Effect of residual Bose-Einstein correlations on the Dalitz plot of hadronic charm meson decays

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Abstract

We show that the presence of residual Bose-Einstein correlations may affect the non-resonant contribution of hadronic charm decays where two identical pions appear in the final state. The distortion of the phase-space of the reaction would be visible in the Dalitz plot. The decay $D^+ \rightarrow K^- \pi^+ \pi^+$ is discussed but results can be generalized to any decay with identical bosons.

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1 Introduction

Hadronic decays of D mesons are an important source of information. The study of pseudoscalar-pseudoscalar and pseudoscalar-vector decays of D mesons, for example, shed light on heavy quark decay and hadronization. The lifetime difference between the $D^+$ and the $D^0$ mesons as well as the role of final state interactions may be understood by carefully studying their hadronic decays.

The fractions of resonant (pseudoscalar-pseudoscalar or pseudoscalar-vector) and non resonant three-body contributions to each decay mode are determined by fitting the Dalitz plot distributions to a coherent sum of amplitudes. The three-body amplitude is normally assumed to be constant over the Dalitz plot but, decays like $D^+ \rightarrow K^-\pi^+\pi^+$ contain identical charged pions in the final state and their interference produce a non-constant behaviour of the phase-space. The first evidence for a non-uniform population on the Dalitz plot for $D^+ \rightarrow K^-\pi^+\pi^+$ was shown in ref. [2]. In [1] an attempt is made to account for the interference by adding an ad hoc term to the likelihood function. The best fit is obtained with a Bose symmetric term of the form $B_3\text{body} = |m^2_{K\pi1} - m^2_{K\pi2}|$. However, they found that this form does not seem to adequately describe the physics.

In a recent analysis [3] the amplitudes of the fit were again unsuccessfully Bose-symmetrized in an attempt to describe the non-resonant contribution. Recently Bediaga et al. [4] studied the problem and found that the non-resonant contribution should actually not be constant over the phase-space. They used a method based on factorization and an effective hamiltonian for the partonic interaction to estimate the distortion of the Dalitz plot.

Here we will try to address the problem from the perspective of Bose-Einstein interference among the identical bosons in the final state and the residual correlation produced. This may help to understand the Dalitz plot of $D^+ \rightarrow K^-\pi^+\pi^+$ in particular, and the effect in any decay with several bosons in general.

2 Bose-Einstein correlations

The study of Bose-Einstein correlations (BEC) in connection with other phenomena [5] in high energy physics reactions became important recently when
it was realized that such correlations may affect measurements of the standard model parameters [6, 7].

Charm mesons travel long enough to neglect the effect of interference among their decay products and the pions produced in the reaction. Effects of the kind as described in [5] will not be considered here. We rather will regard the decay itself as a particle production process in which the interference may arise.

Fig. 1(a) shows the decay with a bubble representing the region in space-time where particles are produced. Decays where particles are produced via fragmentation are not completely coherent. The incoherence present in the decay gives rise to the Bose-Einstein interference.

The Bose-Einstein correlations are commonly described in terms of a two particle correlation function:

$$R_{BE}(p_1, p_2) = \frac{P(p_1, p_2)}{P(p_1)P(p_2)}$$

where $P(p_1, p_2)$ is the joint probability amplitude for the emission of two bosons with momenta $p_1$ and $p_2$, and $P(p_1)$ and $P(p_2)$ are the single production probabilities.

The Bose-Einstein correlation among identical mesons has been used to probe the space-time structure of the intermediate state right before hadrons appear [8, 9] in high energy and nuclear collisions.

Real data are analysed in terms of the ratio of distributions of like charged pairs (identical bosons) to that of an uncorrelated sample

$$R_{BE}(p_1, p_2) = \frac{C(p_1, p_2)}{C_0(p_1, p_2)}$$

The uncorrelated sample $C_0(p_1, p_2)$ with all the features of $C(p_1, p_2)$ except for the Bose-Einstein effect may be obtained from the distribution of unlike charged pions or by using an event mixing technique.

One parametrizes the effect assuming a set of point-like sources emitting bosons. These point like sources are distributed according to a density $\rho(r)$. The correlation function can then be written as,

$$R_{BE}(\vec{p}_1, \vec{p}_2) = \int \rho(\vec{r}_1)\rho(\vec{r}_2) \mid \psi_{BE}(\vec{p}_1, \vec{p}_2) \mid^2 d^3r_1 d^3r_2,$$
where $\vec{p}_1, \vec{p}_2$ are the momenta of the two bosons, $\psi_{BE}$ represents the Bose-Einstein symmetrized wave function of the bosons system and $\int_V \rho(\vec{r}) d^3 r = 1$. Taking plane waves to describe the bosons one obtains the correlation function for an incoherent source:

$$R_{BE}(\vec{p}_1, \vec{p}_2) = 1 + |F(\rho(\vec{r}))|^2,$$

(4)

where $F(\rho)$ represents the Fourier transform of the density function $\rho(\vec{r})$.

In order to describe the quantum interference during the fragmentation in high energy reactions, phenomenological parametrizations of the effect have been proposed. For a recent review see \[10\].

One of the most commonly used parametrizations is given by:

$$R_{BE}(\vec{p}_1, \vec{p}_2) = 1 + \lambda e^{-Q^2 \beta},$$

(5)

where $Q^2$ is the Lorentz invariant, $Q^2 = -(p_1 - p_2)$, which can be written also as, $Q^2 = m^2 - 4m^2_{\pi}$, where $m$ is the invariant mass of the two pions and $m_{\pi} = 0.139 GeV$ the mass of the pion. The parameter $\lambda$ lies between 0 and 1 and reflects the degree of coherence in the pion production. The radius of the pions source will be given by $r = \frac{hc}{\sqrt{\beta}}[fm]$.

The presence of Bose-Einstein correlations will modify not only the invariant mass spectrum of like charged but also that of unlike charged pions. This reflection of BEC known as residual correlation has been studied since long time ago to make sure that the reference sample of unlike charged pions used to subtract the effect from the like charged pions spectra is clean. In particle production processes with high multiplicity, residual correlations easily dissolve. Although some studies \[5\] claim that this is not the case and that using unlike charged pions as a reference sample is questionable. In a particle production process where only three particles are yield (e.g. $D^+ \rightarrow K^- \pi^+ \pi^+$) residual correlations are expected to be larger.

### 3 Simulation of Bose-Einstein correlations

In this letter we want to estimate the effects of BEC on the phase space of a three body decay. We take the particular case of $D^+ \rightarrow K^- \pi^+ \pi^+$ and look at it, as a particle production process with only two identical bosons in the final state. The reduced number of pions makes easy the incorporation of BEC in a Monte Carlo simulation. We will take the approach used in \[5\] where the BEC are simply simulated by weighting each event. The MC generator consists of a three body decay simulation with the appropriate masses of the
decay products. In [5] each event is weighted according to:

\[ W = \prod_{i,j} (1 + \lambda e^{-\beta Q_{ij}^2}), \quad (6) \]

where the product would be taken over all pairs \((i, j)\) of like charged pions. In our case the product is reduced to one pair of like charged pions

\[ W = 1 + \lambda e^{-\beta Q^2}. \quad (7) \]

We did simulations with different values for \(\lambda\) and \(\beta\). Fig. 2 shows the squared invariant mass distributions both for \(K^-\pi^+\) and \(\pi^+\pi^+\) before and after simulation of BEC. The \(Q^2(\pi^+\pi^+)\) distribution can be normalized using its distribution in the absence of BEC. Fig. 3 shows the correlation function \(R_{BE}(\pi^+\pi^+)\) obtained according to eq. (2) and the curve of eq. (5) with \(\lambda = 1\) and \(\beta = 4GeV^{-2}\). One can see that one obtains from the simulation exactly what one puts in.

### 4 Effects on the Dalitz plot

The effect of BEC on the Dalitz plot in some cases would be a straightforward manifestation of the interference among identical pions. An example of this is the Dalitz plot of \(D^+ \rightarrow K^-\pi^+\pi^+\) where the invariant mass squared of the two pions is plotted against the invariant mass squared of the kaon and one of the pions, i.e. \(m^2(\pi\pi)\) vs. \(m^2(K\pi)\). In this case the BEC will be visible directly in \(m^2(\pi\pi)\). The invariant mass spectra of \(m^2(K\pi)\), however, will show residual effects.

The Dalitz plot of \(m^2(K\pi_1)\) vs. \(m^2(K\pi_2)\) will show only residual effects.

Fig. 4 shows the Dalitz plots for the \(D^+ \rightarrow K^-\pi^+\pi^+\) decay once the BEC has been incorporated. The values \(\lambda = 1\) and \(\beta = 4GeV^{-2}\) were used to produce the distributions shown. Increasing \(\beta\) from \(4GeV^{-2}\) to \(100GeV^{-2}\) would make the long bands in Fig. 4 narrower. As mentioned above, the physical meaning of \(\beta = 4, 9, 16, 25, 100GeV^{-2}\) is given in terms of the pions source radius \(r \approx 0.4, 0.6, 0.8, 1.0, 2.0 fm\) respectively. We show the results using \(r = 0.4fm\) just because the effect is more visible. Of course the real value would be extracted from fits to experimental data.

A better fit of the non-resonant part of the decay would be possible once \(r\) has been extracted from data.
As in the case of the parameter $\beta$, we used $\lambda = 1$ because this produces the biggest possible effect. The value $\lambda = 1$ indicates a totally chaotic production in the decay. Of course a particle decay is not necessarily a completely chaotic process. In fact, if fragmentation would not take place during the decay, one may expect a completely coherent process in which $\lambda = 0$ and Bose-Einstein interference would not be present. Fragmentation in the decay introduces some degree of incoherence giving to $\lambda$ a value between 0 and 1. The exact value would be obtained fitting the correlation function as in the case of the source radius. On the other hand, there may be production mechanisms other than fragmentation that introduce some degree of incoherence. The study of BEC in particular decays may help to disentangle the production process.

The hadronic decay $D^+ \rightarrow K^-\pi^+\pi^+$ seems to be dominated by the non resonant contribution [11] and is therefore an interesting laboratory to study the role of BEC in hadronic decays in general. A resonant decay like $D^+ \rightarrow K^*\pi^+$ corresponds to a situation illustrated in Fig. 1(b). The interference between the pions in this case would, according to Bowler [12], reduce and narrow the strength of the correlation.

In hadronic decays where the resonant contribution is significant such effects must be taken into account.

An interesting aspect of the fact that BEC correlations are observable in hadronic decays of charm and eventually beauty mesons is that there must be some relationship between the parametrization of $\rho(r)$ for the pions source and the D meson form factors. Such relationship can be examined and the study of BEC may be a tool to learn more about the decay and fragmentation mechanism.

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References

[1] J. Adler et al. Phys. Lett. 196B (1987)107
[2] R. H. Schindler et al. Phys. Rev. D24 (1981)78
[3] P. L. Frabetti et al. Phys. Lett. **B331** (1994)217

[4] I. Bediaga, C. Göbel and R. Méndez Galain Phys. Rev. Lett. **78** (1997)22

[5] G.D. Lafferty Z. Phys. **C60** (1993)659

[6] A. Bialas and A. Krzywicki, Phys. Lett. **B354** (1995)134

[7] L. Lonnblad and T. Sjostrand Phys. Lett. **B351** (1995)293

[8] G. Goldhaber et al., Phys. Rev. **120** (1960)300

[9] R. Hernández and G. Herrera, Phys. Lett. **B332** (1994)448
   A. Gago and G. Herrera, Mod. Phys. Lett. **A10**(1995)1435

[10] D.H.Boal, C.K.Gelbke, B.K.Jennings, Rev. Mod. Phys. **62** (1990)553

[11] Particle Data Group; Review of Particle Physics Phys. Rev. **D54** (1996)1-720

[12] M. G. Bowler, Z. Phys. **C46** (1990)305
Figure Captions

Fig. 1: Picture of (a) the non-resonant part of the decay in which the partonic processes are overlooked. The bubble described by $\rho(r)$, parametrizes the space time of the production mechanism. In (b) a resonant decay is illustrated.

Fig. 2: Squared invariant mass distributions for $K^{-}\pi^{+}$ and $\pi^{+}\pi^{+}$ with (histogram) and without (crosses) BEC.

Fig. 3: Correlation function $R_{BE}$ obtained normalizing the $Q^{2}(\pi^{+}\pi^{+})$ distribution in the presence of BEC with the $Q^{2}(\pi^{+}\pi^{+})$ distribution without BEC. The curve corresponds to eq. (5) with $\lambda = 1$ and $\beta = 4 GeV^{-2}$.

Fig. 4: Dalitz plots for the $D^{+} \rightarrow K^{-}\pi^{+}\pi^{+}$ decay once the BEC has been incorporated.
Figure 1.
Figure 2.
Figure 4.