We present an improved method for determining the mass of neutron stars in eclipsing X-ray pulsar binaries and apply the method to six systems, namely, Vela X-1, 4U 1538-52, SMC X-1, LMC X-4, Cen X-3, and Her X-1. In previous studies to determine neutron star mass, the X-ray eclipse duration has been approximated analytically by assuming that the companion star is spherical with an effective Roche lobe radius. We use a numerical code based on Roche geometry with various optimizers to analyze the published data for these systems, which we supplement with new spectroscopic and photometric data for 4U 1538-52. This allows us to model the eclipse duration more accurately and thus calculate an improved value for the neutron star mass. The derived neutron star mass also depends on the assumed Roche lobe filling factor $\beta$ of the companion star, where $\beta = 1$ indicates a completely filled Roche lobe. In previous work a range of $\beta$ between 0.9 and 1.0 was usually adopted. We use optical ellipsoidal light-curve data to constrain $\beta$. We find neutron star masses of $1.77 \pm 0.08 M_\odot$ for Vela X-1, $0.87 \pm 0.07 M_\odot$ for 4U 1538-52 (eccentric orbit), $1.00 \pm 0.10 M_\odot$ for 4U 1538-52 (circular orbit), $1.04 \pm 0.09 M_\odot$ for SMC X-1, $1.29 \pm 0.05 M_\odot$ for LMC X-4, $1.49 \pm 0.08 M_\odot$ for Cen X-3, and $1.07 \pm 0.36 M_\odot$ for Her X-1. We discuss the limits of the approximations that were used to derive the earlier mass determinations, and we comment on the implications our new masses have for observationally refining the upper and lower bounds of the neutron star mass distribution.

Key words: methods: numerical – pulsars: individual (Vela X-1, 4U 1538-42, SMC X-1, LMC X-4, Cen X-3, Her X-1) – stars: neutron – X-rays: binaries

1. INTRODUCTION

A neutron star is a compact object that is the remnant of a massive star. The structure of a neutron star depends on the equation of state of nuclear matter under extreme conditions, specifically the relation between pressure and density in the neutron star interior. For a given equation of state, a mass–radius relation for the neutron star and a corresponding maximum mass can be derived. Many such theoretical equations of state exist, ranging from “soft”—a mass upper limit as low as $1.5 M_\odot$ (Brown & Bethe 1994)—to “stiff”—a higher upper mass limit near $3 M_\odot$ (Kalogera & Baym 1996). The accurate measurement of neutron star masses is therefore important for our understanding of the equation of state of matter in such high-density situations (e.g., see the recent review by Kiziltan et al. 2010).

Eclipsing X-ray binary systems where the X-ray source is a pulsar can be ideal systems for a dynamical determination of the neutron star’s mass. The orbital period, the semiamplitude of the optical star’s radial velocity curve, the duration of the X-ray eclipse, and the projected semimajor axis of the pulsar’s orbit (measured from the pulse arrival times) can be used to find the masses of both stars. We consider six systems where the required measurements have been made: Vela X-1, 4U 1538-52, SMC X-1, LMC X-4, Cen X-3, and Her X-1. The first five systems listed have OB supergiant companion stars, and the lattermost system has a somewhat less massive companion ($\sim 2 M_\odot$).

2. ANALYTIC METHOD

2.1. Basic Equations

In this section, we review the analytic method introduced by Rappaport & Joss (1983) and Joss & Rappaport (1984) that is widely used to measure the mass of the neutron star and its optical companion (e.g., see, van Kerkwijk et al. 1995b; van der Meer et al. 2007). To begin, one can write the masses of the optical companion and the X-ray source ($M_{\text{opt}}$ and $M_X$, respectively) in terms of the mass functions:

$$M_{\text{opt}} = \frac{K_X^3 P (1 - e_i^2)^{3/2}}{2\pi G \sin^3 i} (1 + q)^2$$ (1)

and

$$M_X = \frac{K_X^3 P (1 - e_i^2)^{3/2}}{2\pi G \sin^3 i} \left(1 + \frac{1}{q}\right)^2$$ (2)

where $K_X$ and $K_{\text{opt}}$ are the semiamplitudes of the respective radial velocity curves, $P$ is the period of the orbit, $i$ is the inclination of the orbital plane to the line of sight, $e$ is the...
The eclipse half-angle $\theta_e$, or more specifically the semi-eclipse angle of the neutron star, represents half of the eclipse duration.

The values for $K_X$ and $P$ can be obtained very accurately from X-ray pulse timing measurements (the projected semimajor axis of the pulsar’s orbit in light-seconds, $a_X \sin i$, is usually quoted in publications, from which one finds $K_X = 2\pi c a_X \sin i / P$) and optical and/or UV spectra can provide a value for $K_{opt}$.

Assuming a spherical companion star, the inclination of the system is related to the eclipse half-angle $\theta_e$, the stellar radius $R$, and the orbital separation $a$ by

$$\sin i = \frac{\sqrt{1 - (R/a)^2}}{\cos \theta_e}. \quad (4)$$

Following the approach in Rappaport & Joss (1983), the radius of the companion star is some fraction of the effective Roche lobe radius

$$R = \beta R_L, \quad (5)$$

where $R_L$ is the sphere-equivalent Roche lobe radius. We will refer to the fraction $\beta$ as the “Roche lobe filling factor.” Combining Equations (4) and (5) yields

$$\sin i = \frac{\sqrt{1 - \beta^2 (R_L/a)^2}}{\cos \theta_e}. \quad (6)$$

Rappaport & Joss (1983) provide an approximate expression for $R_L/a$, the ratio of the effective Roche lobe radius and the orbital separation:

$$\frac{R_L}{a} \approx A + B \log q + C \log^2 q, \quad (7)$$

where the constants $A$, $B$, and $C$ are

$$A = 0.398 - 0.026\Omega^2 + 0.004\Omega^3 \quad (8)$$

$$B = -0.264 + 0.052\Omega^2 - 0.015\Omega^3 \quad (9)$$

Here, $\Omega$ is the ratio of the rotational frequency of the optical companion to the orbital frequency of the system. In other words, it is a measure of the degree of synchronous rotation, where $\Omega = 1$ is defined to be synchronous. These four expressions give the value of $R_L$ to an accuracy of about 2% over the ranges of $0 \leq \Omega \leq 2$ and $0.02 \leq q \leq 1$ (Joss & Rappaport 1984).

For a given system, one can calculate the neutron star mass using Equations (1) through (10) when given values of $P$, $a_X \sin i$, $\theta_e$, $K_{opt}$, $\Omega$, and $\beta$ (and if the orbit is eccentric, $e$ and $i_0$).

It is possible to estimate $\Omega$ by measuring the projected rotational velocity, $v_{rot} \sin i$, of the optical companion star. This process is described in van der Meer et al. (2007) and is employed there for the systems SMC X-1, LMC X-4, and Cen X-3. Finally, one must assume some value for the Roche lobe filling factor, $\beta$. Many of the wind-fed systems discussed here are thought to be close to filling their Roche lobes, and a range of $0.9$ and $1.0$ is usually adopted (Rappaport & Joss 1983; van der Meer et al. 2007).

To determine neutron star masses in this manner for the six eclipsing systems where all of the necessary quantities are known or estimated (see Tables 1 and 2 and references therein), we assume that any value within the range $0.9 \leq \beta \leq 1.0$ is equally likely. We then use a Monte Carlo technique to derive the most likely values of $M_X$, $M_{opt}$, and $i$, and the corresponding 1σ confidence limits.

Here, the orbital phase of the X-ray eclipse is determined by the argument of periastron $\omega$.
Figure 1. Resulting probability distributions (histograms) from Monte Carlo simulations using the analytic method for all six systems as a function of neutron star mass. We assume that any filling factor $0.9 \leq \beta \leq 1.0$ is equally likely. The mean value ±1σ is given above each peak. All input parameters for these distributions are given in Tables 1 and 2. We note that our mass for Cen X-3 agrees very well with that found by van der Meer et al. (2007), and the masses for SMC X-1 and LMC X-4 also agree exactly when the values of $\theta_e$ given in van der Meer et al. (2007) are used. We assume $e = 0$ for 4U 1538-52.

### Table 2

| System   | $v_{rot} \sin i$ (km s$^{-1}$) | $\omega$ (deg)$^a$ | Ref. |
|----------|-------------------------------|--------------------|------|
| Vela X-1 | 116 ± 6                       | 332.59 ± 0.92      | 1,6  |
| 4U 1538-52 | 180 ± 20                   | 198 ± 14$^b$       | 2,3,4|
| SMC X-1  | 170 ± 30                     | ...                | 5    |
| LMC X-4  | 240 ± 25                     | ...                | 5    |
| Cen X-3  | 200 ± 40                     | ...                | 5    |
| Her X-1  | ...                          | ...                |      |

Notes.

$^a$ $\omega$ is the argument of periastron for the companion star which is defined only for the systems with nonzero eccentricity (see Table 1).

$^b$ This value assumes a constantly changing $\omega$ over time, as discussed in Section 4.2.

References. (1) Bildsten et al. 1997; (2) Clark 2000; (3) Mukherjee et al. 2006; (4) Reynolds et al. 1992; (5) van der Meer et al. 2007; (6) van Kerkwijk et al. 1995b.

Adjusting the argument of periastron $\omega$ and/or the eclipse duration $\theta_e$ can force a solution. However, for 4U 1538-52, no solution exists within the 1σ uncertainties of $\omega$ and $\theta_e$. This discrepancy arises due to a high inclination and is discussed further in Sections 4.1 and 4.2. As a workaround, when employing this analytic technique we use a larger eclipse width ($\theta_e = 33^\circ ± 3^\circ$; from van Kerkwijk et al. 1995a) for Vela X-1 and adopt a circular orbit ($e = 0$) as in Clark (2000) for 4U 1538-52.

2.2. An Examination of the Approximations

The analytic method presented in Section 2.1 is straightforward and is easy to implement on a computer. However, this method relies on the following two approximations:

1. The computation of the effective Roche lobe radius, $R_L/a$, from Equations (7)–(10).
2. The computation of the X-ray eclipse duration, $2\theta_e$, from Equation (6).

We use the Eclipsing Light Curve (ELC) code of Orosz & Hauschildt (2000), which is based on Roche geometry, to test these two approximations, and we discuss each one in turn.

Strictly speaking, “Roche geometry” applies only to binary systems with circular orbits and co-rotating stars. Numerous authors have presented generalizations of the Roche potential to account for situations in which one or both of these assumptions are not met (e.g., Avni & Bahcall 1975; Avni 1976; Wilson 1979; Avni & Schiller 1982). These generalizations do not fully describe complete dynamics of the star, and as a result some small approximations are involved (e.g., Wilson 1979). The
main assumption is that the timescale for the internal motions of the star that are required for the star to adjust to the varying potential is considerably shorter than the orbital period. Given this, one can compute an effective potential locally at each orbital phase without significant inconsistency (Wilson 1979). Although these generalized potentials are widely used, it is not known how well they work in practice. For most of the systems discussed here, the orbits are circular and the stars rotate close to the synchronous rate, so the modified potential we use (the ELC code uses the potential given in Wilson 1979) should be fairly accurate. Finally, we note that Rappaport & Joss (1983) also adopted a modified “Roche potential” in their analysis—their fitting functions are approximations to numerical integrations of the critical potential surface. Therefore, any systematic error that ELC would have owing to improper generalizations of the Roche model would also be present in the work of Rappaport & Joss (1983) and others that used these fitting functions such as van der Meer et al. (2007).

The shape and size of the critical potential surface (hereafter the “Roche lobe”) depend only on the mass ratio \( q \) and the parameter \( \Omega \). When given \( q \) and \( \Omega \), it is straightforward to define the equipotential surface (from the value of the gravitational potential at the inner Lagrangian point) and to numerically integrate its volume. The sphere-equivalent radius \( R_L \) then follows. We define a large grid of points in the \( q-\Omega \) plane and compute values of \( R_L \); and compare them with the values of \( R_L \) found from Equations (7) through (10). The results are shown in Figure 2. Rappaport & Joss (1983) claim an accuracy of their fitting functions of about 2% over the stated range \( 0 \leq \Omega \leq 2 \) and \( 0.02 \leq q \leq 1 \), and our results confirm this.

From Equation (6), one can see that the duration of the X-ray eclipse depends on the inclination \( i \), the Roche lobe filling factor \( \beta \), and \( R_L/a \), which is a function of the mass ratio \( q \) and the parameter \( \Omega \). The ELC code can be used to compute the duration of the X-ray eclipse for a given geometry. Rather than using ray tracing to determine whether a point is eclipsed by the companion star (e.g., see, Chanan et al. 1976), ELC locates the limb of the star to high accuracy by testing the viewing angles of each surface element. ELC then uses bisection to find, at each latitude row, the longitude of the point that has a viewing angle of \( \mu = 0 \). Once found, the points on the limb define a polygon in sky coordinates. At that same phase, the location of the X-ray source (assumed to be a point source) in sky coordinates is determined. A simple test is used to determine if the sky coordinate of the X-ray source is inside or outside the polygon defined by the horizon of the star. ELC uses another bisection routine to find the orbital phase of the X-ray eclipse ingress to high accuracy. If the orbit is circular, the eclipse half-angle \( \theta_e \) is equal to the ingress phase. If the orbit is eccentric, the X-ray eclipse egress phase is also computed, and the eclipse half-angle is computed from both the ingress and egress phases.

Setting \( \Omega = 1 \), we use ELC to compute the full duration of the X-ray eclipse for a wide range of values in the \( q-i \) plane, using \( \beta = 1.0 \) and \( \beta = 0.9 \) and assuming a circular orbit. The full eclipse duration was also computed from Equation (6), and the difference between the numerically computed value of \( 2\theta_e \) and the analytically computed value of \( 2\theta_e \) was determined. Figure 3 shows the differences for \( \beta = 1 \) and Figure 4 shows the differences for \( \beta = 0.9 \). The differences can be quite extreme; they are in excess of 10° for small mass ratios and large inclinations and are less than \(-10°\) for grazing eclipses.

The approximate locations in the \( q-i \) plane for three systems (SMC X-1, LMC X-4, and Cen X-3) are also shown in Figures 3 and 4. Since Her X-1 is a low-mass X-ray binary, its mass ratio does not appear within the limits of the two figures. Vela X-1 and 4U 1538-52 are excluded because of their eccentric orbits. The instantaneous Roche lobe filling factor of an eccentric system during X-ray eclipse will be less than the value of \( \beta \) as calculated for non-eccentric systems. When \( \beta = 1 \), all three systems shown in Figures 3 and 4 are near the contour denoting zero difference. When \( \beta = 0.9 \), all three systems are above the zero-difference contour, which indicates that the eclipse durations computed numerically are longer than those computed analytically.

To illustrate the differences between the analytic and ELC results, Figures 5 and 6 show a system resembling Cen X-3 in sky coordinates, where the Roche lobe filling factor is \( \beta = 0.9 \). Figure 5 demonstrates that the companion star is not spherical. The parts of the star near its equator extend beyond...
the circle denoting the volume-equivalent sphere. Hence, one would expect that the duration of an X-ray eclipse for very high inclinations would be longer than what one would compute from the analytic approximations, and a glance at Figures 3 and 4 confirms this. In a similar manner, Figures 3 and 4 show that for grazing eclipses (i.e., lower inclinations), the numerically computed durations are shorter than the analytic approximations. One can see from Figure 5 that the polar regions of the companion star are inside the circle denoting the volume-equivalent sphere, and as a result there would be inclinations at which the analytic approximations indicate X-ray eclipses when in fact none occur.

Figure 6 shows a magnified view of Figure 5 near the neutron star eclipse. In this example, the egress phase of the X-ray eclipse is very close to 33°, since the neutron star is just crossing the limb of the companion star. The limb of the analytically computed volume-equivalent sphere is well inside the limb of the star. The egress phase of X-ray eclipse would be near 32° when the analytic expressions are used, since the neutron star is just crossing the limb of the sphere at that phase. Thus, in this example, the full duration of the X-ray eclipse computed numerically is a full 2° longer than the duration computed analytically.

Having tested both approximations, we conclude that the ELC code does offer a significantly more accurate representation of the physical system than the analytic method presented in Section 2.1. The size of the effective Roche lobe radius as a function of $q$ and $\Omega$ is relatively well represented by the analytic formulae and ELC offers only a modest improvement. However, the difference in X-ray eclipse duration can be quite extreme (as much as $\pm10^\circ$) and has a direct effect on the calculation of the neutron star mass.

3. NUMERICAL METHOD

We use the ELC code and its various optimizers to analyze the data given in Table 1 and optical light curves (see Section 4) to derive the neutron star masses and their uncertainties. The ELC code has two advantages here. First, Roche geometry is used (i.e., no approximations are used to find the effective Roche lobe radius or the X-ray eclipse duration, as discussed in Section 2.2). Second, when using ELC, one can make use of any number of other sources of information about the system, such
as optical light curves. When the geometry is specified, one can compute various observable properties of the system and compare them with the observed values using a $\chi^2$ or similar test. One can then find the family of geometries that best match the observed quantities, and from those geometries the masses and other system parameters follow.

To begin, the orbital period $P$ (which is known to high accuracy) and the orbital separation $a$ give the total mass of the binary via Kepler’s Third Law. Specifying the mass ratio $q$ then gives the component masses. The shape of the companion star is determined when the Roche lobe filling factor $\beta$ and the parameter $\Omega$ are given. The shape of the orbit, if eccentric, is determined from the eccentricity $e$ and the argument of periastron $\omega$. Finally, when the inclination $i$ is given, it is possible to find the $K$-velocities of the components, the rotational velocity of the companion star, and the duration of the X-ray eclipse. Thus, we initially have an eight-dimensional parameter space of the companion star, and the duration of the X-ray eclipse. Once the fitness of a given model is determined, new parameter sets are constructed using either a Monte Carlo Markov Chain optimizer or a genetic algorithm optimizer. In the latter case, a “breeding” technique is used that is based on the “survival of the fittest” (Orosz et al. 2002). The probability of “breeding” is based on a model’s fitness. Random variations (i.e., “mutations”) are introduced into a small fraction of the breeding events, and the process of breeding a new population and evaluating its members is repeated over many generations.

See Charbonneau (1995) for a more detailed discussion of genetic algorithms. In both the Monte Carlo Markov Chain and the genetic algorithm, the fitness of each new parameter set is determined and the process is repeated until convergence is achieved.

Finally, the notation (mod) means the quantity computed from the model, the notation (obs) means the observed quantities, and from those geometries the masses and other system parameters follow. The parameter values in Tables 1 and 2. The shape of the orbit, if eccentric, is determined from the eccentricity $e$ and the argument of periastron $\omega$. Finally, when the inclination $i$ is given, it is possible to find the $K$-velocities of the components, the rotational velocity of the companion star, and the duration of the X-ray eclipse. Thus, we initially have an eight-dimensional parameter space of the companion star, and the duration of the X-ray eclipse. Once the fitness of a given model is determined, new parameter sets are constructed using either a Monte Carlo Markov Chain optimizer or a genetic algorithm optimizer. In the latter case, a “breeding” technique is used that is based on the “survival of the fittest” (Orosz et al. 2002). The probability of “breeding” is based on a model’s fitness. Random variations (i.e., “mutations”) are introduced into a small fraction of the breeding events, and the process of breeding a new population and evaluating its members is repeated over many generations.

See Charbonneau (1995) for a more detailed discussion of genetic algorithms. In both the Monte Carlo Markov Chain and the genetic algorithm, the fitness of each new parameter set is determined and the process is repeated until convergence is achieved.

Ultimately, we compute hundreds of thousands of models. Each model has an associated value of $\chi^2$ and various derived parameters including the component masses, system inclination, etc. A lower $\chi^2$ value indicates higher fitness. In this particular case, the minimum possible value of $\chi^2$ is zero since there are more linearly independent input parameters than observed quantities. To define the $1\sigma$ limits, the family of models where $\chi^2 \leq 1$ is found and the distributions of the various parameters of interest are constructed.

We note that the value of $\alpha_X \sin i$ given for 4U 1538-52 has a relatively large uncertainty. To account for this, we modified the genetic code to allow for a range of $\alpha_X \sin i$ values drawn from the appropriate Gaussian distribution to be used. Although this modification hardly made a difference in the output parameter distributions, our results do account for the uncertainty in $\alpha_X \sin i$.

We performed a preliminary analysis of the six systems for a range of $0.75 \leq \beta \leq 1$ and those results are shown in Figure 7. The system inclination is inversely correlated with $\beta$, and the lower bound on $\beta$ for each system corresponds roughly to $i = 90^\circ$. Hence, not all systems can have masses for all of the values of $\beta$ considered. Two things are apparent in Figure 7: first, the neutron star mass is highly dependent on the choice of $\beta$ and second, the numerical and analytic results can differ in opposite senses to varying degrees depending on the choice of $\beta$.

4. OPTICAL LIGHT CURVES

To improve upon the technique described in Section 3, we consider optical light curves for five systems in addition to the parameter values in Tables 1 and 2. The shape of the ellipsoidal light variations from the companion star depends on the inclination $i$, the mass ratio $q$, the parameter $\Omega$, and the companion star’s Roche lobe filling factor $\beta$. Since the first three parameters are already well determined from the width of the X-ray eclipse and the $K$-velocities of each component, $\beta$ should be quite well constrained by including optical light curves in the analysis. Such an analysis, which we now present, is trivial to do using ELC and extremely difficult to do using the analytic approximations.

Adding new observables to the ELC analysis adds terms to the $\chi^2$ merit function as originally shown in Equation (14). We
Figure 7. Preliminary neutron star mass vs. the companion Roche lobe filling factor $\beta$ for all six systems. Analytic indicates masses derived using approximations (see Section 2) while Numerical indicates masses derived using the ELC code based on Roche geometry (see Section 3). Optical light curves have not been included in this analysis. Each numerical solution is equally likely in that the eclipse width in the model matches the observed eclipse width exactly. All residuals are in the sense Numerical–Analytic. For simplicity, 4U 1538-52 is approximated as a circular system ($e = 0$).

\[ \chi^2_{\text{new}} = \chi^2 + \sum_{i=1}^{N} \left( \frac{y_{i,\text{mod}} - y_{i,\text{obs}}}{\sigma_i} \right)^2, \tag{15} \]

where the final term incorporates a set of $N$ observations defined by observable quantities $y_i$ (e.g., an optical light curve with $N$ data points). Similar terms may be added for additional sets of observations (e.g., a radial velocity curve).

All of the folded optical light curves discussed below are presented in Figures 8 and 9 with the best-fit model for each system from ELC. For the five systems we analyze in this manner, the best-fit solution includes an accretion disk around the neutron star. Adding a disk increases the depth of the secondary eclipse and results in a better fit in all cases. For comparison, we have plotted both the best-fit model and the same model with the light from the accretion disk subtracted in Figures 8 and 9. Our final neutron star masses are presented in Table 4 and Figure 10 with the corresponding analytic masses.

4.2. 4U 1538-52

4U 1538-52 has a 3.73 day orbital period and most likely an eccentric orbit. Clark (2000) and Mukherjee et al. (2006) give $e = 0.174 \pm 0.015$ and $0.18 \pm 0.01$, respectively, which are in agreement. However, Makishima et al. (1987) give a much lower $e = 0.08 \pm 0.05$ and van Kerkwijk et al. (1995a) adopt $e = 0$ in their analysis; even Clark (2000) provides an alternate set of fit parameters for a circular orbit. As with Vela X-1, we find no physical analytic solution for 4U 1538-52 with an eccentric orbit and its reported eclipse duration ($\theta_e = 33^\circ \pm 3^\circ$) due to the system’s high inclination. Therefore, we follow the approach of Clark (2000) and present numerical results for both an eccentric orbit and a circular orbit.

4.1. Vela X-1

Vela X-1 has an 8.96 day orbital period and an eccentric orbit with $e = 0.0898 \pm 0.0012$ (Barziv et al. 2001). The argument of periastron for the companion star is $\omega = 332\,^\circ \pm 0.92$ (Bildsten et al. 1997), where $\omega = \omega_X + 180$. The optical V light curve shown in the upper left panel of Figure 8 for Vela X-1 is binned data from the All Sky Automated Survey (Pojmansky 2002). In our preliminary analysis of Vela X-1 (e.g., Figure 7), we adopt a semiduration of the X-ray eclipse $\theta_e = 33^\circ \pm 3^\circ$ (van Kerkwijk et al. 1995a). We later use the more precise value from Kreykenbohm et al. (2008), $\theta_e = 34^\circ 135 \pm 0.5$. Unfortunately, the analytic technique from Section 2 does not arrive at a physical solution with this longer eclipse duration due to the system’s high inclination. Specifically, solving for the inclination $i$ is not possible using Equation (6) as $\sin i > 1$ for all the Monte Carlo simulations. The numerical ELC code does not have this same limitation. As a workaround, we keep $\theta_e = 33^\circ \pm 3^\circ$ for all instances of the analytic case instead. Our final derived mass for Vela X-1 is $1.77 \pm 0.08 M_\odot$ with a system inclination of $77^\circ 8 \pm 1^\circ 2$. 
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Figure 8. Optical light curves for four systems. The solid lines represent the best-fit ELC model for each system, all of which include an accretion disk around the neutron star. The dotted lines depict the models with the light from the accretion disk subtracted for comparison. The light curves for Vela X-1, SMC X-1, and Cen X-3 are all V-band data (Pojmansky 2002; Priedhorsky & Holt 1987; van Paradijs et al. 1983), while the curve for LMC X-4 is B-band data (Ilovaisky et al. 1984). All data are phased relative to the time of X-ray eclipse.

Table 4

| System  | Analytic | Numerical |
|---------|----------|-----------|
|         | $M_\star(M_\odot)$ | $i$ (deg) | $M_\star(M_\odot)$ | $i$ (deg) | $M_{\text{eff}}(M_\odot)$ | $R_{\text{eff}}(R_\odot)^b$ | $f^c$ | $\beta$ |
| Vela X-1 | 1.788 ± 0.157 | 83.6 ± 3.1 | 1.770 ± 0.083 | 78.8 ± 1.2 | 24.00 ± 0.37 | 31.82 ± 0.28 | 0.99 ± 0.01 | 1 |
| 4U 1538-52 (ecc) | ... | ... | 0.874 ± 0.073 | 68.0 ± 1.4 | 20.72 ± 2.27 | 15.72 ± 0.52 | 0.88 ± 0.02 | 0.95 |
| 4U 1538-52 (circ) | 1.104 ± 0.177 | 72.6 ± 4.2 | 0.996 ± 0.101 | 76.8 ± 6.7 | 14.13 ± 2.78 | 12.53 ± 2.11 | 0.76 ± 0.02 | 0.88 |
| SMC X-1 | 1.064 ± 0.105 | 67.8 ± 4.2 | 1.037 ± 0.085 | 68.5 ± 5.2 | 15.35 ± 1.53 | 15.70 ± 1.36 | 0.86 ± 0.07 | 0.95 |
| LMC X-4 | 1.249 ± 0.094 | 68.8 ± 3.3 | 1.285 ± 0.051 | 67.0 ± 1.9 | 14.96 ± 0.58 | 7.76 ± 0.32 | 0.86 ± 0.03 | 0.95 |
| Cen X-3 | 1.473 ± 0.143 | 67.5 ± 3.2 | 1.486 ± 0.082 | 66.7 ± 2.4 | 22.06 ± 1.37 | 12.56 ± 0.56 | 0.96 ± 0.05 | 1 |
| Her X-1 | 1.036 ± 0.311 | 80.5 ± 3.8 | 1.073 ± 0.358 | >85.9 | 2.03 ± 0.37 | 3.76 ± 0.34 | ... | 1 |

Notes.

a For consistency, these analytic values are derived using the $\beta$ values returned by the numerical model rather than a distribution of 0.9 $\leq \beta \leq$ 1 as in Table 3 and Figure 1.

b The sphere-equivalent radius of the companion star (e.g., see Figure 5).

c The ELC fill factor, defined as the distance from the companion star’s center of mass to the point of the star closest to L1. A value of $f$ maps directly to a value for $\beta$, which is the Roche lobe filling factor expressed in terms of the sphere-equivalent volume radius. For the eccentric systems, both $f$ and $\beta$ are defined at periastron.

In addition, Clark (2000) and Mukherjee et al. (2006) give significantly different values for $\omega$, the argument of periastron: 244° ± 9° and 220° ± 12° for the optical source, respectively. Following the approach of Zhang et al. (2005), we have extrapolated a linear decrease of $\omega$ based on the time difference between the observations made by Clark (2000) and Mukherjee et al. (2006). We estimate $\omega$ is decreasing by 0.0010 ± 0.0006 per day and that our subsequent observations should therefore have $\omega = 198° ± 14°$. This is the value we adopt in the numerical model. Even though using a lower $\omega$ nudges the parameter space closer to a single physical analytic solution, the reported eclipse duration is still too short to allow $\sin i < 1$ (as in Equation (6)), and we are forced to retain a model where $e = 0$ for the analytic case.

Due to the above discrepancies, the reportedly low neutron star mass ($\sim 1 M_\odot$; van Kerkwijk et al. 1995a), and the lack of a published light curve, we found this system especially worthy of our attention.

Optical light curves in $BVRI$ for 4U 1538-52 were obtained at the Cerro Tololo Inter-American Observatory on the 1.3 m SMARTS telescope with the ANDICAM in 2009 June–September. There are a total of 39 images in each filter.
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Figure 9. Optical $BVI$ light curves and radial velocity curves for 4U 1538-52. The solid lines represent the best-fit ELC model, which includes an accretion disk around the neutron star. The dotted lines depict the model with the light from the accretion disk subtracted for comparison. The left column shows a model with $e = 0.174 \pm 0.015$ and the right column shows a model for $e = 0$. The eccentric orbit model has a lower overall $\chi^2 = 808$ (vs. $\chi^2 = 818$ for the circular orbit), but as discussed in Section 4.2 there is no single physical solution for an eccentric orbit and a sufficiently long eclipse duration. All data are phased relative to the time of X-ray eclipse.

We ran two sets of model fits. We assumed an eccentric orbit with eccentricities in a narrow range around $e = 0.18$ and also a circular orbit. We find $K_{\text{opt}} = 14.1 \pm 1.1$ km s$^{-1}$ for an eccentric orbit and $K_{\text{opt}} = 21.8 \pm 3.8$ km s$^{-1}$ for a circular orbit. The difference between these two measurements is due in part to the noisy data and incomplete phase coverage near phase 0.25. For comparison, Reynolds et al. (1992) found $K_{\text{opt}} = 19.2 \pm 1.2$ km s$^{-1}$ (uncorrected) and $K_{\text{opt}} = 19.8 \pm 1.1$ km s$^{-1}$ (corrected for tidal distortions) assuming a circular orbit. Our final mass for 4U 1538-52 is quite low: $0.87 \pm 0.07 M_\odot$ for an eccentric orbit and $1.00 \pm 0.10 M_\odot$ for a circular orbit. If the orbit is indeed eccentric, this is an extremely low neutron star mass.

SMC X-1

SMC X-1 has a 3.89 day orbital period. It also exhibits a superorbital X-ray cycle that is probably caused by precession of either the accretion disk or neutron star (e.g., Priedhorsky & Holt 1987). The optical $V$ light curve for SMC X-1 shown in the lower left panel of Figure 8 is from van Paradijs & Kuiper (1984). Our final derived mass for SMC X-1 is $1.04 \pm 0.09 M_\odot$ with a system inclination of $68.5 \pm 5.2\degree$. We discuss the implications of this low-mass result and that of 4U 1538-52 in Section 5.

LMC X-4

LMC X-4 has a 1.41 day orbital period and, like SMC X-1, also exhibits a superorbital X-ray cycle. Heemskerk & van Paradijs (1989) performed an extensive study of the long-term variations in X-ray flux of LMC X-4 and concluded that it contained a warped, precessing accretion disk. The optical $B$ light curve for LMC X-4 shown in the upper right panel of Figure 8 is from Ilovaisky et al. (1984). Since no data table was available, the points were extracted from their “X-ray OFF states” plot using DEXTER available via the SAO/NASA Astrophysics Data System. Our final derived mass for LMC X-4 is $1.29 \pm 0.05 M_\odot$ with a system inclination of $67.0 \pm 1.9\degree$.

Cen X-3

Cen X-3 has a 2.09 day orbital period. The optical $V$ light curve for Cen X-3 shown in the lower right panel of Figure 8 is from van Paradijs et al. (1983). Our final derived mass for Cen X-3 is $1.49 \pm 0.08 M_\odot$ with a system inclination of $66.7 \pm 2.4\degree$. 

taken on different nights, each with a 60 s exposure time. Standard pipeline reductions were done on all images, and differential photometry was performed in IRAF for the target and several comparison stars. Light curves are shown in Figure 9, and observations in all three filters were incorporated into our numerical analysis.

Twenty-one high-resolution spectra of 4U 1538-52 were also taken on several nights in 2009 July and August at Las Campanas Observatory on the 6.5 m Clay Magellan telescope with the MIKE spectrograph. Standard pipeline reductions were done on all frames, including heliocentric corrections. The spectroscopic analysis was performed in IRAF using cross-correlation of the blue half of the spectrum (4750–4950 Å) with a model B0 star. Radial velocities were then computed and incorporated into our numerical analysis. The radial velocity curve is plotted in the bottom panels of Figure 9 with the adopted ELC model solution.
in each analytic solution. We assume have different masses that span a range from as low as 0 masses. It is clear from this figure that these six neutron stars where we have incorporated light curves into the analysis, our given in Table 4. For consistency, the fit value of \( \sigma \) for this system is the eccentric orbit mass derived using ELC. All values are derived mass if the system has a circular orbit (van Kerkwijk et al. 2011). In the case of the lattermost system, the systematic errors are potentially large owing to the extreme irradiation suffered by the pulsar’s evaporating companion. The large and secure (low-uncertainty) mass of PSR J1614–2230 rules out many of the so-called soft equations of state (Ozel et al. 2010).

As discussed by Kiziltan et al. (2010), the core mass of a star needs to exceed the Chandrasekhar mass if it is to end up as a neutron star. The exact value of the Chandrasekhar mass depends on the electron fraction, and Kiziltan et al. (2010) gives a plausible range of possible neutron star birth masses of 1.08 \( \lesssim M_{\text{birth}} \lesssim 1.57 M_\odot \). Finally, as discussed by Kiziltan et al. (2010), the millisecond pulsars in the NS–WD binaries must have accreted some mass in order to end up with millisecond spin periods. They give a range of 0.10 \( \lesssim M_{\text{acc}} \lesssim 0.20 M_\odot \) based on angular momentum considerations and on plausible mass transfer rates in the X-ray binary phase. They conclude that neutron stars with masses less than about 1.1 \( M_\odot \) would be unusual since it would be difficult to exceed the Chandrasekhar mass, while neutron stars with masses above about 1.8 \( M_\odot \) must have had either a prolonged stage of mass transfer at an unusually high rate or an unusually high mass when they formed.

Until recently, the neutron star in Vela X-1 has been at the high end of neutron star mass measurements. Unfortunately, the systematic errors in the mass determination are difficult to minimize owing to the non-radial pulsations in the B-giant companion. Using the \( K \)-velocity for Vela X-1 given in Barziv et al. (2001), the mass of its neutron star is 1.77 \( \pm 0.08 M_\odot \) and the companion star fills its Roche lobe at periastron. Quaintrell et al. (2003) derive a slightly higher \( K \)-velocity for Vela X-1 (22.6 \pm 1.5 km s\(^{-1}\)) compared to 21.7 \pm 1.6 km s\(^{-1}\) from Barziv et al. (2001), and consequently they derive a higher mass for the neutron star using the analytic approximations (2.27 \pm 0.17 \( M_\odot \) for \( \beta = 1 \)). When we used this higher \( K \)-velocity in the ELC code, the best fit gave a more conservative neutron star mass of 1.84 \( \pm 0.06 M_\odot \). However, the overall \( \chi^2 \) was a bit worse (\( \chi^2 = 30.96 \) compared to 29.52), and we note that this mass does barely fall within the 1\( \sigma \) uncertainty of our adopted value. In spite of the uncertainties, it appears likely that the neutron star in Vela X-1 has a relatively high mass that is comparable to the masses found for PSR B1516+02B, PSR J1748–2446I, and the Black Widow Pulsar. It is interesting to note that the companion star in Vela X-1 is high mass and will most likely also produce a neutron star. If so, then Vela X-1 cannot end up in a state similar to the Black Widow Pulsar or to the NS–WD binaries PSR B1516+02B and PSR J1748–2446I.

On the lower mass end, we have 4U 1538-52 at 0.87 \( \pm 0.07 M_\odot \) (eccentric) or 1.00 \( \pm 0.10 M_\odot \) (circular) and SMC X-1 at 1.04 \( \pm 0.09 M_\odot \). Further spectroscopic observations are needed to better establish the shape of the orbit in 4U 1538-52 and to better establish the \( K \)-velocity. In the case of SMC X-1, the
to change in order to have a neutron star mass of \( \sim 1 \) can potentially change are the X-ray timing properties of SMC X-1 are very well known, and the only other main quantities that could potentially change are the \( K \)-velocity of the companion star and the duration of the X-ray eclipse. To see what has to change in order to have a neutron star mass of \( \sim 1.2 \, M_\odot \) in SMC X-1, we performed a series of fits to the light curve where the \( K \)-velocity and eclipse duration were fixed at various values. To drive the neutron star mass to higher than \( 1.2 \, M_\odot \), the optical \( K \)-velocity needs to be higher (\( K_{\text{opt}} \geq 22 \, \text{km s}^{-1} \) or almost \( 2 \sigma \) higher than observed) and the eclipse duration needs to be shorter (\( \theta_e < 27:5 \) or about \( 1 \sigma \) smaller than observed). These simulations are presented in Table 5. If confirmed, the low masses for the neutron stars in SMC X-1 and 4U 1538-52 would indeed challenge neutron star formation models.

There are a few ways to modestly improve the accuracy of our neutron star mass determinations. Since the mass of these neutron stars is proportional to \( K_{\text{opt}} \), improvements in the measured velocity curves will improve the mass determinations. For three of the systems (SMC X-1, LMC X-4, and Cen X-3), this would involve the acquisition and analysis of a significant number of additional high-resolution optical spectra which we did perform in the case of 4U 1538-52 (see Section 4.2). Improvements in the \( K \)-velocity for Vela X-1 will be difficult owing to the presence of non-radial pulsations in its B-giant companion (Barziv et al. 2001; Quaintrell et al. 2003). Likewise, improvements in the \( K \)-velocity of Her X-1 will be difficult owing to the relatively strong X-ray heating of its low-mass companion (e.g., Crampton & Hutchings 1974; Reynolds et al. 1997). Modest improvements in the measured eclipse widths and X-ray timing properties for many of the systems could be made by observing the sources with greater time coverage.

M.L.R. and J.A.O. gratefully acknowledge the support of NSF grant AST-0808145. We also thank the anonymous referee who provided constructive feedback and encouraged us to include the full optical light curve analysis in this work.

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