Gauge parameter dependence and gauge invariance in the Abelian Higgs model

Rainer Häußling

Institut für Theoretische Physik und
Naturwissenschaftlich-Theoretisches Zentrum, Universität Leipzig
Augustusplatz 10/11, D-04109 Leipzig, Germany

and

Elisabeth Kraus

Physikalisches Institut, Universität Bonn
Nußallee 12, D-53115 Bonn, Germany

Abstract

We analyze gauge parameter dependence by using an algebraic method which relates the gauge parameter dependence of Green functions to an enlarged Slavnov-Taylor identity. In the course of the renormalization it turns out that gauge parameter dependence of physical parameters is already restricted at the level of Green functions. In a first step we consider the on-shell conditions which we find to be in complete agreement with these restrictions to all orders of perturbation theory. The fixing of the coupling, however, is much more involved outside the complete on-shell scheme. In the Abelian Higgs model we prove that this fixing can be properly chosen by requiring the Ward identity of gauge invariance to hold in its tree form to all orders of perturbation theory.

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1. Introduction

In the perturbative formulation of gauge theories one has to destroy gauge symmetry by fixing the gauge. Thereby arbitrary gauge parameters are introduced into the action which have to be proven to cancel when physical observables are concerned. The Green functions, however, depend non-trivially on these gauge parameters. Already in the early days of non-abelian gauge theories it has been suggested that the S-matrix is a gauge parameter independent quantity \[1\]. After having established the renormalization of gauge theories via the BRS symmetry gauge parameter independence of the S-matrix has been strictly proven in \[2\] for such gauge theories which are completely broken by the spontaneous symmetry breaking mechanism and which therefore do not contain any massless particles. The ingredients of this proof are the complete on-shell scheme fixing the poles and the residua of the particles at the physical masses and a specific on-shell normalization condition for the transversal vector-vector-Higgs vertex. In order to prove gauge parameter independence of the S-matrix within this special set of normalization conditions heavy technical tools as the Wilson operator product expansion are required.

In pure gauge theories with massless gauge bosons the situation is much more involved because the S-matrix cannot be simply constructed due to infrared divergencies. For these theories it has been derived among other things that the $\beta$-functions are independent of the gauge parameter. In this context, the original proof explicitly refers to an invariant scheme, namely the minimal subtraction scheme of dimensional regularization \[3\].

Aiming at the standard model such considerations are unsatisfactory: Due to the masslessness of the photon a complete on-shell scheme, especially for the $W$-propagator, cannot be formulated in a strict sense, and an invariant scheme is not available due to the parity violating interactions in the fermion sector. Furthermore, in concrete calculations one is not only interested in the gauge parameter independence of the S-matrix but also in a knowledge, how the single Green functions depend on the gauge parameters. We therefore analyze gauge parameter dependence by an algebraic method \[4\], which can be generally applied in the context of Green functions. Although we eventually aim at the standard model we present the techniques for the Abelian Higgs model here as a natural first step.

The algebraic method enlarges the usual BRS transformations by varying the gauge parameter into a Grassmann variable. Constructing the Green functions in agreement with the enlarged BRS invariance automatically yields information about gauge parameter dependence. Moreover, it is seen that the physical parameters of the model have to be chosen gauge-parameter-independently in order to be able to prove gauge parameter independence of the S-matrix. For higher orders in perturbation theory these restrictions are extended to restrictions for the normalization conditions and it is seen that the conditions
of ref. \[2\] for spontaneously broken theories and the MS-scheme for symmetric theories \[3\] are just a special set of adequate normalization conditions (cf. sec. 5 and ref. \[4\], \[5\], where the respective analysis has been performed for pure gauge theories).

The paper is organized as follows: In section 2 we present the Abelian Higgs model and also summarize the results of ref. \[6\]. In section 3 the enlarged BRS transformations and Slavnov-Taylor identities are introduced, which give rise to the algebraic control of gauge parameter dependence. In section 4 the classical approximation is solved, thereby finding the restrictions for the physical parameters mentioned above. In section 5 these considerations are continued to higher orders. This is an application of the method and shows in an impressive way, how the algebraic method works in higher orders. Furthermore, one deduces in this context that the on-shell conditions are in complete agreement with the enlarged Slavnov-Taylor identity. In section 6 and 7 Ward identities of rigid and local symmetry are derived including the BRS-varying gauge parameter. As a consequence of the local Ward identity it turns out that gauge parameter dependence of the longitudinal part of the 3-point vertex, which describes the interaction between vector, Higgs and would-be Goldstone, is completely determined by the gauge dependence of the self energies of the scalars. Section 8 contains a summary of the results. In appendix A we give the general classical solution of the Abelian Higgs model including all the free parameters. As an illustration, in appendix B we sketch the diagrams contributing to the gauge parameter variation of the Higgs self energy.

2. The Abelian Higgs model

In order to set the general framework and to fix notation we first give a short exposition of the model treated in this paper.

The Abelian Higgs model contains a vector field $A_\mu$ and two scalar fields $\varphi = (\varphi_1, \varphi_2)$ interacting in such a way that $U(1)$ gauge invariance is broken spontaneously. It can be described by the following classical action (given in conventional normalization):

$$\Gamma_{inv} = \int \left\{ -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} + \frac{1}{2} (D_\mu \varphi)(D^\mu \varphi) - \frac{1}{8} \frac{m^2}{e^2} \left( \varphi_1^2 + 2 \frac{m}{e} \varphi_1 + \varphi_2^2 \right)^2 \right\}$$

(2.1)

with

$$F_{\mu\nu} \equiv \partial_\mu A_\nu - \partial_\nu A_\mu , \quad D_\mu \varphi_1 \equiv \partial_\mu \varphi_1 + eA_\mu \varphi_2 , \quad D_\mu \varphi_2 \equiv \partial_\mu \varphi_2 - eA_\mu (\varphi_1 + \frac{m}{e})$$

(2.2)

$\Gamma_{inv}$ is invariant under the $U(1)$ transformations

$$\delta_\omega A_\mu = \partial_\mu \omega , \quad \delta_\omega \varphi_1 = -e\omega \varphi_2 , \quad \delta_\omega \varphi_2 = e\omega (\varphi_1 + \frac{m}{e}) .$$

(2.3)
The shift \( \frac{m_e}{m} \) of the field \( \varphi_1 \) produces the mass \( m \) for the vector field \( A_\mu \) and \( \varphi_1 \) is the (physical) Higgs field with mass \( m_H \). The field \( \varphi_2 \) then takes the role of the would-be Goldstone boson eaten up by \( A_\mu \).

In order to quantize the model the gauge has to be fixed,

\[
\Gamma_{g.f.} = \int \left\{ \frac{1}{2} \xi B^2 + B(\partial A + \xi A m \varphi_2) \right\}.
\] (2.4)

\( \xi \) denotes the gauge parameter and \( B \) is an auxiliary field with \( \delta_\omega B = 0 \). Please note that the 't Hooft gauge fixing term \( \int B \xi A m \varphi_2 \) has been introduced in order to avoid a non-integrable infrared singularity in the \( \langle \varphi_2 \varphi_2 \rangle \) propagator. It leads to a non-trivial violation of both local and global gauge invariance:

\[
\delta_\omega \Gamma_{g.f.} = \int \left\{ \omega \Box B + e \omega B \xi A m (\varphi_1 + \frac{m}{e}) \right\}
\] (2.5)

Hence one has to enlarge local gauge transformations to BRS transformations thereby introducing the Faddeev-Popov (\( \phi\pi \)) fields \( c, \bar{c} \),

\[
\begin{align*}
sA_\mu &= \partial_\mu c, \quad sc = 0, \\
s\varphi_1 &= -ec\varphi_2, \quad s\varphi_2 = ec(\varphi_1 + \frac{m}{e}), \\
s\bar{c} &= B, \quad sB = 0,
\end{align*}
\] (2.6)

and to postulate BRS invariance of the theory instead of gauge invariance:

\[
s\Gamma_{cl} = 0
\] (2.7)

This leads to extra-terms in the classical action:

\[
\Gamma_{cl} = \Gamma_{inv} + \Gamma_{g.f.} + \Gamma_{\phi\pi} + \Gamma_{e.f.}
\] (2.8)

The \( \phi\pi \)-part

\[
\Gamma_{\phi\pi} = \int \left\{ -\bar{c} \Box c - e\bar{c} \xi A m (\varphi_1 + \frac{m}{e}) c \right\}
\] (2.9)

is chosen such that its BRS variation exactly cancels the BRS variation of \( \Gamma_{g.f.} \).

The external field part

\[
\Gamma_{e.f.} = \int \{ Y_1 (s\varphi_1) + Y_2 (s\varphi_2) \}
\] (2.10)

couples the non-linear BRS transformations \( s\varphi_i \) to the external fields \( Y_i \). It is added in order to allow for a careful definition of the BRS transformation of \( \varphi_i \) in higher orders (cf. (2.14)). BRS invariance has been shown to define the Abelian Higgs model in its quantized version and it is the relevant symmetry for renormalizability and unitarity of the S-matrix [7].
In a foregoing paper [8] rigid and also local gauge symmetry have been formulated to all orders of perturbation theory (see sections 6 and 7 for details). For this purpose the gauge fixing has to be complemented by external scalar fields \( \hat{\phi}_1 \) and \( \hat{\phi}_2 \) which transform under gauge transformations according to

\[
\delta \omega \hat{\phi}_1 = -e \omega \hat{\phi}_2, \quad \delta \omega \hat{\phi}_2 = e \omega (\hat{\phi}_1 - \xi_A \frac{m}{e}).
\]  

(2.11)

Then the enlarged gauge fixing

\[
\Gamma_{g.f.} = \int \left\{ \frac{1}{2} \xi B^2 + B \partial A - eB \left( (\hat{\phi}_1 - \xi_A \frac{m}{e}) \hat{\phi}_2 - \hat{\phi}_2 (\hat{\phi}_1 - \xi_A \frac{m}{e}) \right) \right\}
\]

(2.12)

is invariant under the global gauge transformations (2.11), if one adjusts \( \hat{\xi}_A = -1 \). For vanishing external fields \( \hat{\phi}_i \) the original gauge fixing (2.4) is recovered and these fields have only been introduced in order to be able to manage the breaking of gauge invariance for higher orders Green functions algebraically.

Also in its enlarged form gauge invariance does not describe the \( \phi\pi \)-part, even it does not in the classical approximation. Instead, BRS invariance has to be used for the construction of the model, which involves also the external fields \( \hat{\phi}_i \). Because they are coupled to BRS variations they are transformed into external fields \( q_i \) with \( \phi\pi \)-charge +1:

\[
s\hat{\phi}_i = q_i, \quad sq_i = 0, \quad i = 1, 2
\]

(2.13)

At the level of the vertex functional \( \Gamma = \Gamma_{cl} + O(h) \) the BRS invariance of the theory is encoded in the Slavnov-Taylor (ST) identity

\[
S(\Gamma) \equiv \int \left\{ \partial_\mu c \frac{\delta \Gamma}{\delta A_\mu} + B \frac{\delta \Gamma}{\delta \bar{c}} + \frac{\delta \Gamma}{\delta \bar{c} \delta \phi} + q \frac{\delta \Gamma}{\delta \bar{\phi}} \right\} = 0
\]

(2.14)

where \( E = (F_1, F_2) \). The ST identity is the essential ingredient for the proof of renormalizability and unitarity. It can be proven that (2.14) together with appropriate normalization conditions, invariance under charge conjugation and the gauge condition (2.12) uniquely defines the model to all orders of perturbation theory. (The quantum numbers of all fields are given in table 1.) In the first step one has to show that the classical action (2.8) is uniquely determined as the local solution of the ST identity. The general classical solution has been calculated in ref. [8] and is presented in appendix A explicitly. The free parameters appearing in the general solution are the usual field and coupling renormalizations given by:

\[
\varphi_i \rightarrow \sqrt{z_i} (\varphi_i - x_i \hat{\phi}_i), \quad A_\mu \rightarrow \sqrt{z_A} A_\mu
\]

\[
m \rightarrow \sqrt{z_m} m, \quad m_H \rightarrow \sqrt{z_{m_H}} m_H, \quad e \rightarrow z_e e
\]

(2.15)
| fields     | $A_\mu$ | $B$ | $\tilde{\varphi}_1$ | $\tilde{\varphi}_2$ | $c$ | $\bar{c}$ | $Y_1$ | $Y_2$ | $q_1$ | $q_2$ |
|------------|---------|-----|----------------------|----------------------|-----|----------|-------|-------|-------|-------|
| dim        | 1       | 2   | 1                    | 1                    | 0   | 2        | 3     | 3     | 1     | 1     |
| charge conj.| -       | -   | +                    | -                    | -   | +        | -     | +     | -     | -     |
| $Q_{\varphi\pi}$ | 0       | 0   | 0                    | 0                    | +1  | -1       | -1    | -1    | 1     | 1     |

Table 1: Quantum numbers of the fields ($\tilde{\varphi}_i = \varphi_i, \hat{\varphi}_i$)

The field redefinitions of the remaining fields and parameters are governed by the ST identity and the gauge fixing (2.12). Via the field redefinitions the external fields $\tilde{\varphi}_i$ enter the gauge invariant part $\Gamma_{inv}$ (2.1) of the action, but only in the combination

$$\bar{\varphi}_i = \varphi_i - x_i \hat{\varphi}_i, \quad (2.16)$$

with $x_i, i = 1, 2$, being further free parameters. These parameters appear also in the BRS variations of the fields $\varphi_i$, which modify (2.6) into

$$s\varphi_1 = -e\bar{\varphi}_2c + x_1q_1, \quad s\varphi_2 = e(\bar{\varphi}_1 + \frac{m}{e})c + x_2q_2. \quad (2.17)$$

The free parameters have to be fixed by appropriate normalization conditions order by order in perturbation theory. In order to have a proper field theoretic definition of the S-matrix the masses of the particles have to be fixed at the poles of the 2-point Green functions; these conditions read for the corresponding vertex functions:

$$Re \frac{\partial}{\partial p^2} \Gamma_{\varphi_1 \varphi_1}(p^2 = m_H^2) = 0 \quad \text{fixes} \quad z_{m_H}$$

$$\Gamma^T(p^2 = m^2) = 0 \quad \text{fixes} \quad z_{m}$$

$$\Gamma_{c\bar{c}}(p^2 = m_{\text{ghost}}^2) = 0 \quad \text{fixes} \quad \xi_A \quad (2.18)$$

Thereby we have defined the transversal part of the 2-point function according to

$$\Gamma_{\mu\nu \lambda\lambda'}(p, -p) \equiv \Gamma_{\mu\nu}(p, -p) = (\eta_{\mu\nu} - \frac{p_{\mu}p_{\nu}}{p^2})\Gamma^T(p^2) + \frac{p_{\mu}p_{\nu}}{p^2}\Gamma^L(p^2). \quad (2.19)$$

The wave function renormalizations $z_i$ are determined on the residua of the respective 2-point functions,

$$\frac{\partial}{\partial p^2} \Gamma^T(p^2 = m^2) = 1 \quad \text{fixes} \quad z_A$$

$$Re \frac{\partial}{\partial p^2} \Gamma_{\varphi_1 \varphi_1}(p^2 = m_H^2) = 1 \quad \text{fixes} \quad z_1$$

$$\frac{\partial}{\partial p^2} \Gamma_{\varphi_2 \varphi_2}(p^2 = m_{\text{ghost}}^2) = 1 \quad \text{fixes} \quad z_2 \quad (2.20)$$

whereas the parameters $x_i$ are chosen to be fixed on the external field part at an arbitrary normalization point $\kappa$:

$$\Gamma_{Y_1q_1}(p^2 = \kappa^2) = x_1^{(0)} \quad \text{fixes} \quad x_1$$

$$\Gamma_{Y_2q_2}(p^2 = \kappa^2) = x_2^{(0)} \quad \text{fixes} \quad x_2 \quad (2.21)$$
$x_1^{(0)}$ and $x_2^{(0)}$ are assumed to be pure numbers (e.g. 1 or 0) throughout the paper. Using rigid invariance they are not independent but $x_2^{(0)} = x_1^{(0)} + O(\hbar)$ (cf. (6.4)).

It remains to give an appropriate normalization condition for the coupling $e$. This coupling can be determined at the 3-point function $\Gamma_{A_\mu \phi_1 \phi_2}$ at a normalization momentum $p_{\text{norm}}$:

$$\partial_{p_1} \Gamma_{A_\mu \phi_1 \phi_2}(-p_1 - p_2, p_1, p_2)\big|_{\{p_i\} = p_{\text{norm}}} = -ie\eta^{\mu\nu} \text{ fixes } z_e$$  \hspace{1cm} (2.22)

Finally we have to require the 2-dimensional BRS-invariant scalar field polynomial (see appendix A) to vanish:

$$\Gamma_{\phi_1} = 0 \text{ fixes } \mu$$  \hspace{1cm} (2.23)

Applying these normalization conditions to the tree approximation and requiring the gauge condition (2.12) the original classical action $\Gamma_{cl}$ (2.8) is recovered up to the modifications encountered in (2.16) and (2.17), i.e. $z_a = 1 + \delta z_a$ with $\delta z_a$ of order $\hbar$.

When controlling the gauge parameter algebraically (the subject of this paper) one gets restrictions on the gauge parameter dependence of the free parameters $z_a$. In higher orders these restrictions have to be extended to a proper definition of the normalization conditions. The main result of this paper is the observation that the on shell conditions (2.18) for the physical particles are in complete agreement with the restrictions given on gauge parameter dependence, whereas the definition of the coupling (2.22) has to be modified in an appropriate way. It turns out that the Ward identity of local symmetry (cf. (7.1)) can be used to fix the gauge parameter dependence of the vertex $\Gamma_{A_\mu \phi_1 \phi_2}$ correctly.

### 3. Algebraic control of gauge parameter dependence

Let us now turn to the proper subject of this paper: the control of gauge parameter dependence. We start with the observations that for the classical action $\Gamma_{cl}$ the gauge parameter dependence is given by a BRS variation:

$$\partial_{\xi} \Gamma_{cl} = \frac{1}{2} \int B^2 = \frac{1}{2} s \int \bar{c}B$$  \hspace{1cm} (3.1)

and that the r.h.s. therefore vanishes between physical states, i.e. physical quantities as the S-matrix are gauge parameter independent in the tree approximation as they should. Now we can ask whether it is possible to extend this statement to higher orders. There the differentiation with respect to $\xi$ produces a non-trivial insertion,

$$\partial_{\xi} \Gamma = \left[ \int \frac{1}{2} B^2 + O(\hbar) \right] \cdot \Gamma ,$$  \hspace{1cm} (3.2)

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which cannot be handled in such a simple way as it was the case in the tree approximation. (Note that the vertex $B^2$ can be inserted non-trivially into loop diagrams according to the appearance of the mixed $B$-$A_\mu$ propagator.) To prove $\xi$-independence of physical quantities demands quite a technical effort \[2\], if one does not use specifically adapted tools. Such a tool has been provided by ref. \[4\] and consists in BRS-transforming $\xi$ into a Grassmann variable $\chi$ with $\phi\pi$-charge $+1$:

$$s\xi = \chi, \quad s\chi = 0 \quad (3.3)$$

Accordingly the ST identity (2.14) has to be modified into:

$$S(\Gamma) + \chi \partial_\xi \Gamma = 0 \quad (3.4)$$

Differentiating this last eq. with respect to $\chi$ and evaluating at $\chi = 0$ leads to

$$-s\chi^0 \partial_\chi \Gamma \bigg|_{\chi=0} + \partial_\xi \Gamma \bigg|_{\chi=0} = 0 \quad (3.5)$$

In the model under investigation $s_\Gamma$ is defined in (6.19) and is – roughly speaking – the functional generalization of $s$. Hence (3.3) is nothing else but the functional analog of (3.1), which can be – in contrast to (3.2) – controlled in higher orders. In other words: Proving (3.4) to all orders of perturbation theory automatically yields the $\xi$-dependence of the 1-PI Green functions in an algebraic way.

In theories with spontaneous breaking of the symmetry (as in the Abelian Higgs model) even a second gauge parameter $\xi_A$ has to be introduced via the ’t Hooft term. For the purpose of this paper we restrict ourselves to the consideration of the dependence on the gauge parameter $\xi$ only and do not BRS-transform $\xi_A$. However, we allow that $\xi_A$ might be a function of $\xi$. For this reason the ’t Hooft gauge, which is defined by the choice

$$\xi_A = \xi \quad , \quad (3.6)$$

is automatically included in our discussion.

We want to finish this section by writing down how the classical action (2.8) has to be modified due to the BRS-transforming gauge parameter $\xi$ (3.3). The only place, where $\xi$ appears, is the gauge fixing term (2.12) and – in the ‘t Hooft gauge – the $\phi\pi$-part. Evaluating the BRS variations we end up with

$$\Gamma_{g.f.} + \Gamma_{\phi\pi} \longrightarrow \Gamma_{g.f.} + \Gamma_{\phi\pi} + \chi \int \left\{ \frac{1}{2} \bar{\chi} B + (\partial_\xi \xi_A) m \bar{\chi} \varphi_2 - (\partial_\xi \xi_A) m \bar{\chi} \varphi_2 \right\} . \quad (3.7)$$

Note that $\chi \bar{\chi} B$ as well as $\chi \bar{\chi} \varphi_2$ can be non-trivially inserted into loop diagrams. Therefore the renormalization of the model for $\chi \neq 0$ has to be carried out carefully to all orders looking thereby into the modifications appearing from gauge parameter dependence.
4. Slavnov-Taylor identity for $\chi \neq 0$

Following the remarks of the last section we are able to control the gauge parameter dependence of the Green functions, if we construct them with the help of the $\chi$-enlarged ST identity

\[
S(\Gamma) \equiv \int \left\{ \partial_{\mu} c \frac{\delta \Gamma}{\delta A_{\mu}} + B \frac{\delta \Gamma}{\delta \phi} + \frac{\delta \Gamma}{\delta Y} \frac{\delta \phi}{\delta \phi} + q \frac{\delta \Gamma}{\delta \hat{\phi}} \right\} + \chi \partial_{\xi} \Gamma = 0 .
\]  

(4.1)

Because the ST identity does not prescribe the gauge fixing terms, we can also postulate the gauge condition (2.12),

\[
\left. \frac{\delta \Gamma}{\delta B} \right|_{\chi=0} = \xi B + \partial A - e \left[ (\hat{\phi}_1 - \xi_A m) \phi_2 - \phi_2 (\phi_1 - \hat{\xi}_A m) \right] ,
\]

(4.2)

to hold for the solution $\Gamma$ of (4.1). It is worth to note that (4.2) is well defined and can be integrated in all orders of perturbation theory due to the linearity in the propagating fields.

Because $\chi$ is a Grassmann variable and therefore $\chi^2 = 0$, the vertex functional $\Gamma$ can be decomposed into a $\chi$-independent and an explicitly $\chi$-dependent part,

\[
\Gamma = \hat{\Gamma} + \chi Q ,
\]

(4.3)

where $Q$ is the generating functional of Green functions with $\phi \pi$-charge $-1$. Inserting this ansatz into (4.1), making use of $\chi^2 = 0$, and looking for terms proportional to $\chi^0$ and $\chi^1$ one immediately gets:

\[
\chi^0 : \int \left\{ \partial_{\mu} c \frac{\delta \hat{\Gamma}}{\delta A_{\mu}} + B \frac{\delta \hat{\Gamma}}{\delta \phi} + \frac{\delta \Gamma}{\delta Y} \frac{\delta \phi}{\delta \phi} + q \frac{\delta \Gamma}{\delta \hat{\phi}} \right\} = 0 \quad (4.4)
\]

\[
\chi^1 : \int \left\{ \partial_{\mu} c \frac{\delta (\chi Q)}{\delta A_{\mu}} + B \frac{\delta (\chi Q)}{\delta \phi} + \frac{\delta (\chi Q)}{\delta Y} \frac{\delta \phi}{\delta \phi} + \frac{\delta \Gamma}{\delta Y} \frac{\delta \phi}{\delta \phi} + q \frac{\delta (\chi Q)}{\delta \hat{\phi}} \right\} + \chi \partial_{\xi} \hat{\Gamma} = 0 \quad (4.5)
\]

Equation (4.4) is exactly the ST identity (4.1) for $\chi = 0$, which has been studied in [6]. Therefore we only have to consider (4.5) and to investigate the modifications brought about by the BRS-varying gauge parameter via the $\chi$-dependent part $Q$.

First we have to look for the general classical solution of the ST identity (4.5) in order to find the free parameters of the model. The general solution $\hat{\Gamma}_{cl}^{gen}$ has already been presented in section 2 and is given explicitly in appendix A. In the classical approximation the decomposition (4.3) implies that $Q$ is a local field polynomial with dimension less than

\[ \text{[From here on we use the symbol $S$ for all the differential operators on the l.h.s. of 3.4].} \]
or equal to four, carries $\phi\pi$-charge $-1$ and is even under charge conjugation. The most general ansatz for $Q_{cl}$ is therefore given by (see table of quantum numbers)

$$Q_{cl} = \{ d_1Y_1\varphi_1 + \hat{d}_1Y_1\hat{\varphi}_1 + dY_1 + d_2Y_2\varphi_2 + \hat{d}_2Y_2\hat{\varphi}_2 + f\bar{c}\varphi_2 + \hat{f}\bar{c}\hat{\varphi}_2 + \hat{f}\bar{c}B + fA\bar{c}\partial A + h_1\bar{c}\varphi_1\varphi_2 + h_2\bar{c}\hat{\varphi}_1\varphi_2 + h_3\bar{c}\varphi_1\hat{\varphi}_2 + h_4\bar{c}\hat{\varphi}_1\hat{\varphi}_2 \},$$

(4.6)

and the 13 parameters $d_1, \hat{d}_1, d, \ldots, h_3, h_4$ have to be determined with the help of (4.3). This calculation is straightforward and most easily done by differentiating (4.3) with respect to suitable fields and then comparing coefficients of independent terms. We find:

$$Q_{cl} = Q_{e.f.} + Q_{\phi\pi}$$

(4.7)

with $(x_1^{(0)} = x_2^{(0)}) \equiv x$ classically, see (6.4), and $\varphi_i = \varphi_i - x\hat{\varphi}_i$

$$Q_{e.f.} = \int\left\{\frac{1}{4}(\partial_\xi\ln z_1 + \partial_\xi\ln z_2)(Y_1\varphi_1 + Y_2\varphi_2) + \frac{1}{4}(\partial_\xi\ln z_1 - \partial_\xi\ln z_2)(Y_1\hat{\varphi}_1 - Y_2\hat{\varphi}_2) - \partial_\xi x(Y_1\hat{\varphi}_1 + Y_2\hat{\varphi}_2)\right\},$$

(4.8)

$$Q_{\phi\pi} = \int\left\{-\frac{1}{4}\bar{c}\partial_\xi(\varphi_1 + \frac{\sqrt{z_m}}{\sqrt{z_1}z_e} m)\hat{\varphi}_2 - \varphi_2(\hat{\varphi}_1 - \xi_A m e)\right\},$$

(4.9)

The parameters $z_1, z_2, z_m, z_e, x$ as well as $\xi_A$ in (4.8), (4.9) are the free parameters appearing in the general solution $\hat{\Gamma}$ of the ST identity (4.1) for $\chi = 0$. Please note that the coefficients in $Q_{cl}$ are fully determined as functions of these parameters. The ST identity (4.3) does not only determine $Q_{cl}$ but even restricts the $\xi$-dependence of the free parameters: Whereas the wave function renormalizations $z_1, z_2$ and $x$ are allowed to depend arbitrarily on the gauge parameter, the parameters $z_e, z_A, z_m, z_{mH}$ and $\mu^2$ have to be independent of the gauge parameter $\xi$:

$$\partial_\xi z_e = 0, \quad \partial_\xi z_A = 0, \quad \partial_\xi z_m = 0, \quad \partial_\xi z_{mH} = 0, \quad \partial_\xi \mu^2 = 0$$

(4.10)

This means that only the $\xi$-independent part of these parameters has to be fixed by normalization conditions, the $\xi$-dependent part, however, is completely determined by the $\chi$-enlarged ST identity and one cannot dispose on it by a normalization condition.

Applying the normalization conditions to the general classical solution of the ST identity the constraints (4.10) are trivially fulfilled, because $e, m_H$ and $m$ are $\xi$-independent.
physical parameters of the model. If one would take these parameters depending on $\xi$, then, of course, the S-matrix would depend on the gauge parameter $\xi$. Such a choice is completely unphysical but it would be allowed just looking into the renormalization of the model via the ordinary ST identity. In the tree approximation it is obvious that nobody will introduce such a strange $\xi$-dependence into the theory. In higher orders, however, the couplings have to be fixed at some value of non-local Green functions, and there it is much less obvious and transparent, how one has to deal with the splitting into $\xi$-dependent and $\xi$-independent quantities in the normalization conditions. A wrong adjustment of $\xi$-dependent counterterms will introduce $\xi$-dependence into the S-matrix in the same way as we have illustrated above for the tree approximation. The next section is devoted to the analysis, how the classical restrictions are continued to higher order perturbation theory.

5. Gauge parameter dependence of Green functions

Having completed the characterization of the classical model via the ST identity the next step would be the proof of the $\chi$-dependent ST identity to all orders. We don’t want to go into the details here, but only mention that, according to the considerations in [4], the proof of (4.1) (for $\chi \neq 0$) can be reduced to the proof of the ST identity for $\chi = 0$. It turns out that no anomaly occurs, and once suitable counterterms are adjusted the $\chi$-enlarged ST identity (4.1) is valid in all orders,

$$S(\Gamma) = 0,$$

(5.1)

with $\Gamma$ being the generating functional of 1-PI Green functions, which includes also the $\chi$-dependent contributions. Accordingly the validity of (5.1) will be assumed throughout the following.

In order to extend the restrictions on gauge parameter dependence found in the classical approximation (4.10) to higher orders we have to consider the $\xi$-dependence of the non-local Green functions, on which the normalization conditions are applied. Thereby we want to restrict ourselves to the case, where the Higgs particle is a stable particle, i.e. $m_H^2 < 4m^2$. The imaginary part is then vanishing for the self energy of the Higgs at $p^2 = m_H^2$ as well as for the transversal part of the vector self energy at $p^2 = m^2$ [8].

$\xi$-independence of the parameter $\mu$ is trivially continued to higher orders: The ST identity reads for the $\xi$-derivative of $\Gamma_{\phi_1}$ in momentum space:

$$\partial_\chi \Gamma_Y (0) \Gamma_{\phi_1 \phi_1} (0) + \partial_\chi \Gamma_Y (0) \Gamma_{\phi_1} (0) = - \partial_\xi \Gamma_{\phi_1} (0)$$

(5.2)
Applying the normalization condition \( (2.23) \), which requires vanishing of the vacuum expectation value of the Higgs field, we end up with the condition

\[
(-m_H^2 + O(h))\partial_\chi \Gamma_{Y_1} = 0 \implies \partial_\chi \Gamma_{Y_1} = 0 ,
\]

which is valid to all orders of perturbation theory.

We now turn to the transversal part of the vector 2-point function, which was found to be gauge parameter independent in the classical approximation \( \partial_\xi z_A = 0 \) and \( \partial_\xi z_m = 0 \). This restriction is valid also in higher orders of perturbation theory: Using the result (5.3) one gets from the ST identity:

\[
\partial_\chi \Gamma_{Y_2 A_\mu} (p, -p) \Gamma_{\varphi_2 A_\nu} (p, -p) + \partial_\chi \Gamma_{Y_2 A_\nu} (p, -p) \Gamma_{\varphi_2 A_\mu} (p, -p) = -\partial_\xi \Gamma_{\mu \nu} (p, -p)
\]

(5.4)

Because of Lorentz invariance one has

\[
\partial_\chi \Gamma_{Y_2 A_\mu} (p, -p) = \frac{p_\mu f_1 (p^2)}{m^2} ,
\]

(5.5)

\[
\Gamma_{\varphi_2 A_\nu} (p, -p) = p_\nu m f_2 (p^2) = ip_\nu m + O(h) ,
\]

and the l.h.s. of (5.4) only contributes to the longitudinal part of \( \partial_\xi \Gamma_{\mu \nu} (p, -p) \). Acting with the transversal projector on (5.4) it is easily seen that the transversal part of \( \Gamma_{\mu \nu} \) is independent of \( \xi \) to all orders of perturbation theory:

\[
\partial_\xi \Gamma^T (p^2) = 0 \quad \text{with} \quad \Gamma_{A_\mu A_\nu} \equiv \Gamma_{\mu \nu} = (\eta_{\mu \nu} - \frac{p_\mu p_\nu}{p^2}) \Gamma^T + \frac{p_\mu p_\nu}{p^2} \Gamma^L
\]

(5.6)

This equation has two consequences: It tells us that the non-local contributions to \( \Gamma^T (p^2) \) are \( \xi \)-independent, and also that the counterterms have to be adjusted in such a way, that (5.4) is fulfilled order by order in perturbation theory. Accordingly we have to check whether the normalization conditions that we have imposed on the transversal part of the vector 2-point function \( (2.18), (2.20) \) are compatible with (5.6). Testing (5.6) at the normalization points it is seen that the on-shell condition is in agreement with (5.6):

\[
\partial_{p^2} \partial_\xi \Gamma^T (p^2) \bigg|_{p^2 = \kappa_m^2} = \partial_\xi 1 = 0 ,
\]

(5.7)

\[
\partial_\xi \Gamma^T (p^2) \bigg|_{p^2 = m^2} = \partial_\xi 0 = 0
\]

(5.8)

(Here we have even relaxed the complete on-shell condition by permitting an arbitrary normalization point \( \kappa_m \) for fixing the residuum.) Working in a scheme without explicit normalization conditions like the MS-scheme (5.6) has to be proven in concrete calculations order by order in perturbation theory. The constraints on the vector 2-point function look very simple and obvious in the Abelian Higgs model, but in this simple way they are
only available by the algebraic control of gauge parameter dependence.

Much more interesting, however, is the extension of $\partial_\xi z_{m_H} = 0$ (4.10) to higher orders. First we consider the wave function renormalization of the scalar particles, i.e. the gauge parameter dependence of $\partial_\xi \Gamma_{i\varphi_i}(p^2)$. Testing the ST identity with respect to the Higgs and the would-be Goldstone we get (using again (5.3)):

$$\partial_\chi \Gamma_{Y_1\varphi_1}(p^2) = -\partial_\xi \Gamma_{\varphi_1\varphi_1}(p^2), \quad (5.9)$$

$$\partial_\chi \Gamma_{Y_2\varphi_2}(p^2) = -\partial_\xi \Gamma_{\varphi_2\varphi_2}(p^2)$$

The normalization conditions on the residua (2.20) uniquely fix the Green functions $\partial_\chi \Gamma_{i\varphi_i}(p^2)$, explicitly

$$\partial_\chi \Gamma_{Y_i\varphi_i}(\kappa_i^2) = -(\partial_{p^2} \partial_\chi \Gamma_{Y_i\varphi_i})\big|_{p^2=\kappa_i^2} \Gamma_{\varphi_i\varphi_i}(\kappa_i^2) \quad (5.10)$$

where we have also used a relaxed form of the complete on-shell condition of the Higgs 2-point function. This equation is well-defined order by order in perturbation theory because $\partial_{p^2} \partial_\chi \Gamma_{Y_i\varphi_i}(p^2)$ is non-local and uniquely determined from lower orders. In the complete on-shell scheme (5.10) simplifies to

$$\partial_\chi \Gamma_{Y_i\varphi_i}(m_H^2) = 0 \quad (5.11)$$

Eq. (5.9) completely governs the $\xi$-dependence of the Higgs self-energy. However, $\Gamma_{\varphi_1\varphi_1}(p^2)$ involves two free parameters, namely the wave function normalization and the Higgs mass. Once $\partial_\chi \Gamma_{Y_1\varphi_1}(p^2)$ is fixed one therefore has to adjust the mass counterterm in such a way that eq. (5.9) is identically fulfilled. In contrast to the transversal part of the vector 2-point function the l.h.s. of (5.9) is far from being trivial, because the vertex function $\partial_\chi \Gamma_{Y_1\varphi_1}(p^2)$ is non-local, i.e. it depends on the momentum $p^2$, due to non-trivial insertions of the vertex $\bar{c}B$ (see appendix B for the 1-loop diagrams). These non-local contributions govern the $\xi$-dependence of the non-local contributions to the Higgs self energy. In 1-loop order eq. (5.9) reads

$$\partial_\chi \Gamma^{(1)}_{Y_1\varphi_1}(p^2)(p^2 - m_H^2) = -\partial_\xi \Gamma^{(1)}_{\varphi_1\varphi_1}(p^2). \quad (5.12)$$

Testing at $p^2 = m_H^2$ and using the on-shell condition for the Higgs (2.18) we see that the l.h.s. as well as the r.h.s. vanish. This result does not depend on the normalization one has chosen for the residuum. Therefore the on-shell conditions are shown to satisfy the restrictions dictated on gauge

\[\text{Therefrom it is immediately deduced, that the normalization conditions of ref. [2], which in addition fix the coupling at an on-shell momentum on the transversal part of } \Gamma_{A_\mu A_\nu \varphi_1}, \text{ are in complete agreement with the BRS-varying gauge parameter.}\]
parameter dependence via a BRS-varying gauge parameter. Finally, these arguments can immediately be applied to all orders of perturbation theory. This result is quite remarkable and underlines once more the relevance of the on-shell conditions for the proper field theoretic definition of spontaneously broken theories.

We now come to the last constraint: the gauge parameter independence of \( z_e \) and its compatibility with the normalization condition (2.22). It is quite instructive to write down explicitly the equation which governs the \( \xi \)-dependence of \( \Gamma_{A,\nu \phi_1 \phi_2} \):

\[
\partial_\chi \Gamma_{Y_1 \phi_1}(p_1^2) \Gamma_{\phi_1 \phi_2 A_\mu}(p_1, p_2, p) + \partial_\chi \Gamma_{Y_2 \phi_2}(p_2^2) \Gamma_{\phi_1 \phi_2 A_\mu}(p_1, p_2, p) \\
+ \partial_\chi \Gamma_{Y_2 \phi_2 A_\mu}(p_2, p_1, p) \Gamma_{\phi_1 \phi_2}(p_2^2) + \partial_\chi \Gamma_{Y_1 \phi_2 A_\mu}(p_1, p_2, p) \Gamma_{\phi_1 \phi_2}(p_1^2) \\
+ \partial_\chi \Gamma_{Y_2 \phi_1 \phi_2}(p, p_1, p_2) \Gamma_{\phi_2 A_\mu}(-p, p) = -\partial_\xi \Gamma_{\phi_1 \phi_2 A_\mu}(p_1, p_2, p) \\
\]  

(5.13)

This equation completely determines the \( \xi \)-dependence of the 3-point vertex. In fact, if the residua of the Higgs and the would-be Goldstone are fixed by the normalization conditions (5.9) and (5.10), eq. (5.13) does not involve any free parameters, because all other \( \chi \)-dependent contributions are non-local. It is even not possible to evaluate the l.h.s. at a normalization point as it was the case for the 2-point functions. Therefore, if one insists in fixing the coupling at a normalization point as we have done in (2.22), one is forced to introduce a reference point \( \xi_o \) in order to fix the gauge parameter independent part, and has to govern \( \xi \)-dependence by eq. (5.13) which is quite a troublesome task in explicit calculations (see ref. [4] for details).

Instead we will show that such a treatment is not necessary in the Abelian Higgs model if one agrees to fix the coupling on the local Ward identity as it is elaborated in section 7.

6. Rigid invariance

The \( \chi \)-independent part of the generating functional of 1-PI irreducible Green functions was shown [6] to satisfy a Ward identity of rigid symmetry to all orders of perturbation theory:

\[
\hat{W}^\text{gen} \Gamma \bigg|_{\chi=0} = 0 \\
\]

(6.1)

with

\[
\hat{W}^\text{gen} \equiv \int \left\{ -z^{-1} \frac{\delta}{\delta \phi_2} + z(\phi_1 - \xi_A \frac{m}{e}) \frac{\delta}{\delta \phi_2} - z Y_2 \frac{\delta}{\delta Y_1} + z^{-1} Y_1 \frac{\delta}{\delta Y_2} \\
- z^{-1} \frac{\delta}{\delta \phi_1} + z(\phi_2 - \xi_A \frac{m}{e}) \frac{\delta}{\delta \phi_1} - z^{-1} q_2 \frac{\delta}{\delta q_2} + z q_1 \frac{\delta}{\delta q_1} \right\} \\
\]  

(6.2)
The parameters $z$, $\hat{\xi}_A$ and $\xi_A$ are uniquely determined by the normalization conditions imposed on the residua of the Higgs particle $\varphi_1$ and the Goldstone boson $\varphi_2$ (2.20), and the mass normalization of the ghosts and the Higgs particle (2.18). They are expanded in orders of the coupling $e$ according to the loop expansion,

$$ z = 1 + \delta z , \quad \hat{\xi}_A = -1 + x \xi_A + \delta \hat{\xi}_A , \quad \xi_A = \xi_A^{(0)} + O(h) , $$

with $\delta z = O(h)$ and $\delta \hat{\xi}_A = O(h)$. In addition the rigid Ward identity imposes a relation between the vertex function $\Gamma_{Y_1q_1}$ and $\Gamma_{Y_2q_2}$. Therefore as we have already mentioned, the parameters $x_1$ and $x_2$ are not independent from each other:

$$ x_2^{(0)} = x_1^{(0)} + O(h) , \quad x \equiv x_1^{(0)} $$

In general the parameters appearing in the Ward identity are functions of the gauge parameter $\xi$. In fact, in the 't Hooft gauge $\xi_A$ and $\hat{\xi}_A$ depend on the gauge parameter even in lowest order of perturbation theory:

$$ \xi_A = \xi , \quad \hat{\xi}_A = -1 + x \xi \quad (6.5) $$

We now have the task to study the modifications of (6.1), (6.2) brought about by a BRS transforming gauge parameter $\xi$. First we will look at the classical approximation, then to higher orders.

### 6.1. Classical approximation

The application of the Ward operator $\hat{W}_{\text{gen}}^{\text{gen}}$ (6.2) to the general classical solution $\Gamma_{Y_1q_1}^{\text{gen}}$ (1.3) of the ST identity (1.1) gives a first insight into the modifications expected in higher orders of perturbation theory:

$$ \hat{W}_{Y_1q_2}^{\text{gen}} \Gamma_{Y_1q_1}^{\text{gen}} = \hat{W}_{Y_2q_2}^{\text{gen}} \Gamma_{Y_2q_2}^{\text{gen}} + c Q = \hat{W}_{Y_1q_1}^{\text{gen}} \Gamma_{Y_1q_1}^{\text{gen}} + \chi \hat{W}_{Y_2q_2}^{\text{gen}} Q = \chi \hat{W}_{Y_2q_2}^{\text{gen}} Q \quad (6.6) $$

with ($z = \sqrt{\frac{m}{e}}$ classically)

$$ \hat{W}_{Y_2q_2}^{\text{gen}} Q = \int \left\{ -\partial_\xi z (Y_2 + e \bar{c} (\varphi_1 - \xi_A \frac{m}{e})) (\varphi_1 + \frac{\sqrt{z m}}{\sqrt{z_1 z_e} e}) \right. \\
+ \left. \partial_\xi z^{-1} (Y_1 - e \bar{c} \varphi_2) \varphi_2 \right. \\
+ \left. z \left( \partial_\xi (\xi_A \frac{m}{e}) - x \partial_\xi (\xi_A \frac{m}{e}) \right) Y_2 + \partial_\xi (\xi_A m) \bar{c} \varphi_1 + \frac{\sqrt{z m}}{\sqrt{z_1 z_e} e} \right\} \neq 0 \quad (6.7) $$

The r.h.s. of (6.4) is non-vanishing, because $z$ as well as $\xi_A$ are allowed to depend on the gauge parameter $\xi$. In the tree approximation the 't Hooft gauge (6.3) breaks the naive rigid invariance by

$$ \hat{W} \Gamma_{Y_1q_1} = \chi \int m \bar{c} (\varphi_1 - x \varphi_1 + \frac{m}{e}) \quad (6.8) $$
This breaking of rigid invariance is potentially harmful because of the appearance of non-linear terms in the propagating (and interacting) fields, these terms not being well-defined in higher orders and leading to non-trivial insertions. Therefore one has to absorb these terms into a functional operator $\chi V_{gen}$ which cancels the unwanted terms when acting on $\Gamma_{gen}^{cl}$. A natural choice is given by extending the operators appearing in $\hat{\Gamma}_{gen}$ as far as possible to invariant operators of the enlarged $\chi$-varying BRS transformations, i.e.:

$$
\begin{align*}
\frac{1}{z}(\hat{\phi}_2 + q_2 \frac{\delta}{\delta q_1}) & \rightarrow \frac{1}{z}(\hat{\phi}_2 + q_2 \frac{\delta}{\delta q_1}) + \chi \partial_\xi z^{-1} \hat{\phi}_2 \frac{\delta}{\delta q_1} \\
\frac{1}{z}(\hat{\phi}_1 + q_1 \frac{\delta}{\delta q_2}) & \rightarrow \frac{1}{z}(\hat{\phi}_1 + q_1 \frac{\delta}{\delta q_2}) + \chi \partial_\xi z \hat{\phi}_1 \frac{\delta}{\delta q_2}
\end{align*}
$$

(6.9)

A short calculation shows that the $\chi$-enlarged Ward operator $W_{gen}$,

$$
W_{gen} = \hat{W}_{gen} + \chi V_{gen},
$$

(6.10)

$$
V_{gen} = \partial_\xi \int \left\{ \frac{1}{z}(\hat{\phi}_1 + \hat{\xi} A_m e) \frac{\delta}{\delta q_2} - \frac{1}{z} \hat{\phi}_2 \frac{\delta}{\delta q_1} \right\},
$$

(6.11)

removes the harmful breakings leaving only expressions linear in the propagating fields on the r.h.s., which cannot be inserted non-trivially into higher orders’ loop diagrams:

$$
W_{gen} \Gamma_{gen}^{cl} = \chi \Delta_{br}
$$

(6.12)

In the tree approximation when applying the normalization conditions (i.e. $z = 1$) the ’t Hooft gauge yields:

$$
W \Gamma_{cl} = (\hat{W} - \frac{m}{e} \frac{\delta}{\delta q_2}) \Gamma_{cl} = \chi \int Y_2 \frac{m}{e}
$$

(6.13)

### 6.2. Higher Orders

Our aim is to demonstrate the validity of the ($\chi$-)deformed WI to all orders, when $W_{gen}$ acts on the generating functional of 1-PI Green functions $\Gamma$:

$$
W_{gen} \Gamma = \chi \partial_\xi \int \left\{ \frac{1}{z}(\hat{\phi}_1 + \hat{\xi} A_m e) \frac{\delta}{\delta q_2} - \frac{1}{z} \hat{\phi}_2 \frac{\delta}{\delta q_1} \right\}
$$

(6.14)

In the following we don’t want to rely on special properties of a specific renormalization scheme, instead we will try to work scheme-independently (as far as possible). Accordingly
we only assume that in the scheme used the action principle holds. This action principle tells us that
\[ W^{\text{gen}} = \tilde{\Delta} \cdot \Gamma, \]  
where \( \tilde{\Delta} \) is a local integrated insertion with dimension less than or equal to four, \( \phi\pi \)-charge 0 and odd under charge conjugation. Because the Ward identity has already been established at \( \chi = 0 \) \( (6.1) \) \( \tilde{\Delta} \) has to depend explicitly on \( \chi \),
\[ \tilde{\Delta} = \chi \tilde{\Delta}_- \]  
where \( \tilde{\Delta}_- \) has \( \phi\pi \)-charge \(-1\).

The second ingredient, needed for the proof, is a certain transformation behaviour of \( W^{\text{gen}} \) with respect to BRS invariance: Acting with \( \hat{W}^{\text{gen}} \) \( (6.2) \) on the ST identity \( (4.1) \) yields
\[ \hat{W}^{\text{gen}} \mathcal{S}(\Gamma) = s_\Gamma (\hat{W}^{\text{gen}} \Gamma) - \chi (\partial_\xi \hat{W}^{\text{gen}}) \Gamma = 0. \]  
The operators \( (6.9) \) are chosen in such a way as to satisfy the consistency condition \( (6.18) \) by construction. Therefore one derives for the \( \chi \)-enlarged operator \( W^{\text{gen}} \) \( (6.10) \) the consistency condition
\[ \mathcal{S}(\Gamma) = s_\Gamma (W^{\text{gen}} \Gamma) - \chi \Delta \br \]  
where \( s_\Gamma \) is given by:
\[ s_\Gamma = \int \left\{ \frac{\delta}{\delta A_\mu} + B \frac{\delta}{\delta \bar{c}} + \frac{\delta \Gamma}{\delta \bar{c}} \frac{\delta}{\delta \bar{c}} + \frac{\delta \Gamma}{\delta \bar{c}} \frac{\delta}{\delta Y} + \frac{\delta}{\delta Y} \right\} + \chi \partial_\xi \]  
Furthermore one calculates:
\[ s_\Gamma \chi \Delta \br = -\chi s_\Gamma \partial_\xi \int \left( z^{-1}Y_1 \varphi_2 - zY_2 (\varphi_1 - \hat{\xi} A \frac{m}{e}) \right) \]  
Please note that the derivation of the consistency condition \( (6.18) \) as well as \( (6.20) \) only uses the invariance of the vertex functional \( \Gamma \) with respect to the Slavnov-Taylor identity, and does not rely on any explicit expressions for the Green functions. Combining \( (6.18) \) and \( (6.20) \) one gets
\[ s_\Gamma (W^{\text{gen}} \Gamma - \chi \Delta \br) = 0 \]  
which is valid to all orders of perturbation theory and constrains the breaking of the Ward identity \( (6.15), (6.16) \) to be \( s_\Gamma \)-invariant.
From here on we proceed by induction order by order in perturbation theory in order to prove the \( \chi \)-enlarged Ward identity of rigid symmetry. The Ward identity has been established in the tree approximation (6.13) and according to the quantum action principle the WI is broken at least by a local polynomial in the fields in 1-loop order:

\[
\left( W^{\text{gen}} \right)^{(\leq 1)} - \chi \Delta^{(\leq 1)}_{\text{br}} = \chi \Delta^{(1)}
\]  

(6.23)

The parameters \( z, \xi_A, \xi_A \) appearing in (6.22) are uniquely determined at \( \chi = 0 \) and expanded including 1-loop order. Applying the operator \( s_\Gamma \) to equation (6.23) and using the consistency condition (6.22) gives in 1-loop order

\[
s_\Gamma \chi \Delta^{(1)} = s_{\Gamma, cl} \chi \Delta^{(1)} + O(h^2) = 0.
\]  

(6.24)

Solving (6.24) is a purely classical problem now. The list of independent polynomials constituting \( \Delta^{(1)}_\chi \) can be given in the following form:

\[
\Delta^{(1)}_\chi = \int \left\{ w_1 Y_1 \phi_2 + w_2 Y_1 \phi_2 + w_3 Y_2 + w_4 Y_2 \phi_1 + w_5 Y_2 \phi_1 \\
+ w_6 \bar{c} + w_7 (-x Y_2 + e \bar{c}(\phi_1 + v)) + w_8 \bar{c} \phi_1 + w_9 \bar{c} A^2 \\
+ w_{10} \bar{c} \phi_1 + w_{11} \phi_1 (-x Y_2 + e \bar{c}(\phi_1 + v)) + w_{12} \bar{c} \phi_1 + w_{13} \bar{c} \phi_1 \\
+ w_{14} \bar{c} \phi_2 (-x Y_1 + e \bar{c} \phi_2) + w_{15} \bar{c} \phi_2 \right\}
\]  

(6.25)

All the coefficients \( w_i \) are purely 1-loop.

Acting with \( s_{\Gamma, cl}^{\chi = 0} \) on (6.23) yields:

\[
s_{\Gamma, cl}^{\chi = 0} \Delta^{(1)}_\chi = \int \left( w_1 (\phi_2 \frac{\delta \Gamma_{cl}}{\delta \phi_2} - Y_1 \frac{\delta \Gamma_{cl}}{\delta Y_2}) + w_2 (\phi_2 \frac{\delta \Gamma_{cl}}{\delta \phi_2} - Y_2 \frac{\delta \Gamma_{cl}}{\delta Y_1}) + w_3 \frac{\delta \Gamma_{cl}}{\delta \phi_2} \right. \\
+ w_4 (\phi_2 \frac{\delta \Gamma_{cl}}{\delta \phi_1} + q_2 \frac{\delta \Gamma_{cl}}{\delta q_1}) + w_5 (\phi_1 \frac{\delta \Gamma_{cl}}{\delta \phi_2} + q_1 \frac{\delta \Gamma_{cl}}{\delta q_2}) + w_7 \frac{\delta \Gamma_{cl}}{\delta \phi_2} \\
+ w_7 (\phi_2 \frac{\delta \Gamma_{cl}}{\delta \phi_2} - Y_1 q_2) + w_8 (\phi_1 \frac{\delta \Gamma_{cl}}{\delta \phi_1} + q_2 \frac{\delta \Gamma_{cl}}{\delta q_2}) + w_9 (B \phi_1^2 - \bar{c} s A^2) \\
+ w_{10} (B \phi_1^2 - \bar{c} s A^2) + w_{11} (B \phi_1^2 - \bar{c} s A^2) + w_{12} (B \phi_2^2 - \bar{c} s \phi_1^2) + w_{13} (B \phi_2^2 - \bar{c} s \phi_2^2) + w_{14} (B \phi_2^2 - \bar{c} s \phi_1^2) + w_{15} (B \phi_2^2 - \bar{c} s \phi_2^2)
\]  

(6.26)

The polynomials in (6.26) are not all independent as it is seen from the classical Ward identity

\[
WT_{\Gamma, cl} \big|_{\chi = 0} = 0.
\]  

(6.27)

This means that there is a \( s_1 \)-invariant in the basis of the \( \Delta^{(1)}_\chi \), which could give rise to a \( \chi \)-anomaly of the global Ward identity. The operators of the first two lines just constitute \( \hat{W} \) and therefore one has to eliminate one of these polynomials via the classical Ward
identity, for instance $\phi^2 \frac{\delta \Gamma_{\text{lat}}}{\delta \phi_1} - Y_1 \frac{\delta \Gamma_{\text{lat}}}{\delta Y_2}$. The remaining 14 polynomials are independent and according to the consistency condition (6.24) their coefficients have to vanish. Therefore the breaking of the Ward identity is restricted to one $s_1$-invariant whose coefficient is not available by algebraic consistency:

$$ (W_{\text{gen}} \Gamma)^{(\leq 1)} = \chi \Delta_{\text{br}}^{(\leq 1)} + w_1 \chi \int \left( (Y_1 - e\bar{\phi} \phi_2) \phi_2 - (Y_2 + e\bar{\phi}(\phi_1 - m / e \xi_A)) (\phi_1 + m / e) \right) \quad (6.28) $$

The coefficient $w_1$ has to be determined by an explicit test. Testing eq. (6.28) with respect to $Y_1 \phi_2$ and $Y_2 \phi_1$ at an asymptotic momentum $p_2^2 \gg m_i^2$, where the three point functions disappear, the coefficient $w_1$ is easily determined,

$$ \Gamma^{(1)}_{Y_1 \phi_1}(p_2^2) + \Gamma^{(1)}_{Y_2 \phi_2}(p_2^2) = - \chi w_1, \quad (6.29) $$

$$ \Gamma^{(1)}_{Y_1 \phi_1}(p_2^2) + \Gamma^{(1)}_{Y_2 \phi_2}(p_2^2) = \chi w_1, $$

and therefore

$$ w_1 = 0. \quad (6.30) $$

The equations (6.28) and (6.30) establish the $\chi$-dependent Ward identity of rigid symmetry at 1-loop order as suggested by the classical approximation.

It is clear how the induction proof has to be finished: We assume the validity of the WI (6.14) in order $n$ of $\hbar$. We conclude, passing through exactly the same steps as above, that the WI also holds at order $n + 1$. Hence we have proven to all orders:

$$ W_{\text{gen}} \Gamma = \chi \Delta_{\text{br}} \quad (6.31) $$

### 7. The local Ward identity

We conclude the general treatment of the $\chi$-dependence in the Abelian Higgs model by constructing a local $\chi$-dependent WI which expresses the invariance of Green functions under deformed local gauge transformations and simultaneously governs the $\xi$-dependence of these Green functions. In ref. [6] it was shown that at $\chi = 0$ the 1-PI Green functions satisfy the local Ward identity:

$$ \left. \left( (\epsilon + \delta \epsilon) w_{\text{gen}}(x) - \partial_\mu \frac{\delta}{\delta A_\mu} \right) \Gamma \right|_{\chi=0} = \Box B \quad (7.1) $$

The local ($\chi$-dependent) operator $w_{\text{gen}}$ is defined from the rigid one (6.10) by taking away the integration:

$$ W_{\text{gen}} = \int d^4x \ w_{\text{gen}}(x) \quad (7.2) $$
\(\delta e\) is of order \(\hbar\) and is determined by the normalization condition imposed for fixing the coupling \(e\) in higher order perturbative calculations.

The main result in deriving the local \(\chi\)-dependent Ward identity is the proof, that the overall normalization factor of the matter transformations \(e + \delta e\) has to be independent of the gauge parameter to all orders of perturbation theory. This expresses the constraint we have found for the \(\xi\)-dependence of the vertex function \(\Gamma_{A,\mu}\phi_1\phi_2\) from the ST identity \((5.13)\) at the level of the Ward identity, where it is easily manageable in concrete calculations.

The derivation of the local \(\chi\)-dependent Ward identity relies on the same two ingredients as the one of the rigid Ward identity, namely the action principle and the transformation behaviour of the local Ward operator \(w^{\text{gen}}(x)\) \((7.2)\) under BRS transformations. Combining the information gained about the transformation of the vertex functional under rigid symmetry \((6.14)\) and about the existence of the local Ward identity at \(\chi = 0\) \((7.1)\) the action principle tells us that at \(\chi \neq 0\) the local Ward identity is at least broken by the local polynomial \(\chi D_{br}(x)\) and the divergence of a \(\chi\)-dependent current \(\chi \partial_{\mu} j^\mu\), i.e.:

\[
((e + \delta e)w^{\text{gen}}(x) - \partial_{\mu} \delta A_{\mu})\Gamma = \Box B + (e + \delta e)\chi D_{br}(x) + \chi [\partial_{\mu} j^\mu] \cdot \Gamma \tag{7.3}
\]

\(D_{br}(x)\) is defined to be the non-integrated breaking term \(\Delta_{br}\) \((6.12)\) of the rigid WI:

\[
D_{br}(x) = \partial_{\xi} \left( z^{-1} Y_1 \varphi_2 - z Y_2 (\varphi_1 - \hat{\xi} m \frac{m}{e}) \right) , \quad \Delta_{br} = \int d^4x \ D_{br}(x) \tag{7.4}
\]

The current \(j_{\mu}\) has \(\phi\pi\)-charge \(-1\), dimension 3 and is odd under charge conjugation. Inspecting the quantum numbers of the fields in question it turns out that there is only one field polynomial satisfying the above requirements,

\[
j_{\mu} = u \partial_{\mu} \bar{c} , \tag{7.5}
\]

which is linear in the propagating fields and therefore a trivial insertion to all orders.

Turning now to the consistency conditions for the local Ward identity one derives in direct analogy to \((6.18)-(6.21)\):

\[
0 = w^{\text{gen}}(x) S(\Gamma) = s_{\Gamma}(w^{\text{gen}}(x) \Gamma - \chi D_{br}(x)) , \tag{7.6}
\]

\[
0 = \partial_{\mu} \frac{\delta}{\delta A_{\mu}} S(\Gamma) = s_{\Gamma} \left( \partial_{\mu} \frac{\delta}{\delta A_{\mu}} \Gamma \right) \tag{7.7}
\]

Applying the \(\chi\)-dependent \(s_{\Gamma}\)-operator \((6.19)\) to eq. \((7.3)\) and using the consistency condition \((7.6)\) one finds the following algebraic constraint valid to all orders of perturbation theory:

\[
\chi \left( \partial_{\xi} (e + \delta e) \right) w^{\text{gen}} \Gamma = -\chi s_{\Gamma} u \Box \bar{c} = -\chi u \Box B \tag{7.8}
\]
In the tree approximation, where \( \partial_\xi e = 0 \) is trivially fulfilled, it follows

\[
u^{(0)} = 0 ,
\]

which is in agreement with the explicit determination.

For higher orders we have to note that the two insertions \( w^{\text{gen}} \Gamma \) and \( \Box B \) are independent and have to vanish separately in order to fulfill the consistency condition (7.8). This implies to all orders

\[
u = 0 \quad \text{and} \quad \partial_\xi \delta e = 0 .
\]

The independence of the two insertions is seen immediately by calculating the lowest order of \( w^{\text{gen}} \Gamma \) at \( \chi = 0 \):

\[
w^{\text{gen}} \Gamma \bigg|_{\chi=0} = \partial^\mu [ j_\mu^{\text{matter}}] \cdot \Gamma
\]

with

\[
j_\mu^{\text{matter}} = \tilde{\varphi}_2 \partial_\mu \varphi_1 - (\tilde{\varphi}_1 + \frac{m}{e}) \partial_\mu \varphi_2 + e A_\mu (\tilde{\varphi}_2^2 + (\varphi_1 + \frac{m}{e})^2) + O(\hbar)
\]

\( j_\mu^{\text{matter}} \) is – in contrast to \( \Box B \) – non-trivially inserted into higher orders’ loop diagrams.

Therefore the local Ward-identity

\[
\left( (e + \delta e) w^{\text{gen}}(x) - \partial_\mu \frac{\delta}{\delta A_\mu} \right) \Gamma = \Box B + (e + \delta e) \chi D_{\mu}(x)
\]

with the restriction

\[
\partial_\xi (e + \delta e) = 0
\]

is established to all orders of perturbation theory. Because of the spontaneous symmetry breaking, in \( w^{\text{gen}} \) there appear the derivatives with respect to \( \varphi_2 \) and \( \tilde{\varphi}_2 \). Hence local gauge invariance is non-trivially formulated at the level of Green functions when compared to classical gauge invariance acting on fields.

As we have already mentioned the main result is the \( \xi \)-independence of the factor \( e + \delta e \) appearing in the local Ward identity. This is a highly non-trivial result in higher orders of perturbation theory and can be deduced only with the formalism of a BRS-varying gauge parameter \( \xi \). Finding in an explicit calculation \( \delta e \) to be \( \xi \)-dependent means that the normalization conditions imposed are not in agreement with the \( \chi \)-enlarged Slavnov-Taylor identity and it is suggested that under such circumstances the \( \xi \)-independence of the S-matrix cannot be proven which in turn is a desaster for the definition of the theory.

Therefore it is obvious that we can also use the local Ward identity in order to fix the coupling \( e \), i.e. we require the local Ward identity to be exact to all orders of perturbation theory:

\[
((e w^{\text{gen}}(x) - \partial_\mu \frac{\delta}{\delta A_\mu}) \Gamma \bigg|_{\chi=0} = \Box B
\]

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This last eq. reads for the vertex function in question:

\[
\begin{multline}
    e \left( z^{-1} \Gamma_{\varphi_1 \varphi_1} (p_1^2) - z (\Gamma_{\varphi_2 \varphi_2} (p_2^2) - \hat{\xi}_A \frac{m}{z} \Gamma_{\varphi_2 \varphi_2} (p_1, p_2) - \xi A \frac{\mu}{z} \Gamma_{\varphi_2 \varphi_2} (p, p_1, p_2) - \xi A \frac{\nu}{z} \Gamma_{\varphi_2 \varphi_2} (p, p_1, p_2) \right)
    = i (p_1^\mu + p_2^\mu) \Gamma_{A \varphi_1 \varphi_2} (p, p_1, p_2) \\
    \text{with } p^\mu + p_1^\mu + p_2^\mu = 0
\end{multline}
\]

(7.16)

As long as we do not apply a normalization condition for fixing the coupling \( e \), the vertex \( \Gamma_{A \varphi_1 \varphi_2} (p, p_1, p_2) \) is only determined up to a local contribution, i.e. a finite counterterm, which can be added in each order:

\[
\Gamma_{A \varphi_1 \varphi_2}^{(n)} (p, p_1, p_2) = \Gamma_{A \varphi_1 \varphi_2}^{(n)} (p, p_1, p_2) - ie (p_1^\mu - p_2^\mu) \delta z_e (\xi)
\]

(7.17)

\( \Gamma_{A \varphi_1 \varphi_2} \) and \( \Gamma_{A \varphi_1 \varphi_2}^{(n)} \) are equivalent solutions of the ordinary ST-identity at \( \chi = 0 \) for an arbitrary \( \delta z_e (\xi) \).

It was shown, however, that the dependence on the gauge parameter cannot be arbitrarily adjusted. Instead, this has to be done in such a way that the ST identity holds at \( \chi \neq 0 \).

Equivalently – and this is the remarkable point of the analysis – one can require that the \( \xi \)-independence of the factor \( e + \delta e \) is not ruined. This is trivially fulfilled if we require \( \delta e = 0 \). It is seen that the local Ward identity (7.13) uniquely determines \( \delta z_e \), if all the other normalization conditions are applied, rigid invariance is established and the Slavnov-Taylor identity at \( \chi = 0 \) holds.

Hence taking the on-shell conditions together with the requirement “Ward identity exact to all orders” (7.13) the determination of the gauge parameter dependence is in agreement with the Slavnov-Taylor identity at \( \chi \neq 0 \). In explicit calculations these conditions ensure that gauge parameter dependence is handled correctly.

Due to the spontaneous symmetry breaking the requirement “Ward identity of local invariance being exact” is not immediately related to a physical interpretation of the coupling, as can be done in QED. There the same requirement is even physical, because the coupling is seen to be the fine structure constant in the Thompson limes. Using the Ward identity of rigid symmetry and the complete on-shell conditions for the Higgs particle (2.18), (2.20) one finds:

\[
\frac{p^\mu \Gamma_{A \varphi_1 \varphi_2} (p, p_1, p_2)}{p_1^2 - p_2^2} \bigg|_{p^2 - m^2 = m_H^2} = ez
\]

(7.18)

with \( z \) being determined from the rigid Ward identity. A similar expression can be derived for the on-shell conditions, when the residuum of the Higgs is fixed at an arbitrary normalization point \( \kappa \). In any case the parameter \( e \) has to be adjusted to its physical
value by calculating a physical process, which is manifestly gauge parameter independent by construction.

8. Conclusions

In the present paper we have investigated the renormalization of the Abelian Higgs model including a BRS-varying gauge parameter. The techniques we have presented relate gauge parameter dependence of the Green functions to the evaluation of a class of ghost diagrams via the enlarged Slavnov-Taylor identity. This procedure is well-defined and can be carried out by just implementing additional Grassmann valued vertices into the action. The diagrams, which determine gauge parameter dependence, have a much simpler structure when compared to the original ones, in general they are also less divergent.

The importance of the algebraic method for controlling gauge parameter dependence is founded in the fact that the Green functions constructed with a BRS-varying gauge parameter are determined in a way which ensures all the physical quantities to be gauge parameter independent just by construction. Especially it is seen, that the normalization conditions, which fix the physical parameters of the model (like the masses of the vector and the Higgs and the coupling constant), cannot be chosen arbitrarily concerning gauge parameter dependence. The advantage of such a construction is obvious, because one is forced to adjust the counterterms correctly already in the procedure of renormalizing the 1-PI Green functions. Otherwise, if one does not introduce a BRS-varying gauge parameter, it can happen, that gauge parameter dependence enters the S-matrix by a wrong adjustment of counterterms. Then at the end, this dependence has to be removed order by order in perturbation theory by carefully adjusting the gauge parameter dependent part of the counterterms once again. Such a treatment is not transparent and difficult to control, especially if the model contains several couplings and parameters.

In the Abelian Higgs model, which we have chosen as the simplest example of a spontaneously broken gauge theory, the results of the algebraic method are quite impressive: Inspection of the Slavnov-Taylor identities and the corresponding diagrams tells us that the transversal part of the vector self energy is independent of the gauge parameter to all orders of perturbation theory. Furthermore, it is seen that the on-shell conditions for the physical particles are in complete agreement with the restrictions arising from the enlarged Slavnov-Taylor identity. In the Abelian Higgs model one is able to derive a local Ward identity of gauge symmetry. We have proven, that the construction prohibits the gauge parameter to enter the overall normalization factor of the matter transformations.
(7.13), which in lowest order coincides with the coupling. Vice versa, making the Ward identity exact to all orders, ensures by itself that gauge parameter dependence is treated correctly.

The techniques applied here are universal and can be generalized to more complex models and situations, whenever one is interested in an explicit knowledge about gauge parameter dependence or whenever one wants to construct and to analyze gauge parameter independent quantities. The S-matrix is the most important object for such considerations. In this context, the analysis of gauge parameter dependence in the case when the theory contains unstable particles is outstanding: The Slavnov-Taylor identity holds as it is and it remains to study the consequences thereof in order to arrive at a proper definition of the mass and width of the unstable particles.

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Appendix A

In this appendix we present (without any calculation) the general solution of the gauge condition (4.2) and the (usual, that is \(\chi\)-independent) ST identity

\[ S(\hat{\Gamma}) = \int \left\{ \partial\mu c \frac{\delta \hat{\Gamma}}{\delta A_\mu} + B \frac{\delta \hat{\Gamma}}{\delta \bar{c}} + \frac{\delta \hat{\Gamma}}{\delta \bar{c}} \frac{\delta \hat{\Gamma}}{\delta \bar{c}} + q \frac{\delta \hat{\Gamma}}{\delta \bar{c}} \right\} = 0 \]  

(A.1)

in the classical approximation. It turns out that the classical solution can be decomposed as follows

\[ \hat{\Gamma}_{\text{cl}} = \Lambda(A_\mu, \bar{\varphi}_1, \bar{\varphi}_2) + \Gamma_{g.f.} + \Gamma_{\phi \pi} + \Gamma_{e.f.}, \]  

(A.2)

where

\[ \bar{\varphi}_i = \varphi_i - x_i \hat{\varphi}_i, \ i = 1, 2. \]  

(A.3)

The matter part \( \Lambda = \Lambda(A_\mu, \bar{\varphi}_1, \bar{\varphi}_2) \) is given by

\[ \Lambda = \int \left\{ - \frac{z_A}{4} F_{\mu\nu} F^{\mu\nu} + \frac{1}{2} \bar{z}_1 (\partial_\mu \bar{\varphi}_1) (\partial_\mu \bar{\varphi}_1) + \frac{1}{2} \bar{z}_2 (\partial_\mu \bar{\varphi}_2) (\partial_\mu \bar{\varphi}_2) \right\} \]
+ \frac{1}{2} z_e^2 c^2 z_A (z_1 \varphi_1^2 + z_2 \varphi_2^2) A^\mu + \frac{1}{2} z_A^2 z_2 \varphi_2^2 A^\mu \\
+ \frac{1}{2} z_m m^2 z_A A^\mu - \sqrt{2} z_m m \sqrt{z_A} (\partial_\mu \varphi_2) A^\mu + z_e e \sqrt{z_m m} \sqrt{z_A} \varphi_1 A^\mu A^\mu \\
+ \frac{1}{2} \mu^2 (z_1 \varphi_1^2 + 2 \sqrt{z_1} \frac{z_m m}{z_e e} \varphi_1 + z_2 \varphi_2^2) \\
- \frac{1}{8} z_m m^2 \frac{m_H}{z_e e} \sqrt{2} \varphi_1 (z_1 \varphi_1^2 + 2 \sqrt{z_1} \frac{z_m m}{z_e e} \varphi_1 + z_2 \varphi_2^2)^2 \right\} \quad (A.4)

The gauge fixing part \( \Gamma_{g.f.} \) directly results from integrating the gauge condition (4.2):

\[
\Gamma_{g.f.} = \int \left\{ \frac{1}{2} \xi B^2 + B \partial A - eB \left[ (\varphi_1 - \xi A m/e) \varphi_2 - \varphi_2 (\varphi_1 - \xi A m/e) \right] \right\} \quad (A.5)
\]

The remaining two parts, the external field part \( \Gamma_{e.f.} \) and the \( \phi \pi \)-part \( \Gamma_{\phi \pi} \), have the form

\[
\Gamma_{e.f.} = \int \left\{ Y_1 (-e z_e \sqrt{z_1} \sqrt{z_A} \varphi_2 c + x_1 q_1) + Y_2 (e z_e \sqrt{z_1} \sqrt{z_A} (\varphi_1 + \frac{z_m m}{\sqrt{z_1} z_e e}) c + x_2 q_2) \right\} \quad (A.6)
\]

and

\[
\Gamma_{\phi \pi} = \int \left\{ -\bar{c} \Box c + e \bar{c} (q_1 \varphi_2 - q_2 (\varphi_1 - \xi A m/e)) \\
+ e \bar{c} (\varphi_1 - \xi A m/e) (z_e e \sqrt{z_1} \sqrt{z_A} (\varphi_1 + \frac{z_m m}{\sqrt{z_1} z_e e}) c + x_2 q_2) \\
- e \bar{c} \varphi_2 (-z_e \sqrt{z_1} \sqrt{z_A} \varphi_2 c + x_1 q_1) \right\} . \quad (A.7)
\]

This general solution of the ST identity (A.1) contains quite a number of so far free parameters, namely the wave function normalizations \( z_1, z_2 \) and \( z_A \), the mass renormalizations of the vector and the Higgs-particle, i.e. \( z_m, z_{mH} \), the coupling renormalization \( z_e \), the parameters \( x_1, x_2 \), the t’Hooft gauge parameter \( \xi_A \) and the parameter \( \xi_A \), which are not prescribed by the ST identity (A.1) and which therefore have to be fixed by appropriate normalization conditions to all orders (see section 2).

**Appendix B**

According to the considerations of section 5 (see especially (5.12)) gauge parameter dependence of the Higgs self-energy is completely governed by the non-local contributions to the vertex \( \partial_\lambda \gamma_{\lambda \phi \varphi} \). In the following we sketch the respective one-loop diagrams:
Figure 1: 1-loop order contributions to $\partial_\lambda \Gamma_{Y_1 \phi_1}$

Please note that in the non-local contributions to the vertex $\partial_\lambda \Gamma_{Y_1 \phi_1}$ there appear the mixed propagators

$$G_{BA_\mu}(p, -p) = \frac{-p^\mu}{p^2 - \xi_A m^2},$$
$$G_{B\phi_2}(p, -p) = \frac{-im}{p^2 - \xi_A m^2},$$
$$G_{\phi_2 A_\mu}(p, -p) = \frac{-m(\xi - \xi_A)p^\mu}{(p^2 - \xi_A m^2)^2}. \quad (B.1)$$

In the 't Hooft gauge $\xi_A = \xi$ the last two diagrams of figure 1 are absent (because the
\( \varphi_2 - A_\mu \)-propagator vanishes), instead there is the additional vertex \( \chi \bar{c} \varphi_2 m \) and therefore:

\[
Y_1 \varphi_2 \chi m \bar{c} c \varphi_1
\]

Figure 2: Additional contribution in the 't Hooft gauge
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