Forecasting of products’ technical condition during the production process

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Abstract. This article identifies the sources of uncertainty in the functioning of technological processes (TP) of manufacturing industrial products in science absorbing industry. The limitation of traditional statistical models’ application for forecasting of industrial products’ technical condition (TC) in the case of insufficiency and fuzziness of input data is shown. We have developed the forecasting model using the methods of the theory of fuzzy sets, combining a formal and heuristic approach. The model is adaptive to the change of TP characteristics. On the basis of the received forecasting of the TC it is possible to correct the TP parameters. The proposed approach for forecasting of industrial products’ TC allows to provide the rational development and implementation of new technologies.

1. Introduction
Continuous updating of the product range, change of quantitative and qualitative indicators, improvement of technological and constructive decisions, production structure and management are characteristic features of modern science absorbing industry. Among the most significant features of the process of manufacturing science absorbing products we note the diversity of products, a wide variety of technological and control operations, frequent change and adjustment of process equipment [1].

These features determine the increasing role of technology in ensuring the quality of products. When designing the TP, the developer must anticipate and evaluate basic parameters and characteristics of the product, its TC. Especially the role of forecasting increases in connection with the creation of unique products that are produced in small batches or in small quantities. The increasing role of forecasting in the production of industrial products is also associated with increasing requirements for their quality and reliability.

With the dominance changes of TP over stationary production, as a rule, there is no sufficient amount of statistical data to predict quantitative characteristics of properties of the product (parameters of TC). Forecasting of products’ TC in the conditions of indeterminacy is becoming the topical task.

2. Formulation of a task and analysis of methods of solution
Forecasting the product’s TC means predicting the main parameters of product on the basis of consideration of factors associated with the formation of these parameters in the production process. As
such factors are usually the parameters of the TP. Let \( \mathbf{X} \) be TP parameters (input parameters), \( y \) be the parameter of product’s TC the (output parameter).

In general, the forecasting problem is formulated as follows: on the basis of available data on input and output parameters it is required to construct the function \( F \):

\[
y = F(\mathbf{X}).
\]

Thus, on the basis of the available heuristic and experimental data, it is necessary to find the forecasting function – the approximation \( f(\mathbf{X}) \) of the function \( F(\mathbf{X}) \), such that the given criterion of the approximation quality takes an extreme value. The functional

\[
I = \int [y - f(x)]^2 p(x) dx,
\]

is usually taken as a criterion, where \( p(x) \) – distribution density of the vector \( \mathbf{X} \).

Methods of the maximum likelihood, the least squares, stochastic approximation, and regression and correlation analysis are usually used to solve this problem [2]. In this case, the type of the approximating function is pre-selected. Then we construct estimates of the unknown parameters of the approximating function that minimize the functional \( I \) from the available experimental data.

In real conditions, the form of the function \( F(\mathbf{X}) \) is usually unknown. Among the most common approximation methods, we note the piecewise linear approximation of the function \( F(\mathbf{X}) \) [3]. The area of input variables \( \mathbf{X} \) is divided into a certain number of areas, in each of which the desired function \( F(\mathbf{X}) \) is approximated by a linear function or another function of a fairly simple form. The parameters of the approximating functions are estimated using one of the above methods.

The advantage of piecewise approximation methods is the simplicity of implementation, the ability to refine the model, the ability to obtain adequate models with a relatively small amount of experimental data, which is important for the approximation in conditions of incomplete information. However, all the mentioned advantages appear only when, in the absence of a priori information about the form of the function \( F(\mathbf{X}) \), the partition of the space of input variables assumes the possibility of approximation of the original function by simple ones in separate areas.

The use of regression, correlation and other statistical methods to construct the forecasting function is problematic due to strict limitations on the amount of experimental data, which are due to the need for statistical significance of the results.

Insufficient amount of experimental data in conjunction with other factors make the forecasting task of TC of manufactured products is uncertain, fuzzy task. Among these factors, we note the absence of a formal description of the TP, the lack of quantitative relations linking the parameters of the product’s TC with the parameters of the TP. In addition, some of the information needed to construct the forecasting function is often presented in a qualitative form, namely, in the form of knowledge and experience of a specialist-technologist. The imprecision of input parameters can be caused by inaccuracy of their measurement or their expert evaluation [4].

These factors allow us to attribute the forecasting problem to badly formalized problems. In solving such problems the use of methods of fuzzy sets theory and fuzzy logic is effectively.

3. **Construction the forecasting function**

As indicated in [5], almost all real – world application fuzzy systems are either systems based on fuzzy production rules or systems using fuzzy relations.

However, this approach is rational to apply with low-dimensional vectors of input and output parameters. To construct a forecasting function, we use a generalized parameterized operator that allows us to aggregate the initial experimental data by input variables.

Let \( \mathbf{X} = (x_1, x_2, \ldots, x_n) \) be the vector of input parameters, \( \mathbf{Y} = (y_1, y_2, \ldots, y_m) \) be the vector of output parameters. Fuzzification of input and output parameters is feasible with use of the membership functions (MF):

\[
\mu_{X}(x) = \frac{1}{1 + e^{-a(x-b)}},
\]

where \( a \) and \( b \) are the parameters of the MF.
\[
\mu_{x_i}(x_i) = \begin{cases} 
1 - \left( \frac{|x_i - x_i^0|}{\Delta x_i} \right)^n, & \text{if } |x_i - x_i^0| < \Delta x_i, \\
0, & \text{if } |x_i - x_i^0| \geq \Delta x_i,
\end{cases}
\] (1)

where \( x_i^0 \) – the desired or nominal value of the input parameter (\( i = 1, \ldots, m \)), \( (x_i^0 \pm \Delta x_i) \) – the variation interval of the input parameter; \( x_i^0 \) and \( \Delta x_i \) can be defined subjectively, based on the estimates of experts.

\[
\mu_{y_j}(y_j) = \begin{cases} 
1 - \left( \frac{|y_j - y_j^0|}{\Delta y_j} \right)^{n_j}, & \text{if } |y_j - y_j^0| < \Delta y_j, \\
0, & \text{if } |y_j - y_j^0| \geq \Delta y_j,
\end{cases}
\] (2)

where \( y_j^0 \) – the desired or nominal value of the output parameter (\( j = 1, \ldots, n \)); \( (y_j^0 \pm \Delta y_j) \) – the interval of variation of the output parameter;

\( y_j^0 \) and \( \Delta y_j \) are determined objectively, based on the technical requirements for the product;

\( p_i \) and \( q_j \) in the formulas (1) and (2) allow to reflect the degree of fuzziness in the description of the corresponding variables;

\( p_i = 1 \) and \( q_j = 1 \) corresponds to the symmetric linear relationship;

\( p_i > 1 \) and \( q_j > 1 \) corresponds to the expansion operation of MF;

\( p_i < 1 \) and \( q_j < 1 \) corresponds to the contraction operation of MF.

The general view of the MF of input parameter is shown in figure 1. MF of output parameters are constructed similarly.

![Figure 1. The membership function of fuzzy variable \( x_i \).](image)

To aggregate fuzzy information on a set of input variables, we use the generalized operator [6]:

\[
M_s(\mu_1, \ldots, \mu_m) = \left( \frac{1}{m} \sum_{i=1}^{m} w_i \mu_i^s \right)^{1/s},
\] (3)
where \( \mu_i \) – MF of variable \( x_i \); \( w_i \) – coefficient of significance \( x_i \); \( s \) – parameter that allows to aggregate fuzzy information taking into account the variability of input parameters.

Note that \( s \) may be chosen from the interval \((-\infty, +\infty)\), it satisfied the basic requirement of aggregating operator

\[
M_s(\mu_1, \ldots, \mu_m) \in [\min(\mu_1, \ldots, \mu_m), \max(\mu_1, \ldots, \mu_m)].
\]

The significance coefficients and the corresponding generalizing operator are determined for each output parameter separately.

To determine the significance \( w_i \) under conditions of lack of information, we use the method of successive comparisons of Churchman – Akoff, which is one of the most popular heuristic methods for evaluating alternatives in weakly formalized problems.

The method is used to adjust the estimates specified by experts. At the initial stage, the input parameters are ranked based on the expert opinion on the degree of influence of input parameters on the output parameter under consideration, with each input parameter \( x_i \) corresponding to a preliminary assessment of \( w_i \). The most significant parameter \( x_1 \) is usually assigned a score of 1. The remaining estimates are in the range \((0,1)\) in order of relative importance of the input parameters. Then the most significant parameter \( x_1 \) and the set of parameters \( x_2, \ldots, x_m \) are compared. If the significance of \( x_1 \) is higher than the significance of the others, the estimates are adjusted so that \( w_1 \) is greater than the sum of all the others; otherwise, less than the sum of all the others. Then \( x_1 \) is excluded from consideration, and the coefficients of significance of other parameters are corrected.

Let’s consider a case of one output parameter \( y \) \((n=1)\). Let \( K \) experiments give experimental data \((X^k, y^k)\), where \( X^k = (x^k_1, x^k_2, \ldots, x^k_m) \).

We aggregate the input data using a generalized operator (3).

Denote \( M^k_i = M^k_s(x^k_1, x^k_2, \ldots, x^k_m) = M_s(\mu_{x_1}(x^k_1), \mu_{x_2}(x^k_2), \ldots, \mu_{x_m}(x^k_m)) \), \( k = 1, 2, \ldots, K \).

Without belittling the community, we believe that \( M^1_i < M^2_i < \ldots < M^K_i \).

Using the formula (2) we define \( \mu^i_k = \mu_x(y^k) \).

We construct an interpolation using a cubic spline based on the coordinates of the points \((M^k_x, \mu^k_x)\) [7].

The spline approximates the available experimental data and is a function of forecasting the output parameter.

The forecasting of product’s TC on the basis of the available data on the input parameters is carried out by the algorithm:

- the most informative input parameters \((x^*_1, x^*_2, \ldots, x^*_m)\) are allocated and their importance is estimated;
- input and output parameters are fuzzified according to formulas (1), (2);
- fuzzy input parameters are aggregated according to formula (3);
- a forecasting function, which is approximation by spline from the available fuzzy experimental data is constructed, ;
- \( \mu_y(y^*) \) is determined for output parameter \( y^* \);
- if \( \mu_y(y^*) > 0 \) then the deviation from the preset value of the output parameter is determined by the formula \( \Delta y^* = \Delta y (1 - \mu_y(y^*))^{1/\gamma} \);
- if \( \mu_y(y^*) = 0 \) then product’s TC does not meet the technical requirements.
If several output parameters determine the TC of the product, then in accordance with the developed algorithm for each such parameter, a forecasting function is built. The result is a family of forecasting functions to estimate the parameters of product’s TC.

4. Conclusions

The forecasting model is intended for an assessment of industrial products’ TC under conditions of insufficiency and fuzziness of the initial information. Forecasting accuracy is improved by training the model: the forecasting function is specified when new experimental data are obtained.

The use of the generalized operator allows solving the problem of multidimensionality of the input. We also note that the use of the model is particularly effective in the case of dichotomous evaluation of product’s TC, since the transition to a "fuzzy" formulation of the problem greatly simplifies solution; the result obtained is of acceptable quality. The model is also applicable in the conditions of «definiteness" (sufficiency of initial information), when realization of multivariate statistical methods is problematic.

The proposed approach to forecasting of products’ TC can be used in the development of intelligent control systems of TP.

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