Lepton-Neutron Bound States

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Abstract

We consider lepton-neutron (and lepton-antineutron) bound states and resonances which appear due to spin-spin, spin-orbital interactions, neutron polarization by muon. Our analysis is also true for any system which include one charge and one neutral particle with finite size e.g. $\pi^0\mu^\pm$-bound states. We consider also cylindrically symmetric bound states and resonances of particles with anomalous magnetic moments.
1. Introduction

In this article we consider lepton-neutron (and lepton-antineutron) bound states and resonances which appear due to spin-spin and spin-orbital forces between neutron and lepton (antilepton) neutron polarization by lepton (because neutron is composite particle with finite size which consist of particles with opposite charge and in the field of lepton neutron polarization appear).

At large distances also exist the potentials from $\rho^0 - \gamma$ (see Appendix B below) mixing. Our analysis is also true for any system which include one charge and one neutral particle with finite size e.g. $\pi^0 \mu^\pm, K^0 \mu^\pm$-bound states. As we see below however the attraction of neutron polarization (see Appendix A) is much smaller than spin-spin and spin-orbital interaction and essential only for $\pi$-meson-lepton bound states).

In non-relativistic approximation spin-spin and spin-orbital interactions, (which depend in general on angles) has the form [1]:

$$V(r) = 6\mu_\mu \mu_n \frac{1}{r^3} \left( \hat{S} \hat{r} \hat{S} \hat{r} \right) - \frac{1}{r^2} (\hat{S})^2 + 4\pi\mu_\mu \mu_n \left( \frac{7}{3} (\hat{S})^2 - 2\delta(\hat{r}) \right)$$ (1)

however at $J = 0 \ L = -\hat{S}$ and potential of spin-spin and spin-orbital interactions become spherically symmetric (because $\hat{S} \hat{n} = -\hat{L} \hat{n} = 0$) interaction:

$$V_{SS}(r) = 6\mu_\mu \mu_n \frac{1}{r^3} \left( -\frac{1}{3} (\hat{S})^2 \right) + 4\pi\mu_\mu \mu_n \left( \frac{7}{3} (\hat{S})^2 - 2\delta(\hat{r}) \right)$$ (2)

$$V_{LS}(r) = -6\mu_\mu \mu_n \frac{1}{r^3} L(L+1)$$ (3)

We have the following possible quantum numbers of bound states with $J = 0: L = S = 0, L = S = 1$.

The potentials (3)-(5) are non-relativistic. For relativistic consideration it is convenient to use the Dirac equation in the form:

$$(\gamma \cdot (E - eA_0))^2 - (\gamma \cdot (\hat{p} - e\vec{A}))^2 + e\gamma \cdot \vec{H} - ie\gamma \cdot \vec{E}) \psi = 0$$ (4)
where $\vec{A} = [\vec{r}\sigma_n]F(r)$, $\vec{E} = -\vec{n}A'_0(r)$, $H_i = \vec{\sigma}_n(-rF' - 2F) + rF'(r)\vec{n}(\sigma'_n\vec{n})$.

In (4) we can put $\vec{p}\vec{A} = 0$, $\vec{A}\vec{p} = 0$ because $\vec{A} = [\vec{r}\sigma_n]F(r)$. We would like to notice that term $e\vec{A}^2$ is repulsive.

We find the solution in the following form:

$$\psi^T = (a(r), i(\vec{\sigma}\vec{n})b(r))$$ (5)

Taking into account (9)(10) we obtain the following system of radial equations:

$$\hat{T}a(r) - eA'_0b(r) = 0$$ (6)

$$\hat{T}b(r) + eA'_0a(r) = 0$$ (7)

where

$$\hat{T} = (E - eA_0)^2 - (\vec{p})^2 - (e\vec{A})^2 + \frac{1}{2}(S(S + 1) - 6)(-rF' - 2F) - erF')$$, (8)

$$(\vec{p})^2 = -\frac{1}{r^2} \frac{d}{dr}r^2 \frac{d}{dr} + \frac{L(L+1)}{r^2}$. Of course only $S = 0, 1$ is possible.

From this system of equations we can obtain:

$$\hat{T} \frac{1}{eA'_0} \hat{T}a(r) + eA'_0 a(r) = 0$$ (9)

It was consideration for particle without anomalous magnetic moment (it is actual e.g. for positronium).

In case of $\pi^0$-lepton(antileton) bound states if for simplicity we suggest that only dielectric polarization exist (i.e. only $A_0(r)$ is taken into account) we have the same equations as in [1], and for radial functions ($\psi = (f(r)\Omega_{jlm}, i^{l' - l}, \vec{\sigma}\vec{n}\Omega_{jlm}g(r))$) we obtain the following equations where
however $V(r)$ is potential from polarization of $\pi^0$-meson (see Appendix A below):

\[
(f'(r) + \frac{1 + \kappa}{r} f(r)) - (E + m - V(r))g(r) = 0 \tag{10}
\]

\[
(g'(r) + \frac{1 - \kappa}{r} g(r)) + (E - m - V(r))f(r) = 0 \tag{11}
\]

where $\kappa = \mp(j + \frac{1}{2})$ if $j = l + \frac{1}{2}$.

It is of interest to consider also bound states of composite neutral fermion (scalar)-charged particle, e.g. neutron-ion. If $Z\alpha \sim 1$ polarization effects may be significant. The Dirac equation for particle with anomalous magnetic moment has the following form:

\[
(\hat{k} - m_n + \mu_n(\Sigma\hat{H} - i\widehat{\alpha E}))\psi(k) = 0, \tag{12}
\]

where $\mu_n$ is anomalous magnetic moment of neutron, $\Sigma\hat{H} = F_1(r)$ at $J = 0$ and $\widehat{\alpha E} = \widehat{\sigma nG}(r)$.

For radial equations we obtain the following results:

\[
(f'(r) + \frac{1 + \kappa}{r} - \mu_nG(r))f(r) - (E + m - \mu_nF_1(r))g(r) = 0 \tag{13}
\]

\[
(g'(r) + \frac{1 - \kappa}{r} + \mu_nG(r))g(r) + (E - m - \mu_nF_1(r))f(r) = 0 \tag{14}
\]

where $\kappa = \mp\frac{1}{2}$ (because $J = 0$). If magnetic field is absent ($F_1(r) = 0$) $J$ may be arbitrary.

Thus we have singular potential $\sim r^{-n}$ at $r >> r_n$. At $r < r_n << \frac{1}{m_e}$ singular behaviour is absent. It means the presence of deep levels of $e^{\mp n}$ bound states. In our next paper we will calculate it numerically.

In case of neutron the effect of neutron polarization is of order $\sim \frac{\alpha^2}{m_e}$ is essentially smaller than spin-spin and spin-orbital interaction $\sim \frac{\alpha}{m_e M R^3}$. However in composite scalar-charged particle bound states (see below) spin-spin and
spin-orbital interaction are absent and attraction by polarization is important.

We would like also to stress that the potential is the sum of the potential from polarization+formfactors effect (formula (1) above)+ of spin-spin interaction and from photon-ρ-meson mixing.

It must be noted also exist long-range potential induced by $\pi^0\gamma\gamma$ vertex:

$$V(q) = e^2 g_{\pi NN} \frac{1}{q^2 + m_{\pi}^2} \vec{E} \vec{H}$$

(15)

where $\vec{E}, \vec{H}$ are fields of the lepton.

**Production and decays of $e^\pm n$ atoms**

This bound states may be produced after stop of neutrons (antineutrons) in matter. Resonances may be produced in $e^\pm n$-collisions. One of the decay mode of this bound states and resonances is weak decays:

$$e^\pm n \text{ atoms} \rightarrow \nu p$$

(16)

$e^\pm \pi^0(K^0)$ atoms may be produced by the same way.

Also exist potential connected with neutron formfactor:

$$M = e F_n (q^2) \bar{n} \gamma_\alpha n e \gamma_\beta e$$

(17)

where $F_n(0) = 0$. The formfactor correspond to the some charge density and we obtain:

$$V(r) = -\alpha \int \frac{1}{|\vec{r} - \vec{r}'|} \rho(\vec{r}') d^3 r'$$

(18)

Thus we have singular potential $\sim r^{-n}$ at $r >> r_n$. At $r < r_n << \frac{1}{m_e}$ singular behaviour is absent. It mean the presence of deep levels of $e^\pm n$ bound states. In our next paper we will calculate it numerically.
We would like also to stress that the potential is the sum of the potential from polarization+formfactors effect (formula (1) above)+ of spin-spin interaction and also vector boson-photon mixing which fall as \( r^{-3} \) and the most essential at large \( r \).

In case of composite scalar particle-charged scalar particle bound states and resonances the situation is simplified because \( \pi^0 \) is spinless. For \( e^\pm \) may be used Klein-Gordon equation:

\[
((E - V(r))^2 + \frac{1}{r^2} \frac{d}{dr} r^2 \frac{d}{dr} - \frac{l(l + 1)}{r^2} - m^2)\phi(r) = 0
\]

(19)

Numerical solution of this equation for has been performed in [5]. In the same reference has been considered also task on energy levels of electron in the field of conducting spheres (see below). The dependence of binding energy versus radius of the sphere is shown on the Fig.3 of this reference.

It must be noted that in [5] has been considered only case of homogeneous dielectric sphere, in the future we plane to consider more realistic potential (1).

It is of interest to consider Cooper pairs which consist of electron and neutron (lepton and proton). The Cooper pairs existence in this case is possible due to attractive forces considered above (attraction via polarization and via spin-spin and spin-orbital interactions). The work in this direction is under progress (for some estimates see [5]).

Besides application to \( e^\pm\pi^0(K^0) \) atoms problems may be considered also task about bound states of particles in the attractive field of the dielectric and conducting bodies. Due to symmetry properties the most interesting are sphere and cylinder. In [5] has been considered also charged particle levels in the potential of charge particle interaction with its reflection.
For various case of conducting spheres (see [6]) we have:

\[
V(r) = \frac{e^2 R}{2(r^2 - R^2)}
\]  

(20)

if charge of the conducting spheres \( e' \) (induced on sphere) is nonzero, and

\[
V(r) = \frac{e^2 R^3}{2r^2(r^2 - R^2)}
\]  

(21)

if charge of conducting sphere is zero.

If radius of sphere (dielectric or conducting) much larger than Bohr radius we obtain well known Tamm levels, the numerical results is shown on the Fig.1-3 of the [5] for case 1,2 of the ideal conducting sphere and for case of homogeneous dielectric sphere.

**Appendix A : Attraction from polarization**

If we consider as first approximation neutron as homogeneous dielectric matter inside sphere with radius \( R_n \) we can use the result of [4]. At \( r > R_n \), \( r < R_n \) we have [4] respectively:

\[
V(r) = -e^2(\epsilon_{n,\bar{n}} - 1) \sum_{l=0}^{\infty} \frac{l}{\epsilon_{n,\bar{n}} + l + 1} \frac{R_{n}^{2l+1}}{r^{2l+2}},
\]  

(22)

\[
V(r) = -e^2(\epsilon_{n,\bar{n}} - 1) \sum_{l=0}^{\infty} \frac{l + 1}{\epsilon_{n,\bar{n}} + l + 1} \frac{r^{2l}}{R_{n}^{2l+1}},
\]  

(23)

where \( \epsilon_{n,\bar{n}} \) are dielectric polarizability of neutron(antineutron).

At \( r >> R_n \) only \( l = 1 \) is essential and we obtain:

\[
V(r) = -e^2(\epsilon_{n,\bar{n}} - 1) \frac{1}{\epsilon_{n,\bar{n}} + 2} \frac{R_{n}^3}{r^4},
\]  

(24)

We see that at \( r < R_n \) interaction is repulsive. In polarizability on the neutron and proton was achieved in ref. [3] (see also references therein) \( (\alpha \sim 10^{-3} fm^3) \).
Above was suggested that neutron is homogeneous dielectric sphere, however as known neutron may be treated as nonlocal object with with density $\rho_n(r) \sim \exp -Mr$. In this case we must take into account that $\epsilon(r)$ is function on $r$. If we know $\epsilon(r)$, in this case we can obtain electron-neutron potential via neutron polarization by electron from Maxwell equation:

$$\text{div}(\epsilon(r)\vec{E}) = 4\pi \rho$$

(25)

In particular because $1 - \epsilon(r) \to 0$ at $r \to \infty$ by the same low as density we obtain:

$$\epsilon(r) = (\epsilon(0) - 1) \exp(-Mr) + 1$$

(26)

where $\epsilon(0)$ may be obtained from $e^-n$ elastic scattering at large $\vec{q}^2$. Also $V(q)$ may be obtained from $e^-n$ elastic scattering at all $q^2$. In this case we can obtain the potential $V(r) = \int V(q) \exp(i\vec{q}\cdot\vec{r}) \frac{d^3q}{(2\pi)^3}$ and substitute $V(r)$ in above derived equations for electron.

From experiments on lepton-proton elastic scattering (see [2] and references therein), interaction of leptons and protons has the following form:

$$M = eF_V(q^2)\bar{p}\gamma_\alpha pe\gamma_\beta e$$

(27)

where formfactor $F_V(q^2) = \frac{M^2}{(q^2 + M^2)^2}$ ($M^2 = 0.71 GeV^2$) (see [2] and references therein) are described their charge distribution in the proton.

It mean that in momentum space the potential between electron and proton has the form $\frac{1}{q^2} \to \frac{1}{q^2}F_V(q^2)$.

$$V(r) = \alpha \left(\frac{1}{r} - \frac{\exp(-Mr)}{r} - \frac{M}{2}\exp(-Mr)\right)$$

(28)
Also exist potential connected with neutron formfactor:

\[ M = eF_n(q^2)\bar{p}\gamma_\alpha p e\gamma_\beta e \]  \hspace{1cm} (29)

From this formula we obtain the following short-range potential:

\[ V(r) = \int \exp i\vec{q}\vec{r} \frac{\alpha}{q^2} F_n(q^2) \]  \hspace{1cm} (30)

which decrease as \( \sim \exp(-Mr) \) at large \( r \). where \( F_n(0) = 0 \).

If we suppose e.g. that the formfactor has the form \( F_n(t) = \frac{t^2}{(t+M^2)^2} \) (which correspond to the density \( \rho(r) = \delta(r) - M^2 \frac{\exp(-Mr)}{r} \)) we obtain;

\[ V(r) = -\alpha \left( \frac{\exp(-Mr)}{r} - \frac{M}{2} \exp(-Mr) \right) \]  \hspace{1cm} (31)

Thus we have singular potential \( \sim r^{-n} \) at \( r \gg r_n \).At \( r < r_n \ll \frac{1}{m_e} \) singular behaviour is absent. It mean the presence of deep levels of \( e^\pm n \) bound states. In our next paper we will calculate it numerically.

In case of neutron the effect of neutron polarization is of order \( \sim \frac{\alpha^2}{R} \) is essentially smaller than spin-spin and spin-orbital interaction \( \sim \frac{\alpha}{m_e M R^3} \). However in composite scalar-charged particle bound states (see below) spin-spin and spin-orbital interaction are absent and attraction by polarization is important.

We would like also to stress that the potential is the sum of the potential from polarization+formfactors effect (formula (1) above)+ of spin-spin interaction and from photon-\( \rho \)-meson mixing.

It must be noted also exist long-range potential induced by \( \pi^0\gamma\gamma \) vertex:

\[ V(q) = e^2 g_{\pi nn} \frac{1}{q^2 + m_{\pi}^2} \vec{E}\vec{H} \]  \hspace{1cm} (32)

where \( \vec{E}, \vec{H} \) are fields of the lepton.
Appendix B: $Z^0 - \gamma$ and rho-meson -photon mixing

We consider also potential which appear due to $Z^0 - \gamma$ mixing via electron-positron loop. We obtain the following result for electromagnetic field generated by $Z - \gamma$ mixing (only P-even part) (which act on $Q_{1,2}$):

$$eQ_{1,2}A(r) = \frac{e^2 Q_{1,2} g_{2,1} (-\frac{1}{4} + \sin \theta_W^2)}{\cos^2 \theta_W} \frac{1}{(2\pi)^5} \frac{4\pi^2}{3r} \int_1^\infty dx F(x) \left( \frac{M^2 \exp(-Mr) - 4m_e^2 x^2 \exp(-2m_e r x)}{M^2 - 4m_e^2 x^2} \right)$$

where $g_{2,1} = (\frac{1}{2} T_{2,1} - Q_2 \sin \theta_W^2)$ P-odd part of $Z - \gamma$ mixing also give long range P-odd forces.

At $r >> \frac{1}{m_e}$ integral is suppressed by exponent $\exp(-2m_e r)$ while at $\frac{1}{M} << r << \frac{1}{m_e}$ we have:

$$eQ_{1,2}A(r) = \frac{4\pi^2}{3(2\pi)^5} \frac{e^2 Q_{1,2} g_{2,1} (-\frac{1}{4} + \sin \theta_W^2)}{\cos^2 \theta_W} \frac{1}{M^2 r^3}$$

(34)

At $r << \frac{1}{M}$ we obtain the behaviour $\sim (\frac{1}{r} + O(\alpha ln(m_e r)))$.

This results are also applicable for potential which appear due to $\rho^0 - \text{meson} - \gamma$ mixing via electron-positron loop. At large distances ($\frac{1}{M} << r << \frac{1}{m_e}$) this potential also fall as $\sim r^{-3}$. We can obtain the potential from $\rho^0 - \text{meson} - \gamma$ mixing taking into account that $\rho^0 e^+ e^-$ effective lagrangian may be written from $\rho^0 \rightarrow e^+ e^-$ decays by the following way:

$$L = \sqrt{\frac{24\pi \Gamma(\rho^0)}{M_\rho}} \bar{e} \hat{\rho} e$$

(35)

(this consideration is true at $r > \frac{1}{M_\rho}$) and besides interaction with nucleons is following:

$$L = g_n \bar{n} \hat{\rho} n$$

(36)
This potential may be found from above presented formulas by the following substitution:

\[
\left( \frac{1}{2} T_{2,1} - Q_2 \sin \theta_W^2 \right) \left( -\frac{1}{4} + \sin \theta_W^2 \right) \frac{1}{\cos^2 \theta_W} \rightarrow g_n \sqrt{\frac{24\pi \Gamma(\rho^0)}{M_\rho}} \tag{37}
\]

**Appendix C: Bound States of Particles with anomalous magnetic moment (Cylindrical symmetry case)**

Below we consider cylindrically symmetric bound states and resonances of particles with anomalous magnetic moments. It is of interest to consider several special cases in particular:

1) Particle energy levels in pure electric axial field case \( (\vec{E} = (E_r(r), 0, 0)) \) which created e.g. by homogeneously charged cylinder.

In accordance with [4] we have:

\[
E_r = \frac{2\sigma}{r} \quad \text{at} \quad r > R \tag{38}
\]

and

\[
E_r = \frac{2\sigma r}{R^2} \quad \text{at} \quad r < R \tag{39}
\]

where \( \sigma \) is density of charge of the unit of length of the cylinder.

2) Particle energy levels in pure magnetic field case \( (\vec{H} = (0, H(r), 0)) \) which created e.g. by homogeneously charged cylinder.

In accordance with [1] we have:

\[
H_\phi = \frac{2I}{r} \quad \text{at} \quad r > R \tag{40}
\]

and

\[
H_\phi = \frac{2Ir}{R^2} \quad \text{at} \quad r < R \tag{41}
\]

where \( I \) is the current in the cylinder.
Also we consider particle energy levels in the field of solenoid ($\vec{H} = (0, 0, H_z(r)), \ H_z(r) = \theta(R - r)$.)

Analogously may be considered situation in which both electric and magnetic fields are presented. It may be for example electric and magnetic fields created by charged particles beams.

A. Pure electric field case

The Dirac equation for particle with anomalous magnetic moments in pure electric axial field case takes the form [1]:

$$\left(\hat{k} - m_n + \mu(-i\vec{E})\right)\psi(k) = 0,$$

where $\mu$-is anomalous magnetic moment, and $\vec{E} = \vec{n}G(r) \ (\vec{n} = \left(\frac{x}{r}, \frac{y}{r}\right), r = \sqrt{x^2 + y^2})$. We find the solution in the form $\psi_{1,3} = e^{i\phi(l-1)}e^{ip_z} f_{1,3}(r), \psi_{2,4} = e^{i\phi}e^{ip_z} f_{2,4}(r)$ and in component form we obtain the following system of equations:

\[
\begin{align*}
(\epsilon - m)f_1 - p_3 f_3 + Q_- f_4 &= 0 \\
(\epsilon - m)f_2 + Q_+ f_3 + p_3 f_4 &= 0 \\
p_3 f_1 + P_- f_2 - (\epsilon + m)f_3 &= 0 \\
P_+ f_1 - p_3 f_2 - (\epsilon + m)f_4 &= 0
\end{align*}
\]

where

$$Q_- = i\left(\frac{d}{dr} + \frac{l}{r} - \mu E\right)$$

$$Q_+ = i\left(\frac{d}{dr} - \frac{l - 1}{r} - \mu E\right)$$

$$P_- = -i\left(\frac{d}{dr} + \frac{l}{r} + \mu E\right)$$

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\[ P_+ = -i \left( \frac{d}{dr} - \frac{l - 1}{r} + \mu E \right) \]

At small \( r \) we find the solution in the form:

\[ f_i(r) = a_i r^s, \] (51)

Substituting (51) into (43)-(50) we obtain:

\[
(\epsilon + m)(s^2 - l^2 - 4\sigma^2 \mu^2 + 2\mu \sigma (2s+1)) a_1 - p_z (-s^2 - l^2 - 4\sigma^2 \mu^2 - 4\mu \sigma (l+s-1)) a_2 = 0
\] (52)

\[
(\epsilon - m)(s^2 - l^2 - 4\sigma^2 \mu^2 - 4\mu \sigma s) a_2 + p_z (-s^2 - l^2 - 4\sigma^2 \mu^2 + 4\mu \sigma s) a_2 = 0
\] (53)

The determinant of this system of equations must be equal zero, and that condition gives us the equation for defining \( s \).

B. Pure magnetic field case \( \vec{H} = (0,0,H) \)

The Dirac equation for particle with anomalous magnetic moments in pure magnetic field \( \vec{H} = (0,0,H) \) case takes the form [1]:

\[ (\hat{k} - m_n + \mu \vec{\sigma} H) \psi(k) = 0, \] (54)

In component form we obtain the following system of equations:

\[
(\epsilon - m + \mu H) f_1 - p_3 f_3 - p_- f_4 = 0
\] (55)

\[
(\epsilon - m - \mu H) f_2 - p_+ f_3 + p_3 f_4 = 0
\] (56)

\[
p_3 f_1 + p_- f_2 + (-\epsilon - m + \mu H) f_3 = 0
\] (57)

\[
p_+ f_1 - p_3 f_2 + (-\epsilon - m - \mu H) f_4 = 0
\] (58)

where
\[ p_- = -i\left(\frac{d}{dr} + \frac{l}{r}\right) \quad (59) \]
\[ p_+ = -i\left(\frac{d}{dr} - \frac{l - 1}{r}\right) \quad (60) \]

In case of solenoid homogeneous magnetic field exist only inside cylinder and we obtain:

\[
[(\epsilon^2 - p^2 - (m + \mu H)^2)(\epsilon^2 - p^2 - (m + \mu H)^2) - 4\mu^2 H^2(p^2 - p_z^2)]\psi_4 = 0 \quad (61)
\]

where \( p^2 = p_z^2 - \frac{1}{r} \frac{d}{dr} r \frac{d}{dr} + \frac{l^2}{r} \). At \( r > R \) we have:

\[
\psi_{r>R} = CK_{||} (\sqrt{\epsilon^2 - p_z^2 - m^2 r}) \quad (62)
\]

whereas at \( r < R \):

\[
\psi_{r<R} = C_{1,2} J_{||} ((\mu^2 H^2 - m^2 + \epsilon^2 - p_z^2 \pm 2\sqrt{\epsilon^2 - p_z^2 - m^2})r) \quad (63)
\]

Energy levels are obtained from the condition:

\[
\frac{\psi_{r<R}}{\psi_{r>R}}|_{r=R} = \frac{\psi'_{r>R}}{\psi_{r>R}}|_{r=R} \quad (64)
\]

Besides considered above solenoid it is of interest to consider also magnetic field created by current inside cylinder (\( H = \frac{2I}{r} \) at \( r > R \)). Now this consideration is in progress.

C. Electric and magnetic field case (\( \vec{H} = (0, 0, H_z(r)) \))

If both electric and magnetic fields are presented we obtain the following system of equations:

\[
(\epsilon - m + \mu H)\psi_1 - p_3 \psi_3 + Q_- \psi_4 = 0 \quad (65)
\]
\[
(\epsilon - m - \mu H)\psi_2 + Q_+ \psi_3 + p_3 \psi_4 = 0 \quad (66)
\]
\[ p_3 \psi_1 + P_- \psi_2 - (\epsilon + m - \mu H) \psi_3 = 0 \]  (67)

\[ P_+ \psi_1 - p_3 \psi_2 - (\epsilon + m + \mu H) \psi_4 = 0 \]  (68)

D. Electric and magnetic fields case \((\vec{H} = (0, H_\phi, 0))\)

The system of equation is following:

\[ (\epsilon - m)f_1 - i\mu H f_2 - p_3 f_3 - (p_- - i\mu E)f_4 = 0 \]  (69)

\[ i\mu H f_1 + (\epsilon - m)f_2 - (p_+ - i\mu E)f_3 + p_3 f_4 = 0 \]  (70)

\[ p_3 f_1 + (p_- - i\mu E)f_2 + (-\epsilon - m)f_3 - i\mu H f_4 = 0 \]  (71)

\[ (p_+ - i\mu E)f_1 - p_3 f_2 + i\mu H f_3 + (-\epsilon - m)f_4 = 0 \]  (72)

E. Electric charge and magnetic moment joint consideration

Previously we consider neutral particles with anomalous neutral moment. If we take into account also charge we obtain the following system of equations:

\[ A_1 f_2 + A_2 f_4 = 0 \]  (73)

\[ A_3 f_2 + A_4 f_4 = 0 \]  (74)

where:

\[ A_1 = \frac{d}{dr} \left( \frac{(\epsilon - V + m)}{(l - 1)} \rho_0 + (\epsilon - V + m)(\Omega_0 - \mu^2 E^2 + 2\mu E \frac{d}{dr} + \mu E \frac{1}{r}) \right) \]  (75)

\[ A_2 = p_2 \left( \Omega_0 + \mu^2 E^2 + 2\mu E \frac{d}{dr} + \mu E' + \mu E' \left( \frac{2l - 1}{r} + \frac{dk^{-1}}{dr} \right) \right) \]  (76)
\[ A_3 = (-\epsilon + V + m)(\Omega_0 - \mu^2 E^2 - 2\mu E \frac{d}{dr} - \mu E' - \mu E \frac{1}{r}) - k \frac{d}{dr}(\epsilon - m + V) \left( \frac{1}{dr} + \frac{l - 1}{r} \right) + \mu E \] (77)

\[ A_4 = p_z (-\Omega_0 - \mu^2 E^2 + 2\mu E \frac{d}{dr} + \mu E' + \mu E \frac{1}{r} + \frac{1}{2} \frac{d\epsilon}{dr} - \mu E \left( \frac{d}{dr} + \frac{l - 1}{r} \right) - \mu E \] (78)

If charge is zero (i.e. at \( V = 0 \)) we obtain previous formulas (43)-(50).

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