PREDICTION OF COUPLING CONSTANT RATIO VALUES IN THE OCTET HYPERON EM STRUCTURE
UNITARY AND ANALYTIC MODELS

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Utilizing the SU(3) invariant vector–meson–baryon interaction Lagrangians, the knowledge of the universal vector–meson coupling constants \( f_V \) and the numerical values of nucleon coupling constant ratios, all unknown hyperon coupling constant ratios in hyperon electromagnetic structure Unitary and Analytic models are predicted.

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All \( 1/2^+ \) baryons are compound of 3 quarks, then they have to manifest some space structure, which for the first time was discovered for \( p \) in elastic scattering process \( e^- p \rightarrow e^- p \) almost 70 years ago. As the electrons are dominantly interacting with \( p \) electromagnetically, this structure is called the proton electromagnetic (EM) structure, which has been theoretically generalized also to all other members of \( 1/2^+ \) octet baryons.

The EM structure of baryons is completely described by the Dirac \( F_{1B}^\gamma(s) \) and Pauli \( F_{2B}^\gamma(s) \) form factors (FFs) to be naturally obtained in a decomposition of the baryon matrix element of the EM current \( J_{\mu}^{EM}(0) \) into a maximal number of linearly-independent covariants,

\[
\langle B|J_{\mu}^{EM}(0)|B \rangle = e \bar{u}(p') \left[ \gamma_{\mu} F_{1B}^\gamma(s) + \frac{i}{2m_B} \sigma_{\mu\nu} (p' - p)_{\nu} F_{2B}^\gamma(s) \right] u(p), \quad (1)
\]

constructed from four-momenta, \( \gamma \)-matrices and Dirac bispinors of baryons. They are very suitable for a theoretical description of the baryon EM structure. However, for an extraction of experimental information on it from

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the measured quantities, such as cross sections and polarizations, the Sachs
electric $G_E^B(s)$ and magnetic $G_M^B(s)$ FFs are more suitable. Both systems of
FFs are related as follows:

$$
G_E^B(s) = F_1^B(s) + \frac{s}{4m_B^2} F_2^B(s),
$$

$$
G_M^B(s) = F_1^B(s) + F_2^B(s),
$$

(2)

with normalizations $G_E^B(0) = Q$ and $G_M^B(0) = \mu_B$, $Q$ is the electric charge
of the baryon and $\mu_B$ is its magnetic moment.

Further, in a theoretical description of the EM structure of baryons, the
mixed transformation property of the EM current under rotation in isospin
space, i.e. that a part of it transforms as an isoscalar and the other of it as
a third component of an isovector, is utilized.

As a result, every baryon matrix element of the EM current may be
expressed in terms of the matrix elements of an isoscalar and of an isovector
currents, which lead to a splitting of the Dirac and Pauli FFs on a combina-
tion of the isoscalar part and the isovector part. The sign between them de-
pends on the value of the third component of isospin of the concrete baryon
from the $1/2^+$ baryon octet, which can be found by the relation $T_3 = Q - \frac{B}{2} - \frac{S}{2}$.

Next 9 vector–meson resonance Unitary and Analytic models of the $1/2^+$
octet baryon EM FFs are discussed, which also, like nucleon Unitary and
Analytic model [1], are generally representing an unification of well-balanced
properties like the experimental fact of a creation of unstable vector–meson
resonances $\rho, \omega, \phi, \rho', \omega', \phi', \rho'', \omega'', \phi''$ [2] with photon quantum numbers in $e^+e^-$-annihilation processes into hadrons, very similar hypothetical analytic
properties for all baryon EM FFs in the complex s-plane, and the asymptotic
behavior of baryon EM FFs $G_E^B(s) = G_M^B(s) \sim \frac{1}{s}$ as predicted by the quark
model of hadrons to be also proved [3] in the framework of QCD. Taking into
account a similarity of analytic properties, the explicit formulas for baryon
isoscalar and isovector parts of the Dirac and Pauli FFs can be taken as a
generalization of those for nucleons [1] in the form of

$$
F_{1s}^B[V(s)] = f \left[ s; \left( f_{\omega_BB}^{(1)} / f_\omega \right), \left( f_{\phi_BB}^{(1)} / f_\phi \right), \left( f_{\omega'_BB}^{(1)} / f_{\omega'} \right), \left( f_{\phi'_BB}^{(1)} / f_{\phi'} \right), s_{1s}^{1s} \right],
$$

$$
F_{1v}^B[W(s)] = f \left[ s; \left( f_{\rho_BB}^{(1)} / f_\rho \right), s_{1v}^{1v} \right],
$$

$$
F_{2s}^B[U(s)] = f \left[ s; \left( f_{\omega_BB}^{(2)} / f_\omega \right), \left( f_{\phi_BB}^{(2)} / f_\phi \right), \left( f_{\omega'_BB}^{(2)} / f_{\omega'} \right), \left( f_{\phi'_BB}^{(2)} / f_{\phi'} \right), s_{2s}^{2s} \right],
$$

$$
F_{2e}^B[X(s)] = f \left[ s; s_{2e}^{2e} \right].
$$

(3)

They are functions of squared energy in the c.m. system $w^2 = s$ and depend
on some baryon coupling constant ratios and effective inelastic thresholds $s_{in}^i$,
\( i = 1s, 1v, 2s, 2v \) as free parameters of the model. So, if one knows numerical values of free parameters in \( F_{1s}^B[V(s)], F_{1v}^B[W(s)], F_{2s}^B[U(s)], F_{2v}^B[X(s)] \), one can predict behaviors of electric \( G_E^B(s) \) and magnetic \( G_M^B(s) \) FFs of any baryon from the \( 1/2^+ \) octet.

The simplest way of a determination of the coupling constant ratios is in the case of nucleons, as there is around 600 reliable experimental points on the nucleon EM FFs in the spacelike and timelike regions, dominant for proton and less for neutron. Their values are found in an optimal description of these data by normalized \( F_{1s}^B[V(s)], F_{1v}^B[W(s)], F_{2s}^B[U(s)], F_{2v}^B[X(s)] \) to nucleons, which gives

\[
\begin{align*}
(f_{\rho NN}^{(1)}/f_\rho) &= (0.3747 \pm 0.0022); & (f_{\omega NN}^{(1)}/f_\omega) &= (1.5717 \pm 0.0022); \\
(f_{\phi NN}^{(1)}/f_\phi) &= (-1.1247 \pm 0.0011); & (f_{\omega NN}^{(1)}/f_\omega) &= (0.0418 \pm 0.0065); \\
(f_{\phi NN}^{(1)}/f_\phi) &= (0.1879 \pm 0.0010); & (f_{\omega NN}^{(1)}/f_\omega) &= (-0.2096 \pm 0.0067); \\
(f_{\phi NN}^{(1)}/f_\phi) &= (0.2657 \pm 0.0067); & (f_{\phi NN}^{(1)}/f_\phi) &= (0.1781 \pm 0.0029).
\end{align*}
\]

On the other hand, there are no data on EM FFs of \( 1/2^+ \) octet hyperons \( \Lambda, \Sigma^+, \Sigma^0, \Sigma^-, \Xi^0, \Xi^- \), only some total cross-section experimental points on \( e^+e^- \)-annihilation into hyperon–anti hyperon exist and one cannot repeat a similar to nucleons procedure in determination of the unknown coupling constant ratios in the Unitary and Analytic hyperon EM structure models.

Despite this fact, as it is demonstrated further, one finds a method for their determination, utilizing the nucleon coupling constant ratios (4) and the SU(3) invariant Lagrangians of vector–meson interaction with octet baryons

\[
L_{VBB} = \frac{i}{\sqrt{2}} f^F \left[ \bar{B}_\beta^\alpha \gamma_\mu B_\gamma^\beta - \bar{B}_\gamma^\beta \gamma_\mu B_\beta^\alpha \right] (V_\mu)_\alpha^\gamma \\
+ \frac{i}{\sqrt{2}} f^D \left[ \bar{B}_\beta^\alpha \gamma_\mu B_\beta^\alpha + \bar{B}_\gamma^\alpha \gamma_\mu B_\gamma^\alpha \right] (V_\mu)_\alpha^\gamma + \frac{i}{\sqrt{2}} f^S \bar{B}_\beta^\gamma \gamma_\mu B_\alpha^\beta \omega_\mu^0,
\]

where baryons are represented by the octet matrices

\[
B = \begin{pmatrix}
\frac{\Sigma^0}{\sqrt{2}} + \frac{\Lambda^0}{\sqrt{6}} & \Sigma^+ & p \\
\Sigma^- & -\frac{\Sigma^0}{\sqrt{2}} + \frac{\Lambda^0}{\sqrt{6}} & n \\
\Xi^- & \Xi^0 & \Xi^0 \end{pmatrix},
\]

\[
\bar{B} = \begin{pmatrix}
\frac{\Sigma^0}{\sqrt{2}} + \frac{\Lambda^0}{\sqrt{6}} & \bar{\Sigma}^- & \bar{\Xi} \\
\bar{\Sigma}^+ & -\frac{\Sigma^0}{\sqrt{2}} + \frac{\Lambda^0}{\sqrt{6}} & \bar{\Xi} \\
\bar{p} & \bar{n} & -\frac{2\Lambda^0}{\sqrt{6}}
\end{pmatrix},
\]
and the $1^{--}$ vector–meson nonets are classified into octet matrices and singlets like

$$V = \begin{pmatrix}
\omega_8/\sqrt{6} + \rho^0/\sqrt{2} & \rho^+ & K^{*+} \\
\rho^- & \omega_8/\sqrt{6} - \rho^0/\sqrt{2} & K^{*0} \\
K^{*-} & -\rho^0/\sqrt{2} & -2\omega_8/\sqrt{6}
\end{pmatrix}, \quad \omega_0. \quad (8)$$

Decomposing Lagrangians (5) into a sum of products of single matrix elements, applying at the same time a physically acceptable $\omega$–$\phi$ mixing forms [4], one finds relations between $f_{VNN}$ and the coupling constants $f^F, f^D, f^S$ of SU(3) invariant Lagrangians as follows (for details, see [4]):

1. $f_{\rho NN} = \frac{1}{2} (f^F + f^D)$, $f_{\omega NN} = \frac{1}{\sqrt{2}} f^S \cos \theta + \frac{1}{2\sqrt{3}} (3 f^F - f^D) \sin \theta,$

   $f_{\phi NN} = \frac{1}{\sqrt{2}} f^S \sin \theta - \frac{1}{2\sqrt{3}} (3 f^F - f^D) \cos \theta,$

4. $f_{\rho NN} = \frac{1}{2} (f^F + f^D)$, $f_{\omega NN} = -\frac{1}{\sqrt{2}} f^S \cos \theta - \frac{1}{2\sqrt{3}} (3 f^F - f^D) \sin \theta,$

   $f_{\phi NN} = -\frac{1}{\sqrt{2}} f^S \sin \theta + \frac{1}{2\sqrt{3}} (3 f^F - f^D) \cos \theta,$

5. $f_{\rho NN} = \frac{1}{2} (f^F + f^D)$, $f_{\omega NN} = \frac{1}{\sqrt{2}} f^S \cos \theta + \frac{1}{2\sqrt{3}} (3 f^F - f^D) \sin \theta,$

   $f_{\phi NN} = -\frac{1}{\sqrt{2}} f^S \sin \theta + \frac{1}{2\sqrt{3}} (3 f^F - f^D) \cos \theta,$

8. $f_{\rho NN} = \frac{1}{2} (f^F + f^D)$, $f_{\omega NN} = -\frac{1}{\sqrt{2}} f^S \cos \theta - \frac{1}{2\sqrt{3}} (3 f^F - f^D) \sin \theta,$

   $f_{\phi NN} = +\frac{1}{\sqrt{2}} f^S \sin \theta - \frac{1}{2\sqrt{3}} (3 f^F - f^D) \cos \theta,$

(9)

and similarly for hyperons.

Then from the inverse relations to (9), substituting there numerical values of $f_{VNN}^{(i)}, i = 1, 2$ determined from the known nucleon coupling constant ratios (4) by the values of the universal vector–meson coupling constants $f_V$ to be extracted from lepton widths $\Gamma(V \to e^+e^-)$ of the vector–mesons under consideration, respecting at the same time signs [5] of them caused by physically acceptable $\omega$–$\phi$ mixing forms, one finds numerical values

$$f^F_1 = -4.9916; \quad f^D_1 = +1.2776; \quad f^S_1 = -43.0979 \quad (10)$$

to be fully independent of the used $\omega$–$\phi$ mixing form.
In the same way also

\[ f^F_2 = -7.6976; \quad f^D_2 = -21.0036; \quad f^S_2 = +7.1225; \]  \quad (11) \\
\[ f^{F'}_1 = -5.1607; \quad f^{D'}_1 = -16.4400; \quad f^{S'}_1 = -9.3767; \]  \quad (12) \\
\[ f^{F'}_2 = +29.9610; \quad f^{D'}_2 = +5.7612; \quad f^{S'}_2 = +20.0463 \]  \quad (13)

are determined.

If these SU(3) coupling constants are substituted into similar relations to (9) for \( f^{(i)}_{VYY} \) and \( f^{(i)}_{V'Y'Y} \), with \( i = 1, 2 \) and \( Y = \Lambda, \Sigma, \Xi \), respectively, to be derived from the vector–meson–baryon interaction Lagrangians as functions of these SU(3) coupling constants, and the resultant values are divided, respectively, by \( f_\rho = |4.9582|, \quad f_\omega = |17.0620|, \quad f_\phi = |13.4428| \) and \( f_{\rho'} = |13.6491|, \quad f_{\omega'} = |47.6022|, \quad f_{\phi'} = |33.6598| \) with signs to be dependent on the physically acceptable \( \omega–\phi \) mixing forms, one finally gets unambiguous values of all unknown coupling constant ratios of vector–meson with hyperons in the hyperon Unitary and Analytic EM structure models.

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