3D heterogeneous bin packing framework for multi-constrained problems using hybrid genetic approach

S K Rajesh Kanna¹, *, K C Udaiyakumar², S Dinesh Kumar³, N Lingaraj⁴

Professor, Rajalakshmi Institute of Technology, Chennai, India
Vice Principal, SRM Institute of Science and Technology, Ramapuram campus, Chennai
Professor, Rajalakshmi Institute of Technology, Chennai, India
Professor, Rajalakshmi Institute of Technology, Chennai, India

*Corresponding Author: skrkanna@gmail.com

Abstract. This work presents distinct methodologies in using Genetic Algorithm (GA) for optimizing Three Dimensional (3D) packing of heterogeneous shaped bins with arbitrary sizes into a prismatic container, by considering the major real time packing constraints such as load bearing constraint, placement constraint, stability constraint, overlapping constraint, orientation constraint and weight constraint. The primary aim of this research is focused in optimizing the packing of heterogeneous prismatic bins of arbitrary sizes into standard rectangular commercial containers by obeying the above mentioned packaging constraints. Different genetic approaches adopted to achieve these goals are Binary coded GA, Decimal coded GA with and without penalty fitness function, Constrained GA with maximization and minimization fitness function, Heuristic GA and Hybrid GA. GA has been used to minimize the unused void space in the interior of the container by loading as much heterogeneous bins, by satisfying the packing constraints. Tweaking Algorithm (TA) is an application dependent heuristic algorithm applied in this research and has been used to enhance the genetic output by filling the remaining unused empty space inside the container. TA has also been enhanced in converting the obtained output into packer readable box packing sequence in tabular and diagrammatical format. In general, combination of GA and TA are considerably at par compared with the heuristic techniques for box packing.

1. Introduction

Evolutionary algorithms are based on the principle of biological evolution (Hopper and Turton, 1997). Genetic algorithm is a class of evolutionary algorithm used to optimize a wide range of complex constrained problems in less time. One of the complex problem faced by the industries and logistic firms are to pack and distribute their goods in less time and with less dissemination cost to their customers all around the world. In general, goods are packed into bins and the bins will be loaded into containers for safe, secure and compact distribution. Freight rate of using the containers are also included with the product cost, but not adding any additional credits to those products. Thus it became imperative for the firms to reduce this non-value added cost to the minimum by perfectly utilizing the available container space. This has been achieved in this research by applying genetic approach. Aristide et al (2018) developed algorithm with the focus on overlapping of bins in packing process and
the same considered in this research while calculating the occupied container volume. Ranga et al (2018) considered the rotation of the bins inside the container for optimal packing and in this research, instead of free rotation, orthogonal rotation have been considered and achieved by overloading the mutation operation. Fabrizio and Andrea (2018) carried out packing of two dimensional problems with the heuristic rules, but this research concentrated on 3D packing and more or less the generalized algorithm. Martinez et al (2017) developed bin packing algorithm with free rotation of bins with irregular bins, in this research, arbitrary sizes of bins with four different shapes considered. Xueping and Kaike (2018) utilized the shop floor algorithm for packing of 2d bins, but this research uses 3D packing. Célia et al (2018) attempted to identify the solution for the packing problems by considering the transportation constraints and the same have been considered in this research with penalty function. Gorge and Robinson (1980) introduced the approach for packaging bins into a standard container using layer-by-layer packing approach. Hopper and Turton (1997) applied the concept of GA to the box packaging problem and proved that the performance of GA was satisfactory. Bortfeldt and Gehring (2001) developed by hybridizing GA based on the layer-layer packing concept for container loading problems with packaging constraint and proved that the feasible solution can be obtained by satisfying the packaging constraints. Lodi et al (2002) used Tabu search algorithm with a heuristic to reinforce carton orientation in 3D multiple-container loading problem and showed that the box orientation also plays a major role in optimal packaging. Martine Labbe et al (2003) framed upper bound and enumeration algorithm for packaging maximum number of bins into the container in less computational time. Nihat Kasap and Anurag Agarwal (2004) implemented Augmented artificial neural network algorithm for solving classical bin packaging datasets which combines priority logic approach with iterative trial learning approach. Zhoujing Wang and Kevin Li (2006) developed a tertiary tree algorithm for packaging heterogeneous bins into containers. Wenqi and Kun (2009) incorporated a new caving algorithm in solving 3D single container loading problem. Leonardo et al (2010) developed a mathematical model for the problem of loading rectangular bins into containers by considering cargo stability and load bearing constraint. Rajesh et al (2017) experimented the bin packing optimization using Firefly algorithm. Allen et al (2011) introduced placement of bin strategy in 3D strip type packaging problems, which considers packing of standard available set of cuboids into a container of fixed width, height and unconstrained length. Rajesh et al (2012) developed the genetic algorithm approach to pack the two dimensional rectangular bin packing inside the larger rectangle. Rajesh and Saravanan (2012) incorporated genetic approach for the dynamic bin packing problems. Even-though numerous heuristic and hybrid approaches are existing for 3D bin packing, most of these have simplified the 3D problem into 2D or even to 1D for solving. Also some of the approaches did not consider the major box packing constraints. As the result of the literature survey, it is clear that there is a necessity to optimize 3D box packing problem with ‘n’ number of arbitrary sized heterogeneous bins by satisfying packing constraints.

In this research work, rectangular prismatic container of regular dimension have been considered for experimenting the packing of ‘n’ number of differently shaped prismatic bins namely cubical bins, rectangular prismatic bins, cylindrical bins and spherical bins of varying sizes by satisfying the related packing constraint. The packing constraints considered are load bearing constraint, placement constraint, stability constraint, overlapping constraint, orientation constraint and weight constraint. The user defined box data have been encoded and given as input to GA for optimizing. The developed GA module identifies the best optimal and feasible box packing pattern which yields maximum container volume utilization without violating the constraints.

Many of the researchers didn’t consider the major packing constraints altogether and it has been considered in this research to yield feasible optimal solution which can be directly used by the layman for packing. Also sensitivity analysis carried out to overload the mutation genetic operator, where as the other researchers didn’t try with mutation overloading. In addition, special tuning heristic developed to enhance the genetic results in packing. This paper has been organized as follows.
Background research works carried out by the researchers have been discussed in literature section 2. Section 3 describes the problem formulation followed by the various experimental implementations and the obtained results. The paper concludes with conclusion and future scope.

2. Bin Packing Problem

Bin packing problem is defined as the problem of identifying the best mix of bins and packing pattern which utilizes the maximum container volume by meeting the packing constraints. Inputs are the container specification, number of bins and box specifications along with its related constraints. The user defined data should be checked for its completeness and sorted in descending order based on the weightage values. Improper box packing and arbitrary heterogeneous bins lead to some empty unused space formation in the interior of the container. This void formed inside the container is the primary problem, which results in box instability, utilizing of additional containers, usage of additional airbags/dampers/fillers and inturn, increases the cargo/transportation cost and overall retail cost without adding any additional value to packed components. Hence in this research area, various frameworks had developed for solving the problem of box packing and the developed modules were validated with the second order mathematical equations.

3. GENETIC ALGORITHM – BINARY ENCODING

In the Binary encoded genetic approach (BEGA), the user defined box data have been encoded into binary elements. The encoding format for a box should contain the box dimension, position inside the container and its constraints. A sample of two parents with four bins each is given in the figure 1.

Figure 1. BEGA Parents

From the figure 1, the first nine binary digits represent the 3D box dimension followed by the box position inside the container in next three digits. The box position can be left, right, above, in-front and behind the recently packed bin. The orientation of the box occupies next two digits to denote vertical or horizontal positions. The last two digits are the auxiliary elements to identify the bin. The other packing constraints have been checked in the final stage for making the solution feasible. These binary elements will be feed as input to the developed hybrid genetic module, which in turn identify the optimal sequence of packing pattern inside the container. The various stages of the GA module are the initial parent formulation, parent crossover, mutation of child and fitness function calculation. The fitness function used is to maximize the container volume utilization. The GA output will have the set of bins to be packed along with its position and orientation. These outputs have been checked for packing constraints and the pattern which satisfies the packing constraints is considered to be a feasible solution. Even though the module gives better result, the computational time and memory usage will be increased with the increase in the problem size. So the developed module had enhanced to decimal encoding approach.

4. DECIMAL CODED GENETIC ALGORITHM

In the Decimal coded genetic algorithm (DCGA) module, the computational time and memory usage had reduced using decimal encoding instead of binary encoding. A set of sample decimal coded parents are given in figure 2.
The decimal numbers 1, 2, 3 and 4 represent cube shaped bins, rectangular prismatic shaped bins, cylindrical shaped bins and spherical shaped bins respectively. In figure 2, starting digits ‘2’, ‘4’, ‘2’, ‘1’, ‘3’ … represent first box from rectangular prism database, first box from sphere database, second box from rectangular prism database, first box from cuboid database, first box from cylinder database respectively and so on. These parents have been allowed to operate by the genetic operators to identify the best packing pattern. The constraints need to be checked for identifying the feasible box packing pattern from the set of DCGA outputs. In some cases, the best GA solution was also eliminated by the constraint checking and thereby the computational time was wasted for identifying the optimal solution from the infeasible set of patterns. This has been avoided by enhancing the DCGA module with decimal coded GA with penalty fitness function (DCGAP).

5. DECIMAL CODED GA WITH PENALTY FITNESS FUNCTION

In this DCGAP module, the constraints were checked at the stage of calculating the fitness function value. The pattern which violates the constraints will have lesser fitness value and will be eliminated from the generation. The penalty fitness function is given as follows.

Max. \( f_{obj}(x) = \text{Vol}(B) - \text{Pen}(B) \) (1)

\[
\text{Vol}(B) = \sum_{i=1}^{n} (\text{Len}_i \times \text{Ber}_i \times \text{Hig}_i) \times (\text{Vol}(\text{Ctr})) \tag{2}
\]

\[
\text{Pen}(B) = \text{Plc}(X) + \text{Ovr}(X) + \text{Stb}(X) + \text{Weg}(X) + \text{Ori}(X) + \text{Lber}(X) \tag{3}
\]

whereas, \( f_{obj}(x) \) is the objective function for maximizing fitness function which calculates total volume occupied by the packed bins inside the container, Plc(X) is the Placement (Boundary crossing) constraint with the penalty value of 0.25. Ovr(X), Stb(X), Weg(X), Ori(X) and Lber(X) represent the Overlapping of bins within it and the container, Stability aspect constraint, Weight withstanding capability constraint, Orientation of the bin constraint and maximum Load bearing by the container respectively with the penalty value of 0.1. Even though the DCGAP generates the optimal and feasible packing pattern in less computational time and memory usage, the performance may not satisfied for the larger problem size. This can be solved in this research by modifying the genetic operators for better performance.

6. HEURISTIC GENETIC ALGORITHM

In Heuristic Genetic Algorithm module (HGA) the crossover and mutation operators had enhanced to two-point-double-crossover (2PDC) and overloaded mutation operator (OM) respectively. The implemented 2PDC is explained from the figure 3.
In Figure 3, “Parent-x” are the randomly generated sample parents from a group of randomly formulated population. For crossover, in this research, crossover sites have also been generated at random and the sample sites are 07, 22, 51 and 68. So the strings between 58th position to 102nd position were exchanged between the first and second parents in order to generate temporary offspring denoted by ‘temp offspring’, which in turn again exchanges the strings from 30th to 75th position with the third parent, to give birth to a new child denoted by ‘offspring I’, which is inheriting the properties from three parents. Thus the solution space increases by generating new combination of parameters. In order to avoid stagnation at local optimal points, the generated offspring’s have been permitted to operate over mutation genetic operator.

In this research work, as a application specific heuristic, two-point genetic mutation is overloaded with orientation satisfaction constraint. By overloading, instead of changing the box type, box orientation has been changed by interchanging the dimensional parameters such as length, breadth and height of a box at random. Figure 4 explains the mutation operation. Randomly generated mutation sites are 9 and 65. Decoding the 9th and 65th position gives 4th box in the rectangular prism database and 12th box in the cylinder database. By applying mutation operation, mutated offspring structure remains same, but the dimension value of those bins gets interchanged by considering the user defined orientation constraint and is shown in the figure 4. Thus the stagnation has been avoided and various orientations also checked by overloading operation. Based on many iterative results, it was found that the strings in the starting position of the parents in the population remains same, even after many generations. This can be avoided by using swapping operation.

Before Mutation Operation
Offspring I: 42232444242222422221323342222412341221342321422332344113121412312122444144411232314342113421223234221232232144443244112232313422123
Mutation Site: 9, 65
Bin at 9th Position: Code : 2 & 4th Bin in Rectangular Prism Database. Dimension: Length = 4; Width = 8; and Height = 15.
Bin at 65th Position: Code : 3 & 12th Bin in the Cylinder Database. Dimension: Length = 15; Width = 8 (Dia.) and Height = 8.

After Mutation Operation
Mutated New Offspring I: 4223244424222242222132334222241234122134232142233234411312141231212244414441123231434211342122323422123223214444324411122323134224123
Bin at 9th Position: Code : 2 & 4th Bin in Rectangular Prism Database. Dim.: Length = 15; Width = 8; and Height = 4.
Bin at 65th Position: Code : 3 & 12th Bin in the Cylinder Database. Dim.: Length = 8 (Dia.); Width = 8; Height = 15.

Figure 4. Mutation Overloading

In this research, single point random swapping was implemented and is explained in the figure 5. Randomly generated swapping site is 82 and then the strings beyond 82nd position were swapped to the front to form ‘swapped offspring’. Thus the same set of bins will be retained with different packing sequence.

New Offspring I: 4223244424222242222132334222241234122134232142233234411312141231212244414441123231434211342122323422123223214444324411122323134224123
Swapping Site: 12
Swapped offspring: 2242222132334222241234122134232142233234411312141231212244414441123231434211342232412223324411122323134224123422324442422

Figure 5. Swapping Operation
The HGA produces better results in par with the benchmark problems. But in real time situations, logistics firms are receiving higher heterogeneity bins and bins of various sizes. This will lead to formation of empty spaces near the boundaries and at the same time, the HGA output should be in packer readable format. So in this research, special heuristic algorithm namely Tweaking Algorithm (TA) had developed.

7. HYBRID GENETIC-TWEAKING ALGORITHM

Genetic algorithm in combination with Tweaking algorithm namely Hybrid Genetic Tweaking Algorithm (HGTA) have been used to pack the bins with or without empty space inside the container based on the concept of First-fit algorithm [4] and also to generate the output in packer readable tabular and diagrammatical format. A sample box packing output shown in Table 1 denotes the box type, its dimensions, placement position inside the container along with the volume data.

| S.No. | Chromosome | Box Shape  | Box Dimension | Placement Corner | Box Position | Volume (X 1000 units) | Container Volume | Empty Volume |
|------|------------|------------|---------------|------------------|--------------|------------------------|------------------|-------------|
| 1    | 1          | Cube       | 15 15 15      | CB1              | 0 0          | 15 15 1 1 1            | 3375 3375       | 559125      |
| 2    | 2          | Rect. Prism| 20 15 10      | RP1              | 15 0          | 35 15 1 1 2            | 3000 6375       | 556125      |
| 3    | 1          | Cube       | 15 15 15      | CB2              | 35 0          | 50 15 1 1 3            | 3375 9750       | 552750      |
| 4    | 2          | Rect. Prism| 20 15 10      | RP2              | 50 0          | 70 15 1 1 4            | 3000 12750      | 549750      |
| 5    | 1          | Cube       | 15 15 15      | CB3              | 70 0          | 85 15 1 1 5            | 3375 16125      | 546375      |
| 6    | 2          | Rect. Prism| 20 15 10      | RP3              | 0 15          | 20 30 1 2 1            | 3000 19125      | 543375      |
| 7    | 2          | Rect. Prism| 20 15 10      | RP4              | 20 15         | 40 30 1 2 2            | 3000 22125      | 540375      |
| 8    | 4          | Sphere     | 20 20 20      | SR1              | 40 15         | 60 35 1 2 3            | 4133 30125      | 536242      |
| 9    | 2          | Rect. Prism| 20 15 10      | RP5              | 60 15         | 80 30 1 2 4            | 3000 33125      | 533242      |
| 10   | 2          | Rect. Prism| 20 15 10      | RP6              | 80 15         | 100 30 1 2 5           | 3000 36125      | 530242      |

This tabular data have been converted to the diagrammatical pictorial format for easy understanding of the user and packers. The sample diagrammatical format and the top view/plan of the first bottom layer for the given dataset is shown in the Figure 6. Each box and its numerical value in the figure 6 represent a bin and its serial number respectively.
8. RESULTS

Table 2 compares the utilization percentage of the loaded container for Bischoff and Ratcliff (1995) test instances by different approaches. The data of HGTA are the best value obtained form continuous ten trials.

Table 2. Container Volume Utilization Comparison in Percentage.

| Instance | DCGAP | HGA   | HGTA | Improvement |
|----------|-------|-------|------|-------------|
| BR1      | 72.56 | 84.34 | 92.92| 20.36       |
| BR 2     | 71.23 | 85.61 | 93.92| 22.69       |
| BR 3     | 71.89 | 85.81 | 93.71| 21.82       |
| BR 4     | 70.26 | 87.07 | 93.65| 23.39       |
| BR 5     | 71.74 | 86.46 | 93.68| 21.94       |
| BR 6     | 70.56 | 88.21 | 93.25| 22.69       |
| BR 7     | 70.24 | 85.96 | 93.10| 22.86       |
| BR 8     | 68.23 | 85.96 | 92.98| 24.75       |
| BR 9     | 65.84 | 86.23 | 92.32| 26.48       |
| BR 10    | 64.25 | 85.72 | 92.42| 28.17       |
| BR 11    | 64.32 | 85.85 | 91.62| 27.30       |
| BR 12    | 64.10 | 85.18 | 91.32| 27.22       |
| BR 13    | 62.36 | 85.40 | 91.25| 28.89       |
| BR 14    | 61.38 | 84.87 | 91.32| 29.94       |
| BR 15    | 61.24 | 85.41 | 91.12| 29.88       |
| Average  | 67.35 | 85.87 | 92.57| 25.23       |

Table 2 interprets that, 25.23% of the left out unpacked boxes were re-packed by special heuristic TA into the partially loaded container compared to the GA with penalty function. After applying TA the effectiveness of the box packing increased from 67.35% to 92.57%. Thus it is clear that the effectiveness of the GA is increased by applying TA.

9. CONCLUSION

This research presents a better hybrid genetic approach for tackling 3D packing of different shaped bins with varying sizes by incorporating the real time packing constraints/limitations. The experimental results analyzed reveals the fact, that heuristic genetic enhancements like double point double crossover, orientation mutation and swapping operations increases the solution space. Feasible box packing pattern has been obtained by satisfying the packing constraints. The effectiveness of the genetic output is further enhanced by TA. Further improvements could also be incorporated by considering non-prismatic shaped bins.
REFERENCES

[1] Allen, S.D., Burke, E.K. and Kendall, G. 2011 A hybrid placement strategy for the three-dimensional strip packing problem, European Journal of Operational Research, 209, 219-227.

[2] Bischoff, E.E. and Ratcliff 1995 Issues in the development of approaches to container loading, OMEGA, 23, 377-390.

[3] Bortfeldt, A. and Gehring, H. 2001 A hybrid genetic algorithm for container loading problem, European Journal of Operational Research, 131, 143-161.

[4] George, J.A. and Robinson, D.F 1980 A heuristic for packing bins into a container, Journal of Computers and Operations Research, 7, 147-156.

[5] Rajesh Kanna S.K. and Udaiyakumar K.C. 2017 A complete framework for multi-constrained 3D bin packing optimization using firefly algorithm, International Journal of Pure and Applied Mathematics, 114, 267-282.

[6] Hopper, E. and Turton, B 1997 Application of genetic algorithm to packing problems – a review, Springer Verlag, London, 279-288.

[7] Leonardo Junqueira, Reinaldo Morabito, and Denise Sato Yamashita 2012 Three-dimensional container loading models with cargo stability and load bearing constraints, Journal of Computers & Operations Research, 39, 74-85.

[8] Lodi, Silvano Martello, and Daniele Vigo.2002 Recent advances on two-dimensional box packing problems, Journal of Discrete Applied Mathematics, 123, 379 – 396.

[9] Martine Labbe, Gilbert Laporte, and Silvano Martello.2003 Upper bounds and algorithms for the maximum cardinality box packing problem, European Journal of Operational Research, 149, 490–498.

[10] Nihat Kasap, and Anurag Agarwal 2004 Augmented-Neural-Networks approach for the bin-packing problem, Proceedings of 4th International Symposium on Intelligent Manufacturing Systems, Sakarya University, Department of Industrial Engineering, 348-358.

[11] Rajesh Kanna S K and Saravana Manigandan 2012 3D Arbitrary Sized Bin Packing Optimization Using 2PDC Heuristic: An Adaptive Genetic Approach. ARPN Journal of Science and Technology, 2, 26-31.

[12] Wenqi Huang, and Kun He 2009 A caving degree approach for the single container loading problem, European Journal of Operational Research, 196, 93–101.

[13] Zhoujing Wang and Kevin W. Li 2006 A heuristic algorithm for the container loading problem with heterogeneous boxes, IEEE International Conference on Systems, Man and Cybernetics.

[14] Bremermann, H. J., Rogson M., and Salaff S. 1966 Global properties of Evolution Processes in natural Automata and useful Simulation, Spartan Books.

[15] Rajesh Kanna S.K and Malliga. P. 2012 Multi-Constrained Optimization of Rectangular Bin Packing Problem using Binary coded Evolutionary Algorithm, International Journal of Materials Manufacturing and Optimization, 1, 27-35.

[16] Aristide Grange, Imed Kacem, Sébastien Martin 2018 Algorithms for the bin packing problem with overlapping items, Computers & Industrial Engineering, 115, 331-341

[17] Ranga P. Abeysooriya, Julia A. Bennell 2018 Antonio Martinez-Sykora, Jostle heuristics for the 2D-irregular shapes bin packing problems with free rotation, International Journal of Production Economics, 195, 12-26

[18] Fabrizio Marinelli, Andrea Pizzuti,2018 A Sequential Value Correction heuristic for a bi-objective two-dimensional bin-packing, Electronic Notes in Discrete Mathematics, 64, 25-34.

[19] A. Martinez-Sykora, R. Alvarez-Valdes, J.A. Bennell, R. Ruiz, J.M. Tamarit, 2017 Matheuristics for the irregular bin packing problem with free rotations, European Journal of Operational Research, 258, 440–455.

[20] Xueping Li, Kaike Zhang, 2018 Single batch processing machine scheduling with two-dimensional bin packing constraints, International Journal of Production Economics, 196, 113-121
[21] Célia Paquay, Sabine Limbourg, Michaël Schyns 2018 A tailored two-phase constructive heuristic for the three-dimensional Multiple Bin Size Bin Packing Problem with transportation constraints, *European Journal of Operational Research*, 267, 52-64.