Letter

Quantum state conversion in opto-electro-mechanical systems via shortcut to adiabaticity

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Abstract

Adiabatic processes have found many important applications in modern physics, the distinct merit of which is that accurate control over process timing is not required. However, such processes are slow, which limits their application in quantum computation, due to the limited coherent times of typical quantum systems. Here, we propose a scheme to implement quantum state conversion in opto-electro-mechanical systems via a shortcut to adiabaticity, where the process can be greatly speeded up while precise timing control is still not necessary. In our scheme, by modifying only the coupling strength, we can achieve fast quantum state conversion with high fidelity, where the adiabatic condition does not need to be met. In addition, the population of the unwanted intermediate state can be further suppressed. Therefore, our protocol presents an important step towards practical state conversion between optical and microwave photons, and thus may find many important applications in hybrid quantum information processing.

Keywords: quantum state conversion, opto-electro-mechanical systems, superadiabatic

(Some figures may appear in colour only in the online journal)

1. Introduction

Hybrid quantum systems [1] may consolidate the advantages of different systems, and thus may find many important applications in quantum information processing. Recently, opto-electro-mechanical systems [2, 3] interfacing optical and microwave photons have attracted considerable attention due to the advanced fabrication of superconducting circuits that support microwave photons, and scalable integrated optical photonic circuit techniques. Through opto-electro-mechanical systems, one can efficiently up-convert microwave information to the optical counterpart, and thus enable the transmission of the information through optical fibres in a low-loss way. Therefore, great efforts have been directed to the conversion between microwave and optical fields [4–14]. However, the conversion process usually uses the adiabatic passage, which requires a long operation time to satisfy the adiabatic criteria, and thus decoherence may induce unacceptable loss.

One possible way out of this difficulty is the so-called ‘superadiabatic transitionless driving (SATD)’ [15–17] or ‘shortcut to adiabaticity (STA)’ [18–22] protocol, where the conversion process is speeded up and still keeps the merits of the adiabatic passage. In this protocol, a system is forced to follow the instantaneous eigenstates of its Hamiltonian exactly, by applying an additional, precisely controlled field to cancel nonadiabatic transitions between the instantaneous
electromechanical coupling is induced by a capacitance \( C(x) \) of the microwave cavity \( a_2 \), and \( C_0 \) is the gate capacitance of the microwave cavity \( a_2 \). (b) Coupling configuration in the single excitation subspace of the considered system. (c) Energy level in the dressed picture, where \( \theta(t) \) stand for the nonadiabatic transition between \(|+\rangle \) and \(|-\rangle \).

Figure 1. Illustration of the proposed protocol. (a) Schematic diagram for the two cavity optomechanical system, the electromechanical coupling is induced by a capacitance \( C(x) \) of the microwave cavity \( a_2 \), and \( C_0 \) is the gate capacitance of the microwave cavity \( a_2 \). (b) Coupling configuration in the single excitation subspace of the considered system. (c) Energy level in the dressed picture, where \( \theta(t) \) stand for the nonadiabatic transition between \(|+\rangle \) and \(|-\rangle \).

2. The system and its adiabatic dynamics

We consider an opto-electro-mechanical system, as illustrated in figure 1(a), where a mechanical resonator is simultaneously coupled to an optical cavity and a microwave cavity via dispersive coupling and each cavity mode is under external driving with frequencies \( \omega_i \) \((i = 1, 2)\) [41, 42]. Following the standard linearization procedure [41], the system can be described by

\[
\hat{H} = \omega_m \hat{b}_m^\dagger \hat{b}_m + \sum_{i=1}^{2} \left[ \Delta_i \hat{a}_i^\dagger \hat{a}_i + g_i (\hat{a}_i^\dagger \hat{b}_m + \hat{a}_m \hat{b}_m^\dagger) \right],
\]

where we have assumed \( \hbar = 1 \) (here and hereafter); \( \hat{a}_1, \hat{a}_2 \) and \( \hat{b}_m \) are the annihilation operators for the optical, microwave and mechanical modes; \( \omega_i \) \((i = 1, 2)\) is the frequency of the \( i \)th cavity mode, \( \omega_m \) is the mechanical oscillator’s frequency, and \( \Delta_i = \omega_{\text{m}} - \omega_i \) is the detuning between cavity mode and external driving; \( g_i = G_0 \sqrt{n_i} \) \((i = 1, 2)\) is the effective linear coupling, which, being proportional to the driving amplitude applied to cavity \( i \), is tunable by varying the driving field \cite{44} with \( G_0 \) and \( n_i \) are the effective single-photon coupling strength and photon number inside the cavity, respectively. Note that the Hamiltonian in equation (1) is written in a displaced frame, which has a form readily for QSC, and thus the quantum state to be transferred here sits atop a classical coherent state with large number of photons [6].

We consider that the case of strong coupling with external driving in the first red sideband of the mechanical mode, i.e. \( -\Delta_1 = \omega_m \). Moving to the interaction picture, and using the rotating wave approximation, we obtain that

\[
\hat{H}_{\text{int}} = \begin{pmatrix} 0 & g_1(t) & 0 \\ g_1(t) & 0 & g_2(t) \\ 0 & g_2(t) & 0 \end{pmatrix},
\]

where we have assumed the basis vectors as \(|a_1\rangle = [1, 0, 0]^T\), \(|b_m\rangle = [0, 1, 0]^T\), and \(|a_2\rangle = [0, 0, 1]^T\), corresponding to states with one excitation of the optical cavity, the mechanical oscillator and the microwave cavity respectively. Note that the state transfer protocol is confined to a single excitation subspace formed by \{\(|a_1\rangle, |b_m\rangle, |a_2\rangle\}\} in low temperatures. Three instantaneous eigenvectors of the Hamiltonian in equation (2) can be described by

\[
|+\rangle = \frac{1}{\sqrt{2}} (\sin \theta |a_1\rangle + |b_m\rangle + \cos \theta |a_2\rangle),
\]

\[
|d\rangle = -\cos \theta |a_1\rangle + \sin \theta |a_2\rangle,
\]

\[
|\rangle = \frac{1}{\sqrt{2}} (\sin \theta |a_1\rangle - |b_m\rangle + \cos \theta |a_2\rangle),
\]

and the corresponding three eigenvalues are \( E_- = -g(t) \), \( E_d = 0 \), and \( E_+ = g(t) \), where \( g(t) = \sqrt{g_1^2(t) + g_2^2(t)} \) and \( \tan \theta = g_1(t)/g_2(t) \), as shown in figure 1(c). In this system, \(|+\rangle \) and \(|\rangle \) are the ‘bright’ modes, which are superpositions of the cavity \(|a_1\rangle\), cavity \(|a_2\rangle\) and the mechanical mode \(|b_m\rangle\); while \(|d\rangle\) is the ‘dark’ mode, which decouples from the mechanical mode \(|b_m\rangle\) due to destructive interference. If one initially prepares an excitation in the microwave mode \(|a_2\rangle\) one can, through the ‘dark’ state passage, adiabatically convert the excitation to the optical mode \(|a_1\rangle\), and vice versa [8].

3. Shortcut to adiabatic quantum state conversion

3.1. The protocol

We now consider the case of speeding up the adiabatic QSC process, where the adiabatic condition is not met. In the adiabatic basis \{\(|+\rangle, |d\rangle, |\rangle\}\}, the Hamiltonian in equation (2) becomes

\[
H_{\text{ad}}(t) = U(t)^{\dagger} H_{\text{int}} U(t) + i \frac{dU(t)}{dt} U(t)^{\dagger}
\]

\[
= g_0 \hat{M}_2 - \dot{\theta} \hat{M}_1,
\]

where \( \hat{M}_2 = |d\rangle \langle d| - |\rangle \langle \| \) and \( \hat{M}_1 = |+\rangle \langle +| - |\rangle \langle \rangle \).
where
\[ U(t) = \frac{1}{\sqrt{2}} \begin{pmatrix} \sin \theta & 1 & \cos \theta \\ -\sqrt{2} \sin \theta & 0 & \sqrt{2} \sin \theta \\ \sin \theta & 1 & \cos \theta \end{pmatrix}, \] (5)

\[ \hat{M}_c = (|+\rangle - |\rangle) \langle 0 | \sqrt{2} + \text{H.c.}, \quad \hat{M}_t = (|\rangle + |\rangle) \langle 0 | \sqrt{2} + \text{H.c.}, \] and \( \hat{M}_c = (|\rangle + |\rangle) \langle 0 | \sqrt{2} + \text{H.c.}, \) is the Pauli matrix for a spin 1 system, which obeys the commutation relation \( [\hat{M}_c, \hat{M}_t] = i \varepsilon^{\mu \nu} \hat{M}_\mu \) with \( \varepsilon^{\mu \nu} \) being the Levi–Civita symbol.

The second term of the Hamiltonian in equation (4) corresponds to nonadiabatic transitions among the dark state and the two bright states when the adiabatic condition \( |\hat{\theta}| \ll g(t) \) is not met well, as shown in figure 1(c).

In order to correct the nonadiabatic leakage, a correction Hamiltonian
\[ H'_c = \begin{pmatrix} 0 & 0 & i\hat{\theta} \\ 0 & 0 & 0 \\ -i\hat{\theta} & 0 & 0 \end{pmatrix} \] (6)
is introduced, so that the total Hamiltonian becomes
\[ H'_m = H_m + H'_c, \] (7)
for transitionless quantum driving. Therefore, in the adiabatic basis, the total modified Hamiltonian becomes
\[ H'_m = U(t)[H_{\text{ad}}(t) + H'_c(t)]U^\dagger(t) + i \frac{\text{d}U(t)}{\text{d}t} U^\dagger(t) = g(t) \hat{M}_c, \] (8)
which does not have the nonadiabatic transitions. However, the Hamiltonian in equation (6) refers to the direct coupling between the microwave and optical modes, which is hard to induce directly in practice.

To overcome this obstacle, we look for another correction Hamiltonian \( H''_c \) via the dressed state method [32], so that we can speed up the QSC by modifying only the pulse shape of the coupling strength in equation (2). Here, we take
\[ H''_c = U^\dagger(t) \left[ g_1(t) \hat{M}_c + g_2(t) \hat{M}_f \right] U(t), \] (9)
so that the total Hamiltonian in the adiabatic basis now becomes
\[ \hat{H}(t) = H_m + H''_c = \hat{g}_1(t) \langle a_1 | + \hat{g}_2(t) \langle a_2 | + \text{H.c.}, \] (10)
where the modified couplings are
\[ \hat{g}_1(t) = [g(t) + g_1(t) \sin \theta + g_2(t) \cos \theta], \]
\[ \hat{g}_2(t) = [g(t) + g_1(t) \cos \theta - g_2(t) \sin \theta]. \] (11)

From the discussion in the last section, we know that this cannot be achieved if we use the adiabatic eigenstates as the conversion channel. However, it only requires the initial and the final states are in the adiabatic eigenstates. Therefore, we move to the dressed state picture with respect to \( \tilde{V}(t) = \exp[i\hat{\mu}(t) \hat{M}_c] \). After these two transformations, we find that the total Hamiltonian becomes
\[ H''_m = VH_{\text{ad}}(t)V^\dagger + \frac{\text{d}V(t)}{\text{d}t} V^\dagger. \] (12)

As the modified Hamiltonian \( H''_m \) should be designed to cancel out the unwanted off-diagonal elements, the controlled parameters \( g_1(t), g_2(t) \) should be
\[ g_1(t) = \hat{\mu}(t), \quad g_2(t) = -g(t) + \hat{\theta}/\tan \mu(t). \] (13)

3.2. Numerical simulations

For the time dependence coupling, we choose \( g_1(t) = g_0 \sin^2 (\theta t/2), g_2(t) = g_0 \cos^2 (\theta t/2) \). In order to obtain such coupling in opto-electro-mechanical systems, modulating the external driving field is a feasible way, as \( g_1 \propto \sqrt{n} \) with \( n \) being the photon number inside a cavity. In the strongly-driven condition, external driving field excites large number of photons, \( 10^6 \sim 10^8 \), in the cavity [43]. Therefore, the amplitude and phase of the couplings can be adjusted in a broad range [44]. Note that during the superadiabatic correction process, one should guarantee that each corrected coupling \( \hat{g}_1(t), \hat{g}_2(t) \) cannot exceed its original couplings’ peak amplitude \( g_0 \). i.e. we need to ensure that \( \max \| \hat{g}_1(t), \hat{g}_2(t) \| \leq g_0 \). This constraint implies that we can only spread up the process with a minimal time \( T_0 = 3.24/g_0 \), through our numerical verification. As shown in figure 2(a), the final fidelity reaches approximately 98.4% and 99.9% for \( \tau = 5T_0 \) and \( \tau = 8T_0 \) respectively, without decoherence. Therefore, we set \( \tau = 8T_0 \) as the adiabatic case for our reference.

We now compare the performance of the adiabatic and superadiabatic QSC under dissipation. The performance of the QSC is evaluated by considering the influence of dissipation using the Markovian master equation
\[ \dot{\rho} = -i[H(t), \rho] + \frac{\gamma_1}{2} L(a_1) + \frac{\gamma_2}{2} L(a_2) + \frac{\kappa}{2} L(b_m), \] (14)
where \( \rho \) is the density matrix of the considered system. \( L(A) \) is the Lindblad superoperator \( L(A) = 2 A \rho A^\dagger - A^\dagger A \rho - \rho A^\dagger A \) with \( A \in \{ a_1, a_2, b_m \} \); \( \gamma_1 \) and \( \gamma_2 \) are the decay rate of the optical cavity \( a_1 \) and microwave cavity \( a_2 \) due to the loss of photons inside cavity; \( \kappa \) is the decay rates of the mechanical oscillator \( b_m \). Here, we take the decoherence induced by the mechanics mode for having thermal excitation at low temperatures. We choose \( g_0 = 2 \pi \times 5 \text{ MHz} \) and the decay rates \( \kappa = \gamma_2 = g_0/1000, \gamma_1 = g_0/50 \) has already been demonstrated experimentally [43]. As shown in figure 2(b), when \( \tau = 8T_0 \), we can only obtain a fidelity of 73.80% for the QSC. Correspondingly, when \( \tau = T_0 \), one finds that the conversion fidelity reaches 96.53% as shown in figure 2(c).

3.3. Suppression of the intermediate state population

In most dissipation systems, operation time and decoherence are two major factors influencing the final fidelity. There is a trade-off between operation time and decoherence [14]. When the operation time is long enough to satisfy the
adiabatic condition well, high fidelity can be obtained, while dissipation will destroy it due to the long time integral. When the operation time is too short, non-adiabatic leakage may lead to poor performance during the conversion procedure. Through the superadiabatic correction, there is no need to worry about this trade-off, for we release the adiabatic condition. The decoherence property of the system becomes our major concern.

However, as shown in figure 2(c), the population of the intermediate mechanical mode is quite large, which is what we should try to avoid. The intermediate state may decay to the ground state, and thus reduce the conversion fidelity. Therefore, if the decay of the intermediate mode is large, one of the main issues of QSC mediated by a quantum bus is to find ways of reducing the population of intermediate state. The population of the intermediate level is determined to the ground state, and thus reduce the conversion fidelity. We note that when the operation time is longer, the population of the intermediate state undergoes a significant decrease as we increase $A$, as shown in figure 3(b), which shows our way of reducing the population of the mechanical mode is quite effective. The pulse shape for $A = 0.85$ is plotted in figure 3(c). Under this suppression, we note that the fidelity is deceased instead of increased. This is because the decay of the optical cavity is the main decoherence source in our system; the suppression requires further modification of the pulse shape, which deviates from the optimal one and thus results in slight decrease of the final state fidelity. If the decay rate of the intermediate state is larger, the suppression of the immediate population will be more important, and the suppression will lead to the increase of the final fidelity, as shown in figure 3(d).

We note that when the operation time is longer, the population of the intermediate mechanical mode can also be suppressed. Therefore, we further explore the conversion fidelity for a slower process. For $\tau = 2T_0$, the fidelity is 92.86% in figure 4(a) and the corresponding superadiabatic pulse shapes have been plotted in figure 4(b). Meanwhile, by introducing another auxiliary function $f' = 1 + B \sin^4 (\pi t/\tau)$ with $B = 0.69$, the fidelity we can obtain is 92.81%, as shown in figure 4(c). It is obvious that the fidelity is also slightly
Figure 4. (a) Population dynamics of the superadiabatic process when $\tau = 2T_0$ without suppression ($A = 0$), and the pulse shape is plotted in (b). (c) Population dynamics of (a) with $A = 0.69$. (d) Population dynamics of the intermediate mode for different $A$.

decreased, as explained above. Meanwhile, the fidelity is smaller that the case of $\tau = T_0$. This is quite natural, as the decay rate of the mechanical mode is small, and thus the operation time here is the most important decoherence source. The largest $B$ we can choose for $\tau = 2T_0$ is 0.69, since we still need to guarantee that the peak amplitude of $g_0$ is no larger than the peak amplitude of $g_0$. Hence, when we also choose values of $A$ ranging from 0.69 to 0, the maximum population of the intermediate state also undergoes a significant decrease, as shown in figure 4(d).

4. Conclusion

In conclusion, we have proposed a scheme to realize the superadiabatic QSC process in an opto-electro-mechanical system, which can significantly speed up the adiabatic procedure by using dressed states. Our scheme possesses the following remarkable advantages. Firstly, there is no direct coupling between the target and initial modes in the Hamiltonian, and thus is feasible experimentally. Secondly, during the whole evolution, the adiabatic condition is released, and thus rapidity and high fidelity can still be achieved compared to conventional methods. Therefore, our protocol presents an important step towards practical state conversion between optical and microwave photons, and thus may find many applications in hybrid quantum information processing.

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