FERMIONS UNDER STRONG REPULSION: 
FROM MULTICONNECTED FERMI SURFACES 
TO FERMI CONDENSATION

Yu. G. Pogorelov and V. R. Shaginyan

Centro de Física do Porto, Universidade do Porto, 4169-007 Porto, Portugal
and
Petersburg Nuclear Physics Institute, RAS, 188300 Gatchina, Russia

1. INTRODUCTION

The studies of strongly interacting fermion systems are of considerable interest in solid state physics, nuclear physics, astrophysics, and so on. It is well known that many of the basic properties result already from the notion of the quasiparticles, at temperature $T = 0$, characterized by the step-like occupation function $n(p, \sigma) = n_F(p, \sigma) = \theta(p_F - p)$, so that the number density, $\rho = \sum_\sigma \int n(p, \sigma)dp/(2\pi)^3$, determines the Fermi momentum $p^3_F = 3\pi^2\rho$ [1].

In real physical systems, presence of various interactions between particles (by Coulomb forces, mesonic forces, van der Vaals forces, etc.) can essentially change the ground state energy and can rarely diversify the phenomenology of a Fermi system, preserving it as a Landau Fermi liquid. The Landau theory of normal Fermi liquids is based on the notions of the quasiparticles and the amplitudes $F$ which characterize the effective interaction between the quasiparticles [1]. It permits to describe all the observable physical properties in terms of the first and second variational derivatives of the energy functional $E[n(p, \sigma)]$ with respect to the occupation numbers $n(p, \sigma)$: $\varepsilon(p, \sigma) = \delta E/\delta n(p, \sigma)$ and $F(p, \sigma, p', \sigma') = \delta^2 E/\delta n(p, \sigma)\delta n(p', \sigma')$. As a result, the Landau theory has removed high energy degrees of freedom and kept a sufficiently large number of relevant low energy degrees of freedom to treat liquid’s low energy properties. Usually, it is assumed that the breakdown of the Landau theory is defined by the Pomeranchuk stability conditions and occurs when the Landau amplitudes being negative reach its critical value [2]. The new phase at which the stability conditions are restored can in principle be again described in the framework of the Landau theory. But it is not a general case. The most evident example is the
superconducting transition under arbitrarily weak attraction between fermions with opposite spins $\sigma$ and opposite momenta close to the Fermi momentum $p_F$ [3]. But one can also search for alternative scenarios of such restructuring produced by repulsive interactions, if strong enough. The quasiparticle dispersion law

$$\varepsilon(p,\sigma) = \frac{\delta E}{\delta n(p,\sigma)}$$  

(1)

satisfies the stability criterion: $v_F = [\partial\varepsilon(p,\sigma)/\partial p]_{p=p_F} > 0$. Then the minimum of the energy,

$$\delta \left( E - \mu \sum_{\sigma} \int n(p,\sigma) \frac{dp}{(2\pi)^3} \right) = \int (\varepsilon(p,\sigma) - \mu) \delta n(p,\sigma) \frac{dp}{(2\pi)^3} = 0,$$  

(2)

is reached (although with the chemical potential $\mu$ generally different from $p_F^2/(2m)$) still for $n = n_F$. This quasiparticle occupation should not be confused with the renormalization of real particle occupation function [4].

However one can also search for more general solutions of Eq. (2), $n \neq n_F$. Regarding the Pauli principle restriction $0 \leq n(p,\sigma) \leq 1$, these general solutions must satisfy the equation

$$n(p,\sigma)[1 - n(p,\sigma)](\varepsilon(p,\sigma) - \mu) = 0.$$  

(3)

The possibility for the last factor in the l.h.s. of Eq. (3) to be zero implies a possibility for the so-called Fermi condensate (FC), manifested by a flat dispersion law: $\varepsilon(p,\sigma) \equiv \mu$, and a fractional occupation $0 < n(p,\sigma) < 1$ within a certain finite spherical layer $p_i \leq p \leq p_f$ in the momentum space. Otherwise, the occupation function can only take the values 0 or 1, so that $\varepsilon(p,\sigma) < \mu$, $n(p,\sigma) = 1$ at $p < p_i$ and $\varepsilon(p,\sigma) > \mu$, $n(p,\sigma) = 0$ at $p > p_f$ [3,5-7]. But the full set of alternatives for the ground state includes, besides FC and Fermi ground state (FGS), also the so-called multiconnected Fermi states (MFS). This possibility was first recognized yet in the Hartree-Fock framework [8,9]. We will study such a possibility using model forms for the interaction: $F(p,\sigma,p',\sigma') = g_{\sigma,\sigma'}U_{\sigma,\sigma'}(q)$, with only relevant dependence on the momentum transfer $q = |p - p'|$ and some coupling constants $g_{\sigma,\sigma'}$.

Let us restrict consideration to uniform, isotropic, and spin-symmetric Fermi systems, where $\varepsilon(p,\sigma) = \varepsilon(p)$ (this is also the scope in which all the known FC models were considered). Then any MFS presents a sequence of fully occupied areas (we call them icebergs) where $\varepsilon(p) < \mu$, separated by empty spacers where $\varepsilon(p) > \mu$. Hence it can be uniquely labeled by the number $N$ of interfaces between icebergs and spacers: $\text{MFS}(N)$. When this number is odd, the $\text{MFS}(N = 2n - 1)$ consists of $n$ icebergs: $p_{2i-1} < p < p_{2i}$ ($i = 1, \ldots, n$), and $n$ spacers: $p_{2i-2} < p < p_{2i-1}$ (including a central spherical void $0 < p < p_1$). For even numbers, the $\text{MFS}(N = 2n)$ contains $n$ icebergs (including a central spherical island $0 < p < p_1$) and $n - 1$ spacers. Interfaces $p_i$ are fixed by the equilibrium conditions

$$\varepsilon(p_i) = \mu, \quad i = 1, \ldots, N,$$  

(4a)
and icebergs fit to the normalization condition:

$$\sum_{i=1}^{n} (p_{2i}^3 - p_{2i-1}^3) = 3\pi^2 \rho$$  \hspace{1cm} (4b)$$

(for odd $N$, the analogue for even $N$ is evident). In this context, FGS can be identified as MFS(0).

At least, the choice between the two different types of FGS restructuring with growing repulsion constant $g$ is uniquely defined by the analytic properties of the model potential $U(q)$, so that FGS$\rightarrow$FC scenario is only possible if this function (or its derivatives) have singular points at real axis in the complex $q$-plane, otherwise the infinite sequence of transitions FGS$\rightarrow$MFS(1)$\rightarrow$MFS(2)$\rightarrow$... takes place [10].

Despite such strict dichotomy between the two evolution routes of model fermion systems, several analytic and numerical studies [10,11] showed that their expected physical properties should become very close in this course. If the distinction between analytic and non-analytic potentials is not fully respected, one can even arrive at a wrong conclusion on possible MFS($N$) $\rightarrow$FC transition at some finite $N$ [12]. In fact, as will be shown below, the FC state is only reached in the limit of MFS($N \to \infty$) (for a given analytic $U(q)$). But aside of these formal issues, there remains a clear interest in practical conditions when the difference between the MFS and FC states cannot be ignored, permitting to use the MFS picture for explanation of various actual physical effects related to the behavior of heavy-fermion systems in magnetic fields [13,14]. The present work is just focused on a detailed analysis of common features between FC and advanced MFS($N \gg 1$) states. To this end, we use the analytic model $U(q) = 1/\sqrt{q^2 + q_0^2}$ which tends to the non-analytic FC model $1/q$ in the limit $q_0 \to 0$, to develop an effective approximation for the average occupation function in this limit and to establish the criteria on model parameters ($g$, $q_0$) and external parameters (as temperature) for the two states to be practically indistinguishable.

2. **ANALYTIC SOLUTION FOR THE $1/\sqrt{q^2 + q_0^2}$ MODEL**

Let us start from the model interaction

$$F(p, \sigma, p', \sigma') = \frac{g}{\sqrt{(p - p')^2 + q_0^2}}. \hspace{1cm} (5)$$

This form implies spherically symmetric and spin independent occupation: $n(p, \sigma) \to n(p)$, and we can present the energy functional as a 1D integral:

$$E[n(p)] = \frac{1}{\pi^2} \int p^2 n(p) \left[ \epsilon_0(p) + V(p)/2 \right] dp. \hspace{1cm} (6)$$

The potential energy $V(p)$ also enters the quasiparticle dispersion law:

$$\epsilon(p, \sigma) = \epsilon_0(p) + V(p), \hspace{1cm} (7)$$
here $\varepsilon_0(p) = p^2/(2m)$ is the kinetic energy of system’s particle with the bare mass $m$. $V(p)$ presents a function of $p$ and a functional of $n(p')$ given by a 1D integral:

$$V(p) = \frac{q}{\pi^2} \int p'^2 n(p') v(p, p') dp', \quad (8)$$

where the kernel $v(p, p')$ results from integration of the model potential $U(|p - p'|)$ in the solid angle between the vectors $p$ and $p'$:

$$v(p, p') = \frac{1}{2pp'} \int_{|p-p'|}^{p+p'} tU(t)dt. \quad (9)$$

In the limit $q_0 \to 0$, the model, Eq. (5), passes into the FC model $U_{FC}(q) = 1/q$ [5,7]. Recall briefly its main properties in the present notation. The kernel, Eq. (9), is here simply

$$v_{FC}(p, p') = 1/\max(p, p'), \quad (10)$$

At small enough $g$, when FGS still holds, this gives the explicit potential energy

$$V_{FC}(p) = \begin{cases} 
 3\xi p_F^2/(2m) - \xi p^2/(2m), & p < p_F, \\
 3\xi p_F^3/(mp), & p > p_F.
\end{cases} \quad (11)$$

When $\xi = gm/(3\pi^2) \to 1$, the dispersion in Eq. (7) vanishes for all occupied states, manifesting the onset of FC, which leads to the chemical potential $\mu_{FC}$ that is lower then the corresponding to FGS. For $\xi > 1$, the FC occupation function is: $n_{FC}(p) = \xi^{-1} \theta(p_f - p)$ with $p_f = p_F \xi^{1/3}$, thus keeping the quasiparticle energy constant: $\varepsilon(p) = 3p_f^2/(2m)$, within the whole occupied band $p < p_f$ ($p_i = 0$ in the $1/q$ model).

In the actual model, Eq. (5), the kernel takes a more complicate form

$$v(p, p') = \frac{\sqrt{(p + p')^2 + q_0^2} - \sqrt{(p - p')^2 + q_0^2}}{2pp'},$$

and using it in Eq. (8) at small enough $g$ when FGS holds, we calculate the potential energy produced by the fully occupied Fermi sphere of radius $p_F$:

$$V(p) = V_{p_F}(p) + V_{p_F}(-p), \quad (12)$$

where

$$V_{p_F}(p) = \frac{\xi}{2m} \left[ \frac{\sqrt{(P + p)^2 + q_0^2}}{p} \left( p^2 + q_0^2 + p \frac{P - p}{2} \right) - \frac{3q_0^2}{2} \arcsinh \frac{P + p}{q_0} \right].$$
Figure 1. Instabilities of Fermi states in the $\xi/\sqrt{q^2 + q_0^2}$ model at $q_0 = 0.1p_F$ with growing $\xi$. a) Dispersion law $\varepsilon(0)$ tends to $\mu$ from below at $\xi \rightarrow \xi_1 \approx 1.04293$. b) A void is growing in the center of filled Fermi sphere at $\xi_1 < \xi < \xi_2$, c) an island will appear in the center of the void at $\xi = \xi_2 \approx 1.04356$.

Note that, unlike Eq. (11), the form of Eq. (12) defines analytic behavior of $\varepsilon(p)$ for all real $p$. In presence of finite “screening parameter” $q_0$, the repulsion potential $U(q)$ is reduced compared to $U_{FC}(q)$, hence the critical value of $\xi$, when FGS fails, turns higher than 1. It is easy to see that $V(p)$ is an even function of $p$ with maximum at $p = 0$ and satisfies the inequality: $V(p) > V(0) + p^2V''(0)/2$. Hence the instability of FGS in this model occurs just in the center of the Fermi sphere, when $\varepsilon(0) \rightarrow \varepsilon(p_F) = \mu$ (Fig. 1a). As discussed below, this instability is the first in an infinite series, and the above condition involves the corresponding values of potential energy:

$$V(0) = \frac{3\xi}{2m} \left( p_F \sqrt{p_F^2 + q_0^2} - q_0^2 \arcsinh \frac{p_F}{q_0} \right),$$

and

$$V(p_F) = \frac{\xi}{2m} \left[ (p_F^2 + 2q_0^2) \sqrt{4p_F^2 + q_0^2} - 3q_0^2 \arcsinh \frac{2p_F}{q_0} - \left( \frac{q_0}{p_F} \right)^2 \right].$$

Using them in Eq. (1) we obtain the first critical value of $\xi$ as a function of $q_0$:

$$\xi_1(q_0) = \left[ 3 \sqrt{p_F^2 + q_0^2} - \frac{(p_F^2 + q_0^2) \sqrt{4p_F^2 + q_0^2}}{p_F^3} \right. \left. + \frac{3q_0^2}{2p_F^2} \left( \arcsinh \frac{2p_F}{q_0} - 2 \arcsinh \frac{p_F}{q_0} \right) + \left( \frac{q_0}{p_F} \right)^3 \right]^{-1}. \quad (13)$$

It behaves as

$$\xi_1(q_0) \approx 1 + \left( \frac{3q_0}{2p_F} \right)^2 \ln \left( \frac{p_F}{q_0} \right) \quad (14)$$

in the important limit $q_0 \ll p_F$ and as $\xi(q_0) \approx (q_0/p_F)^3$ at $q_0 \gg p_F$. At $\xi \rightarrow \xi_1(q_0)$, the dispersion law in the center is characterized by a negative effective mass: $m^* =$
$1/\varepsilon''(0) = m/[1 - \xi_1(q_0)]$ (insert to Fig. 1a). When $\xi$ exceeds $\xi_1(q_0)$, a spherical void of small radius $p_1$ opens in the center, followed by the first iceberg $p_1 < p < p_2$ (Fig. 1b). This corresponds to the FGS $\rightarrow$ MFS(1) transition, and the values of $p_1$ and $p_2$ result from the equilibrium and normalization conditions, Eqs. (4). Their numerical solution shows that the void radius extends with $\xi$, and in this course the negative effective mass diminishes and changes to positive. Eventually this again leads to an instability in the center: $\varepsilon(0) \rightarrow \varepsilon(p_1) = \varepsilon(p_2) = \mu$, when $\xi$ attains the next critical value $\xi_2(q_0) > \xi_1(q_0)$ (Fig. 1c). But in this case a filled spherical island (second iceberg) emerges in the center of the void, and the system passes to MFS(2). With growing $\xi$, the alternating formation of voids and islands in the center continues infinitely, and the number of icebergs increases very rapidly, making the numerical analysis of exact conditions, Eqs. (4), practically impossible.

However, an effective asymptotical treatment can be proposed for advanced MFS($N \gg 1$), suggested by the general $\xi - q_0$ phase diagram. Considering the monotonically growing function $\xi_1(q_0)$, Eq. (13), and the fact that $\xi_{N+1}(q_0) > \xi_N(q_0)$, we reasonably expect that MFS($N \gg 1$) are reached with decreasing screening parameter $q_0$ at fixed $\xi > 1$. The averaged characteristics of such state should be close to the FC characteristics for the same $\xi$. Below we develop an effective description of MFS($N \gg 1$) using this closeness and prove by the obtained results that such approximation is asymptotically correct.

3. EFFECTIVE DESCRIPTION OF MFS($N \gg 1$)

We expect that, for an MFS($N \gg 1$) at given $\xi > 1$ and $q_0 \ll p_f$, the average occupation function

$$n_{av}(p) = \int_{p-\Delta}^{p+\Delta} n(p') dp'$$  \hspace{1cm} (15)

(at $p_f \gg \Delta \gg q_0$), is close to the corresponding FC function $n_{FC}(p) = \xi^{-1} \theta(p_f - p)$, constant over the whole range $[0, p_f]$. Therefore we model the “microscopic” MFS($N$) function $n(p)$ by a certain effective function $n_{eff}(p)$. In vicinity of any $0 \leq p \leq p_f$ it consists in a sequence of icebergs of width $\approx \delta(p)/\xi$ with spacers of width $\approx (1 - 1/\xi)\delta(p)$ (Fig. 2). The local period $\delta(p)$ is supposed to be a slow function of $p$ (such that $\delta'(p) \ll 1$) chosen in order to satisfy the equilibrium conditions, Eqs. (4), for the effective dispersion law $\varepsilon_{eff}(p)$. Then the overall multiplicity is asymptotically given by

$$N = \int_{0}^{p_f} \frac{dp}{\delta(p)}.$$  \hspace{1cm} (16)

We present the effective potential energy as:

$$\varepsilon_{eff}(p) - \varepsilon_0(p) = V_{eff}(p) = \sum_{i=1}^{N} V_i(p),$$  \hspace{1cm} (17)

where $i$th iceberg with the center in $P_i$ (Fig. 2) contributes by:

$$V_i(p) = \frac{3\xi}{m} \int_{-d_i/2\xi}^{d_i/2\xi} (P_i + x)^2 v(p, P_i + x) dx,$$  \hspace{1cm} (18)
\[ \delta_i = \delta(P_i). \]  The most important step consists in separating from \( V_{\text{eff}}(p) \) the FC function \( V_{\text{FC}}(p) \), so that \( \varepsilon_{\text{eff}}(p) = \mu_{\text{FC}} + \delta V(p) \). Then the small functional \( \delta V(p) = V_{\text{eff}}(p) - V_{\text{FC}}(p) \) can be naturally approximated by the linear terms in (effectively) small differences \( \delta n(p) = n_{\text{eff}}(p) - n_{\text{FC}}(p) \) and \( \delta v(p, p') = v(p, p') - v_{\text{FC}}(p, p') \):

\[ \delta V(p) = \delta V_1(p) + \delta V_2(p), \]

where

\[
\begin{align*}
\delta V_1(p) &= \frac{3}{m} \int_0^{p_f} p'^2 \delta v(p, p') dp', \\
\delta V_2(p) &= \frac{3 \xi}{m} \left[ \int_0^p p'^2 \delta n(p') dp' + \int_p^{p_f} p' \delta n(p') dp' \right].
\end{align*}
\]

(19)

A direct calculation shows that \( \delta V_1(p) \) is a negative monotonously growing function such that

\[
\delta V_1(0) \approx -\frac{3 q_0^2}{2m} \ln \frac{pf}{q_0}, \quad \delta V_1(p_f) \approx \frac{\delta V_1(0)}{2},
\]

and \( V_1(p) \to 0 \) at \( p \gg p_f \). When integrating in \( \delta V_2(p) \) over each interval \( \delta_i \) centered in \( P_i \), we first suppose that the width of \( i \)th iceberg is exactly \( \delta_i/\xi \). Under this condition, called the geometric equilibrium (Fig. 2a), the simple rules hold:

\[
\begin{align*}
\int_{-\delta_i/2}^{\delta_i/2} \delta n(P_i + x) dx &= 0, \\
\int_{-\delta_i/2}^{\delta_i/2} \delta n(P_i + x) x dx &= 0, \\
\int_{-\delta_i/2}^{\delta_i/2} \delta n(P_i + x) x^2 dx &= \frac{\delta_i^2}{2} - \frac{\xi^3}{2}. \tag{20}
\end{align*}
\]

At a special choice \( \xi = 2^{1/3} \), also the last line in Eq. (20) turns zero, and the contributions to \( V_2(p) \) from all the intervals vanish, except for that containing \( p \) itself. Adopting the above choice, we obtain the only contribution to \( \delta V_2(p) \) from the interval \( P_i - \delta_i \leq p \leq P_i + \delta_i \):

\[
V_{\text{osc}}(p) = \frac{3 \xi}{m} \left[ \frac{P_i + x}{p} \theta(p - P_i - x) + \theta(P_i + x - p) \right] \times \\
\times \left[ \theta\left(\frac{\delta_i^2}{2 \xi^2} - x^2\right) - \frac{1}{\xi} \right] (P_i + x) dx. \tag{21}
\]

The totality of such contributions defines a continuous positive function, reaching zero between the icebergs: \( V_{\text{osc}}(P_i \pm \delta_i/2) = 0 \), and maximum at their centers:

\[
V_{\text{osc}}(P_i) = V_{\text{max}}(p) \approx \frac{3 \delta^2(p)(\xi - 1)}{4m\xi^2},
\]
Figure 2. a) Condition of geometric equilibrium between icebergs and spacers, b) Physical equilibrium between iceberg distribution and dispersion law at slow variation of period.

so that its fast oscillating behavior can be modelled by a simple expression:

$$V_{osc}(p) = \frac{V_{max}(p)}{2} \left[ 1 + \cos \frac{2\pi p}{\delta(p)} \right].$$

(22)

However, the equilibrium for MFS requires that the minima of $\varepsilon_{eff}(p)$ be located within icebergs and its maxima within spacers, opposite to what happens with $V_{osc}(p)$ at geometric equilibrium (Fig. 2a). This misfit can be corrected with small correlated shifts of iceberg boundaries from the geometric equilibrium, described by a finite gradient $\delta'(p)$. Then non-zero contributions to $\delta V_2(p)$ appear from all the intervals, giving rise to its slow component $V_{slow}(p) \sim \delta'(p)p_f^2/m$ while the fast component $V_{osc}(p)$ only obtains a certain phase shift (see below). For this situation, we present the chemical potential as

$$\mu = \mu_{FC} + \delta V_1(p) + V_{slow}(p) + V_b(p),$$

(23)

where $V_b(p) \sim V_{max}(p)$ is the value of $V_{osc}(p)$ at iceberg boundaries. The constancy of $\mu$ is mainly controlled by the condition that $V_{slow}(p)$ scales with $\delta V_1(p)$, leading to the estimate

$$\delta'(p) \sim \frac{q_0^2}{p_f^2} \ln \frac{p_f}{q_0}.$$

(24)

On the other hand, the extrema of $V_{osc}(p)$ estimated, e.g., from differentiation of Eq. (22), are shifted due to $\delta'(p) \neq 0$ by

$$\sim \frac{\delta^2(p)\delta'(p)}{\delta(p) - p\delta'(p)},$$

and they can reach physical equilibrium positions (Fig. 2b) if this shift is $\sim \delta(p)/2$. This implies the condition on the gradient $\delta'(p) \approx \delta(p)/p$, agreeing with the initial
assumption $\delta'(p) \ll 1$. Then the comparison with Eq. (24) gives an estimate for the slow function

$$\delta(p) \sim p \frac{q_0^2}{p_f^2} \ln \frac{p_f}{q_0}.$$  \hspace{1cm} (25)

This approximately linear behavior is valid at $p \gg q_0$ and $p_f - p \gg q_0$, otherwise it is limited by the edge effects, the limiting values being $\delta_{\text{min}} \sim (q_0^2/p_f^2) \ln(p_f/q_0)$ and $\delta_{\text{max}} \sim (q_0^2/p_f) \ln(p_f/q_0)$. The $\propto 1/p$ decrease of the density of icebergs, defined by Eq. (25), reflects the outward pressure on them from the center. Then the estimate for total multiplicity follows from Eq. (16):

$$N \sim \frac{p_f^2}{q_0^2 \ln(p_f/q_0)} \int_{q_0}^{p_f} \frac{dp}{p} = \frac{p_f^2}{q_0^2}. \hspace{1cm} (26)$$

Also, using Eq. (25) in Eq. (22), we confirm that $V_b$ is negligible in Eq. (23). Correspondingly, the chemical potential $\mu$ is shifted downwards with respect to the FC value $\mu_{\text{FC}}$ by

$$\Delta \mu \sim \frac{q_0^2}{m} \ln \frac{p_f}{q_0} \sim \frac{\mu}{N} \ln N,$$ \hspace{1cm} (27)

and, since the deviations $\sim V_{\text{osc}} \ll \Delta \mu$, the energy difference between FC and MFS($N$) is correctly estimated by Eq. (27). We notice that all the considerations after Eq. (23) are equally valid for the general situation, $\xi \neq 2^{1/3}$.

The asymptotic behavior of MFS($N$) resulting from Eqs. (25-27) depends only weakly on the coupling constant $\xi$. However this almost universal regime is only reached for $N$ above some crossover value $N^*$ which essentially depends on $\xi$. This relates to the fact that multiplicity $N$ for given $\xi$ grows with $1/q_0$, but this process starts from $1/q_0 = 1/q_\xi$ such that $\xi = \xi_1(q_\xi)$, Eq. (13). Hence the universal regime is reached already at $1/q_0 > 1/q_\xi$ and corresponds to $N > N^* \sim (p_f/q_\xi)^2$. Thus, for $\xi - 1 \ll 1$, we estimate from Eq. (14) $N^* \sim \ln[1/(\xi - 1)]/(\xi - 1)$.

4. CONCLUDING REMARKS

At $T = 0$, our treatment has shown that the energy difference between FC and MFS($N$) tends to zero when $N$ overcomes the crossover value $N^*, N \geq N^*$. For MFS, the effective mass $M^*$ is defined by the dispersion near the boundaries of icebergs:

$$M^* \sim \frac{p}{\partial \varepsilon_{\text{eff}}/\partial p} \sim m \frac{p}{\delta(p)} \sim \frac{mN}{\ln N}. \hspace{1cm} (28)$$

It is seen from Eq. (28) that at $N \gg N^*$ the effective mass can be extremely large, though finite even at $T = 0$. Thus MFS represents a heavy-fermion system which can be treated as Landau Fermi liquid. At finite temperatures, the system persists to be a Landau Fermi liquid, but there is a crossover temperature $T^*$ at which the difference between FC and MFS vanishes. To calculate $T^*$, we observe that the single-particle spectrum should not be altered when the temperature reaches $T^*$. The effective mass of a system with FC at finite $T$ is given by [15]

$$M^* \approx p_F \frac{p_f - p_i}{4T}. \hspace{1cm} (29)$$
Upon comparing Eqs. (28) and (29), we obtain

\[ T^* \sim T_F \frac{p_f - p_i}{p_F} \ln \frac{N}{N}. \]  

(30)

where Fermi temperature \( T_F = \frac{p_F^2}{2k_Bm} \). At \( T \geq T^* \), the system comes into the state with the effective mass, Eq. (29), and the difference between the MFS and FC states is eliminated. Hence, Eq. (30) defines the crossover temperature \( T^* \) at which the system is caused to pass from a Landau Fermi liquid to a strongly correlated Fermi liquid with FC and temperature dependent effective mass. A more detailed analysis of the physical properties of this and other model Fermi systems and their possible relation to the observed effects in real systems will be the topic of forthcoming studies.

ACKNOWLEDGMENT

The present work was partly supported by the Russian Foundation for Basic Research under Grant No. 01-02-17189.

REFERENCES

[1] L. D. Landau, Sov. Phys. JETP 3, 920 (1957).
[2] I. Ya. Pomeranchuk, Sov. Phys. JETP 8, 361 (1958).
[3] J. Bardeen, L. Cooper, and J. Schrieffer, Phys. Rev. 108, 1175 (1957).
[4] A. B. Migdal, Sov. Phys. JETP 5, 333 (1958).
[5] V. A. Khodel and V. R. Shaginyan, JETP Lett. 51, 553 (1990).
[6] P. Nozieres, J. Phys. France 2, 443 (1992).
[7] V. A. Khodel, V. R. Shaginyan, and V. V. Khodel, Phys. Rep. 249, 1 (1994).
[8] M. de Llano and J. P. Vary, Phys. Rev. C 19, 1083 (1979).
[9] M. de Llano, A. Plastino, and J. G. Zabolitsky, Phys. Rev. C 20, 2418 (1979).
[10] S. A. Artamonov, Yu. G. Pogorelov, and V. R. Shaginyan, JETP Lett. 68, 897 (1998).
[11] M. V. Zverev and M. Baldo, JETP 87, 1129 (1998).
[12] M. V. Zverev, V. A. Khodel, and M. Baldo, JETP Lett. 72, 126 (2000).
[13] P. Gegenwart, J. Clusters, C. Geibel, K. Neumaier, T. Tayama, K. Tenya, O. Trovarelli, and F. Steglich, Phys. Rev. Lett. 89, 056402 (2002).
[14] Yu.G. Pogorelov and V.R. Shaginyan, cond-mat/0209503.
[15] J. Dukelsky, V.A. Khodel, P. Schuck, and V.R. Shaginyan, Z. Phys. B 102, 245 (1997).