Effect of quantum critical fluctuations on the amplitude of de Haas-van Alphen oscillations

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Abstract. A simple model of spinless fermions with nested Fermi surface leading to a charge density wave is considered. A quantum critical point (QCP) is obtained by tuning the nesting mismatch of the Fermi surface so that the critical temperature is zero. The amplitudes of de Haas-van Alphen oscillations are calculated using a Lifshitz-Kosevich expression modified by the quasi-particle selfenergy. The amplitudes are considerably reduced by the interaction. The effects of the QCP extend over a large temperature interval.

1. Introduction
Landau’s Fermi liquid (FL) theory has been successful in describing the low energy properties of most normal metals. Numerous U, Ce and Yb based heavy fermion systems [1] display deviations from FL behavior, which manifest themselves as, e.g., a ln(T)-dependence in the specific heat over T, C/T, a singular behavior at low T of the magnetic susceptibility, χ, and a power-law dependence of the resistivity with an exponent close to one. These deviations from FL are known as non-Fermi liquid (NFL) behavior. The breakdown of the FL can be tuned by alloying (chemical pressure), hydrostatic pressure or the magnetic field. In most cases the systems are close to the onset of antiferromagnetism (AF) and the NFL behavior is attributed to a quantum critical point (QCP) [2, 3, 4, 5, 6, 7, 8, 9].

Recently we studied the pre-critical region of a heavy electron band with two parabolic pockets, one electron-like and the other hole-like, separated by a wave vector Q using (i) a field-theoretical multiplicative renormalization group (RG) approach [7] and (ii) the Wilsonian RG [10]. In the spirit of a Fermi liquid, the interaction is the remaining repulsion between heavy quasi-particles after the heavy particle bands have been formed and is assumed to be weak. The interaction between the electrons induces itinerant AF or charge density waves (CDW) due to the nesting of the Fermi surfaces of the two pockets. For perfect nesting (electron-hole symmetry) an arbitrarily small interaction is sufficient for a ground state with long-range order. The degree of nesting is controlled by the mismatch parameter, \( \frac{1}{2} |k_{F1} - k_{F2}| [k_{F1} (k_{F2}) is the Fermi momentum of the electron (hole) pocket]. In this way the ordering temperature can be tuned to zero, leading to a QCP.

Our main results are the following. In the paramagnetic phase C/T, the effective mass and χ increase logarithmically as T is lowered and diverge at the critical point signaling the breakdown of the FL [7, 10]. There is a crossover from the \(-\ln(T)\) dependence of C/T to constant γ as T is lowered if the QCP is not perfectly tuned, in agreement with experiments on numerous systems. The quasi-particle linewidth, a quantity relevant to the electrical resistivity and the dynamical
susceptibility, shows a crossover from NFL ($\sim T$) to FL ($\sim T^2$) behavior with increasing nesting mismatch and decreasing temperature [11].

These results have been extended to the two-pocket model with spinless fermions and nested Fermi surface [12, 13, 14, 15]. Neglecting the spin simplifies the model reducing the independent vertices to one and the only possible instability is a CDW. In this paper we present results for the amplitude of the de Haas-van Alphen oscillations for the case of spinless fermions [15].

2. Model

The model consists of one particle pocket and one hole pocket separated by a wave-vector $Q$. The kinetic energy is given by [7]

$$H_0 = \sum_{k} \epsilon_1(k) c_{1k}^\dagger c_{1k} + \sum_{k} \epsilon_2(k) c_{2k}^\dagger c_{2k},$$

where $k$ is measured from the center of the pocket, $\epsilon_1(k) = v_F(k - k_{F1})$ and $\epsilon_2(k) = v_F(k_{F2} - k)$. For simplicity we assume that the Fermi velocity is the same for both pockets. The weak remaining interaction between the heavy quasi-particles (after the quasi-particles are formed) is

$$H_V = V \sum_{k,k',q} c_{1k+q}^\dagger c_{1k} c_{2k'-q}^\dagger c_{2k}.$$

The renormalized vertex has the form

$$\tilde{V} = V \rho_F / (1 - V \rho_F \xi),$$

where $\xi = \ln[D/(|\omega| + 2T + \delta)]$ with $\delta = v_F|k_{F1} - k_{F2}|/2$ being the nesting mismatch parameter, $\rho_F$ the constant density of states and $D$ an energy cut-off [10]. A divergent vertex (for $V > 0$) indicates strong coupling and signals a CDW instability.

3. Quasi-particle line-width

In a FL the damping of the quasi-particles, $\Gamma$, is proportional to $T^2$, while the nesting condition changes this behavior to a quasi-linear dependence in $T$. In the disordered phase $\Gamma$ is given by the imaginary part of the electron self-energy, which can be expressed as a convolution of a staggered susceptibility $\chi'_S(\omega/2T)$ with a fermion Green’s function [11],

$$\Gamma_{NFL}(\omega, T) = \frac{T}{2} \int dx \chi'_S(x) |\tilde{V}|^2 \rho_F \left[ \coth(x) - \tanh \left( x - \frac{\omega}{2T} \right) \right];$$

$$\chi'_S(\omega/2T) \approx \frac{\rho_F}{2} \sum_{\sigma = \pm 1} \Im \psi \left( \frac{1}{2} + \frac{\Gamma_{NFL}}{2\pi T} + i \frac{\omega + 2\sigma(\delta - \delta_0)}{4\pi T} \right).$$

Here $\Im \psi$ is the imaginary part of the digamma function, $\omega$ is the external frequency, and $\delta_0$ is the nesting mismatch parameter at the QCP. The frequency in the vertices is $2T|x| + |\omega|/2$ and $i\pi/2$ is added to $\xi$. The frequency of $\Gamma_{NFL}$ in $\Im \psi$ is $2T|x|$. The selfconsistent solution of Eqs. (4) and (5) yields the quasi-particle NFL linewidth as a function of $\omega$ and $T$ [11].

There is also a FL contribution to the quasi-particle linewidth given by[11]

$$\Gamma_{FL}(\omega, T) = \frac{\pi}{8} V^2 \rho_F^2 [\omega^2 + (\pi T)^2],$$

which is added to $\Gamma_{NFL}$. The vertices in $\Gamma_{FL}$ are not dressed.

The $\omega$ and $T$ dependence of $\Gamma_{NFL}$ can be understood from limiting cases. Firstly, consider the perfectly tuned QCP, i.e. $\delta = \delta_0$. Set $\omega = 0$ and neglect $\Gamma_{NFL}$ in the digamma function, as well
as the vertex renormalizations. The integral in Eq. (4) is then independent of $T$ and $\Gamma_{NFL} \propto T$, and not $T^2$ as for a FL. Similarly, as $T \to 0$, neglecting $\Gamma_{NFL}$ in the digamma function and the vertex renormalizations, we obtain that the right-hand side of Eq. (4) is proportional to $|\omega|$, which again differs from the FL behavior ($\propto \omega^2$). The vertex renormalizations yield additional logarithmic corrections, so that to logarithmic order we have approximately

$$\Gamma_{NFL} \propto \tilde{V}^2 \rho_F \max(|\omega|, T). \quad (7)$$

In the presence of an instability the vertex corrections strongly enhance $\Gamma_{NFL}$. The selfconsistent solution, as a consequence of the logarithmic corrections, yields a $\Gamma_{NFL}$ that has a slightly sublinear $T$- and $|\omega|$-dependence [11]. Secondly, for $\delta \neq \delta_0$, on the other hand, neglecting again the selfconsistency and the vertex corrections, $\Gamma_{NFL}$ is exponentially activated at low $T$ and gradually crosses over to a linear $T$-dependence with increasing $T$. At low $T$ then the FL contribution (proportional to $T^2$) dominates, but at higher $T$ there is NFL behavior. At $T = 0$, $\Gamma_{NFL}$ vanishes identically for $|\omega| < 2(\delta - \delta_0)$ and is proportional to $|\omega| - 2(\delta - \delta_0)$ at larger frequencies. Hence, the FL contribution (proportional to $\omega^2$) dominates at low energies. The crossover from NFL to FL is consistent with the one obtained for $C/T$ [11]. Thirdly, for the tuned QCP, the imaginary part of the selfenergy is proportional to $T$ and $|\omega|$ and, hence, the real part of the selfenergy is then proportional to $|\omega| T \ln[\max(|\omega|, T)]$. This is consistent with the logarithmic dependence in $C/T$, which arises from the logarithm in this selfenergy.

4. Amplitude of the de Haas-van Alphen oscillations

There are two circular orbits corresponding to extremal cross-sectional areas of the Fermi surface of radii $k_{F1}$ (electrons) and $k_{F2}$ (holes), respectively, and hence two fundamental frequencies of oscillation. In the absence of interactions the oscillatory part of the thermodynamic potential reduces to $A \omega_n$, where $A$ is the amplitude of the de Haas-van Alphen oscillations.

The overall reduction of the amplitudes is larger close to the QCP. If the bandwidth $D$ of the heavy carriers in Eq. (1) is 1000 K and the unrenormalized effective mass is 20 times that of free electrons, then Fig. 1 corresponds to an external field of 40 T,
Figure 1. de Haas-van Alphen amplitudes for the first five harmonics $r$ as a function of $T$ for fixed $B$ for the tuned QCP, (a) $\delta_0 = 0.07$, and (b) $\delta = 0.11$, respectively. The amplitudes are normalized to that of the noninteracting system. Here $V_{\rho F} = 0.2$, $D = 10$ and if $D$ is equated to 1000 K the magnetic field is 2 T for $m^*$ being the free electron mass. For other values of $m^*$, $B$ has to be renormalized by $m^*/m$, i.e. for $m^*/m = 10$ the field corresponds to 20 T.

and $T/D = 0.001$ to 1K. A very low Dingle temperature is then necessary to observe even the fundamental frequency. The NFL effects can be seen far away from the QCP. The results are similar to those of the spinful model [18].

The dHvA-oscillations in the magnetization or the de Haas-Shubnikov oscillations in the resistivity are periodic as a function of $B^{-1}$. Hence, they are measured over a magnetic field interval and the amplitude of oscillation cannot be associated with a given field. On the other hand, frequently the magnetic field also acts as a tuning parameter for the QCP. Hence, a discussion of the oscillation amplitudes is only meaningful if the magnetic field, within the regime of measurement, does not affect the tuning of the QCP.

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