Development of methods for calculating the solidification time of castings and ingots in metal mold

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Abstract. A technique has been developed for calculating the solidification time of castings and ingots in metal form, taking into account not only the thermophysical properties of the casting materials and molds, but also the heat transfer conditions at the “casting – mold” interface, the dissipation of the heat flux from the crystallization front to the heating of the chill mold and into the environment.

1. Introduction
The manufacture of castings and ingots by gravity casting in metal molds (gravity die casting) is an important industrial method for the production of high-quality engineering blanks [1, 2]. The widespread use of this method in foundry practice is due to a number of obvious advantages, which include a reduction in material consumption and mass of workpieces, an increase in the utilization of metal, an improvement in the mechanical and operational properties of cast products, an increase in efficiency, and improvement in working conditions.

In order to purposefully control the structure and achieve the specified operational characteristics of castings and ingots hardening in metal molds, it is necessary to understand the kinetics of the processes that occur during solidification [3-5]. In this regard, the choice of rational technological parameters of die casting should, to a large extent, be based on the use of reliable calculation and analytical methods for determining all significant physical and thermotechnical parameters characterizing the solidification process.

In this case, one of the most important parameters is the total solidification time of the casting or ingot. In particular, the relationship between the solidification time and the formation of various casting defects, for example, shrinkage porosity in castings was confirmed [6]. Many research activities are devoted to modeling the parameters of alloy crystallization and solidification of castings, including [7, 8], the development of which is the present work.
The purpose of the work is to develop a methodology for calculating the solidification time of castings and ingots in metal form.

2. Methods

It was experimentally recorded [9, 10] that during the solidification of castings in metal mold, the temperature of all points of the system tends to a certain level, which can be defined as the average calorimetric temperature of the system – $T_C$ [9]. The average calorimetric temperature of the system remains constant provided that the loss of heat to the environment from the surface of the chill mold does not significantly affect the solidification process. This is possible when the mass of the chill mold $m_{mo}$ is significantly higher than the mass of the casting $m_O$, that is, it is true for a massive chill mold.

In this case, the heat balance equation for the ‘casting – mold’ system has the form

$$L_{CR} \rho_m V_O + c_m \rho_m V_O (T_P - T_{CR}) + c_m \rho_m V_O (T_{CR} - T_C) = c_{mo} \rho_{mo} V_{Mo} (T_C - T_{mo}^0),$$

(1)

where $L_{CR}$ – the heat of metal crystallization, J/kg; $\rho_m$, $\rho_{mo}$ – density of the alloy and the material of the mold, kg/m$^3$; $V_O$, $V_{Mo}$ – volume of casting and chill mold, m$^3$; $c_m$, $c_{mo}$ – heat capacity of the alloy and the mold material, J/(kg K); $T_P$ – pouring temperature of the alloy, K; $T_{CR}$ – crystallization temperature of the alloy, $T_{mo}^0$ – the initial temperature of the form, K.

From equation (1) it follows that the average calorimetric temperature

$$T_C = \frac{L_{CR} \rho_m V_O + c_m \rho_m V_O T_P + c_{mo} \rho_{mo} V_{Mo} T_{mo}^0}{c_m \rho_m V_O + c_{mo} \rho_{mo} V_{Mo}}.$$  

(2)

For real castings hardening in metal molds, the condition $m_O \ll m_{mo}$, is often not fulfilled, since the chill mold is cooled due to heat transfer to the environment. The amount of heat removed from the outer surface of the chill mold varies with time and can be calculated by the formula

$$Q_{env} = \alpha_3 F_{sur} (T_{sur} - T_{env}) \cdot \tau,$$

(3)

where $\alpha_3$ – the heat transfer coefficient from the outer surface of the chill mold to the environment, W/(m$^2$K); the coefficient $\alpha_3$ affects when the thickness of the mold heated layer is greater than or equal to the thickness of the mold wall ($\delta_{mo} \geq x_{mo}$); $F_{sur}$ – the surface area of the chill mold, m$^2$; $T_{sur}$, $T_{env}$ – the temperature of the outer surface of the mold and the environment, K.

For a casting in the form of an infinite plate

$$F_{sur} = F_O,$$

(4)

for an infinite cylinder

$$F_{sur} = F_O \cdot \left(1 + \frac{x_{mo}}{R}\right),$$

(5)

for a ball

$$F_{sur} = F_O \cdot \left(1 + \frac{x_{mo}}{R}\right)^2,$$

(6)

where $x_{mo}$ – is the thickness of the mold wall, m.

Thus, the average calorimetric temperature of the ‘casting – mold’ system will decrease with time and at time $\tau$ will be equal to

$$T_C^{\tau} = \frac{L_{CR} \rho_m V_O + c_m \rho_m V_O T_P + c_{mo} \rho_{mo} V_{Mo} T_{mo}^0 - Q_{env}}{c_m \rho_m V_O + c_{mo} \rho_{mo} V_{Mo}}.$$  

(7)
The heat flux from the crystallization front is spent on heating the chill mold and released into the environment through the hardened metal rim. Therefore, we can write the equality

$$K_H F_0 (T_P - T_{env}) d\tau = c_{mo} \rho_{mo} F_{Mo} (T_C - T_{mo}^0) d\delta_{Mo} + \alpha_3 F_{sur} (T_{sur} - T_{env}) d\tau,$$

where $$K_H$$ – the heat transfer coefficient for a flat multilayer wall, W/(m²K):

$$K_H = \left( \frac{1}{\alpha_1} + \frac{\varepsilon}{\lambda_m} + \frac{\delta_{mo}}{\lambda_{mo}} + \frac{1}{\alpha_3} \right)^{-1},$$

where $$\alpha_1$$ – the heat transfer coefficient at the crystallization front; $$\delta_{mo} = b \cdot \varepsilon$$ – the thickness of the heated mold layer for a flat casting, m. Here $$b$$ is a dimensionless coefficient showing how many times the thickness of the heated mold layer $$\delta_{mo}$$ is greater than the thickness of the hardened metal rim $$\varepsilon$$.

The heat transfer coefficient at the border ‘casting – mold’ $$\alpha_2$$ can be determined from the conditional heat resistances of the gas gap and the paint layer:

$$\alpha_2 = \left( \frac{x_{gas}}{\lambda_{gas}} + \frac{x_{pa}}{\lambda_{pa}} \right)^{-1},$$

where $$\lambda_{gas}$$, $$\lambda_{pa}$$ – the heat conductivity of the gas layer and paint, W/(mK); $$x_{gas}$$, $$x_{pa}$$ – the thickness of the gas layer and the thickness of the paint layer, m.

$$x_{gas} = \alpha_m C_m (T_{CR} - T_{Mo}^0) X_O + \alpha_m (T_{Mo} - T_{mo}^0) X_O,$$

where $$\alpha_m C_m$$, $$\alpha_m$$ – the linear expansion coefficients of the casting solid metal and the mold material, 1/K; $$X_O$$ – is the wall thickness of the casting in the direction of the heat sink, m.

$$\alpha_3 = Nu \cdot \frac{\lambda_m}{l},$$

where $$l = 2r + 2x_{mo}$$ is the characteristic size of the system; $$Nu$$ is the Nusselt criterion characterizing the intensity of heat transfer by contact.

$$Nu = C_1 (Gr \cdot Pr)^{C_2},$$

where $$C_1$$ and $$C_2$$ – tabular coefficients [10], $$Gr$$ – the Grashof criterion characterizing the intensity of free motion of the medium (natural convection), $$Pr$$ – the Prandtl criterion (similarity criterion for temperature and velocity fields);

$$Gr = \frac{g l^3 \rho}{\mu} \cdot \frac{\Delta T}{T_{env}},$$

where $$g$$ – the acceleration of gravity, $$g = 9.81$$ m/s²; $$\rho$$ – the gas density, kg/m³; $$\mu$$ is the dynamic viscosity of the gas, N s/m²; $$\Delta T = T_{sur} - T_{env}$$ – temperature difference between the surface of the chill mold and the environment, K.

$$Pr = \frac{\nu}{a_{env}},$$

$$\nu$$ is the kinematic viscosity coefficient of the medium, m²/s; $$a_{env}$$ – coefficient of thermal diffusivity of the medium, m²/s.
Since the heat is absorbed by the mold only until it is completely heated, then to determine the thickness of the heated layer of the mold, we write the heat balance equation

\[ L_C R \rho_m F_{CR} d\varepsilon + c_m \rho_m F_{CR} d\varepsilon(T_p - T_{CR}) + c_m \rho_m F_{CR} d\varepsilon(T_{CR} - \bar{T}_C) = c_{m0} \rho_{m0} F_{Mo} d\delta_{mo}(T_{C} - T_{mo}^0) \] (12)

On the left side of equation (12), the heat input is recorded, which is the sum of the heat of crystallization and the heat of overheating, as well as the heat accumulated by the hardened metal rim. On the right side of the equation is the heat consumption, which is the heat spent on heating the mold layer. To take into account the multidimensionality of the problem, the parameter \( F_{CR} \div F_{O} \div F_{Mo} \), is introduced, which takes into account the scattering of the heat flow from the crystallization front (\( F_{CR}, F_{Mo} \) – surface area of the crystallization front and the heated layer of the mold, \( m^2 \), \( F_{O} \) – casting surface area, \( m^2 \)).

For an infinite flat casting
\[ F_{CR} = F_{O} = F_{Mo}. \] (13)

For an infinite cylindrical casting
\[ F_{CR} \div F_{O} \div F_{Mo} = \left(1 - \frac{\varepsilon}{R}\right) \div 1 \div \left(1 + \frac{\delta_{mo}}{R}\right), \] (14)

where \( R \) is the reduced casting size, \( R = V_0/F_0 \) (where \( V_0 \) and \( F_0 \) are the volume and surface area of the casting). For ball casting and casting of finite sizes with an arbitrary shape (plate, cylinder, ball)
\[ F_{CR} \div F_{O} \div F_{Mo} = \left(1 - \frac{\varepsilon}{R}\right)^2 \div 1 \div \left(1 + \frac{\delta_{mo}}{R}\right)^2. \] (15)

Based on the introduced criterion, the thickness of the heated layer of the mold can be expressed as
\[ d\delta_{mo} = b \frac{F_{CR}}{F_{Mo}} d\varepsilon. \] (16)

Thus, the thickness of the heated layer of the mold is determined from equation (16), while
\[ b = \frac{L_C R \rho_m + c_m \rho_m(T_p - \bar{T}_C)}{c_{m0} \rho_{m0}(\bar{T}_C - T_{mo}^0)}. \] (17)

3. Results and discussion

Based on the described transformations, a differential equation is obtained for determining the solidification time of castings in metal molds, which for the general case has the following form:

\[ d\tau = \frac{b \cdot c_{m0} \rho_{m0} (T_{C} - T_{mo}^0)}{K_C(T_p - T_{env}) - \alpha_3 F_{Mo}(T_{surf} - T_{env})} \cdot \frac{F_{CR}}{F_{O}} d\varepsilon, \] (18)

For a casting of an infinite plate type
\[ d\tau = \frac{b \cdot c_{m0} \rho_{m0} (T_{C} - T_{mo}^0)}{K_C(T_p - T_{env}) - \alpha_3 (T_{surf} - T_{env})} \cdot d\varepsilon, \] (19)

for a casting of an infinite cylinder type
\[ d\tau = \frac{b \cdot c_{m0} \rho_{m0} (T_{C} - T_{mo}^0)}{K_C(T_p - T_{env}) - \alpha_3 (1 + \frac{\delta_{mo}}{R})(T_{surf} - T_{env})} \cdot (1 - \frac{\varepsilon}{R}) d\varepsilon, \] (20)
for a casting of a ball type

\[ d\tau = \frac{b \cdot c_{Mo} \rho_{Mo}(T_e^2 - T_{Mo}^2)}{K_H(T_p - T_{env}) - \alpha_3 \left( 1 + \frac{\rho_{Mo}}{\rho_{env}} \right) \left( T_{sur} - T_{env} \right)} \cdot \left( 1 - \frac{e}{R} \right)^2 \cdot d\epsilon . \]  

(21)

The developed mathematical models make it possible to take into account while calculating the solidification time, not only the thermophysical properties of the casting materials and molds, but also the heat transfer conditions at the ‘casting – mold’ boundary, the heat-flux dissipation from the crystallization front for the mold heating and into the environment.

4. Conclusion

The methodology for calculating the solidification time of castings and ingots in metal mold was developed. The method takes into account the thermophysical properties of casting materials and molds, the heat transfer conditions at the ‘casting – mold’ interface, the heat-flux dissipation from the crystallization front for the mold heating and into the environment.

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