Study The Behavior of Pile Group Under Torsional and Horizontal Load

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Abstract. Many structures are subjected to torsional and horizontal loads in addition to vertical load lead that the foundation of these structures may be collapsed. The proposed model for pile groups consists of an elastic beam-column that represents the pile, rigid cap and nonlinear spring to simulate the soil. In the horizontal direction, the p-y curve method is used to calculate the subgrade reaction of soil while the τ-θ curve method and the load transfer method are used in the vertical direction. The effect of pile group and load coupling is considered by the p multiplier method in the pile head. A parametric study is carried out under the combined action for horizontal and torsional loads. The study shows that unequal shear force distribution in the head of piles is very significant. The axial force variation in each pile with different length is obtained. The study is very important to take into account the design pile group under combined loads.

1. Introduction

Some engineering structures such as offshore oil platforms, cross-sea bridges, towers, and trans-sea transmission towers are subjected to horizontal loads by water waves, ship impacts, and wind forces. These forces are transmitted to the foundation in the form of eccentricities loads and cause torsion and complex structural responses. Many Bridges have been subjected to ship collision accidents, and the foundations of these Bridges have been collapsed [1]. Many researchers have carried out theoretical and experimental studies on a pile or pile groups under horizontal loads [2-6], but there are few studies on the effects of pile groups under horizontal and torsional loads. Hu et al. [7] study the experimental behaviour of a single pile subjected to torsion load, the result of the test shows the torsion is reducing the horizontal capacity by 30% to 50%. Abdulameer Q. [8] studied the experimental behaviour of a single pile subjected to pure torsion load for different piles section and different soil type, the result of the tests show the pile section and sand soil is very sensitive to torsion load. Konj et al. [9] carried out many experimental studies and theoretical models on pile groups under twisting load. He was found that the strong coupling effect directly between torsional and horizontal degrees of freedom of all piles, the torsion has little influence on the lateral capacity of the pile, and there are significantly improve in torsional capacity when the pile is trusted. The present work used a calculation method to study the behavior of pile groups under torsional and horizontal loads and discusses the force law of pile groups under combined loads.
2. Calculation Method

2.1. Theoretical Model
Under the torsional and horizontal loads, the pile's cap is subjected to horizontal displacement and rotation, and the piles are subjected to complex forces and moments. The displacements, forces, and moments on the piles will be not the same. Figure 1 shows the piles cap under torsional and horizontal loads.

![Figure 1. The pile's cap under torsional and horizontal loads.](image)

This study presents a new model to calculate the behavior of pile group as shown in Figure (2). The new model assumed the following: Piles are assumed elastic beams, the pile's cap is regarded as rigid and has a large thickness, and the soil action is replaced by nonlinear springs. Horizontal spring (p–y curve), torsion spring (τ–θ curve) and friction vertical spring (τ–z curve) are used in the side of the pile, vertical springs (q–z curves) and torsion springs (Tt –θ curves) are used in the pile end. The p-y curve method depended on the lateral loading at the pile head, and the spring is in the same plane of the pile body. However, the p-y curves method not applicable under horizontal and torsional loads, because the forces and moments are applied in different planes. To solve the above problem, vertical and horizontal springs are used at each node in the pile.

![Figure 2. Group pile Model.](image)

2.2. Cap Forces and Displacements
The total coordinate system of the model at the center of the cap for calculation, the total coordinate system is translated to the top of the pile to form the local coordinate system for each pile. Single pile calculations are done according to the local coordinate system; each coordinate system is defined in the basic physical quantity as shown in Figure (2) \[10\].

Cap center displacement vector \( s^T = [S_x \ S_y \ S_z \ \theta_x \ \theta_y \ \theta_z] \)

External load vector \( F^T = [F_x \ F_y \ F_z \ M_x \ M_y \ M_z] \)

Pile foundation displacement vector \( (s^1)^T = [S_{x1} \ S_{y1} \ S_{z1} \ \theta_{x1} \ \theta_{y1} \ \theta_{z1}] \)

Back pile force vector \( (s^1)^T = [F_{x1} \ F_{y1} \ F_{z1} \ M_{x1} \ M_{y1} \ M_{z1}] \)

Where \( s \) is the displacement, the rotation angle, \( F \) is the force, and \( M \) is the bending moment.

### 2.2.1. Equations of Equilibrium force

The cap is subjected to the external load and the reaction force of each pile is taken to the center of the cap (origin), the balance forces expression is as follows;

\[
\begin{align*}
F_x &= \sum F_{x_i} \\
F_y &= \sum F_{y_i} \\
F_z &= \sum F_{z_i} \\
M_x &= \sum (-F_{y_i} z_i + F_{z_i} y_i + M_{x_i}) \\
M_y &= \sum (F_{x_i} z_i - F_{z_i} x_i + M_{y_i}) \\
M_z &= \sum (-F_{x_i} y_i + F_{y_i} x_i + M_{z_i})
\end{align*}
\]

Where \( F = \Pi(S) = \sum (A^i F^i) \) (1)

Among them, \( \Pi(S) \) is the total reaction force of each test pile head, and \( A_i \) is the \( i \)-th pile conversion \[10\].

The matrix, \( A^i = \begin{bmatrix}
1 & 0 & 0 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 & 0 & 0 \\
0 & 0 & 1 & 0 & 0 & 0 \\
0 & -z_i & y_i & 1 & 0 & 0 \\
z_i & 0 & -x_i & 0 & 1 & 0 \\
-y_i & x_i & 0 & 0 & 0 & 1
\end{bmatrix} \)

### 2.2.2. Displacement equations at each pile head

In the rigid cap, after the displacement of the cap is determined, the displacement of each pile head can be derived through the displacement of the cap, which has the following relationship \[10\]:

\[
\begin{align*}
S_x^i &= S_x + z_i \theta_y - y_i \theta_z \\
S_y^i &= S_y - z_i \theta_x - x_i \theta_z \\
S_z^i &= S_z + y_i \theta_x - x_i \theta_y \\
\theta_{x1}^i &= \theta_x \\
\theta_{y1}^i &= \theta_y \\
\theta_{z1}^i &= \theta_z
\end{align*}
\]

Where \( S^i = (A^i)^T S \) (2)

### 2.3. Stiffness Matrix Definition
The total reaction force $\Pi(S)$ at each pile head is regarding the displacement of the cap [10],

$$
\frac{d\Pi}{d\bar{S}} = \frac{d}{d\bar{S}} \left[ \sum (A_i^T f_i) \right] = \frac{d}{d\bar{S}} \left[ \sum (A_i^T f_i) \right] dS^i
$$

(3)

by substitute equation 2, one can obtain

$$
\frac{dS^i}{d\bar{S}} = (A_i)^T
$$

(4)

by substitute equation 4 in equation 1 yields to,

$$
\frac{d}{d\bar{S}} \left[ \sum (A_i^T f_i) \right] = \sum A_i \frac{d f_i}{dS^i}
$$

(5)

the i-th pile head stiffness matrix:

$$
K_i = \frac{d f_i}{dS^i}
$$

(6)

The simultaneous equations (4) to (6) define the total stiffness matrix $K$:

$$
K = \frac{d\Pi}{d\bar{S}} = \sum A_i K_i \left( A_i \right)^T
$$

(7)

2.4. Solving of The Equation

2.4.1. Newton Iteration Method

Newton iterative method has quadratic convergence characteristics, which brings good convergence speed and is more suitable for nonlinear calculation. According to the load balance equation (1) [7].

$$
\Phi(S) = \Pi(S) - F = 0
$$

(8)

Using the Newton iteration method, In the case where the n-th approximate solution $S(n)$ is obtained, $\Phi(S(n+1))$ can be expressed as a Taylor expansion in which only the linear term is retained near $S(n)$, that is,

$$
\Phi\left( S^{(n+1)} \right) \approx \Phi\left( S^{(n)} \right) + \frac{d\Phi}{dS}\bigg|_{S^{(n)}} \Delta S^{(n)} = 0
$$

(9)

Introducing the total stiffness matrix (7), gives

$$
\frac{d\Phi}{dS}\bigg|_{S^{(n)}} = \frac{d(\Pi(S) - F)}{dS}\bigg|_{S^{(n)}} = \sum A_i K_i \left( A_i \right)^T
$$

(10)

Based on,

$$
\Delta S^{(n)} = \left[ \sum A_i K_i \left( A_i \right)^T \bigg|_{S^{(n)}} \right]^{-1} \left[ \Pi(S) - F \right]
$$

(11)

$$
S^{(n+1)} = S^{(n)} + \Delta S^{(n)}
$$

(12)

2.4.2. Flow Chart

To analyze the nonlinear behaviour of pile group under the horizontal and torsional loads based on the above calculation model, MATLAB program is used to simulate the calculation model. The calculation process is shown in Figure 3.
Figure 3. Flow chart of calculation.
3. Verification Model

Che et al. [10] conducted a large-scale model test on the horizontal and torsional load of a 3×3 pile group. The test pile is made of steel pipe pile; the pile diameter D is 114 mm, the wall thickness is 4.5 mm, the pile length is 5.95 m, and the length above the mud surface is 2.54 m, and the pile end is a cone-shaped pile tip. The cap is a cast-in-place reinforced concrete with dimensions 1026 × 1026 mm and 300 mm thick, and the pile spacing is 3D, as shown in Figure 4. The test soil sample was sand, soil parameters: K=23464 kN/m³, internal friction angle 30°, floating weight = 8.8 kN/m³, ultimate side friction = 8 kPa, ultimate end resistance = 1200 kPa. Two different eccentric loadings were used in the test, 6D, and 11D respectively. In the test, the maximum torsion is 21.8 kN.m, and the maximum force is 31.9 and 17.4 kN in two directions. This paper verifies the eccentricity as a 6D test. In the model, the horizontal reaction force and the stiffness calculation of the sand p–y curve proposed by Reese et al. [11]; the vertical reaction force and stiffness calculation are based on the load transfer curve of the sand pile side and pile end proposed by Coyle et al. [12], the ultimate resistance reduction coefficient of the lateral soil of the pile is 0.5; the torsional reaction force and stiffness are calculated by torsional load calculation method and the load transfer curve is proposed by Kong [13]. the torsional reaction force and stiffness are calculated by the above-mentioned method; the load transfer curve is proposed by Kong [13]. The calculation parameters of the load transfer curve are calculated in the test, the results of a single pile torsion test were obtained, A=4.5×10⁷ N/m², B=8800×0.23z N/m², At=1090 N/m, Bt = 52 N/m, β=2.0.

In this paper, the torsional frictional resistance of the pile side varies linearly with the depth z. The elastic modulus of the pile is E=2.06 MPa, and the shear modulus G=7.9 MPa, see Figure 4.

![Figure 4. (3×3) Pile group test under combined loads.](image)

It can be seen from the analysis that the calculation results of the horizontal load-displacement curve and the torsion-angle curve of the pile cap are in good agreement with the measured results, which reflects the behaviour of the pile group under horizontal and torsional loads, as shown in Figure 5.
Under the horizontal loads, the piles in the front of the pile group are the most stressed, also when the piles are subjected to torsional loads all the piles have the same force state. In Figure 6 (T indicates that the internal force of the pile head is obtained by the test, and S indicates that the internal force of the pile head is calculated; the numbers represent different horizontal force load values). Under the combined action of horizontal and torsional loads, the internal force distribution in each pile head combines the characteristics of the two alone and is more complicated. The maximum internal force in the pile head appears in the corner pile. Among them, the total bending moment and total shearing force of the pile 7 are the largest. The calculation results and measured results are reflecting this fact. Pile 5 has the smallest bending moment and pile 6 head has the smallest shearing force.

Figure 5. Load, Displacement, and Angles curves of cap center.
4. Parameter Study

4.1. Influence of Pile Spacing on Shear Force.

When the pile spacing increases, the force arm of each pile to the center of the cap increases also the horizontal force in the pile head which resists the torsion is reduced so that the total shear force of the pile head is reduced. For smaller spacing, the internal force distribution in the pile head is more significant. The pile 7 in the front row is the most sheared, and the maximum shear force in the pile head is about 1.6 to 2.4 times the nominal average shear force in the pile 1, the non-corner pile 4 and pile 8 are also subject to a large shearing force, as shown in Figure 7. The load change has little effect on the total shear force distribution in the pile head, mainly because the horizontal stiffness of the pile has little change within the horizontal load range.
4.2. Influence of Pile Cap Thickness on Axial Force.

For small pile cap thickness, Horizontal and vertical stiffness for pile become greater, and the inclination of the cup is smaller, when pile cup height 2.6, 2.0, 1.3, 0.7 m the y-direction inclination is 2.9°, 1.9°, 1.0°, 0.4°. When the pile group are affected by the horizontal and torsional loads, the front piles are pressed and the rear piles are taken 30% of the total horizontal force. For increase the cup thickness the axial force becomes greater (upper/lower pressure). Figure (8) show the ultimate pile capacity of the piles under different cap thickness. The change in the cap thickness will not effect on the vertical ultimate bearing capacity of the pile, Therefore, when the horizontal force of the pile is large, the axial force of each pile is the same. In the calculation of the fails of the pile group, the back row reaches the ultimate bearing capacity. Therefore, when analyzing the pile group foundation subjected to complex loads, it must be attention pile pullout performance.

Figure 7. Shear Force Distribution on the pile head with different spacing between piles.

Figure 8. Variation of axial force of pile head in 3×3 pile group with different free lengths.
5. Conclusion

In this paper, a nonlinear proposal model of group piles has been established under the action of horizontal and torsion. The following are the summarized conclusions of the present proposal model:

a. The method proposed in this paper is mainly for the analysis of the pile group under horizontal and torsional loads.

b. The proposed model gave very acceptable results with test result for group piles under combined horizontal and torsional loads.

c. Under the combined action of horizontal and torsional loads, the shear force distribution of each pile in the pile group is very complicated, and when a small spacing between the pile the shear force is more uneven in the distribution.

d. The axial force of each pile in the pile group is change with the applied horizontal force and the thickness of the pile cup.

e. When applied combined load on pile groups, it should be attention to the pile pull resistance.

f. The spacing between the pile can improve the mechanical properties of pile groups.

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