New Method of Solving the Seepage Model for the Multilayer Composite Reservoir with the Double Porosity

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Abstract Aimed at multilayer composite reservoir with the double porosity, meanwhile considering the influence of well bore storage and skin effect, the seepage model for multilayer composite reservoir with the double porosity which the flow was steady from pore to crack was established in different outer boundary (infinite; closed; constant pressure) conditions; the exact solution of reservoir pressure drop and bottom hole pressure drop were obtained by Laplace transform in the Laplace space; the unified expression of solution was obtained by constructing similar kernel functions in different outer boundary conditions, therefore new method which solving this class of reservoir model is put forward, namely similar construction method. This method plays an important guiding role in exploring seepage law of oil and gas reservoir.

Keywords: double porosity, multilayer composite reservoir, similar construction method, similar kernel function, similar structure

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1. Introduction

Reservoir with the double porosity is reservoir that has the natural fractures [1], it is a kind of reservoir with the complex medium, which consist of two kinds of pore medium, namely matrix block medium and fracture medium, and most of reservoirs have the characteristics of multilayer. Composite reservoir is reservoir that consist of two reservoir area with the different nature[1]. Therefore multilayer composite reservoir with the double porosity is the combination of multilayer reservoir with the double porosity and composite reservoir.

The multilayer reservoir has been researched by some domestic and foreign scholars. In 1986, Kuchuk [2] built model of the multilayer connecting reservoir, and analytic expression of the model in the Laplace space was obtained by using superposition principle. In 1999, considering the influence of the well bore storage and skin effect, aimed at multilayer reservoir without the cross-flow between layers, Xianzhi Xu etc [3] provided computing method of the type curve for well test model and fitting method of the layering parameter. In 2002, Shunchu Li etc [4] obtained general solution of pressure distribution by using Laplace transform for multilayer reservoir which well bore storage and skin effect were taken into consideration. In 2009, considering cross-flow between layers, Liehui Zhang [5] built seepage model for the double layer reservoir with the double porosity, and typical curve of flow pressure at well bottom was drawn by using Stehfest numerical inverse and computer programming. In 2011, considering the influence of well bore storage and skin effect, Jingjing Guo etc [6] built model without cross-flow for the multilayer reservoir with the double porosity, analytic solution of bottom hole pressure and production of the layer were obtained in the Laplace space by Laplace transform, and according to method of numerical inverse, the changeable curve of production of the layer was drawn. In 2013, Quanyong Li etc [7] solved exact solution of reservoir pressure and bottom hole pressure by Laplace transform for multilayer reservoir with the double porosity, and unified expression of solution was obtained for different outer boundary conditions. However, so far no one studies the seepage model for the multilayer composite reservoir with the double porosity.

In 2004, Shunchu Li and Minhui Jia [8] put forward an important method which solved a class of differential equation, namely similar construction method, just as any a real number can be expressed by continued fraction. In the next years, many studies [9-19] have been carried out for some boundary value problems of the special differential equation. On the basis of above research, considering the influence of well bore storage and skin effect, this paper firstly built the seepage model for the multilayer composite reservoir with the double porosity. We solved similar structure of reservoir pressure and bottom hole pressure in the Laplace space by Laplace
transform, and this research will further improve the theory of similar structure which solving the seepage model of complex reservoir, and it is also a kind of supplement for theoretical research of the seepage law in reservoir.

2. Establishment of Model

In order to build the model of the multilayer composite reservoir with the double porosity. The basic assumptions are included as follows:

(1) The reservoir with the double porosity is level; equal thickness and isotropy.

(2) N layers are level and equal thickness in multilayer composite reservoir, and inter-layer has not cross-flow. The fluid of every layer is homogeneous and weakly compressible. Formation fluid is single-phase, and fluid obeys Darcy’s law.

(3) Ignoring gravity and capillary force.

(4) Constant production is q (the yield of the j layer is

\[ q_j = \frac{1}{N} \sum_{j=1}^{N} q_j \]  

(5) A well of radius \( r_w \) (the well radius of the j layer is \( r_{w_j} \), and \( r_w = \frac{1}{N} \sum_{j=1}^{N} r_{w_j} \) ) is opened in the center of reservoir, the boundary radius is \( \beta r_{w_j} (\beta > 1) \) in inner region, the boundary radius is \( R \) in outer region.

(6) Initial pressure of every layer is \( p_{oj} \) ( \( j = 1, 2, \ldots, N \)).

According to above assumptions, the model of the multilayer composite reservoir with the double porosity is built as follows:

Seepage differential equations

\[
\frac{\partial^2 R_{jj}}{\partial r^2} + \frac{1}{r} \frac{\partial R_{jj}}{\partial r} + \lambda_j \left( P_{j,m} - P_{I,jf} \right) = \delta_{jj} \left( \frac{\partial R_{jm}}{\partial t} + \lambda_j \left( P_{j,m} - P_{I,jf} \right) \right) 
\]

\[
= 0 \left( r_{w_j} < r < \beta r_{w_j}, t > 0, j = 1, 2, \ldots, N \right) 
\]

\[
\frac{\partial^2 P_{jj}}{\partial r^2} + \frac{1}{r} \frac{\partial P_{jj}}{\partial r} + \lambda_j \left( P_{j,m} - P_{I,jf} \right) = \delta_{jj} \left( \frac{\partial P_{jm}}{\partial t} + \lambda_j \left( P_{j,m} - P_{I,jf} \right) \right) 
\]

\[
= 0 \left( r_{w_j} < r < \beta r_{w_j}, t > 0, j = 1, 2, \ldots, N \right) 
\]

Initial conditions

\[ P_{jj} (r, 0) = P_{I,jm} (r, 0), P_{2,jf} (r, 0) = P_{2, jm} (r, 0) = 0 \]

Internal boundary condition

\[
\begin{cases} 
P_w (t) = \left[ P_{1,jf} - S_j r \frac{\partial P_{1,jf}}{\partial r} \right]_{r=r_{w_j}} \\
2\pi N \sum_{j=1}^{N} \gamma_{jj} \left( r \frac{\partial P_{jj}}{\partial r} \right)_{r=r_{w_j}} = -Bq + C \frac{dp_w}{dt} 
\end{cases} 
\]

Interfacial boundary condition

\[
\begin{cases} 
P_{1,jf} (\beta r_{w_j}, t) = P_{2,jf} (\beta r_{w_j}, t) \\
\frac{\partial P_{jj}}{\partial r} \big|_{r=r_{w_j}} = \gamma_{jj} \frac{\partial P_{jj}}{\partial r} \big|_{r=r_{w_j}} 
\end{cases} 
\]

Outer boundary conditions

When outer boundary is infinity:

\[ P_{2,jf} (x, t) = 0 \]

When outer boundary is closed:

\[ \frac{\partial P_{jj}}{\partial r} \big|_{r=R} = 0 \]

When outer boundary is constant pressure:

\[ P_{2,jf} (R, t) = 0 \]

Where:

\[ P_{oj} = P_{oj} - P_{nik} (n = 1, 2; j = 1, 2, \ldots, N; i = f, m) ; \]

\[ \lambda_{nj} = \frac{\alpha_{nj} k_{njm}}{k_{nj}} ; \quad \gamma_{jj} = \frac{k_{1,jf} h_{jj}}{\mu_j} ; \quad \gamma_{jj} = \frac{k_{2,jf} \mu_j}{k_{1,jf} \mu_{2,j}} ; \]

3. Laplace Form of the Model

Applied to models (1)-(10) by employing the Laplace transform method on dimensionless time \( t \), Let:

\[
\begin{align*}
\bar{p}_{oj} (r, z) &= \int_0^\infty e^{-zt} p_{oj} (r, t) dt (n = 1, 2; j = 1, 2, \ldots, N; i = f, m) ; \\
\bar{p}_w (z) &= \int_0^\infty e^{-zt} p_w (t) dt
\end{align*}
\]

then the boundary value problem of an ordinary differential equation is obtained in the Laplace space as follows:

\[
\frac{d^2 \bar{P}_{jj}}{dr^2} + \frac{1}{r} \frac{d \bar{P}_{jj}}{dr} + \lambda_j \left( \bar{P}_{j,m} - \bar{P}_{I,jf} \right) = \delta_{jj} \left( \frac{d \bar{P}_{jm}}{dt} + \lambda_j \left( \bar{P}_{j,m} - \bar{P}_{I,jf} \right) \right) 
\]

\[
= 0 \left( r_{w_j} < r < \beta r_{w_j}, t > 0, j = 1, 2, \ldots, N \right) 
\]

\[
\delta_{jj} \left( \frac{d \bar{P}_{jm}}{dt} + \lambda_j \left( \bar{P}_{j,m} - \bar{P}_{I,jf} \right) \right) = 0 
\]

\[
\left( r_{w_j} < r < \beta r_{w_j}, t > 0, j = 1, 2, \ldots, N \right) 
\]
\[ d^2 \tilde{P}_{2jf} \over dr^2 + \frac{1}{r} d\tilde{P}_{2jf} + \lambda_{2j} \left( \tilde{P}_{2jm} - \tilde{P}_{2jf} \right) = \delta_{2jf} \tilde{P}_{2jf} \] (13)

\[ \left( r > \beta_{rwj}, t > 0, j = 1, 2 \cdots N \right) \]

\[ \delta_{2jm} \tilde{P}_{2jm} + \lambda_{2j} \left( \tilde{P}_{2jm} - \tilde{P}_{2jf} \right) = 0 \] (14)

\[ \tilde{P}_w(z) = \left[ \tilde{P}_{1j} - S_z \frac{d\tilde{P}_{1j}}{dr} \right]_{r=rwj} = -Bq + Cz \tilde{P}_w \] (15)

\[ 2\pi \sum_{j=1}^{N} \gamma_j \left( r \frac{d\tilde{P}_{1j}}{dr} \right)_{r=rwj} = -Bq + Cz \tilde{P}_w \]

4. Solution of the Model and Similar Structure

After the Laplace transform, equation (12) and (14) plug into the equation (11) and (13) respectively, and then Bessel equation of 0 order is obtained, therefore the general solutions of equation (11)-(14) are obtained as follows:

\[ \tilde{P}_{1jm} (r,z) = A_{j1} K_0 \left[ r \sqrt{f_{1j}(z)} \right] + B_{j1} I_0 \left[ r \sqrt{f_{1j}(z)} \right] \] (18)

\[ \tilde{P}_{2jm} (r,z) = \frac{\lambda_{j1}}{\delta_{2jm} z + \lambda_{nj}} A_{j1} K_0 \left[ r \sqrt{f_{1j}(z)} \right] + B_{j1} I_0 \left[ r \sqrt{f_{1j}(z)} \right] \] (19)

\[ \tilde{P}_{2jm} (r,z) = A_{j2} K_0 \left[ r \sqrt{f_{2j}(z)} \right] + B_{j2} I_0 \left[ r \sqrt{f_{2j}(z)} \right] \] (20)

\[ \tilde{P}_{2jm} (r,z) = \frac{\lambda_{j2}}{\delta_{2jm} z + \lambda_{nj}} A_{j2} K_0 \left[ r \sqrt{f_{2j}(z)} \right] + B_{j2} I_0 \left[ r \sqrt{f_{2j}(z)} \right] \] (21)

Where:

\[ f_{nj}(z) = \frac{\delta_{nj} \delta_{nj} z^2 + \delta_{nj} z \lambda_{nj} + \delta_{nj} z \lambda_{nj}}{\delta_{nj} z + \lambda_{nj}} (n = 1, 2) \]

\[ A_{j1}, B_{j1}, A_{j2}, B_{j2} \] are undetermined coefficients, and they are determined by internal and outer boundary conditions.

The following equations are obtained by putting the formula (18)-(21) plug into the formula (15)-(17).
\[ \psi^*(r,z) = \frac{1}{f_{11}(z)} \psi_{0.1} \left( r, R_j, \sqrt{f_{22}(z)} \right) \]

When outer boundary is constant pressure:

\[ \psi^*(r,z) = \frac{1}{f_{11}(z)} \psi_{0.0} \left( r, R_j, \sqrt{f_{22}(z)} \right) \]

4 With regard to the similar structure of the solution with reservoir pressure distribution in the Laplace space, it provides immensely convenient for solving the seepage model for the multilayer composite reservoir with the double porosity. Similarly, similar structures of the solution with reservoir pressure distribution in the Laplace space are obtained by putting the \( A_{j1}, B_{j1}, A_{j2}, B_{j2} \) plug into the formula (18)-(21) and using similar kernel function.

\[ \bar{P}_{1j} = \frac{Bq}{z} \frac{1}{2\pi} \sum_{j=1}^{N} \gamma_{ij} r_{nj} \sqrt{f_{11}(z)} \left[ \psi \left( r_{nj}, z \right) + S_j r_{nj} \sqrt{f_{11}(z)} \right] \] (27)

\[ \psi \left( r_{nj}, z \right) + S_j r_{nj} \sqrt{f_{11}(z)} \]

\[ \bar{P}_{2j} = \frac{Bq}{z} \frac{1}{2\pi} \sum_{j=1}^{N} \gamma_{ij} r_{nj} \sqrt{f_{11}(z)} \left[ \psi \left( r_{nj}, z \right) + S_j r_{nj} \sqrt{f_{11}(z)} \right] \] (28)

Similar structure of bottom hole pressure is obtained by putting the formula (27) plug into the formula (15) and using the property of differential equation.

\[ \bar{P}_w = \frac{Bq}{z} \frac{1}{z} \frac{1}{2\pi} \sum_{j=1}^{N} \gamma_{ij} r_{nj} \sqrt{f_{11}(z)} \left[ \psi \left( r_{nj}, z \right) + S_j r_{nj} \sqrt{f_{11}(z)} \right] + Cz \] (31)

5. Conclusion and Cognition

1 Observing the reservoir pressure (27) and (29), bottom hole pressure (31), we find that they have the following relation.

\[ \bar{P}_{1j} = \bar{P}_w \frac{1}{\psi \left( r_{nj}, z \right)} \frac{1}{\psi \left( r_{nj}, z \right) + S_j r_{nj} \sqrt{f_{11}(z)}} \] (30)

\[ \bar{P}_{2j} = \bar{P}_w \frac{1}{\psi \left( r_{nj}, z \right) + S_j r_{nj} \sqrt{f_{11}(z)}} \]

2 With regard to the similar structure of the solution with reservoir pressure distribution in the Laplace space (27)-(30), when outer boundary condition changes, the solution with reservoir pressure distribution in the Laplace space is easily obtained by changing similar kernel function.

3 Similar construction method is a kind of primary and algebraic method, it avoids the complex derivation, it is simple and effective for solving the seepage model for the multilayer composite reservoir with the double porosity.

4 With regard to the similar structure of solution in the Laplace space, it provides immensely convenient for compiling corresponding analysis software of well test, and it also provides scientific basis for tremendous numerical simulation of oil-gas reservoir and other engineering application of seepage theory.

**Explanation of Symbols**

\( \rho_{nj} \) is initial pressure of each layer in the reservoir (MPa); \( r_w \) is well radius (m); \( r_{nj} \) is well radius of
the $j$ layer $m$; $p_j$ is pressure of the $j$ layer at time $t$ (MPa); $P_j$ is pressure drop of the $j$ layer (MPa); $S_j$ is skin effect of the $j$ layer (dimensionless); $C_j$ is coefficient of well-bore storage (m$^3$/MPa); $B$ is coefficient of crude oil volume (m$^3$/m$^3$); $k_j$ is permeability of the $j$ layer ($\mu$m$^2$); $C_{ij}$ is coefficient of integrated compression for the $j$ layer (1/MPa); $\varphi_j$ is porosity of the $j$ layer ($\%$); $\mu_j$ is fluid viscosity of the $j$ layer (MPa·s); $h_j$ is thickness of the $j$ layer ($m$); $q_j$ is production of the $j$ layer (m$^3$/d); $q$ is production (m$^3$/d); $R_j$ is outer boundary radius of the $j$ layer ($m$); $R$ is outer boundary radius ($m$); $t$ is time ($h$); $Z$ is variable of Laplace space.

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