Seesaw mechanism, quark-lepton symmetry and Majorana phases

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A previous short analysis of the seesaw mechanism, based on quark-lepton symmetry, experimental data and hierarchical neutrino spectrum, is enlarged to include small but not zero $U_{e3}$, inverted mass hierarchy, and the qualitative effect of Majorana phases. The structure of the heavy neutrino mass matrix obtained in several cases is discussed. We find two leading forms for this matrix. One is diagonal and stands at the unification scale or above. The other is off-diagonal and stands at the intermediate scale.

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I. INTRODUCTION

The SuperKamiokande Collaboration has recently confirmed the oscillation of atmospheric neutrinos [1]. This evidence, as well as the strong indications of oscillation of solar neutrinos too, which could explain the solar neutrino deficit [2,3], lead to nonzero neutrino masses. Although not zero, such masses have to be much smaller than the charged lepton and quark masses, less than few eV [4]. This feature can be explained by means of the seesaw mechanism [5], where the mass matrix $M_L$ of light (left-handed) Majorana neutrinos is given by

$$M_L = M_D M_R^{-1} M_D^T,$$ 

with the Dirac mass matrix $M_D$ of the same order of magnitude of the charged lepton or quark mass matrix, and the eigenvalues of $M_R$, the mass matrix of right-handed neutrinos, much bigger than the elements of $M_D$.

In the Minimal Standard Model plus three right-handed neutrinos, the mass matrix of heavy neutrinos is generated by a Majorana mass term $(1/2)\nu_R M_R (\nu_R)^c$ and hence $M_R$ is not constrained. Instead, in Grand Unified Theories (GUTs) like $SO(10)$, $M_R$ is obtained from the Yukawa coupling of right-handed neutrinos with the Higgs field that breaks the unification or the intermediate group to the Standard Model [6]. When such a field gets a VEV $v_R$, which is the unification or the intermediate scale, the right-handed neutrinos take a mass and $M_R = Y_R v_R$, where $Y_R$ is the matrix of Yukawa coefficients. Actually, this happens when and because at the same stage it is also $B - L$ broken, allowing for Majorana masses. In the supersymmetric case $v_R$ is the unification scale ($v_R \sim 10^{16}$ GeV), while in the nonsupersymmetric case it is the intermediate scale ($v_R \sim 10^9 - 10^{13}$ GeV) [7]. On the other hand, GUTs generally predict $M_D \sim M_u$, where $M_u$ is the mass matrix of up quarks, and $M_l \sim M_d$, where $M_l$ is the mass matrix of charged leptons and $M_d$ the mass matrix of down quarks. This is called quark-lepton symmetry.
From the experimental data on neutrino masses and mixings, and the quark-lepton symmetry, it is possible to infer the heavy neutrino mass matrix $M_R$ by inverting formula (1),

$$M_R = M_D^T M_L^{-1} M_D.$$ (2)

In fact, $M_L$ can be obtained, at least approximately, from experimental data on neutrino oscillations, and quark-lepton symmetry suggests

$$M_D \simeq \frac{m_\tau}{m_b} \text{diag}(m_u, m_c, m_t).$$ (3)

The nearly diagonal form of $M_D$ is due to the fact that mixing in the Dirac sector is similar to the small mixing in the up quark sector [8], and the factor $m_\tau/m_b \equiv k$ is due to approximate running from the unification or intermediate scale, where $m_b = m_\tau$ should hold [9]. As a matter of fact $M_D$ is almost scale independent. Then the Dirac masses of neutrinos are fixed by the values of the up quark masses at the unification scale in the supersymmetric model, and at the intermediate scale in the nonsupersymmetric model. However, in both cases their values are roughly similar [10], namely $M_D \simeq \text{diag}(0.001, 0.3, 100)$ GeV. It is now important to check if the resulting scale of $M_R$ is in accordance with the physical scales of GUTs, and also the structure of $M_R$, which would give further insight towards a more complete theory. This program has been addressed in refs. [11,12] and in the recent papers [13–17].

In this paper we want to extend the analysis of ref. [17], in order to include small but not zero $U_{e3}$, inverted hierarchy of light neutrino masses, approximate effect of Majorana phases, and a discussion on the structure of $M_R$.

In section II we summarize the experimental informations on neutrino masses and mixings, coming mainly from solar and atmospheric oscillations. In sections III and IV the normal and inverted mass hierarchy cases are studied. In section V the effect of Majorana phases is briefly considered and finally we give some concluding remarks.
II. NEUTRINO MASSES AND MIXINGS

We denote by $m_i$ ($i = 1, 2, 3$) the light neutrino masses. The mass eigenstates $\nu_i$ are related to the weak eigenstates $\nu_{\alpha}$ ($\alpha = e, \mu, \tau$) by the unitary matrix $U$,

$$\nu_\alpha = U_{\alpha i} \nu_i. \quad (4)$$

The results on solar oscillations imply for the three solutions of the solar neutrino problem, namely small mixing MSW (SM), large mixing MSW (LM) and vacuum oscillations (VO), the following orders of magnitude for $\Delta m_{\text{sol}}^2$:

$$\Delta m_{\text{sol}}^2 \sim 10^{-6} \text{ eV}^2 \quad (\text{SM}) \quad (5)$$

$$\Delta m_{\text{sol}}^2 \sim 10^{-5} \text{ eV}^2 \quad (\text{LM}) \quad (6)$$

$$\Delta m_{\text{sol}}^2 \sim 10^{-10} \text{ eV}^2 \quad (\text{VO}) \quad (7)$$

On the other hand, atmospheric oscillations give

$$\Delta m_{\text{atm}}^2 \sim 10^{-3} \text{ eV}^2, \quad (8)$$

so that $\Delta m_{\text{sol}}^2 \ll \Delta m_{\text{atm}}^2$. We can set

$$\Delta m_{\text{sol}}^2 = m_2^2 - m_1^2, \quad \Delta m_{\text{atm}}^2 = m_3^2 - m_{1,2}^2, \quad (9)$$

and, assuming without loss of generality $m_3 > 0$, there are three possible spectra for $m_i$:

$$m_3 \gg |m_2|, |m_1| \quad \text{(hierarchical)} \quad (10)$$

$$|m_1| \sim |m_2| \gg m_3 \quad \text{(inverted hierarchy)} \quad (11)$$

$$|m_1| \sim |m_2| \sim m_3 \quad \text{(nearly degenerate)} \quad (12)$$
Moreover, due to the near maximal mixing of atmospheric neutrinos [1] and the smallness of $U_{e3}$ [19], the mixing matrix $U$ can be written as [20]

\[
U = \begin{pmatrix}
c & s & \epsilon \\
-\frac{1}{\sqrt{2}}(s + c\epsilon) & \frac{1}{\sqrt{2}}(c - s\epsilon) & \frac{1}{\sqrt{2}} \\
\frac{1}{\sqrt{2}}(s - c\epsilon) & -\frac{1}{\sqrt{2}}(c + s\epsilon) & \frac{1}{\sqrt{2}}
\end{pmatrix}, \tag{13}
\]

where $\epsilon$ is small and $s = \sin \theta$, $c = \cos \theta$, with $\theta$ the mixing angle of solar neutrinos. The SM solution corresponds to $s \simeq 0$, while the LM and especially the VO solutions correspond to $s \simeq 1/\sqrt{2}$ [21], that is bimaximal mixing [22].

We set $D_L = \text{diag}(m_1, m_2, m_3)$. Since the mixing in the charged lepton sector can be considered small [3] and our experimental informations on neutrinos are approximate, for our analysis we can also set $U^\dagger M_L U^* = D_L$ (exact in the basis where $M_l$ is diagonal), that is

\[
M_L = UD_LU^T, \tag{14}
\]

which gives the light neutrino mass matrix [20], valid up to small corrections of the order $\epsilon^2 \lesssim 0.03$,

\[
M_L = \begin{pmatrix}
\mu & \delta & \delta' \\
\delta & \rho & \sigma \\
\delta' & \sigma & \rho'
\end{pmatrix}, \tag{15}
\]

with

\[
\mu = m_1 c^2 + m_2 s^2 \\
\mu' = m_1 s^2 + m_2 c^2 \\
\delta = \frac{1}{\sqrt{2}}[\epsilon(m_3 - \mu) + (m_2 - m_1)cs] \\
\delta' = \frac{1}{\sqrt{2}}[\epsilon(m_3 - \mu) - (m_2 - m_1)cs] \\
\sigma = \frac{1}{2}(m_3 - \mu')
\]
\[
\begin{align*}
\rho &= \frac{1}{2}[m_3 + \mu' - 2(m_2 - m_1)cs\epsilon] \\
\rho' &= \frac{1}{2}[m_3 + \mu' + 2(m_2 - m_1)cs\epsilon].
\end{align*}
\]

The inverse of \( M_L \) is given by

\[
M_L^{-1} = \begin{pmatrix}
\rho\rho' - \sigma^2 & \sigma\delta' - \delta\rho' & \delta\sigma - \rho\delta' \\
\sigma\delta' - \delta\rho' & \mu\rho' - \delta'^2 & \delta\delta' - \mu\sigma \\
\delta\sigma - \rho\delta' & \delta\delta' - \mu\sigma & \mu\rho - \delta^2
\end{pmatrix} \frac{1}{D},
\]

with \( D = m_1 m_2 m_3 \).

In the following sections we will study the matrix \( M_R \) which is obtained from eqns.\((14),(3),(2)\) by the first two possible neutrino spectra \((10),(11)\) and \( s \approx 0 \) (single maximal mixing) or \( s \approx 1/\sqrt{2} \) (double maximal mixing). We do not consider the nearly degenerate spectrum because it suffers from a number of instabilities \[20\]. Notice that one of the advantages of such spectrum was the possibility of providing a hot dark matter component (with \( m_i \approx 2 \text{ eV} \)), but now we believe that the amount of hot dark matter is probably much smaller, and one neutrino with mass about 0.07 eV, as in the hierarchical case, can be relevant \[23\]. In any case, if one assumes a hierarchical \( M_D \) it is quite difficult to make \( M_L \) having degenerate eigenvalues. Nevertheless, we give here a rough evaluation for the scale of \( M_R \) at the intermediate value \( 10^{12} \text{ GeV} \).

In this paper we do not consider the results of the LSND experiment \[24\], which have not yet been confirmed by other experiments. If confirmed the LSND results would imply a third \( \Delta m^2 \) scale, \( \Delta m^2_{LSND} \approx 1 \text{ eV}^2 \), and thus a fourth (light and sterile) neutrino.
III. HIERARCHICAL SPECTRUM

In this case the light neutrino mass matrix can be written as

$$M_L = \begin{pmatrix} \mu & \delta & \delta' \\ \delta & \frac{m_3}{2} & \frac{m_3}{2} \\ \delta' & \frac{m_3}{2} & \frac{m_3}{2} \end{pmatrix},$$

(17)

with

$$\mu = m_1 c^2 + m_2 s^2$$

$$\delta = \frac{1}{\sqrt{2}} [\epsilon m_3 + (m_2 - m_1) c s]$$

$$\delta' = \frac{1}{\sqrt{2}} [\epsilon m_3 - (m_2 - m_1) c s].$$

The leading form is

$$M_L \sim \begin{pmatrix} 0 & 0 & 0 \\ 0 & 1 & 1 \\ 0 & 1 & 1 \end{pmatrix}.$$

The inverse of $M_L$ is given by

$$M_L^{-1} \simeq \begin{pmatrix} \frac{m_3 \mu'}{2} & \frac{m_3}{2} (\delta' - \delta) & \frac{m_3}{2} (\delta - \delta') \\ \frac{m_3}{2} (\delta' - \delta) & \frac{m_3}{2} (\mu - \delta'^2) & \delta \delta' - \frac{m_3}{2} \mu \\ \frac{m_3}{2} (\delta - \delta') & \delta \delta' - \frac{m_3}{2} \mu & \frac{m_3}{2} (\mu - \delta'^2) \end{pmatrix} \frac{1}{D},$$

(18)

where for the entry 1-1 we have used a better degree of approximation from eqn.(16) than that obtained from eqn.(17). Due to the mass hierarchy (10) we also have $m_3^2 \simeq \Delta m_{atm}^2$, for example we can take $m_3 = 6 \cdot 10^{-2}$ eV. It will be useful to match results obtained for the scale of $M_R$ with the one obtained when $M_L = D_L$, that is

$$M_{R33} \sim \frac{k^2 m_3^2}{m_3^2}.$$

Within the paper we assume that the largest Yukawa coefficient in $Y_R$ is of order 1, as indeed it happens for the up quark Yukawa coefficients.
A. Single maximal mixing

If \( s \simeq 0 \), then \( \delta = \delta' = (1/\sqrt{2})\epsilon m_3 \), so that

\[
M_L^{-1} \simeq \begin{pmatrix}
  m_3 m_2 & 0 & 0 \\
  0 & x & -x \\
  0 & -x & x
\end{pmatrix} \frac{1}{D},
\]

with \( x = m_3(m_1 - \epsilon^2 m_3)/2 \) and hence

\[
M_{R33} \sim \frac{1}{2} \frac{m_1 - \epsilon^2 m_3}{m_1 m_2} k^2 m_t^2.
\]

If \( \epsilon^2 m_3 \ll m_1 \), then

\[
M_{R33} \sim \frac{1}{2} \frac{k^2 m_t^2}{m_2}.
\]

Since \( s \simeq 0 \) corresponds to the SM solution, one has \( m_2 \lesssim 10^{-3} \) eV and \( M_{R33} \gtrsim 10^{15} \) GeV. The scale can be lowered for \( m_1 \simeq \epsilon^2 m_3 \). If this cancellation does not occur, the structure of \( M_R \) is hierarchical, with the leading form

\[
M_R \sim \text{diag}(0, 0, 1),
\]

which is the same as that obtained for \( M_D \) (see eqn.(3)).

B. Double maximal mixing

For \( s \simeq 1/\sqrt{2} \) we have three subcases: \( |m_2| \gg |m_1|, m_2 \simeq m_1 \) and \( m_2 \simeq -m_1 \).

1. We consider now the case with \( |m_2| \gg |m_1| \), where we have

\[
\delta = \frac{1}{\sqrt{2}} (\epsilon m_3 + m_2/2)
\]

\[
\delta' = \frac{1}{\sqrt{2}} (\epsilon m_3 - m_2/2)
\]

and \( \mu = m_2/2 \). If \( 2\epsilon m_3 \ll |m_2| \), then \( \delta = m_2/2\sqrt{2} = -\delta' \) and
\[
M_L^{-1} \simeq \begin{pmatrix}
\frac{m_3 m_2}{2} & -\frac{m_3 m_2}{2 \sqrt{2}} & \frac{m_3 m_2}{2 \sqrt{2}} \\
-\frac{m_3 m_2}{2 \sqrt{2}} & \frac{m_3 m_2}{4} & -\frac{m_3 m_2}{4} \\
\frac{m_3 m_2}{2 \sqrt{2}} & -\frac{m_3 m_2}{4} & \frac{m_3 m_2}{4}
\end{pmatrix} \frac{1}{D}.
\]

The scale of \( M_R \) is given by \[17\]
\[
M_{R33} \sim \frac{1}{4} k^2 \frac{m^2}{m_1},
\]
and \( M_{R33} \gtrsim 10^{16} \text{ GeV (LM)} \) or \( M_{R33} \gtrsim 10^{18} \text{ GeV (VO)} \). If \( \delta \simeq 0 \) or \( \delta' \simeq 0 \) results are similar. We have a hierarchical structure for \( M_R \), reflecting the hierarchy of Dirac masses. The leading form is again eqn.\(22\).

2. If \( m_2 \simeq m_1 \), then \( \delta = \delta' = (1/\sqrt{2}) \epsilon m_3 \) and \( \mu = m_2 \) yielding

\[
M_L^{-1} \simeq \begin{pmatrix}
m_3 m_2 & 0 & 0 \\
0 & y & -y \\
0 & -y & y
\end{pmatrix} \frac{1}{D},
\]

with \( y = m_3 (m_2 - \epsilon^2 m_3)/2 \) and

\[
M_{R33} \sim \frac{1}{2} \frac{m_2 - \epsilon^2 m_3}{m_2^2} k^2 m_1^2.
\]

If \( \epsilon^2 m_3 \ll m_2 \), then \[17\]
\[
M_{R33} \sim \frac{1}{2} \frac{k^2 m_1^2}{m_2}
\]
and \( M_{R33} \gtrsim 10^{15} \text{ GeV (LM and VO)} \). The scale can be lowered if \( m_2 \simeq \epsilon^2 m_3 \). If the cancellation does not occur, \( M_R \) is hierarchical with the leading form \(22\).

3. For \( m_2 \simeq -m_1 \) we have

\[
\delta = \frac{1}{\sqrt{2}} (\epsilon m_3 + m_2)
\]
\[
\delta' = \frac{1}{\sqrt{2}} (\epsilon m_3 - m_2)
\]
and \( \mu \simeq 0 \). Assuming \( \epsilon m_3 \ll |m_2| \), one has \( \delta = m_2/\sqrt{2} = -\delta' \) and
\[
M_L^{-1} \simeq \begin{pmatrix}
0 & -\sqrt{2}m_3m_2 & \sqrt{2}m_3m_2 \\
-\sqrt{2}m_3m_2 & -m_2^2/2 & -m_2^2/2 \\
\sqrt{2}m_3m_2 & -m_2^2/2 & -m_2^2/2 \\
\end{pmatrix} \frac{1}{D},
\]

(28)

\[
M_{R33} \sim \frac{1}{2} \frac{k^2m_1^2}{m_3},
\]

(29)

\[
M_{R13} \sim \sqrt{2} \frac{k^2m_4m_t}{m_2}.
\]

(30)

For \(m_2/m_3 \sim m_u/m_t\), \(M_{R33}\) and \(M_{R13}\) are similar and near the unification scale. Otherwise \(M_R\) is hierarchical. An interesting case is \(\delta \simeq 0\), which is possible if \(m_2 < 0\), when \(\delta' = -\sqrt{2}m_2\) and

\[
M_L^{-1} \simeq \begin{pmatrix}
0 & -\frac{m_3m_2}{\sqrt{2}} & \frac{m_3m_2}{\sqrt{2}} \\
-\frac{m_3m_2}{\sqrt{2}} & 2\epsilon m_3m_2 & 0 \\
\frac{m_3m_2}{\sqrt{2}} & 0 & 0 \\
\end{pmatrix} \frac{1}{D},
\]

(31)

so that the scale is given by

\[
M_{R13} \sim \frac{1}{\sqrt{2}} \frac{k^2m_4m_t}{m_2},
\]

(32)

that is intermediate. In fact \(m_2 \lesssim 10^{-3} \text{ eV}\) gives \(M_{R33} \gtrsim 10^{11} \text{ GeV}\). In this special case the structure of \(M_R\) is roughly off-diagonal with the leading form

\[
M_R \sim \begin{pmatrix}
0 & 0 & 1 \\
0 & 0 & 0 \\
1 & 0 & 0
\end{pmatrix},
\]

(33)

which was obtained for example in refs. [25,26]. If \(\delta' \simeq 0\), \(\delta = \sqrt{2}m_2\) and

\[
M_L^{-1} \simeq \begin{pmatrix}
0 & -\frac{m_3m_2}{\sqrt{2}} & \frac{m_3m_2}{\sqrt{2}} \\
-\frac{m_3m_2}{\sqrt{2}} & 0 & 0 \\
\frac{m_3m_2}{\sqrt{2}} & 0 & 2\epsilon m_3m_2 \\
\end{pmatrix} \frac{1}{D},
\]

(34)

with \(M_{R33} \sim \epsilon k^2m_t^2/m_2\), \(M_{R13} \sim k^2m_4m_t/m_2\), near the unification scale.
IV. INVERTED HIERARCHY

In this case the light neutrino mass matrix is

\[
M_L = \begin{pmatrix}
\mu & \delta & \delta' \\
\delta & \frac{\mu'}{2} & -\frac{\mu'}{2} \\
\delta' & -\frac{\mu'}{2} & \frac{\mu'}{2}
\end{pmatrix},
\tag{35}
\]

with

\[
\mu = m_1 c^2 + m_2 s^2 \\
\mu' = m_1 s^2 + m_2 c^2 \\
\delta = -\frac{1}{\sqrt{2}}[\epsilon \mu - (m_2 - m_1)cs] \\
\delta' = -\frac{1}{\sqrt{2}}[\epsilon \mu + (m_2 - m_1)cs]
\]

\[
M_L^{-1} \simeq \begin{pmatrix}
m_{3\mu'} & -(\delta + \delta')\frac{\mu'}{2} & -(\delta + \delta')\frac{\mu'}{2} \\
-(\delta + \delta')\frac{\mu'}{2} & \frac{\mu'}{2} - \delta^2 & \frac{\mu'}{2} + \delta \delta' \\
-(\delta + \delta')\frac{\mu'}{2} & \frac{\mu'}{2} + \delta \delta' & \frac{\mu'}{2} - \delta^2
\end{pmatrix} \frac{1}{D},
\tag{36}
\]

and \(m_{1,2}^2 \simeq \Delta m_{atm}^2\). The lightest neutrino mass \(m_3\) does not depend on the solar neutrino solution, and can be arbitrarily small.

A. Single maximal mixing

If \(s \simeq 0\), then \(\mu = m_1, \mu' = m_2, \delta = -(1/\sqrt{2})\epsilon m_1 = \delta'\), the leading \(M_L\) is given by

\[
M_L \simeq \begin{pmatrix}
1 & 0 & 0 \\
0 & 1 & 1 \\
0 & 1 & 1
\end{pmatrix}
\]

and the inverse of \(M_L\) by
\[
M_L^{-1} \sim \begin{pmatrix}
    m_3 m_2 & \epsilon \frac{m_1 m_2}{\sqrt{2}} & \epsilon \frac{m_1 m_2}{\sqrt{2}} \\
    \epsilon \frac{m_1 m_2}{\sqrt{2}} & \frac{m_1 m_2}{2} & \frac{m_1 m_2}{2} \\
    \epsilon \frac{m_1 m_2}{\sqrt{2}} & \frac{m_1 m_2}{2} & \frac{m_1 m_2}{2}
\end{pmatrix} \frac{1}{D}
\]  

(37)

so that

\[
M_{R33} \sim \frac{1}{2} \frac{k^2 m_l^2}{m_3},
\]

(38)

which is at or above the unification scale. The structure of \(M_R\) is hierarchical, with the leading form (22).

**B. Double maximal mixing**

For \(s \simeq 1/\sqrt{2}\) we have two cases, corresponding to \(m_2 \simeq m_1\) and \(m_2 \simeq -m_1\).

If \(m_2 \simeq m_1\), then \(\mu = m_{1,2} = \mu', \delta = -(1/\sqrt{2})\epsilon m_{1,2} = \delta'\), the leading \(M_L\) is like for \(s \simeq 0\) and

\[
M_L^{-1} \sim \begin{pmatrix}
    m_3 m_{1,2} & \epsilon \frac{m_{1,2}^2}{\sqrt{2}} & \epsilon \frac{m_{1,2}^2}{\sqrt{2}} \\
    \epsilon \frac{m_{1,2}^2}{\sqrt{2}} & \frac{m_{1,2}^2}{2} & \frac{m_{1,2}^2}{2} \\
    \epsilon \frac{m_{1,2}^2}{\sqrt{2}} & \frac{m_{1,2}^2}{2} & \frac{m_{1,2}^2}{2}
\end{pmatrix} \frac{1}{D}
\]

(39)

with the same result as for \(s \simeq 0\).

If \(m_2 \simeq -m_1\), then \(\mu = \mu' = 0, \delta = (1/\sqrt{2})m_{1,2} = -\delta'\), the leading \(M_L\) is

\[
M_L \sim \begin{pmatrix}
0 & 1 & 1 \\
1 & 0 & 0 \\
1 & 0 & 0
\end{pmatrix}
\]

and the inverse is

\[
M_L^{-1} \sim \begin{pmatrix}
0 & 0 & 0 \\
0 & -\frac{m_{1,2}^2}{2} & -\frac{m_{1,2}^2}{2} \\
0 & -\frac{m_{1,2}^2}{2} & -\frac{m_{1,2}^2}{2}
\end{pmatrix} \frac{1}{D}
\]

(40)
with

\[ M_{R33} \sim 2 \frac{k^2 m_1^2}{m_3}, \quad (41) \]

similar to above, and with a hierarchical \( M_R \), of the leading form \( (22) \).

**V. EFFECT OF PHASES**

In the preceding sections we have considered only real matrices, which is a CP
conserving framework. The signs of \( m_i \) correspond to CP parities of neutrinos,
while the physical masses are \( |m_i| \) \[27\]. Let us now write a more general form of \( M_L \)
\[ 28 \], namely the same as eqn.\((14)\) but with \( U \) parametrized as the ordinary CKM
matrix (with the CP violating phase \( \delta \)) and

\[ D_L = \text{diag}(m_1 e^{i\alpha}, m_2 e^{i\beta}, m_3) \quad (42) \]

with \( m_i > 0 \) \[27\]. We see that the preceding formalism transforms according to

\[ m_1 \rightarrow m_1 e^{i\alpha}, \quad m_2 \rightarrow m_2 e^{i\beta}, \quad \epsilon \rightarrow \epsilon e^{i\delta}. \]

Moreover, in the hierarchical case \( \epsilon \) (or \( \epsilon^2 \)) is often joined to \( m_3 \), while in the inverted
hierarchy case it is joined to \( m_{1,2} \). It is clear that if there is no fine tuning of
the parameters \( m_i, \epsilon \), phases have a minor effect. However, we have found some
important cases where cancellations occur, indicating also small (that is about 0)
or large (that is about \( \pi \)) phase differences. For example eqn.\((31)\) may be obtained
for \( \alpha \simeq 0, \delta \simeq 0, \beta \simeq \pi \). It is to remember that only the phase \( \delta \) affects neutrino
oscillations (see \( \epsilon \) in eqn.\((13)\)), while all three phases appear in the neutrinoless
double-beta decay parameter \( M_{ee} = |U_{ei}^2 m_i| \). If \( \epsilon \simeq 0 \) and \( |m_2| \simeq |m_1| \), a large
phase difference \( \alpha - \beta \simeq \pi \) gives a much smaller \( M_{ee} \) with respect to a small phase
difference \( \alpha - \beta \simeq 0 \).
VI. CONCLUDING REMARKS

We have found two leading forms for $M_R$, namely eqn.(22) ($M_R$ diagonal) and eqn.(33) ($M_R$ off-diagonal). The diagonal form generally is around the unification scale (except for the case of VO with full hierarchy, where the scale goes well above the unification scale, towards the Planck scale [14]), while the off-diagonal form is at the intermediate scale and hence welcome in the nonsupersymmetric model. Moreover, the off-diagonal form is obtained for a particular pattern for the signs of the light neutrino masses, namely $m_2$ opposite to both $m_1$ and $m_3$, with nearly bimaximal mixing. Of course, this pattern gives a smaller $M_{ee}$ with respect to the pattern with all $m_i$ of the same sign.

From the point of view of effective parameters, the off-diagonal form seems related to some suitable cancellations, but all of them lead to the smallness of entry $M_{R33}$ and hence point towards a different underlying theory with respect to the diagonal form, where the largest element is just $M_{R33}$. With regard to this, we would like to refer, for example, to the model [26], where a suitable pattern of horizontal $U(1)$ charges gives

$$M_R \simeq \begin{pmatrix} 0 & \sigma^2 & 1 \\ \sigma^2 & \sigma^2 & 0 \\ 1 & 0 & 0 \end{pmatrix} M_0,$$

with $\sigma = (m_c/m_t)^{1/2}$ and $M_0 \sim 10^{12}$ GeV.

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[1] Y. Fukuda et al. (SuperKamiokande Collaboration), Phys. Rev. Lett. 81 (1998) 1562

[2] R. Davis, D. Harmer and K. Hoffman, Phys. Rev. Lett. 20 (1968) 1205

J.N. Bahcall, N.A. Bahcall and G. Shaviv, Phys. Rev. Lett. 20 (1968) 1209

V. Gribov and B. Pontecorvo, Phys. Lett. B 28 (1969) 493

[3] J.N. Bahcall, P.I. Krastev and A.Yu. Smirnov, Phys. Rev. D 58 (1998) 096016; 60 (1999) 093001

[4] V. Barger, T.J. Weiler and K. Whisnant, Phys. Lett. B 442 (1999) 255

[5] M. Gell-Mann, P. Ramond and R. Slansky, in Supergravity, eds. P. van Nieuwenhuizen and D. Freedman (North Holland, Amsterdam, 1979)

T. Yanagida, in Proceedings of the Workshop on Unified Theory and Baryon Number in the Universe, eds. O. Sawada and A. Sugamoto (KEK, 1979)

S. Weinberg, Phys. Rev. Lett. 43 (1979) 1566

R.N. Mohapatra and G. Senjanovic, Phys. Rev. Lett. 44 (1980) 912

[6] R.N. Mohapatra and P.B. Pal, Massive Neutrinos in Physics and Astrophysics (World Scientific, Singapore, 1991)

[7] N.G. Deshpande and E. Keith, Phys. Rev. D 50 (1994) 3513

[8] P. Ramond, R.G. Roberts and G.G. Ross, Nucl. Phys. B 406 (1993) 19

[9] H. Arason, D.J. Castano, E.J. Piard and P. Ramond, Phys. Rev. D 47 (1993) 232

[10] H. Fusaoka and Y. Koide, Phys. Rev. D 57 (1998) 3986

[11] A. Yu. Smirnov, Nucl. Phys. B 466 (1996) 25

[12] G.K. Leontaris, S. Lola, C. Scheich and J.D. Vergados, Phys. Rev. D 53 (1996) 6381
[13] G. Altarelli and F. Feruglio, Phys. Lett. B 439 (1998) 112

[14] D. Falcone, hep-ph/9909207 (Phys. Rev. D, to be published)

[15] E.Kh. Akhmedov, G.C. Branco and M.N. Rebelo, hep-ph/9911364

[16] T.K. Kuo, G.-H. Wu and S.W. Mansour, hep-ph/9912366

[17] D. Falcone, hep-ph/9912491 (Phys. Lett. B, to be published)

[18] G. Altarelli and F. Feruglio, Phys. Rep. 320 (1999) 295

[19] M. Apollonio et al. (CHOOZ Collaboration), Phys. Lett B 420 (1998) 397

[20] E.Kh. Akhmedov, Phys. Lett B 467 (1999) 95

[21] S.T. Petcov, hep-ph/9907216

[22] F. Vissani, hep-ph/9708483

V. Barger, S. Pakvasa, T.J. Weiler and K. Whisnant, Phys. Lett. B 437 (1998) 107
A.T. Baltz, A.S. Goldhaber and M. Goldhaber, Phys. Rev. Lett. 81 (1998) 5730

[23] G. Gelmini, hep-ph/9904369

[24] C. Athanassopoulos et al. (LSND Collaboration), Phys. Rev. Lett. 75 (1995) 2650; 77 (1996) 3082; 81 (1998) 1774

[25] M. Jezabek and Y. Sumino, Phys. Lett. B 440 (1998) 327

[26] B. Stech, Phys. Lett. B 465 (1999) 219

[27] S.M. Bilenky, C. Giunti and W. Grimus, Prog. Part. Nucl. Phys. 43 (1999) 1

[28] See for example H. Georgi and S.L. Glashow, hep-ph/9808293