Investigation of binary chemical reaction in magnetohydrodynamic nanofluid flow with double stratification

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Abstract
This article addresses MHD nanofluid flow induced by stretched surface. Heat transport features are elaborated by implementing double diffusive stratification. Chemically reactive species is implemented in order to explore the properties of nanofluid through Brownian motion and thermophoresis. Activation energy concept is utilized for nano liquid. Further zero mass flux is assumed at the sheet’s surface for better and high accuracy of the out-turn. Transformations are used to reconstruct the partial differential equations into ordinary differential equations. Homotopy analysis method is utilized to obtain the solution. Physical features like flow, heat and mass are elaborated through graphs. Thermal stratified parameter reduces the temperature as well as concentration profile. Also decay in concentration field is noticed for larger reaction rate parameter. Both temperature and concentration grows for Thermophoresis parameter. To check the heat transfer rate, graphical exposition of Nusselt number are also discussed and interpret. It is noticed that amount of heat transfer decreases with the increment in Hartmann number. Numerical results shows that drag force increased for enlarged Hartmann number.

Keywords
Nanofluid, MHD, linear stretching, thermal stratification, activation energy, chemical reaction

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Introduction
Having amazing heat transfer characteristics in contrast with usual heat transit-fluids, nanofluids with astonishing heat transit characteristics is the most discoursed topic of present time. Nanofluids contain nanoparticles with size under 100 nm. Over traditional heat transit fluids, nanofluids offer special influence. Sensational development in the thermal effects of host fluids is produced when a very slight quantity of nanoparticles suspended thoroughly and dissipates constantly in the base fluids. To generate steady and highly conducted nano-fluids one step and two step methods are applied but creating nanoparticles with both methods experience cluster of nanoparticles. This is the pivotal issue in industrial science including nano-

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powder. Nanofluid was introduces by Choi and Eastman. After that different researchers analysed many properties of nanofluids. Evans et al. discussed the Brownian motion effect on nanofluid thermal conductivity. He investigated that thermal conductivity can be increased. Yacob et al. presented nanofluid flow along with convective boundary condition passing through a stretching surface. Nanofluid flow accompanied by a nonlinear stretchable surface was elaborated by Nadeem and Lee. Influence of viscous dissipation of Copper-water nanofluid along with side by side plates was discussed by Sheikholeslami and Ganji. Hatami and Ganji examined the MHD nanofluid flow with suction between parallel disks. Hayat et al. discussed the thermal radiation effect of Powell-Eyring nanofluid and squeezing flow of carbon nanotubes. Ramana Reddy et al. studied the inclined MHD unsteady flow of a nanofluid with Hall current and soret effects. They observed that enhanced soret number strengthens the momentum boundary layer thickness. Ayub et al. inspected the nanofluid flow with MHD, slip effects and Riga plate. Impact of Brownian motion on MHD nanofluid over variable stretching sheet was discussed by Jayachandra Babu and Sandeep. It is found that velocity slip parameter diminishes the velocity field. Ramana Reddy et al. considered the thermophoresis and slip effects on MHD nanofluid. They used R.K. Fehlberg method for solving this problem and found that unsteadiness parameter decays the concentration profile. Anantha Kumar et al. investigates about slip effects on bioconvective flow of nanofluids passing through a stretching sheet. Surface thickness parameter minimizes the temperature as well as concentration profile. Shah et al. considered the Cattaneo-Christov heat flux model for second grade nanofluid having carbon nanotubes. He investigated that how entropy plays a role in our daily life. Impact of nonlinear radiation on MHD Casson fluid flow was studied by Ramudu et al. They used numerical technique for solving differential equations. It is concluded that brownian motion and thermophoresis parameters strengthens the temperature profile. Consequences of Joule heating on copper and silver nanofluid flow with porous medium was discussed by Shah et al. HAM and shooting methods are compared with the aid of numerical tables.

The process of stratification is illustrated as the formation of assorted layers possessing different densities. Stratification process occurs because of temperature changes, diversity of fluids and differences in concentration. Stratification is the interesting process in the mechanics of convective transportation because it has wide applications in the domains of industrial, natural and engineering processes. These applications include manufacturing processes, in atmosphere involving heterogeneous mixture, industrial food and salinity and thermal stratification mechanisms in oceans, rivers, reservoirs and reservoirs of ground water. Moreover, thermal stratification impedes penetration of oxygen between upper and lower layers of water and thus water be nominated as anoxic by the involvement of biological process and hence, this is illustrated as the disadvantage of it. RamReddy et al. described the impact of non-Darcy porous mediumon nanofluid flow with thermal stratification. Hayat et al. scrutinized the flow of an Oldroyd-B fluid with thermal stratification and stagnation point. Sheremet et al. looked into the consequences of nanofluid filled in a square porous cavity under thermal stratification. Abbasi et al. reported the upshot of double stratification and radiation effects on Jeffrey nanofluid. Muhammad et al. elaborated the features of thermal stratification in ferromagnetic fluid with stagnation point. Rehman et al. flashes the characteristics of flat and cylindrical surfaces on tangent hyperbolic fluid with thermal radiations. Impact of thermal stratification and Joule heating on MHD nanofluid flow was considered by Daniel et al. Rehman et al. investigated the double stratification phenomenon on tangent hyperbolic fluid. Hayat et al. studied the characteristics of heat absorption in Oldroyd-B fluid with convective boundary conditions.

The process of mass transfer along with chemical reaction energy has been observed by many researchers due to its consequences in chemical engineering, geothermal reservoirs, cooling of nuclear reactor and in oil recovery. Activation energy is the most important factor in chemical reaction. It is the fewer amount of energy for atoms or molecules due to which they experience a chemical reaction. Bestman initially investigated the mass transfer of boundary layer flow in a porous medium with suction. Mass transfer flow of MHD thermally radiative fluid with activation energy was discussed by Maleque. Shaﬁque et al. declared the behaviour of activation energy on Maxwell fluid. Influence of thermal reaction on Casson ﬂuid with stagnation point was studied by Abbas et al. Zulfiqar et al. analysed the soret, dufour effects on MHD ﬂuid flow with uniform suction/injection. Mustafa et al. explored the features of chemical reaction on the MHD nanofluid flow passing through a vertical surface. Outcomes of chemical reaction on Casson ﬂuid flow over a rotating cone with Hall effect was inspected by Deebani et al. He concluded that enhanced Brinkman number decays the Bejan number. Dawar et al. considered the Williamson nanofluid with activation energy. It is noted that Williamson parameter decays the drag force.

Magnetohydrodynamic nanofluid flows are very useful in engineering as well as in biomedicines. They have wide applications in nuclear power plant, nanofluid as a coolant, in fuels like Copper-oxide brake nanofluid (CBN), AOBN etc, magnetic nanoparticles.
for cancer therapy and many more. Aforeknown literature surveys designates that researchers highlighted the characteristics of Magnetohydrodynamic nanofluid flows. Although peculiarity of thermally stratified flows because of stretching sheet with chemical reaction have not been explored until now. Therefore our main objective is to investigate the behaviour of chemical reaction on MHD nanofluid flow over a stretching sheet. Thermal stratification is also contemplated. We considered variable temperature both at the surface and away from the wall. The emerging dominating equations are figure out by homotopic technique.\textsuperscript{31,34-42} The consequences of demanding parameters are inspected through graphs. Additionally, drag force and heat transfer rate has been elaborated graphically.

**Formulation**

Two dimensional and steady state flow of MHD nanofluid passing through a stretching sheet is taken with thermal stratification. Energy and concentration equations are used to discuss the temperature and concentration profiles. Velocity is contemplated at the surface of the plate. Variable temperature is considered at wall that is, $T_w = T_0 + bx$. Similarly, surrounding temperature that is, $T_v = T_0 + dx$ is also taken variable. Concentration is assumed to be variable at surface and ambient fluid. After utilizing boundary layer approximations the ruling equations takes the succeeding forms:\textsuperscript{31} see Figure 1 below.

\begin{align}
\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} &= 0, \quad (1) \\
\frac{u}{\partial x} + v \frac{\partial u}{\partial y} &= \frac{\partial^2 u}{\partial y^2} - \frac{\sigma^+}{\rho_f} B_0^2 u, \quad (2)
\end{align}

\begin{align}
\frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} &= \alpha \frac{\partial^2 T}{\partial y^2} + \tau \left[ D_B \frac{\partial C}{\partial y} + \frac{D_T}{T_v} \left( \frac{\partial T}{\partial y} \right)^2 \right]. \quad (3)
\end{align}

\begin{align}
\frac{\partial C}{\partial x} + v \frac{\partial C}{\partial y} &= D_B \frac{\partial^2 C}{\partial y^2} + \frac{D_T}{T_v} \frac{\partial^2 T}{\partial y^2} \\
- k_e^2 (C - C_\infty) \left( \frac{T}{T_v} \right)^n \exp \left( -\frac{E_a}{kT} \right), \quad (4)
\end{align}

Here velocity components are denoted by $u$ and $v$ in $x-$ and $y-$ directions respectively, $U_w(x) (= ax)$ is the stretching velocity, Magnetic field is denoted by $B_0$, $T$ corresponds to fluid temperature, Wall temperature is $T_w(x) (= T_0 + bx)$, $T_v(x) (= T_0 + dx)$ denotes the ambient fluid temperature, Brownian motion parameter is set as $D_B$, $D_T$ figure out thermophoresis parameter, $C$ represents fluid concentration, $k_e$ exhibits reaction rate, $E_a$ corresponds to activation energy, $b$, $d$ and $e$ are the dimensional constants, Boltzmann constant is denoted by $K (= 8.61 \times 10^{-5} eV/K)$, fitted rate constant is represented by $n$ and normally its range is $-1 < n < 1$.

Using

\begin{align}
\theta(\eta) &= \frac{T - T_v}{T_w - T_0}, \quad \Phi(\eta) = \frac{C - C_\infty}{C_v},
\end{align}

\begin{align}
\theta'' + f' \theta' - f'' \theta = S c f' + \frac{N_f}{N_v} \theta', \quad (5)
\end{align}

\begin{align}
\frac{1}{Pr} \theta'' + f \theta' - f'' \theta = S c f' + \frac{N_f}{N_v} \theta', \quad (6)
\end{align}

\begin{align}
\frac{1}{Pr} \theta'' + f \theta' - f'' \theta = S c f' + \frac{N_f}{N_v} \theta', \quad (7)
\end{align}

\begin{align}
\theta'' + f' \theta' = S c f' + \frac{N_f}{N_v} \theta', \quad (8)
\end{align}

\begin{align}
\frac{N_f \Phi'}{N_v} + \frac{N_f \theta'}{N_v} \Phi' = 0, \quad (9)
\end{align}

\begin{align}
\theta(0) = 1, \quad \phi(0) = 0, \quad f'(\infty) = 0, \quad (10)
\end{align}

\begin{align}
\phi(0) = 1 - S_1, \quad \theta(\infty) = 0, \quad (11)
\end{align}

\begin{align}
N_f \Phi'(0) + N_v \theta'(0) = 0, \quad \Phi(\infty) = 0, \quad (12)
\end{align}

In the above equations Hartmann number is represented by $M$, Stratification parameter is denoted by
$S_1, \text{Pr}$ denotes Prandtl number, $Sc$ expresses the Schmidt number, $Nb$ flashes the Brownian motion parameter, $N_b$ stands for Thermophoresis parameter, $\sigma$ appears as dimensionless reaction rate, Non dimensional activation energy corresponds to $E$, $\delta$ exhibits the temperature difference parameter. Mathematical form of these parameters are as follows:

$$
M = \frac{\alpha^2 R_0^2}{\rho c}, \quad \text{Pr} = \frac{\nu}{\alpha}, \quad S_1 = \frac{d}{b}, \quad Sc = \frac{\nu y}{D_b}, \quad N_b = \frac{\tau D_b C_a}{\nu}, \quad N_t = \frac{\tau D_f (T_w - T_c)}{\nu T_c}, \quad \sigma = \frac{k^2}{c}, \quad E = \frac{E_f}{T_c}, \quad \delta = \frac{(T_w - T_c)}{T_c}.
$$

(14)

Mathematical expression of skin friction coefficient is disclosed as follows:

$$
C_f = \frac{\tau_w}{\rho C_f w},
$$

(15)

Undimensioned mode is given as

$$
C_f R_e^{1/2} = f'(0),
$$

(16)

Here, $Re_c = U_0 x / \nu$ denotes local Reynolds number. Nusselt number for the present analysis is given as

$$
Nu R_e^{1/2} = - \frac{\theta'(0)}{1 - S_1}.
$$

(17)

**Homotopic solutions**

Liao $^{34}$ was the first who presented Homotopy analysis method in 1992. We get series solution of highly non-linear problems with this method. This gave us great opportunity to choose initial guesses and linear operators. They are declared as given below:

$$
\begin{align*}
\dot{f}_0(\eta) &= (1 - \exp(-\eta)), \\
\dot{\theta}_0(\eta) &= -\exp(-\eta) (1 + s_1), \\
\dot{\Phi}_0(\eta) &= \exp(-\eta) N_b (1 + s_1) \\
\end{align*}
$$

(18)

$$
\begin{align*}
L_f(f) &= \frac{d^2 f}{d \eta^2} - \frac{d f}{d \eta}, \\
L_{\theta}(\theta) &= \frac{d^2 \theta}{d \eta^2} - \theta, \\
L_{\Phi}(\Phi) &= \frac{d^2 \Phi}{d \eta^2} - \Phi = 0.
\end{align*}
$$

(21)

With

$$
\begin{align*}
L_f([C_1 + C_2 \exp(\eta) + C_3 \exp(-\eta)]) &= 0, \\
L_{\theta}([C_4 \exp(\eta) + C_5 \exp(-\eta)]) &= 0, \\
L_{\Phi}([C_6 \exp(\eta) + C_7 \exp(-\eta)]) &= 0,
\end{align*}
$$

(22)

Here $C_i (i = 1, \ldots, 7)$ denotes the capricious constants.

**Convergence analysis**

Convergence of series solution can be secured by HAM. It depends over an auxiliary parameters $h_f, h_{\theta}$ and $h_{\Phi}$. Figures 2 and 3 declares the $h$-curves for velocity, temperature and mass equations. The permissible ranges of $h_f, h_{\theta}$ and $h_{\Phi}$ are $-1.6 \leq h_f \leq -1.0, -0.5 \leq h_{\theta} \leq -0.2$ and $-0.6 \leq h_{\Phi} \leq -0.1$.

**Discussion**

This section contains graphical results of various opposite parameters for velocity, temperature and concentration distributions. Influence of Hartmann number $M$
on the velocity field is sketched in Figure 4. Decrement in velocity field is noticed for enlarged values of Hartmann number $M$. Also momentum boundary layer thickness decays. It is because with enhanced Hartmann number $M$ produces a large amount of Lorentz force which is subjected to reduction of velocity. The outcomes of Hartmann number on temperature field is displayed in Figure 5. Temperature profile grows clearly when Hartmann number $M$ grows. Thickness of thermal boundary layer also expands. The reason behind this is the production of Lorentz force. Maximum resistance produces because of Lorentz force due to which more heat will be generated and ultimately temperature profile raises. Analysis of Brownian motion parameter $N_b$ on the temperature field is illustrated in Figure 6. Both the temperature field and thermal boundary layer thickness grows with an increment of Brownian motion parameter $N_b$. With the increase in $N_b$ collisions between fluid particles will increase. Due to this more heat will be generated. Figure 7 is plotted for Brownian motion parameter $N_b$ versus concentration field. It is remarked that concentration field shows decaying behaviour for Brownian motion parameter $N_b$. With the increase in Brownian motion collisions between fluid particles will be maximum. Therefore small quantity of mass is relocated and thus downfall in concentration distribution is noticed. Figure 8 is plotted for noticing the consequences of thermophoresis parameter $N_t$ on temperature distribution. Intensified temperature profile is observed when Thermophoresis parameter $N_t$ increases. Increase in thermal boundary layer thickness is noticed too. Actually the particles near the heated plate have maximum temperature rather than the particles away from the plate. Therefore heated particles shifts heat to cold particles and hence temperature enhanced. Figure 9 is portrayed to display the response of Thermophoresis parameter $N_t$ on concentration profile. Both concentration field and solutal boundary layer thickness grows when Thermophoresis parameter $N_t$ expands. Physically, thermophoresis force grows for enhanced thermophoresis parameter. So more heat transfer will occur and leads to higher diffusive effects. Figure 10 describes the significance of thermal stratification parameter $S_1$ on the temperature
field. Decrease in thermal stratification parameter is noticed when temperature field increases. Lower region has maximum density for enlarged thermal stratification parameter $S_1$. As a outcome, heated wall produces resistance in the flow towards the surrounding wall. Consequences of thermal stratification parameter $S_1$ on concentration field is exposed in Figure 11. Concentration field is detected to increase with the enhanced thermal stratification parameter $S_1$. Both thermal and solutal boundary layer thickness decays when thermal stratification parameter $S_1$ increases. With the increase in $S_1$ density difference increases between the layers of fluid particles. This density difference creates a barrier for mass transfer. Hence decrement occurs in concentration field. Figure 12 reflects the change in Prandtl number $Pr$ versus temperature field. Here we examine that temperature field decreases when Prandtl number $Pr$ grows. Thermal diffusivity decreases as Prandtl number enhances and temperature field reduces. Figure 13 flashes the effect of dimensionless reaction rate parameter $\sigma$ on concentration field. It is esteemed that concentration field decreases for growing dimensionless reaction rate parameter $\sigma$. It is revealed that the factor $\sigma(1 + \delta \theta)^n \exp \left( \frac{-E}{RT} \right)$ enhances for dominant $\sigma$. Therefore, concentration gradient enhances at the wall. Hence concentration profile diminishes. Figure 14 reflects the deviation of Schmidth number $Sc$ on concentration field. It is noticed that concentration profile raises for larger values of Schmidt
number. Momentum diffusivity enhanced for increment in Schmidt number $Sc$. Therefore concentration distribution raises. Impact of wall temperature difference parameter $\delta$ on the concentration field is depicted in Figure 15. Concentration field shows increasing trend when temperature difference parameter $\delta$ gradually increases. Dominant values of $\delta$ result in higher energetic fluid particles which consequently grows the mass transfer. Thus concentration field enhances. Figures 16 and 17 are plotted for numerous values of activation energy $E$. Enhanced activation energy results in destructive chemical reaction due to which temperature raises and concentration decreases. Figures 18 and 19 are plotted to check the heat transfer rate corresponding to Hartmann number $M$, thermophoresis parameter $N_t$, Prandtl number $Pr$ and stratification parameter $S_1$. It is perceived that heat transfer rate reduces for dominant Hartmann number $M$ and it grows for higher thermophoresis parameter $N_t$. Physically, Lorentz force produces maximum resistance. Due to this temperature field increases and hence less heat will be transferred to the environment. Therefore heat transfer rate diminishes. Similarly heat transfer rate increases for growing Prandtl number $Pr$ and stratification parameter $S_1$. This is because enhanced Prandtl number and stratification parameter...
weakens the temperature field. So maximum heat will be transferred from fluid to environment. Therefore, heat transfer rate increases.

Table 1 is constructed for the inspection of skin friction ($f''(0)$) with published work of Hayat et al.\textsuperscript{43} in limiting case. Both results are in good manner.

### Table 1. Comparison of drag force for distinct values of Hartman number $M$ when $Pr = 1$, $Nb = Nt = Sc = S_l = E = n = \delta = \sigma = 0$

| $M$ | Hayat et al.\textsuperscript{43} | Present |
|-----|-----------------|---------|
| 0   | 1.00000         | 1.00000 |
| 1   | −1.41421        | −1.41421|
| 5   | −2.44948        | −2.44949|
| 10  | −3.31662        | −3.31662|

It is hoped that the present study subsidized as a motivation for representing supplementary Magnetohydrodynamic nanofluid flows mainly in nuclear reactors, accelerators, flow metres, MHD generators, designing of cooling systems and biomedicines. This article may be used in nano drug delivery, cancer therapy, cell separation, automotive industry and solar energy harvesting.

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