Probing braneworld hypothesis with a \textit{neutron-shining-through-a-wall} experiment

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The possibility for our visible world to be a 3-brane embedded in a multidimensional bulk is at the heart of many theoretical edifices in high energy physics. Probing the braneworld hypothesis is then a major experimental challenge. Following recent theoretical works showing that matter swapping can occur between braneworlds, we propose a \textit{neutron-shining-through-a-wall} experiment.

As suggested by several extensions of the Standard Models of particle physics and cosmology, our world could form a three-dimensional space sheet (a brane) embedded in a larger bulk with more dimensions \cite{11,12}. As for most theories beyond the Standard Model of particle physics, one can search for new effects predicted by the braneworld hypothesis at both the high energy frontier and the precision frontier. Particle colliders such as the Large Hadron Collider attempt at exciting new degrees of freedom at high energies \cite{4,5} - like Kaluza-Klein excitation of particles in the extra dimensions - whereas precision experiments at low energies attempt at finding tiny signals induced by the new physics \cite{6,16}. For instance, in the context of braneworld scenarios, the compactified extra dimensions would modify the inverse square law of gravity at short distance. A great variety of experimental techniques are deployed to search for a possible modification of gravity from subatomic to macroscopic distance scales (see \cite{6} for a recent review).

In the present work, we are interested by some peculiar low energy effects induced by the existence of other branes in the bulk. In recent theoretical works \cite{11,12}, it has been argued that usual matter could leap from our braneworld toward a hidden one, and vice versa (see Fig. 1). This matter swapping between two neighboring braneworlds could be triggered by magnetic vector potentials, either generated in the laboratory or of astrophysical origin. In particular, we have considered neutrons oscillating between a state where the neutron sits in our brane and a state corresponding to a neutron located in the other brane. The matter swapping probability would oscillate at high frequency $\eta$ and small amplitude $p$. Moreover, neutrons are a well-known versatile tool already used to test other concepts such as axion\cite{17} or mirror-particles\cite{18,21}.

In a previous work, we have shown that such an oscillation could have affected experiments measuring the neutron lifetime. The existing experiments set a constraint on an anomalous neutron disappearance at the level of $p < 7 \times 10^{-6}$\cite{16}. It is amusing to note that a small tension has appeared very recently between the neutron lifetime measured with two different methods\cite{22}, that could be interpreted as an anomalous neutron disappearance. In addition, such a constraint on neutron disappearance could be also useful to constrain some approaches related to the big bang nucleosynthesis\cite{23}.

In the present paper, we propose an experiment to probe matter swapping between branes by looking at the appearance of neutrons from the neighboring brane. The concept is similar to light-shining-through-a-wall experiments (see \cite{6} for a recent review of this topic) where photons from an intense light source would convert into a sterile state (a dark photon or an axion-like particle), the sterile state could pass through a wall, then would convert back to photons. In a \textit{neutron-shining-through-a-wall} experiment, one needs a very bright source of neutrons and a low-background neutron detector separated by a wall. Neutrons would swap into a sterile state – the state where the neutron is located in another brane – the sterile state would be free to cross the wall. Then, one looks at the re-appearance of the neutron into the detector situated behind the wall. Although very similar in principle, there is, however, one essential difference between light and neutron shining through walls. In the former case, the oscillation frequency is supposed to be rather slow and the conversion of photon into a sterile.
In section II, we review the theoretical aspects and the phenomenology of the neutron dynamics in a two brane universe at low energy. The conditions leading to matter swapping between branes are given. We discuss the strength of the ambient magnetic vector potential, which drives the matter exchange between branes. The environmental conditions that could preclude the swapping to occur are also discussed. In section III, we describe how the neutron diffusion in the reactor moderator creates a neutron flux in another brane and how this hidden neutron flux can be detected. Finally, the sensitivity of the suggested experiment is estimated, and as a proof of concept, a constraint is derived from previous experiments.

II. PHENOMENOLOGY OF BRANE MATTER SWAPPING

A. Dynamics of swapping

\[ i\hbar \frac{\partial}{\partial t} \begin{pmatrix} \psi_+ \\ \psi_- \end{pmatrix} = (H_0 + H_{\text{cm}}) \begin{pmatrix} \psi_+ \\ \psi_- \end{pmatrix} \]  

(1)

where the indices ± allow to discriminate the two branes. \( \psi_+ \) and \( \psi_- \) are usual Pauli spinors corresponding to the wave functions in the (+) and (−) branes respectively. The unperturbed Hamiltonian \( H_0 \) is diagonal and describes the usual non-relativistic evolution of two uncoupled spin-1/2 particles in the magnetic fields \( B_\pm \) and the gravitational potentials \( V_\pm \), in each brane:

\[ H_0 = \begin{pmatrix} H_+ & 0 \\ 0 & H_- \end{pmatrix}, \]  

(2)

\[ H_\pm = -\frac{\hbar^2}{2m} \Delta + \mu_n \sigma \cdot B_\pm + V_\pm. \]  

(3)

The non-diagonal part \( H_{\text{cm}} \) of the Hamiltonian describes the coupling between the branes:

\[ H_{\text{cm}} = g\mu_n \begin{pmatrix} 0 & -i\sigma \cdot \{A_+ - A_-\} \\ i\sigma \cdot \{A_+ - A_-\} & 0 \end{pmatrix} \]  

(4)

such that \( A_+ \) and \( A_- \) correspond to the magnetic vector potentials in the branes (+) and (−) respectively. The same convention is applied to the magnetic fields \( B_\pm \) and to the gravitational potentials \( V_\pm \). \( \mu_n \) is the magnetic moment of the neutron. \( H_{\text{cm}} \) implies that matter exchange between branes depends on the magnetic moment and on the difference between the local (i.e. on a brane) values of the magnetic vector potentials. \( g \) is the coupling strength between the matter fields of each brane.

In the following, we will consider the case of an ambient magnetic potential with an astrophysical origin (section II B). Let \( A_{\text{amb}} = A_{\text{amb},+} - A_{\text{amb},-} \) be the difference of ambient magnetic potentials of each brane. Assuming that \( \mu_n B_\pm \ll V_\pm \), i.e. one can neglect the magnetic fields in the branes (in particular one assumes that \( \nabla \times A_{\text{amb}} = 0 \)), then by solving the Pauli equation, it can be shown that the probability for a neutron initially localized in our brane to be found in the other brane is \([11],[13] \):

\[ P = \frac{4\Omega^2}{\eta^2 + 4\Omega^2} \sin^2 \left( (1/2)\sqrt{\eta^2 + 4\Omega^2 t} \right) \]  

(5)

where \( \eta = |V_+ - V_-|/\hbar \) and \( \Omega = g\mu_n A_{\text{amb}}/\hbar \). \( P \) is the instantaneous probability for matter swapping between branes. Eq. (5) shows that the neutron in the potential \( A_{\text{amb}} \) undergoes Rabi-like oscillations between the branes. Note that the swapping probability is independent of the neutron spin direction \([13] \). As detailed in previous papers \([13],[15] \), the environmental interactions (related to \( V_\pm \)) are usually strong enough and the oscillations are suppressed by the factor \( \Omega/\eta \ll 1 \). As a consequence, in a nucleus, neutron is fully frozen in its brane due to the large nuclear potential.
B. Ambient magnetic vector potential

Overall ambient astrophysical magnetic vector potential $A_{amb}$ was previously assessed in literature [24]. $A_{amb}$ is the sum of each magnetic vector contributions related to the magnetic fields of astrophysical objects (planets, stars, galaxies, etc.) since $B(r) = \nabla \times A(r)$. As a rule of thumb, $A \approx DB$ where $D$ is the distance from the astrophysical source and $B$ is the typical field induced by the object. At large distances from sources, close to Earth for instance, $A_{amb}$ is almost uniform (i.e. $\nabla \times A_{amb} \approx 0$) and cannot be canceled with magnetic shields [24]. Now, Eqs. (1) and (5) shows the dependence of the swapping effect against $A_{amb} = A_{amb,+} - A_{amb,-}$, i.e. the difference between the vector potentials of the two brane worlds. Since $A_{amb,-}$ depends on unknown sources in the hidden brane, we cannot assess its value. Then, $A_{amb}$ should be considered as an unknown parameter of the model. Nevertheless, the expected order of magnitude of $A_{amb}$ can be roughly constrained by $A_{amb,+}$ in our visible world (since $A_{amb}$ results from a vectorial difference, it seems quite unlikely that $A_{amb}$ can fortuitously fall to zero). Considering the galactic magnetic field variations on local scales towards the Milky Way core $A_{amb} \approx 2 \times 10^{9}$ T m is usually assumed [24]. By contrast, Earth’s magnetic field leads to 200 T m while the Sun contributes for 10 T m [23]. By contrast, intergalactic contributions were expected to be about $10^{12}$ T m [24] (see also our previous papers for more detailed discussion [14–16]). Anyway, for now, $A_{amb}$ is a parameter fairly bounded between $10^{9}$ T m and $10^{12}$ T m.

C. Environmental potential

If one considers free neutrons shielded from magnetic fields, only gravitational contributions are relevant. Because $\eta = |V_+ - V_-|/\hbar$, it is difficult to assess the value of $\eta \hbar$ as it results from a scalar difference involving the unknown gravitational contribution $V_-$ of the hidden world. Therefore, $\eta$ appears as an effective unknown parameter of the model and could reach weak values of about a few eV up to large values around one keV. Indeed, estimations given in previous works [14–16] suggest that $V_+$ could be of the order of 500 eV due to the Milky Way core gravitational influence on neutrons. Note that, the Sun, the Earth and the Moon provide lower contributions of about 9 eV, 0.65 eV and 0.1 meV respectively. At last, it must be emphasized that $\eta$ is time-dependent due the motion of Earth around the Sun at the lab scale. Between the Earth’s aphelion and perihelion, the gravitational energy (due to the Sun) of a neutron varies from 9.12 eV to 9.43 eV. Of course, a time dependence could have many different origins. For instance, the particle motion relatively to an unknown mass distribution in the hidden brane should be considered. However, it is unlikely that the Earth is “close” enough to a hidden mass distribution large enough to induce a significant energy time dependence on a timescale about one year or one day. The time dependence is then mainly induced by the Earth’s motion around the Sun, with a relative variation of $\eta$ about 2 % on one year. Such a large variation could be detected through an annual modulation of the swapping probability.

D. Neutrons as a sensitive probe

With high energy particle colliders, one could probe the braneworld hypothesis if the brane thickness is in the range $\xi \approx 10^{-15}$ m corresponding to the TeV scale ($hc/\xi \approx 1$ TeV). Colliders are blind to the Planck scale $10^{-35}$ m. By contrast, experiments at lower energies using high intensity neutron sources could reveal a multi-brane world through effects induced by the interbrane coupling $g$, which can be approximated by [11]:

$$g \propto (1/\xi) \exp (-kd/\xi)$$  \hspace{1cm} (6)

with $d$ the real distance between each brane in the bulk and where $k$ is a constant of the model [11]. Now, as an illustration, let us consider a coupling constant $g \approx 10^{-3}$ m$^{-1}$ for instance. Such a value is consistent with present experimental bounds [10]. For branes at TeV scale, the above value of $g$ is reached for $d \approx 50\xi$. Now, if one considers branes at Planck scale, the coupling constant remains unchanged for $d \approx 87\xi$. As a consequence, while brane physics could be invisible at colliders, it could be observed in low-energy experiments using neutrons if a second brane exists close enough to ours.

At last we note that neutrons are more suitable for such a purpose than electrons, protons or atoms [14]. Indeed, for a charged particle, the Hamiltonian (3) also contains the usual terms:

$$H_{p,\pm} = -q/mA_\pm \cdot \hat{P} + q^2/2m|A_\pm|^2$$  \hspace{1cm} (7)

with $\hat{P} = -i\hbar\nabla$. This implies that term $\hbar\eta$ in Eq. (3) is then supplemented by large terms proportional to $qA_{amb}$ which strongly freeze the oscillations. Considering, for instance, $A_{amb} \approx 200$ T m (see section [11B] and a proton with a kinetic energy of about 10 eV, the first term in the Hamiltonian (7) contributes 9 MeV to $\hbar\eta$. For such values, the amplitude of the oscillations is suppressed by eight orders of magnitude compared to the neutron case. Now, let us consider atoms. Though they are neutral, atoms are endowed with an instantaneous electric dipole moment (IEDM) $d$. Obviously, according to the time average, we must verify $\langle d \rangle = 0$ and $\langle d^2 \rangle \neq 0$ since IEDM results from quantum fluctuations of atomic orbitals. The London dispersion forces between atoms result from interactions between those instantaneous dipoles. Then, the Hamiltonian (3) must be
supplemented by terms which derive from (7):

$$H_{d,±} \sim -A_± \frac{Δd}{Δt}$$

(8)

where the fluctuation time is about $Δt \sim h/E_i$, with $E_i$ the ionization energy of the atom ($E_i \approx 10$ eV). Provided that $Δt$ is larger than the period $T$ of the Rabi oscillation of an atom between two branes, $h\eta$ in Eq. (3) is then supplemented by a huge term proportional to $(1/Δt)A_{amb}d$. Then, for $d \approx 10^{-30}$ C·m (0.3 D) (a typical value for atoms), and $A_{amb} \approx 200$ T·m only for instance, we still get $h\eta \approx 20$ MeV. With these values, the oscillations are strongly damped by nine orders of magnitude compared to the neutron case. As a result, neutron is a good candidate to test matter swapping between branes, since it is devoid of global charge or any electric dipole moment.

E. Collision-induced neutron swapping probability

As a consequence of the environmental potential $η\hbar$, the neutron oscillations present a weak amplitude and a high angular frequency of the order $η/2$. Due to the fast oscillating behavior, one can approximate the swapping probability by its time averaged value $p = ⟨P⟩$ (see Eq. (5)) such that:

$$p = \frac{2Ω^2}{η^2}.$$  

(9)

When freely propagating, the neutron can be described as a superposition of two states: neutron in our brane vs. neutron in the other brane. When colliding with a nucleus situated in our brane, the interaction acts as a measurement and the neutron collapses either in our brane with a probability $1 - p$ or in the other invisible brane with a probability $p$. In the following, the swapping probability $p$ is considered as the relevant measurable parameter. By contrast, bounds on the coupling parameter $g$ depend on the knowledge of galactic magnetic potential fields and on the ambient gravitational fields.

III. A NEUTRON-SHINING-THROUGH-A-WALL EXPERIMENT

As explained above, in a two-brane universe, neutrons have a non-zero probability to escape from our brane into another brane at each collision. Therefore, a nuclear reactor, where the neutron density is very high, would be a very intense source of hidden neutrons. These neutrons could escape the reactor and be detected, after they swapped back in our brane, with a standard neutron detector located near the reactor (see Fig. 2). In the present section, we discuss the expected magnitude of the hidden neutron flux as a function of the swapping probability and we estimate the sensitivity that can be reached with such an experiment. For the sake of clarity, the details of our calculations appear only in the appendices of the present paper.

A. Induced neutron flux in the hidden world

Let us assume our Universe as made of two braneworlds mutually invisible and let us also consider a neutron flux $Φ_+$ inside a nuclear core. Our aim is to determine the intensity of the hidden neutron flux $Φ_-$ (in the vacuum in the hidden braneworld) in the vicinity of the core. For a given volume element of the reactor, since each neutron collision creates an hidden neutron with probability $p$, the hidden neutron source is proportional to the macroscopic elastic cross section $Σ_E$ of the reactor moderator and to the neutron flux $Φ_+$. More specifically, we get the source term corresponding to the number of generated hidden neutrons per unit volume and unit time (see Appendix A):

$$S_− = \frac{1}{2}ρΣ_EΦ_+$$

(10)

with $p$ given by (9). Equation (10) is derived by using the matrix density approach related to the equation (11) as shown in Appendix A. From this source term, we deduce the hidden neutron flux $Φ_-$ at the position $r$ by considering the solid angle and integrating over the reactor volume $V$:

$$Φ_−(r) = \frac{p}{8π} \int_V \frac{1}{|r - r'|^2} Σ_E(r')Φ_+(r')d^3r'.$$  

(11)

Relation (11) shows that any signal should decay as $1/D^2$ when distance $D$ between reactor and detector increases. This is an important issue to discriminate a hidden neutron signal from the common neutron background in the vicinity of the reactor. Indeed, such a background mainly depends on local secondary sources.
B. Principle of hidden neutron flux detection

We want now to design a detector able to measure the hidden neutron flux. Neutron detectors are based on detection of charged particles emitted after a neutron absorption. In the present case, we suggest the use of a gaseous detector, such as usual helium-3 or boron trifluoride neutron (BF₃) detectors. The mechanism of hidden neutron capture can be described through the approach of Feinberg and Weinberg [25] also used by Demidov, Gorbunov and Tokareva [26] to describe the positronium oscillations in a volume full of gas, in the mirror matter concept. The details of calculation are shown in Appendix B. We can then compute the event rate \( \Gamma \) detected by the detector in our brane against the swapping probability \( p \) (see Appendix B). For monochromatic neutrons, we get the intuitive but not obvious result:

\[
\Gamma = \frac{1}{2} p \Sigma_A \Phi_n V
\]

with \( V \) the volume of the detector, and \( \Sigma_A \) the macroscopic absorption cross-section related to BF₃ or He-3. For a continuous energy spectrum, the event rate is obtained by integrating Eq. (12) over the spectrum. The main difficulty comes from the background which is usually high in such environment. The detector must be shielded from neutrons.

C. Proposal at ILL reactor

For our experiment, we propose to use the reactor at the Institute Laue Langevin (ILL) (see Fig. 2) where the core (diameter of 40 cm and height of 80 cm) is surrounded by a heavy water tank (diameter of 2.5 m and height of 2 m) which will be our hidden neutron source. It can be shown that the major contribution to the hidden neutron flux come from the heavy water tank, while neutrons are strongly absorbed by light water and other surrounding materials. The thermal neutron flux inside the heavy water tank is modeled by a using a point-like source \( \Phi_n(r) = S \exp(-r/L)/(4\pi Dr) \), with \( L = 116 \) cm and \( D = 0.57 \) cm the length and the coefficient of diffusivity respectively, for heavy water. \( S \) is fitted to match with the neutron flux in the heavy water tank [27] with \( S = 6.0 \times 10^{17} \) neutrons/s. For heavy water, the macroscopic elastic cross section is \( \Sigma_E = 0.4 \) cm⁻¹. The present rough model is reliable enough to discuss the experimental concept. Indeed, it is sufficient to assess the correct magnitude of the induced hidden neutron flux. Of course, any discussion of later experimental results will require a detailed computation of the neutron flux in each location in the reactor, as well as to consider the various materials surrounding the core.

We plan to use a cylindrical helium-3 detector (see Fig. 2) with a volume of 36 cm³ with a gas pressure of 4 atm. This detector could be located at 10 meters from the center of the core. The detector will be shielded by a 4π-box made of borated rubber (40% boron content) of 3 cm-thick at least, to capture thermal neutrons, and supplemented by a polyethylene cover with a tunable thickness to moderate epithermal and fast neutrons. Indeed, hidden neutrons should provide a characteristic constant counting rate which does not depend on the shielding thickness.

Using (11) and (12), estimated rates are shown in Fig. 3 (black solid line). Considering the previous constraint \( p < 7 \times 10^{-6} \) [16] (vertical blue solid line) we see that the expected event rate could be high as 36 kHz, which is easily detectable even without shielding. Actually, we think that we can reach an upper rate about 10 mHz or even 1 mHz [28–30] for convenient shielding and detector (horizontal red solid line). We can then expect to reach a new upper constraint for the swapping probability \( p \) about \( 10^{-9} \). Such a constraint would be better than the constraint \( p < 7 \times 10^{-6} \) [16] by at least three orders of magnitude.

![FIG. 3. (Color online). Black solid line: Expected event rate \( \Gamma \) against the swapping probability \( p \). Horizontal red solid line: Expected threshold of the background noise (1 mHz). Vertical blue solid line: Known upper limit of the constraint on the swapping probability \( p < 7 \times 10^{-6} \). Black dashed line: Expected event rate \( \Gamma \) against the swapping probability \( p \) in the experiment of Ref. [28]. Horizontal red dashed line: Background noise (0.3 mHz) in the experiment of Ref. [28]. Vertical blue dashed line: Threshold of the rough constraint on the swapping probability \( p < 5 \times 10^{-8} \) considering the experiment of Ref. [28].]

In Fig. 4, as an example, we show the resulting expected bound on the coupling constant \( g \) between our brane and an invisible one. The values of \( g \) are given against the gravitational constraint \( \eta \) from relation (9). The existing constraint is given [16] (shaded domain above the short-dashed blue line) and is compared with the expected results of the neutron-shining-through-a-wall experiment (black solid line) by assuming \( A_{amb} \approx 2 \times 10^9 \) Tm [16]. For the best rate constraint, we improve the constraint on \( g \) by two orders of magnitude. By contrast, a constraint on the detected signal lower than 10 events per second (see Fig. 3) still allows...
$p < 10^{-7}$, i.e. we improve the constraint on $g$ by one order of magnitude. As a consequence, the present experiment can easily improve the constraint on the coupling between our world and invisible ones.

![Graph](image_url)

**FIG. 4.** (Color online). Expected experimental limits for the coupling constant $g$ against the confinement energy for some constraints on the swapping probability $p$ for $A_{amb} = 2 \times 10^9$ T m. Red-grey domains are excluded. $p = 7 \times 10^{-6}$ corresponds to our previous best experimental constraint. $p = 5 \times 10^{-9}$ corresponds to the expected constraint deduced from the experiment of Ref. [28].

**D. Yearly time-dependent drift of the swapping probability**

Let us now briefly underline the possible consequence of the time evolution of the potential $\eta h$ due to Earth’s revolution around Sun, as described in section II C. From Eq. (9) one sees that $p$ varies in time as $\Delta \rho / \rho = 2 \Delta \eta / \eta$. Now, from Eqs. (11) and (12) one shows that $\Gamma$ varies as $\Delta \Gamma / \Gamma = 2 \Delta \eta / \eta$. Then, we deduce that the event rate $\Gamma$ varies in time as $\Delta \Gamma / \Gamma = 4 \Delta \eta / \eta$, i.e. $\Delta \Gamma / \Gamma \approx 10^{-3}$ on six months if we consider values from section III C. If one expects to detect a time-dependent drift of the swapping probability, for a given duration $\Delta t$ of the experiment, the number of detected neutrons $N = \Gamma \Delta t$ and its instrumental uncertainty $\Delta N \sim \sqrt{N}$ imply that one must detect enough neutrons to allow $\Delta N / N$ to be lower than $4 \Delta \eta / \eta$. In the present case, for month-by-month measurements, we should have $N > 10^6$ detected neutrons by month at least. Assuming $\Delta t \sim 20$ days by month, this leads to $\Gamma \approx 1$ Hz. As a consequence, considering the detector above mentioned, a time-dependent drift of the swapping probability $p$ can be measured provided that $p > 3.7 \times 10^{-8}$.

**E. A constraint as a proof of concept**

It can be fairly noticed that a constraint can be suggested from previous experiments [28, 30]. Indeed, in a context different from the present one, these experiments required low-noise neutron detectors to detect ultracold neutrons generated by convenient sources. But since these detectors were in the neighborhood of the nuclear reactor, they should have been sensitive to hidden neutrons also. As a consequence, considering the recorded background of these experiments, we can already derive a first rough constraint on the swapping probability, and thus on the coupling constant between two adjacent branes. We consider the favorable conditions introduced in [28]. The detector volume is then of $500 \text{ cm}^3$ with a partial helium-3 pressure of 15 mbar. The distance between detector and the nuclear core was of 16.5 m. The background was of 0.3 mHz [28]. The resulting constraint is shown in Fig. 3. Using the reactor model previously introduced, the expected hidden neutron flux $\Gamma$ against the swapping probability $p$ corresponds to the black dashed line. The neutron background is shown by the horizontal red dashed line. As a result, it leads to a new constraint for the swapping probability $p$ such that (see vertical blue dashed line on Fig. 3):

$$p \leq 5 \times 10^{-9}.$$

Such a value can be related to a constraint on the coupling constant between our visible braneworld and a hypothetical invisible one. This constraint is given by the red dashed line on Fig. 4. Obviously, these constraints need to be confirmed or improved by a dedicated experiment such the one described in previous sections. As a consequence, the present constraint on $p$ does not preclude the measurement of a yearly time-dependent drift of the swapping probability. Nevertheless, it shown the feasibility of the experiment introduced in the present work.

**IV. CONCLUSION**

We have proposed an experiment in which neutrons can be used to test the braneworld hypothesis. Using an intense flux of neutrons behind a wall and placing a well shielded neutron detector, reappearing neutrons could be detected as a clue of the existence of hidden braneworlds. Three typical signatures can be examined to verify the reliability of a measured signal as a real exotic phenomenon. First, for thick enough shielding the number of detected neutrons must not depend on the shield thickness. Second, when distance to the core increases, the number of detected neutrons must decay as an inverse square law against the distance between the nuclear core and the detector. At last, a yearly modulated time-dependent drift of the number of detected neutrons could be observed for not to low swapping probability. By contrast, without direct detection, such an experiment allows at least, to constrain the existence of braneworlds. The experimental constraint on the matter swapping probability between branes could be im-
proved by three orders of magnitude with the proposed experiment. Then, a rough constraint on the swapping probability has been proposed to show the feasibility of a neutron-shining-through-a-wall experiment.

Appendix A: Shielded nuclear core as a hidden neutron source

In a previous work [10], it was shown that ultra-cold neutrons colliding with a solid wall can escape into another hidden brane. In this situation, since ultra-cold neutrons present a wavelength roughly larger than the typical distance between atoms of the wall, the wall appears as a continuous medium for neutrons [31]. Such a medium can be described through the Fermi potential [31]. Here, we want to estimate the production rate of hidden neutrons due to collisions between fast (or thermal) neutrons and materials surrounding the nuclear core, i.e. mainly the heavy and light water tanks. In this context, since neutrons cannot be considered in the “optical” domain, media cannot be considered as continuous, and we must consider collision between neutrons and individual nucleus. Then, our aim is to find a relation between the visible neutron flux $Φ_+^{\text{vis}}$ and the hidden neutron flux $Φ_+^{\text{hid}}$ induced by the collisions between visible neutrons and nuclei in our brane. More specifically, we look for a source term:

$$S_- = KΩ_+$$  \hspace{1cm} (A1)

where the constant $K$ in the source term will be assessed in the following. It will be shown that $K = (1/2)\mu n\Sigma_E = (1/2)\rho \Sigma_E$ where $\Sigma_E$ is the macroscopic cross section of the materials of the reactor and $Φ_+$ the neutron flux in the whole reactor. To do this, we consider the matrix density approach, we follow the approach of F einberg and Weinberg [25] also used by Demidov, Gorbunov and Tokareva [26] to describe other kind of oscillations in a volume occupied by matter.

Neutrons are described through the two-brane Pauli equation and we neglect the neutron decay process:

$$i\hbar \partial_t Ψ = HΨ$$  \hspace{1cm} (A2)

such that

$$H = \begin{pmatrix} -\frac{\hbar^2}{2m} \Delta + V_{g,+} & -i\mu_\nu \sigma \cdot A_{amb} \\ ig\mu_\nu \sigma \cdot A_{amb} & -\frac{\hbar^2}{2m} \Delta + V_{g,-} \end{pmatrix}$$  \hspace{1cm} (A3)

where $V_{g,+}$ and $V_{g,-}$ are the gravitational potential felt by the neutron in each brane. We assume that $\sigma \cdot A_{amb} = A_{amb}\sigma_z$ and we restrain ourself to a spin-up state without loss of generality (matter swapping probability does not depend neither on the direction of the magnetic vector potential nor on the spin state [22 14]). Then we can write:

$$H = \begin{pmatrix} -\frac{\hbar^2}{2m} \Delta + V_{g,+} & -iΩh \\ iΩh & -\frac{\hbar^2}{2m} \Delta + V_{g,-} \end{pmatrix}$$  \hspace{1cm} (A4)

with $Ωh = g\mu_n A_{amb}$. The behaviour of the neutron flux is described through the use of the matrix density $ρ$ which follows [25]:

$$\frac{d}{dt}ρ = -i\hbar^{-1} (Hρ - ρH^\dagger) + I_c$$  \hspace{1cm} (A5)

where $I_c$ is the collisional integral which describes the collisions between neutrons and the atoms of the gas, with

$$I_c = nv\int F(θ)ρF^\dagger(θ)dΩ.$$  \hspace{1cm} (A6)

$v$ is the mean relative velocity between neutrons and gas molecules and $n$ is the density of gas particles. We have

$$F(θ) = \begin{pmatrix} f(θ) & 0 \\ 0 & 0 \end{pmatrix}$$  \hspace{1cm} (A7)

which describes the scattering of neutrons by the atoms of the gas and where $θ$ is a scattering angle. The second diagonal term is equal to zero since we assume there is no gas in the hidden brane.

We also define $H = H + C$ with

$$C = \begin{pmatrix} -2πnvf(0)/k & 0 \\ 0 & 0 \end{pmatrix}$$  \hspace{1cm} (A8)

which accounts for the presence of gas in the system with $k$ the neutron vector wave. Let us use the definition:

$$ρ = \begin{pmatrix} ρ_+ & r + is \\ r - is & ρ_- \end{pmatrix}.$$  \hspace{1cm} (A9)

We also define:

$$w_R = 4π\frac{nv}{k}Re(f(0)),$$ and
$$w_I = 4π\frac{nv}{k}Im(f(0))$$  \hspace{1cm} (A10)

with the optical theorem related to the cross-section $σ_{tot}$:

$$σ_{tot} = σ_E + σ_I = 4π\frac{k}{iω}Im(f(0)) = w_I$$  \hspace{1cm} (A11)

and

$$σ_E = \int |f(θ)|^2 dΩ.$$  \hspace{1cm} (A12)

We then obtain the following system from equation (A5):

$$\frac{d}{dt}ρ_+ = -2Ωr - nvσ_Iρ_+,$$  \hspace{1cm} (A13)

$$\frac{d}{dt}ρ_- = 2Ωr,$$  \hspace{1cm} (A14)

$$\frac{d}{dt}r = (η - (1/2)w_R)s - (1/2)r w_I + Ω(ρ_+ - ρ_-),$$  \hspace{1cm} (A15)

$$\frac{d}{dt}s = -(η - (1/2)w_R)r - (1/2)s w_I,$$  \hspace{1cm} (A16)
with $h\eta = V_{g,+} - V_{g,-}$. In the following, we consider a statistical set of neutrons initially localized in our visible brane and emerging from the nuclear core, i.e. $\rho_+(t = 0) = 1$ and $\rho_-(t = 0) = 0$. If we consider Eq. (A13), we obviously deduce that the neutron population in our brane must mainly decrease due to absorption, i.e. neutron leakage into an hidden brane must be very weak to be consistent with known physics. So, we must verify: $2\Omega \ll n\nu\sigma_1\rho_+$. Now, if we consider thermal neutrons in heavy water for instance, and if we value $\Omega$ weak to be consistent with known physics. So, we must deduce from Eq. (A18) that the neutron population following convenient form:

$$
\frac{d}{dt} \rho_+ = -n\nu\sigma_1\rho_+, \tag{A17}
$$

$$
\frac{d}{dt} \rho_- = 2\Omega \rho_+, \tag{A18}
$$

$$
\frac{d}{dt} \sigma = \eta s - (1/2)n\nu\sigma_1\sigma + \Omega \rho_+, \tag{A19}
$$

$$
\frac{d}{dt} \sigma = \eta s - (1/2)n\nu\sigma_1\sigma + \Omega \rho_+, \tag{A20}
$$

At this stage of the problem, it is relevant to underline that from Eqs. (A19) and (A20) we can easily deduce:

$$
\frac{d}{dt} \sigma = -(1/2)n\nu\sigma_1\sigma + \Omega \rho_+, \tag{A21}
$$

where we have set: $r + is = me^{i\theta}$. Without coupling, i.e. if $\Omega = 0$, we deduce from Eq. (A21) that the coherence terms of the density matrix (i.e. its off-diagonal terms $r$ and $s$) must exponentially decay as $\exp(-(1/2)n\nu\sigma_1\sigma t)$. As a consequence, the collisional dynamics should suppress the quantum coherence precluding then neutron swapping between branes. By contrast, Eq. (A21) as well as Eqs. (A19) and (A20) also show us that the coupling $\Omega$ preserves from decoherence. This allows stationary coherences such that $dr/dt = d\sigma/dt = 0$, with $r$ and $s$ different from zero. The steady state is quickly achieved due to the exponential behavior above mentioned. Let us call $X$ the distance covered by the neutron in the medium. The length $X$ cannot be assimilated to the crossed thickness $L$ in the medium. Since neutrons are localized in our visible brane, they can be diffused by nuclei of molecules or atoms. $X$ is the sum of the lengths of the various straight lines covered by the neutron and then $L = X \sqrt{\sigma_1/3\sigma_F}$. Now, for instance, since $n\sigma_{tot} \approx 0.4 \text{ cm}^{-1}$ for heavy water, the stationary coherences occurs at $X = 4 \text{ cm}$, i.e. $L = 0.2 \text{ mm}$ from the neutron source, a distance which must be compared with the 105 cm-thick heavy water slab in the ILL's reactor. By contrast, $n\sigma_I = 4 \times 10^{-5} \text{ cm}^{-1}$ in $\text{D}_2\text{O}$, i.e. $\rho_+$ slowly varies in the heavy water region (see Eq. (A17)). Then, in the stationary coherences hypothesis, Eqs. (A19) and (A20) become a linear system of two equations with two unknowns ($r$ and $s$) which is trivially solved. We get:

$$
\begin{pmatrix}
    r \\
    s
\end{pmatrix}
= \begin{pmatrix}
    1 \\
    (1/4)\nu\sigma_1\Omega + \eta^2 \nu\sigma_1\Omega
\end{pmatrix}
\approx
\begin{pmatrix}
    1 \\
    (1/4)\nu\sigma_1\Omega + \eta^2 \nu\sigma_1\Omega
\end{pmatrix}
\tag{A22}
$$

From Eqs. (A18) and (A22) we deduce:

$$
\frac{d}{dt} \rho_- \approx (1/2)\nu\sigma_1\Omega \rho_+ \tag{A23}
$$

Since the neutron flux $\Phi_- = u_- v$, where $u_-$ is the local neutron density in the hidden brane, then:

$$
\frac{d}{dt} \Phi_- = (1/2)\nu\sigma_1\Omega \Phi_+ \tag{A24}
$$

where we have set $p = 2\Omega^2/\eta^2$ to be consistent with [9]. Let us now consider an initial local neutron flux $\Phi_0 = u_0 v$ where $u_0$ is the initial neutron density. The neutron fluxes in each brane are given by $\Phi_+ = \Phi_0 \rho_+$. Then we deduce from Eq. (A23):

$$
\frac{d}{dt} \Phi_- = (1/2)\nu\sigma_1\Omega \Phi_+ \tag{A25}
$$

Since $u_-$ is now local, Eq. (A25) must be supplemented by a divergence term $\nabla \cdot j_-$ to account of the local behaviour of the neutron current $j_- = u_- v$. We then deduce the continuity equation for neutrons in the second brane:

$$
\nabla \cdot j_- + \frac{\partial}{\partial t} u_- = (1/2)\nu\sigma_1\Omega \Phi_+ \tag{A26}
$$

which is the continuity equation endowed with a source term $S_-$:

$$
S_- = (1/2)\nu\sigma_1\Omega \Phi_+ \tag{A27}
$$

Obviously, for heavy water, $\sigma_{tot} \approx \sigma_F$ and we retrieve Eq. (A1). Now, as striking result, we note that neutrons propagating in the hidden brane are not absorbed since we expect that vacuum rules in this other world. As a consequence, while visible neutrons have been stopped by water surrounding the nuclear core, hidden neutrons can propagate freely far away from the reactor. Then we can fully suppress the visible neutron flux in our brane while keeping a constant flux of hidden neutrons in the invisible brane. One can then build a hidden neutron source from a nuclear core surrounded by a relevant shield.

**Appendix B: Hidden neutron detector**

We consider a gas stored in a vessel as a neutron detector. More specifically, we consider a neutron flux in
the second brane and we expect to detect neutrons that arise from the second brane in our braneworld. The distance between the atoms of the gas is much larger than the wavelength of the neutrons. As a consequence, neutrons see a set of scattering objects instead of a continuous medium. We follow the same approach as previously, and start with equations (A13) to (A16), but now, we consider a statistical set of neutrons initially localized in the invisible brane, such one can write: \( \rho_\pm(t = 0) = 0 \) and \( \rho_\pm(t = 0) = 1 \).

Looking Eq. (A14), we see that the neutron population in the hidden brane mainly decrease due to neutron leakage into an hidden brane. Following similar hypothesis as in the previous section, we can consider that \( 2\sqrt{r} \) must be very weak and we assume that \( \partial_t \rho_- \approx 0 \), i.e. \( \rho_- \) is almost constant at the detector scale. We can also assume: \( \rho_+ \ll \rho_- \) and \( \eta \gg w_R \). Then, using also Eqs. (A13) to (A16), we can recast in the following convenient form:

\[
\frac{d}{dt}\rho_+ = -2\Omega r - n\nu\sigma_1 \rho_+ , \tag{B1}
\]

\[
\rho_- \approx 1, \tag{B2}
\]

\[
\frac{d}{dt}r = \eta s - (1/2)rn\nu\sigma_{tot} - \Omega \rho_- , \tag{B3}
\]

\[
\frac{d}{dt}s = -\eta r - (1/2)sn\nu\sigma_{tot} , \tag{B4}
\]

Now, from Eqs. (B3) and (B4) we can easily deduce:

\[
\frac{d}{dt}m = -(1/2)rn\nu\sigma_{tot}m - \Omega \cos \theta \rho_- \tag{B5}
\]

where we have set: \( r + is = me^{i\theta} \). Without coupling, i.e. if \( \Omega = 0 \), we deduce from Eq. (B5) that the coherence terms of the density matrix (i.e. its off-diagonal terms \( r \) and \( s \)) must exponentially decay as \( \exp(-(1/2)n\nu\sigma_{tot}t) \). As a consequence, the collisional dynamics should suppress the quantum coherence precluding then neutron swapping between branes. By contrast, Eq. (B5) as well as Eqs. (B3) and (B4) also show us that the coupling \( \Omega \) preserves from decoherence. Nevertheless, by contrast to the case described in the previous section, stationary coherences such that \( dr/dt = ds/dt = 0 \), with \( r \) and \( s \) different from zero cannot be achieved here due to the short length of the detector. Indeed, assuming for instance an He-3 detector with a pressure about 4 atm we get \( n\sigma_{tot} \approx 0.53 \text{ cm}^{-1} \) for thermal neutrons. Now, the distance \( X = vt \) crossed by neutron can be assimilated to the length \( L \) of the detector since we can assume that the hidden neutrons are not significantly diffused by nuclei of gas molecules or atoms in our brane. Let us consider for instance a detector about 10 cm. At half length, we get \( \exp(-(1/2)n\nu\sigma_{tot}t) \approx 27 \% \), i.e. the coherences are not negligible in the half part of the detector at least. As a consequence, we cannot apply the stationary coherences hypothesis here.

Nevertheless, Eqs. (B3) and (B4) constitute a simple non-homogeneous first order differential equation easy to solve since \( \rho_- \approx 1 \), i.e. since \( \rho_- \) is constant. Let us set:

\[
\left(\begin{array}{c}
\frac{r}{s}
\end{array}\right) = e^{-(1/2)n\nu\sigma_{tot}t} \left(\begin{array}{c}
\cos(\eta t) & \sin(\eta t) \\
-\sin(\eta t) & \cos(\eta t)
\end{array}\right) \left(\begin{array}{c}
R \\
S
\end{array}\right) , \tag{B6}
\]

If we inject (B6) in Eqs. (B3) and (B4), we obtain

\[
\frac{d}{dt} \left(\begin{array}{c}
R \\
S
\end{array}\right) = e^{(1/2)n\nu\sigma_{tot}t} \left(\begin{array}{c}
\cos(\eta t) & -\sin(\eta t) \\
\sin(\eta t) & \cos(\eta t)
\end{array}\right) \left(\begin{array}{c}
-\Omega \rho_- \\
0
\end{array}\right) , \tag{B7}
\]

from which we deduce:

\[
\left(\begin{array}{c}
R \\
S
\end{array}\right) = -\Omega \rho_- \left(\begin{array}{c}
\int_0^te^{(1/2)n\nu\sigma_{tot}t'}\cos(\eta(t - t'))dt' \\
\int_0^te^{(1/2)n\nu\sigma_{tot}t'}\sin(\eta(t - t'))dt'
\end{array}\right) , \tag{B8}
\]

Using Eqs. (B6) and (B8), Eq. (B1) can be rewrite as:

\[
\frac{d}{dt}\rho_+ = 2\Omega^2\rho_- \int_0^t\int_0^{t'}e^{-(1/2)n\nu\sigma_{tot}(t'-t'')}e^{-n\nu\sigma_1(t'-t'')}\cos(\eta(t' - t''))dt''dt' , \tag{B9}
\]

which leads to:

\[
\rho_+ = 2\Omega^2\rho_- \int_0^t\int_0^{t'}e^{-(1/2)n\nu\sigma_{tot}(t'-t'')}e^{-n\nu\sigma_1(t'-t'')}\cos(\eta(t' - t''))dt''dt' , \tag{B10}
\]

Then, the rate \( \Gamma \) of events recorded per second by the
detector in our brane is given by:
\[
\Gamma = \int_{0}^{L/v} n\nu \sigma I \int_{S_{d}} \Phi_{e} dS dt
\]
\[
= \int_{0}^{L/v} n\nu \sigma_{I} \rho_{+} \int_{S_{d}} \Phi_{-} dS dt
\]
where \( S_{d} \) is the effective area of the detector and \( L \) is the length of the gas vessel such that \( L/v \) is the time during which hidden neutrons travel in the detector volume. The length \( L \) can be assimilated to the length of the detector since we can assume that the hidden neutrons are not significantly diffused by nuclei of gas molecules or atoms. In addition, absorption prevails on diffusion processes \( (\sigma_{I} \gg \sigma_{E} \text{ in helium-3 or BF3}) \).

Assuming that \( \eta \gg n\nu \sigma_{tot} \), and setting \( p = 2\Omega^{2}/\eta^{2} \) and \( x = vt \), we get:
\[
\Gamma = \varphi_{s} \frac{n\sigma_{I} \eta^{2}}{\nu^{2}} \int_{0}^{L} e^{-n\sigma_{I} x} \int_{0}^{x} e^{(1/2)n\sigma_{tot}(x''-x')} e^{n\sigma_{I} x'} \cos((\eta/v)(x'' - x')) dx'' dx' \quad (B12)
\]
with \( \varphi_{s} = \int_{S_{d}} \Phi_{-} dS \).

Due to the fast oscillating cosine term, and since \( \eta \gg n\nu \sigma_{tot} \), Eq. \( (B12) \) reduces to:
\[
\Gamma \sim (1/2)\varphi_{d} \left[ \frac{\sigma_{E}}{\sigma_{I}} - 1 \right] (e^{-n\sigma_{I} L} - 1) + n\sigma_{tot} L \quad (B13)
\]
For pure helium-3, relevant argon-helium-3 mixtures or BF₃, we verify \( \sigma_{I} \gg \sigma_{E} \) and then for long enough detector we simply get the intuitive but not so obvious result:
\[
\Gamma \sim (1/2)pn\sigma_{I} V \Phi_{-} \quad (B14)
\]
with \( V = S_{d}L \) the volume of the detector, and \( n\sigma_{I} = \Sigma_{A} \) the macroscopic absorption cross-section of the gas.

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