Remnant evolution after a carbon–oxygen white dwarf merger

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ABSTRACT

We systematically explore the evolution of the merger of two carbon–oxygen (CO) white dwarfs. The dynamical evolution of a 0.9 M⊙ + 0.6 M⊙ CO white dwarf merger is followed by a 3D smoothed particle hydrodynamics (SPH) simulation. The calculation uses a state-of-the-art equation of state that is coupled to an efficient nuclear reaction network that accurately approximates all stages from helium burning up to nuclear statistical equilibrium. We use an elaborate prescription in which artificial viscosity is essentially absent, unless a shock is detected, and a much larger number of SPH particles than earlier calculations. Based on this simulation, we suggest that the central region of the merger remnant can, once it has reached quasi-static equilibrium, be approximated as a differentially rotating CO star, which consists of a slowly rotating cold core and a rapidly rotating hot envelope surrounded by a centrifugally supported disc. We construct a model of the CO remnant that mimics the results of the SPH simulation using a 1D hydrodynamic stellar evolution code and then follow its secular evolution, where we include the effects of rotation on the stellar structure and the transport of angular momentum. The influence of the Keplerian disc is implicitly treated by considering mass accretion from the disc on to the hot envelope. The stellar evolution models indicate that the growth of the cold core is controlled by neutrino cooling at the interface between the core and the hot envelope, and that carbon ignition in the envelope can be avoided despite high effective accretion rates. This result suggests that the assumption of forced accretion of cold matter that was adopted in previous studies of the evolution of double CO white dwarf merger remnants may not be appropriate. Specifically we find that off-centre carbon ignition, which would eventually lead to the collapse of the remnant to a neutron star, can be avoided if the following conditions are satisfied. (1) When the merger remnant reaches quasi-static equilibrium, the local maximum temperature at the interface between the core and the envelope must be lower than the critical limit for carbon ignition. (2) Angular momentum loss from the central merger remnant should not occur on a time-scale shorter than the local neutrino cooling time-scale at the interface. (3) The mass accretion rate from the centrifugally supported disc must be sufficiently low (M \lessapprox 5 \times 10^{-6} to 10^{-5} M⊙ yr^{-1}). Our results imply that at least some products of double CO white dwarfs merger may be considered good candidates for the progenitors of Type Ia supernovae. In this case, the characteristic time delay between the initial dynamical merger and the eventual explosion would be \sim 10^5 yr.

Key words: accretion, accretion discs – stars: evolution – supernovae: general – white dwarfs.

1 INTRODUCTION

The coalescence of two carbon–oxygen (CO) white dwarfs with a combined mass in excess of the Chandrasekhar limit has long been considered a promising path towards a Type Ia supernova (SN Ia; Iben & Tutukov 1984; Webbink 1984). Indeed, in the last few years, a few massive double CO white dwarf systems have been found that have periods short enough for them to merge within a Hubble time (e.g. Napiwotzki et al. 2002, 2004). This double degenerate scenario can also easily explain the lack of hydrogen and helium lines in most SN Ia spectra and the occurrence of SNe Ia both in old and young star-forming systems (e.g. Branch et al. 1995).
Theoratically, the final fate of double CO white dwarf mergers has been much debated. Previous studies assumed that the dynamical disruption of the Roche lobe filling secondary should lead to the formation of a thick disc around the primary white dwarf (Tutukov & Yungelson 1979; Mochkovitch & Livio 1989, 1990). Therefore, accretion of CO-rich matter from the thick disc on to the central cold white dwarf has been studied for investigating the evolution of such mergers by many authors (Nomoto & Iben 1985; Saio & Nomoto 1985, 1998; Piersanti et al. 2003a,b; Saio & Nomoto 2004). As accretion rates from the thick disc should be close to the Eddington limit \( M \approx 10^{-5} \text{M}_\odot \text{yr}^{-1} \), most of those studies concluded that carbon ignition in the envelope of the accreting white dwarf is an inevitable consequence of such rapid accretion of CO-rich matter. Once carbon ignites off-centre, the burning flame propagates inwards on a relatively short time-scale (\( \approx 5000 \text{yr} \)), and the CO white dwarf is transformed into an ONeMg white dwarf (Saio & Nomoto 1985, 1998). When the mass of the ONeMg white dwarf approaches the Chandrasekhar limit, electron capture on to Ne and Mg is expected to lead to the gravitational collapse of the white dwarf to a neutron star (Nomoto & Kondo 1991; see Dessart et al. 2006; Kitaura, Janka & Hillebrandt 2006 for recent studies of such collapse).

However, the evolution of the remnants of double CO white dwarf mergers is not yet well understood. For instance, it has been debated whether the accretion rate decreases when the accreting white dwarf reaches critical rotation (Piersanti et al. 2003a; Saio & Nomoto 2004). More importantly, the canonical description of the merger remnant as a primary white dwarf + thick disc system is clearly an oversimplification. In previous 3D smoothed particle hydrodynamics (SPH) simulations (Benz et al. 1990; Segretain, Chabrier & Mochkovitch 1997; Guerrero, García-Berro & Isern 2004; see also Section 2), a large fraction of the disrupted secondary and the outermost layers of the primary form an extended hot envelope around the cold core containing most of the primary mass. The rest of the secondary mass becomes a centrifugally supported disc in the outermost layers of the merger remnant. Interestingly, the merger remnant reaches a state of quasi-static equilibrium within a few minutes from the onset of the dynamical disruption of the secondary. As the structure of the cold core plus the hot envelope appears to have a fairly spheroidal shape (see below) rather than the toroidal shape obtained with a zero-temperature equation of state (EOS) (Mochkovitch & Livio 1989, 1990), the merger remnant may be better described as a differentially rotating single CO star consisting of a slowly rotating cold core and a rapidly rotating hot extended envelope surrounded by a Keplerian disc, as illustrated in Fig. 1, than the previously adopted primary white dwarf + thick disc system. The further evolution of the merger must therefore be determined by the thermal cooling of the hot envelope and the redistribution of the angular momentum inside the central remnant, and accretion of matter on to the envelope from the Keplerian disc.

With this new approach to the problem in mind, we here revisit both the dynamical and the secular evolution of double CO white dwarf mergers. In the following section (Section 2), we present the numerical results of an SPH simulation of the dynamical evolution of the coalescence of a 0.9 M_\odot white dwarf and a 0.6 M_\odot CO white dwarf up to the stage of quasi-hydrostatic equilibrium, and we carefully investigate the structure of the merger remnant. In Section 3, we construct models of the central remnant in quasi-static equilibrium state (primary + hot extended envelope) which mimic the SPH result and calculate the thermal evolution of the merger remnant using a hydrodynamic stellar evolution code. In particular, the conditions for avoiding off-centre carbon ignition are systematically explored. In Section 4, we conclude this work by discussing uncertainties in our assumptions, the implications for Type Ia supernovae and future work.

The next section will discuss the subsequent thermal evolution after the coalescence of a double CO white dwarf coalescence binary, we investigate the configuration of the remnant in quasi-static equilibrium in some detail. For this purpose, we have carried out an SPH simulation of the dynamical process of the coalescence of two CO white dwarfs of 0.9 M_\odot and 0.6 M_\odot, respectively. Our simulation uses a 3D SPH code that is an offspring of a code developed to simulate neutron star mergers (Rosswog et al. 2000; Rosswog & Davies 2002; Rosswog & Liebendörfer 2003). It uses an artificial viscosity scheme with time-dependent parameters (Morris & Monaghan 1997). In the absence of shocks, the viscosity parameters have a very low value (\( \alpha = 0.05 \) and \( \beta = 0.1 \); most SPH implementations use values of \( \alpha = 1-1.5 \) and \( \beta = 2-3 \)); if a shock is detected, a source term (Rosswog et al. 2000) guarantees that the parameters rise to values necessary to damp the shock.

2 DYNAMICAL EVOLUTION OF THE MERGER

Before discussing the subsequent thermal evolution after the coalescence of a double CO white dwarf coalescence binary, we investigate the configuration of the remnant in quasi-static equilibrium in some detail. For this purpose, we have carried out an SPH simulation of the dynamical process of the coalescence of two CO white dwarfs of 0.9 M_\odot and 0.6 M_\odot, respectively. Our simulation uses a 3D SPH code that is an offspring of a code developed to simulate neutron star mergers (Rosswog et al. 2000; Rosswog & Davies 2002; Rosswog & Liebendörfer 2003). It uses an artificial viscosity scheme with time-dependent parameters (Morris & Monaghan 1997). In the absence of shocks, the viscosity parameters have a very low value (\( \alpha = 0.05 \) and \( \beta = 0.1 \); most SPH implementations use values of \( \alpha = 1-1.5 \) and \( \beta = 2-3 \)); if a shock is detected, a source term (Rosswog et al. 2000) guarantees that the parameters rise to values necessary to damp the shock.

Figure 1. Schematic illustration of the configuration of the remnant of a double CO white dwarf merger once quasi-static equilibrium has been established.

Table 1. Comparison of SPH simulations of double CO white dwarf mergers. The columns list: \( M_1 \) and \( M_2 \) – the masses of the primary and the secondary, respectively; NoP – the total number of particles used; \( v_{\text{ sph}} \) – the type of artificial viscosity employed; ‘std.’ refers to Monaghan & Varnas (1988); network – type of nuclear network employed; \( t_{\text{ sim}} \) – evolutionary time that has elapsed by the end of the calculation; \( T_{\text{ max}} \) – maximum temperature obtained during the simulation and \( T_P \) – the local peak of temperature at the end of the calculation.

| Reference | \( M_1 \) (\text{M}_\odot) | \( M_2 \) (\text{M}_\odot) | NoP | \( v_{\text{ sph}} \) | Network | \( t_{\text{ sim}} \) | \( T_{\text{ max}} \) (K) | \( T_P \) (K) |
|-----------|----------------|----------------|-----|----------------|----------|----------------|----------------|----------------|
| 1         | 1.2            | 0.9            | ~7 \times 10^5 | std. | None        | 51 s          | ?              | ~10^9          |
| 2         | 0.8            | 0.6            | ~4 \times 10^4 | std. + Balsara-switch | \( \alpha \)-network | 50 s          | 1.4 \times 10^9 | ?              |
| 2         | 1.0            | 0.6            | ~4 \times 10^4 | std. + Balsara-switch | \( \alpha \)-network | 65 s          | 1.6 \times 10^9 | ?              |
| 2         | 1.0            | 0.8            | ~4 \times 10^4 | std. + Balsara-switch | \( \alpha \)-network | 65 s          | 2.0 \times 10^9 | ?              |
| 3         | 0.9            | 0.6            | ~6 \times 10^4 | std. | None        | 1.56 min      | ?              | ~7 \times 10^8  |
| 4         | 0.9            | 0.6            | 2 \times 10^7  | See Rosswog et al. (2000) | QSE-\( \alpha \)-network | 5.3 min       | 1.7 \times 10^9 | 5.6 \times 10^8 |

(1) Benz et al. (1990); (2) Guerrero et al. (2004); (3) Segretain et al. (1997); (4) this study. \(^b\)Quasi-equilibrium (QSE).
that are able to resolve the shock properly without spurious post-shock oscillations. To suppress artificial viscosity forces in pure shear flows, we additionally apply a switch originally suggested by Balsara (1995).

To account for the energetic feedback on to the fluid from nuclear transmutations, we use a minimal nuclear reaction network developed by Hix et al. (1998). It couples a conventional $\alpha$-network stretching from He to Si with a quasi-equilibrium-reduced $\alpha$-network. Although a set of only seven nuclear species is used, this network reproduces the energy generation of all burning stages from helium burning to nuclear statistical equilibrium very accurately. For details and tests we refer to Hix et al. (1998). We use the

Figure 2. Dynamical evolution of the coalescence of a $0.6 \, M_\odot + 0.9 \, M_\odot$ CO white dwarf binary. The panels in the left-hand column show the density in the orbital plane, the panels in the right-hand column the temperature in units of $10^6$ K. Lengths are in code units ($= 10^9$ cm).
Helmholtz EOS, developed by the Centre for Astrophysical Thermonuclear Flashes at the University of Chicago. This EOS allows us to freely specify the chemical composition of the gas and can be coupled to nuclear reaction networks. The electron/positron EOS has been calculated without approximations, that is, it makes no assumptions about the degree of degeneracy or relativity; the exact expressions are integrated numerically to machine precision. The nuclei in the gas are treated as a Maxwell–Boltzmann gas, the photons as blackbody radiation. The EOS is used in tabular form with densities ranging from $10^{-10} \leq \rho y_e \leq 10^{11}$ g cm$^{-3}$ and temperatures

Figure 3. Dynamical evolution of the coalescence of a $0.6 \, M_\odot + 0.9 \, M_\odot$ CO white dwarf binary. Continued from Fig. 2.
from $10^4$ to $10^{11}$ K. A sophisticated, biquintic Hermite polynomial interpolation is used to enforce thermodynamic consistency (i.e. the Maxwell relations) at interpolated values (Timmes & Swesty 2000).

We use a MacCormack predictor–corrector method (e.g. Lomax, Pulliam & Zingg 2001) with individual particle time-steps to evolve the fluid. With our standard parameters for the tree-opening criterion and the integration, this time marching implementation conserves the total energy to better than $4 \times 10^{-3}$ and the total angular momentum to better than $2 \times 10^{-4}$. Note that this could, in principle, be improved even further by taking into account the so-called ‘grad–h’ terms (Monaghan 2002; Springel & Hernquist 2002; Price 2004) and extra terms arising from adapting gravitational smoothing terms (Price & Monaghan 2007).

To avoid numerical artefacts, we only use equal mass SPH particles. For the initial conditions, we therefore stretch a uniform particle distribution according to a function that has been derived from solving the 1D stellar structure equations. This technique is described in detail in Rosswog, Ramirez-Ruiz & Hix (in preparation). This particle set-up is then further relaxed with an additional damping force (e.g. Rosswog, Speith & Wynn 2004) so that the particles can settle into their true equilibrium configuration. The calculations are performed with $2 \times 10^5$ SPH particles, a much larger particle number than could be afforded by previous calculations, and run up to a much longer evolutionary time (5 min) than previous calculations (see Table 1).

For the white dwarf masses we choose 0.6 $M_\odot$, in the peak of the white dwarf mass distribution (e.g. Kepler et al. 2007), and 0.9 $M_\odot$, so that the total binary mass is above the Chandrasekhar limit. Two stars are placed on the $x$-axis at a mutual separation $a_0 = 2.6$ code units (1 code unit corresponds to $10^9$ cm). According to the formula of Plavec & Kratochvil (1964), the distance of the secondary, 0.6 $M_\odot$ star from the Lagrange point $L_1$ is $b_2 = a_0/0.5 + 0.227 \log q \approx 1.2$ code units, where $q$ is the mass ratio of the stars. The radius of the secondary is $R_2 \approx 0.75$, that is, the secondary is not yet filling its Roche lobe at the initial separation. To initiate the merger, the stars are provided with velocity components in positive and negative $y$ direction that correspond to 99 per cent of the Kepler frequency of a point mass binary at separation $a_0$. In addition, we add a very small radial velocity component, so that the relative radial velocity corresponds to about 0.005 of the relative orbital velocities. Viscous dissipation in the stellar material is unlikely to be large enough to lead to tidal synchronization of the stars (Segretain et al. 1997) during the gravitational wave-driven inspiral. For simplicity, we therefore start the simulations with non-rotating white dwarfs.

Figs 2 and 3 show the dynamical evolution of the merging process of the double white dwarf system considered in this study. The panel in the left-hand columns show the densities and the panel in the right-hand columns the temperatures (in units of $10^9$ K) in the orbital plane. The secondary is completely disrupted within 1.7 min, and mass accretion on to the primary induces local heating near the surface of the primary. Fig. 4 shows the evolution of the maximum temperature as a function of time. The peak in temperature reaches $1.7 \times 10^9$ K at $t \approx 1.0$ min, where $t = 0.0$ marks the moment when the simulation starts. Carbon ignites when $T \gtrsim 10^6$ K, but nuclear burning is quenched soon due to the local expansion of the hottest region, as is also observed in the simulations of Guerrero et al. (2004). The total amount of energy released due to nuclear burning is about $10^{56}$ erg.

Segretain et al. (1997) considered the same initial white dwarf masses as in the present study. But they adopted the original artificial viscosity prescription of Monaghan & Varnas (1988), which is known to introduce spurious forces in shear flows, and they did not include nuclear burning (Table 1). By the end of their calculation ($t = 1.56$ min), $T_{\text{max}}$ reached $8 \times 10^8$ K, while in our simulation, it decreases to $8 \times 10^8$ K only when $t \sim 1.7$ min. Interestingly, $T_{\text{max}}$ decreases further afterwards in our calculation, as shown in Fig. 4, and reaches a steady value at $T_{\text{max}} \approx 5.6 \times 10^8$ K when $t \gtrsim 2.5$ min. In the other calculations by Benz et al. (1990) and Guerrero et al. (2004), the dynamical evolution of the merger was not followed for more than 2 min either, and we cannot directly compare our results to theirs. However, we suspect that $T_{\text{max}}$ would also decrease further in the systems they considered if they had continued their calculations for a longer evolutionary time. It should also be noted that energy dissipation by artificial viscosity might lead to overheating, and that thermal diffusion – which may play an important role in the outermost layers – is not considered in the present study. It is thus likely that $T_{\text{max}}$ in the quasi-static equilibrium state may be even lower in reality than in our simulation.

Fig. 5 shows the structure of the merger remnant at quasi-static equilibrium. The central region with $R \lesssim 10^5$ cm ($M_r \lesssim 1.1 M_\odot$) has a fairly spheroidal shape, and a centrifugally supported disc appears at $R \gtrsim 10^6$ cm where the angular velocity is close to the Keplerian value. The fraction of the secondary mass contained in the Keplerian disc is larger in our simulation (about 67 per cent) than in Segretain et al. (1997) (about 41 per cent). The innermost core ($R \lesssim 3 \times 10^5$ cm; $M_r \lesssim 0.6 M_\odot$) is essentially isothermal, and the temperature has its peak value ($T \approx 5.6 \times 10^8$ K) at $R \approx 5 \times 10^5$ cm and $M_r \approx 0.85 M_\odot$. The disc material extends over 4 $\times 10^5$ cm along the $\z$-axis as the temperature is still high; if thermal diffusion were included, the disc would become much thinner on a short time-scale of a few hours. Therefore, our simulation confirms the remnant structure at quasi-static equilibrium that is illustrated in Fig. 1. In the next section, we investigate the secular evolution of the merger from such a quasi-static equilibrium state.

3 SECULAR EVOLUTION OF THE MERGER REMNANT

3.1 Physical assumptions and methods

Our SPH simulation shows that the remnant of the merger of two CO white dwarfs ($0.9 M_\odot + 0.6 M_\odot$) in the state of quasi-static equilibrium has the following features (see Fig. 6).

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Figure 4. The evolution of the local peak of temperature during the merger of two CO white dwarfs of 0.6 $M_\odot$ and 0.9 $M_\odot$, respectively, as a function of time after the onset of the simulation.
Figure 5. Top: Density contour of the merger remnant in the $xz$ plane at $t = 5.3$ min. Here one code unit corresponds to $10^9$ cm. Middle: Thermodynamic structure of the merger remnant at $t = 5.31$ min: shown are the temperature and the density as a function of distance from the centre, along the positive $x$- and $z$-axes, as indicated. Bottom: Angular velocity in units of the local Keplerian value at $t = 5.31$ min, along the positive/negative $x$- and $y$-axes of the merger remnant.

Figure 6. Initial model of the central remnant for sequences Sa1–Sa11. The top and middle panels show temperature as a function of the mass coordinate and radius, respectively. The solid curve in the bottom panel gives the angular velocity profile as a function of radius. The dashed curve denotes the angular velocity in units of the local Keplerian value.

(i) The core is cold and nearly isothermal.
(ii) The local peak of temperature ($T_p$) is located at a mass coordinate slightly less than the primary mass.
(iii) A steep gradient in temperature appears at the interface between the core and the local peak of temperature.
(iv) The interface is rather widely extended into the primary ($\Delta M_{\text{interface}} \approx 33$ per cent of the primary mass), and the mass of the quasi-isothermal cold core ($M_{\text{core}}$) is about 77 per cent of the primary mass.
(v) The mass of the outer envelope above the local peak of temperature contains about 33 per cent of the mass of the secondary, and the rest of the secondary forms a Keplerian disc.

Let us define $T_p$ as the local peak of temperature at quasi-static equilibrium, $M_{\text{CR}}$ as the mass of the central remnant (cold core +...
hot envelope, and $M_p$ as the location of $T_p$ in the mass coordinate (i.e. $M_p = M_{\text{core}} + \Delta M_{\text{interface}}$; see Fig. 6). To construct models of the central remnant, we use a 1D hydrodynamic stellar evolution code. Following Kippenhahn & Thomas (1970), Endal & Sofia (1976) and Heger, Langer & Woosley (2000) (cf. Meynet & Maeder 1997), we include the effect of rotation on the stellar structure by using the correction terms $f_\theta$ and $f_\tau$ in the momentum equation and the radiative temperature gradient:

$$\frac{\partial P}{\partial M} = -\frac{GM_p}{4\pi r^2} f_\theta - \frac{1}{4\pi r^2 \tau} \frac{\partial^2 \rho}{\partial t^2}$$

and

$$\frac{\partial T}{\partial P} = \frac{3 \kappa P L_\odot}{16 \pi c M_p g} \left[ 1 + \frac{r^2 \partial^2 \rho}{GM_p f_\tau \tau} \right]^{-1}.$$  

Table 2. Merger remnant model sequences. Each column lists the following: No. – sequence label; $M_{\text{CR}}$ – mass of the central remnant; $M_{\text{core}}$ – mass of the quasi-isothermal core; $M_p$ – location of the local peak of temperature in the mass coordinate; $T_p$ – the local peak of temperature; $\rho_p$ – density at $M_p$; $\tau_p$ – adopted time-scale for angular momentum loss according to equation (7); $M_{\text{acc}}$ – adopted mass accretion rate from the Keplerian disc; C-ig – off-centre ignition of carbon; $M_{\text{WD,ig}}$ – total mass of the central remnant when off-centre carbon ignition occurs; $M_{\text{ig}}$ – location of off-centre carbon ignition in the mass coordinate.

| No. | $M_{\text{CR}}$ ($M_\odot$) | $M_{\text{core}}$ ($M_\odot$) | $M_p$ ($M_\odot$) | $T_p$ ($10^6$ K) | $\rho_p$ ($10^6$ g/cm$^3$) | $\tau_p$ yr | $M_{\text{acc}}$ ($10^{-6}$ $M_\odot$ yr$^{-1}$) | C-ig | $M_{\text{WD,ig}}$ ($M_\odot$) | $M_{\text{ig}}$ ($M_\odot$) |
|-----|-----------------|-----------------|---------------|-------------|-----------------|-----------|-----------------|-----|-----------------|-------|
| Sa1 | 1.11            | 0.6             | 0.84          | 5.6         | 0.8             | $\infty$  | 0.0             | No  | –               | –     |
| Sa2 | 1.11            | 0.6             | 0.84          | 5.6         | 0.8             | 10$^2$    | 0.0             | Yes | 1.11            | 0.80  |
| Sa3 | 1.11            | 0.6             | 0.84          | 5.6         | 0.8             | 10$^3$    | 0.0             | Yes | 1.11            | 0.80  |
| Sa4 | 1.11            | 0.6             | 0.84          | 5.6         | 0.8             | 10$^4$    | 0.0             | Yes | 1.11            | 0.85  |
| Sa5 | 1.11            | 0.6             | 0.84          | 5.6         | 0.8             | 10$^5$    | 0.0             | No  | –               | –     |

where $r$ is the mean radius of an isobar. See Endal & Sofia (1976) for the details of a numerical evaluation of $f_\theta$ and $f_\tau$. As discussed in Yoon & Langer (2004) (hereafter YL04) and Yoon & Langer (2005), this method can accurately describe the structure of a rotating star for the layers rotating at a velocity less than 60% of the critical value. However, the outermost layers of our central remnant models rotate more rapidly than this limit, and it should be noted that, in our simulations, the effect of the centrifugal force is underestimated in those layers. We discuss implications of this limit for our results in Section 4. Transport of angular momentum and chemical species due to rotationally induced hydrodynamic instabilities is treated as a non-linear diffusive process, as explained in YL04. We also consider the dissipation of the rotational energy following Kippenhahn & Thomas (1978) as
In this way, the temperature profile in the central remnant model and references therein. (see Section 4). More details about the code are described in YL04 and Fricket instability). The effects of magnetic fields are neglected (due to this limitation are critically discussed in Section 4).

In order to mimic the temperature profile of the central remnant as obtained from the SPH simulation, we artificially deposit energy in the envelope, using the following prescription for a white dwarf with $M = M_{CR}$:

$$
\epsilon(M_e) = A(T'(M_e) - T(M_i)) \, \text{erg} \, \text{s}^{-1},
$$

where

$$
T'(M_e) = \left\{ \begin{array}{ll}
3 \times 10^7 \, \text{K} + (7 \times 10^7 - 3 \times 10^7 \, \text{K}) \left( \frac{M_e}{M_{\text{core}}} \right)^2, & \text{if } M_e < M_{\text{core}}, \\
T_p - (T_p - 7 \times 10^7 \, \text{K}) \left( \frac{M_e - M_p}{M_{\text{core}} - M_p} \right)^2, & \text{if } M_{\text{core}} \leq M_e \leq M_p, \\
C - (C - T_p) \left( \frac{\log(M_e/M_p)}{\log(M_{\text{core}}/M_p)} \right)^2, & \text{if } M_e > M_p.
\end{array} \right.
$$

In this way, the temperature profile in the central remnant model follows $T'(M_e)$. Here, $A$, $C$ and $\Omega$ are constants. We use $A = 10^9 \, \text{erg} \, \text{s}^{-1} \, \text{K}^{-1}$ and $C = 2 \times 10^8 \, \text{K}$ in most cases.

A rotational profile is imposed as

$$
\Omega(M_e) = \Omega_0 \left\{ \begin{array}{ll}
(1 - \Omega_0) \left( \frac{M_e - M_{\text{core}}}{M_{\text{core}}} \right)^{0.9}, & \text{if } M_e < M_{\text{core}}, \\
\Omega_0 \sqrt{\frac{GM_e}{r^3}}, & \text{if } M_e \geq M_{\text{core}},
\end{array} \right.
$$

where $\Omega_0 = 0.2 \sqrt{GM_{\text{CR}}/R^3}$ and $\omega_0 = \Omega_0 / \sqrt{GM_{\text{core}}/r_{\text{core}}^3}$. As shown in Fig. 6, this simple assumption gives a rotational velocity profile that is morphologically similar to that found in the SPH simulation: a steep gradient at the interface between the core and the envelope, and a local peak in the envelope. Within our 1D approximation of the effects of rotation, the exact shape of the rotational velocity profile does not affect the main conclusions of the present work for the following reasons. First, the velocity gradient at the interface is adjusted to the threshold value for the dynamical shear instability (DSI) on a very short time-scale (see below, and discussions in YL04). Secondly, our 1D approximation underestimates the effect of the centrifugal force on the stellar structure in layers which rotate more rapidly than about 60 per cent critical (YL04), and uncertainties due to this limit are much greater than due to the shape of $\Omega(r)$ in the outer layers of the envelope. Possible uncertainties due to this limitation are critically discussed in Section 4.

The central remnant may lose angular momentum by outward angular momentum transport into the Keplerian disc (Paczyński 1991; Popham & Narayan 1991) and/or by the gravitational wave radiation, for example, due to the r-mode instability (Andersson 1998; Friedman & Morsink 1998). Our code cannot properly describe any of these effects, and here we consider them simply by assuming a constant time-scale for the angular momentum loss ($\tau_j$; see Knaap 2004; cf. Piersanti et al. 2003a), such that the specific angular momentum of each mass shell decreases over a time-step $\Delta t$ by an amount

$$
\Delta j = j_i \left[ 1 - \exp \left( -\frac{\Delta t}{\tau_j} \right) \right].
$$

Mass accretion from the Keplerian disc is also considered in some model sequences, with different values for the accretion rate ($\dot{M}_{\text{acc}}$). The angular momentum accretion is treated in the same way as in YL04: the accreted matter is assumed to carry angular momentum at a value close to the Keplerian value if the surface velocity of the central remnant is below critical, while no angular momentum accretion is allowed otherwise.

Model sequences with different sets of $M_{CR}$, $M_{\text{core}}$, $M_p$, $\tau_j$, $M$ and $T_p$ are calculated, as summarized in Table 2. The initial model in sequence S is intended to reproduce the result of our SPH simulation, where $M_{CR} = 1.10 \, M_\odot$ and $M_p \approx 0.84 \, M_\odot$ are adopted. We also assume $M_{CR} = 1.25 \, M_\odot$ and $M_p \approx 0.9 \, M_\odot$ in sequence A, and $M_{CR} = 1.34 \, M_\odot$ and $M_p = 0.95 \, M_\odot$ in sequence B, to simulate mergers of $0.9-1.0 \, M_\odot + 0.7-1.0 \, M_\odot$ white dwarf binaries. At a given $M_{CR}$, different sets of $M_{\text{core}}, M_p$ and $T_p$ are marked in the sequence label by minor characters (a, b, c, d, e), while different sets of $\tau_j$ and $M_{\text{acc}}$ are indicated by Arabic numbers. For instance, sequences Sa1–Sa11 have the same initial merger model, but different values for $\tau_j$ and $M_{\text{acc}}$. Rotation is neglected in a test sequence 3a1 (i.e. the models are non-rotating). The temperature and angular velocity profiles in the initial central remnant model of sequences Sa1–Sa11 are shown in Fig. 6. The temperature (a few to several $\times 10^8$ K) and the size ($\sim 10^8$ cm) of the envelope appear to be comparable to those obtained from the SPH simulation (see Fig. 5).

For comparison, we also ran model sequences for classical cold matter accretion with a constant accretion rate of $\dot{M} = 10^{-5} \, M_\odot \, \text{yr}^{-1}$, for both non-rotating and rotating cases, as summarized in Table 3.

### 3.2 Results

#### 3.2.1 Classical models of cold matter accretion

Before discussing the central remnant models, let us first investigate the evolution of classical cold matter accreting white dwarf models in detail. In these models, the accreted matter is assumed to have the same entropy as the surface value of the accreting white dwarf. As shown in previous studies (e.g. Nomoto & Iben 1985), the thermal

| No. | $M_{\text{init}}$ | log $L_{\text{init}}/L_{\odot}$ | $M_{\text{WD,ig}}$ | $M_{\text{ig}}$ |
|-----|------------------|-----------------|-----------------|--------------|
| N0.7 | 0.7 | −2.118 | 0.999 | 0.793 |
| N0.8 | 0.8 | −2.128 | 1.010 | 0.862 |
| N0.9 | 0.9 | −2.188 | 1.039 | 0.939 |
| N1.0 | 1.0 | −2.137 | 1.087 | 1.024 |
| N1.1 | 1.1 | −2.170 | 1.150 | 1.114 |
| N1.2 | 1.2 | −2.119 | 1.225 | 1.207 |
| R0.8 | 0.8 | −2.114 | 1.297 | 1.038 |
| R0.9 | 0.9 | −2.119 | 1.249 | 1.050 |
| R1.0 | 1.0 | −2.082 | 1.207 | 1.069 |
| R1.1 | 1.1 | −2.050 | 1.205 | 1.127 |
Figure 7. (a) Evolution of a non-rotating white dwarf accreting with a constant accretion rate of $M = 10^{-5} \, M_\odot \, \text{yr}^{-1}$ with an initial mass of $0.9 \, M_\odot$ (sequence N0.9) in the density–temperature plane. (b) The local effective accretion rate ($\dot{M}_\text{eff}$, $r$) as a function of density in sequence N0.9, at different evolutionary epochs as indicated by the labels. (c–f) The rates of energy loss/production due to neutrino ($\epsilon_\nu$) cooling, compressional heating ($\epsilon_\text{comp}$), nuclear energy generation ($\epsilon_\text{nuc}$) and thermal diffusion ($\epsilon_\text{th}$) at different evolutionary epochs. Note that here $\epsilon_\nu$, $\epsilon_\text{comp}$ and $\epsilon_\text{nuc}$ represent the values which are used in the evolutionary calculations, while $\epsilon_\text{th}$ is an order-of-magnitude estimate according to equation (9).
temperature is located where the compressional heating rate begins to dominate over the thermal diffusion rate \( (\rho \approx 10^5 \text{ g cm}^{-3}) \), as expected. The neutrino cooling rate also increases as the temperature in the envelope becomes higher, but nuclear energy generation becomes significant before neutrino cooling dominates the thermal evolution, inducing a carbon-burning flash around \( \rho \approx 5.6 \times 10^5 \text{ g cm}^{-3} \).

As Table 3 shows, and consistent with the findings of Nomoto & Iben (1985), such off-centre carbon flashes occur regardless of the initial mass of the white dwarf, if \( \dot{M}_{\text{acc}} \approx 10^{-5} \text{ M}_\odot \text{ yr}^{-1} \). The results with models including rotation show that carbon ignition may be delayed if the effect of rotation is included (Table 3; see also Piersanti et al. 2003a; Saio & Nomoto 2004). The reason is that the local effective mass accretion rate \( (\dot{M}_{\text{eff},3} \approx 4\pi\tau^2\rho v) \) inside the white dwarf at a given mass is lower because of the centrifugal force.

For instance, in sequence NO.9, we have \( \dot{M}_{\text{eff},3} \approx 10^{-5} \text{ M}_\odot \text{ yr}^{-1} \) at around \( \rho = 5 \times 10^5 \text{ g cm}^{-3} \) when \( t \approx 10^7 \text{ yr} \) (Fig. 7b), but \( \dot{M}_{\text{eff},3} \) is lowered by a factor of 2 in the corresponding rotating model at a similar epoch (i.e. \( \dot{M}_{\text{eff},3} \approx 5 \times 10^{-6} \text{ M}_\odot \text{ yr}^{-1} \)), as revealed in Fig. 8. However, carbon ignition occurs well before the white dwarf reaches the Chandrasekhar limit, in all model sequences considered. Thus, rotation by itself cannot change the conclusion of the previous work that the coalescence of double CO white dwarfs should lead to accretion-induced collapse rather than a thermonuclear explosion, unless the accretion rate is significantly lowered, as was also shown by Piersanti et al. (2003a) and Saio & Nomoto (2004).

### 3.2.2 Sequences without angular momentum loss and mass accretion

Having understood the physics of the thermal evolution of CO white dwarfs which accrete cold matter with a rate close to the Eddington limit, we now investigate the evolution of the central remnant model consisting of a cold core and a hot envelope as described in Section 3.1. First, we examine the results of the model sequences where both angular momentum loss and mass accretion from the Keplerian disc are neglected (i.e. \( \tau = \infty \) and \( M = 0 \)); sequences Sa1, Aa1, Ab1, Ac1, Ad1, Ae1, Ba1 and Ta1).

Fig. 9(a) illustrates the evolution of the central remnant for \( M_{\text{eq}} = 1.10 \text{ M}_\odot \) in sequence Sa1 in the density–temperature plane. Note that the local peak of temperature at \( t = 0.00 \) (\( T_p = 5.6 \times 10^8 \text{ K} \)) is significantly below the critical temperature for carbon ignition (\( T_{C-\text{ig}} \); dotted curve in Fig. 9a). It is shown in Fig. 9(b) that the local effective accretion rate \( (\dot{M}_{\text{eff},3}) \) remains relatively high \( (5 \times 10^{-6} - 10^{-5} \text{ M}_\odot \text{ yr}^{-1}) \) around \( \rho = 10^6 \text{ g cm}^{-3} \), where the local peak of temperature is located, for about 5000 yr. Despite such high effective accretion rates, the temperature peak continuously decreases, although the inner core becomes somewhat hotter due to compression, and the central remnant finally becomes a cold white dwarf. A few remarkable differences compared to the standard accreting white dwarf models are found in this regard. First, since the envelope is very hot, neutrino cooling – in particular by photoneutrinos – is significant from the beginning, and even dominant over the thermal diffusion at the interface between the core and the envelope as shown in Fig. 9(c). In cold matter accreting white dwarfs, neutrino cooling becomes important only after a significant amount of mass has been accreted (Fig. 7). Secondly, the compressional heating rate is slightly lower than the neutrino cooling rate around the local peak of temperature. As the contraction of the central remnant is mainly determined by the thermal evolution of the envelope, the local accretion rate is in fact controlled by the cooling process. This explains why we have \( \epsilon_{\text{comp}} \approx \epsilon_c \) around the local peak of temperature for the initial \( \sim 10^5 \) yr, and why the local peak of temperature continuously decreases despite the relatively high effective accretion rate. This conclusion is the same for all other sequences with a \( T_p \) that is significantly lower than \( T_{C-\text{ig}} \) (sequences Aa1, Ab1 and Ba1) including the non-rotating case (sequence Ta1; Table 2).

We find that, in sequence Sa1, the differentially rotating layers at the interface between the core and the envelope in the initial central remnant model are stable against the DSI. They are, however, unstable to the DSI in other sequences, where the interface is more degenerate (see YL04 for discussions on the DSI). Consequently, in sequence Sa1, for example, the rate of rotational energy dissipation \( (\epsilon_{\text{rot}}) \) appears to be very high initially (Fig. 10). The differentially rotating layers are rapidly smeared out by the DSI (see the discussion in Section 2 in YL04), and \( \epsilon_{\text{rot}} \) falls below the thermal diffusion and/or neutrino cooling rate only within 20 yr. Hence we conclude that the rotational energy dissipation does not play an important role for the long-term evolution of the central remnant.

Fig. 11 shows how the evolution of the central remnant changes if the local peak of temperature in the initial model (\( T_p \)) is close to or above the critical limit for carbon burning (\( T_{C-\text{ig}} \)), with sequences Ac1 and Ae1 as examples. In contrast to sequence Sa1 or Aa1, carbon burning dominates the evolution very soon in both sequences, and the temperature increases rapidly. Although the further evolution has not been followed in the present study, it is most likely that the carbon-burning flame propagates inward such that the central remnant is converted into an ONeMg white dwarf within several thousand years as shown by Saio & Nomoto (1998).

As summarized in Table 2, all other sequences follow the same evolutionary pattern: off-centre carbon ignition is avoided in sequences Ab1, Ba1 and Ta1 while carbon ignites off-centre in the other sequences where \( T_p \) is significantly below \( T_{C-\text{ig}} \). It is thus remarkable that the thermal evolution of the central remnant is sensitively determined by the local peak of temperature in the quasi-static equilibrium state.

In conclusion, in the absence of angular momentum loss and mass accretion from the Keplerian disc, the thermal evolution of the central remnant is roughly controlled by neutrino cooling at the interface between the core and the envelope, and off-centre carbon burning may be avoided as long as \( T_p \geq T_{C-\text{ig}} \), while it seems inevitable if \( T_p \leq T_{C-\text{ig}} \).

### 3.3 Effect of angular momentum loss

In sequences Sa2–Sa5, the central remnant has the same initial conditions as in sequence Sa1, angular momentum loss from the white
Figure 9. (a) Evolution of the central remnant in sequence Sa1 in the log ρ−T plane. The dotted curve gives the critical temperature where the nuclear energy generation rate due to carbon burning equals the energy loss rate due to neutrino cooling. (b) The local effective accretion rate ($\dot{M}_{\text{eff}} \equiv 4\pi r^2 \rho v_r$) as a function of density in the merger remnant model of sequence Sa1, at different evolutionary epochs as indicated by the labels. (c–f) The rates of energy loss/gain due to neutrino ($\epsilon_{\nu}$), cooling, compressional heating ($\epsilon_{\text{comp}}$), nuclear energy generation ($\epsilon_{\text{nuc}}$) and thermal diffusion ($\epsilon_{\text{th}}$) as a function of density in the central remnant models of sequence Sa1 at different evolutionary epochs. Note that here $\epsilon_{\nu}, \epsilon_{\text{comp}}$ and $\epsilon_{\text{nuc}}$ represent the values which are used in the evolutionary calculations, while $\epsilon_{\text{th}}$ is an order-of-magnitude estimate according to equation (9).

dwarf with different time-scales $\tau_J$ is considered according to equation (7). Note that off-centre carbon ignition occurs in sequences Sa2, Sa3 and Sa4, where $\tau_J \lesssim 10^4$ yr, while it is avoided in sequence Sa5 where $\tau_J = 10^5$ yr. These results indicate that off-centre carbon ignition should be induced if the angular momentum loss occurs too rapidly for neutrino cooling or thermal diffusion to control the effective mass accretion. For instance, Fig. 12 shows that in sequence Sa4, where $\tau_J = 10^4$ yr, the effective mass accretion rate reaches a few $10^{-4}$ $M_\odot$ yr$^{-1}$ at the interface between the core and the envelope ($\rho \approx 10^7$ g cm$^{-3}$), and the compressional heating rate exceeds the neutrino cooling rate.

It is shown that the critical angular momentum loss time-scale, $\tau_J$, for off-centre carbon ignition ($\tau_{J,\text{crit}}$) is smaller for sequences Ab and Ba than for sequences Sa and Aa: $\tau_{J,\text{crit}} \approx 10^3$ for sequences Ab and Ba, and $\tau_{J,\text{crit}} \approx 10^4$ for sequences Sa and Aa. This is due to the different local thermodynamic properties at the interface between the core and the envelope in different central remnant models. As shown in Fig. 13, higher density and/or temperature at the interface result in a shorter neutrino cooling time, making it possible to avoid local heating for a smaller $\tau_J$. In other words, $\tau_{J,\text{crit}}$ roughly corresponds to the time-scale for neutrino cooling at the local peak of temperature ($\tau_{\nu,p}$).

From this experiment, we conclude that, in the absence of mass accretion from the Keplerian disc, carbon ignition may be avoided in the central remnant, if $T_{\text{max init}} < T_{\text{C-ig}}$, and if $\tau_J > \tau_{\nu,p}$.
3.4 Mass accretion from the Keplerian disc

In reality, mass accretion from the Keplerian disc on to the central remnant is expected. The accretion rate is determined by the viscosity of the disc, which is not well known. However, we expect the accretion rate from a Keplerian disc may be significantly lower than from a pressure-supported thick disc that was assumed in previous studies. Our results, as summarized in Table 2, indicate that even with mass accretion, the central remnant with $T_p < T_{C-ig}$ can avoid off-centre carbon ignition if the accretion rate is sufficiently low (i.e. $\dot{M} < 5 \times 10^{-6}$ to $10^{-5} M_\odot$ yr$^{-1}$), and if $\tau_J > \tau_\nu$ (see Table 2).

The thermal history of the central remnant in those sequences where carbon ignites off-centre is similar to that of the white dwarf in classical accretion model sequences. However, as the central remnant has a rapidly rotating hot envelope, carbon ignition is significantly delayed compared to the case of classical accretion. In sequence N1.2, where $M_{\text{inj}} = 1.2 M_\odot$ and $M_{\text{kep}} = 10^{-3} M_\odot$ yr$^{-1}$, carbon ignites only when about $0.025 M_\odot$ is accreted. In sequence Aa7 more than $0.15 M_\odot$ have to be accreted to induce carbon ignition at the same accretion rate, despite its higher initial mass. On the other hand, the comparison of sequence Aa7 with sequence Aa10 indicates that off-centre carbon ignition is delayed if the central remnant keeps more angular momentum. The critical accretion rate for inducing off-centre carbon ignition is thus difficult to precisely determine, as our 1D models significantly underestimate the effect of the centrifugal force, especially in the envelope where carbon ignites. In addition, the physics of angular momentum loss/gain is not well understood yet, as discussed in Yoon & Langer (2005).

Note that $M_{\text{WD,ig}}$ in sequences Aa10, Ba6 and Ba7 is already very close, or even above the Chandrasekhar limit. However, the central density in those models is still smaller by an order of magnitude than the critical limit for carbon ignition due to the effect of rotation. As the carbon-burning flame will propagate inwards within several thousand years (Saio & Nomoto 1998), only about $0.05 M_\odot$ may be further accreted by the time the burning flame reaches the centre, and the central density may not become high enough to induce a thermonuclear explosion before the whole central remnant is converted into an ONeMg white dwarf. (Super-)Chandrasekhar mass ONeMg white dwarfs produced in this way will eventually collapse to a neutron star (see Yoon & Langer 2005; Dessart et al. 2006).

On the other hand, the white dwarf continuously grows to/above the Chandrasekhar limit ($\approx 1.4 M_\odot$) without suffering carbon ignition (not at the centre nor off-centre) in sequences Sa8, Sa9, Sa10, Sa11, Aa8, Aa9, Ab6 and Ba8. The outcome in these cases is thus the formation of a (super-)Chandrasekhar mass CO white dwarf, which will eventually explode as a Type Ia supernova. The mass of the exploding white dwarf should depend on the amount of angular momentum (Yoon & Langer 2005) and cannot exceed the mass budget of merging white dwarfs. Fig. 14 shows the evolutionary paths of the central remnant for sequences Sa8, Sa9 and Sa11 as examples in the mass–angular momentum plane. Note that the central remnant initially has a large amount of angular momentum ($J = 1.11 \times 10^{50}$ erg s), such that without loss/gain of angular
momentum, it should accrete matter until it reaches $M \approx 1.68 M_\odot$ where it explodes in an SN Ia explosion. In sequences Sa8 and Sa9, the accretion time-scale ($\tau_{\text{acc}}$) is longer than the angular momentum loss time-scale, and the total angular momentum of the white dwarf continuously decreases while the total mass increases. Consequently, carbon ignites at the centre when the white dwarf grows to 1.50 $M_\odot$ and 1.42 $M_\odot$ for sequences Sa8 and Sa9, respectively.

In sequence Sa11, on the other hand, both mass and angular momentum of the central remnant continuously increase, given that $\tau_{\text{acc}} \lesssim \tau_I$, and an SN Ia explosion is expected only when $M \approx 1.70 M_\odot$. Note that this is even larger than the mass budget of the binary system considered for this sequence (i.e. 0.9 $M_\odot + 0.6 M_\odot$).

In nature, the white dwarf must stop growing in mass when $M = 1.5 M_\odot$, and an SN Ia explosion will be induced only when a sufficient amount of angular momentum has been removed, for example, via gravitational wave radiation, as illustrated by the path Sa11–B in Fig. 14. In these cases, the eventual SN Ia explosion would occur only $\gtrsim 10^5$ yr after the initial dynamical merger. Our models in sequence Sa9 have characteristic surface luminosity and temperature of about $\sim 10^3 L_\odot$ and $\sim 4.5 \times 10^5$ K, respectively. Such objects may resemble supersoft X-ray sources (SSS, Kahabka & van den Heuvel 1997), but they should appear 10 times less luminous than a typical SSS (cf. van den Heuvel et al. 1992).

**4 CONCLUSION AND DISCUSSION**

We have explored the dynamical and secular evolution of the merger of double CO white dwarf binaries whose total mass exceeds the Chandrasekhar limit. Based on our new SPH simulation of the coalescence of two CO white dwarfs of 0.9 $M_\odot$ and 0.6 $M_\odot$, we suggest that the immediate post-merger remnant is best described as a differentially rotating CO star consisting of a slowly rotating cold core and a rapidly rotating hot envelope that is surrounded by a Keplerian disc rather than as ‘cold white dwarf + thick disc’ system, as in previous investigations. The evolution of such a CO star is determined by the thermal evolution of the envelope, and the growth of the core is controlled by the cooling due to neutrino emission and thermal
diffusion, which is fundamentally different from the assumption of ‘forced accretion of cold matter’.

Our 1D stellar evolution models of the central remnant, that is, the cold core and the hot envelope, which include the effects of rotation, indicate that there are three necessary conditions for the merger remnant to avoid off-centre carbon ignition such that an SN Ia may be produced.

(i) The local peak of temperature of the merger remnant at the interface between the core and the envelope must be lower than the critical temperature for carbon ignition (\(T_P < T_{C-ig}\)).

(ii) The time-scale for angular momentum loss from the central remnant by must be larger than the neutrino cooling time-scale at the interface (\(\tau_J > \tau_{\nu,p}\)).

(iii) Mass accretion from the Keplerian disc on to the central remnant must be sufficiently slow (\(M_{\text{acc}} \lesssim 5 \times 10^{-6} \text{ to } 10^{-5} \, M_{\odot} \text{ yr}^{-1}\)).

Our new SPH simulation confirms that at least the first condition (\(T_P < T_{C-ig}\)) should be fulfilled in the CO white dwarf binary considered.

As emphasized in Section 3.1, our 1D models significantly underestimate the effect of the centrifugal force on the stellar structure in the rapidly rotating outermost layers. However, since thermal diffusion always dominates over both neutrino cooling and compressional heating in the outer envelope (\(\rho \lesssim 10^9 - 10^{10} \, g \text{ cm}^{-3}\)) above the interface, as shown in Figs 7 and 12, the detailed structure of the rapidly rotating outermost layers above the interface may not significantly affect our results on the thermal evolution of the merger remnant, as long as the angular momentum of the envelope is not lost faster than the local neutrino cooling time-scale at the interface. On the other hand, mass accretion from the Keplerian disc should occur preferentially along the equatorial plane of the envelope. As shown in the SPH simulation, the envelope is more extended along the equatorial plane, where most angular momentum is deposited, than along the polar axis, and the resultant compressional heating must be much weakened, compared to the case of our 1D models. The enhanced role of rotation must thus help to increase the critical mass accretion rate for inducing off-centre carbon ignition, in favour of producing a Type Ia supernova.

We have concluded that the loss of angular momentum on a short time-scale (\(\tau_J \lesssim \tau_{\nu,p} \approx 10^4 - 10^5 \, \text{yr}\)) may induce off-centre carbon ignition even when \(T_{\text{max,init}} < T_{C-ig}\). Rapidly rotating compact stars may experience loss of angular momentum by gravitational wave radiation, due to either the bar-mode instability or the r-mode instability. The onset of the dynamical or secular bar-mode instability requires a very high ratio of the rotational energy to the gravitational energy: \(E_{\text{rot}}/E_{\text{grav}} \gtrsim 0.2\) for the dynamical bar-mode instability, and \(E_{\text{rot}}/E_{\text{grav}} \gtrsim 0.14\) for the secular bar-mode instability (e.g. Shapiro & Teukolsky 1983). As both our 1D models and SPH simulation give a value of \(E_{\text{rot}}/E_{\text{grav}}\) that is much lower (about 0.06–0.07) than those critical limits, the bar-mode instability may not be relevant. The r-mode instability may operate, in principle, even with such a low \(E_{\text{rot}}/E_{\text{grav}}\) (Andersson 1998; Friedman & Morsink 1998). However, we estimate that the growth time of the r-mode instability (\(\tau_r\), using our central remnant models and following Lindblom (1999), is \(\gtrsim 10^6 \, \text{yr}\), which is much longer than the local neutrino cooling time-scale (\(\tau_{\nu,p} \approx 10^3 \, \text{yr}\)). Alternatively, angular momentum might be transported from the accreting star into the Keplerian disc when the accretor reaches critical rotation. Calculations by Saio & Nomoto (2004) indicate, however, that the decrease of the total angular momentum due to such an effect is not significant in accreting white dwarfs. In conclusion, neither gravitational wave radiation nor outward angular momentum transport is likely to lead to a rapid loss of angular momentum from the central remnant such that \(\tau_J < \tau_{\nu,p}\), unless magnetic torques are important.

The central remnant may be enforced to rotate rigidly on a short time-scale in the presence of strong magnetic torques (cf. Spruit 2002). The central remnant in both our SPH simulation and 1D models has \(J_{\text{tot}} > 10^{36} \, \text{erg s}\), which is significantly higher than the maximum limit a rigidly rotating white dwarf can retain, as shown in Fig. 14. This means that if magnetic torques led to rigid rotation, a large amount of angular momentum should be transported into the Keplerian disc (Case ‘a’ in Fig. 14), or mass shedding of supercritically spun-up layers should occur from the central remnant (Case ‘b’ in Fig. 14). In Case ‘a’, the local density around the interface should increase by several factors by the time when the central remnant reaches rigid rotation as implied by Fig. 15. Off-centre carbon ignition might be inevitable in this case due to a resultant high effective accretion rate, if the time for angular momentum redistribution were shorter than the local cooling time due to neutrino losses. In Case ‘b’, on the other hand, the local density at the interface might not increase if mass shedding from the central remnant occurred at a sufficiently high rate. Therefore, the role of magnetic fields in the merger evolution remains uncertain at the current stage and is a challenging subject for future work.

The coalescence of more massive double CO white dwarf binaries is likely to result in a higher maximum temperature due to the enhanced role of gravity. Consequently, given the important role of the maximum temperature in the merger remnant for its final fate, less massive binary CO white dwarfs may be favoured for the production of SNe Ia from such a channel. Our SPH simulation shows that the maximum temperature at steady state in the considered system (\(M_{\text{stab}} = 1.5 \, M_{\odot}\)) is fairly close to the critical limit for carbon ignition. This implies that the total mass of a double CO white dwarf
merger should not be much higher than $1.5 M_\odot$ to produce an SN Ia. However, it is theoretically expected that more than 30 per cent of the massive double degenerate mergers with $M_{\text{total}} \gtrsim 1.4 M_\odot$ should be less massive than $\sim 1.5 M_\odot$ (Yungelson et al. 1994; Han 1998), which is suitable for SNe Ia production.

In this regard, we note that there are a number of potentially important factors that have not been included in either the present study or previous simulations. These include the following points.

(i) The previous and present simulations assumed that white dwarfs are cold prior to the merging process. However, Iben, Tutukov & Fedorova (1998) point out that tidal interactions might heat up the white dwarfs as the orbit shrinks, which could weaken the gravitational potential of the primary. Furthermore, as the temperature of white dwarfs is a function of their age, younger progenitors should have more extended envelopes, which may result in a lower $T_p$.

(ii) A thin hydrogen/helium envelope must be present initially in both the primary and the secondary. As hydrogen or helium should ignite at a much lower temperature than carbon, the influence of the release of nuclear energy during the merger process may be even more important than shown in the existing SPH simulations. Furthermore, as the orbit shrinks, which could weaken the gravitational potential of the primary, younger progenitors should have more extended envelopes, which may result in a lower $T_p$.

(iii) At a given total mass ($M_{\text{tot}} = M_{\text{primary}} + M_{\text{secondary}}$), different mass ratios of the white dwarf components ($q = M_{\text{secondary}}/M_{\text{primary}}$) must result in different merger structures. A lower $q$ at a given $M_{\text{tot}}$ may not only lead to a stronger gravitational potential of the primary, but also to a lower mass accretion rate during the dynamical mass transfer (Guerrero et al. 2004). As the former and the latter will tend to increase and decrease $T_p$, respectively, quantitative studies are necessary to predict how $T_p$ will change with $q$.

Finally, another important ingredient that needs to be considered is thermal diffusion during the dynamical evolution. As shown above, the mass accretion rate from the Keplerian disc on to the envelope of the central remnant is one of the most important factors that critically determine the final fate of double CO white dwarf mergers. The accretion rates depend on the structure of the Keplerian disc at thermal equilibrium, which can be only understood by including thermal diffusion in future simulations. But here we emphasize again that the accretion rates from a centrifugally supported Keplerian disc should be significantly lower than those from a pressure-supported thick disc that was previously assumed, which opens the possibility for at least some double CO white dwarf mergers to produce SNe Ia.

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**REFERENCES**

Andersson N., 1998, ApJ, 502, 708
Balsara D. S., 1995, J. Comput. Phys., 121, 357
Benz W., Cameron A. G. W., Press W. H., Bowers R. L., 1990, ApJ, 348, 647
Branch D., Livio M., Yungelson L. R., Boffi F. R., Baron E., 1995, PASP, 107, 1019
Dessart L., Burrows A., Ott C., Livne E., Yoon S.-C., Langer N., 2006, ApJ, 644, 1063
Endal A. S., Sofia S., 1976, ApJ, 210, 184
Friedman J. L., Morsink S. M., 1998, ApJ, 502, 714
Guerrero J., García-Boiro E., Isern J., 2004, A&A, 413, 257
Han Z., 1998, MNRAS, 296, 1019
Heger A., Langer N., Woosley S. E., 2000, ApJ, 528, 368
Hix W. R., Kehkshov M. A., Wheeler J. C., Thielemann F.-K., 1998, ApJ, 503, 332
Iben I. Jr, Tutukov A., 1984, ApJS, 55, 335
Iben I. Jr, Tutukov A. V., Fedorova A. V., 1998, ApJ, 503, 344
Itoh N., Hayashi H., Hishikawa A., Kohyama Y., 1996, ApJS, 102, 411
Kalabka P., van den Heuvel E. P. J., 1997, ARA&A, 35, 69
Kepler S. O., Kleinman S. J., Nitta A., Koester D., Castanheira B. G., Giovannini O., Costa A. F. M., Althaus L., 2007, MNRAS, 375, 1315
Kippenhahn R., Thomas H.-C., 1970, in Sletteback A., ed., IAU Col. 4, Stellar Rotation. Taylor & Francis, New York, p. 4
Kippenhahn R., Thomas H.-C., 1978, A&A, 63, 265
Kippenhahn R., Weigert A., 1990, Stellar Structure and Evolution. Springer-Verlag, Berlin
Kitaura F. S., Janka H.-Th., Hillebrandt W., 2006, A&A, 450, 345
Knap R. J. C., 2004, Master thesis, Utrecht University
Lomax H., Pulliam T. H., Zingg D. W., 2001, Fundamentals of Computational Fluid Dynamics. Springer-Verlag, Heidelberg
Lindblom L., 1999, Phys. Rev. D, 60, 4007
Meynet G., Maeder A., 1997, A&A, 321, 465
Mochkovitch R., Livio M., 1989, A&A, 209, 111
Mochkovitch R., Livio M., 1990, A&A, 236, 378
Monaghan J. J., 2002, MNRAS, 335, 843
Monaghan J. J., Varnas S. R., 1988, MNRAS, 231, 515
Morris J. P., Monaghan J. J., 1997, J. Comput. Phys., 136, 41
Meynet G., Maeder A., 1997, A&A, 321, 465
Napiwotzki R., et al., 2004, Rev. Mex. Astron. Astrofis. Ser. Conf., 20, 113
Nomoto K., Kondo Y., 1991, ApJ, 370, 604
Popham R., Narayan R., 1991, ApJ, 370, 604
Price D., Monaghan J. J., 2007, MNRAS, 374, 1347
Popham R., Narayan R., 1991, ApJ, 370, 604
Price D., 2004, PhD thesis, Cambridge University
Rosswog S., Liebendörfer M., 2003, MNRAS, 342, 673
Rosswog S., Davies M. B., 2002, MNRAS, 334, 481
Rosswog S., Davies M. B., 2003, MNRAS, 342, 673
Rosswog S., Davies M. B., Thielemann F.-K., Piran T., 2000, A&A, 360, 171
Rosswog S., Speith R., Wynn G., 2004, MNRAS, 351, 1121

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Figure 15. The density profile in the initial model of the central remnant in sequence Sa (solid curve), and in a corresponding hot ($T_p = 10^8$ K) white dwarf model that rotates rigidly at critical rotation at the surface (dashed curve).
Saio H., Nomoto K., 1985, A&A, 150, 21
Saio H., Nomoto K., 1998, ApJ, 500, 388
Saio H., Nomoto K., 2004, ApJ, 615, 444
Segretain L., Chabrier G., Mochkovitch R., 1997, ApJ, 481, 355
Shapiro S. L., Teukolsky S. A., 1983, Black Holes, White Dwarfs and Neutron Stars: The Physics of Compact Objects. Willey Interscience Publication, New York
Springel V., Hernquist L., 2002, MNRAS, 333, 649
Spruit H. C., 2002, A&A, 381, 923
Tutukov A. V., Yungelson L., 1979, Acta. Astron., 29, 665
Timmes F. X., Swesty F. D., 2000, ApJS, 126, 501
van den Heuvel E. P. J., Bhattacharya D., Nomoto K., Rappaport S. A., 1992, A&A, 262, 97
Webbink R. F., 1984, ApJ, 277, 355
Yoon S.-C., Langer N., 2004, A&A, 419, 623 (YL04)
Yoon S.-C., Langer N., 2005, A&A, 435, 967
Yungelson L. R., Livio M., Tutukov A. V., Saffer R. A., 1994, ApJ, 420, 336
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