Phantom Behavior Bounce with Tachyon and Non-minimal Derivative Coupling

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Abstract

The bouncing cosmology provides a successful solution of the cosmological singularity problem. In this paper, we study the bouncing behavior of a single scalar field model with tachyon field non-minimally coupled to itself, its derivative and to the curvature. By utilizing the numerical calculations we will show that the bouncing solution can appear in the universe dominated by such a quintom matter with equation of state crossing the phantom divide line. We also investigate the classical stability of our model using the phase velocity of the homogeneous perturbations of the tachyon scalar field.

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1 Introduction

Cosmological observations have revealed that our universe has experienced a phase of accelerated expansion in the early-time known as inflation [1]. Although inflation has great successes such as solving the flatness, horizon and homogeneity problems [2], but it suffers from the initial singularity problem and hence cannot give complete descriptions of the early universe.

A successful solution of the cosmological singularity problem may be provided by non-singular bouncing cosmology which has been proposed a long time ago [3]. In order to circumvent the Big-Bang singularity problem one needs to choose one of the following approaches: 1) modified gravity [4], 2) using a kind of matter field that violates the null energy condition (see [5] for review). Through the modified gravity approach various scenarios have been constructed such as the brane world models [6], Pre-Big-Bang [7] and the Ekpyrotic models [8], \( f(T) \) gravity in which gravity is described by an arbitrary function of the torsion scalar [9], higher derivative gravity actions [10], non-relativistic gravitational actions [11, 12] or loop quantum cosmology [13]. Bouncing cosmologies can also be inspired by string theory when the string gas cosmology [14, 15] embed in a bouncing universe [16].

Furthermore, Bouncing cosmology for a homogeneous and isotropic universe can lead to the matter bounce scenario. Non-conventional fluids [17, 18], non-minimally coupled scalar fields to gravity [19] and ghost condensates [20-22] are belong to this sort of bouncing cosmology. The simplest possibility to obtain a bounce is a massive scalar field in the presence of a positive scalar curvature [23] and the other type of the matter bounce cosmology is the quintom bounce model [24, 25].

Moreover, various observational data such as SNe Ia Gold data set [26] confirmed that the effective equation of state (EoS) parameter \( \omega \) (the ratio of the effective pressure of the universe to the effective energy density of it) cross \(-1\), namely, the phantom divide line, currently or in the past.

The scenarios with phantom behavior are designed to understand the nature of dark energy with \( \omega \) across \(-1\). To realize a viable scenario with phantom behavior one needs to introduce extra degree of freedom to the conventional theory with a single fluid or a single scalar field. The fact that the phantom behavior can be achieved without ghosts in scalar-tensor gravity was first shown in [27]. Also, quintom scenario which realizes the phantom behavior first introduced in Ref. [28] with two scalar fields (quintessence and phantom). Another attempts for constructing a model that shows phantom behavior are as follows: scalar field model with non-linear kinetic terms [29] or a non-linear higher-derivative one [30], braneworld models [31], phantom coupled to dark matter with an appropriate coupling [32], string inspired models [33], non-local gravity [34], modified gravity models [35] and also non-minimally coupled scalar field models in which scalar field couples with scalar curvature, Gauss-Bonnet invariant or modified \( f(R) \) gravity [36-38] (for a detailed review, see [39]). Crossing of the phantom divide can also be realized with single imperfect fluid [40] or by a constrained single degree of freedom dust like fluids [41].

Moreover, non-minimal couplings are generated by quantum corrections to the scalar field theory and they are essential for the renormalizability of the scalar field theory in curved
space (see [42] and references therein). One can extend the non-minimally coupled scalar tensor theories, allowing for non-minimal coupling between the derivatives of the scalar fields and the curvature [43]. A model with non-minimal derivative coupling was proposed in [43-45] and interesting cosmological behaviors of such a model in inflationary cosmology [46], quintessence and phantom cosmology [47, 48], asymptotic solutions and restrictions on the coupling parameter [49] have been widely studied in the literature. General non-minimal coupling of a scalar field and kinetic term to the curvature as a source of dark energy has been analyzed in [50]. Also, non-minimal coupling of modified gravity and kinetic part of Lagrangian of a massless scalar field has been investigated in [51]. It has been shown that inflation and late-time cosmic acceleration of the universe can be realized in such a model.

In this paper we consider a model with an explicit coupling between the scalar field, the derivative of the scalar field and the curvature and study the bouncing solution in such a model. We are interested in our analysis to the case of tachyon scalar field. The tachyon field in the world volume theory of the open string stretched between a D-brane and an anti-D-brane or a non-BPS D-brane plays the role of scalar field in the context of string theory [52]. What distinguishes the tachyon Lagrangian from the standard Klein-Gordan form for scalar field is that the tachyon action has a non-standard type namely, Dirac-Born-Infeld form [53]. Our motivation for investigating a model with non-minimal derivative coupling and tachyon scalar field is coming from a fundamental theory such as string/superstring theory and it may provide a possible approach to quantum gravity from a perturbative point of view [54-56].

An outline of the present work is as follows: In section 2 we introduce our model in which the tachyon field plays the role of scalar field and the non-minimal coupling between scalar field, the derivative of scalar field and Einstein tensor is also present in the action. Then, we derive the field equations as well as the energy density and pressure of the model in order to study the bouncing behavior of our model. We obtain the condition required for bouncing solution of our model and then we show that such a condition can be satisfied numerically. In Section 3 we discuss the classical stability of our model. Section 4 is devoted to our conclusions.

2 Bouncing behavior of non-minimal derivative coupling of tachyon gravity

In this section we review the field equations of tachyon gravity with non-minimal derivative coupling and provide the cosmological equations in a Friedmann-Robertson-Walker (FRW) universe. Then, we obtain the necessary conditions required for a successful bounce in such a model. Our starting point is the following Born-Infeld type action for tachyon field with non-minimal derivative coupling and also with itself,

$$S = \int d^4x \sqrt{-g} \left[ \frac{1}{2\kappa^2} (R - 2\Lambda) - V(\phi) \sqrt{1 + g^{\mu\nu} \partial_\mu \phi \partial_\nu \phi} + \xi f(\phi) G_{\mu\nu} \partial_\mu \phi \partial_\nu \phi \right], \quad (1)$$
where $\kappa^2 = 8\pi G = \frac{1}{M_{Pl}^2}$ while $G$ is a bare gravitational constant and $M_{Pl}$ is a reduced Planck mass, $\Lambda$ is the cosmological constant and $V(\phi)$ is the tachyon potential which is bounded and reaching its minimum asymptotically. $f(\phi)$ is a general function of the tachyon field $\phi$ and $\xi$ is coupling constant. The models of kind (1) with non-minimal coupling between derivatives of a scalar field and curvature are the extension of scalar-tensor theories. Such a non-minimal coupling may appear in some Kaluza-Klein theories [57, 58]. In Ref. [43], Amendola has considered a model with non-minimal coupling between derivative of scalar field and the Ricci scalar, $\xi R \partial_\mu \phi \partial^\mu \phi$, and by using generalized slow-roll approximations, he has obtained some inflationary solutions of this model.

A general model containing two derivative coupling terms $\xi_1 R \partial_\mu \phi \partial^\mu \phi$ and $\xi_2 R_{\mu \nu} \partial^\mu \phi \partial^\nu \phi$, has been studied in [44, 45]. It was shown in [47] that field equations of this theory are of third order in $g_{\mu \nu}$ and $\phi$, but in the special case where $-2\xi_1 = \xi_2 = \xi$ the order of equations are reduced to the second order. This particular choice of $\xi_1$ and $\xi_2$ leads to the non-minimal coupling between derivative of scalar field and the Einstein tensor, $\xi G_{\mu \nu} \partial_\mu \phi \partial_\nu \phi$. Sushkov in [47] has obtained the exact cosmological solutions of this theory and he has concluded that such a model is able to explain a quasi-de sitter phase as well as an exit from it without any fine-tuned potential.

Let us now present the cosmological equations. Variation of action (1) with respect to the metric tensor $g_{\mu \nu}$ gives,

$$R_{\mu \nu} - \frac{1}{2} g_{\mu \nu} R + g_{\mu \nu} \Lambda = \kappa^2 (T_{\mu \nu} + \xi T'_{\mu \nu}),$$

(2)

where

$$T_{\mu \nu} = V(\phi) \left( \frac{\nabla_\mu \phi \nabla_\nu \phi}{u} - g_{\mu \nu} u \right),$$

(3)

and

$$T'_{\mu \nu} = R(\nabla_\mu \phi \nabla_\nu \phi) - 4 \nabla_\gamma \phi \nabla_\mu \phi \nabla_\nu \phi + G_{\mu \nu} (\nabla \phi)^2 - 2 R_{\mu \nu \gamma \lambda} \nabla^\gamma \phi \nabla^\lambda \phi - 2 \nabla_\mu \nabla_\nu \phi \nabla_\phi \nabla_\phi$$

$$+ 2 \nabla_\mu \nabla_\phi \phi \nabla \phi + g_{\mu \nu} \left( \nabla^\gamma \nabla^\lambda \phi \nabla_\gamma \nabla_\lambda \phi - (\nabla \phi)^2 \right) + 2 R^\gamma \lambda \nabla_\gamma \phi \nabla_\lambda \phi,$$

(4)

where $u = \sqrt{1 + \nabla_\mu \phi \nabla^\mu \phi}$.

One can obtain the scalar field equation of motion by varying (1) with respect to $\phi$,

$$\nabla_\mu \left( \frac{V(\phi) \nabla^\mu \phi}{u} \right) - \frac{dV(\phi)}{d\phi} u - \xi f(\phi) G^{\mu \nu} \nabla_\mu \nabla_\nu \phi + \xi \frac{df(\phi)}{d\phi} G_{\mu \nu} \partial^\mu \phi \partial^\nu \phi = 0.$$

(5)

For a flat FRW metric,

$$ds^2 = -dt^2 + a^2(t)(dr^2 + r^2 d\Omega^2),$$

(6)

the components of the Ricci tensor $R_{\mu \nu}$ and the Ricci scalar $R$ are given by

$$R_{00} = -3(\dot{H} + H^2), \quad R_{ij} = a^2(t)(\dot{H} + 3H^2) \delta_{ij}, \quad R = 6(\dot{H} + 2H^2),$$

(7)
where $H = \frac{\dot{a}(t)}{a(t)}$ is the Hubble parameter and $a(t)$ is the scale factor. The scalar field equation of motion for a homogeneous $\phi$ in FRW background (6) takes the following form

$$\frac{\ddot{\phi}}{1 - \dot{\phi}^2} + 3H \dot{\phi} + \frac{1}{V(\phi)} \frac{dV}{d\phi}$$

$$+ \frac{\sqrt{1 - \dot{\phi}^2}}{V(\phi)} \left( 3\xi H^2 \left( 2f(\phi)\dot{\phi} + \frac{df}{d\phi} \dot{\phi}^2 \right) + 18\xi H^3 f(\phi) \dot{\phi} + 12\xi H \dot{H} f(\phi) \dot{\phi} \right) = 0. \quad (8)$$

From equation (2) the energy density and pressure are as follows respectively,

$$\rho = \frac{V(\phi)}{\sqrt{1 - \dot{\phi}^2}} + 9\xi H^2 f(\phi) \dot{\phi}^2 + \frac{\Lambda}{\kappa^2}, \quad (9)$$

and

$$P = -V(\phi) \sqrt{1 - \dot{\phi}^2} - \xi \left( 3H^2 + 2\dot{H} \right) f(\phi) \dot{\phi}^2 - 2\xi H \left( 2f(\phi)\ddot{\phi} + \frac{df}{d\phi} \dot{\phi}^3 \right) - \frac{\Lambda}{\kappa^2}. \quad (10)$$

Friedmann equation is also as follows,

$$H^2 = \frac{\kappa^2}{3} \left( \frac{V(\phi)}{\sqrt{1 - \dot{\phi}^2}} + 9\xi H^2 f(\phi) \dot{\phi}^2 \right) + \frac{\Lambda}{3}. \quad (11)$$

Moreover, a bouncing universe starts with an initial contraction to a non-vanishing minimal radius, then subsequent an expanding phase [24]. During the contracting phase, the scale factor $a(t)$ is decreasing, i.e., $\dot{a} < 0$, and in the expanding phase we have $\dot{a} > 0$. At the bouncing point, $\dot{a} = 0$, and around this point $\ddot{a} > 0$ for a period of time. Equivalently in the bouncing cosmology the Hubble parameter $H$ runs across zero from $H < 0$ to $H > 0$ and $H = 0$ at the bouncing point. A successful bounce requires around this point,

$$\dot{H} = -\frac{\kappa^2}{2} (\rho + P) = -\frac{\kappa^2}{2} \rho (1 + \omega) > 0. \quad (12)$$

As it is clear from equation (12), a successful non-singular bouncing cosmology violates the null energy condition (NEC) around the bouncing point. In addition, in order to achieve the hot Big-Bang era after the bouncing, the EoS of the universe, $\omega$, must cross the so called phantom divide line $\omega = -1$.

A salient feature of our model (1) is that it realizes the phantom divide line crossing [59]. In Ref. [59] we have presented the conditions required for such a crossing and numerically shown that this model realizes crossing of the $\omega = -1$ for a special case of the Hubble parameter $H = \frac{h_0}{t}$ with constant $h_0$. In the present work we show that crossing over $-1$ can take place in our model for a general Hubble parameter obtained from equation (11).

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3The NEC for a null vector $k^\mu$, implies $T_{\mu\nu}k^\mu k^\nu \geq 0$ which for a perfect fluid reads $\rho + P \geq 0$. 


To study the bouncing behavior of the present model we use the equation (12). From equations (9), (10) and (12) one can see that a successful bounce requires:

$$\frac{V(\phi)}{\sqrt{1 - \dot{\phi}^2}} + 6\xi H^2 f(\phi)\dot{\phi}^2 - 2\xi H f(\phi)\dot{\phi}^2 - 2\xi H \left(2 f(\phi)\ddot{\phi} + \frac{df}{d\phi} \dot{\phi}^3\right) < 0. \quad (13)$$

We will show below that our model can easily fulfilled the above bouncing condition. Note that in the numerical study on the bouncing solution, we have used the following state for Hubble parameter obtained from equation (11):

$$H^2 = \frac{1}{\left(\frac{3}{k^2} - 9\xi f(\phi)\dot{\phi}^2\right)\left(\frac{V(\phi)}{\sqrt{1 - \dot{\phi}^2}} + \frac{\Lambda}{\kappa^2}\right)}. \quad (14)$$

In figures 1 and 2, we show the bouncing solution for two different potentials. Figure 1 shows such a bouncing solution for $V(\phi) = V_0 e^{-\alpha \phi^2}$ with constant $\alpha$. One can see from this figure that the model predicts crossing of the $\omega = -1$, which gives rise to a possible inflationary phase after the bounce. Also, the Hubble parameter evolution as a function of cosmic time has been shown in figure 1. One can read from this figure that the Hubble parameter $H$ running across zero at $t = 0$ which have choosed it, as the bouncing point. In figure 2, we consider the model with potential $V(\phi) = \frac{V_0}{\phi^2}$ and then show another example of the bouncing behavior of the model. This figure show the crossing over $-1$ for the EoS again. In the numerical calculations we have used the function $f(\phi) = b\phi^n$ with constants $b$ and $n$.

![Figure 1: Plots of the evolution of the EoS and the Hubble parameter versus $t$ for the potential $V(\phi) = V_0 e^{-\alpha \phi^2}$, $\phi = \phi_0 t$, $f(\phi) = b\phi^n$ and negative cosmological constant $\Lambda = -0.05$, (with $\xi = 10$, $b = 1$, $n = 8$, $V_0 = 0.04$, $\phi_0 = 0.6$ and $\alpha = 5$).](image-url)
Figure 2: Plots of the evolution of the EoS and the Hubble parameter versus $t$ for the potential $V(\phi) = \frac{V_0}{\phi^2}$, $\phi = \phi_0 t$, $f(\phi) = b\phi^n$ and $\Lambda = -0.05$, (with $\xi = 10$, $b = 1$, $n = 8$, $V_0 = 0.04$ and $\phi_0 = 0.6$).

3 Stability of the model

Now we discuss on the stability of the model. The sound speed, $C_s^2$, is the function appearing before spatial gradients in the scalar field equation of motion and for perturbations around homogeneous solutions, it is a function of time. The sound speed express the phase velocity of the inhomogeneous perturbations of the tachyon field [60]. The absence of gradient instabilities requires $C_s^2 \geq 0$, where

$$C_s^2 = \frac{P'}{\rho'},$$

(15)

where a prim denotes derivative with respect to $\dot{\phi}^2$. By using equations (9), (10) and (14), the sound speed parameter reads,

$$C_s^2 = \frac{2k^2}{3\left(\frac{3}{k^2} - 27\xi f(\phi)\dot{\phi}^2 + 18\xi f(\phi)\right) + \frac{135\xi^2 f^2(\phi)\dot{\phi}^2}{2\left(\frac{3}{k^2} - 9\xi f(\phi)\dot{\phi}^2\right)^2(1 - \dot{\phi}^2)^3}} \left(\frac{3\xi f(\phi)\dot{\phi}}{\left(\frac{3}{k^2} - 9\xi f(\phi)\dot{\phi}^2\right)^2(1 - \dot{\phi}^2)^4} - \frac{21\xi f(\phi)\dot{\phi}^4\dot{\phi}}{4(1 - \dot{\phi}^2)} + 6\xi f(\phi)\ddot{\phi}(1 - \dot{\phi}^2) - \frac{3}{2}\xi f(\phi)\dot{\phi}\dot{\phi}\ddot{\phi}\right)$$

$$+ \frac{135\xi^2 f^2(\phi)\dot{\phi}^2}{2\left(\frac{3}{k^2} - 9\xi f(\phi)\dot{\phi}^2\right)^2(1 - \dot{\phi}^2)^3} \left(\frac{3\xi f(\phi)\dot{\phi}}{\left(\frac{3}{k^2} - 9\xi f(\phi)\dot{\phi}^2\right)^2(1 - \dot{\phi}^2)^4} - \frac{21\xi f(\phi)\dot{\phi}^4\dot{\phi}}{4(1 - \dot{\phi}^2)} + 6\xi f(\phi)\ddot{\phi}(1 - \dot{\phi}^2) - \frac{3}{2}\xi f(\phi)\dot{\phi}\dot{\phi}\ddot{\phi}\right)$$

$$+ 6\xi f(\phi)\ddot{\phi}(1 - \dot{\phi}^2) + 27\xi^2 f^2(\phi)\dot{\phi}^2(1 - \dot{\phi}^2) + 3\xi f(\phi)\left(\frac{3}{k^2} - 9\xi f(\phi)\dot{\phi}^2\right)(1 - \frac{1}{2}\dot{\phi}^2)$$
\[-\frac{1}{2\sqrt{(1 - \dot{\phi}^2)}} + \frac{1}{\sqrt{V(\phi)}} \left( \xi \dot{\phi} \left( \frac{3}{k^2} - 9\xi f(\phi)\dot{\phi}^2 \right)^{\frac{1}{2}} (1 - \dot{\phi}^2)^{\frac{3}{2}} \left( \frac{1}{2} \dot{\phi}^2 \frac{df}{d\phi} + f(\phi)\ddot{\phi} + 2(1 - \dot{\phi}^2) \frac{df}{d\phi} \right) \\
+ 9\xi^2 f(\phi)\dot{\phi} \left( \frac{3}{k^2} - 9\xi f(\phi)\dot{\phi}^2 \right)^{\frac{1}{2}} (1 - \dot{\phi}^2)^{\frac{3}{2}} \left( \dot{\phi}^2 \frac{df}{d\phi} + 2f(\phi)\ddot{\phi} \right) \right) \\
+ \frac{1}{V(\phi)} \left( \frac{\xi f(\phi)\dot{\phi} \frac{dV(\phi)}{d\phi}}{(1 - \dot{\phi}^2)^{\frac{3}{2}}} \right) \left( 1 - \frac{\dot{\phi}^2}{4} + \frac{27\xi f(\phi)\dot{\phi}^2 (1 - \dot{\phi}^2)}{2(\frac{3}{k^2} - 9\xi f(\phi)\dot{\phi}^2)} \right). \tag{16} \]

This relation is very complicated to find exact analytical conditions for the stability of the model. So, we perform the numerical computations to show the evolution of the $C_s^2$. In figure 3, we have plotted the $C_s^2$ for the models considered in this paper for the numerical calculations shown in figures 1 and 2. From this figure, we can see that the sound speed parameter is positive throughout the bouncing phase.

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{sound_speed.png}
\caption{Plots of the sound speeds versus $t$, (left for the potential $V(\phi) = V_0 e^{-\alpha \phi^2}$ and right for the potential $V(\phi) = \frac{V_0}{\phi^2}$), $\phi = \phi_0 t$ and $f(\phi) = b\phi^n$, (with $\xi = 10$, $b = 1$, $n = 8$, $V_0 = 4$, $\phi_0 = 0.5$ and $\alpha = 5$).}
\end{figure}

Moreover, we should expect that the coefficient of $\ddot{\phi}$ in equation (8), is responsible for the presence or absence of ghosts. We show this coefficient by $D$ (for more details see [61]). In order to ensure that the perturbation of $\phi$ are not ghosts, one needs $D > 0$, where $D$ is
as follows,

\[ D = \frac{V(\phi)}{(1 - \dot{\phi}^2)^{\frac{3}{2}}} + 6\xi H^2 f(\phi). \]  

(17)

The evolution of \( D \) have been shown in figure 4 for two different potentials. It is clear from this figure that the function \( D \) is positive during the bouncing phase.

Figure 4: Plots of the \( D \) versus \( t \), (left for the potential \( V(\phi) = V_0 e^{-\alpha \phi^2} \) and right for the potential \( V(\phi) = \frac{V_0}{\phi^2} \)), \( \phi = \phi_0 t \) and \( f(\phi) = b \phi^n \), (with \( \xi = 10, b = 1, n = 8, V_0 = 4, \phi_0 = 0.5 \) and \( \alpha = 5 \)).

Now we study the tensor perturbations of our model. We use the procedure developed in [62] for most general scalar-tensor theories with second-order field equations. It has been shown that the quadratic action for the tensor perturbations is as,

\[ S_T = \frac{1}{8} \int dt \, d^3x \, a^3 \left[ G_T \dot{h}_{ij}^2 - \frac{F_T}{a^2} (\nabla h_{ij})^2 \right], \]  

(18)

\( h_{ij} \) is a tensor perturbation satisfying \( h_{ij} = 0 = h_{ij,j} \) and in case of our model \( G_T \) and \( F_T \) are,

\[ G_T = 2 \left( \frac{1}{2\kappa^2} - \frac{1}{2} \partial_{\mu} \phi \partial^{\mu} \phi \frac{df(\phi)}{d\phi} \right), \]  

(19)

\[ F_T = 2 \left( \frac{1}{2\kappa^2} + \frac{1}{2} \partial_{\mu} \phi \partial^{\mu} \phi \frac{df(\phi)}{d\phi} \right). \]  

(20)

In order to avoid ghost and gradient instabilities in the tensor sector we require the conditions \( F_T > 0 \) and \( G_T > 0 \).
We find the power spectrum of the primordial tensor perturbation as follows,

$$P_T = 8\gamma_T \frac{G_T^2}{F_T^3} \frac{H^2}{4\pi^2} \bigg|_{k\gamma_T=1},$$  

(21)

where $dy_T = \frac{1}{a} \sqrt{\frac{\dot{F}_T}{G_T}} dt$ and $k$ is the Fourier wavenumber. Note that the sound horizon crossing occurs when $k^2 \sim \frac{1}{y_T^2}$. Also in (21) we have

$$\gamma_T = 2^{\nu_T - 3} \left| \frac{\Gamma(\nu_T)}{\Gamma(\frac{3}{2})} \right|^2 \left( 1 - \frac{f_T}{2} + \frac{g_T}{2} \right),$$  

(22)

and $\nu_T = \frac{3 - \epsilon + g_T}{\Delta - f_T + g_T}$, where

$$g_T = \frac{-\dot{\phi} \dot{\xi} \left( 2\phi \frac{d\phi}{d\phi} + \phi \frac{d^2 f(\phi)}{d\phi^2} \right)}{2H \left( \frac{1}{2\kappa^2} - \frac{1}{2} \dot{\xi} \frac{d\phi}{d\phi} \right)},$$  

(23)

$$f_T = \frac{-\dot{\phi} \dot{\xi} \left( 2\phi \frac{d\phi}{d\phi} + \phi \frac{d^2 f(\phi)}{d\phi^2} \right)}{2H \left( \frac{1}{2\kappa^2} + \frac{1}{2} \dot{\xi} \frac{d\phi}{d\phi} \right)},$$  

(24)

and $\epsilon \equiv -\frac{\dot{H}}{H^2}$ that takes the following form for our model

$$\epsilon = -9\xi \dot{\phi} \left( \frac{df(\phi)}{d\phi} \right) \left( \frac{V(\phi)}{\sqrt{1 - \dot{\phi}^2}} + \frac{\Lambda}{\kappa^2} \right) + \frac{\left( \frac{3}{\kappa^2} - 9\xi f(\phi) \dot{\phi}^2 \right) \dot{\phi} \left( \frac{dV(\phi)}{d\phi} + 2V(\phi) \dot{\phi} \right)}{2 \left( \frac{V(\phi)}{\sqrt{1 - \dot{\phi}^2}} + \frac{\Lambda}{\kappa^2} \right) \left( \frac{3}{\kappa^2} + 9\xi f(\phi) \dot{\phi}^2 \right)^{\frac{1}{2}}}.$$

(25)

Finally, the tensor spectral index is given by

$$n_T = 3 - 2\nu_T.$$

(26)

Thus, one can obtain a blue or red spectrum of tensor perturbations by choosing different values of coupling parameter $\xi$ in tachyonic non-minimal derivative coupling model.

Let us focus on scalar perturbations by putting $h_{ij} = 0$. The quadratic action for curvature perturbations is as follows,

$$S_S = \int dt d^3 x a^3 \left[ G_S \dot{\zeta}^2 - \frac{F_S}{a^2} (\vec{\nabla} \zeta)^2 \right],$$  

(27)

here $\zeta$ is scalar perturbation in perturbed metric (for more details see [62]) also $G_S$ and $F_S$ are given by,

$$G_S = \frac{\Xi}{\Theta^2} G_T^2 + 3G_T,$$

(28)
\[ F_S = \frac{1}{a} \frac{d}{dt} \left( \frac{a}{\Theta} G_S^2 \right) - F_T, \]  
(29)

where \( \Xi \) and \( \Theta \) in the case of our model are,

\[ \Xi = \frac{\dot{\phi}^2 V(\phi)}{2(1 - \dot{\phi}^2)^2} - 3H^2 \left( \frac{1}{\kappa^2} + 6\xi \dot{\phi}^2 \frac{df(\phi)}{d\phi} \right), \]  
(30)

\[ \Theta = H \left( \frac{1}{\kappa^2} + 3\xi \dot{\phi}^2 \frac{df(\phi)}{d\phi} \right). \]  
(31)

By substituting above equations as well as (19) and (20) into \( G_S \) and \( F_S \), one can obtain,

\[ G_S = \frac{1}{\left( \frac{1}{\kappa^2} + 3\xi \dot{\phi}^2 \frac{df(\phi)}{d\phi} \right)^2} \left( \frac{\dot{\phi}^2 V(\phi)}{2H^2(1 - \dot{\phi}^2)^{\frac{3}{2}} \left( \frac{1}{\kappa^2} + 3\xi \dot{\phi}^2 \frac{df(\phi)}{d\phi} \right)} - 3 \right) + 3 \left( \frac{1}{\kappa^2} - \xi \dot{\phi}^2 \frac{df(\phi)}{d\phi} \right), \]  
(32)

\[ F_S = \frac{1}{\left( \frac{1}{\kappa^2} + 3\xi \dot{\phi}^2 \frac{df(\phi)}{d\phi} \right)} \left( \frac{\dot{H}^2}{\kappa^2} + 3\xi \left( \dot{H} \dot{\phi}^2 \frac{df(\phi)}{d\phi} + 2H \dot{\phi} \dot{\phi}^2 \frac{df(\phi)}{d\phi} + H \ddot{\phi}^2 \frac{df(\phi)}{d\phi} + H \dot{\phi} \ddot{\phi} \right) \right) \left( \frac{1}{\kappa^2} - \xi \dot{\phi}^2 \frac{df(\phi)}{d\phi} \right)^2 \]  
(33)

As it is mentioned in [62], the analysis of the curvature perturbation hereafter is completely parallel to that of the tensor perturbation. So, ghost and gradient instabilities are avoided as long as \( F_S > 0 \) and \( G_S > 0 \).

The power spectrum of the primordial curvature perturbation is given by,

\[ P_\zeta = \frac{\gamma_S G_S^2}{2} \frac{H^2}{F_S^2} \frac{1}{4\pi^2} \left| \frac{dy_S}{a} \right|_{k_y = 1}, \]  
(34)

where \( dy_S = \frac{1}{a} \sqrt{\frac{F_S}{G_S}} dt \) and \( k \) is the Fourier wavenumber. Note that the sound horizon crossing occurs when \( k^2 \sim \frac{1}{g_S} \). Also in (34) we have

\[ \gamma_S = 2^{\nu_S - 3} \left| \frac{\Gamma(\nu_S)}{\Gamma(\nu_S/2)} \right| \left( 1 - \epsilon - \frac{f_S}{2} \delta_S + \frac{g_S}{2} \right), \]  
(35)

here we have defined \( \nu_S = \frac{3-\epsilon+g_S}{2-2\epsilon-f_S+g_S} \), where

\[ g_S = \frac{G_S}{H G_S}, \]  
(36)

\[ f_S = \frac{\dot{F}_S}{H F_S}. \]  
(37)
The spectral index is,
\[ n_S - 1 = 3 - 2\nu_S. \]  
\hspace{1cm} (38)

The tensor-to-scalar ratio is given by,
\[ r = 16 \left( \frac{F_S}{F_T} \right)^\frac{7}{2} \left( \frac{G_S}{G_T} \right)^{-\frac{1}{2}}. \]  
\hspace{1cm} (39)

4 Conclusion

In this work we investigated the bouncing solution in the universe dominated by the quintom matter. We analyzed the possibility of obtaining a cosmological bounce in a model which contains a scalar field with non-minimal derivative coupling to Einstein tensor and itself too. We used the numerical methods to study the bouncing behavior in this setup where the tachyon field played the role of scalar field. We obtained the bouncing condition as equation (13), then by considering a couple example for potential of the tachyon scalar field we showed that the above mentioned condition can be satisfied. After that we considered the conditions required for the classical stability of our model. These conditions obtained as equations (16) and (17) which reflect the positivity of the sound speed \( C_s^2 \) and \( D \). Our numerical results for such a classical stability plotted in figures 3 and 4 respectively. Finally, we investigated the tensor perturbations of our model using the method of Ref. [62].

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