I discuss anew how arguments about the internal dynamics of galactic disks set constraints on the otherwise ambiguous decomposition of the rotation curves of spiral galaxies into the contributions by the various constituents of the galaxies. Analyzing the two sample galaxies NGC 3198 and NGC 2985 I conclude from the multiplicities of the spiral arms and the values of the $Q$ disk stability parameters that the disks of both galaxies are ‘maximum disks’.

1. Introduction

The rotation curves of spiral galaxies provide the most direct evidence for the presence of dark matter in galaxies. However, taken alone they do not discriminate luminous from dark matter because their decomposition into the contributions from the various constituents of the galaxies is highly ambiguous. Thus further constraints are needed. Considerations of the dynamical state of the resulting disk models can provide such constraints (Bosma 1999, Fuchs 1999). Of particular interest is the question if the much discussed ‘maximum–disk’ models, i.e. disks with their masses chosen at the maximum allowed by the data, are dynamical viable disk models. The degeneracy of the decomposition problem is illustrated in Fig. 1 for the example of NGC 3198. The rotation curve is modelled as

$$v_c^2(R) = v_{c,disk}(R) + v_{c,halo}(R) + v_{c,is,gas}(R),$$

(1)

where $v_{c,disk}$, $v_{c,halo}$, and $v_{c,is,gas}$ denote the contributions due to the stellar disk, the dark halo, and the interstellar gas, respectively. The disk is modelled as an exponential disk and the dark halo is described by a quasi-isothermal sphere. In the left panel of Fig. 1 the maximum–disk model of Broeils (1992) is reproduced, while the right panel shows a submaximal
disk model. The halo model parameters have been changed so that both fits to the observed rotation curve are of the same quality.

2. Dynamical Constraints on the Decomposition of Rotation Curves

The diagnostic tools I use to analyze the dynamical state of the disk models are the Toomre stability parameter of the disks and, following Athanassoula et al. (1987), the predicted multiplicity of the spiral structures. The Toomre stability parameter is given by

\[ Q = \frac{\kappa \sigma_U}{3.36 G \Sigma_d}, \]

where \( \kappa \) denotes the epicyclic frequency, \( \kappa = \sqrt{2} \frac{\Omega}{R} \sqrt{1 + \frac{R}{v_c} \frac{dv}{dR}} \), \( \sigma_U \) is the radial velocity dispersion of the stars, \( G \) the constant of gravitation, and \( \Sigma_d \) the surface density of the disk. The stability parameter must lie in the range \( 1 < Q < 2 \), in order to prevent Jeans instability of the disk, on one hand, and to allow the disks to develop spiral structures, on the other hand. If \( Q \) were less than 1, the disks would develop fierce dynamical instabilities and heat up dynamically on very short time scales as demonstrated, for example, by Fuchs & von Linden (1998). Thus such disk models would be not equilibrium models.

The dynamics of galactic spiral structure is theoretically well understood in the framework of the density wave theory of spiral arms. Density wave theory makes, in particular, a specific prediction for the number of spiral arms. Spiral density waves develop in galactic disks preferentially with a circumferential wavelength of about (Toomre 1981, Fuchs 2001, 2003c)

\[ \lambda \approx X \left( \frac{A}{\Omega_0} \right) \lambda_{\text{crit}}, \]

where \( \lambda_{\text{crit}} \) denotes the critical wavelength

\[ \lambda_{\text{crit}} = \frac{4 \pi^2 G \Sigma_d}{\kappa^2}. \]
The coefficient $X(\frac{A}{10})$ depends on the slope of the rotation curve measured by Oort's constant $A$, $\frac{A}{10} = \frac{1}{2} \left(1 - \frac{R}{R_{vc}} \frac{dR}{dR}\right)$, and has been determined explicitly for various cases by Toomre (1981), Athanassoula (1984), or Fuchs (2001, 2003c). For a flat rotation curve the value is $X(0.5) = 2$. The number of spiral arms is obviously determined by how often the wavelength $\lambda$ fits onto the annulus,

$$m \approx \frac{2\pi R}{X\lambda_{crit}}. \quad (5)$$

The predicted number of spiral arms (5) is based on the local model of a shearing sheet which describes the dynamics of a patch of a galactic disk (Goldreich & Lynden–Bell 1965, Julian & Toomre 1966, Fuchs 2001). Equation (5) can be applied globally to an entire disk strictly only in the case of a Mestel disk, which has an exactly flat rotation curve and a surface density distribution falling radially off as $1/R$. In the exponential disk models used here $m$ varies formally with galactocentric distance. However, the maximum growth factor of the amplitudes of the density waves is not sharply peaked at the circumferential wavelength (3) (Toomre 1981, Fuchs 2001, 2003c) so that density waves with smaller or greater wavelengths can develop as well. Allowing for these side fringes of wavelengths and the corresponding variations of the coefficient $X$ in equation (5) one can derive for the distance range in the galactic disks spanned by the spiral arms a uniform value of $m$.

3. Examples: NGC 3198 and NGC 2985

I demonstrate the implications of the dynamical constraints on the decomposition of the rotation curves with the examples NGC 3198 and NGC 2985. Both galaxies show clear spiral structure and the velocity dispersions of the stars have been measured in both galaxies (Bottema 1988, Gerssen 2000). The expected multiplicity of spiral arms and the $Q$ parameter are shown in Fig. 3 for the maximum disk and the submaximal disk models of NGC 3198, respectively. The maximum disk model predicts a two–armed spiral just as can be seen in the NIR image of the galaxy in Fig. 2. The $Q$ parameter is about 1 in this model. This avoids the the onset of fierce dynamical instabilities, but would imply that the spiral structure grows very rapidly which would lead to a considerable dynamical disk heating. However, the velocity dispersions have been measured by Bottema (1988) in the B band and might be dominated by bright young stars with velocity dispersions lower
than that of the mass carrying stellar populations (Fuchs 1999). Thus
the $Q$ parameter is probably underestimated. The submaximal disk model
predicts a three–armed spiral and the $Q$ parameter is around 1.5 which is
probably underestimated again and thus seems to be too high.

Figure 2. Images of NGC 3198 (left) and NGC 2985 (right) retrieved from the Digitized
Sky Survey (ESO). The image sizes are 5’×5’.

Figure 3. Expected number of spiral arms and $Q$ parameter according to the maximum
disk and submaximal disk models of NGC 3198 shown again in the upper panels. The
radial scale is also indicated in arcmin.

I conclude from this that the maximum disk model is the more realistic
disk model. In a previous paper (Fuchs 1999) I have argued to the contrary.
However, that was judging from an optical image with many filaments which does not reveal the major spiral arms as the NIR image and I revise my opinion here.

Fig. 4 shows the rotation curve of NGC 2985 (Gerssen 2000) and the corresponding maximum disk and submaximal disk models, respectively, which in both cases include a bulge contribution in the inner parts. The velocity dispersions have been measured by Gerssen (2000) in the I band and should allow a more reliable estimate of the $Q$ parameter than in the previous case. As can be seen from Fig. 4 both the expected multiplicity of spiral arms and the $Q$ parameter indicate even clearer than in the case of NGC 3198 that the maximum disk model is the more realistic disk model.

Figure 4. Expected number of spiral arms and $Q$ parameter according to the maximum disk and submaximal disk models of NGC 2985 shown in the upper panels. The radial scale is also indicated in arcmin.

4. Discussion

Arguments for and against maximum disks have been discussed at length in the literature and are reviewed in detail, for instance, by Bosma (1999) or Sellwood (1999). One of the major consequences of maximum disks are the implied large core radii of the dark halos. These challenge the contemporary theory of the formation of galaxies according to CDM cosmology.

Obviously the dynamical constraints on the decomposition of rotation curves must be tried out on a much larger data set than here in order to test the maximum disk hypothesis. This can be easily done with the density wave theory criterion by inspecting images of galaxies for which rotation
curves are available. I have, for instance, analyzed a set of low surface brightness galaxies and found again indications for maximum disks (Fuchs 2003a, b). However, velocity dispersions have been measured in very few galaxies. Thus the $Q$ stability parameter criterion, which is an independent consistency check on the density wave theory criterion, can be applied only in deplorably few cases.

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