Real Terrain Simulation based on Point Cloud of Water Area and Land Area

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Abstract - The joint simulation of land and water integration plays an increasingly important role in surveying and mapping, water conservancy, and land resources supervision. Based on the theory of 3D coordinate transformation, this paper discusses the application of Rodrigues’s matrix and indirect adjustment model in the iterative calculation of point cloud registration parameters. In this method, anti-symmetric matrix elements are used to form a rotation matrix, which avoids the complex trigonometric function operation of traditional methods, and improves the efficiency of calculation while ensuring the accuracy, and can easily calculate the initial value of parameters. Experimental results demonstrate that the proposed method has the advantages of high accuracy, robustness, stability, and reliability.

1. Introduction

In recent years, with the development of new mapping technologies such as RS (Remote Sensing), LiDAR (Light Detection And Ranging) and the integrated development of computer science such as VR (Virtual Reality) and AI (Artificial Intelligence), 3D terrain visualization and simulation have gradually become a hot topic for many scholars to study. At present, terrain simulation is commonly used to fit ground relief based on the digital ground model and combine the corresponding satellite or aerial remote sensing image data to provide a more realistic and intuitive scene experience. For point cloud modeling, 3D model reconstruction of surface buildings, cultural relics, and topography are common, while for point cloud co-simulation of land and water topography, there are relatively little researches[1]. At the same time, with the gradual development of surveying and mapping geographic information science from digitization to informatization, the demand for geospatial data is also developing towards the direction of high precision, massive, current potential, and visualization. The sharing and integrated management of geospatial data in airspace, land, and water area are becoming more and more important for national economic construction, social development, national security, and space science research[2]. Therefore, it is of great practical significance to realize the joint simulation of land and water terrain through multi-source data fusion, and it is also one of the trends of 3D modeling in the field of surveying and mapping in the future.

2. Point cloud collection

At present, the point cloud of water terrain is acquired mainly through a multi-beam sounding system, which adopts multi-array and wide-angle transmitting and receiving, thus forming the banded dense sounding data. Compared with the traditional single-beam sounding technology, the multi-beam sounding system has the advantages of rapid, accurate, and wide range measurement. It extends the depth sounding technique from the traditional point and line to the plane and further develops the three-dimensional map so that the submarine topographic survey technique develops to a higher level.
Working principle of the multi-beam sounding system is wide sectors covered by transmitting transducer array acoustic emission to the sea bed, using a narrow beam of sound wave receiving, receiving transducer array formed by the orthogonally of the transmitting and receiving sector points to illuminate the footprints of submarine topography, the processing of these footprints are appropriate, a probe can be in a vertical plane perpendicular to the direction of hundreds of even more the site of the bottom of the seawater depth value, achieve rapid continuous access to test area within the scope of the surface characteristics of underwater detecting object and change of the ups and downs, get accurate underwater terrain[3].

![Figure 1. The principle of multi-beam sounding system](image)

The land point cloud can be acquired utilizing ground/mobile LiDAR and tilt photogrammetry, etc., while in the reconstruction of refined terrain, the ground 3D laser scanner has certain advantages in precision and operation efficiency. Based on the principle of laser ranging and the non-contact high-speed laser measurement, the ground 3D laser scanner can automatically, continuously and rapidly obtain the array geometry data of terrain and complex objects 3d surface in the form of the point cloud. In the scanner, the scanning control module controls and measures the Angle of each pulse laser[4]. For each scanning point, the oblique distance $S$ from the measuring station to the scanning point can be measured. In addition, along with the horizontal angle and vertical angle of the scanning, the spatial relative coordinates of each scanning point and the measuring station can be obtained. The general internal coordinate system of the instrument: X-axis is in the transverse scanning surface, the Y-axis is perpendicular to the X-axis, and the Z-axis is perpendicular to the transverse scanning surface (as shown in Fig. 2).

![Figure 2. The principle of terrestrial laser scanners system](image)

### 3. Joint registration of water and land point clouds

In general, the point cloud collected in the land area mostly uses an independent coordinate system, while the point cloud collected in the water area mostly uses local or engineering coordinate systems, so the registration between them is the key to realize the joint simulation of land and water. Point cloud registration is a three-dimensional coordinate transformation, and the approximate Bursa seven-
parameter model can be utilized to the small Euler angle[5]. In conventional point cloud registration, it is usually approximate to a rigid body transformation that only involves rotation and translation. However, in this study, scale transformation is considered to improve the accuracy of the solution because different instruments and equipment are used for measurement on the ground and underwater.

3.1. 3D Coordinate transformation model
In the 3D coordinate transformation of the geodetic survey, the Bursa-Wolf model, Molodensky model, and Wu model are widely used. A simplified conversion model is adopted for registration conversion of land and water point clouds.

Let the land point cloud $P$ and water area point cloud $Q$, and their corresponding coordinate systems are $SP$ and $SQ$, respectively. According to the physical process of coordinate conversion, the coordinate conversion model from the coordinate system $SP$ to the coordinate system $SQ$ is as the following equation.

$$\begin{bmatrix} X \\ Y \\ Z \end{bmatrix}_Q = \lambda \begin{bmatrix} X \\ Y \\ Z \end{bmatrix}_P + T$$  \(1\)

In formula (1), $R$ is the rotation matrix consisting of three rotation angles $\alpha$, $\beta$, and $\gamma$, $T$ is the translation matrix consisting of three scalars, and $\lambda$ is the scale factors. Sometimes, the angles of rotation $\alpha$, $\beta$, and $\gamma$ are called Euler angles, and the matrix $R$ that is built is called the Euler angle matrix.

3.2. Construction of Rodrigo matrix
The process of aligning land and water point clouds involves the three-dimensional conversion of large rotation angles, and the use of the simplified Bursa seven-parameter model is not appropriate. The reason is that the traditional seven-parameter coordinate transformation takes three Euler angles rotating around the coordinate axis to form the rotation matrix, which requires a lot of complex trigonometric function operation and linearization process, and the initial value of the parameters is not easy to determine[6]. Based on Rodrigo matrix and the indirect adjustment model of point cloud registration parameter iteration computation, this method involves only simple arithmetic, to a certain extent, improve the efficiency of understanding calculate, can determine the values of initial parameters, through the iterative calculation to ensure understanding of numerical results with good accuracy, besides, the use of indirect adjustment model can easily observation equation and the equation to determine the number listed.

The rotation matrix $R$ of equation (1) is an orthogonal matrix containing only three independent parameters, and an anti-symmetric matrix $S$ can be constructed.

$$S = \begin{bmatrix} 0 & -c & -b \\ c & 0 & -a \\ b & a & 0 \end{bmatrix}$$  \(2\)

In formula (2), $a$, $b$ and $c$ are three independent parameters, and $I$ is set as the third-order identity matrix. Then $R$ can be composed of $S$ and $I$ Rodrigo matrix.

$$R = (I + S)(I - S)^{-1}$$  \(3\)

3.3. Solution of the initial value of the parameter
In the process of using indirect adjustment model to solve 3D coordinate transformation parameters, it is particularly important to select the initial value of the parameters, which directly affects the stability of the adjustment system, the accuracy of the results and the efficiency of the solution, and the unreasonable initial value may even cause the adjustment results to seriously deviate from the truth value. To obtain the initial value of the parameter, firstly calculate the scale factor $\lambda^0$, then solve the rotation matrix $R^0$, and finally calculate the translation parameter $T^0$.

The ratio of the distance between any two pairs of homonyms $i$ and $j$ in coordinate system $SQ$ and coordinate system $SP$ is considered as the initial value of the scale parameter.
\[ \lambda^0 = \sqrt{\Delta X_{Qij}^2 + \Delta Y_{Qij}^2 + \Delta Z_{Qij}^2} / \sqrt{\Delta X_{Pij}^2 + \Delta Y_{Pij}^2 + \Delta Z_{Pij}^2} \]  

(4)

To improve the proximity between the initial value and the true value, it is also possible to combine more pairs of the same name points and calculate the average distance ratio as the initial value of the scale parameter.

For the determination of three independent parameters \( a^0, b^0 \) and \( c^0 \) constituting the rotation matrix \( R \), the difference between the coordinate transformation model (1) satisfying any two pairs of identically named points \( i \) and \( j \) of point cloud \( P \) and \( Q \) can be obtained by combining with the properties of Rodrigo matrix.

\[ \lambda (I + S) \begin{bmatrix} \Delta X_{ij} \\ \Delta Y_{ij} \\ \Delta Z_{ij} \end{bmatrix}_P = (I - S) \begin{bmatrix} \Delta X_{ij} \\ \Delta Y_{ij} \\ \Delta Z_{ij} \end{bmatrix}_Q \]  

(5)

\[ \begin{bmatrix} 0 & -w_{ij} & -v_{ij} \\ -w_{ij} & 0 & u_{ij} \\ v_{ij} & u_{ij} & 0 \end{bmatrix} \begin{bmatrix} a \\ b \\ c \end{bmatrix} = \begin{bmatrix} \Delta X_{Qij} - \lambda \Delta X_{Pij} \\ \Delta Y_{Qij} - \lambda \Delta Y_{Pij} \\ \Delta Z_{Qij} - \lambda \Delta Z_{Pij} \end{bmatrix} \]  

(6)

Equation (8) has only two independent equations, so at least one pair of homonymous point \( k \) is needed to be combined with two points \( i \) and \( j \) to form the following equation.

\[ \begin{bmatrix} 0 & -w_{ij} & -v_{ij} \\ -w_{ij} & 0 & u_{ij} \\ v_{ij} & u_{ij} & 0 \\ 0 & -w_{jk} & -v_{jk} \\ -w_{jk} & 0 & u_{jk} \\ v_{jk} & u_{jk} & 0 \end{bmatrix} \begin{bmatrix} a \\ b \\ c \end{bmatrix} = \begin{bmatrix} \Delta X_{Qij} - \lambda \Delta X_{Pij} \\ \Delta Y_{Qij} - \lambda \Delta Y_{Pij} \\ \Delta Z_{Qij} - \lambda \Delta Z_{Pij} \\ \Delta X_{Qjk} - \lambda \Delta X_{Pjk} \\ \Delta Y_{Qjk} - \lambda \Delta Y_{Pjk} \\ \Delta Z_{Qjk} - \lambda \Delta Z_{Pjk} \end{bmatrix} \]  

(7)

Equation (7) is an overdetermined system of equations. The least-square solutions of the initial values of three independent parameters, \( a^0, b^0, \) and \( c^0 \), can be solved by using the indirect differential. In order to improve the accuracy, multiple points with the same name can also be combined in pairs to obtain multiple equations with the form of Equation (8). Then, according to the initial values of the scale parameters and rotation parameters obtained by the solution, the coordinates of any point are substituted into equation (1) to solve the initial values of the three translational parameters \( T^0_X, T^0_Y \) and \( T^0_Z \), so as to complete the initial values of all parameters.

3.4. Solution of the Adjustment Model

In formula (1), the spatial coordinate transformation model based on Rodrigo’s matrix is used, and the initial values of the above parameters are given, \( R^0 = R(a^0, b^0, c^0) \), \( T^0 = [T^0_X \ T^0_Y \ T^0_Z]^T \), \( \lambda^0 \), and the following linearized observation equation is obtained by indirect adjustment.

\[ F = F^0 + \frac{\partial F}{\partial T_X} dT_X + \frac{\partial F}{\partial T_Y} dT_Y + \frac{\partial F}{\partial T_Z} dT_Z + \frac{\partial F}{\partial a} da + \frac{\partial F}{\partial b} db + \frac{\partial F}{\partial c} dc + \frac{\partial F}{\partial \lambda} d\lambda \]  

(8)

The error equation of Equation (7) was sorted out.

\[ V = B\hat{x} - l \]  

(9)

Where, error equation coefficient matrix

\[ B = \begin{bmatrix} \frac{\partial x_Q}{\partial T_X} & \frac{\partial x_Q}{\partial T_Y} & \frac{\partial x_Q}{\partial T_Z} & \frac{\partial x_Q}{\partial a} & \frac{\partial x_Q}{\partial b} & \frac{\partial x_Q}{\partial c} & \frac{\partial x_Q}{\partial \lambda} \\ \frac{\partial y_Q}{\partial T_X} & \frac{\partial y_Q}{\partial T_Y} & \frac{\partial y_Q}{\partial T_Z} & \frac{\partial y_Q}{\partial a} & \frac{\partial y_Q}{\partial b} & \frac{\partial y_Q}{\partial c} & \frac{\partial y_Q}{\partial \lambda} \\ \frac{\partial z_Q}{\partial T_X} & \frac{\partial z_Q}{\partial T_Y} & \frac{\partial z_Q}{\partial T_Z} & \frac{\partial z_Q}{\partial a} & \frac{\partial z_Q}{\partial b} & \frac{\partial z_Q}{\partial c} & \frac{\partial z_Q}{\partial \lambda} \end{bmatrix} \]  

(10)
The above three error equations are listed for a pair of points with the same name. For \( n \) pairs of points with the same name, a total of \( 3n \) error equations with the same name can be listed, and then the normal equation can be obtained

\[
N_{BB} \hat{x} - W = 0
\]  

(11)

Where, the coefficient matrix of the normal equation \( N_{BB} = B^T P B \), constant term \( W = B^T P I \), \( P \) is the weight matrix of observed values, and the identity matrix is taken here, that is, \( P = I \).

Obtained parameter correction number \( \hat{x} = N_{BB}^{-1} W \), adjustment value \( \hat{x} = X^0 + \hat{x} \), further can calculate the most probable value of the rotation matrix and the translation matrix. Since the stochastic model of indirect adjustment is \( V^T P V = \min \) and follows the least square principle, the most probable value obtained here is the optimal unbiased solution.

After registration, the mean square error of \( i \) point is

\[
\hat{\sigma}_i = \hat{\sigma}_0 \sqrt{Q_{x,i} + Q_{y,i} + Q_{z,i}}
\]  

(12)

Where, \( Q_{x,i} \), \( Q_{y,i} \), \( Q_{z,i} \) is the co-factors of the point \( i \) in the x-coordinate, y-coordinate and z-coordinate directions respectively. Besides, in the process of solving the transformation parameters, the correct number of control parameters can be calculated iteratively within the specified limit difference to solve the optimal valuation of coordinate transformation parameters.

4. Experiment

Embankment projects in flood control, irrigation, water supply, navigation, water conservation, and other aspects of the huge social benefits, economic benefits, and environmental benefits. The safety monitoring of levee includes not only the deformation monitoring of the land area but also the terrain change of the water area. The digital simulation model of the real scene plays an important role in the safety evaluation of levee. In this paper, an area in the Yangtze River section of Nanjing was selected as the experimental object. The water area was scanned with R2SONIC 2024 multi-beam sounding system, and the land lever was scanned with Faro Foucs3D ground 3D laser scanner to obtain point cloud data in the water area and the land area respectively, as shown in Fig. 3.

![Figure 3. Experimental point cloud](image)

(a) Point clouds of water area                                    (b) Point clouds of land area

The ground control network is used to obtain the coordinates of 4 pairs of homonymous feature points in the water area coordinate system and the land area coordinate system respectively, among which the land area uses the independent coordinate system and the water area uses the BJ54 coordinate system.

The rotation matrix \( R \), the translation matrix \( T \), and the scaling factor \( \lambda \) are calculated according to the above algorithm. After conversion, the coordinate difference between the coordinate of each point with the same name and the actual measurement control point in the specified coordinate system can be registered. After 4 times of iteration, calculate the unit mean error \( \hat{\sigma}_0 = 2.1mm \). According to the analysis of the precision index, the median error of unit weight of point cloud registration and the median error of point position of the same name point are both controlled at the millimeter level. The quality of adjustment results is good and meets the expected effect, which verifies the feasibility of the above algorithm.

Coordinate transformation of all coordinate data in the land point cloud is carried out according to the transformation parameters obtained. In this way, registration is realized between the water area and the land point cloud. The combined surface and land point clouds after registration are shown in Fig.4.
It can be seen that the two are located in the same coordinate system, achieving relatively accurate and strict stitching with a good registration effect. On this basis, the pre-processing operations such as simplification, denoising, and so on are carried out for the registered combined surface cloud, and encapsulation and combination are carried out. Finally, the three-dimensional simulation model of integrated surface and land can be obtained through surface reconstruction.

5. Conclusion
With the development of new surveying and mapping technology and computer simulation technology, the real digital terrain expression of water and land integration tends to be more and more real and refined, and this kind of simulation model is more and more widely used in informatization surveying and mapping, flood simulation, submergence analysis, disaster warning, and other work. However, the model construction is only the first step of relevant work, and there are still many key issues to be further explored, such as the coordinate transformation of multi-source data, processing of massive data, application analysis of the 3D model.

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References
[1] Travelletti J., Delacourt C., Allemand P., et al. (2012) Correlation of multi-temporal ground-based optical images for landslide monitoring: Application, potential and limitations, ISPRS Journal of Photogrammetry and Remote Sensing, vol. 70, pp. 39-55.
[2] Gruen A., Akca D. (2005) Least squares 3D surface and curve matching. ISPRS Journal of Photogrammetry and Remote Sensing, vol. 59, pp. 151-174.
[3] Lu J., Guo C., Fang Y., et al. (2017) Fast point cloud registration algorithm using multiscale angle features. Journal of Electronic Imaging, vol. 26, pp. 219-231.
[4] Chuang T., Jaw J. (2015) Automated 3d feature matching. The Photogrammetric Record, vol. 30, pp. 8-29.
[5] Mavridis P., Andreidis A., Papaioannou G. (2015) "Efficient Sparse ICP," Computer Aided Geometric Design, vol. 35-36, pp. 16-26.
[6] Rabbani T., Dijkman S., Heuvel F., et al. (2007) An integrated approach for modeling and global registration of point clouds. ISPRS Journal of Photogrammetry and Remote Sensing, vol. 61, pp. 355-370.