Boson mass spectrum in $SU(4)_L \otimes U(1)_Y$ model with exotic electric charges

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Abstract

The boson mass spectrum of the electro-weak $SU(4)_L \otimes U(1)_Y$ model with exotic electric charges is investigated by using the algebraical approach supplied by the method of solving gauge models with high symmetries. Our approach predicts for the boson sector a one-parameter mass scale to be tuned in order to match the data obtained at LHC, LEP, CDF.

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1 Introduction

In view of new experimental challenges - such as tiny massive neutrinos and their oscillations or extra-neutral gauge bosons, to mention but a few - the Standard Model (SM) [1] - [3] - based on the gauge group $SU(3)_C \otimes SU(2)_L \otimes U(1)_Y$ that undergoes in its electro-weak sector a spontaneous symmetry breakdown (SSB) up to the electromagnetic universal one $U(1)_{em}$ - has to be properly extended.

One of the first and most popular such extensions - designed initially to address another puzzle of the particle physics, namely the parity non-conservation in weak interactions - was proposed in mid 70’s by Pati, Salam, Mohapatra and Senjanovic [4] - [7] and is known as the (minimal) “left-right symmetric $SU(2)_L \otimes SU(2)_R \otimes U(1)_{B-L}$ model” for electro-weak sector. Regarding the quark sector, which is subject to the strong interaction, the “color” gauge group $SU(3)_C$ was replaced by $SU(4)_C$ . The so called Pati-Salam model [4] favors phenomenological consequences, such as violation of the barion - lepton number in quark and proton decays and the appearance of the exotic gauge mesons in semileptonic processes. However, in the last decade, a new direction has emerged in the literature, now being widely exploited. It resides in replacing the gauge group $SU(2)_L$ in the electro-weak sector by the new $SU(4)_L$. Therefore, the gauge symmetry $SU(3)_C \otimes SU(4)_L \otimes U(1)_Y$ (hereafter 3-4-1 model) was championed in a series of papers [8] - [17] with notable results. According to the charge assignment, different classes of 3-4-1 models were obtained by resorting to particular methods [18] - [20] for discriminating among the models.
It is worth noticing that both kinds of extensions above mentioned seem to have common points, as it was recently emphasized in Refs. [21, 22]. Plausible phenomenology can occur since, for instance, the partial unification $SU(4) \otimes U(1)_{B-L}$ is broken down to "minimal left-right" symmetric model $SU(2)_{L} \otimes SU(2)_{R} \otimes U(1)_{B-L}$ either by four-dimensional Higgs mechanism or by inner or outer automorphisms orbifolding in five dimensions [22].

Here we deal with a particular class of 3-4-1 models, namely the one including exotic electric charges [8] - [12] and solve it by using the exact algebraical approach [23] designed for solving gauge models with high symmetries. Our main purpose is to obtain for the boson sector a one-parameter mass scale to be tuned in order to match the data obtained at LHC, LEP, CDF [24]. This goal is achieved by simply imposing a plausible physical criterion on the resulting mass matrix. That is, in the low energy limit of the model, the third neutral (diagonal) boson has to be decoupled by the other two, since it comes from a higher breaking scale.

The paper is organized as follows: Sec.2 briefly presents the main steps of the general method of treating $SU(n)_{L} \otimes U(1)_{Y}$ electro-weak gauge models, in which a particular Higgs mechanism - based on a special parametrization of the scalar sector - is embedded in order to properly break the symmetry.

2 The General Method

In this section we recall the main results of the general method [23] of treating $SU(n)_{L} \otimes U(1)_{Y}$ electro-weak gauge models, in which a particular Higgs mechanism - based on a special parametrization of the scalar sector - is embedded in order to properly break the symmetry.

2.1 $SU(n)_{L} \otimes U(1)_{Y}$ electro-weak gauge models

In our general approach, the basic piece involved in the gauge symmetry is the group $SU(n)$. Its two fundamental irreducible unitary representations (irreps) $\mathbf{n}$ and $\mathbf{n}^{*}$ play a crucial role in constructing different classes of tensors of ranks $(r, s)$ as direct products like $(\otimes_{n}^{r}) \otimes (\otimes_{n}^{s})^{*}$. These tensors have $r$ lower and $s$ upper indices for which we reserve the notation, $i, j, k, \cdots = 1, \cdots, n$. As usually, we denote the irrep $\rho$ of $SU(n)$ by indicating its dimension, $n_{\rho}$. The $su(n)$ algebra can be parameterized in different ways, but here it is convenient to use the hybrid basis of Ref. [23] consisting of $n - 1$ diagonal generators of the Cartan subalgebra, $D_{i}$, labeled by indices $i, j, \ldots$ ranging from 1 to $n - 1$, and the generators $E_{j}^{i} = H_{j}^{i}/\sqrt{2}$, $i \neq j$, related to the off-diagonal real generators $H_{j}^{i}$ [25, 26]. This way the elements $\xi = D_{i}\xi^{i} + E_{j}^{i}\xi^{j} \in su(n)$ are now parameterized by $n - 1$ real parameters, $\xi^{i}$, and by $n(n - 1)/2$ c-number ones, $\xi^{ij} = (\xi^{ji})^{*}$, for $i \neq j$. The advantage of this choice is that the parameters $\xi^{ij}$ can be directly associated to the c-number gauge fields due to the factor $1/\sqrt{2}$ which gives their correct normalization. In addition, this basis exhibit good trace orthogonality...
reducible representation of the gauge group. The Lagrangian density (Ld) of the free
one) which is put in pure left form using the charge conjugation. Consequently this
includes only left components,

\[ \rho \rightarrow U(\xi^0, \xi) L^\rho = e^{-i(g\xi^0 + g' y_{ch} \xi^0)} L^\rho \]  

where \( \xi = \in su(n) \) and \( y_{ch} \) is the chiral hypercharge defining the irrep of the \( U(1)_Y \) group parametrized by \( \xi^0 \). For simplicity, the general method deals with the character \( y = y_{ch} g'/g \) instead of the chiral hypercharge \( y_{ch} \), but this mathematical artifice does not affect in any way the results. Therefore, the irreps of the whole gauge group \( SU(n)_L \otimes U(1)_Y \) are uniquely detemined by indicating the dimension of the \( SU(n) \) tensor and its character \( y \) as \( \rho = (n_\rho, y_\rho) \).

In general, the spinor sector of our models has at least a part (usually the leptonic one) which is put in pure left form using the charge conjugation. Consequently this includes only left components, \( L = \sum_\rho \oplus L^\rho \), that transform according to an arbitrary reducible representation of the gauge group. The Lagrangian density (Ld) of the free spinor sector has the form

\[ \mathcal{L}_{S_0} = \frac{i}{2} \sum_\rho \overline{\mathcal{L}}^\rho \otimes L^\rho - \frac{1}{2} \sum_{\rho \rho'} \left( \overline{\mathcal{L}}^\rho \chi^{0 \rho'} \left( L^\rho \right)^c + h.c. \right). \]

Bearing in mind that each left-handed multiplet transforms as \( L^\rho \rightarrow U^\rho(\xi^0, \xi) L^\rho \) we understand that \( \mathcal{L}_{S_0} \) remains invariant under the global \( SU(n)_L \otimes U(1)_Y \) transformations if the blocks \( \chi^{0 \rho'} \) transform like \( \chi^{0 \rho'} \rightarrow U^\rho(\xi^0, \xi) \chi^{0 \rho'} (U^\rho(\xi^0, \xi))^T \), according to the representations \( (n_\rho \otimes n_{\rho'}, y_\rho + y_{\rho'}) \) which generally are reducible. These blocks will give rise to the Yukawa couplings of the fermions with the Higgs fields. The spinor sector is coupled to the standard Yang-Mills sector constructed in usual manner by gauging the \( SU(n)_L \otimes U(1)_Y \) symmetry. To this end we introduce the gauge fields \( A^0_\mu = (A^0_\mu)^* \) and \( A^{+}_\mu = A^{+}_\mu \in su(n) \). Furthermore, the ordinary derivatives are replaced in Eq. (3) by the covariant ones, defined as \( D_\mu L^\rho = \partial_\mu L^\rho - ig(A^0_\mu + y_\rho A^+_{\mu})L^\rho \) thus arriving to the interaction terms of the spinor sector.

\( 2.2 \) Minimal Higgs Mechanism

The scalar sector, organized as the so called minimal Higgs mechanism (mHm) \[23\], is flexible enough to produce the SSB and, consequently, generate masses for the plethora of particles and bosons in the model. The scalar sector consists of \( n \) Higgs multiplets \( \phi^{(1)}, \phi^{(2)}, \ldots, \phi^{(n)} \) satisfying the orthogonality condition \( \phi^{(i)} + \phi^{(j)} = \phi^2 \delta_{ij} \) in order
to eliminate the unwanted Goldstone bosons that could survive the SSB. $\phi$ is a gauge-invariant real scalar field while the Higgs multiplets $\phi^{(i)}$ transform according to the irreps $(n, y^{(i)})$ whose characters $y^{(i)}$ are arbitrary numbers that can be organized into the diagonal matrix

$$ Y = \text{Diag} \left( y^{(1)}, y^{(2)}, \ldots, y^{(n)} \right) . \quad (4) $$

The Higgs sector needs, in our approach, a parameter matrix

$$ \eta = \text{Diag} \left( \eta^{(1)}, \eta^{(2)}, \ldots, \eta^{(n)} \right) \quad (5) $$

with the property $\text{Tr}(\eta^2) = 1 - \eta_{0}^{2}$. It will play the role of the metric in the kinetic part of the Higgs Ld which reads

$$ \mathcal{L}_H = \frac{1}{2} \eta_{0}^{2} \partial_\mu \phi \partial^\mu \phi + \frac{1}{2} \sum_{i=1}^{n} \left( \eta^{(i)} \right)^{2} \left( D_\mu \phi^{(i)} \right)^{+} \left( D^\mu \phi^{(i)} \right) - V(\phi) \quad (6) $$

where $D_\mu \phi^{(i)} = \partial_\mu \phi^{(i)} - ig(A_\mu + y^{(i)}A_\mu^{0})\phi^{(i)}$ are the covariant derivatives of the model and $V(\phi)$ is the scalar potential generating the SSB of the gauge symmetry [23]. This is assumed to have an absolute minimum for $\phi = \langle \phi \rangle \neq 0$ that is, $\phi = \langle \phi \rangle + \sigma$ where $\sigma$ is the unique surviving physical Higgs field. Therefore, one can always define the unitary gauge where the Higgs multiplets, $\hat{\phi}^{(i)}$ have the components

$$ \hat{\phi}^{(i)}_k = \delta_{ik} \phi = \delta_{ik} \left( \langle \phi \rangle + \sigma \right) . \quad (7) $$

This will be of great importance when the fermion masses will be computed, due to the fact that the fermion mass terms - provided by Eq. (3) via this $m_H^m$ - exhibit the Yukawa traditional form only when the theory is boosted towards the unitary gauge.

### 2.3 Neutral bosons

A crucial goal is now to find the physical neutral bosons with well-defined properties. This must start with the separation of the electromagnetic potential $A_{\mu}^{em}$ corresponding to the surviving $U(1)_{em}$ symmetry. We have shown that the one-dimensional subspace of the parameters $\xi^{em}$ associated to this symmetry assumes a particular direction in the parameter space $\{\xi^0, \xi^i\}$ of the whole Cartan subalgebra. This is uniquely determined by the $n - 1$ - dimensional unit vector $\nu$ and the angle $\theta$ giving the subspace equations $\xi^0 = \xi^{em} \cos \theta$ and $\xi^i = \nu_i \xi^{em} \sin \theta$. On the other hand, since the Higgs multiplets in unitary gauge remain invariant under $U(1)_{em}$ transformations, we must impose the obvious condition $D_i \xi^i + Y \xi^0 = 0$ which yields

$$ Y = -D_i \nu^i \tan \theta \equiv -(D \cdot \nu) \tan \theta . \quad (8) $$

In other words, the new parameters $(\nu, \theta)$ determine all the characters $y^{(i)}$ of the irreps of the Higgs multiplets. For this reason these will be considered the principal parameters of the model and therefore one deals with $\theta$ and $\nu$ (which has $n - 2$ independent components) instead of $n - 1$ parameters $y^{(i)}$. 

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Under these circumstances, the generating mass term
\[
\frac{g^2}{2} \langle \phi \rangle^2 T r \left[ (A_\mu + Y A_0^0) \eta^2 (A_\mu + Y A_0^0) \right],
\]
depends now on the parameters \( \theta \) and \( \nu \). The neutral bosons in Eq. 9 being the electromagnetic field \( A_{\mu}^{em} \) and the \( n-1 \) new ones, \( A_{\mu}^{i} \), which are the diagonal bosons remaining after the separation of the electromagnetic potential (23).

This term straightforwardly gives rise to the masses of the non-diagonal gauge bosons
\[
M_{ij} = \frac{g}{2} \langle \phi \rangle \sqrt{\left[ (\eta^{(i)})^2 + (\eta^{(j)})^2 \right]},
\]
while the masses of the neutral bosons \( A_{\mu}^{i} \) have to be calculated by diagonalizing the matrix
\[
(M^2)^{ij} = \langle \phi \rangle^2 T r (B_i B_j)
\]
where
\[
B_i = g \left( D_i + \nu (D \cdot \nu) \frac{1 - \cos \theta}{\cos \theta} \right) \eta,
\]
As it was expected, \( A_{\mu}^{em} \) does not appear in the mass term and, consequently, it remains massless. The other neutral gauge fields \( A_{\mu}^{i} \) have the non-diagonal mass matrix (11). This can be brought in diagonal form with the help of a new \( SO(n-1) \) transformation, \( A_{\mu}^{i} = \omega^{j} j Z_{\mu}^{j} \), which leads to the physical neutral bosons \( Z_{\mu}^{j} \) with well-defined masses. Performing this \( SO(n-1) \) transformation the physical neutral bosons are completely determined. The transformation
\[
A_{\mu}^{0} = A_{\mu}^{em} \cos \theta - \nu (D \cdot \nu) \sin \theta,
A_{\mu}^{k} = \nu^{k} A_{\mu}^{em} \sin \theta + \left( \delta^{k} i - \nu^{k} (1 - \cos \theta) \right) \omega^{j} j Z_{\mu}^{j},
\]
which switches from the original diagonal gauge fields, \( (A_{\mu}^{0}, A_{\mu}^{i}) \) to the physical ones, \( (A_{\mu}^{em}, Z_{\mu}^{j}) \) is called the generalized Weinberg transformation (gWt).

### 2.4 Electric and Neutral Charges

The next step is to identify the charges of the particles with the coupling coefficients of the currents with respect to the above determined physical bosons. Thus, we find that the spinor multiplet \( L^{\rho} \) (of the irrep \( \rho \)) has the following electric charge matrix
\[
Q^{\rho} = g \left[ (D^{\rho} \cdot \nu) \sin \theta + y_{\rho} \cos \theta \right],
\]
and the \( n-1 \) neutral charge matrices
\[
Q^{\rho}(Z^{j}) = g \left[ D^{\rho}_{k} - \nu_{k} (D^{\rho} \cdot \nu) (1 - \cos \theta) - y_{\rho} \nu_{k} \sin \theta \right] \omega^{k} j
\]
corresponding to the \( n-1 \) neutral physical fields, \( Z^{j}_{\mu} \). All the other gauge fields, namely the charged bosons \( A_{\mu}^{i} \), have the same coupling, \( g/\sqrt{2} \), to the fermion multiplets.
3 Solving the $SU(4)_L \otimes U(1)_Y$ model

The general method - constructed in Ref. [23] and briefly presented in the above section - is based on the following assumptions in order to give viable results when it is applied to concrete models:

(I) the spinor sector must be put (at least partially) in pure left form using the charge conjugation (see for details Appendix B in Ref. [23])

(II) the minimal Higgs mechanism - with arbitrary parameters $(\eta_0, \eta)$ satisfying the condition $Tr(\eta^2) = 1 - \eta_0^2$ and giving rise to traditional Yukawa couplings in unitary gauge - must be employed

(III) the coupling constant, $g$, is the same with the first one of the SM

(IV) at least one $Z$-like boson should satisfy the mass condition $m_Z = m_W / \cos \theta_W$ established in the SM and experimentally confirmed.

Bearing in mind all these necessary ingredients, we proceed to solving the particular 3-4-1 model [8] - [12] by imposing from the very beginning the set of parameters we will work with.

3.1 Fermion representations

In what follows we denote the irreps of the electro-weak model under consideration here by $\rho = (n_\rho, y^\rho_{ch})$ indicating the genuine chiral hypercharge $y_{ch}$ instead of $y$. Therefore, the multiplets of the 3-4-1 model of interest here will be denoted by $(n_{color}, n_\rho, y^\rho_{ch})$.

With this notation, after little algebra involving Eqs. (14) - (15) and the versor setting $\nu_1 = 1, \nu_2 = 0, \nu_3 = 0$ - Case 1 in Ref. [23], one finds the following irreps of the spinor sector:

Lepton families

$$f_{\alpha L} = \begin{pmatrix} e^c_{\alpha} \\ e_{\alpha} \\ \nu_{\alpha} \\ N_{\alpha} \end{pmatrix} \sim (1, 4, 0)$$ (16)

Quark families

$$Q_{iL} = \begin{pmatrix} J_i \\ u_i \\ d_i \\ D_i \end{pmatrix}_L \sim (3, 4^*, -1/3) \quad Q_{3L} = \begin{pmatrix} J_3 \\ -b \\ t \\ T \end{pmatrix}_L \sim (3, 4, +2/3)$$ (17)

$$(b_L)^c, (d_{iL})^c, (D_{iL})^c \sim (3, 1, +1/3) \quad (t_L)^c, (u_{iL})^c, (T_L)^c \sim (3, 1, -2/3)$$

$$(J_{3L})^c \sim (3, 1, -5/3) \quad (J_{iL})^c \sim (3, 1, +4/3)$$ (18)

with $\alpha = 1, 2, 3$ and $i = 1, 2$. With this, the conditions (I) and (III) are fulfilled.

In addition, the connection between the $\theta$ angle of our method and $\theta_W$ (the Weinberg angle from SM) was inferred [20]: $\sin \theta = 2 \sin \theta_W$ along with the coupling relation: $g'/g = \sin \theta_W / \sqrt{1 - 4 \sin^2 \theta_W}$.
In the representations presented above we assumed, like in majority of the papers in the literature, that the third generation of quarks transforms differently from the other two ones. This could explain the unusual heavy masses of the third generation of quarks, and especially the uncommon properties of the top quark. The capital letters \( J \) denote the exotic quarks included in each family. With this assignment the fermion families cancel all the axial anomalies by just an interplay between them, although each family remains anomalous by itself. Note that one can add at any time sterile neutrinos - i.e. right-handed neutrinos \( \nu_{\alpha R} \sim (1, 0) \) - that could pair in the neutrino sector of the \( \text{Ld} \) with left-handed ones in order to eventually generate tiny Dirac or Majorana masses by means of an adequate see-saw mechanism. These sterile neutrinos do not affect anyhow the anomaly cancelation, since all their charges are zero. Moreover, their number is not restricted by the number of flavors in the model.

### 3.2 Boson mass spectrum

Subsequently, we will use the standard generators \( T_\alpha \) of the \( \text{su}(4) \) algebra. Under these circumstances, the Hermitian diagonal generators of the Cartan subalgebra are, in order, \( D_1 = T_3 = \frac{1}{2} \text{Diag}(1, -1, 0) \), \( D_2 = T_8 = \frac{1}{2\sqrt{3}} \text{Diag}(1, 1, -2) \), and \( D_3 = T_{15} = \frac{1}{2\sqrt{6}} \text{Diag}(1, 1, 1, -3) \) respectively. In this basis, the gauge fields are \( A_\mu \) and \( A_\mu \in \text{su}(4) \), that is

\[
A_\mu = \frac{1}{2} \left( \begin{array}{cccc}
D^1_\mu & \sqrt{2} X_\mu & \sqrt{2} X'_\mu & \sqrt{2} K_\mu \\
\sqrt{2} X^*_\mu & D^2_\mu & \sqrt{2} W_\mu & \sqrt{2} K'_\mu \\
\sqrt{2} X'_\mu & \sqrt{2} W^*_\mu & D^3_\mu & \sqrt{2} Y_\mu \\
\sqrt{2} K^*_\mu & \sqrt{2} K'^*_\mu & \sqrt{2} Y^*_\mu & D^4_\mu \\
\end{array} \right),
\]

with neutral diagonal bosons: \( D^1_\mu = A^3_\mu + A^8_\mu / \sqrt{3} + A^{15}_\mu / \sqrt{6}, D^2_\mu = -A^3_\mu + A^8_\mu / \sqrt{3} + A^{15}_\mu / \sqrt{6}, D^3_\mu = -2A^8_\mu / \sqrt{3} + A^{15}_\mu / \sqrt{6}, \) and \( D^4_\mu = -3A^{15}_\mu / \sqrt{6} \) respectively.

Apart from the charged Weinberg bosons, \( W \), there are new charged bosons, \( K, K', X, X' \) and \( Y \). Note that \( X \) is doubly charged coupling different chiral states of the same charged lepton (the so called "bilepton"), while \( Y \) is neutral.

The masses of both the neutral and charged bosons depend on the choice of the matrix \( \eta \) whose components are free parameters. Here it is convenient to assume the following matrix

\[
\eta^2 = (1 - \eta_0^2) \text{Diag} \left( 1 - c, \frac{1}{2} a - b, \frac{1}{2} a + b, c - a \right),
\]

where, for the moment, \( a, b \) and \( c \) are arbitrary non-vanishing real parameters. Obviously, \( \eta_0, c \in (0, 1), a \in (0, c) \) and \( b \in (-a, +a) \). Note that with this parameter choice the condition (II) is accomplished.
Under these circumstances, the mass spectrum of the off-diagonal bosons according to (10) are

\[ m^2(W) = m^2 a, \quad (22) \]
\[ m^2(X) = m^2 \left( 1 - c + \frac{1}{2} a - b \right), \quad (23) \]
\[ m^2(X') = m^2 \left( 1 - c + \frac{1}{2} a + b \right), \quad (24) \]
\[ m^2(K) = m^2 (1 - a), \quad (25) \]
\[ m^2(K') = m^2 \left( c - 1 - a - b \right), \quad (26) \]
\[ m^2(Y) = m^2 \left( c - 1 - a + b \right). \quad (27) \]

while the mass matrix of the neutral bosons is given by Eq. (11)

\[
M^2 = m^2 \begin{pmatrix}
\frac{1}{\cos^2 \theta} & \frac{1}{\sqrt{3} \cos \theta} & \frac{1}{\sqrt{6} \cos \theta} & \frac{1}{\sqrt{3} \cos \theta} \\
\frac{1}{\sqrt{3} \cos \theta} & \frac{1}{\cos^2 \theta} & \frac{1}{\sqrt{3} \cos \theta} & \frac{1}{\sqrt{6} \cos \theta} \\
\frac{1}{\sqrt{6} \cos \theta} & \frac{1}{\sqrt{3} \cos \theta} & \frac{1}{\cos^2 \theta} & \frac{1}{\sqrt{3} \cos \theta} \\
\frac{1}{\sqrt{6} \cos \theta} & \frac{1}{\sqrt{3} \cos \theta} & \frac{1}{\sqrt{3} \cos \theta} & \frac{1}{\cos^2 \theta} + \frac{1}{2} (1 + 8c - 8a)
\end{pmatrix}
\]

(28)

with \( m^2 = g^2 \langle \phi \rangle^2 (1 - \eta_0^2) / 4 \) throughout this paper. In order to fulfil the requirement (IV), the above matrix has to admit \( m^2 a / \cos^2 \theta_W \) as eigenvalue.

Now, one can enforce some other phenomenological assumptions. First of all, it is natural to presume that the third neutral (diagonal boson) \( Z'' \) should be considered much heavier than its companions, so that it decouples from their mixing as the symmetry is broken to \( SU(3) \). For this purpose a higher breaking scale is responsible, that at the same time supplies mass to \( Z'' \).

Therefore one can well consider \( M_{13}^2 = M_{23}^2 = M_{31}^2 = M_{32}^2 \) in the matrix (28).

This gives rise to the condition

\[ b = - \left( \frac{\sqrt{3} - \sqrt{1 - 4 \sin^2 \theta_W}}{\sqrt{3} + 3 \sqrt{1 - 4 \sin^2 \theta_W}} \right) \left( 1 - c - \frac{1}{2} a \right), \quad (29) \]

which naturally leads to

\[ 1 - c \approx \frac{1}{2} a \quad (30) \]

in order to vanish the above terms.

Hence, with the plausible physical condition assumed, Eq. (28) looks like
\[ M^2 = m^2 \begin{pmatrix} \frac{1}{\cos^2 \theta} (a - b) & \frac{1}{2\sqrt{3} \cos \theta} b & 0 \\ \frac{1}{2\sqrt{3} \cos \theta} b & a + b & 0 \\ 0 & 0 & \frac{1}{2} (3 - 4a) \end{pmatrix} \]  

(31)

Let us observe that the condition (IV) -  \[ \text{Det} \left| M^2 - m^2 a / \cos^2 \theta \right| = 0 \]  - is fulfilled if and only if  \[ b = \frac{3}{2} a \tan^2 \theta_W \], resulting from diagonalization of the remaining part of the matrix (31). Therefore, one finally remains with only one parameter - say \( a \).

In addition, for there are terms which become singular for  \( \cos \theta = 0 \) which corresponds to the value  \( \sin^2 \theta_W = 1/4 \) the Weinberg angle is restricted in our model so that  \( \sin^2 \theta_W \) less than  \( 1/4 \), which is in good accord to experimental measurements on it [24].

Obviously, \( Z \) is the neutral boson of the SM, while \( Z' \) is a new neutral boson of this model (also occurring in 3-3-1 models) whose mass comes from  \( \text{Tr}(M^2) = m^2(Z) + m^2(Z') + m^2(Z'') \).

\[ m^2(W) = m^2 a, \]  
\[ m^2(X) = m^2 a \left( 1 - \frac{3}{2} \tan^2 \theta_W \right), \]  
\[ m^2(X') = m^2 a \left( 1 + \frac{3}{2} \tan^2 \theta_W \right), \]  
\[ m^2(K) = m^2 (1 - a), \]  
\[ m^2(K') = m^2 \left[ 1 - a \left( 1 + \frac{3}{2} \tan^2 \theta_W \right) \right], \]  
\[ m^2(Y) = m^2 \left[ 1 - a \left( 1 - \frac{3}{2} \tan^2 \theta_W \right) \right], \]  
\[ m^2(Z) = m^2 a / \cos^2 \theta_W, \]  
\[ m^2(Z') = m^2 a \left[ 1 + \frac{3 \sin^2 \theta_W (1 - 2 \sin^2 \theta_W)}{1 - 4 \sin^2 \theta_W} \right], \]  
\[ m^2(Z'') = m^2 \frac{1}{2} (3 - 4a). \]  

(32) \quad (33) \quad (34) \quad (35) \quad (36) \quad (37) \quad (38) \quad (39) \quad (40)

The mass scale is now just a matter of tuning the parameter \( a \) in accordance with the possible values for \( \langle \phi \rangle \). Obviously, parameter \( a \) has now to be upper limited, so that \( a \in (0, 0.7) \) in order to have only positive values for the above expressions of the squared boson masses.

### 3.3 Numerical estimates

In order to allow for a high breaking scale in the model \( \langle \phi \rangle \geq 1 \text{TeV} \) and keep at the same time consistency with low energy phenomenology of the SM our solution favors
Table 1: Boson masses

| $a$ \ Mass (GeV) | $m = \frac{m(W)}{\sqrt{a}}$ | $m(Z''')$ | $m(Y)$ | $m(K)$ | $m(K')$ |
|------------------|----------------------------|-----------|--------|--------|--------|
| 0.5              | 119.36                    | 59.68     | 100.9  | 84.4   | 63.7   |
| 0.2              | 188.72                    | 207.6     | 177.6  | 168.8  | 159.5  |
| 0.05             | 377.45                    | 528.4     | 372.0  | 367.9  | 363.7  |
| 0.02             | 596.80                    | 871.3     | 593.4  | 590.8  | 588.2  |
| 0.007            | 1008.7                    | 1499.0    | 1006.7 | 1005.1 | 1003.6 |
| 0.005            | 1193.6                    | 1778.5    | 1191.9 | 1190.6 | 1189.3 |
| 0.002            | 1887.2                    | 2823.3    | 1886.1 | 1885.3 | 1884.5 |
| 0.0007           | 3190.0                    | 4780.5    | 3189.3 | 3188.8 | 3188.4 |
| 0.0005           | 3774.5                    | 5658.0    | 3773.9 | 3773.5 | 3773.2 |
| $a \to 0$        | $m$                       | $1.2m$    | $m$    | $m$    | $m$    |

The more heavier bosons are - roughly speaking - in the following hierarchy mass $m(Z''') > m(Y) > m(K) > m(K')$ for small values of the parameter $a$. If one desires a more detailed estimate for the latter bosons, depending on the possible values of the free parameter (and consequently on the mass scale), we present a plot of their masses dependence on $a$ for a given $m$. That is, we represented the functions $m(Z''') \sim \sqrt{1.5 - 2a}$, $m(Y) \sim \sqrt{1 - 0.57a}$, $m(K) \sim \sqrt{1 - a}$ and $m(K') \sim \sqrt{1 - 1.43a}$ (see Figure).

If the mass scale of the model ($m$ in our notation), lies in the TeV region - which is the current energy involved in LHC data - or even in a higher one, the three latter masses become almost degenerate. More precisely, if $m \approx 1$TeV, from Eq. (32) $a \approx 0.00712336$ is inferred. Hence, $m(Z''') \approx 1.485$TeV and $m(Y) \approx 0.998$TeV, $m(K) \approx 0.996$TeV and $M(K') \approx 0.995$TeV. Furthermore, if the mass scale reaches GUT energies (specific to some see-saw mechanism) these bosons - as expected - be-
come degenerate.

We refer the reader to the following Table for some numerical results in the boson sector of the given 3-4-1 model. A more accurate estimate for the masses of these bosons and the relations among them (by a more appropriate tuning of parameter $a$) will come, once the experimental evidence of their phenomenology will be definitely bring to light at LHC, LEP, CDF and other high energy accelerators in a near future.

4 Conclusions

In this work the boson mass spectrum of a 3-4-1 model with exotic electric charges has been worked out within the framework of the general method for solving gauge models with high symmetries [23] and analyzed by just tuning a unique free parameter $a$. In our approach the latter is close related to the mass scale (and thus on the overall breaking scale $\langle \phi \rangle$) in the manner $m \simeq 84.4/\sqrt{a}$ (GeV). Plausible phenomenological predictions were made at a reasonable scale around 1TeV to be confronted by data supplied by LHC, LEP, CDF and other colliders. The method used is flexible enough in order to allow even a different Higgs mechanism which could consist in re-defining the scalar multiplets in the manner $\phi^{(i)} \rightarrow \eta^{(1)} \phi^{(i)}$ in order to match the traditional approach, with each new $\phi^{(i)}$ responsible for one particular step in breaking the symmetry. One can further investigate this model by calculating the neutral currents and specific decays, the oblique corrections $S, T, U$ [21], or conceive a suitable see-saw mechanism in order to generate tiny masses for neutrinos. Breaking down the symmetry to the left-right Pati-Salam proposal [22] is also a worthy way for further theoretical investigations. All these, however, are beyond the scope of this paper and will be treated in a future work.

References

[1] S. Weinberg, *Phys. Rev. Lett.* 19, 1264 (1967).
[2] S. L. Glashow, *Nucl. Phys.* 22, 579 (1961).
[3] A. Salam, in Elementary Particle Theory: Relativistic Groups and Analyticity (Nobel Symposium No.8), ed. N. Svartholm (Almqvist and Wiksell, 1968) p.367.
[4] C. Pati and A. Salam, *Phys. Rev. D* 10, 275 (1974).
[5] R. N. Mohapatra and J. C. Pati, *Phys. Rev. D* 11, 566 (1975).
[6] G. Senjanovic and R. N. Mohapatra, *Phys. Rev. D* 11, 1502 (1975).
[7] R. N. Mohapatra and J. C. Pati, *Phys. Rev. D* 11, 2558 (1975).
[8] R. Foot, H. N. Long and T. A. Tran, *Phys. Rev. D* 50, R34 (1994).
[9] F. Pisano and V. Pleitez, *Phys. Rev. D* 51, 3865 (1995).
[10] A. Doff and F. Pisano, *Mod. Phys. Lett. A* 14, 1133 (1999).
[11] A. Doff and F. Pisano, *Phys. Rev. D* **63**, 097903 (2001).

[12] Fayyazuddin and Riazuddin, *JHEP* **0412**, 013 (2004).

[13] W. A. Ponce, D. A. Gutierrez and L. A. Sanchez, *Phys. Rev. D* **69**, 055007 (2004).

[14] L. A. Sanchez, F. A. Perez and W. A. Ponce, *Eur. Phys. J C* **35**, 259 (2004).

[15] L. A. Sanchez, L. A. Wills-Toro and J. I. Zuluaga, *Phys. Rev. D* **77**, 035008 (2008).

[16] Riazuddin and Fayyazuddin, *Eur. Phys. J C* **56**, (2008).

[17] A. Palcu, arXiv: 0902.3756 [hep-ph].

[18] A. Doff and F. Pisano, *Mod. Phys. Lett. A* **15**, 1471 (2000).

[19] W. A. Ponce and L. A. Sanchez, *Mod. Phys. Lett. A* **22**, 435 (2007).

[20] A. Palcu, arXiv: 0902.1301 - hep-ph (to be published in Mod. Phys. Lett. A).

[21] S. Sen, arXiv: 0901.2240 [hep/ph].

[22] T. Li, F. Wang and J. M. Yang, arXiv: 0901.2161 [hep-ph].

[23] I. I. Cotăescu, *Int. J. Mod. Phys. A* **12**, 1483 (1997).

[24] Particle Data Group (C. Amsler et al.), *Phys. Lett. B* **667**, 1 (2008).

[25] R. Gilmore, *Lie Groups, Lie Algebras and some of their Applications* (Wiley Interscience, 1974).

[26] A. O. Barut and R. Racza, *Theory of Group representations and their Applications* (PWN, 1977).