Abstract

Particle track reconstruction capabilities of the silicon tracking detector system have been studied. As the multiple Coulomb scattering (MCS) induces unavoidable uncertainties on the coordinate measurement, the corresponding error estimates and the associated correlations have been used to find the best track fit parameters and their errors. Finally it permits to find the proper particle characteristics, as vertex position and resolution, flight direction and the error.
1 Introduction

Design and preparation of any components of a detector system must take care of characteristics and detection performances (efficiency, acceptance, position or energy resolution) necessary for a specific process study. The silicon tracking detector system, in our case, must furnish the best information about the coordinate track intercept of the incident particle on every detector layer. Otherwise, for a good track reconstruction, it is necessary to find the best estimate of the track parameters in a specific particle measurement. This is the reason we studied, the particle transport in a silicon tracking system and estimated the multiple Coulomb scattering (MCS) perturbation in particle track measurements.

This study furnishes information on the position resolution we could get from every detector layer, and also its dependence on some physical and geometrical parameters. In the detector system design these parameters could be chosen in an optimal manner, aiming the best track reconstruction possibilities or the best estimate of the track parameters (vertex position, curve radius) directly connected to physical quantities.

The simplified tracking system we used, consists of a 5 layers silicon microstrip detector, 300\(\mu\)m thick, interspaced by 1.5 cm, and the first detector layer located at 130 cm from the interaction point (see Fig.1).

2 The track particle position uncertainties due to multiple Coulomb scattering.

When a charged particle is traversing the detector elements of a tracking system, it undergoes small deviations of the track, due to MCS. The effect is usually described by the theory of Moliere (see for example [1]). It shows that, by traversing detector’s material, thickness \(s\), the particle undergoes successive small-angle deflections, symmetrically distributed about the incident direction. Applying the central limit theorem of statistics to a large number of independent scattering events, the Moliere distribution of the scattering angle can be approximated by a Gaussian one [2]. It is sufficient for many applications to use Gaussian approximation for the central 98% of the plan projected angular distribution. The width of this distribution is the root mean square of the scattering angle [3]

\[
\theta_0 = \frac{13.6 MeV}{p\beta c}z_c\sqrt{\frac{s}{X_L}} \left[1 + 0.038 \ln \left(\frac{s}{X_L}\right)\right]
\]

(1)

where \(p, \beta c\) and \(z_c\) are the momentum, velocity and charge number of the incident particle, and \(X_L\) is the radiation length of the scattering medium. That is, the plane projected angle \(\theta_{\text{plane},x}\) or \(\theta_{\text{plane},y}\) of the deflection angle \(\theta\), onto the xOz and yOz planes, where the x and y axes are orthogonal to the Oz direction of motion, shows an approximately
Gaussian angular distribution

\[ \frac{1}{\sqrt{2\pi\theta_0}} \exp \left[ -\frac{\theta^2_{\text{plane}}}{2\theta^2_0} \right] d\theta_{\text{plane}} \quad (2) \]

Deflections into \( \theta_{\text{plane},x} \) and \( \theta_{\text{plane},y} \) are independent and identically distributed, and \( \theta^2_{\text{space}} = \theta^2_{\text{plane},x} + \theta^2_{\text{plane},y} \).

The angular distribution is translated to a coordinate distribution by particles fly onto every detector layer. The more intersected layers the larger distribution width is. The coordinate distribution is defined by statistical spread due to MCS, and depends on the number and position of the intersected detector layer elements. It has the same form as angular distribution

\[ \frac{1}{\sqrt{2\pi\sigma_{x_i}}} \exp \left[ -\frac{x^2_{i}}{2\sigma^2_{x_i}} \right] dx_i \quad (3) \]

with the mean square deviation (distribution width) as the squares sum of the \((i - 1)\) preceding distribution widths projected onto \(i\)-th detector layer

\[ \sigma^2_{x_i} \equiv < x^2_i > = \theta^2_0 \left[ (z_i - z_1)^2 + (z_i - z_2)^2 + ... + (z_i - z_{i-1})^2 \right] \quad (4) \]

For an oblique incidence \((\theta \neq 0)\) the effective path length in the silicon detector is larger, and the same is the position distribution width on the next detector layers. Nevertheless, in this work we will consider only the minimal width \((4)\), as to emphasize the precision limit in particle position measurement with a given silicon tracking system.

3 The Monte-Carlo particle scattering description.

The change of track parameters is usually \([4]\) parametrized by two mutually orthogonal, uncorrelated scattering angles \(\delta\theta_{\text{plane},x}\) and \(\delta\theta_{\text{plane},y}\) or \((\delta\theta_{\text{space}}\) and \(\delta\varphi)\) which leads to a corresponding displacement \(\delta x_{\text{plane}}\) and \(\delta y_{\text{plane}}\) in the position.

Following the stochastic nature of the MCS, we use the Monte-Carlo study by generating the joint \((\delta x_{\text{plane}}, \delta\theta_{\text{plane},x})\) distribution with independent Gaussian random variables \((w_1, w_2)\) \([4]\)

\[ \delta x_{\text{plane}} = \frac{w_1 s\theta_0}{\sqrt{12}} + \frac{w_2 s\theta_0}{2} \]
\[ \delta\theta_{\text{plane},x} = w_2 \theta_0 \quad (5) \]

The same have been used for the joint \((\delta y_{\text{plane}}, \delta\theta_{\text{plane},y})\) distribution in \(yOz\) plane. Finally we constructed the incidence points distribution (coordinate distribution) on the detector layer no. 2, 3, 4 and 5. (see Fig.2 and Table \([4]\)), for an incident 500 MeV/c muon.
In Monte-Carlo simulated particle transport the particle position uncertainty on every detector layer has been measured as the variance or the mean square deviation of the scattered track incidence points \( x_m \) about the unscattered one \( x_0 \)

\[
\sigma_x^2 = \frac{1}{N} \sum_{n=1}^{N} (x_n - x_0)^2
\]

and similar for \( \sigma_y^2 \) variance, for every detector layer. The \( x_n \) (and \( y_n \)) are the coordinates of the scattered track incidence points on detector layer, and \( N \) is the total number of generated events.

The distribution widths \( \sigma_{x_i} \) of the muon track \( x \)-coordinate points on detector layer \( i=2, 3, 4, 5 \) and for momentum values from 500 to 7500 MeV/c, are presented in Table 1. There are also the analytical estimation of the same widths of the MCS incidence point distribution on each of the \( i \)-detector layer, according (4)

\[
\sigma_{x_i}^2 = \frac{i(i-1)(2i-1)}{6}(l\theta_0)^2
\]

where \( l \) - the distance between detector layers \( (l = 1.5 \text{ cm}) \) and \( \theta_0 \) - the plane r.m.s. scattering angle \([4]\).

This Monte-Carlo particle transport description within silicon detector system will be used as a coordinate data generator for the track reconstruction procedure.

4 Position error correlations.

MCS produces errors correlated from one layer to the next. Clearly a scattering in layer 1 produces correlated position errors in layer 2, 3 and so on (see Fig.3). The proper error matrix is non-diagonal \([3]\), and it must be find out.

Let’s denote \( \delta x_i \) the track deviation \( x \)-coordinate point on the \( i \)-th layer, with respect to the initial incident direction on the detector system, then (see Fig.3)

\[
\begin{align*}
\delta x_1 &= 0 \\
\delta x_2 &= \delta_1 \\
\delta x_3 &= \delta_{31} + \delta_2 \\
\delta x_4 &= \delta_{41} + \delta_{42} + \delta_3 \\
\delta x_5 &= \delta_{51} + \delta_{52} + \delta_{53} + \delta_4 \\
\end{align*}
\]

where the individual contributions due to preceding scatterings are

\[
\begin{align*}
\delta_{31} &= \delta_1 \frac{z_3 - z_1}{z_2 - z_1} \\
\delta_{41} &= \delta_1 \frac{z_4 - z_1}{z_2 - z_1} ; \quad \delta_{42} = \delta_2 \frac{z_4 - z_2}{z_3 - z_2} \\
\delta_{51} &= \delta_1 \frac{z_5 - z_1}{z_2 - z_1} ; \quad \delta_{52} = \delta_2 \frac{z_5 - z_2}{z_3 - z_2} ; \quad \delta_{53} = \delta_3 \frac{z_5 - z_3}{z_4 - z_3}
\end{align*}
\]
Because they are statistical variables it is necessary to find their mean value, and to express it by the independent scattering deviations \( \delta_k \), unaffected by earlier scatterings

\[
\delta_k \equiv \sqrt{\langle \delta_k^2 \rangle} = \theta_0(z_{k+1} - z_k)
\]

(10)

We try to express the \( \delta x_i \) deviations by the independent \( \delta_k \) ones (10), for which

\[
\langle \delta_k^2 \rangle = 0
\]

\[
\delta x_i = \sum_{k=1}^{i-1} \delta_k \frac{z_i - z_k}{z_{k+1} - z_k}
\]

(11)

Now the position error (covariance) matrix \( (V_{ij}) \), defined [4] as the statistical mean of the pair deviation products \( \langle \delta x_i \delta x_j \rangle \) for all possible detection layers is

\[
V_{ij} \equiv \langle \delta x_i \delta x_j \rangle = \theta_0^2 \left[ (z_i - z_1)(z_j - z_1) + (z_i - z_2)(z_j - z_2) + \ldots + (z_i - z_{i-1})(z_j - z_{i-1}) \right]
\]

for \( i \leq j = 1, 2, \ldots, n \)

(12)

The \( (V_{ij}) \) matrix is symmetric. The error correlation matrix is immediately

\[
\rho_{ij} = \frac{\langle \delta x_i \delta x_j \rangle}{\sqrt{\langle \delta x_i^2 \rangle} \sqrt{\langle \delta x_j^2 \rangle}}
\]

(13)

The uncorrelated position errors in the coordinate reading \( \sigma_0 \), have to be added in squares \( (\sigma_0^2) \) into the diagonal terms of the error matrix \( V \).

For example, for a 500 MeV/c MUON track detection \( (\theta_0 = 1.23\text{mrad}) \) by our system configuration \( (z_1 = 130\text{cm}, z_2 = 131.5\text{cm}, z_3 = 133\text{cm}, z_4 = 134.5\text{cm}, z_5 = 136\text{cm} \) and \( \sigma_0 = 10\mu\text{m} \) the matrix elements \( V_{ij} \) (in square microns) and \( \rho_{ij} \) are

\[
V = \begin{pmatrix}
100. & 0. & 0. & 0. & 0. \\
0. & 440.64 & 681.27 & 1021.9 & 1362.5 \\
0. & 681.27 & 1803.2 & 2725.1 & 3747.0 \\
0. & 1021.9 & 2725.1 & 4868.9 & 6812.7 \\
0. & 1362.5 & 3747.0 & 6812.7 & 10319.
\end{pmatrix}
\]

(14)

\[
\rho = \begin{pmatrix}
1 & 0 & 0 & 0 & 0 \\
0 & 1 & .76430 & .69768 & .63898 \\
0 & .76430 & 1 & .91970 & .86865 \\
0 & .69768 & .91970 & 1 & .96114 \\
0 & .63898 & .86865 & .96114 & 1
\end{pmatrix}
\]

(15)

As long as the \( V_{ij} \) matrix elements depend both on kinematical characteristics of the detected particles and on the tracking detector system configuration, the \( \rho_{ij} \) matrix elements are independent on particle characteristics, and is defined only by system configuration.

In the following we will use these matrices in the track reconstruction by a least squares fit procedure.
5 Track reconstruction parameters and the errors

In the absence of the magnetic field, the unscattered track is a straight line. The independent description of \( x \) and \( y \) MCS data permits a separate least squares fit to these data by a linear relationship \[3, 7\]

\[
\begin{align*}
x &= x_0 + \alpha_x z \\
y &= y_0 + \alpha_y z
\end{align*}
\] (16)

With the coordinate and error data \((x_i \pm \sigma_{x_i}), (y_i \pm \sigma_{y_i}), z_i\), along with the corresponding correlation matrix \(\rho_{ij} \quad (V_{ij} = \rho_{ij}\sigma_{x_i}\sigma_{x_i})\) as input data, it is possible to express the \(\chi^2\) in matrix form, for every coordinate data set. For \(x\)-data set it will be

\[
\chi^2 = (X - HA_x)^T V^{-1} (X - HA_x)
\] (17)

where

\[
X = \begin{pmatrix}
x_1 \\
x_2 \\
\vdots \\
x_n
\end{pmatrix} \quad ; \quad H = \begin{pmatrix}
1 & z_1 \\
1 & z_2 \\
\vdots & \vdots \\
1 & z_n
\end{pmatrix} \quad ; \quad A_x = \begin{pmatrix}
x_0 \\
\alpha_x
\end{pmatrix}
\] (18)

the \(V\) matrix is given by \([12]\) and \([14]\).

Least squares criterion impose

\[
\frac{\partial\chi^2}{\partial A_x} = 0 \quad \text{or} \quad H^T V^{-1} (X - HA_x) = 0
\]

By solving the linear system relative to \(A_x\) we get the fit parameters

\[
A_x = (H^T V^{-1} H)^{-1} (H^T V^{-1} X)
\] (19)

and the errors of these parameters

\[
E_{A_x} \equiv \langle \delta A_x \delta A_x^T \rangle = (H^T V^{-1} H)^{-1}
\] (20)

The same procedure we applied to \(y\) coordinate, as to find the fit parameters and errors; \(z_i\) has no any uncertainty, they are detector position coordinates.

Let’s take an example. Using the particle transport simulation data within silicon tracking system, along with the variance matrix data \(V\) \([14]\), we have been obtained the fit parameters and their errors \((x_0 \pm \sigma_{x_0}, \alpha_x \pm \sigma_{\alpha_x}; y_0 \pm \sigma_{y_0}, \alpha_y \pm \sigma_{\alpha_y})\). The results are presented in Table 2.

The found track fit parameters, could be connected with some particle characteristic quantities as \(xy\)-vertex position \((x_0, y_0)\) and the \(xy\)-vertex uncertainty \((\sigma_{xy}^{(vertex)} = \ldots)\)
\sqrt{\sigma_{x0}^2 + \sigma_{y0}^2}$ and the particle production direction \( (\theta \pm \sigma_\theta, \varphi \pm \sigma_\varphi) \). This last one is connected with the slope parameters \( \alpha_x \) and \( \alpha_y \). See Table 3.

$$
\begin{align*}
tg \varphi &= \frac{\alpha_y}{\alpha_x} \\
tg \theta &= \frac{\alpha_x}{\cos \varphi}
\end{align*}
$$

and the corresponding errors

\[
\begin{align*}
\sigma_{tg\varphi}^2 &= \left( \frac{1}{\alpha_x} \right)^2 \sigma_{\alpha_y}^2 + \left( \frac{\alpha_y}{\alpha_x^2} \right)^2 \sigma_{\alpha_x}^2 \\
\sigma_{tg\theta}^2 &= \left( \frac{1}{\cos \varphi} \right)^2 \sigma_{\alpha_x}^2 + \left( \frac{\alpha_x}{\cos^2 \varphi} \right)^2 \sigma_{\cos \varphi}^2
\end{align*}
\]

In Table 3 there are results on the reconstructed \( \theta \) and \( \varphi \) direction values in comparison with the generated ones.

The \( xy \)-vertex position uncertainty, due to MCS in the detector material, have been also calculated as \( \sigma_{xy}^{\text{vertex}} = \sqrt{\sigma_{x0}^2 + \sigma_{y0}^2} \) and depends, of course, on particle momentum. If we want to reduce this error it is necessary to bring closer the detector tracking system relative to interaction point. The \( xy \)-vertex position uncertainty (resolution) dependence on particle (muon) momentum and \( z_1 \) distance to first detector layer is shown in Fig.4. The result is useful in detector system design to find an optimal configuration in preparing the experimental work. Also, to have a choice for vertex resolution as a compromise between the best possible values in the proximity of the interaction point and the worse ones far from this point at the radiation harmless distance. The \( xy \)-vertex position resolution due to MCS errors combined with the intrinsic detector coordinate uncertainties, could not be better than the values shown in Table 4. Nevertheless, from Table 4 and Fig.4 we see also that it is possible to have a better vertex position resolution if the detector tracking system is placed to a smaller distance from the interaction point.

### 6 Conclusions

We have calculated the uncertainties in particle characteristic parameters as vertex position and the particle flight direction, by a linear fitting procedure, using position error correlation due to MCS in successive detector layers of a silicon tracking system.

The most important result shows these uncertainties are independent on individual track coordinates (see for example the errors for \( x_0, y_0 \), and \( \alpha_x, \alpha_y \)) and they depend exclusively (see (21) and (22)) on the detector system configuration (geometry). The results are useful both in particle tracking system design and in experimental data analysis for
pattern track recognition. The former one could use our analytical relations for particle parameter error estimation relative to different tracking system configuration. The later one could use our data fitting procedure, including system configuration dependence on the coordinate covariance matrix, in particle track reconstruction and the associated parameter determination.

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Table 1: MUON x-distribution width on the detector layers of a silicon tracking system due to multiple Coulomb scattering.

These are the correlated uncertainties $\sigma_i$, used in track fitting procedure.

| $p$ (MeV/c) | $\sigma_2$ ($\mu$m) | $\sigma_3$ ($\mu$m) | $\sigma_4$ ($\mu$m) | $\sigma_5$ ($\mu$m) |
|-------------|-------------------|-------------------|-------------------|-------------------|
| 500         | 18.88 / 18.46     | 41.75 / 41.27     | 69.74 / 69.06     | 101.94 / 101.09   |
| 1000        | 9.20 / 9.08       | 20.60 / 20.30     | 34.43 / 33.97     | 50.34 / 49.73     |
| 1500        | 6.10 / 6.03       | 13.55 / 13.49     | 22.73 / 22.58     | 33.42 / 33.05     |
| 2000        | 4.56 / 4.52       | 10.19 / 10.11     | 17.05 / 16.91     | 25.07 / 24.76     |
| 2500        | 3.71 / 3.61       | 8.18 / 8.08       | 13.65 / 13.53     | 19.96 / 19.80     |
| 3000        | 3.08 / 3.01       | 6.84 / 6.73       | 11.36 / 11.27     | 16.57 / 16.49     |
| 3500        | 2.62 / 2.58       | 5.84 / 5.77       | 9.72 / 9.66       | 14.18 / 14.14     |
| 4000        | 2.28 / 2.26       | 5.09 / 5.05       | 8.54 / 8.45       | 12.52 / 12.36     |
| 4500        | 2.03 / 2.01       | 4.50 / 4.49       | 7.48 / 7.51       | 10.90 / 10.99     |
| 5000        | 1.82 / 1.81       | 4.05 / 4.04       | 6.79 / 6.76       | 9.96 / 9.89       |
| 5500        | 1.66 / 1.64       | 3.68 / 3.67       | 6.12 / 6.14       | 8.94 / 8.99       |
| 6000        | 1.53 / 1.51       | 3.39 / 3.37       | 5.67 / 5.63       | 8.31 / 8.24       |
| 6500        | 1.39 / 1.39       | 3.12 / 3.11       | 5.22 / 5.20       | 7.61 / 7.61       |
| 7000        | 1.31 / 1.29       | 2.90 / 2.88       | 4.85 / 4.83       | 7.06 / 7.07       |
| 7500        | 1.21 / 1.20       | 2.69 / 2.69       | 4.50 / 4.50       | 6.58 / 6.59       |
Table 2: Particle transport simulation data within silicon tracking system.

MUON 500 MeV/c, $x_0 = 0, y_0 = 0, z_0 = 0, \theta = 15^\circ, \varphi = 30^\circ$
Silicon layer width = 300$\mu$m, $z_1 = 130 cm, dz = 1.5 cm$

Mean scattering angle (plan projected) $\theta_0 = 1.23 mrad$

| det | $x(cm)$ | $\sigma_0(\mu m)$ | $\sigma_{MCS}(\mu m)$ | $y(cm)$ | $\sigma_0(\mu m)$ | $\sigma_{MCS}(\mu m)$ | $z(cm)$ |
|-----|---------|-------------------|------------------------|---------|-------------------|------------------------|--------|
| 1   | 30.166605 | 10                | 0.0                    | 17.416698 | 10                | 0.0                    | 130.0  |
| 2   | 30.517353 | 10                | 18.5                   | 17.617701 | 10                | 18.5                   | 131.5  |
| 3   | 30.868230 | 10                | 41.3                   | 17.816364 | 10                | 41.3                   | 133.0  |
| 4   | 31.218312 | 10                | 69.1                   | 18.013349 | 10                | 69.1                   | 134.5  |
| 5   | 31.569294 | 10                | 101.1                  | 18.207651 | 10                | 101.1                  | 136.0  |

Linear track reconstructed (parameter) data

$x_0 = -2316.4 \pm 1828.3 \mu m \quad \alpha_x = 0.23383 \pm 0.001404$

$y_0 = 618.69 \pm 1828.3 \mu m \quad \alpha_y = 0.13350 \pm 0.001404$

$\sigma_{xy}^{(vertex)} = \sqrt{2}\sigma_{x_0} = 2585.65 \mu m$

$\theta = 15.070^\circ \pm 0.096^\circ$

$\varphi = 29.723^\circ \pm 0.298^\circ$

Table 3: Track reconstructed direction by least squares fit of the 500 MeV/c MUON Monte-Carlo simulated transport data within silicon tracking system.

| Generated |  | Reconstructed |  |  |
|-----------|  |  |  |  |
| $\theta(\text{deg})$ | $\varphi(\text{deg})$ | $\theta(\text{deg})$ | $\varphi(\text{deg})$ |
| 5 | 270 | 5.07 $\pm$ 0.81 | 269.20 $\pm$ 0.91 |
| 10 | 240 | 10.16 $\pm$ 0.20 | 239.52 $\pm$ 0.45 |
| 15 | 210 | 14.87 $\pm$ 0.10 | 210.20 $\pm$ 0.30 |
| 20 | 180 | 19.85 $\pm$ 0.07 | 179.79 $\pm$ 0.22 |
| 25 | 150 | 25.02 $\pm$ 0.08 | 149.89 $\pm$ 0.17 |
| 30 | 120 | 30.05 $\pm$ 0.16 | 119.85 $\pm$ 0.14 |
| 35 | 90 | 35.04 $\pm$ 0.20 | 89.98 $\pm$ 0.11 |
| 40 | 60 | 39.91 $\pm$ 0.13 | 59.94 $\pm$ 0.10 |
| 45 | 30 | 44.99 $\pm$ 0.05 | 30.04 $\pm$ 0.08 |
Table 4: \textit{xy}-vertex position uncertainty due to multiple Coulomb scattering by track reconstruction procedure.

Table shows the \textit{xy}-vertex uncertainty (\(\mu m\)) obtained by a linear track reconstruction with a 5 detector layers system, interspaced by \(dz=1.5\) cm, for some \(z_1\) target to first detector layer distance (cm) and some MUON momentum (MeV/c) values.

| \(z_1 (cm)\) | 10  | 30  | 50  | 70  | 90  | 110 | 130 |
|---------------|-----|-----|-----|-----|-----|-----|-----|
| \(p (MeV/c)\) |     |     |     |     |     |     |     |
| 500           | 203.414 | 600.261 | 997.307 | 1394.383 | 1791.468 | 2188.558 | 2585.650 |
| 1000          | 116.415 | 336.970 | 557.795 | 778.662  | 999.541  | 1220.428 | 1441.317 |
| 1500          | 86.285  | 245.645 | 405.317 | 565.036  | 724.772  | 884.515  | 1044.262 |
| 2000          | 70.975  | 198.945 | 327.254 | 455.615  | 583.994  | 712.382  | 840.774  |
| 2500          | 61.993  | 171.278 | 280.916 | 390.610  | 500.322  | 610.044  | 719.770  |
| 3000          | 56.277  | 153.481 | 251.041 | 348.659  | 446.296  | 543.943  | 641.594  |
| 3500          | 52.429  | 141.375 | 230.675 | 320.033  | 409.410  | 498.797  | 588.189  |
| 4000          | 49.726  | 132.790 | 216.202 | 299.672  | 383.161  | 466.660  | 550.164  |
| 4500          | 47.761  | 126.498 | 205.575 | 284.709  | 363.863  | 443.026  | 522.194  |
| 5000          | 46.291  | 121.760 | 197.561 | 273.417  | 349.292  | 425.177  | 501.067  |
| 5500          | 45.166  | 118.111 | 191.379 | 264.701  | 338.043  | 411.394  | 484.749  |
| 6000          | 44.287  | 115.246 | 186.519 | 257.846  | 329.191  | 400.546  | 471.905  |
| 6500          | 43.588  | 112.957 | 182.634 | 252.363  | 322.110  | 391.867  | 461.628  |
| 7000          | 43.024  | 111.103 | 179.482 | 247.914  | 316.364  | 384.822  | 453.285  |
| 7500          | 42.562  | 109.581 | 176.894 | 244.258  | 311.640  | 379.031  | 446.426  |
Figure captions

Fig. 1  The track particle position uncertainties due to multiple Coulomb scattering. The uncertainty is defined as the coordinate distribution width $\sigma_i$ on every detector layer $i$ and results as the squares sum of all preceding layer scattering contributions projected onto the $i$-th layer. $\sigma_{x_i}^2 = \theta_0^2 [(z_i - z_1)^2 + (z_i - z_2)^2 + ... + (z_i - z_{i-1})^2]$

Fig. 2  The plan projected x-coordinate point distributions of the scattered 500 MeV/c MUONS, incident on detector layer No. 2, 3, 4, and 5 obtained by Monte-Carlo particle transport simulation.

Fig. 3  Multiple Coulomb scattering error correlations.

Fig. 4  The $xy$-vertex position uncertainty (reconstructed vertex resolution deterioration) due to multiple Coulomb scattering as a function of MUON momentum and distance from the interaction point to first detector layer ($z_1$).