Finiteness results for compact hyperkähler manifolds

Daniel Huybrechts

In [12] Sullivan proved the following finiteness result for simply connected Kähler manifolds.

**Theorem** — The diffeomorphism type of a simply connected Kähler manifold is determined up to finite ambiguity by its integral cohomology ring and its Pontrjagin classes.

The aim of this note is to point out that much less of the topology needs to be fixed in order to determine the diffeomorphism type of a compact hyperkähler manifold (up to finite possibilities). Recall that a hyperkähler manifold is a $4n$-dimensional Riemannian manifold $(M, g)$ with holonomy $Sp(n)$. If such a manifold is compact it is automatically simply connected [1, Prop. 4]. The first Pontrjagin class $p_1(M) \in H^4(M, \mathbb{Z})$ defines a homogeneous polynomial of degree $2n - 2$ on $H^2(M, \mathbb{Z})$ by $\alpha \mapsto \int_M \alpha^{2n-2} p_1(M)$.

**Theorem 3.3** — If the second integral cohomology $H^2$ and the homogeneous polynomial of degree $2n - 2$ on $H^2$ defined by the first Pontrjagin class are given, then there exist at most finitely many diffeomorphism types of compact hyperkähler manifolds of dimension $4n$ realizing this structure.

We will also see that instead of fixing the action of $p_1$ in the above theorem one could as well fix $H^2$ with the top intersection product on $H^2$. This would amount to fixing the action of $p_0(M)$.

The proof of this theorem uses the finiteness result of Kollár and Matsusaka [9], a formula by Hitchin and Sawon [6], the projectivity criterion for hyperkähler manifolds, and the surjectivity of the period map proved in [7].

On any compact hyperkähler manifold $(M, g)$ there exists a $\mathbb{P}1$ of complex structures on $M$ compatible with the metric $g$. With any of these complex structures the manifold $M$ becomes an irreducible holomorphic symplectic manifold. Conversely, any Kähler class on an irreducible holomorphic symplectic manifold is uniquely induced by a hyperkähler metric on the underlying real manifold $M$. Once the diffeomorphism type of $M$ is fixed one wants to know how many deformation types of hyperkähler metrics $g$ or, equivalently, of irreducible holomorphic symplectic complex structures do exist on $M$. Again, there is only a finite number of possibilities.

**Theorem 2.1** — Let $M$ be a fixed compact manifold. Then there exist at most finitely many different deformation types of irreducible holomorphic symplectic complex structures on $M$. 

1
1 Finiteness of polarized manifolds

Let $X$ be a projective manifold over $\mathbb{C}$. If $L \in \text{Pic}(X)$ is an ample line bundle on $X$, then the Hilbert polynomial of the polarized manifold $(X, L)$ is $P(X, L, z) \in \mathbb{Q}[z]$, such that $P(X, L, m) = \chi(X, L^m)$ for all $m \in \mathbb{Z}$. For some high power $m$ the line bundle $L^m$ is very ample and $X$ is naturally embedded into a projective space $\mathbb{P}^N$ of dimension $N = P(X, L, m)$. The Quot-scheme parametrizing all subvarieties of $\mathbb{P}^N$ with given Hilbert polynomial is projective and therefore has only finitely many irreducible components. In particular, the number of deformation types of smooth projective varieties parametrized by this Quot-scheme is finite. In [10] Matsusaka proved that there exists a universal power $m$ as above that depends only on the numerical polynomial $P(X, L, z) \in \mathbb{Q}[z]$ and not on $X$ itself.

On the other hand, for a given $d, a_0, a_1 \in \mathbb{Q}$ there exists only a finite number of numerical polynomials $P_i(z) \in \mathbb{Q}[z]$ of degree $d$ with $P_i(z) = a_0 z^d + a_1 z^{d-1} + \ldots$ that occur as Hilbert polynomials of polarized manifolds $(X, L)$ as above [9].

Note that $P(X, L, z) = \frac{d!}{d(z_d - (K_X.L^d - 1))!} z^{d-1} + \ldots$. Thus one obtains the following result due to Kollár and Matsusaka:

**Theorem 1.1** [9, Thm. 3] — The number of deformation types of projective manifolds $X$ of dimension $d$ that admit an ample line bundle $L$ with fixed intersections $L^d, (K_X.L^{d-1}) \in \mathbb{Z}$ is finite.

For manifolds with trivial canonical bundle $K_X$ this becomes

**Corollary 1.2** — The number of deformation types of projective manifolds $X$ of dimension $d$ with trivial canonical bundle that admit an ample line bundle $L$ with bounded $L^d \in \mathbb{Z}$ is finite.

The result in particular applies to irreducible holomorphic symplectic manifolds. In order to get rid of the supplementary polarization of the manifold one has to show that any irreducible holomorphic symplectic manifold can be deformed to one that admits a polarization $L$ with bounded $L^d$. This will be done in the next section.

2 Complex structures on a fixed manifold

Here we will prove the following

**Theorem 2.1** — Let $M$ be a fixed compact manifold. Then there exist at most finitely many different deformation types of irreducible holomorphic symplectic complex structures on $M$.

Recall that an irreducible holomorphic symplectic manifold is a compact complex Kähler manifold $X$, which is simply connected and admits a nowhere degenerate two-form $\sigma$ such
that the space of global holomorphic two-forms $H^0(X, \Omega^2_X)$ is generated by $\sigma_X$. It will be clear from the proof that ‘deformation type’ could also be taken in the more restrictive sense that just allows families of irreducible holomorphic symplectic manifolds.

If $I$ is a complex structure on $M$ such that $X = (M, I)$ is irreducible holomorphic symplectic, then the Beauville-Bogomolov form $q_X$ is a primitive integral quadratic form on $H^2(M, \mathbb{Z})$ with the property that for some positive constant $c$

$$q_X(\alpha)^n = c \int_X \alpha^{2n}. \quad (1)$$

Clearly, the integral form $q_X$ does not change under deformation of $I$. But in fact $q_X$ is completely independent of the complex structure. The argument to prove this combines results of Fujiki and Nieper and will be given in Sect. 4. We will denote this quadratic form on $H^2(M, \mathbb{Z})$ by $q$. Thus, whenever $X = (M, I)$ is irreducible holomorphic symplectic, then $(H^2(M, \mathbb{Z}), q) \cong (H^2(X, \mathbb{Z}), q_X)$.

Let $\Gamma$ be the lattice $(H^2(M, \mathbb{Z}), q)$. The moduli space of marked irreducible holomorphic symplectic manifolds $\mathcal{M}_\Gamma$ is defined as $\mathcal{M}_\Gamma := \{(X, \varphi)| \varphi : (H^2(X, \mathbb{Z}), q_X) \cong \Gamma \}/ \sim$. Here, $\sim$ is the equivalence relation induced by $\pm f^*$, where $f : X \to X'$ is an arbitrary biholomorphic map. One proves that $\mathcal{M}_\Gamma$ has a natural smooth complex structure, which however is not Hausdorff. The period map is the holomorphic map $\mathcal{P} : \mathcal{M}_\Gamma \to Q_\Gamma \subset \mathbb{P}(\Gamma \otimes \mathbb{C})$, $(X, \varphi) \mapsto \varphi(H^2(X, \mathbb{Z}))$, where $Q_\Gamma$ is the period domain $Q_\Gamma := \{x|q(x) = 0, q(x+\bar{x}) > 0\}$. We will use the following result [7, Thm. 8.1]

**Theorem 2.2** — If $\mathcal{M}_\Gamma^0$ is a non-empty connected component of $\mathcal{M}_\Gamma$, then $\mathcal{P} : \mathcal{M}_\Gamma^0 \to Q_\Gamma$ is surjective.

Let us fix a primitive element $0 \neq \alpha \in \Gamma$ with $q(\alpha) > 0$ and consider the intersection $S_\alpha$ of $Q_\Gamma$ with the hyperplane $H_\alpha$ defined by $q(x, \alpha) = 0$. The assumption that $\alpha$ is primitive is not important, but makes the formulation of some statements more elegant.

**Lemma 2.3** — For any $0 \neq \alpha \in \Gamma$ the intersection $S_\alpha$ is not empty.

**Proof.** Let $V$ be a real vector space with a non-degenerate quadratic form $(\ ,\ )$ of index $(p, q)$. If $p \geq 2$ then one finds orthogonal vectors $y_1, y_2 \in V$, such that $(y_1, y_1) = (y_2, y_2) > 0$. Then $y = y_1 + iy_2 \in V \otimes \mathbb{C}$ satisfies $(y, y) = 0$ and $(y + \bar{y}, y + \bar{y}) > 0$. Apply this to the orthogonal complement $V$ of $\alpha \in \Gamma$ in $\Gamma \otimes \mathbb{Z} \mathbb{R}$. Since $q$ has index $(3, b_2(M) - 3)$, the induced quadratic form on $V$ has two positive eigenvalues. \qed

**Lemma 2.4** — If $y \in S_\alpha$ is generic, then $\{\beta \in \Gamma|q(y, \beta) = 0\} = \mathbb{Z}\alpha$.

**Proof.** Clearly, $\mathbb{Z}\alpha \subset \{\beta \in \Gamma|q(y, \beta) = 0\}$. If for generic $y$ the right hand side were strictly bigger, then $S_\alpha$ would equal $S_{\beta}$ for some $\beta \in \Gamma$ linearly independent of $\alpha$. Thus, the two hyperplanes defined by $\alpha$ and $\beta$, resp., would contain the intersection of the quadric defined by $q$ and $H_\alpha$. Hence, $q$ restricted to $H_\alpha$ would be degenerate, which contradicts $q(\alpha) > 0$. \qed
Proposition 2.5 — If \((Y, \psi) \in \mathcal{M}_\Gamma\) such that \(y := \mathcal{P}(Y, \psi) \in S_\alpha\) is generic, then \(\varphi : \text{Pic}(Y) \cong \mathbb{Z} \alpha\) and \(\text{Pic}(Y)\) is generated by an ample line bundle \(L\). In particular, \(Y\) is projective.

Proof. One knows that \(\text{Pic}(Y) = \{\beta \in H^2(Y, \mathbb{Z})| q_Y(\beta, \sigma_Y) = 0\}\). By the previous lemma, the first assertion follows from this. Thus, \(\text{Pic}(Y)\) is generated by a line bundle \(L\) with \(q_Y(L) > 0\). Hence, the intersection of the positive cone \(\mathcal{C}_Y\) with \(H^2(Y, \mathbb{Z})\) is non-empty and by the projectivity criterion \([7]\) this shows that \(Y\) is projective. Therefore, \(L\) or \(L^\vee\) is ample. \(\blacksquare\)

Remark — The proof of the projectivity criterion in \([7]\) was flawed. A complete proof was given recently in the Erratum \([8]\) using a new result by Demailly and Paun \([4]\).

Thus, for any fixed primitive \(0 \neq \alpha \in \Gamma\) with \(q(\alpha) > 0\) one can deform any \((X, \varphi) \in \mathcal{M}_\Gamma\) to a marked manifold \((Y, \psi) \in \mathcal{M}_\Gamma\) (which then is of course contained in the same connected component of \(\mathcal{M}_\Gamma\)) such that \((Y, L := \psi^{-1}(\alpha))\) is a polarized projective manifold with fixed \(q_Y(L) = q(\alpha)\) (use Thm. 2.2 and Lemma 2.3). In order to apply the boundedness result of Kollár and Matsusaka one has to fix \(L^{2n}\) instead. Since the underlying real manifold for the different components of \(\mathcal{M}_\Gamma\) and thus the constant \(c\) in (1) could be different, fixing \(q_Y(L)\) is a priori not enough in order to fix \(L^{2n}\).

Proof of Thm. 2.4. Let \(\alpha \in H^2(M, \mathbb{Z})\) be fixed such that \(\alpha\) is primitive and \(q(\alpha) > 0\). Let \(I\) be a complex structure on \(M\), such that \(X = (M, I)\) is an irreducible holomorphic symplectic manifold. Then fixing a marking \(\varphi : (H^2(X, \mathbb{Z}), q_X) \cong \Gamma \cong (H^2(M, \mathbb{Z}), q)\) yields a point \((X, \varphi)\) in the moduli space \(\mathcal{M}_\Gamma\). Hence, \(X\) is deformation equivalent to a projective manifold \(Y\) that admits an ample line bundle \(L\) with \(L^{2n} = c^{-1} q_Y(L) = c^{-1} q(\alpha)\). By Kollár and Matsusaka (cf. Cor. 1.2) the number of deformation types of those is finite. \(\square\)

Note that the argument does not show the finiteness of the number of connected components of \(\mathcal{M}_\Gamma\) that parametrizes complex structures on a fixed manifold \(M\) is finite. A priori, this seems only to be the case modulo the action of \(\text{Aut}(\Gamma)\).

The theorem is equivalent to a statement about the number of connected components of the space of all \(\text{Sp}(n)\)-metrics on \(M\). Since the pull-back of an \(\text{Sp}(n)\)-metric under a diffeomorphism of \(M\) yields again an \(\text{Sp}(n)\)-metric on \(M\), one only gets a finiteness result modulo the action of the diffeomorphism group.

Theorem 2.6 — The group \(\text{Diff}(M)/\text{Diff}^0(M)\) acts on the set of connected components of the set of all \(\text{Sp}(n)\)-metrics on \(M\). The quotient by this action is finite. \(\square\)

3 Using the Hitchin-Sawon formula

Applying the theory of Rozansky-Witten invariants, Hitchin and Sawon in \([8]\) proved the following formula
Theorem 3.1 [6, Thm. 3] — Let \((M, g)\) be a compact Riemannian manifold of dimension \(4n\) with holonomy \(\text{Sp}(n)\). Then

\[
\frac{||R||^{2n}}{(192\pi^2 n)^n} = \int_M \sqrt{\hat{A}(M)} \cdot (\text{vol} M)^{n-1}.
\]

Here, \(R\) is the curvature of \((M, g)\) and \(\hat{A}\) is the \(\hat{A}\)-genus of \(M\). By fixing a complex structure \(I\) on \(M\) that is compatible with the hyperkähler metric \(g\) we obtain an irreducible holomorphic symplectic manifold \(X = (M, I)\). Then \(g\) and \(I\) define a Kähler form \(\omega_I\) on \(X\). With respect to this Kähler form the norm of the curvature can be expressed as (cf. [2])

\[
||R||^2 = \frac{\pi^2}{(2n-2)!} \int_X c_2(X) \omega_I^{2n-2}
\]

and for the volume of \(M\) one finds \(\text{vol}(M) = (1/(2n)!)) \int_X \omega_I^{2n}\). Furthermore, \(\hat{A}(M) = \text{td}(X)\). Hence, the Hitchin-Sawon formula can be rewritten as

\[
\left( \int_X c_2 \omega_I^{2n-2} \right)^n = c \int_X \sqrt{\text{td}(X)} \cdot \left( \int_X \omega_I^{2n} \right)^{n-1}
\]

with \(c = \frac{(2n)^n}{(2n)!}\). As is pointed out in [3] this immediately yields

\[
\int_X \sqrt{\text{td}(X)} = \int_M \sqrt{\hat{A}(M)} > 0.
\]

Clearly, \(c \int_X \sqrt{\text{td}(X)}\) is a rational number which can be written as \(p/q\) with positive integers \(p\) and \(q\), where \(q\) is bounded from above by a universal constant \(c_n\) depending only on \(n\). Hence, \(c \int_X \sqrt{\text{td}(X)} \geq 1/c_n\). Since any Kähler class \(\omega\) on an irreducible holomorphic symplectic manifold \(X\) is obtained as \([\omega_I]\), this yields

**Corollary 3.2** — Let \(X\) be an irreducible holomorphic symplectic manifold of dimension \(2n\). If \(\omega\) is any Kähler class on \(X\), then \(\int_X \omega^{2n} < c_n^{1/(n-1)} \cdot (\int_X c_2(X) \omega^{2n-2})^{\frac{n}{n-1}}\), where \(c_n\) depends only on \(n\). \(\square\)

Theorem 3.3 — If the second integral cohomology \(H^2\) and the homogeneous polynomial of degree \(2n - 2\) on \(H^2\) defined by the first Pontryagin class are given, then there exist at most finitely many diffeomorphism types of compact hyperkähler manifolds of dimension \(4n\) realizing this structure.

**Proof.** Let \(M\) be any compact real \(4n\)-dimensional manifold and let \(I\) be a complex structure on \(M\). Then, \(p_1(M) = -2c_2(X)\), where \(X = (M, I)\). In particular, the action of \(c_2(X)\) on \(H^2(X, \mathbb{Z}) = H^2(M, \mathbb{Z})\) does not depend on the chosen complex structure \(I\). By assumption the second cohomology \(H^2\) of \(M\) as an abelian group and the homogeneous polynomial \(\alpha \mapsto \int_X \alpha^{2n-2} p_1\) of degree \(2n - 2\) on \(H^2\) are fixed. If \(X = (M, I)\) is an irreducible holomorphic symplectic manifold that admits an ample line bundle \(L \in \text{Pic}(X)\), then \(L^{2n}\) is bounded from above by \(c_n^{1/(n-1)}(\int_X \frac{\omega}{2}. L^{2n-2})^{\frac{n}{n-1}}\). Thus, by Kollár-Matsusaka the number of deformation
types of these complex manifolds is finite. Using the surjectivity of the period map and arguing as in the proof of Thm. 2.1, one concludes that the number of diffeomorphism types of compact manifolds \( M \) of dimension \( 4n \) carrying a Riemannian metric with holonomy \( \text{Sp}(n) \) is finite.

**Corollary 3.4** — The number of deformation types of irreducible holomorphic symplectic manifolds \( X \) of dimension \( 2n \) with given \( H^2(X, \mathbb{Z}) \) (as abelian group) and given homogeneous polynomial \( H^2(X, \mathbb{Z}) \to \mathbb{Z}, \alpha \mapsto \int_X \alpha^{2n-2}c_2(X) \) is finite.

### 4 Finiteness for fixed quadratic form

Let \( X \) be an irreducible holomorphic symplectic manifold and \( q_X \) the Beauville-Bogomolov form on \( H^2(X, \mathbb{Z}) \). If \( \sigma \) generates \( H^0(X, \Omega^1_X) \) such that \( \int_X (\sigma \bar{\sigma})^n = 1 \), then \( q_X \) is a scalar multiple of the form \( f_X(\alpha) = (n/2) \int_X \beta(\sigma \bar{\sigma})^{n-1} + \lambda \mu \), where \( \alpha = \lambda \sigma + \mu \bar{\sigma} + \beta \) is the Hodge-decomposition of \( \alpha \). Usually, \( q_X \) is chosen such that it becomes a primitive integral form on \( H^2(X, \mathbb{Z}) \). Note that \( q_X \) usually does not define the structure of a unimodular lattice on \( H^2(X, \mathbb{Z}) \). So classification theory of unimodular lattices cannot be applied. Due to a result of Fujiki (cf. [7, 1.11] or [3]) one has

**Theorem 4.1** [5, 4.12] — There exists a constant \( c \in \mathbb{Q} \) such that for all \( \alpha \in H^2(X) \)

\[
c \cdot q_X(\alpha) = \int_X \sqrt{\text{td}(X)\alpha^2}.
\]

Note that \( \sqrt{\text{td}(X)} \) does not depend on the complex structure \( I \). Since all odd Chern classes of \( X \) are trivial, \( \sqrt{\text{td}(X)} \) is a universal expression in the Pontrjagin classes of the underlying real manifold \( M \).

A priori, the constant \( c \) could be trivial or negative. That this is not the case follows from a result of Nieper [11]

**Theorem 4.2** — For any \( \alpha \in H^2(X) \) one has

\[
\int_X \sqrt{\text{td}(X)\exp(\alpha)} = (1 + \lambda(\alpha))^n \int_X \sqrt{\text{td}(X)},
\]

where \( \lambda \) is a positive multiple of the quadratic form \( f_X \) (or equivalently of \( q_X \)).

In fact \( \lambda \) and \( f_X \) differ by \( 12/\int_X c_2(X)(\sigma \bar{\sigma})^{n-1} \). The formula in particular gives back (1).

More generally, it yields

\[
\int_X \sqrt{\text{td}(X)\alpha^{2k}} = (2k)! \binom{n}{k} \lambda(\alpha)^k \int_X \sqrt{\text{td}(X)}.
\]
For $k = 1$ one obtains $\int_X \sqrt{\text{td}(X)} \alpha^2 = 2^k (\int_X \sqrt{\text{td}(X)} \lambda(\alpha))$. Thus the constant $c$ in Fujiki’s result is a positive multiple of $(\int_X \sqrt{\text{td}(X)})^{-1}$. The latter is positive as was observed by Hitchin and Sawon. Note that Nieper’s result also nicely generalizes the Hitchin-Sawon formula (2) to

$$(\int_X \sqrt{\text{td}(X)} \alpha^2)^n = (2k)! \binom{n}{k} \left( \frac{1}{(2n)!} \int_X \alpha^{2n} \right)^k \left( \int_X \sqrt{\text{td}(X)} \right)^{n-k}.$$  

Hence, for two different irreducible holomorphic symplectic structures $X = (M, I)$ and $X' = (M, I')$ on a manifold $M$ the associated quadratic forms $q_X$ and $q_{X'}$ differ by a positive multiple and, therefore, are equal. This was used in Sect. 3.

Let us define the unnormalized Beauville-Bogomolov form $\tilde{q}_X$ on $H^2(X)$ by $\tilde{q}_X(\alpha) := d_n \int_X \sqrt{\text{td}(X)} \alpha^2$, where $d_n$ is a universal integer (of no importance) depending only on $n$ such that $d_n (\int_X \sqrt{\text{td}(X)})^{2n-4}$ is universally an integral class. This quadratic form $\tilde{q}_X$ is a positive integral multiple of the primitive form $q_X$. As one usually does not have much information about the divisibility of Chern classes, it looks, at least from this point of view, more naturally to work with $\tilde{q}_X$ instead of the primitive form $q_X$.

As it is widely conjectured, a global Torelli theorem for higher dimensional hyperkähler manifold should assert that two irreducible holomorphic symplectic manifolds are birational whenever there exists an isomorphism $H^2(X, \mathbb{Z}) \cong H^2(X', \mathbb{Z})$ that respects Hodge structures and the Beauville-Bogomolov quadratic forms $q_X$ and $q_{X'}$, resp. A modified version of it would replace $q_X$ and $q_{X'}$ by the unnormalized Beauville-Bogomolov forms $\tilde{q}_X$ and $\tilde{q}_{X'}$, resp. Note that the modified version is a consequence of the original one, but it is, a priori, weaker. Here one uses that a birational map $X \dasharrow X'$ induces an isomorphism $H^*(X) \cong H^*(X')$ that maps $c(X)$ to $c(X')$.

From this point of view, it seems reasonable to replace $(H^2(X, \mathbb{Z}), q_X)$ by $(H^2(X, \mathbb{Z}), \tilde{q}_X)$ also in the finiteness question we are interested here. The proof of the following theorem is omitted as it uses arguments already explained in the previous sections.

**Theorem 4.3** — There exists only a finite number of deformation types of irreducible holomorphic symplectic manifolds $X$ of fixed dimension such that the lattice $(H^2(X, \mathbb{Z}), \tilde{q}_X)$ is isomorphic to a given one.

It should be clear from the discussion that fixing the lattice $(H^2(X, \mathbb{Z}), \tilde{q}_X)$ is actually too much. Everything that is needed is a class $\alpha \in H^2(X, \mathbb{Z})$ with fixed $\tilde{q}_X(\alpha) > 0$. Thus, for any $k \in \mathbb{Z}_{>0}$ there is only a finite number of deformation types of irreducible holomorphic symplectic manifolds $X$ that admit an element $\alpha \in H^2(X, \mathbb{Z})$ with $\tilde{q}_X(\alpha) = k$.

**Acknowledgement:** I would like to thank Manfred Lehn, Marc Nieper, and Justin Sawon for comments and useful conversations.
References

[1] Beauville, A. Variétés Kählériennes dont la première classe de Chern est nulle. J. Diff. Geom. 18 (1983), 755-782.
[2] Besse, A. Einstein Manifolds. Springer, Berlin (1987).
[3] Bogomolov, F. A. On the cohomology ring of a simple hyperkähler manifold (on the results of Verbitsky). GAFA 6(4) (1996), 612-618.
[4] Demailly, J.-P., Paun, M. Numerical characterization of the Kähler cone of a compact Kähler manifold. math.AG/0105176
[5] Fujiki, A. On the de Rham Cohomology Group of a Compact Kähler Symplectic Manifold. Adv. Stud. Pure Math. 10 (1987), 105-165.
[6] Hitchin, N., Sawon, J. Curvature and characteristic numbers of hyper-Kähler manifolds. Duke Math. J. 106 (2001), 599-615.
[7] Huybrechts, D. Compact Hyperkähler Manifolds: Basic Results. Invent. math. 135 (1999), 63-113.
[8] Huybrechts, D. Erratum to the paper: Compact Hyperkähler Manifolds: Basic Results. math.AG/0106014
[9] Kollár, J., Matsusaka, T. Riemann-Roch type inequalities. Am. J. Math. 105 (1983), 229-252.
[10] Matsusaka, T. Polarized varieties with a given Hilbert polynomial. Am. J. Math. 94 (1972), 1027-1077.
[11] Nieper, M. Hirzebruch-Riemann-Roch formula on irreducible symplectic Kähler manifolds. math.AG/0101062
[12] Sullivan, D. Infinitesimal computations in topology. Publ. Math. 47 (1977), 269-331.

Mathematisches Institut
Universität zu Köln
Weyertal 86-90
50931 Köln, Germany
huybrech@mi.uni-koeln.de