Current Reversals and Synchronization in Coupled Ratchets

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(Dated: August 21, 2009)

Current reversal is an intriguing phenomenon that has been central to recent experimental and theoretical investigations of transport based on ratchet mechanism. By considering a system of two interacting ratchets, we demonstrate how the coupling can be used to control the reversals. In particular, we found that current reversal that exists in a single driven ratchet system can ultimately be eliminated in the presence of a second ratchet. Furthermore, we show that for larger values of the coupling strength, the dynamics of the single driven ratchet completely dominates that of the coupled ratchets, thereby resulting in a fully synchronized state.

PACS numbers: 05.45.Xt, 05.45.Ac, 05.40.Fb, 05.45.Pq, 05.60.Cd

Transport phenomena and, particularly, directed transport occur in many situations ranging from physical systems to chemical and biological systems. Some recent research interest in transport problems is related to ratchet physics where unbiased, noise-induced transport occurs away from thermal equilibrium as a result of the action of Brownian motors.1,2,3 Brownian motors, especially “ratchet” models, have been widely investigated partly due to the challenge to describe and control mechanisms of fundamental biological processes at both the cell level (e.g. transport in ion channels) and body level (muscle operations)4. Another motivation is derived from recent advances in technology wherein devices for guiding tiny particles on nano/micro scales are sought; these include particle separation techniques, smoothing of atomic surfaces during electromigration and control of the motion of vortices in superconductors.2,5 Remarkably, experimental realizations of some of these practical systems have been reported.5,6,8

In this framework, two basic types of ratchet models have commonly been employed, namely: (i) the rocking or correlated ratchet, in which the particle is subject to an unbiased external force or additive noise, and (ii) and the flashing ratchet, where in the particle is periodically kicked. The vast majority of these models are overdamped where the noise plays a vital role in the transport process. However, recent studies have shown that the role of noise can be replaced by deterministic chaos induced by the inertial term.9 In such inertial ratchets, the issue of current reversal has been carefully investigated10,11. Besides, Hamiltonian ratchets have recently seen a breakthrough in the ratchet community12.

In the description of the model transport in general, attention has mostly been paid to single particle ratchets. However many such systems coexist in numbers and do work in cooperation. For instance, molecular motors do not operate as a single particle but in groups - the most prominent example being the actin-myosin system in muscles13. For this reason, the relevance of many interacting particles and possible effects of collective behavior, e.g. transport enhancement, current inversion, clustering and spontaneous current among others have been considered in the overdamped case14 and references therein and the underdamped case15,16,17. Besides a few studies15,16,17 on synchronization behavior and possible connections to transport properties in the underdamped cases, a large number of studies investigate transport properties based on the coupled overdamped case.

In this paper, we study the synchronization dynamics of two coupled driven ratchets and show how the coupling can be used to control current reversals. We further show that the process of transition to full synchrony is preceded by several sudden changes in the current and particularly current reversals.

The model we are interested in is made of two rocking ratchets16, symmetrically perturbed by means of an elastic coupling. Its dynamics (in dimensionless form) can be described by

\[ \ddot{x}_i + bx_i + \frac{dV(x_{1}, x_{2})}{dx_i} = a \cos(\omega t) \quad (i = 1, 2), \]

where the normalized time \(t\) is taken in units of the small resonant frequency \(\omega_0^{-1}\) of the system; \(a\) and \(\omega\) represent the amplitude and the frequency of the driving, respectively; and \(b\) the damping parameter. Here \(V(x_{1}, x_{2})\) is the perturbed two-dimensional ratchet potential given
where coupling, $k = 0$, (a) No interaction, $k = 0$, (b) weak coupling, $k = 0.05$, (c) moderate coupling, $k = 0.15$ and (d) strong coupling, $k = 1.0$.

as:

$$V(x_1, x_2) = 2C - \frac{1}{4\pi\delta} [\phi(x_1) + \phi(x_2)] + \frac{k}{2}(x_1 - x_2)^2,$$

where $\phi(x_i) = \sin 2\pi(x_i - x_0) + \frac{1}{4}\sin 4\pi(x_i - x_0)$, $i = 1, 2$ and $k$ is the coupling strength which determines the dynamics and hence the transport properties of Eq. (1).

The parameter $x_0$ is appropriately chosen such that the minima of $V(x_1, x_2)$ are located at the integers; and $\delta = 1.614324$, $C = 0.0173$, $x_0 = 0.82$, $b = 0.1$ and $\omega = 0.67$ are kept constant throughout.

The 2D ratchet potential is shown in Fig. 1 for four different values of the coupling strengths. The minima and maxima of the potential are marked with blue and red colors, respectively. It is to be noted that as $k$ is increased, the heights of the potential $V(x_1, x_2)$ move outward, opening up a valley along the diagonal, in which the two interacting ratchets may most likely share.

Fig. 2 displays the behavior of a single trajectory analysis. Here, on-off intermittency (OOI) can clearly be observed in Fig. 2(a). Fig. 2(c) is the zoom of Fig. 2(b) that shows typical trajectories of the system. During the earlier dynamics ($t \leq 400$), the particles advance in an asynchronous manner and then get captured in the ratchet potential well, where they achieve temporal synchrony - moving quasiperiodically as a single entity for a while, and suddenly escapes from the well and resume chaotic motion. The temporal synchrony state in which they move with the same displacement along the same path, is short-lived and then followed by unlocking. Thus, the repeated sequence of this motion gives rise to OOI synchronization. To quantify the OOI observed here we have computed the probability distribution of the laminar phases $\Lambda(t)$ of $\Delta x(t)$ and the average laminar lengths $l$ of the trajectories $x_{1,2}(t)$ as function of $\epsilon = a - a_c$ at the critical bifurcation point $a_c = 0.0809474$ as shown in Fig. 3. In Fig. 3(a), the collective dynamics for the coupled ratchets shows a $-3/2$ power law scaling, typical of on-off intermittency, while each subsystem dynamics exhibits type I intermittency similar to the single ratchet dynamics with a $-1/2$ power law scaling as shown in Fig. 3(b). The above picture based on a single trajectory analysis is exact for single attractor systems and may turn out to be misleading for irregular one, where periodic and chaotic attractors could co-exist.

Next, we explore the dynamics of the coupled ratchets as $k$ is varied. Considering that single trajectory dynamics will not suffice for a highly chaotic system, all observables have to be averaged out over a large number of trajectories generated from the entire space $[-1, 1] \times [-1, 1]$ which is the unit cell of the resulting periodic structure. Here we make use of two important indicators, namely the error state $\eta$ as a good measure of the synchronization and the current $J$ as the transport quantifier. For a long time dynamics $T$, the error state for a given trajectory is given by:

$$\eta_j = \frac{1}{T} \int_0^T \left[ (x_2^{(j)} - x_1^{(j)})^2 + (\dot{x}_2^{(j)} - \dot{x}_1^{(j)})^2 \right]^{1/2} dt,$$

(3)

with the full error $\eta = N^{-1}\sum_{j=1}^N \eta_j$, evaluated over the
FIG. 3: (Color online) Distribution of (a) laminar phases $\Lambda(t)$ of $\Delta x(t)$ satisfying a $-3/2$ power law scaling typical of on-off intermittency and (b) average laminar lengths with varying parameter $a$ satisfying the scaling law $\langle \rangle \propto \epsilon^{-0.5}$ with $\epsilon = a_c - a$ and $a_c = 0.0809474$ [x1(red) and x2(green)] showing type-I intermittency. Here, $k = 0.065$.

total number $N$ of trajectories. On the other hand, the current in a subsystem $(i = 1, 2)$ is defined as follows:

$$J_i = \frac{1}{M - n_c} \frac{1}{N} \sum_{i=1}^{M} \sum_{j=1}^{N} x_i^{(j)}(t_i) \quad (i = 1, 2). \quad (4)$$

where $N$ is the total number of trajectories and $t_i$ of $x_i(t_i)$ a given observation time. This gives the average velocity, which is then further time-averaged over the number of observations $M - n_c$ being an empirically obtained cut-off accounting for the transient effect, such that a converged current is obtained [11].

With the amplitude $a = 0.0809472$ of the driving, each independent system $(k = 0)$ exhibits intermittent chaotic dynamics. Fig. 4(a) displays $\eta$ as a function of $k$ in the range $k \in (0, 1)$. Above the threshold value $k > k_{th} \approx 0.5$, $\eta$ approaches zero, indicating a fully synchronized state. In Figs. 4(b,c) we observe, in the same range of $k$, the global dependence of the currents $J_- = J_1 - J_2$ and $J_1$, respectively, on the degree of synchronization. Fig. 4(b) shows that prior to the fully synchronized state, $J_-$ fluctuates around zero and when a full synchrony is achieved, $J_-$ is identically zero. Notice also that for $0.259 \leq k \leq 0.295$, $J_-$ is identically zero. The zero current here is not however due to synchrony, but rather to no transport, see Fig. 4(c). This figure also shows several spikes or jumps in $J_1$ prior to the synchronized regime, in which (i) current-reversals show up for lower values of $k$, (These spikes with or without current-reversals certainly have dramatic changes in the corresponding bifurcation diagrams [11]), (ii) a regime of zero-current occurs in the range $(0.259 \leq k \leq 0.295)$ and (iii) full synchronization takes place and is accompanied by current-reversal, during which the current is marginally zero and then negative for all $k > 0.51$. This remains to be further investigated, if this transition with synchrony is a characteristics feature of such systems.

FIG. 4: (Color online) Transition to full synchronization showing (a) the average error dynamics, $\eta$; (b) the current, $J_- = J_1 - J_2$ and (c) the current, $J_1$ in the same range of $k$. The parameters are set as: $a = 0.0809472$, $b = 0.1$, $x_0 = 0.82$, and $\omega = 0.67$.

FIG. 5: (Color online) Current $J$ vs the amplitude $a$ for (a) strong couplings, $0.4 < k < 1$ and (b) weak coupling regime $0 < k < 0.07$ for fixed $k$. 

\[\text{FIG. 3: (Color online) Distribution of (a) laminar phases } \Lambda(t) \text{ of } \Delta x(t) \text{ satisfying a } -3/2 \text{ power law scaling typical of on-off intermittency and (b) average laminar lengths with varying parameter } a \text{ satisfying the scaling law } \langle \rangle \propto \epsilon^{-0.5} \text{ with } \epsilon = a_c - a \text{ and } a_c = 0.0809474 \text{ [x1(red) and x2(green)] showing type-I intermittency. Here, } k = 0.065.\]

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Finally, special attention is markedly paid to controlling current reversals as these may happen to be undesirable as far as transport is concerned. The case $k = 0$ corresponds to no interactions for which current-reversals have been observed in a single ratchet model \[11, 12\]. The large coupling regime, where full synchronization is reached, corresponds identically to the current reversal observed in one single ratchet model, (see Fig. 5(a)). However as $k$ takes on smaller values, dramatic changes occur on current reversals leading to the rectification of the particles motions - for instance, current reversal observed in a large window of $k$ (Fig. 5(a)) is completely eliminated in Fig. 5(b). Likewise current reversals found for smaller values of $k$ are destroyed as $k$ increases. Interestingly, there are few values of the coupling strength for which the system becomes totally reversals-free. This result clearly demonstrates the importance of the coupling strength for the full control of transport over currents.

To sum up, we have shown that the dynamics of a single particle ratchet can be significantly modified when coupled elastically to a second one. We have thus made use of the coupling strength to systematically rectify the particle motion. In particular, current reversals observed in a single ratchet can completely be annealed by appropriately choosing the coupling strength. Interestingly a current-reversal-free regime has been also detected. The particle-particle interaction thus plays a significant role in the transport properties and acts as a control input that can be used to track current reversals. It thus implies, that the nature of particle-particle interactions could be used to control the transport properties of non-equilibrium dynamical systems. Moreover we found, that for larger values of the coupling strengths, the dynamics of the single particle ratchet predominates, leading to the fully synchronized state.

Acknowledgment:

The work of UEV and DVS is supported by the Alexander von Humboldt Foundation. JK has been supported by NoE BIOSM (EU Contract No. LSHB-CT-2004-005137). UEV is also supported by the Royal Society of London under the Newton International Fellowship scheme. We thank Peter McClintock for carefully reading through the manuscript.

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