Non-adiabatic generation of a pure spin current in a 1D quantum ring with spin-orbit interaction

Marian Niță

National Institute of Materials Physics,
P.O. Box MG-7, Bucharest-Magurele, Romania

D. C. Marinescu

Department of Physics and Astronomy,
Clemson University, Clemson, South Carolina 29634, USA

Andrei Manolescu

School of Science and Engineering, Reykjavik University,
Menntavegur 1, IS-101 Reykjavik, Iceland

Vidar Gudmundsson

Science Institute, University of Iceland,
Dunhaga 3, IS-107 Reykjavik, Iceland

Abstract

We demonstrate the theoretical possibility of obtaining a pure spin current in a 1D ring with spin-orbit interaction by irradiation with a non-adiabatic, two-component terahertz laser pulse, whose spatial asymmetry is reflected by an internal dephasing angle $\phi$. The stationary solutions of the equation of motion for the density operator are obtained for a spin-orbit coupling linear in the electron momentum (Rashba) and used to calculate the time-dependent charge and spin currents. We find that there are critical values of $\phi$ at which the charge current disappears, while the spin current reaches a maximum or a minimum value.
I. INTRODUCTION

Obtaining and controlling spin currents in solid structures has been one of the most important goals of spintronics research. In the recent past, the focus of this endeavor has been on the creative use of the spin-orbit interaction (SOI) that appears in systems with broken inversion symmetry, be that via confinement (Rashba)\(^1\) or in the bulk (Dresselhaus)\(^2\). The well-known properties of quasi-one dimensional rings to support persistent charge currents\(^3–5\) made them particularly appealing for spin-dependent exploration. The underlying physical phenomenon leading to the creation of persistent currents is the imbalance in the left/right charge carrying states realized, initially, in the presence of a magnetic flux threaded through the center of the ring. In a similar experimental setup, the existence of persistent spin-currents was obtained in the presence of SOI\(^6–8\), when electron-electron interaction are neglected. Both analytical\(^6\) and numerical results\(^8\), point out that in the presence of a magnetic flux, the charge and spin currents in the rings are simultaneously present.

In this paper, we revisit this problem from the perspective of creating persistent charge and spin current by non-adiabatic methods. As before, the original exploration of such ideas was focused on the generation of charge currents various mechanisms able to realize an imbalance between left/right momentum-carrying states\(^9–11\). In particular, we are concerned with the application of a ultrashort, terahertz frequency laser pulse, endowed with a spatial asymmetry expressed through an internal dephasing angle. The efficiency of this method for creating persistent charge currents has been explored in Refs.\(^11, 12\). Its extension to a ring endowed with SOI is discussed below. As we demonstrate, the interplay between the spin orbit coupling that rotates the electron-spin around the ring and the spatial asymmetry of the initial excitation generates favorable conditions where the charge current disappears, while the spin current reaches a maximum level. For both a dipolar and a quadrupolar perturbation, we can find the critical values of the dephasing angle for which this situation occurs.

II. THE RING MODEL

We consider a one-dimensional (1D) quantum ring of radius \(r_0\) containing few electrons, endowed with a Rashba interaction, linear in the electron momentum. In a discrete (tight-binding) representation the ring is reduced to \(N\) sites (points) distributed on a circle, whose angular coordinate is given by \(\theta_n = 2n\pi/N\) with \(n = 1, ..., N\) the site index. The Hamiltonian describing the noninteracting electrons is written in terms of the creation and annihilation operators \(c^\dagger_{n\sigma}\) and \(c_{n\sigma}\) associated with the single-particle states \(|n\sigma\rangle\), where \(\sigma = \pm 1\) is the spin index. This Hamiltonian has been extensively discussed in literature\(^6–8, 13\), so here we will write it directly:

\[
H = V \left\{ 2 \sum_{n,\sigma} c^\dagger_{n\sigma} c_{n\sigma} - \sum_{n,\sigma} \left[ c^\dagger_{n\sigma} c_{n+1\sigma} + c^\dagger_{n+1\sigma} c_{n-1\sigma} \right] \right\}
- iV\alpha \sum_{n,\sigma,\sigma'} \left[ \sigma_x(\theta_n,\theta_{n+1}) \right]_{\sigma\sigma'} c^\dagger_{n\sigma} c_{n+1\sigma'} + \text{h.c.}
\]

(1)

The two energy scales of the problem are set by the hopping matrix element \(V = \hbar^2/2m^*a^2\), where \(m^*\) is the effective electron mass and \(a = 2\pi r_0/N\) is the discretization constant, and by the Rashba coupling, of strength \(\alpha\), which generates \(V_\alpha = \alpha/2a\). The spin
operator $\sigma_r(\theta)$ introduced in Eq. (1) represents the local orientation of the electron spin along the radius of the ring and is given by a linear combination of the Pauli operators $\sigma_x, \sigma_y$, written for the azimuthal coordinate $\theta_{n,n+1} = (\theta_n + \theta_{n+1})/2$,

$$\sigma_r(\theta_{n,n+1}) = \sigma_x \cos \theta_{n,n+1} + \sigma_y \sin \theta_{n,n+1}.$$  

The energy spectrum of the Hamiltonian, calculated for an even number of sites, $N = 20$, is shown in Fig. 1 for $V_\alpha = 0$ and for $V_\alpha = 1.0$ units of $V$. The split realized by SOI is described by two eigenvalues, $E_{l+}$ (right arm) and $E_{l-}$ (left arm), with $l = 0, \pm 1, \pm 2, \cdots, \pm (N/2 - 1), N/2$,

$$E_{l\pm} = \frac{E_l + E_{l\pm 1}}{2} + \frac{E_l - E_{l\pm 1}}{2} \sqrt{1 + \tan^2 2\theta_\alpha},$$  

where $E_l = 2V - 2V \cos(2\pi l/N)$ is the degenerate eigenvalue in the absence of SOI and $\theta_\alpha$ is given by

$$\tan 2\theta_\alpha = \frac{V_\alpha}{V \sin(\Delta\theta/2)}.$$  

The corresponding eigenvectors of the Hamiltonian (1) are:

$$|\Psi_{l+}\rangle = \frac{1}{\sqrt{N}} \sum_n e^{i\theta_n} \begin{pmatrix} \cos \theta_\alpha \\ -e^{i\theta_n} \sin \theta_\alpha \end{pmatrix} |n\rangle,$$

$$|\Psi_{l-}\rangle = \frac{1}{\sqrt{N}} \sum_n e^{i\theta_n} \begin{pmatrix} e^{-i\theta_n} \sin \theta_\alpha \\ \cos \theta_\alpha \end{pmatrix} |n\rangle.$$  

The velocity operator is given by

$$v_\theta = r \dot{\theta} = \frac{i}{\hbar} [H, \theta]$$

$$= -\frac{V_\alpha}{\hbar} \left\{ i \sum_{n,\sigma} c^+_n c_{n+1} + \frac{V_\alpha}{V} \sum_{n,\sigma,\sigma'} [\sigma_r(\theta_{n,n+1})]_{\sigma\sigma'} c^+_n c_{n+1} + \text{h.c.} \right\},$$
and its eigenvectors are also $|\Psi_{l\pm}\rangle$, with the associated eigenvalues

$$v_{l\pm} = \frac{v_l + v_{l\pm1}}{2} + \frac{v_l - v_{l\pm1}}{2}\sqrt{1 + \tan^2 2\theta_\alpha},$$

where $v_l = 2(Va/\bar{h})\sin(2\pi l/N)$ is the velocity in the absence of SOI.

It is customary to introduce the tilt-spin operator $S_{2\theta_\alpha}$,

$$S_{2\theta_\alpha} = \cos 2\theta_\alpha S_z - \sin 2\theta_\alpha S_r,$$

which is a linear combination of $S_z$ and $S_r$, the spin operators for $z$ and radial directions, respectively. The spinors $|\Psi_{l\pm}\rangle$ are also eigenvectors of $S_{2\theta_\alpha}$ associated with eigenvalues $\pm\bar{h}/2$.

In the absence of SOI, all quantum states are four-fold degenerate, except those at the edges of the spectrum which are only twice degenerate. This can be seen in Fig. 1(a) where for $V_\alpha = 0$ the two branches $E_{l+}$ and $E_{l-}$ coincide. In this case the spin operator $S_{2\theta_\alpha}$ becomes $S_z$ and $|\Psi_{l\pm}\rangle$ become the $|\uparrow\rangle$ and $|\downarrow\rangle$ eigenstates of $S_z$. For $V_\alpha \neq 0$, the energy spectrum becomes broader and all states are twice degenerate since $E_{l\pm} = E_{-l\mp}$. States at the crossing points in the spectrum shown in Fig. 2 preserve the four-fold degeneracy. Since these degenerate states carry opposite currents $v_{l\pm} = -v_{-l\mp}$, the system can support a nonzero spin current.

III. THE TIME EVOLUTION

At $t = 0$ the quantum ring described above is exposed to a short terahertz two-component pulse,

$$H_n(t) = Ae^{-\Gamma t} [\sin(\omega_1 t) \cos \theta + \sin(\omega_2 t) \cos n(\theta + \phi)],$$

of duration $\sim \Gamma^{-1}$ and amplitude $A$ [12]. $n = 1, 2$ describes the multipole order of the second component, while the dephasing angle $\phi$ between the two components makes the external perturbation asymmetric. The terahertz scale of the excitation frequencies $\omega_{1,2}$ is at least an order of magnitude larger than the spin relaxation rates in InAs semiconductor heterostructures which reaches values from anywhere between tens and hundreds of picoseconds [14, 15].

FIG. 2: Spectrum of the 1D quantum ring with $N=20$ points vs. Rashba parameter $V_\alpha$. The eigenvalues $E_{3\pm}$ and $E_{0\pm}$ are dotted to illustrate the lifting of degeneracy when $V_\alpha \neq 0$. 

![Spectrum of the 1D quantum ring with N=20 points vs. Rashba parameter V_\alpha. The eigenvalues E_{3\pm} and E_{0\pm} are dotted to illustrate the lifting of degeneracy when V_\alpha \neq 0.](image-url)
The time evolution of the system’s observables is determined by using the density operator $\rho(t)$, which at $t>0$ satisfies a quantum Liouville equation
\[ i\hbar \rho(t) = [H + H_n(t), \rho(t)]. \] (10)

The initial condition is such that $\rho(t=0)$ represents the ground-state density operator which is constructed in terms of the eigenvectors $|\Psi_{i\sigma}\rangle$ shown in Eq. (5), of the initial, time-independent Hamiltonian, Eq. (1):
\[ \rho(t=0) = \sum_{i\sigma} \rho_{i\sigma} |\Psi_{i\sigma}\rangle\langle\Psi_{i\sigma}|, \] (11)

where $\rho_{i\sigma}$ are the populations of the degenerate single-particle states ($\sum_{i\sigma} \rho_{i\sigma} = 1$).

For any $t>0$, Eq. (10) is solved numerically and $\rho(t)$ is obtained by using the Crank-Nicolson finite difference method [11] with small time steps $\delta t \ll \Gamma^{-1} f$. The expectation value of any observable $O$ is then calculated as $\langle O \rangle = \text{Tr} (\rho(t)O)$.

IV. RESULTS

We define the charge current as $I^c = e v_0$ and the spin current along a direction $\nu$ as $I^s_{\nu} = \frac{\hbar}{2} (\sigma_\nu v_0 + v_0 \sigma_\nu)$. Since the spin operator $S_{2\theta_0}$ commutes with the unperturbed Hamiltonian [11], we calculate the expectation value of $I^s_{\nu}$, which corresponds to the direction of the spin $e_{2\theta_0} = \cos \theta_0 e_x - \sin 2\theta_0 e_y$. To simplify the notation, we denote $I^s(t) = \langle I^s_{2\theta_0} \rangle$, $v_0(t) = \langle v_0 \rangle$ and $I^c(t) = e v_0(t)$. Since $v_0$ and $I^s_{2\theta_0}$ commute with $H$, the charge and spin currents become constant after the external perturbation vanishes.

To illustrate our results, we consider an InAs (electron effective mass is $m^* = 0.023 m_e$) quantum ring of radius $r_0 = 14$ nm. We choose our Rashba parameter $V_0 = 0.05$, which corresponds to a SOI strength $\alpha = 37.56$ meVnm, within the range of experimentally determined values [16]. The number of sites used in the discretization is $N = 20$, leading to a length unit $a = 4.4$ nm and an energy unit $V = 85.6$ meV. For a system with $n_e = 6$ electrons, in the many-particle, non-interacting ground-state, the occupied states are $\Psi_{0\pm}$, $\Psi_{1\pm}$ and $\Psi_{-1\pm}$, with weights $p_{0\pm} = p_{1\pm} = p_{-1\pm} = 1/n_e$ (and all the other $p_{i\sigma} = 0$). The many-particle ground-state is therefore nondegenerate. In this configuration, i.e. at $t=0$, the average velocity is $v_0(t=0) = 0$ and the spin current $I^s(t=0) = 2\hbar (v_{0+} + v_{1+} + v_{-1+}) = -0.089 V a/h$.

The first external pulse we consider is the superposition of two dipoles, corresponding to $n = 1$ in Eq. (9). For the selected parameters of the ring, in the absence of the SOI, we obtain the Bohr frequencies $\hbar \omega_{01} = 2.89$ meV and $\hbar \omega_{12} = 8.60$ meV (defined as $\omega_{i\nu} = |E_i - E_{i\nu}|/\hbar$). In Figs. 3 and 4 we show the numerical results obtained for frequencies $\hbar \omega_1 = 2.83$ meV, $\hbar \omega_2 = 8.11$ meV, with the attenuation factor $\Gamma = 4 \omega_1$, and amplitude $A = 67.68$ meV. The duration of the pulse is $t_f \approx 0.5$ ps. Since the pulse produces many-particle excited states, in our calculations $\omega_1$ and $\omega_2$ are chosen to be slightly different from the Bohr frequencies, in an effort to create a more realistic algorithm that reproduces closely what happens in an experimental situation. Moreover, for $\omega_1$ and $\omega_2$ exactly equal to the Bohr frequencies, the outcome is preserved.

The time evolution of velocity $v_0(t)$, proportional with the charge current $I^c(t)$, and that of the spin current $I^s(t)$ are illustrated in Fig. 3 for dephasing angles $\phi = 0$, $\pi/2$ and $\pi$. In Fig. 3a, we recover the result of Ref. [11] where it was demonstrated that a charge current can be non-adiabatically generated through the application of a spatially asymmetric terahertz excitation. Thus, for $\phi = 0$, $v_0 = 0$, while for $\phi = \pi/2$, $v_0 \approx 0.37 V a/h$. 

5
In Figure 3(b) we present the time evolution of the spin current, $I_s(t)$, corresponding to the spin projected on the proper axis $e_2θα$. As previously stated, in the presence of SOI, the initial state of the system has a nonzero spin current, $I_s(0) ≠ 0$. On account of the external pulse, the electrons in the ring are non-adiabatically excited, and the spin current evolves towards a new steady-state value, $I_s(t ≫ t_f)$. After the external pulse vanishes, the spin current $I_s(t)$ is constant in time, but its amplitude varies with the dephasing angle.

We call $v_θ$ and $I_s$ the constant values of the velocity and spin current after the perturbation vanished (i.e. $v_θ(t ≫ t_f)$ and $I_s(t ≫ t_f)$ respectively). They are plotted as a function of the dephasing angle $φ ∈ [0, 2π]$ in Fig. 4, but we note that they reflect the periodicity in $φ$ of the applied pulse. For $φ_v = 0.6π$ and $φ_v = 1.4π$ the charge current is maximum and minimum, respectively. For $φ_v$ equal to 0 and $π$ no charge current flows through the ring ($v_θ = 0$) whereas a spin current is present, $I_s ≠ 0$, being maximum for $φ_s1 = 0$ and minimum for $φ_s2 = π$. These are the critical dephasing angles for which only pure spin currents exist. In this case, the numerical results show that the states with opposite velocity, $Ψ_{l+}$ and $Ψ_{l-}$, are equally excited by the external pulse netting zero total velocity and no charge current in the steady state. This is not the case for intermediate angles where states with opposite velocity are asymmetrically excited and consequently, generating nonzero values for both charge and spin currents.

In Fig. 5 we display $v_θ$ and $I_{2θ}$ obtained by exciting the system with a pulse given by Eq. (9) written for $n = 2$, i.e. by a combination of a dipole and phase shifted quadrupole. The frequencies $ω_1, ω_2$, the attenuation factor $Γ$, and the amplitude $A$ remain the same as before. Due to the quadrupolar component of the pulse, the period of $v_θ$ and $I_{2θ}$ is halved to $Δφ = π$. The maximum and minimum values of $v_θ$ are reached for $φ_v1 = 0.28π$ and $φ_v2 = 0.72π$ (nearly half of the previous values). For the critical angles $φ_s1 = 0$ and $φ_s2 = π/2$ no charge current is induced in the ring ($v_θ = 0$), but the induced spin current reaches extreme values, minima or maxima respectively. Again, in these situations only a pure spin current is induced in the ring.
FIG. 4: The stationary values of the velocity $v_0$ and spin current $I_{2\theta}$ after the external pulse vanishes (at $t \gg t_f$) vs. the dephasing angle $\phi$. The external pulse is $H_1(t)$, Eq. (9) with $n = 1$.

FIG. 5: The stationary values of the velocity $v_0$ and spin current $I_{2\theta}$ after the external pulse vanishes (at $t \gg t_f$) vs. the dephasing angle $\phi$. The external pulse is $H_2(t)$, Eq. (9) with $n = 2$.

V. CONCLUSIONS

In conclusion, we studied the non-adiabatic excitation of spin and charge currents in a 1D quantum ring in the presence of Rashba SOI. The ring was subjected to an external pulse that is spatially asymmetric, having two components with a relative dephasing angle. We investigated two models, a dipole plus a rotated dipole and a dipole plus a rotated quadrupole. By numerical calculation, we showed that for certain values of $\phi$ called $\phi_{v1}$ and $\phi_{v2}$ the induced charge current reaches extreme values with $I_c(\phi_{v1}) = -I_c(\phi_{v2})$. Due to the presence of SOI a nonzero spin current is also induced in the ring, for both pulse models, with amplitudes depending on the parameters of the pulse. We found simple critical values of the dephasing angle, $\phi_{c1}$ and $\phi_{c2}$, for which the induced charge current disappears, whereas the spin current reaches maxima or minima. The method may be used in practice to convert an optical signal of variable amplitude into a dissipationless (persistent) spin current for information transfer purposes. Similar results can be obtained with the Dresselhaus SOI instead of the Rashba SOI, due to the equivalence of the corresponding Hamiltonians.
Acknowledgments

This work was supported by the Icelandic Research Fund, DOE grant number DE-FG02-04ER46139, Romanian PNCDI2 program Grant No. 515/2009 and Grant No. 45N/2009. We acknowledge helpful discussions with Alexandru Aldea and Mugurel Tolea. M.N. is thankful to Clemson University, Reykjavik University, and Science Institute - University of Iceland, for hospitality.

[1] Y. A. Bychkov and E. I. Rashba, J. Phys. C 17, 6039 (1984).
[2] G. Dresselhaus, Phys. Rev. 100, 580 (1955).
[3] M. Böttiker, Y. Imry, and R. Landauer, Phys. Lett. 96 A, 365 (1983).
[4] L. Wendler, V. M. Fomin, and A. A. Krokhin, Phys. Rev. B 50, 4642 (1994).
[5] T. Chakraborty and P. Pietiläinen, Phys. Rev. B 50, 8460 (1994).
[6] J. Splettstoesser, M. Governale, and U. Zülicke, Phys. Rev. B 68, 165341 (2003).
[7] S. Souma and B. K. Nikolić, Phys. Rev. B 70, 195346 (2004).
[8] J. S. Sheng and K. Chang, Phys. Rev. B 74, 235315 (2006).
[9] M. Moskalets and M. Böttiker, Phys. Rev. B 68, 075303 (2003).
[10] M. Moskalets and M. Böttiker, Phys. Rev. B 68, 161311 (2003).
[11] V. Gudmundsson, C.-S. Tang, and A. Manolescu, Phys. Rev. B 67, 161301 (2003).
[12] S. S. Gylfadottir, M. Niță, V. Gudmundsson, and A. Manolescu, Physica E 27, 278 (2005).
[13] F. E. Meijer, A. F. Morpurgo, and T. M. Klapwijk, Phys. Rev. B 66, 033107 (2002).
[14] B. N. Murdin, K. Litvinenko, J. Allam, C. R. Pidgeon, M. Bird, K. Morrison, T. Zhang, S. K. Clowes, W. R. Branford, J. Harris, et al., Phys. Rev. B 72, 085346 (2005).
[15] K. C. Hall, K. Gündodu, E. Altunkaya, W. H. Lau, M. E. Flatté, T. F. Boggess, J. J. Zinck, W. B. Barvosa-Carter, and S. L. Skeith, Phys. Rev. B 68, 115311 (2003).
[16] S. D. Ganichev, V. V. Bel’kov, L. E. Golub, E. L. Ivchenko, P. Schneider, S. Giglberger, J. Eroms, J. De Boeck, G. Borghs, W. Wegscheider, et al., Phys. Rev. Lett. 92, 256601 (2004).
velocity $v_0$ \([2\pi V/a]\)
