A Soft Decoding Algorithm with Hard Decision Boundary Factor for High-Order M-APSK Signal

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Abstract. High-order amplitude phase shift keying is adopted in the latest digital satellite broadcasting system, which presents new challenges to the design of a receiver demodulation scheme. This study focuses on the soft decision decoding algorithm. Traditionally, maximum likelihood soft decision based on log likelihood ratio estimation is often used as a decoding scheme. Accordingly, a soft decision algorithm based on hard decision boundary is designed and the corresponding soft decision estimation formula is derived. Numerical analytical results show that the proposed soft decision algorithm can improve the error performance of a system and substantially reduce the complexity of implementation.

1. Introduction
The digital satellite broadcasting is an important part of satellite communication, the industry and its demands have considerably changed over the years. Ultra-high definition television, high-speed IP data dissemination, increase in the number of users, and improvement of service quality requirements have driven continuous innovation in satellite broadcasting technology. These factors have resulted in breakthroughs in high access probability, high spectrum efficiency, and enhanced communication services.

In order to improve the spectrum efficiency of satellite communication systems and performance of the system at extremely low signal-to-noise ratio and expand the conversion efficiency of bandwidth and capacity of satellite communication systems, the DVB standard-setting organization ETSI released the DVB-S2 Extensions standard on March 4, 2014, its main body follows the DVB-S2 standard and is also compatible with the DVB-S[1] and DVB-S2[2] standards. In particular, DVB-S2X[3] provides additional options in coefficients, modulation schemes, and forward error correction, including high-order amplitude phase shift keying(APS), small FIR filter roll-off factors, and considerably advanced filtering techniques. These modifications immensely improve the efficiency of satellite transmission channels. Moreover, the spectrum efficiency of DVB-S2X can be effectively increased by 20%–30%.

The receiver processing algorithm should be improved according to the new technical features of DVB-S2X, particularly the adoption of high-order M-APSK(up to 256APSK). This research introduces a soft decoding algorithm with hard decision boundary factor for the high-order M-APSK signal.

2. System Model
In a DVB-S2X receiver, the demodulation module is consisted by matched filtering, timing offset correction, down sampling, frame synchronization, frequency offset estimation and correction, coarse phase offset estimation and correction, constellation decoding, forward error correction decoding,
descrambling, and cyclic redundancy (CRC) check[3]. The receiver processing block diagram is shown in Figure 1.

![Block diagram of the DVB-S2X Receiver Processing](image)

**Figure 1.** Block diagram of the DVB-S2X Receiver Processing

For low-order modulation, such as QPSK and 8PSK, the small phase and radius deviation in the demodulation process and processing the fine phase offset estimation and correction and DAGC are no longer necessary. We can use the hard decision method to accomplish the decoding given the sparse constellation points and large minimum Euclidean distance between them. However, the following properties are manifested in high-order modulation, particularly for 256 APSK: (1) eight rings are present in the constellation map, (2) the radius of each ring slightly varies, (3) each ring has 32 constellation points, (4) the phase difference between the adjacent constellation points is merely $\pi/16$. Therefore, the residual phase offset should be corrected after phase synchronization and the amplitude automatic control should be considered. The soft decision method should be used in constellation decoding to improve the demodulation performance.

In the high-order demodulation, the residual phase offset must be corrected (after the preceding processing of the coarse frequency offset, fine frequency offset is estimated and corrected and the phase offset estimated and corrected) using the Q-power loop algorithm. Bit decoding (soft demodulation algorithm is designed to improve the system performance for APSK modulation), de-interleaving, LDPC decoding, BCH decoding, descrambling, and CRC decoding are needed upon the completion of the processing in the front-end module of the receiver. Thereafter, the bit and frame error rates are calculated through a comparison with the transmitter.

3. **Low Complexity M-APSK Soft Decoding Algorithm**

In a communication system, the transmitter converts the encoded data into I and Q real signals through constellation mapping. In the receiver, the I and Q real signals should be decoded into bit streams in the hard demodulation. By contrast, the I and Q real signals are decoding into real sequences corresponding to each bit, and then decoded into bit streams through channel decoding in the soft demodulation. Compared with hard demodulation, soft demodulation retains more channel information, and it can improve the posterior probability of data decoding and reduce the bit error rate. The advantage of soft decision is performance improvement, but high complexity follows.

3.1. **Maximum Likelihood Soft Decision Based on Logarithmic Likelihood Ratio Estimation**

The commonly used method of soft decision is the estimation method based on logarithmic likelihood ratio. The maximum likelihood soft decision is used to solve the logarithmic likelihood ratio and retain the soft information of each bit. The transmission probability of 0/1 at the sender is the same. That is, $P(b_i = 0) = P(b_i = 1) = 0.5$. Thereafter, LLR can be written as follows:

$$ L(b_i | y) = \log \left( \frac{\sum_{h_i = 0} \exp \left( \frac{-1}{2\sigma^2} \| y - h_i \| \right)}{\sum_{h_i = 0} \exp \left( \frac{-1}{2\sigma^2} \| y - h_i \| \right)} \right) $$

(1)
where $S$ represents all symbols in the M-APSK modulation. Although the bit error rate (BER) of this method is low, and the performance is excellent, the computational complexity will exponentially increase with an increase in $M$. Therefore, this method will generally be replaced by the maximum logarithmic likelihood ratio method[4]:

$$L(b_i | y) = \ln \left( \frac{\max_{x \in \mathbb{R}^2} \exp \left( \frac{-1}{2\sigma^2} \| y - h \| \right)}{\max_{x \in \mathbb{R}^2} \exp \left( \frac{-1}{2\sigma^2} \| y - h \| \right)} \right)$$

which can be simplified as follow[5]:

$$L(b_i | y) \approx \frac{1}{2\sigma^2} \left[ \min_{x \in \mathbb{R}^2} \| y - h \| - \min_{x \in \mathbb{R}^2} \| y - h \| \right]$$

This estimation method is simple and has good performance for low-order modulation. However, a disadvantage of the high-order modulation M-APSK is the substantial increase in complexity and additional resources will be costed[6]. The following subsection discusses an improved algorithm (i.e., soft decision algorithm based on hard decision boundary) for the MLR estimation.

### 3.2. Soft Decision Algorithm Based on Hard Decision Boundary

A simple BPSK modulation mode is considered. Without loss of generality, we assume that the received signal is composed of I-channel and Q-channel signals. Thereafter, the received signal after channel equalization is provided as follows:

$$r = (h \ast y) / \| h \|^2$$

The soft decision result of $b_1$ based on the LLR maximum likelihood estimation method can be expressed as follows:

$$L(b_1 | y) = \rho R(r) , \rho = 2 \| h \|^2 / y$$

This logarithmic likelihood ratio can be considered $\rho$ multiplying the boundary of a hard decision: $R(r) = 0[7] [8]$. We can obtain the soft decision schemes on the basis of the hard decision boundary of 16, 32, 64, 128 and 256 APSK by extending this idea. Thereafter, we analyse the following specific scheme using the 64APSK as an example.

**Figure 2.** 64APSK signal constellation (4+12+20+28)

The 64 APSK signal constellation is shown in Figure 1. We assume that $b_1$, $b_2$, $b_3$, $b_4$, $b_5$, $b_6$, $b_7$ and $b_8$ represent the constellation points. Given that $b_7$ and $b_8$ determine the quadrant of the
constellation points, the hard decision boundary of b7 is \( R(r) = 0 \) and that of b8 is \( I(r) = 0 \). The constellation points of the second/third/fourth quadrant can be analysed by that in the first quadrant as they are gray codes. Thus, the angle should be normalized to the first quadrant and the conversion of the angle can be completed using \( \theta_r = \arctan(|I(r)/R(r)|) \).

\[ \text{Figure 3. Boundaries of } b_1, b_2, b_3 \text{ and } b_4 \]

Figure 3 is the hard decision boundaries of b1–b2 and b3–b4, respectively. Figure 2 (a) shows that the dotted line is the boundary line of the hard decision of the first bit. We can assess the soft information of the first bit, which is represented by the symbol bit of soft information multiplied by the minimum distance between the signal and the boundary line, according to the different positions of the received signal in the first quadrant. When the outputs of the hard decision are 0 and 1, the symbols are −1 and 1, respectively. For example, the sketch point in the above figure is located in the range of \( R \geq \frac{(R_3 + R_4)}{2}, \frac{(R_3 + R_4)}{2} \geq \frac{3\pi}{14} \) and its boundary is an arc \( R \geq \frac{(R_3 + R_4)}{2} \) and a ray \( \theta \geq \frac{3\pi}{14} \). Thus, the minimum distance between the point and the two boundaries is as follows:

\[ \min \{ R - \frac{(R_3 + R_4)}{2}, R \sin(\theta - \frac{3\pi}{14}) \} \]

(6)

Since that the hard decision output in this range is 0, the final soft decision output is as follows:

\[ -\min \{ R - \frac{(R_3 + R_4)}{2}, R \sin(\theta - \frac{3\pi}{14}) \} \]

(7)

The soft information formulas for all bits of 16, 32, 64, 128, and 256 APSK can be similarly derived on the basis of the preceding method. The formula of 256 APS is provided as follows:

\[ \text{• } \text{LLR}_{b_1} = R \left( \frac{R_4 + R_5}{2} \right) \]

\[ \text{• } \text{LLR}_{b_2} = \begin{cases} R - \frac{(R_2 + R_3)}{2} - R \sin(\frac{(R_2 + R_3)}{2}) & \frac{R_2 + R_3}{2} \leq R < \frac{(R_6 + R_7)}{2} \\ \min \{ R - \frac{(R_2 + R_3)}{2}, \frac{(R_6 + R_7)}{2} - R \sin(\frac{(R_6 + R_7)}{2}) \} & \frac{(R_6 + R_7)}{2} - R < R \sin(\frac{(R_6 + R_7)}{2}) \end{cases} \]

\[ \text{• } \text{LLR}_{b_3} = \begin{cases} -\min \{ R - \frac{(R_3 + R_4)}{2}, \frac{(R_3 + R_4)}{2} - R \sin(\frac{(R_3 + R_4)}{2}) \} & \frac{(R_3 + R_4)}{2} \leq R < \frac{(R_5 + R_6)}{2} \\ \min \{ R - \frac{(R_3 + R_4)}{2}, \frac{(R_5 + R_6)}{2} - R \sin(\frac{(R_5 + R_6)}{2}) \} & \frac{(R_5 + R_6)}{2} - R < R \sin(\frac{(R_5 + R_6)}{2}) \end{cases} \]

\[ \text{• } \text{LLR}_{b_4} = -\text{imag}(r) \]

\[ \text{• } \text{LLR}_{b_5} = \text{real}(r); \]

\[ \text{• } \text{LLR}_{b_7} = \begin{cases} \text{Rsin}(\theta_4 - \frac{\pi}{8}), \theta_4 < \frac{\pi}{8} \\ \min \{ \text{Rsin}(\theta_4 - \frac{\pi}{8}), \text{Rsin}(\frac{3\pi}{8} - \theta_4) \}, \frac{3\pi}{8} \leq \theta_4 < \frac{3\pi}{8} \\ \text{Rsin}(\frac{3\pi}{8} - \theta_4), \theta_4 \geq \frac{3\pi}{8} \end{cases} \]

\[ \text{• } \text{LLR}_{b_8} = \begin{cases} \text{Rsin}(\theta_5 - \frac{\pi}{16}), \theta_5 < \frac{\pi}{16} \\ \min \{ \text{Rsin}(\theta_5 - \frac{\pi}{16}), \text{Rsin}(\frac{3\pi}{16} - \theta_5) \}, \frac{3\pi}{16} \leq \theta_5 < \frac{3\pi}{16} \\ \text{Rsin}(\frac{3\pi}{16} - \theta_5), \theta_5 \geq \frac{3\pi}{16} \end{cases} \]
4. Numerical Analysis

The system performance of the hard and soft decision schemes based on the hard decision boundary of 16, 64, 128 and 256APSK are tested through MATLAB simulations. In the simulations, the minimum number of error bits is 200, the minimum number of simulation frames is 50, the maximum number of simulation frames is 1000, and the LDPC bit rate is 2/3.

![Figure 4. Performance comparison between the M-APSK soft and hard decisions](image)

The advantages of soft demodulation are evident with the increase of SNR (see Figure 4). The BER curves of the soft demodulation algorithms based on the hard decision boundary show rapid fading when SNRs are 8.5, 13.5, 15, 17, and 19dB in 16, 32, 64, 128, and 256APSK, respectively. When BER=$10^{-5}$, the decreases in SNR of the soft demodulation algorithm based on the hard decision boundary are 10, 10.5, 11.5, 15.5, and 20 dB in 16, 32, 64, 128, and 256APSK, respectively, compared with those based on the hard decision boundary. This finding shows substantial improvement in the system performance.

The performance comparison of the soft demodulation based on the hard decision boundary with the soft decision schemes based on the logarithmic likelihood ratio is shown as follows:

![Figure 5. Performance comparison between the 16APSK soft decision schemes](image)

The HDT performance is nearly the same as those of the logarithmic likelihood ratio estimation method and maximum likelihood soft decision method based on the logarithmic likelihood ratio (see Figure 5). When BER reaches $10^{-6}$, the SNR difference is below 0.5dB.

The simulation results manifest that the soft decision algorithm based on the hard decision boundary can immensely improve the system performance. Compared with the maximum likelihood
method based on the logarithmic likelihood ratio estimation, the HDT soft demodulation can substantially reduce the complexity of the system at the expense of small system performance. The required adder and multiplier resources are provided as follows:

| Resource comparison of the three soft decision methods |
|--------------------------------------------------------|
| Adder | HDT | Multiplier |
|-------|------|-------------|
| M-LLR | 16APS K | 192 | 5+m | 128 | 4 |
| ML-LLR | 32APS K | 480 | 6+m | 320 | 5 |
| ML-LLR | 64APS K | 1152 | 12+m | 768 | 12 |
| ML-LLR | 128APS K | 2688 | 10+m | 1792 | 9 |
| ML-LLR | 256APS K | 6144 | 5+m | 4096 | 5 |
| ML-LLR | 16APS K | 192 | 5+m | 128 | 4 |

Table 1 shows that \( m \) is a fixed number of the adder resources occupied by the IQ2AP module. By comparison, the soft decision method based on the hard decision boundary can change the computational complexity from \( O(M \log M) \) to \( O(\log M) \). Accordingly, the consumption of hardware resources is substantially reduced.

5. Summary
This research studied the decoding algorithm for high-order modulation signal. A soft decision algorithm based on hard decision boundary was designed in accordance with the traditional soft decision decoding algorithm, namely, maximum likelihood soft decision algorithm based on logarithmic likelihood ratio estimation. Accordingly, the corresponding soft decision estimation formula was deduced. The numerical results show that the proposed soft decision algorithm can improve the BER performance of the system and substantially reduce the complexity compared with the maximum likelihood soft decision method.

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