Mathematical critical thinking skills profile of high school students in solving linear program word problems

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Abstract. Critical thinking is one of the most important skills of the 21st century. Students’ mathematical critical thinking can develop their concept and knowledge to solve a problem. This research aims to determine the profile and provide information about students’ mathematical critical thinking in solving linear program problems. This study was descriptive-qualitative. The data was collected by a test and followed by an interview. The test was developed based on mathematical critical thinking’s indicators, namely: formulating the problem, analyzing and solving the problem, evaluating, and drawing conclusion. The subjects of this study were thirty two students of senior high school in Jepara. The result shows that among 32 students, those who can formulate the problem are 12.70%, analyze and solve the problem are 12.17%, evaluate are 11.91%, and draw conclusion are 10.16%.

1. Introduction
In the 21st century, students must have a high-level thinking skill in learning mathematics, such as critical thinking. Critical thinking appears to be a higher-order type of reasoning employing cognitive skills and directed by a motivational component in problem solving [1]. Based on their critical thinking, students can develop their concept and knowledge to solve a problem. Critical thinking refers to the use strategies that increase the probability of a desirable outcome. It is purposeful, reasoned, and goal-directed. This kind of thinking involved solving problems, formulating inferences, calculating likelihoods, and making decisions. Critical thinkers use their skills appropriately, without prompting, and usually with conscious intent, in a variety of settings. When studentsthink critically, they evaluate the outcomes of their thought processes—how good a decision is or how well a problem is solved [2].

This skill is considered one of the chief goals and ideals in the education area [3], such as mathematics education. Mathematics has a complete and clear structure and study between the concepts, it plays a key role in shaping how individuals deal with the various spheres of private, social, and civil life [4, 5]. Therefore, students need to be trained in mathematical critical thinking skills to form and develop it well. A routine activities in classroom has an important role in the development of students’ mathematical critical thinking, so teachers should make sure that all students are provided with opportunities to struggle with mathematics for themselves. The activities can be seen from the students’ skills in solving problems with accurately and systematically. In addition, one of the mathematics learning goals of learning mathematics is to train learners to have mathematical critical thinking skills.

In learning activities, it is stated that critical thinking consists of many sub skills such as cause-and-effect relationships [6] by indicators of critical thinking. The indicators are formulating the problem, analyzing and solving the problem, evaluating, and drawing conclusion from mathematical problem [7, 8, 9, 10]. In indicator of formulate the problem, students are expected to be able to understand and express the meaning or purpose a mathematical statement or mathematical are problem. This indicator is indicated by students being able to write down all information that is known and the problem that must be solved. In indicator of analyze and solve the problem, students are expected to be able to identify the relationship between the information provided with the problem to be solved and all the mathematical
concepts needed to solve mathematical problem. This indicator is indicated by students being able to connect information that is known to their resolution strategies.

In indicator of evaluating, students are expected to be able to assess the credibility of statements on mathematical questions from solving mathematical problems that have been done. This indicator is indicated by students being able to check the correctness of the answer whether it is in accordance with the information provided and correct completion. Furthermore, in indicator of drawing conclusion, students are expected to conclude correctly by giving important and reasonable reason.

Students need to understand that developing a critical approach is important if later they become skilled employees who can adapt to new work situations [11]. Now, an expert in education and development agrees that critical thinking must not only be one of the goals of education and development, but also an integral part of education at any level; because, critical thinking is a kind of thought that analyzes, evaluates, and chooses, and provides the best solution for humans, what is the need for the world today [12].

2. Research Method
The subject of this research was 32 students of eleventh grade from SMA Negeri 1 Welahan which is located in Jepara, Central Java, Indonesia. This research used descriptive-qualitative method. The instrument was developed based on the four indicators of mathematical critical thinking skill, namely: formulating the problem, analyzing and solving the problem, evaluating, and drawing conclusion [7, 8, 9, 10]. The instrument consists of three essay questions of linear program word problems. Each student answer had a different scores start from zero to four. A description of the scoring rubric of mathematical critical thinking skill test used in this research was shown in Table 1.

| Mathematical Critical Thinking Indicators | Description of Student Response                   | Score |
|------------------------------------------|---------------------------------------------------|-------|
| Formulating the problem                  | No answer                                         | 0     |
|                                          | Only write down the informations, or only write down the problems that must be solved, but not right | 1     |
|                                          | Only write down the informations, or only write down the problems that must be solved correctly | 2     |
|                                          | Write down the informations and write down the problems that must be solved correctly, but not complete | 3     |
|                                          | Write down the informations and write down the problems that must be solved correctly and completely | 4     |
| Analyzing and solving the problem        | No answer                                         | 0     |
|                                          | Identify the informations and the problems correctly, but cannot make mathematical models | 1     |
|                                          | Identify the informations and the problems correctly, can make mathematical models, but cannot choose the strategy correctly and miscalculate | 2     |
|                                          | Identify the informations and the problems correctly, can make mathematical models, can choose the strategy and calculate correctly | 3     |
|                                          | Identify the informations and the problems correctly, can make mathematical models, can choose the strategy correctly, but miscalculate | 4     |
| Evaluating                               | No answer                                         | 0     |
|                                          | Only find credibility statements but do not detect | 1     |
### Mathematical Critical Thinking Indicators

| Credibility Statements            | Description of Student Response                                      | Score |
|-----------------------------------|-----------------------------------------------------------------------|-------|
| Find and detect credibility       | Find and detect credibility statements, but do not relate to         | 2     |
| statements, but do not relate to  | informations                                                        |       |
| informations                      | Find and detect credibility statements, relate to information, but   | 3     |
|                                   | cannot check the truth                                               |       |
|                                   | Find and detect credibility statements, relate to information, and    | 4     |
|                                   | can check the truth                                                  |       |

| Drawing conclusion                | No answer                                                             | 0     |
|                                  | Make incorrect conclusion and not adapted to the context of           | 1     |
|                                  | mathematical problem                                                 |       |
|                                  | Make incorrect conclusion even though it has been adjusted to the    | 2     |
|                                  | context of mathematical problem                                       |       |
|                                  | Make conclusion correctly but not adjusted to the context of         | 3     |
|                                  | mathematical problem                                                 |       |
|                                  | Make conclusion adjusting to the context of mathematical problem     | 4     |
|                                  |                                                                      |       |

Besides using tests, the data was taken from interview. The confirmed data process was done by comparing information from the result of mathematical critical thinking test and interview. In the final step, researchers drew conclusions from data result which has been done before.

### 3. Results and Discussion

#### 3.1. Results

In this research, 32 students were given 3 essay questions of linear program to solve. Question number 1 is about inequalities that correspond to a graph and the feasible region, question number 2 is about mathematical model that correspond to the problem, and question number 3 is about maximum income in a linear program problem.

**Figure 1.** Three essay questions of linear program.

As many as 32 students answered the questions which were developed based on the mathematical
critical thinking indicators. Each question has 4 indicators, and each indicator has a maximum score of 4 (scale 0-4). For example, in question number 1, only 2 of 32 students can reach the first indicator correctly and get score 4, and only 2 of 32 students could reach the fourth indicator correctly and get score 4. No one can reach second and third indicator correctly and get score 4, dominant students get score 1 and 2. So, it can be concluded from 32 students, only 2 students can formulate the mathematical problem and 2 students can draw conclusion in question number 1 well.

In question number 2, just 4 students get score 4 on first indicator, no one can get score 4 on second, third, and fourth indicator. This is reinforced by many students who get score 1. Otherwise, in question number 3 there are 11 of 32 students get score 4, 3 students can formulate the problem correctly, 2 students can analyze and solve the problem correctly, 2 students in very good evaluation, and 2 students can draw conclusion well. For example, one of 32 students, student (A) answers the question number 3 shown in figure 2, figure 3, figure 5, and figure 6.

a. Write down the information that is known and asked correctly!

![Figure 2. The answer of student (A) on first indicator.](image)

That answer quite correct and the student gets score 3, because the student has not answered completely what is known. The student writes down what is asked incorrectly, it was on the information that is known as Dita’s estimation, and the correct answer for what is asked is “Is Dita’s estimation true? How do you find out?”

b. Suppose the value is known, create a table first, then solve the problem!

![Figure 3. The answer of student (A) on second indicator.](image)
The student is correct in supposing known values, making tables, and using elimination and substitution methods in finding the value of x and y. However, in looking for a feasible region (DP), student shades incorrectly. Therefore, the boundary points obtained are incorrect and student is also not right in determining the optimum value (maximum value). The correct answer for the maximum value is 95,000 with a boundary point of (150,100). The other student’s (B) answer to the question number 3 on second indicator was shown in figure 4.

![Figure 4](image)

**Figure 4.** The answer of student (B) on second indicator.

There are some differences between A and B in answering the question. Student B suppose x and y each as the amount of flour and the amount of sugar. In addition, student B does not make a table first, so she tend to be wrong in supposing x and y. The similarity of two works are determining the feasible region (DP) and the maximum value incorrectly. Next, the answer of student (A) for question number 3 on third indicator was shown in figure 5.

![Figure 5](image)

**Figure 5.** The answer of student (A) on third indicator.

The student does not answer as directed that he must substitute boundary points to the objective function, although he made objective function correctly before. Even though the information on the
question is obvious that Dita’s estimation is the maximum income that will be obtained in more than 100,000.00 IDR, not 100,000.00 IDR. This indicates that the student cannot evaluate the answers he has obtained. Next, the answer of student (A) for question number 3 on fourth indicator was shown in figure 6.

![Figure 6. The answer of student (A) on fourth indicator.]

The student makes conclusion correctly but the student does not adjust to the context of mathematical problem. The student should add what is the error of Dita’s estimation and how it is true. The correct answer is “Dita’s estimation is wrong, Dita’s maximum income is 95,000.00 IDR, and it’s less than 100,000.00 IDR”.

The examples above are some mistakes made by students on one of the mathematical critical thinking questions. Next, the student’s work is corrected by the researcher, then the researcher calculates the total score obtained by the students, after that it is changed to percent form. The test result of 32 students is shown in table 2.

### Table 2. Percentage of students’ mathematical critical thinking skill in Senior High School.

| Indicator                  | Total score each question | Total score each indicator | Percentage |
|----------------------------|---------------------------|-----------------------------|------------|
| Formulating the problem    | Number 1: 57  Number 2: 69  Number 3: 69 | 195                          | 12.70%     |
| Analyzing and solving the problem | Number 1: 56  Number 2: 42  Number 3: 89   | 187                          | 12.17%     |
| Evaluating                 | Number 1: 70  Number 2: 36   Number 3: 77  | 183                          | 11.91%     |
| Drawing conclusion         | Number 1: 60  Number 2: 43  Number 3: 53  | 156                          | 10.16%     |

Based on table 2, it can be seen that the 11th grade students of senior high school have low mathematical critical thinking skills. The percentage obtained by each indicator has quite a bit. Based on indicator of mathematical critical thinking skills, it was shown that students achieve the highest average score on formulating the problem indicator.

Besides using that test, researcher conducted interview with several subjects randomly to find out what their difficulties when finishing the test. The interview illustration summary between researcher (R) and one of subjects (S) can be described as follows.

R : Did you face the difficulties on finishing the problems?
S : Yes
R : What are your difficulties?
S : Even though I can formulate the problems, but I cannot check the answers I have received to be related to the information given. I do not understand, what can I answer for fill this third column.
R : Although you cannot check your answer, but are you sure about the answer you wrote in the second column?
S : Yes, because this question is the same as what my teacher taught me. In these questions, I just can finish it in the usual way, which doesn’t have to be checked.

Based on the interviews that have been conducted, the dominant students have difficulty when evaluating the problem. It has several causes, such as students do not know what to do on evaluate step, students can not connect their answer with the information, and students do not check their answer.

### 3.2. Discussion
Students achieve the highest average score on formulating the problem indicator. This situation indicates that the students have low mathematics skills too, because students were better able to sort out what was known and asked on the question [13]. The evaluating and concluding indicators are the most seldom to do, the “low” categorized students show all indicators in low quality even to formulate has problem to do [14]. Whereas evaluation is useful for make comparisons, reassessing the solution to a problem, and in asking questions to one’s self when after solving a problem in order to consider different possible solutions [15]. Moreover mathematical critical thinking is was constructed from skills, such as spotting conclusions, examining premises, forming conclusions and diagnosing fallacies [16].

The dominant students have difficulty when evaluating the problem, and it is caused by several things: such as students do not understand what should be done when asked to evaluate, in addition students also can not connect their answer with the information and re-check their answer. Furthermore, it causes the students get used to learn and memorize the formula, the students can solve the problem quickly without passing analyzing step, the logical mathematics knowledge is still low, the students cannot apply the concept and have not gotten used to solve the high level difficulty demanding evaluation and analysis [14]. Actually, someone is said to be able to think critically if they are able to interpret and evaluate information or arguments [17].

The students cannot evaluate the problem because the teachers have not assessed the cognitive skill in the level of analyzing, synthesizing, and evaluating classroom activities [12]. The study from the Commission on Teacher Credentialing in California and the Center for Critical Thinking at Sonoma State University initiated a study of school in California to assess current teaching practices and knowledge of critical thinking [18]. They found that only 19% could think critically. This observation is supported by evidence from other sources which shows that many lessons do not encourage the development of critical thinking skills [19]. There is no denying that critical thinking is hard to teach as discrete parts of a course because it is not easy to foster them in the typical classroom pedagogy as meta-cognition is hard to be trained; they must be tightly integrated into the learning to think more deeply, to analyse and then synthesize often apparently contradictory materials and views, applying the familiar to the unfamiliar, and evaluating texts are skills that require plenty of practice to learn, much less to master [2].

4. Conclusion
Based on the result, the students’ mathematical critical thinking skills in the 11th grade students of senior high school in SMA Negeri 1 Welahan Jepara is still low. It is based on the test result analysis. The number of students who can fulfill all mathematical critical thinking indicators are just under 50%, especially in the evaluating and concluding indicators. Therefore, we can recommend for teachers that they should design learning model to improve the skill of the students with low indicator of evaluating, concluding, and remembering its importance in mathematics. Teachers should include analyzing, evaluating, and concluding in classroom activities and assignments so students can think mathematically.

Acknowledgements
The authors would like to thank the referees for improving the quality of this article and also to the Head of SMA Negeri 1 Welahan Jepara for the opportunity to do the research.

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