Prediction of Cutting Forces Experimentally in End Milling Process Using Inverse Analysis

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Abstract. Milling is a versatile manufacturing process which is used in machining many types of complex geometry parts, which has become the prior subject for many research studies. This paper provides study on cutting forces on end milling during machining. The study deals with the indirect measurement of cutting forces during machining using frequency response function (FRF). The problem of determining the unknown force acting on a body or a structure is the inverse problem. The cutting forces acting on a tool can be estimated by the responses measured at different points when the tool comes in contact with the work piece. The accuracy of the method can be improved by using singular value decomposition method and Moore Penrose pseudo inverse method. The experimental method results in better identification of cutting force which helps in optimum design and development of cutting tools. Statistical analysis helps in knowing the level of significance of the proposed method.

1. Introduction.
One of the attentive inverse problem analysis is the force identification which is very much significant for engineering design. The advancements in technology has made possible in designing and developing the cutting tools to carry out the task in efficient way. The increasing tussle between the industries and limited availability of resources has made it essential to utilize the resources to the maximum possible extent. The determination of actual cutting forces [1] is helpful in solving problems like strength, reliability and fatigue of structures and machine members. The actual cutting forces helps in predicting the optimum power consumption by the machine tool and tool life of the cutting tool. Inverse analysis is a technique where force is reconstructed from the response signals to that of the applied or input force coming on to the member. The reconstruction of force is primarily of interest where the input force cannot be directly measured, while the response data or signals can be measured easily using transducers. In spite of the actual application, the force coming on to the member plays a vital role in determining the properties and parameters of the member. The study of force identification on a machine tool helps in the better knowledge of actual cutting force coming on to the cutting tool, which is very useful in estimating the cutting power consumption, which is helpful in selection of power sources at the time of designing the machine tool. The force identification [2] also helps in many ways such as optimum design of structural machine fixture- tool system, evaluating the process and machining parameters for machining such as geometry of tool, material of tool, cutting environment etc. on the cutting forces, helps in understanding and studying the machinability characterization regarding the materials to be machined, monitoring of machine tools and cutting
tools. In most of the practical applications it is difficult to measure the forces directly acting on structures or during machining. In some applications, the force transducer used to measure the dynamic forces are inserted on the force convey path which affects the system properties and obtrude in the load path. More over as an alternative to this, the responses during machining can be conveniently measured using a force transducer such as accelerometer. In such cases indirect measurement of the cutting forces can be done from the measured structural responses, which is treated as an inverse problem. The unknown force determined coming on to cutting tool is the solution to the inverse problem in the force identification which is done from the measured responses. The various examples staging the need for the force identification, one of them is cutting forces during the operation of machine tools, reaction forces encountered in the engine mounts and forces occurring from the supports in case of bearings etc. Determining the unknown force coming on to a structure or machine from the measured responses is an inverse problem.

The cutting tool life of a cutting tool is greatly affected by cutting forces as these forces are not static in nature. During the beginning when the cutting tool comes into contact with the work piece higher magnitude of cutting force, which are of impact type comes on to the tool and are enough to cause plastic deformation or chipping of tool tip which eventually causes tool failure. In case of end mills when the first teeth comes into contact with the work piece the teeth experiences sudden load which may initially deform the cutting tool. Parameters such as depth of cut, cutting speed, feed, cutting fluid, type of material, tool geometry, condition of the machine tool, properties of work material, defects in work material etc. affects the cutting forces on the tool there by affecting the tool life and power consumption of machine tool. The surface finish and the surface accuracy of the machined part is also affected by the cutting forces. The latest developments in the manufacturing industry has improved the efficiency of production plants, but on the other side there are industries which also run on conventional machine tools where premature tool failure is biggest concern and problem, as these cutting tool are been designed for average cutting forces. M. Wan and W.H. Jhang carried out the work on determination of uncut chip thickness for estimation of cutting forces. Where cutting forces were determined at regular intervals of rotational angle of end mill tool. Premature tool failure brings the conventional production plants to a standstill there by hindering major losses to the plant. The measurement of the cutting forces directly during machining is difficult, as it is very much difficult to place the sensor at the tool tip as the tool tip comes in contact with the work piece during machining. The placement of the sensor directly on the tool tip damages the sensor, so in order to measure the cutting force, inverse force identification technique is used to predict the cutting force coming on to the tool. The dynamics of the mechanical structures can be grouped into two types of problem, one is the forward problem and the other is the inverse problem. In case of forward problem prediction of data (output) is done based upon the estimated parameters (input) using mathematical model or physical theory. Where as in inverse problem the parameters are predicated (input) based upon the measured data (output) using mathematical model or physical theory. There are various other methods developed to determine the cutting forces in end milling.

The amount of external force or impact force coming on the structure is significant factor in designing the cutting tool. However it is difficult task to measure the cutting forces directly with the transducer when the force locations are not easily accessible, while vibrations occurring during machining can be easily measured with the help of the responses. That’s why, indirect methods are usually preferred over direct methods, where the forces can be determined based on inverse analysis with the help of sensors. There are numerous force identification methods using indirect technique which have been proposed in recent past years. The indirect techniques used are either in the time domain or in the frequency domain. In frequency domain the force and the response of the structure are related by FRF (frequency response function) which can be determined using Fourier Transform method. The inverse force identification problem requires inverse of FRF matrix or the transfer function matrix, which suffers from ill-posedness. During measuring the responses the noise component could have adverse effect on the stability of the solution which can make the solution instable. The ill-posed problem is converted to well poise problem by regularization process where additional data is added to the obtained
solution. There are various regularization methods used such as singular decomposition method (SVD), Generalised cross validation, Iterative method, Data filtering approach, Tikhonov regularisation method. In the present study SVD technique and Moore Penrose Pseudo inverse method [16] is used for the effective inversion.

2. Force Assessment Techniques
The different types of force estimation techniques that are widely used are (i) Sum of weighted acceleration technique (ii) Pseudo inverse method, (iii) Kaman filter and (iv) Empirical approach for system identification. In SWAT the input force is estimated in time domain using weights which are estimated based on the reference input force. The pseudo inverse technique is based on computing FRF’s which is carried out in frequency domain. In the Kalman filter technique, system parameters are must for estimating the input force where the system is modelled in terms of state space equations. The empirical method used for system identification involves pertaining to obtain the empirical relationship between the response data and input force, using which the latter can be estimated. The forces which are dynamic in nature can be estimated either using (i) time domain (ii) frequency domain.

2.1 Time Domain Method
The Time domain techniques estimate the input forces from measured response in time domain. The structure response is a function of its Impulse Response Function (IRF). The forces acting on the structure is given by the convolution as,

\[ x(n) = h(n) * f(n) + s(n) \]  

(2.1)

where, \( x(n) \) is the system output, \( h(n) \) is the impulse response the system possess, \( f(n) \) is the system input and \( s(n) \) is the unwanted noise component during the measurement of responses. It can be also given by the convolution integral as,

\[ x = \int_{t_0}^{t} h(t-\tau)f(\tau)d\tau \]  

(2.2)

The unknown forces are estimated with the help of measured responses and impulse response. The accuracy of identification of force in time domain is influenced conditioning of system markov parameters. The methods used are direct computation of the condition number of the matrix which involves determination under various sensor location combinations. The estimation of the input force acting on the nonlinear structural system can be determined using extended kalman filter and recursive least squares estimator from the responses measured. Spline curved based interpolation scheme [17] is an alternative to the kalman filter as the kalman filter leads to accuracy loss because of incorrect approximation of the transfer function.

2.2 Frequency Domain Method
The relationship between the applied forces and the measured responses is a linear one as the function of frequency in the frequency domain method. The relationship is the transfer function which is also known as frequency response function. The convolution integral which is given in equation 2.2, Where \( x(t) \) is the \((n_s \times 1)\) response vector, \( f(t) \) is the \((n_f \times 1)\) excitation force vector, \( h(t) \) is the \((n_s \times n_f)\) Impulse Response Function (IRF) matrix.

Taking the Fourier transform of Eqn. (2.2), the relation can be expressed in the frequency domain as:

\[ \{X(\omega)\} = \{H(\omega)\}\{F(\omega)\} \]  

(2.3)

Where \( \omega \) is the angular frequency, \( \{X(\omega)\} \) is the \((n_s \times 1)\) response vector, \( \{H(\omega)\} \) is the \((n_f \times 1)\) excitation force vector, \( \{F(\omega)\} \) is the \((n_s \times n_f)\) Frequency Response Function (FRF) matrix.
The FRF is determined experimentally from measured data, or from reconstruction of modal model of the system, or can be examined using FEM. The responses \( \{X(\omega)\} \) can be measured experimentally considering as any of the physical quantities such as displacement, velocity, acceleration, or strain. Once \( \{X(\omega)\} \) and \( [H(\omega)] \) are known, the problem now remains that of solving for \( \{F(\omega)\} \) and thereby computing the time history of the input forces \( f(t) \) using the inverse Fourier transform. For square \( n = n \) and non-singular \( [H(\omega)] \), Eqn. (2.3) can be inverted to give:

\[
\{F(\omega)\} = [H(\omega)]^{-1} \{X(\omega)\}
\]

But, this inverse problem is not straightforward and easy as it appears as the ill-conditioning should be solved. For the non-minimum phase system [18] the ill conditioning can be solved using the SVD technique for the best inversion of the system matrix which has either singular or near singular values. The information of the phase of the system and ill conditioning is got from the small singular values which are verified with the cantilever beam [19]. The force identification problem is ill-posed, as some of the initial variables are unknown, which can be solved using regularization process.

3. Experimentation

The experiments were performed on end mill tool with cast iron as the work material. The end mill of 12mm diameter made up of high speed steel material having four flutes were chosen, where test were carried out at different depth of cut, different feed rate and at various speeds. The work piece material is cast iron having dimensions of 50mm x 100mm x 3mm. The thickness of the work piece material is 3mm. The end mill tool has the tool geometry as, axial rake angle 30° to 35°, radial rake 12°, radial relief angle 10° and the corner 1 x 45°. The approach angle is the angle between a plane perpendicular to the cutter axis and a plane tangent to the surface of revolution of the cutting edges. In end mills the approach angle is 90°. The accelerometer is mounted on the work piece at a distance of 25mm from the free surface and the impact hammer is impacted at the free edge of the work piece. The responses were measured with the help of accelerometer using data acquisition system. The measured responses are force and acceleration in time domain for three seconds and these responses are measured with the help of LAB VIEW software and stored as lvm file.

![Figure 1. Details of impact force and acceleration in time domain](image1)

![Figure 2. FRF curve for the data of figure.1](image2)

The accelerometer and the impact hammer are of PCB make having 352C68, 10.2 mV/ (m/s²) and 086D05, 0.23 mV/N respectively. The frequency response function (FRF) is determined by writing the algorithm in the MATLAB. The FRF obtained is a function of acceleration and force in frequency domain. The procedure is repeated for various impacted force test on the work piece, where a FRF is chosen for force prediction. Figures 1 and 3 represents the acceleration and force history when impacted at the free edge of the work piece plate. Figures 2 and 4 represent the FRF for the data in
figures 1 and 3. The FRF shown in figure 4 is chosen to determine the unknown forces coming on to the tool.

![Figure 3. Details of impact force and acceleration in time domain](image3.png)

**Figure 3.** Details of impact force and acceleration in time domain

**Figure 4.** FRF curve for the data of figure 3.

Now the work piece is mounted on the vertical milling machine and an end mill tool is attached. The accelerometer is attached at 25mm from the free edge on the work piece and machining is done on the work piece. The accelerometer measures the acceleration during the machining and is stored as lvm file for three seconds. The measured acceleration is in time domain and converted to frequency domain using fast Fourier transform. With this available data i.e. acceleration of end mill tool and frequency response function the unknown force coming on to the tool is determined which is in frequency domain. For analysis to know the correct magnitude of incoming cutting force, inversion from frequency to time domain is done. Figures 5, 6, 7, 8, 9 shows the prediction of cutting forces at various feed rates at 136 rpm having depth of cut as 0.5mm.

![Figure 5. Force for 3mm/min feed.](image5.png)

**Figure 5.** Force for 3mm/min feed.

![Figure 6. Force for 9mm/min feed.](image6.png)

**Figure 6.** Force for 9mm/min feed.
Figure 7. Force for 18mm/min feed.

Figure 8. Force for 27mm/min feed.

Figure 9. Force for 36mm/min feed.

Figure 10. Force for 0.1mm depth of cut.

Figure 11. Force for 0.3mm depth of cut.

Figure 12. Force for 0.5mm depth of cut.
Figures 10, 11, 12, 13, 14 show the prediction of cutting forces at various depth of cut at 136 rpm having feed rate as 18 mm/min.

Statistical analysis was carried in order to determine the level of significance of the experimental method. Sampling distribution is on the basic assumption which may be either normal or approximately normal provided the sample size N is large (N > 30), but if the sample size is less or small samples then the sampling distribution is t-distribution [20] which is used in studies involving small samples. The statistic t is defined by

\[
t = \frac{x - \mu}{s \sqrt{N}}
\]

(3.1)

Where N is the sample size, x is the sample mean, \(\mu\) is the population mean and s is the sample standard deviation. To carry out the statistical analysis experiments were conducted on the cutting tool for five samples. The cutting force, \(F_i\), predicted in Newton for five samples at feed rate of 18 mm/min, 0.5 mm depth of cut and at 136 rpm are 64.39 N, 65.92 N, 65.35 N, 64.91 N, 62.47 N respectively. Where \(F_i\) is experimentally predicted force for five samples.

The sample mean is given by

\[
x = \frac{\sum F}{N}
\]

(3.2)

\(x = 64.60\) N

The sample variance is

\[
s^2 = \frac{\sum (F_i - x)^2}{N}
\]

(3.3)

\(s = 1.186\)

The population mean is \(\mu = 62.24\). From equation 3.1 the statistic is \(t = 4.4487\). But the critical value for t-distribution of sample size five with N-1 is 4.604 for 0.005 level of significance, which means the predicted cutting force by experimentation has confidence level of 99% having 1% of error.

4. Results

The experimental results are validated using theoretical relation. The cutting force in Newton can be theoretical calculated by

\[
F_c = Zs Ks a_s
\]

(4.1)

\(Z_s = \) No. of teeth in simultaneous engagement with the work piece = \((Z/360) \Psi_s\), where \(\Psi_s\) is the angle of contact in degree.

\(Z = \) No. of teeth.

\(K_s = \) Specific cutting force in N/mm² which can be read from graph of specific forces vs. chip thickness corresponding to the given material.

\(a_s = \) chip thickness in mm.
b = t / \sin \lambda \text{ chip width in mm, } t = \text{depth of cut in mm}

\lambda = \text{approach angle in degree i.e. } 90^\circ \text{ for end mills.}

For cast iron material at 136 rpm. For varying feed rate of 3 mm/min to 36 mm/min and constant depth of cut of 0.5 mm the cutting forces are,

**Table 1**: Comparison of experimental and theoretical cutting force at various feed rates showing absolute and relative error.

| Sl. No | Feed in mm/min | Experimental Cutting Force in N | Theoretical cutting Force in N | Absolute error | Relative error |
|--------|-----------------|--------------------------------|-------------------------------|----------------|---------------|
| 1      | 3               | 13.48                          | 13.19                         | 0.29           | 0.0215        |
| 2      | 9               | 33.12                          | 32.17                         | 0.95           | 0.0287        |
| 3      | 18              | 64.39                          | 62.24                         | 2.15           | 0.0333        |
| 4      | 27              | 98.23                          | 96.55                         | 1.68           | 0.0171        |
| 5      | 36              | 145.2                          | 141.51                        | 3.69           | 0.0254        |

For varying depth of cut 0.1 mm to 0.9 mm and constant feed rate of 18 mm/min the cutting forces are,

**Table 2**: Comparison of experimental and theoretical cutting force at various depth of cuts showing absolute and relative error.

| Sl. No | Depth of Cut in mm | Experimental Cutting Force in N | Theoretical cutting Force in N | Absolute error | Relative error |
|--------|--------------------|--------------------------------|-------------------------------|----------------|---------------|
| 1      | 0.1                | 17.15                          | 14.30                         | 2.85           | 0.1666        |
| 2      | 0.3                | 39.81                          | 38.66                         | 1.15           | 0.0288        |
| 3      | 0.5                | 47.56                          | 45.60                         | 1.96           | 0.0412        |
| 4      | 0.7                | 55.70                          | 53.15                         | 2.55           | 0.0457        |
| 5      | 0.9                | 65.43                          | 64.22                         | 1.21           | 0.0185        |

The cutting forces were measured at different cutting parameters and it is clear from the above table 1 and 2 that the magnitude of cutting forces encountered during the machining process are higher than the computed one. The actual cutting forces are greater as there is initial impact or sudden force coming on to the tool or the flute of end mill during the initial instance when tool comes in contact with the work piece during the first machining cut. In comparison the cutting forces for the varied feed rate and varied depth of cut, the cutting forces are higher in magnitude in case of varied feed rate. The amount of cutting forces coming on to the tool are also affected by the condition of machine tool. The position and the location of the accelerometer is important as in some cases the results obtained can be oscillatory in nature. This can be, because the position of the accelerometer can be close to one of the nodal point of the mode shapes. From the present analysis the predicted cutting force are 0.29 to 3.69 times the computed force, which is the deciding factor for the deviation in the tool life of the cutting tool.

In figures 15, 16, 17, 18 series 1 is the experimental data and series 2 is the computed or the theoretical data. Figures 15 and 16 shows the comparisons of cutting force between computed and experimental values. It is found from these figures that there is an obvious fluctuation of the experiment results due to the initial impact of cutting tool with the work piece and other uncertainties of the cutting process.
Figures 17 and 18 shows the comparisons of power consumption during machining between computed and experimental values. It is found from these figures that there is a fluctuation of the experiment results as the amount of actual cutting force coming on to the tool is greater than the computed one.
5. Conclusion
Inverse methods were used for the estimation of cutting force coming on to the cutting tool during the machining process. Inverse method are helpful in determining the unknown force coming on to the tool. The cutting force coming on to cutting tool is greater than the computed one due to initial impact between tool and work piece. The cutting forces during machining operation are important as these forces affect the life of the tool, because of which premature tool failure occurs. These cutting forces help in predicating the power consumption of the machine tool there by predicating the process economy of the cutting tool during its operation. Actually consumed power during machining operation is higher which lowers the efficiency of machine tool. By placing the accelerometer close to the location of unknown forcing function best results are achieved. Prediction of the cutting force during machining process is affected by the noise component and the unwanted signals which are measured during the measurement. The noise component is suppressed by adding suitable condition to the solution there by making the solution stable and prediction of cutting force accurate. Regularization technique such as SVD method helps achieving better results thereby minimizing the oscillations. The method used predicts accurate results there by having higher reliability. The cutting tools are required to design using the actual operating cutting force there by avoiding the premature tool failures. This identification of cutting force also helps in better design developments and optimization of machine tools.

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