The Misaligned Orbit of the Earth-sized Planet Kepler-408b

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Abstract

Kepler-408 is one of the 33 planet-hosting Kepler stars for which asteroseismology has been used to investigate the orientation of the stellar rotation axis relative to the planetary orbital plane. The transiting hot Earth, Kepler-408b, has an orbital period of 2.5 days and a radius of 0.86 R⊕, making it much smaller than the planets for which spin–orbit alignment has been studied using the Rossiter–McLaughlin effect. Because conflicting asteroseismic results have been reported in the literature, we undertake a thorough re-appraisal of this system and perform numerous checks for consistency and robustness. We find that the conflicting results are due to the different models for the low-frequency noise in the power spectrum. A careful treatment of the background noise resolves these conflicts, and shows that the stellar inclination is i∗ = 42°±5° degrees. Kepler-408b is, by far, the smallest planet known to have a significantly misaligned orbit.

Key words: asteroseismology – methods: data analysis – planetary systems – stars: oscillations – stars: rotation – techniques: photometric

1. Introduction

Planets around other stars are occasionally found to have orbits that are misaligned, or even retrograde, relative to the direction of stellar rotation (e.g., Winn & Fabrycky 2015; Triaud 2018). However, all previous detections of misaligned orbits are for planets larger than Neptune. Smaller planets are relatively unexplored because of the difficulty of the relevant measurements.

Three of the techniques for investigating spin–orbit alignment—the Rossiter–McLaughlin effect, the starspot-tracking method, and the gravity-darkening method—require the observation of signals for which the amplitude is proportional to the loss of light during planetary transits. Hence, they are much easier to apply to giant planets than small planets. Two other techniques—the asteroseismic method and the v sin i method—rely on observing signals that are independent of planet size. However, the asteroseismic method has only been applied to 33 stars, because it requires an unusually bright star with large-amplitude oscillations. The v sin i method has been applied to samples of hundreds of stars, but in most cases it only provides weak constraints (Schlaufman 2010; Winn et al. 2017). Due to these limitations, it is unclear whether the misalignments are the result of processes specific to giant planets, or whether they also occur for terrestrial planets.

Kepler-408 (also known as KIC 10963065 and KOI-1612) is one of approximately 150,000 Sun-like stars that were monitored for 4 yr with the NASA Kepler space telescope (Borucki et al. 2010). Its light curves exhibit a periodic transit signal due to an Earth-sized planet with Porb ~ 2.5 days (Marcy et al. 2014). Table 1 summarizes the known characteristics of the system. With a Kepler apparent magnitude of 8.8, the host star is the third brightest of all the Kepler stars with confirmed planets. This unusual brightness enables an investigation of the stellar obliquity using asteroseismology. In particular, it is possible to determine the inclination i∗ of the stellar rotation axis based on the fine structure in the p-mode pulsation spectrum (Toutain & Gouttebroze 1993; Gizon & Solanki 2003).

However, there are conflicting reports in the literature. Campante et al. (2016) found the inclination to be consistent with 90° and set a lower limit of 54°. This was part of a homogeneous study of 25 stars with transiting planets. In contrast, Nielsen et al. (2017) found the inclination to be between 40 and 45 degrees. This finding was incidental to the main purpose of the study, which was to probe the internal rotation profiles of six stars. The authors did not remark on the transiting planet, nor on the conflict with Campante et al. (2016).

We have examined the case of Kepler-408 in greater detail, to try and resolve this conflict. We were also motivated by the numerical simulations of Kamiaka et al. (2018), who established the observational requirements for the reliable inference of the rotational inclination, and found that the characteristics of Kepler-408 should allow for reliable results. Section 2 describes the transit analysis. Section 3 presents some independent checks on the previous measurements of the stellar rotation period, which plays a key role in the asteroseismic analysis. Section 4 describes the asteroseismic analysis and resolves the prior discrepancy by identifying a problem with the analysis by Campante et al. (2016). Section 5 shows that our asteroseismic estimate of i∗ agrees with the constraint that is obtained by combining measurements of the stellar radius, rotation period, and sky-projected rotation velocity. Our findings and some implications are summarized in Section 6. Just for definiteness, the present paper refers to those systems as misaligned if either λ (sky-projected spin–orbit angle) or 90° − i∗ (a proxy for the stellar obliquity in transiting planetary systems) exceeds 30° in 95% confidence.

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2. Transit Modeling

The orbital inclination, $i_{\text{orb}}$, of a transiting planet is always close to 90°. For a precise measurement, we modeled the Kepler transit light curve. We downloaded the short-cadence, pre-search data conditioning (PDC) light curves from the Mikulski Archive for Space Telescopes. The data surrounding each transit were fitted with a standard model for the loss of light (Mandel & Agol 2002), assuming the orbit to be circular and accounting for stellar variability with a locally quadratic function of time. After dividing through by the best-fitting quadratic functions, the transit data were phase-folded and averaged, giving a mean light curve with a higher signal-to-noise ratio (Figure 1).

This light curve was then fitted to obtain our final estimates for the transit parameters (Table 1). Uniform priors were adopted for the logarithm of the planet-to-star radius ratio ($R_p/R_*$), the cosine of the orbital inclination ($\cos i_{\text{orb}}$), the normalization of the light curve, the two coefficients of the quadratic limb-darkening profile, and the logarithm of a noise term to account for the scatter of the residuals between the data and the model. A Gaussian prior was adopted for the mean stellar density ($\rho_* = 0.816 \pm 0.025 \, \text{g cm}^{-3}$) based on the previous asteroseismic analysis of Kamiaka et al. (2018). The posterior distributions for the model parameters were obtained with a nested sampling code (Feroz et al. 2009). The result for the orbital inclination was $i_{\text{orb}} = 81.85 \pm 0.10$ degrees.

In the present analysis, we adopted the circular model because the timescale for tidal orbital circularization is likely short for the 2.5 day orbit. We also checked that the model with non-zero eccentricity does not significantly improve the fit. This is in agreement with the analysis of Van Eylen et al. (2019), who found that the eccentricity was consistent with zero within 95% confidence.

3. Stellar Rotation Period from Photometric Variability

The Kepler photometric time series exhibits quasi-periodic modulation that is presumably due to the rotation of surface inhomogeneities across the star’s visible hemisphere. By computing the autocorrelation function, McQuillan et al. (2013) determined the photometric rotation period to be 12.44 ± 0.17 days. Angus et al. (2018) reported a value of 12.89 ± 0.19 days by modeling the Kepler data as a Gaussian process with a quasi-periodic covariance kernel function.

To perform an independent check on the determination of the stellar rotation period, we analyzed the Kepler data outside of transits. We normalized the data from each quarter by setting the median flux equal to unity. A Lomb–Scargle periodogram of the resulting time series has its most prominent peak at 12.96 ± 0.07 days, and the autocorrelation function shows a series of peaks spaced by 12.94 ± 0.22 days (Figure 2). Previous experience has shown that the strongest photometric periodicity sometimes occurs at harmonics of the true rotation period, presumably because there are several active regions on the star.

In the present case, visual inspection of the light curve confirms that the true period is close to 12.9 days. We were able to identify several time intervals in which a complex pattern of variations repeats nearly exactly after 12.9 days (Figure 3), which would be an unlikely coincidence if the true period were different. We highlighted part of the light curves in

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**Table 1**

| Parameter | Value | Reference |
|-----------|-------|-----------|
| Stellar Parameters | | |
| Effective temperature, $T_{\text{eff}}$ [K] | 6088 ± 65 | Petigura et al. (2017) |
| Surface gravity, $\log (\text{g/cm} \, \text{s}^{-2})$ | 4.318±0.008 | Petigura et al. (2017) |
| Metallicity, [Fe/H] | −0.138±0.043 | Petigura et al. (2017) |
| Mass $M_*$ [$M_\odot$] | 1.05 ± 0.04 | Johnson et al. (2017) |
| Radius $R_*$ [$R_\odot$] | 1.253 ± 0.051 | Berger et al. (2018) |
| Age [Gyr] | 4.7 ± 1.2 | Johnson et al. (2017) |
| Projected rotation rate, $v_{\text{rot}} \sin i_*$ [km s$^{-1}$] | 2.8 ± 1.0 | Petigura et al. (2017) |
| Rotation period $P_{\text{rot}}$ [days] | 12.89 ± 0.19 | Angus et al. (2018) |
| Planetary Parameters | | |
| Planet-to-star radius ratio, $R_p/R_*$ | 0.0063 ± 0.0003 | This work |
| Radius $R_p$ [$R_\odot$] | 0.86 ± 0.04 | This work |
| Time of inferior conjunction [BJD] | 2454965.6804 ± 0.0003 | This work |
| Orbital period $P_{\text{orb}}$ [days] | 2.465024 ± 0.000005 | Thompson et al. (2018) |
| Orbital inclination $i_{\text{orb}}$ [deg] | 81.85 ± 0.10 | This work |
Figure 2. Estimates of the stellar rotation period from the photometric time series. Top: autocorrelation function. The red dashed lines indicate the locations of several peaks. Middle: the Lomb–Scargle periodogram. The location of the most significant peak is marked with a red dashed line and the 1σ uncertainty interval is plotted with orange dashed lines. Bottom: close-up of the Lomb–Scargle periodogram around the most significant peak.

Figure 3 so as to clarify the periodicity, but it should be regarded as a sanity check on the more objective measures with no claim to be objective or complete. For a more systematic comparison between asteroseismic and photometric estimates of stellar rotation periods for other stars, see Suto et al. (2018).

In what follows, we adopt the value $P_{rot} = 12.89 \pm 0.19$ days based on the work of Angus et al. (2018), since their analysis appears to be the most rigorous with regard to the quoted uncertainty. The reciprocal of the rotation period, which is most relevant to the asteroseismic analysis, is $1/P_{rot} = 0.898 \pm 0.013 \mu Hz$.

4. Asteroseismic Analysis

4.1. Brief Description of the Method

The star’s pressure-mode oscillations ($p$ modes) are manifest in the Kepler data as quasi-periodic variations in stellar brightness with amplitudes of a few parts per million (ppm) and frequencies on the order of 2000 $\mu$Hz (periods $\sim$ 10 minutes). The modes can be classified with three integers: the radial order $n$ ($\geq 1$), which depends on the radial dependence of the oscillatory pattern; the angular degree $\ell$ ($\geq 0$), which specifies the variation with the polar angle; and the azimuthal order $m (= -\ell, \ldots, 0, \ldots, \ell)$, which specifies the variation with the azimuthal angle. For a non-rotating star, all of the modes with the same $n$ and $\ell$ would have the same frequency, regardless of $m$. Rotation breaks this degeneracy, producing small frequency shifts,

$$\nu_{n,\ell,m} = \nu_{n,\ell} + m \delta \nu \approx \left( n + \frac{\ell}{2} + \varepsilon_{n,\ell} \right) \Delta \nu + m \delta \nu,$$

where $\Delta \nu$ is the large separation (the spacing between consecutive radial modes), $\varepsilon_{n,\ell}$ is a small correction of order unity (Tassoul 1980, 1990; Mosser et al. 2013), and the rotational splitting $\delta \nu$ is approximately the reciprocal of the stellar rotation period (Appourchaux et al. 2008).

Because the shifts are small ($\delta \nu \sim 1 \mu$Hz), all of the modes within a multiplet are expected to have the same intrinsic amplitude. However, the height of each peak in the observed power spectrum is also proportional to a factor of $E_{\ell,m}$ depending on the rotational inclination $i_*$ as shown in Equations (2) and (4) below. This is because the peak heights are based on the average intensity of the mode pattern across the visible hemisphere, and the modes have different symmetries with respect to the rotation axis.

4.2. Power Spectrum Modeling

We downloaded the Kepler-408 power spectrum from the Kepler Astroseismic Science Operations Center database. We modeled the power spectrum as

$$P(\nu) = \sum_{n=n_{min}}^{n_{max}} \sum_{\ell=0}^{\ell_{max}} \sum_{m=-\ell}^{\ell} \frac{H_{n,\ell} E_{\ell,m}(i_*)}{1 + \left( \frac{\nu - \nu_{n,\ell,m}}{\Gamma_{n,\ell,m}} \right)^2} + N(\nu),$$  \(2\)

where $H_{n,\ell}$ is the intrinsic mode amplitude, $E_{\ell,m}(i_*)$ is the mode visibility (see Equation (4)), $\nu_{n,\ell,m}$ is the line center, $\Gamma_{n,\ell,m}$ is the line width, and $N(\nu)$ is the noise background. The background was modeled as

$$N(\nu) = \frac{A_1}{1 + (\tau_1 \nu)^{p_1}} + \frac{A_2}{1 + (\tau_2 \nu)^{p_2}} + N_0,$$  \(3\)

where $N_0$ is a constant (white noise), and $A_i, \tau_i$, and $p_i$ ($i = 1, 2$) are the height, characteristic timescale, and slope of a Harvey-like profile. Further details are given by Kamiaka et al. (2018).
The most readily observed multiplets are the dipole ($\ell = 1$) and quadrupole ($\ell = 2$) modes, for which

$$E_{1,0} = \cos^2 i_*, \ E_{1,\pm 1} = \frac{1}{2} \sin^2 i_*, $$

$$E_{2,0} = \frac{1}{4} (3 \cos^2 i_* - 1)^2, \ E_{2,\pm 1} = \frac{3}{8} \sin^2 2i_*, $$

$$E_{2,\pm 2} = \frac{3}{8} \sin^4 i_*.$$  

For a star with $i_* = 90^\circ$, the central peak ($m = 0$) is missing, while for $i_* = 0^\circ$, only the central peak is visible (e.g., Gizon & Solanki 2003).

The power spectrum was analyzed using a Markov chain Monte Carlo algorithm based on a Metropolis–Hasting scheme, with parallel tempering. We divided the analysis into three steps: the burn-in phase, training phase, and acquire phase. The burn-in phase (40,000 samples) ensures that we reach the region of interest in the parameter space. The training phase (700,000 samples) employs an adaptive algorithm to optimize the covariance matrix of the Gaussian proposal probability density function to achieve the ideal acceptance rate of 23.4% (Atchade 2006). During the acquire phase ($10^6$ samples), the optimal covariance matrix is used to sample the posterior distribution. Convergence of the posterior distribution is confirmed through the Heidelberg–Welch and Geweke tests.

Because of the large number of free parameters, we fitted the spectrum in two steps. First, we concentrated on fitting the background, using a single Gaussian function to model the envelope of excess power from the oscillation modes. The results for the background model were then used as priors when fitting for the parameters of the oscillation modes. Figure 4 plots the height-to-background ratio (HBR) and the splitting-to-width ratio ($\delta \nu / \Gamma$) as a function of mode frequency $\nu_{n,l=0}$, showing that HBR takes the maximum value $\text{HBR}_{\text{max}}$ at $\nu_{\text{max}}$.

Kamiaka et al. (2018) found that reliable inference of $i_*$...
practically requires at least $\text{HBR}_{\text{max}} \gtrsim 1$ and $\delta \nu_{c}/\Gamma(\nu_{\text{max}}) \gtrsim 1/2$. As indicated by the dashed line in Figure 4, their criteria are satisfied for Kepler-408.

We inspected each line profile visually so as to avoid too noisy modes, and selected the radial orders of $13 \leq n \leq 25$ for $\ell = 0$ and $1 \leq n \leq 24$ for $\ell = 2$, respectively, for the analysis. Figures 5 and 6 give the mode profiles for $\ell = 1$ ($13 \leq n \leq 24$) and for $\ell = 2$ ($12 \leq n \leq 23$), respectively. In those panels our best fits of $i_{c} = 42^{\circ} \pm 5^{\circ}$ degrees and $\delta \nu_{c} = 0.99 \pm 0.10 \mu\text{Hz}$ are plotted with solid green lines (see Figure 7 below).

4.3. Checks for Consistency and Robustness

The measured splitting is in agreement with the value of $1/P_{\text{rot}} = 0.898 \pm 0.013 \mu\text{Hz}$ based on the photometric rotation period, thereby providing a successful consistency check. We also tried using the photometric rotation period as a prior constraint on the asteroseismic analysis, which sharpened the constraint on the stellar inclination angle to $45.9 \pm 2.1$ degrees (see the blue curves in Figure 7).

To allow for a visual inspection, Figure 8 displays the average $\ell = 1$ and $\ell = 2$ profiles, based on the combination of the data from 13 different radial orders. The profile of the average $\ell = 1$ multiplet (the top panel) is centrally peaked, demonstrating the visibility of the $m = 0$ mode, and ruling out an inclination angle near $90^{\circ}$. The signal-to-noise ratio and frequency resolution are high enough that the absence of the $m = 0$ mode would have led to a flat-topped appearance, from the combination of the marginally resolved $m = +1$ and $-1$ modes. On the other hand, the profile of the $\ell = 2$ modes is not centrally peaked, ruling out inclinations near zero. Together, the appearance of the modes suggests an intermediate value of the inclination.

The bottom panel of Figure 8 shows that the $\ell = 2$ multiplet has an asymmetric appearance, with more power at frequencies above the line center than below. This is unexpected because the geometrical factors $\ell_{m}$ do not depend on the sign of $m$. Figure 6 suggests that this asymmetry in power is mainly due to modes of high radial order ($n = 18-21$). Such high-order modes are more sensitive to the conditions near the stellar surface (e.g., Christensen-Dalsgaard & Thompson 1997; Kjeldsen et al. 2008; Ball & Gizon 2014; Sonoi et al. 2015). Thus, the observed asymmetry may arise from the (poorly understood) magnetic and non-adiabatic processes occurring near the surface. We performed the similar stacking analysis for several stars in Kamiaka et al. (2018), all of which do not exhibit any noticeable asymmetry. Thus the asymmetry seems fairly specific to Kepler-408. Because the reason for the asymmetry is not clear, we tried fitting only the $\ell = 1$ modes and found that a low inclination (and a high stellar obliquity) is still preferred as shown in Figure 9(c) below. We repeated the fit after decreasing the amplitude of the (unsmoothed) data by 30% for $-\Gamma_{n,l=2,m=0}/2 < \nu - \nu_{n,l=2,m=1} < \Gamma_{n,l=2,m=0}/2$ for $18 \leq n \leq 21$, and found that $i_{c} = 39^{\circ} \pm 3^{\circ}$ degrees. Since fitting these simulated spectra did not lead to any systematic bias in the results for the inclination, the observed asymmetry for the $\ell = 2$ modes does not affect our conclusion that Kepler-408b has a significantly misaligned orbit.

As further tests of robustness, we repeated the analysis for five different choices of the set of radial orders and angular degrees to be fitted (see Figure 9). This led to larger uncertainties and small systematic changes in the derived parameters. Fitting the $\ell = 1$ modes tends to give lower inclinations, while the $\ell = 2$ modes favor higher inclinations. Such complementary roles of $\ell = 1$ and $\ell = 2$ modes are very useful in constraining $i_{c}$, and $\delta \nu_{c}$ reliably. While Kepler-408 is one of the stars with the clearest pulsation spectrum, its asteroseismic modeling is still subtle and careful individual tests are required for the reliable parameter extraction.

In all cases, though, the results are compatible with a large spin–orbit misalignment, and the splitting is compatible with the photometric rotation period, implying that our asteroseismic inference for the Kepler-408 system is robust.

4.4. Comparison with Previous Results

In reality, our results do not agree with those of Campante et al. (2016), who found $i_{c} > 54^{\circ}$ within $1\sigma$ confidence (we note,
however, that their 95.4% constraint is $i > 36.5^\circ$. In attempting to understand the reason for the discrepancy, we realized that we use an unweighted power spectrum (Lomb–Scargle periodogram), while Campante et al. (2016) computed it using a weighted least-square fitting method. Thus we repeated our analysis using their spectrum, and obtained almost the same inclination angle, implying that the difference of the spectra is not a major reason for the discrepancy. We also noticed that their best-fitting model gave $\delta v_r = 0.50^{+0.20}_{-0.04} \mu Hz$, which is inconsistent with the photometrically measured rotation period. Another difference is related to the chosen model for the background noise in the power spectrum. For the sake of uniformity, Campante et al. (2016)
adopted the same model for all 25 systems of their analysis. Their model was parameterized as

$$B(\nu) = B_0 + \left[ \frac{B_1}{1 + (2\pi\nu/\eta)^2} + \frac{B_2}{\nu^2} \right] \text{sinc}^2\left(\frac{\pi\nu}{2\nu_0}\right), \quad (5)$$

where $\nu_0 = 8496.6 \mu\text{Hz}$ is the Nyquist frequency.

While Equation (5) works reasonably well in general, the residuals from the best fit of their noise model (right panels of Figure 10) shows that it poorly fits the low-frequency part of the noise background of Kepler-408; there is a systematic departure from the zero baseline of up to 10% in the vicinity of the low-frequency modes, suggesting that their noise background fit is not satisfactory. In contrast, our background

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**Figure 6.** Same as Figure 5, but for quadrupole modes ($\ell = 2$) from $n = 12$ to $23$. 

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model, Equation (3), fits much better, as illustrated in the left panels of Figure 10. More specifically, our best-fit model based on our power spectrum with the background model (3) is preferred over that based on the Campante et al. power spectrum with Equation (5) by an odds ratio significantly larger than 100. The odds ratio is the ratio of the two integrated likelihoods; see Section 12.7 of Gregory (2005) for details. According to the Jeffreys (1961) classification, this is decisive evidence in favor of our background noise model.

Incidentally Equation (3) does not include the apodization correction factor $\frac{\sin^2 (\pi/2 \nu_0)}{\nu^2}$, unlike Equation (5). This could affect the estimate of $i_r$. We assessed this possibility by applying the correction for the first two terms in Equation (3) as in Davies et al. (2016), and found that the resulting fit yields $i_r = 41.9^{+3.7}_{-3.6}$ degrees (see green histogram in Figure 7). Thus we confirm that the apodization factor does not change our conclusion.

When we replaced our model for the background with that of Equation (5), we were able to reproduce the result of $i_r > 54^\circ$ reported by Campante et al. (2016). Evidently, it is essential to perform a careful subtraction of the low-frequency noise for each system to obtain an unbiased estimate of $i_r$ from asteroseismology analysis. Appourchaux et al. (2012) have studied the systematic errors in measurements of seismic parameters caused by inaccuracies in the model for background noise. Although they did not examine the implications for inference of the inclination angle, they did note that inaccuracies can greatly impact the inferred mode heights and linewidths, which in turn may bias the measurement of the rotational splitting and inclination. Our work demonstrates that this is indeed the case; systematic errors in the background model can severely bias the measured inclination.

As an additional test, we tried replacing the background model with a simple quadratic function of frequency. By restricting the frequency range to the limited interval spanned by the oscillation modes (1300–2900 $\mu$Hz), we found that the quadratic function also gives a good fit. The results for the inclination were the same as in our original analysis ($i_r = 42^{+5}_{-4}$ degrees), confirming that the exact functional form of the background model does not matter, as long as it fits reasonably well.

5. Projected Rotation Rate

There is also evidence independent of asteroseismology that the rotational inclination is in the neighborhood of $45^\circ$, based on the measured values of the stellar radius, rotation period, and sky-projected rotation velocity (Table 1). The stellar radius ($R_*$) was determined by combining the observed geometric parallax, apparent $K$ magnitude, and spectroscopic effective temperature (Berger et al. 2018). The rotation period ($P_{\text{rot}}$) was determined from the Kepler photometry, as noted above. The combination of these quantities implies $v_{\text{rot}} = 2\pi R_*/P_{\text{rot}} = 4.92 \pm 0.21$ km s$^{-1}$. Meanwhile, the sky-projected rotation velocity ($v_{\text{rot}} \sin i_*$) was found to be $2.8 \pm 1.0$ km s$^{-1}$ by...
modeling the Doppler-rotational contribution to the observed spectral line broadening (Petigura et al. 2017).

Together, these data can be used to place constraints on $\sin i_*$. To obtain the likelihood function for $\sin i_*$, we integrated $p_1(v_{\text{rot}}) \cdot p_2(v_{\text{rot}} \sin i_*)$ over $v_{\text{rot}}$, where $p_1$ and $p_2$ are Gaussian functions representing the constraints $v_{\text{rot}} = 4.92 \pm 0.21 \text{ km s}^{-1}$ and $v_{\text{rot}} \sin i_* = 2.8 \pm 1.0 \text{ km s}^{-1}$. The result is $\sin i_* = 0.70 \pm 0.21$, or $i_* = 44^{\circ} + 13^{\circ}$ degrees, which is consistent with our asteroseismic result.

As another consistency check, we can combine the spectroscopically determined $v_{\text{rot}} \sin i_*$ and $R_*$ to give $\delta v_{\text{rot}} \sin i_* = 0.51 \pm 0.19 \mu\text{Hz}$. The white lines in Figure 7 show the region that is defined by this constraint, which is independent of the asteroseismic analysis. The results are again consistent to within 1$\sigma$.

### 6. Summary

By modeling the power spectrum of $p$ modes, we found the stellar inclination to be $i_* = 41.7^{\circ} + 5.1^{\circ}$ ($i_* = 41.7^{\circ} \pm 13^{\circ}$) degrees with a 68% (95%) credible interval (Section 3). Nielsen et al. (2017) and Kamiaka et al. (2018) previously reported a similar result for Kepler-408, but did not remark on the conflict with the analysis of Campante et al. (2016), nor did they appreciate the importance of this system for understanding the origin of the spin–orbit misalignment (described below). The more
Figure 9. Constraints on $i_s$ and $\delta v_s$, as in Figure 7, after making variations in the analysis procedure. (a) Fitting only the lower radial orders ($13 \leq n \leq 18$) and $\ell = 0, 1, 2$. (b) Fitting only the higher radial orders ($20 \leq n \leq 25$) and $\ell = 0, 1, 2$. (c) Fitting only the radial and dipole modes ($\ell = 0, 1$) of all orders. (d) Fitting only the radial and quadrupole modes ($\ell = 0, 2$) of all orders. (e) Fitting $\ell = 0, 1, 2$ of all orders, with a Gaussian prior of $\delta v_s = 0.898 \pm 0.013 \mu$Hz based on the measured rotation period (Angus et al. 2018). (f) Fitting all orders and modes, after replacing our model for the noise background with the (unsatisfactory) model of Campante et al. (2016). The green solid and dashed lines in the histograms indicate the median and 1σ credible regions.
thorough analysis in the present paper has resolved the conflict by examining the individual and stacked line profiles for different modes, comparing the best fit with and without the photometric rotation period constraint, and exploring different possibilities for the background model. This experience with Kepler-408 and the methodology presented in this paper should

Figure 10. Models of the noise background. Top panels: the entire power spectrum of Kepler-408, along with the best-fitting model and its three separate components. The left panel shows the spectrum and model used in our analysis. The right panel shows those used in Campante et al. (2016), which does not fit the lower envelope of the power spectrum in the vicinity of the oscillation modes well. Bottom panels: close-up of the oscillation modes, after subtracting the best-fitting model for the background.

Figure 11. Sizes and orbital periods of planets for which the stellar obliquity has been constrained, based on the Rossiter–McLaughlin effect (red) and asteroseismology (blue). Misaligned planets (with their $2\sigma$ lower limit of either $\lambda$ or $90^\circ - i$, exceeding $30^\circ$) are marked with bold symbols, based on the compilation of Southworth (2011) and our measurement (Kamiaka et al. 2018).
allow for more robust determinations of $i_0$ in the future, through the precise and accurate combination of asteroseismology, photometry, and spectroscopy.

As for the inclination of the orbital axis, by fitting the Kepler light curve we found $i_{\text{orb}} = 81.85 \pm 0.10$ degrees (Section 2). Knowledge of both the rotational and orbital inclinations is not enough to determine the stellar obliquity, because both measurements are subject to the usual degeneracy $i \leftrightarrow 180^\circ - i$, and because we do not know the position angle on the sky between the two axes. Nevertheless we may set a lower limit on the stellar obliquity of $|i_{\text{orb}} - i_0| = 40 \pm 5$ (deg).

Of all the planets known to have a spin–orbit misalignment, Kepler-408b is the smallest by a factor of six, as illustrated in Figures 11 and 12. As described earlier, we classify systems as misaligned in those plots if either their sky-projected spin–orbit angle $\lambda$ or a proxy for the stellar obliquity in transiting systems, $90^\circ - i_0$, exceeds 30 degrees. The strong selection bias for the Rossiter-McLaughlin measurement toward larger planets and shorter orbital periods is clearly illustrated in the upper and lower panels of Figures 12, in contrast to the homogeneous selection for asteroseismic targets.

Those figures also identify other systems of particular interest. Kepler-56 is an obliquely rotating star ($i_0 \sim 45^\circ$) hosting two transiting planets (Huber et al. 2013). HAT-P-7 and Kepler-25 are the only known systems for which both Rossiter–McLaughlin and asteroseismic measurements have been successful (Benomar et al. 2014; Lund et al. 2014). HAT-P-11b and GJ 436b are the smallest planets previously known to be misaligned (Winn et al. 2010b; Bourrier et al. 2018; Yee et al. 2018).

Kepler-408 provides a clue about the origin of misalignments in general. Stars and their planets are thought to form in a well-aligned state. This is because the star and the protoplanetary disk inherit the same direction of angular momentum from an initial clump of gas that contracts under its own gravity. The observation of a large obliquity is an indication that something torqued the system out of alignment. The circumstances and the timing of the torque are unknown. Since all the previous cases of large obliquities involved planets larger than Neptune, some of the proposed theories have focused on giant planets. The data have often been regarded as evidence that the formation of close-orbiting giant planets, including hot Jupiters, involves processes that tilt the planet’s orbit (Triaud et al. 2010; Winn et al. 2010a).

The case of Kepler-408 shows that orbit-tilting processes are not specific to giant planets and must occur at least occasionally for hot Earths. In a recently proposed theory for the formation
of very short-period terrestrial planets (Petrovich et al. 2018), an inner planet’s orbital angular momentum is reduced through chaotic long-term interactions with more distant planets, leading to spin–orbit misalignments of $10^\circ$–$50^\circ$, as observed here. Another theory involves a secular resonance with a more distant giant planet (Hansen & Zink 2015), although in the case of Kepler-408, no additional transiting planets are known. The existing Doppler data do not show any signals exceeding 4 m s$^{-1}$ on timescales less than a year (Marcy et al. 2014). Other possibilities are that stars and their protoplanetary disks are occasionally misaligned due to torques from neighboring stars (Batygin 2012) or that inner planets become misaligned due to the torque from a wider-orbiting and misaligned giant planet (Lai et al. 2018). To decide among these and other theories will require a larger and more diverse sample of planetary systems for which the stellar obliquity can be probed.

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