A formulation for description of $\pi^+(2\pi^-)$ and $\pi^-(2\pi^+)$ channels in Bose-Einstein correlation by Coulomb wave function

M. Biyajima, a,b,1 T. Mizoguchi c,2 and N. Suzuki d,3

aDepartment of Physics, Faculty of Science, Shinshu University, Matsumoto 390-8621, Japan
bNiels Bohr Institute, University of Copenhagen, Denmark
cToba National College of Maritime Technology, Toba 517-8501, Japan
dMatsumoto University, Matsumoto 390-1295, Japan

Abstract

In order to analyze data on charged pions correlation channels, $\pi^+(2\pi^-)$ and $\pi^- (2\pi^+)$, we propose new interferometry approach using the Coulomb wave function. We show that to describe adequately data we have to introduce new parameter describing the contribution of $\pi^- (k_1)\pi^+ (k_2) \rightarrow \pi^- (k_2)\pi^+ (k_1)$ process. Using this new formula we analyze data on $\pi^+(2\pi^-)$ and $\pi^- (2\pi^+)$ channels at $\sqrt{s} = 91$ GeV by DELPHI Collaboration, and estimate the magnitude of this new parameter as well as the degree of coherence.

Key words: Bose-Einstein Correlation, Coulomb wave function, $e^+e^-$ annihilation

1 Introduction

In 1995 DELPHI Collaboration reported data of the 3rd order Bose-Einstein Correlations (BEC) and concluded that there is a genuine the 3rd order BEC in $3\pi^-$ channel and the effect of the 2nd order BEC in $\pi^+(2\pi^-)$ channel [1,2]. The method used in their analyses is the formulation in terms of the plane wave amplitude.

1 E-mail: biyajima@azusa.shinshu-u.ac.jp
2 E-mail: mizoguti@toba-cmt.ac.jp
3 E-mail: suzuki@matsu.ac.jp
From theoretical point of view, at almost the same time, formulations for the 2nd order BEC by means of the Coulomb wave function had been investigated in Refs. [3,4] 4. Moreover, formulations for the 3rd and 4th order BEC by means of the Coulomb wave function have been recently proposed in Refs. [6,7]. In particular, the formulation of Ref. [7] contains the degree of coherence \((\lambda^{1/2})\) for the magnitude of the BE exchange term. The advantage of this approach is that it can be directly applicable to analyses of the data on \(3\pi^-\) channel with the CERN-MINUIT program. 5

In the present study we investigate whether this approach using formula derived by means of the Coulomb wave function can be extended to an unlike charged combination channel, \(\pi^+(2\pi^-)\). We should stress that:

1) Through our approach, the plane wave amplitude is used as a basic calculation. At the second step the Coulomb wave function is utilized.
2) The interferometry effect for \(\pi^+(2\pi^-)\), i.e., the squared amplitude by the Coulomb wave function, contains the following neutral current:

\[
\pi^-(k_1)\pi^+(k_2) \rightarrow \pi^-(k_2)\pi^+(k_1) .
\]  (1)

If the magnitude of this contribution is zero, we cannot explain the data on \(\pi^+(2\pi^-)\) channel by DELPHI Collaboration. Their data ask for the finite magnitude which is expressed by a new parameter \(\beta\) in this paper.

This paper is organized in the following way: In §2, a simple model for \(\pi^+(2\pi^-)\) channel is investigated. In §3, a model including full amplitudes \((3! = 6)\) is presented. A new formula for \(\pi^+(2\pi^-)\) channel is also derived here. In §4, analyses of the data on \(3\pi^-\) channel are performed. The estimated parameters are compared with those in \(\pi^+(2\pi^-)\) channel. In the final section, concluding remarks are given.

### 2 A simple model

Authors of Ref. [1] stressed that data on \(\pi^+(2\pi^-)\) channel are described by the formulas of the 2nd order BEC. We shall study whether this statement is correct or not using the interferometry approach formulated in terms of the Coulomb wave function. To analyze data on BEC effect in \(e^+e^- \rightarrow \pi^+(2\pi^-) + X\) channel, first of all we consider the simplest diagram shown in Fig. 1.

---

4 For recent presentation of various aspects of BEC in high energy physics one should consult a recent review [5].

5 In application of formulas in Ref. [6] we have to assume the parameter of the interaction region \((R)\) a priori.
Fig. 1. A thick line denotes the positive pion. Thin lines do negative pion. The cross mark (×) means the exchange effect due to the Bose-Einstein statistics, whose magnitude is expressed by the effective degree of coherent \((\lambda^{1/2})\).

The plane wave amplitudes (PWA) for \(\pi^+(2\pi^-)\) channel are given as

\[
PWA(1; + -) = \frac{1}{\sqrt{2}} e^{i(k_1^+ \cdot x_1 + k_2^- \cdot x_2 + k_3^- \cdot x_3)} , \tag{2a}
\]

\[
PWEA(1; + -) = \frac{1}{\sqrt{2}} e^{i(k_1^+ \cdot x_1 + k_2^- \cdot x_3 + k_3^- \cdot x_2)} , \tag{2b}
\]

where (+) stands for the positive pion and \(PWEA\) represents the plane wave exchange amplitude due to the BE statistics.

When the Coulomb wave functions is taken into account because of charged pions \(\pi^+(2\pi^-)\), the above PWA and PWEA should be replaced by \(A(1; + -)\) and \(EA(1; + -)\),

\[
A(1; + -) = \psi_{k_1, k_2}^{C+}(x_1, x_2) \psi_{k_2, k_3}^{C-}(x_2, x_3) \psi_{k_3, k_1}^{C+}(x_3, x_1) , \tag{3a}
\]

\[
EA(1; + -) = \psi_{k_1, k_2}^{C-}(x_1, x_3) \psi_{k_2, k_3}^{C+}(x_3, x_2) \psi_{k_3, k_1}^{C-}(x_2, x_1) , \tag{3b}
\]

where \(\psi_{k_1, k_2}^{C}(x_i, x_j)\) is defined as

\[
\psi_{k_1, k_2}^{C}(x_i, x_j) = \Gamma(1 + i\eta_{ij}) e^{i\pi\eta_{ij}/2} e^{ik_{ij} \cdot r_{ij}} F[-i\eta_{ij}, 1; i(k_{ij} r_{ij} - k_{ij} \cdot r_{ij})] \tag{4}
\]

with \(r_{ij} = x_i - x_j\), \(k_{ij} = (k_i - k_j)/2\), \(r_{ij} = |r_{ij}|\), \(k_{ij} = |k_{ij}|\) and \(\eta_{ij} = \pm m\alpha/k_{ij}\) \((+\text{ and } -\text{ are the like-charge combination and unlike-charge one, respectively})\). \(F[a, b; x]\) and \(\Gamma(x)\) are the confluent hypergeometric function and the gamma function, respectively. The parts of the plane wave of Eq. (3) are given as \([8,9]\)

\[
A(1; + -) \xrightarrow{PWA} e^{(3/2)i[k_1^+ \cdot x_1 + k_2^- \cdot x_2 + k_3^- \cdot x_3]} , \tag{5a}
\]

\[
EA(1; + -) \xrightarrow{PWA} e^{(3/2)i[k_1^+ \cdot x_1 + k_3^- \cdot x_2 + k_2^- \cdot x_3]} , \tag{5b}
\]
Fig. 2. Analyses of data on $\pi^+(2\pi^-)$ and $\pi^-(2\pi^+)$ channels by means of Eq. (6). \[\chi^2/n.d.f. = 3.2/29.\]

where the factor \((3/2)\) is attributed to property of the Coulomb wave function and 3-body problem.

The interferometry effect for the \((\pi^+(2\pi^-))\) channel is calculated as

$$\frac{N_{(\pi^+2\pi^-)/N_{BG}}} = (1 + \gamma Q_3) \prod_{i=1}^{3} \int \rho(x_i) d^3 x_i \left[ F_1^{(--+)} + \lambda F_2^{(--)} \right],$$  \hspace{1cm} (6)

where $\rho(x_i)$ stand for the source functions of particle $i$. We use the Gaussian distribution of the radius $R$, $\rho(x_i) = \frac{1}{(2\pi R^2)^{3/2}} \exp \left[ -\frac{x_i^2}{2R^2} \right]$, and

$$Q_3 = \sqrt{(k_1 - k_2)^2 + (k_2 - k_3)^2 + (k_3 - k_1)^2},$$  \hspace{1cm} (7a)

$$F_1^{(+--)} = \frac{1}{2} [ |A(1; + --)|^2 + |EA(1; + --)|^2 ],$$  \hspace{1cm} (7b)

$$F_2^{(+-+)} = \text{Re}[A(1; + --)EA^*(1; + --)].$$  \hspace{1cm} (7c)

The parameter $\lambda^{1/2}$ is introduced in order to estimate the strength of the BE effect, where the magnitude of the cross mark ($\times$) is expressed by $\lambda^{1/2}$. Of course $\lambda$ should be less than one (1), because it can be interpreted as the degree of coherence in quantum optics [See also Ref. [7]].

Our result is given in Fig. 2 and Table 1. As seen there, the magnitude $\lambda$ is larger than 1. This suggests that we have to consider also additional diagrams (contributions) to Fig. 1. Therefore we should seek other possible schemes for the description of $\pi^+(2\pi^-)$ channel.
Table 1
Analyses of \(\pi^+(2\pi^-)\) and \(\pi^-(2\pi^+)\) BEC by DELPHI Collaboration. The systematic errors for all points are assumed to be \(\pm 0.05\). Small normalizations \((C)\) are attributed to the long range effect \((1 + \gamma Q_3)\).

| formulas | \(\beta\) (fixed) | \(C\) | \(R\) [fm] | \(\lambda\) | \(\gamma\) | \(\chi^2/N_{dof}\) |
|----------|------------------|------|---------|---------|--------|------------------|
| Eq. (6)  | —                | 0.51±0.11 | 0.13±0.01 | 2.53±0.66 | 0.20±0.09 | 3.2/29           |
| Eq. (10) | 0.8              | 0.79±0.09 | 0.16±0.02 | -0.12±0.09 | 0.09±0.05 | 1.3/29           |
|          | 0.5              | 0.74±0.09 | 0.14±0.01 | 0.23±0.12 | 0.11±0.05 | 1.2/29           |
|          | 0.4              | 0.72±0.09 | 0.13±0.01 | 0.39±0.14 | 0.11±0.05 | 1.1/29           |
|          | 0.3              | 0.70±0.09 | 0.13±0.02 | 0.62±0.17 | 0.12±0.05 | 1.1/29           |
|          | 0.2              | 0.66±0.09 | 0.13±0.01 | 0.96±0.23 | 0.13±0.05 | 1.3/29           |
|          | 0.1              | 0.60±0.10 | 0.13±0.01 | 1.51±0.36 | 0.16±0.07 | 1.8/29           |

Fig. 3. Six diagrams for three charged pions, because of \(3! = 6\). The BE statistics is taken into account between \(\pi^-(k_2)\) and \(\pi^-(k_3)\) pions.

3 A model including six amplitudes

As explained in the previous section, we should consider more complex diagrams than Fig. 1. In this case the following PWA and PWEA in addition to PWA(1; +−−) and PWEA(1; +−−) are necessary, because of the increased number of diagrams \((3! = 6)\),

\[
PWA(2; -++ \ t) = e^{\t i (k_1^{(+) \cdot x_2 + k_2 \cdot x_3 + k_3 \cdot x_1)}},
\]

\[
PWEA(2; -++ \ t) = e^{\t i (k_1^{(+) \cdot x_2 + k_2 \cdot x_1 + k_3 \cdot x_3)}},
\]

\[
PWA(3; --+) = e^{\t i (k_1^{(+) \cdot x_3 + k_2 \cdot x_1 + k_3 \cdot x_2)}},
\]

\[
PWEA(3; --+) = e^{\t i (k_1^{(+) \cdot x_3 + k_2 \cdot x_2 + k_3 \cdot x_1)}},
\]

These additional amplitudes described by means of the Coulomb wave function are given as

\[
A(2; -++) = \psi_{k_1 k_2}^{C^-}(x_2, x_3) \psi_{k_2 k_3}^{C^-}(x_3, x_1) \psi_{k_1 k_1}^{C^+}(x_1, x_2),
\]
is given as

\[ E_A(2; - + -) = \psi^{C+ -}_{k_1 k_2}(x_2, x_1) \psi^{C- -}_{k_2 k_3}(x_1, x_3) \psi^{C+ -}_{k_3 k_1}(x_3, x_2), \]  
\[ A(3; - - +) = \psi^{C+ -}_{k_1 k_2}(x_3, x_1) \psi^{C- -}_{k_2 k_3}(x_1, x_2) \psi^{C+ -}_{k_3 k_1}(x_2, x_3), \]  
\[ E_A(3; - - +) = \psi^{C+ -}_{k_1 k_2}(x_3, x_2) \psi^{C- -}_{k_2 k_3}(x_2, x_1) \psi^{C+ -}_{k_3 k_1}(x_1, x_3). \]

The interferometry effect for \( \pi^+(2\pi^-) \) channel described by Eqs. (3) and (9) is given as

\[
\frac{N^{(\pi^+2\pi^-)}}{N_{BG}} = (1 + \gamma Q_3) \prod_{i=1}^{3} \int \rho(x_i) d^3 x_i \\
\times \left[ F_1^{(+--)} + \lambda F_2^{(--+)} + \beta F_3^{(+-+)} + \lambda \beta F_4^{(-+-)} \right], \tag{10}
\]

where

\[
F_1^{(+--)} = \frac{1}{6} \left\{ \sum_{i=1}^{3} A(i; c_1 c_2 c_3) A^*(i; c_1 c_2 c_3) + \sum_{i=1}^{3} E_A(i; c_1 c_2 c_3) E_A^*(i; c_1 c_2 c_3) \right\} \xrightarrow{\text{PWA}} 1, \tag{11a}
\]

\[
F_2^{(--+)} = \frac{1}{6} \left\{ \sum_{i=1}^{3} A(i; c_1 c_2 c_3) E_A^*(i; c_1 c_2 c_3) + \text{c. c.} \right\} \xrightarrow{\text{PWA}} (\text{typical PW}) e^{(3/2)i[(k_2 - k_3) \cdot x_2 + (k_3 - k_2) \cdot x_3]}, \tag{11b}
\]

\[
F_3^{(+-+)} = \frac{1}{6} \left\{ \sum_{i=1}^{3} \sum_{j \neq i} A(i; c_1 c_2 c_3) E_A^*(j; c_1 c_2 c_3) + \text{c. c.} \right\} \xrightarrow{\text{PWA}} (\text{typical PW}) e^{(3/2)i[(k_1 - k_3) \cdot x_1 + (k_3 - k_1) \cdot x_3]}, \tag{11c}
\]

\[
F_4^{(-+-)} = \frac{1}{6} \left\{ \sum_{i=1}^{3} \sum_{j \neq i} A(i; c_1 c_2 c_3) A^*(j; c_1 c_2 c_3) + \sum_{i=1}^{3} \sum_{j \neq i} E_A(i; c_1 c_2 c_3) E_A^*(j; c_1 c_2 c_3) \right\]
Fig. 5. (a) The exchange part between $\pi^+$ and $\pi^-$ contained in $F_3$. (b) A possibly effective interpretation for the shadow region in (a) is a “$\rho^0$-meson-like contribution”.

Fig. 6. Analyses of the data on $\pi^+\pi^-\pi^+$ and $\pi^-\pi^+\pi^+$ channels by means of Eq. (10). $\chi^2/n.d.f. = 1.1/29$.

\[
\text{Eq. (10)} \quad \beta = 0.3 \text{ (fixed)} \quad \lambda = 0.62 \pm 0.17
\]

In the above equations for the sake of simplicity we neglect the suffix (+). In actual analyses, however we should fix the charge assignment of (+) in $Q_3$. Moreover, we have to introduce a new parameter ($\beta$) to describe the strength of the shadow parts in $F_3^{(+--)}$ and $F_4^{(+-+)}$, cf., Fig. 4. A possible interpretation of the role of the shadow region is a “$\rho^0$-meson-like contribution” occurring here in order to satisfy the conservation of the neutral current, cf., Fig. 5.

The results obtained by our new formula, i.e., Eq. (10), are given in Fig. 6 and Table 1.

As seen in Table 1, $\lambda$ becomes negative for large $\beta$ and exceeds unity for small $\beta$. The possibly sets of parameters ($\beta$, $\lambda$) leading to reasonable results
are \((\beta, \lambda) \sim (0.5, 0.2), (0.4, 0.4), (0.3, 0.6)\) and \((0.2, 0.96)\). To estimate the strength of \(\lambda\) we shall now analyze BEC data on \(3\pi^- + 3\pi^+\).

### 4 Analyses of \(3\pi^- + 3\pi^+\) BEC by Coulomb wave function

To analyze data on \(3\pi^-\) BEC we can use the following formula presented in Ref. [7]

\[
\frac{N^{(3\pi^-)}}{N_{BG}} = (1 + \gamma Q_3) \prod_{i=1}^{3} \int \rho(x_i) d^3x_i \left[ F_1^{(3\pi^-)} + \lambda F_2^{(3\pi^-)} + \lambda^{3/2} F_3^{(3\pi^-)} \right],
\]

where

\[
F_1^{(3\pi^-)} = \frac{1}{6} \sum_{i=1}^{6} A(i) A^*(i),
\]

\[
F_2^{(3\pi^-)} = \frac{1}{6} \left[ A(1) A^*(2) + A(1) A^*(3) + A(1) A^*(4) + A(2) A^*(5) + A(2) A^*(6) + A(3) A^*(5) + A(3) A^*(6) + A(4) A^*(5) + A(4) A^*(6) + \text{c. c.} \right],
\]

\[
F_3^{(3\pi^-)} = \frac{1}{6} \left[ A(1) A^*(5) + A(1) A^*(6) + A(2) A^*(3) + A(2) A^*(4) + A(3) A^*(4) + A(5) A^*(6) + \text{c. c.} \right],
\]

and

\[
A(1) = \psi_{k_1 k_2}^C (x_1, x_2) \psi_{k_2 k_3}^C (x_2, x_3) \psi_{k_3 k_1}^C (x_3, x_1)\]

\[
\text{PWA} e^{i k_{12} r_{12}} e^{i k_{23} r_{23}} e^{i k_{31} r_{31}} = e^{(3/2)i((k_1 \cdot x_1 + k_2 \cdot x_2 + k_3 \cdot x_3)},
\]

\[
A(2) = \psi_{k_1 k_2}^C (x_1, x_2) \psi_{k_2 k_3}^C (x_2, x_3) \psi_{k_3 k_1}^C (x_3, x_1)\]

\[
\text{PWA} e^{i k_{12} r_{13}} e^{i k_{23} r_{23}} e^{i k_{31} r_{31}} = e^{(3/2)i((k_1 \cdot x_1 + k_2 \cdot x_3 + k_3 \cdot x_2)},
\]

\[
A(3) = \psi_{k_1 k_2}^C (x_2, x_1) \psi_{k_2 k_3}^C (x_2, x_3) \psi_{k_3 k_1}^C (x_3, x_2)\]

\[
\text{PWA} e^{i k_{12} r_{21}} e^{i k_{23} r_{13}} e^{i k_{31} r_{31}} = e^{(3/2)i((k_1 \cdot x_2 + k_2 \cdot x_3 + k_3 \cdot x_1)},
\]

\[
A(4) = \psi_{k_1 k_2}^C (x_2, x_3) \psi_{k_2 k_3}^C (x_3, x_1) \psi_{k_3 k_1}^C (x_1, x_2)\]

\[
\text{PWA} e^{i k_{12} r_{23}} e^{i k_{23} r_{31}} e^{i k_{31} r_{12}} = e^{(3/2)i((k_1 \cdot x_2 + k_2 \cdot x_1 + k_3 \cdot x_3)},
\]

\[
A(5) = \psi_{k_1 k_2}^C (x_3, x_1) \psi_{k_2 k_3}^C (x_3, x_2) \psi_{k_3 k_1}^C (x_2, x_3)\]

\[
\text{PWA} e^{i k_{12} r_{31}} e^{i k_{23} r_{12}} e^{i k_{31} r_{23}} = e^{(3/2)i((k_1 \cdot x_3 + k_2 \cdot x_1 + k_3 \cdot x_2)},
\]

\[
A(6) = \psi_{k_1 k_2}^C (x_3, x_2) \psi_{k_2 k_3}^C (x_2, x_1) \psi_{k_3 k_1}^C (x_1, x_3)\]

\[
\text{PWA} e^{i k_{12} r_{32}} e^{i k_{23} r_{21}} e^{i k_{31} r_{13}} = e^{(3/2)i((k_1 \cdot x_3 + k_2 \cdot x_2 + k_3 \cdot x_1)}.}
Fig. 7. Analyses of data on $3\pi^+$ and $3\pi^-$ channels by means of Eq. (12).

Table 2
Analyses of $3\pi^-$ and $3\pi^+$ BEC by DELPHI Collaboration. The systematic errors for all points are assumed to be ±0.05. Small normalizations ($C$) are attributed to the long range effect $(1 + \gamma Q_3)$.

| formulas | $\beta$  | $C$       | $R$ [fm]  | $\lambda$ | $\gamma$ | $\chi^2/N_{dof}$ |
|----------|----------|-----------|-----------|-----------|-----------|------------------|
| Eq. (12) | —        | 0.33±0.02 | 0.22±0.01 | 1.0 (fixed)| 0.51±0.05 | 20.6/31          |
| Eq. (12) | —        | 0.49±0.04 | 0.24±0.01 | 0.69±0.06 | 0.22±0.05 | 4.0/30           |

$\pi^+(2\pi^-)$ and $\pi^-(2\pi^+)$ BEC

| Eq. (10) | 0.28±0.06 | 0.69±0.07 | 0.13±0.01 | 0.7 (fixed)| 0.12±0.04 | 1.2/29           |

The result obtained by this formula is given in Fig. 7 and Table 2. Notice that fixed $\lambda = 1$ results in large $\chi^2$. This means that an additional parameter is necessary here, i.e., that we should allow the degree of coherence ($\lambda$) to vary as well. As seen in Table 2 we have found that there is a common region ($\beta, \lambda$) $\sim (0.28, 0.7)$ for $\pi^+(2\pi^-)$ channel and $\lambda \sim 0.7$ for $3\pi^- + 3\pi^+$ channel.

5 Concluding remarks

We have obtained the new formula for $\pi^+(2\pi^-)$ channel in the unlike-3rd order BEC, introducing the degree of coherence ($\lambda$) and the effective magnitude of neutral current ($\beta$). We have analyzed BEC data on $\pi^+(2\pi^-)$ and $\pi^-(2\pi^+)$ channels in $e^+e^-$ collision at $\sqrt{s} = 91$ GeV, using the new formula. (Notice that to compare the obtained here values of $R$ with those obtained by using the plane wave approach, we have to multiply it by the factor $3/2$, i.e., $R^{(3\pi^-)} \rightarrow$
As seen from Table 2, the following choice of parameters is possible which leads to good fit to data,

\[ (\beta, \lambda) \sim (0.28, 0.7) \]

provided that the degree of coherence (\(\lambda\)) is almost the same in both channels. This finite \(\beta\) suggests that there is the genuine 3rd order contribution even in \(\pi^+(2\pi^-)\) channel.\(^6\) To confirm the choice of this set of parameters we need other data at \(\sqrt{s} = 91\) GeV as well as at different energies. Moreover, our formula i.e., Eq. (10) can be also applied to the same kind data from heavy-ion collisions.

Acknowledgements

One of authors (M. B.) would like to thank The Scandinavia-Japan Sasakawa Foundation for financial support, and authors are also thankful for G. Wilk’s reading the manuscript. They are indebted to useful conversations with H. Boggild, T. Csorgo, B. Lorstad and L. Muresan.

References

[1] P. Abreu et al. [DELPHI Collaboration], Phys. Lett. B 355 (1995) 415.
[2] M. Biyajima, A. Bartl, T. Mizoguchi, O. Terazawa and N. Suzuki, Prog. Theor. Phys. 84 (1990) 931 [Addendum-ibid. 88 (1992) 157]; See also, M. Biyajima, Phys. Lett. B 92 (1980) 193.
[3] M. Biyajima, T. Mizoguchi, T. Osada and G. Wilk, Phys. Lett. B 353 (1995) 340; ibid 366 (1996) 394.
[4] T. Osada, S. Sano and M. Biyajima, Z. Phys. C 72 (1996) 285.
[5] G. Alexander, Rept. Prog. Phys. 66 (2003) 481.
[6] E. O. Alt, T. Csorgo, B. Lorstad and J. Schmidt-Sorensen, Phys. Lett. B 458 (1999) 407.
[7] T. Mizoguchi and M. Biyajima, Phys. Lett. B 499 (2001) 245.
[8] L. I. Schiff, Quantum Mechanics, 2nd Ed., (McGraw-Hill, New York, 1955).
[9] T. Sasakawa, Scattering Theory (in Japanese), (Shokabo, Tokyo, 1991).

\(^6\) Hopefully, in future some additional different data on \(\pi^+(2\pi^-)\) channel would be available, in which case the usefulness of Eq. (10) could be checked again.