Weighted Scale-Free Network Properties of Ecological Network

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Abstract. We investigate the scale-free network properties of the bipartite ecological network, in particular, the plant-pollinator network. In plant-pollinator network, the pollinators visit the plant to get the nectars. In contrast to the other complex network, the plant-pollinator network has not only the trophic relationships among the interacting partners but also the complexities of the coevolutionary effects. The interactions between the plant and pollinators are beneficial relations. The plant-pollinator network is a bipartite and weighted network. The networks have two types of the nodes: plant and pollinator. We consider the visiting frequency of a pollinator to a plant as the weighting value of the link. We defined the strength of a node as the sum of the weighting value of the links. We reported the cumulative distribution function (CDF) of the degree and the strength of the plant-pollinator network. The CDF of the plants followed stretched exponential functions for both degree and strength, but the CDF of the pollinators showed the power law for both degree and strength. The average strength of the links showed the nonlinear dependence on the degree of the networks.

1. Introduction
The ecological networks are good examples of the complex networks. The scaling invariance is observed in the ecological networks. There are many types of ecological networks such prey-predator networks, plant-pollinator networks, host-parasite networks, plant-seed disperser networks etc. In the food web the degree distribution function of the network followed some functional forms like the power law, stretched exponential function, exponential function, uniform function etc.[1,2]. Many studied are oriented to the network properties of the binary ecological networks[1,2]. However, some ecological networks are bipartite networks with two types of the components such as the mutualistic network and host-parasite networks [3-11].

In this work we consider the mutualistic networks, in particular, the plant-pollinator networks. In these networks, the pollinators obtain their diet from the plants and help pollinate the plants. Such connections give the two species beneficial mutualistic relations. In these networks, the degree and strength distribution function for the pollinators followed the different functional dependence in comparison to those for the plants. In the plant-pollinator network, a pollinator visits frequently some plants, it visits rarely other plants. The ecologists had recorded the visiting frequencies of a pollinator to a plant at a region in a fixed season by the field experiments [3]. We consider the visiting frequencies as the interacting strength between two species. Therefore, the plant-pollinator networks act like the weighted complex networks.

In this work we analyze the seven plant-pollinator networks. It includes boreal forest, alpine forest, montane forest, beach forest, meadow, and upland grassland food web. We reported the network properties in section 3. We consider the nonlinear dependence of the plant-pollinator networks in section 4. We give the conclusions in section 4.
Table 1. Quantitative properties of the seven mutualistic plant-pollinator networks.

| Food Web  | Type            | $N_{pl}$ | $N_{po}$ | $C$   | $<k>$ | $<s>$ |
|-----------|-----------------|----------|----------|-------|-------|-------|
| Bar1987   | Boreal forest   | 12       | 102      | 0.136 | 27.83 | 91.66 |
| Elb1999   | Alpine forest   | 23       | 118      | 0.104 | 20.7  | 33.3  |
| Ino1988   | Montane forest  | 42       | 91       | 0.073 | 13.38 | 69.48 |
| Kat1999   | Beach forest    | 93       | 679      | 0.019 | 25.94 | 51.44 |
| Mem1999   | Meadow          | 25       | 79       | 0.151 | 23.92 | 174.64|
| Oll2003   | Upland grassland| 9        | 56       | 0.204 | 22.89 | 132   |
| Sma1976   | Montane forest  | 13       | 34       | 0.319 | 21.69 | 152.62|

Figure 1. The cumulative probability distribution functions of the degree for the seven mutualistic ecological networks. The CDFs of the plant-pollinator networks follow the power law, $P_x(k) = k^{-\gamma_d}$, with the average degree exponent, $\gamma_d = 1.14(3)$. The solid line indicates the least square fits.

2. Network Properties of Plant-Pollinator Networks

We consider seven plant-pollinator mutualistic ecological networks as shown in table 1. The plant-pollinator networks are small number of nodes because the food web is limited in an area of the observing field. The seven networks include many types of ecological food web such as boreal forest, alpine forest, montane forest, beach forest, meadow, and upland grassland. The number of pollinators is bigger than the number of the plant in all seven ecological networks. The total number of species, $N = N_{pl} + N_{po}$, lines in the range from $N=45$ to $N=772$ in the seven plant-pollinator networks where $N_{pl}$ is the number of the plants and $N_{po}$ is the number of the pollinators. The pollinators visit the plants to get their diets. We define the visiting frequencies, $w_{ij}$, of a pollinator $j$ to a plant $i$. This visiting frequencies act as the weights in the network. We summarize the network properties of the seven networks in table 1 [11-17]. The connectance of the network is defined by $C = \sum k_i / N_{pl}N_{po}$, where $k_i$ is the total number of connecting links of the node $i$. The average degrees of each network are large and the connectances are large compared to the other networks [1,2]. The food web, Ino1988, has the small average degree $<k>=13.38$ among the networks. The strength of the node $i$ in the
network is defined as $s_i = \sum_{j} w_{ij}$, where $N_i$ is the number of nearest neighbours of the node $i$. The average strength of the networks is also very large. However, the average strength lies in the range between 1.609 for food web, Elb1999 and 7.301 for food web, Mem1999. The average shortest path length between two nodes is around 3.0 in the networks. Figure 1 shows the cumulative probability density function (CDF) of the node degree for seven mutualistic ecological networks. The CDF follows a power law, $P_\gamma \sim k^{-\gamma_d}$, up to the cutoff degree $k < k_c$ with $\gamma_d = 1.1(1)$ for Bar1987, $\gamma_d = 1.33(7)$ for Elb1999, $\gamma_d = 0.9(1)$ for Ino1988, $\gamma_d = 1.25(8)$ for Kat1990, $\gamma_d = 1.04(5)$ for Mem1999, and $\gamma_d = 1.22(7)$ for Oll2003, $\gamma_d = 1.15(8)$ for Sma2002. We obtained the average exponent of the degree distribution as $\gamma_d = 1.14(3)$. When the total numbers of the nodes are increase, the range of the power law extends to the large value of the degree, i.e. the cutoff $k_c$ increases. The plant-pollinator networks show the scale-free network. The complex networks of the ecological bipartite systems give the beneficial relationships among the species and guarantee the stability and the robustness of the ecological networks.

Figure 2. The average weights, $\langle w_{ij} \rangle$, depend on the degree or strength of the end-point node. The average weights followed the power law, $\langle w_{ij} \rangle \sim (k_i k_j)^\theta$ for the degree (circle) and $\langle w_{ij} \rangle \sim (s_i s_j)^\delta$ for the strength (square). We obtained the exponents as $\theta = 1.22(9)$ and $\delta = 0.77(8)$.

3. Nonlinearity of Network

Figure 2 represents the dependence of the average weight, $\langle w_{ij} \rangle$ on the degree of the end-point node $k_i$ and $k_j$, and the strength of the end-point node $s_i$ and $s_j$. We observed the power law dependence of the average weights, $\langle w_{ij} \rangle \sim (k_i k_j)^\theta$ for the degree and $\langle w_{ij} \rangle \sim (s_i s_j)^\delta$ for the strength. The power law of the average weights indicates that the degree and strength
of the end-point node are strongly correlated each other. We observed the average exponents of the power law as $\theta = 1.22(9)$ for the degree and $\delta = 0.77(8)$ for the strength.

4. Conclusions
The network properties of seven plant-pollinator ecological networks were studied. The CDFs followed power law for both degree and strength. Such different distributions are a unique property of ecological networks. The plants adopt a different strategy compared with the pollinators in the interactions between them. The average strengths of the nodes showed nonlinear dependence on the degree or the strength of the mutualistic network.

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