Rare $\Lambda_b \to nl^+l^-$ decays in the relativistic quark-diquark picture

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The form factors of the rare $\Lambda_b \to nl^+l^-$ decays are calculated in the framework of the relativistic quark-diquark picture of baryons with the consistent account of the relativistic effects. Their momentum transfer squared dependence is determined explicitly in the whole accessible kinematical range. The decay branching fractions, forward-backward asymmetries and the fractions of longitudinally polarized dileptons are determined. The branching fraction of the rare $\Lambda_b \to n\mu^+\mu^-$ decay are found to be $\text{Br}(\Lambda_b \to n\mu^+\mu^-) = (3.75 \pm 0.38) \times 10^{-8}$ and thus could be measured at the LHC. Prediction for the branching fraction of the rare radiative $\Lambda_b \to n\gamma$ decay is also given.

Recently significant experimental progress has been achieved in studying the rare decays of the $\Lambda_b$ baryon. In 2011 the CDF Collaboration [1] reported the first observation of the rare $\Lambda_b \to \Lambda\mu^+\mu^-$ decay. Then the LHCb Collaboration performed detailed angular analysis [2] of this decay which allowed to extract not only the total branching fraction but also different decay distributions and asymmetries in several momentum transfer squared bins. This year the LHCb Collaboration observed for the first time the suppressed decay $\Lambda^0_b \to p\pi^-\mu^+\mu^-$, where the muons do not originate from charmonium resonances [3]. In addition contributions from the $\Lambda_b \to \Lambda(\to p\pi^-)\mu^+\mu^-$ decays were removed by requiring $m_{\pi^-} > 1.12$ GeV. Such decays are mediated by the $b \to d$ transition and thus are highly suppressed in the standard model. The measured branching fraction of this decay is of order of $10^{-8}$. This observation indicates that other similar rare decays, such as $\Lambda_b \to n\mu^+\mu^-$, can be observed in the near future.

In the recent paper [4] we considered the rare $\Lambda_b \to \Lambda l^+l^-$ decays in the relativistic quark-diquark picture of baryons [5, 6]. The analytic expressions for the decay form factors as the overlap integrals of the initial and final baryon wave functions were obtained. All relativistic effects including transformations of the baryon wave functions from rest to the moving reference frame and contributions of intermediate negative energy states were taken into account. This allowed us to explicitly determine the momentum transfer squared $q^2$ dependence of the decay form factors in the whole kinematical range without additional assumptions and extrapolations, thus increasing reliability of the obtained predictions. On this basis various $\Lambda_b \to \Lambda l^+l^-$ decay observables were calculated and were found to be consistent with detailed measurements of the LHCb Collaboration [2]. In this paper we extend this analysis to the consideration of the suppressed rare $\Lambda_b \to nl^+l^-$ and $\Lambda_b \to n\gamma$ decays.

The matrix elements of the flavour changing neutral current governing the $b \to d$ transition between baryon states is usually parametrized by the following set of the invariant form factors [7]

$$\langle n(p', s')|\bar{d}\gamma^\mu b|\Lambda_b(p, s)\rangle = \bar{u}_n(p', s')[f_1^V(q^2)\gamma^\mu - f_2^V(q^2)i\sigma^{\mu\nu}q_\nu M_{\Lambda_b} + f_3^V(q^2)\frac{q^\mu}{M_{\Lambda_b}}]u_{\Lambda_b}(p, s),$$
The Cabibbo-Kobayashi-Maskawa (CKM) matrix takes the following form

\[
\left(\begin{array}{ccc}
V_{ud} & V_{us} & V_{ub} \\
V_{cd} & V_{cs} & V_{cb} \\
V_{td} & V_{ts} & V_{tb}
\end{array}\right)
\]

of the form factors is given in the Appendix of Ref. [4]. Substituting in the calculated ones does not exceed 0.5%. Our model form factors are plotted in Fig. 1.

Now we can use the obtained form factors for the calculation of the rare \( \Lambda_b \rightarrow n \) transition.

Using the relativistic quark-diquark picture of baryons with the QCD-motivated interquark potential we obtained expressions for these form factors as the overlap integrals of the baryon wave functions. They are given in the Appendix of Ref. [4]. Substituting in these expressions the wave functions obtained while considering the baryon spectroscopy we calculate the form factors in the whole accessible kinematical range.

We found that the numerically calculated form factors can be approximated with high accuracy by the following analytic expression

\[
F(q^2) = \frac{1}{1 - q^2/M_{\text{pole}}^2} \left\{ a_0 + a_1 z(q^2) + a_2 [z(q^2)]^2 \right\},
\]

where the variable

\[
z(q^2) = \frac{\sqrt{t_+ - q^2} - \sqrt{t_+ - t_0}}{\sqrt{t_+ - q^2} + \sqrt{t_+ - t_0}}
\]

\( t_+ = (M_B + M_\pi)^2 \) and \( t_0 = q_{\text{max}}^2 = (M_{\Lambda_b} - M_n)^2 \). The pole masses have the values:

\( M_{\text{pole}} \equiv M_{B^*} = 5.325 \text{ GeV for } f_{1,2}^V, f_{1,2}^{TV} \); \( M_{\text{pole}} \equiv M_{B_1} = 5.723 \text{ GeV for } f_{1,2}^A, f_{1,2}^{TA} \); \( M_{\text{pole}} \equiv M_{B_0} = 5.749 \text{ GeV for } f_{3}^V \); \( M_{\text{pole}} \equiv M_B = 5.280 \text{ GeV for } f_{3}^A \). The fitted values of the parameters \( a_0, a_1, a_2 \) as well as the values of form factors at maximum \( q^2 = 0 \) and zero recoil \( q^2 = q_{\text{max}}^2 \) are given in Table I. The difference of the fitted form factors from the calculated ones does not exceed 0.5%. Our model form factors are plotted in Fig. I.

Now we can use the obtained form factors for the calculation of the rare \( \Lambda_b \rightarrow nl^+l^- \) decay observables.

The effective Hamiltonian for the \( b \rightarrow dl^+l^- \) transitions, taking into account the unitarity of the Cabibbo-Kobayashi-Maskawa (CKM) matrix takes the following form

\[
\mathcal{H}_{\text{eff}} = -\frac{4G_F}{\sqrt{2}} \left[ V_{td}^* V_{tb} \sum_{i=1}^{10} c_i O_i + V_{ud}^* V_{ub} \sum_{i=1}^{2} c_i (\mathcal{O}_i - \mathcal{O}_i^{(u)}) \right],
\]

where \( u_{\Lambda_b}(p, s) \) and \( u_{\Lambda_b}(p', s') \) are the Dirac spinors of the initial \( \Lambda_b \) and final \( n \) baryon.
FIG. 1: Form factors of the rare $\Lambda_b \to n$ transition.

where $G_F$ is the Fermi constant, $V_{tj}$ and $V_{uj}$ are the CKM matrix elements, $c_i$ are the Wilson coefficients and $\mathcal{O}_i(\mathcal{O}_i^{(u)})$ represent the four-quark operator basis. Then the resulting matrix element of the transition amplitude between baryon states can be written as [9]

$$
\mathcal{M} = \frac{G_F \alpha}{\sqrt{2\pi}} |V_{td}^* V_{tb}| \left\{ \langle n | c_9^{\text{eff}} \gamma_{\mu}(1 - \gamma_5) - \frac{2m_b}{q^2} c_7^{\text{eff}} i\sigma_{\mu\nu} q^\nu (1 + \gamma_5) \rangle | \Lambda_b \rangle (\bar{l} \gamma^\mu l) 
+ c_{10} \langle n | \gamma_{\mu}(1 - \gamma_5) | \Lambda_b \rangle (\bar{l} \gamma^\mu \gamma_5 l) \right\},
$$

(5)

where the values of the Wilson coefficients $c_i$ and of the effective Wilson coefficient $c_7^{\text{eff}}$ are taken from Ref. [10]. The effective Wilson coefficient $c_9^{\text{eff}}$ contains additional perturbative and long-distance contributions coming from hadron resonances

$$
c_9^{\text{eff}} = c_9 + h^{\text{eff}} \left( \frac{m_c}{m_b}, \frac{q^2}{m_b^2} \right) c_0 + \lambda_u \left[ h^{\text{eff}} \left( \frac{m_c}{m_b}, \frac{q^2}{m_b^2} \right) - h^{\text{eff}} \left( \frac{m_u}{m_b}, \frac{q^2}{m_b^2} \right) \right] (3c_1 + c_2) 
- \frac{1}{2} h \left( \frac{q^2}{m_b^2} \right) (4c_3 + 4c_4 + 3c_5 + c_6) 
+ \frac{1}{2} h \left( 0, \frac{q^2}{m_b^2} \right) (c_3 + 3c_4) 
+ \frac{2}{9} (3c_3 + c_4 + 3c_5 + c_6).
$$

(6)
Here \( \lambda_u = \frac{V^*_{ub} V_{ub}}{|V_{td}|^2 V_{tb}} \), the coefficient \( c_0 = 3c_1 + c_2 + 3c_3 + c_4 + 3c_5 + c_6 \) and

\[
\begin{align*}
  h \left( \frac{m_c}{m_b}, \frac{q^2}{m_b^2} \right) & = -\frac{8}{9} \ln \frac{m_c}{m_b} + \frac{8}{27} + \frac{4}{9} x - 2 \left( 2 + x \right) |1 - x|^{1/2} \\
  & \quad + \left\{ \ln \left| \frac{\sqrt{1 - x} + 1}{\sqrt{1 - x} - 1} \right| - i \pi, \quad x = \frac{4m_c^2}{q^2} < 1, \\
  & \quad 2 \arctan \frac{1}{\sqrt{x - 1}}, \quad x = \frac{4m_c^2}{q^2} > 1,
\end{align*}
\]

\[
\begin{align*}
  h \left( 0, \frac{q^2}{m_b^2} \right) & = \frac{8}{27} - \frac{4}{9} \ln \frac{q^2}{m_b^2} + 4 \frac{4}{9} i \pi.
\end{align*}
\]

The function

\[
\begin{align*}
  h^{\text{eff}} \left( \frac{m_c}{m_b}, \frac{q^2}{m_b^2} \right) = h \left( \frac{m_c}{m_b}, \frac{q^2}{m_b^2} \right) + \frac{3\pi}{4} c_0 \sum \Gamma(V_i \to l^+l^-) M_{V_i} \frac{M_{V_i}^2 - q^2 - i M_{V_i} \Gamma_{V_i}}{M_{V_i}^2 - q^2 - i M_{V_i} \Gamma_{V_i}}
\end{align*}
\]

contains additional long-distance contributions originating from the \( c\bar{c} \) resonances \([J/\psi, \psi(2S), \ldots]\). In our study we include contributions of the vector \( V_i(1^-) \) charmonium states: \( J/\psi, \psi(2S), \psi(3770), \psi(4040), \psi(4160) \) and \( \psi(4415) \), with their masses \( (M_{V_i}) \), leptonic \( \Gamma(V_i \to l^+l^-) \) and total \( \Gamma(V_i) \) decay widths taken from PDG \[11\].

Similar expression holds for the function \( h^{\text{eff}} \left( \frac{m_b}{m_b}, \frac{q^2}{m_b^2} \right) \), where the long-distance contributions now come from \( \rho \) and \( \omega \) states.

The differential decay distribution is given by

\[
\begin{align*}
  \frac{d^2 \Gamma(\Lambda_b \to nl^+l^-)}{dq^2 d\cos \theta} = \frac{d\Gamma(\Lambda_b \to nl^+l^-)}{dq^2} \left\{ \frac{3}{8} (1 + \cos^2 \theta)[1 - F_L(q^2)] + A_{FB}(q^2) \cos \theta + \frac{3}{4} F_L(q^2) \sin^2 \theta \right\},
\end{align*}
\]

where \( \theta \) is the angle between the \( \Lambda_b \) baryon and the positively charged lepton in the dilepton rest frame,

\[
A_{FB}(q^2) = \frac{\int_{q^2}^{\Lambda_b} \frac{d\Gamma}{dq^2} \cos \theta \ d\cos \theta - \int_{q^2}^{0} \frac{d\Gamma}{dq^2} \ d\cos \theta}{\int_{0}^{\Lambda_b} \frac{d\Gamma}{dq^2} \ d\cos \theta},
\]

is the forward-backward asymmetry and \( F_L(q^2) \) is the fraction of longitudinally polarized dileptons. The explicit expressions for the differential branching fractions and asymmetries in terms of the form factors are given in Ref. \[4\]. Note that the rare decays \( \Lambda_b \to n\ell^+\ell^- \) are additionally suppressed by the ratio of the CKM matrix elements \((|V_{td}|/|V_{ts}|)^2\) with respect to the \( \Lambda_b \to \Lambda\ell^+\ell^- \) decays. Substituting in these expressions the calculated form factors we get predictions for the differential decay branching fractions, forward-backward asymmetries \( A_{FB}(q^2) \) and the fractions of longitudinally polarized dileptons \( F_L(q^2) \). They are plotted in Figs. \[2\,4\] for the \( \Lambda_b \to n\mu^+\mu^- \) and \( \Lambda_b \to n\tau^+\tau^- \) rare decays. By solid and dashed lines we plot theoretical results obtained without and with inclusion of the long-distance contributions to the Wilson coefficient \( c^{\text{eff}}_9 \). The values of the total branching fractions and averaged forward-backward asymmetries \( \langle A_{FB}(q^2) \rangle \) and fractions of longitudinally polarized dileptons \( \langle F_L(q^2) \rangle \) are given in Table \[11\]. The branching fractions were calculated without inclusion of the hadron resonance contributions, while \( \langle A_{FB}(q^2) \rangle \) and \( \langle F_L(q^2) \rangle \) are presented both without (nonres.) and with (res.) their inclusion. We estimate the theoretical errors of our predictions, which emerge from the uncertainties in the calculation of the decay form factors, to be about 10%. 
FIG. 2: The differential branching fractions for the $\Lambda_b \to n\mu^+\mu^-$ (left) and $\Lambda_b \to n\tau^+\tau^-$ (right) rare decays.

Using calculated values of the rare decay form factors we can also predict the exclusive rare radiative $\Lambda_b \to n\gamma$ decay rate. It is expressed in terms of the decay form factors by

$$\Gamma(\Lambda_b \to n\gamma) = \frac{\alpha}{64\pi^4}G_F^2m_b^2M_{\Lambda_b}^2|V_{td}|^2|c_{\text{eff}}(m_b)|^2(|f_{2T}(0)|^2 + |f_{2A}(0)|^2) \left(1 - \frac{M_{n}^2}{M_{\Lambda_b}^2}\right)^3.$$  \hspace{1cm} (9)

Substituting their calculated values we get the prediction for the branching fraction

$$\text{Br}(\Lambda_b \to n\gamma) = 3.7 \times 10^{-7}.$$  \hspace{1cm} (10)

TABLE II: Predictions for the branching fractions, averaged rare decay forward-backward asymmetries and polarization fractions.

| Decay            | Br ($\times 10^{-8}$) | $\langle A_{FB} \rangle$ | nonres. | $\langle F_L \rangle$ | res. |
|------------------|-----------------------|--------------------------|--------|-----------------------|------|
| $\Lambda_b \to ne^+e^-$ | 3.81                  | -0.294                  | -0.301 | 0.499                 | 0.543 |
| $\Lambda_b \to n\mu^+\mu^-$ | 3.75                  | -0.298                  | -0.299 | 0.504                 | 0.555 |
| $\Lambda_b \to n\tau^+\tau^-$ | 1.21                  | -0.173                  | -0.148 | 0.344                 | 0.339 |
In this paper we calculated the form factors of the rare $b \to d$ transitions between baryon states in the relativistic quark-diquark picture of baryons. These form factors were explicitly determined in the whole accessible kinematical range without extrapolations with the account of the relativistic effects including wave function transformations from rest to the moving reference frame and intermediate contributions of the negative energy states. On this basis predictions for the branching fractions of the rare semileptonic $\Lambda_b \to n l^+ l^−$ and rare radiative $\Lambda_b \to n\gamma$ decays were obtained. The rare semileptonic branching fractions were found to be of order of $10^{-8}$, while the rare radiative decay is predicted to have branching fraction of order of $10^{-7}$. Thus the CKM suppressed rare baryon decays governed by the $b \to d$ transition could be accessible for observation at LHC by the LHCb Collaboration. The predictions for the averaged values of the forward-backward asymmetry $⟨A_{FB}(q^2)⟩$ and the fractions of longitudinally polarized dileptons $⟨F_L(q^2)⟩$ are also given.

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