OPTIMAL PRICING, ORDERING, AND CREDIT PERIOD POLICIES FOR DETERIORATING PRODUCTS UNDER ORDER-LINKED TRADE CREDIT

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ABSTRACT. In the modern global economy, trade credit financing is typical in business transactions for both sellers and buyers. The seller offers a credit period to attract new buyers or stimulate demand, and the buyer takes the opportunity to accumulate revenue. To obtain this benefit, the seller prefers trade credit policies that are dependent on the quantity ordered, referred to as order-linked trade credit. The buyer can obtain the benefits from a fully delayed payment if their order is sufficiently large. Similarly, the seller can sell many products while granting a credit period. Otherwise, the buyer receives only partial trade credit, and the seller can take the opportunity of both cash and credit payments. In this study, an economic order quantity (EOQ) inventory model for deteriorating products, under default risk control-based trade credit, is formulated using a discounted cash flow approach. The seller offers to the buyer order-linked trade credit with price-and credit-period-dependent demand. The optimal selling price, credit period policies, and replenishment cycle time are determined simultaneously, while maximizing the present value of the seller’s total profit. Moreover, this research provides numerical examples and sensitivity analysis to illustrate the theoretical results, solution procedure, and gain managerial insights. 200 words.

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1. Introduction. Tighter market competition and harsh financial conditions have persuaded firms to cut costs, increase profits, and gain the necessary funds to achieve business goals. Effective and efficient supply chain management can help firms attain their goals and create a competitive advantage. According to [27], firms need to consider financial flow together with material and information flow for better cost reduction in supply chain management. In addition, excess inventory is a common problem that accounts for a significant portion of total costs in the supply chain [3]. These issues pose a challenge for firms to unlock funds tied up in inventory, especially for deteriorating products.

Trade credit financing is one of the financial options that affect inventory decisions, including postponed inventory payments and delayed sales collection [19]. It can help firms reduce costs and improve their financial performance [1]. Recently, trade credit schemes have become common in modern business transactions. In the US and UK, approximately 80% of companies offer trade credit on their daily business transactions [32] [38]. In general, there are two types of trade credit schemes: full and partial trade credit. In full trade credit, the seller permits the buyer to delay payment up to a given credit period. In partial trade credit, the buyer pays a fraction of the purchasing cost upon receipt.

Also, in managing inventory, the characteristic of the product is essential to be considered. Some products (e.g., fruits, vegetables, pharmaceuticals, volatile liquids, electronic components, and fashion goods) will have physical phenomena or lose their value called a deterioration in which the quality of the products will be gradually decreasing because of spoilage, physical depletion, obsolescence or decay. Consequently, these products will cause additional costs and an impact on inventory decisions. To the best of our knowledge, no studies have been conducted on deteriorating products under order-linked trade credit from the seller’s perspective.

Generally, an enormous amount of research has been conducted on the inventory model under trade credit policy and deteriorating products [23] [17] [35]. Nevertheless, so far, the researchers mainly focus on the perspective of the buyer. Wang [37] revealed that the literature focusing on the seller’s view on trade credit policy was limited. Mostly, the inventory model is described to examine the inventory decisions under the seller’s credit period. However, the decision on how long the credit period is offered for the seller has received little attention (Section 2). This study will fill this gap.

This study considers an EOQ model for deteriorating products under order-linked trade credit and discounted cash flow (DCF). The contributions of this study are as follows. First, this study is the first to develop an inventory model under order-linked trade credit from the seller’s perspective. The seller offers buyers order-linked trade credit which depends on order quantity, buyers could receive the different kinds of trade credit scheme. With a large order size, full trade credit is offered; otherwise, partial trade credit is provided. In addition, the use of a credit period increases demand but incurs a higher default risk. Thus, sellers need to consider both of credit period and selling price as their decisions. The demand function is also defined as a function of the selling price and credit period in this study. Second, a DCF analysis is used to include the time value of money. The DCF model more frequently used in valuation because of the consistency of long-term value creation and may capture all the elements that alter the company value in an inclusive way, as explain in [16] and [35]. The difference of trade credit applications is the payment time. Thus, a DCF that considers the effect of time on money value is necessary [16].
Third, a non-linear algorithm is developed to determine the optimal selling price, credit period terms, and replenishment cycle time while maximizing the present value of the total annual seller’s profit. Finally, a numerical analysis is conducted to illustrate the model and solution approach. A comparison of four cases: (1) model with ordered-link trade credit; (2) model without ordered-link trade credit; (3) model with full trade credit; and (4) model with partial trade credit is also conducted to see the benefits of the proposed credit scheme.

The remainder of this paper is structured as follows: Section 2 reviews the literature. Section 3 explains the problem definition and establishes the notation, assumption, and proposed mathematical model for the order-linked payment scheme case. Section 4 explains the theoretical results and presents the solution procedure. Section 5 provides numerical examples and a sensitivity analysis to illustrate the problem, solution, and managerial insights. Finally, Section 6 concludes the study.

2. Literature review.

2.1. EOQ model for deteriorating products. Inventory management is an essential aspect of operation management. According to [12], the main objective of inventory management is to strike a balance between inventory investment and customer service. A well-known inventory model for independent demand is an EOQ model for deteriorating products. In the existing literature, in [10], the authors first developed an exponentially deteriorating inventory model. The model assumes that the product has a constant deterioration rate during one cycle. In [2], the authors studied inventory control for a constant deteriorated item under full trade credit. Recently, in [22], the author first introduced an EOQ model with a time-varying deterioration rate by taking fixed maximum lifetime consideration. The model used time-dependent demand to find the optimal cycle time under full trade credit with a price discount policy. In [37], the authors considered an inventory model for the deteriorated items and non-deteriorated items with credit-dependent demand. The model also adds the assumption of a maximum lifetime in which the product has the expiration date. The decision variables are the optimal credit period and the cycle time under full trade credit policy while maximizing the total profit of the inventory system. In [9], the authors extended [37]’ model by proposing a generalized theoretical result and behavior of parameter decisions. Besides, the authors [24] [28] also considered the assumption of time-varying deterioration rate with maximum lifetime in inventory control with quantity dependent trade credit policies. In [24], the author extended [7] by incorporating the price-sensitive demand and time-varying deterioration rate to find the optimal price and cycle time.

2.2. Trade credit payment. Numerous, extensive studies on various trade credit terms in the inventory model have been conducted. Initially, in [11], the author developed an economic order quantity (EOQ) under a permissible delay in payment. This shows that applying permissible payment delays can reduce costs because of the ability to delay payment without paying interest. In [29], the authors examined the partial trade credit terms for an EOQ inventory model under partial backorder and obtained the optimal replenishment cycle to maximize average total annual revenue. In [14], the authors considered fuzzy price-dependent demand and full backorder under full trade credit terms to maximize total profit. In [6], the authors examined an EOQ inventory model under nonlinear, stock-dependent holding costs.
and nonlinear, stock-dependent demand under full trade credit terms to minimize the total cost.

Currently, sellers are encouraged to practice order-linked trade credit because this policy is beneficial for both the seller and buyer. The seller can stimulate demand, attract new buyers, reduce inventory costs, and increase competitive power. Moreover, the buyer can accumulate revenue and earn interest by investing in the banking account during the credit period [20]. First, in [15], the authors studied supplier credit terms, both independently and dependent on order quantity. Meanwhile, in [13], the author established an EOQ model under credit payment, dependent on the order quantity in which the supplier offers partial trade credit if the retailer’s order quantity is below the predetermined amount. Otherwise, full trade credit is permitted. In [8], the authors provided another solution procedure and relaxed some assumptions from Huang’s (2007) model, such as interest charged greater than interest earned and purchase cost the same as selling price. In [7], the authors extended [13]’s model by proposing the arithmetic-geometric mean inequality method to find the closed form of the replenishment cycle and complementing some shortcomings regarding interest earned and charged. In addition, The authors [36] and [33] examined an EOQ model under order-linked trade credit for the shortage case. The objectives are to minimize total cost and obtain the optimal replenishment time and length of time at which the inventory reaches zero. Nevertheless, the researchers described above have mainly focused on the buyer’s perspective. In [37], the authors noted that research focusing on the seller’s view of trade credit term was limited. Notably, the inventory model under the order-linked trade credit policy, explored from the perspective of the seller, is hardly seen in the literature. Mostly, the inventory model is used to examine inventory decisions under the seller’s credit period (see Table 1).

For the seller, granting trade credit terms creates additional costs and increases default risk [34]. The more extended the credit period allowed by the seller, the higher the risk of uncollectible debt due to the buyer failing to make the payment. For example, as explained by [17], a 30-year mortgage has a higher default risk than a 15-year mortgage. Accordingly, it is necessary to conduct research on granting credit payments, considering default risk from the perspective of the seller. Other relevant studies on trade credit terms and default risk from the seller’s perspective can be found in [30] [39] [23] [39], and [18]. This study also considers default risk under the implementation of order-linked trade credit. The more extended the credit period to the buyer, the higher the default risk.

2.3. Discounted cash flow analysis. Generally, some research works used discounted cash flow (DCF) analysis to compute the exact timing of cash-flow and time value of money. In [26], the authors explained that DCF analysis has used to overcome the drawback of the typical cost approach. It provides decent recognition of the financial implication of the opportunity cost and out-of-pocket costs in the inventory analysis. For example, if the annual compound interest rate is \( r \) per dollar per year, then $1 today is worth $e^{-r} a year later. Conversely, $1 a year from now is equivalent to $e^{-r}$ now [16]. Besides, in [18], the authors incorporated price and credit dependent demand to determine the optimal order quantity, selling price, and credit period of the seller for the deteriorating item by using DCF analysis. Two-level credit terms under full trade credit and partial trade credit are presented to maximize the present value of total profit. Furthermore, the following Table 1 presents a summary of the literature review and research gap of this thesis.
### Table 1. Summary of literature review

| Author          | Payment Terms | Time value of money | Perspective | Decision Variables | Demand Function | Deterioration | Credit risk | Customer     |
|-----------------|---------------|---------------------|-------------|--------------------|-----------------|---------------|-------------|--------------|
| Sarkar [22]     | FTC           | No                  | Buyer       | Cycle Time         | Time            | Time-varying  | No          |              |
| Taleizadeh [28]| PTC           | No                  | Buyer       | Cycle Time         | Rate            | No            | No          |              |
| Huang [13]      | Order Linked of FTC and PTC | No | Buyer | Cycle Time | Rate | No | No |              |
| Chen [7]        | Order Linked of FTC and PTC | No | Buyer | Cycle Time | Rate | No | No |              |
| Ting [31]       | Order Linked of FTC and PTC | No | Buyer | Cycle Time | Rate | Constant | No |              |
| Shah [24]       | Order Linked of FTC and PTC | No | Buyer | Cycle Time; Selling Price | Rate | No | No |              |
| Taleizadeh [28]| Order Linked of FTC and PTC | No | Buyer | Cycle Time | Rate | Time-varying | No |              |
| Tiwari [33]     | Order Linked of FTC and PTC | No | Buyer | Cycle Time | Rate | Constant | No |              |
| Wang [37]       | FTC           | No                  | Seller      | Cycle Time; Credit Period | Time-varying | Yes |              |              |
| Shah [23]       | FTC           | No                  | Seller      | Cycle Time; Credit Period | Time | No | Yes |              |
| This Research   | Order Linked of FTC and PTC | Yes | Seller | Cycle Time; Selling Price; Credit Period | Constant | Yes |              |              |

Note: FTC corresponds to full trade credit and PTC corresponds to partial trade credit.

3. Model formulation.

**Figure 1. Conceptual model for deteriorating products under order-linked trade credit**

3.1. **Problem definition.** This study focuses on developing an inventory model to derive the optimal selling price \( p \), credit period policies \( M \), and replenishment cycle time \( T \) to maximize the present value of the seller’s total profit for each cycle. This study considers two echelons, the seller (i.e., wholesaler) and the buyer (i.e., industry or distributor), where the seller permits order-linked trade credit to the buyer. If demand from the buyer exceeds the predetermined quantity \( D_d \), the seller
allows full trade credit. Otherwise, the seller offers partial trade credit. The seller purchases products from the supplier and sells them to the buyer. The seller uses an EOQ inventory model with deterministic demand, dependent on selling price \( p \) and credit period length \( M \), for a constant rate \( \theta \) of deteriorating products (e.g., foods, pharmaceuticals, liquid, etc.).

When payment is received, the seller starts to accumulate revenue by depositing it in an interest-bearing account at a rate \( I_e \). However, during the credit period, the seller may experience revenue loss due to allowing trade credit with a rate \( I_{Loss} \). In this model, there are two primary cases of interest earned and interest revenue loss. First, when full trade credit is allowed, the seller only accumulates revenue from credit payments. Second, when partial trade credit is offered, sellers can accumulate revenue from cash payments and credit payments. For each case, two subcases occur in which either the trade credit length is less than or greater than the replenishment cycle time. Figure 1 illustrates the proposed model and the decision variables involved. The assumptions for developing the mathematical model are as follows:

1. An EOQ inventory model contains a single product with a constant deterioration rate \( \theta \).
2. The demand rate \( D(p, M) \) is a function of the selling price \( p \) and credit period length \( M \), given by Eq. (1), as follows:
   \[
   D(p, M) = a - bp + nM
   \] (1)
   where \( a > 0 \) denotes the demand scale, \( b > 0 \) denotes the coefficient of price, and \( n > 0 \) indicates a trade credit markup. The assumption represents that the demand reduces as the price decreases, however, it increases due to an increase of credit time. This assumption is common in research field such as [17] and [35].
3. It is evident that a 30-year mortgage has a higher default risk than a 15-year mortgage [37]. Likewise, the more extended the credit period to the buyer the higher the default risk, the rate of default risk given the credit period \( M \) follows the same assumption as in [37] and [17], presented by Eq. (2) as follows:
   \[
   F(M) = 1 - e^{-lM}
   \] (2)
   where \( l > 0 \) is the default risk coefficient. Default risk affects sales revenue and interest earned in the model.
4. The replenishment rate is instantaneous, and the lead-time is negligible.
5. The planning horizon of the inventory system is infinite.
6. Shortages are not allowed.
7. There is no replacement, repair, or salvage value for the products in the interval \([0, T]\). The product is withdrawn from the warehouse immediately as it deteriorates.
8. The seller should pay the procurement cost (i.e., the cost of purchasing products) directly (i.e., cash payment) to the supplier.
9. The seller provides a permissible payment delay time \( M \) to the buyer, which is an order-linked credit payment. If the buyer's order quantity that represents the demand \( D(p, M) \) exceeds the minimum order quantity \( D_m \), then the seller provides the buyer full trade credit (FTC), in which the seller receives the revenue \( pD(p, M) \) at time \( M \). Otherwise, the seller offers partial trade credit (PTC) in which the seller receives an amount of \( (1 - \alpha) \) revenue \( pD(p, M) \)
at time \( t = 0 \) (i.e., cash payment), and then obtains the balance revenue \( \alpha p D(p, M) \) at time \( M \) (i.e., credit payment).

10. The seller can accumulate revenue and earn interest \( (I_e) \) from cash payment starting from time \( t = 0 \) to the end of replenishment cycle time, \( t = T \), and credit payment starting from time \( t = M \) to \( t = T + M \) (i.e., the time at which the seller receives the credit payment from the last customer). During the trade credit \( (M) \), the seller suffers an interest loss \( (I_{Loss}) \) with an annual rate. The interest loss \( (I_{Loss}) \) is different from interest earned \( (I_e) \). The authors [24] and [17] also have the same assumption. This assumption is reasonable in practice, for instance, the banks lend you an amount with higher interest than the interest they pay you when you deposit to the banks.

The notations for the proposed inventory model are listed as follows (see Table 2).

3.2. **Mathematical model.** Based on Figure 2, inventory is depleted not only by demand but also by deterioration. Deterioration follows an exponential distribution with parameter, \( \theta \). This assumption is common in previous works such as [21], [40], [17], and [34].

![Inventory Level](image)

**Figure 2.** Inventory level

Let \( I(t) \) be the inventory level at any time \( 0 \leq t \leq T \). Depletion due to demand and deterioration occurs concurrently. The differential equation that describes the instantaneous state of \( I(t) \) over \( (0, T) \) is given by Eq. (3):

\[
\frac{dI(t)}{dt} + \theta I(t) = -D(p, M), 0 \leq t \leq T
\]  

(3)

The solution of the above differential equation (boundary condition at \( t = T \) and \( I(t) = 0 \)) is specified by Eq. (4):

\[
I(t) = \frac{D(p, M)}{\theta} \left( e^{\theta(T-t)} - 1 \right), 0 \leq t \leq T
\]  

(4)

From Eq. (4), with the condition \( t = 0, I(0) = Q \), the result of the quantity order \( Q \), and the beginning inventory level for each cycle is given by Eq. (5).

\[
I(0) = Q = \frac{D(p, M)}{\theta} \left( e^{\theta T} - 1 \right)
\]  

(5)

The relevant present value of cost and revenue of total profit consist of the following elements:

1. **The present value of ordering cost**
Table 2. Notations

| Notation | Description |
|----------|-------------|
| $i$      | Index of case based on demand, $i = [1, 2]$ |
| $j$      | Index of case based on time, $j = [1, 2]$ |
| $A$      | Replenishment cost per order, in dollar |
| $c$      | Procurement cost per unit, in dollar |
| $h$      | The inventory holding cost rate per unit per unit time, in dollar |
| $I_e$    | Interest earned per dollar per unit time |
| $I_{Loss}$ | Interest revenue loss due to offering trade credit per dollar per unit time |
| $r$      | Annual compound interest rate per dollar per unit time |
| $D(p, M)$ | Annual demand rate per unit time, as a function of both $p$ and $M$ |
| $F(M)$   | The rate of default risk |
| $\theta$ | Deterioration rate, $0 \leq \theta \leq 1$ |
| $D_d$    | The specific threshold in which permits the full trade credit |
| $\alpha$ | The fraction of the delay payments is permitted, $0 \leq \alpha \leq 1$ |
| $I(t)$   | Inventory level at time $t$ |
| $Q$      | Seller’s order quantity |
| $OC$     | The present value of ordering cost |
| $HC$     | The present value of holding cost |
| $PC$     | The present value of procurement cost |
| $SR_i$   | The present value of revenue in case $i$, $i = [1, 2]$ |
| $IE_{i,j}$ | The present value of interest earned in case $(i,j)$, $i, j = [1, 2]$ |
| $IL_{i,j}$ | The present value of interest loss in case $(i,j)$, $i, j = [1, 2]$ |
| $T$      | Length of the replenishment cycle (decision variable) |
| $M$      | The credit period policies offered by the seller (decision variable) |
| $p$      | Selling price offered by the seller per unit (decision variable) |
| $PTP(p, M, T)$ | The present value of total annual profit, which is the function of $p$, $M$ and $T$ |

The seller’s ordering time is at the time of delivery 0. Therefore, the present value of the ordering cost ($OC$) at time 0 is given by Eq. (6):

$$OC = A$$  \hspace{1cm} (6)

2. The present value of procurement cost

The seller’s procurement time is at the time of delivery, 0 (i.e., cash payment). Therefore, the present value of the procurement cost ($PC$) at time 0 is calculated by multiplying the procurement cost per unit and quantity order, as shown in Eq. (7):

$$PC = cQ = \frac{cD(p, M)}{\theta}(e^{\theta T} - 1)$$  \hspace{1cm} (7)

3. The present value of carrying inventory cost
The present value of the holding cost ($HC$) is calculated by multiplying the holding cost per unit and the average inventory level, as given by Eq. (8):

$$HC = h \int_{0}^{T} I(t)e^{-rt}dt$$  \hspace{1cm} (8)

4. The present value of sales revenue

Case 1 The seller receives the present value of revenue ($SR_1$) in the full trade credit case, calculated by multiplying the selling price and demand, starting from time $M$ until $T + M$, as shown in Eq. (9):

$$SR_1 = pD(p, M) \int_{M}^{T+M} e^{-rt}dt$$  \hspace{1cm} (9)

Case 2 Meanwhile, the present value of revenue ($SR_2$) in the partial trade credit case is obtained by multiplying the selling price and demand for the fraction $(1 - \alpha)$ from time 0 until $T$ and the $\alpha$ fraction from time $M$ until $T + M$, given by Eq. (10):

$$SR_2 = pD(p, M)((1 - \alpha) \int_{0}^{T} e^{-rt} + \alpha \int_{M}^{T+M} e^{-rt}dt)$$  \hspace{1cm} (10)

5. Interest earned and interest revenue loss

There are two possible cases based on the values of $D_d$ and $D(p, M)$. In case 1, the seller offers full trade credit when $D_d < D(p, M)$. In case 2, the seller permits partial trade credit when $D_d \geq D(p, M)$. Based on these scenarios, the interest earned will be different for each case, as follows (see Table 3):

| Case (1): $D_d < D(p, M)$ | Case (2): $D_d \geq D(p, M)$ |
|--------------------------|-----------------------------|
| Sub-case 1.1: $0 \leq M \leq T \leq T + M$ | Sub-case 2.1: $0 \leq M \leq T \leq T + M$ |
| Sub-case 1.2: $0 \leq T \leq M \leq T + M$ | Sub-case 2.2: $0 \leq T \leq M \leq T + M$ |

**Case 1 Subcase 1.1 $0 \leq M \leq T \leq T + M$**

The interest earned in subcase 1.1 ($IE_{1,1}$) is shown in Figure 3. From the graphical representation, the seller offers full trade credit, in which the revenue can be
accumulated from time $M$ until $T + M$. The present value of interest-earned in subcase 1.1 is given by Eq. (11):

$$IE_{1.1} = I_e p D(p, M) \int_M^{T+M} (T + M - t)e^{-rt} dt$$

(11)

In addition, the seller may experience revenue loss during the credit period, from time 0 until $M$. The present value of the interest revenue loss in subcase 1.1 ($IL_{1.1}$), as shown in Figure 3, is given by Eq. (12):

$$IL_{1.1} = I_{Loss} p D(p, M) \int_0^M te^{-rt} dt$$

(12)

Therefore, the present value of the total profit for subcase 1.1 per cycle time $T$ ($PTP_{1.1}$), is given by Eq. (13):

$$PTP_{1.1}(p, M, T) = \frac{1}{T}(SR_1[1 - F(M)] - OC - PC - HC + IE_{1.1}[1 - F(M)] - IL_{1.1})$$

(13)

**Sub Case 1.2** $0 \leq T \leq M \leq T + M$

![Figure 4. Graphical representation of Sub-case 1.2](image)

The interest earned in subcase 1.2 is shown in Figure 4. From the graphical representation, the seller offers full trade credit, in which the revenue can be accumulated from time $M$ until $T + M$. The present value of the interest-earned ($IE_{1.2}$) in Subcase 1.2 is given by Eq. (14):

$$IE_{1.2} = I_e p D(p, M) \int_M^{T+M} (T + M - t)e^{-rt} dt$$

(14)

In addition, the seller may experience revenue loss during the credit period, from time 0 until $M$. The present value of the interest revenue loss in subcase 1.2 ($IL_{1.2}$), as shown in Figure 4, is given by Eq. (15):

$$IL_{1.2} = I_{Loss} p D(p, M)(\int_0^T te^{-rt} dt + \int_T^M T e^{-rt} dt)$$

(15)

Therefore, the present value of the total profit for subcase 1.2 per cycle time $T$ ($PTP_{1.2}$) is given by Eq. (16), as follows:

$$PTP_{1.2}(p, M, T) = \frac{1}{T}(SR_1[1 - F(M)] - OC - PC - HC + IE_{1.2}[1 - F(M)] - IL_{1.2})$$

(16)

**Case 2**

**Subcase 2.1** $0 \leq M \leq T \leq T + M$
The interest earned in subcase 2.1 is shown in Figure 5. From the graphical representation, the seller offers partial trade credit, in which the revenue can be accumulated from time 0 until $T$ for cash payment and $M$ until $T + M$ for credit payment. The present value of the interest-earned in subcase 2.1 ($IE_{2.1}$) is given by Eq. (17):

$$IE_{2.1} = L(p,D_T,M)((1 - \alpha) \int_0^T te^{-rt} + \alpha \int_M^{T+M} (T + M - t)e^{-rt} dt)$$  

(17)

In addition, the seller may experience revenue loss during the credit period, from time 0 until $M$. The present value of interest revenue loss in subcase 2.1 ($IL_{2.1}$), as shown in Figure 5, is given by Eq. (18):

$$IL_{2.1} = ILoss \cdot L(p,D_T,M) \alpha \int_0^M te^{-rt} dt$$  

(18)

Therefore, the present value of total profit for subcase 2.1 per cycle time $PTP_{2.1}$, is given by Eq. (19):

$$PTP_{2.1}(p,M,T) = \frac{1}{T}(SR_2[1 - F(M)] - OC - PC - HC + IE_{2.1}[1 - F(M)] - IE_{2.1})$$  

(19)

Subcase 2.2 $0 \leq T \leq M \leq T + M$

The interest earned in subcase 2.2 is shown in Figure 6. From the graphical representation, the seller offers partial trade credit in which the revenue can be accumulated from time 0 until $T$ for cash payment and $M$ until $T + M$ for credit payment. The present value of the interest-earned in subcase 2.2 ($IE_{2.2}$) is given by Eq. (20):

$$IE_{2.2} = L(p,D_T,M)((1 - \alpha) \int_0^T te^{-rt} + \alpha \int_M^{T+M} (T + M - t)e^{-rt} dt)$$  

(20)

In addition, the seller may experience revenue loss during the credit period, from time 0 until $M$. The present value of interest revenue loss in subcase 2.2 ($IL_{2.2}$), as shown in Figure 6, is given by Eq. (21):

$$IL_{2.2} = ILoss \cdot L(p,D_T,M) \alpha \int_0^M te^{-rt} dt$$  

(21)

Therefore, the present value of total profit for subcase 2.2 per cycle time $PTP_{2.2}$, is given by Eq. (22):

$$PTP_{2.2}(p,M,T) = \frac{1}{T}(SR_2[1 - F(M)] - OC - PC - HC + IE_{2.2}[1 - F(M)] - IE_{2.2})$$  

(22)
payment. The present value of the interest-earned in subcase 2.2 \((IE_{2.2})\) is given by Eq. (20):

\[
IE_{2.2} = I_p D(p, M)(1 - \alpha) \int_0^T te^{-rt} dt + \alpha \int_M^{T+M} (T + M - t)e^{-rt} dt
\]  

(20)

In addition, the seller has opportunity revenue loss during the credit period from time 0 until \(M\). The present value of the interest revenue loss in subcase 2.2 \((IL_{2.2})\), as shown in Figure 6, is given by Eq. (21):

\[
IL_{2.2} = I_{loss} p D(p, M) \alpha \int_0^T te^{-rt} dt + \int_T^M T e^{-rt} dt
\]  

(21)

Therefore, the present value of the total profit for subcase 2.2 per cycle time \(T\) \((PTP_{2.2})\), is given by Eq. (22):

\[
PTP_{2.2}(p, M, T) = \frac{1}{T}(SR_T[1 - F(M)] - OC - PC - HC + IE_{2.2}[1 - F(M)] - IE_{2.2})
\]  

(22)

4. Theoretical results and algorithm.

4.1. **Theoretical results.** The seller’s objective is to determine the optimal selling price, credit time, and replenishment cycle time while maximizing profit. The profit functions are as follows:

\[
\begin{align*}
\text{Maximize} & \quad PTP_{i,j}(p, M, T) = \begin{cases} 
PTP_{1,1}, & \text{when} \ D_d < D \text{ and } 0 \leq t \leq T \\
PTP_{1,2}, & \text{when} \ D_d < D \text{ and } 0 \leq t \leq T \\
PTP_{2,1}, & \text{when} \ D_d \geq D \text{ and } 0 \leq t \leq T \\
PTP_{2,2}, & \text{when} \ D_d \geq D \text{ and } 0 \leq t \leq T 
\end{cases}
\end{align*}
\]

(23)

where \(i, j = [1, 2]\) and \(0 \leq t \leq T\).

Due to the problem’s complexity, it cannot be proven that the present value of total profit is a joint concave or joint pseudo-concave in \(P, M, T\). However, the following theoretical results demonstrate that the present value of total profit is concave or pseudo-concave under certain conditions.

**Theorem 4.1.** For any given cycle time \(T\) and credit period \(M\), \(PTP_{i,j}(p, M, T)\) is a strictly concave function of selling price \(p\). The proof is presented in Appendix 1.

According to the necessary condition, the optimal value of \(p\) is obtained by taking the first derivative of \(PTP_{i,j}(p, M, T)\) with respect to \(p\) equals zero. Hence, the optimal value of the selling price \(p(M, T)\) for each subcase is given by Eqs. (24) through (27), as follows:

\[
\begin{align*}
p \ast_{1,1}(M, T) &= \left( e^{r(M+T)} - e^{-(M+T)} \right) \frac{I_n M_n (1 + M r)}{r^2} + \frac{e^{-M r} I_{loss} M_n (1 + M r)}{r^2} \\
&\quad + e^{-(M+T)} \frac{M_n (1 + M r)}{r^2} + \frac{e^{-(M+T)} \frac{M_n (1 + M r)}{r^2}}{r^2} - \frac{r + e^{T} \theta}{r + \theta} - \frac{e^{T} \theta}{r + \theta} - \frac{e^{T} \theta}{r + \theta} - \frac{e^{T} \theta}{r + \theta} - \frac{e^{T} \theta}{r + \theta}
\end{align*}
\]  

(24)

\[
\begin{align*}
p \ast_{1,2}(M, T) &= \left( e^{r(M+T)} - e^{-(M+T)} \right) \frac{\frac{M_n (1 + M r)}{r^2}}{r^2} + \frac{e^{r(M+T)} \frac{M_n (1 + M r)}{r^2}}{r^2} + \frac{e^{r(M+T)} \frac{M_n (1 + M r)}{r^2}}{r^2} - \frac{r + e^{T} \theta}{r + \theta} - \frac{e^{T} \theta}{r + \theta} - \frac{e^{T} \theta}{r + \theta} - \frac{e^{T} \theta}{r + \theta} - \frac{e^{T} \theta}{r + \theta}
\end{align*}
\]  

(25)
Theorem 4.2. For any given selling price $p$ and cycle period $M$, $PTP_{i,j}(p, M, T)$ is a strictly pseudo-concave function of cycle time $T$. The proof is presented in Appendix 2.

Based on the generalized concave function in [5], the real-value function can be described by Eq. (28):

$$z(x) = \frac{f(x)}{g(x)}$$

if $f(x)$ is non-negative, differentiable, and (strictly) concave, and $g(x)$ is positive, differentiable, and concave, then $z(x)$ is (strictly) pseudo-concave.

According to the necessary condition, the optimal value of $T$ is obtained by taking the first derivative of $PTP_{i,j}(p, M, T)$ with respect to $T$ equals zero, and the sufficient condition in which the second derivative condition with respect to $T$ is less than zero. However, because $PTP_{i,j}(p, M, T)$ is a complex function, due to its high-powered exponential form, it is not analytically possible to present the validity of the closed form.

Theorem 4.3. For any given selling price $p$ and cycle time $T$, suppose that $\frac{d\text{PTP}_{i,j}(p,M,T)}{dM} > 0$ and $(2n - (a + Mn - bp)(l + r)) > 0$, then $\text{PTP}_{i,1}(p, M, T)$ is a strictly concave function of credit period $M$. The proof is presented in Appendix 3.

Theorem 4.4. For any given selling price $p$ and cycle time $T$, suppose that $\frac{d\text{PTP}_{i,j}(p,M,T)}{dM} > 0$, $(2n - (a + Mn - bp)(l + r)) > 0$, and $2n - (a + Mn - bp)r > 0$ then $\text{PTP}_{i,2}(p, M, T)$ is a strictly concave function of credit period $M$. The proof is presented in Appendix 3.

Theorem 4.5. For any given selling price $p$ and cycle time $T$, suppose that $\frac{d\text{PTP}_{i,j}(p,M,T)}{dM} > 0$, $(-2n + l(-bp + nM)) > 0$, $(-2n + (-bp + nM)(l + r)) > 0$, $(-2n + l(a + Mn - bp)) < 0$, and $(-2n + (a + Mn - bp)(l + r)) < 0$ then $\text{PTP}_{i,3}(p, M, T)$ is a strictly concave function of credit period $M$. The proof is presented in Appendix 3.
Theorem 4.6. For any given selling price \( p \) and cycle time \( T \), suppose that 
\[
\frac{dPTP_{i,j}(p,M,T)}{dM} > 0, \quad -2n + (b - np + nM)(l + r) > 0, \\
-2n + (a + Mn - bp)(l + r) < 0, \quad -2n + (a + Mn - bp)(l + r) < 0, \quad \text{and} \quad -2n + (a + Mn - bp)r < 0, 
\]
then \( PTP_{i,j}(p,M,T) \) is a strictly concave function of credit period \( M \). The proof is presented in Appendix 3.

For Theorems 3 through 6, an optimal value of \( M \) exists for some conditions. When the value of all decisions \((p,M,T)\) are found, we need to substitute the value of \( M \) into the constraints presented in Theorems 3 and 6 and check if they are satisfied or not. If yes, the sufficient condition is satisfied and the value of decisions \((p,M,T)\) are optimal; otherwise, the Algorithm could be used to determine the solution. According to the necessary condition, the optimal value of \( M \) is obtained by taking the first derivative of \( PTP_{i,j}(p,M,T) \) with respect to \( M \) equals zero when all conditions are satisfied. However, because \( PTP(p,M,T) \) is a complex function, due to its high-powered exponential form, it is not analytically possible to present the validity of the closed form.

4.2. Algorithm. By applying Theorems 1, 2, 3, 4, 5, and 6, the optimal selling price \( p(M,T) \) for each subcase (Eqs. (23-26)) is attainable. To find the optimal solution, we substitute the value of each \( p(M,T) \) into the profit function for each subcase; thereafter, the value of total profit becomes \( PTP_{i,j}(M,T)(i,j = [1,2]) \) for all subcases.

The substitution of \( PTP(p,M,T) \) with the present value of total profit converts the model to a non-linear function with two decision variables. However, because of the complexity of the function, the model cannot be reduced to a single variable. Therefore, this research proposes the following algorithm to determine the optimal values of \( p, M, \) and \( T \).

Algorithm

Step 1 for all subcases:
Set \( M_{i,j,q} = 0 \) as the initial value for each sub-case, \( i = j = [1,2] \) and \( q = 1,2,...,n \) represents the iteration. The initial value can be the least possible credit period, based on the case study.

Step 2 for subcase 1.1 \((i = 1,j = 1)\):
Step 2.1: Given \( M = M_{i,j,q} \), calculate \( p_{i,j,q}(T) \) by Eq. (23).
Step 2.2: Calculate \( T_{i,j,q}(M_{i,j,q},p_{i,j,q}) \) that satisfies \( \frac{dPTP_{i,j}(p,M,T)}{dT} = 0 \) and go to Step 2.3.
Step 2.3: Set \( M_{i,j,q+1} = M_{i,j,q} + \epsilon \), where \( \epsilon \) is a small positive number as an increment, and calculate \( p_{i,j,q+1}(T) \) using Eq. (23).
Step 2.4: Calculate \( T_{i,j,q+1}(M_{i,j,q+1},p_{i,j,q+1}) \) that satisfies \( \frac{dPTP_{i,j}(p,M,T)}{dT} = 0 \).
Check if \( \frac{dPTP_{i,j}(p,M,T)}{dT} > 0 \) and \( 2n - (a + Mn - bp)(l + r) > 0 \), go to Step 2.5; otherwise, go to Step 2.6.
Step 2.5: IF \( PTP_{i,j}(p_{i,j,q},M_{i,j,q},T_{i,j,q}) < PTP_{i,j}(p_{i,j,q+1},M_{i,j,q+1},T_{i,j,q+1}) \), let \( M_{i,j,q} = M_{i,j,q+1} \) and go to Step 2.3; ELSE, let \( p_{i,j,q} = p_{i,j,q+1}, M_{i,j,q} = M_{i,j,q+1}, T_{i,j,q} = T_{i,j,q+1} \), and go to Step 2.7.
Step 2.6: Let \( M_{i,q} = M_{i,j,q+1} \) and \( p_{i,q} = p_{i,j,q+1} \), and go to Step 2.3;
Step 2.7: IF \( D(p_{i,j},M_{i,j}) > D_{d} \) and \( 0 \leq M_{i,j} \leq T_{i,j} \leq T_{i,j} + M_{i,j} \) then \( PTP_{i,j}(p_{i,j},M_{i,j},T_{i,j}) \) is at maximum and go to Step 6;
ELSE, let \( PTP_{i,j}(p_{i,j},M_{i,j},T_{i,j}) = -\infty \).

Step 3 for subcase 2.1 \((i = 2,j = 1)\):
Step 3.1: Given \( M = M_{i,j,q} \), calculate \( p_{i,j,q}(T) \) by Eq. (24).
Step 3.2: Calculate $T_{i,j,q}(M_{i,j,q}, p_{i,j,q})$ that satisfies $\frac{dPTP_{i,j}(p,M,T)}{dT} = 0$, and go to Step 3.3.

Step 3.3: Set $M_{i,j,q+1} = M_{i,j,q} + \epsilon$, where $\epsilon$ is a small positive number as an increment and calculate $p_{i,j,q+1}(T)$ using Eq. (24).

Step 3.4: Calculate $T_{i,j,q+1}(M_{i,j,q+1}, p_{i,j,q+1})$ that satisfies $\frac{dPTP_{i,j}(p,M,T)}{dT} = 0$.

Check if $\frac{dPTP_{i,j}(p,M,T)}{dT} > 0$ and $(2n - (a + M_n - bp)(l + r)) > 0$ and $2n - (a + M_n - bp)r > 0$, go to Step 3.5; otherwise, go to Step 3.6.

Step 3.5: IF $PTP_{i,j}(p_{i,j,q}, M_{i,j,q}, T_{i,j,q}) < PTP_{i,j}(p_{i,j,q+1}, M_{i,j,q+1}, T_{i,j,q+1})$, let $M_{i,j,q} = M_{i,j,q+1}$ and go to Step 3.3;
ELSE, let $p_{i,j} = p_{i,j,q}, M_{i,j} = M_{i,j,q}, T_{i,j} = T_{i,j,q}$, and go to Step 3.7.

Step 3.6: Let $M_{i,j} = M_{i,j+1}$ and $p_{i,j} = p_{i,j+1}$, and go to Step 3.3;
ELSE, IF $D(p_{i,j}, M_{i,j}) > D_d$ and $0 \leq T_{i,j} \leq M_{i,j} + M_{i,j}$ then $PTP_{i,j}(p_{i,j}, M_{i,j}, T_{i,j})$ is at maximum and go to Step 6;
ELSE, let $PTP_{i,j}(p_{i,j}, M_{i,j}, T_{i,j}) = -\infty$.

**Step 4** for subcase 2.1 $(i = 1, j = 2)$:

Step 4.1: Given $M = M_{i,j,q}$, calculate $p_{i,j,q}(T)$ by Eq. (25).

Step 4.2: Calculate $T_{i,j,q}(M_{i,j,q}, p_{i,j,q})$ that satisfies $\frac{dPTP_{i,j}(p,M,T)}{dT} = 0$ and go to Step 4.3.

Step 4.3: Set $M_{i,j,q+1} = M_{i,j,q} + \epsilon$, where $\epsilon$ is a small positive number as an increment and calculate $p_{i,j,q+1}(T)$ by Eq. (25).

Step 4.4: Calculate $T_{i,j,q+1}(M_{i,j,q+1}, p_{i,j,q+1})$ that satisfies $\frac{dPTP_{i,j}(p,M,T)}{dT} = 0$. If $\frac{dPTP_{i,j}(p,M,T)}{dT} > 0$ and $-2n + l(-bp + nM)(l + r) > 0$, $-2n + l(a + Mn - bp)(l + r) > 0$, and $-2n + l(a + Mn - bp)(l + r) > 0$, go to Step 4.5; otherwise, go to Step 4.6.

Step 4.5: IF $PTP_{i,j}(p_{i,j,q}, M_{i,j,q}, T_{i,j,q}) < PTP_{i,j}(p_{i,j,q+1}, M_{i,j,q+1}, T_{i,j,q+1})$, let $M_{i,j,q} = M_{i,j,q+1}$ and go to Step 4.3;
ELSE, let $p_{i,j} = p_{i,j,q}, M_{i,j} = M_{i,j,q}, T_{i,j} = T_{i,j,q}$, and go to Step 4.7.

Step 4.6: Let $M_{i,j} = M_{i,j+1}$ and $p_{i,j} = p_{i,j+1}$, and go to Step 4.3;
ELSE, IF $D(p_{i,j}, M_{i,j}) \leq D_d$ and $0 \leq M_{i,j} \leq T_{i,j} + M_{i,j}$, then $PTP_{i,j}(p_{i,j}, M_{i,j}, T_{i,j})$ is at maximum and go to Step 6;
ELSE, let $PTP_{i,j}(p_{i,j}, M_{i,j}, T_{i,j}) = -\infty$.

**Step 5** for subcase 2.2 $(i = 2, j = 2)$:

Step 5.1: Given $M = M_{i,j,q}$, calculate $p_{i,j,q}(T)$ by Eq. (26).

Step 5.2: Calculate $T_{i,j,q}(M_{i,j,q}, p_{i,j,q})$ that satisfies $\frac{dPTP_{i,j}(p,M,T)}{dT} = 0$ and go to Step 5.3.

Step 5.3: Set $M_{i,j,q+1} = M_{i,j,q} + \epsilon$, where $\epsilon$ is a small positive number as an increment and calculate $p_{i,j,q+1}(T)$ using Eq. (26).

Step 5.4: Calculate $T_{i,j,q+1}(M_{i,j,q+1}, p_{i,j,q+1})$ that satisfies $\frac{dPTP_{i,j}(p,M,T)}{dT} = 0$.

Check if $\frac{dPTP_{i,j}(p,M,T)}{dT} > 0$ and $-2n + l(-bp + nM)(l + r) > 0$, $-2n + l(a + Mn - bp)(l + r) < 0$, $-2n + l(a + Mn - bp)(l + r) < 0$, and $-2n + l(a + Mn - bp)(l + r) < 0$, go to Step 5.5; otherwise, go to Step 5.6.

Step 5.5: IF $PTP_{i,j}(p_{i,j,q}, M_{i,j,q}, T_{i,j,q}) < PTP_{i,j}(p_{i,j,q+1}, M_{i,j,q+1}, T_{i,j,q+1})$, let $M_{i,j,q} = M_{i,j,q+1}$ and go to Step 5.3;
ELSE, let $p_{i,j} = p_{i,j,q}, M_{i,j} = M_{i,j,q}, T_{i,j} = T_{i,j,q}$, and go to Step 5.7.

Step 5.6: Let $M_{i,j} = M_{i,j+1}$ and $p_{i,j} = p_{i,j+1}$, and go to Step 5.3;
Step 5.7: IF $D(p_{i,j}, M_{i,j}) \leq D_d$ and $0 \leq T_{i,j} \leq M_{i,j} \leq T_{i,j} + M_{i,j}$ then $PTP_{i,j}(p_{i,j}, M_{i,j}, T_{i,j})$ is at maximum and go to Step 6;
ELSE, let $PTP_{i,j}(p_{i,j}, M_{i,j}, T_{i,j}) = -\infty$.

Step 6 Compare all $PTP_{i,j}(p_{i,j}, M_{i,j}, T_{i,j}), i = j = [1, 2]$ and find $k$, such as $PTP_k(p, M, T) = \max\{PTP_{i,j}(p_{i,j}, M_{i,j}, T_{i,j}), i = j = [1, 2]\}$.

5. Computational analysis. In this section, numerical examples and sensitivity analysis are provided to show the theoretical results, solution procedure, and gain some managerial insights.

5.1. Numerical example. In this section, three examples are conducted. First, our proposal model with order-linked trade credit is considered to see if our model and algorithm are applicable. Second, we keep other parameters and constraints as the first examples, but no order-linked trade credit is considered. Third, the models with full trade credit and partial trade credit are considered separately. Finally, a comparison among the results of all examples is conducted to see what happens.

The data used are as follow. For a deteriorating product, let the annual demand rate $D(p, M) = 1000 - 25p + 100M$ kg per year and the deterioration rate be constant, $\theta = 0.05$. The parameter values are as follows: $A = $20 per order, $c = $10/per unit, $h = $1 per unit per year, $I_{e} = 0.04$ per dollar per year, $I_{Loss} = 0.06$ per dollar per year, $l = 0.1$ per year, $r = 0.04$ per dollar per year, $\alpha = 0.8$, and $D_d = 350$ units. Most of the parameter value are referred from [17].

First, the results of the case with the order-linked trade credit are presented. The optimal solution is that the seller offers full trade credit terms with subcase 1.1. The optimal selling price is $p_{1,1} = $25.3312; the optimal credit period policy is $M_{1,1} = 0.0819213$ years; and the optimal replenishment cycle is $T_{1,1} = 0.27852$ years. In addition, the present value of the seller’s total profit is $PTP_{1,1}(p, M, T) = $5482.41. Figures 7-9 graphically show the profit function. The results show that our proposal model and algorithm could be applicable in case study to find the optimal solution for the seller.

![Figure 7. The graph of $PTP_{1,1}(p, M, T)$](image)

Second, Table 4 presents the results of four cases: (1) model with ordered-link trade credit; (2) model without ordered-link trade credit; (3) model with full trade credit; and (4) model with partial trade credit. According to Table 4, the models with financing schemes always benefit the seller with higher profit. However, the seller obtains the highest profit in the case of partial trade credit with the longest
credit time. On the contrary, the model with order-linked trade credit creates more demand than other schemes. That means the ordered-link trade credit could help the seller increase the demand for sale. Thus, the seller could choose any scheme depended on his/her objective.

Table 4. A comparison of four cases:

| Cases                | \( p \) ($) | \( M \) (years) | \( T \) (years) | Demand (items) | \( PTP_{i,j}(p, M, T) \) ($) |
|----------------------|-------------|-----------------|-----------------|----------------|-------------------------------|
| Ordered-link trade credit | 25.3312     | 0.081921        | 0.27852         | 375            | 5482.41                       |
| No ordered-link trade credit | 26.2325     | 0.25352         | 344             |                | 5398.24                       |
| Full trade credit     | 25.505      | 0.054           | 0.27741         | 368            | 5478.92                       |
| Partial trade credit  | 25.7267     | 0.151           | 0.2979          | 372            | **5485.39**                   |

5.2. Sensitivity analysis. The following analysis studies the change in each parameter at -40%, -20%, +20%, and +40% from Example 1, with \( D_d = 450 \) units (i.e., partial trade credit). A summary of the sensitivity analysis is presented in Tables 5 and 6. Based on Tables 5 6, the following results can be obtained:
Table 5. Summary of sensitivity analysis

| Parameter | \( p \) | \( M \) | \( T \) | \( PTP_{i,j}(p, M, T) \) |
|-----------|--------|--------|--------|------------------|
| \( D_d=350 \) | 25.3312 | 0.0819213 | 0.266878 | 5482.41 |
| \( D_d=450 \) | 25.5224 | 0.149572 | 0.297398 | 5493.47 |
| \( c=6 \) | 23.7634 | 0.260868 | 0.360191 | 7123.23 |
| \( c=8 \) | 24.6262 | 0.196586 | 0.320289 | 6280.3 |
| \( c=10 \) | 25.5224 | 0.149572 | 0.297398 | 5493.47 |
| \( c=12 \) | 26.4370 | 0.112410 | 0.28441 | 4761.02 |
| \( c=14 \) | 27.3616 | 0.081181 | 0.277903 | 4081.91 |
| \( h=0.6 \) | 25.5873 | 0.177748 | 0.360788 | 5518.11 |
| \( h=0.8 \) | 25.5501 | 0.161851 | 0.324659 | 5505.19 |
| \( h=1 \) | 25.5224 | 0.149572 | 0.297398 | 5493.47 |
| \( h=1.2 \) | 25.5012 | 0.139723 | 0.279273 | 5482.67 |
| \( h=1.4 \) | 25.4845 | 0.131596 | 0.258471 | 5472.62 |
| \( \theta=0.03 \) | 25.5515 | 0.162534 | 0.325811 | 5505.41 |
| \( \theta=0.04 \) | 25.5359 | 0.155573 | 0.310662 | 5499.3 |
| \( \theta=0.05 \) | 25.5224 | 0.149572 | 0.297398 | 5493.47 |
| \( \theta=0.06 \) | 25.5106 | 0.144213 | 0.285662 | 5487.86 |
| \( \theta=0.07 \) | 25.5002 | 0.139388 | 0.275185 | 5482.47 |
| \( b=15 \) | 39.6903 | 0.291871 | 0.409115 | 11962.1 |
| \( b=20 \) | 30.7678 | 0.198115 | 0.329033 | 7882.19 |
| \( b=25 \) | 25.5224 | 0.149572 | 0.297398 | 5493.47 |
| \( b=30 \) | 22.0502 | 0.114364 | 0.291447 | 3947.53 |
| \( b=35 \) | 19.5771 | 0.084373 | 0.29072 | 2882.45 |
| \( l=0.06 \) | 27.3895 | 0.790391 | 0.771212 | 5638.77 |
| \( l=0.08 \) | 26.0368 | 0.328971 | 0.397466 | 5537.01 |
| \( l=0.1 \) | 25.5224 | 0.149572 | 0.297398 | 5493.47 |
| \( l=0.12 \) | 25.2051 | 0.0372622 | 0.268559 | 5476.34 |
| \( l=0.14 \) | 25.1003 | 6.15627×10^{-27} | 0.266878 | 5475.06 |

1. As the quantity threshold \( D_d \) increases, the used credit terms should be partial trade credit. The results show that the buyer fails to reach the quantity threshold, \( D_d \). The seller will allow more extended the credit time \( M \), increase the selling price \( p \), order more products, to get a higher \( PTP_{i,j}(p, M, T) \) due to a higher quantity threshold. This result is also similar to [21] which showed that the retailer should take the partially delayed payment due to a higher quantity threshold. In a short time, the seller allows buyers a longer credit time, which means the risk default increases and the sale revenue decreases. Thus, the seller needs to increase the selling price to compensate for reduced revenue and also change to partial trade credit to obtain some interest charges from cash payment; consequently, the present value of total profit is higher. In contrast, in a long term, the buyers are not willing to pay a higher price to receive partial trade credit from the seller. As a result, the demand reduces even the credit time is longer. Thus, the order-linked trade credit cannot benefit the seller as he/she wants. In sum, the seller should consider carefully before setting the quantity threshold. Our model could be used to examine what is the best level of the quantity threshold \( D_d \) among others.
Table 6. Summary of sensitivity analysis (cont')

| Parameter | $p$ | $M$ | $T$ | $PTP_{i,j}(p,M,T)$ |
|-----------|-----|-----|-----|------------------|
| $\alpha=0.48$ | 26.3120 | 0.427639 | 0.398781 | 5544.5 |
| $\alpha=0.64$ | 25.7899 | 0.243915 | 0.329357 | 5510.66 |
| $\alpha=0.8$ | 25.5224 | 0.149572 | 0.297398 | 5493.47 |
| $\alpha=0.96$ | 25.3619 | 0.092792 | 0.281188 | 5484.03 |
| $\alpha=1$ | 25.3312 | 0.081921 | 0.278520 | 5482.41 |
| $r=0.024$ | 25.9844 | 0.307269 | 0.445592 | 5546.65 |
| $r=0.032$ | 25.7022 | 0.211592 | 0.349793 | 5516.32 |
| $r=0.04$ | 25.5224 | 0.149572 | 0.297398 | 5493.47 |
| $r=0.048$ | 25.3915 | 0.103826 | 0.264839 | 5475.44 |
| $r=0.056$ | 25.2871 | 0.067011 | 0.242921 | 5460.93 |
| $I_{Loss}=0.036$ | 25.7848 | 0.242724 | 0.314041 | 5504.49 |
| $I_{Loss}=0.048$ | 25.6232 | 0.185330 | 0.304028 | 5497.72 |
| $I_{Loss}=0.06$ | 25.5224 | 0.149572 | 0.297398 | 5493.47 |
| $I_{Loss}=0.072$ | 25.5224 | 0.149572 | 0.297398 | 5493.47 |
| $I_{Loss}=0.084$ | 25.5224 | 0.149572 | 0.297398 | 5493.47 |
| $A=12$ | 25.4177 | 0.120489 | 0.232408 | 5523.65 |
| $A=16$ | 25.4741 | 0.136219 | 0.26707 | 5507.64 |
| $A=20$ | 25.5224 | 0.149572 | 0.297398 | 5493.47 |
| $A=24$ | 25.5651 | 0.161264 | 0.324661 | 5480.61 |
| $A=28$ | 25.6034 | 0.171715 | 0.349611 | 5468.74 |
| $n=60$ | 25.1003 | 1.57412×10$^{-9}$ | 0.266878 | 5475.06 |
| $n=80$ | 25.1003 | 6.67337×10$^{-9}$ | 0.266878 | 5475.06 |
| $n=100$ | 25.5224 | 0.149572 | 0.297398 | 5493.47 |
| $n=120$ | 25.5224 | 0.149572 | 0.297398 | 5493.47 |
| $n=140$ | 25.5224 | 0.149572 | 0.297398 | 5493.47 |
| $I_e=0.024$ | 25.4698 | 0.131901 | 0.258682 | 5472.69 |
| $I_e=0.032$ | 25.4934 | 0.139910 | 0.270978 | 5482.71 |
| $I_e=0.04$ | 25.5224 | 0.149572 | 0.297398 | 5493.47 |
| $I_e=0.048$ | 25.5589 | 0.161542 | 0.324322 | 5505.12 |
| $I_e=0.056$ | 25.6065 | 0.176891 | 0.359702 | 5517.93 |

2. A higher value of procurement cost $c$ stimulates the selling price $p$ and decreases the credit time $M$, the replenishment cycle time $T$, the order quantity $Q$, and the profit $PTP_{i,j}(p,M,T)$. These results are also found in [16] and [34]. If the procurement cost is high, the seller should offer a shorter credit time $M$ and increase the price $p$ to achieve better cash flow and higher revenue to compensate for a higher procurement cost. He should also reduce the quantity ordered and increase order frequency to reduce the total cost.

3. An increase of holding cost $h$, deterioration rate $\theta$, credit payment fraction $\alpha$, annual compound interest rate $r$, coefficient of price $b$, coefficient of default risk $l$, and interest revenue loss $I_{Loss}$, decrease $p$, $M$, $T$, $Q$ as well as $PTP_{i,j}(p,M,T)$. A simple economic explanation is as follows. If the holding cost is higher, the seller should offer a shorter credit time $M$ to achieve better cash flow, decrease the price $p$ to increase the demand sale, and reduce the inventory holding items. Similarly, if the deterioration rate $\theta$ is high, then
the seller should replenish more often to avoid product loss during the storage period. A higher credit payment fraction $\alpha$ decreases the cash payment ratio, which means the sellers could receive less money at the time $t=0$ and the interest loss during the trade credit time increases. When the interest loss rate $I_{Loss}$ increases, the total cost increases and the profit reduces. In this case, the sellers need to decrease the selling price and credit period to compensate for higher cost and achieve better cash flow. In addition, if the annual compound interest rate $r$ increases, the seller should permit a shorter credit $M$ and decrease the price $p$ because the monetary value is of less worth. If the coefficient of price $b$ is high, the seller should offer a lower price $p$ and a shorter credit period $M$ to stimulate market demand. Furthermore, if the coefficient of default risk $l$ is high, the seller should provide a shorter credit period $M$ to avoid bad debt collection and permit a lower price $p$ to increase and balance the demand sale which has be reduced due to a shorter $M$. Our model and algorithm could help the seller to analyse and make decisions when the market data changes.

4. A higher ordering cost $A$ results in a higher price $p$ and longer credit period $M$ and $T$. The company gets a lower $PTP_{i,j}(p, M, T)$ when the ordering cost increases. The result is different from [24] who found that the selling price and purchase quantity decrease drastically as the ordering cost increases. A simple economic interpretation is as follows: If the ordering cost increases, the seller should raise the selling price to maintain profit. Also, the seller needs to increase the ordered quantity and decrease the replenishment frequency to reduce the total ordering cost. However, in [24], the author considered the retailers’ perspective who want to reduce the selling price to increase their sale. Thus, the retailers could benefit from reducing the ordering frequency.

5. A higher trade credit mark-up $n$ and interest earned $I_e$ causes higher $p$, $M$, $T$, $Q$, and $PTP_{i,j}(p, M, T)$. A simple economic interpretation is as follows. Since the trade credit mark-up $n$ has a positive effect on market demand, the seller can offer a longer credit time $M$, higher price $p$, and orders a higher quantity to enhance revenue. In addition, to take the opportunity of a higher interest rate, the seller should offer a higher $M$ and increase the cycle time $T$. Therefore, the seller can obtain more accumulated revenue through higher interest earned.

Furthermore, the value of the average change is calculated from Table 4 by averaging the shift of each increasing element in each parameter from -40% to +40%. This value is used to examine the average significant change in each parameter on the present value of total profit. The findings are as follows:

1. Quantity threshold $D_d$, interest earned $I_e$, and trade credit mark-up $n$ have a positive impact on total profit. Otherwise, the other parameters have a negative influence on total profit.
2. Procurement cost $c$, coefficient of price $b$, and default risk $l$ have the most significant negative impact on total profit.
3. Trade credit mark-up $n$ has the most significant positive influence on total profit.
4. The deterioration rate $\theta$ and interest revenue loss $I_{Loss}$, have the smallest effect on total profit.
6. Conclusion. This study builds a practical inventory model from the seller’s perspective that considers the following facts: (1) the seller offers customers order-linked trade credit, (2) the credit period affects both increasing demand and default risk, (3) demand is a function of selling price and credit period, and (4) a discounted cash flow analysis is used to include the time value of money. The seller offers full trade credit if customer demand is above a particular level. Otherwise, partial trade credit is offered. This study proves that the present value of total profit is concave or pseudo-concave under certain conditions. In addition, by using Mathematica 7.0 (software), a numerical example and solution procedure are presented to determine the optimal selling price, replenishment cycle time, and credit period policies, while maximizing the present value of the seller’s total annual profit. The example shows that all of the decision variables have an optimal value, and the dimensional graphs are concave. The order-linked trade credit could be applicable in the case study. Besides, a comparison of four cases: (1) model with ordered-link trade credit; (2) model without ordered-link trade credit; (3) model with full trade credit; and (4) model with partial trade credit is also conducted. The results show that the models with financing schemes always benefit the seller with higher profit. However, the seller obtains the highest profit in the case of partial trade credit and the model with order-linked trade credit has more demand than other schemes. Thus, the seller could choose any scheme depended on his/her objective.

Finally, a sensitivity analysis is conducted, and several managerial insights are presented here. The results show that our model could be a reference for the seller to make his decisions (price, credit time, and replenishment cycle time) while maximizing the profit. For example, as the quantity threshold $D_d$ increases, the buyer fails to reach the quantity threshold $D_d$. Thus, the optimal credit period terms become partial trade credit. The seller will allow more extended $M$, increase $p$, order more products, and achieve a higher $PTP_{i,j}(p, M, T)$. However, in a long term, the buyers are not willing to pay a higher price to receive a partial trade credit from the seller, and consequently the demand increases. The benefit of the order-linked trade credit also reduces. Thus, the seller should consider carefully before setting the quantity threshold. Also, the interest earned $I_e$ and trade credit mark-up $n$ have positive influences on total profit. The other parameters have negative influences on total profit. The procurement cost $c$, coefficient of price $b$, and default risk $l$ have the most significant negative influences on total profit. The trade credit mark-up $n$ has the most significant positive influence on total profit. The deterioration rate $\theta$ and interest revenue loss $I_{Loss}$, have the smallest influences on total profit. Thus, the seller can make better decisions using the model in the practical case.

For further research, this study can be extended in several ways. For instance, we may add the maximum lifetime for deteriorating items, quantity discounts, and partial backlogging. It can also be extended to an EPQ inventory model to consider production issues such as maintenance or quality control. Finally, it may incorporate an integrated cooperative win-win policy for both players or a non-cooperative Nash or Stackelberg policy.

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Appendix 1. Proof of Theorem 1 Case 1 Full Trade Credit

Sub-case 1.1 $0 \leq M \leq T \leq T + M$

$$\frac{\partial PTP_{1,1}(p, M, T)}{\partial p} = \frac{1}{T} \left( e^{-M(l+r)}rT(Mn - 2bp)(I_e - r) + e^{-M(l+r)}rT(Mn - 2bp)(-I_e + r + I_e rT) \right) + \frac{be^{-rT}h}{r(r + \theta)} + \frac{be^{T} \theta h}{\theta(r + \theta)} = I_{Loss}Mn\theta + b(r + cr - c(e^{T}T) - 2I_{Loss}p\theta)$$ \quad (29)

$$\frac{\partial^2 PTP_{1,1}(p, M, T)}{\partial p^2} = -\frac{2be^{-M-r(M+T)}}{r^2T}((1 + e^{T})r + I_e(1 + e^{T}(-1 + rT)) + (e^{M+rT}I_{Loss}(1 + Mr) - e^{M+r(M+T)}I_{Loss})) < 0.$$ \quad (30)

Based on equation (30), accordingly, the $PTP_{1,1}(p, M, T)$is a strictly concave function on $p$. Thus, proof of Theorem 1 is completed.
Sub-case 1.2 $0 \leq T \leq M \leq T + M$

$$\frac{\partial PTP_{1.2}(p, M, T)}{\partial p} = \frac{1}{r^2 T (r + \theta)} e^{-M - 2r(M + T)} (e^{r(M + T)} (a + Mn)(I_e + e^{M(r + \theta)} I_{Loss})$$

$$-e^{M + r(M + T)} I_{Loss} - r + e^{rT} r + e^{M + rT} I_{Loss} rT + e^{rT} I_e (-1 + rT) \theta (r + \theta) + b(e^{M + 2r(M + T) + T \theta} \theta r^2 + 2e^{r(M + T)} p (-I_e + r) \theta (r + \theta)$$

$$-2e^{M + r + 2rT} I_{Loss} pr T \theta (r + \theta) - 2e^{r(M + 2T)} p (r + I_e (-1 + rT)) \theta (r + \theta)$$

$$+ e^{M + 2r(M + T)} (r + \theta) (-hr + c (-1 + (e^\theta r)^2) + 2I_{Loss} p \theta),$$

(31)

$$\frac{\partial^2 PTP_{1.2}(p, M, T)}{\partial p^2} \leq \frac{2be^{-M - r(M + T)}(-1 + e^T)}{r^2 T (1 - e^{-T r} - e^{-M r} T)} < 0.$$  

(32)

Based on equation (32), accordingly, the $PTP_{1.2}(p, M, T)$ is a strictly concave function on $p$. Thus, proof of Theorem 1 is completed.

Case 2 Partial Trade Credit

Sub-case 2.1 $0 \leq M \leq T + M$

$$\frac{\partial PTP_{2.1}(p, M, T)}{\partial p} = \frac{1}{T} (\frac{b e^{-M - r(M + T)} (-1 + e^T) p (e^M r (-1 + \alpha) - \alpha)}{r}$$

$$+ b I_{Loss} p (1 - e^{-Mr} (1 + Mr)) \alpha - \frac{I_{Loss} (a + Mn - bp) (1 - e^{-Mr} (1 + Mr)) \alpha}{r^2 T}$$

$$+ e^{-M + r(M + T) - 2rT} (-1 + e^{rT}) (a + Mn - bp) (-e^{r(M + T)} (-1 + \alpha) + e^{rT} \alpha)$$

$$- be^{-M} I_e p (1 - \alpha + e^{-r(M + T)} (e^M r (1 + rT) (-1 + \alpha) + e^{rT} (-1 + rT) \alpha))$$

$$+ \frac{\gamma^{-M} I_e (a + Mn - bp) (1 - \alpha + e^{-r(M + T)} (e^M r (1 + rT) (-1 + \alpha) + e^{rT} (-1 + rT) \alpha)}{r^2 T}$$

$$+ \frac{b c (1 + (e^\theta r)^2)}{r^2 T} - \frac{b h (r - \gamma T r + \theta - e^{-rT} \theta)}{r \theta (r + \theta)},$$

(33)

$$\frac{\partial^2 PTP_{2.2}(p, M, T)}{\partial p^2} = \frac{-2be^{-M} (1 - e^{-rT}) (1 - \alpha)}{r^2 T} + \frac{e^{-r(M + T)} (-1 + 2rT) \alpha}{r^2 T}$$

$$\frac{2b I_{Loss} (e^{-Mr} (1 + Mr) - 1) \alpha}{r^2 T} - \frac{2be^{-M} I_e (1 - e^{-rT} (1 + rT)) (1 - \alpha)}{r^2 T}$$

$$+ \frac{e^{-r(M + T)} (1 + e^T (-1 + rT)) \alpha}{r^2 T} < 0.$$  

(34)

Based on equation (34), accordingly, the $PTP_{2.1}(p, M, T)$ is a strictly concave function on $p$. Thus, proof of Theorem 1 is completed.
Sub-case 2.2 $0 \leq M \leq T \leq T + M$

$$\frac{\partial PTP_{2.2}(p, M, T)}{\partial p} = \frac{1}{T} \left( -e^{-l^M-r(M+T)}(1 - e^{-rT})pD(p, M) + \frac{e^{-l^M-r(M+T)}I_{loss}(a + M - bp)(c^M - e^{rT} + e^{rT}rT)\alpha}{r^2} \right)$$

$$\frac{\partial^2 PTP_{2.2}(p, M, T)}{\partial p^2} = -2be^{-l^M}(1 - e^{-rT})(1 - \alpha)$$

Appendix 2. Proof of Theorem 2 Case 1 Full Trade Credit

Sub-case 1.1 $0 \leq M \leq T \leq T + M$

Let,

$$f_{1.1}(T) = -A + \frac{e^{-l^M-r(M+T)}(1 - e^{-rT})pD(p, M)}{r}$$

$$+ \frac{e^{-l^M-r(M+T)}I_{loss}(a + M - bp)(c^M - e^{rT} + e^{rT}rT)\alpha}{r^2}$$

$$+ \frac{hD(p, M)(r - e^{-rT}r + \theta - e^{-rT}\theta)}{\theta(r^2 + r\theta)} - \frac{I_{loss}pD(p, M)(1 - e^{-M^r(1 + M^r)})}{r^2}$$

and,

$$g_{1.1}(T) = T.$$ 

Hence, for any given selling price $p$ and credit period $M$, 

$$PTP_{1.1}(p, M, T) = f_{1.1}(T)/g_{1.1}(T).$$

Taking the first and second-order derivatives of $f_{1.1}(T)$ respect to $T$, and simplifying terms will result as follows:

$$f_{1.1}'(T) = \frac{1}{r\theta(r + \theta)}e^{-l^M-r(M+T)}D(p, M)(\theta(e^{M^r}hr - e^{M^r+M^r+r^r\theta})hr + e^{rT}I_{loss}(a + M - bp)(c^M - e^{rT} + e^{rT}rT)\alpha)$$

and,

$$f_{1.1}''(T) = -\frac{e^{-l^M-r(M+T)}D(p, M)(\theta(e^{M^r}hr + e^{M^r+M^r+r^r\theta})p\theta - p(I_{loss} - (r + \theta)) + ce^{M^r+M^r}(e^\theta)r(r + \theta)Log[e^\theta])}{\theta(r + \theta)}$$

and,

$$-p(I_{loss} - (r + \theta)) + ce^{M^r+M^r}(e^\theta)r(r + \theta)Log[e^\theta^2] < 0.$$
If \((Ie - r) \leq 0\) and by applying the equation (4.3) based on the result in Cambini and Martein (2009), then \(f_{1.1}''(T) < 0\). Hence the total profit \(PTP_{1.1}(p, M, T)\) is a strictly pseudo-concave function on \(T\). Thus, proof of Theorem 2 is completed.

**Sub-case 1.2** \(0 \leq T \leq M \leq T + M\)

Let,

\[
\begin{align*}
f_{1.2}(T) &= -A + \frac{e^{-lM-r(M+T)}(-1 + e^{rT})pD(p, M)}{r} \\
&\quad + \frac{e^{-lM-r(M+T)}IeD(p, M)(1 + e^{rT}(-1 + rT))}{r^2} \\
&\quad - \frac{c(-1 + (e^\theta)^T)}{\theta}D(p, M)
\end{align*}
\]

\[\text{(42)}\]

and,

\[
g_{1.2}(T) = T.
\]

Hence, for any given selling price \(p\) and credit period \(M\),

\[
PTP_{1.2}(p, M, T) = f_{1.2}(T)/g_{1.2}(T).
\]  

(43)

Taking the first and second-order derivatives of \(f_{1.2}(T)\) with respect to \(T\), and simplifying terms will result as follows:

\[
\begin{align*}
f_{1.2}'(T) &= \frac{1}{r\theta(r + \theta)}e^{-lM-r(M+T)}D(p, M)(\theta(p((1 + r^T)Ie + e^{lM+rT}I_{Loss} + r)(r + \theta) \\
&\quad + e^{M(r+\theta)}(hr - I_{Loss}p(r + \theta)) - e^{lM+r(M+T)}r(\theta T h \theta + c(\theta^T(r + \theta))Log(e^\theta)));
\end{align*}
\]

\[\text{(45)}\]

and,

\[
\begin{align*}
f_{1.2}''(T) &= -\frac{D(p, M)e^{-lM-r(M+T)}}{\theta(r + \theta)}(\theta(e^{lM+r(M+T)(r+\theta)}h \theta - p(Ie - r)(r + \theta) \\
&\quad + e^{M(r+\theta)}(hr - I_{Loss}p(r + \theta)) + ce^{lM+r(M+T)}(\theta^T(r + \theta)Log(e^\theta^2)) < 0).
\end{align*}
\]

(46)

If \((Ie - r) \leq 0\) and by applying the equation (4.3) based on the result in Cambini and Martein (2009), then \(f_{1.2}''(T) < 0\). Hence the total profit \(PTP_{1.2}(p, M, T)\) is a strictly pseudo-concave function on \(T\). Thus, proof of Theorem 2 is completed.

**Case 2 Partial Trade Credit**

**Sub-case 2.1** \(0 \leq M \leq T \leq T + M\)

Let,

\[
\begin{align*}
f_{2.1}(T) &= -A + e^{-lM}pD(M, p)(\frac{1 - e^{-rT}}{r} \big(1 - \alpha\big) \\
&\quad + \frac{e^{-r(M+T)}(-1 + e^{rT})\alpha}{} + \gamma^{-lM}I_pD(M, p)(\frac{1 - \gamma^{-rT}(1 + rT)}{r^2} \big(1 - \alpha\big) \\
&\quad + \frac{\gamma^{-r(M+T)}(1 + e^{rT}(-1 + rT))\alpha}{} - \frac{c(-1 + \gamma^T\theta)D(M, p)}{\theta} \\
&\quad + \frac{hD(M, p)(r - \gamma^{-T}r + \theta - e^{-rT}\theta)}{\theta(r^2 + r\theta)} - \frac{I_{Loss}pD(p, M)(1 - e^{-M}(1 + Mr))\alpha}{r^2},
\end{align*}
\]

(47)
and,

\[ g_{2.1}(T) = T, \]

(48)

Hence, for any given selling price \( p \) and credit period \( M \),

\[ PTP_{2.1}(p, M, T) = f_{2.1}(T)/g_{2.1}(T). \]

(49)

Taking the first and second-order derivatives of \( f_{2.1}(T) \) respect to \( T \), and simplifying terms will result as follows:

\[
\begin{align*}
f'_{2.1}(T) &= D(p, M) e^{-M(l+r) - 2rT} (\gamma^{(M+T)} \theta (e^{IM + \alpha} p - pr(1 + I_e T)(-1 + \alpha)(r + \theta) ) \\
&+ e^{rT} \theta (-e^{IM+r(M+T)} I_e hr + p((1 + e^{rT}) I_e + r) \alpha (r + \theta)) \\
&- ce^{M(l+r) + 2rT} (e^\theta)^T r(r + \theta) Log[e^\theta]/r\theta(r + \theta), \\
\end{align*}
\]

and,

\[
\begin{align*}
f''_{2.1}(T) &= -e^{-M(l+r) - 2rT} D(p, M) (e^{r(M+T)} \theta (e^{IM + \alpha} p - p(-I_e + r + I_e r T)(-1 + \alpha)(r + \theta) ) \\
&+ e^{rT} \theta (e^{IM+r(M+T)} + T \theta) I_e hr + p(-I_e + r + \alpha)(r + \theta)) \\
&+ ce^{M(l+r) + 2rT} (e^\theta)^T (r + \theta) Log[e^\theta]/r\theta(r + \theta) < 0. \\
\end{align*}
\]

(50)

If \((-I_e + r) \geq 0\) and by applying the equation (4.3) based on the result in Cambini and Martein (2009), then \( f''_{2.1}(T) < 0\). Hence the total profit \( PTP_t(p, M, T) \) is a strictly pseudo-concave function on \( T \). Thus, proof of Theorem 2 is completed.

Sub-case 2.2 \( 0 \leq T \leq M \leq T + M \)

Let,

\[
\begin{align*}
f_{2.2}(T) &= -A + e^{-lM} pD(M, p) \frac{(1 - r^T)(1 - \alpha)}{r} \\
&+ \frac{e^{-r(M+T)}(1 + r^T) \alpha}{r} A e^{-lM} pD(M, p) \frac{(1 - r^T)(1 + r T)}{r^2} \\
&+ \frac{e^{-r(M+T)}(1 + e^{rT}(-1 + r T) \alpha}{r^2} A e^{-lM} pD(M, p) \frac{(1 - r^T)(1 + r T)}{r^2} \\
&\frac{c(-1 + e^{rT}) D(M, p)}{\theta} + \frac{hD(M, p) (r - e^{rT} \theta + \theta - e^{-rT} \theta)}{\theta(r^2 + r \theta)} \\
&- I_{Loss} pD(M, p) \frac{(-e^{-M + e^{-rT}} T)}{r} + \frac{1 - e^{-rT}(1 + r T)}{r^2} \alpha, \\
\end{align*}
\]

(52)

and,

\[ g_{2.2}(T) = T. \]

(53)

Hence, for any given selling price \( p \) and credit period \( M \),

\[ PTP_{2.2}(p, M, T) = f_{2.2}(T)/g_{2.2}(T). \]

(54)
Taking the first and second-order derivatives of \( f_{2,2}(T) \) respect to \( T \), and simplifying terms will result as follows:

\[
\begin{align*}
  f'_{2,2}(T) &= D(p, M)e^{-M-T+e+M+M+T)}(\theta(-e^{M+2r(M+T)}+T\theta)hr \\
  &- e^{r(2M+T)}pr(1 + I_eT)(-1 + a)(r + \theta) + e^{r(M+T)}I_e\rho a(r + \theta) \\
  &+ e^{r(i+r)+2r}T_{I_{loss}}p(r + \theta) + e^{r(M+T)}p(-I_e + r)\alpha(r + \theta) \\
  &+ e^{r+2r+T}(h_r - I_{loss}p\rho(r + \theta)) - ce^{r+2r+M+T}(\theta^rT(r + \theta)\log|\theta^rT|/r\theta(r + \theta),
\end{align*}
\]

and,

\[
\begin{align*}
  f_{2,2}''(T) &= -e^{-M(i+r)-2rT}D(p, M)\left(e^{rT}\theta(e^{M+r(M+T)}+T\theta)\theta\right) \\
  &+ p(-I_e + r)\alpha(r + \theta) + e^{r(M+T)}\theta(-p(r + I_e(-1 + RT))(-1 + a)(r + \theta) \\
  &+ e^{rM}h_r - I_{loss}p\rho(r + \theta)) + c\theta^{M(i+r)+2rT}(\theta^rT(r + \theta)\log|\theta^rT|/r\theta(r + \theta) < 0.
\end{align*}
\]

If \((-I_e+r)\geq 0\) and by applying the equation (4.3) based on the result in Cambini and Martein (2009), then \( f_{2,2}''(T) < 0 \). Hence the total profit \( PTP_2(p, M, T) \) is a strictly pseudo-concave function on \( T \). Thus, proof of Theorem 2 is completed.

**Appendix 3. Proof of Theorem 3, Theorem 4, Theorem 5 and Theorem 6 Case 1 Full Trade Credit**

**Sub-case 1.1 0 ≤ M ≤ T ≤ T + M**

\[
\begin{align*}
  \frac{\partial PTP_{1,1}(p, M, T)}{\partial M} &= \frac{e^{-M-r(M+T)}(-1 + e^T)p(n - (a + Mn - bp)(l + r))}{rT} \\
  &+ \frac{e^{-M-r(M+T)}I_e\rho p(n - (a + Mn - p)(l + r))(1 + e^T(-1 + rT))}{r^2T} \\
  &+ \frac{c(-1 + (\theta^rT)n - 1 + e^T\rho + \theta - e^{-rT}\theta)}{rT} \\
  &- \frac{I_{loss}np(1 - \theta^{-M}r(1 + Mr))}{r^2T} \\
  &- \frac{I_{loss}p(a + Mn - bp)(-e^{-M}r + \theta^{-M}r(1 + Mr))}{r^2T}
\end{align*}
\]

For simplicity, let \( \frac{\partial PTP_{1,1}(p, M, T)}{\partial M} = F_{1,1}(M) \) For

\[
\begin{align*}
  F_{1,1}(0) &= \frac{e^{-rT}(-1 + e^T)p(n - (a - bp)(l + r))}{rT} \\
  &+ \frac{e^{-rT}I_e\rho p(n - (a - bp)(l + r))(1 + e^T(-1 + rT))}{r^2T} - \frac{c(-1 + (\theta^rT)n)}{\theta} \\
  &+ \frac{hn(r - e^T\rho + \theta - e^{-rT}\theta)}{rT\theta(r + \theta)}
\end{align*}
\]
Thus, proof of Theorem 3 is completed. M unique value of the Mean Value Theorem into PTP

\[
\begin{align*}
F_{1.1}(\infty) &= \lim_{M \to \infty} e^{-rT} \frac{\partial}{\partial M} \left( -1 + e^{rT} \right) p(n - (a + Mn - bp)(l + r)) \\
&\quad + e^{-rT} \frac{\partial}{\partial M} \left( -1 + e^{rT} \right) (n - (a + Mn - bp)(l + r))(1 + e^{rT}(-1 + rT)) \\
&\quad - \frac{c(-1 + (e^T)_n) + h(n(r - e^{rT} + \theta - e^{-rT}\theta))}{rT} \\
&\quad + \frac{e^{-rT} I_{Loss}(M(-a + bp)r^2 - n(-1 + e^{Mr} + Mr(-1 + Mr))))}{r^2T} = -\infty
\end{align*}
\]

(59)

\[
\frac{\partial^2 PTP_{1.1}(p, M, T)}{\partial M^2} = -\frac{1}{r^2T} e^{-rT} (2n - (a + Mn - bp)(l + r)) \\
\left((-1 + e^{rT})r + I_c(1 + e^{rT}(-1 + rT)) - \frac{2I_{Loss}np(e^{-Mr}M)^2}{r^2T} \\
- I_{Loss}(a + Mn - bp)(e^{-Mr}r^2 - e^{-Mr}Mr)^3\right) < 0
\]

(60)

If \((2n - (a + Mn - bp)(l + r)) > 0\), then \(\frac{\partial^2 PTP_{1.1}(p, M, T)}{\partial M^2} < 0\) and the total profit \(PTP_{1.1}(p, M, T)\) is a strictly concave function on \(M\). However, two conditions will occur. First, if \(\frac{\partial^2 PTP_{1.1}(p, M, T)}{\partial M^2} < 0\) then \(F_{1.1}(M) < 0\) (i.e., \(F_{1.1}(0) < 0\)) for all \(M > 0\). Hence, the total profit \(PTP_{1.1}(p, M, T)\) is a monotonic decreasing function in \(M\) and maximized at \(M_1 \approx 0\). Otherwise, \(\frac{\partial^2 PTP_{1.1}(p, M, T)}{\partial M^2} > 0\) then \(F_{1.1}(0) > 0\). By using the Mean Value Theorem into \(F_{1.1}(0) > 0\) and \(F_{1.1}(\infty) < 0\), there will exist a unique value of \(M_{1.1} > 0\) that satisfy \(F_{1.1}(M) = 0\) and maximized at \(M_{1.1} > 0\).

Thus, proof of Theorem 3 is completed.

Sub-case 2.0 \(0 \leq M \leq T \leq T + M\)

\[
\frac{\partial PTP_{1.2}(p, M, T)}{\partial M} = e^{-rT} \frac{\partial}{\partial M} \left( -1 + e^{rT} \right) p(n - (a + Mn - bp)(l + r)) \\
+ e^{-rT} \frac{\partial}{\partial M} \left( -1 + e^{rT} \right) (n - (a + Mn - bp)(l + r))(1 + e^{rT}(-1 + rT)) \\
- \frac{c(-1 + (e^T)_n) + h(n(r - e^{rT} + \theta - e^{-rT}\theta))}{rT} \\
+ \frac{e^{-rT} I_{Loss}(\frac{e^{-Mr}n - e^{rT}M^2n - e^{-rT}r(-n + (a + Mn - bp)r)r)}{r^2T}}
\]

(61)

For simplicity, let \(\frac{\partial PTP_{1.2}(p, M, T)}{\partial M} = F_{1.2}(M)\). For

\[
F_{1.2}(0) = e^{-rT} \frac{\partial}{\partial M} \left( -1 + e^{rT} \right) p(n - (a - bp)(l + r)) \\
+ e^{-rT} I_c(p(n - (a - bp)(l + r))(1 + e^{rT}(-1 + rT)) \\
- \frac{c(-1 + (e^T)_n) + h(n(r - e^{rT} + \theta - e^{-rT}\theta))}{rT} \\
+ \frac{e^{-rT} I_{Loss}(e^{-rT}n - r^2T + n(1 + e^{rT}(-1 + rT)))}{r^2T}
\]

(62)
\[ F_{1.2}(\infty) = \lim_{M \to \infty} \frac{e^{-1M-r(M+T)}(-1 + e^{rT})p(n - (a + Mn - bp)(l + r))}{rT} \]
\[ + \frac{e^{-1M-r(M+T)}Iep(n - (a + Mn - bp)(l + r))(1 + e^{rT}(-1 + rT))}{r^2T} \]
\[ - \frac{c(-1 + (e^\theta)T)n}{T\theta} + \frac{hn(r - e^{T\theta} + \theta(1 - e^{-rT}))}{rT\theta(r + \theta)} \]
\[ + \frac{e^{-r(M+T)}I_{Loss}p(e^{Mr}n - r(M+T)n - e^{rT}r(-n + (a + Mn - bp)r)T)}{r^2T} = -\infty, \] (63)

\[ \frac{\partial^2 PTP_{1.2}(p, M, T)}{\partial M^2} = -\frac{1}{r^2T} e^{-1M-r(M+T)}p(l+r)(2n - (a + Mn - bp)(l + r)) \]
\[ ((-1 + e^{rT})r + Ic(1 + e^{rT}(-1 + rT))) - e^{-Mr}I_{Loss}p(2n - (a + Mn - bp)r) < 0. \] (64)

If \((2n - (a + Mn - bp)(l + r)) > 0\) and \((2n - (a + Mn - bp)r > 0)\), then \(\frac{\partial^2 PTP_{1.2}(p, M, T)}{\partial M^2} < 0\) and the total profit \(PTP_{1.2}(p, M, T)\) is a strictly concave function on \(M\). However, two conditions will occur. First, if \(\frac{\partial^2 PTP_{1.2}(p, M, T)}{\partial M^2} < 0\) then \(F_{1.2}(M) < 0\) (i.e., \(F_{1.2}(0) < 0\)) for all \(M > 0\). Hence, the total profit \(PTP_{1.2}(p, M, T)\) is a monotonic decreasing function in \(M\) and maximized at \(M_{1.2} \approx 0\). Otherwise, \(\frac{\partial^2 PTP_{1.2}(p, M, T)}{\partial M^2} > 0\) then \(F_{1.2}(0) > 0\). By using the Mean Value Theorem into \(F_{1.2}(0) > 0\) and \(F_{1.2}(\infty) < 0\), there will exist a unique value of \(M_{1.2} > 0\) that satisfy \(F_{1.2}(M) = 0\) and maximized at \(M_{1.2} > 0\). Thus, proof of Theorem 4 is completed.

**Case 2 Partial Trade Credit**

**Sub-case 2.1** \(0 \leq M \leq T \leq T + M\)

\[ \frac{\partial PTP_{2.1}(p, M, T)}{\partial M} = e^{-1M-r(M+T)}(-1 + e^{rT})p(e^{Mr}(-n + D(p, M)))(-1 + \alpha + ...) \]
\[ + \frac{1}{r^2T} e^{-1M}Ip(...) - \frac{c(-1 + (e^\theta)T)n}{T\theta} + \frac{hn(r - e^{T\theta}r + \theta - e^{-rT}\theta)}{rT\theta(r + \theta)} \]
\[ + \frac{e^{-Mr}I_{Loss}p(M(-a + bp)r^2 - n(1 + e^{Mr} + Mr(-1 + Mr))\alpha]}{r^2T}. \] (65)

For simplicity, let \(\frac{\partial PTP_{2.1}(p, M, T)}{\partial M} = F_{2.1}(M)\). For,

\[ F_{2.1}(0) = \frac{e^{-rT}(-1 + e^{rT})p(n - a(l + ra) + bp(l + ra)) + e^{rT}Ip(...)r/r^2T}{rT} \]
\[ - \frac{c(-1 + (e^\theta)T)n}{T\theta} + \frac{hn(r - e^{T\theta}r + \theta - e^{-rT}\theta)}{rT\theta(r + \theta)}, \] (66)
\[ F_{2.1}(\infty) = \lim_{M \to \infty} \frac{1}{rT} e^{-lM-r(M+T)}(-1 + e^{rT}) p(e^{Mr}(-n + D(p, M)l)(-1 + \alpha) + (n - D(p, M)(l + r)\alpha) + \frac{1}{rT} e^{-lM} I_e p(-e^{-r(M+T)}D(p, M)r(1+rT)(-1 + rT))\alpha + n(1 - \alpha + e^{-r(M+T)}(\gamma^M(T)(1 + rT)(-1 + \alpha) + e^{rT}(-1 + rT))\alpha) + e^{rT}(-1 + rT)\alpha)\) + \frac{e^{-Mr} I_{Loss}p((M-a + bp)r^2 - n(-1+M_r+Mr(-1+Mr))\alpha)}{r^2T} \left[ -c(-1 + e^{\theta T})n + \frac{hn(r - e^{T\theta} + \theta(1 - e^{-rT}))}{rT\theta(r + \theta)} \right] = -\infty. \]

(67)

\[ \frac{\partial^2 PTP_{2.1}(p, M, T)}{\partial M^2} = \frac{1}{rT} e^{-lM-r(M+T)} p(-e^{r(M+T)}l(-2n + l(a + Mn - bp)) (I_e + r)(-1 + \alpha) + ae^{Mr} I_e r(-1 + \alpha) + a(I_e - r)(l + r)^2 \alpha + e^{-lM-r(M+T)} p(e^{Mr}l(-2n + l(-bp + Mn))(I_e + r + I_e rT)(-1 + \alpha)) \right] + \frac{e^{-lM-r(M+T)} p((I_e - r)(l + r)(-2n + (a + Mn - bp))(l + r)))(r + I_e(-1 + r T))\alpha) / r^2T}{r^2T} \]

If \((-2n + l(-bp + Mn)) > 0, (-2n + (-bp + Mn)(l + r)) > 0, (-2n + l(a + Mn - bp)) < 0, (-2n + (a + Mn - bp)(l + r)) < 0 and (Ie - r) \leq 0, then \( \frac{\partial^2 PTP_{2.1}(p, M, T)}{\partial M^2} < 0 \) and the total profit \( PTP_{2.1}(p, M, T) \) is a strictly concave function on \( M \). However, two conditions will occur. First, if \( \frac{dPTP_{2.1}(p, M, T)}{dM} < 0 \) then \( F_{2.1}(M) < 0 \) (i.e., \( F_{2.1}(0) < 0 \)) for all \( M > 0 \). Hence, the total profit \( PTP_{2.1}(p, M, T) \) is a monotonic decreasing function in \( M \) and maximized at \( M_{2.1} \approx 0 \). Otherwise, \( \frac{dPTP_{2.1}(p, M, T)}{dM} > 0 \) then \( F_{2.1}(0) > 0 \). By using the Mean Value Theorem into \( F_{2.1}(0) > 0 \) and \( F_{2.1}(\infty) < 0 \), there will exist a unique value of \( M_{2.1} > 0 \) that satisfy \( F_{2.1}(M) = 0 \) and maximized at \( M_{2.1} > 0 \). Thus, proof of Theorem 5 is completed.

Sub-case 2.2 \( 0 \leq T \leq M \leq T + M \)

\[ \frac{\partial PTP_{2.2}(p, M, T)}{\partial M} = \frac{e^{-lM-r(M+T)}(-1 + e^{rT}) p(e^{Mr}(-n + D(p, M)l)(-1 + \alpha) + ...}{rT} + \frac{1}{r^2T} e^{-lM} I_e p(-c(-1 + e^{\theta T})n + \frac{hn(r - e^{T\theta} + \theta - e^{-rT})}{rT\theta(r + \theta)} \right]}{r^2T} + \frac{e^{-r(M+T)} I_{Loss}p((\gamma^M(T)n - e^{r(M+T)}n - e^{rT}(-n + (a + Mn - bp)r)T)\alpha}{r^2T}, \]

(69)
Theorem 6 is completed.

\[ \text{For simplicity, let } \frac{\partial P_T^P}{\partial M} = F_{2.2}(M). \text{ For,} \]
\[ F_{2.2}(0) = e^{-rT}(-1 + e^{rT})p(n - a(l + r\alpha) + bp(l + r\alpha)) \]
\[ + e^{-rT}I_c p(-a - bp)(1 + e^{rT}(-1 + rT)(1 + ...)/r^2T \]
\[ - c(1 + (e^\theta) )n + hn(r - e^{rT}\theta + \theta - e^{-rT}\theta) \]
\[ + \frac{e^{-rT}I_{Loss}p(-e^{rT}(a - bp)r^2T + n(1 + e^{rT}(-1 + rT)))\alpha}{r^2T} \]
\[ F_{2.2}(\infty) = \lim_{M \to \infty} \frac{1}{rT} e^{-1M - r(M+T)}(-1 + e^{rT})p(e^{Mr}(-n + D(p, M)) T(1 + \alpha) + ... \]
\[ + \frac{e^{-1M+T}I_c p(-e^{-(M+T)} D(p, M)) r(1 + e^{-rT})(-1 + rT)\alpha + ... }{r^2T} \]
\[ + \frac{e^{-1M+T}I_{Loss}p(M + a + bp)r^2 - n(1 + e^{Mr} + Mr(-1 + Mr))\alpha}{r^2T} \]
\[ - c(1 + (e^\theta) )n + hn(r - e^{rT}\theta + \theta(1 - e^{-rT})) \]
\[ + \frac{e^{-r(M+T)}I_{Loss}p(e^{Mr} - e^{-(M+T)})(n - e^{rT}r(-n + (a + M_n - bp)r)T)\alpha}{r^2T} = -\infty \]
\[ \frac{\partial^2 P_T^P}{\partial M^2} = \frac{1}{r^2T} e^{-1M - r(M+T)} p(-e^{-(M+T)} l(l(a + M_n - bp))...) \]
\[ + e^{-1M - r(M+T)} p(e^{Mr} l(-2n + l(-bp + nM))(I_c + r + I_c rT)(-1 + \alpha) \]
\[ + \frac{e^{-1M - r(M+T)} p((I_c - r)(l + r)(-2n + (-bp + nM)(l + r)))\alpha}{r^2T} \]
\[ + \frac{e^{-1M - r(M+T)} p(e^{rT}(l + r)(-2n + (a + M_n - bp)(l + r))(r + I_c(-1 + rT))\alpha}{r^2T} \]
\[ + e^{-Mr}I_{Loss}p(-2n + (a + M_n - bp)r)\alpha < 0 \]
\[ \text{(72)} \]

If \((-2n + l(-bp + nM)) > 0, (-2n + (-bp + nM)(l + r)) > 0, (-2n + l(a + M_n - bp)) < 0, (-2n + (a + M_n - bp)(l + r)) < 0, (-2n + (a + M_n - bp)r) < 0 \)
and \((le - r) \leq 0, \text{ then } \frac{\partial^2 P_T^P}{\partial M^2} < 0 \text{ and the total profit } P_T^P \text{ is a strictly concave function on } M. \text{ However, two conditions will occur. First, if } \frac{\partial^2 P_T^P}{\partial M^2} < 0 \text{ then } F_{2.2}(M) < 0 \text{ (i.e., } F_{2.2}(0) < 0 \text{) for all } M > 0. \text{ Hence, the total profit } P_T^P \text{ is a monotonic decreasing function in } M \text{ and maximized at } M_{2.2} \approx 0. \text{ Otherwise, } \frac{\partial^2 P_T^P}{\partial M^2} > 0 \text{ then } F_{2.2}(0) > 0. \text{ By using the Mean Value Theorem into } F_{2.2}(0) > 0 \text{ and } F_{2.2}(\infty) < 0, \text{ there will exist a unique value of } M_{2.2} > 0 \text{ that satisfy } F_{2.2}(M) = 0 \text{ and maximized at } M_{2.2} > 0. \text{ Thus, proof of Theorem 6 is completed.} \]

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