Spacetime and gravity’s emergence from eternally entangled particles

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This work derives a model for systems of entangled particles of which quantized spacetime is an emergent property, thus modeling the emergence of (quantized) spacetime from quantum entanglements. To confirm meaningful compatibility with general relativity, a mathematical model of quantum gravity is derived from this model and shown to be consistent with general relativity at classical scales. This work builds on recent advancements which use similar models to account for the emergence of Minkowski spacetime. However, unlike these other works, this work is based on a distance function that does not imply a universal metric space. Instead, metric inequalities are indicated to be relative to frames of reference, in the same way as special relativity asserts that other coordinate space properties are. In short, this paper advances the unification of the branches of modern physics via a quantum model that accounts for relativity’s core concept, spacetime - complete with quantum and gravitational effects.

I. INTRODUCTION

In pursuit of a quantum theory of gravity, the approaches of loop quantum gravity and emergent spacetime are the most prominent. The basic approach of loop quantum gravity is to quantize spacetime. Whereas, with emergent spacetime, spacetime is viewed as a geometric abstraction of underlying quantum phenomena. Therefore, the basic approach for emergent spacetime is to identify how quantum phenomena can account for spacetime geometry.

Much of the work on emergent spacetime is based on a correspondence, identified by Maldacena, between two types of spaces, quantum’s conformal field theory (CFT) and anti-de Sitter space (AdS) (a special kind of spacetime). Building on this work, and on mathematics from Ryu and Takayanagi, van Raamsdonk postulated a theory which, essentially, claims that spacetime is a geometric abstraction of how a quantum system is entangled.

However, despite supporting evidence, significant challenges remain. For example, the AdS/CFT correspondence solution to gravity relies on the holographic principle, which only applies to a universe, unlike our own, that is static and bounded. Furthermore, our universe does not appear to be an anti-de Sitter space in that it does not appear to have a negative cosmological constant. Therefore, a solution to the AdS/CFT correspondence is insufficient for realizing a quantum theory of gravity.

More recently, Cao et al. began researching this relationship between entanglement and emergent spacetime from a purely quantum-mechanical framework. This approach avoids any dependency on the cosmological constant or the holographic principle.

Their approach is based on a distance function that, effectively, maps any state for a system of entangled particles to a 3D metric space, with well-defined relative positions for all particles. Although these positions are relative, this is sufficient to account for the emergence of a 3D metric space complete with multiple 3D frames of reference. Each of these frames of reference, effectively, defines its own 3D coordinate space, in which all particles have a well-defined position. Ultimately, this accounts for the emergence of Minkowski spacetime from a system of entangled particles.

However, this is insufficient to account for gravitational effects. To account for gravitational effects the approach has been to try to unite a mathematical framework of quantum mechanics (Hilbert space) with that of general relativity (spacetime), an effort that is, so far, incomplete.

This work takes a first principles approach, focusing on foundational principles and concepts of general relativity and quantum mechanics. So, although this work does not unite a mathematical framework of quantum mechanics with that of relativity, it does mathematically unite the branches of modern physics at a fundamental level. Consequently, this mathematically links concepts at various levels across these branches. For example, quantum states are indicated to be as subject to an observer’s frame of reference as object velocities are, thus relating the foundational concept of relativity to quantum’s observer effect. For our present purposes, however, the main benefit of this approach is the reduction of considerable mathematical complexity. Ultimately, this allows for an account of quantized spacetime complete with quantum and gravitational effects via classical mathematics.
II. MODEL DESCRIPTION

Einstein’s aim for general relativity was to realize the principle of general covariance, or the principle that the laws of nature are to be invariant under arbitrary coordinate transformations[29][30]. If, as van Rammsdonk claims[4], spacetime is an abstraction of quantum entanglements then there must be a model that satisfies the principles of general covariance and quantum entanglement. To align with theories of loop quantum gravity[1–3] and quantum mechanics in general, the quantization principle must also be considered. Based on this, the strategy herein is to design and analyze a minimal model that satisfies these three principles: general covariance, entanglement and quantization.

With an aim of developing a quantum theory of gravity, our model must include quantities related to particle distances and motion. With approaches[20–23, 25, 27, 28] that aim to unite a mathematical framework of quantum mechanics with that of relativity, distance information is related to the degree of mutual information for a particle pair. However, without this aim our model can remain more abstract and simply presume there exists some fundamental quantity from which metric distances are derived. For this quantity the term non-metric distance is used.

These non-metric distances are not distances within a metric space. Instead, they are the input for a metric’s distance function. So, although modeled as fundamental, this is compatible with relativity’s assertion that spatial distances are non-invariant[31].

To realize quantum’s entanglement principle, no fundamental quantities can be associated with individual particles[32]. Instead, they are associated with a simple Bell pair, which, herein, is taken to be any arbitrary pair of particles that are entangled. Furthermore, given the quantization principle, all fundamental quantities must be defined in terms of an instant in time[3]. As a basis for a model we can express these conditions as a hypothesis, which I call the quantum-relativity hypothesis.

Quantum-relativity hypothesis (QRH): Particle pair relationships are fundamental, including non-metric distances between particles and related instantaneous quantities such as the rate of change for such distances.

The QRH satisfies quantum’s entanglement and quantization principles. Furthermore, it does not violate the principle of general covariance since its fundamental quantities are not contingent on any particular coordinate system.

Based on the quantities and relationships expressed by the QRH, a system composed of two entangled particles would correspond to a model of a single particle pair with an associated non-metric distance and related instantaneous data.

This model is extend to larger systems of entangled particles by modeling all possible particle pairings such that the overall model is that of an undirected graph as described:

1. Nodes of the graph represent particles.
2. Each edge represents the relationships between its associated particle pair, including the non-metric distance between the particles and related instantaneous quantities.
3. Because all particles are entangled with each other, this is a complete graph (every node connected to every other node with a single edge).

I call this particle-pair graph the quantum-relativity model (QRM). Although expressed in more abstract terms, the QRM is comparable with the metric graph that is fundamental to the approach of Cao et al.[20–23]. The key distinction between these models is that with the QRM the distance between any two particles is defined by their common edge only. This is not the case for the metric graph’s distance function which must consider all edge’s in order to ensure adherence to the triangle inequality[20][23].

More generally, the QRM is not subject to any metric inequalities, or any restrictions that relate its (non-metric) distances to each other. Instead, metric inequalities play a critical role at higher levels of abstraction, where quantum effects and spacetime emerge.

III. EMERGENT SPACETIME

In this section we discuss how frames of reference, Euclidean space and, finally, spacetime are emergent properties of the QRM.

Any node within the QRM can be considered a reference point and any set of reference points a frame of reference. A frame of reference is a valid frame of reference for a metric space iff its edges’ distances (interpreted via the metric’s distance function) conform to the metric’s inequalities. For example, for a Euclidean frame of reference all possible sets of three distances must satisfy the triangle inequality as defined in equation 1

\[ d_a + d_b \geq d_c, \]

where \(d_i\) is a distance for all distinct \(a, b, c \in \{1, 2, 3\}\)

Any frame of reference which includes four reference points that are not coplanar is a 3D frame of reference. Therefore, the QRM can contain numerous 3D Euclidean frames of reference as emergent properties.

For metric spaces, given distances from a set of reference points that span all dimensions a well-defined position can be determined via trilateration. That is, as long as the distances from the reference points, combined with the distances between the reference points, do not
contain any violations to the metric’s inequalities. Consequently, relative to each 3D Euclidean frame of reference, all nodes that do not correspond to such violations have a well-defined position in the corresponding 3D Euclidean space. Therefore, the QRM can contain multiple 3D Euclidean spaces each containing particles with well-defined positions as emergent properties.

To observe such a space merely requires an observer capable of perceiving 3D Euclidean space from a given frame of reference.

Furthermore, the state for multiple moments in time can be derived from the QRM’s distances combined with its related instantaneous quantities. Each moment would be associated with its own emergent 3D Euclidean frames of reference and corresponding spaces. A set of moments would form a universe that is quantized across time.

For (quantized) Minkowski spacetime[24] the relationship between distance and time would need to be consistent with special relativity[31]. Quantized Minkowski spacetime is, therefore, an emergent property when the non-metric distances are scaled based on a frame of reference’s direction and speed relative to the speed of light.

In general, with the QRM the distance function of the target metric space is used. What the QRM does stipulate is that distance functions are only applicable to certain edge’s. Specifically, to edge’s associated with (at least) one node included in the frame of reference. Unlike other quantum mechanical approaches to emergent spacetime[20, 23, 25, 27, 28], this results in metric inequalities being relative to a frame of reference. Consequently, the metric space for one frame of reference may coincide with that of another, but not necessarily.

This is analogous to special relativity’s assertion that coordinate spaces are relative to a frame of reference[24] [31]. Furthermore, since metric inequalities are a property of coordinate spaces, the QRM simply indicates one additional coordinate space property (to those indicated by special relativity) is relative to a frame of reference.

As for general relativity, its foundational principle, general covariance, is already satisfied by the QRM. Therefore, rather than detailing how (general relativity’s) spacetime could emerge, we will focus on whether the QRM accounts for the emergence of gravitational effects in these other types of spacetime. Before discussing that, the emergence of quantum phenomena is covered.

IV. QUANTUM PHENOMENA

Within an emergent metric space (of the QRM), there is no well-defined position for a node which violates an inequality with respect to the space’s reference points. However, as there is a well-defined distance to every node from each reference point, there are many potential positions with respect to each reference point. In geometric terms, these potential positions are located on the surface of a sphere centered at the reference point with a radius matching their distance to the node (see Fig. 1).

Therefore, any node that would cause an inequality violation is in a state of quantum superposition with respect to the given space.

However, since metric inequalities are relative to a frame of reference, such a node need not be in superposition for all emergent spaces. In fact, it could serve as a reference point within multiple emergent spaces. Therefore, quantum states are subject to the frame of reference.

With the QRM the universe is modeled as a set of eternally entangled particles without any restrictions relating particle-pair distances to each other. Normal space emerges for observers capable of, but limited to, perceiving normal space from a given frame of reference.

V. QUANTUM GRAVITY

This section derives a mathematical model of quantum gravity via the mathematical analysis of the relationship between classical motion and relative motion. To highlight the non-fundamentalness of coordinate systems the term ‘perceived’ is used herein to refer to quantities related to a coordinate system. For example, ‘perceived speed’ relates to changes to spatial position within a coordinate system.

For this analysis, a formula is derived for the perceived acceleration, given a constant rate of change in distance between two particles, denoted $p_1$ and $p_2$. Particle $p_1$ is taken to be included in the frame of reference and, therefore, its position is considered fixed. As for $p_2$ it could be perceived as moving in any direction. Regardless of the direction, a geometric relationship between these particles’ positions and $p_2$’s direction is known (see Fig. 2).

Herein, the term ‘direction-line’ refers to the line parallel to a particle’s direction. That is, a direction-line maintains a constant angle with a particle’s direction but extends indefinitely in both directions.

Where $p_2$’s direction-line is closest to $p_1$ there would exist a point $P$. Given that the shortest distance between
The GF depends on the perceived direction, represented by $m$. The lowest value for $m$ is zero, which corresponds to a heading directly towards or away from $p_1$. If $p_2$ is at $P$ then $y$ is zero and, therefore, $m$ equals the hypotenuse, its maximum value. Therefore, $m$ has the following range:

$$0 \leq m \leq x$$

(6)

Considering this range, the direction has a non-trivial influence. Given equation [6] when $m = 0$ the perceived acceleration is 0 but as $m$ approaches $x$ the perceived acceleration approaches $-\infty$.

$$\lim_{m \to x^-} a_p = \lim_{m \to x^-} -\frac{m^2}{(x^2 - m^2)(\frac{1}{2})} = -\infty$$

(7)

Therefore, dependent on the direction, $a_p$ has the following range:

$$-\infty < a_p \leq 0$$

(8)

Only particles exactly on the direction-line of $p_2$ are associated with an $m$ value of 0. Therefore, for any multi-dimensional frame of reference only some of its particles could have an $m$ value of zero, all others would be associated with a negative perceived acceleration.

Because the direction of the perceived acceleration varies for each particle in the frame of reference, the average perceived acceleration can be zero. However, this is only when the particle is surrounded by the frame of reference such that the perceived accelerations cancel out. In all other cases, for a constant rate of change in particle distance, a multi-dimensional frame of reference is sufficient for a (non-zero) perceived acceleration.

As detailed below, given a non-zero perceived acceleration the GF (formula [5]) accounts for the following assertions of general relativity:

1. Gravity is not a force.
2. Gravity results in a perceived acceleration for approaching objects.
3. Gravity results in a perceived deceleration for objects receding from each other.
4. Gravity reduces with distance.
5. Gravity is not limited to a finite distance.

The perceived acceleration is not attributable to a force (assertion 1) because it is not related to changes to the rate of change for a fundamental quantity; instead, it is derived from the relationship between the particle distance’s rate of change and the perceived speed. Furthermore, any non-zero perceived acceleration is negative under all valid conditions (see equation [3]), this indicates a perceived acceleration towards the point closest to $p_1$ on $p_2$’s direction-line. This means, perceived acceleration
when approaching and perceived deceleration when receding (assertions 2 & 3). Finally, the magnitude of the perceived acceleration reduces with distance ($|a_p| \propto \frac{1}{x^2}$) but never reaches zero (given $m > 0$), thus realizing all five assertions.

Given that this analysis was based on Euclidean space, general relativity’s non-linear, curved spacetime is not required for these assertions to hold. Instead, a flat, linear spacetime emerging from the QRM is sufficient.

As for Minkowski spacetime\[24], the difference is its scaling of distances, which for the analysis of the GF’s characteristics is inconsequential. Therefore, the analysis is applicable to Minkowski spacetime.

With either general relativity or the QRH, gravity is seen as an anomaly caused by the mismatch between the fundamentally relativistic structure of the universe and our perception of it\[29].

One fundamental difference with the QRH is that it leads to predictions of quantum states. Consider two objects on a collision course. The first object is taken to be the frame of reference and is composed of two particles $p_1$ and $p_2$. The second object is a single particle $p_3$ which is equidistant from $p_1$ and $p_2$ and headed directly towards the midpoint between them (see Fig. 3). As $p_3$ approaches, its rate of change in distance with both $p_1$ and $p_2$ is constant (resulting in a perceived acceleration towards the midpoint).

![FIG. 3. Particle $p_3$ maintains a constant rate of change in distance of $\Delta x/s$ with both $p_1$ and $p_2$. After reaching the midpoint $p_3$ enters a state of superposition as indicated by $p_{3a}$ and $p_{3b}$.](image)

However, when $p_3$ reaches the midpoint it cannot continue to approach both $p_1$ and $p_2$, in the classical sense; at this point any movement towards $p_1$ would be a movement away from $p_2$ and vice versa. The GF offers no prediction because it is undefined at this point (denominator is zero, since $x = m$ at the midpoint). However, with the QRH/QRM there are no constraints that relate distances to each other. The only constraint is a constant rate of change in the non-metric distances (assuming no forces).

One second after reaching the midpoint, the corresponding QRM representation would include three non-metric distances corresponding to the metric distances shown in equations \[[9] and [10]\\
\[d_{1,2} = 2 \cdot m\\
\]
where $d_{i,j}$ is the distance between $p_i$ and $p_j$.

\[d_{1,3} = d_{2,3} = m - |\Delta x|\\
\]
So, their sum is:

\[d_{1,3} + d_{2,3} = 2 \cdot m - 2|\Delta x|\\
\]
Since $\Delta x \neq 0$:

\[d_{1,3} + d_{2,3} < 2 \cdot m\\
\]
Combining \[[9] and [12]\\
\[d_{1,3} + d_{2,3} \geq d_{1,2}\\
\]
The triangle inequality is violated because the sum of the distances to $p_3$ is less than the distance between $p_1$ and $p_2$ (see equation \[[13]\). Therefore, the prediction is that $p_3$ would continue to approach both $p_1$ and $p_2$ by entering a state of quantum superposition.

Therefore, gravitational effects and transitions into quantum superpositions are both attributable to a system governed by relative motion with coordinate spaces including metric inequalities relating to observers’ perceptions.

**VI. DISCUSSION**

As van Raamsdonk’s theorized\[4], spacetime may be a geometric abstraction of how a system is entangled\[3, 5]. This is accounted for by the QRM which models entanglements at a fundamental level and quantized spacetime as an emergent property.

By not modeling the existence of a universal metric space the QRM readily models quantum states, while associating metric inequalities with frames of reference to realize the emergence of metric spaces, Euclidean spaces and, ultimately, quantized Minkowski spacetime\[24], complete with quantum and gravitational phenomena.

Like the prominent approaches to emergent spacetime, including the recent purely quantum mechanic approaches\[20, 23, 27, 28], spacetime is related to quantum disentanglement. Specifically, space emerges by, effectively, disentangling a set of particles based on a frame of reference. However, this disentanglement is related only to the interpretation of emergent properties. Like the many-worlds interpretation of quantum mechanics\[23, 24], the universe’s underlying state is modeled as eternally entangled.

However, unlike the many-worlds interpretation the determinate perspective associated with each moment of
observation³² is not explained via parallel universes³⁴. Instead, this determinate perspective is associated with a frame of reference which is an emergent property of the universe’s state. Although parallel universes can be reconciled with spacetime³³,³⁵ this requires speculative assertions³⁶ which add complexity to general relativity’s model of the universe; whereas frames of reference are already fundamental to relativity²⁹,³¹.

With relativity, multiple frames of reference coexist within spacetime. These frames of reference provide different perspectives, including potentially disagreeing on the order of events³¹. However, all of these frames of reference share the common coordinate system established by spacetime’s metric³⁰. Therefore, all frames of reference must agree on a particle’s spacetime position(s). Although coordinate transformations for spacetime positions vary between frames of reference, at any given time all frames of reference must agree on the number of positions for a given particle. Therefore, spacetime’s metric ensures that all frames of reference agree whether a particle is in superposition or not.

This is true for any model that includes a universal metric, such as the theories of emergent spacetime²⁰–²³ which are based on a graph. The key distinction with the QRM is that it does not imply a universal metric. Consequently, the (emergent) metric space for one frame of reference may coincide with that of another, but not necessarily. By having independent metric spaces, frames of reference can disagree on whether a particle has a well-defined position or is in superposition. Furthermore, as is a principle of relativity, there is no preferred frame of reference²⁹,³¹. Meaning, all potential metric spaces are equally valid. Therefore, with the QRM there are many independent metric spaces, despite them sharing the same underlying state.

A set of frames of reference that coincide with the same metric space can be considered as one metric space containing the set of frames of reference. This makes such a set analogous to a parallel universe of the many worlds interpretation, in that each parallel universe is an independent metric space containing any number of frames of reference³⁴. This suggests that the key to reconciling relativity with quantum mechanics (via the many worlds interpretation) is to accommodate independent metric spaces.

Current approaches to this include modeling either multiple spacetimes or causally independent regions within spacetime³⁵. What the QRM adds to the former approach is a low-level model from which multiple spacetimes can emerge. Each of these emergent spacetimes is associated with a single frame of reference. For, although a set of frames of reference can coincide with the same metric space at any given moment, they need not continue to coincide across time. This is because metric spaces are emergent and, therefore, do not govern the movements of particles. That is, the universe is not subject to metric inequalities but observations are, resulting in the perception of metric spaces. This is, again, consistent with the many-worlds interpretation where parallel universes are emergent properties related to an observer’s perspective (including their memories) for a moment in time³⁴. With either the QRM or the many-worlds interpretation metric spaces are emergent and relative to a frame of reference (that of the observer).

With a universal metric, one must deal with the complications of a particle having potential positions. Not only the idea that a particle is associated with many positions but that at each of these positions the particle is both there but not really there. The complexity of this is reflected within the interpretations of quantum mechanics which remain a source of controversy³².

The QRM shifts this complexity away from physical systems by making metric inequalities an observer limitation only. The QRM’s fundamental quantities (e.g. non-metric distances) are modeled as well-defined and deterministic. However, this is not the case for particle positions which are emergent properties and are relative to an observer’s frame of reference.

Spacetime’s inequalities do not represent a foundational principle of general relativity²⁹ and based on the analysis herein are not critical for general relativity’s conceptualization of gravity. However, further research is required to determine if the quantum model of gravity derived herein is quantitatively equivalent to general relativity.

For efforts²⁰,²³,²⁵,²⁸ taking a purely quantum mechanical approach towards a quantum theory of gravity, the quantum gravity model derived herein potentially eliminates significant complexity. Specifically, as this quantum gravity model is applicable to (quantized) Euclidean and Minkowski spacetime²⁴, general relativity’s conceptualization of gravity can be accounted for without the complexities of general relativity’s non-quantized, non-linear, curved spacetime.

In general, by realizing quantized and emergent spacetime, the QRM establishes key compatibles with both other emergent spacetime theories⁴–⁶ and loop quantum gravity¹¹–¹³.

Despite its relative simplicity, the QRM realizes a number of key principles: general covariance, entanglement and quantization. Adherence to these principles of general relativity and quantum mechanics allows for the modeling and relating of key concepts across the branches of modern physics.

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