Numerical simulation of fast ions guiding-center orbits in mega ampere spherical tokamak-reactor configuration

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Abstract. Determining the charged particle trajectories in the inhomogeneous electric and magnetic fields is the common problem in the plasma physics. For the tokamak plasma, the orbits of the captured fast ions are affected both by the curvature and gradient of the magnetic field. In the present work, the fast ion trajectories and their guiding-centre orbits in a mega ampere spherical tokamak reactor are found with employing the numerical code, that was elaborated by our team, together with the leap-frog and Runge-Kutta techniques. The magnetic field is calculated by solving the Grad-Shafranov equation at the fixed boundaries on the non-homogeneous mesh grid. The obtained results show that the ions with the energies in the range 20 keV - 80 keV can describe the typical banana orbits as well as the passing trajectories in the poloidal plane. Finally, the numerical simulation scheme stability is tested.

1. Introduction

The most promising system to confine thermonuclear plasmas is the tokamak device whose magnetic field of a torus geometry is formed by many poloidal and toroidal d.c. coils [1]. A modern version of tokamak systems is the spherical tokamak (ST) which is more compact and operates at a lower magnetic field as compared to the conventional ones. The physics of the plasma confined in the spherical tokamak has been making great progress due to the rapid deployment and experimental work in recent years [2]. The most important experiments have been realized at the mega ampere spherical tokamak (MAST) constructed at Culham centre for fusion energy (CCFE) [3]. In the MAST plasma, the orbits of confined particles are classified into two types: those of the trapped ones and the passing guiding-centre orbits [4]. The presence of these type trajectories in the axisymmetric tokamak geometry is a controlling fusion reaction factors [5,6] because they determine the neoclassical and anomalous transports in these confined plasmas [7,8]. It should be noted that the fast ion energies are at least one order of magnitude greater than the tokamak plasma thermal energy [9].

In this work, we have calculated the equilibrium magnetic field lines through solution of the Grad-Shafranov equation and the ion guiding-centre orbits in a tokamak equilibrium plasma which is akin to the MAST reactor plasma. The paper is organized as follows: in section 2 and section 3 we describe the physical and numerical schemes and in section 4 the simulation results concerning the ion closed trajectories in the poloidal plane are presented.
2. Physical scheme

The Grad-Shafranov (GS) equation is derived from the MHD equilibrium, it is one of the most famous equations arising from MHD. It is a second-order partial differential equation that describes equilibrium in an axisymmetric plasma [10]. In cylindrical coordinates \((r,z)\) it take the form presented in Equation (1).

\[
\Delta^* \psi = -r^2 \frac{dP(\psi)}{d\psi} - g \frac{dg(\psi)}{d\psi},
\]

where \(\psi\) is the poloidal magnetic flux, \(P(\psi)\) and \(g(\psi)\) are the equilibrium pressure and the poloidal current function, and \(\Delta^* \equiv \frac{\partial^2}{\partial z^2} - \frac{1}{r} \frac{\partial}{\partial r} + \frac{\partial^2}{\partial r^2}\) the elliptical toroidal operator. Equation (2) show the magnetic and current density fields defined in terms of poloidal magnetic flux.

\[
B_r = -\frac{1}{r} \frac{\partial \psi}{\partial z}, \quad B_z = \frac{1}{r} \frac{\partial \psi}{\partial r}, \quad B_\phi = -g(\psi) r
\]

In these simulations, the external electric field is not considered for the sake of simplicity. It is well known that the dynamics of charge particles in the magnetic and electric fields is totally defined by the Newton-Lorentz equation, Equation (3), which in the relativistic variant is

\[
\frac{d\vec{p}}{dt} = q\left(\vec{E} + \vec{v} \times \vec{B}\right)
\]

with \(\vec{p} = \gamma m\vec{v}\) and \(\vec{v} = \frac{d\vec{r}}{dt}\)

Equation (3) contain the Lorentz force that instantaneously generates rotation around the magnetic field lines, effect produced by the normal velocity component \(\vec{v}_\perp\).

Averaging Equation (3) over the fast Larmor rotations, the drift equations which determine the guiding centre (GC) motion are presented in Equation (4) to Equation (6) that is a relativistic formulation for the guiding-center (GC) model presented by Sommariva and collaborators in [11], which can be deduced in [12,13].

\[
\dot{\vec{X}} = \frac{1}{\vec{b} \cdot \vec{B}^*} \left( q\vec{E} \times \vec{b} - p_\parallel \frac{\partial \vec{b}}{\partial t} \times \vec{b} + \frac{m\mu \vec{b} \times \vec{B}^*}{m\gamma_{GC}} \right)
\]

\[
p_\parallel = \vec{B}^* \cdot \left( q\vec{E} - p_\parallel \frac{\partial \vec{b}}{\partial t} \right)
\]

\[
\gamma_{GC} = \sqrt{1 + \left(\frac{p_\parallel}{mc}\right)^2 + \frac{2\mu B}{mc^2}}
\]

This is a reduced dynamical model, exactly is the first order GC approximation and it is associated to a number of validity conditions presented in [11]. Here \(\vec{X}\) is the GC position vector, \(p_\parallel\) is the particle momentum in the direction of the magnetic field (parallel momentum), \(\mu = \frac{||p_\parallel - \vec{p}||^2}{2mB}\) is the magnetic moment, \(B\) is the magnetic field norm, \(\vec{b} = \frac{\vec{B}}{B}\) the magnetic field direction and \(\vec{B}^* = p_\parallel \nabla \times \vec{b} + q\vec{B}\) is the so called effective magnetic field.

In the plasma transport theory when applied to the tokamak plasmas, the GC approximation is employed to study the fast ion trajectories because the six implicit equations of the full giro-motion model are substituted in the GC model for four differential equations of the first order.

3. Numerical scheme

To obtain the magnetic field line configurations, we solve the Grad-Shafranov equation through an iterative procedure which is very much alike the typical Poisson solver applied to calculate the derivatives on the non-homogeneous meshgrid. The complete description of this solver is
Equation (7) shows the separatrix curve in the D-shape plasma of a MAST reactor.

\[ r(\theta) = R_o + a_o \cos (\theta + \delta \sin(\theta) + \tau_o \sin(2\theta)) \; ; \; \quad z(\theta) = a_o \gamma \sin(\theta) \quad \text{with} \quad 0 \leq \theta \leq 2\pi \quad (7) \]

Here, \( \gamma \) is the ellipticity, \( \delta \) is the triangularity, \( R_o \) and \( a_o \) are the major and minor radius respectively. Note that the system is entirely described in cylindrical coordinates due to the symmetry imposed by the GS-equation.

Figure 1 shows the geometry of the plasma boundary and the representative non-homogeneous rectangular grid adjusted to the separatrix.

The numerical integration of the relativistic Newton-Lorentz differential equation, Equation (3), is fulfilled using the leap-frog method which in the dimensionless variables is represented by Equation (8).

\[ \frac{d\vec{U}}{d\tau} = \vec{g}_0 + \frac{\vec{U}}{\gamma} \times \vec{b}, \quad (8) \]

where \( \vec{U} = \vec{p}/m_e c \) is the ion momentum, \( \vec{g}_0 = q \vec{E}/m_e c \omega \) is the external electric field, \( \vec{b} = \vec{B}/B_0 \) is the magnetic field at the ion position, \( \tau = \omega t \) is the time and finally \( \gamma = (1 + U^2)^{1/2} \) is the relativistic factor. The parameters \( B_0 \) and \( \omega^{-1} = m_e / q B_0 \) represent the characteristic values for the magnetic field and time respectively.

The dimensionless motion equation represent in a finite differences form, can be write as the Equation (9).

\[ \frac{\vec{U}^{n+1/2} - \vec{U}^{n-1/2}}{\Delta \tau} = \vec{g}^n + \frac{\vec{U}^{n+1/2} + \vec{U}^{n-1/2}}{2\gamma^n} \times \vec{b}^n \quad (9) \]

Equation (9) is solved using the Boris-Bunneman algorithm in a leap-frog scheme. New positions in each simulation time step is calculated by Equation (10), where \( \gamma^{n+1/2} = [1 + (U^{n+1/2})^2]^{1/2} \). These positions are normalized with respect to \( l_o = c/\omega \).
\[ r^{n+1} = r^n + \frac{U_n^{n+1/2} \Delta \tau}{\gamma^{n+1/2}} \tag{10} \]

To find out the guide-center trajectories of fast ions, we solve the Equation (4) to Equation (6) in a fourth order Runge-Kutta algorithm, as is represented in Equation (11).

\[
Y^{n+1} \approx Y^n + \frac{\Delta t}{6} \left( k_1 + 2k_2 + 2k_3 + k_4 \right)
\]
with:
\[
\begin{align*}
k_1 &= RH \left( Y^n \right) \\
k_2 &= RH \left( Y^n + \frac{1}{2} k_1 \Delta t \right) \\
k_3 &= RH \left( Y^n + \frac{1}{2} k_2 \Delta t \right) \\
k_4 &= RH \left( Y^n + k_3 \Delta t \right)
\end{align*}
\tag{11}
\]

where \( Y \) denotes the set of the independent variables \( (\vec{X}, p||) \) and RH have the information of the right side in Equation (4) and Equation (5); \( n \) and \( \Delta t \) denotes the temporal index and temporal step respectively.

4. Results

With the aim to obtain a similar equilibrium state profiles like in MAST-reactor, we choose the \( R_o = 0.85 \) m, \( a_o = 0.6 \) m, \( \gamma = 1.1/a_o \), \( \delta = (R_o - 0.62)/a_o \) and \( \tau_o = 0.2 \).

Figure 2 shows the iso-pressure surfaces, a magnetic field line and the radial dependence in the \( z = 0 \) plane of the pressure, the toroidal current density and the safety factor. The pressure at magnetic axis and the total current values in each iteration are \( P_o = 30 \) kPa and \( I_T = 1.6 \) MA respectively.

In the simulations, the fast ions of different velocities and momenta are injected into different points of the reactor volume. Figure 3(a) demonstrates the banana orbits in the poloidal plane of an ion injected into the point with the coordinates \( r = 0.51 \) m and \( z = 0.4 \) m at the kinetic energy \( k_o = 10 \) keV and \( p|| = -0.019 kg \cdot m/s \). This trajectory corresponds to the confined ions whose 3D view is pictured in Figure 3(b). The passing trajectories demonstrated in 3(c) and 3(d) represent the ions injected into the point \((r, z) = (0.4 \) m, \( 0.1 \) m) with \( k_o = 50 \) keV and \( p|| = 0.165 kg \cdot m/s \). The motion trajectory repeats a plasma border geometry in the poloidal plane but the 3D view in Figure 3(d) shows that this ion is not confined in the toroidal direction because of its chaotic motion.
Figure 3. Trapped particle trajectory plotted in (a) poloidal plane and (b) 3D view. Passing particle trajectory plotted in (c) poloidal plane and (d) 3D view.

In the simulations, the GC solutions (blue curve) and the non-averaged solution (red curve) of the motion equation are compared to evidence a close agreement between them. Figure 4(a) presents the paths of the guiding centre motion for the ions injected into the different points with different energies and momenta. It should be noted that near the plasma border the banana orbits are thinner. Figure 4(b) evidences that the passing particles take a plasma shape. In both cases the ions are characterized by the closed trajectories in the poloidal plane.

Figure 4. Fast ions guiding-centre trajectories for (a) trapped and (b) passing particles in the poloidal plane.

The simple test for determining errors and stability of the employed numerical scheme is the relative variation of energy in each time iteration. The calculated values for the trapped ion are shown in Figure 5 (the upper part) and for the passing ion they are also presented in Figure 5 (the lower part). One can see that the maximal error does not exceed 4% and the energy oscillations don’t grow in time. This result is acceptable but requires a further analysis of the error accumulation effect in the simulations.

Figure 5. Time evolution for the numerical error in the energy for a (up) trapped ion and for a (down) passing particle.
5. Conclusions
This is our preliminary study of the fast ion dynamics made with the help of our simulation software which is still in progress. The used simulation code has solved the exact Newton-Lorentz equation, and its GC version reviling a good stability has been tested during 5 poloidal periods.

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