CONSTRAINTS ON EXTENDED QUINTESSENCE FROM HIGH-REDSHIFT SUPERNOVAE

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ABSTRACT

We obtain constraints on quintessence models from magnitude-redshift measurements of 176 Type Ia supernovae. The considered quintessence models are ordinary quintessence with Ratra-Peebles or supergravity (SUGRA) potentials, and extended quintessence with a Ratra-Peebles potential. We compute confidence regions in the \( \Omega_m-\Omega_x \) plane and find that for SUGRA potentials it is not possible to obtain useful constraints on these parameters; for the Ratra-Peebles case, for both the extended and ordinary quintessence we find \( \Omega_x \leq 0.8 \) at the 1 \( \sigma \) level. We also consider simulated data sets for the Supernova/Acceleration Probe (SNAP) satellite for the same models. Again, for a SUGRA potential it is not possible to obtain constraints on \( \Omega_x \), while with a Ratra-Peebles potential, its value is determined, with an error \( \pm 0.6 \). We evaluate the inaccuracy made by approximating the time evolution of the equation of state with a linear or constant \( w(z) \) instead of using its exact redshift evolution. Finally, we discuss the effects of different systematic errors in the determination of quintessence parameters.

Subject headings: cosmological parameters — cosmology: observations — cosmology: theory — supernovae: general

1 INTRODUCTION

Cosmological tests such as the baryon fraction in galaxy clusters (Hrbecky et al. 2000; Allen, Schmidt, & Fabian 2002), the abundance of massive galaxy clusters (see, e.g., Bahcall 2000; Borgani et al. 2001), the magnitude-redshift relation for Type Ia supernovae (SNe Ia) (Riess et al. 1998; Perlmutter et al. 1999; Gott et al. 2001; Tonry et al. 2003), and the statistical analysis of the galaxy distribution in large-redshift catalogs (e.g., Percival et al. 2001; Verde et al. 2002), and cosmic microwave background (CMB) anisotropies (Bennett et al. 2003; Spergel et al. 2003) indicate a low value for the matter density parameter, \( \Omega_m \), probably lying in the interval \( 0.15 \leq \Omega_m \leq 0.4 \). Recent results from the study of CMB anisotropies obtained with the Wilkinson Microwave Anisotropy Probe (WMAP) satellite (Bennett et al. 2003) also provide strong support for a flat (or very nearly flat) universe. Combining these two different indications leads to the hypothesis that a new form of energy, dark energy (DE), fills the gap between \( \Omega_m \) and unity: \( \Omega_m + \Omega_{DE} = 1 \).

One of the main goals of modern cosmology is to explain the nature of dark energy. The simplest solution to this problem is to introduce a cosmological constant \( \Lambda \) in our universe model. This scenario can have a simple theoretical interpretation, as \( \Lambda \) can be related to the energy density of the vacuum state in quantum field theory (Zel’dovich 1968). Unfortunately, this simple explanation results in a very large (formally infinite) value for the vacuum energy density that is larger by tens of orders of magnitude than the observed one (on the order of \( 10^{-127} \) GeV). What emerges is a fine-tuning problem, namely, the "cosmological-constant problem." Another apparently unnatural feature of the cosmological-constant model is that it starts to dominate the evolution of the universe only in the very near past. This issue is usually referred to as the "cosmic-coincidence problem" and reduces to a fine-tuning problem in the choice of the initial conditions, in particular, of the value of \( \Lambda \).

A possible way of alleviating these problems is to allow for a time variation of the dark energy density, which is only constant if due to \( \Lambda \). A very interesting class of models with this property are the quintessence models (Coble, Dodelson, & Frieman 1997; Caldwell et al. 1998; Ferreira & Joyce 1998; Viana & Liddle 1998).

One of the most promising cosmological tests of the properties of the DE component is based on the already-mentioned magnitude-redshift relation for SNe Ia (see, e.g., Brax & Martin 1999; Podariu & Ratna 2000; Podariu, Nugent, & Ratna 2001; Goliath et al. 2001; Weller & Albrecht 2001, 2002; Eriksson & Amanullah 2002; Gerke & Efstathiou 2002; Padmanabhan & Choudhry 2003; Di Pietro & Claeskens 2003; Knop et al. 2003). In fact, these objects can be considered as good standard candles, which makes it possible to determine their luminosity distance, whose dependence on redshift is specific to each particular cosmological framework.

In this paper we focus on three different kinds of quintessence models, whose features are briefly described in § 2. In § 3 we present the constraints obtained on these models using the magnitude-redshift relation for existing SN Ia data. In § 4 we illustrate how the constraints will improve with the projected Supernova/Acceleration Probe (SNAP) satellite, which will be devoted to the discovery and study of SNe Ia (Aldering et al. 2002). We also check the validity of approximating the exact redshift evolution of the equation of state with linear or constant behavior and discuss the possible effects of different systematic errors on the parameter determination. Finally, in § 5 we present our conclusions.

2 THEORETICAL FRAMEWORK

In this paper we focus on the extended quintessence (EQ) models, introduced in Perrotta, Baccigalupi, & Matarrese
(2000) and Baccigalupi, Matarrese, & Perrotta (2000). Extended quintessence and related models have been also considered (Chiba 1999, 2001; Uzan 1999; Bartolo & Pietroni 2000; Boisseau et al. 2000; de Ritis et al. 2000; Faraoni 2000; Fujii 2000; Chen, Scherrer, & Steigman 2001; Bean 2001; Gasperini 2001; Perrotta & Baccigalupi 2002; Riazuelo & Uzan 2002; Torres 2002; Kneller & Steigman 2003).

For these models the evolution of the scale factor $a$ and the scalar field $\phi$, responsible for the quintessence component, can be obtained by solving the set of equations

$$H^2 \equiv \left( \frac{\dot{a}}{a} \right)^2 = \frac{1}{3F} \left( \dot{\phi}^2 + \frac{1}{2} \ddot{\phi}^2 + a^2 V - 3H\dot{F} \right),$$

$$\ddot{\phi} + 2H\dot{\phi} = \frac{a^2}{2} F_{\phi\phi} R - a^2 V_{\phi},$$

where the dots denote derivatives with respect to the conformal time, the subscript $\phi$ on $F$ and $V$ denotes differentiation with respect to the scalar field, $R$ is the Ricci scalar, $\rho_{\text{fluid}}$ represents the energy density associated with all the constituents of the universe except for the quintessence scalar field, $V$ is the quintessence potential, and finally, $F$ is a function specifying the form of the coupling between $\phi$ and gravity. Hereafter we always refer to the non–minimally coupled (NMC) scalar-field models (Perrotta et al. 2000), for which the function $F$ is defined as $F(\phi) = 1/8\pi G + \tilde{F}(\phi) - \tilde{F}(\phi_0)$, where $\tilde{F}(\phi) = \xi \phi^2$. This kind of model has two free parameters: the dimensionless constant $\xi$, parameterizing the amount of coupling, and the present value of the scalar field $\phi_0$.

The coupling between the scalar field and gravity generates a time-varying gravitational constant (see, e.g., the review by Uzan 2003). Upper bounds on this variation come from local laboratory and solar system experiments (Gillies 1997) and from the effects induced on photon trajectories (Reasenberg et al. 1979; Will 1984; Damour 1998). As pointed out by Perrotta et al. (2000), these bounds become constraints on the parameters of the models:

$$32\pi G^2 \phi_0^2 \leq \frac{1}{500}.$$

We use this inequality in the following sections to improve our determination of the cosmological parameters.

For completeness and to allow a comparison with similar analyses, we also consider the case of ordinary quintessence (OQ), i.e., models for which there is no direct coupling between the scalar field and gravity (which is often referred to as the minimal coupling case). OQ can be easily obtained from Eqs. in the limit of $\xi \rightarrow 0$.

If we want to completely specify a quintessence model, we have to choose the analytical form for the potential, $V(\phi)$. One of the main advantages of a time-varying DE density is that it is possible to alleviate the cosmic coincidence problem. This is achieved by assuming particular classes of potentials, the so-called “tracker potentials” (Steinhardt, Wang, & Zlatev 1999), for which one obtains at low redshifts the same time evolution for $a$ and $\phi$, even starting from initial conditions that differ by orders of magnitude; this leads to a DE-dominated era close to the present time, without fine-tuning on the initial conditions. For the following analysis, we consider two different classes of tracker potentials: the inverse power-law Ratra-Peebles potential (RP; Ratra & Peebles 1988; see also Peebles & Ratra 2003 and references therein)

$$V(\phi) = \frac{M^{4+\alpha}}{\phi^\alpha}$$

and the SUGRA potential (Binétruy 1999; Brax & Martin 1999)

$$V = \frac{M^{4+\alpha}}{\phi^\alpha} \exp(4\pi G \phi^2).$$

In particular, we use the RP potential (eq. [4]) in the context of both Eqs and OQ models, while the SUGRA potential (eq. [5]) is considered in the minimal coupling ($\xi = 0$) case only.

We numerically solved equations (1) and (2) in the tracking regime. The behaviors we found for $a(t)$ and $\phi(t)$ (not shown in our figures) are in excellent agreement with those obtained by Baccigalupi et al. (2000), whose analysis also includes the RP case in the framework of EQ. In the next section we use these results to obtain our theoretical estimates of the luminosity distance $d_L$.

3. CONSTRAINTS FROM PRESENT HIGH-REDSHIFT SUPERNOVA DATA

The purpose of this section is to test the possibility of constraining the cosmological parameters describing quintessence models by using the best SN Ia data set presently available. Toward this aim, we build a sample combining data from the literature. As a starting point, we consider the data reported in Table 15 of Tonry et al. (2003). In particular, we use for our analysis the subset that is presented by the authors as the most reliable for cosmological studies. This data compilation, comprising 172 SNe Ia, is obtained by eliminating from the original sample of 230 SNe Ia objects at low redshift ($z < 0.01$) and those with high reddening ($A_V > 0.5$ mag). Then, we consider the data from Table 3 of Knop et al. (2003) but include in the sample only SNe Ia belonging to their “low-extinction primary subset” (seven objects). In their Table 4, Knop et al. (2003) present new fits to data from Perlmutter et al. (1999) that were already included in the Tonry et al. (2003) sample. In this case, we decided to use the magnitudes from Knop et al. (2003), because they are obtained using newly fitted light curves. The two catalogs also have SNe Ia in common in the low-redshift sample, taken from Hamuy et al. (1996) and Riess et al. (1998). For these, we prefer to use the data reported by Tonry et al. (2003), because they are given as a median of different fitting methods. For coherence with our previous choices, we decided to exclude five objects present in the low-redshift and Perlmutter et al. (1999) samples but excluded by the low-extinction primary subset of Knop et al. (2003). Finally, we add two new SNe Ia, SN 2002de and 2002dd, recently studied by Blakeslee et al. (2003). Therefore, the sample of SNe Ia considered here comprises 176 objects.

To constrain the cosmological parameters, we compare, through a $\chi^2$ analysis, the redshift dependence of the observational estimates of $\log(d_L)$ with their theoretical values, which for a flat, matter- or DE-dominated universe can be obtained from

$$d_L(z) = \frac{1 + z}{H_0} \int_0^z \left[ \Omega_{\text{m0}}(1 + z')^3 + \Omega_{\text{de0}} f(z') \right]^{-1/2} dz',$$
where

\[ f(z) = \exp \left[ 3 \int_0^z \frac{1 + w(z')}{1 + z'} dz' \right]. \tag{7} \]

Here \( H_0 \) is the present value of the Hubble constant, and \( \Omega_{m0} \) and \( \Omega_{\phi0} \) represent the contributions to the present density parameter due to matter and scalar field, respectively. The quantity \( w \equiv p_\phi/p_0 \) sets the equation of state relating the scalar field pressure

\[ p_\phi = \frac{\dot{\phi}^2}{2\alpha^2} + V(\phi) - \frac{3H^2}{a^2} \tag{8} \]

and its energy density

\[ \rho_\phi = \frac{\dot{\phi}^2}{2\alpha^2} - V(\phi) + \frac{\dot{\phi}}{a^2} + \frac{H^2}{a^2}. \tag{9} \]

The quantity \( w(z) \) can be computed by using the numerical solutions of the previous equations, once a cosmological model is assumed.

We can now find, for the different quintessence models described in § 2, the parameters (and their confidence regions) that best fit the SN Ia data by using the standard \( \chi^2 \) method. For this analysis, we have to consider the errors on the distance moduli. The objects coming from Tonry et al. (2003) are obtained directly from their Table 15, once a value for \( H_0 \) is assumed. On the other hand, Knop et al. (2003) report errors for the apparent magnitudes of their SNe Ia. We obtain the uncertainty on the distance modulus by adding in quadrature an error of 0.05 in the estimate of the absolute magnitude, as suggested by the data of Hamuy et al. (1996), Riess et al. (1998), and Knop et al. (2003, Table 4). In addition, we include an extra error contribution from the possible uncertainty in the peculiar velocities. In particular, following Tonry et al. (2003) we add 500 km s\(^{-1}\) divided by the redshift in quadrature to the distance error.

We compute the values for \( \chi^2 \) on a regular grid of the considered parameters (\( \Omega_{m0}, \alpha, \) and \( \xi \) for the EQ models; \( \Omega_{m0} \) and \( \alpha \) for the OQ models), looking for its minimum value, \( \chi^2_{\text{min}} \). In particular, we allow \( \Omega_{m0} \) to vary between 0 and 1 with spacing of 0.001, \( \alpha \) between 0 and 12 with spacing of 0.1, and \( \xi \) between 0.001 and 1.00 with spacing of 0.001.

The results in the \( \Omega_{m0}-\alpha \) plane are shown in Figure 1, where we display the contours of constant \( \Delta \chi^2 \equiv \chi^2 - \chi^2_{\text{min}} \) at \( 2.30, 5.99, \) and 11.8, corresponding to 1, 2, and 3 \( \sigma \), respectively, for a Gaussian distribution with two free parameters. The top left panel refers to the OQ model with a RP potential. In this case, we find \( \chi^2_{\text{min}} = 206 \) for \( \Omega_{m0} = 0.30 \) and \( \alpha = 0.0 \). However, from the plot it is evident that there is a strong degeneracy between the two free parameters \( \alpha \) and \( \Omega_{m0} \), and it is not possible to obtain strong constraints on them at the same time. Nevertheless, we can extract some information by considering the \( \chi^2 \) distribution when only a single parameter is allowed to vary. In this case, we obtain the error bars associated with the best-fit value by assuming \( \Delta \chi^2 = 1 \), which corresponds to 1 \( \sigma \) for a Gaussian distribution with a single parameter. The SN Ia data set we used does not allow us to obtain tight constraints on the parameter \( \alpha \): \( \alpha < 0.83 \) at the 1 \( \sigma \) confidence level, with a best-fit value of \( \alpha = 0 \). In addition, we obtain \( \Omega_{m0} = 0.30 \pm 0.03 \) for the matter density parameter. If we impose the Gaussian prior \( \Omega_{m0} = 0.27 \pm 0.04 \), as suggested by the combined analysis of recent CMB observations and large-scale structure data (Spergel et al. 2003), we obtain \( \alpha < 0.47 \) at the 1 \( \sigma \) confidence level (best-fit value, \( \alpha = 0.10 \)). We notice that our results are in agreement with those obtained from a similar analysis carried out by Podariu & Ratra (2000), although, thanks to our improvement in the SN Ia data set, the confidence contours start to converge, at least at the 1 \( \sigma \) confidence level.

In the top right panel of Figure 1, we show the results for the OQ model with the SUGRA potential. The values for \( \chi^2_{\text{min}} \) and the corresponding parameters are the same as obtained for the RP potential. In this case the confidence regions are almost vertical, showing a very small dependence on \( \alpha \), which does not allow us to extract constraints on \( \Omega_{m0} \) and \( \alpha \). Only if we impose the Gaussian prior \( \Omega_{m0} = 0.27 \pm 0.04 \) are we able to obtain \( \alpha < 2.78 \) at the 1 \( \sigma \) level (best-fit value, \( \alpha = 0.24 \)).

In the two bottom panels of Figure 1, we present the results for the model with EQ and a RP potential. In this case, we have three free parameters: in addition to \( \alpha \) and \( \Omega_{m0} \), there is \( \xi \), which parameterizes the strength of the coupling between the scalar field and gravity. In the bottom left panel we show the two-dimensional confidence regions in the \( \Omega_{m0}-\alpha \) plane, obtained by minimizing \( \chi^2 \) with respect to \( \xi \); they appear very similar to the previous case of OQ with RP potential, only slightly larger. Again, it is convenient to discuss the results when a single free parameter is considered. Unfortunately, there are no possibilities of obtaining constraints on \( \xi \) (best-fit value, \( \xi = 0.001 \)), even at the 1 \( \sigma \) confidence level. For the other two parameters, our results are again very similar to those of OQ with RP potential: \( \alpha < 0.82 \) (best-fit value, \( \alpha = 0.02 \)) and \( \Omega_{m0} = 0.28^{+0.05}_{-0.09} \). The bottom right panel shows how the confidence regions change when the upper limits on the time variation of the gravitational constant are taken into account. The estimates of \( \alpha \) and \( \Omega_{m0} \) do not change.
of 176 SNe Ia, being weaker, but we notice that the best-fit results from the Whole sample smallness of this sample, the resulting constraints cannot be only from the Supernova Cosmology Project. Because of the extinction primary subset of Knop et al. (2003) and coming analysis, considering only 54 SNe Ia belonging to the low-redshift primary subset of Knop et al. (2003) and coming only from the Supernova Cosmology Project. Because of the smallness of this sample, the resulting constraints cannot be directly compared with those obtained from the whole sample of 176 SNe Ia, being weaker, but we notice that the best-fit values for $\Omega_{m0}$ and $\alpha$ are only slightly different. Then, we checked the systematic effect due to possible type contamination, by excluding from our sample those objects whose confirmations as SNe Ia are questionable. This new subsample has a smaller number of very high redshift SNe Ia, and consequently the constraint on $\alpha$ becomes much weaker when compared to the whole sample: $\alpha < 1.62$ for OQ with RP potential, and $\alpha < 1.52$ for EQ with RP potential. Finally, we checked that the exclusion of very low redshift objects ($z < 0.03$), for which measurements of distance moduli are possibly affected by peculiar velocities, does not change the resulting confidence levels, which confirms that our treatment of this kind of error is reliable and that the constraining power of the analysis comes from high-redshift SNe Ia.

4. CONSTRAINTS FROM FUTURE HIGH-REDSHIFT SUPERNOVAE DATA

In order to improve the sample of studied SNe Ia, the new satellite SNAP is currently under development. SNAP is expected to perform over 2 years a complete spectroscopic and photometric analysis for approximately 2000 high-redshift SNe Ia, reaching a maximum redshift of $z = 1.7$. Because of the large increase in both the number of observed objects and the covered redshift interval, SNAP should prove extremely useful for cosmological studies.

In order to check the ability of this satellite to constrain the parameters for quintessence models, we generate pairs of distance moduli and redshift for 1,998 SNe Ia, adopting the fiducial distribution proposed by Kim et al. (2004) and shown in Figure 2. In addition, as in Kim et al. (2004), we always include a very low redshift sample of 300 supernovae with redshifts uniformly distributed between $z = 0.03$ and 0.08. Thus, the total number of objects considered in the following analysis is 2298.

The simulated distance moduli, $\mu$, are assumed to have a Gaussian distribution around the true value, with a dispersion $\sigma_{\mu} = 0.15$. The true distance modulus for a given redshift $z$ is known once we specify the background cosmology. For our analysis, we decided to simulate SN Ia data sets with three different background cosmologies (data sets 1, 2, and 3), whose properties and parameter values are reported in Table 1. For all data sets, we use for the Hubble constant $H_0 = 71$ km s$^{-1}$ Mpc$^{-1}$, which is the value suggested by the combined analysis of WMAP and large-scale structure data (Spergel et al. 2003).

Adopting the same $\chi^2$ method described in § 3, we then analyze these simulated data sets, fitting each quintessence model only with the data obtained from the same background cosmology. In computing the value for $\chi^2$, we consider an error of 0.15 mag on the distance moduli, and we neglect errors on the redshifts $z$.

It is also quite important to verify how good the approximation is using a linear [i.e., $w(z) = w_0 + w_1 z$, where $w_0$ and $w_1$ are suitable constants] or a constant ($w = w_0 = constant$) equation of state in the fitting procedure (as is often done in

| Data Set | Model | Potential | $\alpha$ | $\Omega_{m0}$ | $\xi$ |
|----------|-------|-----------|---------|--------------|------|
| 1............ | OQ    | RP        | 1.0     | 0.3          | 0.000|
| 2............ | OQ    | SUGRA     | 1.0     | 0.3          | 0.000|
| 3............ | EQ    | RP        | 1.0     | 0.3          | 0.015|
the literature), rather than the exact redshift evolution (as we did in § 3). For this purpose we apply the following procedure. Let us refer, for simplicity, to one of the OQ models. For each pair of values ($\Omega_m$, $\alpha$), we numerically obtain the redshift evolution of the equation of state, computing $w(z)$ in 1700 equally spaced values of $z$ in the interval [0, 1.7]. Using the least-squares method, we determine the straight line that best fits this ensemble of points. Finally, we use the theoretical distance modulus obtained from this linear approximation for $w$, instead of its exact evolution, to calculate the value of $\chi^2$ with the SN Ia simulated data. Similarly, in order to check the validity of the approximation of a constant equation of state, we follow the same procedure but substitute a straight line with the mean value of $w(z)$ in the considered range. In the figures mentioned below, we illustrate the differences in the confidence regions between these two approximations and the results obtained assuming the exact equation of state.

In Figure 3 we show the confidence regions in the $\Omega_m$-$\alpha$ plane obtained by fitting the OQ model with a RP potential to data set 1. As is also done with the other fits described in this section, we fix the value of the Hubble constant to the “true” value, $H_0 = 71$ km s$^{-1}$ Mpc$^{-1}$. In Figure 3a we show the results obtained with the exact equation of state. The best-fitting parameters are $\alpha = 0.91^{+0.34}_{-0.33}$ and $\Omega_m = 0.30^{+0.03}_{-0.02}$ at the 1 $\sigma$ level. They are consistent with the values assumed in the background cosmology, within the error limits. By comparing these results with those presented in the top left panel of Figure 1, the improvement with respect to the constraints obtained with the present SN Ia data is also evident, since the confidence regions are much narrower. Nonetheless, there is still a little degeneracy between $\alpha$ and $\Omega_m$; to resolve it, it will be useful to combine magnitude-redshift measurements with other cosmological tests (see, e.g., Balbi et al. 2003; Frieman et al. 2003; Jimenez et al. 2003; Caldwell & Doran 2004). The case of an OQ model with a RP potential has already been examined by Podariu et al. (2001) for SNAP simulated data. Their confidence regions in the $\Omega_m$-$\alpha$ plane look quite similar to ours, even though they considered a fiducial cosmology with $\alpha = 4$ and $\Omega_m = 0.2$. A similar analysis has also been carried out by Eriksson & Amanullah (2002), who found an error on $\alpha$ that is approximately 2 times larger than ours. This difference is probably due to the different redshift distribution they adopted.

In Figure 3b we show the confidence regions obtained when $\chi^2$ is computed using the linear approximation for $w(z)$, while in Figure 3c we adopt a constant equation of state. The differences between the three considered cases (exact, linear, and constant $w$) are evident also from the bottom right panel of Figure 3, which shows the superposition of the corresponding confidence regions. The best-fit values we obtain are $\alpha = 0.81^{+0.37}_{-0.29}$ and $\Omega_m = 0.31^{+0.03}_{-0.02}$ for linear $w(z)$ and $\alpha = 0.76^{+0.30}_{-0.26}$ and $\Omega_m = 0.31^{+0.02}_{-0.03}$ for constant $w$, in both cases at the 1 $\sigma$ level. The measured cosmological parameters are thus consistent with the true values within the error limits. However, there seems to be a systematic tendency to underestimate $\alpha$, which increases when we decrease the accuracy used to describe the redshift evolution of $w$. Nevertheless, this systematic offset will be too small to be evident in the SNAP data, even at the 1 $\sigma$ level. On the contrary, the approximations on $w(z)$ do not strongly influence the determination of $\Omega_m$. Finally, we notice that, at the 1 $\sigma$ confidence level, the size of the errors for both $\alpha$ and $\Omega_m$ appear to be smaller, i.e., systematically underestimated, when the linear and constant approximations are used instead of the exact solution.

We then perform the same kind of analysis by fitting OQ models with a SUGRA potential using data set 2. The results are shown in Figure 4. Looking at Figure 4a, which plots the results obtained with exact $w(z)$, it is evident that the SUGRA models present a strong degeneracy in $\alpha$, becoming larger with increasing values of $\alpha$. This was already pointed out by Brax & Martin (1999), who studied the dependence of $w_0$ on $\alpha$ in the same class of models. As a consequence, in this case it is

![Figure 3](image-url)

**Fig. 3.—Confidence regions ($\Delta \chi^2 = 2.30$, 5.99, and 11.8, corresponding to 1, 2, and 3 $\sigma$, respectively, for a Gaussian distribution with two free parameters) for the parameters $\Omega_m$ and $\alpha$, for the OQ model with a RP potential. The results are obtained with the simulated SNAP sample (data set 1). (a) Constraints obtained by assuming the exact equation of state $w(z)$. (b) Linear $w(z)$ approximation. (c) Constant $w$ approximation. The bottom right panel shows the superposition of the cases in (a–c).**

![Figure 4](image-url)

**Fig. 4.—As in Fig. 3, but for the OQ model with a SUGRA potential. The results are obtained with the simulated SNAP sample (data set 2).**
impossible to determine the errors on the cosmological parameters. The confidence regions under the approximation of a linear equation of state are shown in Figure 4b: they appear very similar to those displayed in Figure 4a. If we instead assume a constant $w$ (see Fig. 4c), the 1 $\sigma$ contour converges, and we obtain $\alpha = 0.34^{+0.59}_{-0.33}$ and $\Omega_{m0} = 0.32 \pm 0.02$, which are marginally consistent with the true values $\alpha = 1$ and $\Omega_{m0} = 0.30$. Again, the bottom right panel of Figure 4 shows the superposition of the three cases. The situation slightly improves if we impose a Gaussian prior on $\Omega_{m0}$, namely, $\Omega_{m0} = 0.30 \pm 0.05$. In fact, we obtain $\alpha = 0.95^{+0.25}_{-0.78}$ for the exact $w(z)$, $\alpha = 0.49^{+0.23}_{-0.40}$ for the linear $w(z)$ and $\alpha = 0.38^{+0.59}_{-0.26}$ for the constant $w$, all at the 1 $\sigma$ confidence level. All these values are consistent, within the error limits, with the original assumption of $\alpha = 1$, but we see that by using the exact $w(z)$ the determination is very poor. Moreover, the use of linear or constant equation of state approximations could lead to an artificially high precision in the determination of $\alpha$. For these reasons, we can conclude that even with SNAP, it will not be possible to constrain OQ models with a SUGRA potential.

Finally, we use data set 3 to estimate the best-fit parameters for EQ models with a RP potential. Toward this aim, we determine the values of $\chi^2$ on a three-dimensional grid of values for the parameters $\alpha$, $\Omega_{m0}$, and $\xi$. In particular, we consider 100 values for $\xi$, regularly spaced between 0.001 to 0.100, as done in §3 with the currently available SN Ia data.

In Figure 5 we show the confidence regions in the $\Omega_{m0}$-$\alpha$ plane, obtained by minimizing (for each pair of these parameters) $\chi^2$ with respect to $\xi$. Figure 5a shows the exact $w(z)$ case and can be directly compared with the bottom left panel of Figure 1. Even if the regions become narrower, they are still larger than in the OQ models. It is possible to obtain the one-dimensional distribution separately for each of the three parameters by minimizing $\chi^2$ with respect to the others. In this way, we find that there is very little dependence of the minimized $\chi^2$ on $\xi$. Therefore, it is not possible to obtain useful constraints on this parameter (best-fit value, $\xi = 0.001$). Instead, we have $\alpha = 0.95^{+0.44}_{-0.62}$ and $\Omega_{m0} = 0.30^{+0.02}_{-0.03}$ at the 1 $\sigma$ level. These values are consistent with the true values ($\alpha = 1$ and $\Omega_{m0} = 0.3$) within the errors.

The confidence regions obtained using the approximated (linear and constant) equations of state (Figs. 5b and 5c, respectively) are quite different from the previous ones, as is evident from the superposition of the three cases in the bottom right panel of Figure 5. Again, there is no possibility of constraining $\xi$ [best-fit values: $\xi = 0.001$ for linear $w(z)$ and $\xi = 0.003$ for constant $w$]. For the remaining parameters we obtain $\alpha = 0.84^{+0.37}_{-0.51}$ and $\Omega_{m0} = 0.31^{+0.02}_{-0.03}$ for the linear $w(z)$ and $\alpha = 0.78^{+0.31}_{-0.46}$ and $\Omega_{m0} = 0.32^{+0.01}_{-0.03}$ for constant $w$ at the 1 $\sigma$ confidence level. These values are always consistent with the true values, within the error limits.

As a final point, it is interesting to discuss how the confidence regions change if we impose the constraint coming from the time variation of the gravitational constant. The results are shown in Figure 6, where we draw the confidence regions in the $\Omega_{m0}$-$\alpha$ plane, obtained by minimizing $\chi^2$ with respect to $\xi$ and considering only the combination of the parameters for which the bounds on $G(t)$ are satisfied. This condition excludes combinations of the parameters for which $\alpha$ and $\xi$ are high, and this is the main motivation for the differences with respect to Figure 5. Considering the case in which the exact equation of state is used (Fig. 5a), we have $\alpha = 0.95 \pm 0.43$ and $\Omega_{m0} = 0.30 \pm 0.03$ at 1 $\sigma$; there is no significant change in the determination of these parameters with the imposition of the constraint. However, this time we are able to obtain an upper limit on $\xi$: $\xi < 0.028$ at the 1 $\sigma$ level. This result is consistent with the assumed true value, $\xi = 0.015$. Figure 5b shows the linear $w(z)$ approximation, from which we derive $\alpha = 0.84^{+0.37}_{-0.51}$, $\Omega_{m0} = 0.31^{+0.02}_{-0.03}$ and $\xi < 0.030$ at the 1 $\sigma$ level. Finally, Figure 5c shows the constant $w$ approximation. In this case the resulting constraints are $\alpha = 0.78^{+0.32}_{-0.33}$, $\Omega_{m0} = 0.32^{+0.01}_{-0.03}$ and $\xi < 0.030$ at the 1 $\sigma$ level. Again, all these values are consistent with the true ones.

Up to now, we have discussed the possibility of determining the quintessence parameters in the future data coming from SNAP by assuming intrinsic statistical errors only. Here we...
consider the impact of systematic errors on the previous results. Kim et al. (2004) define two general forms for them: uncorrelated systematic uncertainties and magnitude offsets. The first case represents a random dispersion that cannot be reduced below a given magnitude error over a finite redshift range (here assumed to be $\Delta z = 0.1$): possible examples are calibration errors and imperfect galaxy subtraction. The second case is a coherent shift acting as a bias on all SN Ia magnitudes: selection effects as Malmquist bias or detector problems can produce this kind of effect.

In order to simulate irreducible systematics, we strictly follow Kim et al. (2004). In particular, for each redshift bin, containing $N_{\text{SN}}$ objects (see Fig. 2), we add in quadrature to the intrinsic magnitude dispersion per SN Ia (0.15 mag) an irreducible magnitude error $\Delta m$

$$\sigma_m = \sqrt{\frac{0.15^2}{N_{\text{SN}}} + (\Delta m)^2}.$$  

As in Kim et al. (2004), we consider two different possibilities for $\Delta m$: a constant value equal to 0.02 mag, which is the target error for the SNAP mission, and a linear function increasing with redshift that reaches the target error at the maximum covered redshift $z = 1.7$, $\Delta m = 0.02(z/1.7)$. The resulting constraints on $\Omega_m$ and $\alpha$ are shown in Figure 7 for the RP potential in the case of OQ and EQ (top left and bottom left panels, respectively). In both cases we show the 1 $\sigma$ confidence regions when no priors are considered. The solid contour shows the constraints obtained by considering statistical errors only; they are the same shown in Figures 3a and 5a. Dotted and dashed lines represent the 1 $\sigma$ regions when we include constant and linear irreducible systematic errors. We notice that systematic errors, as expected, extend the confidence regions in the direction of lower values for $\Omega_m$ and higher values for $\alpha$, in particular. This effect is larger when we apply a constant $\Delta m$. In the OQ case, the best-fit values for $\alpha$ are shifted up 0.1, and the errors on the two variables are strongly correlated.

Next we investigate the effect of systematic magnitude offsets. First, we consider a constant shift of 0.02 mag, both positive and negative. The analysis (not shown in our figures) confirms the results obtained by Kim et al. (2004): in this case the best-fit parameters are the true values, with errors that are very similar to the ones obtained by considering statistical errors only, without any systematic biases. Then we consider a magnitude offset that is linearly proportional to the redshift, as $\Delta m = \pm 0.03(z/1.7)$. The results for the same models considered previously (OQ and EQ with RP potential) are shown in the right panels of Figure 7. As discussed by Kim et al. (2004), an offset in magnitude can give best-fit parameters that are wrong but have smaller confidence regions, i.e. it is possible to have very precise but inaccurate answers. This is exactly what is happening in our case. Considering a positive linear offset, we find the tendency to have confidence regions systematically shifted toward the left, while we have the opposite trend for negative magnitude shifts in which the true values of the parameters $\Omega_m$ and $\alpha$ are excluded at a 1 $\sigma$ confidence level. Finally, we consider the presence of a possible mismatch between the calibration of low- and high-redshift SNe Ia. This is done by introducing a constant offset of $\pm 0.02$ mag to the 300 local objects only. The results (not shown in this figure for clarity) show that this kind of effect produces a small enlargement of the confidence regions, without introducing systematic biases in the parameter determination.

5. CONCLUSIONS

The main goal of this paper has been to discuss the possibility of using SN Ia distances and redshifts to obtain reliable constraints on the parameters defining the quintessence models. In particular, we considered extended quintessence models with Ratra-Peebles potential, and, for completeness, ordinary quintessence models with both Ratra-Peebles and SUGRA potentials.

As a first step, we studied the constraints that result from the analysis of the largest SN Ia sample currently available (176 objects, from Tonry et al. 2003, Knop et al. 2003, and Blakeslee et al. 2003). To this purpose, we used the exact redshift evolution of the equation of state $w(z)$ as numerically determined, avoiding any approximation. Our results show that for an ordinary quintessence model with a SUGRA potential, it is not possible to obtain significant limits on the potential exponent $\alpha$, because of the weak dependence of the equation of state on it. For ordinary quintessence with a Ratra-Peebles potential, it is possible to obtain an upper limit on $\alpha$ at the 1 $\sigma$ confidence level we find $\alpha \lesssim 0.83$. Our results, which are in agreement with a similar analysis made by Podariu & Ratra (2000) using an older SN Ia data set, are consistent with a cosmological-constant model for which $\alpha = 0$. We obtained a similar constraint for $\alpha$ when considering the extended quintessence models with Ratra-Peebles potential. Unfortunately, it is not possible to obtain useful constraints on the non-minimal coupling parameter $\xi$ between the scalar field and gravity.
We then discussed the potential improvement on the previous results when future SN Ia samples, with a larger number of objects and more extended redshift coverage, will be available. To this purpose, we simulated SN Ia data sets in different cosmological models, with the characteristics of the expected SNAP satellite observations (almost 2000 objects up to z = 1.7).

For ordinary quintessence with a SUGRA potential, we found that it will still be difficult to constrain the parameters even with the SNAP SN Ia data. Considering models with the Ratra-Peebles potential, both in extended and ordinary quintessence, our results suggest that α can be determined to an error ≤0.6 (at the 1 σ significance level), while Ω_m0 can be constrained with an error of approximately 0.03 (again at 1 σ). In extended quintessence models, even by imposing upper bounds on the time variation of the gravitational constant, it will be possible to obtain only an upper limit on the coupling constant: ξ < 0.028.

As a final issue, we discussed the systematic errors on the parameter estimates that originate if a constant or linear approximation is used for the equation of state, w(z), instead of the exact redshift evolution. For all the considered quintessence models, the confidence regions obtained with these approximations are narrower than the exact ones. As a consequence, these approximations on w(z) lead to a systematic underestimate of the errors. Nevertheless, the set of cosmological parameters determined by the fitting procedure are consistent with those used in the simulations. This means that the systematic errors induced by the assumed approximations are still smaller than the precision allowed on the cosmological parameters by SNAP, even at the 1 σ confidence level.

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