Abstract
This paper addresses the following question: What kind of symmetry breaking would be required to make the gauginos of a supersymmetric U(3)xU(3) theory look like two families of Standard Model left-handed quarks? The answer is: A heavy Higgs mechanism that breaks the group down to SU(3)xSU(2)xU(1) along with an explicit first order supersymmetry breaking term. After this symmetry breaking, the gauginos have the same charge and gauge interactions as two families of left-handed quarks. Right-handed quarks and leptons for the two families are introduced as part of chiral multiplets, and quark masses are generated through interaction terms.

Most reviews of supersymmetry (SUSY) (see for example [1-4]) state that none of the particles that have so far been detected can be supersymmetric partners of each other. This view is supported by a theorem from Haag, Lopuszanski, and Sohnius [5] which showed that for particles to be supersymmetric partners (in N=1 supersymmetry), they would have to be in the same representation of the gauge group. For example, this restriction would rule out left-handed quarks from being the supersymmetric partners of gauge bosons, since the former are in the (3,2) representation of the group, while the latter are in the adjoint representation. In the 1970s, Fayet explored a number of possibilities of matching up existing particles as supersymmetric partners before finally concluding that this could not be done [6,7]. It is argued that if supersymmetry is realized in Nature, every existing particle must have a supersymmetric partner that has not yet been detected.

Although the above argument applies to theories with unbroken supersymmetry, the aim of this paper is to show by example that it does not necessarily apply to a theory with explicitly broken supersymmetry. Like most supersymmetric models that attempt to describe observed particles (e.g. the MSSM [8-10]), the model explored here has explicit supersymmetry breaking terms in the Lagrangian.

In particular, this paper presents an example of explicitly broken supersymmetry that allows the “gaugino” supersymmetric partners of gauge bosons in a U(3)xU(3) Lagrangian to have the same masses and interactions with gauge bosons as the first two families of left-handed quarks in the Standard Model. The supersymmetry breaking terms added to the Lagrangian go
beyond the usual “soft” terms that are known to avoid introducing quadratic divergences \[11\] and even beyond the “maybe soft” terms presented in [2]. However, arguments are provided that at least for the vector multiplet, the supersymmetry breaking terms should not introduce quadratic divergences.

The explicitly SUSY-broken Lagrangian considered in this paper has an interesting mathematical property. It is an “average” of “Half-SUSY” Lagrangians. A Half-SUSY Lagrangian is defined in this paper through the following feature: If one adds two copies of a Half-SUSY Lagrangian, where in the second copy, all fields are replaced by their conjugates, the resulting Lagrangian is supersymmetric. Conjugate in this sense (and throughout the paper) refers to a conjugate representation of the underlying gauge group. As will be shown below, although not supersymmetric in the usual sense, actions written from Half-SUSY Lagrangians are invariant to superspace translations.

The supersymmetric vector Lagrangian for a U(3) gauge field is given by [1-4]:

\[
\mathcal{L}_{\text{v1}} = tr\left(\hat{\mathcal{L}}_{\text{v1}}\right) = tr\left(-\frac{1}{2} f_{1}^{\mu\nu} f_{j,1\mu\nu} + d_{i}^{2} - 2i \bar{\lambda}_{i} \sigma^{\mu} \partial_{\mu} \lambda_{i} - 2 g_{1} \bar{\lambda}_{i} \sigma^{\mu} \left[a_{1\mu}, \lambda_{1}\right]\right), \tag{1}
\]

where \(a_{1\mu} = a_{1\mu}^{a} t^{a}\) are the U(3) gauge fields, \(g_{1}\) is the SU(3) coupling constant, \(\lambda_{1} = \lambda_{1}^{a} t^{a}\) are left-handed gauginos, \(d_{i} = d_{i}^{a} t^{a}\) are auxiliary fields, and \(t^{a}\) are the U(3) fundamental representation matrices, normalized by \(tr\left(t^{a} t^{b}\right) = \frac{1}{2} \delta^{ab}\). The conventions of [1] and standard notations for 2-component Weyl fermions are used throughout. The \(t^{a}\) include not only the eight SU(3) Gell-Mann matrices (divided by 2), they also include the U(1) matrix \(t^{0} = \frac{1}{\sqrt{6}} \text{diag}(1,1,1)\). Although it does not appear in eq. (1) due to cancellation in the commutator, the U(1) coupling constant for \(d_{i}^{0}\) will be denoted by \(g_{1}'\).

Consider a theory that includes a second U(3) vector Lagrangian labelled with a “2” index. \(\hat{\mathcal{L}}_{\text{v1}}\) and \(\hat{\mathcal{L}}_{\text{v2}}\) may be arranged as block diagonal parts of a 6x6 matrix:

\[
\mathcal{L}_{\text{VSUSY}} = Tr\left(\begin{pmatrix} \hat{\mathcal{L}}_{\text{v1}} & 0 \\ 0 & \hat{\mathcal{L}}_{\text{v2}} \end{pmatrix}\right). \tag{2}
\]

In the above matrix, each of the four quadrants represents a 3x3 matrix and the trace (with a capital T) is taken over the full 6x6 matrix. The Lagrangian \(\mathcal{L}_{\text{VSUSY}}\) can be rewritten using the following notation for 6x6 field matrices:
\[ A_\mu = \begin{pmatrix} a_{1\mu} & 0 \\ 0 & a_{2\mu} \end{pmatrix}, \quad F_{\mu\nu} = \begin{pmatrix} f_{1\mu\nu} & 0 \\ 0 & f_{2\mu\nu} \end{pmatrix}, \quad D = \begin{pmatrix} d_1 & 0 \\ 0 & d_2 \end{pmatrix}. \] (3)

To simplify expressions involving the four independent coupling constants, the following abbreviated notation will also be used when a \( g \) is directly in front of a 6x6 gauge field:

\[ gA_\mu = \begin{pmatrix} g_1(a_{1\mu} - a_{1\mu}^0 t^0) + g'_1a_{1\mu}^0 t^0 & 0 \\ 0 & g_2(a_{2\mu} - a_{2\mu}^0 t^0) + g'_2a_{2\mu}^0 t^0 \end{pmatrix}. \] (4)

The “twist” of this paper is to create combinations of the “1” and “2” gaugino fields and rotate those combinations to the 3x3 off-diagonal blocks of a 6x6 matrix:

\[ \lambda^{(1)} = \frac{1}{\sqrt{2}} \left( \lambda_1 + \lambda_2^T \right), \quad \lambda^{(2)} = \frac{1}{\sqrt{2}} \left( \lambda_1 - \lambda_2^T \right) \]

\[ \Lambda^{(1)} = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 & \lambda^{(1)} \\ \lambda^{(1)T} & 0 \end{pmatrix}, \quad \Lambda^{(2)} = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 & \lambda^{(2)} \\ -\lambda^{(2)T} & 0 \end{pmatrix}. \] (5)

Throughout this paper, an upper T index is used to denote a transpose in U(3) space (not spin space); for example: \( \lambda^T = \lambda^a t^{T a} \). An upper index in parentheses is used to denote fermion family, as opposed to the lower “1” and “2” indices that correspond to the two U(3) Lagrangians in eq (2). Using these 6x6 notations, the supersymmetric Lagrangian of eq (2) can be rewritten as follows:

\[ \mathcal{L}_{\text{VSUSY}} = T \text{r} \left( -\frac{1}{2} F^{\mu\nu} F_{\mu\nu} + D^2 - 2i \sum_{f=1,2} \left( \bar{\Lambda}^{(f)} \tilde{\sigma}^\mu \partial_\mu \Lambda^{(f)} \right) - 2U \left( \bar{\Lambda}^{(1)} + \bar{\Lambda}^{(2)} \right) \tilde{\sigma}^\mu \left[ gA_\mu, \left( \Lambda^{(1)} + \Lambda^{(2)} \right) U \right] \right), \] (6)

where from eq (5)

\[ \Lambda^{(1)} + \Lambda^{(2)} = \begin{pmatrix} 0 & \lambda_1 \\ \lambda_2 & 0 \end{pmatrix}, \quad U = \begin{pmatrix} 0 & I \\ I & 0 \end{pmatrix}, \] (7)

and \( I \) is the 3x3 unit matrix. \( \mathcal{L}_{\text{VSUSY}} \) is still supersymmetric; it is just rewritten with different notation.

The next step is to introduce a first-order term \( g\mathcal{L}_{\text{SB}} \) that explicitly breaks supersymmetry such that the resulting Lagrangian \( \mathcal{L}_\nu = \mathcal{L}_{\text{VSUSY}} + g\mathcal{L}_{\text{SB}} \) takes the form:

\[ \mathcal{L}_\nu = T \text{r} \left( -\frac{1}{2} F^{\mu\nu} F_{\mu\nu} + D^2 - 2 \sum_{f=1,2} \left( i\bar{\Lambda}^{(f)} \tilde{\sigma}^\mu \partial_\mu \Lambda^{(f)} + \bar{\Lambda}^{(f)} \tilde{\sigma}^\mu gA_\mu \Lambda^{(f)} + \bar{\Lambda}^{(f)} \tilde{\sigma}^\mu \Lambda^{(f)} gA_\mu^T \right) \right). \] (8)
Usually when explicit supersymmetry breaking terms are included in a Lagrangian, a restriction is imposed that these terms must be “soft”, meaning that they do not introduce quadratic divergences. For theories involving scalar fields, only a few types of supersymmetry breaking terms have been unambiguously proven to meet this requirement [11]. Although the symmetry breaking term $gL_{VSB}$ used here is not one of the proven “soft” terms, the resulting SUSY-broken theory $L_{\psi}$ is still a gauge theory without any scalar fields, so at least in that context, $gL_{VSB}$ should not introduce quadratic divergences.

If a Brout-Englert-Higgs mechanism breaks the U(3)xU(3) gauge symmetry of $L_{\psi}$ down to SU(3)xSU(2)xU(1), the gauginos $\Lambda^{(f)}$ in eq (8) interact with the remaining gauge bosons in the same way as left-handed quarks interact with the gauge bosons of the Standard Model. To make this correspondence more concrete, for the remainder of the paper, the nonAbelian $a_{1\mu}$ gauge bosons in eq (3) will be taken to correspond to gluons, and the top 2x2 block of $a_{2\mu}$ will be taken to correspond to weak gauge bosons (mixed with the U(1) field). Possibilities for the Higgs symmetry breaking will be discussed later in the paper, but for now it will just be assumed that all gauge fields with components in the last row or column of $a_{2\mu}$ obtain very heavy masses.

Potentially following a symmetry breaking scenario like the one presented later in the paper, the U(1) field that survives after that heavy symmetry breaking has the group structure $T^\gamma = \frac{\sqrt{5}}{2k} \text{diag}(-\frac{1}{3}, -\frac{1}{3}, -\frac{1}{3}, \frac{1}{2}, 0)$, equivalent to that of the electroweak hypercharge. Electroweak symmetry breaking then imparts Z-boson-scale masses to gauge fields with components in the second-to-last row and column of $gA_{\mu}$. The U(1) field that remains massless after this symmetry breaking (the photon) has the following group structure:

$$-gA_{\mu}^\gamma T^\gamma = eA_{\mu}^\gamma \text{diag}(-\frac{1}{3}, -\frac{1}{3}, -\frac{1}{3}, 1, 0, 0),$$  \hspace{1cm} (9)

where the electric charge $e$ can be derived from the coupling constants $g_1, g_2, g_1'$ and $g_2'$. It should be noted that the top 5x5 block of the photon in eq (9) has the same group structure as the photon in the SU(5) grand unified theory [4,12].

The similarities of $L_{\psi}$ to SU(5) are not limited to the photon. The gauge-gaugino interaction in $L_{\psi}$ is similar in structure to the interaction between the gauge fields and the 10-
representation of fermions in SU(5). Using this analogy as a guide, the components of \( \Lambda^{(i)} \) can be re-labelled as follows:

\[
\Lambda^{(i)} = \frac{1}{2} \begin{pmatrix}
0 & 0 & 0 & u_1^{(i)} & d_1^{(i)} & X_1^{(i)} \\
0 & 0 & 0 & u_2^{(i)} & d_2^{(i)} & X_2^{(i)} \\
0 & 0 & 0 & u_3^{(i)} & d_3^{(i)} & X_3^{(i)} \\
 u_1^{(i)} & u_2^{(i)} & u_3^{(i)} & 0 & 0 & 0 \\
d_1^{(i)} & d_2^{(i)} & d_3^{(i)} & 0 & 0 & 0 \\
 X_1^{(i)} & X_2^{(i)} & X_3^{(i)} & 0 & 0 & 0
\end{pmatrix},
\]

(10)

where lower indices represent color and upper indices represent the quark family. The components of \( \Lambda^{(2)} \) can be labelled in a similar way, except with a minus sign for all components in the lower left block. From the interaction terms in \( \mathcal{L}_r \), one can see that the left-handed gauginos \( u_i^{(f)} \), \( d_i^{(f)} \), and \( X_i^{(f)} \) have electric charges of \( \frac{2}{3}, -\frac{1}{3}, \) and \( -\frac{1}{3} \), respectively.

Assuming that the \( X_i^{(f)} \) acquire large masses in the heavy symmetry breaking mentioned above, the remaining gauginos have the same charge and gauge interactions as two families of left-handed quarks in the Standard Model.

At this point, it is interesting to look at a mathematical property of the Lagrangian \( \mathcal{L}_r \). As stated in the introduction, \( \mathcal{L}_r \) can be thought of as an “average” of “half-SUSY” Lagrangians, as shown below. Start with the following Lagrangian:

\[
\mathcal{L}_{\text{VH}} \left( A, D, \bar{\Lambda}, \Lambda \right) = \text{Tr} \left( -\frac{1}{2} F_{\mu\nu} F^{\mu\nu} + D^2 - 2i \bar{\Lambda} \bar{\sigma}^\mu \partial_\mu \Lambda - 2 \bar{\Lambda} \bar{\sigma}^\mu g A_\mu \Lambda - 2 \bar{\Lambda}^T \bar{\sigma}^\mu \Lambda^T g A^T_\mu \right),
\]

(11)

where

\[
\Lambda = \Lambda^{(1)} + \Lambda^{(2)} = \begin{pmatrix}
0 & \lambda_1 \\
\lambda_2 & 0
\end{pmatrix} \quad \text{and} \quad \bar{\Lambda} = -\begin{pmatrix}
0 & \lambda_1^T \\
\lambda_2^T & 0
\end{pmatrix}.
\]

(12)

It is straightforward to verify that the supersymmetric Lagrangian \( \mathcal{L}_{\text{VSUSY}} \) of eq (6) may be obtained from \( \mathcal{L}_{\text{VH}} \) through:

\[
\mathcal{L}_{\text{VSUSY}} = \frac{1}{2} \mathcal{L}_{\text{VH}} \left( A, D, \bar{\Lambda}, \Lambda \right) + \frac{1}{2} \mathcal{L}_{\text{VH}} \left( -A^T, -D^T, \bar{\Lambda}, \bar{\Lambda} \right).
\]

(13)
In other words, $\mathcal{L}_{\text{SUSY}}$ is the “average” of two copies of the Lagrangian $\mathcal{L}_v$, where in the second copy, the fields are replaced with their conjugates. A Lagrangian like $\mathcal{L}_v$ that has this property will be referred to as a “Half-SUSY” Lagrangian.

Using eq (12), the functional dependence of $\mathcal{L}_v$ can be expressed in terms of $\Lambda^{(1)}$ and $\Lambda^{(2)}$ independently. Using this notation, the Lagrangian $\mathcal{L}_v$ presented above that allows gauginos to interact like left-handed quarks can be derived through the following “average” of “Half-SUSY” Lagrangians:

$$\mathcal{L}_v = \frac{1}{2}\mathcal{L}_{vH} \left( A_\mu, D, \bar{\Lambda}^{(1)}, \Lambda^{(2)}, \Lambda^{(1)}, \Lambda^{(2)} \right) + \frac{1}{2}\mathcal{L}_{vH} \left( A_\mu, D, \bar{\Lambda}^{(1)}, -\Lambda^{(2)}, \Lambda^{(1)}, -\Lambda^{(2)} \right).$$

(14)

The procedure outlined above provides additional motivation for the symmetry breaking $\mathcal{L}_v = \mathcal{L}_{\text{SUSY}} + g \mathcal{L}_{\text{SB}}$ that was presented previously.

It would be interesting to ascertain which features of supersymmetry are present in Half-SUSY Lagrangians or their averages. Consider the following Half-SUSY Lagrangian that can be used to construct $\mathcal{L}_v$ of eq (11):

$$\mathcal{L}_{vH1} = \text{tr} \left( -\frac{1}{2} f^{\mu\nu} f_{\mu\nu} + d_i^2 - 2i \bar{\lambda}_1 \bar{\sigma}^\mu \partial_\mu \lambda_1 - 4g_i \bar{\lambda}_1 \bar{\sigma}^\mu a_\mu \lambda_1 \right).$$

(15)

Up to a total derivative, $\mathcal{L}_{vH1}$ can be written in terms of a chiral superfield $W_\alpha$ as follows:

$$\mathcal{L}_{vH1} = -\frac{1}{2} \text{tr} \left( \int d^2 \theta W^\alpha W_\alpha + \text{h.c.} \right)$$

(16)

$$W_\alpha = \lambda_\alpha - \sigma^{\mu\nu} \theta_\mu f_{\nu\alpha} + i \theta_\alpha d + \theta^2 \sigma_{\alpha\alpha} \left( i \partial_\alpha \bar{\lambda}_\alpha - 2 \bar{\lambda}^\alpha a_\mu \lambda_\alpha \right),$$

(17)

where “+h.c.” means to add the Hermitian conjugate and the notation conventions of [1] have been used. Since $\mathcal{L}_{vH1}$ involves an integral $\int d^2 \theta$ over chiral superfields, a superspace translation of $W^\alpha W_\alpha$ leaves the action $\int d^4 x \mathcal{L}_{vH1}$ invariant. In addition to this, there may be other features of supersymmetry present in Half-SUSY Lagrangians.

At least some of these features may continue to be present when an average is taken of Half-SUSY Lagrangians. Consider the following sum:

$$\mathcal{L}_{vH1A} = \frac{1}{2}\mathcal{L}_{vH1} \left( a_{1\mu}, d_1, \bar{\lambda}_1, \lambda_1 \right) + \frac{1}{2}\mathcal{L}_{vH1} \left( a_{2\mu}, d_2, \bar{\lambda}_2, \lambda_2 \right).$$

(18)

Since $\mathcal{L}_{vH1}$ is a Half-SUSY Lagrangian, $\mathcal{L}_{vH1A}$ is also a Half-SUSY Lagrangian – just with double the number of independent fields. An average Lagrangian can be obtained from the path
integral formalism by using $\mathcal{L}_S$ in the exponent and inserting functional delta functions like $\delta(\alpha - \beta)$ or $\delta(\lambda)$ into the path integral. In other words, an average of Half-SUSY Lagrangians can be thought of a specific configuration of a more general Half-SUSY Lagrangian.

To generate quark masses for this theory, one must add the remaining quarks (and other fermions) of the first two families. Starting with the supersymmetric vector Lagrangian $\mathcal{L}_S$, one may introduce chiral multiplets and form a supersymmetric Lagrangian involving them: $\mathcal{L}_{SUSY}$. After adding an explicit supersymmetry breaking term and vacuum expectation values for certain scalar fields, it is possible to obtain a Lagrangian with the same interactions and mass terms as the first two families of quarks in the Standard Model.

As was the case for the vector Lagrangian, the form of the supersymmetry breaking term can be derived by taking an average of Half-SUSY Lagrangians. Rather than simply write down the SUSY-broken Lagrangian as was done in the vector section, in this section, the Lagrangian will be derived from a Half-SUSY Lagrangian and a number of averaging procedures. But first, more notation must be defined.

Throughout this paper, a tilde is used to denote the conjugate representation of a group. For example, the fundamental and conjugate-fundamental representation matrices of U(3) are

$$ t_i^a = t^a, \quad \bar{t}_i^a = -t^{T_a}, \quad (19) $$

where the subscript “F” denotes the fundamental representation.

The group U(3) also supports an antisymmetric representation. Consider the following matrix and vector:

$$ \phi_M = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 & \phi_3 & -\phi_2 \\ -\phi_3 & 0 & \phi_1 \\ \phi_2 & -\phi_1 & 0 \end{pmatrix}, \quad \phi_A = \begin{pmatrix} \phi_1 \\ \phi_2 \\ \phi_3 \end{pmatrix}. \quad (20) $$

The antisymmetric matrix above has the same group structure as the $u^c_i$ (left-handed antiparticles to right-handed up quarks) in the 10 representation of SU(5). Noting that $\left( \phi_M \right)^\dagger = \frac{1}{\sqrt{2}} e^{i\phi} \phi_k$, one can see that
\[ \text{tr} \left( \phi_m^* \tau^a \phi_m + \phi_m^* \phi_m \tau^a \right) = \phi_A^* \left( -\tau^a + 3t^0 \delta^{a0} \right) \phi_A. \]  
\hfill (21)

Consequently, the matrices for the antisymmetric and conjugate-antisymmetric representations of U(3) are defined to be:

\[ t^a_A = -\tau^a + 3t^0 \delta^{a0}, \quad \tilde{t}^a_A = t^a - 3t^0 \delta^{a0}, \]  
\hfill (22)

where the subscript “A” denotes the antisymmetric representation. From eqs (19) and (22), it can be seen that the antisymmetric representation is the same as the conjugate-fundamental representation for SU(3), but differs for U(3).

For the remainder of the paper, whenever an “A” or “F” subscript or a tilde is applied to \( A_\mu, A, \) or \( D, \) it means to replace all of the \( t^a \) matrices in their definition with the appropriate matrices from eqs (19) or (22) above. As will be shown later in the paper, if one replaces the \( t^a \) matrices with the \( t^a_A \) matrices of eq (22) when constructing the photon, one finds that in the antisymmetric representation, the photon has the following group structure:

\[ -gA_\mu^T \gamma^\nu = eA_\mu^T \text{diag} \left( -\frac{2}{3}, -\frac{2}{3}, -\frac{2}{3}, 0, 1, 1 \right). \]  
\hfill (23)

The placement of the 0 in this representation of the photon is important, since it facilitates the generation of up and charm quark masses.

Now that four representations of U(3) have been identified, the next step is to add the following four 6-vectors of fermions and scalars into the theory:

\[ \Psi_F = \begin{pmatrix} \psi_{1F} \\ \psi_{2F} \end{pmatrix}, \quad \bar{\Psi}_F = \begin{pmatrix} \bar{\psi}_{1F} \\ \bar{\psi}_{2F} \end{pmatrix}, \quad \Psi_A = \begin{pmatrix} \psi_{1A} \\ \psi_{2A} \end{pmatrix}, \quad \bar{\Psi}_A = \begin{pmatrix} \bar{\psi}_{1A} \\ \bar{\psi}_{2A} \end{pmatrix} \]

\[ \Phi_F = \begin{pmatrix} \phi_{1F} \\ \phi_{2F} \end{pmatrix}, \quad \bar{\Phi}_F = \begin{pmatrix} \bar{\phi}_{1F} \\ \bar{\phi}_{2F} \end{pmatrix}, \quad \Phi_A = \begin{pmatrix} \phi_{1A} \\ \phi_{2A} \end{pmatrix}, \quad \bar{\Phi}_A = \begin{pmatrix} \bar{\phi}_{1A} \\ \bar{\phi}_{2A} \end{pmatrix}, \]  
\hfill (24)

where the top and bottom components of each of the above pairs are 3-vectors associated with one of the U(3) representations in (19) or (22). For the scalar fields, the top and bottom triplets will be referred to as squarks and sleptons, respectively. To make chiral multiplets, the theory also includes auxiliary fields \( F \) and the antiparticles of the chiral fermions. Using these multiplets, one may construct the following Lagrangian:

\[ \mathcal{L}_H = \mathcal{L}_{VH} + \mathcal{L}_{SH} + \mathcal{L}_{WH} + \mathcal{L}_{KH} + \mathcal{L}_{GH} + \mathcal{L}_{HH} \]

\[ \mathcal{L}_{SH} = \sum_{\bar{R}} \left( \Phi^*_{\bar{R}} \left( \partial_{\mu} - igA_{\mu}^{\bar{R}} \right) \Phi_{\bar{R}} + F_{\bar{R}}^* F_{\bar{R}} + \Phi^*_{\bar{R}} gD_{\bar{R}} \Phi_{\bar{R}} \right) \]
\[ \mathcal{L}_{\text{WH}} = \sum_{\tilde{R}} \left( \left( \partial W / \partial \tilde{\Phi}_{\tilde{R}} \right) \tilde{F}_{\tilde{R}} \right) - \frac{1}{2} \sum_{\tilde{R},\tilde{R}'} \left( \partial^2 W / \partial \Phi_{\tilde{R}} \partial \Phi_{\tilde{R}'} \right) \Psi_{\tilde{R}'} + h.c. \]

\[ \mathcal{L}_{\text{KH}} = \sum_{\tilde{R}} \left( -i \overline{\Psi}_{\tilde{R}} \tilde{\sigma}^\mu \partial_{\mu} \Psi_{\tilde{R}} \right) \]

\[ \mathcal{L}_{\text{GH}} = -2 \overline{\Psi}_{\mu} \tilde{\sigma}^\mu g A_{\mu} \overline{\Phi}_{\mu} - 2 \overline{\Psi}_{\Lambda} \tilde{\sigma}^\mu g A_{\mu} \Psi_{\Lambda} \]

\[ \mathcal{L}_{\text{HI}} = 2 \sqrt{2} i \Phi_{\lambda} g A_{\mu} U \overline{\Phi}_{\mu} + 2 \sqrt{2} i \Phi_{\lambda} g A_{\mu} U \overline{\Phi}_{\mu} + h.c. \]

\[ (25) \]

where \( \mathcal{L}_{\text{WH}} \) is defined in eq (11) and the sums over \( \tilde{R} \) (and \( \tilde{R}' \)) mean to sum over four values: the “F” and “A” representations and their conjugates. Also \( U \) is the rotation matrix of eq (7) and \( W \) is the superpotential, a function of the scalar fields.

Just as in eq (4), \( g \) multiplied by \( A_{\mu}, \Lambda, \) or \( D \) is being used as abbreviated notation to include all of the correct coupling constants for each part of the field’s representation. For example:

\[ g \tilde{\Lambda}_{\lambda} = \begin{pmatrix} 0 & g_{1} \left( \lambda_{1} - \lambda_{1}^{0} t_{0} \right) - 2 g_{1} \lambda_{1}^{0} t_{0} \\ g_{2} \left( \lambda_{2} - \lambda_{2}^{0} t_{0} \right) - 2 g_{2} \lambda_{2}^{0} t_{0} & 0 \end{pmatrix}. \]

Using \( \mathcal{L}_{\text{HI}} \), one may construct the following Lagrangian:

\[ \mathcal{L}_{\text{SUSY}} = \frac{1}{2} \mathcal{L}_{\text{HI}} \left( A_{\mu}, D, \Lambda, \Psi_{\mu}, \overline{\Phi}_{\mu}, \Phi_{\mu}, F_{\mu}, \tilde{F}_{\mu} \right) + \frac{1}{2} \mathcal{L}_{\text{HI}} \left( \tilde{A}_{\mu}, \tilde{D}, \tilde{\Lambda}, \overline{\Phi}_{\mu}, \Phi_{\mu}, \overline{F}_{\mu}, \tilde{F}_{\mu} \right). \]

\[ (27) \]

On the right side of the above equation, the conjugate representations are shown explicitly; the subscript \( \tilde{R} \) only designates two values “F” or “A” (as opposed to \( \tilde{R} \) in eq (25) which was used to designate four values: F, A, and their conjugates). In the second term of eq (27), it is also implied that \( \overline{\Lambda}, \overline{\Phi}_{\mu}, \overline{\Psi}_{\mu} \) are replaced by their conjugates \( \tilde{\Lambda}, \tilde{\Phi}_{\mu}, \tilde{\Psi}_{\mu} \). It is straightforward to check that \( \mathcal{L}_{\text{SUSY}} \) is indeed a supersymmetric Lagrangian (see for example eq (5.11) in [1]). As a result, \( \mathcal{L}_{\text{HI}} \) is a Half-SUSY Lagrangian.

Just as for the vector multiplet theory, when \( \mathcal{L}_{\text{HI}} \) is “averaged” over different field values in a certain way, the resulting Lagrangian can be made to have interactions similar to those in the Standard Model. For \( \mathcal{L}_{\text{HI}} \), however, the averaging required is more involved.

One may begin by defining the field combinations:
\[
\tilde{\Psi}^{(1)}_F = \frac{1}{\sqrt{2}} \left( \bar{\Psi}_F + U\Psi_F \right) \quad \tilde{\Psi}^{(2)}_F = \frac{1}{\sqrt{2}} \left( \bar{\Psi}_F - U\Psi_F \right)
\]
\[
\Psi^{(1)}_A = \frac{1}{\sqrt{2}} \left( \Psi_A + U\bar{\Psi}_A \right) \quad \Psi^{(2)}_A = \frac{1}{\sqrt{2}} \left( \Psi_A - U\bar{\Psi}_A \right)
\]

(28)

A first averaging procedure (designated by appending a “1” index to the Lagrangian) is defined by
\[
\mathcal{L}_{HI} \left( \Lambda^{(2)}, \tilde{\Psi}^{(2)}_F, \Psi^{(2)}_A, \ldots \right) = \frac{1}{2} \mathcal{L}_{HI} \left( \Lambda^{(2)}, \tilde{\Psi}^{(2)}_F, \Psi^{(2)}_A, \ldots \right) + \frac{1}{2} \mathcal{L}_{HI} \left( -\Lambda^{(2)}, -\tilde{\Psi}^{(2)}_F, -\Psi^{(2)}_A, \ldots \right),
\]

(29)

where only the fields that change sign are shown; all other fields have the same sign in both terms. For eq (29) and the remainder of the averaging procedures in this paper, it is implied that if a field changes sign (e.g. \( \Lambda^{(2)} \)), then its antiparticle field also changes sign (e.g. \( \bar{\Lambda}^{(2)} \)). With this averaging, \( \mathcal{L}_{KH} \), \( \mathcal{L}_{GH} \) and \( \mathcal{L}_{HI} \) transform to the following:
\[
\mathcal{L}_{KH1} = \sum_{j=1,2} \left( -i\bar{\Psi}^{(f)}_F \sigma^\mu \partial_\mu \Psi^{(f)}_F - i\bar{\Psi}^{(f)}_A \sigma^\mu \partial_\mu \tilde{\Psi}^{(f)}_A \right)
\]
\[
\mathcal{L}_{GH1} = \sum_{j=1,2} \left( \bar{\Psi}^{(f)}_F \sigma^\mu gA^\mu \Psi^{(f)}_F - \bar{\Psi}^{(f)}_A \sigma^\mu gA^\mu \tilde{\Psi}^{(f)}_A \right)
\]
\[
\mathcal{L}_{HI1} = 2i \left( \Phi^{*}_F \left( g\Lambda^{(1)}_F \tilde{\Psi}^{(1)}_F - g\Lambda^{(2)}_F \tilde{\Psi}^{(2)}_F \right) + \Phi^{*}_A \left( g\bar{\Lambda}^{(1)}_A \Psi^{(1)}_A - g\bar{\Lambda}^{(2)}_A \Psi^{(2)}_A \right) \right) + h.c.
\]

(30)

Keeping in mind SU(5), the particle content of the chiral fermions can now be labelled as follows:
\[
\begin{array}{c}
\tilde{\Psi}^{(1)}_F = \left( \begin{array}{c}
d^c_1 \\
d^c_2 \\
d^c_3 \\
e^- \\
v_e \\
N_e 
\end{array} \right) \quad \tilde{\Psi}^{(2)}_F = \left( \begin{array}{c}
s^c_1 \\
s^c_2 \\
s^c_3 \\
\mu^- \\
v_\mu \\
N_\mu 
\end{array} \right) \quad \Psi^{(1)}_A = \left( \begin{array}{c}
u^c_1 \\
u^c_2 \\
u^c_3 \\
N_\mu' \\
C_\mu \\
e^+
\end{array} \right) \quad \Psi^{(2)}_A = \left( \begin{array}{c}
c^c_1 \\
c^c_2 \\
c^c_3 \\
N_\mu' \\
C_\mu \\
\mu^+
\end{array} \right)
\end{array}
\]

(31)

From the form of \( \mathcal{L}_{GH1} \), one can see that the quarks, electrons, and muons in the above vectors have the same charges and gauge interactions as in the Standard Model. The additional leptons \( C, N \), and \( N' \) are assumed to be heavy. One can also see that due to the off diagonal form of the left-handed quarks, there are terms in \( \mathcal{L}_{HI1} \) involving squarks that have only a single lepton field. These terms imply interactions that violate lepton number conservation.

The next stage of averaging is the color averaging defined in the Appendix and notated with double brackets. The Lagrangian after this averaging is labelled with a “2” index:
\[
\mathcal{L}_{n2} = \left[ \left[ \mathcal{L}_{n1} \right] \right].
\]

In particular, from eq (A11), color averaging transforms \( \mathcal{L}_{n1} \) into the following:

\[
\mathcal{L}_{n2} = \sqrt{2} i \phi^*_{2F} \left( \frac{8}{9} g_2 + \frac{1}{9} g_2' \right) \left( \lambda^{(1)T} d^c + \lambda^{(2)T} s^c \right) + \sqrt{2} i \phi^*_{2A} \left( \frac{8}{9} g_2 - \frac{2}{9} g_2' \right) \left( \lambda^{(1)T} u^c + \lambda^{(2)T} c^c \right) + h.c.
\]

where the 3x3 matrix notations of eqs (5) and (31) are used above. There are a couple of things to note about \( \mathcal{L}_{n2} \). First, color averaging has caused the lepton number violating terms to cancel. Second, if symmetry breaking causes the following neutral (see eqs (9) and (23)) sleptons to acquire vacuum expectation values

\[
\langle \phi_{2F} \rangle = \begin{pmatrix}
0 \\
-i M_F \\
0
\end{pmatrix}
\quad \langle \phi_{2A} \rangle = \begin{pmatrix}
-i M_A \\
0 \\
0
\end{pmatrix},
\]

then \( \mathcal{L}_{n2} \) generates quark mass terms. However, at this stage of averaging, there is no difference between quark masses in different families and no Cabibbo mixing [4,13].

To introduce a Cabibbo angle, \( \mathcal{L}_{n2} \) can be re-expressed in terms of the following rotated variables:

\[
\lambda_{\theta}^{(1)T} = \cos \theta_c \lambda^{(1)T} - \sin \theta_c \lambda^{(2)T} \quad \lambda_{\theta}^{(2)T} = \cos \theta_c \lambda^{(2)T} + \sin \theta_c \lambda^{(1)T}
\]

\[
u_{\theta}^c = \cos \theta_c u^c - \sin \theta_c c^c \\
c_{\theta}^c = \cos \theta_c c^c + \sin \theta_c u^c
\]

The third averaging procedure is the following:

\[
\mathcal{L}_{n3} \left( \lambda_{\theta}^{(2)T}, c_{\theta}^c, s^c, \ldots \right) = \frac{1}{2} \mathcal{L}_{n2} \left( \lambda_{\theta}^{(2)T}, c_{\theta}^c, s^c, \ldots \right) + \frac{1}{2} \mathcal{L}_{n2} \left( -\lambda_{\theta}^{(2)T}, -c_{\theta}^c, -s^c, \ldots \right).
\]

Transforming back to the unprimed fields after the average, one has:

\[
\mathcal{L}_{n3} = \sqrt{2} i \left( \frac{8}{9} g_2 + \frac{1}{9} g_2' \right) \cos \theta_c \phi_{2F}^* \left( \left( \cos \theta_c \lambda^{(1)T} - \sin \theta_c \lambda^{(2)T} \right) d^c + \left( \cos \theta_c \lambda^{(2)T} + \sin \theta_c \lambda^{(1)T} \right) s^c \right) \\
+ \sqrt{2} i \left( \frac{8}{9} g_2 - \frac{2}{9} g_2' \right) \phi_{2A}^* \left( \lambda^{(1)T} u^c + \lambda^{(2)T} c^c \right) + h.c.
\]

This averaging has generated the needed Cabibbo angle between the two families.

Additional averaging procedures can be performed to allow different masses for quarks in different families. The following averaging procedures are defined:

\[
\mathcal{L}_{n4} \left( d^c, \ldots \right) = \frac{1}{2} \left( 1 + \xi_d \right) \mathcal{L}_{n3} \left( d^c, \ldots \right) + \frac{1}{2} \left( 1 - \xi_d \right) \mathcal{L}_{n3} \left( -d^c, \ldots \right)
\]

11
\[ \mathcal{L}_{H5} \left( s^c, \ldots \right) = \frac{1}{2} \left( 1 + \xi_s \right) \mathcal{L}_{H4} \left( s^c, \ldots \right) + \frac{1}{2} \left( 1 - \xi_s \right) \mathcal{L}_{H4} \left( -s^c, \ldots \right) \]

\[ \mathcal{L}_{H6} \left( u^c, \ldots \right) = \frac{1}{2} \left( 1 + \xi_u \right) \mathcal{L}_{H5} \left( u^c, \ldots \right) + \frac{1}{2} \left( 1 - \xi_u \right) \mathcal{L}_{H5} \left( -u^c, \ldots \right) \]

\[ \mathcal{L} \left( c^c, \ldots \right) = \frac{1}{2} \left( 1 + \xi_c \right) \mathcal{L}_{H6} \left( c^c, \ldots \right) + \frac{1}{2} \left( 1 - \xi_c \right) \mathcal{L}_{H6} \left( -c^c, \ldots \right) \] (38)

Where the $\xi_q < 1$ and are real. Not only do these constants allow the quark masses to be different from each other, as noted below they also allow the freedom to let the same slepton vacuum expectation values play the role of the Higgs Boson in imparting masses to the W and Z bosons.

The final Lagrangian derived by imposing the above averaging procedures on the Half-SUSY Lagrangian of eq (25) is:

\[ \mathcal{L} = \mathcal{L}_v + \mathcal{L}_s + \mathcal{L}_w + \mathcal{L}_{KH1} + \mathcal{L}_{GH1} + \mathcal{L}_t, \] (39)

where $\mathcal{L}_v$, $\mathcal{L}_{KH1}$, and $\mathcal{L}_{GH1}$ are given by eqs (8) and (30), and

\[ \mathcal{L}_t = \sqrt{2} i \left( \frac{8}{9} g_2 + \frac{1}{9} g^\prime_2 \right) \cos \Theta_C \phi_{2F}^+ \left( \xi_d \lambda_{1i}^{(1)r} d^c + \xi_s \lambda_{1e}^{(2)r} s^c \right) \]

\[ + \sqrt{2} i \left( \frac{8}{9} g_2 - \frac{2}{9} g^\prime_2 \right) \tilde{\phi}_{2A}^+ \left( \xi_u \lambda_{1i}^{(1)r} u^c + \xi_c \lambda_{1e}^{(2)r} c^c \right) + h.c. \] (40)

Due to color averaging, $\mathcal{L}_s$ gets modified such that the only interactions of squarks with $a_{1\mu}$ are through $a_{1\mu}^0 t_{\tilde{R}}^0$ and $tr \left( a_{1\mu} \tilde{a}_{1\mu} \right)$. The superpotential part of the Lagrangian $\mathcal{L}_w$ is also modified by color averaging as discussed below.

Full specification of the superpotential and any additional soft symmetry breaking terms required to generate vacuum expectation values for scalar fields is outside the scope of this paper. Nonetheless a few comments are in order.

Consider a superpotential where there is no mixing between squarks and sleptons, so it can be split into two parts: $W = W_1 + W_2$. One possibility for the squark superpotential would be

\[ W_1 = m_{1F} \phi_{1F} \tilde{\phi}_{1F} + m_{1A} \phi_{1A} \tilde{\phi}_{1A}. \] (41)

Through the F-terms, this choice for $W_1$ would give masses to the squarks, allowing them to be heavy relative to current experimental energies. But due to the half-flipped definitions of eq (28), the second derivative term in $\mathcal{L}_{WH}$ has the danger of creating mass terms that mix leptons and quarks. Fortunately that term vanishes after color averaging.
\[
\left[\Psi_R \left( \partial^2 W_i / \partial \Phi_R \partial \Phi_R \right) \Psi_R \right] = 0,
\]

so there would be no quark-lepton mass mixing from a squark superpotential like \( W_i \).

As for the slepton superpotential, if it includes the term \( \phi_{2F} \phi_{2M} \bar{\phi}_{2F} \) where the “M” index stands for the matrix form of the antisymmetric representation (as in eq (20)), the double derivative term in \( \mathcal{L}_{WH} \) will include \( \phi_{2F} \left( l_{Me} - l_{Mn} \right) \left( l_{Fe} - l_{Fm} \right) \), where notation from eq (31) is being used. After the first round of averaging, this becomes \( \phi_{2F} \left( l_{Me} l_{Fe} + l_{Mn} l_{Fm} \right) \). Due to the nonzero vacuum expectation value \( \langle \phi_{2F} \rangle \), this will generate electron and muon mass terms (with the same mass). Additional averaging similar to that of eq (38) could be used to obtain different electron and muon masses.

It is interesting to sketch possible mechanisms that could lead to the heavy symmetry breaking mentioned at the beginning of the paper and how a third chiral multiplet family could potentially play a role.

One possibility would be for \( U(3) \times U(3) \) to \( SU(3) \times SU(3) \times U(1) \) symmetry breaking to happen first (at the highest mass scale). The two initial \( U(1) \) fields with group structure \( T_1^0 = \frac{1}{\sqrt{6}} \text{diag}(1,1,1,0,0,0) \) and \( T_2^0 = \frac{1}{\sqrt{6}} \text{diag}(0,0,0,1,1,1) \) could be rewritten in terms of rotated fields with group structure \( T_1^0 = \frac{1}{\sqrt{2}} \text{diag}(-1,-1,-1,1,1) \) and another diagonal matrix \( T_H^0 \). For the case \( g_1' = g_2' \), \( T_H^0 \) would be proportional to the 6x6 unit matrix. One could introduce a chiral multiplet and its conjugate that only interact with the gauge field proportional to \( T_H^0 \). A Brout-Englert-Higgs mechanism could then generate a large mass for that gauge field, leaving the field proportional to \( T_H^0 \) still massless at that stage.

A second stage of heavy symmetry breaking (from \( SU(3) \times SU(3) \times U(1) \) to \( SU(3) \times SU(2) \times U(1) \)) could be accomplished by introducing another set of multiplets like those in eq (24). For concreteness, assume all the scalar fields in these multiplets are labelled with a \( (3) \) upper index. If symmetry breaking caused the sleptons in this family to acquire the following vacuum expectation value

\[
\langle \phi_{2F}^{(3)} \rangle = \begin{pmatrix} 0 \\ 0 \\ -iM_H \end{pmatrix},
\]

(43)
that would impart large masses to the gauge bosons in the bottom row and column of $a_{2\mu}$. In addition, the X fermions of eq (10) could acquire large masses in the same way that the down and strange quarks did in eq (33). The diagonal gauge field that would stay massless during this symmetry breaking would be the mixture of $T^0$ and $T_2^8$ proportional to

$$T^V = \frac{1}{\sqrt{2}} \left( 2T^0 + T_2^8 \right) = \frac{\sqrt{5}}{\sqrt{8}} \text{ diag} \left( -\frac{1}{3}, -\frac{1}{3}, -\frac{1}{3}, \frac{1}{2}, \frac{1}{2}, 0 \right)$$

$$T_A^V = \frac{1}{\sqrt{2}} \left( 2T_A^0 + T_{2A}^8 \right) = \frac{1}{\sqrt{8}} \left( 4T^0 - T_2^8 \right) = \frac{\sqrt{6}}{\sqrt{8}} \text{ diag} \left( -\frac{2}{3}, -\frac{2}{3}, -\frac{2}{3}, \frac{1}{2}, \frac{1}{2}, 1 \right). \quad (44)$$

The first expression is the expected group structure of the weak hypercharge in the fundamental representation, and the second is the group structure in the asymmetric representation.

It should be noted that the slepton vacuum expectation value $\langle \phi_{2F} \rangle$ of eq (34) that is required for down and strange quark masses also imparts masses to gauge fields with components in the second row and column of $a_{2\mu}$. As usual in electroweak symmetry breaking, the diagonal gauge fields mix to make one that is massless (the photon) and one that is massive (the Z). The photon has the group structure

$$T^V = \frac{\sqrt{5}}{\sqrt{8}} \left( T^V + \frac{\sqrt{5}}{\sqrt{8}} T_2^3 \right) = \frac{\sqrt{5}}{\sqrt{8}} \text{ diag} \left( -\frac{1}{3}, -\frac{1}{3}, -\frac{1}{3}, 1, 0, 0 \right)$$

$$T_A^V = \frac{\sqrt{5}}{\sqrt{8}} \left( T_A^V + \frac{\sqrt{5}}{\sqrt{8}} T_{2A}^3 \right) = \frac{\sqrt{5}}{\sqrt{8}} \left( T_A^V - \frac{\sqrt{5}}{\sqrt{8}} T_2^3 \right) = \frac{\sqrt{6}}{\sqrt{8}} \text{ diag} \left( -\frac{2}{3}, -\frac{2}{3}, -\frac{2}{3}, 0, 1, 1 \right), \quad (45)$$

where the latter is in the antisymmetric representation. The slepton vacuum expectation value $\langle \tilde{\phi}_{2A} \rangle$ of eq (34) that is required for up and charm quark masses also imparts masses to gauge fields with components in the first row and column of $\tilde{a}_{2A\mu}$. Due to the zero in $T_A^V$, the photon remains massless in the presence of $\langle \tilde{\phi}_{2A} \rangle$.

The masses acquired by the W and Z bosons from the slepton vacuum expectation values are $g_2 \sqrt{M_F^2 + M_A^2}$ and $g_2 \sqrt{M_F^2 + M_A^2}/\cos \theta_W$, respectively. The mixing angle used here is defined in the usual way by

$$\sin^2 \theta_W = \frac{\frac{3}{5} g_Y^2}{\frac{3}{5} g_Y^2 + g_2^2}, \quad (46)$$
where \( \sqrt{\frac{3}{5}} g_Y \) corresponds to the standard normalization of the weak hypercharge. The constants introduced in eq (38) provide the freedom that allows quark masses and W and Z boson masses to all be generated from the same slepton vacuum expectation values.

This paper has presented an example of a Lagrangian with explicitly broken supersymmetry in which the gauginos have the same gauge interactions and masses as the first two families of left-handed quarks in the Standard Model. The motivation behind this work was to identify assumptions that would be needed to revisit the possibility that some currently known particles are superpartners with each other. Furthering that motivation, in the example of this paper, the slepton superpartners of the leptons play the role of the Higgs boson.

For a theory such as the one presented here to be taken seriously, a number of questions must be addressed: Does the explicit symmetry breaking introduce quadratic divergences or problems with renormalizability? What are the details of the “heavy” gauge symmetry breaking from U(3)xU(3) to SU(3)xSU(2)xU(1)? Would the analysis improve if U(3)xU(3) were considered a subgroup of a semi-simple GUT like SO(6)xSO(6)? How exactly is the third family of fermions incorporated, and do they play a role in the heavy gauge symmetry breaking? Does the theory predict interactions or relationships that contradict experimental evidence?

In addition to looking into these questions, further work will also include exploring the features of supersymmetry that are present Half-SUSY Lagrangians and their averages.

Appendix: Color Averaging

In preparation of performing a color averaging procedure, primed versions of the colored gauge and gaugino fields are defined as follows (see eqs (3), (5) and (10)):

\[
a_{\mu}^{i} = \begin{pmatrix} a_{1\mu}^{i1} & \alpha' \beta' a_{1\mu}^{i2} & \alpha' \gamma' a_{1\mu}^{i3} \\ \beta' \alpha' a_{1\mu}^{i4} & a_{1\mu}^{i5} & \beta' \gamma' a_{1\mu}^{i6} \\ \gamma' \alpha' a_{1\mu}^{i7} & \gamma' \beta' a_{1\mu}^{i8} & a_{1\mu}^{i9} \end{pmatrix} \quad \sqrt{2} \lambda^{i} = \begin{pmatrix} \alpha' u_1^{(i)} & \alpha' d_1^{(i)} & \alpha' X_1^{(i)} \\ \beta' u_2^{(i)} & \beta' d_2^{(i)} & \beta' X_2^{(i)} \\ \gamma' u_3^{(i)} & \gamma' d_3^{(i)} & \gamma' X_3^{(i)} \end{pmatrix}, \quad (A1)
\]

where \( \alpha', \beta' \) and \( \gamma' \) are the complex constants specified below whose complex conjugate is also their inverse. Double-primed and triple-primed versions of the fields are also defined in the same way, just with double-primed and triple-primed constants. The constants are:
\[ \alpha' = 1 \quad \beta' = e^{2i\pi/3} \quad \gamma' = e^{-2i\pi/3} \]
\[ \alpha'' = e^{2i\pi/3} \quad \beta'' = e^{-2i\pi/3} \quad \gamma'' = 1 \]
\[ \alpha''' = e^{-2i\pi/3} \quad \beta''' = 1 \quad \gamma''' = e^{2i\pi/3}. \]  

(A2)

For the other fields in the vector Lagrangian \( \mathcal{L}_v \) (\( a_2 \), \( d_1 \), and \( d_2 \)), their primed, double-primed and triple-primed versions are defined to be the same as their unprimed versions. With these definitions, it is straightforward to verify that
\[ \mathcal{L}_v = \mathcal{L}'_v = \mathcal{L}''_v = \mathcal{L}'''_v, \]  

(A3)

where a prime on the Lagrangian means to make all fields in the Lagrangian primed.

Primed versions of the upper 3 components of the vectors in eq (31) are defined in the following way:
\[ d^{c'} = \begin{pmatrix} \alpha^{c'} d_1^c \\ \beta^{c'} d_2^c \\ \gamma^{c'} d_3^c \end{pmatrix} \quad s^{c'} = \begin{pmatrix} \alpha^{c'} s_1^c \\ \beta^{c'} s_2^c \\ \gamma^{c'} s_3^c \end{pmatrix} \quad u^{c'} = \begin{pmatrix} \alpha^{c'} u_1^c \\ \beta^{c'} u_2^c \\ \gamma^{c'} u_3^c \end{pmatrix} \quad c^{c'} = \begin{pmatrix} \alpha^{c'} c_1^c \\ \beta^{c'} c_2^c \\ \gamma^{c'} c_3^c \end{pmatrix}. \]  

(A4)

Again, double-primed and triple-primed versions are defined the same way, just using the corresponding constants in (A2). Like \( \mathcal{L}_v \), \( \mathcal{L}_{gh} \) is unchanged when all fields are replaced with their primed, double-primed, or triple-primed versions.

There are terms in the Lagrangian \( \mathcal{L}_{m1} \) of eq (30) that violate lepton conservation number since they connect a lepton with a left-handed quark and a squark. There are many choices that could be made in defining a color averaging scheme. The following choice is made in order to cause the lepton number violating terms in \( \mathcal{L}_{m1} \) to vanish: For the leptons as well as all components of scalar and auxiliary chiral fields (including the squarks), primed, double-primed and triple-primed versions are defined to be the same as unprimed versions.

With the above definitions, a color averaging procedure can be specified. Color averaging has two steps. The first step is defined through the following:
\[ [f] = \frac{1}{3} \left( f' + f'' + f''' \right), \]  

(A5)

where primes are applied to all of the fields in \( f \). To see how this step of color averaging affects \( \mathcal{L}_{m1} \), it is helpful to first expand the abbreviated notation used in \( \mathcal{L}_{m1} \). For example:
\[ g^\Lambda^{(i)}_A = ( \overline{g} + \frac{1}{2} \overline{g} I_- ) \Lambda^{(i)} - 2 \left( (\overline{g}' + \frac{1}{2} \overline{g}) + \frac{1}{2} (\overline{g}' + \frac{1}{2} \overline{g}) I_- \right) \frac{1}{\sqrt{12}} \lambda^{(i)\nu} U, \]  

(A6)
where eqs (5) and (10) were used and
\[
\bar{g} = \frac{1}{2}(g_1 + g_2), \quad \bar{g} = (g_1 - g_2), \quad \bar{g}' = \frac{1}{2}(g_1' + g_2'), \quad \bar{g}' = (g_1' - g_2')
\]
\[
I_\perp = \begin{pmatrix} I & 0 \\ 0 & -I \end{pmatrix}
\]
\[
\lambda^{(i)p} = \frac{1}{q^3} \left( u_1^{(i)} + d_2^{(i)} + X_3^{(i)} \right)
\]
(A7)
Consistent with eq (A1), one has:
\[
\lambda'^{(i)p} = \frac{1}{\sqrt{2}} \left( \alpha' u_1^{(i)} + \beta' d_2^{(i)} + \gamma' X_3^{(i)} \right)
\]
(A8)
with corresponding definitions for double- and triple-primed versions.

Due to the fact that \(1 + e^{2\pi i/3} + e^{-2\pi i/3} = 0\), all of the terms in \([\mathcal{L}_{\text{tr1}}]\) involving squarks cancel in the color averaging. For example one finds that \([\phi_{1p}^{*} \lambda^{(i)}] = 0\). One also finds that
\[
\left[ \lambda^{(i)T} d^c \right] = \lambda^{(i)T} d^c,
\]
as is true with the other three flavors of quarks. However, one finds
\[
\left[ \frac{1}{\sqrt{2}} \lambda^{(i)p} I d^c \right] = \frac{1}{3} \cdot \frac{1}{2} \begin{pmatrix} u_1^{(i)} d_1^c \\ d_2^{(i)} d_2^c \\ X_3^{(i)} d_3^c \end{pmatrix}.
\]
(A9)
The second step in color averaging is denoted by \([\{ f \}]\) and is defined by averaging over the following 3 cases: (i) original labelling of color indices, (ii) relabeling the color indices \(1 \to 2 \to 3 \to 1\), and (iii) relabeling the color indices \(1 \to 3 \to 2 \to 1\). After this averaging, one finds:
\[
\left[ \frac{1}{\sqrt{2}} \lambda^{(i)p} I d^c \right] = \frac{1}{3} \cdot \frac{1}{2} \begin{pmatrix} u_1^{(i)} d_1^c \\ d_2^{(i)} d_2^c \\ X_3^{(i)} d_3^c \end{pmatrix} = \frac{1}{9} \cdot \frac{1}{2} \lambda^{(i)T} d^c
\]
(A10)
Putting it together, after both steps of color averaging, one has:
\[
\{\mathcal{L}_{\text{tr1}}\} = \sqrt{2} i \tilde{\phi}_{2p}^{*} \left( \frac{2}{9} g_2 + \frac{4}{9} g_2' \right) \left( \lambda^{(1)T} d^c + \lambda^{(2)T} s^c \right) + \sqrt{2} i \tilde{\phi}_{1p}^{*} \left( \frac{2}{9} g_2 - \frac{4}{9} g_2' \right) \left( \lambda^{(1)T} u^c + \lambda^{(2)T} c^c \right) + \text{h.c.}
\]
(A11)
This is used in the main body of the paper.
REFERENCES

[1] R. Argurio, PHYS-F-417 Supersymmetry Course (2017),
http://homepages.ulb.ac.be/~rargurio/susycourse.pdf .
[2] S. Martin, arXiv:hep-ph/9709356v7 (2016).
[3] H. Haber and L. Haskins, arXiv:hep-ph/1712.05926v4 (2018).
[4] P. Binetruy, Supersymmetry, Oxford University Press (2006).
[5] R. Haag, J. Lopuszanski, and M. Sohnius, Nucl. Phys. B88, 257 (1975).
[6] P. Fayet, Phys. Lett. 64B, 159 (1976).
[7] P. Fayet, Phys. Lett. 69B, 489 (1977).
[8] J.F. Gunion and H.E. Haber, Nucl. Phys. B272, 1 (1986); Nucl. Phys. B402, 567 (1993).
[9] H.E. Haber and G.L. Kane, Phys. Rept. 117, 75 (1985).
[10] A. Djouadi, Phys. Rept. 459, 1 (2008).
[11] L. Girardello and M.T. Grisaru, Nucl. Phys. B194, 65 (1982).
[12] H. Georgi and S.L. Glashow, Phys. Rev. Lett. 32, 438 (1974).
[13] N. Cabibbo, Phys. Rev. Lett. 10, 531 (1963).