GPS Positioning Error Compensation Based on Kalman Filtering

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Abstract. GPS is a widely used navigation signal in road traffic, bringing great convenience to transportation. However, when it is used in advanced driving assistance system (ADAS), the positioning accuracy cannot meet the needs of applications and the positioning error could reach to ten meters. In order to improve the positioning accuracy, the Kalman filtering and extended Kalman filtering are adopted. By appropriate kinematic modelling and actual data collection, the approach is shown to be effective to improve positioning accuracy in the sense of probability.

1. Introduction
GPS enables automatic vehicle location and in-vehicle navigation systems that are widely used throughout the world today [1]. But in some applications of ADAS, when the vehicle moves at high speed or have high manoeuvrability, if the drivers use GPS navigation system, it is hardly to detect pedestrian on the road, and it is also difficult for a driver to take effective precautions before a terrible traffic accident comes. The cause of this problem is GPS positioning error. The GPS gives measurements for position, heading and inertial velocity. Under the best conditions, i.e., eight satellites or more, GPS error has a variance of about 30cm. However, when the number of satellites is six or less due to building or trees, the errors are as large as 10 meters. Moreover, at six satellites or less, the error usually has a bias.

This paper aims to design an algorithm which used to improve the accuracy of GPS positioning. To solve the aforementioned challenges, we integrate GPS to some types of Kalman filters to calibrate GPS measurement errors and provide two different Kalman filter models that are used in different scenarios. Kalman filter is a recursive process of continuous prediction and correction, because it does not need to store many observation data. A new data can be calculated at any time when new observation data is obtained, so it is convenient for real-time. Data processing is also widely used in computer implementations and applications that require real-time processing of data. The data in GPS positioning is undoubtedly one kind of data that can be dealt with in this way.

The rest of this paper is organized as follows. First, the way to reduce the civil GPS error and to predict pedestrian trajectories will be presented. Next, the algorithm to improve GPS positioning accuracy will be presented too. Finally, the picture of the real-time road situation and the results of experimental trials with real data are shown.

2. The compensation of GPS error
At present, the use of augmentation systems can reduce GPS positioning errors to m levels, including the Wide Area Augmentation System (WAAS), European Geostationary Navigation Overlay Service (EGNOS), Differential GPS (DGPS), inertial navigation systems (INS) and Assisted GPS. The standard
accuracy of about 15 meters (49 feet) can be augmented to 3–5 meters (9.8–16.4 ft) with DGPS, and to about 3 meters (9.8 feet) with WAAS [2]. But this requires complex systems and expensive equipment to support.

According to the causes of GPS positioning error, domestic and foreign scholars have carried out relevant work mainly from the following aspects:

2.1. Compensation for the effects of atmospheric effects.
Hoetal [3] using Global Ionospheric Scintillation Model (GISM) to mitigate the ionospheric effect. Yue Y et al [4] deal with the tropospheric delay by ordinary kriging interpolation method. Helen Hopfield, Harold Black, Jouko Saastamoinen, and Arthur Niell have published a succession of algorithms during the past 30 years that can be used to predict tropospheric delay with increasing accuracy [5].

2.2. Compensation for the effects of the multipath effect.
Multipath is the major error source in high precision Global Positioning System (GPS) static and kinematic positioning [6]. A wide variety of methods from simple antenna site selection to hardware and software approaches have been adopted to tackle multipath effect in GPS. Hardware approaches usually based on optimal antenna site selection and receiver technology. But methods based on hardware modifications are less popular since the fact that the most of users could not implement hardware methods [7]. In contrast, software methods are more user-friendly. The filter-based approaches have been used to extract or eliminate multipath effects [8], such as wavelet filters [9][10][11], Vondrak filter [12] and adaptive filter [13].

3. The design of the Kalman filters
In this paper, we hope to use only the GPS chip that comes with ordinary mobile phones instead of using other devices to improve GPS positioning accuracy. In the process of investigation, it was found that Kalman filter was deeply loved by domestic and foreign scholars. Because Kalman filtering uses the time domain state space method compared to other filters, and because it has the characteristics of real-time recursion, small storage and easy to implement on the computer, in engineering applications the filter has also been given great importance.

In this paper, we use different filters because of the change of scenes. We use about two kinds of filters: one is a general Kalman filter, and the other is an Extended Kalman Filter (EKF).

3.1. General Kalman Filter Structure
A Kalman-filter-based estimator is one of the most common methods used to estimate or calibrate the pedestrian’s position.

For a continuous dynamic system represented as

\[ X(k+1) = \phi X(k) + \Gamma W(k) \]  

(1)

where is the state vector at time k, \( X(k+1) \) is the state vector at time k+1, \( \phi \) is the state transfer matrix, \( \Gamma \) is the noise drive matrix, \( W(k) \) is Gaussian white noise with a mean of zero and a variance of \( Q \).

The observation equation of the system represented as

\[ Y(k) = HX(k) + V(k) \]  

(2)

In this equation, \( Y(k) \) is the valued observed at time k, \( H \) is the observation matrix and \( V(k) \) is Gaussian white noise with a mean of zero and a variance of \( R \). The update equations are thought of as predictor or corrector equations, which are given in as following:

\[ \hat{X}(k+1|k) = \phi \hat{X}(k|k) \]  

(3)

\[ \xi(k+1) = Y(k+1) - H \hat{X}(k+1|k) \]  

(4)
\[
\hat{X}(k+1|k+1) = \hat{X}(k+1|k) + K(k+1)\xi(k+1)
\] (5)
\[
K(k+1) = P(k+1|k)H^T [HP(k+1|k)H^T + R]^{-1}
\] (6)
\[
P(k+1|k) = \varphi P(k|k)\varphi^T + \Gamma Q \Gamma^T
\] (7)
\[
P(k+1|k+1) = [I_n - K(k+1)H]P(k+1|k)
\] (8)

where vectors used in the filters are defined as follow:
\[
\hat{X}(k+1|k) : \text{prior estimate of the system state at time } k+1
\]
\[
K(k+1) : \text{Kalman Gain at time } k+1
\]
\[
P(k+1|k) : \text{prior error covariance matrix at time } k+1
\]
\[
P(k+1|k+1) : \text{updated error covariance matrix at time } k+1
\]
\[
Y(k+1) : \text{observations of the system at time } k+1
\]
\[
H : \text{observation matrix}
\]
\[
\varphi : \text{state transfer matrix}
\]
\[
\Gamma : \text{noise drive matrix}
\]
\[
R : \text{measurement noise covariance}
\]
\[
Q : \text{process noise covariance}
\]
\[
I_n : N\text{-th order identity matrix}
\]

This Kalman filter structure is used for one of the filters adopted in this paper.

3.2. Extended Kalman Filter Structure

Extended Kalman filter is based on General Kalman filter and applies the general Kalman filter for the linear system to the nonlinear system to expect that the GPS error can be reduced as much as possible in a dynamic scenario. A extended-Kalman-filter-based estimator is one of the most common methods used to estimate or calibrate the pedestrian’s position when the pedestrian is moving at a certain speed. This is used to predict the states of the system at time k based on all measurements till time k-1.

First, the discrete nonlinear dynamic equation is represented as follows:
\[
X(k+1) = f[k, X(k)] + G(k)W(k)
\] (9)

The observation equation of the system represented as
\[
Z(k) = h[k, X(k)] + V(k)
\] (10)
do a first-order Taylor expansion of equation (9) around \( \hat{X}(k) \), we can get equation (11)
\[
X(k+1) \approx f[k, \hat{X}(k)] + \left. \frac{\partial f}{\partial \hat{X}(k)} \right|_{\hat{X}(k) = X(k)} [X(k) - \hat{X}(k)] + \left. \frac{\partial f}{\partial \hat{X}(k)} \right|_{\hat{X}(k) = X(k)} G[k, \hat{X}(k)]W(k)
\] (11)

let
\[
\left. \frac{\partial f}{\partial \hat{X}(k)} \right|_{\hat{X}(k) = X(k)} = \varphi(k+1|k)
\] (12)
\[
\left. \frac{\partial f}{\partial \hat{X}(k)} \right|_{\hat{X}(k) = X(k)} G[k, \hat{X}(k)] = \varphi(k)
\] (13)

Then rewrite equation (9) into equation (14)
\[
X(k+1) = \varphi(k+1|k)X(k) + G(k)W(k) + \varphi(k)
\] (14)
do a first-order Taylor expansion of equation (10) around \( \hat{X}(k) \), we can get equation (15)
\[ Z(k) = h[X(k \mid k-1), k] + \frac{\partial h}{\partial \hat{X}(k)} \lambda_{\hat{X}(k)}(k) \quad \left[ X(k) - \hat{X}(k \mid k-1) \right] + V(k) \] 

(15)

let

\[ \frac{\partial h}{\partial \hat{X}(k)} \lambda_{\hat{X}(k)}(k) = H(k) \] 

(16)

\[ y(k) = h[X(k \mid k-1), k] - \frac{\partial h}{\partial \hat{X}(k)} \lambda_{\hat{X}(k)}(k) \] 

(17)

Then rewrite equation (9) into equation (18)

\[ Z(k) = H(k)X(k) + y(k) + V(k) \] 

(18)

The update equations are thought of as predictor or corrector equations, which are given as following:

\[ \hat{X}(k \mid k+1) = f(\hat{X}(k \mid k)) \] 

(19)

\[ P(k+1 \mid k) = \Phi(k+1 \mid k)P(k \mid k)\Phi^T(k+1 \mid k) + Q(k+1) \] 

(20)

\[ K(k+1) = P(k+1 \mid k)H^T(k+1)\left[H(k+1)P(k+1 \mid k)H^T(k+1) + R(k+1)\right]^{-1} \] 

(21)

\[ \hat{X}(k+1 \mid k+1) = \hat{X}(k \mid k+1) + K(k+1)[Z(k+1) - hX(k+1 \mid k)] \] 

(22)

\[ P(k+1) = [I - K(k+1)H(k+1)]P(k+1 \mid k) \] 

(23)

3.3. Location estimate

3.3.1. Kalman filter design in dynamic scenes

For the second filter, i.e., dynamic Kalman filter, when the pedestrian moves relative to the vehicle at a certain speed \( v \), the kinematic relationship between the latitude and longitude coordinates of the pedestrian's location and its speed can be written as:

\[ x_{k+1} = x_k + \frac{0.5 \cdot (v_x + v_{x+1}) \cdot \sin \theta_{x+1} \cdot T}{l_{\text{Lat}}} \] 

(24)

\[ y_{k+1} = y_k + \frac{0.5 \cdot (v_y + v_{y+1}) \cdot \cos \theta_{y+1} \cdot T}{l_{\text{Lng}}} \] 

(25)

where \( x_{k+1} \) is the minute of the longitude of the pedestrian at time \( k+1 \), \( y_{k+1} \) is the minute of the latitude of the pedestrian at time \( k+1 \). Similarly, \( x_k \) is the minute of the longitude of the pedestrian at time \( k \), \( y_k \) is the minute of the latitude of the pedestrian at time \( k \). \( \theta_{x+1} \) is the direction that the pedestrian moving to, \( l_{\text{Lat}} \) is the distance corresponding to each one minute of movement on the latitude circle, \( l_{\text{Lng}} \) is the distance corresponding to every one minute of movement on the longitude circle, \( T \) is the sampling time.

Now we do a simple proof of equations (26) and (27). We assume that the object moves at a velocity on the latitude circle, the distance corresponding to the longitude of each point is

\[ l_{\text{Lng}} = \frac{2\pi R \cos y_k}{360 \cdot 60} \] 

(26)
In the equation (28), R is the Earth radius, $y_k$ is latitude of this latitude circle. The distance traveled by the object within time $T$ is $v*T$, the longitude of the change corresponding to this distance is $v*T/\lambda_{Lng}$, $x_k$ is the longitude position before time $T$. The longitude position at this moment is

$$x_{k+1} = x_k + \frac{v*T}{\lambda_{Lng}}$$

(27)

If our speed is not along the latitude circle, it is $\theta_{k+1}$ angle with the true north direction, we should change $v$ to $v*\sin\theta$ as described in Fig.1

![Figure 1. Velocity transformation coordinate system](image)

Then we can get equations (26) and (27).

### 3.3.2. System Model in Dynamic Situation

In the dynamic situation, the equation of state is shown in equation (26) and (27). The system’s observation equation is

$$Z_{Lng}(k) = Z_{Lng}(k) + V_1(k)$$

(28)

$$Z_{Lat}(k) = Z_{Lat}(k) + V_2(k)$$

(29)

At the same time, we can also use kinematics equations to build models. For uniform linear motion, the speed of the motion is $v$. The position of the target at time $k$ is, after the sampling time $T$, the target position is

$$s(k+1) = s(k) + vT$$

(30)

In a short distance where collision may occur, the movement of the object can be seen as a movement in Cartesian coordinates. At this time, the target has components in the x and y directions. The state variables of the motion system include the position in the x direction, the position in the y direction, the velocity in the x direction, and the velocity in the y direction. The random perturbation in the target motion is $U(k)$.

Based on the above, the equations of state are

$$x(k+1) = x(k) + v_x(k)T + \frac{1}{2}u_x(k)T^2$$

(31)

$$v_x(k+1) = v_x(k) + u_x(k)T$$

(32)

$$y(k+1) = y(k) + v_y(k)T + \frac{1}{2}u_y(k)T^2$$

(33)

$$v_y(k+1) = v_y(k) + u_y(k)T$$

(34)

The observation equation is

$$Z(k) = \sqrt{(x(k) - x_0)^2 + (y(k) - y_0)^2} + V(k)$$

(35)

where $(x, y)$ is the location of the station, and in our scenario, it is the location of the driver, and $V(k)$ is Gaussian white noise with a mean of zero and a variance of $R$. 


For convenience, we write the state of the system as following,

\[ X(k) = \begin{bmatrix} x_p(k), y_p(k), v_x(k), v_y(k) \end{bmatrix}^T \]  

(36)

\[ U(k) = \begin{bmatrix} u_x(k), u_y(k) \end{bmatrix}^T \]  

(37)

At this time, the equation of state of the system is

\[ X(k+1) = \varphi X(k) + \Gamma U(k) \]  

(38)

where

\[ \varphi = \begin{bmatrix} 1 & 0 & T & 0 \\ 0 & 1 & 0 & T \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}, \Gamma = \begin{bmatrix} T^2/2 & 0 \\ 0 & T^2/2 \\ T & 0 \\ 0 & T \end{bmatrix} \]

where T is the sampling time at GPS system.

From the above equation, the equation of state is linear, and the observation equation is nonlinear. We use this model when the vehicle or pedestrian is in motion.

4. Evaluation

In order to figure out the algorithm capacity, we implemented the Kalman filter algorithm using java and installed the program on the mobile phone. In our experimental environment, two mobile phones were used to simulate ADAS scenarios, one that could be seen as a pedestrian and the other as a driver, which is shown as Figure 2.

![Figure 2. Experiment in a simulated ADAS scenario](image)

In dynamic scenarios, we used two mobile phones, one that remained stationary to represent pedestrians, and the other moving at a constant speed of 5m/s is considered the driver. There are three kinds of trajectory, the red line represents the driver's trajectory, the black line represents the movement track measured by GPS, and the green line represents the trajectory after EKF filtering, which is shown as Figure 3.

![Figure 3. The result of EKF](image)

![Figure 4. Location error estimation](image)
As shown in Figure 4., we can see that most of the errors are within 5m. Compared with the data by GPS, EKF has improved the positioning accuracy of GPS obviously.

5. Conclusion
In this paper, we proposed an algorithm which can compensate for GPS errors. What’s innovation is the reducing GPS errors by means of Kalman filtering instead of traditional physical methods. The results of our experiments and simulations turn out to support our algorithm and system. Although limited by the concern of the risk scenarios of the ADAS may bring, our outdoor experimental data give quite satisfying and reasonable results.

However, there still exist few defects that need to overcome in the future. Firstly, since filtering might not be the best option to improve GPS positioning accuracy, there is still much work to do on that field. Besides, it is worthwhile and necessary to find a better way to process experimental data so as to figure out the algorithm capacity well. And finally, we expect the algorithm could actually be applied to the ADAS and bring great convenience to road traffic.

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