Växjö Interpretation of Quantum Mechanics

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Abstract

We present critical arguments against individual interpretation of Bohr’s complementarity and Heisenberg’s uncertainty principles. Statistical interpretation of these principles is discussed in the contextual framework. We support the possibility to use Statistical Contextual Realist Interpretation of quantum formalism. In spite of all no-go theorems (e.g., von Neumann, Kochen and Specker,..., Bell,...), recently (quant-ph/0306003 and 0306069) we constructed a realist basis of quantum mechanics. In our model both classical and quantum spaces are rough images of the fundamental prespace. Quantum mechanics cannot be reduced to classical one. Both classical and quantum representations induce reductions of prespace information.

"Often we enter the unknown edifice of a new scientific discipline through a lesser gate that leads us into a side passage. It may take us a long while to find our way to the main portal and to view the whole structure in its proper perspective”.

E. Schrödinger

1 Introduction

In this paper we present some critical arguments against individual understanding of Bohr’s complementarity and Heisenberg’s uncertainty principles. It is argued the only possible interpretation of these principles is the statistical contextual one. In particular, in the opposite to orthodox Copenhagen
views such interpretation does not imply the impossibility to assign to elementary particles\(^1\) objective properties; including position and momentum. Of course, such ontic description of physical reality might not be provided by quantum formalism. In particular, it is clear that quantum formalism does not provide the complete description of physical reality.

We argue that the most natural interpretation of physical theories (in particular, quantum formalism) is Statistical Contextual Realist Interpretation.

2 **Contextual statistical realist interpretation of physical theories**

Quantum theory as well as classical physics is used to describe properties of pairs

\[ \pi = (\text{physical system, measuring device}). \] (1)

I do not think that understanding of this fact, contextual structure of physical theories, really was Bohr’s invention. It was clear to everybody that physical observables are related to properties of physical systems as well as measuring devices. The main invention of N. Bohr was not contextuality, but complementarity. Bohr’s greatest contribution was the recognition of the fact that there exist complementary experimental arrangements and hence complementary, incompatible, pairs \(\pi_1, \pi_2\) of form (1). I think nobody can be against the recognition of such a possibility. Why not? Why must all contexts, complexes of physical conditions, be coexisting? Contextuality and complementarity are two well understandable principles (not only of quantum physics, but physics in general).

The real problem was that N. Bohr as well as W. Heisenberg (but merely further generations of their adherents) did not pay attention that quantum complementarity is the experimental fact concerning pairs

\[ \pi = (\text{elementary particle, macroscopic measuring device}) \] (2)

and not elementary particles by their self. It is a pity that the greatest promoters of contextualism forgot about contextual basis of complementarity.

If we remember about contextuality discussing complementarity we see that complementarity of contexts in quantum physics does not imply complementarity of corresponding objective properties (of elementary particles).

\(^1\)The reader may protest against the use of the notion ‘particle’. It might be better to use e.g. ‘quantum system.’ But I dislike to use ‘quantum’ regarding to real physical objects. ‘Quantum’ has become merely the symbol of a rather special mathematical formalism.
contributing into such observables. In particular, contextual complementarity does not imply that elementary particles do not have objective properties at all. In particular, there are no reasons to suppose that it is impossible to provide a kind of hidden variable, HV, description (ontic description, see e.g. [1]) for these objective properties. Mathematically the pair \( \pi \) can be described by two variables \( \lambda_s \) and \( \lambda_d \) representing objective states of a physical system and measuring device. So a physical observable is represented as a function

\[ A = A(\lambda_s; \lambda_d). \]  

(3)

In general, \( \lambda_s \) can be represented as a vector with numerical coordinates. Numerical does not mean ‘real number’. It can be as well e.g. \( p \)-adic number, see [2]. So the possibility of HV-description does not imply continuous infinitely divisible representation of physical reality. In principle, classical physical description is often identified with the description based on the real-number ‘continuous’ model. We would not like to use such a restrictive, XVIIIth, XIXth century and the first half of XXth century, interpretation of classical physics. A classical physical model is every model that provides objective description of phenomena – representation of physical observables in form (3). I think that mixing of ‘classical’ and ‘continuous’ was one of main roots of vague interpreting of results of experiments with elementary particles. Thus the development of alternative (nonreal, noncontinuous) classical models, e.g. \( p \)-adic [3], [4], might play important role in clarification of foundations of quantum theory.

On the other hand, an adherent of N. Bohr would argue that “Such a separation and, hence, the description of (properties of) quantum objects and processes themselves (as opposed to certain effects of their interaction with measuring instruments upon latter) are impossible in Bohr’s interpretation,” [5].

I think that the origin of such an interpretation of complementarity by N. Bohr was the individual interpretation of Heisenberg’s uncertainty principle:

\[ \Delta q \Delta p \geq \frac{\hbar}{2}. \]  

(4)

Close relation between Bohr’s complementarity principle and Heisenberg’s uncertainty principle is well know. For two years (1926-1927) N. Bohr could not present any model that could explain both corpuscular behaviour (black body radiation, photoelectric effect) and wavelike behaviour (two slit experiment, diffraction) of elementary particles. You can read e.g. in the book [6] how heavy this thinking process was for N. Bohr - really a kind of mental disaster. Only after the derivation by W. Heisenberg of the uncertainty
principle, N. Bohr proposed a new model of physical reality based on the principle of **complementarity**. Unfortunately, Heisenberg’s uncertainty relation was interpreted as the relation for an individual elementary particle. The main problem was mixing by W. Heisenberg of *individual* and *statistical* uncertainty. For example, in his famous book [7] he discussed the uncertainty principle as a relation for an individual system, but derived this principle by using statistical methods!

The roots of such individual complementarity can be found already in the first work of W. Heisenberg [8]. At the beginning Heisenberg’s quantization procedure was not statistical one. It seems, see [8], that W. Heisenberg was sure that he found equations for observables related to individual physical systems. It was a rather common point view: in classical mechanics the position and momentum of an individual (!) system are described by real numbers, in quantum mechanics – by matrices. W. Heisenberg rightly underlined that matrice-description could not be used for describing objective position and momentum of electron. There (as everywhere in XVIIth – XXth centuries physics) objective reality was, in fact, identified with the continuous (real number) mathematical model of physical reality. Impossibility to create a continuous real number model for motion of electron in Bohr’s model of atom was considered by Heisenberg (and many others) as impossibility to assign objective properties to electrons. It was a rather strange passage. But we understand that it was the beginning of XXth century and W. Heisenberg used the standard mathematical image of physical reality. However, we would like to remark that even at that time there were attempts to modify continuous mathematical model to reproduce quantum effects, see e.g. Bohr, Kramers, Slater [9] on classical-like quantization based on *difference equations* (instead of differential). In fact, this model stimulated M. Born to introduce the term quantum mechanics. In any case the absence of continuous classical model for motion of electron in Bohr’s atom does not imply impossibility to create other, noncontinuous, classical (causal deterministic) models. Moreover, considerations of W. Heisenberg in [8] even did not imply impossibility to create continuous classical model – as it was claimed by W. Heisenberg and then by N. Bohr. The story is much simpler: first Bohr tried to create such a thing, but could not; then Heisenberg, with the same result. After this it was claimed that such a model did not exist. And what is the most interesting: not only for Bohr’s model of atom (well it might be), but for any other model... I cannot understand this kind of ‘quantum logic’.

Really W. Heisenberg proposed some mathematical model for some class of observations of position and momentum of electron. These observations satisfy to the uncertainty relation. It is not clear why we cannot present other mathematical models for some other observations of position and mo-
mentum of electron that would violate this relation? Of course, if we relate Heisenberg’s position and momentum to individual electron, then such individualization plays the role of objectification of these quantities. It is rather strange logical circle, but it seems that it was done by W. Heisenberg and N. Bohr. Finally, this objectification in combination with the uncertainty relation implies (for W. Heisenberg and N. Bohr) impossibility to consider other position and momentum variables, distinct from Heisenberg’s ones. On the other hand, if we use statistical interpretation of uncertainty relation, then there are no reasons for such NO-GO conclusions. Well, we could not prepare statistical ensembles with small dispersions for two variables introduced by W. Heisenberg. But we should have great imagination to make Heisenberg-Bohr conclusions.

Unfortunately, many clear mind scientists used and still use Heisenberg-Bohr’s ‘quantum logic’. For instance, the fundamental paper of A. Zeilinger [] gives us an excellent example of the modern representation of this logic. In principle, A. Zeilinger is looking for a new quantum paradigm. He correctly underlines that the situation in quantum theory, especially large diversity of interpretations, is not so natural. That in the opposite to e.g. theory of relativity, there is no quantum analogue of the principle of relativity. However, it seems that, for Zeilinger as well as for many other scientists looking for reconsideration of quantum foundations, such a reconsideration could (and moreover should) be performed as some addition to the orthodox Copenhagen. A. Zeilinger started (as always in this story) with the correct statement that we are dealing with a quantum phenomenon as the whole entity which comprises both the observed quantum system and the classical measuring apparata. No doubts! The formalism of quantum mechanics (statistical formalism) deals with such a phenomenon. However, then he continued: It is especially impossible in principle to predict with certainty both through which slit an elementary particle will go and where it will appear in the interference pattern. Well, we still can interpret this statement in the correct way: quantum formalism does not give us such a possibility. Unfortunately, the latter understanding was impossible to orthodox Copenhagen, since (by unclear reasons) it was supposed quantum theory provided the complete description of physical reality. \(^2\) So, Zeilinger continued: I propose that this impossibility to describe the random individual process within quantum mechanics in a complete way is a fundamental limitation of the program of modern science to arrive at a description of the world in every detail. This is the great

\(^2\)I think that the paper of Einstein, Podolsky and Rosen, *Can quantum mechanical description of physical reality be considered complete?*, [], was directed precisely against Heisenberg-Bohr ‘quantum logic’ that was, in fact, based on the idea that quantum theory is complete.
manifestation of Copenhagen NO-GO.

Another important story that stimulated Bohr's complementarity thinking was Shrödinger's quantum story. It is important to recall that E. Shrödinger was sure that he discovered totally new method of quantization [10], "Quantization as the problem for eigenvalues." In his first paper [10] he did not refer to Heisenberg's paper [8]; in the second paper [11] he made a short reference in the sense that W. Heisenberg proposed some other method of quantization that was totally different from Shrödinger's one. It is well known that many famous physicists had the great prejudice against Heisenberg's approach to quantization. So Shrödinger's wave mechanics was considered by many of them as the end of quantum mechanics; as the possibility to describe 'quantum experiments' by using classical theory of partial differential equations (especially strong anti Heisenberg-Bohr comments were done by Einstein and Wien).

Hence, Heisenberg and Bohr must find some strong arguments in favour of Heisenberg approach or disappear from quantum scene. Moreover, the whole quantum spectacle could be ended with quite trivial final: instead great mystery — simply a part of well established theory of partial differential equations. If all those circumstances be taken into account, it would be clear how N. Bohr created complementarity principle with all its NO-GO consequences.

So Bohr's complementarity was a kind of individual complementarity. Complementary features were regarded to individual physical systems. It is a pity that contextualists N. Bohr and W. Heisenberg related the uncertainty relation not to some special class of measurement procedures of the position and momentum described by quantum formalism, but to the position and momentum of an individual elementary particle. This imply the prejudice that the position and momentum even in principle could not be determined simultaneously and, moreover, that it is even in principle impossible to assign such a physical property, e.g. position or momentum, to e.g. electron: "electron does not have trajectory."

In fact, the only possible conscious interpretation of Heisenberg's uncertainty principle is the statistical contextual interpretation, see e.g. [12], [13]. It is impossible to prepare such an ensemble of elementary particles that dispersions of both position and momentum observables would be arbitrary small. Everybody would agree that only this statement can be verified experimentally. Contextualism has to be statistical contextualism and, consequently, complementarity has to be statistical contextual complementarity. Such contextualism and complementarity do not contradict to the possibility of finer description of reality than given by quantum theory.

The complex of experimental physical conditions must be split into two
complexes – a preparation procedure and a measurement procedure, see e.g. [14], [15]. A preparation procedure produces a statistical ensemble of physical system. Then a measurement device produces results – values of a physical observable. Nonzero dispersion of this random variable does not imply that individual physical system does not have objective properties that produce values of the observable via (3). Thus contextual statistical interpretation can be, in principle, extended to contextual statistical realist interpretation.

Of course, individual complementarity can be used as an argument against the possibility to create finer description of physical reality than given by quantum mechanics. But statistical complementarity cannot be used as such an argument. By contextual statistical interpretation it is not forbidden in principle to create such preparation and measurement procedures that position and momentum would be measured with dispersions $\Delta q$ and $\Delta p$ such that $\Delta q \Delta p < \frac{\hbar}{2}$. Of course, such a statement should immediately induce a storm of protests with reference to the principle of complementarity. However, we again recall that the right complementarity principle is contextual and statistical. It is about some class of measurement and preparation contexts described by quantum formalism. In particular, we do not consider quantum formalism as a kind of complete physical theory.

However, as far as we cannot perform such experiments for elementary particles, it is really impossible to reject Bohr’s principle of individual complementarity. What can we do?

We can study consequences of the general statistical contextuality and try to demonstrate that some distinguishing features of quantum theory that are typically associated with individual complementarity, NO-GO complementarity, are, in fact, simple consequences of statistical complementarity, GO-DEEPER complementarity. I did this in a series of papers, see e.g. [16], [17]. The main consequence of these investigations is that ‘waves of probability’ can be produced in the general situation (including macroscopic systems) due to combination of a few preparation contexts. Thus such ‘waves’ are not directly related to some wave features of objects participating in an experiments. Moreover, our investigation demonstrated that in some experiments there can be created other types of probabilistic waves, namely hyperbolic waves of probability.

In particular, our contextual probabilistic investigations demonstrated that contextual complementarity, wave-particle dualism, is not rigidly cou-

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3 Well, we may produce various realistic models underlying quantum formalism, for example, Bohmian mechanics, stochastic electrodynamics. However, it seems that all such models would be more or less automatically rejected by quantum community.

4 In the two slit experiment we consider the screen with slits as a part of an ensemble-preparation device and the screen with photoemulsion as a measurement device.
pled to microworld. Thus we can, in principle, perform experiments with macro systems that would demonstrate ‘wave-particle duality’, but not of macro objects, but contexts.

Such a numerical experiment, a classical analogue of the well known quantum two-slit experiment, was performed in [18]. Charged particles are scattered on flat screen with two slits and hit the second screen. We show that the probability distribution on the second screen when both slits are open is not given by the sum of distributions for each slit separately, but has an extra interference term that is given with the quantum rule of the addition of probabilistic alternatives. We have two theoretical descriptions of this experiment: 1) quantum-like statistical description; 2) Newtonian classical description. Both theories give the same statistical distribution of spots on the registration screen. Quantum-like theory operates with complex waves of probability; there is uncertainty, Heisenberg-like, relation for position and momentum. Of course, this relation is the statistical one.

Suppose now that some observer could not provide the verification of Newtonian description, e.g. such an observer is a star-size observer and its measuring device produces nonnegligible perturbations of our macroscopic charged balls. Such an observer might speculate on impossibility to find objective phase-space description and even about waves features of macroscopic balls. This experiment might be used as (at least indirect) argument against the orthodox Copenhagen NO-GO in experiments.

We recognize that at the moment there is one (and seems just one) argument supporting individual complementarity, namely Bell’s inequality.\(^5\) However, I have great doubts that the experimental violation of Bell’s inequality can be interpreted as an argument against the possibility of HV ontic description (or even against realism) or locality, see e.g. my works [19],[20] (see also papers of L.Accardi, L. Ballentine, W. De Myunck in [21]; see also [22] on contextual statistical realist interpretation of GHZ-paradox.

Finally, we remark that the possibility of (3)–description implies that ‘quantum randomness’ does not differ essentially from ‘classical randomness’ Of course, this contradict to orthodox quantum views to randomness as *fundamental or irreducible randomness*. Unfortunately, I could not understand the latter ideas. Instead of fundamental irreducible quantum randomness, I prefer to consider well understandable theory of context (complex of experimental physical conditions) depending probabilities.

A new fundamental principle that we propose can be called:

**The principle of contextual relativity of probabilities.**

Contextuality means that all probabilities depend on complexes of phys-

\(^5\)Some people use this framework to support quantum nonlocality.
ical conditions, $S$, – contexts: $P(E) \equiv P_S(E)$. It is meaningless to speak about probability without to determine complex of physical conditions. This is clear idea looks even trivial after it has been formulated. However, we remark that the conventional probability theory based on Kolmogorov measure theoretical axiomatics, 1933, [], is not contextual. In Kolmogorov’s theory the probability space can be fixed once for ever. We need not remember that probabilities depend on complexes of physical conditions and can use just the symbol of abstract probability $P$.

3 Citation with comments

In this section we shall present some citations on orthodox quantum theory and our contextual statistical realist comments. We use, in particular, collections of Bohr’s views presented in papers of H. Folse and A. Plotnitsky, see [23], [5].

(S1) “In contrast to ordinary mechanics, the new quantum mechanics does not deal with a space-time description of the motion of atomic particles. It operates with manifolds of quantities which replace the harmonic oscillating components of the motion and symbolize the possibilities of transition between stationary states in conformity with the correspondence principle”, N. Bohr.

This is simply the recognition of the restrictiveness of the domain of applications of quantum theory. I would like to interpret this as the recognition of incompleteness of quantum theory. However, it was not so for N. Bohr:

(S2) “... the quantum postulate implies that any observation of atomic phenomena will involve an interaction with the agency of observation not be neglected. Accordingly, an independent reality in the ordinary physical sense can neither be ascribed to the phenomena nor to the agencies of observation,” N. Bohr.

The first part of this citation is the manifestation of contextuality. However, I cannot understand what kind of logic N. Bohr used to proceed to the

6We remark that A. Kolmogorov by himself understood well contextual dependence of probabilities, see section 2 on experimental applications of his theory in[]. However, this contextuality was not present in his mathematical formalism. It was the terrible mistake of Kolmogorov. In fact, he tried to improve the situation and published paper[] in that he noticed that probabilities are contextual probabilities and even used corresponding symbol. However, the paper was published only in Russian and even in Soviet Union it was forgotten. Recently the pupil of Kolmogorov, A. Shiryaev, paid my attention to this paper.
second part. The second part can be interpreted as the declaration of the impossibility of objective, ontic description of reality.

(S3) "...to reserve the word phenomenon for comprehension of effects observed under given experimental conditions... These conditions, which include the account of the properties and manipulation of all measuring instruments essentially concerned, constitute in fact the only basis for the definition of the concepts by which the phenomenon is described," N. Bohr.

I would agree if the last sentence would be continued as "is described in quantum formalism."

(S4) "...by the very nature of the situation prevented from differentiating sharply between an independent behaviour of atomic objects and their interaction with the means of observation indispensable for the definition of the phenomena," N. Bohr.

I would agree if the last sentence would be continued as "of the phenomena described by quantum formalism."

(S5) "There are two forms in which quantum mechanics may be expressed, based on Heisenberg’s matrices and Schrödinger’s wave function respectively. The second of these is not connected directly with classical mechanics. The first is in close analogy with classical mechanics, as it may be obtained from classical mechanics simply by making the variables of classical mechanics into non-commuting quantities satisfying the correct commutation relations.” P.Dirac, [24].

No Comment...

4 On romantic interpretation of quantum mechanics

Finally, we ask: "Why the realistic interpretation is not so popular in quantum community?"

The common opinion: this is the direct consequence of experiments with elementary particles. Well, I do not think so. Of course, interference experiments with massive particles should induce reconsideration of methods of classical statistical mechanics. As well as discreteness of energy levels should induce reconsideration of classical continuous real model of physics. However, such reconsiderations would be merely mathematical. And it seems (at least for me) that they were merely mathematical. Observables take only discrete values – consider ‘discrete’ number systems instead of ‘continuous’. The standard probability calculus (created for one fixed sample space prepared under stable physical conditions) does not work – create the new one. And this was
done and very successfully. However, there were no reasons to create new quantum philosophy, based on Bohr’s principle of complementarity and the individual interpretation of Heisenberg’s uncertainty relations. Nevertheless, such a new philosophy was invented (merely by N. Bohr) and, moreover, it was recognized as philosophy of modern physics. Since this recognition, all realistic models for experiments with elementary particles were more or less automatically rejected, see e.g. A. Lande’s statistical contextual realistic model for diffraction [25]. Lande’s model looks quite natural; here we need not apply to wave-particle dualism, collapse and so on... It was simply rejected. Bohmian mechanics, see e.g. [26], – well, it has its disadvantages, but merely, mathematical.

Finally, Bell’s inequality arguments were interpreted as they should be interpreted in the orthodox quantum framework, despite very strong counter-arguments. If all these counter-arguments be taken into account, Bell’s inequality activity would look very strange, as a kind of mystification.

I suspect that the main reason for this rather strange situation in modern physics is the great attraction of romantic spirit of orthodox quantum philosophy. All these nonreal things, wave-particle dualism, collapse, nonlocality, irreducible randomness, were attractive for some of creators as well as further quantum generations. These are different stories to discuss merely mathematical modification of real-continuous model of classical statistical mechanics or to declare scientific revolution. So orthodox Copenhagen interpretation was a kind of romantic stage in the development of physics. It is always not easy argue against romanticism. Probably we need not do this. I hope that realism (as the history of literature shows us) would (sooner or later) come to physics.

I would like to thank J. Bub, P. Lahti, W. De Baere, A. Plotnitsky and I. Pitowsky for extended discussions on the interpretation of quantum mechanics. Despite their criticism, these discussions were very important for me; in particular, to understand better Copenhagen Interpretation (or it would be better to say Copenhagen Interpretations) of quantum mechanics. Especially important role played discussions with A. Plotnitsky including deep analysis of Bohr’s and Heisenberg’s views. I would like to thank him for his long ‘private lectures’ on Bohr’s complementarity. On the other hand, it is important to underline that my views presented in this paper strongly differ from his views (that are presented e.g. in [5]).

We underline that our analysis of views of N. Bohr and H. Heisenberg demonstrated that by using the statistical interpretation of the principle of complementarity and uncertainty relations we escape the contradiction between the quantum formalism and realism. Statistical approach to Bohr’s contextualism does not contradict to the possibility to construct a realist
prequantum model. And recently I, finally, constructed such a model (in spite of all no-go theorems – e.g., von Neumann, Kochen and Specker,..., Bell,...), see [27]. In our model both classical and quantum spaces are rough images of the fundamental prespace. Quantum mechanics cannot be reduced to classical one. But the realist prespace model (inducing both classical and quantum representations of reality) can be constructed. In such a model both classical and quantum representations induce reductions of prespace information.

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