Lifts of Almost $r$-Contact and $r$-Paracontact Structures

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Abstract
In this study, taking into considering lifting theory, we shall obtain both almost complex and paracomplex structures on the tangent bundle, based on almost Lorentzian $r$-contact and $r$-paracontact manifold.

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1 Introduction
In modern differentiable geometry, lifting theory has a considerable position. Because, using lift method it is possible to generalize to differentiable structures on any manifold to its extensions. Also, Vertical, complete and horizontal lifts of functions, vector fields, 1-forms and other tensor fields defined on any (complex) manifold $M$ to tangent bundle $TM$ has been obtained and the basic classical results can be traced in the well-known papers [1-8]. It is introduced almost $r$-contact structures on the tangent bundle [9].

The paper is structured as follows. In section 2, we recall complex, paracomplex, contact and Lorentzian differential structures [9-14]. In section 3, we give vertical, complete and horizontal lifts of complex (or paracomplex)
structures [9]. In section 4, we produce almost complex (or paracomplex) structures by means of complete and horizontal lifts of almost Lorentzian r-contact (or r−paracontact) structure on almost Lorentzian r-contact (or r−paracontact) manifold. In the finally section, we see as to what kind of structure is defined on the tangent bundle $T(M)$ when we consider an almost Lorentzian r− contact (or r−paracontact) structure on the base manifold.

Along this paper, all mappings and manifolds will be understood to be of class differentiable and the sum is taken over repeated indices. In this study we denote by $\mathcal{F}_r^s(M)$ the set of all tensor fields of class $C^\infty$ and of type $(r,s)$ in $M$. We now put $\mathcal{F}(M) = \sum_{r,s=1}^{\infty} \mathcal{F}_r^s(M)$; which is the set of all tensor fields in $M$. For example $\mathcal{F}_0^0(M)$, $\mathcal{F}_1^0(M)$, $\mathcal{F}_0^1(M)$ and $\mathcal{F}_1^1(M)$ are the set of functions, vector fields, 1-forms and tensor fields of type (1,1) on $M$, respectively. We give by $F(M)$ to $\mathcal{F}_0^0(M)$, by $\chi(M)$ to $\mathcal{F}_1^0(M)$ and by $\chi^*(M)$ to $\mathcal{F}_0^1(M)$. Similarly, we respectively denote by $\mathcal{F}_r^s(T(M))$ and $\mathcal{F}(T(M))$ the corresponding sets of tensor fields in the tangent bundle $T(M)$. Also $v$, $c$ and $h$ will denote the vertical, complete and horizontal lifts to $TM$ of geometric structures on $M$, respectively.

1.1 Complex and Paracomplex manifolds

Let $TM$ be tangent bundle of $m$- dimensional a differential manifold $M$. A tensor field $J$ on $TM$ is called an almost complex (or paracomplex) structure on $TM$ if at every point $p$ of $TM$, $J$ is endomorphism of the tangent space $T_pM$ such that $J^2 = -I$ (or $J^2 = I$), i.e., shortly it can shown by $J^2 = \epsilon I$. For $\epsilon = -1$ (or $\epsilon = 1$), a manifold $M$ with fixed almost complex (or paracomplex) structure $J$ is called almost complex (or paracomplex) manifold.

Let $(x^i, y^i)$ be a real coordinate system on a neighborhood $U$ of any point $p$ of $TM$. In this case, it is respectively defined by $\{\frac{\partial}{\partial x^i}|_p, \frac{\partial}{\partial y^i}|_p\}$ and $\{dx^i|_p, dy^i|_p\}$ natural bases over $R$ of tangent space $T_p(M)$ and cotangent space $T_p^*(M)$ of $M$. Then $TM$ is called complex (or paracomplex) manifold if there exists an open covering $\{U\}$ of $M$ satisfying the following condition: There is a local coordinate system $(x^i, y^i)$ on each $U$, such that

$$J(\frac{\partial}{\partial x^i}) = \frac{\partial}{\partial y^i}, \quad J(\frac{\partial}{\partial y^i}) = \epsilon \frac{\partial}{\partial x^i},$$

for each point of $U$.

Let $(z^i, \overline{z}^i)$ be an complex (or paracomplex) local coordinate system on a neighborhood $U$ of any point $p$ of $M$ where $z^i = x^i + i y^i$, $i^2 = \epsilon$. Therefore this
coordinates are defined by locally $x^i, y^i : M \to \mathbb{R}^m, \forall A, \overline{A} \in M$, $A = a^i + ib^i, x^i(A) = a^i, y^i(A) = b^i$ and $z^i, \overline{z}^i : M \to \mathbb{C}^m, \overline{z}^i(A) = A, z^i(A) = \overline{A}$, where $\mathbb{R}$ is a real number, $\mathbb{C}_\epsilon$ is a complex (or paracomplex) number.

We define the vector fields
\[
\frac{\partial}{\partial z^i}|_p = \frac{1}{2}\left\{\frac{\partial}{\partial x^i}|_p - i\frac{\partial}{\partial y^i}|_p\right\}, \quad \frac{\partial}{\partial \overline{z}^i}|_p = \frac{1}{2}\left\{\frac{\partial}{\partial x^i}|_p + i\frac{\partial}{\partial y^i}|_p\right\}
\]
(2)

and the dual covector fields
\[
dz^i|_p = dx^i|_p + i\,dy^i|_p, \quad d\overline{z}^i|_p = dx^i|_p - i\,dy^i|_p
\]
(3)

which represent bases of the tangent space $T_p(M)$ and cotangent space $T^*_p(M)$ of $M$ respectively. Then the endomorphism $J$ is given as:
\[
J\left(\frac{\partial}{\partial z^i}\right) = -\epsilon i\frac{\partial}{\partial z^i}, \quad J\left(\frac{\partial}{\partial \overline{z}^i}\right) = i\epsilon \frac{\partial}{\partial \overline{z}^i}.
\]
(4)

The dual endomorphism $J^*$ of the cotangent space $T^*_p(M)$ of manifold $M$ satisfies $J^*J = \epsilon I$ and is defined by
\[
J^*(dz^i) = -\epsilon i dz^i, \quad J^*(d\overline{z}^i) = \epsilon i d\overline{z}^i.
\]
(5)

1.2 Almost Contact and Paracontact Manifolds

Let $\overline{M}$ be an $n$-dimensional differentiable manifold. If there exist on $\overline{M}$ a $(1,1)$-tensor field $\varphi$, a vector field $\xi$ and a 1-form $\eta$ satisfying
\[
\eta(\xi) = 1, \quad \varphi^2 = \epsilon I - \eta \otimes \xi,
\]
where $I$ is the identity, then $\overline{M}$ is said to be an almost contact (or paracontact) manifold for $\epsilon = -1$ (or $\epsilon = 1$). In the almost contact (or paracontact) manifold, the following relations hold good:
\[
\varphi(\xi) = 0, \quad \eta \circ \varphi = 0, \quad rank(\varphi) = n - 1.
\]
(7)

Every almost contact (or paracontact) manifold has a positive definite Riemannian metric $G$ such that
\[
\eta(\overline{X}) = G(\xi, \overline{X}), \quad G(\varphi \overline{X}, \varphi \overline{Y}) = G(\overline{X}, \overline{Y}) - \eta(\overline{X})\eta(\overline{Y}), \quad \overline{X}, \overline{Y} \in \chi(\overline{M})
\]
(8)
where $\chi(M)$ denotes the set of differentiable vector fields on $M$. In this case, we say that $M$ has an almost contact (or paracontact) Riemannian structure $(\varphi, \xi, \eta, G)$ and $M$ is said to be an almost contact (or paracontact) Riemannian manifold.

This structure generalizes as follows.

1.3 Almost r-Contact and r-Paracontact Manifolds

One takes care of a tensor field $F$ of type $(1,1)$ on a manifold $M$ of dimension $(2n + r)$. If there exists on $M$ the vector fields $(\xi_\alpha)$ and the 1-forms $(\eta_\alpha)$ such that

$$\eta_\alpha(\xi_\beta) = \delta_\alpha^\beta, \quad F(\xi_\alpha) = 0, \quad F^2 = \epsilon I + \sum_{\alpha=1}^{r} \xi_\alpha \otimes \eta_\alpha,$$

then the structure $(F, \xi_\alpha, \eta_\alpha)$ is an almost $r$-contact (or $r$-paracontact) structure, where $(\alpha, \beta = 1, 2, ..., r)$ and $\delta_\beta^\alpha$ denotes Kronecker delta. A manifold $M$ endowed with $(F, \xi_\alpha, \eta_\alpha)$-structure is called an almost $r$-contact (or $r$-paracontact) manifold.

1.4 Almost Lorentzian Contact and Paracontact Manifolds

Let $M$ be an $n$-dimensional differentiable manifold equipped with a triple $(\varphi, \xi, \eta)$, where $\varphi$ is a (1,1)-tensor field, $\xi$ a vector field and $\eta$ is a 1-form on $M$ satisfying

$$\eta(\xi) = -1, \quad \varphi^2 = \epsilon I + \eta \otimes \xi,$$

where $I$ is the identity. In manifold $M$, the following relations hold good:

$$\varphi(\xi) = 0, \quad \eta \circ \varphi = 0, \quad rank(\varphi) = n - 1.$$  \hfill (11)

If $M$ has a Lorentzian metric $G$ such that

$$G(\varphi X, \varphi Y) = G(X, Y) + \eta(X)\eta(Y), \quad X, Y \in \chi(M),$$

we say that $M$ has a Lorentzian almost contact (or paracontact) structure $(\varphi, \xi, \eta, G)$ and $M$ is said to be a Lorentzian almost contact (or paracontact) manifold.

This structure extends the following as:
1.5 Almost Lorentzian r-Contact and r-Paracontact Manifolds

One takes care of a tensor field $F$ of type $(1,1)$ on a manifold $M$ of dimension $(2n + r)$. If there exists on $M$ the vector fields $(\xi_\alpha)$ and the 1-forms $(\eta^\alpha)$ such that

$$\eta^\alpha(\xi_\beta) = -\delta_\alpha^\beta, \quad F(\xi_\alpha) = 0, \quad F^\circ = 0, \quad F^2 = \epsilon I - \sum_{i=1}^r \xi_\alpha \otimes \eta^\alpha,$$  

(13)

then the structure $(F, \xi_\alpha, \eta^\alpha)$ is also called a Lorentzian almost $r$-contact (or $r$-paracontact) structure, where $(\alpha, \beta = 1, 2, ..., r)$ and $\delta_\alpha^\beta$ denotes Kronecker delta. A manifold $M$ endowed with $(F, \xi_\alpha, \eta^\alpha)$-structure is called a Lorentzian almost $r$-contact (or $r$-paracontact) manifold.

2 Lifts of Almost Lorentzian r-Contact and r-Paracontact Structures

In this section, we obtain almost complex (or paracomplex) structures on tangent bundle $T(M)$ of almost Lorentzian $r$-contact (or $r$-paracontact) manifold $M$ having the structure $(F, \xi_\alpha, \eta^\alpha)$.

2.1 Complete Lifts

Theorem 4.1. Let $M$ be a differentiable manifold endowed with almost $r$-contact (or $r$-paracontact) structure $(F, \xi_\alpha, \eta^\alpha)$, then

$$\tilde{J} = F^c + \sum_{\alpha=1}^r \xi_\alpha^v \otimes \eta^{\alpha v} - \xi_\alpha^c \otimes \eta^{\alpha c}$$

is almost complex (or paracomplex) structure on $T(M)$.

Proof: From (9) and the vertical and complete lifts of complex (or paracomplex) tensor fields we have

$$(F^2)^c = (\epsilon I + \sum_{\alpha=1}^r \xi_\alpha \otimes \eta_{\alpha})^c,$$  

(14)
\[(\mathbf{F}^c)^2 = \epsilon I + \sum_{\alpha=1}^{r} \xi_{\alpha}^v \otimes \eta^{\alpha c} + \xi_{\alpha}^c \otimes \eta^{\alpha v}, \quad (15)\]

and

\[\mathbf{F}^c(\xi_{\alpha}^v) = 0, \quad \mathbf{F}^c(\xi_{\alpha}^c) = 0, \quad (16)\]

\[\eta^{\alpha v} \circ \mathbf{F}^c = 0, \quad \eta^{\alpha c} \circ \mathbf{F}^v = 0, \quad \eta^{\alpha c} \circ \mathbf{F}^c = 0, \quad (17)\]

\[\eta^{\alpha v}(\xi_{\beta}^v) = 0, \quad \eta^{\alpha v}(\xi_{\beta}^c) = \delta_{\beta}^{\alpha}, \quad \eta^{\alpha c}(\xi_{\beta}^v) = \delta_{\beta}^{\alpha}, \quad \eta^{\alpha c}(\xi_{\beta}^c) = 0. \quad (18)\]

Consider a structure \(\tilde{J}\) of \(\mathfrak{I}_1(T\bar{M})\) given by

\[\tilde{J} = \mathbf{F}^c + \sum_{\alpha=1}^{r} (\xi_{\alpha}^v \otimes \eta^{\alpha v} - \xi_{\alpha}^c \otimes \eta^{\alpha c}). \quad (19)\]

Using (15) and (19) we find equation

\[(\tilde{J})^2 = (\mathbf{F}^c + \sum_{\alpha=1}^{r} (\xi_{\alpha}^v \otimes \eta^{\alpha v} - \xi_{\alpha}^c \otimes \eta^{\alpha c}))^2 \quad (20)\]

\[= \epsilon I + \sum_{\alpha=1}^{r} \left[ \xi_{\alpha}^v \otimes \eta^{\alpha c} + \xi_{\alpha}^c \otimes \eta^{\alpha v} + \mathbf{F}^c(\xi_{\alpha}^v)\eta^{\alpha v} - \mathbf{F}^c(\xi_{\alpha}^c)\eta^{\alpha c} + \eta^{\alpha v} \circ \mathbf{F}^c \xi_{\alpha}^v - (\eta^{\alpha c} \circ \mathbf{F}^v)\xi_{\alpha}^c + \xi_{\alpha}^v \otimes (\eta^{\alpha v} \circ (\xi_{\alpha}^v))\eta^{\alpha v} - \xi_{\alpha}^c \otimes (\eta^{\alpha v} \circ (\xi_{\alpha}^c))\eta^{\alpha c} - \xi_{\alpha}^c \otimes (\eta^{\alpha c} \circ (\xi_{\alpha}^v))\eta^{\alpha v} + \xi_{\alpha}^c \otimes (\eta^{\alpha c} \circ (\xi_{\alpha}^c))\eta^{\alpha c} \right]. \]

By means of (16), (17) and (18), we have

\[(\tilde{J})^2 = \epsilon I. \quad (21)\]

So, \(\tilde{J}\) is an almost complex (or paracomplex) structure in \(T(\bar{M})\). Hence the proof is completed.

Thinking lift properties of tensor fields and the equation (19), we

\[\tilde{J}X^v = (\mathbf{F}X)^v - (\eta^{\alpha}(X))^{\nu}U_{\alpha}^c\]

\[\tilde{J}X^c = (\mathbf{F}X)^c + (\eta^{\alpha}(X))^{\nu}U_{\alpha}^v - (\eta^{\alpha}(X))^{\nu}U_{\alpha}^c.\]

where \(X \in \chi(\bar{M}), X^v, X^c \in \chi(T\bar{M})\).

For example, if \(\eta^{\alpha}(X) = 0\), we obtain

\[\tilde{J}X^v = (\mathbf{F}X)^v, \quad \tilde{J}X^c = (\mathbf{F}X)^c.\]
In view of the structure \((F, \xi_\alpha, \eta^\alpha)\) and the equations (16), (18) and (19) it follows

\[
\tilde{J} \xi^v_\alpha = -\delta^\alpha_\beta \xi^c_\beta, \quad \tilde{J} \xi^c_\alpha = \delta^\alpha_\beta \xi^v_\beta, \quad \alpha, \beta = 1, 2, ..., r.
\]

**Theorem 4.2.** Let \(\overline{M}\) be a differentiable manifold endowed with almost Lorentzian \(r\)–contact (or \(r\)–paracontact) structure \((F, \xi_\alpha, \eta^\alpha)\), then an almost complex (or paracomplex) structure on \(T(\overline{M})\) is calculated by

\[
\hat{J} = F^c - \sum_{\alpha=1}^{r} \xi^v_\alpha \otimes \eta^{av} - \xi^c_\alpha \otimes \eta^{ac}.
\]

**Proof:** By means of the equation (13) and the vertical and complete lifts of complex (or paracomplex) tensor fields we have

\[
(F^2)^c = (\epsilon I - \sum_{\alpha=1}^{r} \xi_\alpha \otimes \eta_\alpha)^c, \quad (22)
\]

\[
(F^c)^2 = \epsilon I - \sum_{\alpha=1}^{r} \xi^v_\alpha \otimes \eta^{ac} + \xi^c_\alpha \otimes \eta^{av}, \quad (23)
\]

and

\[
F^c(\xi^v_\alpha) = 0, \quad F^c(\xi^c_\alpha) = 0, \quad (24)
\]

\[
\eta^{av} \circ F^c = 0, \quad \eta^{oc} \circ F^c = 0, \quad (25)
\]

\[
\eta^{av}(\xi^v_\alpha) = 0, \quad \eta^{ac}(\xi^v_\alpha) = -\delta^\alpha_\beta, \quad \eta^{ac}(\xi^c_\beta) = 0. \quad (26)
\]

Take a structure \(\hat{J}\) of \(\mathfrak{g}_1^1(T\overline{M})\) defined by

\[
\hat{J} = F^c - \sum_{\alpha=1}^{r} (\xi^v_\alpha \otimes \eta^{av} - \xi^c_\alpha \otimes \eta^{ac}) \quad (27)
\]

Similarly proof of **Theorem 4.1**, using (15), (16), (17) and (18) and (19) we have the equation

\[
(\hat{J})^2 = \epsilon I. \quad (28)
\]

Thus, for \(\hat{J}\) is an almost complex (or paracomplex) structure in \(T(\overline{M})\), the proof is completed.

Considering lift properties of tensor fields and the equation (27), we have

\[
\hat{J}X^v = (FX)^v + (\eta^\alpha(X))^v \xi^c_\alpha
\]

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\[ \hat{J} X^c = (FX)^c - (\eta^\alpha(X))^v \xi^v_\alpha + (\eta^\alpha(X))^c \xi^c_\alpha. \]

where \( X \in \chi(\overline{M}), X^v, X^c \in \chi(T\overline{M}). \)

For example, if \( \eta^\alpha(X) = 0 \), we obtain

\[ \hat{J} X^v = (FX)^v, \hat{J} X^c = (FX)^c. \]

In view of the structure \((F, \xi_\alpha, \eta^\alpha)\) and the equations \((24),(26)\) and \((27)\) it follows

\[ \hat{J} \xi^v_\alpha = \delta^\alpha_\beta \xi^c_\beta = \xi^c_\alpha, \quad \hat{J} \xi^c_\alpha = \delta^\alpha_\beta \xi^v_\beta, \quad \alpha, \beta = 1, 2, ..., r. \]

### 2.2 Horizontal Lifts

**Theorem 4.3.** Let \((F, \xi_\alpha, \eta^\alpha)\) be an almost \(r\)-paracontact (or \(r\)-paracontact) structure in \(\overline{M}\) with an affine connection \(\nabla\). Then an almost complex (or paracomplex) in \(T(\overline{M})\) is given by

\[ \tilde{J}^* = F^h + r \sum_{\alpha=1}^{r} (\xi^h_\alpha \otimes \eta^{\alpha v} - \xi^v_\alpha \otimes \eta^{\alpha h}). \]

**Proof:** Taking into consideration the equation given by \((9)\) and the horizontal lifts of complex (or paracomplex) tensor fields, we have

\[ (F^2)^h = (\epsilon I + \sum_{\alpha=1}^{r} \xi^h_\alpha \otimes \eta^{\alpha h}), \quad (29) \]

\[ (F^h)^2 = \epsilon I + \sum_{\alpha=1}^{r} \xi^h_\alpha \otimes \eta^{\alpha v} + \xi^v_\alpha \otimes \eta^{\alpha h}, \quad (30) \]

and

\[ F^h(\xi^h_\alpha) = 0, \quad F^h(\xi^v_\alpha) = 0, \quad (31) \]

\[ \eta^{\alpha h} \circ F^h = 0, \quad \eta^{\alpha v} \circ F^h = 0. \quad (32) \]

\[ \eta^{\alpha h}(\xi^h_\beta) = 0, \quad \eta^{\alpha v}(\xi^v_\beta) = \delta^\alpha_\beta, \quad \eta^{\alpha v}(\xi^h_\beta) = \delta^\alpha_\beta. \quad (33) \]

Given an element \(\tilde{J}\) of \(\mathfrak{Z}_1(T\overline{M})\) defined by

\[ \tilde{J}^* = F^h + \sum_{\alpha=1}^{r} (\xi^v_\alpha \otimes \eta^{\alpha v} - \xi^h_\alpha \otimes \eta^{\alpha h}). \quad (34) \]

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Thinking of the above equations, it is clear that

\[(\tilde{J}^*)^2 = \epsilon I.\] (35)

Finally, \(\tilde{J}^*\) is an almost complex (or paracomplex) structure in \(T(\overline{M})\). Thus the theorem is proved.

Taking care of lift properties of tensor fields and the equation (34), we

\[\tilde{J}^*X^v = (FX)^v - (\eta^\alpha(X))^v\xi^h_\alpha\]

\[\tilde{J}^*X^h = (FX)^h + (\eta^\alpha(X))^h\xi^v_\alpha - (\eta^\alpha(X))^h\xi^h_\alpha.\]

where \(X \in \chi(\overline{M}), X^v, X^h \in \chi(T\overline{M}).\)

For example, if \(\eta^\alpha(X) = 0\), we obtain

\[\tilde{J}^*X^v = (FX)^v, \tilde{J}^*X^h = (FX)^h.\]

In view of the structure \((F, \xi_\alpha, \eta^\alpha)\) and the equations (31), (33) and (35) it follows

\[\tilde{J}^*\xi^v_\alpha = \delta^\alpha_\beta\xi^h_\beta = \xi^h_\alpha, \quad \tilde{J}^*\xi^h_\alpha = \delta^\alpha_\beta\xi^v_\beta = \xi^v_\alpha, \quad \alpha, \beta = 1, 2, ..., r.\]

**Theorem 4.4.** Let \((F, \xi_\alpha, \eta^\alpha)\) be an almost Lorentzian \(r\)–paracontact (or \(r\)–paracontact) structure in \(\overline{M}\) with an affine connection \(\nabla\). Then structure

\[\tilde{J}^* = F^h - \sum_{\alpha=1}^{r} \xi^v_\alpha \otimes \eta^\alpha v - \xi^h_\alpha \otimes \eta^\alpha h.\]

is an almost complex (or paracomplex) in \(T(\overline{M})\).

**Proof:** It can easily shown to taking care of the proofs of the above.

3 Corollary

Taking into consideration the above theorems, we conclude that when we consider an almost Lorentzian \(r\)–contact (or \(r\)-paracontact) structure on the base manifold, the structure defined on the tangent bundle \(T(\overline{M})\) is an almost complex (or paracomplex).
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