Four-dimensional $M$-theory and supersymmetry breaking

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Abstract

We investigate compactifications of $M$-theory from $11 \to 5 \to 4$ dimensions and discuss geometrical properties of 4-d moduli fields related to the structure of 5-d theory.

We study supersymmetry breaking by compactification of the fifth dimension and find that an universal superpotential is generated for the axion-dilaton superfield $S$. The resulting theory has a vacuum with $\langle S \rangle = 1$, zero cosmological constant and a gravitino mass depending on the fifth radius as $m_{3/2} \sim R_5^{-2}/M_{Pl}$.

We discuss phenomenological aspects of this scenario, mainly the string unification and the decompactification problem.
1 Introduction

In trying to describe four-dimensional physics from compactified string theories, it has soon appeared [1] that the ten-dimensional string can hardly be weakly coupled, leading to a far too large Newton’s constant. So phenomenological attention must be paid to strongly coupled strings. Recently [2]–[3], many progresses have been made in understanding this new physics: the strongly coupled regime is now viewed as the low energy limit of $M$-theory and in particular, the strongly coupled $E_8 \times E_8$ heterotic string, traditionally considered as the most relevant one for phenomenology, can be described, in the low energy limit, by the eleven-dimensional supergravity with the two $E_8$ gauge factors living each on a 10-d boundary. The radius of the eleventh dimension is related to the string coupling by $R_{11} \sim \lambda_{st}^{2/3}$. So in the strongly coupled regime, $R_{11}$ has to be large, in particular, could be larger than the typical radius of the six other compact dimensions. Therefore, it appears in that case that the eleventh dimension has to be compactified after the Calabi-Yau internal manifold ([4],[5],[7]). Thus describing four-dimensional physics from $E_8 \times E_8$ heterotic strongly coupled string should be equivalent to compactify the eleven-dimensional supergravity on a Calabi-Yau and then compactify the fifth dimension on $S^1/Z_2$. Our goal is to compactify the Lagrangian to 4-d in a way compatible with $N=1$ 4-d supersymmetry. As shown in [3], the presence of the boundaries and the interaction between the boundary fields and the bulk fields make this task difficult, as the 7-d internal space is not really a direct product $Q \times S^1/Z_2$. However, we shall be mainly concerned in compactifying the (bulk) gravitational sector of the theory, where this difficulty do not appear and simply add the kinetic terms for the gauge fields on the boundaries. We ignore all the matter fields in 4-d and their interactions. This is the point of view adopted here and we compare this pattern of compactification with the previous one studied in [8] and which corresponds to $11 \to 10 \to 4$.

Performing an explicit compactification on a CY manifold can be rather difficult, so we adopt an alternative way, by truncating with a symmetry of the compact space such that to maintain $N=1$ supersymmetry in 4-d. Of course, in this case we can describe only the analog of untwisted fields of string theories, originating from 11-d and 10-d fields.

In the section 2 of the paper, we identify, for different projections, the Kähler structure for the moduli fields describing the shape of the internal manifold. We will observe very interesting geometrical properties, namely in all the cases the size of the compactified manifold is contained exclusively
in the dilaton-axion superfield $S$, all the other moduli fields being invariant under dilatations of the 6-d compactified space.

Our main goal, to be studied in section 3, is the $N = 1$ spontaneous supersymmetry breaking in four dimensions by compactification from 5-d to 4-d, by using the Scherk-Schwarz mechanism \[8\]. We argue that we obtain results that look like non-perturbative from the perturbative heterotic string point of view, like an universal superpotential generation for $S$. The corresponding model spontaneously breaks supersymmetry with a zero cosmological constant, has the invariance $S \to 1/S$ and the minimum is reached for $S = 1$.

Section 4 shows that the use of the eleventh (or fifth, after the CY compactification) dimension to break supersymmetry offers a new perspective on the decompactification problem and the unification problem of perturbative string theories. The dependence of the gravitino mass on the fifth radius $R_5$ (seen from 5-d in Einstein units) is

$$m_{3/2} \sim \frac{R_5^{-2}}{M_{Pl}^{(4)}}.$$  \hspace{1cm} (1)

By using this result, we argue that a value of the fifth radius of the order $10^{12} \text{GeV}$ could solve both above-mentioned problems.

We end with some conclusions and prospects.

## 2 Dimensional reduction

The bosonic part of the strongly coupled $E_8 \times E_8$ heterotic string Lagrangian is (we neglect for the moment higher-derivative terms) \[3\]

$$S^{(11)} = \frac{1}{\kappa_{11}^2} \int_{M^{11}} d^{11}x \sqrt{g} \left( -\frac{1}{2} R^{(11)}_{11} - \frac{1}{48} G_{IJKL} G^{IJKL} \right)$$

$$- \frac{\sqrt{2}}{3456 \kappa_{11}^2} \int_{M^{11}} d^{11}x \varepsilon^{I_1 \ldots I_{11}} C_{I_1 I_2 I_3} G_{I_4 \ldots I_7} G_{I_8 \ldots I_{11}} - \frac{1}{8\pi (4\pi \kappa_{11}^2)^{2/3}} \int_{M^{10}} d^{10}x \sqrt{g} \text{tr} F_{AB} F^{AB},$$  \hspace{1cm} (2)

where $I, J, K, L = 1 \ldots 11$ and $G_{IJKL}$ is related to the field-strength of the three-form $C_{IJK}$ by (in differential form language)

$$G = 6dC + \frac{\kappa_{11}^{2/3} \delta^{(x^{11})}}{2\sqrt{2\pi (4\pi)^{2/3}}} dx^{11} \omega_3,$$  \hspace{1cm} (3)
which is similar to the ten-dimensional relation \( H = dB - \frac{\omega_3}{2} \). In (2), \( \kappa_{11} \) is the 11-d gravitational coupling which defines the 11-d Newton constant \( \kappa_{11} = M_{11}^{-2/3} \), which is the \( M \)-theory scale. For the purposes of this section we set \( M_{11} = 1 \). The mass units will be discussed in detail later on.

The Lagrangian above must be supplemented by the Horava-Witten \( \mathbb{Z}_2 \) projection, which project out one gravitino and part of the other fields on the boundary. It acts in the following way:

\[
x_{11} \rightarrow -x_{11}, \quad \Psi_I(-x_{11}) = \Gamma_{11} \Psi_I(x_{11}),
\]

where \( \Psi_I \) is the 11-d gravitino and \( \Gamma_{11} = \Gamma_1 \cdots \Gamma_{10} \) is the 10-dim chirality matrix, while the three form \( C \) is odd and the metric tensor is even.

Throughout the paper, \( x^6, \cdots, x^{11} \) will denote coordinates on the CY manifold \( Q \), \( x^5 \), the extra eleventh dimension, and \( x^1, \cdots, x^4 \), the ordinary four-dimension space-time. We introduce a complex structure on \( Q \) by defining

\[
y^i = \frac{x^{2i+4} + i x^{2i+5}}{\sqrt{2}}, \quad \bar{y}^i = \frac{x^{2i+4} - i x^{2i+5}}{\sqrt{2}}, \quad i = 1, 2, 3.
\]

In the following, \( \mu, \nu, \cdots \) will refer to 4-d and 5-d Lorentz indices (to be distinguished whenever necessary) and \( i, j, k \cdots \) to compact indices.

We truncate the Lagrangian (2) in order to obtain \( \mathcal{N}=2 \) models\(^2\) in 5-d in the supergravity (Einstein) units, which we argue to be the natural units in \( M \)-theory. Then we impose the Horava-Witten projection (which acts by substituting \( x_{11} \rightarrow x_5 \) and \( \Gamma_{11} \rightarrow \Gamma_5 \) in (4)), which gives \( \mathcal{N}=1 \) models in 4-d.

A simple dimensional reduction can be performed to obtain an action in five dimensions. Our way of truncation (see below) is such that, in the metric tensor, there is no mixing between compact and non-compact indices. So, going into supergravity coordinates in 5-d, we take

\[
g^{(11)}_{\mu \nu} = G^{-1/3} g^{(5)}_{\mu \nu}, \quad g^{(11)}_{ij} = g_{ij},
\]

where \( G \) is the determinant of the metric in the compact space \( g \) (to be distinguished form \( g^{(5)} \)).

Similarly, the relevant components of the metric in the three form are

\[
C_{\mu \nu \rho}, \quad C_{\mu ij} = A^{(ij)}_\mu, \quad C_{ijk} = a \epsilon_{ijk}, \quad C_{ijk},
\]

\(^2\)We call \( \mathcal{N}=2 \) supersymmetry the smallest possible supersymmetry in 5-d.
where $a$ and $C_{ijk}$ are complex scalars. Moreover we shall put $C_{ijk} = 0$ in the following, these fields being irrelevant for our analysis.

After algebraic manipulations, we obtain the desired 5-d bosonic action (see [9])

$$ S^{(5)} = -\frac{1}{2} \int d^5x \sqrt{g^{(5)}} \left[ R^{(5)} + \frac{1}{12} tr(g^{-1}\partial_\mu g)tr(g^{-1}\partial^\mu g) + \frac{1}{4} tr(g^{-1}\partial_\mu gg^{-1}\partial^\mu g) 
+ \frac{1}{24} G_{\mu\nu\rho\sigma} G^{\mu\nu\rho\sigma} + 18 G_{i}^{1/3}(g^{i\bar{i}} - g^{i\bar{i}}) F^{(i\bar{j})} F_{\mu\nu}^{(i\bar{j})} F_{\mu\nu}^{(i\bar{j})} 
+ 36 \left( \det g^{ij} (\partial_\mu a)(\partial^\mu a) + 2 \det g^{ij}(\partial_\mu a)(\partial^\mu a) + \det g^{ij}(\partial_\mu a)(\partial^\mu a) \right) \right] 
- 36\sqrt{2} \int d^5x \epsilon_{\mu\nu\rho\sigma\tau} \left[ iC_{\mu\nu\rho}(\partial_\sigma a)(\partial_\tau a) + \frac{1}{4} \epsilon_{ijk}\epsilon_{\bar{i}\bar{j}\bar{k}} A^{(i\bar{j})}_{\mu} F_{\nu\rho}^{(j\bar{k})} F_{\mu\nu\rho\sigma}^{(i\bar{k})} \right] 
- \frac{1}{2\pi(4\pi)^{2/3}} \int d^4x \sqrt{g^{(4)}} G_{\mu\nu}^{1/2} tr F_{\mu\nu} F_{\mu\nu} . \quad (8) $$

In the following, we perform the rescalings $(a, A_{\mu}^{(i\bar{j})}) \rightarrow (a, A_{\mu}^{(i\bar{j})})/6$. We now consider particular truncations and also compactify the fifth dimension to obtain an 4-d action. In each case we discuss, the gauge group on the observable boundary is broken by the embedding of the spin connection into the gauge group.

To begin with, consider the simplest truncation, corresponding to a compactified space with just one radius $e^{\sigma/2}$, the "breathing" mode\[\footnote{This simple truncation in $M$-theory regime has been recently performed also in [10].}\]. Following Witten [10], we pick up an $SU(3)$ subgroup of the $SU(4) \simeq SO(6)$ rotational group acting on $x^6, \cdots, x^{11}$. We define this $SU(3)$ such that $(y^1, y^2, y^3)$ transforms like a representation 3 and $(\bar{y}^1, \bar{y}^2, \bar{y}^3)$, like a representation $\bar{3}$. The 11-d fields have to respect this $SU(3)$ symmetry and also the $Z_2$ symmetry acting on $x^5$. The observable gauge group is $E_6$. The massless spectrum in 5-d is the universal hypermultiplet $(C_{\mu\nu}, e^{4\sigma}, a, a^\dagger)$ and the gravitational multiplet, corresponding to a CY manifold with $h_{(1,1)} = 1$ and $h_{(2,1)} = 0$. The resulting 11-d tensor metric is written

$$ g^{(11)}_{i\bar{j}} = e^{\sigma} \delta_{i\bar{j}}, \quad g^{(11)}_{\mu\nu} = e^{-2\sigma} g^{(5)}_{\mu\nu} , 
\quad g^{(5)}_{\mu\nu} = e^{2\gamma}, \quad g^{(5)}_{\mu\nu} = e^{-\gamma} g^{(4)}_{\mu\nu} . \quad (9) $$

We have adopted here supergravity coordinates in which the 5-d and 4-d actions have a canonical supergravity expression. The scalar field $e^{\sigma}$ is related to the radius of the CY manifold, while $e^{\gamma}$ is related to the radius of
the fifth dimension. Similarly, in 4-d and after the Horava-Witten ($Z_2^{HW}$) projection, the three-form has only the massless components

$$C_{\mu\nu}^{(11)} = C_{\mu\nu\delta}, \quad C_{5i\bar{j}}^{(11)} = B_{\delta i\bar{j}}, \quad C_{ijk}^{(11)} = \epsilon_{ijk}a, \quad C_{i\bar{j}k}^{(11)} = \epsilon_{i\bar{j}k}a^\dagger.$$  

(10)

$C_{\mu\nu\delta}$ is a two-form in the 4-d space-time while $B$ is a 4-d scalar field.

The dimensional truncation of $S^{(5)}$ is easily performed and leads to

$$S^{(4)} = -\int d^4x \sqrt{g^{(4)}} \left\{ \frac{1}{2} R^{(4)} + \frac{3}{4} (\partial_\mu \gamma)(\partial^\mu \gamma) + \frac{9}{4} (\partial_\mu \sigma)(\partial^\mu \sigma) ight\} + \frac{1}{12} \epsilon^{6\sigma} G_{\mu\nu\rho\delta} G^{\mu\nu\rho\delta} + \frac{3}{4} \epsilon^{-2\gamma} (\partial_\mu B)(\partial^\mu B) + \frac{1}{2\pi (4\pi)^{2/3}} \epsilon^{3\sigma} F_{\mu\nu} F^{\mu\nu}.$$  

(11)

To be consistent with a true CY compactification, this 4-d action has to derive from a Kähler potential; the supergravity coordinates are the appropriate ones for the identification of this Kähler structure. The complex fields are indeed easily identified as

$$S = e^{3\sigma} + ia_1,$$

$$T = e^{\gamma} + iB,$$

(12)

and the corresponding Kähler potential is $K = -\ln(S + S^\dagger) - 3\ln(T + T^\dagger)$.

The imaginary part of $S$ is obtained from a Hodge duality of $G$

$$\epsilon^{6\sigma} G_{\mu\nu\rho\delta} = \epsilon_{\mu\nu\rho\sigma} \partial^\sigma a_1.$$  

(13)

Notice, first of all, the well-known exchange of roles $S - T$ compared to string compactifications. Moreover, unlike in the direct compactification of ten-dimensional string case, the definitions of $T$ and $S$ are completely decoupled, $S$ being related to the volume of the 6-d compactified space and $T$ to the radius of the eleventh dimension, but seen from 5-d. We call it the radius of the fifth dimension $R_5$ in the following, in $M$-theory units (denoted by $R_5^{(M)}$ in section 4). We shall see in the last section, how, with another interpretation of this radius, we can reproduce the Kähler structure of usual string compactifications.

The $SU(3)$ invariance requirement on the CY manifold allows for one single field $\sigma$ in the metric. We can in fact mimicke compactification on orbifolds by restriction to discrete subgroups of $SU(3)$ that act non-trivially on the representation 3 and $\bar{3}$ in such a way that we maintain $N = 2$ in five dimensions, by using the methods employed in [11]. We shall consider three cases (the action is on the representation 3):
a. $Z_{12}$ symmetry acting like $(i e^{2i\pi/3}, -i e^{2i\pi/3}, e^{2i\pi/3})$;

b. $Z_3$ symmetry acting like $(e^{2i\pi/3}, e^{2i\pi/3}, e^{2i\pi/3})$;

c. $Z_2 \times Z'_2$ symmetry acting like $(-1, 1, -1) \times (1, -1, -1)$.

**a. $Z_{12}$ symmetry.** This model has a massless spectrum in 5-d consisting of the universal hypermultiplet, 2 vector multiplets and the gravitational multiplet, corresponding to $h_{(1,1)} = 3, h_{(2,1)} = 0$. The observable gauge group is $E_6 \times U(1) \times U(1)$. In the supergravity coordinates, the metric tensor takes then the form (written directly from 11-d $\rightarrow$ 4-d)

$$g^{(11)}_{\mu\nu} = e^{-\gamma}(1 + \sigma_1 + \sigma_2 + \sigma_3)/3, \quad g^{(11)}_{55} = e^{2\gamma}(1 + \sigma_1 + \sigma_2 + \sigma_3)/3, \quad g^{(11)}_{ij} = e^{\sigma_i \delta_{ij}},$$

where the shape of the orbifold is here described by the three radii $\sigma_1, \sigma_2,$ and $\sigma_3$. After the $Z_{2}^{HW}$ projection, the massless modes of the three form in 4-d are

$$C_{\mu\nu}, \quad C_{5ij} = B_i \delta_{ij}.$$  

It is easy to perform the dimensional reduction of $S^{(5)}$ and, as previously, identify the Kähler structure. The 4-d action now derives from

$$K = -\ln(S + S^\dagger) - \sum_{1 \leq k \leq 3} \ln(T_k + T_k^\dagger),$$

and the complex fields are identified as

$$S = e^{\sigma_1 + \sigma_2 + \sigma_3} + i a_1,$$

$$T_k = e^{\gamma} e^{-(\sigma_1 + \sigma_2 + \sigma_3)/3 + \sigma_k} + i B_k,$$

where $a_1$ is still defined by a Hodge duality $e^{2(\sigma_1 + \sigma_2 + \sigma_3)}G_{\mu\nu\rho\delta} = \epsilon_{\mu\nu\rho\sigma} \Theta^\sigma a_1$. Notice here the relations $t_i / t_j = R_i^2 / R_j^2, \quad t_1 t_2 t_3 = R_3^2, \quad s = R_1 R_2 R_3^2$, where $R_i$ denote the three CY radii. An interesting fact is that under a dilatation of the 6-d compactified space $g_{ij} \rightarrow \lambda g_{ij}$, the only field which changes is $S$. This will be valid for the more general truncations discussed below and is related to the special geometry structure in 5-d.

**b. $Z_3$ symmetry.** The massless fields in 5-d are the gravitational multiplet, the universal hypermultiplet and eight vectors multiplets, corresponding to $h_{(1,1)} = 9$ and $h_{(2,1)} = 0$ and the observable gauge group is $E_6 \times SU(3)$.
In the supergravity coordinates, the $Z_3$-invariant metric tensor takes the form

$$g_{\mu\nu}^{(11)} = e^{-\gamma G^{-1/3}} g_{\mu\nu}^{(4)}, \quad g_{55}^{(11)} = e^{2\gamma G^{-1/3}}, \quad g_{ij}^{(11)} = g_{ij}. \quad (18)$$

The shape of the orbifold is here described by the nine parameters appearing in the unrestricted $g_{ij}$. The massless modes of the three form in 4-d are $C_{\mu\nu 5}, C_{5i\bar{j}} = B_{i\bar{j}}$. The resulting compactified 4-d action derives now from the following Kähler potential

$$\mathcal{K} = -\ln(S + S^\dagger) - \ln \det(T_i + T_i^\dagger), \quad (19)$$

and the complex fields are identified as

$$S = G^{1/2} + i a_1,$$
$$T_{ij} = e^{\gamma G^{-1/6}} g_{ij} + i B_{ij}, \quad (20)$$

where $G^2 G_{\mu\nu\rho\delta} = \epsilon_{\mu\nu\rho\sigma} \theta^\sigma a_1$. Notice the relation $\det t_{ij} = e^{3\gamma} = R_3^2$ which is again related to special geometry in 5-d.

c. $Z_2 \times Z'_2$ symmetry. Here, in contrast with previous cases, we obtain three additional hypermultiplets (in addition to the universal one), containing the bosonic fields $(g_{ii}, C_{ijk})$. The model is described by $h_{(1,1)} = 3, h_{(2,1)} = 3$ and the observable gauge group on the boundary is $E_6 \times U(1) \times U(1)$. In the supergravity coordinates, the $Z_2 \times Z'_2$-invariant metric tensor takes now the form

$$g_{\mu\nu}^{(11)} = e^{-\gamma G^{-1/3}} g_{\mu\nu}^{(4)}, \quad g_{55}^{(11)} = e^{2\gamma G^{-1/3}} \quad (21)$$

and

$$g_{ij}^{(11)} = \begin{pmatrix} g^{(1)} & g^{(2)} \\ g^{(2)} & g^{(3)} \end{pmatrix}, \quad (22)$$

where $g^{(i)}$ are $2 \times 2$ symmetric matrices, $G_i$ their determinant and $G$ the global determinant.

The 4-d massless modes of the three form are $C_{\mu\nu 5}, C_{5i\bar{j}} = B_{i\bar{j}}$. The corresponding 4-d Kähler potential is now

$$\mathcal{K} = -\ln(S + S^\dagger) - \sum_{1 \leq k \leq 3} \ln(T_k + T_k^\dagger) - \sum_{1 \leq k \leq 3} \ln(U_k + U_k^\dagger), \quad (23)$$
where

\begin{align}
S &= G^{1/2} + i a_1 , \\
T_k &= e^\gamma G^{-1/6} G_k^{1/2} + i B_k , \\
U_k &= \frac{(G_k)^{1/2}}{g_{22}^{(k)}} + i \frac{g_{12}^{(k)}}{g_{22}^{(k)}} ,
\end{align}

(24)

with \( G G_{\mu \rho \nu} = \epsilon_{\mu \rho \sigma} \partial^\sigma a_1 \).

Notice that all the models studied above are particular cases of no-scale models \([12]\). So the no-scale structure seems to be present in both the weak-coupling limit and strong-coupling limit of superstring compactifications \([4], [7]\).

Recently, it was shown that the anomaly cancellation in \(M\)-theory ask for another term in the \(M\)-theory Lagrangian \([3], [13]\), which can be viewed as a term cancelling fivebrane worldvolume anomalies \([14]\) or, by compactifying one dimension, as a one-loop term in the type \(IIA\) superstring \([15]\). In our conventions and by using differential form language, it reads

\[ W_5 = -\frac{1}{2^{11/6}(2\pi)^{10/3}} \int_{M_{11}} C \wedge X_8 , \]

(25)

where \(X_8 \equiv -\frac{1}{8} tr R^4 + \frac{1}{32} (tr R^2)^2\). This term can be compactified to 4-d in different compactification schemes by using the field definitions given above and the quantization rule \(\frac{1}{8\pi^2} \int tr R \wedge R = m\), where \(m\) is an integer and the integration is over a 4-cycle in the compactified space. Without entering into details and restricting to one overall radius, we find in 4-d a higher-derivative term of the type \(T R^2\) in superfield language (recently, similar 4-d terms were studied \([16]\) and claimed to be relevant for supersymmetry breaking).

Armed with the above field definitions, we can now study our main concern, supersymmetry breaking from \(5 \to 4\) dimensions by compactification.

3 The spontaneous breaking of \(N = 1\) supersymmetry in four dimensions by compactification

We are interested in the phenomenology of the \(M\)-theory compactifications, in particular we would like to break the \(N = 1\) supersymmetry in 4-d. To achieve this aim, we perform a mechanism proposed by Scherk and Schwarz.
in a supergravity context and then generalized to superstrings in \[ \text{[4]} \]
(another possibility for breaking supersymmetry is the gaugino condensation of one of the $E_8$ gauge group \[ \text{[8]} \]). As for the moment there is no quantum description of $M$-theory, we use its effective low energy description at the level of supergravity.

The Scherk-Schwarz mechanism is a generalized dimensional reduction which allows for the fields a dependence in the compact coordinates. Nevertheless, to avoid ghost particles and to include ordinary dimensional reduction, this dependence must satisfy some properties: it has to be in a factorisable form and has to correspond to a R-symmetry of the theory. The simplest example is the use of the compact $SO(6)$ symmetry of the 6-d compact manifold, which is readily applicable to the superstrings case. In this case, any component of a tensor with $p$ compact indices and $q$ non-compact indices takes the form

\[
\hat{T}_{i_1 \cdots i_p \mu_1 \cdots \mu_q}(x, y) = \prod_{n=1}^{p} U_{i'_{in}}(y) T_{i_1' \cdots i'_n \mu_1 \cdots \mu_q}(x),
\]

where $y$ denotes compact coordinates and $x$ non compact ones. This tensor decomposition is stable under product and exterior derivation.

This extended dimensional reduction, when applied to the curvature term, generates a potential for the scalar fields corresponding to the metric in the compact space. The requirement for this scalar potential to be positive imposes further restrictions on the form of $U$. A solution was proposed by Scherk and Schwarz, by taking

\[
U = e^{My^1},
\]

where $M$ is an antisymmetric matrix in the compact space with zeros in the row and the column corresponding to $y^1$. Then the 4-d scalar potential, in supergravity units, reads \[ \text{[8]} \]

\[
V_0 = \frac{1}{4 \sqrt{G}} \left( g^{1,1} tr \left( M^2 - MgMg^{-1} \right) - \left( g^{-1} M^t gMg^{-1} \right)^{1,1} \right),
\]

where in \[ \text{[25]} \] and the rest of this section, $g$ is the metric in the 7-d compact space and $G$ its determinant.

When applied to the kinetic term for the 11-d spin-$\frac{3}{2}$ field, this extended dimensional reduction also generates, through the spin connection for compact indices, masses for the resulting 4-d gravitini.
More important for our purposes are symmetries which mix fields of different parities, for reasons which will become transparent below. For the truncations we are discussing here, these must be subgroups of the $Sp(8)$ symmetry present in the $N = 8$ supergravity in 5-d (see third reference in [8]).

We shall perform this mechanism in two cases

a. with $y^1$ being the extra eleventh dimension, $x^5$;

b. with $y^1$ being a Calabi-Yau or orbifold coordinate.

We use for this purpose the truncations derived in the preceding section. The projections $P$ we use have to commute with the Scherk-Schwarz matrix $U$, which means explicitly

$$[P, M] = 0 \text{ if } Py^1 = y^1,$$

$$\{P, M\} = 0 \text{ if } Py^1 = -y^1. \quad (29)$$

a. An eleventh dimension coordinate dependence. The characteristics of this case is the anticommutation relation

$$\{Z_{2}^{HW}, M\} = 0. \quad (30)$$

We consider here two different possibilities. The first uses an $SU(2)$ R-symmetry present in the 5-d theory, which we argue to be the subgroup of $Sp(8)$ which commutes with all the projections. The second uses the usual symmetry of the 6-d internal manifold, which is relevant for the type IIA supergravity in 4-d.

1. $SU(2)$ R-symmetry.

To keep things as simple as possible, consider for the moment the simplest truncation (3) corresponding to an $SU(3)$ invariance in the compactified space. In this case, in 5-d the only matter multiplet is the universal hypermultiplet, whose scalar fields parametrize the coset $SU(2,1)/SU(2)\times U(1)$. This structure can be simply viewed from 4-d by a direct truncation (without the $Z_{2}^{HW}$ projection). The truncated dependence on the universal hypermultiplet can be derived from the Kähler potential [19]

$$K = -\ln(S + S^\dagger - 2a^\dagger a), \quad (31)$$

where the Hodge duality in 5-d is $G_{\mu\nu\rho\sigma} = \epsilon_{\mu\nu\rho\sigma\delta}(-\partial^\delta a_1 + i a_1^\dagger \partial^\delta a/\sqrt{2})$ and $S = e^{3\sigma} + a^\dagger a + ia_1$. The $SU(2)$ symmetry acts linearly on the redefined
fields

\[ z_1 = \frac{1 - S}{1 + S}, \quad z_2 = \frac{2a}{1 + S}, \quad (32) \]

which form a doublet \((z_1, z_2)\). The \(Z_2^{HW}\) projection acts as \(Z_2^{HW} S = S, \ Z_2^{HW} a = -a\), which translates on the \(SU(2)\) doublet in the obvious way

\[ Z_2^{HW} \begin{pmatrix} z_1 \\ z_2 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \begin{pmatrix} z_1 \\ z_2 \end{pmatrix}. \quad (33) \]

The Scherk-Schwarz decomposition in this case reads explicitly

\[ \begin{pmatrix} \hat{z}_1 \\ \hat{z}_2 \end{pmatrix} = \begin{pmatrix} \cos mx_5 & \sin mx_5 \\ -\sin mx_5 & \cos mx_5 \end{pmatrix} \begin{pmatrix} z_1 \\ z_2 \end{pmatrix}, \quad (34) \]

corresponding to the matrix defined in \((27)\) \(M = im\sigma_2\) and where \(m\) is a mass parameter. Notice that, thanks to the anticommutation relation \((30)\) which is clearly verified, the fields \(\hat{z}_i\) have the same \(Z_2^{HW}\) parities as the fields \(z_i\). The resulting scalar potential in 4-d in Einstein metric is computed from the kinetic terms of the \((\hat{z}_1, \hat{z}_2)\) fields derived form \((31)\). After putting \(z_2 = 0\) corresponding to the projection \(Z_2^{HW}\), it is easily worked out and it is positive semi-definite. Expressed in terms of \(S\) and \(T\), the result is

\[ V = \frac{4m^2}{(T + T^\dagger)^3} \frac{|1 - S|^2}{S^2 + S^\dagger}. \quad (35) \]

This result can be seen as a superpotential generation for \(S\). The 4-d theory is completely described by\[4\]

\[ \mathcal{K} = -\ln(S + S^\dagger) - 3\ln(T + T^\dagger), \]

\[ W = m(1 + S). \quad (36) \]

Notice that this superpotential for \(S\) should correspond to a non-perturbative effect from the heterotic string point of view. The minimum of the scalar potential corresponds to \(S = 1\) and a spontaneously broken supergravity model with a zero cosmological constant. The order parameter for supersymmetry breaking is the gravitino mass \(m_{3/2}^2 = e^\mathcal{K}|W|^2 = 2m^2/(T + T^\dagger)^3\).

\[4\]This result is in agreement with general results on no-scale models (second ref. in \([13]\)).
Notice that the theory (36) is symmetric under the inversion $S \to 1/S$, easily checked also on the scalar potential (35). This corresponds to a subgroup of the $SL(2, \mathbb{Z})$ acting on $S$ as $S \to (aS - ib)/(icS + d)$, where $ad - bc = 1$, restricted here to $a = d = 0, b = -c = 1$. This already suggest a weak coupling - strong coupling symmetry of our resulting model, as in the $S$-duality models proposed some time ago [20].

We stress out that this 4-d model presents some general features, independent of the details of the compactification. The most general R-symmetry present in 5-d in N=8 supergravity is $Sp(8)$. This symmetry is reduced once we impose truncations, and the truncation to N=2 leave an $SU(2)$ symmetry appearing in the supersymmetry algebra. The scalar fields from the universal hypermultiplet, present in any compactification, transform as a doublet under $SU(2)$. Scherk-Schwarz mechanism uses only the antisymmetric part ($O(2)$) of it, so we are led to (34). On the other hand, the $O(2)$ part is exactly which is required by the Horava-Witten $Z^H_W$ projection which defines the $E_8 \times E_8$ heterotic string. In more general CY models in 5-d, it is maybe possible that the $SU(2)$ symmetry acts also on other scalar fields than the universal hypermultiplet, but the piece (36) we computed should always exist and is universal.

ii. Symmetry of the compactified space.

Since $P = SU(3), Z_3, Z_{12}$ or $Z_2 \times Z_2'$ doesn’t act on $x^5$, the commutation relation (29) simply reads

$$[P, M] = 0.$$  

(37)

For $P = SU(3)$, there is no antisymmetric matrix $M$ respecting this condition, so we can’t perform the Scherk-Schwarz’s mechanism. For $P = Z_2 \times Z_2'$, the allowed M matrices involve three parameters and, in the basis $(x^5, x^6, \ldots, x^{11})$, read

$$M = \begin{pmatrix}
0 & \cdots & 0 \\
0 & m_1 & 0 \\
-1 & 0 & m_1 \\
\vdots & \ddots & \vdots \\
-1 & 0 & m_2 \\
0 & m_2 & 0 \\
0 & 0 & m_3 \\
0 & -m_3 & 0
\end{pmatrix}.$$  

(38)

The following arguments were developed in collaboration with R. Minasian.
In this case, the Scherk-Schwarz matrix doesn’t anticommute with $Z^2_{HW}$, so the mechanism is better adapted to type $IIA$ supergravity in 4-d. We are only interested here in field spectrum common to type $IIA$ and heterotic string. As we will see in the next section, the qualitative features of this case concerning the decompactification problem are however similar to that of the previous example.

The scalar potential (28) in this case reads

$$V_0 = \frac{1}{4} e^{-\gamma} \sum_{i=1}^{3} \frac{m_i^2}{G_i} \left( (g_{11}^{(i)} - g_{22}^{(i)})^2 + 4g_{12}^{(i)^2} \right),$$

(39)

which, expressed in terms of moduli fields (24), takes the simple expression

$$V_0 = \frac{1}{t_1 t_2 t_3} \sum_{i=1}^{3} m_i^2 \frac{U_i^2 - 1}{U_i + U_i^\dagger}.$$

(40)

This scalar potential has interesting consequences. In analogy with our previous example, the scalar potential is invariant under $U_i \to 1/U_i$. The three $U$-fields develop a vev $< U_i > = 1$ (the self-dual points of duality transformations) and acquire masses

$$m_{U_i}^2 = \frac{m_i^2}{< t_1 t_2 t_3 >}.$$

(41)

At the same time, the single gravitino which remains after imposing the $Z_2 \times Z'_2 \times Z^2_{HW}$ symmetries, also acquires a mass which is computed to be

$$m_{3/2} = \frac{m_1 + m_2 + m_3}{2\sqrt{< t_1 t_2 t_3 >}}.$$

(42)

b. A Calabi-Yau or orbifold coordinate dependence. We choose here $y^1$ to be $x^6$, a Calabi-Yau or orbifold coordinate. Then the commutation relation (23) reads

$$[P, M x^6] = 0, \ [Z^2_{HW}, M x^6] = 0.$$

(43)

For $P = SU(3)$, there is no antisymmetric matrix $M$ respecting this condition, so we can’t perform the Scherk-Schwarz’s mechanism. For $P = Z_2 \times Z'_2$, the situation is different; since $x^6$ is now odd under the first $Z_2$ and even under the second one and under $Z^2_{HW}$, the commutation relation simplifies again

$$\{ Z_2, M \} = 0, \ [Z'_2, M] = 0, \ [Z^2_{HW}, M] = 0.$$

(44)
Thus we can always choose $M$ to be of the off-diagonal following form, in the basis $(x^5, x^6, \ldots, x^{11})$,

$$M = \begin{pmatrix}
0 & m & 0 & 0 \\
m & 0 & m' & 0 \\
0 & m' & 0 & m \\
-m & 0 & 0 & -m'
\end{pmatrix}. \quad (45)$$

Then the scalar potential, written directly in term of moduli fields, reads

$$V_0 = \frac{1}{4st_1u_1t_2u_2t_3u_3} \left( m^2|t_2U_2 - t_3U_3|^2 + m'^2|t_2U_3 - t_3U_2|^2 ight) - \frac{1}{2} \left( (m - m')^2t_2t_3 - mm'(t_2 - t_3)^2 \right) (U_2 - U_2^\dagger)(U_3 - U_3^\dagger). \quad (46)$$

The minimum (for $m \neq m'$) of the scalar potential corresponds to the flat directions

$$< t_2 >= < t_3 >, \quad < u_2 >= < u_3 > \quad \text{and} \quad < \Im mU_2 >= < \Im mU_3 > = 0. \quad (47)$$

We don’t write here the scalar mass matrix, but just notice that the leading matrix elements, which fix the physical masses, are of the type (consider $m \sim m'$)

$$m_0^2 \sim \frac{m^2}{st_1}, \quad (48)$$

where $t_1$ is the modulus field corresponding to the Scherk-Schwarz coordinate. This case is the analog of the supersymmetry breaking by compactification in the weakly coupled superstrings [17]. Notice that in the superstring case, the mass parameters $m, m'$ in (45) are discrete, being related to automorphisms of the compactification lattice.

All the masses we computed above are measured in $M$-theory units. In the following section, we translate these results in 4-d supergravity units.

### 4 Phenomenological implications of $M$-theory compactifications

We shall discuss in this section some phenomenological issues of $M$-theory compactifications focussing essentially on the four dimensional Newton’s
constant and the gravitino mass. For simplicity reasons and without affecting our conclusions, we restrict ourselves to the simplest case \( t_i = e^\gamma \).

String compactifications don’t easily allow [1], [21] to adjust the string mass scale, the vev of the dilaton and the volume of the compact space to tune the three four-dimensional observable quantities that are the Planck scale, the GUT scale and the gauge coupling constant. For instance, in the weakly coupled regime of the \( E_8 \times E_8 \) heterotic string, the well-known relation

\[
G_N \sim \lambda^{2/3} \alpha_{\text{GUT}}^{4/3} M_{\text{GUT}}^2 \tag{49}
\]

disagrees with experimental values by a factor of order 20. Witten has shown [21] how in strongly coupled regime the compatibility between string predictions and experimental values could be restored. It is interesting to worry about this issue in our patterns of compactification.

In the 11-d action appears only the \( M \)-theory scale, which is the eleven-dimensional Planck mass \( M_{11} \), such that

\[
S^{(11)} \supset -\frac{1}{2} \int d^{11}x \sqrt{g^{(11)}} M_{11}^3 R^{(11)} - \frac{1}{8\pi(4\pi)^{2/3}} \int_{x^5=0} d^{10}x \sqrt{g^{(10)}} M_{11}^6 F_{\mu\nu} F^{\mu\nu}. \tag{50}
\]

Now we compactify this action in the \( M \)-theory coordinates in which, like in the five-brane units for the 10-d string, there is no kinetic term for the "radius" of the extra-dimension, that is we adopt

\[
g_{\mu\nu}^{(11)} = e^{-2\sigma} g_{\mu\nu}^{(5)}, \quad g_{ij}^{(11)} = e^\sigma \delta_{ij},
\]

\[
g_{55}^{(5)} = e^{2\gamma}, \quad g_{\mu5}^{(5)} = g_{\mu\nu}^{(4)} \tag{51}
\]

It is transparent that these are the natural units of 5-d supergravity. By performing the Hodge transformation (using the field definitions [12])

\[
\frac{s}{t} G_{\mu\nu5} = \epsilon_{\mu\nu\rho\sigma} \partial^\rho a', \tag{52}
\]

the 4-d low energy effective action contain the terms

\[
S^{(4)} \supset -\frac{1}{2} \int d^4x \sqrt{g^{(4)}} \left[ t M_{11}^2 \left( R^{(4)} + \frac{1}{2 s^2} (\partial_\mu s)^2 + (\partial_\mu a')^2 \right) + \frac{1}{(4\pi)^{5/3}} s F_{\mu\nu} F^{\mu\nu} \right]. \tag{53}
\]

We can pass from the \( M \)-theory action (53) to the Einstein action (11) by the Weyl rescaling \( g_E^{(4)} = t g_M^{(4)} \). Notice the absence of a kinetic term for \( t \)
in (53), the clear indication of a no-scale structure. This can therefore be associated with the $M$-theory (or 5-d supergravity) units.

The Lagrangian (53) allows us to identify the gauge coupling and the 4-d reduced Planck mass to be

$$\frac{1}{\alpha_{\text{GUT}}} \sim \frac{1}{(4\pi)^{5/3}} s, \quad t M^2_{11} = M_{P_l}^{(4)} t_{M_1}^2.$$  \hspace{1cm} (54)

These quantities are related to the radii of the Calabi-Yau space and the extra fifth dimension expressed in terms of the $M$-theory units

$$e^{-\sigma/2} = s^{1/6} = \frac{M_{\text{GUT}}}{M_{11}}, \quad t = R_5^{(M)} M_{11}.$$  \hspace{1cm} (55)

However the radius of the fifth dimension expressed in $M$-theory units is not convenient for physics in four dimensions, so we prefer to express it in terms of $M_{P_l}^{(4)}$, the four-dimensional Planck scale, by defining

$$t = \frac{M_{P_l}^{(4)}}{R_5}.$$  \hspace{1cm} (56)

Combining these equalities, we obtain the $M$-theory equivalent of (49)

$$G_N \sim \frac{R_5^{-1} \alpha_{\text{GUT}}^{1/3}}{M_{P_l}^{(4)} M_{\text{GUT}}^2},$$  \hspace{1cm} (57)

and, in order to be compatible with experimental constraints, the "radius" of the fifth dimension has an intermediate scale value, as already suggested by [4], [5]

$$R_5^{-1} \sim 10^{12-13}\text{GeV}.$$  \hspace{1cm} (58)

The fifth radius also appears in the gravitino mass generated by the Scherk-Schwarz mechanism, so we have to take care that $R_5^{-1}$ is compatible with phenomenological preferred value for the gravitino mass. In the expressions for the gravitino mass from (36) and (42), the mass parameters

\footnote{In weakly-coupled regime, the gauge coupling constant can have big threshold corrections. If this holds true in strongly coupled regime, some of our considerations below will be modified.}
that appear in the Scherk-Schwarz mechanism using the fifth dimension are naturally of the order of the $M$-theory scale, so we obtain\[^7\]

$$m_{3/2} \sim \frac{M_{11}}{\sqrt{<t^3>}} = \frac{R_5^{-2}}{M_{Pl}^{(4)}} , \quad (59)$$

where to obtain the last identity we have used the identification of the moduli fields \[^{12}\] and the definitions \[^{24}\], \[^{56}\]. This expression is similar to a usual supergravity form in which an auxiliary field would have developed a vev of the order of $R_5^{-2}$. Phenomenologically, the gravitino mass should be of the order of $1\, TeV$. This leads to a value for the extra-radius rather compatible with the one deduced from Newton’s constant,

$$R_5^{-1} \sim 10^{11-12} GeV. \quad (60)$$

Consequently, the presence of the extra-dimension could offer a solution to the decompactification puzzle usually encountered in string compactifications. Indeed generically, Scherk-Schwarz mechanism in strings leads to \[^{17}\]

$$m_{3/2} \sim \frac{1}{R_{CY}} \sim M_{GUT}. \quad (61)$$

This is actually similar to the result we got in the last section in the case where a CY coordinate was used for the Scherk-Schwarz mechanism. To check it, rewrite the masses \[^{18}\] in 4-d supergravity units as $m_0^2 \sim M_{Pl}^{(4)^2} / (s t^2) \sim g^2 / R_5^2$, where $g$ is the gauge coupling.

This relation is phenomenologically hard to accommodate. Two issues have been proposed to solve this problem:

- compactify six dimensions on an asymmetric Calabi-Yau space with two or more different radii \[^{22}\]. However to agree with phenomenological values for the gravitino mass, these two (or more) radii must have hierarchical values by about fifteen orders of magnitude.

- construct models of strings such that the gravitino mass is $m_{3/2} \sim 1 / (R_{CY}^{n+1} M_{Pl}^{(4)^n}) \sim (M_{GUT}/M_{Pl}^{(4)})^n M_{GUT}$. However, phenomenology asks for $n = 4, 5$, and is difficult to explicitly construct such models.

\[^7\]A similar formula was conjectured by Antoniadis and Quiros \[^{3}\], but replacing $R_5$ with the eleventh radius. We stress out that in our expression, $R_5$ is the radius of the eleventh dimension seen from 5-d in the 5-d supergravity (Einstein) units.
So the extra-dimension brings a new and more satisfying alternative to the decompactification problem of compactified strings.

To conclude this phenomenological study, we would like to compare our pattern of compactification (11 $\rightarrow$ 5 $\rightarrow$ 4) with another one already studied \[1\] in which the extra-dimension was compactified first (11 $\rightarrow$ 10 $\rightarrow$ 4). From a strongly coupled 10-d point of view, the radius of the eleventh dimension $x^5$ is $R_{11}^2 \sim g_{55}^{(11)} \sim e^{2\gamma-2\sigma}$, and this radius is related to the string dilaton by \[21\]

$$R_{11} \sim e^{2\phi/3}. \quad (62)$$

Combining those relations, the expressions of the real part of $T$ becomes $t \sim e^{2\phi/3} e^\sigma$, which is the expression obtained by Binétruy \[23\] by compactifying the 10-d heterotic string in the five-brane units which are supposed to be appropriate to the strongly coupled regime.

Actually we can, in the same way, reproduce the moduli’s identification obtained by string compactification down to 4-d in the various units. Suppose that we are working in units characterized, in 10-d, by the tensor metric $g_{\hat{\mu}\hat{\nu}}^{(10)}$. These units are related to the five-brane ones by a Weyl rescaling

$$g_{\hat{\mu}\hat{\nu}}^{(10)} \equiv \lambda g_{\hat{\mu}\hat{\nu}}^{(10) B}. \quad (63)$$

So the typical radius of the Calabi-Yau space, measured in five-brane units, is now $R_{CY} \sim (\lambda^{-1} e^\sigma)^{1/2} \equiv (e^{\sigma B})^{1/2}$ and the radius of the extra-dimension, viewed from 10-d in five-brane units, is $R_{11} \sim e^{\gamma-\sigma B}$ which is still related to string dilaton by \[62\]. The real parts of the fields are now identified as

$$s = e^{3\sigma B} = \lambda^{-3} e^{3\sigma}, \quad t = e^\gamma = \lambda^{-1} e^{\sigma+2\phi/3}. \quad (64)$$

For instance, string (S) and supergravity (E) units are related to five-brane (B) units by

$$g_{\hat{\mu}\hat{\nu}}^{(10) B} = e^{-2\phi/3} g_{\hat{\mu}\hat{\nu}}^{(10) S} = e^{-\phi/6} g_{\hat{\mu}\hat{\nu}}^{(10) E}. \quad (65)$$

Eqs. \[64\] reproduce the results of string compactification summarized in tab.1.

However from a five-dimensional point of view, \textit{i.e.} after the compactification of the CY manifold, the radius of the eleventh dimension should be
$R_{11} \sim e^\gamma$, so the identification of the moduli fields is now different, leading to the previous conclusions.

From eleven-dimensional point of view in $M$-theory coordinates (51), the $11 \rightarrow 5 \rightarrow 4$ pattern of compactification is consistent if $R_5 > R_{CY}$ which reads $t > s^{1/2}$.

From ten-dimensional point of view in five-brane coordinates (63), in the $11 \rightarrow 10 \rightarrow 4$ pattern of compactification, the radius of the Calabi-Yau space and of the extra-dimension are now $R_{CY} \sim e^{\sigma / 2}$ and $R_{11} \sim e^{2\phi / 3}$, while the consistency condition of compactification is $R_{CY} > R_{11}$, which, in term of moduli fields (tab.1), becomes now $t < s^{1/2}$.

In both cases, the ten-dimensional string strongly coupled regime condition $e^\phi > 1$ simply reads $t > s^{1/3}$. So we can draw, on the $< t >$ line, the different consistent patterns of compactification (we assume that $s > 1$).

We have seen that phenomenology asks for large values of $t$, so our pattern of compactification is self-consistent.

5 Conclusions

We studied truncations of $M$-theory from $11 \rightarrow 5 \rightarrow 4$ dimensions, applying our results to the supersymmetry breaking mechanism by compactification.

\[
\begin{array}{cccc}
\text{five-brane units} & \text{string units} & \text{supergravity units} \\
\hline
\lambda & 1 & e^{2\phi / 3} & e^{\phi / 6} \\
\phi & e^{3\sigma} & e^{3\sigma - 2\phi} & e^{3\sigma - \phi / 2} \\
t & e^{\sigma + 2\phi / 3} & e^{\sigma} & e^{\sigma + \phi / 2} \\
\end{array}
\]

Table 1: identification of the real parts of moduli fields in string compactification in various units [23].
from 5-d to 4-d. The geometric properties of the moduli fields, related to the special geometry of the 5-d theory are peculiar and important for phenomenological purposes. \( \Re e S \) is the volume of 6-d internal manifold, the various \( h_{1,1} \) moduli fields \( T \) are related to the radius of the fifth-dimension and are invariant under dilatations of the 6-d manifold and the \( h_{2,1} \) moduli \( U \) characterize, as usual, the complex structure of the 6-d manifold.

Using different symmetries of the 5-d theory, we are able to break supersymmetry by compactification in various ways. The most interesting one is by using an \( SU(2) \) symmetry related to the universal hypermultiplet of the 5-d theory. In this case, we obtain (for the simplest possible truncation) a model described by

\[
\begin{align*}
K &= -\ln(S + S^\dagger) - 3\ln(T + T^\dagger), \\
W &= m(1 + S).
\end{align*}
\] (66)

This universal superpotential for \( S \) should correspond to a non-perturbative string effect. The minimization gives \( S = 1 \) and a spontaneously broken supergravity model with a zero cosmological constant. We hope that this result will shed some light on problems like dilaton stabilization and supersymmetry breaking in effective strings. Another example, using symmetries of the compactified 6-d space gives rise to a potential for the \( U \) moduli, the corresponding vacuum is \( U_i = 1 \) with a zero cosmological constant.

Defining 4-d scales and couplings we find that the resulting gravitino mass in supergravity units is \( m_{3/2} \sim R_5^{-2}/M_{Pl}^{(4)} \), which ameliorate the usual decompactification limit \( (R_5 \to 0) \) problem and correlate it with the unification problem in a way which look phenomenologically promising.

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