Search for center-of-mass excitation free states in the SU(3) no-core shell model space

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Abstract. We address the removal of states with center-of-mass excitation from the SU(3) no-core shell model [SU(3)-NCSM] space. After these spurious states have been removed, the remaining center-of-mass excitation free (CMF) states form a suitable model space for describing the intrinsic dynamics of nuclei. We formulate the problem of finding the CMF states as a null-space problem for the center-of-mass harmonic oscillator lowering operator $\tilde{B}_{01}^{(0)}$, acting on the SU(3)-NCSM many-body space. We outline the construction process for the matrix representation of $\tilde{B}_{01}^{(0)}$ and provide a simple application of the method to the deuteron.

1. Introduction

The ability to carry out no-core configuration interaction calculations of light nuclei, in the no-core shell model (NCSM) [1–3], has made a significant contribution to recent progress in the ab initio description of nuclei. However, the dimensionality of the nuclear model space becomes computationally prohibitive as the number of active nucleons and orbitals increases. Symmetry can play a significant role in addressing this problem, by assisting in the selection of the physically relevant portions of the model space. Elliott [4–6] explored the U(3) ⊃ SU(3) symmetry of the harmonic oscillator, which serves as an organizational scheme for quadrupole deformation and rotation in the nuclear shell model. The U(3) algebra of the oscillator is contained in a larger Sp(3, $\mathbb{R}$) algebra, which serves as an organizational scheme for quadrupole deformation and rotation in the nuclear shell model. The SU(3) algebra of the oscillator is contained in a larger Sp(3, $\mathbb{R}$) algebra, which is found to have a close connection both to the dominant components of the nuclear Hamiltonian and to nuclear collective motion [7–9]. Following the discovery of evidence for SU(3) and Sp(3, $\mathbb{R}$) symmetry in conventional NCSM calculations for light nuclei [10, 11], the NCSM has been reformulated in terms of an SU(3)-based model space, in the SU(3) no-core shell model [SU(3)-NCSM] [12]. This development provides the foundations for future realization of a symplectic no-core shell model (Sp-NCSM) [13], making full use of Sp(3, $\mathbb{R}$) symmetry.

In predicting physical properties of the nucleus, only the intrinsic dynamics of the nucleus is of interest, not the center-of-mass motion. In principle, the center-of-mass motion may be eliminated from the problem by explicitly changing variables to relative coordinates. However, the nuclear many-body state must be antisymmetrized, and with increasing nucleon number this process rapidly becomes prohibitive in relative coordinates [3]. On the other hand, for a many-body basis constructed from antisymmetrized products of single-particle states, as in the NCSM, antisymmetrization is straightforward, but both center-of-mass (spurious) and...
intrinsic excitations are included in the model space [14, 15]. Consequently many of the nuclear eigenstates obtained by diagonalizing the Hamiltonian will carry center-of-mass excitation, presenting challenges for the study of the intrinsic excitations.

The usual approach to addressing the problem of spurious states consists of modifying the nuclear Hamiltonian, by adding a Lawson term [16] proportional the harmonic oscillator Hamiltonian $H_{\text{c.m.}}$ for the center-of-mass coordinate. After diagonalization, the spurious states remain in the spectrum but are shifted to an excitation energy above the low-lying intrinsic states of interest. However, the other possibility is to modify the model space, so as to explicitly remove the spurious states, before the Hamiltonian is diagonalized. This approach may be made feasible through the use of SU(3) symmetry [17–19]. After the spurious states are removed, the remaining center-of-mass excitation free (CMF) states form a suitable model space for describing the intrinsic dynamics of nuclei.

In this contribution, we establish an algorithm for identifying the CMF subspace within the SU(3)-NCSM space. The CMF states are associated with the lowest eigenvalue of the center-of-mass Hamiltonian $H_{\text{c.m.}}$. However, they also constitute the null space of the center-of-mass harmonic oscillator lowering operator $B_{\text{c.m.}}^{(01)}$, which serves as the basis for a simpler numerical approach developed here. It should also be noted that the SU(3)-NCSM model space is the starting point for constructing the Sp-NCSM model space [13]. The present results are intended to provide the foundation for obtaining Sp(3, R) irreducible representations (irreps) which are free of spurious excitations.

We first outline the construction of the SU(3)-NCSM basis (section 2). We then introduce an algorithm for identification of the CMF subspace within an SU(3)-NCSM model space, based on constructing the matrix representation of $B_{\text{c.m.}}^{(01)}$ with respect to the SU(3)-NCSM basis and solving for the null space (section 3). Finally, we provide a simple application of the present method to the SU(3)-NCSM model space for the deuteron (section 4).

2. SU(3)-NCSM basis states
The many-body basis states for the SU(3)-NCSM have good SU(3) $\times$ SU(2) quantum numbers, where the SU(3) symmetry label $(\Lambda \mu)$ characterizes the spatial degrees of freedom, according to the Elliott classification, while the SU(2) label $S$ describes the total intrinsic spin angular momentum. The creation operator for a nucleon in a given major oscillator shell $\eta$ comprises an SU(3) $\times$ SU(2) tensor $a_{(\eta \mu)1/2}^\dagger$, where the labels denote $(\Lambda \mu) S = (\eta \mu) S_{\eta} \text{[20]}$. The operators $a_{(\eta \mu)1/2}^\dagger$ are then used as the fundamental units in building up an SU(3) $\times$ SU(2)-coupled nuclear state.

Specifically, each SU(3)-NCSM basis state is characterized by a definite distribution of nucleons over the major shells. First, all nucleons in the each major shell $\eta$ are combined to form a configuration of the type \[ |a_{(\eta \mu)1/2}^\dagger \times a_{(\eta \mu)1/2}^\dagger \times \cdots |^{(\Lambda \mu) S_{\eta}}, \]
with good SU(3) $\times$ SU(2) coupling [21], separately for the protons and neutrons. Then, such configurations for individual major shells are coupled successively to form a total proton state and total neutron state carrying good SU(3) $\times$ SU(2) quantum numbers. Finally, the proton state and neutron state are coupled, to give the SU(3)-NCSM basis state. Since the major shells have definite occupations, the state may be classified, as usual in the oscillator basis for the NCSM [3], by the number $N_{\text{ex}}$ of harmonic oscillator excitations, taken relative to the minimal number of oscillator quanta possible for the given number of protons and neutrons.

1 Briefly, an Sp(3, R) irrep is labeled by its extremal SU(3) state [13]. Other Sp-NCSM states within this irrep are constructed by acting on the extremal state with an SU(3)-coupled product $|A^{(20)} \times A^{(20)} \cdots A^{(20)}|^{(\Lambda \mu)}$, where $A^{(20)}$ is the Sp(3, R) raising operator. Since the action of $A^{(20)}$ does not introduce any center-of-mass excitation, an Sp(3, R) irrep is CMF as long as its extremal SU(3) state is CMF.
The resulting SU(3)-NCSM basis state has the form, with all coupling labels shown explicitly,

\begin{equation}
\left| \left( \left( \gamma_{p,0} \times \gamma_{p,1} \right) \rho_{p,0} \omega_{p,0} \times \gamma_{p,2} \right) \rho_{p,1} \omega_{p,1} \times \gamma_{p,3} \rho_{p,2} \omega_{p,2} \ldots \times \gamma_{p,\eta_{\text{max}}} \rho_{p,\eta_{\text{max}}} \right)_{\rho_{p} \omega_{p}} \right| \times \left| \left( \left( \gamma_{n,0} \times \gamma_{n,1} \right) \rho_{n,0} \omega_{n,0} \times \gamma_{n,2} \right) \rho_{n,1} \omega_{n,1} \times \gamma_{n,3} \rho_{n,2} \omega_{n,2} \ldots \times \gamma_{n,\eta_{\text{max}}} \rho_{n,\eta_{\text{max}}} \right)_{\rho_{n} \omega_{n}} \right| \right. ^{\rho_{\omega}}. \tag{1}
\end{equation}

Here the symbol \( \gamma_{s,n} \) represents the labels \( \gamma = [f_1, f_2, \ldots, f_N] \alpha(\lambda \mu) S \) needed to completely specify the coupling of nucleons of type \( s \) (i.e., protons or neutrons) within major shell \( \eta \) [21]. Specifically, each major shell has associated with it a U(\( N \)) algebra \[ N = (\eta + 1)(\eta + 2) \] consisting of bilinears of creation and annihilation operators, for which the irreps are labeled by \( [f_1, f_2, \ldots, f_N] \), where we consider only antisymmetric irreps, and \( \sum_{i=1}^{N} f_i \) equals the occupation of the shell. Within a U(\( N \)) irrep, a multiplicity index \( \alpha \) is required to distinguish SU(3) \( \times \) SU(2) irreps with the same quantum numbers \( (\lambda \mu) S \), yielding the labeling scheme

\begin{equation}
U(N) \supset SU(3) \times SU(2) \tag{2}
\end{equation}

\( \alpha = (\lambda \mu) S \)

The symbols \( \omega_{s,n} \) in (1) then indicate the SU(3) \( \times \) SU(2) coupling labels \( |\omega = (\lambda \mu) S \rangle \) of successive shells, and \( \rho_{s,n} \) denotes the multiplicity index for this coupling. Finally we have total couplings \( \omega_{p} \equiv (\lambda_{p} \mu_{p}) S_{p} \) for the protons, \( \omega_{n} \equiv (\lambda_{n} \mu_{n}) S_{n} \) for the neutrons, \( \omega \equiv (\lambda \mu) S \) for the entire basis state, with corresponding multiplicity indices \( \rho_{p} \), \( \rho_{n} \), and \( \rho \), respectively.

The expression in (1) represents not just a single state but an entire set of states, with various values for the quantum numbers associated with the branching of SU(3) \( \times \) SU(2) into angular momentum subalgebras: the orbital (spatial) angular momentum \( L \), the inner multiplicity \( \kappa \) for this \( L \) within the SU(3) irrep \( (\lambda \mu) \), and the total angular momentum \( J \), as well as its z-projection \( M \). However, these states share the same “internal” microscopic structure, given by the same couplings of the particles at the level of SU(3) \( \times \) SU(2). Therefore, they may be thought of as a single reduced state for certain purposes, in particular, evaluation of reduced matrix elements under the SU(3) \( \times \) SU(2) Wigner-Eckart theorem and, as we shall see, identification of CMF linear combinations. The analogy is to angular momentum theory, where one may consider the states \( |JM \rangle \), for different \( M \), to be substates of a single state \( |J \rangle \), more formally, a tensorial set or SU(2) irrep.

3. Construction of CMF states

The separation of the many-body space into CMF and spurious parts simplifies in the context of an SU(3) \( \times \) SU(2)-coupled basis, since the process may be carried out independently within subspaces characterized by definite SU(3) \( \times \) SU(2) quantum numbers. To start with, the center-of-mass Hamiltonian \( H_{\text{c.m.}} \) does not connect states involving different numbers \( N_{\text{ex}} \) of oscillator excitations. Thus, the separation of CMF states may be carried out separately within the space of states with each specific number of oscillator quanta, which we denote by \( W_{N_{\text{ex}}} \). Furthermore, \( H_{\text{c.m.}} \), commutes with the SU(3) generators and may therefore be diagonalized within a subspace with good SU(3) quantum numbers [18]. As an operator acting only on spatial degrees of freedom, \( H_{\text{c.m.}} \) also commutes with the total spin operators for protons and neutrons, as well as their combined spin. Thus, collecting these properties, the separation of CMF states may be carried out separately within subspaces of given \( N_{\text{ex}} \), \( (\lambda \mu) \), \( S_{p} \), \( S_{n} \), and \( S \), which we denote by \( W_{N_{\text{ex}}} [(\lambda \mu) S_{p} S_{n} S] \).

2 It is due to this property that the usual \( N_{\text{max}} \) truncation scheme for the NCSM [3], based on the total number of oscillator quanta for the many-body state, permits an exact factorization of center-of-mass and intrinsic wavefunctions.
In considering how to extract CMF states, we note that these states are defined by their relation to the center-of-mass harmonic oscillator raising and lowering operators. The center-of-mass raising operator $B_{\text{c.m.}}^{(1)}$, which is an $L = 1$ operator, furthermore constitutes an SU(3) (10) tensor, with components $B_{\text{c.m.},L=1,M}^{(1)}$. This operator may be written in terms of single-particle harmonic-oscillator raising operators $b_i^{(10)}$ as [13, 20]

$$B_{\text{c.m.}}^{(10)} = \frac{1}{\sqrt{A}} \sum_{i=1}^{A} b_i^{(10)}.$$  

(3)

The corresponding lowering operator $\tilde{B}_{\text{c.m.}}^{(01)}$ similarly has the form

$$\tilde{B}_{\text{c.m.}}^{(01)} = \frac{1}{\sqrt{A}} \sum_{i=1}^{A} \tilde{b}_i^{(01)}.$$  

(4)

The center-of-mass harmonic oscillator Hamiltonian is built from these operators as

$$H_{\text{c.m.}} = \hbar \omega \left( B_{\text{c.m.}}^{(10)} \cdot B_{\text{c.m.}}^{(01)} + \frac{3}{2} \right),$$  

(5)

where the dot indicates a spherical tensor scalar product. The CMF states are then defined by the property that they have no center-of-mass excitations, i.e., they have zero eigenvalue for the center-of-mass number operator $N_{\text{c.m.}} = B_{\text{c.m.}}^{(10)} \cdot B_{\text{c.m.}}^{(01)}$. Equivalently, however, they are identified by the property that they are annihilated by the center-of-mass lowering operator $\tilde{B}_{\text{c.m.}}^{(01)}$.

Either of these criteria allow the problem of identifying CMF states to be formulated as a null-space problem. We seek the subspace $W_{\text{CMF}}^{\text{ex}} [(\lambda \mu)S_p S_n S]$ of $W_{\text{ex}} [(\lambda \mu)S_p S_n S]$ consisting of states $|\Psi_{\text{CMF}}\rangle$ such that $N_{\text{c.m.}} |\Psi_{\text{CMF}}\rangle = 0$ or, equivalently, $B_{\text{c.m.},L=1,M}^{(01)} |\Psi_{\text{CMF}}\rangle = 0$. In practice, this means first representing the operator as a matrix with respect to the SU(3)-NCSM basis and then solving for a complete set of null vectors of this matrix. The result yields new basis states, for $W_{\text{ex}}^{\text{CMF}} [(\lambda \mu)S_p S_n S]$, as linear combinations of the original basis states, for $W_{\text{ex}} [(\lambda \mu)S_p S_n S]$.

Although we could in principle search for the null space of either $N_{\text{c.m.}}$ or $B_{\text{c.m.}}^{(01)}$, there is a practical advantage to working with $B_{\text{c.m.}}^{(01)}$. While $N_{\text{c.m.}}$ is a two-body operator, $B_{\text{c.m.}}^{(01)}$ is simply a one-body operator. Consequently, evaluation of matrix elements is computationally less involved. Note that $N_{\text{c.m.}}$, acts within the space $W_{\text{ex}}$, conserving the number of oscillator excitations, while $B_{\text{c.m.}}^{(01)}$ instead connects the space $W_{\text{ex}}$ to the next lower space $W_{\text{ex}} - 1$.

The problem of identifying CMF states is simplified by the realization that it may be formulated entirely at the level of reduced states. An SU(3) × SU(2)-reduced state is CMF if and only if its substates, labeled by $\kappa L J$, are all CMF. [Recall $H_{\text{c.m.}}$ commutes with the SU(3) generators, which connect states within an irrep.] We need thus only find a basis of SU(3) × SU(2)-reduced states, for $W_{\text{ex}}^{\text{CMF}} [(\lambda \mu)S_p S_n S]$, independent of $\kappa L J$. We also need only consider the SU(3) × SU(2)-reduced matrix elements of the operator $B_{\text{c.m.}}^{(01)}$, rather than the matrix elements among individual $\kappa L J$ substates, which are much greater in number.

To explicitly relate the null space problem to SU(3) × SU(2)-reduced matrix elements, let us return for a moment to $\kappa L J$ states and observe that any CMF state $|\Psi_{\text{CMF}}^{\text{ex}} [(\lambda \mu)S_p S_n S; \kappa L J M]\rangle$ within the $W_{\text{ex}} [(\lambda \mu)S_p S_n S]$ subspace must satisfy

$$\langle \Psi_{\text{ex}}^{\text{ex}} - 1 [(\lambda \mu)' S_p' S_n' S'; \kappa' L' J' M'] | B_{\text{c.m.,L}=0-1,M_0}^{(01)} | \Psi_{\text{ex}}^{\text{CMF}} [(\lambda \mu)S_p S_n S; \kappa L J M]\rangle = 0,$$

(6)

for every state $|\Psi_{\text{ex}}^{\text{ex}} - 1 [(\lambda \mu)' S_p' S_n' S'; \kappa' L' J' M']\rangle \in W_{\text{ex}} - 1$. Since we are working with states of good angular momentum, we can immediately rewrite the condition in terms of a reduced matrix element as

$$\langle \Psi_{\text{ex}}^{\text{ex}} - 1 [(\lambda \mu)' S_p' S_n' S'; \kappa' L' J'] | B_{\text{c.m.,L}=0}^{(01)} | \Psi_{\text{ex}}^{\text{CMF}} [(\lambda \mu)S_p S_n S; \kappa L J]\rangle = 0.$$  

(7)
Note that, since $\tilde{B}_{c.m.}$ acts only on spatial degrees of freedom, we actually need only consider the case $(S'_S S'_S S') = (S_p S_n S)$. The ordinary SU(2)-reduced matrix element in (7) is related to the SU(3) × SU(2)-reduced matrix element of $\tilde{B}_{c.m.}$ by the SU(3) Wigner-Eckart theorem (and LS-coupling relations), as

$$\langle \Psi_{N_{ex}-1}[(\lambda' \mu') S_p S_n S; \kappa' L' J']|\tilde{B}_{c.m.,L_{0}=1}|\Psi_{N_{ex}}^{CMF}[(\lambda \mu) S_p S_n S; \kappa LJ]\rangle$$

$$= (-)^{J+L'+1+S} \hat{L}' \hat{J}' \left[ \begin{array}{ccc} L & J & S \\ J' & L' & 1 \end{array} \right] \left( \begin{array}{c} (\lambda \mu) \\ (01) \end{array} \right) \left( \begin{array}{c} (\lambda' \mu') \\ (01) \end{array} \right)$$

$$\times \langle \Psi_{N_{ex}-1}[(\lambda' \mu') S_p S_n S]|\tilde{B}_{c.m.}^{(01)}||\Psi_{N_{ex}}^{CMF}[(\lambda \mu) S_p S_n S]\rangle,$$

(8)

where $\hat{J} \equiv (2J + 1)^{1/2}$, and the quantity in parentheses is an SU(3) Clebsch-Gordan coefficient [22]. The condition that the complete set of SU(2)-reduced matrix elements appearing on the left hand side of (8) vanish is equivalent to the condition that the single SU(3) × SU(2)-reduced matrix element on the right hand side vanish.

Thus, for the CMF states, we seek SU(3) × SU(2)-reduced states $|\Psi_{N_{ex}}^{CMF}[(\lambda \mu); S_p S_n S]\rangle$ such that

$$\langle \Psi_{N_{ex}-1}[(\lambda' \mu') S_p S_n S]|\tilde{B}_{c.m.}^{(01)}||\Psi_{N_{ex}}^{CMF}[(\lambda \mu) S_p S_n S]\rangle = 0,$$

(9)

for all possible reduced states $|\Psi_{N_{ex}-1}[(\lambda' \mu') S_p S_n S]\rangle \in W_{N_{ex}-1}[(\lambda' \mu') S_p S_n S]$. The subspaces $W_{N_{ex}-1}[(\lambda' \mu') S_p S_n S]$ which may be linked with $W_{N_{ex}}[(\lambda \mu) S_p S_n S]$ through $\tilde{B}_{c.m.}^{(01)}$ are restricted by the SU(3) tensor character of $\tilde{B}_{c.m.}$. Specifically, $(\lambda' \mu')$ must be contained in the product $(\lambda \mu) \times (01)$ which, from the general rules of SU(3) coupling [23], may be seen to consist of

$$(\lambda \mu) \times (01) = \begin{cases} (01) & \lambda = \mu = 0 \\ (0 \mu + 1) \oplus (1 \mu - 1) & \lambda = 0, \mu \geq 1 \\ (\lambda 1) \oplus (\lambda - 1 0) & \lambda \geq 1, \mu = 0 \\ (\lambda \mu + 1) \oplus (\lambda + 1 \mu - 1) \oplus (\lambda - 1 \mu) & \lambda \geq 1, \mu \geq 1. \end{cases}$$

(10)

The problem of finding reduced states which satisfy (9) can be converted into searching for the null space of a matrix, the entries of which are the SU(3) × SU(2)-reduced matrix elements of $\tilde{B}_{c.m.}^{(01)}$ between the SU(3) × SU(2)-reduced basis states for the $W_{N_{ex}}[(\lambda \mu) S_p S_n S]$ subspace and the SU(3) × SU(2)-reduced basis states for each of the possible subspaces $W_{N_{ex}-1}[(\lambda' \mu') S_p S_n S]$. The resulting matrix has the form illustrated in figure 1, where the horizontal dashed lines delimit submatrices corresponding to the different final spaces with $(\lambda' \mu') = (\lambda' \mu'_1), (\lambda' \mu'_2), \ldots$, as allowed by the selection rule (10). (There will be at most three such submatrices.) The entries of the null vectors then give the expansion coefficients for the basis states for the CMF space $W_{N_{ex}}^{CMF}[(\lambda \mu) S_p S_n S]$ in terms of the original basis states of $W_{N_{ex}}[(\lambda \mu) S_p S_n S]$.

The SU(3)-reduced matrix elements of $\tilde{B}_{c.m.}$ entering into the matrix of figure 1 can be calculated numerically using methods from [24]. In particular, the SU(3) × SU(2) matrix elements of a one-body operator such as $\tilde{B}_{c.m.}^{(01)}$ (or an n-body operator in general) can readily be computed, once the operator is expressed in second-quantized form in terms of SU(3) × SU(2)-coupled products of creation operators $\hat{a}_{n}\hat{a}_{m}$ and annihilation operators $\hat{a}_{n'}\hat{a}_{m'}$. For the center-of-mass annihilation operator $\tilde{B}_{c.m.}^{(01)}$, the second-quantized form is obtained as

$$\tilde{B}_{c.m.}^{(01)} = \frac{1}{\sqrt{A}} \sum_{\eta} \sqrt{\frac{(\eta + 1)(\eta + 2)}{3}} \langle \eta || \tilde{b}^{(01)} || \eta + 1 \rangle \left[ \hat{a}_{(\eta 01/2)} \hat{a}_{(0,\eta+1/2)} \right]^{(01)\eta},$$

(11)

where $\langle \eta || \tilde{b}^{(01)} || \eta + 1 \rangle = \sqrt{\eta + 3}$. 

\[5\]
4. Application to the deuteron

The case of deuteron provides a simple illustration of the methods described in section 3. The full SU(3)-NCSM space for the deuteron may be broken into subspaces \( W_{N_{\text{ex}}}[(\lambda\mu)S_pS_nS] \), with \( S = 0 \) and 1. The dimensions of these subspaces — specifically, the number of SU(3) \( \times \) SU(2)-reduced basis states, not the total number of \( \kappa L J \) basis states, which would be much higher — are indicated by the areas of the outer, light circles in figure 2. Then, for each value of \( N_{\text{ex}} \), it is found that solution of the null space problem for the center-of-mass annihilation operator yields two CMF reduced states. Both have \( (\lambda\mu) = (N_{\text{ex}}0) \), one with \( S = 0 \) and one with 1. These CMF subspaces are indicated by the inner, dark circles in figure 2.

The deuteron provides a particularly illuminating test case, since the quantum numbers of the CMF spaces obtained in figure 2 may be understood through simple arguments. Let \( N_{\text{c.m.}} \) and \( N_{\text{rel}} \) denote the number of oscillator excitations of the center-of-mass and relative degrees of freedom, respectively, so \( N_{\text{ex}} = N_{\text{c.m.}} + N_{\text{rel}} \). The CMF condition imposes \( N_{\text{c.m.}} = 0 \) and thus \( N_{\text{rel}} = N_{\text{ex}} \). For the two-particle system, the transformation between single-particle and relative coordinates is straightforward. There is only a single relative coordinate vector, and the harmonic oscillator in this coordinate is equivalent to the harmonic oscillator for a single particle in three dimensions. For a given \( N_{\text{ex}} \), it will therefore carry SU(3) quantum numbers \((N_{\text{ex}}0)\). The center-of-mass oscillator carries \((00)\) for a CMF state. Therefore the CMF state as a whole will have \( (\lambda\mu) = (N_{\text{ex}}0) \times (00) = (N_{\text{ex}}0) \), as well. Since the deuteron consists of distinguishable particles, the coupling of spins is independent of the spatial degrees of freedom, and both \( S = 0 \) and 1 are obtained.

5. Conclusions

The elimination of spurious center-of-mass excitations is essential to the problem of determining the intrinsic structure of nuclei. If an SU(3)-coupled harmonic oscillator basis is used for the many-body problem, as in the SU(3)-NCSM, the separation may be carried out at the level of SU(3) irreps, in particular, within subspaces of fixed number of oscillator quanta and \( (\lambda\mu)S_pS_nS \) labels. We have formulated the problem of finding the CMF subspace as a matrix null-space problem, based on the SU(3) \( \times \) SU(2)-reduced matrix elements of the center-of-mass annihilation operator. It is therefore possible to remove spurious contributions from the SU(3)-NCSM model space prior to diagonalization of the Hamiltonian, rather than through a Lawson term. In the

\[
\begin{bmatrix}
\langle \Psi_{N_{\text{ex}}-1,1}^{(\lambda\mu')_1} | \hat{B}_{\text{c.m.}} | \Psi_{N_{\text{ex},1}}^{(\lambda\mu)} \rangle \\
\langle \Psi_{N_{\text{ex}}-1,2}^{(\lambda\mu')_1} | \hat{B}_{\text{c.m.}} | \Psi_{N_{\text{ex},1}}^{(\lambda\mu)} \rangle \\
\vdots \\
\langle \Psi_{N_{\text{ex}}-1,1}^{(\lambda\mu')_2} | \hat{B}_{\text{c.m.}} | \Psi_{N_{\text{ex},1}}^{(\lambda\mu)} \rangle \\
\langle \Psi_{N_{\text{ex}}-1,2}^{(\lambda\mu')_2} | \hat{B}_{\text{c.m.}} | \Psi_{N_{\text{ex},1}}^{(\lambda\mu)} \rangle \\
\vdots \\
\end{bmatrix}
\]

Figure 1. Form of the matrix representation of \( \hat{B}_{\text{c.m.}}^{(01)} \), as reduced matrix elements between the SU(3)-NCSM basis states for the \( W_{N_{\text{ex}}}[(\lambda\mu)S_pS_nS] \) subspace (identified with the columns) and the SU(3)-NCSM basis states for each of the possible subspaces \( W_{N_{\text{ex}}-1}[(\lambda\mu')S_pS_nS] \) (identified with the rows).
context of the SU(3)-NCSM, this raises the possibility of significant reductions in dimensionality. However, this work is more broadly motivated by the goal of constructing nonspurious irreps for an Sp(3, \mathbb{R}) symmetry-adapted no-core shell model.

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References

[1] Navrátil P, Vary J P and Barrett B R 2000 Phys. Rev. Lett. 84 5728
[2] Navrátil P, Vary J P and Barrett B R 2000 Phys. Rev. C 62 054311
[3] Navrátil P, Quaglioni S, Stetcu I and Barrett B R 2009 J. Phys. G 36 083101
[4] Elliott J P 1958 Proc. R. Soc. London A 245 128
[5] Elliott J P 1958 Proc. R. Soc. London A 245 562
[6] Elliott J P and Harvey M 1963 Proc. R. Soc. London A 272 557
[7] Rosensteel G and Rowe D J 1977 Phys. Rev. Lett. 38 10
[8] Rosensteel G and Rowe D J 1980 Ann. Phys. (N.Y.) 126 343
[9] Draayer J P, Weeks K J and Rosensteel G 1984 Nucl. Phys. A 413 215
[10] Dytrych T, Sviratcheva K D, Bahri C, Draayer J P and Vary J P 2007 Phys. Rev. C 76 014315
[11] Dytrych T, Sviratcheva K D, Bahri C, Draayer J P and Vary J P 2007 Phys. Rev. Lett. 98 162503
[12] Dytrych T, Draayer J P, Launey K D, Caprio M A and Langr D in these proceedings
[13] Dytrych T, Sviratcheva K D, Draayer J P, Bahri C and Vary J P 2008 J. Phys. G 35 123101
[14] Elliott J P and Skyrme T H R 1955 Proc. R. Soc. London A 232 561
[15] Baranger E and Lee C W 1961 Nucl. Phys. 22 157
[16] Gloeckner D H and Lawson R D 1974 Phys. Lett. B 53 313
[17] Kretzschmar M 1960 Z. Phys. 158 284
[18] Verhaar B J 1960 Nucl. Phys. 21 508
[19] Hecht K T 1971 Nucl. Phys. A 170 34
[20] Escher J and Draayer J P 1998 J. Math. Phys. 39 5123
[21] Draayer J P, Leschber Y, Park S C and Lopez R 1989 Comput. Phys. Commun. 56 279
[22] Akiyama Y and Draayer J P 1973 Comput. Phys. Commun. 5 405
[23] O’Reilly M F 1982 J. Math. Phys. 23 2022
[24] Bahri C and Draayer J P 1994 Comput. Phys. Commun. 83 60