Dark Matter candidate in a Heavy Higgs Model - Direct Detection Rates

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Abstract

We investigate direct detection rates for Dark Matter candidates arise in a $SU(2)_L \times U(1)_Y$ with an additional doublet Higgs proposed by Barbieri, Hall and Rychkov. We refer this model as ‘Heavy Higgs Model’. The Standard Model Higgs mass comes out in this model very heavy adopting the few per cent chance that there is no Higgs boson mass below 200 GeV. The additional Higgs boson develops neither any VEV due to the choice of coefficient of the scalar potential of the model nor it has any coupling with fermions due to the incorporation of a discrete parity symmetry. Thus, the neutral components of the extra doublet are stable and can be considered as probable candidate of Cold Dark Matter. We have made calculations for three different types of Dark Matter experiments, namely, $^{76}$Ge (like GENIUS), DAMA (NaI) and XENON ($^{131}$Xe). Also demonstrated the annual variation of Dark Matter detection in case of all three detectors considered.

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A 'Heavy Higgs Model' has been proposed [1] within the framework of $SU(2)_L \times U(1)_Y$ symmetry through the inclusion of another doublet and a discrete parity symmetry. This model is motivated to give an alternate way to control the quadratic divergence in the Higgs sector and to solve the naturalness problem. Again, results of electroweak precision tests are in favour of light Standard Model (SM) Higgs boson, $m_h < 186$ GeV at 95\% c.l [2] with a central value too low from the lower bound (114 GeV) obtained from direct search experiments whereas, the present model is based on adopting the few per cent chance that there is no Higgs boson mass below 200 GeV. The basic feature of the model is as follows.

The additional doublet neither develops VEV nor it has any coupling with matter, however, the doublet representation necessarily admits weak interaction and also scalar self interaction. Due to absence of any coupling with matter of the extra doublet its neutral components are stable, and, therefore, those Lightest Stable Higgs (LSH) may be considered as probable candidate of cold dark matter. The zero VEV of the extra doublet is assured through the choice of coefficients of the scalar potential of the model and the discrete parity symmetry proposed in the model prohibits the extra doublet to couple with fermions.

In the present work, we investigate Direct detection rates of those Dark Matter particles in view of ongoing and future Dark Matter experiments.

Let us first discuss relevant parts of the model. The model contains two doublet scalars $H_1$ and $H_2$. The $H_1$ scalar is the usual standard model doublet and the additional $H_2$ scalar is fermiophobic. The discrete parity symmetry imposed to achieve decoupling of $H_2$ field is given by

$$H_2 \rightarrow -H_2$$

while keeping all other fields invariant. The VEV of the Higgs fields are

$$H_1 = (0, v), \quad H_2 = (0, 0)$$

and expanding around their minima we get

$$H_1 = \left(v + (h + i\chi)\right), \quad H_2 = \left(H^+ / (S + iA)/\sqrt{2}\right)$$

The scalar potential of the model is given by

$$V = \mu_1^2|H_1|^2 + \mu_2^2|H_2|^2 + \lambda_1|H_1|^4 + \lambda_2|H_2|^4 + \lambda_3|H_1|^2|H_2|^2 + \lambda_4|H_1^*H_2|^2 + \lambda_5\frac{1}{2} \left[(H_1^*H_2)^2 + h.c\right].$$
The mass of the usual Higgs boson $h$ in the present model is set around $m_h \sim 400 - 600$ GeV and the three additional scalars as

$$
m^2_I = \mu^2_I + \lambda_I v^2, I = H, S, A
\lambda_H = \lambda_3
\lambda_S = \lambda_3 + \lambda_4 + \lambda_5
\lambda_A = \lambda_3 + \lambda_4 - \lambda_5
$$

and from the minimization of the scalar potential one obtains conditions for which the potential is stable, as

$$
\lambda_{1,2} > 0 \text{ and } \lambda_3, \lambda_L > -2(\lambda_1 \lambda_2)^{1/2}
$$

where

$$
\lambda_L = \lambda_3 + \lambda_4 - |\lambda_5|
$$

The Higgs fields $S, A$ are stable as long as the parity symmetry (1) is unbroken and those stable neutral scalars appear to be a candidate for Cold Dark Matter in the universe. In order to see the prospect of direct detection of such a Dark Matter candidate, we consider two processes of scalar-nucleon interaction i) via $Z$ boson exchange and ii) via Higgs scalar $h$ exchange.

The amplitude comes out due to $Z$ boson exchange is too large (about 8-9 orders) compared to the existing CDMS collaboration experimental limits [3] on spin-independent wimp-nucleon interactions from the two-tower run of the cryogenic dark matter search. Hence, to neglect this contribution, a mass splitting between the two scalars $S$ and $A$ are considered which is greater than the kinetic energy of the dark matter in our galactic halo, so that the process due to $Z$ exchange is kinematically forbidden. We consider the other process due to $h$ exchange at the tree level for our analysis in the present work.

In Ref. [1], the possible range of mass, $m_L$ for the LSH, to account for the Dark Matter content of the universe, is discussed. For $m_L > m_W$ (mass of $W$ boson), the Dark Matter density $\Omega_{DM}$ falls much below the acceptable value due to dark matter annihilation to $W$ bosons ($LL \rightarrow WW$, where $L$ is generic representation of $S$ and $A$ fields). Therefore, massive LSHs can contribute to only a fraction of Dark Matter content. While LSH mass $m_L \lesssim m_W$, the annihilation to $W$ bosons are kinematically disfavoured and in particular $m_L \sim 60 - 70$ GeV can account for the Dark Matter content of the universe with $\Delta m$, the mass difference between the LSH and NLSH (next lightest stable Higgs), $\sim 8$ GeV. Therefore, we consider three LSH masses, in the range $60 - 70$ GeV in the present work. We
also consider, as an example, a value of $m_L$ in the $m_L \gg m_W$ regime, which as mentioned before, can only account for just a fraction of the total Dark Matter.

The scalar-nucleon cross-section due to the later case comes out as

$$\sigma(LN \rightarrow LN) = \frac{m_r^2}{4\pi} \left( \frac{\lambda_L}{m_L m_h^2} \right)^2 f_N^2 m_N^2 \quad (8)$$

where $m_r$ is the reduced nucleon mass, $\lambda_L$ is the combination of scalar couplings defined in eqn.(4), $m_h$ is the mass of the Standard Model like Higgs scalar and $f_N$ is the usual nuclear matrix element given by

$$< N|\Sigma m_q q\bar{q}|N > = f_N m_N < N|N > \quad (9)$$

and for the present analysis we set $f_N \sim 0.3$.

In the present work, we calculate the possible direct detection rates for such Lightest Stable Particles (or LSH) Dark Matter candidates discussed above, in the experiments like GENIUS (target material $^{76}$Ge) [4, 5], DAMA (target material NaI) [6, 7, 8] and XENON (target material $^{131}$Xe) [9, 10].

The direct detection of Dark Matter with a terrestrial detector uses the elastic scattering of Dark Matter candidate off the detector nuclei. As this cross-section is very small, the energy deposited by a Dark Matter candidate of mass in the range 1 GeV to 1 TeV on a detector nucleus is not generally more than 100 keV. Hence to perform this difficult task of Dark Matter detection a very low threshold detector condition is required.

Differential detection rate of Dark Matter per unit detector mass can be written as

$$\frac{dR}{d|q|^2} = N_T \Phi \frac{d\sigma}{d|q|^2} \int f(v)dv \quad (10)$$

where $N_T$ denotes the number of target nuclei per unit mass of the detector, $\Phi$ - the Dark Matter flux, $v$ - the Dark Matter velocity in the reference frame of earth with $f(v)$ - its distribution. The integration is over all possible kinematic configurations in the scattering process. In the above, $|q|$ is the momentum transferred to the nucleus in Dark Matter-nucleus scattering. Nuclear recoil energy $E_R$ is

$$E_R = \frac{|q|^2}{2m_{nuc}} = m_r^2 v^2 (1 - \cos \theta)/m_{nuc} \quad (11)$$

$$m_{red} = \frac{m_L m_{nuc}}{m_L + m_{nuc}} \quad (12)$$
where $\theta$ is the scattering angle in Dark Matter-nucleus centre of momentum frame, $m_{\text{nuc}}$ is the nuclear mass and $m_L$ is the mass of the Dark Matter.

Now expressing $\Phi$ in terms of local Dark Matter density $\rho_\chi$, velocity $v$ and mass $m_L$ and writing $|q|^2$ in terms of nuclear recoil energy $E_R$ with noting that $N_T = 1/m_{\text{nuc}}$, Eq. (10) takes the form

$$\frac{dR}{dE_R} = \frac{2\rho_\chi}{m_L} \frac{d\sigma}{d|q|^2} \int_{v_{\text{min}}}^{\infty} v f(v) dv;$$

$$v_{\text{min}} = \left[ \frac{m_{\text{nuc}} E_R}{2 m_{\text{red}}^2} \right]^{1/2} \quad (13)$$

Following Ref. [11] the Dark Matter-nucleus differential cross-section for the scalar interaction can be written as

$$\frac{d\sigma}{d|q|^2} = \frac{\sigma_{\text{scalar}}}{4 m_{\text{red}}^2 v^2} F^2(E_R). \quad (14)$$

In the above $\sigma_{\text{scalar}}$ is Dark Matter-nucleus scalar cross-section and $F(E_R)$ is nuclear form factor given by [12, 13]

$$F(E_R) = \left[ \frac{3j_1(qR_1)}{qR_1} \right] \exp \left( \frac{q^2 s^2}{2} \right) \quad (15)$$

$$R_1 = (r^2 - 5s^2)^{1/2}$$

$$r = 1.2 A^{1/3}$$

where thickness parameter of the nuclear surface is given by $s \simeq 1$ fm, $A$ is the mass number of the nucleus and $j_1(qR_1)$ is the spherical Bessel function of index 1.

The distribution $f(v_{\text{gal}})$ of Dark Matter velocity $v_{\text{gal}}$ with respect to Galactic rest frame, is considered to be of Maxwellian form. The velocity $v$ (and $f(v)$) with respect to earth rest frame can then be obtained by making the transformation

$$v = v_{\text{gal}} - v_{\oplus} \quad (16)$$

where $v_{\oplus}$ is the velocity of earth with respect to Galactic rest frame and is given by

$$v_{\oplus} = v_\odot + v_{\text{orb}} \cos \gamma \cos \left( \frac{2\pi(t - t_0)}{T} \right) \quad (17)$$

In Eq. (17), $T = 1$ year, the time period of earth motion around the sun, $t_0 = 2^{nd}$ June, $v_{\text{orb}}$ is earth orbital speed and $\gamma \simeq 60^o$ is the angle subtended by earth orbital plane at Galactic
plane. The speed of solar system $v_\odot$ in the Galactic rest frame is given by,

$$v_\odot = v_0 + v_{pec}$$  \hspace{1cm} (18)

where $v_0$ is the circular velocity of the Local System at the position of Solar System and $v_{pec}$ is speed of Solar System with respect to the Local System. The latter is also called peculiar velocity and its value is 12 km/sec. Although the physical range of $v_0$ is given by [14, 15] $170 \text{ km/sec} \leq v_0 \leq 270 \text{ km/sec}$ (90 % C.L.), in the present work we consider the central value of $v_0 = 220 \text{ km/sec}$. Eq. (17) gives rise to annual modulation of Dark Matter signal reported by DAMA/NaI experiment [6, 7, 8].

Defining a dimensionless quantity $T(E_R)$ as,

$$T(E_R) = \frac{\sqrt{\pi}}{2} v_0 \int_{v_{min}}^{\infty} \frac{f(v)}{v} dv$$  \hspace{1cm} (19)

and noting that $T(E_R)$ can be expressed as [11]

$$T(E_R) = \frac{\sqrt{\pi}}{4v_{\odot}} v_0 \left[ \text{erf} \left( \frac{v_{\min} + v_{\odot}}{v_0} \right) - \text{erf} \left( \frac{v_{\min} - v_{\odot}}{v_0} \right) \right]$$  \hspace{1cm} (20)

we obtain from Eqs. (13) and (14)

$$\frac{dR}{dE_R} = \frac{\sigma_{\text{scalar}} \rho_{\chi}}{4v_{\odot} m_L m_{\text{red}}^2} F^2(E_R) \left[ \text{erf} \left( \frac{v_{\min} + v_{\odot}}{v_0} \right) - \text{erf} \left( \frac{v_{\min} - v_{\odot}}{v_0} \right) \right].$$  \hspace{1cm} (21)

The total local Dark Matter density $\rho_{\chi}$ is taken to be 0.3 GeV/cm$^3$. The above expression for differential rate is for a monoatomic detector like Ge but it can be easily extended for a diatomic detector like NaI as well.

The measured response of the detector by the scattering of Dark Matter off detector nucleus is in fact a fraction of the actual recoil energy. Thus, the actual recoil energy $E_R$ is quenched by a factor $q_{n_X}$ (different for different nucleus $X$) and we should express differential rate in Eq. (21) in terms of $E = q_{n_X} E_R$. For $^{76}\text{Ge}$, $q_{n_{\text{Ge}}} = 0.25$ [16], for $^{23}\text{Na}$, $q_{n_{\text{Na}}} = 0.3$ [17], for $^{127}\text{I}$, $q_{n_{\text{I}}} = 0.09$ [17] and for $^{131}\text{Xe}$, $q_{n_{\text{Xe}}} = 0.8$ [16].

Thus the differential rate in terms of the observed recoil energy $E$ for a monoatomic detector like Ge detector can be expressed as

$$\frac{\Delta R}{\Delta E}(E) = \int_{E/q_{n_{\text{Ge}}}}^{(E+\Delta E)/q_{n_{\text{Ge}}}} dR_{\text{Ge}}(E_R) \frac{dE_R}{\Delta E}$$  \hspace{1cm} (22)
and for a diatomic detector like NaI, the above expression takes the form

$$\frac{\Delta R}{\Delta E}(E) = a_{Na} \int_{E/q_{Na}}^{(E+\Delta E)/q_{Na}} \frac{dR_{Na}(E_R)}{dE_R} \frac{dE_R}{\Delta E} + a_{I} \int_{E/q_{I}}^{(E+\Delta E)/q_{I}} \frac{dR_{I}(E_R)}{dE_R} \frac{dE_R}{\Delta E}$$

(23)

where $a_{Na}$ and $a_{I}$ are the mass fractions of Na and I respectively in a NaI detector and are given by (see Table 2)

$$a_{Na} = \frac{m_{Na}}{m_{Na} + m_{I}} = 0.153 \quad a_{I} = \frac{m_{I}}{m_{Na} + m_{I}} = 0.847$$

The differential detection rates $\Delta R/\Delta E$ (/kg/day/keV) in case of LSH Dark Matter for different values of observed recoil energies are calculated using Eqs (10 - 22) for monoatomic detectors like Ge and Xe and using Eqs (10 - 23) for diatomic detector like NaI with $\Delta E = 1$ keV. For calculation of the differential rates, we put $t = t_0$ in Eq. (17) for all three types of detectors considered here.

In Fig. 1 we plot LSH Dark Matter detection rate for 76 Ge detector such as in GENIUS experiment at Gran Sasso. For demonstrative purpose we calculate the rates for three different LSH masses in $m_L < m_W$ regime, namely $m_L = 60$ GeV, 65 GeV and 70 GeV. For all the calculations the value of the coupling $|\lambda|$ is fixed at 0.5. It appears from Fig. 1 that the differential rate decreases with the increase of LSH mass. This can be understood from Eq. (21) where the LSH mass $m_L$ appears at the denominator. We wish to make a comment here that while trying to plot this variation in logscale, it reveals that although the rates are more for lower mass of LSH Dark Matter and decreases with the increase of mass (as is evident form Fig. 1), the situation becomes reversed for high recoil energies as the rates show an oscillatory behaviour which becomes prominent at higher recoil energies. This phenomenon is due to the oscillatory nature of the Bessel function used in Eqs. (14) and (15). But at those high recoil energies where the oscillatory nature becomes prominent, the yields are virtually nil. Hence we do not plot them in linear scale instead.

The variation of rates with detector recoil energies for diatomic NaI detector (used in DAMA experiment), for three different LSH masses mentioned above, is shown in Fig. 2. The coupling $|\lambda|$ is fixed at the same value of 0.5.

Similar results for Xenon (131 Xe) detector are plotted in Fig. 3.

One very positive signature of Dark Matter detection by direct detection method is the periodic annual variation of the detected Dark Matter. This periodicity arises due
to the periodic motion of the earth around the sun by which the directionality of earth’s motion changes continually throughout the year. Due to this, the amount of Dark Matter encountered by the earth varies annually which can be understood from Eq. (17). This is the annual modulation of the detected Dark Matter and detecting this annual variation serves as a confirmatory test for Dark Matter detection. In order to see the possible annual variation of the detected LSH Dark Matter in three types of detectors discussed here, we have calculated the total events per day for a whole year for each of these three types of detectors. Thus the value of $t$ in Eq. (17) varies from 1 to 365 while $t_0 = 153$ (2nd June). For this purpose we have chosen a LSH mass of 65 GeV and the value of coupling to be $|\lambda| = 0.5$. The results are plotted in Figs. 4, 5 and 6 for $^{76}$Ge, NaI and $^{131}$Xe detectors respectively. The plots clearly show the sinusoidal behaviour of daily yield over a year. It peaks in June and is minimum in December as expected.

For comparison, we calculate the rate and annual variation for one sample case for $m_L \gg m_W$ regime and as discussed earlier this LSH cannot account for the total Dark Matter. Hence the local Dark Matter density $\rho_\chi$ for this case is taken to be 0.003 GeV/cm$^3$ rather than the usual total local Dark Matter density of 0.3 GeV/cm$^3$. The coupling $|\lambda|$ for this case is also kept at 0.5. Both the rate (Fig. 7) and the annual variation (Fig. 8) for this LSH clearly orders of magnitude lower than the cases considered in $m_L \lesssim m_W$ regime (Figs 1 - 6).

In summary, we investigate direct detection rates for a possible Dark Matter candidate in a Heavy Higgs model. The model contains an additional Higgs which neither couples with matter nor it has developed any VEV. This can be achieved through the choice of model parameters and discrete parity symmetry. Thus, the neutral part of the extra doublet becomes stable and can be a possible candidate for Cold Dark Matter. The LSH with mass around 60 - 70 GeV (in the regime $m_L \lesssim m_W$) can explain the Dark Matter content of the universe. On the other hand if $m_L > m_W$, the LSH candidate may explain only a fraction of Dark Matter. We calculated direct detection rates of those Dark Matter candidates in the context of three experiments namely $^{76}$Ge (as in GENIUS), DAMA and XENON. We have also shown the annual variation of the Dark Matter detection due to periodic motion of earth, for all the three experiments considered here. For comparison, the results for one case with $m_L = 300$ GeV ($>> m_W$) is also shown.
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Figure Captions

**Fig. 1** Differential detection rates $\Delta R/\Delta E$ vs recoil energy $E$ for three different values of LSH mass ($m_L$) namely 60 GeV, 65 GeV and 70 GeV. The topmost plot corresponds to $m_L = 60$ GeV and the lowermost plot corresponds to $m_L = 70$ GeV and the plot in between is for $m_L = 65$ GeV. The coupling constant is kept fixed at 0.5 (see text).

**Fig. 2** Same as Fig. 1 but for NaI.

**Fig. 3** Same as Fig. 1 but for $^{131}Xe$.

**Fig. 4** Annual variation of LSH Dark Matter direct detection signal (events/kg/day vs each day in a year) for $^{76}$Ge detector with $m_L = 65$ and $|\lambda| = 0.5$.

**Fig. 5** Same as Fig. 4 but for NaI.

**Fig. 6** Same as Fig. 4 but for $^{131}Xe$.

**Fig. 7** Differential detection rates $\Delta R/\Delta E$ vs recoil energy $E$ for LSH mass $m_L = 300$ GeV for $^{76}$Ge.

**Fig. 8** Annual variation for LSH mass $m_L = 300$ at $^{76}$Ge detector.