The complete bispectrum of COBE-DMR Four Year Maps

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Abstract
We extend a previous bispectrum analysis of the COBE-DMR 4 year maps, allowing for the presence of correlations between all possible angular scales. We find that the non-Gaussian signal found earlier for bispectrum components including adjacent modes does not extend to triplets of modes with larger separations. Indeed for all separations $\Delta \ell > 1$ we find that the COBE-DMR data is very Gaussian. The implication seems to be that the previously detected non-Gaussian scale-scale correlation falls off very quickly with mode separation.

1 Introduction

The issue of whether or not the primordial fluctuations are Gaussian is of fundamental importance in theories of structure formation. Gaussian large-scale primordial fluctuations are believed to be the hallmark of the simplest inflationary scenarios. In these theories vacuum quantum fluctuation, with Gaussian statistics, are stretched by a period of exponential expansion to give rise to the large scale fluctuations we observe today. In contrast, topological defect theories, which are also possible candidates for structure formation [Contaldi 00, Contaldi et al 99a], are believed to produce non-Gaussian density fields. Non-Gaussianity is also believed to be associated with some non-minimal inflationary models [Salopek 1992, Linde and Mukhanov 1997, Contaldi et al 99b], such as those generating isocurvature fluctuations [Peebles 97].

Several recent [Ferreira, Magueijo & Górski (1998), Pando et al (1998), Magueijo (2000)] papers have shown indications of non-Gaussianity in the cosmic microwave background (CMB) temperature fluctuations, as measured by the COBE-DMR experiment. The first of these detections
[Ferreira, Magueijo & Górski (1998)] was later found to be due to an experimental systematic [Banday et al 99], and the second [Pando et al (1998)] to an error of method [Barreiro et al 00]. The third claim [Magueijo (2000)], however, remains unassailable. In that work a bispectrum analysis was carried out, using adjacent scales \( \{\ell - 1, \ell, \ell + 1\} \) as components of the bispectrum. A significant deviation from Gaussianity was found, with a high confidence level.

An obvious question is whether this non-Gaussianity extends to components correlating scales with a larger separation, say \( \{\ell - \Delta \ell, \ell, \ell + \Delta \ell\} \), with \( \Delta \ell > 1 \). The purpose of this paper is to answer this question. As we shall see the non-Gaussian signal found in [Magueijo (2000)] does not appear to extend to larger separations \( \Delta \ell \).

\section{Method}

In this letter we examine the possibility of deviations from Gaussianity in terms of the bispectrum, which should be zero for a Gaussian process. Let us expand the temperature fluctuations \( \frac{\Delta T}{T}(n) \) in Spherical Harmonic functions:

\[
\frac{\Delta T}{T}(n) = \sum_{\ell m} a_{\ell m} Y_{\ell m}(n) \tag{1}
\]

A Gaussian probability distribution function is completely described by the first two moments, \( \langle a_{\ell m} \rangle \) and \( \langle a_{\ell m} a_{\ell' m'} \rangle \) i.e. all higher moments can be obtained from the mean value and the variance. This does not hold for Non-Gaussian functions. The Bispectrum, a rotationally invariant cubic form associated with the third moment, \( \langle a_{\ell m} a_{\ell' m'} a_{\ell'' m''} \rangle \), is therefore a possible test of Gaussianity. It is given by

\[
\hat{B}_{\ell_1 \ell_2 \ell_3} = \alpha_{\ell_1 \ell_2 \ell_3} \sum_{m_1 m_2 m_3} \left( \frac{\ell_1 m_1}{m_1} \frac{\ell_2 m_2}{m_2} \frac{\ell_3 m_3}{m_3} \right) a_{\ell_1 m_1} a_{\ell_2 m_2} a_{\ell_3 m_3}
\]

\[
\alpha_{\ell_1 \ell_2 \ell_3} = \frac{1}{(2\ell_1 + 1)^{\frac{1}{2}}(2\ell_2 + 1)^{\frac{1}{2}}(2\ell_3 + 1)^{\frac{1}{2}}} \left( \begin{array}{ccc} \ell_1 & \ell_2 & \ell_3 \\ 0 & 0 & 0 \end{array} \right)^{-1} \tag{2}
\]

where the (…) is the Wigner 3J symbol. In [Magueijo (2000)] correlations between multipoles separated by \( \Delta \ell = 1 \), \( \{\ell - 1, \ell + 1\} \) were investigated. In this letter we complete this work by using the other possible separations, \( \Delta \ell = 2, 3, 4, 5, \) and \( 6 \), which fit within the signal dominated range of \( \ell \) probed by the COBE-DMR experiment. Selection rules require \( \ell_1 + \ell_2 + \ell_3 \leq 2\)
to be even, $|\ell_i - \ell_j| \leq \ell_k \leq \ell_i + \ell_j$ and $m_1 + m_2 + m_3 = 0$. From these constraints we see that a suitable chain of correlators is $\hat{A}_{\ell}^{\Delta \ell} = B_{\ell - \Delta \ell \ell \ell + \Delta \ell}$, with $\ell$ even.

We then follow precisely the same procedure documented in [Magueijo (2000)] and [Ferreira, Magueijo & Górski (1998)], apart from considering the more general ratio

$$J_{\ell}^{\Delta \ell} = \frac{\hat{A}_{\ell}^{\Delta \ell}}{(\hat{C}_{\ell - \Delta \ell})^{1/2}(\hat{C}_{\ell})^{1/2}(\hat{C}_{\ell + \Delta \ell})^{1/2}}$$

(3)

where $\hat{C}_\ell = \frac{1}{2\pi+1} \sum_m |a_{\ell m}|^2$. This quantity is dimensionless, and is invariant under rotations and parity. The same data is examined; the inverse noise-variance-weighted average maps of the 53A,53B,90A and 90B COBE-DMR channels, with monopole and dipole removed, at resolution 6, in galactic and ecliptic pixelization. We use the extended galactic cut of [Banday et al 1997], and [Bennet et al 1996] to remove most of the emission from the plane of the Galaxy. To estimate the $J_{\ell}^{\Delta \ell}$s we set the value of the pixels within the galactic cut to 0 and the monopole and dipole of the cut map to zero. We then integrate the map multiplied with spherical harmonics to obtain the estimates of the $a_{\ell m}$s and apply equations 2 and 3.

The observed $J_{\ell}^{\Delta \ell}$s are to be compared with their distributions $P(J_{\ell}^{\Delta \ell})$ as inferred from Monte Carlo simulations in which Gaussian maps are subject to DMR noise and galactic cut. In simulating DMR noise we take into account the full noise covariance matrix, as described in [Lineweaver et al 1994]. This includes correlations between pixels 60$^\circ$ degrees apart.

3 Results and conclusions

The comparison between $P(J_{\ell}^{\Delta \ell})$ and the observed $J_{\ell}^{\Delta \ell}$ for various $\Delta \ell$ is displayed in figs. 1 - 4. The observed values agree well with the probability distributions from the Monte Carlo distributions: implying that as far as $\Delta \ell > 1$ bispectrum components are concerned the data supports Gaussianity.

To quantify just how well the observations agree with Gaussianity, we do a goodness of fit statistic on the data. We calculate the $\chi^2$ value for the different $\ell$ separations, and compare with the respective distributions $F(\chi^2)$. The rationale behind considering separate $\chi^2$ was explained in [Magueijo (2000)] and for the distributions $P(\chi^2)$ found (which are very

\footnote{This is to be contrasted with more dubious philosophy towards combining separate tests expressed in [Bromley and Tegmark 99, Phillips & Kogut 00]}.
close to Gaussians) it makes sense to use the standard $\chi^2$:

$$\chi^2 = \frac{1}{N} \sum_{i=1}^{N} \frac{(x_i - \mu_i)}{\sigma^2_i}$$

(4)

where $N$ is the number of different observables and $\mu_i$ and $\sigma^2_i$ are the expectation value and the variance of the $i$th observable respectively. This reduces to the expression proposed in [Ferreira, Magueijo & Górski (1998)] in our case. The $F(\chi^2)$ are then calculated through further Monte Carlo simulations, so that possible correlations between the different $\ell$ may be taken into account. The $\chi^2$ distributions and corresponding observed values are shown in fig 5.

As we can see $\chi^2 \approx 1$ for all separations $\Delta \ell > 1$, with deviations well within the spread expected from $F(\chi^2)$. This is to be contrasted with the results obtained for $\Delta \ell = 1$, that is bispectrum components measuring correlations between adjacent modes. For these $\chi^2 \ll 1$, meaning that the observed $J_\ell$ do not display the scatter around zero expected from a Gaussian distribution. We now found that this distinctive non-Gaussian signal does not extend to bispectrum components with $\Delta \ell > 1$. The implication seems to be that the inter-mode correlations suggested by the findings of [Magueijo (2000)] fall off quickly with the scale separation, something which conforms to theoretical prejudice.

In a recent paper [Phillips & Kogut 00] a variety of components of the bispectrum was studied in the context of a semi-analytical texture model. Such a model is at the very least oversimplified, and probably has little to do with real-life textures. In any case that work stresses the importance of analysing off-diagonal components, as we have done in this paper.

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Figure 1: The vertical thick dashed lines represent the values of the observed $J_{\ell}^{\Delta \ell=2}$, for different values of $\ell$. The solid lines are the corresponding probability distribution functions of the $J_{\ell}^{\Delta \ell=4}$'s for a Gaussian sky with extended galactic cut and DMR noise, as inferred from 10000 realizations.
Figure 2: The vertical thick dashed lines represent the values of the observed $J_{1}^{\Delta \ell=3}$, for different values of $\ell$. The solid lines are the corresponding probability distribution functions of the $J_{\ell}^{\Delta \ell=4}$s for a Gaussian sky with extended galactic cut and DMR noise, as inferred from 10000 realizations.
Figure 3: The vertical thick dashed lines represent the values of the observed $J^\Delta l = 4$, for different values of $\ell$. The solid lines are the corresponding probability distribution functions of the $J^\Delta l = 4$'s for a Gaussian sky with extended galactic cut and DMR noise, as inferred from 10000 realizations.
Figure 4: The vertical thick dashed lines represent the values of the observed $J^\Delta \ell$s, $\Delta \ell = 5$, and 6 for different values of $\ell$. The solid lines are the corresponding probability distribution functions of the $J^\Delta \ell$s for a Gaussian sky with extended galactic cut and DMR noise, as inferred from 10000 realizations.
Figure 5: The dashed lines are observed values of $\chi^2$ from the COBE data for different $\ell$ separations $\Delta \ell = 2, 3, 4$ and 5. The solid lines are corresponding probability distribution functions for $\chi^2$ for a Gaussian sky with extended galactic cut and DMR noise, as inferred from 10000 realisations.
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