COMPARISON OF PLANAR PARALLEL MANIPULATOR ARCHITECTURES BASED ON A MULTI-OBJECTIVE DESIGN OPTIMIZATION APPROACH

Damien CHABLAT, Stéphane CARO, Raza UR-REHMAN, Philippe WENGER
Institut de Recherche en Communications et Cybernétique de Nantes
UMR CNRS n° 6597
1 rue de la Noé, 44321 Nantes, France
Email: {chablat, caro, ur-rehman, wenger}@irccyn.ec-nantes.fr

ABSTRACT

This paper deals with the comparison of planar parallel manipulator architectures based on a multi-objective design optimization approach. The manipulator architectures are compared with regard to their mass in motion and their regular workspace size, i.e., the objective functions. The optimization problem is subject to constraints on the manipulator dexterity and stiffness. For a given external wrench, the displacements of the moving platform have to be smaller than given values throughout the obtained maximum regular dexterous workspace. The contributions of the paper are highlighted with the study of 3-PRR, 3-PRR and 3-BRR planar parallel manipulator architectures, which are compared by means of their Pareto frontiers obtained with a genetic algorithm.

INTRODUCTION

The design of parallel kinematics machines is a complex subject. The fundamental problem is that their performance heavily depends on their geometry [1] and the mutual dependency of almost all the performance measures. This makes the problem computationally complex and yields the traditional solution approaches inefficient. As reported in [2], since the performance of a parallel manipulator depends on its dimensions, the latter depend on the manipulator application(s). Furthermore, numerous design aspects contribute to the Parallel Kinematics Machine (PKM) performance and an efficient design will be one that takes into account all or most of these design aspects. This is an iterative process and an efficient design requires a lot of computational efforts and capabilities for mapping design parameters into design criteria, and hence turning out with a multiobjective design optimization problem. Indeed, the optimal geometric parameters of a PKM can be determined by means of a the resolution of a multiobjective optimization problem. The solutions of such a problem are non-dominated solutions, also called Pareto-optimal solutions. Therefore, design optimization of parallel mechanisms is a key issue for their development.

Several researchers have focused on the optimization problem of parallel mechanisms the last few years. They have come up either with mono- or multi-objective design optimization problems. For instance, Lou et al. [3, 4] presented a general approach for the optimal design of parallel manipulators to maximize the volume of an effective regular-shaped workspace while subject to constraints on their dexterity. Hay and Snyman [1] considered the optimal design of parallel manipulators to obtain a prescribed workspace, whereas Ottaviano and Ceccarelli [5, 6] proposed a formulation for the optimum design of 3-Degree-Of-Freedom (DOF) spatial parallel manipulators for given position and orientation workspaces. They based their study on the static analysis and the singularity loci of a manipulator in order to optimize the geometric design of the Tsai manipulator for a given free-singularity workspace. Hao and Merlet [7] discussed a multi-criterion optimal design methodology based on interval analysis to determine the possible geometric parameters satisfying two compulsory requirements of the workspace and accuracy. Similarly, Ceccarelli et al. [8] dealt with the multi-criterion op-
timum design of both parallel and serial manipulators with the
focus on the workspace aspects, singularity and stiffness prop-
ties. Gosselin and Angeles [9, 10] analyzed the design of a
3-DOF planar and a 3-DOF spherical parallel manipulator by
maximizing their workspace volume while paying attention to
their dexterity. Pham and Chen [11] suggested maximizing the
workspace of a parallel flexible mechanism with the constrains
on a global and uniformity measure of manipulability. Stamper et
al. [12] used the global conditioning index based on the integral
of the inverse condition number of the kinematic Jacobian matrix
over the workspace in order to optimize a spatial 3-DOF transla-
tional parallel manipulator. Stock and Miller [13] formulated
a weighted sum multi-criterion optimization problem with ma-
nipulability and workspace as two objective functions. Menon
et al. [14] used the maximization of the first natural frequency
as an objective function for the geometrical optimization of the
parallel mechanisms. Similarly, Li et al. [15] proposed dynam-
ics and elastodynamics optimization of a 2-DOF planar parallel
robot to improve the dynamic accuracy of the mechanism. They
proposed a dynamic index to identify the range of natural fre-
quency with different configurations. Krefli [16] also formulated
a multi-criterion elastodynamic optimization problem for parallel
mechanisms while considering workspace, velocity transmis-
sion, inertia, stiffness and the first natural frequency as optimiza-
tion objectives. Chablat and Wenger [17] proposed an analytical
approach for the architectural optimization of a 3-DOF transla-
tional parallel mechanism, named Orthoglide 3-axis, based on
prescribed kinetostatic performance to be satisfied in a given
Cartesian workspace.

Most of the foregoing research works aimed to improve the
performance of a given manipulator and the comparison of vari-
ous architectures for a given application or performance has not
been considered. In this paper, the mechanisms performance are
improved over a regular shaped workspace for given specifica-
tions. As a result, we propose a methodology to deal with the
multiobjective design optimization of PKMs. The size of the
regular shaped workspace and the mass in motion of the mech-
anism are the objective functions of the optimization problem.
Its constraints are determined based on the mechanism accuracy,
assembly and the conditioning number of its kinematic Jacobian
matrix. The proposed approach is applied to the optimal design
of Planar Parallel Manipulators (PPMs) with the same mobility
and set of design parameters. The non-dominated solutions, also
called Pareto-optimal solutions, are obtained by means of a ge-
netic algorithm for the three architectures and finally a compari-
son is made between them.

MANIPULATORS UNDER STUDY

Figure 1(a)–(c) illustrate the architectures of the planar par-
allel manipulators (PPMs) under study, which are named 3-PRR,
3-RPR and 3-RRR PPMs, respectively. Other families of PPMs
are described in [2]. Here and throughout this paper, R, P, R
and P denote revolute, prismatic, actuated revolute and actuated
prismatic joints, respectively. The manipulators under study are
composed of a base and a moving platform (MP) connected by
means of three legs. Points A1, A2 and A3, (C1, C2 and C3, respec-
tively) lie at the corners of a triangle, of which point O (point P,
resp.) is the circumcenter. Each leg of the 3-PRR PPM is com-
pared to a P, a R and a R joint in sequence. Each leg of the 3-
RPR PPM is composed of a R, a P and a R joint in sequence.
Likewise, each leg of the 3-RRR PPM is composed of three R
joints in sequence. The three P joints of the 3-PRR and the 3-
RPR PPMs are actuated while the first R joint of each leg of the
3-RRR PPM is actuated.

Fb and Fp are the base and the moving platform frames
of the manipulator. In the scope of this paper, Fb and Fp are
supposed to be orthogonal. Fb is defined with the orthogonal
dihedron (Ox, Oy), point O being its center and Ox parallel to
segment A1A2. Likewise, Fp is defined with the orthogonal di-
hedron (PX, PY), point C being its center and PX parallel to seg-
ment C1C2. The manipulator MP pose, i.e., its position and its
orientation, is determined by means of the Cartesian coordinates
vector p = [px, py, z]T of operation point P expressed in frame Fb
and angle φ, namely, the angle between frames Fb and Fp.

The geometric parameters of the manipulators are defined
as follows: (i) R is the circumradius of triangle A1A2A3 of cir-
cumcenter O, i.e., R = OA1; (ii) r is the circumradius of triangle
C1C2C3 of circumcenter P, i.e., r = PCi, i = 1, . . . , 3; (iii) Lb is
the length of the intermediate links, i.e., Lb = B1C1 for the 3-
PRR PPM. Lb is also the maximum displacement of the prismatic
joints of the 3-RPR PPM. Similarly, Lh is the length of the two
intermediate links of the 3-RRR PPM, i.e., Lh = A1B1 = B1C1;
(iv) rj is the cross-section radius of the intermediate links; (v) rj:
the cross-section radius of links of the moving platform, the latter
being composed of three links.

Stiffness Modeling

The stiffness models of the three manipulators under study are
obtained by means of the refined lumped mass modeling de-
scribed in [18]. Figures 2 to 4 illustrate the flexible models of
the legs of the 3-PRR, 3-RPR and 3-RRR PPMs, respectively.
The actuator control loop compliance is described with a 1-dof
virtual spring in each flexible model denoted F. Bes-
ides, the moving platform of the manipulators is supposed to be
composed of three links of length r connected to its geometric
center P.

From Fig. 2 the flexible model of the legs of the 3-
PRR PPM contains sequentially: (i) a rigid link between the
manipulator base and the ith actuated joint (part of the base plat-
form) described by the constant homogeneous transformation
matrix Tbase; (ii) a 1-dof actuated joint, defined by the homo-

(a) 3-PPR PPM
(b) 3-RRP PPM
(c) 3-RPR PPM

FIGURE 1. THE THREE PLANAR PARALLEL MANIPULATORS UNDER STUDY

FIGURE 2. FLEXIBLE MODEL OF THE 3-PPR PPM’S KINEMATIC CHAINS

geneous matrix function \( V_a(\theta^0) \) where \( \theta^0 \) is the actuated coordinate; (iii) a 1-dof virtual spring describing the actuator mechanical stiffness, which is defined by the homogeneous matrix function \( V_s(\theta^0) \) where \( \theta^0 \) is the virtual spring coordinate corresponding to the translational spring; (iv) a 1-dof passive R-joint at the beginning of the leg allowing one rotation angle \( q^2 \), which is described by the homogeneous matrix function \( V_r(q^2) \); (v) a rigid leg of length \( L \) linking the foot and the movable plat-
form, which is described by the constant homogeneous transformation matrix \( T_i \); (vi) a 6-dof virtual spring describing the leg stiffness, which is defined by the homogeneous matrix function \( V_{x2} (\theta_1^i \cdots \theta_6^i) \), with \( \theta_1^i, \theta_2^i, \theta_3^i \) and \( \theta_4^i, \theta_5^i, \theta_6^i \) being the virtual spring coordinates corresponding to the small-displacement screw of the moving-platform. (vii) a 1-dof passive R-joint between the leg and the platform, allowing one rotation angle \( q_3^i \), which is described by the homogeneous matrix function \( V_{x2} (q_3^i) \); (viii) a rigid link of length \( r \) from the manipulator leg to the geometric center of the mobile platform, which is described by the constant homogeneous transformation matrix \( T_i \); (ix) a 6-dof virtual spring describing the stiffness of the moving platform, which is defined by the homogeneous matrix function \( V_{x3} (\theta_1^i \cdots \theta_{12}^i) \), \( \theta_1^i, \theta_2^i, \theta_3^i \) and \( \theta_4^i, \theta_5^i, \theta_6^i, \theta_7^i, \theta_8^i, \theta_9^i \), \( \theta_{10}^i, \theta_{11}^i, \theta_{12}^i \) being the virtual spring coordinates corresponding to the small-displacement screw of the moving-platform. Similarly, the mathematical expressions associated with the \( X \)-PPMs can be reduced to a system of two matrix equations, namely

\[
T_i = T_{Baseline} V_{x2} (q_0^i) V_{x1} (\theta_0^i) V_r (q_1^i) T_r V_{x2} (\theta_1^i \cdots \theta_6^i)
\]

\[
V_{x2} (q_2^i) T_r V_{x3} (\theta_1^i \cdots \theta_{12}^i) T_{End}
\]  

(1)

Similarly, the mathematical expressions associated with the kinematic chains of the \( 3-\text{RRP} \) and \( 3-\text{RRR} \) PPMs are obtained.

From [13], the kinetostatic model of the \( i^{th} \) leg of the \( X \)-PPMs can be reduced to a system of two matrix equations, namely

\[
\begin{bmatrix}
S_{\theta}^i & J_q^i \\
J_q^i & 0_{2 \times 2}
\end{bmatrix}
\begin{bmatrix}
f_i \\
\delta q_i
\end{bmatrix}
= \begin{bmatrix}
\delta t_i \\
0_2
\end{bmatrix}
\]  

(2)

where \( X \) stands for \( 3-\text{RRP} \), \( 3-\text{RRR} \) or \( 3-\text{RRR} \). The sub-matrix

\[
S_{\theta}^i = J_{q}^{i} \frac{\partial K_i}{\partial \theta} \frac{1}{J_{q}^{i}}
\]

is the Jacobian matrix related to the virtual springs and \( J_q^i \) is the one related to the passive joints. \( K_i \) describes the compliance of the virtual springs.

\[
K_i^{-1} =
\begin{bmatrix}
0_1 & 0_{6 \times 1} & 0_{6 \times 6} \\
0_{6 \times 1} & 0_{6 \times 6} & 0_{6 \times 6} \\
0_{6 \times 6} & 0_{6 \times 6} & 0_{6 \times 6}
\end{bmatrix}
\]

(3a)

\[
K_i^{-1} =
\begin{bmatrix}
0_1 & 0_{6 \times 1} & 0_{6 \times 6} \\
0_{6 \times 1} & 0_{6 \times 6} & 0_{6 \times 6} \\
0_{6 \times 6} & 0_{6 \times 6} & 0_{6 \times 6}
\end{bmatrix}
\]

(3b)

\[
K_i^{-1} =
\begin{bmatrix}
0_1 & 0_{6 \times 1} & 0_{6 \times 6} \\
0_{6 \times 1} & 0_{6 \times 6} & 0_{6 \times 6} \\
0_{6 \times 6} & 0_{6 \times 6} & 0_{6 \times 6}
\end{bmatrix}
\]

(3c)

where \( K_{act} \) is the \( 1 \times 1 \) stiffness matrix of the \( i^{th} \) actuator, \( K_{link} \) is the \( 6 \times 6 \) stiffness matrix of the intermediate link for the \( 3-\text{RRP} \) and \( 3-\text{RRR} \) PPMs while \( K_{link1} \) and \( K_{link2} \) are the \( 6 \times 6 \) stiffness matrices of the first and second intermediate links of the \( i^{th} \) leg of the \( 3-\text{RRR} \) PPM. \( K_{pf} \) is the \( 6 \times 6 \) stiffness matrix of the \( i^{th} \) link of the moving platform. The compliance matrix of each link is expressed by means of the stiffness model of a cantilever beam, namely

\[
K_i^{-1} =
\begin{bmatrix}
\frac{L^3}{3EI} & 0 & 0 & 0 & 0 & 0 \\
0 & \frac{L^3}{3EI} & 0 & 0 & 0 & 0 \\
0 & 0 & \frac{L^3}{6EI} & 0 & 0 & 0 \\
0 & 0 & 0 & \frac{L^3}{6EI} & 0 & 0 \\
0 & 0 & 0 & 0 & \frac{L^3}{6EI} & 0 \\
0 & 0 & 0 & 0 & 0 & \frac{L^3}{6EI}
\end{bmatrix}
\]

(4)

\( L \) being the length of the corresponding link, \( A \) is its the cross-sectional area, i.e., \( A = \pi r^2 \) for the links of the manipulators legs and \( A = \pi r^2 \) for the links of the moving platform. \( I_y \) and \( I_z \) are the polar moments of inertia about the \( y \) and \( z \) axes, resp. \( I_y = I_z = \pi r^4/4 \) for the links of the manipulators legs and \( I_y = I_z = \pi r^4/4 \) for the links of the moving platform. \( I_e = I_e + I_z \) is the polar moment of inertia about the longitudinal axis of the link. \( E \) and \( G \) are the Young and shear moduli of the material.

Accordingly, the Cartesian stiffness matrix \( K_i \) of the \( i^{th} \) link defining the motion-to-force mapping is obtained from Eq. (2).

\[
f_i = K_i \delta t_i
\]

(5)

with \( f_i \) being the wrench exerted on the \( i^{th} \) leg of the manipulator and at the geometric center of the moving platform while \( \delta t_i \) is the small-displacement screw of the moving-platform.
Finally, the Cartesian stiffness matrix $K$ of the manipulator is found with a simple addition of the three $K_i$ matrices, namely,

$$K = \sum_{i=1}^{3} K_i$$

(6)

MULTIOBJECTIVE OPTIMIZATION PROBLEM

A multiobjective optimization problem (MOOP) is formulated in this section in order to compare 3-P$	ext{RR}$, 3-RPR and 3-R$	ext{RR}$ PPMs. In scope of this study, the manipulators are compared with regard to their mass in motion and their regular workspace size, i.e., the two objective functions of the MOOP, defined below. Moreover, the MOOP is subject to constraints on the manipulator dexterity and stiffness. It means that for a given external wrench, the displacements of the moving platform have to be smaller than given values throughout the obtained maximum regular dexterous workspace.

Objective Functions

Mass in Motion of the Manipulators

The components in motion of the manipulators are mainly their moving platform and the links of their legs. As a consequence, the mass in motion for the three PPMs under study is expressed as follows:

$$m_{\text{PRR}} = 3m_{\text{link}} + m_{\text{pf}}$$

(7a)

$$m_{\text{RPR}} = 3m_{\text{link}} + m_{\text{pf}}$$

(7b)

$$m_{\text{RRR}} = 6m_{\text{link}} + m_{\text{pf}}$$

(7c)

$m_{\text{link}}$ is the mass of links of the legs and are supposed to be the same while $m_{\text{pf}}$ is the mass of the moving platform. The mass of the prismatic or revolute actuators does not appear in Eqs. (7a)-(c) as it is supposed to be fixed for the 3-P$	ext{RR}$ PPM and close to the base for the 3-R$	ext{PR}$ PPM.

$$m_{\text{pf}} = \pi r_p^2 \nu$$

(8a)

$$m_{\text{link}} = \pi r_j^2 L \nu$$

(8b)

where $\nu$ is the material density.

Finally, the first objective function of the MOOP is expressed as:

$$f_1(x) = m_X \rightarrow \text{min}$$

(9)

$x$ being the vector of design variables, i.e., the geometric parameters of the manipulator at hand, and $X$ stands for 3-P$	ext{RR}$, 3-R$	ext{PR}$ or 3-R$	ext{RR}$.

Regular workspace size

The quality of the manipulator workspace is of prime importance for the design of Parallel Kinematics Machines (PKMs). It is partly characterized by its size and shape. Moreover, the lower the amount of singularities throughout the workspace, the better the workspace for continuous trajectory planning. The workspace optimization of parallel manipulators can usually be solved by means of two different formulations. The first formulation aims to design a manipulator whose workspace contains a prescribed workspace and the second one aims to design a manipulator, of which the workspace is as large as possible. However, maximizing the manipulator workspace may result in a poor design with regard to the manipulator dexterity and manipulability [12, 19]. This problem can be solved by properly defining the constraints of the optimization problem. Here, the multiobjective optimization problem of PPMs is based on the formulation of workspace maximization, i.e., the determination of the optimum geometric parameters in order to maximize a regular-shaped workspace.

In the scope of the paper, the regular-shaped workspace is supposed to be a cylinder of radius $R_w$, for which at each point a rotation range $\Delta \phi = 20^\circ$ of the moving-platform about the Z-axis has to be reached. Figure 5 illustrates such a regular-shaped workspace, whose $x_c$, $y_c$ and $\phi_c$ are its center coordinates and the rotation angle of the moving-platform of the manipulator in the home posture.

FIGURE 5. A REGULAR-SHAPED WORKSPACE

Consequently, in order to maximize the manipulator workspace, the second objective of the optimization problem can be written as:

$$f_2(x) = R_w \rightarrow \text{max}$$

(10)

Constraints of the Optimization Problem

The constraints of the optimization problem deals with the geometric parameters, the dexterity and the accuracy of the manipulators. Moreover, the constraints have to be defined in order to obtain a singularity-free regular-shaped workspace.
Constraints on the Geometric Parameters  For the three PPMs under study, the kinematic constraints are handled with their inverse kinematics. It means that the inverse kinematics is solved in order for the postures of the PPM to belong to the same working mode throughout the manipulator regular-shaped workspace. Besides, for the 3-PRR PPM, the lower and upper bounds of the prismatic lengths $p_i$ are defined such as $0 \leq p_i \leq \sqrt{3}R$ in order to avoid collisions. To obtain feasible displacements of the prismatic joints, the range of the 3-PRR PPM is defined such that $L/2 \leq p_i \leq L$.

Constraint on the Manipulator Dexterity  The manipulator dexterity is defined by the condition number of its kinematic Jacobian matrix. The condition number $\kappa_F(M)$ of a $m \times n$ matrix $M$, with $m \leq n$, based on the Frobenius norm is defined as follows

$$\kappa_F(M) = \frac{1}{m} \sqrt{\text{tr}(M^T M) \text{tr}([M^T M]^{-1})}$$

Here, the condition number is computed based on the Frobenius norm as the latter produces a condition number that is analytic in terms of the posture parameters whereas the 2-norm does not. Besides, it is much costlier to compute singular values than to compute matrix inverses.

The terms of the direct Jacobian matrix of the three PPMs under study are not homogeneous as they do not have same units. Accordingly, its condition number is meaningless. Indeed, its singular values cannot be arranged in order as they are of different nature. However, from [20] and [21], the Jacobian can be normalized by means of a normalizing length. Later on, the concept of characteristic length was introduced in [22] in order to avoid the random choice of the normalizing length. For instance, the previous concept was used in [23] to analyzethe kinesthetic performance of manipulators with multiple inverse kinematic solutions, and therefore to select their best working mode.

Accordingly, for the design optimization of the three PPMs, the minimum of the inverse condition number $\kappa^{-1}(J)$ of the kinematic Jacobian matrix $J$ is supposed to be higher than a prescribed value, say 0.1, throughout the regular-shaped workspace, for any rotation of its moving-platform, i.e.,

$$\min (\kappa^{-1}(J)) \geq 0.1$$

Constraints on the moving-platform pose errors  The position and orientation errors on the moving-platform are evaluated by means of the stiffness models of the manipulators. Let $(\delta x, \delta y, \delta z)$ and $(\delta \phi_x, \delta \phi_y, \delta \phi_z)$ be the position and orientation errors of the moving-platform subject to external forces $(F_x, F_y, F_z)$ and torques $(\tau_z, \tau_y, \tau_z)$. The constraints on the pose errors on the moving-platform are defined as follows:

$$\delta x \leq \delta x_{\text{max}} \quad \delta y \leq \delta y_{\text{max}} \quad \delta z \leq \delta z_{\text{max}}$$

$$\delta \phi_x \leq \delta \phi_x_{\text{max}} \quad \delta \phi_y \leq \delta \phi_y_{\text{max}} \quad \delta \phi_z \leq \delta \phi_z_{\text{max}}$$

$(\delta x_{\text{max}}, \delta y_{\text{max}}, \delta z_{\text{max}})$ being the maximum allowable position errors and $(\delta \phi_x_{\text{max}}, \delta \phi_y_{\text{max}}, \delta \phi_z_{\text{max}})$ the maximum allowable orientation errors of the moving-platform. These accuracy constraints can be expressed in terms of the components of the mechanism stiffness matrix and the wrench applied to the moving-platform. Let us assume that the accuracy requirements are:

$$\sqrt{\delta x^2 + \delta y^2} \leq 0.0001 \text{m}$$

$$\delta z \leq 0.001 \text{m}$$

$$\delta \phi_z \leq 1 \text{deg}$$

If the moving-platform is subject to a wrench whose components are $\|F_{xy}\| = F_z = 100 \text{N}$ and $\tau_z = 100 \text{Nm}$, then the accuracy constraints can be expressed as:

$$k_{\text{xy}}^{\text{min}} \geq \frac{\|F_{xy}\|}{\sqrt{\delta x^2 + \delta y^2}} = 10^6 \text{N.m}^{-1}$$

$$k_z^{\text{min}} \geq F_z / \delta z = 10^5 \text{N.m}^{-1}$$

$$k_{\phi_z}^{\text{min}} \geq \tau_z / \delta \phi_z = \frac{10}{\pi / 180} \text{N.m.rad}^{-1}$$

Design Variables of the Optimization Problem  Along with the above mentioned geometric parameters $(R, r, L_b)$ of the PPMs, the radius $r_j$ of the circular-cross-section of the intermediate bars defined and the radius $r_p$ of the circular-cross-section of the platform bars are considered as design variables, also called decision variables. As a remainder, the moving-platform is supposed to composed of three circular bars of length $r$.

As there are three PPMs under study, the PPM type is another design variable that has to be taken into account. Let $d$ denote the PPM type: $d = 1$ stands for the 3-PRR PPM; $d = 2$ stands for the 3-PRR PPM; and $d = 3$ stands for the 3-PRR PPM.

As a result, the optimization problem contains one discrete variable, i.e., $d$, and five continuous design variables, i.e., $R$, $r$, $L_b$, $r_j$ and $r_p$. Hence, the design variables vector $x$ is given by:

$$x = [d \ R \ L_b \ r_j \ r_p]^T$$

Formulation of the Optimization Problem  The Multiobjective Design Optimization Problem of PPMs can be stated as: Find the optimum design variables $x$ of PPMs
in order to minimize the mass of the mechanism in motion and to maximize its regular shaped workspace subject to geometric, kinematic and accuracy constraints.

Mathematically, the problem can be written as:

\[
\begin{align*}
\text{minimize} & \quad f_1(x) = m_X \\
\text{maximize} & \quad f_2(x) = R_w \\
\text{over} \quad & x = [d \ R \ L_b \ r_j \ r_p]^T \\
\text{subject to:} & \quad g_1 : L_b + r \geq \frac{R}{2} \\
& \quad g_2 : 0 < \rho_i < \sqrt{3}R \\
& \quad g_3 : \kappa^{-1}(J) \geq 0.1 \\
& \quad g_4 : k_{x y}^{\text{min}} \geq \frac{F_{x y}}{\sqrt{\delta x^2 + \delta y^2}} = 10^6 \\
& \quad g_5 : k_{z z}^{\text{min}} \geq \frac{F_z}{\delta z} = 10^5 \\
& \quad g_6 : k_{z z}^{\text{min}} \geq \frac{\tau_z}{\delta \phi_z} = \frac{10}{\pi/180} \\
& \quad x_l \leq x \leq x_u
\end{align*}
\]

where \(x_l\) and \(x_u\) are the lower and upper bounds of \(x\), respectively.

### RESULTS AND DISCUSSIONS

The multiobjective optimization problem (17) is solved by means of modeFRONTIER [24] and by using its built-in multiobjective optimization algorithms. MATLAB code is incorporated in order to analyze the system and to get the numerical values for the objective functions and constraints that are analyzed in modeFRONTIER for their optimality and feasibility. The lower and upper bounds of the design variables are given in Tab. 1. The components of the PPMs are supposed to be made up of steel, of material density \(d = 7850 \text{ kg/m}^3\) and Young modulus \(E = 210 \times 10^9 \text{ N/m}^2\). For each iteration, the regular-shaped workspace is evaluated for the corresponding design variables and a discretization of this workspace is performed. The constraints of the optimization problem are also evaluated at each grid point of the regular-shaped workspace to check whether they are satisfied or not. A multiobjective genetic algorithm (MOGA) is used to solve MOOP (17) and to obtain the Pareto frontier in the plane defined by the mechanism mass and the workspace radius. modeFRONTIER scheduler and Design Of Experiments (DOE) parameters are given in Tab. 2. MATLAB is used to evaluate each individual of the current population (generated by the modeFRONTIER scheduler). MATLAB returns the output variables that are analyzed by modeFRONTIER for the feasible solutions according to the given constraints. At the end, the Pareto-optimal solutions are obtained from the generated feasible solutions.

### TABLE 1. LOWER AND UPPER BOUNDS OF THE DESIGN VARIABLES

| Design Variable | \(d\) [m] | \(R\) [m] | \(L_b\) [m] | \(r_j\) [m] | \(r_p\) [m] |
|-----------------|---------|---------|---------|---------|---------|
| Lower Bound     | 1       | 0.5     | 0.5     | 0       | 0       |
| Upper Bound     | 3       | 4       | 4       | 0.1     | 0.1     |

### TABLE 2. modeFRONTIER ALGORITHM PARAMETERS

| Parameter                  | Value          |
|----------------------------|----------------|
| Scheduler                  | MOGA-II        |
| Number of iterations       | 200            |
| Directional cross-over probability | 0.5  |
| Selection probability      | 0.05           |
| Mutation probability       | 0.1            |
| DNA (DeoxyriboNucleic Acid) string mutation ratio | 0.05 |
| DOE algorithm              | Sobol          |
| DOE number of designs      | 30             |
| Total number of iterations | \(30 \times 200 = 6000\) |

![FIGURE 6. PARETO FRONTIER OF MOOP (17)](image-url)
The Pareto frontier, solution of MOOP (17), is depicted in Fig. 6 whereas the design parameters and the corresponding objective functions for two extreme and one intermediate Pareto optimal solutions, as shown in Fig. 6 are given in Tab. 3. The CAD designs illustrating the three foregoing solutions are also shown in Fig. 8.

It appears that all Pareto-optimal solutions of MOOP (17) are 3-PRR PPMs. Accordingly, Fig. 7 illustrates the Pareto Frontiers associated with the three planar parallel manipulator architectures. It is noteworthy that the Pareto-optimal solutions associated with the 3-PRR PPM architectures are better than the Pareto-optimal solutions associated with the 3-RPR and 3-RRR PPM architectures.

FIGURE 7. PARETO FRONTIERS ASSOCIATED WITH THE 3-PRR, 3-RPR, AND 3-RRR PLANAR PARALLEL MANIPULATOR ARCHITECTURES

Figures 9(a)–(c) and 10(a)–(c) show the evolution of the design variables as a function of $R_w$, along the Pareto Frontier associated with each PPM architecture. It is noteworthy that the higher $R_w$, the higher the design variables. It is apparent that the variations in variables $R, r, L_b, \alpha$, and $r_j$ with respect to (w.r.t.) $R_w$ are almost linear whereas the variations in $r_p$ w.r.t. $R_w$ is rather quadratic. This is due to the fact that the higher the size of the mechanism the higher the bending of the moving platform links whereas the intermediate links are mainly subjected to tension and compression.

CONCLUSIONS

In this paper, the problem of dimensional synthesis of parallel kinematics machines was addressed. A multiobjective design optimization problem was formulated in order to determine optimum structural and geometric parameters of any parallel kinematics machine. The proposed approach is similar to that used in [25] but we took into account the mass and the regular workspace instead of considering the entire volume of the manipulator. The proposed approach was applied to the optimum design of three planar parallel manipulators with the aim to minimize the mass in motion of the mechanism and to maximize its regular shaped workspace. Other performance indices can be used as constraints. However, they cannot necessarily be used as objective functions as the latter are usually formulated as a sum of an index over all the manipulator workspace. As another constraint, we could use the collisions between the legs of the manipulator.

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### TABLE 3. THREE PARETO OPTIMAL SOLUTIONS

| Design ID | Design Variables | Objectives |
|-----------|------------------|------------|
|           | $d$ | $R$ [m] | $r$ | $L$ | $b$ | $r$ | $p$ | $m$ | $R_w$ |
| I         | 1   | 1.412 | 0.319 | 0.620 | 0.026 | 0.023 | 44.5 | 0.110 |
| II        | 1   | 3.066 | 1.283 | 1.896 | 0.036 | 0.056 | 484.8 | 1.207 |
| III       | 1   | 3.872 | 1.947 | 1.977 | 0.039 | 0.096 | 1545.6 | 1.609 |

**FIGURE 8. CAD DESIGNS OF THREE PARETO-OPTIMAL SOLUTIONS OF MOOP [17]**

**FIGURE 9. DESIGN VARIABLES $R$, $r$, $L_b$ AS A FUNCTION OF $R_w$ ALONG THE PARETO FRONTIER ASSOCIATED WITH THE MANIPULATOR AT HAND**

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FIGURE 10. DESIGN VARIABLES $r_j, r_p$ AS A FUNCTION OF $R_w$ ALONG THE PARETO FRONTIER ASSOCIATED WITH THE MANIPULATOR AT HAND

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