Solving Manifold Ambiguity by Sliding the Array in MIMO Radar

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Abstract. Arbitrary sparse array could extend the array aperture. And the position of sparse array elements has little constraint. But, the manifold ambiguity of sparse array will cause ambiguity angle estimation. In order to avoid manifold ambiguity in Multiple Input Multiple Output (MIMO) radar with sparse array, a new solving ambiguity method is proposed based on sliding the array. By employing reduced multiple signal classical algorithm(RE-MUSIC), the true direction of departure (DOD) and direction of arrival (DOA) are obtained. The main idea of the proposed method is that get DODs and DOAs separately by RE-MUSIC algorithm. When transmitter exists the ambiguity, we compare the RE-MUSIC spectrum of pre-sliding transmitting array with the RE-MUSIC spectrum of post-sliding transmitting array to obtain the true DODs. Then we obtain the true DOAs by the RE-MUSIC spectrum of receiving array. It is similar to receiver exists the ambiguity. Simulation results verify the validity of the proposed method.

1. Introduction
MIMO radar employs multiple transmitting and receiving elements, and has the ability to plan transmissions and process receiving signals jointly. It has important performance enhancement compared to the conventional phased-array radar, so it has been the focus of radar research[1]-[2]-[3]. Compared to MIMO radar with uniform array, MIMO radar with sparse array has a lot of advantages, such as larger array aperture, more degrees of freedom and more effective array elements[4]-[5]-[6]. Typically larger array aperture produces more accurate angle estimation. And the sparse array can greatly reduce the system cost.

However, sparsity will cause manifold ambiguity due to the distance increase between elements. Few methods have been presented to eliminate the ambiguity angle in MIMO radar with sparse array. ESPRIT polynomial root finding and ESPRIT spectral search method was proposed in [7]. The method of virtual interpolation was presented for MIMO radar with sparse array in [8]. In [9], Alternating Least Squares(ALS) method for tensor decomposition was proposed to get the angles estimation of targets which is used to the minimum redundant array. Aiming at nested array and co-prime array, the angle estimation algorithms were presented for MIMO radar in [10] and [11].

We propose a new method to solve the ambiguity of MIMO radar with sparse array in this paper. The two receiving signals of pre-sliding transmitting array and post-sliding transmitting array are got by sliding the array. The true DODs estimation can be obtained by comparing with the RE-MUSIC spectrum of the two receiving signals. Then the DOAs estimation can be got by searching the RE-MUSIC spectrum of the receiving array. Finally, the DODs and DOAs can be auto-paired.
2. Data model of RE-MUSIC algorithm in MIMO radar

2.1. Data model of MIMO radar

As Figure 1, the transmitting array is a sparse array and the receiving array is a uniform linear array, they have M array elements and N array elements respectively. Assuming that the target is far away from the transmitting array and the receiving array. The transmitting signal is a narrowband signal, and M transmitting signals are orthogonal to each other. Assuming that P non-coherent targets located in the same range bin. The DOD of the $p$th target is $\theta_p$, and the DOA of the $p$th target is $\phi_p$.

Therefore, the receiving signal is

$$y(l) = A\Phi(l) + n(l) \quad (1)$$

$A = [a_1, a_2, \ldots, a_P]$ is a $M \times P$ matrix consists of $P$ steering vectors. $a_p$ is the Kronecker product of the receiving steering vector and the transmitting steering vector of the $p$th target

$$a_p = a_r(\phi_p) \otimes a_t(\theta_p) \quad (2)$$

The expression of transmitting steering vector is

$$a_t(\theta_p) = \left[1, e^{j2\pi d_{T,1}\sin \theta_p / \lambda}, \ldots, e^{j2\pi d_{T,M-1}\sin \theta_p / \lambda}\right]^T \quad (3)$$

where $d_T = [d_{T,0}, d_{T,1}, \ldots, d_{T,M-1}]$ is the position vector of the transmitting element.

Similarly, the receiving steering vector is expressed as

$$a_r(\phi_p) = \left[1, e^{j2\pi d_{R,1}\sin \phi_p / \lambda}, \ldots, e^{j2\pi d_{R,N-1}\sin \phi_p / \lambda}\right]^T \quad (4)$$

where $d_R = [d_{R,0}, d_{R,1}, \ldots, d_{R,N-1}]$ is the position vector of the receiving array element. In equation (1), $\Phi(l) = [\Phi_1(l), \Phi_2(l), \ldots, \Phi_P(l)]$. The column elements consist of the amplitude and phase of $P$ target sources. $n(l)$ is the noise term which is supposed to be additive Gaussian white noise. Its mean is 0, and its variance is $\sigma^2$.
2.2. RE-MUSIC algorithm in MIMO radar
The maximum likelihood estimation of the covariance matrix of the receiving signal \( y(l) \) is
\[
R = \frac{1}{L} \sum_{l=1}^{L} y(l)y^H(l)
\]  
(5)

\( L \) is the snapshot number, and \( (\cdot)^H \) is the Hermitian transpose. The RE-MUSIC spectrum of the DOD is[12]
\[
\hat{\theta} = \arg \min_{\theta} \frac{1}{e_i^H Q^{-1}(\theta)e_i} = \arg \max_{\theta} e_i^H Q^{-1}(\theta)e_i
\]  
(6)

where \( Q(\theta) = [I_N \otimes a(\theta)]^H E_{n} E_{n}^H [I_N \otimes a(\theta)] \) and \( e_i = [1, 0, \ldots, 0]^T \in \mathbb{R}^{N_1} \). \( E_{n} \) is the noise subspace involving the eigenvectors corresponding to the smallest \( M - P \) eigenvalues by singular decomposition of \( R \).

By searching \( \theta \in [-90^\circ, 90^\circ] \) through the equation(6), the DODs estimation of targets \( \hat{\theta} = [\hat{\theta}_1, \hat{\theta}_2, \ldots, \hat{\theta}_p, \ldots \hat{\theta}_{p,K}] \) can be obtained, there are \( P \) true angle estimates and \( K \) ambiguity angle estimates. In order to remove the ambiguity angle estimation and obtain the true angle estimation, the solving ambiguity method based on RE-MUSIC algorithm by sliding the array is proposed.

2.3. The solving ambiguity method based on RE-MUSIC algorithm by sliding the array
Based on the analysis of literature[13], the value of \( d_i/l \lambda \) can be changed by sliding the array element, and the RE-MUSIC spectrum of the two data sets of the pre-sliding and post-sliding arrays can be compared to remove the pseudo-peaks. Therefore, the true angle estimate can be obtained. The minimum value of the sliding distance is
\[
\min \Delta d_i \approx \text{step} \times \frac{(d_i)^2}{\lambda} \approx \text{step} \times \frac{(d_i)^2}{\lambda}
\]  
(7)

where \( \text{step} \) is small, it satisfies \( \sin(\text{step}) \approx \text{step} \). Assuming that the sliding distance vector of the transmitting array is \( \Delta d = [\Delta d_0, \Delta d_1, \ldots, \Delta d_{M-1}] \). The new transmitting steering vector after sliding the array is \( a'(\theta) = \left[e^{j2\pi(d_0\sin(\theta)/\lambda)}, e^{j2\pi(d_1\sin(\theta)/\lambda)}, \ldots, e^{j2\pi(d_{M-1}\sin(\theta)/\lambda)} \right]^T \).

So the new receiving signal is
\[
y'(l) = A'\Phi(l) + n(l)
\]  
(8)

where \( A' = [a'_1, a'_2, \ldots, a'_P] \) is a \( M \times P \) matrix consists of \( P \) new steering vectors. \( a'_p \) is the Kronecker product of the receiving steering vector and the new transmitting steering vector of the \( p \)th target
\[
a'_p = a_p(\varphi_p) \otimes a'_p(\theta_p)
\]  
(9)

Thus, the maximum likelihood estimation of the covariance matrix of the new receiving signal \( y'(l) \) is
\[
R' = \frac{1}{L} \sum_{l=1}^{L} y'(l)y'^H(l)
\]  
(10)

So the new RE-MUSIC spectrum of post-sliding the transmitting array can be expressed as
\[
\hat{\theta}' = \arg \min_{\theta} \frac{1}{e_i^H Q^{-1}(\theta)e_i} = \arg \max_{\theta} e_i^H Q^{-1}(\theta)e_i
\]  
(11)
where \( \mathbf{Q}'(\theta) = [I_N \otimes \mathbf{a}'_i(\theta)]^H \mathbf{E}'_n \mathbf{E}^H_n [I_N \otimes \mathbf{a}'_i(\theta)] \), \( \mathbf{E}'_n \) is the noise subspace involving the eigenvectors corresponding to the smallest \( M - P \) eigenvalues by singular decomposition of \( \mathbf{R}' \).

By searching \( \theta \in [-90^\circ, 90^\circ] \) through the equation (11), the new DODs estimation of targets \( \hat{\theta}' = [\theta'_1, \theta'_2, \ldots, \theta'_{P+K}] \) can be obtained. By comparing the two data sets \( \hat{\theta} \) and \( \hat{\theta}' \), the two RE-MUSIC spectrum have \( P \) spectrum peaks overlapped, and \( K \) spectrum peaks separated. The overlapped \( P \) spectrum peaks are the true angle estimation, and other \( K \) spectrum peaks are the ambiguity angle estimation. We get \( P \) true DODs estimation \( \hat{\theta} = [\hat{\theta}_1, \hat{\theta}_2, \ldots, \hat{\theta}_P \] .

So the pre-sliding transmitting steering vector is \( \mathbf{a}_i(\hat{\theta}) \) . According to the literature[12], the RE-MUSIC spectrum of the receiving array can be expressed as

\[
\phi = \arg \min_\phi \mathbf{Q}(\phi)^H \mathbf{Q}(\hat{\theta}) \mathbf{a}_i(\phi)
\]

(12)

where \( \mathbf{Q}(\hat{\theta}) = [I_N \otimes \mathbf{a}_i(\hat{\theta})]^H \mathbf{E}_n \mathbf{E}^H_n [I_N \otimes \mathbf{a}_i(\hat{\theta})] \). By searching \( \theta \in [-90^\circ, 90^\circ] \) through the equation (12), the estimation of DOAs \( \hat{\phi} = [\hat{\phi}_1, \hat{\phi}_2, \ldots, \hat{\phi}_P \] can be obtained. Thus we get the true estimation of paired DODs and DOAs.

2.4. Experimental simulation and result analysis

The minimum discrimination is defined as

\[
D = \min |\theta_{i_1} - \theta_{i_2}| - \max |\theta_{j_1} - \theta_{j_2}|
\]

(13)

where \( \theta_{i_1} \) denotes the \( i \)th ambiguity angle estimation obtained by pre-sliding array, \( \theta_{i_2} \) denotes the \( i \)th ambiguity angle estimation obtained by post-sliding array. \( \theta_{j_1} \) represents the \( j \)th true angle estimation obtained by pre-sliding array, \( \theta_{j_2} \) represents the \( j \)th true angle estimation obtained by post-sliding array. The minimum discrimination reflects the distinction between the true angle and the ambiguity angle.

In order to verify the validity of the proposed method, the following simulation is done.

Three target sources are in position \( (\theta_1, \phi_1) = (10^\circ, 15^\circ) \) , \( (\theta_2, \phi_2) = (20^\circ, 60^\circ) \) , \( (\theta_3, \phi_3) = (50^\circ, 35^\circ) \) . The number of transmitting array elements is 4, and the position vector of the transmitting array is \( d_r = (\lambda/2) [0, 4, 6, 8] \) . The receiving array is uniform linear array; the number of element is 4. The snapshot number is 100 and the SNR is 10dB. The sliding distance is the minimum sliding distance.

![Figure 2. The RE-MUSIC spectrum of original transmit array and sliding transmit array](image-url)
In Figure 2, the real line is the RE-MUSIC spectrum of pre-sliding transmitting array, and the dashed line is the RE-MUSIC spectrum of post-sliding transmitting array. From the figure, it can be seen that the true angle peaks of pre-sliding transmitting array coincides with the true angle peaks of post-sliding transmitting array, while the ambiguity angle peaks are separated. The DODs estimation obtained by the RE-MUSIC spectrum of pre-sliding transmitting array are (-55.74°, -41.06°, -13.51°, 9.99°, 20.07°, 50.03°); The DODs estimation obtained by the RE-MUSIC spectrum of post-sliding transmitting array are (-53.78°, -39.73°, -12.5°, 10.01°, 19.99°, 49.89°). The absolute value of the difference between the two data sets are (1.89°, 1.33°, 1.01°, 0.02°, 0.08°, 0.14°). So the true DODs are corresponding to three smallest absolute values. And the true DODs estimation are (10°, 20.03°, 50.03°) which is mean value of the two data sets. The rest DODs estimation is ambiguity angle. Thus the true DODs estimation is obtained. And the minimum discrimination is 0.87°. It can be seen from Figure 2 and the parameter that the performance of eliminating ambiguity angle is excellent. Then the DOAs estimation (15.11°, 60.25°, 35.07°) can be obtained by the RE-MUSIC spectrum of the receiving array as Figure 3. Figure 4 shows the auto-pairing result of the DODs and DOAs.

3. Conclusion
For solving the manifold ambiguity in MIMO radar, a new method is proposed based on PE-MUSIC algorithm by sliding the array. The true angle estimation can be got by comparing with the RE-MUSIC spectrum of the two receiving signals. As the simulation, it is simple to distinguish between true and ambiguity angle estimation from the results by the proposed method. And the DODs and
DOAs of targets can be automatically matched. The validity of the proposed method is verified. However, this method is not applicable to the case where both the transmitting and receiving arrays are sparse arrays.

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