Instability of strong regular reflection and
counterexamples to the detachment criterion

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Abstract

We consider a particular instance of reflection of shock waves in self-similar compressible flow. We prove that local self-similar regular reflection (RR) cannot always be extended into a global flow. Therefore the detachment criterion is not universally correct. More precisely, consider the following angle condition: the tangent of the strong-type reflected shock meets the opposite wall at a sharp or right downstream side angle. In cases where the condition is violated and the weak-type reflected shock is transonic, we show that global RR does not exist. Combined with earlier work we have shown that none of the classical criteria for RR→MR transition is universally correct. A new criterion is proposed. Moreover, we have shown that strong-type RR is unstable, in the sense that global RR cannot persist under perturbations to one side. This yields a definite answer to the weak-strong problem because earlier work shows stability of weak RR in the same sense.

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1 Introduction

Consider compressible flow. In regular reflection (RR; see Figure 1) an incident shock wave meets a solid wall in a reflection point and continues as a second, reflected shock. In many circumstances the effects of viscosity, heat conduction,

Figure 1: Left: local regular reflection (RR). Center: shock polar ($\tau < \tau_*$, local RR possible). Right: $\tau > \tau_*$, local RR impossible).
boundary roughness etc. are negligible, so that inviscid models are appropriate; in this article we focus on compressible polytropic potential flow. Then shocks are sharp jumps satisfying Rankine-Hugoniot conditions and the **slip condition** is used at walls: \( \vec{v}_1, \vec{v}_3 \) must be tangential.

Consider a fixed constant state (velocity, density and sound speed) in the 2-sector and vary the angle of the reflected shock. Each angle yields a different 3-sector state. The curve of possible \( \vec{v}_3 \) is called **shock polar**. The maximum angle between \( \vec{v}_2 \) and \( \vec{v}_3 \) is called **critical angle**. If it is larger than \( \tau \) (Figure 1 center), the angle between \( \vec{v}_2 \) and wall, then there are two possible reflected shocks satisfying the slip condition, called **weak-type** (W) and **strong-type** (S).

There is no local argument to rule out one type; the Rankine-Hugoniot and slip conditions allow both. At least in initial-value problems we expect uniqueness, in nature and in good mathematical models. For this we need to consider the **global flow** that contains the reflection, in particular domain shape and far-field/boundary conditions far from the reflection point. Of course there is an infinite variety of such flows, but some observations and arguments apply to most if not all of them.

If \( \tau > \tau_* \), on the other hand, then even locally RR is theoretically impossible because none of the reflected shock angles can make \( \vec{v}_3 \) parallel to the wall (Figure 1 right). Around 1875, Ernst Mach [10] discovered another pattern, now named **Mach reflection** (MR; see Figure 6 left), where incident and reflected shock meet off the wall in a **triple point** with a third shock, the Mach stem. For some parameters both RR and MR are possible. Starting with John von Neumann [11], many researchers have tried to predict the precise parameters at which the RR→MR transition takes place (see [1, 2] for a survey of this and other problems in shock reflection).

There are three classical transition criteria. The **von Neumann criterion** does not apply in potential flow at all. The **detachment criterion** predicts global RR whenever a local RR exists. The **sonic criterion**, in contrast, predicts global RR if and only if there is a local RR with supersonic reflected shock. All three criteria are motivated by local considerations and well-defined for any global problem; of course the same criterion need not be correct for all global problems. However, we make a stronger observation: in a particular global problem, **none** of the classical criteria is correct, so that an entirely new criterion must be found. (The most promising candidates are modifications of the detachment criterion.)

To define our problem we add a second solid wall that meets the original wall right of the reflection point (Figure 2 left), enclosing a corner angle \( 180^\circ - \theta \). To satisfy the slip condition in the constant-state 2-sector, the opposite wall has to move with horizontal speed \( \vec{w} = \vec{w}(\theta) \) so that \( \vec{v}_2 \cdot \vec{n} = \vec{w} \cdot \vec{n} \) (\( \vec{n} \) wall normal).

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1 for experimental examples see [14, p. 142f]
2 The names refer to their relative strength, but the absolute strength can be arbitrarily small or large.
3 In fact almost all flows with shocks include some form of shock reflection.
4 Even in Euler flow it applies only for sufficiently high Mach numbers.
5 which is necessarily weak-type
There is exactly one $\theta$ so that the new wall is perpendicular to the reflection-point tangent of the strong-type reflected shock (Figure 2 left). In this case, $\vec{v}_1 \cdot \vec{n} = \vec{w} \cdot \vec{n}$ as well, so the fluid in the 1-sector is also compatible with the wall. The result is what we call a trivial RR.

However, for any other $\theta$ the reflected shock would have to be curved (and border a non-constant region on its right), because its reflection point tangent does not form a right angle with the new wall. So there is a large variety of nontrivial cases; each has the same incident and reflected shock, but $\theta$ and $\vec{w}$ vary.

Alternatively, we may consider the coordinate system of an observer travelling in the wall-wall corner. He observes steady walls but moving shocks (Galilean invariance). Moreover, use reflection so that the new (opposite) and old (reflection) wall change places (Figure 3).

Let $\alpha$ be the counterclockwise angle from incident shock to opposite wall in Figure 3 left. We have a family of problems, with parameter space consisting of triples $(M_1, \alpha, \theta)$. At time $t = 0$ the incident shock starts in the wall-wall corner (Figure 3 left).

Such reflections occur in practice (Figure 4). Experimentally a (nearly) straight vertical shock could be produced by breaking diaphragms or detonating small charges. This shock (Figure 4 left) travels to the right through a tube, meeting the lower corner at some time. A first reflection occurs (Figure 4 left center). It is the classical case $\alpha = 0^\circ$, $\theta < 90^\circ$ which has been studied extensively [4, 9, 14, 3]. The reflected shock travels up the wall, reaching a second corner at $t = 0$. In that instant, the local flow near the upper corner is the same as the initial data in Figure 3 left.

[5] has already obtained global weak-type transonic RR for small perturbations of the trivial $\theta$, in the following class:

**Definition 1.** Consider self-similar potential flow (see Section 2). A transonic (or sonic) global RR (see Figure 5 left) has a straight incident shock extending to infinity, meeting the reflected shock in a single reflection point on the reflection wall. The incident shock separates the 1- and 2-sector, two regions of constant
fluid state $\rho, c, \vec{v}$. The reflected shock is $C^1$ including the endpoints, separating the 2- from the 3-sector, meeting the opposite wall in a right angle. Flow in the interior of the 3-sector is elliptic (pseudo-Mach number $L < 1$, see (10)), with continuous fluid variables.

The sonic criterion, in any reasonable precise formulation, predicts non-existence (and appearance of MR), so \[5\] demonstrates that it cannot be universally correct. The present paper considers the case of strong-type RR.

**Definition 2.** We say $\theta$ satisfies the angle condition if the reflection-point tangent of the strong-type reflected shock forms an angle $\leq 90^\circ$ (Figure 2 left and center) on its downstream side with the opposite wall.

**Theorem 1.** Consider parameters $M_1, \alpha, \theta$ so that local weak-type transonic RR exists, but the angle condition (Definition 2) is violated. Then global RR solutions of the kind in Definition 4 do not exist (neither weak-type nor strong-type).

**Corollary 3.** The following version of the detachment criterion is not universally correct:

\[6\]

It is violated for a subset of the parameter space which is open and nonempty, hence
Generically\(^6\), when local RR exists, either weak- or strong-type can be extended into a global RR.

Proof. It is sufficient to give a rigorous proof of existence of local RR satisfying the conditions of Theorem \(^1\) take \(\vec{x}/t = 0\) to be the reflection point. Choose some supersonic 1-sector state. Then for sufficiently small \(\tau\) (Figure \(^\|\) left) we can find a weak-type incident shock and a 2-sector state with \(M_2 > 1\), as well as a strong-type reflected shock with \(\vec{v}_3\) parallel to the wall. Choose an opposite wall whose extension to a line passes through the point \(\vec{x}/t = \vec{v}_2\) (so that the slip condition \((\|\) in the 2-sector is satisfied). If the angle between the two walls is chosen small enough, then the angle condition is violated. We can choose this local RR transonic as follows: by Proposition \(^5\) applied to the incident shock polar, for sufficiently large \(\tau\), \(M_2 \downarrow 1\), so \(\tau_+ \downarrow 0\) for the corresponding reflected shock polar (Proposition \(^5\)). For \(\tau \approx \tau_+\), \(M_3 < 1\) which necessarily happens as \(\tau\) grows.

Corollary 4. Strong-type trivial RR is not always structurally stable.

Proof. Given \(M_1, \alpha, \theta\) for a strong-type trivial RR, perturb to \(\alpha - \delta, \theta + \delta\) for some small \(\delta > 0\). Then we are in the situation of Theorem \(^1\) where global RR cannot exist.

In summary, we have obtained two separate results. First, the detachment criterion is not universally correct. Note however that we have discussed only some cases with \(\theta > 90^\circ\). It would be interesting to find extensions to the classical case \(\theta < 90^\circ = \alpha\); in that case, the detachment criterion is probably correct. A new RR→MR transition criterion is proposed in Section \(\#\).

Second, while weak-type transonic trivial RR is structurally stable, strong-type is not (Corollary \(^3\)). This provides an important new answer to the weak-strong problem. Note that historically, dynamic stability, i.e. under perturbation of the initial data, has been considered. \(\|\) observes numerically that both weak- and strong-type reflection are dynamically stable, so any mathematical result to the contrary appears to use an overly restrictive definition of stability.

2 Potential flow

Self-similar potential flow is the second-order quasilinear PDE

\[
\nabla \cdot (\rho \nabla \chi) + 2\rho = 0. \tag{1}
\]

Here

\[
\chi = \psi - \frac{1}{2} |\xi|^2. \tag{2}
\]

"generic" by any reasonable definition.
\[ \vec{\xi} = (\xi, \eta) = \vec{x}/t \] are similarity coordinates. \( \chi \) is called pseudo-potential. \( \psi \) is the velocity potential: physical velocity is
\[ \vec{v} = \nabla \psi. \]

Moreover, density is
\[ \rho = \pi^{-1}(-\chi - \frac{1}{2}|\nabla \chi|^2). \tag{3} \]
\( \pi \) satisfies
\[ \frac{d\pi}{d\rho} = \rho^{-1} \frac{dp}{d\rho} = \rho^{-1}c^2, \quad (\pi^{-1})' = \rho c^{-2} \tag{4} \]
where
\[ p(\rho) = \rho_0 c_0^2 \left( \frac{\rho}{\rho_0} \right)^\gamma \]
is the equation of state \((\rho_0, c_0 \text{ free parameters})\). The ratio of heat is restricted to \( \gamma \in (1, \infty) \). Differentiation of (1) yields the non-divergence form
\[ (c^2 I - (\nabla \psi - \vec{\xi})^2) : \nabla^2 \psi = 0. \tag{5} \]

Here \( A : B \) is the Frobenius product \( \text{tr}(A^T B) \), \( \vec{w}^2 := \vec{w} \otimes \vec{w} = \vec{w} \vec{w}^T \) (as opposed to \( |\vec{w}|^2 = \vec{w} \cdot \vec{w} \)) and \( \nabla^2 \) is accordingly the Hessian. In coordinates:
\[ (c^2 - (\psi_\xi - \xi)^2)\psi_\xi\xi - 2(\psi_\xi - \xi)(\psi_\eta - \eta)\psi_\eta + (c^2 - (\psi_\eta - \eta)^2)\psi_\eta\eta = 0. \tag{6} \]

\( c \) is the sound speed, defined by
\[ c^2 = c_0^2 + (1 - \gamma)(\chi + \frac{1}{2}|\nabla \chi|^2). \tag{7} \]

It is sometimes more convenient to use the form
\[ (c^2 I - \nabla \chi^2) : \nabla^2 \chi + 2c^2 - |\nabla \chi|^2 = 0. \tag{8} \]

This form is manifestly translation-invariant. Translation is nontrivial: in \((t, x, y)\) coordinates it corresponds to a change of inertial frame
\[ \vec{v} \leftarrow \vec{v} - \vec{w}, \quad \vec{\xi} \leftarrow \vec{x}/t \leftarrow \vec{\xi} - \vec{w}, \tag{9} \]
where \( \vec{w} \) is the velocity of the new frame relative to the old one. Obviously the pseudo-velocity
\[ \vec{z} := \nabla \chi = \nabla \psi - \vec{\xi} \]
do not change.

\[^7\text{also: isentropic coefficient}\]
Self-similar potential flow is mixed-type; the local type is determined by the coefficient matrix \( c^2 I - \nabla \chi \) which is positive definite if and only if \( L < 1 \), where

\[
L := \left| \frac{\tilde{z}}{c} \right| = \frac{|\tilde{v} - \tilde{x}/t|}{c} \tag{10}
\]

is called pseudo-Mach number. For \( L > 1 \) the equation is hyperbolic; parabolic is \( L = 1 \). \( L \) and \( \tilde{x} \) are the Mach number and velocity perceived by an observer traveling on the ray \( \tilde{x} = t\tilde{\xi} \).

On a solid wall the slip condition

\[
\nabla \chi \cdot \tilde{n} = 0 \tag{11}
\]

holds; for an observer traveling on the wall it corresponds to the usual

\[
\tilde{v} \cdot \tilde{n} = \nabla \psi \cdot \tilde{n} = 0 \tag{12}
\]

### 3 Shock conditions

The weak solutions of potential flow are defined by \( [\psi] \). The corresponding Rankine-Hugoniot condition is

\[
\rho u z^n_u = \rho d z^n_d \tag{13}
\]

where \( u, d \) indicate the limits on the upstream and downstream side and \( z^n, z^t \) are the normal and tangential component of \( \tilde{z} \). As the equation is second-order, we must additionally require continuity of the potential:

\[
\psi_u = \psi_d. \tag{14}
\]

By taking a tangential derivative, we obtain

\[
z^t_u = z^t_d =: z^t. \tag{15}
\]

Observing that \( \sigma = \tilde{\xi} \cdot \tilde{n} \) is the shock speed, we obtain the more familiar form

\[
\rho u v^n_u - \rho d v^n_d = \sigma (\rho_u - \rho_d), \tag{16}
\]

\[
v^t_u = v^t_d =: v^t. \tag{17}
\]

Fix the unit shock normal \( \tilde{n} \) so that \( z^n_u > 0 \) which implies \( z^n_d > 0 \) as well. To avoid expansion shocks we must require the admissibility condition \( z^t_u \geq z^t_d \), which is equivalent to

\[
v^n_u \geq v^n_d. \tag{18}
\]

We choose the unit tangent \( \tilde{t} \) to be \( 90^\circ \) counterclockwise from \( \tilde{n} \).

By \( [\tilde{v}] \) the tangential components of the velocity are continuous across the shock, so the velocity jump is normal. Assuming \( v^n_u > v^n_d \) (positive shock strength), we can express the shock normal as

\[
\tilde{n} = \frac{\tilde{v}_u - \tilde{v}_d}{|\tilde{v}_u - \tilde{v}_d|} \tag{19}
\]
Figure 5: Left: transonic global RR, angle condition satisfied. Self-similar potential flow is elliptic in $\Omega$, hyperbolic elsewhere. Right: Shock polar argument.

4 Nonexistence of some global RR

We start with some facts about the shock polar.

**Proposition 5.** Consider arbitrary $c_u, \rho_u > 0$ and $M_u \in (1, \infty)$ and set $\vec{v}_u = (M_u c_u, 0)$. For each $\beta \in (-90^\circ, 90^\circ)$ there is a steady shock with downstream unit normal $\vec{n} = (\cos \beta, \sin \beta)$. Its downstream data depends smoothly on $\beta$. Let $\tau$ be counterclockwise angle from $\vec{v}_u$ to $\vec{v}_d$. We restrict $|\beta| < \arccos \frac{1}{M_u}$ so that the shock is admissible.

Then the shock polar $\beta \mapsto \vec{v}_d$ is smooth and strictly convex, with $\partial_\beta \vec{v}_d$ nowhere zero.

There is an angle $\tau_\ast \in (0, 90^\circ)$ so that each $\tau \in (-\tau_\ast, \tau_\ast)$ is attained for two different $\beta$. The one with smaller $|\vec{v}_d|$ yields a strong-type shock, the other one weak-type. For $|\tau| = \tau_\ast$ they are identical and critical-type.

There is a $\tau_\ast \in (0, \tau_\ast)$ so that the weak-type shocks are supersonic for $|\tau| > \tau_\ast$, transonic for $|\tau| < \tau_\ast$. The other types are always transonic.

If $M_u \downarrow 1$ with $\rho_u, c_u$ fixed, then $\tau_\ast \downarrow 0$.

**Proof.** Most has been shown in [4, Theorem 1] and [8, Proposition 2.10]; we only need to prove the last statement. Admissible shocks are those for $|\beta| \leq \arccos \frac{1}{M_u}$. As $M_u \downarrow 1$, this range shrinks to $\{0\}$. By continuity, all points on the shock polar approach $\vec{v}_u$. In particular $\tau_\ast \downarrow 0$. \qed

Let $\Omega$ be the 3-sector excluding boundary (Figure 5 left), $A$ opposite wall, $B$ reflection wall, $S$ reflected shock, each not containing its endpoints.

**Proposition 6.** Consider the setting of Theorem 4, with $(0, 0)$ the wall-wall corner (see Figure 5). The vertical straight shock with upstream data $\vec{v}_2, \rho_2, c_2$ through the reflection point $\xi_R$ has a downstream velocity $\vec{v}_0 = (v_0^x, 0)$ with $v_0^x > 0$. The same holds for all vertical shocks to the right of it.
Proof. First change to a coordinate frame with origin in the reflection point. For this observer the 2-sector velocity is \( \vec{v_2} - \vec{\Xi_R} \), which points into the reflection wall (Figure 5 right). Consider shocks with that upstream velocity and upstream density \( \rho_2 \) and sound speed \( c_2 \); let \( \vec{v_d} - \vec{\Xi_R} \) be the downstream velocity. Let \( \beta \) be the counterclockwise angle from \( \vec{v_2} - \vec{\Xi_R} \) to shock downstream normal \( \vec{n} \); by (19) \( \vec{n} \) is a positive multiple of \( \vec{v_2} - \vec{v_d} \).

By design (slip condition), the velocities \( \vec{v_w} - \vec{\Xi_R} \) and \( \vec{v_s} - \vec{\Xi_R} \) for weak-type and strong-type reflected shock are on the extension of the reflection wall into a line (see Figure 5 right). By assumption of Theorem 1 the angle condition is violated, so the strong-type and therefore the weak-type reflected shock tangent in the reflection point are down and strictly right. Thus the vertical shock through the reflection point has smaller \( |\beta| \) than either type, so by strict convexity of the shock polar (Proposition 5) \( \vec{v_0} - \vec{\Xi_R} \) points into the reflection wall (see Figure 5). Therefore \( v_x^0 > 0 \), since \( \vec{v_2} \) and thus \( \vec{v_0} \) are horizontal.

By [8, Proposition 2.9], vertical shocks more to the right have \( v_x^d - \xi_R > v_x^0 - \xi_R \) (because they are weaker, so \( \vec{v_d} \) is closer to \( \vec{v_2} \)). Hence \( v_x^d > v_x^0 > 0 \) as well.

**Proof of Theorem 1.** Consider the same coordinates as in the statement of Proposition 6. Restrict \( \psi \) to \( \bar{\Omega} \) (taking its \( \Omega \)-side limits on \( \partial \Omega \)). Let \( \psi_0 \) be the value of \( \psi \) in the reflection point \( \vec{\Xi_R} \). Let \( S_0 \) be the straight vertical shock through the reflection point; let \( \sigma_0 \) be its \( \xi \) coordinate.

Consider a transonic global RR. Again by assumption the angle condition does not hold, so the reflection point shock tangent points down and strictly right (as in Figure 3 right, as opposed to Figure 5 left). The upstream velocity \( \nabla \psi = \vec{v_2} \) has \( \psi_x = v_x^2 > 0 \), so necessarily \( \psi > \psi_0 \) at the shock near the reflection point \( \vec{\Xi_R} \). Therefore, the global maximum of \( \psi \) over \( \bar{\Omega} \) (which must be attained since \( \bar{\Omega} \) is compact and \( \psi \) continuous) is greater than \( \psi_0 \) and not attained in \( \vec{\Xi_R} \).

Consider a maximum \( \psi \geq \psi_0 \) in a point \( \xi \in S - \{\vec{\Xi_R}\} \). The shock tangent is vertical in \( \vec{\xi} \) (by \( \psi_t = 0 \) for a maximum at \( S \); by the slip condition (12) at \( A \) for a maximum in the point where \( S \) meets \( A \)). Moreover, \( \psi > \psi_0 \) implies the shock is right of the vertical reflection point shock because \( \psi \) is continuous across the shock and \( \psi_x = v_x^2 > 0 \) on the upstream side. Hence by Proposition 6 \( \psi_t = v_x^2 \geq \psi_0^2 > 0 \). This is incompatible with a local maximum.

Hence \( \psi \) does not attain its \( \bar{\Omega} \)-maximum anywhere on \( \bar{S} \). Then the same is true for

\[
\hat{\psi} := \psi + \delta \xi
\]

if we choose \( \delta > 0 \) sufficiently small. By linearity

\[
(I - c^{-2} \nabla \chi^2) : \nabla^2 \hat{\psi} = (I - c^{-2} \nabla \chi^2) : \nabla^2 \psi = 0,
\]

so by the strong maximum principle the maximum is not attained in \( \bar{\Omega} \) either (if we choose \( \delta > 0 \) so small that \( \psi \), like \( \psi \), cannot be constant). Moreover

\[
\nabla \hat{\psi} \cdot \vec{n} = \nabla \psi \cdot \vec{n} + \delta \hat{n}^x = \delta n^x \geq 0
\]

9
on $A$ and $B$, so the Hopf lemma rules out local maxima there.

Finally, the boundary conditions on $A$ and $B$, combined with $C^1$ continuity in 0 (Definition 1), imply $\nabla \psi(0) = 0$, so $\hat{\psi}_\xi(0) = \psi_\xi(0) + \delta > 0$, thus a local maximum in 0 is impossible.

We have ruled out every possible global maximum point in $\Omega$. The contradiction demonstrates that no $\psi$ with the desired properties exists.

\section{Numerical comparison}

Theorem 1 concerns the range of parameters with transonic weak-type RR, which is so narrow (see Figure 7 right) that numerics and experiments have not been able to settle questions for these flows. However, the range with supersonic weak-type RR violating the angle condition is much larger and certainly interesting by itself.

For $\gamma = 7/5$, $M_1 \approx 3$ and $\alpha = 0^\circ$, $\theta = 142.9^\circ$ corresponds exactly to a strong-type trivial RR (i.e. strong-type shock perpendicular to opposite wall). We change $\theta$ by $5^\circ$ to $147.9^\circ$ without changing $\theta + \alpha$ or $M_1$. This way the opposite wall angle changes, but not the local RR parameters. The numerical results in Figure 6 left show an MR.

We also study the opposite perturbation, to $\theta = 137.9^\circ$ (see Figure 6 right). As expected there is still a local RR. The shock is essentially the strong-type shock, except in a small neighbourhood of the reflection point where it is weak-type and slightly hyperbolic. As $\theta \uparrow 142.9^\circ$, this neighbourhood shrinks to zero; it appears that the pattern converges to the trivial strong-type RR in this manner. This is why strong-type reflections are observed at a large scale sometimes. Note that there is a MR as well: at the transition from hyperbolic to elliptic.

The calculations were made with a second-order scheme on an unstructured grid; other choices have no influence on the qualitative structure (RR vs. MR).
Figure 7: Proposed RR→MR transition, for $\gamma = 7/5$ potential flow with $\alpha = 0^\circ$. Right: detail. Weak-type reflection is supersonic above the “sonic” curve, transonic below; neither type exists below the “detach” curve. Theorem 1 rules out global RR below the solid curve.

In principle Definition 1 and Theorem 1 could be extended to the supersonic cases. But while in the transonic case the flow is simple and predictable, at least for small perturbations from trivial RR, the supersonic cases can have several different qualitative structures. [4, 6] construct self-similar RR with a continuous transition from hyperbolic to elliptic in the 3-sector, but Figure 6 right shows a MR, i.e. a discontinuous transition; double Mach reflection and other more complicated flows are possible too. Proving nonexistence in function classes large enough to accommodate all these structures is far beyond present-day techniques.

6 Interpretation

Despite the theorem and numerical examples, it is likely that the detachment criterion is still valid over a large part of the parameter space. In particular, the author believes that it is correct in the classical case $\alpha = 90^\circ$, $\theta < 90^\circ$. We propose the following new criterion:

The global flow is RR if and only if local RR exists and angle condition is satisfied.

Strong-type RR would appear only in the trivial right-angle borderline case separating global RR and global MR.

Figure 7 shows the regions predicted by this criterion for $\gamma = 7/5$ and $\alpha = 0^\circ$. Cases that have already been treated by construction of an exact solution or another rigorous method:

1. Nonexistence of global RR below the detachment criterion is trivial.
2. Nonexistence of global RR below the solid curve is done in this article for transonic weak-type RR by Theorem 1.
3. Existence of global transonic RR is done by [5] for some neighbourhood of each point on the transonic part of the “weak trivial” curve in Figure 7 (excluding endpoints).

4. [4, 6] construct global supersonic RR for some of the supersonic parameters, in particular some neighbourhood of each point of the supersonic part of the “weak trivial” curve in Figure 7 (excluding endpoints).

In principle, [4, 8, 6, 5] go a long way towards constructing global RR in all cases not covered so far. In comparison, global MR is very difficult: the triple point, well-known to be theoretically impossible (von Neumann paradox), has a very complicated detail structure, according to numerical results of Hunter/Tesdall ([9, 13]), see also [15, 12]).

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