A study on a new method for flexoelectric coefficient estimation of the flexoelectric unimorph sensing element

Seol Ryung Kwon and Yongrae Roh

1Korea Institute for Robot Industry Advancement, Daegu, Korea, 2Kyungpook National University, Daegu, Korea

Abstract In the flexoelectric sensing element using the bending mode, the estimation of the flexoelectric coefficient was investigated using 3-D stress/strain analysis and experiments. The proposed method uses the results (deformation and strain) from the finite element analysis (FEA). The estimated flexoelectric coefficients were compared with those obtained via the conventional method (Euler’s beam theory) under the assumption of the quasi 1-D stress field. The results show that the RMS value and standard deviation of the estimated flexoelectric coefficient for the 3-D stress-field case of the sensing element are 31.51 µC/m and 0.24 %, respectively. In addition, it is found that the flexoelectric coefficient obtained from the results of the 3-D stress analysis is 1.8 % smaller than that of the quasi-1-D stress analysis. Therefore, in order to obtain a more reliable flexoelectric coefficient in the sensing element, the results of the 3-D numerical stress analysis should be used for accurate estimation of the flexoelectric coefficient.

1. Introduction

In order to meet the severe requirements of the rapidly developing industry in different fields, a sensing mechanism using the flexoelectric effect was first introduced and researched by Kogan [1], as an alternative to the sensing mechanism using the piezoelectric effect. The piezoelectric effect produces the electric charge according to the induced stress/strain generated by mechanical inputs such as force or pressure. By contrast, the flexoelectric effect generates the electric charge according to the induced strain gradient.

The flexoelectric effect has several unique properties over other sensing mechanisms, such as the lack of aging problems and broad material choice. In particular, the scaling effect is the most outstanding feature of this sensing mechanism. This effect enables sensors to be micro/nano miniaturized, as the charge output is generated based on the strain gradient [2]. In general, there are three modes in which the flexoelectric effect can be exploited, namely, longitudinal, shear and bending modes [3]. Among these, the structure of the longitudinal and shear modes is quite robust, so these modes are more suitable for sensing high-strength incident signals. However, since the induced strain gradient is small, the charge output generated at the sensing element is also small.

On the other hand, structures in the bending mode can induce a high strain gradient, leading to a large flexoelectric output and high sensitivity [4]. In this respect, the bending mode is typically used to design several types of sensors where the amplitude of the incident signal is infinitesimal. As an example, the flexoelectric accelerometer and microphone [5, 6] use the bending mode for their sensing ability. In addition, Huang et al. [7] investigated a flexoelectric curvature sensor using the bending mode by attaching the sensor on the side surface of a target structure.

It is well known that the accuracy of a sensor depends on the linearity between the input signal strength and electric output. Until recently, for the sake of simplicity, the flexoelectricity of a sensing element was calculated using the analytical results of the stress and strain obtained...
under the assumption of the quasi 1-D stress field for the sensing element [8].

The following equation describes the flexoelectric effect.

\[ P_i = \mu_{12} \frac{\partial \varepsilon_i}{\partial x_j} \]  

(1)

where, \( P_i \) is the flexoelectric polarization, \( \mu_{12} \) is the polar tensor flexoelectric coefficient, \( \varepsilon_i \) is the elastic strain and \( x_j \) is the position coordinate.

As it can be deduced from the Eq. (1), the charge output is linearly proportional to the input pressure when it is under a 1-D stress field. However, the stress field in the sensing element is a 3-D problem in practical cases. So, it is natural that the actual distributions of stress and strain in the sensing element are quite different from those of the quasi-1-D analytical analysis, which can result in the discrepancy between the incident signal strength and measured charge output [9].

In these connections, for the sensing element of the unimorph mode, the present work proposes a new method for estimating the flexoelectric output and flexoelectric coefficient, which consists of using the results of the 3-D structural analysis and experiments with a given input displacement of the element. Furthermore, this work aims to determine the standard deviation of the flexoelectric coefficient with an initial displacement \( \delta \). This paper will be beneficial to obtain the flexoelectric coefficient \( \mu_{12} \) of the bending mode in a more accurate way. Consequently, it will be able to estimate the inputs with the accurate coefficient as a sensor.

Barium strontium titanate (Ba\(_{0.65}\)Sr\(_{0.35}\)TiO\(_3\), here referred to as BST) was used as the material for the sensing element [10]. BST is known to exhibit the highest flexoelectric coefficient [11-15] ever reported.

For the experiment, a laser vibrometer, piezoelectric actuator, charge amplifier and lock-in amplifier were used. The structural stress analysis was performed using ANSYS Workbench. The distribution of the strain gradient in the 3-D case was calculated using the results of stress and strain from FEA. The number of nodes and mesh type of the element was set to 3420 (38×18×5) and hexahedral, respectively.

### 2. Structural analysis

#### 2.1 Flexoelectricity and the sensing element

The flexoelectric effect is defined as the linear coupling between the mechanical strain gradient and electric polarization in dielectric materials. For the bending mode, it is expressed as [16].

\[ P_y = \mu_{12} \frac{\partial \varepsilon_y}{\partial y} \]  

(2)

where \( P_y \) is the flexoelectric polarization, \( \mu_{12} \) is the transverse flexoelectric coefficient, and \( \varepsilon_y \) is the elastic strain in the \( x \) direction, as shown in Fig. 1. Eq. (2) can be used to derive the charge output as.

\[ q_0 = \mu_{12} \int_0^L P_y dA \]  

(3)

where \( q_0 \) is the charge output induced by the flexoelectric effect.

Table 1. Material properties of BST and dimensions of the BST unimorph.

| Property               | Value |
|------------------------|-------|
| Young’s modulus (GPa)  | 153   |
| Poisson’s ratio         | 0.33  |
| \( l_x \) (mm)          | 10.7  |
| \( l_y \) (mm)          | 1     |
| \( l_z \) (mm)          | 2.54  |

In Table 1, the properties of the BST material [5] used as the sensing element and dimensions of the BST unimorph are listed. The coordinate system and boundary condition used in the numerical analysis of the unimorph-type sensing element with the bending mode are shown in Fig. 1. In particular, the boundary conditions for the numerical analysis are set at the interface of the unimorph sensing element.

Fig. 2 shows the arrangement of the experimental setup used to find the flexoelectric coefficient in the unimorph sensing element of the bending mode. Experiments are necessary to find the flexoelectric coefficient \( \mu_{12} \) for an arbitrary concentrative force acting at the center tip of the element. The procedure to obtain the flexoelectric coefficient which plays a key role in the estimation of the charge output for each displacement of the sensing element can be summarized as follows. Firstly, the displacement \( \delta \) generated by a piezoelectric actuator is measured via a laser vibrometer. Secondly, for the case of the measured displacement, a 3-D stress and strain analysis on the given unimorph sensing element is performed; next, the values of the integrated strain gradient term are calculated using the results of the stress analysis. Thirdly, the total charge output obtained applying the displacement \( q_0 \) is measured via a lock-in amplifier. Finally, the flexoelectric coefficients \( \mu_{12} \) for the unimorph sensing element can be obtained from the results...
of the numerical analyses in Eq. (3). Upon repeating all the steps of this process, the flexoelectric coefficients for the sensing element of the respective stress fields with various values can be obtained. Finally, the flexoelectric coefficient is calculated as the RMS value of the respective coefficients.

As an example, for the case of δ = 7.5 µm, the distributions of the stress σ along the x axis at various z values along the upper surface of the element are shown in Fig. 4. Note that ξ, η and ζ are normalized coordinates (ξ = x/L, η = y/L, and ζ = z/L). For simplicity, the bars on the variables will be omitted here. The average stress along the x axis on the upper surface is \( \sigma_{\text{avg}} = \frac{1}{n} \sum_{i=1}^{n} \sigma_i \), where n is the maximum number in the nodal point of z at an arbitrary x. The 1-D result shows the stress obtained under the assumption of the quasi 1-D stress field in the sensing element. As can be seen from the figure, regardless of the z value, the stress σ decreases with the increase of x. Moreover, for the same x value, due to the influence of the distance from the acting point of the concentrative force, the stress along the z direction decreases with the increase of z. On the other hand, the trends for both the 1-D and average 3-D stresses fields in the sensing element are quite similar. However, for x ≤ 0.1 the stress distribution obtained under the assumption of the quasi-1-D stress field is quite different from the respective stress for z obtained via 3-D numerical analysis. Specifically, the stress at z = 0.0 is 26 % larger than that at z = 1.0 on the upper surface. It is thus possible to infer that the charge outputs for the quasi-1-D and actual-3-D stress fields must be in disagreement.

2.3 Strain

Fig. 5 shows the distributions of the strain \( \varepsilon_x \) along the x axis on the upper surface of the element under the same conditions of Fig. 4. Regardless of the z value, the strain \( \varepsilon_x \) increases with increasing x at first, reaches a maximum, and
then decreases with a further increase of $x$. In agreement with the stress distributions in Fig. 4, the maximum strain $\varepsilon$ increases with decreasing $z$. Due to the large differences in strain obtained under the assumptions of the quasi-1-D and actual-3-D stress analyses, it can be reconfirmed that, for the sake of accuracy, the 3-D numerical stress analysis should be used to determine the flexoelectricity coefficient.

### 2.4 Strain gradient

For $x = 0.1$, due to the influence of the distance from the acting point of the concentrative force, the strain gradient increases with decreasing $z$. As an example, the strain gradient for $z = 0.0$ and $x = 0.25$ is 16% larger than that for $z = 1.0$. It can thus be concluded that the effect of the deformation of the element should be taken into account for the exact calculation of the charge output.

Fig. 7 shows the estimated flexoelectric coefficients using both methods, quasi 1-D and 3-D stress fields. Table 2 shows the experimental results of the charge output for given displacements. In addition, with these experimental results, the coefficients estimated by two methods are listed. As can be seen from the figure and table, for the same experimental charge output (which corresponds to $\delta$), the average flexoelectric coefficient for the 3-D case is 1.8% smaller than that of the 1-D case. On the other hand, it is found that the standard deviations in the flexoelectric coefficients. It may be caused by measurement errors, however, from the fact that the standard deviation of 3-D case is smaller than that of 1-D case, it can be deduced that the proposed method for estimating the flexoelectric coefficient is more accurate.

### 3. Conclusions

A new method was proposed to estimate more accurately the flexoelectric coefficient of the unimorph typed sensing elements. This method relies on finite element analysis (FEA) and numerical calculation. Compared to the previously used method which is based on the calculation under the assumption of 1-D stress/strain field, the present method offers more accurate flexoelectric coefficient estimation. Estimated flexoelectric coefficient using quasi 1-D and 3-D stress fields calculated from the experimental results are compared. The average flexoelectric coefficient of the 3-D case is 1.8% smaller than that of the 1-D case. It can be deduced that this
discrepancy comes from the fact that Euler’s beam theory cannot reflect the real stress field. In addition to that, in spite of the measurement error, less standard deviation (0.24 %) of the coefficient represents the fact that the present method offers more accuracy in the estimation of the flexoelectric coefficient. From these findings, it can be concluded that to obtain a more reliable flexoelectric coefficient, the results of the 3-D numerical stress analysis should be used.

Acknowledgments

This work was supported by Basic Science Research Program through the National Research Foundation of Korea (NRF) funded by the Ministry of Science, ICT & Future Planning (NRF-2017R1A2B4009049).

Nomenclature

\( \delta \) : Displacement at the center tip of the element
\( l_x \) : Length of the unimorph
\( l_y \) : Thickness of the unimorph
\( l_z \) : Half width of the unimorph
\( q_0 \) : Charge output
\( P_y \) : Polarization in y direction
\( \mu_{12} \) : Transverse flexoelectric coefficient

References

[1] S. M. Kogan, Piezoelectric effect during inhomogeneous deformation and acoustic scattering of carriers in crystals, Soviet Physics—Solid State [Translation of Fizika Tverdogo Tela (Leningrad)], 5 (10) (1964) 2069-2070.
[2] W. Huang, K. Kim, S. Zhang, F.-G. Yuan and X. Jiang, Scaling effect of flexoelectric (Ba,Sr)TiO3 microcantilevers, *Physica Status Solidi Rapid Research Letters*, 5 (9) (2011) 350-352.
[3] X. Jiang, W. Huang and S. Zhang, Flexoelectric nanogenerator: materials, structures and devices, *Nano Energy*, 2 (6) (2013) 1079-1092.
[4] S. R. Kwon, W. Huang, S. Zhang, F.-G. Yuan and X. Jiang, Study on a flexoelectric microphone using barium strontium titanate, *Journal of Micromechanics and Microengineering*, 26 (4) (2016) 045001.
[5] W. Huang, S. R. Kwon, F.-G. Yuan, S. Zhang and X. Jiang, A flexoelectric micro-accelerometer, *ASME 2012 International Mechanical Engineering Congress and Exposition*, American Society of Mechanical Engineers (2012) 597-603.
[6] S. R. Kwon, W. Huang, S. Zhang, F. G. Yuan and X. Jiang, Study on a flexoelectric microphone using barium strontium titanate, *Journal of Micromechanics and Microengineering*, 26 (4) (2016) 045001.
[7] W. Huang, X. Yan, S. R. Kwon, S. Zhang, F.-G. Yuan and X. Jiang, Flexoelectric strain gradient detection using Ba 0.64 Sr 0.36 TiO3 for sensing, *Applied Physics Letters*, 101 (25) (2012) 252903.
[8] S. R. Kwon, W. Huang, S. Zhang, F.-G. Yuan and X. Jiang, Flexoelectric sensing using a multilayered barium strontium titanate structure, *Smart Materials and Structures*, 22 (11) (2013) 115017.
[9] S. R. Kwon, A study on the flexoelectric coefficient in the frustum pyramid sensing element with the rigid laminates at the upper and bottom surfaces, *Transactions of the Korean Society of Mechanical Engineers*, 44 (1) (2020) 1-6.
[10] W. Ma and L. E. Cross, Flexoelectric polarization of barium strontium titanate in the paraelectric state, *Applied Physics Letters*, 81 (18) (2002) 3440-3442.
[11] B. Chu, W. Zhu, N. Li and L. Eric Cross, Flexure mode flexoelectric piezoelectric composites, *Journal of Applied Physics*, 106 (10) (2009) 104109.
[12] W. Ma and L. E. Cross, Flexoelectric effect in ceramic lead zirconate titanate, *Applied Physics Letters*, 86 (7) (2005) 072905.
[13] W. Ma and L. E. Cross, Flexoelectricity of barium titanate, *Applied Physics Letters*, 88 (23) (2006) 232902.
[14] W. Ma and L. E. Cross, Observation of the flexoelectric effect in relaxor Pb(Mg1/3Nb2/3)O3 ceramics, *Applied Physics Letters*, 78 (19) (2001) 2920-2921.
[15] W. Zhu, J. Y. Fu, N. Li and L. Cross, Piezoelectric composite based on the enhanced flexoelectric effects, *Applied Physics Letters*, 89 (19) (2006) 192904.
[16] S. R. Kwon, Study on the flexoelectric characteristics in the sensing element of a duplex frustum pyramid, *Journal of Mechanical Science and Technology*, 32 (12) (2018) 5839-5843.

Seol ryung Kwon received the Ph.D. degree in mechanical engineering from North Carolina State University in 2014. Currently, she is a senior researcher in Korea Institute for Robot Advancement, Daegu, Korea. Her major research interests include design and fabrication of flexoelectric sensors, mechatronics and micro manufacturing and standardization for intelligent robots.

Yonggare Roh received his B.S. and M.S. degrees from Seoul National University, Korea, in 1984 and 1986, respectively. He got his Ph.D. degree in Engineering Science and Mechanics from the Pennsylvania State University, USA, in 1990. From 1990 to 1994, he worked in the Research Institute of Industrial Science & Technology, Korea, as a senior research scientist. He joined Kyungpook National University, Korea, in 1994 and now is a Professor in the School of Mechanical Engineering. His major research area includes development of piezoelectric devices, medical ultrasonic transducers, and acoustic transducers for underwater SONAR systems.