Multipole correction of stretched-wire measurements of field-gradients in quadrupole accelerator magnets

P. Arpaia\textsuperscript{a,1}, M. Buzio\textsuperscript{b}, C. Petrone\textsuperscript{a,b}, S. Russenschuck\textsuperscript{b} and L. Walckiers\textsuperscript{b}

\textsuperscript{a}Department of Engineering, University of Sannio, Corso Garibaldi 107, 82100 Benevento, Italy
\textsuperscript{b}CERN, Department of Technology, Group of Magnets, Superconductors and Cryostats (MSC), section of Magnetic Measurements, 1211 Geneva 23, Switzerland

E-mail: arpaia@unisannio.it

ABSTRACT: A correction of field gradients in quadrupole accelerator magnets measured by stretched-wire methods is described. The gradient is first measured by means of the single-stretched-wire method. By using the same experimental setup, the relative multipole-field errors of the quadrupole are then measured by means of the oscillating-wire technique and used for the correction scheme. Results of the experimental validation are presented for a prototype quadrupole for the CLIC accelerator study at CERN.

KEYWORDS: Instrumentation for particle accelerators and storage rings - high energy (linear accelerators, synchrotrons); Hardware and accelerator control systems; Acceleration cavities and magnets superconducting (high-temperature superconductor; radiation hardened magnets; normal-conducting; permanent magnet devices; wigglers and undulators); Beam-line instrumentation (beam position and profile monitors; beam-intensity monitors; bunch length monitors)

\textsuperscript{1}Corresponding author.

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1 Introduction

Quadrupole magnets are installed to focus particle beams in accelerators [1]. Their focusing strength is proportional to the distance between the beam and the magnetic axis [2]. However, due to manufacturing errors, the magnetic axis (defined as the locus of points in the magnet aperture with zero magnetic flux density) may not coincide with the geometrical axis [3].

The magnetic axis is commonly measured in quadrupole magnets, together with the magnetic field strength, by the Single-Stretched Wire (SSW) method [4]. However, these measurements are affected by nonlinear field errors in the magnet. These errors are expressed as multipole coefficients, obtained by a Fourier series expansion of the measured (or calculated) field in the aperture of the magnet [5]. Usually these multipoles are referred to as non-allowed multipoles, because they arise from manufacturing tolerances and deviations from the quadrupolar symmetry.

A classical approach to measure higher-order multipole coefficient in quadrupole magnets is the rotating coil [6]. Coils mounted on a support shaft, driven by a motor/encoder unit, deliver voltage signals proportional to the magnetic field. As the coil rotates, a digital integrator records the magnetic flux increments, related to the angular position of the coil by an the encoder [7]. The accuracy of the higher harmonics is improved by a compensated (bucked) coil configuration for rejecting effects of geometrical imperfections. In the configurations for quadrupole-magnets, a probe contains two or more off-axis coils, connected such that the voltage signals induced by the main field are cancelled out.

Rotating coil systems are the reference for measuring multipoles in straight magnets with circular apertures [8]. However, the small apertures of quadrupole magnets, e.g. for the CLIC
study [9] at CERN, pose challenges for building coil configurations with the required accuracy and for stabilizing the system against vibrations during the measurement.

Using wire systems yields more flexibility and avoids the construction of dedicated coils for small-aperture magnets. However, stretched-wire measurements of the field gradient in quadrupole magnets are affected by the abovementioned nonlinear field errors, expressed as higher-order field multipoles [10].

A method based on an oscillating wire for measuring the field quality in accelerator magnets was proposed in [11]. The wire is positioned step-by-step on the generatrix of a cylindrical surface inside the magnet aperture and the amplitudes of the wire’s forced oscillations are measured and related to the field harmonics by a suitable analytical model.

In this paper, a correction for the field gradient measurement by the stretched-wire method is proposed. By keeping the same measurement setup (stages, wires, integrators, and data acquisition system), the multipoles are measured by the oscillating-wire method and then used to correct the field gradient measured by the stretched wire.

2 Magnetic measurements with wire systems

In the following, the classical stretched-wire and the oscillating-wire techniques are briefly described. Detailed information can be found in [10] and [11].

2.1 The single-stretched wire method

The stretched-wire method consists of moving a single conducting wire of length $L$, suspended between two stages $A$ at $z = 0$ and $B$ at $z = L$ sufficiently far away from the fringe field of the magnet, along a path in the magnet aperture. The surface $A$ spanned by the wire in its motion intercepts the magnetic flux:

$$\psi = \int \int_A \vec{B} \cdot d\vec{a}, \quad (2.1)$$

where $\vec{B}$ is the magnetic flux density and $d\vec{a}$ is an element of the surface $A$. The flux through the surface can be expressed by means of the field harmonics of the average magnetic flux density along the entire wire length, in the notation $C_n := B_n + iA_n$ on the complex plane $z = x + iy$:

$$\vec{B}_y + i\vec{B}_x = \sum_{n=1}^{\infty} (B_n + iA_n) \left( \frac{z}{r_0} \right)^{n-1}, \quad (2.2)$$

where $r_0$ is an arbitrary reference radius, usually chosen at 2/3 of the magnet’s aperture [5]. By substituting (2.2) into (2.1), the intercepted flux can be calculated:

$$\psi = L \text{Re} \left\{ \int_{z_1}^{z_2} (\vec{B}_y + i\vec{B}_x) \, dz \right\} = L \text{Re} \left\{ \int_{z_1}^{z_2} \sum_{n=1}^{\infty} (B_n + iA_n) \left( \frac{z}{r_0} \right)^{n-1} \, dz \right\}, \quad (2.3)$$

where $L$ is the length of the wire between the two stages. By solving the integral (2.3), the flux is expressed as:

$$\psi = L \text{Re} \left\{ \sum_{n=1}^{\infty} \frac{(B_n + iA_n) z^n}{(r_0)^{n-1} \, n} \right\} \left\{ \frac{z_2}{z_1} \right\}. \quad (2.4)$$
Easier relations result if the wire is moved in the horizontal plane, that is, from the centre \( z_1 = (0 + i0) \) to \( z_2 = (x + i0) \):

\[
\psi = L \sum_{n=1}^{\infty} \frac{B_n}{(r_0)^{n-1}} \frac{x^n}{n}. \tag{2.5}
\]

Repeating the measurement with a movement from \( z_1 = (0 + i0) \) to \( z_2 = (-x + i0) \) and averaging the results yields:

\[
\psi = L \sum_{n=1}^{\infty} \frac{B_n}{(r_0)^{n-1}} \frac{(x^n + (-x)^n)}{2n}. \tag{2.6}
\]

### 2.2 The oscillating-wire method

The ends of the suspended wire are moved stepwise along circular trajectories so that the wire sweeps a cylindrical surface of radius \( r_0 \) inside the aperture of the magnet. A measurement system relates the wire-oscillation amplitudes collected at \( K \) positions on a circular trajectory to the integrated magnetic flux density. The wire displacement is proportional to the Lorentz force, which is proportional to the flux density normal to the wire \([11]\).

The oscillation amplitudes at \((r_0, \phi_k)\) are measured at a fixed position \( z_0 \) near one of the stages by means of two phototransistors arranged in x and y-directions. Thus the wire displacements \( d_k^x = \max_i |u_i (r_0, \phi_k, z_0, t)| \) and \( d_k^y = \max_i |u_i (r_0, \phi_k, z_0, t)| \) are obtained, where \( \Phi_k \) are the phase differences between the wire current and the wire oscillation \([11]\).

The discrete Fourier transform can then be performed directly on the oscillation amplitudes:

\[
\hat{A}_n = \frac{2}{K} \sum_{k=0}^{K-1} d_k^x \cos n \phi_k, \quad \hat{B}_n = \frac{2}{K} \sum_{k=0}^{K-1} d_k^y \sin n \phi_k. \tag{2.7}
\]

The tilde is used to distinguish these from the coefficients obtained by the series expansion of the flux density in eq. (2.2). Furthermore, by normalizing these Fourier coefficients \( \hat{A}_n (r_0) \) and \( \hat{B}_n (r_0) \) with respect to the main component, the relative multipoles\(^1\) \( \alpha_n = b_n + i a_n \) can be obtained:

\[
a_{n+1} (r_0) = \frac{\hat{A}_n (r_0)}{B_N (r_0)}, \quad b_{n+1} (r_0) = \frac{\hat{B}_n (r_0)}{B_N (r_0)} \tag{2.8}
\]

where \( N = 2 \) for the quadrupole. The index \( n + 1 \) stems from the fact that the Cartesian components of a multipole of order \( n \) oscillate sinusoidally \( n-1 \) times around a circle. In a straightforward way, eqs. (2.8) relate the relative field multipoles to the normalized coefficients of the Fourier series of the wire oscillation amplitudes collected on the circular trajectory. Owing to the holomorphic properties of the field (namely, complex differentiability in a neighbourhood of each point of its domain), the field harmonics calculated separately on the set of displacements \( d_k^x \) and \( d_k^y \) must yield the same results. The only limitation is represented by the fact that the set \( d_k^x \) is not sensitive to the normal dipole \( B_1 \), while conversely \( d_k^y \) is not sensitive to the skew dipole \( A_1 \). Both the x and y-components of the displacement are therefore acquired independently at little additional effort, and the redundancy on the harmonics \( n > 1 \) can be used to improve the measurement precision.

\(^1\)The relative multipoles are derived by \( B_n + i a_n = B_N (b_n + i a_n) \), where \( B_N \) is the main field component, e.g. \( B_2 \) for the quadrupole.
3 The multipole correction scheme

In the following, first the measurement problem of higher-order multipole components in quadrupole field gradients is explained, and then the basic idea and the analytical description of the proposed gradient correction are discussed.

3.1 Measurement problem

According to eq. (2.6), the wire displacement in x must be small for an accurate measurement of the local field gradient. However, this produces only a weak integrated signal, giving rise to low measurement sensitivity. Conversely, if the displacement in x is increased, the higher-order terms become more important, leading to a worse estimation of the field gradient.

This problem is clarified by the example of figure 1: in figure 1(a), a simulated magnetic field distribution along the x-axis, given as the superposition of a quadrupole component $B_2$ and higher-order multipoles ($B_6$ and $B_{10}$ weighted by factors of 5 and 50 for illustration purposes) is shown. figure 1(b) illustrates the corresponding relative error on the gradient. For the sake of comparison, the typical relative $1 - \sigma$ uncertainty of the SSW measurement is also shown. It is inversely proportional to the displacement level and its typical values are $\pm 5 \cdot 10^{-3}$ for displacement smaller than 1 mm and $\pm 5 \cdot 10^{-4}$ for larger displacements.

3.2 Basic idea

The correction method is based on two measurements (stretched and oscillating wire) carried out on the same measurement-bench set-up, without dismounting the magnet or the wire stages.

The flow diagram of the correction procedure is shown in figure 2. The position of the quadrupole magnetic axis is determined by means of the vibrating wire method [12]. The wire has the highest response to the transversal magnetic field when excited at its resonance frequency; thus the magnetic axis position is found by minimizing the oscillation amplitude of the wire.

By exploiting this result, a classical stretched-wire measurement of the magnetic field strength is then carried out. Measurements on x- and y-directions are processed independently because the higher-order field harmonics affect the integrated fluxes differently in the two directions. This increases the method significance by increasing the degrees of freedom for the estimate precision.
The magnetic centre found by the vibrating wire is also the starting point for the oscillating-wire method (OW). Acquiring the oscillation amplitudes and processing this data with a Fourier series expansion [11] determines the magnetic multipole field errors.

The possibility to carry out measurements at different reference radii by means of the same hardware yields a high flexibility useful to validate magnet prototypes of different apertures.

3.3 Gradient correction

The gradient measurement is corrected by determining the field harmonics by a rotating-coil or oscillating-wire measurement. With the field gradient defined by \( \vec{B}_y = \vec{g}x = \frac{\vec{B}_2(r_0)}{r_0}x \), eq. (2.6) can be written as:

\[
\psi = \frac{Lg}{B_2} \sum_{n=1}^{\infty} \frac{B_n}{2n(r_0)^{n-1}} ((x)^n + (-x)^n) = \frac{1}{2} x^2 + \sum_{n=4,6,...} b_n \frac{x^n}{n} (r_0)^{n-2},
\]

(3.1)

where the relative multipole field errors are defined by

\[
c_n = \left( \frac{C_n}{B_2} \right) = \left( \frac{B_n + iA_n}{B_2} \right) = b_n + ia_n.
\]

(3.2)

The correction factor for the gradient can be derived from

\[
\frac{1}{2} g^c x^2 = g^c \left( \frac{1}{2} x^2 + \sum_{n=4,6,...} \frac{b_n}{n} \frac{x^n}{(r_0)^{n-2}} \right),
\]

(3.3)
that is,

\[ \bar{g}^c = \bar{g}^m \left( 1 + \frac{b_4}{2} \left( \frac{x}{r_0} \right)^2 + \frac{b_6}{3} \left( \frac{x}{r_0} \right)^4 + \frac{b_8}{4} \left( \frac{x}{r_0} \right)^6 + \ldots \right)^{-1}, \]  

(3.4)

where the indices \( m \) and \( c \) denote the measured and the corrected gradients. Moving the wire in the vertical plane, that is, from \( z_1 = (0 + i0) \) to \( z_2 = (0 + iy) \) and \( z_2 = (0 - iy) \) respectively, results in a sign change in the real part of expression (2.4) for the even numbered multipoles. The corresponding equation for the correction of the measured gradient is then

\[ \bar{g}^c = \bar{g}^m \left( 1 - \frac{b_4}{2} \left( \frac{y}{r_0} \right)^2 + \frac{b_6}{3} \left( \frac{y}{r_0} \right)^4 - \frac{b_8}{4} \left( \frac{y}{r_0} \right)^6 + \ldots \right)^{-1}. \]  

(3.5)

4 Experimental validation

In the following, an experimental proof demonstration of the correction effectiveness is presented for a model magnet [13] of the final focus transport line for the Compact Linear Collider (CLIC) at CERN. CLIC is a site-independent feasibility study for an electron-positron linear collider in the multi-TeV centre-of-mass-range [14]. The high-gradient, hybrid quadrupole under test has a yoke length of 0.1 m and an aperture of 8.3 mm. Permanent magnet blocks provide a gradient of 85.6 T/m corresponding to an integrated gradient of about 9 T. The gradient can be increased to 509 T/m when the trim coils are excited to 18.6 A. The magnet on the stretched wire measurement bench is shown in figure 3.

The stretched wire can be considered free of martensitic contaminations, thus avoiding a position-dependent magnetic force and the corresponding measurement uncertainty [10]. The wire is passed through the magnet aperture and kept taut by means of a motor. A Fast Digital Integrator (FDI) [7] is used to acquire the voltage signal.

In figure 4, the architecture of the measurement system is shown. For the oscillating wire, measurement phototransistors are positioned in x- and y-directions on the stage in order to transduce the
corresponding wire displacements into an electrical voltage. The transducer output, pre-amplified and low-pass filtered by an antialiasing filter, is then sent to the 18-bit acquisition system NI6289 from National Instruments. The same system also acquires the wire current on a reference resistor. An alternating current generator Keithley 6221 is used to excite the wire. The measurements require a current frequency lower than the first natural resonance of the system. Although giving rise to higher measurement sensitivity, the amplitude of the wire vibration in resonance condition depends on the damping conditions and is therefore strongly nonlinear. A motor controller Newport ESP7000 is used for moving the stages, and the position is measured through a linear encoder with a precision of ±1 µm.

The test software is implemented by means of the Flexible Framework for Magnetic Measurements (FFMM). The field multipoles are finally computed according to the procedure described above, implemented in Matlab. The multipoles shown in figure 5 at different current intensities were measured at a reference radius of 3.0 mm. All the tests were carried out after centring the magnet within ±0.01 mm, in order to minimize the difference between the magnetic centre and the centre of the circular trajectory. The allowed multipoles $b_6$, $b_{10}$, $b_{14}$ show relatively large values; the small values of the non-allowed multipoles are affected by large relative measurement inaccuracy.

In table 1, the $1-\sigma$ repeatability computed on 30 consecutive measurements without current in the four magnet coils is shown. The repeatability of the oscillating wire measurements is better than ±500 ppm (5 units).

The reproducibility was analyzed, by dismounting and remounting the experimental setup (magnet and wire system) for three times by achieving a level of ±500 ppm.

The duration of the single measurement cycle is 1200 s, including acquisition (15 s for each point) and analysis. A settling time of 7 s is required before the acquisition lasting 3 s.

In table 2, the total gradients $L\bar{g}$ are given as a function of the excitation current, for a ramp from zero to 15.5 A and back to zero. Using the multipole field measurements up to order 10, the

![Figure 4. Architecture of the wire system [11].](image-url)
**Figure 5.** Normal relative multipoles ($b_n$, in units $10^{-4}$) in log scale as a function of current at $r_0 = 3$ mm (average on $30x$ and $y$ phototransistor measurements).

**Table 1.** $1 - \sigma$ repeatability (in units $10^{-4}$) on 30 consecutive measurements for the magnet only excited by the permanent magnets, i.e. null current in the trim coils.

|   | b3  | b4  | b5  | b6  | b7  | b8  | b9  | b10 | b11 | b12 | b13 | b14 | b15 |
|---|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|
|   | 2.3 | 2.9 | 2.8 | 3.1 | 2.7 | 2.7 | 3.3 | 2.8 | 2.8 | 2.9 | 2.5 | 2.8 | 3.0 | 2.8 |

|   | a3  | a4  | a5  | a6  | a7  | a8  | a9  | a10 | a11 | a12 | a13 | a14 | a15 |
|---|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|
|   | 2.5 | 2.5 | 4.2 | 2.9 | 2.8 | 2.7 | 4.5 | 3.4 | 2.6 | 2.7 | 3.1 | 2.6 | 3.2 |

gradients are corrected by means of eqs. (3.4)–(3.5). Notice the reduction in the residuals:

$$R^m = \frac{|\bar{g}^m(-x,x) + \bar{g}^m(-y,y)|}{|\bar{g}^m(-x,x)|},$$

$$R^c = \frac{|\bar{g}^c(-x,x) + \bar{g}^c(-y,y)|}{|\bar{g}^c(-x,x)|}$$

(4.1)

where the arguments $(-x,x)$ and $(-y,y)$ indicate the measurements along the $x$ and $y$ axes; see eqs. (3.4) and (3.5). The relative difference is reduced by a factor of about 2 at high field and by about one order of magnitude, on average, at low field. The improvement at low field, where relative errors are much larger, is quite noticeable. High field measurements, on the other hand, seem to be affected by another undetermined source of uncertainty of approximately equal magnitude, which remains to be investigated.

### 5 Conclusions

A method based on wire techniques for correcting higher-order field errors in field-gradient measurement is proposed. On the same setup, and without realigning the magnet and the wire stages,
Table 2. Comparison between integrated gradients before and after the correction, versus the excitation current in the hybrid quadrupole magnet.

| I (A) | $L\tilde{g}^m(-x,x)$ (T) | $L\tilde{g}^m(-y,y)$ (T) | $R^m$ (T) | $L\tilde{g}^c(-x,x)$ (T) | $L\tilde{g}^c(-y,y)$ (T) | $R^c$ (T) |
|-------|--------------------------|--------------------------|-----------|--------------------------|--------------------------|-----------|
| 0.0   | 9.245                    | -9.327                   | 0.009     | 8.886                    | -8.907                   | 0.002     |
| 6.2   | 32.686                   | -32.786                  | 0.003     | 32.557                   | -32.618                  | 0.002     |
| 7.6   | 39.174                   | -39.274                  | 0.003     | 39.144                   | -39.177                  | 0.001     |
| 15.5  | 51.358                   | -51.462                  | 0.002     | 51.504                   | -51.430                  | 0.001     |
| 7.6   | 39.523                   | -39.618                  | 0.002     | 39.498                   | -39.530                  | 0.001     |
| 6.2   | 33.077                   | -33.164                  | 0.003     | 32.959                   | -33.002                  | 0.001     |
| 0.0   | 9.238                    | -9.307                   | 0.007     | 8.880                    | -8.883                   | 0.000     |

three different techniques are applied: (i) vibrating wire, for the magnetic axis position, (ii) oscillating wire, for the multipole field errors, and (iii) the single-stretched wire, for the field strength along the horizontal and vertical planes.

The method is based on general-purpose instrumentation (sensing coil arrays and stages) in order to achieve high flexibility for different magnet lengths and magnet apertures. The method has been validated with the measurements of a hybrid quadrupole magnet for the CLIC study. The residual inaccuracy of the correction method can be estimated on the order of 0.1% with respect to the main field.

The method can be applied without significant performance loss to magnets with (i) apertures ranging from 1 mm up to 15 cm, where the upper limit is imposed by the moving stages; and (ii) length from 45 mm up to a theoretical upper limit of 15 m, provided that additional sensors are placed over the length and that oscillating wire is validated for such long magnets; and (iii) integral gradients with low levels limited by the electronics signal-to-noise ratio (e.g. for 0.01 T, the measurement is affected by significant noise), and without specific upper limits because the wire speed can be reduced to decrease the signal amplitude. In any case, for low integral gradients (0.01 T) combined with small apertures (< 10 mm), uncertainty increases beyond the limits explored in this paper.

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