PQ state space representation and its application to electromagnetic compatibility/incompatibility degree, influence degree, and PQ performance assessment

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Abstract: This study introduces power quality (PQ) state space representation to describe continuous PQ disturbances. A tolerance boundary is designed to represent the integrated disturbance tolerance of all equipment connected to electric power network, and the uncertainty of integrated disturbance tolerance is described with the cloud model. Electromagnetic compatibility and incompatibility degree indices are proposed to reflect the severity of PQ disturbances affecting the equipment. The influence degree index is defined to quantify the severity and impact scope of PQ disturbances. The uncertainty of equipment fault state is expressed by a fuzzy membership function. Subsequently, PQ performance is assessed according to equipment compatibility/incompatibility degree to PQ disturbances. Case studies on a 14-bus distribution system are performed and analysed. Results reveal the effectiveness and reasonableness of the proposed indices in assessing equipment compatibility/incompatibility degree to PQ disturbances, influence of PQ disturbances on the equipment, and PQ performance.

1 Introduction

Power system loads present complication and diversification because of the development of power electronics and new energy technology. The extensive use of power electronic devices, renewable energy generation, and storage equipment in electric power networks has resulted in a large number of non-linear fluctuating loads, which in turn cause serious power quality (PQ) pollution [1–3]. Control devices, adjustable speed drive systems, computer systems, and precision production lines are extensively applied in power systems, which are highly sensitive to PQ disturbances [4–6]. The effect of PQ disturbances on electric equipment should be assessed to avoid device insulation breakdown, power interruptions, or damages. Moreover, reasonable spot pricing of electricity is an important ancillary service in competitive electricity markets [7]. Adequate assessment of the influence of PQ disturbances on equipment is essential to quality-based pricing.

A large amount of PQ data is available from power quality monitor (PQM) devices. Different methodologies have been proposed to extract useful information from the massive data used to evaluate PQ performance. In [8], PQ disturbance indices were expressed by membership degree functions, and a fuzzy inference method was utilised to assess the PQ level. A fuzzy index to represent the displacement, transmission efficiency and oscillation power factors was developed in [9]. A fuzzy total demand distortion index was proposed in [10] to evaluate current harmonic distortion. PQ disturbance indicators were redefined based on wavelet packet transform, and a fuzzy inference system was introduced for PQ evaluation in [11, 12]. For PQ evaluation and pricing, a quantitative index was developed based on artificial neural network and fuzzy logic in [13]. To evaluate PQ levels in the presence of distributed generation, probabilistic indices that consider the PQ variation and use weighting functions were developed in [14].

Most PQ assessment methodologies focus on the synthesis algorithms of disturbance indices and do not consider the interactions between PQ disturbances and electric equipment. PQ is the demand of equipment for quality electric energy; it can be directly reflected by the severity of PQ disturbances affecting the equipment. In essence, PQ disturbances are electromagnetic disturbances generated in electric power networks. The interactions between electromagnetic disturbances and electric equipment are described as electromagnetic compatibility (EMC) [15]. The degree of EMC or incompatibility reflects the severity of PQ disturbances that affect the equipment. In [16], four scenarios were considered to assess EMC and incompatibility statuses from the perspective of the power supply system and equipment. However, further study of the quantisation of the compatibility and incompatibility degree was not provided. To assess equipment compatibility/incompatibility degree to multiple PQ disturbances and the influence of disturbances on equipment, the integrated disturbance tolerance of equipment should be clearly established. However, existing standards and test specifications only provide the tolerance of equipment to single disturbance. Although the level of each disturbance satisfies the requirement of relevant standards, the comprehensive effect of PQ disturbances may exceed the tolerance of equipment. Moreover, the actual integrated disturbance tolerance of all equipment connected to electric power network has uncertainty because of the difference in types, sensitivities, service lives, and operating conditions of the equipment. Therefore, an adequate PQ disturbances representation model is required to assess equipment compatibility/incompatibility degree to disturbances and the influence of disturbances on equipment.

In this study, multidimensional PQ state space is introduced to express continuous PQ disturbances. A tolerance boundary is designed to represent the integrated disturbance tolerance of equipment, and the uncertainty of integrated disturbance tolerance is described with a cloud model. EMC degree $D_{EMC}$, electromagnetic incompatibility degree $D_{EMIC}$, fuzzy EMC degree $D_{FEMC}$, and fuzzy electromagnetic incompatibility degree $D_{FEMIC}$ indices are proposed to reflect the severity of PQ integrated disturbance affecting the equipment. Influence degree index $D_I$ is defined to quantify the severity and impact scope of PQ disturbances. The uncertainty of the equipment fault state is expressed by a fuzzy membership function. Subsequently, PQ performance is assessed according to equipment compatibility/incompatibility degree...
incompatibility degree to PQ disturbances. The EMC/incompatibility degree and influence degree indices not only reflect the degree of compatibility/incompatibility of equipment to PQ disturbances and the influence of PQ disturbances on equipment, but also consider the uncertainty of equipment tolerance and fault state. The assessment result is reasonable and accurate.

This paper is organised as follows. Section 2 describes the establishment of PQ state space representation and the calculation of the proposed indices. In Section 3, PQ performance is comprehensively assessed based on the EMC and incompatibility degree indices. Section 4 presents the case studies and results. The conclusions are presented in Section 5.

2 PQ state space representation

2.1 EMC from the perspective of continuous PQ disturbances and equipment

The International Electrotechnical Commission has established a series of standards and normative legislation on the coordinated operation between a power supply system and the electrical equipment connected to it. In [15], EMC is defined as ‘the ability of an equipment or system to function satisfactorily in its electromagnetic environment without introducing intolerable electromagnetic disturbances to anything in that environment.’ In the same reference, immunity (to a disturbance) was explained as ‘the ability of a device, equipment or system to perform its functions without degradation in the presence of an electromagnetic disturbance’. Continuous PQ disturbances are essentially electromagnetic disturbances. Various continuous disturbances are usually generated in power systems simultaneously. The effects of these electromagnetic disturbances on the equipment are vector superposing. From the perspective of continuous PQ disturbances and equipment, EMC is suited for an overall analysis of the interactions between the power supply system and all connected electrical equipment because of the possible concurrency of disturbances. Therefore, the EMC investigated in this study reflects the ability of equipment normal operation affected by several continuous PQ disturbances simultaneously, which is determined by the integrated disturbance level and system immunity. System immunity differs from the definition in [15], which states that system immunity represents the integrated disturbance tolerance of all equipment connected to electric power network rather than a single disturbance tolerance.

In [17], ‘compatibility level’ is defined as ‘the specified electromagnetic disturbance level used as a reference level in a specified environment for coordination in the setting of emission and immunity levels.’ For continuous PQ disturbances, compatibility level is an integrated disturbance level, which is a set of minimum requirements necessary for normal operation of all the equipment in an electric power network. Compatibility level represents the integrated disturbance tolerance of the equipment. Obviously, EMC is present when the integrated disturbance level is within the tolerance of the equipment. All the equipment perform their functions normally. Conversely, electromagnetic incompatibility exists when the integrated disturbance level exceeds the tolerance of the equipment. The equipment perform their functions with malfunctions.

Compatibility level distinguishes EMC and incompatibility, which only reflect two general stages of equipment operation (i.e. normal and malfunction). Quantisation of the EMC and incompatibility degree is required to assess the severity of PQ disturbances affecting the equipment and the influence of PQ disturbances on the equipment.

2.2 Construction of PQ state space and standard tolerance boundary

To assess the degree of EMC and incompatibility, the integrated disturbance level and integrated disturbance tolerance of equipment should be determined. This study introduces PQ state space representation to analyse various continuous disturbances as a whole.

In reference to international standards [18–21], the deviation of supply voltage, voltage fluctuation, three-phase voltage unbalance, total harmonic distortion of voltage, and inter-harmonic ratio of voltage are selected as state variables that are expressed as $x_1, x_2, \ldots, x_5$, respectively. Flicker is not individually selected as a disturbance state variable because it is a cumulative effect of voltage fluctuations over a period of time, which is another expression of voltage fluctuation.

A multi-dimensional PQ state space is constructed with five coordinate axes that represent the disturbance state variables corresponding to $x_1, x_2, \ldots, x_5$. Given the differences in the properties of the five disturbance state variables, the coordinate axes are mutually orthogonal. The PQ state space lies in the first quadrant of the multi-dimensional coordinate system because the disturbance state variables are non-negative.

In the PQ state space, the single disturbance tolerance limit of equipment corresponds to a point on a positive axis, which is available from the relevant standards and test specifications. When equipment is suffering from various PQ disturbances simultaneously, although each disturbance level satisfies the requirement of relevant standards, the comprehensive effect of PQ disturbances may exceed the tolerance of the equipment. In expressing the integrated disturbance tolerance limit of equipment in PQ state space, a surface is complicated and difficult to establish based on each disturbance tolerance limit. Therefore, the single disturbance tolerance limit of equipment is selected as a base value to transform disturbance state variables into per unit values. A per unit value state space is constructed to make the state variables comparable. The per unit value is calculated as

$$x_i^* = \frac{x_i}{x_{i,\max}}, \quad i = 1, 2, \ldots, 5,$$

where $x_i^*$ is the per unit value of a state variable, and $x_{i,\max}$ is the disturbance tolerance limit of the equipment.

The per unit values $x_1^*, x_2^*, \ldots, x_5^*$ are obtained by with (1). Each disturbance tolerance limit of equipment is 1 pu in the per unit value state space. A hyper-sphere with a radius of 1 is established to represent the standard tolerance boundary. In the case of three disturbance state variables, the 3D per unit value state space and the standard tolerance boundary are constructed based on the positive axes of the coordinate system, as shown in Fig. 1.

![Fig. 1 Schematic of per unit value state space and standard tolerance boundary](image)
2.3 Description of boundary fuzziness using the cloud model

In engineering practice, the single disturbance tolerance of some equipment are different from the tolerance recommended in standards because of the difference in types, sensitivities, service lives, and operating conditions of the equipment. Therefore, the value of the integrated disturbance tolerance of all the equipment connected to electric power network has fuzziness and uncertainty, which should be an uncertain interval. The cloud model is utilised in this study to describe the fuzziness and uncertainty of the tolerance boundary.

The cloud model is an effective tool in the uncertain transition between a qualitative concept and its quantitative representation [22]. The numerical characteristics of the cloud model are expressed by expectation $E_x$, entropy $E_y$, and hyperentropy $H_y$. The $E_x$ is the expectation of the cloud droplet distribution and the position that best represents the qualitative concept. The $E_x$ is the uncertainty measurement of the qualitative concept. The $E_y$ reflects the discrete degree of cloud droplets that represents the concept and the interval of cloud droplets accepted by the concept. The $H_y$ is the uncertainty measurement of $E_y$, i.e. the entropy of $E_x$, which is determined by the randomness and fuzziness of $E_x$. An example of a normal cloud with 2500 droplets is shown in Fig. 2 [23].

Actual single disturbance tolerances of some equipment differ from the tolerance recommended in standards could be obtained from testing and empirical data provided by manufacturers. The backward cloud generator is utilised to convert the data of single disturbance tolerances of all equipment to cloud digital features $(E_{nx}, E_{ny}, H_{nx})$. The per unit values of the disturbance $x_i$ tolerance of all the equipment are expressed as $(y_{1i}, y_{2i}, …, y_{ni} 1)$, where the value 1 represents the discrete degree of disturbance $x_i$ tolerances conform to standard requirement, $y_{1i}, y_{2i}, …, y_{ni}$ represents the per unit values of disturbance $x_i$ tolerances differ from the tolerance recommended in standard, and $y_{ni} ≠ 1$. $E_{nx}$ represents the mean value of the disturbance $x_i$ tolerance of all equipment. $E_{ny}$ represents the discrete degree of the disturbance $x_i$ tolerance of all equipment. $H_{nx}$ represents the discrete degree of $E_{ny}$. The backward cloud generator is implemented as follows.

(i) Mean value $\bar{y}_i$ and variance $S^2$ of disturbance $x_i$ tolerance per unit values $(y_{1i}, y_{2i}, …, y_{ni} 1)$ are calculated with the following equations.

\[
\bar{y}_i = \frac{1}{n+1} \left( \sum_{j=1}^{n} y_{ij} + 1 \right) \quad i = 1, 2, …, 5, \quad (2)
\]

\[
S^2 = \frac{1}{n} \left[ \frac{\sum_{j=1}^{n} (y_{ij} - \bar{y}_i)^2 + (1 - \bar{y}_i)^2}{n} \right] \quad (3)
\]

(ii) Expectation $E_{nx,i}$ is calculated.

\[
E_{nx,i} = y_i, \quad (4)
\]

(iii) Entropy $E_{ny,i}$ is calculated.

\[
E_{ny,i} = \frac{1}{\sqrt{2 \pi}} \times \frac{1}{n+1} \left[ \sum_{j=1}^{n} y_{ij} - E_{nx,i} \right] + \left[ 1 - E_{nx,i} \right]. \quad (5)
\]

(iv) Hyperentropy $H_{nx,i}$ is calculated.

\[
H_{nx,i} = \sqrt{S^2 - E_{nx,i}^2}. \quad (6)
\]

An integrated cloud is fused by the cloud digital features of different disturbance tolerances, which is utilised to describe the fuzziness and uncertainty of the integrated disturbance tolerance boundary. The cloud fusion modelling is shown in Fig. 3. The data fusion rule uses the weighted average method based on vectorisation.

\[
E_x = (E_{x,1} + E_{x,2} + E_{x,3} + E_{x,4} + E_{x,5})^{-1}E_1^T E_2, \quad (7)
\]

\[
E_y = \frac{E_1^T E_2}{5}, \quad (8)
\]

\[
H_x = \frac{1}{5} \sqrt{\sum_{i=1}^{5} H_{nx,i}^2}, \quad (9)
\]

where $E_1$ comprises expectations $E_y,i$ of different disturbance tolerances, $E_1 = [E_{x,1}, E_{x,2}, E_{x,3}, E_{x,4}, E_{x,5}]^T$; $E_y$ is the weight values of the expectations of different disturbances tolerance, $E_2 = [E_{nx,1}, E_{nx,2}, E_{nx,3}, E_{nx,4}, E_{nx,5}]^T$; $E_x$ is the expectation of the tolerance boundary cloud model, which represents the mean value of integrated disturbance tolerance of all equipment; $H_x$ is the entropy of the tolerance boundary cloud model, which represents the discrete degree of integrated disturbance tolerance range; and $H_y$ is the hyperentropy of the tolerance boundary cloud model, which represents the discrete degree of $E_y$. Tolerance boundary cloud digital feature $(E_x, E_y, H_x)$ is a fuzzy correction value of the tolerance boundary.

2.4 Electromagnetic compatibility and incompatibility degree assessment

In PQ state space, various continuous disturbances are expressed as a vector that corresponds to a point. In Fig. 4, EMC exists when the point is within the tolerance boundary; electromagnetic incompatibility exists when the point is outside of the tolerance boundary. If the point is in the tolerance boundary cloud, it may be compatible or incompatible. Two types of EMC and incompatibility degree indices are provided in this study for both standard tolerance boundary and tolerance boundary cloud.

Standard EMC degree $D_{EMC}$ and incompatibility degree $D_{IMC}$ indices are defined according to the shortest distance from a point in PQ state space.
to the standard tolerance boundary.

\[
d = \sqrt{\sum_{i=1}^{6} x_i^2}, \quad i = 1, 2, \ldots, 6, \quad (10)
\]

\[
D_{\text{EMC}} = \begin{cases} 1 - d, & x_1 \leq x_{i_{\text{max}}} \\ d - 1, & x_1 > x_{i_{\text{max}}} \end{cases}, \quad (11)
\]

where \( d \) represents the shortest distance from a point to the origin of the coordinate system.

EMC degree \( D_{\text{EMC}} \), electromagnetic incompatibility degree \( D_{\text{EMIC}} \), fuzzy EMC degree \( D_{\text{EMFC}} \), and fuzzy electromagnetic incompatibility degree \( D_{\text{EMFC}} \) indices are defined according to a point deviation degree to the tolerance boundary cloud. For one point in PQ state space, when the \( E_x \) of the boundary cloud is known, the larger \( E_x \) and \( H_t \) are, the larger the boundary cloud discrete degree is, i.e. the EMC and incompatibility are smaller.

According to the influence rules of cloud digital features on EMC and incompatibility degree, the calculation of \( D_{\text{EMC}}, D_{\text{EMIC}}, D_{\text{EMFC}} \) indices is expressed as

\[
\begin{align*}
D_{\text{EMC}} &= (E_x - d) \exp \left( -\left( \frac{E_x^2 + H_t^2}{2} \right) \right), \quad d \in [0, E_x - 3E_n] \\
D_{\text{EMIC}} &= (E_x - d) \exp \left( -\left( \frac{E_x^2 + H_t^2}{2} \right) \right), \quad d \in [E_x - 3E_n, E_x] \\
D_{\text{EMFC}} &= (d - E_x) \exp \left( -\left( \frac{E_x^2 + H_t^2}{2} \right) \right), \quad d \in [E_x, E_x + E_n] \\
D_{\text{EMFC}} &= (d - E_x) \exp \left( -\left( \frac{E_x^2 + H_t^2}{2} \right) \right), \quad d \in [E_x + E_n, \infty]
\end{align*}
\]

\[(12)\]

where \((E_x, E_n, H_t)\) is the cloud digital feature of the boundary cloud, \(|E_x - d|\) represents the point deviation degree from the boundary cloud expectation, and \(\exp\left(-\sqrt{(E_x^2 + H_t^2)/2}\right)\) represents the point deviation coefficient to the boundary cloud.

### 2.5 Influence degree assessment

The EMC/incompatibility degree and fuzzy EMC/incompatibility degree indices reflect the severity of PQ disturbances affecting the equipment. In incompatibility status, the influence of continuous disturbances on the equipment should be comprehensively assessed in consideration of both the severity and impact scope of continuous disturbances.

The definition of the influence degree index mainly considers three factors, namely, electromagnetic incompatibility degree, total equipment capacity, and equipment fault state. The electromagnetic incompatibility degree reflects the severity of PQ disturbances affecting the equipment, and total equipment capacity reflects the impact scope of PQ disturbances on the equipment. When the electromagnetic incompatibility degree and impact scope of disturbances are large, the influence of PQ disturbances on the equipment is serious. Moreover, the equipment operation status exhibits uncertainty because a fault may occur in the equipment while a point is in the tolerance boundary cloud. Therefore, the calculation of influence degree index \( D_i \) is expressed as subsection functions as follows

\[
D_i = \begin{cases} D_{\text{EMC}} \times P_e \times \mu_F, & d \in [E_x - 3E_n, E_x] \\
D_{\text{EMIC}} \times P_e \times \mu_F, & d \in [E_x, E_x + 3E_n] \\
D_{\text{EMFC}} \times P_e \times \mu_F, & d \in (E_x + 3E_n, \infty)
\end{cases}, \quad (13)
\]

where \( P_e \) is the total equipment capacity in MW and \( \mu_F \) is the membership function of the equipment fault state, \( \mu_F \in [0,1] \).

\( \mu_F \) can be defined based on the membership function of the fuzzy safety event [24]. Fig. 5 shows expectation curve \( \mu(x) \) of the tolerance boundary cloud.

\[
\mu(x) = \exp \left( \frac{-(x - E_n)^2}{2E_n^2} \right), \quad (14)
\]

When \( 0 \leq d < E_x - 3E_n \), the equipment is in normal operation, and the membership degree of the fault state is equal to 0. When \( E_x - 3E_n \leq d \leq E_x + 3E_n \), the equipment may exhibit a fault, and the membership function of the equipment fault state is defined as the area ratio of the shaded part to the part enclosed by expectation curve \( \mu(x) \) and the x-axis. When \( E_x + 3E_n < d \leq \infty \), a fault should occur in the equipment, and the membership degree of the fault state is equal to 1. The membership function of the equipment fault state is expressed as

\[
\mu_F = \begin{cases} 0, & d \in [0, E_x - 3E_n] \\
\frac{d}{\int_{E_x - 3E_n}^{E_x} \mu(x)dx}, & d \in [E_x, E_x + 3E_n] \\
1, & d \in (E_x + 3E_n, \infty)
\end{cases}, \quad (15)
\]

![Fig. 5 Schematic of the membership function of the equipment fault state](image-url)
where \( \int_{E_{	ext{m}}}^{E_{	ext{max}}} \mu(x) \, dx \) is the area of the shaded part and \( \int_{E_{	ext{m}}}^{E_{	ext{max}}} \mu(x) \, dx \) is the area of the part enclosed by expectation curve \( \mu(x) \) and the x-axis.

3 PQ comprehensive assessment

3.1 PQ assessment of buses based on the electromagnetic compatible and incompatible degree

The quality of the product should be assessed according to users demand for the performance of the product. Electric energy is a special type of product. PQ is the demand of the equipment for quality electric energy. The EMC/incompatibility degree indices proposed in this study reflect the severity of PQ disturbances affecting the equipment. Therefore, PQ performance can be assessed based on the indices.

In PQ state space representation, the \( D_{\text{EMC}}, D_{\text{EMIC}}, D_{\text{EMC}}', D_{\text{EMIC}}' \) indices reflect the degree of deviation from a point to the tolerance boundary cloud. In EMC and fuzzy EMC statuses, when \( D_{\text{EMC}} \) and \( D_{\text{EMIC}} \) are large (the point deviation degree to the boundary cloud is large), PQ is good. In electromagnetic incompatibility and fuzzy electromagnetic incompatibility statuses, when \( D_{\text{EMC}}' \) and \( D_{\text{EMIC}}' \) are large (the point deviation degree to the boundary cloud is large), PQ is poor. In this study, PQ is divided into several levels for intuitive recognition. The interval \( [0, D_{\text{EMC}} \text{ max}] \) is averagely divided into three sections corresponding to 0–3 levels that represent normal operation. The interval \( [0, D_{\text{EMIC}} \text{ max}] \cup [0, D_{\text{EMC}}' \text{ max}] \) corresponds to 3–4 levels that represent the possible occurrence of a fault in the equipment. The interval \( (0, D_{\text{EMC}} \text{ max}) \) is averagely divided into two sections corresponding to 4–6 levels that represent the situation where a fault must occur in the equipment. The \( D_{\text{EMC}}' \) index can be infinite in theory, but this is impossible in engineering practice because the PQ disturbances are not infinite. The interval partition of \( D_{\text{EMIC}} \) is looser than the interval partition of \( D_{\text{EMC}} \) in actual situations. The interval \( (0, D_{\text{EMC}} \text{ max}) \) is set to 3 times the width of \( [0, D_{\text{EMC}} \text{ max}] \) for small PQ disturbances. A high PQ level is also large. Comparison and judgment matrix \( N \) is expressed as

\[
\begin{align*}
n_{ij} &= \frac{L_i}{L_j} > 0, & i, j = 1, 2, \ldots, m, \quad (17)
\end{align*}
\]

where \( n_{ij} \) is the element of comparison and judgment matrix \( N \); \( L_i \) and \( L_j \) are the PQ levels of bus \( i \) and \( j \), respectively, which are obtained with (16). The weights are the normalised eigenvector corresponding to eigenvalue \( m \) of matrix \( N \), which are represented as \( p = (p_1, p_2, \ldots, p_m)^T \).

3.2 Comprehensive assessment of a bus system based on the combined weighting model

The EMC and incompatibility degree indices of a bus system can be obtained by weighting calculation for bus assessment results. The weighting coefficients are determined by the severity and impact scope of PQ disturbances. Severity is represented by EMC and incompatibility degree indices and is ultimately reflected by bus PQ level calculated by these indices. The impact scope is reflected by total equipment capacities \( P_e \) of the buses. A combined weighting model with comprehensive consideration of bus PQ level and bus equipment capacity is utilised to assess the PQ performance of a bus system.

The eigenvalue weighted method is utilised for weighting calculation from the perspective of continuous disturbance severity. The core of the eigenvalue weighted method is the establishment of a reasonable comparison and judgment matrix [25]. The relative importance of evaluation objects is determined by the PQ assessment result of each bus. A high PQ level indicates serious PQ pollution; hence, power operators and users should pay attention to it. The weight corresponding to a high PQ level is also large. Comparison and judgment matrix \( N \) is expressed as

\[
\begin{align*}
n_{ij} &= \frac{L_i}{L_j} > 0, & i, j = 1, 2, \ldots, m, \quad (17)
\end{align*}
\]

where \( n_{ij} \) is the element of comparison and judgment matrix \( N \); \( L_i \) and \( L_j \) are the PQ levels of bus \( i \) and \( j \), respectively, which are obtained with (16). The weights are the normalised eigenvector corresponding to eigenvalue \( m \) of matrix \( N \), which are represented as \( p = (p_1, p_2, \ldots, p_m)^T \).

\[
N = \begin{bmatrix}
L_1 & L_2 & \cdots & L_m
\end{bmatrix},
\]

where \( L_i \) and \( L_j \) are the weighting coefficients of adjacent vector indices, \( P_{e,i,k} \) and \( P_{e,j,k} \) are the total equipment capacities of adjacent vector indices, and \( r_k \) is the ratio of \( w_{k-1} / w_k \).

The synthetic weights \( w_i \) of the evaluation objects are expressed as

\[
\begin{align*}
w_i &= \sum_{j=1}^{m} \left( p_j + q_j \right), & i = 1, 2, \ldots, m. \quad (21)
\end{align*}
\]

By considering the relativity between compatibility and incompatibility index, index vectorisation is conducted. EMC degree vector \( \overrightarrow{D_{\text{EMC}}} \) and fuzzy EMC degree vector \( \overrightarrow{D_{\text{EMC}}} \) have the same direction. Electromagnetic incompatibility degree vector \( \overrightarrow{D_{\text{EMC}}} \) and fuzzy electromagnetic incompatibility degree vector \( \overrightarrow{D_{\text{EMC}}} \) have the same direction and exhibit a 180° phase difference with the first two. The EMC/incompatibility assessment result of the bus system is obtained by weighting calculation of the vectorisation indices of different buses. The influence degree index of the bus system is calculated with (13)–(15). The PQ level of the bus system is calculated with (16).

4 Case studies and results

Comprehensive assessment of EMC/incompatibility degree, influence degree, and PQ performance is conducted with 14-bus distribution systems, as shown in Fig. 6. The nominal voltage of this distribution system is set to 35 kV. The PQ disturbance data
and the total equipment capacities $P_i$ of buses are given in Table 2.

The following two cases are considered.

Case 1: PQ state space representation is introduced to assess the EMC/incompatibility degree, influence degree, and PQ performance of integrated disturbance.

Case 2: The EMC/incompatibility degree and influence degree of various disturbances are assessed based on single disturbance tolerance provided in standard.

### 4.1 Case 1

PQ state space representation is introduced to analyse continuous PQ disturbances $x_1$, $x_2$, $\ldots$, $x_5$ as a whole. EMC/incompatibility degree and influence degree indices are assessed based on two types of tolerance boundary: standard tolerance boundary and tolerance boundary cloud.

The Chinese standard disturbances limits and an example of some equipment disturbance tolerances are provided as inputs of backward cloud generator [27], as shown in Tables 3 and 4. (The example is not typical but a sample). The cloud digital features of disturbance tolerance in the sample is slightly larger than 1. This shows that the EMC degree is slightly larger than the standard EMC degree and the electromagnetic incompatibility degree is slightly smaller than the standard electromagnetic incompatibility degree. Fig. 7b shows that the actual influence degree is slightly smaller than the standard influence degree. Taking the Table 5 as the reference, the expectation of integrated disturbance tolerance in the sample is slightly larger than 1. This result indicates that the actual tolerance of the equipment is larger

### Table 1 PQ grading based on the electromagnetic compatibility and incompatibility degree indices

| Interval | $0, \frac{1}{3} D_{\text{EMC}}^{\max}$ | $\frac{1}{3} D_{\text{EMC}}^{\max}, \frac{2}{3} D_{\text{EMC}}^{\max}$ | $\frac{2}{3} D_{\text{EMC}}^{\max}, D_{\text{EMC}}^{\max}$ | $0, \frac{1}{2} D_{\text{EMC}}^{\max}$ | $\frac{1}{2} D_{\text{EMC}}^{\max}, D_{\text{EMC}}^{\max}$ |
|----------|-----------------------------------|-----------------------------------|-----------------------------------|-----------------------------------|-----------------------------------|
| PQ level | 0-1 excellent                     | 1-2 good                          | 2-3 medium                        | 3-4 unsatisfactory                | 4-5 bad                           |

### Table 2 PQ disturbance data and total equipment capacities of buses

| Node | $x_7$, % | $x_2$, % | $x_3$, % | $x_4$, % | $x_5$, % | $P_e$, MW |
|------|----------|----------|----------|----------|----------|-----------|
| 1    | 2.09     | 1.06     | 0.68     | 1.22     | 0.021    | 0         |
| 2    | 2.23     | 1.23     | 0.71     | 1.51     | 0.027    | 6.3       |
| 3    | 2.35     | 1.25     | 0.78     | 1.43     | 0.025    | 16.2      |
| 4    | 2.49     | 1.4      | 0.86     | 1.59     | 0.028    | 9.6       |
| 5    | 2.26     | 1.28     | 0.76     | 1.56     | 0.026    | 4.5       |
| 6    | 3.16     | 1.48     | 0.8      | 2        | 0.031    | 2.2       |
| 7    | 2.52     | 2.56     | 1.16     | 2.82     | 0.039    | 0         |
| 8    | 3.22     | 1.46     | 0.88     | 1.79     | 0.03     | 0         |
| 9    | 5.95     | 3.05     | 1.24     | 3.47     | 0.044    | 8.7       |
| 10   | 5.83     | 3.01     | 1.15     | 3.32     | 0.041    | 5.6       |
| 11   | 5.05     | 2.55     | 1.09     | 2.93     | 0.037    | 7.2       |
| 12   | 2.56     | 1.48     | 0.75     | 1.61     | 0.032    | 14.4      |
| 13   | 3.27     | 2        | 0.97     | 2.43     | 0.036    | 4.3       |
| 14   | 5.67     | 2.89     | 1.21     | 3.2      | 0.038    | 5.8       |

### Table 3 Sample of disturbance tolerances of some equipment in a 35 kV bus system

| Equipment | $y_1$, % | $y_2$, % | $y_3$, % | $y_4$, % | $y_5$, % |
|-----------|----------|----------|----------|----------|----------|
| 1         | 10.6     | 3.96     | 2.06     | 2.91     | 0.141    |
| 2         | 10.7     | 3.88     | 2.04     | 2.95     | 0.144    |
| 3         | 11       | 3.84     | 2.04     | 3.12     | 0.147    |
| 4         | 10.4     | 3.92     | 2.1      | 3.18     | 0.134    |
| 5         | 9.9      | 3.96     | 1.94     | 3.09     | 0.15     |
| 6         | 9.8      | 4.12     | 1.98     | 3.09     | 0.152    |
| 7         | 9.6      | 4.28     | 1.84     | 3.27     | 0.156    |
| 8         | 10.2     | 3.96     | 1.9      | 3.09     | 0.154    |
| 9         | 10.3     | 3.86     | 2.2      | 2.79     | 0.157    |
| 10        | 11       | 3.52     | 2.05     | 2.82     | 0.16     |
| 11        | 9.9      | 3.88     | 2.04     | 3.45     | 0.163    |
| 12        | 10.2     | 4.12     | 2.08     | 3.33     | 0.171    |
| 13        | 10.5     | 3.96     | 2.06     | 2.94     | 0.155    |
| 14        | 10.1     | 4.16     | 2.12     | 3.21     | 0.149    |
| 15        | 10.7     | 4.32     | 2.04     | 3.21     | 0.158    |
| 16        | 10.9     | 3.92     | 2.06     | 3.69     | 0.163    |
| 17        | 9.4      | 3.84     | 2.02     | 3.23     | 0.168    |
| 18        | 9.8      | 4.04     | 2.08     | 3.27     | 0.15     |
| 19        | 9.9      | 3.8     | 2.12     | 3.15     | 0.149    |
| 20        | 10.3     | 3.92     | 1.98     | 3.09     | 0.147    |

### Table 4 Standard limits of disturbances in a 35 kV bus system

| $x_{1\text{max}}$, % | $x_{2\text{max}}$, % | $x_{3\text{max}}$, % | $x_{4\text{max}}$, % | $x_{5\text{max}}$, % |
|----------------------|----------------------|----------------------|----------------------|----------------------|
| 10                   | 2                    | 3                    | 0                    | 0.16                 |

### Table 5 Cloud digital features of disturbance tolerances

| Disturbance | $E_s$ | $E_s$ | $H_s$ |
|-------------|-------|-------|-------|
| $x_1$       | 0.02  | 0.05  | 0.01  |
| $x_2$       | 0.09  | 0.04  | 0.02  |
| $x_3$       | 0.02  | 0.04  | 0.02  |
| $x_4$       | 0.05  | 0.07  | 0.03  |
| $x_5$       | 0.06  | 0.06  | 0.01  |
| integrated disturbance | 1.01 | 0.05 | 0.04 |

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than the standard tolerance. The obtained results from Fig. 7 conform to the actual situation of the sample and thus verify that the proposed EMC degree, electromagnetic incompatibility degree, and influence degree indices are reasonable and effective. The PQ levels at each node and in the entire system are obtained by derivation and calculation based on the EMC/incompatibility degree indices and total equipment capacities. The calculation process verifies the feasibility of the assessment method.

### 4.2 Case 2

Continuous PQ disturbances \( x_1, x_2, \ldots, x_5 \) are analysed separately. The EMC and incompatibility degree indices of single disturbance are obtained with (22), and the influence degree indices of PQ disturbance \( x_4 \) (total harmonic distortion of voltage) at nodes 9, 10, and 14 are obtained from (13)–(15), as shown in Fig. 8.

\[
\begin{align*}
D_{\text{EMC}} &= 1 - \frac{x_i}{x_{i,\text{max}}}, \quad x_i \leq x_{i,\text{max}} \\
D_{\text{EMIC}} &= \frac{x_i}{x_{i,\text{max}}} - 1, \quad x_i \geq x_{i,\text{max}} \\
\end{align*}
\] (22)

Comparison of Figs. 7a and 8a shows that the EMC degree of integrated disturbance is lower than the EMC degree of each disturbance. The electromagnetic incompatibility degree of integrated disturbance is higher than the electromagnetic incompatibility degree of \( x_4 \) (THDU). The integrated disturbance is in incompatibility status, although each disturbance is in compatibility status at nodes 7, 11, and 13. Comparison of Figs. 7b and 8b shows that the integrated disturbance influence on the equipment is more serious than the influence of \( x_4 \) (THDU). Undoubtedly, integrated disturbance tolerance is poorer than single disturbance tolerance because the comprehensive influence of disturbances is more serious than the influence of a single disturbance. The comparison results are in accordance with reality, which also verifies the rationality and effectiveness of the proposed EMC degree, electromagnetic incompatibility degree, and influence degree indices.

Analysis of Figs. 7 and 8 reveals that PQ state space representation and application for the assessments can achieve the equipment compatibility/incompatibility degree to the integrated disturbance,

### Table 6  
| Node | \( p_i \) | \( q_i \) | \( w_i \) | Combined weight |
|------|---------|---------|---------|----------------|
| 1    | 0.03    | 0.03    | 0.06    | 0.06           |
| 2    | 0.05    | 0.07    | 0.08    | 0.08           |
| 3    | 0.05    | 0.07    | 0.08    | 0.08           |
| 4    | 0.05    | 0.07    | 0.08    | 0.08           |
| 5    | 0.07    | 0.07    | 0.08    | 0.08           |
| 6    | 0.09    | 0.09    | 0.10    | 0.10           |
| 7    | 0.11    | 0.11    | 0.12    | 0.12           |
| 8    | 0.11    | 0.11    | 0.12    | 0.12           |
| 9    | 0.12    | 0.12    | 0.13    | 0.13           |
| 10   | 0.12    | 0.12    | 0.13    | 0.13           |
| 11   | 0.13    | 0.13    | 0.14    | 0.14           |
| 12   | 0.14    | 0.14    | 0.15    | 0.15           |
| 13   | 0.15    | 0.15    | 0.16    | 0.16           |
| 14   | 0.16    | 0.16    | 0.17    | 0.17           |

### Table 7  
| \( D_{\text{EMIC}} \) | \( D_i \) | \( L \) |
|-----------------------|---------|-------|
| 0.11                  | 9.14    | 3.9   |

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the influence of the integrated disturbance on the equipment, and PQ performance. It also indicates that the interaction between PQ disturbances and equipment should not be ignored, and the integrity of continuous PQ disturbances should not be separated.

5 Conclusions
In this study, PQ state space representation was introduced to assess equipment compatibility/incompatibility degree to PQ disturbances, influence of PQ disturbances on equipment, and PQ performance. EMC degree $D_{EMC}$, electromagnetic incompatibility degree $D_{EMIC}$, fuzzy EMC degree $D_{FEMC}$, fuzzy electromagnetic incompatibility degree $D_{FEMIC}$, and influence degree $D_I$ indices were proposed for application to the assessment.

Case studies on a 14-bus distribution system were performed to show the usefulness and reasonableness of these indices.

The main outcomes of the paper are as follows.

(i) The proposed $D_{EMC}$, $D_{EMIC}$, $D_{FEMC}$, $D_{FEMIC}$, and $D_I$ indices are an effective tool to assess EMC degree, electromagnetic incompatibility degree, influence degree and PQ performance from the perspective of electric equipment.

(ii) The introduction of PQ state space representation to the assessment allows the integrated disturbance tolerance of equipment to be easily determined. The assessment results can be expressed intuitively and accurately.

(iii) The problems of uncertainty of equipment tolerance and fault state are considered, and the cloud model and fuzzy membership function are applied to solve the problems.

(iv) The comprehensive assessment of the EMC degree, electromagnetic incompatibility degree, influence degree, and PQ performance of the entire bus system considers the severity and impact scope of PQ disturbances in the weighting process.

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