Formation of the separation characteristic of ore material thickening based on the model predictive control method

V Morkun¹, N Morkun¹, V Tron¹, O Serdiuk¹, I Haponenko¹ and A Haponenko¹
¹ Kryvyi Rih National University, 11 Vitalii Matusevych Str., Kryvyi Rih, 50027, Ukraine
E-mail: morkunv@knu.edu.ua, nmorkun@knu.edu.ua, vtron@knu.edu.ua, serdiuk@knu.edu.ua, haponenko@protonmail.com, a.haponenko@protonmail.com

Abstract. The aim of the investigation is to form an optimal separation characteristic of beneficiation processes on the basis on operational information on time-varying of their parameters exemplified by the process of thickening ore raw materials. Methods of research. In the research work, the following methods are used: analysis of scientific research and practical experience; statistics methods and the probability theory for estimation of experiment results; methods of analytical synthesis and numerical simulation; methods of model predictive control for developing control algorithms of the thickening process; numerical simulation methods for synthesizing and analyzing a mathematical model. The scientific novelty of the investigation is in finding optimal values of the control horizon and the prediction horizon in terms of quality control for a single-channel system of model predictive control of ore raw material thickening. Practical significance involves development of methods and software for determining the control horizon and the prediction horizon values of the single-channel system of model predictive control of the process of ore raw material thickening that are optimal from the point of view of quality control, this enabling optimization of separation characteristics of ore raw material thickening. Results To form a separation characteristic of the process of ore raw material thickening based on model predictive control for the single-channel control system of the thickening process, satisfactory control results are achieved by setting the control horizon equal to one interval. For this value, the quadratic control error does not exceed 0.1452-0.1474. A further increase in the prediction horizon is not feasible since it does not allow significant reduction of the quadratic control error. At the same time, the value of 3-5 intervals is sufficient for prediction horizons. These values are determined by an increase in computational complexity of prediction by 10-20 intervals, which causes a slight decrease in the quadratic control error.

1. The problem and its connection with scientific and practical tasks
Optimization of the technological line of ore beneficiation requires determination of a resulting separation characteristic with the operating point within the optimal separation limit. Yet, in flows of raw materials of the ore processing line, significantly different varieties of raw ore as for their mineralogical, physical, chemical and mechanical characteristics can be processed at different stages [1–3].

Ore processing plants involve such multistage technological processes to prepare ore for subsequent separation as crushing and grinding [4, 5]. The main goal of these operations is in releasing useful ore aggregates and reduce the size of mineral grains to 0.1 mm or less to
separate particles of different minerals. The size of inclusions of the useful component in some cases exceeds several classes of particles by its size used to assess particle size distribution of beneficiation products at ore beneficiation plants [6, 7]: “+3”, “-3 + 1”, “-1 + 0.5”, “-0, 5 + 0.25”, “-0.25 + 0.125”, “-0.125 + 0.071”, “-0.071 + 0.056”, “-0.056 + 0.044”, “-0.044 + 0”.

Therefore, it is necessary to study calculation parameters of separation characteristics of technological processes of ore processing, in particular ore raw material thickening, based on operational information about dynamics of their parameters.

2. Analysis of researches and publications

According to the research results [4, 8, 9] for quantitative evaluation of mineral products besides distribution of mineral particles in fractions with varied physical properties $\xi$, it is advisable to use the index of distribution of useful components. Indices and also allow quantitative evaluation of ore materials. To quantify efficiency of technological units, it is proposed to use separation characteristics which determine the degree of mineral fractions release $\epsilon$ into final products of ore processing. The releasing of useful components from initial raw ore mass and moving them into the final product becomes possible due to differences in physical properties $\xi$ of a given useful component. The action of physical forces in the working zones of ore processing units separates particles of raw ore in such way: certain physical properties ($\xi > \xi_p$) of some part of ore particles causes them to move in the different way than other part of ore particles with other physical properties ($\xi < \xi_p$).

A separation characteristic is a continuous function that determines a dependence of release of elementary fractions $[\xi, \xi + \Delta\xi]$ into the concentrate on the physical property $\xi$ [4, 9]. The perfect separation characteristic is noted for a step-like appearance with a jump at a point $\xi_p$. This point corresponds to the elementary fraction, which is half released into the concentrate, half – into tailings. Real technological units have an imperfect separation characteristic with some inclination $\tan\alpha = d\epsilon(\xi)/d\xi$ at the working point, which corresponds to the separation boundary $\xi - \xi_p$. The value of the useful component content $\beta$ in it correlates with the physical property $\xi$ of each mineral particle: $\beta$ increases with increasing $\xi$ (or vice versa).

In [10], flocculation of iron ore tailings in a thickener (thickener) at the ore beneficiation plant is investigated. There are obtained dependencies of flocculation efficiency (sediment volume, settling rate, turbidity) on dosage of flocculant. The disadvantage is in that operational control of thickening becomes more complicated as a result of the chemical agents usage.

The research [11] proved that the sediment flow and also the depth of the sediment layer caused increasing residence time for particles in the thickening process. Simultaneously, the disadvantage here is operational control methods for characteristics of ore particles that the work does not suggest.

The research results of ultrafine particles slurry dehydration with the solid phase are presented in [12]. Presented results does not consider formation of control actions directly during the ore processing to increase thickening quality indices.

The research [13] applies methods of calculating fluid dynamics to improving quantity and quality indices of thickening process. In the research a model for balancing the number of particles was used. It is noted that simulation of the feed source enables forecasting solid particle flows and fluids, yet sufficiently accurate results are not provided. Therefore, the authors apply a model for balancing the number of particles. The research results of the balance between hydrodynamic and physicochemical requirements for flocculation are described. There are presented some basic conclusions on optimization of the feed channel design and potential application of the obtained results to controlling the thickener. That’s why further research in area of information support of the control system is required.

The research [14] proves that when concentrating a flocculant solution, there appears significant differences in concentration of sands with and without displacement. In the process of
the proposed dependencies implementation to control methods of ore thickening must be taken into account the differences in mineral’s varieties characteristics.

Methods of cleaning of technological water, used in iron ore processing was proposed in [15]. According to this approach usage of chemical methods of cleaning in thickening process was proposed.

The research [16] suggests neural network approach for creation of the model of ore processing plant. In this case additional investigation in the field of information support is required. Results of flocculation characteristics research is proposed in [17]. According to this approach usage of chemical methods in thickening process was proposed.

The research [18] considers the influence of ultrasonic radiation characteristics on final concentration of thickener sands and flocculation processes in this unit. It is established that ultrasound is able to significantly improve concentration of overflow, while its frequency and capacity are the most important factors. The research [19] investigates into slurry treatment by means of ultrasound and its influence on electrochemical and flocculation designated to increase efficiency of precipitation processes.

3. Problem statement
The results of the analysis of scientific research and practical developments have shown that application of the ultrasonic methods to thickening in developing a method of automated control of ore raw material thickening requires additional research.

This research is aimed at developing a method of automated control of ore raw material thickening to optimize a separation characteristic of this technological process when concentrating several mineral-technological varieties of ore based on the model predictive control method.

4. Presentation of materials and results
An example of comparing the useful component content in a certain grain-size class to the yield of this class distributed along the technological line of beneficiation provided on figure 1. The results of testing the technological line are obtained under the supervision of professor Oliynyk [20,21].

Figure 1. Useful component content in the grain-size class and the yield of this class distributed along the technological line.

The final fractions are released by dividing the total range \([\xi_{\text{min}}, \xi_{\text{max}}]\) of changes in the physical property \(\xi\) of ore particles by a certain number of fractions [4,22]. The solid fraction in the concentrate is equal to the ratio of the solid yield of a given fraction in the concentrate and the initial material:

\[
\bar{\epsilon}_{ik} = \frac{P_{ik}}{P_{in}} = \frac{Q_k^{\xi_i}}{Q_{in}^{\gamma_i}} = \frac{Q_k^{\gamma_i}}{Q_{in}^{\gamma_i}} \frac{(\xi_i)\Delta\xi_i}{\Delta\xi_i} \tag{1}
\]
where \( Q_k, Q_{in} \) is the yield of solids in the concentrate and initial materials respectively, t/h; \( \bar{\gamma}_{ik}, \bar{\gamma}_{in} \) is the yield of the fraction in the concentrate and the initial material respectively; \( \gamma_{ik}(\xi_i), \gamma_{in}(\xi_i) \) is distribution of solids in fractions in the concentrate and the initial material respectively. Application of this formula to each fraction allows obtaining a set of released fractions. When the condition \( \Delta \xi_i \to 0, n \to \infty \) is fulfilled, the mentioned set becomes a continuous function, i.e. the separation characteristic [4].

The values of separation points of the first sizing stage calculated by formula (1) and the data of experimental studies [20,21] are presented in table 1.

### Table 1. Results of calculating the separation characteristic of the first sizing stage.

| Grain-size class, mm | Input fraction yield, % | Output fraction yield | Fraction size | Separation characteristic |
|----------------------|-------------------------|-----------------------|---------------|--------------------------|
| -1+0.5               | 23.40                   | 1.50                  | 0.500         | 0.0000                   |
| -0.5+0.25            | 19.10                   | 8.00                  | 0.250         | 0.0000                   |
| -0.25+0.125          | 8.90                    | 10.90                 | 0.125         | 0.0224                   |
| -0.125+0.071         | 5.70                    | 11.00                 | 0.054         | 0.1461                   |
| -0.071+0.056         | 4.10                    | 9.40                  | 0.015         | 0.4272                   |
| -0.056+0.044         | 2.40                    | 6.00                  | 0.012         | 0.6731                   |
| -0.044+0             | 23.00                   | 53.20                 | 0.044         | 0.7997                   |
| **Total**            | **100.00**              | **100.00**            |    -          |    -                     |

On figure 2 it is shown separation characteristics of sizing units of the technological line at the ore beneficiation plant.

Numerical values of separation characteristics are given in table 2.

### Table 2. Results of calculating separation characteristics of sizing units.

| Grain-size class, mm | Classifier 1 (overflow) | Classifier 2 (sands) | Thickener 1 (sands) | Classifier 3 (overflow) | Thickener 2 (sands) |
|----------------------|-------------------------|----------------------|---------------------|-------------------------|---------------------|
| +3                   | 0.0000                  | 0.0000               | 0.0000              | 0.0000                  | 0.0000              |
| -3+1                 | 0.0000                  | 0.0000               | 0.0000              | 0.0000                  | 0.0000              |
| -1+0.5               | 0.0224                  | 0.1376               | 0.0000              | 0.0000                  | 0.0000              |
| -0.5+0.25            | 0.1461                  | 0.2016               | 0.0000              | 0.0000                  | 0.0000              |
| -0.25+0.125          | 0.4272                  | 0.2510               | 0.0000              | 0.0000                  | 0.0000              |
| -0.125+0.071         | 0.6731                  | 0.3445               | 0.9171              | 0.0000                  | 0.0000              |
| -0.071+0.056         | 0.7997                  | 0.4354               | 0.9395              | 0.1154                  | 0.8846              |
| -0.056+0.044         | 0.8720                  | 0.4408               | 0.9892              | 0.1306                  | 0.8846              |
| -0.044+0             | 0.8068                  | 0.1669               | 0.8756              | 0.3594                  | 0.9379              |

To apply separation characteristics of technological units to controlling beneficiation processes, it is necessary to approximate them. Basic results of approximation are presented in table 3.
Figure 2. Separation characteristics of sizing units of the technological line at the ore beneficiation plant: classifier overflow (a), classifier sands (b), thickener sands (c).

Parametric models (table 3) are evaluated by the following indicators. The sum of squares due to error (SSE) is calculated by the formula [23]:

$$SSE = \sum_{i=1}^{n} w_i (y_i - \hat{y}_i)^2. \quad (2)$$

The closer to 0 the value obtained is, the smaller the value of the random error component is, this indicating high accuracy of the model. The multiple coefficient of determination (R-Square) indicates what fraction of result dispersion is caused by the influence of independent variables. This indicator is determined as a ratio of the sum of regression squares (SRS) and the total sum of squares (TSS). SRS and TSS are determined as follows [23]:
Table 3. Results of approximating separation characteristics of sizing units.

| Function  | Number of coefficients | SSE  | R-square | Adjusted R-square | RMSE  |
|-----------|------------------------|------|----------|-------------------|-------|
| Gaussian  | 3                      | 0.01177 | 0.9897 | 0.9863            | 0.04429 |
| Linear    | 3                      | 0.09543 | 0.9169 | 0.8892            | 0.1261  |
| Polynomial| 2                      | 0.1013  | 0.9118 | 0.8992            | 0.1203  |
| Polynomial| 3                      | 0.1012  | 0.9118 | 0.8824            | 0.1299  |
| Power     | 2                      | 0.4844  | 0.578   | 0.5177            | 0.2631  |
| Power     | 3                      | 0.09881 | 0.9139 | 0.8852            | 0.1283  |

\[ SSR = \sum_{i=1}^{n} w_i (\hat{y}_i - \bar{y})^2, \quad SST = \sum_{i=1}^{n} w_i (y_i - \bar{y})^2. \]  

At the same time, these indicators are related by the ratio \( TSS = SRS + SSE \). The multiple determination coefficient (R-Square) is defined as follows [23]:

\[ R - \text{Square} = \frac{SSR}{SST} = 1 - \frac{SSE}{SST} \] (4)

The value obtained is in the range from 0 to 1. The closer the value obtained to 1, the greater dispersion fraction is taken into account by the model. The Adjusted R-Square is used to ensure correctness of the models with different number of factors, so that the number of fixed variables (factors) does not affect the results [24].

A radial thickener is a low cylindrical tank (3.6m–4.2 m) with a conical bottom and an annular chute for draining the clarified slurry [25]. A truss is installed in the tank with the rakes fixed for continuous mixing of settled materials to the unloading opening at the bottom of the conical part of the tank. The truss makes 0.01-0.3 revolutions per minute. The feed slurry is continuously supplied to the middle of the thickener tank through the recessed feeder. The clarified fluid is poured through the threshold into the annular chute and removed through a special pipeline to the tailings or used as circulating water. The thickened slurry formed from sediments is removed through the discharge openings and pumped out. The main input signal – a controlled value for automatic stabilization of the thickener – is the density of the thickener overflow slurry (or initial concentration), and the density of the overflow slurry (or initial concentration) is the main signal to optimize the unit [25,26]. Main disturbances of the process are changes in concentration and volume of the feed slurry. Changes in the valve opening of the overflow of the thickened slurry at the output from the thickener and coagulant consumption are control actions.

The control of the thickening process is formed using the method of model predictive control (MPC). The MPC controller uses the following mathematical model [27,28]:

\[ x_p(k+1) = A_p x_p(k) + B S_i u_p(k) \]
\[ y_p(k) = S_0^{-1} C x_p(k) + S_0^{-1} D S_i u_p(k) \] (5)

where \( S_i, S_0 \), So are the diagonal matrix of input and output scale factors in technical units, \( x_p \) is the state vector, \( u_p \) is the vector of dimensionless values of control coefficients, \( y_p \) is the vector of dimensionless values of the controlled object output. When synthesizing model predictive control, the controller solves an optimization problem at each control interval. The result of the solution is the values of controlled variables to be applied to the object in the next control interval.
The formation of model predictive control of the thickening process by the channel “area of the overflow valve opening - initial concentration” is depicted on figure 3.

![Diagram](image)

**Figure 3.** System of forming model predictive control over thickening.

On the basis of this model predictive control scheme (figure 3), the influence of values of the control horizon and the prediction horizon on control quality of the ore thickening process is investigated. The duration of the control interval is assumed to be equal to 1 min. The simulation results for different control and prediction horizons are presented in figure 4. The research methodology enables the fixed control horizon (1, 2 and 5 intervals) and different values of the prediction horizon set (3, 5, 10 and 20 intervals).

Total results of calculating control quality indicators of iron ore thickening at the set values of the control horizon and the prediction horizon are presented in table 4.

| Control horizon | Prediction horizon | Overcontrol | Mean error | Quadratic error |
|-----------------|-------------------|-------------|------------|-----------------|
| 1               | 3                 | 1.2349      | -0.0715    | 0.1474          |
| 1               | 5                 | 0.005       | -0.072     | 0.1472          |
| 1               | 10                | 0.0001      | -0.0733    | 0.1467          |
| 1               | 20                | 0.0016      | -0.0778    | 0.1452          |
| 2               | 3                 | 1.6074      | -0.0715    | 0.1475          |
| 2               | 5                 | 0.5218      | -0.0717    | 0.1473          |
| 2               | 10                | 0.1553      | -0.0719    | 0.1473          |
| 2               | 20                | 0.2301      | -0.0718    | 0.1473          |
| 5               | 3                 | 1.6558      | -0.0715    | 0.1475          |
| 5               | 5                 | 0.9032      | -0.0716    | 0.1474          |
| 5               | 10                | 0.8679      | -0.0716    | 0.1474          |
| 5               | 20                | 0.7708      | -0.0716    | 0.1474          |

The obtained results allow concluding that when forming model predictive control for a single-channel control system of the thickening process (figure 3), the control horizon equal to 1 interval is sufficient. For this value, the quadratic error varies within 0.1452-0.1474. Further increase in the prediction horizon does not significantly reduce the quadratic control error. Yet, the value of 3-5 intervals will be sufficient for the prediction horizon as increased computational complexity of prediction by 10-20 intervals leads to a slight decrease in the quadratic control error.
5. Conclusions and further research

To form the separation characteristic of the ore raw material thickening process based on the model predictive control method for a single-channel control system, satisfactory control results are provided by setting the control horizon equal to 1 interval with the prediction horizon of 3-5 intervals. Further research is aimed at studying the possibility of controlling the thickening process as a multidimensional object with variable parameters considering, in particular changes in characteristics of mineral and technological varieties of processed iron ore.

ORCID iDs
V Morkun https://orcid.org/0000-0003-1506-9759
N Morkun https://orcid.org/0000-0002-1261-1170
V Tron https://orcid.org/0000-0002-6149-5794

Figure 4. Output variable for different control and prediction horizons: 1 step prediction horizon (a); 2 steps prediction horizon (b); 5 steps prediction horizon (c).
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