Abstract—The development of renewable energy and the increasing peak-valley difference of load demand lead to an increasing requirement of spinning reserve (SR). However, the traditional operation reliability analysis of power system mainly focuses on the up SR and neglects the down SR. Therefore, the operation reliability of power system considering the impacts of down SR is investigated in this paper. Firstly, the constraints of down SR are incorporated into the day-ahead unit commitment (UC) model to obtain the generation scheduling and reserve allocation of power systems. Based on the dispatch results of UC model, the re-dispatched energy and load interruption can be determined using optimal power flow (OPF) model in the real-time operation in various contingency states. Operation reliability indices are calculated based on the load curtailment to represent the reliability performances of power systems. The proposed approaches are validated using the modified IEEE reliability test system. Case studies demonstrate that down SR can improve the operation reliability of power systems.

Index Terms—Down spinning reserve, operation reliability evaluation, unit commitment (UC), optimal power flow (OPF).

I. INTRODUCTION

The penetration of renewable energy sources (especially wind power) in power system is increasing around the world. In the United States, 20% of the energy production will be provided by wind power at 2030 [1]. According to plan of European Union, renewable power will account for 20% of the total energy production by 2020 [2]. The increasing integration of thermostatically controlled load such as air conditioner has also increased the peak-valley difference of power system load [3]. Both the high penetration of fluctuating renewable energy and the huge peak-valley difference of load have largely increased the difficulties for system operators (SOs) to balance the generation and demand during the real-time operation [4]. In Denmark, more than half of the system imbalances are caused by wind power fluctuation [5], which leads to an increasing requirement of spinning reserve (SR). SR is an important resource utilized by SOs to compensate for the power imbalances and to maintain the reliable operation of power system [6]. Generally, SR can be divided into up SR and down SR according to the adjustment direction of power output. The up SR is utilized to provide an increasing rate of generation and compensate for the deficit of production. In contrast, the down SR is used to provide a decreasing rate of generation output without shutting down the units, when there is a surplus of generation [7].

Traditional operation of power system usually takes account of the effects of up SR more than the effects of down SR. As an important resource to power system operation, down SR can increase the flexibility to re-dispatch power system, relieve congestion and increase renewable energy consumption. In some extreme circumstances, there might be a huge decrease of system load, following with a rapid increase of system load during a very short period in power systems such as “duck curve” [8]. Without enough down SR, the SO must firstly switch off units and rapidly switch on units afterward to maintain the simultaneous balance between power generation and demand. However, it is costly to frequently switch on/off generators than implementing down SR [9] and there might not be enough time to switch on/off generators. Furthermore, down SR can improve the consumption of renewable energy by decreasing the output of traditional generators when renewable energy increases suddenly. In the northwest power grid of China, down SR is implemented to increase the consumption of renewable energy [10]. Therefore, down SR should be incorporated into the unit commitment (UC) model to schedule the generation and guarantee the reliable operation of power system.

The results of generation scheduling and reserve allocation obtained from UC model may vary greatly with different levels of SR requirement, which further affect the operation reliability of power system. A number of proposals to incorporate up SR constraints into UC model have been proposed for the joint scheduling of generation and SR. In [11], the unavailability of the generation units and the uncertainty of load forecasting are incorporated into the UC model to schedule up SR. Reference [12] proposes a method to schedule up SR more reasonably based on probabilistic criterions. Reference [13] proposes a method to schedule up SR considering the reliability choice of customer. On the basis of these UC models, the influence of up SR on power system reliability is widely studied. References [14], [15] show that
the power system must increase the amount of up SR or will face a measurable decrease in reliability as wind capacity increases. Reference [16] shows that the up SR is necessary for keeping power system reliability under emergencies, whereas the cost of up SR should be traded off with the cost of energy not served. However, most of the previous researches mainly focus on the correlation between the up SR and the power system reliability without consideration of the effects of down SR. Thus, it is necessary to evaluate the impacts of down SR on power system reliability.

The researches on reliability analysis for power systems have been extensively done, including Monte Carlo simulation (MCS) approaches [17], [18] and analytical methods [19]-[22]. However, the previous researches mainly focus on the long-term reliability evaluation of power system based on the steady-state probabilities of system components. Due to the high fluctuation of wind power generation in short periods, operation or short-term reliability of power system should be investigated.

This paper proposes an evaluation method for operation reliability of power system considering down SR and the uncertainty of wind power output. The main contributions of this paper are threefold:

1) The impacts of down SR on the operation reliability of power system are analyzed, which are seldom considered in other researches.
2) A modified UC model in the day-ahead market is proposed, which incorporates constraints of down SR provided by both generation units and loads.
3) Based on the results of the day-ahead market, the optimal power flow (OPF) model is proposed to re-dispatch power system considering random failures and output uncertainty of wind power during the real-time operation. Linearization methods for the OPF model are also implemented.

The remainder of this paper is organized as follows. Section II introduces the UC and OPF model considering down SR and the analytical method for operation reliability of power system. Illustrative examples using IEEE reliability test system are presented in Section III. Conclusions are given in Section IV.

II. MATHEMATICAL MODEL FORMULATION

The proposed method is formulated using a modified UC model considering down SR to co-optimize the generation scheduling and the dispatch of up and down reserves in the day-ahead electricity market. The dispatch results of UC model are utilized as the basic inputs of the OPF model during real-time operation, which is formulated to re-dispatch the power system and calculate the load interruption under different contingency states. With the results of load interruption, the operation reliability can be evaluated using expected energy not supplied (EENS) and loss of load probability (LOLP) indices. The structure for the simulation model is presented in Fig. 1.

A. Uncertainty Modeling of Wind Power Output

In this section, the model of multi-state wind power generation is developed to describe the uncertainty of wind power output. The generated power of a wind turbine varies with the wind speed, which can be formulated mathematically with:

\[
\begin{align*}
V_{w}^{k} = V_{w}^{0} + \frac{5\delta_{w}}{3}(k_{w} - 3) \\
p_{w}^{k} = \operatorname{Pr}(V_{w}^{k})
\end{align*}
\]

where \(k_{w} = 1, 2, \ldots, 6\); \(V_{w}^{0}\) and \(\delta_{w}\) are the mean value in the normal state and the variance of wind speed distribution of wind turbine \(w\) at time period \(t\), respectively; and \(V_{w}^{k}\) and \(p_{w}^{k}\) are the wind speed and the corresponding probability of
wind turbine $w$ in state $k_w$ at time period $t$, respectively.

The output power of a wind turbine can be determined from its power curve, which is a plot of output power against wind speed and can be expressed as:

$$
P_{w}^{k_w} = \begin{cases} 
0 & 0 \leq V_{w}^{k_w} < V_{c}^{w} \\
\frac{1}{2} \rho A \lambda_{c} \frac{V_{w}^{k_w} - V_{c}^{w}}{\lambda_{c}}^{2} \frac{\lambda_{c} + \frac{1}{2} \frac{V_{w}^{k_w} - V_{c}^{w}}{\lambda_{c}}}{\lambda_{c}} & V_{c}^{w} \leq V_{w}^{k_w} < V_{r}^{w} \\
\frac{1}{2} \rho A \lambda_{c} \frac{V_{r}^{w}}{V_{c}^{w}} \left( \frac{V_{r}^{w}}{V_{c}^{w}} \right)^{2} \left( \frac{\lambda_{c} + \frac{1}{2} \frac{V_{r}^{w}}{V_{c}^{w}}}{\lambda_{c}} \right)^{2} & V_{r}^{w} \leq V_{w}^{k_w} \leq V_{r}^{w}
\end{cases}
$$

where $V_{c}^{w}$, $V_{r}^{w}$, and $V_{r}^{w}$ are the cut-in speed, cut-out speed and rated speed of wind turbine $w$, respectively; $P_{w}^{k_w}$ is the rated power of wind turbine $w$ when the wind speed is between the rated speed and the cut-out speed; and $A_w, B_w, C_w$ are the constants of wind turbine $w$. These parameters in (2) can be found in [23]. Thus, the wind power output uncertainty can be discretized and expressed with multi-state models.

B. UC Model Considering Down SR for Day-ahead Electricity Market

Based on the day-ahead forecasted load demand and wind power output, the multi-period UC model is implemented in the day-ahead electricity market for determining the states of generation units, hourly scheduling of units and the SR dispatch of units. The objective is to minimize the total cost, including no-load cost, energy production cost, start-up and shut-down cost per unit of each unit. Generally, the cost is provided in quadratic form. For the simplification of calculation, the energy production cost is piecewise-linearized to reduce the complexity of calculation in various electricity markets [24]. Traditional UC model without considering the constraints of down SR is presented as follows:

$$
\text{min} \sum_{i \in T} \sum_{t \in S} \left( GC_{g}(P_{g,t}^{0},x_{g,t}) + RU_{g,t} \cdot PRU_{g,t} + RU_{d,t} \cdot PRU_{d,t} \right)
$$

where $RU_{g,t}$ and $RU_{d,t}$ are the up SRs dispatched for unit $g$ and load $d$ at time $t$, respectively; $PRU_{g,t}$ and $PRU_{d,t}$ are the costs of up SR dispatched for unit $g$ and load $d$ at time $t$, respectively; $T$ is the set of time periods; $NG_t$ is the set of generations on bus $i$; $N$ is the set of buses; $P_{g,t}^{0}$ is the real power output of unit $g$ at time $t$ in the normal state; and $GC_{g}(\cdot)$ is the generation cost of unit $g$, which can be expressed as:

$$
GC_{g}(P_{g,t}^{0},x_{g,t}) = NLC_{g} \cdot x_{g,t} + \sum_{i \in S} \lambda_{g,i} \cdot P_{g,i}^{0} + SU_{g,t} \cdot y_{g,t} + SD_{g,t} \cdot z_{g,t}
$$

where $SU_{g,t}$ and $SD_{g,t}$ are the startup and shutdown costs of unit $g$ at time $t$, respectively; $x_{g,t}, y_{g,t}, z_{g,t}$ are the binary variables, and if unit $g$ is online at time $t$, $x_{g,t}$ is equal to 1, if unit $g$ is start-up at time $t$, $y_{g,t}$ is equal to 1, if unit $g$ is shut-down at time $t$, $z_{g,t}$ is equal to 1; $NLC_{g}$ is the no-load cost of unit $g$; $\lambda_{g,i}$ is the incremental cost of unit $g$ for segment $s$; $P_{g,i}^{0}$ is the real power output of unit $g$ at time $t$ at segment $s$ in the normal state; and $N$ is the set of cost curve segments.

The UC model is subject to the following constraints.

1) Generation Constraints

$$
P_{g,t}^{\text{max}} x_{g,t} \leq P_{g,t}^{0} \leq \sum_{i \in S} P_{g,i}^{0} \leq P_{g,t}^{\text{max}} x_{g,t}
$$

where $P_{g,t}^{\text{max}}$ and $P_{g,t}^{\text{min}}$ are the maximum and minimum real power outputs of unit $g$, respectively; and $P_{g,t}^{\text{max}}$ is the maximum real power output of unit $g$ for segment $s$.

Constraint (5) bounds the generation by the minimum power output, and the maximum available power output of unit $g$. Constraint (6) specifies that the output of generation units in each linear segment $s$ should be inferior to the output range of the segment.

2) Power Flow Constraints

$$
\sum_{g \in N_{G}} P_{g,t}^{0} + \sum_{w \in N_{W}} P_{w,t}^{0} - \sum_{d \in N_{D}} P_{d,t}^{0} = \sum_{k \in N} V_{k,t}^{0} V_{k,t}^{0} |Y_{k,t}^{0}| \left( \cos(\theta_{k,t}^{0} - \theta_{k,t}^{0} - \delta_{k,t}^{0}) \right)
$$

$$
\sum_{g \in N_{G}} Q_{g,t}^{0} + \sum_{w \in N_{W}} Q_{w,t}^{0} - \sum_{d \in N_{D}} Q_{d,t}^{0} = \sum_{k \in N} V_{k,t}^{0} V_{k,t}^{0} |Y_{k,t}^{0}| \left( \sin(\theta_{k,t}^{0} - \theta_{k,t}^{0} - \delta_{k,t}^{0}) \right)
$$

where $V_{k,t}^{0} = \left| V_{k,t}^{0} \right| \angle \theta_{k,t}^{0}$ and $V_{k,t}^{0} = \left| V_{k,t}^{0} \right| \angle \theta_{k,t}^{0}$ are the bus voltages at bus $i$ and bus $k$, respectively; $Y_{k,t}^{0} = \left| Y_{k,t}^{0} \right| \angle \delta_{k,t}^{0}$ is the element of admittance matrix in the normal state; $Q_{g,t}^{0}$ is the reactive output of unit $g$ at time $t$ in the normal state; $P_{g,t}^{0}$, and $Q_{g,t}^{0}$ are the active and reactive wind power outputs of wind turbine $w$ in the normal state, respectively; $P_{g,t}^{0}$, and $Q_{g,t}^{0}$ are the active and reactive power demands of load $d$ at time $t$ in the normal state, respectively; $NW_{i}$ is the set of wind turbines on bus $i$; and $NL_{i}$ is the set of loads on bus $i$.

3) Up SR Constraints

$$
0 \leq RU_{g,t} + P_{g,t}^{0} \leq P_{g,t}^{\text{max}} x_{g,t}
$$

$$
0 \leq RU_{g,t} + (P_{g,t}^{\text{max}} - P_{g,t}^{\text{min}}) x_{g,t}
$$

$$
RU_{g,t} \leq \text{min}(A_{g,t}^{p} \cdot P_{g,t}^{\text{max}} - P_{g,t}^{0})
$$

$$
0 \leq RU_{d,t} \leq P_{d,t}^{0} - P_{d,t}^{\text{min}}
$$

$$
\sum_{i \in N_{G}} \sum_{g \in N_{G}} RU_{g,i} + \sum_{i \in N_{G}} \sum_{d \in N_{D}} RU_{d,i} \geq RU_{t}^{\text{req}}
$$

where $P_{g,t}^{\text{min}}$ is the minimum active power demand of load $d$; $A_{g,t}^{p}$ is the maximum up ramping rate of unit $g$ at time $t$; and $RU_{t}^{\text{req}}$ is the up SR requirement at time $t$.

Constraint (9) specifies that the summation of assigned up SR and output of each unit should be inferior to the maximum output. Constraint (10) specifies that up SR can be provided by unit $g$ during period $t$ only if it is in operation. Constraint (11) specifies that the up SR provided by generation units is limited by its maximum generation level and up ramping rates. Constraint (12) specifies that the up SR provided by load sector is limited by the minimum load demand. Constraint (13) specifies that the total up SR should be superior to the requirement of up SR in the whole system.

4) Ramping Up/Down Constraints

$$
P_{g,t+1} - P_{g,t} \leq A_{g,t}^{p}
$$

$$
-A_{g,t}^{d} \leq P_{g,t} - P_{g,t-1}
$$

where $P_{g,t}$ is the real power output of unit $g$ at time $t$; and $A_{g,t}^{d}$ is the maximum down ramping rate of unit $g$ at time $t$.

Constraints (14) and (15) are inter-temporal constraints,
which specify the ramping limits of generation units from time interval \( t-1 \) to time interval \( t \).

5) **Minimum Up and Down Time Constraints**

\[
(u_{g,t-1} - U_T)(x_{g,t-1} - x_{g,t}) \geq 0 \quad (16)
\]

\[
(u_{g,t} - D_T)(x_{g,t} - x_{g,t-1}) \geq 0 \quad (17)
\]

where \( U_T \) and \( D_T \) are the minimum up time and down time of unit \( g \), respectively; and \( u_{g,t} \) is the number of hours that unit \( g \) has been on or off at the end of time \( t \).

Constraints (16) and (17) represent the minimum time limits of generation units to switch on and off.

6) **Line Flow Limits**

\[
|S_i^L| \leq S_i^{max} \quad (18)
\]

where \( S_i^L \) is the apparent power of line \( l \) at time \( t \) in the normal state; and \( S_i^{max} \) is the maximum apparent power of line \( l \).

7) **Voltage Limits**

\[
V_{i,t}^{min} \leq |V_{i,t}^{0}| \leq V_{i,t}^{max} \quad (19)
\]

where \( V_{i,t}^{max} \) and \( V_{i,t}^{min} \) are the upper and lower limits of voltage at bus \( i \), respectively; and \( V_{i,t}^{0} \) is the voltage of bus \( i \) at time \( t \) in the normal state.

8) **Reactive Power Output Limits**

\[
Q_{g,t}^{min} x_{g,t} \leq Q_{g,t}^{0} \leq Q_{g,t}^{max} x_{g,t} \quad (20)
\]

where \( Q_{g,t}^{max} \) and \( Q_{g,t}^{min} \) are the upper and lower output limits of reactive power of unit \( g \), respectively.

9) **Integer Constraints**

\[
y_{g,t} - z_{g,t} = x_{g,t-1} - x_{g,t} \quad (21)
\]

\[
y_{g,t} + z_{g,t} \leq 1 \quad (22)
\]

\[
x_{g,t}, y_{g,t}, z_{g,t} \in [0,1] \quad (23)
\]

Traditional UC models usually neglect the effects of down SR, which are considered in this paper. The cost of down SR should be supplemented into the objective of UC model, which is given as follows:

\[
\min \sum_{i=1}^{N} \sum_{j=1}^{N} (GC_i(P_{g,i}^0) + RU_{g,i} + PRU_{g,i} + RU_{d,i} + PRD_{d,i}) 
\]

\[
RD_{g,i} \cdot PRD_{g,i} + RD_{d,i} \cdot PRD_{d,i} \quad (24)
\]

where \( RD_{g,i} \) and \( RD_{d,i} \) are the down SRs dispatched for unit \( g \) and load \( d \) at time \( t \), respectively; and \( PRD_{g,i} \) and \( PRD_{d,i} \) are the costs of down SR dispatched for unit \( g \) and load \( d \) at time \( t \), respectively.

The constraints of down SR should also be supplemented into the UC model, which are given as:

\[
P_{g,t}^{min} \leq P_{g,t}^0 - RD_{g,t} \quad (25)
\]

\[
0 \leq RD_{g,t} \leq (P_{g,t}^{max} - P_{g,t}^{min}) \quad (26)
\]

\[
RD_{g,t} \leq \min (A_{g,t}, P_{g,t}^{max} - P_{g,t}^{min}) \quad (27)
\]

\[
0 \leq RD_{d,i} \leq P_{d,i}^{max} - P_{d,i}^{min} \quad (28)
\]

\[
\sum_{i=1}^{N} RD_{g,i} + \sum_{i=1}^{N} RD_{d,i} \geq RD_{eq} \quad (29)
\]

Constraint (26) specifies that down SR can be provided by unit \( g \) during period \( t \) only if it is in operation. Constraint (27) specifies that the down SR provided by generation units is limited by its minimum generation level and down ramping rates. Constraint (28) specifies that the down SR provided by load sector is limited by the maximum demand. Constraint (29) specifies that the total down SR should be superior or to the requirement of down SR in the whole system.

**C. Energy Re-dispatching Model Considering Down SR**

Despite down SR and up SR are allocated in the day-ahead market, customer loads may still be interrupted in contingency states if wind power generation decreases or the generation units failed to produce electricity. In real-time operation, the OPF model is implemented to re-dispatch generation within the reserve limits to maintain the system balance, as shown in Fig. 2, where \( P_{g,t}^r \) and \( P_{d,t}^r \) are the re-dispatched power of unit \( g \) and the re-dispatched demand of load \( d \) in real-time operation in contingency state \( j \), respectively; and \( LI_{d,t,j}^l \) is the load interruption for load \( d \) at time \( t \) in contingency state \( j \).

![Fig. 2. Operation reserve in real-time market. (a) Up and down reserves provided by generation units. (b) Up and down reserves provided by loads.](attachment:image)

The obtained results from the UC model such as the generation dispatching results, the up and down SR allocations, are used as the basic inputs for the OPF model. The objective function of OPF model is to minimize the total system operation cost considering the network and market constraints during the real-time operation [25], [26]. The operation cost includes the energy production cost and customers’ interruption cost. If customers are called to provide reserve, they should change their consumption behavior without further remuneration. This is reasonable because the probability of contingency occurrence is low and customers already profit from their willingness to provide the corrective actions [5]. For contingency state \( j \) and wind power output state \( k \) at time \( t \), the objective function can be expressed as:

\[
\min \left[ \sum_{i=1}^{N} \sum_{j=1}^{N} GC_i(P_{g,i}^0 + \Delta P_{g,i}^{k,j} \cdot C_{g,i}^l) + \sum_{i=1}^{N} \sum_{d=1}^{N} IC_d(LI_{d,i,j}^l) \right] \quad (30)
\]

where \( IC_d(t) \) is the interruption cost function of load \( d \); \( LI_{d,i,j}^l \) is the load interruption for load \( d \) at time \( t \) in contingency state \( j \) and wind output state \( k \); \( N_a \) is the set of load buses; \( \Delta P_{g,i}^{k,j} \) is the active power generation re-dispatched from the reference point in the day-ahead market for unit \( g \) at time \( t \) in contingency state \( j \) and wind output state \( k \); and \( C_{g,i}^l \) is
the binary variable, which is equal to 0 if the unit fails in contingency state \( j \).

The interruption cost is a function of load curtailment and corresponding interruption characteristics, which can be represented as:

\[
IC_d(L_t^{j,k}) = L_t^{j,k} \cdot CDF_d
\]

where \( CDF_d \) is the customer damage function, which can be used to evaluate the interruption cost of different load sectors [25].

The objective function (28) is subject to the following constraints.

1) Transmission Constraints

\[
\sum_{g \in N_g} (\Delta P_{g,t}^i + P_0^g) + \sum_{w \in N_w} \sum_{d \in N_d} P_{d,t}^{j,k} = \sum_{v \in N_v} \sum_{d \in N_d} V_{v,t}^{j,k} \cos(\theta_{v,t}^{j,k} - \theta_{v,t}^{i,k} - \delta_{v,t}^{j,k})
\]

\[
\sum_{g \in N_g} (\Delta Q_{g,t}^i + Q_0^g) + \sum_{w \in N_w} \sum_{d \in N_d} Q_{d,t}^{j,k} = \sum_{v \in N_v} \sum_{d \in N_d} V_{v,t}^{j,k} \sin(\theta_{v,t}^{j,k} - \theta_{v,t}^{i,k} - \delta_{v,t}^{j,k})
\]

where \( \Delta Q_{g,t}^i \) is the reactive power generation re-dispatched for unit \( g \) at time \( t \) in contingency \( j \) and wind output state \( k_s \); \( V_{v,t}^{j,k} = |V_{v,t}^{j,k} - \theta_{v,t}^{j,k} - \delta_{v,t}^{j,k}| \) and \( V_{v,t}^{j,k} = |V_{v,t}^{j,k} - \theta_{v,t}^{j,k} - \delta_{v,t}^{j,k}| \) are the bus voltages at bus \( i \) and bus \( j \) in contingency state \( j \) and wind output state \( k_s \), respectively; \( Y_{v,t}^{j,k} = |Y_{v,t}^{j,k} - \theta_{v,t}^{j,k} - \delta_{v,t}^{j,k}| \) is the element of admittance matrix in contingency state \( j \) and wind output state \( k_s \); and \( P_{d,t}^{j,k} \) and \( Q_{d,t}^{j,k} \) are the re-dispatched active and reactive power demands of load \( d \) in contingency state \( j \) and wind output state \( k_s \), respectively.

2) Generation Constraints

\[
P_0^g - RU_{d,t}^g \leq P_{g,t}^{j,k} \leq P_0^g + \Delta P_{g,t}^{j,k}
\]

\[
P_0^g + \Delta P_{g,t}^{j,k} - RU_{d,t}^g \leq P_{g,t}^{j,k} \leq P_0^g + \Delta P_{g,t}^{j,k} + RU_{d,t}^g
\]

\[
Q_{g,t}^{j,k} = Q_{g,t}^{j,k} - RU_{d,t}^g \leq Q_{g,t}^{j,k} \leq Q_{g,t}^{j,k} + RU_{d,t}^g
\]

Constraints (34), (35) and (36) specify that unit \( g \) can be re-dispatched within the limits of up and down SRs during the period \( t \) in contingency state \( j \) only if it is on.

3) Load Interruption Constraints

\[
P_{min}^{j,k} \leq P_{d,t}^{j,k} \leq P_{max}^{j,k}
\]

\[
LI_{d,t}^{j,k} = (P_0^{d,t} - P_{d,t}^{j,k} - RU_{d,t}^g) F(0 \leq P_0^{d,t} - P_{d,t}^{j,k} - RU_{d,t}^g)\]

where \( F(0 \leq P_0^{d,t} - P_{d,t}^{j,k} - RU_{d,t}^g) \) is equal to 1 when \( 0 \leq P_0^{d,t} - P_{d,t}^{j,k} - RU_{d,t}^g \) is true, and 0 otherwise.

Other constraints include line flow constraints, voltage constraints, etc.

In the proposed model, constraint (38) is the product of continuous and binary variables and should be converted into linear expression. Assuming that \( r \) is the product of a binary variable \( a \) and a bounded continuous variable \( \beta \in [-M,M] \), where \( M \) is a large positive number. The product can be replaced with:

\[-aM \leq \beta r \leq aM\]

\[\beta - M(1 - a) \leq r \leq \beta + M(1 - a)\]

Clearly, if \( a \) is equal to zero, \( \beta \) will vanish in constraint (39), while constraint (40) is inactive. Otherwise, if \( a \) is equal to one, \( r \) must be equal to \( \beta \), while constraint (40) is inactive. Thus, constraint (38) can be replaced with the following linear expression:

\[
y_{d,t}^{j,k} = F(0 \leq P_0^{d,t} - P_{d,t}^{j,k} - RU_{d,t}^g)\]

\[
(P_0^{d,t} - P_{d,t}^{j,k} - RU_{d,t}^g)/M \leq y_{d,t}^{j,k}\]

\[
y_{d,t}^{j,k} \leq 1 + (P_0^{d,t} - P_{d,t}^{j,k} - RU_{d,t}^g)/M\]

\[-M y_{d,t}^{j,k} \leq LI_{d,t}^{j,k} \leq My_{d,t}^{j,k}\]

\[
P_0^{d,t} - P_{d,t}^{j,k} - RU_{d,t}^g - M(1 - y_{d,t}^{j,k}) \leq LI_{d,t}^{j,k}\]

\[
LI_{d,t}^{j,k} \leq P_0^{d,t} - P_{d,t}^{j,k} - RU_{d,t}^g + M(1 - y_{d,t}^{j,k})\]

D. Reliability Indices and Evaluation Process of Power Systems

The obtained results from the analysis of normal state and contingency state can be used to evaluate the power system reliability, according to the following steps.

Step 1: implement the multi-period UC model in the day-ahead electricity market for determining the states of generation units and the corresponding generation scheduling, as well as the up and down SRs for each operation time interval. Forward the obtained results as the reference points to the energy re-dispatch model.

Step 2: for each contingency state, solve the energy re-dispatch model formulated by the single-period OPF model to determine the generation re-dispatching results and the load interruption.

Step 3: evaluate the probability of power system contingency state \( j \) and wind power output state \( k_s \), with the following equations:

\[
p_t^{j,k} = \prod_{g \in G, r \in R_{g,t}^j} \prod_{g \in G, r \in R_{g,t}^j} \prod_{g \in G} (1 - ORR_{g,t}^j) \prod_{g \in G} p_{g,0}^{j,k}
\]

\[
ORR_{g,t}^j = 1 - e^{-MTTF_g^j} \approx \text{t/MTTF}_g^j
\]

where \( ORR_{g,t}^j \) is the failure probability of the unit \( g \) during the time interval \( t \); \( B_j \) is the set of failed units in contingency state \( j \) during the time interval \( t \); and \( \text{MTTF}_g^j \) is the mean time to failure (MTTF) of unit \( g \).

Step 4: evaluate the two reliability indices \( LOLP_\text{t} \) and \( EENS_\text{t} \), which are defined as LOLP and EENS in the power system for the time interval \( t \), respectively. They can be calculated with the following equations:

\[
LOLP_{\text{t}} = \sum_{j \in J} \sum_{d \in D} p_{d,0}^{j,k} F(0 < LI_{d,t}^{j,k}, \forall d)
\]

\[
EENS_{\text{t}} = \sum_{j \in J} \sum_{d \in D} (p_{d,0}^{j,k} \Delta t \sum_{d \in D} \sum_{d \neq d} LI_{d,t}^{j,k})
\]

where \( F(0 < LI_{d,t}^{j,k}, \forall d) \) is equal to 1 when \( 0 < LI_{d,t}^{j,k}, \forall d \) is true, and 0 otherwise; and \( \Delta t \) is the duration of the time interval \( t \).

III. ILLUSTRATIVE EXAMPLES

To further illustrate the proposed models and techniques, the IEEE reliability test system [27], as shown in Fig. 3, has
been studied to demonstrate the proposed models and techniques.

![Diagram of IEEE reliability test system](image)

The ramping limits, minimum on/off time, piecewise linear cost, start-up and shut-down costs as well as the no-load cost are taken from [28]. The transmission capacity of each line is set to be 60% of the initial capacity to increase the value of EENS and LOLP. The costs of providing up and down SRs at demand side are 100 $/MWh [7]. The minimum of the load is set to be zero and the maximum of the load is set to be 10% higher than the original load level. Besides, the minimum and maximum outputs of units are increased to emphasize the impact of down SR when the load demand decreases rapidly. The minimum and maximum outputs of units and the reserve prices are given in Table I.

![Table I: Parameters of Generation Units](image)

| Generation unit | Output level (MW) | Reserve price ($/MWh) |
|-----------------|------------------|-----------------------|
|                 | Lower | Upper | Up SR | Down SR |
| I1, I2, I5, I6, I15-I19, I24, I26-I29 | 12    | 69    | 48    | 48    |
| I3, I4, I7, I8, I23 | 24    | 120   | 65    | 65    |
| I9-I11, I20, I21, I30, I32 | 35    | 150   | 70    | 70    |
| I12-I14          | 65    | 180   | 76    | 76    |
| I22, I23, I32    | 150   | 350   | 88    | 88    |

To decrease the failure time of units and increase the values of EENS and LOLP, MTTF of each generation unit is decreased and provided in Table II.

![Table II: MTTF of Each Generation Unit](image)

| Generation unit | MTTF (hour) |
|-----------------|-------------|
| I1, I2, I5, I6  | 450         |
| I12-I14, I20, I21 | 900        |
| I29, I30, I32   | 1100        |
| I9-I11          | 1200        |
| I3, I4, I7, I8, I12-I14, I15-I19, I22, I23, I31 | 1900 |
| I15-I19, I22, I23, I31 | 2900 |

In the real-time operation, the compensation for load interruption is required in the energy re-dispatch model. Loads at the same node are assumed to have the same priority of interruption cost, which is given in Table III.

![Table III: Load Interruption Cost](image)

| Load location | Interruption cost ($/MWh) |
|---------------|----------------------------|
| Buses 2-4, 7-10, 19 | 200                        |
| Buses 5, 14-16, 18, 20 | 220                        |
| Buses 1, 6, 13 | 240                        |

Considering that the operation reliability of power system is closely related to load levels, different load curves including the traditional load curve (case 1) and duck curve (case 2) are considered.

A. Case 1

In case 1, a typical load curve is utilized to illustrate the proposed models and techniques. The whole duration of study period is 24 hours. The peak load is 1850 MW and the valley load is 750 MW. The load increases rapidly between hour 6 and hour 8 and the load decreases rapidly between hour 22 and hour 24. The load curve is shown in Fig. 4.

![Load curve of case 1](image)

With the data mentioned above, the results of UC considering different levels of down SRs (scenario 1, 0 MW; scenario 2, 200 MW; scenario 3, 400 MW) can be calculated.
In scenario 1, the constraint of down SR is not considered. In the study, the up SR is set to be 200 MW, which is about 10% of the maximum load. Based on the UC results of different down SR levels, the reliability indices during different simulation periods can be calculated. The results of EENS and LOLP with different levels of down SRs are shown in Fig. 5 and Fig. 6, respectively.

The total minimum generation capacity of the three units (I22, I23 and I32) is 450 MW and the load demand is just 820 MW. Hence, one of the units of (I22, I23 and I32) is shut down, and other two smaller generation units are switched on. During the real-time operation, failed generators are not able to provide SR. With more on-line generation units, the expected up SRs of all contingency states \( \sum_j p_j \sum_g (RU_g, C'_g) \) in scenario 1 are less than those in scenario 2 and scenario 3. As a result, the whole power system is more reliable with more expected up SRs.

Furthermore, it can also be observed in Fig. 7 that between hour 19 and hour 22 when the demand decreases rapidly, the units are shut down less rapidly in scenario 3 (2 units are shut down) than that in scenario 1 (6 units are shut down) and scenario 2 (5 units are shut down). This is because there are more down SRs in scenario 3 and the units are not necessary to shut down immediately as the load demand decreases. With more online units, the power system of scenario 1 is more reliable than those of the other two scenarios. It can be proved from Fig. 5 and Fig. 6 that EENS increases less rapidly in scenario 3 than those in scenario 1 and scenario 2, which means scenario 3 is more reliable than scenario 1 and scenario 2.

The down SR can improve power system operation reliability in two aspects. On the one hand, down SR leads to a higher number of small units to replace large units, which further leads to less loss of up SR under contingency states and indirectly improves the operation reliability of power system. On the other hand, down SR enables the power system to slowly shut down generation units, when the load decreases rapidly. This leads to a higher number of online generation units and a higher level of up SR in real-time operation, which improves the operation reliability of power system.

**B. Case 2**

With large amount of renewable energy integrated into power systems, the shape of net load curve may change dramatically into duck curve, where the peak of electricity production occurs at the night and the valley of electricity production occurs in the day time. In case 2, a typical duck curve is considered to illustrate the proposed models and techniques. Due to the uncertainty of wind power generation, the realistic net load may vary greatly from the fore-
casted net load demand. The forecasted net load curve and the realistic load curve is shown in Fig. 8.

![Realistic and forecasted net load curves](image)

Fig. 8. Forecasted net load curve and realistic net load curve.

Based on the data mentioned above and the proposed simulation method, the UC results of different scenario with different levels of down SRs (scenario 1, 0 MW; scenario 2, 100 MW; scenario 3, 200 MW) can be calculated. The up SR is set to be 200 MW. The number of online units during hour 7 and hour 14 in two load demand curves are shown in Table IV.

| Time (hour) | Scenario 1 | Scenario 2 | Scenario 3 |
|------------|------------|------------|------------|
|            | Forecasted | Realistic  | Forecasted | Realistic  | Forecasted | Realistic  |
| 7          | 7          | 5          | 8          | 6          | 8          | 8          |
| 8          | 5          | 3          | 6          | 4          | 5          | 5          |
| 9          | 5          | 3          | 6          | 4          | 5          | 5          |
| 10         | 5          | 3          | 6          | 3          | 5          | 5          |
| 11         | 5          | 3          | 5          | 3          | 5          | 5          |
| 12         | 4          | 3          | 4          | 3          | 5          | 5          |
| 13         | 4          | 3          | 4          | 3          | 5          | 5          |
| 14         | 4          | 3          | 4          | 3          | 5          | 5          |

As can be seen from Table IV, it can be observed that with the increasing requirement of down SR, there are more on-line generation units in scenario 3 than those in scenario 2 and scenario 1 between hour 7 and hour 14. Besides, it can also be observed that the number of online units during hour 7 and hour 14 with the day-ahead forecasted load demand and with realistic load demand of scenario 3 is identical. However, in scenario 1, the number of online units during hour 7 and hour 14 with realistic load demand is less than that with day-ahead forecasted load demand. This is because the generation units in scenario 3 are not necessary to be shut down and can decrease the output to follow the sudden net load demand decrease with sufficient down SRs. On the contrary, several generation units are forced to be shut down to keep the power balance. As a result, the overall up SR provided by online units is reduced due to the shut down of generation units and the operation reliability of power system will worsen. Figure 9 presents the online up SRs and EENSs of scenario 1 and scenario 3 in real-time operation.

As can be seen from Fig. 9, EENS in scenario 1 is higher than that in scenario 3, which means that the operation reliability of scenario 3 is better. Therefore, when system load demand decreases suddenly to a very low level, the down SR enables the power system to keep the units operate at a low output level rather than being shut down. This indirectly maintains the quantity of up SR in power system, which can improve the operation reliability of power systems.

C. Case 3: Impacts of Wind Power Uncertainty on Operation Reliability of Power Systems

The effect of the uncertainty of wind power output on the operation reliability of power systems is studied. Six wind turbines have been added on buses 2, 14, 16, 17, 18, 19. The capacities of these wind turbines are all set to be 300 MW. Their outputs and the corresponding probabilities of the multi-state models are taken from [19]. The forecasted net load is the same as in case 2, while the realistic net load fluctuates around the forecasted net load due to the uncertainty of wind power output. Three scenarios with different down SR requirements are studied, as shown in Fig. 10.

![EENS with different down SRs](image)

Fig. 9. Comparison of up SR and EENS with different down SRs.

It can be observed from Fig. 10 that the wind power uncertainty reduces the operation reliability of power system. This is because the load may be curtailed if the wind power output is smaller than the forecasted value. By increasing the quantity of down SR in power system that indirectly maintains the quantity of up SR, the operation reliability of pow-
system with large uncertainty of wind power output can be improved.

D. Case 4: Comparison of Proposed Method and MCS

To validate the accuracy of the proposed method, the MCS approach is also developed to compare the results obtained by the proposed method. The convergence error for MCS is set to be 0.05. The proposed method and MCS approach are tested based on different scenarios of case 2, and the EENS results of the two methods are given in Fig. 11.

![Comparison of proposed method and MCS](image-url)

**Fig. 11.** Comparison of proposed method and MCS.

Figure 11 shows that the results of the proposed method are close to those of the MCS approach. The average percentage error of the proposed method and MCS is 3.9%, which is relatively low.

IV. CONCLUSION

This paper proposes an operation reliability evaluation method of power system considering the down SR and wind power uncertainty. Case studies show that the uncertainty of wind power output reduces the operation reliability of power systems, which also demonstrate that down SR can improve the operation reliability of power systems with large uncertainty of wind power output. The constraints of down SR enforce the power system to switch on smaller units instead of large units and enable a slower down rate of the generation units when the load decreases rapidly, which increases the number of online units. With the increased number of online units, the loss of up SR in contingency states will decrease and the operation reliability of power system will be improved. Furthermore, when load demand decreases to a very low level, the down SR enables the power system to keep the units operate at a low output level rather than being shut down. Thus, the quantity of up SR is maintained and the operation reliability of power systems is kept.

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