Curious Variables Experiment (CURVE): SDSS J210014.12+004446.0 - dwarf nova with negative and positive superhumps

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ABSTRACT

We report the results of 67 hours of CCD photometry of the recently discovered dwarf nova SDSS J210014.12+004446.0 (SDSS J2100). The data were obtained on 24 nights spanning a month. During this time we observed four ordinary outbursts lasting about 2-3 days and reaching an amplitude of $\sim 1.7$ mag. On all nights our light curve revealed persistent modulation with the stable period of 0.081088(3) days ($116.767 \pm 0.004$ min) and large amplitude of 0.5-0.6 mag in quiescence reduced to 0.1-0.2 mag during outbursts.

These humps were already observed on one night by Tramposch et al. (2005), who additionally observed superhumps during a superoutburst. Remarkably, from scant evidence at their disposal they were able to discern them as negative and positive (common) superhumps, respectively. Our period in quiescence clearly different from their superhump period confirmed this. Our discovery of an additional modulation, attributed by us to the orbital wave, completes the overall picture. Lack of superhumps in our data indicates that all eruptions we observed were ordinary outbursts. The earlier observation of the superhumps combined with the presence of the ordinary outbursts in our data enables classification of SDSS J2100 as an active SU UMa dwarf nova with two types of outbursts.

Additionally, we have promoted SDSS J2100 to the select group of cataclysmic variables exhibiting three periodic modulations of light from their accretion discs. We updated available information on positive and negative superhumps and thus provided enhanced evidence that their properties are strongly correlated mutually as well as with the orbital period. By recourse to these relations we were able to remove an alias ambiguity and to identify the orbital period of SDSS J2100 of 0.083304(6) days ($119.958 \pm 0.009$ min). SDSS J21000 is only third SU UMa dwarf nova showing both positive and negative superhumps. Their respective period excess and deficit equal to $4.99 \pm 0.03\%$ and $-2.660 \pm 0.008\%$, yielding the mass ratio $q \approx 0.24$.

Key words: stars: dwarf novae – stars: individual: SDSS J210014.12+004446.0

1 INTRODUCTION

In the zoo of variable stars SU UMa stars arguably belong to the most intriguing exhibits. They are cataclysmic binaries, consisting of a non-magnetic white dwarf and a late main sequence secondary filling its Roche lobe and loosing mass through the inner Lagrangian point. Their orbital periods usually are less than the period gap at 2.5 hours. The transferred matter forms an accretion disc around the primary (Warner 1995, Hellier 2001). In many cataclysmic variables accretion rate varies considerably resulting in brightness increase by several magnitudes recurring in a semi-regular fashion after weeks or months intervals. The bright phases are called outbursts and faint ones quiescence.

The SU UMa stars are unique in that they simultaneously exhibit two patterns of re-brightening: the ordinary outbursts, with amplitudes typically 2-6 magnitudes, and superoutbursts brighter by about one magnitude than the ordinary ones. In most cases the superoutbursts recur more regularly at intervals several times longer than the ordinary
outbursts. All superoutbursts studied sufficiently well reveal characteristic periodic tooth-shaped modulation called superhumps. Their periods are couple hours and amplitudes are of order 0.1 magnitude.

The peculiar behavior of SU UMa stars can be understood within the frame of the thermal and tidal instability model (see Osaki 1996 for review). The superhumps period is slightly longer than the orbital period of the binary star. Most likely they are caused by prograde rotation of the line of the apsides of a disk elongated by tidal perturbation from the secondary. The perturbation is most effective when disk particles moving in eccentric orbits enter the 3:1 resonance with the binary orbit. Then the superhump period is simply the beat period between orbital and precession rate periods (Whitehurst 1988).

Apart from the ordinary- and super-outbursts with the superhumps SU UMa stars exhibit often additional types of periodic modulation. An orbital wave and/or eclipses may occur for inclination $i \geq 60^\circ$. In the late stages of superoutbursts and during the early quiescence may appear late superhumps, shifted by 0.5 in phase in respect to the ordinary superhumps. In some systems negative superhumps occur with a period slightly shorter than the orbital period. They are explained by invoking the classical retrograde precession of the disc tilted with respect to the orbital plane (Wood & Burke 2007).

On one hand the positive and negative superhumps were discovered together in just two SU UMa stars, namely V503 Cyg (Harvey et al. 1995) and BF Ara (Kato et al. 2003b, Olech et al. 2007). On the other hand, such phenomena were already observed in AM CVn variables (AM CVn itself - Skillman et al. 1999), classical novae (V1974 Cyg - Retter et al. 1997 and V603 Aql - Patterson et al. 1997), VV Scl stars (TT Ari - Skillman et al. 1998), SW Sex stars (V795 Her and DW UMa - Patterson et al. 2002), nova-like variables (TV Col - Retter et al. 2003) and even in the low mass X-ray binaries (V1405 Aql - Wood & Burke 2007).

### 2. SDSS J210014.12+004446.0

After examining objects from Sloan Digital Sky Survey (SDSS) Szkody et al. (2004) listed SDSS J2100 as a candidate dwarf nova because of its spectrum characteristic for dwarf novae in outburst. The follow-up photometric and spectroscopic observations of SDSS J2100 were obtained by Tramposch et al. (2005). Their photometry was obtained on 3 nights spanning 3 months and on each occasion extended over 3 cycles. On two nights the star was bright and clearly revealed the tooth-shaped modulation with period of 2.099(2) hours (0.08746 days) and amplitude of 0.3 mag. On third night it was in quiescence and pulsating sinusoidally with period 1.96(2) hours (0.0817 days) and amplitude reaching 0.5 mag. Tramposch et al. (2005) argued that because of overall similarity of SDSS J2100 to V503 Cyg (Harvey et al. 1995) the modulation in quiescence corresponds to the negative superhumps. However, light curves of cataclysmic variables are known to suffer from red noise due to flickering. Because of that any derived periods have errors underestimated, with no regard whether they come from the least squares or the white noise simulations (Schwarzenberg-Czerny, 1991). This combined with scant evidence at hand prompted us to look closer at this star.

### 3. OBSERVATIONS AND DATA REDUCTION

Curious Variables Experiment (CURVE) is a long-term project of photometric observations of interesting variable stars in the Galaxy and its clusters (Olech et al. 2008, Pietrukowicz et al. 2008). In order to study southern objects we applied for time on the 1m telescope of the South African Astronomical Observatory (SAAO). The time allocated to our project lasted from August 15 till September 11, 2007.

The telescope was equipped with the STE3 camera with SITe CCD chip of size $512 \times 512$ pixels, back illuminated, cooled with liquid nitrogen. At the focal length of 8.5m the scale was 0.31 arcsec/pix providing the field of view of $158 \times 158$ arcsec. The camera design aims to minimize the noise from the bias and dark current hence we skipped obtaining bias and dark frames. Although Johnson-Cousins $UBVRI$ filters were fitted into filter wheel, most of our observations were obtained through clear opening, in white light. This enabled us to obtain good quality photometry of faint objects and to study their short time variations. The exposure times ranged from 100 to 200 sec, depending on weather and actual brightness of the star. Our data were reduced in a standard way using procedures from the

| Date of 2007 | HJD-2454000 Start | HJD-2454000 End | Length | No. exp. |
|-------------|------------------|----------------|--------|---------|
| Aug 16      | 329.31619        | 329.41052      | 2.264  | 76      |
| Aug 17      | 330.30653        | 330.41395      | 2.578  | 79      |
| Aug 18      | 331.33241        | 331.42551      | 2.234  | 41      |
| Aug 19      | 332.32616        | 332.44182      | 2.776  | 31      |
| Aug 20      | 333.33906        | 333.43797      | 0.934  | 75      |
| Aug 21      | 334.32155        | 334.42363      | 2.450  | 89      |
| Aug 22      | 335.33009        | 335.41185      | 1.962  | 29      |
| Aug 24      | 337.27465        | 337.34748      | 1.748  | 30      |
| Aug 25      | 338.28106        | 338.37153      | 2.171  | 59      |
| Aug 27      | 340.32293        | 340.41477      | 2.204  | 34      |
| Aug 28      | 341.30609        | 341.40181      | 2.297  | 24      |
| Aug 29      | 342.31350        | 342.47927      | 3.978  | 99      |
| Aug 30      | 343.32932        | 343.47112      | 3.403  | 45      |
| Aug 31      | 344.30701        | 344.45856      | 3.637  | 121     |
| Sep 01      | 345.32769        | 345.33317      | 0.132  | 5       |
| Sep 02      | 346.30706        | 346.36444      | 1.377  | 35      |
| Sep 03      | 347.30015        | 347.44513      | 3.480  | 67      |
| Sep 04      | 348.30153        | 348.47059      | 4.057  | 52      |
| Sep 05      | 349.29878        | 349.46284      | 3.937  | 64      |
| Sep 06      | 350.30580        | 350.46436      | 3.805  | 41      |
| Sep 07      | 351.30375        | 351.49162      | 4.509  | 58      |
| Sep 08      | 352.31904        | 352.46207      | 3.433  | 47      |
| Sep 09      | 353.30317        | 353.46041      | 3.774  | 49      |
| Sep 10      | 354.30464        | 354.46907      | 3.946  | 64      |

Total - - 67.086 1314
The profile photometry has been derived using the DAOPhotII package (Stetson 1987). The differential photometry yielded typical accuracy of 0.01 mag or less.

Table 1 presents the journal of our observations of SDSS J2100. In total, we obtained 1314 exposures in 67.1 hours of observations during 24 nights.

4 PERIOD ANALYSIS

The global light curve, spanning almost one month of systematic monitoring, is shown in Fig. 1. It demonstrates that SDSS J2100 is an active dwarf nova with frequent ordinary outbursts. We observed four outbursts with a typical amplitude of ~1.7 mag and duration of 2-3 days. In Fig. 2 we plot expanded sample light curves from eight nights. They demonstrate behaviour of the star both in outbursts (on Aug 16, Aug 21 and Aug 31) and during quiescence (Aug 29, Sep 5-9). Inspection of the light curve reveals clearly presence of the modulation with a period of around two hours and with the amplitude changing from 0.1-0.2 mag in outburst to 0.5-0.6 mag during quiescence.

To facilitate our period analysis, we transformed the light curve of SDSS J2100 in two ways. First, we converted it from the magnitudes into corresponding intensity units. In this way the amplitudes of modulation on all nights became comparable. Next, each night light curve was detrended by subtraction of the best fit parabola. In this way we obtained a light curve expressed in counts with its mean intensity set at zero. The ANOVA periodogram (Schwarzenberg-Czerny 1996) computed for the result light curve is shown in Fig. 3 and referred as $A$.

The periodogram $A$ is dominated by a window pattern with its peak centered at frequency $f_0 = 12.3321(40)$ c/d accompanied with 1 c/d aliases. The peak value corresponds to the period of 0.08109(3) days and agrees with the period of 0.0817(8) days reported by Tramposch et al. (2005) during quiescence.

The fact that the 0.0811-day modulation revealed changes both in shape and amplitude raised suspicion of presence in our light curve of yet another periodicity. To check this hypothesis we prewhitened our data with the main periodicity and its two harmonics. The resulting periodogram $B$ looks rather complex but clearly reveals presence of some remaining oscillation.

In the periodogram $B$ there are at least two overlapping window patterns. At high frequencies peaks cluster close to- but not quite at- the $2f_0$ harmonic. Still, much power is left at low frequencies. A pronounced pattern centered near the removed frequency $f_0$ remains. It contains a stong side-band doublet centered exactly at $f_0$. This is consistent with remains of original peak at $f_0$ broadened due to modulation of amplitude and/or phase and demonstrates insufficient prewhitening in the periodogram $B$.

We turn back to the original data to better remove any interference from the $f_0$ modulation. In our second try we prewhiten the original data with a sine function of fixed frequency $f_0$ and of a modulated amplitude. See Appendix B for the description of our procedure. The periodogram $C$ resulting from our second attempt of prewhitening of $f_0$ is shown in Fig. 4. Now the pattern at low frequencies is markedly fainter and the periodogram is dominated at high frequency by a pattern centered around its peak at $f_1 = 24.008(2)$ c/s. The ±1 cycle/day aliases at $f_2 = 23.010(2)$ and $f_3 = 25.007(2)$ c/d are prominent and we could not exclude at this stage that one represents the true frequency. The detailed values were obtained by least squares fit. The corresponding periods are $P_1 = 0.041652(3)$ days, $P_2 = 0.043460(3)$ days and $P_3 = 0.039989(3)$ days, respectively. In the periodogram $B$ the dominant peak corresponded to $f_2$. One could argue that in that case because of the presence of the strong low frequency pattern, the relative height of the high frequency peaks was distorted due to interference with the residues of the $2f_0$ harmonic. This argument is further supported by consideration of errors of sine fits of the new frequencies. For the case $B$ errors of $f_2$ and $f_3$ were factor 4 larger than those quoted above for the case $C$. Prewhitening of $f_1$ leaves no evidence of any remaining periodic modulation.

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It may be informative to present our conclusion in purely statistical terms. For our periodogram $C$ the peak values of the ANOVA statistics exceed 80 for 3 and 1314 degrees of freedom. Accordingly, the null hypothesis stating that our data reveals no coherent periodic modulation must be rejected at the confidence level of $>10\sigma$. In that sense our detection of the secondary periodicity in SDSS J2100 is secure. The remaining alias ambiguity concerns of the frequency value and not of the reality of the observed modulation.

5 O-C ANALYSIS

The light curve of SDSS J2100 from August and September 2007 contains 32 maxima and 30 minima. To check for any modulation of the light curve shape we examined them separately. The cycle numbers $E$, times and errors and $O - C$ values are listed in Table 2. The residuals $O - C$ are computed using the ephemerides given below.

The moments of maxima and minima can be fitted with their respective linear ephemerides of the form:

$$\text{HJD}_{\text{max}} = 2454329.4183(16) + 0.081083(7) \cdot E \quad (1)$$

$$\text{HJD}_{\text{min}} = 2454329.3702(17) + 0.081109(8) \cdot E \quad (2)$$
a sinusoid, yielding a hump period of SDSS J2100 we fitted the light curve with the mentioned formulae are presented in Table 2 and also plotted in Fig. 6. The 0.08746 day period corresponds to the positive superhump discussed in the next section. To make this picture clear, we subtracted values observed in other SU UMa stars of the similar period. This period differs only by 0.000006 days from the weighted mean of the values from Eqs. 1 and 2, suggesting that its error is not underestimated by more than a factor of 2 (Schwarzenberg-Czerny, 1991).

### Table 2. Times of maxima and minima in the light curve of SDSS J2100.

| Maxima | Minima |
|--------|--------|
| E      | HJD<sub>max</sub> | O − C | E      | HJD<sub>min</sub> | O − C |
| 0      | 29.4040(50) | −0.176 | 0  | 29.3750(40) | 0.059 |
| 12     | 30.3970(35) | 0.070 | 12 | 30.3475(60) | 0.049 |
| 24     | 31.3645(40) | 0.003 | 24 | 31.4000(45) | 0.026 |
| 48     | 32.3110(50) | 0.009 | 25 | 32.3570(35) | −0.093 |
| 61     | 34.3600(25) | −0.054 | 62 | 34.4100(50) | 0.025 |
| 73     | 35.3420(40) | 0.057 | 74 | 35.3725(35) | 0.003 |
| 110    | 38.3480(45) | 0.131 | 98 | 37.3240(60) | 0.063 |
| 135    | 40.3660(45) | 0.018 | 111 | 38.3675(40) | −0.071 |
| 147    | 41.3500(60) | 0.154 | 135 | 40.3270(35) | 0.087 |
| 160    | 42.3870(30) | −0.056 | 136 | 40.4045(70) | 0.043 |
| 161    | 42.4670(40) | −0.070 | 148 | 41.3760(55) | 0.021 |
| 172    | 43.3635(30) | −0.013 | 160 | 42.3500(60) | 0.029 |
| 173    | 43.4460(40) | 0.004 | 173 | 43.4055(45) | 0.042 |
| 184    | 44.3510(40) | 0.166 | 185 | 44.3670(50) | −0.103 |
| 185    | 44.4280(50) | 0.115 | 186 | 44.4530(30) | −0.043 |
| 209    | 46.3640(50) | −0.008 | 209 | 45.3250(50) | 0.037 |
| 210    | 46.4500(40) | 0.053 | 210 | 46.4070(50) | 0.048 |
| 221    | 47.4345(30) | 0.091 | 221 | 47.3695(50) | −0.085 |
| 222    | 47.4245(30) | 0.071 | 234 | 48.3420(45) | −0.095 |
| 234    | 48.3820(45) | −0.120 | 234 | 48.2820(70) | −0.035 |
| 246    | 49.3570(40) | −0.095 | 246 | 49.3265(30) | 0.043 |
| 247    | 49.4410(35) | −0.059 | 247 | 49.4050(35) | 0.011 |
| 258    | 50.3410(35) | 0.041 | 259 | 50.3750(45) | −0.030 |
| 259    | 50.4190(55) | 0.003 | 271 | 51.3500(30) | −0.009 |
| 271    | 51.3860(20) | −0.071 | 271 | 51.4232(30) | −0.107 |
| 272    | 51.4690(20) | −0.048 | 283 | 52.3257(40) | 0.021 |
| 283    | 52.3620(25) | −0.034 | 284 | 52.4805(40) | 0.035 |
| 284    | 52.4430(25) | −0.035 | 296 | 53.3800(40) | 0.019 |
| 295    | 53.3450(30) | 0.089 | 308 | 54.4370(40) | 0.064 |
| 296    | 53.4280(30) | 0.113 | 309 | 54.4395(35) | 0.082 |
| 307    | 54.3140(50) | 0.040 | 308 | 54.3960(40) | 0.051 |

The O − C values computed according to the above-mentioned formulae are presented in Table 2 and also plotted in Fig. 5. They are consistent with the 0.0811-day period remaining constant during our run.

To calculate the final value of the principal negative hump period of SDSS J2100 we fitted the light curve with a sinusoid, yielding \( P_h = 0.081088(3) \) days (116.767 ± 0.004 min). This period differs only by 0.000006 days from the weighted mean of the values from Eqs. 1 and 2, suggesting that its error is not underestimated by more than a factor of 2 (Schwarzenberg-Czerny, 1991).

### 6 PROPERTIES OF SDSS J2100

#### 6.1 The superhump period

In classical SU UMa stars, the superhumps are observed during supermaxima, their shape resembles shark teeth and their period is few percent longer than the orbital one while typical amplitudes are in the range 0.1-0.3 mag. Curiously, this was exactly what Tramposch et al. (2005) have observed in SDSS J2100 during the July 2003 outburst. Namely, during two nights the star was bright (\( V \approx 16.3 \pm 16.5 \) mag) and revealed teeth-shaped oscillations with the period of 2.099 ± 0.002 hours i.e. 0.08746(8) days and the amplitude of 0.2-0.3 mag. This and their lack during our outbursts confirms they were common (positive) superhumps arising from the apsidal precession of the eccentric accretion disc.

#### 6.2 The negative superhumps

Some cataclysmic variables reveal the so-called negative superhumps. They are thought to occur due to the nodal precession of a tilted accretion disc. According to Wood & Burke (2007) for the tilted accretion disk the stream often misses its rim, penetrating deeper inwards on most orbital phases. When the stream eventually collides with the disk, it releases more energy in the ensuing hot spot. Early collision occurs only when the stream sweeps across the disk rim (the nodal line). Depending whether the stream hits or misses the rim, as the hot spot migrates in and out, and its brightness is modulated accordingly, roughly twice per each binary revolution. However, the observer sees only one event per cycle, from the hot spot occurring on the visible side of the disk. To complete the scenario, the tilted disc is subject to a slow retrograde precession, resulting in the modulation period slightly shorter than the orbital one.

The negative superhumps were detected so far just in two classical SU UMa stars namely in V503 Cyg (Harvey et al. 1995) and BF Ara (Olech et al. 2007). In both stars the modulation persists over long period of time covering several cycles and supercycles and its amplitude in quiescence is large, reaching up to 0.5-1 mag. Unlike in ordinary superhumps, their decline branches are often steeper than the rise.

Th 0.0811 day oscillation of SDSS J2100 described in Sect. 4 has all properties of the negative superhump. This is best appreciated by inspection of the light curve of SDSS J2100 phased and binned with this period (Fig. 6). Each bin contains about 20 measurements and small scatter indicates excellent phasing. To make this picture clear, we subtracted from the light curve the high frequency modulation, to be discussed in the next section.

Note, that as pointed by Tramposch et al. (2005) if the 0.08746 day period corresponds to the positive superhump it would be wrong to attribute the 0.0811 day period to the orbital motion. The corresponding period excess \( \epsilon \), defined as \( P_h / P_{orb} - 1 \), of 8% would exceed by a factor of two the values observed in other SU UMa stars of the similar period. As discussed in Sect. 7, among SU UMa stars the period
excess is strongly correlated with the orbital period. In the next section we confirm this argument by detection of the separate orbital modulation.

6.3 The orbital wave

In quiescence dwarf novae of intermediate inclination often reveal orbital modulation of their light curve. The modulation is caused by changing aspect of the non-transparent hot spot. Depending on the transparency of the spot, the resulting light curve is complex, often double humped. The double humped light curve reveals more power in its $P_{orb}/2$ harmonics than in the fundamental period. In this regard note that our light curve of SDSS J2100 after prewhitening of the negative superhump yields another modulation of frequency $f_1 = 24.008 \text{ c/d}$, about right for the orbital harmonics. Thus $f_1/2 = 12.004 \text{ c/d}$ would correspond to the orbital period of $P_{orb} = 0.083304(6)$ days. Recalling large power in 1 c/d alias $f_2$ it seems justified to consider it as an alternative orbital harmonics. The hitch is that the orbital frequency $f_2/2 = 11.5049 \text{ c/d}$ corresponding to $P_2 = 0.08692$ days would yield a period excess of only 0.6% i.e. much, much smaller than observed in the typical SU UMa stars of the orbital period close to $P_2$.

For comparison in Fig. 7 we plot the light curve of SDSS J2100 prewhitened with the negative superhump modulation, phased and binned with the periods of 0.08330 and 0.08692 days. Note that the former period yields the largest peak-to-peak modulation, while the latter one yields a smoothed-out light curve with similar maxima. This suggests that for the period 0.08692 alternate maxima of differing shapes were averaged out and it lends further support to our orbital period of 0.08330 days. The asymmetry in the heights of the maxima of the orbital light curve is consistent with the orbital wave hypothesis for a semi-transparent hot spot. The higher maximum arises from the hot spot visible face-on. The secondary maximum is produced half orbital cycle later while only a fraction of radiation penetrates through the back of the hot spot moved to the opposite side of the disc.

7 SDSS J2100 AND ITS RELATIVES

7.1 SDSS J2100 place in the bi-hummers family

Summarizing conclusions from previous sections, SDSS J2100 reveals typical characteristic of an active SU UMa stars. Tramposch et al. (2005) likely caught it during superoutburst and demonstrated presence of superhumps with the period of $P_{sh} = 0.08746(8)$ days (125.94 ± 0.12 min). We observed frequent normal outbursts lasting 2-3 days and with amplitudes of ~1.7 mag.

Except for the superoutbursts, the light curve is dominated by the combination of two oscillations with periods $P_{ns} = 0.081088(3)$ days (116.767 ± 0.004 min) and $P_{orb} = 0.083304(6)$ days (119.958 ± 0.009 min), interpreted as the negative superhumps and the orbital period of the binary, respectively. Already Tramposch et al. (2005) speculated that the three humps seen by them on one night in quiescence were negative superhumps. We refined their period and discovered the orbital modulation, thus securing interpretation of all three periods. Their presence allow us to determine both the period excess and the period deficit, respectively for the positive and negative humps, as equal to $\epsilon = 0.0499(32)$ and $\epsilon_- = -0.02660(8)$.

The most recent compilation of the properties of stars showing superhumps were made by Patterson (1998), Patterson et al. (2005) and Pearson (2006). However, the last two papers contain only tables of values without particular references for each object. This compelled us to redo the literature survey again and to update it till the end of 2008. The survey returned 112 cataclysmic variables with known orbital and any superhump period (no matter positive, negative or both). The complete material will be discussed elsewhere (Olech 2009, in preparation). Its subset for the systems revealing simultaneously positive and negative superhumps is discussed in Sect. 7.2. Here in Fig. 8 we present only an updated graph of the period excesses/deficits, for all surveyed stars with $P_{orb} > 0.05$, plotted against their orbital period.

The big solid triangles in Fig. 8 correspond to the values of period excess and deficit for SDSS J2100, $\epsilon$ and $\epsilon_-$, calculated for the orbital period determined in Sect. 6.3. Their excellent match with other points renders strong support to our choice of the true orbital period among its aliases. For alias periods, the triangles would be located far away from the general trend.

Pearson (2006) fitted a simple formula to the empirical relation between period excess and mass ratio:

$$\epsilon = -4.1 \times 10^{-4} + 0.2076q$$  \hspace{1cm} (3)

By employing it we estimated the mass ratio for SDSS J2100 as equal to $q = 0.24$. Next, assuming that the secondary size is consistent with the main sequence value we could employ the empirical mass-period relation (Warner 1995):

$$M_2 = 0.065 P_{orb}^{3/4} \text{[h]}$$ \hspace{1cm} (4)

to derive masses. This yields tentative mass estimates $M_2 = 0.15$ and $M_1 = 0.64M_\odot$. This values are consistent with
7.2 Properties of the bi-humpers

Retter et al. (2002) listed seven cataclysmic variables exhibiting both positive and negative superhumps. Since then the number of such objects doubled, hence we felt compelled to update their Table 2 with the object studied till 2008. These updated results are summarized in our Table 3.

Apart from the orbital and superhump periods we also list period excesses and deficits and their ratio $\phi$. Retter et al. (2002) already suggested that $\phi$ correlates with the orbital period. Our Fig. 9, containing twice as much points confirms their hypothesis. A straight line fitted to the plotted points yields the following empirical formula:

$$\phi = 0.318(6) \cdot \log P - 0.161(10)$$  \hfill (5)

As was pointed out by Retter et al. (2002) existence of this relation poses some challenge to the theory as it fits binaries with rather different components. It is obeyed by the double-degenerate systems of AM CVn type, by the classical red dwarf - white dwarf cataclysmic binaries and by the low-mass X-ray binary containing a neutron star. This star-type independence seems to confirm beliefs that the positive and negative superhumps are pure accretion disk phenomena, independent of any stellar influence except for the gravity.

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REFERENCES

Araujo-Betancor, S., Knigge, C., Long, K.S., et al. 2003, ApJ, 583, 437
Chou, Y., Grindlay, J. E., Bloser, P. F. 2001, ApJ, 549, 1135
Gao, W., Li, Z., Wu, X., Zhang, Z., & Li, Y. 1999, ApJ Letters, 527, L55
Harvey, D. A., Skillman, D. R., Patterson, J., & Ringwald, F. A. 1995, PASP, 107, 551
Harvey, D. A., Skillman, D. R., Kemp, J., Patterson, J., Vannmunster, T., Fried, R. E., & Retter, A. 1998, ApJ Letters, 493, L105
Hellier, C. 2001, Cataclysmic Variable Stars, Springer, 2001
Kato, T., Nogami, D., & Masuda, S. 2003a, PASJ, 55, L7
Kato, T., Bolt, G., Nelson, P., et al. 2003b, MNRAS, 341, 933
Olech, A. 2002, Acta Astronomica, 52, 273
Olech, A., Rutkowski, A., & Schwarzenberg-Czerny, A. 2007, Acta Astronomica, 57, 331
Olech, A., Wisniewski, M., Zloczewski, K., Cook, L. M., Mularczyk, K., & Kedzierski, P. 2008, Acta Astronomica, 58, 131
Osaki, Y. 1996, PASP, 108, 39
Patterson, J., Jablonski, F., Koen, C., O’Donoghue, D., & Skillman, D. R. 1995, PASP, 107, 1183
Patterson, J., Kemp, J., Saad, J., et al. 1997, PASP, 109, 468
Patterson, J. 1998, PASP, 110, 1132
Patterson, J. 1999, Disk Instabilities in Close Binary Systems. 25 Years of the Disk-Instability Model. Proceedings of the Disk-Instability Workshop, Eds. S. Mineshige and J.C. Wheeler. Frontiers Science Series No. 26. Universal Academy Press, Inc., 1999., p. 61
Pearson, K. J. 2006, MNRAS, 371, 235
Table 3. Properties of the systems that have two kinds of superhumps. Stars marked by asterisk have uncertain detections. For explanation of $S$, $E$, $C$ and $L$ symbols see Appendix A.

| Object     | Orbital Period [d] | Positive Superhump [d] | $\epsilon$ | Negative Superhump [d] | $\epsilon_-$ | $\phi = \epsilon_-/\epsilon$ | Ref. |
|------------|---------------------|-------------------------|-------------|------------------------|--------------|----------------------------|------|
| AM CVn     | 0.011906623(3)$^E$  | 0.0121667(13)$^S$       | 0.0218(11)  | 0.01170613(35)$^E$     | −0.01684(3)  | −0.771(4)                  | 1.2  |
| V1405 Aql  | 0.034729754(11)$^E$ | 0.035041999(60)$^E$     | 0.00965(18) | 0.034483(13)$^E$       | −0.0071(4)   | −0.793(42)                 | 3.4  |
| V1159 Ori* | 0.06217801(13)$^C$  | 0.0641674(6)$^E$        | 0.0320(6)   | 0.0583(3)$^S$          | −0.0624(50)  | −1.95(15)                  | 5.6  |
| ER UMa*    | 0.06366(12)$^E$     | 0.065552(25)$^E$        | 0.030(2)    | 0.0589(7)$^E$          | −0.075(11)   | −2.52(41)                  | 6.78 |
| V503 Cyg   | 0.0777(8)$^E$       | 0.08104(7)$^E$          | 0.043(11)   | 0.07569(7)$^S$         | −0.026(10)   | −0.60(28)                  | 9    |
| V1974 Cyg  | 0.08125873(23)$^E$  | 0.0849(1)$^S$           | 0.0448(12)  | 0.07911(5)$^S$         | −0.0264(6)   | −0.590(21)                 | 10,11,12,27 |
| SDSS J2100 | 0.083304(6)$^E$    | 0.087462(27)$^E$        | 0.0499(32)  | 0.081088(3)$^E$        | −0.02660(8)  | −0.533(35)                 | 13,14 |
| BF Ara     | 0.084176(21)$^L$   | 0.087971(1)$^E$         | 0.0451(3)   | 0.0825(1)$^S$          | −0.0247(12)  | −0.547(27)                 | 15,16 |
| V592 Cas   | 0.115063(5)$^E$    | 0.12228(1)$^E$          | 0.06272(10) | 0.11193(5)$^E$         | −0.0272(4)   | −0.434(7)                  | 17   |
| DW UMa     | 0.136606499(3)$^E$ | 0.14539(13)$^E$         | 0.0643(95)  | 0.13259(5)$^E$         | −0.0294(4)   | −0.457(9)                  | 18,19 |
| TT Ari     | 0.13755040(37)$^{S+E}$ | 0.14926(5)$^E$      | 0.0851(4)   | 0.1329(3)$^S$          | −0.0338(22)  | −0.397(26)                 | 20,21,28,29 |
| V603 Aql   | 0.1351(1)$^E$     | 0.1460(7)$^S$           | 0.0572(51)  | 0.1343(3)$^S$          | −0.0275(3)   | −0.481(59)                 | 22   |
| RR Cha*    | 0.1401(1)$^E$     | 0.1444(3)$^E$           | 0.0307(23)  | 0.1363(3)$^E$          | −0.0271(23)  | −0.883(98)                 | 23   |
| PX And     | 0.1463527(1)$^E$  | 0.1595(2)$^E$           | 0.0898(14)  | 0.141(1)$^S$           | −0.0366(68)  | −0.407(76)                 | 24,25 |
| TV Col     | 0.22860(1)$^S$    | 0.2639(35)$^S$          | 0.154(15)   | 0.2160(5)$^S$          | −0.0551(22)  | −0.357(38)                 | 26   |

Refs: 1 - Harvey et al. (1998), 2 - Skillman et al. (1999), 3 - Chou et al. (2001), 4 - Retter et al. (2002), 5 - Patterson et al. (1995), 6 - Thorstensen et al. (1997), 7 - Kato et al. (2003a), 8 - Gao et al. (1999), 9 - Harvey et al. (1996), 10 - Retter et al. (1997), 11 - Skillman et al. (1997), 12 - Olech et al. (2002), 13 - Tramposch et al. (2005), 14 - this work, 15 - Kato et al. (2003b), 16 - Olech et al. (2007), 17 - Taylor et al. (1998), 18 - Patterson et al. (2002), 19 - Arauj-Betancor et al. (2003), 20 - Thorstensen et al. (1985), 21 - Skillman et al. (1998), 22 - Patterson et al. (1997), 23 - Woudt & Warner (2002), 24 - Patterson (1998), 25 - Stanishev et al. (2002), 26 - Retter et al. (2005), 27 - Patterson (1999), 28 - Wu et al. (2002), 29 - Semeniuk et al. (1987)
flickering. For the orbital periods derived from the eclipses we adopt $\sigma_\phi = 0.02$. For the radial velocity (RV) curves of the emission lines in CVs we adopt $\sigma = 0.15$. The latter estimate reflect both random errors of measurements of broad emission lines of varying profile and possible systematic RV deviations to be discussed later. The errors consistent with Eq. (A1) are coded (E) in Table 3. We caution here that the above values are rough guesses to be used only as last resorts.

A1.2 Correlation or Red Noise Effects

Direct least squares (LSQ) fit of CVs light curves result in claims of period errors substantially less than in Eq. (A1). These are often artefacts resulting from improper treatment of intrinsic random flickering of a typical time scale $t_f$ of several minutes. In the time and frequency domains the flickering manifests respectively as correlation of consecutive observations and red noise. Thus any measurements repeated on a time scale $t_m$ short compared to $t_f$ yield no additional information on period (Schwarzenberg-Czerny, 1991). After correction for the correlation/red noise effects the LSQ errors increase by $\sqrt{t_f/t_m}$ and they became comparable to those of Eq. (A1). Similar problems may arise if two different modulations in the light curve are difficult to separate from each other. The estimates reflect both random errors of measurements of broad emission lines and they seem to depend on the luminosity/accretion status of a CV binary (Thorstensen et al. 1985, hereafter TSH). On one hand such shifts may prevent reliable orbital cycle count over gaps factor 3 larger (GF3L) than the base time interval. On the other hand consistent appearance of alias envelopes in the periodograms from different data sets suggests that likely cycle count errors do not exceed several over the whole data interval. Such errors are of negligible astrophysical consequence in the present context.

Our point is well illustrated for TT Ari, a bright and well observed novalike CV. TSH derived its spectroscopic ephemeris by carefully attempting to eliminate any systematic effects and using datasets of good quality spanning over 6 years. However, their stated period error was consistent with $\sigma_\phi$ as small as 0.016. After 21 years Wu et al. (2002) obtained another good sample of RV observations. They found TSH ephemeris shifted in phase by 0.35, indicating period error underestimated and insecure cycle count. The updated period shift is consistent with $\sigma_\phi \geq 0.09$ for TSH data. The new ephemeris by Wu et al. (2002) also does suffer from GF3L hence a $\pm 1$ cycle count error over 21 years may not be entirely excluded. We stress again that such problems arise for a well observed star and careful analysis. The true culprit are systematic effect in the accretion disc light sources. Insecure cycle count due to GF3L determinations are coded with (C).

A2 Systematic errors

A2.1 Period changes

In CVs systematic period modulations result from evolution of accretion disc and/or from solar cycle of the secondary star. Sparse observations of these effects often yield apparently semi-random scatter of the derived periods. To prevent over-interpretation we must caution here that neither the underlying physics nor telescope time allocation are random processes so that strictly speaking the resulting period estimates are not random variables. This said, we handled them as if they were random. Thus our prefered procedure was to derive periods by averaging the results from different observing runs and quoting half of their spread interval as their error. Occasionally period derivative was used to estimate the spread interval. The errors derived from the spread interval are coded (S).

A2.2 Orbital period changes

Studies of eclipses revealed that orbital periods of CVs fluctuate over time scales from several years to several decades. Possible explanations involve solar-type cycle of the secondary star (Warner, 1995). However, these period changes $\delta_\phi < 0.02$ for reasonable coverage should not affect orbital cycle count and they may be neglected in our analysis. In spectroscopic observations large random errors combine with large systematic orbital phase shifts, e.g. manifested by RV and eclipse phase differences. The shifts reach $\delta_\phi < 0.15$. They are likely caused by non-axisymmetric disk sources of the emission lines and they seem to depend on the luminosity/accretion status of a CV binary (Thorstensen et al. 1985, hereafter TSH). On one hand such shifts may prevent reliable orbital cycle count over gaps factor 3 larger (GF3L) than the base time interval. On the other hand consistent appearance of alias envelopes in the periodograms from different data sets suggests that likely cycle count errors do not exceed several over the whole data interval. Such errors are of negligible astrophysical consequence in the present context.

Our point is well illustrated for TT Ari, a bright and well observed novalike CV. TSH derived its spectroscopic ephemeris by carefully attempting to eliminate any systematic effects and using datasets of good quality spanning over 6 years. However, their stated period error was consistent with $\sigma_\phi$ as small as 0.016. After 21 years Wu et al. (2002) obtained another good sample of RV observations. They found TSH ephemeris shifted in phase by 0.35, indicating period error underestimated and insecure cycle count. The updated period shift is consistent with $\sigma_\phi \geq 0.09$ for TSH data. The new ephemeris by Wu et al. (2002) also does suffer from GF3L hence a $\pm 1$ cycle count error over 21 years may not be entirely excluded. We stress again that such problems arise for a well observed star and careful analysis. The true culprit are systematic effect in the accretion disc light sources. Insecure cycle count due to GF3L determinations are coded with (C).

A2.3 Period changes of disc humps

Negative and/or positive humps display periods close to the orbital one. They are observed as permanent features in some novalike stars or occasionally in dwarf novae, mainly during superoutbursts. They periods are known to vary in time. The variations may be systematic during superoutbursts or they manifest as period fluctuations between different superoutbursts and in permanent humps. The humps are thought to arise solely in an accretion disc. Their clocks are tied to the dynamical effects while any period changes reflect varying residual pressure effects. For sufficient data we estimated periods using (S) procedure and otherwise we had to rely on (E) values.

APPENDIX B: PREWHITENING WITH AMPLITUDE MODULATION

While our prewhitening technique is rather simple and efficient, it seems to be rarely used hence we devote space to its brief description. We assume that the exact value of frequency $\omega$ is known, e.g. from a constant amplitude sine fit. We adopt time zero so that it corresponds the to maximum of the fitted sinusoid. Let $f(t) = \cos \omega t$ denotes a fixed cosine function and $P = 2\pi/\omega$ is its period. For each data point $x_i$ obtained at time $t_i$ we select a running window extending over $t_i \pm P/2$. Within that window the data are fitted with the function $a + b f(t)$ by least squares adjustment of the shift and scale parameters, $a$ and $b$ respectively. For the value of the modulated oscillation at time $t_i$ we adopt $y_i = a + b f(t_i)$. For the next moment of time $t_{i+1}$ we select a new window and obtain new values of $a$ and $b$ and so on.
The above procedure may be efficiently implemented by noting that

$$y_i = \frac{(n[f] - [f][x]) f(t_i) + [x][f^2] - [x][f]}{n[f^2] - [f^2]} \quad (B1)$$

where $[\cdot]$ denote sums of the corresponding values over the running window and $n$ is number of points in the window. These sums are easily adjusted after moving of the window by adding contributions from the new points and subtracting those from the omitted ones. Our particular selection of the window width of $P$ or its integer multiple is optimal for roughly uniform distribution of observations. Namely, it may be demonstrated that then the parameters $a$ and $b$ are uncorrelated, hence they suffer less from statistical errors.