String duality and massless string states

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ABSTRACT

We are discussing the $S$ & $T$ duality for special class of heterotic string configurations. This class of solutions includes various types of black hole solutions and Taub-NUT geometries. It allows a self-dual point for both dualities which corresponds to massless configurations. As string state this point corresponds to $N_R = 1/2$ and $N_L = 0$. The string/string duality is shortly discussed.

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Duality symmetries have played an important role for a better understanding of different string backgrounds. They are given by field redefinition and usually relates different geometries or even different topologies what is unknown from point particle physics. Mainly we have two types, one which are valid off-shell (as symmetry of the low energy action), e.g. $T$ duality, or which are valid on-shell only (as symmetry of the equation of motion), e.g. $S$ duality. Both dualities relate one heterotic background to another one. But there are also duality transformations of completely different kind. One example is the string/string duality in $D = 6$ which transforms a heterotic background to type IIA background, and hence relates different types of string theories to each other. For a nice review of this topic see Ref. 1. To be correct one has to admit that as well the string/string duality as the $S$-duality are still conjectures, although the evidence for the latter one is overwhelming.

The usual starting point in discussing dualities is the 10-dimensional effective action

$$S_{10} = \int d^{10}x \sqrt{G} e^{-2\phi} \left[ R + 4(\partial \phi)^2 - \frac{1}{12} \hat{H}^2 \right] + \mathcal{O}(\alpha') + \text{(higher genus terms)} .$$

(1)

Assuming that the fields do not depend on 6 coordinates (internal) we can reduce this action down to 4 dimensions (for details see e.g. Ref. 2)

$$S_4 = \int d^4x \sqrt{G} e^{-2\phi} \left[ R + 4(\partial \phi)^2 - \frac{1}{12} H^2 + \text{KK-gauge fields + KK-scalars} \right] + \mathcal{O}(\alpha') + \text{(higher genus terms)} .$$

(2)

The orign of the $T$-duality is a space time symmetry of the 10d theory, namely the existence of a Killing vector. Let us assume that $u$ is the symmetry direction and $I, J$ are the other coordinates, then the $T$-duality can be written as

$$
\begin{align*}
G_{uu} &\rightarrow 1/G_{uu} , \quad G_{uI} \rightarrow B_{uI}/G_{uu} , \quad B_{uI} \rightarrow G_{uI}/G_{uu} \\
G_{IJ} &\rightarrow G_{IJ} - (G_{uI}G_{uJ} - B_{uI}B_{uJ})/G_{uu} \\
B_{IJ} &\rightarrow B_{IJ} - (G_{uI}B_{Ju} - G_{uJ}B_{Iu})/G_{uu} , \quad 2\phi \rightarrow 2\phi - \log G_{uu} .
\end{align*}
$$

(3)

On the other side the $S$-duality is implemented in the 4d theory. First we have to go to the canonical frame by $G_{\mu\nu}^c = e^{-2\phi}G_{\mu\nu}$, express the torsion by the axion $a(x)$ ($H = e^{4\phi} * da$) and define a complex scalar $S = a + ie^{-2\phi}$. Then the $S$-duality says
that the equations of motion of (2) are invariant under\(^b\)

\[
S \rightarrow -\frac{1}{S} ,
\]

\[
F^{(m)}_{\mu\nu} \rightarrow -Re S F^{(m)}_{\mu\nu} - Im S (ML)_{mn} * F^{(n)}_{\mu\nu}
\]

where the definition of the matrices \( M \) and \( L \) can be found in the paper of Sen\(^2\) and the dual of the KK-gauge fields are

\[
* F^{(m)}_{\mu\nu} = \frac{1}{2} (\det G^c)^{-\frac{1}{2}} G^c_{\mu\nu' \lambda \rho} \epsilon^{\nu' \nu \lambda \rho} F^{(m)}_{\lambda \rho} .
\]

Since the canonical metric remains unchanged under this transformation the masses are the same too. An \( S \)-self-dual point is given by \(|S|^2 = 1\), where the axion changes only the sign and the dilaton remains unchanged.

Both \( S \) & \( T \) duality are symmetries of one string model. In recent times a complete different type of dualities is increasingly investigated. That is the string/string duality in 6 dimension. This duality is a special case of the string/p-brane duality which maps the string (1-dimensional objects) on p-branes (p-dimensional objects) and acts in \( p + 5 \) dimensional space. In 10 dimensions it is just the string/5-brane duality and in 6 dimensions the string/string duality. There several aspects which make the string/string duality attractiv to investigate. First it is an essential part in the “unification” of the five string theories (type I, IIA, IIB, both heterotic models), see also Ref.\(^4\). Secondly, it interchanges classical (\( \sim \alpha' \)) and quantum corrections (\( \sim e^{2\phi} \)). Therefore, if one has a string model where the \( \alpha' \) corrections are under controll one knows the quantum corrections of the dual model. Implemented is this duality by a field transformation in \( D = 6 \)

\[
H_{MNP} \rightarrow e^{-2\phi} * H_{MNP} \]

\[
G_{MN} \rightarrow e^{-2\phi} G_{MN} , \quad \phi \rightarrow -\phi .
\]

All dualities are expected to be symmetries of the string theory. But usually they are not respected by special backgrounds. Instead, they break this symmetry. Nevertheless we can use these transformations to find new dual backgrounds, which are equivalent from the string point of view. Some special configurations, however, are explicitly invariant under these dualities and these points in the field space correspond to points of enlarged symmetries and often related to additional massless modes.

In this paper we are going to discuss the \( T \) and \( S \) duality for a special background which allow self-dual points and includes many known black hole and monopole

\(^b\)In general the \( S \) duality is given by an \( SL(2, \mathbb{R}) \) transformation. We are considering here only the nontrivial \( SL(2, \mathbb{Z}) \) part.
solutions as special limits. We will find that in the self-dual limit our black holes or monopoles become massless. The model is defined by the following 10d metric, antisymmetric tensor and dilaton

\[ ds^2 = 2F(x) \, du[ dv - \frac{1}{2} K(x) du + \omega_I(x) dx^I ] - dx^I dx^I \]

\[ \hat{B} = 2F(x) \, du \wedge [ dv + \omega_I(x) dx^I ] \] , \[ e^{2\phi} = F(x) . \]

It can be shown that if \( \partial^2 F^{-1} = \partial^2 K = 0, \partial^2 \partial_{[j} \omega_{i]} = 0 \) and \( 2\hat{\phi} = \log F \) it is a solution of (7) and do not receive \( \alpha' \) corrections in a proper renormalization scheme. Furthermore, this model possesses unbroken supersymmetries and be embedded in \( N = 1, D = 10 \) supergravity. It is the natural generalization of the fundamental string (\( K = \omega_I = 0 \)) and the gravitational wave (\( F = 1, \omega_I = 0 \)) background. Since it is independent on \( u \) and \( v \) we can dualize any non-null direction in the \((u, v)\) plane. A suitable form is that we first shift \( v \) by \( v \to v + cu \) (\( c = \text{const.} \)), then dualize \( u \) and afterwards revers the shift. Doing this and using (3) we find that the model does not change, only the harmonic functions \( K \) and \( F^{-1} \) are mixing

\[ F^{-1} \to 2c - K \] , \[ K \to 2c - F^{-1} \] (8)

and the model is explicitly \( T \)-self-dual iff

\[ K + F^{-1} = 2c . \] (9)

Now let us come to the 4d solution. We choose \( v \) as time coordinate and three of the transversal coordinates as spatial part, i.e. \( x^M = (v = t, x^i|x^r, u) \) and get

\[ ds^2 = e^{4\phi}(dt + \omega_i dx^i)^2 - dx^i dx^i \] , \[ \partial_i a = \epsilon_{ijm} \partial_j \omega_m \]

\[ \tilde{A}_{\mu}^{(+)} = -\frac{1}{\sqrt{8}} e^{-2\sigma}(0 , 1 + K F) V_\mu \] , \[ \tilde{A}_{\mu}^{(-)} = -\frac{1}{\sqrt{8}} e^{-2\sigma}(2 F \omega_r , 1 - K F) V_\mu \] (10)

\[ e^{2\sigma} = K F - F^2 | \omega_r |^2 \] , \[ e^{-2\phi} = \sqrt{K F^{-1} - | \omega_r |^2} \]

with \( V_\mu = F(1, \omega_i), a \) the axion and the vector field \( \tilde{A}_{\mu}^{(+)} \) is sum of the KK gauge fields coming from the metric and the antisymmetric tensor whereas \( \tilde{A}_{\mu}^{(-)} \) is just the difference. So, the 4d solution has 7 gauge fields, one graviphoton \( \tilde{A}_{\mu}^{(+)} \), 6 gauge fields belonging to the vector multiplet \( \tilde{A}_{\mu}^{(-)} \) and 8 scalars correlated to the 8 harmonic functions \((K, F^{-1}, a, \omega_r)\). These 8 harmonic functions determine the solution completely. The simplest choice is

\[ K = 1 + \frac{2m}{r} \] , \[ F^{-1} = 1 + \frac{2\bar{m}}{r} \] , \[ \omega_m = \frac{2q^m}{r} \] , \[ a = \frac{2n}{r} \] (11)
where \( r^2 = x^2 + y^2 + z^2 \). Inserting this we get for the metric and dilaton
\[
 ds^2 = e^{4\phi} \left( dt + 2n \cos \theta d\phi \right)^2 - \left( dr^2 + r^2 d\Omega^2 \right), \quad e^{4\phi} = \frac{1}{(1 + \frac{r_+}{r})(1 + \frac{r_-}{r})} 
\]  
with \( r_\pm = m + \tilde{m} \pm \sqrt{(m - \tilde{m})^2 + 4|q_m|^2} \). This is a Taub-NUT geometry, which is asymptotically not flat and has to have a periodic time in order to avoid a white singularity along the axes \( \theta = 0, \pi \). All solutions are extremely charged solutions. Let us therefore ignore this point in the further discussion.

The next possibility is that the harmonic functions are not singular in one point \( (r = 0) \) but along a ring. This can be done by choosing the real and imaginary part of complex harmonic functions
\[
 K = \text{Re} \left( 1 + \frac{2m}{r_\alpha} \right), \quad F^{-1} = \text{Re} \left( 1 + \frac{2\tilde{m}}{r_\alpha} \right), \\
 \omega^r = \text{Re} \left( \frac{2\alpha}{r_\alpha} \right), \quad a = \text{Im} \left( \frac{2\alpha}{r_\alpha} \right), 
\]
with \( r_\alpha^2 = x^2 + y^2 + (z - i\alpha)^2 \). These functions have a singularity in the plane \( z = 0 \) along the circle \( x^2 + y^2 = \alpha^2 \). Then the solution is a rotating black hole
\[
 ds^2 = e^{4\phi} \left( dt + \frac{2n\alpha \sin^2 \theta}{R} d\phi \right)^2 - d\bar{x}^2, \quad e^{-4\phi} = \left( 1 + \frac{r_+}{R} \right) \left( 1 + \frac{r_-}{R} \right), \quad R = \frac{r^2 + \alpha^2 \cos^2 \theta}{r} 
\]
This solution is asymptotically flat and we can define the charges and get for the mass, angular momentum and charges
\[
 M = \frac{1}{2}(r_+ + r_-) = \frac{1}{2}(m + \tilde{m}), \quad J = n \alpha \\
 \bar{Q}^{(+)} = \sqrt{2}(0, M), \quad \bar{Q}^{(-)} = \frac{1}{\sqrt{2}}(-2q^r, (\tilde{m} - m)) .
\]
As expected, we see that the right-handed part (graviphoton) saturates the Bogomol'nyi bound.

Now we have the 4d solutions and can look on the self-dual cases. From (9) and (13) it follows that at the \( T \)-self-dual point (with \( c = 1 \)) \( m + \tilde{m} = 0 \) and therefore \( M = 0 \), i.e. the black hole becomes massless and therefore the right-handed sector is uncharged \( (\bar{Q}^{(+)} = 0) \). Note that the \( T \) duality here corresponds not to an isometry direction in the internal space. Instead, it is a mixing between the time \( (v = t) \) and the internal coordinate \( u \). Thus it is different from the \( T \) duality correlated to \( O(6, 6) \) moduli space. Furthermore, we see that in this case we have an additional singularity at \( r = -r_- \). This singularity is a consequence of the dimensional reduction and exhibits a gravitational repulsion. The 10d theory is non-singular at this point. In addition, we have still the naked singularity at
$R = 0$. But this singularity cannot be removed by uplifting to higher dimensions. However, by assuming that the fields depend on more than three coordinates and making them periodic the naked singularity disappears\cite{10}.

We can also identify this configuration as state in the string spectrum. As usual we consider the mass formula

$$M^2 = \frac{1}{2} \left( |\bar{Q}_e^{(+)}|^2 + 2(N_R - \frac{1}{2}) \right) = \frac{1}{2} \left( |\bar{Q}_e^{(-)}|^2 + 2(N_L - 1) \right) \quad (16)$$

where $N_{R/L}$ are the oscillator modes. For our self-dual configuration we have $M = Q^{(+)} = 0$ and therefore $N_R = \frac{1}{2}$. Since $Q^{(-)} \neq 0$ the only possibility for the left moving sector is given by $N_L = 0$. This elementary state is electrically charged. The magnetic counterpart\cite{11} appear then as soliton excitation related to the electric solution.

Next, we look on the $S$-self-dual point, which was given by $|S|^2 = a^2 + e^{-4\phi} = 1$. Looking on the harmonic functions, we see that this case is only possible for the Taub-Nut geometry. We obtain for the parameter the restrictions

$$m + \tilde{m} = 0 \quad , \quad n^2 = \tilde{m}^2 + |q|^2 \quad (17)$$

and thus it coincides with the $T$-self-dual point with an additional restriction for the parameter $n$.

Finally let me add an remark concerning the string/string dual of this solution. We know\cite{13} that the scalar fields are interchanging, e.g. the dilaton and modulus field. The KK gauge fields from metric remains unchanged and the KK gauge field from the antisymmetric tensor becomes magnetic. But the crucial point is that the canonical 4d metric remains unchanged and thus the dual masses are the same, which means that this massless state is maped again on a massless state on the type IIA side. We will come back in more detail about this solution in a forthcoming paper\cite{12}.

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