Maximum power for a power plant with $n$ Carnot-like cycles

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Abstract. A stationary power plant with $n$ Carnot-like cycles is optimized. Each cycle has the following irreversibilities: finite rate heat transfers between the working fluid and the external heat sources, internal dissipation of the working fluid, and heat leak between reservoirs. In a previous work, a plant with two cycles of this type was optimized, with respect to the isentropic temperature ratio, applying the optimal allocation or effectiveness of the heat exchangers for the power plant by two design rules, alternatively: fixed internal thermal conductance or fixed areas. Also, in an above work the optimal allocation or effectiveness of the heat exchangers were extended to a power plant with $n$ Carnot-like cycles. In this work, these optimal relations obtained are substituted in the power and the maximum power is calculated, with respect to the isentropic temperature ratios corresponding to each one of the Carnot-like cycles of the power plant with $n$ Carnot-like cycles. Also, the efficiency to maximum power for both design rules is presented.

1. Introduction

Recently in [1], we have obtained the universal optimal relations of the allocation or effectiveness of the heat exchangers for a power plant with $n$ Carnot-like cycles. These optimal relations for this power plant proved to be invariant to the power and efficiency and to the heat transfer law. Each one of these cycles, that correspond to a standard irreversible Carnot-like cycle, have been studied at length for many objective functions, different transfer heat laws and several characteristic parameters (see [2]-[9] for further details). Also, recently in [8] we have found these optimal relations for the same constraints but for a power plant with $n$ Carnot-like cycles. In this way, we have calculated the maximum power and the efficiency of maximum power for this stationary power plant with two Carnot-like cycles [9]. On the other hand, [10-13] have presented the optimization of a $n$-stage combined Carnot-like cycle. In [12] the specific power and efficiency was obtained for this cycle using only the law of heat conduction and considering uniquely the effectiveness.

In this work, we extend our works [8-9] to a power plant with $n$ Carnot-like cycles and calculate the maximum power and the efficiency to maximum power using the Bellman’s Principle [11] and Mathematical Induction [14]. Also, the work [12] is extending to two of the design rules fundamentals. As instance, the optimization is carried using the law of heat conduction. A section of Conclusions is presented.
2. Power plant with \( n \) Carnot-like cycles

The stationary power plant with \( n \) Carnot-like cycles is shown in Figure 1. Each cycle satisfies the conditions expressed in [12]: leak heat \( Q \) and finite heat transfer rates \( \dot{Q}_i \) \((i = 1, \ldots, n)\), and internal dissipations of the working fluid expressed by constants \( I_i \) \((i = 1, \ldots, n)\), such that

\[
I_i = \frac{\Delta S_i}{\Delta v_i} \geq 1; \quad i = 1, \ldots, n
\]

in which the Claussius inequality becomes equality. Each cycle of the power plant consists of two isothermal and two irreversible adiabatic processes, denoting, for each cycle the temperatures of the working fluid during the hot and cold isothermal processes are \( T_{2i} < T_{2i-1} \) \((i = 1, \ldots, n)\); for the temperature of the hot reservoir and temperature of the cold reservoir, respectively, and the end temperatures \( T_L < T_H \) (Figure 1).

Following [1, 5 and 8], the thermal efficiency of the power plant is given by:

\[
\eta = \frac{P}{1 - Ix + Q}
\]

where \( I = \prod_{i=1}^n I_i \) and \( x = \prod_{i=1}^n x_i; \quad x_i = \frac{T_{2i}}{T_{2i-1}} \) correspond to the isentropic temperature ratio for each cycle \( i \) \((i = 1, \ldots, n)\), \( P \) is the power of the plant, \( Q \) is the leak heat.

Equation (1) was obtained in [1] and generalizes to equation (6) of [2]. In [1], we have found invariant optimal relations, by following the spirit of Carnot’s work, for other characteristic parameters, which can be independent from the heat transfer law, of the irreversible power plant. This is because of: when the parameter \( z \neq x_i \) \((i = 1, 2, \ldots, n)\), then, the following is fulfilled:

“The power \( P \) achieves a maximum value in \( z_{wp} \) if and only if the efficiency \( \eta \) achieves a maximum value in the value: \( z_{wp} = z_{me} \)”

Thus, in optimizing of the power plant with \( n \) Carnot-like cycles, with respect to \( z \), it is enough to find the maximum power by,

\[
\frac{\partial P}{\partial z} \bigg|_{z_{wp}=z_{me}} = 0 \quad \text{and} \quad \frac{\partial^2 P}{\partial z^2} \bigg|_{z_{wp}=z_{me}} < 0
\]

and the optimization performed, with respect to \( z_i \) is a property independent from the heat transfer law. Furthermore, \( z = \phi \) or \( \psi \) depending on which design rule is applied. In the following section we present briefly the optimal relations obtained in [1].

3. Optimal relations for the allocation and effectiveness of the power plant’s heat exchangers

In this section, \( x_i \) \((i = 1, 2, \ldots, n)\) will be fixed and we will assume that the law of heat transfer can be any law, including heat leak. Next, we will discuss the following two design rules: fixed internal thermal conductance or fixed areas for the heat exchangers, which will be alternately applied.
Figure 1: A power plant stationary with \( n \) Carnot-like cycles with linear leak heat and finite heat transfer rates, and internal dissipations of the working fluid.

The first design rules are:

- The internal conductance of the Carnot-like cycle is constrained to: \( \sum_{i=1}^{n} \alpha_i = \Gamma \), where \( \Gamma \) is a constant applied to the allocation of the heat exchangers from the hot and cold sides.
- The same overall heat transfer coefficient \( \bar{U} \) by unit of area \( A \) in both ends of the cycle \( i (i = 1, 2, ..., n) \)
- \( \alpha_i, \alpha_{i+1} (i = 1, 2, ..., n-1) \), the thermal conductance correspondent to the finite heat transfers of the hot/cold sides for this cycle respectively. Thus, \( \sum_{i=1}^{n} U A_i = \Gamma \) where \( A_i, A_{i+1} \) are heat transfer areas on hot/cold sides of the cycle \( i (i = 1, 2, ..., n-1) \).

From [1], the optimal relation for the first design rule is given by:

\[
\alpha_n = \sqrt[n]{\bar{U}} \alpha_1, \quad n = 2, 3, .... \tag{3}
\]
and \( a_n = \sum_{j=0}^{n-1} \sqrt{I_j}; I_0 = 1 \).

And, also of [1], the second design rule is that the total area is constrained by: \( \sum_{i=1}^{n} A_i = A \); where \( A_i, A_{i+1} \) are the heat transfer areas on the hot and cold sides for the cycle \( i \), respectively. Now, the total area \( A \) is fixed but, when distributed, it has different overall heat transfer coefficients and hence different effectiveness on each one of the hot and cold sides. How \( \alpha_i = U_i/A_i \) (see [1, 10]), then \( A = \sum_{i=1}^{n} A_i = \sum_{i=1}^{n} \frac{\alpha_i}{U_i} \), where \( U_i, U_{i+1} \) are the overall heat transfer coefficients on the hot and cold sides of the cycle \( i \) \((i = 1, ..., n)\), respectively.

From [1], the optimal relation for the second design rule is given by:

\[
A_n = \frac{1 + \sqrt{u_{n-1}}(b_{n-1} - 1)}{1 + b_{n-1} \sqrt{u_{n-1}}} A_1 \tag{4}
\]

With:

\[
u_{n-1} = \frac{U_i}{U_n}; b_{n-1} = b_2 + b_{n-2}, n > 2; b_2 = 1 + \frac{1}{\sqrt{b_1 U_i}}; b_1 = 1
\]

Next, applying the Bellman’ Principle [11]: “to state that every part of an optimum path is optimal”; we can determine the maximum power of the power plant.

4. Maximum power and efficiency to maximum power for each one of the design rules

If \( P_j \) is the power of the cycle \( j \) \((j = 1, ..., n)\); see Figure 1. Then, for the first design rule, the maximum power of the power plant will be given by:

\[
\left( \sum_{j=1}^{n} P_j \right)_{\text{max}} = \frac{\sqrt{I_{n-1}}UAT_{H} \left( 1 - \sqrt{\mu} \right)^2}{a_{n+1} \left( I_{n-1} + I^* a_n \right)} \tag{5}
\]

Where:

\[
a_{n+1} = \sum_{j=0}^{n} \sqrt{I_j}; I_0 = 1; I^* = \prod_{j=1}^{n-1} I_j; \mu = \frac{T_L}{T_H}
\]

For the second design rule, the maximum power will be given by:

\[
\left( \sum_{j=1}^{n} P_j \right)_{\text{max}} = \frac{b_{n+1} b_n (b_n - 1) A U T_{H} \left( 1 - \sqrt{\mu} \right)^2}{(I_{n-1} b_n (b_n - 1) + b_{n+1})(b_n (b_n - 1) + b_{n+1})} \tag{7}
\]

Where

\[
b_{n+1} = 1 + \frac{\sqrt{I_{n+1} + u_n (b_n - 1)}}{\sqrt{I_{n+1} + u_n \sqrt{u_n}}}; u_{n-1} = \frac{U_n}{U_1}\]  

With: \( I = \prod_{j=1}^{n} I_j; \mu = \frac{T_L}{T_H} \)

(8)

The case \( n = 2 \) was presented in [8] for both cases. For the first design rule this case is:
(P_1 + P_2)_{\text{max}} = \sqrt{I_{1z}UA \left( 1 - \sqrt{I_{1z} I_{2}} \right)^2 a_3 \left( I_{1z}a_2 + \sqrt{I_{2}} \right)}

With UA = \alpha_1 + \alpha_2 + \alpha_3 = \alpha_1 \left( 1 + \sqrt{I_{1z} + \sqrt{I_{2}}} \right); \text{ so, } \alpha_i = \frac{UA}{a_3}

Now, applying Mathematical Induction [14], we can assume that the equation (5) is valid for n=k-1 and we show that also is valid for n=k (cfr. with the proof of the equation (8) de [8]). Indeed,

\[
P_1 + \sum_{j=2}^{k} P_j \frac{1}{\alpha_i T_H} = \left( 1 - \sqrt{I_{1z} I_{11}} \right)^2 + \frac{\sqrt{I_{k-1} I_{12}}}{I_{1z}a_{k-1}} \frac{T_{k2}}{I_{2}} \left( 1 - \sqrt{\prod_{j=2}^{k-1} I_{j} I_{1}} \right)^2 \]

(9)

using the equation (3), we have:

\[
P_1 + \sum_{j=2}^{k} P_j \frac{1}{\alpha_i T_H} = \left( 1 - \sqrt{I_{1z} I_{11}} \right)^2 + \frac{\alpha_k}{I_{1z}a_{k-1}} \left( \frac{\sqrt{I_{1z} I_{11}}}{I_{1z}a_{k-1} \alpha_1} - \sqrt{\prod_{j=1}^{k-1} I_{j} I_{1}} \right)^2 \]

(10)

In optimizing with respect to \sqrt{I_{1z} I_{11}} , the equation (5) is obtained for n=k. Therefore, the equation (5) is valid for any n. Moreover, \alpha_i = \frac{UA}{\sum_{j=0}^{n} \sqrt{I_{j}}}

Similarly, for the second design rule, the equation (7) for n = 2 is:

\[
(P_1 + P_2)_{\text{max}} = \frac{T_{H} b_{1} b_{2} (b_{2} - 1) AU_{1} \left( 1 - \sqrt{I_{1} I_{1}} \right)^2}{(I_{1} b_{1} b_{2} (b_{2} - 1) + b_{3} (b_{2} - 1) + b_{3})}.
\]

Now, applying Mathematical Induction and the procedure used for obtain the equation (10) from the equation (9). The equation (7) is valid for any n.

Finally, the efficiencies to maximum power are obtained of substitute the equations (5) or (7) in the equation (1):

\[
\eta_{mpC} = \frac{(1 - \sqrt{I_{1} I_{1}})^2}{(1 - \sqrt{I_{1}})^{a_{1} + \left( \frac{1}{\sqrt{I_{11}}} + I_{11} \right) - k(1 - \mu)}} \quad \text{or} \quad \eta_{mpA} = \frac{(1 - \sqrt{I_{1} I_{1}})^2}{(1 - \sqrt{I_{1}}) \left( \frac{b_{1} (b_{1} + b_{2} + b_{3})}{b_{1} + b_{2} + b_{3}} \right) \left( \frac{b_{1} (b_{1} + b_{2} + b_{3})}{b_{1} + b_{2} + b_{3}} \right) k(1 - \mu)}
\]

where \( K = \frac{K}{\alpha_i} \); K is the thermal conductance of the leak heat and the constants of the expressions of the efficiencies are given by the equations (6 and 8).

5. Conclusions
In this work, the optimal relations for the allocation and effectiveness of heat exchangers of a power plant with \( n \) Carnot-like cycles have been applied. This was carried out by the substitution of the equations (3 and 4) in the expression of power of the power plant with \( n \) cycles Carnot-like. And thus the maximum power, with respect to isentropic temperature ratio, and the efficiency to maximum power were obtained for each one of the rules of design. The optimization was performed using, as an example the conduction heat transfer law; however, the application to other laws of heat transfer is immediate. Equations (3 and 4) are enough substitution after optimizing the power with respect to the
isentropic temperature ratios that depend on the heat transfer law and the operation regime of the power plant with $n$ Carnot-like cycles, with the added imposition of the power conditions (equation (2)). Thus, we have proposed in this work a methodology of optimization taking as base the optimal relations (equations (3 and 4)) which are valid for any heat transfer law and are satisfied for other operation regimes, e.g. algebraic combination of power and/or efficiency that have thermodynamic meaning and comply equation (2). Finally, the maximum efficiency can be calculated by the substitution of the equations (3 and 4) and applying the criterion of [4]. Further work is underway.

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