Constraining Large Extra Dimensions Using Dilepton Data from the Tevatron Collider

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Abstract

We use the invariant mass distribution of Drell-Yan dileptons as measured by
the CDF and DØ Collaborations at the Fermilab Tevatron and make a careful
analysis to constrain Kaluza-Klein models with large extra dimensions. The
combined data from both collaborations lead to a conservative lower bound on
the string scale $M_S$ of about 1 TeV at 95% confidence level.

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Recently, the idea that gravity could become strong at scales of the order of a few TeV has attracted a great deal of attention\[1\]. This is made possible if we allow for large compactified dimensions at the TeV scale. While such ideas can be fitted in within the scheme of quantum field theories\[2\], a more natural construction\[3, 4\] involves string theories with all Standard Model (SM) fields living on a three-dimensional D-brane (or 3-brane) embedded in a space of $(4+d)$ dimensions (bulk). Of course, the original suggestion that we live in a spacetime continuum with more than the three canonical spatial dimensions was made early in this century \[5\], but these Kaluza-Klein (KK) theories, as they are called, have not been able to satisfactorily reproduce the observed mass spectrum. Such ideas, however, have always formed a basic ingredient of string theories\[6\]. In fact, models having extra dimensions with compactification scales of the order of a few TeV have been proposed\[7\] from time to time in the literature with various motivations. However, it is the discovery of D-branes\[8\] which has provided the rather venerable KK theories with a new lease of life over the past year.

In a nutshell, the ideas proposed by Arkani-Hamed, Dimopoulos and Dvali (ADD)\[2\] and by Antoniadis et al.\[3\] are as follows. They suggest — as all KK theories do — that spacetime consists of $(4+d)$ dimensions. The extra (spatial) $d$ dimensions are compactified, typically on a $d$-dimensional torus $T^d$ with radius $R$ each way. Since gravity experiments have not really probed the sub-millimetre regime, it is proposed that $R$ can be as large as $\sim 0.1 - 1$ mm, a very large value when compared with the Planck length $\approx 10^{-33}$ cm. Though the actual value of Newton’s constant $G_N^{(4+d)}$ in the bulk is of the same order as the electroweak coupling, its value $G_N^{(4)}$ in the effective 4-dimensional space at length scales $\gg R$ is the extremely small one measured in gravity experiments. This is described by a simple relation derived\[10\] from Gauss’ Law,

$$\left[ M_{Pl}^{(4)} \right]^2 \sim R^d \left[ M_{Pl}^{(4+d)} \right]^{(d+2)}$$

where $M_{Pl} \sim 1/\sqrt{G_N}$ denotes the Planck mass. If $M_{Pl}^{(4+d)} \sim 1$ TeV, then $R \sim 10^{30/d-19}$ m. This means that for $d = 1$, $R \sim 10^{11}$ m, which, in turn, means that deviations from Einstein gravity would occur at solar system scales; since these have not been seen, we are constrained to take $d \geq 2$. For these values $R < 1$ mm, hence there is no conflict with known facts. It is also perhaps worth mentioning
that we would normally require \( d < 7 \), since that is the largest number allowed if the string theory is derivable from M-theory, believed to be the fundamental theory of all interactions. In the ADD model the smallness of Newton’s constant is a direct consequence of the compactification-with-large-radius hypothesis and hence there is no hierarchy problem in this theory.  

In traditional KK theories, the mass-spectrum of nonzero KK modes arising from compactification of fields living in the bulk is driven to the Planck scale \( M_{pl}^{(4)} \). This problem is avoided in the ADD model by having the SM particles live on a ‘surface’ with negligible width in the extra \( d \) dimensions, which we identify with the 3-brane. The SM particles may then be thought of as excitations of open strings whose ends terminate on the brane; gravitons correspond to excitations of closed strings propagating in the bulk. Thus, the only interactions which go out of the 3-brane into the bulk are gravitational ones. We thus have a picture of a 4-dimensional ‘surface’ embedded in a \( (4 + d) \)-dimensional space, where SM fields live on the ‘surface’, but gravitons can be radiated-off into the bulk. Noting that the SM fields are confined to the 3-brane, it is obvious that the only new effects will be those due to exchange of gravitons between particles on the 3-brane. To construct an effective theory in 4 dimensions, gravity is quantized in the usual way, taking the weak-field limit, assuming that the underlying string theory takes care of ultraviolet problems. The interactions of gravitons now follow from the \( (4 + d) \)-dimensional Einstein equations in the compactification limit. Feynman rules for this effective theory have been worked out in detail in Refs. \[1\] and \[2\]. We use their prescriptions in our work. On the 3-brane, the couplings of the gravitons to the SM particles will be suppressed, as is well-known, by the Planck scale \( M_{pl}^{(4)} \simeq 1.2 \times 10^{19} \) GeV. This is offset, however, by the fact that, after compactification, the density of massive KK graviton states in the effective theory is very high, being, indeed proportional to \( M_{pl}^{(4)} / M_{pl}^{(4+d)} \). The Planck mass dependence cancels out, therefore, leaving a suppression by the string scale \( M_s \equiv M_{pl}^{(4+d)} \simeq M_{EW} \). In the ADD theory, therefore, the tower of KK graviton states leads to effective interactions of electroweak strength. A further assumption made in our work — and in other phenomenological studies — is that \( Y \)-particles,  

\footnote{A related problem, that of stabilization of the compactification scale, exists, however; this has been discussed in Ref. \[4\].}
excitation modes of the 3-brane itself in the bulk — are heavy and do not affect the processes under consideration. This corresponds to a static approximation for the brane. It is also relevant to mention that the dilaton field associated with the graviton couples only to the trace of the energy-momentum tensor, \textit{i.e.} to the mass of the SM particles at the vertex. For light fermions, as we have in the Drell-Yan process, this means that the interactions of the dilaton can be safely neglected.

Using these Feynman rules, it has been possible to explore a number of different processes where the new interactions could cause observable deviations from the SM. Only two new parameters enter the theory: one is the string scale \( M_S \equiv M_{(4+d)}^{\text{pl}} \). The other is a factor \( \lambda \), of order unity and indeterminate sign, which arises when we sum over all possible KK modes of the graviton. As the amplitudes for virtual graviton exchange (with which we are concerned in this work) are always proportional to \( \lambda/M_S^4 \), it is usual to absorb the magnitude of \( \lambda \) into \( M_S \); this reduces the uncertainty to \( \lambda = \pm 1 \). Obviously this determines whether the graviton exchanges interfere constructively or destructively with the SM interactions.

Remembering that the gravitons couple to any particle with a non-vanishing energy-momentum tensor, it is possible to make a variety of phenomenological studies of the new interactions and to test the workability of the ADD model. Though the phenomenology of this model has not yet been fully explored, several important results are already available in the literature. These can be classified into two types: those involving real KK graviton production, and those involving virtual graviton exchange. A real KK mode of the graviton will have interactions with matter suppressed by the Planck scale \( M_{\text{pl}}^{(4)} \) and will therefore escape the detector. One can, therefore, see signals with large missing momentum and energy if an observable particle is produced in association with a KK graviton mode. However, cross-sections for these depend explicitly on \( d \), the number of extra dimensions, and bounds derived from data reflect this dependence. Some of the processes examined so far include single-photon final states at \( e^+e^- \) colliders\([11, 13, 14]\) as well as hadron colliders\([11]\), monojet production at hadron colliders\([11, 13]\), two-photon processes at \( e^+e^- \) colliders\([13]\), single-Z production at \( e^+e^- \) colliders\([14]\) and the neutrino flux from the supernova SN1987A\([10, 16]\).
Each process can be used to obtain a bound on the string scale $M_S$ for a given number $d$. The most dramatic of these bounds is $M_S > 50$ TeV for $d = 2$ and it comes from a study\[16\] of neutrinos from the supernova SN1987A. However, this last bound drops to about a TeV as soon as we go to $d = 3$. Most of the other processes lead to lower bounds of about 1–1.1 TeV on the string scale for $d = 2$, but these bounds become much weaker for $d > 3$.

Virtual (KK) graviton exchanges lead to extra contributions to processes involving SM particles in the final state and can be observed as deviations in the cross-sections and distributions of these from the SM prediction. After summation over all the KK modes of the graviton, the final result is proportional to $\text{sgn}(\lambda)/M_S^4$, with \textit{practically no dependence on the number of extra dimensions}.\footnote{This is really because the density of graviton KK modes is approximated by a continuum, as a result of which mass degeneracies due to the number of extra dimensions are lost, at least to the leading order. In a sense, therefore, bounds from virtual graviton exchange are more general.}

Each process can be used to obtain a bound on the string scale $M_S$ for a given sign of $\lambda$. Some of the processes examined include Bhabha and Møller scattering at $e^+e^-$ colliders\[11, 17\], photon pair-production in $e^+e^-$ \[11, 22\] and hadron colliders\[11\], fermion pair production in $\gamma\gamma$ colliders\[17\], Drell-Yan production of dileptons\[18\], dijet\[19\] and top-quark\[20\] pair production at hadron colliders, deep inelastic scattering at HERA\[21, 17\], massive vector-boson pair production in $e^+e^-$ collisions \[22, 23\] and pair production of scalars (Higgs bosons and squarks) at both $e^+e^-$ and $\gamma\gamma$ colliders\[17\]. Among the best of these bounds is $M_S > 920$ (980) GeV for $\lambda = +1(−1)$ which comes from a study\[18\] of experimental data on Drell-Yan leptons at the Tevatron. We make a more elaborate analysis if the same data in this work.

The contributions to the Drell-Yan production of dileptons at hadron colliders from graviton exchanges have been considered by Hewett\[18\]. Some of her findings relevant to the Tevatron are:

- There is very little difference between the cases $\lambda = +1$ and $\lambda = −1$ for the dilepton invariant mass distribution.

- The $\lambda = ±1$ cases differ, however, in the angular distribution; therefore, widely differing forward-backward asymmetries may be predicted.
• There are large deviations between the SM and the ADD model for large invariant masses.

• The gluon-gluon contribution to the Drell-Yan process (see below) is much suppressed compared to the quark-initiated process.

• The bounds can increase to about 1.15 (1.35) TeV for \( \lambda = +1(-1) \) in Run-II of the Tevatron.

We agree with most of these results at the generator level. However, in the absence of published details about the angular distribution of dileptons observed by the CDF and DØ Collaborations, we confine our analysis to the invariant mass distributions only. Hence we do not make a separate analysis for the two signs of \( \lambda \).

![Feynman diagrams for the contribution to the Drell-Yan process from (a) the Standard Model and (b,c) exchange of a Kaluza-Klein graviton.](image)

**Figure 1.** *Feynman diagrams for the contribution to the Drell-Yan process from (a) the Standard Model and (b,c) exchange of a Kaluza-Klein graviton.*

The Drell-Yan cross-section, including the effects of Kaluza-Klein graviton exchanges, is given by the above Feynman diagrams. The Standard Model diagrams (a) involving exchange of a photon or a Z-boson in the s-channel, interfere with the diagram with s-channel exchange of a Kaluza-Klein graviton (b), while the diagram (c) has no Standard Model analogue.

Evaluating these leads to the result

\[
\sigma_{DY}(p\bar{p} \rightarrow \ell^+\ell^-) = \int dx_1 dx_2 \ f_{g/p}(x_1) \ f_{g/p}(x_2) \ \hat{\sigma}(gg \rightarrow \ell^+\ell^-) \\
+ \sum_{q=u,d,s} \int dx_1 dx_2 \ [f_{q/p}(x_1) \ f_{\bar{q}/\bar{p}}(x_2) + f_{\bar{q}/p}(x_1) \ f_{q/\bar{p}}(x_2)] \ \hat{\sigma}(q\bar{q} \rightarrow \ell^+\ell^-) \ , \tag{1}
\]
where \( f_{a/b}(x) \) denotes the flux of a parton \( a \) in a beam of particles \( b \),

\[
\hat{\sigma}(q\bar{q} \text{ or } gg \rightarrow \ell^+\ell^-) = \frac{1}{16\pi \hat{s}^2} |\mathcal{M}(q\bar{q} \text{ or } gg \rightarrow \ell^+\ell^-)|^2 ,
\]

and \( |\mathcal{M}|^2 \) represents the squared Feynman amplitude summed over final spins and averaged over initial spins and colours.

Evaluation of the Feynman diagrams gives, for the gluon-induced process (which has no Standard Model analogue):

\[
|\mathcal{M}(gg \rightarrow \ell^+\ell^-)|^2 = \left( \frac{\pi}{2M_s^4} \right)^2 \hat{s}^4 + 2\hat{t}\hat{u} (\hat{t} - \hat{u})^2 ,
\]

when all the graviton Kaluza-Klein modes have been summed over.

Evaluation of the Feynman diagrams gives for the quark-induced process (including interference terms):

\[
|\mathcal{M}(q\bar{q} \rightarrow \ell^+\ell^-)|^2 = T_{\text{SM}}^{qq} + T_{\text{KK}}^{qq} ,
\]

where the Standard Model contribution is given below. We adopt the convention that \( T_a \) denotes the contribution from exchange of a particle \( a \) and \( T_{ab} \) denotes the interference term between diagrams with exchange of \( a \) and \( b \) respectively.

With these, we get

\[
T_{\text{SM}}^{qq} = T_\gamma^{qq} + T_Z^{qq} + T_{Z\gamma}^{qq} ;
\]

\[
T_\gamma^{qq} = \frac{32}{3} (\pi\alpha Q_q)^2 \left[ \frac{\hat{t}^2 + \hat{u}^2}{\hat{s}^2} \right] ;
\]

\[
T_Z^{qq} = \frac{1}{3} \left( \frac{\pi\alpha}{4\sin^2\theta_W \cos^2\theta_W} \right)^2 |D_Z(\hat{s})|^2 \left[ (L_\ell^2 L_q^2 + R_\ell^2 R_q^2) \hat{t}^2 + (L_\ell^2 R_q^2 + R_\ell^2 L_q^2) \hat{u}^2 \right] ,
\]

\[
T_{Z\gamma}^{qq} = -\frac{2}{3} Q_q \left( \frac{\pi\alpha}{\sin\theta_W \cos\theta_W} \right)^2 |D_Z(\hat{s})|^2 \left( 1 - \frac{M_Z^2}{\hat{s}} \right) \times \left[ (L_\ell L_q + R_\ell R_q) \hat{t}^2 + (L_\ell R_q + R_\ell L_q) \hat{u}^2 \right] ,
\]

defining

\[
D_Z(\hat{s}) = [\hat{s} - M_Z^2 + iM_Z\Gamma_Z]^{-1}
\]

and

\[
L_\ell = 4\sin^2\theta_W - 2 , \quad R_\ell = 4\sin^2\theta_W ,
\]

\[
L_q = 4(T_{3q} - Q_q \sin^2\theta_W) , \quad R_q = -4Q_q \sin^2\theta_W ,
\]

\[
\frac{1}{16\pi \hat{s}^2} |\mathcal{M}(q\bar{q} \text{ or } gg \rightarrow \ell^+\ell^-)|^2 ,
\]

and \( |\mathcal{M}|^2 \) represents the squared Feynman amplitude summed over final spins and averaged over initial spins and colours.

Evaluation of the Feynman diagrams gives, for the gluon-induced process (which has no Standard Model analogue):

\[
|\mathcal{M}(gg \rightarrow \ell^+\ell^-)|^2 = \left( \frac{\pi}{2M_s^4} \right)^2 \hat{s}^4 + 2\hat{t}\hat{u} (\hat{t} - \hat{u})^2 ,
\]

when all the graviton Kaluza-Klein modes have been summed over.

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\]

\[
T_\gamma^{qq} = \frac{32}{3} (\pi\alpha Q_q)^2 \left[ \frac{\hat{t}^2 + \hat{u}^2}{\hat{s}^2} \right] ;
\]

\[
T_Z^{qq} = \frac{1}{3} \left( \frac{\pi\alpha}{4\sin^2\theta_W \cos^2\theta_W} \right)^2 |D_Z(\hat{s})|^2 \left[ (L_\ell^2 L_q^2 + R_\ell^2 R_q^2) \hat{t}^2 + (L_\ell^2 R_q^2 + R_\ell^2 L_q^2) \hat{u}^2 \right] ,
\]

\[
T_{Z\gamma}^{qq} = -\frac{2}{3} Q_q \left( \frac{\pi\alpha}{\sin\theta_W \cos\theta_W} \right)^2 |D_Z(\hat{s})|^2 \left( 1 - \frac{M_Z^2}{\hat{s}} \right) \times \left[ (L_\ell L_q + R_\ell R_q) \hat{t}^2 + (L_\ell R_q + R_\ell L_q) \hat{u}^2 \right] ,
\]

defining

\[
D_Z(\hat{s}) = [\hat{s} - M_Z^2 + iM_Z\Gamma_Z]^{-1}
\]

and

\[
L_\ell = 4\sin^2\theta_W - 2 , \quad R_\ell = 4\sin^2\theta_W ,
\]

\[
L_q = 4(T_{3q} - Q_q \sin^2\theta_W) , \quad R_q = -4Q_q \sin^2\theta_W ,
\]
for the couplings. The non-Standard part, using the same convention, is given by

\[
T_{q\bar{q}}^{KK} = T_{q\bar{q}}^{G} + T_{q\bar{q}}^{G\gamma} + T_{q\bar{q}}^{GZ},
\]

(7)

\[
T_{q\bar{q}}^{G} = \frac{\lambda^2}{3} \left( \frac{\pi}{2M_S^4} \right)^2 \left[ \hat{s}^4 - 4\hat{s}^2 (\hat{t} - \hat{u})^2 + (\hat{t} - \hat{u})^2 (5\hat{t}^2 - 6\hat{t}\hat{u} + 5\hat{u}^2) \right],
\]

\[
T_{q\bar{q}}^{G\gamma} = -\frac{4}{3} \lambda Q_q \left( \frac{\pi}{M_S^4} \right) \left[ \hat{s}^2 - 2(\hat{t} + \hat{u}) - (\hat{t} - \hat{u})^2 \right],
\]

\[
T_{q\bar{q}}^{GZ} = -\lambda \left( \frac{\alpha}{3} \right) \left( \frac{\pi}{2\sin \theta_W \cos \theta_W M_Z^2} \right)^2 \left( \hat{s} - M_Z^2 \right) |D_Z(\hat{s})|^2 \times \left[ (L_{\ell L} L_q + R_{\ell R} R_q) \hat{t}^2 (\hat{t} - 3\hat{u}) - (L_{\ell R} R_q + R_{\ell L} L_q) \hat{u}^2 (\hat{u} - 3\hat{t}) \right],
\]

when, as before, all the Kaluza-Klein modes have been summed over.

The above formulae represent the lowest order (LO) calculation in perturbation theory. The calculation of higher-order effects, especially next-to-leading order (NLO) and next-to-NLO (NNLO) QCD corrections has been done in detail[26] for the SM process represented by \( T_{q\bar{q}}^{SM} \). No corresponding calculations have been attempted as yet for the KK parts, \( T_{q\bar{q}}^{KK} \) and \( M(gg \rightarrow \ell^+ \ell^-) \). In the absence of such a calculation, we make the assumption that the change in the LO cross-section due to QCD corrections — the ‘K-factor’ — is identical for the SM and KK parts. Our results are, therefore, correct only within this approximation[7]. However, we do not expect a proper calculation of NLO effects to make a drastic change in our rough-and-ready results, because the dominant contribution to dilepton production at the Tevatron comes from quark-induced processes. Since the SM and KK results both arise from colour-singlet exchange, the actual ‘K-factor’ is likely to be rather similar in both cases. For gluons, this is not true, but the gluon-induced process makes only a minor contribution at Tevatron energies.

In keeping with this philosophy, therefore, we have extracted, for each value of the dilepton invariant mass \( M \equiv M_{\ell^+ \ell^-} \), a ‘K-factor’ by taking the ratio of the LO SM cross-section calculated using the above formulae with that calculated using the full NNLO calculation of Ref. [26]. This set of ratios is then used to scale the entire differential cross-section when the KK effects are included. It

\[\text{footnote}{\text{This places our results on an equal footing with a large number of experimental bounds on new physics scenarios, such as those involving quark and lepton compositeness[27], for which the QCD corrections are not available.}}\]
is worth pointing out that this procedure also takes care of the leading effects arising from initial-state radiation. Finally, it is relevant to mention that we have used the CTEQ-4M set of structure functions\cite{25} to calculate the initial state parton luminosities.

We now describe our analysis in some detail. The DØ Collaboration has presented\cite{27} the $e^+e^-$ invariant mass distribution in 9 bins starting from 120 GeV till 1 TeV using the di-electron data collected with 120 pb$^{-1}$ of luminosity. The cuts relevant for the cross-section calculation are given below. No distinction is made between the electron and the positron.

- The transverse momentum of both the isolated electrons must satisfy $p_T > 25$ GeV.
- The electrons are called CC (for Central Calorimeter) if they satisfy $|\eta| < 1.1$, $\eta$ being the pseudorapidity; they are called EC (for End Cap) if they satisfy $1.5 < |\eta| < 2.5$.

Only those events are considered in which there is at least one CC electron, while the other can be CC or EC. The acceptances described above are taken into account while estimating our Monte Carlo cross-sections. These cross-sections need to be further convoluted with efficiencies\cite{27} which are $(74.1 \pm 0.6)$% when both electrons are CC and $(52.6 \pm 1.0)$% when one of them is EC. Multiplying by the luminosity now gives us a prediction for the number of di-electron events expected in each mass bin, which is then compared with the DØ data.

The CDF Collaboration has presented\cite{28} results for dimuon samples, using 107 pb$^{-1}$ of data. The relevant cuts are given below.

- The reconstructed rapidity $y$ of the virtual $s$-channel state (‘boson rapidity’) is required to satisfy $|y| < 1$ for all events.
- Both muons are required to satisfy $|\eta| < 1$, which confines the analysis to the central region.
- A back-to-back cut $|\eta_1 + \eta_2| \geq 0.2$ is imposed: this gets rid of cosmic ray backgrounds.
Both muons are required to satisfy a ‘loose’ transverse momentum cut of \( p_T > 17 \text{ GeV} \) and at least one is required to satisfy a ‘tight’ cut of \( p_T > 20 \text{ GeV} \).

These cuts are applied in our Monte Carlo generator to estimate the cross-section times acceptance for the 6 mass bins in the range 120 GeV to 500 GeV presented in Ref. [28] (Table X). These are convoluted with the experimental efficiencies (Table VI of Ref. [28]). We then obtain an additional correction factor for each mass bin by normalising our SM expectation to the numbers given in Ref. [28]. This may be expected to take care of the effect of other detector-specific cuts like triggers, etc. Finally, we use this correction factor along with our generator-level acceptance and the experimental efficiencies to estimate the number of events in each mass bin for various values of \( M_S \). The choice of only 6 mass bins in the range 120 GeV to 500 GeV is because the ADD model predicts wider deviations from the SM in the higher mass bins (see Fig. 2). We also take note of the fact that no events are seen at CDF in the mass bin 500 GeV to 1 TeV.

![Graph](image.png)

**Figure 2.** Illustrating the effects of TeV scale quantum gravity on the invariant mass distributions of dileptons seen at the Tevatron by the DØ and CDF Collaborations respectively. Solid lines show the SM prediction; dashed lines show the predictions of the ADD model for marked values of \( M_S \).

In Fig. 2 we show the differential cross-section as a function of the invariant mass \( M \) of the dilepton, compared to the DØ and CDF data. We have set \( \lambda = +1 \),
but $\lambda = -1$ will not make a discernible change in the figure. Solid lines show the SM prediction; dashed lines show the predictions of the ADD model for $M_S = 0.5, 0.75, 1, 1.25$ and $1.5$ TeV respectively. The data points correspond to those used in our analysis and do not represent the full set of available points. Error bars are presented at 68% confidence level (C.L.) if there are events in the relevant mass bin and a 95% C.L. upper bound if there are no events in the relevant mass bin. The DØ numbers correspond to a differential cross-section $d\sigma/dM$: this is obtained by dividing the cross-section (modulo cuts) in the mass bin by the width of that bin. The CDF numbers correspond to a double differential cross-section $d^2\sigma/dMdy$ in both $M$ and $y$: this is obtained by dividing the cross-section (modulo cuts) in the mass bin by the width of that bin as well as by a factor $\Delta y = 2$.

As is apparent from the figure, the string scale cannot be anywhere near 500 GeV, since that would show extreme deviations from the observed data. This is just one of the arguments which tells us that quantum gravity effects must lie at scales of a TeV or more. On the other hand, as $M_S$ approaches 1 TeV, the differentiation between signal and background is less striking. This is partly because the deviations arise only in the high mass bins, where no events are expected with the current luminosities.

The actual limits on the string scale $M_S$ are calculated using a Bayesian analysis of the shape of the mass distribution of events. For a value $M_S$ of the string scale, the expected number of events in the $k$th mass bin can be written as:

$$N^k(M_S) = b_k + \mathcal{L} \epsilon_k \sigma^k(M_S)$$  \hspace{1cm} (8)

where $\mathcal{L}$ is the data luminosity, $b_k$ is the expected background, $\epsilon_k$ is the dilepton detection efficiency and $\sigma^k(M_S)$ is the expected dielectron cross section with inclusion of the effect due to large extra dimension.

The posterior probability density for the string scale to be $M_S$, given the observed data distribution ($D$), is given by

$$P(M_S|D) = \frac{1}{A} \int db \, d\mathcal{L} \prod_{k=1}^n \left[ \frac{e^{-N^k(M_S)} N^k(M_S)^{N_k}}{N_0^k!} \right] P(b, \mathcal{L}) P(M_S).$$  \hspace{1cm} (9)
In the above equation the term in square brackets is the likelihood for the data distribution to be from a model with string scale $M_S$. The prior probability $P(b, L \epsilon)$ is taken to be a product of independent Gaussian distributions in $b$, $L$ and $\epsilon$, with the measured value in each bin defining the mean and the uncertainty defining the width. The overall factor $1/A$ is just a normalisation. Since the excess cross-sections due to graviton exchanges are combinations of direct terms proportional to $1/M_S^8$ and interference terms proportional to $1/M_S^4$, we consider a prior distribution $P(M_S)$ uniform in (a) $1/M_S^4$ and (b) $1/M_S^8$ separately. The limit on the string scale from a prior uniform in $1/M_S^8$ represents a conservative estimate; using a prior uniform in $1/M_S^4$ provides more stringent limits. From the above posterior probability, the cumulative probability $= \int_{M_S}^{\infty} P(M_S|D) dM_S$ can be calculated. The $M_S$ value at which the cumulative probability equals 0.95 is, then, the 95% C.L. limit. We also combine the data using the simple expedient of treating the CDF probability as a prior for the D0 analysis (and vice versa).

![Figure 3](image.png)

**Figure 3.** Showing the (cumulative) posterior probability for the ADD model with different values of the string scale $M_S$ assuming a prior probability which is (a) uniform in $1/M_S^4$ and (b) uniform in $1/M_S^8$.

In Fig. 3, we have plotted the cumulative posterior probability for the ADD model with string scale $M_S$ as a function of (a) $1/M_S^4$ and (b) $1/M_S^8$. The dashed (dash-dot) lines indicate the results of considering the DØ (CDF) data alone, while the solid lines show the result of a combined fit. A glance at the figure will
show that the horizontal (dotted) lines correspond to 95% C.L. limits, while the fact that the curve saturates for higher values of $1/M_s^{(4,8)}$ shows that the SM values ($M_s \to \infty$) constitute the best hypothesis to fit the data. In a more quantitative idiom, we may interpret the vertical (dotted) lines as lower bounds on the string scale $M_s$. It is clear that the bound of 927 GeV assuming a prior probability of $1/M_s^4$, and using the CDF data, is consistent with that reported in Ref. [18], while the value 874 GeV assuming a prior probability of $1/M_s^8$ represents a more conservative estimate. The DØ data provide an improvement\(^8\) in the bound by about 100 GeV in both cases. Since the cross-section varies principally as $1/M_s^8$ in the region around $M_s = 1$ TeV (this is reflected in the fact that it depends very weakly on the sign of $\lambda$), this corresponds to an increase in the sensitivity by a factor of about 2.6. Combining the data increases the sensitivity by another factor of about 1.2, which takes the bound to 1080 (1016) GeV, depending on the choice of prior probability. Increasing the energy to 2 TeV and the luminosity to 2 fb\(^{-1}\), which may be expected with the commissioning of the Main Injector in Run-II, improves the bounds by a further 200–300 GeV; this corresponds to an improvement in the sensitivity by a factor close to 4.

To conclude then, we have used published dilepton data from the DØ and CDF Collaborations to put bounds on the string scale $M_s$. This is the fundamental scale of the ADD model, which envisages large compact dimensions in addition to the known (noncompact) ones and predicts strong quantum gravity effects at TeV scales. Only the invariant mass distribution has been used and not the angular distribution. The latter might show some sensitivity to the sign of $\lambda$. For the current analysis, however, there is hardly any such sensitivity. Our result is also independent of the number of extra dimensions $d$. We obtain a bound on $M_s$ of 900 GeV (900 GeV – 1 TeV) using CDF (DØ) data alone and a bound of around 1.0 – 1.1 TeV using the combined data from both experiments. This is one of the most stringent bound obtained from collider studies at the present time and is likely to be improved (to about 1.3 TeV) in Run-II of the Tevatron.

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\(^8\)This is for two reasons: (a) the published results from DØ use a slightly higher integrated luminosity and (b) the DØ Collaboration presents more data in the higher mass bins — where most of the deviations lie — than the CDF Collaboration.
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