From soft to hard regime
in elastic pion-pion scattering
above resonances

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Abstract

We discuss the onset of the dominance of the Glauber-Gribov-
Landshoff (GGL) component of pQCD hard two-gluon (2G) exchange
contribution to hard elastic $\pi\pi$ scattering at moderate energies. Such
a hard $\pi\pi$ scattering could via final state interaction on the $\gamma\gamma \rightarrow
\pi^+\pi^-$ reaction in which pQCD quark-exchange contribution is known
to be short of strength. While in the nonrelativistic approximation
the GGL amplitude is known to be free of suppression by the pion
form factor, we show that in the relativistic light-cone approach it ac-
quires a residual, albeit a weak, suppression. Furthermore, the same
mechanism it is free of the end-point contributions. We evaluation the
GGL amplitude with a model light-cone wave function consistent with
the pion charge form factor data. The soft contribution to elastic $\pi\pi$
scattering is estimated based on the $NN$ and $\pi N$ total cross section
data and Regge factorization, which gives the $\pi\pi$ total cross sections
consistent with the ones deduced earlier from the absorption model
analysis of the $\pi N \rightarrow XN, X\Delta$ data. We evaluate the large-$|t|$ tail
of the soft amplitude within the Regge absorption models. We find
that while in the same sign $\pi^+\pi^\mp$ scattering the hard GGL mechanism
takes over at $|t| \gtrsim 3 \text{ GeV}^{-2}$, in the opposite-sign $\pi^\pm\pi^\mp$ scattering the
hard GGL mechanism $|t| \lesssim 4 \text{ GeV}^{-2}$. 
1 Introduction

The pion-pion scattering, although not directly accessible experimentally, is of special theoretical interest. At low energy it is the fundamental testing ground of chiral perturbation theory \[1, 2, 3\]. For the extension into the resonance region one includes explicit resonance fields in conjunction with suitable unitarization models \[4\] or invokes meson-exchange interactions \[5\] tested in low and intermediate energy \(NN\) and \(\pi N\) interactions. Soft, i.e., small angle, pion-pion scattering at moderate energies above the prominent resonances falls into the domain of the Regge theory. In the soft region the scattering amplitudes fall rapidly with \(|t|\) because of the form factor effects. Large-angle, hard, scattering will eventually be dominated by pQCD mechanisms.

At low and resonance energies one of important sources on \(\pi\pi\) scattering is the extraction of the pion exchange contribution to \(\pi + p \rightarrow \pi + \pi + N\) (see e.g.\[6\] and references therein). There is only very limited information on \(\pi\pi\) scattering above resonances. Here the information about the \(\pi\pi\) total cross section comes from the absorption model analysis of the experimental data on \(\pi N \rightarrow XN, X\Delta\) reactions \[7\].

There are no direct experimental data on hard \(\pi\pi\) scattering which is of special interest within pQCD. At low \(|t|\) the \(t\)-dependence of the scattering amplitude is controlled by the size of the beam and target hadrons. For instance, within the Glauber-Gribov multiple scattering theory of composite objects, the impulse approximation amplitude is proportional to the product of one-body form factors of the beam and target \[3, 4\]. Within the same multiple scattering theory, elastic scattering of the \(n\)-body beam on the \(n\)-body target receives the special contribution form the \(n\)-fold rescattering in which different constituents of the beam scatter off different constituents of the target. This special contribution does not depend on the size of the beam and target, i.e. is free of the form factor suppression. In the realm of pQCD such a three-gluon exchange mechanism of \(pp, \bar{p}p\) scattering has been discussed by Landshoff \[10\]. In scattering of the two-body pions the related Glauber-Gribov-Landshoff (GGL) mechanism emerges already at the level of the two-gluon exchange, i.e., in the Born term for the pQCD hard pomeron exchange. For this reason, one can expect a precocious dominance of the GGL mechanism in hard \(\pi\pi\) scattering.

In this communication we discuss the onset of the dominance of the GGL mechanism in \(\pi\pi\) scattering at \(\sqrt{s} \lesssim 10\) GeV. Such an analysis requires an understanding of the large-\(|t|\) tail of soft elastic scattering, which we evaluate within the Regge absorption models. The Glauber-Gribov arguments for the absence of the form factor suppression of the GGL amplitude were implicitly
based on the nonrelativistic (NR) approximation for composite systems. We find that in contrast to NR approximation, in the light-cone approach there is a residual, albeit weak, form factor suppression of the GGL amplitude. Although the direct experimental study of hard $\pi\pi$ elastic scattering is not feasible, this process can contribute via final state interaction to hard charge exchange reaction $\gamma\gamma \rightarrow \pi^+\pi^-$ which has been observed experimentally \cite{11} with the cross section much larger than perturbative QCD quark-exchange predictions \cite{12}. New, preliminary data from DELPHI \cite{13} suggest even stronger excess with respect to pQCD. Our interest in hard $\pi\pi$ scattering has been motivated by this missing strength of pQCD predictions, because soft $\gamma\gamma \rightarrow \pi\pi$ process followed by hard $\pi\pi$ elastic scattering could account for the missing strength.

The presentation of the paper is organized as follows. We start with the evaluation of the pQCD 2G exchange amplitude in section 2. In section 3, assuming Regge factorization, we derive parameters of soft $\pi\pi$ scattering, based on experimental data for $\pi N$ and $N N$ scattering. The role of multiple soft and hard rescatterings on angular distributions of pions is discussed in section 4.

\section{The pQCD two-gluon exchange}

Let us start with evaluation of the pQCD two-gluon contribution to the elastic pion-pion scattering. We treat the pion as the quark-antiquark state. The relevant pQCD diagrams which contribute to the pion impact factor are shown in Fig.4. A calculation of Born amplitudes in the non-relativistic approximation for the target and beam hadrons can be found elsewhere \cite{14}. In the present communication we use both nonrelativistic and the light-cone description of the pion.

Making use of the Sudakov technique \cite{15}, one readily obtains the impact factor representation of the pion-pion scattering amplitude

$$A(q) = is \cdot \frac{2}{9} \cdot \frac{1}{(2\pi)^2} \cdot \int d^2\kappa \, g_s^2(\kappa_1^2)g_s^2(\kappa_2^2)\Phi^{2G}_{\pi\rightarrow\pi}(q,\vec{\kappa})\Phi^{2G}_{\pi\rightarrow\pi}(q,\vec{\kappa}) \frac{1}{(\vec{q}/2 + \vec{\kappa})^2} \frac{1}{(\vec{q}/2 - \vec{\kappa})^2}, \quad (1)$$

where $\vec{\kappa}_{1/2} = \frac{\vec{q}}{2} \pm \vec{\kappa}$ are the exchanged-gluon momenta, which are purely transverse, $2/9$ is the QCD color factor for the $\pi\pi$ scattering process considered and $g_s$ is the QCD strong charge\footnote{In our practical calculations the QCD coupling constant is frozen in the infra-red region.}. Please note that the coupling...
constants $g_s$ have been taken out from the impact factors. We use the standard normalization of amplitudes such that

$$\frac{d\sigma}{dt} = \frac{1}{16\pi s^2}|A(t, u, s)|^2. \quad (2)$$

We approximate the pion by its $q\bar{q}$ Fock state. A relatively straightforward calculation of the diagrams which define two-gluon pionic impact factor gives \[16\]

$$\Phi_{\pi \rightarrow \pi}^{2G}(\vec{q}, \vec{\kappa}) = \int_0^1 \frac{dz d^2k}{z^2(1-z)^2} \left\{ \Psi(z, \vec{k})\Psi(z, \vec{k} + z\vec{q}) \cdot [m_Q^2 + \vec{k} \cdot (\vec{k} + z\vec{q})] + \Psi(z, \vec{k})\Psi(z, \vec{k} - (1-z)\vec{q}) \cdot [m_Q^2 + \vec{k} \cdot (\vec{k} - (1-z)\vec{q})] - \Psi(z, \vec{k})\Psi(z, \vec{k} + (z - \frac{1}{2})\vec{q} + \vec{\kappa}) \cdot [m_Q^2 + \vec{k} \cdot (\vec{k} + (z - \frac{1}{2})\vec{q} + \vec{\kappa})]ight\}. \quad (3)$$

Here the pion-quark-antiquark vertex is taken of the form $i\Gamma_\pi(M^2)\bar{\Psi}\gamma_5\Psi$. In terms of the quark & antiquark helicities $\lambda$ the $\pi q(k)\bar{q}(-k)$ vertex has the form ([17], for the related discussion see Jaus [[18]])

$$\bar{\Psi}(k)\gamma_5 q(\lambda)(-k) = \frac{\lambda}{\sqrt{z(1-z)}}[m_Q\delta_{\lambda,-\lambda} - k(\lambda)\delta_{\lambda,\lambda}], \quad (4)$$

where $m_Q$ is the quark mass, taken equal for the up and down quarks, $k(\lambda) = \sqrt{2k \cdot \epsilon_\lambda}$ and $\epsilon_\lambda = \frac{1}{\sqrt{2}}(\lambda\epsilon_x + i\epsilon_y)$ is the familiar polarization vector for the state of helicity $\lambda$. In transitions of spin-zero pions into $q\bar{q}$ states with the sum of helicities $\lambda + \bar{\lambda} = \pm 1$ the latter is compensated by the orbital momentum of quark and antiquark.

The radial wave function of the pion in the momentum space is defined in terms of the $\pi q\bar{q}$ vertex function as

$$\psi_\pi(z, k) = \frac{N,\Gamma_{\pi}(R, M^2)}{4\pi^3 z(1-z)(M^2 - m_\pi^2)} \quad (5)$$

where $M$ is the invariant mass of the constituent quark-antiquark system,

$$M^2 = \frac{\vec{k}^2 + m_Q^2}{z(1-z)}.$$
\( \vec{k} \) is the transverse momentum of the quark, \( z \) and \( 1 - z \) are fractions of the lightcone momentum of the pion carried by a quark and antiquark, respectively.

The first two terms in (3) corresponding to the impulse approximation (IA) diagrams (a) and (b) in Fig.1 are equal to the one-body pion form factor,

\[
G_\pi(q^2) = \int_0^1 \frac{dz}{z^2(1-z)^2} \int d^2k [m_Q^2 + \vec{k} \cdot (\vec{k} + z\vec{q})] \Psi(z, \vec{k}) \bar{\Psi}(z, \vec{k} + (z\vec{q})) .
\]  

(6)

Such contributions, when exchanged gluons couple to one constituent, are typical for additive quark models and are suppressed at large transverse momentum \( q \) by the form factor.

The last two terms in (3) correspond to the diagrams (c) and (d) in Fig.1 and in conjunction with their counterpart in the target impact factor give the GGL contribution to elastic scattering amplitude. Now notice that in the NR approximation \( z = \frac{1}{2} \), so that the contribution of the diagrams (c) and (d) would not depend on \( \vec{q} \) altogether. Then a comparison with the one-body form factor (3) evaluated in the same approximation would give the impact factor of a simple form

\[
\Phi^{2G}_{\pi \to \pi}(q, \vec{\kappa}) = 2[G_\pi(q^2) - G_\pi(4\kappa^2)] ,
\]

(7)

where the factor 2 in front is the number of constituents in the pion.

Beyond the NR approximation, the terms \( (z - \frac{1}{2})q \) in the arguments of the last two terms in Eq. (3) are substantial at large \( q \) and make them decreasing with \( q \). Furthermore, at very large \( q \) the dominant contribution to the GGL amplitude would come from \( (z - \frac{1}{2}) \approx \frac{1}{R_\pi q} \), where \( R_\pi \) is the pion radius, so that the GGL amplitude is free of contributions from the end points \( z \to 0 \) and \( z \to 1 \) (for the discussion of the end-point properties of the GGL contribution to the pp scattering see \[19\]).

For the calculation of pQCD 2G-exchange in the NR case it is sufficient to know the form factor of the pion. We take here simply the \( \rho \)-dominance monopole parameterization. In the light-cone approach, the Ansatz for the WF must be subjected to the normalization condition,

\[
\int_0^1 \frac{dz}{z(1-z)} \int d^2k M^2 |\Psi(z, \vec{k})|^2 = 1 ,
\]

(8)

and its parameters, i.e., the effective quark mass \( m_Q \) and the pion radius \( R_\pi \) must be constrained by the \( \pi \to \mu \nu \) decay constant (we use the PDG convention \( F_\pi = 131 \) MeV \[20\])

\[
F_\pi = \int d^2k dz m_f \psi_\pi(z, k) ,
\]

(9)
the charge radius of the pion as defined by eq. (6), the slope of the form factor of the $\pi^0 \rightarrow \gamma \gamma^*$ transition and the decay width $\Gamma_{\pi^0 \rightarrow \gamma \gamma}$ [10, 13, 17]. The results for these low-energy parameters for the pion from the WF parameterization [17] with $m_Q = 0.215$ GeV and $R_\pi = 2.2$ GeV$^{-1}$ are cited in table 1. This WF provides a satisfactory description of the charge form factor of the pion into the semihard region of $q^2$, as seen from fig 2, in which we compare the prediction from eq. (4) to the recent experimental data from the Jefferson Laboratory [20].

The above discussion of the 2G exchange holds in the pQCD domain of $|t| \gg 1$ GeV$^2$. In order to extrapolate the hard pQCD scattering contribution to the forward scattering amplitude and to the total cross section, one needs to impose an infrared (IR) regularization to mimic the finite propagation radius of color fields. We model this by the “Debye” screening in the gluon propagator,

$$\frac{1}{\kappa^2} \rightarrow \frac{1}{\kappa^2 + m_g^2},$$

where $m_g = R_c^{-1}$. It was found in [22] that the pQCD 2G exchange with the Debye screening radius $R_c \approx 0.27$ fm provides a viable boundary condition for the color dipole BFKL description of the proton structure function. More recently, very close values of the screening radius $R_c$ were found from an analysis [23] of color field correlations in lattice QCD, (see also a recent review [24] on the effective mass of the gluon). The different analyses lead to $m_g$ of about 0.75 GeV. The band between the results for $m_g = 0.5$ GeV and $m_g = 1.0$ GeV can be regarded as a conservative estimate for uncertainties of the two-gluon exchange amplitude at subasymptotic $|t|$.

Table 1: Low energy parameters of the pion from the Ansatz [17] for the pion WF.

| observable       | experiment | WF Ansatz [17] |
|------------------|------------|----------------|
| $<r^2>_{\pi}$ [fm$^2$] | 0.451 ± 0.05 | 0.442 |
| $f_\pi$ [MeV]   | 130.7 ± 0.15 | 121 |
| $\Gamma_{\pi^0 \rightarrow \gamma \gamma}$ [eV] | 7.8 ± 0.6 | 7.67 |
| $\Lambda_{\pi^0}$ [MeV] | 750 ± 30 | 640 |

The numerical results from pQCD 2G exchange for $\pi \pi$ scattering are shown in Fig.3 for different values of the IR screening parameter $m_g$, for the nonrelativistic (left panel) and light-cone (right panel) approach. The

6
hatched area gives an idea of the uncertainties due to the not precisely known correlation radius. For a comparison, we show also the soft cross section evaluated in the Regge impulse approximation, i.e., the single pomeron and reggeon exchange. At $|t| \gtrsim 2 \text{ GeV}^2$ the hard contribution takes over the soft contribution evaluated in the impulse approximation, for the discussion of how the soft-hard interplay is affected by absorption correction see in more detail below.

The anatomy of the pQCD 2G exchange is shown in more detail in fig. 4. Here the dashed line shows the result found if only the IA components of result obtained if only the impulse approximation diagrams (a) and (b) of fig. 1 are kept in the impact factor of both pions. The dotted line shows the pure GGL contribution, when only the diagrams (c) and (d) of fig. 1 are kept in the impact factor of both pions. The contribution from the impulse approximation components of the impact factors dominates at small to moderate values of $|t| \lesssim 0.5 \text{ GeV}^2$, where the nonrelativistic and light-cone amplitudes are nearly identical. The slight variation from NR to LC cases is due to the fact that the one-body form factor given by the LC Ansatz decreases somewhat faster than the $\rho$-pole formula. The GGL mechanism starts taking over at $|t| \gtrsim 1.0 \text{ GeV}^2$. Here a comparison of the NR and LC cases shows clearly a substantial suppression of the GGL contribution by the $q-z$ correlations inherent to the LC case. One should note, however, that destructive interference of the impulse approximation and GGL components of the impact factor remains noticeable even at large $|t| \sim 4 \text{ GeV}^2$.

3 Soft Regge-pole amplitudes

The understanding of the onset of hard pQCD regime requires evaluation of the large-$|t|$ tail from soft non-perturbative interactions. In the nonperturbative region of small transferred momenta in baryon-baryon and meson-baryon elastic scattering one is bound to phenomenological parameterizations such as the seasoned absorption Regge model [25, 26]. The powerful Regge factorization enables one to estimate the $\pi\pi$ scattering amplitudes from the experimental data on $\pi N$ and $NN$ scattering. The absorption corrections model the large-$|t|$ tail of the scattering amplitude.

In the case of pion-pion scattering in the considered region of energies the soft pomeron exchange must be supplemented by the subleading isoscalar ($f$) and isovector ($\rho$) reggeon exchanges. For the purposes of our analysis we
resort to the simplest Regge-inspired phenomenological form:

\[ A_{\Pi}(t) = i C_{\Pi} \cdot (s/s_0)^{\alpha_{\Pi}(t)} \cdot F_{\Pi}^2(t), \]

\[ A_f(t) = -\eta_f(t) C_f \cdot (s/s_0)^{\alpha_f(t)} \cdot F_f^2(t), \]

\[ A_\rho(t) = -\eta_\rho(t) C_\rho \cdot (s/s_0)^{\alpha_\rho(t)} \cdot F_\rho^2(t), \]

(10)

where \( \eta_f \) and \( \eta_\rho \) are somewhat simplified signature factors:

\[ \eta_R = \exp(i\phi_R(t)), \]

(11)

with

\[ \phi_R(t) = \begin{cases} -\frac{\pi}{2} \alpha_R(t) & \text{for positive signature,} \\ -\frac{\pi}{2} [\alpha_R(t) - 1] & \text{for negative signature.} \end{cases} \]

(12)

The pion-reggeon(pomeron) vertex form factor \( F_i \) in (10) is not calculable from the first principles and one is bound to parameterizations driven by the educated guess and plausible restrictions on the large-\( |t| \) behaviour of the form factors. One the one hand, one would like soft amplitudes to decrease at large-\( |t| \) faster than the perturbative ones, and one customarily uses the exponential parametrization \( F(t) = \exp(B_4 t) \). On the other hand, the differential cross section of the elastic scattering exhibits at small \( |t| \) a curvature which is somewhat better described if one would take the inverse power parameterization \( F(t) = 1/(1 - B_{mon}) \). Such a monopole form factor for the pion-reggeon vertex should not be extended indiscriminately to large-\( |t| \), though, otherwise soft and hard form factors would have had unwanted similar asymptotic behaviour. In what follows, we shall evaluate the absorption corrections for the exponential soft pion-reggeon(pomeron) form factor.

In the Regge-pole exchange approximation, the powerful Regge factorization allows to relate the residues \( C_i \) of the Regge pole contributions to different scattering processes. are related by the Regge factorization. In our case of \( \pi\pi \) scattering residues at \( t = 0 \) can be evaluated from those for \( \pi N \) and \( NN \) scattering as:

\[ C_{i \pi\pi} = \frac{(C_{i \pi N})^2}{C_{i NN}} \]

(13)

for each reggeon considered \( i = \Pi, f, \rho \). Although absorption corrections, i.e., the Regge cut contributions, violate the exact Regge factorization, it still remains a viable approximation for small \( t \), as well documented in many reactions [23, 26]. Specifically, although the absorption corrections to the Regge-pole approximation can be as large as 10-20 %, they are believed not
to change strongly from one reaction to another [25, 26]. For our purpose of evaluating the soft background to hard pQCD pion-pion scattering, we are content with the 10-20% accuracy. Then, based on the known Regge phenomenology of \( \pi N \) and \( NN \) scattering [27], from (13) we find for \( \pi \pi \) scattering 
\[ C_{IP} = 8.56 \text{ mb}, \quad C_f = 13.39 \text{ mb} \quad \text{and} \quad C_\rho = 16.38 \text{ mb}. \]

We take for the pomeron trajectory \( \alpha_{IP}(0) = 1 \) and \( \alpha_{IP}^1 = 0.25 \text{ GeV}^{-2} \) and for both subleading trajectories \( \alpha_R(0) = 0.5 \) and \( \alpha_R^1 = 0.9 \text{ GeV}^{-2} \), i.e. values well known from the Regge phenomenology [26]. In the above evaluation we can safely neglect a small, see fig. 3, pQCD 2G exchange contribution to the \( \pi N \) and \( NN \) total cross section (see also [28]). For the diffraction slope, the same Regge factorization entails

\[ B_{\pi\pi} \approx 2B_{\pi N} - B_{NN}, \]  

which suggests \( B_{\pi\pi} \sim (6 - 8) \text{ GeV}^{-2} \). The diffraction slopes are fairly sensitive to the absorption corrections, though, see the discussion below. For the purposes of the present exploratory analysis, it is sufficient to a universal slope for all the pion-reggeon(pomeron) vertex form factors, \( B_{IP} = B_f = B_\rho = B \). This slope is the main adjustable parameter of our soft amplitudes.

The total single-reggeon exchange amplitude is now

\[ A_{1^{st}\text{soft}}(t) = A_{IP}(t) + A_f(t) + \xi A_\rho(t), \]  

where \( \xi = -1 \) for \( \pi^+\pi^- \), \( \xi = 0 \) for \( \pi^+\pi^0 \) and \( \xi = 1 \) for two identical pions.

We note in passing that QCD motivated models for soft pomeron exchange were discussed in [28, 29]. To a crude approximation, such models respect the quark additivity and in their extension to the \( \pi\pi \) scattering are similar to the Regge factorization approach.

The role of the soft Regge amplitude at small \( t \) is illustrated in fig. 3, where the dashed line shows the soft Regge contribution to the differential cross section evaluated with the exponential form factor \( F \) and \( B = 6 \text{ GeV}^{-2} \), as suggested by (14). As anticipated the soft-reggeon exchanges dominate at small \( |t| \) over the discussed in the previous section two-gluon exchange. The situation reverses at larger \( |t| \), but as we shall see below, the exact pattern of the soft-to-hard transition details depends on somewhat on the absorption corrections.

Now we are in the position to evaluate the total cross section for \( \pi^+\pi^- \) scattering. The results for the Regge-pole approximation, including small hard two-gluon component, are shown in Fig. 5a by the dashed line. To the

\footnote{Here we did not include yet absorption corrections, which allowance for which the preferred slope of the Regge-pole amplitudes is rather close to \( B \approx 4 \text{ GeV}^{-2} \).}
extent that the absorption corrections are small and depend only weakly on
the diffraction slope, the Regge factorization evaluation of $\sigma_{\pi\pi}^{\text{tot}}$ is parameter
free. This can be judged from the thick solid line in which we include in ad-
inclusion the absorption corrections evaluated in the double-scattering approx-
imation to be discussed in more detail in the next section \(^3\). Our predictions
well coincide with total cross sections extracted in \([7]\) from the absorption
Regge model analysis of $\pi N \to XN, X\Delta$ reactions. This extraction of the
pion exchange contribution is not entirely parameter free and is subject to $\sim$
(10-20)% uncertainties. The low-energy extrapolation of this parameter-free
Regge model results joins smoothly with the low energy data on the $\pi^+\pi^-$
total cross section in the resonance region \([30, 31]\), in close analogy to to the
situation in $\pi N, NN, K N, K\bar{N}$ scattering, see the total cross section plots in
PDG \([20]\).

In Fig.\(5\) we compare our predictions for same-sign pion-pion scatte-
ring with the quasi-data from \([7]\). While the opposite-sign pion-pion total
cross section depends strongly on energy, the same-sign pion-pion total cross
section is almost independent of energy. In the spirit of duality, the near
cancellation of contributions from crossing-even and crossing-odd Regge ex-
changes in the $\pi^+\pi^+, \pi^-\pi^-$ channels is not accidental and is consistent with
the absence of isotensor $s$-channel resonances, in close analogy to the flat-
ness of the $pp$ total cross section. Exactly the same effect can be seen in the
quasi-data from \([7]\).

For the sake of completeness we show in Fig.\(6\) also the ratio of the real-
to-imaginary parts of the scattering amplitude:
\begin{equation}
\rho = \frac{\text{Re}(A(t = 0, W))}{\text{Im}(A(t = 0, W))}.
\end{equation}

In close analogy to the $p\bar{p}, pp$ system, the destructive interference of crossing-
odd and crossing-even amplitudes in the total cross section corresponds to
the constructive interference in the real part and vice versa. For this reason
$\rho$ is large in the $\pi^+\pi^+$ and $\pi^-\pi^-$ channels and nearly negligible in the $\pi^+\pi^-$
channel.

4 Absorption corrections and multiple soft
and hard exchanges

When going to the region of intermediate $|t|$ one has to include absorption cor-
rections, which model the Regge cuts due to multiple pomeron and reggeon

\(^3\)Adding absorption corrections to the Regge-factorization estimates for $C_i$ is not en-
tirely consistent, here we only want to give an idea on the size of the absorption effects.
exchanges. The salient feature of these multiple exchange amplitudes is that at large $|t|$ they decrease slower than the Regge-pole amplitude. At high energies, when the pomeron exchange dominates the leading absorption corrections are known to have the sign opposite to single reggeon exchange. This is the origin of diffractive dip in, e.g., proton-proton scattering which is partly filled due to a smaller real part of the scattering amplitude. At intermediate energies, we consider here, the situation is somewhat more complicated. The absorption corrections affect also substantially the diffraction slope and its $t$-dependence. In this section we shall discuss such effects for pion-pion scattering.

In the evaluation of absorption corrections one usually resorts to the so-called eikonal approximation (see for instance [32] and references therein; a more recent discussion and references can be found e.g. in [33]). Here we restrict ourselves to the dominant double-scattering corrections which read

$$A^{(2)}_{ij}(s,\vec{k}) = \frac{i}{32\pi^2s} \int d^2\vec{k}_1d^2\vec{k}_2 \delta^2(\vec{k} - \vec{k}_1 - \vec{k}_2) A^{(1)}_{i}(s,\vec{k}_1) A^{(1)}_{j}(s,\vec{k}_2). \quad (17)$$

In general, the single scattering amplitudes $A^{(1)}_{k}$ in (17) are not restricted to soft reggeon exchanges and hard two-gluon exchanges should be included too. Consequently in the following we shall include the (soft $\otimes$ soft), (soft $\otimes$ hard)+(hard $\otimes$ soft) and (hard $\otimes$ hard) double-scattering amplitudes. The double-scattering involving hard mechanism is expected to be small, at least at forward angles, compared to the leading (soft $\otimes$ soft) absorption correction. In the (hard $\otimes$ hard) case the eikonal amplitude sums only a certain subset of possible four-gluon exchange amplitudes, but numerically this contribution is entirely negligible.

First, let us focus on salient features of double-scattering contributions to the imaginary part of the pion-pion elastic scattering amplitude shown in Fig.7. In this calculation $m_g = 0.75$ GeV and the slope parameter $B = 4$ GeV$^2$ is adjusted to have a reasonable slope of the forward diffractive peak. We observe that at large $|t|$ the mixed (soft $\otimes$ hard) + (hard $\otimes$ soft) terms are of a size comparable to that of the (soft $\otimes$ soft) terms. Very small (hard $\otimes$ hard) terms can at best contribute at the diffraction dip, otherwise it is negligible small for all the practical purposes. We observe a huge difference between the opposite-sign and same-sign pion-pion scattering for the (soft$\otimes$soft component, which is caused by a substantial contribution from the secondary reggeon exchange. This difference is further illustrated in Fig.8, where we present for the decomposition of the imaginary part of the scattering amplitude for elastic $\pi^+\pi^-$ scattering (left panel) and the same-sign pion scattering (right panel) at $W = 4$ GeV (solid lines) into single- (dash-dotted line) and double-scattering contributions (dashed line) terms. A destructive
Table 2: Global characteristics of elastic pion-pion scattering for exponential ($B=4 \text{ GeV}^{-2}$) and monopole ($B_{\text{mon}} = 1 \text{ GeV}^{-2}$) form factors.

| Reggeon f.f. | W (GeV) | $\sigma_{el}^1$ (mb) | $\sigma_{el}$ (mb) | $B_{eff}$ (GeV$^{-2}$) | $\sigma_{el}^1$ (mb) | $\sigma_{el}$ (mb) | $B_{eff}$ (GeV$^{-2}$) |
|--------------|---------|----------------------|-------------------|----------------------|----------------------|-------------------|----------------------|
| exp.         | 3       | 2.87                 | 1.38              | 7.59                 | 1.05                 | 0.91              | 8.62                 |
|              | 4       | 2.10                 | 1.20              | 7.91                 | 0.86                 | 0.72              | 8.81                 |
|              | 5       | 1.71                 | 1.06              | 8.08                 | 0.77                 | 0.64              | 8.73                 |
| mon.         | 3       | 3.13                 | 1.38              | 7.77                 | 1.13                 | 0.96              | 8.63                 |
|              | 4       | 2.28                 | 1.20              | 8.07                 | 0.93                 | 0.76              | 8.85                 |
|              | 5       | 1.84                 | 1.08              | 8.21                 | 0.85                 | 0.68              | 8.79                 |

Interference of the single- and double-scattering contributions is much weaker for the same-sign pions than for the opposite-sign pions. This property is also seen in the table 2, where we show the total elastic cross section calculated for several energies in the Regge impulse approximation without, $\sigma_{el}^1$, and including, $\sigma_{el}$, absorption corrections (we notice in passing that the effect of the pQCD 2G-exchange on total elastic cross section is marginal and does not exceed 10 per cent). The two evaluations do practically coincide for the same-sign pions, but for the opposite-sign pions the effect of absorption on elastic cross section is strong. That will lead to a pronounced difference for the corresponding cross sections in the soft-hard interference region of intermediate $|t|$.

In Fig.9 we show the decomposition of the differential cross sections of elastic scattering (solid line) into the contributions of single- , including both the sift and 2G exchange contributions, and double-scattering terms, shown by the dashed and dotted lines, respectively (the interference term is not shown). One observes quite a different pattern of destructive interference of single- and double-scattering for the opposite-sign (left panel) and the same-sign (right panel) pion-pion scattering. The diffractive dips for the opposite-sign pion-pion scattering occur at values of $t$ at which the contributions to differential cross section from single- (dashed line) and double-scattering (dotted line) are about identical. Whereas the origin of different diffractive structures for the same-sign and opposite-sign pions is clear and the onset of pQCD hard regime in the same-sign pion scattering is substantiated by this analysis, one must conclude that model-dependence of the double-scattering amplitude makes the large-$|t|$ results and the dip positions for the opposite-sign case numerically unstable. This point is corroborated...
by the energy dependence of differential cross section shown in Fig. 10: while for the same-sign pions (the right panel) it is very weak in accordance to the small contribution from secondary reggeons to the imaginary part of the scattering amplitude, for the opposite-sign $\pi^+\pi^-$ case (the left panel) the substantial energy dependence of the secondary reggeon contribution entail very strong energy dependence at large-$t$. Only at very large $t \sim 4 \text{ GeV}^2$ the $\pi^+\pi^-$ indicates an onset of weak energy dependence. This masking effect of subleading reggeon exchanges persists up to rather high $W \sim 10\text{-}20 \text{ GeV}$.

The above shown results were for the exponential reggeon-pion vertices with $B = 4 \text{ GeV}^{-2}$. The sensitivity of the forward diffraction peak to the parameterization of the reggeon-pion vertices is shown in Fig. 11, where we plot the effective diffraction slope

$$B_{eff} = -\frac{\log(d\sigma/dt)}{dt}$$

for the $\pi^+\pi^-$ scattering. These plots demonstrate clearly a substantial enhancement of the diffraction slope by absorption corrections: systematically $B_{eff} > B$, see also the results for the diffraction slope shown in Table 2. Notice the rise of $B_{eff}$ with increasing $|t|$ for the exponential parameterization, which is driven by destructive interference of the single- and double-scattering amplitudes (the same trend is obvious from the convex shape of the $\pi^+\pi^-$ differential cross section in Fig. 10). On the other hand, the diffraction slope obtained with the monopole parameterization for the reggeon-pion vertex decreases with rising $|t|$ in close semblance to what has been observed experimentally in the proton-proton and pion-proton scattering \[34, 35\]. Still, at large $|t|$ the monopole form factor gives a soft amplitude which does not vanish much faster than pQCD asymptotic predictions \[36\] and, consequently, should not be applied beyond the small $t$ region. This shows that there is no simple one-parameter functional form which would be preferable for both small and large $|t|$.

5 Conclusions

In the present analysis we have explored the onset of pQCD hard mechanism for elastic pion-pion scattering in the region of intermediate energies $W = 3\text{-}5 \text{ GeV}$. The Glauber-Gribov-Landshoff mechanism is shown to dominate hard two-gluon scattering at large $|t|$. We have shown that while in the non-relativistic approximation for the pion the GGL amplitude is free of the form factor suppression at large $|t|$, this is not so in the relativistic lightcone approach. Furthermore, the correlation between the transverse and longitudinal motion of quarks inherent to the lightcone treatment makes the GGL
amplitude free of the end-point contributions. Assuming dominance of soft physics and Regge factorization at small $|t|$ we have predicted the total cross section for pion-pion scattering consistent with experimental values extracted in the literature.

We analysed the impact of large-$|t|$ tail of soft hadronic scattering on the onset of pQCD hard mechanism. Within the conventional Regge absorption models, there emerges a rather complex interplay of soft-hard interference. Specifically, pQCD hard scattering is found to dominate elastic scattering of the same sign pions at $|t| \gtrsim 3$ GeV$^2$. However, in the case of the opposite-sign pions the destructive soft-hard interference remains strong up to at least $|t| \gtrsim 4$ GeV$^2$.

The effects discussed here are important in the context of a recently reported deficit of pQCD result at large-angle scattering in $\gamma\gamma \rightarrow \pi^+\pi^-$ as compared to the experimental data measured at electron-positron colliders. The discussion of the latter goes beyond the scope of the present analysis and will be presented elsewhere.

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Figure 1: The pQCD diagrams for the pion impact factors.
Figure 2: Charged pion electromagnetic form factor. The experimental data are from [21]. The solid line is the result of calculation based on Eq. (6) with the light-cone wave function. The dashed line is the monopole parametrization with the $\rho$-meson mass.
Figure 3: Angular distribution of elastic pion-pion scattering for the two-gluon exchange model with different values of the infrared regularization parameter $m_g = 0.5, 0.75, 1.0$ GeV. The dashed line shows the soft $\pi^+\pi^-$ elastic scattering evaluated in the Regge-pole approximation with $B_{eff} = 6$ GeV$^{-2}$. 
Figure 4: The emergence of GGL dominance in elastic pion-pion scattering for the two-gluon exchange model for $m_g = 0.75$ GeV. The dashed line is for pure IA contributions (a) and (b) for the impact factor in Fig. 1, whereas the dotted line corresponds to the pure GGL terms (c) and (d) for the impact factor in Fig. 1. The thick solid line corresponds to the full result with all terms for the impact factor.
Figure 5: Total cross section for $\pi^+\pi^-$ (left panel) and $\pi^+\pi^+$ or $\pi^-\pi^-$ (right panel) scattering as a function of center-of-mass energy $W$. The experimental data are from [7]. The experimental data for $\pi^+\pi^-$ scattering (left panel) were extracted from $\pi^+p \rightarrow X\Delta^{++}$ (open circles) and from $\pi^+n \rightarrow Xp$ (full circles). The experimental data for $\pi^-\pi^-$ scattering (right panel) were extracted from $\pi^-p \rightarrow X\Delta^{++}$ (open circles) and from $\pi^-n \rightarrow Xp$ (full circles). The single pomeron and subleading reggeon exchanges are given by the dashed lines. The solid line is obtained from the dashed line after including the absorption corrections to be discussed in the next section.
Figure 6: The ratio of the real to imaginary part of the forward elastic scattering amplitude ($\rho$) as a function of center-of-mass energy $W$. The meaning of the curves is the same as in the previous figure.

Figure 7: The imaginary part of the amplitude of isolated different double-scattering terms for elastic $\pi\pi$ scattering: (soft $\otimes$ soft) - dashed line, (soft $\otimes$ hard) or (hard $\otimes$ soft) - dash-dotted line and (hard $\otimes$ hard) - dotted line for $W = 4$ GeV for the opposite-sign pions (left panel) and for the same-sign pions (right-pions). The sum of all contributions is given by the thick solid line.
Figure 8: Imaginary part of the single-scattering (dashed) and double-scattering (dotted) terms. The resulting imaginary part of the full amplitude is shown by the solid line.
Figure 9: The effect of the absorption corrections on the $t$-dependence of the elastic $\pi\pi$ cross sections for opposite-sign pions (left panel) and same-sign pions (right panel) for $W = 4$ GeV. In this calculation the slope parameter $B = 4$ GeV$^{-2}$. The cross section for single-exchange is shown by the dashed line, while the cross section which includes double-scattering effect is shown by the solid line. For the discussion in the text by the dotted line we show the cross section calculated from the double-scattering amplitude alone.
Figure 10: The $t$-dependence of the pion-pion elastic cross section with the inclusion of all single- and double-exchange contributions for different center of mass energies 3 (dash-dotted), 4 (dashed), 5 (solid) GeV. In this calculation: $B = 4 \text{ GeV}^{-2}$, $m_g = 0.75 \text{ GeV}$.

Figure 11: The effective slope parameter for elastic $\pi^+\pi^-$ scattering at $W = 4 \text{ GeV}$ as a function of the Mandelstam variable $t$ for exponential ($B = 2, 3, 5 \text{ GeV}^{-2}$) and monopole ($B_{\text{mon}} = 1, 1.5, 2 \text{ GeV}^{-2}$) form factors.