Power-law cosmological solution derived from DGP brane with a brane tachyon field

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By studying a tachyon field on the DGP brane model, in order to embed the 4D standard Friedmann equation with a brane tachyon field in 5D bulk, the metric of the 5D spacetime is presented. Then, adopting the inverse square potential of tachyon field, we obtain an expanding universe with power-law on the brane and an exact 5D solution.

Keywords: Brane cosmology, tachyon field.

1. Introduction

An increasing number of people believe that extra dimensions can be probed by gravitons and eventually non-standard matter. These models usually yield the correct Newtonian (1/r)-potential at large distances since the gravitational field is quenched on sub-millimeter transverse scales. This quenching appears either due to finite extension of the transverse dimensions1–6 or sub-millimeter transverse curvature scales derived by negative cosmological constants.7–12 A common feature to these models is that they predict deviations from the 4D Einstein gravity at short distances. Over large distances, a model which predicts deviations from the standard 4D gravity is proposed by Dvali, Gabadadze and Porrati (DGP).13,14,15 There are the brane and bulk Einstein terms in the action of DGP model. It was shown that the DGP model allows for an embedding of the standard Friedmann cosmology in the sense that the cosmological evolution of the background metric on the brane can entirely be described by the standard Friedmann equation plus energy conservation on the brane.16,17 Recently, K. Atazadeh and H. R. Sepangi study the DGP brane with a scalar field and propose curvature corrections in DGP brane cosmology.18,19 A comprehensive review on DGP cosmology is dished up in Ref. 20.

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The observable universe is presently undergoing an accelerating expansion basing on the observations of Type Ia supernovae. It is possible that such an accelerated expansion could be the result of a modification to the Einstein-Hilbert action. Recently, it has been proposed that accelerated expansion of universe is driven by a tachyon field. This field is derived from string-brane physics and describes the lowest energy level of an unstable Dp-brane or that of a brane-antibrane system. The notable characteristic of tachyon field is that the tachyon field has a negative squared mass. Moreover, there are several papers about the brane-world model with a tachyon field.

In Ref. DGP brane cosmology with a scalar field is introduced. Now, we use a tachyon field take the place of the scalar field on the brane. In this paper, it is found that standard Friedmann equation with a brane tachyon field embeds in 5D bulk; then, by adopting a common potential of tachyon field, we obtain the metric which satisfies a power-law expansion on the brane. At the same time, an exact 5D solution is derived.

2. DGP model with a brane tachyon field

The action for the DGP model with a scalar field on the brane is written as

\[ S = \frac{M_5^3}{2} \int d^5x \sqrt{-g}R + \int d^4x \sqrt{-q} \left[ \frac{M_{pl}^2}{2} R - \frac{1}{2} (\nabla \phi)^2 - V(\phi) \right] + S_m[q_{\mu\nu}, \psi_m], \] (1)

where the first term in (1) describes the Einstein-Hilbert action in 5D bulk for a five-dimensional metric \( g_{AB} \) with Ricci scalar \( R \). And the second term is the Einstein-Hilbert action for the induced metric \( q_{\mu\nu} \) on the brane with a scalar field \( \phi \) and \( R \) is the scalar field of brane. \( M_5 \) is the Planck mass in five dimensions and \( M_{pl} \) is the induced 4D Planck mass. The last term \( S_m \) is the matter action on the brane with matter field \( \psi \). The metric \( q_{\mu\nu} \) is induced from the bulk metric \( g_{AB} \) via

\[ q_{\mu\nu} = \delta^A_{\mu} \delta^B_{\nu} g_{AB}. \] (2)

Now assuming a tachyon field instead of the scalar field on the brane, the action is rewritten as

\[ S = \frac{M_5^3}{2} \int d^5x \sqrt{-g}R + \int d^4x \sqrt{-q} \left[ \frac{M_{pl}^2}{2} R - V(T) \sqrt{1 + \partial_\mu T \partial^\mu T} \right] + S_m[q_{\mu\nu}, \psi_m]. \] (3)

From the action (3), the Einstein equations are derived as

\[ M_5^3 \left( R_{AB} - \frac{1}{2} g_{AB} R \right) + M_{pl}^2 \delta^A_\alpha \delta^B_\beta \left( R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R \right) \delta(y) = \delta^A_\alpha \delta^B_\beta (T_{\mu\nu} + \mathcal{T}_{\mu\nu}) \delta(y), \] (4)

here, \( T_{\mu\nu} \) and \( \mathcal{T}_{\mu\nu} \) are the energy-momentum tensor in the tachyon field and matter field respectively. The corresponding junction conditions (16) become

\[ \lim_{\epsilon \to +0} [K_{\mu\nu}]^\pm = \frac{1}{M_5^3} \left( T_{\mu\nu} + \mathcal{T}_{\mu\nu} - \frac{1}{3} q_{\mu\nu} q^{\alpha\beta} (T_{\alpha\beta} + \mathcal{T}_{\alpha\beta}) \right) \bigg|_{y=0} \]
Power-law cosmological solution derived from DGP brane with a brane tachyon field

\[-\frac{M^2_{pl}}{M^5_5} \left( R_{\mu\nu} - \frac{1}{6} g_{\mu\nu} q^{\alpha\beta} R_{\alpha\beta} \right) \bigg|_{y=0}.\]

From the Lagrangian of the tachyon field, the density and pressure are given

\[\rho_T = \frac{V(T)}{\sqrt{1 - T^2}},\]

\[p_T = -V(T) \sqrt{1 - T^2}.\]

Upon variation of the action, the equation of the motion for the tachyon field can be written

\[\frac{V(T)}{1 - T^2} + 3 \frac{\dot{a}}{a} V(T) \dot{\dot{T}} + \frac{dV(T)}{dT} = 0.\]

The form of the line element is written as in the brane gravity

\[ds^2 = -n^2(y, t) dt^2 + a^2(y, t) \gamma_{ij} dx^i dx^j + b^2(y, t) dy^2,\]

here, \(\gamma_{ij}\) is a maximally symmetric 3D metric where \(k = 0, \pm 1\) parameterizes the spatial curvature. Now, we will follow the work in Ref. [16], adopting the Gaussian normal system gauge \(b^2(y, t) = 1\), the Einstein tensors in the bulk are

\[G_{00} = 3n^2 \left( \frac{\ddot{a}^2}{a^2} - \frac{a'^2}{a^2} + \frac{k}{a^2} \right) - 3n^2 \frac{a''}{a},\]

\[G_{ij} = \left( \frac{a'^2}{a^2} - \frac{\ddot{a}^2}{n^2a^2} - \frac{k}{n^2a^2} \right) g_{ij} + 2 \left( \frac{n' a'}{na} + \frac{\ddot{a}}{n^2a} + \frac{\dot{n} a}{n^3a} \right) g_{ij} + \frac{n''}{n} g_{ij},\]

\[G_{05} = 3 \left( \frac{n' a'}{na} - \frac{\dot{n} a}{n^2a} \right),\]

\[G_{55} = 3 \left( \frac{a'^2}{a^2} - \frac{\ddot{a}^2}{n^2a^2} - \frac{k}{n^2a^2} \right) + 3 \left( \frac{n' a'}{na} + \frac{\ddot{a}}{n^2a} - \frac{\dot{n} a}{n^3a} \right).\]

The matching condition [14] for the perfect fluid on the brane

\[T_{00} = \rho, \quad T_{11} = T_{22} = T_{33} = p\]

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\[\lim_{\epsilon \to +0} [n']^\pm = \frac{n}{3M^3_5} \left( 2(\rho + p_T) + 3(p + p_T) \right) \bigg|_{y=0}
+ \frac{M^2_{pl}}{M^5_5} 2n \left( \frac{\ddot{a}}{n^2a} - \frac{\dot{a}^2}{2n^2a^2} - \frac{\dot{n} a}{n^3a} - \frac{k}{2a^2} \right) \bigg|_{y=0},\]

\[\lim_{\epsilon \to +0} [a']^\pm = \frac{M^2_{pl}}{M^5_5} \left( \frac{\ddot{a}^2}{n^2a} + \frac{k}{a} \right) \bigg|_{y=0} - \frac{(\rho + p_T)a}{3M^3_5} \bigg|_{y=0}.\]

Since \(T_{05} + T_{50} = 0\) from [12], we get \(n' = \dot{a}' / \dot{a}\). Then, in order to simplify, adopting the gauge \(n(0, t) = 1\), it is obtained

\[n(y, t) = \frac{\dot{a}(y, t)}{\dot{a}(0, t)}.\]
Therefore, from (10), (13) and (16), we have
\[
\lim_{\varepsilon \to 0} \left[ \frac{a'}{a} + \left( t \right) \right] = M^2_{\text{pl}} M_3^5 \left[ \dot{a}^2(0,t) + k a^2(y,t) \right]_{y=0},
\]
(18)

\[
I^+ = \left[ \dot{a}^2(0,t) - a^2(y,t) + k \right] a^2(y,t)_{y>0},
\]
(19)

\[
I^- = \left[ \dot{a}^2(0,t) - a^2(y,t) + k \right] a^2(y,t)_{y<0}.
\]
(20)

By taking \( I^+ = I^- \), the embedding of standard Friedmann cosmology with a tachyon field is given as follows:
\[
\dot{a}^2(0,t) + k = \rho + \rho_T \frac{a(0,t)}{3 M^2_{\text{pl}}},
\]
(21)

\[
I = \left[ \dot{a}^2(0,t) - a^2(y,t) + k \right] a^2(y,t).
\]
(22)

From (17) and (22), we can obtain the components of metric for \( I = 0 \), that is
\[
a(y,t) = a(0,t) + \sqrt{\dot{a}^2(0,t) + k y},
\]
(23)

\[
n(y,t) = 1 + \frac{\dot{a}(0,t)}{\sqrt{\dot{a}^2(0,t) + k}} y.
\]
(24)

Therefore, given a specific \( a(0,t) \), the exact solution of 5D metric is determined by equations (23,24). Then, it is presented the mathematic configuration of 5D brane world. This metric is similar to the one in space-time-matter theory and brane model. There are some link between them. \(^{52} \)

3. The power-law cosmological solution for a given \( V(\phi) \)

From (8) and (21) for flat FRW metric, we have
\[
3 H_0^2 = \frac{1}{M^2_{\text{pl}}} (\rho + \rho_T),
\]
(25)

and
\[
\frac{\dot{T}}{1 - T^2} + 3 H_0 \dot{T} + \frac{1}{V(T)} \frac{dV(T)}{dT} = 0
\]
(26)

where \( H_0 = \dot{a}(0,t)/a(0,t) \). Then, we assume the tachyon field dominates the universe, i.e. \( \rho = 0 \). Considering the tachyon field, the potential of this field is arbitrary. But most adopt the inverse square potential. We utilize this potential as
\[
V_T = 2n M^2_{\text{pl}} (1 - \frac{2}{3n})^{1/2} T^{-2}
\]
(27)

with \( n \geq 2/3 \), which is shown in Ref. \(^{52} \) in order to obtain an accelerated expansion of the universe driven by tachyonic matter. Substituting this potential into (25) and (26) with \( M^2_{\text{pl}} = 1 \), in the brane the evolution of the tachyon field is
\[
T \propto \left( \frac{2}{3n} \right)^{1/2} t,
\]
(28)
and the evolution of the scale factor is obtained

$$a(0, t) \propto t^n.$$  \hfill (29)

Therefore, the deceleration parameter $q$ is rewritten as

$$q = -1 + \frac{1}{n}. \hfill (30)$$

The deceleration parameter $q$ is plotted with $n$. From the Fig[1] we find that when

$$2/3 \leq n < 1,$$

this predicts deceleration on the brane; when $n = 1$, there is an uniform speed expansion on the brane; and when $n > 1$, an accelerated universe is obtained on the brane. Substituting (29) into (23) and (24), for $k = 0$ we derive

$$a(y, t) = C \left[ t^n + n t^{n-1} y \right]$$  \hfill (31)

and

$$n(y, t) = C \left[ 1 + (n - 1) \frac{y}{t} \right], \hfill (32)$$

where $C$ is a constant. When $y = 0$, Eq. (31) and (32) return to (29) and $n(0, t) = 1$. There appears coordinate singularities on the space-like hypercone $y = \pm t/(n - 1)$. This is presumably a consequence of the fact that the orthogonal geodesics emerging from the brane (which we used to set up $b^2 = 1$) do not cover the full five-dimensional spacetime. Therefore, the exact solution is

$$ds^2 = -[1 + (n - 1) \frac{y}{t}] dt^2 + [t^n + nt^{n-1} y] \gamma_{ij} dx^i dx^j + dy^2.$$  \hfill (33)

This solution can lead to arbitrarily velocity expansion with $n$ on the brane. For getting the uniform speed expansion universe $n = 1$, this solution is rewritten as

$$ds^2 = -dt^2 + [t+y] \gamma_{ij} dx^i dx^j + dy^2,$$  \hfill (34)

which is a critical situation.
To simplify, we choose a more simple form of potential as
\[ V_T = M^2 T^{-2}. \] (35)
Substituting this potential into (25) and (26), the evolution of the tachyon field on the brane is
\[ T \propto \left( \frac{4}{2 + \sqrt{9M^2 + 4}} \right)^{1/2} t. \] (36)
Therefore, when \( M^2 = 1 \), we obtain the tachyon field is
\[ T \propto \left( \frac{4}{2 + \sqrt{13}} \right)^{1/2} t, \] (37)
and the evolution of the scale factor is
\[ a(0, t) \propto t^{(1/3 + \sqrt{13}/6)}. \] (38)
Substituting (29) into (23) and (24), for \( k = 0 \) we derive
\[ a(y, t) = C \left[ t^{(1/3 + \sqrt{13}/6)} + \left( \frac{1}{3} + \frac{\sqrt{13}}{6} y \right) t^{(\sqrt{13}/6 - 2/3)} \right], \] (39)
and
\[ n(y, t) = C \left[ 1 + \left( \frac{\sqrt{13}}{6} - \frac{2}{3} \frac{y}{t} \right) \right], \] (40)
where \( C \) is a constant. However, when \( a(0, t) \propto t^{(1/3 + \sqrt{13}/6)} \), \( (1/3 + \sqrt{13}/6) \approx 0.93 < 1 \). So, the deceleration parameter \( q > 0 \). Therefore, we get an exact 5D cosmological solution which leads to decelerated expansion on the brane.

4. Conclusion
In this paper, we consider the DGP model with a brane tachyon field. In order to make the standard Friedmann cosmology with the tachyon field embed in the 5D bulk, the metric of 5D spacetime is derived. In this form of the metric, we find if the scale factor \( a \) is given the exact solution of the 5D bulk will be obtained. Then, it is known that if the potential of the tachyon field is given, the scale factor \( a \) will be described. For a general form of the inverse square potential, an arbitrarily velocity expanding universe with power-law is obtained on the brane. Meanwhile the corresponding 5D solution is derived. The velocity of expansion is related to \( n \). If \( 2/3 \leq n < 1 \), the universe is decelerating expansion; if \( n = 1 \), the universe is uniform speed expansion; and if \( n > 1 \), the accelerated expansion universe is obtained. For example, giving specific potential \( V_T = M^2 T^{-2} \), when \( M^2 = 1 \), we have the decelerated expansion universe and the exact solution of 5D bulk.
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