Acquire information about neutrino parameters by detecting supernova neutrinos

Ming-Yang Huang\(^{1}\)*, Xin-Heng Guo\(^{1}\), and Bing-Lin Young\(^{2,3} \)‡

\(^{1}\)College of Nuclear Science and Technology, Beijing Normal University, Beijing 100875, China
\(^{2}\)Department of Physics and Astronomy, Iowa State University, Ames, Iowa 5001, USA
\(^{3}\)Institute of Theoretical Physics, Chinese Academy of Sciences, Beijing 100190, China

We consider the supernova shock effects, the Mikheyev-Smirnov-Wolfenstein (MSW) effects, the collective effects, and the Earth matter effects in the detection of type II supernova neutrinos on the Earth. It is found that the event number of supernova neutrinos depends on the neutrino mass hierarchy, the neutrino mixing angle \(\theta_{13}\), and neutrino masses. Therefore, we propose possible methods to identify the mass hierarchy and acquire information about \(\theta_{13}\) and neutrino masses by detecting supernova neutrinos. We apply these methods to some current neutrino experiments.

PACS numbers: 14.60.Pq, 13.15.+g, 25.30.Pt, 26.30.-k

I. INTRODUCTION

Supernovas (SNs) are extremely powerful explosions in the universe which terminate the life of some stars [1]. They make the catastrophic end of stars more massive than 8 solar masses, leaving behind compact remnants such as neutron stars or black holes. The SN explosion, such as SN1987A [2], is one of the most spectacular comic events and the source of new physics ideas [3]. Detecting SN neutrinos on the Earth has been a subject of active investigation in astroparticle physics [1]-[8]. We may obtain some information about the explosion mechanism of SN [5] and the parameters of neutrinos [4][6][7][8] by detecting SN neutrinos on the Earth. Several neutrino experiments, including the Daya Bay reactor neutrino underground laboratory [9], can be used to detect neutrino events from a SN explosion and serve as a SN Early Warning System [10].

Recently, several methods to obtain information on neutrino parameters by the study of SN neutrinos have been proposed. In Ref. [6], Dasgupta et al. proposed a method to identify the neutrino mass hierarchy for very small \(\theta_{13}\) (\(\sin^{2} \theta_{13} \leq 10^{-5}\)) through the Earth matter effects. In Ref. [7], Skadhaug et al. proposed methods to identify the neutrino mass hierarchy for larger \(\theta_{13}\) (\(\sin^{2} \theta_{13} \geq 10^{-4}\)) and obtain information about \(\theta_{13}\) by SN neutrinos. In Ref. [8], using the event number of the delayed SN neutrinos, a method to measure neutrino masses was proposed by Beacom et al. In these works, the SN shock effects and the collective effects were not taken into account. Furthermore, uncertainties of some of the parameters appearing in SN neutrino fluxes, such as the luminosities \(L_{0}\), temperatures \(T_{\alpha}\), and pinching parameters \(\eta_{\alpha}\), were not considered.

In Ref. [11], we studied the Earth matter effects in the detection of SN neutrinos at the Daya Bay experiment and proposed a possible method to acquire information about the neutrino mixing angle \(\theta_{13}\) smaller than 1.5° which is difficult to access in reactor neutrino experiments. We defined the ratio \(R\) as the event number of \(\nu_{e}\) over that of \(\bar{\nu}_{e}\). Using the relation between the event number of SN neutrinos \(N\) and \(\theta_{13}\), we can obtain information about small \(\theta_{13}\) by measuring \(R\). In that paper, a simplified picture for the collective effects was used [12] and the neutrino flavor transformation due to the SN shock effects was not considered. Furthermore, it was assumed that different flavor SN neutrinos have the same time-independent luminosity. Hence in that calculation there are several uncertainties which we will address in the present work.

In this paper, we study the collective effects more accurately [13][14] and consider the neutrino flavor transformation arising from the SN shock effects [15][17][17]. In addition, according to current Monte Carlo simulations on SN neutrino spectra [18], we allow different flavor neutrinos to have different time-dependent luminosities. Furthermore, in order to reduce the uncertainties of unknown parameters in SN neutrino fluxes such as luminosity, temperature, and pinching parameter, we will define the ratio of the event number of SN neutrinos for the charged-current reaction detected after one second over that detected before one second (this is different from the ratio considered in Ref. [11]). With this ratio, we may identify the mass hierarchy, \(\Delta m^{2}_{31} > 0\) vs \(\Delta m^{2}_{31} < 0\), and acquire information about \(\theta_{13}\). This method can be applied to neutrino detectors of various types. Examples are the liquid scintillator detector of the Daya Bay experiment [9] under construction, the decommissioned heavy water detector of the previous Sudbury Neutrino
The other main purpose of the present work is to propose a method to obtain some information about neutrino masses from SN neutrinos. Since neutrinos have small masses, their transmit velocities are smaller than that of light. Therefore, a massive neutrino will have an energy and mass dependent delay relative to a massless neutrino after traveling over a long distance. We will define another ratio, for the neutral-current reactions, as the event number of the delayed SN neutrinos over the total event number. With the relation between this ratio and neutrino masses, we will propose a possible method to acquire information about neutrino masses. This method will also be applied to Daya Bay and SNO for examples.

The paper is organized as follows. In Sec. II, we review the detection of SN neutrinos on the Earth. In Sec. III, we propose a possible method to identify the mass hierarchy and acquire information about \( \theta_{13} \) and apply the method to the Daya Bay and SNO experiments. In Sec. IV, a method to acquire information about neutrino masses is given and also applied to Daya Bay and SNO. A summary is given in section V.

II. DETECTION OF SN NEUTRINOS ON THE EARTH

In the explosion a typical type II SN generates intensive neutrinos which carry about 99% of the total energy and the explosion itself consumes about 1% of the total energy. The vast amount of neutrinos are produced in two bursts. In the first burst which lasts for only a few milliseconds, electron neutrinos are generated by the electron capture by nuclei \( e^- + N(Z, A) \rightarrow N(Z-1, A) + \nu_e \) via the inverse beta decay \( e^- + p \rightarrow n + \nu_e \) which leads to a neutron rich star. In the second burst which lasts longer (about 10 seconds), neutrinos of all flavors are produced through the electron-positron pair annihilation \( e^- + e^+ \rightarrow \nu_e + \bar{\nu}_e \), the electron-nucleon bremsstrahlung \( e^\pm + N \rightarrow e^\pm + N + \nu_\alpha + \bar{\nu}_\alpha \), the nucleon-nucleon bremsstrahlung \( N + N \rightarrow N + N + \nu_\alpha + \bar{\nu}_\alpha \), the plasmon decay \( \gamma \rightarrow \nu_\alpha + \bar{\nu}_\alpha \), and the photoannihilation \( \gamma + e^\pm \rightarrow e^\pm + \nu_\alpha + \bar{\nu}_\alpha \).

We assume a "standard" SN explosion at a distance \( D = 10kpc \) from the Earth, releasing a total energy \( E_B = 3 \times 10^{53} \text{erg} \) (similar to SN1987A) and the luminosity flux of the SN neutrino \( \alpha \), \( L_\alpha \), distributed in time as

\[
L_\alpha(t) = L_\alpha^0 e^{-t/\tau},
\]

where \( L_\alpha^0 \) is the luminosity of the neutrino \( \alpha \) at \( t = 0 \) and \( \tau \) the luminosity decay time. The following range of \( \tau \) was obtained by fitting the the experimental data of SN1987A with the exponentially decaying luminosity:

\[
\tau = 1.74 - 4.19 \text{s}.
\]

The typical relations among \( L_\alpha^0 \) obtained from numerical simulations are

\[
\frac{L_\nu_e^0}{L_\nu_x^0} = (0.5 - 2), \quad L_\nu_e^0 = L_{\bar{\nu}_e}^0, \quad L_{\nu_x}^0 = L_{\bar{\nu}_x}^0, \quad (x = \mu, \tau).
\]

There are two parametrizations of SN neutrino fluxes used in general simulations, one used by the Livermore group and the other by the Garching group. In the present work, we use the Livermore parameterization which is similar to the Fermi-Dirac distribution. For the SN neutrino of flavor \( \alpha \), the time-integrated neutrino energy spectra is given by

\[
F_\alpha^{(0)}(E) = \frac{L_\alpha}{F_{\alpha,0} T_\alpha} \frac{E^2}{\exp\left(E/T_\alpha - \eta_\alpha\right) + 1},
\]

where \( E \) is the neutrino energy, \( T_\alpha \) the temperature of the neutrino \( \alpha \), \( \eta_\alpha \) the pinching parameter of the spectrum, \( L_\alpha \) the luminosity given by Eq. (1), and \( F_{\alpha,j} \) is defined by

\[
F_{\alpha,j} = \int_0^{\infty} \frac{x^j}{\exp(x - \eta_\alpha) + 1} \, dx,
\]

where \( j \) is an integer. The spectra obtained from numerical simulations can be well fitted by

\[
T_{\nu_e} = 3 - 4 \text{MeV}, \quad T_{\bar{\nu}_e} = 5 - 6 \text{MeV}, \quad T_{\nu_x} = T_{\bar{\nu}_x} = 7 - 9 \text{MeV}, \quad \eta_{\nu_e} \approx 3 - 5, \quad \eta_{\nu_x} \approx 2.0 - 2.5, \quad \eta_{\bar{\nu}_e} = \eta_{\bar{\nu}_x} \approx 0 - 2.
\]

While neutrinos propagate outward to the surface of the SN they could be subjected to the SN shock effects, the MSW effects, and the collective effects. Then, after travelling the cosmic distance to reach the Earth, they go through a part of the Earth and are subjected to the Earth matter effects. In the following, we will consider the above four types of effects in detail.
A. Time dependence of the signal: SN shock effects

As was pointed out in Ref. [16], the shock propagating inside the SN may reach the region of densities relevant for neutrino conversion during the period of neutrino emission. It modifies the density profile of the star, thus affecting neutrino flavor transitions after the core bounce [16,17]. The time-dependent variations of the neutrino potential in matter along with the SN shock profile can leave distinctive imprints on the energy and time structure of the neutrino signal. The study of such effects may provide us with additional constraints for the determination of the neutrino mass hierarchy, the neutrino mixing angle $\theta_{13}$, and neutrino masses.

In Ref. [17], the authors introduce a simplified, empirical parametrization of the shock density profile $\rho$, which reproduces the main features of the graphical profile of Ref. [16] and is continuous in both $t$ and the supernova radial coordinate $r$. In this way, numerical and analytical calculations of the crossing probability at the high resonance region $P_H$ can continuously cover the relevant parameter space. We follow the notation of Ref. [17] in the discussion below.

For post-bounce time $t < 1 s$ [44] before the shock effects begin, the SN matter density is

$$\rho_0(r) \simeq 10^{14} \cdot \left(\frac{r}{1 \text{ km}}\right)^{-2.4} \text{ g/cm}^3. \quad (6)$$

For $t \geq 1 s$ the shock effects set in and the matter density is given by

$$\rho(r,t) = \rho_0(r) \cdot \begin{cases} \xi \cdot f(r,t) & (r \leq r_s), \\ 1 & (r > r_s), \end{cases} \quad (7)$$

where $f(r,t)$ is defined as

$$f(r,t) = \exp\{[0.28 - 0.69 \ln(r_s/km)][\arcsin(1 - r/r_s)]^{1.1}\}, \quad (8)$$

and $\xi$ is a typical ratio of the potential across the shock front,

$$\xi = V_+/V_- \simeq 10, \quad (9)$$

which measures the SN matter potential $V(r)$ drop from

$$V_+ = \lim_{r \to r_+^{-}} V(r) \quad (10)$$

to

$$V_- = \lim_{r \to r_+^{+}} V(r). \quad (11)$$

The SN matter potential is related to the SN electron density $N_e(r)$ by

$$V(r) = \sqrt{2} G_F N_e(r) = \sqrt{2} G_F N_A \rho(r) Y_e, \quad (12)$$

where $N_A$ is the Avogadro’s number and $Y_e$ is the electron fraction. In the numerical calculations, we assume $Y_e = 0.5$.

In Eqs. (6)-(9), a slightly accelerating shock-front position $r_s$ is assumed with the explicit time dependence,

$$r_s(t) = -4.6 \times 10^3 + 1.13 \times 10^4 \cdot t + 1 \times 10^2 \cdot t^2, \quad (13)$$

where $r_s$ is in units of $km$ and $t$ in units of $s$.

Using Eqs. (6)-(9), we plot in Fig. 1 the density profile of the SN as a function of the radius at different times. These curves reproduce reasonably well the main features of the shock profiles as shown in Ref. [16]. The straight line labelled by $t = 0$ is the unmodified density profile given by Eq. (6). Once the shock effects begin, at a given time the shock radius $r_s$ is determined by Eq. (13). The density profile is suppressed more and more in the region $r < r_s$ as the time and therefore $r_s$ increase. The density reaches a minimum and then rises relatively quickly to the value $\xi \rho_0(r_s)$ as $r \to r_s^{-}$. In the region $r > r_s$ the density profile is the unperturbed original form $\rho_0$. The density suffers a sizable discontinuity at $r_s$. For neutrinos emitted in the region $r > r_s$, due to their much higher velocities, which is the order

1 It can be shown that $f(r_s,t) = 1$ and therefore the discontinuity is $(\xi - 1)\rho_0(r_s) \approx 9\rho_0(r_s)$. 

of speed of the light, than that of the shock wave propagation, which is the order of $10^4$ km/s, they are not influenced by the shock effects. Hence the majority of SN neutrinos which are emitted within a 10 second period will be able to semble the whole range of the profile of the SN density under the influence of the shock. An important consequence of the density modification by the shock wave is that some of the neutrinos can undergo multiple resonance flavor conversion when they traverse the SN. We consider them below.

Let us first consider the neutrino flavor conversion due to the density discontinuity at the shock front $r_s$. This crossing probability of the neutrino flavor transformation which we denote as $P_s$ can be determined by the conservation of flavor across the matter potential discontinuity. By a simple calculation, we have

$$P_s = \sin^2(\theta_m^+ - \theta_m^-),$$

(14)

where $\theta_m^+$ and $\theta_m^-$ are respectively the effective mixing angles in matter immediately before and after the shock front position $r_s$. Such angles are defined by

$$\cos 2\theta_m^\pm = \frac{\Delta m^2_{31} \cos 2\theta_{13} - 2E\gamma}{\sqrt{(\Delta m^2_{31} \cos 2\theta_{13} - 2E\gamma)^2 + (\Delta m^2_{31} \sin 2\theta_{13})^2}},$$

(15)

B. MSW effects

The MSW effects is the adiabatic or partially adiabatic neutrino flavor conversion in media with locality or time varying matter density. The main feature is that in a high density medium a flavor is also a mass state which propagates without flavor oscillation, until crossing a resonance where there is a finite probability for flavors change, i.e., a jump from one mass eigenstate to another. For $\theta_{13}$ smaller than $3^\circ$, using the Landau-Zener formula, one can obtain the crossing probability, $P_{HI}$, for neutrinos to jump from one mass eigenstate to another at the high resonance layer ($r = r_i$)

$$P_{HI} = \exp(-\frac{\pi}{2} \gamma),$$

(16)

with

$$\gamma = \frac{|\Delta m^2_{31}| \sin^2 2\theta_{13}}{2E} \cos 2\theta_{13} \frac{1}{|\ln N_e/\Delta r_{res}|},$$

(17)

where $N_e$ is the electron density and $|\Delta m^2_{31}| \simeq |\Delta m^2_{21}| \simeq 2.4 \times 10^{-3} eV^2$. Similarly, one can calculate the expression of the crossing probability $P_L$ at the low resonance region inside the SN. However, due to the large angle solution of the solar neutrino, $P_L \approx 0$.

In the presence of a propagating shock wave, the calculation of the global crossing probability, $P_H$, is not straightforward. The shock modifies the density profile as discussed in the preceding subsection. Within the shock wave and near the shock front the matter density is no longer monotonically decreasing, but rather goes through a minimum and a maximum. So the resonance condition can be satisfied possibly as many as three points:

$$\Delta m^2_{31} \cos 2\theta_{13} = 2\sqrt{2}G_F N_e(r_i)E, \quad (i = 1, 2, 3).$$

(18)

It is, of course, possible that there are only two or one resonance points, or even none at all. In the case of three resonance points, which we denote as $r_i$, $i = 1, 2, 3$, two of the points lie below the shock front and the third one beyond the shock front, i.e., $r_1 < r_2 < r_s < r_3$. The crossing probability $P_{HI}$ at $r_i$, $i = 1, 2, 3$, can be calculated from Eq. (16). Note that although the matter densities are the same at all resonance points as required by Eq. (18), their crossing probabilities, as given by Eq. (17), may be different because the slop of the matter density may be different at different resonance points. The global crossing probability can be expressed in terms of four crossing probabilities $P_{H1}$, $P_{H2}$, $P_s$, and $P_{H3}$.

To obtain the expressions of the global crossing probabilities, we first define four densities: $\rho_{res}$ is the resonance density which satisfies Eq. (18), $\rho_+$ the density at $r = r_s$ corresponding to the matter potential $V_+$, $\rho_-$ the density at $r = r_s$ corresponding to the matter potential $V_-$, and $\rho_0$ the density at the bottom of the camber which is the minimum of the density profile below the shock front. From Fig. 1, it can be seen that $\rho_+ > \rho_-$, $\rho_+ > \rho_0$.

In the pre-bounce time period $t < 1s$, the resonance condition occurs at only one point and there are no neutrino shock effects. Then,

$$P_H = P_{H3}.$$  

(19)
In the post-shock period \( t \geq 1 \) s we have four different situations:

(1) For \( \rho^- < \rho_{\text{res}} < \rho_b \), the resonance condition doesn’t occur and then

\[
P_H = P_s. \tag{20}\]

(2) For \( \rho_{\text{res}} > \rho^+ \), or \( \rho_{\text{res}} = \rho_b \) and \( \rho_{\text{res}} > \rho^- \), or \( \rho_{\text{res}} < \rho_b \) and \( \rho_{\text{res}} \leq \rho^- \), the resonance condition occurs at only one point, \( r = r_1 \) or \( r = r_3 \), and then

\[
P_H = P_H + P_s - 2P_{Hk}P_s + 2\sqrt{P_{Hk}P_s(1 - P_{Hk})(1 - P_s)} \cos \phi_{ks}, \quad (k = 1, 3). \tag{21}\]

(3) For \( \rho_b < \rho_{\text{res}} \leq \rho^+ \) and \( \rho_{\text{res}} > \rho^- \), or \( \rho_{\text{res}} = \rho_b \) and \( \rho_{\text{res}} \leq \rho^- \), the resonance condition occur at two points, \( r = r_1, r_2 \) or \( r = r_1, r_3 \), and then

\[
P_H = \left( P_{H1} + P_{H2} + P_s + P_{H3} \right) - 2( P_{H1}P_{H2} + P_{H1}P_{H3} + P_{H1}P_s + P_{H2}P_{H3} + P_{H2}P_s + P_{H3}P_s )
+ 4P_{H1}P_{H2}P_s + 2(1 - 2P_s - 2P_{H3} + 4P_{H3}P_s)\sqrt{P_{H1}P_{H2}(1 - P_{H1})(1 - P_{H2})} \cos \phi_{12}
+ 2(1 - 2P_{H1} - 2P_{H3} + 4P_{H1}P_{H3})\sqrt{P_{H2}P_s(1 - P_{H2})(1 - P_s)} \cos \phi_{2s}
+ 2(1 - 2P_{H2} - 2P_{H3} + 4P_{H2}P_{H3})\sqrt{P_{H1}P_s(1 - P_{H1})(1 - P_s)} \cos \phi_{1s}
+ 2(1 - 2P_{H1} - 2P_{H3} + 4P_{H1}P_{H3})\sqrt{P_{H3}P_s(1 - P_{H3})(1 - P_s)} \cos \phi_{3s}
+ 2(1 - 2P_{H2} - 2P_{H3} + 4P_{H2}P_{H3})\sqrt{P_{H1}P_{H2}(1 - P_{H1})(1 - P_{H2})} \cos \phi_{23}
+ 2(1 - 2P_{H1} - 2P_{H3} + 4P_{H1}P_{H3})\sqrt{P_{H3}P_s(1 - P_{H3})(1 - P_s)} \cos \phi_{31}
- 8\sqrt{P_{H1}P_{H2}(1 - P_{H1})(1 - P_{H2})} \cos \phi_{12}\sqrt{P_{H3}P_s(1 - P_{H3})(1 - P_s)} \cos \phi_{31}. \tag{23}\]

In Eqs. \([21, 22, 23]\), \( \phi_{ij} \) is defined as

\[
\phi_{ij} \approx \int_{r_s}^{r_f} \frac{dx}{2E} \sqrt{[\Delta m_{31}^2 \cos 2\theta_{13} - 2EV(r)]^2 + (\Delta m_{31}^2 \sin 2\theta_{13})^2}, \tag{24}\]

with \( V(r) \) being the SN matter potential. A summary for neutrino flavor conversions due to the neutrino shock effects and the MSW effects in various density regions is given in Table I. Due to the SN matter effects and the SN shock effects, the expressions of \( P_H \), Eqs. \([19, 20, 21, 22, 23]\), are different from Eq. (25) in Ref. \[17\] and Eq. (34) in Ref. \[33\].

Because of the various effects they have been subjected to, the SN neutrinos reaching the Earth have a very rich structures to be explored in their spectra in time, energy, and the mixing angle \( \theta_{13} \). In Fig. 2, we plot the crossing probability \( P_H \) as a function of the neutrino energy \( E \) at \( t = 4 \) s for two very small mixing angles \( \sin^2 \theta_{13} = 10^{-5} \) and \( 10^{-4} \). It is seen that there are large variations in \( P_H \) as the neutrino energy varies. These forms of behavior are different from those given in Refs. \[11\] and \[17\]. In Fig. 3, we plot \( P_H \) as a function of \( t \) for \( \sin^2 \theta_{13} = 10^{-4} \) at a fixed \( E \) for three different values of \( E \). It can be seen that the shapes of the curves depend strongly on \( E \) and there are many oscillations in \( P_H \) in an interval around \( t = 5 \) s. The interval enlarges and moves to higher \( t \) value as \( E \) increases. This is different from that given in Ref. \[17\]. In Fig. 4, we plot \( P_H \) as a function of \( \theta_{13} \) for different values of \( E \) and \( t \). It is found that at some values of \( \theta_{13} \), which depends on the energy of SN neutrinos, \( P_H \) begins to change very rapidly with \( \theta_{13} \) at \( t = 4 \) s, and the behavior of the \( P_H \) dependence on \( \theta_{13} \) is different from that given in Ref. \[11\]. These differences are attributable to the different SN matter effects when the shock effects is included in the consideration. The relation between \( P_H \) and \( \theta_{13} \) will provide a way to obtain information on \( \theta_{13} \) and the study of the shock wave effects.

C. Neutrino-neutrino interactions: collective effects

Because of the neutrino-neutrino forward scattering or neutrino self-interaction, neutrinos can experience collective flavor transformation in the SN near the neutrinosphere where the neutrino number densities are very large \[55\]. This
phenomenon is different from the MSW effects in that the flavor evolution histories of neutrinos in the collective oscillations are coupled together and must be solved simultaneously.

Recently, a significant amount of studies on collective effects have been made by a number of authors, e.g., Duan et al. [33, 34, 35, 36], Dasgupta et al. [6, 12], Hannestad et al. [37], Raffelt et al. [38], and Fogli et al. [39]. Defining $P_{\nu \nu}(\bar{P}_{\bar{\nu} \bar{\nu}})$ as the probability that the (anti-)neutrino $\nu(\bar{\nu})$ remains as $\nu(\bar{\nu})$ after the collective effects, Dasgupta et al. [6] introduced a much simplified picture to explain the characteristics of the collective effects which we followed in our earlier paper [11], i.e. $P_{\nu \nu} = P_{\bar{\nu} \bar{\nu}} = 1$ in the case of normal mass hierarchy; and $\bar{P}_{\bar{\nu} \bar{\nu}} = 1$.

in the case of inverted mass hierarchy, where $E_C$ is a critical energy across which a stepwise flavor conversion of $\nu_e$ develops. However, from the numerical simulations of the collective effects obtained by Duan et al. [14], it is found that $P_{\nu \nu}(\bar{P}_{\bar{\nu} \bar{\nu}})$ depends on the neutrino (anti-neutrino) energy $E$, the SN radial coordinate $r$, the SN neutrino emission angle, and it is very difficult to obtain an analytical expression for $P_{\nu \nu}(\bar{P}_{\bar{\nu} \bar{\nu}})$. It can be seen from Movies 5 and 6 in Ref. [14] that when the SN radial coordinate $r$ increases, there are oscillations between $\nu_e$ ( $\bar{\nu}_e$) and $\nu_\tau$ ( $\bar{\nu}_\tau$) due to the collective effects. When $r \geq 220km$, which is far away from the neutrinosphere, the collective effects for the neutrino amount to a swap between the $\nu_e$ and $\nu_\tau$ fluxes, irrespective of the mass hierarchy being normal or inverted. For the anti-neutrino, in the case of normal mass hierarchy, the fluxes of $\bar{\nu}_e$ and $\bar{\nu}_\tau$ are nearly unchanged and there is no swap between them. In the case of inverted mass hierarchy, the behaviors of the $\bar{\nu}_e$ and $\bar{\nu}_\tau$ fluxes are very complex and, for simplicity, they are generally approximated as that there is no swap between them. In this way, we apply the following new simplified picture to describe the characteristics of the collective effects:

$$P_{\nu \nu} = \begin{cases} 1 & (E < E_C), \\ 0 & (E > E_C), \end{cases}$$

for neutrinos and

$$\bar{P}_{\bar{\nu} \bar{\nu}} = 1,$$

for antineutrinos. It will be seen from our numerical calculations that the results obtained in Secs. III and IV hardly change for $E_C$ varying between 6MeV and 10MeV. Therefore, similar to Ref. [13], we take $E_C = 10MeV$.

D. Earth matter effects

Neutrinos from a SN may travel through a significant portion of the Earth before reaching the detector and are subjected to the matter effects [40]. Therefore, we need to consider the Earth matter effects in studying SN neutrinos. Suppose a neutrino reaches the detector at the incident angle $\theta$ (see Fig. 5). Then the distance that the neutrino travels through the Earth to the detector $L$ and the distance of the neutrino to the center of the Earth $\hat{x}$ are given by

$$L = (-R_E + h) \cos \theta + \sqrt{R_E^2 - (R_E - h)^2 \sin^2 \theta},$$
$$\hat{x} = \sqrt{(-R_E + h)^2 + (L - x)^2 + 2(R_E - h)(L - x) \cos \theta},$$

where $h$ is the depth of the detector in the Earth, $x$ the distance that the neutrino travels into the Earth, and $R_E$ the radius of the Earth.

Let $P_{\nu e}$ be the probability that a neutrino mass eigenstate $\nu_i$ enters the surface of the Earth and arrives at the detector as an electron neutrino $\nu_e$, we have [42]

$$P_{\nu e} = \sin^2 \theta_{12} + \frac{1}{2} \sin^2 2\theta_{12} \int_{x_0}^{x_1} dx V(x) \sin \phi^m_{x-x_1},$$

Movies 5 and 6 which describe the full simulation of the collective effects [14] are available at [http://iopscience.iop.org/1749-4699/1/1/015007/media](http://iopscience.iop.org/1749-4699/1/1/015007/media)
where \( V(x) \) is the potential \( \nu_e \) experiences due to the matter density \( \rho(x) \) inside the Earth
\[
V(x) = \sqrt{2} G_F N_A \rho(x) Y_e, \tag{28}
\]
and \( \phi_{a \rightarrow b}^m \) is defined as
\[
\phi_{a \rightarrow b}^m = \int_a^b dx \Delta_m(x),
\]
where
\[
\Delta_m(x) = \frac{\Delta m_{21}^2}{2E} \sqrt{(\cos 2\theta_{12} - \varepsilon(x))^2 + \sin^2 2\theta_{12}}, \tag{29}
\]
with \( \theta_{12} \approx 34.5^\circ, \Delta m_{21}^2 = 7.7 \times 10^{-5} eV^2, \varepsilon(x) = 2EV(x)/\Delta m_{21}^2 \).

E. Summary of all matter effects

When all the effects, including the shock, MSW, collective, and the Earth matter, are taken into account, the SN neutrino fluxes at the detector can be written as
\[
F_{\alpha}^D = pF_{\alpha e}^{(0)} + (1 - p)F_{\alpha x}^{(0)},
\]
where the survival probabilities \( p \) and \( \bar{p} \) are given by
\[
p = P_{2e}[P_H P_{\nu e} + (1 - P_H)(1 - P_{\nu e})],
\bar{p} = (1 - P_{2e})\bar{P}_{\nu e}, \tag{31}
\]
for the normal mass hierarchy and
\[
p = P_{2e} P_{\nu e},
\bar{p} = (1 - P_{2e})[\bar{P}_H \bar{P}_{\nu e} + (1 - \bar{P}_H)(1 - \bar{P}_{\nu e})], \tag{32}
\]
for the inverted mass hierarchy.

Therefore, the event numbers \( N(i) \) of SN neutrinos in the various reaction channels ”i” can be calculated by integrating over the neutrino energy, the product of the target number \( N_T \), the cross section of the given channel \( \sigma(i) \), and the neutrino flux function at the detector \( F_{\alpha}^D \),
\[
N(i) = N_T \int dE \cdot \sigma(i) \cdot \frac{1}{4\pi D^2} \cdot F_{\alpha}^D, \tag{33}
\]
where \( \alpha \) stands for the neutrino or antineutrino of a given flavor, and \( D \) is the distance between the SN and the Earth. In the following section, we propose a method to identify the mass hierarchy and acquire information about \( \theta_{13} \) by detecting SN neutrinos.

III. IDENTIFY THE MASS HIERARCHY AND ACQUIRE INFORMATION ABOUT \( \theta_{13} \) BY DETECTING SN NEUTRINOS

In the above section, we have discussed the SN shock effects, the MSW effects, the collective effects, and the Earth matter effects in detail for the detection of SN neutrinos. From Eqs. (14)-(17) and (19)-(33), it can be seen that the event number of SN neutrinos depends on the mixing angle \( \theta_{13} \) for different mass hierarchies. Therefore, we may identify the mass hierarchy and acquire information about \( \theta_{13} \) by detecting SN neutrinos. In Ref. 11, we defined a ratio of the event number of \( \nu_e \) over that of \( \bar{\nu}_e \) and studied the method to acquire information about \( \theta_{13} \). However, in that paper, we ignored the SN shock effects and only a very simple model for the collective effects was considered.
In addition, with the naive luminosity of SN neutrinos employed there, the uncertainties in $\theta_{13}$ will be very large. In the present work, we take into account the SN shock effects and consider the collective effects more realistically. In order to further reduce the influence of the uncertainties in the parameters of the SN neutrino fluxes, we define a more appropriate ratio as the event number of SN neutrinos for the charged-current reaction detected after 1s over that detected before 1s. Using this new ratio, we can better acquire information about $\theta_{13}$ and identify the neutrino mass hierarchy. From Ref. [11] and detailed numerical studies related to the present work, we found that the ratios of event numbers of SN neutrinos are nearly dependent of the SN neutrino incident angle. Therefore, in the following we will only consider the case for the incident angle of 30°.

There will be eight detectors located at the near and far sites of the Daya Bay experiment [9]. The total detector mass is about 300 tons and the depth of the detector $h \approx 400$ m. The Daya Bay Collaboration has decided to use Linear Alkyl Benzene (LAB) as the main part of the liquid scintillator. LAB has a chemical composition including $H$ and $C$ and the ratio of the number of $C$ and $H$ is about 0.6. Then, the total numbers of target protons, electrons, and $^{12}C$ are

$$N_T^{(p)} = 2.20 \times 10^{31}, \quad N_T^{(e)} = 1.01 \times 10^{32}, \quad N_T^{(C)} = 1.32 \times 10^{31}. $$

In the Daya Bay experiment, there are three reactions which can be used to detect SN neutrinos: the inverse beta decay, neutrino-electron reactions, and neutrino-carbon reactions. However, since the event number observed in the channel of neutrino-electron reactions at Daya Bay is very small [11], these reactions will not be considered in the present work. Therefore, we only consider the inverse beta decay and the neutrino-carbon reactions, their effective cross sections are given as follows [18] [49]:

1. The inverse beta decay is

$$\sigma(\bar{\nu}_e p) = 9.5 \times 10^{-44}(E(MeV) - 1.29)^2 cm^2, $$

where the reaction threshold is $E_{th} = 1.80 MeV$.

2. The neutrino-carbon reactions are

$$\langle \sigma(12C(\nu_e, e^-)12N) \rangle = 1.85 \times 10^{-43} cm^2, $$

$$\langle \sigma(12C(\bar{\nu}_e, e^+)12B) \rangle = 1.87 \times 10^{-42} cm^2, $$

for the charged-current capture, and

$$\langle \sigma(\nu_{12}C) \rangle = 1.33 \times 10^{-43} cm^2, $$

$$\langle \sigma(\bar{\nu}_{12}C) \rangle = 6.88 \times 10^{-43} cm^2, $$

$$\langle \sigma(\nu_{\mu,\tau}(\bar{\nu}_{\mu,\tau})12C) \rangle = 3.73 \times 10^{-42} cm^2, $$

for the neutral-current capture. The effective cross sections in Eq. (35) are valid for SN neutrinos without oscillations. When neutrino oscillations are taken into account, the oscillations of higher energy $\nu_e$ into $\nu_\tau$ result in an increased event rate since the expected $\nu_e$ energies in the absence of oscillations are just at or below the charged-current reaction threshold. This leads to an increase by a factor of 35 for the cross section $\langle \sigma(12C(\nu_e, e^-)12N) \rangle$ if we average it over a $\nu_e$ distribution with $T = 8 MeV$ rather than 3.5 MeV. Similarly, the cross section $\langle \sigma(12C(\bar{\nu}_e, e^+)12B) \rangle$ is increased by a factor of 5.

In order to reduce the uncertainties of the unknown parameters appearing in the SN neutrino fluxes, including those in the luminosities $L_0^\alpha$, temperatures $T_\alpha$, and pinching parameters $\eta_\alpha$, we define the ratio $R_1^{(+(-)}$ as the event number of $\nu_e(\bar{\nu}_e)$ for the charged-current reaction detected after 1s over that detected before 1s,

$$R_1^{(+(-)} = \frac{N_{CC}^{\nu_e(\bar{\nu}_e)}(1s < t < 10s)}{N_{CC}^{\nu_e(\bar{\nu}_e)}(0 < t < 1s)}. $$

With Eqs. (11), (12), (14) - (17), (19) - (33), and (37), we can see that $R_1^{(+(-)}$ depends on the mixing angle $\theta_{13}$ and the luminosity decay time $\tau$. Form a detailed numerical calculation, we found that $R_1^{\pm}$ which changes with $\theta_{13}$ has also a significant dependence on $\tau$. This can lead to large uncertainties in $R_1^{\pm}$ when $\tau$ varies in the range given in Eq. (2). Therefore, it will be difficult to identify the mass hierarchy and acquire information on $\theta_{13}$ from $R_1^{\pm}$ when $\tau$ has a significant uncertainty. However, if the value of $R_1^{\pm}$ is determined through SN neutrinos, we can obtain a rather tight relation between $\tau$ and $\theta_{13}$. We demonstrate this relation in Fig. 6, for the neutrino-carbon reactions. In Fig. 6 we
plot $\tau$ as a function of $\theta_{13}$ for a fixed value of $R^+_1$. We take $R^+_1 = 2.4$ and $L^0_{\nu_e}/L^0_{\bar{\nu}_e} = 1$ as an example$^3$. From this figure, we see that the value of $\tau$ lies in a small range, i.e., 2.68 - 3.00 when $\theta_{13}$ varies from 0 to 3$^\circ$ for the normal mass hierarchy. This is much smaller than the presently allowed range given in Eq. (2). It should be remarked that several authors$^{13 }$ have typically used the best fit value, i.e., $\tau = 3.8$ in their study of SN neutrinos.

First consider the channel of the inverse beta decay in which only $R^-_1$ is relevant. We plot $R^-_1$ in Fig. 7 as a function of $\theta_{13}$ for the two mass hierarchies, and for $L^0_{\nu_e}/L^0_{\bar{\nu}_e}$, $T_\alpha$, and $\eta_\alpha$ taking their allowed extreme values in the ranges given in Eqs. (3) and (5). For the normal mass hierarchy $R^-_1$, having the value of 2.4, is independent of $\theta_{13}$. So it is impossible to obtain any information on $\theta_{13}$ for the normal mass hierarchy in the inverse beta decay process. For the inverted mass hierarchy, for a given value of $L^0_{\nu_e}/L^0_{\bar{\nu}_e}$, $R^-_1$ lies in a band bounded by the curves marked as "inverted & max" and "inverted & min", which are obtained by setting the values of both $T_\alpha$ and $\eta_\alpha$ at their upper and lower bounds, respectively. From this plot, it can be seen that if the value of $R^-_1$ is away from 2.4, that is statically significant, the mass hierarchy must be inverted and a band of values of $\theta_{13}$ and $L^0_{\nu_e}/L^0_{\bar{\nu}_e}$ can be determined. But it is difficult to determine $\theta_{13}$ from $R^-_1$ alone. It is interesting to observe that, since the value of $R^-_1$ falls bound in the limited range of 2.1-2.8, it can serve as a test of the validity of the overall parametrization of the SN explosion mechanism.

For the neutrino-carbon reactions, both $R^+_1$ and $R^-_1$ can be used to identify the mass hierarchy and obtain information on $\theta_{13}$. In Fig. 8a, we plot $R^+_1$ vs $\theta_{13}$ similar to Fig. 7. Here for the inverted mass hierarchy, $R^+_1$ has the constant value 2.4 independent of $\theta_{13}$. If $R^+_1$ is larger than 2.4, the mass hierarchy must be normal. It can also be seen from Fig. 8a that $R^+_1$ depends on $\theta_{13}$ sensitively for the normal mass hierarchy and the uncertainties of $R^+_1$ due to $L^0_{\nu_e}$, $T_\alpha$, and $\eta_\alpha$ are small. Therefore, in the case of normal mass hierarchy, we can restrict $\theta_{13}$ to a small range of values by measuring $R^+_1$. Similar plots are made for $R^-_1$ in Fig. 8b. Now for the normal mass hierarchy, $R^-_1$ is independent of $\theta_{13}$. For the inverted mass hierarchy, $R^-_1$ varies with $\theta_{13}$ and is smaller than 2.4. Hence we may also be able to identify the mass hierarchy by $R^-_1$. However, as shown in the plot, in the case of inverted mass hierarchy the large uncertainties of $R^-_1$ due to $L^0_{\nu_e}$, $T_\alpha$, and $\eta_\alpha$ make it difficult to constrain $\theta_{13}$ by $R^-_1$.

The Daya Bay experiment has the sensitivity limit of 0.01 for $\sin^2 2\theta_{13}$, i.e., 3$^\circ$ for $\theta_{13}$. However, if a SN explosion takes place during the operation of the experiment, it is possible to reach a much smaller value of $\theta_{13}$, with the benefit of determining the mass hierarchy by measuring $R^+_1$ for the neutrino-carbon reactions.

Now let us consider the heavy water detector, for an example, used in the previous SNO experiment$^{[10]}$. The detector mass was made of 1000 tons of heavy water located in the depth of $h = 6000$ m.w.e.$^4$.$^{[10]}$. We consider only the neutrino-deuterium reactions. The total number of deuterium is $N_T^{(d)} = 6.02 \times 10^{31}$. Scaling the experimentally measured energy values from the decay of the muon at rest to the energy scale of SN neutrinos, one can obtain the effective cross sections of neutrino-deuterium reactions in analogy with Figs. 6a and 6b. The total number of deuterium is $N_T^{(d)} = 6.02 \times 10^{31}$. Scaling the experimentally measured energy values from the decay of the muon at rest to the energy scale of SN neutrinos, one can obtain the effective cross sections of neutrino-deuterium reactions as follows$^{[49]}$$^{[51]}$:

$$\langle \sigma(d(\nu_e,e^-)pp) \rangle = (3.35T_{\nu_e}^{2.31} - 3.70) \times 10^{-43} \text{ cm}^2,$$
$$\langle \sigma(d(\bar{\nu}_e,e^+)nn) \rangle = (3.05T_{\bar{\nu}_e}^{2.08} - 7.82) \times 10^{-43} \text{ cm}^2,$$

(38)

for the charged-current capture, and

$$\langle \sigma(d(\nu_\alpha,\nu_\alpha)np) \rangle = (1.63T_{\nu_\alpha}^{2.26} - 2.78) \times 10^{-43} \text{ cm}^2,$$
$$\langle \sigma(d(\bar{\nu}_\alpha,\bar{\nu}_\alpha)np) \rangle = (2.03T_{\bar{\nu}_\alpha}^{2.05} - 3.76) \times 10^{-43} \text{ cm}^2,$$

(39)

for the neutral-current capture, where $\alpha = e, \mu, \tau$.

In Figs. 9a and 9b we plot $R^+_1$ and $R^-_1$ of the neutrino-deuterium reactions in analogy with Figs. 8a and 8b. Similar to the neutrino-carbon case given in Fig. 8a, Fig. 9a for $R^-_1$ can be used to determine the neutrino mass hierarchy. But it is difficult to pin down the value of $\theta_{13}$. Figure 9b for $R^+_1$ is similar to Fig. 7 and the discussion can be applied to the present situation, with the allowed value of $R^+_1$ between 1.9 to 2.6 in the case of inverted mass hierarchy.

In the above discussions, we have applied our method to three reactions: inverse beta decay, neutrino-carbon, and neutrino-deuterium. A summary is given in Table II. From this table, it can be seen that the mass hierarchy may be identified by measuring $R^+_1$ in the neutrino-carbon reactions and the neutrino-deuterium reactions, and by measuring $R^-_1$ in the inverse beta decay, the neutrino-carbon reactions, and the neutrino-deuterium reactions. In addition, in the case of normal mass hierarchy, $R^+_1$ measured in the channel of neutrino-carbon reactions can provide the most accurate information about $\theta_{13}$. Similarly, the present method can also be applied to other neutrino experiments, such as KamLAND$^{[52]}$, MinBooNE$^{[53]}$, Borexino$^{[54]}$, and Double-Chooz$^{[55]}$ in the channel of neutrino-carbon.

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$^3$ We have checked that when $R^+_1$ and $L^0_{\nu_e}/L^0_{\bar{\nu}_e}$ take other values, the qualitative results do not change.

$^4$ m.w.e. means meter-water-equivalent and 1 m of rock is about 2.7 m of water.
reactions. In the future, if a SN explosion takes place within the cosmic distance considered here, it is possible to identify the mass hierarchy and reach a much smaller value of $\theta_{13}$ than current neutrino experiments through the measurement of $R^+_i$ in the channel of neutrino-carbon reactions.

IV. CHECK NEUTRINO MASSES UNDER COSMIC SETTING

All of our understanding of neutrino properties has been derived in the terrestrial environment, with input from solar neutrinos, and our study of SN neutrinos has been made under the reasonable assumption that the terrestrial properties can be applied unaltered to the extreme conditions of the cosmos, such as those in the SN, e.g., high temperature, high density, etc.. If an observation of SN neutrinos deviates from the prediction, it will be argued that our picture of SN requires modification. However, it would be prudent to check if some aspects of the properties of neutrinos may have been modified under the unusual cosmic conditions. In this section we propose a measurement, the observation of which, can only be attributed to some change of the neutrino property, albeit a very specific one.

Since neutrinos, at least some of the species, have finite masses, their transmitting velocities are smaller than that of light. Then, neutrinos from a SN explosion, while traveling through a large distance to the Earth, can experience a delay in the arrival time, which depends on their energies and masses and may be observable [8][50][56].

Suppose a neutrino from a SN, of mass $m$ (in eV) and energy $E$ (in MeV), travels over a distance $D$ (in 10kpc) to the Earth, it will experience an energy-dependent time delay of $\Delta t$ (in second) relative to a massless neutrino [8]:

$$\Delta t(s) \approx 0.5143 \left( \frac{m(eV)}{E(MeV)} \right)^2 D(10kpc), \quad (40)$$

where only the lowest order term in the neutrino mass expansion has been kept. In Ref. [8], a method of measuring the neutrino mass is proposed, using the detection of the event number of the delayed SN neutrinos. In the papers, a simple model of the luminosities of SN neutrinos was assumed, in the absence of various effects considered in recent SN neutrino studies. The neutrino luminosities were assumed to have a common step-function cutoff at a simple model of the luminosities of SN neutrinos was assumed, in the absence of various effects considered in the neutrino mass is proposed, using the detection of the event number of the delayed SN neutrinos. Therefore, the luminosities, which are constant before $t_{BH}$ and vanishes abruptly afterward, take the form $L(t) = L^0 \theta(t_{BH} - t)$. In the present work, we take into account the full complexity of the neutrino flavor transformation due to the effects of the shock, MSW, collective, and the Earth matter. Also, we consider the time-dependence of the SN neutrino luminosities given in Eq. (11) and the uncertainties in the SN neutrino flux parameters given in Eqs. (3) and (5). Since the time delay $\Delta t$ depends on the neutrino mass $m$, we can express the event number of the delayed SN neutrinos as a function of $m$. Therefore, information about $m$ can be obtained by detecting the event number of the delayed SN neutrinos.

From Ref. [57] there are the following neutrino mass bounds:

$$m_{\nu_e} < 2eV, \quad m_{\nu_\mu} < 0.19MeV, \quad m_{\nu_\tau} < 18.2MeV. \quad (41)$$

For the SN electron antineutrino the averaged energy is $\langle E_{\bar{\nu}_e} \rangle \approx 16MeV$, which together with $D = 10kpc$, gives $\Delta t_{\bar{\nu}_e} < 0.01s$ from Eq. (40). This time delay of the $\bar{\nu}_e$ is much smaller than the time scale of the SN explosion of about 10s. And it is also too short to be observable. Similarly, for the electron neutrino $\langle E_{\nu_e} \rangle \approx 11MeV$, we find $\Delta t_{\nu_e} < 0.02s$, which is again very short and much smaller than the time scale of SN explosion. Therefore, unless their effective mass is somehow significantly enhanced, $\nu_e$ and $\bar{\nu}_e$ can be considered as massless for the time delay observation. For the muon and tau neutrinos, according to the oscillation experiments their effective masses in the terrestrial setting will also be bound by 2eV because of the much smaller values of $\Delta m^2_{21}$ and $\Delta m^2_{31}$, their time delay will be very small too. In the following, we will tentatively ignore these terrestrial bounds to form the basis of our proposal for a test of neutrino properties under the cosmic setting of extreme conditions.

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5 Note that these bounds are obtained from direct measurements of neutrino masses and they do not take into account neutrino oscillation experiments. The latter will relate neutrino effective masses of different flavors in a very tight range.

6 The effective masses of the flavor neutrinos, $m_\alpha$, $\alpha = e, \mu,$ and $\tau$, to those of the regular neutrino masses, $m_j$, $j = 1, 2,$ and $3$, by

$$m_\alpha = \sum_{j=1,2,3} |U_{\alpha j}|^2 m_j,$$

where $U_{\alpha j}$ is the neutrino mixing matrix.
After the SN explosion the emission of different flavors of neutrinos lasts for a period of about 10s. The arrival to the Earth of those flavors of neutrinos and antineutrinos whose effective masses are not affected will last about the same period of time. For other flavors which are affected there may be a delay in the arrival time, depending on the values of their effective masses. We look for measurements which are sensitive to both charged and neutral current events, such as the detector types of the Daya Bay [6], LVD [58], and SNO [59] experiments, which involve the neutrino-carbon and neutrino-deuterium reactions. The experimental information that these detectors can potentially provide include the event numbers of the electron antineutrino from charged-current reactions and the total event number of all the three flavors of neutrinos from the neutral-current reactions. We shall assume that the electron neutrino mass is not affected so as to avoid any possibility of influencing the existing result of nucleosynthesis. But the effective masses of the muon and tau neutrinos might be affected. Then, there might be a time delay $\Delta t_x$ between the charged-current events and some of the neutral-current events if the muon and tau flavors of SN neutrinos appear.

Denoting the effective mass of the charged-current events and some of the neutral-current events if the muon and tau flavors of SN neutrinos appear, the effective masses of the muon and tau neutrinos might be affected. Then, there might be a time delay $\Delta t_x$ between the charged-current events and some of the neutral-current events if the muon and tau flavors of SN neutrinos appear to have sufficiently sizable effective masses. Denoting the effective mass of $\nu_x$ ($\nu_\mu$, $\bar{\nu}_\mu$, $\nu_\tau$, or $\bar{\nu}_\tau$) by $m_{\nu_x}$, we have

$$\Delta t_x = \langle t \rangle_{NC} - \langle t \rangle_{CC} \cong 0.5143 \left( \frac{m_x}{E} \right)^2 D,$$

where we have dropped the units for $\Delta t_x$, $m_x$, $E$, and $D$ as indicated in Eq. (40). $< t >_{NC}$ is the average time of SN neutrinos of the neutral-current reactions to arrive at the detector and $< t >_{CC}$ that of the charged-current reactions. A nonvanishing $\Delta t_x$ signals a significant larger effective mass for muon or tau neutrino, independent of parameters of the SN and neutrinos.

To gain a sense of how $\Delta t_x$ may look like and how sensitive it is to the effective mass of the muon and tau neutrinos, we have to do some model calculation for quantities which can be measured in a SN neutrino experiment. We found that such a quantity is available as described below. We use the charged-current events which are from the electron antineutrino to monitor the events that are not delayed, and the neutral-current events which involve all flavors to measure the possible time delay. We divide the total neutral-current events into two groups: $N_{NC}^{(r)}(0)$ and $N_{NC}(0)$, where $N_{NC}^{(r)}(0)$ is the total neutral-current events in accompanying with the charged-current events and are therefore not delayed, and $N_{NC}(0)$ are all the total neutral-current events including both delayed and not delayed. We define the event number of the delayed $\nu_x$ plus $\bar{\nu}_x$ from the neutral-current events as

$$N_{NC}^{(delay)} = N_{NC}(0) - N_{NC}^{(r)}(0).$$

The quantity to be measured is the new ratio,

$$R_2 = \frac{N_{NC}^{(delay)}}{N_{NC}(0)} = \frac{N_{NC}(0) - N_{NC}^{(r)}(0)}{N_{NC}(0)}.$$

This ratio is very insensitive to the values of $\Delta m^2_{21}$ and $\Delta m^2_{31}$. It should also be remarked that because of the ratio, the effect of the various uncertainties involved, such as the neutrino and SN parameters, will be reduced in $R_2$. Using the charged current events to monitor the undelayed neutral current events will bypass the question of the uncertainty in the total emission time of SN neutrinos.

Since we can not tag the flavor of neutral-current events the information on the masses of the muon and tau neutrinos are correlated. For the purpose of illustration we assume $\nu_\mu$ and $\nu_\tau$ to have the same effective mass. We keep all other features of neutrinos and the SN unchanged. For the neutrino-carbon reactions at Daya Bay, we plot in Fig. 10 two sets of curves which describe $R_2$ as a function of $m_{\nu_x}$, each for a fixed value of $\theta_{13}$. In Fig. 10(a) for $\theta_{13} = 0$, it can be seen that the curves are insensitive to $l_{\nu_x}^F / l_{\nu_x}^D$ and the mass hierarchy. For $m_{\nu_x} < 200eV$, $R_2$ is sensitive to $m_{\nu_x}$. A measurement of $R_2$ can restrict $m_x$ in a distinctive range of values determined by the uncertainties in the SN neutrino temperature and chemical potential. In Fig. 10(b) for $\theta_{13} = 3^\circ$, $R_2$ is again shown to be sensitive to $m_x$ for $m_{\nu_x} < 200eV$. The two mass hierarchies overlap for small $m_x$ but fall into well separated regions for $m_x > 200eV$.

In Figs. 11 we plot $R_2$ for the neutrino-deuterium reactions in analogy to Figs. 11 of the neutrino-carbon reactions. Figure 11(a) is very similar to Fig. 10(a). Figure 11(b) is also similar to Fig. 10(b) for $m_{\nu_x}$ smaller than 150eV, but there are some differences at larger $m_x$. In this latter case, unlike Fig. 10(b), Fig. 11(b) shows that the two mass hierarchies can no long be separated even for $m_{\nu_x}$ greater than 200eV.

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7 Our numerical simulation shows that $R_2$ is insensitive to variations of $\Delta m^2_{21}$ and $\Delta m^2_{31}$ in a range of 10 and 8 orders of magnitude respectively.
The method can also be applied to KamLAND, MinBooNE, Borexino, and Double-Chooz in the channel of neutrino-carbon reactions, and the results are similar to those given for the Daya Bay experiment.

V. DISCUSSION AND SUMMARY

In this paper, we considered the SN shock effects, the MSW effects, the collective effects, and the Earth matter effects in the detection of type II SN neutrinos. Also, we considered the uncertainties in the neutrino luminosities $L_{\nu}$, their temperatures $T_{\nu}$, and their pinching parameters $\eta_{\nu}$ in the calculation of different flavors of SN neutrinos. We found that quantities suitably defined in terms of event numbers of different flavors of SN neutrinos are sensitive to the neutrino mass hierarchy, the mixing angle $\theta_{13}$, and neutrino masses. Therefore, it is possible to identify the mass hierarchy, acquire information about $\theta_{13}$, and look into certain neutrino mass patterns of SN neutrinos.

Firstly, we defined the ratio $R_{1}^{+(-)}$ as the event number of $\nu_{e}(\bar{\nu}_{e})$ for the charged-current reaction detected after one second over that detected in the first one second, and discussed the dependence of $R_{1}^{+(-)}$ on $\theta_{13}$ for different mass hierarchies. With this, we may identify the mass hierarchy and obtain information about $\theta_{13}$. This method can be applied to several detector types relying on specific physical processes, including the inverse beta decay (Fig. 7), neutrino-carbon reactions (Figs. 8(a) and 8(b)), and neutrino-deuterium reactions (Figs. 9(a) and 9(b)). From our numerical calculations, the mass hierarchy may be identified by measuring $R_{1}^{+}$ in the neutrino-carbon reactions and the neutrino-deuterium reactions, $R_{1}^{+}$ in the inverse beta decay, the neutrino-carbon reaction, and the neutrino-deuterium reactions. In addition, in the case of normal mass hierarchy, $R_{1}^{+}$ measured in the neutrino-carbon reactions provides the most restrictive information about $\theta_{13}$. Therefore, we can both identify the mass hierarchy and measure $\theta_{13}$ from $R_{1}^{+}$ in the channel of neutrino-carbon reactions at several experiments such as Daya Bay, KamLAND, MinBooNE, Borexino, and Double-Chooz.

Secondly, we defined another ratio, $R_{2}$, for the neutral-current reactions, as the event number of the delayed neutrinos over the total event number, with the charge current events as a monitor of undelayed events. The significance of $R_{2}$ is that it offers a possibility to detect possible difference between the electron neutrino and its other flavors counterparts, in particular, any significant difference in their effective masses under the cosmic setting of extreme conditions. With the plot of $R_{2}$ vs $m_{\nu_{e}}$, we can study $m_{\nu_{e}}$ vs $m_{\nu_{\mu}}$. This method is applied to both neutrino-carbon and neutrino-deuterium reactions and the result is given respectively in Figs. 10 and 11. What we have presented for $R_{2}$ is a case study with rather restricted input. In case there is an indication of non-vanishing $R_{2}$ a detailed refined calculation should be made.

In conclusion, if a SN explosion takes place within the cosmic distance considered here, it is possible to identify the mass hierarchy and obtain information on $\theta_{13}$. With suitable set up of the detector, a study of some unusual properties of neutrinos under the cosmic setting is possible. A summary of the detector types and their capabilities in association with the studies presented in this article is given in Table II.

VI. ACKNOWLEDGMENTS

We would like to thank F.-G. Cao and C.-G. Yang for helpful discussions. This work was supported in part by National Natural Science Foundation of China (Project Numbers 10535050, 10675022, 10975018, and 10890092) and the Special Grants from Beijing Normal University.

[1] K. Kotake, K. Sato, and K. Takahashi, Rept. Prog. Phys. 69, 971 (2006) arXiv:astro-ph/0509456.
[2] K. Hirata et al., Phys. Rev. Lett. 58, 1490 (1987); R.M. Bionta et al., Phys. Rev. Lett. 58, 1494 (1987); V. Trimble, Rev. Mod. Phys. 60, 859 (1988); W.D. Arnett, J.N. Bahcall, R.P. Kirshner, and S.E. Woosley, Ann. Rev. Astron. Astrophys. 27, 629 (1989); W. Hillebrandt and P. H"{o}flich, Rep. Prog. Phys. 52, 1421 (1989); D.N. Schramm, Phys. Rep. 189, 89 (1990); M. Koshiba, Phys. Rep. 220, 229 (1992); R. McCray, Ann. Rev. Astron. Astrophys. 31, 175 (1993).
[3] A. Dighe, J. Phys. Conf. Ser. 136, 022041 (2008) arXiv:0809.2977. AIP Conf. Proc. 981, 75 (2008) arXiv:0712.4386.
[4] X.-H. Guo, M.-Y. Huang, and B.-L. Young, Chin. Phys. C 34, 257 (2010) arXiv:0906.1534.
[5] G.G. Raffelt, Stars as Laboratories for Fundamental Physics (The University of Chicago Press, Chicago 1996).
[6] B. Dasgupta, A. Dighe, and A. Mirizzi, Phys. Rev. Lett. 101, 171801 (2008) arXiv:0802.1481.
[7] S. Skadhauge and R.Z. Funchal, J. Cosmol. Astropart. Phys. 04, 014 (2007) arXiv:hep-ph/0611194.
[8] J.F. Beacom, R.N. Boyd, and A. Mezzacappa, Phys. Rev. Lett. 85, 3568 (2000) arXiv:hep-ph/0006015. Phys. Rev. D 63, 073011 (2001) arXiv:astro-ph/0010398.
New York, 1977), 2nd ed..

[48] L. Cadonati, F.P. Calaprice, and M.C. Chen, Astropart. Phys. 16, 361 (2002) [arXiv:hep-ph/0012082].

[49] A. Burrows, S. Reddy, and T.A. Thompson, Nucl. Phys. A 777, 356 (2006) [arXiv:astro-ph/0404432].

[50] J.F. Beacom and P. Vogel, Phys. Rev. D 58, 093012 (1998) [arXiv:hep-ph/9806311]; J.F. Beacom and L.E. Strigari, Phys. Rev. C 73, 035807 (2006) [arXiv:hep-ph/0508202].

[51] N. Tatara, Y. Kohyama, and K. Kubodera, Phys. Rev. C 42, 1694 (1990); S. Ying, W.C. Haxton, and E.M. Henley, Phys. Rev. D 40, 3211 (1989), Phys. Rev. C 45, 1982 (1992).

[52] J. Busenitz et al., (KamLAND Collaboration), Proposal for US Participation in KamLAND, March 1999; J. Busenitz, Int. J. Mod. Phys. A 16, 742 (2001); K. Eguchi et al., Phys. Rev. Lett. 90, 021802 (2003) [arXiv:hep-ex/0212021]; T. Araki et al., Nature 436, 499 (2005).

[53] S.J. Brice, Nucl. Instrum. Methods Phys. Res., Set. A 562, 97 (2006); A.A. Aguilar-Arevalo, et al., Phys. Rev. Lett. 98, 231801 (2007) [arXiv:0704.1500].

[54] G. Alimonti et al., Astropart. Phys. 16, 205 (2002) [arXiv:hep-ex/0012030]; M. Balata et al., Eur. Phys. J. C 47, 21 (2006) [arXiv:hep-ex/0002027].

[55] F. Ardellier et al., Double-Chooz, a search for the neutrino mixing angle $\theta_{13}$, arXiv:hep-ex/0606025 M. Apollonio et al., Eur. Phys. J. C 27, 331 (2003) [arXiv:hep-ex/0301017].

[56] J.F. Beacom and P. Vogel, Phys. Rev. D 58, 053010 (1998) [arXiv:hep-ph/9802404]; H. Yüksel, S. Ando, and J.F. Beacom, Phys. Rev. C 74, 015803 (2006) [arXiv:astro-ph/0509297].

[57] [Particle Data Group] C. Amsler et al., Phys. Lett. B 667, 1 (2008).

[58] N.Y. Agafonova et al., Astropart. Phys. 27, 254 (2007) [arXiv:hep-ph/0609305].

[59] Q.R. Ahmad et al., Phys. Rev. Lett. 89, 011301 (2002) [arXiv:nucl-ex/0204008]; S.N. Ahmed et al., Phys. Rev. Lett. 92, 181301 (2004) [arXiv:nucl-ex/0309004] B. Aharmim et al., Phys. Rev. C 75, 045502 (2007) [arXiv:nucl-ex/0610020].
Figure Captions

Fig. 1 The density of the SN as a function of the radius $r$ at different times.

Fig. 2 The crossing probability at the high resonance region $P_{H}$ as a function of the neutrino energy $E$ for two very small neutrino mixing angles $\theta_{13}$ at $t = 4s$.

Fig. 3 The crossing probability at the high resonance region $P_{H}$ as a function of time $t$ for three different neutrino energies at $\sin^{2}\theta_{13} = 10^{-4}$.

Fig. 4 The crossing probability at the high resonance region $P_{H}$ as a function of the neutrino mixing angle $\theta_{13}$: (a) for three different times at $E = 11MeV$. The solid, dashed, and dotted curves are for $t = 2s, 4s, 6s$, respectively; (b) for three different neutrino energies at $t = 4s$. The solid, dashed, and dotted curves are for $E = 11MeV, 16MeV, 25MeV$, respectively.

Fig. 5 Illustration of the path of the SN neutrino reaching the detector in the Earth. $D$ is the location of the detector, $\theta$ is the incident angle of the neutrino, $O$ is the center of the Earth, $L$ is the distance the neutrino travels through the Earth matter, and $\tilde{x}$ is the instantaneous distance of the neutrino to the center of the Earth.

Fig. 6 The luminosity decay time $\tau$ as a function of the mixing angle $\theta_{13}$ in the channel of neutrino-carbon reactions at Daya Bay when $R_{I}^{+} = 2.4$ and $L_{\nu_{e}}^{0}/L_{\nu_{x}}^{0} = 1$. The ”max” (”min”) corresponds to the maximum (minimum) values of $T_{\alpha}$ and $\eta_{\alpha}$.

Fig. 7 The ratio $R_{I}^{-}$ of the event number of $\bar{\nu}_{e}$ for the charged-current reaction detected after 1s over that detected before 1s, as a function of the mixing angle $\theta_{13}$ in the channel of inverse beta decay at Daya Bay. The solid curves correspond to $L_{\nu_{e}}^{0}/L_{\nu_{x}}^{0} = 1/2$, the dashed curves $L_{\nu_{e}}^{0}/L_{\nu_{x}}^{0} = 1$, and the dot-dashed curves $L_{\nu_{e}}^{0}/L_{\nu_{x}}^{0} = 2$. The ”max” (”min”) corresponds to the maximum (minimum) values of $T_{\alpha}$ and $\eta_{\alpha}$.

Fig. 8 (a) Similar to Fig. 7 but the electron neutrinos $\nu_{e}$ are observed in the channel of neutrino-carbon reactions at Daya Bay; (b) similar to Fig. 7 but the anti-electron neutrinos $\bar{\nu}_{e}$ are observed in the channel of neutrino-carbon reactions at Daya Bay.

Fig. 9 (a) Similar to Fig. 7 but the electron neutrinos $\nu_{e}$ are observed in the channel of neutrino-deuterium reactions at SNO; (b) similar to Fig. 7 but the anti-electron neutrinos $\bar{\nu}_{e}$ are observed in the channel of neutrino-deuterium reactions at SNO.

Fig. 10 For the neutral-current reactions $R_{2}$ in the channel of neutrino-carbon reactions at Daya Bay, the ratio of the event number of delayed neutrinos over the total event number, as a function of $m_{\nu_{e}}$ at two values of $\theta_{13}$, each has the two mass hierarchies with $L_{\nu_{e}}^{0}/L_{\nu_{x}}^{0}$, $T_{\alpha}$, and $\eta_{\alpha}$ taking their limiting values. The solid curves correspond to $L_{\nu_{e}}^{0}/L_{\nu_{x}}^{0} = 1/2$, the dashed curves $L_{\nu_{e}}^{0}/L_{\nu_{x}}^{0} = 1$, and the dot-dashed curves $L_{\nu_{e}}^{0}/L_{\nu_{x}}^{0} = 2$. The ”max” (”min”) corresponds to the maximum (minimum) values of $T_{\alpha}$ and $\eta_{\alpha}$. Figure (a) shows that $R_{2}$ is nearly independent of $L_{\nu_{e}}^{0}/L_{\nu_{x}}^{0}$ and the mass hierarchy.

Fig. 11 Similar to Fig. 10 but in the channel of neutrino-deuterium reactions at SNO.
Fig. 1

(a) $\sin^2 \theta_{13} = 10^{-5}$

(b) $\sin^2 \theta_{13} = 10^{-4}$

Fig. 2

(a) $E = 11\text{MeV}$

(b) $E = 16\text{MeV}$

(c) $E = 25\text{MeV}$

Fig. 3
Fig. 4

Fig. 5

Fig. 6
Fig. 7

(a) electron neutrinos

(b) anti-electron neutrinos

Fig. 8
Fig. 9

Fig. 10
Fig. 11
TABLE I: Summary of neutrino flavor conversions due to the neutrino shock effects and the MSW effects in various density regions.

| $t < 1s$ | $t \geq 1s$ | $t \geq 1s$ |
|-----------|-------------|-------------|
| $\rho_\nu > \rho_\nu$ | $\rho_\nu > \rho_\nu$ | $\rho_\nu > \rho_\nu$ |
| Resonance in flavor conversion region involved | Resonance in flavor conversion region involved | Resonance in flavor conversion region involved |
| $\rho_{res} > \rho_\nu$ | $P_{H1}, P_s$ | $P_{H1}, P_s$ |
| $\rho_\nu > \rho_\nu > \rho_{res}$ | $P_{H1}, P_{H2}, P_s$ | $P_{H1}, P_{H2}, P_s$ |
| $\rho_{res} = \rho_\nu$ | $P_{H1}, P_s$ | $P_{H1}, P_s$ |
| $\rho_{res} > \rho_\nu$ | $P_{H1}, P_s$ | $P_{H1}, P_s$ |
| $\rho_\nu > \rho_{res} > \rho_\nu$ | $P_s$ | $P_{H1}, P_s$ |
| $\rho_{res} \geq \rho_\nu > \rho_\nu$ | $P_s$ | $P_{H1}, P_s$ |

TABLE II: Summary of measurement of the neutrino parameters by SN neutrinos in three reactions: the inverse beta decay, neutrino-carbon reactions, neutrino-deuterium reactions. "√" and "×" represents respectively the information can and can not be obtained.

| Reaction | normal hierarchy $|\Delta m^2_{31}| > 0$ | inverted hierarchy $|\Delta m^2_{31}| < 0$ | $\theta_{13} \leq 3^\circ$ | cosmic setting $(m_{\nu_e}, m_{\nu_\mu}, m_{\nu_\tau}, m_{\bar{\nu}_e})$ | figures |
|-----------|-----------------|-----------------|----------------|-----------------|-----------------|
| $\bar{\nu}_e + p$ | × | $\sqrt{(R^*_1 \neq 2.4)}$ | × | × | Fig. 7 |
| $\nu_\alpha + C$ | $\sqrt{(R^+_1 > 2.4)}$ | × | $\sqrt{(R^-_1 < 2.4)}$ | × | Figs. 8(a), 10(a), 10(b) |
| $\bar{\nu}_\alpha + C$ | × | $\sqrt{(R^-_1 < 2.4)}$ | × | $\sqrt{(R^-_1 < 2.4)}$ | Figs. 8(b), 10(a), 10(b) |
| $\nu_\alpha + d$ | $\sqrt{(R^+_1 > 2.4)}$ | × | $\sqrt{(R^-_1 \neq 2.4)}$ | × | Figs. 9(a), 11(a), 11(b) |
| $\bar{\nu}_\alpha + d$ | × | $\sqrt{(R^-_1 \neq 2.4)}$ | × | $\sqrt{(R^-_1 \neq 2.4)}$ | Figs. 9(b), 11(a), 11(b) |