Abstract. In [4], the existence of the solution is proved for a scalar linearly growing backward stochastic differential equation (BSDE) if the terminal value is $L \exp \left( \mu \sqrt{2 \log (1 + L)} \right)$-integrable with the positive parameter $\mu$ being bigger than a critical value $\mu_0$. In this note, we give the uniqueness result for the preceding BSDE.

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1 Introduction

Let $\{W_t, t \geq 0\}$ be a standard Brownian motion with values in $\mathbb{R}^d$ defined on some complete probability space $(\Omega, \mathcal{F}, \mathbb{P})$, and $\{\mathcal{F}_t, t \geq 0\}$ its natural filtration augmented by all $\mathbb{P}$-null sets of $\mathcal{F}$. Let us fix a nonnegative real number $T > 0$. The $\sigma$-field of predictable subsets of $\Omega \times [0,T]$ is denoted by $\mathcal{P}$.

For any real $p \geq 1$, denote by $L^p$ the set of all $\mathcal{F}_T$-measurable random variables $\eta$ such that $E|\eta|^p < \infty$, by $S^p$ the set of (equivalent classes of) all real-valued, adapted and càdlàg processes $\{Y_t, 0 \leq t \leq T\}$ such that

$$||Y||_{S^p} := E \left[ \sup_{0 \leq t \leq T} |Y_t|^p \right]^{1/p} < +\infty,$$

by $L^p$ the set of (equivalent classes of) all real-valued adapted processes $\{Y_t, 0 \leq t \leq T\}$ such that

$$||Y||_{L^p} := E \left[ \int_0^T |Y_t|^p \, dt \right]^{1/p} < +\infty,$$

and by $M^p$ the set of (equivalent classes of) all predictable processes $\{Z_t, 0 \leq t \leq T\}$ with values in $\mathbb{R}^{1 \times d}$ such that

$$||Z||_{M^p} := E \left[ \left( \int_0^T |Z_t|^2 \, dt \right)^{p/2} \right]^{1/p} < +\infty.$$
Consider the following Backward Stochastic Differential Equation (BSDE):

\[ Y_t = \xi + \int_t^T f(s, Y_s, Z_s)ds - \int_t^T Z_s dW_s, \quad t \in [0, T]. \]  

(1.1)

Here, \( f \) (hereafter called the generator) is a real valued random function defined on the set \( \Omega \times [0, T] \times \mathbb{R} \times \mathbb{R}^{1 \times d} \), measurable with respect to \( \mathcal{P} \otimes \mathcal{B} (\mathbb{R}) \otimes \mathcal{B} (\mathbb{R}^{1 \times d}) \), and continuous in the last two variables with the following linear growth:

\[ |f(s, y, z) - f(s, 0, 0)| \leq \beta |y| + \gamma |z|, \quad (s, y, z) \in [0, T] \times \mathbb{R} \times \mathbb{R}^{1 \times d} \]

with \( f_0 := f(\cdot, 0, 0) \in L^1, \beta \geq 0 \) and \( \gamma > 0 \). \( \xi \) is a real \( \mathcal{F}_T \)-measurable random variable, and hereafter called the terminal condition or terminal value.

**Definition 1.1** By a solution to BSDE (1.1) we mean a pair \( \{(Y_t, Z_t), 0 \leq t \leq T \} \) of predictable processes with values in \( \mathbb{R} \times \mathbb{R}^{1 \times d} \) such that \( \mathbb{P} \)-a.s., \( t \mapsto Y_t \) is continuous, \( t \mapsto Z_t \) belongs to \( L^2(0, T) \) and \( t \mapsto f(t, Y_t, Z_t) \) is integrable, and \( \mathbb{P} \)-a.s. \((Y, Z)\) verifies (1.1).

By BSDE \((\xi, f)\), we mean the BSDE with generator \( f \) and terminal condition \( \xi \).

It is well known that for \((\xi, f_0) \in L^p \times L^p \) (with \( p > 1 \)), BSDE (1.1) admits a unique adapted solution \((y, z)\) in the space \( \mathcal{S}^p \times \mathcal{M}^p \) if the generator \( f \) is uniformly Lipschitz in the pair of unknown variables. See e.g. [3, 6, 3, 1] for more details. For \((\xi, f_0) \in L^1 \times L^1 \), one needs to restrict the generator \( f \) to grow sub-linearly with respect to \( z \), i.e., with some \( q \in (0, 1) \),

\[ |f(t, y, z) - f_0(t)| \leq \beta |y| + \gamma |z|^q, \quad (t, y, z) \in [0, T] \times \mathbb{R} \times \mathbb{R}^{1 \times d} \]

for BSDE (1.1) to have a unique adapted solution (see [1]) if the generator \( f \) is uniformly Lipschitz in the pair of unknown variables.

In [3], the existence of the solution is given for a scalar linearly growing BSDE (1.1) if the terminal value is \( L \exp(\mu \sqrt{2 \log (1 + L)}) \)-integrable with the positive parameter \( \mu \) being bigger than a critical value \( \mu_0 = \gamma \sqrt{T} \), and the preceding integrability of the terminal value for a positive parameter \( \mu \) less than critical value \( \mu_0 \) is shown to be not sufficient for the existence of a solution. In this note, we give the uniqueness result for the preceding BSDE under the preceding integrability of the terminal value for \( \mu > \mu_0 \).

We first establish some interesting properties of the function \( \psi(x, \mu) = x \exp \left( \mu \sqrt{2 \log (1 + x)} \right) \).

We observe that the obtained solution \( Y \) in [3] has the nice property: \( \psi(|Y|, a) \) belongs to the class \((D)\) for some \( a > 0 \), which is used to prove the uniqueness of the solution by dividing the whole interval \([0, T]\) into a finite number of sufficiently small subintervals.

### 2 Uniqueness

Define the function \( \psi \):

\[ \psi(x, \mu) := x \exp \left( \mu \sqrt{2 \log (1 + x)} \right), \quad (x, \mu) \in [0, +\infty) \times (0, +\infty). \]

We denote also \( \psi(\cdot, \mu) \) as \( \psi_{\mu}(\cdot) \).

The following two lemmas can be found in Hu and Tang [3].

**Lemma 2.1** For any \( x \in \mathbb{R} \) and \( y \geq 0 \), we have

\[ e^x y \leq e^{\frac{x^2}{2a^2}} + e^{2\mu^2} \psi(y, \mu). \]  

(2.2)
Lemma 2.2 Let $\mu > \gamma \sqrt{T}$. For any $d$-dimensional adapted process $q$ with $|q_t| \leq \gamma$ almost surely, for $t \in [0,T]$,

$$
E \left[ e^{\frac{1}{4\mu} \int_t^T q_s dW_s} \mid \mathcal{F}_t \right] \leq \frac{1}{\sqrt{1 - \frac{\gamma^2}{\mu^2} (T-t)}}.
$$

(2.3)

Proposition 2.3 We have the following assertions on $\psi$:

(i) For $\mu > 0$, $\psi(\cdot, \mu)$ is convex.

(ii) For $c > 1$, we have $\psi_\mu(cx) \leq \psi_\mu(c) \psi_\mu(x)$ for any $x \geq 0$.

(iii) For any triple $(a, b, c)$ with $a > 0, b > 0$ and $c > 0$, we have

$$
\psi(\psi(x, a), b) \leq e^{\frac{ax^2}{c}} \psi(x, a + b + c).
$$

Proof. The first assertion has been shown in [4]. It remains to show the Assertions (ii) and (iii).

We prove Assertion (ii).

$$
\psi_\mu(cx) = cx \exp \left( \mu \sqrt{2 \log (1 + cx)} \right) \\
\leq cx \exp \left( \mu \sqrt{2 \log [(1 + c)(1 + x)]} \right) \\
= cx \exp \left( \mu \sqrt{2 \log (1 + c) + 2 \log (1 + x)} \right) \\
\leq cx \exp \left( \mu \sqrt{2 \log (1 + c)} + \mu \sqrt{2 \log (1 + x)} \right) \\
= \psi_\mu(c) \psi_\mu(x).
$$

We now prove Assertion (iii). We have

$$
(\psi_b \circ \psi_a)(x) = \psi_a(x) \exp \left( b \sqrt{2 \log (1 + \psi_a(x))} \right) \\
= x \exp \left( a \sqrt{2 \log (1 + x)} \right) \exp \left( b \sqrt{2 \log (1 + xe^{a \sqrt{2 \log (1+x)}})} \right) \\
\leq x \exp \left( a \sqrt{2 \log (1 + x)} \right) \exp \left( b \sqrt{2 \log (1 + xe^{a \sqrt{2 \log (1+x)}})} \right) \\
= x \exp \left( a \sqrt{2 \log (1 + x)} \right) \exp \left( b \sqrt{2 \log (1 + x) + 2a \sqrt{2 \log (1 + x)}} \right).
$$

In view of the following elementary inequality:

$$
2a \sqrt{2 \log (1 + x)} \leq \frac{a^2 b^2}{c^2} + \frac{2c^2}{b^2} \log (1 + x),
$$

we have

$$
(\psi_b \circ \psi_a)(x) \\
\leq x \exp \left( a \sqrt{2 \log (1 + x)} \right) \exp \left( b \sqrt{2 \log (1 + x) + \frac{a^2 b^2}{c^2} + \frac{2c^2}{b^2} \log (1 + x)} \right) \\
\leq x \exp \left( a \sqrt{2 \log (1 + x)} \right) \exp \left( b \sqrt{2 \log (1 + x) + \frac{a^2 b^2}{c^2} + \frac{2c^2}{b^2} \log (1 + x)} \right) .
$$
Therefore,

\[(\psi_b \circ \psi_a) (x)\]

\[\leq x \exp \left( a \sqrt{2 \log (1 + x)} \right) \exp \left( b \sqrt{2 \log (1 + x)} + \frac{ab^2}{c} + c \sqrt{2 \log (1 + x)} \right)\]

\[\leq x e^{\frac{a^2}{c}} \exp \left( (a + b + c) \sqrt{2 \log (1 + x)} \right).\]

Consider the following BSDE:

\[Y_t = \xi + \int_t^T f(s, Y_s, Z_s) \, ds - \int_t^T Z_s \, dW_s, \quad (2.4)\]

where \(f\) satisfies

\[|f(s, y, z) - f_0(s, 0, 0)| \leq \beta|y| + \gamma|z|, \quad (2.5)\]

with \(f_0 := f(\cdot, 0, 0) \in L^1, \beta \geq 0 \text{ and } \gamma > 0.\)

**Theorem 2.4** Let \(f\) be a generator which is continuous with respect to \((y, z)\) and verifies inequality \((2.5)\), and \(\xi\) be a terminal condition. Let us suppose that there exists \(\mu > \gamma \sqrt{T}\) such that \(\psi([\xi] + \int_0^T |f_0(t)| \, dt, \mu) \in L^1(\Omega, \mathbb{P})\). Then BSDE \((2.4)\) admits a solution \((Y, Z)\) such that

\[|Y_t| \leq \sqrt{1 - \frac{2}{\mu^2}(T - t)} e^{\beta(T-t)} + e^{2\mu^2+\beta(T-t)} \mathbb{E} \left[ \psi_\mu \left( [\xi] + \int_t^T |f_0(s)| \, ds \right) \bigg| \mathcal{F}_t \right].\]

Furthermore, there exists \(a > 0\) such that \(\psi(Y, a)\) belongs to the class \((D)\).

**Proof.** Let us fix \(n \in \mathbb{N}^*\) and \(p \in \mathbb{N}^*\). Set

\[\xi^{n,p} := \xi^+ \wedge n - \xi^- \wedge p, \quad f_0^{n,p} := f_0^+ \wedge n - f_0^- \wedge p, \quad f_0^{n,p} := f - f_0 + f_0^{n,p}.\]

As the terminal value \(\xi^{n,p}\) and \(f_0^{n,p}(\cdot, 0, 0)\) are bounded (hence square-integrable) and \(f_0^{n,p}\) is a continuous generator with a linear growth, in view of the existence result in [5], the BSDE \((\xi^{n,p}, f_0^{n,p})\) has a (unique) minimal solution \((Y^{n,p}, Z^{n,p})\) in \(S^2 \times M^2\). Set

\[\bar{f}_n^{p}(s, y, z) = |f_0^{n,p}(s)| + \beta y + \gamma |z|, \quad (s, y, z) \in [0, T] \times \mathbb{R} \times \mathbb{R}^{1 \times d}.\]

In view of Pardoux and Peng [6], the BSDE \(\left(|\xi^{n,p}|, \bar{f}_n^{p}\right)\) has a unique solution \((\bar{Y}^{n,p}, \bar{Z}^{n,p})\) in \(S^2 \times M^2\).

By comparison theorem,

\[|Y_t^{n,p}| \leq \bar{Y}_t^{n,p}.\]

Letting \(q_s^{n,p} = \gamma \text{ sgn}(Z_s^{n,p})\) and

\[\mathbb{P}_{q^{n,p}} = \exp \left\{ \int_0^T q_s^{n,p} dW_s - \frac{1}{2} \int_0^T |q_s^{n,p}|^2 \, ds \right\} \mathbb{P},\]

we obtain,

\[|Y_t^{n,p}| \leq \bar{Y}_t^{n,p}
\]

\[= \mathbb{E}_{q^{n,p}} \left[ e^{\beta(T-t)} |\xi^{n,p}| \bigg| \mathcal{F}_t \right] + \int_t^T e^{\beta(s-t)} |f_0^{n,p}(s)| \, ds
\]

\[\leq e^{\beta(T-t)} \mathbb{E}_{q^{n,p}} \left[ |\xi^{n,p}| + \int_t^T |f_0^{n,p}(s)| \, ds \bigg| \mathcal{F}_t \right]
\]

\[\leq \frac{1}{\sqrt{1 - \frac{2}{\mu^2}(T - t)}} e^{\beta(T-t)} + e^{2\mu^2+\beta(T-t)} \mathbb{E} \left[ \psi_\mu \left( [\xi] + \int_t^T |f_0(s)| \, ds \right) \bigg| \mathcal{F}_t \right].\]
Since $Y^{n,p}$ is nondecreasing in $n$ and non-increasing in $p$, then by the localization method in [2], there is some $Z \in L^2(0,T;\mathbb{R}^{1\times d})$ almost surely such that $(Y := \inf_p \sup_n Y^{n,p}, Z)$ is an adapted solution. Therefore, we have for $a > 0$, using Jensen’s inequality and the convexity of $\psi_a(\cdot) := \psi(\cdot, a)$ together with Assertion (ii) of Proposition 2.3, we have
\[
\psi_a(\{Y_t^{n,p}\}) \leq \psi_a \left( e^{\beta(T-t)} \mathbb{E}_{q^{n,p}} \left[ |\xi_{n,p}| + \int_t^T |f_0^{n,p}(s)| \, ds \right] |\mathcal{F}_t \right)
\]
\[
\leq \psi_a \left( e^{\beta(T-t)} \right) \mathbb{E}_{q^{n,p}} \left[ |\xi_{n,p}| + \int_t^T |f_0^{n,p}(s)| \, ds \right] |\mathcal{F}_t \right)
\]
\[
\leq \psi_a \left( e^{\beta(T-t)} \right) \mathbb{E}_{q^{n,p}} \left[ |\xi_{n,p}| + \int_t^T |f_0^{n,p}(s)| \, ds \right] |\mathcal{F}_t \right)
\]
\[
\leq \psi_a \left( e^{\beta(T-t)} \right) \mathbb{E} \left[ \exp \left( \int_t^T q_n^{n,p} \, dW_s \right) \psi_a \left( |\xi_{n,p}| + \int_t^T |f_0^{n,p}(s)| \, ds \right) \right] |\mathcal{F}_t \right).
\]
For $b > \gamma \sqrt{T}$, applying Lemma 2.1 we have
\[
\psi_a(\{Y_t^{n,p}\}) \leq \psi_a \left( e^{\beta(T-t)} \right) \mathbb{E} \left[ \exp \left( \int_t^T q_n^{p,n} \, dW_s \right) \psi_a \left( |\xi_{n,p}| + \int_t^T |f_0^{n,p}(s)| \, ds \right) \right] |\mathcal{F}_t \right).
\]
Using Lemma 2.2 and Assertion (iii) of Proposition 2.3 we have for any $c > 0$,
\[
\psi_a(\{Y_t^{n,p}\}) \leq \psi_a \left( e^{\beta(T-t)} \right) \left( \frac{1}{1 - \frac{a^2}{b^2}(T-t)} + e^{2\beta^2 + \frac{2a^2}{c}} \mathbb{E} \left[ \psi_{a+b+c} \left( |\xi_{n,p}| + \int_t^T |f_0^{n,p}(s)| \, ds \right) \right] |\mathcal{F}_t \right).
\]
For $\mu > \gamma \sqrt{T}$, we can choose $a > 0, b > \gamma \sqrt{T}$, and $c > 0$ such that $a + b + c = \mu$. Then, we have
\[
\psi_a(\{Y_t^{n,p}\}) \leq \psi_a \left( e^{\beta(T-t)} \right) \left( \frac{1}{1 - \frac{a^2}{b^2}(T-t)} + e^{2\beta^2 + \frac{2a^2}{c}} \mathbb{E} \left[ \psi_{\mu} \left( |\xi| + \int_t^T |f_0(s)| \, ds \right) \right] |\mathcal{F}_t \right).
\]
Letting first $n \to \infty$ and then $p \to \infty$, we have
\[
\psi_a(\{Y_t\}) \leq \psi_a \left( e^{\beta(T-t)} \right) \left( \frac{1}{1 - \frac{a^2}{b^2}(T-t)} + e^{2\beta^2 + \frac{2a^2}{c}} \mathbb{E} \left[ \psi_{\mu} \left( |\xi| + \int_t^T |f_0(s)| \, ds \right) \right] |\mathcal{F}_t \right).
\]
Consequently, we have $\psi_a(\{Y\})$ belongs to the class $(D)$.

Now we state our main result of this note.
Theorem 2.5 Assume that the generator $f$ is uniformly Lipschitz in $(y, z)$, i.e., there are $\beta > 0$ and $\gamma > 0$ such that for all $(y^i, z^i) \in \mathbb{R} \times \mathbb{R}^{1 \times d}$, $i = 1, 2$, we have
\[ |f(t, y^1, z^1) - f(t, y^2, z^2)| \leq \beta |y^1 - y^2| + \gamma |z^1 - z^2|. \]
Furthermore, assume that there exists $\mu > \gamma \sqrt{T}$ such that $\psi(\xi) + \int_0^T f(t, 0, 0)|dt, \mu) \in L^1(\Omega, P)$. Then, BSDE (2.4) admits a unique solution $(Y, Z)$ such that $\psi(Y, a)$ belongs to the class $(D)$ for some $a > 0$.

Proof. The existence of an adapted solution has been proved in the preceding theorem. It remains to prove the uniqueness.

For $i = 1, 2$, let $(Y^i, Z^i)$ be a solution of BSDE (2.4) such that $\psi_a(Y^i)$ belongs to the class $(D)$ for some $a > 0$. Define
\[ a := a^1 \wedge a^2, \quad \delta Y := Y^1 - Y^2, \quad \delta Z := Z^1 - Z^2. \]
Then both $\psi_a(Y^1)$ and $\psi_a(Y^2)$ are in the class $(D)$, since $\psi(x, \mu)$ is nondecreasing in $\mu$, and the pair $(\delta Y, \delta Z)$ satisfies the following equation
\[ \delta Y_t = \int_t^T [f(s, Y^1_s, Z^1_s) - f(s, Y^2_s, Z^2_s)]ds - \int_t^T \delta Z_s dW_s, \quad t \in [0, T]. \]

By a standard linearization we see that there exists an adapted pair of processes $(u, v)$ such that $|u_s| \leq \beta, |v_s| \leq \gamma$, and $f(s, Y^1_s, Z^1_s) - f(s, Y^2_s, Z^2_s) = u_s \delta Y_s + \delta Z_s v_s$.

We define the stopping times
\[ \tau_n := \inf\{t \geq 0 : |Y^1_t| + |Y^2_t| \geq n\} \wedge T, \quad n = 1, 2, \ldots, \]
with the convention that $\inf \emptyset = \infty$. Since $(\delta Y, \delta Z)$ satisfies the linear BSDE
\[ \delta Y_t = \int_t^T (u_s \delta Y_s + \delta Z_s v_s) ds - \int_t^T \delta Z_s dW_s, \quad t \in [0, T], \]
we have the following formula
\[ \delta Y_{t \wedge \tau_n} = E\left[ e^{\int_{t \wedge \tau_n} u_s ds + \int_{t \wedge \tau_n} \langle v_s, dW_s \rangle} - \frac{1}{2} \int_{t \wedge \tau_n} |v_s|^2 ds \delta Y_{\tau_n} \bigg| \mathcal{F}_t \right]. \]
Therefore,
\[ |\delta Y_{t \wedge \tau_n}| \leq E\left[ e^{\int_{t \wedge \tau_n} u_s ds + \int_{t \wedge \tau_n} \langle v_s, dW_s \rangle} |\delta Y_{\tau_n}| \bigg| \mathcal{F}_t \right] \leq e^{\beta T} E\left[ e^{\int_{t \wedge \tau_n} \langle v_s, dW_s \rangle} |\delta Y_{\tau_n}| \bigg| \mathcal{F}_t \right]. \]

Now we show that the family of random variables $e^{\int_{t \wedge \tau_n} \langle v_s, dW_s \rangle} \delta Y_{\tau_n}$ is uniformly integrable. For this note that, thanks to Lemma 2.1,
\[ e^{\int_{t \wedge \tau_n} \langle v_s, dW_s \rangle} |\delta Y_{\tau_n}| \leq e^{2\alpha T} (e^{\int_{t \wedge \tau_n} \langle v_s, dW_s \rangle})^2 + 2e^{2\alpha} \psi_a(|\delta Y_{\tau_n}|). \]
For $t \in [T - \frac{a^2}{4\gamma^2}, T]$, we have from Lemma 2.2
\[ E \left[ \frac{1}{e^{2\alpha^2 T}} (e^{\int_{t \wedge \tau_n} \langle v_s, dW_s \rangle})^2 \right]^2 \leq E \left[ \frac{1}{e^{2\alpha^2 T}} (e^{\int_{t \wedge \tau_n} \langle v_s, dW_s \rangle})^2 \right] \leq \frac{1}{\sqrt{1 - \frac{2\alpha^2}{a^2}(T - t)}} \leq \sqrt{2}, \]
and, thus, the family of random variables $e^{\frac{1}{2a^2}(\int_{\tau_n}^{\tau_n} \langle v_s, dW_s \rangle)^2}$ is uniformly integrable.

On the other hand, since $\psi_a$ is nondecreasing and convex, we have thanks to Proposition 2.3 (ii)

$$
\psi_a(\delta Y_{\tau_n}) \leq \psi_a(|Y_{\tau_n}^1| + |Y_{\tau_n}^2|) = \psi_a\left(\frac{1}{2} \times 2|Y_{\tau_n}^1| + \frac{1}{2} \times 2|Y_{\tau_n}^2|\right)
$$

$$
\leq \frac{1}{2} \psi_a(2|Y_{\tau_n}^1|) + \frac{1}{2} \psi_a(2|Y_{\tau_n}^2|) \leq \frac{1}{2} \psi_a(2[\psi_a(|Y_{\tau_n}^1|) + \psi_a(|Y_{\tau_n}^2|)]).
$$

From (2.7) it now follows that, for $t \in [T - \frac{a^2}{4\gamma^2}, T]$, the family of random variables $e^{\int_{\tau_n}^{\tau_n} \langle v_s, dW_s \rangle} |\delta Y_{\tau_n}|$ is uniformly integrable.

Finally, letting $n \to \infty$ in inequality (2.6), we have $\delta Y = 0$ on the interval $[T - \frac{a^2}{4\gamma^2}, T]$. It is then clear that $\delta Z = 0$ on $[T - \frac{a^2}{4\gamma^2}, T]$. The uniqueness of the solution is obtained on the interval $[T - \frac{a^2}{4\gamma^2}, T]$. In an identical way, we have the uniqueness of the solution on the interval $[T - \frac{a^2}{2\gamma^2}, T - \frac{a^2}{4\gamma^2}]$. By a finite number of steps, we cover in this way the whole interval $[0, T]$, and we conclude the uniqueness of the solution on the interval $[0, T]$.

□

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