Compactification driven Hilltop Inflation in Einstein-Yang-Mills.

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**Abstract**

Starting from Einstein-Yang-Mills in higher dimensions with an instanton on a compact sphere, we dimensionally reduce to find an effective four-dimensional action describing “hilltop” inflation. Using recent CMB data, we analyse the parameter space of this model to search for viable set-ups. One unique feature of this class of inflationary models is that the value of the inflaton field, or alternatively, the size of the compact sphere, is stabilised dynamically during the inflationary process.
1 Introduction

With the recent advent of precision cosmology and some imaginative astrophysical projects, we have gained considerable knowledge about the structure of our universe. In particular, we now appreciate that the universe is spatially flat, and the latest CMB data from WMAP5 narrows down the curvature perturbation spectral index to $n_s = 0.96 \pm 0.013$ \cite{1}. This data can be plausibly accounted for by an epoch of cosmic inflation in the early universe \cite{2}, modelled by a suitable inflaton field $\phi$, whose positive scalar potential $V(\phi)$ powers the nearly exponential expansion.

The observed temperature fluctuations in the CMB suggest that the shape of the potential of the inflaton is very flat, at least long before the end of inflation when the observed fluctuations probe the primordial perturbations generated during inflation. For a single scalar field driving inflation then, a slow-roll inflationary set-up is consistent with the current observational data. Fast-roll inflation may also be entertained \cite{3}, though one requires at least one additional field to liberate the inflaton from the task of making a dominant contribution to the curvature perturbation \cite{4}. There are other mechanisms to generate the primordial curvature perturbation (see for example \cite{5}), but in this paper we focus on the two mentioned above. Other key constraints on these potentials may come from the observation that the present universe is in a state of accelerated expansion \cite{6}. As a result, a local minimum of the potential with a small constant vacuum energy $V > 0$ corresponding to an asymptotic de Sitter (dS) phase is expected after the end of inflation. Finally, the inflationary stage should last about 60 efolds, so that scales corresponding to observation have a chance to leave the horizon during inflation. (See \cite{4} and references therein for an extensive review of inflation.)

Although $\phi$ may be a fundamental field, evoking extra-dimensions, other viable candidates for the inflaton emerge, including the location of branes or supersymmetric moduli in a string theory setting. With extra dimensions, recovering our observable universe, involves typically reducing on a compact space, with the proviso that the extra dimensions are hidden from macroscopic view. Despite the unimaginably large zoo of possible four-dimensional theories one encounters in this process \cite{7}, vibrant developments in observational cosmology, provide a means to whittle them down. On that basis, we have already witnessed many fascinating studies of moduli stabilisation in the string theory literature \cite{8}, where CMB data has been used as a powerful probe.

\footnote{It was argued in \cite{3} that an Anti-de Sitter vacuum cannot be the end result of inflation. For a minimum at $V(\phi) < 0$ the field oscillates about the minimum with positive energy density until the universe reaches its maximum size. After that the universe contracts to eventually collapse into a singularity.}
of the fundamental theory of gravity in the early universe.

This paper builds on the work initiated in [9], where it was demonstrated that compactification in a similar spirit to Cremmer-Scherk “spontaneous compactification” [10] could arise in a set-up incorporating a non-trivial soliton on the compactified space. In these models, the time-dependent radius of the compactified space may settle down to a finite value, instead of decompactifying to infinity. One bonus feature of this study was the observation that a description of inflation could be incorporated through the compactification process.

In this paper we will focus on the simpler case where we have a Yang-Mills instanton on $S^4$ and consider compactifications from a $d=8$ Einstein-Yang-Mills action with gauge-coupling $q_8$ and a positive cosmological constant $\Lambda_8 > 0$. We will then dimensionally reduce on the $S^4$, before conducting a study of the effective potential and its local maximum/minimum pairing which constitute a “hilltop” inflation model [12, 13]. Finally, we explore the stability of the de Sitter vacuum to general metric fluctuations to see if tachyonic instabilities further constrain the parameter space.

## 2 Set-up

We start from considerations of the following action in $d=8$ space-time,

$$S_8 = \int d^8x \sqrt{-g_8} \left( \frac{\mathcal{R}}{16\pi G_8} - \frac{\Lambda_8}{2} - \frac{1}{4}(F^a_{mn}F^a{}^{mn}) \right),$$  \hspace{1cm} (2.1)$$

where $\Lambda_8$ and $G_8$ are the $d=8$ cosmological constant and Newton’s constant respectively, while $a = 1, 2, 3$ are the indices on the $SU(2)$-valued instanton field strength $F^a$ on the $S^4$. $F$ may in turn be expressed in terms of the gauge potential as

$$F = dA + q_8 A \wedge A,$$  \hspace{1cm} (2.2)$$

where $q_8$ is the gauge coupling constant. In terms of projective coordinates $y^m, m = 1, \ldots, 4$, we may express the metric on the $S^4$ as

$$ds^2(S^4) = \frac{\delta_{mn} dy^m dy^n}{(1 + |y|^2/4)^2},$$  \hspace{1cm} (2.3)$$

with $|y|^2 = y_m y^m$.

We may now introduce a gauge configuration which satisfies the Bogomol’nyi duality relation $*_{4}F = \pm F$. We consider a static gauge configuration on the $S^4$ with

\footnote{The Abelian case was presented in [11]. They have considerable overlap, though the effective potential varies through the flux terms.}
gauge potential

\[ A = \frac{1}{4q_8 (1 + |y|^2/4)} U^\dagger dU, \]  

where we define \( U \) to be

\[ U = \frac{1}{|y|} Y, \quad Y = y^4 1_2 + y^1 \tau_1 + y^2 \tau_2 + y^3 \tau_3. \]  

The matrices \( \tau_a \) are related to the usual Pauli-matrices, \( \tau_a = -i \sigma_a \), and satisfy the multiplication relation \( \tau_a \tau_b = -\delta_{ab} 1_2 + \epsilon_{abc} \tau_c \). The field strength corresponding to (2.4) may be written

\[ F = \frac{1}{4q_8 (1 + |y|^2/4)^2} dY^\dagger \wedge dY, \]  

or, alternatively, in terms of components \( F = F^a \tau_a \):

\[ F^a = \frac{1}{4q_8} \frac{1}{(1 + |y|^2/4)^2} \epsilon^{abc} dy^b \wedge dy^c + 2 dy^a \wedge dy^4. \]  

One may readily check that the gauge configuration is self-dual: \( F = *_4 F \). The equations of motion are then ensured via the Bianchi identity, which is satisfied by construction.

2.1 The \( d=4 \) effective potential

Although we may consider the equations of motion in \( d=8 \), a better view of the dynamics of the system may be gleaned by reducing on the \( S^4 \) and looking at the effective potential in \( d=4 \) in Einstein frame. For simplicity, we just exhibit the low-energy effective action for the lowest Kaluza-Klein mode of \( \phi \), with higher modes being discussed in section 4. To dimensionally reduce, we introduce the following time-dependent ansatz for the metric

\[ ds^2 = e^{-4\phi(t)} g^{(4)}_{\mu\nu} dx^\mu dx^\nu + e^{2\phi(t)} ds^2(S^4), \]  

where the \( g^{(4)}_{\mu\nu} \) denotes the usual metric of a Friedmann-Robertson-Walker (FRW) spacetime,

\[ g^{(4)}_{\mu\nu} dx^\mu dx^\nu = -dt^2 + a^2(t) [dr^2 + r^2 d\Omega^2]. \]  

Denoting the radius of \( S^4 \) by \( L_0 \), the reduced action then becomes

\[ S_4 = \int d^4 x \sqrt{-g_4} \left( \frac{1}{16\pi G_4} [R_4 + 4 \nabla_\mu \nabla^\mu \phi - 12 \partial_\mu \phi \partial^\mu \phi] - V_4 \right), \]  

where

\[ V_4 = \frac{\Lambda_4}{2} e^{-4\phi} + \frac{3}{4q_8^2 L_0^4} e^{-8\phi} - \frac{3}{4\pi G_4 L_0^2} e^{-6\phi}, \]  

(2.11)
and we have redefined new d=4 parameters in terms of the volume of $S^4$, $V_{S^4} = \pi^2 L_0^4/2$:

$$G_4 = \frac{G_8}{V_{S^4}}, \quad \Lambda_4 = V_{S^4} \Lambda_8, \quad q_4 = \frac{q_8}{\sqrt{V_{S^4}}}. \quad (2.12)$$

Note above that the second term in the action is a total derivative term, and may be integrated out. Finally, bringing the action to the canonical form, requires the rescaling $\phi \to \frac{1}{2\sqrt{3} M_p} \phi$,

$$S = \int d^4x \sqrt{-g_4} \left[ \frac{1}{16\pi G_4} R - \frac{1}{2} \partial_\mu \phi \partial^\mu \phi - V_4 \right], \quad (2.13)$$

where now

$$V_4 = \frac{\Lambda_4}{2} e^{-2\phi/\sqrt{3} M_p} + \frac{3}{4q_4^2 L_0^2} e^{-4\phi/\sqrt{3} M_p} - \frac{3}{4\pi G_4 L_0^2} e^{-2\phi/\sqrt{3} M_p}.$$

As is customary in the literature, we have used the reduced Planck mass $M_p = (8\pi G_4)^{-1/2}$ to absorb factors appearing in the Einstein equations. In preparation for later numerical work, we also use this opportunity to introduce the dimensionless parameters

$$b \equiv \frac{4\pi G_4}{q_4^2 L_0^2}, \quad c \equiv 4\pi G_4 \Lambda_4 L_0^2, \quad (2.15)$$

so that the potential becomes

$$V_4 = \frac{M_p^2}{L_0^2} \left[ ce^{-2\phi/\sqrt{3} M_p} + \frac{3b}{2} e^{-4\phi/\sqrt{3} M_p} - 6e^{-2\phi/\sqrt{3} M_p} \right]. \quad (2.16)$$

From now on we will refer to $V_4$ simply as $V$.

We now proceed to examine this effective potential. $\phi \to \infty$ corresponds to the $S^4$ decompactifying, so we ignore this possibility. Also, in the opposite direction, as $b > 0$, $V$ goes to infinity for small $\phi$. In order to stabilise $\phi$ and ensure that it doesn’t run away, we require $V$ to have two extrema. This means that the solution to $V' = 0$,

$$\phi = -\sqrt{3} M_p \ln \left[ \frac{3}{2b} \left( 1 \pm \sqrt{1 - \frac{4bc}{27}} \right) \right], \quad (2.17)$$

should correspond to two roots, which in turn dictates the upper bound on the product of $b$ and $c$ is $bc < \frac{27}{4}$. These roots now correspond to a minimum $\phi_{\text{min}}$ and a maximum $\phi_{\text{max}}$ respectively, and offer the ideal place to look for an inflationary model, in which inflation takes place at the top of the potential: hilltop inflation \cite{12, 13}. For later use, we also here record the distance between the two roots

$$\phi_{\text{max}} - \phi_{\text{min}} = -2\sqrt{3} M_p \ln \left[ \sqrt{\frac{27}{4bc}} \left( 1 - \sqrt{1 - \frac{4bc}{27}} \right) \right]. \quad (2.18)$$

\[3\]This may be easily seen by bearing in mind that the covariant derivative the scalar density $\sqrt{-g_4}$ may be given as $\nabla_\mu \sqrt{-g_4} = \partial_\mu \sqrt{-g_4} - \Gamma^\nu_{\mu\nu} \sqrt{-g_4} = 0.$
and also note explicitly the combination $bc$ in terms of the original $d = 8$ parameters of the model

$$bc = \frac{(4\pi G_8)^2 \Lambda_8}{q_8^2}.$$  \hfill (2.19)

Another constraint on the $(b, c)$ parameter space comes from looking at the roots of the potential $V = 0$:

$$\phi = -\sqrt{3}M_p \ln \left[ \frac{b}{2} \left( 1 \pm \sqrt{1 - \frac{bc}{6}} \right) \right].$$  \hfill (2.20)

Avoiding a potential with a negative minimum, one corresponding to an Anti-de Sitter vacuum, necessitates $bc \geq 6$, making the final range for compactification to a Minkowski/de-Sitter vacuum to be

$$6 \leq bc < \frac{27}{4}.$$  \hfill (2.21)

Within this range, the effect of varying $bc$ may be noted from Fig. (2.1), where the vacuum raises from Anti-de Sitter ($bc < 6$) to Minkowski ($bc = 6$) and de-Sitter ($bc > 6$).

![Figure 2.1](image)

**Figure 2.1:** The figure shows the effective 4d potential $V$ for $b = 1$ and different values of the product $bc$. The effect of increasing $bc$ corresponds to the raising the de Sitter vacuum.

Within the permitted range for the parameters (2.21), we note from Fig. (2.1) that there are three channels for the dynamics of the scalar field $\phi$ depending on its initial conditions. Denoting $\phi_{\text{min}}$ and $\phi_{\text{max}}$ to be the value of the field at the local minimum and maximum respectively, the three channels correspond to $\phi < \phi_{\text{min}}$, $\phi_{\text{min}} < \phi < \phi_{\text{max}}$, and $\phi > \phi_{\text{max}}$. 


\( \phi_{\text{min}} < \phi < \phi_{\text{max}} \), and \( \phi > \phi_{\text{max}} \), where we have omitted the classically stable extrema from the quoted ranges. Our focus in this paper will be the second channel, though some amount of inflation may occur in the other two.

We ignore the third possibility, as from the reduction ansatz, the run-away direction for \( \phi \) corresponds to the extra dimensions decompactifying. The first channel is also constrained and there is an lower bound on the value of \( \phi \) in order to avoid a scenario where the field \( \phi \) reaches the minimum with enough kinetic energy to surmount the local maximum and also escape to infinity. We show solutions for the evolution of \( \phi \) via this channel later in the text.

Therefore, in order to safely stabilise the compactified extra dimensions, we focus on the case where the inflaton rolls down to the minimum (stable vacuum) of the potential from somewhere near the maximum (unstable vacuum) giving rise to inflation. In contrast to work initiated in \([9]\) where compactification in these models was studied at the level of the equations of motion in higher dimensions, the reduced action and effective potential, offer a more intuitive picture, and also allow an estimation of the primordial perturbations produced during inflation to compare with CMB observational data.

### 3 Inflation

In this section, we analyse the effective potential in \( (2.10) \) in terms of the slow-roll parameters \( \epsilon \) and \( \eta \), to determine how the values of the parameters \( b \) and \( c \) affect the resulting nature of the inflation. Our purpose here is to explore the effective action as a viable model of inflation, identify any potential shortcomings that may arise, and address those concerns in future work. Before proceeding to the analysis, we summarise the basics of slow-roll inflation and highlight the current observational bounds that are the ultimate judges of any inflationary model.

For a homogeneous inflaton field, \( \nabla \phi = 0 \), the equation of motion for the field \( \phi \) is

\[
\ddot{\phi} + 3H \dot{\phi} + V'(\phi) = 0,
\]

where \( H^2 = \rho / 3M_p^2 \) and \( \rho \) is the energy density of the inflaton field. In the slow-roll approximation, \( \dot{\phi}^2 \ll V(\phi) \), the equation of motion for the field is approximately given by \( 3H \dot{\phi} \simeq -V'(\phi) \) with \( H^2 \simeq V(\phi)/3M_p^2 \). A necessary condition for the slow-roll approximation to be valid is that the slow-roll parameters \( \epsilon \) and \( \eta \), defined by

\[
\epsilon = \frac{M_p^2}{2} \left( \frac{V'}{V} \right)^2, \quad \eta = M_p^2 \frac{V''}{V},
\]

(3.2)
are small, $\epsilon \ll 1$ and $|\eta| \ll 1$ [14] [15]. In our hilltop model if the slow-roll conditions are satisfied, then $\epsilon \ll |\eta|$. The end of inflation coincides with $\epsilon = 1$, with $\epsilon < 1$ being a necessary condition for inflation to occur. As a result, by relaxing the slow-roll conditions, fast-roll inflation [3] is also permitted when $|\eta| \sim O(1)$. We investigate both of these possibilities later.

The vacuum fluctuations of the inflaton field generate, at horizon exit, the curvature perturbation $R$. Its amplitude and scale dependence are typically described in terms of the power spectrum and the spectral index. At the time of horizon exit during inflation, these are given respectively by

$$P_R = \frac{1}{24\pi^2 M_p^4} \left( \frac{V}{\epsilon} \right),$$

and

$$n_s - 1 = 2\eta - 6\epsilon \simeq 2\eta_0.$$  \hspace{1cm} (3.4)

The approximate expression for the spectral index is the result of the slow-roll approximation for inflation in the proximity of the maximum of the potential. The subscript ‘0’ denotes the evaluation near the maximum of the potential. In addition to density perturbations, tensor perturbations or gravitational waves are produced during the inflationary epoch. The scalar to tensor fraction is given by $r = 16\epsilon$.

According to five-year WMAP data [1], the observed amplitude of the curvature perturbation and spectral index are

$$n_s = 0.960 \pm 0.013,$$

$$P_R = 2.5 \times 10^{-9}.$$  \hspace{1cm} (3.5)

Neglecting any fine tuned cancellations between $\epsilon$ and $\eta$, and the absence of secondary fields, the observed value of the spectral index can only be realised in a scenario where both of these parameters are small. Additionally, the absence of tensor perturbations in the CMB places an upper bound on $r$, $r < 0.2$. This upper bound constrains the scale of inflation to be

$$V^{1/4} < 2 \times 10^{16} \text{ GeV}.$$  \hspace{1cm} (3.7)

If we adopt the maximum inflation scale and consider instant reheating, we require about $N = 60$ e-folds of inflation for the scales corresponding to observation to leave the horizon during inflation. Reducing the scale of inflation and delaying reheating after the end of inflation reduces the number of e-folds required [16].

As we have a hilltop model, a great deal may be understood from expanding the potential around the local maximum. Expanding the potential to second order about
the local maximum, one may write

\[ V(\phi) \simeq V_0 - \frac{m^2}{2} (\phi - \phi_{\text{max}})^2 + \cdots, \quad (3.8) \]

where the remaining terms are deemed to be negligible. Here \( V_0 \) denotes the potential energy at the maximum, and the vacuum expectation value of \( \phi \) is of the order of \( M_\phi \). In the context of our model, the above constants depend on \( b \) and \( c \):

\[ V_0 = \frac{M_\phi^2}{L_0^2} \left( -9 + \sqrt{81 - 12bc} \right)^2 \frac{(2bc - 9 + \sqrt{81 - 12bc})}{144b^3}, \]

\[ m^2 = \frac{1}{L_0^2} \left( -9 + \sqrt{81 - 12bc} \right)^2 \frac{(4bc - 27 + 3\sqrt{81 - 12bc})}{108b^3}. \quad (3.9) \]

To this degree of approximation, the slow-roll parameters become

\[ \epsilon = \left( \frac{\phi - \phi_{\text{max}}}{2M_\phi^2} \right)^2 \eta_0^2, \]

\[ \eta_0 = \frac{4}{3} \frac{(4bc - 27 + 3\sqrt{81 - 12bc})}{(2bc - 9 + \sqrt{81 - 12bc})}. \quad (3.10) \]

At the top of the potential \( |\eta_0| \) varies as \( bc \) moves in its permitted range \( 6 \leq bc < 27/4 \) as

\[ \frac{4}{3} \geq |\eta_0| = \frac{4}{3} \frac{(4bc - 27 + 3\sqrt{81 - 12bc})}{(2bc - 9 + \sqrt{81 - 12bc})} > 0, \quad (3.11) \]

with \( |\eta_0| \approx 4/3 \) correspond to small \( bc \), with increasing values of the product \( bc \) leading to smaller and smaller values of \( \eta \). As may be seen above, \( |\eta_0| = 1 \) corresponds to \( bc = 162/25 = 6.48 \), thus delineating where the slow-roll approximation ends and a fast-roll description becomes valid.

One more special value of \( bc \) requires mention. As can be seen from \( 2.18 \), as \( bc \) increases, the difference between the extrema decrease until one reaches \( bc = 27/4 \) where they coalesce. For large enough \( bc \), as may be appreciated from \( 3.10 \), \( \epsilon \) may never reach unity to end inflation. To the quadratic order above, once \( bc \) exceeds 6.1 inflation fails to end.

The above values \( bc \approx 6.1 \) and \( bc \approx 6.48 \) we have produced by expanding the potential to quadratic order. However, alternatively one may work directly with the potential \( 2.16 \) and drop the approximation. Taking into account that \( \epsilon, \eta \) are curves with maximum and a minimum respectively, it is possible to repeat the simplified analysis. In general, inflation ends provided \( bc \lesssim 6.21 \) and \( bc \approx 6.49 \) separates slow-roll from fast-roll inflation.

In the next two subsections we take a closer look at these regions. In each case we may determine the relevant quantities analytically when it is possible for a comparison with numerical results.
3.1 Slow-roll Inflation

In this subsection, we consider slow-roll solutions for the effective potential in \((2.16)\). In this class of models \(|\eta| \gg \epsilon\) along the 60 or so efolds of inflation that occur close to the maximum of the potential.

The slow-roll parameter \(\eta\) is less than the unity at the maximum of the potential for values of the parameters \(b\) and \(c\) such that \(bc \gtrsim 6.49\) As we increase the value of \(bc\) for \(b\) fixed, the potential energy between the stable and unstable vacua and the distance between the two extrema decrease. As \(bc\) approaches the upper limit \(bc = 27/4\), \(\eta\) becomes negligibly small and tends to zero. It is close to this value of \(bc\) that we obtain slow-roll solutions.

Numerical solutions for the equation of motion of \(\phi\), \((3.1)\), may be found by introducing the following dimensionless variables

\[
\bar{\phi} = \frac{\phi}{M_p}, \quad \bar{t} = \frac{t}{L_0}, \quad V = \frac{L_0^2}{M_p^2} V. \tag{3.12}
\]

In Fig.\((3.2)\) we show the number of efolds \(N\), the Hubble parameter \(H\) and the evolution of the field \(\phi\) for the case \(bc = 6.74\). The field starts rolling down in the proximity of the maximum of the potential with negligible velocity to ensure that the slow-roll conditions are satisfied. The Hubble parameter remains practically constant as the field \(\phi\) slowly rolls close to the maximum of the potential. The number of efolds of inflation at the top of the potential is about \(N = 70\) in this particular case shown in Fig.\((3.2)\). The field eventually falls into the stable vacuum, but without sufficient kinetic energy to enter a stage of coherent oscillations and reheating. The change in \(H\) as the field evolves from the top of the potential to the minimum is very small, \(\dot{H}/H^2 \ll 1\), and therefore the slow-roll conditions are never violated throughout the evolution of \(\phi\). Once the field settles down at the minimum of the potential we obtain de Sitter accelerated expansion with \(V\) constant.

The dynamics of the field for the case \(bc = 6.74\) described above, is similar to all the models of our effective potential with values of the parameters \(b\) and \(c\) such that \(bc \gtrsim 6.49\). Since \(\epsilon\) never reaches unity as the field evolves to its stable vacuum inflation continues at a different scale, determined by the actual value of \(bc\), after the field reaches the minimum of the potential. The introduction of a secondary waterfall field with a large mass would ensure that reheating occurs very rapidly while the slow-roll conditions for the dynamics of the field \(\phi\) hold at the end of inflation, like in hybrid inflationary models \([17]\). Ideally, the extra waterfall field would come from dimensional reduction of the d=8 Einstein-Yang-Mills theory. The analysis of this possibility for our extra-dimensional theory is however beyond the scope of this preliminary paper. We hope to address it in follow-up work.
Despite this model of slow-roll inflation from our extra dimensional theory not being in itself a complete realistic model, it is interesting to investigate the relation between the size of the extra dimensions and the observational bounds from the CMB. To do so we assume that inflation occurs at the top of the potential, and that it ends about 60 efolds after the cosmological scales corresponding to the observed CMB leave the horizon during inflation.

For inflation at the top of the potential, $H$ is practically constant. The equation of motion for $\phi$ is easily solved expanding the potential to quadratic order and taking $H$ constant. The evolution of $\phi$ is given by

$$\Delta \phi_N = \Delta \phi_f e^{-FN},$$

(3.13)

where $\Delta \phi_N \equiv \phi(N) - \phi_{\text{max}}$, and

$$F \equiv \frac{3}{2} \left( \sqrt{1 + \frac{4}{3}|\eta_0|} - 1 \right).$$

(3.14)

$\phi_f$ here denotes where the approximation $\eta(t) \simeq \eta_0$ breaks down and the quadratic expansion is not longer valid. The analytical approximation agrees very well with our numerical estimations. For $bc = 6.74$, $|\eta_0| = 0.276$. From Fig. (3.2) we get $|\Delta \phi_f| \simeq 0.011M_p$ and $N = 73$ for $t = 80L_0$, before $H$ and $\eta$ change substantially. For this values we get $|\Delta \phi_N| \simeq 10^{-10}M_p$, which is precisely the value we used in our numerical computation as the initial conditions for $\phi(t)$.

In these models, we get a small $|\eta_0|$ compatible with the observed spectral index for values of $bc$ very close to the limit $bc = 27/4$. Taking a small increment $\delta$ such that $bc = 27/4 - \delta$, the observational constraint on $n_s$ requires values of $\delta$ in the range $\delta = 2 \times 10^{-5} - 8 \times 10^{-5}$. Then the variation of the field $\phi$ is approximately given by $\Delta \phi_N = \Delta \phi_f e^{-|\eta_0|N}$, where $|\eta_0| \simeq M_p^2m^2/3V_0$ with $F \simeq |\eta_0| \ll 1$. On the other hand, using the observed value of $\mathcal{P}_R$, the scale of inflation is given by

$$V_0^{1/4} = 2.3 \times 10^{-2}M_p \left( \frac{\Delta \phi_f}{M_p} \right)^{1/2} e^{-|\eta_0|N/2}.$$  

(3.15)

In any event, the quantity $|\Delta \phi_f|$ can not be larger than the distance between the maximum and minimum for inflation occurring at the maximum of the potential. An approximate estimate then for $V_0^{1/4}$, given the allowed range of values of $\delta$ which gives about $|\Delta \phi_f| < 0.01M_p$, is

$$V_0^{1/4} < 1.8 \times 10^{-4}M_p = 2 \times 10^{15} \text{ GeV},$$

(3.16)

where we have used $N = 60$ in (3.13) for an estimate of $\Delta \phi_N$. The scale of inflation in this case is below the upper bound in (3.7) from the absence of tensor modes in the CMB, as it was shown in [12] for $|\Delta \phi_f| \lesssim M_p$. 

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3.2 Fast-roll inflation

Apart from the slow-roll solutions for the 4-dimensional effective action, there is a broad range of values of $bc$ for which we get inflationary solutions with $\eta \simeq O(1)$. This regime corresponds to fast-roll inflation [3, 17]. In fast-roll inflation values of $\eta \simeq O(1)$ are too large to match the observed spectral index in the CMB. Then to have a viable model of inflation, it is necessary that some additional field releases the inflaton field from generating the curvature perturbation (see e.g [4]). The inflaton may still contribute to the total amplitude of the curvature perturbation, but the spectral index would not be related to the perturbations generated by the inflaton. For $bc \lesssim 6.21$, in addition to getting values $\eta_0 \simeq O(1)$, inflation ends when $\Delta \phi_e \equiv (\phi_e - \phi_{\text{max}}) \simeq M_p$ corresponding to $\epsilon \simeq O(1)$. Below we focus on this particular case.

Again, we can reproduce the dynamics of the field $\phi$ numerically. In Fig.(3.3) we show numerical solutions of the equations of motion for $bc = 6.01$. As in the case in the previous section, the Hubble parameter remains practically constant as the field $\phi$ rolls down in the proximity of the maximum of the potential. The field eventually falls into the stable vacuum as well, but then it enters a regime of coherent oscillations. The condition for coherent oscillation for the field at the minimum of the potential is $V'' \gg H^2$ or $\eta \gg 1$, which is satisfied in this case. Then one expects that during the coherent oscillations, the field $\phi$ decays due to quantum particle creation of other fields that might couple to $\phi$. The damping of the oscillations leads to the reheating of the universe [18]. We find about $N = 55$ efolds of inflation on the top of the potential before the field enters the coherent oscillations period in this case. Then it behaves as matter domination, as it is shown in Fig. (3.3).
The variation of $V$, or alternatively $H$, from the top of the potential to the minimum increases as $bc$ approaches to the limit value 6. For $bc = 6$ we have $V = 0$ at the minimum, which corresponds to the Minkowski vacuum. Expanding the potential at the minimum about the value $bc = 6$, we get

$$\frac{L_0^2}{M_p^2} V_{\text{min}} \simeq 4 \delta + O(\delta^2),$$

(3.1)

where $\delta > 0$ and it represents a small quantity. A stable vacuum energy similar to $\Lambda \sim 10^{-120} M_p^4$ to reproduce the observed dark energy requires tuning the value of $\delta$ very close to 0.

The analytical solution for fast-roll is similar to the ones we derived in (3.13). That is

$$\Delta \phi_N = \Delta \phi_e e^{-FN},$$

(3.2)

where $\Delta \phi_N \equiv \phi(N) - \phi_{\text{max}}$ and in this case $\phi_e$ denotes the end of inflation. One can in principle take the classical value of $\Delta \phi_N$ as close as necessary to the value of the field at the maximum to get an arbitrarily large number of efolds of inflation. In the numerical estimation in Fig. (3.3) the initial value of the field is $\Delta \phi_N = 10^{-24}$. The end of inflation is determined by $\epsilon = 1$, which gives $\Delta \phi_e \simeq \sqrt{2} M_p$. For $bc = 6.01$ $\eta_0 = 1.34$. Then the number of efolds of fast-roll inflation is

$$N \simeq \frac{1}{F} \ln \left( \frac{\Delta \phi_N}{\Delta \phi_e} \right) \simeq 55,$$

(3.3)

which is approximately the number of efolds we have got in the numerical estimation shown in Fig. (3.3). In this case the analytical approximation is quite accurate reproducing our numerical results.

### 3.3 Some remarks

It would be nice if the size of the extra dimensions were constrained sufficiently by the CMB data so that we could make a prediction. They are insufficient, but the size of the extra dimensions, $L_{ex}$, can be determined from (2.8) and $\phi_{\text{min}}$ in terms of a single parameter:

$$L_{ex} \simeq \sqrt{b} L_0 \simeq \frac{1}{M_p \eta_4}.$$  

(3.4)

Some added assumptions are required to go further.

As explained in the introduction, we have opted to focus on inflation arising from the inflaton traveling between the extrema i.e. $\phi_{\text{min}} < \phi < \phi_{\text{max}}$. However, referring

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*4However, as argued in [3], if the classical displacement of the field is subject to quantum fluctuations, there is a lower bound for $\Delta \phi_N$ that amounts to $\Delta \phi_N \sim m$ for $|\eta| \sim O(1)$*
Figure 3.3: In the figures above we show the number of efolds $N$, the Hubble parameter $H$, and the evolution of the field $\phi$ as a function of time $t$ in $L_0$ units for $b = 1$ and $c = 6.74$. $H$ is given in $L_0^{-1}$ units and $\phi$ in $M_p$ units. In this case we obtain fast-roll at the top of the potential with $|\eta_0| \approx O(1)$. We set the initial conditions close enough to the maximum of the potential to ensure that we get about 70 efolds of inflation about the top of the potential.

As our inflaton starts off close to the top of a hill, $\epsilon$ is very small, and eternal inflation [19] is a possibility in all these models. As explained in a recent review [20], as the inflaton field $\phi$ rolls down the hill, the change in its classical value $\Delta \phi_{cl}$ in time $\Delta t$, is subject to additional quantum fluctuations i.e. $\Delta \phi = \Delta \phi_{cl} + \Delta \phi_q$. In the ordinary, non-eternal regime, quantum fluctuations are just small corrections to the classical story and may be ignored. However, if the classical $\dot{\phi}$ becomes very small - smaller than $H^2$ - the classical inflaton change drops below the typical size of quantum fluctuations. As a result, there is finite probability that the inflaton field will move up the hill, meaning that over the interval $\Delta t$, regions with these larger values for $\phi$ will expand more than the average. As this process is repeated for successive time intervals $\Delta t$, one ends up with an infinite number of universes, providing an echo of the multiverse [7] appearing in string theory.
Figure 3.4: In these figures we the existence of a critical value of $\phi$ in the left branch of inflation. In the first figure, the inflaton rolls over the “bump” presented by the local maximum and escapes, whereas, in the second figure, we see compactification at work. As in the previous figures $\phi$ is given in $M_p$ units whereas $t$ in units of $L_0$.

4 Vacuum stability

In general there is no guarantee that the vacuum at the end of inflation is stable against tachyons. Here, we address the constraints that arise from demanding that the final vacuum is stable against metric fluctuations to linear order in the d=8 Einstein equations. As the compactified vacuum corresponds to $\dot{\phi} = 0$, the overall spacetime becomes a direct product of a FRW spacetime and a four-sphere and there is some simplification.

To begin with, the d=8 equations of motion may be re-written

$$
\frac{R_{MN}}{16\pi G} = F_{a\,MP}^a F^P_{\,N} - \frac{g_{MN}}{12} \left( \frac{1}{2} F_{MN}^{a} F^{a\,MN} - \Lambda \right),
$$

(4.1)

where $M, N = 1, ..., 8$.

Now we consider fluctuations in the metric $\delta g_{MN}$, but will assume for simplicity that the gauge field is not varied $\delta A = 0$. We will follow the treatment which parallels work in [21, 22], using Greek indices for the non-compact and Roman for the compact space. The basic idea is to vary the Einstein equations above to the linear level in $\delta g_{MN}$. These fluctuations may be expanded in terms of the spherical harmonics on $S^4$, $Y^I$. The idea then is to check that there is no tachyonic instability by ensuring that the mass squared term in the resulting equations

$$
(\Box_{FRW} - m^2) Y^I = 0,
$$

(4.2)

is non-negative, and thus of the correct sign. If $m^2 < 0$, this would herald the onset of a tachyonic instability.
Proceeding, we firstly consider the following linearised metric fluctuations

\[
\delta g_{\mu\nu} = H_{\mu\nu} - \frac{1}{2} g_{\mu\nu} h_c^c, \quad H_{\mu\nu} = H_{(\mu\nu)} + \frac{1}{4} g_{\mu\nu} H_\rho^\rho,
\]
\[
\delta g_{\mu a} = h_{\mu a}, \quad \delta g_{ab} = h_{(ab)} + \frac{1}{4} g_{ab} h_c^c,
\]

where \(g^\mu_{\nu} H_{(\mu\nu)} = g^{ab} h_{(ab)} = 0\) and we have performed a linearised Weyl shift to decouple the FRW graviton from the internal scalars. We fix the internal gauge freedom by imposing the de Donder-type gauge conditions

\[
\nabla^a h_{(ab)} = 0.
\]

With these gauge choices, we can expand the metric fluctuations in spherical harmonics as

\[
H_{(\mu\nu)} = H^I_{(\mu\nu)} Y^I, \quad H_{\mu}^\mu = H^I Y^I_{(\mu}), \quad h_{(ab)} = \Phi^I Y^I_{(ab)},
\]
\[
h_a^a = \pi^I Y^I, \quad h_{\mu a} = B^I_{\mu} Y^I_{a},
\]

with the summation over \(I\) denoting a sum over all possible spherical harmonics.

When \(\dot{\phi} = 0\), the \(S^4\) becomes independent of the FRW space and the background Riemann tensors may be written as

\[
R_{\mu\nu\rho\sigma} = e^{A\phi} H_c^2 (g_{\mu\nu} g_{\rho\sigma} - g_{\mu\rho} g_{\nu\sigma}), \quad R_{abcd} = e^{-2\phi} L_0^{-2} (g_{ac} g_{bd} - g_{ad} g_{bc}),
\]

where we have used \(\dot{a}(t) = H_c a(t)\), with \(H_c\) denoting the constant Hubble parameter at the end of inflation. In general, the linearised Ricci tensor may be written

\[
\delta R_{MN}(h_{MN}) = -\frac{1}{2} \left[ \square h_{MN} + \nabla_M \nabla_N h_P^P - \nabla_M \nabla^P h_{PN} - \nabla_N \nabla^P h_{PM} - 2 R_{MPQN} h_{PQ} - R_M^P h_{NP} - R_N^P h_{MP} \right].
\]

As we are interested in examining the stability at the minimum when \(\dot{\phi} = 0\), we can write the d’Alembertian operator in \(d=8\) in terms of the sum of d’Alembertian operators on the FRW spacetime and \(S^4\), \(\square = \square_{FRW} + \square_{S^4}\), where \(\square_{FRW} \equiv g^\mu_{FRW} \nabla_\mu \nabla_\nu\) and \(\square_{S^4} \equiv g^a_{ab} \nabla_a \nabla_b\). We will also use \(-e^{2\phi} L_0^2 \square_{S^4} \equiv \lambda^I Y^I\), where the eigenvalues \(\lambda^I\) of the ordinary Laplacian on \(S^4\) for the various tensor harmonics are

| Harmonic | \(\lambda^I\) | Range of \(k\) |
|----------|----------------|----------------|
| \(Y^I\)  | \(k(k+3)\)     | \(k \geq 0\)   |
| \(Y^I_{a}\) | \(k(k+3) - 1\) | \(k \geq 1\)   |
| \(Y^I_{(ab)}\) | \(k(k+3) - 2\) | \(k \geq 2\)   |
Note as $\Box_{S^4}$ always makes a contribution of the correct sign, we focus on the stability of the lowest mode (smallest $k$) in each case.

We start by considering the $S^4$ variation $\delta R_{ab} = 8\pi G \delta T_{ab}$. The left hand side is

$$- \frac{1}{2} \left[ (\Box_{FRW} + \Box_{S^4}) h_{ab} + \frac{1}{4} g_{ab} (\Box_{FRW} + \Box_{S^4}) h_c^c + \nabla_a \nabla_b H^\rho_\rho - \nabla_a \nabla^\rho h_{\rho b} - \nabla_b \nabla^\rho h_{\rho a} ight] + \frac{8}{L_0^2 e^{2\phi}} h_{(ab)} - \frac{3}{2} \nabla_a \nabla_b h_c^c,$$

while $8\pi G \delta T_{ab}$ is

$$16\pi G \left[ \frac{1}{12} \Lambda h_{(ab)} + \frac{1}{48} g_{ab} h_c^c \left( \Lambda - \frac{3}{q^2 L_0^4 e^{4\phi}} \right) \right].$$

Decomposing in terms of spherical harmonics, one immediately sees that

$$\left[ (\Box_{FRW} + \Box_{S^4} - \frac{8}{3} \pi G \left( \frac{3}{q^2 L_0^4 e^{4\phi}} - \Lambda \right) ) \pi^I \right] Y^I = 0, \quad \left( H^I - \frac{3}{2} \pi^I \right) \nabla_{(a} \nabla_{b)} Y^I = 0, \quad \left( \nabla^\mu B^I_\mu \right) \nabla_{(a} Y^I_{b)} = 0, \quad \left[ (\Box_{FRW} + \Box_{S^4} - \frac{8}{L_0^2 e^{2\phi}} + \frac{8}{3} \pi G \Lambda \right] \Phi^I Y^I_{(ab)} = 0.$$

For certain low-lying scalar harmonics, some or all of their derivatives appearing above may vanish. As our concern here is stability and establishing a window in parameter space, we consider the generic case where all the derivatives of $Y^I$ are non-zero. In effect, the coefficients in the above equations must all vanish independently. From (4.11)-(4.14) we find that $H^I$ and $\pi^I$ are not independent, $B^I_\mu$ is transverse, and that the mass squared $m^2$ associated to $\pi^I$ and $\Phi^I$ are non-tachyonic provided

$$\Lambda \leq \frac{3 e^{-4\phi}}{q^2 L_0^4}, \quad \text{or} \quad bc \leq \frac{27}{4} \left[ 1 + \sqrt{1 - 4bc/27} \right]^2,$$

$$\Lambda \leq \frac{6 e^{-2\phi}}{4\pi G L_0^2}, \quad \text{or} \quad bc \leq 36 \left[ 1 + \sqrt{1 - 4bc/27} \right].$$

Neither of these constraints reduce the window of allowable parameters (2.21) noted earlier.

We now turn our attention to the mixed components of the Einstein equation $\delta R_{\mu a} = 8\pi G \delta T_{\mu a}$. We find these become

$$- \frac{1}{2} \left[ (\Box_{FRW} + \Box_{S^4}) h_{\mu a} + \frac{3}{4} \nabla_\mu \nabla_a H^\rho_\rho - \frac{3}{4} \nabla_\mu \nabla_a h^d_\rho - \nabla_\mu \nabla^\nu h_{\nu a} - \nabla_a \nabla^\nu H_{(\mu \nu)} \right]$$

$$- 3 \left( e^{4\phi} H_e^2 + \frac{e^{-2\phi}}{L_0^2} \right) h_{\mu a} = - \frac{4}{3} \frac{g_{\mu a}}{2q^2 L_0^4 e^{4\phi} - \Lambda} h_{\mu a}.$$
This in turn means
\[
\left[ (\Box_{FRW} + \Box_{S^4}) - \nabla_\mu \nabla_\nu \delta_\mu^\nu - 3 \left( e^{4\phi} H^2_c + \frac{e^{-2\phi}}{L_0^2} \right) - \frac{8\pi G}{3} \left( \frac{3e^{-4\phi}}{2q^2 L_0^2} - \Lambda \right) \right] B^I_\mu Y^I_a = 0,
\]
\[-\nabla^\nu H^I_{(\mu\nu)} \nabla_a + \frac{3}{4} \nabla_\mu H^I \nabla_a - \frac{3}{4} \nabla_{\mu} \pi^I \nabla_a \right] Y^I = 0.
\]
The upper equation above, when evaluated at the compactification radius, places
looser stability constraint derived from the graviphoton fluctuation \( B^I_\mu \)
\[
\Lambda \leq \frac{3e^{-4\phi}}{2q^2 L_0^2} + \frac{9e^{-2\phi}}{4\pi G L_0^2} + \frac{9e^{4\phi} H^2_c}{8\pi G}.
\]
(4.18)
Even when \( H_c = 0 \), this condition may be re-written in terms of \( b \) and \( c \) as
\[
bc \leq \frac{27}{8} [1 + \sqrt{1 - 4bc/27}] [5 + \sqrt{1 - 4bc/27}].
\]
(4.19)
This again, does not constrain the parameters considered earlier.
Finally we consider the FRW part \( \delta R_{\mu\nu} = 8\pi G \delta T_{\mu\nu} \)
\[
-\frac{1}{2} \left[ (\Box_{FRW} + \Box_{S^4}) H_{\mu\nu} + \nabla_\mu \nabla_\nu H^\rho_\rho - \nabla_\mu \nabla^\rho H_{\mu\rho} - \nabla_\nu \nabla^\rho H_{\rho\mu} - 2R_{\mu\rho\nu\sigma} H^{\rho\sigma} \right] - R^\rho_\mu H_{\rho\nu} - R_\nu^\rho H_{\rho\mu} + \frac{1}{4} g_{\mu\nu}(\Box_x + \Box_y) h_c^e
\]
\[= 16\pi G \left[ -\frac{1}{12} \left( H_{\mu\nu} - \frac{1}{2} g_{\mu\nu} h_c^e \right) \left( \frac{3e^{-4\phi}}{2q^2 L_0^2} - \Lambda \right) - \frac{1}{12} g_{\mu\nu} \left( -\frac{3e^{-4\phi}}{4q^2 L_0^2} h_c^e \right) \right],
\]
(4.20)
or in terms of spherical harmonics that
\[
\left[ \delta R_{\mu\nu}(H^I_{(\rho\sigma)}) - \frac{1}{2} \Box_{S^4} H^I_{(\mu\nu)} + \frac{4\pi G}{3} \left( \frac{3e^{-4\phi}}{2q^2 L_0^2} - \Lambda \right) H^I_{(\mu\nu)} + \frac{1}{4} g_{\mu\nu}(\Box_{FRW} + \Box_{S^4}) \pi^I \right.
\]
\[-\frac{2\pi G}{3} g_{\mu\nu} \left( \frac{3e^{-4\phi}}{2q^2 L_0^2} - \Lambda \right) \pi^I - \frac{\pi Ge^{-4\phi}}{q^2 L_0^4} g_{\mu\nu} \pi^I \right] Y^I = 0.
\]
(4.21)
We now find ourselves in a position to establish the existence of the FRW graviton.
The graviton comes from the constant \( Y^I \) mode of the equation (4.21). Using (4.11),
it may be brought to the form
\[
\delta R_{\mu\nu}(H^I_{(\rho\sigma)}) + \frac{4\pi G}{3} \left( \frac{3e^{-4\phi}}{2q^2 L_0^2} - \Lambda \right) H^I_{(\mu\nu)} = 0.
\]
(4.22)
One may readily check that it is the correct equation for the linearised graviton in
FRW (4.6), by using the equations of motion (3.1) to recast it as
\[
\delta R_{\mu\nu}(H^I_{(\rho\sigma)}) - 3e^{4\phi} H^2_c H^I_{(\mu\nu)} = 0.
\]
(4.23)
One may then simply proceed as in [21] by defining a shifted massive tensor field
\( \varphi^I_{(\mu\nu)} = H^I_{(\mu\nu)} + C(\nabla(\mu \nabla_\nu) \pi^I) \), where one may determine the constant \( C \) so that \( \varphi^I_{(\mu\nu)} \)
is transverse \( \nabla^\mu \varphi^I_{(\mu\nu)} = 0 \). The contribution to the mass from the spherical harmonic
eigenvalues have the correct sign, so these modes are also stable.
5 Conclusions

Evoking extra-dimensional Einstein gravity with a positive cosmological constant and a reduction on a compact space with a Yang-Mills instanton, we have obtained an effective potential dependent on two parameters $b, c$ which offers much promise as an inflationary model. One neat feature is that the size of the compact space is dynamically stabilised during the inflationary process.

We have explored the parameter space for models of inflation consistent with current observational bounds from CMB data, and have identified some attractive features. In particular, as the inflaton starts off rolling from a local maximum, our model falls into the class of hilltop models, and as such, any number of efolds may be accommodated classically.

We have also looked at metric fluctuations to see how the requirement that the final vacuum at the local minimum be stable, affects the allowed window of the parameters $b, c$. We find that it does not constrain the permissible values further. In general, the fluctuations on the Yang-Mills instanton should also be incorporated into the analysis, but treating spherical harmonics for non-Abelian gauge fields is quite involved. We recognise that the analysis here is not complete and would like to see if stability is still guaranteed in the presence of both gauge and metric fluctuations in future work.

To fulfill the potential of this set-up as a viable model of inflation, we need to consider the introduction of secondary fields for both slow-roll ($bc \gtrsim 6.5$) and fast-roll inflation ($bc \lesssim 6.5$). In the former case, we require a waterfall field to ensure reheating, while in the latter the inflaton has to be liberated from the role of generating the curvature perturbations, so that observational bounds are not violated. Modifying the higher dimensional action by including higher order corrections, or including a warp-factor in the reduction ansatz, offer means of introducing extra scalars. It would be interesting to address these in future work.

Another exciting area of immediate future investigation is generalisations to different dimensions. For simplicity, we considered the Yang-Mills instanton in a $(4+4)$-dimensional set-up. In general, to stabilise a topological configuration of a Yang-Mills field in dimensions greater than four, we need higher order terms of the gauge field strength. Any meaningful comparison between compactifications from different dimensions would hopefully identify a mechanism for picking out the four-dimensions of our observed universe.
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References

[1] E. Komatsu et al. [WMAP Collaboration], “Five-Year Wilkinson Microwave Anisotropy Probe (WMAP) Observations:Cosmological Interpretation,” Astrophys. J. Suppl. 180, 330 (2009) [arXiv:0803.0547 [astro-ph]].

[2] A. H. Guth, “The Inflationary Universe: A Possible Solution To The Horizon And Flatness Problems,” Phys. Rev. D 23, 347 (1981), A. D. Linde, “A New Inflationary Universe Scenario: A Possible Solution Of The Horizon, Flatness, Homogeneity, Isotropy And Primordial Monopole Problems,” Phys. Lett. B 108, 389 (1982), A. J. Albrecht and P. J. Steinhardt, “Cosmology For Grand Unified Theories With Radiatively Induced Symmetry Breaking,” Phys. Rev. Lett. 48, 1220 (1982).

[3] A. D. Linde, “Fast-Roll Inflation,” JHEP 0111, 052 (2001) [arXiv:hep-th/0110195].

[4] D. H. Lyth and A. Riotto, “Particle physics models of inflation and the cosmological density perturbation,” Phys. Rept. 314, 1 (1999) [arXiv:hep-ph/9807278]; A. R. Liddle and D. H. Lyth, “Cosmological inflation and large-scale structure,”

[5] M. Bastero-Gil, V. Di Clemente and S. F. King, “Preheating curvature perturbations with a coupled curvaton,” Phys. Rev. D 70 (2004) 023501 [hep-ph/0311237]; E. W. Kolb, A. Riotto and A. Vallinotto, “Curvature perturbations from broken symmetries”, [astro-ph/0410546] G. Dvali, A. Gruzinov and M. Zaldarriaga, “A new mechanism for generating density perturbations from inflation,” Phys. Rev. D 69, 023505 (2004) [astro-ph/0303591]; G. Dvali, A. Gruzinov and M. Zaldarriaga, “Cosmological Perturbations From Inhomogeneous Reheating, Freeze-Out, and Mass Domination,” Phys. Rev. D 69, (2004) 083505 [astro-ph/0305548]; L. Kofman, “Probing string theory with modulated
D. H. Lyth and D. Wands, “Generating the curvature perturbation without an inflaton,” Phys. Lett. B 524, (2002) 5 [hep-ph/0110002]; T. Moroi and T. Takahashi, “Effects of cosmological moduli fields on cosmic microwave background,” Phys. Lett. B 522 (2001) 215 [Erratum-ibid. B 539 (2002) 303] [hep-ph/0110096]. D. H. Lyth, C. Ungarelli and D. Wands, “The primordial density perturbation in the curvaton scenario,” Phys. Rev. D 67 (2003) 023503 [astro-ph/0208055].

[6] A. G. Riess et al. [Supernova Search Team Collaboration], “Observational evidence from supernovae for an accelerating universe and a cosmological constant,” 11619981009 [arXiv:astro-ph/9805201], S. Perlmutter et al. [Supernova Cosmology Project Collaboration], “Measurements of Ω and Λ from 42 high-redshift supernovae,” 5171999565 [arXiv:astro-ph/9812133].

[7] L. Susskind, “The anthropic landscape of string theory,” arXiv:hep-th/0302219

[8] S. Kachru, R. Kallosh, A. Linde and S. P. Trivedi, “De Sitter vacua in string theory,” 682003046005 [arXiv:hep-th/0301240]; S. Kachru, R. Kallosh, A. Linde, J. M. Maldacena, L. P. McAllister and S. P. Trivedi, “Towards inflation in string theory,” JCAP 0310 (2003) 013 [arXiv:hep-th/0308055]; S. B. Giddings, S. Kachru and J. Polchinski, “Hierarchies from fluxes in string compactifications,” 662002106006 [arXiv:hep-th/0105097]; O. DeWolfe, A. Giryavets, S. Kachru and W. Taylor, “Type IIA moduli stabilization,” 05072005066 [arXiv:hep-th/0505160]. J. P. Derendinger, C. Kounnas, P. M. Petropoulos and F. Zwirner, “Superpotentials in IIA compactifications with general fluxes,” 7152005211 [arXiv:hep-th/0411276]; P. G. Câmara, A. Font and L. E. Ibáñez, “Fluxes, moduli fixing and MSSM-like vacua in a simple IIA orientifold,” 05092005013 [arXiv:hep-th/0506066]; G. Villadoro and F. Zwirner, “N = 1 effective potential from dual type-IIA D6/O6 orientifolds with general fluxes,” 05062005047 [arXiv:hep-th/0503169]; C. Caviezol, P. Koerber, S. Kors, D. Lust, T. Wrase and M. Zagermann, “On the Cosmology of Type IIA Compactifications on SU(3)-structure Manifolds,” JHEP 0904, 010 (2009) arXiv:0812.3551 [hep-th]].

[9] H. Kihara, M. Nitta, M. Sasaki, C. M. Yoo and I. Zaballa, “Dynamical Compactification and Inflation in Einstein-Yang-Mills Theory with Higher Derivative Coupling,” arXiv:0906.4493 [hep-th].

[10] E. Cremmer and J. Scherk, “Spontaneous Compactification Of Space In An Einstein Yang-Mills Higgs Model,” Nucl. Phys. B 108, 409 (1976).
[11] S. M. Carroll, M. C. Johnson and L. Randall, “Dynamical compactification from de Sitter space,” [arXiv:0904.3115 [hep-th]].

[12] L. Boubekeur and D. H. Lyth, “Hilltop inflation,” JCAP 0507, 010 (2005) [arXiv:hep-ph/0502047].

[13] K. Kohri, C. M. Lin and D. H. Lyth, “More hilltop inflation models,” JCAP 0712, 004 (2007) [arXiv:0707.3826 [hep-ph]].

[14] A. R. Liddle and D. H. Lyth, “Cobe, Gravitational Waves, Inflation And Extended Inflation,” Phys. Lett. B 291, 391 (1992) [arXiv:astro-ph/9208007].

[15] A. R. Liddle and D. H. Lyth, “The Cold dark matter density perturbation,” Phys. Rept. 231, 1 (1993) [arXiv:astro-ph/9303019].

[16] L. Alabidi and D. H. Lyth, “Inflation models and observation,” JCAP 0605, 016 (2006) [arXiv:astro-ph/0510441].

[17] A. D. Linde, “Hybrid inflation,” Phys. Rev. D 49, 748 (1994) [arXiv:astro-ph/9307002].

[18] E. W. Kolb and M. S. Turner, “The Early universe,” Front. Phys. 69 (1990) 1.

[19] P. J. Steinhardt, “Natural Inflation,” Invited talk given at Nuffield Workshop on the Very Early Universe, Cambridge, England, Jun 21 - Jul 9, 1982; A. Vilenkin, “The Birth Of Inflationary Universes,” Phys. Rev. D 27, 2848 (1983); A. D. Linde, “Eternal Chaotic Inflation,” Mod. Phys. Lett. A 1, 81 (1986); A. D. Linde, “Eternally Existing Selfreproducing Chaotic Inflationary Universe,” Phys. Lett. B 175, 395 (1986);

[20] A. H. Guth, “Eternal inflation and its implications,” J. Phys. A 40, 6811 (2007) [arXiv:hep-th/0702178].

[21] O. DeWolfe, D. Z. Freedman, S. S. Gubser, G. T. Horowitz and I. Mitra, “Stability of \( \text{AdS}(p) \times \text{M}(q) \) compactifications without supersymmetry,” Phys. Rev. D 65, 064033 (2002) [arXiv:hep-th/0105047].

[22] R. Bousso, O. DeWolfe and R. C. Myers, “Unbounded entropy in spacetimes with positive cosmological constant,” Found. Phys. 33, 297 (2003) [arXiv:hep-th/0205080]; J. E. Martin and H. S. Reall, “On the stability and spectrum of non-supersymmetric AdS(5) solutions of M-theory compactified on Kahler-Einstein spaces,” JHEP 0903, 002 (2009) [arXiv:0810.2707 [hep-th]].