Impact of a Binary System Common Envelope on Mass Transfer through the Inner Lagrange Point

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ABSTRACT

Results of numerical simulations of the impact of a common envelope on the matter flow pattern near the outflowing component in a semidetached binary system are presented. Three-dimensional modeling of the matter transfer gas dynamics in a low-mass X-ray binary X1822–371 enable investigation of the structure of flows in the vicinity of the inner Lagrange point $L_1$. Taking into account the common envelope of the system substantially changes the flow pattern near the Roche surface of the outflowing component. In a stationary regime, accretion of common envelope gas is observed over a significant fraction of the donor star’s surface, which inhibits the flow of gas along the Roche surface to $L_1$. The change in the flow pattern is particularly significant near $L_1$, where the stream of common envelope gas strips matter off the stellar surface. This, in turn, significantly increases (by an order of magnitude) the gas flow from the donor surface in comparison with the estimates of standard models.

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1. Introduction

The observational manifestations of interaction in semidetached binary systems (cataclysmic binaries, low-mass X-ray binaries, and supersoft X-ray sources) are extremely interesting. Comparisons of computational results from numerical simulations of matter flows in these systems with observational data provide a basis for studying and understanding the physical processes occurring in these binaries. As a rule, numerical simulations of semidetached systems have been performed using the generally well-justified assumptions that the orbits of the system components are circular and the components’ rotation is synchronous with their orbital motion. In this case, the standard scenario for mass transfer in the system is as follows. In the course of its evolution, the donor star fills its critical surface (which, under the outlined assumptions, coincides with the Roche surface in the restricted three-body problem), and mass begins to flow through the vicinity of the inner Lagrange point $L_1$, where the pressure gradient is not balanced by the gravitation force.

The flow pattern in semidetached systems has been investigated using both analytical [1–3] and numerical [4–15] models. However, in all these studies, the impact of the common envelope on the structure of gaseous flows was either not taken into account at all, or was taken into account not entirely correctly. The morphology of flows in binary systems allowing for the presence of a common envelope was first considered by us in [16] (henceforth, Paper I). There, as well as in this paper, the “common envelope” of the system refers to the gas filling the space between the two components of the system and not involved in the accretion process (i.e., not belonging to the accretion disc). Our results of three-dimensional numerical simulations of a low-mass X-ray binary X1822–371 in Paper I showed the contribution of the common envelope to the formation of flow structures in the system to be significant.

The presence of a common envelope also influences the flow pattern in the vicinity of the donor star, which, in turn, is reflected in the mass exchange rate in the system. This problem is extremely important, because though the observational manifestations of interaction primarily depend on the general flow pattern studied in Paper I, the evolution of the system is determined by the mass exchange parameters. Here, we present the results of an analysis based on the calculations described in Paper I, but with the main accent on a detailed consideration of the influence of the common envelope on the structure of gas flows near the Roche surface of the outflowing component.

2. Flow Structure in the Vicinity of $L_1$. Results of Standard Models

Analytical studies of the mass transfer process in semidetached systems make it possible to draw conclusions concerning the parameters of flows of matter (streams) in such systems. The first detailed analysis of matter flows in the vicinity of $L_1$ was carried out by Lubow and Shu [1], who estimated the basic characteristics of the stream using a perturbation method. In later papers, the stream parameters were refined using Bernoulli integral analysis, and the dependence of the mass transfer rate on the degree of overflow of the Roche lobe by the donor star was found [2,3]. The basic characteristics of the streams obtained in various studies (including numerical calculations, e.g., [17]) differ only in details, and are currently widely used as standard results in analyses of mass transfer in semidetached binary systems (see, e.g., the reviews [18, 19]). Unfortunately, none of these studies took into account the impact of the common envelope of the system on the flow pattern near the donor star. The aim of our study is to consider the contribution of the common envelope and to obtain new stream parameter estimates using the results of three-dimensional numerical simulations of the flow in a low-mass X-ray binary system (these will be presented in Section 3). However, when analyzing changes in the flow pattern associated with the presence of a common envelope, it is expedient to first consider the generally accepted values for flow parameters in semidetached systems.

We will first define the reference frame for our analysis. We place the origin of the coordinate frame at the donor star center of mass. In our Cartesian coordinate system $(x, y, z)$, we will direct the $X$ axis along the line connecting the centers of the stars, the $Z$ axis along the rotation axis, and the $Y$ axis so that the resulting system is right-handed. We will set the characteristic scale of the coordinate system by placing the secondary star at the point $(A, 0, 0)$, where $A$ is the distance between the two components. This means that the center of mass of the system is at the point $(\mu A, 0, 0)$, where $\mu = M_2 / (M_1 + M_2)$ is the ratio of the accretor’s mass to the total mass of the system.
It is assumed in standard analytical models that the matter at the surface of the donor star (which coincides with the Roche lobe) is characterized by some temperature (or local sound speed $c_0$) and density. The kinetic energy of this matter ($\sim c_0^2$) determines the degree of overflow of the Roche lobe. The local sound speed of the gas for typical semidetached binaries is much less than the orbital velocity of the system components $v_{orb} \sim \Omega A$, and their ratio is, as a rule, used as a small parameter in analyses. Proceeding from consideration of the energetic conditions of the gas at the Roche surface, it is not difficult to estimate the characteristic dimensions of the matter stream that forms in the vicinity of $L_1$. The store of kinetic energy of the gas allows it to move in the $YZ$ plane passing through $L_1$. Equating the potential energy difference and the specific kinetic energy, we can derive an equation for the shape of the stream in the vicinity of $L_1$, similar to the corresponding equation from [1]. Following the adopted scheme for determining the $YZ$ cross section of the stream passing through $L_1$, we can write:

$$\Phi - \Phi_{(x_1,0,0)} = c_0^2,$$  

(1)

where $\Phi$ is the force field potential in a reference frame rotating with the angular velocity $\Omega$ of the system’s rotation about the common center of mass, in the absence of other external forces, excluding gravity. The potential in the Roche approximation has the form:

$$\Phi(x,y,z) = -\frac{GM_1}{\sqrt{x^2 + y^2 + z^2}} - \frac{GM_2}{\sqrt{(x-A)^2 + y^2 + z^2}} - \frac{\Omega^2}{2} ((x-\mu A)^2 + y^2),$$

where the term $-\frac{\Omega^2}{2} ((x-\mu A)^2 + y^2)$ is associated with the centrifugal force.

Expanding expression (1) into a Taylor series in $y$ and $z$ in the vicinity of $L_1$ and using the condition

$$\text{grad } \Phi|_{(x_1,0,0)} = 0,$$

it is not difficult to show that the stream cross section is an ellipse with semiaxes

$$a = \sqrt{g_y(\mu)\frac{c_0}{\Omega}} \sim \frac{c_0}{\Omega},$$

(2)

and

$$b = \sqrt{g_z(\mu)\frac{c_0}{\Omega}} \sim \frac{c_0}{\Omega},$$

(3)

where $g_y(\mu) \sim 1$ and $g_z(\mu) \sim 1$ are the coefficients of the expansion of the Roche potential in the Taylor series in $y$ and $z$ in the vicinity of $L_1$, determined by the relation:

$$\Phi = \Phi_{L_1} + g_y(\mu)\Omega^2 y^2 + g_z(\mu)\Omega^2 z^2.$$  

Apart from considering the stream dimensions at the point $L_1$, it is also of interest to determine its deflection as a result of the Coriolis force. According to the analysis of Lubov and Shu [1], the stream deflection angle is

$$\cos(2\theta_s) = -\frac{4}{3g_\theta} + \left(1 - \frac{8}{9g_\theta}\right)^{1/2},$$

where

$$g_\theta(\mu) = \frac{\mu}{(x_{L_1}/A)^3} + \frac{1 - \mu}{(1 - x_{L_1}/A)^3}.$$  

The rotation angle of the stream depends on the mass ratio of the components, and, in the simplest case (without taking into account the stream’s interaction with the common envelope gas), it lies in the range from $-28.37^\circ$ to $-19.55^\circ$.

In more thorough analytical models (see, e.g. [2,3]), the Bernoulli integral for the matter leaving the donor star surface is used to determine the stream parameters. The conservation of this integral along a stream line,

$$\Phi + \frac{u^2}{2} + \frac{c^2}{\gamma - 1} = \Phi_{L_1} + \frac{u_0^2}{2} + \frac{c_0^2}{\gamma - 1},$$

and the assumption that the gas flow velocity is equal to the sound speed (the Mach number is 1) can be used to derive both the stream dimensions and the cross-sectional distribution of parameters across the stream. The stream dimensions obtained in this approximation differ from (2) and (3) by a factor of $\sqrt{\frac{1}{\gamma - 1}}$, however, analysis of the density distribution across the stream shows that the stream profile consists of a dense core whose characteristic dimensions
are close to expressions (2) and (3) and a rarefied peripheral region.

The dimensions of the stream are of fundamental importance for determining the characteristics of the stream of matter flowing from the vicinity of $L_1$. The values for the total mass flow from the system in various approaches are essentially equal (within a factor of two), and can be determined from the relation

$$\dot{M} = \rho_0 c_0 S,$$  \hspace{1cm} (4)

where $\rho_0$ and $c_0$ are the gas density and sound speed at the surface of the outflowing star and

$$S = \pi a b = \pi \sqrt{g_\mu g_\rho} \left(\frac{c_0^2}{\Omega^2}\right) \sim \frac{c_0^2}{\Omega^2}$$

is the area of the stream cross section. Since for a polytropic gas $\rho = \left(\frac{c_0^2}{\gamma R}\right)^{\frac{1}{\gamma-1}}$, the final expression for the dependence of the mass flow on $c_0$ is:

$$\dot{M} \sim c_0^{3+\frac{2}{\gamma}}.$$  \hspace{1cm}

If we transform the "energetic" overflow of the Roche lobe to "geometrical" overflow using the formula

$$c_0^2 = \Delta \Phi = \frac{GM_1}{R_1^2} \Delta R,$$

($R_1$ is the effective radius of the donor star; $\Delta R = R_1 - R_{L_1}$, where $R_{L_1}$ is the radius of a sphere with volume equal to the volume of the Roche lobe), then for $\gamma = 5/3$, we obtain for the mass flow the widely-used expression

$$\dot{M} \sim (\Delta R)^3.$$

These expressions for the mass flow into the system were derived in the framework of simplified models. Their common and, obviously most serious, disadvantage is the lack of allowance for the system’s common envelope, which appreciably influences the flow pattern along the Roche surface and, correspondingly, the mass exchange parameters in the system. This effect has unfortunately previously been neglected, due to the fact that a correct study of this phenomenon is possible only using three-dimensional numerical gas dynamics models.

3. Numerical Simulation Results

We base our analysis of the flow pattern near the donor star filling its Roche lobe on the results described in Paper I. The formulation of the problem is laid out in detail in Paper I, so that we repeat here only the basic features of the model, referring the reader to Paper I for details.

(1) We considered a low-mass X-ray binary similar to X1822–371 in which the mass of the outflowing component $M_1 = 0.28 M_\odot$, the temperature of the gas on the surface of this component $T = 10^4 K$, the mass of the secondary (a compact object with radius $0.05R_\odot$) $- M_2 = 1.4M_\odot$, the system orbital period $P_{orb} = 1.78^d$, and the distance between the component centers $A = 7.35R_\odot$.

(2) A three-dimensional system of gas dynamics equations closed by the ideal gas equation of state was used to describe the flow.

(3) To take into account radiative losses in the system, we took the adiabatic index to be close to unity, namely $\gamma = 1.01$, which is close to the isothermal value [20].

(4) We assumed that the outflowing star fills its Roche lobe and that the velocity of the gas at the surface is directed normal to the surface. We took the gas velocity to be equal to the local sound speed $v_0 = c_0 = 9km/s$. The density at the surface of the outflowing component $\rho$ was taken to be $\rho_0$. Note that the boundary value of the density does not affect the solution, due to the scaling of the system of equations in $\rho$ and $P$. We used an arbitrary value for $\rho_0$ in the calculations; when a specific system with a known rate of mass loss is considered, the real densities in the system can be determined simply by adjusting the calculated density values using the scale indicated by the ratio of the real and simulated densities at the surface of the outflowing component.

(5) We adopted the conditions of free mass outflow both at the accretor and at the outer boundary of the calculation region.

(6) The calculated region was the parallelepiped $(-A..2A) \times (-A..A) \times (0..A)$; by virtue of the problem’s symmetry relative to the equatorial plane, the calculations were performed only for the upper half-space.

(7) We used a high-order Total Variation Diminishing scheme to solve the system of equations on a non-uniform (finer along the line connecting the com
ponent centers) difference grid with $78 \times 69 \times 35$ nodes.

The system of equations was solved starting from arbitrarily chosen initial conditions up to the establishment of a stationary flow regime. To ensure that this flow regime was established, we continued the calculations over a rather long time interval (exceeding ten orbital periods) after the initiation of the stationary regime.

The overall flow pattern in the equatorial plane of the system considered is presented in Fig. 1, which depicts density and velocity field contours in the area from $-5$ to $13R_\odot$ along the $X$ axis and from $-6$ to $6R_\odot$ along the $Y$ axis. Figure 1 also shows four stream lines (marked with the letters A, B, C, D), which illustrate the directions of flows in the system. Analysis of the results in Fig. 1 shows that a significant fraction of the common envelope gas (stream lines A and B) approaches the surface of the outflowing star (Roche lobe) in the course of its motion, and is accreted by this surface, preventing the formation of flows along the stellar surface. Some of the envelope material (stream lines C and D) approaches the donor star and, stripping gas off its surface, participates in the formation of the stream in the vicinity of $L_1$.

Details of the flow pattern in the vicinity of the inner Lagrange point are shown in Fig. 2, which shows the same flow parameters as in Fig. 1 in a small region of the equatorial plane from -1 to $5R_\odot$ along the $X$ axis and from -2 to $2R_\odot$ along the $Y$ axis. We can see from Fig. 2 that the common envelope gas substantially changes the flow structure in the vicinity of $L_1$, in particular, it strips matter off the surface of the donor star. The asymmetry of the action of the envelope gas on the outflowing stream is also visible in Fig. 2. Gas flowing in from above, along the direction of the orbital motion, is accreted by the donor star, and only in a small region near $L_1$ is this matter taken up from the surface and transported to the stream. The envelope gas, approaching to $L_1$ from below, strips matter off a significant fraction of the donor star surface.

This effect of "stripping" matter off the surface of the donor star substantially changes the generally accepted views of the formation of the matter stream flowing into the system and of the corresponding mass exchange parameters. According to the standard models, the structure of the atmosphere near the surface of the donor star is determined by the equations of hydrostatic equilibrium (see, e.g., [1, 18]). For the adopted system parameters and gas temperature

Fig. 1.— Density isolines and velocity vectors in the equatorial plane of the system. Roche equipotentials are shown with dashed lines. Four stream lines marked with the letters A, B, C, D are also shown, illustrating directions of the flow of the common envelope material. The location of the accretor is marked with the dark circle. The vector in the upper right corner corresponds to a velocity of 800 km/s.

Fig. 2.— Density isolines and velocity vectors in the equatorial plane of the system in the vicinity of the inner Lagrange point. Roche equipotentials are shown with dashed lines. The vector in the upper right corner corresponds to a velocity of 300 km/s.
(sound speed) at the donor star surface, the gas energy is not large enough for the gas to escape directly from the stellar surface, so that the mass flow associated with thermal escape will be negligible small compared to the flow through the vicinity of $L_1$. Gas located in the near-surface layer (with characteristic height of the order of the atmospheric scale height) can flow along the stellar surface, but, in this case, also, the total mass flow into the system does not change significantly [1].

The situation fundamentally changes only when the system’s common envelope is taken into account, because, in this case, gas from the surface layer can be "stripped" from the star and carried into the system. Our calculations have shown that this is the flow regime that is realized in the system considered here. The momentum of the rarefied common envelope gas along the surface of the star is large enough to pick up matter from the surface layer over a significant fraction of the stellar surface. Further, this material and the gas flowing out from the vicinity of $L_1$ form the stream of matter flowing into the system, and also determine the total mass flow.

The effect of the envelope gas on the flow structure near $L_1$ is especially visible in Fig. 3, which shows a three-dimensional projection of stream lines originating on the stellar surface and entering the system with the gas stream. The area containing the points through which the stream lines enter the system represents that part of the surface providing mass exchange in the system. We can clearly see the asymmetry of this area in Fig. 4, which also shows stream lines leaving the stellar surface (also in 3-D projection), but for an observer located on the line joining the component centers. The size of this area substantially exceeds the size of the stream in standard models (shown by the ellipses near $L_1$ in Figs. 3 and 4). Analysis of these results indicates that the stream forms in a region is more than order of magnitude larger than predicted by theoretical estimates obtained without allowance for the common envelope and with the same values for the gas parameters at the donor star surface. The matter flow injected into the system can now be represented as the sum of two terms, and, accordingly, we obtain instead of (4)

$$\dot{M} = \dot{M}_{L_1} + \dot{M}_{E_nvel},$$

where the first term, derived from standard estimates, is an order of magnitude smaller than the second.
term, determined by the common envelope gas.

![Fig. 5a.— Density isolines and velocity vectors in the stream section formed by the YZ plane passing through the inner Lagrange point. The stream cross-section obtained in standard models is shown by the bold ellipse. The vector in the upper right corner corresponds to a velocity of 200 km/s. The maximum density (at the stream center) is $1.2\rho_0$, and the minimum density (outer contour) is $0.01\rho_0$.](image)

We will now consider the distributions of gas parameters across the stream cross section. Figure 5 shows density and velocity vector contours in the stream cross section formed by the YZ plane passing through $L_1$. The maximum density (in the center) exceeds the density at the stellar surface $\rho_0$ by a factor of 1.2; the minimum density, depicted by the outer contour, corresponds to $0.01\rho_0$. We can see from Fig. 5 that the matter stripped off the stellar surface by the common envelope gas collects in a small zone centered on the equatorial plane. The center of this zone, which is determined by the common envelope gas flows considered above, does not coincide with $L_1$ (the point $(0,0)$ in the figure); moreover, it lies beyond the matter stream inferred from standard models (shown by the ellipse near $L_1$ in Fig. 5). The two-humped nature of the density distribution can be seen particularly well in Fig. 6, which shows the density distribution (normalized to its value at the stellar surface) along the $Y$ axis for the section of the stream under consideration. The stream boundaries obtained in standard models are shown in the figure by the vertical lines. The effects described above can be interpreted as the presence of two well-defined streams in the system, one associated with the gas flowing from $L_1$, and the other forming under the action of the common envelope gas. Further analysis of the calculation results indicates that the distributions of gas parameters across the stream change with distance from the inner Lagrange point, and the stream becomes a single flow at a distance of the order of $0.1R_\odot$ ($0.013A$) from $L_1$. This picture is supported by the results in Fig. 6, which shows density contours in the $YZ$ cross section of the stream $0.1R_\odot$ from the inner Lagrangian point. The maximum density in Fig. 6 is $1.4\rho_0$, and the minimum (outer contour) is $0.01\rho_0$.

4. Conclusion

Analysis of the three-dimensional numerical simulation results presented here indicates that taking into account the impact of the common envelope in semidetached binary systems radically changes the mass exchange parameters obtained. For the low-mass X-ray binary X1822–371 that we have considered, the overall mass inflow to the system increased
Fig. 6.— Density isolines and velocity vectors in the stream section formed by the YZ plane passing 0.1\(R_\odot\) from the inner Lagrange point. The vector in the upper right corner corresponds to a velocity of 200 km/s. The maximum density (at the stream center) is 1.4\(\rho_0\), and the minimum density (outer contour) is 0.01\(\rho_0\).

by a factor of about 50 compared to the estimates of standard models for the same gas parameters values at the surface of the donor star. In addition, the common envelope gas changed the flow pattern near the surface of the outflowing component, which ultimately affected the overall structure of the gas flows in the system, and, thus, the expected observational manifestations of these flows.

Unfortunately, the quantitative estimates we have obtained are valid only for the specific object considered, and do not allow us to draw more general conclusions about changes in the mass exchange parameters expected for other semidetached systems. This is due to the fact that the mass flow increase depends not only on the parameters of the binary system, but also on the properties of the common envelope gas, which can only be determined using three-dimensional numerical simulations. Nevertheless, given the results presented here, it is clear that an adequate description of the mass flows in any semidetached system with a common envelope is possible only in models that take into consideration the influence of this envelope.

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REFERENCES

[1] Lubow, S.H., Shu, F.H., 1975, ApJ, 198, 383
[2] Paczyński, B., Sienkiewicz, R., 1972, Acta Astron., 22, 73
[3] Savonije, G.L., 1978, A&A, 62, 317
[4] Warner, B., Peters, W.L., 1972, MNRAS, 160, 15
[5] Flannery, B., 1975, MNRAS, 170, 325
[6] Sawada, K., Matsuda, T., Hachisu, I., 1986, MNRAS, 219, 75
[7] Sawada, K., Matsuda, T., Inoue, M., Hachisu, I., 1987, MNRAS, 224, 307
[8] Taam, R.E., Fu, A., Fryxell, B.A., 1991, ApJ, 371, 696
[9] Molteni, D., Belvedere, G., Lanzafame, G., 1991, MNRAS, 249, 748
[10] Sawada, K., Matsuda, T., 1992, MNRAS, 255, 17
[11] Lanzafame, G., Belvedere, G., Molteni, D., 1992, MNRAS, 258, 152
[12] Belvedere, G., Lanzafame, G., Molteni, D., 1993, A&A, 280, 525
[13] Lanzafame, G., Belvedere, G., Molteni, D., 1994, MNRAS, 267, 312
[14] Blondin, J.M., Richards, M.T., Malinowski, M.L., 1995, ApJ, 445, 939
[15] Armitage, P.J., Livio, M., 1996, ApJ, 470, 1024
[16] Bisikalo, D.V., Boyarchuk, A.A., Kuznetsov, O.A., Chechetkin, V.M., 1997, Astron. Reports, 41, 786 (Paper I; also available as [astro-ph/9802004](http://arxiv.org/abs/astro-ph/9802004))
[17] Nazarenko, V.V., 1993, Astron. Zh., 70, 101
[18] Pringle, J.E., Wade, R.A. (eds) 1985, Interacting Binary Stars. Cambridge Univ. Press, Cambridge
[19] Livio, M., 1992, Topics in the theory of cataclysmic variables and X-ray binaries, STScI Preprint Series, No. 659.
[20] Landau, L.D., Lifshitz, E.M., 1959, Fluid Mechanics. Pergamon, Elmsford

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