Output entanglement and squeezing of two-mode fields generated by a single atom

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I. INTRODUCTION

One of the most intriguing features of quantum mechanics is entanglement, which has been recognized as a valuable resource for quantum information process. Discontinuous variables and continuous variables entanglement, as two kinds of entanglement resource, both have been concentrated much more attention. Continuous variables entanglement, compared with its partner discontinuous variables, has many advantages in quantum-information science [1] and can be used to efficiently implement quantum information process by utilizing the continuous quadrature variables of the quantized electromagnetic fields.

Conventionally, two-mode squeezed state emerging from the nonlinear optical interaction of a laser with a crystal (from parametric amplification or oscillation ) is a typical continuous variables entanglement. Recently, it has been shown that correlated spontaneous emission laser can also work as continuous variables entanglement producer and amplifier [2-7]. Guzmán [8] proposed a method of generating unitary single and two-mode field squeezing in an optical cavity with an atomic cloud. As a result of realization of a single atom laser in experiment [9, 10], people began to interest in generating two-mode entanglement via single-atom system [11-16]. Morigi [11, 12] et al have shown that a single trapped atom allows for the generation of entangled light under certain conditions. One of our authors Zhou [13] has proposed generating unitary two-mode field squeezing in a single three-level atom interacting dispersively with two classical fields inside a doubly resonant cavity, which can produce a macroscopic entangled light. Our group also proposed schemes to generate continuous variables entanglement in a single atom system [2, 14]. Most recently, based on the same atomic level scheme as single-atom laser experiment [10], Kuffner [16] investigated a single atom system to generate a two-mode entangled laser via standard linear laser theory.

Although output entanglement and squeezing have been studied extensively in other system, the existence of a squeezing operator in the system which is similar to that of the single atom laser experiment [10] has never been exhibited before. In this paper, we study a similar atomic level as the experiment in [10] ( but with two-mode fields ). However, there they studied one mode laser, here we concentrate on the output entanglement of the cavity. Under large detuning condition, we deduce unitary squeezing operator of two-mode fields. By means of the input-output theory, we show that entanglement and squeezing of two-mode fields can be achieved at the output. This paper differ from [16] in these aspects: We use effective Hamiltonian method to obtain a squeezing field operator decoupled from the atomic degrees of freedom rather than by tracing the atomic degrees of freedom. Instead of studying intracavity fields, we show output entanglement.

II. SYSTEM DESCRIPTION AND CALCULATIONS

We consider a single four-level atom interacting with two nondegenerate cavities. The first cavity mode couples to atomic transition $|a\rangle \leftrightarrow |c\rangle$ with the detuning $\Delta_1$ and the second mode interacts with the atom on $|b\rangle \leftrightarrow |d\rangle$ with detuning $\Delta_2$. The two classical laser fields with Rabi frequencies $\Omega_3$ and $\Omega_4$ drive the transitions $|a\rangle \leftrightarrow |d\rangle$ and $|b\rangle \leftrightarrow |c\rangle$ with detunings $\Delta_3$ and $\Delta_4$, respectively. The atomic configuration is the same as that in [10]. In the interaction picture, the Hamiltonian is

$$H_1 = g_1a_1e^{-i\Delta_1 t}|a\rangle\langle c| + g_2a_2e^{-i\Delta_2 t}|b\rangle\langle d| + \Omega_3|a\rangle\langle d|e^{-i\Delta_3 t} + \Omega_4|b\rangle\langle c|e^{-i\Delta_4 t} + h.c.$$  \hfill (1)

Under large detuning condition $|\Delta_k|$ $\gg$ $\{|g_j|, |\Omega_k|\}$ ($k = 1...4, j = 1, 2, l = 3, 4$), we can adiabatically eliminate the excited level $|a\rangle$ and $|b\rangle$ and obtain the effective Hamiltonian

$$H_2 = (\frac{|g_1|^2}{\Delta_1}a_1^\dagger a_1 + \frac{|\Omega_3|^2}{\Delta_3})|c\rangle\langle c| + (\frac{|g_2|^2}{\Delta_2}a_2^\dagger a_2 + \frac{|\Omega_4|^2}{\Delta_4})|d\rangle\langle d| + ([\frac{\Omega_3 g_1}{\Delta_13}a_1e^{i\delta_1 t} + \frac{\Omega_4 g_2}{\Delta_24}a_2e^{-i\delta_2 t})]|d\rangle\langle c| + h.c.,$$  \hfill (2)
we can perform adiabatic elimination once more and have the new Hamiltonian

\[
H = \frac{\Omega_1^2}{\Delta_3^2} |c\rangle\langle c| + \frac{\Omega_3^2}{\Delta_3} |d\rangle\langle d| + \frac{\delta_1 + \delta_2}{2}(a_1^\dagger a_1 + a_2^\dagger a_2),
\]

we have the new Hamiltonian

\[
H_3 = \frac{\delta_1 + \delta_2}{2}(a_1^\dagger a_1 + a_2^\dagger a_2)
+ \frac{\Omega_1^2}{\Delta_3} a_1^\dagger a_1 |c\rangle\langle c| + \frac{\Omega_3^2}{\Delta_3} a_2^\dagger a_2 |d\rangle\langle d|
+ \left[\frac{\Omega_1^2}{\Delta_3} a_1 + \frac{\Omega_3^2}{\Delta_3} a_2\right] e^{\delta t} |d\rangle\langle c| + h.c.,
\]

where \( \delta = \frac{\Omega_1^2}{\Delta_3} - \frac{\Omega_2^2}{\Delta_4} + \frac{\delta_1 - \delta_2}{2} \). If \( |\delta| \gg \{|\Omega_1^2/\Delta_3\rangle |\Omega_2^2/\Delta_4\rangle\} \), we can perform adiabatic elimination once more and have

\[
H_4 = \frac{\delta_1 + \delta_2}{2}(a_1^\dagger a_2 + a_2^\dagger a_1)
+ \frac{\Omega_1^2}{\Delta_3} a_1^\dagger a_1 |c\rangle\langle c| + \frac{\Omega_3^2}{\Delta_3} a_2^\dagger a_2 |d\rangle\langle d|
+ \frac{1}{\delta}\left[\frac{\Omega_1^2 g_1}{\Delta_3} a_1 + \frac{\Omega_3^2 g_2}{\Delta_3} a_2\right] (a_1^\dagger a_1 + a_2^\dagger a_2) |d\rangle\langle d|
- \left[\frac{\Omega_1^2 g_1}{\Delta_3} a_1 + \frac{\Omega_3^2 g_2}{\Delta_3} a_2\right] (a_1^\dagger a_1 + a_2^\dagger a_2) |c\rangle\langle c|.
\]

If the atom is initially in state \(|d\rangle\), we finally have the effective Hamiltonian taken on the atomic state \(|d\rangle\) as

\[
H_{eff} = \lambda_1 a_1^\dagger a_1 + \lambda_2 a_2^\dagger a_2
+ \eta a_1 a_2 + \eta^* a_2 a_1,
\]

with

\[
\lambda_1 = \frac{\Omega_3 g_1^2}{\delta \Delta_3} - \frac{\delta_1 + \delta_2}{2},
\]

\[
\lambda_2 = \frac{\Omega_2 g_2^2}{\delta \Delta_2} + \frac{|g_2|^2}{\Delta_2} - \frac{\delta_1 + \delta_2}{2},
\]

\[
\eta = \frac{g_1 g_2 \Omega_3^* \Omega_1^*}{\delta \Delta_2 \Delta_3}.
\]

In Eq.(6), we have thrown off a constant which does not affect the dynamics of the system. Because the initial atomic state is \(|d\rangle\), only the terms which take action on \(|d\rangle\) survive. The stark shift \(\frac{|g_2|^2}{\Delta_2} a_2^\dagger a_2 |d\rangle\) has no contribution and \(\frac{g_1 g_2 \Omega_3^* \Omega_1^*}{\delta \Delta_2 \Delta_3} |d\rangle\) remain (see the second line in Eq.(5)). Thus, \(\lambda_1\) and \(\lambda_2\) are asymmetric in form. We will show the effect of the asymmetry on the output squeezing and entanglement.

If the initial cavity fields are in coherent state \(|\epsilon_1, \epsilon_2\rangle\) (with the help of two laser pumping, we can easily obtain the initial two-mode coherent state), we can use \(SU(1, 1)\) algebra to obtain evolution of wave function of the fields with \(|\Psi_f(\tau)\rangle = e^{-i H f + r} |\Psi_f(0)\rangle\). The exact expression of the fields evolution is a two-mode coherent-squeezed state as

\[
|\Psi_f(\tau)\rangle = S(\vartheta) |\epsilon_1, \epsilon_2\rangle,
\]

where \(\vartheta = r e^{i \varphi}\), and the squeeze parameter \(r (\varphi)\) is determined by \(r = \tan^{-1} |\eta^* \epsilon_0 \sin \varphi| (\tan \varphi = \Im(-\eta^* \epsilon_0 \sin \varphi) / \Re(-\eta^* \epsilon_0 \sin \varphi)) \) with \(\varphi^2 = [|\eta|^2 - \left(\frac{\Delta_1 + \Delta_2}{2}\right)^2 r^2, b_0 = [\psi \cos \phi + i \tau (\lambda_1 + \lambda_2) / 2 \sin \phi]^{-1}\).

The evolution time \(\tau\) is limited by the \(\tau_{\text{diss}} = \min\left(\frac{1}{\kappa_1}, \frac{1}{\kappa_2}\right)\) where \(\kappa_1\) and \(\kappa_2\) are the decay rates of modes 1 and 2. So, the intensity of the fields can not be increased into infinity with time evolution although the initial coherent state can effectively enhance the intensity of the cavity fields. Actually, the intensity of fields can not be increased largely due to the loss of the cavity and the large detunings condition. Consequently, the adiabatic elimination still can be used within \(\tau_{\text{diss}}\), only if the intensity of the quantum fields is not larger than the intensity of two classical fields \(\Omega_3\) and \(\Omega_4\). The decay effects will be discussed in next section where we do not need narrow the evolution time because physical quantities are automatically limited by time evolution after considering the decays. On the other hand, from Eq.(7) we see that the enhanced intensity of the fields do not affect the entanglement of the two modes because the entanglement results from the squeeze parameter.

### III. OUTPUT SQUEEZING AND ENTANGLEMENT

We now concentrate on the squeezing properties of the outgoing cavity fields which can be detected and used as entanglement source. To evaluate the entangled light
outside the cavity, we employ the input-output theory. We assume that the two cavity modes are driven by external laser fields besides the interaction with the atom in Eq.(6). The classical laser drive the cavity modes with strengths $\mu_1$ and $\mu_2$, respectively. The Langyevi equations of motion for the two-mode fields are given by

$$
\dot{a}_1 = -i\lambda_1 a_1 - i\mu_1^* - i\eta^* a_1^\dagger - \frac{\kappa_1}{2} a_1 - \sqrt{\kappa_1} a_{1in},
$$

$$
\dot{a}_2 = -i\lambda_2 a_2 - i\mu_2^* - i\eta^* a_2^\dagger - \frac{\kappa_2}{2} a_2 - \sqrt{\kappa_2} a_{2in},
$$

where $a_{1in}$ and $a_{2in}$ are annihilation operators associated with the input fields, and $\kappa_1$ and $\kappa_2$ are the cavity decay rates of modes $a_1$ and $a_2$. Using the transformation

$$
a_1 = a'_1 + a_0,
$$
$$
a_2 = a'_2 + \beta_0,
$$
we can rewrite the Eq. (9) as

$$
\dot{a}'_1 = -i\lambda_1 a'_1 - i\eta^* a_1^\dagger - \frac{\kappa_1}{2} a'_1 - \sqrt{\kappa_1} a_{1in},
$$

$$
\dot{a}'_2 = -i\lambda_2 a'_2 - i\eta^* a_2^\dagger - \frac{\kappa_2}{2} a'_2 - \sqrt{\kappa_2} a_{2in},
$$

Here, $\alpha_0 = \frac{-2i\mu_1^2(\kappa_1 + 2i\lambda_1) - 4\mu_1^2\eta^*}{(\kappa_1 + 2i\lambda_1)(\kappa_1 + 2i\lambda_2) + 4\eta^*}$. Performing Fourier transformation, we can solve the above equation and then use the relation $a_{jout} = a_{jin} + \sqrt{\kappa_j} a_j$ ($j = 1, 2$) to obtain the output fields as

$$
a_{1out}(\omega) = \sqrt{\kappa_1} a_0 \delta(\omega) + \frac{-(a_0^* a_2 + |\eta|^2) a_{1in}(\omega) + i\eta^* \sqrt{\kappa_1 \kappa_2} a_{2in}^\dagger(-\omega)}{\alpha_1 a_2 - |\eta|^2},
$$

$$
a_{2out}(\omega) = \sqrt{\kappa_2} \beta_0 \delta(\omega) + \frac{i\eta^* \sqrt{\kappa_1 \kappa_2} a_{1in}^\dagger(-\omega) - (\beta_1^* \beta_2 + |\eta|^2) a_{2in}(\omega)}{\beta_1 \beta_2 - |\eta|^2},
$$

where

$$\alpha_1 = \frac{\kappa_1}{2} + i(\lambda_1 - \omega),
$$
$$\alpha_2 = \frac{\kappa_2}{2} - i(\lambda_2 + \omega);$$
$$\beta_1 = \frac{\kappa_2}{2} + i(\lambda_2 - \omega),$$
$$\beta_2 = \frac{\kappa_1}{2} - i(\lambda_1 + \omega).$$

Observing that $|\alpha_0|$ and $|\beta_0|$ are in proportion to $|\mu_1|$ and $|\mu_2|$, therefore, we see that the driving parameters $|\mu_1|$ and $|\mu_2|$ yield effective displacements to the two mode output fields.

Now, we discuss the output entanglement of the fields. Define $I_+ = \frac{1}{\sqrt{3}}(a_1^\dagger a_1 + a_2^\dagger a_2 - a_1^\dagger a_2 - a_2^\dagger a_1)$, $I_- = \frac{1}{\sqrt{3}}(a_1^\dagger a_1 - a_2^\dagger a_2 + a_1^\dagger a_2 + a_2^\dagger a_1)$. The squeezing spectrum can be defined as [19]

$$
\langle I_\pm(\omega)I_\pm(\omega') + I_\pm(\omega')I_\pm(\omega) \rangle = 2S(\omega)\delta(\omega + \omega'),
$$

where $S(\omega)$ is Fourier transformation of $I_\pm$. With the definition of $I_\pm$, we have $S_+(\omega) = S_-(\omega)$ for uncorrelated vacuum input noise. The squeezing spectrum has been connected with entanglement criterion [19]. The “sum” criterion of Duan et al. [20] can be rewritten with $S_\pm(\omega)$ as

$$
S_+(\omega) + S_-(\omega) < 2.
$$

So, the two output modes are entangled if [19]

$$
S_\pm(\omega) < 1.
$$

Thus, the time evolution of entanglement is transformed into frequency domain. The spectrum $S_\pm(\omega)$ will be not only squeezing but also entanglement judge.

We assume that the input field is in the vacuum. From Eq.(12), we have

$$S_+(\omega) = \frac{|\eta|^2 + 2\alpha_2 a_1^* + 2|\eta|^2 \kappa_1 \kappa_2}{2|\alpha_2 a_1 - |\eta|^2},
$$

$$+ \alpha_j \rightarrow \beta_j.
$$

We also find that the squeezing spectrum has not been affected by the displacements $\alpha_0$ and $\beta_0$ because Eq.(17) has no relation with $\alpha_0$ and $\beta_0$. $S_+(\omega)$ is connected with the squeezing parameters $\eta$, decay rate $\kappa$, as well as $\lambda_1$ and $\lambda_2$.

Fig. 2 shows that the squeezing $S_+(\omega)$ change with $\kappa$ and $\omega$ where we choose $\kappa_1 = \kappa_2 = \kappa$. With the group of the parameters, $\Delta_k$ is about ten times the values of $\{g_1, \Omega_1\}$, which means that the first adiabatic elimination condition $\Delta_k \gg \{|g_1|, |\Omega_1|\}$ is fulfilled. With the parameters used in Fig. 2, we have $\delta = 1.48g$, $|\Omega_2| = 0.14g$, and $|\Omega_2| = 0.13g$, so the second adiabatic elimination condition $|\delta| \gg \{|\Omega_2|, |\Omega_2| \}$ satisfies. Therefore, all of the approximation conditions are fulfilled for the parameters in Fig. 2. As presented in Fig. 2, we see that
the entanglement is achievable and $S_+(\omega)$ changes with the leakage rate. For small value of $\kappa$, we cannot obtain ideal squeezing outside the cavity. For large value of $\kappa$, the degree of squeezing will be decreased. That is to say, there is a suitable value of $\kappa$ for achieving maximum degree squeezing.

In Fig. 3, we show the squeezing spectrum for several values of leakage rate $\kappa$. For $\kappa = 0.05g$, we observe two minimum values in the squeezing spectrum which also can be seen in Fig. 2. However, usually the squeezing spectrum should have one valley if $\kappa_1 = \kappa_2 = \kappa$, the split from one valley into two minima is similar to the effect of asymmetric loss for each mode [18] where if the loss of each mode differs, the squeezing spectrum shows two minima. Here, although we set $\kappa_1 = \kappa_2 = \kappa$, we can still observe the interesting split. Actually, the split originates from the nonzero and asymmetric $\lambda_1$ and $\lambda_2$ ($\lambda_1 \neq \lambda_2$ seen Eq. (7)]. If $\lambda_1 = \lambda_2 = 0$, we will have only one valley even for small value $\kappa$. With the increasing of $\kappa$, the split disappears. Following the relation Eq. (13), we know that because of the larger value of $\kappa$, i.e. $\kappa \gg |\lambda_1|$, the difference in $\lambda_1$ and $\lambda_2$ will have little effect so that we have one minimum squeezing. Physically, the asymmetry originates from the asymmetric detuning and asymmetric atomic initial state. In addition, we can also observe the existence of a appropriate value of $\kappa$ where squeezing are better than others. For example the squeezing for $\kappa = 0.5g$ is better than that for $\kappa = 0.05g, 2g$. Moreover, one can see that the bandwidth of the squeezing spectrum becomes wide when the minimum values of squeezing are increased.

We now discuss the correlation between the output fields amplitude. With Eq. (12), we have

$$
(a_{1\text{out}}^\dagger(\omega)a_{1\text{out}}(\omega')) = \kappa_1|\alpha_0|^2\delta(\omega)\delta(\omega')
$$

$$
+ \frac{|\eta|^2\kappa_1\kappa_2}{|\alpha_2\alpha_1 - |\eta|^2|^2}\delta(\omega - \omega'),
$$

$$
(a_{2\text{out}}^\dagger(\omega)a_{2\text{out}}(\omega')) = \kappa_2|\beta_0|^2\delta(\omega)\delta(\omega')
$$

$$
+ \frac{|\eta|^2\kappa_1\kappa_2}{|\beta_2\beta_1 - |\eta|^2|^2}\delta(\omega - \omega').
$$

We let $N_1(\omega) = \frac{|\eta|^2\kappa_1\kappa_2}{|\alpha_2\alpha_1 - |\eta|^2|^2}$, which is one of the contributors in intensity spectrum. In Fig. 4, we plot $N_1(\omega)$ as a function of $\omega$. We see that for resonance $\omega = 0$ (the output frequency equal to the frequency of the cavity fields), $N_1(\omega)$ achieve its maximum value but the intensity is relative smaller than that in [16]. This might be because there [16] the detuning $\Delta_i$ is not so larger and the input-output effect is not considered. However, in this paper, we consider the atom disperively interacts with the cavity as well as the input-output effect, so the output fields are decreased much more. From Eq. (17), we know that when $\omega = \omega' = 0$, the output fields are enhanced by $\kappa_1|\alpha_0|^2$ and $\kappa_2|\beta_0|^2$ with $\delta$ function. With the same parameters with Fig. 4, we have $\kappa_1|\alpha_0|^2 \approx 19$, $\kappa_2|\beta_0|^2 \approx 8$ if $\mu_1 = \mu_2 = 0.8g$. So, the fields will be enhanced largely.

IV. CONCLUSION

In summary, we study a single four-level atom interacting with two-mode cavity system. We deduce unitary squeezing operator of the two-mode fields via adiabatic elimination technique. By means of the input-output theory, we show that two-mode entanglement and squeezing can be achieved at the output fields in frequency domain. The squeezing spectrum reveals that asymmetric detuning and asymmetric atomic initial state split the squeezing into two minimum values, and appropriate leakage of the cavity is needed for obtaining output entangled fields.
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