Non-intercommuting Cosmic Strings

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We perform the numerical field evolution for the collision of two Abelian type I cosmic strings. We present evidence that, for collisions at small but characteristic relative velocities and angles, these cosmic strings do not exchange ends and separate. Rather, local higher winding number bound states are formed close to the collision point, which promote multiple local scatterings at right angles and prevent intercommutation from happening. This constitutes the simplest example of the breakdown of the intercommutation rule, usually assumed in the construction of effective models for cosmic string network evolution.

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Scenarios based on cosmic strings, formed at a Grand Unified Theory (GUT) phase transition are important candidates to explain the origin of the primordial perturbations responsible for the formation of structure in the Universe. Cosmic Strings may also be associated with many other important cosmological phenomena. After being formed at a GUT symmetry breaking phase transition, a network of cosmic strings is thought to evolve so as to approach a universal scaling behavior, characterized by a given mean length of string per Hubble volume. In all implementations to date, this complicated evolution is assumed to be well described by a Nambu-Goto action governing the dynamics of each string, together with a rule for the outcome of the collisions between them, deduced from the original field theory of which strings are classical solutions. Our present ignorance of the details of GUTs and their string solutions makes the latter task impossible. So far detailed studies of string collisions have been limited to the simplest field theory exhibiting strings, the Abelian Higgs model. Cosmic strings formed at a GUT transition may not be Abelian, even though these solutions are the simplest.

The study of string collisions amounts to solving an infinite degree of freedom non-linear dynamical system, which can only be done numerically. Numerical scattering experiments in the Abelian Higgs model, for type II and global strings, have confirmed the usual assumption that strings intercommute, i.e., they exchange ends at every collision. In this region of parameter space, when the Higgs mass is larger than the mass of the gauge field, the interactions between two strings with the same orientation are repulsive, leading to their separation after the collision.

Type I strings are more interesting because the static potential between them is always attractive. As a consequence, higher winding number bound states can be formed. In particular these bound states prevent an ordered Abrikosov lattice from existing in laboratory experiments involving type I superconductors. Nevertheless, a network of type I strings is thought to be viable in the early Universe as long as the string density at formation is sufficiently low. All numerical studies concerning the outcome of type I string collisions performed to date were targeted at showing that, at high approach center-of-mass velocities ($v = 0.75$ with $c = 1$), two high winding number strings will form a bridge of lower winding number connecting them. This bridge then grows, promoting the peeling of the original high winding number configurations onto lower ones.

At a phase transition, one expects to form predominantly unit winding number strings. Consequently, it is interesting to investigate the converse process, i.e., if and when the collision of two strings can result in the formation of a higher winding number bridge between them and for what range of model and dynamical parameters. In this Letter, we present evidence for the existence of...
such bound states. They constitute the simplest example of the breakdown of the intercommutation rule. We expect an initially growing bridge solution, or zipper (see Fig. 1), to exist for small approach velocities $v$ and angles $\alpha$. The meaning of small depends, in turn, on the ratio of the Higgs to gauge masses in the model. We define the Abelian Higgs model by the Lagrangian

$$\mathcal{L} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \frac{1}{2}((\partial_\mu + ieA_\mu)\phi)^2 - \frac{\lambda}{8}(|\phi|^2 - \eta^2)^2. \quad (1)$$

Type I strings correspond to $\beta \equiv e/\sqrt{\lambda} > 1$, and type II to $\beta < 1$. For $\beta = 1$, the static attractive and repulsive potentials exactly cancel, resulting in no interactions between vortices in 2D.

Under the simple assumption that the zipper grows with constant velocity, $\kappa$, it is possible to find a solution for the Nambu-Goto equations relating $\kappa$ to the collision parameters $F$, $\kappa = (\xi - 1/\epsilon)/(\epsilon - \xi)$, where $\epsilon > 1$ is the ratio of twice the energy per unit length of a unit winding number, $n = 1$, string to that of an $n = 2$ string, and $\xi = \cos(\alpha/2)/\sqrt{1 - v^2}$. A growing zipper can only exist for $\xi > 1/\epsilon$, i.e., for small enough angles $\alpha$ and/or small approach velocities $v$. If the zipper’s growth $\kappa$ is constant, it could in principle continue forever. However, this is a simplistic scenario. It not only assumes that the strings away from the zipper remain straight and travel at constant velocity, but this picture also neglects the attractive interaction between string segments close to junctions. In realistic circumstances, a more complicated motion should take place, namely changes in the relative velocity and/or angles between the interacting strings. Our results show that as long as the angle at the junction is small enough, optimal conditions exist for the zipper to grow.

In our numerical experiment, we chose as initial conditions two $n = 1$ straight strings at a relative angle $\alpha$ and approaching each other with initial center of mass velocity $v$. The dynamical parameter space is given by these two variables and the model parameter space by $\beta$. We keep $\beta = 2$ throughout. The boundary conditions correspond to boosted, unperturbed free vortex configurations. A typical set of events is presented in Fig. 2. Initially, we observe the two strings approaching each other, Fig. 2a, and the colliding segments at the center re-emerging at $90^\circ$, Fig. 2b. This is expected because, at very small separations, the interactions between colliding string segments are unimportant and the geometry of the collision can be understood on topological grounds. Other segments of string, that have not collided head-on but have impact parameter smaller than twice its width, are also affected by the interactions. Their orbits are curved inwards, in a manner similar to what has been observed in 2D studies. The result of the first scatter is the configuration of Fig. 2c. The two strings appear to have been twisted relative to each other. The velocity of the string segments in the central region is now predominantly orthogonal to the original one. At the collision point, in particular, the velocity has no component in the original incoming direction. Globally, the magnitude of the velocity in the orthogonal direction diminishes as one goes outwards along the strings, while the component in the original direction increases approaching its asymptotic value on the boundaries. Meanwhile, the segments with original impact parameter smaller than twice the width of the string remain under the effect of the interactions. The attractive forces are strongest at the center and bring together the string segments once more, now more softly. These then scatter again at $90^\circ$ but lack the initial large kinetic energy to re-emerge as individual $n = 1$ segments, Fig. 2d. While these events take place in the central region, the segments of string immediately further out orbit slowly around each other and fall inwards. This configuration creates optimal conditions for the $n = 2$ zipper to form.

The zipper then rapidly grows outwards at a large $\kappa$ speed, as seen in Figs. 2d and 2e. Figure 2e shows the configuration when the kinks at $\sigma = \pm \epsilon$ (see Fig. 1) reach the boundaries of the computational domain. After that point, because of the boundary conditions (boosted, unperturbed strings), the opening angle of the zipper increases. The angle formed by the two $n = 1$ segments of string at the $n = 2$ junction in Fig. 2e is $\approx 60^\circ$. This is larger than the small angle necessary to sustain the growth of the zipper. The motion at the junctions decelerates and ultimately stops. The result is an approximately static $n = 2$ bridge joining the original $n = 1$ strings. It is important to point out that because the $n = 2$ bridge has reached an approximately static state, there is no memory of the origin of its two $n = 1$ components. At this point, the unperturbed vortices at the edge of the computational domain act as the dominant force determining the dynamics of the zipper. Although in the present case, this is the result of our boundary conditions, in a realistic string network evolution, the forces acting on the zipper are due the dynamics of string segments much longer than the effective length participating in the collision. The consequence is the unpeeling of the $n = 2$ bridge, in such a way that the two original $n = 1$ strings re-emerge from the scatter as if they had gone through each other without having exchanged ends (see Fig. 2f). There is effectively no intercommutation!

We verified that intercommutation always takes place for type II strings, in identical circumstances. The onset of the zipper was also tested by performing several string collision simulations for different computational domain sizes. The results presented in Fig. 2 were obtained on a $128^2 \times 256$ mesh. We have performed identical evolutions on meshes of up to $180^2 \times 810$, maintaining the lattice spacing constant. The result of increasing the size of the computational domain was only a longer growth of the zipper. This is because the arrival of the string kinks at the boundaries, and the subsequent increase in the opening angle, is more delayed the larger the computational domain. Moreover, we explicitly observed the transition from the sequence of events involving the zipper...
per to the usual intercommutation by increasing the initial approach velocity and/or the angle $\alpha$. For an initial velocity $v = 0.1$, two strings effectively do not intercommute for angles smaller than $25^\circ$. For larger approach velocities the angle for which the transition between the two outcomes happens is smaller, e.g., about $\alpha = 20^\circ$ for $v = 0.15$. These results are in good agreement with the original estimates for the zipper formation [10].

The main source on uncertainty in [10] was that, due to the extended nature of the strings, it was difficult to estimate how much kinetic energy participates in the collision. The amount of string carrying relevant kinetic energy was then parameterized by a characteristic length, $l_{\text{eff}}$. Fig. 2, makes it possible to measure the length of string in the zipper. In units of the width of the string, it is about 30. Adopting this value for $l_{\text{eff}}$ we find transient velocities and angles very similar to the ones measured here. For $v = 0.15$ the resulting angle indicates that a smaller length of string (about $l_{\text{eff}} = 20$) carries relevant kinetic energy. This is compatible with a faster collision and the observed extent of the zipper.

In conclusion, our numerical experiments indicate that the outcome of the collision of two type I cosmic strings with sufficiently low scattering energy at small angles follows two stages. First, the two strings collide, and, due to the attractive forces between them, after a few local scatters at right angles, settle down to a local bound state, the zipper. Second, the zipper grows under the effect of the attractive interaction between segments of string at its two ends. The final fate of the zipper depends on whether the condition for zipper formation, $\xi > 1/e$, is violated (large collision angle and/or velocity). In our simulations, this violation was the result of the boundary conditions, but in a string network evolution it could be due to the string dynamics itself. In any case, if a zipper stops growing, its free ends will pull the strings apart and unzip the $n = 2$ bridge in the energetically most favorable way, namely the configuration that minimizes the overall string length. In our simulations, such configuration is that in which strings do not exchange ends. Whenever the kinetic energy involved in the collision is larger than the attractive potential between the strings the zipper cannot settle in and usual intercommuting takes place.

In closing, we note that the results obtained above correspond to very mildly type I strings, $\beta = 2$. It is very interesting that large relative couplings are not necessary for the zipper to form. The extent of the space of $v$ and $\alpha$ for which a zipper and no intercommutation will occur is dependent on the relative strength of the interactions $\beta$. The fraction of non-intercommuting collisions is a measure of how strongly type I a theory is. Type I theories are thought to display first order transitions, which in standard cosmological scenarios seem to be necessary at the GUT scale in order to account for the baryonic asymmetry of the Universe.

The consequences of the breakdown of the intercommutation rule for cosmic string network dynamics can be very important. Intercommutation, leading to the formation of small loops from long strings that can subsequently decay, is the feature that makes cosmic strings viable cosmologically in contrast to other topological defects. Loop production and decay provides the string network with an effective means of dissipation, preventing it from dominating the energy density of the Universe. In practice, given that $v$ and $\alpha$ must be small (the string coherent velocity measured in network evolutions, is $v \sim 0.15$ [13]), it seems likely that in a network of type I strings only a fraction of all collisions will result in no intercommutation. Then the string network will possess globally a less effective mechanism of producing loops and thereby losing energy. A slower approach to the expected universal scaling regime would probably follow, allowing for more string to be present at later times. It would be interesting to determine whether such a string network would permit viable structure formation scenarios, and, in what sense their predictions would be different.

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FIG. 2. Snapshots of the numerical evolution of the scattering of two type I cosmic strings with parameters $v = 0.1$, $\alpha = 25^\circ$ and $\beta = 2$. Contour levels represent 65% of the energy at the core of the strings. The viewing angles, with respect to the colliding velocity direction, are $30^\circ$ for (a-c), $60^\circ$ for (d) and (e), and $85^\circ$ for (f). The stars label the two ends of one of the two original strings.