Quadratic Quantum Measurements

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We develop a theory of quadratic quantum measurements by a mesoscopic detector. It is shown that quadratic measurements should have non-trivial quantum information properties, providing, for instance, a simple way of entangling two non-interacting qubits. We also calculate output spectrum of a quantum detector with both linear and quadratic response continuously monitoring coherent oscillations in two qubits.

The problem of quantum measurements with mesoscopic solid-state detectors attracts considerable current interest (see, e.g., chapters on quantum measurements in [1]). This interest is motivated in part by important role of measurement in quantum computing, and in part by the possibility, provided by the mesoscopic structures, to study directly both in theory and experiment transition between quantum and classical behavior in systems that are large on atomic scale. Although mesoscopic detectors can be quite different and include, e.g., quantum point contacts (QPC) [2–8], normal and superconducting SET transistors [9–15], SQUID magnetometers [16] and generic mesoscopic conductors [17,18], the operating principle of almost all of them is the same. Measured quantum system controls transmission amplitude of some particles (electrons, Cooper pairs, or magnetic flux quanta) between the two reservoirs, so that the flux of these particles provides information on the state of the system [19]. In a generic situation the amplitude $t$ varies together with some control operator $x$, and for sufficiently weak detector-system coupling, the dependence $t(x)$ can be approximated as linear. Dynamics of such linear measurements is well understood (see, e.g., [20,21]).

At some special bias points, however, the linear response coefficient of the $t(x)$ dependence vanishes and this dependence becomes quadratic. This can happen, for instance, if the amplitude $t$ is formed by more than one interfering tunneling trajectories. Known examples of such situation include dc SQUIDs and superconducting SET transistors. In this work, we show that quantum detector operating at such a special point should enable measurements of product operators referring to separate systems and have non-trivial quantum information processing properties, e.g., create simple entanglement mechanism for non-interacting qubits. Specifically, we consider the measurement of two qubits (Fig. 1) which is the simplest system that reveals non-trivial characteristics of quadratic detection. (Quadratic detectors can not measure individual qubits since dynamic variables of one qubit are given by the Pauli matrices $\sigma$ for which $\sigma^2 = 1$.)

The two qubits (indexed by $j = 1, 2$) are assumed to be coupled to one detector through their basis-forming variables, i.e., $x = c_1 \sigma_z^1 + c_2 \sigma_z^2$ so that

$$t(x) = t_0 + \sum_j \delta_j \sigma_z^j + \lambda \sigma_z^1 \sigma_z^2. \quad (1)$$

The last term in this equation appears due to non-linearity of $t(x)$. If $\delta_j = 0, \lambda \neq 0$, one has purely quadratic detector. For the two qubits, Eq. (1) represents the most general dependence of $t$ on $\sigma_z^j$, whereas for measurements of other systems, expansion in the measured operators similar to Eq. (1) can be justified as Taylor’s expansion in weak detector-system coupling.

![FIG. 1. Diagram of a mesoscopic detector measuring two qubits. The qubits modulate amplitude $t$ of tunneling of detector particles between the two reservoirs.](image)

The Hamiltonian of the detector-qubit system is:

$$H_t = H_0 + H_d + t(\{\sigma_z^j\}) \xi + t^\dagger(\{\sigma_z^j\}) \xi^\dagger, \quad (2)$$

where $H_0 = -(1/2) \sum_{j=1,2} (\varepsilon_j \sigma_z^j + \Delta_j \sigma_z^j) + (\nu/2) \sigma_z^1 \sigma_z^2$. Here $\Delta_j$ is the tunnel amplitude and $\varepsilon_j$ is the bias of the $j$th qubit, $\nu$ is the qubit interaction energy, $H_d$ is the detector Hamiltonian, and $\xi^\dagger, \xi$ are the detector operators that create excitations when a particle is transferred, respectively, forward and backward between the detector reservoirs. For instance, for the QPC detector, $\xi^\dagger, \xi$ describe excitation of electron-hole pairs in the QPC electrodes. The qubit interaction is not essential to us, and we will frequently take $\nu = 0$ below.

We make two assumptions about the detector: the tunneling between reservoirs is weak, so that the evolution of
the system can be described in the lowest non-vanishing order in $t$; characteristic time scale of tunneling is much shorter than that of the qubit evolution due to $H_0$. In the example of the QPC detector these assumptions mean that the QPC operates in the tunneling regime and that the voltage across it is much larger than the qubit energies. Under these assumptions, precise form of the detector Hamiltonian $H_d$ is not important and dynamics of measurement is defined by the correlators

$$
\gamma_+ = \int_0^\infty dt \langle \xi(t)\xi^\dagger(t) \rangle, \quad \gamma_- = \int_0^\infty dt \langle \xi^\dagger(t)\xi(t) \rangle.
$$

In Eq. (3), the angled brackets denote averaging over internal degrees of freedom of the detector reservoirs which are taken to be in a stationary state. The correlators $\langle \xi(t)\xi^\dagger(t) \rangle, \langle \xi^\dagger(t)\xi(t) \rangle$ that do not conserve the number of particles are assumed to vanish.

The measurement contribution to the evolution of the qubit density matrix $\rho$ is obtained by standard perturbation theory in tunneling and can be written down conveniently in the “measurement” basis of eigenstates of the $\sigma_z$ operators, $|↑↑⟩, |↓↓⟩, |↑↓⟩,$ and $|↓↑⟩$. Each state $|k⟩$ of this basis is characterized by the value $t_k$ of the transmission amplitude (1): $t_1 = t_0 + \sum_j \delta_j + \lambda$, $t_2 = t_0 + \delta_j - \delta_j - \lambda$, $t_3 = t_0 - \delta_j + \delta_j - \lambda$, $t_4 = t_0 - \sum_j \delta_j + \lambda$. To describe qubit dynamics conditioned on particular outcome of measurement, we keep in the evolution equation the number of particles transferred through the detector. Since the correlators that do not conserve $n$ vanish, only terms diagonal in $n$ contribute to the evolution, and in the lowest order in tunneling, measurement contribution to $\dot{\rho}$ is:

$$
\dot{\rho}_{kl} = -(1/2)(\Gamma_+ + \Gamma_-)(|t_k|^2 + |t_l|^2)\rho_{kl}^n + \Gamma_- t_k t_l\rho_{kl}^{n+1} + \Gamma_+ t_k t_l\rho_{kl}^{n-1} - i[\delta H, \rho_{kl}^n].
$$

Here $\delta H = \sum_j \delta_j \sigma_z^j + \delta\sigma_z^1 \sigma_z^2$ is the renormalization of the qubit Hamiltonian due to coupling to the detector: $\delta\sigma_z = \text{Re}(\delta_j \sigma_0^z + \delta_j \lambda)\text{Im}(\gamma_+ + \gamma_-)$ and $\delta_\nu = \text{Re}(\delta_j \sigma_0^x + \delta_j \lambda)\text{Im}(\gamma_+ + \gamma_-)$. Equation (4) is the basis for our quantitative discussion of quadratic measurements. It generalizes to arbitrary detector and two qubits, the equation obtained in [5] for a qubit measured with the QPC in the tunnel regime.

Disregarding the index $n$ in Eq. (4), we obtain equation for the measurement-induced evolution of the qubit density matrix averaged over different measurement outcomes. Together with the evolution due to the qubit Hamiltonian $H_0$ this equation is:

$$
\dot{\rho}_{kl} = -\gamma_{kl}\rho_{kl} - i[H_0, \rho_{kl}].
$$

Here $\gamma_{kl} \equiv (1/2)(\Gamma_+ + \Gamma_-)|t_k - t_l|^2$, with $\Gamma_\pm \equiv 2\text{Re}\gamma_\pm$, and we included in $H_0$ two renormalization terms: $\delta H$ (4) and $\delta H'$ due to phases $\varphi_{kl} \equiv \text{arg}(t_k t_l^*)$ of the transfer amplitudes in Eq. (4) defined by: $[\delta H', \rho_{kl}] = (\Gamma_+ - \Gamma_-)|t_k t_l|\sin\varphi_{kl}\rho_{kl}$.

Evolution (5) of the qubit density matrix is reflected in the detector current. The form of the current $I$ operator in the qubit space is obtained by the same lowest-order perturbation theory in tunneling that leads to Eq. (5):

$$
I = (\Gamma_+ - \Gamma_-)t^t t.
$$

This equation can be used to calculate both the dc current $\langle I \rangle = \text{Tr}\{I\rho_0\}$, where $\rho_0$ is the stationary solution of Eq. (5), and the current spectral density

$$
S_I = S_0 + 2\int_0^\infty \int_0^\infty d\tau \cos \omega \tau \text{Tr}\{I e^{i\omega \tau}\{I\rho_0\} - \langle I \rangle^2\}.
$$

Here $S_0 = (\Gamma_+ + \Gamma_-)\text{Tr}\{t^t t\} \rho_0$, and $e^{i\omega \tau\{A\}}$ denotes the evolution of the matrix $A$ during time interval $\tau$ governed by Eq. (5).

Decay of the off-diagonal matrix elements Eq. (5) is the result of averaging over the measurement outcomes. However, since $n$ is the classical detector output, it is legitimate to ask a question what is the qubit evolution for a specific measurement outcome $n$. Such a “conditional” description of the measurement dynamics (see, e.g., [21]) is convenient for calculation of more complicated correlators involved, for instance, in problems of feed-back control of the dynamics of the measured system. For measurement process governed by Eq. (4) such a description is obtained by first solving this equation in terms of $n$. Noticing that Eq. (4) coincides in essence with the recurrence relations for the modified Bessel functions $I_n$ and assuming initial condition $\rho_{kl}^n(0) = \rho_{kl}(0)\delta_{n,0}$ we get:

$$
\rho_{kl}^n(\tau) = \rho_{kl}(0)(\Gamma_+ + \Gamma_-)^{n/2}I_n(2\tau|t_k t_l|\sqrt{\Gamma_+ - \Gamma_-}) \exp\left\{-\left(\Gamma_+ + \Gamma_-\right)(|t_k|^2 + |t_l|^2)\tau - i\varphi_{kl}\right\}.
$$

The qubit density matrix conditioned on the particular “observed” number $n$ of transferred particles is obtained then by selecting the term with this $n$ in Eq. (8) and normalizing the resulting reduced density matrix.

Quantitatively, conditional “Bayesian” equations for qubit evolution can be derived starting from Eq. (8) and following the same steps as in [21]. In particular, for weak detector-qubit coupling, $|\delta_j|, |\lambda| \ll |t_0|$, when individual tunneling events do not provide significant information on the qubit state, it is convenient to condition the evolution on the quasicontinuous current $I(t)$ in the detector. Then, conditional equation for the evolution of the qubit density matrix is:

$$
\dot{\rho}_{kl} = -i[H_0, \rho_{kl}] - \gamma_{kl}\rho_{kl} + I_f(t)\rho_{kl}\left(\frac{1}{2S_0}\sum_j \rho_{jj}(I_k + I_l - 2I_j) - i\varphi_{kl}\right).
$$

where $S_0$ is the background current noise (see Eq. (7)), variation of which with the qubit state can be neglected in the weak-coupling limit, $S_0 = (\Gamma_+ + \Gamma_-)|t_0|^2$. Also, $I_k = (\Gamma_+ - \Gamma_-)|t_k|^2$ is the average detector current in the
qubit state \( k \), and \( I_f(t) = I(t) - \sum_k \rho_{kk} I_k \) is the fluctuation component of the detector current. Equation (9) is written in the Itô form, in which averaging over \( I_f(t) \) can be done by simply omitting the terms with it. In the weak-coupling regime, \( \gamma_{kl} = (1/2)(\Gamma_+ + \Gamma_-)[(|t_k|-|t_l|)^2 + \varphi_k^2|t_0|^2] \). It is the same ensemble-averaged decoherence rate as in Eq. (5), but in Eq. (9), it leads to decoherence only after averaging over \( I_f(t) \).

We now use equations obtained above to discuss several quantitative characteristics of quadratic measurements. We start with the purely quadratic detectors, when \( \delta_j = 0 \), so that \( I_1 = I_4 = (\Gamma_+ - \Gamma_-)|t_0 + \lambda|^2 \equiv I_{\uparrow\uparrow} \) and \( I_2 = I_3 = (\Gamma_+ - \Gamma_-)|t_0 - \lambda|^2 \equiv I_{\uparrow\downarrow} \). In this case, if the qubits are stationary, \( H_0 = 0 \), the detector effectively measures the product operator \( \sigma_1^2 \sigma_2^2 \) of the two qubits. I.e., on the time scale of measurement time \( \tau_m = 4S_0/(I_{\uparrow\uparrow} - I_{\uparrow\downarrow})^2 \), the subspace \( \{|1\rangle, |4\rangle\} \) in which the states of the two qubits are the same and the average detector current is \( I_{\uparrow\uparrow} \), is distinguished from the subspace \( \{|2\rangle, |3\rangle\} \) in which the states of the two qubits are opposite and the current is \( I_{\uparrow\downarrow} \), while the states within these subspaces are not distinguished. This property of quadratic measurements can be used to design simple error-correction scheme for dephasing errors [22].

Next, we consider the case of identical, unbiased, non-interacting qubits with non-vanishing Hamiltonian, \( H_0 = -(\Delta/2) \sum \sigma^2_z \). In this case the two degenerate zero-energy eigenstates of \( H_0 \) can be chosen as \( \{|\uparrow\uparrow\rangle - |\downarrow\downarrow\rangle, |\uparrow\downarrow\rangle + |\downarrow\uparrow\rangle\} \). In the remaining subspace that will be denoted \( D_+ \), in the basis \( \{|\uparrow\uparrow\rangle + |\downarrow\downarrow\rangle, |\uparrow\downarrow\rangle + |\downarrow\uparrow\rangle\} \), \( H_0 \) reduces to \( -\Delta \sigma_z \) and mixes the states with similar and opposite states of the two qubits. Accordingly, there are three possible measurement outcomes characterized by the different dc currents \( I \) in the detector, \( I_{\uparrow\uparrow}, I_{\uparrow\downarrow}, \) and \( (I_{\uparrow\uparrow} + I_{\uparrow\downarrow})/2 \). These outcomes can be interpreted as measurement of the operator \( \sigma_1^2 \sigma_2^2 + \sigma_1^2 \sigma_2^2 \). Conditional equation (9) can be used to simulate how the qubits, on the time scale \( \tau_m \approx 4S_0/I_{\uparrow\uparrow}^2 \), \( I_0 \equiv (I_{\uparrow\uparrow} - I_{\uparrow\downarrow})/2 \), are driven into one of the three outcomes driven by the specific realization of the detector current. The probabilities of different outcomes depend on the initial qubit state. In the first two outcomes, the initial state is projected on one of the fully entangled states of the two qubits, e.g.,

\[
|I\rangle = I_{\uparrow\uparrow} \Leftrightarrow |\psi\rangle = (|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle)/\sqrt{2}.
\]

(10)

Thus, quadratic measurements of two symmetric qubits provides a simple way of generating entangled states of qubits that in contrast to linear measurements [23] is based only on monitoring dc current instead of spectrum.

In the third scenario, when \( |I\rangle = (I_{\uparrow\uparrow} + I_{\uparrow\downarrow})/2 \), the two qubits are confined to the subspace \( D_+ \) and perform coherent quantum oscillations. Equation for the density matrix (5) reduced to \( D_+ \) is:

\[
\dot{\rho}_{kl} = i\Delta[\sigma_*^z, \rho]_{kl} - \gamma \begin{pmatrix} 0 & \rho_{12} \\ \rho_{21} & 0 \end{pmatrix},
\]

(11)

where \( \gamma = 2(\Gamma_+ + \Gamma_-)|\lambda|^2 \). Solving this equation and using the fact that the current operator (6) is reduced in \( D_+ \) to \( I_0 \sigma_z \), we find the current spectral density (7):

\[
S_I(\omega) = S_0 + \frac{8\Delta^2 I_0^2 \gamma}{(\omega^2 - 4\Delta^2)^2 + \gamma^2 \omega^2},
\]

(12)

where \( S_0 = (\Gamma_+ + \Gamma_-)(|t_0|^2 + |\lambda|^2) \). Qualitatively, for \( \gamma \ll \Delta \), spectral density (12) describes coherent oscillations of the two qubits with the frequency \( 2\Delta \), twice the oscillation frequency in one qubit. Similarly to the case of linear measurements [24], the maximum of the ratio of the oscillation peak versus noise \( S_0 \) is 4. As one can see from Eq. (12), this maximum is reached in the case of weak measurement \( |\lambda| \ll |t_0| \) by the “ideal” detector for which \( \arg(t_0 \lambda^*) = 0 \) and only \( \Gamma_+ \) or \( \Gamma_- \) is non-vanishing.

For different qubit tunnel amplitudes, transitions (with the rate \( (\Delta_1 - \Delta_2)^2/2\gamma \) for small \( \Delta_1 - \Delta_2 \)) between the states \( |\uparrow\uparrow\rangle - |\down\down\rangle \) and \( |\up\down\rangle - |\down\up\rangle \) mix the measurement outcomes \( I_{\uparrow\uparrow} \) and \( I_{\up\down} \). This means that for \( \Delta_1 \neq \Delta_2 \) there are only two outcomes that have the same dc current and differ by current spectral densities. In one, the qubits are again in the subspace \( D_+ \) and the spectral density is given by Eq. (12) where now \( 2\Delta \to \Delta_1 + \Delta_2 \). In the other, the qubit dynamics is confined to the subspace orthogonal to \( D_+ \), the basis of which can be chosen as \( \{|\up\up\rangle - |\down\down\rangle, |\up\down\rangle - |\down\up\rangle\} \). Evolution of the qubit density matrix is described in this basis by the same Eq. (11) with \( \Delta \to (\Delta_1 - \Delta_2)^2/2 \), and as a result the current spectral density is given by the same Eq. (12) with \( 2\Delta \to \Delta_1 - \Delta_2 \) and describes the current peak at difference of qubit oscillation frequencies.

As the last application of the general theory we consider two identical qubits measured by a weakly and symmetrically coupled detector with arbitrary non-linearity. It is convenient to discuss this situation in terms of the total effective spin \( S \) of the two qubits which determines the amplitude (1) of detector tunneling:

\[
t = t_0 + 2S_z \lambda(2S^2_z - 1).
\]

(13)

The \( S = 0 \) state (10) does not evolve in time under the qubit Hamiltonian and represents one of the measurement outcomes characterized by the dc detector current \( I_{\uparrow\down} \) and flat current spectral density \( S_I(\omega) = (\Gamma_+ + \Gamma_-)(|t_0|^2 + |\lambda|^2) \). Three other, \( S = 1 \), states are mixed by measurement and represent the second measurement outcome. We take the basis of the \( S = 1 \) subspace as three energy eigenstates \( S_\pm = -1, 0, 1 \) with energies \( \Delta, 0, -\Delta \). Interaction with the detector induces transitions between these states with the rate independent of the transition’s direction, so that the stationary qubit density matrix is \( \rho_0 = 1/3 \). Equations (6) and (13) show that the dc detector current for this outcome is:

\[
\langle I \rangle = (\Gamma_+ - \Gamma_-)[2(|t_0|^2 + |\lambda|^2) + |t_0 + \lambda|^2 + 8|\delta|^2]/3,
\]
and can be written as \( \langle I \rangle = (I_{++} + I_{\pm\pm} + I_{\pm\mp})/3 \), where the currents \( I_{++}, ... \) are introduced in the same way as before, e.g. \( I_{++} = (\Gamma_+ - \Gamma_-)|t_0 + 2\delta + \lambda|^2 \). The background current noise \( S_0 \) coincides with \( \langle I \rangle \) with \( \Gamma_+ - \Gamma_- \) replaced by \( \Gamma_+ + \Gamma_- \).

The system performs oscillations at frequencies \( \Delta \) and \( 2\Delta \) whose spectral peaks should have Lorentzian form for weak detector-qubit coupling. Evaluating the current matrix elements from Eqs. (5) and (13), and evolution of the density matrix from Eq. (5) reduced to the subspace, we obtain parameters of these Lorentzians in the spectral density (7):

\[
\omega \simeq j \Delta, \quad S_{\nu}(\omega) = S_0 + \frac{2a_j^2\gamma_j}{3(\omega-j\Delta)^2 + \gamma_j^2}, \quad (14)
\]

\[ j = 1, 2, \quad \gamma_1 = (\Gamma_+ + \Gamma_-)(|\delta|^2 + |\lambda|^2/2), \quad \gamma_2 = 2\gamma_1, \]

\[ a_1 = 4(\Gamma_+ - \Gamma_-)\text{Re}[t_0 + \lambda \delta^*] I_{++} - I_{\pm\pm}] / 2, \]

\[ a_2 = 2(\Gamma_+ - \Gamma_-)\text{Re}[t_0 \lambda^* + |\delta|^2] (I_{++} + I_{\pm\pm} - 2I_{\pm\mp}) / 4. \]

Note that condition \( \rho_0 = 1/3 \), used in Eq. (14) and Fig. 2, implies that the coefficient \( \delta \) of linear measurement mixing all three \( S = 1 \) states, does not vanish identically. Otherwise, only two states are mixed and \( \rho_0 = 1/2 \), as assumed in Eq. (12). In general, there is also a spectral peak at \( \omega = 0 \) caused by switching between states with different average currents. For small but finite \( \delta, \delta \ll \lambda \), this peak can be very high, e.g., in the case of an “ideal” detector \( \lambda = \arg t_0 \lambda^* = \arg t_0 \delta^* = 0 \) its height and half-width are, respectively, \((8\lambda^2/27\delta^2)S_0 \) and \(6\delta^2\Gamma_+ \).

Figure 2 shows current spectral density calculated from Eqs. (5) and (7) for an ideal detector without making weak-coupling approximation. Figure 2a illustrates the transition between “single-qubit” oscillations at \( \omega \simeq \Delta \) in the case of linear measurement and oscillations at \( \omega \simeq 2\Delta \) for quadratic measurement. One can see that in agreement with Eq. (14) the \( \omega \simeq \Delta \) peak is typically higher than the quadratic peak at double frequency. It is at the same time more sensitive to qubit-qubit interaction as illustrated in Fig. 2b. Even weak interaction \( \nu \ll \Delta \) first suppresses and then splits the peak at \( \omega \simeq \Delta \) in two while affecting the quadratic peak only slightly.

In summary, we discussed quadratic quantum measurements and have shown that they should have non-trivial properties providing, for instance, simple method of entangling non-interacting qubits. We also calculated output spectra of the quadratic detector measuring coherent oscillations in two qubits. Consistent with the case of classical oscillations, quadratic measurements results in the spectral peak at frequency that is twice the frequency of individual qubit oscillations. Quadratic measurements should be an interesting and potentially useful tool in solid-state quantum devices.
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