1. Introduction

Performing winter road maintenance (WRM) is an important activity, necessary to keep the traffic on our roads in winter period safe and smooth. The appropriate authorities at all levels supposed to make sure roads are passable are the nation-wide authorities that administer the roads of the categories II and III, and the municipal authorities that are in charge of the roads and walkways in municipalities. Yearly, they all spend not a little money on the job. The finance they have to spend is not unlimited, of course, and the companies carrying out WRM often face the problem of having winter road maintenance machinery that is not functioning properly. This sometimes means the companies are not able to react to calamities sufficiently enough, which might result in dangerous traffic situations.

When WRM is well planned and organized, it is possible to achieve remarkable cost and energy savings, and at the same time to keep or even improve the operation level and quality of WRM.

2. Problem formulation

Let us have a road network consisting of individual road segments. The road network is divided into several regions. In the network there are fixed maintenance centres. Each of them has a given number of available winter maintenance vehicles (gritting vehicles, ploughs), as well as a given number of sufficiently dimensioned gritting material depots.

The WRM operation plan may be formally defined as follows: after a calamity emergency has been announced, the road network needs to become passable within a given time limit. The road categories determine the maintenance priority. Vehicle routings need to be planned and synchronized, so that the given time limit can be kept. The individual routings are represented by a sequenced list of road segments. The starting and finishing points of the list are identical – the maintenance centre. The routings do not need to be planned to be mutually excluding, which means some of the road segments can be part of several routings. Thus the routings of individual vehicles consist of two types of road segments: those the vehicle does WRM on, and those the vehicle just uses for transit.

The goal of the solution is to propose the minimal number of WRM vehicles necessary to keep the operation plan time limit. The proposal does not deal with changing the locations of the winter maintenance centres and gritting material depots, or with changing the road segments allocation to the individual regions.

3. Proposed solution

We are going to describe the road network of one region as a connected oriented graph \( G = (V, E) \), where \( G = \{v_1, v_2, ..., v_n\} \) is a vertex set and \( E = \{(v_i, v_j) : v_i, v_j \in V, i \neq j\} \) is an edge set. With every edge \((v_i, v_j)\) is associated a non-zero length \( c_{i,j} \). Let \( d_{i,j} \) be the length of the shortest path connecting the vertex \( v_i \) with the vertex \( v_j \) in the graph \( G \). Let \( D \subset V \) be a set of the centres, each of them with a given number of vehicles.

The analysis of the problem has determined the type of the task to be solved: it is a modification of CPP (Chinese postman problem). The modification is a result of the gritting vehicles capacity limitation, which leads to CCPP (Capacitated Chinese postman problem).
as well as of the fact that not all of the edges require to be attended, which leads to RPP (Rural postman problem) [2].

The modifications named above make the task NP-difficult. That is why it was suggested to use a heuristic approach to solve the task. The approach is based on the primary Clark-Wright heuristic method [1] modified to attend the edges in the oriented graph. The method is further modified in dependence on the number of additional and limiting conditions [3].

The routings of individual gritting vehicles are calculated independently for each of the regions. We might describe the solution method in a simplified way as follows:
1. Selecting a random not gritted edge \((v_i, v_j)\) with the highest priority, we will call it the primary edge.
2. Allocating the primary edge \((v_i, v_j)\) to the nearest centre \(v_d \in D\) with a gritting vehicle available.
3. Transferring the gritting vehicle from the centre \(v_d \in D\) to the primary edge \((v_i, v_j)\) using the shortest path.
4. Gritting the primary edge and transferring to the centre, using the shortest path. The primary routing \(\{v_d, ..., v_3, v_2, v_1, ..., v_d\}\) has originated.
5. Selecting a not gritted edge \((v_j, v_j)\) with the highest priority, the one closest to the primary edge.
6. Gritting the edge \((v_j, v_j)\) and inserting it into the routing \(\{v_d, ..., v_j, v_j, ..., v_d\}\).
7. If the routing takes less time than the time limit is, returning to the point 5. Otherwise as follows:
8. Closing the routing.
9. If there is a not gritted edge remaining, returning to the point 1. Otherwise as follows:
10. Saving the solution and returning to the point 1. Repeating the steps \(n\)-times.
11. Selecting the best solution which is the one than requires the lowest number of vehicles within the region, and the sum of the routing times of all vehicles is minimal.
12. The end of calculation within one region.

4. Processing the documentation and creating the input data

An important, and often also a much time consuming part of solving the task was to prepare the input data of good quality for the calculations [3]. We used the road network of Žilina self-governing region divided into five regions as the source of the input data for the solution. When transforming the road network into the graph, we identified the road segments from the map documentation as the graph edges, and they were allocated with the starting and ending nodes. The total number of the road segments is 727, each of them is a two-way, which means the number of the edges in the graphs equals 1454. Table 4.1 shows the number of edges for the individual regions.

| Region          | Number of edges | Number of centres | Number of vehicles |
|-----------------|-----------------|-------------------|-------------------|
| Horna Povazie  | 326             | 5                 | 21                |
| Liptov          | 344             | 4                 | 21                |
| Turiec          | 310             | 2                 | 15                |
| Orava           | 270             | 3                 | 25                |
| Kysuce          | 204             | 4                 | 14                |

Except the length there are more data defined for each of the edges:
- **Routing** – it is supposed to differentiate the roads of various width arrangements and ways of maintenance. It distinguishes two and more lane two-way roads with only one lane to be gritted in each direction, one-way roads with one lane to be gritted, four and more lane two-way roads where two vehicles move simultaneously in each of the directions during the maintenance, two and more lane one-way roads where two vehicles move simultaneously in one direction.
- **Priority** – a numerical entry which distinguishes the importance of the individual road segments transportation-wise; it depends on the category which the road the segment is a part of falls into.
- **Demandingness** – it takes into consideration the consumption of gritting material according to the type and complexity of a road segment. From the viewpoint of their demandingness segments are divided into critical (very dangerous), to be gritted with the dosage of 300g/m² of mixed gritting material, dangerous with the dosage of 150g/m², others with the dosage of 80g/m², and not maintained segments.
• Gritting speed of the vehicle – it takes into consideration how much the demandingness of a segment enables the vehicle to move. For critical segments, the speed was set to 25 kms/hr, for dangerous segments it is 30kms/hr, and for other segments it is 40 kms/hr.
• Transit speed of the vehicle – it takes into consideration how much the demandingness of a segment enables the vehicle to move. The speed values for critical and dangerous segments are identical with the gritting speeds, the transit speed for flat segments is set to 50kms/hr.
• Number of the road – it helps identify a given segment.
• Name – identification entry (text description of an edge) which makes it possible to identify the exact position of a segment on a road map.

Vertex of the graph represent the vehicle centres, and they are allocated with the existing numbers of vehicles. Table 4.3 is an example for the region of Kysuce.

Numbers of vehicles in the region of Kysuce Table 4.3

| Centre                   | Number of gritting vehicles |
|--------------------------|----------------------------|
| Kysucke Nove Mesto       | 5                          |
| Makov                    | 5                          |
| Stara Bystrica           | 2                          |
| Podvysoká                | 2                          |

5. Assessing the current situation and proposing the required number of vehicles

We did calculations for the input entries above, and processed several of their outputs into tables. They provide an overview of numbers of gritting vehicles in the regions (Table 5.1) and in the

Proposed number of vehicles for the individual regions Table 5.1

| Region        | Current situation | Proposal | Surplus | Shortage |
|---------------|-------------------|----------|---------|----------|
| Horne Povazie | 21                | 21       | -       | -        |
| Liptov        | 21                | 26       | -       | 5        |
| Turiec        | 15                | 15       | -       | -        |
| Orava         | 25                | 23       | 2       | -        |
| Kysuce        | 14                | 16       | -       | 2        |

Proposed number of vehicles for the centres of the region of Kysuce Table 5.2

| Region       | Current situation | Proposal | Surplus | Shortage |
|--------------|-------------------|----------|---------|----------|
| Kysuce N. Mesto | 5               | 8        | -       | 3        |
| Makov        | 5                 | 6        | -       | 1        |
| Stara Bystrica | 2              | 2        | -       | -        |
| Podvysoká    | 2                 | 0        | 2       | -        |
| SPOLU        | 14                | 16       | 2       | 4        |

Gritting vehicle routing Table 5.3

| Seq number | Road number | Starting vertex | Ending vertex | Action          | Name of edge                          |
|------------|-------------|-----------------|---------------|-----------------|----------------------------------------|
| 1.         | III / 520003 | 4–53            | 4–32          | gritting        | ST BYSTRICA – RADOŠTKA, connection     |
| 2.         | II / 520    | 4–32            | 4–33          | gritting        | RADOŠTKA, connection – NOV BYSTRICA    |
| 3.         | II / 520    | 4–33            | 4–34          | gritting        | NOV BYSTRICA – VYCHYLOVKA, connection  |
| 4.         | II / 520    | 4–34            | 4–35          | gritting        | VYCHYLOVKA, connection – NOV BYSTRICA, east |
| 5.         | II / 520    | 4–35            | 4–77          | gritting        | NOV BYSTRICA, east – water reservoir BYSTRICA |
| 6.         | II / 520    | 4–77            | 5–118         | gritting        | nadzr BYSTRICA – ORAV LESNA, west      |
| 7.         | II / 520    | 5–118           | 4–77          | gritting        | ORAV LESNA, west – water reservoir BYSTRICA |
| 8.         | II / 520    | 4–77            | 4–35          | gritting        | water reservoir BYSTRICA – NOV BYSTRICA, east |
| 9.         | II / 520    | 4–35            | 4–34          | gritting        | NOV BYSTRICA, east – VYCHYLOVKA, connection |
| 10.        | II / 520    | 4–34            | 4–33          | gritting        | VYCHYLOVKA, connection – NOV BYSTRICA  |
| 11.        | II / 520    | 4–33            | 4–32          | gritting        | NOV BYSTRICA – RADOŠTKA, connection    |
| 12.        | III / 520003| 4–32            | 4–53          | gritting        | RADOŠTKA, connection – ST BYSTRICA     |
| 13.        |              |                 |               |                 | loading                                |
| 14.        | III / 520003| 4–53            | 4–32          | transit         | ST BYSTRICA – RADOŠTKA, connection     |
| 15.        | II / 520    | 4–32            | 4–31          | gritting        | RADOŠTKA, connection – KLUBINA         |
| 16.        | II / 520    | 4–31            | 4–10          | gritting        | KLUBINA – KRAŠNO NAD K                 |
| 17.        | II / 520    | 4–10            | 4–31          | gritting        | KRAŠNO NAD K – KLUBINA                 |
| 18.        | II / 520    | 4–31            | 4–32          | gritting        | KLUBINA – RADOŠTKA, connection         |
| 19.        | III / 520003| 4–32            | 4–53          | transit         | RADOŠTKA, connection – ST BYSTRICA     |
individual centres (Table 5.2) that are necessary to keep the given 120-minute time period for WRM.

Apart from proposing the number of vehicles, we designed a routing for each vehicle, defined by the sequence of the edges and with the information whether the given edge is supposed to be gritted or just transited without gritting. There is an example in Table 5.3 with a routing proposed for one of the vehicles of the centre Stara Bystrica in the region of Kysuce.

6. Conclusion

The proposed solution suggests such numbers of vehicles that are able to carry out WRM in the required quality. We suggested routings for WRM vehicles for the time limits set for carrying out WRM.

To a great extent, the results are influenced by the values of the input data – gritting material consumption, WRM vehicles speed when gritting the road segments and when transiting the road segments that do not require to be gritted. Last but not least the results are also influenced by the division of the road segments into critical (very dangerous), dangerous and others. Specifying them will increase the reliability of the calculations.

Also the number of simplifications of the suggested solution has an impact on the accuracy of the results. Moreover, it is possible to improve the proposed heuristic, too, and more research is being done into the possibility to combine proposed routings within one centre, re-calculate them and look for potential improvements.

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