Decay widths of resonances and pion scattering lengths in a globally invariant sigma model with vector and axial-vector mesons

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We calculate low-energy meson decay processes and pion-pion scattering lengths in a two-flavour linear sigma model with global chiral symmetry, exploring the scenario in which the scalar mesons $f_0(600)$ and $a_0(980)$ are assumed to be $\bar{q}q$ states.

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1. Introduction

Effective field theories provide a very efficient means to describe Quantum Chromodynamics (QCD) at low energies. They possess the same global symmetries as QCD - e.g., the chiral $SU(N_f)_L \times SU(N_f)_R$ symmetry, where $N_f$ is the number of flavours - and are expressed in terms of hadronic degrees of freedom rather than in terms of quarks and gluons. Spontaneous breaking of the chiral symmetry leads to the emergence of low-mass pseudoscalar Goldstone bosons and their chiral partners, large-mass scalar states.

In this paper we present a linear sigma model with global chiral invariance, similar to the one of Ref. [6]. The model contains scalar and pseudoscalar as well as vector and axialvector mesons. The global invariance is motivated by the results of Refs. [8, 9] where it has been shown that a locally invariant linear sigma model fails to describe simultaneously pion-pion scattering lengths and some important decay widths. For the globally invariant model additional terms appear in the Lagrangian which introduce new coupling constants that can in principle be adjusted to improve the agreement with the experimental data. In this paper we show the first results from this approach for the case of $N_f = 2$.

As outlined in Ref. [3], there are two possibilities to interpret the scalar $\sigma$ and $a_0$ fields contained in the model where they are $\bar{q}q$ states ($\sigma = \frac{1}{\sqrt{2}}(\bar{u}u + \bar{d}d)$, $a_0 = \frac{1}{\sqrt{2}}(\bar{u}u + \bar{d}d)$) (i) they are identified with $f_0(600)$ and $a_0(980)$ which form a part of a larger nonet that consists of $f_0(980)$, $a_0(980)$, $k(800)$ and $f_0(600)$(resonances below 1 GeV); (ii) they are identified with the $f_0(1370)$ and $a_0(1450)$ resonances forming a part of a nonet that consists of $f_0(1370)$, $f_0(1500)$, $f_0(1710)$, $a_0(1450)$, $K_0(1430)$ - i.e., resonances above 1 GeV (see Ref. [4]). In the assignment (ii), scalar mesons below 1 GeV, whose spectroscopic wave functions possibly contain a dominant tetraquark or mesonic molecular contribution [5], may be introduced as additional scalar fields.

In this paper, we describe briefly the consequences of assignment (i); the consequences of assumption (ii) as well as more detailed results in assignment (i) may be found in Ref. [6].

2. The Linear Sigma Model with Global Chiral Symmetry

The Lagrangian of the globally invariant linear sigma model with $U(2)_R \times U(2)_L$ symmetry reads [3, 8, 10]:

$$\mathcal{L} = \text{Tr}[(D^\mu \Phi) (D^\mu \Phi)] - m_0^2 \text{Tr}(\Phi^3 \Phi) + \lambda_1 \left[ \text{Tr}(\Phi^4) \right]^2 + \lambda_2 \text{Tr}(\Phi^3 \Phi)^2$$

$$+ \frac{1}{4} \text{Tr} [(L^{\mu\nu})^2 + (R^{\mu\nu})^2] + \frac{m_t^2}{2} \text{Tr} [(L^{\mu})^2 + (R^{\mu})^2] + \text{Tr} [H](\Phi + \Phi^\dagger)$$

$$+ c (\text{det} \Phi + \text{det} \Phi^\dagger) + 2ig_2 (\text{Tr}FL_{\mu\nu}[L^\mu, L^\nu]) + \text{Tr}FR_{\mu\nu}[R^\mu, R^\nu]$$

$$+ 2g_3 \varepsilon \text{Tr}[(\partial_\mu L^\nu - \partial_\nu L^\mu)\varepsilon L^\mu L^\nu] + \text{Tr}[(\partial_\mu R^\nu + \partial_\nu R^\mu)\varepsilon R^\mu R^\nu]$$

$$+ \frac{h_1}{2} \text{Tr}(\Phi^3 \Phi) \text{Tr}[L^{\mu\nu} L^{\mu\nu}] + h_2 \text{Tr}[\Phi R^{\mu\nu}]^2 + (L^{\mu\nu})^2 + 2h_3 \text{Tr}(\Phi R_{\mu\nu} \Phi^\dagger L^\mu) \quad (2.1)$$

with $\Phi = (\sigma + i \eta_N) t^0 + (a_0 + i \pi) \tau$ (scalar and pseudoscalar mesons; our model is valid for $N_f = 2$ and thus our eta meson $\eta_N$ contains only non-strange degrees of freedom); $L^\mu = (\omega^\mu f^\mu_1) t^0 + (\rho^\mu a^\mu_1) \tau$ and $R^\mu = (\omega^\mu + f^\mu_1) t^0 + (\rho^\mu + a^\mu_1) \tau$ (vector and axialvector mesons), where $t^0, \tau$ are the generators of $U(2); D^\mu \Phi = \partial^\mu \Phi + ig_1 (\Phi L^\mu - R^\mu \Phi), L^{\mu\nu} = \partial^\mu L^\nu - \partial^\nu L^\mu, R^{\mu\nu} = \partial^\mu R^\nu - \partial^\nu R^\mu$. 

and \( \mathcal{L}_4 \) containing all the vertices with four vector and axialvector mesons. Explicit breaking of the global symmetry is described by the term Tr \([H (\Phi + \Phi^\dagger)] h \sigma (h = \text{const})\). The chiral anomaly is described by the term \( c (\det \Phi + \det \Phi^\dagger) \) \([8]\).

Irrespectively of \( \mathcal{L}_4 \), the model contains 13 parameters - 12 parameters from the Lagrangian \((2.1)\), plus the wave function renormalisation constant of the pseudoscalar mesons \([9]\), \(Z\). However, only seven of those \((Z, g_{12}, h_{123}, \lambda_2)\) are relevant for the decays that will be considered in the following. The parameters \(g_1, h_3 \) and \( \lambda_2 \) are expressed in terms of \(Z\):

\[
g_1 = g_1(Z) = \frac{m_{\rho}}{Z f_\pi} 1 \frac{1}{Z^2}; \quad h_3 = h_3(Z) = \frac{1}{Z^2 f_\rho^2} m_\rho^2 \frac{m_{a_0}^2}{Z^2}; \quad \lambda_2 = \lambda_2(Z) = \frac{Z^2 m_{a_0}^2}{Z^4 f_\rho^2} m_\pi^2; \]

and thus the number of independent relevant parameters is decreased to four. Additionally, \(m_\sigma\) (which is a function of \(m_0, \lambda_1, \lambda_2, c \) and \(Z\)) is taken as a parameter that can be determined from the pion-pion scattering lengths yielding five independent parameters for the meson decay modes and scattering lengths described below.

### 2.1 Relevant Decay Modes and \(\pi \pi\) Scattering Lengths

The following decay modes of two-flavour low-energy mesons have been taken into account [parameter dependence in brackets]: \(\rho \pi \pi [Z, g_{2}] f_1 \pi \pi a_{0} \pi [Z, h_{2}] a_{1} \pi \pi \eta \pi [Z, h_{2}] \sigma \pi \pi [Z, h_{1}, h_{2}] a_{1} \sigma \pi \pi [h_{1}, h_{2}, Z] a_{1} \rho \pi \pi [a_{2}, Z]\). We have also considered the pion-pion scattering lengths \(a_0^h (h_1, h_2, Z, m_\sigma)\) and \(a_2^h (h_1, h_2, Z, m_\sigma)\).

Given that the decay widths for the channels \(\sigma \pi \pi [Z, h_{1}, h_{2}] a_{1} \sigma \pi \pi [h_{1}, h_{2}, Z]\) and \(a_1 \rho \pi \pi [a_{2}, Z]\) are poorly known, we have not taken any numerical values for these decay widths to fit our parameters - these decay widths will be calculated as a consistency check on the basis of the results obtained from the other decay widths and the scattering lengths.

Here, we will present formulas that have been used to fit the parameters; for all other formulas, see Ref. \([9]\).

**Decay width for \(\rho \pi \pi\).** The decay width reads

\[
\Gamma_{\rho \pi \pi} = \frac{m_\rho^5}{48 \pi m_{a_1}^3} \frac{1}{2} \frac{2 m_\pi}{m_\rho} 2^\frac{-3}{2} \frac{\lambda_0}{h_1} g_1 \frac{g_2}{2} \frac{Z^2}{2} + \frac{g_2}{2} \frac{i_2}{2};
\]

The experimental value is \((149.4 \pm 1.0)\) MeV \([10]\).

**Decay width for \(f_1 a_{0} \pi\).** The following formula for the decay width is obtained:

\[
\Gamma_{f_1 a_{0} \pi} = \frac{g_2^2 Z^2}{2 \pi} \frac{k^3}{m_{f_1}^2 m_{a_1}^2} \frac{m_\rho^2}{2} \frac{1}{2} (h_2 + h_3) \phi^2; \quad j = \frac{1}{2} \frac{m_{f_1}^2}{m_{f_1}^2} \frac{2 (m_{a_0}^2 + m_{a_1}^2)}{m_{a_1}^2} + \frac{(m_{a_0}^2 + m_{a_1}^2)^2}{m_{f_1}^2}
\]

where \(\phi\) \(Z f_\pi\) is the vacuum expectation value of the \(\sigma\) field.

**Decay width for \(a_{1} \pi \gamma\).** The Lagrangian leading to the formula for the decay width \(\Gamma_{a_{1} \pi \gamma}\) is obtained from the Lagrangian \((2.1)\) by coupling the photon to the relevant part of the axial
current $J_\mu = ig_1 Z f_\pi (a_{1\mu}^+ \pi - a_{1\mu} \pi^+) \cdot g_\pi (a_{1\mu}^+ \partial^\nu \pi - a_{1\mu} \partial^\nu \pi^+)$, where $w = g_1 \phi / m_{a_1}^2$, and reads

$\mathcal{L}_{a_1 \pi \gamma} = e J_\mu A^\mu$.

The decay width reads

$$\Gamma_{a_1 \pi \gamma} = \frac{e^2}{96\pi} (Z^2 \Phi m_{a_1}^2 \frac{m_\pi}{m_{a_1}} 1) m_{a_1} 1 \frac{m_\pi}{m_{a_1}} 2 \phi^2$$

Note that the sole dependence of the $a_1 \pi \gamma$ decay width on the parameter $Z$ may in principle lead to an accurate determination of this parameter. However, the experimental value of the $a_1 \pi \gamma$ decay width is not very precise ($\Gamma_{a_1 \pi \gamma} = 0.640 \pm 0.246$ MeV [10]) and thus we have used the $\chi^2$ method to determine all the parameters from the decay widths and scattering lengths.

Decay amplitude for $a_0 \eta \pi$. The mass of the $\eta$ meson can be calculated using the well-known mixing of strange and non-strange contributions in the physical fields $\eta$ and $\eta^0$ (958) yielding $\eta = \eta_S \cos \phi + \eta_S \sin \phi$; $\eta^0 = \eta_S \sin \phi + \eta_S \cos \phi$, where $\eta_S$ denotes a pure $\bar{s}s$ state and $\phi \approx 36 \frac{1}{10}$ [10]. Then we obtain $m_{\eta} = 716$ MeV.

Note that we have used the decay amplitude for the $a_0 \eta \pi$ decay instead of the decay width as quoted by the PDG [10] in order to fit the parameters of the model. The experimental value of the decay amplitude is known from the Crystal Barrel data: $A_{a_0 \eta \pi} = (3330 \pm 150)$ MeV [12] which for our purposes has to be divided by $\cos \phi$; the formula for the decay amplitude obtained from Eq. (2.1) is

$$A_{a_0 \eta \pi} = \frac{g_{1\pi}^2 \Phi}{m_{a_1}^2} + \frac{\phi^2 m_{a_1}^2}{m_{a_1}^2} \cdot \frac{1}{2} Z^2 \phi^2 (h_2 + h_3) \cdot (m_{a_1}^0 m_{a_1}^1 m_{a_1}^2 + Z^2 m_{a_1}^0)$$

Scattering length $a_0^0$. The formula for $a_0^0$ is calculated using the partial wave decomposition [13] which leads to

$$a_0^0 = \frac{1}{4\pi} \frac{m_{a_1}^2}{m_{a_1}^2} Z^2 \frac{m_{a_1}^2}{m_{a_1}^2} m_{a_1}^2 + \frac{\phi^2}{m_{a_1}^2} \cdot \frac{1}{2} Z^2 \phi^2 (h_2 + h_3) \cdot (14 h_3) \cdot \frac{5}{8} \frac{Z^2 m_{a_1}^2}{m_{a_1}^2} \cdot \frac{m_{a_1}^2}{m_{a_1}^2}$$

$$= \frac{3}{2} g_{1\pi}^2 \frac{Z^2 \phi m_{a_1}^2}{m_{a_1}^2} 1 + \frac{m_{a_1}^2}{2} \cdot \phi \cdot \frac{1}{2} \frac{Z^2 m_{a_1}^2}{m_{a_1}^2} \cdot \frac{m_{a_1}^2}{m_{a_1}^2} \cdot \frac{1}{2} \frac{Z^2 m_{a_1}^2}{m_{a_1}^2}$$

We are using the result $a_0^0 = 0.233 \pm 0.023$ (normalised to the pion mass) in accordance with data published by the NA48/2 collaboration [14].

Scattering length $a_0^2$. An analogous calculation as in the case of the scattering length $a_0^0$ leads to

$$a_0^2 = \frac{1}{4\pi} \frac{Z^2 m_{a_1}^2}{m_{a_1}^2} m_{a_1}^2 + \frac{g_{1\pi}^2 Z^2 m_{a_1}^2}{m_{a_1}^2} m_{a_1}^2 \cdot \frac{\phi^2}{2} (h_1 + h_2 + h_3)$$
3. Results

In order to fit the relevant parameters of our model \((Z, g_2, h_1, m_\sigma)\) to experimental data (for the aforementioned decay widths, \(a_0\) \(\eta\pi\) decay amplitude and scattering lengths) we have used the \(\chi^2\) method. The error for the mixing angle \(\varphi\) is neglected in this first case study.

Our best fit yields the minimal value of \(\chi^2_{\text{min}} = 0.752516\) per degree of freedom which leads to the following values of parameters: \(Z = 1.5217, g_1 = 6.59, g_2 = 0.3365, h_1 = 100.7, h_2 = 106.045, h_3 = 2.63, m_\sigma = 330\) MeV.

It is interesting to note that, although new parameters have been introduced in the globally invariant model, the values of \(Z = 1.5217, g_1 = 6.59, \) and \(m_\sigma = 330\) MeV are virtually the same as those obtained in the locally invariant model where the corresponding values were \(Z = 1.586, g_1 = 6.51, \) and \(m_\sigma = 315\) MeV [3]. Note also that the value of \(h_1\) does not appear to be large-\(N_C\) suppressed, although the parameter \(h_1=2\) is the prefactor to a term consisting of a product of two traces \(\text{Tr}(\Phi^\dagger \Phi \text{Tr}[L^\mu L^\mu] + \langle R^\mu R^\mu \rangle)\) - in fact, the modulus of the corresponding prefactor \(h_1=2\) is by about a factor of ten larger than the prefactor to the term \(\text{Tr}(\Phi R_\mu \Phi^\dagger L^\mu)\) (i.e., \(2h_3 = 5.26\)).

Using the parameters above leads to the following consequences: (i) \(\Gamma_{a_1!} \sigma_{\pi} = 90.163\) MeV; (ii) given that in the globally invariant model the \(\rho\) mass term is \(m_\rho^2 = m_1^2 + \frac{1}{2} (h_1 + h_2 + h_3) = 2\), it is possible to calculate the contribution of the bare mass \(m_1^2\) to the total mass \(m_\rho^2\) - the result \(m_1 = 758\) MeV is obtained, leading to a very small contribution of the quark condensate to the \(\rho\) mass; (iii) the \(\sigma \pi\pi\) decay width has a value of less than \(10\) MeV - it is thus too small - and the \(a_1\) \(\rho\pi\) decay width has the value of \(1.4\) GeV - it is thus too large.

Hence, in the light of our results we conclude that the \(\bar{q}q\) assignment of the light scalar mesons leads to contradictions to experiment. For a definite conclusion, the errors of the parameters in the model should be evaluated (see Ref. [3]), but it is already clear from our current results that the assignment of \(f_0(600)\) and \(a_0(980)\) as \(\bar{q}q\) states may be problematic.

A possible way to resolve the aforementioned problem is to redefine \(\sigma\) and \(a_0\) mesons in the model as \(f_0(1370)\) and \(a_0(1450)\), respectively, and hence assign the scalar meson states to the energy region above \(1\) GeV [3]. Then, the mixing of quarkonia and tetraquark states [10] needs to be examined.

4. Conclusions and Outlook

A globally invariant linear sigma model with vector and axial vector mesons and its consequences for low-energy meson decay channels and pion-pion scattering lengths have been presented. Results obtained in the assignment in which scalar mesons are identified as states under \(1\) GeV indicate contradictions to experimental data, hence raising questions about the justification of the mentioned assignment. Thus, a detailed study of the other possible assignment for scalar mesons (in which those states are located in the energy region above \(1\) GeV) is necessary. In the
future, other relevant issues in connection with vacuum phenomenology will be addressed such as the inclusion of the nucleon field together with its chiral partner [15] as well as extending the work of Refs. [9, 16] to consider chiral symmetry restoration at nonzero temperature.

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