Solution of seepage field in different soil layers of concrete dam foundation by flow net method

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Abstract. Because of the seepage problem of concrete dam foundation, the water head, pore water pressure, hydraulic gradient and seepage velocity of each point in the seepage field are obtained by drawing flow net, and finally the seepage flow is obtained to predict the seepage of dam foundation. In this paper, the flow net diagram is used to solve the pore water pressure at the fixed point of the dam foundation soil layer under three different conditions: homogeneous isotropy, homogeneous anisotropy and layered soil. The calculation accuracy of the flow net method is verified through the calculation with the finite difference software FLAC3D numerical analysis. The results indicated that the pore water pressure of the three kinds of dam foundation soil layers calculated by the flow net method is more similar from that calculated by the numerical method, which indicates that the flow net method has high accuracy and can be used in some engineering practice with difficult modeling.

1. Introduction

Dam foundation seepage is a common engineering geological problem of the dams [1]. It is very essential to study the seepage of the dam foundation for the stability and seepage of dam foundation. Leakage of the dam foundation leads to poor water storage capacity, even dam break accidents. Seepage calculation is very important in the design of the dam foundation, which needs to be paid more attention.

At present, the analysis and calculation methods of the dam foundation seepage field mainly include theoretical analysis method, numerical method, and graphic method. In general, theoretical analysis can only solve simple boundary conditions, and involves the calculation of complex partial differential equations. In recent years, because of the rapid development of computer technology, the numerical method has been widely used in the calculation of the seepage field. Common numerical methods include finite element method, finite difference method, and boundary element method [2-3]. But numerical modeling is more complex, which is difficult for engineers to master, so it is limited in practical application. Flow net is a kind of graphic method, which is an old method. It has the advantages of simple theory and high calculation accuracy, so it can be widely used in engineering practice. Mining Yueming et al [4] used a drawing flow net to calculate the two-dimensional seepage field in the heterogeneous anisotropic domain of geotechnical engineering. Nie Zhengfan [5] uses the flow net method and the finite element method to analyze the isotropic seepage field, and Rena [6] uses the seepage theory and the Galerkin finite element method to draw the flow net of the heterogeneous anisotropic seepage field. The above research results play a positive role in promoting the flow net calculation of the dam foundation seepage field, but these are mainly aimed at a single soil layer, which are generally regarded as a homogeneous isotropic body, lacking systematic drawing...
of flow network of different soil layers and discussion of calculation accuracy.

For this reason, this paper takes the seepage problem of three different types of concrete dam foundation soil layers as homogeneous isotropic, homogeneous anisotropic and heterogeneous isotropic. The flow net of three types of soil layers is drawn to calculate the pore water pressure, and the calculation results are compared with the FLAC3D calculation results to discuss the calculation accuracy. The research results can provide a reference for the solution of the seepage field of similar dam foundation.

2. Solving seepage field of the dam foundation with flow net

2.1 Two dimensional stable seepage theory

When the water level difference between the upstream and downstream of the reservoir keeps constant, the seepage is stable. The total head \( h \) and seepage velocity \( v \) in the stable seepage field are only functions of location. According to Darcy's law, the flow equation of two-dimensional stable seepage field in homogeneous and isotropic aquifer is obtained, which is (Laplace) equation [7-8]:

\[
\frac{\partial^2 h}{\partial x^2} + \frac{\partial^2 h}{\partial y^2} = 0
\]

In (1), \( h \) is the total head, equal to the sum of the pressure head \( h_u \) and the position head \( h_z \).

At present, there are analytic method, numerical method and graphic method to solve the equation. Graphic method is used to solve the equation in this paper. The solution of Laplace equation of stable seepage field has two orthogonal curve clusters, which represent potential function and flow function respectively. According to the boundary conditions, the corresponding flow net is drawn to obtain the specific solution [8]. In the drawing of flow net, firstly determine the boundary, draw the boundary streamline and head tail equipotential line; secondly determine the equal head value, draw the equal head line according to the differential interpolation of upstream and downstream water head; finally, repeat drawing according to the orthogonality of flow net and get the flow net with high accuracy.

2.2 Calculation of pore water pressure in different soil layers

The upstream water level of the dam is 5m, the downstream water level is 0m, the dam length is 12m, and the aquifer thickness is 15m. In the seepage field, points a(18,-7), b(18,-3), c(22,-7), d(22,-3), e(26,-7), f(26, -3), g(30,-7), h(30,-3), i(34,-7), j(34,-3)are arranged. The following will give the pore water pressure of each point in the way of flow net for different types of dam foundation soil.

1) Homogeneous isotropy

When the soil layer is homogeneous isotropic soil, determine the required streamline and equipotential line through the given boundary conditions of the seepage field, draw the flow net diagram of dam foundation seepage and show the elements of the layout point, as shown in figure1.

![Figure 1. Flow net of homogeneous isotropic seepage](image)

According to the flow net diagram of homogeneous isotropic soil drawn in figure1, equipotential lines are divided into 26, and the head drop between each equipotential line is \( \Delta h = \frac{5}{26} = 0.19m \).

2) Homogeneous anisotropy

When the soil layer is homogeneous and anisotropic, permeability coefficient is, \( k_x \neq k_z \), the basic
The differential equation of stable seepage field is:
\[ k_x \frac{\partial^2 h}{\partial x^2} + k_z \frac{\partial^2 h}{\partial z^2} = 0 \]  \hspace{1cm} (2)

This equation is not Laplace's equation. Multiply the horizontal coordinate \( x \) by \( \sqrt{\frac{k_z}{k_x}} \), the new coordinate \( x' \) is obtained, and formula (2) is transformed into the same form of Laplace equation:
\[ \frac{\partial^2 h}{\partial x'^2} + \frac{\partial^2 h}{\partial z^2} = 0 \]  \hspace{1cm} (3)

According to the isotropic soil to draw the flow net, the flow net drawn at this time is abnormal flow net. Multiply the horizontal coordinates of the abnormal flow net by \( \frac{1}{\sqrt{k_z/k_x}} \), the actual flow net of anisotropic soil can be drawn. When the soil layer is anisotropic, the flow net of dam foundation seepage and each element of the point are drawn as shown in figure 2.

Figure 2. Flow net of homogeneous anisotropic seepage

According to the flow net diagram of homogeneous anisotropic soil drawn in figure 2, equipotential lines are divided into 22, and the head drop of each equipotential line interval is \( \Delta h = \frac{h_{22}}{22} = 0.23 \text{m} \).

3) Stratified soil

Due to the different permeability coefficient of each layer of layered soil, at the interface of each layer, the streamline will turn, and the shape of flow net in different layers will be different. When drawing the multi-layer soil seepage flow net, the transformation conditions are as follows[7]-[8]: at the interface of different soil layers, the flow line should deflect according to the seepage refraction law; the width \( \Delta s \) of the flow channel of the upper and lower soil layers changes with the deflection angle. With the change of hydraulic gradient, the shape of grids on both sides of the interface changes with the change of permeability coefficient.

When the soil layer is three layers of soil, the permeability coefficient of each layer of soil is different. It is assumed that the permeability coefficient of three layers of soil from top to bottom is \( k_1,k_2,k_3 \) (\( k_1=2\times10^{-7}\text{m/s}, k_2=3\times10^{-7}\text{m/s}, k_3=1\times10^{-7}\text{m/s} \)). Under the condition that the dam foundation soil is layered soil, the flow net drawn and the conditions of points a-j 10 are shown in figure 3.

Figure 3. Flow net of three layers of soil.

According to the layered soil flow net diagram drawn in figure 3, equipotential lines are divided into 28, the head drop \( \Delta H = \frac{h_{28}}{28} = 0.18 \text{m} \) between each equipotential line. According to the flow net diagram in three cases, it can be seen that the location head \( h_z \), the reduced head value is the number of equipotential lines where the point is multiplied by the head drop of each equipotential line interval,
the pressure head $h_u$ subtracts the reduced head value from the original pressure head value, and the pore water pressure is calculated by the formula $u = h_u - y_w$, Table 1 gives the calculation results of pore water pressure at 10 points of a-j in three cases.

| Soil layer | Coordination (m) | $h_u$ (m) | $u$ (kN/m$^2$) |
|------------|------------------|-----------|----------------|
| Homogeneous isotropy | a(18,-7) | 11.24 | 110.2 |
| | b(18,-3) | 7.62 | 74.7 |
| | c(22,-7) | 10.48 | 102.7 |
| | d(22,-3) | 6.67 | 65.5 |
| | e(26,-7) | 9.53 | 93.4 |
| | f(26,-3) | 5.33 | 54.2 |
| | g(30,-7) | 8.58 | 84.1 |
| | h(30,-3) | 4.39 | 43.0 |
| | i(34,-7) | 7.82 | 76.6 |
| | j(34,-3) | 3.44 | 33.7 |

| Homogeneous anisotropy | a(18,-7) | 11.31 | 110.8 |
| | b(18,-3) | 7.77 | 76.1 |
| | c(22,-7) | 10.39 | 101.8 |
| | d(22,-3) | 6.62 | 64.9 |
| | e(26,-7) | 9.47 | 92.8 |
| | f(26,-3) | 5.47 | 53.6 |
| | g(30,-7) | 8.55 | 83.8 |
| | h(30,-3) | 4.32 | 42.3 |
| | i(34,-7) | 7.63 | 74.8 |
| | j(34,-3) | 3.17 | 31.1 |

| Stratified soil | a(18,-7) | 10.74 | 105.3 |
| | b(18,-3) | 7.28 | 71.3 |
| | c(22,-7) | 10.02 | 98.2 |
| | d(22,-3) | 6.38 | 62.5 |
| | e(26,-7) | 9.48 | 92.9 |
| | f(26,-3) | 5.48 | 53.7 |
| | g(30,-7) | 8.94 | 87.6 |
| | h(30,-3) | 4.58 | 44.9 |
| | i(34,-7) | 8.22 | 80.6 |
| | j(34,-3) | 3.68 | 36.1 |

### 3. Numerical solution and accuracy verification of flow net calculation

#### 3.1 Numerical solution of seepage field of three types of soil layers

FLAC3D software has been widely used in many fields such as tunnel engineering, water conservancy engineering, mine engineering and slope stability evaluation [9]. FLAC3D software is used to calculate the seepage field of dam foundation based on the seepage theory of porous continuous medium, and the seepage equation is solved by finite difference method [10]. The plane strain model is adopted in the numerical calculation. The impermeable boundary is set at the dam base, the upper and lower reaches of the dam are set as the constant head boundary, and the two sides of the dam base are impermeable boundary. The seepage calculation parameters are shown in Table 2.

| Permeability coefficient (m/s) | Porosity | Poisson ratio | Modulus of elasticity (GPa) | Density of dam foundation soil (kg/m$^3$) |
|-----------------------------|---------|--------------|----------------------------|-------------------------------------------|
| $k_x=2\times10^{-7}$ | 0.4 | 0.25 | $1\times10^9$ | 1.82$\times10^3$ |
| $k_y=3\times10^{-7}$ | | | | |
| $k_z=1\times10^{-7}$ | | | | |

#### 1) Homogeneous isotropy

Set the permeability coefficient: $k_x=k_y=k_z$, and calculate the pore water pressure through FLAC3D software, as shown in figure 3(a). It can be seen from figure 4(a) that the pore water pressure value tends to be stable after 0.4s calculation, indicating that the seepage field tends to be stable, the maximum pore water pressure of point a is 110.2 kN/m$^2$, and the minimum pore water pressure of point j is 33.7 kN/m$^2$; when the abscissa is the same, the greater the burial depth, the greater the pore water pressure; when the point is at the same depth, the closer the infiltration area, the greater the pore water pressure.
2) Homogeneous anisotropy

Take the permeability coefficient \( k_x = k_1 \), \( k_z = k_3 \), as shown in table 2. The pore water pressure calculated by FLAC\(^{3D} \) software numerical method is shown in figure 4(b), it can be seen that the pore water pressure value of point a-j tends to be stable after 0.5s calculation, the maximum pore water pressure of point a is 110.8 kN/m\(^2\), and the minimum pore water pressure of point j is 31.1 kN/m\(^2\).

3) Homogeneous stratified soil

When the dam foundation is composed of three layers of soil layers, the permeability coefficients \( (k_1, k_2, k_3) \) of each layer should be set respectively, seepage parameters are shown in table 2, and the results of seepage calculation by FLAC\(^{3D} \) are shown in figure 5.

![Figure 5. Calculation result of pore water pressure of layered soil.](image)

From figure 5, it can be seen that the pore water pressure value of point a-j tends to be stable after 0.5s calculation, the maximum pore water pressure of point a is 105.3 kN/m\(^2\), and the minimum pore water pressure of point j is 36.1 kN/m\(^2\).

3.2 Calculation accuracy verification of flow net method

Figure 6 shows the histogram of the comparison of the pore water pressure calculated by flow net method and numerical method for homogeneous, anisotropic and layered soil layers. In the case of homogeneous isotropic soil in figure 6(a), the difference between the flow net method and the numerical method is large at point j, point i and point h, the difference is 9%, 6% and 5% respectively, and the difference values of other points are all within 5%, which shows that the accuracy of the flow net method for calculating the seepage field of homogeneous isotropic soil is high.

Figure 6(b) shows that in homogeneous anisotropic soil, the difference between point i and point j is relatively high, reaching 10% and 22% respectively. The difference between point c and point e is relatively small, 0.7% and 2% respectively. It indicated that the accuracy of flow net method is high.

Figure 6(c) shows that in stratified soil. It can be seen that the histogram of each point of the flow net method and the numerical method is nearly the same height, the difference between the larger difference point c and point a is 4%, the difference between the other points is kept within 3%, and the difference between point g and point i is only within 1%. This shows that the accuracy of flow net method for calculating the seepage field of homogeneous isotropic soil is high.
4. Conclusion
Based on the seepage theory, this paper systematically draws the flow net of three soil layers (homogeneous isotropy, homogeneous anisotropy and layered soil) under the dam foundation, calculates seepage field and discusses the calculation accuracy of flow net. Through the flow net diagram of three soil layers, the pore water pressure value of fixed point in the seepage field is obtained, and then the flow field is solved by FLAC3D software to verify the calculation accuracy of the flow net method. It is concluded that the accuracy of drawing flow net for calculating seepage field is close to that of numerical method, which shows that the flow net method has high accuracy, and it can be used to make up for the limitations of numerical modeling difficulties in engineering practice.

Acknowledgments
This study was partially supported with the major projects of natural science research in Higher Education Institutions (KJ2019ZD11) and the research grants from the National Natural Science Foundation of China (41977253).

Reference
[1] Chu, C.X. (2017) Numerical analysis of seepage treatment of aquifer dam foundation. J. Water Conservancy Planning and Design., 7: 97-99.
[2] Li, D., Bai, Y., Wei, A.H. (2016) Analysis of seepage and seepage stability of dam foundation based on three-dimensional seepage simulation -- a case study of Wanyan reservoir. J. Hydrology, 5: 46-49.
[3] Li, K.H., Chai, J.R. (2003) Comparison of two numerical methods for seepage analysis of dam foundation. J. HongshuiHE, 4: 14-17.
[4] Cai, Y.M., He J., Shao, J.D. (1994) Numerical calculation method for drawing flow net. J. Scientific Research On Water Conservancy and Water Transport, Z1: 119-125.
[5] Nie, Z.F., Cheng, T. (2009) Seepage analysis of dam foundation of Jiangkou hydropower station. J. Journal of Huangshi Institute of technology, 3: 45-48.
[6] Lei, N., Wang, X.J. (2012) Drawing of seepage flow net based on numerical calculation method. J. Journal of water conservancy and construction engineering, 4: 113-117.
[7] Xue, Y.Q., Wu, J.C. (2010) Groundwater dynamics (Third Edition), Geological Publishing House, Beijing.
[8] Chen, Z.Y., Zhou, J.X., Wang, H.J., (2013) Soil Mechanics, Tsinghua University Press, Beijing.
[9] Huang, L.M., Chen, Z.Z., Zhang, C.B., Zhang, J. (2015) Preliminary study on calculation method of seepage stability of layered medium slope. J. Hydrogeological engineering geology, 3: 59-63 + 91
[10] Luo, Q., Liu, W.S., (2018) Slope seepage simulation and stability analysis based on FLAC3D J. Shanxi architecture, 13: 84-86.