The complete search for the supersymmetric Pati-Salam models from intersecting D6-branes

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Abstract: We construct a systematic method to build all the possible three-family \( \mathcal{N} = 1 \) supersymmetric Pati-Salam models from Type IIA orientifolds on \( T^6/(\mathbb{Z}_2 \times \mathbb{Z}_2) \) with intersecting D6-branes, in which the \( SU(4)_C \times SU(2)_L \times SU(2)_R \) gauge symmetry can be broken down to the \( SU(3)_C \times SU(2)_L \times U(1)_Y \) Standard Model gauge symmetry by the D-brane splitting and supersymmetry preserving Higgs mechanism. This is essentially achieved by solving all the common solutions for three generation conditions, RR tadpole cancellation conditions, and \( \mathcal{N} = 1 \) supersymmetry conditions with deterministic algorithm. We find that there are 202752 possible supersymmetric Pati-Salam models in total, and show that there are only 33 independent models with different gauge coupling relations at GUTs scale after modding out equivalent relations, such as T-dualities, etc. In particular, there is one and only one independent model which has gauge coupling unification. Furthermore, one can construct other types of intersecting D-brane models utilizing such deterministic algorithm, and therefore we suggest a brand new method for D-brane model building.

Keywords: D-Branes, String and Brane Phenomenology
1 Introduction

Supersymmetric Pati-Salam models has been investigated intensively with the purpose of realizing $\mathcal{N} = 1$ supersymmetric Standard Models (MSSM) and Standard Models (SM). It provides consistent constructions of four-dimensional supersymmetric $\mathcal{N} = 1$ chiral models with non-Abelian gauge symmetry on Type II orientifolds for the open string sectors. The chiral fermions on the worldvolume of the D-branes are located at orbifold singularities [1–9], and/or at the intersections of D-branes in the internal space [10] with a T-dual description in terms of magnetized D-branes as shown in [11, 12]. Explicit models have been constructed from Type IIA string theory with orientifolds $T^6/(\mathbb{Z}_2 \times \mathbb{Z}_2)$ such as in [13]. These gauge symmetries are constructed from intersecting D6-branes, with the Pati-Salam gauge symmetries $SU(4)_C \times SU(2)_L \times SU(2)_R$ breaking down to $SU(3)_C \times SU(2)_L \times U(1)_{B-L} \times U(1)_{I3R}$ via D6-brane splittings. It further breaks down to the SM via the Higgs mechanism. This provides a way to realize the SM without any additional anomaly-free $U(1)$’s around the electroweak scale. Note that there are also hidden sectors containing $USp(n)$ branes parallel to the orientifold planes or to their $\mathbb{Z}_2$ images. These models are normally constructed with at least two confining gauge groups in the hidden sector. For these models, the gaugino condensation triggers supersymmetry breaking and (some) moduli stabilization. In particular, one of these types of models admits a realistic...
phenomenology found by in [14, 15]. Its variations have been also visited in [16]. Moreover, there are a few other potentially interesting constructions with possible massless vector-like fields that might lead to SM [13]. These vector fields are not arising from a $\mathcal{N} = 2$ subsector, but can break the Pati-Salam gauge symmetry down to the SM or break $U(1)_{B-L} \times U(1)_{I_{3R}}$ down to $U(1)_Y$. For such construction, large wrapping numbers are required because the increased absolute values of the intersection numbers between the $U(4)_C$ stack of D-branes and the $U(2)_R$ stack (or its orientifold image). Therefore, more powerful search methods reaching large wrapping numbers are required. Many non-supersymmetric three-family SM-like models and generalized unified models have been constructed with intersecting D6-brane models on Type IIA orientifolds, for example in [17–19]. These models typically suffer from the large Planck scale corrections at the loop level which results in gauge hierarchy problem. A large number of the supersymmetric Standard-like models and generalized unified models have been constructed in [13–16, 20–36], solving the aforementioned problem. For a pedagogical introduction to phenomenologically interesting string models constructed with intersecting D-Branes, we refer to [37].

In [38], it was argued that there are a finite number of intersecting brane models on the orientifold $T^6/(\mathbb{Z}_2 \times \mathbb{Z}_2)$ with supersymmetry and tadpole conditions satisfied. An estimated number for models with gauge group $SU(4)_C \times SU(2)_L \times SU(2)_R$ was given. In [39, 40], we constructed supersymmetric Pati-Salam models and generalized Pati-Salam models on Type IIA $T^6/(\mathbb{Z}_2 \times \mathbb{Z}_2)$ orientifolds with D6-branes intersecting at generic angles to obtain the massless open string state spectra. In particular, in [39], we systematically studied the three-family $\mathcal{N} = 1$ supersymmetric Pati-Salam model building in Type IIA orientifolds on $T^6/(\mathbb{Z}_2 \times \mathbb{Z}_2)$ with intersecting D6-branes in which the $SU(4)_C \times SU(2)_L \times SU(2)_R$ gauge symmetries arise from $U(n)$ branes. We also found that the Type II T-duality in the previous study in [13] is not an equivalence relation in Pati-Salam model building as most of the models are not invariant under $SU(2)_L$ and $SU(2)_R$ exchange. In what follows, by swapping the $b-$ and $c-$ stacks of D6-branes, gauge couplings can be redefined and refined, making the gauge coupling unification possible. The search for supersymmetric Pati-Salam models in the previous works mentioned above are mostly based on random scanning methods. More recently, powerful machine learning methods have also been employed, such as in [41] and [42] with realistic intersecting D-brane models revisited. However, the complete landscape of concrete D-brane models such as supersymmetric Pati-Salam models remains to be drawn.

Distinct from the scanning methods we employed in ref. [39], in [40] by explicitly solving the conditions of generalized version of Pati-Salam models, we for the first time systematically discuss the $\mathcal{N} = 1$ supersymmetric $SU(12)_C \times SU(2)_L \times SU(2)_R$ models, $SU(4)_C \times SU(6)_L \times SU(2)_R$ models, and $SU(4)_C \times SU(2)_L \times SU(6)_R$ models from the Type IIA orientifolds on $T^6/(\mathbb{Z}_2 \times \mathbb{Z}_2)$ with intersecting D6-branes. These gauge symmetries can be broken down to the Pati-Salam gauge symmetry $SU(4)_C \times SU(2)_L \times SU(2)_R$ via three $SU(12)_C/ SU(6)_L/ SU(6)_R$ adjoint representation Higgs fields, and further down to the Standard Model (SM) via the D-brane splitting and Higgs mechanism.

In this paper, we develop a systematic method to obtain the complete landscape of supersymmetric Pati-Salam models. By solving the RR tadpole cancellation conditions,
supersymmetry conditions and three generation conditions step by step, we managed to find a total of all 202752 possible supersymmetric Pati-Salam models, with 33 independent gauge coupling relations. We achieve gauge coupling unification in 6144 models with three generations obtained from the brane intersection of $a$- and $b$- stacks of branes, of $a$- and $b'$- stack of branes, of $a$- and $c$- stack of branes, and of $a$- and $c'$- stack of branes. This gauge symmetry can be further broken down to the Standard Model via D-brane splitting and Higgs mechanism. This method may also be utilized for other string model buildings, especially for brane intersecting models.

The paper is organized as follows. We will first review the basic rules for supersymmetric intersecting D6-brane model building on Type IIA $T^6/(\mathbb{Z}_2 \times \mathbb{Z}_2)$ orientifolds in section 2 and 3, and introduce our systematic solving method to obtain all the supersymmetric Pati-Salam models in section 4. In section 5, we present one representative supersymmetric Pati-Salam model for each of the 33 different gauge coupling relations. In section 6, we discuss the phenomenology behaviors of supersymmetric Pati-Salam models. Finally, section 7 is dedicated to a conclusion and outlook.

2 $T^6/(\mathbb{Z}_2 \times \mathbb{Z}_2)$ orientifolds with intersecting D6-branes

In refs. [21, 23], supersymmetric models are constructed on Type IIA $T^6/(\mathbb{Z}_2 \times \mathbb{Z}_2)$ orientifolds with D6-branes intersecting at generic angles. One can identify the six-torus $T^6$ to the product of three two-tori $T^6 = T^2 \times T^2 \times T^2$. The corresponding complex coordinates on the two-tori are denoted by $z_i$, $i = 1, 2, 3$ respectively. The generators $\theta$ and $\omega$ of the orbifold group $\mathbb{Z}_2 \times \mathbb{Z}_2$, which are respectively associated with the twist vectors $(1/2, -1/2, 0)$ and $(0, 1/2, -1/2)$, act on the complex coordinates $z_i$ as below

$$
\theta : (z_1, z_2, z_3) \mapsto (-z_1, -z_2, z_3),
\omega : (z_1, z_2, z_3) \mapsto (z_1, -z_2, -z_3).
$$

We implement the orientifold projection by gauging the $\Omega R$ symmetry, where $\Omega$ is the world-sheet parity, and $R$ is the map defined by

$$
R : (z_1, z_2, z_3) \mapsto (\bar{z}_1, \bar{z}_2, \bar{z}_3).
$$

It follows that, we have four kinds of orientifold 6-planes (O6-planes) for each of the actions of $\Omega R$, $\Omega R\theta$, $\Omega R\omega$, and $\Omega R\theta\omega$. To cancel the RR charges of O6-planes, stacks of $N_a$ D6-branes wrapping on the factorized three-cycles are introduced. Also, on each two-torus, there are two possible complex structures consistent with orientifold projection: rectangular or tilted [15, 18, 21, 23]. The homology classes of the three cycles wrapped by the D6-brane stacks can be written as $n_a^i[a_i] + m_a^i[b_i]$ and $n'_a[a'_i] + m'_a[b_i]$ for rectangular and tilted tori respectively, with $[a'_i] = [a_i] + \frac{1}{2}[b_i]$. Therefore, in both cases, a generic one cycle can be labelled by two wrapping numbers $(n'_a, l'_a)$, where $l'_a = m'_a$ on a rectangular two-torus and $l'_a = 2m'_a = 2m'_a + n'_a$ on a tilted two-torus. As a consequence, $l'_a - n'_a$ must be even for a tilted two-torus.
In addition, for a stack of \( N_a \) D6-branes along the cycle \((n^i_a, l^i_a)\), we introduce their \( \Omega R \) images as \( a' \)-stack of \( N_a \) D6-branes with wrapping numbers \((n^i_a, -l^i_a)\). Their homology three-cycles are

\[
[\Pi_a] = \prod_{i=1}^{3} (n^i_{a}[a_i] + 2^{-\beta_i}l^i_{a}[b_i]) \quad \text{and} \quad [\Pi_{a'}] = \prod_{i=1}^{3} (n^i_{a}[a_i] - 2^{-\beta_i}l^i_{a}[b_i]) ,
\]

where \( \beta_i = 0 \) if the \( i \)-th torus is rectangular and or \( \beta_i = 1 \) otherwise. The homology three-cycles wrapped by the four O6-planes, can be written as

\[
\Omega R : [\Pi_{\Omega R}] = 2^{\beta_i}[a_i] \times [a_2] \times [a_3] ,
\]

\[
\Omega R\omega : [\Pi_{\Omega R\omega}] = -2^{3-\beta_2-\beta_3}[a_1] \times [b_2] \times [b_3] ,
\]

\[
\Omega R\theta\omega : [\Pi_{\Omega R\theta\omega}] = -2^{3-\beta_1-\beta_3}[b_1] \times [a_2] \times [b_3] ,
\]

\[
\Omega R\theta : [\Pi_{\Omega R\theta}] = -2^{3-\beta_1-\beta_2}[b_1] \times [b_2] \times [a_3] .
\]

It follows that the intersection numbers between different stacks can be expressed in terms of wrapping numbers as follows

\[
I_{ab} = [\Pi_a][\Pi_b] = 2^{-k} \prod_{i=1}^{3} (n^i_{a}l^i_{b} - n^i_{b}l^i_{a}) ,
\]

\[
I_{ab'} = [\Pi_a] \left[ \Pi_{b'} \right] = -2^{-k} \prod_{i=1}^{3} (n^i_{a}l^i_{b'} + n^i_{b'}l^i_{a}) ,
\]

\[
I_{aa'} = [\Pi_a] \left[ \Pi_{a'} \right] = -2^{3-k} \prod_{i=1}^{3} (n^i_{a}l^i_{a}) ,
\]

\[
I_{aO6} = [\Pi_a][\Pi_{O6}] = 2^{3-k} (-l^1_{a}l^2_{a}l^3_{a} + l^1_{a}n^2_{a}n^3_{a} + n^1_{a}l^2_{a}n^3_{a} + n^1_{a}n^2_{a}l^3_{a}) ,
\]

where \( k = \beta_1 + \beta_2 + \beta_3 \) is the total number of tilted two-tori, and \([\Pi_{O6}] = [\Pi_{\Omega R}] + [\Pi_{\Omega R\omega}] + [\Pi_{\Omega R\theta\omega}] + [\Pi_{\Omega R\theta}]\) is the sum of four O6-plane homology three-cycles. The generic massless particle spectrum for intersecting D6-branes at general angles can be expressed in terms of the intersection numbers as listed in table 1, which is valid for both rectangular and tilted two-tori.

### 2.1 The RR tadpole cancellation conditions

Moreover, as discussed in [21, 43, 44], the tadpole cancellation conditions directly result in the \( SU(N_a)^3 \) cubic non-Abelian anomaly cancellation, and the cancellation of \( U(1) \) mixed gauge and gravitational anomaly or \([SU(N_a)^2]\ U(1) \) gauge anomaly can be achieved by Green-Schwarz mechanism which mediated by untwisted RR fields [21, 43, 44]. The D6-branes and orientifold O6-planes give rise of RR fields, and thus are restricted by the Gauss law in a compact space. To be precise, the sum of the RR charges of D6-branes and O6-planes must be zero because of the conservations of the RR field flux lines. The conditions for RR tadpole cancellations can be written as

\[
\sum_a N_a[\Pi_a] + \sum_a N_a \left[ \Pi_{a'} \right] - 4[\Pi_{O6}] = 0 ,
\]

[\text{JHEP08(2022)044}].
| Sector | Representation |
|--------|----------------|
| aa     | $U(N_a/2)$ vector multiplet |
|        | 3 adjoint chiral multiplets |
| $ab + ba$ | $I_{ab} (\square, \square)$ fermions |
| $ab' + b'a$ | $I_{ab'} (\square, \square)$ fermions |
| $aa' + a'a$ | $\frac{1}{2}(I_{aa'} - \frac{1}{2}I_{a,O6})$ fermions |
|        | $\frac{1}{2}(I_{aa'} + \frac{1}{2}I_{a,O6})$ fermions |

Table 1. General massless particle spectrum for intersecting D6-branes at generic angles. The second column lists the representations of the resulting gauge symmetry group $U(N_a/2)$ under the $\mathbb{Z}_2 \times \mathbb{Z}_2$ orbifold projection [21]. The chiral supermultiplets include both scalars and fermions in this supersymmetric constructions. And we choose the positive intersection numbers to denote the left-handed chiral supermultiplets as convention.

where the last terms come from the O6-planes with $-4$ RR charges in D6-brane charge unit.

To make the following discussion less cumbersome, we introduce the following variables which are products of wrapping numbers

$$
A_a \equiv -n_a^1 n_a^2 n_a^3, \quad B_a \equiv n_a^1 l_a^2 n_a^3, \quad C_a \equiv l_a^1 n_a^2 n_a^3, \quad D_a \equiv l_a^1 l_a^2 n_a^3, \\
\tilde{A}_a \equiv -l_a^1 l_a^2 l_a^3, \quad \tilde{B}_a \equiv l_a^1 n_a^2 n_a^3, \quad \tilde{C}_a \equiv n_a^1 l_a^2 n_a^3, \quad \tilde{D}_a \equiv n_a^1 n_a^2 l_a^3. 
$$

In order to cancel the RR tadpoles, we define an arbitrary number of D6-branes wrapping cycles along the orientifold planes, which is the so-called “filler branes”, contributing to the RR tadpole cecallation conditions. Moreover, this also trivially satisfy the four-dimensional $N = 1$ supersymmetry conditions. The tadpole conditions then can be represented by

$$
-2^k N^{(1)} + \sum_a N_a A_a = -2^k N^{(2)} + \sum_a N_a B_a = -2^k N^{(3)} + \sum_a N_a C_a \\
= -2^k N^{(4)} + \sum_a N_a D_a = -16, 
$$

where $2N^{(i)}$ is the number of filler branes wrapping along the $i$-th O6-plane, as given in table 2. The filler branes representing the USp group carry the same wrapping numbers as one of the O6-planes as shown in table 2. We denote the USp group as the $A$-, $B$-, $C$- or $D$-type USp group according to non-zero $A$, $B$, $C$ or $D$ filler branes, respectively.

### 2.2 Conditions for four-dimensional $N = 1$ supersymmetric D6-brane

In four-dimensional $N = 1$ supersymmetric models, 1/4 supercharges from ten-dimensional Type I T-dual need to be preserved. That is, these 1/4 supercharges remain preserved under the orientation projection of the intersecting D6-branes and the $\mathbb{Z}_2 \times \mathbb{Z}_2$ orbifold projection on the background manifold. One can show that for the four-dimensional $N = 1$ supersymmetry to survive the orientation projection, the rotation angle of any D6-brane with respect to the orientifold plane must be an element of $SU(3)$ [10]. Or equivalently,
Among $\theta_i$ which one is positive, we can classify the NZ-type branes into the $\mathbb{Z}$-type and NZ-type D6-branes. Each type can be further classified into two subtypes with the wrapping numbers in table 2, the gauge symmetry is USp group. In this case, there are two negative and two zero values among $\theta_i$ are the complex structure moduli for the $i$-th two-torus. Because the $\theta_i$ are non-zero, we call the corresponding D6-brane a NZ-type D6-brane. According to which one is positive, we can classify all possible D6-brane configurations into three types (which preserve the four-dimensional $\mathcal{N} = 1$ supersymmetry):

1. If the filler brane has the same wrapping numbers as one of the O6-planes shown in table 2, the gauge symmetry is USp group. In this case, one and only one wrapping number product $A$, $B$, $C$ and $D$ has non-zero and negative value. According to which one is non-zero, we call the corresponding USp group as the $A$-, $B$-, $C$- or $D$-type USp group, as already mentioned in the last section.

2. If there is a zero wrapping number, we call the D6-brane a Z-type D6-brane. In this case, there are two negative and two zero values among $A$, $B$, $C$ and $D$.

3. If there is a no zero wrapping number, we call the D6-brane a NZ-type D6-brane. Among $A$, $B$, $C$ and $D$, three of them are negative and one of them is positive. According to which one is positive, we can classify the NZ-type branes into the $A$-, $B$-, $C$- and $D$-type NZ branes. Each type can be further classified into two subtypes with the wrapping numbers taking the form as follows

\[
\begin{align*}
A1 &: (\cdot, \cdot) \times (\cdot, \cdot) \times (\cdot, \cdot), \\
B1 &: (\cdot, \cdot) \times (\cdot, \cdot) \times (\cdot, \cdot), \\
C1 &: (\cdot, \cdot) \times (\cdot, \cdot) \times (\cdot, \cdot), \\
D1 &: (\cdot, \cdot) \times (\cdot, \cdot) \times (\cdot, \cdot).
\end{align*}
\]

Table 2. The wrapping numbers for four O6-planes.
3 Supersymmetric Pati-Salam model building

To construct SM or SM-like models by intersecting D6-branes model, in addition to the U(3)C and U(2)L gauge symmetries from stacks of branes, we also need at least two extra U(1) gauge groups in both supersymmetric and non-supersymmetric models so as to obtain the correct quantum number for right-handed charged leptons [21–23, 44]. One is the lepton number symmetry U(1)L, while the other is similar to the third component of right-handed weak isospin U(1)L3R. Then the hypercharge can be expressed as

$$Q_Y = Q_{I_{3R}} + \frac{Q_B - Q_L}{2},$$

(3.1)

where U(1)B is the overall U(1) of U(3)C. In general, the U(1) gauge symmetry, coming from a non-Abelian SU(N) gauge symmetry, is anomaly free and hence its gauge field is massless. In our model, U(1)B−L and U(1)L3R arise respectively from SU(4)C and SU(2)R gauge symmetries. Thus, they are both anomaly free and their gauge fields are massless.

We introduce three stacks of D6-branes, called a-, b-, c-stacks with respective D6-brane numbers 8, 4, and 4. Their respective gauge symmetries are U(4)C, U(2)L and U(2)R. Thus, the Pati-Salam gauge symmetries we obtain is SU(4)C × SU(2)L × SU(2)R. Via splitting of the D6-branes and Higgs Mechanism, as discussed in [13], the Pati-Salam gauge symmetry can be broken down to the SM following the chain

$$\text{SU}(4) \times \text{SU}(2)_{L} \times \text{SU}(2)_{R} \xrightarrow{a \rightarrow a_1 + a_2} \text{SU}(3)_{C} \times \text{SU}(2)_{L} \times \text{SU}(2)_{R} \times \text{U}(1)_{B-L}$$

$$\xrightarrow{c \rightarrow c_1 + c_2} \text{SU}(3)_{C} \times \text{SU}(2)_{L} \times \text{U}(1){}_{I_{3R}} \times \text{U}(1)_{B-L}$$

$$\xrightarrow{\text{Higgs Mechanism}} \text{SU}(3)_{C} \times \text{SU}(2)_{L} \times \text{U}(1)_{Y}.$$  

(3.2)

Moreover, to have three families of the SM fermions, the intersection numbers must satisfy

$$I_{ab} + I_{ab'} = 3,$$

$$I_{ac} = -3, \ I_{ac'} = 0.$$  

(3.3)

The quantum numbers under the SU(4)C × SU(2)L × SU(2)R gauge symmetries are (4, 2, 1) and (4, 1, 2). To satisfy the Iac′ = 0 condition in (3.3), the stack a D6-branes must be parallel to the orientifold (ΩR) image c′ of the c-stack along at least one torus. Instead of (3.3), we could also require the intersection number to satisfy

$$I_{ac} = 0, \ I_{ac'} = -3.$$  

(3.4)

But this actually entirely equivalent due to the symmetry transformation c ↔ c′, the model with intersection numbers are equivalent to that with I_{ac} = -3 and I_{ac'} = 0. The gauge kinetic function for a generic stack x of D6-branes can be expressed as [27]

$$f_x = \frac{1}{4} \left[ n^1_x n^2_x n^3_x S - \left( \sum_{i=1}^{3} 2^{-\beta_j - \beta_k} n^1_x n^2_x n^3_x \beta_i \bar{U} U_i \right) \right],$$

(3.5)
where the real parts of dilaton \( S \) and moduli \( U^i \) respectively are
\[
\text{Re}(S) = \frac{M_s^3 R_1^1 R_1^2 R_1^3}{2\pi g_s}, \quad \text{(3.6)}
\]
\[
\text{Re}(U^i) = \text{Re}(S) \chi_j \chi_k, \quad \text{(3.7)}
\]
in which \( i \neq j \neq k \), \( g_s \) is the string coupling. The gauge coupling constant associated with \( x \) can be related with the kinetic function by
\[
g_{D6_x}^{-2} = |\text{Re} (f_x)|. \quad \text{(3.8)}
\]

By our convention, the holomorphic gauge kinetic functions for SU(4)\(_C\), SU(2)\(_L\) and SU(2)\(_R\) are respectively identified with the \( a\)-, \( b\)-, and \( c\)- stacks. Moreover, the holomorphic gauge kinetic function for U(1)\(_Y\) is a linear combination of those for SU(4) and SU(2)\(_R\). Namely, by [15, 18], we have
\[
f_Y = \frac{3}{5} \left( \frac{2}{3} f_a + f_c \right). \quad \text{(3.9)}
\]

In addition, we can write the tree-level MSSM gauge couplings as
\[
g_a^2 = \alpha g_b^2 = \beta g_c^2 = \gamma \left[ \pi e^{\phi_s} \right] \quad \text{(3.10)}
\]
where \( g_a^2 \), \( g_b^2 \), and \( \frac{2}{3} g_c^2 \) are the strong, weak and hypercharge gauge couplings and \( \alpha, \beta, \gamma \) are the ratios between them. Moreover, the Kähler potential takes the form of
\[
K = -\ln(S + \bar{S}) - \sum_{I=1}^{3} \ln(U^I + \bar{U}^I). \quad \text{(3.11)}
\]

According to the four-dimensional \( \mathcal{N} = 1 \) supersymmetry conditions, three stacks of D6-branes, carrying the U(4)\(_C\) × U(2)\(_L\) × U(2)\(_R\) gauge symmetry, generically determine the complex structure moduli \( \chi_1, \chi_2 \) and \( \chi_3 \). For this reason, we only have one independent modulus field. In order to stabilize these moduli, one usually has at least two USp groups with negative \( \beta \) functions allowing for gaugino condensations [45–47]. In general, the one-loop beta function for the \( 2N^{(i)} \) filler branes, which are on top of \( i \)-th O6-plane and carry USp\( (N^{(i)}) \) group, is given by [13]
\[
\beta_{\phi_s}^i = -3 \left( \frac{N^{(i)}}{2} + 1 \right) + 2 |I_{ai}| + |I_{bi}| + |I_{ci}| + 3 \left( \frac{N^{(i)}}{2} - 1 \right)
\]
\[
= -6 + 2 |I_{ai}| + |I_{bi}| + |I_{ci}|, \quad \text{(3.12)}
\]
which will be considered in our model building also.

4 Deterministic algorithm

For the searching method we developed in this section, we consider all the above mentioned three generation conditions. Moreover, we do not restrict the tilted two-torus to be the third one, but consider this can be the first and second two-torus also. Furthermore, we
will also include all the possible constructions in terms of wrapping numbers, including the models that can be related via T-duality and D6-brane Sign Equivalent Principle as this might lead to models with different gauge couplings as discussed in [13, 39].

Recall that each Pati-Salam model is determined by 18 integer parameters, called wrapping numbers $n_1^a, \ldots, n_c^3, l_1^a, \ldots, l_c^3$. These wrapping numbers are required to satisfy the tadpole condition (2.14), the supersymmetry condition (2.15) and the three generation conditions (3.3) and (3.4). Moreover, $l_l^a - n_l^a$ must be an even number if the corresponding torus is a tilted one, while it must be an odd number if the corresponding torus is rectangular. We refer to this condition as the parity condition. The goal here is to describe a method to find all 18-uplets satisfying these conditions.

Observe that all the conditions in question can be expressed as polynomial or rational equations or inequalities with the wrapping numbers as variables. Hence mathematically, the system consisting of (2.14), (2.15), (3.3), (3.4) together with the parity condition is equivalent to a Diophantine equation. We shall call this equation the “Pati-Salam equation”.

On the one hand, there does not exist any algorithm to solve a general Diophantine equation, as implied by the negative solution to Hilbert’s tenth problem. On the other hand, because of the large number of variables, it is unrealistic to use a brute-force search to complete the full landscape of the models, even though an estimated upper bound on the absolute values of the wrapping numbers was discussed in [38]. Previously, in [39], random search method has been implemented and some particular solutions to the Pati-Salam equation have been found.

In the present work, we devised a deterministic algorithm which allowed us to find the whole solution set of the Pati-Salam equation, constituting the complete picture of the Supersymmetric Pati-Salam landscape. Note that our algorithm does not rely on the estimated wrapping number bound given in [38], while we reconfirm the finiteness argued therein.

Let us describe the main strategy of our algorithm.

**First step.** We list all the possible combinations of the signs of the twelve wrapping number products $A_a, B_a, \ldots, C_c, D_c$. As observed previously in [23] and recalled in section 2.2, because of the supersymmetry condition, $(A_a, B_a, C_a, D_a)$ contains three negative numbers and one positive number, or two negative numbers and two zeroes, or with one negative number and three zeros. Same applies to the $b-$ stack and the $c-$ stack.

**Second step.** For each possibility listed in the first step, we append the twelve corresponding inequalities (for example, $A_a > 0, B_a < 0, C_a < 0, D_a < 0$) to our system and try to solve the new system. Inside the system, we look for the following four kinds of equations or subsystems due to the requirements from three generation conditions (3.3) and (3.4) for Subsystem 1, 2 and 3, and tadpole condition (2.14) for Subsystem 4.

- **Subsystem 1.** Equation of the form $v_1 \cdots v_j = 0$ where $v_1, \ldots, v_j$ are variables. This includes for example $0 = 2I_{ac} = \prod_{i=1}^{3} (n_i^a l_i^c - l_i^a n_i^c)$ where we view $n_i^a l_i^c - l_i^a n_i^c$, $i = 1, 2, 3$ as variables. An equation of this form, obviously, can be solved as $v_1 = 0$ or $v_2 = 0$ or $\ldots$ or $v_j = 0$. 

• **Subsystem 2.** Equation of the form \( v_1 \cdots v_j = p \) where \( v_1, \ldots, v_j \) are variables and \( p \) is an nonzero integer. Since \( p \) has only finitely many factors, an equation of this form can also be solved, leading to finitely many choices for the variables \((v_1, \ldots, v_j)\).

• **Subsystem 3.** A system of linear equations of full rank. This includes for example

\[
\begin{cases}
 n^1_a l^1_c - l^1_a n^1_c = 0, \\
 n^1_a l^1_c + l^1_a n^1_c = 0,
\end{cases}
\]

where \( n^1_a \) and \( l^1_c \) are viewed as variables. A system of this kind has unique solution.

• **Subsystem 4.** A system of linear inequalities according to \( A_a, A_b, A_c \) which has finitely many integer solutions. Thanks to the first step, it is easier to find such system. For example, we could often see subsystem of the following form

\[
\begin{cases}
 4 + 2 A_a + A_b + A_c \geq 0, \\
 A_a < 0, \\
 A_b < 0, \\
 A_c = 0.
\end{cases}
\]

To find such subsystem, we list all linear inequalities in our system. Then for each subset of these inequalities, we can determine whether its integer solution set is finite. A subsystem of linear inequalities over the real numbers corresponds to a polyhedron, and there are well-known algorithms (e.g. Fourier-Motzkin elimination) that determine whether the polyhedron volume is finite. If the polyhedron has a finite volume, then the corresponding subsystem has finitely many integer solutions, while if the volume is infinite, there will be infinitely many integer solutions. We only deal with the first case in this Subsystem, and consider the case with infinitely many solutions in the Fourth step.

When we find inside our system a subsystem of the above four types, we proceed to solve this subsystem. For each solution, we substitute the solution back to the remaining equations in the system, leading to a new system of equations with less unknown variables. Such new system will be called a subcase.

**Third step.** We iterate the second step for each subcase with the focus on the unknown variables left from the second step and repeat. We stop the iteration until there are no more subsystem of these four kinds. Till here, we use three generation conditions and tadpole conditions to the most extend.

**Fourth step.** This step is devoted to the restrictions from supersymmetry conditions. Recall that for supersymmetry equality condition in (2.15), as we discussed in [40], \( x_A, x_B, x_C, x_D \) are solutions to the linear system

\[
\begin{align*}
 x_A \tilde{A}_a + x_B \tilde{B}_a + x_C \tilde{C}_a + x_D \tilde{D}_a &= 0, \\
 x_A \tilde{A}_b + x_B \tilde{B}_b + x_C \tilde{C}_b + x_D \tilde{D}_b &= 0, \\
 x_A \tilde{A}_c + x_B \tilde{B}_c + x_C \tilde{C}_c + x_D \tilde{D}_c &= 0.
\end{align*}
\]
Crámer’s rule tells us, provided that the linear system has rank 3, the solution of \((x_A, x_B, x_C, x_D)\) is proportional to \((y_A, y_B, y_C, y_D)\) where

\[
y_A = \begin{vmatrix} \bar{B}_a & \bar{C}_a & \bar{D}_a \\ \bar{B}_b & \bar{C}_b & \bar{D}_b \\ \bar{B}_c & \bar{C}_c & \bar{D}_c \end{vmatrix}, \quad y_B = - \begin{vmatrix} A_a & \bar{C}_a & \bar{D}_a \\ A_b & \bar{C}_b & \bar{D}_b \\ A_c & \bar{C}_c & \bar{D}_c \end{vmatrix}, \quad y_C = - \begin{vmatrix} A_a & \bar{B}_a & \bar{D}_a \\ A_b & \bar{B}_b & \bar{D}_b \\ A_c & \bar{B}_c & \bar{D}_c \end{vmatrix}, \quad y_D = - \begin{vmatrix} A_a & \bar{B}_a & \bar{C}_a \\ A_b & \bar{B}_b & \bar{C}_b \\ A_c & \bar{B}_c & \bar{C}_c \end{vmatrix}.
\]

More precisely, the solution of supersymmetry equality conditions in terms of the linear system \((4.1)\) can be solved by

\[
\begin{align*}
x_A &= \lambda, \\
x_B &= \lambda y_B / y_A, \\
x_C &= \lambda y_C / y_A, \\
x_D &= \lambda y_D / y_A,
\end{align*}
\]

where \(x_A = \lambda, \ x_B = \lambda^2 \beta_2 + \beta_3 / \lambda \chi_2 \chi_3, \ x_C = \lambda^2 \beta_1 + \beta_3 / \lambda \chi_1 \chi_3, \ x_D = \lambda^2 \beta_1 + \beta_2 / \lambda \chi_1 \chi_2\), in which \(\chi_i = R_i^2 / R_i^3\) represent the complex structure moduli for the \(i\)-th two-torus, and the values of \(x_A, x_B, x_C, x_D\) are required to be all positive with positive constant factor \(\lambda\). It follows that, \((y_A, y_B, y_C, y_D)\) in \((4.2)\) have to have same sign, and thus impose restrictions of the wrapping numbers for the inequality conditions as well.

Now we make use of supersymmetry inequality conditions \((2.15)\). We list all the possible signs of the remaining variables from the third step and then we determine whether the inequality condition in question can be achieved, taking into consideration of supersymmetry equality conditions. For example, say, we are looking for solutions with \(A_a < 0, B_a < 0, C_a < 0, D_a > 0, A_b < 0, B_b < 0, C_b = 0, D_b = 0, A_c < 0, B_c > 0, C_c < 0, D_c < 0\) (by the choice made in the first step). After running the second and third steps, we find \(\nu_a^1 = 1, \nu_a^2 = -1, \nu_a^3 = 1, \nu_b^1 = -1, \nu_b^2 = -1, \nu_b^3 = 1, \nu_c^1 = -1, \nu_c^2 = -1, \nu_c^3 = 1, \nu_a^1 = -1, l_a^1 = -1, l_a^2 = 0, l_a^3 = 1, l_b^2 = 2\) as a potential partial solution. The remaining variables are \(l_a^2, l_b^2, l_c^2\) and \(l_b^3\). Then \(B_a < 0, B_b < 0, B_c > 0\) implies that \(l_b^2 < 0, l_c^2 < 0\) and \(l_b^3 > 0\). It follows that

\[
\frac{x_A}{x_C} = \frac{-3l_a^2 + l_a^2 l_b^3 + l_a^2 l_c^2}{-l_b^2 (l_a^2 + 2l_c^2)} < 0.
\]

Hence \(x_A\) and \(x_C\) have opposite signs and one must be negative, which contradicts with the supersymmetry equality condition as described under \((4.3)\) that \(x_A, x_B, x_C, x_D\) are required to be all positive with positive overall factor \(\lambda\). Note that in some cases it is more convenient to check whether \(y_A, y_B, y_C, y_D\) in \((4.2)\) have the same sign for the exclusion.

After running the whole algorithm, we find that intriguingly all the cases left from the third step are eliminated in the fourth step, and thus we complete the landscape exploration. We found that in total there are 202752 different supersymmetric Pati-Salam models, constituting the full landscape of Pati-Salam models. The largest allowed wrapping number is 5, e.g. as shown in table 6 and 7. In particular, there are 6144 models with gauge coupling unification at GUTs scale i.e. \(g_a^2 = g_b^2 = g_c^2 = \left(\frac{5}{3}\right)^2\).
Among the total possible 202752 supersymmetric Pati-Salam models, we find there are 33 types of gauge coupling relations offered from supersymmetric Pati-Salam models as shown in table 5–37 with one representative model for each type in appendix A. For each type of 33 gauge coupling relations, there are 6144 models with the tilted two-tori (for which $l_a^b - n_a^b$ must be even) to be the first, second, and third one, while the other two-tori are rectangular. This can be naturally understood since in our type Type IIA and $b$-stacks and $c$-stacks of D6-branes (with non-trivial T-dualities involved). This corresponds to exchanging the gauge couplings of $SU(2)$ and $SU(2)_R$ at near string scale as Model 6 and 7 presented. Note that this construction is not simply swapping the $b$- and $c$-stack of D6-branes, but non-trivial Type II T-dualities also
need to be performed. In [48], we present that models with wrapping number 5 can obtain gauge coupling unification at string scale with two-loop renormalization group equation running by introducing seven pairs of vector-like particles from $N = 2$ sector.

There are several classes of models with one, two, three or four USp groups. In particular, such as in Model 5, there are four confining USp(2) gauge groups, similar as in Model XVIII in [39] and Model I-Z-10 in [13]. Distinct from the random models one can obtain from random scanning, we obtain all the allowed 6144 supersymmetric Pati-Salam models with gauge coupling unification, i.e. $g^2_a = g^2_b = g^2_c = \left( \frac{5}{3} g^2_Y \right)$, where $g^2_a, g^2_b$, and $\frac{5}{3} g^2_Y$ denote the strong, weak and hypercharge gauge couplings respectively.

We observe there are three classes of gauge coupling unification models with the large wrapping number in the first, second, and third torus respectively. In which the large wrapping number appears in the first, second, and third torus, e.g. in Model 38, Model 39 and Model 40. This is easy to understand due to the democratic position of three tori in the orientifold construction $\mathbb{T}^6/\mathbb{Z}_2 \times \mathbb{Z}_2$. For all these 6144 models with gauge coupling unification, there are four confining USp(2) gauge groups allowing the moduli of these supersymmetric Pati-Salam models stabilized via gaugino condensation. Moreover, such as Model 40 and 41 both with the large wrapping appear in the third torus, are related by swapping their $b$- and $c$-stacks of branes. As we discussed in [39] and [40], when $g^2_a = g^2_b = g^2_c = \left( \frac{5}{3} g^2_Y \right) = 4$, the $b$- and $c$-stacks of branes swapping preserves the gauge coupling unification at near string scale.

Furthermore, according to the sources of three generations of the SM fermions, there are four classes of models whose quantum numbers under $SU(4)_C \times SU(2)_L \times SU(2)_R \times USp(2)$ with gauge symmetries are $(4, 2, 1)$ and $(\overline{4}, 1, 2)$ in supersymmetric Pati-Salam construction. For these models in appendix B, e.g. Model 38 with $I_{ab} = 3$ and $I_{ac} = -3$, Model 41 with $I_{ab} = 3$ and $I_{ac} = -3$, Model 42 with $I_{ab} = 3$ and $I_{ac} = -3$, Model 43 with $I_{ab} = 3$ and $I_{ac} = -3$ achieve three generations of the SM fermions.

6 Phenomenological studies

In this section, we shall discuss the phenomenological features of our models. In these four models in appendix B, the gauge symmetry is $U(4) \times U(2)_L \times U(2)_R \times USp(2)^4$. The $\beta$ functions of USp(2) group are negative, with four confining gauge groups in the hidden sector, thus we can break supersymmetry via gaugino condensation. For the other type of models shown in appendix A, there are models with one, two, three and four confining groups. For these models with two confining USp(N) gauge groups, one can perform a general analysis of the non-perturbative superpotential with tree-level gauge couplings, there can exist extrema with the stabilizations of dilaton and complex structure moduli as discussed in [27]. However, these extrema might be saddle points and thus will not break the supersymmetry. If the models have three or four confining USp(N) gauge groups, the non-perturbative superpotential allows for moduli stabilization and supersymmetry breaking at the stable extremum in general.

Now we take Models 41 as examples to show explicitly the full spectrum of the models with gauge coupling unification from appendix B in table 3. At the near string scale, we
Table 3. Chiral spectrum in the open string sector—Model 41.

| Model | SU(4) × SU(2)\_L × SU(2)\_R × USp(2)⁴ | Field |
|-------|--------------------------------------|-------|
|       | Q₄ | Q₂ \_L | Q₂ \_R | Q_{em} | B - L |
| ab'   | 1 | 1 | 0 | \(-\frac{1}{3}, \frac{2}{3}, -1, 0\) | \(\frac{1}{3}, -1\) | \(Q\_L, L\_L\) |
| ac'   | 3 × (\(\mathbb{T}, 1, \mathbb{Z}, 1, 1, 1\)) | -1 | 0 | -1 | \(\frac{1}{3}, -\frac{2}{3}, 1, 0\) | \(\frac{1}{3}, 1\) | \(Q\_R, L\_R\) |
| a2    | 1 × (4, 1, 1, 2, 1, 1) | 0 | 0 | 0 | \(\frac{1}{6}, -\frac{1}{2}\) | \(\frac{1}{3}, -1\) | \(\mathbb{1}, L\_L\) |
| a3    | 1 × (4, 1, 1, 1, 2, 1) | 0 | 0 | 0 | \(\frac{1}{6}, -\frac{1}{2}\) | \(\frac{1}{3}, -1\) | \(\mathbb{1}, L\_L\) |
| b1    | 3 × (1, 2, 1, 2, 1, 1) | 0 | 1 | 0 | \(\pm \frac{1}{2}\) | \(0\) | \(\mathbb{1}, L\_L\) |
| b2    | 1 × (1, 2, 1, 1, 2, 1) | 0 | 1 | 0 | \(\pm \frac{1}{2}\) | \(0\) | \(\mathbb{1}, L\_L\) |
| c2    | 1 × (1, 1, 2, 1, 2, 1) | 0 | 0 | 1 | \(\pm \frac{1}{2}\) | \(0\) | \(\mathbb{1}, L\_L\) |
| c4    | 3 × (1, 1, 2, 1, 1, 2) | 0 | 0 | -1 | \(\pm \frac{1}{2}\) | \(0\) | \(\mathbb{1}, L\_L\) |
| b     | 2 × (1, 1, 1, 1, 1, 1) | 0 | -2 | 0 | 0, ±1 | \(0\) | \(\mathbb{1}, L\_L\) |
| c     | 2 × (1, 1, 1, 1, 1, 1) | 0 | 2 | 0 | 0, ±1 | \(0\) | \(\mathbb{1}, L\_L\) |

Table 4. The composite particle spectrum of Model 41, which is formed due to the strong forces from hidden sector.

have SU(3)\_C × SU(2)\_L × U(1)\_Y gauge coupling unification. Its composite states are shown explicitly in table 4.

Model 41 has four confining gauge groups. Thereinto, both USp(2)\_2 and USp(2)\_3 have two charged intersections. Thus for them, besides self-confinement, the mixed-confinement between different intersections also exist, which yields the chiral supermultiplets (4, 1, 2, 1, 1, 1, 1) and (4, 1, 1, 1, 2, 1). As for USp(2)\_1 and USp(2)\_4, they have only one charged intersection. Therefore, there is no mixed-confinement, their self-confinement leads to 6 tensor representations for each of them. In addition, one can check from the spectra that no new anomaly is introduced to the remaining gauge symmetry, namely, such models are anomaly-free [24].

7 Conclusions and outlook

In this paper, we developed a systematic method for $\mathcal{N} = 1$ supersymmetric Pati-Salam models, and managed to obtain all the possible models from this construction. In such a way, we complete the landscape of supersymmetric Pati-Salam models constructed from Type IIA string theory with orientifolds $T^6/(\mathbb{Z}_2 \times \mathbb{Z}_2)$ for the first time.

We found that there are in total 33 types of allowed gauge coupling relations for supersymmetric Pati-Salam models, with in total 6144 models with gauge coupling unification among total 202752 models. In particular, these 202752 models are obtained by solving the common solutions of three generation conditions, RR tadpole cancellation conditions, iteratively. And intriguingly supersymmetry conditions eliminated the other possibilities. Thus all the possible supersymmetry Pati-Salam models are found. The largest allowed wrapping number is 5 in this system. Took the class of models with gauge coupling unification at near string scale as example, we studied their phenomenology features, and observe well anomaly-free behaviors with self- and mixed-confinements.

Distinct from the earlier investigations, we considered all the intersection conditions to have three families of the SM fermions, and allow any of the three tori to be tilted. We specified all the possible 33 gauge coupling relations by modding out the equivalent relations from 202752 supersymmetric Pati-Salam models. With vector-like particles from $\mathcal{N} = 2$ sector introduced, in [48], we obtain string scale gauge coupling unification with precise energy scale for these vector-like particles given for possible predictions.

We expect that our searching method for landscape of supersymmetric Pati-Salam models can be generalized to other string model buildings, especially brane intersecting models with RR tadpole cancellation conditions, supersymmetry conditions, and three generation conditions required simultaneously.

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A  Independent supersymmetric Pati-Salam models

In this section, we present the supersymmetric Pati-Salam models with 33 types of allowed gauge coupling relations on the landscape of supersymmetric Pati-Salam model building.\footnote{The full data of wrapping numbers \((n_i, l_i)\) with \(x = a, b, c\) and \(i = 1, 2, 3\) for 202752 models are listed in http://newton.kias.re.kr/~sunrui/files/finaldata.csv.}

### Table 5. D6-brane configurations and intersection numbers of Model 5, and its MSSM gauge coupling relation is \(g_a^2 = g_b^2 = g_c^2 = \left(\frac{5}{3}g_Y^2\right) = 4\sqrt{\frac{2}{3}\pi e^\phi}\).

| model 5 | \(U(4) \times U(2)_L \times U(2)_R \times USp(2)^4\) |
|---------|--------------------------------------------------|
| stack   | \(N\) \(n^1, l^1\) \(n^2, l^2\) \(n^3, l^3\) | \(n\) \(n\) \(b\) \(b'\) \(c\) \(c'\) \(1\) \(2\) \(3\) \(4\) |
| \(a\)   | 8 \((1, 1) \times (1, 0) \times (1, -1)\) | 0 | 0 | 3 | 0 | -3 | 0 | 1 | 0 | -1 |
| \(b\)   | 4 \((-1, 0) \times (-1, 3) \times (1, 1)\) | 2 | -2 | — | — | — | — | -3 | 1 | 0 |
| \(c\)   | 4 \((0, 1) \times (-1, 3) \times (-1, 1)\) | 2 | -2 | — | — | — | — | -3 | 1 | 0 |
| 1       | 2 \((1, 0) \times (1, 0) \times (2, 0)\) | \(x_A = \frac{1}{3} x_B = x_C = \frac{1}{3} x_D\) |
| 2       | 2 \((1, 0) \times (0, -1) \times (0, 2)\) | \(\beta_1^q = -3, \beta_2^q = -3, \beta_3^q = -3, \beta_4^q = -3\) |
| 3       | 2 \((0, -1) \times (1, 0) \times (0, 2)\) | \(\chi_1 = 1, \chi_2 = \frac{1}{3}, \chi_3 = 2\) |
| 4       | 2 \((0, -1) \times (0, 1) \times (2, 0)\) |

### Table 6. D6-brane configurations and intersection numbers of Model 6, and its MSSM gauge coupling relation is \(g_a^2 = g_b^2 = g_c^2 = \left(\frac{5}{3}g_Y^2\right) = 4\sqrt{\frac{2}{3}\pi e^\phi}\).\footnote{The full data of wrapping numbers \((n_i, l_i)\) with \(x = a, b, c\) and \(i = 1, 2, 3\) for 202752 models are listed in http://newton.kias.re.kr/~sunrui/files/finaldata.csv.}

| model 6 | \(U(4) \times U(2)_L \times U(2)_R \times USp(2)^2\) |
|---------|--------------------------------------------------|
| stack   | \(N\) \(n^1, l^1\) \(n^2, l^2\) \(n^3, l^3\) | \(n\) \(n\) \(b\) \(b'\) \(c\) \(c'\) \(1\) \(4\) |
| \(a\)   | 8 \((1, -1) \times (-1, 1) \times (1, -1)\) | 0 | 4 | 0 | 3 | 0 | -3 | -1 | 1 |
| \(b\)   | 4 \((0, 1) \times (-2, 1) \times (-1, 1)\) | -1 | 1 | — | — | 0 | -1 | -1 | 0 |
| \(c\)   | 4 \((-1, 0) \times (5, 2) \times (-1, 1)\) | 3 | -3 | — | — | — | — | 0 | -5 |
| 1       | 2 \((1, 0) \times (1, 0) \times (2, 0)\) | \(x_A = 2 x_B = \frac{14}{3} x_C = 7 x_D\) |
| 2       | 2 \((0, -1) \times (0, 1) \times (2, 0)\) | \(\beta_1^q = -3, \beta_2^q = 1\) |
|          |                                                  | \(\chi_1 = \frac{7}{\sqrt{5}}, \chi_2 = \sqrt{5}, \chi_3 = \frac{4}{\sqrt{5}}\) |
### Table 7. D6-brane configurations and intersection numbers of Model 7, and its MSSM gauge coupling relation is $g_a^2 = \frac{5}{8}g_b^2 = \frac{7}{9}g_c^2 = \frac{35}{32}\,\sqrt{\frac{2}{3}}\,e^{\phi}$.

| stack | $N$ | $(n^1, l^1) \times (n^2, l^2) \times (n^3, l^3)$ | $n$ | $n'$ | $b$ | $b'$ | $c$ | $c'$ | 1 | 4 |
|-------|-----|---------------------------------|-----|-----|-----|-----|-----|-----|----|----|
| $a$   | 8   | $(-1, -1) \times (1, 1) \times (1, 1)$ | 0   | -4  | 0   | 3   | 0   | -3  | 1  | -1 |
| $b$   | 4   | $(-5, 2) \times (-1, 0) \times (1, 1)$ | -3  | 3   | --- | --- | 0   | 1   | 0  | 5  |
| $c$   | 4   | $(-2, -1) \times (0, 1) \times (1, 1)$ | 1   | -1  | --- | --- | --- | 1   | 0  |    |
| 1     | 2   | $(1, 0) \times (1, 0) \times (2, 0)$   | $x_A = \frac{14}{5}x_B = 2x_C = 7x_D$  |     |     |     |     |     |     |    |
| 4     | 2   | $(0, -1) \times (0, 1) \times (2, 0)$   | $\beta_1^B = -3, \beta_1^A = 1$      |     |     |     |     |     |     |    |
|       |     |                                               | $\chi_1 = \sqrt{5}, \chi_2 = \frac{7}{\sqrt{5}}, \chi_3 = \frac{4}{\sqrt{5}}$ |     |     |     |     |     |     |    |

### Table 8. D6-brane configurations and intersection numbers of Model 8, and its MSSM gauge coupling relation is $g_a^2 = \frac{5}{3}g_b^2 = (\frac{2}{3})g_c^2 = 4\sqrt{\frac{2}{3}}\,e^{\phi}$.

| stack | $N$ | $(n^1, l^1) \times (n^2, l^2) \times (n^3, l^3)$ | $n$ | $n'$ | $b$ | $b'$ | $c$ | $c'$ | 2 | 4 |
|-------|-----|---------------------------------|-----|-----|-----|-----|-----|-----|----|----|
| $a$   | 8   | $(1, 1) \times (1, 0) \times (1, -1)$ | 0   | 0   | 3   | 0   | 0   | -3  | 1  | -1 |
| $b$   | 4   | $(-2, 1) \times (-1, 1) \times (1, 1)$ | -2  | -6  | --- | --- | 4   | 0   | -1  | 2  |
| $c$   | 4   | $(0, 1) \times (-1, 3) \times (-1, 1)$ | 2   | -2  | --- | --- | --- | 1   | 0  |    |
| 2     | 4   | $(1, 0) \times (0, -1) \times (0, 2)$   | $x_A = \frac{1}{3}x_B = x_C = \frac{1}{3}x_D$ |     |     |     |     |     |     |    |
| 4     | 4   | $(0, -1) \times (0, 1) \times (2, 0)$   | $\beta_2^g = -2, \beta_1^g = -2$      |     |     |     |     |     |     |    |
|       |     |                                               | $\chi_1 = 1, \chi_2 = \frac{1}{3}, \chi_3 = 2$ |     |     |     |     |     |     |    |

### Table 9. D6-brane configurations and intersection numbers of Model 9, and its MSSM gauge coupling relation is $g_a^2 = \frac{5}{7}g_b^2 = g_c^2 = (\frac{2}{3})g_d^2 = \frac{12}{5}\sqrt{2}\,e^{\phi}$.

| stack | $N$ | $(n^1, l^1) \times (n^2, l^2) \times (n^3, l^3)$ | $n$ | $n'$ | $b$ | $b'$ | $c$ | $c'$ | 1 | 2 | 3 | 4 |
|-------|-----|---------------------------------|-----|-----|-----|-----|-----|-----|----|----|----|----|
| $a$   | 8   | $(1, 1) \times (1, 0) \times (1, -1)$ | 0   | 0   | 2   | 1   | 0   | -3  | 0  | 1  | 0  | -1 |
| $b$   | 4   | $(-1, 0) \times (-1, 1) \times (1, 3)$ | -2  | 2   | --- | 4   | 4   | 0   | 0  | -1  | 3  |    |
| $c$   | 4   | $(0, 1) \times (-1, 3) \times (-1, 1)$ | 2   | -2  | --- | --- | -3  | 1   | 0  |    |    |    |
| 1     | 2   | $(1, 0) \times (1, 0) \times (2, 0)$   | $x_A = \frac{1}{5}x_B = \frac{1}{5}x_C = \frac{1}{5}x_D$ |     |     |     |     |     |     |    |
| 2     | 2   | $(1, 0) \times (0, -1) \times (0, 2)$   | $\beta_1^g = -3, \beta_2^g = -3, \beta_3^g = -5, \beta_4^g = -1$ |     |     |     |     |     |     |    |
| 3     | 2   | $(0, -1) \times (1, 0) \times (0, 2)$   | $\chi_1 = \frac{1}{3}, \chi_2 = 1, \chi_3 = \frac{2}{3}$ |     |     |     |     |     |     |    |
| 4     | 2   | $(0, -1) \times (0, 1) \times (2, 0)$   |     |     |     |     |     |     |     |    |    |    |
### Table 10. D6-brane configurations and intersection numbers of Model 10, and its MSSM gauge coupling relation is $g_a^2 = 2g_b^2 = g_c^2 = \frac{5}{2} g_Y^2 = \left(\frac{\sqrt{2}}{\sqrt{2}}\right)^2 \pi e^\phi$.

| model 10 | U(4) × U(2)_L × U(2)_R × USp(2)^3 |
|----------|-----------------------------------|
| stack    | N \((n^1, t^1) \times (n^2, t^2) \times (n^3, t^3)\) | \(n\) | \(n\) | \(b\) | \(b'\) | \(c\) | \(c'\) | \(2\) | \(3\) | \(4\) |
| \(a\)   | 8 \((1, 1) \times (1, 0) \times (1, -1)\) | 0 | 0 | 3 | 0 | 0 | -3 | 1 | 0 | -1 |
| \(b\)   | 4 \((1, 0) \times (-2, 3) \times (1, 1)\) | 1 | -1 | - | - | 3 | 0 | 0 | -3 | 2 |
| \(c\)   | 4 \((0, 1) \times (-1, 3) \times (1, 1)\) | 2 | -2 | - | - | - | - | 1 | 0 | 0 |

### Table 11. D6-brane configurations and intersection numbers of Model 11, and its MSSM gauge coupling relation is $g_a^2 = g_b^2 = g_c^2 = \frac{25}{19} (\frac{5}{2} g_Y^2) = 4 \sqrt{\frac{2}{3}} \pi e^\phi$.

| model 11 | U(4) × U(2)_L × U(2)_R × USp(4)^2 |
|----------|-----------------------------------|
| stack    | N \((n^1, t^1) \times (n^2, t^2) \times (n^3, t^3)\) | \(n\) | \(n\) | \(b\) | \(b'\) | \(c\) | \(c'\) | \(2\) | \(4\) |
| \(a\)   | 8 \((1, 1) \times (1, 0) \times (1, -1)\) | 0 | 0 | 3 | 0 | 0 | -3 | 1 | -1 |
| \(b\)   | 4 \((1, 0) \times (1, -3) \times (1, 1)\) | 2 | -2 | - | - | -4 | 0 | 0 | 1 |
| \(c\)   | 4 \((1, 2) \times (-1, 1) \times (-1, 1)\) | -2 | -6 | - | - | - | - | 2 | -1 |

$x_A = \frac{1}{3} x_B = \frac{1}{3} x_C = \frac{1}{3} x_D$

$\beta_2^a = -3$, $\beta_3^a = -3$, $\beta_4^a = -2$

$\chi_1 = \frac{1}{\sqrt{2}}$, $\chi_2 = \frac{\sqrt{2}}{\sqrt{3}}$, $\chi_3 = \sqrt{2}$
Table 12. D6-brane configurations and intersection numbers of Model 12, and its MSSM gauge coupling relation is \( g_a^2 = \frac{11}{3} g_b^2 = \frac{5}{3} g_c^2 = \frac{28}{27}(\frac{5}{3} g_Y) = \frac{8 \sqrt{2} \pi^4 \epsilon^6}{\sqrt{3}} \cdot\)

| model 12 | \( U(4) \times U(2)_L \times U(2)_R \times \text{USp}(2) \) |
|---|---|
| stack | \( N \) | \( n^1, l^1 \) \( \times \) \( n^2, l^2 \) \( \times \) \( n^3, l^3 \) |
| \( a \) | 8 | \((1, 1) \times (1, 0) \times (1, -1)\) |
| \( b \) | 4 | \((2, -1) \times (1, -1) \times (1, 1)\) |
| \( c \) | 4 | \((-2, 5) \times (0, 1) \times (-1, 1)\) |
| 3 | 2 | \((0, -1) \times (1, 0) \times (0, 2)\) |
| \( x_A = \frac{1}{6} x_B = \frac{5}{6} x_C = \frac{1}{6} x_D \) |
| \( \beta_3^0 = -2 \) |
| \( \chi_1 = \sqrt{\frac{2}{5}} \), \( \chi_2 = 1 \sqrt{\frac{2}{5}} \), \( \chi_3 = 2 \sqrt{\frac{2}{5}} \) |

Table 13. D6-brane configurations and intersection numbers of Model 13, and its MSSM gauge coupling relation is \( g_a^2 = \frac{10}{3} g_b^2 = \frac{4}{3} g_c^2 = \frac{16}{27}(\frac{5}{3} g_Y) = \frac{16 \sqrt{2} \pi^4 \epsilon^6}{\sqrt{3}} \cdot\)

| model 13 | \( U(4) \times U(2)_L \times U(2)_R \times \text{USp}(2)^2 \times \text{USp}(4) \) |
|---|---|
| stack | \( N \) | \( n^1, l^1 \) \( \times \) \( n^2, l^2 \) \( \times \) \( n^3, l^3 \) |
| \( a \) | 8 | \((1, -1) \times (-1, 1) \times (1, -1)\) |
| \( b \) | 4 | \((0, 1) \times (-2, 1) \times (-1, 1)\) |
| \( c \) | 4 | \((-1, 0) \times (4, 1) \times (-1, 1)\) |
| 1 | 4 | \((1, 0) \times (1, 0) \times (2, 0)\) |
| 2 | 2 | \((1, 0) \times (0, -1) \times (0, 2)\) |
| 4 | 2 | \((0, -1) \times (0, 1) \times (2, 0)\) |
| \( x_A = 2 x_B = 5 x_C = 10 x_D \) |
| \( \beta_1^0 = -3 \), \( \beta_2^0 = -2 \), \( \beta_4^0 = 0 \) |
| \( \chi_1 = \sqrt{\frac{5}{2}} \), \( \chi_2 = 2 \sqrt{\frac{2}{3}} \), \( \chi_3 = \sqrt{2} \) |

Table 14. D6-brane configurations and intersection numbers of Model 14, and its MSSM gauge coupling relation is \( g_a^2 = g_b^2 = \frac{5}{3} g_c^2 = \frac{5}{17}(\frac{5}{3} g_Y) = \frac{16 \pi^4 \epsilon^6}{\sqrt{3}} \cdot\)

| model 14 | \( U(4) \times U(2)_L \times U(2)_R \times \text{USp}(2) \times \text{USp}(4) \) |
|---|---|
| stack | \( N \) | \( n^1, l^1 \) \( \times \) \( n^2, l^2 \) \( \times \) \( n^3, l^3 \) |
| \( a \) | 8 | \((1, 1) \times (-1, 0) \times (-1, 1)\) |
| \( b \) | 4 | \((1, 0) \times (1, -3) \times (1, 1)\) |
| \( c \) | 4 | \((-1, 4) \times (0, 1) \times (-1, 1)\) |
| 1 | 2 | \((1, 0) \times (1, 0) \times (2, 0)\) |
| 3 | 2 | \((0, -1) \times (1, 0) \times (0, 2)\) |
| \( x_A = \frac{1}{12} x_B = \frac{1}{12} x_C = \frac{1}{12} x_D \) |
| \( \beta_1^0 = -2 \), \( \beta_3^0 = -2 \) |
| \( \chi_1 = \frac{1}{2} \), \( \chi_2 = \frac{1}{3} \), \( \chi_3 = 1 \) |
Table 15. D6-brane configurations and intersection numbers of Model 15, and its MSSM gauge coupling relation is \( g_u^2 = \frac{35}{64} g_b^2 = \frac{7}{8} g_c^2 = \frac{35}{82} (\frac{5}{4} g_Y^2) = \frac{8 \sqrt{2} \pi e^{\phi}}{U(2)} \).

| stack | model 15 | U(4) \times U(2)_L \times U(2)_R \times U(2)_U(2)
|-------|----------|--------------------------------|
|       | stack | \( N \) \( (n^1, l^1) \times (n^2, l^2) \times (n^3, l^3) \) | \( n \) \( n \) \( b \) \( b' \) \( c \) \( c' \) | 3 |
| \( a \) | 8 | \((1, -1) \times (1, 0) \times (1, 1)\) | 0 | 0 | -3 | 6 | 0 | -3 | 0 |
| \( b \) | 4 | \((-2, 1) \times (0, 1) \times (-5, 1)\) | -9 | 9 | — | — | 0 | 8 | 10 |
| \( c \) | 4 | \((-2, 1) \times (-1, 1) \times (1, 1)\) | -2 | -6 | — | — | — | — | -2 |
| 3 | 2 | \((0, -1) \times (1, 0) \times (0, 2)\) | \(x_A = \frac{5}{6} x_B = 10 x_C = \frac{5}{6} x_D\) |
| | | \( \beta_3^g = 6\) |
| | | \(\chi_1 = \sqrt{10}, \chi_2 = \sqrt{\frac{5}{2}}, \chi_3 = 2 \sqrt{10}\) |

Table 16. D6-brane configurations and intersection numbers of Model 16, and its MSSM gauge coupling relation is \( g_u^2 = \frac{35}{64} g_b^2 = \frac{10}{37} g_c^2 = \frac{50}{87} (\frac{5}{4} g_Y^2) = \frac{16 \sqrt{2} \pi e^{\phi}}{USp(4)} \).

| stack | model 16 | U(4) \times U(2)_L \times U(2)_R \times U(2)_USp(4) |
|-------|----------|--------------------------------|
| 1 | \( a \) | 8 | \((1, -1) \times (1, 1) \times (1, 1)\) | 0 | -4 | 0 | 3 | 0 | -3 | 1 | -1 | -1 |
| 3 | \( b \) | 4 | \((-4, 1) \times (0, 1) \times (1, 1)\) | -3 | 3 | — | — | 0 | 2 | 0 | 0 | 4 |
| 4 | \( c \) | 4 | \((-2, -1) \times (0, 1) \times (1, 1)\) | 1 | -1 | — | — | — | 1 | -2 | 0 |
| 1 | 4 | \((0, 1) \times (0, 1) \times (0, 2)\) | \(x_A = \frac{5}{2} x_B = 2 x_C = 10 x_D\) |
| | | \( \beta_3^g = -3, \beta_4^g = -2, \beta_4^g = 0\) |
| | | \(\chi_1 = 2 \sqrt{2}, \chi_2 = \frac{5}{\sqrt{2}}, \chi_3 = \sqrt{2}\) |

Table 17. D6-brane configurations and intersection numbers of Model 17, and its MSSM gauge coupling relation is \( g_u^2 = 2 g_b^2 = \frac{4}{3} g_c^2 = \frac{40}{47} (\frac{5}{4} g_Y^2) = \frac{16 \sqrt{2} \pi e^{\phi}}{USp(4)} \).

| stack | model 17 | U(4) \times U(2)_L \times U(2)_R \times U(2)_USp(4) |
|-------|----------|--------------------------------|
| 1 | \( a \) | 8 | \((1, 1) \times (-1, 0) \times (-1, 1)\) | 0 | 0 | 3 | 0 | 0 | -3 | 0 |
| 3 | \( b \) | 4 | \((1, 0) \times (2, -3) \times (1, 1)\) | 1 | -1 | — | — | 8 | 0 | -3 |
| 4 | \( c \) | 4 | \((-4, 1) \times (0, 1) \times (-1, 1)\) | 3 | -3 | — | — | — | — | 1 |
| 3 | 4 | \((0, -1) \times (1, 0) \times (0, 2)\) | \(x_A = \frac{1}{4} x_B = \frac{1}{4} x_C = \frac{1}{4} x_D\) |
| | | \( \beta_3^g = -2\) |
| | | \(\chi_1 = \frac{1}{2}, \chi_2 = \frac{1}{3}, \chi_3 = 1\) |
Table 18. D6-brane configurations and intersection numbers of Model 18, and its MSSM gauge coupling relation is $g_a^2 = g_b^2 = g_c^2 = \frac{10}{\sqrt{3}} \frac{\phi_{\pi e}}{\sqrt{g_Y}} = \frac{10}{\sqrt{3}} \frac{\phi_{\pi e}}{\sqrt{g_Y}}$.

| model 18 | U(4) x U(2)_L x U(2)_R x USp(2)^3 |
|----------|-------------------------------------|
| stack    | N (n^1, l^1) x (n^2, l^2) x (n^3, l^3) | n b b' c c' 1 2 4 |
| a        | 8 (1,1) x (1,0) x (1,-1)               | 0 0 3 0 -3 0 1 -1 |
| b        | 4 (-1,0) x (-2,3) x (1,1)             | 2 -2 -3 0 0 0 0 1 |
| c        | 4 (0,1) x (-2,3) x (1,-1)             | 1 -2 -3 0 0 0 0 1 |
| 1        | 2 (1,0) x (1,0) x (2,0)               | $x_A = \frac{3}{2} x_B = 2 x_C = \frac{3}{2} x_D$ |
| 2        | 2 (1,0) x (0,1) x (0,2)               | $\beta_1^q = -3, \beta_2^q = -2, \beta_3^q = -3$ |
| 4        | 2 (0,-1) x (0,1) x (2,0)              | $\chi_1 = \sqrt{2}, \chi_2 = \frac{\sqrt{2}}{3}, \chi_3 = 2 \sqrt{2}$ |

Table 19. D6-brane configurations and intersection numbers of Model 19, and its MSSM gauge coupling relation is $g_a^2 = g_b^2 = g_c^2 = \frac{10}{\sqrt{3}} \frac{\phi_{\pi e}}{\sqrt{g_Y}} = \frac{8 \sqrt{2} \pi e}{\sqrt{3}}$.

| model 19 | U(4) x U(2)_L x U(2)_R x USp(2)^2 |
|----------|-------------------------------------|
| stack    | N (n^1, l^1) x (n^2, l^2) x (n^3, l^3) | n b b' c c' 2 4 |
| a        | 8 (1,1) x (1,0) x (1,-1)               | 0 0 3 0 -3 1 -1 |
| b        | 4 (-1,0) x (-2,3) x (1,1)             | 1 -1 -3 0 0 0 2 |
| c        | 4 (0,1) x (-2,3) x (1,-1)             | 1 -1 -3 0 0 0 2 |
| 2        | 2 (1,0) x (0,1) x (0,2)               | $x_A = \frac{3}{2} x_B = x_C = \frac{3}{2} x_D$ |
| 4        | 2 (0,-1) x (0,1) x (2,0)              | $\beta_1^q = -2, \beta_2^q = -2$ |
|          |                                     | $\chi_1 = 1, \chi_2 = \frac{2}{3}, \chi_3 = \chi_3 = 2$ |

Table 20. D6-brane configurations and intersection numbers of Model 20, and its MSSM gauge coupling relation is $g_a^2 = g_b^2 = g_c^2 = \frac{10}{\sqrt{3}} \frac{\phi_{\pi e}}{\sqrt{g_Y}} = \frac{48 \sqrt{2} \pi e}{\sqrt{3}}$.
### Table 21. D6-brane configurations and intersection numbers of Model 21, and its MSSM gauge coupling relation is \( g_a = \frac{7}{8}g_b = \frac{11}{6}g_c = \frac{11}{5}g_\chi = \frac{135}{4}\sqrt{\frac{3}{5}}\pi e^{\phi} \).}

| stack | \( N \) | \((n^1, l^1) \times (n^2, l^2) \times (n^3, l^3)\) | \( n \times n \times b \times b' \times c \times c' \times 3 \) |
|-------|------|---------------------------------|------------------|
| \( a \) | 8 | \((1, -1) \times (1, 0) \times (1, 1)\) | 0 \( 0 \) 3 \( 0 \) 0 \(-3\) 0 |
| \( b \) | 4 | \((-2, 5) \times (0, 1) \times (-1, 1)\) | 3 \(-3\) \(-\) \(-\) \(-8\) 0 2 |
| \( c \) | 4 | \((-2, 1) \times (-1, 1) \times (1, 1)\) | \(-2\) \(-6\) \(-\) \(-\) \(-\) \(-2\) |
| 3 | 2 | \((0, -1) \times (1, 0) \times (0, 2)\) | \( x_A = \frac{1}{7}x_B = \frac{1}{5}x_C = \frac{1}{6}x_D \) \( \beta_3^2 = -2 \) \( \chi_1 = \sqrt{\frac{2}{5}} \), \( \chi_2 = \frac{\sqrt{2}}{6} \), \( \chi_3 = 2\sqrt{\frac{2}{5}} \) |

### Table 22. D6-brane configurations and intersection numbers of Model 22, and its MSSM gauge coupling relation is \( g_a = \frac{7}{8}g_b = \frac{11}{6}g_c = \frac{11}{5}g_\chi = \frac{135}{4}\sqrt{\frac{3}{5}}\pi e^{\phi} \).}

| stack | \( N \) | \((n^1, l^1) \times (n^2, l^2) \times (n^3, l^3)\) | \( n \times n \times b \times b' \times c \times c' \times 1 \times 4 \) |
|-------|------|---------------------------------|------------------|
| \( a \) | 8 | \((1, -1) \times (-2, 1) \times (1, -1)\) | 0 \( 8\) 0 3 \( 0\) \(-3\) \(-1\) 2 |
| \( b \) | 4 | \((-2, 1) \times (1, 1) \times (-1, 1)\) | \(-2\) \(-6\) \(-\) \(-\) \(-8\) \(-1\) \(-2\) |
| \( c \) | 4 | \((-4, -1) \times (1, 0) \times (-1, 1)\) | \(3\) \(-3\) \(-\) \(-\) \(-\) \(-8\) \(-4\) |
| 1 | 4 | \((1, 0) \times (1, 0) \times (2, 0)\) | \( x_A = \frac{13}{2}x_B = \frac{13}{5}x_C = 26x_D \) \( \beta_1^2 = -3 \), \( \beta_2^2 = 4 \) \( \chi_1 = \sqrt{\frac{13}{2}} \), \( \chi_2 = 2\sqrt{26} \), \( \chi_3 = \frac{\sqrt{26}}{2} \) |
| 4 | 2 | \((0, -1) \times (0, 1) \times (2, 0)\) | \( \chi_1 = \sqrt{\frac{13}{2}} \), \( \chi_2 = 2\sqrt{26} \), \( \chi_3 = \frac{\sqrt{26}}{2} \) |

### Table 23. D6-brane configurations and intersection numbers of Model 23, and its MSSM gauge coupling relation is \( g_a = \frac{7}{8}g_b = \frac{11}{6}g_c = \frac{11}{5}g_\chi = \frac{135}{4}\sqrt{\frac{3}{5}}\pi e^{\phi} \).}

| stack | \( N \) | \((n^1, l^1) \times (n^2, l^2) \times (n^3, l^3)\) | \( n \times n \times b \times b' \times c \times c' \times 2 \times 3 \times 4 \) |
|-------|------|---------------------------------|------------------|
| \( a \) | 8 | \((1, 1) \times (1, 0) \times (1, -1)\) | 0 \( 0\) 3 \( 0\) \(-3\) \(1\) 0 \(-1\) |
| \( b \) | 4 | \((2, -1) \times (1, 1) \times (1, 1)\) | \(-2\) \(-6\) \(-\) \(-\) \(7\) 0 \(-1\) \(-2\) 2 |
| \( c \) | 4 | \((-1, 4) \times (0, 1) \times (-1, 1)\) | \(3\) \(-3\) \(-\) \(-\) \(-\) \(-8\) \(-4\) |
| 2 | 2 | \((1, 0) \times (0, -1) \times (0, 2)\) | \( x_A = \frac{1}{5}x_B = \frac{1}{5}x_C = \frac{1}{3}x_D \) \( \beta_3^2 = -3 \), \( \beta_2^2 = -3 \), \( \beta_2^2 = -2 \) |
| 3 | 2 | \((0, -1) \times (1, 0) \times (0, 2)\) | \( \chi_1 = \frac{1}{2} \), \( \chi_2 = \frac{2}{5} \), \( \chi_3 = 1 \) |
| 4 | 2 | \((0, -1) \times (1, 0) \times (2, 0)\) | \( \chi_1 = \frac{1}{2} \), \( \chi_2 = \frac{2}{5} \), \( \chi_3 = 1 \) |
Table 24. D6-brane configurations and intersection numbers of Model 24, and its MSSM gauge coupling relation is $g_a^2 = \frac{1}{3} g_b^2 = g_c^2 = (\frac{5}{8} g_Y^2) = \frac{16\pi e^4}{5\sqrt{3}}$.

| model 24 | U(4) × U(2)ₓ × U(2)ᵧ × USp(2) × USp(4) |
|-----------|------------------------------------------|
| stack     | N $(n^1, l^1) \times (n^2, l^2) \times (n^3, l^3)$ | n | b | b’ | c | c’ | 1 | 3 |
| a         | 8 $(-1, 1) \times (-1, 0) \times (1, 1)$     | 0 | 0 | 3 | 0 | 0 | -3 | 0 | 0 |
| b         | 4 $(-1, 4) \times (0, 1) \times (-1, 1)$     | 3 | -3 | — | — | -4 | 0 | -4 | 1 |
| c         | 4 $(1, 0) \times (1, -3) \times (1, 1)$     | 2 | -2 | — | — | — | 0 | -3 | — |
| 1         | 2 $(1, 0) \times (1, 0) \times (2, 0)$     | $x_A = \frac{1}{12} x_B = \frac{1}{3} x_C = \frac{1}{12} x_D$ |
| 3         | 4 $(0, -1) \times (1, 0) \times (0, 2)$     | $\beta_1^g = -2, \beta_2^g = -2$ |
|           |                                           | $\chi_1 = \frac{1}{2}, \chi_2 = \frac{4}{3}, \chi_3 = 1$ |

Table 25. D6-brane configurations and intersection numbers of Model 25, and its MSSM gauge coupling relation is $g_a^2 = \frac{13}{21} g_b^2 = \frac{1}{2} g_c^2 = (\frac{5}{8} g_Y^2) = \frac{16\pi e^4}{5\sqrt{3}}$.

| model 25 | U(4) × U(2)ₓ × U(2)ᵧ × USp(4) |
|-----------|------------------------------------------|
| stack     | N $(n^1, l^1) \times (n^2, l^2) \times (n^3, l^3)$ | n | b | b’ | c | c’ | 4 |
| a         | 8 $(-2, 1) \times (-1, 0) \times (1, 1)$     | -1 | 1 | 3 | 0 | 0 | -3 | 2 |
| b         | 4 $(-1, 2) \times (-1, 1) \times (-1, 1)$     | 2 | 6 | — | — | 4 | 0 | 1 |
| c         | 4 $(1, 0) \times (1, -3) \times (1, 1)$     | 2 | -2 | — | — | — | 1 |
| 4         | 4 $(0, -1) \times (0, 1) \times (2, 0)$     | $x_A = \frac{4}{7} x_B = 8 x_C = \frac{8}{3} x_D$ |
|           |                                           | $\beta_1^g = 0$ |
|           |                                           | $\chi_1 = 4, \chi_2 = \frac{4}{3}, \chi_3 = 4$ |

Table 26. D6-brane configurations and intersection numbers of Model 26, and its MSSM gauge coupling relation is $g_a^2 = \frac{27}{37} g_b^2 = 2 g_c^2 = (\frac{5}{8} g_Y^2) = \frac{48\pi e^4}{5\sqrt{3}}$.

| model 26 | U(4) × U(2)ₓ × U(2)ᵧ × USp(2)² |
|-----------|------------------------------------------|
| stack     | N $(n^1, l^1) \times (n^2, l^2) \times (n^3, l^3)$ | n | b | b’ | c | c’ | 1 | 2 | 4 |
| a         | 8 $(1, 1) \times (1, 0) \times (1, -1)$     | 0 | 0 | 2 | 1 | 0 | -3 | 0 | 1 | -1 |
| b         | 4 $(-1, 0) \times (-1, 1) \times (1, 3)$     | -2 | 2 | — | — | 2 | 5 | 0 | 0 | 3 |
| c         | 4 $(0, 1) \times (-2, 3) \times (-1, 1)$     | 1 | -1 | — | — | — | -3 | 2 | 0 | — |
| 1         | 2 $(1, 0) \times (1, 0) \times (2, 0)$     | $x_A = \frac{2}{3} x_B = \frac{2}{3} x_C = \frac{2}{3} x_D$ |
| 2         | 2 $(1, 0) \times (0, -1) \times (0, 2)$     | $\beta_1^g = -3, \beta_2^g = -2, \beta_3^g = -1$ |
| 4         | 2 $(0, -1) \times (0, 1) \times (2, 0)$     | $\chi_1 = \sqrt{2}, \chi_2 = \sqrt{2}, \chi_3 = \frac{2\sqrt{2}}{3}$ |
### Table 27

| model 27 | $U(4) \times U(2)_L \times U(2)_R \times USp(2)^2$ | stack | $N$ | $(n^1, l^1) \times (n^2, l^2) \times (n^3, l^3)$ | $a$ | $b$ | $b'$ | $c$ | $c'$ | 1 | 4 |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| a | 8 | $(-1, -1) \times (1, 1) \times (1, 1)$ | 0 | -4 | 3 | 0 | 0 | -3 | 1 | -1 |
| b | 4 | $(-2, 1) \times (2, 1) \times (-1, 1)$ | -3 | -13 | - | - | 7 | 0 | -1 | -4 |
| c | 4 | $(1, -4) \times (1, 0) \times (1, 1)$ | 3 | -3 | - | - | - | 0 | 1 |

$x_A = 28x_B = \frac{28}{23}x_C = 7x_D$

$\beta_1^2 = -3, \beta_3^2 = 1$

$\chi_1 = \sqrt{\frac{2}{23}}, \chi_2 = \sqrt{161}, \chi_3 = 8\sqrt{\frac{2}{23}}$

### Table 28

| model 28 | $U(4) \times U(2)_L \times U(2)_R \times USp(4)$ | stack | $N$ | $(n^1, l^1) \times (n^2, l^2) \times (n^3, l^3)$ | $a$ | $b$ | $b'$ | $c$ | $c'$ | 3 |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| a | 8 | $(-1, 1) \times (-1, 0) \times (1, 1)$ | 0 | 0 | 3 | 0 | 0 | -3 | 0 |
| b | 4 | $(-1, 4) \times (0, 1) \times (-1, 1)$ | 3 | -3 | - | - | -8 | 0 | 1 |
| c | 4 | $(1, 0) \times (2, -3) \times (1, 1)$ | 1 | -1 | - | - | - | -3 |

$x_A = \frac{1}{3}x_B = \frac{1}{3}x_C = \frac{1}{3}x_D$

$\beta_3^3 = -2$

$\chi_1 = \frac{1}{2}, \chi_2 = \frac{3}{2}, \chi_3 = 1$

### Table 29

| model 29 | $U(4) \times U(2)_L \times U(2)_R \times USp(2) \times USp(4)$ | stack | $N$ | $(n^1, l^1) \times (n^2, l^2) \times (n^3, l^3)$ | $a$ | $b$ | $b'$ | $c$ | $c'$ | 1 | 3 |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| a | 8 | $(1, 1) \times (-1, 0) \times (-1, 1)$ | 0 | 0 | 2 | 1 | 0 | -3 | 0 | 0 |
| b | 4 | $(1, 0) \times (1, -1) \times (1, 3)$ | -2 | 2 | - | - | 8 | 4 | 0 | -1 |
| c | 4 | $(-1, 4) \times (0, 1) \times (-1, 1)$ | 3 | -3 | - | - | - | -4 | 1 |

$x_A = \frac{2}{3}x_B = \frac{1}{3}x_C = \frac{2}{3}x_D$

$\beta_1^4 = -2, \beta_3^4 = -4$

$\chi_1 = \frac{1}{2}, \chi_2 = \frac{3}{2}, \chi_3 = 1$
### Table 30

D6-brane configurations and intersection numbers of Model 30, and its MSSM gauge coupling relation is $g_a^2 = \frac{18}{5} g_b^2 = 2 g_c^2 = \frac{10}{7} (\frac{5}{3} g_Y^2) = \frac{24 \pi e^\prime}{5}$.

| model 30 | $U(4) \times U(2)_L \times U(2)_R \times USp(2)$ |
| --- | --- |
| stack | $N$ | $(n^1, l^1) \times (n^2, l^2) \times (n^3, l^3)$ | $n$ | $b$ | $b'$ | $c$ | $c'$ | 2 | 4 |
| a | 8 | $(1, 1) \times (1, 0) \times (1, -1)$ | 0 | 0 | 2 | 1 | 0 | -3 | 1 | -1 |
| b | 4 | $(-1, 0) \times (-2, 1) \times (1, 3)$ | -5 | 5 | — | — | 8 | 0 | 0 | 6 |
| c | 4 | $(0, 1) \times (-2, 3) \times (-1, 1)$ | 1 | -1 | — | — | — | 2 | 0 | — |
| 2 | 2 | $(1, 0) \times (0, -1) \times (0, 2)$ | $x_A = \frac{2}{7} x_B = \frac{1}{5} x_C = \frac{2}{7} x_D$ | $\beta_2^f = -2$, $\beta_2^o = 2$ |
| 4 | 2 | $(0, -1) \times (0, 1) \times (2, 0)$ | $\chi_1 = \frac{1}{3}$, $\chi_2 = 2$, $\chi_3 = \frac{2}{3}$ |

### Table 31

D6-brane configurations and intersection numbers of Model 31, and its MSSM gauge coupling relation is $g_a^2 = \frac{21}{10} g_b^2 = \frac{7}{2} g_c^2 = \frac{7}{4} (\frac{5}{3} g_Y^2) = \frac{114}{5} \pi e^\prime$.

| model 31 | $U(4) \times U(2)_L \times U(2)_R \times USp(4)$ |
| --- | --- |
| stack | $N$ | $(n^1, l^1) \times (n^2, l^2) \times (n^3, l^3)$ | $n$ | $b$ | $b'$ | $c$ | $c'$ | 4 |
| a | 8 | $(2, 1) \times (1, 0) \times (1, -1)$ | 1 | -1 | 2 | 1 | 0 | -3 | -2 |
| b | 4 | $(1, 0) \times (1, -1) \times (1, 3)$ | -2 | 2 | — | — | 0 | 4 | 3 |
| c | 4 | $(-1, 2) \times (-1, 1) \times (-1, 1)$ | 2 | 6 | — | — | — | 1 |
| 4 | 4 | $(0, -1) \times (0, 1) \times (2, 0)$ | $x_A = 2 x_B = \frac{4}{7} x_C = 4 x_D$ | $\beta_4^f = 2$ |
| | | | $\chi_1 = 2 \sqrt{\frac{2}{5}}$, $\chi_2 = \sqrt{6}$, $\chi_3 = 2 \sqrt{\frac{2}{5}}$ |

### Table 32

D6-brane configurations and intersection numbers of Model 32, and its MSSM gauge coupling relation is $g_a^2 = \frac{24}{15} g_b^2 = \frac{17}{9} g_c^2 = \frac{85}{64} (\frac{5}{3} g_Y^2) = \frac{32 \pi e^\prime}{15}$.

| model 32 | $U(4) \times U(2)_L \times U(2)_R \times USp(2)^3$ |
| --- | --- |
| stack | $N$ | $(n^1, l^1) \times (n^2, l^2) \times (n^3, l^3)$ | $n$ | $b$ | $b'$ | $c$ | $c'$ | 2 | 3 | 4 |
| a | 8 | $(1, -1) \times (1, 0) \times (1, 1)$ | 0 | 0 | 3 | 0 | 0 | -3 | 1 | 0 | 1 |
| b | 4 | $(-1, 4) \times (0, 1) \times (-1, 1)$ | 3 | -3 | — | — | -7 | 0 | 0 | 1 | 0 |
| c | 4 | $(-2, 1) \times (-1, 1) \times (1, 1)$ | -2 | -6 | — | — | — | 1 | -2 | 2 |
| 2 | 2 | $(1, 0) \times (0, -1) \times (0, 2)$ | $x_A = \frac{1}{9} x_B = \frac{1}{4} x_C = \frac{1}{9} x_D$ | $\beta_2^f = -3$, $\beta_2^o = -3$, $\beta_2^o = -2$ |
| 3 | 2 | $(0, -1) \times (1, 0) \times (0, 2)$ | $\chi_1 = \frac{1}{2}$, $\chi_2 = \frac{2}{5}$, $\chi_3 = 1$ |
| 4 | 2 | $(0, -1) \times (0, 1) \times (2, 0)$ | $\beta_4^f = -2$, $\beta_4^o = 2$, $\beta_4^o = 3$ |

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Table 33. D6-brane configurations and intersection numbers of Model 33, and its MSSM gauge coupling relation is \( g_a = \frac{1}{2} g_b^2 = \frac{1}{6} g_c^2 = \frac{5}{44} (\frac{2}{3} g_Y^2) = \frac{16}{3} \sqrt{\frac{2}{3}} \pi \phi^4 \).

| model 33 | \( U(4) \times U(2)_L \times U(2)_R \times USp(4) \) |
|---|---|
| stack | \( N \) | \( (n^1, l^1) \times (n^2, l^2) \times (n^3, l^3) \) | \( n \) | \( n' \) | \( b \) | \( b' \) | \( c \) | \( c' \) | 1 | 4 |
| \( a \) | 8 | \( (2, 1) \times (1, 0) \times (1, -1) \) | 1 | 2 | -1 | 3 | 0 | 0 | -3 | -2 |
| \( b \) | 4 | \( (1, 0) \times (1, -3) \times (1, 1) \) | 2 | 6 | -2 | -3 | -4 | 0 | 1 |
| \( c \) | 4 | \( (1, 1) \times (1, -1, 1) \times (1, -1, 1) \) | 2 | 6 | -2 | -3 | -4 | 0 | 1 |

\[ x_A = \frac{4}{5} x_B = 8 x_C = \frac{8}{5} x_D \]
\[ \beta_1^3 = 0 \]
\[ \chi_1 = 4, \chi_2 = \frac{2}{3}, \chi_3 = 4 \]

Table 34. D6-brane configurations and intersection numbers of Model 34, and its MSSM gauge coupling relation is \( g_a = \frac{5}{8} g_b^2 = \frac{1}{6} g_c^2 = \frac{11}{44} (\frac{3}{2} g_Y^2) = \frac{8}{63} \sqrt{\frac{3}{2}} \pi \phi^4 \).

| model 34 | \( U(4) \times U(2)_L \times U(2)_R \times USp(2)^2 \) |
|---|---|
| stack | \( N \) | \( (n^1, l^1) \times (n^2, l^2) \times (n^3, l^3) \) | \( n \) | \( n' \) | \( b \) | \( b' \) | \( c \) | \( c' \) | 1 | 4 |
| \( a \) | 8 | \( (1, -1) \times (1, 1) \times (1, 1) \) | 0 | -4 | 6 | -3 | 0 | -3 | 1 | -1 |
| \( b \) | 4 | \( (1, 2) \times (1, 0) \times (5, 1) \) | 9 | -9 | -3 | 0 | -10 | -9 | 0 | 1 |
| \( c \) | 4 | \( (1, 1) \times (1, 0) \times (1, 1) \) | 1 | -1 | -3 | -3 | -4 | -3 | 1 | 0 |

\[ x_A = 22 x_B = 2 x_C = \frac{11}{5} x_D \]
\[ \beta_1^3 = -3, \beta_2^3 = -3 \]
\[ \chi_1 = \frac{1}{\sqrt{5}}, \chi_2 = \frac{11}{\sqrt{5}}, \chi_3 = 4 \sqrt{5} \]

Table 35. D6-brane configurations and intersection numbers of Model 35, and its MSSM gauge coupling relation is \( g_a = \frac{1}{5} g_b^2 = \frac{7}{6} g_c^2 = \frac{35}{133} (\frac{3}{3} g_Y^2) = \frac{16}{133} \sqrt{\frac{2}{3}} \pi \phi^4 \).

| model 35 | \( U(4) \times U(2)_L \times U(2)_R \times USp(2) \times USp(4) \) |
|---|---|
| stack | \( N \) | \( (n^1, l^1) \times (n^2, l^2) \times (n^3, l^3) \) | \( n \) | \( n' \) | \( b \) | \( b' \) | \( c \) | \( c' \) | 1 | 4 |
| \( a \) | 8 | \( (1, -1) \times (2, 1) \times (1, 1) \) | 0 | -4 | 6 | -3 | 0 | -3 | 1 | -2 |
| \( b \) | 4 | \( (1, 0) \times (1, 0) \times (1, 1) \) | 2 | 6 | -2 | -3 | -4 | 0 | 1 |
| \( c \) | 4 | \( (1, 0) \times (1, 1) \times (1, 1) \) | 0 | -4 | 6 | -3 | 0 | -3 | 1 | -2 |

\[ x_A = \frac{13}{2} x_B = \frac{13}{8} x_C = 26 x_D \]
\[ \beta_1^3 = -3, \beta_2^3 = -3 \]
\[ \chi_1 = \sqrt{\frac{13}{2}}, \chi_2 = 2 \sqrt{26}, \chi_3 = \frac{1}{2} \sqrt{26} \]
| model 36 | U(4) × U(2) _L_ × U(2) _R_ × USp(4) |
|---------|------------------------------------|
| stack   | N | \( (n^1, l^1) \times (n^2, l^2) \times (n^3, l^3) \) | \( n \) | \( n_b \) | \( n_c \) | \( b \) | \( b' \) | \( c \) | \( c' \) | 3 |
| a       | 8 | \((1, 1) \times (-1, 0) \times (-1, 1)\) | 0 | 0 | 2 | 1 | 0 | -3 | 0 |
| b       | 4 | \((1, 0) \times (2, -1) \times (1, 3)\) | -5 | 5 | — | — | 16 | 8 | -1 |
| c       | 4 | \((-1, 4) \times (0, 1) \times (-1, 1)\) | 3 | -3 | — | — | — | 1 |
| 3       | 4 | \((0, -1) \times (1, 0) \times (0, 2)\) |  |  |  |  |  |  |  |
|         |   | \( x_A = \frac{2}{3} x_B = \frac{1}{4} x_C = \frac{3}{2} x_D \) |  |  |  |  |  |  |  |
|         |   | \( \beta_3^g = -4 \) |  |  |  |  |  |  |  |
|         |   | \( \chi_1 = \frac{1}{2}, \chi_2 = 3, \chi_3 = 1 \) |  |  |  |  |  |  |  |

Table 36. D6-brane configurations and intersection numbers of Model 36, and its MSSM gauge coupling relation is \( g_a^2 = \frac{26}{\pi} g_b^2 = 6 g_c^2 = 2(\frac{5}{3} g_Y^2) = \frac{16}{\pi} \sqrt{6} \pi e^{\delta} \).

| model 37 | U(4) × U(2) _L_ × U(2) _R_ × USp(2)² |
|----------|--------------------------------------|
| stack    | N | \( (n^1, l^1) \times (n^2, l^2) \times (n^3, l^3) \) | \( n \) | \( n_b \) | \( n_c \) | \( b \) | \( b' \) | \( c \) | \( c' \) | 1 | 4 |
| a        | 8 | \((1, -1) \times (-1, 1) \times (1, -1)\) | 0 | 4 | 3 | 0 | 0 | -3 | -1 | 1 |
| b        | 4 | \((1, -4) \times (1, 0) \times (1, 1)\) | 3 | -3 | — | — | -7 | 0 | 0 | 1 |
| c        | 4 | \((-2, 1) \times (2, 1) \times (-1, 1)\) | -3 | -13 | — | — | — | — | -1 | 4 |
| 1        | 2 | \((1, 0) \times (1, 0) \times (2, 0)\) |  |  |  |  |  |  |  |  |
| 4        | 2 | \((0, -1) \times (0, 1) \times (2, 0)\) |  |  |  |  |  |  |  |  |
|         |   | \( x_A = 28 x_B = \frac{28}{27} x_C = 7 x_D \) |  |  |  |  |  |  |  |  |
|         |   | \( \beta_1^g = -3, \beta_2^g = 1 \) |  |  |  |  |  |  |  |  |
|         |   | \( \chi_1 = \sqrt{\frac{27}{216}}, \chi_2 = \sqrt{161}, \chi_3 = 8 \sqrt{\frac{1}{27}} \) |  |  |  |  |  |  |  |  |

Table 37. D6-brane configurations and intersection numbers of Model 37, and its MSSM gauge coupling relation is \( g_a^2 = \frac{5}{6} g_b^2 = \frac{11}{6} g_c^2 = \frac{11}{8} (\frac{5}{3} g_Y^2) = \frac{8}{305} \sqrt{2161} \frac{3/4}{\pi} e^{\delta} \).
B Gauge coupling unified supersymmetric Pati-Salam models

In this section, we present in particular some examples of supersymmetric Pati-Salam models with gauge unification at GUTs scale.

| model 38 | U(4) × U(2)\text{L} × U(2)\text{R} × USp(2)^4 |
|---------|-----------------------------------------------|
| stack   | N (n^1, l^1) × (n^2, l^2) × (n^3, l^3) | n | n | b | b' | c | c' | 1 | 2 | 3 | 4 |
| a       | 8 (1, 0) × (−1, 1) × (−1, −1) | 0 | 0 | 3 | 0 | 0 | −3 | 0 | 0 | −1 | 1 |
| b       | 4 (1, −3) × (−1, −1) × (−1, 0) | 2 | −2 | — | — | 0 | 0 | 0 | −3 | 1 | 0 |
| c       | 4 (1, −3) × (−1, 1) × (0, −1) | 2 | −2 | — | — | — | −3 | 0 | 0 | 1 | 1 |

Table 38. D6-brane configurations and intersection numbers of Model 38, and its MSSM gauge coupling relation is \( g_a^2 = g_b^2 = g_c^2 = (\frac{5}{4} g_Y^2) = 4 \sqrt{\frac{7}{4} \pi e^4} \).

| model 39 | U(4) × U(2)\text{L} × U(2)\text{R} × USp(2)^4 |
|---------|-----------------------------------------------|
| stack   | N (n^1, l^1) × (n^2, l^2) × (n^3, l^3) | n | n | b | b' | c | c' | 1 | 2 | 3 | 4 |
| a       | 8 (−1, 1) × (0, 1) × (−1, −1) | 0 | 0 | 3 | 0 | 0 | −3 | 0 | 0 | 1 | 0 |
| b       | 4 (0, −1) × (−3, −1) × (−1, −1) | 2 | −2 | — | — | 0 | 0 | 1 | −3 | 0 | 0 |
| c       | 4 (1, 0) × (−3, −1) × (1, −1) | 2 | −2 | — | — | — | −3 | 0 | 0 | 1 | −3 |

Table 39. D6-brane configurations and intersection numbers of Model 39, and its MSSM gauge coupling relation is \( g_a^2 = g_b^2 = g_c^2 = (\frac{5}{4} g_Y^2) = 4 \sqrt{\frac{7}{4} \pi e^4} \).
Table 40. D6-brane configurations and intersection numbers of Model 40, and its MSSM gauge coupling relation is $g_a^2 = g_b^2 = g_c^2 = (\frac{5}{3}g_Y^2) = 4\sqrt{\frac{2}{3}}\pi e^\phi$.

Table 41. D6-brane configurations and intersection numbers of Model 41, and its MSSM gauge coupling relation is $g_a^2 = g_b^2 = g_c^2 = (\frac{5}{3}g_Y^2) = 4\sqrt{\frac{2}{3}}\pi e^\phi$.

Table 42. D6-brane configurations and intersection numbers of Model 42, and its MSSM gauge coupling relation is $g_a^2 = g_b^2 = g_c^2 = (\frac{5}{3}g_Y^2) = 4\sqrt{\frac{2}{3}}\pi e^\phi$. 
\begin{table}[h!]
\centering
\begin{tabular}{|c|c|c|c|c|c|c|c|c|c|}
\hline
model 43 & \multicolumn{1}{c|}{U(4) \times U(2)_L \times U(2)_R \times USp(2)^4} \\
\hline
stack & \multicolumn{1}{c|}{N} & (n^1, l^1) \times (n^2, l^2) \times (n^3, l^3) & n & u & b & b' & c & c' & 1 & 2 & 3 & 4 \\
\hline
a & 8 & (-1, 1) \times (-1, -1) \times (1, 0) & 0 & 0 & 3 & 0 & -3 & 0 & 0 & -1 & 1 & 0 \\
b & 4 & (-1, -1) \times (-1, 0) \times (1, -3) & 2 & -2 & - & - & 0 & 0 & 0 & 1 & 0 & -3 \\
c & 4 & (1, 1) \times (0, 1) \times (-1, -3) & -2 & 2 & - & - & - & 3 & 0 & 0 & -1 & 0 \\
\hline
1 & 2 & (2, 0) \times (1, 0) \times (1, 0) & & & & & & & & & \multicolumn{1}{c}{x_A = \frac{2}{3}x_B = \frac{1}{3}x_C = x_D} & \\
2 & 2 & (2, 0) \times (0, -1) \times (0, 1) & & & & & & & \multicolumn{1}{c}{\beta_1^0 = -3, \beta_2^0 = -3, \beta_3^0 = -3, \beta_4^0 = -3} & \\
3 & 2 & (0, -2) \times (1, 0) \times (0, 1) & & & & & & & \multicolumn{1}{c}{\chi_1 = 1, \chi_2 = 1, \chi_3 = \frac{2}{3}} & \\
4 & 2 & (0, -2) \times (0, 1) \times (1, 0) & & & & & & & \multicolumn{1}{c}{\chi_1 = 1, \chi_2 = 1, \chi_3 = 2} & \\
\hline
\end{tabular}
\caption{D6-brane configurations and intersection numbers of Model 43, and its MSSM gauge coupling relation is $g_a^2 = g_b^2 = g_c^2 = (\frac{5}{3}g_Y^2) = 4\sqrt{\frac{2}{3}\pi e^s}$.}
\end{table}

\begin{table}[h!]
\centering
\begin{tabular}{|c|c|c|c|c|c|c|c|c|c|}
\hline
model 44 & \multicolumn{1}{c|}{U(4) \times U(2)_L \times U(2)_R \times USp(2)^4} \\
\hline
stack & \multicolumn{1}{c|}{N} & (n^1, l^1) \times (n^2, l^2) \times (n^3, l^3) & n & u & b & b' & c & c' & 1 & 2 & 3 & 4 \\
\hline
a & 8 & (1, 0) \times (-1, 1) \times (-1, 1) & 0 & 0 & 3 & 0 & -3 & 0 & 0 & 0 & -1 & 1 \\
b & 4 & (1, -3) \times (-1, -1) \times (-1, 0) & 2 & -2 & - & - & 0 & 0 & 0 & -3 & 1 & 0 \\
c & 4 & (-1, -3) \times (1, 1) \times (0, 1) & -2 & 2 & - & - & - & 3 & 0 & 0 & -1 & 0 \\
\hline
1 & 2 & (1, 0) \times (2, 0) \times (1, 0) & & & & & & & & & \multicolumn{1}{c}{x_A = x_B = \frac{2}{3}x_C = \frac{1}{3}x_D} & \\
2 & 2 & (1, 0) \times (0, -2) \times (0, 1) & & & & & & & \multicolumn{1}{c}{\beta_1^0 = -3, \beta_2^0 = -3, \beta_3^0 = -3, \beta_4^0 = -3} & \\
3 & 2 & (0, -1) \times (2, 0) \times (0, 1) & & & & & & & \multicolumn{1}{c}{\chi_1 = \frac{2}{3}, \chi_2 = 1, \chi_3 = 2} & \\
4 & 2 & (0, -1) \times (0, 2) \times (1, 0) & & & & & & & \multicolumn{1}{c}{\chi_1 = \frac{2}{3}, \chi_2 = 1, \chi_3 = 2} & \\
\hline
\end{tabular}
\caption{D6-brane configurations and intersection numbers of Model 44, and its MSSM gauge coupling relation is $g_a^2 = g_b^2 = g_c^2 = (\frac{5}{3}g_Y^2) = 4\sqrt{\frac{2}{3}\pi e^s}$.}
\end{table}
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