Stellar GADGET: A smooth particle hydrodynamics code for stellar astrophysics and its application to Type Ia supernovae from white dwarf mergers

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ABSTRACT
Mergers of two carbon-oxygen white dwarfs have long been suspected to be progenitors of Type Ia Supernovae. Here we present our modifications to the cosmological smoothed particle hydrodynamics code GADGET to apply it to stellar physics including but not limited to mergers of white dwarfs. We demonstrate a new method to map a one-dimensional profile of an object in hydrostatic equilibrium to a stable particle distribution. We use the code to study the effect of initial conditions and resolution on the properties of the merger of two white dwarfs. We compare mergers with approximate and exact binary initial conditions and find that exact binary initial conditions lead to a much more stable binary system but there is no difference in the properties of the actual merger. In contrast, we find that resolution is a critical issue for simulations of white dwarf mergers. Carbon burning hotspots which may lead to a detonation in the so-called violent merger scenario emerge only in simulations with sufficient resolution but independent of the type of binary initial conditions. We conclude that simulations of white dwarf mergers which attempt to investigate their potential for Type Ia supernovae should be carried out with at least $10^6$ particles.

Key words: stars: supernovae: general – hydrodynamics – binaries: close – methods: numerical

1 INTRODUCTION
Mergers of two white dwarfs have first been proposed as progenitor systems of Type Ia Supernovae (SNe Ia) by Iben & Tutukov (1984) and Webbink (1984). More than 25 years later, however, we are neither able to confirm them nor rule them out theoretically nor observationally.

On the theory side there is a long tradition of numerical simulations of white dwarf mergers to determine the fate of those systems after the merger. With time, these simulations have become better and more and more sophisticated both in terms of the numerical resolution used and the treatment of the input physics.

The merger of two white dwarfs is an inherently three-dimensional problem. There is no intrinsic symmetry that can be exploited to simulate it in less than three dimensions. Starting with the pioneering work by Benz et al. (1990) most simulations of white dwarf mergers used smoothed particle hydrodynamics (SPH) codes. This results from several advantages SPH codes have compared to finite volume codes for this particular problem, including the fact that SPH conserves angular momentum very well, does not need a description of the volume surrounding an object and offers a simple way of implementing additional physics. Nevertheless, there are a few attempts to use finite volume codes (D’Souza et al. 2006; Motl et al. 2007). A problem of these simulations, however, has been shown to be the use of a polytropic equation of state which suppresses shocks and leads to an artificial increase of the orbital separation (Dan et al. 2011). Qualitatively, their results were not so different from Benz et al. (1990), although they used a resolution of only 7000 particles. They showed that the merger of two white dwarfs will lead to the destruction of the secondary, less massive, white dwarf. Its material forms a hot envelope and an accretion disk around the remaining primary white dwarf.

The result of the long-term evolution of such a merger remnant is then likely to be the conversion of the central core into an oxygen-neon white dwarf by a slowly inward propagating flame (e.g. Nomoto & Iben 1985; Saio & Nomoto 1998). If this oxygen-neon white dwarf accretes enough material from its envelope and the accretion disk to reach the Chandrasekhar mass, it will undergo an accretion-induced...
collapse, rather than a thermonuclear explosion (Nomoto & Kondo 1991).

Following Benz et al. (1990), merger simulations have been repeated with more and more particles (i.e. Guerrero et al. (2004) using $4 \times 10^5$ particles, Yoon et al. (2007) using $2 \times 10^6$ particles, and Loren-Aguilar et al. (2009) using $4 \times 10^6$ particles) confirming previous results. Starting with Guerrero et al. (2004) a nuclear reaction network has been coupled to the equations of hydrodynamics to account for the energy release from nuclear reactions during the merger.

Only recently, indications have been found that those results may not be the full picture. Pakmor et al. (2010) who simulated the merger of two equal-mass white dwarfs with a mass ratio of one with an unprecedented resolution of $2 \times 10^6$ particles found that in the final phase of the merger, just before the disruption of the secondary white dwarf, a hotspot develops on the surface of the primary white dwarf. In this hotspot carbon burning starts and a detonation may form that directly consumes the whole merging binary system leading to a thermonuclear explosion.

This violent merger scenario, of course, stands and falls with the assumption that a detonation forms. Further work showed that these hotspots form even with lower mass ratios of the merging white dwarfs, down to 0.8 (Pakmor et al. 2011). Doubts on the formation of those hotspots have been raised by Dan et al. (2011) who introduced a method to obtain more relaxed (“exact”) binary initial conditions. They found that this leads to a significantly more stable binary system and without hotspots at a resolution of $2 \times 10^5$ particles. In contrast, Raskin et al. (2012) and Pakmor et al. (2012) found hotspots to be present in merger simulations with exact binary initial conditions and a resolution of $10^6$ particles and more. Note, however, that all these simulations have been run with different codes, different parameters, and at least partially different input physics.

In this paper, we therefore describe in detail our methodology, which is based on the GADGET code. While we emphasize that the modifications of the code, that originally was designed to address problems in cosmology, are useful for a potentially wide range of problems in stellar astrophysics, we use our implementation to test the role of initial conditions and resolution in mergers of white dwarfs here.

In Sec. 2 we describe in detail the modification we made to the GADGET code and test our new time-stepping mechanism in Sec. 3. We present our new nuclear reaction network in Sec. 4. We discuss our new method to create initial conditions for stars in SPH in Sec. 5 and demonstrate that it works well. In addition, we describe our implementation to create “exact” binary initial conditions following Dan et al. (2011). We then show results of simulations of merging white dwarfs in Sec. 6 studying in particular the influence of binary initial conditions and resolution on the merger of two massive white dwarfs using the same code and parameters for all simulations. We finish with a summary of the paper and some conclusions in Sec. 7.

2 CODE MODIFICATIONS

A detailed description of the GADGET code can be found in Springel (2005) and Springel et al. (2001). Our changes include additional physics as well as technical changes to adapt the code to different conditions. The changes are as follows.

(i) Gravitational softening

To calculate the gravitational forces we use the standard tree algorithm implemented in GADGET. Instead of a fixed gravitational softening length as usually applied in cosmological simulations, we use the individual smoothing lengths of gas particles as their gravitational softening (Bate & Burkert 1997). This choice significantly improves the stability of objects in hydrostatic equilibrium. However, it can lead to errors in the total energy budget of the simulation when the smoothing length of a particle changes, as the gravitational softening and therefore the local gravitational potential also changes. As described by Price & Monaghan (2007), it is possible to compensate for this by adding extra terms to the evaluation of the gravitational force (for an implementation into the GADGET code see Iannuzzi & Dolag (2011)). The improved energy conservation, however, comes at the cost of additional noise in the velocity field. Therefore, we refrain from using it. For most applications the errors introduced are small anyway, because the smoothing lengths of the particles do not change significantly during the simulation.

(ii) Wakeup mechanism

In the GADGET code, individual time steps are assigned to all particles, which depend only on the local conditions near the particles. The particles are then evolved on their own time steps. While this greatly reduces the computational costs to run a simulation, it can cause severe problems if very fast particles run into a medium in which the sound speed is much smaller then the velocity of the fast particles. In this case, a fast particle on a small time step can pass through a particle on a much larger time step without being noticed. As this behavior can obviously lead to completely unphysical results, we implement a “wakeup mechanism” for the time-stepping. Similar to the method proposed by Saitoh & Makino (2009), it should activate inactive particles as other particles approach them which evolve on much shorter time scales. The hydrodynamical time step of an SPH particle is calculated as

$$\Delta t_i = \frac{C_{\text{Courant}}}{\max_j \left( v_{\text{signal}}^{ij} \right)}$$

where $C_{\text{Courant}}$ is the Courant factor, $h_i$ is the smoothing length of the particle and the denominator finds the maximum of the signal velocities $v_{\text{signal}}^{ij}$ from particle $i$ and all its neighbors $j$ as defined in equation 13 of Springel (2005). The resulting maximum signal velocity of a particle is stored. In each time step we check for all inactive neighbors of all active particles whether their stored maximum signal velocity is smaller by some factor than the new signal velocity calculated for the active particle and the inactive neighbor,

$$v_{\text{signal}}^{ij} < w \cdot v_{\text{signal}}^{ij}.$$  

If fulfilled, we flag the particle to be woken up. Our usual choice for the wakeup factor $w$ is 4.1; thus an active particle can be in range of an inactive particle for three time steps at most. The check can be done very efficiently, since the active particle has to loop over all its neighbors anyway to
calculate the local pressure force. After finishing the time step, we change the time step of all particles that have been flagged to wake up such that they become active in the next possible time step. When a particle was active in a time step, its properties had been predicted half a time step into the future using its actual rates of change of these properties. Thus, this extrapolation needs to be corrected when we shorten the time step. The correction for the difference between the estimated time step and the time step the particle really experienced is done consistently for all quantities.

(iii) Energy equation
In contrast to the original implementation of GADGET that uses the entropy equation, we solve the energy equation. This is convenient choice because evolving the internal energy simplifies combining hydrodynamics with the nuclear reaction network.

(iv) Equation of state
The equation of state (EoS) captures all intrinsic properties of the material. It is used to calculate local pressure and speed of sound from the primary thermodynamical variables evolved in the code. The original GADGET code only implements an ideal gas equation of state. We replace it with the Helmholtz-EoS (Timmes & Swesty 2000). This EoS describes an arbitrarily degenerate, arbitrarily relativistic electron–positron gas together with an ideal gas of completely ionized ions. It also includes radiation from a black body with the local gas temperature. Since internal energy is our thermodynamic variable of choice, most calls of the EoS have the internal energy as input. Because it is in general not possible to invert the EoS, we use an iterative procedure to find a valid temperature for each time step. In this rare case, when the temperature drops below the minimum temperature tabulated by the EoS, we assign a temperature of 1000K to the particle. All other thermodynamical quantities apart from temperature, density and composition are calculated from internal energy. Due to numerical errors (e.g. when the value of the internal energy becomes smaller than the minimum energy of the degenerate electron), it may not be possible to find a valid temperature for a given internal energy. In this case, or when the temperature drops below the minimum temperature tabulated by the EoS, we assign a temperature of 1000K to the particle. All other thermodynamical quantities apart from internal energy are then calculated for this temperature.

(v) Nuclear reaction network
Because SPH is a purely Lagrangian method, the composition of a particle can only change by nuclear reactions. The nuclear network is calculated at the hydrodynamical time step, and the nuclear reaction network is integrated for all active particles for the duration of their timestep. During this integration, the density is assumed to be constant. Changes of the temperature as a result of energy release or consumption by nuclear reactions are taken into account in the network. The nuclear reaction network and its integration are described in detail in Section IV. We restrict evolving the nuclear network to particles with temperatures higher than 10^6 K. For most of our applications this is a very conservative estimate, as carbon burning starts only at around 10^8 K.

After evolving the nuclear network for the duration of the hydrodynamical time step, the composition of the particle is changed according to the nuclear reaction network. From the rate of change of the abundances of the different species we then calculate the amount of energy that is released or consumed by nuclear reactions.

\[
\dot{e}_{\text{nuc}} = \sum_{j=1}^{N_{\text{species}}} N_A M_j c^2 \frac{dY_j}{dt} \quad (3)
\]

Here, \( N_A \) is Avogadro’s constant, \( M_j c^2 \) is the rest mass energy of species \( j \), and \( Y_j \) is its number fraction. This change of energy is included as a source term in the energy equation.

3 WAKEUP TEST: SEDOV PROBLEM WITH LOCAL TIMESTEPPING
To check the accuracy of our wakeup mechanism we apply it to the Sedov problem (Sedov 1959), which is a simple point explosion leading to a strong shock wave expanding in a surrounding medium. We start from a uniform initial condition (White 1994) in a periodic box of size 10 in arbitrary units that contains \( 10^6 \) particles with a mass of \( 10^{-3} \) each. Then we inject an internal energy of \( 10^5 \) equally into the 33 particles closest to the center of the box. The internal energy of the surrounding particles is chosen to be \( 10^{-5} \). For this configuration there is a large spread of local timesteps. Particles which are influenced by one of the hot particles (i.e. one of them is closer than their smoothing length) are evolved on a timestep of \( 10^{-5} \). All other particles, however, start with a timestep of \( 5 \times 10^2 \), more than a million times larger. Therefore, these particles obviously decouple completely from the hot particles and their close neighbours. Although the conditions we use are quite extreme, the same problem also occurs for realistic problems, i.e. the interaction of the ejecta of a supernova explosion with a companion star in a binary system (e.g. Pakmor et al. 2008). One way to...
circumvent this problem is to use the same global timestep for all particles, which guarantees correct behavior. However, it makes the simulation significantly more expensive, because we then have to evolve all particles on a very small timestep from the beginning rather than from the time when the shock actually hits them. Another approach is discussed in Sec. 2, which reduces the timestep of a particle as soon as the shock actually hits it. Fig. 1 compares the radial density profile for the Sedov problem for a global timestep and a local timestep combined with this wakeup mechanism. It clearly shows that using local timesteps plus the wakeup mechanism recovers exactly the same result we get from using a global timestep.

4 THE NUCLEAR NETWORK

The nuclear reaction network calculates the change of the composition with time. The change of the abundance \( Y_i \) of one species depends on the reaction rates \( R_k \) that destroy or create nuclei of this species.

\[
Y_i = \sum_k R_k (\rho, T, Y)
\]  

Each reaction rate depends on the local density, temperature and abundance of the reactants. The reaction rates are tabulated and taken from the latest (2009) release of the REACLIB database (Rauscher & Thielemann 2000), which includes experimental data as well as theoretical rates when experimental data are not available. Additionally, the weak interaction rates of Langanke & Martínez-Pinedo (2001) are included. Neutrino losses are neglected.

Under the conditions relevant for nucleosynthesis atoms are usually fully ionized. Nevertheless, the assumption of bare nuclei colliding with each other is not valid, because the electron background shields part of the charge of the nuclei. Thus, since the Coulomb repulsion is reduced, the nuclei can interact more easily and the reaction rate increases. We treat the effects of electron screening on the reaction rates as described by Wallace et al. (1982). This description discriminates between regimes of strong (Alastuey & Jancovici 1978) and weak (Dewitt et al. 1973) screening and an intermediate region. In contrast to the original paper, we use a different interpolation method in the intermediate regime and a corrected coefficient for strong screening, both taken from Wallace et al. (1983).

Since the reaction rates depend on temperature to a high power and temperature can change quickly due to the energy release of nuclear reactions, the temperature has to be evolved along with the abundances for correct integration. This is taken care of by an additional equation for the evolution of temperature

\[
\ddot{T} = \left. \frac{\partial T}{\partial E} \right|_{\rho=\text{const}} + \sum_i \dot{Y}_i \frac{\partial T}{\partial Y_i}.
\]  

It should be noted that \( \dot{T} \) is completely determined by the abundances \( Y_i \) and the density, if the energy release is given by equation (3). Therefore it is not necessary to include it explicitly as an additional variable. The problem with this approach is that the Jacobian of the right side of equation (4) would be a dense matrix. By separating the temperature evolution into an additional equation, the Jacobian becomes sparse, with non-zero entries at the respective input and output nuclei of the corresponding reactions. Therefore, we choose to include the extra equation for \( T \) to be able to use significantly faster algorithms for inverting the Jacobian.

We assume the density to be constant over one timestep. Once this approximation is not valid anymore, an evolution equation for the density can be implemented the same way as for the temperature. Some kind of thermodynamic constraint (e.g. constant pressure) is needed in this case.

If electron screening is accounted for, the same method is used for the quantity \( \sum_{j=1}^{N_{\text{species}}} Z_j Y_j \) on which the screening factors depend. The same would apply to \( Y_\rho = \sum_{j=1}^{N_{\text{species}}} Z_j Y_j \) but because \( Y_\rho \) only changes slowly over a typical timestep it can safely be assumed be constant for the computation of the Jacobian.

Thus, we end up with a system of \( N + 1 \) \((N + 2, \text{with screening})\) ordinary differential equations for \( N + 1 \) \((N + 2, \text{with screening})\) variables in a nuclear network of \( N \) species. Because this system of equations is very stiff, it needs to be integrated implicitly. To this end we apply the variable-order Bader–Deuflhard method (Bader & Deuflhard 1983) as suggested by Timmes (1999). Depending on the number of nuclei we use a full direct matrix solver (for small networks) or the sparse matrix solver SuperLU (Demmel et al. 1999).

5 INITIAL CONDITIONS FOR COMPACT OBJECTS

Models of stars and compact objects usually assume hydrostatic equilibrium and spherical symmetry. When we want to simulate such objects in GADGET, we therefore have to start from one-dimensional profiles of density, composition, pressure, etc. in hydrostatic equilibrium. We then have to find a way to construct a stable three-dimensional particle distribution, that reproduces these profiles. Because the local density is coupled to the local particle distribution, noise in the density estimate of any non-trivial initial particle configuration cannot be avoided. It is essential to reduce this noise to maintain the initial configuration as accurate as possible.

5.1 Setup

The idea of our method to construct the particle distribution of a single star is to divide the star into spherical shells and these shells into subvolumes of roughly cubical size to which we attribute one SPH particle each. In addition, each of the subvolumes should contain the same mass, to fulfill the constraint of equally massive particles. With this method we obtain a rather regular particle distribution that resembles any given radial density profile with only very small noise compared to a random sampling of the density profile.

In our implementation we use the HEALPix library (Górski et al. 2005) to tessellate the surface of a sphere into \( 12 \cdot N^2 \) approximately quadratical pieces of the same area where the index \( N \) is a non-negative integer. We then construct the star out of several shells starting from the center. The width of the shells is chosen such that it is of about the same size as the edges of the pieces. As we show below, this condition can be satisfied using a simple method to determine the width of the shells. We express the constraints
Figure 2. Comparison of the initial conditions of a 1.0 M\odot white dwarf generated by the HEALPix method (see text). Black points show the SPH density estimate of all particles depending on their radial coordinate. The red line shows the initial one-dimensional density profile. Left and right column show the same data, but use a linear/logarithmic scale for the density. The top row shows the initial setup, the bottom row shows the configuration after completion of the relaxation.

Let us assume we know the lower radius $r_{\text{lower}}$ of a shell and want to find its upper radius $r_{\text{upper}}$. For a given upper radius, the width of a shell and its mass are given by

$$d_{\text{shell}} = r_{\text{upper}} - r_{\text{lower}}$$

and

$$m_{\text{shell}} = 4\pi \int_{r_{\text{lower}}}^{r_{\text{upper}}} \rho(r) r^2 dr,$$

respectively. The mass increases with increasing $r_{\text{upper}}$. Fixing the uniform particle mass in the beginning, we can calculate the number of particles that need to be placed in this shell to

$$n_{\text{particles}} = \frac{m_{\text{shell}}}{m_{\text{particle}}}.$$  (8)

This is equivalent to an index $n_1$ of

$$n_1 = \sqrt{\frac{m_{\text{shell}}}{12 \cdot m_{\text{particle}}}},$$

which increases with increasing width of the shell.

The second constraint on our shell is the requirement of cubical volumes. The surface area of a shell is roughly given by

$$S = 4\pi r^2 = 4\pi [0.5 \cdot (r_{\text{lower}} + r_{\text{upper}})]^2.$$  (9)

As each shell contains $12 \cdot n_2^2$ particles, the edge size of one piece on the surface can be written as

$$a_{\text{particle}} = \sqrt{\frac{S}{12 \cdot n_2^2}} = \sqrt{\frac{\pi}{12} (r_{\text{lower}} + r_{\text{upper}})} \frac{1}{n_2}.$$  (11)
This edge size should be equal to \(d_{\text{shell}}\) and solving for \(n_2\) leads to

\[
n_2 = \sqrt{\frac{\pi}{12}} \frac{r_{\text{lower}} + r_{\text{upper}}}{r_{\text{upper}} - r_{\text{lower}}}.
\]  

(12)

While increasing the upper radius of the shell, the second index decreases. Since \(n_1 = 0\) and \(n_2 \to \infty\) for \(r_{\text{upper}} = r_{\text{lower}}\), it is always possible to increase the upper radius until \(n_1\) equals \(n_2\). Having found the upper radius, we place a shell of particles at \(r = 0.5 \cdot (r_{\text{upper}} + r_{\text{lower}})\) using the coordinates from the HEALPix library and continue with the next shell. Internal energy and composition of the particle are chosen according to the radial coordinate of the particles. The initial velocity of all particles is set to zero.

5.2 Relaxation

To damp out spurious numerical noise introduced by the setup, we relax the object before we start the actual simulation. To this end we evolve it for several dynamical timescales while applying a time-dependent damping force similar to that used by [Rosswog et al. (2004)]. The specific force is given by

\[
\tilde{v}_i = -\frac{v_i}{\tau}
\]

and applied together with gravitational and hydrodynamical forces. The damping timescale \(\tau\) controls the strength of the damping term. The smaller it is, the stronger is the damping. We start with a large damping force that decreases with time and is eventually switched off completely. Afterwards we continue the simulation for a short time. If the relaxation has been successful, the object will remain in equilibrium and particle motions will stay close to zero. Otherwise, assuming that the initial model is not intrinsically unstable, the relaxation parameters have to be adjusted. We vary the damping timescale with time as

\[
\tau = \begin{cases} 
\frac{1}{\tau_0} & t \leq 0.2 \ t_{\text{max}} \\
\frac{1}{\tau_0} \cdot \frac{0.2 \ t_{\text{max}}}{t_{\text{max}} - t} & 0.2 \ t_{\text{max}} \leq t \leq 0.8 \ t_{\text{max}} \\
0 & t > 0.8 \ t_{\text{max}}
\end{cases}
\]  

(14)

We run the relaxation for a total time \(t_{\text{max}}\), which is chosen to be at least several dynamical timescales. The initial damping timescale \(\tau_0\) should be much smaller than the dynamical timescale. Typical choices for a white dwarf mass of \(1.0 M_\odot\) are \(\tau_0 = 0.002s\) and \(t_{\text{max}} = 100s\). Figure 2 shows the configuration of the initial setup and after the relaxation step. The final relaxed configuration agrees nearly perfectly with the initial profile. Only for the very outermost particles of the star the density is overestimated systematically, because these particles only have neighbors which are at smaller radii and have a higher density.

5.3 Binary initial conditions

Having obtained equilibrium models for individual stars we need to join two of them to form a binary system. For this we use two different approaches, which we will label approximate and exact binary initial conditions, following [Dan et al. (2011)]. For approximate initial conditions both stars are set on a circular orbit with an orbital period to render the binary system marginally unstable. The initial orbital period is not determined exactly and can be chosen in different ways. Those include an iterative procedure until the system is stable for a certain number of orbits or using an approximation for the distance at the onset of Roche lobe mass transfer (Eggleton 1983). To construct exact binary initial conditions we implement the approach used by [Dan et al. (2011)]. We first set both stars on a stable, circular, synchronized orbit with a large orbital period such that there is no mass transfer and the binary system is stable. Then we evolve the binary system in the co-rotating frame and add an artificial damping force to remove all residual velocities. This is done by adding an additional acceleration \(a_{\text{ext},i}\) on each particle that is given by

\[
a_{\text{ext},i} = -\omega \times (\omega \times r_i) - 2 \omega \times v_i - \frac{v_i}{\tau_{\text{damp}}},
\]

(15)

where \(r_i\) and \(v_i\) are position and velocity of particle \(i\) in the co-rotating frame, \(\omega\) the orbital frequency of the binary system, and \(\tau_{\text{damp}}\) the damping timescale. The orbital frequency is updated in each global timestep, i.e. when all particles are active at the same time. It is calculated as the average of the orbital frequencies necessary to exactly balance the total force on the two individual stars. The difference between the two individual orbital frequencies calculated for both stars is typically smaller than one percent. For the white dwarf merger simulations presented in this work we set \(\tau_{\text{damp}}\) to 0.005s, about a factor of five larger than the smallest timesteps in the high resolution run with \(1.6 \times 10^6\) particles.

We then apply an artificial radial drift velocity to both stars such that the center of mass of the binary system does not move. The drift velocity is estimated as described in Equation (8) of [Dan et al. (2011)]. We continue the simulation until the first particle of the less massive white dwarf crosses the inner Lagrange point. At this point we switch off damping and radial drift, and use the current orbital frequency to transform the binary system into a non-rotating frame. The resulting binary system then gives us the initial conditions for simulations of merging white dwarfs as presented in Section 6.

6 APPLICATION TO DOUBLE WHITE DWARFS MERGERS

6.1 Setup

To study the influence of resolution and initial conditions on the properties of the merger simulation we pick a specific binary system of a 1.1 \(M_\odot\) primary and a 0.9 \(M_\odot\) secondary carbon-oxygen white dwarf. This binary system is of particular interest, because it has been shown to lead to observables that closely resemble normal SNe Ia under the assumption that a detonation forms during the merger ([Pakmor et al. (2012)].

Here we perform four merger simulations of this binary system with low (\(1.8 \times 10^6\) SPH particles) and high (\(1.8 \times 10^6\) SPH particles) resolution and approximate and exact binary initial conditions for each, respectively. The properties of these models are summarized in Table 1. The individual
Table 1. Model parameters and properties of the four merger simulations. Tabulated are the total number of particles in the simulation $N_p$, the type of binary initial conditions used (IC Type), the orbital period $T$, orbital separation $a$ and total angular momentum $L$ as properties of the initial binary system. In addition, several properties of the binary system are shown at the time $t$ when the separation is $6 \times 10^8$ cm. These are the number of particles $N_{hot}$ on the surface of the primary with a temperature larger than $2 \times 10^9$ K, the central densities of the primary and the secondary white dwarf ($\rho_{sec,max}$ and $\rho_{prim,max}$, respectively) and an estimate of the smoothing length in the hotspot on the surface of the primary, if present.

| Number | $N_p$ [$10^5$] | IC Type | $T_{initial}$ [s] | $a_{initial}$ [$10^9$ cm] | $L_{initial}$ [$10^{50}$ g cm$^2$ s$^{-1}$] | $t$ [s] | $N_{hot}$ | $\rho_{prim,max}$ [$10^7$ g cm$^{-3}$] | $\rho_{sec,max}$ [$10^6$ g cm$^{-3}$] | $H_{sml_{hotspot}}$ [$10^7$ cm] |
|--------|----------------|---------|------------------|------------------|------------------|-------|---------|------------------|------------------|------------------|
| 1      | 1.8            | approx. | 28.0             | 1.74             | 7.18             | 73    | 8       | 5.6              | 7.0              | 5                |
| 2      | 18             | approx. | 28.0             | 1.74             | 7.18             | 77    | 81      | 6.1              | 4.2              | 3                |
| 3      | 1.8            | exact   | 32.6             | 1.93             | 7.57             | 469   | --      | 5.2              | 7.1              | --               |
| 4      | 18             | exact   | 32.6             | 1.93             | 7.55             | 615   | 44      | 5.5              | 4.2              | 3                |

Figure 3. Initial conditions for the merger simulations. Shown is a density slice in the plane of rotation with a size of $4 \times 10^9$ cm.

white dwarfs in these models are first constructed as one-dimensional models in hydrostatic equilibrium with a uniform temperature of $10^5$ K and composition of equal parts by mass of $^{12}$C and $^{16}$O. We then create three-dimensional particle distributions resembling the one-dimensional profiles as described in Sec. 5.1. The mass of the particles is chosen such that all particles of a particular binary system have the same mass. We relax the individual white dwarfs for $10$ s as discussed in Sec. 5.2. Two white dwarfs are then combined to a binary system as described in Sec. 5.3. To obtain exact initial conditions we use a relaxation timescale of $5 \times 10^{-3}$ s and a radial velocity of $1.0 \times 10^7$ cm s$^{-1}$ for the $0.9 M_\odot$ white dwarf and $0.82 \times 10^7$ cm s$^{-1}$ for the $1.1 M_\odot$ white dwarf. We start with an initial orbital period of $50$ s and stop when the first particle of the secondary white dwarf crosses the inner Lagrange-point of the binary system. The orbital separation and the orbital period at this time are given in Table 1. We then remove the radial and add the orbital velocity and restart the simulation in a fixed frame.

The density distribution of the initial conditions is shown in Fig. 3. In the models with exact initial conditions, the secondary white dwarf is slightly deformed and fills its Roche-lobe. The initial separation of the models with exact initial conditions is a bit larger compared to the models with approximate initial conditions, which also gives them
Figure 5. Density slice through the binary system when the separation distance is $6 \times 10^8$ cm. The individual plots are centered on the center of mass of the binary system and rotated to have the centers of mass of both white dwarfs on the x-axis. The box has a height and width of $4 \times 10^9$ cm.

Figure 4. Evolution of the orbital separation for the merger models until the binary system has merged. The dashed black line shows the separation at which we compare the different simulations in detail. The resolution has no significant effect on the initial conditions. The configuration of the GADGET code is mostly identical for all four simulations. We use the nuclear reaction network described in Sec. 4 for the 13 $\alpha$-element isotopes from $^4$He to $^{56}$Ni. It is active only for particles with a temperature larger than $5 \times 10^8$ K, because carbon burning is negligible at lower temperatures. In the high resolution simulations we limit the smoothing lengths of the particles to be smaller or equal to $2 \times 10^8$ cm. This only affects less than 1% of the particles, in particular the particles which are ejected from the binary system during the inspiral phase.

6.2 Inspiral

The main difference between the simulations in the inspiral phase is in the stability of the binary systems and thus the time it takes until the binary system merges. Fig. 4 shows the evolution of the orbital separation for all four simulations. Obviously, there is a much larger difference between models with different binary initial conditions than between models with different resolution. In particular, binary systems with exact initial conditions are significantly more stable than binary systems with approximate initial conditions, in agreement with previous results by Dan et al. (2011). In our case, the binary systems with approximate initial con-
Figure 6. Temperature slice through the binary system when the separation distance is $6 \times 10^8$ cm. The individual plots are centered on the center of mass of the binary system and rotated to have the centers of mass of both white dwarfs on the x-axis. The box has a height and width of $4 \times 10^9$ cm.

6.3 The last binary orbit

The most important stage for the fate of the binary system is the final orbit of the merger when the secondary white dwarf is destroyed. At this time, it is decided whether a detonation forms which leads to the violent merger scenario (Pakmor et al. 2010, 2011, 2012) or whether the system ultimately forms a cold primary white dwarf surrounded by an accretion disk and a hot envelope made from the material of the disrupted secondary white dwarf (see, e.g., Dan et al. 2011).

To investigate the effect on resolution and binary initial conditions on this phase, we compare all simulations at the time when their separation first becomes as small as $6 \times 10^8$ cm. This distance is approximately the distance at which the system described in Pakmor et al. (2012) forms its first hotspot.

Fig. 5 shows a density slice at this time. Qualitatively, all four simulations are very similar. In all cases, the primary white dwarf remains mostly unaffected and is surrounded by a very thin envelope. There are, however, some quantitative differences in the central density of the secondary white dwarf which is about to be disrupted. In particular, for the low resolution it is about $7 \times 10^6$ g cm$^{-3}$, but only about $4 \times 10^6$ g cm$^{-3}$ for the high resolution simulations. Although this difference is dynamically not important, it will change the burning products of the secondary white dwarf slightly in the violent merger scenario.

At the place where the material from the secondary impacts the primary, a denser region forms. The most interesting region is located between this dense region and the surface of the primary white dwarf. Here, material is
compressed and heats up enough to start carbon burning which can be interpreted as an indication that a detonation can form there. A temperature slice of the binary system is shown in Fig. 6, clearly demonstrating the hot region on the surface of the primary white dwarf. The plot already shows that the temperature is significantly higher for the high resolution simulations than for the low resolution runs.

To quantify this difference, we count the number of particles that can form there. A temperature slice of the binary system worked out that the type of binary initial conditions has no effect on the properties of the actual merger.

Comparing the simulations in the last binary orbit, when they reach the same separation between primary and secondary white dwarf, we found that resolution is the dominating factor. It decides whether carbon burning starts in hotspots or not, regardless of the type of binary initial conditions. We stress that merger simulations should be carried out with at least $10^6$ particles to avoid incorrect conclusions due to under-resolved hotspots.

Unfortunately, our simulations also confirmed again that even for our best-resolved runs the resolution around and at the hotspots is far from sufficient to really confirm or rule out the formation of a detonation. Given our results it seems rather unlikely that this will be possible in the near future using an SPH code. Thus, the best way to investigate this might be to try to model the accretion stream on a high resolution grid as done for helium white dwarf secondaries in Guillochon et al. (2010).

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7 SUMMARY AND CONCLUSION

In this paper we presented the modifications to the GADGET code that are necessary to apply it to problems of stellar physics. These modifications include a general equation of state, a new efficient nuclear reaction network and a mechanism to avoid errors from integrating particles on local timesteps.

We then described a new method to arrange the SPH particles in the initial conditions according to a one-dimensional density and pressure profile as commonly given in problems of stellar astrophysics. We showed that the configuration we obtain is perfectly stable and retains its radial profiles. We then discussed two different ways of setting up a marginally stable binary system from two white dwarfs using approximate and exact (Dan et al. 2011) initial conditions.

As an example demonstrating the use of the "Stellar Gadget" code, we investigated the effects of binary initial conditions and resolution on the properties of the merger of a $1.1 M_\odot$ and with a $0.9 M_\odot$ carbon-oxygen white dwarf (Pakmor et al. 2012). We found that exact initial conditions lead to a significantly more stable binary system, confirming previous results by Dan et al. (2011). However, we also worked out that the type of binary initial conditions has no effect on the properties of the actual merger.

This result is consistent with other previous simulations with low resolution and exact initial conditions that did not show any carbon burning particles (Dan et al. 2011, 2012) and simulations with both types initial conditions and high resolutions which found “hot” particles (Pakmor et al. 2010, 2011, Raskin et al. 2012). Therefore, we conclude that the resolution of the simulation is the deciding factor which determines whether hotspots form with particles which started carbon burning. The initial conditions, in contrast, do not seem to have a significant effect on this phase of the merger. This result can be understood in terms of timescales. The sound crossing time for the secondary white dwarf with a mass of $0.9 M_\odot$ is only about 7 s. Therefore, even with approximate initial conditions for which the binary system becomes unstable after a bit more than 2 full orbits, the secondary white dwarf has enough time to adjust its structure to the gravitational potential of the primary white dwarf. Moreover, the difference in the total angular momentum of the systems with approximate and exact binary initial conditions is too small to significantly affect this phase of the merger.
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