Kinematic Analysis of Treatment Table Robot in Proton Radiotherapy Project

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Abstract. This paper introduces a multi-link robot, named treatment table robot (TTR), which has been developed as an important part in proton therapy system SC200. TTR is aimed to carry and move patient’s body during treatment, so as to change the space location and posture of patient’s tumour to ensure the proton beam can always focus on it. To realize the requirement, the TTR has been designed as a 6-DOF robot. Based on the built D-H model, the forward and inverse kinematics are studied. Then the joint trajectory planning has been researched. Finally, ADAMS software was employed to simulate the kinematic of the TTR, which verified the feasibility of the kinematic algorithm.

1. Introduction

Proton therapy is thought as a better way to cure cancer compared with X-Ray, the advantage of this method is its accuracy and lesser side effect on normal tissue[1]. SC200 is a integrated proton therapy system developed by Hefei ion medical center. Proton therapy contains the following steps: (1)Superconducting Cyclotron, create and accelerate proton beams,(2) Beam Transport System, which focuses and shapes the beam and guides it to the treatment room,(3) Treatment system, the rotating gantry and the treatment table robot adjusted position and pose to cooperatively complete established stratified scanning irradiation plan of the tumour,(4) Control system, handle various data and manage the whole system.

As shown in figure 1, The robot, named treatment table robot (TTR), as an important component of treatment system, is installed on the ground of Patient Treatment Room. In order to associatively adjust the position and pose of patient’s tumour with the rotation of gantry, the TTR should have the ability of moving along and rotating around all the xyz-axis, based on this request, the TTR has been designed as a 6 DOF serial-link robot, so, the accompanying issue, such as the structure and mathematic model of TTR, the motion planning method of robot to response the instruction of control system, is what we need investigate in this paper[2].
2. Structure and D-H model

As shown in figure 2, the TTR contains seven arms and six joints, the base arm is fixed on ground, the terminal arm is designed as a platform for patient to lie flat, all of the arms connected one by one to form the special configuration of joints. Joints 1 and 2 make the TTR can move on the horizontal plane widely, the joints 4 and 6 modify the heading and roll, the joints 3 and 5 change both the position of TTR in vertical direction and the pitch. The unique structure makes the TTR can fulfill the adjustment of position and posture of patient’s tumour during proton medical therapy.

Specialy, the z-axis of joint 4, 5 and 6 has been designed as perpendicularly intersected with eachother, so that accord with the Pieper norm.as we know, when the 6 D.O.F manipulator meet the Pieper principle, the rotate degree of joint 1, 2, 3 mainly influence the final coordinate point’s location, and the rotate degree of joint 4, 5 and 6 have more impact on the ultimate arm’s posture. Based on this feature, we can do some reasonable adjustment of the joint’s coordinate system when we set up the D-H model of TTR to make inverse kinematic problem more easy.

As shown in figure 3, the D-H model of TTR is established, each joint’s Cartesian coordinate system has been formulated by referring to D-H convention [3]. If we want to adjust the location and posture of tumour’s coordinate system, the first step is identified the location and posture of tumour’s coordinate frame in global reference coordinate system, then the homogeneous coordinate transition model of joints just like a bridge to realize it.
The homogeneous coordinate transition model of joints have been accomplished in the order of \( \theta \rightarrow \alpha \rightarrow d \rightarrow a \). What’s more, the frame 2 has been moved some distance along its z axes from joint 2, and moved the frame 4, 6 to joint 5 along its z axes, that ensured the homogeneous coordinate transferred in certain order, and made the D-H parameters more simplified, so as to derive the forward and inverse kinematics more easy. The standard D-H parameters[4] of TTR as shown in Table 1.

### Table 1. Standard D-H parameters of TTR.

| Link  | Theta(\(^\circ\)) | d(mm) | Alpha(\(^\circ\)) | a(mm) | Range(\(^\circ\)) |
|-------|------------------|-------|-------------------|-------|-------------------|
| base  | 0                | \( d_0 \) | 0                 | 0     | NULL             |
| 1     | \( \theta_1 \) (180) | \( d_1 \) | 0                 | \( a_1 \) | +/- 220          |
| 2     | \( \theta_2 \) (180) | 0     | -90               | \( a_2 \) | +/- 220          |
| 3     | \( \theta_3 \) (-90) | \( d_3 \) | -90               | 0     | -45~18           |
| 4     | \( \theta_4 \) (180) | 0     | -90               | 0     | +/- 10           |
| 5     | \( \theta_5 \) (90) | 0     | -90               | 0     | +/- 28           |
| 6     | \( \theta_6 \) (180) | 0     | 0                 | 0     | +/- 165          |
| tumour|                  | \( a_7 \) | 0                 | \( d_7 \) | NULL             |

\[(a_1 = 1200, a_2 = 360, d_1 = 563, d_3 = 1165.5, a_7 = 1581 \text{ and } d_7 = 604.5)\]

### 3. Kinematic analysis

#### 3.1. Forward kinematic

The objective of forward kinematic is identified the location and posture of tumour’s coordinate frame in global reference coordinate system(GCS) when we already knew the rotation angle of each joint. According to D-H convention and transformation theory between neighboring D-H coordinate system, the tumour’s position and posture in GCS can be ascertained as equation 1.

\[
\begin{align*}
\mathbf{T}_{\text{base}} = A_{\text{base}}(A_i)^nA_{\text{tumour}} = A_{\text{base}}T_{\text{table}}A_{\text{tumour}} \\
& (i=1,2,3...6)
\end{align*}
\]

The transformation matrix of the base and tumour are varied with different installation site and patient, so we focus on the constant part—the robot. as mentioned, the transformation order between neighboring D-H coordinate system is theta-alpha-d-a, then the transformation matrix of two joints could be expressed as equation 2 in which \( \theta_i, \alpha_i, d_i \) and \( a_i \) are the D-H parameters of link i, \( c \equiv \cos \) and \( s \equiv \sin \), the same abbreviations are used in the following formulae.

\[
A_i = \text{Rot}(z, \theta_i)\text{Rot}(x, \alpha_i)\text{Trans}(0, 0, d_i)\text{Trans}(a_i, 0, 0)
\]
\[ A = \begin{bmatrix} C(\theta) & -C(\alpha)S(\theta) & S(\alpha)S(\theta) & aC(\theta) + dS(\alpha)S(\theta) \\ \alpha S(\theta) & C(\alpha)C(\theta) & -S(\alpha)C(\theta) & aS(\theta) - dS(\alpha)C(\theta) \\ 0 & S(\alpha) & C(\alpha) & dC(\alpha) \\ 0 & 0 & 0 & 1 \end{bmatrix} \] (2)

With the parameters in Table 1, the forward kinematics can be solved by multiplying each homogeneous transformation matrix between consecutive links. The formula shown as equation 3

\[ T_{n\rightarrow k} = (A)^k = \begin{bmatrix} n_x & a_x & a_y & a_z & p_x & p_y & p_z \\ 0 & 0 & 1 & 0 & 0 & 0 \end{bmatrix} \] (3)

where \( \theta \) is the (6x1) vector of joint variables, and \( n, o, a \) are the unit vectors of the tumour frame and \( p \) is the position vector respect to the origin of the base frame, details as equation 4.

\[
\begin{align*}
  n_x &= C6(C12S3S5 + S12C5S4 + C12C3C4C5) - S6(S12C4 - C12C3S4) \\
  o_x &= -C6(S12C4 - C12C3S4) - S6(C12S3S5 + S12C5S4 + C12C3C4C5) \\
  a_x &= C12C5S3 - S5(S12S5 + C12C3C4) \\
  n_y &= S6(C12C4 + S12C3S4) + C6(S12S3S5 - C12C5S4 + S12C3C4C5) \\
  o_y &= C6(C12C4 + S12C3S4) - S6(S12S3S5 - C12C5S4 + S12C3C4C5) \\
  a_y &= S5(S12C4 - S12C3C4) + S12C5S3 \\
  n_z &= C6(C3S5 - C4C5S3) - S3S4S6 \\
  o_z &= -S6(C3S5 - C4C5S3) - C6S3S4 \\
  a_z &= C3C5 + C4S3S5 \\
  p_x &= a_xC12 + a_xC1 - d_xC12S3 \\
  p_y &= a_yS12 + a_xS1 - d_yS12S3 \\
  p_z &= d_z - d_zC3 
\end{align*}
\] (4)

3.2. Inverse Kinematic

The inverse kinematics is required to find the angles of the six revolute joints while the tumour’s position and posture in GCS is given, and the inverse kinematics is the foundation of trajectory planning.

Based on the Eq.4, the analytic solution can be derived, the inverse kinematics of the TTR is solved in two stages[5], in the first stage, we get the theta 1, 2, 3 using the position vector \([p_x, p_y, p_z]\). The steps are as follow:

1. From the equation \( p_x \), we can infer that

\[ \theta_1 = -a \cos \left( \frac{d_x - p_x}{d_x} \right) \] (5)

2. Extracted \( \cos(\theta_1) \) and \( \sin(\theta_1) \) from the equation \( p_x, p_y, \) and obtained sum of the squares, then we get

\[
\begin{align*}
  k_1 &= S12 + k_2C12 = k_3 \\
  k_2 &= 2p_x(d_xS3 - a_x) \\
  k_3 &= a_x^2 - p_x^2 - p_y^2 - (d_xS3 - a_x)^2 \\
  \theta_1 + \theta_2 &= \arctan2(k_3, k_2)\pm\left(\frac{k_2^2 + k_3^2 - k_1^2}{2}\right) - \arctan2(k_2, k_1)
\end{align*}
\] (6)

3. Plug the \( \theta_1 + \theta_2 \) into \( p_x, p_y \), we get

\[
\begin{align*}
  k_4 &= \sin(\theta_1) = \left| p_y - a_xS12 + d_yS12S3 \right|/a_x \\
  k_5 &= \cos(\theta_1) = \left( p_x - a_xC12 + d_xC12S3 \right)/a_x \\
  \theta_1 &= \arctan2(k_4, k_5) \\
  \theta_2 &= (\theta_1 + \theta_2) - \theta_1
\end{align*}
\] (7)
In the second stage, defined the matrix \( R = A_4 A_5 A_6 \) and matrix \( L = \text{inv}(A_3) \text{inv}(A_2) \text{inv}(A_1) T_{table} \), then each item of \( L \) and \( R \) are corresponding equal.

4. Based on \( L_{33} = R_{33} \), we get
   \[
   \begin{align*}
   k_6 &= C5 = a_r S1S2S3 - a_s C1C2S3 - a_s C1S2S3 - a_s C2S1S3 - a_r C3 \\
   \theta_6 &= a \cos(-k_6)
   \end{align*}
   \]

5. Based on \( L_{11} + L_{12} = R_{11} + R_{12} \), we get
   \[
   \begin{align*}
   k_7 S6 - k_r C6 &= k_y \\
   k_7 &= S5 \\
   k_8 &= a_r S1S2S3 - a_s C1C2S3 - a_s C1S2S3 - a_s C2S1S3 - a_r C3 - n_r C3 \\
   \theta_8 &= \sin\left[k_y / \sqrt{2k_7}\right] + \pi / 4
   \end{align*}
   \]

6. Based on \( L_{13} + L_{23} = R_{13} + R_{23} \), we get
   \[
   \begin{align*}
   k_9 S4 + k_r C4 &= k_y \\
   k_9 &= a_r C1C2C3 - a_s S3 + a_s C1C3S2 + a_s C2C3S1 - a_r C3S1S2 + a_r S12 - a_r C12 \\
   \theta_9 &= \sin\left[k_y / \sqrt{2k_9}\right] - \pi / 4
   \end{align*}
   \]

Normally, the inverse kinematic could be solved from the formula 5-10 when \( a \ o \ n \ p \) are given, each angle of rotation can directly obtain a closed-form solution.

3.3. Joint trajectory planning

Regarding the motion of robots, we often need to discuss the trajectory planning of joint space and the motion planning of Cartesian space [6]. For TTR, there are no other special requirements except speed limit about the movement in space, the rotation angle range limit has already prevented the collision of TTR with gantry, what we need to do is connect the two discrete joint angle values which obtained by the inverse solution algorithm with smooth curves, to drive TTR movement from one point to another. In this paper, the joint angle space has been planned by 5th polynomial as equation 11.

\[
\theta(t) = a_0 + a_1 t + a_2 t^2 + a_3 t^3 + a_4 t^4 + a_5 t^5
\]

For given beginning and end points, when we defined the speed and acceleration limit of rotation, the 5th polynomial’s parameters can be derived by equation 12.

\[
\begin{align*}
\theta_0 &= \hat{\theta}_\text{begin} ; & a_1 &= \hat{\theta}_\text{begin} ; & a_2 &= \ddot{\theta}_\text{begin} / 2 \\
a_3 &= \left[ 20 \dddot{\theta}_\text{end} - 20 \ddot{\theta}_\text{begin} - (8 \dddot{\theta}_\text{end} + 12 \ddot{\theta}_\text{begin}) t_{\text{total}} - (3 \dddot{\theta}_\text{begin} - \ddot{\theta}_\text{end}) t_{\text{total}}^2 \right] / 2 t_{\text{total}}^3 \\
a_4 &= \left[ 30 \dddot{\theta}_\text{end} - 30 \ddot{\theta}_\text{begin} + (14 \dddot{\theta}_\text{end} + 16 \ddot{\theta}_\text{begin}) \dot{\theta}_\text{total} + (3 \dddot{\theta}_\text{begin} - 2 \ddot{\theta}_\text{end}) \dot{\theta}_\text{total}^2 \right] / 2 t_{\text{total}}^4 \\
a_5 &= \left[ 12 \dddot{\theta}_\text{end} - 12 \ddot{\theta}_\text{begin} - (6 \dddot{\theta}_\text{end} + 6 \ddot{\theta}_\text{begin}) \dot{\theta}_\text{total} + (3 \dddot{\theta}_\text{begin} - 2 \ddot{\theta}_\text{end}) \dot{\theta}_\text{total}^2 \right] / 2 t_{\text{total}}^5
\end{align*}
\]

Furthermore, we need determined the time \( t_{\text{total}} \) of motion between two points by calculating the interval of the joints with the rotation speed limit of the joints.

4. Kinematic simulation

4.1. Joint position curve

In order to test the kinematics algorithm, selected the motion between two position for simulation analysis. As shown in figure 4, the position 1 is assigned for patient to get on the table, the position and pose in GCS is \( [R(z, -60^\circ), -1245, -2020, 930] \), and the position 2 is assigned for patient to complete CBCT imaging, the position and pose in GCS is \( [E, 2700, 0, 1400] \).
Based on the given \([\mathbf{a o n p}]\) and the inverse kinematic algorithm, the corresponding joint angle can be calculated by the use of Matlab, as shown in Table 2.

**Table 2. Joint angle of position.**

| position | \(\theta_1(\circ)\) | \(\theta_2(\circ)\) | \(\theta_3(\circ)\) | \(\theta_4(\circ)\) | \(\theta_5(\circ)\) | \(\theta_6(\circ)\) |
|----------|-------------------|-------------------|-------------------|-------------------|-------------------|-------------------|
| 1        | 156.26            | 74.16             | -72.39            | 180               | 72.39             | 249.57            |
| 2        | 81.8              | 226.79            | -95.79            | 180               | 95.79             | 231.41            |

According to Table 2, we can find that rotational stroke of joint 2 is the biggest, to ensure smooth movement and take care of patient’s feeling, limited the speed of joint rotation with no more than 10 deg/s, simply assumed that the joint rotate at average speed 5 deg/s, the whole motion time is nearly 31 second. What’s more, the speed and acceleration of each joint’s rotation must be zero to reduce the impact of starting and stopping on TTR. So, the 5th polynomial of each joint can be derived as equation 13.

\[
\begin{align*}
\theta_1(t) &= 156.2628 - 0.02499t^3 + 0.001209t^4 - 0.0000156t^5 \\
\theta_2(t) &= 74.1637 + 0.05123t^3 - 0.002479t^4 + 0.00003199t^5 \\
\theta_3(t) &= -72.3932 - 0.00785t^3 + 0.000385t^4 - 0.0000049t^5 \\
\theta_4(t) &= 72.3932 + 0.00785t^3 - 0.000385t^4 + 0.0000049t^5 \\
\theta_5(t) &= 249.569 - 0.00669t^3 + 0.0002949t^4 - 0.00000385t^5 \\
\theta_6(t) &= 249.569 - 0.00669t^3 + 0.0002949t^4 - 0.00000385t^5
\end{align*}
\]  

(13)

Plot the joint position curve (a), the velocity (b) and acceleration (c) in figure 5. As shown, the curve of velocity and acceleration are continuous without mutation, the velocity of joint’s rotation is less than 10 deg/s, and the speed and acceleration of joint rotation start and stop are zero, it meet the planning expectation.

**Figure 5. Rotation related curve of joints**

### 4.2. Joint position curve
In order to test whether the joint position curve can drive TTR from position 1 to position 2 as we want, a kinematic model is established in ADAMS software as shown in figure 6 [7].

![Kinematic model in ADAMS](image)

**Figure 6.** Kinematic model in ADAMS

Applied curves in equation 13 as the drive function for each joint to the kinematic model, the entire exercise process took 31s. Then, tracked the position change in each axis direction of the assumed tumor coordinate system in the GCS. As figure 7 shown, the track curve is continuous and smooth, and the position of the beginning and the end exactly reached our planned location, so the PTP movement can be considered successful.

![Coordinate track during the simulation from position 1 to 2](image)

**Figure 7.** Coordinate track during the simulation from position 1 to 2

5. Conclution
A 6-DOF robot TTR has been presented as an indispensable component of proton therapy system SC200, to carrying patient’s body to adjust location and posture in GCS as proton therapy plan requested. The structure and D-H model are established, and the whole kinematic models are coded in Matlab. Two position points are selected to test the motion planning algorithm, then, the drive function of each joint are added to the kinematic model which is setted up in ADAMS to implement motion simulation. The TTR achieves point-to-point motion smoothly and accurately, so the kinematic planning algorithm this paper offered is practical and reliable. Further research as dynamic studies and structure strength analysis are in the works.

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