Abstract

Heavy baryon chiral perturbation theory is extended to include the effects of quenching. In this framework the leading nonanalytic dependence of the heavy baryon masses on the light quark masses is studied. The size of quenching effects is estimated by comparing the results of quenched and ordinary chiral perturbation theories. It is found that in general they can be large. This estimate is relevant to lattice simulations of the heavy baryon masses.
I. INTRODUCTION

Recent technological developments have made it possible to perform lattice simulations of QCD with increased accuracy. At the same time the precision of the lattice calculations depends on the understanding of the errors due to various approximations used in lattice simulations. One of the most important errors is due to quenching. The modifications induced by quenching at short distances have been studied in systems of quarkonia and are more or less well understood. Because of its nonperturbative nature, the long distance effects of quenching are more difficult to quantify. One approach is to use Quenched Chiral Perturbation Theory (QChPT). The idea was first proposed by Sharpe, with the formalism further developed by Bernard and Golterman. The advantage of using the quenched version of Chiral Perturbation Theory (ChPT) is that its predictions follow from the basic properties of the underlying theory, and ChPT describes the low energy dynamics of ordinary QCD very well.

More recently, QChPT has been extended to describe the interactions of soft pions with baryons, heavy mesons and vector mesons. Here it is further extended to describe the interactions with heavy baryons. Ordinary chiral perturbation theory for heavy baryons has been formulated by a number of authors. We adapt their formalism to quenched QCD.

As every effective theory, ChPT contains a number of undetermined couplings all of which must be fixed from experiment. Only for the self-interactions of pions are there enough data to carry out this procedure beyond the leading order. The situation is even worse in the quenched case, where the only “data” available are extracted from lattice simulations. For this very reason we cannot obtain any accurate quantitative predictions from our calculations. Nevertheless, there are certain loop corrections whose existence is predicted unambiguously by the lowest order Lagrangian, in particular, terms that have nonanalytic dependence on the light quark masses. Contributions of these terms to the matrix elements cannot be absorbed into the higher order terms of the chiral expansion. Furthermore, since the nonanalytic dependence arises from the infrared region of the loops, it is particularly sensitive to the low energy behavior of the theory.

In this paper the nonanalytic corrections to heavy baryon masses are computed in the framework of QChPT. Comparison with the analogous corrections obtained in ChPT shows the qualitative differences between the quenched and unquenched cases. It is also found that these nonanalytic terms could in general be large. This is important for the lattice calculations of the heavy baryon masses, where results are extrapolated linearly in the light quark masses.

Since quenched QCD is not a unitary theory, it is possible that QChPT does not describe its behavior at low energies correctly. However, the most important qualitative feature of QChPT, the presence of quenched logarithms of the form $M_0^2 \ln m_q^2$, was first derived in strong-coupling perturbation theory. Moreover, the coefficients of these terms obtained by the two different approaches are the same. We thus expect that the description of the low energy limit of quenched QCD by QChPT is valid.
II. CHIRAL PERTURBATION THEORY FOR HEAVY BARYONS

Heavy Quark Effective Theory (HQET) and ChPT are combined to describe the interactions of heavy baryons with soft Goldstone mesons. Here we give a brief review of the formalism.

To leading order of chiral expansion, self-interactions of soft pseudoscalar mesons are described by the Lagrangian

\[ \mathcal{L} = \frac{f_\pi^2}{8} \left( \text{Tr}[\partial_\mu \Sigma \partial^\mu \Sigma] + 4 B_0 \text{Tr}[M_+] \right), \tag{2.1} \]

where \( \Sigma = \xi^2 \) and \( \xi = e^{i\phi/f} \). Eight light pseudoscalar mesons are combined in a matrix \( \phi \),

\[
\phi = \begin{pmatrix}
\frac{1}{\sqrt{2}} \pi^0 + \frac{1}{\sqrt{6}} \eta \\
\pi^- \\
-\frac{1}{\sqrt{2}} \pi^0 + \frac{1}{\sqrt{6}} \eta \\
K^- \\
K^0 \\
-\sqrt{\frac{2}{3}} \eta
\end{pmatrix}. \tag{2.2}
\]

The normalization is such that \( f_\pi \approx 130 \text{ MeV} \). The leading symmetry breaking term depends on

\[ M_+ = \frac{1}{2} (\xi^\dagger M \xi^\dagger + \xi M \xi), \tag{2.3} \]

where \( M \) is the quark mass matrix:

\[
M = \begin{pmatrix}
m_u & 0 & 0 \\
0 & m_d & 0 \\
0 & 0 & m_s
\end{pmatrix}. \tag{2.4}
\]

The mass term of (2.1) breaks the \( SU(3)_L \times SU(3)_R \) symmetry which is restored in the limit \( m_u, m_d, m_s \to 0 \). In this limit the eight pseudoscalar mesons become massless. Note that the meson masses are nonlinear in the quark masses.

Heavy baryons contain one heavy quark, \( c \) or \( b \), and two light quarks. According to HQET, in the leading order of heavy quark mass expansion, properties of the heavy baryons depend entirely on the configuration of light quarks and are independent of the spin and the flavor of the heavy quark. The three light quarks, \( u \), \( d \), and \( s \), that form the fundamental representation of flavor \( SU(3) \) can be combined in pairs to form an \( SU_f(3) \) antitriplet and an \( SU_f(3) \) sextet. There is a correlation between the spin wavefunction and the flavor wavefunction of the light quarks; those in the \( SU_f(3) \) antitriplet combine in a spin 0 state and those in the \( SU_f(3) \) sextet have spin 1. The former, combined with a spin 0/2 heavy quark, form a triplet of baryons with \( J^P = \frac{1}{2}^+ \). The spin 1 combination of light quarks together with the heavy quark forms two sextets of baryons with \( J^P = \frac{3}{2}^+ \) and \( J^P = \frac{1}{2}^+ \), which are degenerate in the infinite heavy quark mass limit, \( M_Q \to \infty \). Since the configuration of light quarks in the triplet baryons differs from that in the sextet baryons, the two are not degenerate even in the \( M_Q \to \infty \) limit.

The two sextets of baryons can be described with one field \( S^i_{\mu} \).
\[ S_{ij}^{ij} = S_{3/2}^{ij} + S_{1/2}^{ij} \]

\[ S_{1/2}^{ij} = \frac{1}{3} (\gamma_\mu + v_\mu) \gamma^5 \frac{1 + \gamma^\mu}{2} B_6^{ij} \]

\[ S_{3/2}^{ij} = \frac{1 + \gamma^\mu}{2} B_6^{ij} \],

where \( v^\mu \) is the 4-velocity of the baryon. The baryons with \( J^P = \frac{1}{2}^+ \) are combined in a matrix \( B_6^{ij} \)

\[ B_6^{ij} = \begin{pmatrix} \Sigma_Q^+ & \sqrt{\frac{1}{2}} \Sigma_Q^0 & \sqrt{\frac{1}{2}} \Sigma_Q^{1/2} \\ \sqrt{\frac{1}{2}} \Sigma_Q^0 & \Sigma_Q^{-1} & \sqrt{\frac{1}{2}} \Sigma_Q^{-1/2} \\ \sqrt{\frac{1}{2}} \Sigma_Q^{1/2} & \sqrt{\frac{1}{2}} \Sigma_Q^{-1/2} & \Omega_Q \end{pmatrix}. \]

The matrix \( B_6^{ij} \) is similar to \( B_6 \), except that it contains \( J^P = \frac{3}{2}^+ \) baryons. Here the notation of Ref. [13] is used, where the superscripts denote the \( I_3 \) projection of the isospin of the baryons.

The baryons in the three dimensional representation of \( SU_f(3) \) are described by a field \( T^{ij} \)

\[ T^{ij} = \frac{1 + \gamma^\mu}{2} B_3^{ij}, \]

\[ B_3^{ij} = \begin{pmatrix} 0 & \Lambda_Q & \Xi_Q^{+1/2} \\ -\Lambda_Q & 0 & \Xi_Q^{-1/2} \\ -\Xi_Q^{+1/2} & -\Xi_Q^{-1/2} & 0 \end{pmatrix}. \]

To lowest order in the \( 1/M_Q \) expansion, strong interactions of the heavy baryons with the soft pions are described by the chiral Lagrangian \[ \mathcal{L} \]

\[ \mathcal{L}_{\text{kin}} = \frac{i}{2} \text{tr} \left[ T (v \cdot D) T \right] - i \text{tr} \left[ S^\mu (v \cdot D) S_\mu \right] \]

\[ \mathcal{L}_{\text{mass}} = \frac{\Delta_0}{2} \text{tr} [TT] + \lambda_1 \text{tr} [S^\mu M_+ S_\mu] + \lambda_2 \text{tr} [S^\mu S_\mu] \text{tr} [M_+] \]

\[ + \frac{\lambda_3}{2} \text{tr} [T M_+ T] + \frac{\lambda_4}{2} \text{tr} [TT] \text{tr} [M_+] \]

\[ \mathcal{L}_{\text{int}} = g_3 \left( \text{tr} [TA^\mu S_\mu] + \text{h.c.} \right) + ig_2 \text{tr} [S^\mu A^\rho S^\sigma] v^{\nu} \epsilon_\mu_{\rho\sigma}. \]

Here

\[ D_\mu S_{ij}^{ij} = \partial_\mu S_{ij}^{ij} + (V_\mu)^i_j S_{k}^{kj} + (V_\mu)^j_i S_{k}^{ik} \]

is a covariant derivative, with a vector current

\[ \text{\footnotesize The normalization and the notation for coupling constants are the same as in [16].} \]
\[ V_\mu = \frac{1}{2} \left( \xi \partial_\mu \xi^\dagger + \xi^\dagger \partial_\mu \xi \right), \quad (2.11) \]

and the axial current, \( A_\mu \), is defined as follows:

\[ A_\mu = \frac{i}{2} \left( \xi \partial_\mu \xi^\dagger - \xi^\dagger \partial_\mu \xi \right). \quad (2.12) \]

The covariant derivative of \( T^{ij} \) has the same form.

We chose to absorb the mass of the sextet baryons into the static phase of the heavy baryon fields. The mass splitting between the sextet and triplet baryons, finite as \( M_Q \to \infty \), is denoted by \( \Delta_0 \). The propagators for \( J^P = \frac{3}{2}^+ \) and \( J^P = \frac{1}{2}^+ \) sextet baryons are

\[ i \frac{1 + \gamma^\mu - \frac{1}{2} \eta \gamma^\mu}{v \cdot k} \left( \gamma^\nu + \frac{1}{3} (\gamma^\mu - \gamma^\mu) \right) \quad (2.13) \]

and

\[ -i \frac{1 + \gamma^\mu - \frac{1}{2} \eta \gamma^\mu}{v \cdot k} \left( \gamma^\nu + \frac{1}{3} (\gamma^\mu - \gamma^\mu) \right), \quad (2.14) \]

respectively, while the propagator of the triplet baryon is

\[ \frac{1 + \gamma^\mu \gamma^\nu + \frac{1}{2} \eta \gamma^\mu}{v \cdot k + \Delta_0}. \quad (2.15) \]

### III. QUENCHED QCD

Quenched QCD is obtained from the ordinary theory by removing the disconnected fermion loops. Formally this is done by introducing a bosonic “ghost” partner \( \tilde{q}^i \) for each quark \( q^i \), which has the same mass and couples to the gluons the same way that the quark does. The sign difference between the fermionic and the bosonic loops results in cancellation of the two. The symmetry of the quenched theory is enlarged from \( SU(3)_L \times SU(3)_R \) to the semi-direct product \( (SU(3)_L \times SU(3)_R) \otimes U(1) \).

In this larger theory the Goldstone matrix becomes a supermatrix,

\[ \Pi = \begin{pmatrix} \pi \\ \chi^\dagger \\ \tilde{\pi} \end{pmatrix}, \quad (3.1) \]

where the quark/ghost content of the fields is \( \pi \sim q\bar{q}, \chi \sim \bar{q}q, \chi^\dagger \sim q\bar{q} \) and \( \tilde{\pi} \sim q\bar{q} \). Each of these is an \( 3 \times 3 \) matrix; for example

\[ \pi = \begin{pmatrix} \frac{1}{\sqrt{2}} \pi^0 + \frac{1}{\sqrt{6}} \eta & \pi^+ & K^+ \\ \pi^- & -\frac{1}{\sqrt{2}} \pi^0 + \frac{1}{\sqrt{6}} \eta & K^0 \\ K^- & \bar{K}^0 & -\frac{1}{\sqrt{3}} \eta I_3 \end{pmatrix} + \frac{1}{\sqrt{3}} \eta I_3. \quad (3.2) \]

Note that \( \chi \) and \( \chi^\dagger \) are fermionic fields, while \( \pi \) and \( \tilde{\pi} \) are bosonic.
Notice one important difference from ordinary ChPT: the inclusion of the $SU(3)$ singlet in the theory. In ordinary QCD the anomaly pushes the mass of the $\eta'$ up to the chiral scale and it decouples from the low energy effective theory. In the quenched case, because of the absence of disconnected quark loops, this decoupling does not occur and the $\eta'$ has to be included in the theory. In the quenched case additional terms are required to describe the dynamics of the anomalous field. The lowest order Lagrangian is similar to Eq. (2.1):

$$L_{Q\chi} = \frac{f^2}{8} \left( \text{Str}[\partial_\mu \Sigma \partial^\mu \Sigma^\dagger] + 4B_0 \text{Str}[\mathcal{M}_+] \right) + \frac{1}{2} \left( A_0 \text{Str}[\partial_\mu \Pi] \text{Str}[\partial^\mu \Pi] - M_0^2 \text{Str}[\Pi] \text{Str}[\Pi] \right),$$

with $\Sigma = \xi^2$, $\xi = e^{i\Pi/f}$ and

$$\mathcal{M} = \begin{pmatrix} M & 0 \\ 0 & M \end{pmatrix}. \quad (3.4)$$

$\mathcal{M}_+$ is defined analogously to $M_+$. The “supertrace” Str is defined with a minus sign for the ghost-antighost fields. Normalization of $A_0$ and $M_0$ is such that they have no implicit dependence on $N_f$, the number of flavors.

The presence of the $\text{Str}[\Pi] = N_f^{1/2} (\eta' - \tilde{\eta}')$ term leads to a double-pole structure for the propagators of the flavor-neutral mesons. In the basis where these mesons correspond to $q_i\bar{q}_i$ and $\tilde{q}_i\bar{q}_i$ this propagator takes the form

$$G_{ij}(p) = \frac{\delta_{ij} \xi_i}{p^2 - m_{ii}^2} + \frac{-A_0 p^2 + M_0^2}{(p^2 - m_{ii}^2)(p^2 - m_{jj}^2)}, \quad (3.5)$$

where $m_{ii}^2 = 2B_0 m_i$, and $\epsilon_i = 1$ if $i$ corresponds to a quark and $\epsilon_i = -1$ if $i$ corresponds to a ghost. The second term in the propagator can be treated as a vertex, called a hairpin, with the rule that it can be inserted only once on a given meson line.

Inclusion of quenching effects in the heavy baryon sector is straightforward. The baryon matrices are promoted to supermatrices and $\text{Tr}$ is replaced by $\text{Str}$. The kinetic and the mass terms of the quenched Lagrangian of the heavy baryons are

$$L_{Q\chi}^{\text{kin}} = \frac{i}{2} \text{Str}[\mathcal{T} (v \cdot D) \mathcal{T}] - i \text{Str}[\overline{\mathcal{S}}^\mu (v \cdot D) \mathcal{S}_\mu],$$

$$L_{Q\chi}^{\text{mass}} = \frac{\Delta_0}{2} \text{Str}[\mathcal{T} \mathcal{T}] + \lambda_1 \text{Str}[\overline{\mathcal{S}}^\mu \mathcal{M}_+ \mathcal{S}_\mu] + \lambda_2 \text{Str}[\overline{\mathcal{S}}^\mu \mathcal{S}_\mu] \text{Str}[\mathcal{M}_+] + \lambda_3 \text{Str}[\mathcal{T} \mathcal{M}_+ \mathcal{T}] + \frac{\lambda_4}{2} \text{Str}[\mathcal{T} \mathcal{T}] \text{Str}[\mathcal{M}_+]. \quad (3.6)$$

where

$$\mathcal{S}_\mu = \begin{pmatrix} S_\mu \\ L_\mu \\ \bar{S}_\mu \end{pmatrix}, \quad (3.7)$$

with $K_\mu$ and $L_\mu$ describing the heavy baryons obtained by replacing one of the quarks by its “ghost” partner and $\bar{S}_\mu$ is the heavy baryon with both light quarks replaced by “ghosts”.

The superfield $\mathcal{T}$ is defined the same way,
\[ \mathcal{T} = \begin{pmatrix} T & R \\ P & \bar{T} \end{pmatrix}. \] (3.8)

The covariant derivative, the vector and the axial currents are defined analogously to (2.10) – (2.12).

In the quenched case, \( \text{Str} \mathcal{A}_\mu \) does not vanish and the symmetry allows the baryon fields to couple to the flavor singlet Goldstone mesons through this term. Thus the interaction part of the quenched Lagrangian has one additional coupling that is absent in the unquenched case:

\[
\mathcal{L}_{\text{int}}^{Q\chi} = g_3 \left( \text{Str}[\mathcal{T} \mathcal{A}^\mu S_\mu] + \text{h.c.} \right) \\
+ i g_2 \text{Str}[S^\mu A^\rho S^\sigma] v^{\nu} \epsilon_{\mu\nu\rho\sigma} \\
+ i g_1 \text{Str}[S^\mu S^\sigma] \text{Str}[A^\rho] v^{\nu} \epsilon_{\mu\nu\lambda\sigma}. \] (3.9)

IV. NONANALYTIC CORRECTIONS TO HEAVY BARYON MASSES

We encounter two types of integrals in our calculations. One, \( J_{1}^{\mu\nu} \), arises from the chiral loops without the hairpin vertex and the other, \( J_{2}^{\mu\nu} \), from the loops with the hairpin vertex on Goldstone meson line:

\[
J_{1}^{\mu\nu}(m, \Delta_0, v \cdot k) = i \int \frac{d^np}{(2\pi)^n f^2 (p^2 - m^2 + i\epsilon)(v \cdot (p + k) + \Delta_0 + i\epsilon)} \] (4.1)

and

\[
J_{2}^{\mu\nu}(x, y, \Delta_0, v \cdot k) = \frac{1}{x^2 - y^2} \left[ (M_0^2 - A_0 x^2)J_{1}^{\mu\nu}(x, \Delta_0, v \cdot k) - (M_0^2 - A_0 y^2)J_{1}^{\mu\nu}(y, \Delta_0, v \cdot k) \right]. \] (4.2)

We truncate the series to include terms of order \( m_q, m_q \log m_q \) and \( m_q^{3/2} \) but not those of order \( m_q^2 \) and higher. If not otherwise indicated, dimensionful parameters \( \Delta_0 \) and \( M_0^2 \) are considered to be of order \( m_q \). With these assumptions, terms contributing to the baryon mass corrections are:

\[
J_{1}^{\mu\nu}(m, 0, 0) = (v^\mu v^\nu - g^{\mu\nu}) I_{1}(m) \] (4.3)

\[
J_{2}^{\mu\nu}(x, y, 0, 0) = (v^\mu v^\nu - g^{\mu\nu}) I_{2}(x, y), \] where

\[
I_{1}(x) = -\frac{x^3}{12\pi f^2} \] (4.4)

\[
I_{2}(x, y) = \frac{1}{x^2 - y^2} \left[ (M_0^2 - A_0 x^2)I_{1}(x) - (M_0^2 - A_0 y^2)I_{1}(y) \right]. \]

\[\text{Setting } \Delta_0 = 0 \text{ in (4.1) and (4.2) means that we neglect terms of order } \Delta_0 m_q \ln m_q. \]
$I_2(x, y)$ has the limit

$$I_2(x, x) = -\frac{x^3}{12\pi f^2} \left( \frac{3}{2} M_0^2 x - \frac{5}{2} A_0 x^3 \right). \tag{4.5}$$

Here we only compute corrections to masses arising from the chiral loops and not from the $1/M_Q$ expansion. Chiral corrections do not lift the degeneracy between the $B_6$ and $B_6^*$ baryons. However, the baryon masses inside the sextets as well as inside the triplet are split. Mass corrections to the baryons that have the same flavor light quarks differ from those that have light quarks of two different flavors. The latter get contributions from loops which contain triplet baryons. This is different from the unquenched case, where the loops with the triplet baryons contribute in both cases. The reason for this difference is that the diagrams with the triplet baryons which do not contain the hairpin vertex require a closed quark loop, and in the quenched theory they vanish. On the other hand, the hairpin vertex appears only on the flavor diagonal meson line and these mesons only couple the triplet baryons with the off-diagonal sextet baryons. The diagrams that modify sextet baryon masses are shown in Fig. 1 and Fig. 2. The contributions from these diagrams are:

$$\delta M_{ii}^6 = 2g_2g_1I_1(m_{ii}) + g_2^2I_2(m_{ii}, m_{ii}), \tag{4.6}$$

for the diagonal baryons, and
\[ \delta M^6_{ij} = g_2g_1 (I_1(m_{ii}) + I_1(m_{jj})) + \frac{1}{4} g_2^2 (I_2(m_{ii}, m_{ii}) + I_2(m_{jj}, m_{jj}) + 2I_2(m_{ii}, m_{jj})) \\
+ \frac{1}{4} g_3^2 (I_2(m_{ii}, m_{ii}) + I_2(m_{jj}, m_{jj}) - 2I_2(m_{ii}, m_{jj})) \] (4.7)

for the off-diagonal sextet baryons. \( I_1(x) \) and \( I_2(x,y) \) are defined in Eq.(4.4). In the limit of exact isospin symmetry the term proportional to \( g_3^2 \) in Eq.(4.7) vanishes when the baryon contains up and down quarks.

As was discussed above, in the case of triplet baryons the only diagrams that contribute to the mass corrections are those with the hairpin vertex (Fig. 3). The triplet baryon mass corrections have the following form:

\[ \delta M^3_{ij} = \frac{3}{4} g_3^2 (I_2(m_{ii}, m_{ii}) + I_2(m_{jj}, m_{jj}) - 2I_2(m_{ii}, m_{jj})) \] (4.8)

This expression has the same form as the correction to the off-diagonal sextet baryon masses that arise from the diagrams with the triplet baryons. Thus in the limit of exact isospin symmetry the mass corrections to \( \Lambda_Q \) baryon vanish.

There is no mixing between the triplet and the sextet baryons in the leading order of the \( 1/M_Q \) expansion. Mixing would require the breaking of heavy baryon spin symmetry, which is exact at this order of the heavy quark expansion.

Finally, we note that these results are readily generalized to an arbitrary number of light flavors.

\textbf{V. PHENOMENOLOGY}

As was mentioned in the introduction, there are many undetermined parameters in the theory. In the quenched case there are additional parameters that are absent from the Lagrangian of ordinary ChPT. Of course the parameters of the quenched and unquenched theories, even those that are present in both cases, might not be the same. Here couplings that are present in ordinary ChPT are assumed to have the same value in the quenched case. Other couplings we assume to be of order one. Calculations are done for two sets of values for \( M_0 \) and \( A_0 \): \( M_0 = 400 \) MeV, \( A_0 = 0 \) and \( M_0 = 100 \) MeV, \( A_0 = 0.2 \). For more detailed discussion of the values of these parameters we refer the reader to [11].

There is a nontrivial relation between the sextet baryon masses that holds in both quenched and unquenched cases in the leading order of \( SU_f(3) \) breaking. Namely, in the limit of exact isospin symmetry:

\[ \text{FIG. 3. Diagram that modifies the masses of triplet baryons.} \]
\[ \delta M = M_{\Sigma Q} + M_{\Omega Q} - 2M_{\Xi' Q} = 0. \] (5.1)

This relation gets corrections in the next order of the chiral expansion. In the limit of vanishing \( \Delta_0 \), this correction is \[ M_{\Sigma Q} + M_{\Omega Q} - 2M_{\Xi' Q} = \frac{1}{24\pi f^2} (g_2^2 - g_3^2) (4m_K^3 - 3m_\eta^3 - m_\pi^3), \] (5.2)
assuming the tree level relation between the Goldstone mesons, \( 4m_K^2 - 3m_\eta^2 - m_\pi^2 = 0 \). In the quenched case the equivalent relation takes the form

\[ M_{\Sigma Q} + M_{\Omega Q} - 2M_{\Xi' Q} = \frac{1}{2} \frac{(g_2^2 - g_3^2)}{12\pi f^2} \left[ I_2(m_{uu}, m_{uu}) + I_2(m_{ss}, m_{ss}) - 2I_2(m_{uu}, m_{ss}) \right]. \] (5.3)

To simplify the last relation let us consider the limit \( m_u = m_d = 0 \). Then Eq. (5.3) becomes:

\[ M_{\Sigma Q} + M_{\Omega Q} - 2M_{\Xi' Q} = \frac{1}{48\pi f^2} (g_2^2 - g_3^2) \left[ M_0^2 m_{ss} + A_0 m_{ss}^3 \right]. \] (5.4)

The main difference between the two cases is the presence of the term proportional to \( m_q^{1/2} \) in the quenched case. The Goldstone meson masses are linear in \( m_q^{1/2} \). Fig. 4 shows the dependence of these corrections on \( m_\pi^2 \) while keeping the value of the strange quark mass fixed. (This is the extrapolation used in lattice calculations to extract the values of the baryon masses.) In both cases the corrections to \( \delta M \) are small but they have different signs. The plot indicates the very different behavior of the nonanalytic corrections in the two cases.

We also investigate the mass splittings between the sextet and the triplet baryons. The splittings are calculated in the limit of exact isospin symmetry. As was discussed in the previous section, in this limit \( \Lambda_Q \) does not receive corrections at the first order of the chiral expansion. The results for the mass splittings are:

\[ \Sigma_Q - \Lambda_Q = \Delta_0 - 2g_2g_1 \frac{m_\pi^3}{12\pi f^2} - g_2^2 \frac{1}{12\pi f^2} \left( \frac{3}{2} M_0^2 m_\pi - \frac{5}{2} A_0 m_{\pi}^3 \right) \] (5.5)

and

\[ \Xi'_Q - \Xi_Q = \Delta_0 + g_2g_1 (I_1(m_{\pi}) + I_1(m_{ss})) + \frac{1}{4} g_2^2 (I_2(m_{\pi}, m_{\pi}) + I_2(m_{ss}, m_{ss}) + 2I_2(m_{\pi}, m_{ss})) \]
\[ - \frac{1}{2} g_2^2 (I_2(m_{\pi}, m_{\pi}) + I_2(m_{ss}, m_{ss}) - 2I_2(m_{\pi}, m_{ss})) . \] (5.6)

\(^3\)In this limit \( m_{uu} = m_{dd} = m_\pi \).

\(^4\)Not included in the following expressions are the contributions from the mass terms of the Lagrangian (3.6) that are linear in the light quark masses. These terms depend on the undetermined parameters, \( \lambda_i \). We found that these contributions are small if one considers the linear combinations of \( \lambda_i \)’s to be of order 1.
FIG. 4. Dependence of $\delta M$ on $m_{\pi}^2$ in the quenched and unquenched cases. The solid line corresponds to the unquenched case, the dashed line to the quenched case with $M_0 = 400\text{ MeV}$ and $A_0 = 0$, and the dashed-dotted line to the quenched case with $M_0 = 100\text{ MeV}$ and $A_0 = 0.2$.

The last relation simplifies in the $m_u = m_d = 0$ limit:

$$
\Xi_Q' - \Xi_Q = \Delta_0 - \frac{M_0^2 m_{ss}}{48\pi f^2} \left( \frac{7}{2} g_2^2 + g_3^2 \right) - g_2 g_1 \frac{m_{ss}^3}{12\pi f^2}.
$$

(5.7)

For simplicity of the analytic expressions only the cases with $A_0 = 0$ are shown.

We would like to point out two things. First, the corrections to the mass splittings, as the correction to $\delta M$, display nonanalytic dependence on the light quark masses that is different from ordinary ChPT. In the quenched case the leading nonanalytic term is $m_q^{1/2}$. Notice that the coefficient of the term linear in the Goldstone meson mass depends, apart from the $M_0^2$ term, on the parameters that exist in the unquenched theory. One can estimate the size of this term assuming that the corresponding parameters in the quenched and unquenched theories are the same. Second, there are terms proportional to $g_1$, the coupling that is absent from the ordinary theory. In general this term gives a large contribution to the mass corrections, which depends on the value of $g_1$, of course. This makes the estimate of quenching errors even more uncertain. The dependence of the mass corrections on $m_{\pi}^2$ for the fixed strange quark mass is shown in Fig. 4. Here $g_1$ is taken to be 0.5 and $\Delta_0 = 200\text{ MeV}$. Note that, by contrast, the terms proportional to $g_1$ are absent from the corrections to $\delta M$. In this particular combination of the heavy baryon masses these terms cancel. This indicates that one has to be careful when using such mass relations for the estimate of quenching errors.

Recently these splittings, as well as the heavy baryon masses, were calculated on the lattice [17]. In that analysis, calculations are performed for the light quark masses of order of the strange quark mass, and then the results are linearly extrapolated to the chiral limit of the up and down quark masses, while the strange quark mass is fixed to its phenomenological value. In particular, the following relation for the mass of the heavy baryon is assumed:
FIG. 5. Dependence of the sextet–triplet mass splittings on $m^2_{\pi}$. The solid and dashed lines correspond to the $\Xi'_{Q} - \Xi_{Q}$ splitting with $M_0 = 400 \text{ MeV}$, $A_0 = 0$ and with $M_0 = 100 \text{ MeV}$, $A_0 = 0.2$ respectively. The line with small dashing and the dashed-dotted line correspond to the $\Sigma_{Q} - \Lambda_{Q}$ splitting with $M_0 = 400 \text{ MeV}$, $A_0 = 0$ and with $M_0 = 100 \text{ MeV}$, $A_0 = 0.2$ respectively. $g_1$ is taken to be 0.5.

$$M_{Qij} = \mu(M_Q) + C(m_i + m_j), \quad (5.8)$$

where $M_Q$ is the mass of the heavy quark and $m_i$ and $m_j$ denote the masses of the light quarks. $\mu(M_Q)$ is the mass of the baryon in the chiral limit, $m_u = m_d = m_s = 0$. $\mu$ and $C$ are different for the sextet and triplet baryons. The mass splitting between the sextet and triplet baryons is:

$$\Delta M_{Qij} = \Delta_0(M_Q) + D(m_i + m_j). \quad (5.9)$$

From Eq. (5.9) it follows that in the chiral limit for up and down quarks $m_u = m_d = 0,$

$$M_{\Sigma_{Q}} - M_{\Lambda_{Q}} = \Delta_0, \quad (5.10)$$

and this agrees with Eq. (5.5) when $m_\pi = 0$. In ChPT $\Delta_0$ is a free parameter and has to be determined from the fit to the experimental data. For our choice of $\Delta_0 = 200 \text{ MeV}$ the quenched result is in agreement with the lattice result.

The situation is different for the $\Xi'_{Q} - \Xi_{Q}$ mass splitting. The quenched result is given by Eq. (5.7). The lattice result is

$$M_{\Xi'_{Q}} - M_{\Xi_{Q}} = \Delta_0(M_Q) + Dm_s \quad (5.11)$$

In this case the finite value of the strange quark mass results in the modification of the tree level splitting even in the limit $m_u = m_d = 0$. In QChPT the $\Xi'_{Q} - \Xi_{Q}$ splitting, in contrast to the $\Sigma_{Q} - \Lambda_{Q}$ splitting, has different values for the two sets of $M_0$ and $A_0$ parameters in this
limit. As Fig. 5 indicates, the corrections to the $\Xi_Q^\prime - \Xi_Q$ splitting could in general be large. The plot also shows that the size of the corrections strongly depends on the parameters $M_0$ and $A_0$.

The most important result, however, is the different dependence of the lattice and quenched results on the light quark masses. The nonanalytic terms, present in QChPT, are not accounted for in lattice simulations. The presence of these nonanalytic terms is unambiguously predicted by the quenched theory and they can be calculated from the tree level Lagrangian. Because of the presence of several undetermined parameters, it is possible that the coefficients of the nonanalytic terms are small. The only way to determine the size of these terms is to take them into account in lattice simulations and perform the nonlinear extrapolations.

VI. SUMMARY

Let us summarize briefly the results of our calculation. The main qualitative result is the dependence of the heavy baryon masses on the square root of the light quark mass $m_q^{1/2}$. This is different from ordinary ChPT where the leading nonanalytic dependence goes as $m_q^{3/2}$. The presence of these terms is unambiguously predicted. Their coefficients are calculated from the tree level Lagrangian and they can not be modified by the inclusion of the higher order terms in the chiral expansion. Because the coefficients in the Lagrangian for QChPT are not well constrained, it is difficult to make numerical predictions of the size of the nonanalytic terms. However, it is certain that lattice simulations should be modified to account for this leading nonanalytic behavior.

VII. ACKNOWLEDGMENTS

I am greatly indebted to Adam Falk, Michael Booth and Thomas Mehen for many useful discussions on the subject. This work was supported by the National Science Foundation under Grant No. PHY–9404057.
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