Friedmann equations in braneworld scenarios from emergence of cosmic space

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Recently, it was argued that the spacetime dynamics can be understood by calculating the difference between the degrees of freedom on the boundary and in the bulk in a region of space. In this Letter, we apply this new idea to braneworld scenarios and extract the corresponding Friedmann equations of \((n - 1)\)-dimensional brane embedded in the \((n + 1)\)-dimensional bulk with any spacial curvature. We will also extend our study to the more general Gauss-Bonnet braneworld with curvature correction terms on the brane and in the bulk, and derive the dynamical equation in a nonflat Universe.

I. INTRODUCTION

The emergence properties of gravity has a long history since the original proposal made by Sakharov in 1968 [1]. Recent investigations supports the idea that gravitational field equations in a wide range of theories can be recast as the first law of thermodynamics on the boundary of space [2–6]. Among various proposal on the connection between thermodynamics and gravity, the so called entropic origin of gravity proposed by Verlinde [7], has got a lot of attentions [8–15]. According to Verlinde, gravity can be identified with an entropic force caused by changes in the information associated with the positions of material bodies. Verlinde considers the gravitational field equations as the equations of emergent phenomenon and leaves the spacetime as a background geometric which has already exist.

A new insight to the origin of spacetime dynamics, was recently suggested by Padmanabhan [16] who claimed that the cosmic space is emergent as the cosmic time progresses. Using this new idea, Padmanabhan [16] derived the Friedmann equation of a flat Friedmann-Robertson-Walker (FRW) Universe. Following [16], further investigations have been carried out to extract the Friedmann equations of a FRW Universe in various gravity theories [17–20]. In these investigations ([17–20]), following [16], the authors could only derive the Friedmann equations of a flat FRW Universe and they failed to obtain the dynamical equations describing the evolution of the Universe with any spacial curvature in other gravity theories. Very recently, an interesting modification of Padmanabhan’s proposal, which works in a nonflat Universe, was suggested by Sheykhi [21]. Using this modified proposal one is able to derive the corresponding dynamical equations governing the evolution of the Universe with any spacial curvature not only in Einstein gravity, but also in Gauss-Bonnet and more general Lovelock gravity [21]. See also [22] for some application and extension of [21]. In this paper, we will address the question on the connection between the degrees of freedom and the spacetime dynamics by investigating whether and how the relation can be found in braneworld models.

Let us briefly review the proposal of [21]. According to Padmanabhan in an infinitesimal interval \(dt\) of cosmic time, the increase \(dV\) of the cosmic volume, in a flat Universe, is given by [16]

\[
\frac{dV}{dt} = L_p^2 (N_{\text{sur}} - N_{\text{bulk}}),
\]

where \(L_p\) is the Planck length, \(N_{\text{sur}}\) is the number of degrees of freedom on the boundary and \(N_{\text{bulk}}\) is the number of degrees of freedom in the bulk. Through this paper we set \(k_B = 1 = c = \hbar\) for simplicity. Inspired by (1), an improved extension for \(n \geq 4\)-dimensional Universe with spacial curvature was found as [21]

\[
\beta \frac{dV}{dt} = L_p^{n-2} H \tilde{r}_A (N_{\text{sur}} - N_{\text{bulk}}),
\]

where \(H = \dot{a}/a\) is the Hubble parameter, \(a\) is the scale factor, \(\beta = (n-2)/2(n-3)\) and \(\tilde{r}_A\) is the apparent horizon radius of FRW Universe given by

\[
\tilde{r}_A = \frac{1}{\sqrt{H^2 + \kappa/a^2}}.
\]

Motivated by the area law of the entropy, we assume the number of degrees of freedom on the apparent horizon is

\[
N_{\text{sur}} = \frac{\beta A}{L_p^{n-2}},
\]

where \(A = (n-1) \Omega_{n-1} \tilde{r}_A^{n-2}\) is the area of the apparent horizon with \(\Omega_{n-1}\) is the volume of a unit \((n - 1)\)-sphere. The volume of the \((n - 1)\)-sphere with radius \(\tilde{r}_A\) is \(V = \Omega_{n-1} \tilde{r}_A^{n-1}\). We assume the energy content inside the \(n\)-dimensional bulk is in the form of Komar energy [17]

\[
E_{\text{Komar}} = \frac{(n-3) \rho + (n-1) p}{n-3} V,
\]

where \(\rho\) and \(p\) are the energy density and pressure of the perfect fluid inside the Universe, respectively. Hence according to the equipartition law of energy, the bulk
degrees of freedom is obtained as
\[ N_{\text{bulk}} = \frac{2 |E_{\text{Komar}}|}{T} = -4\pi \Omega_{n-1} \hat{r}_A^2 \frac{(n-3)\rho + (n-1)p}{n-3}, \]  
(6)
where \( T = 1/(2\pi \hat{r}_A) \) is the Hawking temperature associated with the apparent horizon. Substituting Eqs. (4) and (6) in relation (11), we arrive at
\[ H^{-1} \hat{r}_A^2 \hat{r}_A^2 - \hat{r}_A^{-2} = \frac{8\pi L_p^{n-2}}{(n-1)} \times \frac{(n-3)\rho + (n-1)p}{n-2}, \]  
(7)
Multiplying both hand side of by factor \( 2 \dot{a}a \), and using the \( n \)-dimensional continuity equation:
\[ \dot{\rho} + (n-1)H(\rho + p) = 0, \]  
(8)
we obtain
\[ \frac{d}{dt} \left[ a^2 \left( H^2 + \frac{k}{a^2} \right) \right] = \frac{16\pi L_p^{n-2}}{(n-1)(n-2)} \frac{d}{dt}(\rho a^2). \]  
(9)
After integrating and setting the constant of integration equal to zero, we find
\[ H^2 + \frac{k}{a^2} = \frac{16\pi L_p^{n-2}}{(n-1)(n-2)} \rho. \]  
(10)
This is the Friedmann equation of \( n \)-dimensional FRW Universe with any spacial curvature [3].

**II. Emergence of Friedmann Equations in RS II Braneworld**

In the remaining part of paper, we want to extend the study to the braneworld scenarios. Gravity on the brane does not obey Einstein theory, thus the usual area formula for the holographic boundary get modified on the brane [3, 4]. Two well-known scenarios in braneworld are Randall-Sundrum (RS) II [23, 24] and Dvali, Gabadadze, Porrati (DGP) [25, 26] models. In the first scenario an \( (n-1) \)-dimensional brane embedded in an \( (n+1) \)-dimensional AdS bulk. In this case, the extra dimension has a finite size and the localization of gravity on the brane occurs due to the negative cosmological constant in the bulk. In the second scenario which is called DGP model, an \( (n-1) \)-dimensional brane is embedded in a spacetime with an infinite-size extra dimension, with the hope that this picture could shed new light on the standing problem of the cosmological constant as well as on supersymmetry breaking [27]. In the original DGP model the bulk was assumed to be a Minkowskian spacetime with infinite size. In this case the recovery of the usual gravitational laws on the brane is obtained by adding an Einstein-Hilbert term to the action of the brane computed with the brane intrinsic curvature. The so-called warped DGP model corresponds to the case where both the intrinsic curvature term on the brane and the negative cosmological constant in the bulk are taken into account.

In order to apply the proposal (11) to braneworld scenarios, we modify it a little by replacing \( L_p^{n-2} \) with \( G_{n+1} \), namely
\[ \beta \frac{dV}{dt} = G_{n+1} H \hat{r}_A (N_{\text{sur}} - N_{\text{bulk}}). \]  
(11)
First of all, we consider the RS II scenario. The apparent horizon entropy for an \( (n-1) \)-brane embedded in an \( (n+1) \)-dimensional bulk in RS II model is given by [3]
\[ S = \frac{2\Omega_{n-1} \hat{r}_A^{n-1}}{4G_{n+1}} \times 2F_1 \left( \frac{n-1}{2}, \frac{n+1}{2}, \frac{\hat{r}_A^2}{\ell^2} \right), \]  
(12)
where \( 2F_1(a, b, c, z) \) is a hypergeometric function, and \( \ell \) is the bulk AdS radius,
\[ \ell^2 = -\frac{n(n-1)}{16\pi G_{n+1} A_{n+1}}, \quad \Omega_{n-1} = \frac{\pi^{(n-1)/2}}{\Gamma((n+1)/2)}. \]  
(13)
In the above relation, \( A_{n+1} \) represents the \( (n+1) \)-dimensional bulk cosmological constant. The entropy expression (12) can be written in the form [3]
\[ S = \frac{(n-1)\Omega_{n-1}}{2G_{n+1}} \int_0^{\hat{r}_A} \frac{\hat{r}_A^{n-2}}{\sqrt{\hat{r}_A^2 + \ell^2}} d\hat{r}_A, \]  
(14)
and hence we define the effective area as
\[ \tilde{A} = 4G_{n+1} S = 2(n-1)\Omega_{n-1} \int_0^{\hat{r}_A} \frac{\hat{r}_A^{n-2}}{\sqrt{\hat{r}_A^2 + \ell^2}} d\hat{r}_A. \]  
(15)
Now we calculate the increasing in the effective volume as
\[ \frac{dV}{dt} = \frac{\hat{r}_A}{(n-2)} \frac{d\tilde{A}}{dt} = \frac{2\Omega_{n-1} (n-1)}{(n-2)G_{n+1}} \hat{r}_A^{n-1} \frac{\hat{r}_A}{\sqrt{\hat{r}_A^2 + \ell^2}} \]  
(16)
\[ = -2\Omega_{n-1} (n-1) \int_0^{\hat{r}_A} \frac{d}{dt} \left( \frac{\hat{r}_A^2 - 1}{\ell^2} \right) \]  
(17)
Motivated by (17), we assume the number of degrees of freedom on the boundary is given by
\[ N_{\text{sur}} = \frac{4\beta(n-1)\Omega_{n-1}}{(n-2)G_{n+1}} \hat{r}_A \frac{\hat{r}_A^{n-1}}{\sqrt{\hat{r}_A^2 + \ell^2}} \]  
(18)
Inserting Eqs. (9), (17) and (18) in relation (11), after multiplying both hand side by factor \( \dot{a}a \), we get
\[ \hat{r}_A^{-3} \frac{\dot{a}a^2 + 2\dot{a}a}{\sqrt{\hat{r}_A^2 + \ell^2}} = -4\pi G_{n+1} \dot{a} a \frac{(n-3)\rho + (n-1)p}{n-1}. \]  
(19)
Using the continuity equation \[8\], after some simplification, we arrive at
\[
\frac{d}{dt} \left( a^2 \sqrt{\hat{r}_A^{-2} + \frac{1}{\ell^2}} \right) = \frac{4\pi G_{n+1}}{(n-1)} \frac{d}{dt} \left( \rho a^2 \right) .
\]
(20)

Integrating and dividing by \(a^2\), we find
\[
\sqrt{\hat{r}_A^{-2} + \frac{1}{\ell^2}} = \frac{4\pi G_{n+1}}{(n-1)} \rho ,
\]
where we assumed the integration constant to be zero. Substituting the apparent horizon radius from relation \[3\], we get
\[
\sqrt{H^2 + \frac{k}{a^2} + \frac{1}{\ell^2}} = \frac{4\pi G_{n+1}}{(n-1)} \rho .
\]
(22)

In this way we derive the Friedmann equation of higher dimensional FRW Universe in RS II braneworld by calculating the difference between the number of degrees of freedom on the boundary and in the bulk. This coincides with the result obtained in \[3\] from the field equations.

III. FRIEDMANN EQUATIONS IN WARPED DGP BRANEWORLD

Next we consider an \((n-1)\)-dimensional warped DGP brane embedded in an \((n+1)\)-dimensional AdS bulk. The entropy associated with the apparent horizon is given by \[3\]
\[
S = (n-1)\Omega_{n-1} \hat{r}_A^{-2} + \frac{2\Omega_{n-1} \hat{r}_A^{-2}}{G_{n+1}} \times F_1 \left( \frac{n-1}{2}, \frac{1}{2}, \frac{n+1}{2}, -\frac{\hat{r}_A^{-2}}{\ell^2} \right) .
\]
(23)

It is important to note that in DGP braneworld, the entropy expression of the apparent horizon consists two terms. The first term which satisfies the area formula on the brane is the contribution from the Einstein-Hilbert term on the brane. The second term is the same as the entropy of RS II braneworld and therefore obeys the \((n+1)\)-dimensional area law in the bulk \[3\].

One can write the entropy associated with the apparent horizon on the brane as \[3\]
\[
S = (n-1)\Omega_{n-1} \int_0^{\hat{r}_A} \left( \frac{(n-2)\hat{r}_A^{-3}}{4G_n} + \frac{\ell}{2G_{n+1} \sqrt{\hat{r}_A^{-2} + \ell^2}} \right) \hat{r}_A d\hat{r}_A .
\]
(24)

We define the effective surface as
\[
\tilde{A} = 4G_{n+1} S = 4G_{n+1} (n-1)\Omega_{n-1} \times \int_0^{\hat{r}_A} \left( \frac{(n-2)\hat{r}_A^{-3}}{4G_n} + \frac{\ell}{2G_{n+1} \sqrt{\hat{r}_A^{-2} + \ell^2}} \right) d\hat{r}_A .
\]
(25)

We also obtain the rate of increase in the effective volume as
\[
\frac{d\tilde{V}}{dt} = \frac{\tilde{r}_A}{(n-2)} \frac{d\tilde{A}}{dt} = \Omega_{n-1} (n-1) \tilde{r}_A \hat{r}_A^{-2} \times \left( \frac{(n-2)G_{n+1}}{G_n} + \frac{2}{\sqrt{\tilde{r}_A^{-2} + \ell^2}} \right) \]
\[
= -2\Omega_{n-1} (n-1) \hat{r}_A^{-2} \times \frac{d}{dt} \left( \frac{(n-2)G_{n+1} \hat{r}_A^{-2}}{4G_n} + \sqrt{\hat{r}_A^{-2} + \frac{1}{\ell^2}} \right) .
\]
(26)

Inspired by \[20\], we suppose the number of degrees of freedom on the apparent horizon in warped DGP model is given by
\[
N_{sur} = \frac{2\Omega_{n-1} (n-1) \hat{r}_A^{-2}}{G_{n+1} (n-3)} \left( \frac{G_{n+1} (n-2)\hat{r}_A^{-2}}{4G_n} + \sqrt{\hat{r}_A^{-2} + \frac{1}{\ell^2}} \right) .
\]

Combining Eqs. \[6\], \[20\] and \[27\] with relation \[11\], it is a matter of calculation to find
\[
\frac{d}{dt} \left( a^2 \sqrt{\hat{r}_A^{-2} + \frac{1}{\ell^2}} \right) = \frac{-G_{n+1}}{4G_n} (n-2) \frac{d}{dt} \left( \hat{r}_A^{-2} a^2 \right) + \frac{4\pi G_{n+1}}{(n-1)} \frac{d}{dt} \left( \rho a^2 \right) .
\]
(28)

Integrating and dividing by \(a^2\) we obtain
\[
\sqrt{\hat{r}_A^{-2} + \frac{1}{\ell^2}} = \frac{G_{n+1}}{4G_n} (n-2)\hat{r}_A^{-2} + \frac{4\pi G_{n+1}}{(n-1)} \rho .
\]
(29)

Substituting the apparent horizon radius from relation \[3\], we have
\[
\sqrt{H^2 + \frac{k}{a^2} + \frac{1}{\ell^2}} = \frac{G_{n+1}}{4G_n} (n-2) \left( H^2 + \frac{k}{a^2} \right)
\]
\[= \frac{4\pi G_{n+1}}{(n-1)} \rho .
\]
(30)

This equation is indeed the Friedmann equation of FRW Universe in warped DGP braneworld derived in \[3\] from the field equations. If we define, as usual, the crossover length scale between the small and large distances in DGP braneworld as \[27\]
\[
r_c = \frac{G_{n+1}}{2G_n} ,
\]
then one can easily check that for \(r_c \to \infty\), the standard Friedmann equation in \(n\)-dimensional FRW Universe presented in \[10\] is recovered. On the other hand, when \(r_c \to 0\), Eq. \[30\] reduces to the Friedmann equation in RS II braneworld obtained in the previous section.
IV. EMERGENCE OF SPACETIME DYNAMICS IN GAUSS-BONNET BRANeworld

Finally, we apply the method developed in the previous sections to investigate the emergence properties of the spacetime dynamics in general braneworld with curvature correction terms including a 4D scalar curvature from induced gravity on the brane, and a 5D Gauss-Bonnet curvature term in the bulk. With these correction terms, especially including a Gauss-Bonnet correction to the 5D action, we have the most general action with second-order field equations in 5D [28], which provides the most general models for the braneworld scenarios. The entropy of apparent horizon in general Gauss-Bonnet braneworld embedded in a 5D bulk, can be written as [4]

\[
S = \frac{3\Omega_3 \rho^3}{4G_4} + \frac{2\Omega_3 \rho^3}{4G_5} \times 2F_1\left(\frac{3}{2}, \frac{5}{2}, \frac{5}{2}, \Phi_0 \rho^2\right)
+ \frac{6\alpha \Omega_3 \rho^3}{G_5} \left(\Phi_0 \times 2F_1\left(\frac{3}{2}, \frac{5}{2}, \frac{5}{2}, \Phi_0 \rho^2\right)
+ \sqrt{1 - \Phi_0 \rho^2}\right),
\]

where \(\Phi_0 = \frac{1}{4\pi} \left(-1 + \sqrt{1 - \frac{8\rho}{\ell^2}}\right)\) = constant [4], and \(\alpha\) is the Gauss-Bonnet coefficient with dimension (length)^2. When \(\alpha \to 0\) we have \(\Phi_0 = -\ell^{-2}\) and the above expression reduces to the entropy of warped DGP braneworld presented in [23] for \(n = 4\). Expression (32) can be written as [4]

\[
S = \frac{3\Omega_4}{2G_4} \int_0^{\tilde{r}_A} \tilde{r}_A d\tilde{r}_A + \frac{3\Omega_3}{2G_5} \int_0^{\tilde{r}_A} \sqrt{\tilde{r}_A - \Phi_0 \tilde{r}_A} d\tilde{r}_A
+ \frac{6\alpha \Omega_3}{G_5} \int_0^{\tilde{r}_A} \frac{2 - \Phi_0 \tilde{r}_A}{\sqrt{1 - \Phi_0 \tilde{r}_A}} d\tilde{r}_A.
\]

We define the effective area of the apparent horizon corresponding to the above entropy as

\[
\tilde{A} = 4G_5 S = \frac{6G_5 \Omega_3}{G_4} \int_0^{\tilde{r}_A} \tilde{r}_A d\tilde{r}_A + 6\Omega_3 \int_0^{\tilde{r}_A} \sqrt{\tilde{r}_A - \Phi_0 \tilde{r}_A} d\tilde{r}_A
+ 24\alpha \Omega_3 \int_0^{\tilde{r}_A} \frac{2 \tilde{r}_A - \Phi_0 \tilde{r}_A}{\sqrt{\tilde{r}_A - \Phi_0 \tilde{r}_A}} d\tilde{r}_A,
\]

and therefore the increase of the effective volume is obtained as

\[
\frac{dV}{dt} = \frac{\tilde{r}_A d\tilde{A}}{2} = \frac{3G_5 \Omega_3}{G_4} \tilde{r}_A^2 + 3\Omega_3 \tilde{r}_A \sqrt{\tilde{r}_A^2 - \Phi_0 \tilde{r}_A}
+ 12\alpha \Omega_3 \tilde{r}_A \sqrt{\tilde{r}_A^2 - \Phi_0 \tilde{r}_A}.
\]

Motivated by [33], we write the number of degrees of freedom on the boundary in general Gauss-Bonnet braneworld as

\[
N_{sur} = \frac{3\Omega_3 \tilde{r}_A^2}{G_4} + \frac{6\Omega_3 \tilde{r}_A^4}{G_5} \sqrt{\tilde{r}_A^2 - \Phi_0 \tilde{r}_A}
+ \frac{16\alpha \Omega_3}{G_5} \tilde{r}_A \left(\frac{\tilde{r}_A^2}{2} + \Phi_0 \tilde{r}_A \right) \sqrt{\tilde{r}_A^2 - \Phi_0}.
\]

Substituting Eqs. (6), (35) and (36) into (11) and setting \(n = 4\), after some mathematic simplification, one obtains

\[
\frac{d}{dt} \left\{ \frac{3G_5}{4} \frac{d}{dt} (a^2 \tilde{r}_A^2) + 6 \frac{d}{dt} (a^2 \sqrt{\tilde{r}_A^2 - \Phi_0 \tilde{r}_A})
+ \frac{d}{dt} \left\{ 16\alpha a^2 \left(\tilde{r}_A^2 - \Phi_0 \tilde{r}_A \right) \sqrt{\tilde{r}_A^2 - \Phi_0} \right\} \right\} = 8\pi G_5 \frac{d}{dt} (\rho a^2).
\]

Integrating, dividing by \(a^2\) and then using the definition [33], we find

\[
\left[ 1 + \frac{8}{3} \alpha \left( H^2 + \frac{k}{a^2} - \frac{1}{2\ell^2} \right) \right] \sqrt{H^2 + \frac{k}{a^2} + \frac{1}{\ell^2}} = \frac{4\pi G_5}{3} \rho - \frac{G_5}{2G_4} \left( H^2 + \frac{k}{a^2} \right).
\]

This is the Friedmann equation governing the evolution of the Universe in general Gauss-Bonnet braneworld with curvature correction terms on the brane and in the bulk. This result is exactly the same as one obtains from the field equation of Gauss-Bonnet braneworld [29]. Here we arrived at the same result by using the novel proposal of [21]. When \(\alpha = 0\), the above result reduces to the Friedmann equation of warped DGP model obtained in Eq. (30) for \(n = 4\).

V. SUMMERY AND DISCUSSION

Recently, Padmanabhan [16] argued that the spacetime dynamics can be considered as an emergent phenomenon and the cosmic space is emergent as the cosmic time progresses. An improved version of Padmanabhan proposal which is applicable to a nonflat Universe was found by one of the present author [21]. In this paper, we extended the study to other gravity theory such as braneworld scenarios. Gravity on the brane does not obey the Einstein theory and therefore the usual area formula for the entropy does not hold on the brane. We have discussed several cases including whether there is or not a Gauss-Bonnet curvature correction term in the bulk and whether there is or not an intrinsic curvature term on the brane. We found that one can always derive the Friedmann equations of FRW Universe with any spacial curvature, by calculating the difference between the horizon degrees of freedom and the bulk degrees of freedom regardless of the existence of the intrinsic curvature term on the brane and the Gauss-Bonnet correction term in the bulk.
The result obtained here in RS II, warped DGP and the general Gauss-Bonnet braneworld scenarios further supports the novel idea of Padmanabhan [11] and its extension as [1], and show that this approach is powerful enough to extract the dynamical equations describing the evolution of the Universe in other gravity theories with any spacial curvature.

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