Mathematical Modeling of Synovial Joints with Chemical Reaction

R. VijayaKumar 1 and Nirmala P. Ratchagar 2

1 Mathematics Section, FEAT, Annamalai University, Annamalainagar - 608 002, India
Department of Mathematics, Periyar Government Arts College, Cuddalore, Tamil Nadu - 607 001, India
2 Department of Mathematics, Annamalai University, Annamalai Nagar, Tamil Nadu - 608 002, India
E-mail: 1 rathirath_viji@yahoo.co.in

Abstract. This paper, an effort has been made to examine the impact of some distinct parameters, namely magnetic field, porous parameter, reaction rate parameter and viscoelastic parameters of an unsteady convective diffusion through a rectangular channel. The momentum equation and concentration equation of the model have been derived by utilizing the perturbation technique and generalized dispersion model incorporated with BJ slip condition. To get an insight into various parameter of the problem on the velocity and dispersion coefficient has been analysed through different graphs.

1. Introduction
In mathematical physiology, we study the applications of mathematical modelling and mathematical techniques to get an insight into the models of physiology. The biomechanics of the knee joint, the space between the cartilages extremities of the bones, known as joint cavity is filled with a viscous non-Newtonian fluid called synovial fluid and it plays essential role in humans and animal movement. Tandon et al.,[1] explored the some important characteristics for a knee joint in normal and pathological states. Rudraiah et al.,[2] noted the dispersion of nutrients in synovial fluid it is advantageous to determine the dispersion coefficient using the generalized dispersion model of Gill and Sankarasubramanian[10].

Sing et al [4] explored the impact of magnetic field on young(old) synovial fluid, osteoarthritic synovial fluid through poro-elastic region, load carrying capacity and pressure. Bali and Sharma[5] have discussed the influence of magnetic field on the lubrication of synovial joint. Chiu-On Ng et al.[6] explored the impact of nonuniform electric field on the electrohydrodynamic dispersion of hyaluronic acid, glycoprotein from Synovial Fluid to nature or artificial joints. AmbreenAfsar Khan et al.[7]studied the influence of induced magnetic field of synovial fluid with peristaltic flow in an asymmetric channel. Nagara et al.[8] have reported the influence of electric field and magnetic field is find the dispersion of nutrients from synovial fluid to cartilages.

In this paper, we have the above discussion and its application is to motivation to investigate the importance of nutrition transport in knee joint with impact of inclined magnetic field. The obtained governing equations are then computed numerically using perturbation technique and generalized dispersion model of Gill and Sankarasubramanian[10].
2. Formulation of the Problem

The Physical configuration of the model assumed in this paper shows as incompressible synovial fluid flowing two parallel surfaces and it is separated by 2h. The synovial fluid has been represented by viscoelastic fluid and its elasticity is pertinent in joint lubrication and bounded by porous layer and externally applied inclined magnetic fields.

![Figure 2. Physical Configuration](image)

The synovial fluid lies between porous cartilageous surfaces. Using the above assumption the governing equations of fluid film region are given by

**Fluid Film Region**

Equation of motion

\[
0 = \frac{\partial p^*}{\partial x^*} + \eta_0 \frac{\partial}{\partial y^*} \left[ \frac{\partial u^*}{\partial y^*} - K_0 \left( \frac{\partial u^*}{\partial y^*} \right)^3 \right] - B_0^2 \sigma_0 u^* \cos^2 \gamma
\]  

(1)

\[
0 = \frac{\partial p^*}{\partial y^*}
\]

(2)

Equation of continuity

\[
\frac{\partial u^*}{\partial x^*} + \frac{\partial v^*}{\partial y^*} = 0
\]

(3)

Introducing a slug of concentration \( c \) with chemical reaction which is a function of time(\( t^* \)), coordinate \( x^* \) and coordinate \( y^* \). The concentration \( c \) satisfies convective diffusion equation

\[
\frac{\partial c}{\partial t^*} + u^* \frac{\partial c}{\partial x^*} = D \left( \frac{\partial^2 c}{\partial x^*^2} + \frac{\partial^2 c}{\partial y^*^2} \right) - k_1 c
\]

(4)

The required conditions for \( u^* \) and \( c \) are

\[
\frac{\partial u^*}{\partial y^*} = \mp \frac{\alpha}{\sqrt{k}} u^* \quad \text{at} \quad y^* = \pm h,
\]

(5)
where, $u^*$ and $v^*$ are the components of the synovial fluid velocity along $x^*$ and $y^*$ directions, $\mu$ is the viscosity, $k$ is the permeability of the porous medium, $\alpha$ is the slip parameter. Equation (5) is Beavers and Joseph\[9\] slip condition at the lower and upper permeable surfaces, $B_0$ is the magnetic induction, $\sigma_0$ is the electrical conductivity, $c$ the instantaneous concentration of the solute species of the kinematics viscosity, $C_0$ is the initial concentration of the initial slug input of length $x_s^*$, $K_0$ the viscoelastic parameter, $D$ the diffusion coefficient and $k_1$ is the first order reaction rate parameter.

We define the non-dimensional quantities

$$u = \frac{u^*}{\bar{u}}; \quad x = \frac{x^*}{hPe}; \quad y = \frac{y^*}{h}; \quad p = \frac{p^*}{\rho \bar{u}^2}; \quad x' = \frac{x^*}{hPe}; \quad t = \frac{Dt^*}{h^2};$$

$$Pe = \frac{h \bar{u}^*}{D}; \quad C = \frac{c}{C_0};$$

where, $\bar{u}$ and $h$ are the characteristic velocity and characteristic length.

Equations (1) and (8) in non-dimensional form are

**Fluid Film Region**

$$\frac{\partial^2 u}{\partial y^2} - 3 \epsilon_v \left( \frac{\partial u}{\partial y} \right)^2 \frac{\partial^2 u}{\partial y^2} - M^2 u \cos^2 \gamma = P_1$$

and

$$\frac{\partial C}{\partial t} + u \frac{\partial C}{\partial x} = \frac{1}{Pe^2} \left( \frac{\partial^2 C}{\partial x^2} + \frac{\partial^2 C}{\partial y^2} \right) - \alpha^2 C$$

We define the axial coordinate moving with the average velocity of flow as $x_1 = x^* - t^* \bar{u}$ which is in dimensionless form $x = x' - t$. Then equation (10) gives

$$\frac{\partial C}{\partial t} + U \frac{\partial C}{\partial x} = \frac{1}{Pe^2} \left( \frac{\partial^2 C}{\partial x'^2} + \frac{\partial^2 C}{\partial y^2} \right) - \alpha^2 C$$

with $U = \frac{u - \bar{u}}{\bar{u}}$ (non-dimensional velocity in a moving coordinate system)

The non-dimensional initial and boundary conditions are

$$\frac{\partial u}{\partial y} = \mp \alpha \sigma u \quad \text{at} \quad y = \pm 1$$

$$C(0, x, y) = \begin{cases} 
1, |x| \leq \frac{x_s}{2} \\
0, |x| > \frac{x_s}{2}
\end{cases}$$

$$\frac{\partial C}{\partial y} (t, x, -1) = \frac{\partial C}{\partial y} (t, x, 1) = 0$$
where \( P_1 = \frac{R_e}{Re} \frac{\partial \phi}{\partial x}, \epsilon_e = \frac{k_0 \omega^2}{k} \) is the viscoelastic parameter, \( M = B_0 h \sqrt{\frac{\sigma}{\mu}} \) is the Hartmann number, \( \gamma \) is the inclination angle of the magnetic field parameter, \( R_e = \frac{\rho \omega h}{\mu} \) is the Reynolds number, \( P_e = \frac{\alpha h}{T} \) is the Peclet number, \( \sigma = \frac{h}{\sqrt{k}} \) is the porous parameter and \( \alpha_e^2 = \frac{h^2 k_1}{D} \) is the chemical reaction rate coefficient.

3. Method of solution

3.1. Velocity distribution

Introduce the perturbation technique in equation (9) with appropriate boundary conditions, let

\[
u = U_0 + \epsilon U_1 + O(\epsilon^2)
\]

Substituting equation (16) in (9), we get the zeroth and first order solution of equation with boundary conditions (12) as follows

\[
u = 2A_1 \cosh(M \cos \gamma \eta) - \frac{P_1}{(M \cos \gamma)^2} + 2A_2 \cosh(M \cos \gamma \eta)
- \frac{3}{2} A_3^2 (M \cos \gamma)^3 \left( \frac{\cosh(3M \cos \gamma \eta)}{4(M \cos \gamma)} - \eta \sinh(3M \cos \gamma \eta) \right)
\]

The normalized axial components of velocity obtained from (17) is

\[ar{U} = \frac{u - \bar{u}}{\bar{u}}
\]

where, the average velocity \( \bar{u} = \frac{1}{2} \int_{-1}^{1} u(y)dy = \frac{-24P - 12A_3^2 (M \cos \gamma)^3 \epsilon \sinh 3(M \cos \gamma) + 48(M \cos \gamma)(A_1 + A_2 + 3\epsilon) \sinh(M \cos \gamma) + A_2 (M \cos \gamma)^3 \epsilon \sinh 3(M \cos \gamma)}{12(M \cos \gamma)^2}, \)

\[
a_1 = \epsilon(M \cos \gamma + \alpha \sigma), a_2 = \epsilon(M \cos \gamma - \alpha \sigma), a_3 = \frac{\alpha \sigma P}{(M \cos \gamma)^2},
\]
\[
a_4 = \frac{3}{2} A_3^2 (M \cos \gamma)^3 \left[ \left( \frac{7}{4} + \alpha \sigma \right) \sinh 3(M \cos \gamma) + \left( 3M \cos \gamma + \frac{\alpha \sigma}{4M \cos \gamma} \right) \right],
\]
\[
A_1 = \frac{a_3}{a_1 - a_2}, A_2 = \frac{a_4}{a_1 - a_2}.
\]

3.2. Generalized Dispersion model

To obtaining the mean concentration valid for all time \( \tau \), we introduce the generalized dispersion model of Gill and Sankarasubramanian[10], formulated as a series expansion in the form

\[
C(\tau, \xi, y) = C_m(t, x) + \sum_{k=1}^{\infty} f_k(t, y) \frac{\partial^k C_m}{\partial x^k}
\]

where, \( C_m \) is average concentration

\[
C_m(t, x) = \frac{1}{2} \int_{-1}^{1} C(t, x, y)dy
\]
Equation is obtained by integrating equation (11), we get
\[
\frac{\partial C_m}{\partial t} = \frac{1}{P_c^2} \frac{\partial^2 C_m}{\partial x^2} + \frac{1}{2} \int_{-1}^{1} \frac{\partial^2 C}{\partial y^2} \, dy - \frac{1}{2} \frac{\partial}{\partial x} \int_{-1}^{1} U \, C \, dy - \alpha_e^2 C_m
\]  
(20)

Substituting value of \( C \) from equation (18) in (20), we obtain
\[
\frac{\partial C_m}{\partial t} = \frac{1}{P_c^2} \frac{\partial^2 C_m}{\partial x^2} - \frac{1}{2} \frac{\partial}{\partial x} \int_{-1}^{1} U \left(C_m(t, x) + f_1(t, y) \frac{\partial C_m}{\partial x}(t, x) + \ldots \right) \, dy - \alpha_e^2 C_m
\]  
(21)

In this model we write
\[
\frac{\partial C_m}{\partial t} = \sum_{k=1}^{\infty} K_k(t) \frac{\partial^k C_m}{\partial x^k}
\]  
(22)

where the dispersion coefficient, \( K_k(t) \) Substituting the Equation (22) in (21) we obtain
\[
K_1 \frac{\partial C_m}{\partial x} + K_2 \frac{\partial^2 C_m}{\partial x^2} + K_3 \frac{\partial^3 C_m}{\partial x^3} + \ldots = \frac{1}{P_c^2} \frac{\partial^2 C_m}{\partial x^2} - \frac{1}{2} \frac{\partial}{\partial x} \int_{-1}^{1} U(C_m(t, x)
+f_1(t, y) \frac{\partial C_m}{\partial x}(t, x) + f_2(t, y) \frac{\partial^2 C_m}{\partial x^2}(t, x) + \ldots dy - \alpha_e^2 C_m
\]  
(23)

Equating the coefficient \( \frac{\partial C_m}{\partial x}, \frac{\partial^2 C_m}{\partial x^2} \ldots \) we get,
\[
K_i(t) = \frac{\delta_{ij}}{P_c^2} - \frac{1}{2} \int_{-1}^{1} U f_{i-1}(t, y) \, dy, \quad (i = 1, 2, 3, \ldots \text{ and } j = 2)
\]  
(24)

where, Kronecker delta \( \delta_{ij} = \begin{cases} 1, & \text{if } i = j \\ 0, & \text{if } i \neq j \end{cases} \)

Substituting equation (18) in (11), we get
\[
\frac{\partial}{\partial t} \left(C_m(t, x) + f_1(t, y) \frac{\partial C_m}{\partial x}(t, x) + f_2(t, y) \frac{\partial^2 C_m}{\partial x^2}(t, x) + \ldots \right)
+ U \frac{\partial}{\partial x} \left(C_m(t, x) + f_1(t, y) \frac{\partial C_m}{\partial x}(t, x) + f_2(t, y) \frac{\partial^2 C_m}{\partial x^2}(t, x) + \ldots \right)
= \frac{1}{P_c^2} \frac{\partial^2}{\partial x^2} \left(C_m(t, x) + f_1(t, y) \frac{\partial C_m}{\partial x}(t, x) + \ldots \right)
+ \frac{\partial^2}{\partial y^2} \left(C_m(t, x) + f_1(t, y) \frac{\partial C_m}{\partial x}(t, x) + \ldots \right)
- \alpha_e^2 \left(C_m(t, x) + f_1(t, y) \frac{\partial C_m}{\partial x}(t, x) + \ldots \right)
\]  
(25)

Rearranging the terms and using
\[
\frac{\partial^{k+1} C_m}{\partial t \partial x^k} = \sum_{i=1}^{\infty} K_i(t) \frac{\partial^{k+i} C_m}{\partial x^{k+i}}
\]
we obtain

\[ \frac{\partial f_1}{\partial t} = \frac{\partial^2 f_1}{\partial y^2} - U - K_1(t) - \alpha_c^2 f_1 \]  
\[ \frac{\partial f_2}{\partial t} = \frac{\partial^2 f_2}{\partial y^2} + f_1 U + K_1(t) f_1 + K_2(t) - \frac{1}{P_e} + \alpha_c^2 f_2 \]  
\[ + \sum_{k=1}^{\infty} \left[ \frac{\partial f_{k+2}}{\partial t} - \frac{\partial^2 f_{k+2}}{\partial y^2} + f_{k+1} U + f_{k+1} K_1(t) + \left( K_2(t) - \frac{1}{P_e} \right) f_k \right] 
\[ + \sum_{i=3}^{k+2} K_i f_{k+2-i} + \alpha_c^2 f_{k+2} \right] \frac{\partial^{k+2} C_m}{\partial x^{k+2}} = 0 \]  

(26)

with \( f_0 = 1 \).

Comparing the coefficients of \( \frac{\partial^k \theta_m}{\partial x^k} \) \( (k = 1, 2, 3, \ldots) \) in (26) and equating to zero, we get

\[ \frac{\partial f_1}{\partial t} = \frac{\partial^2 f_1}{\partial y^2} - U - K_1(t) - \alpha_c^2 f_1 \]  
\[ \frac{\partial f_2}{\partial t} = \frac{\partial^2 f_2}{\partial y^2} + f_1 U - K_1(t) f_1 - K_2(t) + \frac{1}{P_e} - \alpha_c^2 f_2 \]  
\[ \frac{\partial f_{k+2}}{\partial t} = \frac{\partial^2 f_{k+2}}{\partial y^2} - f_{k+1} U - K_1(t) f_{k+1} - \left( K_2(t) - \frac{1}{P_e} \right) f_k - \sum_{i=3}^{k+2} K_i f_{k+2-i} - \alpha_c^2 f_{k+2} \]  

(29)

To find \( K_i \)'s we know the \( f_k \)'s and its corresponding initial and boundary conditions are

\[ f_k(0, y) = 0 \]  
\[ \frac{\partial f_k}{\partial y}(t, \pm 1) = 0 \]  
\[ \int_{-1}^{1} f_k(t, y) dy = 0, \text{for } k = 1, 2, 3, \ldots \]  

(30-32)

From equation (24) for \( i = 1 \), we get

\[ K_1(t) = 0 \]  

(33)

From equations (24) for \( i = 2 \), we get \( K_2 \) as,

\[ K_2(t) = \frac{1}{P_e} - \frac{1}{2} \int_{-1}^{1} U f_1 dy \]  

(34)

let \( f_1 = f_{10}(y) + f_{11}(t, y) \)  

(35)

where, \( f_{10}(y) \) is independent of \( t \) and corresponds to an infinitely wide slug and \( f_{11} \) is \( t \)- dependent satisfying

\[ \frac{df_{10}}{dy} = 0 \text{ at } y = \pm 1 \]  
\[ \int_{-1}^{1} f_{10} dy = 0 \]  

(36-37)
Using the (35) in (27) gives

$$\frac{d^2 f_{10}}{dy^2} - \alpha_y^2 f_{10} = U(y) \quad (38)$$

$$\frac{\partial f_{11}}{\partial t} = \frac{\partial^2 f_{11}}{\partial y^2} - \alpha_x^2 f_{11} \quad (39)$$

Solving the equation (38) with conditions (36) and (37) is

$$f_{10} = A_3 e^{\beta y} + A_4 e^{-\beta y} + s_2 \cosh[y M \cos \gamma] + s_3 + s_4 e^{-M \cos \gamma} \left( \frac{2M \cos \gamma}{s_1} + y \right) + s_4 e^{M \cos \gamma} \left( \frac{2M \cos \gamma}{s_1} - y \right) \quad (40)$$

where,

$$s_1 = (M \cos \gamma)^2 - \alpha_x^2, s_2 = \frac{2A_1}{s_1} + \frac{2A_2 e^{-3A_1(M \cos \gamma)^2} e^{-\beta y}}{s_2}, A_3 = \frac{P}{\alpha_x M \cos \gamma}, A_4 = \frac{s_4 M \cos \gamma e^{-M \cos \gamma} \left( \frac{2M \cos \gamma}{s_1} + 1 \right) + s_4 M \cos \gamma e^{-M \cos \gamma} \left( 1 - \frac{2M \cos \gamma}{s_1} \right)}{s_3}, s_4 = \frac{2s_4 \cosh[M \cos \gamma]}{s_5}$$

Equation (39) is heat conduction type and its solution satisfying condition \(f_{11}(t, y) = -f_{10}(y)\) of the form

$$f_{11} = \sum_{n=1}^{\infty} B_n e^{-\left(\lambda_n^2 - \alpha_x^2\right)t} \cos(\lambda_n y) \quad (41)$$

where,

$$B_n = -2 \int_0^1 f_{10}(y) \cos(\lambda_n y) dy$$

Substituting (40) and (41) in equation (35) we get,

$$f_{11} = -2e^{-(\beta^2 + \alpha_y^2)t} \cos \pi y \left( -\frac{M \cos \gamma s_2 \sinh[M \cos \gamma]}{\pi^2 + (M \cos \gamma)^2} + \frac{e^{M \cos \gamma} (\pi^2 - 1 - e^{M \cos \gamma} + M \cos \gamma) + (M \cos \gamma)^2 (1 + e^{M \cos \gamma} + M \cos \gamma) s_4}{\pi^2 + (M \cos \gamma)^2} \right)$$

$$- \left( (-1 + e^{M \cos \gamma} (-1 + M \cos \gamma)) (M \cos \gamma)^2 + 4 \pi^2 (1 + e^{M \cos \gamma} (1 + M \cos \gamma)) s_4 \right) + \frac{2(M \cos \gamma)^2 s_4 (1 + e^{M \cos \gamma})}{\pi^2 + (M \cos \gamma)^2} + \frac{A_4 \alpha_x (1 + e^{-\alpha_x}) - A_3 \alpha_x (1 + e^{\alpha_x})}{\pi^2 + \alpha_x^2}$$

$$- 2e^{-(4\pi^2 + \alpha_x^2)t} \cos 2\pi y \left( -\frac{M \cos \gamma s_2 \sinh[M \cos \gamma]}{4\pi^2 + (M \cos \gamma)^2} + \frac{e^{M \cos \gamma} (4\pi^2 - 1 - e^{M \cos \gamma} + M \cos \gamma) + (M \cos \gamma)^2 (1 + e^{M \cos \gamma} + M \cos \gamma) s_4}{4\pi^2 + (M \cos \gamma)^2} \right)$$

$$- \left( (-1 + e^{M \cos \gamma} (-1 + M \cos \gamma)) (M \cos \gamma)^2 + 4 \pi^2 (1 + e^{M \cos \gamma} (1 + M \cos \gamma)) s_4 \right) + \frac{2(M \cos \gamma)^2 s_4 (1 + e^{M \cos \gamma})}{4\pi^2 + (M \cos \gamma)^2} + \frac{A_4 \alpha_x (1 + e^{-\alpha_x}) - A_3 \alpha_x (1 + e^{\alpha_x})}{4\pi^2 + \alpha_x^2}$$

$$- 2e^{-(4\pi^2 + \alpha_x^2)t} \cos 3\pi y \left( -\frac{M \cos \gamma s_2 \sinh[M \cos \gamma]}{9\pi^2 + (M \cos \gamma)^2} \right)$$

$$- 2e^{-(9\pi^2 + \alpha_x^2)t} \cos 3\pi y \left( -\frac{M \cos \gamma s_2 \sinh[M \cos \gamma]}{9\pi^2 + (M \cos \gamma)^2} \right)$$
In the present analysis, we have discussed a theoretical model established an generalised dispersion mechanism of Gill and SankaraSubramanian[10] to investigate the effects of magnetic field on dispersion coefficient modeling the synovial fluid. The impact of magnetic field is to decreases the velocity profile and decrease the dispersion coefficient. It is seen that the dispersion coefficient increased with raise in inclination angle of magnetic field. we can conclude that are very useful for understanding dispersion of macromolecular components in biological bearing in BJ-slip condition of the physical model.

4. Results and Discussion

In this section, we discussed the part of numerical results of the problem through graphs and expression for nondimensional velocity and dispersion coefficient are calculated numerically by utilizing MATHEMATICA Software. Equations 17 and 34 are calculated and it depicted through Figures 2-12 to show the velocity and dispersion coefficient for distinct values of inclination angle of magnetic field ($\gamma = 0, \frac{\pi}{6}, \frac{\pi}{4}$), Hartmann number ($M = 1, 3, 5, 7$), porous parameter ($\sigma = 10, 10^2, 10^3, 10^4$), viscoelastic parameter ($\epsilon = 0, 0.5, 1, 1.5$), slip parameter ($\alpha = 0.1, 0.2, 0.3, 0.4$) and reaction rate parameter ($\alpha_c = 0.5, 1, 1.5, 2$) are shown in Figures 2 to 12 with fixed values such as $Pe = 100, \alpha = 0.1$ and $Re = 0.05$.

Figure 2 depicts that the influence of the angle of inclination $\gamma$ on the velocity profile of a synovial fluid flow between cartilage. It is examined that from figure 1 the velocity profile increases for large values of $\gamma$ and also it is parabolic in nature due to inclination of magnetic field. Figures 3 to 6 describe the variation of velocity profile with $y$ for various values of the $M, \sigma, \epsilon$, and $\alpha$ respectively. It is clearly shows that the velocity distribution raises near the boundaries and flattens in the middle of fluid region.

Figures 7 to 12 depict the $K_2(t) - Pe^{-2}$ with nondimensional time $t$ for distinct values of $M, \sigma, \epsilon, \alpha$ and $\alpha_c$. It shows that the dispersion coefficient reduces with increasing nondimensional time $t$. From the figure 7-9 illustrate inclination angle of magnetic field($\gamma$), slip parameter($\alpha$) and porous parameter ($\sigma$) on the $K_2(t) - Pe^{-2}$. It is evident that the dispersion coefficient increased with an increase in inclination angle of magnetic field, slip parameter and porous parameter ($\sigma$). This results is noticed that the insight the phenomena of nutritional transport to synovial joint.

In diseases states, figures 10-12 shows that the applied magnetic field($M$) are helps in sustaining greater loads. In this manner,$K_2(t) - Pe^{-2}$ increases with raise in viscoelastic parameter and reaction rate parameter. From these result it is very helpful to analyses the transport of solute at various times. The movement of suspending medium of the synovial fluid through the cartilage plays a vital role in the mechanism.

5. Conclusions

In the present analysis, we have discussed a theoretical model established an generalised dispersion mechanism of Gill and SankaraSubramanian[10] to investigate the effects of magnetic field on dispersion coefficient modeling the synovial fluid. The impact of magnetic field is to decreases the velocity profile and decrease the dispersion coefficient. It is seen that $K_2(t) - Pe^{-2}$ increases with large value of an inclination angle of magnetic field. we can conclude that are very useful for understanding dispersion of macromolecular components in biological bearing in BJ-slip condition of the physical model.
Figure 2. Variation of $\gamma$ with Velocity

Figure 3. Variation of $M$ with Velocity

Figure 4. Variation of $\sigma$ with Velocity

Figure 5. Variation of $\epsilon$ with Velocity
Figure 6. Variation of $\alpha$ with Velocity

Figure 7. Variation of $\gamma$ with Dispersion coefficient

Figure 8. Variation of $\sigma$ with Dispersion coefficient.

Figure 9. Variation of $\alpha$ with Dispersion coefficient
Figure 10. Variation of $\varepsilon$ with Dispersion coefficient.

Figure 11. Variation of $M$ with Dispersion coefficient.

Figure 12. Variation of $\alpha_c$ with Dispersion coefficient.

References
[1] Tandon P N, Nirmala P, Pal T S and Agarwal R 1988 Int. J. Appl. Math. Modelling 12 72
[2] Rudraiah N, Kasiviswanathan S R and Kaloni P N 1991 Biorheology 28 207
[3] Gill W N and Sankarsubramanian R 1970 Proc. Roy. Soc. Lond. A, 316 341
[4] Singh N P, Ajay Kumar Singh, Usha Singh and Atul Kumar Singh 2005 Indian J. pure appl. Math. 385 385
[5] Bali R and Sharma S K 2005 Tribology Letters 19 281
[6] Chiu-On Ng, Rudraiah N, Nagaraj C, and Nagaraj H N 2005 Journal of Energy, Heat and Mass Transfer 27 39
[7] AmbreenAfsar Khan, Arfa Farooq and Kambiz Vafai 2017 Journal of Magnetism and Magnetic Materials 446 54
[8] Nagaraj C, Dinesh P A and Kalavathi G K 2018 Defect and Diffusion Forum 388 361
[9] Beavers G S and Joseph D D 1967 J.Fluid 30 197
[10] Sankar R 1995 Prentice-Hall of India 3