Stimulation of instability of flow of endothermic reacting and ionized gas past curved surfaces by sonic waves

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Abstract. Stability of plane-parallel and weak curved chemically reacting gas flow is studied. It is shown that for endothermic processes instability is possible both for high-frequency disturbances and low frequencies. Instability of flow is not associated with viscous properties of matter and has a baroclinic character. Comparison with experiment and with previous similar theoretical investigations is made. Instability of flow with ionization in air can be stimulated by an external ultrasonic radiation. Research is important for control the transfer coefficients in gas during the transition to turbulent flow and for control the flow and aerodynamics of wrapping.

1. Introduction

It is well known that turbulent flow in many ways much more preferable than laminar. So, for example, it is improving the profile shape due to later detachment of the turbulent flow wrapping around the bodies. Flows inside the channels have more filled profile, and heat transfer on a wall is much higher.

This is connected with the fact that in turbulent flow transport coefficients (viscosity, thermal conductivity) is higher than in laminar [1]. However, it is important that turbulent flow would be developed, i.e. a characteristic scale, wavelength of perturbation would be low and the perturbation frequency would be high.

Study of influence of chemical processes in gas on flow stability and especially endothermic processes such as dissociation and ionization emerged from experimental and theoretical study of instability of flow behind bow shock wave in some polyatomic gases [2,3].

In the process of studying this effect it was understood, that in addition to influence of chemical processes on the effect, influence of several other factors should be taken into account. The first of these is the curvature of the current, because for fairly large curvature flow instability will happen even in the ideal gas.

The second factor is reduction of the ratio of specific heats of gas, as for specific heats equal to 1 the flow and shock wave become unstable [4]. Of course, perturbations wavelength should be taken into account also because for some wavelengths instability can manifest itself, while for others - can’t.

Boundary conditions on the surface of solids, such as surface roughness certainly affect the stability of gas movement, as in other tasks of mathematical physics. However, in the problem stated the internal cause of flow instability explores, for example, state of the matter, chemical reactions in it. In the investigated problem it is important to find out exactly how the internal processes may affect the appearance of instability. Setting recalls Orr - Summerfield task of flow instability owing to viscous
properties of gas [5]. However accounting boundary conditions only will complicate the understanding of the causes of instability.

2. Setting of the problem
The proposed formulation of the problem considers plane layer of gas flow. The transverse distribution of gas dynamic parameters and their transverse derivative in it, as well as the power of release or absorb energy through chemical-physical reactions are characteristic parameters, changing from task to task. Boundary conditions are lacking.

Viscosity of the gas is ignored so, because its accounting could complicate understanding, as accounting boundary conditions. Thus, the internal causes of instability are explored, which are not related to the influence of boundary conditions and viscosity. In this formulation, it is naturally to explore dispersion equation. This problem, of course, is easier than the general setting, with accounting the boundary conditions [6,7,8].

Incorporation of boundary conditions corresponds to the incorporation of the transverse velocity gradient in the layer, which occurs due to the zero velocity of the flow at the border. Without taking into account the transverse gradient, thus, emergence of instability cannot be realized. Curvature flow in persistent Cartesian coordinates also leads to a deviation from the vertical velocity profile. Therefore parameter of crookedness should be proportional to the deviation of transverse derivative speed profile from zero.

Of course, flow curvature should be weak so flat-parallel flow approximation could be applied for. This condition is feasible if the member that contains the parameter of crookedness will be a lot less than the rest members of the equations. It is clear that wavelength of perturbations must be among the task parameters.

Finally, the influence of the course of physical-chemical processes in gas on the emergence of the instability is most important for researched problem factor. The greatest influence on gas flow these processes have through heat exchange between internal and kinetics degrees of freedom, i.e. through gas heating or cooling as a result of physical-chemical processes. Therefore, the study should take into account the speed of release (release power) or absorption of heat as a result of processes in gas, more precisely, the partial derivatives of the power on temperature and density of gas.

In the below setting task is linearized by the curvature (by the transverse gradient of velocity profile), because investigation of a large curvature involves the necessity of direct numerical simulation of occurrence of instability, which today is nearly impossible in real flows [9].

3. Used equations
The problem statement is based on equations derived in [10]. It is known that the instability of plane-parallel flows occurs when the Reynolds number \(Re\) is big enough and the source system of linearized hydrodynamics equations can be written in a shorter form [10]

\[
\rho[i(U-c)f + U'\phi] = -\left(\frac{i}{\gamma M^2}\right)\pi
\]

\[
\rho i\alpha^2(U-c)\phi = -\left(\frac{i}{\gamma M^2}\right)\pi'
\]

\[
i(U-c)r + \rho'\varphi + \rho(if + \varphi') = 0
\]

\[
\rho[i(U-c)\theta + T'\phi] = -(\gamma-1)(if + \varphi')P + \theta \frac{dQ}{dT} + r \frac{dQ}{d\rho}
\]

\[
\pi = \frac{r}{\rho} \frac{\theta + \frac{\theta}{T}}{T}
\]

Derivatives denoted with strokes are taken on the variable \(\gamma\); \(\rho\) is the density of the gas; \(U\) is the velocity of the gas, \(p\) - pressure, \(T\) is temperature, \(c\) is the speed of sound; \(M\) is the Mach number of the flow, \(\gamma\) is the ratio of specific heats, \(\mu\) is dynamic shear viscosity; \(f\), \(\varphi\), \(r\), \(\theta\), \(\pi\)-amplitude of enthalpy, gas flow rate, density, temperature and pressure respectively.

One takes into account derivatives of the volumetric capacity of heat source (or heat absorption) as
on the temperature and gas density \(dTQ, dpQ\). Crookedness of current lines is modeled by specifying the speed gradient across layer for flat-parallel currents - \(U'\). Transverse direction \(y\) is selected inside the layer, i.e., the direction of propagation of pressure perturbations. With this choice of coordinate system convect flow is modeled by positive gradient \(U'\), and concave - negative. The original system of equations for small perturbations of the flow is reduced, as it is known [6,7], to differential equation of the second order:

\[
\pi'' = \pi' \frac{2U'}{U - c} + \pi \alpha^2 \left\{ 1 - \frac{(U - c)^2 \gamma M^2}{\tau} \cdot \frac{\alpha(U - c) + i\delta Q}{\gamma(K_s)} \right. \]

(1)

Here pressure perturbations \(\delta p = \pi(y) \exp[i\alpha(x - ct)]\) and \(T(y), U(y), \rho(y), P(y)\) - distributions of temperature, velocity, density, and pressure across the layer; \(c\) is the speed of sound in the gas, \(\alpha\) and \(\beta\) - wave numbers of disturbances along the flow (\(x\)-coordinate) and transverse the flow (\(y\) coordinate). \(M\) - Mach number, calculated through characteristic values of \(U\) and \(T\), \(d = (\rho d_p Q)(Td_Q)^{-1}\); \(i\) - the imaginary unit. As in the theory of boundary layer stability [10,11], in this work it is accepted that instability occurs when \(\Im \delta p > 0\). Solution of (1) is looked up as usual as follows [10,11]:

\[
\pi'' = -\beta^2 \pi_0 \exp(i\beta y).
\]

Substituting these expressions as well as expressions for \(\pi\) in equation (1) and reducing the exponent, we get:

\[
-\beta^2 = \left( \frac{2U'}{U - c} \right) \beta + \alpha^2 - \left( \frac{\gamma M^2}{\tau} \right) \left[ \frac{i\beta+\delta Q}{\gamma(K_s)} \right].
\]

So, the study on stability is reduced to dispersion equation of 4-th degree relative \(Z = i\alpha (U - c)(K_s)^{-1}\):

\[
Z^4 + \kappa Z^3 + Z^2 + \left[ \frac{k(1-d)}{\gamma} + g \right] Z + g \kappa(1-d) = 0
\]

(2)

that depends on the 4 parameters: \(\gamma, d = (\rho d_p Q)(Td_Q)^{-1}\) \((K^2 = \alpha^2 + \beta^2, s^2 = PpM^{-2} = \gamma P^2 U^{-2})\). Weak crookedness of flow means that \(|g| \leq 0.1\). I.e. wavelength of perturbations must be more than 20 times less than the typical distance of influence different areas of flow each to other, that justifies the using of the local flat-parallel setting in the problem statement.

4. Results of computations.

Complex roots of the equation of the fourth degree (2) were calculated by the method of Ferrari. Four complex roots were obtained in the calculations. However, three of them are trivial-obvious.

One solution is proved to be more informative. For endothermic physical-chemical processes with normal dependence of reactions speed on the pressure and temperature \(\kappa > 0, 1 > d > 0\) and for convex currents area of stability \((\Re Z < 0)\) are realized for average values of \(\kappa\) (figure 1). Reduction of \(\gamma\) and \(d\) or increasing of curvature parameter \(g\) as expected leads to a decrease in the area of stability. Calculations showed that the equation (2) for the specified parameters has a "wave" solution (2 complex conjugate solution) and 2 purely real solutions, one of which \(\approx -g\) for any \(\kappa, \gamma\) and \(d\) (it could be called the "gradient" solution). The width of the field of stability depends on values of \(d, \gamma\) and \(g\). Increase in \(\gamma\), and \(d\) (for constant \(g\)) leads to expand the range of stability. A particularly sharp increase can occur if \(d\) is little. For instance, stability range increases almost in 2 times with the growth of \(\gamma\) from 1.2 (polyatomic gases) to 1.4 (air) when \(d \leq 0.1\). For zero \(g\), as can be seen from figure 1, instability disappears.

It should be noted that the instability of the flow has inviscid nature and does not require that the second derivative of the velocity profile equals zero. This means that instability is not barotropic and has baroclinic nature [5].
Figure 1 Real part of complex roots of equation (2) as function of dimensionless perturbations wavelength $\kappa$ in depending parameters $d$, $\gamma$ and $g$. Number under curves are values of $d$. Solid curves $- \gamma = 1.2$. Dashed lines $- \gamma = 1.4$. Instability corresponds $Re(Z) > 0$. Shaped white arrows show areas of wavelength $\kappa$, for which unstable gas motion is realized in the plane-parallel weak curved flows for $g = 0.1$ and $d = 0.1$ as well as for $g = 0.01$ and $d = 0.4$.

The results of the calculations are consistent with calculations of L M Mack [6] and G V Petrov [7] that have nothing to do with crookedness flows. In those calculations, the gradient of the transverse profile speed was greater than zero due to the boundary conditions. In these works, the instability of plate-parallel flow of chemically dissociating gas around a plate is discovered when posting boundary conditions. They are second and third modes according to the classification of L M Mack [6] and not the primary viscous mode of perturbations. Perturbation increments are consistent with increments in the above presented calculations.

5. Evaluation of instability regimes

The equation parameters should be expressed through the basic values of medium. The parameter $d = (\rho d p Q)(T d p Q)^{-1} = n/u$ is proportional to the reaction order, $n$, and is inversely proportional to the relative energy of activation, $u = \Delta E(R_g T)^{-1}$ because absorption of energy $Q$ is proportional to the rate of reaction $Q = (\Delta E(R_g)^{-1}) A \rho^n \exp(-u)$, where $\Delta E$ is the warmth of one act of reaction, $R_g$ - gas constant. The parameter $\kappa$ is also easy to find: $\kappa = d r Q(K s)^{-1} = (M(P\rho)^{-1/2}) A u^2 \rho^n e^{-u} \lambda (K \lambda^{-1}, \lambda$ is wavelength of perturbations, $s = (P\rho)^{1/2}(M)^{-1}, M$ is the Mach number of the flow and $A$ - preexponent of constant of reaction.

Consider plasma flow in the neck of the air intake. The radius of curvature of the surface $R_k$ equals 0.17 m and $g = 0.01$. It appears from calculations for $g = 0.01$ that instability occurs when $\kappa < 0.02$ or when $\kappa > 10^4$. For weak shock waves in the neck (flow Mach number $M \approx 0.5$) pressure $p \approx 6 \cdot 10^5$ Pa, density $\rho \approx 0.1$ kg·m$^{-3}$, the temperature of the atoms and ions $T_a \approx 10^3$K, and the temperature of electrons $T_e \approx 1.2 \cdot 10^4$K. At this regime, the ionization process goes through associative ionization, involving electronically-excited levels [12]:

$$N + O + 2,8 \text{ ev} \rightarrow NO^+ + e$$
\[ k_r = 0.43 \cdot 10^{-11} \exp \left( -\frac{32535}{T} \right) \text{m}^3(\text{kg s})^{-1} \]

It could be considered that \( \Delta E \) equal to difference between associative ionization energy of NO and electronically-excited levels energy of \( \text{O}_2(\text{a}_1\Delta) \), so \( u = \Delta E (R_gT)^{-1} \approx 22.3 \), and for binary reaction parameter \( \delta \approx 0.1 \). Then: \( \kappa = "B" \cdot M \lambda \) (\( \lambda \) – in meters), \( g = "C" \cdot M^2 \lambda / R_k \). Coefficient "C" depends on the direction of propagation of the perturbation in the layer of the flow. It is maximal at the direction of 45° to the direction of flow. Herewith "C" = 1.41. At sound speed 600 m s\(^{-1}\) behind the shock wave, for reaction constant of associative ionization \( k_r \) and density \( \rho = 0.1 \text{ kg m}^{-3} \) coefficient "B" is equal to 0.75 m\(^{-1}\).

Thus, the instability of the air plasma flow in the neck of the air intake should occur for short waves (wavelength \( \lambda < 0.05 \text{ m} \), frequency \( f > 8 \text{ kHz} \)), which is close to the boundary of the range of ultrasound in air (20 kHz). Another boundary corresponds to infrasound vibrations and rise increment (\( \text{ReZ} \)) for it grows very slowly with increasing wavelength, as can be seen from figure 1 (right branch).

Ultrasound disturbances in gas and plasma rapidly damped, so to stimulate shortwave instability one should use external exposure by ultrasonic waves.

Consider the flow instability behind bow shock wave when gaseous \( \text{CF}_4 \) flows around segmental body [2]. Constant of decomposition of \( \text{CF}_4 \) can be found in [13]:

\[ \text{CF}_4 + M \rightarrow \text{CF}_3 + F + M; \]

\[ K_{f1} = 6.15 \cdot 10^{31} \cdot R \cdot T^{-4.64} \exp \left( -\frac{122421}{RT} \right) \text{cm}^3(\text{mol} \cdot \text{s})^{-1} \]

The radius of curvature of blunting segmental body \( R_k = 0.04 \text{ m} \), so \( g = 0.1, \Delta E = 122 \text{kcal m}^{-1} \). Then constant "B" equals 1.64·10\(^5\) m\(^{-1}\). Gas temperature \( T = 1800 \text{ K} \), gas density behind the shock wave is equal to 8 kg m\(^{-3}\). The parameter \( u = 30.6 \). The process of dissociation is of the second order in density, so the \( d \) parameter is 0.066. For \( g = 0.1 \) of high-frequency instability corresponds to \( \lambda < 1.2 \cdot 10^{-5} \text{ m} \) (frequency 3·10\(^8\) Hz or more) that is unrealistic.

More realistic case is low-frequency instability. For \( g = 0.1, d = 0.1, \gamma = 1.2 \) according to figure 1 border instability: \( \kappa = 5 \) (wavelength \( \lambda = 5\cdot10^{-5} \text{ m} \)). But for this \( \lambda \) we have \( g = 4.4 \cdot 10^{-4} \) and border instability \( \kappa = 1000 \), not value 5 estimated for \( g = 0.1 \). It is easy to verify that the maximum value of wavelength \( \lambda \) where \( g \) will remain constant, equal to 10 mm, which corresponds to the observed perturbations in the experiment [2]. The parameter \( \kappa \) depends both on the wavelength and on the parameter \( g \), therefore, reducing of the wavelength border will increase border of \( g \). Unstable wave length of perturbations must be defined for each task by selection of appropriate parameters, if this selection will converge.

6. Conclusions.
One of the solutions to the dispersion equation of disturbances in flat-parallel weak curved reacting gas flow shows the instability for endothermic reactions, as for shortwave disturbances and for longwise disturbances, which disappear with the disappearance of crookedness.

The detected effect depends on the ratio of specific heats of gas and on the curvature of the streamlined surfaces, as well as on the speed of release or absorption of energy due to physical and chemical processes. Instability has baroclinic nature, and for example, flows of air plasma unstable at high-frequency of disturbances, order 8 kHz or more, regardless of whether there is a bend of speed profile or not (\( U'' = 0 \)), as it is required for the barotropic instability in the theory of Orr-Summerfield.

Because the attenuation of ultrasonic frequency perturbations in plasma is great, stimulation of plasma flow instability needs to be realized through the outside influence of ultrasonic waves.

As possible application it should be noted three positions. All of them are only hypothetical opportunities. The first is rising of compression in hypersonic intake, because shock wave in neck should be unstable due to instability of plasma flow. The second is possibility of reduction of
aerodynamic drug due to the later separation of boundary layer of turbulent plasma flow. And third is an improvement of efficiency of plasma-chemical technologies due to more uniform conditions of turbulent plasma flow across the section of tube and greater thermo flux to reactor walls.

References

[1] Loitsyanskii L G 1987 Mechanics of Liquids and Gases, Int. Series of Monographs in Aeronautics and Astronautics: Division II: Aerodynamics ed. R T Jones, W P Jones (Oxford: Pergamon)
[2] Baryshnikov A S, Basargin I V, Bobashev S V, Monakhov N A, Popov P A, Sakharov V A and Chistyakova M V 2016 J. Eng. Phys. Thermophys. 89 1232
[3] Baryshnikov A S 1996 Tech. Phys.Lett. 22 667
[4] Gurevich L E and Rumyntsev A A 1970 J. Exp. Theor. Phys. 58 1395
[5] Monin A S 1986 Sov. Phys. Usp. 29 843
[6] Mack L M 1975 AIAA J. 13 278
[7] Petropv G V 1979 Development of perturbations in boundary layer (Novosibirsk: ITPM, Siberia Dep. of RAS) p104 [in Russian]
[8] Molevich N E 1999 Izvestia RAS, Mechanics of fluids and gases №5 82 [in Russian]
[9] Schumann U, Grötzbach G, and Kleiser L 1980 Prediction methods for turbulent flows ed. V Kolman (Washington, Hemisphere publishing corporation) p103
[10] Gaponov C A and Maslov A A 1980 Development of perturbations in compressd fluxes (Novosibirsk: Nauka) [in Russian]
[11] Schlichting H 1974 Grenzschicht-Theorie. (Karlsruhe: Verlag G.Braun)
[12] Gladkov A A, Polyanskiy O Yu, Agafonov V P, Vertushkin V K 1972 Nonequilibrium physicochemical processes in aerodynamics ed. G I Maykapara (Moscow: Mashinostroenie) [in Russian]
[13] Modica A P and Sillers S J 1968 J. Chem. Phys. 48 3283