The jerk and the vertical fall of a shuttlecock

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Abstract
Jerk, the derivative of acceleration with respect to time, is a physical concept of great practical significance. However, this concept is rarely mentioned in textbooks and is often neglected in physics education. This paper describes how the concept of jerk can be easily introduced in the study of the dynamics of falling bodies, which are significantly affected by air resistance. In this regard, the vertical fall of two different feather shuttlecocks, a standard and a miniature one, is studied. In this simple vertical fall experiment, air resistance is significant and measurable, implying that the acceleration changes, and thus, the jerk can be determined. The velocity, acceleration, and jerk measurements with time during the vertical fall are described and compared with those from different standard air resistance models. The proposed setup can help initiate a discussion of well-known basic physics concepts and modeling approaches, such as displacement, velocity, acceleration, and particularly the often-neglected jerk concept.

Keywords: vertical fall, air drag, terminal velocity, jerk, shuttlecock, modeling

(Some figures may appear in colour only in the online journal)

1. Introduction

The time rate of change of the acceleration, the third time derivative of the distance, is typically referred to as jerk [1, 2]. A variable force always produces variable acceleration and hence, a non-zero jerk. As a high jerk can substantially affect the human body, numerous interpretations have been proposed, e.g., the coefficient of ‘fun’ in roller coasters [3], ‘fear’ when cats fall [4] or ‘discomfort’ when an electric elevator starts and stops [5]. Although motions

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with non-constant acceleration are typical phenomena, the third derivative of the position with respect to time is not typically discussed in physics education, and the concept of jerk is rarely mentioned in physics textbooks [2]. An example of a common phenomenon in which the non-negligible jerk discussion is overlooked is the vertical fall where air resistance changes the acceleration.

The study of falling objects is central to all physics curricula. Experimental studies performed by students include falling coffee filters [6–8], toy cats [4], balloons [9–12], and balls [11, 13–15]. Similar to objects in nature, such as leaves, raindrops, or soot particles, none of these objects move through the air with constant acceleration. However, to easily observe the measurable effect of the air resistance, the drag force must be of the same order of magnitude as the weight of the falling object. In particular, the effect of air resistance can be manipulated in various ways. For example, by experimenting in liquid instead of air [13], changing the surface of the object [15], or changing the mass using lightweight objects such as a coffee filter [6–8], muffin cup [16] or balloon [9–12].

Moreover, various ways have been proposed for the experimental study of falling objects. Horvat and Jecmenica [17] presented an experiment in which the time-of-flight of small metal spheres along measured vertical distances was used to determine the gravitational acceleration. Cross and Lindsey [13] dropped different balls and filmed every vertical drop with a video camera. The authors showed that estimates of the drag coefficient of a sports ball, such as tennis, basket, ping-pong, or baseball, can be obtained in a student laboratory dropping the balls from a height of 2 m. Studnička et al [4] described theoretical calculations and measurements of the terminal velocity of a falling cat. The study used a scaled absolute value of the cat jerk to define the ‘coefficient of the cat’s fear’.

The drag force for almost all air drag phenomena occurring in everyday situations is quadratic with respect to the velocity of the moving object, except for a few engineered objects [17, 18]. Although the typical approach to present air resistance in many basic physics texts is qualitatively correct, this approach can lead to misconceptions. The reason is that using the air drag force linear model with respect to the velocity of the object is overemphasizing, thereby underemphasizing the quadratic air force model. This overuse of the linear model is probably due to its computational simplicity rather than its accuracy and appropriateness in describing everyday physical phenomena. Model evaluations and comparisons are at the heart of all studies. Progress in science has been made by constructing models and comparing them to observations and measurements to decide which model best explains the data. Recently, model comparisons and data evaluations have been suggested to provide students with a more enriching laboratory experience [19]. Furthermore, studies has shown that physics labs are more effective when the purpose is to train experimental methods rather than to enhance classroom instruction. Even small elements of open-ended reasoning in labs can improve students’ attitudes towards experimental physics [20–22].

The aerodynamic properties of feather shuttlecocks differ largely from those of other balls, rackets, and projectile sports. Because of the high initial velocity, which decreases rapidly due to the high air resistance, the parabolic flight path after the racket stroke is generally highly skewed, implying that in the game, the fall of the shuttlecock has a steeper angle than the rise. This asymmetrical trajectory ending with an almost vertical fall is known as the ‘aerodynamic wall’ [23]. Understanding the aerodynamic properties of the feather shuttlecock can significantly impact the outcome of the game and, more importantly, can lead to classroom discussions that can reinforce physics learning.

A feather shuttlecock (figure 1) is a good object for studying air resistance during a vertical fall [24]. It is a conical object characterized by its small mass (approximately 5 g), resulting in a small gravitational force. However, the object has a large cross-sectional area
A standard shuttlecock is made of a cork tip and 16 overlapping goose feathers. The cork tip has a diameter \(d\) and a mass \(m\) of approximately 25 mm and 3 g, respectively. The diameter \(D\) of the skirt, total length \((L + d)\), and total mass are approximately 60 mm, 85 mm, and 5 g, respectively.

(approximately 28 cm²), resulting in a large air drag force and a high drag coefficient. This small mass but relatively large surface area enable measuring its position with respect to time using standard laboratory equipment. In addition, the shuttlecock is aerodynamically stable because of its conical shape. Whatever the initial orientation, it always turns to fly cork first and remains in this way [25]. As Peastrel et al [24] pointed out, this particular aspect makes the badminton shuttlecock an almost perfect object for studying air-resistance effects in laboratory physics courses at various levels. Nevertheless, the shuttlecock is also ideal for studying the time rate of change in acceleration in the laboratory. To the best of our knowledge, this study is the first to consider this aspect.

An artificial object instead of a shuttlecock might be used to teach air resistance in a vertical fall. However, students are prone to be interested if the physics curriculum relates to something they are familiar with and possibly already interested in, such as sports. In particular, the lack of data and information on the aerodynamic behavior of the badminton shuttlecock also provides an opportunity for open-ended student activities.

This article discusses a vertical fall experiment in which students can investigate drag force by studying a falling shuttlecock. The students can determine which model better explains the data by contrasting the experimental results with different models describing the drag force. Finally, this study provides a simple example of how the concept of jerk can be introduced in physics education and how it can be used to improve learning, thereby the understanding of fundamental physics concepts.
2. Theory

2.1. The equation of motion

Three vertical forces act on a fully submerged object falling vertically into a fluid: the weight of the object \( F_g \), buoyancy \( F_B \), and fluid resistance \( F_R \) \[26,27\]. By applying Newton’s second law \( F_T = F_g - F_B - F_R \), the equation of motion of the object can be written as follows:

\[
m \frac{d^2 y}{dt^2} = mg - m_B g - F_R(v).
\]

This is a second-order ordinary differential equation with initial conditions \( y = 0 \) and \( v = 0 \) for \( t = 0 \), where \( y \) is the distance traveled at time \( t \), \( m \) is the mass, \( m_B \) is the mass of the displaced volume of the fluid, and \( g \) is the acceleration due to gravity (assumed to be constant). Recall that for a shuttlecock falling in the air, the contribution of buoyancy can be neglected because the dimensions of the shuttlecock are small. Therefore, the displaced mass of air is minimal compared to the mass of the shuttlecock.

2.2. Reynolds number

Fluid resistance can be divided into two components, namely frictional (viscous) and pressure (inertial) forces, and it depends on numerous factors, such as the speed of the object, its shape and size, whether it rotates, and the properties of the fluid. Because the relative velocity of the liquid has to be zero at the interface with a solid surface, a fluid flowing over a solid surface will experience a velocity gradient, that is, a region close to the surface where the fluid velocity increases gradually from zero to the relative velocity far from the surface. The shearing occurring in this velocity gradient is counteracted by the viscosity, and the counteracting viscous force is given by the ratio between the shear stress and the area of the object in contact with the flow \[28\]. Viscosity also indirectly contributes to the pressure force. The viscosity near the surface slows down the fluid at the front of the object. Thus, the fluid does not have enough momentum to move all around the back. The flow is separated from the object before it moves all around, and the fluid immediately behind the object between the stagnation points builds a low-pressure wake. The difference between the low pressure at the back and higher pressure at the front of the object causes a pressure force. Pressure forces are strongly dependent on the shape of the object; when pressure forces dominate, the object is called a bluff body.

The nondimensional Reynolds number \[29\] is the ratio of the inertial to the viscous forces and describes the dynamic flow properties around the object, and hence the fluid forces to which it is subjected. This can be written as follows:

\[
Re = \frac{\text{inertial forces}}{\text{viscous forces}} = \frac{\rho v^2}{\mu} = \frac{Lv}{\eta}
\]

where \( \rho \) is the density of the liquid, \( v \) is the velocity of the object relative to the fluid, \( \mu \) is the dynamic viscosity, \( \eta = \mu/\rho \) is the kinematic viscosity of the fluid, and \( L \) is the characteristic length (in this case, the shuttlecock diameter). In general, the Reynolds number play a decisive role in determining whether a flow is laminar, turbulent, or neither \[30\]. Viscous forces dominate when \( Re < 1 \) and inertial forces when \( 10^3 < Re < 10^5 \), whereas both are significant when \( 1000 > Re > 1 \).

The Reynolds number decreases with viscosity but increases with speed and size. When a small object moves at a low speed through a viscous medium, the molecules initially in front of the object are pushed aside and resume their original position behind the object as it passed.
The viscous forces dominate, Re is small, and the flow is laminar. In contrast, for a large object moving rapidly through a low-viscosity medium, the moving object creates a wake, where the fluid molecules in the wake region swirl around the eddies behind the object. In this case, the inertial forces dominate, Re is large, and the flow is turbulent.

2.3. The fluid resistance

The magnitude of the viscosity and pressure forces vary substantially with respect to each other because the magnitude of the viscosity forces is proportional to the flow speed $v$. In contrast, the magnitude of pressure forces is proportional to $v^2$ [28]. However, fluid-like air does not require much disturbance to become turbulent, implying that the air resistance forces are complex functions of speed. Nevertheless, in numerous cases, a good approximation can be obtained using a combination of linear and quadratic terms, as follows:

$$F_R(v) = k_1v + k_2v^2$$  \hspace{1cm} (3)

where $k_1$ and $k_2$ are proportionality constants that depend on the shape and size of the object and the properties of air [28]. In particular, for small objects moving at low speeds in the air, the linear term is dominant. For large objects moving at high speeds, the air resistance is roughly equal to the square term and is referred to as air drag or simply drag. However, the assumption that the resistive force is proportional to $v$ for laminar flows and $v^2$ for turbulent flows is incorrect. Laminar flow can be proportional to $v$, $v^2$, or more complex functions [31].

2.4. The terminal velocity

An object falling through a fluid in a gravitational field accelerates for a transition period until it reaches a constant final velocity ($v_T$), that is, the steady state of the fall. This terminal velocity is the maximum velocity reached by the falling object and corresponds to $dv/dt = 0$. Thus, the left-hand side of equation (1) vanishes. This occurs when the resistive fluid force ($F_R$) and buoyancy ($F_B$) exactly balance the weight of the object ($F_g$), that is,

$$mg = m_Bg + F_R(v_T).$$  \hspace{1cm} (4)

When the linear term in equation (3) is dominant, the well-known simplification of linear terminal velocity is obtained as follows:

$$v_T \approx \frac{m_{eff}g}{k_1}$$  \hspace{1cm} (5)

where $m_{eff}$ is the effective mass, which is defined as $m_{eff} = (m - m_B)$. Accordingly, the terminal velocity when the quadratic term in equation (3) is dominant is as follows:

$$v_T \approx \sqrt{\frac{m_{eff}g}{k_2}}.$$  \hspace{1cm} (6)

For a shuttlecock falling in air, the contributions due to buoyancy can be neglected, thereby $m_{eff} \approx m$.

2.5. The aerodynamic drag coefficient

The aerodynamic drag coefficient ($C_D$) is a dimensionless quantity defined as follows:

$$F_D(v) = \frac{1}{2}C_D\rho Sv^2$$  \hspace{1cm} (7)
where $F_D$ is the force acting on the shuttlecock in the opposite direction to its motion, $\rho = 1.20 \text{ kg m}^{-3}$ is the density of the fluid, $S$ is the projected frontal area of the shuttlecock without deformation on a plane perpendicular to the direction of motion, and $v$ is the velocity of the shuttlecock relative to the fluid.

Using equation (4), the drag coefficient can be determined from the terminal velocity as follows:

$$C_D = \frac{mg}{\frac{1}{2} \rho S v_T^2}. \quad (8)$$

2.6. The closed form solutions

If the resistive fluid force can be approximated proportional to the velocity of the object, equation (1) can be expressed as follows:

$$\frac{dv}{dt} = g - k_1 m v. \quad (9)$$

A closed-form solution can be obtained by replacing $mg/k_1$ with $v_T$, considering that $v = 0$ when $t = 0$, and solving equation (7). Thus, the jerk ($j$), acceleration ($a$), velocity ($v$), and distance ($y$) as a function of time ($t$) can be expressed as follows:

$$j = -g^2 v_T e^{-(g/v_T)t} \quad (10)$$

$$a = g e^{-(g/v_T)t} \quad (11)$$

$$v = v_T \left[ 1 - e^{-(g/v_T)t} \right] \quad (12)$$

$$y = v_T \left[ t - \frac{v_T}{g} \left( 1 - e^{-(g/v_T)t} \right) \right]. \quad (13)$$

If the drag force is proportional to the square of the object velocity, equation (1) becomes

$$\frac{dv}{dt} = g - k_2 \frac{v^2}{m}. \quad (14)$$

In this case, $\sqrt{mg/k_2}$ is replaced by $v_T$, and $j$, $a$, $v$, and $y$ can be expressed as follows:

$$j = 2g^2 v_T \sinh \left( \frac{g}{v_T} \right) \left( v_T \cosh \left( \frac{g}{v_T} \right) \right)^{-1} \quad (15)$$

$$a = 2g \left( \cosh \left( \frac{2g}{v_T} t \right) + 1 \right)^{-1} \quad (16)$$

$$v = v_T \tanh \left( \frac{g}{v_T} t \right) \quad (17)$$

$$y = \frac{v_T^2}{g} \ln \left[ \cosh \left( \frac{g}{v_T} t \right) \right]. \quad (18)$$

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Figure 2. Photograph of the standard and miniature feather shuttlecocks.

Figure 3. Schematic drawing of the experimental set-up.
Figure 4. Flowchart schematically illustrating the experimental and computational steps.

Note that the above equation can be applied provided that the object is sufficiently large to \( \text{Re} \) reach a value greater than \( 10^3 \) almost immediately after release and sufficiently small to \( \text{Re} \) be less than \( 10^5 \) at the terminal velocity.

Both pressure and viscous drags are significant for \( 1000 > \text{Re} > 1 \), and neither of the models for the limiting cases can be applied. Therefore, the motion equation must be solved numerically.

3. Experiment

A standard and a miniature feathered shuttlecock were selected for this study (figure 2). Their effective mass was measured by weighing them (immersed in air) on a standard spring balance, and their size was measured using a caliper.

The experimental setup (figure 3) consisted of a vertical pole, timer, tape measure, movable rod, electromagnet attached to a household clothespin, and a collision platform equipped with a piezoelectric sensor. The time-of-flight was measured by students in an open stairwell inside the physics building. The time \( (t) \) required to fall a given distance \( (y) \) was measured using a timer, and the distance was measured using a tape measure. The opening of the electromagnet simultaneously started the timer and released the shuttle initially attached to the clothespin.
### Table 1. Physical characteristics of the standard and miniature feather shuttlecocks.

|                           | Standard | Miniature | Scaling factor |
|---------------------------|----------|-----------|----------------|
| Total length (mm)         | 85 ± 2   | 48 ± 2    | 0.56           |
| Length of cock tip (mm)   | 25 ± 1   | 14 ± 1    | 0.56           |
| Cock tip diameter (mm)    | 25 ± 2   | 14 ± 2    | 0.56           |
| Skirt diameter (mm)       | 60 ± 5   | 34 ± 3    | 0.57           |
| Effective mass (g)        | 4.9 ± 0.2| 2.7 ± 0.2 | 0.55           |

The shuttlecock fell freely in the air over the measured distance until it hit the platform, which was equipped with a piezoelectric sensor that stopped the timing device.

Each shuttlecock was released from rest at several altitudes, and for each altitude, the average time-of-flight was calculated using at least five measurements. The standard uncertainties were <0.01 s for all measurements. It was noted that after a fall of about 2 m, all shuttlecocks rotated around their major axes.

Finally, the data points were fitted to the closed-form solution functions in equations (13) and (18) using the terminal velocity ($v_T$) and acceleration due to gravity ($g$) as free parameters. In this regard, standard least-squares curve-fitting routines in MATLAB (MathWorks Inc., Natick, MA, USA) were implemented. The workflow is illustrated in the flowchart in figure 4.

### 4. Results and discussion

The physical dimensions of the two shuttlecocks are listed in table 1. The dimensions of the miniature shuttlecock are 0.56 ± 0.01 times smaller than the standard one, but everything else is identical. Thus, the miniature shuttlecock is an almost perfectly scaled-down version of the standard version.

Because the miniature and standard shuttlecocks are identical in shape but different in size, one may suspect that they are geometrically similar [32]. For example, if the two shuttlecocks are geometrically similar, the ratio of their skirt diameters should be equal to one-third of the power of their mass ratios

$$\frac{D_{\text{Miniature}}}{D_{\text{Standard}}} = \left(\frac{m_{\text{Miniature}}}{m_{\text{Standard}}}\right)^{\frac{1}{3}}.$$  \hspace{1cm} (19)

Although the miniature shuttlecock could have a relatively large projected cross-sectional area-to-volume ratio, this ratio did not hold due to the relatively large mass of the miniature shuttlecock. Therefore, the miniature shuttlecock might not necessarily travel slower towards the ground than the standard shuttlecock.

Because $\eta = 1.50 \times 10^{-5}$ m$^2$ s$^{-1}$ and $\rho = 1.20$ kg m$^{-3}$ at room temperature (20 °C), the Reynolds number for a standard falling shuttlecock in terms of velocity ($v$) and skirt diameter ($D$) can be approximately determined as follows:

$$\text{Re} = \frac{Lv}{\eta} \approx \frac{Dv}{\eta} \approx \frac{0.060 \cdot v}{1.50 \times 10^{-5}} \approx 4 \times 10^3 \cdot v.$$  \hspace{1cm} (20)

This result implies that Re ≫ 1000 almost immediately when the shuttlecock begins to fall. Thus, it can be expected that the quadratic rather than the linear force law applies. This is consistent with previous results [24, 33] and with the results of this analysis, showing that
the best model is the quadratic air resistance force at the instantaneous speed of the two falling shuttlecocks. The shuttlecock is a *bluff body*; the airflow around it cannot completely surround the base, creating a low-pressure wake that increases the total air resistance [34].

Figure 5(a) shows the experimental data and results of the two least-squares fits. The obtained accelerations due to gravity and terminal velocities and the corresponding standard uncertainties are listed in table 2. The results for acceleration due to gravity agree well with the accepted value for the city of Lund in Sweden, which is 9.81 m s\(^{-2}\). Furthermore, the terminal velocity result of the standard feather shuttlecock agrees well with the findings of Peastrel *et al*
Figure 6. Experimental results and results of tree theoretical models for the standard shuttlecock (a) distance fallen (y), (b) velocity (v), (c) acceleration (a), and (d) jerk (j) versus time (t).

[24] using electronic timing gates, Cohen et al [23] using a vertical wind tunnel, and Post et al [33] using a radar gun obtained values of 6.80, 6.7 and 6.79 m s$^{-1}$ respectively. Chen et al [35] used a video camera to study the shuttlecock flight path and found that the terminal velocity varied between 6.51 and 6.87 m s$^{-1}$. The difference might be related to the different methods, variations in the shuttlecock material and design, and the accuracy of the instruments used. Even if the size, mass, and shape are within the range specified by the International Badminton Federation, the shuttlecocks can still behave aerodynamically differently [36]. Cohen et al [23] and Post et al [33] used synthetic shuttlecocks, which are prone to differ from the professional quality feather shuttlecocks used in this study. In particular, the air jets might pass through the gaps in the synthetic skirt, causing skirt deformation. Consequently, the projected area, and hence the drag coefficient, of a synthetic shuttlecock was lower than that of the more stable feather shuttlecock [34, 36]. Compared with the velocity of feathered shuttlecocks, that of synthetic shuttlecocks should be higher, which is consistent with the experience of professional players [36]. However, as Cooke [34] pointed out, at very high speeds, the gaps in the skirt can
increase drag due to the strong axial jet of air, causing a jet-pump effect. This result agreed with Alam et al [37], who showed that the feather shuttlecock exhibited lower drag coefficients at low speeds and significantly higher drag coefficients at high speeds. In contrast, the synthetic shuttlecock demonstrated the opposite trend.

The aerodynamic drag coefficient ($C_D$) at the terminal speed using equation (8) resulted in 0.66 and 0.73 for the standard and miniature shuttlecock, respectively. At the terminal speed, the Reynolds numbers for the two shuttlecocks are approximately $2.5 \times 10^4$ and $1.5 \times 10^4$, respectively. Thus, the standard shuttlecock result is consistent with the results of Alam et al [37].

The displacement ($y$) versus time ($t$), the velocity ($v$), acceleration ($a$), and jerk ($j$) as functions of time ($t$) can be determined using the parameters determined from the least-square curve fit analysis and equations (15)–(17), respectively. The results for the two shuttlecocks are shown in figures 5(b)–(d). As shown in figures 5(b) and (c), the velocity asymptotically approaches the terminal velocity as the acceleration decreases and approaches zero. This occurs
in less than 2 s for both shuttlecocks. As expected, the miniature has the largest terminal velocity.

A changing force always produces a changing acceleration and, thus, a jerk. As shown in figure 5(c), the acceleration decreases faster for the standard shuttlecocks than for the miniature shuttlecocks. This results in a larger absolute value of the corresponding jerk. Moreover, owing to the inflection point of the acceleration (where the acceleration changes the most), both jerks have a distinct minimum, as shown in figure 5(d).

The experimental results were compared with the results of the theoretical models (equations (10)–(13) and (15)–(18)) using the parameters listed in table 2 (figures 6(a)–(d) and 7(a)–(d)). As shown in figures 6(a)–(d), the curves from the quadratic models are always closer in cases where no air resistance is present. This is because, in a vertical fall from rest, the quadratic resistance will initially always be less than the linear resistance [24]. As shown in figures 6(c) and 7(c), the modeled acceleration derived from the quadratic model is initially the largest; however, the two models provide the same results at 0.8 and 1.0 s for the standard and miniature shuttlecocks, respectively. Thereafter, the linear model yielded the highest acceleration. Nevertheless, as shown in figures 6(b) and 7(b), it takes more than a second for the velocity to catch up, and then both shuttlecocks hit the ground.

Finally, as shown in the figures, the most striking characteristic of the quadratic drag model is the existence of the maximum absolute value of the jerk. Studnička et al [4] interpreted this as the maximum fear of a cat during free fall. Recall that a jerk system is a dynamical system modeled by an ordinary differential equation of the third order describing the time evolution of a single variable, for example, the distance a feather shuttlecock has fallen [39]. Because jerk varies as the shuttlecock falls, we also have non-zero higher derivatives of motion, such as the snap. Because higher derivatives are unfamiliar concepts to most people, the falling shuttlecock can stimulate further discussions and insights that can enhance learning in both physics and engineering.

5. Conclusions

A simple and low-cost free-fall experiment using a shuttlecock was described. The experiment is well suited for physics education at undergraduate and graduate levels. The experimental results can be used to initiate a discussion of well-known basic physics concepts, such as displacement, velocity, and acceleration. Moreover, the proposed experiment is particularly well designed to introduce and discuss the often-neglected jerk concept. The experiment could be further developed by using a sensor (e.g. an ultrasonic sensor) that measures distance as a function of time.

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