Strength effects in an imploding cylinder with constant mass-to-explosive loading

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Abstract. High explosives were used to implode thin-walled metal cylinders of different strengths (Al 6061-O, Al 6061-T6, mild steel, and stainless steel) at a constant mass-to-explosive ($M/C$) ratio. The velocity history of the inner surface of the imploding cylinder was measured via photonic Doppler velocimetry. These histories and maximum velocities were compared to an imploding Gurney model that used a detonation pressure-based time constant, giving good agreement with the experiments. The deceleration caused by strength effects was modeled via a simplified stress-strain curve, which was then used to predict the entire velocity history.

1. Introduction

The implosion of metal cylinders has been studied since the beginning of the 20\textsuperscript{th} century, with aspects such as wall velocity, buckling instabilities, and transition to hydrodynamic behavior all well-represented within the literature. Building on the work of Gurney \cite{1}, Chou et al. and Hirsch developed closed-form analytic solutions to the momentum and energy balance equations for an imploding cylinder \cite{2, 3}. Photonic Doppler velocimetry (PDV) \cite{4} has been used to investigate explosively-imploded tubes, measuring inward wall velocities nearing 60 km/s by Dolan et al. \cite{5}. However, experimental results at low explosive loadings, where material strength effects will significantly alter the implosion trajectory, are absent in the literature, prompting the current study.

2. Methodology

A selection of four metals were machined into cylinders of approximate outside diameter ($D$) 102 mm and length 153 mm. These cylinders were first wrapped with 0.3 mm polyethylene film to prevent spallation, and then with two explosive loadings of constant thicknesses 2 mm or 13 mm of Primasheet 1000 (a PETN-based rubberized sheet explosive). The thickness ($t$) of each cylinder varied based on the material density to maintain a constant liner-mass to explosive-mass ratio ($M/C$) of approximately 4 for the 2 mm loading and 0.6 for the 13 mm loading. A wrapped cylinder was placed on an intermediate spacer ring (to allow the detonation to run down the entire length with minimal edge effect), which itself was joined to a polyvinyl chloride (PVC) cap. Within the PVC cap, a central bore was made to accept an aluminum support which centered an optical collimator and 45° mirror with the test cylinder. The grazing detonation was initiated with a waveshaper made of the same explosive to ensure wave planarity and implosion...
Figure 1. Cut-away rendering a) and actual charge with waveshaper b).

Table 1. Matrix of experiments; material properties as in [6 - 9].

| No. | Material       | Rockwell Hardness | $\rho$ [g/cc$^{-1}$] | $E$ [GPa] | $E_h$ [GPa] | $\sigma_Y$ [MPa] | $\sigma_{Ult}$ [MPa] | $D$ [mm] | $t$ [mm] | $M/C$ |
|-----|----------------|-------------------|-----------------------|-----------|-------------|------------------|------------------------|----------|-----------|-------|
| 1   | Steel 1018     | 40.1 B            | 7.87                  | 200       | 2.69        | 236              | 354                    | 101.5    | 1.5       | 3.93  |
| 2   | Steel 304      | 85.3 B            | 8.10                  | 190       | 2.43        | 275              | 393                    | 101.7    | 1.5       | 4.08  |
| 3   | Al 6061-T6     | 56.2 B            | 2.69                  | 69        | 0.625       | 275              | 310                    | 101.6    | 5.0       | 4.31  |
| 4   | Al 6061-O      | 57.4 H            | 2.69                  | 69        | 0.564       | 55               | 125                    | 101.6    | 5.0       | 4.31  |
| 5   | Al 6061-O      | 57.4 H            | 2.69                  | 69        | 0.564       | 55               | 125                    | 100.9    | 4.9       | 0.59  |

symmetry. Planarity of the detonation wave was verified with two pairs of self-shorting time-of-arrival gauges at two axial locations, the lower of which lay directly over the spot of velocity measurement. A cut-away rendering and photo are presented in figure 1, followed by the test matrix presented as table 1.

The PDV interferometer used incorporated a 1550 nm erbium-amplified C band laser whose return signal (processed by a Miteq DR-125G-A-FA optical detector) was sampled by a 13 GHz LeCroy WaveMaster at full bandwidth. For each experiment, a sliding-window Fast Fourier Transform (FFT) was applied to the output data with a window size of either 200 ns (for $M/C \approx 4$) or 100 ns ($M/C \approx 0.6$). The sequence of dominant frequencies, once multiplied by the interferometer’s conversion factor, yielded the velocity history for the inner surface of a cylinder during implosion.
3. Results

3.1. Results and Modeling of Low Loading ($M/C \approx 4$)
All four materials were subjected to the same loading of 2 mm of explosive about their diameters. The peak velocities and characteristic times to reach that velocity corresponded well to Chou et al. and Hirsch’s Gurney methodology [2, 3]. However, each experimental curve bowed over and dropped below the predicted terminal velocity.

To reconcile the discrepancies between the predictions of Gurney methods and the experimental data, the influence of hoop strength was considered. Beginning with a force balance applied to a differential segment of wall and using a simplified stress-strain curve as in figure 2, an analytic solution can be derived. Summing along the radial direction and taking the small-angle approximation, the following equation of motion is obtained:

$$\sigma = \rho r \ddot{r}$$  \hspace{1cm} (1)

where stress is related to strain linearly by Young’s modulus ($E$), by the strain hardening modulus ($E_h$), or held constant at ultimate strength. Strain as a function of radial coordinate is:

$$\varepsilon = \frac{r - r_o}{r_o} r^{-1}$$  \hspace{1cm} (2)

Coupling these equations with the initial condition provided by Chou et al. and Hirsch [2, 3] — that of the expression for an exponentially-decaying acceleration to a terminal inward velocity — equation (1) is solved and plotted alongside the experimental data in figure 3. The model’s solution qualitatively reproduces the observed trends in wall deceleration, although features that might be attributed to multidimensional instability (e.g., buckling) are missed.

3.2. Results and Modeling of High Loading ($M/C \approx 0.6$)
A single experiment was performed with aluminum (6061-O) at a radial explosive thickness of 13 mm. Its velocity trace is shown in figure 4, with poor agreement with the aforementioned Gurney-strength model, specifically due to its high estimate for the time constant.

Unlike Experiments 1-4, this trace does not decelerate, apart from what may be a shock interaction at 2 $\mu$s. The inflection point and subsequent increase in acceleration at 10 $\mu$s may be indicative of a wall thickening event, due to conservation of mass in a geometry with decreasing radius. Assuming Gurney velocities are constant at the center of mass of a differential segment of wall (as in figure 2) and noting the following expression is always valid,

$$dm = 2\rho \pi (r_{CM}^2 - r_i^2) d\theta = \rho \pi (r_{CM}^2 - r_i^2) d\theta$$  \hspace{1cm} (3)
Equation (3) can then be differentiated with respect to time as below.

\[ v_{\text{Gurney}} r_{CM} = v_i r_i \]  

(4)

An expression below for inner wall velocity \((v_i)\) as a function of instantaneous center-of-mass position \((r_{CM})\) may be solved after an integration of the final Gurney velocity over the time period of interest.

\[ v_i = \frac{v_{\text{Gurney}} r_{CM}}{r_i} = v_{\text{Gurney}} r_{CM} \left( \frac{2r_{CM}^2 - r_{oi}^2 + r_{ii}^2}{2} \right)^{-\frac{1}{2}} \]  

(5)

This expression is plotted with the experimental data, along with the Gurney-strength model described in section 3.1, in figure 4. These two models provide a higher- and lower-bound on the actual behavior seen in the experimental trace.
4. Conclusion

Five cylinders of four materials of varying strength were imploded with two distinct explosive loadings. The wall velocities during implosion were captured and compared to Gurney-type estimates of flyer velocity and ramp-up time. A strength-based analytic model was derived, which correlated material strength with deceleration at low loading. At high loading, wall thickening was directly observed via PDV and then confirmed with an analytic model based on conservation of mass.

As both these models only qualitatively capture the trends within the two loading cases, a finite-element simulation of the experiments may be able to further explain the detailed features observed in the PDV traces. These simulations should be conducted in both axisymmetric and planar geometries, so as to determine the role of multidimensional processes on the problem.

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