The $B-L$ Scotogenic Models for Dirac Neutrino Masses

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We construct the one-loop and two-loop scotogenic models for Dirac neutrino mass generation in the context of $U(1)_{B-L}$ extensions of standard model. It is indicated that the total number of intermediate fermion singlets is uniquely fixed by anomaly free condition and the new particles may have exotic $B-L$ charges so that the direct SM Yukawa mass term $\bar{\nu}_L \nu_R \phi^0$ and the Majorana mass term $(m_N/2)\bar{\nu}_R \nu_R$ are naturally forbidden. After the spontaneous breaking of $U(1)_{B-L}$ symmetry, the discrete $Z_2$ or $Z_3$ symmetry appears as the residual symmetry and give rise to the stability of intermediated fields as DM candidate. Phenomenological aspects of lepton flavor violation, DM, leptogenesis and LHC signatures are discussed.

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I. INTRODUCTION

The standard model (SM) needs extensions to incorporate two important missing pieces: the tiny neutrino masses and the cosmological dark matter (DM) candidates. The scotogenic model, proposed by Ma[1], has recently became an attractive and economical scenario to accommodate the above two issues in a unified framework. The main idea is based on the assumption that the DM candidates can serve as intermediate messengers propagating inside the loop diagram in neutrino mass generation. Classical examples are the Ma’s one-loop model[1] and two-loop model[2]. Some representative variations are found in Refs. [3–31]. In these models, the stability of DM is usually guaranteed by imposing the odd parity under ad hoc $Z_2$ or $Z_3$ symmetry. The origin of discrete symmetry is still unknown. An attractive scenario, known as Krauss-Wilczek mechanism [32], is that the discrete symmetry appears as the residual symmetry which originates from the spontaneous symmetry breaking (SSB) of a continuous gauge symmetry at high scale. The simplest and well-studied gauge extension of SM is that of $U(1)_{B-L}$, which was first realized within the framework of left-right symmetric models [33–36]. Following this spirit, several loop-induced Majorana neutrino mass models were constructed based on gauged $U(1)_{B-L}$ symmetry [37–44]. In these works, exotic $B-L$ charges are assigned to new particles to satisfy the anomalies cancelation condition. By taking appropriate charge assignment, the residual discrete $Z_2(Z_3)$ symmetry arises after the SSB of $U(1)_{B-L}$ symmetry. Then the lightest particles with odd $Z_2(Z_3)$ parity can not decay into SM ingredients, becoming a DM candidate.

On the other hand, the evidences establishing whether neutrinos are Majorana or Dirac fermion is still missing. If neutrinos are Dirac fermions, certain new physics beyond the SM should exist to account for the tiny neutrino mass. Several scotogenic models for Dirac neutrino masses were proposed in Refs. [45–51]. The generic one-loop topographies are discussed in Ref. [52] and subsequently, specific realizations with $SU(2)_L$ multiplets fields are presented in Ref. [53]. In these models, two ad hoc discrete symmetries were introduced, one is responsible for the absence of SM Yukawa couplings $\bar{\nu}_{L/R}\phi\theta$ and the other for the stability of intermediate fields as dark matter (DM). The symmetries could be discrete $Z_2$[46, 52, 53], $Z_3$[49, 54], or $Z_4$[55, 56].

It is natural to ask if the $B-L$ symmetry also shed light on Dirac neutrino mass generation and DM phenomena. Recently several efforts were made at tree level[54, 57–59], and a specific one-loop realization was also proposed based on left-right symmetry scheme[51]. In this brief article,
we propose the $U(1)_{B-L}$ extensions of scotogenic Dirac neutrino mass models with intermediate Dirac fermion singlets. We will systematically discuss the one- and two-loop realizations for Dirac neutrino masses with typical topographies respectively. In these models, a singlet scalar $\sigma$ is responsible for the SSB of gauged $U(1)_{B-L}$ symmetry as well as masses of the heavy intermediate Dirac fermions. To get the Dirac type neutrino mass term, we introduce three right-handed components $\nu_R$ and assume that they share the same $B-L$ charges. The intermediate Dirac fermions are SM singlets but carry $B-L$ quantum numbers. This implies that the anomaly cancelations of $[SU(3)_c]^2 \times U(1)_{B-L}$, $[SU(2)_L]^2 \times U(1)_{B-L}$ and $U(1)_Y \times [U(1)_{B-L}]^2$ are automatically satisfied. Thus we only need to consider the $[U(1)_{B-L}] \times [Gravity]^2$ and $[U(1)_{B-L}]^3$ anomaly conditions. Then the effective Dirac neutrino mass term $m_D \bar{\nu}_L \nu_R$ is induced by SSB of $U(1)_{B-L}$. As we shall see, the discrete $Z_2$ or $Z_3$ symmetry could appear as a remnant symmetry of gauged $U(1)_{B-L}$ symmetry, naturally leading to DM candidates.

In Sec.II, we construct the one/two-loop diagrams for Dirac neutrino mass generation and discuss their validity under $B-L$ anomaly free condition. We consider the phenomenology of the models in Sec.III. A summary is given in Sec. IV.

II. MODEL BUILDING

A. One-loop Scotogenic Model

Consider first the one-loop scotogenic realization of Dirac neutrino masses. In the $B-L$ extended scotogenic models, the particle content under $SU(2)_L \times U(1)_Y \times U(1)_{B-L}$ symmetry is listed as follow

\begin{align}
L &\sim (2, -1/2, -1), \quad \nu_{R1,2,3} \sim (1, 0, Q_{\nu_R}), \quad F_{L/Ri} \sim (1, 0, Q_{F_{L/R}}) \\
\Phi &\sim (2, 1/2, 0), \quad \eta \sim (2, 1/2, Q_{\eta}), \quad \chi \sim (1, 0, Q_{\chi}), \quad \sigma \sim (1, 0, Q_{\sigma})
\end{align}

where several Dirac fermion singlets are added with their chiral components denoted as $F_{Ri}$ and $F_{Li}(i = 1 \cdots n)$ respectively. In the scalar sector, we further add one doublet scalar $\eta$ and one singlet scalar $\chi$.

In the original $Z_2$ model [45, 46], $Z_2$ odd parity is assigned to $\nu_R$ and intermediated particle fields running in the loop. As a warm up, we start from the simplest $U(1)_{B-L}$ extension. We denote it as $A_1$ model with the corresponding Feynman diagrams illustrated as the first diagram in Fig. [1]
The relevant interactions for radiative Dirac neutrino mass generation are given as

$$\mathcal{L} \supset y_1 \ell F_L \bar{e}_2 \eta + y_2 \bar{\nu}_R F_L \chi + f F_L F_R \sigma + \mu (\Phi^\dagger \eta) \chi^* + \text{h.c.,}$$  \hspace{1cm} (2)$$

where \(L\) is the SM lepton doublet and we omit the summation indices. In terms of gauged \(U(1)_{B-L}\) symmetry, one should consider the \([U(1)_{B-L}] \times [(\text{Gravity})^2\] and \([U(1)_{B-L}]^3\) anomaly free conditions

$$-3 - 3Q_{\nu_R} - nQ_{F_R} + nQ_{F_L} = 0,$$  \hspace{1cm} (3)

$$-3 - 3Q_{\nu_R}^3 - nQ_{F_R}^3 + nQ_{F_L}^2 = 0,$$

which, using relevant interactions given in Eq.(2), can be solved exactly as

$$n = 3, \quad Q_{F_R} = -Q_{\nu_R}, \quad Q_{F_L} = 1$$  \hspace{1cm} (4)$$

Given the interactions in Eq.(2), the charge assignments for other particles are listed in the \(A_1\) row in Table. I. Therefore the total number of heavy fermions is fixed by the anomaly free conditions and the \(B - L\) charge assignments for all new particles are determined in terms of free parameter \(Q_{\nu_R}\). Let us now discuss precisely what values \(Q_{\nu_R}\) can be taken. First, the condition \(Q_{\nu_R} \neq -1\) should also be imposed to forbid the SM direct Yukawa coupling term \(\bar{\nu}_L \nu_R \phi_0\). Second, forbidding Majorana mass terms \((m_R)^{\sigma^{\dagger}} \nu_R \sigma^{\dagger} \nu_R\) and \(\sigma^{\dagger} \nu_R \nu_R\) requires \(Q_{\nu_R} \neq 0, -1/3, 1\) respectively (note that \(Q_{\sigma} = Q_{\nu_R} + 1\) for \(A_1\) model). Third, to generate a purely loop-induced neutrino mass term, \(Q_{\sigma}\) and \(Q_{\chi}(= Q_{\nu_R} - 1)\) appropriately assigned so that \(\sigma^k \chi\) and \((\sigma^*)^k \chi(k = 1, 2, 3)\) terms, which cause the VEV of \(\chi\), are forbidden. This further requires \(Q_{\nu_R} \neq 0, -1/3, -1/2, -2\) and \(-3\). Similarly, the \((\Phi^\dagger \eta)^{\sigma^k}\) and \((\Phi^\dagger \eta)(\sigma^*)^k (k = 1, 2)\) should also be avoid to generate the VEV of \(\eta\), leading to \(Q_{\nu_R} \neq 0, -1/3, -3\). Once an appropriate \(Q_{\nu_R}\) is taken, the residual \(Z_2\) symmetry appears in Eq.(2), under which the parity is odd for inert particles \((\eta, \chi, F_{L/R})\) and even for all other particles.

We now consider other possible realizations. In the scalar sector, the interactions relevant to radiative neutrino mass generation are given by

$$\mathcal{L}_S \supset (\Phi^\dagger \eta) \chi, \quad (\Phi^\dagger \eta)^* \chi^*, \quad (\Phi^\dagger \eta) \chi \sigma, \quad (\Phi^\dagger \eta)^* \chi^* \sigma,$$  \hspace{1cm} (5)

$$(\Phi^\dagger \eta) \sigma^*, \quad (\Phi^\dagger \eta)^* \sigma^*, \quad \chi^2 \sigma, \quad \chi^2 \sigma^*, \quad +\text{h.c.}$$

Taking appropriate charge assignment, at least one \(\eta - \chi\) mixing term given in Eq. (5) should be selected to build the model. All the seven possible topological diagrams(denoted as \(A_1 - A_7\)) are depicted in Fig.[1], where we have already discussed the specific model \(A_1\) above.
Under the gauged $U(1)_{B-L}$ symmetry, the quantum numbers of new particles are required to satisfy the anomaly free conditions. We summarize the $B-L$ quantum number assignments for each diagram in Table I. We have checked that among the seven models, five of them ($A_1, A_2, A_4, A_5$ and $A_6$) are suitable for the gauged $B-L$ extension. For each available model, the total number of intermediate fermions $F_R/L$ is uniquely determined by the anomaly free condition of $[U(1)_{B-L}] \times [\text{Gravity}]^2$. The $B-L$ quantum number of $A_1$ and $A_2$ model can not be uniquely fixed and we choose $Q_{\nu_R}$ as the variable. If $\chi$ linear terms are forbidden by appropriate $Q_{\nu_R}$ assignment, the residual $Z_2$ symmetry arises after the SSB of $U(1)_{B-L}$. Thus the lightest particle with odd $Z_2$ parity can serve as a DM candidate.

Compared with $A_1$ and $A_2$, for models $A_4, A_5$ and $A_6$, the $B-L$ quantum numbers for new particles are fixed uniquely. This is due to the fact that the interaction $\chi^2 \sigma (\chi^2 \sigma^*)$ contributes an
Table I. $B - L$ charge assignments for new particles in each one-loop models. In $A_2$ model, we set $z \equiv (5x^2 - 6x + 5)^{1/2}$. The symbol “×” means that no appropriate charge assignment are available to meet the requirement of anomaly cancellation

additional constraint on $Q_{\chi}$ and $Q_{\sigma}$, i.e.,

$$2Q_{\chi} \pm Q_{\sigma} = 0. \quad (6)$$

The existence of $\chi^2 \sigma (\chi^2 \sigma^*)$ term has two-fold meanings: (i) that it automatically forbids the $\chi$ linear terms and guarantee the existence of residual $Z_2$ symmetry after the SSB of $U(1)_{B - L}$; (ii) that it induces a mass splitting $\Delta M = |M_{\chi_R} - M_{\chi_I}|$ between the real ($\chi_R$) and imaginary part ($\chi_I$) of $\chi$. Provided $\Delta M$ is larger than the DM kinetic energy $K E_D \sim O(100)$ KeV, the tree-level DM-nucleon scattering via the $U(1)_{B - L}$ gauge boson $Z'$ and SM $Z$ boson exchange (due to the mixing between $\eta$ and $\chi$) are kinematically forbidden, thus a $\chi_R/\chi_I$ dominated DM is expectable through the scalar singlet $\sigma$ or SM Higgs portal.

One recalls that in the prototype scotogenic Dirac model [46] with sizable Yukawa couplings, a relatively small coupling constants of $\eta - \chi$ mixing terms is required to reproduce the scale of neutrino masses. To rationalize such a unnaturally small coupling, an extra soften broken symmetry is added[46]. We emphasize that the fine tuning can be relaxed in $A_4 - A_6$ models with the help of double suppression from $\eta - \chi$ and $\chi_R - \chi_I$ mixing interactions. Takeing $A_5$ model as an example, with scalar interactions $\lambda(\Phi^\dagger \eta)\chi^* \sigma$ and $\mu \chi^2 \sigma^*$, the radiative neutrino mass is evaluated as

$$m_\nu \sim \frac{\lambda y_1 y_2 f}{16\pi^2} \left( \frac{\langle \Phi \rangle \langle \sigma \rangle^3}{\Lambda^4} \right) \mu_\chi, \quad (7)$$

where $\Lambda \sim m_\eta, m^R_\chi, m^I_\chi$ denotes the scale of new physics, usually taken to be $\Lambda \sim \langle \sigma \rangle \sim O(1)\text{TeV}$. Then for $\lambda \sim y_1 \sim y_2 \sim f \sim 10^{-2}$ and $\mu_\chi \sim O(10)\text{GeV}$, the neutrino mass scale ($0.1 \text{eV}$) can be reproduced.
B. Two-loop Scotogenic Models

Now let us discuss the two-loop scotogenic realizations of Dirac neutrino masses. The simple model with $Z_3$ discrete symmetry was proposed recently\cite{49} where two classes of Dirac fermion singlets are added. Here we denote the corresponding chiral components as $F_{R,Li}(i = 1, 2 \cdots n)$ and $S_{R,Lj}(j = 1, 2 \cdots m)$ respectively. In the scalar sector, we add one scalar doublet $\eta$, two scalar singlets $\chi$ and $\xi$. In order to accomplish the $U(1)_{B-L}$ extension, a scalar singlet $\sigma$ is also added to play the role as $B - L$ symmetry breaking. The particle content and quantum number assignments under $SU(2)_L \times U(1)_Y \times U(1)_{B-L}$ gauge symmetry are summarized as follow

\begin{align}
L &\sim (2, -1/2, -1), \quad \nu_{R1,2,3} \sim (1, 0, Q_{\nu R}) \\
F_{L/Ri} &\sim (1, 0, Q_{F_{L/R}}), \quad S_{L/Rj} \sim (1, 0, Q_{S_{L/R}}) \\
\Phi &\sim (2, 1/2, 0), \quad \eta \sim (2, 1/2, Q_\eta), \quad \chi \sim (1, 0, Q_\chi), \quad \sigma \sim (1, 0, Q_\sigma)
\end{align}

Similar as the one-loop cases, the two-loop model can be realized though various pathways. As an illustration, we start from a simple $U(1)_{B-L}$ extension (denoted as $B_1$) with topology depicted by the first diagram in Fig. 2. The relevant interactions are

\begin{equation}
L \supset y_1 LF_{Ri} \tau_2 \eta^* + y_2 \nu_R S_L \xi + f_1 F_L F_R \sigma + f_2 S_L S_R \sigma + h S_R F_L \chi^* + \lambda_1 (\Phi^\dagger \eta) \chi^* \sigma + \lambda_2 \chi^3 \sigma^* + \mu_3 \xi \chi \sigma + \text{h.c.}
\end{equation}

Under gauged $U(1)_{B-L}$ symmetry, the condition of cancelation for $[U(1)_{B-L}] \times [\text{Gravity}]^2$ anomaly is given by

\begin{equation}
-3 - 3Q_{\nu R} - nQ_{F_R} + n(Q_{F_R} + Q_{\nu R} + 1) - mQ_{S_R} + m(Q_{S_R} + Q_{\nu R} + 1) = 0
\end{equation}

Notice that $Q_{\nu R} \neq -1$ is required to forbid $\nu_L \nu_R \phi^0$ term. From Eq.\cite{10}, one obtains

\begin{equation}
n + m = 3.
\end{equation}

Clearly, only $(n, m) = (1, 2)$ and $(2, 1)$ patterns are allowed for model $B_1$. In this scenario, the rank of effective neutrino mass matrix is two, implying a vanishing neutrino mass eigenvalue. Hence the models with condition $n + m = 3$ are the minimal two-loop realizations allowed phenomenologically. The anomaly free condition of $[U(1)_{B-L}]^3$ is given by

\begin{equation}
-3 - 3Q_{\nu R}^3 - nQ_{F_R}^3 + n(Q_{F_R} + Q_{\nu R} + 1)^3 - mQ_{S_R}^3 + m(Q_{S_R} + Q_{\nu R} + 1)^3 = 0
\end{equation}
Taking the interaction terms in Eq.(9) into account and solving Eq.(11), (12), we find

\[ Q_{\nu_R} = \frac{5n - 17}{3n + 5}. \] (13)

Subsequently, the $B - L$ charges of other particles are obtained, which are shown explicitly in Table II.

Now we investigate other viable realizations. Without loss of generality, we focus on the minimal models with three intermediate fermions, i.e., $n + m = 3$. To generate a residual $Z_3$ discrete symmetry, the $\chi^3\sigma$ or $\chi^3\sigma^*$ is needed. After the SSB of $U(1)_{B-L}$, $\chi$ transforms as $\omega = e^{i2\pi/3}$ under the residual $Z_3$ symmetry. It is found that only four models are available under the anomaly free condition. The corresponding topological diagrams are shown in Fig. 2. Besides $B_1$, we denote rest of models as $B_2$, $B_3$ and $B_4$ respectively. Following the same methodology in one-loop case, the $B - L$ charge assignments of new particles for each model are obtained. The main results are listed in Table II. Obviously, after $B - L$ breaking, the residual $Z_3$ symmetry arises with

\[ F_{L,Ri} \sim \omega, \quad S_{L,Ri} \sim \omega, \quad \eta, \chi \sim \omega, \quad \xi \sim \omega^2 \] (14)
Table II. $B - L$ quantum number assignments and relevant scalar interactions for two-loop models with $n + m = 3$.

| $(n, m)$ | $\nu_{R1,2,3}$ | $F_{Ri}$ | $F_{Li}$ | $S_{Ri}$ | $S_{Li}$ | $\eta$ | $\chi$ | $\xi$ | $\sigma$ | Scalar interactions |
|----------|----------------|----------|----------|----------|----------|--------|--------|--------|--------|---------------------|
| $B_1$    | (1, 2)         | $-\frac{1}{11}$ | $-\frac{13}{3}$ | $\frac{17}{3}$ | $\frac{7}{3}$ | $\frac{37}{3}$ | $-\frac{20}{3}$ | $\frac{10}{3}$ | $-\frac{40}{3}$ | $\frac{10}{11}$ | $(\Phi^\dagger \eta) \chi^* \sigma, \chi^3 \sigma, \chi \xi \sigma$ |
|          | (2, 1)         | $0$      | $-\frac{1}{4}$ | $\frac{2}{3}$ | $\frac{3}{4}$ | $\frac{4}{3}$ | $-\frac{2}{3}$ | $\frac{1}{3}$ | $-\frac{4}{3}$ | $1$ | $(\Phi^\dagger \eta) \chi^* \sigma, \chi^3 \sigma, \chi \xi \sigma$ |
| $B_2$    | (1, 2)         | $\times$ | $\times$ | $\times$ | $\times$ | $\times$ | $\times$ | $\times$ | $\times$ | $\times$ | \null |
|          | (2, 1)         | $-11$ | $\frac{37}{3}$ | $\frac{7}{3}$ | $\frac{17}{3}$ | $-\frac{13}{3}$ | $-\frac{40}{3}$ | $-\frac{10}{3}$ | $-\frac{20}{3}$ | $-10$ | $(\Phi^\dagger \eta) \chi^* \sigma, \chi^3 \sigma, \chi \xi \sigma^*$ |
| $B_3$    | (1, 2)         | $-\frac{1}{3}$ | $-\frac{12}{9}$ | $-\frac{7}{9}$ | $-\frac{5}{9}$ | $\frac{4}{9}$ | $-\frac{2}{9}$ | $-\frac{4}{9}$ | $\frac{2}{9}$ | $\frac{1}{3}$ | $(\Phi^\dagger \eta) \chi^* \sigma, \chi^3 \sigma, \chi \xi \sigma$ |
|          | (2, 1)         | $-3$ | $\frac{1}{3}$ | $-\frac{5}{3}$ | $-\frac{7}{3}$ | $-\frac{13}{3}$ | $-\frac{4}{3}$ | $\frac{2}{3}$ | $\frac{4}{3}$ | $-2$ | $(\Phi^\dagger \eta) \chi^* \sigma, \chi^3 \sigma, \chi \xi \sigma$ |
| $B_4$    | (1, 2)         | $-\frac{3}{11}$ | $-\frac{31}{42}$ | $\frac{23}{42}$ | $\frac{13}{42}$ | $\frac{23}{21}$ | $-\frac{11}{42}$ | $-\frac{11}{42}$ | $-\frac{55}{42}$ | $\frac{11}{42}$ | $(\Phi^\dagger \eta) \chi^* \sigma, \chi^3 \sigma, \chi \xi \sigma^2$ |
|          | (2, 1)         | $-\frac{1}{5}$ | $-\frac{17}{42}$ | $\frac{1}{5}$ | $\frac{11}{21}$ | $\frac{4}{7}$ | $-\frac{7}{7}$ | $-\frac{7}{21}$ | $-\frac{35}{21}$ | $\frac{1}{5}$ | $(\Phi^\dagger \eta) \chi^* \sigma, \chi^3 \sigma, \chi \xi \sigma^2$ |

III. PHENOMENOLOGY: A CASE STUDY

In the following, we consider some phenomenological aspects of the gauged $B - L$ scotogenic Dirac models. From Table I, we can see that besides the $B - L$ charge and some scalar interactions being different, all the one-loop models have same interactions as in Eq. (2). Therefore, we can concentrate on the simplest one, i.e., model $A_1$. As for the two-loop models, phenomenon will be similar provided the additional $\xi$ and $S_{L,R}$ are heavy enough.

In model $A_1$, the $B - L$ charges of all the additional particles are determined by $B - L$ charge of right-handed neutrino $Q_{\nu_R}$. To make sure a residual $Z_2$ symmetry after the breaking of $B - L$, we fix $Q_{\nu_R} = 1/6$ in the following discussion. The complete gauge invariant scalar potential for model $A_1$ is

$$V = -\mu_\Phi \Phi^\dagger \Phi + \mu_\eta \eta^\dagger \eta + \mu_\chi \chi^* \chi - \mu_\sigma^* \sigma + \lambda_\Phi (\Phi^\dagger \Phi)^2 + \lambda_\eta (\eta^\dagger \eta)^2 + \lambda_\chi (\chi^* \chi)^2$$

$$+ \lambda_\sigma (\sigma^* \sigma)^2 + \lambda_{\Phi \eta} (\Phi^\dagger \Phi)(\eta^\dagger \eta) + \lambda_{\Phi \chi} (\Phi^\dagger \Phi)(\chi^* \chi) + \lambda_{\Phi \sigma} (\Phi^\dagger \Phi)(\sigma^* \sigma) + \lambda_{\eta \chi} (\eta^\dagger \eta)(\chi^* \chi) + \lambda_{\eta \sigma} (\eta^\dagger \eta)(\sigma^* \sigma) + \lambda_{\chi \sigma} (\chi^* \chi)(\sigma^* \sigma) + [\mu(\Phi^\dagger \eta)\chi^* + \text{h.c.}] .$$

For the $Z_2$ even scalars, $\phi_R^0$ and $\sigma_R$ mix into physical scalars $h$ and $H$ with mixing angle $\alpha$. Here, we regard $h$ as the discovered 125 GeV scalar at LHC [60–62]. In order to escape various direct and indirect searches for the scalar $H$ [63], a small mixing angle $\sin \alpha = 0.01$ is assumed in this work. Meanwhile, for the $Z_2$ odd scalars $\eta^0$ and $\chi$, they will mix into physical scalars $H_2^0$ and $H_1^0$ with mixing angle $\beta$. As shown in Refs. [64, 65], a small mixing angle, e.g., $\sin \beta \lesssim 0.01$ is preferred in case of scalar DM $H_1^0$. In this paper, we take $\sin \beta = 10^{-6}$, mainly aiming to interpret tiny neutrino
masses. And we also have one pair of $Z_2$ odd charged scalar $H^\mp_2 (= \eta^\mp)$.

Given the interactions in Eq. [2] the one-loop induced neutrino mass for model $A_1$ is

$$m_{ij}^\nu = \frac{\sin 2\beta}{32\pi^2} \sum_k y^i_k y^j_k M_{F_k} \left[ \frac{M^2_{H_0^2}}{M^2_{H_2^0} - M^2_{F_k}} \ln \left( \frac{M^2_{H_0^2}}{M^2_{F_k}} \right) - \frac{M^2_{H_1^0}}{M^2_{H_2^0} - M^2_{F_k}} \ln \left( \frac{M^2_{H_1^0}}{M^2_{F_k}} \right) \right].$$

To give some concrete prediction, we present one promising benchmark point (BP) for model $A_1$

$$\sin \beta = 10^{-6}, |y^1_{1,2}| = 10^{-6}, |y_{2,3}^{i2,13}| = 0.007,$$
$$M_{H_1^0} = 45 \text{ GeV}, M_H = 100 \text{ GeV}, M_{H_2^0}, H_0^2 = 600 \text{ GeV},$$
$$M_{F1} = M_{F2,F3}/2 = 5 \text{ TeV}, M_{Z'} = 4 \text{ TeV}, g_{BL} = 0.1,$$

which could realise $m_\nu \sim 0.1 \text{ eV}$. For simplicity, we denote $|y_{1,2}^{i2,13}| = y$ in the following.

![Figure 3. BR($\mu \to e\gamma$) as a function of $M_{H_2^\pm}$](image)

Firstly, the existence of Yukawa interaction $\bar{L}F_Ri\tau_2\eta^*$ will induce various lepton flavor violation (LFV) processes. Detail studies on LFV processes in scotogenic models can be found in Ref. [66]. Here, we take the current most stringent one, i.e., the MEG experiment on the radiative decay $\mu \to e\gamma$ with $\text{BR}(\mu \to e\gamma) < 4.2 \times 10^{-13}$ [67], for illustration. The future limit might be down to $6 \times 10^{-14}$ [68]. In the scotogenic Dirac models, the analytical expression for branching ratio of $\mu \to e\gamma$ is calculated as [66]

$$\text{BR}(\mu \to e\gamma) = \frac{3\alpha}{64\pi G_F^2} \left| \sum_i \frac{(y^i_1)(y^i_2)^*}{M^2_{H_2^0}} F \left( \frac{M^2_{F_i}}{M^2_{H_2^0}} \right) \right|^2,$$

where the loop function $F(x)$ is

$$F(x) = \frac{1 - 6x + 3x^2 + 2x^3 - 6x^2 \ln x}{6(1 - x)^4}.$$
In Fig. 3, we show the BR(\(\mu \to e\gamma\)) as a function of \(M_{H^\pm_2}\) for \(y = 0.01, 0.007\). Our BP in Eq. 17 predicts BR(\(\mu \to e\gamma\)) \(\approx 4 \times 10^{-16}\), which is far below current and even future experimental limits.

Secondly, we briefly discuss the phenomenology of dark matter (DM). In this paper, we mainly consider scalar DM candidate, since for the fermion singlet, \(M_F = f \langle \sigma \rangle\) is naturally around TeV-scale and it is more interesting to realize successful leptogenesis. We emphasis that the \((\Phi^\dagger \eta)^2\) term is not allowed in \(U(1)_{B-L}\) extensions to generate a mass splitting between \(\eta_R^0\) and \(\eta_I^0\), rendering the \(\eta\) dominated component \(H^0_2\) unsuitable as a DM candidate to escape the direct detection bound. Therefore, we concentrate on the \(\chi\) dominated component \(H^0_1\) as the DM candidate.

![Figure 4. \(\Omega h^2\) as a function of \(M_{H^0_1}\). Here, we also fix \(\lambda_{\Phi\chi} = \lambda_{\chi\sigma} = 0.001\).

With heavy \(F\) and relatively small Yukawa couplings, i.e., \(|y_2| \lesssim 0.01\), the contribution of \(F\) to \(H^0_1\) annihilation is negligible. To generate the correct relic density, the possible annihilation channels are: 1) SM Higgs \(h\) portal; 2) scalar singlet \(H\) portal; 3) gauge boson \(Z'\) portal. For case 1), the extensive researches imply that \(M_{H^0_1} \lesssim M_h/2\) is the only allowed region under tight constraints from relic density and direct detection [69, 70]. For case 2), \(M_{H^0_1} \sim M_H/2\) is needed, and electroweak scale \(H^0_1\) DM is allowed [71]. Notably, when \(M_H \sim 100\) GeV thus \(M_{H^0_1} \sim 50\) GeV, the observed excess in gamma-ray flux by Fermi-LAT can be interpreted [72, 73]. For case 3), it requires \(M_{H^0_1} \sim M_{Z'/2}\), and \(M_{H^0_1}\) is usually around TeV-scale [74]. In Fig 4, we show the relic density \(\Omega h^2\) as a function of \(M_{H^0_1}\). The Higgs \(h/H\) portal could easily acquire the correct relic density, while the \(Z'\) portal could not due to too small \(g_{BL}\). Note that the process \(H^0_1 H^0_{1\ast} \to HH\) could also realise correct relic density provided \(M_{H^0_1} \sim M_H\).

Thirdly, we consider Dirac leptogenesis. It is well known that the leptogenesis can be accomplished in Dirac neutrino models [75, 76]. In model \(A_1\), the heavy Fermion singlet \(F\) can decay into
$\nu_{R\chi}$ to generate lepton asymmetry in the left-handed $\epsilon_L$ and right-handed sector $\epsilon_R$. Due to the fact that the sphaleron processes do not have direct effect on right-handed fields, the lepton asymmetry in the left-handed sector can be converted into a net baryon asymmetry via sphaleron processes, as long as the one-loop induced effective Dirac Yukawa couplings are small enough to prevent the lepton asymmetry from equilibration before the electroweak phase transition\cite{77}.

\begin{figure}[h]
\centering
\includegraphics[width=0.5\textwidth]{figure5.png}
\caption{Y$_B$ as a function of M$_{F_1}$. The blue bound corresponds to 2\sigma range of Planck result.}
\end{figure}

Under the assumption $y_1 = y_2$, the final lepton asymmetry is calculated as \cite{45}

$$\epsilon_{F_1} \simeq -\frac{1}{8\pi} \frac{1}{(y^\dagger_1 y_1)_{11}} \sum_{j \neq 1} \frac{M_{F_1}}{M_{F_j}} \text{Im} \left( (y^\dagger_1 y_1)_{1j} \right)^2.$$ \hspace{1cm} (20)

Define the parameter $K = \Gamma_{F_1} / H(T = M_{F_1})$, where $\Gamma_{F_1}$ is the tree-level decay width of $F_1$ and $H(T) = \sqrt{8\pi^3 g_*/90} T^2 / M_{Pl}$ wit $g_* \simeq 114$ and $M_{Pl} = 1.2 \times 10^{10}$ GeV. As in our case $K \gtrsim 1$, the final baryon asymmetry is estimated as \cite{77}

$$Y_B = -\frac{28}{79} Y_{\nu_R} \approx -\frac{28}{79} \frac{\epsilon_{F_1}}{\sqrt{8\pi^3 g_*}} \frac{0.12}{90}.$$ \hspace{1cm} (21)

In Fig. 5 we depict $Y_B$ as a function of $M_{F_1}$. It is clear that the BP in Eq. 17 could predict the correct value of $Y_B$, as well as satisfy the out of equilibration condition

$$\frac{|y_1|^2 |y_2|^2}{M_{F_1}} \lesssim \frac{1}{M_{Pl}} \sqrt{\frac{8\pi^3 g_*}{90}}.$$ \hspace{1cm} (22)

Then we turn to the collider phenomenology. The DM candidate $H_1^0$ will contribute to invisible Higgs decay. The corresponding decay width for $h \to H_1^0 H_1^{0*}$ is calculated as

$$\Gamma(h \to H_1^0 H_1^{0*}) = \frac{g_h^2 H_1^0 H_1^{0*}}{16\pi M_h} \left[ 1 - 4 \frac{M_{H_1^0}^2}{M_h^2} \right]^2.$$ \hspace{1cm} (23)
where $g_{hH_0^0H_1^0} = \lambda_{\Phi\chi}v\cos\alpha + \lambda_{\chi\sigma}v_{\sigma}\sin\alpha$ is the effective trilinear $hH_0^0H_1^0$ coupling and $v = 246$ GeV, $v_{\sigma} = M_{Z'}/(g_{BL}Q_{\sigma})$. So the invisible branching ratio is $\text{BR}_{\text{inv}} = \Gamma_{\text{inv}}/(\Gamma_{\text{inv}} + \Gamma_{\text{SM}})$ with $\Gamma_{\text{SM}} = 4.07$ MeV at $M_h = 125$ GeV [73]. Our BP in Eq. 17 with $\lambda_{\Phi\chi} = \lambda_{\chi\sigma} = 0.001$ predicts $\text{BR}_{\text{inv}} \sim 0.01$, which can escape the most stringent bound comes from fitting to visible Higgs decays, i.e., $\text{BR}_{\text{inv}} < 0.23$ [79]. As for the light scalar $H$, the dominant visible decay is $H \to b\bar{b}$ and invisible decay is $H \to H_1^0H_1^0$. The possible promising signatures are $e^+e^- \to ZH$ at future lepton colliders [80]. Meanwhile, due to the doublet nature of $H_2^\pm$ and $H_0^2$, they can be pair produced at LHC via Drell-Yan processes as $pp \to H_2^\pm H_2^\mp, H_0^2H_1^0, H_2^0H_2^0$. In the case of light $H_1^0$ DM, the most promising signature is

$$pp \to H_2^\pm H_2^0 \to W^\pm Z + H_1^0H_1^0,$$ (24)

then leptonic decays of $W$ and $Z$ will induce trilepton signature as $2l^\pm l^- + E_T$. The direct searches for such trilepton signature at LHC have excluded $M_{H_2^\pm, H_2^0} \lesssim 350$ GeV when $M_{H_1^0} \sim 50$ GeV [81, 82]. In Fig 6 we show the cross section of trilepton signature at 13 TeV LHC. The cross section of our BP in Eq. 17 is about 0.02 fb.

The gauged $U(1)_{B-L}$ symmetry predicts $Z'$ boson with mass $M_{Z'} = Q_{\sigma}g_{BL}v_{\sigma}$. Since $\sigma$ scalar is SM singlet and $\Phi$ do not transform under $U(1)_{B-L}$, there is no mixing between $Z$ and $Z'$ boson. The LEP II data requires that [83]

$$\frac{M_{Z'}}{g_{BL}} = Q_{\sigma}v_{\sigma} \gtrsim 6 \sim 7 \text{ TeV}.$$ (25)

And the direct searches for $Z'$ with SM-like gauge coupling in the dilepton final states have excluded $M_{Z'} \lesssim 4$ TeV [84]. Recasting of these searches in gauged $U(1)_{B-L}$ has been performed in Ref. [74].

![Figure 6. Trilepton signature $2\ell^\pm \ell^\mp + E_T$ as a function of $M_{H_2^\pm}$ at 13 TeV LHC.](image)
where the exclusion region in the $M_{Z'} - g_{BL}$ is obtained. In this paper, we consider $M_{Z'} = 4$ TeV and $g_{BL} = 0.1$ to respect these bounds. In the limit that masses of SM fermions $f(f \equiv q,l,\nu_{L,R})$ are small compared with the $Z'$ mass, the decay width of $Z'$ into fermion pair $f\bar{f}$ is given by

$$\Gamma(Z' \rightarrow f\bar{f}) = \frac{g_{BL}^2 M_{Z'}}{24\pi} C_f (Q_{fL}^2 + Q_{fR}^2)$$

(26)

where $C_{l,\nu} = 1, C_q = 3$. Then the branch ratios of $Z'$ decay into each final states take the ratios as

$$\text{BR}(Z' \rightarrow q\bar{q}) : \text{BR}(Z' \rightarrow l^- l^+) : \text{BR}(Z' \rightarrow \nu\bar{\nu}) = 4 : 6 : 3(1 + Q_{\nu_R}^2),$$

(27)

where $l = e,\mu$. Thus, the $B-L$ nature of $Z'$ can be confirmed when $\text{BR}(Z' \rightarrow b\bar{b})/\text{BR}(Z' \rightarrow \mu^+ \mu^-) = 1/3$ is measured [39]. In addition, the decay width of $Z'$ into scalar pair $SS^*$ is given by

$$\Gamma(Z' \rightarrow SS^*) = \frac{g_{BL}^2 M_{Z'} Q_S^2}{48\pi}$$

(28)

in the limit $M_S \ll M_{Z'}$ as well. In case of $H^0_i$ DM with the special mass spectrum $M_{H^0_i} < M_H < M_{H^\pm} < M_{Z'} < M_F$ as we discussed above, the dominant invisible decays of $Z'$ are $Z' \rightarrow \nu\bar{\nu}$ and $Z' \rightarrow H^0_i H^0_{i*}$, and the subdominant contributions are coming from cascade decays as $Z' \rightarrow HH$ with $H \rightarrow H^0_i H^0_{i*}$ and $Z' \rightarrow H^0_2 H^0_{2*}$ with $H^0_{2*} \rightarrow Z(\rightarrow \nu\bar{\nu}) H^0_i$. In Table III, we show the branching ratio of $Z'$ predicted by our BP. Due to different values of $B-L$ charges for the

| $q\bar{q}$ | $l^\pm$ | $\nu\bar{\nu}$ | $HH$ | $H^0_i H^0_{i*}$ | $H^0_2 H^0_{2*}$ | $H^0_2 H^0_{2*}$ |
|---|---|---|---|---|---|---|
| 0.27 | 0.41 | 0.21 | 0.05 | 0.02 | 0.02 | 0.02 |

Table III. Decay branching ratio of $Z'$.

new particles in all the possible models present in Table I and Table II, they can be distinguished by precise measurement of the invisible decays of $Z'$.

IV. CONCLUSION

In conclusion, we propose the $U(1)_{B-L}$ extensions of the scotogenic models with intermediate fermion singlets added. The Dirac nature of neutrinos is protected by $B-L$ symmetry while the DM stability is guaranteed by the residual symmetry of $B-L$ SSB. Under gauged $U(1)_{B-L}$, the values of $B-L$ quantum numbers for new particles are assigned to satisfy the anomaly free condition. We first present the topological diagrams of one-loop $Z_2$ realizations and subsequently
check their validity under anomaly free condition. Among the seven one-loop realizations, five of them is available ($A_1, A_2, A_4, A_5$ and $A_6$). It is found that the total number of intermediate fermion singlets is uniquely fixed by anomaly free condition. Especially, the $B - L$ charge assignments for $A_4, A_5$ and $A_6$ models can also be uniquely fixed due to the mass splitting terms in scalar sector. We emphasis the implications of such terms on alleviating the fine tuning in the model and also permitting intermediate scalar singlet as a DM candidate. Then we study the two-loop $Z_3$ realizations where $n F_{R/L}$ and $m S_{R/L}$ fermion singlets are added. Doing the same in one-loop model, we found $n + m$ and $B - L$ charge assignments of all new particles is uniquely determined by anomaly free condition. With out loss of generality, we consider the minimal realizations with $n + m = 3$ and found four viable models (denoted as $B_1, B_2, B_3$ and $B_4$).

By considering phenomenology on lepton flavor violation, dark matter, leptogenesis and LHC signatures, we consider the benchmark point in Eq. 17. In addition to generate tiny neutrino mass via scalar DM mediator, this BP can also interpret the gamma-ray excess from the galactic center, and realize successful leptogenesis. As for collider signatures, the scalar DM $H_1^0$ will contribute to invisible Higgs decay as $h \to H_1^0 H_1^{0*}$. The scalar singlet $H$ might be testable via $e^+e^- \to ZH$ with $H \to b\bar{b}/H_1^0 H_1^{0*}$ at lepton colliders. Meanwhile, the promising signature at LHC is the trilepton signature as $pp \to H_2^\pm H_2^{0(*)} \to W^\pm Z + H_1^0 H_1^{0*}$ with leptonic decays of $W/Z$. The new $B - L$ gauge boson is expected discovered via the dilepton signature $pp \to \ell^+\ell^-$ at LHC [86]. And in principle, the constructed models in Table [I] and Table [II] can be distinguished by precise measurement of the invisible decays of $Z'$.

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