Precision measurements constraints on the number of Higgs doublets

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We consider an extension of the Standard Model (SM) with arbitrary number $N$ of Higgs doublets (NHDM), and calculate their contribution to the oblique parameters $S$ and $T$. We examine possible limitations on $N$ from precision measurements of these parameters. In view of complexity of the general case of NHDM, we analyze several benchmark scenarios for the Higgs mass spectrum, identifying the lightest CP-even Higgs with the Higgs-like particle recently observed at the LHC with the mass of $\sim 125$ GeV. The rest of the Higgses are put above the mass scale of $\sim 400$ GeV, below which the LHC experiments do not detect any Higgs-like signals except for the former famous one. We show that in a scenario, with all the heavy Higgses degenerate at any scale, there are no limitations on the number $N$ of the Higgs doublets. However, upper limits appear for certain not completely degenerate configurations of the heavy Higgses.

I. INTRODUCTION

The recent discovery of the $\sim 125$ GeV scalar particle at the Large Hadron Collider (LHC) \cite{1, 2} perfectly fills the vacancy of the Higgs boson necessary for the completion of the Standard Model (SM) at the Fermi scale. Surprisingly, the SM with the Higgs boson in this mass range becomes formally self consistent up to the Planck scale. In the absence of any signal of physics beyond the SM this fact drastically strengthens the position of this model as the theoretical basis of particle physics.

Although the new observed scalar state has so far all the properties expected of the SM Higgs boson, it is still possible that it could be a light scalar in a multi Higgs extensions of the SM, or a light Supersymmetric Higgs boson, or a Higgs boson coming from a strongly interacting dynamics, where the theory becomes non-perturbative above the Fermi scale and the breaking is achieved through some condensate. Now the priority of the LHC experiments will be to measure precisely the couplings of the observed scalar to fermions and gauge bosons, and to establish its quantum numbers in order to identify it with one of these or some other options. On the other hand, searches for new particles beyond the SM are an essential task of the LHC experiments \cite{3–8}.

In this paper we consider a multi Higgs extension of the SM, with an arbitrary number $N$ of the Higgs electroweak doublets. Our goal is to study possible bounds on the number of Higgs doublets from the precision measurements of the oblique $T$ and $S$ parameters.

We assume that the $N$ Higgs $SU(2)$ doublets are identical, with hypercharge equal to $1$. Some features such as the relation between the mass and gauge eigenstates in the scalar sector and the relation of the Higgs VEVs with the symmetry breaking scale $v \approx 246$ GeV presented in the Two Higgs Doublet model are still fulfilled when the number of Higgs Doublets is increased \cite{3}.

The paper is organized as follows. In Section II we briefly describe the theoretical structure of the $N$ Higgs Doublet Model (NHDM). In Section III we compute the one loop contribution to the $T$ and $S$ parameters in the NHDM. The bounds on the number of Higgs doublets coming from $T$ and $S$ parameter constraints at 95%CL are computed in Section IV. In Section V we summarize our results.

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II. THE MODEL

We consider an extension of the SM with $N$ copies of the complex $SU(2)_L$ weak doublet scalar Higgs fields with hypercharge $Y = 1$ (NHDM). The model scalar potential, invariant with respect to the SM gauge group is

$$ V = \frac{1}{2} \sum_{i,j=1}^{N} \mu_{ij}^2 \Phi_i^\dagger \Phi_j + \frac{1}{4} \sum_{i,j,k,l=1}^{N} \lambda_{ij,kl} \left( \Phi_i^\dagger \Phi_j \right) \left( \Phi_k^\dagger \Phi_l \right) + \sum_{i,j,k,l=1}^{N} \sigma_{ij,kl} \left( \Phi_i \tau^2 \Phi_j \right) \left( \Phi_k \tau^2 \Phi_l \right)^\dagger. \quad (1) $$

where $\tau^2$ is a Pauli matrix in the $SU(2)_L$ space and

$$ \sigma_{ij,kl} = -\sigma_{ji,kl} = -\sigma_{ij,lk}, \quad (2) $$

For simplicity we assume all the parameters in the scalar potential to be real. Then the hermiticity of the scalar potential implies

$$ \lambda_{ij,kl} = \lambda_{ji,kl}, \quad \sigma_{ij,kl} = \sigma_{ji,kl}, \quad \mu_{ij} = \mu_{ji}, \quad (3) $$

The minimum of the scalar potential is parametrized by $N$ vacuum expectation values

$$ \langle \Phi_i \rangle = \begin{pmatrix} 0 \\ \frac{v_l}{\sqrt{2}} \end{pmatrix}, \quad l = 1, 2, \ldots, N. \quad (4) $$

We decompose the Higgs fields around this minimum as

$$ \Phi_l = \left( \frac{1}{\sqrt{2}} \phi_l^+ \right) = \left( \frac{1}{\sqrt{2}} (v_l + \rho_l + i\eta_l) \right) \quad (5) $$

where

$$ \langle \rho_l \rangle = \langle \eta_l \rangle = \langle \omega_l \rangle = \langle \xi_l \rangle = 0, \quad l = 1, 2, \ldots, N. \quad (6) $$

Then the covariant derivative acting on the Higgs doublets takes the form:

$$ D_\mu \Phi_l = \partial_\mu \Phi_l - i \frac{g}{2} W^{\alpha}_\mu \tau^\alpha \Phi_l - i \frac{g}{2} Y_l B_\mu \Phi_l $$

$$ = \left( \frac{1}{\sqrt{2}} \partial_\mu \omega_l + \frac{1}{2\sqrt{2}} \left[ g W^{1}_\mu \eta_l - g W^{2}_\mu (v_l + \rho_l) + \left( g W^{3}_\mu + g Y_l B_\mu \right) \xi_l \right] \right) $$

$$ + \frac{i}{2\sqrt{2}} \left( 2 \partial_\mu \xi - \left( g W^{3}_\mu + g Y_l B_\mu \right) \omega_l - \left[ g W^{1}_\mu (v_l + \rho_l) + g W^{2}_\mu \xi_l \right] \right), \quad (7) $$

where the $\tau^\alpha$ are the ordinary $SU(2)_L$ Pauli matrices and $Y_l = 1$.

The NHDM Scalar-Gauge boson interactions are given by:

$$ \sum_{l=1}^{N} \left( D_\mu \Phi_l \right) \left( D^{\mu} \Phi_l \right)^\dagger = \frac{1}{8} \sum_{l=1}^{N} \left\{ 2 \partial_\mu \omega_l + \left[ g W^{1}_\mu \eta_l - g W^{2}_\mu (v_l + \rho_l) + \left( g W^{3}_\mu + g Y_l B_\mu \right) \xi_l \right] \right\}^2 $$

$$ + \frac{1}{8} \sum_{l=1}^{N} \left\{ 2 \partial_\mu \xi_l - \left( g W^{3}_\mu + g Y_l B_\mu \right) \omega_l - \left[ g W^{1}_\mu (v_l + \rho_l) + g W^{2}_\mu \xi_l \right] \right\}^2 $$

$$ + \frac{1}{8} \sum_{l=1}^{N} \left\{ 2 \partial_\mu \rho_l + \left[ g W^{1}_\mu \xi_l + g W^{2}_\mu \omega_l - \left( g W^{3}_\mu - g Y_l B_\mu \right) \right] \eta_l \right\}^2 $$

$$ + \frac{1}{8} \sum_{l=1}^{N} \left\{ 2 \partial_\mu \eta_l - g W^{1}_\mu \omega_l + g W^{2}_\mu \xi_l - \left( -g W^{3}_\mu + g Y_l B_\mu \right) (v_l + \rho_l) \right\}^2. \quad (8) $$

The connection between the interaction and mass scalar eigenstates is explained in what follows. The charged scalar fields of Eq. (5) are linear combinations of the charged Goldstone bosons and the charged physical scalars. The
imaginary part of the neutral component of the scalar doublets of Eq. (6) are linear combinations of the neutral Goldstone bosons and of the CP odd neutral scalar fields. The real part of the neutral component of the scalar doublets of Eq. (5) are linear combinations of the CP odd neutral scalar fields. Within this framework we consider a scenario where the interaction and mass eigenstates are related in the way analogous to the Two Higgs Doublet Model (2HDM) [9].

\[
\rho_l = \sum_{j=1}^{N} R_{lj} H_j^0, \quad \eta_l = Q_{l1} \pi^0 + \sum_{j=2}^{N} Q_{lj} A_j^0, \quad l = 1, 2, \ldots, N. \tag{9}
\]

\[
\omega_l = Q_{l1} \pi^1 + \sum_{j=2}^{N} Q_{lj} H_j^1, \quad \xi_l = Q_{l1} \pi^2 + \sum_{j=2}^{N} Q_{lj} H_j^2, \quad l = 1, 2, \ldots, N. \tag{10}
\]

where:

\[
v_l = v Q_{l1}, \quad l = 1, 2, \ldots, N, \quad v^2 = \sum_{i=1}^{N} v_i^2, \quad \sum_{i=1}^{N} R_{ii} = \delta_{ii}, \quad \sum_{i=1}^{N} Q_{ii} Q_{lj} = \delta_{ij}. \tag{11}
\]

Here \( v \approx 246 \text{ GeV} \) is the conventional electroweak symmetry breaking scale. The fields \( H_j^0 (i = 1, 2, \ldots, N) \) and \( A_j^0 (j = 1, 2, \ldots, N-1) \) are the CP even and CP odd neutral Higgs bosons, respectively. Similarly to the \( W^\pm \) gauge bosons which are defined in terms of \( W^1 \) and \( W^2 \), the charged Higgs and Goldstone bosons are related to the component fields in [10] as

\[
H_j^\pm = \frac{H_j^1 \pm i H_j^2}{\sqrt{2}}, \quad \pi^\pm = \frac{\pi^1 \mp i \pi^2}{\sqrt{2}} \tag{12}
\]

Thus we assumed the following:

1. The rotation matrix \( Q \), which relates the neutral Goldstone boson \( \pi^0 \) and the CP odd neutral Higgses \( A_j^0 \), with the interaction eigenstate scalars \( \eta_l \) (\( l = 1, 2, \ldots, N \)) in Eq. (9), is the same as the one that relates the components of the charged Goldstone bosons \( \pi^{1,2} \) and Higgses \( H_j^{1,2} \) with the corresponding interaction eigenstates \( \omega_l, \xi_l \), (\( l = 1, 2, \ldots, N \)) in Eqs. (10), (12).

2. The vacuum expectation values of \( N \) Higgs fields \( v_l \) (\( l = 1, 2, \ldots, N \)) are related to the common symmetry breaking scale \( v \approx 246 \text{ GeV} \) through the first relation in Eq. (11).

Both assumptions are generalizations of the corresponding relations of the 2HDM [9]. In the case of NHDM these relations are not true everywhere in the parametric space, but only in a certain part of it. Adopting the above assumptions we limit ourselves to a region in the parametric space of the NHDM, which is motivated (hinted) by the 2HDM.

### III. 1-LOOP CONTRIBUTION TO THE T AND S PARAMETERS.

In this section we calculate 1-loop contributions to the oblique parameters \( T \) and \( S \) defined as [10,15]:

\[
T = \frac{\Pi_{33} (q^2) - \Pi_{11} (q^2)}{\alpha_{EM} (M_Z) M_W^2} \bigg|_{q^2=0}, \quad S = \frac{2 \sin 2\theta_W}{\alpha_{EM} (M_Z)} \frac{d \Pi_{30} (q^2)}{dq^2} \bigg|_{q^2=0}. \tag{13}
\]

Here \( \Pi_{11} (0), \Pi_{33} (0) \) and \( \Pi_{30} (q^2) \) are the vacuum polarization amplitudes with \( \{ W_{\mu}, W_{\mu} \}, \{ W_{\mu}, W_{\mu} \} \) and \( \{ W_{\mu}, B_{\mu} \} \) external gauge bosons, respectively, where \( q \) is their momentum.
\[ L_{int}^{(T)} = \frac{g g_f v}{2} \pi^1 W^{3\mu} B_\mu + \frac{g g_f v}{2} \sum_{i=1}^{N} P_i H_i^{0} W^{3\mu} B_\mu + \frac{g}{2} \sum_{i=1}^{N-1} (A_i^{0} \partial_\mu H_i^{1} - H_i^{1} \partial_\mu A_i^{0}) W^{3\mu} \]

\[ \sum_{i=1}^{N} \sum_{j=1}^{N-1} T_{(H_i^{0} A_j^{0})} = \frac{1}{16 \alpha_{EM}(M_Z)^2 v^2} \sum_{i=1}^{N} \sum_{j=1}^{N-1} P_{i,j+1}^2 F \left( \Lambda^2, m^2_{H_i^{0}}, m^2_{A_j^{0}} \right), \]

\[ \sum_{i=1}^{N} \sum_{j=1}^{N-1} T_{(H_i^{1} H_j^{1})} = -\frac{1}{16 \alpha_{EM}(M_Z)^2 v^2} \sum_{i=1}^{N} \sum_{j=1}^{N-1} P_{i,j+1}^2 F \left( \Lambda^2, m^2_{H_i^{1}}, m^2_{H_j^{1}} \right), \]

\[ \sum_{i=1}^{N} \sum_{j=1}^{N-1} T_{(H_i^{1} A_j^{0})} = -\frac{1}{16 \alpha_{EM}(M_Z)^2 v^2} \sum_{i=1}^{N} \sum_{j=1}^{N-1} P_{i,j+1}^2 F \left( \Lambda^2, m^2_{H_i^{1}}, m^2_{A_j^{0}} \right). \]

The subscripts in \( T_{ab} \) denote the internal lines of the diagrams in Fig. I. The functions \( F \left( \Lambda^2, m^2_{1}, m^2_{2} \right) \) and \( G \left( \Lambda^2, m^2 \right) \) are defined as

\[ F \left( \Lambda^2, m^2_{1}, m^2_{2} \right) = \Lambda^2 - \frac{m_{1}^4}{m_{1}^4 - m_{2}^4} \ln \left( \frac{\Lambda^2 + m_{1}^2}{m_{1}^2} \right) - \frac{m_{2}^4}{m_{2}^4 - m_{1}^4} \ln \left( \frac{\Lambda^2 + m_{2}^2}{m_{2}^2} \right), \]
Figure 1: 1-loop Feynman diagrams contributing to the $T$ parameter.

\[ G (\Lambda^2, m^2) = \lim_{m_1, m_2 \to m} F (\Lambda^2, m_1, m_2) = \Lambda^2 - 2m^2 \ln \left( \frac{m^2 + \Lambda^2}{m^2} \right) + \frac{\Lambda^2 m^2}{\Lambda^2 + m^2}. \]  

(24)

Collecting all the contributions together, we find the 1-loop contribution to the $T$ parameter coming from the scalar
sector of the NHDM:

\[
T = \sum_{ab} T_{ab} \simeq -\frac{3}{16\pi\cos^2\theta_W} \sum_{i=1}^{N} P_{i}^2 \ln \left( \frac{m_{h_i}^2}{m_W^2} \right) + \frac{1}{16\alpha_{EM}(M_Z)^2\pi^2} \sum_{i=1}^{N-1} \left[ m_{H_i^+}^2 - h \left( m_{A_i^0}, m_{H_i^+}^2 \right) \right] + \frac{1}{16\alpha_{EM}(M_Z)^2\pi^2} \sum_{i=2}^{N} \sum_{j=1}^{N-1} P_{i,j+1}^2 \left[ h \left( m_{H_i^+}^2, m_{A_j^0}^2 \right) - h \left( m_{H_i^+}^2, m_{H_j^+}^2 \right) \right]
\]

where we identified the lightest CP-even Higgs $H_1^0 = h$ with the LHC Higgs-like particle with the mass $m_h = 125$ GeV.

The function $h \left( m_{1}^2, m_{2}^2 \right)$ is given by:

\[
h \left( m_{1}^2, m_{2}^2 \right) = \frac{m_{1}^2 m_{2}^2}{m_{1}^2 - m_{2}^2} \ln \left( \frac{m_{1}^2}{m_{2}^2} \right), \quad \lim_{m_2 \to m_1} h \left( m_{1}^2, m_{2}^2 \right) = m_{1}^2.
\]

We can split the $T$ parameter as $T = T_{SM} + \Delta T$, where $T_{SM}$ is the contributions from the SM, while $\Delta T$ contain all the contributions involving the heavy scalars:

\[
T_{SM} = -\frac{3}{16\pi\cos^2\theta_W} \ln \left( \frac{m_{h}^2}{m_{W}^2} \right),
\]

\[
\Delta T \simeq -\frac{3}{16\pi\cos^2\theta_W} \sum_{i=2}^{N} P_{i}^2 \ln \left( \frac{m_{H_i^0}^2}{m_{H_i^0}^2} \right) + \frac{1}{16\pi^2\alpha_{EM}(M_Z)^2} \sum_{i=1}^{N-1} \left[ m_{H_i^+}^2 - h \left( m_{A_i^0}, m_{H_i^+}^2 \right) \right] + \frac{1}{16\pi^2\alpha_{EM}(M_Z)^2} \sum_{i=2}^{N} \sum_{j=1}^{N-1} P_{i,j+1}^2 \left[ h \left( m_{H_i^+}^2, m_{A_j^0}^2 \right) - h \left( m_{H_i^+}^2, m_{H_j^+}^2 \right) \right].
\]

### B. S parameter

The interaction Lagrangian relevant for the computation of 1-loop contribution to the $S$ parameter in Eq. (13) is:

\[
\mathcal{L}^{(S)}_{int} = \frac{g}{2} \left( \pi^2 \partial_{\mu} \pi^1 - \pi^1 \partial_{\mu} \pi^2 \right) W^{3\mu} + \frac{g}{2} \sum_{i=1}^{N-1} \left( H_i^2 \partial_{\mu} H_i^1 - H_i^1 \partial_{\mu} H_i^2 \right) W^{3\mu}
\]

\[
+ \frac{g}{2} \left( \pi^2 \partial_{\mu} \pi^1 - \pi^1 \partial_{\mu} \pi^2 \right) B^{\mu} + \frac{g}{2} \sum_{i=1}^{N-1} \left( H_i^2 \partial_{\mu} H_i^1 - H_i^1 \partial_{\mu} H_i^2 \right) B^{\mu}
\]

\[
+ \frac{g}{2} \sum_{i=1}^{N} P_{i} \left( H_i^0 \partial_{\mu} H_i^0 - \pi^0 \partial_{\mu} H_i^0 \right) W^{3\mu} + \frac{g}{2} \sum_{i=1}^{N} \sum_{j=1}^{N-1} P_{i,j+1} \left( H_i^0 \partial_{\mu} A_j^0 - A_j^0 \partial_{\mu} H_i^0 \right) W^{3\mu}
\]

\[
- \frac{g}{2} \sum_{i=1}^{N} P_{i} \left( H_i^0 \partial_{\mu} \pi^0 - \pi^0 \partial_{\mu} H_i^0 \right) B^{\mu} - \frac{g}{2} \sum_{i=1}^{N} \sum_{j=1}^{N-1} P_{i,j+1} \left( H_i^0 \partial_{\mu} A_j^0 - A_j^0 \partial_{\mu} H_i^0 \right) B^{\mu}.
\]
Their partial contributions, assuming the cutoff $\Lambda$ to be much larger than the masses of the scalar particles, are:

\[
S_{(\pi^1\pi^2)} \simeq \frac{1}{12\pi} \ln \left( \frac{\Lambda^2}{m^2_W} \right),
\]

\[
\sum_{i=1}^{N-1} S_{(H^1_i H^2_i)} \simeq \frac{1}{12\pi} \sum_{i=1}^{N-1} \ln \left( \frac{\Lambda^2}{m^2_{H^\pm_i}} \right),
\]

\[
\sum_{i=1}^{N} S_{(H^0_{\pi^0 \pi^0})} \simeq -\frac{1}{12\pi} \sum_{i=1}^{N} P^2_{i,1} \ln \left( \frac{\Lambda^2}{m^2_{H^0_i}} \right),
\]

\[
\sum_{i=1}^{N} \sum_{j=1}^{N-1} S_{(H^0_i A^0_j)} \simeq \left\{-\frac{1}{12\pi} \sum_{i=1}^{N} \sum_{j=1}^{N-1} P^2_{i,j+1} \left( m^2_{A^0_j} - m^2_{H^0_i} \right)^3 \left\{ m^6_{A^0_j} \left[ \ln \left( \frac{\Lambda^2}{m^2_{A^0_j}} \right) + \frac{5}{6} \right] - m^6_{H^0_i} \left[ \ln \left( \frac{\Lambda^2}{m^2_{H^0_i}} \right) + \frac{5}{6} \right] \right. \right. \\
+ 3m^2_{H^0_i} m^2_{A^0_j} \left[ m^2_{H^0_i} \left[ \ln \left( \frac{\Lambda^2}{m^2_{H^0_i}} \right) + \frac{3}{2} \right] - m^2_{A^0_j} \left[ \ln \left( \frac{\Lambda^2}{m^2_{A^0_j}} \right) + \frac{3}{2} \right] \right] \left. \right\} \right\}
\]

As before, the subscripts in $S_{ab}$ denote the internal lines of the diagrams in Fig. 2. Then, the 1-loop Higgs contribution

Figure 2: One loop Feynman diagrams contributing to the $S$ parameter.
to the $S$ parameter in the NHDM is:

$$S = \sum_{ab} S_{ab} \simeq \frac{1}{12\pi} \left[ \sum_{i=1}^{N} P_{i1}^2 \ln \left( \frac{m_{H_i^0}^2}{m_W^2} \right) + \sum_{i=1}^{N-1} \sum_{j=i+1}^{N} P_{i,j+1}^2 K \left( m_{H_i^0}^2, m_{H_j^0}^2, m_{H_j^0}^2 \right) \right]$$

$$= \frac{1}{12\pi} \ln \left( \frac{m_h^2}{m_W^2} \right) + \frac{1}{12\pi} \left[ \sum_{i=2}^{N} P_{i1}^2 \ln \left( \frac{m_{H_i^0}^2}{m_h^2} \right) + \sum_{i=1}^{N-1} \sum_{j=i+1}^{N-1} P_{i,j+1}^2 K \left( m_{H_i^0}^2, m_{H_j^0}^2, m_{H_j^0}^2 \right) \right],$$

(34)

where we identified the lightest CP-even Higgs $H_1^0 = h$ with the LHC Higgs-like particle with the mass $m_h = 125$ GeV. We defined a function

$$K \left( m_{1}^2, m_{2}^2, m_{3}^2 \right) = \frac{1}{(m_{2}^2 - m_{1}^2)^3} \left\{ m_{1}^4 (3m_{2}^2 - m_{1}^2) \ln \left( \frac{m_{1}^2}{m_{3}^2} \right) - m_{2}^4 (3m_{2}^2 - m_{3}^2) \ln \left( \frac{m_{2}^2}{m_{3}^2} \right) \right\} - \frac{1}{6} \left[ 27m_{2}^2m_{3}^2 (m_{1}^2 - m_{2}^2) + 5 (m_{2}^2 - m_{1}^2) \right],$$

(35)

with the properties

$$\lim_{m_{1} \to m_{2}} K(m_{1}^2, m_{2}^2, m_{3}^2) = K_1(m_{2}^2, m_{3}^2) = \ln \left( \frac{m_{2}^2}{m_{3}^2} \right),$$

$$\lim_{m_{2} \to m_{3}} K(m_{1}^2, m_{2}^2, m_{3}^2) = K_2(m_{1}^2, m_{2}^2) = \frac{-5m_{1}^6 + 27m_{1}^4m_{2}^2 - 27m_{1}^2m_{3}^4 + 6 (m_{2}^6 - 3m_{1}^4m_{3}^2) \ln \left( \frac{m_{2}^2}{m_{3}^2} \right) + 5m_{3}^6}{6 (m_{1}^2 - m_{3}^2)^3},$$

$$\lim_{m_{1} \to m_{3}} K(m_{1}^2, m_{2}^2, m_{3}^2) = K_2(m_{2}^2, m_{3}^2).$$

(36)

We can split the $S$ parameter as $S = S_{SM} + \Delta S$, where $S_{SM}$ is the contributions from the SM, while $\Delta S$ contain all the contributions involving the heavy scalars:

$$S_{SM} = \frac{1}{12\pi} \ln \left( \frac{m_h^2}{m_W^2} \right),$$

(37)

$$\Delta S \simeq \frac{1}{12\pi} \left[ \sum_{i=2}^{N} P_{i1}^2 \ln \left( \frac{m_{H_i^0}^2}{m_h^2} \right) + \sum_{i=1}^{N-1} \sum_{j=i+1}^{N} P_{i,j+1}^2 K \left( m_{H_i^0}^2, m_{H_j^0}^2, m_{H_j^0}^2 \right) \right].$$

(38)

IV. T AND S BOUNDS ON NHDM

The inclusion of the extra scalar particles modifies the SM predictions for the oblique parameters $T$ and $S$, and therefore their values extracted from high precision measurements can be used to constrain the N-Higgs extension of the SM. Our goal is to examine if these measurements are able to restrict the number $N$ of Higgs electroweak doublets. The experimental results on $T$ and $S$ restrict the deviations $\Delta T$ and $\Delta S$ from the SM predictions to lie inside a region in the $\Delta S - \Delta T$ plane. At the 95% C.L. this region is the elliptic contour shown in Figs. 3 taken from Ref. 16. The reference point $\Delta S = \Delta T = 0$ is conventionally taken to be the SM value of $\Delta S$ and $\Delta T$ at $m_h = 125.7$ GeV and $m_t = 173.18$ GeV. In view of the complexity of the general case of the N Higgs doublet model, we consider several benchmark scenarios described below.

A. All the heavy Higgses are degenerate.

This is the most simple case of the Higgs spectrum with the lightest CP-even Higgs $H_1^0 = h$ identified with the LHC Higgs-like particle, with a mass $m_h = 125.7$ GeV and all the other heavier Higgses degenerate having a common mass.
Figure 3: The interior of the ellipse in $\Delta S - \Delta T$ plane is the experimentally allowed region at 95\% CL from Ref. [16]. The reference point $\Delta S = \Delta T = 0$ is conventionally taken to the Standard Model value of $\Delta S$ and $\Delta T$, at $m_h = 125.7$ GeV and $m_t = 173.18$ GeV.

$m_H$. Thus

\[
m_{H^\pm} = m_{A_0^j} = m_H, \quad j = 1, 2, \cdots, N - 1, \tag{39}
\]
\[
m_{H_0^i} = m_h = 125.7 \text{ GeV}, \quad m_{H^0} = m_H, \quad i = 2, \cdots, N, \tag{40}
\]
\[
m_H > m_h
\]

For this spectrum Eqs. (28) and (38) for the $\Delta T$ and $\Delta S$ parameters are drastically simplified and take the form:

\[
\Delta T = - \frac{3 (1 - P_{11}^2)}{16 \pi \cos^2 \theta_W} \ln \left( \frac{m_H^2}{m_h^2} \right), \tag{40}
\]
\[
\Delta S = \frac{1 - P_{11}^2}{12 \pi} \left[ \ln \left( \frac{m_H^2}{m_h^2} \right) + K_2 \left( m_h^2, m_H^2 \right) \right]. \tag{41}
\]

Thus in this scenario neither of the two parameters $S$ and $T$ depends on $N$. Therefore the spectrum in Eq. (39) do not constrain the number of Higgs doublets.

**B. Degeneracy inside the groups of the heavy CP even, CP odd and Charged Higgses.**

**Subscenario B1: The CP even and CP odd neutral Higgs are degenerate.**

The next-to-simplest scenario that we consider has the following Higgs spectrum:

\[
m_{A_0^j} = m_H, \quad j = 1, 2, \cdots, N - 1, \tag{42}
\]
\[
m_{H_0^i} = m_h = 125.7 \text{ GeV}, \quad m_{H^0} = m_H, \quad i = 2, \cdots, N, \tag{43}
\]
\[
m_H > m_h
\]

This spectrum, using Eqs. (28) and (38), leads to the expressions:
Figure 4: Upper bound $N \leq N_{\text{max}}$ on the number $N$ of Higgs doublets, obtained from $T$ and $S$ at 95\%CL, using the experimental constraints indicated in Fig. 3 for the Higgs spectrum in Eq. 42.

\[ \Delta T = -\frac{3}{16\pi} \cos^2 \theta_W \ln \left( \frac{m_H^2}{m_h^2} \right) + \frac{N - 1}{16\pi^2 v^2 \alphaEM(M_Z)} \left[ m_H^2 + m_H^2 - 2h(m_H^2, m_H^2) \right] \]

\[ + \frac{1}{16\pi^2 v^2 \alphaEM(M_Z)} \left[ h(m_h^2, m_H^2) - h(m_h^2, m_H^2) - m_H^2 + h(m_h^2, m_H^2) \right], \]

\[ \Delta S = \frac{1}{12\pi} \left\{ (1 - P_{11}) \left[ \ln \left( \frac{m_H^2}{m_h^2} \right) + K(m_h^2, m_A^2, m_H^2) - K_1(m_H^2, m_H^2) \right] + (N - 1) K_1(m_H^2, m_H^2) \right\}, \]

with

\[ P_{11} = \sum_{l=1}^{N} R_{l1} Q_{l1} = \frac{1}{v} \sum_{l=1}^{N} R_{l1} v_l, \] (44)

where we used the first relation in Eq. 11.

Now, using Eqs. (43), we find the maximal values $N_{\text{max}}$ of the Higgs doublets $N$ compatible with the precision data in Fig 3. We scan the parameter space within

\[ 0 \leq P_{11} \leq 1, \quad 600\text{GeV} \leq m_H \leq 100\text{TeV} \]

(45)

In Fig. 4 we show the resulting values $N_{\text{max}}$ in function of the splitting parameter $\Delta$. It is noteworthy that the maximum number of Higgs doublets decreases when the mass splitting $\Delta$ between the heavy physical scalars is increased. This behavior follows from the fact that increasing the number of Higgs doublets yields an increase of the $T$ and $S$ oblique parameters.

In the limit $\Delta \to 0$ we find $N_{\text{max}} \to \infty$, corresponding to no limits on $N$, which is consistence with the scenario (39).
Subscenario B2: The CP even neutral and charged Higgs are degenerate:

\begin{align}
  m_{H^\pm}^j &= m_H, \\
  m_{A^0}^j &= m_H + \Delta, \\
  m_{H^0}^i &= m_h, \\
  m_{H_i^0} &= m_h + \Delta, \\
  m_H &> m_h
\end{align}

(46)

\[ m_{H_0}^{1} = m_h = 125.7 \text{ GeV}, \quad m_{H_i^0} = m_h \, i = 2, \cdots , N, \]

Subscenario B3: The CP odd neutral and charged Higgs are degenerate:

\begin{align}
  m_{H^\pm}^j &= m_H, \\
  m_{A^0}^j &= m_H + \Delta, \\
  m_{H^0}^i &= m_h, \\
  m_{H_i^0} &= m_h + \Delta, \\
  m_H &> m_h
\end{align}

(48)

From Eqs. (28) and (38) we find for this spectrum:

\[ \Delta T \simeq \frac{3 (1 - P_{11}^2)}{16\pi \cos^2 \theta_W} \ln \left( \frac{m_H^2}{m_h^2} \right) + \frac{1 - P_{11}^2}{16\pi^2 v^2 \alpha_{EM} (M_Z)} \left[ h \left( m_h^2, m_A^2 \right) - h \left( m_h^2, m_H^2 \right) + m_H^2 - h \left( m_H^2, m_A^2 \right) \right], \]

\[ \Delta S \simeq \frac{1}{12\pi} \left\{ (1 - P_{11}^2) \left[ \ln \left( \frac{m_H^2}{m_h^2} \right) + K \left( m_h^2, m_A^2, m_H^2 \right) \right] + (N - 2 + P_{11}^2) K_2 \left( m_H^2, m_A^2 \right) \right\}. \]

(47)

Scanning the parameter space in the region (45) we find the maximal values on the number \( N \) of Higgs doublets compatible with the data in Fig. 3. The results are shown in Fig. 5. As seen, this spectrum is significantly less restrictive for \( N \) than that in Eq. (42). This is mainly because of the fact that only the \( S \) parameter depends on \( N \) in the present case, while in the case of the spectrum (42) both \( T \) and \( S \) are \( N \)-dependent.

Subscenario B3: The CP odd neutral and charged Higgs are degenerate:

From Eqs. (28) and (38) we find for this spectrum:

\[ \Delta T \simeq \frac{3 (1 - P_{11}^2)}{16\pi \cos^2 \theta_W} \ln \left( \frac{m_H^2}{m_h^2} \right) + \frac{1 - P_{11}^2}{16\pi^2 v^2 \alpha_{EM} (M_Z)} \left[ h \left( m_h^2, m_A^2 \right) - h \left( m_h^2, m_H^2 \right) + m_H^2 - h \left( m_H^2, m_A^2 \right) \right], \]

\[ \Delta S \simeq \frac{1}{12\pi} \left\{ (1 - P_{11}^2) \left[ \ln \left( \frac{m_H^2}{m_h^2} \right) + K \left( m_h^2, m_A^2, m_H^2 \right) \right] + (N - 2 + P_{11}^2) K_2 \left( m_H^2, m_A^2 \right) \right\}. \]

(49)
Applying the same procedure as previously, we scan the parameter space in the region \( (45) \), and find the maximal

\[
N_{\text{max}}
\]

values on the number \( N \) of the Higgs doublets compatible with the data of Fig. 3. The results are shown in Fig. 6. Again, as in the case \((46)\), the parameter \( T \) is independent of \( N \). As a consequence the limits on \( N \) for the spectrum \((48)\) are significantly weaker than for \((42)\).

**Subscenario B4: Split groups with the degenerate interior:**

\[
\begin{align*}
\text{Case 1 : } & \quad m_{A_0} = m_H + \Delta, \quad m_{H^0} = m_h = 125.7 \text{ GeV}, \quad m_{H^0_i} = m_H, \quad j = 1, 2, \cdots, N - 1, \quad i = 2, \cdots, N, \\
& \quad m_H > m_h \\
\text{Case 2 : } & \quad m_{H^0} = m_H + \Delta, \quad m_{H^0} = m_h = 125.7 \text{ GeV}, \quad m_{A_0} = m_H, \quad j = 1, 2, \cdots, N - 1, \quad i = 2, \cdots, N, \\
& \quad m_H > m_h \\
\text{Case 3 : } & \quad m_{H^0} = m_H + \Delta, \quad m_{A_0} = m_h = 125.7 \text{ GeV}, \quad m_{H^0_j} = m_H, \quad j = 1, 2, \cdots, N - 1, \quad i = 2, \cdots, N
\end{align*}
\]
For all these cases we find from Eqs. (28) and (38):

\[ \Delta T = -\frac{3}{16\pi \cos^2 \theta_W} \ln \left( \frac{m_{H^0}^2}{m_h^2} \right) + \]

\[ + \frac{1 - P_{11}^2}{16\pi^2 v^2 \alpha_{EM}(M_Z)} [h \left( m_{H^0}, m_{H^\pm} \right) - h \left( m_{H^0}, m_A^2 \right) + h \left( m_h^2, m_A^2 \right) - h \left( m_h^2, m_{H^\pm} \right)] + \]

\[ + \frac{N - 1}{16\pi^2 v^2 \alpha_{EM}(M_Z)} \left[ m_{H^\pm}^2 - h \left( m_A^2, m_{H^\pm}^2 \right) + h \left( m_{H^0}^2, m_A^2 \right) - h \left( m_{H^0}^2, m_{H^\pm}^2 \right) \right], \]

\[ \Delta S = \frac{1}{12\pi} \left\{ (1 - P_{11}^2) \left[ \ln \left( \frac{m_{H^0}^2}{m_h^2} \right) + K \left( m_h^2, m_A^2, m_{H^\pm}^2 \right) \right] + (N - 2 + P_{11}^2) K \left( m_{H^0}^2, m_A^2, m_{H^\pm}^2 \right) \right\}. \] (53)

As seen from Eq. (53), now both parameters \( T \) and \( S \) depend linearly on \( N \), the number of Higgs doublets. Scanning of the parameter space in the region (45) we find the results for several sample values of the parameter \( a = 0.5, 1, 2 \) shown in Figs. 7, 9. Note the general tendency: the large splitting, corresponding to the larger values of \( \Delta \) and \( a \), leads to more stringent constraints on \( N \).

Figure 7: The same as in Fig. 4 but for the spectrum in Eq. (50). The left panel is for \( a = 1 \); The right panel is for \( a = 0.5 \) (dashed line), \( a = 2 \) (solid line).

C. No degeneracy with a particular structure of the spectrum

Finally let us consider a benchmark scenario in which all the Higgses are non-degenerate. Since the general case can hardly be analyzed, we consider a particular structure of the Higgs spectrum:

\[ m_{H^0} = m_h = 125.7 GeV, \quad m_{H^\pm} = m_h + 2(i - 2)\Delta, \quad i = 2, \cdots, N \] (54)

\[ m_{A^0} = m_H + (2j - 1)\Delta, \quad m_{H^\pm} = m_H + 2(j - 1)\Delta + \delta, \quad j = 1, 2, \cdots, N - 1 \]

\[ m_H > m_h \]

This is a spectrum equidistant in each of the three groups of Higgses \( \{ A^0 \}, \{ H^0_i \}, \{ H^\pm_j \} \) with a step \( 2\Delta \). The bands of these groups overlap with each other. The spectrum is characterized by two parameters \( \Delta \) and \( \delta \).
For simplicity we assume

\[ P_{i,j} = \delta_{ij}, \quad i, j = 1, 2, \ldots, N \]

(55)

Then from Eqs. (28) and (38) we get

\[ \Delta T \simeq \frac{1}{16\pi^2 v^2 \alpha_{EM}(M_Z)} \sum_{i=1}^{N-1} \left[ m_{H^\pm_i}^2 - h \left( m_{A_{0_i}}^2, m_{H^+_i}^2 \right) + h \left( m_{A_{0_{i+1}}^2}, m_{H^+_i}^2 \right) - h \left( m_{A_{0_{i+1}}}, m_{H^+_i}^2 \right) \right], \]

(56)

\[ \Delta S \simeq \frac{1}{12\pi} \sum_{i=1}^{N-1} K \left( m_{H^0_{i+1}}^2, m_{A^0_i}, m_{H^+_i}^2 \right). \]

(57)

Scanning of the parameter space in the region (45) we find the results for several sample values of the parameter \( \delta = \Delta/n \) shown in Fig. 10. The curves from the bottom to the top correspond to \( n = 20, 10, 2 \), respectively. With larger value \( n = 50 \) we find \( N_{\text{max}} \sim 570 \) for \( \Delta = 20 \) GeV.

V. SUMMARY AND CONCLUSIONS

We have considered a \( N \) Higgs \( SU(2) \) Doublet Model (NHDM) with an arbitrary number \( N \). In this model we calculated the one-loop contribution, \( \Delta S \) and \( \Delta T \), of the Higgs doublets to the electroweak oblique parameters \( S \) and \( T \). The calculated contribution depends on the number \( N \) of Higgs doublets and, therefore, our results can be used to constrain \( N \) from the data on the precision measurements of the parameters \( T \) and \( S \). Within the generic case of the NHDM, due to large number of the free parameters, this program can hardly be realized. For this reason we have analyzed several benchmark scenarios with particular mass spectra, Eqs. (39), (42), (46), (48), (50), (52), (54), of the physical scalars of the NHDM, including some other simplifying assumptions, inspired by the well-known case of the 2HDM, about the physical Higgs mixing and the structure of the vacuum of the model. These scenarios correspond to certain domains of the NHDM parameter space. We have shown that, except for a very particular “fine-tuned” case with all the physical heavy Higgses degenerate, Eq. (39), these scenarios imply constraints on the number of Higgs doublets \( N \), to be compatible with the existing data on the precision measurements of \( T \) and \( S \).
We presented our results on $N \leq N_{\text{max}}$ in Figs. 4-10 as function of the mass splitting parameter $\Delta$. The general feature of our results is that the maximal number $N_{\text{max}}$ of Higgs doublets is a monotonically increasing function for small values of the splitting $\Delta \leq 20$ GeV and a monotonically decreasing one for $\Delta \geq 70$ GeV. Thus the data on $T$ and $S$ are able to accommodate an arbitrary large number $N$ with decreasing splitting between the masses of the physical scalars, and vice versa, $N$ becomes stringently constrained in those parts of the NHDM parameter space with large mass splitting in the scalar sector. The same tendency is demonstrated by the plots with respect to their dependence on the two other additional parameters $a$ and $\delta$ characterizing the Higgs mass spectrum: the smaller mass splitting, the larger number $N$ is compatible with the analyzed data and visa versa. This is the main message of the present study. Our analysis can not exclude a deviation from this tendency in certain parts of the NHDM parameter space, however, in our opinion, they are related with certain “fine-tuning” of the parameters, as in the case of the spectrum (39). Naturally, the concrete limit on $N$ depends on a particular scenario within the generic NHDM framework. We hope our results will help examine such scenarios regarding their consistence with the present and future data on the precision measurements of the electroweak oblique parameters $T$ and $S$.

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Figure 10: The same as in Fig. [4] but for the spectrum Eq. (54). The curves from the bottom to the top correspond to $n = 20, 10, 2$, respectively.

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