On exact polytropic equilibria of self-gravitating gaseous and radiative systems: their application to molecular cloud condensation

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\begin{abstract}
We propose a novel mathematical method to construct an exact polytropic sphere in self-gravitating hydrostatic equilibrium, improving the non-linear Poisson equation. The central boundary condition for the present equation requires a ratio of gas pressure to total one at the centre, which is uniquely identified by the whole mass and molecular weight of the system. The special solution derived from the Lane-Emden equation can be reproduced. This scheme is now available for modelling the molecular cloud cores in interstellar media. The mass-radius relation of the first core is found to be consistent with the recent results of radiation hydrodynamic simulations.

\textbf{Key words:} equation of state — methods: numerical — ISM: clouds — ISM: molecules
\end{abstract}

1 INTRODUCTION

Fundamental research on the structure and evolution of stars, molecular clouds, and galaxies is still very important \cite{Longair 1994}. As a rule, a family of the Lane-Emden (LE) equations classified by the polytrope indices \( n \) is known to be the most powerful tool to investigate the self-gravitating polytropic equilibria \cite{Binney & Tremaine 1985}. So far, many numerical and semi-analytical works have been devoted to seeking solutions \cite{Liu 1987, Roxburgh & Stockman 1999, Goenner & Havas 2000, Hunter 2003, Medvedev & Rybicki 2001}, which include the non-singular function with \( n \rightarrow \infty \) for an isothermal polytropic sphere \cite{Natarajan & Lynden-Bell 1997}. Removing infinite divergence of the enclosed mass, some modified models are used to depict the globular clusters, intergalactic media, clusters of galaxies, and so forth \cite{Jones & Forman 1984}.

Prior to modern numerical methods \cite{VandenBerg & Bell 1983, VandenBerg 1983}, the pioneering stellar model proposed by Eddington, which is based on the solution of the LE equation with the polytrope index of \( n = 3 \), could be useful to envisage stellar interiors, although the complexities such as transport processes and non-ideal equation of state (EOS) have been omitted. Regarding the study of the interstellar medium (ISM), it seems that some classes of the LE equations shed light on dark molecular clouds and their condensations \cite{Mizuno et al. 1994}. However, in the general case that the pressure which supports the celestial objects is provided largely by the thermal motions of particles, we need to pay attention to the extensive use of the LE equation with a specified index. For heuristic ways, we explain this crucial point below.

Let us suppose that the object sustains its own self-gravity due to the total pressure of \( P = p_g + p_r \), where the partial pressures of gas and radiation field are described by the EOS of \( p_g = \beta P = [k_B/(\mu m_p)](T/V) \) and \( p_r = (1 - \beta)P = \frac{4}{3}aT^4 \), respectively. Here, \( m_p \) is the proton mass, \( V = \rho^{-1} \) is the specific volume, \( a = 8\pi^2k_B^4/(15c^3h^3) \) is the Stefan-Boltzmann constant, and \( \mu \geq 1 \) denotes the mean molecular weight. \footnote{For plasmas, \( 1/2 < \mu < 2/(1 + 3X + 0.5Y) < 2 \), where the mass fractions contained in the unit mass of the medium are \( X \) g hydrogen, \( Y \) g helium, and \( Z = 1 - X - Y \) g heavy elements.}

For \( 0 < \beta < 1 \), the relation between temperature and density is given by \( T = ([k_B/(\mu m_p)](3/a)[(1-\beta)/\beta])^{1/3}V^{-1/3} \). Hence, we obtain the polytropic relation as follows:

\begin{equation}
P(r) = \left( \frac{k_B}{\mu m_p} \right)^4 \frac{3}{a} \beta^4 \left( \frac{1 - \beta}{\beta} \right) V^{-4/3} (r),
\end{equation}

If we assume \( \beta = \text{const} \) \cite{Chandrasekhar 1967}, equation (1) has the form of \( P = K \rho^{4/3} \), where \( K = ([k_B/(\mu m_p)]^4(3/a)[(1-\beta)/\beta])^{1/3} \) is the adiabatic constant. In this aspect, such a radiative polytropic leads to the LE
equation of index \( n = 3 \) for gravitational equilibria. Hereafter, we refer to this scheme, namely, the Eddington model, as ‘LE3’, without further notice. Apparently, for gaseous objects of \( \beta \simeq 1 \), equation (1) does not match with the relation of \( P(r) \propto \rho^{3/2}(r) \) involving \( n = 3/2 \) for a perfect monatomic gas.

The possible scenario that recovers the physical consistency is to allow the spatial variation of polytropic index. In equation (1) such changes can be achieved by invoking the density dependence of \( \beta = \beta[V(r)] \), and a trivial solution of \( d\mu/\mu = 0 \) (not shown). Of course, these are not mathematical artefacts. In the astronomical context, the present scheme is applicable to study, especially, the high-density adiabatic cores deeply immersed in the isothermal molecular cloud, which are so-called ‘first cores’ (Masunaga, Miyama & Inutsuka 1998). As yet, there has been no distinct detection of the first cores. We hope for future-planed observations of molecular cloud cores in the range of millimetre and submillimetre wavelengths with high angular resolution (Hasegawa 2001).

In this paper, we present a self-consistent solution of a polytropic sphere for self-gravitating gaseous and radiative systems, going beyond the LE framework. In the following, we derive a generic equation for hydrostatic equilibria. In the present scheme, if we know the mass and the mean molecular weight of the system, then the pressure ratio of gas/total pressure at the centre, which contributes to the boundary condition, is identified. It is shown that the polytrope varies radially, modifying the density and pressure profiles derived from the LE3 model. In the limit that the gaseous pressure is asymptotically close to the total one, the solution exactly agrees with the LE solution of index \( n = 3/2 \).

## 2 THE NONLINEAR POISSON EQUATION INCLUDING A SELF-CONSISTENT VARIABLE POLYTROPE

We begin by defining the adiabatic exponent as \( \Gamma_1 = \frac{dP}{d(lnP)} \). Introducing the total derivative of \( dP = d(p_k + p_v) \) and \( d(T/T) \), we obtain

\[
(p_k + p_v) \frac{dT}{T} - (p_k + p_v) \Gamma_1 \frac{dV}{V} = 0. \tag{2}
\]

For a quasi-static and adiabatic change, the thermodynamical principle requires

\[
\frac{dQ}{V} = \left(1 - \frac{1}{\gamma} \right) p_k \frac{dT}{T} + (p_k + p_v) \frac{dV}{V} \simeq 0, \tag{3}
\]

where \( \gamma = c_p/c_v \) is the specific-heat ratio. Combining equation (2) with equation (3), we find the relation between \( \Gamma_1 \) and \( \beta \) in the form of

\[
\Gamma_1[\beta(V), \gamma] = \beta(V) + \frac{\gamma - 1}{\gamma} \frac{\beta(V) - 1}{\beta(V) + 1} \beta(V) \tag{4}
\]

It should be noticed that equation (4) holds the asymptotic properties of \( \Gamma_1 \to 4/3 \) and \( \gamma \to 0 \) and 1, respectively.

On the other hand, taking the derivative of equation (1), we obtain

\[
\frac{dP}{P} + \frac{4dV}{3V} + \frac{4}{3} \left( \frac{\beta}{1 - \beta} \right) \frac{d\beta}{\beta} = 0. \tag{5}
\]

According to the relation of \( dP/P + \Gamma_1 dV/V = 0 \), therefore, we obtain the new relation

\[
\frac{d\beta}{\beta} = \left( \frac{\beta}{1 - \beta} \right) \frac{d\beta}{\beta} + 4 \frac{d\beta}{\beta} \tag{6}
\]

which effectively changes the polytrope index of equation (1).

Now we consider a hydrostatic equilibrium of a self-gravitating system, described by the spherically symmetric Poisson equation, namely, \( \nabla \Phi = r^{-2}(d/dr)(r^2 \Phi) = 4\pi G \rho \), where \( \Phi(r) = -\rho^{-1}(dP/dr) \) and \( G \) is the gravitational constant. By using equations (1) and (6), the non-linear equation can be cast to

\[
\frac{1}{4\pi G} \left[ \frac{k_B}{\mu m_\rho} \right] \frac{4}{3} \frac{1}{a} \left[ \frac{1}{\beta(V)} \right] \frac{dP}{P} \frac{1}{\mu m_\rho} = \frac{1}{r^2} \left[ \frac{1}{\beta(V)} \right] \frac{d\beta}{\beta} \frac{1}{\mu m_\rho} = \rho(r). \tag{7}
\]

In order to normalize equation (7), we introduce \( \tilde{\rho}(\xi) = \rho(\xi)/\rho_c \), where \( \rho_c \) is the central mass density, and we properly choose the dimensionless frame of \( \xi = r/\alpha \). Here, the characteristic scalelength \( \alpha \) is given by

\[
\alpha(\rho_c, \mu) = \left\{ \frac{1}{(4\pi G)^2 \rho_c^2} \left( \frac{k_B}{\mu m_\rho} \right) \frac{4}{3} \frac{1}{a} \right\}^{1/6}. \tag{8}
\]

Then, we newly find a set of ordinary differential equations in the following form:

\[
\frac{d\tilde{m}(\xi)}{\xi^2} = -\tilde{\rho}(\xi), \tag{9}
\]

\[
\tilde{m}(\xi) = \frac{\xi^2 \frac{d\tilde{\rho}(\xi)}{d\xi}}{\left( \frac{1}{\beta(\xi)} \right)^{1/3} \frac{d\beta(\xi)}{d\xi}} = \frac{4 - 3\beta(\xi)}{\left( \beta(\xi) \right)^{1/3} \frac{d\beta(\xi)}{d\xi}}. \tag{10}
\]

Note that the enclosed mass defined by \( m(r) = 4\pi \int_0^r \rho(r') r'^2 dr' \) is transformed into \( m(\xi) = -4\pi \rho_c \alpha^3(\rho_c, \mu) \tilde{m}(\xi) \geq 0 \). In the special case of \( d\beta/d\tilde{\rho} \simeq 0 \), equation (9) reduces to the LE equation with an index of \( n = (\gamma - 1)^{-1} \) and 3 for \( \beta \simeq 1 \) and 0, respectively.

## 3 NUMERICAL SOLUTIONS OF THE GENERALIZED EQUATION

To survey the inner structure of the polytropic sphere, we numerically integrate equation (9) for \( \gamma = 5/3 \), and the boundary conditions of \( \tilde{\rho}(\xi)|_{\xi=0} = 1 \), \( \tilde{m}(\xi)|_{\xi=0} = 0 \), and \( \beta(\xi)|_{\xi=0} = \beta_c \), where \( \beta_c = (p_k/P)_c \) is a parameter given at the centre. It is noted that the solutions of \( \tilde{\rho}(\xi) \) and \( \tilde{m}(\xi) \) tend to monotonically decrease outwards, while \( \beta(\xi) \) monotonically increases. At \( \xi = \xi_1 \), \( \tilde{\rho}(\xi) \) and \( d\tilde{m}(\xi)/d\xi \) vanish, indicating a well-defined radius and mass of the object, i.e.,

\[
R = \alpha(\rho_c, \mu) \xi_1(\beta_c), \tag{10}
\]

\[
M = m[\xi_1(\beta_c)] = -4\pi \rho_c \alpha^3(\rho_c, \mu) \tilde{m}[\xi_1(\beta_c)]. \tag{11}
\]
Table 1. Values of $\xi_1$, $\tilde{m}$, $\mu$, and $\rho_c$ as a function of $\beta_c$.

| $(1 - \beta_c)$ | $\xi_1$ | $\tilde{m}(\xi_1)$ | $\mu \sqrt{(\frac{M_c}{M})^a}$ | $\rho_c(\frac{M_c}{M})(\frac{\tilde{m}}{\rho})$ au$^{-3}$ $\times 10^{-13}$ g cm$^{-3}$ |
|-----------------|--------|-----------------|------------------|---------------------------------|
| 0.01            | 2.72   | -1.10           | 1.11             | 8.62                           |
| 0.05            | 3.77   | -2.76           | 1.75             | 9.17                           |
| 0.1             | 4.55   | -4.49           | 2.24             | 9.95                           |
| 0.3             | 7.57   | -14.2           | 3.98             | 14.4                           |
| 0.5             | 12.2   | -38.6           | 6.57             | 22.1                           |
| 0.7             | 21.6   | -1.36 $\times 10^2$ | 12.3          | 35.3                           |
| 0.9             | 56.8   | -1.48 $\times 10^3$ | 40.7          | 58.6                           |

For $\mu = 2$, see also Fig. 3 (dotted curve).

Table 1 lists the typical values of $\xi_1$ and $\tilde{m}(\xi_1)$ as a function of $\beta_c$.

3.1 Inner structure of gaseous and radiative polytropic sphere

Substituting equation (8) into equation (11), we obtain the explicit notation of the mean molecular weight:

$$\mu = \left[\frac{4\pi G m_f}{M} \right]^{3/4} \left(\frac{-\tilde{m}[\xi_1(\beta_c)]}{\xi_1} \right)^{1/2}.$$

Taking the allowed parameter region into consideration, this can be expressed as

$$\mu(M, \beta_c) = 1.06 \left(\frac{M_c}{M}\right)^{1/2} \left\{ -\tilde{m}[\xi_1(\beta_c)] \right\}^{1/2} \geq 1,$$

which corresponds to the LE3 solution of $\rho_{c,LE3} = 4.25(M_c/M)^{1/2}(1 - \beta_c)^{1/4}/\beta_c$. Furthermore, using equations (8) and (10), we obtain

$$\rho_c = \left\{\frac{4\pi G R^3}{M_f} (\frac{\tilde{m}}{\rho_f})^{1/4} \frac{\xi_1}{\xi_1(\beta_c)} \right\}.$$

Making use of expression (12), this can be written as

$$\rho_c(M, R, \beta_c) = 4.73 \times 10^{-14} \left(\frac{M_c}{M}\right)^3 \left(\frac{100 \text{ au}}{R}\right)^3 \frac{\xi_1}{\xi_1(\beta_c)} \frac{-\tilde{m}[\xi_1(\beta_c)]}{\tilde{m}[\xi_1(\beta_c)]} \text{ g cm}^{-3}.$$

Thus, for given $M$ and $\mu$, the fraction of gas pressure $\beta_c$ is identified by equation (12). In addition, for given $R$, the central mass density is determined by equation (13), in contrast to the LE3 solution of $\rho_{c,LE3} = 7.68 \times 10^{-12}(M_c/M)(100 \text{ au}/R)^3$ cm$^{-3}$, which is independent on $\beta_c$. These dependencies are summarized in Table 1.

In Fig. 1 we plot equations (12) and (13) as a function of $(p_r/P_c) = (1 - \beta_c)$. It is found that, for $M = M_c$ as an example, the inequality of equation (12) requires the fraction of radiation pressure at the centre to take its parameter region of $(1 - \beta_c) \geq 6.7 \times 10^{-3}$. If we specify the mean molecular weight, then the fraction can be fixed numerically. For $\mu = 2$ (hydrogen molecule), we obtain $(1 - \beta_c) = 7.4 \times 10^{-2}$. In such a gas dominant regime of $(1 - \beta_c) \ll 1$, the dimensionless quantity $\sim \rho_c(R^3/M)$ does not largely depend on $\beta_c$, as seen in the figure. For $R = 100 \text{ au}$, the numerical solution reveals the central density of $\rho_c = 8.6 \times 10^{-13}$ g cm$^{-3}$, which is about 90 per cent lower than that of $\rho_{c,LE3} = 7.68 \times 10^{-12}$ g cm$^{-3}$. For massive objects of $M > M_c$, the radiation dominant regime appears. In the limit of $\beta_c \to 0$, both $\rho_c$ and $\mu$ are asymptotically close to $\rho_{c,LE3}$ and $\mu_{LE3}$, respectively. This characteristic is a result of the behavior of $c_{LE3}$ in equation (4).

In Fig. 1 for $0.01 \leq (1 - \beta_c) \leq 0.9$, we show the radial profiles of the normalized mass density $\rho(r)/\rho_c$, enclosed mass $m(r)/M$, total pressure $P(r)/P_c$, as well as the adiabatic exponent $\Gamma_1(r)$, comparing the LE solutions of index $n = 3$ and $3/2$. Here the radius $r = \alpha x$ is normalized by $R$. It is shown in Fig. 2(a) that the density profile gradually deviates from that of the LE3 solution as $(1 - \beta_c)$ decreases. Concerning the case that the parameters of $M$ and $R$ are fixed, in the lower $\beta_c$ the central density decreases, because the density varies sufficiently slowly from the centre to the envelope. In the limit of $\beta_c \to 1$, the present solution shows an exact agreement with that of the LE equation with a polytropic index of $n = 3/2$. The radial profiles of the enclosed mass are shown in Fig. 2(b). For higher $(1 - \beta_c)$ including the LE3, the mass is more condensed in the central region. For example, the radius confining the half-mass indicates $R_{M/2} = 0.235R$. As $(1 - \beta_c)$ decreases, this radius gradually shifts outward, up to $R_{M/2} = 0.521R$. It is confirmed that the gaseous polytropic effects are likely to flatten the density profile significantly.

In Fig. 2(c), we show the radial profiles of $\Gamma_1(\beta(x), \gamma = 5/3)$, defined by equation (4). As the argument $\beta(x)$ monotonically increases outwards, the adiabatic exponent also monotonically increases, yielding the self-consistent variable polytrope. This is one of the most important results in the present paper. For all $\beta_c$ being considered, the exponent varies quite slowly near the central region, while for

$$\delta = 4.4 \times 10^{-4} \leq (1 - \beta_c) \leq 7.4 \times 10^{-2},$$

corresponding to the allowed parameter region of $1/2 \leq \mu \leq 2$ (see footnote 1).
Notice that, in contrast with the LE3 solution of $P_{c,\text{LE3}} = 0.582 (M/M_\odot)^2 (100 \text{ au}/R_\odot)^3 \text{ ergs cm}^{-3}$, equation (14) does depend on $\beta_c$, but weakly for $1 - \beta_c \ll 1$. For $M = M_\odot$, $\mu = 2$, and $R = 100 \text{ au}$, we obtain the central pressure of $P_c = 4.6 \times 10^{-2} \text{ ergs cm}^{-3}$, which is an order of magnitude smaller than $P_{c,\text{LE3}}$.

Invoking the EOS, the temperature profile can be described as $T(\xi) = 3G M^2/(4\pi r^4) \times \xi(1 - \beta(\xi))/\beta(\xi) \times \beta(\xi)$ (not shown in figure). At $\xi = 0$, it takes the peak value of

$$T_c = 1.13 \times 10^3 \left( \frac{M}{M_\odot} \right)^{1/2} \left( \frac{100 \text{ au}}{R_\odot} \right)^{1/2} K,$$

whereas the LE3 solution reads $T_{c,\text{LE3}} = 3.90 \times 10^3 (M/M_\odot)^{1/2} (100 \text{ au}/R_\odot) (1 - \beta(\xi))^{1/4} K$; both having the dependence of $\beta_c$. For $M = M_\odot$, $\mu = 2$, and $R = 100 \text{ au}$, the central temperature of equation (15) is found to be $T_c = 1.1 \times 10^3 K$, which is again lower than $T_{c,\text{LE3}} = 2.0 \times 10^3 K$. It is found that the decrease of the temperature is relatively small, within a factor of 2 for each $\beta_c$.

3.2 Their application to the molecular cloud condensation: the critical radius of the ‘first core’

In the context of the study of molecular cloud condensation in the ISM, the present quasi-stationary model is now available to provide an insight into the complicated dynamics (Penston 1969; Larson 1969; Shu 1977; Saigo & Hanawa 1998), in particular, the formation of the first core (Massana et al. 1998). For such an application, the effects of magnetic fields and turbulence might be taken into account, and the additional pressure $\delta P$ might be effectively included in equation (1) by replacing $\mu$ with $\mu' = \mu \beta_p/(1 + \beta_p)$, where $\beta_p = p_p/\delta P > 0$ (Bludman & Kennedy 1996); e.g. for pure magnetic pressure, $\beta_p = (8\pi p_p)/B^2$. Moreover, for $T_c \gtrsim 2000 K$, the dissociation of hydrogen molecules triggers gravitational contraction of the quasi-stationary first core. By invoking the scaling of $T_c \propto 1/\mu'$ and equation (15), therefore, we find the relation between the mass and radius of the first core as follows:

$$R \gtrsim 13 \left( \frac{1 + \beta_p}{\beta_p} \right) \left( \frac{M}{0.05 M_\odot} \right)^{1/2} \left( \frac{1 - \beta_c}{\beta_c} \right)^{1/3} \frac{\xi}{\beta_0} \left( \frac{M}{M_\odot} \right)^{1/2} \left( \frac{100 \text{ au}}{R_\odot} \right)^{1/2} \left( \frac{0.05 M_\odot}{M}\right)^{1/2} \left( \frac{100 \text{ au}}{R_\odot} \right)^{1/2} \left( \frac{1 - \beta_c}{\beta_c} \right)^{1/3} \frac{\xi/\beta_0}{\beta_0},$$

(16)

for $\mu = 2$. Note that the constraint of equation (12) gives the dependence of $\beta_c = \beta_c(M, \mu = 2)$ in equation (16). On the other hand, in the LE3 we obtain the mass-radius $(M - R)$ relation of $R \gtrsim 4.6 (M/0.05 M_\odot) \beta_c$ au, where $\beta_c = \beta_c(M, \mu = 2)$.

In Fig. 3 for $\beta_p \gg 1$, we plot the numerical solution of equation (16), that is, the allowable parameter region in the $M - R$ plane. For convenience sake, we also plot the fractional radiation pressure at the centre as a function of the mass of the core: $\left( p_\delta / P_c \right) = 1 - \beta_c(M, \mu = 2)$. For
M ∝ R, corresponding to β_c ≈ 1 for fixed μ (compare Table 1), the minimum core radius of equation (16) can be well approximated by that from the LE3: $R_{\text{min}} \approx 1.9 \times 10^{-3} (1 + \beta_p) \left( \frac{\mathcal{M}}{100 M_\odot} \right)^{1/2}$ au. On the other hand, for $M \ll M_\odot$, we numerically find that the minimum possible radius of equation (16) scales as

$$R_{\text{min}} \approx 3.0 \left( \frac{1 + \beta_p}{\beta_p} \right) \left( \frac{\mathcal{M}}{0.05M_\odot} \right)^{1/2},$$

and $(1 - \beta)_c \approx 2.8 \times 10^{-4} \left( \frac{\beta_p}{1 + \beta_p} \right)^2 (\mathcal{M}/0.05M_\odot)^2$. The estimation of (17) reasonably supports the results of radiation hydrodynamic calculations by Masunaga et al. (1998). They show that the mass and radius of the first core are $M \sim 0.05M_\odot$ and $R \sim 5$ au > $R_{\text{min}}$, respectively, and the results do not largely depend on the initial conditions of wide parameter ranges. If $R \ll R_{\text{min}}$, the core tends to be gravitationally unstable and evolves along the dynamically-contracting track, to self-organize the 'second core' as a protostar (Masunaga & Inutsuka 2001).

### 4 CONCLUDING REMARKS

In conclusion, we have developed a generic scheme to construct an exact polytropic sphere of self-gravitating gaseous and radiative medium. The particular results derived from the newly modified Poisson equation for hydrostatic equilibria are:

(i) the numerical solutions show that for all cases, the central density and pressure are lower than those from the LE function with the index of $n = 3$;

(ii) the adiabatic exponent monotonically increases radially outwards; and

(iii) in the special case that the central polytrope is gaseous closely, the present solution reproduces the properties of the LE function with $n = 3/2$.

Within this framework, the whole mass of the system is connected with the central density, temperature, and the mean molecular weight.

For an application to modelling the molecular cloud condensation in the ISM, we have newly found the scaling law of the critical radius of the first core. The preliminary result is in consistent with that of the radiation hydrodynamic simulations. We expect that the major consequence can be also referred to, for example, mutatis mutandis, the study of the passive phase of protoplanetary discs (Honda & Nakagawa 1999), stellar modelling (Basu, Pinsonneault & Bahcall 2000), and so on.

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Figure 3. Minimum possible radius of the first core of hydrogen molecular clouds (solid curve) and fractional radiation pressure at the centre (dotted curve) as a function of the mass of the core. The upper-left side of the solid curve ($R > R_{\text{min}}$) corresponds to the stable region, while the opposite side ($R < R_{\text{min}}$) corresponds to unstable region, concomitant with the dissociation of hydrogen molecules. For comparison, the $R - \beta$ relation from radiation hydrodynamic simulations is indicated by the shaded inset (Masunaga et al. 1998).
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