A time dependent solution for the operation of ion chambers in a high ionization background.

Christos Velissaris\textsuperscript{a,1}

\textsuperscript{a}University of Wisconsin, 1150 University Avenue, Madison WI, 53706

Abstract

We have derived a time dependent solution describing the development of space charge inside an ion chamber, subjected to an externally caused ionization rate $N$. The solution enables the derivation of a formula that the operational parameters of the chamber must satisfy for saturation free operation. This formula contains a correction factor to account for the finite duration of the ionization rate $N$.

Key words: Ionization chambers, Saturation, Time dependent solution

1 Introduction

It is a well known effect that the operation of ion chambers in a high ionization environment is limited by the space charge accumulated in the chamber active volume. This space charge is dominated by the positive ions since the electrons move with a drift velocity 1,000 times larger. The resulting electric

\textsuperscript{1} Address for Correspondence: 1150 University Avenue, University of Wisconsin, Madison WI, 53706
field inside the chamber is no more uniform and it takes its least absolute value near the anode.

When this value reaches zero the electron collection stops and the chamber becomes saturated. It is well known [1,2,3,4] that a distinction between long and short pulses should be made when attempting to calculate saturation effects. In the short pulse approximation the duration of ionization and charge collection time is short with respect to the time the positive ions need to move appreciably with respect to the distance d between the chamber electrodes. In this approximation the positive ions are considered practically immovable throughout the whole electron collection time, thus the space charge density inside the chamber is uniform and constant. In the long pulse approximation the duration of ionization is long enough that a steady state is reached. In this state the number of collected positive ions per unit time is equal to the number of generated by the external ionization factor. The space charge is no more depending on time but only on the position x between the chamber electrodes.

In this paper we derive a time dependent solution of the space charge accumulated in the chamber. We have made the assumption that the positive ions are moving with a constant drift velocity $V_{\text{drift}} = \mu \cdot \frac{V_0}{d}$ throughout the whole ionization and charge collection time. This solution collapses into the short pulse approximation for ionization times much less than the characteristic time of the chamber $T_0$, and yields the steady state solution for ionization times larger or equal to $T_0$. We derive a simple expression for the quantity $T_0$, the characteristic time of the ion chamber, as well as for the electric fields inside the chamber as a function of space and time.

Finally we derive a modified saturation equation, which defines the conditions that the operational parameters of the ion chamber should satisfy in order for
the detector to operate free of saturation effects. This equation is a function not only of the positive ion mobility $\mu$, the electrode gap $d$ and the externally applied voltage $V_0$, but a function of the duration $T$ of the external ionization as well.

Throughout the discussion we have assumed that the cathode plate is kept at potential $-V_0$ at $x=0$ and the anode is grounded at $x=d$. In our calculations we have used the esu system of units.

2 The Steady State

The accumulation of space charge inside the ion chamber, subjected in a constant external ionization rate $N$, is governed by the continuity equations:

$$\int_{x}^{d} \frac{\partial p(x',t)}{\partial t} dx' = N(d-x) - p(x,t)u_p(x,t)$$ for the positive ions, and:

$$\int_{0}^{x} \frac{\partial n(x',t)}{\partial t} dx' = Nx - n(x,t)u_n(x,t)$$ for the electrons.[1,4,9]

Here $u_p$ is the positive ion and $u_n$ the electron drift velocities. For $x=d$ $p(d,t)=0$, and for $x=0$ $n(0,t)=0$ that is the concentration of positive ions next to the anode and the concentration of electrons next to the cathode are always zero. Also at $t=0$ $p(x,0)=n(x,0)=0$. In the steady state:

$p(x) = \frac{N(d-x)}{u_p(x)}$ and $n(x) = \frac{Nx}{u_n(x)}$

Since the mobility of the electrons is 1,000 larger than the positive ions we can ignore the electron contribution to the spacecharge and thus we will write for the steady state:

$$\rho(x) = \frac{N(d-x)}{\mu|E(x)|}$$
where $E(x)$ is the electric field in the active volume of the chamber in the steady state and $\mu$ is the positive ion mobility. By solving the Maxwell equations we get for the electric field:

$$|E(x)| = \sqrt{E(d)^2 + \frac{4\pi N}{\mu} \cdot (d - x)^2}$$

If we imposing the initial conditions $V(0)=-V_0$ and $V(d)=0$ we get the relationship:

$$\frac{V_0}{d^2} \cdot \sqrt{\frac{\mu N}{\pi}} = \sqrt{1 + z^2 + z^2 \ln\left(\frac{1 + \sqrt{1 + z^2}}{z}\right)}$$

with $z=\sqrt{\frac{\mu E(d)^2}{4\pi N d^2}}$

For real values of $z$ the second part of the equation is greater than 1, so in order for $E(d)$ to exist:

$$\frac{V_0}{d^2} \geq \sqrt{\frac{N\pi}{\mu}}$$

This is a condition the operating parameters of the ion chamber must satisfy in order for the ion chamber to operate free of saturation effects, provided that the steady state has been reached.

If we assume that the positive ions are moving with constant drift velocity $\mu \cdot \frac{V_0}{d}$ then the steady state space charge and electric field become:

$$\rho(x) \approx \frac{Nd(d-x)}{\mu V_0}$$

$$E(x) \approx -\frac{V_0}{d} + \frac{2\pi N d}{\mu V_0} \left( -\frac{2d^2}{3} - x^2 + 2dx \right).$$

Under this approximation the steady state saturation condition yields:
\[
\frac{V_0}{d^2} \geq \sqrt{\frac{2\pi N}{3\mu}}.
\]

This approximate condition based on the assumption of constant positive ion drift velocity differs from the one based on the exact positive ion drift velocity, by a correction factor \( \sqrt{\frac{2}{3}} \approx 1.22 \).

3 The Short Pulse Approximation

When the ion chamber is subjected to an intense short lived ionization pulse the short pulse approximation is more suitable to describe the space charge effects in the operation of the detector. We can model the space charge density inside the chamber at the end of the ionization pulse as \( \rho \), constant throughout the chamber volume. Solution of Maxwell equations for the electric field yields:

\[
E(x) = 4\pi \rho x - 2\pi \rho d \frac{V_0}{d} = -4\pi (x_{sat} - x) \text{ where } x_{sat} = \frac{d}{2} + \frac{V_0}{4\pi \rho d}
\]

The electric field becomes zero at \( x = x_{sat} \). Thus, in order for the chamber to work free of saturation effects \( x_{sat} \geq d \) which leads to the condition:

\[
\frac{V_0}{d^2} \geq 2\pi \rho.
\]

An interesting consequence of the short pulse approximation is that always, a portion of the fast moving electrons is collected. If we suppose that the whole ionisation happens instantaneously the fast electron swarm will keep moving (and collected at the anode) from \( x = 0 \) to \( x = x_{sat} \) where the electric field be-
comes zero. We have considered the electron drift velocity as a product of the electron mobility times the Electric field. If we suppose that all the electrons from x=0 to x=x_{sat} have been collected and electrons between x=x_{sat} and x=d have been lost (since the drift velocity has become zero), the collection efficiency can be estimated as

\[ \epsilon_{\text{coll}} = \frac{x_{\text{sat}}}{d} = \frac{1}{2} + \frac{V_0}{4\pi\rho d^2} \text{ if } x_{\text{sat}} \leq d \text{ otherwise } \epsilon_{\text{coll}} = 1 \]

We have ignored all recombination effects during the electron movement. We see that for extremely fast pulses the collection efficiency is at least 0.5, even with the chamber operating under saturation conditions. The collection efficiency also increases linearly with the applied voltage \( V_0 \) and it is inversely proportional to the charge density \( \rho \). Recently conducted experiments have observed this effect.[10,11]

4 A Time Dependent Solution.

The discussion below is based on the assumption that the positive ions are moving with constant drift velocity \( \mu \cdot \frac{V_0}{d} \) throughout the whole chamber volume and duration of charge collection.

We have derived a solution describing the time dependence of the accumulated space charge inside the chamber active volume. It is described by the equation:

\[
\rho(x, t) = \frac{N d}{\mu V_0} \cdot x_0 \text{ when } 0 \leq x \leq d - x_0 \text{ and } \\
\rho(x, t) = \frac{N d}{\mu V_0} \cdot (d - x) \text{ when } d - x_0 \leq x \leq d \\
\text{with } x_0 = \frac{\mu V_0}{d} \cdot t
\]
At time $t$ the accumulated space charge is constant for values of $x$ less than $d-x_0$ and a linear function of $x$ for values greater than $d-x_0$, where $x_0 = \frac{\mu V_0}{d} \cdot t$.

The steady state is reached when $x_0=d$, that is, when $t = T_0 = \frac{d^2}{\mu V_0}$. We called $T_0$ the characteristic time of the ion chamber.

In Figure 1 we present the space charge distribution inside the ion chamber for various times. By solving the Maxwell equations we can calculate the electric field inside the ion chamber as a function of $x$ and $t$.

![Figure 1](image-url)

Fig. 1. In this plot we present the space charge distribution inside the ion chamber for various times $t_1, t_2, t_3, t_4, t_5$. At $t=t_5$ the steady state is reached and the space charge distribution becomes stationary thereafter.
\[ E(x,t) = E(d) - \frac{2\pi N d}{\mu V_0} \cdot x_0^2 - \frac{4\pi N d}{\mu V_0} \cdot x_0(d - x_0 - x) \text{ for } 0 \leq x \leq d-x_0 \text{ and} \]
\[ E(x,t) = E(d) - \frac{2\pi N d}{\mu V_0} \cdot (d-x)^2 \text{ for } d-x_0 \leq x \leq d \]

\( E(d) \) is the electric field value at the anode at \( x=d \). It can be calculated from the boundary values \( V(x=0,t)=-V_0 \) and \( V(x=d,t)=0 \). We find:

\[ E(d) = -\frac{V_0}{d} + \frac{2\pi N}{\mu V_0} \cdot x_0 \cdot (d^2 - x_0d + \frac{x_0^2}{3}) \text{ with } x_0 = \frac{\mu V_0}{d} \cdot t \]

In order for the chamber to work free from saturation effects \( E(d) \leq 0 \), so we arrive at the saturation condition:

\[
\begin{align*}
\frac{d V_0}{d\tau} &\geq \sqrt{\frac{2\pi N}{3\mu} \cdot \tau (\tau^2 - 3\tau + 3)} \text{ for } \tau \leq 1 \text{ and} \\
\frac{d V_0}{d\tau} &\geq \sqrt{\frac{2\pi N}{3\mu}} \text{ for } \tau \geq 1
\end{align*}
\]

with \( \tau = \frac{T}{T_0} \) and \( T_0 = \frac{d^2}{\mu V_0} \) the characteristic time of the ion chamber. In Figure 2 we present the upper limit of the quantity \( \frac{d V_0}{d\tau} \) as a function of the parameter \( \tau \) for \( 0 \leq \tau \leq 1 \). For \( \tau \geq 1 \) the steady state has been reached and this upper limit remains constant, independent of \( \tau \). For simplicity we have assumed \( \frac{2\pi N}{3\mu}=1 \).

For a pulsed beam of duration \( T \) we have, \( N=\frac{\rho}{T} \). Here, \( N \) is the ionization charge density rate and \( \rho \) is the total charge density having been produced inside the ion chamber by the pulse after time \( T \). We can then write:

\[
\begin{align*}
\frac{d V_0}{d\tau} &\geq \sqrt{\frac{2\pi \rho}{3\mu T_0} \cdot \left( \frac{T^2}{T_0^2} - 3\frac{T}{T_0} + 3 \right)} \text{ for } T \leq T_0 \text{ and} \\
\frac{d V_0}{d\tau} &\geq \sqrt{\frac{2\pi \rho}{3\mu T_0}} \text{ for } T \geq T_0.
\end{align*}
\]

For \( \tau \geq 1 \) (\( T \geq T_0 \)) that is for times greater than the characteristic time of ion
Fig. 2. In this plot we present the lower limit \( f(\tau) = \sqrt{\frac{2\pi N}{3\mu} \cdot \tau (\tau^2 - 3\tau + 3)} \) of the quantity \( \frac{V_0}{d^2} \) in order for an ion chamber to operate saturation free as a function of the parameter \( \tau = \frac{t}{T_0} \) for \( 0 \leq \tau \leq 1 \). For simplicity we have taken \( \frac{2\pi N}{3\mu} = 1 \).

For times \( t \) much less than \( T_0 \) if we approximate \( \tau^2 - 3\tau + 3 \approx 3 \) we get:

\[
\frac{V_0}{d^2} \geq 2\pi N t.
\]

Since \( N \) is the ionization density rate in the chamber, \( Nt \) is the total charge density \( \rho \) produced by the pulse after time \( t \). We then retrieve the short pulse approximation condition:

\[
\frac{V_0}{d^2} \geq 2\pi \rho.
\]
5 Conclusion

After a brief review of the operation of ion chambers in an intense ionization environment we presented a formula describing the time development of the space charge inside the chamber. The formula collapses into the short pulse approximation for short enough ionization pulses with respect to the characteristic time of chamber $T_0$ and reproduces the steady state solution if the externally caused ionization lasts long enough. Although the formula has been derived under the assumption that the positive ions are moving always with a constant drift velocity $\frac{\mu V_0}{d}$, it helps us to understand (qualitatively and up to some degree quantitatively) the operation of ion chambers. More specifically we deduce:

- In order to account for the finite duration of an externally caused ionization density rate $N$ in an ion chamber we have derived the following modified saturation condition:

$$\frac{V_0}{d^2} \geq \sqrt{\frac{2\pi N}{3\mu}} \cdot \tau (\tau^2 - 3\tau + 3)$$

for $\tau \leq 1$ and

$$\frac{V_0}{d^2} \geq \sqrt{2\pi N \mu}$$

for $\tau \geq 1$

with $\tau = \frac{t}{T_0}$ and $T_0 = \frac{d^2}{\mu V_0}$ the characteristic time of the ion chamber.

For a pulsed beam of duration $T$, $N = \frac{\rho}{T}$ where $\rho$ is the total charge density produced in the chamber by the pulse after time $T$. We can then write:

$$\frac{V_0}{d^2} \geq \sqrt{\frac{2\pi \rho}{3\mu T_0}} \cdot \left(\frac{T^2}{T_0^2} - 3\frac{T}{T_0} + 3\right)$$

for $T \leq T_0$ and

$$\frac{V_0}{d^2} \geq \sqrt{\frac{2\pi \rho}{3\mu T_0}}$$

for $T \geq T_0$.

- Both, the short pulse approximation and the steady state represent valid solutions in judging saturation effects in ion chambers.
• The steady state is reached throughout the whole chamber volume within a finite time $T_0$ (the characteristic time of the ion chamber) after the onset of the external ionization process. However, as we showed in this paper, different points reach the steady state at different times.

• For short enough pulsed beams with respect to the ion chamber characteristic time the positive ion mobility $\mu$ does not play any role in judging saturation effects. [10]

• Saturation effects should be judged by taking into account not only the chamber operating voltage, gap and ion mobility but the duration of the ionization process as well.

• Whether we assume that the positive ions move with constant drift velocity or their speed is affected by the variation of the electric field due to the space charge, for short pulses both solutions collapse into the short pulse approximation and no distinction is made. Corrections may be important when the steady state is formed due to the different space distribution of the positive ion charge density inside the chamber. The maximum correction must be applied when steady state is reached. As we have seen that factor is $\sqrt{\frac{2}{\pi}} \approx 1.22$. An effort to solve the continuity equations, that describe the time development of the positive ion distribution in the chamber, by using computational techniques is presented in a recent paper[8].

References

[1] J. Sharpe, “Nuclear radiation Detectors”, John Wiley.

[2] J. W. Boag “Ionization Measurements at very high Intensities”. I. Pulsed Radiation Beams. Brit. J. Radiol. 23, 601 (1950)
[3] J. W. Boag “The Saturation Curve for Ionization Measurements in Pulsed radiation Beams” Brit. J. Radiol. 25, 649 (1952)

[4] J. W. Boag “Space Charge Distortion of the Electric Field in a Plane parallel Ionization Chamber” Phys. Med. Biol. 8, 461 (1963)

[5] J. W. Boag and T. Wilson, “The Saturation curve at High Ionization Density” Brit. J. Appl. Phys., vol. 3, pp. 222-229 (1952)

[6] J. W. Boag in F. H. Attix, W.C. Roesch “Radiation Dosimetry”, Academic Press.

[7] “Ionization Dosimetry at High Intensity”. In proceedings of the International School of Physics E. Fermi, Course XXX, “Radiation Dosimetry”, p. 70.

[8] S. Palestini, G. D. Barr, C. Biino, P.O. Calafiura, A. Ceccucci, C.Cerri et. al., “Space Charge in Ionisation Detectors and the NA48 Electromagnetic Calorimeter” Nuclear Instruments and Methods in Physics Research A421 (1999) pp. 75-89.

[9] C. Velissaris. “Principles of ionization chamber operation under intense ionization rates.” NuMI-NOTE-BEAM-0717, unpublished

[10] J. McDonald, C. Velissaris, B. Viren, M. Diwan, A. Erwin, D. Naples, H. Ping “Ionization chambers for monitoring at high intensity charged particle beams.” Nucl. Instrum. Meth. A496:293-304, 2003

[11] R. Zwaska et.al.“Beam Tests of Ionization chambers for the NuMI neutrino beam” IEEE Trans. Nucl. Sci.50:1129-1135, 2003