Majorana states and longitudinal NMR absorption in $^3$He-B film

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The topological superfluid $^3$He-B supports massless Dirac spectrum of surface bound states which can be described in terms of the self-conjugated Majorana field operators. We discuss here the possible signature of surface bound states in nuclear magnetic resonance absorption spectrum in a $^3$He-B film. It is shown that transitions between different branches of the surface states spectrum lead to the non-zero absorption signal in longitudinal NMR scheme when the frequency is larger than the Larmour one.

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I. INTRODUCTION

Recently much attention has been devoted to the investigation of bound fermion states on surfaces and interfaces of topological superfluid $^3$He-B. The presence of surface states in $^3$He-B can be observed through anomalous transverse sound attenuation, surface specific heat measurements. Gapless fermion states are supported by the non-zero value of the topological invariant in $^3$He-B and have two-dimensional relativistic massless Dirac spectrum. Such massless fermions can be described in terms of the Majorana self-conjugated field operators which have been intensively studied recently in a number of condensed matter systems.

It has been suggested that NMR technique can be employed to study the spectrum of surface states. The NMR measurements were implemented recently on superfluid $^3$He films demonstrating various frequency shifts associated with the order parameter dynamics both at high-temperature $A$ phase as well as with two nonequivalent low temperature $B$ phase states characterized by the different values of the Ising variable.

In this paper we focus on the contribution of the surface bound states to the ac magnetic susceptibility of $^3$He-B film. We demonstrate that the transitions between different branches of surface states spectrum result in dissipation manifested in non-zero imaginary part of the component of magnetic susceptibility, where $z$ is the axis normal to the film surface. This effect should provide the contribution to the longitudinal NMR absorption signal absent both in the normal state and bulk $^3$He-B phase. Under the action of the $z$ component of magnetic field which destroys the self-conjugating Majorana states the magnetic absorption is suppressed at the frequencies smaller than the Larmour one.

II. SPECTRUM OF SURFACE STATES IN $^3$HE-B FILM

At first we introduce the basic formalism for treating the spectrum of fermionic quasiparticles. We consider $^3$He-B film confined in a slab at $z > 0$ and homogeneous in $x$ and $y$ directions. The $^3$He-B surface mode is derived from the quasiclassical BdG Hamiltonian

$$\hat{H} = -i\hbar V z \hat{\gamma}_3 \partial_z + \hat{\gamma}_1 \hat{\Delta} - \frac{\gamma}{2} \mathbf{H} \cdot \mathbf{\hat{\sigma}},$$ (1)

where $\hat{\Delta} = A_{ij} q_i \hat{\sigma}_j$, and $\mathbf{q} = \mathbf{k}/k_F$ where $\hat{\gamma}_i$ are Pauli matrices of Bogolyubov-Nambu spin, $\hat{\sigma}_a$ are Pauli matrices of $^3$He nuclear spin, $\gamma$ is the gyromagnetic ratio of the $^3$He atom.

The order parameter in $^3$He-B is $3 \times 3$ matrix $A_{ij}$ where the Greek and Latin indices correspond to the spin and orbital variables. The geometrical confinement induced by the walls destroys the isotropy of the B phase order parameter. As a result For the distorted 3He-B it has the form

$$A_{ij} = \begin{pmatrix} \Delta_{ij} & 0 & 0 \\ 0 & \Delta_{||} & 0 \\ 0 & 0 & \Delta_{\perp} \end{pmatrix}$$

where $\Delta_{ij}$ is the gap for quasiparticles propagating in direction along the normal to the wall and $\Delta_{||}$ is the gap for quasiparticles propagating in directions parallel to the wall.

Upon the specular reflection from the surface the quasiparticle momentum projection $q_z$ and therefore the part of the order parameter changes the sign which leads to the formation of surface bound states. To find the spectrum of this states we employ the usual procedure, considering the two parts of hamiltonian $\hat{H} = H_0 + H_1$ so that

$$\hat{H}_0 = -i\hbar V z \hat{\gamma}_3 \partial_z + \hat{\gamma}_1 \hat{\Delta}$$ (2)

$$\hat{H}_1 = \Delta_{||} \hat{\gamma}_1 (\hat{\sigma}_z q_z + \hat{\sigma}_y q_y) - \frac{\gamma}{2} \mathbf{H} \cdot \mathbf{\hat{\sigma}},$$ (3)

where $\hat{\Delta}(z) = \Delta_{\perp}(z) \hat{\sigma}_z q_z$ and $\mathbf{q} = \mathbf{p}/p_F$. The hamiltonian has zero energy eigenvalues corresponding to the degenerate surface bound states. To get into account correction from the perturbation terms $\hat{H}_1$ we find the eigenfunctions as a superposition

$$\psi = \sum_{j=1}^2 X_j \varphi_j(z),$$ (4)
where \( X_k \) are the arbitrary coefficients and the generic terms \( \varphi_j(z) \) are the eigenfunctions of hamiltonian \( \hat{H}_0 \) corresponding to the zero energy \( \varepsilon = 0 \)

\[
\varphi_j(z) = A^{-1/2} \alpha_j \beta_j \exp[-K(z)], \quad (5)
\]

where \( A = \langle \varphi_1 | \varphi_1 \rangle = \langle \varphi_2 | \varphi_2 \rangle \) is the normalizing coefficient and

\[
K(z) = \frac{1}{hV_F} \int_0^z \Delta(z) ds.
\]

The Pauli and Nambu spinors \( \beta_j \) and \( \alpha_j \) satisfy the relations \( \sigma_z \beta_{1,2} = \mp \beta_{1,2} \) and \( \sigma_2 \alpha_{1,2} = \pm \alpha_{1,2} \).

Following the standard method we substitute the solution in the form (5) into the equation \( (\hat{H}_0 + H_1) \psi = \varepsilon \psi \) multiply by \( \psi^*_j(z) \) from the left and integrate over \( z \). Then for the spinor \( X = (X_1, X_2)^T \) we obtain the two-dimensional Dirac equation

\[
[C \hat{\sigma} p + \hat{\sigma}_z M] X = \varepsilon X, \quad (6)
\]

with the ‘light velocity’ given by

\[
C = \frac{1}{A_p F} \int_0^\infty dz \Delta(z) \exp[-2K(z)] \sim \Delta / p_F
\]

and ‘mass’ determined by the Larmor frequency \( M = \hbar \omega_H / 2 \).

For the massless particles we can choose the eigenfunctions of Eq.(6) satisfying the relation

\[
X_\varepsilon = i \hat{\sigma}_y X_{-\varepsilon}. \quad (7)
\]

As a result the quasiparticle field operators are self-conjugated analogously to the Majorana fermions in relativistic quantum field theories. In the external magnetic field the particles described by Eq.(6) become massive so that the property (7) does not hold and therefore the quasiparticles are no longer self-conjugated Majorana fermions.

The Dirac equation (6) determines the equation for the energy levels in the following form

\[
\varepsilon_{1,2} = \pm \sqrt{(C p)^2 + (\hbar \omega_H)^2 / 4}. \quad (8)
\]

The energy spectrum of surface bound states is sensitive only to the \( z \) projection of the magnetic field. It can result in a large anisotropy of magnetic susceptibility if the magnetic field is much smaller then the effective dipole field. However the larger magnetic field will reorient the spin axes eliminating the magnetic anisotropy. Note that deriving the spectrum (5) we have neglected the finite thickness of the slab. The size effect due to the overlap of quantum states localized at the opposite surfaces of the slab leads to the splitting of Dirac cone even in zero magnetic field. For sufficiently strong magnetic field this modification can be neglected.

III. MAGNETIC SUSCEPTIBILITY AND NMR ABSORPTION SPECTRUM

Now we consider the contribution of surface bound states to the imaginary part of ac magnetic susceptibility component \( \chi_{zz} \) which determines the power absorption under the experimental conditions of the longitudinal scheme of magnetic resonance when both the total magnetic field is directed along the \( z \) axis.

To find the magnetic susceptibility let us use a conventional Kubo formula:

\[
\chi_{ij} = \frac{\gamma^2}{4} T \sum_{\omega_n} \int \frac{d^2 p}{(2\pi \hbar)^2} Tr \{ \hat{G}(p_+ \sigma_j \hat{G}(p_-) \}, \quad (9)
\]

where

\[
\hat{G}(\omega_M, p) = \sum_{k=1,2} \frac{|\psi_k\rangle \langle \psi_k|}{i \omega_M - \varepsilon_k(p)}
\]

is a temperature Green function and \( p_{\pm} = (\omega_n \pm \varepsilon_i / 2, p) \). Here \( \omega_n = \pi(2n + 1)T \) is a fermionic and \( \varepsilon_i = 2n\hbar T \) is a photonic Matsubara frequencies. The normalized wave functions \( \psi_{1,2} \) are given by the superpositions (5) and correspond to the energy branches \( \varepsilon_{1,2} \).

We use the value of the matrix element \( \langle \psi_1 | \sigma_j | \psi_2 \rangle = C_p / \varepsilon_1 \) and the formula for the sum over fermionic frequencies

\[
\sum_{\omega_n} \frac{T}{\pi} \frac{\hbar \omega}{\hbar \omega_H + \varepsilon_2} = f_0(\varepsilon_1) - f_0(\varepsilon_2),
\]

where \( \varepsilon_{1,2} = \varepsilon_{1,2}(p) \). We use now the dispersion relation (5) and obtain that when the frequency is larger than the Larmor frequency \( \omega > \omega_H \) the susceptibility given by Eq.(10) has a non-zero imaginary part

\[
Im \chi_{zz} = \gamma^2 C \frac{2}{8\pi \hbar^2} \int d^3 p \int d^3 p \frac{\varepsilon_2 - \varepsilon_1}{\varepsilon_1^2 ([\hbar \omega^2 - (\varepsilon_2 - \varepsilon_1)^2]}
\]

(10)

Note that the non-zero dissipation \( Im \chi_{zz} \neq 0 \) occurs only when the frequency is larger than the threshold value \( \omega_H \) which is similar to the threshold behavior of absorption rate in semiconductors where the absorption edge frequency is determined by the band gap energy. However in the vicinity of the threshold the frequency dependence of absorption rate Eq.(11) is completely different from that of the electromagnetic wave absorption in the physics of semiconductors.

The estimate of imaginary magnetic susceptibility (11) of a unit film area yields

\[
Im \chi_{zz} \sim \chi_n \frac{\hbar \omega}{\Delta_H} f_0 \left( \frac{\hbar \omega}{4T} \right)
\]

where \( \chi_n \sim \gamma^2 k_F^3 / E_F \) is a normal state susceptibility and \( \xi = \hbar V_F / \Delta_H \) is a coherence length.
IV. EFFECT OF SURFACE ROUGHNESS

In general the quasiparticle energy levels are broadened due to the statistical fluctuation of the film surface which affect the boundary conditions for the wave functions. Different models of surface roughness related to the surface effects in $^3$He were developed including the diffusive surface layer,18,19 randomly rippled wall (RRW), randomly oriented mirrors (ROM) model,18,19 and random scattering matrix model.18 Under the conditions of diffusive scattering the acoustic impedance data demonstrate the presence of surface bound states in $^3$He-B film18,19 although there is no evidence of the relativistic massless Dirac spectrum. On the other hand the surface conditions can be varied in the experiments by coating the surface of several layers of random and random facets. For increased specular factor the new features on the temperature dependence of acoustic impedance were observed indicating the formation of 2D Dirac energy spectrum.20

Here we employ the ROM model assuming that the surface consists of small randomly oriented specularly scattering facets. This model is applicable to describe the fluctuations with the scale much larger than $k_F^{-1}$ of the $^4$He coated surface. Within the framework of ROM model the important characteristic is the angle $\alpha_s$ which constitutes the local normal vector to the wall $\mathbf{n}_s$ with the $z$ axis. Let us use the new coordinate system rotated by the angle $\alpha_s$ with respect to the axis defined by $\nu = z \times \mathbf{n}_s$. Then we obtain in the new coordinate system the expression for the order parameter matrix $\tilde{A}_{ij} = A_{ik}R_{kj}$, where $R$ is the corresponding rotation matrix. Let us assume without loss of generality that the rotation axis coincide with the $y$ axis. Then in the Eq. (12) for the hamiltonian $\tilde{H}_0$ we obtain $\tilde{F} = (\Delta_\perp \alpha_s \hat{\sigma}_z - \Delta_\parallel \sin \alpha_s \hat{\sigma}_x)q_\alpha$ and the perturbation term is given by

$$\tilde{H}_1 = \tilde{\gamma}_1 q_\alpha (\Delta_\parallel \alpha_s \hat{\sigma}_z + \Delta_\perp \hat{\sigma}_z \sin \alpha_s) - \frac{1}{2} \tilde{H} \cdot \mathbf{\sigma}. \quad (12)$$

To proceed further with analytical calculations we assume that the order parameter does not depend on the space coordinates so that $\Delta_\perp, \Delta_\parallel = \text{const}$. Then we obtain easily the zero energy eigenvectors of the hamiltonian $\tilde{H}_0$ in the form of Eq. (3) with

$$\beta_1 = \left( \begin{array}{c} \Delta_\parallel \sin \alpha_s \\ \Delta_\perp \cos \alpha_s + \tilde{\Delta} \end{array} \right)^T \quad (13)$$

$$\beta_2 = \left( \begin{array}{c} 1 \\ - \Delta_\parallel \sin \alpha_s \\ \Delta_\perp \cos \alpha_s + \tilde{\Delta} \end{array} \right)^T, \quad (14)$$

where $\tilde{\Delta} = \sqrt{\Delta_\parallel^2 + \Delta_\perp^2}$. Correspondingly in the Eq. (1) for the zero order wave functions we obtain

$$K(z) = \frac{1}{\hbar V_F} \int_0^z \tilde{\Delta} ds. \quad (15)$$

The quasiparticle spectrum obtained along the perturbation theory scheme described above yields the spectrum in the form $\tilde{\mathcal{S}}$ but with modified parameters. We will study the modification of the absorption threshold which is determined by the ‘mass’ term and does not depend on the ‘light velocity’. We therefore will neglect the modification of ‘light velocity’ and focus on the ‘mass’ term which is given by

$$\tilde{\omega}_\alpha = \omega_\alpha \frac{(\Delta_\perp \cos \alpha_s + \tilde{\Delta})^2 - (\Delta_\parallel \sin \alpha_s)^2}{(\Delta_\perp \cos \alpha_s + \Delta_\parallel \sin \alpha_s)^2 + (\Delta_\perp \sin \alpha_s)^2}. \quad (16)$$

To proceed further and calculate statistical average over the surface roughness we assume that $|\alpha_s| \ll 1$ and obtain to the leading order

$$\tilde{\omega}_\alpha = \omega_\alpha \left( 1 - \frac{\alpha_s^2 \Delta_\perp^2}{2 \Delta_\parallel^2} \right). \quad (17)$$

The above Eq. yields the fluctuating correction to the absorption edge in Eq. (11). It leads to the smoothing out the sharp absorption edge at the Larmour frequency. To estimate this effect we assume the Gaussian distribution of angle $\alpha_s$ with the zero average value $<\alpha_s> = 0$ and the dispersion $\langle \alpha_s^2 \rangle = \sigma_\alpha^2$. After that the average value of the susceptibility in the vicinity of Larmour frequency is given by

$$\langle \chi_{zz} \rangle = -\frac{\gamma}{4\hbar C} \left[ \frac{\Delta_\parallel}{\Delta_\perp} \right]^2 \hbar \omega_\alpha \int_0^{\omega/\omega_H} S(\omega_\alpha, \sigma_\alpha)$$

$$= -\frac{\gamma}{4\hbar C} \left[ \frac{\Delta_\parallel}{\Delta_\perp} \right]^2 \hbar \omega_\alpha \int_0^{\infty} \left( \omega - \omega_\alpha \right) \frac{\exp[-(\omega - \omega_\alpha)^2/\sigma_\alpha^2]}{\sigma_\alpha \sqrt{\pi}} d\omega$$

$$= -\frac{\gamma}{4\hbar C} \left[ \frac{\Delta_\parallel}{\Delta_\perp} \right]^2 \hbar \omega_\alpha \sigma_\alpha \sqrt{\frac{\pi}{\omega_H}}.$$

The plots of the function (17) for the different values of $\sigma_\alpha$ are shown in Fig. (1) demonstrating smoothing of absorption edge with increasing dispersion of surface ripples. Although in general for $\sigma_\alpha > 0$ the absorption signal is non-zero at the whole frequency domain however it is exponentially decaying for $\omega \ll \omega_H$. The size of the crossover domain in Fig. (1) is determined by the dispersion $\delta \omega = \omega_H \sigma_\alpha$. Therefore in general we can conclude that the absorption edge should well observed provided these fluctuations are small so that $\sigma_\alpha \ll 0$.

V. CONCLUSION

To conclude we have calculated the contribution of fermionic surface bound states to the ac magnetic susceptibility of $^3$He-B film. We have shown that in the longitudinal NMR scheme the non-zero absorption signal appears provided the frequency is larger than the threshold one determined by the Larmour frequency $\omega > \omega_H$. Such absorption is absent in the normal state of $^3$He and can not occur due to the dynamics of the order parameter spin either. In zero magnetic field there is no frequency
threshold for the dissipation which can be considered as the fingerprint of the gapless Majorana surface bound states. The surface fluctuations are shown to smooth the threshold behavior out providing the small absorption in the frequency domain $\omega < \omega_H$.

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