Could Bitcoin enhance the portfolio performance?

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Abstract. This study analyses the effect of adding bitcoin into the portfolio by exploiting the Long Only investment strategy. The Portfolio consists of five assets: bitcoin, crude oil price index, stock exchange of Thailand (SET) price index, the exchange rate between Thai and USD and Thai government bond compound with treasurer bill. The model used for modelling the return of all asset is Multivariate t-copula based on GARCH and also measure the risk of the portfolio using the Value-at-risk (VaR) under the condition of minimizing the variance of return. We find that when adding more bitcoin into the portfolio, the return and risk of asset increased. If we only invest in bitcoin, we will face the risk at 16.90% and gain 6.27%. When comparing the effectiveness of portfolio by using Return-risk ratio, it found that portfolio with bitcoin shows the higher return rate than portfolios without bitcoin. Therefore, it can conclude that bitcoin could indeed increase the effectiveness of portfolio.

1. Introduction
With progress in financial and capital markets, investors can own various assets to gain returns that are expected by considering possible risks through portfolio allocation [1]. To obtain an appropriate proportion of portfolios, we need to measure the Value at Risk and Expected Shortfall (ES) by using the mean-variance method and choose the portfolio with the minimum variance mean. It found that investors can manage their investment portfolios to achieve the lowest risk. As for the measurement of the efficiency of the investment portfolio, according to the definition that the most efficient portfolio is one with the maximized rate of return at particular risk or one at the lowest risk with a particular rate of return.

Nowadays, it has one word that many people especially investors interested in "Cryptocurrency" In the aftermath of the Financial Crisis in 2008, or the Hamburger Crisis. Many investors lose their faith in the traditional currency as the central banks had the authority to produce money without any backup asset. Therefore, investors and financial institutions took more of an interest in investing alternative assets leading to the occurrence of cryptocurrency which considers as an alternative asset which gives a higher return than traditional assets [2]. In the recent, one of the most popular cryptocurrency is Bitcoin [3]. There are many literatures studying about the aspect of cryptocurrencies. We found that cryptocurrencies have a relatively low relationship with traditional assets [8].

According to the
existing study; the asset, which has low dependence on the other assets, is considered as a good alternative asset for investors as it is viewed as a safe-haven asset. As cryptocurrencies have a low relationship with other assets, they may be viewed as a safe-haven asset as well; therefore, cryptocurrency is one interesting asset for investors. As we highlighted earlier so many literatures interest to include the bitcoin into the portfolio for gaining higher return [9].

In the academic world, bitcoin have become an interesting issue as it has a high return and also high volatility. Many research papers tried to study many aspects of this virtual currency such as price structure and prediction bitcoin price and volatility. However, it is interesting to consider a bitcoin as an alternative asset as it is free from the inflationary influence of the stimulation of monetary policy. In recent years, this issue is shaded light by many papers. The initial papers is mean-variance approach which proposed by Markowitz [1] and it is widely used for asset allocation in the portfolio. However, it still has some drawbacks when an asset is the non-normal distribution i.e. existing of large excess kurtosis and positive skewness. The classical mean-variance is not appropriate and is likely to underestimate the potential loss resulting from additional tail-risk and thereby leading to sub-optimal portfolio decisions. After that there has some papers attempted to state more reliable portfolio optimization. They exploited the Conditional-Value at risk (CVaR) which is regarded as better risk measurement method. (see, [10] and [11]). Although the risk measurement determinant is enhanced, the financial data appears various patterns of tail behavior which may not captured by the traditional methods. In the other way, the copula approach is found to be very useful in risk analysis. A copula is a multivariate probability distribution whose marginal distributions uniformly distributed on the interval [0, 1]. This allows us to analyze non-linear dependencies between assets, and so copula-based models can be used to incorporate complex interdependencies between assets implicitly, and thus offer an improvement over isolated simulation method. In this study, we exploit multivariate t copula since it has been used extensively in the context of modeling multivariate financial return data and have been shown the superior to the normal copula. Thus, applying multivariate t copula generates a more accurate CVaR or expected shortfall result in reliable for measuring the risk of portfolio investment.

In this study, we state a research question whether adding bitcoins into an investment portfolio makes the portfolio more efficient and how they affect the value at risk. This study employs copula-based on the GARCH model to estimate the dependency of the assets and further compute the Value at Risk (VaR) of portfolios. [12] suggested that this model allows us to construct a flexible multivariate distribution with different margins, which allows the joint distribution of the portfolio to be free from any normality and linear correlation. In this study, we consider the multivariate t-copula model as it is widely used to model the financial returns and it was proved to be more efficient than other typical copula models. To the best of our knowledge, the empirical investigations that employ in finance are those by [13 and 14].

In the next section, the concept and model used in the study are explained, followed by information used in the study and empirical result. Conclusion and critique provide in Section 5.

2. Methodology
In this study, we can summarize the method for calculating the suitable proportion of individual assets following these steps. Firstly, we filter log-return daily data using univariate GARCH model to obtain standard residuals and construct the marginal distributions. It is vital to choose a suitable conditional distribution for the innovations $\epsilon_t$. This distribution conditionally describes the nature of the return series and captures its behaviour. Since a financial series is typically closer to a student's t-distribution, an excellent choice for this conditional distribution is a student's t-distribution.

Secondly, a student’s t copula is used to join the estimated marginal distributions obtained from the first step. After that, we use it for calculating the expected shortfall using the simulate data from our Copula based GARCH model. Finally, we calculate the appropriate proportion of individual assets under the condition that the investor needs to minimize portfolio expected shortfall for expected return.
2.1. The generalized autoregressive conditional heteroskedasticity (GARCH)

In empirical applications, the use of ARCH model is quite uncommon, and the basic model has been generalized in many different ways. The most popular volatility model is the generalized autoregressive conditional heteroskedasticity (GARCH) [15]. In this study, we consider ARMA (p, q) and GARCH (k, l) which is defined by

\[ r_t = \mu + \sum_{i=1}^{p} \phi_i r_{t-i} + \sum_{i=1}^{q} \psi_i e_{t-i} + e_t, \]

\[ \sigma_t^2 = \omega + \sum_{i=1}^{k} \alpha_i \epsilon_{t-i}^2 + \sum_{i=1}^{l} \beta_i \sigma_{t-i}^2, \]

where \( \sum_{i=1}^{p} \phi_i < 1, \omega > 0, \alpha_i, \beta_i \geq 0 \) and \( \sum_{i=1}^{k} \alpha_i + \sum_{i=1}^{l} \beta_i \leq 1 \). \( e_t \) is the standardized residual of a chosen innovation. In this case, we use \( t \)-distribution since the financial time series data mostly non-normal distribution but, it seems to have a skewness and heavy tail.

2.2. Copula

Copula function was proposed by [16]. The function has a feature that can estimate dependence between bi-variables or multivariable in case of using multivariate copula. The Copula is a way to construct a joint distribution function in marginal forms which has different distributions. The joint of difference distribution marginals is called “Copula” which can be defined by

\[ C(u_1, u_2) = C(F_1(x_1), F_2(x_2)), \]

where \( C \) is copula of the marginal distribution of \( n \) random variables \( F(i) \) is a distribution function. If the marginal distribution is a continuous function

\[ C(u_i, K, u_j) = C(F_i^{-1}(u_i), K, F_j^{-1}(u_j)), \]

where \( u_i \) is the cumulative distribution function (CDF) with uniform marginal on an interval \([0,1]\)

This study employs student’s \( t \) copula because it is suitable for financial time series and the market dependence structure. Moreover, it can estimate a tail dependence both in the lower and upper, and thus the extreme event captured and detected. The multivariate \( t \) distribution with degrees of freedom \( \nu \), mean vector \( \mu \) and a positive definite dispersion matrix \( \Sigma \), has density written as

\[ f(x) = \frac{\Gamma((\nu+n)/2)}{\Gamma(\nu/2)\sqrt{\pi\nu}\sqrt{\det(\Sigma)}} \left( 1 + \frac{1}{\nu}(x-\mu)^T \Sigma^{-1}(x-\mu) \right)^{-(\nu+n)/2}. \]

And the correlation matrix defines by

\[ \rho_{ij} \]

where \( \Gamma: \alpha > 0 \rightarrow \Gamma(\alpha) = \int_{0}^{\alpha} x^{\alpha-1} e^{-x} dx \). In the same line as, Gaussian random vectors. The random vectors are the \( t \) distributions have the stochastic representation as

\[ X = \mu + \sqrt{\nu} \frac{Z}{S}, \]

where \( S \) is distributed as \( \chi^2(\nu) \), \( Z : N(0, \Sigma) \) and \( S \) and \( Z \) are independent. Thus, the \( t \)-copulas can be defined as

\[ C_{\nu, \Sigma} = \int_{-\infty}^{\infty} L \int_{-\infty}^{\infty} f(x)dx, \]
2.3. Value-at-risk
Value at risk (VaR) is the popular statistical technique for financial risk measurement since it can specify the value at risk that probably happens with understandable numeral [3]. The way to calculate Value at risk has 2 ways. One is Local-valutation methods and the other is Full-valuation methods. In this case, we choose the Full-valuation methods by Monte Carlo stimulation due to this method is the most sophisticated and allows for any distribution and non-linear securities

2.4. Expected shortfall (ES)
Expected Shortfall (ES) is a risk measurement which shows the expected return on the portfolio in the worst cases at each confident interval. Expected shortfall estimates the risk of an investment in a conservative way, focusing on the less profitable outcomes. So, expected shortfall can be defined by

\[
ES = E \left[ \frac{W_i - W_0}{W_0} \right] \left[ r_{t+1} \leq -VaR \right],
\]

\[
= E \left[ r_{t+1} | r_{t+1} \leq -VaR \right]
\]

where \( W_i, t = 0,1 \) is the value of an asset which 1 and 0 indicate the range of time for example 1 day or 1 week. The conditional expected shortfall can define by

\[
ES_{t+1} = E \left[ r_{t+1} | r_{t+1} \leq -VaR_{t+1} \right]
\]

where \( r_{t+1} \) is the return of investment at \( t+1 \) and \( t \) time period which indicates the range of time for example 1 day or 1 week.

2.5. Portfolio optimization
Portfolio optimization is the process for selecting the most appropriate portfolio which subject to the objective that minimize variance of return. In this case, we stimulate samples by multivariate \( t \)-copula and exploiting Monte Carlo stimulation for estimating expected shortfall (ES).The result is provided proportion of each asset of the most efficient portfolio under condition minimization expected shortfall (ES) and maximization return. The method for calculating the expected shortfall can be summarized into three steps. First, we use multivariate \( t \)-copula to simulate events which length is sample size \( N \), then we plug the random number into inverse functions of the uniforms of the probability distributions. In this study, we employed the mean and variance equations of the ARMA-GARCH to get the \( N \) values of each variable at period \( t+1 \). Finally, the investor need to minimize her portfolio \( P \) with respect to her expected returns given by:

\[
MinES = E \left[ r^p | r^p \leq r^p_\alpha \right],
\]

subject to

\[
r^p = \omega_t r_t + \cdots + \omega_n r_n
\]

\[
\omega_t + \omega_n + \cdots + \omega_n = 1,
\]
where $0 \leq \omega_i \leq 1$, $i = 1, 2, \ldots, n$ and $\alpha^*_i$ is the return on individual asset at $\alpha$ quantile and $r_i$ is the return of asset $i$.

3. Data

In this study, the monthly Bitcoin price (Bit), Crude oil price index (Oil), Stock exchange of Thailand (SET) price index, Exchange rate (Ex), Thai government bond (Bond) and treasure bill yield at 10 years maturity (TB) are collected from October 2010 to December 2017. The data is collected from Financial Times, SET, Bank of Thailand, and www.bitstamps.com.

**Table 1. Descriptive statistics of price data.**

|       | Bit    | Oil    | SET    | EX     | TB     |
|-------|--------|--------|--------|--------|--------|
| Mean  | 1,190.175 | 59.0284 | 648.851 | 33.472 | 3.773  |
| Variance | 6,773.925 | 1,114.022 | 227,390.3 | 39.757 | 0.964  |
| Standard deviation | 2,602.67 | 33.37 | 476.85 | 6.30 | 0.98  |
| Skewness | 3.325 | 0.428 | 0.593 | 0.183 | 0.268  |
| Kurtosis | 13.657 | -1.010 | -0.850 | -0.898 | -0.371 |
| Variation coefficient | 2.186 | 0.565 | 0.735 | 0.188 | 0.26  |
| Maximum | 3.190 | 9.871 | 77.57 | 24.515 | 1.72  |
| Minimum | 13889 | 133.379 | 1826.86 | 53.613 | 6.4   |

From the descriptive statistic (table 1), bitcoin is the most fluctuating asset. Its standard deviation is high as 2,602.676 where the number is 5.5 times higher than that of SET. Also, with the coefficient of variation having disposed of the effects of mean information, it can be seen that bitcoin has the highest coefficient of variation which is 2.187, while that of petroleum price—an alternative asset—is 0.565. As for the skewness value, it found that bitcoin with positive skewness value shows that the enumeration of returns skew to the right. Before numerical estimation, all data is converted into returns. Then Augmented Dickey Fuller test is employed, and it was found that all returns are stationary with statistical significance.

4. Result

**Table 2. ARMA-GARCH estimates.**

|       | Bit    | Oil    | SET    | EX     | TB     |
|-------|--------|--------|--------|--------|--------|
| AR(1) | 0.3634 | 0.2614 | 0.2004 | 0.2371 | 0.1528 |
| C     | (2.1958)** | (1.9606)** | (1.814)* | (1.6452)** | (1.3049) |
| $\alpha$ | 0.0061 | 0.0001 | -1.42E-06 | 9.28E-06 | 0.0002 |
| $\beta$ | (1.2641) | (1.0933) | (-0.1679) | (0.4071) | (0.6181) |
| $\beta$ | 0.6313 | 0.6584 | -0.0942 | -0.0835 | 0.1350 |
| t-DIST.DOF. | (2.006)** | (2.177)** | (-0.83) | (0.5402) | (0.7151) |
| Log-likelihood | 7.2437 | 489774.5 | 1472.855 | 12.9697 | 11.3441 |
| AIC | (0.8673) | (8.59E-06) | (0.0030) | (0.4410) | (0.5415) |

**Table 2. ARMA-GARCH estimates.**

This section presents the estimated result starting from the estimated relationship among the assets using Multivariate t-copula followed by risk measurement in terms of value at risk and expected shortfall and portfolio optimization, respectively. Finally, the performance of portfolios with- and
without bitcoin are compared using return-risk ratio. Table 2 shows the estimated results of ARMA-GARCH for observe the volatility persistence of the model. By summing alpha and beta, it is found that oil prices present the highest volatility persistence while exchange rate shows the lowest volatility.

4.1. Multivariate t-copula
From table 3, bitcoin has strong relationship with petroleum price, followed by currency exchange rate. Bitcoin goes in the same direction with petroleum price in London market with the degree of dependence as 17.65%, but it has an opposite direction with currency exchange rate with a degree of dependence as 16.32%. Bitcoin has the weakest positive relationship with SET index with a degree of dependence as 10.29%. According to this result, we can conclude that there is a relationship between bitcoin and other financial assets especially petroleum price and currency exchange rate. Nonetheless, one thing that should consider carefully is that the dependence estimates are relatively low corresponding to the work of [17].

| Bit  | Oil    | SET   | EX    | TB    |
|------|--------|-------|-------|-------|
| Bit  | 1.0000 | 0.1765| 0.1029| -0.1632| 0.1050 |
| Oil  | 0.1765 | 1.0000| 0.0396| -0.0808| 0.3029 |
| SET  | 0.1029 | 0.0396| 1.0000| -0.1877| -0.1044|
| EX   | -0.1632| -0.0808| -0.1877| 1.0000 | 0.1052 |
| TB   | 0.1050 | 0.3029| -0.1044| 0.1052 | 1.0000 |

4.2. Risk measurement and portfolio optimization
Further estimation of the VaR and ES report in table 4. We calculate the average expected values of 1%, 5%, and 10% VaR and ES on an equally weighted portfolio base on the Multivariate T copula-based GARCH. We can see the volatility persistence of the model. By summing alpha and beta, it is found that oil prices present the highest volatility persistence while exchange rate shows the lowest volatility.

Table 6 shows the proportions of each asset in portfolios without bitcoin. It is found that the lowest expected risk in portfolio 1 is 1.038% while the expected return is 0.0755%. On the other hand, the highest expected risk in portfolio 10 is 3.256% while the expected return is 0.339%. In portfolio 1, it can be seen that the exchange rate shows the highest proportion at 85.9% of portfolios. The proportion of other assets are the followings, petroleum (0.6%), SET stocks (11.7%), and treasury bills and government bonds (1.7%).

We then compare the efficiency of these two portfolios using a return-risk ratio approach. According to table 7, we can observe that the portfolios with bitcoin give a better investment efficiency. For example, Risk-return ratio of portfolios with and without bitcoin in Portfolio 1 are, respectively, 0.1630 and 0.0723. This implies that the portfolios with bitcoins provide a higher return accounting 16.30% compared to 7.2% of portfolios with bitcoins. For the rest of Portfolios, we also obtained the same interpretation. The average difference in returns between portfolio with and without bitcoin is about 25%. The greatest one is in the range of investing bitcoin 11-22%.

Furthermore, our results suggest that the inclusion of Bitcoin into diversified portfolio may be profitable, serving therefore as risk diversifiers. Nevertheless, the investors who hold portfolios containing stocks, bonds, and others asset. May face great losses during bear states. We should not forget to mention the major challenges facing investors in digital monies. Given the short track record
of these assets, there is not a standard valuation tool that is mostly accepted to predict the trading prices of Bitcoin, and there is no consensus on the best method able to estimate the price trend. Moreover, the cryptocurrency market exposes to severe speculations, and new players enter the market every day, making the application of any valuation method problematic.

Table 4. Value at risk and expected shortfall.

| Portfolio with bitcoin | Expected Value |
|------------------------|----------------|
|                        | 1%  | 5%  | 10% |
| VaR                   | -0.0566 | -0.0339 | -0.0272 |
| ES                    | -0.0646 | -0.0462 | -0.0375 |

Table 5. Portfolio with Bitcoin.

| Portfolio | Return | Risk  | Asset |
|----------|--------|-------|-------|
|          |        |       | Bit   | OIL  | SET  | EX   | TB   |
| 1        | 0.15%  | 0.92% | 1.15% | 0.44% | 10.29% | 87.62% | 0.41% |
| 2        | 0.83%  | 2.09% | 11.59%| 0.00% | 20.53% | 67.88% | 0.00% |
| 3        | 1.51%  | 3.81% | 22.48%| 0.00% | 21.54% | 55.97% | 0.00% |
| 4        | 2.19%  | 5.63% | 33.36%| 0.00% | 22.87% | 43.77% | 0.00% |
| 5        | 2.87%  | 7.48% | 44.22%| 0.00% | 24.65% | 31.13% | 0.00% |
| 6        | 3.55%  | 9.35% | 55.10%| 0.00% | 25.78% | 19.12% | 0.00% |
| 7        | 4.23%  | 11.22%| 65.92%| 0.00% | 28.55% | 5.53%  | 0.00% |
| 8        | 4.91%  | 13.10%| 77.11%| 0.00% | 22.89% | 0.00%  | 0.00% |
| 9        | 5.59%  | 15.00%| 88.55%| 0.00% | 11.45% | 0.00%  | 0.00% |
| 10       | 6.27%  | 16.90%| 100.00% | 0.00% | 0.00%  | 0.00%  | 0.00% |

Table 6. Portfolio allocation without Bitcoin.

| Portfolio | return | risk | asset |
|----------|--------|------|-------|
|          |        |      | OIL   | SET   | EX   | TB   |
| 1        | 0.075% | 1.038% | 0.6% | 11.7% | 85.9% | 1.7% |
| 2        | 0.105% | 1.088% | 0.0% | 20.8% | 78.0% | 1.2% |
| 3        | 0.134% | 1.218% | 0.0% | 30.1% | 69.7% | 0.2% |
| 4        | 0.163% | 1.404% | 0.0% | 40.0% | 60.0% | 0.0% |
| 5        | 0.192% | 1.635% | 0.0% | 50.0% | 50.0% | 0.0% |
| 6        | 0.222% | 1.908% | 0.0% | 60.0% | 40.0% | 0.0% |
| 7        | 0.251% | 2.211% | 0.0% | 70.0% | 30.0% | 0.0% |
| 8        | 0.280% | 2.542% | 0.0% | 80.0% | 20.0% | 0.0% |
| 9        | 0.310% | 2.893% | 0.0% | 90.0% | 10.0% | 0.0% |
| 10       | 0.339% | 3.256% | 0.0% | 100.0% | 0.0% | 0.0% |

Table 7. Return-risk ratio.

| Portfolio | 1    | 2    | 3    | 4    | 5    | 6    | 7    | 8    | 9    | 10   |
|----------|------|------|------|------|------|------|------|------|------|------|
| With Bitcoin | 0.163 | 0.397 | 0.396 | 0.389 | 0.383 | 0.379 | 0.377 | 0.374 | 0.372 | 0.371 |
| Without Bitcoin | 0.072 | 0.096 | 0.110 | 0.1161 | 0.117 | 0.116 | 0.113 | 0.110 | 0.107 | 0.104 |
5. Conclusions and critique
We study the portfolio allocation using the variation principle of the minimum variance return and the dependency of returns in a portfolio; it found that bitcoins still have dependencies in other assets such as petroleum and SET stock price index. However, the testing of portfolio allocation comparing the portfolios with and without bitcoins shows that for bitcoins, if increase the proportion of bitcoin, the risk is higher. If we allocate all of the investment in bitcoins, the risk can reach 16.90%. The ratio test result, risk-return ratio, signifies that the investment efficiency of portfolios with bitcoins are better. This result corresponds to the result of the efficiency of investment frontier which indicates that investing in bitcoins can efficiently increase returns. In conclusion, this study suggests that one can invest in bitcoins should one be able to bear the risks as the theory goes “High risk, high return.” Realizing the nature of bitcoin, whether one should invest in it depends on the type of investor one is. If an investor can bear the risk, so he should invest bitcoin into the portfolio.

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