Dark matter pair-production in $b \rightarrow s$ transitions.

C. Bird∗, R. Kowalewski†, M. Pospelov‡
Department of Physics and Astronomy, University of Victoria, Victoria, BC, V8P 1A1, Canada

Received (Day Month Year)
Revised (Day Month Year)

The flavour-changing neutral current transition $b \rightarrow s$ can serve as a sensitive probe of WIMP dark matter models, if the WIMP mass is under 2 GeV. In this work we extend our earlier analysis to a generic class of models where the interaction between the dark matter sector and the Standard Model matter sector is mediated by the Higgs boson(s). We show that experimental limits on the decays of $B$-mesons to $K(K^*)$ and missing energy provide stringent constraints on the parameter space of such models, but do not rule out sub-GeV WIMPs in a model-independent way. We find that in the context of the NMSSM with light pseudoscalar Higgs, the WIMP masses under a few hundred MeV are generically excluded with the exception of few highly tuned points in the parameter space.

Keywords: dark matter; $B$-decays; cosmology.

PACS Nos.: include PACS Nos.

1. Introduction

The dark matter content of the total energy density in the Universe is now well-measured, with several experiments determining an abundance of $\Omega_{DM} h^2 \sim 0.12$. So far the presence of dark matter was deduced only through its gravitational interaction, and thus its identity remains a mystery.

Among numerous candidates for dark matter suggested in the literature, the weakly interacting massive particles (WIMPs) are the most interesting from the point of view of particle physics. The residual abundance of such particles in the early Universe is well understood, with the observed dark matter abundance providing a measure of the WIMP couplings through their annihilation cross section at freeze-out (for a review, see Ref. 2). Furthermore the inferred couplings of WIMPs to the Standard Model fields are sufficiently large that detection of WIMP pair-production via a missing energy signal is possible in current particle physics experiments.

∗cbird@uvic.ca
†kowalews@uvic.ca
‡pospelov@uvic.ca
The mass of the WIMP can be constrained from above and below using certain theoretical arguments. The assumptions that the mass scale of the particles that mediate interactions between Standard Model and WIMPs is not lighter than the weak scale, and couplings are perturbative produce the WIMP annihilation cross sections that vanish both in the limits of very large and very small WIMP masses. Comparison with \( \sigma_{\text{ann}} v_{\text{rel}} \sim 0.1 \text{ pb} \), which is required to reduce the dark matter abundance to an acceptable level during freeze-out, produces the allowed mass range, referred to as the Lee-Weinberg window \(^{15,16}\). Obviously, such a window is very model dependent. In many popular models this limit requires the WIMP mass to be larger than a few GeV. However it is possible to construct simple models in which the required masses can be as low as 100 MeV (e.g. Ref. \(^{18,19,20}\)). The Lee-Weinberg limit is usually quoted for fermions, in which case the annihilation cross-section is suppressed by a factor of \( m_{\text{DM}}^2 / M^2 \), where \( M \) is the mass of the mediator particle, which requires large couplings for light WIMPs. However if the cross section is enhanced, or if scalars are used instead of fermions, the lower limit on WIMP masses can be significantly reduced.

Light dark matter models have several interesting features. The dedicated dark matter searches, such as DAMA and CDMS, rely on measuring the recoil of nuclei from WIMP scattering and as such are less sensitive to WIMPs which are significantly lighter than the nuclei used. Light WIMPs produce small recoil energy, which is usually well under the background that peaks at small \( E \). As indicated in Figure 1 this insensitivity to sub-GeV WIMPs will not be solved with the next generation of underground experiments, although the CRESST experiment is expected to provide an improved probe of dark matter as light as \( O(1 \text{ GeV}) \) due to its low recoil threshold. Interestingly enough, there is a possibility that a light WIMP could explain the positive signal observed in the annual modulation at DAMA without violating the bounds from other experiments \(^3\).

It has also been suggested that the annihilation of light WIMPs could explain the galactic positron excess \(^{21}\). The positron excess has been measured through the flux of 511 KeV \( \gamma \)-rays measured by the INTEGRAL/SPI experiment \(^{22,23}\), and seems to require non-standard sources of positrons. Although there are other possible astrophysical explanations (see for example Ref. \(^{24,25,26}\)), positron production by annihilations \(^3\) or decays of light dark matter particles \(^5\) remains as one of the more attractive solutions due to its natural explanation of the nonlocalized nature of the observed \( \gamma \)-rays. It has also been demonstrated that the annihilation of light dark matter does not conflict with measurements of either the galactic \( \gamma \)-ray flux \(^{27,28,29}\) or the extragalactic \( \gamma \)-ray background \(^{30}\), and may also provide an explanation for the observed flux of \( \gamma \)-rays in the 1-20 MeV range \(^{31}\).

As we recently demonstrated, an intriguing possibility of a sub-GeV WIMPs is the opportunity of detecting a pair of WIMPs as a missing energy signal in \( B \) meson decays \(^{17}\). Indeed, as dedicated underground searches for dark matter remain largely insensitive to sub-GeV WIMPs, their production is possible in experiments such as BABAR and BELLE, which produce large numbers of \( B \)-mesons, and can
study their rare decay modes. As a result such facilities provide a new opportunity to search for light dark matter. For the minimal scalar WIMP model these experiments have already excluded most of the parameter space with $m_S \lesssim 1$ GeV, while future data from $B$ factories will be able to probe as high as $m_S \sim 2$ GeV.

The purpose of this paper is to explore the question of how generic the limits on light WIMPs derived in Ref. 17 are, and whether all dark matter models with sub-GeV WIMPs can be efficiently constrained by $B$-physics. To answer these questions we study the class of models where the interaction between Standard Model sector and WIMPs is mediated by one or more Higgs particles. We demonstrate that $b \rightarrow s$ decays with missing energy provide important constraints on the parameter space of such models. We also point out the possibility, based on the two-Higgs doublet model (2HDM) at large $\tan \beta$, that these constraints can be circumvented.

In Section 2 we review our previous results on the minimal scalar model and extend the result for more general scalar models with an additional singlet scalar that mixes with the Higgs boson. In Section 3 we apply the same techniques to a related model with two Higgs doublets and calculate the branching ratios of WIMP-producing decays of $B$-mesons. This model has the additional benefit of relaxing the fine tuning condition required for a sub-GeV scalar WIMP in the minimal model. In Section 4 we introduce some simple models of fermionic dark matter, calculate the WIMP production in $B$-decays, and discuss the limitations on such models from the Lee-Weinberg limit. We also address the case of NMMSM (next-to-minimal
supersymmetric Standard Model) where we show that WIMPs under 200 MeV are generically excluded except special points in the parameter space with $b \to s$ transitions being fine-tuned well below its natural scale. In Section 6, we review the experimental bounds on the decay $B \to K + missing energy$, and discuss the potential for future searches.

Before we present our calculations, we would like to make several important comments. In our discussions of the Lee-Weinberg limit, it should be noted that there is no clear division between the perturbative and non-perturbative regions of parameter space and therefore no definitive lower bound on the WIMP mass. However in this paper we will require the WIMPs to be sufficiently heavy that the abundance constraints satisfy $\kappa^2 \lesssim 4\pi$, with $\kappa$ representing an effective coupling in each model.

As it is done in Ref. [17], we treat the significant hadronic uncertainties in the annihilation cross section using the most "optimistic" and "pessimistic" scenarios. That results in the certain range for predicted WIMP-producing branching ratios. In most of the models we present, the largest uncertainty exists for a WIMP with mass $m_{DM} \sim 500$ MeV that could annihilate through the $f_0$ resonance potentially enhancing the cross section and resulting in weaker couplings. However, it is unclear whether such resonances exist at the freeze-out temperature of $T \sim O(50$ MeV). The heavier resonances are not expected to contribute to the annihilation cross section as the higher freeze-out temperatures will significantly reduce the strength of the resonances. We also will assume that WIMPs heavier than $\sim 1$ GeV will freeze-out before the transition from quarks to hadrons.

We will also limit our discussion to the decay channel $B \to K + missing energy$. While it is possible to produce WIMPs in other decays, such as $B \to missing energy$ or $B \to \gamma + missing energy$, the experimental limits on such decays are inferior to $B \to K + missing energy$. Heavier WIMPs could be produced in $\Upsilon \to \gamma + missing energy$, however a SM decay of $\Upsilon$ is due to strong interactions, which greatly reduces the branching ratio of WIMP production compared to weak $B$-meson decays.

Finally, it should be noted that the collection of models presented in this paper is not exhaustive. There are other possibilities which could result in light dark matter that generically require relatively light particle mediating the interaction between the dark matter sector and the SM sector. In this paper, we will briefly touch a possibility of a lighter pseudoscalar Higgs particle discussed in Ref. [21] and omit the models with an extra $U(1)$ gauge group that have an additional $Z'$ in the sub-GeV range. The latter possibility is interesting, and should such $Z'$ have axial-vector couplings to fermions, one expects large flavour-changing $b - s - Z'$ amplitudes induced by radiative corrections. However, should this new force couple to a pure vector current (with e.g. $B - L$ quantum numbers), the constraints from flavour-changing decays will be greatly relaxed. The detailed analysis of models with light WIMPs supplemented by light $Z'$ goes outside the scope of the present paper.
Dark matter pair-production in $b \to s$ transitions

Fig. 2. Feynman diagrams which contribute to B-decay with missing energy in the minimal scalar model of dark matter.

2. Minimal Scalar Models

The simplest WIMP model is a singlet scalar which interacts with the Standard Model through exchange of the Higgs:

$$
-L_S = \frac{\lambda S^4}{4} + \frac{m_0^2}{2} S^2 + \lambda S^2 H^\dagger H
$$

where $H$ is the SM Higgs field doublet, $v_{EW} = 246$ GeV is the Higgs vacuum expectation value (vev) and $h$ is the corresponding physical Higgs, $H \equiv (0, (v_{EW} + h)/\sqrt{2})$.

The physical mass of the scalar $S$ receives contributions from two terms, $m_S^2 = m_0^2 + \lambda v_{EW}^2$, and requires significant fine-tuning to provide a sub-GeV mass. In this section we will calculate the branching ratio for the pair production of scalars in the decay $B \to K + SS$, which contributes to $\text{Br}(B^+ \to K^+ + \text{missing energy})$. Being minimal, this model obviously possesses maximum predictivity, and the branching ratio of WIMP production can be calculated as a function of dark matter mass only.

It should be noted that the decay $B \to K + \text{missing energy}$ is actually expected to occur regardless of the existence or nature of light dark matter. As shown in Figure 2a and 2b, the Standard Model predicts the transition $b \to s + \bar{\nu}\nu$ at one loop, so that the $B$-meson can decay to neutrinos with $\text{Br}(B^+ \to K^+ + \text{missing energy}) \simeq (4 \pm 1) \times 10^{-6}$. However as demonstrated before, the decay $B \to K + SS$ (resulting from the $b \to s$ transition shown in Figure 2c) can enhance the missing energy signal by up to two orders of magnitude.

The transition $b \to s + h$ occurs as a loop process, which at low momentum transfer can be calculated by differentiation of the $b \to s$ self-energy operator with respect to $v_{EW}$,

$$
L_{bsh} = \left(\frac{3g_{W}m_{b}m_{t}^{2}V_{ts}^{*}V_{tb}}{64\pi^{2}M_{W}^{2}v_{EW}}\right)S_{L}b_{R}h + (h.c.).
$$

As the Higgs is significantly heavier than the other particles involved in the process, it can be integrated out leaving an effective Lagrangian for the $b \to s$ transitions
with missing energy:

\[ \mathcal{L}_{b \to s E} = \frac{1}{2} C_{DM} m_b \bar{s} L b R S^2 - C_\nu \bar{s} L \gamma_\mu b L \bar{\nu} \gamma_\mu \nu + (h.c.). \]  

(3)

Leading order Wilson coefficients for the transitions with dark matter scalars or neutrinos in the final state are given by

\[ C_{DM} = \frac{\lambda m^2_h}{16\pi^2} \frac{3g_W^2 V_{ts}^* V_{tb}}{x_t} \] 

(4)

\[ C_\nu = \frac{g_W^2}{M_W^2} \frac{g_Y V_{ts}^* V_{tb}}{16\pi^2} \left[ \frac{x_t^2 + 2x_t}{8(x_t - 1)} + \frac{3x_t^2 - 6x_t}{8(x_t - 1)^2} \ln x_t \right], \]

where \( x_t = m^2_t / M^2_W \). From this effective theory and the hadronic form factors calculated in the light-cone sum rules \(^{25,26}\), the partial width for decays with missing energy can be calculated

\[ \text{Br}_{B^+ \to K^+ + E} = \text{Br}_{B^+ \to K^+ \nu \bar{\nu}} + \text{Br}_{B^+ \to K^+ SS} \]

\[ \simeq 4 \times 10^{-6} + 2.8 \times 10^{-4} \kappa^2 F(m_S). \]  

(5)

where we use the parametrization

\[ \kappa^2 \equiv \lambda^2 \left( \frac{100 \text{ GeV}}{m_h} \right)^4, \]  

(6)

and the available phase space as a function of the unknown \( m_S \),

\[ F(m_S) = \int_{\hat{s}_{\text{min}}}^{\hat{s}_{\text{max}}} f_0(\hat{s})^2 I(\hat{s}, m_S) \, d\hat{s} \left[ \int_{\hat{s}_{\text{min}}}^{\hat{s}_{\text{max}}} f_0(\hat{s})^2 I(0, \hat{s}) \, d\hat{s} \right]^{-1}, \]

with

\[ I(\hat{s}, m_S) = [\hat{s}^2 - 2\hat{s}(M^2_B + M^2_K) + (M^2_B - M^2_K)^2]^{\frac{1}{2}} [1 - 4m^2_S / \hat{s}]^{\frac{1}{2}}. \]

For comparison, BABAR recently reported\(^{35}\) a limit of \( \text{Br}(B^+ \to K^+ + \text{missing energy}) < 5.2 \times 10^{-5} \) at 90% c.l. which is well above the SM prediction. Belle’s preliminary results were reported at \( \text{Br}(B^+ \to K^+ + \text{missing energy}) < 3.5 \times 10^{-5} \) level\(^{36}\). Similar calculations can be used for the decay \( B \to K^* SS \),

\[ \text{Br}_{B^+ \to K^* SS} \simeq 1.3 \times 10^{-5} + 3.0 \times 10^{-4} \kappa^2 F(m_S). \]  

(7)

with an analogous form factor.

Even before we perform the freeze-out abundance calculation and extract \( \kappa \), it is obvious that the decay into WIMPs would dominate the Standard Model rate to neutrinos by one-to-two orders of magnitude near the Lee-Weinberg bound of
(a) Constraints on $\kappa$ from dark matter abundance. The parameter space above A produces a density too small to explain dark matter, while the space below B would lead to a higher than observed dark matter abundance. (b) Predicted branching ratios for the decay $B^+ \rightarrow K^+ + \text{missing energy}$, with current limits from CLEO (I) $^{37}$, BaBar (II) $^{35}$, Belle (III) $^{36}$ and expected results from BaBar (IV). Parameter space above curves I, II and III is excluded. The grey bar shows the expected $B \rightarrow K\nu\bar{\nu}$ signal. Parameter space to the left of the vertical dashed line can also be probed with $K^+ \rightarrow \pi^+ + \text{missing energy}$. 

$Dark matter pair-production in b \rightarrow s transitions$
κ ∼ O(1). To make this statement more precise, we extract the parameter κ using the relation of the WIMP annihilation cross section,

\[ \sigma_{\text{ann}} v_{\text{rel}} = \frac{8v_{\text{rel}}^2\lambda^2}{m_h^4} \times \lim_{m_h \to 2m_S} \left( \frac{\Gamma_{h \to X}}{m_h} \right) \]  

(8)

with the observed dark matter abundance \( \Omega_{DM} h^2 \sim 0.12 \). Here \( \Gamma_{h \to X} \) is the total decay width for a virtual Higgs of mass \( m_h \sim 2m_S \), and in our calculation we utilize the zero temperature decay widths which were studied previously in early searches for a light Higgs \[27,28,29\] as explained in our previous paper \[17\]. The allowed range of \( \kappa \) is plotted in Figure 2a, where the main uncertainty comes from \( \Gamma_{h \to \text{hadrons}} \). It should be kept in mind that the domain of \( \kappa \)'s below curve B gives an over-production of dark matter and therefore is firmly excluded. Inserting \( \kappa - m_S \) domain allowed by the relic abundance into the result for the branching ratio gives the prediction of \( Br(B^+ \to K^+ + \text{missing energy}) \) as a function of \( m_S \), Figure 2b.

These results can be easily generalized to models of scalar dark matter coupled to the SM via an additional singlet Higgs particle \( U \). The simplest model of this type has the potential

\[ -L_S = \frac{\lambda_S}{4} S^4 + \frac{m_u^2}{2} S^2 + (\mu_1 U + \mu_2 U^2) S^2 + V(U) + \eta U^2 H^\dagger H \]

(9)

where in the second line we retained only mass terms and relevant interaction terms. \( u \) denotes the excitation around the vev of \( U \) and \( \mu \) and \( \eta \) stand for dimensionful parameters presumably of order the electroweak scale. The last term in the second line of (9) gives the mixing between scalars \( u \) and \( h \). If such mixing is significant, the existing bounds on the higgs mass would also place a lower bound on the mass of the \( u \)-boson. The effective Lagrangian for \( b \to s + \bar{e} \) transitions is the same as Eq. (8) with the Wilson coefficient given by

\[ C_{DM} = \frac{\mu \eta}{m_u^2 m_h^2} \frac{3g_W^2 V_{ts}^* V_{tb}}{32\pi^2} x_t \]

(10)

which, with the redefinition

\[ \kappa^2 \equiv \frac{\mu^2 \eta^2}{m_u^4} \left( \frac{100 \text{ GeV}}{m_h} \right)^4 \]

results in the same abundance constraints and the same branching ratio for \( B \to K + \text{missing energy} \), as plotted in Figure 2. Thus, we see that the model with more parameters in the singlet sector \[9\] in the limit of light WIMPs gives identical
predictions to the minimal model \( (1) \), and in this sense the predictions of Figure \( (2) \) are generic.

3. Scalar dark matter in 2HDM Model

The singlet scalar model of WIMPs coupled to the SM is the most economical model of dark matter, and for sub-GeV WIMPs can be well constrained with existing B-factory experiments. However, the light masses of WIMPs in this model are not natural, as it requires significant fine-tuned cancelation between \( m_0^2 \) and \( \lambda v_E^2 \). However, if the electroweak sector of the SM is modified, this fine-tuning can be significantly relaxed.

In this section, we consider a singlet scalar WIMP which interacts with two Higgs doublets, \( H_u \) and \( H_d \).

\[
-\mathcal{L} = \frac{m_0^2}{2} S^2 + \lambda_1 S^2 (|H_d^0|^2 + |H_d^+|^2) + \lambda_2 S^2 (|H_u^0|^2 + |H_u^+|^2) + \lambda_3 S^2 (H_d^- H_u^+ - H_d^+ H_u^-) 
\]

(11)

We assume the most conservative flavour arrangement in which the up- and down-types of quarks originated from the expectation values of different Higgses, exactly as it happens in supersymmetric models (see e.g. Ref. 30). This model differs from the SM in a significant way: due to two different vev’s of the two Higgs doublets, there is an additional parameter \( \tan \beta \equiv v_u/v_d \), which is large when Yukawa couplings in the down-sector are enhanced. Indeed, there are many theories which attempt to unify the Yukawa couplings of the third generation of the Standard Model (eg see Ref. 31, 32, 33). Such unification typically requires \( \tan \beta \approx m_t/m_b \) or \( \tan \beta \approx m_t/m_\tau \) and thus \( v_d \sim O(\text{few GeV}) \). Therefore in the regime of large \( \tan \beta \) the mass corrections which depend only on \( v_d \) will in general be small and may lead to minimal fine tuning.

For example, the physical mass of the WIMP scalar in the model given above is

\[
m_S^2 = m_0^2 + \lambda_1 v_d^2 + \lambda_2 v_u^2 - \lambda_3 v_u v_d.
\]

If \( \tan \beta \sim O(1) \), sub-GeV WIMPs would require the same amount of fine tuning as before. However in the special case of large \( \tan \beta \) and the hierarchy of \( \lambda \)'s, \( \lambda_1 \gg \lambda_2, \lambda_3 \), the mass correction depends only on \( v_d \), resulting in \( \frac{\delta m_S^2}{m_S^2} \sim O(1 - 10) \) for a GeV-scale WIMP mass, which then does not require significant fine tuning. In the case of \( \lambda_3 \gg \lambda_1, \lambda_2 \), the fine-tuning is also relaxed, especially if the abundance constraint allows for \( \lambda_3 \sim O(\tan^{-1} \beta) \). In the rest of this section, we analyze the WIMP production in the model, assuming large \( \tan \beta \).

3.1. \( \lambda_1 \) dominant

The additional diagrams which contribute to the decay \( B \rightarrow K + SS \) and give the leading-order contributions in \( \tan \beta \) are given in Figure 3. The effective Lagrangian

\[
-\mathcal{L} = \frac{m_0^2}{2} S^2 + \lambda_1 S^2 (|H_d^0|^2 + |H_d^+|^2) + \lambda_2 S^2 (|H_u^0|^2 + |H_u^+|^2) + \lambda_3 S^2 (H_d^- H_u^+ - H_d^+ H_u^-) 
\]

(11)

We assume the most conservative flavour arrangement in which the up- and down-types of quarks originated from the expectation values of different Higgses, exactly as it happens in supersymmetric models (see e.g. Ref. 30). This model differs from the SM in a significant way: due to two different vev’s of the two Higgs doublets, there is an additional parameter \( \tan \beta \equiv v_u/v_d \), which is large when Yukawa couplings in the down-sector are enhanced. Indeed, there are many theories which attempt to unify the Yukawa couplings of the third generation of the Standard Model (eg see Ref. 31, 32, 33). Such unification typically requires \( \tan \beta \approx m_t/m_b \) or \( \tan \beta \approx m_t/m_\tau \) and thus \( v_d \sim O(\text{few GeV}) \). Therefore in the regime of large \( \tan \beta \) the mass corrections which depend only on \( v_d \) will in general be small and may lead to minimal fine tuning.

For example, the physical mass of the WIMP scalar in the model given above is

\[
m_S^2 = m_0^2 + \lambda_1 v_d^2 + \lambda_2 v_u^2 - \lambda_3 v_u v_d.
\]

If \( \tan \beta \sim O(1) \), sub-GeV WIMPs would require the same amount of fine tuning as before. However in the special case of large \( \tan \beta \) and the hierarchy of \( \lambda \)'s, \( \lambda_1 \gg \lambda_2, \lambda_3 \), the mass correction depends only on \( v_d \), resulting in \( \frac{\delta m_S^2}{m_S^2} \sim O(1 - 10) \) for a GeV-scale WIMP mass, which then does not require significant fine tuning. In the case of \( \lambda_3 \gg \lambda_1, \lambda_2 \), the fine-tuning is also relaxed, especially if the abundance constraint allows for \( \lambda_3 \sim O(\tan^{-1} \beta) \). In the rest of this section, we analyze the WIMP production in the model, assuming large \( \tan \beta \).

3.1. \( \lambda_1 \) dominant

The additional diagrams which contribute to the decay \( B \rightarrow K + SS \) and give the leading-order contributions in \( \tan \beta \) are given in Figure 3. The effective Lagrangian
Fig. 3. Diagrams contributing to the decay $b \to s + SS$ in the 2HDM plus scalar dark matter model when $\lambda_1$ is dominant and $\tan \beta$ is large. Inside the loops, $H_u$ and $H_d$ denote the two charged Higgs bosons, with the mixing of the two doublets denoted by a cross.

is of the same form as Eq 3 and in the limit of large $\tan \beta$ and $M^2_{H_d} \gg M^2_W$ the Wilson coefficient is

$$C_{DM} = \frac{\lambda_1 g_W^2 V_{tb}^* V_{ts}}{32\pi^2} \left( \frac{1 - a_t + a_t \ln a_t}{(1 - a_t)^2} \right)$$

where $a_t = m_t^2/M^2_H$. The branching ratio is calculated as before,

$$\text{Br}_{B \to K + E} = 4.0 \times 10^{-6} + 3.2 \times 10^{-5} \kappa^2 \left( \frac{1 - a_t + a_t \ln a_t}{(1 - a_t)^2} \right)^2 F(m_S)$$

Here we use the parameterization,

$$\kappa = \lambda_1 \left( \frac{100 \, GeV}{M_H} \right)^2$$

and unlike the minimal case there is an additional rather mild dependence on $m_H$ through $a_t$. It disappears in the limit $m_H \gg m_t$. We also make a safe assumption that additional diagrams with charged Higgses are not going to alter the branching to neutrinos. As before, $F(m_S)$ is constructed such that

$$F(m_S) = \int_{s_{min}}^{s_{max}} f_0(s)^2 I(s, m_S) ds \left[ \int_{s_{min}}^{s_{max}} f_0(s)^2 I(s, 0) ds \right]^{-1}$$

and $F(0) = 1$, and $F(m_S) = 0$ for $m_S > \frac{1}{2}(m_B - m_K)$. The branching ratio is plotted in Figure 4b with the current experimental bounds from BABAR,\(^\text{35}\), BELLE,\(^\text{36}\) and CLEO.\(^\text{37}\)

The abundance constraint on $\kappa$ is calculated as in Section 2 with one important difference. When $\tan \beta$ is large, the scalars predominantly couple to leptons and down-type quarks. Consequently, the charm and top loops do not contribute in the effective couplings to gluons, leading to the reduction in the cross-sections for $SS \to gg, \pi \pi$ by a factor of $O(1/N^2_H) \sim 0.1$, where $N_H$ is the number of heavy quark flavors that convert virtual Higgs to hadrons. $N_H = 3$ in the minimal model, and $N_H = 1$ in the 2HDM with large $\tan \beta$. The charm threshold also does not offer
additional annihilation channels. As a result, the values of $\lambda_1$ that would fit the observed abundance of dark matter are higher than the corresponding values for $\lambda$ in the minimal scalar model studied previously. The resulting $\kappa$, Figure 4a, clearly corresponds to a strong interaction regime below WIMP masses of 500 MeV.
Fig. 5. Diagrams contributing to the decay $b \rightarrow s + SS$ in the 2HDM plus scalar dark matter model when $\lambda_3$ is dominant and $\tan \beta$ is large.

3.2. $\lambda_2$ dominant

The case in which the scalars couple predominantly to $H_u$ produces results similar to the minimal model of Section 2. In the large $\tan \beta$ limit $v_u \approx v_{SM}$ and therefore the fine-tuning of the scalar mass is significant, while the branching ratio for $B \rightarrow K + SS$ and constraints on $\lambda_2$ are not expected to be significantly different from the minimal model with one Higgs doublet.

3.3. $\lambda_3$ dominant

The third possibility is that the dark matter scalars couple to the $H_uH_d$ combination of the two Higgs fields. In this case the mass of the scalar is

$$m_S^2 = m_0^2 - \frac{\lambda_3 v_{EW}^2}{\tan \beta}$$

which in the large $\tan \beta$ limit requires less fine-tuning to produce sub-GeV WIMPs than in the minimal scalar model.

We find that in this model the branching ratio of WIMP pair production in $B$-meson decays is suppressed. It turns out that two $b \rightarrow s + SS$ diagrams that have $\tan \beta$ enhancement shown in Figure 5 exactly cancel each other so that $C_{DM} \sim O(\tan^0 \beta)$.

At the same time, the abundance constraint is similar to the $\lambda_1$ dominant case, except that the scalar annihilation cross-section is enhanced by a factor of $\tan^2 \beta$, resulting in $\lambda_3$ being $\tan \beta$ times smaller than $\lambda_1$ from the previous example. A combination of the enhanced annihilation cross section and cancellation of $b \rightarrow s + SS$ amplitude at leading order in $\tan \beta$, results in $\tan^{-2} \beta \sim O(10^{-3} - 10^{-4})$ suppression of $b \rightarrow s + SS$ branching ratio relative to previous examples. Consequently,

$$Br_{B \rightarrow K + SS} \ll Br_{B \rightarrow K + \bar{\nu}\nu} \quad \text{for } \lambda_3 \text{ dominant}$$

and therefore this model cannot be constrained by $B$-decays. It is then clear that a generic case with both couplings, $\lambda_1$ and $\lambda_3$, being important can create a large range for $Br_{B \rightarrow K + SS}$ even for a fixed value of $m_S$, and therefore the model of scalar dark matter coupled to 2HDM is far less predictive than the minimal model.
Dark matter pair-production in $b \to s$ transitions

Fig. 6. The Feynman diagrams which contribute to $b \to s + \not{E}$ in the 2HDM plus fermionic WIMP model.

It also interesting to note that the elastic scattering of dark matter on nucleons is enhanced by $\tan^2 \beta$ when $\lambda_3$ is dominant, and therefore this model will indeed create a measurable signal for detectors with low recoil energy threshold such as CRESST. Detailed analysis of such constraints goes outside the scope of the present paper.

4. Fermionic Dark Matter

In the previous two sections the WIMPs were presumed to be scalar fields. This is common in models of light dark matter, as the Lee-Weinberg limit on the WIMP mass is in general lower for scalar WIMPs. However fermionic dark matter could still be light if its annihilation cross section is enhanced. In this section we will present some simple models of fermionic dark matter, and review the constraints from $B$-decays.

The simplest renormalizable model of fermionic dark matter is the analogue of the scalar model given in Eq 9,

$$-\mathcal{L}_f = \frac{m_\chi}{2} \bar{\chi} \chi + \frac{m_u^2}{2} U^2 + \mu U \bar{\chi} \chi + \eta U^2 H^\dagger H$$

$$= \frac{m_\chi}{2} \bar{\chi} \chi + \frac{m_u^2}{2} U^2 + \mu U \bar{\chi} \chi + \eta v_{ew} w U h + \lambda U U^4$$

(15)

where $\chi$ is a Majorana fermion and $H$ is the Standard Model Higgs field. As before, $w \equiv <U> \sim O(v_{ew})$. However the annihilation cross-section, at freeze out, in this model is suppressed relative to the scalar model by

$$\frac{\sigma_{\text{fermion}}}{\sigma_{\text{scalar}}} \sim \frac{m_\chi v_{rel}^2}{m_u^2} \sim O(10^{-5})$$

There is a possibility of producing light fermionic WIMPs in this model if $m_u \ll v_{ew}$, and $\eta \ll \mu < 1$, though such a model would require significant fine-tuning to avoid being detected in direct searches of light(er) Higgs.
Fig. 7. Abundance constraints on $\kappa$ in the 2HDM plus fermionic WIMP model. It is expected that most of the parameter space near curve B is also excluded, however a detailed calculation of the annihilation cross-section $\sigma_{\tilde{\chi}\tilde{\chi} \to 3\pi}$ is beyond the scope of this paper, and as such we have utilized the perturbative formula for production of gluons and unbound quarks. However except for possible resonances the decay is expected to be significantly smaller, and therefore the abundance constraints would require the coupling constant to be larger.

for $m_\chi \sim O(1 \text{ GeV})$ and $m_u \sim O(v_{ew})$. As a result of this suppression, the fermions cannot annihilate efficiently in the early Universe and therefore the couplings would have to be non-perturbative to explain dark matter.

The second model we will consider is the analogue of the model presented in Section 3. For the model to be renormalizable, the Majorana fermions cannot couple directly to the higgs fields, but must instead couple through an intermediate scalar.

\[
-L = \frac{m_\chi^2}{2} \overline{\chi} \chi + \frac{m_U^2}{2} U^2 + \mu U \overline{\chi} \chi + \lambda_1 U^2 (|H_d^0|^2 + |H_d^-|^2) \\
+ \lambda_2 U^2 (|H_u^0|^2 + |H_u^+|^2) + \lambda_3 U^2 (H_d^- H_u^+ - H_d^0 H_u^0) + \lambda_U U^4
\]  

(16)

As in the previous model, the inclusion of fermions in this model requires an increase in the coupling constants by a factor of $\sim m_u^2/(wm_\chi)$, with $w = <U>$, to produce the same abundance of dark matter. However in the special case of $\lambda_3 \gg \lambda_1, \lambda_2$ the annihilation cross section is also enhanced by a factor of $\tan^2 \beta$, and so $\chi$ could be light without requiring large $\lambda_3$. The abundance constraints are given in Figure 7 with

$$\kappa^2 \equiv 4\lambda_3^2 \mu^2 \left( \frac{v_{ew} m_u}{m_u^2} \right)^2 \left( \frac{100 \text{ GeV}}{M_H} \right)^4 \left( \frac{\tan \beta}{100} \right)^2.$$
As in the $\lambda_3$ dominant case presented in Section 3, the $O(\tan \beta)$ contributions (see Figure 6) to the process $b \to s + \chi \chi$ cancel and as a result

$$\text{Br}(B \to K + \chi \chi) \ll \text{Br}(B \to K + \nu \nu).$$

The final model we will consider is a Majorana fermion $\chi$ coupled to higgs-higgsino pairs. By "higgsinos" we mean an $SU(2) \times U(1)$ charged fermionic fields with the same quantum numbers as the Higgs fields but do not impose supersymmetry requirements on the size of the couplings. By the same token $\chi$ can be called the neutralino. The relevant part of the Lagrangian is given by

$$-\mathcal{L}_f = \frac{1}{2} M \bar{\psi} \psi + \mu \bar{H}_d H_u + \lambda_d \bar{\psi} H_d + \lambda_u \bar{\psi} H_u,$$

where $M \ll \mu, \lambda_u v_u$, and $\tan \beta$ is large. In this model the dark matter candidate is taken to be the lightest mass eigenstate,

$$\chi = -\psi \cos \theta + \bar{H}_d \sin \theta \quad \sin^2 \theta \equiv \frac{\lambda_d^2 v_u^2}{\lambda_u^2 v_u^2 + \mu^2}$$

(17)

$$m_1 = M \left(1 - \frac{\lambda_d^2 v_u^2}{\lambda_u^2 v_u^2 + \mu^2}\right)$$

It should be noted that this model is constrained by the Z-boson invisible decay width, and requires $\sin^2 \theta \lesssim 0.15$.

The $b \to s$ transition in this model is calculated from a single diagram, given in Figure 8. The effective Lagrangian is

$$\mathcal{L}_{b \to s E} = \frac{1}{2} C_{DM} m_b \bar{s}_L b_R \chi \chi - C_{\nu} \bar{s}_L \gamma_{\mu} b_L \bar{\nu}_{\mu} \nu + (h.c.)$$

(18)

with Wilson coefficient

$$C_{DM} = \frac{V_{tb}^* V_{ts} \tan \beta}{32 \pi^2 v_{\text{im}}^3} \left(\frac{\lambda_d \lambda_u v_u \mu}{\lambda_d^2 v_u^2 + \mu^2}\right) \frac{a_t}{1 - a_t}$$

(19)
Fig. 9. (a) Constraints on \( \kappa \) from the observed dark matter abundance. (b) Branching Ratios for \( B \to K + \text{ missing energy} \) in the neutralino model. Current limits from BABAR, BELLE, and CLEO are indicated as in Figure 2b. The Standard Model predictions are given in the gray box.

where \( a_t \equiv m_t^2/M_H^2 \), and the branching ratio is

\[
BR_{B \to K + \text{E}} = 4.0 \times 10^{-6} + 9.8 \times 10^{-5} \kappa^2 \left( \frac{\ln a_t}{1 - a_t} \right)^2 F(m_1), \tag{20}
\]

where
Dark matter pair-production in $b \to s$ transitions

\[ \kappa^2 \equiv \left( \frac{\lambda_d \lambda_u v_u \mu}{\lambda_d^2 v_u^2 + \mu^2} \right)^2 \left( \frac{100 \text{ GeV}}{M_H} \right)^4 \left( \frac{\tan \beta}{100} \right)^2. \]

The phase space integral for fermions is defined as

\[ F(m_\chi) = \int_{\hat{s}_{\text{min}}}^{\hat{s}_{\text{max}}} f_0(\hat{s})^2 (\hat{s} - 2m_{\chi}^2/M_B^2) I(\hat{s}, m_\chi) \, d\hat{s} \left[ \int_{\hat{s}_{\text{min}}}^{\hat{s}_{\text{max}}} f_0(\hat{s})^2 \hat{s} I(\hat{s}, 0) \, d\hat{s} \right]^{-1}. \]  

(21)

The cosmological abundance and the branching ratio are plotted in Figure 9 with the current experimental limits. Although it appears that the effects of dark matter in this model could be observed, the abundance constraints require $\kappa$ to be in the non-perturbative region for $m_\chi \lesssim 1$ GeV. Furthermore, constraints on $\lambda_u$ from $Z$-decays and on $M_{H_d}$ require the original coupling constant $\lambda_d$ to be a few orders of magnitude larger than $\kappa$ and therefore the model is non-perturbative for $m_\chi \lesssim 2$ GeV. Heavier WIMPs may still be produced in $B$-decays, though the branching ratio is significantly smaller than the Standard Model signal.

5. NMSSM with light dark matter

One of the most popular dark matter candidates is the lightest neutralino present in supersymmetric models. The existence of supersymmetry is well motivated in particle physics, and the cosmological abundance of neutralinos is comparable to the dark matter abundance for a large region of parameter space. In this section we will demonstrate that light neutralinos, if kinematically allowed, can be produced in $B$-meson decays and will provide a significant contribution to the branching ratio $\text{Br}(B \to K + \ell \ell)$.

In discussions of supersymmetric dark matter, it is common to work with the minimal supersymmetric standard model (MSSM). However in that case the lightest neutralino is expected to be heavier than a few tens of GeV and therefore too heavy to produce in $B$-meson decays. Although it is possible to lower this limit to 6 GeV, it requires extreme tuning of the parameters, while sub-GeV WIMPs seem to be impossible even with the fine tuning. However it is possible to produce sub-GeV neutralinos in extensions of MSSM by introducing new resonances significantly lighter than $M_W$.

In this section we present a case study of NMSSM with light neutralino dark matter. The lightness of $\chi$ in this model is made possible, without significant fine-tuning, by one of the pseudoscalar Higgs bosons $a$ being chosen to have a mass of a few GeV. A comprehensive analysis of dark matter in this model was attempted in Ref. 21 while the constraints from $B$-physics were considered in Ref. 10. None of these papers, however, considers the decays of $B$-mesons with missing energy. As mentioned in the previous section, the neutralino annihilation cross section in the non-perturbative QCD regime has not been adequately addressed. A proper
account of $m_\chi < 1$ GeV would require the calculation of $a \to gg \to PPP$ and $a \to q\bar{q} \to PPP$, where $P$ is a pseudoscalar meson such as $\pi$ and $K$, plus an extensive scan over the NMSSM parameter space, which we will not attempt here. Instead, we choose to analyze the domain of neutralino masses of $m_\mu < m_\chi < 1.5 m_\pi$ where the annihilation proceeds primarily via muons in the final state, and is free from hadronic uncertainties:

$$\sigma_{ann} v_{rel} \simeq \frac{1}{2\pi} \left( \frac{\lambda_{a\chi} \sin \gamma \tan \beta}{m_a^2} \right)^2 \left( \frac{m_f}{v_{EW}} \right)^2 m_\chi^2 \sqrt{1 - m_\chi^2 / m_\mu^2},$$

(22)

In this formula, the coupling constants $\lambda_{a\chi}$ and mixing angles $\gamma$ originate from the tree-level interaction Lagrangian of $a$ with the SM fermions and neutralinos,

$$- \mathcal{L}_{int} = \frac{1}{2} \lambda_{a\chi} \bar{\chi} \gamma_5 \gamma_5 \chi + \sin \gamma \tan \beta \sum m_f v_{EW} a \bar{f} i \gamma_5 f.$$  

(23)

Here the summation goes over the charged leptons and down-type quarks, $\gamma$ parametrizes the admixture of $\text{Im} H_d$ in $a$, and $\tan \beta$ is assumed to be a large parameter. Specific relations between $\gamma$, the neutralino-pseudoscalar coupling $\lambda_{a\chi}$ and the parameters of the fundamental NMSSM Lagrangian can be worked out, but are of no interest in this discussion since they bear no consequences for our predictions.

The SUSY diagrams that can play an important role in the $b \to s + \chi \chi$ decay are shown in Figure 10. These are two Higgsino-stop exchanged diagrams that give the following Wilson coefficient in the effective Lagrangian defined in (18),

$$C_{DM} = \frac{V_{ts}^* V_{tb}^* \lambda_{a\chi} \sin \gamma \tan \beta}{32\pi^2 v_{EW}} \left( \frac{\lambda_{a\chi} \sin \gamma \tan \beta}{m_a^2} \right) I_{SUSY}.$$ 

(24)

In this expression, we adhere to the minimal flavour-violation scheme of the soft-breaking sector and treat the stop mixing as a mass insertion. The loop function $I_{SUSY}$ depends on the trilinear soft-breaking parameter $A_t$, masses of left- and right-handed stops, which for simplicity can be chosen equal, $m_{tLL}^2 \simeq m_{tRR}^2 \simeq m_t^2$, and $\mu$ parameter:

$$I_{SUSY} = \frac{A_t \mu}{m_t^2} \left( \frac{1 - a_t + a_t \ln a_t}{(1 - a_t)^2} \right) \left[ 1 + \frac{\lambda_{aH}}{\tan \beta \sin \gamma} \right],$$

(25)
where \( a_t = \mu^2 / m_{\tilde{t}}^2 \). Note that in the context of NMSSM the Higgsino-pseudoscalar coupling \( \lambda a_{\tilde{H}} \) and \( \mu \)-parameter are often chosen to be related via the singlet’s vev. It is important that \( I_{SUSY} \) does not decouple even in the limit of large SUSY masses, provided that the relative size of \( A_t, \mu \) and \( m_{\tilde{t}} \) remains fixed. When all SUSY masses are equal, \( A_t = \mu = m_{\tilde{t}} \), the expression in front of the square bracket in (25) becomes 1/2, which we would consider as a natural value for \( I_{SUSY} \). Notice an additional power of \( \tan \beta \) in the expression for \( C_{DM} \) [24], a very well-known enhancement in the SUSY models [21][22][43]. By introducing an effective coupling \( \kappa \),

\[
\kappa^2 \equiv \left( \frac{\tan \beta}{10} \right)^2 \left( \frac{10 \text{ GeV}}{m_a} \right)^4 (\lambda_{a\chi} \sin \gamma)^2 ,
\]

we express the \( B \)-decay missing energy signal as

\[
\text{Br}_{B \to K^+ + E} = 4.0 \times 10^{-6} + 0.98\kappa^2 \left( \frac{\tan \beta}{10} I_{SUSY} \right)^2 F(m_\chi) \tag{27}
\]

with

\[
F(m_\chi) = \int_{\hat{s}_{\text{min}}}^{\hat{s}_{\text{max}}} f_0(\hat{s})^2 \hat{s} I(\hat{s}, m_\chi) \, d\hat{s} \left[ \int_{\hat{s}_{\text{min}}}^{\hat{s}_{\text{max}}} f_0(\hat{s})^2 \hat{s} I(\hat{s}, 0) \, d\hat{s} \right]^{-1} . \tag{28}
\]

Using the freeze-out abundance to constrain \( \kappa \),

\[
\kappa \lesssim \frac{2.4 \text{ GeV}}{m_\chi \left(1 - m_\mu^2/m_\chi^2\right)^{1/4}} \tag{29}
\]

and imposing the experimental bound of \( \text{Br}(B^+ \to K^+ + E) < 3.5 \times 10^{-5} \) on the branching ratio, we arrive at the following constraint on \( I_{SUSY} \),

\[
\left( \frac{\tan \beta}{10} I_{SUSY} \right) < 3.0 \times 10^{-4} \quad \text{for } m_\chi = 150 \text{ MeV} \tag{30}
\]

This is a requirement on \( I_{SUSY} \) to be suppressed relative to its natural value by three orders of magnitude. We conclude that unless an unnatural cancellation of \( I_{SUSY} \) occurs, the possibility of a \( \sim 100 \text{ MeV} \) neutralino in NMSSM is excluded. However, even if the SUSY contribution vanishes for some reasons, the next-to-leading order in \( \tan \beta \) contribution mediated by charged Higgs loops is still capable of producing an observable signal, which from existing bounds still excludes a \( \sim 100 \text{ MeV} \) neutralino.

6. Experiment

The decays of the type \( B^+ \to K^+ + E \) discussed here involve missing energy but provide no constraint on the associated missing mass. This is problematic experimentally, even for a hypothetical detector with full solid angle coverage, unless one of the two \( B \) mesons in the event is fully reconstructed. Similarly, it is hard to
imagine that a sensitive search can be done without the well-constrained initial state provided by experiments at the threshold for $B\bar{B}$ production. This discussion will therefore focus on experiments at the $\Upsilon(4S)$.

The $B$ reconstruction can be done using either hadronic or semileptonic decays. After the decay products of one $B$ are removed from consideration, signal events should contain only a single kaon while background events will generally contain additional charged or neutral particles. In reality, beam-related backgrounds tend to leave additional energy in the electromagnetic calorimeter and interactions of particles from the fully reconstructed $B$ with material in the detector can lead to additional, spurious charged or neutral particles. The selection criteria balance the loss of efficiency from strict vetoes on additional activity against the suppression of background. After removing particles associated with the fully reconstructed $B$, no charged tracks are allowed beyond the identified kaon, and the neutral electromagnetic energy deposition is restricted to be less than 200-300 MeV. The background is negligible for $K$ energies near $m_B/2$ and rises steadily toward lower $K$ momenta. As a result, the experiments place a requirement on the minimum accepted $K$ momentum; this is further discussed below.

The CLEO measurement reconstructs $B$ mesons in hadronic decays and requires $p_K > 0.7$ GeV, providing sensitivity to dark matter particle masses up to 2.1 GeV and placing a limit of $\text{Br}(B^+ \to K^+\nu\bar{\nu}) < 24 \times 10^{-5}$ for the SM decay. The BBar and BELLE measurements each require $p_K > 1.2$ GeV, thereby restricting sensitivity to dark matter particle masses below 1.9 GeV. BBar reconstructs $B$ mesons in both hadronic and semileptonic modes with comparable sensitivity in each, and determines $\text{Br}(B^+ \to K^+\nu\bar{\nu}) < 5.2 \times 10^{-5}$ based on a sample of $88 \times 10^6 \Upsilon(4S) \to B\bar{B}$ decays. BELLE uses only events with one $B$ reconstructed in a hadronic decay mode and find the preliminary result $\text{Br}(B^+ \to K^+\nu\bar{\nu}) < 3.5 \times 10^{-5}$ based on a sample of $275 \times 10^6 \Upsilon(4S) \to B\bar{B}$ decays.

The expected background from normal $B$ decays with two or more missing particles is several times higher than the signal expected from the SM process with neutrino pair. This background has contributions from semileptonic decays (e.g. $\bar{B} \to D(\pi\nu)$ with $D \to K^+\ell^-\nu$ or $D \to K^+X$) where the the detectable particles other than the $K^+$ lie outside the detector acceptance, as well as from hadronic $B$ decays involving one or more $K^0_L$, which have a non-negligible probability of traversing the electromagnetic calorimeter without depositing significant energy. The latter decays in some cases still have poorly known branching ratios, but these can be measured (or more stringently bounded) with larger $B$-factory data sets. The sensitivity can be expected to improve as $1/\sqrt{N}$ as more data are analyzed.

The decay $B^0 \to K^0 + \bar{\nu}E$ doesn’t add much sensitivity due to the smaller reconstruction efficiency for both the $B^0$ (about 1/2 that of $B^+$) and the $K^0$ (only 1/3 decay to $\pi^+\pi^-$). The decays $B \to K^+ + \bar{\nu}E$ have sensitivity comparable to $B^+ \to K^+ + \bar{\nu}E$ (the branching ratios for both the SM and New Physics transitions are higher due to the higher probability of producing a $K^*$ in the fragmentation of the $s$ quark, offsetting the lower experimental efficiency). The $B \to K^* + \bar{\nu}E$ channel
should be sensitive to dark matter masses only slightly smaller than those explored in the $B^+ \to K^+ + \bar{\nu} \nu$ channel, allowing for an improvement in overall sensitivity to light dark matter. This, coupled with the $\sim 1$ ab$^{-1}$ data samples that will soon be available from the $B$ factories, should push the sensitivity to missing energy processes to $\sim 2$ times the level of the SM branching ratio.

7. Conclusions

In conclusion, we have demonstrated that $B$-decays with missing energy provide a viable method for exploring light dark matter models. This alternative to dedicated underground experiments has considerable sensitivity to light WIMPs, and can probe dark matter using existing experimental results. While this result has been previously demonstrated in Ref [17] this paper extends the result to include several different models.

In particular it has been demonstrated that the 2HDM+scalar model, in the case of $\lambda_1$ dominant, can satisfy the abundance constraint with $\text{Br}(B^+ \to K^+ + SS) > \text{Br}(B^+ \to K^+ + \bar{\nu} \nu)$. This model has an additional benefit, as it requires only minimal fine tuning to achieve sub-GeV WIMPs in the large $\tan \beta$ limit. Data from BABAR and BELLE excludes the parameter space below $m_S \sim 1.3$ GeV, while future measurements are expected to probe up to $m_S \sim 1.9$ GeV. The case of $\lambda_3$ dominant also provides sub-GeV WIMPs with minimal fine-tuning. However in that case the branching ratio $\text{Br}(B^+ \to K^+ + SS)$ is suppressed by factor of $\tan^2 \beta$ relative to the $\lambda_1$ dominant case, and therefore is significantly smaller than the Standard Model signal. In this case, only the direct searches with the low recoil energy threshold has a chance of detecting these WIMPs.

Furthermore we have demonstrated that it is possible to search for fermionic dark matter in $B$-decays, though in the models presented the couplings are required to be non-perturbative to meet the abundance constraint for $m_\chi \lesssim 2$ GeV. While it is possible that heavier fermions could be produced in $B$-decays, the branching ratio for such a process is significantly smaller than the Standard Model predictions. It is also possible to lower the Lee-Weinberg limit in these models by allowing other light particles which are very weakly coupled to the Standard Model, such as in the NMSSM with one light pseudoscalar Higgs. In such a model, it would be possible to have weak couplings and still produce an observable increase in $\text{Br}(B \to K + \text{missing energy})$. Our analysis shows that indeed $O(100)$ MeV neutralinos of NMSSM are generically excluded by the $B$ decay data, while heavier (above 1 GeV) neutralinos would probably be compatible with existing constraints.

Except for the case of NMSSM, we have limited the discussion to models which do not include new forces below the electroweak scale. Models with new resonances below $M_W$ can be explored using $B$-decays, but in addition they might be well constrained by existing experiments and therefore should be introduced with caution. Such models can be analyzed in exactly the same manner as presented in this paper.
References

1. D. N. Spergel et al. [WMAP Collaboration], Astrophys. J. Suppl. 148, 175 (2003) arXiv:astro-ph/0302209.

2. E. W. Kolb and M. S. Turner, The Early Universe, Addison-Wesley, 1990.

3. P. Gondolo and G. Gelmini, Phys. Rev. D 71, 123520 (2005) arXiv:hep-ph/0504010.

4. C. Boehm, D. Hooper, J. Silk, M. Casse and J. Paul, Phys. Rev. Lett. 92, 101301 (2004) arXiv:astro-ph/0309686.

5. C. Picciotto and M. Pospelov, Phys. Lett. B 605, 15 (2005) arXiv:hep-ph/0402178.

6. J. Knodlseder et al., Astron. Astrophys. 411, L457 (2003) arXiv:astro-ph/0309442.

7. P. Jean et al., Astron. Astrophys. 407, L55 (2003) arXiv:astro-ph/0309484.

8. G. Bertone, A. Kusenko, S. Palomares-Ruiz, S. Pascoli and D. Semikoz, arXiv:astro-ph/0405005.

9. S. Schanne, M. Casse, B. Cordier and J. Paul, arXiv:astro-ph/0404492.

10. P. A. Milne, M. D. Leising and L. S. The, arXiv:astro-ph/0404499.

11. C. Boehm, D. Hooper, J. Silk, M. Casse and J. Paul, Phys. Rev. Lett. 92, 101301 (2004) arXiv:astro-ph/0309686.

12. J. F. Beacom and H. Yuksel, arXiv:astro-ph/0512411.

13. Y. Rasera, R. Teyssier, P. Sizun, B. Cordier, J. Paul, M. Casse and P. Fayet, arXiv:astro-ph/0507707.

14. K. Ahn and E. Komatsu, arXiv:astro-ph/0506520.

15. B. W. Lee and S. Weinberg, Phys. Rev. Lett. 39, 165 (1977).

16. M. I. Vysotsky, A. D. Dolgov and Y. B. Zeldovich, JETP Lett. 26, 188 (1977) [Pisma Zh. Ekspt. Teor. Fiz. 26, 200 (1977)].

17. C. Bird, P. Jackson, R. Kowalewski and M. Pospelov, Phys. Rev. Lett. 93, 218003 (2004) arXiv:hep-ph/0401195.

18. C. P. Burgess, M. Pospelov and T. ter Veldhuis, Nucl. Phys. B 619, 709 (2001) arXiv:hep-ph/0011335.

19. P. Fayet, Phys. Rev. D 70, 023514 (2004) arXiv:hep-ph/0403226.

20. P. Fayet, arXiv:hep-ph/0408357.

21. J. F. Gunion, D. Hooper and B. McElrath, arXiv:hep-ph/0509024.

22. V. Silveira and A. Zee, Phys. Lett. B 161, 136 (1985).

23. J. McDonald, Phys. Rev. D 50, 3637 (1994).

24. G. Buchalla, G. Hiller and G. Isidori, Phys. Rev. D 63, 014015 (2001) arXiv:hep-ph/0006136.

25. A. Ali, P. Ball, L. T. Handoko and G. Hiller, Phys. Rev. D 61, 074024 (2000) arXiv:hep-ph/9910221.

26. C. Bobeth, T. Ewerth, F. Kruger and J. Urban, Phys. Rev. D 64, 074014 (2001) arXiv:hep-ph/0104284.

27. M. B. Voloshin, Sov. J. Nucl. Phys. 44, 478 (1986) [Yad. Fiz. 44, 738 (1986)].

28. S. Raby and G. B. West, Phys. Rev. D 38, 3488 (1988).

29. T. N. Truong and R. S. Willey, Phys. Rev. D 40, 3635 (1989).

30. S. P. Martin, arXiv:hep-ph/9709356.

31. M. Carena, M. Olechowski, S. Pokorski and C. E. M. Wagner, Nucl. Phys. B 426, 269 (1994) arXiv:hep-ph/9402253.

32. L. J. Hall, R. Rattazzi and U. Sarid, Phys. Rev. D 50, 7048 (1994) arXiv:hep-ph/9306309.

33. R. Rattazzi and U. Sarid, Phys. Rev. D 53, 1553 (1996) arXiv:hep-ph/9505428.

34. J. F. Gunion, H. E. Haber, G. L. Kane and S. Dawson, SCIPP-89/13.

35. B. Aubert et al. [BABAR Collaboration], Phys. Rev. Lett. 94, 101801, (2005) arXiv:hep-ex/0411061.
Dark matter pair-production in $b \rightarrow s$ transitions

36. K. Abe et al. [Belle Collaboration], arXiv:hep-ex/0507034
37. T. E. Browder et al. [CLEO Collaboration], Phys. Rev. Lett. 86, 2950 (2001) arXiv:hep-ex/0007057.
38. A. Bottino, F. Donato, N. Fornengo and S. Scopel, Phys. Rev. D 68, 043506 (2003) arXiv:hep-ph/0304080.
39. R. Dermisek and J. F. Gunion, Phys. Rev. Lett. 95, 041801 (2005) arXiv:hep-ph/0502105.
40. G. Hiller, Phys. Rev. D 70, 034018 (2004) arXiv:hep-ph/0404220.
41. R. Hempfling, Phys. Rev. D 49, 6168 (1994) arXiv:hep-ph/9504364.
42. T. Blazek, S. Raby and S. Pokorski, Phys. Rev. D 52, 4151 (1995) arXiv:hep-ph/9807350.
43. C. Hamzaoui, M. Pospelov and M. Toharia, Phys. Rev. D 59, 095005 (1999) arXiv:hep-ph/9504364.