Electroweak Bremsstrahlung in Dark Matter Annihilation

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A conservative upper bound on the total dark matter (DM) annihilation rate can be obtained by constraining the appearance rate of the annihilation products which are hardest to detect. The production of neutrinos, via the process $\chi\chi \rightarrow \bar{\nu}\nu$, has thus been used to set a strong general bound on the dark matter annihilation rate. However, Standard Model radiative corrections to this process will inevitably produce photons which may be easier to detect. We present an explicit calculation of the branching ratios for the electroweak bremsstrahlung processes $\chi\chi \rightarrow \bar{\nu}\nu Z$ and $\chi\chi \rightarrow \bar{\nu}\nu W$. These modes inevitably lead to electromagnetic showers and further constraints on the DM annihilation cross-section. In addition to annihilation, our calculations are also applicable to the case of dark matter decay.

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I. INTRODUCTION

The identity of the dark matter (DM) is one of the great unresolved questions in particle physics and cosmology [1, 2, 3]. An important method of probing DM properties is via indirect detection, whereby we look for the appearance of particles produced via DM annihilation or decay. We can search for such a signal emanating from the dark matter concentration in our own galaxy, other galaxies (satellite, dwarf, or clustered), or for an isotropic flux from the dark matter distributed throughout the Universe [4]. Investigated signals include positrons, gamma-rays, x-rays, and even microwaves (the “WMAP haze” [5]).

If we make the reasonable assumption that DM decay or annihilation products must be Standard Model (SM) particles (i.e. we assume the dark matter is the lightest stable particle in the beyond-SM sector) then it is possible to set a conservative upper bound on the total DM annihilation rate by looking for the annihilation products which are the hardest to detect, namely, neutrinos [6]. All other possible final states would lead to the production of gamma rays, for which more stringent bounds apply. For example, quarks and gluons hadronize, producing pions and thus photons via $\pi^0 \rightarrow \gamma\gamma$; the decays of $\gamma\gamma$ and $Z\gamma$ also produce $\nu\nu$. Charged particles produce photons via electromagnetic radiative corrections [7, 8], while energy loss processes for $e^{\pm}$ also produce photons [9]. By calculating the cosmic diffuse neutrino flux produced via the DM annihilation process $\chi\chi \rightarrow \bar{\nu}\nu$ in all halos throughout the Universe, a strong and general bound on the total DM annihilation cross section has recently been derived [10]. The corresponding signal from our own galaxy can be used to set a comparable limit (and improves upon the cosmic bound in some mass ranges) [10] while the technique has been extended to low (MeV) masses in Ref. [11]. Analogous bounds have been derived for the DM decay rate [12].

The general upper bound on the total DM annihilation cross section defined via the limit on $\chi\chi \rightarrow \bar{\nu}\nu$ is surpris-ingly strong. (See Ref. [13] for a comparison between photon-based and neutrino-based limits.) However, a scenario in which neutrinos alone are produced in the final state is technically impossible. Even leaving aside the theoretical issue that a direct coupling of DM to only neutrinos violates the $SU(2)$-invariance of the weak interaction, electroweak radiative corrections imply indirect couplings to states other than neutrinos. For example, for energies above $M_W$, electroweak bremsstrahlung of $W$ or $Z$ bosons can occur at sizeable rates [14, 15], see Fig. 1. The hadronic decays of these gauge bosons produce neutral pions, which decay to gamma rays. Even for energies below $M_W$, processes involving virtual electroweak gauge bosons will lead to particles with electromagnetic interactions, though the rate for such processes is suppressed at low energy.

Kachelriess and Serpico have estimated the constraints on the cross section for $\chi\chi \rightarrow \bar{\nu}\nu$ (and hence on total DM annihilation cross section) by considering gamma rays produced via the accompanying process $\chi\chi \rightarrow \bar{\nu}\nu Z$ [14]. We present here an explicit calculation of the branching ratios for the electroweak bremsstrahlung processes $\chi\chi \rightarrow \bar{\nu}\nu Z$ and $\chi\chi \rightarrow \bar{\nu}\nu W$.

Note that the expected magnitude of the DM total annihilation cross section varies enormously between specific models. For LSP (lightest supersymmetric particle) DM, $s$-wave annihilation to fermions is helicity sup-
Lorentz-Invariant Phase Space (LIPS) is given by

\[ \text{LIPS}^{(2)}(x, y, z) = \frac{1}{8\pi} \frac{\sqrt{\lambda(x, y, z)}}{x} \left( \frac{d\Omega}{4\pi} \right)^2, \]

where \( d\Omega \) is the CoM solid angle, and \( \lambda(x, y, z) \) is the triangle function given by

\[ \lambda(x, y, z) = x^2 + y^2 + z^2 - 2xy - 2xz - 2yz. \] 

The phase space may be written in a form useful for our calculation,

\[ \text{LIPS}^{(4)} = \frac{1}{16} \frac{1}{(2\pi)^4} \int d^2 q^2 \int dQ^2 \frac{(P^2 - q^2)(q^2 - Q^2)}{P^2 q^2} \times \left( \frac{d\Omega_P}{4\pi} \right) \left( \frac{d\Omega_q}{4\pi} \right) \int d\text{LIPS}^{(2)}(Q^2, 0, 0). \]

II. \textsc{W}-\textsc{Strahlung}

We shall first consider DM annihilation to four body final states via W-strahlung, an example of which is the \( \chi\chi \rightarrow e^+ e^- \bar{\nu} \bar{\nu} \) process shown in Fig. 2 and calculate the ratio of the cross section for this process, to that for the lowest order tree level process. The phase space then factorizes into a product of the two virtual particle momenta:

\[ \text{LIPS}^{(4)} = (2\pi)^4 \int dk_2 \int dk_1 \int dl_2 \int dl_1 \times \delta^4(P - l_1 - l_2 - k_1 - k_2), \]

where \( dk \equiv (2\pi)^{-3} d^3 k / 2k_0 \), etc. We integrate over the momenta of the virtual particles by inserting

\[ d^3 q d^4 Q \delta^4(q - k_2 - Q) \delta^4(Q - l_1 - l_2), \]

which ensures momentum conservation for the virtual processes. The phase space then factorizes into a product of three separate two-body phase space factors convolved over the two virtual particle momenta:

\[ \text{LIPS}^{(4)} = \int dQ^2 \int dk_1 \text{LIPS}^{(2)}(q^2, k_1^2) \times \int d\text{LIPS}^{(2)}(Q^2, k_2^2) \int d\text{LIPS}^{(2)}(Q^2, l_2^2, l_1^2). \]

Each two-body differential phase space factor is easily evaluated in the respective two-body center of momentum (CoM) frame, using the expression

\[ d\text{LIPS}^{(2)}(x, y, z) = \frac{1}{8\pi} \frac{\sqrt{\lambda(x, y, z)}}{x} \left( \frac{d\Omega}{4\pi} \right)^2, \]

where \( d\Omega \) is the CoM solid angle, and \( \lambda(x, y, z) \) is the triangle function given by

\[ \lambda(x, y, z) = x^2 + y^2 + z^2 - 2xy - 2xz - 2yz. \] 

We now multiply this partial result by the remaining part of the phase space in Eq. (3), and perform the integrations over \( d\Omega_P, d\Omega_q, \) and \( q^2 \), to obtain the rate \( \Gamma(\rightarrow \nu_\ell \bar{\nu}_\ell e^+ e^-) \). We wish to compare this rate with that for the \( B^+ \rightarrow \nu_\ell \bar{\nu}_\ell, \) for which the lowest-order tree-level expression is \( d\text{LIPS} \sum_{\text{spins}} |\mathcal{M}|^2 = g_B^2 s / (4\pi). \) The
The resulting expression for the ratio of rates for these two processes is
\[ \frac{\Gamma(\bar{\nu}_e e^- W^{\pm} \rightarrow \nu_e \bar{\nu}_e e^+ e^-)}{\Gamma(\nu_e \bar{\nu}_e \rightarrow Z)} = \frac{g^4}{3^2 2^9 (2\pi)^4} x_W^2 \] (10)
expressed in terms of dimensionless scaling variables \( y \equiv Q^2/s \) and \( x_W \equiv s/m_W^2 \).

In Fig. 3 we plot the integrand of Eq. (10) versus \( y = Q^2/s \); in ascending order of the curves, the values of \( x_W = s/m_W^2 \) are 0.9, 1.0, 1.1, 1.5, 2.0, and 5.0. The figure reveals that, for \( y \) expressed in terms of dimensionless scaling variables, i.e., for \( m = 1.1, 1.5, 2.0, \) and 5.0, the “9” comes from adding the \( W^- \)-strahlung channel to the \( W^+ \) channel. Note that the two amplitudes do not interfere since the charges of the produced \( W \)'s, and therefore of the pairs they produce with invariant mass \( M \sim m_W \), are distinguishable. The “9” comes from summing over all decay channels available to the decaying \( W \). We have three leptonic channels, and two quark flavor channels, the latter multiplied by three for color channels.

We may also evaluate the \( W \)-width. At the level of our calculation, we have for this quantity
\[ \Gamma_W = (9) \times \frac{g^2}{48\pi} m_W \] (13)
Here we have dropped the flavor subscript on the (anti)neutrinos since this ratio remains the same when flavors are summed, and multiplied by a prefactor of \( 2 \times 9 \), which we now explain. The “2” comes from adding the \( W^- \)-strahlung channel to the \( W^+ \) channel. Note that the two amplitudes do not interfere since the charges of the \( W \)'s, and therefore of the pairs they produce with invariant mass \( M \sim m_W \), are distinguishable. The “9” comes from summing over all decay channels available to the decaying \( W \). We have three leptonic channels, and two quark flavor channels, the latter multiplied by three for color channels.

We may also evaluate the \( W \)-width. At the level of our calculation, we have for this quantity
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The “9” here is the same final state count that appeared in Eq. (12). Inputting this width into Eq. (12) and using \( g^2 = 4\pi\alpha/\sin^2\theta_w \), we arrive at our final expression,
\[ R_W = \left( \frac{\alpha}{4\pi \sin^2 \theta_w} \right) \frac{x_W}{48} \times \left[ 1 + \frac{17}{x_W} - \frac{9}{x_W} - \frac{9}{x_W} + \left( \frac{6}{x_W} + \frac{18}{x_W^2} \right) \ln(x_W) \right] \] (14)

We note that this expression may also be obtained by directly computing the production of real (on-shell) gauge bosons. However, in that case one must choose unitary gauge, where all degrees of freedom are physical, in order to reproduce Eq. (14).

For our numerical work, we will take \( \sin^2 \theta_w = 0.231 \) (\( \sin^{-2} \theta_w = 4.33 \)), and \( \alpha = 1/128 \) as appropriate for physics at the electroweak scale. This latter choice is especially appropriate in light of the accuracy of the NWA; \( q^2 \) of the virtual neutrino will itself have a value near the threshold for on-shell \( W \) production, i.e., at \( \sim m_W^2 \).

### III. Z-STRAHLUNG

The cross section for the Z-strahlung process, \( \chi\chi \rightarrow \bar{\nu}\nu Z^0 \rightarrow All \), may be calculated similarly to that for W-strahlung. For a scalar coupling as assumed in Fig. 2 there is no interference between diagrams in which the \( Z \) is radiated by the \( \nu \) and \( \bar{\nu} \). Thus the cross section for the \( Z \) channel is simply obtained from that for the \( W \) channel given in Eq. (14) by dividing by a factor of \( 2\cos^2\theta_W \), viz.
\[ R_Z(x_Z) = \frac{1}{2\cos^2 \theta_W} R_W(x_Z) \] (15)

### IV. DISCUSSION

Figure 11 shows the ratios \( R_W \) and \( R_Z \) as functions of \( m_\chi \). We choose to plot rates versus \( m_Z \) rather than
$x_W$ and $x_Z$ to make the presentation more physical, and to better illustrate the difference between the $W$ and $Z$ rates. To convert from the scaling variables to $m_\chi$, we have used the expressions $r_G = s/m_G^2 \approx 4(m_\chi^2/m_G^2)$, $G = W, Z$, appropriate for non-relativistic dark matter. The curve for $R_Z$ may be directly compared to the points from Ref. [14], which we show. Qualitative but not quantitative agreement is evident.

Let us discuss some general features of $R_G$, $G = W, Z$. At large $s \gg m_G^2$, we expect a leading term linear in the dimensionless variable $x_G = s/m_G^2$. The factor of $s$ arises from the ratio of 3-body to 2-body phase space, while the numerator is provided by the only other dimensionful quantity in the process. From Fig. 4 we see that the leading linear term indeed dominates above $x_G \sim 10$, which corresponds to $m_\chi \gtrsim 1.5m_G$. Thus, we may write a very simple expression for the width ratio at $x_G \gtrsim 10$. It is

$$R_W = \left(\frac{\alpha}{4\pi \sin^2 \theta_w}\right) \left(\frac{x_W}{48}\right)^2,$$

and likewise times $(2 \cos^2 \theta_W)^{-1}$ for $R_Z$. It is unsurprising that the inequality $s \gtrsim 10m_G^2 \Leftrightarrow m_\chi \gtrsim 1.5m_G$ has appeared twice, earlier to put the $W$ or $Z$ on-shell, and here to impose the dominance of the leading term in the expression for $W$- or $Z$-strahlung process.

In the very large $s$ (or equivalently, the very large $m_\chi$) limit, the branching ratio for multi $W/Z$ production will become sizeable. We may estimate the onset of double-$W/Z$ production. The general formula for $n$-body massless phase space is

$$LIPS^{(n)} = \frac{1}{8\pi} \left(\frac{s}{16\pi^2}\right)^{n-2} \frac{1}{(n-1)! (n-2)!}.$$ 

Neglecting combinatoric factors, the perturbative expansion parameter for additional $W/Z$ bosons is then $(s/16\pi^2)^2 \sim \alpha m_\chi^2/m_G^2$. Thus, perturbation theory becomes unreliable and multiple $W/Z$ production occurs when $x_G \sim 2\pi/\alpha$, or $m_\chi \sim \alpha^{-1/2}m_G \sim \text{TeV}$. Resummations in the very large $s$ regime, involving ordered $\ln^2(x_G)$ terms from emission of (nearly) massless or collinear $W/Z$’s, are discussed in Ref. [15]. Co-emission of a hard photon will also occur, at a rate comparable to double $W/Z$ emission [16].

Finally, we note that our results are easily applied to the DM decay process $\chi \rightarrow \nu \bar{\nu}$, where $\chi$ is now a boson, with $x_G = s/m_G^2 \rightarrow m_\chi^2/m_G^2$. Indeed, similar expressions will hold for any neutrino production mechanism in which the invariant mass of the $\nu \bar{\nu}$ exceeds $m_W$.

V. CONCLUSIONS

The decay of $W$ and $Z$ bosons produced via electroweak bremsstrahlung will lead to neutral pions and thus photons. One may constrain this DM annihilation signal by considering its contribution to the Galactic or extragalactic diffuse gamma ray background. This was considered in Ref. [14], where it was shown that the contribution of the process $\sigma(\chi \chi \rightarrow \nu \nu Z)$ to the galactic gamma ray background imposed limits on the lowest order process, $\sigma(\chi \chi \rightarrow \nu \nu)$, comparable to those obtained directly with neutrinos. The branching ratio expressions we have derived differ quantitatively, through not qualitatively, from the cross-section estimates in Ref. [14]. Thus, our results lead to similar bounds. Future Galactic gamma ray observations, such as those to be made by GLAST, have the potential to somewhat reduce the diffuse backgrounds through better point source identification, and to measure the background more precisely. In turn, this will strengthen the electromagnetic constraint on DM annihilation, and increase the utility of the quantitative results we have presented herein.

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VI. APPENDIX: INITIAL ($\chi\chi$) OR FINAL ($\nu\nu$) STATE MAJORANA FERMIONS

Although our goal in this paper has been to present radiative corrections to the no-neutrino tree-level process in a manner as model-independent as possible, it is nevertheless interesting to ask what constraints would arise if the particles of either the initial $\chi\chi$ state or the final $\nu\nu$ state (or both) are Majorana particles. It is quite possible that neutrinos are Majorana particles. It is also possible that the DM is Majorana. Although the LSP in supersymmetric extensions of the SM is not a neutrinos-only DM candidate, it provides a popular example of Majorana DM. SUSY examples of Majorana fermions include the neutralino and the photino. (On the other hand, Kaluza-Klein DM, with mass reflecting the length scale of extra dimensions, is a popular example of non-Majorana DM. Typically the LKP is the bosonic recurrence of the photon. For a no-neutrinos model, the DM would be different again, and currently unknown.)

Two identical fermions comprise a Majorana pair. A fermion pair can have total spin $S$ in the symmetric state $S = 1$ or in the antisymmetric state $S = 0$. The parity of the two-fermion state is $P = (-)^{L+1}$, where $L$ is the orbital angular momentum of the pair. This parity formula holds for both Dirac and Majorana pairs. The negative intrinsic parity of the pair, independent of the orbital parity $(-)^{L}$, is the same for Dirac and Majorana pairs for different reasons. In the Dirac case, the $u$ and $v$ spinors (equivalently, the positive and negative energy states) are independent and have opposite parity corresponding to the $\pm 1$ eigenvalues of the parity operator $\gamma^0$. Reinterpreting the two spinor types, or positive and negative energy states, as particle and antiparticle, then leads directly to opposite intrinsic parity for the particle-antiparticle pair. In the Majorana case, the fermion has intrinsic parity $\pm i$, and so the two-particle state has intrinsic parity $(\pm i)^2 = -1$.

On general grounds, the $L$th partial wave contribution to the annihilation rate is suppressed as $v^{2L}$, where $v$ is the relative velocity between the heavy, non-relativistic $\chi\chi$ pair. The virial velocity in our Galactic halo is only $v \sim 300$ km/s $\sim 10^{-3}$c, so even for $L = 1$ the suppression is considerable. Thus only the $L = 0$ partial wave gives an unsuppressed annihilation rate in today’s Universe. The $L \geq 2$ states are too suppressed to contribute to observable rates.

A Majorana pair is even under charge-conjugation (particle-antiparticle exchange), and so from the general relation $C = (-)^{L+S} = +1$ one infers that $L$ and $S$ must be either both even, or both odd for the pair. The origin of the $C = (-)^{L+S} = +1$ rule is as follows: Under particle-antiparticle exchange, the spatial wave function contributes $(-)^L$, and the spin wave function contributes $(+1)$ if in the symmetric triplet $S = 1$ state, and $(-1)$ if in the antisymmetric $S = 0$ singlet state, i.e., $(-)^{S+1}$. In addition, there is an overall $(-1)$ from anticommutation of the two particle-creation operators $b^\dagger b^\dagger$ for the Dirac case, and $b^\dagger b^\dagger$ for the Majorana case.

Consider the $L \leq 2$ states. In spectroscopic notation $(2S+1)L_J$ and spin-parity notation $(J\,^P\!\!C)$, the vector $^3S_1 (1^-)$, C-odd axial vector $^1P_1 (1^+)$, and assorted $^3D_J (J^-)$ states are all C-odd and therefore disallowed. The pseudoscalar $^3S_0 (0^-)$, scalar $^3P_0 (0^+)$, axial vector $^3P_1 (1^+)$, C-even tensor $^3P_2 (2^+)$, and pseudotensor $^3D_2 (2^-)$ are all C-even and therefore allowed. In particular, the sole $L = 0$ state, with no $v^{2L}$ suppression, is the pseudoscalar $^1S_0 (0^-)$.

Incidentally, at threshold, defined by $s = (2M_\chi)^2$ or $v = \sqrt{1 - 4M_\chi^2/s} = 0$, the orbital angular momentum $L$ is necessarily zero. With two identical Majorana fermions, the two-particle wave function must be antisymmetric under particle interchange. Since $L = 0$ at threshold, the $\chi\chi$ spatial wave function is even, and the wave function must be antisymmetrized in its spin. The antisymmetric spin wave function is the $S = 0$ state. Thus, the only contributing partial wave at threshold is the $^1S_0$ state. We have just seen that this is also the only state with no $v^{2L}$ suppression, so one may expect an unsuppressed Majorana annihilation rate at threshold if and only if there is a $^1S_0$ partial wave.

One may also invoke CP invariance to note that the spin $S$ of initial and final two-fermion states, Dirac or Majorana, are the same. This follows simply from $CP = (\chi\chi) = (-)^{L+S}(-)^{L+1} = (-)^{S+1}$, and the fact that $S = 0, 1$ are the only possibilities for a pair of spin $1/2$ particles.

What does this all mean for a model with an s-channel exchange particle coupling to Majorana bilinears? It means that among the basis fermion bilinears, the candidates are just the pseudoscalar $\bar{\psi}\gamma_5\psi (0^-)$, the scalar $\bar{\psi}\psi (0^+)$, and the axial vector $\bar{\psi}\gamma_\mu\gamma_5\psi (1^+)$. The vector $\bar{\psi}\sigma^{\mu\nu}\psi (1^-)$, tensor $\bar{\psi}\sigma^{\mu\nu}\psi (2^-)$, and pseudotensor $\bar{\psi}\gamma_5\sigma^{\mu\nu}\psi (2^-)$ bilinears are C-odd and therefore disallowed. The only s-channel particles which may couple to these candidate bilinears are the pseudoscalar, scalar, or axial vector.

There is some subtlety associated with the s-channel exchange of an axial-vector. The axial-vector is an $L = 1$ mode, and we have seen that this mode elicits a $v^2$ suppression in the rate. However, the exchange particle is off-shell (away form resonance) and so has a time-like pseudoscalar piece in addition to the axial three-vector piece. This pseudoscalar coupling is effectively $\partial_\mu (\bar{\psi}\gamma_\mu\gamma_5\psi)$. The weak interaction coupling of the pion to the axial vector current provides a familiar example of such a coupling. The axial current is not conserved, and so the pseudoscalar coupling is nonzero. One has $\partial_\mu (\bar{\psi}\gamma_\mu\gamma_5\psi) = 2i\partial_\mu\bar{\psi}\gamma_5\psi - \partial_\mu\bar{\psi}(\gamma^{\alpha\beta}\gamma_5\lambda_\alpha)k_\beta\lambda_\beta(k)$. The first term shows an $m_\omega$-dependence in the amplitude, leading to $(m_\omega/M_\chi)^2$ helicity-suppression of the $L = 0$ piece, while the second term is the famous anomalous VVA coupling. It offers $W^+W^-$ and $ZZ$ production (with momenta $k$, $\lambda$ and helicities $\lambda(k)$, $\lambda(k)$), but at higher order $\sigma_\omega = g_\omega^2/4\pi$ in the electroweak $W\nu\nu$ or $Z\nu\nu$ coupling $g_\nu$. The linear combination of a $v^2$-
suppressed $L = 1$ piece and a $m_L^2$-suppressed $L = 0$ piece to the rate from axial-vector exchange was first noticed by Goldberg [17].

For $s$-channel exchange of a true pseudoscalar or scalar particle, there is no helicity suppression. In addition, it was shown several years ago [5] that helicity suppression in the axial-vector case may be avoided when the two-body final state is replaced by a three-body final state. In the work of [5], a charged pair was produced and a photon was radiated from one of the charged particles. Radiation of the photon changes the Dirac structure of the current; it also allows one fermion to become virtual, with a large $k^2$ replacing its small mass. It was found that bremsstrahlung from the final state was dominated by collinear and infrared emission, which left the emitting fermion nearly on-shell, and helicity suppressed. However, photon emission from the accompanying $t$-channel particle did give an unsuppressed amplitude. In our work, it is a massive $W$ or $Z$-boson that is radiated. There are no collinear of infrared singularities, and the virtual fermion is necessarily off-shell by $k^2 \sim M_W^2$. It seems likely to us that this $2 \to 3$ rate for the radiatively corrected $s$-channel axial-vector exchange will exceed the helicity-suppressed $2 \to 2$ rate by $(k/2M)^2$, which is many orders of magnitude. We do not pursue this feature of the axial-vector exchange further here, for the premise of the neutrino-only model, investigated in this work, is that the tree-level annihilation rate to neutrinos is unsuppressed.

So far we have discussed $s$-channel exchange processes. We turn now to a brief discussion of $t$-channel exchange annihilation models. The implications of a Majorana $\chi\chi$ pair are best recognized by Fierz transforming the two fermion bilinears to “charge-retention” order, i.e., to a $\chi$-bilinear and a $\nu$-bilinear. If the Fierz’d bilinears contain a pseudoscalar, there is no suppression of the rate. Otherwise, there is a $v^2$ rate suppression. If the Fierz’d bilinears contain an axial vector piece, it contributes to the rate a $v^2$-suppressed piece, and a $(m_v/M)^2$ helicity-suppressed piece. (However, as just explained above, a $W$ or $Z$ radiated in the final state may well lead to a much enhanced rate.)

It is illuminating to explain in this context the often seen remark that LSP-annihilation has a helicity-suppressed rate to fermions. This is true for SUSY extensions of the SM. It is not true in general for models of dark matter. For SUSY extensions of the SM, the annihilation graphs consist of $t$-channel scalar exchanges, and from crossing the identical Majorana fermions, also $u$-channel scalar exchanges; in addition, there are scalar, pseudoscalar and axial-vector $s$-channel exchanges. Fierzing the $t$- and $u$-channel scalar exchanges yields $s$-channel axial-vector bilinears, with the concomitant helicity-suppressed $L = 0$ contribution and $v^2$-suppressed $L = 1$ contribution to the annihilation rate. The only contributions that are potentially large come from the $s$-channel pseudoscalars. However, the scalars and pseudoscalars are Higgs particles, whose Yukawa couplings $g_Y$ to the SM fermion are all proportional to $(m_f/\text{vev})$, thereby giving the same effect as a true helicity suppression. Recall that the Higgses are assigned the double burden of providing mass to the electroweak gauge bosons and to the fermions. The $g_Y \propto \text{vev}^{-1}$ relation can be traced back to mass generation of the gauge bosons, while the $g_Y \propto m_f$ relation comes from mass generation for the fermions. There are other possibilities for the Yukawa coupling to neutrinos. To give one example, if the neutrino mass is small as a result of a see-saw mechanism, then the coupling of the Higg doublet to the neutrino field will be $\sim \text{GeV}/\text{vev}$, similar to the coupling of the Higgs to most quarks. And importantly, a more general scalar or pseudoscalar field, not complicit in fermion mass generation, would couple with an arbitrary $g_Y$.

The upshot of all this for our investigation is the following: With an $s$-channel scalar or pseudoscalar $B$-meson exchange, there is no helicity suppression. The Yukawa coupling of the $B$-meson to $\nu\nu$ (and to $\chi\chi$) is arbitrary. The scalar exchange proceeds in the $L = 1$ partial wave, which suppresses the $\chi\chi$ annihilation rate by $v^2$. On the other hand, the pseudoscalar exchange proceeds in the $L = 0$ partial wave, with no $v^3$ suppression of the rate. These deductions from partial wave analysis hold true for Dirac or Majorana $\chi$.

In the rate ratio that we investigate, any $v^2$ rate suppression factors out. Moreover, when our assumed scalar exchange is replaced with a pseudoscalar exchange, there is no change in the rate ratio. This is easily seen by noting that placement of an $i\gamma_5$ in the amplitude of Eq. (7) just before the spinor $v(k_1)$ alters this amplitude by just the overall phase $-i$ (neglecting the neutrino mass). Thus, our results as presented hold also for the unsuppressed $L = 0$ $s$-channel exchange, all the way down in energy to the $\chi\chi$ annihilation threshold.

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[16] While unitarity of the DM annihilation cross-section does not directly impact the results in this paper, we take this opportunity to summarize the unitarity constraints. The velocity-averaged annihilation rate must satisfy $\langle v_{\text{rel}} \sigma_{\text{ann}} \rangle \leq \frac{4\pi}{m_\chi^2 v_{\text{rel}}}$. In terms of the spinless $B$ exchange model assumed in this work, the unitarity constraint becomes (for $m_B^2 \gg m_\chi^2$)

$$\left( \frac{M_B}{M_B} \right)^2 \left( \frac{\Gamma(B \rightarrow \chi \chi)}{M_B} \right) \left( \frac{\Gamma(B \rightarrow \text{SM})}{M_B} \right) \leq 1,$$

where SM refers to Standard Model final states. There is no unitarity constraint on a decay width, a remark relevant for the case where DM decay dominates DM annihilation.

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A detailed calculation of the related amplitude $e^+ e^- \rightarrow \tilde{\gamma} \tilde{\gamma}$ involving two identical Majorana particles is available in App. E (as well as a lucid and complete presentation of Feynman rules for Majorana fermions in App. D) of H. E. Haber and G. L. Kane, Phys. Rept. 117, 75 (1985). Another lucid listing of Feynman rules for Majorana fermions is available in Chapter 49 of “Quantum Field Theory”, by M. Srednicki, Cambridge University Press.

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