Restudy of the color-allowed two-body nonleptonic decays of bottom baryons $\Xi_b$ and $\Omega_b$ supported by hadron spectroscopy

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In this work, we calculate the branching ratios of the color-allowed two-body nonleptonic decays of the bottom baryons, which include the $\Xi_b \to \Xi^0$ and $\Omega_b \to \Omega^0$ weak transitions by emitting a pseudoscalar meson ($\pi^-, K^-, D^-, \text{ and } D_s^-$) or a vector meson ($\rho^-, K^{*-}, D^{*-}, \text{ and } D_s^{*-}$). For achieving this aim, we adopt the three-body light-front quark model with the support of hadron spectroscopy, where the spatial wave functions of these heavy baryons involved in these weak decays are obtained by a semirelativistic potential model associated with the Gaussian expansion method. Our results show that these decays with the $\pi^-, \rho^-$, and $D_s^{*-}$-emitted mode have considerable widths, which could be accessible at the ongoing LHCb and Belle II experiments.

I. INTRODUCTION

The investigation of bottom baryon weak decay has aroused the attentions from both theorist and experimentalist. It is not only an important approach to deepen our understanding to the dynamics of the weak transition, but also is the crucial step of searching for new physics beyond the Standard Model (SM).

Taking this opportunity, we want to introduce several recent progresses. As we know, the lepton flavor universality (LFU) violation has been examined in various $b \to c$ weak transitions [1–7] in the past decade. The measurement of the ratio $R_{D^{(*)}} = |B(B \to D^{(*)}τντ)/B(B \to D^{(*)}eμνeμ)|$ [1–7] shows the discrepancy with the prediction of the SM [8], which indicates the possible evidence of new physics. Inspired by the anomalies of $R_{D^{(*)}}$ existing in the $b \to c$ weak transitions, it is interesting to study the corresponding ratios for the bottom baryon weak decays like $\Xi_b \to \Xi_c τντ$ and $\Omega_b \to \Omega_c τντ$, where the key point is to calculate the form factors involved in the corresponding weak transition of the bottom baryon into the charmed baryon. For the nonleptonic decays of the bottom baryon, a series of intriguing measurements were performed, which include the observation of charmed and charmless modes [9–12], the discovery of the hidden-charm pentaquark states $P_c(4312)$, $P_c(4380)$, $P_c(4440)$, and $P_c(4457)$ in the $Λ_b \to J/\psi pK$ process [13, 14], and $P_{cs}(4459)$ in the $Ξ_b \to J/\psi ΛK$ process [15]. These efforts make us gain a deeper understanding of the dynamics involved in the heavy-flavor baryon weak decays.

Although great progress had been made, continuing to explore new allowed decay modes of the bottom baryons is a research issue full of opportunity [see the Particle Data Group (PDG) [12] for learning the present experimental status]. With the accumulation of experimental data, the LHCb experiment shows its potential to explore the allowed decays of the bottom baryons like the $\Xi_b$ and $\Omega_b$ states, which is still missing in the PDG. Besides, with the KEKB upgrading to the SuperKEKB, the center-of-mass energy of the $e^+e^-$ collision may reach up to 11.24 GeV. The ongoing Belle II [16] should be a potential experiment to perform the study on the bottom-flavor physics.

Facing this exciting status, we have reason to believe that it is suitable time to investigate the two-body nonleptonic decays of the $\Xi_b$ and $\Omega_b$ baryons, which is the main task of this work.

The bottom baryon weak decays have been widely studied by various approaches including the quark models [17–24], the flavor symmetry method [25], the light-front approach [25–30], and the quantum chromodynamics (QCD) sum rules [31–34]. For these theoretical studies, how to estimate the form factors of the weak transition is the key issue. Additionally, for the bottom baryon weak decays, how to optimize the three-body problem is also a challenge. Usually, the quark-diquark scheme as an approximate treatment was widely used in previous theoretical works [25–29, 35]. And the spatial wave functions of these hadrons involved in the bottom baryon weak decays are approximately taken as a simple harmonic oscillator wave function, which makes the results dependent on the parameter of the harmonic oscillator wave function. For avoiding the uncertainty from these approximate treatments mentioned above, in this work we calculate the weak transition form factors of the $\Xi_b \to \Xi^0_c$ and $\Omega_b \to \Omega^0_c$ transitions with emitting a pseudoscalar meson ($\pi^-, K^-, D^-, \text{ and } D_s^-$) or a vector meson ($\rho^-, K^{*-}, D^{*-}, \text{ and } D_s^{*-}$) in the three-body light-front quark model. Here, $\Xi^0_c$ denotes the ground state $\Xi_c$ or its first radial excited state $\Xi_c(2970)$, while $\Omega^0_c$ represents the ground state $\Omega_c$ or its first radial excited state $\Omega_c(2S)$. In the realistic calculation, we take the numerical spatial wave functions of these involved bottom and charmed baryons as input, where the semirelativistic potential model [30, 36] associated with the Gaussian expansion method (GEM) [37–40] is adopted. By fitting the mass spectrum of these observed bottom and charmed baryons, the parameters of the adopted semirelativistic potential model can be fixed. Comparing with former approximation of taking a simple harmonic oscillator wave function [25–29, 35], the treatment given in this work can avoid the uncertainties resulting from the selection of the spatial wave function of the heavy baryon. Thus, the color-
allowed two-body nonleptonic decays of bottom baryons $\Xi_b$ and $\Omega_b$ with the support of hadron spectroscopy as a development. In the following sections, more details will be illustrated.

This paper is organized as follows. After the introduction, the formula of the form factors of the weak transitions $\Xi_b \rightarrow \Xi_c^{(*)}$ and $\Omega_b \rightarrow \Omega_c^{(*)}$ is given in Sec. II. For getting the numerical spatial wave functions of these involved heavy baryons, we introduce the adopted semirelativistic potential model and GEM. With these results as input, the calculated concerned form factors are displayed. In Sec. III, we study the color-allowed two-body nonleptonic decays with emitting a pseudoscalar meson ($\pi^0$, $K^-$, $D^-$, and $D_s^-$) or vector meson ($\rho^-$, $K^{*-}$, $D^{*-}$, and $D_s^{*-}$) in the naive factorization assumption. Finally, the paper ends with a short summary.

II. THE TRANSITION FORM FACTORS OF THE BOTTOM BARYON TO THE CHARMED BARYON

In this section, we briefly introduce how to calculate the form factors discussed in this work. Given that the quarks are confined in hadron, the weak transition matrix element cannot be calculated in the framework of perturbative QCD. Usually, the weak transition matrix element can be parametrized in terms of a series of dimensionless form factors [28, 30]

$$
\langle \mathcal{B}_b(1/2^+)(P', J')|\bar{c}\gamma^\mu(1-\gamma_5)b|\mathcal{B}_b(1/2^+)(P, J)\rangle = \bar{u}(P', J') \left[ f_1(q^2)\gamma^\mu + i f_2(q^2)\sigma_{\mu\nu} q^\nu + \frac{f_3(q^2)}{M} q^\mu \right] \gamma_5 u(P, J) 
$$

as the transitions of the bottom baryon to the charmed baryon. Here, $M(P)$ and $M'(P')$ are the mass(four-momentum) for the initial and final baryons, respectively, $\sigma_{\mu\nu} = i[\gamma^\mu, \gamma^\nu]/2$, and $q = P - P'$ denotes the transferred momentum between the initial and final baryons.

The vertex function of a single heavy-flavor baryon $\mathcal{B}_Q$ ($Q = b, c$) with the spin $J = 1/2$ and the momentum $P$ is

$$
|\mathcal{B}_Q(P, J, J_a)\rangle = \sqrt{2(2\pi)^3} \int d^3p_1 d^3p_2 d^3p_3 \frac{\psi(P, J, J_a)\bar{\xi}_b(p_1, p_2, p_3)}{2(2\pi)^3} \psi(Q, J, J_a, J_b)\bar{\xi}_c(p_1, p_2, p_3) \langle \xi_b(p_1, p_2, p_3)|\mathcal{B}_b(1/2^+)(P, J)\rangle \langle \xi_c(p_1, p_2, p_3)|\mathcal{B}_c(1/2^+)(P', J')\rangle 
$$

for the $\mathcal{B}_b(3_f) \rightarrow \mathcal{B}_c(3_f)$ and $\mathcal{B}_b(6_f) \rightarrow \mathcal{B}_c(6_f)$ transitions, respectively. Here, $\tilde{P} = p_1 + p_2 + p_3$ and $\tilde{P}' = p_1 + p_2 + p_3'$ are the light-front momenta for initial and final baryons, respectively, considering $p_1 = p_1'$ and $p_2 = p_2'$ in the spectator scheme, while

$$
\tilde{p}_i = (p^+_i, p_{i\perp}), \quad p^+_i = p^+_0 + p^+_3, \quad p_{i\perp} = (p^1_i, p^2_i). 
$$
respectively.

In the previous references [25–29, 35], the wave functions for baryon are usually treated as a simple harmonic oscillator forms with the oscillator parameter $\beta$, which results in the $\beta$ dependence of the result. For avoiding this uncertainty, in this work, we adopt the numerical spatial wave functions for these involved baryons calculated by solving the three-body Schrödinger equation with the semirelativistic quark model.

To calculate the form factors defined in Eq. (2.1) from Eqs. (2.5)-(2.6), $V^+, \Lambda^+, \bar{q}_\perp \cdot \bar{V}, \bar{q}_\perp \cdot \bar{A}, \bar{n}_\perp \cdot \bar{V},$ and $\bar{n}_\perp \cdot \bar{A}$ are applied within a special gauge $q^*_\perp = 0$. The details can be found in Ref. [27]. Finally, the form factors are expressed as [30]

\[
\begin{align*}
\hat{f}_1'(q^2) &= \int \mathcal{D}S_0 \frac{1}{8\vec{P}^* \cdot \vec{P}^*} \text{Tr}[ \hat{P} + M_0 \gamma^\mu (\hat{P}^* + M'_0) \gamma^\nu (\hat{p}_3^* + m_3)]. \\
\hat{f}_2'(q^2) &= \int \mathcal{D}S_0 \frac{i M}{8\vec{P}^* \cdot \vec{P}^* q^2} \text{Tr}[ \hat{P} + M_0 \gamma^\mu q_\mu (\hat{P}^* + M'_0) \gamma^\nu (\hat{p}_3^* + m_3)]. \\
\hat{f}_3'(q^2) &= \int \mathcal{D}S_0 \frac{M}{M + M'} \left( \int \mathcal{D}S_0 \frac{1}{4 \sqrt{P^* \cdot P^*_q}} \text{Tr}[ \hat{P} + M_0 \gamma^\nu (\hat{P}^* + M'_0) \gamma^\nu (\hat{p}_3^* + m_3)] \right). \\
\hat{g}_1'(q^2) &= \int \mathcal{D}S_0 \frac{1}{8\vec{P}^* \cdot \vec{P}^*} \text{Tr}[ \hat{P} + M_0 \gamma^\mu (\hat{P}^* + M'_0) \gamma^\nu (\hat{p}_3^* + m_3) \gamma^\nu (\hat{p}_3^* + m_3)], \\
\hat{g}_2'(q^2) &= \int \mathcal{D}S_0 \frac{i M}{8\vec{P}^* \cdot \vec{P}^* q^2} \text{Tr}[ \hat{P} + M_0 \gamma^\mu q_\mu (\hat{P}^* + M'_0) \gamma^\nu (\hat{p}_3^* + m_3) \gamma^\nu (\hat{p}_3^* + m_3)], \\
\hat{g}_3'(q^2) &= \int \mathcal{D}S_0 \frac{M}{M + M'} \left( \int \mathcal{D}S_0 \frac{1}{4 \sqrt{P^* \cdot P^*_q}} \text{Tr}[ \hat{P} + M_0 \gamma^\nu (\hat{P}^* + M'_0) \gamma^\nu (\hat{p}_3^* + m_3) \gamma^\nu (\hat{p}_3^* + m_3)] \right). \\
\end{align*}
\]

(2.7)

\[
\begin{align*}
\mathcal{D}S_0 &= \frac{d \vec{x}_1 d \vec{k}_{\perp 1} d \vec{x}_2 d \vec{k}_{\perp 2} \phi^\prime(x_i^\prime, \vec{k}_{\perp i}) \phi(x_i, \vec{k}_{\perp i})}{2(2\pi)^3} \frac{1}{16 \sqrt{x_3^3 x_M^3 M_0^3}} \sqrt{(e_1 + m_1)(e_2 + m_2)(e_3 + m_3)(e_i^\prime + m_i^\prime)(e_i + m_i)}.
\end{align*}
\]

and

\[
\begin{align*}
\hat{f}_1'(q^2) &= \int \mathcal{D}S_1 \frac{1}{8\vec{P}^* \cdot \vec{P}^*} \text{Tr}[ \hat{P} + M_0 \gamma^\mu (\hat{P}^* + M'_0) \gamma^\alpha \gamma^\beta (\hat{p}_3^* + m_3) \gamma^\nu (\hat{p}_3^* + m_3) \gamma^\nu \gamma^\nu], \\
\hat{f}_2'(q^2) &= \int \mathcal{D}S_1 \frac{i M}{8\vec{P}^* \cdot \vec{P}^* q^2} \text{Tr}[ \hat{P} + M_0 \gamma^\mu q_\mu (\hat{P}^* + M'_0) \gamma^\alpha \gamma^\beta (\hat{p}_3^* + m_3) \gamma^\nu (\hat{p}_3^* + m_3) \gamma^\nu \gamma^\nu], \\
\hat{f}_3'(q^2) &= \int \mathcal{D}S_1 \frac{M}{M + M'} \left( \int \mathcal{D}S_1 \frac{1}{4 \sqrt{P^* \cdot P^*_q}} \text{Tr}[ \hat{P} + M_0 \gamma^\nu (\hat{P}^* + M'_0) \gamma^\nu (\hat{p}_3^* + m_3) \gamma^\nu (\hat{p}_3^* + m_3) \gamma^\nu \gamma^\nu] \right), \\
\hat{g}_1'(q^2) &= \int \mathcal{D}S_1 \frac{1}{8\vec{P}^* \cdot \vec{P}^*} \text{Tr}[ \hat{P} + M_0 \gamma^\mu (\hat{P}^* + M'_0) \gamma^\alpha \gamma^\beta (\hat{p}_3^* + m_3) \gamma^\nu (\hat{p}_3^* + m_3) \gamma^\nu \gamma^\nu], \\
\hat{g}_2'(q^2) &= \int \mathcal{D}S_1 \frac{i M}{8\vec{P}^* \cdot \vec{P}^* q^2} \text{Tr}[ \hat{P} + M_0 \gamma^\mu q_\mu (\hat{P}^* + M'_0) \gamma^\alpha \gamma^\beta (\hat{p}_3^* + m_3) \gamma^\nu (\hat{p}_3^* + m_3) \gamma^\nu \gamma^\nu], \\
\hat{g}_3'(q^2) &= \int \mathcal{D}S_1 \frac{M}{M + M'} \left( \int \mathcal{D}S_1 \frac{1}{4 \sqrt{P^* \cdot P^*_q}} \text{Tr}[ \hat{P} + M_0 \gamma^\nu (\hat{P}^* + M'_0) \gamma^\nu (\hat{p}_3^* + m_3) \gamma^\nu (\hat{p}_3^* + m_3) \gamma^\nu \gamma^\nu] \right), \\
\mathcal{D}S_1 &= \frac{d \vec{x}_1 d \vec{k}_{\perp 1} d \vec{x}_2 d \vec{k}_{\perp 2} \phi^\prime(x_i^\prime, \vec{k}_{\perp i}) \phi(x_i, \vec{k}_{\perp i})}{2(2\pi)^3} \frac{1}{48 \sqrt{x_3^3 x_M^3 M_0^3}} \sqrt{(e_1 + m_1)(e_2 + m_2)(e_3 + m_3)(e_i^\prime + m_i^\prime)(e_i + m_i)(e_i^\prime + m_i^\prime)(e_i + m_i)}.
\end{align*}
\]

(2.8)

for the $B_0(\bar{3}_f) \to B_0(\bar{3}_f)$ and $B_0(6_f) \to B_0(6_f)$ transitions, respectively.

III. THE SEMIRELATIVISTIC POTENTIAL MODEL FOR CALCULATING BARYON WAVE FUNCTION

In this section, we illustrate how to obtain the concerned spatial wave functions by the semirelativistic quark model with the help of the GEM. Different from the meson system, baryon is a typical three-body system. Thus, its wave function can be extracted by solving the three-body Schrödinger equa-
\[ \mathcal{H} = K + \sum_{i<j} (S_{ij} + G_{ij} + V^\text{pot}(i) + V^\text{pot}(v) + V^\text{ten} + V^\text{con}) \]  (3.1)

with \( K, S, G, V^\text{pot}(i), V^\text{pot}(v), V^\text{ten} \) and \( V^\text{con} \) representing the kinetic energy, the spin-independent linear confinement piece, the Coulomb-like potential, the scalar type-spin-orbit interaction, the vector type-spin-orbit interaction, the tensor potential, and the spin-dependent contact potential, respectively. Their concrete expressions are listed here [36, 42–44]:

\[ K = \sum_{i=1,2,3} \sqrt{m_i^2 + p_i^2}, \]  (3.2)

\[ S_{ij} = -\frac{3}{4} \left( \frac{e^\text{r}_{ij} + e^{-\text{r}_{ij}}}{\sqrt{\pi} \sigma_{r_{ij}}} + \left( 1 + \frac{1}{2\alpha^2 r_{ij}^2} \right) \frac{2}{\sqrt{\pi}} \right) \times \int_0^{\sigma_{r_{ij}}} e^{-x^2} dx \]  (3.3)

\[ G_{ij} = \sum_k \frac{\alpha_k}{r_{ij}} \left[ \frac{2}{\sqrt{\pi}} \int_0^{\sigma_{r_{ij}}} e^{-x^2} dx \right] F_i \cdot F_j \]  (3.4)

for the spin-independent terms with

\[ \sigma^2 = \sigma_0^2 \left( 1 + \frac{1}{2} \left( \frac{4m_im_j}{m_i + m_j} \right)^2 + s^2 \left( \frac{2m_im_j}{m_i + m_j} \right)^2 \right), \]  (3.5)

and

\[ V^\text{pot}(i) = -\frac{r_{ij} \times p_i \cdot S_i}{2m_i^2 r_{ij}} \frac{\partial S_{ij}}{\partial r_{ij}}, \]

\[ V^\text{pot}(v) = \frac{r_{ij} \times p_i \cdot S_i}{2m_i^2 r_{ij}} \frac{\partial S_{ij}}{\partial r_{ij}} + \frac{r_{ij} \times p_j \cdot S_j}{2m_j^2 r_{ij}} \frac{\partial S_{ij}}{\partial r_{ij}}, \]

\[ V^\text{ten} = -\frac{1}{m_im_j} \left[ (S_i \cdot \mathbf{r}_{ij}) (S_j \cdot \mathbf{r}_{ij}) - \frac{S_i \cdot S_j}{3} \right] \left( \frac{\partial G_{ij}}{\partial r^2} - \frac{\partial G_{ij}}{r_j \partial r_{ij}} \right), \]

\[ V^\text{con} = \frac{2S_i \cdot S_j}{3m_im_j} \sqrt{2} G_{ij} \]

for the spin-dependent terms, where \( m_i \) and \( m_j \) are the masses of quark \( i \) and \( j \), respectively. And, we take \( (F_i \cdot F_j) = -2/3 \) for quark-quark interaction.

In the following, a general potential which relies on the center-of-mass of interacting quarks and momentum are made up for the loss of relativistic effects in the nonrelativistic limit [36, 42, 45–47], that is,

\[ G_{ij} \rightarrow \left( 1 + \frac{p_{ij}^2}{E_i E_j} \right)^{1/2} G_{ij} \left( 1 + \frac{p_{ij}^2}{E_i E_j} \right)^{1/2}, \]

\[ \frac{V_{ij}^k}{m_im_j} \left( \frac{m_im_j}{E_i E_j} \right)^{1/2 + \epsilon_k} \]  (3.6)

with \( E_i = \sqrt{p_i^2 + m_i^2} \), where subscript \( k \) was applied to distinguish the contributions from the contact, tensor, vector spin-orbit, and scalar spin-orbit terms. In addition, \( e_k \) represents the relevant modification parameters, which are collected in Table I.

The total wave function of the single heavy baryon is composed of color, flavor, spatial, and spin wave functions, i.e.,

\[ \Psi_{J,LM} = \chi^* \left( \begin{array}{c} \psi_{S,M} \\ \psi_{L,M_L} \end{array} \right) \]  (3.7)

where \( \chi^* = (rbg - rbg + grb - bgr - bgr) / \sqrt{6} \) is the color wave function, which is universal for the baryon. For the \( \Xi^{(s)}_Q \) baryon, its flavor wave function is \( \psi_{S,M} = (ns - sn)Q/\sqrt{2} \), while for the \( \Omega_Q^{(s)} \) baryon, its flavor wave function denotes \( \psi_{S,M} = ssQ \), where \( Q = b, c \) and \( n = u, d \). Besides, \( S \) denotes the total spin and \( L \) is the total orbital angular momentum. \( \psi_{L,M_L} \) is the spatial wave function which is composed of \( \rho \) and \( \lambda \) mode, that is,

\[ \psi_{L,M_L}(\bar{\rho}, \bar{\lambda}) = \left( \psi_{L,M_L}^\rho(\bar{\rho}) \psi_{L,M_L}^\lambda(\bar{\lambda}) \right) \]  (3.8)

where the subscripts \( \rho \) and \( \lambda \) are the orbital angular momentum quanta for \( \rho \) and \( \lambda \) mode, respectively, and the internal Jacobi coordinates are chosen as

\[ \bar{\rho} = \bar{r}_1 - \bar{r}_2, \quad \bar{\lambda} = \bar{r}_3 - \frac{m_1 \bar{r}_1 + m_2 \bar{r}_2}{m_1 + m_2}. \]  (3.9)

\[ ^1 \text{A brief introduction about the classification of the single heavy baryons is helpful to the reader to understand how to construct their wave functions. The single heavy baryons with one heavy-flavor quark and two light-flavor quarks belong to the symmetric 6f or antisymmetric 3f flavor representations based on the flavor SU(3) symmetry. The total color-flavor-spin wave functions for the S-wave members must be antisymmetric. Considering the color wave function is antisymmetric invariably, hence the spin of the two light quarks is \( S = 1 \) for \( 6f \) (e.g. \( \Sigma_Q \) and \( \Xi_Q \)) or \( S = 0 \) for \( 3f \) (e.g. \( \Lambda_Q \) and \( \Xi_Q \)). More details about the classification of the single heavy baryons can be found in Refs. [48, 49]. For \( \Xi'_Q \), its flavor wave function is \( \psi_{S,M} = (ns - sn)Q/\sqrt{2} \).}
In this work, the Gaussian basis [38–40],
\[
\phi_{nm}(\hat{r}) = e^{i \frac{2}{\sqrt{2}} \sqrt{(2l+1)!!}} \lim_{r \to 0} \rho_{n} \sum_{k} C_{lmk} e^{-\rho_{n}(\hat{r})^2} \rho_{n,\max}^{l},
\]
(3.10)
is adopted to expand the spatial wave functions \(\phi_{l,m,l}^n\) and \(\phi_{l,m,l}^{n+1}\) (\(n = 1, 2, \ldots, n_{\max}\)). Here, a freedom parameter \(\rho_{\max}\) should be chosen from positive integers, and the Gaussian size parameter \(\rho_{n}\) is settled as a geometric progression as
\[
\rho_{n} = 1/\rho_{\max}, \quad \rho_{n} = \rho_{\max} a^{n-1}
\]
(3.11)
with
\[
a = \left( \frac{\rho_{\max}}{\rho_{\min}} \right)^{1/(n_{\max} - n_{\min})}.
\]
Meanwhile, in our calculation, the values of \(\rho_{\min}\) and \(\rho_{\max}\) are chosen as 0.2 and 2.0 fm, respectively, and \(n_{\max} = 6\). For \(\lambda\) mode, we also use the same Gaussian sized parameters.

The Rayleigh-Ritz variational principle is used in this work to solve the three-body Schrödinger equation
\[
\mathcal{H} \Psi_{l,m,l} = E \Psi_{l,m,l}.
\]
(3.12)
Finally, by solving Schrödinger equation, the masses and wave functions of the baryons are obtained, which are collected in Table II.

As collected in the PDG [12], there are ten states in the \(\Xi_{c}\) family, where the ground states include \(\Xi_{c}^{+}\) and \(\Xi_{c}^{-}\) with the quark flavor \(usc\) and \(dsc\), respectively. \(\Xi_{c}^{+}\) was first reported by SPEC [50], and then confirmed in Ref. [51] by analyzing the \(\Xi_{c}^{+}\) final state, while the neutral one \(\Xi_{c}^{0}\) was first discovered by CLEO [52] in the \(\Xi_{c}^{-}\) mode. The masses fitted by the PDG are 2467.71 \pm 0.23 and 2470.44 \pm 0.28 MeV for charged \(\Xi_{c}^{+}\) and neutral \(\Xi_{c}^{0}\), respectively. And then, the Belle Collaboration found \(\Xi_{c}^{+}(2970)\) and \(\Xi_{c}^{0}(2970)\) in the \(\Lambda_{c}\) family, where the masses of the charged and neutral \(\Xi_{c}^{0}\) states are measured to be 2964.3 \pm 1.5 and 2967.1 \pm 1.7 MeV, respectively. As indicated by our calculation shown in Table II, the observed \(\Xi_{c}^{+}(2970)\) are good candidate of \(\Xi_{c}^{+}(2536)\). The ground \(\Omega_{c}\) state, denoted as \(\Omega_{c}(1/2^{+})\), was firstly observed in the \(\Xi_{c}^{-}\) channel by WA62 [54], and then was confirmed in ARGUS [55] by checking the same mode. Its mass was fitted as 2695.2 \pm 1.7 MeV by the PDG. Our result given in Table II indeed supports this assignment since the calculated mass of \(\Omega_{c}(1/2^{+})\) is 2.692 GeV consistent with the experimental data. For the \(\Omega_{c}(1/2^{+})\) state, which is the first radial excitation of \(\Omega_{c}(0^{+})\), its mass is calculated to be 3.149 GeV.2

In Table II, we also collected the numerical spatial wave functions corresponding to these charged baryons, which will be applied to the following study.

In 2017, the LHCb Collaboration [56] announced that five narrow excited \(\Omega_{c}\) states, \(\Omega_{c}(3000)\), \(\Omega_{c}(3050)\), \(\Omega_{c}(3066)\), \(\Omega_{c}(3090)\), and \(\Omega_{c}(3119)\), were

IV. THE FORM FACTORS AND COLOR-ALLOWED TWO-BODY NONLEPTONIC DECAYS

A. The weak transitions form factors

With the input of these obtained numerical wave functions of bottom (see Table II) and charmed baryons, and the expressions of the form factors [see Eqs. (2.7)-(2.8)], we present the numerical results for the weak transition form factors of \(\Xi_{b} \rightarrow \Xi_{c}^{0}(1/2^{+})\) and \(\Omega_{c} \rightarrow \lambda_{c}^{0}(1/2^{+})\) processes. Since the expressions of form factors in Eqs. (2.5)-(2.8) are working in the spacelike region \((q^{2} < 0)\), we need to extend them to the timelike region \((q^{2} > 0)\). The dipole form [26–28, 30]
\[
F(q^{2}) = \frac{F(0)}{1 - \frac{q^{2}}{M^{2}}} \left[ 1 - b_{1}(q^{2}/M^{2})^{2} + b_{2}(q^{2}/M^{2})^{4} \right]
\]
(4.1)
is applied in this work, where \(F(0)\) is the form factor at \(q^{2} = 0\), \(b_{1}\), and \(b_{2}\) are obtained by computing each form factor by Eqs. (2.7)-(2.8) from \(q^{2} = -q_{\max}^{2}\) to \(q^{2} = 0\), and fit them by Eq. (4.1).

With the spatial wave functions obtained in the last subsection, we can calculate out the form factors numerically in the framework of the three-body light-front quark model. In this way, all free parameters of the semirelativistic potential model can be fixed by reproducing the mass spectrum of observed heavy baryons. In the previous work [25–29, 35] on baryon weak transitions, simple hadronic oscillator wave function with the oscillator parameter \(\beta\) was widely used to simulate the baryon spatial wave function. This treatment makes the results dependent on \(\beta\) value. In this work, our study is supported by hadron spectroscopy. Thus, we can avoid the above uncertainty resulted by the selection of spatial wave functions of heavy baryons involved in these discussed transitions.

The extended form factors of \(\Xi_{b} \rightarrow \Xi_{c}^{0}\) are collected in Table III. The \(q^{2}\) dependence of \(F_{1,2,3}^{c}(2970)\) and \(g_{A,2,3}^{c}(2970)\) for the \(\Xi_{b} \rightarrow \Xi_{c}^{0}\) and \(\Xi_{b} \rightarrow \Xi_{c}^{0}(2970)\) transitions are plotted in Fig. 1.

For the \(\Xi_{b} \rightarrow \Xi_{c}^{0}\) transition, the corresponding transition matrix element can be rewritten as [27, 64, 65]
\[
\langle \Xi_{c}^{0}(1/2^{+})|(\nu')(\bar{c}\nu)\Gamma_{\nu}\Xi_{b}(1/2^{+})\rangle = \zeta(\omega)|\bar{u}(\nu')\Gamma_{u}(\nu),
\]
(4.2)
in the heavy quark limit at the leading order, so the form factors have more simple behaviors as

\[ f_1^V(q^2) = g_1^A(q^2) = \xi(\omega), \]
\[ f_2^V = f_3^V = g_2^A = g_3^A = 0, \]

(4.3)

where \( \omega = \nu \cdot \nu' = (M^2 + M'^2 - q^2)/(2MM') \) with \( \nu' = p'/M' \) and \( \nu = p/M \) denoting the four velocities for \( \Xi_c \) and \( \Omega_c \), respectively. \( \xi(\omega) \) is the well-known Isgur-Wise function (IWF) and usually expressed as a Taylor series expansion as

\[ \xi(\omega) = 1 - \xi_1(\omega - 1) + \frac{\xi_2}{2}(\omega - 1)^2 + \cdots, \]

(4.4)

where \( \xi_1 = \left. \frac{d \xi(\omega)}{d \omega} \right|_{\omega=1} \) and \( \xi_2 = \left. \frac{d^2 \xi(\omega)}{d \omega^2} \right|_{\omega=1} \) are two shape parameters depicting the IWF. The most obvious character is in the point \( q^2 = q^2_{\text{max}} = (M - M')^2 \) (or \( \omega = 1 \)),

\[ f_1^V(q^2_{\text{max}}) = g_1^A(q^2_{\text{max}}) = \xi(1) = 1. \]

It provided one strong restriction for our result. Besides, when comparing our results with the predictions in heavy quark limit (HQL), we can conclude that our results can well match the requirement from heavy quark effective theory, i.e.,

1. \( f_1^V \) and \( g_1^A \) are close to each other, and dominate over \( f_{1,2,3}^V \) and \( g_{1,2,3}^A \).
2. At \( q^2 = q^2_{\text{max}} \), \( f_1^V(q^2_{\text{max}}) = 1.015 \) and \( g_1^A(q^2_{\text{max}}) = 0.978 \) are very approach to 1.

In addition we also extract the two IWF’s shape parameters \( \xi_1 \) and \( \xi_2 \) in Eq. (4.4) by fitting \( \xi(\omega) \) from \( f_1^V(q^2) \) and \( g_1^A(q^2) \), respectively. The concrete results and other theoretical predictions are listed in Table IV.

For the \( \Xi_c \rightarrow Q \) (2970) transition, the HQL requires \( f_1^V = g_1^A = 0 \) at \( q^2 = q^2_{\text{max}} \) since the wave functions of the low-lying \( \Xi_c \) and the radial excited state \( \Xi^*(2S) \) are orthogonal [27]. Evidently, our results well embody this prediction according to Fig. 1.

Additionally, the extended form factors of \( \Omega_c \rightarrow \Omega_c^{(*)} \) are collected in Table V. The \( q^2 \) dependence of \( f_{1,2,3}^A \) and \( g_{1,2,3}^A \) for the \( \Omega_b \rightarrow \Omega_c \) and \( \Omega_c \rightarrow \Omega_c(3900) \) transitions are plotted in Fig. 2. For the \( \Omega_b \rightarrow \Omega_c \) transition, the corresponding transition matrix element can be rewritten as [27, 64, 65]

\[ \langle \Omega_c(1/2^+) | v \rangle \xi_1 \Gamma E\Omega_b | \Omega_b(1/2^+) | v \rangle = - \frac{1}{3} (g^{\mu \nu} \xi_1 - v^\mu v^\nu \xi_2) \bar{u}(v) (\gamma_\mu - v_\mu) \Gamma (\gamma_\nu - v_\nu) u(v) \]

(4.5)
in HQL at the leading order. Thus, the form factors in HQL have more simple behaviors as

\[
\begin{align*}
 f_1^V(q^2_{\text{max}}^2) &= \frac{1}{3} + \frac{1}{3} \frac{M^2 + M'^2}{MM'} = 1.23, \\
 f_2^V(q^2_{\text{max}}^2) &= \frac{1}{3} \frac{M + M'}{M'} = 1.08, \\
 f_3^V(q^2_{\text{max}}^2) &= -\frac{1}{3} \frac{M - M'}{M'} = -0.41, \\
 g_1^A(q^2_{\text{max}}^2) &= -\frac{1}{3}, \\
 g_2^A(q^2_{\text{max}}^2) &= g_3^A(q^2_{\text{max}}^2) = 0,
\end{align*}
\tag{4.6}
\]

at \( q^2 = q^2_{\text{max}} \) point by substituting the involved masses. Obviously, our results located in the third column of the Table V match well the requirement from the HQL as shown in Eq. (4.6), which can be as a direct test to the HQL.

\[\Xi_b \to \Xi_c\]

\[\Xi_b \to \Xi_c(2970)\]

\[\Omega_b \to \Omega_c\]

\[\Omega_b \to \Omega_c(2S)\]

FIG. 1: The \( q^2 \) dependence of the form factors \( f_{1,2,3}^V(q^2) \) and \( g_{1,2,3}^A(q^2) \) for the \( \Xi_b \to \Xi_c \) (left) and \( \Xi_b \to \Xi_c(2970) \) (right) transitions. Here, the solid and dashed lines represent the vector-type and pseudoscalar-type form factors denoting by the subscripts \( V \) and \( A \), respectively, while the blue, red, and purple lines (both solid and dashed lines) represent the \( i \)th form factors denoting by the subscripts respectively for each types.

FIG. 2: The \( q^2 \) dependence of the form factors \( f_{1,2,3}^V(q^2) \) and \( g_{1,2,3}^A(q^2) \) for \( \Omega_b \to \Omega_c \) (left) and \( \Omega_b \to \Omega_c(2S) \) (right) transitions, in which the solid and dashed lines represent the vector or pseudoscalar-types form factors denoting by the subscripts \( V \) and \( A \), respectively, while the blue, red and purple lines (both solid and dashed lines) represent the \( i \)th form factors denoting by the subscripts, respectively, for each types.

B. The color-allowed two-body nonleptonic decays

With the preparation of the obtained form factors, we further calculate the color-allowed two-body nonleptonic decays of \( \Xi_b \) and \( \Omega \) with emitting a pseudoscalar meson (\( \pi^- \), \( K^- \), \( D^- \), and \( D_s^- \)) or a vector meson (\( \rho^- \), \( K^{*-} \), \( D^{*-} \), and \( D_s^{*-} \)). In this work, the decay rates are investigated by the naïve factorization approach

\[^3\] The naïve factorization approach works well for the color-allowed dominated processes. But, there exists the case that the color-suppressed and penguin dominated processes can not be explained by the naïve factorization, which may show important nonfactorizable contributions to nonleptonic decays [29]. As indicated in Refs. [26, 27, 66], the nonfactorizable contributions in bottom baryon nonleptonic decays are considerable comparing with the factorized ones. Since a precise study of nonfactorizable contributions is beyond the scope of the present work, we still adopt the naïve factorization approximation.
TABLE IV: Our results for the IWF’s shape parameters of the standard light front quark model. Here, we adopt the form defined in Eq. (4.1) for analyzing these form factors.

| $F(0)$ | $F(q_\max^2)$ | $b_1$ | $b_2$ |
|--------|----------------|-------|-------|
| $f_1^\gamma$ | 0.481 | 1.015 | 0.970 | 0.233 |
| $f_2^\gamma$ | -0.127 | -0.312 | 1.380 | 0.578 |
| $f_3^\gamma$ | -0.046 | -0.097 | 1.187 | 0.875 |
| $g_1^\gamma$ | 0.471 | 0.978 | 0.929 | 0.226 |
| $g_2^\gamma$ | -0.026 | -0.068 | 1.318 | 0.122 |
| $g_3^\gamma$ | -0.154 | -0.377 | 1.493 | 0.947 |

TABLE V: The form factors for the $\Omega_b \to \Omega_c^{(*)}$ transitions in the standard light front quark model. We use a three parameter form defined in Eq. (4.1) for these form factors.

| $F(0)$ | $F(q_\max^2)$ | $b_1$ | $b_2$ |
|--------|----------------|-------|-------|
| $f_1^\gamma$ | 0.493 | 1.232 | 1.765 | 1.272 |
| $f_2^\gamma$ | 0.436 | 1.075 | 1.658 | 1.001 |
| $f_3^\gamma$ | -0.255 | -0.620 | 1.628 | 1.005 |
| $g_1^\gamma$ | -0.161 | -0.329 | 1.053 | 0.337 |
| $g_2^\gamma$ | 0.011 | 0.018 | 0.822 | 1.526 |
| $g_3^\gamma$ | 0.055 | 0.137 | 1.680 | 1.052 |

Generally, in the naïve factorization assumption, the hadronic transition matrix element is factorized into a product of two independent matrix elements [28]

$$\langle B_c^{(*)}(P', J'_2) | H_{\text{eff}} | B_b(P, J_2) \rangle = \frac{G_F}{\sqrt{2}} c_{V_{qq}} \langle M^- | \bar{q} \gamma_\mu (1 - \gamma_5) q | 0 \rangle \times \langle B_c^{(*)}(P', J'_2) | \gamma_\mu (1 - \gamma_5) b | B_b(P, J_2) \rangle,$$

where the meson transition term is given by

$$\langle M^- | \bar{q} \gamma_\mu (1 - \gamma_5) q | 0 \rangle = \begin{cases} i f_p q_{\mu}, & M = P, \\ i f_V q_{\mu} m_V, & M = V. \end{cases}$$

Here, $P$ and $V$ denote pseudoscalar and vector mesons, respectively. The baryon transition term can be obtained by Eq. (2.1). The corresponding Feynman diagram (taking the $\Xi_b \to \Xi_c^* M^-$ as an example here) is displayed in Fig. 3.

Finally, the decay width and asymmetry parameter are given by [28]

$$\Gamma = \frac{|p_c|}{8\pi} \left( \frac{(M + M')^2 - m^2}{M^2} |A|^2 + \frac{(M - M')^2 - m^2}{M^2} |B|^2 \right),$$

$$\alpha = \frac{2\kappa \text{Re}(A^* B)}{|A|^2 + \kappa^2 |B|^2}.$$

for the cases involved in the pseudoscalar and vector meson final state, respectively, where $p_c$ is the momentum of the daughter baryon in the rest frame of the parent baryon and $\kappa = |p_c|/(E' + M')$. Besides, $M(E)$ and $M'(E')$ are the masses (energies) of the parent (daughter) baryons, respectively, while $m(E_m)$ denotes the mass(energy) of the meson in the final state.

$A$ and $B$ in Eqs. (4.9) are given by

$$A = \frac{G_F}{\sqrt{2}} c_{V_{qq}} a_f |f_P(M - M')| f_1^\gamma (m^2),$$

$$B = -\frac{G_F}{\sqrt{2}} c_{V_{qq}} a_f |f_P(M + M')| g_1^\gamma (m^2).$$

FIG. 3: The diagram for depicting the color-allowed two-body nonleptonic decay $\Xi_b^0 \to \Xi_c^0 M^-$ in the tree level.

$$\Gamma = \frac{|p_c|}{4\pi M} \left( 2(|S|^2 + |P_2|^2) + \frac{E_m^2}{m^2} (|S| + |D|^2 + |P_2|^2) \right),$$

$$\alpha = \frac{4m^2 \text{Re}(S^* P_2) + 2E_m^2 \text{Re}(S + D)^* P_1}{2m^2 (|S|^2 + |P_2|^2) + E_m^2 (|S| + |D|^2 + |P_2|^2)}.$$
and $S$, $P_{1,2}$ and $D$ in Eqs. (4.10) are expressed as

\[
S = A_1,
\]

\[
P_1 = -\frac{|p|}{E_m}\left(\frac{M+M'}{E'+M'}B_1+MB_2\right),
\]

\[
P_2 = \frac{|p|}{E'+M'}B_1,
\]

\[
D = \frac{|p|^2}{E_m(E'+M')}(A_1-MA_2)
\]

with

\[
A_1 = \frac{G_F}{\sqrt{2}}V_{cb}V_{qf}^*a_1f_{vm}\left(g_1^A(m^2)+g_2^A(m^2)\frac{M-M'}{M}\right),
\]

\[
A_2 = \frac{G_F}{\sqrt{2}}V_{cb}V_{qf}^*a_1f_{vm}\left(2g_2^A(m^2)\right),
\]

\[
B_1 = \frac{G_F}{\sqrt{2}}V_{cb}V_{qf}^*a_1f_{vm}\left(f_1^D(m^2)-f_2^D(m^2)\frac{M+M'}{M}\right),
\]

\[
B_2 = \frac{G_F}{\sqrt{2}}V_{cb}V_{qf}^*a_1f_{vm}\left(2f_2^D(m^2)\right),
\]

where $a_1 = c_1+c_2/N \approx 1.018$ and $a_2 = c_2+c_1/N \approx 0.170$ [27].

With the naïve factorization, the color-allowed two-body nonleptonic decays by emitting one pseudoscalar meson or vector meson are presented. The lifetimes of $\Xi_b^0$ and $\Omega_b^-$ were reported by the LHCb [67-69] and CDF [70] collaborations. In this work, we use the central values as

\[
\tau_{\Xi_b^0} = 1.480 \text{ fs}, \quad \tau_{\Xi_b^-} = 1.572 \text{ fs}, \quad \tau_{\Omega_b^-} = 1.65 \text{ fs},
\]

averaged by the PDG [12]. Besides, the masses of the concerned baryons are from the GEM calculation and the Cabibbo-Kobayashi-Maskawa matrix elements

\[
V_{cb} = 0.0405, \quad V_{ud} = 0.9740, \quad V_{us} = 0.2265, \quad V_{cd} = 0.2264, \quad V_{cs} = 0.9732,
\]

are taken from the PDG [12]. The decay constants of pseudoscalar and vector mesons include [27, 71]

\[
f_\pi = 130.2, \quad f_K = 155.6, \quad f_D = 211.9, \quad f_{D_s} = 249.0, \quad f_{D_c} = 216, \quad f_{K^*} = 210, \quad f_{D^*} = 220, \quad f_{D_c^*} = 230,
\]

in the unit of MeV.

By substituting our numerical results of the form factors from the three-body light-front quark model and the presented decay parameters into Eqs. (4.9)-(4.10), the branching ratios and asymmetry parameters can be further obtained, which are collected in Tables VI-VII for the $\Xi_b^0 \to \Xi_b^{0*}$ and $\Omega_b^- \to \Omega_b^{0*}$ transitions with emitting a pseudoscalar meson ($\pi^-, K^-, D^-$, and $D_c^-$) or a vector meson ($\rho^-, K^{*-}, D^{*-}$, and $D_c^{*-}$), respectively.

In Table VIII, we compare our results of $B(\Xi_b^{0,0*} \to \Xi_b^{0,0*}M^-)$ and $B(\Omega_b^- \to \Omega_b^{0*}M^-)$ with other theoretical results from the nonrelativistic quark model [21], the relativistic three-quark model [22, 23], the light-front quark model [25, 27], and the covariant confined quark model [24]. Our results are comparable with those calculated from other approaches. We also notice that the concerned transitions with emitting $\pi^-, \rho^-$, and $D_c^{*-}$-meson have considerable widths, which are worthy to be explored in future experiment like LHCb and Belle II.

V. SUMMARY

With the accumulation of experimental data from LHCb and Belle II [16], experimental exploration of weak decay of the bottom baryons $\Xi_b$ and $\Omega_b$ is becoming possible. Facing this opportunity, in this work we study the color-allowed two-body nonleptonic decay of the bottom baryons $\Xi_b$ and $\Omega_b$, i.e., the $\Xi_b \to \Xi_b^{(*)}M$ and $\Omega_b \to \Omega_b^{(*)}M$ decay with emitting a pseudoscalar meson ($\pi^-, K^-, D^-$, and $D_c^-$) or a vector meson ($\rho^-, K^{*-}, D^{*-}$, and $D_c^{*-}$).

We adopt the three-body light-front quark model to calculate these form factors depicting these discussed bottom baryon to the charmed baryon transitions under the naïve factorization framework. We also improve the treatment of the spatial wave function of these involved heavy baryons in these decays, where the semirelativistic three-body potential model [30, 36] is applied to calculate the numerical spatial wave function of these heavy baryons with the help of the GEM [37-40]. We call that the study of color-allowed two-body nonleptonic decay of bottom baryons $\Xi_b$ and $\Omega_b$ is supported by hadron spectroscopy. Our result shows that these color-allowed two-body nonleptonic decays $\Xi_b^{0,-} \to \Xi_b^{(*)\pi,0}$ and $\Omega_b^- \to \Omega_b^{(*)\rho,0}$ with the $\pi^-, \rho^-$, and $D_c^{(*)\rho,0}$-emitted modes have considerable widths.

We suggest to measure these discussed color-allowed two-body nonleptonic decay of the bottom baryons $\Xi_b$ and $\Omega_b$, which will be good chance for the ongoing LHCb and Belle II experiments.

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TABLE VI: The branching ratios and asymmetry parameters of the $\Xi_b \to \Xi^{(*)} M$ transitions with $M$ denoting a pseudoscalar or vector meson, where the branching ratios out of or in brackets correspond to the $\Xi_b^0 \to \Xi_b^0$ and $\Xi_b^\pm \to \Xi_b^\mp$ transitions, respectively.

| Mode | $\mathcal{B} \times 10^{-3}$ | $\alpha$ | Mode | $\mathcal{B} \times 10^{-3}$ | $\alpha$ |
|------|-----------------|--------|------|-----------------|--------|
| $\Xi_b^0 \to \Xi_b^0 \pi^-$ | 7.92 | 0.61 | $\Omega_b \to \Omega_b^0 \pi^-$ | 7.92 | 0.61 |
| $\Xi_b^0 \to \Xi_b^0 K^-$ | 0.41 | 0.62 | $\Omega_b \to \Omega_b^0 K^-$ | 0.41 | 0.62 |
| $\Xi_b^0 \to \Xi_b^0 D^-$ | 0.48 | 0.69 | $\Omega_b \to \Omega_b^0 D^-$ | 0.48 | 0.69 |
| $\Omega_b \to \Omega_b^0 D^0$ | 9.73 | 0.70 | $\Omega_b \to \Omega_b^0 (2S) D^+$ | 0.70 | 0.60 |
| $\Omega_b \to \Omega_b^0 (2S) K^-$ | 0.00 | 0.00 | $\Omega_b \to \Omega_b^0 (2S) K^0$ | 0.00 | 0.00 |
| $\Omega_b \to \Omega_b^0 (2S) D^0$ | 0.02 | 0.65 | $\Omega_b \to \Omega_b^0 (2S) D^+$ | 0.02 | 0.65 |
| $\Omega_b \to \Omega_b^0 (2S) D^0$ | 0.03 | 0.65 | $\Omega_b \to \Omega_b^0 (2S) D^+$ | 0.03 | 0.65 |

TABLE VII: The branching rates and asymmetry parameters of $\Omega_b \to \Omega_b^* M$ transitions with $M$ denoting a pseudoscalar or vector meson.

| Mode | $\mathcal{B} \times 10^{-3}$ | $\alpha$ | Mode | $\mathcal{B} \times 10^{-3}$ | $\alpha$ |
|------|-----------------|--------|------|-----------------|--------|
| $\Omega_b \to \Omega_b^{*+} \pi^-$ | 0.40 | 0.40 | $\Omega_b \to \Omega_b^{*+} K^-$ | 0.40 | 0.40 |
| $\Omega_b \to \Omega_b^{*+} D^-$ | 0.40 | 0.40 | $\Omega_b \to \Omega_b^{*+} D^0$ | 0.40 | 0.40 |
| $\Omega_b \to \Omega_b^{*+} (2S) D^+$ | 0.40 | 0.40 | $\Omega_b \to \Omega_b^{*+} (2S) K^0$ | 0.40 | 0.40 |
| $\Omega_b \to \Omega_b^{*+} (2S) D^+$ | 0.40 | 0.40 | $\Omega_b \to \Omega_b^{*+} (2S) D^+$ | 0.40 | 0.40 |

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TABLE VIII: Comparison of theoretical predictions for $\mathcal{B}(\Xi_c^0 \rightarrow \Xi_c^{(0)} M^-)$ and $\mathcal{B}(\Omega_c^0 \rightarrow \Omega_c^0 M^-)$. Here, all values should be multiplied by a factor of $10^{-3}$.

| Process                    | This work | Cheng [21] | Ivanov et al. [22, 23] | Zhao [25] | Gutsche et al. [24] | Chu [27] |
|-----------------------------|-----------|------------|------------------------|-----------|---------------------|----------|
| $\Xi_c^0 \rightarrow \Xi_c^{(0)} \pi$ | 4.03 (4.29) | 4.9 (5.2)  | 7.08 (10.13)           | 8.37 (8.93) | –                   | 3.66\(^{+2.25}_{-1.19}\) (3.88\(^{+2.43}_{-1.69}\)) |
| $\Xi_c^0 \rightarrow \Xi_c^{(0)} \rho$ | 13.3 (14.1) | –          | –                     | 24.0 (25.6) | –                   | 10.88\(^{+7.25}_{-4.26}\) (11.56\(^{+7.04}_{-4.26}\)) |
| $\Xi_c^0 \rightarrow \Xi_c^{(0)} K^-$ | 0.31 (0.33) | –          | –                     | 0.667 (0.711) | –                   | 0.28\(^{+0.10}_{-0.11}\) (0.29\(^{+0.15}_{-0.13}\)) |
| $\Xi_c^0 \rightarrow \Xi_c^{(0)} K^0$ | 0.71 (0.76) | –          | –                     | 1.23 (1.31) | –                   | 0.56\(^{+0.25}_{-0.26}\) (0.60\(^{+0.27}_{-0.26}\)) |
| $\Xi_c^0 \rightarrow \Xi_c^{(0)} D^-$ | 0.58 (0.62) | –          | –                     | 0.949 (1.03) | 0.45     | 0.43\(^{+0.25}_{-0.26}\) (0.45\(^{+0.31}_{-0.21}\)) |
| $\Xi_c^0 \rightarrow \Xi_c^{(0)} D_s^-$ | 1.51 (1.60) | –          | –                     | 1.54 (1.64) | 0.95     | 0.77\(^{+0.50}_{-0.53}\) (0.82\(^{+0.53}_{-0.57}\)) |
| $\Omega_c^0 \rightarrow \Omega_c^0 D_+^*$ | 14.8 (15.7) | 14.6       | –                     | 24.6 (26.2) | –                   | 10.87\(^{+7.51}_{-5.05}\) (11.54\(^{+7.05}_{-5.34}\)) |
| $\Omega_c^0 \rightarrow \Omega_c^0 D_{s+}^*$ | 32.4 (34.4) | 23.1       | –                     | 36.5 (39.0) | –                   | 16.24\(^{+10.55}_{-7.75}\) (17.26\(^{+10.12}_{-7.70}\)) |

$\Omega_c^0 \rightarrow \Omega_c^0 \pi$, $\Omega_c^0 \rightarrow \Omega_c^0 \rho$, $\Omega_c^0 \rightarrow \Omega_c^0 K^-$, $\Omega_c^0 \rightarrow \Omega_c^0 K^0$, $\Omega_c^0 \rightarrow \Omega_c^0 D^-$, $\Omega_c^0 \rightarrow \Omega_c^0 D_s^-$, $\Omega_c^0 \rightarrow \Omega_c^0 D_{s+}^*$.
