Gauge invariant coupling of fields to torsion: a string inspired model

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In a consistent heterotic string theory, the Kalb-Ramond field, which is the source of spacetime torsion, is augmented by Yang-Mills and gravitational Chern-Simons terms. When compactified to 4-dimensions and in the field theory limit, such additional terms give rise to interactions with interesting astrophysical predictions like rotation of plane of polarization for electromagnetic and gravitational waves. On the other hand, if one is also interested in coupling 2 or 3-form (Abelian or non-Abelian) gauge fields to torsion, one needs another class of interaction. In this paper, we shall study this interaction and offer some astrophysical and cosmological predictions. We also comment on the possibility of such terms in loop quantum gravity where, if the Barbero-Immirzi parameter is promoted to a field, acts as a source for torsion.

I. INTRODUCTION

The low energy physics of particle interactions is satisfactorily described by the standard model and general relativity. At higher energies available at the early universe or at astrophysical processes, it is expected that new degrees of freedom will emerge to play important role. Otherwise inaccessible at the present energy scale, these fields might interact with degrees of freedom of the standard model leading to some interesting theoretical predictions and observational signatures. Since string theory is a candidate for a unified description of field interactions even up to the Planck scale, we envisage that nature and the specific form of interaction of new fields with known degrees of freedom can be extracted from this theory in an unambiguous way. In this paper, we shall look for gauge invariant interactions of gauge fields (electromagnetic, gravitational and 2 and 3-form gauge fields) to torsion. In string theory, since the Kalb-Ramond (KR) field acts as a source of torsion, we shall have a look at possible gauge and gravitational interactions of a this KR field. The KR field is generic to any closed string spectrum but is not a degree of freedom of the standard model. One can anticipate that any observational effect involving the KR field, obtained using standard fields as probes, is then a window into the otherwise inaccessible world of very high energy physics supposedly predicted by string theories. On the other hand, loop quantum gravity (LQG) is also a candidate for quantum theory of gravity. In LQG, the Barbero-Immirzi parameter is a one-parameter ambiguity which describes various topological sectors. This parameter also comes up in the area spectrum and consequently in entropy of black holes wherefrom its value is ascertained by comparing with the Bekenstein-Hawking entropy formula. If the Barbero-Immirzi parameter is promoted to a field, it acts as a source for torsion. It is then interesting to compare and contrast various interactions of fields with these two sources of torsion that arise in these two theories of quantum gravity. Since there are observational implications, the issue is even more satisfying.

In the context of the heterotic string theory, electromagnetic and gravitational interactions of KR fields arise quite naturally from the requirements of consistency. As is well known [1], the $(E_8 \otimes E_8)$ or $SO(32)$ heterotic strings are two anomaly free gauge groups which can be coupled to $N = 1$ supergravity in 10 dimensions. Anomaly cancellation (the Green-Schwarz mechanism) requires that the KR 3-form field strength is augmented by addition of $(E_8 \otimes E_8)$ Yang-Mills Chern-Simons 3-form and local Lorentz Chern-Simons 3-form [1]. This augmentation induces electromagnetic and gravitational interactions of the KR field which lead to potentially interesting physical effects showing up in the Maxwell and Einstein equations, when the theory is compactified to four dimensions. The electromagnetic effect mainly comprise a rotation of the polarization plane of electromagnetic waves from large redshift sources, upon scattering from a homogeneous KR background [2–6]. This rotation is independent of the wavelength of the electromagnetic wave and cannot be explained by Faraday effect where the plane of polarization of the electromagnetic wave rotates depending quadratically on the wavelength while passing through some magnetized plasma. Similarly, the gravitational interaction leads to the result that the plane of polarization of gravitational waves rotate through an angle that is proportional to (a power of) the KR field strength component [7]. Predictions of this kind can then be useful if some deviations from the traditional expectations are observed. For example, such interactions have been studied within the framework of the five dimensional Randall-Sundrum braneworld model. When compactified to four dimensions, they lead to huge deviations from the expected results [7–11] which can be used to put bounds on the various
parameters in the theory [12–14]. On the other hand, if the predictions are non-observable, they lead to upper bounds on the presence of new fields which is important in our search for new theories and their couplings. To exemplify, in the present case of rotation of plane of polarization of electromagnetic waves, the magnitude of the effect is sensitive to the dimensional compactification of the underlying theory. For toroidal compactification (as well as for the Calabi-Yau compactification) of the theory (in the zero slope limit), the predicted rotation is proportional to the appropriate KR field strength component (scaled by the inverse scale factor in a Friedmann universe), so that bounds on the observed rotation translate into a stringent upper bound on the size of the KR field strength component. Moreover, if one uses the bounds on the KR field strength obtained from the cosmic optical activity, the order of magnitude of the similar effect for gravitational waves can be calculated.

The interactions which give rise to the above-mentioned predictions arise very naturally in string theory and they have been well studied. Interestingly, one can also perceive of another class of interactions which has not been discussed in this context except for in [15, 16], where only the electromagnetic interaction was considered. In this paper, we shall extend the study to non-Abelian gauge fields and discuss the effects of these possible new interactions in detail. Let us discuss the motivation for introducing such structures in brief (details will be seen in section 2). The issue originally arose during the study of Einstein-Cartan (EC) spacetime. The idea was to construct a gauge invariant coupling of electromagnetic field ($A_{\mu}$) to torsion- which is another geometrical property of the EC spacetime along with the metric. The field strength ($F_{\mu\nu}$) for such a spacetime also depends on torsion [17]. However, because the torsion does not have a transformation under $U(1)$ gauge transformation, the electromagnetic field strength is not gauge invariant. This is dissatisfactory since we expect that field strengths must be measurable even in spacetimes with torsion. This requirement on the field strength demands that the torsion must also stay invariant under $U(1)$ gauge transformation. This situation implies that there is a non-gravitational field, possibly massless, to function as the source of the torsion [2]. Since that field must be bosonic, one can opt for the KR antisymmetric second rank tensor field $B_{\mu\nu}$ as a possible candidate. $B_{\mu\nu}$, being a massless antisymmetric field, is expected to be a gauge connection, as indeed it is, with the following gauge transformation $\delta U_{(1)} B_{\mu\nu} = \partial_{[\mu} A_{\nu]}$ and this leaves its field strength $H_{\mu\nu\lambda}$ gauge invariant. Moreover, for anomaly-free quantum theory, $H_{\mu\nu\lambda}$ must be modified with the addition of an electromagnetic Chern Simons three tensor and if $B_{\mu\nu}$ is endowed with a non-trivial electromagnetic gauge transformation along with Kalb-Ramond gauge transformations, the KR field strength remains invariant under $U(1)$ gauge transformation. This is precisely what was needed: the torsion field is gauge invariant. Interactions of this type gives rise to interactions in the form of rotation of plane of polarization of electromagnetic (and gravitational) waves as discussed in the previous paragraph.

What if one wants to couple a 2-form or a 3-form gauge field to torsion? Such fields arise in the perturbative and non-perturbative sector (D-branes of string theory compactified to four dimensions and in supergravity. Again, field strengths for such higher rank tensor field are also not invariant under their respective gauge transformations in presence of spacetime torsion. Once we take the KR field as a source for torsion, there is a possible way out. We again demand the field strengths of 2- or 3-form gauge fields to be observable so that one again has to modify $H_{\mu\nu\lambda}$, but is a peculiar way. This extra term, instead being of the form $A \wedge F$ for the ($U(1)$) case above, is $A \wedge *F$, where * denotes the Hodge dual and $A$ is a one, two or a three form field. Again, if the field $B_{\mu\nu}$ has a non-trivial transformation under the gauge transformation of the form fields, its field strength ($H_{\mu\nu\lambda}$) and hence torsion remains invariant under gauge transformations, as required (for this case, we shall work in order $O(\sqrt{G})$). It is also interesting to note that addition of such terms ($A \wedge *F$) not only works for 2 and 3 form fields, but also for a 1-form field. Moreover, one gets an additional set of interaction for the electromagnetic fields and $H_{\mu\nu\lambda}$ field with observable consequences. These issues were first discussed in [15] and a possible embedding of such terms in $N = 1$ supersymmetric theory was discussed in [16]. In this paper, we shall extend the formalism for non-Abelian gauge fields and also for gravity waves and look for observational predictions. Interestingly, because of the presence of the Hodge dual, interactions of the later kind violate spatial parity. With the CMB data and the Planck data available, it might be interesting to look for such ideas now. Indeed, observational implications of such terms have already been discussed [18–22]. However, basis of terms have not been discussed in details and the coupling constant for such interactions are usually not pinned down.

The interest in LQG for such interactions and consequently it’s relation or differences with string theory/supergravity is due some recent studies [23–29]. These papers deal with the consequences of promoting the Barbero-Immirzi (BI) parameter to a field. It turns out that the derivative of the BI field is the source for torsion. Moreover, since the BI field is pseudo-scalar, it is natural to compare and contrast this BI field with the axion [24]. If the BI field is an axion, its derivative is dual to the $H_{\mu\nu\lambda}$ field alluded to above and such fields might have interactions with electromagnetic and gravitational fields in the way very similar to the one discussed above in the context of string theory. We shall discuss these issues in detail below and point out to some observational implications.

The plan of the paper is as follows: In section 2, we discuss the gauge invariant coupling of various form fields to torsion and show how this can be achieved with special reference to electromagnetism and gravity. In the next section, we shall review the consequences of such interaction for the Maxwell fields and extend them to gravity in the next section. In section 5, we compute
the quantum effective-potential (Coleman-Weinberg potential) [30] for a theory of gravity by including the modified interactions. We will see that inclusion of a parity violating scalar field (the BI field/axion) doesn’t have any effect in the one-loop effective potential of a theory where higher curvature terms are present. We conclude in section 6.

II. GAUGE INVARIANT INTERACTIONS OF FIELDS WITH TORSION

In the standard Einstein-Maxwell theory, the electromagnetic field-strength, reduces to the flat space expression on account of the symmetric nature of the Christoffel connection. However, in the theory of gravity described by Einstein-Cartan theory, i.e. in case where one has spacetime torsion, the situation changes quite drastically, because the electromagnetic field strength is no longer gauge invariant [17]. Indeed, it is easy to see that

\[ F_{\mu\nu} = \partial_{[\mu} A_{\nu]} - T_{\mu\nu}^\rho A^\rho, \tag{1} \]

where, \( T_{\mu\nu}^\rho A^\rho \) is the torsion (antisymmetric combination of the Christoffel connection), is obviously not invariant under \( U(1) \) gauge transformation \( \delta_\lambda A = d\lambda, \lambda \) being the gauge function. Since \( F_{\mu\nu} \) and any field strengths must be measurable quantities even in a curved spacetime with torsion, the torsion tensor, a purely geometric quantity like curvature must also be gauge invariant. However, this implies that one must also have another geometrical quantity which might compensate for the loss of gauge invariance due to torsion. In absence of such compensating fields, it is natural to look for non-gravitational fields to act as a source for torsion [2]. In the context of string theory, the Kalb-Ramond (KR) field seems to be an ideal candidate source [2]. Indeed, it also has all the desired gauge transformation properties required of torsion.

In this section, we shall first review the basic facts about the KR field as is known from string theory with special emphasis on it’s gauge transformation properties. The KR field is characterized by a 2-form potential \( B \) which has a 3-form field strength \( H \equiv dB \); the field strength is invariant under the KR gauge transformation \( \delta \lambda B = d\lambda, \lambda \) is a one-form gauge parameter. Immediately, one obtains the Bianchi identity for the KR field:

\[ dH = 0 \tag{2} \]

In 4 dimensional spacetime, the free KR action is given by

\[ S_H = \int_{M_4} H \wedge *H, \tag{3} \]

where, \(*H\) is the Hodge-dual of the field strength \( H \). Varying this action w.r.t. \( B \) yields the KR field equation

\[ d*H = 0 \tag{4} \]

which has the local solution

\[ *H = d\Phi_H, \tag{5} \]

where, \( \Phi_H \) is a scalar. Substituting this in the one obtains for the field \( \Phi_H \)

\[ d* d\Phi_H = 0 . \tag{6} \]

Thus, on-shell the Bianchi identity for the field \( B \) is the equation of motion for it’s Hodge dual field. This is not surprising and is a feature of all Hodge-dual related fields.

Let us now point to the string theory connection. \( B \) occurs in the massless spectrum of the free string in ten dimensional heterotic string theory. In the zero slope limit, this theory reduces to ten dimensional \( N = 1 \) supergravity coupled to \( N = 1 \ E_8 \otimes E_8 \) super-Yang-Mills theory. The requirement of ten dimensional supersymmetry and that the quantum theory be free of all anomalies implies that the KR field strength \( H \) be augmented as [1]

\[ H = dB - \frac{1}{M_P} (\Omega_{YM} - \Omega_L) , \tag{7} \]

where

\[ \Omega_{YM} \equiv \text{tr}(A \wedge dA + \frac{2}{3} g A \wedge A \wedge A) \tag{8} \]

is the Yang-Mills Chern-Simons 3-form with \( A \) the gauge connection 1-form and \( M_P \) is the Planck mass in 4- dimensional spacetime. \( \Omega_L \) is the gravitational Chern-Simons 3-form obtained by replacing the Yang-Mills gauge connection \( A \) by the
spin connection 1-form $\omega$, and the trace is taken over the local Lorentz indices. The augmentation in eq. (7) has important consequences. The field $H$, being a field strength, must remain gauge invariant under both Yang-Mills gauge transformations and under local Lorentz transformations. This implies that $B$ must now transform non-trivially under both gauge transformations inspite of $B$ being neutral. To simplify and to set the notations for the remaining part of the paper, let us say that the gauge field $A$ is $U(1)$ valued. Then, the transformation of $A$ is given by

$$\delta_\lambda A = d\lambda,$$

(9)

where, $\lambda$ is the gauge parameter. The Chern-Simons term now only contains $A \wedge dA$. We shall now denote $\Omega_{YM}$ by $\Omega_{EM}$ and this term varies as

$$\delta_\lambda \Omega_{EM} = d\lambda \wedge dA$$

(10)

Thus, to achieve gauge invariance for the $H$ field, the transformation law for $B$ should include the 2-form in (10) so that under Yang-Mills gauge transformation

$$\delta_\lambda B = -\frac{1}{M_P}(\lambda dA)$$

(11)

Also, the gravitational field in the vielbein formalism can be treated very similarly to the Yang-Mills field. Specifically the Yang-Mills potential $A$ is analogous to the spin connection 1-form $\omega_{AB}$, where $A, B$ are Lorentz indices. Under an infinitesimal Lorentz transformation with parameters given by an $SO(D-1,1)$ matrix $\Theta$, the transformation of $\omega$ is

$$\delta_L \omega = d\Theta + [\omega, \Theta],$$

(12)

The Lorentz Chern-Simons term varies as

$$\delta_L \Omega_L = \text{tr}(d\Theta \wedge d\omega)$$

(13)

Similar to the argument above, transformation law for $B$ should include the 2-form in (13) so that under Lorentz transformation

$$\delta_L B = -\frac{1}{M_P}\text{tr}(\Theta d\omega)$$

(14)

Retaining the form of the KR action (3), it follows that the KR field equation does not change. Therefore, $^*H$ still has the local solution (5). However, the KR Bianchi identity certainly changes, leading to

$$d^7 d\Phi_H = \frac{1}{M_P} \text{tr}(F \wedge F - R \wedge R),$$

(15)

where $F(R)$ is the Yang-Mills (spacetime) curvature 2-form. The Yang-Mills and Einstein equations change non-trivially. We shall consider these below in special situations viz., the Maxwell part of the gauge interaction and linearized gravity.

This scenario works well for 1-form gauge fields. How about if we want a gauge invariant coupling of higher form fields to torsion? In [15], it was proposed that one needs additional terms to be augmented to the KR field strength. For $U(1)$ gauge fields, it was proposed that the following additional augmentation is needed (the argument is obviously not based on any requirements arising from string theory)

$$H \rightarrow H + \frac{1}{M_P}(A \wedge ^*F)$$

(16)

However, the origin of such terms remained obscure. In the appendix (see section (VII)), we indicate the origin of such terms in this context. It is also clear that in presence of such terms, the gauge transformation of $B$ field changes from that obtained in equation (11)²:

$$\delta_\lambda B = -\frac{1}{M_P}(\lambda F + \lambda^* F)$$

(17)

² An immediate consequence of this gauge transformation is that the $H_{\mu\nu\lambda}$ now can no longer be thought of as a parity eigenstate, and thus neither is its dual $\Phi_H$. In other words, one can decompose $\Phi_H = \Phi_H^+ + \Phi_H^-$ where $+$ indicates even parity and $-$ is for odd parity. However, we shall continue to use the generic term $\Phi_H$ for this field.
We can also proceed further and add to equation (16) the spin-connection terms so that the augmentation takes the following form:

\[ H \to H + \frac{\zeta}{M_P} (A \wedge \ast F + \omega \wedge \ast R), \tag{18} \]

where, \( \zeta \) is a parameter which takes values +1 or -1. We have introduced this parameter since we don’t quite fix the coefficient. Now, instead of equation (15), the result of such additional terms in equation (18) is (we consider terms only up to order \( M_P^{-3} \))

\[ d^4 \Phi_H = \frac{1}{M_P} \text{tr}(F \wedge F + \zeta F \wedge \ast F - R \wedge R - \zeta R \wedge \ast R), \tag{19} \]

In short, the upshot of the above analysis is that one can consider a gauge invariant action of the following form [2, 3]:

\[ S[g, T] = \int_{M_4} d^4 x \left[ R(g, T) - \frac{1}{4} F_{\mu \nu} F^{\mu \nu} - \frac{1}{2} H_{\mu \nu \lambda} H^{\mu \nu \lambda} + T_{\mu \nu \lambda} H^{\mu \nu \lambda} \right] \tag{20} \]

where, \( H_{\mu \nu \lambda} \) is defined through equation (7) and the torsion tensor \( T_{\mu \nu \lambda} \) is an auxiliary field satisfying the constraint \( T_{\mu \nu \lambda} = H_{\mu \nu \lambda} \). Putting the local solution \( H = -\frac{1}{2} d \Phi_H \) from equation (5) in the action (20), we get the effective equation for the field \( \Phi_H \):

\[ S[g, A, \Phi_H] = \int_{M_4} d^4 x \left[ R(g, T) - \frac{1}{4} F_{\mu \nu} F^{\mu \nu} - \frac{1}{2} \partial_\mu \Phi_H \partial^\mu \Phi_H \right] + \Phi_H(F \wedge F + \zeta F \wedge \ast F - R \wedge R - \zeta R \wedge \ast R) \tag{21} \]

which is precisely the action for a pseudo-scalar (\( \Phi_H \)) coupled to gravity\(^3\). Note that the extra interaction contributes to the action in case of electromagnetism while is a higher derivative term for gravity. Without the \( \Phi_H \) term, the higher derivative gravity terms \( R \wedge R \) and \( R \wedge \ast R \) are the Pontryagin and the Euler invariants. They are related to the gravitational axial current anomaly and stress-tensor anomaly respectively [31–33]. The equation of motion for this pseudo-scalar is however given by equation (19). If the Barbero-Immirzi parameter is promoted to a field, the torsion is dual to the derivative of that pseudo-scalar field (just like the equation (5)). In that case, one gets an effective action same as the first part of the action above [23, 28]. In the following sections, we study the consequences of such interactions.

### III. ELECTROMAGNETIC INTERACTIONS OF KR FIELD

In this section, we shall confine our study to the electromagnetic interactions of the KR field in four dimensional Minkowski spacetime. Let us first restrict ourselves to the interaction of the type \( \Phi_H F_{\mu \nu} \ast F^{\mu \nu} \). Observe that since the field \( \Phi_H \) is a pseudo-scalar, the interaction is parity conserving. The relevant four dimensional field equations are

\[ \partial_\mu H^{\mu \rho} = 0 \]
\[ \partial_\mu F^{\mu \nu} = M_P^{-1} H^{\rho \eta} F_{\rho \eta}. \tag{22} \]

The corresponding Bianchi identities are

\[ \Box \Phi_H = M_P^{-1} F^{\mu \nu} \ast F_{\mu \nu} \]
\[ \partial_\ast F^{\mu \nu} = 0. \tag{23} \]

To simplify, let us assume that the ‘axion’ field \( \Phi_H \) is homogenous and provides a background with which the Maxwell field interacts. We restrict our attention to lowest order in the inverse Planck mass \( M_P \), so that terms on the RHS of the axion field equation (23) are ignored to a first approximation. Consequently, \( \Phi_H \equiv d \Phi_H / dt = \dot{f}_0 \) where \( \dot{f}_0 \) is a constant of dimensionality of \((\text{mass})^2\). Under these conditions, the Maxwell equations can be combined to yield the inhomogeneous wave equation for the magnetic field \( B \)

\[ \Box B = -\frac{2 f_0}{M_P} \nabla \times B. \tag{24} \]

\(^3\) Now, because \( \phi_H \) can be both parity violating as well as parity conserving, each interaction is both parity conserving and parity violating. In what follows, we shall only consider the case where \( \Phi_H \) is parity violating.
With the ansätz for a plane wave travelling in the z-direction, \( B(x,t) = B_0(t) \exp ikz \), we obtain, for the left and the right circular polarization states \( B_{0\pm} \equiv B_{0z} \pm iB_{0y} \),

\[
\frac{d^2 B_{0\pm}}{dt^2} + (k^2 \mp \frac{2f_0 k}{M_P}) B_{0\pm} = 0 .
\]  

(25)

Similarly, we can obtain the wave equation for the left and right circularly polarization states for the electric field which has exactly the same form as that of magnetic field. We concentrate on the equation for magnetic field as the conclusions will be same for that of electric field. Thus, the right and left circular polarization states have different angular frequencies (dispersion)

\[
\omega_\pm^2 = k^2 \mp \frac{2f_0}{M_P}
\]

(26)

so that over a time interval \( \Delta t \), the plane of polarization undergoes a rotation (for large \( k \))

\[
\Delta \Psi_{op} \equiv |\omega_+ - \omega_-| \Delta t \simeq 2\frac{f_0}{M_P} \Delta t .
\]

(27)

If we assume, just like in the case of Faraday rotation, the existence of a coherent electromagnetic wave over a time \( \Delta t \) (in general, cosmologically, any vector perturbation tends to thermalize with time scales typically smaller than \( \Delta t \)), in FRW spacetime, the value of observed angle of rotation also incorporates the scale factor [3]

\[
\Delta \Psi_{op} \equiv |\omega_+ - \omega_-| \Delta t \simeq 2\frac{f_0}{a^2(t)M_P} \Delta t,
\]

(28)

where, \( a(t) \) is the scale factor and \( \Delta t \) is now to be taken as the look-back time. This means that \( \Delta \Psi = \Delta \Psi(z) \), where \( z \) is the redshift, and increases with redshift. This rotation also differs from the better-understood Faraday rotation in that it is achromatic in the limit of high frequencies. Observationally, even for large redshift sources, the angle of rotation is less than a degree, which imposes the restriction on the dimensionless quantity \( f_0/M_P^2 < 10^{-20} \). In regard to astrophysical observations of optical activity, it appears that there is no definite evidence that the rotation of the plane of polarization travelling over cosmologically large distance is not entirely attributable to Faraday rotation due to magnetic fields present in the galactic plasma [34]. However, it is therefore not unlikely that the axion field will endow observable effect in CMB.

In contrast, if we consider only the extra augmentation, i.e. the interaction \( \Phi_H F_{\mu\nu} F^{\mu\nu} \), the resulting wave equation for the \( B \) field lead to entirely different results. Observe that this interaction violates spatial parity. The wave equation is simple to determine:

\[
\frac{d^2 B}{dt^2} - 2\nabla B + \frac{\zeta}{M_P} f_0 \frac{dB}{dt} = 0
\]

(29)

which eventually leads to the following equation for the left/right circularly polarised light [15]:

\[
\frac{d^2 B_{\pm(-)}}{dt^2} + \frac{\tilde{f}_0}{M_P} \frac{dB_{\pm(-)}}{dt} + k^2 dB_{\pm(-)} = 0,
\]

(30)

where, \( \tilde{f}_0 = \zeta f_0 \). The effect of parity violation is confined to the second term, which signifies either an enhancement or an attenuation, of the intensity of the observed electromagnetic wave, depending on the sign of \( \tilde{f}_0 \) [15]. We shall not go into the details of this calculation. Instead, we shall show that a similar effect also exists for gravity waves which might lead to some observational effects.

IV. BEHAVIOUR OF GRAVITATIONAL WAVES

First, let us discuss in some detail the gravitational analogue of the rotation of plane of polarisation (optical activity, equation (26)) discussed above [7]. This arises due to the parity conserving term of the form \( \Phi_H \text{tr}(R \wedge R) \) in equation (15). First note that the augmentation of \( H \) in (7) implies that the \( \text{tr}(R \wedge R) \) term contributes an additional term to the Einstein equation over and above the energy-momentum tensor of the KR field. Formally,

\[
G_{\mu\nu} = \frac{8\pi}{M_P^2} T_{\mu\nu} + \frac{16\pi}{M_P^2} \frac{1}{\sqrt{-g}} \frac{\delta}{\sqrt{g}} \int d^4x' \sqrt{-g(x')} \Phi_H(x') R_{\rho\lambda\sigma\eta(x')} R^{\rho\lambda\sigma\eta(x')},
\]

(31)

where,

\[
T_{\mu\nu} = H_{(\mu|\tau\rho} H_{\nu)}^{\tau\rho} - \frac{1}{6} g_{\mu\nu} H^2.
\]

(32)
Since our focus is on gravitational waves, it is adequate to consider the Einstein equation in a linearized approximation. To this effect, we decompose the metric $g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}$ with the fluctuation $h_{\mu\nu}$ being considered small so that one need only retain terms of $O(h)$ in the Einstein equation. We further impose on the fluctuations $h_{\mu\nu}$ the Lorenz gauge $h_{\mu\nu,\nu} = \frac{1}{2} h_{\mu\nu}$. In this gauge, the linearized Einstein equation from (31) becomes

$$-\Box h_{\mu\nu} = \frac{16\pi}{M_P^2} T_{\mu\nu} - \frac{128\pi}{M_P^2} \epsilon_{(\mu}^\sigma \epsilon_{\nu)\beta} \left[ \Phi_{H,\lambda\sigma} \left( h_{\beta[\nu],\alpha}^\lambda + h_{\beta,\alpha[\nu]}^\lambda \right) - \Phi_{H,\alpha} \Box h_{\lambda[\nu],\sigma} \right]$$  \hfill (33)

Again we regard the axion field $\Phi_H$ as a homogeneous background satisfying eq. (23) and consider its effect on a plane gravitational wave and we restrict to the lowest inverse power of the Planck mass for which a nontrivial effect is obtained. We ignore terms on the RHS of the axion field equation

$$\Box \Phi_H = M_P^{-1} R^{\mu\nu\lambda\sigma} * R_{\mu\nu\lambda\sigma}$$  \hfill (34)

We have chosen the Lorenz gauge and not all components of $h_{\mu\nu}$ are independent. In fact, the only physical degrees of freedom of the spin 2 field are contained in $h_{ij}$, for which we choose a plane wave ansatz travelling in the $z$-direction,

$$h_{ij} = \epsilon_{ij}(t) \exp - ikz.$$  \hfill (35)

The Latin indices above correspond to spatial directions. The other components of $h_{\mu\nu}$ can be gauged away, so that their field equation need not be considered. The only non-vanishing polarization components can be chosen to be $\epsilon_{11} = \epsilon_{22} = \epsilon_\perp$; from these the circular polarization components can be constructed as in the Maxwell case: $\epsilon_{\pm} = \epsilon_{11} \mp i \epsilon_{12}$. Further, we write the energy momentum tensor in eq. (32) in terms of $\Phi_H$ using eq. (5). Then, under the approximation of homogeneous axion field, these polarization components satisfy the inhomogeneous differential equation

$$\left[ \frac{d^2}{dt^2} + k^2 + \mathcal{F}_\pm \right] \epsilon_{\pm} = - \mathcal{F}_\pm,$$  \hfill (36)

where,

$$\mathcal{F}_\pm = \frac{8\pi f_0^2}{M_P^2 \left( 1 + 128\pi k f_0 / M_P^3 \right)}.$$  \hfill (37)

The difference between (36) and the analogous equation (25) is that the former has a forcing term absent in the latter; this forcing term is dependent on the wave number $k$ and controlled by the constant $f_0$ which characterizes the strength of the KR field coupling. We are interested in large $k$, but we would still remain within the Planckian regime $k < M_P$ so that the quantity $16\pi k f_0 / M_P^3 << 1$ and can serve as an expansion parameter, leading to

$$\left[ \frac{d^2}{dt^2} + k^2 + 8\pi f_0^2 / M_P^2 \mp 1024\pi^2 k f_0^3 / M_P^5 \right] \epsilon_{\pm} \simeq - 8\pi f_0^2 (1 \mp 16\pi k f_0 / M_P^3) / M_P^5.$$  \hfill (38)

We can now read off the dispersion relation

$$\omega_{\pm}^2 = k^2 + 4\pi f_0^2 / M_P^2 \mp 1024\pi^2 k f_0^3 / M_P^5$$  \hfill (39)

whence the group velocity is $v_{g\pm} = 1 + O(k^{-2})$ and the phase velocity is given by $v_{p\pm} = 1 \mp 512\pi^2 (f_0^3 / M_P^5)k$. As in the electromagnetic case, the rotation of the polarization plane for gravitational waves is given by

$$\Delta \Psi_{grav} \simeq 1024\pi^2 f_0^3 / M_P^5 \Delta t.$$  \hfill (40)

With the limits on $f_0$ given in the previous subsection, it is very small $O(10^{-30})$. However, since the tensor perturbations characterizing the gravitational wave do not get randomized, so the effect is in principle observable.

Let us now restrict ourselves to the parity violating term of the form $\Phi_H tr(R \wedge \ast R)$. The electromagnetic analogue of this term has been discussed in [15, 16] and reviewed in equation (30). In contrast to the rotation of plane of polarisation for gravity waves as observed above, equation (39), we expect some new consequences. In fact, we expect to observe modulation for gravity waves. First, the effective action can be written as:

$$G_{\mu\nu} = \frac{8\pi}{M_P^2} T_{\mu\nu} + \frac{16\pi}{M_P^2} \frac{1}{\sqrt{-g}} \delta_{\mu\nu} \int d^4x' \sqrt{-g(x')} \Phi_H(x') R_{\rho\lambda\sigma\eta}(x') R^{\rho\lambda\sigma\eta}(x'),$$  \hfill (41)
where,
\[ T_{\mu\nu} = H_{(\mu|\tau\rho} H_{\nu)\tau\rho} - \frac{1}{6} g_{\mu\nu} H^2. \]  

(42)

Again, we shall assume that the scalar field is homogeneous and it has only time dependence so that \( d\Phi/dt =: f_0 \) is a constant. The spin 2 field has only two degrees of freedom and the physical degrees of freedom are only contained in \( h_{ij} \). The equation of motion for the \( h_{ij} \) can be determined in a straightforward manner:

\[ \Box h_{ij} = \frac{16\pi}{M_P^2} \left[ -(\eta_{ij} + h_{ij}) f_0^2 \right] - \frac{16\pi}{M_P^2} \xi \left[ f_0 \Box h_{ij}, t + \Phi_H \Box \Box h_{ij} \right] \]

(43)

We choose a plane wave ansatz for \( h_{ij} \), travelling in the \( z \)-direction:

\[ h_{ij} = \epsilon_{ij}(t) \exp(-ikz) \]

(44)

As in the previous case, we shall assume that the only non-vanishing components of polarisation are \( \epsilon_{11} = -\epsilon_{22} \) and \( \epsilon_{12} = \epsilon_{21} \). Now, to facilitate the calculation, let us make some simplified assumption and notations. First, as seen from the previous section, let us define the dimensionless quantity \( \alpha := \left( f_0/M_P^2 \right) << 1 \). Secondly, we shall remain in the Planckian regime but the wave number \( k \) is such that the dimensionless quantity \( \beta := k/M_P \) is small (let us say \( O(10^{-5}) \)). The modulus of the field \( \Phi_H \) is taken to be order 1. The previous equation now reduces to:

\[ \frac{d^2\epsilon_{ij}}{dt^2} + 16\pi \alpha \zeta \beta k \frac{d\epsilon_{ij}}{dt} + k^2 \left( 1 - \frac{16\pi \alpha^2}{\beta} \right) \epsilon_{ij} = \frac{16\pi f_0^2}{M_P^2} \eta_{ij} \]

(45)

This is an equation for a damped oscillator with a forcing term. The system can get damped or can sustain gravity waves. This depends on the value of the "\((b^2 - 4ac)\)" term which here is given by:

\[ 2ik \left[ 1 - \frac{16\pi \alpha^2}{\beta} + \left( \frac{16\pi \alpha \zeta \beta}{4} \right)^2 \right]^{1/2} \]

(46)

Let us list the various possible cases. First, when \( \alpha^2/\beta \geq 1 \), \( i.e. \) small values of \( k \) (note that the third term in (46) is very small, with the value of \( \beta \), it is of the order of \( 10^{-15} \) smaller compared to the second term and will not contribute appreciably), we get the scenario where the gravity waves dampen and is not observed:

\[ h_{ij}(t, z) = \exp \left( -\frac{16\pi \alpha \zeta k}{M_P} \right) \left[ A_{ij} \exp(ikt - i{kz}) + B_{ij} \exp(-ikt - i{kz}) \right] \]

(47)

Second, consider the case when \( \alpha^2/\beta < 1 \) \( i.e. \) large values of \( k \). Then, the solutions of the equation (45) are:

\[ h_{ij}(t, z) = \exp \left( -\frac{16\pi \alpha \zeta k}{M_P} \right) \left[ A_{ij} \exp(ikt - i{kz}) + B_{ij} \exp(-ikt - i{kz}) \right] \]

(48)

This is the standard solution where the wave proceeds sinusoidally. It is clear that the solution to this equation can give attenuation/amplification of amplitude of gravity waves. To see this, choose \( \zeta = +1 \) then the equation (48) leads to attenuation of gravity waves whereas for \( \zeta = -1 \), we get amplification of gravity waves. In short, in this case we do not see any rotation of plane of polarisation of gravity wave, rather the attenuation/amplification of the wave during propagation is the result of such an interaction. Such phenomena for gravity waves was suggested in [18] which however was largely phenomenological. If such effects are present, they have implications for CMB spectrum. They lead to non-zero cross-correlation in multipole moments \( C_l^{BB} \) and \( C_l^{EB} \). Such effects cannot be induced by Faraday rotation (if there is any intervening magnetic fields). This is because it is an anisotropic effect which will also change \( l \). With the Planck data coming up, we expect to see some of these effects or if these are not seen, the experiments can be used to put bounds on the coupling constants for these interactions.

V. EFFECTIVE POTENTIAL FOR HIGHER DERIVATIVE GRAVITY

In this section we will study the effects of quantum fluctuations of different fields for a theory governed by the action (21) by calculating the one-loop effective potential using loop-expansion scheme [35]. We will concentrate on the gravitational part of the action only. Effective-potential serves as a useful tool to investigate the vacuum structure of such a theory where one can define the theory to be valid up to an energy scale (Planck energy) through cut-off and make predictions treating it as an effective theory. To keep the matters very general, we shall consider a theory of gravitation coupled with three different kinds of matter.
fields. The Einstein term is minimally coupled with a massive/massless scalar field $\phi_S$ which has a self interacting potential. The action also contains an (axion) field $\phi_A$ coupled with a CP-odd term $R_{\mu\nu\alpha\beta}^* R^{\mu\nu\alpha\beta}$ and another field $\phi$ which is coupled to the CP-even term $R_{\mu\nu\alpha\beta} R^{\mu\nu\alpha\beta}$. In Euclidean signature, the Lagrangian of the theory is

$$\mathcal{L} = \mathcal{L}_{g1} + \mathcal{L}_{g2} + \mathcal{L}_{g3} + \mathcal{L}_m$$

$$= -\frac{1}{\kappa^2} R + a \phi R_{\mu\nu\alpha\beta} R^{\mu\nu\alpha\beta} + b \phi_A R_{\mu\nu\alpha\beta}^* R^{\mu\nu\alpha\beta} + \frac{1}{2} g^{\mu\nu} \partial_\mu \phi \partial_\nu \phi + \frac{1}{2} g^{\mu\nu} \partial_\mu \phi_A \partial_\nu \phi_A + \frac{1}{2} g^{\mu\nu} \partial_\mu \phi_S \partial_\nu \phi_S + V(\phi_S)$$

(49)

where $\kappa^2 = 16\pi G$ and $a, b$ are coupling constants which can be specified later (they are $M_p^{-3}$). Let us now turn to calculate the effective potential. For that purpose, we first expand the metric $g_{\mu\nu}$ around a flat background:

$$g_{\mu\nu} = \delta_{\mu\nu} + \kappa h_{\mu\nu},$$

(50)

where $\delta_{\mu\nu}$ is a flat background and the fluctuations $h_{\mu\nu}$ are small, $|h_{\mu\nu}| < 1$. For the decomposition (50), the inverse of the metric is

$$g^{\mu\nu} = \delta^{\mu\nu} - \kappa h^{\mu\nu} + \kappa^2 h^{\mu\lambda} h^{\lambda\nu} + \ldots$$

(51)

Furthermore, the determinant of the metric, which will be needed in the following, will be given by:

$$(g)^{\frac{1}{2}} = 1 + \frac{1}{2} h^\alpha_\alpha - \frac{1}{4} h^\alpha_\beta h^{\beta\alpha} + \frac{1}{8} (h^{\alpha\alpha})^2 + \ldots$$

(52)

To calculate one-loop effective potential we need to expand the Lagrangians only upto quadratic order in the $h_{\mu\nu}$. The expansions are listed below:

$$\sqrt{g} \mathcal{L}_{g1} = \sqrt{g} R = -\frac{1}{4} \partial_\alpha h_{\mu\nu} \partial^\alpha h^{\mu\nu} + \frac{1}{4} \partial_\alpha h \partial^\alpha h - \frac{1}{2} \partial_\alpha h \partial_\beta h^{\alpha\beta}$$

$$+ \frac{1}{2} \partial_\alpha h_{\mu\beta} \partial^\beta h^{\alpha\mu} + \text{total derivatives}$$

(53)

The expressions for the other two terms are long. However, we give them below. First,

$$\sqrt{g} \mathcal{L}_{g2} = \sqrt{g} a \phi R_{\mu\nu\alpha\beta} R^{\mu\nu\alpha\beta}$$

$$= a \kappa^2 (\partial_\rho \partial_\sigma \phi h_{\mu\nu} \partial^\rho \partial^\sigma h^{\mu\nu} + \partial_\rho \phi h_{\mu\rho} \partial^\rho h^{\mu\nu} + \phi h_{\mu\nu} \square h^{\mu\nu})$$

$$+ \partial_\rho \phi h_{\mu\rho} \partial^\rho h^{\mu\nu} + \partial_\rho \phi h_{\mu\rho} \partial^\nu \partial_\sigma h^{\rho\sigma} + \phi h_{\mu\nu} \partial^\nu \partial_\sigma h^{\rho\sigma} - 2 \partial_\rho \partial_\sigma \phi h_{\mu\nu} \partial^\rho \partial^\nu h^{\sigma\sigma} - 2 \partial_\rho \partial_\sigma h_{\mu\rho} \partial^\rho \partial^\nu h^{\sigma\sigma}$$

(54)

and

$$\sqrt{g} \mathcal{L}_{g3} = 2 b \kappa^2 \left( \partial_\lambda \partial_\sigma \phi_A \partial_\alpha \partial_\beta h^{\lambda\sigma}_{\rho\eta} + \partial_\rho \phi_A h_{\rho\eta} \square h^{\lambda\sigma} - \partial_\lambda \partial_\sigma h_{\rho\eta} \partial_\alpha \partial_\beta h^{\lambda\sigma} - \partial_\lambda \partial_\sigma \phi_A h_{\rho\eta} \partial_\alpha \partial_\beta h^{\lambda\sigma} \right)$$

(55)

Note that due to the presence of a Levi-civita tensor which is completely anti-symmetric in it’s indices, only three terms will survive in the expansion of $\mathcal{L}_{g3}$. Since we are calculating one-loop effective potential, terms of order 2 in fluctuations will only contribute. To obtain one-loop effect, it is sufficient to choose spacetime independent saddle points for the scalar (and pseudo-scalar) fields;

$$\phi(x) = \phi_0 + \Phi(x); \ \phi_A(x) = \phi_{A0} + \Phi_A(x); \ \phi_S(x) = \phi_{S0} + \Phi_S(x)$$

With these choices, the derivative terms of the scalar fields will not contribute to the resulting Lagrangian (expanded about the saddle points). The Lagrangian relevant for calculating one loop effective potential is by invoking the transverse-traceless gauge [36, 37]. With $\partial_\rho h^{\mu\nu} = 0$ and $h = 0$, the relevant part of the lagrangian becomes:

$$\mathcal{L}_{rel} = \frac{1}{4} h_{\mu\nu} (-\square h) + a \kappa^2 \phi_0 h_{\mu\nu} \square h + b \phi_A h_{\mu\nu} \square h - \frac{1}{2} \Phi_S (-\square h + V'(\phi_{S0})) \Phi_S - V(\phi_{S0})$$

$$- \frac{1}{4} \kappa^2 h_{\mu\nu} V h^{\mu\nu} + \frac{1}{2} \Phi (-\square E)^2 + \frac{1}{2} \Phi_A (-\square E)^2$$

(56)
where $\Box_E$ is the operator in Euclidean space. Since we are perturbing around a flat background, ghost doesn’t appear in this gauge [38]. Also, it is important to note here that the (axion) field $\Phi_A$ has no contribution to the one-loop effective potential. Now, eqn (56) may be conveniently written as

$$\mathcal{L}_{rel} = \frac{1}{2} h_{\mu\nu} \mathcal{O}^{\mu\nu\alpha\beta} h_{\alpha\beta} + \frac{1}{2} \Phi_S (-\Box_E + V''(\phi_S)) \Phi_S + \frac{1}{2} \Phi (-\Box_E) \Phi + \frac{1}{2} \Phi_A (-\Box_E) \Phi_A$$  

(57)

where the operator

$$\mathcal{O}^{\mu\nu\alpha\beta} = \frac{1}{2} \delta^{\mu\alpha} \delta^{\nu\beta} \left[ -\Box_E + 2ak^2 \phi_0 \Box_E - \kappa^2 V(\phi_S) \right]$$

Now, we rewrite the Lagrangian in terms $\Psi_i$ where $i = 1, 2, \ldots 10$ denotes ten independent components of $h_{\mu\nu}$ [39].

$$\mathcal{L}_{rel} = \frac{1}{2} \Phi (-\Box_E + V''(\phi_S)) \Phi + \frac{1}{2} \Psi_i M_{ij} \Psi_j,$$  

(58)

where we have have employed the following index correspondence: $\mu\nu \rightarrow i$ and $\alpha\beta \rightarrow j$. To get the one-loop effective potential we need to calculate the determinants of differential operators which in this case reduces to calculate the eigenvalues of the $10 \times 10$ matrix $M_{ij}$ [39]. The operator for scalar field is trivial. We write down the eigenvalues,

$$\lambda_i = \frac{1}{2} (k^2 + 4ak^2 \phi_0 k^4 - \kappa^2 V); (1 \leq i \leq 4)$$

$$\lambda_i = (k^2 + 4ak^2 \phi_0 k^4 - \kappa^2 V); (5 \leq i \leq 10)$$  

(59)

The one-loop effective potential is given by

$$V^{(1)}_{eff} = V(\phi_S) + \frac{1}{2} \text{Tr} \ln (k^2 + V'') + \sum_{i=1}^{10} \frac{1}{2} \text{Tr} \ln \lambda_i,$$  

(60)

where Tr is the functional trace. Performing the momentum space integrals and introducing a cut-off we obtain the unrenormalized one-loop effective potential

$$V^{(1)}_{eff}(\phi_S, \phi_0) = \frac{5}{16\pi^2} \left[ \frac{\Lambda^4}{2} - \frac{1 - 2eg}{4e^2} \right] \ln \left( \frac{\Lambda^4}{g} + \frac{\Lambda^2}{2e} + \frac{g}{2e} - \frac{1}{4e^2} + \frac{\sqrt{1 - 4eg}}{4e^2} \ln \left( \frac{1 + \sqrt{1 - 4eg}}{1 - \sqrt{1 - 4eg}} \right) \right]$$

$$+ \frac{\Lambda^2 V''}{32\pi^2} + \frac{V'''}{32\pi^2} \ln \left( \frac{V''}{\Lambda^2} - \frac{1}{2} \right) + V(\phi_S)$$

(61)

where $e = 4\phi_0 a\kappa^2$ and $g = -\kappa^2 V$. $\Lambda^2$ is the momentum cutoff. If we put the expressions of $e$ and $g$ back into the above expression the effective potential is seen to have an imaginary part:

$$V^{(1)}_{eff}(\phi_S, \phi_0) = \frac{5}{16\pi^2} \left[ \frac{1 + 8\kappa^4 \phi_0 a V}{64\kappa^4 \phi_0^2 a^2} - \frac{\Lambda^4}{2} \right] \ln \left( \frac{V}{\Lambda^4} + \frac{\Lambda^2}{8\kappa^4 \phi_0^2 a^2} + \frac{V}{2} - \frac{1}{64\kappa^4 \phi_0^2 a^2} \right)$$

$$+ \sqrt{1 + 8\kappa^4 \phi_0 a V} \ln \left( \frac{1 + \sqrt{1 + 8\kappa^4 \phi_0 a V}}{1 - \sqrt{1 + 8\kappa^4 \phi_0 a V}} \right) + \frac{5i}{16\pi} \left( 1 + 8\kappa^4 \phi_0 a V \right) \left( \frac{\Lambda^4}{2} \right)$$

$$+ \frac{\Lambda^2 V''}{32\pi^2} + \frac{V'''}{32\pi^2} \ln \left( \frac{V''}{\Lambda^2} - \frac{1}{2} \right) + V(\phi_S)$$

(62)

It is interesting to see here that an imaginary part is generated in the effective potential. Similar kind of result was found in [36] for a theory where a single scalar field is coupled to gravity. The imaginary part of the effective potential signifies that we have chosen an unstable vacuum, in fact flat space is not a stable vacuum of this theory. The value of $V_{eff}$ at the asymmetric minimum serves as a cosmological constant at the tree level [36, 40]. This interpretation can be explained as follows: Let $V_{eff}$ develops a symmetry breaking minima at the value of $\phi_S = \phi_{min}$ and $V_{eff}(\phi_{min}) \neq 0$ then $V_{eff}(\phi_{min})$ will act as a cosmological constant at the tree level. Now we include a cosmological constant to this theory, so now we have a different vacuum state not a flat space but a de- Sitter space. The Lagrangian reads as

$$\mathcal{L} = -\frac{1}{\kappa^2} (R - 2C) + a \phi R_{\mu\nu\alpha\beta} R^{\mu\nu\alpha\beta} + b \phi_A R_{\mu\nu\alpha\beta} R^{\mu\nu\alpha\beta}$$

$$+ \frac{1}{2} g^{\mu\nu} \partial_\mu \phi \partial_\nu \phi + \frac{1}{2} g^{\mu\nu} \partial_\mu \phi_A \partial_\nu \phi_A + \frac{1}{2} g^{\mu\nu} \partial_\mu \phi_S \partial_\nu \phi_S + V(\phi_S)$$

(63)
where $C$ is the cosmological constant. If we repeat the calculation for the effective potential from (63), the imaginary part of the potential will be

$$\text{Im}[V_{eff}(\phi S_0, \phi_0)] = \frac{5}{16\pi} \left( \frac{1 + 2(\kappa^2 V + 2C) \phi_0 a \kappa^2}{64\kappa^2 \phi_0^2 a^2} - \frac{\Lambda^4}{2} \right)$$

(64)

It is now obvious that we can fine tune the cosmological constant $C$ such that the imaginary part of $V_{eff}$ and the cosmological constant both vanish

$$\frac{1}{2} \kappa^2 V(\phi_{S_{inv}}) + \frac{1}{4\phi_0 a \kappa^2} + C = 0$$

(65)

This makes the flat background a solution of the Einstein equation at the vacuum state. The calculation of effective potential here done in conventional approach which is not devoid of gauge ambiguities. However, it is well known that Vilkovisky-DeWitt (VD) [41, 42] approach of deriving effective potential is free from any ambiguities related to gauge-fixing condition or parameterization of the theory. We don’t employ the method of VD here, although quite a number of papers have already been in the literature which calculate the effective potential in VD approach for ordinary and higher derivative gravity [43–45]. VD effective potential for the theory under consideration may be taken as a future project.

VI. CONCLUSIONS

Let us first recall the results of the paper. In string theory, the Kalb-Ramond field acts as a source term for torsion which has various interactions with gauge fields. In order that the interactions are gauge invariant, the Kalb-Ramond field $B_{\mu \nu}$ must be endowed with non-trivial transformations under gauge fields. This leads to some interesting interactions with observable consequences. One of them is the rotation of plane of polarisation for electromagnetic and gravity waves. These had been studied earlier and have been matched with experimental results. However, these interactions are not the only possible ones. One can have additional ones which arise from the gauge invariant coupling of higher form fields to torsion. Such interaction was proposed in [15]. We give a theoretical basis for such terms and extend the formalism for gravity waves. Observational consequences of such interactions are altogether different. They lead to amplification/attenuation of electromagnetic or gravity waves and have important implications for anisotropy of the Cosmic Microwave Background (CMB) by spatial parity violation [18]. For such parity breaking term, one can get certain non-vanishing multipole moment correlations between the temperature anisotropy and polarization of the CMB. In the CMB data, one usually observes correlations like $C_{TT}$, $C_{EE}$, $C_{BB}$ and $C_{TE}$ which arise from parity conserving interactions. On the other hand, cross-correlations like $C_{EB}$ and $C_{TB}$ arise from parity violating interactions from which bounds on the strength of such parity violating terms can be ascertained. We also study the Coleman-Weinberg mechanism for such extended theory. This leads to a potential which might have some significance in the early universe and inflation. Initial studies with this potential show that one can generate the requisite number of e-foldings from such a theory near the Planck scale. other consequences from such a potential requires further study.

VII. APPENDIX

In this appendix, we shall show the existence of the extra term of the form $(A \wedge *F)$ added to the KR field in equation . The question is: where to look for such terms? To motivate, let us recall that the usual Chern-Simons term $(\Omega_{YM})$ augmented to the KR field strength $H$ in equation is actually a boundary term. In the $U(1)$ version for example, the Chern-Simons term $(\Omega_{YM})$ reduces to $(A \wedge F)$ which is precisely the contribution to boundary term corresponding to $(F \wedge F)$ in $U(1)$ gauge theory. In the same token, let us look for the boundary terms for the action itself. Consider the Lagrangian 4-form for the free Yang-Mills theory

$$L = \text{tr}(F \wedge *F)$$

(66)

The on-shell variation of the Lagrangian gives

$$\delta L = 2\text{tr} d(\delta A \wedge *F) := d\Theta(\delta)$$

(67)

The term $\Theta(\delta)$ is a three form and is often called the symplectic potential. Now, consider the variation of the one-form $A$ through a parameter $\mu$, $0 \leq \mu \leq 1$ and define:

$$\delta_{\mu} A := A \delta_{\mu} \quad \text{and} \quad A(\mu) := \mu A \quad \text{so that}$$

$$*F(\mu) = \mu^* F + (\mu^2 - \mu)^*(A \wedge A)$$

(68)
This implies that
\[ \Theta(\delta \mu) = 2 \text{tr} (A \wedge^* F(\mu)) \delta \mu \] (70)

Thus, on-shell, the above equation (70) is equivalent to:
\[ \frac{\delta}{\delta \mu} \text{tr}(F \wedge^* F) = 2 d \text{tr} [\mu A \wedge^* F + (\mu^2 - \mu) A \wedge^* (A \wedge A)] \] (71)

Integrating with respect to \( \mu \), we get
\[ \text{tr}(F \wedge^* F) = d \text{tr} [A \wedge^* F - \frac{1}{3} A \wedge^* (A \wedge A)] \] (72)
\[ = d \text{tr} [A \wedge^* dA + \frac{2}{3} A \wedge^* (A \wedge A)] \]

Note that this term arises from a boundary contribution and is valid only on-shell. In contrast, the usual Chern-Simons term, which can be derived in a similar fashion from the other boundary term \( \text{tr}(F \wedge F) \) only requires the Bianchi identity. In standard treatments, the boundary term vanishes by the boundary conditions on the fields. The above derivation is merely to show the existence of such terms in general when the field has all possible configurations.

Two comments are in order. Firstly, in the equation above, we have considered only the free Yang-Mills theory. Now suppose that the Yang-Mills field is also coupled to other fields as the KR field \( H_{\mu\nu\lambda} \) in the present paper. In that case, the equation of motion for the Yang-Mills field is not merely \( D^* F^i = 0 \), but has contributions from the KR fields too. One then needs to look for the modification due to presence of such terms also. Secondly, as mentioned in the paper, we want not only to couple 1-form field to \( H \) field but also 2 and 3-form fields. In those cases, the term \( [A \wedge^* (A \wedge A)] \) does not arise (and in not a 3-form). For this reason, in what follows, we discard that term altogether.

Let us now comment on the effect of the equation of motion. From above construction, we get the following:
\[ \text{tr}(F \wedge^* F) = d \text{tr} [A \wedge^* F] + \text{tr}(\delta A \wedge D^* F) \] (73)

For \( \delta A^i = d\lambda^i + [A, \lambda]^i \), another term needs to be added to the first term. Thus in total, we get the contribution to the total derivative to be:
\[ \text{tr}(F \wedge^* F) = d \text{tr} [A \wedge^* F + \lambda D^* F] \] (74)

To understand the effect of this term, let us restrict to \( U(1) \) gauge theory for simplicity. For \( U(1) \) gauge fields, the effect of this augmentation leads to:
\[ H \rightarrow H = dB + \frac{1}{M^2} (A \wedge^* F + \lambda d^* F) \] (75)

We want that \( H \) remain gauge invariant under \( U(1) \) gauge transformation. Then, \( B \) must transform under \( U(1) \) gauge transformation. This can be easily found from the above equation:
\[ \delta \lambda B = \lambda^* F \] (76)

Thus, the gauge transformation of \( B \)-field comes out cleanly only when the contribution from the equation of motion is taken into account. Then, why have we not added the term \( \lambda d^* F \) in equation? That is because we want to look only for effects of order \( M^{-1} \) while the contribution of second term is of order \( M^{-2} \).

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[1] M. B. Green, J. H. Schwarz and E. Witten, Cambridge, Uk: Univ. Pr. ( 1987) 596 P. ( Cambridge Monographs On Mathematical Physics)
[2] P. Majumdar and S. SenGupta, Class. Quant. Grav. 16 (1999) L89 [arXiv:gr-qc/9906027].
[3] S. Kar, P. Majumdar, S. SenGupta and A. Sinha, Eur. Phys. J. C 23 (2002) 357 [arXiv:gr-qc/0006097].
[4] P. Das, P. Jain and S. Mukherji, Int. J. Mod. Phys. A 16 (2001) 4011 [arXiv:hep-ph/0011279].
[5] S. Kar, P. Majumdar, S. SenGupta and S. Sur, Class. Quant. Grav. 19 (2002) 677 [arXiv:hep-th/0109135].
[6] Y. Itin and F. W. Hehl, Phys. Rev. D 68 (2003) 127701 [arXiv:gr-qc/0307063].
