Modeling of vapor bubble growth at subatmospheric pressures

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Abstract. In this paper, the results of numerical calculations of a vapor bubble growth in superheated water at different pressures are presented. Modeling is based on a previously developed by the authors semi-analytical solution. The results are verified by experimental data obtained at atmospheric and subatmospheric pressures. The presented simulation results and experimental data are in good agreement. The advantage of the solution over the earlier ones (based on the thermal growth model) is shown.

1. Introduction

Boiling is a process wide spread both in common life and technology. It can be found not only in energy applications (various kinds of heat exchangers, where an efficient heat removal is the main problem), but also in modern medical techniques. For example, the medical technologies include the technique of minimally invasive laser surgery, where the explosive boiling occurs. The cumulative jet, which carry out a destructive effect on pathological formations, appears due to the boiling of a biological fluid heated by laser radiation [1]. The basic act of boiling is the nucleation and growth of a vapor / vapor-gas bubble in an overheated (or locally overheated under general subcooling) liquid. Without detailed knowledge about the mechanism of basic act a correct description of the boiling process remains impossible.

In addition to many experimental studies there are a number of theoretical works, in the literature, where the vapor bubble growth is described for various system conditions. It should be noted that the theoretical description of the vapor bubble growth mechanism is a complex nonlinear moving boundary problem. So, the complete solution of this problem is not yet available. Most of the known models, which became classical, and solutions obtained on their basis are either asymptotic or empirical. This fact does not allow us to call them to be a complete solution of the problem for a wide range of the system parameters.

Obviously, it is impossible to find general analytical solution. Therefore, most researchers use significant assumptions that limit the scope of the obtained solutions. This applies both to classical works [2-5] and to relatively modern studies [6-8], where the inertial and thermal stages of bubble growth are considered separately. Some attempts to take into account the dynamic and thermal effects simultaneously were undertaken in [9-13]. However, the transitional stage of bubble growth is still insufficiently described. This work is devoted to numerical modeling of the vapor bubble growth, based on the semi-analytical solution obtained earlier by the authors of the work [14, 15], for a wide
range of Jacob numbers and all stages of the process, including the transition stage, which is extremely necessary to take into account in some cases.

2. Numerical simulation

Our calculations are based on the theoretical model presented in [14, 15] and the semi-analytical solution found under assumptions of this model. The assumptions are the following. The bubble is considered to be spherically symmetric during the entire process. Liquid around the bubble is incompressible. The bubble growth is determined only by external factors, which takes into account both the dynamic and thermal effects. To do this, the Rayleigh equation and the boundary value thermal problem with a moving boundary are jointly solved. Furthermore, the laws of conservation of mass, momentum, and energy at the bubble boundary are taken into account. For vapor in the bubble, the conditions of uniformity of temperature and pressure are met. The vapor is stationary and in a saturated state (the pressure and temperature of the vapor are functions of time). Heat released by viscous dissipation is assumed to be negligible. The liquid is initially uniformly superheated. The latter is generally not universal, but in the zero approximation, it is proposed to introduce a certain superheat averaged over the allocated volume of liquid and use it as a variable parameter of the problem, determining it from the experiment.

To perform numerical simulation of the system, the 4th order Runge-Kutta method and the implicit Krank-Nicholson scheme is used. To find the time functions of vapor parameters given implicitly, the Brent method is used. The calculations were performed using dimensionless variables with the following characteristic values of time, velocity, and size, respectively: \( t_0 = a_1 \rho_1 / \Delta \rho, \quad w_0 = (\Delta \rho / \rho_1)^{1/2}, \quad R_0 = a_1 (\rho_1 / \Delta \rho)^{1/2}, \) where \( a_1 \) is the thermal conductivity coefficient, \( \rho_1 \) liquid density, \( \Delta \rho \) the initial deviation of the liquid pressure from the saturated vapor pressure. The dimensionless phase transition criteria here is the Jacob number \( \text{Ja} = \frac{\rho_1 c_i \Delta T_i}{\rho_v L} \), where \( \rho_1 \) is saturated vapor density; \( c_i \) heat capacity of liquid; \( L \) specific heat of the phase transition; \( \Delta T_i \) initial overheating of the liquid. During comparing the calculation results with the experimental curves of bubble growth, the Jacob number was used as a fitting parameter.

3. Experimental setup

For verification and analysis of the modeling results, the experimental data obtained during heterogeneous boiling of saturated water under subatmospheric pressures in range of 8.8-103 kPa (the saturation temperature in this case varies in range 43.3–100.4 °C) were used [16,17].

The scheme and detailed description of the experimental setup can be found in [16]. The working volume was a sealed chamber connected to a vacuum pump to control pressure. The system pressure was controlled using the “Manometer” BO1227 vacuum gauge. To maintain the constant temperature during the experiment, the working volume was mounted in the isothermal bath.

To observe boiling process the experimental setup has two sealed windows on the side of the chamber. The main feature of the setup is a transparent heater, which is a thin ITO (indium-tin oxide) film with 1 \( \mu \)m thickness, vacuum deposited on a 3-millimeter sapphire substrate. This allows to carry out the high-speed visualization from the bottom side of the heater surface. In addition, opacity of the ITO film in IR spectrum makes it possible to measure integral value of the heater temperature using an IR camera, located under heater.

Figure 1 (a) shows a frame of side view high-speed visualization of the bubble growth at \( p_{sat} = 8.8 \) kPa. As shown in [16], the bubble has a flattened shape during its growth, so it is necessary to operate with the value of the equivalent diameter \( D_{eq} = (D_1 D_2)^{1/3} \). That is why the high-speed visualisation was performed from the side of the transparent heater. Frequency and spatial resolution of the high-speed visualization was 5 kHz and 0.072 mm/pix respectively.
Figure 1. (a) A frame of high speed visualization and measurements of bubble diameter; (b) Scheme for recovering the temperature of a sapphire surface in direct contact with water.

To obtain the temperature of the surface in direct contact with water, a stationary solution of the onedimensional heat equation is used (see figure 1b): \( T_1 - T_2 = q\delta / \lambda \), where \( q \) is the heat flux density, \( \delta \) is the sapphire substrate thickness, \( \lambda \) is the thermal conductivity of sapphire, \( T_2 \) is the temperature on sapphire surface in contact with water, \( T_1 \) is the temperature of the ITO film.

4. Results and discussion

The results of numerical simulation are compared with experimental data on the growth of a vapor bubble at pressures 8.8, 42.1, 72.2 and 102.9 kPa (figure 2). For comparative analysis, the graphs show the Labuntsov-Yagov solution obtained in the framework of the thermal growth model (ignoring the inertial effects) [18] and Rayleigh model [2] developed within framework inertial scheme. As can be seen from the figure, the developed model works well at all stages of the process under consideration in contrast to the limiting laws of growth, which describe the inertial and thermal stages of growth separately and are characterized by power dependences of the form \( R \sim t^n \), where \( n = 1/2 \) and 1 respectively. Both accordance to the laws of growth and to the experimental data confirm the success of the model. Thus, this model can be applied in a wide range of parameters of the process, in particular, in a wide range of initial superheats and pressures (Ja numbers). It should be noted here that at large superheats, the kinetics of the phase transition can play a significant role (at least at the initial stage of the process). This can be expressed in limiting the mass flow due to the phase transition leading to the fact that the vapor in the bubble will be in an unsaturated state. However, this point requires further research.

Noteworthy, there is a deviation between the calculations and the experimental data (the figure shows that the experimental points lie slightly below the theoretical curve especially for low Ja numbers) over long periods of time This fact can be clearly explained. This deviation is observed at the thermal stage of growth when the growth of the bubble is mainly determined by heat transfer to the interphase boundary which directly depends on the overheating of the liquid away outside the bubble (at distances much greater than the characteristic thickness of the temperature boundary layer formed around the bubble during its growth at least). In the model used for the calculation, it is assumed that the overheating of the liquid around the bubble at the initial moment of time is uniform. In contrast, in the experiment at the time of nucleation of the bubble there is always a certain temperature gradient directed from the heating surface deep into the liquid (of course, this implies a negative gradient; the overheating of the liquid decreases away from the heated wall). Hence the bubble, as it grows, comes into contact with the colder layers of the liquid, and its rate of growth becomes less than in case of growing under the condition of uniform overheating. It should also be noted here that the deviation is
less the lower pressure in the medium. This is explained by the fact that the lower pressure, the longer waiting time for an appearance of the vapor phase embryo (which is directly observed in the experiment), and with it the longer the heating time of the liquid adjacent to the heated surface. This is before the appearance of vapor bubble, a layer of liquid with a larger thickness warms up, and the bubble grows at the observed times under the conditions that are closest to our conditions.

![Figure 2](image_url)

**Figure 2.** Dependence of bubble radius on time, calculated for different pressures: solid line - modeling carried out within the presented model; dashed lines - modeling carried out within the inertial model [2] and the thermal model [18]; points – the experimental data.

### 5. Conclusion

Thus, the present study shows a good agreement between the calculation results and experimental data on the dynamics vapor bubble growth during boiling of water in pressure range of 8.8-103 kPa. The calculations are carried out on the basis of the semi-analytical solution found earlier by the authors of the study, which simultaneously takes into account both inertial and thermal effects that determine the process of vapor bubble growth. The obtained solution is compared with the classical solutions found under certain assumptions (within the framework of inertial and thermal growth schemes) and representing power-low dependences of the form $R \sim t^n (n = 1/2, 1)$. It was shown that, in contrast to the classical solutions, which are valid only at a certain stage of the process, the found solution...
correctly describes the experimental data during the entire bubble growth time: from the initial inertial to “asymptotic” thermal one. Further development of the model implies taking into account kinetic effects, which can be significant at large Jacob numbers.

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