Spin-Precession Vortex and Spin-Precession Supercurrent Stability in $^3$He–B

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The stability of the spin-precession currents in superfluid $^3$He–B is analyzed for the precession angle very close to $104^\circ$. In this limit, a spin-precession vortex has a very large core, and the barrier that blocks motion of these large-core vortices across the current streamlines (phase slip) disappears at precession-phase gradients much smaller than critical gradients estimated from the Landau criterion. Nevertheless, spin-precession currents remain stable up to the Landau-critical gradients, since, in this case, there is a barrier, which blocks the phase slip at a very early stage of the vortex-core nucleation. The second-order phase transition between the parity-symmetric and parity-asymmetric spin-precession vortex cores at the precession angle of $126.5^\circ$ is also predicted.

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The phenomenon of the spin superfluidity was intensively studied experimentally and theoretically in the 1970s and 1980s (see reviews [1–3] and references therein). Nowadays, there is a revival of interest in the superfluidity theory [1].

One year later, Fomin [13] suggested that the vortex core radius becomes $r_c \sim \xi_{sd}$ (see below). Thus, the latter is valid only far from the critical angle, where $\beta - \beta_c \sim 1$.

Since no barrier impedes the vortex expansion across a channel if the gradient is on the order of $1/r_c$, the large core $r_c \sim \xi_{sd}$ at $\beta \rightarrow \beta_c$ leads to a strange (from the point of view of the conventional superfluidity theory) conclusion: the instability with respect to the vortex expansion occurs at the phase gradients $\sim 1/r_c$, essentially less than the Landau-critical gradient $\sim 1/\xi_{sd}$ obtained in [11] for any $\beta > \beta_c$. The present letter suggests a resolution of this paradox. It demonstrates that, at precession angles close to $104^\circ$...
and at phase gradients less than the Landau-critical gradient but larger than the inverse core radius, no barrier impedes the phase slips at the stage of the vortex motion across streamlines, but there is a barrier, which blocks the phase slips on the very early stage of nucleation of the vortex core. Thus, for these gradients, the stability of the current states is determined not by the vortices but by the vortex-core nuclei.

This analysis also addresses the possible symmetries of the vortex core. It was expected that the parity symmetry (its definition is given below) is always broken [14]. The present letter presents a numerical calculation demonstrating the second-order transition between a parity-symmetric and a parity-asymmetric vortex at the precession angle 126.5°. In the past, the first-order transition in cores of 3He–B mass vortices was detected in NMR experiments upon rotating 3He–B [15]. It was theoretically explained in [16, 17] in terms of the transition between the axisymmetric and nonaxisymmetric cores. Later, this theory was confirmed experimentally by the direct observation of the nonaxisymmetric core in one of the two vortices [18].

The spin dynamics of the superfluid phases of 3He is described by the theory of Leggett and Takagi [19]. Following Fomin [2], the Euler angles α, β, and γ are introduced in the spin space of the 3He–B order parameter. The angle β is the precession angle, and α is the precession phase. The angle Φ = α + γ characterizes the resultant rotation of the order parameter in the laboratory frame, and in the limit β → 0 (no precession) becomes the angle of rotation about the z axis. The moments canonically conjugate to the angles α, β, and Φ are, respectively, P = M_z = M_β, and M_Φ, where M_z is the z component of the magnetization M in the laboratory frame, M_β is the projection of M on the ξ axis of the rotating coordinate frame, and M_Φ is the projection of M on the axis perpendicular to the ζ and ξ axes.

For phenomena observed experimentally, only one degree of freedom is essential, which is connected with the conjugate pair “precession phase α–precession moment P.” The Hamilton equations for the precession mode are

\[
\frac{\partial\alpha}{\partial t} = \gamma \frac{\partial F}{\partial P}, \quad \frac{\partial P}{\partial t} = -\gamma \frac{\partial F}{\partial \alpha}.
\]

Since the degree of freedom connected with the conjugate pair M–Φ is not active, the angle Φ is determined from the minimization of the energy: δF/δΦ = 0. The free energy \( F = F_z + F_V + V \) includes the Zeeman energy \( F_z = \mathbf{M} \cdot \mathbf{H} = -MHu = -\chi_0^2 Hu^2 \), where \( \mathbf{H} = H \hat{z} \) is an external constant magnetic field, the gradient energy (the spin current is assumed to be normal to the magnetic field \( H \hat{z} \)),

\[
F_V = \frac{\chi_0^2 H^2}{2}\left[A(u)\frac{\partial u^2}{2} - \frac{c_0^2}{c_0^2} \frac{\partial u^2}{(1 - u^2)^2} + \frac{\partial u^2}{2(1 - u^2)^2}\right],
\]

where

\[
A(u) = \frac{c_0^2}{c_0^2}(1 - u^2) + 1 - u^2,
\]

and the dipole energy \( V = \chi c_0^2 \gamma^2 u \), where

\[
\gamma = \frac{2}{15}\left(1 + \cos\Phi\right)u + \cos\Phi - \frac{1}{2}u^2.
\]

Here, \( \chi \) is the magnetic susceptibility, \( \gamma \) is the gyromagnetic ratio, \( \omega_L = \gamma H \) is the Larmor frequency, \( u = \cos\beta \), and the ratio \( c_0^2/c_1^2 \) of the velocities of the longitudinal and transversal spin waves will be chosen to be \( \sqrt{3} \) [13]. In the state of the stationary precession, the precession angular velocity is constant: \( \partial\alpha/\partial t = -\omega_L \). This state corresponds to the extremum of the Gibbs thermodynamic potential, which is obtained from the free energy with the Legendre transformation \( G = F + \omega_L P/\gamma \). Thus, the precession frequency \( \omega_L \) plays the role of the “chemical potential” conjugate to the precession moment density \( P \). The distribution of the parameters \( u = \cos\beta \) and \( \Phi \) is determined from the two Euler–Lagrange equations \( \delta G/\delta u = 0 \) and \( \delta G/\delta \Phi = 0 \).

For uniform precession, the minimization with respect to \( u \) yields the relation

\[
\frac{\chi(\omega_L - \omega_0)}{\gamma^2} u \Delta + \frac{\partial v}{\partial u} \frac{1}{\zeta_0^2} + \frac{1}{\zeta_0^2} \frac{\partial v}{\partial u} = 0,
\]

and the minimization with respect to \( \Phi \) (only the dipole energy depends on \( \Phi \)) gives the equation

\[
(1 + \cos\Phi)u + \cos\Phi - \frac{1}{2}(1 + u)\sin\Phi = 0.
\]

The solution of this equation yields

\[
\cos\Phi = 1/2 - u/(1 + u), \quad \nu(u, \Phi) = 0
\]

for \( \beta < 104^\circ \ (u > -1/4) \) and

\[
\cos\Phi = 1, \quad \nu(u, \Phi) = \nu_0(u) = \frac{8}{15}\left(1 + u^2\right)^2
\]

for \( \beta > 104^\circ (u > -1/4) \).

The spin-precession vortex state is nonuniform, and the gradient energy becomes essential. For an axially symmetric vortex with \( 2\pi \) circulation of the precession phase \( \alpha (\nabla \alpha = 1/r) \), the Euler–Lagrange equations are

\[
\frac{1}{\zeta_0^2} \frac{4 - u}{3^2} - \frac{4 - u}{3^2} \frac{d^2u}{dr^2} = \frac{1}{1 - u^2}\left[\frac{1}{r} \frac{d^2u}{dr^2} + \frac{d^2u}{dr^2}\right]
\]

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