Pure spin current in graphene NS structures

D. Greenbaum, 1 S. Das, 1 G. Schwiete, 1 and P. G. Silvestrov 2

1 Department of Condensed Matter Physics, Weizmann Institute of Science, 76100 Rehovot, Israel
2 Theoretische Physik III, Ruhr-Universität Bochum, 44780 Bochum, Germany
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We demonstrate theoretically the possibility of producing a pure spin current in graphene by filtering the charge from a spin-polarized electric current. To achieve this effect, which is based on the recently predicted property of specular Andreev reflection in graphene, we propose two possible device structures containing normal-superconductor (NS) junctions.

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I. INTRODUCTION

The recent experimental realization of conducting two dimensional monolayers of graphite [1, 2, 3, 4], also known as graphene, offers the promise for new electronic devices. One conceivable use for graphene is in spintronics [5], where the lack of nuclear spin interaction ( 12C has no nuclear spin) could offer the ability to maintain spin coherence over larger distances than in conventional semiconductors. For progress towards this goal, it is essential to have simple and reliable means to transport spin in graphene.

We address this issue by proposing a prescription for producing a pure spin current in ballistic bulk graphene. Spin currents have already been predicted to arise in graphene due to spin-orbit coupling [6] and the Quantum Hall Effect [7]. In both cases, the spin currents are due to counter-propagating edge states of opposite spin. Our proposal makes use of the recently predicted specular [8] of Andreev reflection [9] in graphene to produce a pure spin current in bulk. This is accomplished using structures containing normal-superconducting (NS) boundaries, which filter the charge out of a current of spin-polarized quasiparticles, leaving behind a pure spin current. To be specific, we consider two device paradigms: 1) a V-junction geometry with opening angle appropriately tuned, 2) a channel with a superconductor at one boundary and a normal edge at the other. An advantage of such devices is the large number of transmitting channels in the bulk, offering the possibility of rapid spin accumulation. Also, our proposal does not require a magnetic field, nor does it rely on the spin-orbit gap. In the present case, however, it is crucial to first generate a spin-polarized electric current, which could conceivably be done by contacting the system to a ferromagnetic lead, as has been done with carbon nanotubes [10].

Our description of proximity effects in graphene follows that of Ref. [8]. Later publications, Refs. [11, 12], used this approach to discuss the Andreev spectrum and Josephson effect in NS (SNS) structures in graphene. A very recent paper [13] makes explicit use of the specularity of Andreev reflection in graphene by considering the neutral excitations propagating along a narrow SNS channel. These authors propose a device that is similar to the one considered below in Sec. IV, but suggest investigating the thermoelectric effect to observe the chargeless excitations. In contrast, we analyze the spin transport. Our approach also lends itself to an analysis of the deviations from perfect charge filtering in a channel, which is done in Sec. V. Finally, an elegant method to produce pure spin currents in conventional semiconductors was suggested in Ref. [14], where the spatial separation of electron and hole trajectories is caused by tunneling through a superconductor.

II. ANDREEV REFLECTION IN GRAPHENE

The electron wave function in graphene is described by a two component (pseudo-)spinor \( \psi \). Its spin-up and spin-down components correspond to the quantum mechanical amplitudes of finding the particle on one of the two sublattices of the honeycomb lattice. The low energy physics of graphene is governed by two so-called Dirac points in the spectrum, located at the two inequivalent corners \( \vec{K}, \vec{K}' \) of the Brillouin zone. The spinor wave function for low energy excitations in (lightly-doped) graphene decomposes into a sum of two waves oscillating with different wave vectors \( e^{i\vec{K} \cdot \vec{r}} \phi_\pm + e^{i\vec{K}' \cdot \vec{r}} \phi_- \). The smooth envelope functions \( \phi_\pm \) satisfy the two-dimensional Dirac equation [15] described by the Hamil-
tonian

\[ H_\pm = c(\sigma_x p_x \pm \sigma_y p_y) + U(x), \]  

where \( c \) is the Fermi velocity and \( \vec{p} = -i\hbar \vec{\nabla} \). In regions of constant \( U \) this equation defines a conical energy band, or valley, \( \varepsilon - U = \pm c|p| \). The Pauli matrices \( \sigma_{x,y} \) permute electrons between two triangular sublattices of the honeycomb lattice. The two signs in Eq. (1) (+) and (−) correspond to the two valleys \( \vec{K} \) and \( \vec{K}' \).

We consider a graphene sheet in the \( x - y \) plane, with the region \( x < 0 \) covered by a conventional superconductor (Fig. 1). Following Ref. [8] we assume that in this configuration a pair potential \( \Delta(x) = \Delta_0 \Theta(-x) \) can be induced [16] in the graphene sheet by the proximity effect, accompanied by a sufficiently strong shift in the scalar potential \( U(x) = -U_0 \Theta(-x) \), so that \( U_0 \gg \Delta_0 \). The reflection at the NS interface (\( x = 0 \)) is described by a separate four-dimensional Dirac-Bogoliubov-de Gennes equation [8] for each valley

\[
\begin{pmatrix}
H_\pm - E_F \\
\Delta(x) \\
E_F - H_\pm
\end{pmatrix}
\begin{pmatrix}
\begin{pmatrix}
u \\ u
\end{pmatrix}
\end{pmatrix}
= \varepsilon
\begin{pmatrix}
\begin{pmatrix}
u \\ u
\end{pmatrix}
\end{pmatrix},
\]  

where the two spinors \( u \) and \( v \) represent the electron and hole components of the \( \phi_\pm \) wavefunctions for \( H_\pm \), respectively.

In addition to carrying pseudo-spin, electron-hole, and valley indices, the quasiparticle wave function should also describe the usual spin. Since for \( \varepsilon < \Delta_0 \) no spin can be injected into the superconductor, the spin of the incident electron is transferred to the reflected particle.

For several decades it was considered a basic feature of Andreev reflection [9] that the hole produced upon electron-hole conversion retraces the incident electron’s trajectory. Even in the traditional materials, however, this repetition of trajectories is exact only for zero excitation energy \( \varepsilon \) in Eq. (2). At finite excitation energy the two trajectories do not quite coincide, leading to interesting effects in semiclassical dynamics [17] and the spectrum [18] of Andreev billiards. In graphene, the Andreev reflection described by Eq. (2) has the standard form [10]

\[
\begin{pmatrix}
u \\ u
\end{pmatrix}_{\text{out}} = R_A
\begin{pmatrix}
u \\ u
\end{pmatrix}_{\text{in}} = \begin{pmatrix}
\begin{pmatrix}r \\ r_A
\end{pmatrix}
\end{pmatrix}
\begin{pmatrix}
u \\ u
\end{pmatrix}_{\text{in}}.
\]  

With the notation \( \alpha = \arctan|p_y/p_x| \), \( \varepsilon = c\sqrt{p_x^2 + p_y^2} \), and \( \xi = \sqrt{\Delta_0^2 - \varepsilon^2} \) one has [10]

\[
r = \frac{-\varepsilon \sin \alpha}{\varepsilon + i \xi \cos \alpha}, \quad r_A = \frac{\Delta_0 \cos \alpha}{\varepsilon + i \xi \cos \alpha}.
\]  

III. NS INTERFACE IN V-JUNCTION GEOMETRY

Characteristic of specular Andreev reflection in graphene is the spatial separation of incident and reflected electron-hole beams. The simplest device that makes use of this property is a V-junction, as shown in Fig. 1. Suppose one can inject a spin-polarized collimated monoenergetic beam of electrons through one arm of the junction. [Monoenergetic beams may potentially be produced by resonant transport through a graphene quantum dot (QD) [20], as was done for semiconductor QD’s e.g. in Ref. [21].] The injected electron will be either normally or Andreev reflected at the NS interface and the created quasiparticle will escape through the second arm. Since no spin can penetrate through the superconductor, all the polarization of the injected beam is transferred to the second arm. On the other hand, the average charge of the quasiparticles reflected towards the second arm depends on the excitation energy. One obtains zero reflected total charge by setting \( |r| = |r_A| \). This constrains the incident particle energy to be

\[
\varepsilon = \Delta_0 \cot \alpha.
\]  

Alternatively, one may consider the injection of a collimated beam of electrons with all possible energies in the range \( 0 < \varepsilon < eV \). For Dirac particles and a fixed angular spread of the beam, cancellation of the reflected charge requires

\[
\int_0^{eV} |r|^2 \varepsilon d\varepsilon = \int_0^{eV} |r_A|^2 \varepsilon d\varepsilon.
\]  

One may think here about electrons leaving the biased (\( eV \)) graphene half-plane through a narrow slit and then both angle- and width-collimated by a second slit. Substitution of Eq. (4) into Eq. (6) leads to the transcendental equation

\[
\ln \left[ 1 + \left( \frac{eV}{\Delta_0 \tan \alpha} \right)^2 \right] = \frac{1}{2} \left( \frac{eV}{\Delta_0 \tan \alpha} \right)^2, \quad (7)
\]  

with the solution

\[
eV = 1.59 \Delta_0 \cot \alpha.
\]  

(8)
Experimental realization of pure spin current in a graphene V-junction is limited by the requirement of a collimated electron beam. This difficulty is avoided automatically in the setup proposed below, where the filtering of charge current takes place because of multiple Andreev reflections in a NS channel.

IV. CHARGE FILTERING IN A LONG NS CHANNEL

We consider a normal graphene strip of width \( W \) and length \( L \gg W \), which is formed by covering the region \( x < 0 \) of a graphene half-plane \( -\infty < x < W \) with a superconductor (see Fig. 2). Spin-polarized electrons are injected at \( y = 0 \). We restrict our discussion to the geometric optics limit, \( \lambda \ll W \), where the transmission through the channel may be considered in terms of individual particle trajectories. Here \( \lambda = 2\pi\hbar c/eV \) is the de Broglie wavelength for Dirac electrons with energy \( eV \).

In addition we consider the low-voltage limit \( eV \ll \Delta_0 \), since in this limit the normal reflection at the NS interface is suppressed (see Eq. (4))

\[
|r/r_A|^2 \sim (eV/\Delta_0)^2 \ll 1, \tag{9}
\]

and we may assume a perfect electron-hole (hole-electron) conversion by Andreev reflection.

The number of transmitting channels \( N(\varepsilon) \) for the energy \( \varepsilon \) is determined by the width of the strip \( W \) and the range of variation of transverse momentum \( |p_x| < \varepsilon/c \),

\[
N(\varepsilon) = 2eW/c\pi. \tag{10}
\]

Here the first factor of 2 accounts for the two valleys in graphene. We do not add another 2 for spin, since we assume injection of polarized current. [In the limit given in Eq. (9), the possible inter-valley mixing upon charge-and spin-preserving reflection at the normal edge, see e.g. Refs. [22, 23], is not important for our calculation.] As usual, each open channel adds \( G_0 = e^2/h \) to the differential conductance.

The total current injected into the strip is [24]

\[
I = \int_0^V N(\varepsilon)V'G_0dV' = \frac{e^2}{h} V^2W = \frac{2e^2V^2W}{ch}, \tag{11}
\]

Associated with each incoming electron is a certain quasi-particle trajectory in the channel as depicted in Fig. 2. In the low-voltage limit, Eq. (11), the outgoing particle is an electron if the number of reflections from the NS interface is even, and a hole if the number of reflections is odd. The crucial point is that for a long channel the contributions to the total current of these two kinds of trajectories effectively equilibrate, so that the total charge transfer vanishes after averaging over initial angles. While the charge current upon such equilibration escapes to the superconductor, the spin current continues to flow along the strip. For a 100\% polarized injected beam the rate of spin transfer is simply found as

\[
\frac{dS}{dt} = \frac{\hbar I}{2e}. \tag{12}
\]

V. SOURCES OF INCOMPLETE CHARGE FILTERING

For a channel of finite length the numbers (weights) of trajectories with even and odd numbers of Andreev reflections do not quite coincide, leading to a finite charge current. In this section we first determine the fraction of charge per quasiparticle \( \langle Q \rangle \) remaining in the beam after passage through the NS channel, due to this purely geometrical effect. Since the normal boundary acts as a perfect mirror, we may effectively consider a strip of doubled width, \( 0 < x < 2W \), having NS-interfaces on both sides. The fluxes of electrons injected into this doubled strip with \( p_x < 0 \) and \( p_x > 0 \) are physically equiva-
lent. The charge per quasiparticle with incident angle $\alpha$ changes linearly along the channel from $-e$ at $y = 0$ to $+e$ at

$$y = y_0 = 2W\tan \alpha.$$  

(13)

At $y = y_0$ all electrons injected at angle $\alpha$ are converted to holes. At $y_0 < y < 2y_0$ the holes are converted back to electrons. Charge here changes back (linearly) from $+e$ to $-e$, and so on.

Figure 3 shows the average charge per quasiparticle for electrons injected at a fixed angle $\alpha$ at various initial positions, $0 < x < 2W$, as a function of $p_x c/\varepsilon = \cos \alpha$ for $L = 10W$. The segments of the function $Q(p_x)$ are given by ($n = 0, 1, 2, \ldots$)

$$Q(p_x) = e(-1)^n \left[ \frac{L}{W} \frac{c|p_x|}{\sqrt{e^2 - (cp_x)^2}} - (2n + 1) \right].$$  

(14)

Averaging (integrating) further over $p_x$ and introducing a new variable $\tau = cp_x/\sqrt{e^2 - (cp_x)^2}$, we find the charge fraction $\langle Q \rangle$ per quasiparticle remaining in the beam to be

$$\langle Q \rangle = \frac{e}{2} \int_{-\infty}^{\infty} F \left( \frac{L}{W} \tau \right) \frac{d\tau}{(1 + \tau^2)^{3/2}}.$$  

(15)

Here we have introduced the sawtooth function $F(x + 4) \equiv F(x)$ and $F(x) = |x| - 1$ for $|x| < 2$. Keeping only the first harmonic in the Fourier transform of $F(x)$ in the integral in Eq. (15) gives

$$\langle Q \rangle \approx -\frac{4e}{\pi} \frac{L}{W} \exp \left( -\frac{L\pi}{2W} \right).$$  

(16)

Remarkably, the corrections to charge neutrality are exponentially small in the limit $L \gg W$. Since this incomplete cancellation of charge is of geometric origin, Eq. (16) does not depend on the voltage bias $eV$.

Another source of incomplete charge filtering in the NS channel is the coexistence of Andreev and normal reflection for finite quasiparticle energies ($\varepsilon \sim eV \leq \Delta_0$). To find the charge transfer in this case we add to the reflection matrix, Eq. (3), a part describing particle propagation in graphene between Andreev reflections

$$R = R_0 R_A.$$  

(17)

The form of $R_0$ is sensitive to the details of quasiparticle reflection at the normal side of the strip. Depending on the microscopic structure of the graphene edge, reflection from it may or may not introduce transitions between the two valleys $K, K'$. Here we consider only the latter case as an example. This also means that $R_0$ is a $2 \times 2$ diagonal matrix.

In the case of decoupled valleys the boundary conditions may only have the form (see Ref. [22] for the general situation)

$$(a\psi_1 + b\psi_2)|_{x=W} = 0 , \ a, b = \text{const}$$  

(18)

where $\psi_1$ and $\psi_2$ are the up and down components of either the particle ($u$), or hole ($v$) wave function. Particle number conservation imposes certain restrictions on the allowed values of $a$ and $b$. Below we consider two examples of such boundary conditions, demonstrating both existence and absence of corrections to charge filtering due to finite voltage.

1. The first option is $a/b = \pm i$ (for the edge along the $y$ axis). This boundary is realized if the particle confinement is achieved by adding a term with large mass $\sigma_x Mc^2$ to the Dirac equation (1) at $x > W$ [26, 27]. Straightforward calculation for such a boundary gives

$$R_0 = ie^{ip_cW/h}\text{diag}(1, -1) , \ (R_0R_A)^2 \propto I.$$  

(19)

The particle-hole superposition produced upon Andreev reflection of an electron returns to the pure electron state after the second reflection from the superconductor. The expectation value of the quasiparticle charge along a given trajectory according to Eq. (19) switches from $-e \leftrightarrow (|r_A|^2 - |r|^2)e$ and back after each reflection from the NS-interface. [The average charge of the beam with initial angle $\alpha$ now changes linearly between these two values, not between $-e$ and $e$ as in Eqs. (14,15).] Averaging over angles and energy using the exact Eq. (1) gives the charge per quasiparticle transmitted through the graphene channel for $eV \ll \Delta_0$ as

$$\langle Q \rangle = \frac{\pi eV}{3 \Delta_0^2 e}.$$  

(20)

We note that according to Eq. (20) the condition $|r/r_A| \ll 1$ is violated for grazing trajectories, having $\pi - \alpha \sim \varepsilon/\Delta_0$ (4). The small statistical weight of these grazing trajectories is the source of the small value of $\langle Q \rangle$.

The result Eq. (20) was found in the limit $L \gg W$. However, further increase of the channel length does not lead here to charge relaxation, in contrast with the geometric correction, Eq. (16). The possibility for finite charge current, Eq. (20), to flow without leaking along a (arbitrarily) long NS-interface is very counterintuitive.

2. The second possibility consistent with particle conservation Eq. (18) is $a \times b = 0$ (i.e. either $a = 0$, or $b = 0$). Boundary conditions of this type describe the bulk envelope functions in graphene with a zigzag edge [7]. Reflection from the normal edge is now described by a pure phase matrix $R_0 \propto \text{diag}(1, 1)$. Therefore the product of $n$ reflections reduces to

$$(R)^n \propto (R_A)^n \propto I \cos n\beta + i\sigma_x \sin n\beta,$$  

(21)

where $\cot \beta = -\varepsilon\tan \alpha/\Delta_0$ and the matrix $\sigma_x$ interchanges particles and holes. Eq. (21) leads to a uniform (to exponential accuracy, as in Eq. (19)) mixing of particles and holes after many reflections at the channel boundaries for any value of $\beta$, i.e. at any $\varepsilon < \Delta_0$. In contrast to the previous example, the charge per quasiparticle transmitted through the graphene channel for $L >> W$ is zero even at finite values of $\varepsilon/\Delta_0$. 


VI. CONCLUSIONS

In summary, we have proposed two basic device concepts for conversion of polarized electric current into pure spin current in ballistic bulk graphene. The second device, where the filtering of charge originates simply from the equilibration of the number of trajectories experiencing an even or odd number of Andreev reflections, seems especially promising. We expect this idea to be easily generalized beyond ballistic transport and for different geometries. We stress that neither proposal requires a 100% polarization of the incident beam, although the spin transfer rate will of course depend on the initial polarization.

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[1] K.S. Novoselov, A.K. Geim, S.V. Morozov, D. Jiang, Y. Zhang, S.V. Dubonos, I.V. Grigorieva, and A.A. Firsov, Science, 306, 666 (2004).
[2] K.S. Novoselov, A.K. Geim, S.V. Morozov, D. Jiang, M.I. Katsnelson, I.V. Grigorieva, S.V. Dubonos, and A.A. Firsov, Nature 438, 197 (2005).
[3] Y. Zhang, Y.-W. Tan, H.L. Stormer, and P. Kim, Nature 438, 201 (2005).
[4] C. Berger, Z. Song, X. Li, X. Wu, N. Brown, C. Naud, D. Mayou, T. Li, J. Hass, A.N. Marchenkov, E.H. Conrad, P.N. First, and W.A. de Heer, Science 312, 1191 (2006).
[5] S.A. Wolf, D.D. Awschalom, R.A. Buhrman, J.M. Daughton, S. von Molnar, M.L. Roukes, A.Y. Chtchelkanova, D.M. Treger, Science 294, 1488 (2001).
[6] C. L. Kane and E. J. Mele, Phys. Rev. Lett. 95, 226801 (2005).
[7] D. A. Abanin, P. A. Lee, and L. S. Levitov, Phys. Rev. Lett. 96, 176803 (2006).
[8] C.W.J. Beenakker, Phys. Rev. Lett. 97, 067007 (2006).
[9] A.F. Andreev, Sov. Phys. JETP 19, 1228 (1964).
[10] K. Tsukagoshi, B. W. Alphenaar, and H. Ago, Nature 401, 572 (1999).
[11] M. Titov and C.W.J. Beenakker, Phys. Rev. B 74, 041401(R) (2006).
[12] S. Bhattacharjee and K. Sengupta, Phys. Rev. Lett. 97, 217001 (2006).
[13] M. Titov, A. Ossipov, and C. W. J. Beenakker, Phys. Rev. B 75, 045417 (2007).
[14] N. M. Chtchelkatchev, JETP Letters 78, 230 (2003).
[15] D. P. DiVincenzo and E. J. Mele, Phys. Rev. B 29, 1685 (1984).
[16] For a case of planar junction between a two-dimensional metal and a superconductor see: A.F. Volkov, P.H.C. Magnee, B.J. van Wees, and T.M. Klapwijk, Physica C 242, 261 (1995).
[17] I. Kosztin, D.L. Maslov, and P.M. Goldbart, Phys. Rev. Lett. 75, 1735 (1995).
[18] P.G. Silvestrov, M.C. Goorden, and C.W.J. Beenakker, Phys. Rev. Lett. 90, 116801 (2003); P.G. Silvestrov, Phys. Rev. Lett. 97, 067004 (2006).
[19] The four plane-wave solutions in the normal region used to derive Eq. (4) are (up to normalization)

\[ (u, v)^+_p = \left(\exp(\mp i0/2), \pm \exp(\pm i0/2), 0, 0\right) \]
\[ (u, v)^-_p = \left(0, 0, \exp(\mp i0/2), \mp \exp(\pm i0/2)\right) \]

where (+) denotes \( p_x > 0 \) and (−) denotes \( p_x < 0 \). More precisely, the \( R \)-matrix in Eqs. (3) and (4) defines the transformation of a linear combination of incoming (+) electron and hole solutions into a superposition of outgoing (±) electron and hole solutions.
[20] P.G. Silvestrov and K.B. Efetov, Phys. Rev. Lett. 98, 016802 (2007).
[21] F. Hohls, M. Pepper, J.P. Griffiths, G.A.C. Jones, and D.A. Ritchie, Appl. Phys. Lett. 89, 016802 (2006).