Cuprate is not a strongly-interacting material

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A renormalizable gauge theory is constructed to identify the pseudogap phase in cuprates as a Stueckelberg phase. The pseudogap transition is a Berezinski-Kosterlitz-Thouless-like transition, where the gauge field combines with the Stueckelberg field and acquires a mass. Consequently, the electronic spectrum opens a gap at the pseudogap transition without breaking the time-reversal and the translational symmetry.

In 1986, Bednorz and Müller discovered a new type of superconductivity in the transition-metal oxides [1]. The transition temperature soon boosted up to above the boiling point of liquid nitrogen [2], fostering new excitement in condensed matter physics. A common feature of the compounds with high transition-temperature (high-$T_c$) superconductivity is the layered structure of the copper-oxygen planes. As the system is cooled from the high temperature, the electronic spectrum opens a gap at $T = T^*$, higher than the superconducting transition temperature $T_c$ [3]. The pseudogap opens around the anti-nodal region in the momentum space without exhibiting any signature of conventional phase transition. In earlier days, this ambiguous transition is regarded as a crossover. After almost three decades from its discovery, the central debates still focuses on the formation of the pseudogap phase and its relevancy to the superconductivity in the lower temperature range. In this Letter, we address the first one and comment the second one briefly.

Intensive studies have been done both experimentally and theoretically to identify whether or not the pseudogap phase belongs to a broken-symmetry state. Recently, experimental data are accumulated to indicate that the pseudogap phase breaks time-reversal symmetry and preserves the translational symmetry [4–8]. However, in most of those data, the time-reversal symmetry begins to fluctuate precursory to the pseudogap transition. The connection is still unclear between the time-reversal symmetry breaking and the pseudogap phase.

Considering a conductor with a Fermi surface, the Cooper instability leads the system to the BCS ground state. A state is Mott insulating because the screening interaction between electrons remains to play an important role. Therefore, understanding the pseudogap state is the most radical task to solve the high-$T_c$ problem.

By chemical doping, mobile charge carriers are introduced in the Mott insulator. As the electrons become more and more mobile, the electron scattering process becomes more and more important. In particular, we consider the interaction vertex shown in Fig. (1) and introduce the current-current interaction in the one-band Hubbard model

$$
\mathcal{H} = -t \sum_{\langle ij \rangle, \sigma} (c_{i, \sigma}^\dagger c_{j, \sigma} + h.c.) + U_0 \sum_i n_{i \uparrow} n_{i \downarrow} + U_1 \sum_q \vec{J}_\uparrow(q) \cdot \vec{J}_\downarrow(-q),
$$

(1)

where $c_{i, \sigma}^\dagger (c_{i, \sigma})$ is the electron creation (annihilation) operator, $n_{i, \sigma} = c_{i, \sigma}^\dagger c_{i, \sigma}$, and $\vec{J}_\sigma(q)$ is the current operator

$$
\vec{J}_\sigma(q) = \sum_p c_{q+p, \sigma}^\dagger c_{p, \sigma} (\vec{p} + \frac{\vec{q}}{2}).
$$

(2)

The scattering process can occur in the lattice level, where the vertex represents a copper site. Due to the strong on-site repulsion, only electrons with different spins can occupy at the same site and scatter. Taking the extended Hubbard model to the continuous limit, the theory can be written in the path integral formalism

$$
Z = \int \mathcal{D} \psi \mathcal{D} \bar{\psi} e^{i \int dt dx \mathcal{L}},
$$

$$
\mathcal{L} = \sum_\sigma \psi_\sigma^\dagger(x) (i \partial_0 + \frac{\nabla^2}{2m}) \psi_\sigma(x) - u_0 \rho_\uparrow(x) \rho_\downarrow(x) - u_1 \vec{J}_\uparrow(x) \cdot \vec{J}_\downarrow(x),
$$

(3)

where $\hbar = 1$, $\partial_0 = \frac{\partial}{\partial t}$, and $\rho_\sigma(x)$ and $\vec{J}_\sigma(x)$ are the charge and the current density respectively. The theory described by Eq. (3) is, however, not renormalizable since the dimension of the coupling constants $u_0$.
and \( u_1 \) is \( [u_0] = [u_1] = L \). This situation is similar to Fermi’s \( \beta \)-decay theory, where a non-renormalizable four-fermion vertex is introduced to produce a finite matrix element between a neutron, a proton, an electron, and a neutrino \([9]\). The \( \beta \)-decay problem is completely solved by Glashow, Weinberg, and Salam \([10,11,12]\), who constructed the Standard Model for the electroweak interaction, where \( W^\pm \) and \( Z \) gauge bosons were introduced accounting for the weak interaction, meanwhile making Fermi’s vertex renormalizable.

Here, we play the same trick. We introduce a fictitious gauge field \( (a_0, \vec{a}) \) accounting for the effective interaction between electrons.

\[
\mathcal{L} = \sum \psi^\dagger(x)(i\partial_0 + \frac{\nabla^2}{2m})\psi(x) + i\bar{\psi}g^2 \rho_\sigma(x) = \sum_{\sigma}\frac{i}{k^2 - M_0^2 + i\eta} \rho_\sigma(x) \]

\[
+ \frac{g^2}{2} \sum_{\sigma,\sigma'} \bar{\psi}_{\sigma'}(x) i\bar{\psi}_\sigma(x) \frac{k^2 - M_0^2 + i\eta}{k^2 - M_1^2 + i\eta} \psi_{\sigma'}(x), \quad (5)
\]

where \( k^2 = k_0^2 - k^2 \), \( \eta \) is a small parameter, and terms with higher order of \( O(g^4) \) are ignored. Using the Grassmann variable, terms with the same spin at the same spatial location automatically vanish. Comparing Eq. (3) and Eq. (5), \( u_0 \) and \( u_1 \) can be obtained

\[
u_0 = \frac{g^2}{M_0^2} \]

\[
u_1 = -\frac{g^2}{M_1^2} \quad (6)
\]

in the low-energy and long-wavelength limit. Now, the coupling constant \( g \) has a dimension \([L^{-\frac{3}{2}}] \). The theory becomes renormalizable. Moreover, \( u_0 > 0 \) represents the repulsive interaction, and \( u_1 < 0 \) represents the attractive interaction. It is consistent with our physical intuition of electromagnetism. Namely, the charge interaction between two electrons is repulsive due to the Coulomb interaction. The current interaction is attractive since two conducting wires with the same direction of electric current attract to each other due to the Lorentz force. Similar situation of the current interaction between spinons was considered by Lee et al. before \([13]\). Different from other mechanism by exchanging vector bosons, for example phonons or magnons, the exchange of gauge boson produces both repulsive and attractive interactions.

The gauge theory in Eq. (4) is not gauge invariant due to the finite mass of the gauge boson. To make it gauge invariant, we consider the Stueckelberg mechanism \([14,15,16]\), which is a prevailing one to use to generate mass in the context of the string theory \([17]\). The Stueckelberg field is a phase field \( \phi(t, \vec{x}) = \frac{e^{i\sigma(t, \vec{x})}}{q} \). We add the Stueckelberg Lagrangian

\[
\mathcal{L}_S = \frac{1}{2} M_0^2 (D_0\phi)^\dagger(D_0\phi) - \frac{1}{2} M_1^2 (D_i\phi)^\dagger(D_i\phi)
\]

\[
- \frac{1}{2} M_0^2 (\frac{1}{q} \partial_0 \sigma + a_0)^2 - \frac{1}{2} M_1^2 (\frac{1}{q} \nabla \sigma - \vec{a})^2, \quad (7)
\]

where \( D_0 = i\partial_0 - qa_0 \) and \( D_i = -i\partial_i - qa_i \) are the covariant derivative, and \( q \) is the gauge coupling for the Stueckelberg field. The gauge transformation is defined by

\[
\vec{a} \rightarrow \vec{a}' = \vec{a} + \frac{1}{q} \nabla \lambda
\]

\[
a_0 \rightarrow a'_0 = a_0 - \frac{1}{q} \partial_0 \lambda
\]

\[
\sigma \rightarrow \sigma' = \sigma + \lambda. \quad (8)
\]

Then, the total Lagrangian is manifestly gauge invariant

\[
\mathcal{L} = \sum \psi^\dagger(i\partial_0 + \frac{\nabla^2}{2m})\psi(x) + \frac{1}{2} \bar{\psi}g^2 \rho_\sigma(x) = \sum_{\sigma}\frac{i}{k^2 - M_0^2 + i\eta} \rho_\sigma(x) \]

\[
+ \frac{g^2}{2} \sum_{\sigma,\sigma'} \bar{\psi}_{\sigma'}(x) i\bar{\psi}_\sigma(x) \frac{k^2 - M_0^2 + i\eta}{k^2 - M_1^2 + i\eta} \psi_{\sigma'}(x) + \frac{1}{2} M_0^2 (D_0\phi)^\dagger(D_0\phi) - \frac{1}{2} M_1^2 (D_i\phi)^\dagger(D_i\phi). \quad (9)
\]

The Lagrangian in Eq. (9) contains the correct description for the pseudogap transition. In the high temperature region, the gauge field is massless. Since the Stueckelberg field is fluctuating, the vacuum expectation value \( \langle \phi \rangle = 0 \). The Stueckelberg field is like a needle in a clock. The needle has a finite length, but different directions cancel one another in the high temperature region. As the temperature goes down, the Stueckelberg field tends to be static. The vacuum expectation value of the Stueckelberg field is frozen to \( \langle \phi \rangle = e^{i\sigma_0} \). Now, we can simply gauge away the Stueckelberg field by choosing \( \lambda = -\sigma_0 \). In this case, \( \langle \sigma' \rangle = 0 \) and \( \langle \sigma' \rangle = 1 \). The last two terms in Eq. (9) soon become the last two terms in Eq. (4), and the gauge bosons acquire a mass. Similar to the Anderson-Higgs mechanism, the gauge boson combines with the Goldstone mode of the Stueckelberg field and becomes massive. In fact, the Stueckelberg mechanism was sometimes regarded as a special Higgs mechanism with an infinite Higgs mass.
On the other hand, the Stueckelberg mechanism gives the correct degrees of freedom. In the high temperature phase, there are two degrees of freedom. The massless gauge boson has 1 degree of freedom and the Stueckelberg field contributes to another one. In the low temperature phase, the Stueckelberg field becomes the longitudinal mode of the massive gauge field. The total number of degrees of freedom remains two.

Different from the usual Higgs mechanism, the Stueckelberg mechanism in 2+1 dimensions does not need a condensate. Since the Stueckelberg field is a phase field, the phase transition from \( \phi \rightarrow 0 \) to \( \phi' \rightarrow 1 \) belongs to a 2D XY universality class. Therefore, at finite temperature, the phase transition is a Berezinski-Kosterlitz-Thouless (BKT) transition. The transition temperature is \( T^* = \frac{\pi M_0^2}{2 \eta} \) for the simplest case when \( M_0 \approx M_1 \). The BKT transition is a phase transition of infinite order. In the high temperature phase, it is a disordered phase with an exponential correlation, and in the low temperature phase, it is a quasi-ordered phase with a power-law correlation. In our case, the pseudogap is a BKT-like transition. In the low temperature phase, the Stueckelberg field was eaten by the gauge field. Therefore, propagator-wisely speaking, it does not appear in any physical process, nor can the power-law correlation be discussed. Namely, it is not measurable in the pseudogap phase.

Let us now consider the symmetry property of the Stueckelberg field. Since \( a_0 \) is even and \( \tilde{a} \) is odd under time-reversal transformation, the Stueckelberg field \( \phi \rightarrow \phi^* \) under time-reversal transformation. In the high temperature phase, \( \phi' \rightarrow 0 \), so it is time-reversal symmetric. In the low temperature phase, the Stueckelberg field can always be gauged away and becomes \( \phi' \rightarrow c \). Both time-reversal symmetry and translational symmetry is preserved in the pseudogap phase.

Across the pseudogap transition, the band structure is also modified. To demonstrate, we compute the self energy of the electron. We consider one electron problem in the conduction band and see how the massive gauge field modifies the band structure. To begin with, we should quantize the theory. Using the Feddeev-Popov quantization and applying the unitary gauge, the Green’s function for the electron \( G^\psi \), the Stueckelberg field \( G^\phi \), and the gauge field \( G^a \) can be computed.

\[
G^\psi = \frac{1}{\omega - \varepsilon_k + i\eta},
\]

\[
G^\phi = 0,
\]

\[
G_{00}^a = \frac{\omega^2 - M_1^2}{\omega^2 M_0^2 - (k^2 + M_0^2) M_1^2 + i\eta},
\]

\[
G_{01}^a = G_{10}^a = \frac{\omega k_1}{\omega^2 M_0^2 - (k^2 + M_0^2) M_1^2 + i\eta},
\]

\[
G_{11}^a = G_{21}^a = \frac{k_1 k_2 (\omega^2 - k^2 - M_0^2)}{\omega^2 - k^2 - M_1^2},
\]

where \( \varepsilon_k = \frac{k^2}{2m} \) for the conduction band.

\[
\Sigma(\omega, \tilde{\omega}, \vec{p}) = \Sigma_1(\omega, \tilde{\omega}) + \Sigma_2(\omega, \tilde{\omega}),
\]

where \( \Sigma_1(\omega, \tilde{\omega}) \) and \( \Sigma_2(\omega, \tilde{\omega}) \) are the amplitudes for the diagrams in Fig. (2a) and Fig. (2b) respectively. After some algebra, \( \Sigma_1(\omega, \tilde{\omega}) = \frac{g^2}{2m} (\frac{M_1}{4\pi} + \frac{M_0^2}{M_1}) \) and \( \Re \Sigma_2(0, 0) = 0 \). Namely, the diagram in Fig. (2a) contributes to a finite energy gap and the one in Fig. (2b) does not. We can do the same calculation for the hole in the valence band. Then, we obtain a finite energy gap

\[
\Delta = \frac{g^2}{4\pi m} (M_1 + \frac{1}{3} M_0^2) M_1^2,
\]

which vanishes when \( M_0 = M_1 = 0 \). Therefore, at the pseudogap transition, the gauge field acquires a mass, opening a finite energy gap in the electronic spectrum, which is the origin of the pseudogap formation.

Recently, with considerably-improved sample quality, a nodeless pseudogap structure was confirmed experimentally in the highly-underdoped \( La_{2-x}Sr_xCuO_4 \) (LSCO) systems. While the pseudogap is nodeless in the low and zero temperature, the Fermi-arc feature is restored and robust as either temperature or doping level increases. In fact, the generic Fermi-arc structure is observed only at finite temperature. Therefore, the pseudogap is intrinsically nodeless, since even in \( La_{2-x}Ba_xCuO_4 \) (\( x = 1/8 \)), the failed high-\( T_c \) superconductor, a gap is opened in the nodal direction when the system is approaching to zero temperature. While the robustness of the Fermi arc at finite temperature remains intriguing, further studies with the consideration of the lattice symmetry and the \( d \)-wave superconducting instability should clarify this issue.
Estimating the parameters in the theory, we are able to reproduce the fundamental properties in cuprates. Taking \( M_0 \approx M_1 \approx 2.5 \times 10^5 \text{ eV}, g^2 \approx 50 \text{ eV}, \) and \( q^2 \approx 4 \times 10^8 \text{ eV}, \) we obtain the Hubbard term \( U \approx 8 \text{ eV}, \) the pseudogap magnitude \( \Delta \approx 20 \text{ meV} \ [19-22], \) and the pseudogap transition temperature \( T^* \approx 1.5 \times 10^2 \text{ K}, \) Besides, the fitting is self-consistent in many respects. For example, the mass of the gauge field determines the interaction length scale. The values of \( M_0 \) and \( M_1 \) imply the length scale to be \( \sim \Lambda, \) which explains why the on-site Coulomb repulsion is most relevant. Furthermore, the small dimensionless interaction vertex, proportional to \( \frac{g^2}{2m} \approx 10^{-5}, \) guarantees the validity of the current perturbative analysis.

As a remark, our theory does not imply that the real photon acquires a mass in the pseudogap phase, since the real photon couples to the electron but does not couple to the Stueckelberg field. The gauge field and the Stueckelberg field are simply the complications representing the effective interaction between electrons in the strongly-correlated systems. If the effect by an external electromagnetic source is considered, for example, one should further introduce the photon field \( A_\mu \) in the Lagrangian.

Finally, let us comment the issue about the superconductivity. The mass parameters of the gauge boson \( M_0 \) and \( M_1 \) are temperature dependent. In the pseudogap phase, the repulsion interaction dominates. As the temperature is lowered further, the attractive interaction may win the competition, and then the system becomes a superconductor. Therefore, there is no obvious theoretical correlation between the \( T^* \) and the \( T_c. \) Furthermore, the high-\( T_c \) material is a \( d \)-wave superconductor. Considering the pairing with different spins, \( \phi_k = \langle c_{-k\uparrow} c_{k\uparrow} \rangle, \) the Hamiltonian in Eq. (1) becomes

\[
\mathcal{H} = -t \sum_{<ij>,\sigma} (c_{i\downarrow} c_{j\uparrow} c_{j\sigma}) + \text{h.c.} + \sum_{p,p'} [U_0 + \frac{U_1}{4} (\mathbf{p} + \mathbf{p}')^2 (\phi_p^* c_{-p\uparrow} c_{p\uparrow} + \phi_{p'}^* c_{p\uparrow} c_{-p\downarrow})].
\]

When \( U_1 = 0, \) the system favours the \( d \)-wave pairing in large \( U_0 \) limit due to the antiferromagnetic correlation. The question is whether or not the introduction of the \( U_1 \) term impairs the \( d \)-wave pairing? The superconductor gap equation

\[
\Delta_p = -\sum_{p'} V_{pp'} \frac{\Delta_{p'}}{2E_{p'}}.
\]

where \( \Delta_p = -\sum_{p'} V_{pp'} \phi_{pp'} \) is usually difficult to solve. However, we can do a consistent check to see whether or not \( \Delta_{(\pi,0)} \Delta_{(0,\pi)} \) changes sign when \( V_{pp'} = \frac{U_1}{4} (\mathbf{p} + \mathbf{p}')^2. \) Using the gap equation and by a simple change of the summation variables, we found

\[
\Delta_{(\pi,0)} \Delta_{(0,\pi)} = -\sum_{p'} V_{(0,\pi)p'} V_{(0,\pi)p''} \frac{(\cos p_x' + \sin p_y') (\cos p_x'' + \sin p_y'')}{4E_{p'} E_{p''}} < 0,
\]

(15)

if \( \Delta_p = \cos p_x - \sin p_y. \) That \( \Delta_{(\pi,0)} \) and \( \Delta_{(0,\pi)} \) have different sign implies a \( d \)-wave pairing. Therefore, the \( U_1 \) term can also host the \( d \)-wave pairing symmetry.

In summary, starting from an extended Hubbard model, we construct a renormalizable gauge theory for cuprates. The pseudogap transition is a Berezinski-Kosterlitz-Thouless-like phase transition. It happens when the gauge field accounting for the effective interaction of the electrons acquires a mass. Consequently, the massive gauge field modifies the band structure and introduces a finite energy gap through the process in Fig. (2). We believe that the high-\( T_c \) problem is not a "strong-interaction" problem but a "weak-interaction" problem. Insufficient screening leading to the Hubbard on-site Coulomb repulsion implies that the interaction between electrons has to be treated as a gauge field. Consideration of a more complete Hamiltonian makes the theory closer to the reality, and a weak-coupling theory for the high-\( T_c \) materials becomes possible. Our theory provides a unified framework for the pseudogap phenomena in the strongly-correlated condensed matter systems. The realization of the Stueckelberg phase may stimulate interests in high-energy physics community, too. Further research directions include the experimental signature of the Stueckelberg field and the explanation of a large volume of the present experimental data.

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