Models of Warm Jupiter Atmospheres: Observable Signatures of Obliquity

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Abstract

We present three-dimensional atmospheric circulation models of a hypothetical “warm Jupiter” planet, for a range of possible obliquities from 0° to 90°. We model a Jupiter-mass planet on a 10 day orbit around a Sun-like star, since this hypothetical planet sits at the boundary between planets for which we expect that tidal forces should have aligned their rotation axes with their orbital axes (i.e., ones with zero obliquity) and planets whose timescale for tidal alignment is longer than the typical age of an exoplanet system. In line with observational progress, which is pushing atmospheric characterization for planets on longer orbital periods, we calculate the observable signatures of obliquity for a transiting warm Jupiter: in orbital phase curves of thermal emission and in the hemispheric flux gradients that could be measured by eclipse mapping. For both of these predicted measurements, the signal that we would see depends strongly on our viewing geometry relative to the orientation of the planet’s rotation axis, and we thoroughly identify the degeneracies that result. We compare these signals to the predicted sensitivities of current and future instruments and determine that the James Webb Space Telescope should be able to constrain the obliquities of nearby warm Jupiters to be small (if ≤10°) or to directly measure them if significantly non-zero (≥30°) using the technique of eclipse mapping. For a bright target and assuming photon-limited precision, this could be done with a single secondary eclipse observation.

Key words: eclipses – hydrodynamics – infrared: planetary systems – planets and satellites: atmospheres – planets and satellites: gaseous planets

Supporting material: animation, tar.gz-File

1. Introduction

Exoplanet atmospheric characterization began with a hot Jupiter (Charbonneau et al. 2002) and the majority of all measurements so far have been of the atmospheres of these Jupiter-mass planets that orbit less than 0.1 au from their host stars. In addition to being inherently interesting, hot Jupiters are the brightest, biggest transiting planets and so the easiest to detect and characterize. However, we are entering an era where transit searches are finding increasing numbers of cooler planets on longer orbital periods and, importantly, ones around stars bright enough that we can extend atmospheric characterization beyond hot Jupiters. This is also facilitated by a better understanding of how to best use current instruments for atmospheric measurements, as well as the much-anticipated launch of new facilities, especially the James Webb Space Telescope. As a field we are poised to expand our understanding of exoplanets to the increasingly diverse possibilities we should expect as we study planets on longer orbital periods.

As of this writing, there are almost three dozen warm Jupiters known to transit their host stars,1 where we define “warm Jupiter” somewhat arbitrarily to mean a planet with a radius greater than 0.5 Jupiter radii2 and a zero-albedo equilibrium temperature between 500 and 1000 K. These planets are all around stars bright enough for us to estimate their effective temperatures (necessary to estimate the planets’ equilibrium temperatures). The current brightest planet of this population is WASP-69b, a 1 Jupiter-radius, 0.3 Jupiter-mass planet with an equilibrium temperature of ~960 K around a V = 9.87 mag star (Anderson et al. 2014). The next two brightest, with V magnitudes between 10.5 and 11, are HAT-P-17b (1 RJ, 0.5 MJ, Teq ~ 790 K, and e = 0.34; Howard et al. 2012) and WASP-84b (0.9 RJ, 0.7 MJ, and Teq ~ 800 K; Anderson et al. 2014). We expect that this population of known warm Jupiters should grow as current transit searches continue and new ones begin. In particular, missions such as the Transiting Exoplanet Survey Satellite (TESS) and the PLAnetary Transits and Oscillations of stars (PLATO) satellite should identify more planets that transit bright stars, improving our ability to characterize these worlds.

A significant difference between the populations of “warm” and “hot” Jupiters is that the overwhelmingly strong tidal forces experienced by hot Jupiters will no longer be so overwhelming for the longer orbital period, warm population. As a consequence, whereas we generally assume that the rotation rates of hot Jupiters are synchronized with their orbital rates and their rotation axes are aligned with their orbital axes, these assumptions no longer hold for warm Jupiters and we should expect a range of possible values for these parameters. One observable expression of this decreasing influence of tidal forces on longer period planets is the increase in their range of orbital eccentricities (see the review by Winn & Fabrycky 2015). As we expand observational and theoretical efforts out from hot Jupiters, there are three planetary properties that were fixed for the hot Jupiter population, but that we should expect to create additional diversity among the longer period “warm Jupiters”: eccentricity, rotation rate, and obliquity.3 There have been previous works studying different eccentricities (Kataria et al. 2013) and rotation rates (Showman et al. 2015) for warm Jupiters; here, we focus on non-zero obliquities.

1 NASA Exoplanet Archive, as of 2017 March 20 (https://exoplanetarchive.ipac.caltech.edu/).
2 Uranus and Neptune have radii of ~0.35 RJupiter.
3 Here we use the term “obliquity” to refer to the angle of the planet’s rotation axis away from its orbital axis. This is different from the widely studied (and more easily observable) obliquity between a star’s rotation axis and the exoplanet’s orbital axis (see the review by Winn & Fabrycky 2015).
Previous works studying non-zero obliquities of exoplanets have generally fallen into one of two main categories. The first includes papers that study the influence of obliquity on atmospheric circulation patterns, often with an emphasis on Earth-like planets and their habitability (e.g., Williams & Kasting 1997; Williams & Pollard 2003; Spiegel et al. 2009; Ferreira et al. 2014; Linsenmeier et al. 2015; Shields et al. 2016). The second category contains papers that identify means by which we may be able to measure or constrain the obliquities of exoplanets, either using a presumed atmospheric state or analytic models that allow for any general spatial pattern. Seager & Hui (2002) demonstrated that the oblateness of a planet, due to rotation, can impart a small signal to its transit light curve, and the obliquity of the planet influences the shape and strength of this feature. This method was subsequently used to observationally constrain the oblateness of several exoplanets and a brown dwarf companion (Carter & Winn 2010; Zhu et al. 2014). There have also been studies of ways to constrain a transiting or directly imaged planet’s obliquity with future instrumentation, including signatures of obliquity within scattered light curves (Kawahara & Fujii 2010; Kawahara 2016), polarimetric observations (de Kok et al. 2011), and the planet’s Rossiter–McLaughlin effect during secondary eclipse (Nikolov & Sainsbury-Martinez 2015).

In a more general approach, Cowan et al. (2013) and Schwartz et al. (2016) presented useful analytic frameworks for identifying how the intrinsic properties of exoplanets (e.g., their rotation rates, inclinations, obliquities, albedos, etc.) couple with the extrinsic viewing orientation (related to the planet’s orbital parameters) in order to shape the observed orbital and rotational photometric variations for emitted or reflected light. They explicitly identified how the spatial information on the planet is expressed in the temporal information of the light curve, as well as which spatial patterns are necessarily (i.e., mathematically) invisible in these light curves. Gaidos & Williams (2004) is a rare work that fits within both categories of obliquity research; they simulated the influence of non-zero obliquity (as well as non-zero eccentricity) on a planet’s atmospheric structure, using energy balance models (considering a range of thermal inertias), and also predicted observable consequences, calculating thermal phase curves as a function of viewing orientation. While these authors focused on terrestrial planets with Earth-like temperatures, some of their results regarding the influence of viewing orientation on observed seasonality are reproduced in our predictions, as we will describe below. They also found significant degeneracy between the effects of obliquity, thermal inertia, and viewing orientation; we uniquely identify secondary eclipse maps as a means by which we may be able to break these degeneracies. In addition, our atmospheric model is more complex, and the thermal inertia is not an input but a prediction (albeit influenced by the other, more basic, planetary parameters we choose). Cowan et al. (2012) expanded on this by using two detailed three-dimensional circulation models for the Earth (one similar to current-day conditions and one representative of a snowball state) to simulate orbital thermal phase curves, as viewed from different orientations. While they did not vary the obliquity or eccentricity, their models did include self-consistent heat transport and so could test the influence of thermal inertia by comparing their temperate and snowball models.

In this work, we identify the next set of observable planets whose obliquities may be measurable (“warm Jupiters”) and present a joint analysis of their atmospheric state and observability (both via thermal phase curves and eclipse maps). We start our exploration of non-zero obliquities by modeling a hypothetical Jupiter-like planet on a 10 day orbit around a Sun-like star. We choose this orbital period because it corresponds to where the timescales for tidal circularization and axial alignment (which are of similar magnitude; e.g., Peale 1999) may be comparable to the typical lifetime of an exoplanet system. Starting with the timescale for tidal spin-down of a planet (Guillot et al. 1996), $\tau$, and solving for the planet’s orbital period, we find

$$P \sim \left( \frac{16\pi^3 R_p^3}{Q \, G \omega_p M_p} \right)^{1/4}.$$  \hspace{1cm} (1)

This formulation disguises any complexity of the planet’s tidal dissipation by using a single parameter, $Q$, and we are also ignoring any orbital changes that may happen over time, so we can use this to estimate the orbital period at which the timescale for tidal effects may be comparable to the age of the system (setting that equal to $\tau$), but this is not a strict prediction for a specific boundary in physical conditions. Nevertheless, using the mass, radius, and current spin of Jupiter (as an estimate for the initial spin rate, $\omega_p$), an estimate for the tidal dissipation factor of $Q \sim 10^9$, and assuming an age on the order of Gyr, we solve for an orbital period of $P \sim 10$ days. This is then roughly the orbit at which we would not know whether or not to expect that tidal forces should have aligned a planet’s obliquity, and so this presents an interesting potential target for characterization.

The equilibrium temperature for a planet on a 10 day orbit around a solar analog is $\sim 900$ K. Interestingly, this is near the $\sim 1000$ K boundary identified by Thorngren et al. (2016); for exoplanets that are cooler than this we may be able to assess their metallicities from their bulk densities, without the (still unknown) hot Jupiter radius inflation mechanism confusing matters. In other words, the planets that we may want to study in order to answer fundamental questions about planet bulk compositions are also those planets for which we should expect a range of non-zero obliquities.

In this paper, we compare models for a hypothetical warm Jupiter at different obliquities using a three-dimensional General Circulation Model (GCM). In Section 2, we discuss our modeling framework, including the treatment of non-zero obliquity. In Section 3, we present the results from the GCM and discuss the variation in circulation patterns between models. We show the observable properties of each model in Section 4: the thermal phase curves (Section 4.1) and eclipse maps (Section 4.2). We also compare these predicted signals to the sensitivities of current and future instruments (Section 4.3). Finally, in Section 5, we summarize our results and the potential for observationally constraining the obliquities of warm Jupiters.

2. Atmospheric Circulation Model

We present three-dimensional atmospheric circulation models for a hypothetical planet with the mass, radius, and rotation rate of Jupiter, on a 10 day period orbit around a Sun-like star. We calculated the circulation patterns using a numerical code that solves the primitive equations of meteorology, with pressure as the vertical coordinate, and uses a double-gray scheme for the radiative transfer, as described thoroughly in...
Rauscher & Menou (2012). This code has been updated to allow for the variable stellar irradiation patterns appropriate for planets with non-zero obliquity, as described in Section 2.1 below. We ran models for obliquities from 0° to 90°, with all other physical parameters remaining the same (as listed in Table 1). Specifically, we simulated planets with obliquities of 0°, 3°, 5° (equal to Jupiter’s obliquity), 10°, 30° (equal to the obliquities of Earth, Saturn, and Neptune), 60°, and 90° (close to Uranus’ obliquity). We chose values for the gray optical and infrared absorption coefficients such that the analytic solution (Guillot 2010) roughly matches the temperature—pressure profile for a hypothetical Jupiter-like planet at 0.1 au, as calculated from a one-dimensional atmospheric model with detailed radiative transfer (Fortney et al. 2007).

Our standard model runs were performed at a horizontal spectral resolution of T42 (corresponding to ~3° resolution at the equator) and with 30 vertical levels evenly sampled in log P from 100 bar to 1 mbar. We applied numerical hyperdissipation as an eighth-order operator on the wind and temperature fields (as described in Rauscher & Menou 2012), with a dissipation timescale of 0.02 Prot, which we found to adequately remove noise at the smallest scales without overdamping the kinetic energy spectrum of the circulation. We initialized the models using a horizontally uniform temperature profile, whose vertical (pressure) dependence was set to match the globally averaged analytic solution for our double-gray opacities (Guillot 2010). We started each run with the winds at rest and used 360 time steps per Prot to simulate each planet for 3000 orbital periods (=72552 Prot), by which point the winds throughout the observable atmosphere had finished accelerating (for yearly averaged values).

We can calculate the expected scale of dynamical features in our planet’s global circulation using the Rossby deformation radius (e.g., Showman et al. 2010) and find that at the infrared photosphere, this scale is ~8°. While T42 resolution should be sufficient to capture these features, we tested some limited models at T63 and T85, corresponding to ~2° and ~1.4° resolutions, respectively. We found very similar circulation patterns and temperature structures, without any smaller scale detail emerging in these higher resolution runs, giving us confidence that the T42 resolution is sufficient to model the global circulation pattern of this planet.

One unavoidable concern when modeling planetary atmospheres is whether there could be subgrid physics, not captured within the numerical simulation, that could influence the global circulation of the planet. Although this may be of particular concern for hot Jupiters, where the intense stellar irradiation can drive supersonic winds, potentially triggering subgrid shocks and hydrodynamic instabilities (e.g., Li & Goodman 2010; Fromang et al. 2016), these particular effects should be less important in the calmer atmospheric circulation of warm Jupiters. (To preview our results, the maximum wind speeds we find are about 1 km s$^{-1}$, whereas the sound speeds in the atmosphere range from 2 to 3 km s$^{-1}$.) Nevertheless, we are fundamentally not resolving whatever form of subgrid dissipation exists in these atmospheres, instead capturing it through our hyperdissipation parameter. As there is no a priori physical basis for calculating this parameter (Cho & Polvani 1996), our uncertainty in the correct value to use could, for example, result in errors in our maximum wind speeds (Heng et al. 2011). Mayne et al. (2014) also has a nice discussion of the implications of this and other assumptions used in hot Jupiter circulation modeling. While issues of subgrid physics are not unique to our models, and perhaps may be better behaved than in the hot Jupiter context, this is an important caveat that should be kept in mind.

### 2.1. Stellar Flux Pattern for Non-zero Obliquities

Our atmospheric circulation code has been used before to model the atmospheres of planets with zero obliquity, for a variety of stellar heating patterns. In the case of hot Jupiters, the stellar heating pattern does not change with time (in the frame rotating with the planet). The flux has a maximum at the substellar point, falls off as the cosine of the angle from the substellar point, and is zero everywhere on the planet’s perpetual nightside. In the case of non-synchronous rotation (as in Rauscher & Kempton 2014), the same hemispheric forcing pattern is used, but the longitude of the substellar point varies with time. If the planet is rotating quickly enough that the time between consecutive star-rises is shorter than the radiative timescale of the atmosphere, it is more appropriate to use a diurnally averaged stellar heating pattern, such that the equator receives more flux than the poles (for zero obliquity). This was the forcing pattern we used for the models presented in May & Rauscher (2016).

Here we have updated our code to calculate the appropriate stellar forcing patterns for planets with non-zero obliquities. For these planets, the latitude of the substellar point (ϕₛ) has a yearly seasonal dependence given by

$$\sin \phi_s = \sin \psi \sin(2\pi t/P_{orb}),$$

where ψ is the planet’s obliquity and the zero-point for time in the orbit (t = 0) is set to be the vernal equinox in the northern hemisphere, such that the maximum northern excursion of the substellar latitude (ϕₛ = +ψ) occurs at the northern summer solstice (t = 0.25Porb). If the planet is rotating quickly enough that it is appropriate to use a diurnally averaged stellar flux pattern to heat the atmosphere, the downward flux at the top of

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4 A “star-rise” is the exoplanet version of a sunrise.

5 A diurnally averaged heating pattern is appropriate for most of the planets in the solar system.
the atmosphere can be calculated as (e.g., Liou 1980)

\[ F(\phi) = \frac{F_0}{\pi} (\sin \phi \sin \phi_a H + \cos \phi \cos \phi_a \sin H), \] (3)

where \( F_0 \) is the incident flux at the substellar point and \( H \) is the length of half a day (from star-rise to noon), measured in radians, for the given latitude, \( H = \cos^{-1}(-\tan \phi \tan \phi_a) \). The inverse cosine function is only defined when its argument is within \([-1, 1]\). For latitudes greater/less than the substellar latitude during northern/southern summer, \( H = \pi \) and those regions of the planet experience perpetual daylight for the season, while similarly there are latitudes in the other hemisphere that will have \( H = 0 \) and experience seasonally perpetual nighttime.

Since we assume Jupiter’s rotation rate for our hypothetical planet and a 10 day orbit around a Sun-like star, we can use the prescription in Showman et al. (2015) to determine the appropriateness of using a diurnally averaged stellar heating pattern in this case. We do find that the radiative timescale at the optical photosphere (the \( \tau = 2/3 \) surface for absorbed starlight)\(^6\) is greater than the rotation period (favoring a diurnal average), but only by a factor of \( \sim 2 \). We tested the impact of using a diurnal stellar heating pattern by performing limited versions of our 0° and 90° obliquity models in which we explicitly move the heating pattern with time. In order to correctly model the movement of the substellar point, it should not advance more than one grid point in one time step. For the case of zero obliquity, this is equivalent to requiring that the number of time steps per \( P_{\text{orb}} \) is greater than the number of longitude points (128 at our standard T42 resolution) times the number of time steps between each time the radiative transfer routine is calculated (10), times the number of rotation periods in an orbital period minus one (24.184 – 1). This minimum requirement forces us to use 30,000 time steps per rotation period, a factor of almost 100 greater than (and so computationally slower than) our standard runs. We found that the atmospheric structure in these models was still mostly axisymmetric, with wind and temperature patterns that look very similar to those from the models with diurnally averaged heating patterns. The deviations from axisymmetry were very small; for example, at the infrared photosphere, the temperatures in a single latitude circle are always within 2% of the average at that latitude. This supports the use of a diurnal average for this planet’s heating pattern.

3. Comparison of Circulation Patterns

Our models with relatively low obliquity (0°, 3°, and 10°) all exhibit atmospheric structures where the equator is warmer than the poles and the circulation is eastward throughout most of the atmosphere, dominated by a jet at high latitude in each hemisphere. The \( \psi = 0^\circ \) and \( \psi = 3^\circ \) models are largely indistinguishable, so much so that we do not find it worthwhile to present results from the 3° model. Between the 0° and 10° models, there are only small differences in wind speed and jet width. In Figure 1, we show plots of the zonally averaged zonal wind (in the east–west direction) for each model; the plots for the 0° and 10° models are yearly averages, since little-to-no seasonal variation is observed, while for the higher obliquity models several snapshots are shown throughout the planet’s year.

For this hypothetical planet, we find that significant seasonal differences emerge once the obliquity reaches 30°, with a clear shift in wind patterns and temperature structure as the northern and southern hemispheres receive more or less direct heating. The circulation pattern for the \( \psi = 30^\circ \) model resembles the patterns for lower obliquity models, with a dominant eastward flow and high-latitude jets, but the jet structure is slightly distorted between the two hemispheres, with a seasonal dependence. The flow patterns for the \( \psi = 60^\circ \) and \( \psi = 90^\circ \) models also show a seasonal dependence, but the winds in these models are significantly different from the lower obliquity cases. For \( \psi = 60^\circ \), there is a more even distribution between the eastward and westward winds, and the flow is characterized by an eastward jet that remains near the equator, but shifts up and down in latitude seasonally. The 90° obliquity model is dominated by westward flow, with a strong jet at the equator that shifts slightly in latitude throughout the year. These planets are clearly in a different circulation regime from the lower obliquity planets.

In Figure 2, we show how the temperature structures of the \( \psi \geq 30^\circ \) models respond to the changing seasons. We plot the temperature at the infrared photosphere for each of these models as a function of time throughout one orbit of the planet. For each model, we plot the latitudinal temperature structure by taking an azimuthal average (as is appropriate since our diurnal heating pattern results in no significant variation of the properties with longitude). On top of these temperature contours we plot a dashed black line to show the latitude of the substellar point as a function of time (Equation (2)), so that we can compare the spatial and temporal response of the atmosphere to the changing irradiation pattern.

For the \( \psi = 30^\circ \) model, the temperature structure is similar to that of lower obliquity models, in that the equator remains warmer than the poles, but the latitude of the maximum temperature shifts above and below the equator with time. The stark change in atmospheric regime between the \( \psi \leq 30^\circ \) and \( \psi \geq 60^\circ \) models seen in the wind patterns (Figure 1) is also reflected in the temperature structures. At most times during the year, one hemisphere is significantly warmer than the other, such that the main temperature gradient is between the summer and winter hemispheres, with short times of relatively uniform temperature near the equinoxes.

From Figure 2, we can also see that the atmospheres of the \( \psi = 30^\circ \), 60°, and 90° models all respond to the changing irradiation pattern with a similar response time. This is observed as a lag in the latitude of the maximum temperature, relative to the substellar latitude, by \( \sim 1/8 \) \( P_{\text{orb}} \). This response time is about an order of magnitude longer than the radiative timescale at the infrared photosphere (\( \sim 0.014 \) \( P_{\text{orb}} \)), evidencing a complex radiative and dynamical response of the atmosphere as it transports energy from the heating at the optical photosphere (where the radiative timescale is also shorter than the response time, at \( \sim 0.05 \) \( P_{\text{orb}} \)).

The focus of this paper is the observable consequences of non-zero obliquities and so a more detailed analysis of these circulation patterns and the shift between regimes will have to wait for future work. The takeaway point from this section is that seasonal effects are insignificant for our models with obliquities \( \leq 10^\circ \), but influence the temperature structure at the

\(^6\) More accurately, Heng et al. (2014) showed that in the purely absorbing limit (as we assume here), most of the starlight is absorbed at \( \tau = 0.63 \).
Figure 1. Zonally averaged zonal (east–west) winds for models with obliques of 0° (top left), 10° (top right), 30° (third row from the bottom), 60° (second row from the bottom), and 90° (bottom row). The plots for $\psi = 0^\circ$ and $\psi = 10^\circ$ are annual averages; the other plots are snapshots at (northern) vernal equinox, summer solstice, autumnal equinox, and winter solstice (left to right). The color scale is the same for each plot and the black line is the boundary between the eastward (positive) and westward (negative) winds.
from the simulation \((t/P_{\text{orb}})\), or orbital phases. A full phase curve observation occurs over a planet’s year \((\Delta t = P_{\text{orb}})\) and requires a choice of the viewing orientation.

For an eclipse mapping observation \((\gamma = 0.5)\), we need to know not just the sub-observer latitude, but in which direction the pole of the planet is pointed. Using the spherical law of cosines \((\text{following the example of Schwartz et al. }2016)\), we find that the angle, \(C\), through which the northern pole should be rotated \((\text{in a clockwise direction})\) around a line that passes from the center of the planet to the observer \((\text{through the sub-observer point})\), is given by

\[
\cos C = \cos \psi / \cos \phi_{\text{obs}}.
\]

Cosine is an even function and the solution for \(C\) could be a positive or negative value; these correspond to the two points during a planet’s orbit when an observer might be oriented such that secondary eclipse occurred at that value of \(\phi_{\text{obs}}\). From a consideration of the orbital geometry \((\text{using Equation }4)\), it is apparent that the positive solutions correspond to \(-1/4 < t/P_{\text{orb}} < 1/4\) and the negative solutions to \(1/4 < t/P_{\text{orb}} < 3/4\).

4.1. Orbital Thermal Phase Curves

In the simulated orbital phase curves presented here, we assume that our hypothetical planet is of interest because it transits and we choose to view the system exactly edge-on. This allows us to isolate the observable consequences of non-zero obliquities, but for actual systems these effects would be convolved with the effect of viewing at an orbital inclination slightly less than exactly 90°. This will be a minor effect and in practice the orbital inclination is measurable from the shape of the transit curve and so the reference frame could be adjusted to account for this angle.

Since we force the orbital inclination of our hypothetical system to be exactly edge-on to the observer, this constrains the possible viewing geometry such that the planet must be seen with a sub-observer latitude ranging from zero to \(\pm \psi\). From the zero obliquity model, this means that the planet is always viewed exactly along its equator. We calculate a set of simulated phase curves for each of our models by assuming various orientations \((\text{i.e., using different values for the sub-observer latitude in Equation }4)\), and then integrate the flux emitted from the hemisphere facing the observer, as a function of time throughout one orbit, as the planet undergoes seasonal changes. It is necessary to simulate a set of curves for each non-zero obliquity model because the random orientation of the observer relative to the system will directly affect the measured phase curve, as seen in Figure 3. Figure 4 shows the maps of the flux emitted from each of our models, oriented for the different viewing geometries used to produce the curves in Figure 3. Each map is for a snapshot of the planet at an orbital phase of zero; full-orbit movies showing the seasonal flux variation for the cases shown in Figure 4 are available as a .tar.gz package associated with Figure 5, which shows a representative animation for the case of \(\psi = 60°\) and a sub-observer latitude of 30°.

If our hypothetical planet has zero obliquity, then its orbital phase curve should be completely flat. Its temperature structure is axisymmetric \((\text{due to its assumed Jupiter-like rotation rate and diurnally averaged stellar heating pattern})\), and it experiences no seasonal variations, so the emitted flux is constant with time, as seen in Figure 3. We can compare the
flux level of this curve to the flux level that we would expect if the heating from the star was evenly re-radiated from the entire planet surface, \( F_{\text{int}} / 4 = 3.4 \times 10^4 \text{ W m}^{-2} \). We find that the flux from our \( \psi = 0^\circ \) model is higher than this global average, which is not surprising. Atmospheric transport does not completely homogenize the equator-to-pole temperature difference from differential heating, leaving the equator hotter than the global average, and we preferentially observe this brighter region of the planet.\(^7\) If we could independently constrain the obliquity of a planet to be zero, then the flux level of the orbital phase curve would be a measure of the efficiency of equator-to-pole heat transport, with values closer to the globally averaged flux corresponding to more efficient transport.

In the case where the obliquity of the planet cannot be measured independently, both it and the equator-to-pole heat transport have degenerate effects on the observed flux level, for equatorial viewing geometries. Figure 3 shows that—at equatorial viewing orientations—a lower level of emitted flux is associated with a higher value of planetary obliquity. Values less than the globally averaged incident flux \( (3.4 \times 10^4 \text{ W m}^{-2}) \) are observed for planets with obliquities \( \geq 60^\circ \), a result of the poles of these planets receiving more flux than the equator, integrated over an orbit. Figure 2 shows that these high-obliquity models have equatorial temperatures that are always equal to or less than the planet’s average temperature. When the planet is observed along the equator, we preferentially see these cooler regions and so the average hemispheric flux is decreased relative to the lower obliquity models.

The phase curves for equatorial viewing geometry are not completely flat; there is some small amount of seasonal variation (similar to the semi-orbital variations in the blackbody planet models of Gaidos & Williams 2004). Even though an observer would see an equal amount of each hemisphere, Figure 2 shows that there are times of the year when one hemisphere is much hotter than average and times when both hemispheres are both only warm, leading to an integrated flux level that shows a seasonal variation. However, this signal would likely be lost in the noise of an actual phase curve measurement (see Section 4.3) and so these curves are effectively flat.

The takeaway point from the solid curves in Figure 3 is that if we observe a flat phase curve for the thermal emission from a transiting planet, we are probably observing the planet along its equator. If the measured flux level is above the globally averaged value \( (F_{\text{int}} / 4 = \sigma T_{\text{star}}^4 (R_{\text{star}} / 2a)^2) \), then the planet’s equator is hotter than its poles and the obliquity is likely less than \( 54^\circ \). In this case, the measured flux level is giving us information about the efficiency of equator-to-pole heat transport. If the observed flux is below the globally averaged value, then the obliquity is likely greater than \( 54^\circ \), and our results show that lower levels correspond to higher obliquities. An important caveat to this is that the albedo of the planet may be unknown, in which case the globally averaged flux value would need to be adjusted down by an unknown amount, confusing the interpretation of a flat phase curve.

The set of flat (or effectively flat) solid phase curves shown in Figure 3 is for the particular viewing orientation of the sub-observer point being along the planet’s equator. Plotted as dashed lines in Figure 3 are the other extreme viewing orientations, where the planet is observed at the maximum possible sub-observer latitude (for a transiting system), \( |\phi_{\text{sub}}| = \psi \). For these cases, we observe the maximum variation in emitted flux, since the full seasonal response of a hemisphere is preferentially in view. Without spatially resolving the planet from the star, we cannot know the orientation of the orbital angular momentum vector, leading to a north–south degeneracy in our observed properties. We do not know whether our sub-observer latitude is positive or negative (northern or southern), but since both hemispheres respond to the seasonal cycle symmetrically, it is only the absolute value of \( \phi_{\text{obs}} \) that is relevant anyway.

\(^7\) Note that the internal heat flux of the planet is several orders of magnitude less than the incident stellar irradiation and so is largely irrelevant (see Table 1).

\(^8\) Mathematically, \( \psi = 54^\circ \) is the limit between illumination patterns where the equator or poles receive more stellar flux, integrated over the planet’s year.
Figure 4. Maps of the flux emitted from models with various obliquities (from left to right, as labeled), and as observed with different viewing orientations. In all cases, the planet is assumed to be in a transiting orbit, and these images are for the time when $\gamma = 0$, meaning that the planet would be at the center of transit. The top row shows the planet as it would look if the equator of the planet was aligned toward the observer. The middle row shows the planet as if the observer were seeing the planet at a latitude equal to half of the planet’s obliquity (so at 15° for the $\psi = 30°$ model, 30° for the $\psi = 60°$ model, and 45° for the $\psi = 90°$ model). The bottom row shows the planet oriented such that its rotation axis is pointed toward the observer, so that the observer views a latitude equal to the obliquity. Movies showing the time variation of this flux pattern throughout the planet’s orbit, for all models and viewing orientations represented here, are available in the .tar.gz package associated with Figure 5.

Figure 5. Map of the flux emitted from our model with an obliquity of 60°, oriented from the perspective of an observer who would see this planet in transit, and with a viewing geometry such that the sub-observer point is at a latitude of 30°. The animation shows how the pattern of flux from the planet changes throughout one orbit; the seasonal cycle heats and cools the observed hemisphere, making it much brighter and then dimmer again. This is representative of the animations available in the associated .tar.gz bundle (one for each of the subpanels in Figure 4), all of which go through one orbital period of the planet, with the orbital phase indicated in the upper left of each frame (0 and 1 are transit, 0.5 is secondary eclipse). Since no significant orbit-to-orbit variation is seen in our simulations, each animation can be looped continuously for an accurate depiction of the planet’s yearly seasonal cycle. (A .tar.gz file containing animations of this figure is available.)

For the $\psi = 10°$ model, which does not show significant seasonal variation, the $\phi_{\text{obs}} = \pm 10°$ viewing orientation results in a slightly diminished flux level. This is simply explained by the higher latitudes on the planet being cooler than the equator and so shifting those latitudes to greater visibility diminishes the disk-integrated flux observed. For the models that do exhibit seasonality ($\psi = 30°, 60°, 90°$), the amplitude of variation increases for increasing obliquity, while the orbital phase of the maximum flux is the same for all models. This trend of increasing variation with obliquity was also found for Earth-like planets in Gaidos & Williams (2004), and their results can also be used to make sense of the constant phase we see for maximum flux, since the thermal inertia of the atmosphere is the same for all models.

When we observe the planet at $\phi_{\text{obs}} = \psi$, then according to Equations (2) and (4), the solstice for the hemisphere facing us occurs at $\gamma = 0.5$, and so the phase difference between this and the peak flux is a measure of the atmospheric response time to the changing irradiation pattern. This agrees with the peak of the curves in Figure 3 being at $0.5 + \sim 1/8 \approx 0.625$, matching the timescale for atmospheric response determined above (see Section 3).

In Figure 3, we also show intermediate viewing angles for the highest obliquity models ($\psi = 60°, 90°$)—those with the strongest seasonal variation. When $\phi_{\text{obs}} \neq \psi$, two solutions exist for the orientation of the observer relative to the system. When $\phi_{\text{obs}} = 0°$, as in the solid curves of Figure 3, these two solutions produce overlapping phase curves because of hemispheric symmetry, but at intermediate values they do not, as is evident in the $\phi_{\text{obs}} = \psi/2$ curves shown as dotted lines. For one solution, the planet is observed such that solstice in the primarily observed hemisphere occurs before $\gamma = 0.5$ and in the other solution it occurs after $\gamma = 0.5$, resulting in phase shifts in either direction. The phase at which peak flux occurs depends on a combination of the atmospheric response timescale and the sub-observer latitude, with a positive or negative shift depending on which orientation we are observing from, relative to the seasonal cycle.

In general, we learned that the orbital phase curves of emitted light from this hypothetical planet contain information about the planet’s intrinsic properties of obliquity and either equator-to-pole heat transport (for low obliquity) or the atmospheric timescale for seasonal response (for high obliquity). The expression of these properties is regulated by the viewing orientation of the system, which could make it difficult to disentangle the intrinsic information without some other independent constraint on the viewing or inherent properties (confirming the conclusions of Gaidos & Williams 2004). However, if the planet obliquity or viewing geometry were known, then we could constrain atmospheric properties through...
the type of analysis demonstrated here. In the next section, we discuss another type of measurement that could be done instead or could be used to break the degeneracies in the phase curves.

4.2. Eclipse Maps

Orbital phase curves have been the primary means by which we have measured spatial information about exoplanets but, as we just saw, this type of observation should become less informative (and much more time consuming) for planets on longer orbital periods. Conveniently, another type of observational technique, known as eclipse mapping, should be increasingly useful for longer period planets. In this method, the detailed shape of the flux curve as the planet passes into secondary eclipse (ingress) or comes out from behind the star (egress), can be inverted to construct a two-dimensional map of the planet’s dayside brightness pattern (Williams et al. 2006; Rauscher et al. 2007; de Wit et al. 2012; Majeau et al. 2012). The advantages of using this method for longer period planets are:

1. It only requires observation during the time surrounding secondary eclipse, requiring much less telescope time than a full orbit (which would be 10 days for our hypothetical planet).
2. The slower orbital speed of the planet means that the ingress and egress take longer, providing a better opportunity to characterize the detailed shape of this part of the light curve.
3. The two-dimensional information retrieved by this method can constrain multiple properties of the planet. In addition to the magnitude of the equator-to-pole or north-to-south temperature gradient, the orientation of the flux gradient can reveal information about the planet’s obliquity.

The primary measurement made during secondary eclipse is the total decrement in light when the planet is blocked from view, behind the star. The total depth of the eclipse then gives us the integrated flux from the dayside hemisphere of the planet. From Figure 3, we can see that the amount of planetary flux that would be eclipsed by the star (the value of each curve at an orbital phase of 0.5) can depend strongly on the obliquity of the planet and the viewing orientation, but degeneracies exist such that these two properties cannot be uniquely retrieved. For example, in Figure 3, some of the curves for the \( \psi = 60^\circ \) model would be observationally indistinguishable from some curves for the \( \psi = 90^\circ \) model at 0.5 orbital phase; many more overlapping possibilities exist for the full range of obliquities and viewing geometries. The secondary eclipse depth alone does not provide a strong constraint on the planet’s obliquity.

Eclipse mapping provides the additional information required to discriminate between the temperature structures of low- or high-obliquity planets, in the case where they have the same hemispherically integrated flux. In Figure 6, we show the temperature map at the infrared photosphere for the \( 60^\circ \) obliquity model at several snapshots throughout its orbit. For each snapshot, we have oriented the planet image to the perspective of an observer who happens to see the planet in secondary eclipse at that point in its orbit (using Equations (4) and (5); the viewing orientation changes between each snapshot). From this careful accounting for the geometry of the secondary eclipse at various viewing orientations, we can simulate different possible eclipse mapping measurements. In Figure 6, we only show snapshots from the first half of the orbit because the images from the remainder of the planet’s year are mirror images, flipped vertically: the \( t/P_{\text{orb}} = 4/8 \) snapshot is the flipped image of the \( t/P_{\text{orb}} = 0 \) snapshot, the \( t/P_{\text{orb}} = 5/8 \) snapshot the flipped image of \( t/P_{\text{orb}} = 1/8 \), and so on.

There are two important degeneracies that influence the orientation of the brightness structure that can be measured by eclipse mapping: the unknown north–south orbital orientation of the system and a seasonal hemispheric degeneracy. Figure 7 shows schematic examples of these degeneracies. Since we are not spatially resolving the planet from the star, but rather using the time domain to separate out its signal, we do not know the orientations of the orbital or rotational angular momentum vectors relative to us, the latter of which defines which hemisphere is the northern one. We may be able to determine that one hemisphere is hotter than the other, but cannot label it as “north” or “south.” This degeneracy was also discussed above in Section 4.1, as it relates to the phase curves only being sensitive to the absolute value of the sub-observer latitude.

The second degeneracy diagrammed in Figure 7 is a physical degeneracy in the seasonal response between the two hemispheres. There are no sources of north–south asymmetry for our idealized hypothetical planet and so we see identically changing heating and cooling patterns in each hemisphere’s temperature structure. This explains why times separated by one half of an orbit have mirror-image temperature patterns. Since the north–south viewing degeneracy also exists, this means that we cannot discriminate between these seasonally identical conditions. For the example diagrammed in Figure 7, we cannot differentiate between a planet that is passing into eclipse in northern winter or northern summer, because in either case the hotter hemisphere will be eclipsed first. This degeneracy is also implicitly found in Figure 3, in that each curve is actually two overlapping predictions: the \(+\phi_{\text{obs}}\) and \(-\phi_{\text{obs}}\) solutions with times separated by half an orbit.
Even with those degeneracies, there still are several important pieces of information that we can measure with eclipse mapping. We calculate these from each of our models and plot them in Figure 8:

1. On the vertical axis, we plot the secondary eclipse depth, which is the disk-integrated flux emitted from the hemisphere facing the observer.
2. The color of each point shows the amplitude of the flux gradient, meaning the difference in flux between the brightest and dimmest regions of the planet, for the observed hemisphere.
3. On the horizontal axis, we plot the projected axial tilt, meaning the orientation of the flux gradient relative to the direction of motion of the stellar limb. Specifically, we calculate the projected angle of the observed flux gradient and take the difference between this and the planet’s orbital angular momentum (which is perpendicular to the direction of motion of the stellar limb). Due to the degeneracies discussed above and diagrammed in Figure 7, we only calculate the absolute value of this tilt (i.e., not whether it is tipped clockwise or counterclockwise) and only characterize it with values ≤90° (i.e., do not differentiate between the northern or southern hemisphere).

The third property listed, the projected tilt of the flux gradient, is the mostly directly observable indication of the planet’s obliquity. This is because our axisymmetric models always have flux gradients aligned with the rotation axis and so we are effectively measuring the projected obliquity. This will, however, be a function of the (arbitrary) viewing orientation of the seasons relative to our line of sight and so may take on a range of values. Results for each of our obliquity models (plotted using different symbols) were calculated at evenly spaced snapshots in time throughout the planet’s orbit in order to explore the possible range of observed properties. In Figure 8, we also choose several representative data points for the different obliquity models and show the planet flux map that corresponds to each of those cases.

The zero obliquity model has no seasonal variation, and its temperature structure remains unchanging throughout the year. Its eclipse depth, flux gradient, and projected axial tilt will always be the same value, independent of viewing orientation. These values are all nearly degenerate with the results for the ψ = 10° model, whose temperature structure is very similar and shows only minimal seasonal variation. The only slight difference is that the project tilt for this model could be observed as anything between 0° and 10°, depending on the orientation of the observer relative to the seasonal cycle. It is unlikely that observations could discriminate between these low-obliquity cases for our hypothetical planet.

When we compare the predicted eclipse mapping data for models with higher obliquities and stronger seasonal variation, we do see observable differences. There are some times during its orbit when the ψ = 90° model would be observed to have the same eclipse depth and flux gradient amplitude as the ψ = 0° model, but its projected axial tilt is always at 90° (always aligned with the direction of motion of the stellar limb and so perpendicular to the orbital angular momentum); this would be a clear way to discriminate between these two extremes in obliquity.

Another piece of information from eclipse mapping that would help to differentiate between planets with obliquities of 0° or 90° is that the flux gradient for the 0° case is peaked at...
the equator and drops off toward the poles, while the 90° case has a north–south gradient that stretches across the whole planet. This type of spatial information is retrieved in eclipse mapping observations, although we have not quantified it in our analysis here. Nevertheless, it is worth emphasizing that the planets with equators hotter than their poles (our ψ = 0°, 10°, and 30° models) should be observed to have a different pattern of emission than planets where one hemisphere is hotter and brighter than the other (our ψ = 60° or 90° models). However, this will depend somewhat on the viewing orientation; for example, a high-latitude viewing orientation for a planet with a hotter equator and cooler poles could perhaps mimic the hemispheric flux difference of a higher obliquity planet, depending on the level of detail that can be retrieved with eclipse mapping.

Our models with obliquities of 30° and 60° show intermediate eclipse mapping properties in Figure 8. They both have possible projected axial tilt values from zero up to their full obliquities, but observing a non-zero tilt only constrains the possible obliquities to be greater than or equal to that value.9 However, here we can see eclipse mapping’s power to constrain a planet’s obliquity. Although models with different obliquities could produce the same projected axial tilt, the same eclipse depth, and/or the same flux gradient amplitude, these properties can generally be used in combination to break degeneracies between the different possible planet obliquities. In the next section, we estimate the ability of current and future instrumentation to perform precise enough measurements to be able to constrain a warm Jupiter’s obliquity.

4.3. Current and Future Instrument Sensitivity

We have predicted two types of observations that could potentially be used to discern a planet’s obliquity: orbital phase curves and eclipse maps. Here we compare the signals predicted by our models to the instrumental sensitivity of current and future missions. We use rough estimates, since our modeled planet is hypothetical and we have to wait for a specific system to be targeted before more detailed estimates can be made. First, we go through the exercise of estimating the signal-to-noise ratio, assuming that the planet orbits a bright star and that we can achieve photon-limited precision, before a discussion of how to scale to less optimistic circumstances.

4.3.1. Predicted Planet-to-star Flux Ratio

We already have predictions for the flux emitted from the planet as a function of obliquity and viewing orientation (Figures 3 and 8), but the actual observable is the planet-to-star flux ratio, which we estimate as follows. The double-gray radiative transfer scheme used in our circulation model separates the optical absorbed stellar light from the thermally re-emitted infrared light, with the result that the predicted emission shown above is a general “infrared” emission, and in fact encompasses all emission from the planet (for details, see Rauscher & Menou 2012). If we view this in a bolometric sense, then the planet-to-star flux ratio is \( \sim 6 \times 10^{-4} \), based on an average planetary emission of \( \sim 3.6 \times 10^4 \) W m\(^{-2}\) (from Figures 3 and 8) and assuming Sun-like properties of the host star.

In comparing our predictions to specific instruments, it is important to consider the strong wavelength dependence of the planet-to-star flux ratio. Assuming Planck functions for the planetary and stellar emission, their flux ratio at some wavelength can be calculated as \( \left( R_p/R_* \right)^2 \times B(\lambda, T_p)/B(\lambda, T_*) \). Using the planet’s equilibrium temperature of \( \sim 880 \) K, which is close to the average temperature at the planet’s infrared photosphere (Figure 2), we can estimate the planet-to-star flux ratio for Hubble (WFC3 G141, 1.4 μm) as \( \sim 4 \times 10^{-7} \) and for Spitzer (IRAC1, 3.6 μm) or JWST (NIRCam F356W, 3.5 μm) as \( \sim 1 \times 10^{-4} \).

4.3.2. Estimates of Precision

We base our precision estimates on the values from Table 3 of Cowan et al. (2015), which assume the photon-counting limit for a target 20 pc away around a 5000 K host star. We adjust these predictions from an assumed integration time of 1 hr to the \( \sim 4 \) hr length of the secondary eclipse for our hypothetical planet \( (=2R_*/[2\pi a/P_{orb}]) \). Since our various measurements are all differential (in- versus out-of-eclipse, amplitude at one phase versus another) and so require the comparison of two different flux-level measurements, we apply a \( \sqrt{2} \) penalty and obtain optimistic precisions of 1.6 ppm, 11 ppm, and 1.6 ppm (parts per million) for Hubble (WFC3 G141), Spitzer (IRAC1), and JWST (NIRCam F356W), respectively. We can only make educated guesses as to JWST’s actual performance until it begins operations, but based on lab tests of its detectors (noise floor estimates, quantum efficiencies, etc.), as well as previous experience with Hubble and Spitzer, we expect that JWST will achieve excellent photometric precision for exoplanet transits and eclipses (Beichman et al. 2014).

4.3.3. Detection of Secondary Eclipse

The largest, easiest signal to detect is that of the secondary eclipse of the planet. Using the planet-to-star flux ratios and precisions estimated in the previous two sections, we expect that with a single observation (assuming photon-limited precision), the secondary eclipse of our hypothetical warm Jupiter would not be detected by Hubble, but could have been detected by Spitzer at 9σ, and should be detectable by JWST at 80σ.

This optimistic prediction implies that JWST should be able to measure the eclipse depths plotted in Figure 8 to a precision of \( \sim 0.2 \times 10^4 \) W m\(^{-2}\) at 3σ. The eclipse depths of the models with \( \psi = 0° \) and 10° will be indistinguishable, the lowest possible values of the \( \psi = 30° \) model may be barely distinguishable from the lower obliquity cases, and much of the range of values for the \( \psi = 60° \) or 90° models should be easily distinguishable. Since these are single-eclipse estimates, we could further improve the precision by collecting multiple observations, since we see no significant orbit-to-orbit variability on the planet.

4.3.4. Detection of Variation in Orbital Phase

According to Figure 3, the amplitude of the orbital flux variation we should observe depends both on the planet’s obliquity and our viewing orientation, with the \( (a \text{ priori unknown}) \) viewing orientation having a strong influence on the phases of maximum and minimum flux. Since the orbital period is 10 days, it would be observationally very expensive to monitor a continuous, full orbit of the system, and so a few shorter integration observations would probably be preferred (Krick et al. 2016). If there were already a secondary eclipse...
observation, that would help to constrain possible combinations of obliquity and viewing orientation and to intelligently schedule snapshots throughout the orbit.

We will sidestep the issue of timing and instead just assume that there is one 4 hr observation near the peak in flux and one 4 hr observation near the minimum, in order to determine what level of variation could be measured.\textsuperscript{10} From our estimates of signal and precision, we determined above that \textit{JWST} should be able to measure flux differences of \(~0.2 \times 10^4\) W m\(^{-2}\) for this planet at 3\(\sigma\). This means that all of the phase curves for equatorial viewing geometries (the solid lines in Figure 3) would be considered flat, as would the maximally variable curve for the \(\psi = 10^{\circ}\) model. The rest of the curves shown in Figure 3 (non-equatorially viewed models with \(\psi > 30^{\circ}\)) would have measurable variation, at the 9\(\sigma\) to 40\(\sigma\) level with \textit{JWST}.

\[\text{\textbf{4.3.5. Detection of Flux Gradient via Eclipse Mapping}}\]

The detection of a gradient in the flux across the planet’s disk can be measured during the ingress and egress times of secondary eclipse, given a high-enough precision. This limits the integration time of the observation to the time that it takes the planet disk to move behind the star (or re-emerge), \(2R_p/(2\pi a/f_{\text{orb}})\) = 0.4 hr. Whereas for the eclipse depth measurement we assume an integration time of 4 hr and apply a \(\sqrt{2}\) penalty for comparison with a baseline measured out of eclipse, here all of the information are contained within the times of ingress and egress. We thus use only 0.4 hr as the integration time, instead of 0.8 for the full ingress plus egress, and still apply the \(\sqrt{2}\) penalty for a differential measurement. We could simplistically think of this as a comparison in flux between the first/second half of ingess/egress and the second/first half of ingress/egress. Although the details of eclipse mapping are actually more complex (Williams et al. 2006; Rauscher et al. 2007; de Wit et al. 2012; Majeau et al. 2012), this method still relies on comparing the differential flux during some time(s) of ingress/egress to other times. This adjusts our predicted precision, assuming a single-eclipse mapping observation with \textit{JWST}, to 4 ppm.

The signal we are trying to measure in eclipse mapping is the difference in shape between an ingress/egress curve for a planet disk with uniform brightness and one with a flux gradient. If the gradient stretches across the globe, so that one hemisphere is above average and the other below average, then we can simplistically assume that the maximum signal occurs when the planet is half-occulted by the star (in reality this depends on the projected angle of the gradient relative to the stellar limb).\textsuperscript{11} In this case, we apply a penalty of 1/2 to the expected signal, since only half of the planet is in view. For planets with an equator-to-pole flux gradient, the same simplistic assumption would imply that the greatest signal is when one-fourth of the planet is occulted or left in view. The ingress and egress signals would then average out to the same factor of 1/2. The amplitude of the signal we are trying to measure is then roughly \((1/2)F_{\text{grad}}/F_\text{eq}\). (If the gradient is equal to the eclipse depth, then halfway through ingress/egress the flux would be \((1/2) F_{\text{eq}}\) different from the flux of a uniformly bright disk; if there is no gradient, the signal is zero.)

Note that one underlying assumption of our analysis is that our predicted \textit{JWST} observations are at sufficient cadence to resolve the shape of the light curve during ingress and egress well enough to retrieve the spatial information. This is a safe assumption for two reasons. First, \textit{Spitzer} has demonstrated sufficient cadence for mapping a similarly bright target (de Wit et al. 2012; Majeau et al. 2012), albeit with stacked observations, and the time-series imaging mode for NIRCam has been specifically designed for high-precision and high-cadence observations of bright sources,\textsuperscript{12} a significant improvement over \textit{Spitzer}. Second, in the simplified estimate here, we are effectively binning down to two points during ingress and two during egress, since we are only trying to estimate an amplitude for the flux gradient, rather than its spatial shape. As such, the observational cadence of \textit{JWST} is not a concern here.

We can then take the planet-to-star flux ratios estimated above, multiply by 0.5\(F_{\text{grad}}/F_{\text{eq}}\), and divide by our estimated \textit{JWST} precision for this type of measurement to determine the detectability of our predicted flux gradients. From the eclipse depths and flux gradients presented in Figure 8, we determine that the lower end of the \(F_{\text{grad}}/F_{\text{eq}}\) range for the \(\psi = 60^{\circ}\) model is not accessible to eclipse mapping with a single observation, the lower end of that range for the \(\psi = 90^{\circ}\) model is marginally mappable (3\(\sigma\)), and all other possible views of all other models could be mapped with a single \textit{JWST} observation. The minimum for the \(\psi = 30^{\circ}\) model is 6\(\sigma\) and the maximum values for the \(\psi = 0^{\circ}\), 10\(^{\circ}\), 30\(^{\circ}\), 60\(^{\circ}\), and 90\(^{\circ}\) models are 13\(\sigma\), 12\(\sigma\), 9\(\sigma\), and 12\(\sigma\), respectively.

The technique of eclipse mapping has only successfully been applied to a single exoplanet (HD 189733b; de Wit et al. 2012; Majeau et al. 2012) and does not yet have a standardized method of analysis, making it difficult for us to predict the precision to which axial tilts could be measured. Both de Wit et al. (2012) and Majeau et al. (2012) presented multiple possible maps that could be reconstructed, depending on the particular form of equations used to describe the planet map. This motivates work such as E. Rauscher et al. (2017, in preparation), in which orthonormal basis sets of maximally informative light curves and the corresponding “eigenmaps” that can be combined to reconstruct the planet map are presented. However, our ability to accurately use the method of eclipse mapping is limited by the precision with which we know other properties of the system, namely, the planet’s eccentricity, its impact parameter (or alternatively its orbital inclination), and the global density of the star (which influences the orbit of the planet), as demonstrated by de Wit et al. (2012). It is left to more detailed future work to precisely quantify the sensitivity of eclipse mapping measurements to both the amplitudes of flux gradients and their projected tilts.

\[\text{\textbf{4.3.6. A Less Optimistic Signal-to-noise Estimate}}\]

For the estimates in the sections above, we have optimistically assumed that \textit{JWST} will be able to reach photon-limited

\textsuperscript{10} Four hours is \textasciitilde0.02 in phase and so we would not expect much seasonal variation in thermal emission during that integration time.

\textsuperscript{11} This signal estimate also requires that the transit does not occur exactly edge-on (with an impact parameter of \(b = 0\)), in contradiction to the edge-on assumption used to create the plots of the predicted planet emission above. If the eclipse were perfectly edge-on, then a planet with a projected axial tilt of 0\(^{\circ}\) would have no eclipse mapping signal, due to perfect symmetry. However, the chances of an exactly edge-on orbit are infinitesimal and so in practice this is not a significant concern. This assumption is also not strongly at odds with our previous analysis, since for a planet to transit at all, the orbital inclination must be such that \(b = (a \cos i)/R_* < 1\), which for our hypothetical planet translates to \(i > 87^{\circ}\).

\textsuperscript{12} See https://jwst-docs.stsci.edu/display/JTI/NIRCam+Time-Series+Imaging.
precision and that our hypothetical planet orbits a nearby bright star. Here we briefly consider how our estimated signal-to-noise ratio decreases if we make less generous assumptions. While none of these factors can actually be known before JWST is in operation or before a warm Jupiter system to study is identified, it is possible to adjust our expectations for less desirable circumstances.

The hypothetical target star used by Cowan et al. (2015) to estimate instrument precision, which we use as a basis for our signal-to-noise estimate, has similar properties to the bright hot Jupiter host star HD 189733 (signal-to-noise estimate, has similar properties to the bright hot star. Here we brieﬂy consider how our estimated signal-to-noise ratio decreases if we make less generous assumptions. While none of these factors can actually be known before JWST is in operation or before a warm Jupiter system to study is identified, it is possible to adjust our expectations for less desirable circumstances.

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The expected yield from TESS includes about 20 gaseous exoplanets (meaning radii larger than 4 Earth radii) with periods less than 20 days, orbiting stars with magnitudes as faint as ~12.5, which is roughly the magnitude we would expect for the HD 189733-like host star used in the calculations above. So, if we target a warm Jupiter from the TESS survey, we may expect our optimism about a bright host star to be fair. If, however, we are limited to the currently known warm Jupiter population (see the Introduction), then a host-star V magnitude of 10 would make our precision ~3 times worse. This would still allow for most of our predicted flux gradients to be detected at greater than 3σ with a single JWST observation.

On the instrument side, while it may be overly optimistic to assume that JWST will reach photon-limited precision, the realized performance may not be too far off from it. Cowan et al. (2015) showed that their photon-limited precision estimates for HST and Spitzer were within a factor of 2 to 3 of the precisions actually achieved for exoplanet transits and eclipses. This is encouraging, especially since the NIRCam, NIRISS, and NIRSpec instruments will all use the same type of detector as HST. A factor of 2–3 worse precision is the same as our above consideration of a dimmer stellar host, meaning that we would still be able to measure most of the possible flux gradients to better than 3σ.

We also have reason for some optimism in that, even if the precision of JWST is not as spectacular as one might hope, we should still be able to stack together multiple eclipses to achieve improved precision. The original eclipse mapping measurement was achieved by stacking together seven Spitzer eclipses (de Wit et al. 2012; Majeau et al. 2012). Even if the performance of JWST were an order of magnitude worse than the photon-limited precision, stacking ~4–9 eclipses would still allow us to measure the maximum flux gradients predicted for each obliquity model (and thus help to constrain a warm Jupiter’s obliquity) at 3σ, although ~25 eclipses would be needed to get these to 5σ detections.

5. Summary and Conclusions

We have presented a set of three-dimensional atmospheric models for a hypothetical Jupiter-like planet on a 10 day period orbit around a Sun-like star, testing possible planetary obliquities (ψ) of 0°, 10°, 30°, 60°, and 90°. We find that seasonal variations are negligible for ψ ≤ 10° and important for ψ ≥ 30°, to a level that should be measurable with the James Webb Space Telescope.

The circulation pattern of the ψ ≤ 30° models is characterized by two high-latitude eastward jets (one in each hemisphere) and an equator that is hotter than the poles. For the ψ = 30° model, the hot region near the equator shifts slightly up and down in latitude, as a delayed response to the seasonally shifting irradiation pattern. This atmospheric response time of ~1/8 P_{ecl}, which is about an order of magnitude longer than the radiative timescale at the infrared photosphere, is also seen in the seasonal variations of the models with ψ ≥ 60°. For those models, the temperature pattern is characterized by one hemisphere being hotter during its summer and then a more uniform temperature distribution during equinoxes.

The observable features of each model depends not only on its obliquity, but also on the orientation with which the system is viewed, relative to the seasonal cycle. This results in observational degeneracies between inherent and coincidental properties; however, by combining multiple measurements, it may be possible to constrain a planet’s obliquity. From observing the light emitted by the planet as a function of orbital phase, we can determine:

1. the efficiency of the equator-to-pole heat transport, if the phase curve is flat and is at a level higher than the global average for re-emission of absorbed starlight,
2. the obliquity of the planet, if the phase curve is flat and at a level below the global value for re-emitted starlight, or
3. degenerate information about the planet’s obliquity, our viewing orientation, and the atmospheric seasonal response time, if the phase curve shows measurable variation.

Eclipse mapping, which resolves the dayside of the planet when it is eclipsed by the star, can provide more information than orbital phase curves and will require significantly less telescope time. We present quantitative predictions for the three main features measured by eclipse mapping: the total flux emitted from the observed hemisphere, the gradient in emitted flux across the hemisphere, and the orientation of that gradient relative to the direction of motion of the stellar limb. While none of these parameters can uniquely constrain the planet’s obliquity on its own, their combination can usually only match the predictions from a single obliquity model. The exceptions are the ψ = 0° and 10° models, which have very similar values for each of these predicted observables. Thus, eclipse mapping can either constrain a planet’s obliquity to be small (if ψ ≤ 10°) or can measure it if signiﬁcantly non-zero (≥30°).

The phase curve and eclipse mapping methods are also complementarily informative. For example, independent of obliquity, the smallest phase curve variations occur for equatorial viewing geometries, but if we observe the system from this orientation, the projected axial tilt measured by eclipse mapping will be equal to the true obliquity of the planet. In this way, the multiple parameters that can be observed with eclipse mapping could be combined with phase curve measurements to provide even better constraints on the planet’s obliquity. If we first obtain the (observationally cheaper) eclipse map measurement, we could use the information contained there to identify which orbital phases might be the most informative (providing information about the atmospheric response time and/or efﬁciency of meridional heat transport) and sidestep the need for continuous observation (Krick et al. 2016).

Finally, we compare our various predicted signals to the sensitivity of current and future instruments. We find that Hubble observes at wavelengths too short to be useful for observing this hypothetical planet; however, JWST will be an amazing instrument for this type of work. If we assume

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photon-limited precision and a host star as bright as HD 189733, then JWST should be able to detect the secondary eclipse of this hypothetical warm Jupiter at $\sim 80\sigma$ in a single observation, which is sensitive enough to eclipse-map the planet, measuring the hemispheric flux gradient and its orientation.

We have determined that the expected non-zero obliquities of warm Jupiters should both influence the observational characterization of these planets and also be measurable with JWST. Future work will need to determine the observational degeneracies between the diversity of possible values we expect for the obliquities, eccentricities (Gaidos & Williams 2004; Kataria et al. 2013), and rotation rates (Showman et al. 2015) of warm Jupiters. Since the obliquities (along with eccentricities and rotation rates) are presumably markers of the formation, evolution, and tidal processes that influence these planets, this adds an extra dimension of study for warm Jupiters beyond the lessons we can learn from their bulk compositions (e.g., Thorngren et al. 2016).

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