\( O(\alpha_s) \) Corrections to Longitudinal Spin-Spin Correlations in \( e^+e^- \rightarrow q\bar{q} \)

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Abstract

We calculate the \( O(\alpha_s) \) corrections to longitudinal spin-spin correlations in \( e^+e^- \rightarrow q\bar{q} \). For top quark pair production the \( O(\alpha_s) \) corrections to the longitudinal spin-spin asymmetry amount to less than 1\% in the \( q^2 \)-range from above \( t\bar{t} \)-threshold up to \( \sqrt{q^2} = 1000 \) GeV. In the \( e^+e^- \rightarrow b\bar{b} \) case the \( O(\alpha_s) \) corrections reduce the asymmetry value from its \( m = 0 \) value of \(-1\) to approximately \(-0.96\) for \( q^2 \)-values around the \( Z \)-peak. This reduction can be traced to finite anomalous contributions from residual mass effects which survive the \( m \rightarrow 0 \) limit. We discuss the role of the anomalous contributions and the pattern of how they contribute to spin-flip and no-flip terms.

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Recently there has been renewed interest in the role of quark mass effects in the production of quarks and gluons in $e^+e^-$ annihilations. Jet definition schemes, event shape variables and heavy flavour momentum correlations are affected by the presence of quark masses for charm and bottom quarks even when they are produced at the scale of the $Z$-mass. A careful investigation of quark mass effects in $e^+e^-$ annihilations may even lead to an alternative determination of the quark mass values. There is obvious interest in quark mass effects for $t\bar{t}$ production where quark mass effects cannot be neglected in the envisaged range of energies to be covered by the Next Linear Collider (NLC). In the calculation of radiative corrections to quark polarization variables residual mass effects change the naive $m = 0$ pattern of polarization results that results from the absence of spin-flip contributions in the $m \to 0$ case.

In this report we calculate the $O(\alpha_s)$ radiative corrections to longitudinal spin-spin correlations of massive quark pairs produced in $e^+e^-$ annihilations. The longitudinal polarization of massive quarks affects the shape of the energy spectrum of their secondary decay leptons. Thus longitudinal spin-spin correlation effects in pair produced quarks and antiquarks will lead to correlation effects of the energy spectra of their secondary decay leptons and antileptons. As a byproduct of our calculation we discuss the $m \to 0$ limit and the role of the $O(\alpha_s)$ residual mass effects. We delineate how residual mass effects contribute to the various spin-flip and no-flip terms in the $m \to 0$ limit.

Let us begin with by defining the differential joint quark-antiquark density matrix $d\sigma = d\sigma_{\lambda_1 \lambda_2; \lambda_1' \lambda_2'}$ where $\lambda_1$ and $\lambda_2$ denote the helicities of the quark and antiquark, respectively. In this paper our main interest is in the longitudinal polarization of the quark and antiquark, and in particular, in their longitudinal spin-spin correlations. Thus we specify to the diagonal case $\lambda_1 = \lambda_1'$ and $\lambda_2 = \lambda_2'$.

The diagonal part of the differential joint density matrix can be represented in terms of its components along the products of the unit matrix and the $z$-components of the Pauli matrix $\sigma_3$ ($\sigma_3 = \vec{p}_1 \cdot \vec{p}$ for the quark and $\sigma_3 = \vec{p}_2 \cdot \vec{p}$ for the antiquark, $\hat{p}_i = \vec{p}_i/|\vec{p}_i|$). One has

$$d\sigma = \frac{1}{4} \left( d\sigma \mathbb{1} \otimes \mathbb{1} + d\sigma^{(\ell_1)} \sigma_3 \otimes \mathbb{1} + d\sigma^{(\ell_2)} \mathbb{1} \otimes \sigma_3 + d\sigma^{(\ell_1 \ell_2)} \sigma_3 \otimes \sigma_3 \right).$$

An alternative but equivalent representation of the longitudinal spin contributions can be written down in terms of the longitudinal spin components $s_1^\ell, s_2^\ell = \pm 1$ (or $s_1^\ell, s_2^\ell \in \{\uparrow, \downarrow\}$). One has

$$d\sigma(s_1^\ell, s_2^\ell) = \frac{1}{4} \left( d\sigma + d\sigma^{(\ell_1)} s_1^\ell + d\sigma^{(\ell_2)} s_2^\ell + d\sigma^{(\ell_1 \ell_2)} s_1^\ell s_2^\ell \right).$$

From CP invariance one knows that $d\sigma$ and $d\sigma^{(\ell_1 \ell_2)}$ obtain contributions from the parity-even $VV$- and $AA$-current products, whereas $d\sigma^{(\ell_1)}$ and $d\sigma^{(\ell_2)}$ are contributed to by the parity-odd $VA$ and $AV$-current products. The parity-even terms $d\sigma$ and $d\sigma^{(\ell_1 \ell_2)}$ are $C$-even and thus symmetric under $q \leftrightarrow \bar{q}$ exchange, whereas the parity-odd terms are $C$-odd and thus one has $d\sigma^{(\ell_1)}(p_1, p_2) = -d\sigma^{(\ell_2)}(p_2, p_1)$ for the single-spin dependent contributions.
Eq. (2) is easily inverted. One has

\[
\begin{align*}
\sigma & = \sigma(\uparrow \uparrow) + \sigma(\uparrow \downarrow) + \sigma(\downarrow \uparrow) + \sigma(\downarrow \downarrow) \\
\sigma^{(e_1)} & = \sigma(\uparrow \uparrow) - \sigma(\uparrow \downarrow) - \sigma(\downarrow \uparrow) - \sigma(\downarrow \downarrow) \\
\sigma^{(e_2)} & = \sigma(\uparrow \uparrow) - \sigma(\uparrow \downarrow) - \sigma(\downarrow \uparrow) + \sigma(\downarrow \downarrow) \\
\sigma^{(e_1 e_2)} & = \sigma(\uparrow \uparrow) - \sigma(\uparrow \downarrow) - \sigma(\downarrow \uparrow) + \sigma(\downarrow \downarrow).
\end{align*}
\] (3)

\(O(\alpha_s)\) radiative corrections to the rate component \(d\sigma\) have been discussed before \[\text{[3, 4]}\] including beam polarization effects \[\text{[3]}\] and beam-event correlation effects \[\text{[3, 7]}\]. The \(O(\alpha_s)\) radiative corrections to the longitudinal spin component \(d\sigma^{(e_1)}\) have been recently calculated \[\text{[4]}\] including also beam polarization and beam-event correlation effects \[\text{[3]}\]. As concerns the longitudinal spin-spin correlation component \(d\sigma^{(e_1 e_2)}\) the \(O(\alpha_s)\) tree graph contributions have been determined in \[\text{[10]}\]. Here we calculate the \(O(\alpha_s)\) radiative corrections to the fully integrated spin-spin correlation component \(\sigma^{(e_1 e_2)}\) where we average out beam-event correlation effects.

As before we write the electro-weak cross section \(e^+e^- \rightarrow q(p_1)\bar{q}(p_2)\) and \(e^+e^- \rightarrow q(p_1)\bar{q}(p_2)g(p_3)\) in modular form in terms of two building blocks \[\text{[3]}\]. Thus we write (beam polarization effects not included and beam-event correlations averaged out)

\[
\begin{align*}
\sigma(s_1, s_2) = & \frac{1}{4}(g_{11}(\sigma_{11} + \sigma_{12}^{(e_1 e_2)}s_1 s_2) + g_{12}(\sigma_{21} + \sigma_{22}^{(e_1 e_2)}s_1 s_2) \\
& + g_{14}(\sigma_{41}^{(e_1)} s_1 + \sigma_{42}^{(e_2)} s_2)).
\end{align*}
\] (4)

The index \(i = 1, 2, 4\) on the rate components is explained later on.

The first building block \(g_{ij}\) \((i, j = 1, 2, 4)\) specifies the electro-weak model dependence of the \(e^+e^-\) cross section. For the present discussion we need the components \(g_{11}, g_{12}, g_{14}, g_{41}, g_{42}\) and \(g_{44}\). They are given by

\[
\begin{align*}
g_{11} & = Q_f^2 - 2Q_f v_e v_f Re\chi_x + (v_e^2 + a_e^2)(v_f^2 + a_f^2)|\chi_x|^2, \\
g_{12} & = Q_f^2 - 2Q_f v_e v_f Re\chi_x + (v_e^2 + a_e^2)(v_f^2 - a_f^2)|\chi_x|^2, \\
g_{14} & = 2Q_f v_e a_f Re\chi_x - 2(v_e^2 + a_e^2)v_f a_f |\chi_x|^2, \\
g_{41} & = 2Q_f a_e v_f Re\chi_x - 2v_e a_e (v_f^2 + a_f^2)|\chi_x|^2, \\
g_{42} & = 2Q_f a_e v_f Re\chi_x - 2v_e a_e (v_f^2 - a_f^2)|\chi_x|^2, \\
g_{44} & = -2Q_f a_e a_f Re\chi_x + 4v_e a_e v_f a_f |\chi_x|^2.
\end{align*}
\] (5)

where, in the Standard Model, \(\chi_x(q^2) = gM_Z^2 q^2/(q^2 - M_Z^2 + iM_Z \Gamma_Z)^{-1}\), with \(M_Z\) and \(\Gamma_Z\) the mass and width of the \(Z^0\) and \(g = G_F(8\sqrt{2}\pi\alpha)^{-1} \approx 4.49 \cdot 10^{-5}\) GeV\(^{-2}\). \(Q_f\) are the charges of the final state quarks to which the electro-weak currents directly couple; \(v_e\) and \(a_e\), \(v_f\) and \(a_f\) are the electro-weak vector and axial vector coupling constants. For example, in the Weinberg-Salam model, one has \(v_e = -1 + 4\sin^2\theta_W\), \(a_e = -1\) for leptons, \(v_f = 1 - \frac{8}{3}\sin^2\theta_W\), \(a_f = 1\) for up-type quarks \((Q_f = \frac{2}{3})\), and \(v_f = -1 + \frac{4}{3}\sin^2\theta_W\), \(a_f = -1\) for down-type quarks \((Q_f = \frac{1}{3})\).
for down-type quarks ($Q_f = -\frac{1}{3}$). In this paper we use Standard Model couplings with $\sin^2 \theta_W = 0.226$.

The second building block is determined by the hadron dynamics, i.e. by the current-induced production of a quark-antiquark pair which, in the $O(\alpha_s)$ case, is followed by gluon emission. In the $O(\alpha_s)$ case one also has to add the one loop-contribution. We shall work in terms of unpolarized and polarized hadron tensor components $H_{U+L}$, $H_{U+L}^{(\ell_1)}$, $H_{U+L}^{(\ell_2)}$ and $H_{U+L}^{(\ell_1,\ell_2)}$ where the spin decomposition is defined in complete analogy to Eq. (2). In the two body case $e^+e^- \rightarrow q\bar{q}$ the unpolarized rate components are given by

$$\sigma = \frac{\pi \alpha^2 v}{3q^4} H_{U+L}^i.$$  \hfill (6)

In the three-body case $e^+e^- \rightarrow q\bar{q}g$ the unpolarized differential rate components and the unpolarized hadron tensor components $H_{U+L}^i$ are related by

$$\frac{d\sigma}{dydz} = \frac{\alpha}{48\pi q^2} H_{U+L}^i(y,z) \quad (i = 1, 2).$$  \hfill (7)

As kinematic variables we use the two energy-type variables $y = 1 - 2p_1q/q^2$ and $z = 1 - 2p_2q/q^2$. The same relations hold for polarized production.

The index $i = 1, 2$ in Eqs. (3) and (4) specifies the current composition in terms of the two parity-even products of the vector and the axial vector currents according to (dropping all further indices on the hadron tensor)

$$H_{\mu\nu}^1 = \frac{1}{2}(H_{\mu\nu}^{VV} + H_{\mu\nu}^{AA}) \quad H_{\mu\nu}^2 = \frac{1}{2}(H_{\mu\nu}^{VV} - H_{\mu\nu}^{AA}).$$  \hfill (8)

In the parity-odd case one has

$$H_{\mu\nu}^4 = \frac{1}{2}(H_{\mu\nu}^{VA} + H_{\mu\nu}^{AV}).$$  \hfill (9)

The notation closely follows the one in [4]. Thus the nomenclature $(U + L)$ in Eqs. (4) and (5) denotes the total rate ($U$: unpolarized transverse, $L$: longitudinal) after averaging over the relative beam-event orientation.

The generalization of the above cross section expressions to the case where one starts with longitudinally polarized beams is straightforward and amounts to the replacement

$$g_{14} \rightarrow [(1 - h^-h^+)g_{14} + (h^- - h^+)g_{44}]$$

$$g_{1i} \rightarrow [(1 - h^-h^+)g_{1i} + (h^- - h^+)g_{4i}] \quad (i = 1, 2)$$  \hfill (10)

$$\text{where} \ h^- \text{ and } h^+ \ (-1 \leq h^\pm \leq +1) \text{ denote the longitudinal polarization of the electron and the positron beam.}$$

Let us begin with by listing the Born term contributions to the various polarized and unpolarized two-body hadron tensor components. One has ($\xi = 4m_q^2/q^2$, $v = \sqrt{1-\xi}$)

$$H_{U+L}^{(t_1\ell_2)}(\text{Born}) = (4 - \xi)C_Nq^2, \quad H_{U+L}^{(t_2\ell_2)}(\text{Born}) = 3\xi C_Nq^2,$$

$$H_{U+L}^{(t_1,\ell_2)}(\text{Born}) = -(4 - 3\xi)C_Nq^2, \quad H_{U+L}^{(t_2,\ell_2)}(\text{Born}) = -\xi C_Nq^2,$$

$$H_{U+L}^{\ell_1}(\text{Born}) = 4vC_Nq^2, \quad H_{U+L}^{\ell_2}(\text{Born}) = -4vC_Nq^2.$$  \hfill (11)
The $O(\alpha_s)$ spin dependent hadronic tree-body tensor
\[
H_{\mu\nu}(p_1, p_2, p_3, s_1, s_2) = \sum_{\text{gluon spin}} \langle q\bar{q}g|j_{\mu}|0\rangle\langle 0|j^\dagger_{\nu}|q\bar{q}g \rangle
\] (13)
can easily be calculated from the relevant Feynman diagrams. The $(U+L)$-component is then obtained by contraction with the four-transverse metric tensor $(-g_{\mu\nu} + q_\mu q_\nu/q^2)$. Finally, the longitudinal spin components of the quark and antiquark can be projected out with the help of the respective longitudinal spin vectors. They read
\[
(s^\ell_1)^\mu = \frac{s^\ell_1}{\sqrt{\xi}}(\sqrt{(1-y)^2 - \xi}; 0, 0, 1-y) \quad (14)
\]
\[
(s^\ell_2)^\mu = \frac{s^\ell_2}{\sqrt{\xi}}(\sqrt{(1-z)^2 - \xi}; (1-z)\sin\theta_{12}, 0, (1-z)\cos\theta_{12})
\]
with
\[
\cos\theta_{12} = \frac{yz + y + z - 1 + \xi}{\sqrt{(1-y)^2 - \xi}\sqrt{(1-z)^2 - \xi}}. \quad (15)
\]
The resulting spin-independent and single-spin dependent components of the hadron tensor have been given before (see e.g. [5, 6, 9]). Here we list the spin-spin dependent piece. One has $(v_y := \sqrt{(1-y)^2 - \xi}, v_z := \sqrt{(1-z)^2 - \xi})$
\[
H_{U+L}^{1(\ell_1 \ell_2)}(y, z) = \frac{1}{v_y v_z} \left[ -4(12 - 10\xi + \xi^2) + (1 - \xi)(4 - 3\xi)\xi \left( \frac{1}{y^2} + \frac{1}{z^2} \right) 
+ (4 - 3\xi)(8 - 7\xi) \left( \frac{1}{y} + \frac{1}{z} \right) + 2(12 - 5\xi)(y + z) 
- 2(4 - \xi)(y^2 + z^2) - (4 - 3\xi)\xi \left( \frac{y}{z^2} - \frac{y^2}{z^2} + \frac{z}{y^2} - \frac{z^2}{y^2} \right) 
- 2(1 - \xi)(2 - \xi)(4 - 3\xi) \left( \frac{1}{yz} \right) - (4 - \xi)(6 - 5\xi) \left( \frac{y}{z} + \frac{z}{y} \right) 
+ 2(4 - 5\xi) \left( \frac{y^2}{z} + \frac{z^2}{y} \right) + 4\xi y z \right] \quad (16)
\]
\[
H_{U+L}^{2(\ell_1 \ell_2)}(y, z) = \frac{\xi}{v_y v_z} \left[ -4\xi + (1 - \xi)\xi \left( \frac{1}{y^2} + \frac{1}{z^2} \right) + (8 - 7\xi) \left( \frac{1}{y} + \frac{1}{z} \right) 
- 6(y + z) - 2(y^2 + z^2) - \xi \left( \frac{y}{z^2} - \frac{y^2}{z^2} + \frac{z}{y^2} - \frac{z^2}{y^2} \right) 
- 2(2 - \xi)(1 - \xi) \left( \frac{1}{yz} \right) - (6 + \xi) \left( \frac{y}{z} + \frac{z}{y} \right) + 2 \left( \frac{y^2}{z} + \frac{z^2}{y} \right) - 4yz \right]. \quad (17)
\]
What remains to be done is to perform the phase space integrations and to add in the one-loop contributions. In this calculation we perform the requisite two-fold phase
space integration over the full \((y, z)\) phase space. As in [3, 4] the infrared singularities are regularized by introducing a gluon mass. The infrared singularities in the tree graph and one-loop contributions cancel and one remains with finite remainders. For the sake of completeness we include in our results also the unpolarized hadron tensor components which are needed for the normalization of the longitudinal spin-spin asymmetry. The \(O(\alpha_s)\) corrections (tree plus loop) read

\[
H_{U+L}^1(\alpha_s) = N \left[ \frac{3}{2} (4 - \xi)(2 - \xi)v + \frac{1}{4} (192 - 104\xi - 4\xi^2 + 3\xi^3)t_3 \\
-2(4 - \xi)((2 - \xi)(t_8 - t_9) + 2v(t_{10} + 2t_{12})) \right],
\]

(18)

\[
H_{U+L}^2(\alpha_s) = N \xi \left[ \frac{3}{2} (18 - \xi)v + \frac{3}{4} (24 - 8\xi - \xi^2)t_3 \\
-6((2 - \xi)(t_8 - t_9) + 2v(t_{10} + 2t_{12})) \right],
\]

(19)

\[
H_{U+L}^{1(t_1t_2)}(\alpha_s) = N \left[ \frac{1}{2v} (88 - 78\xi - 5\xi^2 + 3\xi^3) - 40 + 32\sqrt{\xi} - 14\xi + 12\xi\sqrt{\xi} + 3\xi^2 + 3\xi^2\sqrt{\xi} \\
- \left(16 - 42\xi + 31\xi^2 - 4\xi^3 + 8(4 - 3\xi)v^3\right)t_3 \\
+2(4 - 3\xi)((2 - \xi)(t_8 - t_{16}) + 2v(t_{10} + 2t_{12}) - (4 - \xi)(8 - 3\xi - \xi^2)t_{13} \\
-2(8 - 10\xi + \xi^2)t_{14} + (32 - 88\xi + 76\xi^2 - 19\xi^3)t_{15} \right],
\]

(20)

\[
H_{U+L}^{2(t_1t_2)}(\alpha_s) = N \xi \left[ - \frac{1}{2v} (54 - 65\xi + 3\xi^2) - 58 - 56\sqrt{\xi} - 3\xi - 3\xi\sqrt{\xi} \\
+ \left(2 + \xi - 4\xi^2 - 8v^3\right)t_3 \\
+2\left((2 - \xi)(t_8 - t_{16}) + 2v(t_{10} + 2t_{12}) \right) \\
+(96 - 140\xi + 35\xi^2 - 3\xi^3)t_{13} \\
-2(10 + 3\xi)t_{14} + (8 - 20\xi + 13\xi^2)t_{15} \right],
\]

(21)

where we have used an overall normalization factor \(N = \alpha_s N_C C_F q^2 / 4\pi v\). The unpolarized hadron tensor components \(H_{U+L}^1(\alpha_s)\) and \(H_{U+L}^2(\alpha_s)\) including the \(O(\alpha_s)\) rate functions \(t_i\) \((i = 3, 8, 9, 10, 12)\) have been calculated before in [4]. They are listed here for completeness. In addition to the rate functions calculated in [4] the spin-spin contributions bring in a set of new rate functions \(t_i\) \((i = 13, 14, 15, 16)\). The complete set of rate functions needed in the present application is given by

\[
t_3 = \ln \left( \frac{1 + v}{1 - v} \right),
\]

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The mean orientation variables. equivalently the q

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dinal spin-spin correlation functions. In the piece (indicated by square brackets). The normal piece proportional to (1

simplifies and one has (H

We are now in the position to discuss the normalized longitudinal spin-spin correlation function \( \langle P^{\ell\ell} \rangle \) which is defined as

\[
\langle P^{\ell\ell} \rangle = \frac{\sigma(\uparrow\uparrow) - \sigma(\uparrow\downarrow) - \sigma(\downarrow\uparrow) + \sigma(\downarrow\downarrow)}{\sigma(\uparrow\uparrow) + \sigma(\uparrow\downarrow) + \sigma(\downarrow\uparrow) + \sigma(\downarrow\downarrow)}.
\]

The mean \( \langle P^{\ell\ell} \rangle \) is taken with regard to all phase-space variables including the beam-event orientation variables.

Before we present our numerical results we briefly pause to discuss the \( m \to 0 \) limit (or equivalently the \( q^2 \to \infty \) limit) of the various hadron tensor components and the longitudinal spin-spin correlation functions. In the \( m \to 0 \) limit the hadron tensor considerably simplifies and one has \( (H^1_{U+L} \to 0) \)

\[
H^1_{U+L}(s_1^\ell, s_2^\ell) = \frac{1}{4}(H^1_{U+L} + H^1_{U+L} s_1^\ell s_2^\ell)
\]

\[
= N_C q^2 \left( (1 - s_1^\ell s_2^\ell) \left( 1 + \frac{3\alpha_s}{4\pi} C_F \right) + \left[ \frac{4}{3} \cdot \frac{3\alpha_s}{4\pi} C_F s_1^\ell s_2^\ell \right] \right).
\]

By hindsight the \( O(\alpha_s) \) contribution has been split into a normal piece and an anomalous piece (indicated by square brackets). The normal piece proportional to \( (1 - s_1^\ell s_2^\ell) \) is defined
Table 1: $O(\alpha_s)$ corrections to specific spin configurations in QCD ($m = 0$) and QCD ($m \to 0$). Entries are given in terms of contributions to the hadron tensor components $H^{VV}_{LL}(s_1, s_2)$ and $H^{VA}_{LL}(s_1, s_2)$ in units of $3\alpha_s C_F N_C q^2 / 4\pi$. Anomalous contributions are shown in square brackets.

| spin config. | $V V$ | $V A$ |
|-------------|-------|-------|
|             | $m = 0$ | $m \to 0$ | $m = 0$ | $m \to 0$ |
| $(\uparrow \uparrow)$ | 0 | $\frac{1}{3} = 0 + \frac{1}{3}$ | 0 | 0 |
| $(\uparrow \downarrow)$ | 2 | $\frac{2}{3} = 2 - \frac{4}{3}$ | 2 | $\frac{2}{3} = 2 - \frac{4}{3}$ |
| $(\downarrow \uparrow)$ | 2 | $\frac{2}{3} = 2 - \frac{4}{3}$ | $-2$ | $-\frac{2}{3} = -2 + \frac{4}{3}$ |
| $(\downarrow \downarrow)$ | 0 | $\frac{1}{3} = 0 + \frac{1}{3}$ | 0 | 0 |

In terms of the usual massless QCD ($m = 0$) no-flip $O(\alpha_s)$ result whereas the anomalous piece is a residual mass effect which survives the $m \to 0$ limit.

Let us complete our discussion of the $m \to 0$ limit by also listing the corresponding $m \to 0$ limit of the longitudinal single-spin hadron tensor components. One has

$$H^4_{LL}(s_1, s_2) = \frac{1}{4} \left( H^{(\ell_1)}_{LL} s_1 + H^{(\ell_2)}_{LL} s_2 \right)$$

$$= N_C q^2 \left( 1 + \frac{3\alpha_s}{4\pi} C_F - \left[ \frac{2}{3} \cdot \frac{3\alpha_s}{4\pi} C_F \right] \right) (s_1 - s_2).$$

In Table 1 we list all $m \to 0$ contributions to the various spin configurations where again we have split off the $m = 0$ no-flip contributions.

At threshold the production cross section is dominated by the $s$-wave vector current contribution. One thus has $H^1_{LL}(s_1, s_2) = H^1_{LL}(s_1, s_2)$ and $H^1_{LL}(\uparrow \uparrow) = \frac{1}{2} H^1_{LL}(\uparrow \downarrow) = \frac{1}{2} H^1_{LL}(\downarrow \downarrow) = H^1_{LL}(\downarrow \downarrow)$. From Eq. (23) one then obtains a threshold value of $\langle P^{\ell \ell} \rangle = -1/3$.

Let us now present our numerical results. In Fig. 1 we plot the mean longitudinal spin-spin asymmetry $\langle P^{\ell \ell} \rangle$ against the c.m. energy $\sqrt{q^2}$ for top quark pair production. The longitudinal spin-spin asymmetry rises from its threshold value of $\langle P^{\ell \ell} \rangle = -1/3$ to around $\langle P^{\ell \ell} \rangle = -0.9$ at $\sqrt{q^2} = 1000$ GeV. The $O(\alpha_s)$ correction to the asymmetry amounts to less than 1% in the $q^2$-range from above $t\bar{t}$ threshold to $\sqrt{q^2} = 1000$ GeV.

In Fig. 2 we present our results on $\langle P^{\ell \ell} \rangle$ for bottom quark pair production starting from $b\bar{b}$ threshold (where $\langle P^{\ell \ell} \rangle = -1/3$) up to $\sqrt{q^2} = 100$ GeV. For the lower $q^2$-values from threshold to about 30 GeV the $O(\alpha_s)$ corrections are quite small. Starting at around $\sqrt{q^2} = 30$ GeV the $O(\alpha_s)$ correction become larger. The Born term contribution very quickly acquires its asymptotic limiting value $\langle P^{\ell \ell} \rangle = -1$ due to the fact that the corrections to the leading term are quadratic in the ratio $m/\sqrt{q^2}$. Contrary to this the $O(\alpha_s)$ curve
remains below the naive limiting value of $-1$. From the limiting formula

$$\langle P^{\ell\ell} \rangle = -\frac{1 + \frac{\alpha_s}{\pi} - \left[\frac{4\alpha_s}{3\pi}\right]}{1 + \frac{\alpha_s}{\pi}}$$  \hspace{1cm} (27)$$

one concludes that a large part of the deviation is made up by the anomalous contributions. For example, at the position of the $Z$-pole the limiting value of the anomalous contribution to $\langle P^{\ell\ell} \rangle$ amounts to $\langle P^{\ell\ell} \rangle^{(\text{anom})} = 0.048$ ($\alpha_s(m_Z) = 0.118$). From the full calculation one finds $\langle P^{\ell\ell} \rangle^{(\text{Born})} = -0.996$ and $\langle P^{\ell\ell} \rangle^{(\alpha_s)} = -0.964$. Thus the deviation of $\langle P^{\ell\ell} \rangle$ from its naive value of $\langle P^{\ell\ell} \rangle = -1$ can be seen to arise to a large part from the anomalous contribution.

**Note added in proof:** When preparing this manuscript for publication we became aware of a preprint on the same subject by M.M. Tung, J. Bernabéu and J. Peñarrocha [11].

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Figure Captions

Fig. 1: Energy dependence of $O(1)$ and $O(\alpha_s)$ mean longitudinal spin-spin correlations $\langle P^{\ell\ell} \rangle$ in $e^+e^- \to t\bar{t}(g)$

Fig. 2: Energy dependence of $O(1)$ and $O(\alpha_s)$ mean longitudinal spin-spin correlations $\langle P^{\ell\ell} \rangle$ in $e^+e^- \to b\bar{b}(g)$. The vertical line indicates the $b\bar{b}$-threshold
Figure 1

Longitudinal spin–spin correlation

$m_t = 175$ GeV

--- Born result

--- $O(\alpha_s)$ result

Energy $\sqrt{q^2}$ (in GeV)
Fig 2