A REAL-TIME PRICING SCHEME CONSIDERING LOAD UNCERTAINTY AND PRICE COMPETITION IN SMART GRID MARKET

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Abstract. As a powerful tool of Demand Response (DR) techniques in smart grid market, Real-time Pricing (RTP) may optimize the electricity consumption pattern of users and improve the efficiency of electricity market. In this paper, a multi-leader-follower Stackelberg Game (SG) based on RTP is established to model the strategic interaction behavior between multiple electricity retailers and multiple users while simultaneously considering the power load uncertainty of users and the price competition among electricity retailers. In the game model, electricity retailers aim to seek their revenue maximization while the optimal power consumption competition among the users is taken into account. Lagrange multiplier method is utilized to solve the Nash Equilibriums (NE) of two non-cooperative games, and the closed-form optimal solution is obtained, then the Stackelberg Equilibrium (SE) consisting of the optimal real-time prices of electricity retailers and the power consumption of users is given. Finally, the numerical analysis results verify that the proposed scheme can reduce the real-time electricity price and increase the users satisfaction under feasible constraint, which shows the effectiveness and better performance of proposed RTP scheme.

1. Introduction. Smart grid tends to be the development direction of modern power grid in term of its multiple advantages including high efficiency of energy utilization, operation stability, and low power loss [16]. Demand Response Management (DRM) is one of the key solutions to managing the electricity price and the electricity consumption pattern of users as well as the generation capacities of energy providers in smart grid market. DR is mainly divided into Price-based DR and Incentive-based DR. In Price-based DR, the time slot with high price may be avoided by the users through adjusting the electricity price. There are many pricing

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schemes in Price-based DR, such as Flat Pricing (FP), Time-of-Use (TOU) pricing, Critical Peak Pricing (CPP), Peak Load Pricing (PLP), RTP, et al [6]. Each of the above Price-based DR program has good performance under certain circumstances, among which the RTP scheme needs the maximum communication capabilities of the smart grid to encourage users to consume electricity more effectively, which are determined before the start of each time slot [14, 17, 27, 31]. The RTP scheme has been applied into large industrial and commercial customers as well as some residential customers in view of making the benefit for both electricity providers and consumers maximum [27]. At the same time, some algorithms are proposed to solve the RTP problems in smart grid area [12,20–22,31].

We propose a Stackelberg game for analyzing the strategic interaction between multiple retailers and multiple users considering the power load uncertain factor $\delta_{k}^{ij}$ and the price competition among electricity retailers in this paper. $\delta_{k}^{ij}$ satisfying the Gaussian distribution is used to indicate the uncertainty of user’s power load in the future. At the same time, two non-cooperative game models are constructed respectively for power consumption behavior among the users and price competition among the electricity retailers. In the process of solving, Lagrange multiplier method is utilized to solve the power consumption NE of each user, and the closed-form optimal solution is obtained. After that, the optimal power demand of users is transmitted to the electricity retailers by smart meters, and then the price competition among electricity retailers is also formulated as a non-cooperative game. Similarly, each retailer sets the optimal real-time price according to the optimal power demand of users. When the users and the retailers reach their equilibriums and the equilibriums keep stable in the sequential competition between the electricity retailers and the users, the SE is also achieved. Finally, the number simulation results show the impact of load uncertainty and price competition on the real-time electricity price in terms of supply and demand. The major contributions of this work are summarized as follows:

- We formulate a RTP problem between multiple electricity retailers and multiple users as a Stackelberg game. At the same time, two non-cooperative game models are constructed respectively for the power consumption behavior among users and the price competition among electricity retailers.

- An uncertain factor satisfying Gaussian distribution is used to indicate the uncertainty of users power load in the future. We analyze the effect of uncertain factor on RTP in the proposed Stackelberg game considering the price competition among multiple electricity retailers.

The rest of this paper is organized as follows. Section 2 reviews RTP-related literature in Stackelberg game and power load uncertainty. In Section 3, we formulate a Stackelberg game model considering power load uncertainty and price competition among the electricity retailers. Section 4 gives the analysis of SE solution. We provide numerical results and discuss the performance of the proposed pricing scheme in Section 5. The last Section concludes this paper. All proofs are included in the Appendix.

2. Literature review. In this section, we review the literatures that study RTP in SG and electricity load uncertainty.

2.1. RTP in Stackelberg game. Most of existing works on RTP consider only one electricity supplier [2,9,13,19,29], however, the current retail electricity market is gradually going through the change from one or two monopoly electricity retailers
into the coexistence state of multiple electricity retailers [18]. As a result of competition among different electricity retailers, the electricity users enjoy better service and more flexible electricity access by using smart meters and other equipments. The interaction characteristic between electricity retailers and users is typical a SG problem [26].

Stackelberg game, excelling at analyzing sequential decision problems in competition environment, has been used to analyze the DR problems which mainly confined to one-leader-one-follower or one-leader-multi-follower game models in smart grid [10, 11, 34, 35]. But less attention is given to the scenarios with multiple electricity purchasing sources and imperfect competition [15]. Thus, some sophisticated hierarchical game models are leveraged to shed light on RTP in the complex power network with multiple electricity retailers, see for [1, 18, 24]. The authors of [18] studied the RTP scheme in smart grid with multiple retailers and multiple residential users using Stackelberg game. Additionally, the price competition among electricity retailers and the coordination among the residential users are also formulated into different game models. In [24] a Stackelberg game between utility companies and end-users was proposed and derived some analytical results for the SE. The power demand of users is accurately described by solving an optimization problem. [1] studied the multi-stage interaction behavior between the electricity retailers and the users based on [24], and found the multi-stage scenario can provide more incentive to users than the single stage in [24]. In order to study deregulated electricity spot markets, [3] used quadratic bid functions together with the transmission losses in the multi-leader-follower game. In the two islands type market, the existence of equilibrium was shown and the explicit formulae for the optimal solutions were obtained. It is a pity that above works based their studies on the case that the power consumption of users in the future is accurately predicted. However, there exists a large amount of power load uncertainty in real power system.

2.2. Power load uncertainty. The power load uncertainty has been analyzed depending on power load forecasting or statistical model of power demand in traditional power system [4], but some problems such as information circulation and measurement errors are often encountered [25, 28]. The exploring study of power load uncertainty has become one interesting problem in recent years, and some results have been obtained [7, 32]. [32] proposed a RTP problem based on the utility in smart grid considering power load uncertainty, and obtained the optimal real-time prices under different power load models, which revealed how the uncertainty affects power consumption behavior of users and supply capacity of electricity suppliers. In [7], a Stackelberg game model with a electricity supplier was set up to handle power DR scheduling and RTP problems when facing residential electrical load uncertainty. Considering the power load forecast for power demand in the future, [30] designed an optimization algorithm for scheduling users power demand and reducing users electricity expenditure, and analyzed the algorithm complexity as well. [23] studied the coordination of large-scale elastic loads in deregulated electricity markets under MCP and PSP auctions and explored the performances of these auctions in the underlying problems. It is found that the existing literatures related to power load uncertainty consider just the optimal electricity consumption and RTP problem in terms of a single electricity supplier, but the competition between multiple electricity suppliers isn’t taken into account.
3. Stackelberg game model. Now we consider a smart power network with multiple electricity retailers and multiple users consisting of a set $M = \{1, 2, ..., m\}$ of electricity retailers and a set $N = \{1, 2, ..., n\}$ of users. Each user schedules power consumption with the smart meters. The smart meter of each user is equipped with Energy Management Controller (EMC) which receives real-time prices from the retailers through two-way communication network and arranges the use of power, so as to enable the user to choose the retailer reasonably for purchasing power and controlling the power consumption behavior. The electricity retailers procure electricity from the electricity wholesale market, and sell it to the users. Thus the retailers and the users may exchange the information of real-time prices and power demand through the two-way communication network, while the user’s power consumption cost and the retailer’s procurement constraints are also considered.

We take one day as a period and the period is divided into $K$ time slots. $K$ is the set of time slots, and $k$ denotes each time slot, where $k \in K$ (1 hour is used as a sample time slot in this paper). Let $p_{ij}^k$ be the price of retailer $j$ in the $k$th time slot, and $p^{k} = (p_{ij}^k, ..., p_{ij}^k, ..., p_{mn}^k)$ be the price strategy vector. All retailers purchase power from the electricity generator and set the real-time electricity prices to maximize their revenues according to the real-time power demand of all users. At the same time, the retailers compete with each other to maximize their profits by adjusting the real-time electricity unit prices.

User $i$ selects different electricity retailers to serve himself at time slot $k$, the real-time power demand from retailer $j$ is denoted by $x_{ij}^k$, $x_i^k = (x_{ij}^k, x_{ij}^k)$ denotes the power demand vector of user $i$, where $x_{ij}^k$ denotes the power purchasing strategies of user $i$ from retailers excluding $j$, $x_i^k$ denotes the total power demand of user $i$, $m_i^k \leq x_{ij}^k \leq M_i^k$, $x_i^k = \sum_j x_{ij}^k$, $m_i^k$ and $M_i^k$ represent the minimum and maximum electricity consumption of user $i$, respectively. $x_i^k = (x_i^1, ..., x_i^k, ..., x_i^n)$ is the real-time power demand vector of all users at time slot $k$.

The exchange of information between retailers and users proceeds at the beginning of each time slot. After exchanging information, the electricity amount provided by the retailers is fixed, but each user’s power consumption may deviate from the determined power amount because of the load uncertainty. It is assumed that each user can buy enough electricity from retailers in this paper. We use $L_j^k$ to denote the general power amount supplied by retailer $j$ at time slot $k$, where $L_j^k \in [L_{jmin}^k, L_{jmax}^k]$, $L_{jmin}^k$ and $L_{jmax}^k$ represent the minimum and maximum electricity amount supplied by retailer $j$, respectively.

3.1. Utility function and welfare function of user. Utility reflects the satisfaction degree of decision makers, which can be described by utility function based on the strategy. Since the power demand of each user varies from time to time, the different behavior of users may be depicted by different utility functions. The user’s utility function is usually non-decreasing with respect to the amount of electricity consumption whose marginal benefit is a non-increasing function [31]. Similar to [10, 11, 24, 31], the utility function of user $i$ is still adopted as follow:

$$U_i^k(x_{ij}, \omega_i) = \begin{cases} \frac{\omega_i}{2} x_{ij}^k - \frac{\alpha}{2} (x_{ij}^k)^2, & 0 \leq x_{ij}^k \leq \frac{\omega_i}{\alpha}, \\ \frac{\omega_i}{2} x_{ij}^k - \frac{\alpha}{2} x_{ij}^k, & x_{ij}^k > \frac{\omega_i}{\alpha}, \end{cases}$$

where $\omega_i^k > 0$ is time-varying parameter that may vary among users at different time slot, and reflects power users’ wishes of increasing consumption. $0 < \alpha \leq 1$ is
a pre-determined parameter. We may set appropriate parameters, e.g., the larger $\omega_i^k$ and smaller $\alpha$ to reflect higher power demand of the users. It is noted that $\frac{(\omega_i^k)^2}{4\alpha}$ is a constant for user $i$ at time slot $k$ when $x_{ij}^k > \frac{\omega_i^k}{\alpha}$ in equation (1), then the preference behavior of user $i$ may be accurately modeled by $\omega_i^k x_{ij}^k - \frac{\alpha}{2} (x_{ij}^k)^2$, $0 \leq x_{ij}^k \leq \frac{\omega_i^k}{\alpha}$ [7], so we take

$$U_{ij}(x_{ij}^k, \omega_i^k) = \omega_i^k x_{ij}^k - \frac{\alpha}{2} (x_{ij}^k)^2, m_i^k \leq x_{ij}^k \leq M_i^k. \quad (2)$$

Obviously, the utility function obtains the dynamic changes of the power demand at different time slot according to different values of $\omega_i^k$ and $\alpha$. After the $m$ retailers announce their price vector $p_k^j = (p_{1j}^k, ..., p_{mj}^k)$ ($p_{mj}^k$ denotes the real-time power price announced by retailer $j$ at time slot $k$) at time slot $k$, user $i$ adjusts the purchasing power amount. Then the total welfare function of user $i$ is presented as

$$U_i(p_j^k, x_{ij}^k) = \sum_{j=1}^{m} [U_{ij}(x_{ij}^k, \omega_i^k) - p_j^k x_{ij}^k] = \sum_{j=1}^{m} [\omega_i^k - p_j^k x_{ij}^k - \frac{\alpha}{2} (x_{ij}^k)^2].$$

But it is worth noting that the power consumption may be different from the optimal power consumption decided at the beginning of each time slot in actual power system. Therefore, the power load uncertainty, a common phenomenon needs to be considered in the process of power consumption. Considering the load uncertainty, we use $\tilde{x}_{ij}^k = x_{ij}^k + \delta_{ij}^k$ [7, 32] to denote the actual power consumption of user $i$, where $\delta_{ij}^k$ is a random variable representing the power load uncertainty, which is used to reflect the random error in actual scenario. The model of $\delta_{ij}^k$ has different forms, it is assumed that $\delta_{ij}^k$ is a zero-mean random variable related to the user $i$ but not with electricity retailer $j$, and the variance of $\delta_{ij}^k$ is $\sigma^2$ [7]. Hence, when power purchased from retailer $j$ is $x_{ij}^k$ at time slot $k$ considering the load uncertainty, the expected utility function of user $i$ is modeled as

$$E(U_{ij}(\tilde{x}_{ij}^k, \omega_i^k)) = E(U_{ij}(x_{ij}^k + \delta_{ij}^k, \omega_i^k)) = \omega_i^k x_{ij}^k - \frac{\alpha}{2} (x_{ij}^k)^2 - \frac{\alpha}{2} \sigma^2. \quad (3)$$

Mathematically, $E(\delta_{ij}^k) = 0, D(\delta_{ij}^k) = \sigma^2$. It is easy to see that the load uncertainty reduces the expected utility of users in (3).

3.2. Cost and revenue function of electricity retailer. The cost function $C_j^k$ of power retailer $j$ is defined as the cost for procuring the real-time demand power of $j$ at time slot $k$, and $L_j^k$ denotes the power amount procured by retailer $j$ at time slot $k$. Hence, the revenue function of retailer $j$ is given by

$$\sum_{i=1}^{n} [\tilde{x}_{ij}^k p_j^k - C(L_j^k)] = \sum_{i=1}^{n} [(x_{ij}^k + \delta_{ij}^k) p_j^k - C(L_j^k)]. \quad (4)$$

All retailers hope to maximize their own revenues by choosing the optimal real time electricity prices. If the electricity price is too high, the users may switch to other retailers for buying electricity, which reduces the current power load. On the other hand, if the electricity price is too low, the retailers’ profits will decrease even if the electricity supplied to the users is served at full load. So the retailers must choose suitable electricity retail prices. Assume that the optimal price of retailer $j$ is $p_j^*$, the maximum revenue of retailer $j$ satisfies $max R_j(p_j^*, p_j^{k*}, x^{k*})$, where $p_j^{k*}$ and $x^{k*}$ express other participants’ optimal unit prices and the optimal power demand of all users at time slot $k$, respectively.
3.3. **Formation of SG.** Smart grid transmits information such as electricity price and power demand by two-way communication network. After the real-time price data are received by smart meters through communication network, EMC will control the power consumption of users according to the actual circumstance. We take 24 hours as a period, the period is divided into equal time slots, one hour is used as sample time slot in this paper. After the load uncertainty is taken into account, the power consumption of user $i$ from retailer $j$ is turned into $\tilde{x}_{ij}^k = x_{ij}^k + \delta_{ij}^k$.

Additionally, for keeping strictly increasing and concave of $U_{ij}(x_{ij}^k)$, and finding the SE for $m_i^k \leq x_{ij}^k \leq M_i^k$ as well, we give the following assumptions:

1) $\frac{\omega^k_i}{2n} \leq m_i^k \leq x_{ij}^k \leq M_i^k \leq \frac{\omega^k_i}{\alpha}$, which keeps $U_{ij}(x_{ij}^k)$ strictly increasing and concave for $m_i^k \leq x_{ij}^k \leq M_i^k$;

2) $\omega_1^k - \alpha m_1^k > \cdots > \omega_n^k - \alpha m_n^k > \omega_1^k - \alpha M_1^k > \cdots > \omega_n^k - \alpha M_n^k$, which is essential condition for obtaining the SE in Section 4. Following different forms of constraint conditions, $\delta_{ij}^k$ is assumed to obey Gaussian distribution based on [7] while the competition among multiple electricity retailers is also considered in this paper. After that, multi-leader-multi-follower Stackelberg game is adopted to model the strategic interactions with constraints between multiple retailers and multiple users. As a leader, each retailer aims to find the optimal real-time price to maximize his revenue at each time slot, which is obtained by solving the following optimization problems:

**Problem 1.**

$$\max_{p_j^k \geq 0} \sum_{i=1}^n E[(x_{ij}^k + \delta_{ij}^k)p_j^k - C(L_j^k)]$$

s.t. $$\sum_{i=1}^n (x_{ij}^k + \delta_{ij}^k) \leq L_j^k.$$  \hspace{1cm} (5)

For the announced electricity price of retailers, user $i$ determines the optimal power consumption $x_{ij}^k$ to maximize his expected payoff at each time slot. It is denoted as follows:

**Problem 2.**

$$\max_{x_{ij}^k} \sum_{j=1}^m E[U_i(x_{ij}^k + \sigma_{ij}^k) - (x_{ij}^k + \sigma_{ij}^k)p_j^k]$$

s.t. $m_i^k \leq x_{ij}^k \leq M_i^k.$  \hspace{1cm} (8)

Problems 1 and 2 together formulate a Stackelberg game with multiple leaders and multiple followers, so we try to solve the Stackelberg game in the next section.

4. **Analysis of equilibrium solution.**

**Definition 1.** Let $p_j^k$ be a solution to Problem 1 and $x_{ij}^k$ be a solution to Problem 2. The vector $(p_j^k, x_{ij}^k)$ is SE from Problem 1 and Problem 2 if $(p_j^k, x_{ij}^k)$ satisfies the following conditions for any $p_j^k \geq 0$, $m_i^k \leq x_{ij}^k \leq M_i^k$:

1) $\sum_{i=1}^n E[(x_{ij}^k + \sigma_{ij}^k)p_j^k - C(L_j^k)] \geq \sum_{i=1}^n E[(x_{ij}^k + \sigma_{ij}^k)p_j^k - C(L_j^k)];$

2) $\sum_{j=1}^m E[U_i(x_{ij}^k + \sigma_{ij}^k) - (x_{ij}^k + \sigma_{ij}^k)p_j^k] \geq \sum_{j=1}^m E[U_i(x_{ij}^k + \sigma_{ij}^k) - (x_{ij}^k + \sigma_{ij}^k)p_j^k].$
Next we use backward induction to obtain the SE. Firstly, we need to solve the optimization problem of each follower in Problem 2, namely, each user’s optimal power consumption, then we solve the optimization problem of each leader, namely, each retailer’s optimal real-time electricity price.

4.1. Optimal power consumption of user. Since $\delta_{ij}^k$ is a random variable in line with the Gaussian distribution representing the load uncertainty, and satisfies $E(\delta_{ij}^k) = 0$, $D(\delta_{ij}^k) = \sigma^2$, we are interested in the probabilistic constraint of the power consumption sum [30], i.e., we have the following constraint:

$$\Pr \left\{ \sum_{i=1}^{n} \tilde{x}_{ij}^k - L_j^k \geq \eta \right\} \leq \varepsilon, \quad (9)$$

where $\eta$ is seen as a threshold to the amount of exceeding power consumption sum, $\varepsilon$ is a small positive value. Next tail probability of Gaussian distribution is used to rewrite the probability constraint as a deterministic constraint. Above all, (9) is rewritten as

$$\Pr \left\{ \sum_{i=1}^{n} \tilde{x}_{ij}^k - L_j^k \geq \eta \right\} = \Pr \left\{ \frac{\sum_{i=1}^{n} \sigma_{ij}^k}{\sqrt{n} \sigma} \geq \frac{\eta - \sum_{i=1}^{n} x_{ij}^k + L_j^k}{\sqrt{n} \sigma} \right\}.$$ 

Let $Z = \frac{\sum_{i=1}^{n} \sigma_{ij}^k}{\sqrt{n} \sigma}$. Then $Z$ conforms to the standard normal distribution, i.e., $Z \sim N(0, 1)$. Above equation is changed to

$$\Pr \left\{ Z \geq \frac{\eta - \sum_{i=1}^{n} x_{ij}^k + L_j^k}{\sqrt{n} \sigma} \right\} = Q(\eta - \sum_{i=1}^{n} x_{ij}^k + L_j^k),$$

where $Q(x) = \int_{x}^{\infty} \frac{1}{\sqrt{2\pi}} e^{-\frac{t^2}{2}} dt$. Thus (9) is changed to $Q(\eta - \sum_{i=1}^{n} x_{ij}^k + L_j^k) \leq \varepsilon$, $\forall k \in K, j \in M$, whose new form is given in

$$\sum_{i=1}^{n} x_{ij}^k \leq L_j^k + \eta - \sqrt{n} \sigma Q^{-1}(\varepsilon), \forall k \in K, j \in M. \quad (10)$$

Let $l_j^k = L_j^k + \eta - \sqrt{n} \sigma Q^{-1}(\varepsilon)$, (10) is changed to $\sum_{i=1}^{n} x_{ij}^k \leq l_j^k$.

In what follows we discuss the problem in terms of two power retailers. After that we obtain corresponding results to $m$ retailers. Problem 2 may be given as follows:

$$\begin{align*}
\max_{x_{ij}^k, \delta_{ij}^k} \sum_{i=1, j=1,2}^{n} &E[U_i(x_{ij}^k + \delta_{ij}^k) - (x_{ij}^k + \delta_{ij}^k)p^k_j] \\
\text{s.t.} & m_k^l \leq x_{ij}^k \leq M_k^l, j = 1, 2, \quad (11)
\end{align*}$$

where

$$U_i(x_{ij}^k + \delta_{ij}^k) = -\frac{\alpha}{2}(x_{ij}^k)^2 + (\omega_i^k - \alpha \delta_{ij}^k)x_{ij}^k + \omega_i^k \delta_{ij}^k - \frac{\alpha}{2}(\delta_{ij}^k)^2, j = 1, 2.$$
We have
\[ E(U_i(x_{ij}^k + \delta_{ij})) = -\frac{\alpha}{2} (x_{ij}^k)^2 + \omega_i^k x_{ij}^k - \frac{\alpha}{2} E[(x_{ij}^k)^2] = -\frac{\alpha}{2} (x_{ij}^k)^2 + \omega_i^k x_{ij}^k - \frac{\alpha}{2} \sigma^2, j = 1, 2. \]

Finally, (11) has the following form:
\[
\begin{align*}
\max_{x_{i1}^k, x_{i2}^k} & \left[-\frac{\alpha}{2} (x_{i1}^k)^2 - \frac{\alpha}{2} (x_{i2}^k)^2 + (\omega_i^k - p_i^k)x_{i1}^k + (\omega_i^k - p_i^k)x_{i2}^k - \alpha \sigma^2 \right] \\
\text{s.t.} & \left\{ \begin{array}{l}
x_{i1}^k - m_i^k \geq 0, \\
x_{i2}^k - M_i^k \leq 0, \end{array} \right. \quad j = 1, 2.
\end{align*}
\] (12)

This is a convex optimization problem. Through introducing Lagrangian multipliers, the above optimization problem can be conversed by constructing the corresponding Lagrangian function as follows [33]:
\[
L_i = \left[-\frac{\alpha}{2} (x_{i1}^k)^2 - \frac{\alpha}{2} (x_{i2}^k)^2 + (\omega_i^k - p_i^k)x_{i1}^k + (\omega_i^k - p_i^k)x_{i2}^k - \alpha \sigma^2 \right] \\
+ \lambda_{ij} (x_{i1}^k - m_i^k) - \lambda_{ij} (x_{i2}^k - M_i^k) + \lambda_{ij} (x_{i1}^k - M_i^k) + \lambda_{ij} (x_{i2}^k - m_i^k),
\] (14)
where \( \lambda_{ij1}, \lambda_{ij2}, \lambda_{ij3}, \lambda_{ij4} \) are non-negative Lagrangian multipliers associated with the constraint (13). Then, the complementary slackness conditions are given as follows:
\[
\begin{align*}
\lambda_{ij1} (x_{i1}^k - m_i^k) &= 0, \\
\lambda_{ij2} (x_{i1}^k - M_i^k) &= 0, \\
\lambda_{ij3} (x_{i2}^k - m_i^k) &= 0, \\
\lambda_{ij4} (x_{i2}^k - M_i^k) &= 0,
\end{align*}
\] (15)
\[
\lambda_{ij1}, \lambda_{ij2}, \lambda_{ij3}, \lambda_{ij4}, x_{i1}^k, x_{i2}^k \geq 0.
\] (16)

The first-order optimality condition for the above optimization problem is \( \nabla L_i = 0 \), i.e., \( \frac{\partial L_i}{\partial x_{ij}^k} = 0 \), \( \forall i \in N, j = 1, 2 \), it follows:
\[
\begin{align*}
\begin{cases}
-\alpha x_{i1}^k + \omega_i^k - p_i^k + \lambda_{i1} - \lambda_{i2} = 0, \\
-\alpha x_{i2}^k + \omega_i^k - p_i^k + \lambda_{i3} - \lambda_{i4} = 0.
\end{cases}
\end{align*}
\] (17)

Regardless of the extreme situation, the optimal electricity consumption for all users is expressed as
\[
x_{ij}^k = \frac{\omega_i^k - p_j^k}{\alpha}, \quad \forall i \in \mathbb{N}, j = 1, 2,
\]
where \( \omega_i^k - \alpha M_i^k < p_j^k < \omega_i^k - \alpha m_i^k \). Proof can be seen in Appendix A.

According to the linear features of the above formulae, we have
\[
x_{ij}^k = \frac{\omega_i^k - p_j^k}{\alpha}, \quad \forall i \in \mathbb{N}, j \in \mathbb{M},
\] (18)
where \( \omega_i^k - \alpha M_i^k < p_j^k < \omega_i^k - \alpha m_i^k \).
4.2. Optimal electricity prices of retailers. Based on the optimal power consumption obtained in (18), electricity retailer \( j \) aims to find the optimal electricity price for maximizing his revenues. Since \( L^k_j \) and \( C(L^k_j) \) are independent of \( p^k_j \), Problem 1 is rewritten as

\[
\max_{p^k_j \geq 0} \sum_{i=1}^{n} x^k_{ij} p^k_j \tag{19}
\]

subject to

\[
\sum_{i=1}^{n} x^k_{ij} \leq t^k_j (t^k_j = L^k_j + \eta - \sqrt{n\sigma Q^{-1}(\varepsilon)}),
\]

where the constraint condition is transmitted to

\[
\sum_{i=1}^{n} x^k_{ij} - l^k_j \leq 0. \tag{20}
\]

As \( x^k_{ij} = \frac{\omega^k_i - p^k_j}{\alpha} \) is substituted into (19) and (20), above optimal problem (19) is equivalent to

\[
\min_{p^k_j \geq 0} \frac{n}{\alpha} (p^k_j)^2 - \sum_{i=1}^{n} \frac{\omega^k_i}{\alpha} p^k_j \tag{21}
\]

subject to

\[
\sum_{i=1}^{n} \omega^k_i - np^k_j - \alpha l^k_j \leq 0. \tag{22}
\]

(21) is a convex optimization problem. By [5], we know the above problem may be solved by Lagrange function.

The Lagrange function of (21) is

\[
L_j(p^k_j, \lambda^k_{1j}, \lambda^k_{2j}) = \frac{n}{\alpha} (p^k_j)^2 - \sum_{i=1}^{n} \frac{\omega^k_i}{\alpha} p^k_j + \lambda^k_{1j} (\sum_{i=1}^{n} \omega^k_i - np^k_j - \alpha l^k_j) - \lambda^k_{2j} p^k_j, \tag{23}
\]

where \( \lambda^k_{1j} \) and \( \lambda^k_{2j} \) are non-negative dual variables associated with the constraint (22) and \( p^k_j \geq 0 \). Then, the KKT condition is written as

\[
\frac{\partial L_j(p^k_j, \lambda^k_{1j}, \lambda^k_{2j})}{\partial p^k_j} = \frac{2n}{\alpha} p^k_j - \sum_{i=1}^{n} \frac{\omega^k_i}{\alpha} - \lambda^k_{1j} n - \lambda^k_{2j} = 0, \tag{24}
\]

\[
\lambda^k_{1j} (\sum_{i=1}^{n} \omega^k_i - np^k_j - \alpha l^k_j) = 0, \tag{25}
\]

\[
\lambda^k_{2j} p^k_j = 0, \tag{26}
\]

\[
p^k_j \geq 0, \tag{27}
\]

\[
\lambda^k_{1j} \geq 0, \tag{28}
\]

\[
\lambda^k_{2j} \geq 0, \tag{29}
\]

\[
\sum_{i=1}^{n} \omega^k_i - np^k_j - \alpha l^k_j \leq 0. \tag{30}
\]

Before solving the above convex optimization problem, first of all, we give the following lemma:

**Lemma 1.** \( \lambda^k_{2j} = 0 \).

**Proof.** See Appendix B. ☐
Lemma 2. $\lambda_{ij}^k \neq 0$.

Proof. See Appendix B.

We have
$$\sum_{i=1}^{n} \omega_i^k - np_j^k - \alpha l_j^k = 0.$$ According to (25) and Lemma 2, so the optimal real-time electricity prices of retailers are given as
$$p_j^{k*} = \frac{\sum_{i=1}^{n} \omega_i^k - \alpha l_j^k}{n}.$$ (31)

Proposition 1. The electricity price given by (31) is the optimal solution to (21) where $L_j^k$ satisfies that
$$\sum_{i=1}^{n} \frac{\omega_i^k}{\alpha} - \frac{n}{\alpha} \omega_n^k + nm_n^k - \eta + \sqrt{n} \sigma Q^{-1}(\varepsilon) < L_j^k$$
$$< \sum_{i=1}^{n} \frac{\omega_i^k}{\alpha} - \frac{n}{\alpha} \omega_1^k + n M_1^k - \eta + \sqrt{n} \sigma Q^{-1}(\varepsilon).$$ (32)

Proof. See Appendix B.

Theorem 1. The SE for the Stackelberg game formulated in this paper is $(p_j^{k*}, x_{ij}^{k*})$, where $p_j^{k*}$ is given by (31) and $x_{ij}^{k*}$ is given by (18) under the constraint (32).

5. Numerical analysis. In this section, a smart grid system is considered as the number simulation scenario which evaluates the performance of proposed RTP scheme with power load uncertainty and price competition among multiple electricity retailers. The smart grid system includes two electricity retailers and six users. Each day is divided into 24 time slots, $\omega_i^k$ is selected randomly from $\{1, 2, 3, 4\}$. The power consumption of different users with different $\omega_i^k$ at different time slots will be different, then the electricity costs of users and the revenues of retailers will also make a big difference. The parameter $\alpha = 0.5$, and for the load uncertainty $\delta_{ij}^k$, we assume that $\sigma^2 = 0.01$, $\eta = 0.1, \varepsilon = 0.001$, $m_1^k = \frac{\omega_1^k}{2\alpha}$, $M_1^k = \frac{\omega_1^k}{\alpha}$. Since RTP scheme is considered in this paper, we adopt the online pricing data provided by the Commonwealth Edison Company (ComEd) [8] for comparing with the proposed RTP scheme. Figure 1 shows the real-time price data for 24h on June 20, 2016 from [8].

Firstly, to evaluate the performance of proposed RTP scheme in this paper, we examine the effects of proposed RTP scheme on the retailers. Figure 2 gives the real-time electricity prices of two electricity retailers in 24 hours when considering power load uncertainty (R1 and R2) and ignoring power load uncertainty (R10 and R20), respectively. The electricity prices are also compared with RTP data from ComEd (Figure 1), the fixed pricing scheme (FP) proposed in [31], and the RTP scheme (NP) proposed in [18] as well. It may be seen that the real-time electricity price varies at different time slots, but the total change trend is consistent with the actual electricity change trend of power peak and valley under the real-time electricity prices from ComEd, thus the rationality of the proposed RTP scheme is verified. The reason which the price considering power load uncertainty is lower than that of ignoring the uncertainty is that the proposed pricing scheme in this
paper can effectively reduce the supply risk and power waste, so that the retailers sell electricity at a more reasonable and lower price. At the same time, the price competition among electricity retailers is beneficial to reduce the electricity prices. On the other hand, Figure 2 shows also that the proposed RTP scheme is very remarkable in cutting down the real-time price compared to [18,31]. Therefore, the users are able to benefit from the proposed pricing scheme in saving payment.

Next we change this uncertain parameter to other values. Figure 3 shows the simulation result with $\sigma = 0.1, 0.2, 0.3$. It can be seen the real-time price under proposed RTP scheme is always in a lower level than actual price data, which further verifies that our proposed RTP scheme has better performance than the pricing scheme in literatures [18,31].

From Figure 4, we find that the revenues of retailers (R1 and R2) considering power load uncertainty and the price competition among retailers are below the
Figure 3. Optimal real-time pricing of retailers when $\sigma = 0.1, 0.2, 0.3$.

revenues of retailers (R10 and R20) ignoring power load uncertainty in the 24 time slots, which means the retailers get less revenue under the lower real-time power prices while considering power load uncertainty and the price competition.

Figure 4. Expected revenue of electricity retailers.

From Figure 5, we can conclude that the power consumption amount of two users (U1 and U2) when considering the power load uncertainty and the price competition among retailers is more than that of two users (U10 and U20) ignoring power load uncertainty in the 24 time slots, which is because the power load uncertainty of users is considered in the proposed pricing scheme. For maximizing their own utilities, the users increase the power consumption when considering the power load uncertainty. Besides, the lower real-time price stimulates the users to consume more power for improving their satisfaction. So two users (U1 and U2) have more expenses, as is showed by Figure 6.

6. Conclusion. In this paper, we propose a RTP scheme with multiple electricity retailers and multiple users considering the user’s power load uncertainty and the
price competition among electricity retailers in smart grid. The interactive behavior between the retailers and the users is modeled as a Stackelberg game while two non-cooperative game models are constructed respectively for the power consumption behavior among users and the price competition among the retailers. With the backward induction, we obtain the SE. Numerical analysis results show that the proposed RTP scheme is in line with the real peak valley electricity consumption.
trends in smart grid while can reduce the real-time electricity prices and increase the users satisfaction under feasible constraint. Especially, the proposed pricing scheme always keeps the electricity price in a lower level with the changes of uncertain parameter, which is better than the real price data and pricing scheme in \([18,31]\).

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**Appendix A. Discussion on the optimal power demand of users.**

We now discuss the optimal power demand of users by considering five cases.  
1) Case 1. \(m_i^k < x_{i1}^k, m_i^k < M_i^k\). In this case \(\lambda_{i4} = \lambda_{i3} = \lambda_{i4} = 0\), substituting \(\lambda_{i1}, \lambda_{i2}, \lambda_{i3}, \lambda_{i4}\) into (15), we have

\[
x_{i1}^k = \frac{\omega_i^k - p_i^k}{\alpha}, \forall i \in \mathbb{N}, j = 1, 2.
\]

2) Case 2. \(m_i^k < x_{i1}^k < M_i^k, x_{i2}^k = m_i^k\). In this case \(\lambda_{i1} = \lambda_{i2} = \lambda_{i4} = 0\), substituting \(\lambda_{i1}, \lambda_{i2}, \lambda_{i4}\) into (15), we have

\[
x_{i1} = \frac{\omega_i^k - p_i^k}{\alpha}, \forall i \in \mathbb{N}, \lambda_{i3} = -\omega_i^k + p_i^k + \alpha m_i^k.
\]

3) Case 3. \(m_i^k < x_{i1}^k < M_i^k, x_{i2}^k = M_i^k\). In this case \(\lambda_{i1} = \lambda_{i2} = \lambda_{i3} = 0\), substituting \(\lambda_{i1}, \lambda_{i2}, \lambda_{i3}\) into (15), we have

\[
x_{i1} = \frac{\omega_i^k - p_i^k}{\alpha}, \forall i \in \mathbb{N}, \lambda_{i4} = \omega_i^k - p_i^k - \alpha M_i^k.
\]

4) Case 4. \(x_{i1}^k = m_i^k, m_i^k < x_{i2}^k < M_i^k\). In this case \(\lambda_{i2} = \lambda_{i3} = \lambda_{i4} = 0\), substituting \(\lambda_{i2}, \lambda_{i3}, \lambda_{i4}\) into (15), we have

\[
x_{i2} = \frac{\omega_i^k - p_i^k}{\alpha}, \forall i \in \mathbb{N}, \lambda_{i1} = -\omega_i^k + p_i^k + \alpha m_i^k.
\]

5) Case 5. \(x_{i1}^k = M_i^k, m_i^k < x_{i2}^k < M_i^k\). In this case \(\lambda_{i1} = \lambda_{i3} = \lambda_{i4} = 0\), substituting \(\lambda_{i2}, \lambda_{i3}, \lambda_{i4}\) into (15), we have

\[
x_{i2} = \frac{\omega_i^k - p_i^k}{\alpha}, \forall i \in \mathbb{N}, \lambda_{i2} = \omega_i^k - p_i^k - \alpha M_i^k.
\]

To sum up, regardless of the extreme situation, the optimal power consumption for all users is expressed as

\[
x_{ij}^k = \frac{\omega_i^k - p_j^k}{\alpha}, \forall i \in \mathbb{N}, j = 1, 2,
\]

where \(\omega_i^k - \alpha M_i^k < p_j^k < \omega_i^k - \alpha m_i^k\). According to the linear features of the above formulae, we have

\[
x_{ij}^k = \frac{\omega_i^k - p_j^k}{\alpha}, \forall i \in \mathbb{N}, j \in \mathbb{M},
\]

where \(\omega_i^k - \alpha M_i^k < p_j^k < \omega_i^k - \alpha m_i^k\).
Appendix B. Proofs of lemmas and proposition 1. Proof of lemma 1.

Proof. Suppose that \( \lambda_{ij}^k \neq 0 \), i.e., \( \lambda_{ij}^k > 0 \), we have \( p_j^k = 0 \) from (26). Thus, it leads to \( \frac{\sum \omega_i^k}{\alpha} p_j^k - \sum \omega_i^k - \lambda_{ij}^k n - \lambda_{ij}^k \lambda_{ij}^k < 0 \) according to \( \omega_i^k > 0, \alpha > 0, \lambda_{ij}^k \lambda_{ij}^k > 0 \) and \( \lambda_{ij}^k > 0 \), which contradicts (24). Therefore, the assumption \( \lambda_{ij}^k \neq 0 \) does not hold, thus \( \lambda_{ij}^k = 0 \).

Proof of Lemma 2.

Proof. Suppose that \( \lambda_{ij}^k = 0 \), we obtain \( p_j^k = \frac{\sum \omega_i^k}{2n} \) from (24) and Lemma 1.

According to \( \omega_i^k - \alpha M_i^k < p_j^k < \omega_i^k - \alpha m_i^k \) and Assumption 2, we have \( \frac{\sum \omega_i^k}{2n} < \omega_i^k - \alpha m_i^k \). It leads to \( \sum \omega_i^k < 2n(\omega_i^k - \alpha m_i^k) < 2 \sum \omega_i^k - \alpha m_i^k \). Then, \( \sum m_i^k < \sum \omega_i^k \) is obtained, which contradicts \( m_i^k \geq \omega_i^k \frac{1}{2n} \) in Assumption 1. Therefore, \( \lambda_{ij}^k \neq 0 \).

Proof of Proposition 1.

Proof. Substituting (31) into \( \omega_i^k - \alpha M_i^k < p_j^k < \omega_i^k - \alpha m_i^k \), we have

\[
\omega_i^k - \alpha M_i^k < \frac{\sum \omega_i^k}{n} < \omega_i^k - \alpha m_i^k.
\]

Its equivalent deformation is given as

\[
n \omega_i^k - \alpha M_i^k < \sum \omega_i^k - \alpha l_j^k \Rightarrow l_j^k < \sum \frac{\omega_i^k}{\alpha} - \frac{n}{\alpha} \omega_i^k + n M_i^k,
\]

\[
\sum \omega_i^k - \alpha l_j^k < n \omega_i^k - \alpha m_i^k \Rightarrow l_j^k > \sum \frac{\omega_i^k}{\alpha} - \frac{n}{\alpha} \omega_i^k + nm_i^k.
\]

Since \( l_j^k = L_j^k + \eta - \sqrt{n} \sigma Q^{-1}(\varepsilon) \), we have \( L_j^k = l_j^k - \eta + \sqrt{n} \sigma Q^{-1}(\varepsilon) \), it naturally follows

\[
\sum \frac{\omega_i^k}{\alpha} - \frac{n}{\alpha} \omega_i^k + nm_i^k - \eta + \sqrt{n} \sigma Q^{-1}(\varepsilon) < L_j^k \leq \sum \frac{\omega_i^k}{\alpha} - \frac{n}{\alpha} \omega_i^k + n M_i^k - \eta + \sqrt{n} \sigma Q^{-1}(\varepsilon).
\]

So Proposition 1 is proved. Now we find the optimal power price \( p_j^{k^*} \) and solve the Stackelberg game proposed in this paper.

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