Quantum preprocessing for information-theoretic security in two-party computation

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In classical two-party computation, a trusted initializer who prepares certain initial correlations, known as one-time tables, can help make the inputs of both parties information-theoretically secure. We propose some bipartite quantum protocols with possible aborts for approximately generating such bipartite classical correlations with varying degrees of privacy, without introducing a third party. For the security level to be interesting for secure two-party computation, it suffices to require that one particular party is conservative, meaning that he values the privacy of his data higher than the learning of the other party’s data, or to require that the other party is honest-but-curious. We show that the security is usually dependent on the noise level, but not for some party in one of the protocols. We show how to use the generated one-time tables to achieve nontrivial information-theoretic security in generic two-party classical or quantum computation tasks.

I. INTRODUCTION

The security of two-party computation is a main research topic in classical cryptography. The goal is usually to correctly compute some function of the inputs from the two parties, while keeping the inputs as private from the opposite party as possible. This has been studied using classical homomorphic encryption techniques [1, 2] or through implementing Yao’s “Garbled Circuit” solution [3]. Another possibility is to introduce a trusted third party, who may sometimes interact with the two parties for multiple rounds. To lower the requirement on the trusted third party, a “trusted initializer” has been proposed [4]. Such trusted initializer only prepares some initial correlations between the two parties, and does not interact with any party afterwards. A trusted initializer who prepares certain initial correlations, referred to as “one-time tables”, can help make the bipartite computation secure.

Secure two-party quantum computation is the corresponding problem in quantum computing and quantum cryptography. The two parties wish to correctly compute an output according to some public or private program while keeping their (quantum) inputs as secure as possible. Special cases of this general problem include quantum homomorphic encryption (QHE) [5–17], secure assisted quantum computation [18, 19], and computing on shared quantum secrets [20], and physically motivated secure computation (e.g. [21]). In the study of QHE, it is found that secure computation of the modulo-2 inner product of two bit strings provided by the two parties is a key task, and the one-time tables mentioned above turn out to be helpful for this task.

In this work, we propose two-party quantum protocols with aborts as replacements for the trusted initializer in preparing the one-time tables, and show that the prepared one-time tables can help achieving some degree of information-theoretic security in bipartite classical or quantum computation. Our main protocols are based on Protocol 1 which implements the following task approximately with partial privacy: it takes as input two locally-generated random bits $x$ and $y$ from Alice and Bob, respectively, and outputs two bits ($x$ AND $y$) XOR $r$ and $r$ on the two parties, where $r$ is a random bit. The one-time table contains four bits: two input bits and two output bits. By putting the possible aborts in the preprocessing which does not involve useful data, we avoid the problem of early terminations with leaking of data in other possible protocols with aborts.

We propose Protocol 2 to select some one-time tables generated by Protocol 1. It allows Bob to abort during the protocol when he finds that Alice is cheating. When Protocol 2 is used in a generic interactive bipartite classical computation with the roles of Alice and Bob switched, the data leakage of Alice is asymptotically vanishing for noiseless physical systems, but for noisy physical systems, the leakage is linearly related to the noise level. The data privacy of Bob is partial: the leakage is about half of his input bits, but the privacy is better in the case that the function is a many-to-one map for Bob’s input, including the case that the function effectively evaluates universal circuits.

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We then propose Protocol\textsuperscript{3} which includes checks from both sides to ensure that the average rate of cheating by any party is asymptotically vanishing. So for the bipartite computation task, the data leakage of any party is asymptotically vanishing for noiseless systems, while for noisy systems, the leakage of both parties are linearly related to the noise level.

We then propose Protocol\textsuperscript{1} which combines several one-time tables generated by Protocol\textsuperscript{2} or\textsuperscript{9} into one. When Protocol\textsuperscript{4} based on Protocol\textsuperscript{2} is used in bipartite classical computation, the data leakage of Alice is asymptotically vanishing, and almost independent of the physical errors, as discussed in Sec.\textsuperscript{VI} while the data privacy of Bob is worse than that in Protocol\textsuperscript{2} under comparable resource cost.

The Protocols\textsuperscript{2},\textsuperscript{3} and\textsuperscript{4} are secure in the honest-but-curious model. An honest-but-curious party is one who follows the protocol while possibly making measurements which do not affect the final computation result. In our protocols, an honest-but-curious party does not learn anything about the other party’s data, while the privacy of his or her own data is guaranteed to reach the targeted level.

The following remarks are for the general malicious case. The security of Bob’s data is dependent on that he is conservative, meaning that he values the privacy of his data higher than the possibility to learn Alice’s data; Alice needs to be weakly cooperating for the protocols not to abort, meaning that she does not cheat much in some batch of the instances of Protocol\textsuperscript{1}. For Alice’s data security to be enhanced by her verifications in Protocol\textsuperscript{3} she should be conservative in the sense described above. Although Protocol\textsuperscript{4} is quite effective when there is no noise (including errors), it may not be better than Protocol\textsuperscript{2} or\textsuperscript{3} when there is some non-negligible level of noise. In the noisy case, we propose just using Protocol\textsuperscript{4}.

We show some applications in general two-party classical computation, and the cheat-sensitive implementations of oblivious transfer and bit commitment under some assumptions. To enjoy some quantum speedup together with the security benefit brought about by our preprocessing, we propose a quantum homomorphic encryption scheme which uses the generated one-time tables as a resource, but with more rounds of communication than usual. Such scheme is then generalized to general two-party quantum computation with a publicly known circuit and private inputs on both parties.

The rest of the paper is organized as follows. Sec.\textsuperscript{III} contains some introduction of the background. In Sec.\textsuperscript{IV} we introduce the quantum protocols for generating the one-time tables. Sec.\textsuperscript{V} shows applications in general two-party classical computation. Sec.\textsuperscript{VI} shows applications in general two-party quantum computation. Sec.\textsuperscript{VII} contains some discussions about the security in the noisy case, and physical implementations. Sec.\textsuperscript{VIII} contains the conclusion and some open problems.

II. PRELIMINARIES

On computing two-party classical functions with quantum circuits, Lo\textsuperscript{22} studied the data privacy for publicly known classical functions with the output on one party only. Buhrman et al.\textsuperscript{23} studied the security of two-party quantum computation for publicly known classical functions in the case that both parties know the outcome, although with some limitations in the security notions. These and other results in the literature\textsuperscript{24} suggest that secure bipartite classical computing cannot be generally done by quantum protocols where the two parties have full quantum capabilities. In the current work, the protocols allow aborts in the quantum preprocessing (Bob may abort when he detects that Alice has cheated), so the scenario considered here does not fit into the assumptions in the works mentioned above. We assume that one party values the privacy of his data higher than the possibility to learn the other party’s data. Under such assumption, we do not require the parties in the main bipartite computation stage to be entirely classical.

Next, we introduce the simplest case in the one-time tables\textsuperscript{4}. The bipartite AND gate with distributed output is a gate that takes as input two distant bits $a$ and $b$, and outputs $(a \cdot b) \oplus r$ and $r$ on the two parties, respectively, where $r$ is a uniformly random bit. (XOR is denoted as $\oplus$; AND is denoted as $\cdot$ symbol.) It is sufficient for secure two-party classical computation, although there may be other constructions. Theoretically, the bipartite AND gate with distributed output on two distant input bits $a$ and $b$ can be computed while keeping both input bits completely private, with the help of a precomputed ideal one-time table of the nonlocal-AND type. Later we present methods for approximately generating such one-time tables. The one-time table has two locally-generated uniformly random bits $x$ and $y$ on the two parties, respectively, and also has $(x \cdot y) \oplus r$ and $r$ on the two parties, respectively, where $r$ is a uniformly random bit. The steps for the bipartite AND-gate com-
putation with distributed output are as follows:

1. Alice announces $a' = a \oplus x$. Bob announces $b' = b \oplus y$.

2. Each party calculates an output bit according to the one-time table and the received message. Alice’s output is $x \cdot b' \oplus (x \cdot y) \oplus r$. Bob’s output is $a' \cdot b \oplus r$.

The XOR of the two output bits is $(x \cdot b') \oplus (x \cdot y) \oplus r \oplus (a' \cdot b) \oplus r = a \cdot b$, while each output bit is a uniformly random bit when viewed alone, because $r$ is a uniformly random bit. Since the messages $a'$ and $b'$ do not contain any information about $a$ and $b$, the desired bipartite AND gate is implemented while $a$ and $b$ are still perfectly private.

III. THE QUANTUM PROTOCOLS FOR GENERATING ONE-TIME TABLES

The main quantum protocols to be introduced later are based on Protocol [1] which is the revised version of a subprocedure of a protocol from [25]. The Protocol [1] effectively computes an AND function on two remote classical bits from the two parties, with the output being a distributed bit, i.e. the XOR of two bits on the two parties. The security is not ideal: the plain use of such protocol would give rise to non-ideal security in (interactive) quantum homomorphic encryption [25], and the security is such that some additional verification need to be added in the protocol for it to be nontrivial. Later we propose protocols that check and sometimes combine the one-time tables generated from Protocol [1] to be used as a preprocessing stage for a bipartite classical or quantum computation task. The security level in those quantum protocols are higher than that achievable from the plain use of Protocol [1].

In Protocol [1] denote $|+\rangle := \frac{1}{\sqrt{2}}(|0\rangle + |1\rangle)$, and $|\rangle := \frac{1}{\sqrt{2}}(|0\rangle - |1\rangle)$, and the random bits are unbiased and independent of other variables by default. An EPR pair is two qubits in the state $\frac{1}{\sqrt{2}}(|00\rangle + |11\rangle)$. The protocol involves teleportations [20] with partial information about the outcomes withheld by the sending party, and this can alternatively be implemented by the direct sending of quantum states that have been subject to certain Pauli operators just before sending. The teleportation approach allows Alice and Bob to do operations simultaneously. See the discussions in Sec. [11]

We explain the last parts of Protocol [1]. Even if Alice did a $Z$ correction on the first qubit corresponding to Bob’s sent bit (in fact she does not do it, but only ad-

justs the final outcome), she receives the two qubits with a possible $\sigma_y$ mask for each qubit (which are related to Bob’s final outcome), together with possible simultaneous $\sigma_z$ masks on both qubits (which does not affect any party’s final outcome). Bob’s outcome can alternatively be viewed as being obtained in the following way: if a $\sigma_y$ correction is needed for one received qubit, Bob regards his mask bit for that qubit as 1, and otherwise he regards it as 0. Bob’s output bit in the protocol is the XOR of the two mask bits.

In Protocol [II] Alice’s input bit has partial privacy even for a cheating Bob, while Bob’s input bit is secure for an honest-but-curious Alice, but is not secure at all for a cheating Alice. The privacy of Alice’s input bit $x$ can be quantified using the accessible information or the trace distance. The accessible information, i.e. the maximum classical mutual information corresponding to Bob’s possible knowledge about Alice’s input, is exactly $\frac{1}{2}$ bits, which happens to be equal to the Holevo bound in the current case. For a cheating Bob to get the maximum amount of information, his best measurement strategy in the current case is to use a fixed projective measurement: to measure the first qubit in the $Z$ basis, and the second qubit in the $X$ basis. The trace distance of the two density operators for Alice’s two possible input values is $\frac{1}{2}$, by direct calculation (see also [25]). Thus, the probability that Bob guesses Alice’s input bit correctly is $(1 + \frac{1}{2})/2 = \frac{3}{4}$. Note that with this particular measurement just mentioned, he cannot make the distributed output of the one-time table correct. In other words, Bob cannot learn the other party’s input without consequences. We will see below that the similar remark can be said for Alice as well, implying that the protocol is secure in the honest-but-curious model.

To learn about Bob’s input bit, a cheating Alice may use an entangled state $\frac{1}{2}(|00\rangle + |01\rangle + |10\rangle - |11\rangle)$. From Bob’s returned state, and after correcting for the Pauli operator indicated by Bob’s returned classical message, Alice may find out Bob’s input bit with certainty.

In the following we present protocols which check or combine the one-time tables generated in Protocol [II]. The first one has partial security for Alice and near-perfect security for Bob, while the second one involves checking by both parties, and aims for near-perfect security for both parties. The third one aims for near-perfect security for both parties with emphasis on the security of one party.

In Protocol [2] Alice’s input bit has partial privacy,
**Protocol 1** A quantum protocol for generating one-time tables of the nonlocal-AND type with partial privacy

**Input:** A random bit $x$ from Alice and a random bit $y$ from Bob.

**Output:** $(x \cdot y) \oplus r$ and $r$ on the two sides, where $r$ is a random bit.

The input and output together form the one-time table.

1. The two parties initially share some EPR pairs to be used in the teleportations.
2. (The steps 5 and 6 performed by Bob can be done concurrently with the Steps 2 and 3 performed by Alice.) Alice generates a random bit $s$. If $s = 0$, she encodes $x$’s value into $|x\rangle|0\rangle$; if $s = 1$ and $x = 0$, she prepares the state $|+\rangle|+\rangle$; in the remaining case $s = x = 1$, she prepares the state $|+\rangle|-\rangle$.
3. She teleports the pair of qubits to Bob without telling him about the Pauli corrections except the following bit $q$: when $s = 0$, she tells him the bit for $X$ correction on the first qubit; otherwise she tells him the bit for $Z$ correction on the second qubit. When $s = 0$, she records the bit for $X$ correction on the second qubit as $t$, otherwise she records the bit for $Z$ correction on the first qubit as $t$. Then $t$ is a random bit.
4. Instead of doing a Pauli correction corresponding to $q$, Bob sets a temporary bit $q' := q \cdot (1 - y)$ to be used later.
5. If $y = 0$, Bob does a CNOT gate on the two qubits, with the first qubit being the control qubit.
6. Bob teleports the resulting qubits to Alice, while withholding part of the information about the measurement outcomes: he calculates the XOR of the four correction bits, and sends the resulting bit to Alice. Bob calculates the XOR of the two bits for $X$ corrections (although they actually correspond to $\sigma_y$ corrections due to the sending of a bit above) on the two qubits, and takes XOR of the result with $q'$, and regards the resulting bit as his output.
7. If $s = 0$, Alice measures the two received qubits in the $Z$ basis, otherwise she measures them in the $X$ basis. She calculates the XOR of both outcomes, and take the XOR with $t$, and obtains a bit. She flips this bit if $s$ and Bob’s sent bit are both 1. The obtained bit is her output.

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**Protocol 2** A partly-secure quantum protocol for checking the one-time tables

1. Alice and Bob perform many instances of Protocol 1 (sequentially or in parallel) to generate some one-time tables, and exchange messages to agree on which instances were successfully implemented experimentally. Suppose $m$ one-time tables were implemented. The one-time tables labeled by $j$ has inputs $a_j$ and $b_j$, and outputs $e_j$ and $f_j$.
2. Bob randomly selects $q$ integers in $\{1, \cdots, m\}$, which are labels for which one-time table. He tells his choices to Alice. The integer $q$ satisfies that $m - q$ is an upper bound on the number of required one-time tables in the main bipartite computing task, and the ratio $\frac{q}{m}$ is related to the targeted security level of the overall computation.
3. Alice sends the bits $a_j$ and $e_j$ to Bob for all chosen labels $j$.
4. For any chosen label $j$, Bob checks whether $a_j$ and $e_j$ satisfy that $a_j \cdot b_j = e_j \oplus f_j$. If the total number of failures is larger than some preset number of Bob’s (e.g. 0, or a small constant times $m$), he aborts the protocol, or restarts the protocol to do testing on a new batch of instances of Protocol 1 if the two parties still want to perform some secure two-party computation. Otherwise, the remaining one-time tables are regarded as having passed the checking and will be used later in the two-party classical computing task. They may repeat the steps above to prepare more one-time tables on demand.

which is the same as in the analysis of Protocol 1 above. When the ratio $\frac{q}{m}$ is near one, the nonlocal correlations in the remaining unchecked one-time tables can be regarded as almost surely correct. This is because of Bob’s checking. We require Alice to be weakly cooperating, that is, she does not cheat in some of the batches of instances, since otherwise no one-time table may pass the test. Some degree of weak cooperation is required for two parties to perform a computation anyway, and the above assumption of Alice has no effect on the data security of any party when Bob satisfies the assumption below, thus we may ignore the assumption above and just state the following assumption on Bob as the requirement of our protocols. In the following we assume that Bob is conservative, which means that he values the privacy of his data higher than the possibility to learn Alice’s data. Later in Sec. [IV](#) we will see that it effectively implies that he indeed does the checking. For an honest-but-curious Alice, the resulting correlation is correct, and she does not learn anything about Bob’s input bit $y$ (using the notations in Protocol 1 same below). In the following we discuss the case that Alice cheats.
If Alice cheats and gets at least partial information about Bob’s input bit \( y \), the state sent from Alice to Bob must be different from what is specified in the protocol; her best choice of state for cheating is mentioned previously. To pass Bob’s test while learning about Bob’s input \( y \), she should know both \( y \) and \( r \), or know both \( y \) and \( y \oplus r \). (The two conditions are equivalent in the exact case, but not necessarily equivalent in the partial-information case.) In the following, let \( I_y^M \) denote the classical mutual information learnable by Alice about the bit \( y \) if she uses the measurement \( M \). The \( I_y^M \) and \( I_{y\oplus r}^M \) are defined similarly, but note that they are conditioned on the uniform distribution for \( y \).

**Proposition 1.** The following inequalities hold:

\[
\begin{align*}
I_y^M + I_r^M &\leq 1, \\ I_y^M + I_{y\oplus r}^M &\leq 1, \\ I_y^M + \max(I_r^M, I_{y\oplus r}^M) &\leq 1.
\end{align*}
\]

where the two \( M \) are the same in each equation.

**Proof.** The overall communication from Bob to Alice in Protocol \([1]\) is only one classical bit (note the entangled states used in the teleportations are fixed and do not count towards the amount of communication), then, by the Holevo bound (also noting that there are effectively no other prior correlations between the two parties besides the fixed entangled state, so the locking of information [27] does not occur here), the amount of information that Alice learns about the joint distribution of \( y \) and \( r \) is upper bounded by 1 bit. The bits \( y \) and \( r \) are independent when Bob produces them, so the \( y \) and \( r \) are independent prior to Alice’s measurement. Thus the inequality (1) holds. The bits \( y \) and \( y \oplus r \) jointly determine \( y \) and \( r \), so the amount of information that Alice learns about the joint distribution of \( y \) and \( y \oplus r \) is upper bounded by 1 bit. And since the bits \( y \) and \( y \oplus r \) are independent prior to Alice’s measurement, we have that the inequality (2) holds. The inequalities (1) and (2) together imply (3).

The probability that Alice passes Bob’s test at a particular instance is related to the \( \max(I_r^M, I_{y\oplus r}^M) \) in Eq. (3). When the probability of passing approaches 1, such maximum approaches 1, then it must be that one of them approaches 1. Then, Prop. (4) implies that Alice can learn almost nothing about \( y \) if she measured in the same basis, but in fact a cheating Alice knows which instances are remaining and will not be checked later (although it is conceivable that some checks may be done after the main computation, see Sec. (M) below), so she can choose to do any measurement on the received states in these remaining instances. Such measurement may not be the same as \( M \) in the other term in Eq. (3). This implies that Eq. (3) alone is not sufficient for proving the security of Protocol [2].

**Theorem 1.** In Protocol [2] Bob’s input is asymptotically secure.

**Proof.** We first consider the case that Alice’s operations are independent among different instances of Protocol [1] and at last comment that the non-independent case still satisfy the extreme case of the inequalities above, giving rise to the security of Protocol [2].

Due to the freedom of measurement basis choice mentioned above, the Holevo bounds, which are upper bounds of the information quantities, are more relevant for proving the security of Protocol [2]. Under the condition that Alice’s operations are independent among the instances, we need only consider the Holevo bounds for a single instance of Protocol [1]. Let \( \chi_y \) be the Holevo quantity which is the upper bound for \( I_y^M \). It is defined as

\[
\chi_y = S(\rho) - \sum_{j=1}^{2} S(\rho_j),
\]

where \( \rho_j \) is the density operator that Alice receives from Bob for the case of \( y = j \) after Pauli corrections determined by Bob’s sent bit, and \( \rho = \frac{1}{2}(\rho_1 + \rho_2) \). The \( S \) represents the von Neumann entropy. The quantities \( \chi_r \) and \( \chi_{y\oplus r} \) are defined similarly, but note that they are conditioned on the uniform distribution for \( y \). We claim that the following inequality holds for small positive \( \epsilon \) and nonnegative continuous function \( f(\epsilon) \),

\[
\chi_y + \max(\chi_r, \chi_{y\oplus r}) \leq 1 + f(\epsilon),
\]

for \( \max(\chi_r, \chi_{y\oplus r}) \geq 1 - \epsilon, \)

where \( f \) is continuous and \( f(0) = 0 \).

This has been confirmed by numerical calculations. The analytical reason is as follows. The Holevo quantities in Eq. (4) satisfy uniform continuity, because that the ancilla is effectively at most 4 dimensions due to the Schmidt decomposition, and that the Holevo quantity (4) is continuous as a function of \( \rho_1 \) and \( \rho_2 \) and therefore is a continuous function of Alice’s initial state. Together with the fact that

\[
\max(\chi_r, \chi_{y\oplus r}) = 1 \implies \chi_y = 0,
\]


which is by the construction of the protocol, we recover Eq. (4).

Alice may cheat in some instances of Protocol $\mathcal{P}_1$ so we may define a rate of cheating. Partial cheating in a instance is converted into a fractional number of cheating instances in calculating such rate. Alice’s cheating probabilities among different instances may be correlated, but that does not affect the following argument since Bob randomly chooses which instances to check. It is sort of subjective for Bob to determine the average rate of cheating from the number of wrong results and the total number of tests in Protocol $\mathcal{P}_2$ since it depends on the a priori knowledge about the probability distribution for Alice’s average rate of cheating, and also depends on the correlations between rates of cheating among different instances of Protocol $\mathcal{P}_1$. Suppose that after some checking, Bob estimates that Alice’s average rate of cheating is $\epsilon$, which is a small positive constant near 0, then the following approximation holds for the uniform distribution of $y$ and $r$: $\max(\chi_y, \chi_{y^{\oplus} r}) \geq 1 - \epsilon$. Hence, $\chi_y \leq \epsilon + f(\epsilon)$ according to Eq. (5). This shows that the expected amount of information about $y$ learnable by a cheating Alice in the remaining instances of Protocol $\mathcal{P}_1$ is arbitrarily near zero for sufficiently small $\epsilon$, even if she measures in different bases from those for the tested instances. The word “expected” means that even if $L\epsilon < 1$, where $L$ is the total number of one-time tables to be used for the main computation, Alice may sometimes learn about one or a few bits of Bob’s input by chance, but on average, she learns not more than $L\epsilon$ bits. Since the information about $y$ is linearly related to the information learnable by Alice in the later classical computation stage (see Sec. II), this shows the security of Protocol $\mathcal{P}_2$ for use in the later classical computation in the case that Alice’s operations are independent among instances of Protocol $\mathcal{P}_1$.

In the following we consider the general case that Alice’s operations are not necessarily independent among instances of Protocol $\mathcal{P}_1$. If Alice initially prepares some correlated quantum states among $m$ instances, the generalization of Eq. (6) should hold, due to the construction of the protocol. Then the generalization of Eq. (6) for the corresponding Holevo bounds should hold approximately near such extreme point, due to the uniform continuity of the Holevo bounds. This shows that the argument for the security for the case of independent operations of Alice can be extended to the general case.

We mention some numerical results. Numerical calculations confirm the inequalities (1) through (5). Note the same $\mathcal{M}$ occurs twice in each inequality. The calculations assume that Alice uses some general quantum pure state as input, allowing an ancillary system of 4 dimensions, the maximum required by the Schmidt decomposition. The calculations use projective measurements, although POVM measurements may give rise to a larger sum on the left-hand-side, and such weakness is remedied by the calculation of the Holevo bound below. Numerical calculations suggest the following inequalities:

$$\chi_y + \chi_r \leq c, \quad (7)$$
$$\chi_y + \chi_{y^{\oplus} r} \leq c, \quad (8)$$

where $c$ is a constant somewhat larger than 1.388 and is yet to be precisely determined. This implies that

$$\chi_y + \max(\chi_r, \chi_{y^{\oplus} r}) \leq c. \quad (9)$$

Numerics suggest that near the ends of the tradeoff curve indicated by Eqs. (7) and (8) one quantity approaches 1 bit while the other quantity approaches 0. For some input state of Alice’s that approaches the numerically found maximal value of the left-hand-side, the two terms on the left-hand-side of Eq. (9) are about equal, and the corresponding sum in the left-hand-side of Eq. (8) under projective measurements is numerically found to be not greater than 1 bit. The latter sum is observed to have the same property for initial states satisfying $\max(\chi_r, \chi_{y^{\oplus} r}) \approx 1$. When there is no ancilla, numerics suggest that the left-hand-side of Eq. (9) is not greater than 1 bit. As quantitative examples for Eq. (5), we have $f(0.1) \approx 0.3$, and $f(0.01) \approx 0.06$.

To improve Alice’s security in the protocol above, we propose the following Protocol $\mathcal{P}_3$ in which Alice also does some checking about Bob’s behavior.

By noting that there is only one bit of classical communication from Alice to Bob in Protocol $\mathcal{P}_1$ the analysis for Protocol $\mathcal{P}_2$ about Bob’s data privacy can basically be used for analyzing Alice’s data privacy in Protocol $\mathcal{P}_3$. There are analogues of Proposition $\mathcal{P}_1$ and Theorem $\mathcal{P}_1$ for Alice instead of Bob. The quantitative security level is somewhat different: Alice’s data privacy has a nonzero lower bound here; with the same resource cost, Bob’s data privacy is somewhat weaker than that in Protocol $\mathcal{P}_2$.

When one party’s data privacy is very important, and the other party’s data privacy is not too important, we propose the following Protocol $\mathcal{P}_4$. It improves the privacy of Alice’s input in the later main computation task, while that of Bob’s input is somewhat compromised.
Protocol 3 A quantum protocol for checking the one-time tables by both parties

1. Alice and Bob perform many instances of Protocol 1 to generate some one-time tables, and exchange messages to agree on which instances were successfully implemented experimentally. Suppose $m$ one-time tables were implemented. The one-time tables labeled by $j$ has inputs $a_j$ and $b_j$, and outputs $e_j$ and $f_j$.

2. (The steps 2 to 4 can be done concurrently with the steps 5 to 7.) Bob randomly selects $q$ integers in $\{1, \cdots, m\}$, which are labels for which one-time table. He tells his choices to Alice.

3. Alice sends the bits $a_j$ and $e_j$ to Bob for all chosen labels $j$.

4. For any chosen label $j$, Bob checks whether $a_j$ and $e_j$ satisfy $a_j \cdot b_j = e_j \oplus f_j$. If the total number of failures is larger than some preset number of Bob’s (e.g. 0, or a small constant times $m$), he aborts the protocol, or asks Alice to restart the protocol to do testing on a new batch of instances of Protocol 1 if the two parties still want to perform some secure two-party computation.

5. Alice randomly chooses $p$ integers in $\{1, \cdots, m\}$, and tells Bob her choices. The chosen set of integers may overlap with the set chosen by Bob.

6. Bob sends the bits $b_j$ and $f_j$ to Alice for the chosen labels $j$.

7. For any chosen label $j$, Alice checks whether $a_j \cdot b_j = e_j \oplus f_j$ holds. If the total number of failures is larger than some preset number of Alice’s, she aborts the protocol, or asks Bob to restart the protocol if needed.

8. The remaining one-time tables are regarded as having passed the checking and will be used later in the two-party classical computing task. They may repeat the steps above to prepare more one-time tables on demand.

Protocol 4 A quantum protocol for generating improved one-time tables with combinations

1. Alice and Bob perform Protocol 2 or Protocol 3 to obtain some one-time tables after checking. Suppose the instance labeled by $j$ has inputs $a_j$ and $b_j$, and outputs $e_j$ and $f_j$.

2. Bob determines which remaining one-time tables are to be combined into one new instance of one-time table, and tells Alice his decision. Each new instance corresponds to a set $S$ of old instances which satisfy that Bob’s input bits are equal (denoted as $c_0$). A new instance is constructed from the given set of old instances as follows: $a' := \sum_{j \in S} a_j \mod 2$, $b' := c_0$, $e' := \sum_{j \in S} e_j \mod 2$, $f' := \sum_{j \in S} f_j \mod 2$.

In Protocol 1 the privacy of Alice’s bit $x$ for the combined one-time table is quite good: The accessible information for Bob is exactly $\frac{1}{k}$ bits, where $k$ is the size of $S$ in protocol description. It is because the different one-time tables from the first step are independent. The Holevo bound coincides with the accessible information in the current case.

For the privacy of Bob’s input bit $y$ in the combined one-time table, it is possible for a cheating Alice to do a joint measurement on $k$ received states from Bob, to learn the information about $y$ and $r$ simultaneously as much as possible (or $y$ and $y \oplus r$). Bob can deal with this by testing more one-time tables. The resource usage (the amount of entanglement needed and the amount of communication) is estimated to be about $O(tk^2)$ times that of Protocol 2 to achieve the similar level of privacy for Bob, where $t$ is the total number of one-time tables required for the later main computation, and $k$ is the size of $S$ in Protocol 1. In such factor $tk^2$, one $k$ is for the size of $S$, and the additional $tk$ factor means that about $O(tk)$ one-time tables are used in the instance of Protocol 2 in the first step of Protocol 1. This factor appears because Alice may use techniques similar to Grover’s algorithm to increase the amount of information she may learn about $y$, and the same input variable of Bob’s may appear in the original circuit for at most $t$ times. But in the case that the function to be evaluated is for evaluating a program provided by Bob on Alice’s data, it is possible that each variable of Bob’s appears only once, then the $t$ factor can be omitted, so that the overhead becomes only $O(k^2)$ compared to the plain use of Protocol 2.

The Protocol 1 differs from the previous protocols in that it has an extra step of combining the one-time tables, and its usage in the later bipartite computation task may be different by a switch of the roles of Alice and Bob. The verifications in these protocols take hint from similar procedures for quantum key distribution [28]. The success of these quantum protocols is not guaranteed if Alice cheats very often (or Bob cheats very often in Protocol 3), but this does not cause much problem since cheating is
caught with high probability, and this is a preprocessing stage anyway so the useful data is not leaked. The failures in the quantum gates, measurements, and entanglement generation or qubit transmissions in the preprocessing stage can be tolerated by trial-and-error. These failures are required to be reported in the protocols, so they have no effect for Bob’s testing and later computations. In some experimental implementations the failures might not be reported and might appear as errors, and this would affect the security.

IV. APPLICATIONS IN TWO-PARTY CLASSICAL COMPUTATION

The following Protocol 5 is for evaluating a linear polynomial with distributed output using the quantum preprocessing protocols introduced above. The linear polynomial is of the form \( z = (c + \sum_j a_j b_j) \mod 2 \), where \( c \) is a constant bit known to Bob, and \( a_j \) and \( b_j \) are bits on Alice and Bob’s side, respectively. The output is the XOR of two bits on different sides.

If Protocol 2 is used in Protocol 5, the data privacy of one party is partial. The leakage is about half of his or her input bits. See also the comments after Protocol 6 below. Generally, we suggest using Protocol 3 in Protocol 5 since it at least aims for near-perfect security, although the actual security level is linearly related with noise.

For a generic boolean circuit, we propose Protocol 6. The main computation after the preprocessing does not include any aborts, and only requires the number of communication rounds to be about equal to the circuit depth. The circuit is assumed to be known to both parties, except for some initial local gates, which may be known only to the local party.

If Protocol 2 is used in Protocol 6 with the roles of Alice and Bob switched in the preprocessing only, the data privacy of Bob is partial. The leakage is about half of his input bits in each polynomial. But the privacy is better in the case that the function allows many different inputs of Bob to give rise to the same result. In the case that the function effectively evaluates a universal circuit with data given by Alice and the logical circuit given by Bob, his input has partial privacy which is acceptable due to possible recombinations of Bob’s logical circuit. If Protocol 4 is used instead of Protocol 2 it is suggested that Alice always be the first party, to save the required number of one-time tables when Alice’s data privacy is more important than Bob’s data privacy. Then Alice’s data in the main computation is asymptotically secure because of the property of Protocol 4. The remarks above are for the noiseless case. For the case with noise, see Sec. VI where it is suggested that simply using Protocol 3 may be a good solution.

The Protocol 4 has a good property that cheating would usually give rise to wrong results. If some party (partially) cheated in generating some of the one-time tables, so that some but not all of the one-time tables used in Protocol 4 are not secure, then the insecure one-time tables are wrong with some significant probability according to Eq. 9: the calculation results for a particular nonlocal AND gate would often be incorrect after the distributed output bits are recombined. This implies that the final computation result has large probability to be wrong. But if that party cheated in all the generated one-time tables and passed the other party’s test, the computation result could be calculated by the cheating party alone with the help of the messages sent from the other party in the main computation stage. The latter case is not likely to happen, since the other party could set a low threshold in the testing.

Some protocol similar to Protocol 6 could be used for evaluating a public circuit on shared classical secrets between Alice and Bob, when each effective input bit is the XOR of two remote bits. The steps are quite similar except for some initial local gates, so we abbreviate the protocol here.

In the following we discuss the security assumptions. We define Bob to be “conservative”, if he values the privacy of his input data higher than the possibility to learn Alice’s data.

First, let us assume that Bob honestly does the testing in the Protocols 2 and 4. There could be superpositions in the input and the output of these quantum protocols, but in the later classical computation task, the parties may do computational-basis measurements to force the received superposed states to collapse. Note that one party may insist on using the superposed output from some instance of the one-time table, but when the other party does some later gate using such output as an input, the latter party may do computational-basis measurements to force the collapse of the superposition.

Next, we discuss the case out of the assumption, that is, Bob cheats in the quantum protocols. He may cheat by not aborting after finding that Alice is cheating. This way of cheating is not powerful by itself, but see the fol-
Protocol 5 A protocol for evaluating classical linear polynomials with distributed output using one-time tables

1. Alice and Bob perform Protocol 2 or 3 or 4 to obtain some one-time tables.

2. For evaluating the linear polynomial \( z = (c + \sum_j a_j b_j) \mod 2 \), Alice and Bob perform the evaluation of the nonlocal AND gate for \( a_j \) and \( b_j \) using the procedure in Sec. 11 with the output being distributed. They locally calculate the XOR of all bits from the outputs, and Bob additionally takes the XOR with \( c \). Each party obtains a bit as the output.

Protocol 6 A protocol for evaluating classical boolean circuits using one-time tables

1. Alice and Bob decompose the two-party circuit to be evaluated into some local circuits with AND, XOR gates, and some linear polynomials with bipartite input, while adding possible ancillary bits with fixed initial values. Any nonlocal AND gate in the original circuit is a special case of the linear polynomial.

2. For each AND gate not in the initial stage, the inputs are distributed, i.e., each bit is the XOR of two remote bits. Alice and Bob decompose such gate into the XOR of the outputs of two local AND gates and two nonlocal AND gates, the latter being a special case of the linear polynomial. For the XOR gates with distributed input, they are expressed using two local XOR gates, with the output being distributed.

3. They perform the gates in the resulting circuit in pre-arranged order. The linear polynomials are evaluated using Protocol 5 with distributed output.

4. At the end of the circuit, one party sends some bits to the other party so that the distributed bits for the output are recombined to form the correct output; if there are output on two parties, both parties need to send messages.

Following for discussion about his combined ways of cheating. The second way for him to cheat is to use general quantum input (allowing superpositions and entanglement) for the one-time tables, which also allows general quantum output for the one-time tables. In such case, Alice may do computational-basis measurements in the main bipartite computation stage to force the collapse of superpositions. The case that he uses general quantum output for the one-time tables is discussed in the previous paragraph. For the case that Bob combines the two cheating methods above, if Alice is honest, Bob cannot get more information about Alice’s data compared to the case of him not cheating in this way. If Alice also cheats, then it is possible that Bob’s knowledge about Alice’s data on average is better (e.g., when they discard some one-time tables, so that Bob obtains more information about Alice’s input in the remaining one-time tables). But that comes at the expense of the higher possible leakage of Bob’s data. So a conservative Bob should do such combined cheating. The third way for Bob to cheat is by using superposed states in the main computation but not the preprocessing. This has no effect since Alice may make a computational-basis measurement on the state received from Bob in the main computation. Note that Alice’s data leakage is limited by design of the quantum protocols, except in the case of non-conservative Bob discussed above. In conclusion, if we assume Bob to be conservative, the quantum protocols are asymptotically secure; if we assume Alice to be honest-but-curious, the Protocol 4 is asymptotically secure for Alice (as mentioned in Sec. 11), and in such case it does not make much sense for Bob to cheat since he cannot gain from cheating.

In the following we consider implementing some cryptographic primitives such as oblivious transfer and bit commitment. The usual oblivious transfer (as opposed to 1-out-of-2 oblivious transfer) 29, 30, can be implemented in a cheat-sensitive way as follows. Again, it requires that one of the parties be conservative, in order for the one-time tables to be successfully and securely generated.

Protocol 7 A protocol for approximate cheat-sensitive oblivious transfer

1. Alice and Bob perform Protocol 4 to obtain a one-time table.

2. Alice sends Bob the output bit she has obtained, and Bob takes the XOR of such bit and his output bit to obtain the output \( a' \cdot b' \), where \( a' \) and \( b' \) are inputs of the one-time table from Alice and Bob, respectively.

In Protocol 7, if \( b' = 0 \), the resulting product \( a' \cdot b' \) is zero and does not carry any information about \( a' \); otherwise, the result is \( a' \), so it transfers Alice’s input bit to Bob. The protocol is cheat-sensitive in the sense that if Alice cheated in Protocol 4 it would have been detected
and the protocol would have aborted. The protocol has approximate security, since the precomputed one-time tables are approximately secure.

We then consider 1-out-of-2 oblivious transfer. By choosing the linear polynomial of the type \( z = \sum_{j=1}^{2} a_j b_j \), where \( b_1 + b_2 = 1 \), and if Bob asks Alice to send him the output she obtained from running the Protocol 8 Bob may accomplish the same function as 1-out-of-2 oblivious transfer, with the additional property that Bob may ask Alice to send \( a_1 \oplus a_2 \) if he cheats by setting \( b_1 = b_2 = 1 \). The last point makes it different from the definition of 1-out-of-2 oblivious transfer [29, 30].

**Protocol 8** A protocol for approximate cheat-sensitive quantum bit commitment

1. Alice and Bob perform Protocol 4 to obtain some one-time tables with the degree of security dependent on resource usage. They decide on a large integer \( m \) related to the intended security of the current bit commitment protocol.

2. Suppose Alice wants to commit a bit \( b \). She asks Bob to together calculate \( m \) nonlocal AND gates using the method in Sec. [II] with her input bits being always \( b \), but Bob’s inputs are random bits chosen by himself. They obtain some distributed bits as the outcomes. This completes the commit phase.

3. (Reveal phase) Alice sends Bob her output in the instances of the nonlocal AND gates in the previous step. Bob takes the XOR for the corresponding pairs of bits to recover the results of the nonlocal AND gates. From these results, Bob finds out \( b \), or decides that Alice has cheated by sending him some random bit string so he cannot recover \( b \).

There are some no-go theorems for quantum bit commitment [31, 32]. Since our quantum preprocessing protocols allow aborts, and there are some requirements on the players in those protocols, it is still possible that bit commitment can be implemented with the help of the one-time tables generated by the quantum preprocessing protocols. In the Protocol 8 we propose a bit commitment protocol using the above idea, inspired by a computationally-secure construction based on quantum one-way permutations [33]. Here, instead of using the quantum one-way permutations, we use a special bipartite classical computation with distributed output, with the help of quantum preprocessing. Our scheme is cheat-sensitive and subject to some other assumptions similar to those for the generic Protocol 8. The protocol 8 requires that one of the parties be conservative, in order for the one-time tables to be successfully and securely generated.

In the last step of Protocol 8 if Alice sends Bob some random bit string, the results obtained by Bob are generally not consistent with any input value of \( b \). For large \( m \), it is hard for Alice to guess the appropriate bit string to send to Bob to make him believe the input was \( 1 - b \).

V. APPLICATIONS IN TWO-PARTY QUANTUM COMPUTATION

The methods in this work can be applied in two-party secure quantum computing tasks. When such tasks have classical input and output, they also serve as classical tasks of the type discussed in Sec. [IV] but with quantum implementations. In this way, classical computational tasks are completed with quantum speedup and quantum security advantage. But this requires at least one party to have quantum capabilities beyond those required by Protocol 8. A typical problem in two-party quantum computation is quantum homomorphic encryption (QHE). We will consider both interactive protocols and constant-round protocols for QHE. Note that the initial preparation of the one-time tables with checking and preparation of entanglement involve a constant number of legs of communication. The communication in preparation of the one-time tables should be counted in the total number of rounds, while that for entanglement generation is usually not counted in, since it can be absorbed into the first communication in the main part of the protocol (although there may be security problems which should be tolerable in a strong protocol).

In the protocols below, there are some linear polynomials with at least \( 2n \) variables, where \( n \) is the size of input. The \( 2n \) variables correspond to Pauli masks in Alice’s teleportation of the input data to Bob. The way Bob changes the coefficients of the linear polynomials is by some key-update rules. The key-update rules for the first \( 2n \) variables (and other variables mentioned below) under the action of Clifford gates can be easily obtained from the following relations:

\[
PX = iXZP, \quad PZ = ZP, \\
HX = ZH, \quad HZ = XH, \\
\text{CNOT}_{12}(X_1^aZ_1^b \otimes X_2^cZ_2^d) = (X_1^aZ_1^b \otimes X_2^cZ_2^d) \text{CNOT}_{12},
\]

where the \( \oplus \) is addition modulo 2, and in the gate \( \text{CNOT}_{12}, \) the qubit 1 is the control. The effective key-
update rules for the variables under the \( T \) gate can be obtained from the relations

\[
TZ = ZT, \quad TX = e^{-\pi i / 4}PXZT.
\] (11)

More details about the key-update rules are in [10, 11].

An interactive QHE scheme with almost optimal information-theoretic data privacy and circuit privacy is obtainable by using the method in Protocol 5 to evaluate classical linear polynomials, and using the latter as a subprocedure in the Scheme 4 in [25].

To reduce the rounds of communication to a constant, we propose a scheme which is modified from the Scheme 3 in version 12 of [25], while adding the use of precomputed one-time tables. A main technique of the scheme is to use the garden-hose gadget from [11] (and attached as Appendix A), instead of the corresponding simplified gadget in the newer versions of [25]. Some steps “at some scheduled time” by Alice in the mentioned scheme are combined here, since under our way of evaluating linear polynomials, there is no need for multiple rounds of interactive communication in the main part of the scheme. The main part of the scheme has three stages of classical communication: from Bob to Alice, and from Alice to Bob, and a final teleportation from Bob to Alice. The steps are briefly described as follows. The linear polynomials are of the form in Protocol 5.

**Scheme 1 (A three-message QHE scheme using precomputed one-time tables)**

1. Bob’s coefficients of the linear polynomials, except the constant term, do not depend on Alice’s original Pauli mask bits or her measurement outcomes in the garden-hose gadgets. The constant terms in the linear polynomials depend on his local measurements in his part of the garden-hose gadgets. So he calculates the XOR of some coefficients with random inputs of the precomputed one-time tables, and sends the resulting bits to Alice.

2. Alice performs the initial teleportation of her \( n \) input data qubits without telling Bob any Pauli corrections. The \( 2n \) Pauli masks are part of the variables in the linear polynomials to be evaluated. With the received message, Alice computes her part of the output of the linear polynomials, in order to decide what measurements to do in the garden-hose gadgets (shown in Appendix A), and she does the Bell-state measurements with possible \( P^\dagger \) gates before the measurements. The measurement outcomes, and some dummy outcomes of the value zero for those measurements not actually done are also part of the variables of the linear polynomials. She sends Bob the XOR of the variables and the inputs of some one-time tables.

3. Bob receives Alice’s message and calculates his output for the first linear polynomial, and decide which measurements to do in the first garden-hose gadget. He performs the Clifford gates and the \( T \) gates before the first garden-hose gadget in his circuit, and the measurements in the first garden-hose gadget. The outcomes of those measurements help determine the constant term in the next linear polynomial, so they help determine his measurement choices in the next garden-hose gadget, together with the help from Alice’s message. He continues to do the next batch of gates and measurements, so on. At last, he obtains some quantum output with Pauli masks known to Alice. He teleports his output state to Alice.

4. Alice corrects the received state from teleportation with the corresponding Pauli masks, to obtain the final quantum output.

In the scheme above, the number of variables in a linear polynomial is at most \( 2n + 12R \), where \( R \) is an upper bound on the number of \( T \) gates in the circuit to be evaluated. This is because each Bell-state measurement has two outcome bits, and there are six of these measurements, although Alice does three of them at a time. As there are \( R + 2n \) linear polynomials to be evaluated, the total number of consumed one-time tables is \( O(n^2 + R^2) \).

There are two points on which the security of the quantum computation may be somewhat weaker than in the classical case. First, it is less natural in the quantum protocol to impose classicality of the output of the one-time table. Imposing classicality of course helps security, but it is not necessary given our assumptions about the players in the preprocessing stage. In practice, we may assume that the output of the one-time tables have decohered prior to the use in the main computation. Second, in the current framework for quantum bipartite computation such as in [25], the Pauli masks for the original input qubits are used as the variables in all the linear polynomials involved, this means the data privacy is worse than in the case of classical bipartite computation, in which
the intermediate variables replace the roles of the initial variables in many of the linear polynomials. But the use of the quantum preprocessing in this work would give rise to better data privacy than some of the schemes in later parts of [25], because those schemes require correlated encoding of the different variables, while the variables in the current work are encoded independently by the one-time tables.

In the following we extend from QHE to general two-party quantum computation with publicly known circuit and private quantum inputs. The description of the steps are almost the same as for the QHE scheme above, except that Bob has also some (quantum) input data. If the final output is on Bob’s side, the last teleportation to Alice can be omitted, and Alice needs to send Bob some Pauli masks. We omit the description here. The number of legs of communication in the scheme can be reduced to a constant, by the similar method as that in the QHE scheme above. In the case of classical input, the initial teleportation can be replaced with classical communication with withheld bit-flip masks. In the case of classical output, if it needs to be sent to Alice’s side, the final teleportation can just be replaced with classical communication without any masks.

VI. DISCUSSIONS

The Protocol together with other protocols for checking them effectively implement the PR-box (Popescu-Rohrlich box [34]) type of correlations. The implementation needs time in communication, and involves sending of some classical messages which do not contain useful information about the inputs. So it is not a direct implementation of the PR box, which must be instantaneous. Rather, it is a check-based implementation of the PR-box type of correlations. However, in Protocol after the initial entanglement has been established, the two directions of teleportation and partial sending of the measurement outcomes can be done simultaneously. This does have some partial flavor of “instantaneous” implementation. The PR-box is no-signaling, so a natural question is whether there is a no-signaling theory that could exist as a physical theory while allowing implementation of the PR boxes. Some new principles have been suggested in the literature that may exclude a large potential set of no-signaling theories [35]. Whether our protocols provide any insight on such principles is currently unclear.

The qubit-based quantum protocols in this work can be generalized to work for qudits in principle. This is inspired by the classical case in [3]. This requires some changes in the classical usage of the generated correlations.

The methods in this work are extendable to multipartite classical computation in principle. Some pairs of parties (possibly including some server) may prepare one-time tables using the quantum protocols in this work.

A method of enhancing the security by some additional checks after the computation is as follows. If one party, say Alice, does not require the long-term security of her input in the main computation, Bob may ask her to do additional checking of the one-time tables used in the main computation, at a time such that her input data is no longer sensitive, to make sure that she has not cheated by a lot. Of course, in some practical applications, the final computation result provides some check against Alice’s cheating, since Alice usually has to cheat all the way to the end for a generic computation to be correct (provided that the final result is on her side, not distributed as the XOR of remote bits), and always cheating successfully is unlikely to happen because of the inequalities in Sec. [III].

The following are some considerations about physical implementations. The Protocol makes use of teleportation [2], with certain classical messages withheld by the sending party. Such step in the protocol is equivalent to the direct sending of quantum states that have been subject to certain Pauli operators just before sending. Using teleportations allows entanglement to be prepared by a fixed entanglement generating device, allowing for failures in preparation. This may also help getting rid of the issue of multiple photons in direct communication, which harms the privacy in Protocol. As for detector inefficiencies and dark counts, the fact that the Protocol can be redone after failure can help mitigate the effects of these issues. Another interesting thing is that the two directions of teleportations in Protocol can be done simultaneously, with the help of some classical processing, due to the special form of the states to be sent. If direct sending is used instead, Bob has to wait for Alice’s qubits before he can do operations on them.

We now consider how physical noise (errors are counted as noise here) affect the protocols. We consider the case that the main computation is classical, since the quantum case is similar in that it also involves evaluating classical linear polynomials. When Protocol with noise is used for a bipartite classical computation task, and if Alice’s
data privacy is more important than Bob’s, we suggest that Alice who is the first party in the main computation be the second party in the preprocessing. Then the data leakage of Alice is about the product of the circuit size (the number of the one-time tables) and a small constant indicating the noise level. This is because in Protocol\[2\] the physical errors and the first party’s cheating look about the same for the second party in the verifications (the “first party” in this sentence is the Bob in the main computation). For circuits with a high level of parallelism, the data leakage of Alice per input bit is about the product of circuit depth and the error constant described above. So the allowed circuit depth is a constant, which is inverse proportional to the error constant. Similar remarks can be said for Protocol\[3\] for both sides.

If Protocol\[4\] based on Protocol\[2\] is used for a bipartite classical computation task, we suggest that Alice be the first party both in the preprocessing and the main computation. The noise level is not directly related to the data privacy of Alice, as it mainly affects the correctness of the computation, and Bob’s data privacy. If Bob only requires only partial privacy (such as in the case that many input values of Bob correspond to the same output value of the function), then Bob’s data privacy may be acceptable even with noise. The correctness may be improved by repeating some instances of Protocol\[4\] using the same inputs, but that would require using more instances of Protocol\[1\] and would affect both parties’ data privacy in a potential manner. The word “potential” means that to learn the other party’s data requires one party to do some measurement in the preprocessing stage which would affect the correctness. Suppose that the size of $S$ in Protocol\[4\] is $k$, which means Alice has $k$ independent input bits, and each is repeated for $m$ times, then $m = O(k)$, which is necessary to guarantee that Alice’s data is not completely leaked. By adopting majority voting as the decoding method, we may set $m$ to be much smaller than $k$ and still be able to achieve the correctness. The method of “majority voting” involves Alice sending Bob the XOR of some of her output bits with her first output bit, since directly sending all of them reveals the output to be generated. Bob’s data privacy is indeed affected by that $m > 1$ is generally necessary for the correctness of the computation to be at an acceptable level. Hence, when there is non-negligible noise, the advantage of Protocol\[4\] over Protocol\[2\] is weaker than that in the noiseless case, but is still not negligible if Bob’s data privacy is regarded as less important than Alice’s data privacy and the correctness of the computation.

In the noisy case, to avoid the complications in using Protocol\[4\] as mentioned above, simply using Protocol\[3\] may be a good solution. An alternative would be using Protocol\[2\] with “recompilation”, that is, using some new publicly-known function instead of the original function, with Bob’s input changed accordingly, while Alice’s input is unchanged, so that the result is the same as the original function with the original input of Bob. If the new function is chosen so that it encodes universal classical circuits, and the possible new inputs of Bob are long enough, we can achieve a good level of security for Bob’s input. Such recompilation can be done by classical preprocessing.

There have been studies of the effects of noise in classical cryptographic tasks, and noise is not always bad for security \[36\]. Note that adding some assumptions about quantum capabilities may improve the security in bit commitment \[37\]. Adding similar assumptions on top of our quantum preprocessing protocols may improve the security in the applications.

Since the protocol requires the use of entanglement (or quantum communication instead), the allowed distance between the two parties is expected to be not greater than that for quantum key distribution (QKD). In the case that the two parties are located at longer distance than entanglement generation would allow, we suggest the following two methods: the first method is that one party (Bob) tells his input to the one-time tables to a quantum agent near Alice via some computationally-secure method, and the agent cooperates with Alice in running the quantum protocols to generate one-time tables, and the output of the one-time tables belonging to Bob are sent to Bob via a computationally-secure method. The second method is that some trusted intermediate nodes be added, so that Bob could tell his input to the one-time tables to a node nearest to Alice, via the channel established by relayed QKD of the trusted nodes. The one-time tables are generated by such node and Alice. The output of the one-time tables belong to Bob’s side are sent back to Bob via the channel established by relayed QKD of the trusted nodes. In the main two-party classical computation between Alice and Bob, most of the communication (except for the last combining of results) can be via public classical channels, but to make it secure against the trusted nodes mentioned above, the communication can also be via relayed QKD by some
other trusted nodes, or by some computationally-secure method via public classical channels. The last combining of results should be via one of the two methods just mentioned to keep this part of information secure from the prior set of trusted nodes. For two-party quantum computation with quantum inputs, the situation is more difficult, since an initial teleportation is needed, but if the input of one party is fully classical, we may let that party send the bits to the other party with X masks, without using quantum teleportation, and if the final output on that classical-input party is also classical, the final result with unknown Pauli masks can be sent back via a classical channel. So we have that the main two-party quantum computation with classical input and output on one party can be implemented classically, overcoming the difficulty caused by the distance between the two parties, provided that the one-time tables are available with the help of trusted nodes, or by a trusted agent with the help of computationally-secure communication.

To make the protocols secure against eavesdroppers, we suggest using QKD for sending of the classical messages in the protocols. But there is also the use of entanglement in Protocol 1 and the generation of entanglement may be subject to interception by the eavesdropper. The quality of entanglement generation in Protocol 1 may be tested using the standard techniques such as the two parties measuring locally in different bases and compare notes, but that depends on the degree of honesty of the two parties. Note that Alice’s honesty can be verified in Protocols 2 and 4 and Bob’s cheating can be verified in Protocol 5. Bob’s honesty may alternatively be guaranteed by the assumption of conservative Bob. Even if there is a reduction of the quality of entanglement, the leakage of Alice is still upper-bounded by the protocols, although the correctness of the two-party computation would be affected.

VII. CONCLUSION

We have proposed some quantum protocols for approximately generating a certain type of classical correlations (a special case of the one-time tables [4]) with varying degrees of privacy, to be used in bipartite secure computation tasks. We have shown how to use the generated one-time tables in evaluating linear polynomials and generic boolean circuits, and in cheat-sensitive oblivious transfer and cheat-sensitive bit commitment, as well as in quantum homomorphic encryption and general two-party secure quantum computation. In the discussions we have mentioned that our method gives a check-based implementation of the PR-box type of correlations, but with some communication time cost, and involves sending of classical messages which do not contain useful information about the inputs, so it is not a direct implementation of the PR box, but rather the PR-box type of correlations. Open problems include: applications in cheat-sensitive quantum implementation of other cryptographic primitives, which may be weaker than the plain version of the primitives; a refined analysis of the protocols, taking into account the physical errors in quantum states and operations; fault-tolerance; application to special classes of circuits or functions; design of experimental schemes.

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Appendix A: The garden-hose gadget that corrects an unwanted $P$ gate

The Fig. 11 shows a construction from [11] for correcting an unwanted $P$ gate due to a $T$ gate in the circuit with certain prior Pauli corrections. The input qubit starts from the position “in”, and ends up in a qubit on Bob’s side which is initially maximally entangled $[\sqrt{2}(|00\rangle + |11\rangle)]$ with the qubit in the position “out”. The unwanted $P$ on this qubit is corrected, but
FIG. 1: A gadget from [11] for applying a $P^†$ to a qubit initially at the position “in” if and only if $p+q = 1 \pmod{2}$, using the “garden-hose” method. The dots connected by wavy lines are EPR pairs. The curved lines are for Bell-state measurements. For example, if $p = 0$ and $q = 1$, the qubit is teleported through the first and the fourth EPR pairs, with a $P^†$ applied to it by Alice in between. The input qubit always ends up in a qubit on Bob’s side which is initially maximally entangled with the qubit in the position “out”. The bit values of $p$ and $q$ determine which pairs of qubits are subject to Bell-state measurements.

some other Pauli corrections are now needed because of the Bell-state measurements. In each use of this gadget, some of the Bell-state measurements are not actually performed, dependent on the value of $p$ and $q$. The outcomes for those Bell-state measurements not actually performed are recorded as zero.