Evolution of the spin of the nucleon

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Abstract

We compare momentum sum rules from unpolarized electroproduction and the spin sum rule for $g_1$ in polarized electroproduction, and their $Q^2$ evolution in the framework of the operator product expansion. Second order effects in $\alpha_s$ are included. We show that in comparing the evolution of the spin sum rule with the momentum sum rule one is not overly sensitive to using first or second order, even when going to the extreme low $Q^2$ limit in which gluons carry no momentum. Our results show that in that limit there is no need to include any contribution of strange quarks.

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I. INTRODUCTION

Deep inelastic scattering (DIS) is an important tool for studying the structure of hadrons. Through sum rules the experiments provide values of specific quark and gluon operator matrix elements. The framework for this is the operator product expansion (OPE) \[1\]. Experimentally measurable sum rules are expressed as the product of matrix elements and coefficient functions. Examples are the momentum sum rules measured in unpolarized deep inelastic scattering and the Bjorken sum rule \[2\] and Ellis-Jaffe sum rule \[3\] in polarized deep inelastic scattering \[4\]. Initial measurements of the latter, showing deviations from the Ellis-Jaffe prediction, have been interpreted as an indication of a surprisingly small contribution of the quark spin to the nucleon spin \[5\]. One of the points relevant for the interpretation is the scale dependence of the matrix elements and the coefficient functions, which can be calculated in perturbation theory. The QCD corrections to the Bjorken sum rule up to order $\alpha_s^3$ have now been calculated in leading twist \[6–8\], and the higher twist corrections have been estimated \[4\]. Recently, the order $\alpha_s^2$ corrections to the Ellis-Jaffe sum rule in leading twist and massless quark approximation have been completed as well \[8\]. These corrections provide powerful means to further study the $Q^2$ evolution of the spin structure of the nucleon.

For matrix elements that have no scale dependence deep inelastic measurements immediately provide us with interpretable results that occasionally can be compared with other experimental measurements in a completely different domain, e.g. the Bjorken sum rule. It is well known \[5\] that the singlet part of the first moment of the spin structure function $g_1$ is not of this type and has an anomalous $Q^2$ evolution. It is also well known \[10\] that the leading term in $\alpha_s$ in the axial anomalous dimension vanishes, and for this reason some authors dismiss this $Q^2$ evolution as insignificant. Roughly speaking, corrections to the singlet first moment arising from the anomalous dimension can be argued to behave like $\alpha_s \log Q^2$, and hence appear \textit{approximately} $Q^2$ independent. In an earlier paper, \[11\] we argued that comparing momentum sum rules from unpolarized electroproduction and the spin sum rule...
for $g_1$, including their $Q^2$ evolution, showed that DIS spin measurements are consistent with a low energy valence quark picture, where the valence quarks carry a substantial part of the spin of the proton, namely of the order of $G_A/(5/3) \approx 0.75$. In this paper, we extend our earlier calculations to fully include the next higher order QCD corrections in leading twist. In this way we are able to get a feeling for the sensitivity to the use of first and second order in the evolution of the spin sum rule. We also consider the possible effects of a strange quark threshold at very low momentum scales. With the results of more recent experiments we can give an estimate of the “spin carried by quarks”.

II. FORMALISM

A. The momentum sum rules

As a typical example of an (unpolarized) “quark momentum sum rule”, the second moment of $F_2$ is given by

$$
\int_0^1 dx F_2(x, Q^2) = \sum_{i=1}^{n_f} e_i^2 \int x [q_i(x, Q^2) + \bar{q}_i(x, Q^2)] \, dx
= \sum_{i=1}^{n_f} e_i^2 \epsilon_i(Q^2) = \sum_{i=1}^{n_f} e_i^2 \epsilon_i^{NS}(Q^2) + \langle e^2 \rangle \Sigma_2(Q^2),
$$

where $q_i(x, Q^2)$ is the quark distribution function and $\epsilon_i = \epsilon_i(Q^2)$ is the momentum fraction carried by quarks and antiquarks of flavor $i$ ($n_f$ is the number of flavors), which is separated into nonsinglet (NS) and singlet contributions. The quantity $\Sigma_2 \equiv \sum_i \epsilon_i$ is the total momentum fraction carried by the (valence+sea) quarks, which can be expressed as the matrix element of the quark part of the energy momentum tensor. The quantity $\epsilon_i^{NS} \equiv \epsilon_i - \Sigma_2/n_f$ is the nonsinglet part of the second moment for flavor $i$, and $\langle e^2 \rangle$ is the average quark charge.

There is no unique way to define parton distributions beyond leading order, but we follow Buras’ conventions, including renormalization in the $\overline{\text{MS}}$ scheme. This results in the following formulae for the (unpolarized) momentum sum rules, including next to leading order corrections [13]. For the (nonsinglet) valence quark momentum sum rule
\[ V_2 = \sum_{i=1}^{n_f} \int_0^1 x[q_i(x, Q^2) - \bar{q}_i(x, Q^2)] \, dx \]  

(2)

the evolution is given by

\[ V_2(Q^2) = \exp \left( - \int_{\alpha_s(Q_0^2)}^{\alpha_s(Q^2)} d\alpha' \frac{\gamma_{NS}(\alpha')}{2\beta(\alpha')} \right) V_2(Q_0^2), \]

(3)

with the anomalous dimension given by

\[ \gamma_{NS}(\alpha_s) = \gamma_{0}^{NS} \left( \frac{\alpha_s}{4\pi} \right) + \gamma_{1}^{NS} \left( \frac{\alpha_s}{4\pi} \right)^2 + \cdots, \]

(4)

with \( \gamma_{0}^{NS} = 64/9 \) and \( \gamma_{1}^{NS} = 96.6584 - 6.32 \, n_f \). The beta function governs the behavior of the strong coupling constant,

\[ \mu^2 \frac{d\alpha_s}{d\mu^2} = \beta(\alpha_s) = -\beta_0 \frac{\alpha_s^2}{4\pi} - \beta_1 \frac{\alpha_s^3}{16\pi^2} - \cdots, \]

(5)

with \( \beta_0 = 11 - 2 \, n_f/3 \), \( \beta_1 = 102 - 38 \, n_f/3 \).

The leading order (LO) solution for the strong coupling constant is

\[ \frac{4\pi}{\beta_0 \alpha_s(Q^2)} - \log(Q^2) = \text{invariant}, \]

(6)

while the next to leading order (NLO) is

\[ \frac{4\pi}{\beta_0 \alpha_s(Q^2)} - \log(Q^2) = -\frac{\beta_1}{\beta_0} \log \left( 1 + \frac{\beta_0^2}{\beta_1 \beta_0 \alpha_s(Q^2)} \right) = \text{invariant}. \]

(7)

We use these expressions to calculate the running coupling constant, i.e. we make no further expansion.

The leading order solution for the valence quark momentum sum rule \( V_2(Q^2) \) using only the leading term in the \( \gamma \)-function reads

\[ V_2(Q^2) = \left( \frac{\alpha_s(Q^2)}{\alpha_s(Q_0^2)} \right)^{\frac{\gamma_{NS}}{\gamma_{10}}^{NS}} V_2(Q_0^2). \]

(8)

The next to leading order result reads

\[ V_2(Q^2) = \left( \frac{\alpha_s(Q^2)}{\alpha_s(Q_0^2)} \right)^{\frac{\gamma_{NS}}{\gamma_{10}}^{NS}} \left( \frac{4\pi \beta_0 + \beta_1 \alpha_s(Q^2)}{4\pi \beta_0 + \beta_1 \alpha_s(Q_0^2)} \right)^{\frac{\beta_0 \gamma_{11}^{NS} - \beta_1 \gamma_{10}^{NS}}{4\beta_0 \beta_1}} V_2(Q_0^2). \]

(9)
The NLO solution can be rewritten as the LO solution times a polynomial in $\alpha_s$. This is the result which we will refer to as truncated NLO. It reads

$$V_2(Q^2) = V_2(Q_0)^2 \left[ \frac{\alpha_s(Q^2)}{\alpha_s(Q_0^2)} \right]^{d_{NS}^{(2)}} \left( 1 + \frac{\alpha_s(Q^2) - \alpha_s(Q_0^2)}{4\pi} Z_{NS} \right),$$

where the values for $d_{NS}^{(2)}$ and $Z_{NS}$ can be found in Table I.

For the singlet part the second moment of the distribution function for quarks, $\Sigma_2(Q^2)$, mixes with the second moment of the gluon distribution, $G_2(Q^2)$. One has, however, $\Sigma_2(Q^2) + G_2(Q^2) = 1$, which makes it possible to write the evolution as

$$G_2(Q^2) = M_S(\alpha_s(Q^2)) \left[ G_2(Q_0^2) + \int_{\alpha_s(Q_0^2)}^{\alpha_s(Q^2)} d\alpha' \frac{\gamma_{qq}(\alpha')}{2\beta(\alpha')} M_S(\alpha') \right],$$

with

$$M_S(\alpha_s) = \exp \left( - \int_{\alpha_s(Q_0^2)}^{\alpha_s} d\alpha' \frac{(\gamma_{qq}(\alpha') + \gamma_{GG}(\alpha'))}{2\beta(\alpha')} \right).$$

Here $\gamma_{qq}(\alpha_s)$ and $\gamma_{GG}(\alpha_s)$ are elements of the singlet anomalous dimension matrix, expanded in $\alpha_s$ in the same way as the nonsinglet anomalous dimension function. For the second moment the anomalous dimensions obey $\gamma_{qq} = -\gamma_{Gq}$ and $\gamma_{GG} = -\gamma_{qG}$. The expansion coefficients are $\gamma_{qq,0} = 64/9$, $\gamma_{qq,1} = 96.6584 - 10.2716 \ n_f$, $\gamma_{GG,0} = 4n_f/3$ and $\gamma_{GG,1} = 15.0864 \ n_f$.

The solution for the function $M_S(\alpha_s)$, appearing in the evolution of the singlet quark momentum sum rule is in leading order given by

$$M_S(\alpha_s) = \left( \frac{\alpha_s}{\alpha_s(Q_0^2)} \right) \left( \frac{\gamma_{qq,0} + \gamma_{GG,0}}{2\pi_0} \right)^{\frac{\gamma_{qq,0} + \gamma_{GG,0}}{2\pi_0}} \left( \frac{4\pi_0 + \beta_1 \alpha_s(Q^2)}{4\pi_0 + \beta_1 \alpha_s(Q_0^2)} \right).$$

The NLO solution for $M_S(\alpha_s)$ is given by

$$M_S(\alpha_s) = \left( \frac{\alpha_s}{\alpha_s(Q_0^2)} \right) \left( \frac{\gamma_{qq,0} + \gamma_{GG,0}}{2\pi_0} \right)^{\frac{\gamma_{qq,0} + \gamma_{GG,0}}{2\pi_0}} \left( \frac{4\pi_0 + \beta_1 \alpha_s(Q^2)}{4\pi_0 + \beta_1 \alpha_s(Q_0^2)} \right)^{\frac{\gamma_{qq,1} + \gamma_{GG,1} - \beta_1 (\gamma_{qq,0} + \gamma_{GG,0})}{2\pi_0 \beta_1}}.$$

This leads to the following LO solution for $G_2$,

$$G_2(Q^2) = \frac{\gamma_{qq,0}}{\gamma_{qq,0} + \gamma_{GG,0}} + \left( \frac{\alpha_s}{\alpha_s(Q_0^2)} \right) \left( \frac{\gamma_{qq,0} + \gamma_{GG,0}}{2\pi_0} \right)^{\frac{\gamma_{qq,0} + \gamma_{GG,0}}{2\pi_0}} \left[ G_2(Q_0^2) - \frac{\gamma_{qq,0}}{\gamma_{qq,0} + \gamma_{GG,0}} \right].$$
For the NLO solution we do not have an analytic expression, but using the result for $M^S$ a numerical solution is easily obtained. The truncated NLO result is given by the coupled equations

\[
\Sigma_2(Q^2) = \left[ (1 - \tilde{\alpha}) \Sigma_2(Q_0^2) - \tilde{\alpha} G_2(Q_0^2) \right] \left[ \frac{\alpha_s(Q^2)}{\alpha_s(Q_0^2)} \right]^{\frac{d(2)}{4\pi}} \left( 1 + \frac{\alpha_s(Q^2) - \alpha_s(Q_0^2)}{4\pi} Z_+ \right) + \tilde{\alpha} \left[ 1 + \left\{ \frac{\alpha_s(Q_0^2)}{4\pi} \left[ \frac{\alpha_s(Q^2)}{\alpha_s(Q_0^2)} \right]^{\frac{d(2)}{4\pi}} - \frac{\alpha(Q^2)}{4\pi} \right\} K^\psi \right],
\]

(16)

\[
G_2(Q^2) = \left[ (1 - \tilde{\alpha}) \Sigma_2(Q_0^2) + \tilde{\alpha} G_2(Q_0^2) \right] \left[ \frac{\alpha_s(Q^2)}{\alpha_s(Q_0^2)} \right]^{\frac{d(2)}{4\pi}} \left( 1 + \frac{\alpha_s(Q^2) - \alpha_s(Q_0^2)}{4\pi} Z_+ \right) + (1 - \tilde{\alpha}) \left[ 1 + \left\{ \frac{\alpha_s(Q_0^2)}{4\pi} \left[ \frac{\alpha_s(Q^2)}{\alpha_s(Q_0^2)} \right]^{\frac{d(2)}{4\pi}} - \frac{\alpha(Q^2)}{4\pi} \right\} K^G \right].
\]

(17)

The $d$'s are the relevant anomalous dimensions for this moment, here evaluated to first order. Higher order corrections come from the $Z$’s and $K$’s, which are ($Q^2$ independent) coefficients tabulated in table I. Note also that conservation of momentum requires $\Sigma_2(Q^2) + G_2(Q^2) = 1$, which the above moments satisfy due to the relation between $\tilde{\alpha}, K^G$, and $K^\psi$.

**B. The singlet spin sum rule**

The singlet piece of the first moment of $g_1$ has recently been computed to next to leading order in $\alpha_s$ [8]. This includes both the singlet coefficient function $C^S$, as well as the anomalous dimension, $\gamma^S$ of the singlet axial current in the $\overline{MS}$ scheme, using dimensional regularization. In the adopted normalization, this yields for the sum rule expressed in terms of the singlet axial matrix element

\[
\Gamma_1^S(Q^2) = \int_0^1 g_1^S(x, Q^2) dx = C^S(\alpha_s(Q^2)) \Delta \Sigma(Q^2),
\]

(18)

with

\[
\Delta \Sigma(\mu^2)s_\sigma = \langle p, s | J_\sigma^5 | p, s \rangle = \langle p, s | \sum_{i=1}^{n_f} \bar{q}_i \gamma_\sigma \gamma_5 q_i | p, s \rangle \equiv (\Delta u + \Delta d + \Delta s + \cdots) s_\sigma,
\]

(19)

the quantity sometimes interpreted as the spin carried on the quarks. The coefficient function is given by
\[ C^S(\alpha_s) = 1 - \frac{\alpha_s}{\pi} + \frac{\alpha_s^2}{\pi^2} (-4.5833 + 1.16248 n_f). \] 

(20)

Not included in the sum rule for \( g_1 \) in Eq. \[15 \] are higher twist contributions. The matrix element in Eq. \[15 \] is scale dependent,

\[ \Delta \Sigma(Q^2) = \exp \left( - \int_{\alpha_s(Q_0^2)}^{\alpha_s(Q^2)} d\alpha' \frac{\gamma^S(\alpha')}{2\beta(\alpha')} \right) \Delta \Sigma(Q_0^2), \]

(21)

governed by the anomalous dimension, which with our normalization is

\[ \gamma^S(\alpha_s) = \gamma_1^S \left( \frac{\alpha_s}{4\pi} \right)^2 + \gamma_2^S \left( \frac{\alpha_s}{4\pi} \right)^3 + \cdots \]

(22)

with \( \gamma_1^S = 16 n_f \) and \( \gamma_2^S = 314.67 n_f - 3.556 n_f^2 \).

The LO solution for the singlet axial charge \( \Delta \Sigma \) reads

\[ \Delta \Sigma(Q^2) = \exp \left( \frac{\gamma_1^S}{8\pi\beta_0} (\alpha_s(Q^2) - \alpha_s(Q_0^2)) \right) \Delta \Sigma(Q_0^2), \]

(23)

while the NLO solution reads

\[ \Delta \Sigma(Q^2) = \left( \frac{4\pi\beta_0 + \beta_1 \alpha_s(Q^2)}{4\pi\beta_0 + \beta_1 \alpha_s(Q_0^2)} \right)^{2\beta_1} \exp \left( \frac{\gamma_2^S}{8\pi\beta_1} (\alpha_s(Q^2) - \alpha_s(Q_0^2)) \right) \Delta \Sigma(Q_0^2). \]

(24)

Finally, the truncated NLO solution for \( \Delta \Sigma \) is

\[ \Delta \Sigma(Q^2) = \left( 1 + \frac{\gamma_1^S}{8\pi\beta_0} (\alpha_s(Q^2) - \alpha_s(Q_0^2)) \right) + \left( \frac{\beta_0 \gamma_2^S - \beta_1 \gamma_1^S}{64\pi^2\beta_0^2} \right) (\alpha_s^2(Q^2) - \alpha_s^2(Q_0^2)) + \frac{(\gamma_1^S)^2}{128\pi^2\beta_0^2} (\alpha_s(Q^2) - \alpha_s(Q_0^2))^2 \right) \Delta \Sigma(Q_0^2). \]

(25)

Experimental results are mostly given for \( \Delta \Sigma(Q^2) \), which is obtained from the experimental sum rule by explicitly factoring out the coefficient function \( C^S(\alpha_s(Q^2)) \), but not factoring out the \( Q^2 \) dependence in the matrix element, given in Eq. \[21 \]. Note that \( C^S(Q^2) \) differs at second order and beyond from the analogous function \( C^{NS}(Q^2) \),

\[ C^{NS}(\alpha_s) = 1 - \frac{\alpha_s}{\pi} + \frac{\alpha_s^2}{\pi^2} (-4.5833 + n_f/3). \]

(26)

which appears in the Bjorken sum rule. Being nonsinglet, the Bjorken sum rule has no analogous anomalous dimension correction.
III. RESULTS

In our previous work [11], we proposed fixing a quark model scale, \( Q_0^2 \), where e.g. \( G_2(Q_0^2) = 0 \), and/or \( V_2(Q_0^2) = 1 \). This can be obtained by evolving from experimental values at high \( Q^2 \) [15]. The spin sum rule was considered in the same way, with a starting point \( \Delta \Sigma(Q_0^2) \) taken from a quark model value, and then evolved up to \( Q^2 \) relevant to DIS experiments. In the bag model the estimate for \( \Delta \Sigma(Q_0^2) \approx 0.65 \) [11], the reduction from unity coming from the lower components of the (relativistic) quark spinors, the same source which reduces the axial charge in the bag model from its nominal value of 5/3.

The most important improvement of the results of [11] is the inclusion of the effects of corrections in the next order in \( \alpha_s \). This of course does not justify the use of perturbation theory in the domain where we are using it, going to rather large values of the strong coupling constant, \( \alpha_s \sim 2 \). On the other hand, we can get some feeling for the convergence or nonconvergence of our approach by comparing first and second order evolution for the various moments.

It is well-known that for the running coupling constant the difference between the first and second order results is large when one looks at the functional dependence of \( \alpha_s \) on \( Q^2 \). Similarly the evolution of the moments as a function of \( \alpha_s \) can also be strongly dependent on the order. Evolving down from \( \alpha_s(M_Z^2) = 0.117 \) [14,15], and the starting value \( G_2(4 \text{ GeV}^2) = 0.44 \) [15], using \( \text{LO} \) order equations, gives \( G_2 = 0 \) when \( \alpha_s = 1.80 \). Using \( \text{NLO} \) order equations gives \( G_2 = 0 \) when \( \alpha_s = 1.79 \). Much larger is the difference for the valence momentum sum rule. Here one finds that starting from \( V_2(4 \text{ GeV}^2) = 0.40 \), using \( \text{LO} \) order equations, gives \( V_2 = 1 \) when \( \alpha_s = 2.77 \). Using \( \text{NLO} \) order equations gives \( V_2 = 1 \) when \( \alpha_s = 2.21 \).

When we plot moments against each other as done in Fig [1], we notice that the NLO calculations (dashed line) do not exhibit a drastically different behavior as compared to the LO calculations (solid). When we show NLO calculations, the dashed line shows the exact solution to the evolution equations. The dotted line shows the NLO truncated expansions.
in $\alpha_s$ given in Eqs 10, 16 and 17. The dot-dashed line is the same truncated expansion, but any terms involving higher order corrections in $\alpha_s$ are evaluated using the leading order expression for $\alpha_s$, as suggested in ref [12]. Comparison of the dotted, dot-dashed and solid line indicate typical uncertainties in the NLO result. The differences between them is one order in $\alpha_s$ higher.

The same comparison of LO (solid) and three approximations for the NLO results can be seen in Figs 2 and 3, which show $\Delta \Sigma$ plotted against $V_2$ and $G_2$ respectively. For the spin sum rule, we have compared our results with $\Delta \Sigma(Q_{\exp}^2 = 10 \text{ GeV}^2)$ because we note from Eq. (21) that the results remain proportional to the starting value at all $Q^2$. This makes it easy to consider any scenario, e.g. starting from a world average such as $\Delta \Sigma(4 \text{ GeV}^2) = 0.31$ or starting from a low-energy value [11].

In Fig. 1, $\alpha_s$ is increasing down and to the right. There is no single value of $Q_0^2$ where both $V_2 = 1$ and $G_2 = 0$, in either LO or NLO perturbation theory. In both cases $G_2$ vanishes earlier (at higher $Q^2$), which is consistent with an intuitive picture that at the quark model scale, meson-cloud effects result in some residual $q\bar{q}$ sea. Note that if one plots e.g. $V_2$ vs $\alpha_s$, there is a stronger dependence on the order of perturbation theory used, as the evolution of $\alpha_s$ is itself highly modified at these low $Q^2$. It is encouraging that when these observables are plotted against one another, the trends are similar. Furthermore, varying the value of $\alpha_s(M_Z^2)$ within current experimental limits (e.g. using values ranging from 0.11 to 0.12) has negligible effect on these curves.

In Fig. 2, we show the relative value of the singlet axial matrix element $\Delta \Sigma$ versus $V_2$, normalized to the value at $Q_{\exp}^2 = 10 \text{ GeV}^2$, the characteristic scale of the SMC experiment. The right edge corresponds to a value of $Q_0^2$ where $V_2(Q_0^2) = 1$. We see that $\Delta \Sigma(Q_0^2)/\Delta \Sigma(10)$ increases from about 1.71 (LO) to 1.98 (NLO). This increase of the NLO evolution is, unlike that in Fig. 1, sensitive to the value of $\alpha_s(M_Z^2)$. Increasing the value of $\alpha_s(M_Z^2)$ by 5%, gives ratios of 1.83 (LO) and 2.21 (NLO).

In Fig. 3, $\Delta \Sigma$ is plotted against $G_2$, again normalized to its value at $Q_{\exp}^2 = 10 \text{ GeV}^2$. Evolving from 10 GeV$^2$ to $Q_0^2$, this time fixed from $G_2(Q_0^2) = 0$, gives for the
ratio $\Delta \Sigma(Q_0^2)/\Delta \Sigma(Q^2_{\text{exp}})$ values of 1.39 (LO) to 1.68 (NLO). The ratios are smaller than in Fig. 2, because $G_2 = 0$ corresponds to a value of $V_2 < 1$. Again a 5% larger value of $\alpha_s(M_Z^2)$ results in somewhat larger ratios, 1.45 (LO) and 1.85 (NLO). The persistent enhancement of $\Delta \Sigma$ at the low-energy scale, also in NLO, suggest that a valence picture with $\Delta \Sigma$ of the order of 0.5 - 0.8 is consistent with the experimental result in DIS of the order of 0.3 - 0.5. Clearly, the scale dependence cannot be neglected in interpreting the results in deep inelastic experiments.

The evolution from $Q^2 = 1$ GeV$^2$ to 10 GeV$^2$ is presumably more reliably in the perturbative regime, and in this case we find a ratio $\Delta \Sigma(1)/\Delta \Sigma(10)$ of 1.031 (LO), or 1.068 (NLO) when $\alpha_s(M_Z^2) = 0.117$. Especially here, we note a sensitivity to $\alpha_s(M_Z^2)$. Taking its value to be 5% higher, the ratios becomes 1.040 (LO) and 1.110 (NLO). The modification of the actual singlet moment $\Gamma_1^S$, which includes $C^S(Q^2)$ as well, shows a decrease for $\Gamma_1^S$ when going to lower momenta, the ratios being $\Gamma_1^S(1)/\Gamma_1^S(10) = 0.980$ (LO) and 0.956 (NLO). (Taking $\alpha_s(M_Z^2)$ 5% higher gives 0.973 (LO) and 0.921 (NLO).) Although fairly small in this region, the contribution to the evolution from the anomalous dimension clearly can and should be taken into account, and goes beyond the standard QCD effect arising purely from $C^S(Q^2)$, which is sometimes all that is taken into account. Note furthermore that in $\Gamma_1^S(Q^2)$, higher twist contributions proportional to $1/Q^2$ could contribute. These have not been included in the above estimate for $\Gamma_1^S$, which refers purely to the twist two part.

Another point that deserves discussion is the inclusion of the effects of the strangeness threshold. If one considers the $K\overline{K}$ threshold, i.e. $Q^2 \approx 1$ GeV$^2$, as an appropriate value, a large part of the evolution down to $Q_0^2$ involves $n_f = 2$. If the strangeness content of the nucleon has not become zero, the decoupling of strangeness from the evolution leads to ambiguities in the treatment. It would require a global analysis, which takes carefully into account existing inequalities such as $\Delta s(x) \leq s(x)$ and the consequences for the moments. The evolution equations with $n_f = 2$ instead of $n_f = 3$ in general tend to somewhat slow down the increase of $\Delta \Sigma$ at lower momentum scales.

Finally, if we assume that (i) $\Delta \Sigma(Q_0^2) = 0.65$ as an appropriate value at the low mo-
mentum scale, e.g. from an effective low energy model \[1\], and (ii) pQCD continues to work down to low \(Q^2\), and (iii) the asymptotic values of the nonsinglet combinations of the axial matrix elements are known from weak decays, \(\Delta u - \Delta d = 1.257\) and (from low energy hyperon decays) \(\Delta u + \Delta d - 2\Delta s = 0.58 \pm 0.05\), then we are able to calculate at any scale the axial matrix elements for each of the quark flavors. The values at \(Q^2_0\) (corresponding to \(G_2 = 0\)), 1 and 10 GeV\(^2\) are given in Table \[\] . Assuming three active flavors at \(Q^2_0\) gives a positive value for \(\Delta_s\). In this case we have the possibility to incorporate the strangeness threshold in a natural way, using only two active flavors to evolve \(\Delta\Sigma\) down to 0.58, then continuing with three active flavors. The numbers in this scenario for NLO are given in Table \[\]. The strangeness threshold required is \(Q^2_s = 0.286\) GeV\(^2\). We note that decoupling of the strange quarks in the momentum sumrule at this same threshold implies that at 4 GeV\(^2\) the momentum carried by the strange antiquarks as compared to nonstrange antiquarks is \(2\bar{s}/(\bar{u} + \bar{d}) = 0.57\).

In conclusion, using NLO equations, a very satisfactory picture is obtained running all the way from a low-energy-scale proton without gluons but with some nonstrange sea, acquiring nonzero values for strangeness matrix elements only starting at the strangeness threshold which is slightly above the scale where \(G_2 = 0\). We have analyzed the errors arising from an uncertainty in the strong coupling constant \(\alpha_s(M_Z) = 0.117 \pm 0.005\) and those coming from an uncertainty in the octet axial charge, \(0.58 \pm 0.05\). These are indicated in Table \[\]. Note that the results for \(\Delta u\) and \(\Delta d\) are not sensitive to the octet axial charge if this is taken to coincide with the strangeness threshold. Using these results and including second order QCD corrections everywhere, we find that

\[
\Gamma_p^p(10\ \text{GeV}^2) = (0.109 \pm 0.002) + (0.062 \pm 0.007 \pm 0.009)\Delta\Sigma(Q^2_0) = 0.149 \pm 0.006 \pm 0.006, \tag{27}
\]

\[
\Gamma_n^n(10\ \text{GeV}^2) = (-0.080 \pm 0.001) + (0.062 \pm 0.007 \pm 0.009)\Delta\Sigma(Q^2_0) = -0.040 \pm 0.003 \pm 0.006, \tag{28}
\]

\[
\Gamma_d^d(10\ \text{GeV}^2) \equiv 0.5(\Gamma_p^p + \Gamma_n^n)(1 - 1.5\omega_D)
\]
\[(0.013 \pm 0.001) + (0.056 \pm 0.006 \pm 0.008) \Delta \Sigma(Q_0^2)\]
\[= 0.049 \pm 0.004 \pm 0.005 \quad (29)\]

(Using the usual D-state admixture of 6\% in the latter). The first error in each term arises from our assumed uncertainties in \(\alpha_s(M_Z)\) and from the octet part of the sum rule, here added in quadrature. The second error bar (if shown) comes from our estimate of the prescription dependence associated with evolving \(\Delta \Sigma\) from 10 GeV\(^2\) to \(Q_0^2\). This includes e.g. differences in truncation schemes (see Figs. 2 and 3), choice of s-quark threshold mechanism, and determination of \(Q_0^2\) from \(V_2 = 1\) rather than \(G_2 = 0\). These have all been discussed above. We conservatively estimate \(\Delta \Sigma(Q_0^2)/\Delta \Sigma(10) = 1.65 \pm 0.25\), and the final numbers above correspond to this choice, with \(\Delta \Sigma(Q_0^2) = 0.65\). The relations between the experimental sum rules and the "spin carried by the quarks", i.e. \(\Delta \Sigma(Q_0^2)\) are illustrated in Fig. 4.

**IV. CONCLUSIONS**

In this paper we have investigated the extent to which measurements of the spin sum rule at high energies should be interpreted, in view of the role of their scale dependence. We have investigated the spin sum rule together with the momentum sum rules in a systematic way, extending our earlier results that used purely leading order evolution to results that use next to leading order evolution. This has become possible in part due to the recent work of Larin [8]. We have estimated uncertainties arising from scheme dependence and higher order QCD effects in the evolution, by using several truncation prescriptions. Our results indicate that many qualitative features present in the leading order remain the same. Quantitative differences show up, but do not upset the picture. In particular, we have considered momentum sum rules for valence quarks and gluons compared with one another or spin sum rules compared with the momentum sum rules. In general, we have not made any *interpretation* of the relative gluonic contribution to the results for the singlet moment, which helps avoid obvious scheme-dependent assumptions. (see, e.g. reference [16], which
shows that the gluon contribution is zero in certain renormalization schemes). We are simply using the operator product expansion for the moments of interest, at next to leading order.

The results for evolution from $1 \text{ GeV}^2$ to $10 \text{ GeV}^2$ are quite reliable, and the fractional change evolving down to $Q_0^2$ is apparently reasonably stable to next to leading order corrections too. Of course, the moderate sensitivity in going from first to second order is no proof of the reliability of the perturbative expansion. We have noted that while the sensitivity to using first or second order is moderate, the sensitivity to the value of $\alpha_s(M_z)$ is quite strong. We find that the inclusion of a strangeness threshold is not very important for the rate of evolution. Much more important is the fact that there is a large change in $\Delta \Sigma$ running from $10 \text{ GeV}^2$ to a low-momentum scale $Q_0^2$, large enough that it can easily reach a point where $\Delta \Sigma = \Delta u + \Delta d - 2\Delta s$, i.e. $\Delta s = 0$, a natural point for the strangeness threshold. We have made this more explicit by starting with a value of $\Delta \Sigma(Q_0^2) = 0.65$, although any assumption that one can estimate $\Delta \Sigma(Q_0^2)$ from a low energy nucleon model is very clearly subject to debate. This scenario implies that at present there is no reason to require an anomalously large strange contribution in the proton at a low-momentum scale. Of course, independent measurements of the strangeness content at this scale (e.g. from elastic neutrino - nucleon scattering) are important to confirm this.

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FIGURES

FIG. 1. Plot of $G_2$ vs $V_2$. Solid (dashed) curve shows first(second) order calculations. We started from experimental values at roughly $4 \text{ GeV}^2$ and evolve downwards. Dotted curve is 2nd order, but truncated. Dash-dotted is again 2nd order, truncated, but with $\alpha_s$ in all higher order terms replaced with its leading order expression.

FIG. 2. Same as Fig. 1, but plotting $\Delta \Sigma/\Delta \Sigma(Q_0^2)$ versus $V_2$. Curves are labeled as before.

FIG. 3. Same as Fig. 3, but plotting $\Delta \Sigma/\Delta \Sigma(Q_0^2)$ versus $G_2$.

FIG. 4. Experimental sum rules as a function of the spin carried by the quarks, $\Delta \Sigma(Q_0^2)$, including uncertainties from $\alpha_s$, octet axial charge, and scheme dependence are given by the shaded areas. The areas enclosed by the dashed lines are the results as a function of $\Delta \Sigma(10 \text{ GeV}^2)$. 
TABLE I. Numerical values [12,13] used for the various parameters appearing in the truncated
next to leading order solutions for the momentum sum rules, Eqs 10 through 17.

| n_f | ˜α | d_{NS}^{(2)} | d_+^{(2)} | Z_+ | Z_{NS} | K^{d} | K^{G} |
|-----|-----|--------------|-----------|-----|--------|-------|-------|
| 2   | 0.2727 | 0.3678 | 0.5058 | 1.486 | 1.428 | 0.4544 | -0.1704 |
| 3   | 0.36 | 0.3951 | 0.6173 | 1.783 | 1.507 | 2.121 | -1.193 |
| 4   | 0.4286 | 0.4267 | 0.7467 | 2.355 | 1.654 | 5.895 | -4.421 |
| 5   | 0.4839 | 0.4638 | 0.8986 | 3.341 | 1.904 | 22.604 | -21.191 |

TABLE II. Values for the axial matrix elements of the quarks using a starting value of ∆Σ(Q_0^2) = 0.65 at the scale where G_2(Q_0^2) = 0 using NLO results and a strangeness threshold at the point where ∆s = 0. The errors refer to uncertainties in α_s(M_Z) = 0.117 ± 0.005, and in the octet axial charge, 0.58 ± 0.05 (underlined errors).

| Q^2 [GeV^2] | ∆u | ∆d | ∆s | ∆Σ |
|-------------|-----|-----|-----|-----|
| Q_0^2       | 0.954 | -0.304 | - | 0.650 |
| Q_0^2 = 0.3 ± 0.1 | 0.919 ± 0.025 | -0.339 ± 0.025 | 0.00 | 0.580 ± 0.05 |
| 1           | 0.873 ± 0.01 | -0.384 ± 0.01 | -0.045 ± 0.01 | 0.445 ± 0.03 ± 0.025 |
| 4           | 0.866 ± 0.01 | -0.391 ± 0.01 | -0.052 ± 0.01 | 0.423 ± 0.03 ± 0.025 |
| 10          | 0.864 ± 0.01 | -0.393 ± 0.01 | -0.055 ± 0.01 | 0.417 ± 0.03 ± 0.025 |
This figure "fig1-1.png" is available in "png" format from:

http://arxiv.org/ps/hep-ph/9412287v1
$G_2$

$V_2$

1$^{\text{st}}$ order (solid)
2$^{\text{nd}}$ order (dashed)
2$^{\text{nd}}$ order truncated (dotted)
2$^{\text{nd}}$ order mixed truncated (dot-dashed)

figure 1/Mulders & Pollock
This figure "fig1-2.png" is available in "png" format from:

http://arxiv.org/ps/hep-ph/9412287v1
\[ \frac{\Delta \Sigma}{\Delta \Sigma} (10 \text{ GeV}^2) \]

- 1\text{st} order (solid)
- 2\text{nd} order (dashed)
- 2\text{nd} order truncated (dotted)
- 2\text{nd} order mixed truncated (dot-dashed)

figure 2/Mulders & Pollock
This figure "fig1-3.png" is available in "png" format from:

http://arxiv.org/ps/hep-ph/9412287v1
$\Delta \Sigma/\Delta \Sigma(10 \text{ GeV}^2)$

$G_2$

$1^{\text{st}}$ order (solid)
$2^{\text{nd}}$ order (dashed)
$2^{\text{nd}}$ order truncated (dotted)
$2^{\text{nd}}$ order mixed truncated (dot-dashed)

figure 3/Mulders & Pollock
This figure "fig1-4.png" is available in "png" format from:

http://arxiv.org/ps/hep-ph/9412287v1
$\Gamma_1(10 \text{ GeV}^2)$

$\Delta \Sigma$

figure 4/Mulders & Pollock