Neutrino Oscillations Induced by Chiral Phase Transition in a Compact Star

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ABSTRACT

Electric charge neutrality in a compact star provides an important relationship between the chiral dynamics and neutrino propagation in the star. Since the sudden drop of the electron density at the critical point of the first-order chiral phase transition, the oscillation for low energy neutrinos is significant and can be regarded as a signature of chiral symmetry restoration in the core of the star.

Subject headings: dense matter—neutrinos—stars: neutron

1. Introduction

It is generally believed that there are two important QCD phase transitions in hot and dense nuclear matter. One of them is related to the deconfinement process in moving from a hadron gas to a quark-gluon plasma, and the other one describes the transition from the chiral symmetry breaking phase to the phase in which it is restored. Furthermore, it appears from lattice simulations of QCD that both transitions coincide at a temperature of about \( T = 165 \text{ MeV} \) at zero baryon density (Hwa 1990). The transitions may happen in the early universe and the early stage of relativistic heavy-ion collisions where the temperature is expected to be extremely high. The other often considered systems to realize the transition condition are compact stars where the temperature is low but the baryon density can reach several times the normal nuclear density (Glendenning 2000). From the recent lattice QCD simulations, the phase structure at high density is much more rich than that at high temperature. Apart from the quark deconfinement and chiral restoration, there may be pion superfluidity (Kogut & Sinclair 2002; He, Jin, & Zhuang 2005) and even color superconductivity (Alford, Rajagopal, & Wilczek 1998) in compact stars. Different from the temperature effect which leads to a second order phase transition or even only a crossover, the QCD phase transitions at high baryon density are all of first order (Karsch 2002) which can help us to extract signatures in experiments easily.
The signatures of a QCD phase transition that happened in a compact star depend strongly on the structure of the star, namely the equation of state of the star. Normally, a compact star is considered to be completely in a new state of matter, and a bag constant is needed to balance the pressure on the surface of the star (Glendenning 2000). In this case, there is no phase transition inside the star, and the signatures reflect only the properties of the new state of matter. If a phase transition occurs inside a compact star, the sudden change in the equation of state at the critical point of the first order phase transition may lead to some easily observable signatures. These signatures are connected to the phase transition itself and different from those related only to the new state of matter.

Neutrinos play an important role in the study on compact stars (Bethe 1990; Qian et al. 1993; Prakash et al. 1997; Yakovlev et al. 2001). In this paper, we investigate the neutrino conversion in a compact star in the frame of an effective QCD model with chiral symmetry. While it is generally believed that the neutrino oscillation is difficult to occur in compact stars (Bethe 1990), the situation may be changed when the transition from chiral restoration phase to chiral breaking phase is taken into account. We will show that a sudden drop of the electron density associated with the first order chiral phase transition leads to an remarkable neutrino oscillation.

The paper is organized as follows. In Section 2 we describe the chiral thermodynamics in the Nambu–Jona-Lasinio (NJL) model in mean field approximation and then by considering the electric charge neutrality constraint we associate the chiral properties with the inner structure of a compact star via the Tolman-Oppenheimer-Volkoff (TOV) equation. In Section 3 we study the effect of chiral phase transition on the neutrino oscillation through the neutrino propagation in the star. We summarize and discuss the obtained results in Section 4. We use the natural unit of $c = \hbar = k_B = 1$ through the paper.

2. Chiral Thermodynamics

The study on QCD phase diagrams depends on lattice QCD calculations and effective models with QCD symmetries. Since there is not yet precise lattice result at finite baryon density due to the fermion sign problem (Karsch 2002), the structure of chiral phase transition at moderate baryon density is mainly investigated in many low energy effective models. One of the models that enables us to see directly how the dynamical mechanisms of chiral symmetry breaking and restoration operate is the NJL model (Nambu & Jona-Lasinio 1961) applied to quarks (Vogl & Weise 1991; Klevansky 1992; Volkov 1993; Hatsuda & Kunihiro 1994; Buballa 2005). Recently, the trapped neutrino effect on the structure of neutron star was investigated in this model (Menezes, Providencia, & Melrose 2006; Ruster et al. 2006).
We describe the equation of state of the start with the two flavor N JL model defined by the Lagrangian density (Vogl & Weise 1991; Klevansky 1992; Volkov 1993; Hatsuda & Kunihiro 1994; Buballa 2005)

\[
\mathcal{L} = \bar{q} (i\gamma^\mu \partial_\mu - m_0 + \bar{\mu} \gamma_0) q + G \left[ (\bar{q}q)^2 + (\bar{q}i\gamma_5 \vec{\tau}q)^2 \right],
\]

where \( m_0 \) is the current quark mass, \( G \) the effective coupling constant, and \( \bar{\mu} \) the chemical potential matrix in flavor space, \( \bar{\mu} = \text{diag}(\mu_u, \mu_d) = \text{diag}(\mu_B/3 - 2\mu_e/3, \mu_B/3 + \mu_e/3) \) with \( \mu_B \) and \( \mu_e \) being the baryon and electron chemical potential, respectively.

The essential quantity characterizing a system in a grand canonical ensemble can be taken to be the thermodynamical potential \( \Omega \). In mean field approximation, it can be expressed in terms of the effective quarks (Zhuang, Hufner, & Klevansky 1994),

\[
\Omega_q = \frac{(m_q - m_0)^2}{4G} - 6 \int \frac{d^3p}{(2\pi)^3} \left[ 2E_q + T \ln \left( \left( 1 + e^{-(E_q + \mu_u)/T} \right) \left( 1 + e^{-(E_q - \mu_u)/T} \right) \right) \right]
\]

\[
\Omega_e = -2T \int \frac{d^3p}{(2\pi)^3} \ln \left( \frac{1 + e^{-(E_e + \mu_e)/T}}{1 + e^{-(E_e - \mu_e)/T}} \right)
\]

with the electron energy \( E_e = \sqrt{m_e^2 + \vec{p}^2} \) and electron mass \( m_e \), the total thermodynamic potential of the system is

\[
\Omega = \Omega_q + \Omega_e.
\]

The physical order parameter \( \langle \bar{q}q \rangle \) or the effective quark mass \( m_q \) should correspond to the minimum of the thermodynamic potential,

\[
\frac{\partial}{\partial m} \Omega(T, \mu_B, \mu_e, m_q) = 0.
\]

With the known thermodynamical potential, the pressure \( P \) and the energy density \( \epsilon \) of the system are related to \( \Omega \) by

\[
P = -\Omega,
\]

\[
\epsilon = -P - T \frac{\partial \Omega}{\partial T} - \mu_u \frac{\partial \Omega}{\partial \mu_u} - \mu_d \frac{\partial \Omega}{\partial \mu_d}.
\]
where we have considered the electric charge neutrality condition, namely that we choose the electron chemical potential \( \mu_e \) such that the system has zero net electric charge (Huang, Zhuang, & Chao 2003),

\[
    n_e = -\frac{\partial}{\partial \mu_e} \Omega (T, \mu_B, \mu_e, m_q) = 0. \tag{7}
\]

The coupled set of gap equation and neutrality equation determines self-consistently the quark mass \( m_q \) and the electron chemical potential \( \mu_e \) as functions of temperature \( T \) and baryon chemical potential \( \mu_B \). Since we focus on compact stars, the temperature effect can be safely neglected.

To simplify the calculation, we consider in the following chiral limit of the NJL model with \( m_0 = 0 \). There are only two parameters in the model, the coupling constant \( G \) and the hard momentum cutoff \( \Lambda \) to regulate the model. By fitting the pion decay constant and chiral condensate in the vacuum, they are fixed to be \( G = 5.02 \text{ GeV}^{-2} \) and \( \Lambda = 653 \text{ MeV} \) (Zhuang et al. 1994). With these values, the effective quark mass \( m_q \) and electron chemical potential \( \mu_e \) are shown in Fig. 1 as functions of \( \mu = \mu_B/3 \) at zero temperature. Since \( m_q \) can be considered as the order parameter of chiral phase transition, its jump from 294 MeV to zero at \( \mu_c = 332 \) MeV means a first order chiral phase transition. The system is in the chiral symmetry breaking phase at \( \mu < \mu_c \) and the symmetry restoration phase at \( \mu > \mu_c \). The corresponding critical energy density at the transition is \( \epsilon_c = 0.3 \text{ GeV/fm}^3 \). Due to the electric charge neutrality constraint, \( \mu_e \) jumps up at the transition point and is approximately proportional to \( \mu \) in the symmetric phase.

![Fig. 1.— The effective quark mass \( m_q \) (solid line) and electron chemical potential \( \mu_e \) (dashed line) as functions of \( \mu = \mu_B/3 \) at zero temperature in chiral limit of the NJL model.](image)

With the above equation of state with chiral phase transition, the structural of a non-
rotating compact star can be obtained by integrating the TOV equations (Glendenning 2000),
\[
\frac{dP}{dr} = -G_N \frac{(\epsilon + P)(M + 4\pi r^3 P)}{r (r - 2G_NM)},
\]
\[
\frac{dM}{dr} = 4\pi r^2 \epsilon,
\]
where \( G_N = 6.707 \times 10^{-39} \text{ GeV}^{-2} \) is the universal constant of gravitation and \( M(r) \) the gravitational mass enclosed within the radius \( r \). The chiral properties of the star is reflected in the pressure \( P(r) \) and energy density \( \epsilon(r) \). Giving a central pressure \( P(0) \) at \( r = 0 \) with \( \mu > \mu_c \) which ensures that the center of the star is in the chiral restoration phase, one obtains the \( r \)-dependence of \( P \), and the radius \( R \) of the star is determined by the condition \( P(R) = 0 \). If the star is assumed to be completely in the chiral restoration phase, one needs a bag constant to balance the nonzero pressure on the surface of the star. Taking the bag constant \( B = 75 \text{ MeV/fm}^3 \), the maximum radius of the star is \( R = 7.1 \text{ km} \) corresponding to the initial energy density \( \epsilon(0) = 1.5 \text{ GeV/fm}^3 \) and final energy density \( \epsilon(R) = 0.6 \text{ GeV/fm}^3 > \epsilon_c \). In our case with chiral phase transition inside the star, the core of the star is in the chiral restoration state and it is surrounded by the chiral breaking state. Taking into account the fact from the lattice simulations that the deconfinement and chiral restoration happen at the same critical point, the core can be regarded as a quark matter and the surrounding a hadron matter. For the initial energy density \( \epsilon(0) = 0.32 \text{ GeV/fm}^3 \), the chiral phase transition occurs at the critical radius \( R_c = 5.2 \text{ km} \).

3. Neutrino Oscillations

The neutrino oscillation in matter is very different from the oscillation in vacuum (Caldwell, Fuller, & Qian 2000; Wolfenstein 1978; Mikheyev & Smirnov 1985; Rosen & Gelb 1986; Bilenky & Petcov 1987; Gelb, Kwong, & Rosen 1997; Bargett, Phillips, & Whisnant 1986; Kuo & Pantaleone 1989; Qian & Fuller 1995; Savage, Malaney, & Fuller 1991; Notzold & Raf- felt 1988; Botella et al. 1987). We discuss in this paper only the \( \nu_e \rightarrow \nu_\tau \) oscillation in a compact star with chiral phase transition. In this case, the neutrino propagation in the star can be expressed as (Kuo & Pantaleone 1989)
\[
i \frac{d}{dr} \begin{pmatrix} \nu_e \\ \nu_\tau \end{pmatrix} = \frac{1}{4E} \left[ (\Sigma + A) + \begin{pmatrix} A - \Delta \cos 2\theta & \Delta \sin 2\theta \\ \Delta \sin 2\theta & -A + \Delta \cos 2\theta \end{pmatrix} \right] \begin{pmatrix} \nu_e \\ \nu_\tau \end{pmatrix}
\]
where \( \Sigma = m_2^2 + m_1^2 \) and \( \Delta = m_2^2 - m_1^2 \) are related to the neutrino mass eigenvalues \( m_1 \) and \( m_2 \) in vacuum, and \( \theta \) is the mixing angle between the neutrino mass eigenvalue and the
weak interaction eigenvalue. Since $\Sigma$ contributes only a global phase factor to the neutrino eigenstate and does not affect the neutrino conversion (Fukugita & Yanagida 2003), we omit it in the following. The matter dependence of the conversion is reflected in the factor $A$ defined as

$$A = 2\sqrt{2}G_FE N_e,$$

where $E$ is the neutrino energy, $G_F = 1.166 \times 10^{-5}$ GeV$^{-2}$ the Fermi constant, and $N_e$ the free electron density

$$N_e(r) = 2\int \frac{d^3p}{(2\pi)^3} \frac{1}{1 + e^{(E_e - \mu_e(r))/T}}.$$  

the $r-$dependence of the electron chemical potential is given by the TOV equation together with the equation of state. Suppose only electron neutrinos are created at the center of the star, the combination of the propagation equation (9), the TOV equation (8) and the chiral equation of state (6) controls self-consistently the space evolution of the transition from electron to tau neutrinos.

To find the most suitable electron density $N_e$ where there is remarkable neutrino oscillation, we first consider the case with space-independent $\mu_e$. In this case, the propagation equation (9) can be analytically solved, and we obtain the explicit surviving probability

$$P_e(r) = 1 - \frac{1}{2} \sin^2 2\theta_m \left( 1 - \cos \frac{2\pi r}{\lambda_m} \right),$$

where $\sin^2 2\theta_m$ and $\lambda_m$ depend on $\mu_e$ via

$$\sin^2 2\theta_m = \frac{(\Delta \sin 2\theta)^2}{(-A + \Delta \cos 2\theta)^2 + (\Delta \sin 2\theta)^2},$$

$$\lambda_m = \frac{4\pi E}{\sqrt{(\Delta \sin 2\theta)^2 + (A - \Delta \cos 2\theta)^2}}.$$  

In vacuum with $N_e = 0$, the surviving probability is reduced to

$$P_e(r) = 1 - \frac{1}{2} \sin^2 2\theta \left( 1 - \cos \frac{\Delta r}{2E} \right).$$

It is easy to see that the biggest transition in matter happens at $\sin^2 2\theta_m = 1$ which corresponds to the matter environment with resonant electron density

$$N_{er} = \frac{\Delta \cos 2\theta}{2\sqrt{2}G_FE},$$

where $\Delta, \theta$ and $G_F$ are parameters fixed in the vacuum, the only adjustable parameter is the neutrino energy $E$. For low energy neutrinos, the resonant electron density $N_{er}$, or
equivalently, the resonant electron chemical potential $\mu_{er}$ is large, and the matter induced oscillation is remarkably different from the oscillation in vacuum. With increasing neutrino energy, $N_{er}$ approaches to zero, and the difference in the oscillation between in matter and in vacuum disappears gradually.

While the resonant electron chemical potential $\mu_{er}$ is independent of the chiral thermodynamics and the star structure, the matter effect to the neutrino oscillation is sensitive to the initial condition of the TOV equation, namely the equation of state at $r = 0$. For a smooth $r$—dependence of $\mu_e(r)$ around $\mu_{er}$ characterized by the TOV equation, the $r$—integrated matter contribution to the neutrino oscillation is visible, while a sharp $\mu_e(r)$ around $\mu_{er}$ will make the matter effect to the neutrino oscillation difficult to be seen.

Taking the vacuum parameters $\Delta = 2.6 \times 10^{-3}$ eV$^2$ and $\sin^2 \theta = 0.03$ (Maltoni 2004; Apollonio 1999), the effective neutrino mixing angle in matter is shown in Fig. 2 as a function of radius $r$ for low energy neutrinos with $E = 0.1$ MeV. For the initial energy density $\epsilon(0) = 0.32$ GeV/fm$^3$, the peak $\sin^2 2\theta_m = 1$ is located at the resonant radius $R_r \sim 35$ km $> R_c$ in the chiral symmetry breaking phase. This means that a remarkable matter induced neutrino oscillation can not happen in a compact star without chiral phase transition, since the electron density $N_e$ in chiral restoration phase is too large and far away from the resonant electron density $N_{er}$.

To see the initial energy density dependence of the neutrino oscillations, we plot the electron chemical potential $\mu_e$ around $\mu_{er}$ as a function of $r$ in Fig. 3 for different initial energy densities. The slope is very sensitive to the value of $\epsilon(0)$. For $\epsilon(0) = 0.32$ GeV/fm$^3$, $\mu_e$ varies smoothly in the neighborhood of $\mu_{er}$, and therefore, the space integrated matter contribution to the neutrino oscillation can be seen clearly. For $\epsilon(0) = 0.39$ GeV/fm$^3$, $\mu_e$ changes very fast around $\mu_{er}$, and the matter contribution occurs only in a very thin spherical shell of the star. In this case, the $r$—integrated matter effect is negligible.

In Fig. 4 we show the surviving probability (solid lines) of the electron neutrinos and their transition probability to tau neutrinos (dashed line) as functions of $r$. From our assumption of no tau neutrinos at the center of the star, the surviving probability $P_e(r)$ starts propagation with the initial value $P_e(0) = 1$. In the core with chiral symmetry restoration, there is almost no neutrino conversion due to the too dense electrons. At the critical radius $R_c$ where the phase transition from chiral restoration to chiral breaking happens, the electron chemical potential and in turn the electron density drops down suddenly to a small value, and the visible conversion starts gradually. The biggest change occurs around the resonant radius $R_r$. For low energy neutrinos with $E = 0.1$ MeV, almost all the electron neutrinos are converted into tau neutrinos around $R_r$, and this conversion is kept in the further propagation. Clearly, the conversion probability is $P_\tau(r) = 1 - P_e(r)$. With increasing neutrino
Fig. 2.— The effective neutrino mixing angle $\sin^2 2\theta_m$ as a function of radius $r$ of the star with initial energy density $\epsilon(0) = 0.32 \text{ GeV/fm}^3$ for low energy neutrinos with energy $E = 0.1 \text{ MeV}$.

Fig. 3.— The electron chemical potential $\mu_e$ scaled by its resonant value $\mu_{er}$ as a function of $r - R_r$ with $R_r$ being the resonant radius where the biggest neutrino transition occurs. The solid and dashed lines correspond, respectively, to the initial energy density $\epsilon(0) = 0.32$ and $0.39 \text{ GeV/fm}^3$.

energy, the conversion is suppressed by the too large resonant radius in the chiral breaking state, as we explained above. For $E \sim 0.3 \text{ MeV}$, the maximum change from electron to tau neutrino is reduced to about 50%, and for $E > 0.8 \text{ MeV}$, there is almost no difference between the neutrino oscillations in matter and in vacuum.
4. Summary and Discussion

We have examined the $\nu_e \rightarrow \nu_\tau$ neutrino oscillation induced by chiral phase transition in a compact star. Due to the constraint of electric charge neutrality, the chiral properties controlled by baryon chemical potential $\mu_B$ and the neutrino conversion governed by electron chemical potential $\mu_e$ are related to each other. The star contains a dense core with chiral symmetry restoration rounded by the chiral breaking phase. When neutrinos propagate in the radius direction, the transition from chiral restoration to chiral breaking inside the star leads to significantly oscillations for low energy neutrinos. When the neutrino energy is low enough, the conversion probability is nearly 100%. Therefore, it is possible to detect the inner structure of compact stars in terms of low energy neutrino oscillations.

We emphasize that the remarkable oscillation is resulted from the first-order chiral phase transition. For a second-order phase transition, due to the lack of the sudden drop of the electron chemical potential, the resonant radius $R_r$ will be several hundreds or even several thousands kilometers which are certainly beyond the reasonable radius of a compact star. In our calculation, $R_r$ is about 35 kilometers for initial energy density $\epsilon(0) = 0.32$ GeV/fm$^3$ and is reduced to about 13 kilometers for $\epsilon(0) = 0.39$ GeV/fm$^3$. Since the first-order chiral phase transition is a general consequence of lattice simulations and model calculations at high baryon density, our results obtained in the frame of NJL model is still qualitatively correct in general case.

When other phase transitions at high baryon density are taken into account, what is their effect on the neutrino oscillations? Since the other transitions such as color superconductivity occur at extremely high density where the chiral symmetry is already restored, the electron
chemical potential $\mu_e$ in these new phases is much higher than the resonant value $\mu_{er}$, and therefore, these new phases will not lead to remarkable changes in the neutrino oscillations. Similarly, the nonzero current quark mass $m_0$ will change the behavior of $\mu_e$ in the chiral restoration phase considerably, but it changes the neutrino conversion slightly. The neutrino oscillation in compact star is characterized by the first-order chiral phase transition.

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