Low computational cost method for online parameter identification of Li-ion battery in battery management systems using matrix condition number

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Abstract—Monitoring the state of health for Li-ion batteries is crucial in the battery management system (BMS), which helps end-users use batteries efficiently and safely. Battery state of health can be monitored by identifying parameters of battery models using various algorithms. Due to the low computation power of BMS and time-varying parameters, it is very important to develop an online algorithm with low computational cost. Among various methods, Equivalent circuit model (ECM) - based recursive least squares (RLS) parameter identification is well suited for such difficult BMS environments. However, one well-known critical problem of RLS is that it is very likely to be numerically unstable unless the measured inputs make enough excitation of the battery models. In this work, A new version of RLS, which is called conditioned memory recursive least squares (CMRLS) is developed for the Li-ion battery parameter identification to solve such problems and to take advantage of RLS at the same time by varying forgetting factor according to condition numbers. In CMRLS, exact condition numbers are monitored with simple computations using recursive relations between RLS variables. The performance of CMRLS is compared with the original RLS through Li-ion battery simulations. It is shown that CMRLS identifies Li-ion battery parameters about 100 times accurately than RLS in terms of mean absolute error.

Index Terms—Battery management system, Condition memory recursive least squares, Condition number, Recursive least squares.

I. INTRODUCTION

Li-ion batteries have been used in many applications. Accordingly, for efficient and safe battery usage, monitoring state of health (SOH) and state of charge (SOC) has been very important in the battery management system (BMS). In the industrial and academic fields, battery parameters such as capacity and internal resistance have been used as SOH very commonly, which is very crucial parameters for end-users. These parameter values can also used to estimate SOC because the parameters determine the dynamics of SOC. Therefore, monitoring battery parameters is crucial for SOH and SOC estimation and thus for BMS. This can be done by applying parameter estimation algorithms to mathematical models for Li-ion batteries.

Li-ion battery parameter identification methods can be categorized into two classes, methods based on electrochemical battery models and those based on equivalent circuit models (ECMs).

Many researchers have studied parameter identification methods for the electrochemical battery models. Electrochemical battery model parameter identifications have been mostly carried out using meta-heuristic algorithms such as genetic algorithms [1]–[3], adaptive exploration harmony search [4], and particle swarm optimization [5] because meta-heuristic algorithms have a capability to find a global optima of such complex battery models consisting of nonlinear algebraic equations. The meta-heuristic algorithms, however, take a lot of time because of their repeated error evaluation process until the convergence. To solve this problem, some researchers have used machine learning techniques for the parameter identification of electrochemical models [6]–[8]. However, trained machine learning models is too heavy to be embedded on the BMS microprocessor chips that is usually made cheap for competitiveness in the markets related to the Li-ion batteries. For real-time parameter monitoring for the Li-ion batteries in the BMSs, light and fast parameter identification algorithms are need.

Parameter identification based on ECMs are more suitable than electrochemical models for such purposes because parameter identification algorithms for electrochemical models are very likely to be heavy or time-consuming to deal with complex nonlinear algebraic equations of them, which is not the case for ECMs consisting of simple linear equations. ECM-based parameter identification has been usually carried out using Kalman filter (KF)-based algorithms because most ECMs consist of linear equations [9]–[12]. KF-based methods are specialized to track parameters varying fast due to their mathematically well organized prediction and update phases of each iteration. Although KF-based methods are more simple and practical than above-mentioned electrochemical model-based methods from the perspective of usage in the BMSs, there are still heavy computations such as matrix inversion and tuning of covariance matrices. Recursive least squares (RLS) is a better solution because it does not need to inverse matrices or to tune covariance matrices. Although KF-based methods can track fast-varying parameters better than RLS, RLS can show similar accuracy compared to KF-based methods in the battery parameter identification because the dynamics of the battery parameters are usually slow.

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For these reasons, RLS-based algorithms have been used in the battery parameter identifications [13]–[16]. However, one big problem of RLS-based algorithms is their well-known numerical instability problem, which is called a wind-up problem. The wind-up problem is caused when excitation of the system dealt with is poor and thus the covariance matrix of the parameter estimation in the RLS become very large [17], which leads to very high sensitivity of solved parameter values to the truncation error and the error of sensors. To solve the wind-up problem, some researchers have developed modified version of RLS by using a variable forgetting factor according to the parameter estimation errors [17], [18] or to the trace of the covariance matrix [19]. However, these methods do not measure numerical stability directly. One of the well-known direct measurement of numerical stability of the parameter identification is measure condition number of the covariance matrix.

In this paper, a new version of RLS, called condition memory recursive least squares (CMRLS), is developed for real-time Li-ion battery equivalent circuit model (ECM) parameter identification in the BMS hardware with low computing power. The newly developed CMRLS adaptively changes a forgetting factor according to the condition number of the covariance matrix so that the wind-up problem is prevented. Computing a condition number of a matrix requires an inversion operation of the matrix so that the wind-up problem is prevented. Computing a condition number of a matrix requires an inversion operation. However, these methods do not measure numerical stability directly. One of the well-known direct measurement of numerical stability of the parameter identification is measure condition number of the covariance matrix.

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II. MATRIX CONDITION NUMBER

The matrix condition number of a square matrix $A \in \mathbb{R}^{n \times n}$ is defined as follows:

$$\kappa(A) = \|A\| \|A^{-1}\|$$

where $\|A\|$ means the matrix norm of $A$. If $x$ is a solution of a linear equation $Ax = b$ where $b \in \mathbb{R}^{n \times 1}$ and $x + \Delta x$ is a solution of a linear equation $(A + \Delta A)(x + \Delta x) = (b + \Delta b)$, it can be shown that the following inequality holds with some assumptions:

$$\frac{\|\Delta x\|}{\|x\|} \leq \kappa(A)$$

Therefore, $\kappa(A)$ is a measurement of how sensitively the solution of a linear equation $Ax = b$ changes according to the perturbation of $A$ and $b \in \mathbb{R}^{n \times 1}$. The proof of (4) is presented in [20].

III. RECURSIVE LEAST SQUARES

Recursive least squares (RLS) is an recursive algorithm for solving the least squares (LS) problem of finding the parameters $\theta \in \mathbb{R}^{n \times 1}$ of a linear regression model

$$d_t = \theta^T \phi_t$$

where $d \in \mathbb{R}^{1 \times 1}$ is an output and $\phi \in \mathbb{R}^{n \times 1}$ is an input of the linear model when time series data $[\phi_0, d_0], [\phi_1, d_1], [\phi_2, d_2], ..., [\phi_t, d_t], ...$ are given. Specifically, the problem solved by RLS is to find $\theta$ minimizing the output error $e(t)$ at time step $t$ defined as

$$e(t) = \sum_{k=0}^{t} \lambda^{-k}(d_k - \theta^T \phi_k)^2$$

where $\lambda$ is a real number between 0 and 1, which is called the forgetting factor. $\lambda$ assigns bigger weights to the present data compared to the past data when computing the output error. With small lambda, a large amount of the past data is forgotten when finding optimal $\theta$ so that RLS can deal with time-varying $\theta$. It can be easily shown that the optimal solution $\theta_t$ minimizing $e(t)$ is a solution of the following linear equation:

$$\Phi_t \theta_t = \Psi_t$$

where

$$\Phi_t = \sum_{k=0}^{t} \lambda^{t-k} \phi_k \phi_k^T$$

$$\Psi_t = \sum_{k=0}^{t} \lambda^{t-k} \phi_k d_k$$

Applying the matrix inversion formula, the LS problem (7) can be formulated as a recursive algorithm which is called RLS expressed as following :

$$k_t \leftarrow \frac{P_{t-1} \phi_t}{\lambda + \phi_t^T P_{t-1} \phi_t}$$

$$\alpha_t \leftarrow d_t - \phi_t^T P_{t-1}$$

$$P_t \leftarrow \frac{P_{t-1} - k_t \phi_t^T P_{t-1}}{\lambda}$$

$$\theta_t \leftarrow \theta_{t-1} + k_t \alpha_t$$

where $P_t = [\Phi_t]^{-1}$ is covariance matrix for parameter $\theta$ estimation from the perspective of Kalman filter [21].

IV. CONDITION MEMORY RECURSIVE LEAST SQUARES FOR THE PARAMETER IDENTIFICATION OF THE LI-ION BATTERIES

A. Equivalent circuit model of the Li-ion batteries

In this paper, One of the popular equivalent circuit models (ECMs) called “1RC model” is used, which is shown in Figure 1 where $I$ is current and $V$ is terminal voltage of the battery. OCV means open circuit voltage which is assumed to be a function of SOC. The OCV-SOC relationship used in this paper is obtained from slow charge or discharge experiments(Figure 2). OCV is assumed to be linear function of SOC, which means $OCV = \beta_1 SOC + \beta_2$ where $\beta_1$ and $\beta_2$
are parameters of the ECM. Although this assumption seems unrealistic, it does not prevent the newly developed CMRLS from identifying accurate parameter values because the RLS-based methods can deal with time-varying parameters and \( \beta_1 \) and \( \beta_2 \) vary with time in the real world. Setting \( x = \begin{bmatrix} V_i \\ SOC \end{bmatrix} \) as a state vector, a state-space model for this ECM can be expressed as follows:

\[
\frac{dx}{dt} = Ax + Bu \\
y = Cx + Du
\]  

(14)  

(15)

where

\[
A = \begin{bmatrix} -\frac{1}{R_1C_1} & 0 \\ 0 & 0 \end{bmatrix}, \quad B = \begin{bmatrix} \frac{1}{C_1} \\ \frac{1}{C_1 Cap} \end{bmatrix}, \quad C = \begin{bmatrix} 1 & \beta_1 \end{bmatrix},
\]

\[
D = R_0, \quad y = V - \beta_2, \quad u = I
\]

Cap is the capacity of the battery. The state-space model in (14) and (15) can be converted into discrete-time state-space model as follows:

\[
x_{t+1} = A_dx_t + B_du_t \\
y_t = C_dx_t + D_du_t
\]  

(16)  

(17)

where

\[
A_d = e^{\Delta t A} = \begin{bmatrix} \exp \left( -\frac{\Delta t}{R_1C_1} \right) & 0 \\ 0 & 1 \end{bmatrix},
\]

\[
B_d = \left( \int e^{\Delta t B} \right) B = \begin{bmatrix} R_1 \left( 1 - \exp \left( -\frac{\Delta t}{R_1C_1} \right) \right) \end{bmatrix},
\]

\[
C_d = C = \begin{bmatrix} 1 & \beta_1 \end{bmatrix},
\]

\[
D_d = D = R_0,
\]

\[
y_t = V_t - \beta_2, \quad u_t = I_t
\]

where \( \Delta t \) is a time step. Applying z-transform to the discrete-time state-space model equations in (16) and (17) can be represented as follows:

\[
\frac{V(z) - \beta_2}{I(z)} = C_d(zI - A_d)^{-1}B_d + D_d
\]

\[
= \frac{a_3 + a_4 z^{-1} + a_5 z^{-2}}{1 + a_1 z^{-1} + a_2 z^{-2}}
\]

(18)

where \( V(z) \) is the z-transform of \( V_t \), \( I(z) \) is the z-transform of \( I_t \) and

\[
a_1 = -\exp \left( -\frac{\Delta t}{R_1C_1} \right) - 1,
\]

\[
a_2 = \exp \left( -\frac{\Delta t}{R_1C_1} \right),
\]

\[
a_3 = R_0,
\]

\[
a_4 = R_1 - R_1 a_2 + \frac{\beta_1 \Delta t}{Cap} - R_0 a_2 - R_0,
\]

\[
a_5 = -R_1 + R_1 a_2 - \frac{a_2 \beta_1 \Delta t}{Cap} + R_0 a_2
\]

Using (18), a linear regression model can be formulated as follows:

\[
V_t - V_{t-1} = \begin{bmatrix} a_2 \\ a_3 \\ a_4 \\ a_5 \end{bmatrix}^T \begin{bmatrix} V_{t-1} - V_{t-2} \\ I_{t-1} \\ I_{t-1} \\ I_{t-2} \end{bmatrix}
\]

(19)

where \( V_t - V_{t-1} \), \( \begin{bmatrix} a_2 \\ a_3 \\ a_4 \\ a_5 \end{bmatrix} \), and \( \begin{bmatrix} V_{t-1} - V_{t-2} \\ I_{t-1} \\ I_{t-1} \\ I_{t-2} \end{bmatrix} \) corresponds to \( d_t \), \( \theta_t \), and \( \phi_t \) in (5), respectively.

B. Parameter identification using condition memory recursive least squares

The strategy of the newly proposed parameter identification algorithm, which is called condition memory recursive least squares (CMRLS) in this paper, is simple. As mentioned in the section II, when given a linear equation \( Ax = b \), small condition number of \( A \) means high numerical stability for the solution of a linear equation. Because the RLS solves the linear equation in (7), the condition number of covariance matrix \( P_t \) is the important number determining how much numerically stable the solution \( theta_t \) is. Therefore, CMRLS memorizes RLS variables \( (k_t, a_t, P_t, \theta_t) \) when \( \kappa(P_t) \) is small, which are used when \( \kappa(P_t) \) becomes too big so that CMRLS
There are several design parameters of the CMRLS:

- $c_{\text{rem}}$: RLS variables are memorized $\kappa(P)$ crosses this number, this number have to be set small enough for the CMRLS memorize to variables which are numerically stable enough.
- $c_{\text{upper}}$: Memorized RLS variables is used to obtain $\theta_t$ when $\kappa(P_t)$ becomes bigger than this number. If this number is small, the solution $\theta_t$ is very likely to be stable because memorized RLS variables are frequently used but parameter tracking performance is bad because the past data is frequently used. Therefore, this number have to be tuned properly considering this trade-off.
- $\lambda_{\text{for}}$: This number is the same as forgetting factor in the original RLS.
- $\lambda_{\text{rem}}$: This number is set to be bigger than 1. The forgetting factor is replaced with this number to give large weight to the memorized RLS variables when they are used to obtain the solution $\theta_t$.

Infinity norm is used to calculate the condition number. One of the remarkable things of CMRLS is that condition number computation does not require matrix inversion which have high computational complexity. Although the condition number computation originally requires matrix inversion, in CMRLS, using the relation between $P$ and $\Phi$ ($P^{-1} = \Phi$), the computation of the condition number $\kappa(P)$ does not require matrix inversion as follows:

$$\kappa(P) = \|P\| \|P^{-1}\| = \|P\| \|\Phi\|$$  (20)

Therefore, CMRLS can be practically used in the BMS with low computation power hardware.

V. RESULTS AND DISCUSSIONS

Not only can the proposed CMRLS be practically used in the BMS, but it is shown that it also has high accuracy in identifying parameters. The validation is carried out using random pulse simulation data (Figure 4 and Figure 5). CMRLS is carried out with time step of 10 sec which is 1000 times longer than the simulation model computation time step. Considering such long time step for the CMRLS time step, it can be known that the proposed CMRLS can identify battery parameters accurately even with low sampling frequency, which motivates to use CMRLS in the BMS.

The performance of the proposed CMRLS is compared with original RLS algorithm. As shown in Figure 4 and Figure 5 CMRLS has much less parameter identification error than RLS. Table 1 represents mean estimated values and mean absolute errors of RLS and CMRLS for the result show in Figure 5.

VI. CONCLUSION

In this paper, a new version of RLS, which is called condition memory recursive least squares (CMRLS), is proposed to identify the parameters of Li-ion batteries. The proposed CMRLS can accurately identify parameters with low computational complexity compared to the other parameter
identification algorithms, which is very suitable for the usage in the commercial BMS hardware with low computing power.

| Parameter | RLS | CMRLS |
|-----------|-----|-------|
| $R_0$ [Ω] | $6.193 \times 10^{-3}$ | $2.724 \times 10^{-3}$ |
| $R_1C_1$ [sec] | $3.194 \times 10^{-3}$ | $1.615 \times 10^{-3}$ |
| $C_1$ [F] | $2.029 \times 10^{-4}$ | $1.748 \times 10^{-4}$ |
| $R_1$ [Ω] | $-5.741 \times 10^{-1}$ | $1.085 \times 10^{-1}$ |

**TABLE I**

Validation results of the proposed CMRLS.

CMRLS solves a numerical stability problem of the original RLS, one of the critical problems of it. One of the limitation is that this algorithm cannot reflect the real physical phenomena inside the Li-ion batteries. Therefore, the future work will be to identify parameters which have much more physical meanings then ECM parameters using data-driven algorithms.

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