Charmed $\Omega_c$ weak decays into $\Omega$ in the light-front quark model

Yu-Kuo Hsiao\textsuperscript{1,a}, Ling Yang\textsuperscript{1,b}, Chong-Chung Lih\textsuperscript{2,c}, Shang-Yuu Tsai\textsuperscript{1,d}

\textsuperscript{1} School of Physics and Information Engineering, Shanxi Normal University, Linfen 041004, China
\textsuperscript{2} Department of Optometry, Central Taiwan University of Science and Technology, Taichung 40601, Taiwan

Received: 3 October 2020 / Accepted: 29 October 2020 / Published online: 19 November 2020
© The Author(s) 2020

Abstract More than ten $\Omega_c^0$ weak decay modes have been measured with the branching fractions relative to that of $\Omega_c^0 \to \Omega^- \pi^+$. In order to extract the absolute branching fractions, the study of $\Omega_c^0 \to \Omega^- \pi^+$ is needed. In this work, we predict $B_\pi \equiv B(\Omega_c^0 \to \Omega^- \pi^+) = (5.1 \pm 0.7) \times 10^{-3}$ with the $\Omega_c^0 \to \Omega^- \pi^+$ transition form factors calculated in the light-front quark model. We also predict $B_\rho \equiv B(\Omega_c^0 \to \Omega^- \rho^+) = (14.4 \pm 0.4) \times 10^{-3}$ and $B_e \equiv B(\Omega_c^0 \to \Omega^- e^+ \nu_e) = (5.4 \pm 0.2) \times 10^{-3}$. The previous values for $B_\rho/B_\pi$ have been found to deviate from the most recent observation. Nonetheless, our $B_\rho/B_\pi = 2.8 \pm 0.4$ is able to alleviate the deviation. Moreover, we obtain $B_e/B_\pi = 1.1 \pm 0.2$, which is consistent with the current data.

1 Introduction

The lowest-lying singly charmed baryons include the anti-triplet and sextet states $B_c = (\Lambda_c^+, \Sigma_c^0, \Xi_c^-)$ and $B'_c = (\Sigma_c^{0,+,++}, \Xi_c^{0,+}, \Omega_c^0)$, respectively. The $B_c$ and $\Omega_c^0$ baryons predominantly decay weakly [1–5], whereas the $\Sigma_c$ ($\Xi_c$) decays are strong (electromagnetic) processes. There have been more accurate observations for the $B_c$ weak decays in the recent years, which have helped to improve the theoretical understanding of the decay processes [6–14]. With the lower production cross section of $\sigma(e^+e^- \to \Omega_c^0 X)$ [4], it is an uneasy task to measure $\Omega_c^0$ decays. Consequently, most of the $\Omega_c^0$ decays have not been reanalyzed since 1990s [15–23], except for those in [24–29].

One still manages to measure more than ten $\Omega_c^0$ decays, such as $\Omega_c^0 \to \Omega^- \rho^+$, $\Sigma_c^0 \bar{K}^{0(\prime)}$ and $\Omega^- \ell^+ \nu_\ell$, but with the branching fractions relative to $B(\Omega_c^0 \to \Omega^- \pi^+)$ [5]. To extract the absolute branching fractions, the study of $\Omega_c^0 \to \Omega^- \pi^+$ is crucial. Fortunately, the $\Omega_c^0 \to \Omega^- \pi^+$ decay involves a simple topology, which benefits its theoretical exploration. In Fig. 1, $\Omega_c^0 \to \Omega^- \pi^+$ is depicted to proceed through the $\Omega_c^0 \to \Omega^-$ transition, while $\pi^+$ is produced from the external $W$-boson emission. Since it is a Cabibbo-allowed process with $V_{us}^2 V_{ud} \approx 1$, a larger branching fraction is promising for measurements. Furthermore, it can be seen that $\Omega_c^0 \to \Omega^- \pi^+$ has a similar configuration to those of $\Omega_c^0 \to \Omega^- \rho^+$ and $\Omega_c^0 \to \Omega^- \ell^+ \nu_\ell$, as drawn in Fig. 1, indicating that the three $\Omega_c^0$ decays are all associated with the $\Omega_c^0 \to \Omega^-$ transition. While $\Omega$ is a decuplet baryon that consists of the totally symmetric identical quarks $ss$, behaving as a spin-3/2 particle, the form factors of the $\Omega_c^0 \to \Omega^-$ transition can be more complicated, which hinders the calculation for the decays. As a result, a careful investigation that relates $\Omega_c^0 \to \Omega^- \pi^+$, $\Omega^- \rho^+$ and $\Omega_c^0 \to \Omega^- \ell^+ \nu_\ell$ has not been given yet, despite the fact that the topology associates them together.

Based on the quark models, it is possible to study the $\Omega_c^0$ decays into $\Omega^-$ with the $\Omega_c^0 \to \Omega^-$ transition form factors. However, the validity of theoretical approach needs to be tested, which depends on if the observations, given by

\[
\frac{B(\Omega_c^0 \to \Omega^- \rho^+)}{B(\Omega_c^0 \to \Omega^- \pi^+)} = 1.7 \pm 0.3 \text{[4]} (\sim 1.3 \text{[5]}),
\]

\[
\frac{B(\Omega_c^0 \to \Omega^- e^+ \nu_\ell)}{B(\Omega_c^0 \to \Omega^- \pi^+)} = 2.4 \pm 1.2 \text{[5]}, \tag{1}
\]

can be interpreted. Since the light-front quark model has been successfully applied to the heavy hadron decays [27, 30–46], in this report we will use it to study the $\Omega_c^0 \to \Omega^-$ transition form factors. Accordingly, we will be enabled to calculate the absolute branching fractions of $\Omega_c^0 \to \Omega^- \pi^+ (\rho^+)$ and $\Omega_c^0 \to \Omega^- \ell^+ \nu_\ell$, and check if the two ratios in Eq. (1) can be well explained.

\textsuperscript{a} e-mail: yukuohsiao@gmail.com
\textsuperscript{b} e-mail: yangling@ihep.ac.cn
\textsuperscript{c} e-mail: cclih@phys.nthu.edu.tw
\textsuperscript{d} e-mail: shangyu@gmail.com (corresponding author)
Fig. 1 Feynman diagrams for (a) $\Omega^0_c \rightarrow \Omega^- \pi^+(\rho^+)$ and (b) $\Omega^{\prime 0}_c \rightarrow \Omega^- \ell^+ \nu_\ell$ with $\ell^+ = e^+$ or $\mu^+$

2 Theoretical framework

2.1 General formalism

To start with, we present the effective weak Hamiltonians $\mathcal{H}_{H,L}$ for the hadronic and semileptonic charmed baryon decays, respectively [47]:

$$\mathcal{H}_H = \frac{G_F}{\sqrt{2}} V_{cd}^* V_{ud} [c_1 (\bar{u}d)(\bar{s}c) + c_2 (\bar{s}d)(\bar{u}c)],$$

$$\mathcal{H}_L = \frac{G_F}{\sqrt{2}} V_{cs}^* (\bar{s}c)(\bar{u}_\ell u_\ell),$$

where $G_F$ is the Fermi constant, $V_{ij}$ the Cabibbo–Kobayashi–Maskawa (CKM) matrix elements, $c_{1,2}$ the effective Wilson coefficients, $(\bar{q}_1 q_2) \equiv \bar{q}_1 \gamma_\mu (1 - \gamma_5) q_2$ and $(\bar{u}_\ell u_\ell) \equiv \bar{u}_\ell \gamma_\mu (1 - \gamma_5) u_\ell$. In terms of $\mathcal{H}_{H,L}$, we derive the amplitudes of $\Omega^0_c \rightarrow \Omega^- \pi^+(\rho^+)$ and $\Omega^{\prime 0}_c \rightarrow \Omega^- \ell^+ \nu_\ell$ as [48,49]

$$\mathcal{M}_h \equiv \mathcal{M}(\Omega^0_c \rightarrow \Omega^- h^+) = \frac{G_F}{\sqrt{2}} V_{cd}^* V_{ud} a_1 (\Omega^- |(\bar{s}c)|\Omega^0_c) (h^+ |\bar{u}d| 0),$$

$$\mathcal{M}_l \equiv \mathcal{M}(\Omega^0_c \rightarrow \Omega^- \ell^+ \nu_\ell) = \frac{G_F}{\sqrt{2}} V_{cs}^* (\bar{s}c)(\Omega^- |(\bar{s}c)|\Omega^0_c) (\bar{u}_\ell u_\ell),$$

where $h = (\pi, \rho), \ell = (e, \mu), \text{and } a_1 = c_1 + c_2 / N_c$ results from the factorization [50], with $N_c$ the color number.

With $\mathbf{B}^0_c (\mathbf{B}')$ denoting the charmed sextet (decuplet) baryon, the matrix elements of the $\mathbf{B}^0_c \rightarrow \mathbf{B}'$ transition can be parameterized as [28,45]

$$\langle T^{\mu} \rangle \equiv \langle \mathbf{B}' (P', S', S'_\ell); (\bar{q} \gamma^\mu (1 - \gamma_5) q)|\mathbf{B}^0_c (P, S, S'_\ell) \rangle = \bar{u}_\ell (P', S'_\ell) \left[ \frac{p^\mu}{M} F^V_1 + \frac{p^\mu}{M} F^F_2 + \frac{p^\gamma}{M} F^F_3 \right] \gamma_5 u(P, S),$$

where $(M, M')$ and $(S, S') = (1/2, 3/2)$ represent the masses and spins of $(\mathbf{B}', \mathbf{B})$, respectively, and $F^{V,A}_i$ $(i = 1, 2, \ldots, 4)$ the form factors to be extracted in the light-front quark model. The matrix elements of the meson productions are defined as [5]

$$\langle \pi (p)|(\bar{u}d)|0 \rangle = i f_\pi q^\mu,$$

$$\langle \rho (\lambda)|(\bar{u}d)|0 \rangle = m_\rho f_\rho \epsilon_{\mu}^{\lambda},$$

where $f_{\pi(\rho)}$ is the decay constant, and $\epsilon_{\mu}^{\lambda}$ is the polarization four-vector with $\lambda$ denoting the helicity state.

2.2 The light-front quark model

The baryon bound state $\mathbf{B}^0_c (P, S, S'_\ell)$ contains three quarks $q_1, q_2$, and $q_3$, with the subscript $c$ for $q_1 = c$. Moreover, $q_2$ and $q_3$ are combined as a diquark state $q_{1,2}$, behaving as a scalar or axial-vector. Subsequently, the baryon bound state $\mathbf{B}^0_c (P, S, S'_\ell)$ in the light-front quark model can be written as [31]

$$\Psi_{SS'\ell}(P, q_1, q_2, q_3) = \int (d^3 p_1 d^3 p_2) (2\pi)^3 \delta^3 (\vec{P} - \vec{p}_1 - \vec{p}_2) \times \sum_{\lambda_1, \lambda_2} \Psi_{SS'\ell}(\tilde{p}_1, \tilde{p}_2; \lambda_1, \lambda_2) q_{1,2}(p_1) q_{1,2}(p_2, \lambda_2),$$

where $\Psi_{SS'\ell}$ is the momentum-space wave function, and $(p_i, \lambda_i)$ stand for momentum and helicity of the constituent (d)quark, with $i = 1, 2$ for $q_1$ and $q_{1,2}$, respectively. The tilde notations represent that the quantities are in the light-front frame, and one defines $P = (P^-, P^+, P_\perp)$ and $\tilde{P} = (P^+, P^-)$, with $P^\pm = P^0 \pm P^3$ and $P_\perp = (P^1, P^2)$. 

[51x422]H
[51x125]⟨
[51x292]M
[51x572]2 Theoretical framework

Fig. 1 Feynman diagrams for a $\Omega^0_c \rightarrow \Omega^- \pi^+(\rho^+)$ and b $\Omega^{\prime 0}_c \rightarrow \Omega^- \ell^+ \nu_\ell$ with $\ell^+ = e^+$ or $\mu^+$
Besides, $\tilde{p}_i$ are given by
\[
\tilde{p}_i = (p_i^+, p_{i\perp}) , \quad p_{i\perp} = (p_i^1, p_i^2, \ldots, p_i^{2\perp}) , \quad p_i^- = \frac{m_i^2 + p_i^{2\perp}}{p_i^+} ,
\]  
(7)
with
\[
m_1 = m_{q_1} , \quad m_2 = m_{q_1} + m_{q_2} , \quad p_1^+ = (1 - x)P^+ , \quad p_2^+ = xP^+ ,
\]
\[
p_{1\perp} = (1 - x)P - k_{\perp} , \quad p_{2\perp} = xP + k_{\perp} ,
\]  
(8)
where $x$ and $k_{\perp}$ are the light-front relative momentum variables with $k_{\perp} = (k_{\perp}, k_z)$, ensuring that $P^+ = p_1^+ + p_2^+$ and $P_{\perp} = p_{1\perp} + p_{2\perp}$. According to $e_i \equiv \sqrt{m_i^2 + k_z^2}$ and $M_0 \equiv e_1 + e_2$ in the Melosh transformation [30], we obtain
\[
x = \frac{e_2 - k_z}{e_1 + e_2} , \quad 1 - x = \frac{e_1 + k_z}{e_1 + e_2} , \quad k_z = \frac{xM_0}{2} - \frac{m_1^2 + k_z^2}{2xM_0} ,
\]
\[
M_0^2 = \frac{m_1^2 + k_z^2}{1 - x} + \frac{m_2^2 + k_z^2}{x} .
\]  
(9)
Consequently, $\Psi^{SS'}$ can be given in the following representation [41–45]:
\[
\Psi^{SS'}(\vec{p}_1, \vec{p}_2, \lambda_1, \lambda_2) = \frac{A^{(0)}}{\sqrt{2(p_1 \cdot \vec{P} + m_1M_0)}} \tilde{u}(p_1, \lambda_1)\Gamma^{(a)}_{S,A} u(\bar{P}, S_c)\phi(x, k_{\perp}) ,
\]  
(10)
with
\[
A = \sqrt{\frac{3(m_1M_0 + p_1 \cdot \bar{P})}{3m_1M_0 + p_1 \cdot \bar{P} + 2(p_1 \cdot p_2)(p_2 \cdot \bar{P})/m_2^2}} ,
\]
\[
\Gamma_S = 1 , \quad \Gamma_A = -\frac{1}{\sqrt{3}}\gamma_5\xi^{\bar{q}q} \phi(p_2, \lambda_2) ,
\]
and
\[
A' = \sqrt{\frac{3m_2^2M_0^2}{2m_1^2M_0^2 + (p_2 \cdot \bar{P})^2}} , \quad \Gamma_A' = \epsilon^{\bar{q}q} \phi(p_2, \lambda_2) ,
\]  
(11)
where the vertex function $\Gamma_{S(A)}$ is for the scalar (axial-vector) diquark in $B'_s$, and $\Gamma_{A}'$ for the axial-vector diquark in $B'$. We have used the variable $\bar{P} \equiv p_1 + p_2$ to describe the internal motions of the constituent quarks in the baryon [32], which leads to $(\bar{P}_i \gamma^\mu - M_0)u(\bar{P}, S_c) = 0$, different from $(P_i \gamma^\mu - M)u(P, S_c) = 0$. For the momentum distribution, $\phi(x, k_{\perp})$ is presented as the Gaussian-type wave function, given by
\[
\phi(x, k_{\perp}) = 4 \left( \frac{\pi}{\beta^2} \right)^{3/4} \frac{e_1e_2}{\sqrt{x(1 - x)M_0}} \exp \left( -\frac{k_z^2}{2\beta^2} \right) ,
\]  
(12)
where $\beta$ shapes the distribution.

Using $|\mathbf{B}'_s(P, S, S_c)\rangle$ and $|\mathbf{B}'(P', S', S'_c)\rangle$ from Eq. (6) and their components in Eqs. (10), (11) and (12), we derive the matrix elements of the $B_c \rightarrow B'$ transition in Eq. (4) as
\[
\langle \bar{T}^\mu \rangle \equiv \langle \mathbf{B}'(P', S', S'_c)\rangle \bar{q}_y \gamma^\mu (1 - \gamma_5)c|\mathbf{B}'_s(P, S, S_c)\rangle
\]
\[
= \int \langle d^3p_2 \rangle \phi'(x', k_{\perp}' )\phi(x, k_{\perp})
\]
\[
\times \sum_{\lambda_2} \bar{u}_a(\bar{P}', S'_c) \bar{\Gamma}_F^{\beta \mu}(p_1', m_1') \times \gamma^\mu (1 - \gamma_5)(p_1' + m_1')\Gamma_A u(\bar{P}, S_c) ,
\]  
(13)
with $m_1 = m_c, m_1' = m_q$ and $\bar{\Gamma} = \gamma^0\Gamma^+\gamma^0$. We define $J^\mu_{S,j} = \bar{u}(\Gamma^{\beta \mu}_{S, j}) u_\beta$ and $J^\mu_{5,j} = \bar{u}(\Gamma^{\beta \mu}_{5, j}) u_\beta$ with $j = 1, 2, \ldots, 4$, where
\[
(\Gamma^{\beta \mu}_{S, j}) = (\gamma^\mu P^\beta, P^{\mu} P^\beta, P^{\mu} P^\beta, P^{\mu} \lambda^\beta) \gamma_5 ,
\]
\[
(\Gamma^{\beta \mu}_{5, j}) = (\gamma^\mu \bar{P}^\beta, \bar{P}^{\mu} P^\beta, \bar{P}^{\mu} P^\beta, \bar{P}^{\mu} \lambda^\beta) \gamma_5 .
\]  
(14)
Then, we multiply $J_{S,j} \langle \bar{T} \rangle (\langle \bar{T} \rangle) as F_{S,j} \equiv J_{S,j} \cdot \langle T \rangle$ and $F_{5,j} \equiv J_{5,j} \cdot \langle T \rangle$ with $T$ and $\langle \bar{T} \rangle$ in Eqs. (4) and (13), respectively, resulting in [45]
\[
F_{S,j} = Tr \left[ -u_\beta \bar{u}_a \left[ \frac{p_0}{2M} \left( \gamma^\nu F_1^V + \frac{p^\mu F_2^V}{M^2} + \frac{p^\nu F_3^V}{M^3} \right) \right] + g^{\mu\nu} F_4^V \right] \gamma_5 \bar{u}(\Gamma^{\beta \mu}_{S, j}) u_\beta ,
\]
\[
\tilde{F}_{S,j} = \int \langle d^3p_2 \rangle \phi'(x', k_{\perp}' )\phi(x, k_{\perp})
\]
\[
\times \sum_{\lambda_2} Tr \left[ u_\beta \bar{u}_a \left[ \bar{\Gamma}_A^{\mu \nu}(p_1' + m_1')\gamma^\nu (p_1' + m_1')\Gamma_A \right] u(\Gamma^{\beta \mu}_{5, j}) \right] .
\]  
(15)
In the connection of $\tilde{F}_{S,j} = \bar{F}_{S,j}$, we construct four equations. By solving the four equations, the four form factors $F_1^V, F_2^V, F_3^V$ and $F_4^V$ can be extracted. The form factors $F_i^V$ can be obtained in the same way.

2.3 Branching fractions in the helicity basis

One can present the amplitude of $\Omega^{0}_c \rightarrow \Omega^{-}h^+(\Omega^{-}\epsilon^+\nu_e)$ in the helicity basis of $H_{\lambda\gamma\lambda'\ell}(t)$ [28, 45], where $\lambda_\Omega = \pm 3/2, \pm 1/2$ represent the helicity states of the $\Omega^{-}$ baryon, and $\lambda_h, \ell$ those of $h^+$ and $\epsilon^+\nu_e$. Substituting the matrix elements in Eqs. (3) with those in Eqs. (4) and (5), the amplitudes in the helicity basis now read $\sqrt{2}M_h = (i) \sum_{\lambda_h \ell} G_{F} V_{c}^{*} V_{ud} a_{1m_{h}} b_{h} H_{\lambda\gamma\lambda'\ell}$ and $\sqrt{2}M_{\ell} = \sum_{\lambda_h \ell} G_{F} V_{c}^{*} H_{\lambda\gamma\lambda'\ell}$, where $H_{\lambda\gamma\lambda'\ell} = H_{\lambda\gamma\lambda'\ell} - H_{\lambda\gamma\lambda'\ell}^{A}$ with $f = (h, \ell)$. Explicitly, $H_{\lambda\gamma\lambda'\ell}^{A}$ is written as [28]
\[
H_{\lambda\gamma\lambda'\ell}^{V(A)} \equiv \langle \Omega^{-} [\bar{s} y_\mu] (\gamma_5) c |\Omega^{0}_c \rangle \epsilon^\mu_{f} ,
\]  
(16)
with \( \epsilon^{\mu}_{h} = (q^{\mu} / \sqrt{q^{2}}, \epsilon^{\mu}_{\rho}) \) for \( h = (\pi, \rho) \). For the semi-leptonic decay, since the \( \ell^{+}\nu_{\ell} \) system behaves as a scalar or vector, \( \epsilon^{\mu}_{\ell} = q^{\mu} / \sqrt{q^{2}} \) or \( \epsilon^{\mu}_{\rho} \). The \( \pi \) meson only has a zero helicity state, denoted by \( \lambda_{\pi} = 0 \). On the other hand, the three helicity states of \( \rho \) are denoted by \( \lambda_{\rho} = (1, 0, -1) \). For the lepton pair, we assign \( \lambda_{\ell} = \lambda_{\pi} \) or \( \lambda_{\rho} \). Subsequently, we expand \( H_{\lambda_{\alpha} \lambda_{f}} \) as

\[
H_{\lambda_{\alpha} \lambda_{f}}^{V(A)} = \frac{2}{\sqrt{3}} \sqrt{q^{2}} \left( \frac{Q_{\lambda}^{2}}{2M_{M}M_{+}} \right) \left( F_{1}^{V(A)} M_{\pm} \right) + \frac{2}{\sqrt{3}} \left( F_{2}^{V(A)} \tilde{M}_{+} + F_{3}^{V(A)} \tilde{M}_{-} + F_{4}^{V(A)} M_{\mp} \right),
\]

(17)

for \( \epsilon^{\mu}_{\ell} = q^{\mu} / \sqrt{q^{2}} \), where \( M_{\pm} = M \pm M', Q_{\lambda}^{2} = M_{\pm}^{2} - q^{2} \), and \( \tilde{M}_{\pm} = (M_{+}M_{-} - q^{2})/(2M_{M}) \). We also obtain

\[
H_{\lambda_{\alpha} \lambda_{f}}^{V(A)} = \frac{2}{\sqrt{3}} \sqrt{q^{2}} \left( \frac{Q_{\lambda}^{2}}{2M_{M}M_{-}} \right) \left( F_{1}^{V(A)} M_{-} \right) + \frac{2}{\sqrt{3}} \left( F_{2}^{V(A)} \tilde{M}_{-} + F_{3}^{V(A)} \tilde{M}_{+} + F_{4}^{V(A)} M_{+} \right),
\]

(18)

for \( \epsilon^{\mu}_{\ell} = q^{\mu} / \sqrt{q^{2}} \), and \( \left| \tilde{P}_{\ell} \right| = \sqrt{q^{2}Q_{\lambda}^{2} / (2M)} \). Note that the expansions in Eqs. (17) and (18) have satisfied \( \lambda_{\lambda_{\alpha}} = \lambda_{\lambda_{f}} \) for the helicity conservation, with \( \lambda_{\lambda_{\alpha}} = \pm 1/2 \). The branching fractions then read

\[
B_{h} = B(\Omega_{c}^{0} \rightarrow \Omega^{-} h^{+}) = \frac{\tau_{\Omega_{c}^{0}} G_{F}^{2} |V_{ct}V_{ud}|^{2} a_{1}^{2} m_{h}^{2} f_{h}^{2}}{32 \pi m_{c} \Omega_{c}},
\]

\[
B_{\ell} = B(\Omega_{c}^{0} \rightarrow \Omega^{-} \ell^{+} \nu_{\ell}) = \frac{\tau_{\Omega_{c}^{0}} G_{F}^{2} |V_{ct}V_{ud}|^{2}}{192 \pi^{3} m_{c} \Omega_{c}} \int_{m_{t}^{2}}^{(m_{c}^{2} - m_{\ell}^{2})^{2}} dq^{2} \left( \frac{\left| \tilde{P}_{\ell} \right|^{2} (q^{2} - m_{\ell}^{2})^{2}}{q^{2}} \right) H_{\ell}^{2},
\]

(19)

where

\[
H_{\pi}^{2} = \left| H_{\frac{1}{2},0}^{0} \right|^{2} + \left| H_{-\frac{1}{2},0}^{0} \right|^{2},
\]

\[
H_{\rho}^{2} = \left| H_{\frac{1}{2},1}^{0} \right|^{2} + \left| H_{\frac{1}{2},-1}^{0} \right|^{2} + \left| H_{-\frac{1}{2},0}^{0} \right|^{2} + \left| H_{-\frac{1}{2},-1}^{0} \right|^{2} + \left| H_{-\frac{1}{2},1}^{0} \right|^{2} + \left| H_{-\frac{1}{2},-1}^{0} \right|^{2},
\]

\[
H_{\ell}^{2} = \left( 1 + \frac{m_{\ell}^{2}}{2q^{2}} \right) H_{\rho}^{2} + \frac{3m_{\ell}^{2}}{2q^{2}} H_{\pi}^{2},
\]

(20)

with \( \tau_{\Omega_{c}} \) the \( \Omega_{c}^{0} \) lifetime.

### Table 1 The \( \Omega_{c}^{0} \rightarrow \Omega^{-} \) transition form factors with \( F(0) \) at \( q^{2} = 0 \), where \( \delta \equiv \delta m_{c}/m_{c} = \pm 0.04 \) from Eq. (21)

| \( F(0) \) | \( a \) | \( b \) |
|---|---|---|
| \( F_{1}^{V} \) | 0.54 + 0.138 | -0.27 | 1.65 |
| \( F_{1}^{A} \) | 0.35 - 0.365 | -30.00 | 96.82 |
| \( F_{3}^{V} \) | 0.33 + 0.598 | 0.96 | 9.25 |
| \( F_{4}^{V} \) | 0.97 + 0.228 | -0.53 | 1.41 |
| \( F_{4}^{A} \) | 2.05 + 1.385 | -3.66 | 1.41 |
| \( F_{4}^{A} \) | -0.06 + 0.338 | -1.15 | 71.66 |
| \( F_{4}^{A} \) | -1.32 - 0.328 | -4.01 | 5.68 |
| \( F_{4}^{A} \) | -0.44 + 0.116 | -1.29 | -0.58 |

### 3 Numerical analysis

In the Wolfenstein parameterization, the CKM matrix elements are adopted as \( V_{ct} = V_{ud} = 1 - \lambda^{2} / 2 \) with \( \lambda = 0.22453 \pm 0.00044 \) [5]. We take the lifetime and mass of the \( \Omega_{c}^{0} \) baryon and the decay constants \((f_{\pi}, f_{\rho}) = (132, 216) \text{ MeV} \) from the PDG [5]. With \((c_{1}, c_{2}) = (1.26, -0.51) \) at the \( m_{c} \) scale [47], we determine \( a_{1} \). In the generalized factorization, \( N_{c} \) is taken as an effective color number with \( N_{c} = (2, 3, \infty) \) [28, 29, 46, 50], in order to estimate the non-factorizable effects. For the \( \Omega_{c}^{0} (c\bar{s}s) \rightarrow \Omega^{-} (s\bar{s}s) \) transition form factors, the theoretical inputs of the quark masses and parameter \( \beta \) in Eq. (15) are given by \([34, 40]\):

\[
m_{1} = m_{c} = (1.35 \pm 0.05) \text{ GeV}, \quad m_{1} = m_{s} = 0.38 \text{ GeV},
\]

\[
m_{2} = 2m_{s} = 0.76 \text{ GeV}, \quad \beta_{c} = 0.60 \text{ GeV}, \quad \beta_{s} = 0.46 \text{ GeV},
\]

(21)

where \( \beta_{c(s)} \) is to determine \( k_{c(s)}^{(0)}(\rho_{c(s)}) \) for \( \Omega_{c}^{0} (\Omega^{-}) \). We hence extract \( F_{V}^{\rho} \) and \( F_{A}^{\rho} \) in Table 1. For the momentum dependence, we have used the double-pole parameterization:

\[
F(q^{2}) = \frac{F(0)}{1 - a(q^{2}/m_{F}^{2}) + b(q^{4}/m_{F}^{4})},
\]

(22)

with \( m_{F} = 1.86 \text{ GeV} \).

Using the theoretical inputs, we calculate the branching fractions, whose results are given in Table 2.

### 4 Discussions and conclusions

In Table 2, we present \( B_{h} \) and \( B_{\ell} \) with \( N_{c} = (2, 3, \infty) \). The errors come from the form factors in Table 1, of which the uncertainties are correlated with the charm quark mass. By comparison, \( B_{h} \) and \( B_{\ell} \) are compatible with the values in Ref. [28]; however, an order of magnitude smaller than those in Refs. [20, 22], whose values are obtained with the total decay widths \( \Gamma_{\pi(\rho)} = 2.09 a_{1}^{2}(11.34 a_{1}^{2}) \times 10^{11} \text{ s}^{-1} \).
Branching fractions of (non-)leptonic $B_{\Delta c}(2, 3, \infty)$. The three numbers in the parenthesis correspond to $N_c = (2, 3, \infty)$, and the errors come from the uncertainties of the form factors in Table 1. In our work, $R_{\rho/\pi} = 2.8 \pm 0.4$ is able to alleviate the inconsistency between the previous value and the most recent observation. We obtain $R_{\rho/\pi} = 1.1 \pm 0.2$ with $N_c = 2$ to be consistent with the data, which indicates that $(B_{\pi}, B_{\rho}) = (5.1 \pm 0.7, 14.4 \pm 0.4) \times 10^{-3}$ with $N_c = 2$ are more favorable.

The helicity amplitudes can be used to better understand how the form factors contribute to the branching fractions. With the identity $H_{-\lambda\alpha-\lambda_f}^{V(A)} = \mp H_{\lambda\alpha-\lambda_f}^{V(A)}$ for the $B_c(J^P = 1/2^+)$ to $B(J^P = 3/2^+)$ transition [28], $H_{\pi}^2$ in Eq. (20) can be rewritten as $H_{\pi}^2 = 2(H_{V}^{20}|^2 + |H_{A}^{20}|^2)$. From the pre-factors in Eq. (17), we estimate the ratio of $|H_{V}^{20}|^2/|H_{A}^{20}|^2 \simeq 0.05$, which shows that $H_{A}^{20}$ dominates $B_{\pi}$, instead of $H_{V}^{20}$.

More specifically, it is the $F_2^A$ term in $H_{20}^A$ that gives the main contribution to the branching fraction. By contrast, the $F_{1,3}^A$ terms in $H_{20}^A$ largely cancel each other, which is caused by $F_1^AM_1 \simeq F_2^AM_2$ and a minus sign between $F_3^A$ and $F_4^A$ (see Table 1); besides, the $F_2^A$ term with a small $F_2^A(0)$ is ignorable.

Likewise, we obtain $H_{20}^Z = 2(|H_{V}^{20}|^2 + |H_{A}^{20}|^2)$ for $B_{\rho}$, where $|H_{V}^{20}|^2 = |H_{V}^{21}|^2 + |H_{V}^{21}|^2 + |H_{V}^{20}|^2$. We find that $|H_{V}^{20}|^2$ is ten times larger than $|H_{V}^{20}|^2$. Moreover, $H_{A}^{20}$ is similar to $H_{A}^{20}$, where the $F_{1,3}^A$ terms largely cancel each other, $F_2^A$ is ignorable, and $F_4^A$ gives the main contribution. While $F_2^A$ and $F_4^A$ in $H_{20}^A$ have a positive interference, giving 20% of $B_{\rho}$, $F_4^A$ in $H_{20}^A$ singly contributes 35%. In Eq. (20), the factor of $m_\pi/m_l^2$ with $m_l \simeq 0$ should be much suppressed, such that $H_{\rho}^2 \simeq H_{\pi}^2$. Therefore, $B_\ell$ receives the main contributions from the $F_4^A$ terms in $H_{A}^{20}$, $H_{A}^{21}$ and $H_{V}^{21}$, which is similar to the analysis for $B_{\rho}$.

In summary, we have studied the $\Omega_{c}^{0} \rightarrow \Omega^{-}\pi^{+}, \Omega^{-}\rho^{+}$ and $\Omega_{c}^{0} \rightarrow \Omega^{-}\ell^{+}\nu_{\ell}$ decays, which proceed through the $\Omega_{c}^{0} \rightarrow \Omega^{-}$ transition and the formation of the meson $\pi^{+}(\rho^{+})$ or lepton pair from the external $W$-boson emission. With the form factors of the $\Omega_{c}^{0} \rightarrow \Omega^{-}$ transition, calculated in the light-front quark model, we have predicted $B(\Omega_{c}^{0} \rightarrow \Omega^{-}\pi^{+}, \Omega^{-}\rho^{+}) = (5.1 \pm 0.7, 14.4 \pm 0.4) \times 10^{-3}$ and $B(\Omega_{c}^{0} \rightarrow \Omega^{-}\ell^{+}\nu_{\ell}) = (5.4 \pm 0.2) \times 10^{-3}$. While the previous studies have given the $R_{\rho/\pi}$ values deviating from the most recent observation, we have presented $R_{\rho/\pi} = 2.8 \pm 0.4$ to alleviate the deviation. Moreover, we have obtained $R_{\rho/\pi} = 1.1 \pm 0.2$, consistent with the current data.
Acknowledgements YKH was supported in part by National Science Foundation of China (No. 11675030). CCL was supported in part by CTUST (No. CTU109-P-108).

Data Availability Statement This manuscript has associated data in a data repository. [Authors’ comment: The data supporting the findings of this study are available at https://doi.org/10.1103/PhysRevD.97.032001 and https://doi.org/10.1103/PhysRevD.98.030001.]

Open Access This article is licensed under a Creative Commons Attribution 4.0 International License, which permits use, sharing, adaptation, distribution and reproduction in any medium or format, as long as you give appropriate credit to the original author(s) and the source, provide a link to the Creative Commons licence, and indicate if changes were made. The images or other third party material in this article are included in the article’s Creative Commons licence, unless indicated otherwise in a credit line to the material. If material is not included in the article’s Creative Commons licence and your intended use is not permitted by statutory regulation or exceeds the permitted use, you will need to obtain permission directly from the copyright holder. To view a copy of this licence, visit http://creativecommons.org/licenses/by/4.0/.

Funded by SCOAP3.

References

1. D. Cronin-Hennessy et al., CLEO Collaboration. Phys. Rev. Lett. 86, 3730 (2001)
2. R. Ammar et al., CLEO Collaboration. Phys. Rev. Lett. 89, 171803 (2002)
3. B. Aubert et al., BaBar Collaboration. Phys. Rev. Lett. 99, 062001 (2007)
4. J. Yelton et al., Belle Collaboration. Phys. Rev. D 97, 032001 (2018)
5. M. Tanabashi et al., Particle Data Group. Phys. Rev. D 98, 030001 (2018)
6. C.D. Lu, W. Wang, F.S. Yu, Phys. Rev. D 93, 056008 (2016)
7. C.Q. Geng, Y.K. Hsiao, Y.H. Lin, L.L. Liu, Phys. Lett. B 776, 265 (2017)
8. C.Q. Geng, Y.K. Hsiao, C.W. Liu, T.H. Tsai, Phys. Rev. D 97, 073006 (2018)
9. C.Q. Geng, Y.K. Hsiao, C.W. Liu, T.H. Tsai, Phys. Rev. D 99, 073003 (2019)
10. Y.K. Hsiao, Y. Yu, H.J. Zhao, Phys. Lett. B 792, 35 (2019)
11. H.J. Zhao, Y.L. Wang, Y.K. Hsiao, Y. Yu, JHEP 2002, 165 (2020)
12. J. Zou, F. Xu, G. Meng, H.Y. Cheng, Phys. Rev. D 101, 014011 (2020)
13. Y.K. Hsiao, Q. Yi, S.T. Cai, H.J. Zhao, arXiv:2006.15291
14. P.Y. Niu, J.M. Richard, Q. Wang, Q. Zhao, arXiv:2003.09323
15. M. Avila-Aoki, A. Garcia, R. Huerta, R. Perez-Marcial, Phys. Rev. D 40, 2944 (1989)
16. R. Perez-Marcial, R. Huerta, A. Garcia, M. Avila-Aoki, Phys. Rev. D 40, 2955 (1989)
17. R.L. Singleton, Phys. Rev. D 43, 2939 (1991)
18. F. Hussain, J. Korner, Z. Phys. C 51, 607 (1991)
19. J. Korner, M. Kramer, Z. Phys. C 55, 659 (1992)
20. Q. Xu, A. Kamal, Phys. Rev. D 46, 3836 (1992)
21. H.Y. Cheng, B. Tseng, Phys. Rev. D 48, 4188 (1993)
22. H.Y. Cheng, Phys. Rev. D 56, 2799 (1997)
23. M.A. Ivanov, J. Korner, V.E. Lyubovskij, A. Rusetsky, Phys. Rev. D 57, 5632 (1998)
24. M. Pervin, W. Roberts, S. Capstick, Phys. Rev. C 74, 025205 (2006)
25. R. Dhir, C. Kim, Phys. Rev. D 91, 114008 (2015)
26. C.Q. Geng, Y.K. Hsiao, C.W. Liu, T.H. Tsai, JHEP 1711, 147 (2017)
27. Z.X. Zhao, Chin. Phys. C 42, 093101 (2018)
28. T. Gutsche, M.A. Ivanov, J.G. Korner, V.E. Lyubovskij, Phys. Rev. D 98, 074011 (2018)
29. S. Hu, G. Meng, F. Xu, Phys. Rev. D 101, 094033 (2020)
30. H.J. Melosh, Phys. Rev. D 9, 1095 (1974)
31. H.G. Dosch, M. Jamin, B. Stech, Z. Phys. C 42, 167 (1989)
32. W. Jaus, Phys. Rev. D 44, 2851 (1991)
33. F. Schlumpf, Phys. Rev. D 47, 4114 (1993); Erratum: [Phys. Rev. D 49, 6246 (1994)]
34. C.Q. Geng, C.C. Lih, W.M. Zhang, Mod. Phys. Lett. A 15, 2087 (2000)
35. C.R. Ji, C. Mitchell, Phys. Rev. D 62, 085020 (2000)
36. B.L.G. Bakker, C.R. Ji, Phys. Rev. D 65, 073002 (2002)
37. B.L.G. Bakker, H.M. Choi, C.R. Ji, Phys. Rev. D 67, 113007 (2003)
38. H.Y. Cheng, C.K. Chua, C.W. Hwang, Phys. Rev. D 69, 074025 (2004)
39. H.M. Choi, C.R. Ji, Few Body Syst. 55, 435 (2014)
40. C.Q. Geng, C.C. Lih, Eur. Phys. J. C 73, 2505 (2013)
41. H.W. Ke, X.H. Yuan, X.Q. Li, Z.T. Wei, Y.X. Zhang, Phys. Rev. D 86, 114005 (2012)
42. H.W. Ke, N. Hao, X.Q. Li, J. Phys. G 46, 115003 (2019)
43. X.H. Hu, R.H. Li, Z.P. Xing, Eur. Phys. J. C 80, 320 (2020)
44. H.W. Ke, N. Hao, X.Q. Li, Eur. Phys. J. C 79, 540 (2019)
45. Z.X. Zhao, Eur. Phys. J. C 78, 756 (2018)
46. Y.K. Hsiao, S.Y. Tsai, C.C. Lih, E. Rodrigues, JHEP 2004, 035 (2020)
47. G. Buchalla, A.J. Buras, M.E. Lautenbacher, Rev. Mod. Phys. 68, 1125 (1996)
48. Y.K. Hsiao, C.Q. Geng, Eur. Phys. J. C 77, 714 (2017)
49. Y.K. Hsiao, C.Q. Geng, Phys. Lett. B 782, 728 (2018)
50. Y.K. Hsiao, S.Y. Tsai, E. Rodrigues, Eur. Phys. J. C 80, 565 (2020)