Non Scale-Invariant
Topological Landau-Ginzburg Models

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Abstract

The Landau-Ginzburg formulation of two-dimensional topological sigma models on the target space with positive first Chern class is considered. The effective Landau-Ginzburg superpotential takes the form of logarithmic type which is characteristic of supersymmetric theories with the mass gap. The equations of motion yield the defining relations of the quantum cohomology ring. Topological correlation functions in the $CP^{n-1}$ and Grassmannian models are explicitly evaluated with the use of the logarithmic superpotential.
It is well-known that the target space of $N = 2$ supersymmetric sigma models in two dimensions is a Kähler manifold \([1]\). The coefficient of the one-loop $\beta$-function is then fixed by the first Chern class $c_1$ of Kähler manifolds. For $c_1 > 0$, which is the case of the $CP^{n-1}$ and the Grassmannian, the $N = 2$ sigma models are asymptotically free and possess the dynamically generated mass gap. As emphasized in \([2]\) the $N = 2$ models with the mass gap reduce at large distances to topological field theories which capture a non-trivial dynamics of the supersymmetric vacua. An important manifestation of such property is the quantum cohomology ring of Kähler manifolds \([3, 4]\). Another characteristic aspect of supersymmetric theories with the mass gap is observed in the effective superpotential. It is seen that the logarithmic superpotential is ubiquitous in the low-energy effective description of supersymmetric, asymptotically free theories \([5]\).

In this paper we study the two-dimensional topological sigma model with $c_1 > 0$, which is obtained by twisting the corresponding $N = 2$ sigma model \([3, 6]\). Our point is to employ the logarithmic effective superpotential upon evaluating correlation functions. We first discuss the $CP^{n-1}$ model for which the logarithmic effective superpotential has been known for a long time \([7]\). In a more conventional approach, on the other hand, the same class of topological correlation functions has already been calculated in the Landau-Ginzburg (LG) formulation of perturbed $N = 2$ superconformal theories \([8]\). These theories are completely characterized by polynomial superpotentials. Comparing the results obtained in both ways we shall discuss the relation between the two approaches. Then we extend our considerations to the Grassmannian models, and will clarify how logarithmic superpotentials serve as effective superpotentials in topological field theory.

We start with the $N = 2$ sigma model whose target space is the complex $(n - 1)$-dimensional projective space $CP^{n-1}$. The Lagrangian of the $CP^{n-1}$ model is given by

$$L = \int d^4 \theta \left[ \sum_{i=1}^{n} \bar{\Phi}_i e^{-V} \Phi_i + \frac{n}{2g} V \right],$$

in two-dimensional $N = 2$ superspace, where $\Phi_i$ are chiral superfields and $g$ is a dimen-
sionless coupling constant. \( V \) is an auxiliary \( N = 2 \) vector superfield by virtue of which we see manifest \( U(1) \) gauge symmetry which is otherwise hidden.

If we eliminate the auxiliary field \( V \) by means of its equation of motion we have the standard form of \( N = 2 \) sigma model. In an interesting paper [8] (see also [9]), however, the matter fields \( \Phi_i \) are integrated out. Then one ends up with the effective action

\[
S_{\text{eff}} = \frac{1}{4\pi} \int d^2x \left[ \int d^2\theta W_{\text{log}}(\lambda) + h.c. + \cdots \right],
\]

(2)

where \( \lambda \) is a gauge invariant, twisted chiral superfield expressed in terms of the superderivative \( \lambda = D_L\bar{D}_RV \) and the ellipses stand for possible \( D \)-term contributions whose detail is irrelevant to our consideration in topological field theory. The explicit form of the superpotential reads [4, 6]

\[
W_{\text{log}}(\lambda) = \lambda \left( \log \frac{\lambda^n}{\mu^n} - n \right),
\]

(3)

where \( c_1 = n \) and \( \mu \) is a dynamically generated mass scale.

The logarithmic superpotential (3) has the following two important properties: First the effective action (2) produces the correct anomaly structure of the \( N = 2 \) \( CP^{n-1} \) model. An axial, conformal and \( \gamma \) trace anomaly are all reproduced by taking appropriate variations of \( S_{\text{eff}} \) [7]. Second the equation of motion \( \partial_{\lambda}W_{\text{log}}(\lambda) = 0 \) for \( \lambda \) gives rise to

\[
\lambda^n = \mu^n.
\]

(4)

This is nothing but the relation for the quantum cohomology ring of \( CP^{n-1} \) under the identification of \( \lambda \) with the harmonic one-form of \( CP^{n-1} \) [3, 4]. Thus we observe that the potential (3) is equipped with the desired property required to be the effective potential for the topological \( CP^{n-1} \) model.

Before turning to the calculation of topological correlation functions with the use of (3) we review how a suitably perturbed superpotential in the LG formulation of \( N = 2 \) superconformal field theory works in describing the \( CP^{n-1} \) quantum cohomology ring.
The $A_n$-type LG superpotential is a polynomial of a single chiral superfield $X$ and takes the form \[ W(X) = \frac{X^{n+1}}{n+1}, \] \[ (5) \]
The equation of motion for $X$ obtained from (5) may be interpreted as the relation for the classical cohomology ring of $CP^{n-1}$ if we again identify $X$ with the harmonic one-form of $CP^{n-1}$. To generate the quantum cohomology ring of $CP^{n-1}$ the potential (5) needs to be perturbed by the most relevant operator \[ W_{pol}(X) = \frac{X^{n+1}}{n+1} - \beta X, \] \[ (6) \]where $\beta$ is a perturbation parameter. The equation of motion $\partial_{X}W_{pol}(X) = 0$ then yields the relation \[ X^n = \beta. \] \[ (7) \]Hence, as long as the cohomology ring structure is concerned, both potentials $W_{log}$ in (3) and $W_{pol}$ in (6) work well and we see the correspondence $\lambda \leftrightarrow X$, $\mu^n \leftrightarrow \beta$.

Let us now evaluate topological correlation functions defined on the genus $g$ Riemann surface using the residue formula [12, 13]
\[ \langle F \rangle = \sum_{dW=0} FH^{g-1}, \] \[ (8) \]where $F$ is a function of topological observables and $H$ is the Hessian of superpotential. Here the sum is taken over all the critical points of $W$ where $dW = 0$. It is trivial that the critical points are located at $\lambda = \mu e^{\frac{2\pi i}{n}j}$ ($X = \beta^{1/n} e^{\frac{2\pi i}{n}j}$) for $W_{log}$ ($W_{pol}$) with $j = 0, 1, \ldots, n - 1$. Thus both potentials have $n$-fold degenerate vacua. This is the correct degeneracy since the Witten index of $CP^{n-1}$ is equal to the Euler characteristic $\chi$ which is known to be $\chi = n$ [14]. Substituting the Hessians given by $\partial^2_{X}W_{log} = n/\lambda$ and $\partial^2_{X}W_{pol} = nX^{n-1}$ it is straightforward to obtain
\[ \langle \lambda^m \rangle_{log} = \sum_{dW_{log}=0} \lambda^m(n/\lambda)^{g-1} = n^g \mu^{n(k+1-g)}, \] \[ (9) \]
\[ \langle X^m \rangle_{\text{pol}} = \sum_{dW_{\text{pol}}=0} X^m(nX^{n-1})^{g-1} = n^g \beta^k, \]

where \( m = nk + (n - 1)(1 - g) \). Notice that \( \langle \lambda^m \rangle = \langle X^m \rangle = 0 \) if \( m \neq nk + (n - 1)(1 - g) \).

In both results correlation functions are nonvanishing only when \( m = nk + (n - 1)(1 - g) \) where \( k \) is understood as the degree of holomorphic maps from the genus \( g \) Riemann surface to \( CP^{n-1} \). This is the well-known \( U(1) \) charge conservation law in the topological sigma model (without coupling to topological gravity); \( \sum_\alpha q_\alpha = d(1 - g) + kc_1 \) where \( q_\alpha \) are \( U(1) \) charges and \( d \) is the complex dimension of target manifold [3, 6]. Under the identification \( \beta = \mu^n \), however, we observe that (9) and (10) differ by a factor of \( \mu^{n(1-g)} \). Otherwise the logarithmic potential yields the right result derived by using \( W_{\text{pol}} \).

In order to understand this discrepancy properly we have to remember that the Hessian is identified as the top element \( \phi_{\text{top}} \) of the chiral ring [15]. Since \( \phi_{\text{top}} \) is the operator corresponding to the spectral flow, the insertion of \( H \) in (8) is responsible for realizing the correct \( U(1) \) charge conservation [13]. For \( W_{\text{pol}} \) we have \( H_{\text{pol}} = nX^{n-1} = \phi_{\text{top}} \), whereas for \( W_{\text{log}} \) we have \( H_{\text{log}} = n/\lambda \) which is not quite the form of \( \phi_{\text{top}} = \lambda^{n-1} \). Noting the relation (4), however, we find \( H_{\text{log}} \simeq \mu^{-n}\lambda^{n-1} = \mu^{-n}\phi_{\text{top}} \). This is a simple, but key observation in the present paper, based on which we understand why (9) and (10) differ by a factor of \( \mu^{n(1-g)} \). Notice also that \( n \) is the first Chern class of \( CP^{n-1} \) which determines the one-loop \( \beta \)-function, and hence controls the size of scaling violation.

Let us now turn to the \( N = 2 \) Grassmannian model. The Grassmannian \( Gr(N, M) \) is defined to be the set of complex \( N \)-dimensional linear subspaces of a \( (N+M) \)-dimensional complex vector space, and as a homogeneous space, \( Gr(N, M) \) is expressed as \( U(N+M)/U(N) \times U(M) \), which is a natural extension of \( CP^{n-1} \). Actually \( Gr(1, n-1) \) is nothing but \( CP^{n-1} \). In \( N = 2 \) superspace formalism, the Lagrangian of the Grassmannian model
takes the form

$$L = \int d^4 \theta \left[ \sum_{i=1}^{N+M} \sum_{a,b=1}^N \bar{\Phi}_i^a (e^{-V})^{ab} \Phi_i^b + \frac{N+M}{2g} \text{Tr} V \right],$$

(11)

where $\Phi$ is a matrix chiral superfield and $V$ is an $N \times N$ matrix-valued $U(N)$ vector superfield.

In contrast to the $CP^{n-1}$ model, the explicit calculation of an effective superpotential for the Grassmannian is a formidable task, though (11) is quadratic with respect to $\Phi$. We have to look for possible indirect ways to find the effective potential. One way is to examine to what extent one can control the form of the superpotential under the requirement of $N = 2$ supersymmetry, $U(N)$ gauge symmetry and the correct anomaly structure. We were able to write down a few candidate superpotentials which are $N = 2$ supersymmetric as well as $U(N)$ gauge invariant and yield desired anomalies. However these trial potentials do not have the correct Witten index, i.e. the Euler characteristic for $Gr(N,M)$.

To overcome this difficulty we follow another route as discussed at length in [9]. A fundamental field variable to describe the effective action is the field-strength superfield $\Lambda$ which is gauge covariant, rather than gauge invariant, in the non-abelian case. The cohomology ring, on the other hand, should be generated by the gauge invariant objects $X_i$ ($i = 0, 1, \cdots, N$) with $X_0 = 1$. The relation between $\Lambda$ and $X_i$ is given by [9]

$$\det(1 + t\Lambda) = 1 + \sum_{i=1}^N X_i t^i.$$  \hspace{1cm} (12)

Furthermore, under the assumption that $\Lambda$ and $\bar{\Lambda}$ commute, $\Lambda$ may be reduced to the diagonal matrix, $\Lambda = \text{diag}(\lambda_1, \lambda_2, \cdots, \lambda_N)$. This is the idea of abelianization in [9]. Then the effective superpotential is written as [9, 2]

$$W_{\log} = \sum_{a=1}^N \log \left( \lambda_a \mu^{N+M} \right) - N - M,$$

(13)
where \( N + M \) is the first Chern class of \( Gr(N, M) \). In view of (3) the effective potential \( W_{\text{eff}} \) looks like \( N \) copies of the \( CP^{N+M-1} \) model. The equation of motion \( \partial_{\lambda_a} W_{\text{log}} = 0 \) gives us

\[
\lambda_a^{N+M} = \mu^{N+M}, \quad a = 1, 2, \cdots, N.
\]  

The quantum cohomology ring of \( Gr(N, M) \) is generated by \( \{X_i\} \) which are symmetric functions of \( \lambda_a \) subject to (14).

Let us check the Witten index using (13). When \( \Lambda \) is reduced to the diagonal matrix the path integral measure for \( \Lambda \) becomes 

\[
\prod_a d\lambda_a \triangle(\lambda)^2
\]

where the Vandermonde determinant \( \triangle(\lambda) = \prod_{a<b}(\lambda_a - \lambda_b) \) is the Jacobian arising from the angular part. Thus we see that \( \lambda_a \)'s repel each other, and hence in the vacuum configuration \( \{\lambda_a\} \) we have \( \lambda_a \neq \lambda_b \) where \( \lambda_a = \mu e^{\frac{2\pi j_a}{N+M}} \) (\( j_a = 0, 1, \cdots, N + M - 1 \)) from (14). Moreover \( X_i \) is expressed as a symmetric polynomial in \( \lambda_a \), so we have the permutation symmetry of \( N \) objects. Therefore the number of degenerate vacua turns out to be

\[
\frac{(M+N)(M+N-1)\cdots(M+1)}{N!} = \binom{N+M}{N}.
\]  

This is the Euler characteristic for \( Gr(N, M) \), so we find that (13) possesses the desirable vacuum structure.

We briefly recall here the cohomology ring of \( Gr(N, M) \) which is generated by \( X_0, X_1, \cdots, X_N \) where \( X_j \) is a \((j, j)\) form \( [16] \). First introduce the following polynomials

\[
X^{(N)}(t) = 1 + \sum_{i=1}^{N} X_i t^i, \quad Y^{(N)}(t) = (X^{(N)}(-t))^{-1} = \sum_{n \geq 0} Y_n^{(N)} t^n.
\]  

Then the ideal of the classical \( Gr(N, M) \) ring is given by

\[
Y_n^{(N)} = 0, \quad n = M + 1, \cdots, M + N.
\]  

Furthermore, let \( \mathcal{L} \)

\[
W_i^{(N)}(t) = -\log(X^{(N)}(-t)) = \sum_{i \geq 0} W_i^{(N)} t^i,
\]
then we have
\[
\frac{\partial W^{(N)}(t)}{\partial X_i} = -Y^{(N)}(t)(-t)^i, \quad i = 1, 2, \ldots, N,
\] (19)
thereby
\[
\frac{\partial W^{(N)}_{N+M+1}}{\partial X_i} = (-1)^{i+1}Y^{(N)}_{N+M+1-i}, \quad i = 1, 2, \ldots, N.
\] (20)
Hence the generating function for the classical relation (17) is given by \(W^{(N)}_{N+M+1}(X_i)\) which is also regarded as the (unperturbed) LG superpotential for \(Gr(N, M)\) \(N = 2\) superconformal theory [15, 17].

To consider the quantum ring of \(Gr(N, M)\) the superpotential \(W^{(N)}_{N+M+1}\) is perturbed by the most relevant operator \(X_1\) [8]. The deformed potential reads
\[
W_{pol} \equiv W_{N+M+1}(X_i) - \beta X_1.
\] (21)
The equation of motion \(\partial_{X_i}W_{pol} = 0\) gives
\[
(-1)^{i+1}Y^{(N)}_{N+M+1-i} = \beta \delta_{i,1},
\] (22)
which is the defining relation of the \(Gr(N, M)\) quantum cohomology ring.

Let us write [17, 18, 8]
\[
X^{(N)}(t) = \prod_{a=1}^{N} (1 + tq_a),
\] (23)
then (21) is rewritten as
\[
W_{pol} = \sum_{a=1}^{N} \left( \frac{q_a^{N+M+1}}{N + M + 1} - \beta q_a \right).
\] (24)
Using the chain rule \(\partial_{X_i}W_{pol} = 0 = (\partial_{X_i}q_a)\partial_{q_a}W_{pol}\) it is seen that (22) is obtained from
\[
q_a^{N+M} = \beta, \quad a = 1, 2, \ldots, N,
\] (25)
where \(q_a \neq q_b\) has been assumed so that \(\det(X_i/q_a) = \prod_{a<b}(q_a - q_b)\) is nonvanishing.

Note here that (24) takes the form of \(N\) copies of the deformed \(A_{N+M}\)-type LG theory. Moreover, from (14) and (25) we see the important correspondence \(\lambda_a \leftrightarrow q_a, \mu^{N+M} \leftrightarrow \beta\).
Having two types of the potential (13) and (21) for the Grassmannian, we now wish to calculate topological correlation functions with the aid of (8). For $W_{\text{pol}}$ we substitute $H_{\text{pol}}(X) = (-1)^{N(N-1)/2} \det(\partial_i \partial_j W_{\text{pol}})$ with $\partial_i \equiv \partial / \partial X_i$, whereas for $W_{\text{log}}$ we have to take into account the Jacobian factor as pointed out before, and thus the “effective” Hessian $H_{\text{log}}$ is given by $H_{\text{log}}(\lambda) = (-1)^{N(N-1)/2} \Delta(\lambda)^{-2} \det(\partial_a \partial_b W_{\text{log}})$ with $\partial_a \equiv \partial / \partial \lambda_a$. Let us examine the following two simple examples:

**Gr(2,2) model**

For the logarithmic potential, we get

$$\langle X_1^n X_2^m \rangle_{\text{log}} = (-1)^k 2^k - m + 1 + g \beta k 4(k+2(1-g))$$

(26)

for $n + 2m = 4k + 4(1-g)$ with $n \neq 0$, and

$$\langle X_2^m \rangle_{\text{log}} = [2^{2g-1} + (-1)^k 2^{3g-1}] \beta k 4(k+2(1-g))$$

(27)

for $2m = 4k + 4(1-g)$, where $X_1 = \lambda_1 + \lambda_2$ and $X_2 = \lambda_1 \lambda_2$. On the other hand, employing the polynomial potential

$$W_{\text{pol}} = \frac{X_1^5}{5} - X_1^3 X_2 + X_1 X_2^2 - \beta X_1,$$

(28)

we evaluate correlation functions as

$$\langle X_1^n X_2^m \rangle_{\text{pol}} = (-1)^k 2^{2k-m+1+g} \beta^k$$

(29)

for $n + 2m = 4k + 4(1-g)$ with $n \neq 0$, and

$$\langle X_2^m \rangle_{\text{pol}} = [2^{2g-1} + (-1)^k 2^{3g-1}] \beta^k$$

(30)

for $2m = 4k + 4(1-g)$. In (26)-(30) correlation functions vanish if $n + 2m \neq 4k + 4(1-g)$. For both types of the potential we confirm the correct $U(1)$ charge selection rule with degree $k \equiv$ instantons of the topological Grassmannian model, though there exists a
discrepancy by a factor of $\mu^{8(1-g)}$ (putting $\beta = \mu^4$) as in the case of $CP^{n-1}$. Let us next proceed to more interesting example.

**Gr(2, 3) model**

After some algebra we obtain correlation functions for the logarithmic potential

$$\langle X^n \rangle_{\log} = 5(5\sqrt{5})^{g-1}\mu^{5(k+2(1-g))} \left[ \left( \frac{\sqrt{5} - 1}{2} \right)^{5(k-g+1)} + (-1)^k \left( \frac{\sqrt{5} + 1}{2} \right)^{5(k-g+1)} \right]$$

(31)

for $n = 5k + 6(1 - g)$, where $X_1 = \lambda_1 + \lambda_2$. For the polynomial potential

$$W_{pol} = \frac{X_1^6}{6} - X_1^4X_2 + \frac{3}{2}X_1^2X_2^2 - \frac{X_3}{3} - \beta X_1,$$

(32)

we get

$$\langle X^n \rangle_{pol} = 5(5\sqrt{5})^{g-1}\beta^k \left[ \left( \frac{\sqrt{5} - 1}{2} \right)^{5(k-g+1)} + (-1)^k \left( \frac{\sqrt{5} + 1}{2} \right)^{5(k-g+1)} \right]$$

(33)

for $n = 5k + 6(1 - g)$. Replacing $\beta$ by $-\beta$ reproduces the result derived earlier in [8]. In (31) and (33) $\langle X^n \rangle = 0$ if $n \neq 5k + 6(1 - g)$. Thus, the situation is similar to the $Gr(2, 2)$ and $CP^{n-1}$ models, but this time the difference is by a factor of $\mu^{10(1-g)}$ (putting $\beta = \mu^5$).

The origin of the discrepancy observed above is figured out by examining the relation between the Hessian $H$ and the top element $\phi_{top}$ of the chiral ring as in the previous case of $CP^{n-1}$. For $W_{\log}$ we have

$$H_{\log}(\lambda) = (-1)^{\frac{N(N-1)}{2}} \Delta(\lambda)^{-2} \det \left( \frac{\partial^2 W_{\log}}{\partial \lambda_a \partial \lambda_b} \right) = \frac{(-1)^{\frac{N(N-1)}{2}} (N + M)^N}{\prod_{a<b} (\lambda_a - \lambda_b)^2 \prod_{a=1}^{N} \lambda_a},$$

(34)

while for $W_{pol}$ we get

$$H_{pol}(X)_{|dW=0} = (-1)^{\frac{N(N-1)}{2}} \det \left( \frac{\partial^2 W_{pol}}{\partial X_i \partial X_j} \right)_{|dW=0} = \left( -1 \right)^{\frac{N(N-1)}{2}} \frac{(N + M)^N \prod_{a=1}^{N} q_a^{N+M-1}}{\prod_{a<b} (q_a - q_b)^2}.$$

(35)

Under the correspondence $\lambda_a \leftrightarrow q_a$ we find

$$\frac{H_{pol}(X)}{H_{\log}(\lambda)}_{|dW=0} \approx \prod_{a=1}^{N} \lambda_a^{N+M} = \mu^{N(N+M)},$$

(36)
where we have used the relation (14) of the quantum ring. Since $H_{pol}(X) = \phi_{top}$ in the polynomial-type LG description, it follows from (36) that $H_{log}(\lambda) \simeq \mu^{-N(N+M)} \phi_{top}$. This explains why we encountered the extra factor $\mu^{8(1-g)} \mu^{10(1-g)}$ for the $Gr(2, 2) (Gr(2, 3))$ model with $W_{log}$. Thus, whenever we use the topological residue formula in the theory with logarithmic potential which is characteristic of scaling violation, we learn that $H_{log}$ is replaced by $\mu^{rc_1}H_{log}$ where $r$ is the number of fundamental LG fields and $c_1$ is the first Chern class.

We have presented a LG formulation of the topological sigma model. The logarithmic effective superpotential we have used was obtained by exact path-integral computations for sigma models [7, 9]. Thus our LG model is exactly equivalent to the sigma model as stressed in [9]. It will be interesting to develop the LG description of $N = 2$ sigma models on other homogeneous spaces than Grassmannian. In concluding this paper let us finally remark the following two points:

1) We have seen that the topological $CP^{n-1}$ model without coupling to topological gravity is well formulated as LG models in terms of either $W_{log}$ (3) or $W_{pol}$ (6). When coupling to topological gravity, however, the situation changes drastically. After coupling to gravity the LG model with the polynomial superpotential (6) describes the minimal model with the topological central charge $\hat{c} < 1$, rather than the $CP^{n-1}$ model with $\hat{c} = d = n - 1$. In fact, for the $CP^1$ model coupled to gravity, it was found in [19] that a suitable LG superpotential takes the form of exponential interactions; $\mu(e^X + e^{-X})$ with $X$ being the LG field. Then it might be asked if one could find any place where the logarithmic superpotential plays a role. At present we cannot answer to this question, but it is worth pointing out that the action of the $CP^1$ matrix model looks analogous to $W_{log}$ [20, 19]. Precise reasoning for this similarity is desirable.

2) There exists another interesting class of non scale-invariant topological field theo-
ries. Let us take the superpotential with exponential interactions

\[ W_{\text{exp}} = \mu \left( \frac{1}{n - 1} e^{(n-1)X} + e^{-X} \right), \]  

(37)

where \( \mu \) is a mass scale and \( n = 2, 3, \cdots \). Let \( \mu = e^{t/c_1} \) with \( c_1 = n/(n - 1) \) and make a shift of the LG field \( X \to X - t/n \), then we have

\[ W_{\text{exp}} = \frac{1}{n - 1} p^{n-1} + e^{t} p^{-1}, \]  

(38)

where we have put \( p = e^{X} \). The superpotential in this form can be regarded as the Lax operator of a particular reduction of the dispersionless Toda lattice hierarchy \([21, 22, 23]\).

Turning on every interaction term \( p^j \) \((0 \leq j \leq n-2)\) and using the technique of the pseudo differential operator, it is shown that the \( U(1) \) charge \( q_\alpha \) spectrum of chiral primary fields \( \phi_\alpha \) becomes \( \{q_\alpha = \alpha/(n - 1) \mid \alpha = 0, 1, \cdots, n - 1\} \). This charge spectrum agrees with that conjectured in \([9]\). In particular we have a marginal operator \( \phi_{n-1} \) conjugate to the coupling \( t \) in (38). Furthermore, inspecting the \( U(1) \) charge conservation we observe that the topological central charge is \( \hat{c} = 1 \) irrespective of \( n \) and \( c_1 = n/(n - 1) \) plays a similar role to “the first Chern class”. When \( n = 2 \) we actually recover the \( CP^1 \) model. This class of non scale-invariant LG models can be coupled to topological gravity without any difficulty \([22, 23]\). However, we are still in the regime \( \hat{c} (= d) \leq 1 \).

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