Comparison of parameters of q-switching saturable absorbers estimated by different models and the impact of accuracy of input data on the results of the estimation

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ABSTRACT

Descriptions of the main classical models used to estimate the principal parameters of q-switching saturable absorbers are presented. On the basis of these models comparative analysis of q-switching saturable absorbers was done and the best one chosen. Additionally the impact of the accurate determination of the input data such as transmission, thickness and refractive index on the principal parameters was analysed. In order that the results of this analysis are useful not only from theoretical but also from practical point of view a complete setup for investigation of saturation of absorbers was built and investigations of three types of q-switching saturable absorbers Cr^{2+}:YAG, V^{3+}:YAG and Co^{3+}:YAG were carried out. According to the knowledge of the authors the principal parameters of Co^{3+}:YAG saturable absorber at 1.33 μm wavelength are presented herein for the first time.

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1. Introduction

Q-switching saturable absorbers, for the sake of their simplicity, are one of the principal ways of gain modulation in lasers at wide range of wavelengths [1–11]. The output characteristics of such lasers strongly depend on the principal parameters of saturable absorbers such as the ground state absorption and the excited state absorption cross section, the concentration of active ions and the unsaturated losses [12–14]. Thus to design a laser with desired temporal and power characteristics of generated pulses these parameters have to be known with high accuracy. These parameters can vary over wide ranges even for the absorbers characterized by the same small signal transmission and thickness so they should be known for each single sample before applying it to a laser [15].

Most of the principal parameters which characterise a particular q-switching saturable absorber can be determined by measuring its saturation versus the incident energy density or power density and by applying classical models to fit the measured data. Many authors estimated these parameters using different modifications of the models, but the accuracy of the additional parameters that constitute the input data for the approximation was not thoroughly analysed [1,5,9,16–18]. These parameters are thickness and refractive index which should be known in advance with appropriate accuracy. Moreover unsaturated losses can be estimated from the absorption spectra in a spectral region close to but outside the laser wavelength which is more accurate than determining it from the approximation [19]. Knowing the unsaturated losses in advance and fixing it into the model results in determination of the remaining principal parameters with higher accuracy.

Herein short descriptions of the main classical models used to estimate the principal parameters of q-switching saturable absorbers are presented. On the basis of these models comparative analysis of q-switching saturable absorbers was done. Additionally the impact of the accurate determination of values of the input data such as thickness and refractive index on the principal parameters was analysed. To make this analysis useful not only from theoretical but also from practical point of view a complete setup for investigation of saturation of absorbers was built and investigations of three types of q-switching saturable absorbers were carried out. The investigated samples Cr^{2+}:YAG and V^{3+}:YAG were intentionally chosen to constitute the main types of absorbers used for gain modulation at 1.06 and 1.33 μm. Moreover Co^{3+}:YAG saturable absorber was investigated at 1.33 μm supplementing the world literature in this field. The results of the investigations can be used as a reference of the principal parameters of q-switching saturable absorbers by laser designers as well as an indication of how accurate the input data used for analysis should be so as to evaluate the principal parameters with enough accuracy.
2. Models

In the literature three main models can be found which are used to estimate the principal parameters of q-switching saturable absorbers. They are LambertW, Frantz–Nodvik, and Avizonis–Grotbeck models.

2.1. LambertW (L-W) model

If the investigated saturable absorber is a fast absorber (relaxation time is much shorter than the pulse duration of the incident laser beam) and there is no excited state absorption the relationship between transmission and power density of incident radiation can be described by special function LambertW according to [20–23]:

\[
T = \frac{I}{I_0} \text{LambertW} \left( \frac{T_0}{I_0} \cdot \exp \left( \frac{I}{I_0} \right) \right)
\]

where \( T \) is the transmission of the absorber, \( I \) is the power density of the incident radiation, \( T_0 \) is the small signal transmission of the absorber, \( I_0 \) is the saturation intensity density.

The small signal transmission \( T_0 \) can be measured by a spectrometer and inserted in the equation while the saturation density \( I_0 \) is evaluated by approximation of the measured data \((T, I)\). On the basis of \( I_0 \) the ground-state absorption cross section \( \sigma_{GSA} \) can be calculated using the expressions:

- in case of two level quantum model:
  \[
  \sigma_{GSA} = \frac{hv}{2T_if_i}
  \]
- in case of three level quantum model:
  \[
  \sigma_{GSA} = \frac{hv}{T_if_i}
  \]

where \( h \) is the Planck's constant and \( v \) is the frequency of the electromagnetic field.

The concentration of active ions \( N_0 \) can be determined by the equation [5]:

\[
N_0 = -\frac{\ln(T_0)}{\sigma_{GSA}/d}
\]

where \( d \) is the thickness of the saturable absorber.

The unsaturable losses \( \alpha \) can be estimated from the absorption spectra in a spectral region close to but outside the laser wavelength using the following formula:

\[
\alpha = -\frac{\ln(T_r)}{d}
\]

where \( T_r \) is the transmission of the saturable absorber close to but outside the laser wavelength.

For this model we can also evaluate the excited state absorption cross section \( \sigma_{ESA} \) which can be estimated by the equation [5]:

\[
\sigma_{ESA} = \frac{\ln(1 - (T_s - T_{sa}))}{\ln(T_0 - T_{sa})} \cdot \sigma_{GSA}
\]

where \( T_s \) is the saturated transmission of the saturable absorber (transmission at very high power density).

2.2. Frantz–Nodvik (F–N) model

If the investigated saturable absorber is a slow absorber (relaxation time is much longer than the pulse duration of the incident laser beam \( \tau_f \gg \tau_s \)) and there is no excited state absorption the relationship between transmission and energy density of incident radiation can be described by Frantz–Nodvik equation [24]:

\[
T = \frac{E}{E_s} \ln \left( 1 + \left[ \exp \left( \frac{E}{E_s} - 1 \right) \right] \cdot T_0 \right)
\]

where \( E \) is the energy density of the incident radiation and \( E_s \) is the saturation energy density which can be expressed by the following formulas:

in case of two level quantum model:

\[
E_s = \frac{hv}{2\sigma_{GSA}}
\]

in case of three level quantum model:

\[
E_s = \frac{hv}{\sigma_{ESA}}
\]

As in the previous model the small signal transmission \( T_0 \) can be measured by spectrometer and inserted in the equation while saturation energy density \( E_s \) is evaluated by approximation of the measured data \((T, E)\). On the basis of \( E_s \) ground-state absorption cross section \( \sigma_{GSA} \) can be calculated using the expressions 8 or 9. The concentration of active ions \( N_0 \), the unsaturable losses \( \alpha \) and the excited state absorption cross section \( \sigma_{ESA} \) can be determined by Eqs. (4)–(6) respectively.

2.3. Avizonis–Grotbeck (A–G) model

If the investigated saturable absorber is characterized by the excited state absorption the evolution of the energy density \( E(z) \) of incident radiation inside saturable absorber can be described by the differential equation [25]:

\[
\frac{dE(z)}{dz} = -h_N \left[ \left( 1 - \frac{\sigma_{GSA}}{\sigma_{ESA}} \right) \ln \left( 1 - \exp \left( -\frac{\sigma_{GSA}E(z)}{hv} \right) \right) + \frac{\sigma_{ESA}E(z)}{hv} \right] - \alpha E(z)
\]

Using this equation it is possible to evaluate \( \sigma_{GSA}, \sigma_{ESA}, N_0 \) and \( \alpha \) at the same time by fitting the measured data of \( T \) and \( E \). This can be done by non-linear optimization procedure. Moreover some of the parameters can be inserted into this equation as a fixed value. A good example is \( z \) that can be measured by a spectrometer with high accuracy. The Avizonis–Grotback model seems to be the most useful of all the models which was confirmed by many articles [19,26,27].

3. Experimental

Three types of q-switching saturable absorbers Cr\(^{3+}\):YAG, V\(^{3+}\):YAG and Co\(^{2+}\):YAG were investigated. The parameters of the samples are presented in Table 1. Cr\(^{3+}\):YAG was examined at the wavelength 1064 nm while V\(^{3+}\):YAG and Co\(^{2+}\):YAG at the wavelength 1332 nm. The samples were polished but they were not coated. Thus to apply any of the above described models multiple reflection inside the samples have to be taken into account. Before measuring the saturation of the absorbers the small signal transmission spectra were measured. \( T_0 \) and \( T_{sa} \) were calculated from \( T_{sa} \) and \( T_{sa} \) which are the small signal transmission measured by the spectrometer at the investigated wavelength and small the signal transmission in the region close to but outside the investigated wavelength (2000 nm), respectively. Moreover for each of the samples refractive index was measured.

Investigations of saturation of the absorbers were carried out using the experimental setup shown in Fig. 1. As a sources of radiation two solid-state lasers were used generating at 1064 nm and 1332 nm. The lasers operated in active gain modulation mode and generated laser pulses of 4 ns length at half-height with repetition rate 10 Hz. The lasers beams were linearly polarized and characterized by Gaussian distribution. The waist diameters of...
the beams were equal to 1 mm. The parameters of the laser beams are shown in Table 2.

To avoid any uncertainty connected with the laser power fluctuation a plate with 50% transmission and 50% reflection was used to measure energy of the same pulse before and after the sample. During the investigation the saturable absorbers were placed at the beam waist. To change the energy density filters with different transmission were used. The laser beam diameter on the surface of the saturable absorber was measured by a laser beam analyzer. From the measured beam parameters a local energy density was calculated as a ratio of the transmitted energy to the incident energy. The transmitted as well as incident energy was measured during the analysis. The hyphen means that the parameter was not assumed but calculated during the approximation.

4. Results and discussion

Using the method presented in Section 3 saturation of the absorbers was measured. The saturable absorber Cr^{4+}:YAG was investigated at wavelength 1064 nm while the saturable absorbers V^{3+}:YAG and Co^{2+}:YAG were investigated at wavelength 1332 nm. Because of the laser beam polarization the samples were positioned so that the transmission was maximal by rotating the crystals around their axes. The transmission of the absorbers versus the incident energy density and the small signal transmission \( T_0 \) were calculated taking into consideration multiple reflections inside the samples. They are shown in Fig. 2.

These results were fitted using the models described in Section 2. For this purpose a special software was developed in MATLAB environment. The method of approximation used here was based on finding the minimum of function applying downhill simplex method [28]. The function considered here was the maximum singular value of the matrix defined as the difference between the measured transmission and the transmission being calculated during the approximation. The saturable absorbers Cr^{4+}:YAG and V^{3+}:YAG were analysed using F–N model because they are slow absorbers. The relaxation time of Cr^{4+}:YAG is 3.8 µs [5] and of V^{3+}:YAG is 22 ns [29] which is longer than the pulse duration of the incident laser beam 4 ns. It was assumed that these absorbers are characterized by three level quantum model [5]. The saturable absorber Co^{2+}:YAG was analysed using L-W model because it is a fast absorber with relaxation time 1 ns [30]. Here it was also assumed that this absorber is characterized by three level quantum model [9]. Using F–N and L-W models it was assumed that there is no excited state absorption (ESA) in the absorbers [19]. However many papers proves the existence of ESA [5,15] so the absorbers were also analysed by A–G model. The results of the analysis are summarized in Table 3. The symbol NA means that the parameter was not assumed but calculated during the analysis. The hyphen means that the parameter was not taken into consideration during the analysis as a result of the model that was used.

F–N and L-W models does not seem to be appropriate because during approximation they do not take into account dissipative losses which is not physical because they surely exist. Analysing the results achieved by A–G model one can notice that for analysis marked as 1b, 2b, 3b dissipative losses pursue zero while at the same time \( \sigma_{ESA} \) reaches very high value. These results does not seem to be good because \( \sigma \) cannot be equal to zero. On the other hand if we assume that there is no exited state absorption (analysis 1c, 2c and 3c) \( \sigma \) achieves much higher values than calculated on the basis of measurements curried out using a spectrometer (Eq. (5)). Assuming that \( \sigma_{ESA} \) and \( N_0 \) are equal to the values calculated by F–N or L-W model (analysis 1d, 1e, 2d, 2e, 3d, 3e) it is not good because the calculations take into account \( \sigma_{ESA} \) which in turn was calculated assuming \( \sigma \) equal zero (Eqs. (4) and (6)). Thus the analysis by A–G model with the assumption of \( \sigma \) calculated from the absorption spectra in a spectral region close to but outside the laser wavelength (Eq. (5)) and allowing for a possibility that there is excited state absorption seems to be the most appropriate way of approximating the measured data (analysis 1f, 2f, 3f). The results of this analysis were intentionally highlighted in Table 3. Calculating \( \sigma \) in this way is much more accurate then evaluating it by approximation. The approximations of the measured data of saturation for the highlighted model are presented in Fig. 3. To illustrate that this model best fits the experimental data approximations by F–N and L-W models are also presented. In case of Cr:YAG and V:YAG the difference between the models is very clear while in case of Co:YAG the visible difference appears for higher energy density.

Table 1

| Sample       | Thickness \( d \) (mm) | Refractive index \( n \) | \( T_{\text{max}} \) (%) at 1064 nm | \( T_{\text{max}} \) (%) at 1332 nm |
|--------------|------------------------|--------------------------|------------------------------------|------------------------------------|
| Cr^{4+}:YAG  | 1.55                   | 1.815                    | 57.0                               | 82.5                               |
| V^{3+}:YAG   | 2.99                   | 1.810                    | 56.8                               | 80.9                               |
| Co^{2+}:YAG  | 1.09                   | 1.810                    | 43.7                               | 79.7                               |

Table 2

| Wavelength (nm) | Repetition rate (Hz) | Pulse length (ns) | Pulse energy (mJ) | Waist diameter (mm) | Beam quality \( \text{M}^2 \) | Polarization |
|-----------------|----------------------|-------------------|-------------------|---------------------|-----------------------------|--------------|
| 1064            | 10                   | 4                 | 5.50              | 1                   | 1.25                        | Linear       |
| 1332            | 10                   | 4                 | 4.10              | 1                   | 1.25                        | Linear       |
Very small $\sigma_{ESA}$ obtained for Co$^{2+}$:YAG saturable absorber (almost 20 times smaller then $\sigma_{GSA}$) may suggest that there is indeed no exited state absorption in this absorber.

Moreover comparing the most important parameter $r_{ESA}$ with results obtained by other authors ($2.11 \times 10^{-18}$ for V$^{3+}$:YAG [5] and $2.2 \times 10^{-18}$ cm$^2$ for V$^{3+}$:YAG [5]) it seems that the suggested model is the most accurate. Also the other principal parameters are in accordance with the results presented in the mentioned papers ($\sigma_{GSA}$ was calculated to be $0.167 \times 10^{-18}$–$1.120 \times 10^{-18}$ cm$^2$ [5] and $0.337 \times 10^{-18}$–$0.872 \times 10^{-18}$ cm$^2$ [15] for Cr$^{3+}$:YAG and $0.200 \times 10^{-18}$–$0.550 \times 10^{-18}$ cm$^2$ [5] for V$^{3+}$:YAG).

As there is a difficulty to find any data concerning parameters of Co$^{2+}$:YAG saturable absorber at wavelength 1.33 µm the results obtained herein may serve laser designers as a reference.

Because the calculation of $z$ strongly depends on the accuracy of determining $T_{in}$ and $d$ the impact of the accurate determination of values of the input parameters on the calculated parameters of saturable absorbers was analysed. The input parameters were changed with a small step in a short range around the real value of a specific parameter and the analysis was repeated. The steps of the input parameters were chosen to be the accuracy of their evaluation. Thus for $T_{in}$ and $T_{om}$ the step was 0.2% which was the accuracy of measuring the transmission by the spectrometer used for these measurements, for $n$ the step was 0.005 which was the accuracy of determining this parameter and for $d$ the step was 0.01 mm which is the accuracy of measurement of most vernier calliper gauge. In Table 4 the accuracy of calculated parameters as a percentage change are presented. The symbol NC means that the input parameter was not changed but its value was taken from
The results of the investigations can be used as a reference of the principal parameters of q-switching saturable absorbers by laser designers as well as an indication of how accurate the input parameters used for analysis should be so as to evaluate the principal parameters with enough accuracy.

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Table 3. The symbol NI means that the calculated parameter was not influenced by the change of the input parameter. The signs + and – designate whether the value of the calculated parameters increases or decreases with the increase of the input parameter by assumed step. Thus when we take into account Cr4+:YAG saturable absorber analysed by F–N model when only n was changed we can see that the increase of n by 0.005 results in the decrease N0 and τ by 1.14% and 6.49% while at the same time σGSA increases by 0.75% and 2.63% respectively. In case of Co2+:YAG the exited state population increases or decreases with the increase of the input parameter by assumed step. Thus when we take into account Cr6+:Ga2+:YAG saturable absorber was analysed. Applying the method of approximation suggested herein relatively big attention should be paid to the accuracy of determining τ. If this condition is fulfilled the method seems to be good enough to calculate the principal parameters of saturable absorbers.

5. Conclusion

On the basis of different models used to fit the measured data of saturation comparative analysis of q-switching saturable absorbers was carried out. It was stated that the A–G model is the most accurate. It is also reasonable to calculate τ from the absorption spectra in a spectral region close to but outside the laser wavelength before doing the analysis. Moreover in case of Cr4+:YAG and V3+:YAG the assumption that there is excited state absorption should be applied to the model. In case of Co2+:YAG the exited state absorption does not seem to exist which is in accordance with the statements presented in paper [15]. Additionally the impact of the accurate determination of the input data such as the transmission, the thickness and the refractive index on the calculated principal parameters of q-switching saturable absorbers was analysed. Applying the method of approximation suggested herein relatively big attention should be paid to the accuracy of determining τ. If this condition is fulfilled the method seems to be good enough to calculate the principal parameters of q-switching saturable absorbers.

Moreover a sample of Co2+:YAG saturable absorber was investigated at 1.33 μm supplementing the world literature in this field.
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