Energy dependence of $K^-\cdot \text{“pp”}$ effective potential derived from coupled-channel Green’s function

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Abstract We investigate the energy dependence of a single-channel effective potential between the $K^-$ and the “$pp$”-core nucleus, which can be obtained as an $K^-\cdot \text{“pp”}$ equivalent local potential from a coupled-channel model for $\bar{K}(NN)-\pi(\Sigma N)$ systems. It turns out that the imaginary part of the resultant potential near the $\pi\Sigma N$ decay threshold can well approximate the phase space suppression factor of $K^- pp \rightarrow \pi\Sigma N$ decay modes. The effects on the pole position of the $\pi(\Sigma N)$ state in the $\pi\Sigma N$ channel are also discussed.

Keywords Kaonic nuclei · Equivalent local potential

1 Introduction

In order to elucidate the nature of antikaon-nucleon ($\bar{K}N$) interaction in high density nuclear matter, it is an important subject to clarify whether the “deeply-bound kaonic nucleus” exists or not. In particular, the $|K \otimes \{NN\}_{f=1/2}, J^p = 0^- \rangle$ bound state, which is called “$K^- pp$” here, is suggested to be the lightest and most fundamental kaonic nucleus [1]. A new experimental search of $K^- pp$ via the $^3\text{He}(\text{in-flight} K^- , n)$ reaction has been planned at J-PARC as E15 experiment [2]. We theoretically have discussed the expected inclusive and semi-exclusive spectra for the $^3\text{He}(\text{in-flight} K^- , n)$ reaction within the framework of the distorted-wave impulse approximation (DWIA). We have employed a phenomenological single-channel $K^-\cdot \text{“pp”}$ (complex) effective potential between the $K^-$ and the “$pp$”-core nucleus [3,4], which has the form of

$$U_{\text{eff}}(E;r) = (V_0 + iW_0 f(E)) \exp[-(r/b)^2],$$

(1)

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where $V_0$ and $W_0$ are strength parameters in real and imaginary parts, respectively, and $b$ is the range parameter. $f(E)$ is the phase space suppression factor of the $K^-pp \to \pi\Sigma N$ decay modes \[5\] where $E$ is the energy measured from the $K^-pp$ threshold.

In the single-channel framework, we have shown that behavior of the $^3$He(in-flight $K^-$, $n$) spectrum can be understood in the “moving pole” picture \[4\]; a pole of the $K^-pp$ bound state moves in the complex energy plane as a function of the energy $E$ on the real axis because the $K^-pp$ effective potential is considerably energy-dependent. A trajectory of its moving pole governs the shape of the spectrum. Thus, the validity of our calculations relies in part on whether the energy dependence of Eq.\((1)\) is appropriate near the $\pi\Sigma N$ decay threshold or not. To examine this subject, we have extended the previous $K^-pp$ single channel description to the $\tilde{K}(NN)$-$\pi(\Sigma N)$ coupled-channel (CC) description because the energy dependence of Eq.\((1)\) should originate from eliminating the $\pi\Sigma N$ channel in such a CC scheme.

In this article, we investigate a single-channel $K^-pp$ equivalent local potential which is derived from CC Green’s functions for $K^-pp$-$\pi(\Sigma N)$ systems, and evaluate the energy dependence of this effective potential to be compared with a phenomenological one \[4\] which is determined within Eq.\((1)\).

2 Coupled-channel model

Let us consider the following CC Green’s function $G_{ij}$ for $K^-pp$-$\pi(\Sigma N)$ systems;
\[
\begin{bmatrix}
E_1 - T_{11}^{(l)} - U_{11}(r) & -U_{12}(r) \\
-U_{21}(r) & E_2 - T_{22}^{(l)} - U_{22}(r)
\end{bmatrix}
\begin{bmatrix}
G_{11}^{(l)}(r,r') & G_{12}^{(l)}(r,r') \\
G_{21}^{(l)}(r,r') & G_{22}^{(l)}(r,r')
\end{bmatrix}
= \delta(r'-r)1, \tag{2}
\]
where the channel 1 (2) refers to the $K^-$ and $pp$-core system (the $\pi$ and $\Sigma$-core system); $T_{12}^{(l)}$ denote the kinetic energies for 1, 2, and the energy $E_2 = E_1 - E_{th}(\pi\Sigma N) \approx -100$ MeV below the $K^-pp$ threshold. $U_{11}$, $U_{22}$ and $U_{12}$ denote the diagonal and coupling potentials, respectively. For simplicity, we assume an energy-independent Gaussian form as
\[
\begin{bmatrix}
U_{11}(r) & U_{12}(r) \\
U_{21}(r) & U_{22}(r)
\end{bmatrix}
= \begin{bmatrix}
V_1 + iW_1 & V_c \\
V_c & V_2 + iW_2
\end{bmatrix}
\exp\left[-(r/b)^2\right], \tag{3}
\]
where $V_{1,2}$ and $V_c$ denote the strength parameters of the diagonal and coupling potentials, respectively, and $W_{1,2}$ describe effects of the other decay modes. These parameters in Eq.\((3)\) should be determined so as to reproduce the proper values of the binding energy and width which are obtained by single-channel calculations with $U_{eff}$ in Eq.\((1)\). If we use CC Green’s functions in Eq.\((2)\), we expect to confirm the previous results \[3\],\[4\] for the $^3$He(in-flight $K^-$, $n$) spectrum within the DWIA calculation.

Now we define the $K^-$“$pp$” equivalent local potential for channel 1, $U_{11}^{(l)}$ as
\[
\{E_1 - T_{11}^{(l)} - U_{11}^{(eff)}(E;r)\} \ G_{11}^{(l)}(E;r,r') = \delta(r'-r), \tag{4}
\]
where $G_{11}^{(l)}$ is the (1,1) component of a solution of Eq.\((3)\). Then, we get
\[
U_{11}^{(eff)}(E;r) \ G_{11}^{(l)}(E;r,r') = U_{11}(r) \ G_{11}^{(l)}(E;r,r') + U_{12}(r) \ G_{21}^{(l)}(E;r,r'). \tag{5}
\]
By multiplying the initial wave function $\phi_1(r')$ as a bound state in channel 1, and by integrating over $r'$, we obtain the expression of the equivalent local potential as

$$\tilde{U}_{11}^{\text{eff}}(E;r) = U_{11}(r) + U_{12}(r) \frac{\int_{0}^{\infty} G_{21}^{(l)}(E;r,r') \phi_1(r') dr'}{\int_{0}^{\infty} G_{11}^{(l)}(E;r,r') \phi_1(r') dr'},$$

which is defined under the boundary condition at every $E$ on the physical axis, because Green’s functions can be calculated numerically at every points in the complex energy plane. This is the advantage that we used Green’s functions rather than wave functions.

### 3 Results and discussion

Figure 1 shows the calculated equivalent local potential $\tilde{U}_{11}^{\text{eff}}$ with $V_1 = -300$ MeV, $V_2 = -150$ MeV, $V_c = -100$ MeV, $W_1 = W_2 = 0$ MeV and $b = 1.09$ fm, in which the potential parameters are determined so that the binding energy and width of $K^-pp$ bound state are consistent with those calculated by Yamazaki and Akaishi. We find that the strength of the imaginary part of $\tilde{U}_{11}^{\text{eff}}$ becomes shallower as $E$ goes...
Fig. 2 Energy dependence of the imaginary part $W_{\text{eff}}(E)$ of the $K^-\,(pp)$ equivalent local potential $\tilde{U}_{11}^{\text{eff}}$. The solid and dashed lines denote the cases with $V_2 = -150 \text{ MeV}$ and $-300 \text{ MeV}$ in Eq. (3), respectively. The dotted line denotes values obtained by $W_0 \times f(E)$ with $W_0 = -65 \text{ MeV}$.

This behavior is approximately equivalent to that of the energy dependence obtained from the phase space factor in Eq. (1), whereas the shape of $\tilde{U}_{11}^{\text{eff}}$ is not a Gaussian function exactly. Therefore, we estimate the potential strength of $(V_{\text{eff}}(E), W_{\text{eff}}(E))$ which is obtained with the help of the volume integrals;

$$V_{\text{eff}}(E) + iW_{\text{eff}}(E) = \int_0^\infty \tilde{U}_{11}^{\text{eff}}(E;r) r^2 dr \int_0^\infty \exp\left[-(r/b)^2\right] r^2 dr. \quad (7)$$

Figure 2 shows values of $W_{\text{eff}}(E)$ as a function of the energy $E$, together with the fitted curve of $W_0 \times f(E)$ in Eq. (1). The results are as follows:

- If no bound state exists in the $\pi(\Sigma N)$ channel ($V_2 = -150 \text{ MeV}$), $\text{Im} \tilde{U}_{11}^{\text{eff}}$ approximates to the phase space factor $f(E)$ that is used in Eq. (1) near the $K^-pp \rightarrow \pi\Sigma N$ decay threshold.
- If a bound state exists in the $\pi(\Sigma N)$ channel ($V_2 = -300 \text{ MeV}$), the energy dependence of $\text{Im} \tilde{U}_{11}^{\text{eff}}$ considerably differs from that of Eq. (1) due to modification of the phase volume via a pole which is located near the $\pi\Sigma N$ threshold.

In summary, we have investigated the energy dependence of the single-channel $K^-\sim pp$" equivalent local potential $\tilde{U}_{11}^{\text{eff}}$ derived from the $\bar{K}(NN)-\pi(\Sigma N)$ model. It has turned out that the imaginary part of $\tilde{U}_{11}^{\text{eff}}$ near the $\pi\Sigma N$ decay threshold can well approximate the phase space suppression factor of $K^-pp \rightarrow \pi\Sigma N$ decay modes. If potential parameters in Eq. (3) are replaced by the energy-dependent ones, as obtained from chiral $\bar{K}N-\pi Y$ dynamics [6], their energy dependence would additionally contribute to the effective potential. The further detailed investigation is now in progress.

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