Quantum Interference Effects Among Helicities at LEP-II and Tevatron

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A completely model-independent method of obtaining information on the spin using the quantum interference effect among various helicity states was proposed in a recent paper. Here we point out that this effect should be demonstrable in the existing data on \(e^-e^+ \rightarrow W^+W^-\) at LEP-II and \(p\bar{p} \rightarrow Z^0+j\) at Tevatron.

There are many reasons to expect that new particle degrees of freedom will be discovered at the TeV energy scale (Terascale), starting with the Large Hadron Collider (LHC) coming online later this year. The fact that the Terascale must have interesting physics has been known since Fermi’s 1933 theory of nuclear beta decay which introduced a dimensionful constant \(G_F \approx (0.3 \text{ TeV})^{-1}\). In its more modern incarnation, this constant represents the size of the Bose–Einstein condensate that makes the universe a gigantic superconductor. The analog of the Meissner effect then makes the range of the weak force as short as a billionth of a nanometer.

At the least we expect the gap excitation of the superconductor, the Higgs boson, to be discovered at the LHC. In addition, the quantum instability of this energy scale suggests new particles below a TeV in order to protect it from diverging to infinity. Many theoretical frameworks have been proposed in the literature: new strongly coupled gauge theory (technicolor \(^1\)\(^2\)), fermionic dimensions of spacetime (supersymmetry \(^3\)), bosonic dimensions of spacetime (extra dimensions \(^4\)\(^5\)), new hidden extra symmetries (little Higgs \(^6\)), Higgless theories \(^7\)\(^8\) etc. Many of these also provide candidates for the mysterious dark matter of the universe. With great anticipation the community awaits the imminent discovery of such exotic new particles in the upcoming LHC experiments.

Once new particles are discovered, determining what theoretical framework they belong to is of foremost importance. For this purpose truly basic measurements will be required: mass, parity, and spin of the new particles. Among these, the spin measurement is both the key and the most challenging. Numerous studies exist that try to formulate strategies for spin measurements at the LHC \(^9\)\(^10\)\(^11\)\(^12\)\(^13\)\(^14\)\(^15\)\(^16\). Unfortunately, it is very difficult to avoid model-dependent assumptions in the proposed measurement strategies.

In a recent paper \(^17\), three of us (M.B., W.K., H.M.) proposed a completely model-independent way of obtaining information about spin at collider experiments.\(^1\) The key element is quantum interference among various helicity states of the new particle, which, to our surprise, has not been discussed in the modern literature (see, however, \(^19\)). We discussed how this method may work to discriminate the smuon in supersymmetry or the Kaluza–Klein muon in extra dimensions at the proposed International Linear Collider (ILC).

In this letter, we point out that the effectiveness of our proposed method should be demonstrable in the existing data. In particular, \(e^-e^+ \rightarrow W^+W^-\) at LEP-II and \(p\bar{p} \rightarrow Z^0+j\) at Tevatron should allow highly significant studies of the quantum interference among helicities, and demonstrate the spin-one nature of the \(W\) and \(Z\) bosons without any model assumptions. As discussed in \(^17\), this method works particularly well close to the production threshold. This is good news for the LHC, as new physics there will likely be dominated by the energy range just above threshold.

The proposed strategy is extremely simple. In order to obtain model-independent information about spin, or angular momentum in general, we resort to the general principles of quantum mechanics. The angular momentum operators generate spatial rotations; the unitary operator \(U(\phi) = e^{ij\phi/h}\) rotates space around the axis \(\phi\) by the angle \(|\phi|\). If we choose the rotation axis to be the momentun vector of a free particle, it isolates the spin component because the orbital angular momentum is always orthogonal to the momentum vector \(L \cdot p = (\vec{x} \times \vec{p}) \cdot \vec{p} = 0\). Therefore, the angular momentum along the momentum vector is nothing but its helicity, \(h = (\vec{s} \cdot \vec{p})/|\vec{p}|\). The rotation around the momentum axis by an angle \(\phi\) therefore gives the phase \(e^{ih\phi}\) to the quantum mechanical amplitudes.

Obviously a single phase factor does not lead to a physical observable since the probability does not depend on phases. However, an interference effect may pick up the differences in phases among interfering amplitudes. Fortunately, particles produced in collisions are often in a linear superposition of various helicity states, which interfere when they decay into a common final state. This interference of different helicity states produces a cross section dependent on the coherent sum of individual ma-

\(^1\) This possibility was originally suggested in \(^15\).
trix elements squared:

$$\sigma \propto \left| \sum_h M_{\text{prod}}(h) M_{\text{decay}}(h, \phi) \right|^2$$  \hfill (1)

$$M_{\text{decay}}(h, \phi) = e^{ih\phi}.$$  

Here $M_{\text{prod}}(h)$ and $M_{\text{decay}}(h, \phi = 0)$ are the production and decay matrix elements, which depend in detail on the helicity state $h$. However, all $\phi$ dependence has been factored out into the exponential. It is clear from this sum that the azimuthal angular dependence of the event distributions $N = \sigma \times L$ (where $L$ is the luminosity) is

$$\frac{dN}{d\phi} = \frac{d\sigma}{d\phi} \times L = A_0 + A_1 \cos \phi + \cdots + A_n \cos(n\phi),$$  \hfill (2)

where $n = \Delta h$ is the difference between the highest and lowest helicity states contributing to the sum in Eq. (1).

In this way, we obtain an unambiguous lower limit on the spin of the particle, $s \geq (\Delta h)/2$. As we will see, this limit is saturated, $s = (\Delta h)/2$, in the examples below, and the presence of the highest mode is clearly visible in collider data given sufficient statistics.

In the cases of $e^-e^+ \rightarrow W^+W^-$ with leptons plus jets final states and $p\bar{p} \rightarrow Z^0 + j$ with decays to electrons, spin-1 particles are produced in a superposition of helicity states. In both cases, the event is fully reconstructable using the visible momentum in the event, and hence the angle $\phi$ can be fully determined from data.

The angle $\phi$ is defined in the lab frame of the event as the angle between the production plane described by the $W^+W^-$ or $Z^0 + j$ and the decay plane containing the lepton decay products from the vector bosons. We define the positive $z$ axis in the lab frame of LEP-II (Tevatron) as the direction of $e^-$ (proton) beam, then the cosine of $\phi$ at LEP-II can be calculated as follows:

$$n_{\text{prod}} = \frac{\hat{z} \times \vec{p}_{W^\pm}}{|\hat{z} \times \vec{p}_{W^\pm}|}, \quad n_{\text{decay}} = \frac{\vec{p}_{W^\pm} \times \vec{p}_{\ell^\pm}}{|\vec{p}_{W^\pm} \times \vec{p}_{\ell^\pm}|}.$$  \hfill (3)

where $\vec{p}_{\ell^\pm}$ is the charged lepton from the decay of the $W^\pm$ boson. The definition of $\phi$ at Tevatron is the same as in Eq. (3) with the substitution of $Z^0$ for $W^\pm$. An arbitrary (but consistent) choice must be made to define which side of the production plane will contain positive $\phi$. For LEP-II, we chose this positive direction to be in the direction of $\hat{z}$ crossed with the momentum of the leptonically decaying $W^\pm$. Similarly, we chose the direction of the proton beam crossed with that of the $Z^0$ at Tevatron (see Fig. 1). Based on our argument above, we expect to see cross sections for these events as in Eq. (4) with $n = 2$.  

The LEP-II luminosity from the years 1997-2000 are reported in Table I. The OPAL collaboration has observed 1574 events identified as $q\bar{q}\mu\nu$ and an additional 1573 $q\bar{q}\tau\nu$ events. Due to the low purity of the $q\bar{q}\tau\nu$ sample, we ignore those events. Similar data sets are available from the ALEPH, DELPHI, and L3 collaborations.

The CDF collaboration has data for $Z^0 + j$ consisting of 6203 events after selection cuts from 1.7 $fb^{-1}$ of luminosity at 1.96 TeV beam energy. DO has a similar data set available. A total luminosity of 8 $fb^{-1}$ is expected to be available from Tevatron at the conclusion of data collection.

Parton level matrix elements for $W^+W^-$ and $Z^0 + j$ (where the jet consists of a gluon or first generation (anti) quark at the parton level) production were calculated

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2 It is for this reason we cannot consider $p\bar{p} \rightarrow Z$ without jets. In such events, the $Z$ is produced in only one spin state, depending on the spin of the initial state quarks. While the cross section would contain a sum over $Z$ helicity, the sum would be incoherent.

3 It should be noted that if the collider beams are identical, this choice of positive $\phi$ suffers from an ambiguity which maps $\phi \rightarrow \phi + \pi$. This may, for example, introduce difficulties in measuring $A_n$ (n odd) parameters at LHC.
in HELAS 26, while the numerical integration program BASES 27 was used to determine the differential cross section and integrate over all other kinematic variables. For the simulation of the Tevatron results, a K factor of 1.4 was used to correct for higher order QCD effects, in accordance with 24, and CTEQ5L PDFs were implemented using LHAPDF 28. The Tevatron results and fits were confirmed using ALPGEN 29.

![Differential distribution of events $d\sigma/d\phi \times L$ for a) $e^-e^+ \rightarrow W^+W^- \rightarrow q\bar{q}l^\pm\nu$ using the LEP-II run data in Table I and b) $p\bar{p} \rightarrow Z^0 + j \rightarrow e^-e^+ + j$ with luminosity $L = 1.7 \text{ fb}^{-1}$. No cuts are applied on the LEP-II simulation, Tevatron results have $p_T > 30 \text{ GeV}$ and $|\eta| < 2.1$ on the jet.](image)

**Table II:** Fits to the parameters $A_n$ in Eq. (2) for the differential distributions of $e^-e^+ \rightarrow W^+W^- \rightarrow q\bar{q}l^\pm\nu$ (LEP-II) using the integrated luminosity in Table I and $p\bar{p} \rightarrow Z^0 + j \rightarrow e^-e^+ + j$ (Tevatron) using $L = 1.7 \text{ fb}^{-1}$. Errors for each parameter are obtained by marginalizing over the other four parameters in the fit. No cuts are applied on the LEP-II simulation, Tevatron results have $p_T > 30 \text{ GeV}$ and $|\eta| < 2.1$ on the jet.

| Parameter | LEP-II | Tevatron |
|-----------|--------|----------|
| $A_1/A_0$ | $-0.267\pm0.023$ | $0.036\pm0.009$ |
| $A_2/A_0$ | $-0.085\pm0.025$ | $0.100\pm0.009$ |
| $A_3/A_0$ | $0.000\pm0.025$ | $0.000\pm0.009$ |
| $A_4/A_0$ | $0.000\pm0.026$ | $0.000\pm0.010$ |

The generated histograms are assigned Gaussian statistical error bars based on the realistic experimental luminosities. However, no statistical fluctuations are assigned to the central values. As a consequence, the fit results correspond to an average experiment 20.

Before the application of cuts, the differential cross sections for the two processes of interest show a clear $\cos \phi$ and $\cos 2\phi$ dependence, as expected for the decays of spin-1 bosons. These distributions are shown in Fig. 2. We then fit the parameters $A_0, A_1, A_2, A_3$, and $A_4$ in Eq. (2) to the event distributions. For each of the five parameters $A_n$, 1-$\sigma$ error bars are calculated after marginalizing over the other four. Results for the LEP-II and Tevatron simulations are shown in Table II in order to compare simulations with different numbers of events, values of $A_n/A_0$ are reported rather than $A_n$. It is clear at this stage that the results are consistent with the decay of spin-1 bosons.

![Differential distribution of events $d\sigma/d\phi \times L$ for a) $e^-e^+ \rightarrow W^+W^- \rightarrow q\bar{q}l^\pm\nu$ using the LEP-II run data in Table I and b) $p\bar{p} \rightarrow Z^0 + j \rightarrow e^-e^+ + j$ with luminosity $L = 1.7 \text{ fb}^{-1}$. No cuts are applied on the LEP-II simulation, Tevatron results have $p_T > 30 \text{ GeV}$ and $|\eta| < 2.1$ on the jet.](image)

**Table III:** Event selection cuts imposed by the CDF collaboration on $e^-e^+ \rightarrow W^+W^- \rightarrow q\bar{q}l^\pm\nu$ events. Energy fraction is defined as $R_\alpha \equiv E_\alpha/\sqrt{s}$, where $\alpha$ is either the neutrino or the total visible energy. The lepton isolation cut was implemented using $\sqrt{(\Delta \phi)^2 + (\Delta \eta)^2} \equiv \Delta R$ 23.

| Cut Description | Value |
|-----------------|-------|
| Lepton isolation | $\Delta R > 0.75, 0.5, 0.2$ |
| Lepton isolation | $\Delta R > 0.25$ |
| Polar angle $\theta$ | $|\cos \theta| < 0.95$ |
| Visible energy fraction | $R_{vis} > 0.3$ |
| Neutrino transverse momentum | $p_{T,\nu} > 16 \text{ GeV}$ |
| Neutrino energy fraction | $R_\nu > 0.07$ |
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**Table IV:** Event selection cuts imposed by the OPAL collaboration on $e^-e^+ \rightarrow W^+W^- \rightarrow q\bar{q}l^\pm\nu$ events. Energy fraction is defined as $R_\alpha \equiv E_\alpha/\sqrt{s}$, where $\alpha$ is either the neutrino or the total visible energy. The lepton isolation cut was implemented using $\sqrt{(\Delta \phi)^2 + (\Delta \eta)^2} \equiv \Delta R$ 23.

| Cut Description | Value |
|-----------------|-------|
| Jet transverse momentum | $p_T > 30 \text{ GeV}$ |
| Invariant mass of lepton pair | $66 < m_{e\bar{e}} < 116 \text{ GeV}$ |
| Central electron $\eta$ | $|\eta| < 1$ |
| Second electron $\eta$ | $|\eta| < 1$ |
| Electron $E_T$ | $E_T > 25 \text{ GeV}$ |
| Electron isolation cuts | $\Delta R_{e\nu} > 0.7$ |

**Table V:** Event selection cuts imposed by the OPAL collaboration on $e^-e^+ \rightarrow W^+W^- \rightarrow q\bar{q}l^\pm\nu$ events. Energy fraction is defined as $R_\alpha \equiv E_\alpha/\sqrt{s}$, where $\alpha$ is either the neutrino or the total visible energy. The lepton isolation cut was implemented using $\sqrt{(\Delta \phi)^2 + (\Delta \eta)^2} \equiv \Delta R$ 23.

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| Visible energy fraction | $R_{vis} > 0.3$ |
| Neutrino transverse momentum | $p_{T,\nu} > 16 \text{ GeV}$ |
| Neutrino energy fraction | $R_\nu > 0.07$ |
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| Neutrino energy fraction | $R_\nu > 0.07$ |

4 These fits are to the numerically integrated differential cross-section, not generated events.
responding non-zero $A_3$ and $A_4$ components may naively be confused for evidence of spin-2 particles. However, the cuts are responsible for introducing new $\phi$ dependence by selecting out new directions relative to the production axis of the gauge bosons.

We illustrate this effect for the case of cuts in the forward direction (large $|\eta|$ and $|\cos \theta|$) in Fig. 3. Here we see two decays which are kinematically identical in the boson rest frame save for a rotation in $\phi$. The event a) survives the cuts, while the event b) fails. The event a) survives the cuts, while the event b) fails.

Since this $\phi$ dependence did not arise from the quantum interference of helicity amplitudes, we cannot expect the $\phi$ dependence of the cross section to accurately reflect the spin of the decaying particles. Thus non-zero $A_3$ and $A_4$ components do not indicate a higher spin state, but rather a breakdown of the proposed spin-measurement technique.

The solution to this problem is relatively straightforward. For new azimuthal dependences to be avoided, the cuts cannot pick out ‘special’ directions relative to the original momentum of the decaying boson. Therefore we impose ‘rotationally invariant cuts’ in which we require that each event not only passes the experimental cuts but continues to do so when the decay plane is rotated around the boson production axis. This avoids the introduction of a new directional dependence since we restrict ourselves to only those events which could never overlap the forbidden regions of the detector regardless of orientation. However, these cuts are very inefficient: the cuts on LEP-II data preserve only 12% of the original events, while the cuts for the Tevatron leave less than 1%.

The CDF cuts are very inefficient due to the small allowed $|\eta|$ region for the central electron (see Table III). Recent preliminary CDF measurements have demonstrated that the cuts can be relaxed while still maintaining a background level of less than 5%. These loosened cuts are identical to those in Table III for $p_T$ and $\eta$ of the jet and the invariant mass of $m_{l\ell}$. However, the central lepton is allowed $E_T > 20$ GeV and $|\eta| < 2.6$, while the second electron must have $E_T > 10$ GeV and $|\eta| < 2.6$. If both leptons have $2.6 > |\eta| > 1.0, E_T$ must be greater than 25 GeV. Finally, $\Delta R_{e\ell}$ must be greater than 0.4. With these relaxed numbers, the total number of events in the simulated sample is 5821 and the efficiency of the rotationally invariant cuts is 18%.

The result of these rotationally invariant cuts on the

| Cut Region | $A_1/A_0$ | $A_2/A_0$ | $A_3/A_0$ | $A_4/A_0$ |
|------------|-----------|-----------|-----------|-----------|
| Luminosities are as in Table II. | $0.082\pm0.025$ | $0.302\pm0.027$ | $0.117\pm0.028$ | $0.099\pm0.028$ |
| $\Delta R = 0.75$ | $-0.082\pm0.026$ | $-0.308\pm0.026$ | $0.114\pm0.028$ | $-0.099\pm0.029$ |
| $\Delta R = 0.5$ | $-0.082\pm0.026$ | $-0.302\pm0.027$ | $0.117\pm0.028$ | $-0.096\pm0.029$ |
| $\Delta R = 0.2$ | $-0.082\pm0.025$ | $-0.302\pm0.027$ | $0.117\pm0.028$ | $-0.096\pm0.029$ |

TABLE V: Fits of the differential distribution of $e^-e^+ \rightarrow W^+W^- \rightarrow q\bar{q}l^+l\nu$ (LEP-II) with the cuts in Table IV and $p\bar{p} \rightarrow Z^0 + j \rightarrow l^-l^+ + j$ (Tevatron) with the cuts in Table III to parameters $A_i$ in Eq. 2. Luminosities are as in Table II. 1-$\sigma$ errors for each parameter are obtained by marginalizing over the other four parameters in the fit.
the data is clearly consistent with the

TABLE VI: Fits of the differential distribution of

LEP-II and Tevatron data are shown in Fig. 5 (compare to Fig. 2). Table VI confirms that this technique restores the \( \phi \) dependence expected by the interference argument.

In the case of the Tevatron results with loosened cuts, the data is clearly consistent with the \( Z \) being a spin-1 vector boson. The \( A_1 \) parameter is non-zero at 1.8\( \sigma \), the \( A_2 \) parameter is non-zero at nearly 4\( \sigma \), and the higher modes are consistent with zero. It is important to recall that a lower bound on the spin is obtained from the highest non-zero mode, therefore the 4\( \sigma \) signal in \( A_2 \) is far more important than the 1.8\( \sigma \) deviation from zero in \( A_1 \).

| LEP-II                      | \( \Delta R = 0.75 \) | \( \Delta R = 0.5 \) | \( \Delta R = 0.2 \) |
|----------------------------|------------------------|------------------------|------------------------|
| \( A_1/A_0 \)              | \( -0.215 \pm 0.069 \) | \( -0.214 \pm 0.060 \) | \( -0.207 \pm 0.053 \) |
| \( A_2/A_0 \)              | \( -0.068 \pm 0.071 \) | \( -0.071 \pm 0.062 \) | \( -0.072 \pm 0.055 \) |
| \( A_3/A_0 \)              | \( 0.000 \pm 0.073 \)  | \( 0.000 \pm 0.064 \)  | \( 0.000 \pm 0.057 \)  |
| \( A_4/A_0 \)              | \( 0.000 \pm 0.075 \)  | \( 0.000 \pm 0.065 \)  | \( 0.000 \pm 0.058 \)  |

| Tevatron                    | \( A_1/A_0 \) | \( 0.039 \pm 0.022 \) |
|----------------------------|---------------|------------------------|
| \( A_2/A_0 \)              | \( 0.083 \pm 0.021 \) |
| \( A_3/A_0 \)              | \( 0.000 \pm 0.022 \) |
| \( A_4/A_0 \)              | \( 0.000 \pm 0.023 \) |

TABLE VI: Fits of the differential distribution of \( e^{-} e^{+} \rightarrow W^{+} W^{-} \rightarrow q\bar{q}l^{\pm} \nu \) (LEP-II) and \( p\bar{p} \rightarrow Z^{0} + j \rightarrow e^{-} e^{+} + j \) (Tevatron) to the parameters \( A_n \) in Eq. (2), requiring events that pass the cuts in Tables IV and III (with relaxed \( E_T, [\eta] \), and \( \Delta R \) cuts as described in the text) after rotation about the momentum axis of the decaying vector boson. The luminosities are the same as in Tables II and IV, 1-\( \sigma \) errors for each parameter are obtained by marginalizing over the other four parameters in the fit.

From these results there is always the possibility that the parent \( Z \) is a higher spin particle and that some conspiracy amongst the matrix elements in Eq. (1) prevents the \( A_3 \) and \( A_4 \) terms from appearing in the sum. In this interpretation, we can still state unambiguously that the \( Z \) is at least spin-1, and that the data suggest it is not of higher spin.

Higher statistics would allow a reduction of error bars and increase our confidence in the result correspondingly. Using, for example, the estimated total integrated luminosity of 8 \( \text{fb}^{-1} \) for the Tevatron, the parameters have the values shown in Table VII. Another possibility is to use the muon decays of the \( Z^{0} \). However, the rotationally invariant cuts will likely take a high toll on such events, as the muon tracking system at CDF extends only up to \(|\eta| = 1.5 \).

The situation with the LEP-II simulation is more complicated. While the \( A_1 \) parameters are non-zero at over 3\( \sigma \), the \( A_2 \) parameters differ from zero by only one standard deviation. A larger data set would of course solve this problem. As all four LEP-II experiments (ALEPH, DELPHI, L3, and OPAL) have approximately equal statistics available, a two-fold increase in the statistical significance could be achieved by combining the events from these collaborations; the resulting ratios \( A_n/A_0 \) are shown in Table VII.

Another possibility is that some reduction in required cuts would increase the efficiency of the rotationally invariant cuts without greatly degrading the sample purity. A likely candidate for this in our analysis is the \( \Delta R \) cut, which was introduced as a stop-gap measure to approximate the jet-lepton proximity cut used in the OPAL analysis. However, even with the value of \( \Delta R = 0.2 \), the efficiency of the cut is lower than the 85% reported by OPAL. Setting \( \Delta R = 0 \) is clearly an unrealistic cut, but as demonstrated in Table VII indicates the possibilities offered by higher statistics.

In conclusion, we have demonstrated that the quantum interference among the matrix elements of different helicity states provides model-independent probe of particle spin. Using realistic data sets, rotationally invariant cuts can be implemented which correct for the spurious high-frequency noise introduced by the cuts imposed by detector geometry and background reduction. Though these techniques come at a price in terms of efficiency, it seems possible to relax the cuts in such a way that the weak gauge boson spins can be measured at sufficient significance at current colliders.

Measurements of the spin of new particles is expected to be a critical discriminator of new physics at the LHC. As a result, techniques such as the one proposed here are very important. Though the spins of the \( W \) and \( Z \) bosons are not in doubt, we find it encouraging that this new method can be tested on the available data. Such work would be of great use in the coming LHC era.

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|                  | LEP-II                  | Tevatron                  |
|------------------|-------------------------|---------------------------|
|                  | Combined | $\Delta R = 0$ | $\mathcal{L} = 8$ fb$^{-1}$ |
| $A_1/A_0$        | $-0.207\pm0.027$       | $-0.211\pm0.050$          | 0.039$\pm0.010$  |
| $A_2/A_0$        | $-0.072\pm0.028$       | $-0.081\pm0.052$          | 0.083$\pm0.010$  |
| $A_3/A_0$        | 0.000$\pm0.028$        | 0.000$\pm0.053$           | 0.000$\pm0.010$  |
| $A_4/A_0$        | 0.000$\pm0.029$        | 0.000$\pm0.054$           | 0.000$\pm0.010$  |

TABLE VII: Fits of the differential distribution to the parameters $A_n$ in Eq. (2) for: $e^-e^+ \rightarrow W^+W^- \rightarrow q\bar{q}\ell\bar{\ell}\nu$ with the jet-lepton cut parameter $\Delta R = 0.2$ and combining the data sets of ALEPH, DELPHI, L3, and OPAL (LEP-II, Combined), $e^-e^+ \rightarrow W^+W^- \rightarrow q\bar{q}\ell\bar{\ell}\nu$ with the OPAL data set and $\Delta R$ set to zero (LEP-II, $\Delta R = 0$), and $p\bar{p} \rightarrow Z^0 + j \rightarrow \ell^-\ell^+ + j$ with 8 fb$^{-1}$ integrated luminosity (Tevatron). We require all events pass the cuts in Tables IV (with $\Delta R$ as indicated) and III (with relaxed $E_T$ and $|\eta|$ cuts as described in the text) after rotation about the momentum axis of the decaying vector boson. 1-$\sigma$ errors for each parameter are obtained by marginalizing over the other four parameters in the fit.