Mathematical modeling of the toroidal vortex structures interaction

Skladchikov S A 1, Savenkova N P 1, Kuzmin R N 2

1Faculty of Computational Mathematics and Cybernetics, Lomonosov Moscow State University, Moscow, Russia
2Faculty of Physics, Lomonosov Moscow State University, Moscow, Russia

E-mail: skladtchikov@mail.ru

Abstract. The paper presents the investigation of the toroidal vortex structures (TVS) interaction in various media. The proposed mathematical model makes it possible to study in details the stages of interaction of the toroidal vortices and allows to conclude what processes occur in the real collision of vortex structures, and to predict the result of this collision.

1. Introduction

The paper presents the research of toroidal vortex structures (TVS) interaction in various media. We introduce the 3D hydrodynamic mathematical model and demonstrate the results of numerical experiments that allow us to make the conclusions listed below.

The toroidal vortex formation process has three main stages: 1) the beginning; 2) the formation of the toroidal spiral jet in emerging vortex; 3) the establishing of the azimuthal rotation velocity after termination of the inlet. The laws of motion of the toroidal vortex and the surrounding substance, not engaged in the vortex motion, become different when the solid rotational core appears inside the vortex. From this fact we derive one of the main conclusions of our research: the toroidal vortex exists as an independent hydrodynamic structure with its own laws of motion after the rotational solid core has emerged. Such vortex is stable due to the low frontal resistance coefficient of the vortex in its translatory motion through the surrounding medium. This coefficient may be dozens of times smaller than the resistance coefficient of the water drop, which is the most streamline object. The stable toroidal vortex has its own properties similar to the properties of a solid body. If we place the origin in the center of the moving vortex, the velocity field of circumfluent medium will look like the flow around the solid obstacle.

2. Mathematical model

For simulating the process of the TVS formation we use the continuity equation, the Navier-Stokes equations of motion, the heat conductivity equation and the Mendeleev-Clapeyron equation. Our mathematical model in cylindrical coordinates looks as follows:

\[ \frac{\partial \rho}{\partial t} + \frac{1}{r} \frac{\partial r V_r \rho}{\partial r} + \frac{1}{r} \frac{\partial V_\phi \rho}{\partial \phi} = 0, \]

\[ \frac{\partial \rho V}{\partial t} + (\rho V V) V = -\nabla P + \mu \Delta \rho V - \frac{V_0^2}{r} \rho \partial e, \]

\[ -2 \rho [\Omega V], \]

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Here \( V_r \) and \( V_\phi \) are the radial and angular flow velocity components; \( P, T, \rho, M, \mu \) and \( \chi \) are respectively the pressure, temperature, density, molar mass, dynamic viscosity and coefficient of thermal diffusivity; \( R \) is the universal gas constant; \( er \) is the unit vector. This model has been proposed and discussed in details in [1-6].

3. Numerical results

The TVS and the necessary conditions for its formation are described in [1-5]. Here we discuss the results of computer simulation of several TVS interactions in the gaseous and liquid media.

Let us consider the collision of two toroidal vortices in the air when both structures are formed by the same vortex generator and move in the same direction. The second vortex has higher velocity than the first one; that’s why it finally overtakes the latter.

\[
\frac{\partial \rho \theta}{\partial t} + (\rho V \nabla) \theta = \chi \Delta (\rho \theta),
\]

\[
V = \begin{bmatrix} V_r \\ V_\phi \end{bmatrix}, \quad \Omega = \frac{V_\phi}{r}, \quad \theta = \frac{T - T_0}{T_0}, \quad P = \frac{\rho}{M} RT.
\]

Here \( V_r \) and \( V_\phi \) are the radial and angular flow velocity components; \( P, T, \rho, M, \mu \) and \( \chi \) are respectively the pressure, temperature, density, molar mass, dynamic viscosity and coefficient of thermal diffusivity; \( R \) is the universal gas constant; \( er \) is the unit vector. This model has been proposed and discussed in details in [1-6].

Figure 1. Collision stages for two TVS moving in the same direction

In Fig. 1 we present the process of formation and interaction of two toroidal vortices in the air. For convenience we demonstrate the evolution of the temperature distribution in time. The ambient temperature for this computation is 300 K, while the initial temperature of the emerging vortices is 299 K. The initial velocities of the first and of the second vortices are 3 m/s and 5 m/s respectively. As one can see from Fig. 1, the second vortex overtakes the first one, passes through it and then both vortices merge into a single large TVS.

Now let us consider the frontal collision of two toroidal vortices in the water. Two water structures approach each other along the horizontal axis and finally meet. The distance between the vortex generators is 1 m, and the vortices are equal in all aspects except for the initial velocity. The ambient temperature for this computation is 300 K, while the initial temperature of the emerging vortices is 299 K.
In Fig. 2 we present the process of formation and interaction of two toroidal vortices approaching each other. The initial velocities of the left and of the right vortices are 1 m/s and 3 m/s respectively. As one can see from Fig. 2, during the collision the right TVS, which has higher initial velocity, engages the left one into its structure. Together they make a new, more powerful and stable TVS. This unified structure continues moving in the same direction as the faster TVS.

Let us see what happens if the initial velocities are equal.

Figure 3. Collision stages for the frontal interaction of two TVS. Both initial velocities are 3 m/s.
As it is shown in Fig. 3, both vortices are equally strong and stable. That’s why after the collision both TVS disappear; two new structures, moving perpendicularly to the original direction, are formed instead. This is the most interesting case, and for this reason we shall study carefully not only the temperature, but also the velocity field of both structures.

**Figure 4.** Velocity field of frontal collision of two TVS. Both initial velocities are 3 m/s.

In Fig. 4 we see that the maximal velocity at the moments 1 and 2 is observed in the cores of both vortices. At the moments 3 and 4 the maximal velocity changes its direction and becomes perpendicular to the initial one. It means that the vortex transformation has begun, and the new cores emerge. At the moments 5 and 6 the cores of new TVS that move perpendicularly to the original direction are being formed.

4. **Conclusion**

As a result of mathematical modeling we demonstrate the stages of interaction of two stable toroidal vortex structures. The interaction scenarios depend on masses and initial velocities of vortices and may look as follows:

a) Absorption of a vortex by another one and further movement of the joint vortex in the direction defined by the stronger initial vortex.

b) Disintegration of both vortices and formation of new structures moving perpendicularly to the initial motion.

Our mathematical model allows to distinguish the details of described interaction stages. This model makes it possible to conclude what processes occur in a real collision of vortex structures and to predict the result of the collision.

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