Heat blanketing envelopes vs cooling of neutron stars

M V Beznogov\textsuperscript{1}, A Y Potekhin\textsuperscript{2,3}, M Fortin\textsuperscript{4}, P Haensel\textsuperscript{4}, J L Zdunik\textsuperscript{4} and D G Yakovlev\textsuperscript{2}

\textsuperscript{1} Institute of Astronomy, National Autonomous University of Mexico, Mexico D.F. 04510, Mexico
\textsuperscript{2} Ioffe Institute, Politekhnicheskaya 26, St Petersburg 194021, Russia
\textsuperscript{3} Central Astronomical Observatory at Pulkovo, Pulkovskoe Shosse 65, St Petersburg 196140, Russia
\textsuperscript{4} Nicolaus Copernicus Astronomical Center of the Polish Academy of Sciences, Bartycka 18, Warsaw 00-716, Poland

E-mail: mikavb89@gmail.com

Abstract. We discuss the effect of the chemical composition of heat blanketing envelopes of neutron stars (NSs) on the interpretation of the observations of these stars. First we analyze the diffusive fluxes of ions in non-isothermal and non-ideal Coulomb plasmas. Then we outline models of diffusively-equilibrated heat blanketing envelopes composed of binary ionic mixtures and finally we study their effect on the cooling of isolated NSs.

1. Introduction

The interpretation of the observations of isolated NSs is a difficult task for many reasons. One of them is our poor knowledge of the chemical composition of the outer heat blanketing NS envelopes. The uncertainties in their chemical composition create an obstacle in the theoretical inference of the internal structure of NSs from the observations of their thermal radiation.

Let us study different chemical compositions of heat blanketing envelopes. Such studies have been mostly conducted relying on the simplified “onion”-like (PCY97, [1]) model of envelopes (several shells of pure chemical H, He, C, and Fe with abrupt boundaries between the shells). To extend it, we constructed new models with a proper treatment of ion diffusion [2]. In this latter work (which was based on our previous work on diffusion in isothermal dense stellar plasmas [3]) we have considered non-isothermal systems and diffusively-equilibrated or non-equilibrated heat blanketing envelopes composed of binary ionic mixtures (BIMs), H–He, He–C or C–Fe, with different amounts of lighter ion species. Then we have applied the new models to investigate their effect on the cooling of isolated NSs [4]. Here we summarize these results.

2. Diffusion in non-isothermal dense stellar plasmas

Consider a multicomponent Coulomb plasma of several fully ionized ion species ($\alpha = j, j = 1, 2, 3 \ldots$) immersed in a neutralizing electron background ($\alpha = e$) (neglecting magnetic fields). The detailed derivation of the general expressions for the diffusive fluxes of ions is given in [2]; here we present a brief sketch.
The generalized thermodynamic forces acting on given particles $\alpha$ can be written as
\[ \tilde{f}_\alpha = f_\alpha - \left( \nabla \mu_\alpha - \frac{\partial \mu_\alpha}{\partial T} \right) \nabla T, \]
(1)
where $f_\alpha$ is the total force acting on the particles $\alpha$, $\mu_\alpha$ is their chemical potential and $T$ is the temperature. In the outer envelope,
\[ f_\alpha = Z_\alpha e E + m_\alpha g, \]
(2)
$m_\alpha$ and $Z_\alpha e$ being the mass and charge of the particles $\alpha$; $Z_e = -1, e > 0$; $g$ is the surface gravity. Possible deviations from the diffusion equilibrium are characterized by the quantities
\[ d_\alpha = \rho_\alpha \sum_\beta n_\beta \tilde{f}_\beta - n_\alpha \tilde{f}_\alpha, \]
(3)
where $\rho_\alpha$ is the mass density of the particles $\alpha$ and $\rho$ is the total mass density; then $\sum_\alpha d_\alpha = 0$. Using (1) and (2), the Gibbs-Duhem relation $\sum_\alpha n_\alpha \nabla \mu_\alpha = \nabla P - S \nabla T$ ($S$ being the entropy density) and the electric neutrality condition, we obtain
\[ \sum_\alpha n_\alpha \tilde{f}_\alpha = \rho g - \nabla P. \]
(4)
If we assume that the envelope as a whole is in hydrostatic equilibrium (i.e. $\rho g = \nabla P$), then $d_\alpha = -n_\alpha \tilde{f}_\alpha$ and
\[ d_\alpha = -\frac{\rho_\alpha}{\rho} \nabla P - Z_\alpha n_\alpha e E + n_\alpha \left( \nabla \mu_\alpha - \frac{\partial \mu_\alpha}{\partial T} \right) \nabla T. \]
(5)
Using the adiabatic (Born-Oppenheimer) approximation we can factor the electrons out of the problem of ion transport, $d_e = 0$, implying that the electrons are in mechanical quasi-equilibrium in response to the motion of ions. Assuming also $m_e \to 0$, we obtain $f_e = 0$ and
\[ e E = - \left( \nabla \mu_e - \frac{\partial \mu_e}{\partial T} \right) \nabla T. \]
(6)
The final phenomenological expression for the diffusive fluxes becomes
\[ J_\alpha = \frac{nm_\alpha}{\rho k_B T} \sum_\beta m_\beta D_{\alpha \beta} d_\beta - D^T_{\alpha} \nabla T, \]
(7)
where $D_{\alpha \beta}$ is the generalized diffusion coefficient for the particles $\alpha$ with respect to the particles $\beta$, $D^T_{\alpha}$ is the thermal diffusion coefficient of the particles $\alpha$, and the coefficient before the sum is chosen so as to match the conventional definition of $D_{\alpha \beta}$, e.g., [2, 3, 5].

3. Diffusively-equilibrated heat blanketing envelopes of neutron stars

Now we apply (7) to model heat blanketing envelopes. Consider envelopes composed of the BIMs $^1\text{H} - ^4\text{He}$, $^4\text{He} - ^{12}\text{C}$ or $^{12}\text{C} - ^{56}\text{Fe}$. In all examples we employ a model of ‘canonical’ NS (with a mass $M = 1.4M_\odot$ and radius $R = 10$ km). We neglect the thermal diffusion (it is usually slower compared to ordinary diffusion [2]) and assume that the electrons have a little impact on the transport of ions (as ions are much heavier [6]), so $J_e \to 0$ and $J_2 = -J_1$. From (7) we obtain
\[ J_2 = -J_1 = \frac{nm_1 m_2}{\rho k_B T} D_{12} d_1. \]
(8)
The diffusion equilibrium implies the absence of diffusive fluxes, \( \mathbf{J}_1 = -\mathbf{J}_2 = \mathbf{0} \), which leads to \( \mathbf{d}_1 = \mathbf{d}_2 = \mathbf{0} \). Then from (5) we have a system of three linear first-order differential equations,

\[
\nabla \mu_e = -eE, \quad \nabla \mu_j = m_j g + Z_j eE,
\]

where

\[
\nabla \mu_\alpha = \sum_j \frac{\partial \mu_\alpha}{\partial n_j} \nabla n_j + \frac{\partial P}{\partial T} \sum_j n_j \frac{\partial \mu_\alpha}{\partial n_j} \left( \sum_k n_k \frac{\partial P}{\partial n_k} \right)^{-1} \nabla T; \quad (10)
\]

\( j, k = 1, 2 \) enumerate the ion species. The unknowns are \( \nabla n_1, \nabla n_2 \) and \( E \). Note that here we do not need an explicit expression for \( D_{12} \), owing to the assumptions made in the beginning of this section. To solve (9), one needs to add the heat transport equation, e.g. [1, 2].

Fig. 1 shows a schematic cross-section of the heat blanketing envelope in the plane-parallel approximation as well as the expected distribution of ion species. The effective density \( \rho^* \) of the transition between the light and heavy ions is determined by the total mass \( \Delta M \) of light ions in the envelope [2].

Fig. 2 presents the internal temperature (at the bottom of the envelope) \( T_b \) as a function of the effective transition density \( \rho^* \) for the He – C mixture and a fixed surface temperature \( T_s = 1.47 \) MK. The different curves correspond to different possible densities \( \rho_b \) at the bottom of the envelope; after Fig. 4 in [2].

4. Cooling of neutron stars with diffusively-equilibrated envelopes

Now, following [4], we outline how the chemical composition of the outer envelopes affects the cooling of isolated NSs. To this aim, we employ the envelope models from the previous section.

Consider the famous Vela pulsar as an example. Its characteristic age is \( t \approx 11 \) kyr and the redshifted effective surface temperature (inferred using the ‘canonical’ NS model) \( T_s^\infty = 0.68 \pm 0.03 \) MK [7]; its non-redshifted temperature is \( T_s = T_s^\infty / \sqrt{1 - (2GM)/(Rc^2)} \approx 0.89 \) MK. Using

![Figure 1: Schematic cross-section of a heat blanketing envelope composed of a BIM in the plane-parallel approximation vs depth \( z \) from the surface.](image1)

![Figure 2: Internal temperature \( T_b \) vs effective transition (He ⇔ C) density \( \rho^* \) in a He – C mixture with a fixed surface temperature \( T_s \) at three values of \( \rho_b \).](image2)
$T_s - T_b$ relations from the previous section, we can calculate $T_b$ and $\tilde{T}_b = T_b \sqrt{1 - (2GM)/(Rc^2)}$ (the redshifted internal temperature that should be constant over the Vela’s interior). The pulsar has a magnetic field $B \sim 3 \times 10^{12}$ G which has no strong influence on the $T_s - T_b$ relations [8].

Fig. 3 demonstrates the dependence of the redshifted internal Vela’s temperature $\tilde{T}_b$ on the chemical composition of the blanketing envelope. $\tilde{T}_b$ is presented as a function of the accumulated mass $\Delta M$ of lighter ion species in He – C and C – Fe envelopes. The solid black curve is for the PCY97 model, where $\Delta M$ is the total accumulated mass of all light elements (H+He+C). One sees that the variations in the chemical composition cause up to $\sim 2.5$ times changes in the internal temperature; the maximum variation occurs in the PCY97 model which takes into account a wider range of elements than the binary envelope models. The different maximum values of $\Delta M$ for the various mixtures are explained in [2].

It is widely accepted that the Vela pulsar is cooling mainly via neutrino emission from its core (the so-called ‘neutrino cooling stage’, e.g., [9, 10]). The thermal evolution of such NSs is mostly regulated by the neutrino cooling function $\ell(T_b) = L_\nu^\infty(\tilde{T}_b)/C(\tilde{T}_b)$, where $L_\nu^\infty$ is the total redshifted neutrino luminosity of the star and $C$ is its total heat capacity (e.g., [11]). The function $\ell(\tilde{T}_b)$ is determined by the internal temperature; $\ell(\tilde{T}_b)$ is a fundamental quantity of superdense matter in NS cores. It is convenient to introduce the so-called ‘standard candle’ $t_{SC}$, which corresponds to a non-superfluid star cooling via the modified Urca process. Then one can express $\ell$ in units of $t_{SC}$, $f_\ell = \ell/t_{SC}$. Using the fits to $f_\ell$ from [12] which are almost independent of the equation of state (EOS) for nucleon NS cores, one can calculate $f_\ell(\tilde{T}_b)$ for the Vela pulsar. As shown in [4], a factor of $\sim 2$ uncertainty in $\tilde{T}_b$ translates into a factor of $\sim 10^2$ uncertainty in $f_\ell$, because $f_\ell \propto \tilde{T}_b^{-2}$ (mainly due to the strong temperature dependence of the neutrino emissivity; see [13]).

![Figure 3: Vela’s redshifted internal temperature as a function of the accumulated mass of light elements in different blanketing envelopes.](image)

![Figure 4: Cooling of a standard NS candle with different blanketing envelopes as compared with observations; after Fig. 4 in [4].](image)
the observational data on known isolated NSs. The thick curves correspond to the envelopes composed of pure He, C, Fe and for the fully accreted PCY97 envelope. The filled areas are covered with cooling curves obtained for envelopes with different $\Delta M$: the C – Fe envelopes between the Fe and C curves, the He – C envelopes between the C and He curves, and the PCY97 envelopes between the Fe and PCY97 ones. One can also see that despite the ‘broadening’ of the curve for the iron envelope owing to the presence of light elements, it is impossible to explain all the data. In order to explain the sources that are warmer or cooler than the presented cooling curves one needs to consider deviations from standard neutrino candles [4] (see also [9, 10] and references therein).

5. Conclusions
We have derived general expressions for the diffusive fluxes in non-isothermal Coulomb plasmas with an arbitrary Coulomb coupling of ions (in gaseous and liquid states). We have employed these expressions to construct models of diffusively-equilibrated heat blanketing envelopes of NSs and studied how the chemical composition of these envelopes affects the thermal evolution of isolated NSs. We have shown that for a fixed surface temperature (known from observations) the internal temperature can vary up to $\sim 2.5$ times depending on the chemical composition of the heat blanket. This uncertainty translates into a factor of $\sim 200$ uncertainty in the neutrino cooling function, creating a strong obstacle to study the internal structure of NSs. More work should be done to overcome this obstacle [4].

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References
[1] Potekhin A Y, Chabrier G and Yakovlev D G 1997 Astron. Astrophys. 323 415–28
[2] Beznogov M V, Potekhin A Y and Yakovlev D G 2016 MNRAS 459 1569–79
[3] Beznogov M V and Yakovlev D G 2013 Phys. Rev. Lett. 111 161101
[4] Beznogov M V, Fortin M, Haensel P, Yakovlev D G and Zdunik J L 2016 MNRAS 463 1307–13
[5] Hirschfelder J O, Curtiss C F and Bird R B 1954 Molecular Theory of Gases and Liquids (Wiley, New York)
[6] Paquette C, Pelletier C, Fontaine G and Michaud G 1986 Astrophys. J. Suppl. 61 177–95
[7] Pavlov G G, Zavlin V E, Sanwal D, Burwitz V and Garmire G P 2001 Astrophys. J. Lett. 552 L129–33
[8] Potekhin A Y, Yakovlev D G, Chabrier G and Gnedin O Y 2003 Astrophys. J. 594 404–18
[9] Yakovlev D G and Pethick C J 2004 Annu. Rev. Astron. Astrophys. 42 169–210
[10] Potekhin A Y, Pons J A and Page D 2015 Space Sci. Rev. 191 239–91
[11] Yakovlev D G, Ho W C G, Shternin P S, Heinke C O and Potekhin A Y 2011 MNRAS 411 1977–88
[12] Ofengeim D D, Kaminker A D, Klochkov D, Suleimanov V and Yakovlev D G 2015 MNRAS 454 2668–76
[13] Yakovlev D G, Kaminker A D, Gnedin O Y and Haensel P 2001 Phys. Rep. 354 1–155
[14] Potekhin A Y, Fantina A F, Chamel N, Pearson J M and Goriely S 2013 Astron. Astrophys. 560 A48