Feasibility of the experimental study of $D_s^* \rightarrow \phi \pi$ decay

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Abstract The current knowledge on the $D_s^*$ meson are very limited. Besides the dominant electromagnetic decays, the $D_s^*$ weak decays are legal and offer the valuable opportunities to explore the wanted $D_s^*$ meson. In this paper, the $D_s^* \rightarrow \phi \pi$ decay was studied with the factorization approach. It is found that the branching ratio $B(D_s^* \rightarrow \phi \pi) \sim O(10^{-7})$, which corresponds to several thousands of events at the $e^+e^-$ collider experiments including STCF, SuperKEKB, CEPC and FCC-ee, and several millions of events at the hadron collider experiments, such as LHCb@HL-LHC. It is feasible to experimentally study the $D_s^* \rightarrow \phi \pi$ weak decay in the future, even considering the identification efficiency.

1 Introduction

The first evidence for the charmed strange mesons $D_s^*$ was observed in the exclusive reaction $e^+e^- \rightarrow F \bar{F}^*$ by the DASP collaboration in the year of 1977 [1], where the symbols of $F$ and $F^*$ were formerly used to denote the $D_s$ and $D_s^*$ particles, respectively. According to the $SU(4)$ quark model assignments, the vector mesons $D_s^*$ are assumed to have the same quark compositions as their twin pseudoscalar partners $D_s$. Both the $D_s^{*+}$ and $D_s^{*+}$ mesons are consisting of a quark–antiquark pair $c\bar{s}$, and have the same additive quantum numbers of Charm, Strangeness and Charge, i.e., $C = S = Q = +1$. The different spin configurations of interquark potential make the mass of the ground spin-triplet $1^3S_1$ state for the $D_s^*$ mesons to be above that of the ground spin-singlet $1^1S_0$ state for the $D_s$ mesons [2].

Compared with the pseudoscalar meson $D_s$, the experimental information on the properties of the vector meson $D_s^*$ is still very limited by now [2]. Although there were many measurements of the mass of the $D_s^*$ meson (such as in Refs. [1,3–9]), only one measurement was solemnly quoted by the Particle Data Group (PDG) until now [2]. The measurement was carried out by the Mark III collaboration in 1987 [6], 35 years ago. And the errors of the measurement of mass, $m_{D_s^*} = 2109.3 \pm 2.1 \pm 3.1$ MeV [6], are significantly larger than those of current values of the $D_s$ meson, $m_{D_s} = 1968.35 \pm 0.07$ MeV [2]. For the full width of the $D_s^*$ meson, only the upper limit was given by different experimental groups [2] and the latest and minimal upper limit on the decay width of the $D_s^*$ meson was given by the CLEO Collaboration in 1995 [9], 27 years ago. The natural spin-parity of the $D_s^*$ meson was analyzed to be most likely $J^P = 1^-$ [9], but has not been unambiguously determined experimentally [2].

The experimental data on the $D_s^*$ mesons are accumulating increasingly. The quantitative study on the $D_s^*$ mesons is coming. Inspired by the potential prospects of high-luminosity-frontier flavor experiments, more and more data of the $D_s^*$ mesons will be available, so more accurate information and more detailed knowledge of the properties of the $D_s^*$ mesons will be accessible. In the $e^+e^-$ colliders, it is promisingly expected that there will be a total of about $5 \times 10^{10}$ $c\bar{c}$ pairs at the SuperKEKB [10], about $10^{11}$ $c\bar{c}$ pairs from $10^{12}$ $Z^0$ boson decays at the Circular Electron Positron Collider (CEPC) [11], about $6 \times 10^{11}$ $c\bar{c}$ pairs from $5 \times 10^{12}$ $Z^0$ boson decays at the Future Circular Collider (FCC-ee) [12], where the branching fraction for the $Z^0$ boson decay into the $c\bar{c}$ pair is $B(Z^0 \rightarrow c\bar{c}) = (12.03 \pm 0.21)%$ [2]. Considering the fraction of the charmed quark fragmenting into the $D_s^*$ meson $f(c \rightarrow D_s^*) \simeq 5.5%$ [13], these high statistical $c\bar{c}$ pairs correspond to some $6 \times 10^9$, $10^{10}$ and $6 \times 10^{10}$ $D_s^*$ mesons at the SuperKEKB, CEPC and FCC-ee, respectively. In addition, about $10^{10}$ $D_s^*$ mesons are expected above the $\psi(4040)$ threshold (see Fig. 6 of Ref. [14]) at both the super $\tau$-charm factory (STCF) in China [15] and the super charm-tau factory (SCTF) in Novosibirsk, Russia [16], based on an integrated luminosity of $10 \text{ab}^{-1}$ [15]. In the high-energy hadron colliders, about $4 \times 10^{13}$ $D_s^*$ mesons [14] are expected to be observable with a data sample of target lum-
The fit mass of $D_s^*$ meson is $m_{D_s^*} = 2112.2 \pm 0.4$ MeV [2], just below the mass threshold of the $D \bar{K}$ pair and above the mass threshold of the $D_s \pi$ pair and, i.e., the mass relations $m_{D_s \pi} + m_K > m_{D_s} > m_{D_s} + m_\pi$. Thus the hadronic decays $D_s^* \rightarrow D \bar{K}$ are strictly forbidden by the law of conservation of energy. The hadronic decay $D_s^* \rightarrow D_s \pi$ is permissible kinematically, but violates the isospin conservation in the strong interactions.\(^1\) The absence of decay modes induced by the strong interactions make the $D_s^*$ meson to be very narrow. The natural width of the $D_s^*$ meson is significantly less than the best experimental resolution. Here, it should be noted that the $D_s^* \rightarrow D_s \pi$ decay is suppressed not only by the phenomenological Okubo–Zweig–Iizuka (OZI) rule [22–24] but also by the extremely limited phase spaces, due to $m_{D_s \pi} - m_{D_s} - m_\pi < 6$ MeV. Thus the electromagnetic decay $D_s^* \rightarrow D_{\gamma\gamma}$ is dominant, with the branching ratio $B(D_s^* \rightarrow D_{\gamma\gamma}) = (93.5 \pm 0.7)\%$ exceeding that of hadronic decay $B(D_s^* \rightarrow D_s \pi) = (5.8 \pm 0.7)\%$ [2]. In addition, for the $D_s^* \rightarrow D_s \pi^0, D_s \gamma$ decays,\(^2\) the final photons are seriously polluted by those from bremsstrahlung radiation, which will significantly affect the identification efficiency of the accident photon. Besides, the $D_s^*$ meson can also decay via the weak interactions, although with a very small probability. The weak decays of the $D_s^*$ meson provide another platform and opportunities to explore and understand the properties of the $D_s^*$ mesons. In this paper, we will evaluate the feasibility of experimentally investigating the $D_s^*$ meson through the weak decay $D_s^* \rightarrow \phi \pi$.

Theoretically, the charm-flavor-changing decay $D_{s}^{*+} \rightarrow \phi \pi^{+}$ is actually induced by the quark transition $c \rightarrow s + W^{+}$ at the tree level in the standard model (SM) of elementary particles. Here, it is assumed that the vector $\phi$ meson consists of the pure $s \bar{s}$ quark pair with neither possible $u \bar{u}$ nor $d \bar{d}$ components, i.e., that the mixing between the $\phi$-omega system is ideal. Clearly, this decay mode is the Cabibbo-favored one and its amplitudes are proportional to the Cabibbo–Kobayashi–Maskawa (CKM) matrix elements $|V_{cs}| \sim O(1)$. This decay would have a relatively large branching ratio among the $D_s^*$ meson weak decays, and hence should have a high priority to be studied. In addition, the charm quark is somewhat massive and can be regarded as one bridge between the perturbative and nonperturbative regimes. The charm quark decays offer a laboratory to test various phenomenological models and study the behaviors of the strong interactions near the scale of $O(m_c)$.

Experimentally, the curved tracks of the charged pion and kaon plunged into magnetic field will be unambiguously detectable by the highly sensitive detectors. So, the final states are easily identified for the $D_s^* \rightarrow \phi \pi$ decays, where $\phi$ and $\pi$ mesons with a definite momentum are back-to-back in the center-of-mass frame of the $D_s^*$ meson, and the $\phi$ meson can be well reconstructed from the kaon pairs. It is expected to have a higher signal-to-background ratio and a better identification efficiency, and have a big competitive advantage over both the pure leptonic decays $D_s^* \rightarrow \ell \bar{\nu}$ and semileptonic decays $D_s^* \rightarrow \phi \ell \bar{\nu}$ which suffer from the additional complications caused by the final neutrinos.

In this paper, we will study the $D_s^* \rightarrow \phi \pi$ decay within SM by using the phenomenological factorization approach [27], and estimate the branching ratio in order to provide a ready reference for future experimental analysis. This paper is organized as follows. The amplitudes for the $D_s^*$ decay in question using the factorization approximation is given in Sect. 2. Branching ratio and event numbers of the $D_s^* \rightarrow \phi \pi$ decay are listed in Sect. 3. Section 4 devotes to a summary.

2 The theoretical framework

At the quark level, the effective Hamiltonian responsible for the nonleptonic decay $D_s^* \rightarrow \phi \pi$ can be written as [28],

$$H_{\text{eff}} = \frac{G_F}{\sqrt{2}} V_{cs} \bar{V}_{ud} \left\{ C_1 O_1 + C_2 O_1 \right\} + \text{h.c.}, \tag{1}$$

where the Fermi constant $G_F$ is the weak interaction coupling coefficient, $G_F \approx 1.166 \times 10^{-5}$ GeV$^{-2}$ [2]. $V_{cs}$ is the product of CKM matrix elements, which has been determined precisely by experiments, $|V_{cd}| = 0.97370(14)$ and $|V_{cs}| = 0.987(11)$ [2]. The Wilson coefficients $\tilde{C} = \{ C_1, C_2 \}$ can be obtained with the renormalization group equation,

$$\tilde{C}(\mu_c) = U_4(\mu_c, m_b) M(m_b) U_5(m_b, m_W) \tilde{C}(m_W), \tag{2}$$

where $\mu_c \sim O(m_c)$ is the scale for the charm quark decays. $m_b$ and $m_W$ are the mass of the bottom quark and the charged $W$ gauge boson, respectively. $U_4(\mu_f, \mu_c)$ and $M(m_b)$ are the evolution matrix and threshold matching matrix, respectively. The expressions of $\tilde{C}(m_W)$, $U_4(\mu_f, \mu)_{f = c}$ and $M(m_b)$ can be found in Ref. [28]. The effective operators are defined as follows.

\(^1\) Within the chiral perturbative theory, it is usually taken for granted that the $D_s^* \rightarrow D_s \pi$ decay can also decay through the strong interactions via the $\pi^0$-$\eta$ mixing by assuming a small isoscalar $\eta$ meson component in the physical $\pi^0$ meson, because the $\eta$ meson can couple to the strange quark in the charmed strange mesons [18–21].

\(^2\) The neutral pion decay predominantly through $\pi^0 \rightarrow \gamma \gamma$ with a branching ratio of 98.8% [2].
\( O_1 = \left[ \bar{s}_a \gamma^{\mu} \left( 1 - \gamma_5 \right) c_a \right] \left[ \bar{u}_\beta \gamma_\mu \left( 1 - \gamma_5 \right) d_\beta \right], \)  \( O_2 = \left[ \bar{s}_a \gamma^{\mu} \left( 1 - \gamma_5 \right) c_a \right] \left[ \bar{u}_\beta \gamma_\mu \left( 1 - \gamma_5 \right) d_\beta \right], \)

where \( \alpha \) and \( \beta \) are the color indices. Because the \( D_1^+ \rightarrow \phi \pi \) decay is an external \( W \) emission process, there are only two current-current operator \( O_{1,2} \) and without the penguin operators, and the contributions from new physics beyond SM to this decay are negligible.

The initial and final states are hadrons, while the operators are the specific combinations of four quarks. The influence of the long-distance strong interactions on the transitions between quarks and hadrons makes the predictions of nonleptonic decays notoriously difficult. To obtain the decay amplitudes for the \( D_1^+ \rightarrow \phi \pi \) decay, the remaining work is to evaluate the hadronic matrix elements (HMEs) \( \langle \phi \pi | O | D_1^+ \rangle \).

Phenomenologically, one of the most frequently used methods to deal with HME is the naive factorization (NF) approach [27]. The NF approach is based on the color transparency hypothesis [29] that a nearly collinear and relativistic parton may be a good approximation to neglect the final state interactions and energetic in the heavy quark limit, and fly away far from each other before the interference with the soft gluons. It may be a good approximation to neglect the final state interactions for the moment. In addition, the asymptotic freedom property of the strong interactions implies that the creation of quark pairs of high energy from the vacuum by hard virtual gluon is highly suppressed [31], i.e., it is believed that the \( W \)-annihilation amplitudes for the nonleptonic heavy-flavored hadron decays might be much smaller than the \( W \)-emission amplitudes.

Under the assumption of factorization, the decay amplitudes are written as,

\[
A(D_1^+ \rightarrow \phi \pi) = \langle \phi \pi | H_{\text{eff}} | D_1^+ \rangle = \frac{G_F}{\sqrt{2}} \langle \bar{c}_s \gamma_\mu \left( 1 - \gamma_5 \right) c_s \rangle \left[ \bar{u}_\beta \gamma_\mu \left( 1 - \gamma_5 \right) d_\beta \right] = \frac{G_F}{\sqrt{2}} V_{cs} V_{ud} a_1 \langle \bar{c}_s \gamma_\mu \left( 1 - \gamma_5 \right) c_s \rangle \langle \bar{u}_\beta \gamma_\mu \left( 1 - \gamma_5 \right) d_\beta \rangle,
\]

where \( \langle \bar{c}_s \gamma_\mu \left( 1 - \gamma_5 \right) c_s \rangle \) and \( \langle \bar{u}_\beta \gamma_\mu \left( 1 - \gamma_5 \right) d_\beta \rangle \) are the color singlet \( V-A \) hadron currents, and the subscript \( H \) is introduced to indicate the change to hadron currents and distinguish with quark currents of Eqs. (3) and (4). The effects from the color exchanges are embodied into the coefficient \( a_1 = C_1 + \xi C_2 \). It is expected \( \xi = 1/N_c = 1/3 \) from color matching. \( \xi \) or \( a_1 \) sometimes is regarded as a parameter for different factorization approaches, because of the uncertain contributions of color octet current product and nonfactorizable contributions. The approximation of \( a_1 \approx 1.1 \) is frequently used in many phenomenological studies of nonleptonic decays for charmed hadron mesons, such as Refs. [30–41].

Using the parameterization for amplitude in Eq. (5), the decay widths can be given in terms of measurable physical HMEs. The HMEs of hadron currents in Eq. (5) are related to the decay constants and hadron transition form factors. The one-body HMEs are relevant to decay constants of hadrons,

\[
\langle 0|\bar{d}_\gamma \gamma_\mu \bar{u} \pi^+ (p) \rangle = 0,
\]

\[
\langle 0|\bar{d}_\gamma \gamma_\mu \bar{s} \pi^+ (p) \rangle = i f_{\pi} p_\mu.
\]

The charged pion decay constant has been well determined from numerical lattice QCD simulations, \( f_{\pi} = 130.2 \pm 1.2 \) MeV (see Ref. [2] for a summary review). With the conventions of Ref. [42], the form factors are defined as,

\[
\langle \phi (\epsilon_2, p_2) | \bar{s}_\gamma \gamma_\mu c | D_1^+ (\epsilon_1, p_1) \rangle = -(\epsilon_1 \cdot \epsilon_2) \left[ P_\mu V_1 (q^2) - q_\mu V_2 (q^2) \right] - \frac{1}{m_{D_1^+}^2 - m_{\phi}^2} \left[ P_\mu - \frac{m_{D_1^+}^2 - m_{\phi}^2}{q^2} q_\mu \right] V_3 (q^2) + \frac{m_{D_1^+}^2 - m_{\phi}^2}{q^2} q_\mu V_4 (q^2),
\]

where \( \epsilon_i \) denotes the polarization vector of the vector mesons. The momentum \( P = p_1 + p_2 \) and \( q = p_1 - p_2 \). At the pole \( q^2 = 0 \), there is,

\[
V_3 (0) = V_4 (0), \quad A_1 (0) = A_2 (0).
\]

The values of form factors for the \( D_1^+ \rightarrow \phi \) transition have been obtained with the light front approach [42], for example,

\[
A_1 (0) = 0.65, \quad V_1 (0) = 0.71, \quad V_4 (0) = 0.28, \quad V_3 (0) = 1.54, \quad V_2 (0) = 0.86.
\]

Finally, the decay amplitude can be expressed by three invariable amplitudes. They are defined by the decomposition,
\[
A(D_s^* \to \phi \pi) = a (\epsilon_{D_s^*} \epsilon_\phi^*) + \frac{b}{m_{D_s^*} m_\phi} (\epsilon_{D_s^*} \cdot p_\pi) (\epsilon_\phi^* \cdot p_\pi) + \frac{c}{m_{D_s^*} m_\phi} \epsilon_{\mu \nu \alpha \beta} \epsilon_{D_s^*}^{\mu \nu} \epsilon_\phi^{\alpha \beta} \frac{p_\pi^\mu (PD_s^* + p_\phi)^\nu}{(PD_s^* + p_\phi)^2},
\]
and the invariant amplitudes \(a\), \(b\), and \(c\) describe the s-, d-, and p-wave contributions.

\[
a = -i \frac{G_F}{\sqrt{2}} V_{cs}^* V_{ud} f_\pi (m_{D_s^*}^2 - m_\phi^2) a_1 V_1(0),
\]

\[
b = -i \frac{G_F}{\sqrt{2}} V_{cs}^* V_{ud} f_\pi m_{D_s^*} m_\phi a_1 \times \{V_3(0) - V_6(0) - V_4(0)\},
\]

\[
c = - \frac{G_F}{\sqrt{2}} V_{cs}^* V_{ud} f_\pi m_{D_s^*} m_\phi a_1 A_1(0).
\]

In the rest frame of the \(D_s^*\) meson, branching ratio is defined as,

\[
B(D_s^* \to \phi \pi) = \frac{1}{24 \pi} \frac{p_{c.m.}}{m_{D_s^*}^2} \frac{\Gamma(D_s^* \to \phi \pi)}{\Gamma(D_s^*)} |A(D_s^* \to \phi \pi)|^2
\]

\[
= \frac{1}{24 \pi} \frac{p_{c.m.}}{m_{D_s^*}^2} |a|^2 \left(2 + x^2\right) + |b|^2 \left(x^2 - 1\right) + |c|^2 \left(2x^2 - 1\right) - 2Re(a b^\ast) x (x^2 - 1),
\]

where the center-of-mass momentum of final states is of magnitude,

\[
p_{c.m.} = \frac{\lambda^{\frac{1}{2}} (m_{D_s^*}^2, m_\phi^2, m_{\pi}^2)}{2m_{D_s^*}^2},
\]

the parameter \(x\) is defined as,

\[
x = \frac{p_{D_s^*} \cdot p_\phi}{m_{D_s^*} m_\phi} = \frac{E_\phi}{m_\phi} = \frac{m_{D_s^*}^2 + m_\phi^2 - m_\pi^2}{2m_{D_s^*} m_\phi}.
\]

\[
\lambda(x, y, z) = x^2 + y^2 + z^2 - 2x y - 2y z - 2z x
\]

\[
p_{c.m.}^2 = m_\phi^2 (x^2 - 1).
\]

3 Numerical results and discussion

The total decay width \(\Gamma(D_s^*) < 1.9\) MeV was set at the 90% confidence level by the CLEO collaboration in 1995 [9]. A quantitative and concrete result currently comes from theoretical estimations. Because of the lion’s share \(B(D_s^* \to \gamma D_s) = (93.5 \pm 0.7)\%\) [2], an approximation

\[
\Gamma(D_s^* \to \gamma D_s) \approx \Gamma(D_s^* \to \gamma D_s),
\]

\[
\Gamma(D_s^* \to D_s \gamma) = \frac{4}{3} \alpha_{em} k_\gamma^3 \mu_{D_s^* D_s}^2 \approx 0.36\text{ keV},
\]

\[
B(D_s^* \to \phi \pi) \approx 2.4 \times \frac{\Gamma(D_s^*)}{\text{0.36 keV}} \times 10^{-7},
\]

and the corresponding partial decay width, \(\Gamma(D_s^* \to \phi \pi) \approx 0.86 \times 10^{-13}\) GeV, is more than twice as large as the recent estimate using the QCD light cone sum rules in Ref. [43] where a relatively smaller coefficient \(a_1 \approx 1.0\) is used.

We will make two comments on branching ratio. (1) There are many factors which influence the numerical results, such as the final state interactions. It is foreseeable that there will be very large theoretical uncertainties. For example, using a much smaller decay width \(\Gamma(D_s^*) \approx 0.07\) keV from the lattice QCD simulations [44], branching ratio will be increased five times. Our focus is whether there is feasible to explore the \(D_s^*\) meson via the \(\phi \pi\) final states at the future experiments. A rough estimate rather than precise calculation on branching ratio is enough. (2) For the tree-dominated and color-favored nonleptonic heavy flavored meson decays arising from the external \(W\) emission weak interaction, there is a consensus that NF approximation does hold and can give a reasonable and correct magnitude order estimation on branching ratio. In this sense, \(B(D_s^* \to \phi \pi) \sim O(10^{-7})\) seems credible.

Based on the above analysis, it can be conclude that the \(D_s^* \to \phi \pi\) decay should be measurable in the future experiments, such as STCF, SuperKEKB, CEPC, FCC-ee and LHCb. The potential event numbers of the \(D_s^*\) mesons and the \(D_s^* \to \phi \pi\) decays are listed in Table 1. It is clearly seen from Table 1 that the natural properties of the \(D_s^*\) meson can be investigated via the \(D_s^* \to \phi \pi\) weak decays, particularly in the future FCC-ee and LHCb experiments.

4 Summary

Inspired by the inadequate understanding of the properties of \(D_s^*\) meson, and the promisingly experimental prospects of investigating the \(D_s^*\) meson in the future high-luminosity experiments, the \(D_s^* \to \phi \pi\) decay was studied by using the NF approach within SM. The nonleptonic \(D_s^* \to \phi \pi\) weak decay offers a fresh arena and a tempting opportunity to explore the wanted \(D_s^*\) meson, although with a very tiny occurrence probability of \(\sim O(10^{-7})\). The final states of the \(D_s^* \to \phi \pi\) decay have the relatively larger momenta than those of the predominant electromagnetic decays \(D_s^* \to\)

\[
\sum \text{Springer}
\]
Table 1 The potential event numbers of the $D_s^*$ meson available and the $D_s^* \to \phi \pi$ decays in future experiments, with the branching ratio $B(Z^0 \to c\bar{c}) \approx 12\%$ [2] and $B(D_s^* \to \phi \pi) \approx 3 \times 10^{-7}$, the fragmentation fraction $f(c \to D_s^*) \approx 5.5\%$ [13] and the identification efficiency $\epsilon \sim 20\%$

| Experiment          | $N_{D_s^*}$ | $N_{D_s^*\to\phi\pi}$ | $\epsilon \times N_{D_s^*\to\phi\pi}$ | Remarks                  |
|---------------------|------------|------------------------|---------------------------------------|--------------------------|
| STCF [15,16]        | 10$^{10}$  | 3000                   | 600                                   | With 10$ab^{-1}$ data    |
| SuperKEKB [10]      | 5.5 × 10$^9$ | 1600                  | 300                                   | With 5 × 10$^{10}$ charm quark pairs |
| CEPC [11]           | 1.3 × 10$^{10}$ | 4000                  | 800                                   | From 10$^{12}$ Z$^0$ boson decays |
| FCC-ee [12]         | 6.6 × 10$^{10}$ | 2 × 10$^4$             | 4000                                  | From 5 × 10$^{12}$ Z$^0$ boson decays |
| LHCb@HL-LHC [17]    | 4 × 10$^{13}$ | 10$^7$                 | 2 × 10$^6$                            | With 300 fb$^{-1}$ data |

$D_s \gamma$ and $\rightarrow D_s \pi$, and can be more easily identified by the sensitive detectors. It is found that several thousands of events for the $D_s^* \to \phi \pi$ decay are expected to be accessible at the STCF, SuperKEKB, CEPC and FCC-ee experiments, several millions of events at LHCb@HL-LHC experiments. It is practicable to experimentally study the $D_s^* \to \phi \pi$ weak decay in the future.

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Data Availability Statement This manuscript has no associated data or the data will not be deposited. [Authors’ comment: This is a theoretical study. All data generated during this are contained in this published article.]

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