THE TRANSITION MASS-LOSS RATE: CALIBRATING THE ROLE OF LINE-DRIVEN WINDS IN MASSIVE STAR EVOLUTION

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ABSTRACT

A debate has arisen regarding the importance of stationary versus eruptive mass loss for massive star evolution. The reason is that stellar winds have been found to be clumped, which results in the reduction of unclumped empirical mass-loss rates. Most stellar evolution models employ theoretical mass-loss rates which are already reduced by a moderate factor of \(\sim 2-3\) compared to non-corrected empirical rates. A key question is whether these reduced rates are of the correct order of magnitude, or if they should be reduced even further, which would mean that the alternative of eruptive mass loss becomes necessary. Here we introduce the transition mass-loss rate \(M_{\text{trans}}\) between O and Wolf–Rayet stars. Its novelty is that it is model independent. All that is required is postulating the spectroscopic transition point in a given data set, and determining the stellar luminosity, which is far less model dependent than the mass-loss rate. The transition mass-loss rate is subsequently used to calibrate stellar wind strength by its application to the Of/ WNh stars in the Arches cluster. Good agreement is found with two alternative modeling/theoretical results, suggesting that the rates provided by current theoretical models are of the right order of magnitude in the \(\sim50\,M_\odot\) mass range. Our results do not confirm the specific need for eruptive mass loss as luminous blue variables, and current stellar evolution modeling for Galactic massive stars seems sound. Mass loss through alternative mechanisms might still become necessary at lower masses, and/or metallicities, and the quantification of alternative mass loss is desirable.

Key words: stars: early-type – stars: evolution – stars: mass-loss – stars: winds, outflows

1. INTRODUCTION

Mass loss via stellar winds is thought to play a dominant role in the evolution of massive O-type stars, because of the loss of mass, as winds “peel off” the star’s outermost layers (Conti 1976), as well as through the associated loss of angular momentum (e.g., Langer 1998; Meynet & Maeder 2002). However, during the last decade, large uncertainty has been pointed out regarding our quantitative knowledge of the mass-loss rates of massive stars, as stellar winds have been revealed to be clumped, resulting in empirical rates that have been overestimated.

Although it had been known for decades that O-type winds are clumped (Lupie & Nordsieck 1987; Eversberg et al. 1998), the severity did not appear to be fully recognized until Bouret et al. (2005) and Fullerton et al. (2006) claimed mass-loss reductions of factors \(\sim 3-7\) and \(\sim 20-130\), respectively, in comparison to unclumped \(H\alpha\) and radio mass-loss rates (e.g., Lamers & Leitherer 1993). The \(H\alpha\) diagnostics depend on the density squared, and are thus sensitive to clumping, while ultraviolet P Cygni lines such as \(Pv\) are insensitive to clumping as these depend linearly on the density. The above-mentioned Bouret et al. and Fullerton et al. analyses were based on models where the wind is divided into a portion of the wind containing all the material with a volume filling factor \(f_V\) (the reciprocal of the clumping factor), while the remainder of the wind is assumed to be void. This pure \textit{micro}-clumping approach is probably an oversimplification of the real situation, but it provides interesting insights into the potential mass-loss rate reductions.

In reality, clumped winds are likely porous, with a range of clump sizes, masses, and optical depths. \textit{Macro}-clumping and porosity have been investigated with respect to both the spectral analyses (e.g., Oskinova et al. 2007; Sundqvist et al. 2010; Surlan et al. 2012) as well as the radiative driving (Muijres et al. 2011). The upshot from these studies is that O star mass-loss rates may only be reduced by a moderate factor of \(\sim 3\) (Repolust et al. 2004; Puls et al. 2008), which would bring their clumping properties in agreement with those of Wolf–Rayet (WR) winds, for which similar moderate clumping factors have been derived (Hamann & Koesterke 1998). The latter are based on the analysis of emission line wings due to electron scattering, which have the advantage that they do not depend on detailed ionization fractions and abundances of trace elements. These moderate clumping factors would imply that massive star evolution modeling is not affected, as current state-of-the-art rotating stellar models (e.g., Georgy et al. 2011; Brott et al. 2011) already employ moderately reduced rates via the theoretical relations of Vink et al. (2000).

In light of the severe mass-loss reductions claimed, e.g., by Fullerton et al. (2006), Smith & Owocki (2006) argued that the integrated mass loss from stationary stellar winds for very massive stars (VMSs) above \(\gtrsim 50\,M_\odot\) may be vastly insufficient to explain their role as the progenitors of WR stars and stripped-envelope Ibc supernovae. Instead, Smith & Owocki argued that the bulk of VMS mass loss is likely of an eruptive rather than a stationary nature. In particular, they highlighted the alternative option of eruptive mass loss during the luminous blue variable (LBV) phase.

In view of the new porosity results, the arguments of Smith & Owocki (2006), however, seem to have lost weight. Furthermore, quantitative estimates on the integrated amount of eruptive mass loss are hard to obtain as both the eruption frequency and the amounts of mass lost per eruption span a wide range (of a factor 100) with LBV nebular mass estimates varying from \(\sim 0.1\,M_\odot\) in P Cygni to \(\sim 10\,M_\odot\) in \(\eta\) Car, as discussed by Smith & Owocki (2006). Moreover, the energies required to produce such giant mass eruptions are very high (\(\sim 10^{50}\) erg), and their energy source is unknown. Soker (2004) discussed that the energy and angular momentum required for \(\eta\) Car great eruption cannot be explained within a single-star scenario.
While stationary winds in O and WR stars are ubiquitous, it is not at all clear if LBV-type objects like η Car have encountered a special evolution (such as a merger) or if all massive stars go through eruptive mass-loss phases. On the other hand, for a special evolution (such as a merger) or if all massive stars mass-loss indicator, the transition mass-loss rate by presenting a methodology that involves a model-independent mass loss could be inaccurate by factors of 10, and possibly mass loss, it would be beneficial to be able to calibrate either become negligible and alternatives such as eruptive mass loss would need to be considered.

In summary, the relevant roles of eruptive versus stationary mass loss seem rather uncertain and unsettled at the current time. There are ongoing debates as to whether wind clumping reduces the mass-loss rates by moderate factors of ~2–3, such that stellar evolution would not be affected, or by more severe factors of the order of ~10. In the latter case, line driving would become negligible and alternatives such as eruptive mass loss would need to be considered.

In order to address the relative role of wind versus eruptive mass loss, it would be beneficial to be able to calibrate either one of them. At the moment both stellar wind and eruptive mass loss could be inaccurate by factors of 10, and possibly even more. In this Letter, we attempt to alleviate this problem by presenting a methodology that involves a model-independent mass-loss indicator, the transition mass-loss rate $M_{\text{trans}}$—located right at the transition from optically thin to optically thick stellar winds. Martins et al. (2008) found two mass-loss relations for VMS Arches cluster stars, one for the Of stars and one for the late-type WNh stars, respectively. The fact that WR stars with WNh spectral classification have optical depth larger than one has already been discussed in the literature (e.g., Gräfener & Hamann 2008), and one might thus expect to witness a transition from optically thin O-type winds to optically thick WR winds.

Vink et al. (2011) discovered a sudden change in the slope of the mass loss versus $\Gamma$ relation at the transition from O-type (optically thin) to WR-type (optically thick) winds. Interestingly, this transition was found to occur for a wind efficiency parameter $\eta = Mv_\infty/(L/c)$ of order unity. This key result from Monte Carlo modeling that the transition from O to WR-type mass loss coincides with $\eta \sim 1$ can also be found analytically (Section 2). And the result can be utilized to “calibrate” wind mass loss in an almost model-independent manner (Section 3).

2. THE TRANSITION MASS-LOSS RATE

Netzer & Elitzur (1993) and Lamers & Cassinelli (1999; hereafter LC99) give general momentum considerations for dust-driven winds (see LC99, pp. 152–153) that can also be applied to line-driven winds. The integral form of the momentum equation contains four terms (Equation 7.5 of LC99). Because hydrostatic equilibrium is a good approximation for the subsonic part of the wind, and the gas pressure gradient is small beyond the sonic point, LC99 argue that the second and third terms are negligible compared to the first and fourth, resulting in

$$\int_{r_s}^{\infty} 4\pi r^2 \rho v \frac{dv}{dr} dr + \int_{r_s}^{\infty} \frac{GM}{r^2} (1 - \Gamma) \rho 4\pi r^2 dr = 0. \tag{1}$$

Employing the mass-continuity equation $M = 4\pi r^2 \rho v$, one obtains

$$\int_{r_s}^{\infty} M \frac{dv}{dr} dr = \dot{M} v_\infty = 4\pi G M \int_{r_s}^{\infty} (\Gamma(r) - 1) \rho dr, \tag{2}$$

where $r_s$ denotes the sonic radius and $\Gamma(r)$ is the Eddington factor with respect to the total flux-mean opacity $\kappa_F$:

$$\Gamma(r) = \frac{\kappa_F L}{4\pi c GM}. \tag{3}$$

Using the optical depth $\tau = \int_{r_s}^{\infty} \kappa_F \rho dr$, one obtains

$$\dot{M} v_\infty \simeq 4\pi G M \kappa \Gamma(\Gamma - 1) = \frac{L}{c} \frac{\Gamma - 1}{\Gamma} \tau. \tag{4}$$

Where it is assumed that $\Gamma$ is significantly larger than one, and the factor $(\Gamma - 1)/\Gamma$ is thus close to unity (LC99’s second assumption), resulting in

$$\dot{M} v_\infty = \frac{L}{c} \tau. \tag{5}$$

One can now derive a key condition for the wind efficiency number $\eta$,

$$\eta = \frac{\dot{M} v_\infty}{\frac{L}{c}} = \tau = 1. \tag{6}$$

The key point of our Letter is that we can employ the unique condition $\eta = \tau = 1$ right at the transition from optically thin O-star winds to optically thick WR winds, and obtain a model-independent $M$. In other words, if we were to have an empirical data set available that contains luminosity determinations for O and WR stars, we can obtain the transition mass-loss rate $M_{\text{trans}}$ simply by considering the transition luminosity $L_{\text{trans}}$ and the terminal velocity $v_\infty$ representing the transition point from O to WR stars:

$$M_{\text{trans}} = \frac{L_{\text{trans}}}{v_\infty c}. \tag{7}$$

We note that this transition point can be obtained by purely spectroscopic means, independent of any assumptions regarding wind clumping.

2.1. Testing the Assumptions

In the above analysis we made two assumptions that we wish to check with numerical tests involving sophisticated hydrodynamic wind models (from Gräfener & Hamann 2008) and simpler $\beta$-type velocity laws, commonly used in O/WR wind modeling. The results are compiled in Table 1. We first confirm LC99’s first assumption through the comparison of $\eta$, determined from $M$ and $v_\infty$, to the approximate $\eta'$ values as computed from the right-hand side of integral in Equation (1), where $\eta' L/c = 4\pi G M \int_{r_s}^{\infty} (\Gamma(r) - 1) \rho dr$. Evidently, the values of $\eta$ and $\eta'$ agree at the few percent level, and the first LC99 approximation is verified.

Second, we investigate the assumption that the term $(\Gamma - 1)/\Gamma$ in Equation (4) is close to unity by numerical integration of $\tau = \int_{r_s}^{\infty} \kappa \rho dr$. We obtain a correction factor $f$, which we define by

$$\dot{M} v_\infty \equiv f \frac{L}{c} \tau. \tag{8}$$

With this definition Equation (6) becomes

$$\eta = \frac{\dot{M} v_\infty}{\frac{L}{c}} = f \tau. \tag{9}$$

To compute the integral numerically, we need to obtain the density $\rho(r)$, and the flux-mean opacity $\kappa_F(r)$ in the stellar wind $(\Gamma(r)$ follows from Equation (3). The hydrodynamic wind
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Table 1: Wind Models

| $\beta$ | log(M) (M$_\odot$ yr$^{-1}$) | $v_\infty$ (km s$^{-1}$) | $\eta$ | $\eta'$ | $\tau$ | $f$ |
|---|---|---|---|---|---|---|
| WR 22, $M = 78.1 M_\odot$, log(L/L$_\odot$) = 6.3 |
| HYD | $-4.868$ | 979.5 | 0.326 | 0.315 | 1.198 | 0.272 |
| $1.0$ | $-4.868$ | 979.5 | 0.326 | 0.310 | 1.160 | 0.281 |
| $1.0$ | $-4.607$ | 1785.0 | 1.085 | 1.062 | 1.968 | 0.551 |

$v_\infty / v_{esc} = 2.5 \rightarrow (\Gamma - 1)/\Gamma \sim 2.5/3.5 = 0.714$

| $\beta$ | log(M) (M$_\odot$ yr$^{-1}$) | $v_\infty$ (km s$^{-1}$) | $\eta$ | $\eta'$ | $\tau$ | $f$ |
|---|---|---|---|---|---|---|
| WR 22, $M = 78.1 M_\odot$, log(L/L$_\odot$) = 6.3 |
| HYD | $-4.905$ | 2289.6 | 1.4 | 1.385 | 1.681 | 0.833 |
| $1.0$ | $-5.148$ | 2289.6 | 0.8 | 0.793 | 0.956 | 0.837 |
| $1.0$ | $-5.750$ | 2289.6 | 0.2 | 0.200 | 0.238 | 0.840 |
| WR 22, $M = 78.1 M_\odot$, log(L/L$_\odot$) = 6.3 |
| HYD | $-4.905$ | 2289.6 | 1.4 | 1.383 | 1.938 | 0.722 |
| $1.0$ | $-5.148$ | 2289.6 | 0.8 | 0.793 | 1.091 | 0.733 |
| $1.0$ | $-5.750$ | 2289.6 | 0.2 | 0.198 | 0.280 | 0.713 |
| $1.5$ | $-4.905$ | 2289.6 | 1.4 | 1.386 | 2.293 | 0.610 |
| $1.5$ | $-5.148$ | 2289.6 | 0.8 | 0.794 | 1.299 | 0.616 |
| $1.5$ | $-5.750$ | 2289.6 | 0.2 | 0.199 | 0.330 | 0.605 |

$v_\infty / v_{esc} = 1.5 \rightarrow (\Gamma - 1)/\Gamma \sim 1.5/2.5 = 0.600$

Note. If not stated otherwise, the adopted stellar parameters are $T_\ast = 35$ K and log(L/L$_\odot$) = 6.0 (with $M = 60 M_\odot$).

models of Gräfener & Hamann (2008) have these quantities directly available. We have performed a direct computation for the first model (HYD) in Table 1 for the Galactic WNh

star WR 22 (Gräfener & Hamann 2008). For this model we obtain $f = 0.272$. This value is lower than, but of the order of, unity. The terminal wind speed in this model is significantly lower than the observed value for WR 22 (980 km s$^{-1}$ versus 1785 km s$^{-1}$). Consequently, our derived $f$ is likely on the low side. We expect $\Gamma = g_{rad}/g$ to be connected to the ratio $(v_\infty + v_{esc})/v_{esc} = v_\infty/v_{esc} + 1$, and $f \simeq (\Gamma - 1)/\Gamma$.

Here we follow a model-independent approach, adopting $\beta$-type velocity laws. The mean opacity $\kappa_F$ then follows from the resulting radiative acceleration $g_{rad}$.

$$g_{rad}(r) = \kappa_F(r) \frac{L}{4\pi c r^2}.$$ (10)

$g_{rad}$ follows from the prescribed density $\rho(r)$ and velocity structures $v(r)$ via the equation of motion

$$\frac{dv}{dr} = g_{rad} - \frac{1}{\rho} \frac{dp}{dr} - \frac{GM}{r^2},$$ (11)

where we assume a gray temperature structure to compute the gas pressure $p$. We note that these results are completely independent of any assumptions regarding wind porosity, or the chemical composition of the wind material. The only assumption that goes into these considerations is that the winds are radiatively driven. The resulting mean opacity $\kappa_F$ consequently captures all physical effects that could potentially affect the radiative driving.

The obtained values for the correction factor $f$ are summarized in Table 1. The first three models in Table 1 represent a consistency test with the hydrodynamic model for WR 22. Using a beta law with $\beta = 1$, and the same $v_\infty$, we obtain almost exactly the same $f$ as for the hydrodynamic model, justifying our $\beta$-law approach, which we employ in the following. Now employing the observed—and therefore likely close to correct—value of $v_\infty$ and a correspondingly increased $M$, we obtain $f = 0.55$.

To get a handle on the overall behavior of this factor $f$, we computed a series of wind models for a range of stellar parameters $17.5 < T_\ast/kK < 70$ and $5.7 < \log(L/L_\odot) < 6.3$, with wind efficiencies around the transition region $(0.2 < \eta < 1.4)$. Remarkably, the resulting values of $f$ depend only on the adopted values of $v_\infty/v_{esc}$ and $\beta$. For $v_\infty/v_{esc} = 2.5$, we obtain $f \sim 0.8, 0.7, 0.6$, respectively, for $\beta = 0.5, 1.0, 1.5$, where the last value is probably most appropriate (Vink et al. 2011). Overall, we derive values of $f$ in the range 0.4–0.8, with a mean value of 0.6. We note that the error on this number $f$ is within the uncertainty of the luminosity determinations described in the next section.

For transition objects with $\tau \simeq 1$ we thus expect that $f \lesssim 0.6$, i.e., the transition between O and WR spectral types should occur at mass-loss rates of

$$\dot{M} = f \frac{L_{\text{trans}}}{v_{\infty}^2} \lesssim 0.6 \dot{M}_{\text{trans}}.$$ (12)

The fact that the correction factor is within a factor of two of our idealized approach (Equation (7)) is highly encouraging. We stress that this number is independent of any potential model deficiencies, as we have used the observed values of $v_\infty$ in this analysis.

3. THE TRANSITION MASS-LOSS RATE IN THE ARCHESS CLUSTER

Martins et al. (2008) analyzed 28 VMSs in the Arches cluster, with equal numbers of O-type supergiants and nitrogen-rich Wolf–Rayet (WNh) stars (sometimes called “O stars on steroids”). For the O-type supergiants, we expect the winds to be optically thin, while the WNh stars should have optically thick winds. Here we postulate that the O4-6If+ represent the transition point where the optical depth crosses unity.

In Table 2, we compiled a subset of 20 stars, skipping those objects with a He-enriched surface composition. The objects are sorted with respect to their spectral subtypes, and within
Table 2
The Transition from O to WR Stars for the Most Massive Stars in the Arches Cluster

| Star  | Subtype   | log($L$) | log($M$) | $v_{\infty}$ | log($M_{\text{trans}}$) |
|-------|-----------|----------|----------|--------------|-------------------------|
|        |           | ($L_\odot$) | ($M_\odot$ yr$^{-1}$) | (km s$^{-1}$) | ($M_\odot$ yr$^{-1}$) |
| F9    | WN8-9     | 6.35     | -4.78    | 1800         | -4.60                  |
| F1    | WN8-9     | 6.30     | -4.70    | 1400         | -4.54                  |
| F14   | WN8-9     | 6.00     | -5.00    | 1400         | -4.84                  |
| F1    | WN8-9     | 5.95     | -5.00    | 1600         | -4.95                  |
| F16   | WN8-9     | 5.91     | -5.11    | 1400         | -4.94                  |
| F15   | O4-6If+   | 6.15     | -5.10    | 2400         | -4.92                  |
| F10   | O4-6If+   | 5.95     | -5.30    | 1600         | -4.95                  |
| F18   | O4-6I     | 6.05     | -5.35    | 2150         | -4.98                  |
| F21   | O4-6I     | 5.95     | -5.49    | 2200         | -5.09                  |
| F28   | O4-6I     | 5.95     | -5.70    | 2750         | -5.18                  |
| F20   | O4-6I     | 5.90     | -5.42    | 2850         | -5.25                  |
| F26   | O4-6I     | 5.85     | -5.73    | 2600         | -5.26                  |
| F32   | O4-6I     | 5.85     | -5.90    | 2400         | -5.22                  |
| F33   | O4-6I     | 5.85     | -5.73    | 2600         | -5.26                  |
| F22   | O4-6I     | 5.80     | -5.70    | 1900         | -5.17                  |
| F23   | O4-6I     | 5.80     | -5.65    | 1900         | -5.17                  |
| F29   | O4-6I     | 5.75     | -5.60    | 2900         | -5.41                  |
| F34   | O4-6I     | 5.75     | -5.77    | 1750         | -5.19                  |
| F40   | O4-6I     | 5.75     | -5.75    | 2450         | -5.53                  |
| F35   | O4-6I     | 5.70     | -5.76    | 2150         | -5.53                  |

Notes. Designations, subtypes, luminosities ($L$), mass-loss rates ($M$), and terminal wind velocities ($v_{\infty}$) according to Martins et al. (2008). The sixth column indicates the mass-loss rate where $\eta = 1$. For the Arches cluster, we obtain log($M_{\text{trans}}$) $\sim$ -4.95.

4. DISCUSSION

How does our determination of the transition mass-loss rate compare to other methods? Let us first compare our transition mass-loss value to the mass-loss rates of Martins et al. Martins et al. use the non-LTE CMFGEN code by Hillier & Miller (1998), employing a micro-clumping approach with a volume filling factor $f = 0.1$ for their $K$-band analysis. Their values are in good agreement with our transition mass-loss rates for the objects at the boundary between O and WR (see their Table 2). This is unlikely to be a coincidence. According to our findings in Section 2.1, we expect mass-loss rates of the order of $M \simeq 0.6 \times M_{\text{trans}}$ for the transition objects, i.e., log($M/M_\odot$ yr$^{-1}$) $\sim$ -5.2.

We also compare the transition mass-loss rate to the oft-used theoretical mass-loss relation of Vink et al. (2000), for which we find log($M_{\text{Vink}}$) $\simeq$ -5.14 $M_\odot$ yr$^{-1}$ for an assumed stellar mass $M = 60 M_\odot$. This number is within 0.2 dex from the transition mass-loss rate log($M_{\text{trans}}$) $\simeq$ -4.95 $M_\odot$ yr$^{-1}$. The comparison is hardly compromised as a result of the Vink et al. dependence on stellar mass; as for masses in the range 40–80 $M_\odot$, the Vink et al. mass-loss rate varies by at most 0.04 dex.

In summary, we have three independent mass-loss rate determinations that agree within a factor of two. This means that our concept of the transition mass-loss rate has indeed been able to test the accuracy of current mass-loss estimates by stellar winds.

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reason to assume that they are still overestimated.\textsuperscript{1} The results presented here certainly boost confidence in the mass-loss rates currently in use, although they remain uncalibrated for the lower mass regime. One should also realize when working down the mass range, starting from our 60 $M_\odot$ calibrator star, that the mass-loss rates drop significantly below $10^{-5} \ M_\odot \ yr^{-1}$. Its effects on stellar evolution remain significant due to the longer evolutionary timescales for lower mass objects and the fact that it is the multiplication of the mass-loss rate times the duration that is relevant. This is especially relevant for angular momentum evolution, possibly down to stellar masses as low as 10–15 $M_\odot$ (Vink et al. 2010).

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\textsuperscript{1} Recently claimed in the context of core-collapse supernovae by Smith et al. (2011). However, the references quoted in that paper only involve micro-clumping studies, and do not consider subsequent work on macro-clumping, as well as several alternative studies that aim to calibrate stellar wind mass loss (e.g., Voss et al. 2010).