NEUTRINO EMISSION FROM HIGH-ENERGY COMPONENT GAMMA-RAY BURSTS

JULIA K. BECKER1, FRANCIS HALZEN2, AONGUS Ó MURCHADHA2, AND MARTINO OLIVO1,3
1 Fakultät f. Physik und Astronomie, Ruhr-Universität Bochum, D-44780 Bochum, Germany; olivo@tp4.rub.de
2 Department of Physics, University of Wisconsin, Madison, WI-53706, USA; omurchadh@wisc.edu
3 Department of Physics and Astronomy, Uppsala University, Box 516, S-751 20 Uppsala, Sweden

Received 2009 November 11; accepted 2010 August 6; published 2010 September 14

ABSTRACT

Gamma-ray bursts (GRBs) have the potential to produce the particle energies (up to $10^{21}$ eV) and energy budget ($10^{44}$ erg yr$^{-1}$ Mpc$^{-3}$) to accommodate the spectrum of the highest energy cosmic rays; on the other hand, there is no observational evidence that they accelerate hadrons. The Fermi Gamma-ray Space Telescope recently observed two bursts that exhibit a power-law high-energy extension of a typical (Band) photon spectrum that extends to $\sim 30$ GeV. On the basis of fireball phenomenology we argue that these two bursts, along with GRB941017 observed by EGRET in 1994, show indirect evidence for considerable baryon loading. Since the detection of neutrinos is the only unambiguous way to establish that GRBs accelerate protons, we use two methods to estimate the neutrino flux produced when they interact with fireball photons to produce charged pions and neutrinos. While the number of events expected from the two Fermi bursts discussed is small, should GRBs be the sources of the observed cosmic rays, a GRB941017-like event that has a hadronic power-law tail extending to several tens of GeV will be detected by the IceCube neutrino telescope.

Key words: gamma-ray burst: general – gamma-ray burst: individual (GRB941017, GRB090510, GRB090902b)

1. INTRODUCTION

Sources of extragalactic cosmic rays with energies in excess of $\sim 3 \times 10^{18}$ eV remain a mystery, but one of the best-motivated candidates are gamma-ray bursts (GRBs). Large cosmic-ray energies can be achieved in the prompt phase of the GRB fireball where internal shocks have the potential to accelerate charged particles up to $\sim 10^{21}$ eV (Vetri 1995; Waxman 1995). Additionally, the total energy density of cosmic rays in the universe must be matched by a sufficiently high hadronic energy density in a GRB with $\rho_{\text{had,GRB}} \approx \rho_{\text{CR}}$. GRB observations identify synchrotron photons produced by electrons accelerated in the fireball with an energy $\epsilon_e E_{\text{TOT}} \sim 10^{53}$ erg, where $E_{\text{TOT}}$ is the total isotropic energy released by the burst. GRB fireballs also carry energy $\epsilon_B E_{\text{TOT}}$ in the form of magnetic fields and, if they are the sources of cosmic rays, energy $\epsilon_p E_{\text{TOT}}$ in non-thermal protons. Further energy is carried by the thermal leptonic and hadronic parts, which is not discussed here.

GRBs emerge as credible sources for the ultra-high-energy cosmic rays because their observed flux can be accommodated with an energy density in protons that is similar to that in electrons. Recent estimates of the local rate of GRBs yield a maximum of $n_0 \sim 1$ Gpc$^{-3}$ yr$^{-1}$ assuming that GRBs follow the star formation rate. For a stronger evolution with redshift, the local rate can be as low as 0.05 Gpc$^{-3}$ yr$^{-1}$ (Asano et al. 2009b).

Using this result, we estimate the electromagnetic energy density from GRBs to be in the range $\rho_{\text{em,GRB}} \approx n_0 \epsilon_e E_{\text{em}} = 5 \times 10^{42} - 10^{44}$ erg Mpc$^{-3}$ yr$^{-1}$. In order for GRBs to be the sources of cosmic rays, their hadronic energy density $\rho_{\text{had,GRB}}$ needs to produce the observed cosmic-ray energy density $\rho_{\text{had,GRB}} = n_0 \epsilon_p E_{\text{TOT}} \approx 10^{44}$ erg Mpc$^{-3}$ yr$^{-1}$. We therefore conclude that $\epsilon_p/\epsilon_e \approx 1$–20. Discussion in, for example, Waxman & Bahcall (1997), Waxman (2004), and Murase et al. (2008) agrees that the ratio must lie around those values. As can be seen from the rough estimate above, the actual number strongly depends on the local rate of GRBs, which is still quite uncertain. In addition, the non-thermal extragalactic spectrum is expected to extend to energies below the knee, but remains unobserved due to the larger contribution of galactic cosmic rays (see, e.g., Ahlers et al. 2005). If this is the case, the average fraction of proton to electron energy needs to be larger.

The Fermi Gamma-ray Space Telescope recently observed two bursts, GRB090510 and GRB090902b, that show a statistically significant deviation from the typical GRB spectrum described by the Band function (Abdo et al. 2009, 2010). A flux of high-energy events is detected that extends to energies of $\sim 30$ GeV following a power-law spectrum (Table 1). This is similar to the much more luminous burst observed by EGRET in 1994, GRB941017 (González et al. 2003). There has been much discussion on the possible origin of these high-energy tails. A leptonic origin, interpreting the high-energy component as synchrotron self-Compton emission, is discussed by Granot & Guetta (2003), Pe'er & Waxman (2004), and Stern & Poutanen (2004). Hadronic processes are also connected to the emission of high-energy photons. Proton synchrotron emission is discussed in Dermer & Atoyan (2004), Razzaque et al. (2010), and Asano et al. (2009a), while the production of neutral pions in photon-hadronic reactions, which lead to the emission of high-energy photons, is discussed in, e.g., Alvarez-Muñiz et al. (2004) for GRB941017. It is noted by Abdo et al. (2010) that at least in the case of GRB090510, all models show difficulties in explaining the emission and there is as yet no model that perfectly fits the observation. In this paper, we will investigate the possibility that the high-energy component results from $\pi^0$-decays and is therefore a signature of proton acceleration. In particular, this model requires relatively high baryonic loading of the jet. We will discuss this fact in detail in this paper.

The main contribution to the initial opacity of the fireball comes from the annihilation of photons into $e^\pm$ pairs. The Fermi observation of a non-thermal spectrum up to an energy $E_{\text{max}}$ of tens of GeV can be used to constrain the minimum bulk Lorentz factor $\Gamma_{\text{min}}$ required to make the source optically thin at the time of the gamma-ray display. For all photons with energy $E \leq E_{\text{max}}$, the condition $\tau_{\gamma\gamma}(E) < 1$ must be fulfilled, where $\tau_{\gamma\gamma}$ is the opacity. The observation of photons with energies of tens of GeV requires highly relativistic outflows with $\Gamma \approx 10^3$. Because, on the other hand, the observed energy flux of order $10^{-4}$ erg cm$^{-2}$ s$^{-1}$ is typical of an average burst, the large boost...
In this paper, we will first discuss the properties of the bursts. We subsequently compute the neutrino flux inevitably produced when the protons interact with fireball photons. Their observation would provide incontrovertible evidence for the pionic origin of the additional high-energy component in the burst and support the speculation that GRBs are the sources of the highest energy cosmic rays.

Is a kilometer-scale neutrino telescope such as IceCube sensitive enough to shed light on these questions? High-energy neutrinos are produced in the fireball when protons produce pions in interactions with the photon field. Using the $\Delta$-resonance approximation,

$$p \gamma \to \Delta^+ \to \begin{cases} n \pi^+ & 1/3 \text{ of the cases} \\ \mu^+ & 2/3 \text{ of the cases} \end{cases}$$

This gives pion ratios of $\pi^+:\pi^0 = 1:2$. The neutral pions decay as $\pi^0 \to 2\gamma$ and the charged pions decay as $\pi^+ \to \mu^+ \nu_\mu \bar{\nu}_\mu$ and $\pi^- \to \mu^- \nu_\mu \bar{\nu}_\mu$. Here, a single neutrino carries approximately 1/4 of the $\pi^+$ energy and a photon carries $1/2$ of the $\pi^0$ energy. The calculation of the neutrino flux (Waxman & Bahcall 1997) has been performed in detail for the BATSE bursts (Guetta et al. 2004; Becker et al. 2006) with the following results: whereas an average burst produces only $\sim 10^{-2}$ neutrinos, bursts that are unusually energetic or nearby may produce an observable flux in a kilometer-scale neutrino telescope of order 10 events per year. We suggest that the power-law high-energy spectral feature can identify such bursts.

We will compute the neutrino fluxes expected in IceCube using two methods: the standard fireball model and the bolo- metric method which relates the energy in neutrinos from the decay of charged pions to the observed photon energy assuming that it is of pionic origin. We will conclude that while the neutrino rates from the *Fermi* bursts are unexceptional, a burst like GRB941017 extending to tens of GeV energy may be detected by IceCube if its power-law component is due to the decay of mesons produced by cosmic rays. IceCube observes cosmic neutrinos in a background of neutrinos produced in the atmosphere. Given that neutrinos of GRB origin are relatively energetic and that the direction and time of the events can be correlated to satellite alerts, the atmospheric background is suppressed and few neutrino events may still represent a conclusive detection. We note that relating neutrinos to an observed gamma-ray flux using the $\Delta$-resonance approximation is extremely conservative in this context and the potential neutrino flux could be as much as a factor 4 greater (Rachen & MeŽaros 1998; Murase & Nagataki 2006). We also note that a non-detection of neutrinos from a GRB941017-like event will not rule out the possibility of GRBs being the sources of cosmic rays, if the observed photons are due to proton synchrotron emission or if the neutrinos are only produced in external shocks. In the latter case the neutrinos would be associated with the afterglow phase, timing of secondary particles would be difficult to estimate, and the background may become too large for a conclusive detection.

### 2. DETECTION OF GRBs AT HIGH PHOTON ENERGIES

The main contribution to the opacity of GRBs fireballs comes from the annihilation of pairs of photons into $e^\pm$ pairs. The observation of a non-thermal spectrum with maximum energy $E_{\text{max}}$ requires the fireball to be optically thin to photons at the time of emission and therefore can be used to constrain the bulk

---

**Table 1**

Spectral Parameters of *Fermi* GRBs with Power-law Components

| GRB090510 | GRB090902b |
|-----------|-----------|
| $z$       | 0.903     | 1.822     |
| $T_{90}$  | 2.1 s     | 21.9 s    |
| $F^{TOT}_{\gamma}$ | $5.02 \times 10^{-5}$ erg cm$^{-2}$ | $4.36 \times 10^{-4}$ erg cm$^{-2}$ |
| $F^{HE}_{\gamma}$  | $1.84 \times 10^{-5}$ erg cm$^{-2}$ | $1.05 \times 10^{-4}$ erg cm$^{-2}$ |
| $\alpha_\gamma$    | $-0.58$   | $-0.61$   |
| $\beta_\gamma$     | $-2.83$   | $-3.80$   |
| $E^{\gamma}_{\max}$ | 30.53 GeV | 33.40 GeV |
| $\Gamma$            | 1260      | 1000      |
| $\delta t$          | 10 ms     | 53 ms     |

**Notes.** The total fluence in gamma rays, $F^{TOT}_{\gamma}$, is the sum of the Band fluence $F^B_{\gamma}$ and the power-law fluence $F^{HE}_{\gamma}$. The fluence for GRB090510 is calculated over the energy range 10 keV–30 GeV, and for GRB090902b is calculated over the range 10 keV–10 GeV. The parameters for GRB941017 are tabulated in González et al. (2003).

---

The factor implies that the photon density in the rest frame of the burst is low. This is a strong effect as the photon density is suppressed by $\Gamma^{-4}$. From $\tau_{\gamma\gamma}(E) = 1$, we can determine $\epsilon_\gamma$ by finding the electromagnetic energy over the volume of the fireball as a fraction of the total GRB energy. The optical depth is defined as

$$\tau_{\gamma\gamma} = \frac{\Delta R}{\lambda_{\gamma\gamma}}. \quad (1)$$

Here, $\Delta R$ is the thickness of the fireball shell in its rest frame and $\lambda_{\gamma\gamma}$ is the photon mean free path. From the definition of the mean free path and the photon number density $n_\gamma$ as given, e.g., in Guetta et al. (2004), we then obtain

$$\tau_{\gamma\gamma} = \Delta R \sigma_{\gamma\gamma} n_\gamma = \Delta R \sigma_{\gamma\gamma} \left( \frac{N_\gamma}{V_{\text{shell}}} \right)$$

$$= \Delta R \sigma_{\gamma\gamma} \left( \frac{L_\gamma}{16\pi c^2 \Gamma^4 \Delta E_{\gamma}} \right). \quad (2)$$

Therefore, since the isotropic luminosity $L_\gamma = \epsilon_\gamma E^{TOT}_{\gamma}/T_{90}$, the condition $\tau_{\gamma\gamma} = 1$ (for some photon energy $E_\gamma$ that characterizes the burst) gives

$$\epsilon_\gamma \approx (1-5) \times 10^{-2} \left( \frac{\Gamma}{300} \right)^4 \left( \frac{\delta t}{10 \text{ ms}} \right) \left( \frac{E_\gamma}{1 \text{ MeV}} \right) \times \left( \frac{T_{90}}{100 \text{ s}} \right) \left( \frac{10^{51} \text{ erg}}{E^{TOT}_{\gamma/100}} \right), \quad (3)$$

where $\Gamma$ is the bulk Lorentz factor of the fireball, $\delta t$ is the variability timescale, $E_\gamma$ is the characteristic gamma-ray energy (which we take to be the peak energy of the event), $T_{90}$ is the duration of the burst, and $E^{TOT}_{\gamma/100}$ is the total isotropic energy of the burst. We therefore estimate $\epsilon_\gamma$ for GRB090510 to be $\sim 0.05$ and for GRB090902b to be $\sim 0.02$. From the low values of $\epsilon_\gamma$ thus obtained protons seem to dominate the fireball. Of course, this discussion is merely to illustrate the argument that follows in this paper. For instance, since photons of energy $E_\gamma$ pair-produce most often on photons with energies around $E^{\text{thresh}}_{\gamma\gamma} = \Gamma^2 m_e^2/E_\gamma$, approximating GRBs as monoenergetic is a poor assumption. It must also be noted that Equation (3) depends on the value for $E^{TOT}_{\gamma/100}$, which cannot be measured.
Lorentz factor $\Gamma$. By generalizing Lithwick & Sari (2001) to account for arbitrary photon spectra and the energy dependence of the pair production cross section, we can express the optical depth $\tau$ for the most energetic photon in the GRB fireball (with observed energy $E_{\text{max}}$) as

$$\tau = \frac{4\pi d_L^2 \delta t}{4\pi (\Gamma^2 c \delta t)^2} \int_{-1}^{1} d(\cos \theta) \frac{(1 - \cos \theta)}{2} \times \int_{0}^{E_{\text{max}}} dE_{\gamma} \sigma_{\gamma\gamma} \left( \frac{(1 + z)E_{\gamma}}{\Gamma}, \frac{(1 + z)E_{\text{max}}}{\Gamma}, \cos \theta \right) \frac{dN_{\gamma}}{dE_{\gamma}}(E_{\gamma}),$$

(5)

where $d_L(z)$ is the luminosity distance to the source assuming the same $\Lambda$CDM cosmological parameters used in Abdo et al. (2010), $\sigma_{\gamma\gamma}(E_1, E_2, \cos \theta)$ is the cross section for two real photons colliding at an angle $\theta$ with energies $E_1$ and $E_2$ to produce an electron–positron pair (Gould & Schrédér 1967), and $dN_{\gamma}/dE_{\gamma}$ is the observed photon spectrum (on Earth) from the GRB. The collision kinematics must take into account the boost factor of the fireball $\Gamma$ and the fact that the observed photons have nonzero redshift $z$. The additional factors of $\Gamma$ and $(1 + z)$ modifying the arguments of $\sigma_{\gamma\gamma}$ therefore transform the observed photon energies to the energies in the center-of-mass frame and at the source, respectively. The variability timescale $\delta t$ relates to the thickness of the shell $r = 2 \Gamma^2 c \delta t$ and therefore affects the density of target photons. For bursts where it is not known, we take a standard value of 10 ms (see Tables 1 and 2), a source of considerable uncertainty in the calculation. We note that in principle, rather than fixing the variability time to determine the shell thickness, one can use the opacity argument to constrain $r$ rather than $\Gamma$ (Zhang & Pe’er 2009).

We consider only the contribution to the opacity due to pair production and neglect other processes such as inverse Compton scattering and synchrotron self-absorption. The opacity due to Compton scattering is usually taken to be small in a GRB fireball (Razzaque et al. 2004; although there exist models where this assumption fails, e.g., Rees & Meśzaros 2005) and the opacity below $\sim 10^{16}$ eV is not greatly affected by synchrotron self-absorption (Murase 2009). The pair-creation opacity decreases at very high energies and can become smaller than unity at $\sim 10^{35}$--$10^{36}$ eV since photons below the synchrotron self-absorption energy are not important.

The first clear detection of a GRB with a power-law component in addition to the standard Band spectral form (Band et al. 1993) was made by the EGRET satellite with the detection of GRB940107 with a spectrum that extended to 200 MeV, reaching the limit of EGRET’s sensitivity (González et al. 2003). Estimates of the corresponding neutrino flux and the expected event rate in km$^{-2}$-scale neutrino telescopes are presented in Alvarez-Muñiz et al. (2004). Of particular importance is the fact that since EGRET did not detect a cutoff of the power-law component, the flux can be modeled as extending to potentially very high-energy $E_{\text{max}}$. Using Equation (5) with the total observed photon spectrum (Band + power law), we may then find the boost factor $\Gamma$ for which $\tau = 1$. We call this $\Gamma_{\text{min}}$, the minimum boost factor for which the fireball is transparent to all photons up to energy $E_{\text{max}}$, and is shown in Figure 1 for time bins 2–5 (time bin 1 showed no significant power-law component).

$\textit{Fermi}$ has been collecting very energetic photons (8 keV–300 GeV) from GRBs since 2008 June. As of 2009 September, at the end of the first year of observations, 10 bursts have been detected by the Gamma-ray Burst Monitor (GBM) and the Large Area Telescope (LAT) simultaneously with photon energies above $\sim 100$ MeV. $\textit{Fermi}$ LAT detections have constrained the values of the jet boost factors to values of $\Gamma \sim 10^3$ for GRB080916c, GRB090510, and GRB090902b thus confirming the highly relativistic nature of the outflows. Furthermore, the time-integrated spectra of the recently observed GRB090510 and GRB090902b show statistically significant deviations from the Band function that extend up to $\sim 30$ GeV.

GRB090510 is a short burst ($T_{90} = 2.1$ s) that shows a spectral high-energy component up to $\sim 30$ GeV with a fluence of $\sim 1.8 \times 10^{-5}$ erg cm$^{-2}$ ($\sim 40\%$ of the total fluence in the energy range 10 keV–30 GeV; Abdo et al. 2010). The isotropic energy release is estimated to be $\epsilon_e E_{\text{ISO}} \sim 1.08 \times 10^{53}$ erg. The $\sim 31$ GeV photon is detected $\sim 1$ s after the trigger time and sets the highest lower limit on a GRB Lorentz factor ($\Gamma \sim 1200$) thus proving the outflows in short GRBs to be as highly relativistic as those in long GRBs.

The extra component of the long ($T_{90} = 21.9$ s) GRB090902B extends up to 11.2 GeV and its fluence accounts for $\sim 24\%$ of the total fluence over the energy range 10 keV–10 GeV in the first 25 s of the prompt emission (Abdo et al. 2009). The corresponding isotropic energy release is measured to be $\epsilon_e E_{\text{ISO}} \sim 3.63 \times 10^{54}$ erg. The delay of the highest energy photon with respect to trigger time is $\sim 80$ s and the Lorentz factor of the jet is estimated to be $\Gamma \sim 1000$ from opacity considerations involving the highest energy observed photon (11.16 GeV) during the prompt emission phase. In Figure 2, a comparison between the fluences of the $\textit{Fermi}$ bursts and the fluence of GRB941017 as a function of $E_{\text{max}}$ is presented and the relative weakness of the $\textit{Fermi}$ bursts appears evident. In Table 1, the spectral parameters from the $\textit{Fermi}$ observations of GRB090510 and GRB090902b are summarized.

Deviations from the Band-only fit in the spectra are particularly interesting in the context of hadronic acceleration within the fireball and relate closely to predictions of neutrino fluxes detectable on Earth with km$^3$ telescopes. If these extra components originate from $\pi^0$-decay photons they provide an optimal benchmark for testing models of hadronic acceleration in GRB engines.
3. FIREBALL NEUTRINOS

In the hadronic fireball, a burst of high-energy neutrinos is expected to accompany the observed prompt flux of gamma-ray photons. Assuming that electrons and protons are shock-accelerated in the same region, the neutrino spectrum can be calculated from the observed spectrum in gamma rays using conventional fireball phenomenology as described in Guetta et al. (2004) and Abbasi et al. (2009). For a typical GRB, the gamma-ray spectrum is usually well described by a Band function:

$$
\frac{dN_{\gamma}}{dE_{\gamma}} = A \cdot \left\{ \left( \frac{E_{\gamma}}{100 \text{ GeV}} \right)^{\alpha} \exp \left( - \frac{E_{\gamma}}{\Gamma} \right) \right\} \frac{dN_{\gamma}}{dE_{\gamma}} + A \cdot \left\{ \left( \frac{E_{\gamma}}{100 \text{ GeV}} \right)^{\alpha - \beta} \exp \left( \beta - \alpha \right) \left( \frac{E_{\gamma}}{100 \text{ GeV}} \right)^{\beta} \right\} \right. \left. \text{if } (\alpha - \beta)E_{\gamma} \geq E_{\gamma} \right\}
$$

The interaction of accelerated protons with a power-law distribution with GRB photons results in a broken power-law spectrum according to

$$
\alpha_v = 3 - \beta_v, \quad \beta_v = 3 - \alpha_v, \quad \gamma_v = \beta_v + 2.
$$

The first break energy $\epsilon_1$ is determined by the production threshold for the $\Delta$-resonance, where

$$
\epsilon_1 = 7.5 \times 10^5 \text{ GeV} \frac{1}{(1 + z)^2} \left( \frac{\Gamma}{10^{2.5}} \right)^2 \left( \frac{\text{MeV}}{\epsilon_\gamma} \right).
$$

$\epsilon_\gamma$ is the break energy of the Band function, $\Gamma$ is the jet boost factor, and $z$ is the redshift of the source. The spectrum steepens when pions lose energy due to synchrotron radiation prior to decay; the second break energy $\epsilon_2$ is given by

$$
\epsilon_2 = 10^7 \text{ GeV} \frac{1}{1 + z} \sqrt{\frac{\epsilon_e}{\epsilon_B} \left( \frac{\Gamma}{10^{2.5}} \right)^4 \left( \frac{\delta t}{10 \text{ ms}} \right) \sqrt{\frac{10^{52} \text{ erg s}^{-1}}{L_B^\gamma}}}.
$$

$\epsilon_e$ is the fraction of the total burst energy available to electrons, $\epsilon_B$ is the fraction of total energy going into the magnetic field $\vec{B}$, and $\delta t$ is the variability timescale. $L_B^\gamma$ is the isotropic luminosity of the Band function, given by

$$
L_B^\gamma = \frac{E_B^\gamma}{\Delta t} = \frac{4\pi d_L^2(z) F_B^\gamma}{\Delta t (1 + z)}
$$

with $\Delta t = T_{90}$ and $d_L$ the luminosity distance for $\Omega_m = 0.27$, $\Omega_\Lambda = 0.73$, and $h = 0.71$. The Band gamma-ray energy fluence $F_B^\gamma$ is defined as

$$
F_B^\gamma = \Delta t \int_0^\infty dE_\gamma E_\gamma \frac{dN_{\gamma}}{dE_{\gamma}},
$$

where the gamma-ray spectrum $dN_{\gamma}/dE_{\gamma}$ is the Band spectrum given in Equation (6).

The normalization of the neutrino flux is determined by the efficiency of pion production. The relation between the gamma-ray and neutrino spectra is given by

$$
F_\nu = x \cdot F_B^\gamma, \quad \text{where } x = \frac{1}{8} \frac{\epsilon_\gamma}{\epsilon_p} \left[ 1 - (1 - (\tau_{\pi \rightarrow \mu})^{\Delta R/\tau_{\pi \rightarrow \mu}}) \right]
$$

and

$$
\Delta R = \frac{L_B^\gamma}{10^{52} \text{ erg s}^{-1}} \left( \frac{10 \text{ ms}}{\delta t} \right) \left( \frac{10^{2.5}}{\Gamma} \right) \frac{\text{MeV}}{\epsilon_\gamma}.
$$

In Equation (13), $\tau_{\pi \rightarrow \mu} = 0.2$ is the fraction of proton energy transferred to a pion in a single interaction and $\epsilon_\gamma$ is the break energy of the Band function. The neutrino energy fluence $F_\nu$ is defined analogously to $F_B^\gamma$ (Equation (12)).

The values used to calculate the expected fireball neutrino rates from the Fermi bursts, using only the Band component of the photon spectra, are listed in Table 3 together with the typical Waxman–Bahcall (WB) model parameters. The numbers of neutrino events in IceCube estimated using the fireball formalism with the Band photon spectrum are reported in Table 3.
4. NEUTRINOS FROM POWER-LAW GRB SPECTRA

4.1. Fluences of Secondary Particles

If we assume that the observed power-law components of the bursts discussed here are due to the decay of neutral pions produced in interactions between protons accelerated by shocks in the jet and photons, we can predict the accompanying flux of muon neutrinos from the production and decay of charged pions. The observed power-law spectral components are quite flat ($E^{-1} - E^{-1.6}$), a spectral behavior that is compatible with synchrotron radiation from highly relativistic electrons and positrons which are produced when high-energy photons scatter in the fireball photon field. For interactions of protons accelerated in shocks with fireball photons, we assume that the scatter in the fireball photon field. For interactions of protons and positrons which are produced when high-energy photons scatter with synchrotron radiation from highly relativistic electrons, the final neutrino spectrum will be of the form of Equation (7). If we find the charged pion spectrum that, upon decay, produces the correct neutrino spectrum, we can determine the corresponding neutral pion spectrum and thus the gamma-ray spectrum. Assuming that the cascading process conserves energy, we can normalize the fluence of gamma rays to the fluence of the measured power-law tail assuming its extension to some maximum energy $E_{\text{max}}$.

$$\int_{0}^{E_{\text{max}}} E_{\gamma} \frac{dN_{\gamma}}{dE_{\gamma}} dE_{\gamma} = \int_{E_{\text{min}}}^{E_{\text{max}}} E_{\gamma} \frac{dN_{\text{HE}}}{dE_{\gamma}} dE_{\gamma},$$

where $E_{\text{min}}$ is the minimum measured photon energy and $E_{\text{max}}$ is the proposed upper limit of the measured power-law photon spectrum $dN_{\text{HE}}/dE_{\gamma}$. The upper limit of $10^{19}$ eV for the uncascaded gamma-ray spectrum assumes that the parent protons extend to $10^{20}$ eV and that gamma rays take on average 1/10 of the parent proton energy. Due to the flatness of the measured power-law spectra, the precise value of $E_{\text{min}}$ is unimportant and the energy going into hadronic gamma rays is determined purely by $E_{\text{max}}$. Since we assume that the fireball is transparent for photons of energy $E_{\text{max}}$, the boost factor $\Gamma$ will vary as we vary the total energy in hadronic photons.

The procedure for obtaining the decay particle spectra from the pion spectra, Equations (16) and (17), is described in Section 4.2 below. Finally, using Equation (15) with the factor $\eta$ found from Equation (18), we can find the actual amount of energy going into neutrinos relative to gamma rays:

$$\int_{0}^{E_{\text{max}}} E_{\gamma} \frac{dN_{\gamma}}{dE_{\gamma}} dE_{\gamma} = \eta \int_{0}^{10^{19}} E_{\gamma} \frac{dN_{\gamma}}{dE_{\gamma}} dE_{\gamma}. \quad (18)$$

The algorithm for finding the flux of neutrinos is as follows. For a given $E_{\text{max}}$, the measured parameters of the GRB will determine the minimum boost factor $\Gamma_{\text{min}}$ (Equation (5)). These parameters and boost factor determine the neutrino spectral indices and break energies (Equations (8)–(10)), giving us the unnormalized neutrino spectrum. Assuming that the neutrinos take on average 0.25 of the pion energy, we define the charged pion spectrum to be

$$\frac{dN_{\pi^{\pm}}}{dE_{\pi}} = f_{\pi^{\pm}} \cdot \begin{cases} \frac{\epsilon_{1,\pi}^{\gamma} - \epsilon_{2,\pi}^{\gamma}}{\epsilon_{1,\pi}^{\pi} - \epsilon_{2,\pi}^{\pi}}, & \text{if } E_{\pi} < \epsilon_{1,\pi}^{\pi} \\ \frac{\epsilon_{1,\pi}^{\gamma} - \epsilon_{2,\pi}^{\gamma}}{\epsilon_{1,\pi}^{\pi} - \epsilon_{2,\pi}^{\pi}}, & \text{if } \epsilon_{1,\pi}^{\pi} < E_{\pi} < \epsilon_{2,\pi}^{\pi} \\ \frac{\epsilon_{1,\pi}^{\gamma} - \epsilon_{2,\pi}^{\gamma}}{\epsilon_{1,\pi}^{\pi} - \epsilon_{2,\pi}^{\pi}}, & \text{if } E_{\pi} > \epsilon_{2,\pi}^{\pi} \end{cases} \quad (16)$$

with pion break energies $\epsilon_{i,\pi}$ relating to the neutrino break energies $\epsilon_{i}$ (Equations (9) and (10)) via $\epsilon_{i,\pi} = 4\epsilon_{i}$. From the charged pion spectrum, we derive the neutral pion spectrum:

$$\frac{dN_{\pi^{0}}}{dE_{\pi}} = 2 \cdot \begin{cases} \frac{dN_{\pi^{\pm}}}{dE_{\pi}} \left( \frac{\epsilon_{2,\pi}^{\gamma} - \epsilon_{1,\pi}^{\gamma}}{\epsilon_{2,\pi}^{\pi} - \epsilon_{1,\pi}^{\pi}} \right)^{2}, & \text{if } E_{\pi} < \epsilon_{2,\pi}^{\pi} \\ \frac{dN_{\pi^{\pm}}}{dE_{\pi}}, & \text{if } E_{\pi} > \epsilon_{2,\pi}^{\pi} \end{cases} \quad (17)$$

We assume that the photohadronic interaction takes place at the $\Delta$-resonance, giving twice as many neutral as charged pions. Moreover, the second break in the neutrino and charged pion spectra is due to the cooling of long-lived charged pions in the fireball and will not be present in the neutral pion spectrum.

From the pion spectra we can find the decay spectra of gamma rays and neutrinos, normalized relative to the arbitrary factor of $f_{\pi^{\pm}}$ in Equation (16) (see Figure 3). Integrating over the neutrino and gamma-ray spectra then gives the amount of energy going into neutrinos relative to gamma rays:

$$\int_{0}^{5 \times 10^{18}} E_{\gamma} \frac{dN_{\gamma}}{dE_{\gamma}} dE_{\gamma} = \eta \int_{0}^{10^{19}} E_{\gamma} \frac{dN_{\gamma}}{dE_{\gamma}} dE_{\gamma}, \quad (19)$$

Since neutrinos originating from pion decay have the flavor ratio $\nu_{\mu}:\nu_{\tau}:\nu_{\tau} = 1:2:0$, over an astronomical baseline oscillations will transform the beam into the flavor ratio 1:1:1 if $\theta_{23}$ is small. Therefore, the flux of muon-type neutrinos that reach the detector is half of the emitted flux and the final neutrino spectrum must be multiplied by a factor 0.5.

In this method, it is clear that higher observed maximum gamma-ray energies lead to more energy in neutrinos. This does not contradict the discussion in Section 2 that describes...
how a higher \( E_{\text{max}} \) implies a larger boost factor \( \Gamma \) and hence a more transparent firewall. While a larger boost factor will imply a smaller photonson production efficiency, we can relate \( E_{\text{max}} \) to the proton fraction \( \epsilon_p/\epsilon_e \) via Figure 1 and Equation (3) (see Figure 5). For GRB941017, this gives us \( \epsilon_p/\epsilon_e \sim 10,000 \) for \( E_{\text{max}} \sim 50 \) GeV, to keep the total energy of the burst constant. A proton loading of this magnitude would likely compensate for the lowered photonson production due to increased boost factor. The question of the baryonic loading is discussed in detail in Section 5.

Throughout this paper, it has been assumed that the observed high-energy photons from the Fermi bursts are from the prompt phase of the emission rather than from the afterglow. We note that this assumption may not be true and have calculated neutrino event rates for maximum photon energies below the observed maximum, down to 200 MeV in all cases.

We also assume in this section that the observed power-law components correspond to all the energy emitted in hadronic gamma rays. There is, however, the possibility that some part of the high-energy gamma-ray tail will escape the GRB without interaction if the opacity is not large. These photons would cascade outside of the source and are typically not observed (Razzaque et al. 2004; Murase et al. 2008). In that case, estimating the total hadronic gamma-ray energy using the observed photons would result in an underestimation of the energy and hence the neutrino flux. As a result, the neutrino fluxes derived in this section are from a minimum hadronic energy and are therefore conservative.

If GRBs are optically thin to photohadronic interactions, synchrotron self-Compton (Granot & Guetta 2003; Pe’er & Waxman 2004, Stern & Poutanen 2004) or proton synchrotron radiation (Dermer & Atoyan 2004; Razzaque et al. 2010; Asano et al. 2009a) could dominate. Here, however, we posit that the sources are not optically thin to photohadronic interactions. It is clear that the observation of neutrinos with a full IceCube detector will resolve this issue.

Finally, we make a number of assumptions regarding the relevant particle physics. We assume that the observed photons and neutrinos are due to the decays of pions only and neglect the contributions of the more massive \( \eta \) and \( K^{0,\pm} \) mesons. We also assume that between the pion decaying into a muon and a neutrino the muon’s subsequent decay, the muon’s polarization does not change due to interactions with photons or the magnetic field.

### 4.2. Pion Decay Spectra

The muon neutrino and gamma-ray spectra are determined from pion spectra as follows: both gamma rays are the product of the decay of the neutral pion \( \pi^0 \to \gamma + \gamma \). The two muon neutrinos are due to the decay chain \( \pi^\pm \to \bar{\nu}_\mu + \mu^\pm \), \( \mu^\pm \to \bar{\nu}_\mu + \bar{\nu}_e + e^\pm \).

The gamma rays and the first neutrino are found from standard two-body decay kinematics (Stecker 1971; Dermer 1986; Gaisser 1990):

\[
\phi_i(E_i) = A_i \int_{E_{\text{min}}(E_i)}^{E_{\text{max}}(E_i)} \frac{\phi_\pi(E_\pi)}{p_\pi} dE_\pi
= A_i \int_{E_{\text{min}}(E_i)}^{E_{\text{max}}(E_i)} \frac{\phi_\pi(E_\pi)}{\sqrt{E_\pi^2 - m_\pi^2}} dE_\pi.
\]

For both particle types \((i = \nu, \gamma)\),

\[
E_{\pi,\text{min}} = \frac{E_i}{1 - r_i} (1 - r_i) \frac{m_\pi^2}{4E_i}.
\]

where \( r_\gamma = 0 \) (neutral pion decay), \( r_\nu = (m_\nu/m_\pi)^2 \) (charged pion decay), \( A_\nu = 2 \), and \( A_\gamma = (1 - r_\nu)^{-1} \). Due to the \( E + E^{-1} \) form of \( E_{\text{min}} \), the two-body spectra are symmetric (on a log–log scale) around a peak set by the pion mass and by the decay kinematics scale factor \( r \). This peak is at \( m_\nu/2 \approx 70 \) MeV for neutral pion decay and \((1 - r_\nu)m_\nu/2 \approx 30 \) MeV for charged pion decay. For a power-law distribution of pions with energies much larger than the pion mass the spectrum is a power law of the same slope.

The second neutrino is due to the decay of the muon from the charged pion \( \mu^\pm \to e^\pm + \nu_e + \bar{\nu}_\mu \). This is a three-body decay of a particle with a two-body decay energy distribution. The spectrum is (Gaisser 1990)

\[
\phi_{\nu,2}(E_\nu) = \int_{E_{\text{min}}(E_\nu)}^{E_{\text{max}}(E_\nu)} dE_\mu \int_{E_{\text{min}}(E_\pi)}^{E_{\text{max}}(E_\pi)} dE_\pi \frac{\phi_\pi(E_\pi)}{1 - r_\nu} \sqrt{E_\pi^2 - m_\pi^2} E_\pi dE_\pi.
\]

The full limits on the integrals, valid at all energies, are

\[
E_{\mu,\text{min}} = E_\nu + \frac{m_\nu^2}{4E_\nu}
\]

\[
E_{\pi,\text{max}} = \frac{E_\mu}{r_\nu} - (1 - r_\nu) \frac{m_\pi^2}{4E_\mu}
\]

\[
E_{\pi,\text{min}} = E_\mu + (1 - r_\nu) \frac{m_\pi^2}{4E_\mu}.
\]

Gaisser (1990) considers only the spectrum at high energy and therefore omits the second term of each limit. The last term in the integrand \((1/E_\mu) dE/dy \) is the muon decay distribution. It is given, for \( y = E_\nu/E_\mu \), by

\[
\frac{dE}{dy} = \frac{1}{\beta_\mu} \int_{E_{\text{min}}(1)}^{E_{\text{max}}(1)} \left[ f_0(x) \pm P_\mu f_1(x) \frac{2y - x}{\beta_\mu x} \right] dx,
\]

where

\[
f_0 = 2x^2(3 - 2x)
\]

\[
f_1 = 2x^2(1 - 2x)
\]

\[
P_\mu = \frac{1}{\beta_\mu} \left( \frac{2E_\mu r_\nu}{E_\mu(1 - r_\nu)} - \frac{1 + r_\nu}{1 - r_\nu} \right).
\]

\( \pm P_\mu \) corresponds to the decay of \( \mu^\pm \), respectively. The limits of the integral are

\[
x_{\text{min}} = \frac{2y}{1 + \beta_\mu}
\]

\[
x_{\text{max}} = \min \left[ 1, \frac{2y}{1 - \beta_\mu} \right].
\]

### 4.3. Neutrino Spectra and Event Rates

Folding the derived neutrino fluxes with the IceCube effective area (Gonzalez-Garcia et al. 2009), we find the number of neutrino events as a function of neutrino energy and \( E_{\text{max}} \) of the observed gamma-ray fluence. In Figure 4, we show the number of detected neutrinos with energy greater than detector threshold as a function of \( E_{\text{max}} \). We call attention to the fact that a burst with the same parameters as GRB941017, extending to \( E_{\text{max}} > 50 \) GeV, will produce > 1 neutrino event in IceCube. Given the absence of background events over such a short time interval, only a few events would be needed to constitute
The dependence of the pair-creation opacity $\tau_{\gamma\gamma}$ on the jet boost factor $\Gamma$ was presented in Equation (2) where it was shown that $\tau_{\gamma\gamma} \propto \Gamma^{-4}$. The source must be therefore more transparent when photons with increasing energy $E_{\text{max}}$ are detected. In addition to this, the efficiency of pion production, $f_{\pi}$, was shown to be proportional to $\tau_{\gamma\gamma}$ (Waxman & Bahcall 1997). The predicted flux of neutrinos is therefore believed to be suppressed when $\Gamma$ becomes large. Here, we argue that this problem can be overcome by the requirement that GRBs are proton dominated. The parameter $\epsilon_p/\epsilon_e$ (an intrinsic burst parameter, hence a constant during the evolution of the burst) has to be $\gg 1$ to compensate for the decrease of the efficiency of pion production at large boost factors. However, since this is only the high-energy, nonthermal part of the energy balance, extremely high values of $\epsilon_p/\epsilon_e$ cannot be ruled out.

In our discussion we have assumed that the high-energy component of all time bins of GRB941017 originates from the interaction of UHE CRs with photons of the spectral region described with the Band function. Here, we sum up the fluences of time bins 2–5 to get the total (time-integrated) fluence of the observed high-energy component:

$$ F_{\text{TOT}}^{\text{HE}}(E_{\text{max}}) = \sum_{i=2}^{5} F_{\text{HE}}^{i}(E_{\text{max}}) $$

where

$$ F_{\text{HE}}^{i}(E_{\text{max}}) = \int_{30 \text{ keV}}^{E_{\text{max}}} E_{\gamma} \frac{dN_{i,\gamma}}{dE_{\gamma}} dE_{\gamma} $$

and $dN_{i,\gamma}/dE_{\gamma}$ is taken from González et al. (2003). The index $i$ runs over time bins. The value of $E_{\text{max}}$ is treated as a free parameter in all time bins since no break was detected by BATSE up to 200 MeV. Therefore, we let it vary up to 1 TeV and we calculate the corresponding observed total fluence of the high-energy power law. We then compare $F_{\text{TOT}}^{\text{HE}}(E_{\text{max}})$, which is proportional to $\epsilon_p/\epsilon_e$:

$$ F_{\text{HE}}^{\text{TOT}} \propto \frac{\epsilon_p}{\epsilon_e} $$

with the theoretical flux of photons from $\pi^0$-decay, $F_{\text{HE}}^{\text{Theory}}$. In this way, the hadronic model can reproduce the observed high-energy component for a given value of $\epsilon_p/\epsilon_e$. In Figure 5, the ratio $\epsilon_p/\epsilon_e$ is plotted against $E_{\text{max}}$. It is important to stress that

Figure 4. Neutrino events in IceCube from 3 GRBs as a function of maximum observed gamma-ray energy.

Figure 5. $\epsilon_p/\epsilon_e$ as a function of observed $E_{\text{max}}$. 

5. PHOTOMESON PRODUCTION EFFICIENCY

The discovery of proton acceleration in GRBs. While very high $E_{\text{max}}$ would require an (possibly) unphysically large boost factor and proton fraction, the detectability of a GRB941017-like burst with $E_{\text{max}}$ within Fermi’s energy range is highly encouraging.

These neutrino event numbers are very different from the numbers derived with the standard fireball phenomenology using only the Band spectrum (Table 3). While the two numbers of neutrino events for GRB941017 are similar if $E_{\text{max}} \sim 200$ MeV, the fireball event numbers for the Fermi bursts are both more than an order of magnitude smaller than the numbers calculated using the bolometric method, showing the large contribution of the GeV power-law component gamma rays relative to the lower-energy Band spectrum photons.
the correct relation between $\Gamma$ and $E_{\text{max}}$ has been used. We also note that for this GRB the values of $\epsilon_p/\epsilon_e$ shown in Figure 5 refer to the time-integrated fluence defined in Equation (26). This is due to the assumption that $\epsilon_p/\epsilon_e$ is an intrinsic property of the burst and that therefore it should not vary with time. For each value of $\epsilon_p/\epsilon_e$ a lower limit for the beamed energy is then inferred using the relation

$$E_{\text{beam}} \sim \frac{\theta_j^2}{2} \epsilon_p E_{\text{iso}}^{\text{TOT}} \geq \frac{1}{2\Gamma^2} \epsilon_p E_{\text{iso}}^{\text{TOT}} \quad (27)$$

since $\theta_j \geq 1/\Gamma$ (Piran 2004). Results are shown in Figure 6 and a physical limit can be inferred from the figure. For comparable electron and proton number densities in GRB shock waves, the ratio of proton–electron energy density scales with the ratio of the masses, $m_p/m_e \sim 1800$. If the number densities of the two particle species differ for some reason, though, energy densities can also be larger or smaller in principle. Very recently pair instability SNe have been shown to release $> 10^{52}$ erg (Gal-Yam et al. 2009). A similar energy release can be realistic in GRBs. Thus, we conclude that if beaming effects are considered the total energy release implied by the GRBs discussed here can be accommodated.

Applying the same argument to the time-integrated spectra for GRB090510 and GRB090902b up to the observed $E_{\text{max}}$ yields $\epsilon_p/\epsilon_e \sim 170$ and $\sim 63$, respectively, and the beamed energy release using Equation (27) with the corresponding numerical value of the jet boost factor (Table 3) is estimated to be $\sim 3.66 \times 10^{48}$ erg and $\sim 8.70 \times 10^{49}$ erg (Figure 6).

6. SUMMARY AND CONCLUSIONS

Three GRBs with statistically significant high-energy power-law spectral components have been detected thus far. While both leptonic and hadronic models have been proposed to explain the additional components, all models show some difficulties in explaining the observations. In this paper, we discuss the possibility of proton-dominated bursts with photodisintegration interactions being responsible for the production of the high-energy component. We calculate the associated fluxes of neutrinos for all three bursts. First, the event rates of neutrinos are derived from standard fireball phenomenology assuming that the total energy in protons is 10 times more than in electrons. In a second approach, we take the existence of the high-energy power-law components as indicative of the decay of $\pi^0$-mesons and keep the ratio of the energy going into non-thermal electrons and protons as a free parameter. This allows us to calculate the magnitude of the neutrino flux from the related charged pions, and we find that a burst like GRB941017 will produce at least one neutrino event and hence be detectable by IceCube if its power-law component extends to energies in excess of $\sim 50$ GeV. While the event rates for the Fermi bursts are small, this is due to their large boost factors and redshifts as opposed to a burst like GRB941017, which is nearby with apparently low boost factor. IceCube and the future observatory Km3NeT will be able to help determine if the high-energy components in GRBs are indeed due to photodisintegration interactions or if other scenarios are more viable. Finally, non-detection of high-energy prompt neutrinos would not exclude UHECR acceleration in GRBs. In fact neutrino production may occur at external shocks only and may therefore be related to the afterglow phase.

J.K.B. and M.O. acknowledge support from the Research Department of Plasmas with Complex Interactions (Bochum). F.H. and A.Ó.M were supported in part by the National Science Foundation under Grant No. OPP-0236449 and in part by the University of Wisconsin Alumni Research Foundation. The authors thank an anonymous referee for extensive comments.

REFERENCES

Abbasi, R., et al. 2009, ApJ, 701, 1721
Abdo, A. A., et al. 2009, ApJ, 706, L138
Abdo, A. A., et al. 2010, ApJ, in press (arXiv:1005.2141)
Ahlers, M., Anchordoqui, L., Goldberg, H., Halzen, F., Ringwald, A., & Weiler, T. 2005, Phys. Rev. D, 72, 023001
Alvarez-Muñiz, J., Halzen, F., & Hooper, D. 2004, ApJ, 604, L85
Asano, K., Guiriec, S., & Mészáros, P. 2009a, ApJ, 705, L191
Asano, K., Inoue, S., & Mészáros, P. 2009b, ApJ, 699, 953
Band, D., et al. 1993, ApJ, 413, 281
Becker, J. K., Stamatikos, M., Halzen, F., & Rhode, W. 2006, Astropart. Phys., 25, 118
Dermer, C. 1986, ApJ, 307, 47
Dermer, C., & Atoyan, A. 2004, A&A, 418, L5
Gaisser, T. 1990, Cosmic Rays and Particle Physics (Cambridge: Cambridge Univ. Press)
Gal-Yam, A., et al. 2009, Nature, 462, 8579
Gonzalez-Garcia, M. C., Halzen, F., & Mohapatra, S. 2009, Astropart. Phys., 31, 437
Gould, R. J., & Schréder, G. P. 1967, Phys. Rev., 155, 1404
Granot, J., & Guetta, D. 2003, ApJ, 598, L11
Guetta, D., Hooper, D., Alvarez-Muñiz, J., Halzen, F., & Reuvini, E. 2004, Astropart. Phys., 20, 429
Lithwick, Y., & Sari, R. 2001, ApJ, 555, 540
Murase, K. 2009, Phys. Rev. Lett., 103, 081102
Murase, K., Ioka, K., Nagataki, S., & Nakamura, T. 2008, Phys. Rev. D, 78, 023005
Murase, K., & Nagataki, S. 2006, Phys. Rev. D, 73, 063002
Pe'er, A., & Waxman, E. 2004, ApJ, 603, L1
Piran, T. 2004, Rev. Mod. Phys., 76, 1162
Rachen, J. P., & Mészáros, P. 1998, Phys. Rev. D, 58, 123005
Razzaque, S., Dermer, C., & Finke, J. 2010, Open Astron. J., 3, 150
Razzaque, S., Mészáros, P., & Zhang, B. 2004, ApJ, 613, 1072
Rees, M. J., & Mészáros, P. 2005, ApJ, 628, 947
Stecker, F. 1971, Cosmic Gamma Rays (Baltimore, MD: Mono Book Co.)
Stern, B. E., & Poutanen, J. 2004, MNRAS, 352, L35
Vietri, M. 1995, ApJ, 453, 883
Waxman, E. 1995, Phys. Rev. Lett., 75, 386
Waxman, E. 2004, ApJ, 606, 988
Waxman, E., & Bahcall, J. 1997, Phys. Rev. Lett., 78, 2292
Zhang, B., & Pe'er, A. 2009, ApJ, 700, L65