Capacity Enhancement with Meta-Multiplexing

C. Ji†*, R. Liu†, and D. Li†

Kuang-Chi Institute of Advanced Technology, Shenzhen, China

† These authors contributed equally to this work.
* To whom correspondence should be addressed. E-mails: chunlin.ji@kuang-chi.org.

Multiplexing services as a key communication technique to effectively combine multiple signals into one signal and transmit over a shared medium. Multiplexing can increase the channel capacity by requiring more resources on the transmission medium. For instance, the space-division multiplexing accomplished through the multiple-input multiple-output (MIMO) scheme achieves significant capacity increase by the realized parallel channel, but it requires expensive hardware resources. Here, we present a novel multiplexing methodology, named meta-multiplexing, which allows ordinary modulated signals overlap together to form a set of “artificial” parallel channels; meanwhile, it only requires similar resources as ordinary modulation schemes. We prove the capacity law for the meta-multiplexing system and disclose that under broad conditions, the capacity of a single channel increases linearly with the signal to noise ratio (SNR), which breaks the conventional logarithmic growth of the capacity over SNR. Numerous simulation studies verify the capacity law and demonstrate the high efficiency of meta-multiplexing. Through proof-of-concept hardware experiments, we tested the proposed method in communica-
tion practices and achieved a spectral efficiency of 81.7 bits/s/Hz over a single channel, which is significantly higher than the efficiency of any existing communication system.
In the celebrated 1948 paper (1), Shannon derived the seminal formula for the capacity of the additive linear white Gaussian noise (AWGN) channel: $C = W \log_2(1 + \text{SNR})$, where the capacity $C$ is the tight upper bound of the rate at which information can be reliably transmitted over the channel, the capacity $C$ is determined by the bandwidth $W$ and the ratio of signal power to noise power SNR. People have made tremendous efforts to approach the ideal capacity limit (2). The capacity of a single channel is hard to break, except for channels with nonlinearity (3, 4). People also appeal to the multiplexing techniques (5, 6) to increase the capacity of a communication system by allocating more resources on the transmission medium: MIMO communication systems (see Fig. S1 in (7)) establish a spatial parallel channel between the transmitter and receiver using multiple antenna pairs. The space-division multiplexing through the parallel channel significantly increases the capacity of MIMO systems. For instance, in an ideal situation, the capacity of a MIMO system is $N$ times larger than a single channel’s capacity, where $N$ is the smaller of the number of transmitter or receiver antennas. However, MIMO requires extra expensive radio frequency infrastructures and the availability of multiple independent channels, which may limit its applications. This paper introduces a novel multiplexing framework that incorporates ordinary modulated signals to construct an artificial parallel channel for a single physical channel without requiring much extra time or frequency resources. The proposed system requires more computation efforts but leads to a dramatic increase in the capacity. Under surprisingly broad conditions, such as various parameter setting for communication systems or even with interference from neighboring frequency bands, the system possesses a region in which the capacity increases linearly with the SNR. We named the system the meta-multiplexing, which is aligned to the “artificial” material “metamaterial” (8), as both utilize ordinary elements to form an “artificial” entirety and achieve extraordinary performance.

Recall that the enhanced capacity of the MIMO system is accomplished by the realized parallel channel derived from the channel matrix (see Fig. S1 in (7)). However, the channel matrix
is leverage on the physical scattering propagation environment (independent uncorrelated spatial paths) and is difficult for humans to manipulate. In the meta-multiplexing, we construct the parallel channel artificially: assuming that the data is transmitted by symbols with symbol time $T$, Fig. 1A shows the naive idea of the meta-multiplexing scheme, in which we accelerate and parallel the transmission to $K$ streams and introduce a carefully designed time delay $T/K$ for each chain of the data stream. The multiple delays between different data streams are predefined; thus, the signals can be superposed together and propagate over a single physical channel, whereas at the receiver, unlike MIMO, we can decode them without requiring multiple antennas to capture different copies of the signal. We can further simplify the structure and process the multiplexing in the digital part as shown in Fig. 1B, which further reduces the cost of the implementation. We illustrate the meta-multiplexing process through a simple example shown in Fig. 2A. The binary phase-shift keying (BPSK) signal is paralleled and transmitted with a delay of $T/K$ for each stream. The summation of energy levels at each time slot $T/K$ produces the multiplexed signal, which is actually a waveform varying in one symbol time. Straightforwardly, the multiplexing process is equivalent to a convolution process in which the BPSK signal is convoluted with the rectangle waveform. Moreover, the pulse shaping filter $h(t)$ can be further extended to represent all of the impulse responses of the entire communication system, including the pulse shaping filter, the channel impulse response and the matched filter if available. Intuitively, the benefit of this scheme is that the data stream has been accelerated $K$ times and transmitted in $K$ parallel chains, and as a consequence, the information is carried by a novel format of waveform without requiring a great amount of extra transmission resources: no multiple antenna, no or limited extra bandwidth (discussed later) and only a negligible extra transmission time $(K - 1)/K \cdot T$.

To explore the capacity benefits of the meta-multiplexing precisely, we derive the capacity law for this communication scheme. As shown in Fig. 1C, the system can be represented by
the matrix format $y = Hx + z$, where vector $x$ and $y$ denotes the transmitted and received signal respectively, and $z$ denotes the white Gaussian noise with variance $N$; $H$ is a matrix in which each row is the impulse response coefficient $h(t)$ shifted with a delay of $T/K$. By applying a singular value decomposition, $H = U\Lambda V$, with unitary matrix $U$ and $V$, and diagonal matrix $\Lambda$, the matrix expression of the communication system is reformed as $\tilde{y} = \Lambda \tilde{x} + \tilde{z}$, where $\tilde{x}$, $\tilde{y}$, and $\tilde{z}$ are coordinate transformations of $x$, $y$, and $z$ (6, 7). The reformed expression clearly shows that the transmission system is equivalent to a parallel channel (6, 9). Thus, by following the deduction of the capacity of the parallel channel (9), the capacity of the meta-multiplexing system can be obtained: $C = \frac{1}{2} \sum_{i=1}^{K} \log_2 (1 + \frac{P_i^* \lambda_i^2}{N})$ (bits/symbol), where $P_i^*$ are the waterfilling power allocations $P_i^* = \left( \mu - \frac{N}{\lambda_i^2} \right)^+$ in which $\mu$ is chosen to satisfy the power constraint $\sum_{i=1}^{K} P_i^* = P$ and $P$ is the total power allocated in one symbol time, and $\lambda_1, \ldots, \lambda_K$, are the singular values of the matrix $H$. A more detailed proof is presented in (7). According to the formula of the capacity law, we obtain two important properties: 1) the capacity increases with the overlap factor $K$ because a number of $K$ non-zero $\lambda_i$ exists in the meta-multiplexing system (7), and each non-zero $\lambda_i$ corresponds to a sub-channel that can support the transmission of one data stream; 2) the capacity depends on the chosen waveform, because different waveforms generate significant different values for $\lambda_i$.

In communication practices, the energy per bit to noise power spectral density ratio $E_b/N_0$ (also known as the “SNR per bit”) is preferred to measure the condition of the communication without taking bandwidth into account. Denote the sampling period by $T_s$, and assume each data stream carries $\eta_i$ bits of information; then, the capacity over $E_b/N_0$ is $C = \frac{1}{2} \sum_{i=1}^{K} \log_2 (1 + \eta_i \frac{T_s E_{b,i}}{N_0} \lambda_i^2)$, where $E_{b,i}^* = P_i^* T / \eta_i$ and $P_i^*$ is the waterfilling power allocation (7). In practice, one simple but common setting is to divide the power evenly for each stream, $P_i = P/K$. In this case, the capacity becomes, $C_I = \frac{1}{2} \sum_{i=1}^{K} \log_2 (1 + \eta_i \frac{T_s E_{b,i}}{N_0} \lambda_i^2)$. Regarding to the capacity law, we first investigate the capacity increasement with $K$. For simplicity, we choose a complex
BPSK signal \((+1, -1, +j, -j)\) as the input for the meta-multiplexing system; thus, \(\eta_i = 2\). The capacity \(C\) and \(C_I\) are calculated and shown in Fig. 3A. The Shannon capacity curve is a special case of \(C\) that \(K = 1\). To emphasize the linear relationship between capacity and \(E_b/N_0\), the capacity is also displayed in a logarithmic scale. The numerical evaluation shows that \(C_I\) is quite close to the ideal capacity \(C\). Additionally, dividing the power evenly reduce the complexity and the cost of implementation of the meta-multiplexing scheme; thus, we use \(C_I\) in the following discussion. To illustrate the capacity under different waveforms, two Taylor waveforms, a Gaussian waveform and a Hamming waveform, are evaluated and shown in Fig. S5 in (7). It is obvious that the capacity of these waveforms is much smaller than that of the rectangle wave because (except for a few major ones) most of their singular values attenuate much faster than singular values of the rectangle wave. Therefore, the efficiency of the corresponding parallel channel is smaller than the rectangle wave. However, in any circumstance, when \(K\) is sufficiently large, the capacity in a high SNR region can still increase linearly with the SNR.

The structure of this communication is not that complex, why does the community ignore it? One reason is that the meta-multiplexing scheme completely breaks the Nyquist intersymbol interference (ISI) criterion. The example depicted in Fig. 2A clearly shows the intersymbol interference. Generally, when ISI occurs due to various reasons such as the multipath propagation or bandlimit of a channel, people tend to mitigate the effects (6). An alternative perspective is to utilize intersymbol interference to help the communication system, which dates back to the early 1970s (10). However, few later works have had significant success. One noticeable work is the faster-than-Nyquist (FTN) signaling (11,12), which intentionally builds a controlled amount of ISI into the communication system. A small amount of overlap on the symbols can carry more information with an increased sample rate. However, this strategy limits the overlap to less than half of a symbol duration. The overlapped multiplexing principle proposed in (13),
further confirms that the overlapping between adjacent symbols can be a beneficial constraint with coding gain, but the overlap factor $K$ is limited to a small value. In this work, we push the limit of intersymbol “interference” so that we not only achieve impressive capacity when the overlap factor $K$ is sufficiently large but also disclose a capacity law that reveals the reason why the capacity increases with $K$.

To design an entire communication system, conventional modules like synchronization and channel estimation are required; meanwhile, moderate modifications are needed to fit them into the meta-multiplexing system. The conventional FEC code scheme can be intergraded straightforwardly. One primary challenge lies in the decoding process of the multiplexed signal: how to decode the symbols, as they are heavily overlapped together. With regard to the convolution structure of the meta-multiplexing, a Viterbi-style algorithm is proposed. The trellis graph shows that the unknown information bits are one-to-one mappings of the trellis nodes, so decoding involves a dynamic search on the trellis graph (see Fig. 2B for an example of the decoded path on the trellis graph). However, the Viterbi-style algorithm can only deal with small $K$, as the decoding complexity increases exponentially with $K$. We further propose a Bayesian Monte Carlo (14, 15) based approximate decoding method. The proposed algorithm uses simulation-based techniques to simulate from a sequence of probability distributions for sequential inference of the posterior distribution of unknown information bits. The proposed algorithm provides a MAP decoding, whereas the complexity is independent of $K$ (7). The algorithm has also been designed for parallel computing to support hardware implementation.

To simulate the transmission rate (bits per symbol) of the meta-multiplexing over different overlap factor $K$, we choose the complex BPSK signal as the input and the rectangle waveform as the pulse shaping filter. We set a series value for the overlap factor $K = [2, 4, 8, 10, 20, 30, 50, 60, 100, 200, 300, 450, 600, 900, 1200, 1800]$, and run the simulation study for $10^6$ bits. The Bayesian Monte Carlo decoding can support decoding even when $K$ is even larger than 1000.
For each $K$, we evaluate the level of $E_b/N_0$ that the system can gain the transmission rate of $2K$ bits/symbol with the bit error rate (BER) $\leq 10^{-5}$. The simulated transmission rate in bits/symbol over $E_b/N_0$ is shown in Fig. 3A. Obviously, the simulation results support that the realized transmission rate is linearly proportional to the $E_b/N_0$. The discrepancy between the realized transmission rate and the theoretical capacity can be further reduced with the help of a near-capacity forward error correcting (FEC) coding scheme, such as turbo code (16) and LDPC (17, 18). A twofold FEC coding mechanism is applied in our meta-multiplexing system (see Fig. S1 in (7)): the convolution structure of the meta-multiplexing serves as the inner “convolutional” coding, which is a noticeable advantage of our system. For outer FEC coding, we choose a $3/5$ code rate LDPC from the DVB-S.2 standard. An interleaving algorithm is inserted between the inner and outer coding. Simulation studies in Fig. 3A illustrate that with the help of LDPC coding, the realized transmission rate moves approximately 4dB closer to the theoretical capacity curve.

The symbol, mentioned in previous discussion, actually represents a two-dimensional resource containing both the time duration and the frequency bandwidth. However, the dilemma is that for any spectrum bandlimited signal, its waveform is not realizable, and for any realizable waveform, its absolute bandwidth is infinite. In engineering practices, various definitions of the bandwidth for the realizable waveform have been proposed to describe the power distribution of the signal (7). In the meta-multiplexing system, one important property is that the multiplexed signal follows the occupied bandwidth of the pulse shaping filter: the convolution structure of the meta-multiplexing decides that the bandwidth of the multiplexed signal is concentrated and generally less than the bandwidth of the pulse shaping filter. Fig. S4 verifies the spectrum property of the meta-multiplexing signal (7). Given the symbol’s occupied bandwidth $B$, the capacity of the meta-multiplexing can be expressed as $\eta = \frac{1}{2BT} \sum_{i=1}^{K} \log_2(1 + \frac{P_i}{N_i})$ (or $\frac{1}{2BT} \sum_{i=1}^{K} \log_2(1 + \eta_i \frac{T}{T} \frac{E_b}{N_0} \lambda_i^2)$ for the measurement by $E_b/N_0$) (bits/s/Hz) (7). The normalized
capacity (or spectral efficiency) is proportional to the capacity with a coefficient $1/BT$. In a similar setting as discussed before, we simulated the spectral efficiency $E_b/N_0$. The resulting spectral efficiency (in Fig. 4A) shows that the curve moves to the right compared with the capacity curve because that the occupied bandwidth is always larger than $1/T$, and thus, the coefficient $1/BT < 1$.

According to the Nyquist-Shannon sampling theorem (1), the processing bandwidth (the reciprocal of the sampling rate) of a communication signal is at least two times larger than the occupied bandwidth. In the meta-multiplexing, the processing bandwidth becomes much larger than the occupied bandwidth, particularly when $K$ is large (shown in Fig. S5 in (7)). A concern of this multiplexing scheme is how to understand the “problematic” spectrum region that is outside of the occupied bandwidth but inside of the processing bandwidth. Here, we provide a convincing way to eliminate this concern: when the “problematic” spectrum region is occupied by other communication signals, either meta-multiplexing signals or ordinary ones, the meta-multiplexing system can still work properly. Fig. S6 in (7) shows an example in which one 256-QAM signal exits in the processing bandwidth of a meta-multiplexing signal. We define the bounded PSD at 35 dB as the occupied bandwidth; then, the meta-multiplexing signal occupies a bandwidth of 31.6 KHz, while the QAM occupies 750 KHz approximately 75% of the entire processing bandwidth of 1 MHz. A joint decoding strategy is proposed to decode the signal (see (7) for details). The simulated spectral efficiency requires approximately 2 dB higher $E_b/N_0$ than the one without any neighboring signal in the processing bandwidth, but the linear relation with the $E_b/N_0$ still holds (see the spectral efficiency with spectrum sharing for different $K$ in Fig. 4A). Consequently, we can measure the spectral efficiency of the meta-multiplexing only on its occupied bandwidth, and then, the resulting spectral efficiency is consistent linear increase with the SNR, as we discussed previously.

Moreover, to verify that the meta-multiplexing works in real-world physical channels, we
implemented the entire communication system on a standard verification system, the universal software radio peripheral (USRP). The placement of the connection of the USRP devices is shown in Fig. S7 in [7]. All of the algorithms were implemented on the onboard high-performance field-programmable gate array (FPGA). We set $K = 128$ and choose the Taylor waveform with an attenuation level of -35dB for the pulse shaping filter [7]. The artificial delay $T/K$ is 1 millisecond. The meta-multiplexing signal is up-converted to radio frequency at 2.4 Ghz. The spectrum of the channel signal is measured by the spectrum analyzer and displayed in Fig. S8: the measured occupied bandwidth of the channel signal is 24.48 KHz. The noise generator add the Gaussian noise to the channel to evaluate the BER performance. A pilot signal is utilized to estimated the channel conditions for equalization before decoding. We engineer the Bayesian Monte Carlo based approximate decoding algorithm in a parallel style, which can fit the FPGA implementation. Accordingly, all of the defects of a real physical communication system, like analog-to-digital converter (ADC)/digital-to-analog converter (DAC) quantization errors and nonlinearity of the power amplifier, exist in this verification. As shown in Fig. 4B, the dispersion between the simulation and hardware implementation is caused by the effective number of bits (ENOB) of ADC and DAC in the system. The BER of this hardware implementation, which is highly consistent with the simulation with ENOB=12, confirms that the proposed meta-multiplexing works in practical communication systems. We utilized a 24.48 KHz bandwidth but realized a reliable transmission ($\text{BER} < 10^{-5}$) of 2M bits/sec in the $E_b/N_0$ region of 45 dB, so the spectral efficiency is up to 81.7 bits/s/Hz, which is significantly higher than the efficiency of any existing communication system. Actually, such a high spectral efficiency is generally infeasible for conventional communication methods like high-order QAM modulations: the required ADC and DAC ENOB should be larger than 40, and the required $E_b/N_0$ should be more than 230 dB, all of which are beyond the physical ability of conventional communication systems.
The meta-multiplexing strategy can also be applied in other domains, such as the frequency
domain and joint time-frequency domain, to create other novel multiplexing methods. Take
meta-multiplexing in frequency as an example: the superposition in frequency would save
bandwidth and benefit broadband communication applications. The proposed communication
framework is suitable for different channel conditions. For example, in the multipath Rayleigh
fading channel, the random time dispersion of the channel can be treated as part of the en-
tire pulse response $h(t)$, and blind deconvolution techniques can be employed for the decoding
when part of $h(t)$ is unknown. As a general multiplexing technique offering significant capac-
ity enhancement, meta-multiplexing will bring new possibilities for modern communications
jointly with current communication technologies.

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**List of Figures**
Figure 1: The meta-multiplexing scheme. (A) In meta-multiplexing, the information bits are paralleled to $K$ streams, and an artificial time delay is introduced for each stream of the signal, then transmitted over a MISO system; (B) The combination of the signal in multiple antennas can be further simplified and processed digitally; the meta-multiplexing then requires one single transmitter antenna. (C) The system corresponds to a matrix expression with an artificial channel matrix $H$. 
Figure 2: (A) A simple case of the meta-multiplexing scheme in which $K=3$ and $h(\cdot)$ is a rectangle waveform. The modulated signal is a waveform with maximally $K + 1$ levels in each symbol time $T$. The trellis graph (B) shows the idea of a Viterbi-style maximum likelihood sequence detection method for decoding of the signal in (A): in each time step, the survived path is found in which the nodes have the minimum Euclidean distance with the received signal.
Figure 3: (A) The capacity of meta-multiplexing over different $K$ values, $C$ in dashed lines and $C_I$ in dash-dot lines, show the linear relation between the capacity and the $E_b/N_0$. The simulation of the realized transmission rate of the meta-multiplexing system, solid lines with triangle, also supports the linear relation. With a 3/5 LDPC coding, the simulated transmission rate comes much closer to the theoretical capacity curve; compared with (A), plots in (B) show the capacity and simulated transmission rate with the Taylor waveform (with an attenuation level of -35 dB) and demonstrate that the capacity of the meta-multiplexing scheme is highly dependent on the designed waveforms. The linear relation between the capacity and $E_b/N_0$ appears in a higher $E_b/N_0$ region.
Figure 4: (A) Simulation and compare study of the normalized meta-multiplex capacity and the simulated spectral efficiency of the system with the Taylor waveform; in addition, the spectral efficiency with spectrum sharing with the QAM signal is also drawn on the plot. Both the spectral efficiency and the one with spectrum sharing is reasonably close to the theoretical capacity curve, and both posses a linear increase region when $E_b/N_0$ is sufficiently large; (B) Hardware verification of a meta-multiplexing system in which $K = 128$ and $h(\cdot)$ is a Taylor waveform. The transmission rate is 256 bits/symbol, and the realized spectral efficiency is 81.7 bits/s/Hz. A comparison study with simulated spectral efficiency shows that given the current hardware setting, the major influence of the BER performance is the limitation of the ENOB of ADC and DAC. Moreover, the BER of a high order $2^{16}$-QAM is presented. According theoretical calculations, to achieve a spectral efficiency of 81 bits/s/Hz, the QAM needs an extra 195 dB in the SNR.
Supporting Online Material

The capacity of the parallel Gaussian channel in the meta-multiplexing

In the information theory, the concept of the parallel channel, which refers to a set of non-interfering sub channels, has been well accepted and leverages the capacity theory of MIMO systems [1]. For meta-multiplexing, we have shown in Fig. 1 that the system can be expressed as

\[ y = Hx + z, \]  

(S1)

where vector \( x \in \mathbb{C}^{L_t} \) denotes the transmitted signal, vector \( y \in \mathbb{C}^{L_r} \) denotes the received signal, and \( z \) denotes the additive white Gaussian noise that \( z \sim N(0, N\mathbb{I}_{L_r}) \) at a sample time. \( L_t \) and \( L_r \) denotes the length of transmitted and received signal, and \( \mathbb{C} \) denotes the complex field. The matrix \( H \in \mathbb{C}^{L_r \times L_t} \) represents the entire meta-multiplexing process, in which each row is the impulse response coefficient \( h \) shifted with a delay of \( 1/K \) symbol time (equal to a sample time). Due to the special structure of \( H \), it is easy to verify that the rank of matrix \( H \) is \( L_t \).

To compute the capacity of this vector transmission model, we decompose it into a set of parallel, independent Gaussian sub-channels [1]. Apply the singular value decomposition, \( H = U \Lambda V^* \) where \( U \in \mathbb{C}^{L_r \times L_r} \) and \( V \in \mathbb{C}^{L_t \times L_t} \) are unitary matrices. \( \Lambda \in \mathbb{R}^{L_r \times L_t} \) is a rectangular matrix, the diagonal elements of which are non-negative real numbers and the off-diagonal elements of which are zero. The diagonal elements of \( \Lambda \), \( \lambda_1 \geq \lambda_2 \geq \ldots \geq \lambda_{\min} \), are the ordered singular values of the matrix \( H \). Denote \( \tilde{x} = V^*x \), \( \tilde{y} = U^*y \), and \( \tilde{z} = U^*z \) and then, we obtain the transformed expression

\[ \tilde{y} = \Lambda\tilde{x} + \tilde{z} \]  

(S2)

Two properties hold [1]: \( \tilde{z} \sim N(0, N\mathbb{I}_{L_r}) \) has the same distribution as \( z \), due to the Gaussian variable property; the transmission energy is preserved, that \( ||\tilde{x}||^2 = ||x||^2 \). Therefore, the meta-
The multiplexing system (S1) has an equivalent representation as the parallel Gaussian channel:

\[
\tilde{y}_i = \lambda_i \tilde{x}_i + \tilde{z}_i, \ i = 1, 2, ... n_{\text{min}} \quad \text{(S3)}
\]

The equivalence is depicted in Fig. S3.

We consider a simple case first, in which the number of information bits \( L_t = K \), then \( L_r = 2K - 1 \), and \( n_{\text{min}} = K \). Now, the rank of \( \mathbf{H} \) is \( K \); thus, the number of nonzero singular values is also \( K \). The capacity of the parallel channel (S3) is the maximum of mutual information:

\[
C = \max \sum_{E[\tilde{x}_i^2] \leq P} I(\tilde{x}_1, \tilde{x}_2, \ldots \tilde{x}_K; \tilde{y}_1, \tilde{y}_2, \ldots \tilde{y}_K), \quad \text{(S4)}
\]

where \( P \) is the power constraint. Firstly, we need to prove that the capacity of the meta-multiplexing system (S1) has the same mutual information of the parallel channel (S3). Regards to the equation (S2), when \( L_t = K \), only \( n_{\text{min}} = K \) channel exists which means \( \lambda_i = 0 \) for \( i > K \). Thus, \( \tilde{y}_i = \tilde{z}_i \) for \( i > K \). By utilizing the chain rule for the entropy [2], we obtain that

\[
\begin{align*}
    h(\tilde{y}_1, \tilde{y}_2, \ldots, \tilde{y}_K, \tilde{y}_{K+1}, \ldots, \tilde{y}_{L_r}) &= h(\tilde{y}_1, \tilde{y}_2, \ldots, \tilde{y}_K) + h(\tilde{y}_{K+1}, \ldots, \tilde{y}_{L_r} | \tilde{y}_1, \tilde{y}_2, \ldots, \tilde{y}_K) \\
    &= h(\tilde{y}_1, \tilde{y}_2, \ldots, \tilde{y}_K) + h(\tilde{y}_{K+1}, \ldots, \tilde{y}_{L_r}) \\
    &= h(\tilde{y}_1, \tilde{y}_2, \ldots, \tilde{y}_K) + h(\tilde{z}_{K+1}, \ldots, \tilde{z}_{L_r}), \quad \text{(S5)}
\end{align*}
\]

For the noise \( \tilde{z} \), the chain rule also holds:

\[
\begin{align*}
    h(\tilde{z}_1, \tilde{z}_2, \ldots \tilde{z}_K, \tilde{z}_{K+1}, \ldots, \tilde{z}_{L_r}) &= h(\tilde{z}_1, \tilde{z}_2, \ldots, \tilde{z}_K) + h(\tilde{z}_{K+1}, \ldots, \tilde{z}_{L_r}), \quad \text{(S6)}
\end{align*}
\]

A theorem about the entropy of a vector variable with matrix operation is also needed here (chapter 8 in [2]):

\[
h(\tilde{y}) = h(U^\ast \mathbf{y}) = h(\mathbf{y}) + \log |\det(U^\ast)| = h(\mathbf{y}), \quad \text{(S7)}
\]
where \( \log |\det(U^*)| \) equals to 0, as \( U^* \) is a unity matrix. Similarly, we have \( h(\tilde{z}) = h(z) \). Given equations (S5), (S6), and (S7), we can obtain that
\[
I(x_1, x_2, ..., x_K; y_1, y_2, ..., y_L) = h(y_1, y_2, ..., y_L) - h(y_1, y_2, ..., y_L, x_1, x_2, ..., x_K)
\]
\[
= h(y_1, y_2, ..., y_L) - h(z_1, y_2, ..., z_L, x_1, x_2, ..., x_K)
\]
\[
= h(y_1, y_2, ..., y_L) - h(z_1, z_2, ..., z_L)
\]
\[
= h(\tilde{y}_1, \tilde{y}_2, ..., \tilde{y}_L) - h(\tilde{z}_1, \tilde{z}_2, ..., \tilde{z}_L)
\]
\[
= [h(\tilde{y}_1, \tilde{y}_2, ..., \tilde{y}_L) + h(\tilde{z}_{K+1}, ..., \tilde{z}_L)] - h(\tilde{z}_1, \tilde{z}_2, ..., \tilde{z}_L)
\]
\[
I(\tilde{x}_1, \tilde{x}_2, ..., \tilde{x}_K; \tilde{y}_1, \tilde{y}_2, ..., \tilde{y}_K)
\]

Now \( I(x_1, x_2, ..., x_K; y_1, y_2, ..., y_L) = I(\tilde{x}_1, \tilde{x}_2, ..., \tilde{x}_K; \tilde{y}_1, \tilde{y}_2, ..., \tilde{y}_K) \) holds, then we need to find the upper bound of the mutual information. A well known corollary in [2] that \( h(a_1, a_2, ..., a_L) \leq \sum_{i=1}^{L} h(a_i) \) with equality iff (if and only if) \( a_1, a_2, ..., a_L \) are independent is needed. Thus,
\[
h(\tilde{y}_1, \tilde{y}_2, ..., \tilde{y}_K) \leq \sum_{i=1}^{K} h(\tilde{y}_i), \tag{S9}\]

The Gaussian channel noise \( z = z_1, z_2, ..., z_L \) are independent and identically distributed. Here, we prove that \( \tilde{z} = \tilde{z}_1, \tilde{z}_2, ..., \tilde{z}_L \) are also independent: according to the covariance matrix,
\[
Cov(\tilde{z}) = Cov(U^*z) = U^*Cov(z)U = NU^*U = NI_L = Cov(\tilde{z}_i, \tilde{z}_j) = 0 \text{ for } i \neq j; \text{ then, due to the property of Gaussian random variables that the uncorrelation is equivalent to independence,}
\]
so \( \tilde{z} = \tilde{z}_1, \tilde{z}_2, ..., \tilde{z}_L \) are independent [3]. Therefore, the equality holds,
\[
h(\tilde{z}_1, \tilde{z}_2, ..., \tilde{z}_K) = \sum_{i=1}^{K} h(\tilde{z}_i), \tag{S10}\]
To derive the closed-form expression of the capacity for Gaussian channels, we need another
theorem about entropy that given the random vector $a \in \mathbb{R}^n$ with zero mean and covariance
$\Sigma = E[aa^T]$, then $h(a) \leq \frac{1}{2} \log(2\pi e)^n |\Sigma|$ with equality iff $a \sim N(0, \Sigma)$ [2]. Denote the
allocated power to each $x_i$ by $P_i$ and the noise power by $N$. Thus, by definition, $P_i = E[x_i^2]$, $N = E[z_i^2] = E[\tilde{z}_i^2]$. Moreover, because $\tilde{y}_i = \lambda_i \tilde{x}_i + \tilde{z}_i$ and because $\tilde{x}_i$ and $\tilde{z}_i$ are independent,
the average power $E[\tilde{y}_i^2] = \lambda_i^2 P_i + N$. Thus,

$$h(\tilde{y}_i) \leq \frac{1}{2} \log[2\pi e(\lambda_i^2 P_i + N)], \quad (S11)$$

and

$$h(\tilde{z}_i) = \frac{1}{2} \log[2\pi e N]. \quad (S12)$$

Finally, given (S8)-(S12), the mutual information is bounded as follows:
\[ I(\ldots, x_N; y_1, y_2, \ldots, y_L) = I(\ldots, x_N; \tilde{y}_1, \tilde{y}_2, \ldots, \tilde{y}_K) \]
\[ = h(\tilde{y}_1, \tilde{y}_2, \ldots, \tilde{y}_K) - h(\tilde{y}_1, \tilde{y}_2, \ldots, \tilde{y}_K | \tilde{x}_1, \tilde{x}_2, \ldots, \tilde{x}_K) \]
\[ = h(\tilde{y}_1, \tilde{y}_2, \ldots, \tilde{y}_K) - h(\tilde{z}_1, \tilde{z}_2, \ldots, \tilde{z}_K | \tilde{x}_1, \tilde{x}_2, \ldots, \tilde{x}_K) \]
\[ = h(\tilde{y}_1, \tilde{y}_2, \ldots, \tilde{y}_K) - h(\tilde{z}_1, \tilde{z}_2, \ldots, \tilde{z}_K) \]
\[
\leq \sum_{i=1}^K h(\tilde{y}_i) - \sum_{i=1}^K h(\tilde{z}_i) \tag{S13}
\]
\[ = \sum_{i=1}^K h(\tilde{y}_i) - \frac{1}{2} \sum_{i=1}^K \log[2\pi e N] \]
\[ \leq \frac{1}{2} \sum_{i=1}^K \log[2\pi e (P_i \lambda_i^2 + N)] - \frac{1}{2} \sum_{i=1}^K \log[2\pi e N] \]
\[ = \frac{1}{2} \sum_{i=1}^K \log(1 + \frac{P_i \lambda_i^2}{N}) \]
\[ \leq \frac{1}{2} \sum_{i=1}^K \log(1 + \frac{P_i^* \lambda_i^2}{N}) \]

where \( P_1^*, P_2^*, \ldots, P_K^* \) are the waterfilling power allocations [1,2]:
\[ P_i^* = (\mu - \frac{N}{\lambda_i^2})^+ \tag{S14} \]
where \( \mu \) is chosen to satisfy the total power constraint \( \sum_{i=1}^K P_i^* = P \), and \( P \) is the total power allocated in one symbol time.

Moreover, note that when \( L_t = K \), \( L_r = 2K - 1 \) samples are transmitted in the channel, while \( K \) samples form a symbol; therefore, we actually transmit \( 1 + (K - 1)/K \) symbol. With respect to the factor, the capacity is refined as
\[ C = \frac{1}{2} \frac{1}{1 + \frac{K-1}{K}} \sum_{i=1}^K \log(1 + \frac{P_i^* \lambda_i^2}{N}) \] (bits per symbol) \tag{S15}
For the case when $L_t$ is larger than $K$, it is easy to obtain that

$$C = \frac{1}{2} \frac{L_t}{K} \sum_{i=1}^{K} \log(1 + \frac{P_i^* \lambda_i^2}{N})(\text{bits per symbol}) \quad (S16)$$

Thus, when $L_t$ is sufficiently large, we can conclude that the capacity of the meta-multiplexing system is

$$C = \frac{1}{2} \sum_{i=1}^{K} \log(1 + \frac{P_i^* \lambda_i^2}{N})(\text{bits per symbol}), \text{ as } L_t \rightarrow \infty \quad (S17)$$

**Derivation of the capacity over $E_b/N_0$**

We utilize the energy per bit to noise power spectral density ratio $E_b/N_0$ to measure the condition of the communication channel. For each stream, denote the number of information bits per symbol by $\eta_i$, which might be influenced by the size of the modulation alphabet or the code rate of an error-control code in each stream. By definition, $P_i^* = \frac{\eta_i E_{b,i}}{T}$, in which $E_{b,i}$ is the bit energy in each sub-channel. For a complex communication signal, we have $N = W_n N_0 = N_0/T_s$, where $W_n$ is the noise bandwidth, $N_0$ is the noise power spectral density, and $T_s$ is the sampling period. According to the previous discussion, when each stream has the same power, the capacity becomes

$$C = \frac{1}{2} \sum_{i=1}^{K} \log(1 + \frac{P_i^* \lambda_i^2}{N})$$

$$= \frac{1}{2} \sum_{i=1}^{K} \log(1 + \eta_i T_s E_{b,i}^* \frac{1}{N_0} \lambda_i^2). \quad (S18)$$

In practices, one simple setting is that the power for each stream is equally divided, thus $P_i = \frac{P}{K}$.

$$C_I = \frac{1}{2} \sum_{i=1}^{K} \log(1 + \frac{P_i \lambda_i^2}{N})$$

$$= \frac{1}{2} \sum_{i=1}^{K} \log(1 + \eta_i T_s E_b \frac{1}{N_0} \lambda_i^2). \quad (S19)$$
Evaluation of the capacity for different K

We take $ (+1, -1, +j, -j) $ as the input of each stream of the meta-multiplexing, and the information carried by each symbol of each stream is $ \eta_i = 2 $. Given the overlap factor $ K $, then the meta-multiplexing system consists of $ K $ parallel sub-channels and the total information carried by a symbol is $ 2K $. By using equation (S18), we can then evaluate the required $ E_b/N_0 $ to achieve this capacity $ C = 2K $ (or $ C_I = 2k $). Consequently, we can produce the curve of the capacity over different $ K $.

Definition of bandwidths

It is common knowledge that for strictly bandlimited signals, the waveforms are not realizable because the time duration is infinite, whereas for realizable waveforms, the absolute bandwidth is infinite. Therefore, in practices, the bandwidth for the realizable waveform has been proposed to describe the power distribution of the communication signal. In this study, we consider two common definitions of the occupied bandwidth due to different considerations [4]: 1) Bounded power spectral density (Bounded PSD). This criterion states that everywhere outside of the occupied band, the spectrum of the signal $ H(f) $ must have fallen to a specified level below the level at the band center. Typical attenuation levels used in this study are 35 dB and 50 dB; 2) Fractional power containment bandwidth (FPCB). This bandwidth criterion states the percent of the signal power inside the occupied bandwidth. It has been adopted by the Federal Communications Commission. Typical percentages commonly used are 99%. Waveforms and their bandwidths used in this paper are listed as follows:

$ h_1(t) $: Rectangular waveform with time duration $ T $. The fall-off rate of the sidelobe of the rectangular wave is very low, so its occupied bandwidth is much larger than $ 1/T $: By numerical calculation, the 35dB bounded PSD bandwidth is $ B_{BPSD} = 49.30/T $, the 50dB bounded PSD bandwidth cannot be achieved within $ 200/T $, and the 99% FPCB bandwidths are $ 18.28/T $ re-
spectively. Rectangular wave is only employed to demonstrate the idea of the meta-multiplexing and the decoding algorithm, and is not suitable for real communication systems because its occupied bandwidth is unacceptable in practice;

\[ h_2(t) : \text{Taylor waveform with time duration and the sidelobe level at 35dB, the 35dB bounded PSD bandwidth is } B_{BPSD} = 3.16/T \text{ (see Fig. S4), and the 99% fractional power FPCB bandwidths is } B_{FPCB} = 2.35/T; \]

\[ h_3(t) : \text{Taylor waveform with time duration and the sidelobe level at 50dB, the 50dB bounded PSD bandwidth is } B_{BPSD} = 3.90/T \text{(see Fig. S4), and the 99% fractional power FPCB bandwidths is } B_{FPCB} = 2.74/T. \]

**The normalized capacity**

Besides, generally the communication signal is sampled at a higher frequency than the occupied bandwidth. When a Fourier transform applied, the signal is distributed on a much wider bandwidth, which is named processing bandwidth, while the bandwidth in which most of the signal energy are concentrated is defined as the occupied bandwidths. Commonly, to measure the spectral efficiency of a communication system, we use the occupied bandwidths rather than the processing bandwidth. Given the occupied bandwidth \( B \) and time duration \( T \) of a meta-multiplexing symbol, the limit on spectral efficiency of this communication system, measured by bits/s/Hz, is

\[
\eta = \frac{1}{2BT} \sum_{i=1}^{M} \log(1 + \frac{P_i \lambda_i^2}{N})(\text{bits/s/Hz}), \tag{S20}
\]

\[
= \frac{1}{2BT} \sum_{i=1}^{M} \log(1 + \eta \frac{T_s}{T} \frac{E_b}{N_0} \lambda_i^2)(\text{bits/s/Hz}). \tag{S21}
\]
Bayesian Monte Carlo based approximate decoding

First, the meta-multiplexing system must be reformed as a state-space model by the following two processes: the state equation, which acutely represents the convolution process of information bits passing through the meta-multiplexing system,

\[ s_t = \sum_{k=0}^{K-1} h_k x_{t-k} \quad (t = 1, 2, \ldots), \]

in which \( x_t \) represents the information bits and \( h = [h_0, \ldots, h_{k-1}] \) represents the impulse response coefficients of the entire communication system, including the pulse shaping filter, channel impulse response, matched filter, and etc. The second process is the observation equation, which is

\[ y_t = s_t + z_t, \]

in which \( z_t \) is the additive channel noise. Given the state-space model, Bayesian sequential inference can provide the estimation of the posterior of \( p(x_t|y_{1:t}) \), which further leads to the maximum a posterior (MAP ) estimation of \( x_t \). Sequential Monte Carlo (SMC), also known as particle filter, is a class of importance sampling and resampling techniques designed to solve the Bayesian inference of \( p(x_t|y_{1:t}) \). Here, we present one typical SMC method, the sequential importance resampling (SIR) method. We need to design the proposal distribution \( \pi(x_t|x_{1:t-1}, y_{1:t}) \), a probability density function from which it is easy to sample, whereas in our case, we can simply use the prior distribution \( Ber(1/2) \), as \( x_t \) is subject to a Bernoulli distribution and independent of \( x_{1:t-1} \). With a simple initiation, the SIR can be summarized as follows:

At iteration \( t \geq 1 \), for all \( i = 1, \ldots, M \):

**Step 1**: Sample \( x_t^{(i)} \sim Ber(1/2) \) and set \( x_{1:t}^{(i)} = [x_{1:t-1}^{(i)}, x_t^{(i)}] \);

**Step 2**: Compute the weights \( \tilde{\omega}_t^{(i)} \sim \tilde{\omega}_{t-1}^{(i)} (p(y_t|x_t^{(i)}) \), where \( \tilde{\omega}_t^{(i)} \) is normalized as \( \sum_{i=1}^M \tilde{\omega}_t^{(i)} = 1 \);

**Step 3**: If the effective number of particles \( N_{\text{eff}} = \frac{1}{\sum_{i=1}^M (\tilde{\omega}_t^{(i)})^2} \) is less than a threshold, then perform resampling \( x_{1:t}^{(i)} \sim \sum_{i=1}^M \tilde{\omega}_t^{(i)} \delta(x_{1:t}^{(i)}) \).

According to the above process, it is easy to notice that the SIR algorithm is suitable for parallel computing, which significantly benefits the hardware implementation. Moreover,
the computational complexity of the SIR algorithm depends only on the number of particles, so it is suitable for cases when the overlap factor is large in the meta-multiplexing. In the practice of communication systems, the coefficient $h$ may be unknown or with uncertain. In this case, we can either utilize some pilot signal to estimate the coefficients or extend the SMC to jointly estimate the state $x_{1:t}$ and the coefficient $h$ [9].

**Joint decoding strategy**

The joint decoding strategy attempts to demodulate all of the signals in the processing bandwidth of the meta-multiplexing. Generally, we assume that all other communication signals in the processing bandwidth are cooperative and we can demodulate the signal when the SNR is high enough. Without a loss of generality, we assume only one QAM modulated signal coexists with the meta-multiplexing signal in the processing bandwidth. Then, the decoding can be processed as follows:

**Step 1**: Use the coherent demodulation technique to move the QAM signal to baseband, and then filter out the signal outside of the occupied bandwidth of the QAM signal;

**Step 2**: Demodulate the QAM signal by a series of processes: matched filtering, down sampling, and detection;

**Step 3**: In the Bayesian approximate decoding process for the meta-multiplexing, use the demodulated QAM bits and the proposed bits $x_t^{(i)}$ for meta-multiplexing to generate the multiplexing signal $s_t^{(i)}$ (where $i$ is the index of each particle);

**Step 4**: Measure the distance between the observation and each particle by the likelihood function $p(y_t|s_t^{(i)})$, and perform the reweighting and resampling in the SIR algorithm;

**Step 5**: Obtain the MAP estimate of the meta-multiplexing based on the particles $\{x_t^{(i)}\}_{i=1}^M$. 
Hardware verification aperture and settings

The USRP devices become a common tool to verify communication methods and prototypes. In this study, we utilize a pair of USRP devices with programmable FPGA modules and configurable radio-frequency modules. The placement of the hardware connectivity is shown in Fig. S7: the transmitter (USRP Tx) and receiver (USRP Rx) are connected by a radio frequency (RF) cable, and a combiner is inserted to connect the noise generator and spectrum analyzer. Fig. S8 shows the spectrum of the meta-multiplexing signal measured by the spectrum analyzer. All of the communication algorithms are implemented on the FPGA, so the system can run stand-alone. Moreover, both the transmitter and receiver USRP devices are connected to a computer to verify the reliability of the communication system and calculate the bit error rate (BER).

References and Notes

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Fig. S1. Schematic of a MIMO system: the information bits are formatted into multiple streams, transmitted through transmission antennas and a multipath channel, and received by receiver antenna. The MIMO system can be expressed as, where the matrix represents the physical paths between the transmitter and the receiver [1].
Fig. S2. The framework of a meta-multiplexing communication system. Conventional communication modules like FEC coding, synchronization and channel estimation requires only moderate modification to fit into the meta-multiplexing system.

Fig. S3. The meta-multiplexing system can be converted into a parallel channel through singular value decomposition [1].
Fig. S4. (A) Basic waveforms in time domain. (B) Spectrum of the basic waveforms; dotted lines show the bound PSD bandwidth of Taylor waveforms. (C) Spectrum of the meta-multiplexing signal with and data length; dotted lines show the bound PSD bandwidth, while dash lines the processing bandwidth.
Fig. S5. (A) The capacity of meta-multiplexing systems with different waveforms. (B) The capacity and the realized transmission rate of the meta-multiplexing system, which uses the Taylor waveform with an attenuation level of -50 dB as the pulse shaping filter.
Fig. S6. Plot of the spectrum sharing scheme. The meta-multiplexing use an overlap factor K=100 and a Taylor waveform with an attenuation level of -35 dB. One 256-QAM signal occupies approximately 75% of the processing bandwidth of the meta-multiplexing signal.

Fig. S7. The connectivity of the USRP devices for hardware verification.
Fig. S8. Measure the frequency response of the meta-multiplexing signal in the real physical channel by a spectrum analyzer. The signal is rolled off to -35dB in a bandwidth of 24.48 KHz.