Criteria for spin instability based on the node distribution in Trace-transform of the SHS combustion wave chronogram

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Abstract. The article presents the results of the Trace analysis of three types of experimental chronograms of the SHS combustion wave in ignition, extinction, and stable propagation modes. It is shown that the use of the differential chronoscopy (DCS) method allows one to reliably determine the moment of occurrence of the spin instability of the SHS wave by the presence of local maxima of the T-functional, when the trace line exactly coincides with the image of the combustion wave front on the DCS map. It was found that in the stable spin combustion mode, the DCS map is a set of periodic isoclines, and this leads to the appearance of point nodes in the 2D image of the Trace transform. As a criterion for recognizing spin instability, we selected the feature of the appearance of local maxima in the central transverse region of the Trace transform spectrum. It is concluded that spin instability is characteristic of the transition from thermal to diffusion instability in the presence of a hysteretic dependence of the burning rate on temperature.

1. Introduction
Modern experimental methods of research and computer visualization of fast processes of technological combustion \([1-3]\), using optoelectronic measuring systems of nanosecond resolution \([4, 5]\), allow continuous monitoring of the stability of the thermal regime of self-propagating high-temperature synthesis (SHS) of materials \([6-9]\). The urgency of solving such problems in real time is due to the constant need to introduce inert additives into the composition of the initial combustion products, which impart special properties to the final synthesis product \([10, 11]\). It is possible to ensure high-speed processing of large volumes of video data recording the propagation of the SHS combustion wave by compressing information using differential chronoscopy (DCS) and using the Trace transform \([13]\) to recognize local instability of the structure of the microheterogeneous combustion front \([14]\). The spatial shape and structure of the SHS front is largely determined by the random structure of the powder mixture of the starting products and the diffusion-thermal instability of the combustion process \([15]\). The more classes of objects that need to be identified, the more features are required, especially when objects undergo complex transformations in which their appearance can change significantly. The formal principles of constructing triple features used in Trace-analysis \([13]\) allow you to create them for an unlimited number of objects, simply increasing the number of trace-functionals used. These signs may not have any physical meaning and correspondence with human perception, but at the same time they may have the correct mathematical properties, which will make it possible to distinguish objects with a certain group of transformations.
The aim of this work is to select and design an algorithm for calculating the formal trace-transform criterion for recognizing the critical combustion regime at the time of changing the direction of spin combustion.

2. Theoretical model and experimental procedure

The initial experimental data in the form of DCS-maps of the temperature of the combustion front of the SHS wave [12] were obtained using the original thermal imaging micropyrometric complex [14, 16] based on the high-speed video camera “VideoSprint NG” with a NanoGate nanosecond shutter (NPO Videoscan, Russia) and processing the video data stream, similarly to that described in [5, 10, 12], the "Fiji - ImageJ" image analysis and processing program (NIH, USA) [2, 5]. Significant differences of this work include the high-speed video recording mode of chronograms with a frame frequency of up to 2000 fps and the electromechanical camera movement system following the SHS combustion wave [14]. A typical example of a DCS map is shown in figure 1, where three characteristic combustion zones are conventionally identified: 1 - time interval of an unstable ignition mode of a combustion wave; 2 - interval stationary and stabilized propagation of the combustion wave; 3 - area of change of direction of spin combustion.

![Figure 1. High Speed DCS Map (2D Chronogram) SHS.](image)

Figure 1 shows that the trajectory of the combustion front is displayed in the DCS as a thin line with a constant slope to the coordinate axes. A change in the sign of the angle of inclination relative to the axis OX means a change in the direction of spin combustion to the opposite. At the level of microheterogeneity scales, a local “jitter” of the combustion front line is everywhere observed and this characterizes the fundamental instability of the combustion wave at these scales — in the transition from “collective” combustion to “individual”. Features of instability here additionally appear in the form of “branches” of SHS wave propagation with a change in the direction of spin combustion. Thus, with a good degree of certainty, we can assume that the stationary trajectory of the combustion front on the high-speed DCS map has the properties of equal slope lines, the thickness of which can be neglected as a first approximation. A Features consisting in changing the sign of the angle of inclination of this line relative to the vertical axis OX has a well-understood physical meaning of the spin instability of the combustion wave. The inclination to the left corresponds to the right spin combustion screw in the direction of the SHS reaction along the sample, and the inclination to the right corresponds to the opposite. As was shown earlier [8, 16], the spin instability of the combustion wave is characteristic of the transition from the thermal to diffusion instability in the presence of a hysteretic dependence of the burning rate on temperature [9]. The formal constructed procedure and the algorithm for calculating the set of independent trace-features for identifying a large number of
recognizable classes of objects consists of three successive stages and is described in [13]. In our case, the “reference” DCS-maps corresponding to the regulated “permissible” or “unacceptable” technological regimes of combustion act as classes of objects for recognition, for example, this is illustrated by the 3D map of the DCS of the combustion wave, which is shown below in figure 2.

![3D image of the differential chronogram map of the SHS process in the test sample.](image)

Figure 2. 3D image of the differential chronogram map of the SHS process in the test sample.

Step 1. At the first stage, the image of the DCS card undergoes all Trace transformations to calculate Trace direct images in the form of matrices, in accordance with the existing set of Trace functionals (TFs), the total number of which is denoted by $N_T$. In our case, the standard Trace transforms were taken to obtain $T_r$-matrices, with $N_T=7$ and the calculations were performed using the TFs formulas from table below.

| Trace transform | Functional used |
|-----------------|-----------------|
| $T_1$           | $T(f(x)) = \int_{x-c}^{x+c} r f(r) dr$ where $r = x-c$, and $c=\text{median}_x\{x,f(x)\}$ |
| $T_2$           | $T(f(x)) = \int_{x-c}^{x+c} r^2 f(r) dr$ where $r = x-c$, and $c=\text{median}_x\{x,f(x)\}$ |
| $T_3$           | $T(f(x)) = \text{median}_{r>0}[f(r), (f(r))^\frac{1}{3}]$ where $r = x-c$, and $c=\text{median}_x\{x,f(x)\}$ |
| $T_4$           | $T(f(x)) = \text{median}_{r>0}[f(r), (f(r))^\frac{1}{2}]$ where $r = x-c$, and $c=\text{median}_x\{x,f(x)\}$ |
| $T_5$           | $T(f(x)) = \int_{x-c}^{x+c} e^{ik\log r} f(r) dr$, ($p=0.5, k=4$) where $r = x-c$, and $c=\text{median}_x\{x,f(x)\}$ |
| $T_6$           | $T(f(x)) = \int_{x-c}^{x+c} e^{ik\log r} r^2 f(r) dr$, ($p=0, k=3$) where $r = x-c$, and $c=\text{median}_x\{x,f(x)\}$ |
| $T_7$           | $T(f(x)) = \int_{x-c}^{x+c} e^{ik\log r} r f(r) dr$, ($p=1, k=5$) where $r = x-c$, and $c=\text{median}_x\{x,f(x)\}$ |

The first of the above functionals is the Radon transform, and of the rest for the problems of SHS analysis, $T_3$, $T_4$ and $T_5$ are of interest, which use power coefficients corresponding to the laws of diffusion. Figure 3b shows the image of the Trace-transform matrix for $T_3$ obtained for the SHS combustion front line in the mode of steady propagation. As is done in the Radon transform, the Trace transform for the DCS-map of the combustion wave is a 2D representation of the image in coordinates.
φ and ρ with the value of the integral of the image computed along the corresponding line, placed at cell (φ, r). Figure 3(a) shows how the Trace transform calculates functional T over parameter t along the tracing line, which is not necessarily the integral. The value of this functional T is maximum when the tracing line exactly coincides with the image of the combustion wave front on the DCS-card. In the stable spin combustion mode, the DCS map represents a set of periodic isolines and this leads to the appearance of point nodes in the 2D image of the Trace transform, as shown in figure 3(b).

Step 2. At the second stage, the image of the Trace transform matrix \( T(x, \varphi) \) is transformed into “circus function” \( \Phi(\varphi) \) by integrating the values of \( T(x_i, \varphi) \) along the columns \( x_i \) using the diametral functionals \( P \), the total number of which is denoted by \( N_P \). This number cannot exceed the dimension of the Trace matrix \( T(x, \varphi) \) in the number of directions of the circular projections \( \varphi_j \). Thus, a histogram of the T-matrix power distribution along each of the selected angles \( \varphi \) is formed. Figure 4 shows an example of such a histogram for one diametral functional \( P \).

Figure 4 shows that the functional \( P \) is sensitive to the angle of rotation \( \varphi \) and is invariant to shear and also to scaling. This feature manifests itself in view of the above-mentioned properties of the spatial structure of the DHS map. The two-mode distribution of the histogram makes it easy to find the distance \( \varphi \) between the median C and the local maximum of the histogram at point A with the largest weight coefficient \( \rho \). Thus, the end result of the second stage is a sequence of numerical values of statistical weights \{\rho\}, which is a histogram of the diametral functional \( P \).

Step 3. At the third stage, a triple feature \( \Pi \) is created in the form of the most convenient statistic for calculating statistics, usually it comes down to determining the statistical center of the histogram.
{ρ_j}. In the general case, it is possible to create several functionals Φ, the total number of which is denoted by N_Φ. The method of constructing triple trace-signs makes it easy to create many objects. If for each stage of construction only 10 functionals are used (that is, 10 T functionals, 10 P functionals and 10 Φ functionals), it is possible to construct 10×10×10 = 1000 objects. In the general case, this number is determined by the product (N_T × N_P × N_Φ). In our case, the maximum number of “reference” DCS cards is limited by the resolution of the analyzed image (512×512) and N_T=7, which gives an estimate of the order of 10^6.

3. Results and Discussions
To test the methodology described above for constructing formal triple trace-feature corresponding to regulated technological combustion conditions, albums were compiled that included the median regions of the Trace spectrum matrix, which are most sensitive to the occurrence of thermal instability, as well as histograms of all diametrical functionals P. Below are three albums corresponding to three regulated combustion modes: “ignition” (figure 5); “Extinction” (figure 6) and “sustainable” (figure 7). In these examples, the nodes corresponding to the regime of spin combustion with the right screw are indicated by the symbols A, and the nodes with the left screw are indicated by the symbols B.

![Figure 5. Triple trace-feature of the “ignition” mode: on the left is the median region of the T-matrix; on the right the corresponding histogram of statistical weights {ρ_j} for 512 diametral functionals P.](image)

In the “ignition” (figure 5) and “extinction” (figure 6) modes, nodes of type A and type B can be seen at the same time, but the difference is in the different statistical weights of these nodes, which indicates the competition of combustion with the right and left screw. In the “ignition” mode, the probability of burning by type A is slightly higher than burning with the opposite spin, but the value of the “circular function” ϕ can be almost 2 times greater, which means periodic branching of the combustion front at large angles from the initial direction. Such “branches” arise frequently, but quickly degenerate at the initial stage of ignition.

In the “extinction” mode, the node B has a 2 times smaller statistical weight, but the branching of the combustion wave occurs at lower values of the spin direction deviation. This situation can be explained by the fact that, during thermal degradation during the burnup of the SHS reaction products, random spatial inhomogeneities in the form of porosity and non-stoichiometry of the mixture of the initial combustion products begin to influence the choice of the combustion direction.

In the “stable” combustion mode (figure 7), type A knots predominate and the appearance of type B reverse spin combustion knots is typical for cases when an obstacle in the form of a large pore or a large particle of one of the components of the initial mixture is encountered in the propagation of the combustion wave. It should be noted that depending on the inclination of the “branch”, which is determined by the direction of spin combustion, the position of the “node” will be to the left or right of
the axis of symmetry of the Trace matrix, more precisely, the “nodes” for the left and right directions of spin combustion are conjugate in the spatial-frequency spectrum Trace conversion. Thus, the sign of spin instability can be used to choose the moment of the appearance of conjugate maxima in the central transverse region of the spectrum of the Trace transformation.

Figure 6. Triple trace-feature of the "extinction" mode: on the left is the median region of the T-matrix; on the right the histogram of statistical weights \( \{\rho_j\} \) for 512 diametral functionals \( P \).

Figure 7. Triple trace-feature of the "steady" mode: on the left is the median region of the T-matrix; on the right the histogram of statistical weights \( \{\rho_j\} \) for 512 diametral functionals \( P \).

From the analysis of the histograms displayed in figures 5-7, the physical meaning of the amplitude-dispersion empirical trace criterion proposed as a working hypothesis earlier in [15] becomes clear. For example, if all the local maxima of the histograms \( A_i \), located to the left of the median line \( C \), as shown in figure 7, correspond to nodes with the same burning spin, then the combustion is stable and the dispersion of the “total” distribution is less than when the right-handed nodes are present in the general sample \( B_i \) with the inverse spin number of the combustion wave (figure 6-7). An indicator of instability can be the deviation of the \( i \)-th maximum from the median axis of symmetry \( C \) by the angular quantity \( \phi \). The Triple trace-feature can be given in the form

\[
\Pi = \sum (\phi_j \times \rho_j)
\]

where \( \rho_j \) is the statistical weight of the local maximum \( A_i \) or \( B_i \) in the histogram of the corresponding diametral functional \( P \).

An additional result in the analysis of the instability of the combustion wave can be obtained by examining in detail single DCS maps of the combustion front at the moment of formation of a new “branch” of the wave with the opposite spin, as shown in figure 8.

A study of repeating profiles at moments of spin instability, for example, in the vicinity of points 1 and 2 in figure 8, it can be noted that the motion of the combustion front isotherm is higher than the “branch” point than after it. Taking into account the fact that the thermal diffusivity of substances
exceeds the diffusion coefficient, in our case we can conclude: at points 1 and 2 in figure 8, spin instability is characteristic of the transition from thermal to diffusion instability with hysteresis by combustion speed from temperature [14].

Figure 8. Spin instability: 1 - node of "strong" instability; 2 - node “weak” instability; 3 - a branch of a part of the combustion front in the direction of the opposite spin.

4. Conclusions
1. The study of the thermal structure of the combustion wave of SHS using DCS methods, using the example of the Ni-Al system, demonstrates the possibility of estimating the criterion of thermal diffusion instability of the front in accordance with the equation in the Zeldovich-Barenblatt model.
2. The use of high-frequency DCS maps made it possible for the first time to detect the presence of “nodal” points in the central median region of T-matrices, the displacement of which relative to the axis of symmetry to the left or right indicates a change in the direction of spin combustion.
3. By analogy with the Plancherel theorem on convolution of functions, in spatial-frequency Fourier analysis, for using Trace-analysis in SHS stability control systems, it is necessary to accumulate an experimental database of Trace-matrix at images DCS of the combustion wave in the obviously unstable modes and select the corresponding "masks transmittance" in the Trace-spectrum.
4. The simplest statistical criteria, to recognize the critical combustion mode, the best is the algorithm for calculating the triple trace-features in the form of a weighted average value of the angular deviation of all the “nodal” Trace-spectrum of points for all histograms of the allowed diametrical functionals P.

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6. References
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