TORSION ENERGY

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Abstract

In the present work, torsion energy is defined. Its law of conservation is given. It is shown that this type of energy gives rise to a repulsive force which can be used to interpret supernovae type Ia observations, and consequently the accelerating expansion of the Universe. This interpretation is a pure geometric one and is a direct application of the geometrization philosophy. Torsion energy can also be used to solve other problems of General Relativity especially the singularity problem.

1 Introduction

It is well known that gravity is one of the four fundamental interactions, usually used to study and to interpret physical phenomena. Although gravity is the weakest among these four interactions, it is alone responsible for controlling the large scale structure and evaluation of the Universe. Gravity has been used, for a long time, to interpret most of large scale phenomenon of our Universe. But recently, observations of supernovae (SN) type Ia [1] show clearly that gravity, as we understand it today, cannot account for such observations. These observations indicate very clearly that the Universe is now in an accelerating expansion phase. This implies the existence of a repulsive force which is playing an important role in the structure and evolution of the Universe. One can deduce that, either gravity is not well understood and an important ingredient is missing in gravity theories, or a new interaction is about to be discovered, in order to account for such observations.

Gravity theories are usually constructed to give a better understanding of gravitational interactions, from both quantitative and qualitative points of view. Newton’s theory deals with gravity as a force acting at a distance. This theory has suffered from some problems when applied to the motion in the solar system (the advance of perihelion of Mercury’s orbit). In addition, the theory has shown its non-invariance under Lorentz transformations. These problems motivated Einstein to construct a new theory for gravity, the ”General theory of Relativity” (GR). In addition to the solution of the problems of Newton’s theory, GR gives very successful interpretation of gravitational phenomena in the solar system, binary star system and many other systems in the Universe. Also, GR has predicted the existence of a number of phenomenon which has been confirmed observationally, afterwards. Both Newton’s theory and Einstein’s theory give rise to an attractive force. So, neither of these theories, in their orthodox forms, can account for supernovae type Ia observations, since this needs the existence of repulsive force which is missing in both theories.

After the appearance of the problem of interpretation of SN observations, many authors have returned back to a modified version of GR, in which the cosmological constant...
existed in the theory (cf. [2]). Although this constant can solve the problem, since it gives rise to a repulsive force, it suffers from a big problem the ”cosmological constant problem” [3]. However, GR still has its attractive feature i.e. its philosophical basis, the ”geometrization philosophy”. This philosophy still deserves further investigations. It may provide solutions to gravity problems especially those connected to SN observations.

The aim of the present work is to re-examine the geometrization philosophy, seeking a solution for gravity problems.

2 The Geometrization Philosophy

In constructing his theory of GR, Einstein has invented an ingenious idea that geometry can be used to solve physical problems. This idea is known as the ”Geometrization of Physics”. It comprises a philosophical principle which can be summarized in the following statement.

"To understand Nature one has to start with Geometry and end with Physics ".

Einstein has applied this philosophy using the following guide lines:

1. Laws of nature are identities in the chosen geometry.
2. Each physical quantity has a geometric representative.
3. Physical trajectories of test particles are geometric paths (curves in the chosen geometry).

Einstein has chosen the 4-dimensional Riemannian geometry with the following identifications corresponding to the 3-guide lines given above [4]:

1. Conservation, as a law of nature, corresponds to the second contracted Bianchi identity,

\[(R^{\mu\nu} - \frac{1}{2} g^{\mu\nu} R)_{;\nu} \equiv 0\]  

where \(R^{\mu\nu}\) is Ricci tensor, \(R\) is Ricci scalar and the semicolon is used for covariant differentiation.

2. The conserved quantity (the quantity between brackets) ,

\[G^{\mu\nu} \overset{\text{def}}{=} R^{\mu\nu} - \frac{1}{2} g^{\mu\nu} R,\]  

is constructed from the curvature tensor (built using Christoffel symbol). This quantity corresponds to a type of energy that causes the curvature of the space. For this reason, geometrically speaking, we are going to call the quantity defined by (2) ”The Curvature Energy”.

3. Taking the third guide line into consideration, Einstein has used the geodesic equation

\[\frac{d^2 x^\mu}{ds^2} + \left\{ \frac{\mu}{\alpha\beta} \right\} \frac{dx^\alpha}{ds} \frac{dx^\beta}{ds} = 0,\]  

(3)

to represent the trajectory of any test particle in the field and null geodesic equation,

\[\frac{d^2 x^\mu}{d\lambda^2} + \left\{ \frac{\mu}{\alpha\beta} \right\} \frac{dx^\mu}{d\lambda} \frac{dx^\nu}{d\lambda} = 0,\]  

(3)
to represent the trajectory of photons.

On the above scheme, we have the following comments:

1. The choice of Riemannian geometry represents a special case, since this geometry has a vanishing torsion. This choice would not give a complete description of the physical world including space-time. Einstein has realized this fact in his subsequent attempts to construct unified field theories [4], [5] and has used geometries with torsion in these attempts.

2. The use of equation (3) or (4) to describe motion implies the application of the equivalence principle. This may represent a desirable feature in describing the motion of a scalar test particle (particle defined completely by its mass (energy)). Note that, most of elementary constituents of the Universe are fermions (particles with mass, spin, charge, ...), for which (3) and (4) are no longer applicable.

3. The most successful results of GR, which confirm the theory, are those obtained using the field equations for empty space (pure geometric). The use of full field equations of GR (equations containing a phenomenological matter tensor) is almost problematic.

For these comments, we are going to examine the application of the geometrization philosophy to geometries with torsion and curvature.

### 3 Geometries with Torsion and Curvature

Torsion tensor is the antisymmetric part of any non-symmetric linear connection. Einstein has used two types of geometries, with non-vanishing torsion, in his attempts to unify gravity and electromagnetism. The first type is the Absolute Parallelism (AP) geometry with non-vanishing torsion but vanishing curvature [5]. The second is of Riemann-Cartan type with simultaneously non-vanishing torsion and curvature [4]. Calculations in first type are more easier than in the second type. In what follows, we are going to review very briefly a version of AP-geometry giving a version in which both torsion and curvature are simultaneously non-vanishing. We have chosen this type since calculations in its context are very easy.

The structure of a 4-dimensional space is defined completely by a set of 4-contravariant linearly independent vector fields \( \lambda^\mu \). The covariant components of such vector fields are defined such that (for further details cf. [6]),

\[
\lambda^\mu \lambda_\nu = \delta^\mu_\nu, \quad (5)
\]

\[
\lambda^\mu \lambda_\mu = \delta_{ij} \quad (6).
\]

The second order symmetric tensors,

\[
g_{\mu\nu} \overset{\text{def}}{=} \lambda^\mu \lambda_\nu, \quad (7)
\]

\[
g^{\mu\nu} \overset{\text{def}}{=} \lambda^\mu \lambda_\nu, \quad (8)
\]
can play the role of the metric tensor of the Riemannian space associated with the AP-space. Using (7) and (8) one can define a linear connection (Christoffel symbol of the second kind), using which one can define covariant derivatives, as usual. Another linear connection can be defined as a consequence of AP-condition,\[ \lambda^\mu_{\nu i} = 0 \] (9)

which can be solved to give (the (+)sign is used to characterize covariant derivative using the connection \( \Gamma^\alpha_{\mu \nu} \))\[ \Gamma^\alpha_{\mu \nu} \overset{\text{def}}{=} \lambda^\alpha_\mu \lambda^\mu_\nu = \left\{ \frac{\alpha}{\mu \nu} \right\} + \gamma^\alpha_{\mu \nu} \] (10)

where \( \left\{ \frac{\alpha}{\mu \nu} \right\} \) is Christoffel symbol of second the second kind and \( \gamma^\alpha_{\mu \nu} \) is the contortion tensor, given by\[ \gamma^\alpha_{\mu \nu} \overset{\text{def}}{=} \lambda^\alpha_\mu \lambda^\mu_\nu \] (11)

where the semicolon is used to characterize covariant differentiation using Christoffel symbol. Now the torsion of the space is defined by\[ \Lambda^\alpha_{\mu \nu} = \Gamma^\alpha_{\mu \nu} - \Gamma^\alpha_{\mu \nu} \] (12)

The relation between (11) and (12) can be written as [7],\[ \gamma^\alpha_{\mu \nu} = \frac{1}{2} (\Lambda^\alpha_{\mu \nu} - \Lambda^\alpha_{\mu \nu} - \Lambda^\alpha_{\nu \mu}) \] (13)

The relations (12) and (13) implies that the vanishing of the torsion is a necessary and sufficient condition for the vanishing of the contortion. The curvature tensor corresponding to the linear connection (10) can be written, in the usual manner, as\[ B^\alpha_{\mu \nu \sigma} \overset{\text{def}}{=} \Gamma^\alpha_{\mu \sigma, \nu} - \Gamma^\alpha_{\mu \nu, \sigma} + \Gamma^\epsilon_{\mu \sigma} \Gamma^\alpha_{\epsilon \nu} - \Gamma^\epsilon_{\mu \nu} \Gamma^\alpha_{\epsilon \sigma}, \] (14)

This tensor vanishes identically because of (9). Although this appears as a disappointing feature, but we are going to show that it represents a cornerstone in the present work. The curvature tensor (14) can be written in the form,\[ B^\alpha_{\mu \nu \sigma} = R^\alpha_{\mu \nu \sigma} + Q^\alpha_{\mu \nu \sigma} \equiv 0, \] (15)

where,\[ R^\alpha_{\mu \nu \sigma} \overset{\text{def}}{=} \left\{ \frac{\alpha}{\mu \sigma} \right\}_{, \nu} - \left\{ \frac{\alpha}{\mu \nu} \right\}_{, \sigma} + \left\{ \frac{\epsilon}{\mu \sigma} \right\}_{, \nu} \left\{ \frac{\alpha}{\epsilon \nu} \right\} - \left\{ \frac{\epsilon}{\mu \nu} \right\}_{, \nu} \left\{ \frac{\alpha}{\epsilon \sigma} \right\}, \] (16)

is the Riemann-Christoffel curvature tensor and the tensor \( Q^\alpha_{\mu \nu \sigma} \) is defined by\[ Q^\alpha_{\mu \nu \sigma} \overset{\text{def}}{=} \gamma^\alpha_{\mu + i + \nu} - \gamma^\alpha_{\mu + i - \nu} - \gamma^\beta_{\mu \sigma} \gamma^\alpha_{\beta \nu} + \gamma^\beta_{\mu \nu} \gamma^\alpha_{\beta \sigma}, \] (17)

where the (−)sign is used to characterize covariant derivatives using the dual connection \( \Gamma^\alpha_{\mu \nu} (= \Gamma^\alpha_{\nu \mu}) \).
Now to get a non-vanishing curvature, we have to parameterize (10) by replacing it with
\[ \nabla_{\alpha} = \left\{ \frac{\alpha}{\mu} \right\} + b \gamma_{\alpha\mu\nu}. \]  
(18)
where \( b \) is a dimensionless parameter. The curvature tensor corresponding to (18) can be written as,
\[ \hat{B}_{\alpha\mu\nu\sigma} = R_{\alpha\mu\nu\sigma} + b Q_{\alpha\mu\nu\sigma}, \]  
(19)
which is, in general, a non-vanishing tensor. The version of the AP-geometry built using (18) is non as the PAP-geometry [9]. The path equations corresponding to (18) can be written as
\[ \frac{d^2 x^\mu}{d\tau^2} + \left\{ \frac{\mu}{\alpha\beta} \right\} \frac{dx^\alpha}{d\tau} \frac{dx^\beta}{d\tau} = -b \Lambda_{\alpha\beta\mu} \frac{dx^\alpha}{d\tau} \frac{dx^\beta}{d\tau}. \]  
(20)
where \( \tau \) is a scalar parameter. It can be easily shown that (18) defines a non-symmetric, metric linear connection.

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Using equation (15) we can deduce that
\[ R_{\alpha\mu\nu\sigma} = -Q_{\alpha\mu\nu\sigma}. \]  
(21)
Although the tensors \( R_{\alpha\mu\nu\sigma} \) and \( Q_{\alpha\mu\nu\sigma} \) appear to be mathematically equivalent, they have the following differences:
1- The Riemann-Christoffel curvature tensor is made purely from Christoffel symbols (see (16)), while the tensor \( Q_{\alpha\mu\nu\sigma} \) (17) is made purely from the contortion (11) (or from the torsion using (13)). The first tensor is non-vanishing in Riemannian geometry while the second vanishes in the same geometry.
2- The non-vanishing of \( R_{\alpha\mu\nu\sigma} \) is the measure of the curvature of the space, while the addition of \( Q_{\alpha\mu\nu\sigma} \) to it causes the space to be flat. So, one is causing an inverse effect, on the properties of space-time, compared to the other. For this reason we call \( Q_{\alpha\mu\nu\sigma} \) "The Curvature Inverse of Riemann-Christoffel Tensor". Note that both tensors are considered as curvature tensor but one of them cancels the effect of the other.

Now, in view of the above two differences, we can deduce that these two tensors are not, in general, equivalent. In other words, if we consider gravity as curvature of space-time and is represented by \( R_{\alpha\mu\nu\sigma} \), we can consider \( Q_{\alpha\mu\nu\sigma} \) as representing anti-gravity! The existence of equal effects of gravity and anti-gravity in the same system neutralizes the space-time, geometrically. This situation is similar to the existence of equal quantities of positive and negative electric charges in the same system, which neutralizes the system electrically.

If we assume that gravity and anti-gravity effects are not equally existing in the same system, then space-time curvature can be represented by the tensor (19). The existence of anti-gravity gives rise to a repulsive force, which can be used to interpret SN type Ia...
observation. This can be achieved by adjusting the parameter $b$. The above discussion gives an argument on the production of a repulsive force by torsion.

It is well known that the L.H.S. of (21) satisfies the second Bianchi Identity, so using (21) we can easily show that

$$\Sigma_{\alpha\beta\gamma} \equiv 0 \quad (22)$$

where,

$$\Sigma_{\alpha\beta} \overset{\text{def}}{=} Q_{\alpha\beta} - \frac{1}{2} g^{\alpha\beta} Q, \quad (23)$$

$$Q_{\alpha\beta} \overset{\text{def}}{=} Q^{\sigma}_{\alpha\beta\sigma}, \quad (24)$$

$$Q \overset{\text{def}}{=} g^{\alpha\beta} Q_{\alpha\beta}. \quad (25)$$

It is clear from (22) that the physical quantity represented by the tensor $\Sigma_{\alpha\beta}$ is a conserved quantity. We are going to call it the "Torsion Energy", since $Q^{\alpha}_{\mu\nu\sigma}$ is purely made of the torsion as mentioned above (see (17)).

As a second argument on the existence of a repulsive force, corresponding to the torsion of space-time, consider the linearized form of (20) which can be written as [8],

$$\Phi_{T} = \Phi_{N}(1 - b) = \Phi_{N} + \Phi_{\Sigma}, \quad (26)$$

where,

$$\Phi_{\Sigma} \overset{\text{def}}{=} - b \Phi_{N}. \quad (27)$$

$\Phi_{N}$ is the Newtonian gravitational potential and $\Phi_{T}$ is the total gravitational potential due the presence of gravity and anti-gravity. It is clear from (26) that the Newtonian potential is reduced by a factor $b$ due to the existence of the torsion energy. It is obvious from (27) that $\Phi_{\Sigma}$ and the Newtonian potential have opposite signs($b \geq 0$, [8]). Then one can deduce that $\Phi_{\Sigma}$ is a repulsive gravitational potential.

5 Discussion and Concluding Remarks

In the present work we have chosen a version of the 4-dimensional AP-geometry to represent the physical world including space and time. This geometry, PAP, is more wider than the Riemannian geometry. The version of this geometry is characterized by the parameter $b$. If $b = 0$ this geometry becomes Riemannian, while if $b = 1$ it recovers AP-geometry. We can draw the following remarks:

1- We have applied the geometrization philosophy and its guide lines mentioned in section 2, to a geometry with curvature and torsion. One can summarize the results of this application in the following points, corresponding to the three guide lines given in section 2.

(i) A conservation law, as a law of nature, giving conservation of torsion energy is represented by the differential identity (22).

(ii) The quantity called the "torsion energy" is represented by the tensor (23), which is
a part of a geometric structure. This tensor gives rise to anti-gravity and consequently a repulsive force.

(iii) Trajectories of test particles affected by the torsion of the space-time is represented by the path equation (20), which is a curve in the geometric structure used.

The path equation (20) has been used to study trajectories of spinning elementary test particles in a background with torsion and curvature. The R.H.S. of this equation is suggested to represent a type of interaction between torsion and the quantum spin of the moving particle. The application of this equation to the motion of the thermal neutrons in the Earth’s gravitational field removes the discrepancy in the COW-experiment [10].

2- It is clear that torsion energy, defined in section 4., can solve the problem of SN type Ia observations, since it gives rise to a repulsive force. This can be achieved by adjusting the parameter \( b \). One can now replace the term dark energy by the term torsion energy. The later has a known origin (a pure geometric origin). This shows the success of the geometrization scheme in dealing with physical problems.

3- Both terms ”dark energy and the ”cosmological constants” are used as if they are metaphysical terms. They have neither a geometric origin nor a well defined physical origin.

4- Many authors consider \( R^\alpha_{\mu\nu\sigma} \) and \( Q^\alpha_{\mu\nu\sigma} \) as identical tensors from both physical and mathematical points of view. For this reason, they are using the term Teleparallel equivalent of GR for theories built using \( Q^\alpha_{\mu\nu\sigma} \). In view of the present work, such authors are, in fact, constructing anti-gravity theories. But since gravitational and anti-gravitational fields are phenomenologically equivalent, no discrepancy would appear. Discrepancies would appear only if both fields exist in the same system, as in the case of SN type Ia observations.

5- In the context of the present work, we can extend the geometrization philosophy by adding the following postulate.

\textit{Physical phenomenae are just interactions between the space-time structure and the intrinsic properties of the material constituents.}

6- Torsion energy can be used to give a reasonable solution for the singularity problem in GR.

References

[1] Tonry, J.L., Schmdit, B.P. et al. (2003) Astrophys. J. \textbf{594}, 1.
[2] Mannheim, P.D. (2006) Prog.Part.Nucl.Phys. \textbf{56} 340; gr-qc/0505266
[3] Carroll, S.M. (2001) ”The Cosmological Constant”, http://livingreviews.org/Irr-2001-1.
[4] Einstein, A. (1955) ”The Meaning of Relativity”, Princeton, 5th ed.
[5] Einstein, A. (1930) Math. Annal. \textbf{102}, 685.
[6] Wanas, M.I. (2001) Cercet.Stiin.Ser.Mat. \textbf{10}, 297-309; gr-qc/0209050.
[7] Hayashi, K. and Shirafuji, T. (1979) Phys.Rev. D19, 3524.
[8] Wanas, M.I. (1998) Astrophys. Space Sci., 258, 237; gr-qc/9904019
[9] Wanas, M.I. (2000) Turk. J. Phys. 24, 473; gr-qc/0010099
[10] Wanas, M.I., Melek, M. and Kahil, M.E. (2000) Gravit. Cosmol.,6, 319.