Reliability analysis of RC beam bridge considering relevant random variables

Pei Ye1*

1 college of Civil Engineering and Architecture, Shandong University of Science and Technology, Qingdao, Shandong Province, 266590, China
* Pei Ye e-mail: zggzyp123@163.com

Abstract. In order to test the analysis of the influence of non-normally related random variables on the reliability of RC bridge structures, the conversion process and method of Rosenblatt transform to non-normal random variables are systematically introduced. First, the basic principles and applicable conditions of Rosenblatt transformation are introduced. On this basis, based on the standard design expression and the design live dead load ratio as the basic parameter, the reliable index corresponding to different correlation coefficients is calculated according to the Rosenblatt transformation method. The results show that when the FORM method is used for calculation, the reliability index of Rosenblatt transformation calculation is greatly affected by the order of different variables; the structural reliability index of the RC bridge with relevant random variables increases with the increase of the correlation coefficient, and different live dead load ratios also have a more obvious effect on the reliability index; the shear bearing capacity is larger than the calculation result of the reliable index of the bending capacity, which does not meet the actual requirements. It provides a basis for the RC beam bridge to choose the bending capacity as the reliability of the resistance calculation.

1. Introduction
In the existing bridge structure reliability study, the assumption that the basic random variables are independent of each other is introduced [1-2], but this is inconsistent with the actual project. For example, there is a certain correlation between the resistance of concrete structures and the cross-sectional dimensions [3]. Therefore, in the reliability analysis of bridge structures, the influence of the correlation of variables must be considered.

For the structure of related random variables, the related non-normal random variables should be converted to independent standard normal random variables. Commonly used equivalent conversion methods include Nataf transform [4], Rosenblatt transform [5], Orthogonal transform [6], Hermite polynomial transform [7] and Winterstein approximate formula [8]. Among them, Rosenblatt transformation is the most general method and the method with the highest accuracy of calculation results. It mainly uses conditional probability to convert non-normally related random variables to independent standard normal random variables.

The above analysis shows that most of the existing transformation methods remain at the theoretical level, and no research results have been carried out in conjunction with actual bridge design specifications. This article intends to use the current RC beam bridge design criteria as the basis, the live constant load ratio used in the design as the basic parameter, and the FORM (First Order Reliability Method) to calculate the reliable index. The system analyzes the correlation among bending resistance, shear resistance and dead load effect. Taking the ratio of live load to dead load as the basic
parameter of structural reliability, the necessity of considering the correlation of variables in the reliability calculation of bridge structure is verified

2. The basic principle of Rosenblatt transformation

It is known that $X = (X_1, X_2, ..., X_n)^T$ is an $n$-dimensional random variable, and the structural function is expressed as

$$ Z = g_X(X) = g(X_1, X_2, ..., X_n) \quad (1) $$

Assuming that the joint probability density function of the random variable $X$ is $f_X(x) = f_{X_1, X_2, ..., X_n}(x)$, the failure probability of the structure can be expressed as

$$ P_f = \int_{g_X(x) < 0} \cdots \int f_X(x_1, x_2, ..., x_n) dx_1 dx_2 \cdots dx_n \quad (2) $$

From the concept of conditional probability, formula (1) can be expressed as:

$$ P_f = \int_{g_X(x) < 0} \cdots \int f_{X_1, X_2, ..., X_n}(x_1, x_2, ..., x_n) dx_1 dx_2 \cdots dx_n \quad (3) $$

Further, if the standard normal random vector $Y = (Y_1, Y_2, ..., Y_n)^T$, and let

$$ \varphi(y_n) dy_n = f_{X_1, X_2, ..., X_n}(x_1, x_2, ..., x_n) dx_n \quad (4) $$

Integrating the two sides of equation (3) gives:

$$ \Phi(Y_1) = F_{Y_1}(X_1) \quad \Phi(Y_2) = F_{Y_2}(X_2|X_1) \quad \cdots \quad \Phi(Y_n) = F_{Y_n}(X_n|X_1, X_2, ..., X_{n-1}) \quad (5) $$

The expected independent standard normal random variable $Y$ that can be obtained by the inverse transformation of equation (5)

$$ Y_1 = \Phi^{-1}[F_{X_1}(X_1)] \quad Y_2 = \Phi^{-1}[F_{X_2|X_1}(X_2|X_1)] \quad \cdots \quad Y_n = \Phi^{-1}[F_{X_n|X_1, X_2, ..., X_{n-1}}(X_n|X_1, X_2, ..., X_{n-1})] \quad (6) $$

Substituting equation (3) into equation (2), the probability of structural failure represented by standard normal random variables is:

$$ P_f = \int_{g_Y(y) < 0} \cdots \int \varphi(y_1) \varphi(y_2) \cdots \varphi(y_n) dy_1 dy_2 \cdots dy_n \quad (7) $$

The above is the Rosenblatt transformation process. The basic idea is to realize the transformation of related non-normal random variables to independent standard normal random variables through conditional probability.

3. Bridge structure reliability calculation

After Rosenblatt transformation, the reliable index of the relevant random variable structure can be calculated according to the FORM method [9]. The main calculation steps are as follows:

(1) Set the convergence error $\varepsilon$
(2) Assuming the initial value of the checkpoint, the average value of each basic random variable can usually be taken as $x^* = \mu_i$.

(3) Calculate the initial checkpoint $y^*$ by equation (6).

(4) Calculate the Jacobian matrix $J_{yx}$ and its inverse matrix $J_{xy}^{-1}$ by:

$$
\frac{\partial Y}{\partial X_j} = \begin{cases} 
1 & \text{if } i = j \\
\frac{1}{\phi(Y)} \frac{\partial F_{Y_i|Y_i=x_j}}{\partial X_j} & \text{if } i < j \\
\frac{1}{\phi(Y)} f_{Y_i|Y_i=x_j}(X_i|X_1,X_2,\cdots,X_{j-1}) & \text{if } i > j \\
0, & 
\end{cases}
$$

(5) Calculate the gradient of the function $g_Y(Y) = J_{xy} \nabla g_Y(X)$.

(6) Calculate direction cosine $\cos \theta$

$$
\cos \theta_i = \frac{\nabla g_Y(Y)}{\left( \sum_{i=1}^{n} [\nabla g_Y(Y)]^2 \right)^{1/2}}
$$

(7) Calculate $\beta$

$$
\beta = \frac{g_X(x^*) - \sum_{i=1}^{n} [\nabla g_Y(Y)] y_i^*}{\left( \sum_{i=1}^{n} [\nabla g_Y(Y)]^2 \right)^{1/2}}
$$

(8) Calculate new checkpoint $x^*$

$$
y^* = \beta \cos \theta_i
$$

$$
x^* = F_{X_i}^{-1} \left[ \Phi(y^*) \right] \quad i = 1, 2, \ldots
$$

(9) Determine whether the two checkpoints before and after satisfy $\|x^*\| < \varepsilon$, otherwise repeat steps (2) to (8) until the convergence condition is satisfied.

4. **RC beam bridge example**

For medium and small span reinforced concrete bridges, if the resistance $R$, dead load effect $S_G$ and vehicle load effect $S_Q$ are known, and the random variables follow the lognormal distribution, normal distribution, and Gumbel distribution, then the corresponding structural function $g_Y(Y)$ can be expressed as:

$$
Q_G - S_G - S_Q = g(R, S_G, S_Q) = R - S_G - S_Q
$$

Further, assuming that the resistance $R$ is related to the dead load effect $S_G$, the correlation coefficient is represented by $\rho_{RS_G}$, and the resistance $R$ is independent of the vehicle load $S_Q$, and the constant load $S_G$ and the vehicle load $S_Q$ are independent of each other. According to the [13], the relationship between the standard resistance value and the combined design value of the load effect can be expressed as:
\[
R_k = \gamma_k (\gamma_G S_{Gk} + \gamma_Q S_{Qk}) \tag{15}
\]

In the formula, \(R_k\) is the standard value of resistance; \(\gamma_k\) is the partial coefficient of resistance (\(\gamma_k = 1.125\)); \(\gamma_G\) is the partial coefficient of permanent load effect (\(\gamma_G = 1.2\)); \(\gamma_Q\) is the partial coefficient of variable load effect (\(\gamma_Q = 1.4\)); \(S_{Gk}\) and \(S_{Qk}\) are the standard values of constant load and live load respectively.

The purpose of introducing the live constant load ratio \(\kappa = S_{ok} / S_{Gk}\) is to simplify the calculation process. The reliability calculation is only related to the live constant load ratio, but to the specific value of the load standard value. According to [10] (hereinafter referred to as "Uniform Standards")-Class II, the statistical parameters of each basic random variable are shown in Table 1.

| Variable types       | Distribution type | \(\mu_k\) | \(\delta\) |
|----------------------|-------------------|-----------|------------|
| Bending resistance   | Lognormal distribution | 1.226     | 0.141      |
| Shear resistance     | Lognormal distribution | 2.1798    | 0.2230     |
| Dead load effect     | Normal distribution | 1.015     | 0.043      |
| Live load effect     | Gumbel distribution | 0.686     | 0.157      |

4.1. Analysis of Rosenblatt transformation

The exact joint probability density function of related random variables \(R\) and \(S_G\) is [11]:

\[
f_{RS_{G}}(R,S_G) = \frac{1}{2\pi\sigma_{R}\sigma_{S_G}\sqrt{1-\rho_{R,S_G}^2}} \times \exp\left\{-\frac{1}{2} \left[ \ln\frac{\ln R - \mu_{ln R}}{\sigma_{ln R}} - \frac{S_G - \mu_{S_G}}{\sigma_{S_G}} \right]^2 \right\} \tag{16}
\]

According to the joint probability density function of equation (16), the structural function function of equation (6) can be converted into an independent standard normal random variable by using related non-normal random variables, and then the reliable index of the RC beam bridge is calculated by the FORM method.

It is not difficult to see from equation (6) that when conditional probabilities are used to transform related random variables, the transformation order of the variables can be changed. The following transformations can be made for the resistance \(R\) and the dead load effect \(S_G\):

\[
Y_i = \Phi^{-1}(F_{X_i}(X_R)) \quad \text{or} \quad Y_i = \Phi^{-1}(F_{X_i}(X_S))
\]

The Rosenblatt transformation is processed by equation (17) to investigate the influence of different transformation orders of variable variables on the reliability index during the Rosenblatt transformation process. The calculation results are shown in Figure 1.
It can be seen from Figure 1 that when the Rosenblatt transform is used to calculate the reliable index, there are obvious differences in the calculation results of the two transform sequences (R, SG) and (SG, R). At the same time, it can be found that with the increase of the correlation coefficient, the difference of the reliability index gradually becomes larger. At that time, the relative difference of the reliability index was 0.0165 and 1.0966, respectively. When highly correlated, the reliability index differed by 21.37%.

The Rosenblatt transformation should theoretically have nothing to do with the order of the variables, because the joint nonnormal joint distribution function will produce a highly nonlinear constraint function near the checkpoint, and the FORM method cannot accurately estimate the highly nonlinear function after the transformation, so the Rosenblatt transformation is different. Variable order reliable index results are not unique [11].

4.2. Consider the impact of multiple parameters on reliable indicators

Based on the above research and analysis, the influence of multi-parameters on reliability index in Rosenblatt transformation is investigated, and the transformation order is (R, SG). Five different ratios of live to dead load were used to calculate the mean value of basic random variables[], which were 0.1, 0.5, 1.0, 1.5 and 2.5 respectively. The results are shown in Table 2.

| κ   | $\mu_R$ (Bending state) | $\mu_{SG}$ (Shear state) | $\mu_{Q}$ (Shear state) |
|-----|-------------------------|--------------------------|-------------------------|
| 0.1 | 1.8482                  | 3.2860                   | 1.015                   |
| 0.5 | 2.6206                  | 4.6593                   | 1.015                   |
| 1.0 | 3.5860                  | 6.3759                   | 1.015                   |
| 1.5 | 4.5515                  | 8.0925                   | 1.015                   |
| 2.5 | 6.4824                  | 11.5256                  | 1.015                   |

Table 2 Calculate the mean value of each basic random variable based on the ratio of live and dead load by using statistical parameters. The reliability index calculation program is compiled based on Matlab. When the resistance and dead load effect are independent of each other, the influence of bending resistance and shear resistance on the reliability index is investigated. The results are shown in Figure 2.

It can be seen from Figure 2 that after the ratio of live and dead load participates in the calculation, both the bending resistance and the shear resistance have a great influence on the reliability index. When the ratio of live load to dead load is 0.5, the reliability index reaches its peak, while the bending resistance reaches its peak at 1.0. In addition, the reliability index calculated under shear resistance is 6 orders of magnitude larger than that under bending resistance. And considering the shear resistance,
the reliability index fluctuates between 7 and 8, which is far greater than the reliability index 4.2 of the automobile-classⅡspecified in the Unified Standard. Therefore, when investigating the reliability of RC beam bridges at present, most literatures only consider the bending resistance.

The above analysis analyzes the influence of bending resistance and shear resistance on the reliability index, but it is based on the condition that the resistance and dead load effects are independent of each other. The following analysis takes the ratio of live dead load as the basic parameter and considers the correlation of resistance to the reliability index. The results are shown in Table 3 and Table 4.

Table 3 Change of Reliability Index of Bending Resistance

| \( \kappa \) | \( \rho = 0 \) | \( \rho = 0.8 \) |
|---|---|---|
| 0.1 | 3.5994 | 4.8083 |
| 0.5 | 4.3418 | 5.0065 |
| 1.0 | 4.5006 | 4.7887 |
| 1.5 | 4.4902 | 4.6654 |
| 2.5 | 4.4470 | 4.5418 |

Table 4 Change of reliable index of shear resistance

| \( \kappa \) | \( \rho = 0 \) | \( \rho = 0.8 \) |
|---|---|---|
| 0.1 | 7.5496 | 10.1838 |
| 0.5 | 7.8051 | 8.4443 |
| 1.0 | 7.3921 | 7.6687 |
| 1.5 | 7.1531 | 7.3258 |
| 2.5 | 6.9099 | 7.0072 |

It can be seen from Table 3 that when the bending resistance is highly correlated, the reliability indicators are larger than the independent cases, but this difference decreases with the increase of the live dead load ratio, for example, when \( \kappa=0.1 \) and \( \kappa=2.5 \), the corresponding The difference between the reliable indicators of is 1.2089 and 0.0948 respectively. It shows that the larger the ratio of live dead load, the smaller the influence of correlation on the reliable index.

Compared with Table 3, Table 4 shows the influence of shear resistance on the reliability index. It is also seen that the correlation has a greater impact on the reliability index. The difference of the reliability index corresponding to the same live dead load ratio is 2.6342 and 0.0973. It shows that when the ratio of live dead load is small, the reliable index for calculation of shear resistance is more affected by correlation than bending resistance.

According to the above analysis, with the ratio of live dead load as the basic parameter, the influence of different resistances on the reliability index with the correlation changes is investigated. The calculation results are shown in Figure 3(a) and Figure3(b).

![Figure 3 Influence of bending resistance and shearing resistance on reliability index](image-url)
Figure 3 show the influence of the change of the correlation coefficient on the reliability index in the case of bending and shear resistance, respectively. It can be seen that different ratios of live dead load increase as the correlation increases. The difference is that when the live dead load ratio is smaller, the increase is larger. Conversely, as the ratio of live dead load increases, the increase in the reliability index decreases significantly. What is more obvious is that when k=2.5, the reliability index hardly increases.

5. Conclusion
This paper systematically introduces the theoretical basis of Rosenblatt transformation, takes the ratio of live dead load as the basic parameter and calculates the structural reliability index corresponding to different correlation coefficients and variable order according to JC method. After analysis, the following conclusions can be drawn:

1) The Rosenblatt transform calculation reliability index is affected by the order of variables, and it has a greater impact. The relative error of the calculation of the reliability index of different variable sequences is proportional to the correlation coefficient. The relative error can be as high as 21% when highly correlated.

2) When the basic random variables are independent of each other, the reliable index is greatly affected by the ratio of live dead load. Under the same ratio of live load to dead load, the reliable index of bending resistance calculation is far less than that of shear resistance calculation.

3) The calculation of the reliability index of the bending resistance and shear resistance at different live load ratios is greatly affected by the correlation coefficient, where K=0.1 has the greatest influence, and when k=2.5 the reliability index is hardly affected by the correlation coefficient.

References
[1] Ghosn M. (2000) Development of truck weight regulations using bridge reliability model Journal of Bridge Engineering, 5:293-303.
[2] L. M. FERREIRA,A. S. NOWAK,M. K. EL DEBS. (2008)Development of truck weight limits for concrete bridges using reliability theory.IBRACON Structures and Materials Journal,1:421-435.
[3] ZHOU Kerong,XIAO Xiaosong,WU Xiaohan.(1996)Fractal behavior in size effect of compressive strength of concrete cubes. Journal of Fuzhou University (Natural Science Edition), 24: 63-68.
[4] Der Kiureghian A,Liu P L. (1986)Structural reliability under incomplete probability information. Journal of Engineering Mechanics,112: 85-104.
[5] Rosenblatt M. (1952) Remarks on a multivariate transformation. Annals of Mathematical Statistics,23: 470-472.
[6] Rackwitz R,Fiessler B. (1978)Structural reliability under combined load sequence. Computer and Structures,114(12): 2195-2199.
[7] Schoutens W. (2000) Stochastic processes and orthogonal polynomials. Springer, New York.
[8] Winterstein S R. (1988) Nonlinear vibration models for extremes and fatigue [J]. Journal of Engineering Mechanics, 114: 1772—1790.
[9] ZHANG Ming. (2009) Structural reliability analysis-methods and procedures. Science Press, Beijing.
[10] China Academy of Building Research. (1999) Unified standard for reliability design of highway engineering structure. Beijing Kewen Book Industry Information Technology Co., Ltd, Beijing.
[11] Noh Y J,Choi K K,Du L.(2009)Reliability-based design optimization of problems with correlated input variables using a Gaussian Copula.Structural and Multidisciplinary Optimization,38:1-16.
[12] China Communications Highway Planning and Design Institute. (2018) Code for Design of Highway Reinforced Concrete and Prestressed Concrete Bridges and Culverts. China Communications Press, Beijing.