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In this study, we analyze the properties of Bitcoin as a diversifier asset and hedge asset against the movement of international market stock indices: S&P500 (US), STOXX50 (EU), NIKKEI (Japan), CSI300 (Shanghai), and HSI (Hong Kong). For this, we use several copula models: Gaussian, Student-t, Clayton, Gumbel, and Frank. The analysis period runs from August 18, 2011 to June 31, 2019. We found that the Gaussian and Student-t copulas are best at fitting the structure dependence between markets. Also, these copulas suggest that under normal market conditions, Bitcoin might act as a hedge asset against the stock price movements of all international markets analyzed. However, the dependence on the Shanghai and Hong Kong markets was somewhat higher. Also, under extreme market conditions, the role of Bitcoin might change from hedge to diversifier. In a time-varying copula analysis, given by the Student-t copula, we found that even under normal market conditions, for some markets, the role of Bitcoin as a hedge asset might fail on a high number of days.

1. Introduction

Bitcoin is a cryptocurrency or virtual currency that was introduced on October 31, 2008, by a group of programmers under the pseudonym Satoshi Nakamoto (Nakamoto, 2008). Since then, it has shown an unpredicted growth, in both volume and value, and has now become a standard media payment over the internet (European Central Bank (ECB), 2012). An increasing number of providers of goods and services, -legal and illegal- trade in Bitcoins (Brière et al., 2015). On the other hand, the high yield offered by this asset in some periods, for instance, from 2014 to 2017, where the growth was almost exponential, has attracted the attention of many investors. So, the demand for this currency has increased not only because of its use in transactions, but as a form of savings. Some authors note out that Bitcoins are primarily regarded as assets rather than currency (Baek and Elbeck, 2015; Cheah and Fry, 2018). Bitcoin is considered the first large-scale implementation of blockchain technology. The Bitcoin blockchain was designed to allow fast and secure transactions and, at the same time, maintain the anonymity of users, using a public record book that authenticates transactions between economic agents without the need for a central entity that proves the movement of funds (Lopez-Martin et al., 2019).

According to the traditional definition, a currency has three main properties: (1) it serves as a medium of exchange, (2) it is used as a unit of account, and (3) it allows to store value (Bariviera et al., 2017). This last characteristic let us consider Bitcoin as an asset.

The high return offered by the Bitcoin is also accompanied by high risk, so the investment in this currency is classified by the ECB as very risky (European Central Bank (ECB), 2012).

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The spectacular growth of Bitcoin’s price since its introduction has attracted significant attention from the academic world. A large number of papers have been published focusing on descriptive analysis of the Bitcoin network (Zang and Lee, 2019; Heilman et al., 2015; Jim-Bon et al., 2019). Other studies have analyzed the efficiency of the Bitcoin market (see, for instance, Bariviera et al. (2017); Braunies and Mestel (2018); Bariviera (2017), and Nadarajah and Chu (2017)). These authors note that the Bitcoin market was not weakly efficient in the initial years, from 2011 to 2013, but tends to be efficient since then. Other studies have analyzed the determinants of the Bitcoin price. The academic literature in this area has identified several factors that affect Bitcoin’s price: (i) market forces of Bitcoin supply and demand, (ii) Bitcoin attractiveness for investors, (iii) macroeconomic and monetary factors, and (iv) factors related to security. Over this topic, many studies conclude that Bitcoin’s price neither depends on economic factors nor the monetary factors, but rather on speculative and supply and demand factors (see Ciaian et al., 2016; Bouoiyour and Selmi, 2015, Chead and Fry, 2015, Eom et al., 2019; Baek and Elbeck, 2015).

Other studies have focused on analyzing the statistical characteristics of the Bitcoin returns. On this topic, the literature notes that Bitcoin’s return probability distribution is different from the traditional currency distribution, which is more similar to the normal distribution. However, Bitcoin’s return probability distribution is skewed and exhibits a high degree of kurtosis. In this sense, the Bitcoin distribution is more similar to the distribution of traditional assets (stocks, bonds, and commodities), although it exhibits a higher average return, higher volatility, and fatter tail, which means that investing in Bitcoin involves greater risk than investing in traditional assets (European Central Bank (ECB), 2012).

There is also a group of studies that have analyzed the links between the Bitcoin market and the stock markets. These studies have produced two strands of the literature. The first strand posits the strong relationship between the Bitcoin and stock markets (Isah and Raheem, 2019; Bouri et al., 2018a). The second strand of the literature describes a weak relationship between Bitcoin and stock markets, so that Bitcoin may act as a hedge asset against the stock price movements (Klierber et al., 2019; Kang et al., 2019; Klein et al., 2018; Feng et al., 2018; Bouri et al., 2017a, b; Brière et al., 2015; Eisl et al., 2015; and Dyhrberg, 2015). Therefore, as argued by Tiwari et al. (2019b), the literature to this regard is not only immature but also not conclusive.

In this study, we analyze the ability of Bitcoin to act as a diversification asset and hedge against stock assets risk. The motivation for this study is that, as the literature points out, stock markets are exposed to macroeconomic factors, such as government fiscal or monetary policy. However, Bitcoin’s price might not depend on such factors, but rather by speculative and supply and demand factors. The fact that these markets depend on factors so different opens the possibility for the Bitcoin market to be a source of diversification against the risk of the stock markets.

The above-cited papers study the ability of Bitcoin to act as a diversifying or hedging asset using the Dynamic Conditional Correlation (DCC) model. This model might not be appropriate for measuring dependence on whether the bivariate normality assumption on the joint distribution does not hold. In addition, this method helps us in examining the dependence structure between markets when there exists a linear relationship between the marginals of the series under study. However, when the relationship between the marginals is not linear, this model will not be able to return the correct results. Taking this into account, we conduct our study through a copula analysis that appropriately describes the dependence structure between financial assets (e.g., Cherubini and Luciano, 2001; Frey and McNeil, 2003; Jondeau and Rockinger, 2006; Junker et al., 2006; Luciano and Marena, 2002). Moreover, we conduct a constant and time-varying copula model, which allows us to assess the time-varying nature of the diversifier and hedge properties of Bitcoin.

There are at least three advantages to using copulas for analyzing the dependence: 1) the copula method can capture the complex and non-linear dependence structure of a multivariate distribution; 2) the marginal behavior and the dependence structure are separated by the framework of copulas, facilitating both the model specification and the model estimation (the estimation can be performed in separate steps for the marginal models and copula functions); and 3) copulas are invariant to increasing and continuous transformations (Ning, 2010), such as the scaling of logarithm returns, which are commonly used in economics and finance studies.

The objectives of the study are: first, to understand the relationship, if any, of the Bitcoin market with the major stock markets in the world; second, to establish the importance of copula functions with respect to linear correlation coefficients in understanding this relationship; and third, to analyze the possibilities of diversification and hedge that Bitcoin market offers to investors.

Our paper contributes to the literature in several ways. This paper is one of the first studies to use time-varying copula models for assessing the properties of Bitcoin as a diversifier and hedge asset. Further, the study is very comprehensive, since it includes a large number of international stock markets from different geographic areas, including the Chinese stock market.

The rest of the paper is structured as follows. In Section 2, we describe the methodology used to assess the study. The data set is introduced in Section 3. Section 4 presents the empirical results. Finally, Section 5 concludes this paper.

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4 The Efficient Market Hypothesis (EMH) is the cornerstone of financial economics, as most theories developed for asset valuation assume this hypothesis.

5 Although overall most papers conclude that Bitcoin’s price does not depend on macroeconomic factors a recent paper has provided support against this hypothesis (Kristoufek, 2019).

6 See Zhang et al. (2018); Feng et al. (2018); Osterrieder et al. (2017); Chu et al. (2017), and Chu et al. (2015) among others.

7 The time-varying nature of the Bitcoin’s properties as a diversifier and hedge has also been studied by Tiwari et al. (2019b) and Boako et al. (2019).
2. Methodology

To analyze the structure dependence between Bitcoin and the stock market indices, we have used a copula analysis. This section provides a brief review of the methodology.

To estimate the parameters of the copula, we follow Rong and Trück (2014) and implement a two-stage procedure. In the first stage, we fit an APARCH model to the univariate return series and obtain the standardized residuals for each series. In the second stage, we use standardized residuals to estimate the different copula functions. In the following lines, we first review the volatility specification APARCH, and then we review the copula models.

2.1. APARCH model

The APARCH model (Asymmetric Power ARCH model) was proposed by Ding et al. (1993). This model can well express volatility clustering, fat tails, excess kurtosis, the leverage effect, and the Taylor effect. The latter effect is named after Taylor (1986) who observed that the sample autocorrelation of absolute returns was usually larger than that of squared returns. The APARCH equation is,

\[ \sigma_t^2 = \omega + \sum_{i=1}^{q} \alpha_i (|\varepsilon_{i-1}| + \gamma \varepsilon_{i-1})^\delta + \sum_{j=1}^{p} \beta_j \sigma_{i-j}^2 \]

where \( \omega, \alpha_i, \gamma, \beta_j \) and \( \delta \) are additional parameters to be estimated. The parameter \( \gamma \) reflects the leverage effect \((-1 < \gamma < 1)\). A positive (resp. negative) value of \( \gamma \) means that past positive (resp. negative) shocks have a deeper impact on current conditional volatility than past negative (resp. positive) shocks. The parameter \( \delta \) plays the role of a Box-Cox transformation of \( \sigma_t^2 \) (\( \delta > 0 \)).

The APARCH equation is supposed to satisfy the following conditions, i) \( \omega > 0 \) (since the variance is positive), \( \alpha_i \geq 0, \ i = 1, 2, \ldots, q, \ \beta_j \geq 0, \ j = 1, 2, \ldots, p \). When \( \alpha_i = 0, \ i = 1, 2, \ldots, q, \ \beta_j = 0, \ j = 1, 2, \ldots, p \), then \( \sigma_t^2 = \omega \), ii) \( 0 \leq \sum_{i=1}^{q} \alpha_i + \sum_{j=1}^{p} \beta_j \leq 1 \). The APARCH is a general model because it has great flexibility, having as special cases, among others, GARCH and GJR-GARCH models.

2.2. Copulas

We derive the dependence structure between two time series via copula modeling, examining the dependence between the marginal distribution of the standardized innovations, that we assume ST-APARCH(1,1).

Let \( X_i, X_2, \ldots, X_n \) be a set of random variables with a marginal distribution function given by \( F_i(x_i) \), where \( F_i(x_i) = \Pr(X_i \leq x_i) \) for \( i = 1, \ldots, n \). A copula is a function that joins (or couples) the univariate distribution functions to a multivariate distribution function \( F \), as denoted by \( C \) in the equation:

\[ F(x_1, x_2, \ldots, x_n) = C(F_1(x_1), F_2(x_2), \ldots, F_n(x_n)) \]

(2)

Alternatively, a copula can be defined as the multivariate distribution, \( C \), of a vector of random variables with uniformly distributed marginals \( U(0,1) \).

\[ C(u_1, u_2, \ldots, u_n) = F^{-1}(u_1), F^{-1}(u_2), \ldots, F^{-1}(u_n) \]

(3)

where the \( u_i = F(x_i) \) and \( F^{-1} \)’s are the quantile functions of the marginals. A copula extracts the dependence structure from the joint distribution, independent of marginal distributions.

Copula modeling has found many successful applications, especially in actuarial science, survival analysis, and hydrology. In finance, copula modeling has also been applied in a great number of contexts, such as asset pricing, credit risk management, aggregation risks and the study of dependence between markets (e.g., Cherubini et al., 2004; McNeil et al., 2005; Rajwani and Kumar, 2019; Wang et al., 2011; Wen et al., 2012; Nguyen et al., 2017).

In the following lines, we present the functional forms of the five copulas used in this paper, which are the most commonly used in this kind of study: (i) Gaussian, (ii) Student-t, (iii) Clayton, (iv) Gumbel, and (v) Frank.

(i) Gaussian copula. The Gaussian copula is given by

\[ C(u_1, u_2, \ldots, u_n; \sum_{i=1}^{n}) = \phi_n(\varphi^{-1}(u_1), \ldots, \varphi^{-1}(u_n)) \]

(4)

where \( \varphi \) is the standard multivariate normal with linear correlation matrix \( \sum \) and \( \varphi^{-1} \) is the inverse of the standard univariate Gaussian. Deriving the Eq. (4), we obtain the copula density \( c \). For \( n = 2 \), this expression is given by:

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8 Although it is possible to model the dependence structure using original returns some studies point that it is more appropriate to study the dependence structure after filtering out the autoregressive and heteroskedastic behavior of the data (see Gregoire et al., 2008; Jondeau and Rockinger, 2006, and Patton, 2006).

9 In this study we assume a Student-t distribution for modelling return because as we’ll show later, in a comparison with others fat tail distribution the Student-t is the best in fitting data.
\[ c(u_1, u_2; \rho) = \frac{1}{\sqrt{1 - \rho^2}} \exp\left( -\frac{1}{2(1 - \rho^2)} (x_1^2 - 2\rho x_1 + x_2^2) \right) \]

where \( \rho \) is simply the linear correlation coefficient between the two random variables. For \( \rho = 0 \) describes the independence copula, while for \( \rho = 1 \) describes the comonotonicity copula, and for \( \rho = -1 \) the countermonotonicity copula. The normal copula is symmetric and does not exhibit tail independence.

(ii) Student-t copula. The Student-t copula is given by

\[
C(u_1, u_2, ..., u_n; \sum_i t_i^{-1}(u_i), ..., t_n^{-1}(u_n)) = T_n(v, \rho)
\]

where \( T_n \) is the standard multivariate Student-t with \( v \) degrees freedom and with linear correlation matrix \( \Sigma \) and \( t_i^{-1} \) is the inverse of the standard univariate Student-t with \( v_i \) degree of freedom. Deriving the Eq. (5), we obtain copula density for the Student-t. For \( n = 2 \), this expression is given by

\[
c(u_1, u_2; \rho, v) = \frac{K}{\sqrt{1 - \rho^2}} \left[ 1 - \frac{1}{v(1 - \rho^2)} \left( \xi_1^2 - 2\rho \xi_1 \xi_2 + \xi_2^2 \right) \right]^{v+2} \left[ (1 + v^{-1}\xi_2^2)(1 + v^{-1}\xi_2^2) \right]^{v+2}
\]

where \( \rho \) is the linear correlation coefficient of the bivariate Student-t distribution with \( v \) degrees of freedom. Ultimately, \( \xi_i = t_i^{-1}(u_i) \) and \( K = \Gamma\left(\frac{v+1}{2}\right) \Gamma\left(\frac{v}{2}\right) \Gamma\left(\frac{v+2}{2}\right) \Gamma\left(\frac{v+3}{2}\right) \).

The Student-t copula has symmetric but nonzero tail dependence and nests the normal copula. The coefficient of tail dependence is given by:

\[
\lambda = 2\lambda \left( \frac{\sqrt{v+1}}{\sqrt{1 + \rho}} \right)
\]

(iii) Clayton copula. The Clayton copula is given by

\[
C(u_1, u_2, ..., u_n; \alpha) = \left( \sum_{i=1}^{n} u_i^{-\alpha} - n + 1 \right)^{-1/\alpha}
\]

where \( \alpha \) is the parameter of the copula and captures structure dependence among the \( n \) variables \( X_1, X_2, ..., X_n \). When \( \alpha \to 0 \) it describes the independence copula, while for \( \alpha \to \infty \) it describes the comonotonicity copula. Deriving Eq. (6), we get the copula density. For \( n = 2 \), this expression is given by

\[
c(u_1, u_2; \alpha) = (1 + \alpha)(u_1^{-\alpha} + u_2^{-\alpha} - 1)^{-1/\alpha} (u_1u_2)^{-\alpha-1}
\]

The Clayton copula has asymmetric tail dependence. The dependence in the upper tail is zero (\( \lambda_U = 0 \)) while the dependence in the lower tail is given by

\[
\lambda_L = \begin{cases} 2^{-1/\alpha}, & \alpha > 0 \\ 0, & \text{otherwise} \end{cases}
\]

(iv) Gumbel copula. The Gumbel copula distribution is given by

\[
C(u_1, u_2, ..., u_n; \delta) = \exp\left( -\sum_{i=1}^{n} \left( -\ln(u_i) \right) \delta^{1/\delta} \right)
\]

where \( \delta \) is the copula parameter that might take any real value, it is to say \( \delta \in (-\infty, +\infty) \). This copula permits the modeling of positive as negative dependence in the data. The independence case will be attained when \( \delta \) approaches zero. Deriving the Eq. (8),

\[
c(u_1, u_2; \delta) = (A + \delta - 1)A^{-\delta} \exp(-A)(u_1u_2)^{-1}(-\ln u_1)^{\delta-1}(-\ln u_2)^{\delta-1}
\]

where \( A = [(-\ln u_1)(-\ln u_2)]^{1/\delta} \).

The Gumbel copula has asymmetric tail dependence. The dependence in the lower tail is zero (\( \lambda_L = 0 \)) while the dependence in the upper tail is given by

\[
\lambda_U = \begin{cases} 2 - 2^{1/\delta}, & \delta > 1 \\ 0, & \text{otherwise} \end{cases}
\]

(v) Frank copula. The Frank copula distribution is given by

\[
C(u_1, u_2, ..., u_n; \theta) = -\frac{1}{\theta} \ln\left( 1 + \frac{\sum_{i=1}^{n} \exp(-\theta u_i) - 1}{\exp(-\theta) - 1} \right)
\]

where \( \theta \) is the copula parameter that might take any real value, it is to say \( \theta \in (-\infty, +\infty) \). This copula permits the modeling of positive as negative dependence in the data. The independence case will be attained when \( \theta \) approaches zero.
we obtain the density copula. For $n = 2$, this expression is given by
\[
c(u_1, u_2; \theta) = \frac{\theta[1 - \exp(-\theta)]\exp(-\theta u_1 u_2)}{([1 - \exp(-\theta)] - (1 - \exp(-\theta u_1))(1 - \exp(-\theta u_2)))^2}
\]

The Frank copula has neither lower nor upper tail dependence $\lambda_L = \lambda_U = 0$. The Frank copula is thus suitable for modeling data characterized by weak tail dependence. Table 1 presents the main tail dependence features in the set of employed copulas.

The dependence structure between variables might be characterized by a copula, but might also be expressed using synthetic scalar measures derived from the same copula, for example, the measures of rank correlation Kendall’s $\tau$. This rank correlation measure is useful because it allows comparative analyses of global dependence structures when copulas are different and, consequently, the copulas’ dependence parameters are non-comparable. Table 2 represents the relationships between Kendall’s $\tau$ and copula parameters for the set of employed copulas.

To specify the dynamics of the copula dependence parameter, we follow Patton (2006), that proposes observation-driven copula models where the time-varying dependence parameter is a parametric function of transformations of the lagged data and an autoregressive term. Using the marginal distributions of the standardized residuals $u_{t,1}$ and $u_{t,2}$, the dynamics of the parameters for the Gaussian, Student-t, Gumbel, Clayton, and Frank copulas can be specified.

For the Gaussian copula parameter, we apply the following model
\[
\rho_t = \Lambda_t \left\{ \omega + \beta_1 \rho_{t-1} + \alpha \frac{1}{20} \sum_{j=1}^{20} \Phi^{-1}(u_{t-1,j}) \Phi^{-1}(u_{t-2,j}) \right\}
\]

Similarly, the model for the Student-t copula can be specified as
\[
\rho_t = \Lambda_t \left\{ \omega + \beta_1 \rho_{t-1} + \alpha \frac{1}{20} \sum_{j=1}^{20} t^{-1}_\nu(u_{t-1,j}) t^{-1}_\nu(u_{t-2,j}) \right\}
\]

where $\Lambda_t(x) = (1 - e^{-x})(1 + e^{-x})^{-1} = \tanh(x/2)$ is the modified logistic transformation, designed to keep $\rho_t$ in $[-1,1]$ at all times. Eqs. (9) and (10) reveal the assumption that the copula parameter follows an ARMA(1,20)-type process. Note that in these specifications $\rho_{t-1}$ is used as a regressor to capture the persistence in the dependence parameter, and the mean of the product of the last 20 observations of the transformed variables $\Phi^{-1}(u_{t,1})$ and $\Phi^{-1}(u_{t,2})$, and $t^{-1}_\nu(u_{t,1})$ and $t^{-1}_\nu(u_{t,2})$, respectively, to capture any variation in dependence.\(^\text{10}\)

For the Clayton, Gumbel and Frank copula, the model for the evolution of the dependence parameter is given by

\(^\text{10}\) For the Student-t copula we assume that the number of degrees of freedom is constant (Elliott and Timmermann, 2013, p. 932, and Reboredo and Ugolini, 2016) and only the correlation parameter is time-varying.
\[
\bar{z}_i = \lambda_0 + \beta_i - \frac{1}{20} \sum_{j=1}^{20} [u_{i,j} - u_{2,j}]
\] (11)

2.3. Estimation copula parameters

We estimate the copula parameters using the Maximum Likelihood applied to the theoretical joint distribution function. We estimate in a first stage, the marginal parameters and, in a second stage, the copula parameters. This approach is called the inference in the margins (IFM) estimation method. Joe et al. (1997) demonstrates that under standard regularity conditions, this two-stage estimate is consistent, and the parameter estimates are asymptotically efficient and normal. In the following lines, we describe this method.

Let be \(X_1, X_2, \ldots X_T\) bidimensional vectors independent and identically distributed (iid) \(X_i \sim F(x)\) for \(i = 1, 2, \ldots T\). The joint density function is given by

\[
f(X_1, X_2, \ldots X_T) = \prod_{i=1}^{T} f(X_{i,1}, X_{i,2} | \alpha, \theta)
\]

and the log-likelihood function

\[
\ln L(X_1, X_2, \ldots X_T | \alpha, \theta) = \sum_{n=1}^{T} \ln f(X_{i,1}, X_{i,2} | \alpha, \theta)
\]

(13)

where \(f(X_{i,1}, X_{i,2} | \alpha, \theta)\) is the joint density function of the bidimensional vector \(X_i\) and \(\ln\) is the natural log. By the Sklar theorem the joint density of this vector can be decomposed into the marginal distributions and a copula density, which is obtained when differentiating Eq. (2):

\[
f(X_{i,1}, X_{i,2} | \alpha, \theta) = c(F_1(X_{i,1}, \alpha), F_2(X_{i,2}, \alpha); \theta) \prod_{j=1}^{T} f_j(X_{j,i}, \alpha)
\]

(14)

where \(f_j(X_{j,i}, \alpha)\) is the marginal density function of variable \(X_{j,i}\) for \(j = 1, 2\); \(c(\cdot)\) is the density copula; \(\alpha\) parameters vector of the marginal density function of variable \(X_i\) and \(\theta\) is the parameter vector of the copula function. For example, \(\theta = [\rho, \nu]\) for the Student-t copula and \(\alpha = \{\mu, \omega, \alpha, \beta\}\) for the marginal.

We first take log in Eq. (14)

\[
\ln f(X_{i,1}, X_{i,2} | \alpha, \theta) = \ln c(F_1(X_{i,1}, \alpha), F_2(X_{i,2}, \alpha); \theta) + \sum_{j=1}^{T} \ln f_j(X_{j,i}, \alpha)
\]

(15)

and then substituting in (13)

\[
\ln L(X_1, X_2, \ldots X_T | \alpha, \theta) = \sum_{i=1}^{T} [\ln c(F_1(X_{i,1}, \alpha), F_2(X_{i,2}, \alpha); \theta) + \sum_{j=1}^{T} \ln f_j(X_{j,i}, \alpha)]
\]

(16)

The optimal parameter vector is the one that maximizes the previous expression

\[
\theta^* = \arg \max \sum_{i=1}^{T} [\ln c(F_1(X_{i,1}, \alpha), F_2(X_{i,2}, \alpha); \theta) + \sum_{j=1}^{T} \ln f_j(X_{j,i}, \alpha)]
\]

(17)

As the marginal density of each variable does not depend on the \(\varepsilon\), to maximize Eq. (17) is the same as to maximize the first term

\[
\theta^* = \arg \max \sum_{i=1}^{T} [\ln c(F_1(X_{i,1}, \alpha), F_2(X_{i,2}, \alpha); \theta)]
\]

(18)

Thus, it is possible to maximize the likelihood function in two stages. First, we estimate the parameters of the marginal densities individually, using maximum likelihood

\[
\alpha^*_i = \arg \max \sum_{i=1}^{T} \ln f_j(X_{j,i}, \alpha)
\]

(19)

Second, the copula parameters can be estimated by resolving the following problem

\[
\theta^* = \arg \max \sum_{i=1}^{T} \ln c(u_{i,1}, u_{i,2}; \theta)
\]

To go from the first stage to the second stage we must calculate the residual \(z_{j,i}\), to which we use the estimated \(\hat{\alpha}_{j,i}\) and \(\hat{\theta}_{j,i}\):

\[
z_{j,i} = \frac{x_{j,i} - \hat{\mu}_{j,i}}{\hat{\theta}_{j,i}}
\]

for \(i = 1, 2, \ldots T\). The residuals are then transformed into uniformly distributed variables by inserting them in the univariate distribution of the marginals:

\[
u_{j,i} = F_j(z_{j,i}, \alpha)
\]
2.4. Model selection criteria

A typical problem that arises when fitting copulas to data is how to decide for the best fitting model. In this study, we use three methods for that end: (i) graphic methods, (ii) information criteria, and (iii) goodness of fit tests.

The first step in the model selection process is a visual inspection of the scatter plot of the data. For instance, those proposed by Genest and Rivest (1993) rely on two estimates for the distribution function of the copula \( C \). One is a parametric estimate \( (\hat{C}_p) \) obtained by MLE, the other is the empirical copula \( (\hat{C}_e) \). A scatter plot of these copula distributions (parametric and empirical) should yield a straight line.

Second, we use several information criteria as the Akaike information criterion [AIC] (Akaike, 1974, 1976), Bayesian information criterion [BIC] (Schwarz, 1978), Hannan-Quinn information criterion [HQIC] (Hannan and Quinn, 1979), and Shibata information criterion [SIC] (Shibata, 1976, 1980). These criteria are defined as follows

\[
\begin{align*}
AIC &= -2 \cdot LL + 2k \\
BIC &= -2 \cdot LL + k \cdot \log(T) \\
HQIC &= -2 \cdot LL + 2 \cdot k \cdot \log(\log(T)) \\
SIC &= -2 \cdot LL + \log(T + 2 \cdot k)
\end{align*}
\]

where \( T \) is the number of data, and \( k \) is the number of parameters used in the model. The best-fitting model is the one with the lowest AIC, BIC, HQIC, and SIC.

Third, for testing the validity of the null hypothesis \( H_0: C \in C_\theta \) that the dependence structure of a multivariate distribution is well represented by a specific parametric family \( C_\theta \) of copulas, we use several goodness-of-fit tests: (i) one test based on the empirical copula \( (S_T) \); and (ii) two tests based on Kendall’s transform \((S_T^F, T_T^F)\). The test based on copula empirical \( (S_T) \) consists of comparing a “distance” between the empirical copula \( C_T \) and an estimation \( \hat{C}_T \) of \( C \) obtained under \( H_0: C \in C_\theta \).

\[
D_T = \sqrt{T} (C_T - \hat{C}_T)
\]

\[
S_T = \int_{[0,1]^n} D_T(u)^2 \, dc_T(u)
\]

where the empirical copula \( (C_T) \) can be calculated as

\[
C_T(u) = \frac{1}{T} \sum_{i=1}^{T} 1\left[ U_{i,1} \leq u_1, U_{i,2} \leq u_2, ..., U_{i,n} \leq u_n \right]
\]

where \( U_i = (U_{i,1}, ..., U_{i,n}) \) for \( i = 1, ..., T \) are known as pseudo observations which are obtained from \( U_{ij} = \frac{(T/(T + 1)) \hat{F}_i}{\hat{F}_i} \). Genest and Rémillard (2008) establish the convergence of the latter under appropriate regularity conditions on the parametric family \( C_\theta \) and the sequence \( \hat{\theta} \) of estimators. They also show that the tests based on \( S_T \) are consistent; i.e., if \( C \) does not belong to \( C_\theta \), then \( H_0 \) is rejected with probability 1 as \( T \to \infty \).

The tests based on Kendall’s transform consists of basing a test of \( H_0 \) on a probability integral transformation of the data (Genest and Rivest, 1993; Genest et al., 2006).

Let \( X \to V = H(X) = C(U_1, U_2, ..., U_n) \), where \( U_i = F_i(x_i) \) for \( i = 1, 2, ..., n \) and the joint distribution of \( U = (U_1, U_2, ..., U_n) \) is \( C \). Let \( K \) denote the (univariate) distribution function of \( V \). Genest and Rivest (1993) have shown that \( K \) can be estimated nonparametrically by the empirical distribution function of a rescaled version of the pseudo-observations \( V_{i} = C_T(U_i) \) para \( i = 1, ..., n \).

Now, under \( H_0 \), the vector \( U = (U_1, U_2, ..., U_n) \) is distributed as \( C_\theta \) for some \( \theta \in Q \), and hence the Kendall transform \( C_\theta (U) \) has distribution \( K_\theta \). Through a measure of the distance between the empirical distribution function of the rescaled pseudo-observations \((K_T)\) and a parametric estimation \( K_{\hat{\theta}} \), one can test

\[
H_0^T: K \in K_0 = \{ K_\theta: \theta \in Q \}
\]

As the \( K_T \) and \( H_0 \) are equivalent, Genest et al. (2006) propose two test analog of the Crámer von Mises and Kolmogorov-Smirnov test:

\[
S_T^F = \int_{[0,1]^n} K_T(v)^2 \, dk_T(v)
\]

\[
T_T^F = \sup |K_T(v)|
\]

where \( K_T(v) = \sqrt{T} (K_T - K_{\hat{\theta}}) \).

As the asymptotic distributions of \( S_T^F \) and \( T_T^F \) depend both on the unknown copula \( C_\theta \) and on \( \theta \), approximate p-values for these statistics must be found via simulation.

11 The scaling factor \((T/(T + 1))\) is only introduced to avoid potential problems with \( C_T \) blowing up at the boundary of \([0,1]^n\).
3. Data

We used daily price on Bitcoin and five international stock market indices: S&P500 (US), STOXX50 (EU), NIKKEI (Japan), CSI300 (Shanghai), and Hang Seng (Hong Kong). We choose stock market indices from different geographic areas with the idea that Bitcoin might play multiple roles in investment portfolios depending on specific market conditions. The analysis period runs from August 18, 2011 to June 31, 2019. These data were extracted from Datastream. The returns are calculated as the log differences in prices multiplied by 100. Figs. 1 and 2 illustrate the daily price and returns of the data set, respectively.

As we can see in Fig. 1, along the period analyzed, the Bitcoin price has shown a spectacular growth going from $11 in August 2011 to around $12,300 currently. This growth was especially strong between 2014 and 2017, which was almost exponential. In these 3 years, the price increased by 1700 %, from $1000 at the beginning of 2014 to $18,000 at the end of 2017. Since then it has shown a downturn, although in recent months it is again increasing. The five stock market indices also show an uptrend, but the cumulative growth of these assets is lower than that of Bitcoin: 158 % S&P500, 47 % STOXX50, 138 % NIKKEI, 35 % CSI300 and 43 % HSI. The cumulative growth of the Bitcoin price in this period was 113243 %.

Table 3 displays the descriptive statistics. The average daily return of Bitcoin is much higher than the stock market indices’ daily average returns. For instance, concerning the STOXX50, the Bitcoin daily return was 17-times higher and 34-times higher than the CSI300. The volatility, measured by the standard deviation, is also higher for Bitcoin, but in this case, the differences relative to the stock market indices are smaller, so that overall, Bitcoin shows a higher return/risk ratio (Sharpe ratio). This means that although Bitcoin is very risky, the yield offered by this asset is so high that it offsets the risk.

The return distribution of the stock market indices is negatively skewed and exhibits an important excess kurtosis that suggests leptokurtic behavior. The Bitcoin return distribution is also skewed to the left and exhibits a high degree of kurtosis, even higher than the stock market indices distribution. In all cases, the value of the Jarque-Bera statistic indicates the departure from normality. The value of the ARCH statistic for conditional heteroskedasticity confirms that there exists an ARCH effect in the return of the Bitcoin and the rest of the assets, which justify the appropriateness of using a GARCH framework to model the conditional volatility.

Traditionally, portfolio decision and asset allocation have been made based on the Pearson coefficient correlation. Table 3 reports this measure for all pairs of returns considered. This coefficient is very low for all international markets considered, moving in a range of -0.009 for the Japanese market, represented by the NIKKEI index to 0.038 for the Chinese market (CSI300). According to these data, Bitcoin might be considered as a hedge asset against the risk of the international stock markets considered.

4. Empirical results

4.1. Modeling the marginals

First, we studied what distribution best fits the data set. To assess this issue, we fit several fat tail distributions, symmetric and asymmetric, including the (i) Student-t, (ii) generalized error distribution (GED), (iii) the skewed Student-t distribution, and (iv) the skewed GED. For all the stock market index returns, including Bitcoin returns, the Student-t distribution provides the highest Log-Likelihood, indicating that this distribution is the best at fitting the data. The Akaike Information Criterion (AIC) and Bayesian Information Criterion (BIC) corroborate this result (Table 4).

To check the goodness of fit of the Student-t distribution, we made a visual inspection of the QQ plot, which is reported in Fig. 3. For all data series, we can observe that the Student-t distribution fits well the empirical distribution, as the central part of the distribution as the tails, left and right.

4.2. Modeling the conditional variance

In the first step, we estimate the univariate volatility of the five assets to which we apply an APARCH model. This model can depict long memory and asymmetric effects. We estimate the parameters of the APARCH model with a Student-t distribution with $\nu$ degrees of freedom. The results of the estimations are reported in Table 5.

As we can observe, the estimations for the stock market indices are quite similar. We find that the persistence of the volatility, measured by the parameter $\beta_1$, is slightly higher for the stock market indices than Bitcoin. The parameter $\gamma$, which captures the leverage effect, is negative for all the stock market indices, indicating that volatility tends to be higher after negative returns. On the contrary, for the Bitcoin, this parameter is positive, which means that volatility tends to be higher after positive returns. This is known as an inverse leverage effect, a prominent feature of commodities, such as gold (Baur, 2012).12

12 Conlon et al. (2020) point out that there are reasons to believe the properties of the Bitcoin as a safe haven or hedge asset may vary internationally. They argue that there have been a number of papers investigating the relationship between Bitcoin returns and the movement in global economic uncertainty (see, Wu et al., 2019; Wang et al., 2019; Bouri et al., 2018b, and Demir et al., 2018). As policy uncertainty are country-specific, these studies motivate an analysis of the properties of Bitcoin internationally.

13 This behavior is typical of a speculative asset.

14 These percentages are calculated as $100 \times (I_{t+1} - I_{t+4}/I_t - I_{t+4}/I_{t+1})$, where $I_t$ represents the value of the Index in the time $t$.

15 This paper studies the volatility of gold and demonstrates that there is an inverted asymmetric reaction to positive and negative shocks, i.e. positive shocks increase the volatility by more than negative shocks. The paper argues that this effect is related to the safe-haven property of gold.
Fig. 1. Prices of Bitcoin and some international stock market indices from USA (S&P500), Europa (STOXX50), Japan (NIKKEI), China (CSI300), and Hong Kong (HSI).
Fig. 2. Log-returns of Bitcoin and some international stock market indices from USA (S&P500), Europa (STOXX50), Japan (NIKKEI), China (CSI300), and Hong Kong (HSI).
The inverse leverage effect could be explained by the fact that, in this period, Bitcoin behaved as a safe-haven asset, whose demand increases when uncertainty increases in traditional markets. If investors fleeing the risk of traditional markets seek Bitcoin as a safe-haven asset, then the price of Bitcoin will grow, thus transferring the volatility of the equity markets to the Bitcoin market. This argument was given by Baur (2012) to justify that gold volatility increases with positive surprises. These authors use this same argument to justify what is observed in the Bitcoin market. The idea is not far-fetched, since in many articles, the role of Bitcoin as a risk-hedging asset has been proposed, playing a role similar to that played by gold.

On whether or not Bitcoin has a leverage effect and whether it is inverse, there are many controversies. Some authors find results similar to ours (see Klein et al., 2018 and Bouri et al., 2017c), but other studies did not detect a significant leverage effect (Dyhrberg, 2016; Tiwari et al., 2019a; Katsiampa, 2017; Takaishi, 2018). Regarding this, Klein et al. (2018) argue that their results are because they use the Student-t distribution to account for heavy tails present in the distribution of the returns. When they assume a normal distribution, they confirm and replicate the results of Dyhrberg (2016). The sensitivity of the results regarding the underlying distribution is also discussed in Baur et al. (2018), whose results are in line with those presented by Klein et al. (2018).

As an experiment, we also estimate the APARCH model below a normal distribution, and we find the same results as Baur et al. (2018) and Klein et al. (2018). By assuming a normal distribution for the returns of the Bitcoin, the leverage effect is not significant. However, this assumption is not reasonable; as is shown in Table 3, the Bitcoin return distribution is far from being Gaussian.

On the other hand, the power parameter $\delta$ is around 1 for S&P500, STOXX50, and NIKKEI, which means that modeling the standard deviation appears better rather than the variance. However, for the Bitcoin and CSI300, the contrary is the case as the $\delta$ parameter takes a value around 2. In all cases, the ARCH test reveals that the APARCH model adequately captures the volatility of the innovations.

### 4.3. Modeling the dependence structure using copulas

Estimation results for constant copula models are reported in Table 6. In this table, we also include Kendall’s $\tau$ derived from the copula functions. This rank correlation measure is useful because it allows comparative analyses of global dependence structures when copulas are different and, consequently, the copulas’ dependence parameters are non-comparable. This measure is invariant to

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**Table 3**

|                  | Bitcoin | S&P500 | STOXX50 | NIKKEI | CSI300 | HSI |
|------------------|---------|--------|---------|--------|--------|-----|
| Mean             | 0.34    | 0.05   | 0.02    | 0.04   | 0.01   | 0.01|
| Median           | 0.27    | 0.03   | 0.02    | 0.00   | 0.00   | 0.00|
| Max              | 48.47   | 4.84   | 4.09    | 7.42   | 6.49   | 5.52|
| Min              | -66.39  | -4.18  | -6.93   | -8.25  | -9.15  | -6.02|
| Std. Dev.        | 5.90    | 0.85   | 0.95    | 1.26   | 1.44   | 1.10|
| Skewness         | -0.96   | -0.28  | -0.32   | -0.37  | -0.69  | -0.27|
| Kurtosis         | 19.89   | 3.35   | 3.85    | 4.45   | 5.88   | 2.80|
| Jarque-Bera      | 34220***| 994*** | 1308*** | 1750***| 3133***| 702***|
| ARCH (3)         | 35.09***| 75.28***| 28.94***| 7.48** | 5.62   | 0.52|
| Coef. Corr.      | 1.00    | 0.012  | 0.016   | -0.002 | 0.038  | 0.018|
| Sharpe ratio     | 0.058   | 0.059  | 0.021   | 0.032  | 0.007  | 0.009|

**Note:** Std. Dev. is the standard deviation, Min and Max are the minimum and maximum of the time series. ARCH(3) is the test for autoregressive conditional heteroskedasticity by Engle (1982) at the 3 lag. Coef. Corr. is the Pearson coefficient correlation. Sharpe ratio is the Mean and Std. Dev. ratio.

**Table 4**

The goodness of fit for marginal distributions.

|                  | Student-t | GED | Skew Student-t | Skew GED |
|------------------|-----------|-----|----------------|----------|
|                  | Log L     | AIC | BIC            | Log L    | AIC | BIC |
| Bitcoin          | 2312      | -4619 | -4601          | 2275     | -4544 | -4527 |
| S&P500           | 2725      | -5444 | -5427          | 2683     | -5360 | -5343 |
| STOXX50          | 2761      | -5517 | -5499          | 2754     | -5502 | -5485 |
| NIKKEI           | 2750      | -5494 | -5477          | 2715     | -5424 | -5407 |
| CSI300           | 2619      | -5232 | -5215          | 2545     | -5064 | -5067 |
| HSI              | 3006      | -5607 | -5989          | 2978     | -5950 | -5933 |

**Table 5**

The goodness of fit for marginal distributions.

|                  | Log L     | AIC | BIC |
|------------------|-----------|-----|-----|
| Student-t        |           |     |     |
| GED              |           |     |     |
| Skew Student-t   |           |     |     |
| Skew GED         |           |     |     |

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16 We calculate the Kendall’s $\tau$ following the relationships presented in Table 2.
non-linear transformations, as long as it is monotonic, as is the case of probability integral transforms of marginal variables in the context of the copula theory (Horta et al., 2010). Taking this into account, in this study, we use Kendall’s $\tau$ to assess global dependence structures between Bitcoin and the stock markets.

The structure dependence between Bitcoin and the US stock market (S&P500) given by the five copula models considered is reported in Panel (a). As we can see, the dependence parameter of the Gaussian copula is positive and statistically significant, but it returns a very small value (0.002). The corresponding Kendall’s $\tau$ for this parameter is 0.001. The parameter of the Student-t copula is also very small, but unlike the Gaussian copula, it returns a negative sign (-0.058). Its corresponding Kendall’s $\tau$ is -0.036. Both models suggest that the structure dependence between Bitcoin and the US stock market is null and even negative. On the other hand, the Archimedean copulas indicate a dependence degree higher than the suggested by the ellipticals.

Fig. 3. QQ-plots of Bitcoin and some international stock market indices from USA (S&P500), Europa (STOXX50), Japan (NIKKEI), China (CSI300), and Hong Kong (HSI).
The Clayton copula and the Gumbel copula provide Kendall’s $\tau$ of 0.195 and 0.300, respectively, between Bitcoin and the US stock market (S&P500). These results have very different implications for the portfolio selection decision. While the elliptical copula indicates that Bitcoin could act as a hedge asset against US stocks market performance, the Archimedean copulas suggest that Bitcoin

| Parameters | Bitcoin | S&P500 | STOXX50 | NIKKEI | CSI300 | HSI |
|-----------|---------|--------|---------|--------|--------|-----|
| $\mu$     | 0.003 (0.001) | 0.001 (0.000) | 0.000 (0.000) | 0.000 (0.000) | 0.000 (0.000) | 0.000 (0.011) |
| $\varphi$ | 0.00 (0.000) | 0.00 (0.000) | 0.00 (0.000) | 0.00 (0.000) | 0.00 (0.000) | 0.00 (0.000) |
| $\alpha_1$| 0.18 (0.029) | 0.12 (0.013) | 0.10 (0.015) | 0.11 (0.016) | 0.06 (0.015) | 0.03 (0.015) |
| $\gamma_1$| 0.14 (0.048) | -1.00 (0.007) | -1.00 (0.003) | -0.85 (0.115) | -0.11 (0.398) | -0.99 (0.005) |
| $\beta_1$ | 0.82 (0.031) | 0.88 (0.021) | 0.88 (0.023) | 0.86 (0.022) | 0.93 (0.055) | 0.95 (0.011) |
| $\delta$ | 2.16 (0.412) | 0.90 (0.182) | 1.20 (0.176) | 0.86 (0.175) | 1.87 (0.331) | 1.37 (0.450) |
| $\nu$    | 3.04 (0.128) | 5.57 (0.713) | 6.64 (0.973) | 4.73 (0.563) | 3.67 (0.439) | 5.34 (0.662) |
| ARCH(3)  | 0.008 | 0.260 | 0.158 | 0.003 | 3.795 | 6.543 |
| ARCH(5)  | 0.015 | 0.267 | 0.408 | 0.074 | 4.355 | 10.820 |

Standard deviations reported in parentheses. ARCH test for squared standardized innovations. The critical values at a 5% significance level for ARCH (3) and ARCH(5) are 7.8147 and 11.0705, respectively.

The Clayton copula and the Gumbel copula provide Kendall’s $\tau$ of 0.195 and 0.300, respectively, between Bitcoin and the US stock market (S&P500). These results have very different implications for the portfolio selection decision. While the elliptical copula indicates that Bitcoin could act as a hedge asset against US stocks market performance, the Archimedean copulas suggest that Bitcoin

| Copula | Copula parameter | Kendall’s $\tau$ | Lower Tail | Upper tail |
|--------|------------------|-----------------|------------|------------|
| Panel (a) US stock market index (S&P500) | | | | |
| Gaussian ($\varphi$) | 0.002*** | 0.001 | 0 | 0 |
| Student-t ($\varphi$, $v$) | -0.058*** | 2.01*** | -0.036 | 0.165 | 0.165 |
| Clayton ($\gamma$) | 0.487*** | - | 0.195 | 0.261 | 0 |
| Gumbel ($\delta$) | 1.492*** | - | 0.300 | 0 | 0.415 |
| Frank ($\theta$) | 2.932*** | - | 0.301 | 0 | 0 |
| Panel (b) European stock market (STOXX50) | | | | |
| Gaussian ($\varphi$) | 0.012*** | - | 0.007 | 0 | 0 |
| Student-t ($\varphi$, $v$) | -0.032*** | 2.01*** | -0.020 | 0.164 | 0.164 |
| Clayton ($\gamma$) | 0.443*** | - | 0.181 | 0.209 | 0 |
| Gumbel ($\delta$) | 1.395*** | - | 0.283 | 0 | 0.356 |
| Frank ($\theta$) | 2.245*** | - | 0.238 | 0 | 0 |
| Panel (c) Japanese stock market (NIKKEI) | | | | |
| Gaussian ($\varphi$) | -0.013*** | - | -0.008 | 0 | 0 |
| Student-t ($\varphi$, $v$) | -0.505*** | 2.01*** | -0.337 | 0.056 | 0.056 |
| Clayton ($\gamma$) | 0.978*** | - | 0.328 | 0.492 | 0 |
| Gumbel ($\delta$) | 1.805*** | - | 0.445 | 0 | 0.532 |
| Frank ($\theta$) | 3.508*** | - | 0.357 | 0 | 0 |
| Panel (d) Chinese stock market (CSI300) | | | | |
| Gaussian ($\varphi$) | 0.020*** | - | 0.012 | 0 | 0 |
| Student-t ($\varphi$, $v$) | 0.576*** | 2.01*** | 0.393 | 0.428 | 0.428 |
| Clayton ($\gamma$) | 1.883*** | - | 0.489 | 0.555 | 0 |
| Gumbel ($\delta$) | 1.912*** | - | 0.476 | 0 | 0.563 |
| Frank ($\theta$) | 5.179*** | - | 0.468 | 0 | 0 |
| Panel (e) Hong Kong stock market (HSI) | | | | |
| Gaussian ($\varphi$) | 0.004*** | - | 0.002 | 0 | 0 |
| Student-t ($\varphi$, $v$) | 0.105*** | 2.01*** | 0.066 | 0.216 | 0.216 |
| Clayton ($\gamma$) | 0.631*** | - | 0.239 | 0.335 | 0 |
| Gumbel ($\delta$) | 1.547*** | - | 0.353 | 0 | 0.435 |
| Frank ($\theta$) | 3.307*** | - | 0.333 | 0 | 0 |

*Significance at 10 % level, ** significance at 5% level, and *** significance at 1% level.

The Clayton copula and the Gumbel copula provide Kendall’s $\tau$ of 0.195 and 0.300, respectively, between Bitcoin and the US stock market (S&P500). These results have very different implications for the portfolio selection decision. While the elliptical copula indicates that Bitcoin could act as a hedge asset against US stocks market performance, the Archimedean copulas suggest that Bitcoin

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Clayton and Gumbel copula cannot capture negative dependence. So, in order to explore that possibility we have estimated the rotated Clayton 90 copula and rotated Gumbel 90 copula which can capture independence and negative dependence. The domain of rotated Clayton 90 copula parameter, denoted by $\hat{\alpha}$, is $(-\infty, 0]$. If $\hat{\alpha} = 0$ we have independence while $\hat{\alpha} < 0$ we have negative dependence. The domain of rotated Gumbel 90 copula parameter, denoted by $\hat{\gamma}$, is $(-\infty, -1]$. If $\hat{\gamma} = -1$ we have independence while if $\hat{\gamma} < -1$ we have negative dependence. In all cases, the parameter of the rotated Clayton copula is equal to zero while the parameter of the rotated Gumbel copula is equal to -1.01. Both estimations exclude the possibility of a negative dependence between all indices-Bitcoin returns pairs. We do not include the parameter estimations for space reason but they are available for any request.
might just act as a **diversifier** asset. Here, we adopt the definition of hedge assets given by Baur and Lucey (2010). They define a hedge asset as an asset that has a negative dependence or null with another asset or portfolio on average. A diversifier is defined as an asset that has a positive dependence structure (but not perfectly) with another asset in a portfolio.

For the European stock market and the Hong Kong stock market, we find similar results to that found for the US stock market. Elliptical copulas suggest an almost null dependence with the Bitcoin market, while the Archimedean copulas predict a stronger dependence.

The results obtained for the Japanese market are similar to those obtained for the markets already discussed above, but in this case, the differences between elliptical copula and Archimedean copulas are more pronounced (see Panel c). The elliptical copulas suggest a negative dependence, with Kendall’s \( r \) equal to -0.008 for Gaussian copula and -0.337 for Student-t copula. However, for the Archimedean copulas, Clayton, Gumbel, and Frank, predict a positive dependence with Kendall’s \( r \) equal to 0.328, 0.445, and 0.357, respectively. Again, these results have different implications for portfolio selection decisions, while the elliptical indicates that Bitcoin could act as a hedge asset against stocks Japanese market performance, the Archimedean copulas suggest that Bitcoin may just act a weak diversifier asset.

Ultimately, for the Chinese market, all copula models, except Gaussian copula, suggest a relatively high dependence. The Kendall’s \( r \) varies depending on the copula, moving between 0.393 and 0.489.

Unlike the traditional methods used to measure dependence between markets the copula function lets us know the dependence in tails distribution. If there is no tail dependence among the returns in a portfolio, then there is little risk of simultaneous very negative/positive returns, and therefore the probability of occurrence of an extreme negative/positive return on the portfolio will be lower. However, if there exist tail dependence, then the probability of occurrence of extreme negative/positive returns simultaneously can be high. Hence, it is important to consider tail dependence when assessing the diversification benefit and risk of a portfolio (Rajwani and Kumar, 2019).

In columns 5 and 6 of Table 6, we report the lower and upper tail dependencies for all copula models considered. The lower tail dependence gives us the probability of a joint crash between the pairs markets analyzed; meanwhile, the upper tail dependence gives us the probability of a joint boom in these markets. According to the Student-t copula, the probability of observing a joint crash in the Bitcoin market and the US stock market is higher than 0.15, which means that in a market extreme conditions, Bitcoin does not behave as a hedge asset against the performance of US stock market. Moreover, the Clayton suggests a higher probability of joint crashes, 0.261. Similar results are found for the European stock market and Hong Kong market.

Regarding the Japanese market, Student-t copula predicts that the probability of observing a joint crash is almost null (0.05), indicating that even in extreme market conditions, Bitcoin might affect behavior as a hedge asset. However, the Clayton copula predicts a joint crash with a probability of 0.492, which is relatively high. In the case of the Chinese market, the lower tail dependence given by the Student-t copula and Clayton copula is around 0.5, which indicates that in the stress period, the probability of these markets moving jointly is relatively high, reducing the possibilities of diversification when it is more necessary.

The results obtained are as follows: (i) the elliptical copulas suggest that the structure dependence between the Bitcoin and the stock markets analyzed, except the Chinese market, is very low, so that Bitcoin might act as a hedge asset against these markets; (ii) the Archimedean copulas predict a dependence higher than the detected by the elliptical copulas. According to this family copula model, Bitcoin might only act as a diversifier asset; (iii) both family copula models (elliptical and Archimedean) predict an increase of the dependence in extreme market conditions, so that, in such conditions, Bitcoin might only act as a diversifier asset; and (iv) in the case of the Chinese market, Bitcoin acts only as a diversifier asset. This result is robust to the copula model.

Given the disparity of results provided by the copula models analyzed, it is necessary to identify the suitable model before drawing definitive conclusions. The first step in the model selection process is a visual inspection of the scatter plot of the data. If the dependence structure of a multivariate distribution is well represented by a specific parametric family \( C_0 \) of copulas then the scatter plot of the empirical copula \( C_T \) and the parametric copula \( C_0 \) should yield a straight line. Fig. 4 illustrates the scatter plots of the empirical copula and the parametric copula for Normal, Student-t, and Gumbel. In the Appendix of the paper, we present these plots for Clayton and Frank copulas.

The visual inspection of these plots suggests that the Archimedean copulas do not fit the data well. This result does not depend on the market analyzed. However, elliptical copulas provide a good fit. For the US, the European, and Hong Kong markets, both Gaussian and Student-t copulas seem to fit the data well; meanwhile, for the Japanese and Chinese markets, the Gaussian copula seems to outperform the Student-t.

Furthermore, we focus on analyzing several information criteria, such as the Akaike information criterion [AIC] (Akaike, 1974, 1976), Bayesian information criterion [BIC] (Schwarz, 1978), Hannan-Quinn information criterion [HQIC] (Hannan and Quinn, 1979), and Shibata information criterion [SIC] (Shibata, 1976, 1980). These criteria are reported in Table 7. In this table, we also include the Log-Likelihood for all stock markets considered. For all markets analyzed, the lowest value of these criteria is obtained for the Student-t copula. Thus, the information criteria also rule out Archimedean copulas as appropriate models to describe the dependence between the Bitcoin market and the international stock markets analyzed.

For testing the validity of the null hypothesis \( H_0: C \in C_0 \) that the dependence structure of a multivariate distribution is well represented by a specific parametric family \( C_0 \) of copulas, we also use several goodness-of-fit tests: (i) one test based on the empirical copula \( S_T \); and (ii) two tests based on Kendall’s transform \( (S_T^F, T_T^F) \). These tests are reported in Table 8. In this table, we also include

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18 Again, the parameters of the rotated Clayton and Gumbel copulas 90 exclude the possibility of a negative dependence between Bitcoin and NIKKEI.
Fig. 4. Scatter plots between the empirical copula (S&P500, STOXX50, NIKKEI, CSI300, and HSI) and the parametric copula (Normal, Student-t, and Gumbel).
the sum of the square differences between the empirical copula and the parametric copula (SSE) and the mean of the absolute differences (MAE).

The elliptical copulas (Gaussian and Student-t) provide the lowest mean absolute error (MAE) and the lowest sum of the squared error (SSE). In the case of the US, European, and Hong Kong markets, the differences between Gaussian and Student-t copula are reduced. However, for the Japanese and Chinese stock markets, the differences between Gaussian and Student-t copula are notable. The tests based on Kendall’s transform ($S_T^G$, $S_T^T$) do not reject the null hypothesis that the dependence structure of a multivariate distribution is well represented by a Gaussian copula. However, for the rest of the copulas considered (Student-t, Clayton, Gumbel, and Frank), this hypothesis is rejected at all significance levels. Surprisingly, the $S_T$ test rejects the null hypothesis in all cases.

We find that the best copula model depends on the criterion used. According to the graphic methods, Gaussian and Student-t copula is the best at fitting data. However, according to the information criteria, the proper model is the Student-t copula. The goodness of fit test identifies the Gaussian copula as the best at the fitting data.\footnote{Despite the fact that in this work we use several criteria to select the best copula model, the results are not conclusive. So additional research in this area should be done.}

In summary, all the criteria rule out Archimedean copulas as appropriate models to describe the dependency between the Bitcoin market and stock markets analyzed. In the case of the US, the European, and the Hong Kong markets, the elliptical copulas, both, Gaussian and Student-t, seem to describe the dependence structure adequately with the Bitcoin. In the case of the Japanese and Chinese market the copula that seems to best describe the dependence structure with the Bitcoin is the Gaussian copula (see Table 9).

According to these results, we can conclude that in normal market conditions the Bitcoin might act as a hedge asset against the movement in all international stock markets considered, although, in the case of Hong Kong and China, the dependence is somewhat stronger than the observed for the rest of the markets. This result could be explained because China has become the world’s leader in Bitcoin mining, with 70% of its computing power located in the country. This country also has more patents related to blockchain and cryptocurrency than any other and has three times as many as the United States, the nation with the second most patents. The number of blockchain and crypto patents coming from China has almost quintupled since 2016.\footnote{The stronger relationship detected between the Bitcoin market and the Chinese stock market could also be explained by the high popularity that Bitcoin has in the Asian country. This popularity can be explained in turn by factors such as the government’s stringent control of the yuan, the instability of the yuan, and the growth of private wealth, leading some to look for alternative currencies.}

The elliptical copulas (Gaussian and Student-t) provide the lowest mean absolute error (MAE) and the lowest sum of the squared error (SSE). In the case of the US, European, and Hong Kong markets, the differences between Gaussian and Student-t copula are reduced. However, for the Japanese and Chinese stock markets, the differences between Gaussian and Student-t copula are notable. The tests based on Kendall’s transform ($S_T^G$, $S_T^T$) do not reject the null hypothesis that the dependence structure of a multivariate distribution is well represented by a Gaussian copula. However, for the rest of the copulas considered (Student-t, Clayton, Gumbel, and Frank), this hypothesis is rejected at all significance levels. Surprisingly, the $S_T$ test rejects the null hypothesis in all cases.

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Table 8
The goodness of fit test for copula models.

| Copula   | MAE   | SSE   | \(S_T\) p-value | \(S_K^p\) p-value | \(T_K^p\) p-value |
|----------|-------|-------|------------------|--------------------|-------------------|
| Panel (a) US stock market (S&P500) |       |       |                  |                    |                   |
| Gaussian (\(\phi\)) | 0.2 % | 0.07 | 0.00             | 0.65               | 0.90              |
| Student-t (\(\phi, v\)) | 0.6 % | 0.53 | 0.00             | 0.01               | 0.01              |
| Clayton (\(\xi\))     | 2.4 % | 8.33 | 0.00             | 0.00               | 0.00              |
| Gumbel (\(\xi\))     | 3.9 % | 21.9 | 0.00             | 0.00               | 0.00              |
| Frank                | 3.6 % | 19.9 | 0.00             | 0.00               | 0.00              |
| Panel (b) European stock market (STOXX50) |       |       |                  |                    |                   |
| Gaussian (\(\phi\)) | 0.2 % | 0.10 | 0.00             | 0.58               | 0.37              |
| Student-t (\(\phi, v\)) | 0.4 % | 0.30 | 0.00             | 0.01               | 0.01              |
| Clayton (\(\xi\)) | 2.2 % | 7.50 | 0.00             | 0.00               | 0.00              |
| Gumbel (\(\xi\)) | 3.4 % | 16.4 | 0.00             | 0.00               | 0.00              |
| Frank                | 2.9 % | 12.88| 0.00             | 0.00               | 0.00              |
| Panel (c) Japanese stock market (NIKKEI) |       |       |                  |                    |                   |
| Gaussian (\(\phi\)) | 0.3 % | 0.14 | 0.00             | 0.07               | 0.53              |
| Student-t (\(\phi, v\)) | 3.6 % | 18.6 | 0.00             | 0.00               | 0.00              |
| Clayton (\(\xi\)) | 4.1 % | 24.9 | 0.00             | 0.00               | 0.00              |
| Gumbel (\(\xi\)) | 5.3 % | 51.14| 0.00             | 0.00               | 0.00              |
| Frank                | 4.4 % | 30.1 | 0.00             | 0.00               | 0.00              |
| Panel (d) Chinese stock market (CSI300) |       |       |                  |                    |                   |
| Gaussian (\(\phi\)) | 0.3 % | 0.17 | 0.00             | 0.11               | 0.09              |
| Student-t (\(\phi, v\)) | 4.5 % | 28.9 | 0.00             | 0.00               | 0.00              |
| Clayton (\(\xi\)) | 5.5 % | 45.3 | 0.00             | 0.00               | 0.00              |
| Gumbel (\(\xi\)) | 5.4 % | 42.8 | 0.00             | 0.00               | 0.00              |
| Frank                | 5.4 % | 44.6 | 0.00             | 0.00               | 0.00              |
| Panel (e) Hong Kong stock market (HSI) |       |       |                  |                    |                   |
| Gaussian (\(\phi\)) | 0.2 % | 0.10 | 0.00             | 0.11               | 0.33              |
| Student-t (\(\phi, v\)) | 0.8 % | 0.98 | 0.00             | 0.00               | 0.00              |
| Clayton (\(\xi\)) | 2.9 % | 12.6 | 0.00             | 0.00               | 0.00              |
| Gumbel (\(\xi\)) | 4.1 % | 24.1 | 0.00             | 0.00               | 0.00              |
| Frank                | 4.0 % | 23.4 | 0.00             | 0.00               | 0.00              |

MAE denotes the mean absolute error, SSE denotes the sum of the squared error, \(S_T\) denotes the rank-based versions of the Cramér–von Mises test based on empirical copula, \(S_K^p\) and \(T_K^p\) denote the rank-based versions of the Cramér–von Mises and Kolmogorov-Smirnov tests based on Kendall's transform. All goodness of fit tests were run with 1000 simulations. The shadow cells indicate copulas with the lowest values of MAE and SSE. The bold figures indicate the cases in which the null hypothesis \(H_0: C \in C_0\) that the dependence structure of a multivariate distribution is well represented by a specific parametric family \(C_0\) of copulas.

Table 9
The best copula models according to the model selection criteria.

|                | Graphical methods | MAE and SSE | Information criterion | Goodness of fit tests |
|----------------|-------------------|-------------|-----------------------|-----------------------|
| S&P500         | Normal / Student-t| Normal / Student-t | Student-t             | Normal                |
| STOXX50        | Normal / Student-t| Normal / Student-t | Student-t             | Normal                |
| NIKKEI         | Normal            | Normal       | Student-t             | Normal                |
| CSI300         | Normal / Student-t| Normal / Student-t | Student-t             | Normal                |
| HSI            | Normal            | Normal       | Student-t             | Normal                |

Graphical methods: scatter plot of the parameter copula and empirical copula. MAE denotes the mean absolute error and SSE denotes the sum of the squared error. Information criterion: Akaike information criterion (AIC), Bayesian information criterion (BIC), Hannan-Quinn information criterion (HQIC), and Shibata information criterion (SIC). The goodness of fit tests: one test based on the empirical copula (\(S_T\)) and two tests based on Kendall's transform (\(S_K^p\), \(T_K^p\)).
The Student-\( t \) distributions, which fit well the data in many markets, indicate that the dependence between markets increases in extreme market conditions suggesting that Bitcoin’s role as a hedge asset might not work in a crisis period, becoming a simple diversifying asset.

The results provided by the elliptical constant copulas are similar to those provided by the linear correlations analysis with the difference that copula analysis (Student-\( t \)) provides additional information about the tail dependence.

This result is coherent with the facts observed in the recent crisis caused by the COVID-19 pandemic. During this period, the financial markets, including the Bitcoin market, moved in lockstep, with major joint declines. In fact, recent studies (Conlon et al., 2020; Conlon and McGee, 2020) show that incorporating Bitcoin into a well-diversified portfolio during the COVID-19 crisis has increased the risk assumed by investors.\(^{21}\)

\subsection{4.3.1. Time-varying copula models}

In the above section, we use constant copula models to assess the dependence structure between Bitcoin and some international stock markets. As the dependence might change along the period it would be interesting to estimate copula parameters daily to which we use the Eqs. (9) to (11). The parameter estimations for the elliptical copula are reported in Table 10. In this table, we do not include the parameters associated with the Archimedean copulas (Clayton, Gumbel, and Frank), as we show in the above section, we focus our study on Kendall’s \( \tau \) associated with these parameters.

Although the copula parameter of the elliptical copulas is directly comparable, because of the similitary with the results reported in the above section, we focus our study on Kendall’s \( \tau \) associated with these parameters.

Fig. 5 illustrates the dynamic of Kendall’s \( \tau \) given by the Gaussian copula. The first thing that catches our attention is that overall, Kendall’s \( \tau \) are very stable, remaining close to the value estimated by the constant copula. Therefore, we do not observe a changing trend in the dependence structure estimated between these markets throughout the period. Besides, in this Figure, no significant changes in the level of dependency common to all markets are observed.

In the particular case of the Chinese and European markets Kendall’s \( \tau \) move in a range of (0.00, 0.02) and (0.00, 0.01), respectively.\(^{22}\) For the US, Japanese, and Hong Kong markets, the dependence structure is somewhat more volatile, alternating positive and negative values, although this moves in a very narrow range: (± 0.06) for HSI, (−0.05, 0.02) for NIKKEI, and (−0.03, 0.04) for SP500.

Although we observe that the dependence structure is time-varying, the range of fluctuations is so narrow that the economic implications given by this model do not change regarding these given by the constant copula model. Thus, according to the time-varying Gaussian copula Bitcoin might act as a hedge asset against the movement in all the international stock markets.

Fig. 6 illustrates the dynamics of the Kendall’s \( \tau \) derivated from the time-varying Student-\( t \) copula for the US, Europe, and Hong Kong markets.\(^{23}\) In the case of the US stock market, the dependence parameter derivated from time-varying Student-\( t \) copula, unlike

\begin{table}
\centering
\caption{Time-varying copula specification and estimation.}
\begin{tabular}{lcccccc}
\hline
 & \text{Gaussian copula} & & & \text{Student-\( t \) copula} & & \\
 & \( \omega \) & \( \beta \) & \( \alpha \) & \( \omega \) & \( \beta \) & \( \alpha \) & \( \nu \) \\
\hline
S&P500 & 0.001 (0.004) & 1.813 (0.280) & 0.020 (0.028) & –0.120 (0.257) & –1.501 (0.639) & 2.368 (1.036) & 2.010 (0.293) \\
STOXX50 & 0.007 (0.054) & –0.279 (8.445) & –0.038 (0.217) & –0.010 (0.019) & 1.738 (0.218) & 0.143 (0.112) & 2.010 (0.296) \\
NIKKEI & –0.006 (0.010) & 1.696 (0.377) & 0.034 (0.045) & –2.263 (0.039) & 2.125 (0.331) & –0.541 (0.385) & 2.010 (0.000) \\
CSI300 & 0.036 (0.084) & 0.420 (3.332) & –0.048 (0.168) & 0.424 (0.531) & 1.945 (0.959) & 0.643 (0.361) & 2.010 (0.000) \\
HSI & 0.022 (0.068) & –1.076 (0.752) & –0.422 (0.222) & 3.429 (0.070) & –0.475 (0.020) & 6.374 (0.191) & 2.010 (0.000) \\
\hline
\end{tabular}
\end{table}

Notes: The table provides information on maximum likelihood parameter estimates and standard deviation (in parentheses) for the copula models in Eqs. (9) and (10). The standard deviation of copula parameters has been computed following the sandwich form presented in Patton (2013).

\begin{table}
\centering
\caption{Notes}
\begin{tabular}{l}
\hline
21 Curiously, the results obtained by Goodell and Goutteb (2020) move in another direction. Using continuous wavelet power spectrum and coherence methodologies, they examine the relationship between daily deaths from COVID-19 and the price of Bitcoin, finding a negative relationship between those two events.
\hline
22 These values correspond to the empirical percentiles 1% and 99% of the Kendall’s \( \tau \) series, respectively.
\hline
23 We exclude the analysis for Chinese and US markets because for these markets the Gaussian copula outperformed clearly the Student-\( t \) copula.
\hline
\end{tabular}
\end{table}

Cryptocurrencies is not exclusive to China, in other countries such as Hong Kong, Singapore, and South Korea, trading in Bitcoin has grown enormously.

The Student-\( t \) distributions, which fit well the data in many markets, indicate that the dependence between markets increases in extreme market conditions suggesting that Bitcoin’s role as a hedge asset might not work in a crisis period, becoming a simple diversifying asset.

Although we observe that the dependence structure is time-varying, the range of fluctuations is so narrow that the economic implications given by this model do not change regarding these given by the constant copula model. Thus, according to the time-varying Gaussian copula Bitcoin might act as a hedge asset against the movement in all the international stock markets.

The results provided by the elliptical constant copulas are similar to those provided by the linear correlations analysis with the difference that copula analysis (Student-\( t \)) provides additional information about the tail dependence.

This result is coherent with the facts observed in the recent crisis caused by the COVID-19 pandemic. During this period, the financial markets, including the Bitcoin market, moved in lockstep, with major joint declines. In fact, recent studies (Conlon et al., 2020; Conlon and McGee, 2020) show that incorporating Bitcoin into a well-diversified portfolio during the COVID-19 crisis has increased the risk assumed by investors.\(^{21}\)

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the Gaussian copula, experiments important changes along the period moving in a range of ± 0.33. Sixty percent of the days return a
negative value, and the rest positive. The dependence degree estimated for the European market is also very volatile, moving in a
range of (-0.15, 0.13) with 58 % of the days taking negative values. According to this model, the role of the Bitcoin as a
hedge asset against these two markets might fail on a high percentage of days.

In the case of the Hong Kong stock market, Kendall's $\tau$ exhibits a high degree of volatility moving in a range of (-0.17, 0.33),
being positive the majority of the days (86 %). This indicates that Bitcoin only acts as a diversifier asset against the movement of the
Hong Kong stock market. As noted above, this might be due to the strong presence of the Bitcoin industry in this country.

Fig. 5. Kendall’s tau from time-varying Gaussian copula.
Fig. 6. Kendall’s tau from time-varying Student-t copulas.
The time-varying copula analysis reveals that the dependence varies with time meaning that investors frequently change their portfolio structure. In addition, the time-varying Gaussian copula corroborates the results returned by the Gaussian constant copula. According to this copula, Bitcoin might act as a hedge asset against the stock price movements of all international stock markets analyzed. However, the time-varying student-t copula analysis reveals that for some markets, the role of the Bitcoin as a hedge asset might fail on a high number of days.

5. Conclusion

In this study, we analyzed the time-varying properties of the Bitcoin as a diversifier asset and a hedge asset against some international stock market indices movements. For this study, we have used daily data of Bitcoin and five stock market indices: S&P500 (US), STOXX50 (EU), NIKKEI (Japan), CSI300 (Shanghai), and HSI (Hong Kong). We choose stock market indices from different geographic areas with the idea that Bitcoin might play different roles in investment portfolios depending on specific market conditions. The analysis period runs from August 18, 2011 to June 31, 2019.

First, we analyzed some statistical properties of Bitcoin. The harvest results corroborate those reported in the literature. The Bitcoin return distribution is skewed to the left and exhibits a high degree of kurtosis, even higher than the stock market indices distribution. In all cases, the value of the Jarque-Bera statistic indicates the departure from normality. In a comparison of several distributions, Student-t, skewed Student-t, general error distribution (GED), and skewed GED, we find that the Student-t distribution is the best in fitting Bitcoin returns. Also, we observe that Bitcoin volatility tends to be higher after positive returns which is a characteristic of a safe haven asset such as gold. Addressing this question, we believe that additional research must be conducted.

To model the dependence structure between Bitcoin and the stock market indices, we examine several constant copula models: Gaussian, Student-t, Clayton, Gumbel, and Frank. We find that the elliptical copulas (Gaussian and Student-t) provide the best fit to the dependence structure between the considered return series.

According to the elliptical copula, we can conclude that in normal market conditions the Bitcoin might act as a hedge asset against the movement in all international stock markets considered, although, in the case of Hong Kong and China, the dependence is somewhat stronger than that observed for the rest of the markets. The Student-t copula, which fits well the data in many markets, indicates that the dependence between markets increases in extreme market conditions, indicating that Bitcoin’s role as a hedge asset might not work in a crisis period, becoming a simple diversifying asset. The synchronicity shown by Bitcoin and the international stock markets during an ongoing COVID-19 pandemic, where all have collapsed, is an example of this.

The results provided by the elliptical constant copulas are similar to those provided by the correlations analysis with the difference that copula analysis (Student-t) provides additional information about the tail dependence. Moreover, the time-varying copula analysis reveals that the dependence varies with time, meaning that investors frequently change their portfolio structure. In addition, the time-varying Gaussian copula corroborates the results returned by the Gaussian constant copula. According to this copula, Bitcoin might act as a hedge asset against the stock price movements of all international stock markets analyzed. However, the time-varying Student-t copula analysis reveals that for some markets, the role of the Bitcoin as a hedge asset might fail on a high number of days. These results should be taken into account by investors when making portfolio selections and planning asset allocation.

Last, we want to mention that this study has been done with pre-COVID-19 crisis data. To corroborate the findings obtained in this study, as part of future research, it will be interesting to explore the properties of Bitcoin during the COVID-19 pandemic, keeping in mind that this is the first period of significant turmoil in the financial markets since Bitcoin began trading.

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Appendix A

Scatter plots between the empirical copula (S&P500, STOXX50, NIKKEI, CSI300, and HSI) and the parametric copula (Clayton and Frank).
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