Nonsingular Terminal Sliding Mode Control for Flexible Spacecraft Attitude Control

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Abstract. In this paper, a nonsingular terminal sliding mode control method is applied to design an attitude control law for flexible spacecraft. First, the Lagrange method is applied to establish the dynamic equation of the flexible spacecraft and the vibration equation of the flexible appendages. Then, by designing a nonlinear terminal sliding surface function, the spacecraft terminal sliding mode control law is designed. In order to solve the singular problem, a nonsingular terminal sliding mode control law is further designed. At the same time, the saturation function is introduced to replace the sign function to suppress chattering problem of flexible appendages. The Lyapunov method is used to prove that the involved nonsingular terminal sliding mode control law can realize the precise attitude control of the flexible spacecraft. Finally, a numerical simulation example verifies the effectiveness of the control law designed in this paper.

1. Introduction

With the development of space technology, the structural and mechanical problems of spacecraft have increased, and the requirements for control have become higher and higher. Spacecraft equipped with large flexible structures (such as large solar panels, large flexible antennas, and large space truss structures, etc.) have become a significant feature of modern complex spacecraft in terms of structural design. The modal damping of this kind of large-scale flexible structure is small. When operating in space, once it is subjected to a certain excitation force, its large-scale vibration will last for a long time, which will not only affect the operation of the spacecraft (such as attitude problem, stability problem and shorter service life), and also severely cause damage to the instrument. Therefore, it is of great significance to carry out research on precise control of spacecraft with flexible structure and to make it robust against high and low frequency multi-source interference.

In the field of the spacecraft attitude control, many scholars have proposed various control schemes, most of which are still designed by classical control theory, supplemented by interference compensation schemes [1]. Wertz and many scholars co-edited the classic "Spacecraft Attitude Determination and Control" at the end of the 1970s, which summarized various methods of spacecraft attitude determination and control at that time. The design method of the control system was described in detail [2]. A monograph jointly compiled by domestic aerospace experts [3] covered all aspects of the attitude control system. The main methods were PID control. The potential of traditional control schemes to improve control performance was limited, so scholars have begun to focus on new control schemes, such as feedback linearization, sliding mode control, adaptive control and various combined control methods.
Sliding Mode Control is widely used in the attitude control of spacecraft due to its simple calculation, fast response speed, and robustness to parameter uncertainty and interference. Zhu et al. [4] used the dynamic switching function related to the first-order time derivative of the control input to design the attitude control law of the flexible spacecraft. Through simulation verification, the attitude can be stabilized by this dynamic sliding mode control. Yu et al. [5], designed a second-order sliding mode variable structure controller with super spiral algorithm in order to eliminate the high-frequency chattering. The simulation results show that the algorithm is effective for attitude maneuvering. Wu et al. [6] constructed a flexible modal observer and obtained a sliding mode control law based on the observer. The simulation results show that the method has good robustness to uncertainty of inertial moment. Because the tracking error in sliding mode control cannot converge to zero in a finite time, some scholars have proposed a terminal sliding mode control method [7-9], in which nonlinear surface function makes the tracking error of the system converge to zero in a finite time.

This paper studies the attitude control of flexible spacecraft. First, Lagrange method and finite element method are used to obtain the attitude dynamics equation and vibration equation of the flexible spacecraft. Then, the nonsingular terminal sliding surface function is used to design the attitude control law. The simulation results show that the controller designed in this paper can achieve high accuracy and effectively suppress the chattering of the flexible appendages.

2. Flexible Spacecraft Model and Description
The flexible spacecraft is composed of a rigid body and flexible appendages, and its kinematics is determined by the body's attitude. According to the paper [10], Lagrange method can be used to obtain the dynamics of the rigid body posture maneuver and the vibration of the flexible appendages.

\[
\begin{align*}
J \ddot{\theta} + \delta \dot{\theta} \dot{\eta} &= u + d \\
\dot{\eta} + C \eta + K \eta + \delta \dot{\theta} &= 0
\end{align*}
\]

(1)

where \( \delta \) is the spacecraft attitude angle, \( J \) is the total moment of inertia of the flexible spacecraft, \( \delta \in R^4 \) is the rigid-flexible coupling coefficient vector, \( u \) is the control moment acting on the central rigid body, and \( d \) is the disturbance to central rigid body. \( \dot{\eta} \) is the modal acceleration of the flexible attachment. \( C = diag \{2 \xi, \omega \} \in R^{4 \times 4} \) and \( K = diag \{\omega \} \in R^{4 \times 4} \) are the damping matrix and stiffness matrix of the flexible accessory system, where \( \xi \) and \( \omega \) are the damping ratio and natural frequency of the i-th mode, respectively.

Therefore, the target in this paper is to design the attitude control moment \( u \) for the flexible spacecraft described above to achieve precise pointing from the initial attitude angle \( \theta_0 = 0^\circ \) to the target attitude angle \( \theta_d = 10^\circ \). That is, the attitude angle, the angle error and angular velocity error tend to zero as much as possible, while suppressing the vibration of the flexible part as much as possible.

3. Controller Design

3.1. Sliding Mode Control (SMC)
Sliding mode control is essentially a special kind of non-linear control. The control strategy is to continuously change the system structure according to the current state of the system, such as deviation, to make the system move in a predetermined sliding mode. Sliding mode movement includes approaching movement and sliding mode movement. Approaching movement refers to the movement of the system from any initial state to the switching surface. Sliding mode movement refers to the movement in the sliding mode area, that is, on the switching surface \( s = 0 \). Since all moving points on the sliding mode area must be end points, there are \( \lim_{s \to 0^+} s \leq 0 \) & \( \lim_{s \to 0^-} s \geq 0 \)

moving points near the switching surface \( s = 0 \), that is \( \lim_{s \to 0} s \dot{s} \leq 0 \).

Sliding mode control problem can be described as: For the system \( \dot{x} = f(x, u, t) \), switching surface function \( s(x) \) and control function \( u(x) = \begin{cases} u^+(x) & s(x) > 0 \\
 u^-(x) & s(x) < 0 \end{cases} \), need to be designed, so that the sliding mode exists and the movement points outside the switching surface will reach the surface and move stably in the sliding mode area.
The approaching law is used to improve the quality of approaching motion. The commonly used approaching laws mainly include isokinetic approaching law and exponential approaching law. In this paper, the isokinetic approaching law $\dot{s} = -\varepsilon \text{sign}(s)$ ($\varepsilon > 0$) is used, where $\text{sign}(s) = \begin{cases} 1 & s > 0 \\ 0 & s = 0 \\ -1 & s < 0 \end{cases}$ is the sign function. The constant $\varepsilon$ represents the speed at which the system approaches the switching surface $s = 0$. The larger the $\varepsilon$, the faster the approach speed, but the greater the speed when the moving point reaches the switching surface, the greater the jitter caused.

Considering the flexible spacecraft model (1) established above, let $\begin{cases} x_1 = \theta \\ x_2 = \dot{\theta} \end{cases}$ then the state space equation is

$$
\begin{aligned}
\dot{x}_1 &= \dot{\theta} \\
\dot{x}_2 &= \dot{\dot{\theta}} = \frac{1}{j_b}[u + \delta^T(C\dot{\eta} + K\eta) + d]
\end{aligned}
$$

where $j_b = J - \delta^T\delta$ and the disturbance $d(t)$ is assumed as $|d(t)| < D$.

Assume the target attitude angle is $\theta_d$, then the angle error and velocity error are $e = \theta_d - \theta$, $\dot{e} = \dot{\theta}_d - \dot{\theta}$. Sliding surface function is designed as

$$
s = ce + \dot{e} \tag{3}
$$

where $c > 0$. Then $\dot{s} = c\dot{e} + \dot{e} = c(\dot{\theta}_d - \dot{\theta}) + (\dot{\theta}_d - \dot{\theta}) = c(\dot{\theta}_d - \dot{\theta}) + (\dot{\theta}_d - \frac{1}{j_b}[u + \delta^T(C\dot{\eta} + K\eta) + d])$.

The isokinetic approaching law $\dot{s} = -\varepsilon \text{sign}(s)$ is used here, then sliding mode control is

$$
u = j_b(c\dot{e} + \dot{\theta}_d) - \delta^T(C\dot{\eta} + K\eta) + \varepsilon \text{sign}(s) \tag{4}
$$

where $\varepsilon > |D|$.

Considering the following Lyapunov function $V(t) = \frac{1}{2}s^2 \geq 0$, then $\dot{V}(t) = s\dot{s} = s(c\dot{e} + \dot{e}) = -\frac{1}{j_b}(\varepsilon s + ds) \leq -\frac{1}{j_b}|s|(\varepsilon - |D|)$.

Because $\varepsilon > |D|$, only when $e = \dot{e} = 0$, $\dot{V}(t) = 0$. So the law (4) satisfying the Lyapunov function.

3.2. Terminal Sliding Mode Control (TSMC)

In traditional sliding mode control, a linear sliding mode surface is usually designed. After the system reaches the sliding mode state, the tracking error gradually converges to zero, and the speed of the progressive convergence is controlled by adjusting the parameters of the sliding mode surface. The terminal sliding mode control proposed in the paper [11] introduces a nonlinear function into the design of the sliding mode surface to construct the terminal sliding mode surface so that the tracking error of the system on the sliding mode surface converges to zero in a finite time.

Establishing the Terminal sliding mode control law, considering the flexible spacecraft model (1) established above, let $\begin{cases} x_1 = e = \theta_d - \theta \\ x_2 = \dot{e} = \dot{\theta}_d - \dot{\theta} \end{cases}$ then state space function is

$$
\begin{aligned}
\dot{x}_1 &= \dot{e} = \dot{\theta}_d - \dot{\theta} \\
\dot{x}_2 &= \ddot{e} = \ddot{\theta}_d - \frac{1}{j_b}[u + \delta^T(C\dot{\eta} + K\eta) + d] \tag{5}
\end{aligned}
$$

where $j_b = J - \delta^T\delta$.

Terminal sliding surface function is

$$
s = \dot{e} + \beta e^{ \frac{q}{p}} \tag{6}
$$

where, $\beta > 0, p, q (p > q)$ are positive odd numbers, then $\dot{s} = \dot{e} + \beta \frac{q}{p} e^{ \frac{q-1}{p}} \dot{e} = \ddot{\theta}_d - \frac{1}{j_b}[u + \delta^T(C\dot{\eta} + K\eta) + d] + \beta \frac{q}{p} e^{ \frac{q-1}{p}} \dot{e}$

Terminal sliding mode law is

$$
u = j_b\ddot{\theta}_d - \delta^T(C\dot{\eta} + K\eta) + j_b \cdot \beta \frac{q}{p} e^{ \frac{q-1}{p}} \dot{e} + \varepsilon \text{sign}(s) \tag{7}
$$

Where $\varepsilon > |D|$.
Considering the following Lyapunov function $V(t) = \frac{1}{2}s^2 \geq 0$, then $\dot{V}(t) = s\dot{s} = s(\ddot{e} + \beta \frac{p}{q} \dot{e}^{p-1} \dot{e}) = -\frac{1}{j_b} s(sign(s) + d) = -\frac{1}{j_b} p \frac{e}{q} (sign(s) + d) \leq -\frac{1}{j_b} p \frac{e}{q} (|s| + (\varepsilon - |D|))$. When $e = 0, \dot{e} \neq 0$, there is singular problems, so a nonsingular terminal mode control is raised [12].

3.3. Nonsingular Terminal Sliding Mode Control (NTSMC)

Considering the model (1) established above, the Terminal sliding surface function is

$$s = e + \frac{1}{p} \frac{e}{q} \dot{e}^{p-1}$$

where, $\beta > 0, p, q(p > q)$ are positive odd numbers, then $\dot{s} = \dot{e} + \frac{1}{p} \frac{e}{q} \dot{e}^{p-1} \dot{e}$.

Nonsingular terminal sliding mode law is

$$u = f_b \ddot{\theta}_d - \delta^T(C\dot{\eta} + K\eta) + f_b \cdot p \frac{e}{q} \dot{e}^{p-1} + \varepsilon sign(s)$$

where $\varepsilon > |D|$. 

Considering the following Lyapunov function $V(t) = \frac{1}{2}s^2 \geq 0$, then

$$\dot{V}(t) = s\ddot{s} = s(\ddot{e} + \beta \frac{p}{q} \dot{e}^{p-1} \dot{e}) = s\left[-\frac{1}{j_b} \frac{1}{p} \frac{e}{q} \dot{e}^{p-1} (sign(s) + d)\right] = -\frac{1}{j_b} \frac{1}{p} \frac{e}{q} \dot{e}^{p-1} (|s| + (\varepsilon - |D|))$$

When $\dot{e} \neq 0$ and $p > q$ are positive odd numbers, then $\frac{p}{q} \dot{e}^{p-1} > 0$. When $\beta > 0$ and $j_b > 0$, then $\frac{1}{j_b} \frac{1}{p} \frac{e}{q} \dot{e}^{p-1} > 0$ with $\varepsilon > |D|$. So when $\dot{e} \neq 0$, NTSMC satisfies Lyapunov function.

Put law (9) into state space function (5), then $\ddot{e} = -\beta \frac{p}{q} \dot{e}^{p-1} - \frac{1}{j_b} (sign(s) + d). When \dot{e} = 0, \ddot{e} = -\frac{1}{j_b} (sign(s) + d)$; when $s > 0$, $\ddot{e} = -\frac{1}{j_b} (\varepsilon - |D|)$, that is $\dot{e}$ falling; when $s < 0$, $\ddot{e} \geq \frac{1}{j_b} (\varepsilon - |D|)$, that is $\dot{e}$ rising.

Due to the existence of the switching function in the above-designed control law, interference will cause high-frequency chattering during operation. When the interference is severe, it may resonate with the flexible accessories of the spacecraft, which greatly affects the structural safety of the flexible spacecraft. In order to prevent the negative impact of buffeting on the flexible spacecraft, a continuous saturation function $sat(s) = \begin{cases} \begin{align*} 1 & \text{ if } s > \Delta \\ \frac{s}{\Delta} & \text{ if } |s| \leq \Delta \\ -1 & \text{ if } s < -\Delta \end{align*} \end{cases}$ is used to replace the sign function in the above control law. $\Delta$ is the boundary layer thickness of the sliding surface. Switching control is used outside the boundary layer to make the system quickly approach the sliding surface. Within the boundary layer, feedback control is used to weaken the chattering generated by the rapid switching of the sliding module. The arrival time of the sliding mode can be adjusted by changing the boundary layer.

4. Simulation

In order to verify the non-singular terminal sliding mode control law designed in this paper, the flexible spacecraft parameters in the paper [13] are selected for simulation, and the total moment of inertia of the flexible spacecraft system is $J = 24.62 kg \cdot m^2$. Take the first four modes of the system for verification. The damping coefficient, rigid-flexible coupling coefficient and vibration frequency of the system are respectively $\xi_1 = \xi_2 = \xi_3 = \xi_4 = 0.02; \delta_1 = 3.3617 kg^2 \cdot m, \delta_2 = 0.4198 kg^2 \cdot m, \delta_3 = 0.1384 kg^2 \cdot m, \delta_4 = 0.0677 kg^2 \cdot m$ and $\omega_1 = 25809 rad/s, \omega_2 = 193296 rad/s, \omega_3 = 57.9383 rad/s, \omega_4 = 117.9715 rad/s$. Assume the external disturbance as $d = 0.001 sin(0.1\pi + 0.1) N \cdot m$.

The initial attitude angle is $\theta_0 = 0^\circ$ and the target attitude angle is $\theta_d = 10^\circ$.

In SMC, parameters are $c=1, k=5, \varepsilon=0.01, \delta=0.001$.

In TSMC and NTSMC, parameters are $\beta=1, p=5, q=3, \varepsilon=0.01, \delta=0.001$.

The simulation results are as follows:
Figure 1. Trajectory of attitude angle $\theta$.

Figure 2. Trajectory of attitude angle error $e$. 
Figure 3. Moral trajectory of flexible appendages $\eta$.

Figure 4. Trajectory of control moment $u$.

Figure 1 and Figure 2 respectively show the response curves of the attitude angle $\theta$ and the attitude angle error $e$ in 100 seconds. It can be seen that three control laws designed in this paper can all converge quickly, reach a stable state within 10 seconds and achieve a high-precision steady state error. Among them, the steady-state error under SMC is the largest, near $10^{-4}$, while the steady-state error under TSMC or NTSMC is within $10^{-5}$. Although the response time is the fastest under TSMC, due to the existence of singular problems, the steady-state error oscillation is larger than that under NTSMC. Figure 3 shows the response of the first four-order flexural mode $\eta$ within 100 seconds. The second, third, and fourth-order modal responses are all less than $5 \times 10^{-4}$, while the first-order modal response is at $-0.2 \sim 0.2$. It can be seen from the first-order flexible mode response that NTSMC can more effectively suppress the flexible mode vibration. Figure 4 shows the change curve of the control torque. The control torque under SMC and TSMC are within 6N-m, while within 1N-m under NTSMC. In addition, the control torque under SMC has a strong chattering phenomenon, which will cause safety hazards to the structure of flexible spacecraft.

As shown in Table 1, to verify the effectiveness of the algorithm proposed in this article, the following three performance indicators are considered:
(1) The time to enter the error band: time passed when the errors of attitude angle and angular velocity both within $10^{-5}$;
(2) Pointing accuracy: the root mean square value of attitude angle error from 30s to 100s;
(3) Stability: the root mean square value of attitude angular velocity error from 30s to 100s.

| Table 1. Performance of three sliding mode controller. |
|------------------------------------------------------|
| SMC | TSMC | NTSMC |
| Time to enter the error band(s) | 2.5 | 12 |
| Pointing accuracy(°) | 6.43×10^{-4} | 8.95×10^{-6} | 3.31×10^{-6} |
| Stability(°/s) | 0.0013 | 0.0012 | 7.98×10^{-5} |

5. Conclusion
This paper considers the precise attitude control of flexible spacecraft. In traditional sliding mode control, a linear sliding mode surface is designed. The tracking error gradually converges to zero, but it cannot converge to zero in a finite time. Therefore, this paper introduces terminal sliding mode control and designs a nonlinear sliding surface, so that the system tracking error can converge to zero in a finite time. Due to the singular problem of terminal sliding mode control, a nonsingular terminal sliding mode controller is designed. At the same time, in order to effectively suppress the chattering of the flexible appendages, a saturation function is used to replace the sign function. The simulation results show that the nonsingular terminal sliding mode control law designed in this paper can accurately control the attitude of the flexible spacecraft, and at the same time effectively suppress the chattering of the flexible attachment compared with the traditional sliding mode control.

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