Performance of Free Space Optical Communication Base on Orbital Angular Momentum with Pointing Errors

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Abstract. Due to the misalignment of the receiver, the spiral spectrum appears to be extended when the Laguerre-Gaussian beam is applied to atmospheric laser communication. This paper investigates the spiral spectrum spread of the Laguerre-Gaussian beam in the case of receiver misalignment. The analytical expression of Laguerre-Gaussian spiral spectrum under the influence of alignment error is studied by mathematical derivation and simulation. The results show that the detection probability of the original orbital angular momentum (OAM) state varies with the lateral displacement distance and the inclination angle. The bit error rate and channel capacity of each channel were studied to verify the effects of misalignment in multiple OAM communication systems.

1.Introduction
Since Allen [1] discovered the beam carrying OAM in 1992, more and more researches have been made on this type of beam. This type of beam is called a vortex beam and is mainly used in quantum information coding, optical traps, optical manipulation, biomedical and optical communication. The Laguerre-Gaussian (LG) beam is one of the most common vortex beams that can carry angular momentum in different modes. Angular momentum can provide a good carrier for mode multiplexing in atmospheric laser communication.

The OAM mode is an arbitrary integer, which can be regarded as a component of the infinite Hilbert space, which constitutes the physical basis of OAM in the field of information transmission [2]. The communication transmission scheme that first proposed with OAM was derived in [3]. This scheme proves the large capacity and security of OAM information transmission. However, when an OAM beam is applied to a free-space optical communication system, atmospheric turbulence causes amplitude fluctuations of the optical signal, and the phase of the signal will vary randomly on the one hand, resulting in degradation of communication quality. system. On the other hand, random jitter occurs due to the swing of the platform during transmission and reception [4]. Alignment errors caused by these factors can also have a large impact on the communication link. Due to the effects of turbulence and alignment errors, the OAM mode of the vortex beam changes in the atmosphere, which affects the performance of the system.

To clearly describe the changes in the OAM mode, Torner et al. used a linear superposition of spiral harmonic functions to describe the spiral spectrum of a vortex beam [5]. At present, there are some studies on the influence of atmospheric turbulence on the spiral spectrum of vortex beams [6, 7]. Reference [8] studied the transmission characteristics of LG beam in turbulent atmosphere, and derived the analytical formula of beam spiral spectrum. When the LG beam is transmitted in a weak turbulent flow, the reference [9] mainly analyzes the spiral spectrum spread of the LG beam, as well as the
probability of detection, the probability of crosstalk, etc. in communication. In [10], the effect of OAM beams on the helical spectrum spread in non-Kolmogorov turbulence is studied. When there is an alignment error between the transmitting end and the receiving end of the vortex beam, the spiral spectrum of the beam is studied in [11]. In [12], the influence of distance is analyzed based on the research of [11], and the analytical formula of spiral spectrum expansion is given.

This paper mainly studies the performance of OAM transmission under atmospheric turbulence and alignment error. Based on the Rytov approximation, the LG beam spiral spectrum model of turbulence effect and alignment error is established. Considering the four factors of turbulence intensity, propagation distance, lateral displacement and angular dip angle, the channel capacity and bit error rate of free-space optical communication system based on vortex are studied.

2. Spiral spectrum of LG beam with pointing error

2.1. Spiral spectrum

As the most common light carrying OAM, the LG beam’s electric field function in cylindrical polar coordinates can be described as

$$u_0(r,\varphi,z) = A \frac{w_0}{w(z)} \exp\left[-\frac{r^2}{w^2(z)}\right] \left[\frac{\sqrt{2}r}{w(z)}\right]^{\frac{3}{2}} \times L_p^s \left[\frac{2r^2}{w^2(z)}\right] \exp(is\varphi) \times \exp\left[i(2p+s+1)\tan^{-1}\left(\frac{z}{z_0}\right) - i\left(\frac{r^2}{w(z)}\right)\left(\frac{z}{z_0}\right) + ikz\right].$$

(1)

where $w(z) = w_0\sqrt{1 + (z/z_0)^2}$. $z$ is transmission distance and $w_0$ is beam waist, $\varphi$ is the azimuthal angle, $r$ is the radius of the cylindrical coordinate, $s$ and $p$ are the azimuthal and radial mode indices respectively, $z_0$ is Rayleigh distance, and $L_p^s$ is associating Laguerre polynomials. In this paper, $s=0$. Using the spiral spectrum harmonic function, the equation (1) can be decomposed into following,

$$u_0(r,\varphi,z) = \frac{1}{\sqrt{2\pi}} \sum_{m=-\infty}^{\infty} a_m(r,z) \exp(im\varphi).$$

(2)

Where

$$a(r,z) = \frac{1}{\sqrt{2\pi}} \int_0^{2\pi} u_0(r,\varphi,z) \exp(-im\varphi) d\varphi.$$  

(3)

The energy of the beam can be expressed as $U = 2\varepsilon_0 \sum_{m=-\infty}^{\infty} C_m$, where $C_m = \int_0^{\infty} |a_m(r,z)|^2 rdr$. The normalized intensity of the spiral spectrum of the $m$ mode can be obtained as

$$A_m = \frac{C_m}{\sum_{-\infty}^{\infty} C_q}.$$  

(4)

2.2. Lateral displacement error

Keeping the LG beam axis and the receiving antenna completely coaxially aligned is difficult for an actual communication system. Especially for long distance transmissions, this requirement is extremely difficult to satisfy. The error in alignment first causes loss of received optical power, and more importantly, it causes expansion of the spiral spectrum. For the $l$-order LG beam, the lateral offset of the beam is $(x_0, y_0)$. Equation (1) can be expressed as,
where $d$ is the lateral offset distance, and $\rho$ denotes the beam axis offset direction. Since
$$
\exp(-\pi\rho) = \sum_{m=-\infty}^{\infty} J_m(x)e^{im\phi},
$$
and $I_m(x) = \exp(-\frac{\pi}{2}i)J_m(ix)$, where $J_m(\cdot)$ is the first kind Bessel function
with $m$ order, and $I_m(\cdot)$ is the modified first kind Bessel function with $m$ order, the above equation can
be decomposed into
$$
u_l(r, \phi, z) = A_l \left[ 1 + \left( \frac{z}{z_0} \right)^2 \right]^{-0.5} \times \exp \left[ -\frac{r^2 + d^2 - 2rd\cos(\phi - \rho)}{w^2(z)} \right] \\
\times \left[ \sqrt{2} \left( \frac{re^{i\rho} - de^{i\phi}}{w(z)} \right) \right] \times L_p \left[ 2 \frac{r^2 + d^2 - 2rd\cos(\phi - \rho)}{w^2(z)} \right] \times \exp \left[ i(s+1)\arctan \left( \frac{z}{z_0} \right) - i \frac{r^2 + d^2 - 2rd\cos(\phi - \rho)}{w^2(z)} \right] z \\
\times \exp \left[ \sum_{m=-\infty}^{\infty} I_m \left( \frac{2rd}{w^2(z)} \right) \exp \left[ im(\phi - \rho + \pi/2) \right] \right].
$$
(5)

2.3. Angular deflection error

Lateral offset and angular tilt are usually generated at the same time. In order to describe the angular
deviation effect, it is equivalent to the beam passing through a phase wedge with a projection function
$$
exp(i\beta r \cdot \cos(\phi - \eta)), \quad \beta = k \cdot \sin \gamma, \quad \eta \text{ denotes the beam azimuth angle and } \gamma \text{ denotes the tilt angle}[12].$$
The expression of the light beam at the propagation distance $z$ is
$$
u_l'(r, \phi, z) = \sum_{m=-\infty}^{\infty} D_{l,m}(r, d, z) \exp \left[ im\phi - i(m-s)\rho \right].
$$
(7)

$$
D_{l,m}(r, d, z) = A_l \left[ 1 + \left( \frac{z}{z_0} \right)^2 \right]^{-0.5} \times \left[ \sqrt{2} \left( \frac{re^{i\rho} - de^{i\phi}}{w(z)} \right) \right] \times \exp \left[ i(s+1)\arctan \left( \frac{z}{z_0} \right) \right] \times \left[ \sum_{m=-\infty}^{\infty} I_m \left( \frac{2rd}{w^2(z)} \right) \exp \left[ im\phi - i(m-s)\rho \right] \right] \times L_p \left[ 2 \frac{r^2 + d^2 - 2rd\cos(\phi - \rho)}{w^2(z)} \right] \times \exp \left[ \sum_{m=-\infty}^{\infty} J_m \left( \frac{2rd}{z_0w^2(z)} \right) \exp \left[ im\phi - i(m-s)\rho \right] \right] \\
\times \sum_{q=0}^{\pi} C_q r^q (-d)^{s-q} \sum_{m=-\infty}^{\infty} J_q \left( \frac{2rd}{z_0w^2(z)} \right) \exp \left[ im\phi - i(m-s)\rho \right] \times \exp \left[ \frac{i\pi}{2} \right] \\
\times \sum_{\rho=-\infty}^{\infty} J_m(-q-n-p) \left( \frac{2rd}{w^2(z)} \right) J_p(\beta r) \exp \left[ ip(\rho - \eta + \pi/2) \right].
$$
(8)
3. Numerical simulation

3.1. Bit-Error Rate analysis

The pointing errors will cause the OAM state dispersion, which can take crosstalk into the communication. In this section, the detection probability of the received state is called $p_{mn}$, where $m$ is the transmitting state and $n$ is the receiving state.

The signal-noise ratio (SNR) is defined as the ratio between the transmit power times the squared detection probability $p_{mn}$ and the total noise power. The noise power contains two parts. The crosstalk signal modelled as independent Gaussian source is the one part, and the other part is the received noise power $N$. The SNR can be expressed as

$$ SNR = \frac{p_{mn}^2}{\sum_{n \neq m} p_{mn}^2 + N / P}. $$

(9)

For OOK or binary PPM modulation format, the bit-error rate (BER) can be written as

$$ b_{l} = \frac{1}{2} \text{erfc}\left(\sqrt{\text{SNR}} / 2\right). $$

(10)

In Fig.1, the BER performances under different lateral distance errors and tilt angle errors are shown. The $\Delta$ is the distance between the original center and lateral offset center. $\Delta^2 = (x_0^2 - x^2) + (y_0^2 - y^2)$.

![Fig. 1 BER versus SNR with $l = 1, 2, 3, 4, 5$](image)

(a) $\Delta = 0.2 \omega_{0}$, $z = 1000m$, (b) $\Delta = 0.4 \omega_{0}$, $z = 1000m$, (c) $\gamma = 2 mrad$, $z = 1000m$, (d) $\gamma = 4 mrad$, $z = 1000m$

It is evident that when the lateral offset distance error is small, the BER of each channel has a good performance, which is shown in Fig.1 (a) and Fig.1 (b). As the lateral distance increases, the BER of each OAM state would be worse. For the same lateral offset distance, the smaller states show better performance. Similar conclusions can be drawn for tilt errors from Fig.1 (c) and Fig.1 (d).

3.2. Channel capacity analysis

Channel capacity is the measure of the whole system performance. In this section, the channel capacity of the mode multiplex communication system is analyzed. The capacity of each channel can be presented as

$$ C(b_m) = 1 + b_m \log_2 b_m + (1 - b_m) \log_2 (1 - b_m). $$

(11)
The channel capacity can be presented as
\[ C_M = \text{arg max}_{C(b_m)} \sum_{m \in s} C(b_m) \] (12)
where \( M \) is the number of the desired states, and \( S \) is the number of all the channels. In Fig.2, the BER performances under different lateral offset distance errors and tilt angle errors are shown.

In Fig.2 (a) and Fig.2 (c), the lateral offset distance errors and tilt angle errors are small, the crosstalk among OAM states is small, and the channel capacities show a good growth with SNR. In contrast, in Fig.2 (b) and Fig.2 (d), when the lateral offset distance error and the tilt angle error increase, the OAM state crosstalk increases, and the increase in channel capacity becomes slower. Therefore, in a multiplexed system, the larger the misalignment error, the larger the SNR required to achieve the same channel capacity.

4. Conclusion
In this paper, the system performance of the OAMFSO communication system with the alignment error is analyzed. The lateral offset error and the angular tilt error are considered respectively, and the analytical expression of the detection probability is given in these two cases. Then, through numerical simulation, the BER of the system and the channel capacity of the multiplexing system under the influence of two kinds of errors are analyzed.

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