) + kIx,$$
$$s = x(1 + I).$$ (4)

An authorized receiver has an identical copy of the time-delay system which is made synchronized with $x$ by the following coupling:

$$\dot{y} = f(y, y_{\tau_0}) + k(s - y).$$ (5)

In this communication system, the message $I$, often much lower in amplitude than the chaotic signal $x$, is injected into the transmitter to modulate the time-delay system. The injection of
the message has effectively altered the transmitter dynamics and has been considered as a way to improve the security [3, 14] compared with methods where the message is directly added to the chaotic carrier [2]. The masking Scheme II is expected to produce secure masking because the message and the chaotic signal couple with each other in a more sophisticated manner. $s$ is the signal transmitted to the receiver to achieve synchronization with a proper coupling parameter $k$. As a third party, we do not have a receiver system $y$, but have the time series of the transmitted signal $s$ sampled by a time interval $h$.

Our message extraction approach consists of the following steps:

1. We project the time series $\{s^i\}$ to the three-dimensional space $(s^i, s^i, \dot{s}^i)$ with $\dot{s}^i$ estimated as $\dot{s}^i = (s^{i+1} - s^{i-1})/2h$.

2. We investigate the complexity of the projected trajectory in the $(s^i, s^i, \dot{s}^i)$ space by the measure of the smoothness. First, we apply a local linear approximation

   $$\hat{s} = a_i + b_is + c_is_\tau$$  \hspace{1cm} (6)

   to a small neighborhood $U_i$ of a point $(s^i, s^i)$. The fitting parameters $a_i$, $b_i$ and $c_i$ are computed by least square fit, and the local fitting error is

   $$e_i = \frac{1}{M_{U_i}} \sum_{j \in U_i} (s^j_i - a_i - b_is^j - c_is^j_\tau)^2,$$ \hspace{1cm} (7)

   where $M_{U_i}$ is the number of the neighbor points. The average $E$ of $e_i$ over a number of points $(x^i, x^i)$ provides a measure of the smoothness of the structure in the projected space. If $\tau = \tau_0$, the trajectory is restricted to close vicinity of the smooth hypersurface for small enough message $I$, and $\dot{s}$ can be rather small if the size $\epsilon$ of neighborhood is sufficiently small; otherwise, $E$ can be quite larger because there is no unique functional relationship between $\dot{s}$ and $(s^i, s^{i})$ for $\tau \neq \tau_0$. We can expect a minimum of $E$ at $\tau = \tau_0$. By examining $E$ as a function of embedding delay $\tau$, we can detect the time-delay $\tau_0$ of the system by the minimum of $E$.

3. After correct identification of the value of $\tau_0$, we use the local linear approximation

   $$\hat{x}^i = a_i + b_is^i + c_is^i_\tau$$ \hspace{1cm} (8)

   as an estimation of $\dot{x} = f(x, x_{\tau_0})$ of the time-delay system in the absence of message $I$. From Eqs (3,4) we have $\dot{s} = f(x, x_{\tau_0}) + kI + \dot{I}$ for the masking Scheme I, and $\dot{s} = f(x, x_{\tau_0}) + kIx + \dot{x}I + x\dot{I}$ for Scheme II. For the conditions $|I| \ll |x|$, $|\dot{I}| \ll |kI|$, and $|\dot{x}| \ll |kx|$ with $|\cdot|$ denoting the amplitude, the extracted message can be estimated as

   $$kI^i = \begin{cases} s^i - \hat{x}^i, & \text{Scheme I}, \\ (\dot{s}^i - \hat{s}^i)/s^i, & \text{Scheme II}. \end{cases}$$ \hspace{1cm} (9)

To illustrate the message extraction procedure, we employ the Mackey-Glass equation as in Ref. [14],

$$x = f(x, x_{\tau_0}) = -bx + \frac{ax_{\tau_0}}{1 + x^c_{\tau_0}}.$$ \hspace{1cm} (10)

With parameter $b = 0.1$, $a = 0.2$, and $c = 10$, the system is chaotic for $\tau_0 > 16.8$. In the chaotic regime, the number of positive Lyapunov exponents increases with $\tau_0$, and is about 15 for $\tau_0 = 300$, and the chaotic attractor dimension increases almost linearly with $\tau_0$, for example, the Kaplan-Yorke dimension is roughly 30 for $\tau_0 = 300$ [14]. In our simulation, we take $\tau_0 = 300$ and $k = 1.0$ [14].

In all the following examples, we record $N = 50000$ points with sample interval $h = 0.5$. The size of neighborhood is set by $\epsilon = 0.01$. 

First, let us consider a simple message signal of sine wave \( I(t) = A \sin(2\pi t/T) \) with \( A = 0.005 \) and \( T = 200 \). Fig. 1 shows the measure of smoothness \( E \) as a function of \( \tau \). A pronounced minimum at \( \tau = 300 \) enables us to identify the time-delay of the system correctly although the system is modulated by the injected message \( I \). This is also true for our other examples in the following where the results of \( E \) are not presented to save space.

With the correct value of time-delay, the message can be extracted successfully, as illustrated in Fig. 2 for the masking Scheme I and Fig. 3 for the masking Scheme II, respectively. A comparison between the time series of \( s \) in Fig. 3(a) and that of \( \Delta s = \dot{s} - \dot{\hat{s}} \) in Fig. 3(b) reveals that when \( s \) is close to zero in a certain period of time, the corresponding \( \Delta s \) is also close to zero in this period of time, indicating that \( \Delta s \) is modulated by \( s \). The demodulated signal \( \Delta s/s \) is shown in Fig. 3 as the extracted message. The results show that the masking Scheme II, although can result in larger distortion to the extracted messages, does not produce drastic improvement of the security.

Now let us consider an example of more complicated message signal. In our simulation, we construct a message

\[
I(t) = \frac{A}{m} \sum_{i=1}^{m} B_i \sin(2\pi t/T_i),
\]

(11)

where \( B_i \) and \( T_i \) are random numbers uniform on \((0, 1)\) and \((50, 500)\) respectively.

Fig. 4 and 5 are results of message extraction for an realization of such a complicated message with \( A = 0.01 \) and \( m = 100 \). Again it is seen that the quality of the recovered message deteriorates more when masking Scheme II is employed. However, a comparison between the power spectra of the original and extracted messages has shown that unmasking is successful for both masking schemes.

We should point out that the identification of the time-delay \( \tau_0 \) and the quality of the recovered message is not sensitive to the choice of \( N \), \( h \) and \( \epsilon \). In general, \( \epsilon \) should be small enough to apply local linear approximation, but large enough to average out the fluctuations induced by the message. As a result, if the amplitude of the message is too large, the quality of the recovered message can be quite poor, and the message extraction becomes more difficult. However, the chaotic signals may not provide enough masking for messages with quite large amplitude.

For the M-G time-delay system studied above, the frequency of the message should be rather low, because the power spectrum of the chaotic signal is very low at high frequency and is not enough to mask messages with high frequency. A low frequency of the message means \( |\dot{I}| \ll |I| \), which is an advantage for a third party to recover the message with high quality.

In summary, we present a simple method to extract messages masked by chaotic signal of a time-delay system, which has a very high dimensionality and many positive Lyapunov exponents. Using a special embedding space, the infinite dimensional phase space of the time-delay system is projected to a three-dimensional space, independent of the actual dimension and the number of positive Lyapunov exponents of the chaotic attractor. The time-delay of the system is correctly identified even in the presence of the message, which enables us to extract the message successfully using a simple local reconstruction of the time-delay system in the three-dimensional space.

We come to the conclusion based on our analysis that communication using time-delay system is not as secure as intuitively expected. In general, the security of chaos communication may be spoiled if any reconstruction of the dynamics of the system is possible in some appropriate space, even for very high dimensional dynamics.

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Figure Captions

Fig. 1 As a measure of the smoothness, the average fitting error $E$, as a function of the embedding delay $\tau$, has a pronounced minimum at the value of the time-delay of the system.

Fig. 2 Illustration of message extraction for a sine wave message masked by Scheme I. (a) The original message signal $kI$, (b) the extracted message, and (c) the power spectrum of the extracted message.

Fig. 3 Illustration of message extraction for the sine wave message masked by Scheme II. (a) A time series of the transmitted signal $s$, (b) $\Delta s = \dot{s} - \dot{x}^i$, (c) the extracted message, and (d) the power spectra of the extracted message.

Fig. 4 Illustration of message extraction for a complicated message. (a) The original message, (b) the extracted message for the masking Scheme I, and (c) the extracted message for the masking Scheme II.

Fig. 5 The power spectra of the original message and the extracted messages in Fig. 4.
Fig. 2

(a) Waveform of $k_I$ vs. $t$

(b) Waveform of $k_{I_e}$ vs. $t$

(c) Spectrogram of $s(f)$
Fig. 3
Fig. 5