Coupled Topological Surface Modes in Gyrotropic Structures: Green’s Function Analysis

S. Ali Hassani Gangaraj Member, IEEE, and Francesco Monticone Member, IEEE

Abstract—At a transition in a wave-guiding structure (a step, taper, etc.), part of the incident energy is transmitted, coupled into the modes existing upstream from the discontinuity, and part of the energy is reflected. When the waveguide has non-trivial topological properties, however, the transition may occur with no backscattering, and with unusual modal coupling/transitions. Within this context, in this Letter we discuss the response of a nonreciprocal topological structure composed of two nearby interfaces between oppositely-biased gyrotropic media and an isotropic medium, which support unidirectional surface modes. We provide an exact Green’s function analysis of this structure, and we show how the topological surface modes are modified when the two interfaces are brought closer and eventually merged. Our analysis and results may provide relevant insight into a broad class of nonreciprocal topological systems with coupled unidirectional surface modes.

Index Terms—Topological electromagnetics, Nonreciprocity, Green’s function, Gyrotropic medium, Biased plasma.

I. INTRODUCTION

Topological materials and metamaterials are currently a topic of large interest in different domains of wave physics [1], [2], [3]. Of particular relevance is their ability to support back-scattering-immune surface waves on an interface with a material having different topological properties. The simplest realization of a topological material for electromagnetic waves is a nonreciprocal gyrotropic medium: when an interface with a conventional opaque medium is introduced, under certain conditions, a unidirectional surface mode emerges, completely immune to the presence of defects along its propagation [4]. While the study of unidirectional surface waves on gyrotropic materials dates back to the 1960s (see, e.g., [5]), only recently it has been shown that the existence of scattering-immune unidirectional surface modes originates from the non-trivial topological properties provided by the bulk eigenmodes of the gyrotropic medium [6], [7], [8].

A key factor necessary to realize a topologically-protected surface mode is to have a material with a band degeneracy in the bulk dispersion that is then lifted when a relevant symmetry of the system is broken (space-inversion, or time-reversal symmetry). In particular, if time-reversal symmetry is broken, a non-trivial bandgap opens, associated with a non-zero integer number known as gap Chern number (a topological invariant number), which can be used to classify the material in terms of its topological properties (the gap Chern number is the sum of the Chern numbers of all modes below the bandgap, $C_{\text{gap}} = \sum_{n < n_{\text{gap}}} C_n$) [16], [17]. This happens, for example, via the gyrotropy effect in biased plasmas or ferrites [6], [10], [9], making these materials the electromagnetic equivalent of quantum-Hall electronic insulators [11]. Opening or closing a non-trivial bandgap corresponds to a change of the global topological properties of a material. As a result, when two media with different topological properties share a common bandgap, at their interface the bandgap must completely close to allow for a modification of the gap Chern number from one medium to the other; therefore, unidirectional surface modes emerge [11], [12], [13], [14], [15] (the modes may be unidirectional due to the nonreciprocal nature of the system). The number of one-way edge states is equal to the difference of gap Chern numbers between the materials [16], [17] (a principle known as “bulk-edge correspondence”).

Considering the case of gyrotropic materials mentioned above as a model for continuous electromagnetic topological insulators, it was reported in [7] that, at an interface between a magnetized plasma and a topologically-trivial opaque medium (i.e., $C_{\text{gap}} = 0$), the difference of the gap Chern numbers is unity, which predicts the existence of a single one-way surface mode. In this context, we have recently studied more complex situations, in which, for example, the topological surface mode is a leaky wave, hence obtaining nonreciprocal radiation effects [18], as well as the case of topological materials with gain and loss, which enable the emergence of peculiar branch-point-like modal degeneracies, known as “exceptional points” [19]. Another particularly interesting situation occurs when two topologically-protected surface modes, existing on different lossless interfaces, are forced to couple due to a change in wave-guiding geometry. This situation is
shown in Fig. 1 two oppositely-biased gyrotropic materials ($C_{\text{gap}1} = 1$ and $C_{\text{gap}2} = -1$) are separated by a topologically-trivial central layer ($C_{\text{gap}1} = 0$), whose thickness is reduced gradually (Fig. 1a) or abruptly (Fig. 1b), such that the two material interfaces are brought closer to each other until they merge into a single interface. On this common interface, the gap in Chern number becomes $C_{\text{gap}1} - C_{\text{gap}2} = 2$, which predicts the existence of two topological edge modes. Interestingly, our numerical simulations in Fig. 1 show that, when two unidirectional surface waves are separately launched on the two original interfaces, they seamlessly merge into a single interface with different wavenumber. Due to its topological nature, this modal transformation occurs with zero reflection even in the case of abrupt step transition in Fig. 1(b). Moreover, it seems that, in both cases considered in Fig. 1(a-b), only one of the two available modes of the common interface is excited by the waveguide transition. This behavior raises interesting questions: why is the second surface mode predicted by the bulk-edge correspondence not excited at all? How does the waveguide transition control this topological modal coupling and transformation? In order to answer these questions rigorously (not based on perturbation methods such as coupled-mode theory), in this Letter we develop an exact electromagnetic Green’s function analysis of the considered wave-guiding structure that allows elucidating this interesting behavior.

A magnetic line source, $J_m = \hat{z} \delta(x) \delta(y - d)$, is assumed to exist in the dielectric region. The source and the geometry of problem are independent of the $z$-axis, which implies that all electromagnetic field components are invariant along this axis. By solving Maxwell’s equations containing a magnetic source, $\nabla \times E = i\omega \mu_0 H - J_m$, $\nabla \times H = -i\omega \epsilon_0 \epsilon_r E$, it can be shown that for the TM mode, the radiated $H_z(x, y)$ satisfies the following source-dependent wave equation

$$\left[\partial^2/\partial x^2 + \partial^2/\partial y^2 + k_0^2 \epsilon_r\right] H_z(x, y) = -i\omega \epsilon_0 \epsilon_r \delta(y - d) \tag{1}$$

where $k_0 = \omega/c$ and $c$ is the vacuum speed of light. The $x$-invariant geometry suggests the following integral representation for the Green’s function $H_z(x, y)$

$$H_z(x, y) = \frac{1}{2\pi} \int_{-\infty}^{+\infty} \mathcal{P}_z(k_x, y) e^{ik_x x} dk_x \tag{2}$$

where $\mathcal{P}_z(k_x, y)$ is the spatial spectrum (Fourier transform) of the magnetic field in wavenumber space (we assume a time-harmonic convention $e^{-i\omega t}$). It follows from (1) and (2) that

$$\left[\partial^2/\partial y^2 + k_y^2\right] \mathcal{P}_z(k_x, y) = -i\omega \epsilon_0 \epsilon_r \delta(y - d) \tag{3}$$

with $k_y = \sqrt{k_0^2 \epsilon_r - k_x^2}$. The solutions of (3) in the central region are

$$\mathcal{P}_z(k_x, y) = \begin{cases} B \cos(k_y y) + C \sin(k_y y), & 0 < y < d \\ D \cos(k_y y) + E \sin(k_y y), & d < y < h \end{cases} \tag{4}$$

with the following jump condition at the source point

$$\frac{d\mathcal{P}_z(k_x, d^+)}{dy} - \frac{d\mathcal{P}_z(k_x, d^-)}{dy} = -i\omega \epsilon_0 \epsilon_r \tag{5}$$

where $B$, $C$, $D$ and $E$ are unknown field amplitudes in the isotropic region. The electric field can be found from Maxwell’s equations.

At this point, in order to apply boundary conditions at the two material interfaces, one has to make some assumptions for the field distribution inside the gyrotropic layers. As mentioned in the Introduction, topologically-protected surface modes appear when we operate in the bandgap of the bulk modes of the gyrotropic material (i.e., the bandgap obtained by breaking time-reversal symmetry in Fig. 2c). The nature of the bandgap is important. In reciprocal media and structures exhibiting a frequency gap for the real solutions of their dispersion equation, such as in periodic structures near the Bragg condition, the bandgap is typically trivial, which means that the bands below/above the gap cannot be associated to any topological invariant number. Conversely, when a non-trivial bandgap is opened (e.g., in the bulk of a biased plasma), the eigenmode profile exhibits a topologically-non-trivial “rotation” as a function of wave momentum, leading to non-zero Chern number [3]. In order to have such a rotation in terms of momentum, the eigenmode envelope function should be complex (a real eigenmode envelope leads to trivial topological properties [8]). Based on these considerations, it is possible to recognize that, among different configurations of bulk-mode propagation in a biased plasma, only the TM mode propagating orthogonal to the bias provides non-trivial

II. GREEN’S FUNCTION AND DISPERSION EQUATION

As shown in Fig. 2(a), we consider a lossless multi-layered structure with interfaces normal to the $y$-axis and translationally invariant in the $x - z$ plane. An isotropic homogeneous material with permittivity $\epsilon_r$ occupies the region $0 < y < h$, and two different gyrotropic media described by permittivity and permeability tensors $\epsilon_i = \epsilon_0 (\epsilon_{i\epsilon} \hat{I} + \epsilon_{i\gamma} \hat{z} \hat{z} + i\epsilon_{i\delta} \hat{z} \times \hat{I})$, $\mu_i = \mu_0 \hat{I}$, $i = 1, 2$, fill the regions $y > h$ and $y < 0$ respectively, where $\hat{I} = \hat{I} - \hat{z} \hat{z}$ ($\hat{I}$ is the unit tensor).

Fig. 2. (a) Geometry considered for Green’s function extraction: a two-dimensional multi-layered structure with a magnetic line source embedded in the central isotropic region. (b-c) TM bulk-mode dispersion diagram for a plasma with plasma frequency $\omega_p = 0.9\omega_0$ and cyclotron frequency (b) $\omega_c = 0$ (unbiased) and (c) $\omega_c/\omega_p = 0.28$ (biased in the $z$ direction; wave propagation orthogonal to the bias).
topological properties. The dispersion equation of this mode is
\[ k_x^2 + k_y^2 = \epsilon_{eff}(\omega/c)^2, \]
where \( \epsilon_{eff} = (\epsilon_x^2 - \epsilon_y^2)/\epsilon_\ell \) is the effective permittivity. The solutions of this dispersion equation are shown in Fig. 3(b,c): when there is no bias (reciprocal system), \( \omega = 0 \) (\( \epsilon_y = 0 \)), a degeneracy occurs at \( k = \sqrt{k_x^2 + k_y^2} = 0 \) and \( \omega = \omega_p \); once we turn on the bias, the degeneracy is lifted and a non-trivial bandgap opens in the dispersion diagram. In this case, the gap Chern numbers becomes non-zero (the details are provided in [6], [8]), and a unidirectional surface mode is expected to emerge at frequencies within the gap.

For a generic TM surface mode existing on one of the interfaces in Fig. 2(a), the tangential components of the fields inside the two gyrotropic layers have the following form:

\[ y > h \rightarrow \begin{pmatrix} \mathbf{E}_{p1,x,y} \mathbf{k}_x \mathbf{k}_y \\ \mathbf{H}_{p1,z}(k_x,y) \end{pmatrix} = A e^{-\alpha_{p1} y} \begin{pmatrix} \mathbf{E}_{p2,x,y} \mathbf{k}_x \mathbf{k}_y \\ \mathbf{H}_{p2,z}(k_x,y) \end{pmatrix}, \]
\[ y < 0 \rightarrow \begin{pmatrix} \mathbf{E}_{p2,x,y} \mathbf{k}_x \mathbf{k}_y \\ \mathbf{H}_{p2,z}(k_x,y) \end{pmatrix} = F e^{\alpha_{p2} y} \begin{pmatrix} \mathbf{E}_{p1,x,y} \mathbf{k}_x \mathbf{k}_y \\ \mathbf{H}_{p1,z}(k_x,y) \end{pmatrix}, \]

\[ (6) \]

where \( \alpha_{p1} = \sqrt{k_x^2 - k_y^2 \epsilon_{eff,i}}, i = 1, 2 \), such that \( \text{Re}(\alpha_{p1}) > 0 \) is the vertical attenuation constant inside the gyrotropic media, and \( A \) and \( F \) are field amplitudes to be determined. Applying the boundary conditions for the continuity of tangential electric/magnetic fields at \( y = 0 \) and \( y = h \), and considering the continuity of the tangential component of the magnetic field at the source point and the jump condition in (5), one can solve the equations simultaneously for the six unknown field amplitudes, which yields

\[ H_z(x,y) = \frac{-i\omega \epsilon_{eff}}{2\pi} \int_{-\infty}^{+\infty} \cos(k_y d) + \gamma_3 \sin(k_y d) k_y (\gamma_3 - \gamma_1) \times \left( \cos(k_y y) + \gamma_1 \sin(k_y y) \right) e^{ik_x x} dk_x, \]

for \( 0 < y < d \)

\[ H_z(x,y) = \frac{i\omega \epsilon_{eff}}{2\pi} \int_{-\infty}^{+\infty} \cos(k_y d) + \gamma_1 \sin(k_y d) k_y (\gamma_3 - \gamma_1) \times \left( \cos(k_y y) + \gamma_3 \sin(k_y y) \right) e^{ik_x x} dk_x, \]

for \( d < y < h \)

\[ (7) \]

with the following parameters

\[ \gamma_1 = \frac{\epsilon_x \epsilon_y \epsilon_{p2} k_x - \epsilon_0^2 \epsilon_{p2} k_y}{\epsilon_y \epsilon_{p2} k_x + \epsilon_0^2 \epsilon_{p1} k_y}, \]
\[ \gamma_2 = \frac{\epsilon_{x} - \alpha_{p1} k_x - k_x^2 \epsilon_{p1}}{\epsilon_y \epsilon_{p1} k_x + \epsilon_0^2 \epsilon_{p1}}, \]
\[ \gamma_3 = \frac{N_2 \cos(k_y h) + D_2 \sin(k_y h)}{D_2 \cos(k_y h) - N_2 \gamma_2 \sin(k_y h)}, \]

\[ (8) \]

III. The Dynamic of Coupled Topological Modes

A. Coupling through an opaque medium

When the two interfaces in Fig. 2(a) are sufficiently far from each other, and the spacer layer is opaque (e.g., its permittivity is negative), the surface waves supported by the individual interfaces decay fast inside the central region, such that there is no coupling between them (their evanescent tails have negligible overlap). If the two gyrotropic materials have the same properties (same plasma frequencies \( \omega_{p1} = \omega_{p2} = \omega_p \)) and equal but opposite biasing fields \( \omega_{c1} = -\omega_{c2} = \omega_c \), the structure supports two identical topologically-protected and unidirectional surface modes, localized on each interface separately without any interference. The coupling between the two modes begins by decreasing the thickness of the opaque spacer. Fig. 3(a) shows the evolution of the two solutions of the dispersion equation \( D(k_x,\omega) = 0 \), at a given frequency, as the gap thickness decreases. For large gap thicknesses, the two uncoupled surface waves have the exact same propagation constant. For smaller thicknesses, the coupling perturbs the system, and the surface-wave solutions are “forced apart”, forming two distinct unidirectional surface modes with different wavenumber. In the limit of \( h \rightarrow 0 \), one of the solutions converges to a finite value of \( k_x \) smaller than \( k_p = \omega_p/c \) (the specific value depends on the plasma parameters), while the second solution asymptotically tends to infinity.

Fig. 3(b) shows the dispersion diagram of the structure in Fig. 2(a) with an opaque spacer of finite thickness. The solid blue lines indicate the TM bulk modes of the plasmas, \( k_x^2 + k_y^2 = \epsilon_{eff}(\omega/c)^2 \). Our analysis in Section II yields two additional dispersion curves crossing the entire bandgap, indicating the presence of two unidirectional topological surface modes with positive group velocities \( v_g = \partial\omega/\partial k_x > 0 \). A clear consequence of reciprocity and topological protection is the fact that, in this non-trivial bandgap, there are no solutions to the dispersion equation for negative \( k_x \) (no dispersion curves at the negative side of the \( k_x \)-axis), which results in the absence of any backward-propagating mode, and zero backscattering at discontinuities. If the opaque gap thickness is reduced, the dispersion line of the first surface mode moves toward left, converging to a finite values of \( k_x > 0 \) for zero thickness, whereas the second dispersion line shifts rapidly toward positive and very large values, \( k_x \rightarrow \infty \).
These findings answer one of the questions raised in the Introduction: when the two original interfaces merge into a single interface between “opposite” gyrotrropic media, there are still two unidirectional surface modes, in agreement with the bulk-edge correspondence; however, one of the two modes has extremely large (in theory, infinite) wavenumber \( k_x \), which leads to a very large wave-impedance for this TM mode, \( Z_{TM}/\eta \propto k_x \). For this reason, the mode is completely impedance-mismatched from the surface modes existing before the transition region in Fig. 1, hence the waveguide transition will not excite this mode at all. This fact, combined with the absence of backscattering at the topological protection, forces all the energy from the two uncoupled modes to be transferred to a single unidirectional mode, independently of the geometry of the waveguide transition.

Our formulation also allows investigating the spatial distribution of the magnetic field, \( H_z(x) \), produced by a line source that excites the two coupled topological surface modes. Fig. 3(c) shows the field distribution for a separation \( h = 0.4\lambda_0 \), at which the two unidirectional surface waves are weakly coupled and have similar wavenumber [the source is located at \( x = 0 \), and the field profile is calculated thorough both the exact solution in \( 7 \) and the residue evaluation in \( 9 \)]. As clearly seen in Fig. 3(c), there is no backward propagation along the negative \( x \)-axis, confirming the unidirectional nature of the topological modes. The two surface waves form an interference pattern as they propagate along the positive \( x \)-axis. The inset in this panel shows the two unidirectional surface waves separately, obtained from the residue calculation: the two modes have different wavelengths and are excited with slightly different amplitudes. Fig. 3(d) shows the case of smaller gap thickness \( h = 0.1\lambda_0 \), at which the difference in wavenumber of the two surface waves is larger, leading to a different pattern of interference. As seen in the inset, the second mode oscillates faster and has smaller amplitude. Additional numerical tests verify that by decreasing the spacer thickness further, the amplitude of the highly oscillating mode becomes smaller and smaller; in the limit of \( h \to 0 \), the slowly oscillating mode completely dominates the field distribution. The second mode still exists but with a vanishing wavelength and amplitude.

B. Coupling through a transparent medium

Topological mode coupling through a transparent dielectric gap with \( \epsilon_r > 0 \) leads to a different modal behavior. The dispersion curves of the two topological surface modes of this structure are shown in Fig. 4(a), considering vacuum as the transparent spacer between two oppositely-biased plasmas. Interestingly, in this situation the dispersion band of one of the surface modes crosses the vertical axis at \( k_x = 0 \) (black dot), while the other dispersion band is entirely on the positive side of the wavenumber axis. At this point, the phase velocity of the first topological surface mode is infinite, whereas the group velocity is non-zero and finite.

![Fig. 3. (a) Evolution of the two unidirectional TM surface-mode solutions of the dispersion equation (poles of Green’s function), for the geometry in Fig. 2, as a function of thickness of an opaque isotropic spacer with \( \epsilon_r = -1 \). The gyrotrropic media have the same properties as in Fig. 1. \( \lambda_0 \) is the free-space wavelength at the source frequency. (b) Dispersion bands for the bulk TM modes of the biased gyrotrropic media (plasmas) (solid blue) and for the unidirectional TM surface modes (solid red) for a spacer thickness of \( h = 0.63c/\omega_p \). The two horizontal purple lines indicate the lower/upper edges of the bulk-mode bandgap. Black arrows indicate the movement of the bands as \( h \) is reduced. (c) Magnetic field distribution in an opaque isotropic spacer of thickness \( h = 0.4\lambda_0 \), for a magnetic line source at \( x = 0, y = d = 0.7h \). Solid blue and red lines are the real and imaginary parts of \( H_z(x, y = 0.9d) \) obtained from the exact solution, Eq. 7. Dashed green and black lines are the real and imaginary parts obtained from residue evaluation, Eq. 9. Inset shows the field distribution for the two modes separately, calculated with Eq. 9. Solid and dotted lines indicate the real and imaginary parts of the first and second modes. (d) Same as (c) but for a spacer thickness of \( h = 0.1\lambda_0 \).](image1)

![Fig. 4. (a) Dispersion diagram, similar to Fig. 3(b) but for the case of transparent dielectric spacer with \( \epsilon_r = 1 \) and \( h = 0.36c/\omega_p \). (b) Magnetic field distribution, \( H_z(x, y = 0.9d) \), calculated by residue evaluation, Eq. 9, for a magnetic line source at \( x = 0, y = d = 0.7h \). (c) Locus of the two unidirectional TM surface-mode solutions of the dispersion equation in the complex wavenumber plane, for \( \omega/\omega_p = 1.078 \), from lossless to lossy case, i.e., \( \delta = 0 \to 0.05\omega_p \).

Different cases of interest can be identified in this scenario, depending on the value of frequency with respect to the crossing point of the first band at \( k_x = 0 \): (i) For frequencies above the crossing point, where two \( k_x \)-positive solutions exist,
the two surface waves propagates unidirectionally along the positive $x$-axis and interfere, similar to the opaque-spacer case considered above. (ii) At the frequency of the crossing point, a unidirectional $k_x^+$-positive surface wave “interferes” with a surface wave with diverging phase velocity and wavelength, which leads to a spatially constant shift in the real part of the field distribution excited by a line source, as shown in Fig. [4]b). (iii) For frequencies below the crossing point, there are two solutions with opposite sign of $k_x$; the phase velocity of the first mode is negative, $v_p = \omega/k_x < 0$, whereas the slope of the dispersion curve is always positive, leading to a positive group velocity, $v_g = \partial\omega/\partial k_x > 0$. It should be noted that, in this case, the mode is still unidirectional, and if the source is located at $x = 0$, the excited fields are zero for $x < 0$ (if the mode existed for negative values of $x$, the positive group velocity would indicate energy flowing toward the source, which is an unphysical situation). The correct picture for this situation is that the excited surface wave propagates toward right with positive group velocity (energy flows away from the source) but with negative phase velocity. To validate this scenario, Fig. [4]c shows the locus of the dispersion equation solutions, at $\omega_0/\omega_p = 1.078$. When the plasmas are lossless, $\delta = 0$, we have two solutions on the real axis of the wavenumber complex plane, $k_x,1 = -0.131k_0$ (red circle) and $k_x,2 = 0.406k_0$ (black circle). By introducing a small level of loss into the plasmas, $\delta = 0.052\omega_p$, the two solutions migrate toward the upper half plane, $k_x,1 = (-0.06 + i0.265)k_0$ and $k_x,2 = (0.465 + i0.12)k_0$, which is indeed the proper Riemann sheet for right-going surface modes (corresponding to decaying waves for propagation along $+x$).

We have also verified that, by reducing the thickness of the transparent dielectric, the second dispersion curve (SW2) moves toward left, converging to a finite value of $k_x > 0$ for zero thickness, whereas the first dispersion curve (SW1) moves completely to the negative $k_x$-axis, and asymptotically tends toward negative infinity for $h \to 0$, hence converging to the same case analyzed in the previous section of an individual interface between two opposite gyrotropic media.

IV. CONCLUSION

In summary, we have presented an exact Green’s function analysis of coupled topological surface modes in a lossless wave-guiding structure composed of oppositely-biased gyrotropic media separated by an isotropic medium. This formalism allowed us to rigorously study the unusual response (modal coupling/transformation) observed when two interfaces supporting topological surface modes are merged into a single interface. Our analysis provides relevant insight into a broad class of topologically-protected modal transitions. Our results and considerations, while derived for the case of biased gyrotropic materials, are expected to qualitatively apply to any time-reversal-broken topological wave-guiding system with coupled unidirectional surface modes.

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