Phenomenological model explaining Hubble Tension origin

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Abstract

One of the problem revealed recently in cosmology is a so-called Hubble tension (HT), which is the difference between values of the present Hubble constant, measured by observation of the universe at redshift $z \lesssim 1$, and by observations of a distant universe with CMB fluctuations originated at $z \sim 1100$.

In this paper we suggest, that this discrepancy may be explained by deviation of the cosmological expansion from a simple Friedman model of a flat dusty universe during the period after recombination at $z \lesssim 1100$, due to action of additional small component of a dark energy of a different origin.

We suppose, that a dark matter (DM) has a common origin with a small component of a dark energy (DEV).

DE presently may have two components, one of which is the Einstein constant $\Lambda$, and another, smaller component DEV ($\Lambda_V$) comes from the remnants of a scalar fields responsible for inflation. Due to common origin and interconnections the ratio between DEV and DM densities is supposed to remain constant during, at least, the time after recombination, when we may have $\rho_{DEV} = \alpha \rho_{DM}$. 

This part of the dark energy is not connected with the cosmological constant \( \Lambda \), but is defined by existence of scalar fields with a variable density. Taking into account the influence of DEV on the universe expansion we find the value of \( \alpha \) which should remove the HT problem.

*Keywords* dark energy, dark matter, Hubble constant

## 1 Introduction

Recently a challenge in cosmology was formulated, due to different values of the present quantity of the Hubble constant. There is a significant discrepancy (tension) between the *Planck* measurement from cosmic microwave background (CMB) anisotropy, where the best-fit model gives [1],[13],

\[
H_0^{P18} = 67.36 \pm 0.54 \ \text{km s}^{-1} \text{Mpc}^{-1},
\]

and measurements using type Ia supernovae (SNIa) calibrated with Cepheid distances [15],[17],[16],

\[
H_0^{R19} = 74.03 \pm 1.42 \ \text{km s}^{-1} \text{Mpc}^{-1}.
\]

Measurements using time delays from lensed quasars [22] gave the value \( H_0 = 73.3^{+1.7}_{-1.8} \ \text{km s}^{-1} \text{Mpc}^{-1} \), while in Ref. [23] it was found \( H_0 = 72.4\pm1.9 \ \text{km s}^{-1} \text{Mpc}^{-1} \) using the tip of the red giant branch applied to SNIa, which is independent of the Cepheid distance scale. Analysis of a compilation of these and other recent high- and low-redshift measurements shows [21] that the discrepancy between *Planck* [13], and any three independent late-Universe measurements is between 4\( \sigma \) and 6\( \sigma \). Different sophisticated explanations for appearance of HT have been proposed [6, 8, 12], and new experiments have been proposed for checking the reliability of this tension [3].
Dark matter (DM) and dark energy (DE) represent about 95% of the universe constituents [13, 11, 18], but their origin is still not clear. The present value of DE density may be represented by Einstein cosmological constant $\Lambda$ [5], but also may be a result of the action of the Higgs-type scalar fields, which are supposed to be the reason of the inflation in the early universe [4], see also [19, 9, 7]. The value of the induced $\Lambda$, suggested for the inflation, is many orders of magnitude larger, than its present value, and no attempts have been done, to find a connection between them. The origin of DM is even more vague. A number of variants of its possible origin is very high [2], but no of these possibilities were confirmed experimentally or observationally, and many of them were disproved.

One exciting observational feature of the modern cosmology is the fact that we live just in the period, where densities of DE and DM are comparable to each other. The equality by the order of magnitude between DE and DM densities may take place not by chance, but because of their common origin and evolution. If we use an analogy with electromagnetic field, we could suppose, that massive DM particles are born by the massless scalar field, and their energy densities remain comparable in a wide region of parameters.

In this paper, in order to explain the origin of the Hubble Tension, we introduce a small variable part of the cosmological ”constant” $\Lambda V$, proportional to the matter density $\rho_{DM} = \alpha \rho_{DEV}$. This part of $\Lambda$ influence the cosmological expansion at large redshifts, where the influence of the real Einstein constant $\Lambda$ is negligible. The value of $\Lambda V$ is represented by a small component of DE, which we call as DEV. We suppose here, without knowledge of physical properties of DM particles, that there is a wide spectrum of DM particle, which are produced by DEV until the present time. The existence of particles with a very low rest mass (axions [20]) is considered often as a candidate for DM.

We consider a model of the universe after recombination, at $z < \sim 1100$, with a fixed ratio $\alpha$ of energy densities between DEV, connected with a scalar field,
and DM, during expansion at stages, when the action of the real cosmological constant $\Lambda$ is negligible. It could be supported during the long period of universe evolution, until now. We suggest, that a birth of the ordinary matter in the process of inflation takes place also, but DM is born more effectively. If the mass spectrum of DM particles prolongs to very small masses, then we may expect the present DEV-DM connection. It will be stopped, if there is a cutoff of DM masses at the lower side. In the inflation model of the universe, only a scalar field was born at the very beginning, and a matter was created in the process of expansion from the dynamic part of the scalar field density.

Here we show, that in presence of DEV the Hubble value $H$ is decreasing with time slower. This create a larger present value of $H_0$, removing the Hubble tension at $\alpha \sim 114$.

2 Universe with common origin of DM and DE

The scalar field with the potential $V(\phi)$, $\phi$ is the intensity of the scalar field, is considered as the main reason of the inflation [4], but see [19]. The equation for the scalar field in the expanding universe is written as [10]

$$\ddot{\phi} + 3 \frac{\dot{a}}{a} \dot{\phi} = -\frac{dV}{d\phi}. \quad (3)$$

Here $a$ is a scale factor in the flat expanding universe [24]. The density $\rho_V$, and pressure $P_V$ of the scalar field are defined as [10]

$$\rho_V = \frac{\dot{\phi}^2}{2} + V, \quad P_V = \frac{\dot{\phi}^2}{2} - V. \quad (4)$$

Consider the universe with the initial scalar field, at initial intensity $\phi_{in}$ and initial potential $V_{in}$, and at zero derivative $\dot{\phi}_{in} = 0$. The derivative of the scalar field is growing on the initial stage of inflation.

\[^1\text{In most equations below it is taken } c = 1.\]
Let us suggest, that after reaching the relation
\[ \dot{\phi}^2 = 2\alpha V, \] (5)
it is preserved during farther expansion. The kinetic part of the scalar field is transforming into matter, presumably, dark matter, and the constant \( \alpha \) determines the ratio of the the dark energy density, represented by \( V \), to the matter density, represented by the kinetic term. As follows from farther consideration, the main part of DE is represented presently by the Einstein constant \( \Lambda \), and formally we have this relation equal to \( \alpha \sim 1/3 \). At earlier times the input of constant \( \Lambda \) is very small, and we neglected it, considering only \( \Lambda V \) with quite different \( \alpha \). We obtain below the solution for arbitrary \( \alpha \).

Let us consider an expanding flat universe, described by the Friedmann equation \[ \frac{\dot{a}^2}{a^2} = \frac{8\pi G}{3} \rho + \frac{\Lambda}{3}. \] (6)
Suggest \( \Lambda = 0 \), but the part of the scalar field represented by \( V \) have properties similar to \( \Lambda \). Introduce
\[ \rho_\phi = V, \quad P_\phi = -V, \quad \rho_m = \frac{\dot{\phi}^2}{2}, \quad P_m = \beta \frac{\dot{\phi}^2}{2}, \quad \text{with} \quad \rho_m = \alpha \rho_\phi. \] (7)
We suggest, that only part \( \beta \) of the kinetic term make the input in the pressure of the matter, so it follows from \[ \text{(5),(7)} \]
\[ \rho = \rho_\phi + \rho_m = (1 + \alpha)V, \quad P = P_\phi + P_m = -(1 - \alpha \beta)V. \] (8)
The adiabatic condition
\[ \frac{d\rho}{\rho + P} = -\frac{dV}{V} = -3\frac{da}{a}. \quad V \text{ is the volume,} \] (9)
may be written as

\[ \dot{\rho} = -3 \frac{\dot{a}}{a} (\rho_\phi + \rho_m + P_\phi + P_m) = -3 \frac{\dot{a}}{a} (\rho_m + P_m) = -3 \alpha \frac{1 + \beta}{1 + \alpha} \frac{\dot{a}}{a} \rho. \] (10)

The expression for the total density \( \rho \), scaling factor \( a \), and Hubble “constant” \( H \) follows from (6)-(10) as

\[ \frac{a}{a_*} = (6 \pi G \rho_* t_*^2)^{\frac{1 + \alpha}{3(1 + \beta)}} \left( \frac{\alpha(1 + \beta)}{1 + \alpha} \right)^{\frac{2(1 + \alpha)}{3(1 + \beta)}} = \left( \frac{\rho_*}{\rho} \right)^{\frac{1 + \alpha}{3(1 + \beta)}} = \left( \frac{t}{t_*} \right)^{\frac{2(1 + \alpha)}{3(1 + \beta)}}, \]
\[ \rho = \left( \frac{1 + \alpha}{\alpha(1 + \beta)} \right)^2 \frac{1}{6 \pi G t_*^2}, \quad H = \frac{\dot{a}}{a} = \frac{2(1 + \alpha)}{3 \alpha(1 + \beta)t}. \] (11)

Here \( \rho_* = \rho(t_*), \quad a_* = a(t_*), \quad t_* \) is an arbitrary time moment. Write the expressions for particular cases. For \( \beta = 1/3 \) (radiation dominated universe) it follows from (11)

\[ \frac{a}{a_*} = (6 \pi G \rho_* t_*^2)^{\frac{1 + \alpha}{3\alpha}} \left[ \frac{4\alpha}{3(1 + \alpha)} \right]^{\frac{1 + \alpha}{2\alpha}} = \left( \frac{\rho_*}{\rho} \right)^{\frac{1 + \alpha}{2\alpha}} = \left( \frac{t}{t_*} \right)^{\frac{1 + \alpha}{2\alpha}}, \]
\[ \rho = \left( \frac{3(1 + \alpha)}{4\alpha} \right)^2 \frac{1}{6 \pi G t_*^2}, \quad H = \frac{\dot{a}}{a} = \frac{1 + \alpha}{2\alpha t}. \] (12)

For the value of \( \beta = 0 \) (dusty universe, \( z < 1100 \)) we have

\[ \frac{a}{a_*} = (6 \pi G \rho_* t_*^2)^{\frac{1 + \alpha}{3\alpha}} \left[ \frac{\alpha}{1 + \alpha} \right]^{\frac{2(1 + \alpha)}{3\alpha}} = \left( \frac{\rho_*}{\rho} \right)^{\frac{1 + \alpha}{3\alpha}} = \left( \frac{t}{t_*} \right)^{\frac{2(1 + \alpha)}{3\alpha}}, \]
\[ \rho = \left( \frac{4\alpha}{1 + \alpha} \right)^2 \frac{1}{6 \pi G t_*^2}, \quad H = \frac{\dot{a}}{a} = \frac{2(1 + \alpha)}{3\alpha t}. \] (13)

3 Potential of the scalar field

Let us consider in a simplified way a possible connection between DEV with a scalar field responsible for the inflation. For this purpose we look for the solution
with a power-law dependencies \( V = V_0 \phi^k, \phi = \phi_0 t^n \). The values in (3) are written as

\[
\begin{align*}
\phi &= \phi_0 t^n, \\
\dot{\phi} &= n\phi_0 t^{n-1}, \\
\ddot{\phi} &= n(n-1)\phi_0 t^{n-2}, \\
V &= V_0 \phi^k, \\
\frac{dV}{d\phi} &= kV_0 \phi^{k-1}.
\end{align*}
\]

(14)

Using (14) and (11) for \((\dot{a}/a)\), we obtain from (3)

\[
\left( n^2 + \frac{2 + (1 - \beta)\alpha}{\alpha(1 + \beta)} n \right) \phi_0 t^{n-2} = -kV_0 \phi_0^{k-1} t^{(k-1)}.
\]

(15)

The solution of this equation exists at a unique relation between \( k \) and \( n \), and has a form

\[
k = 2\frac{n - 1}{n}, \quad V_0 = -\frac{n^2}{2(n-1)} \left[ n + \frac{2 + (1 - \beta)\alpha}{\alpha(1 + \beta)} \right] \phi_0^{2/n}.
\]

(16)

The intensity of the scalar field is decreasing in the expanding universe, so we should consider only negative \( n < 0 \). The self-consistent solution for the universe with relation (5) exists in the power-law form only at positive \( \left( n^2 + \frac{2+(1-\beta)\alpha}{\alpha(1+\beta)} n \right) \).

As an example, let us consider the solution with \( \beta = 0 \), what is supposed for present state of the cold dark matter, and \( n = -1 \). In this solution we obtain the following dependence of parameters on time

\[
\beta = 0, \quad n = -1, \quad k = 4, \quad V_0 = \frac{1}{2\alpha \phi_0^4}, \quad \phi = \frac{\phi_0}{t}, \quad V = V_0 \phi^4 = \frac{\phi_0^2}{2\alpha t^4}.
\]

(17)

The effective value of \( \Lambda_V \) in presence of the scalar field is determined as

\[
\Lambda_V = \frac{8\pi G \rho_\phi}{c^2}.
\]

(18)

If the present value of \( \Lambda_V \) and the corresponding value in the inflation epoch have the same origin, then from Eq. (17) we obtain the connection of these two values in the form
\[ \Lambda_{V,\text{inf}} = \Lambda_{V,\text{pres}} \left( \frac{t_{\text{pres}}}{t_{\text{inf}}} \right)^4, \]  
(19)

where \( t_{\text{pres}} \sim 10^{17} \text{ s} \) is the present age of the universe, and \( t_{\text{inf}} \sim 10^{-40} \text{ s} \) is the time of the inflation epoch. From Eq. (19) we obtain decrease of \( V \), and of the value of \( \Lambda V \), on 230 orders of the magnitude to the present time. Note, that these estimations have an illustrative meaning, and are related to comparison of the effective \( \Lambda V \) at the inflation epoch with the present effective \( \Lambda V \), which connected with the scalar field. It does not exclude the existence of the Einstein real constant \( \Lambda \), which influence on the universe expansion started only ”recently”.

### 4 Removing Hubble tension

As follows from above consideration, a linear connection between densities \( \rho_\phi \) and \( \rho_m \) leads to a change of rate of the universe expansion. In the inverse dependence of the Hubble ”constant” on time \( H = \lambda/t \), the value of \( \lambda \) occupies the interval \( \left( \frac{2}{3(1+\beta)} < \lambda < \frac{8}{3(1+\beta)} \right) \) for \( (\infty > \alpha > 1/3) \), respectively. Decrease of the matter (DM) input into the density, in comparison with the DE, leads to increasing of the speed of the universe expansion, so that, according to (11), we have formally \( \lambda \to \infty \) at \( \alpha \to 0 \), what means that the expansion is becoming of the exponential de Sitter type at \( \alpha = 0 \).

The main idea of removing the tension is the following. The CMB measurements give the value of the Hubble constant \( H_r \), at the redshift \( z \sim 1100 \), close to the moment of recombination. This value is used for calculation of the present value of \( H_0 \).

For analysis of the Hubble tension it is more convenient to use logarithmic variables, so that from (11), (2) we have

\[
\log H_0^{P18} = \log 67.36 = 1.83; \quad \log H_0^{R19} = \log 74.03 = 1.87;
\]

\[
\log \frac{H_0^{R19}}{H_0^{R18}} = \Delta \log H_0 = 0.04.
\]

(20)
The Planck value $H_{P}^{r}$ was measured at the moment of recombination $z_{r} \approx 1100$, and extrapolated to the present time using dusty flat Friedmann model as \cite{24}

$$z + 1 = \frac{\omega}{\omega_{0}} = \frac{a_{0}}{a} = \left(\frac{t_{0}}{t}\right)^{\frac{2}{3}}, \quad H = \frac{\dot{a}}{a} = \frac{2}{3t}, \quad \frac{H_{0}}{H} = \frac{t}{t_{0}} = (z + 1)^{-\frac{3}{2}}. \quad (21)$$

In the case of equipartition universe the extrapolation should be done using Eq. (11).

Numerical modeling of large scale structure formation give the preference to the cold dark matter model, corresponding to $P_{m} \approx 0, \quad \beta = 0$. We may suppose, that the dynamical part of scalar field give birth to dark energy matter in the form of a mixture of massive DM particles with massless DM quanta. During the expansion the role of DM quanta is decreasing rapidly, like the input of photons in the transparent expanding universe after recombination, so the pressure of DM may be considered as zero, therefore we may use Eq. (13) for the extrapolation. In this procedure the Hubble tension is connected with incorrect extrapolation by Eq. (21). From the value of Hubble tension in Eq. (20) we may estimate $\alpha$ from the condition, that both measurements are correct, but the reported value $H_{0}^{P18}$ is coming from the incorrect extrapolation, and the actual present epoch value of the Hubble constant is determined by $H_{0}^{R19}$.

$$\log H_{r}^{P} = \log H_{0}^{P18} + \frac{3}{2} \log z_{r} \approx 1.83 + \frac{3}{2} \log 1101;$$

$$\log H_{r}^{P} = \log H_{0}^{R19} + \frac{3\alpha}{2(1 + \alpha)} \log z_{r} \approx 1.87 + \frac{3\alpha}{2(1 + \alpha)} \log 1101; \quad (22)$$

$$\frac{1}{1 + \alpha} = \frac{2 \log H_{0}^{R19} - \log H_{0}^{P18}}{3 \log 1101} \approx 0.0088; \quad \alpha \approx 114.$$
5 Discussion

In order to explain the origin of the Hubble Tension we have introduced a small variable part of the cosmological constant $\Lambda_V$, proportional to the matter density. It influences the cosmological expansion at large redshifts, where the influence of the real Einstein constant $\Lambda$ is still negligible. Presently the situation is opposite, $\Lambda \gg \Lambda_V$, because decreasing of a matter density during cosmological expansion determines the transition from the quasi-Friedmann to quasi-de Sitter stage. The estimation of the density $\rho_{\Lambda V}$ at present epoch corresponds to $\Omega_{\Lambda V} \approx 0.0026$, what is much less than the input of $\Omega_\Lambda \approx 0.7$, $\Omega_{DM} \approx 0.26$, $\Omega_b \approx 0.04$, but is larger than the input of the $\Omega_{CMB} \approx 4 \cdot 10^{-5}$.

We have solved the Friedmann equation in presence of the relation (5), and have found the value of $\alpha$, at which the HT disappeared.

We have used for estimations a constant ratio of $\rho_{\Lambda V}/\rho_m = 1/\alpha$ for the universe expansion after recombination, at $z < 1100$, but deviations from this law should not change the conclusion, that a small average contribution of the variable $\Omega_{\Lambda V} \sim 0.0026$ may explain the difference of Hubble constant measure in local and high $z$ distances. In present model the DM should be represented by a wide mass spectrum particles, and not by a unique mass CDM particles, usually considered presently.

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