Heat and mass transfer modelling for storage of food bulk raw materials under active ventilation

O A Egorova, G V Alekseev, I P Yukhnik and A A Sychev

ITMO University, 9, Lomonosova str., St. Petersburg, 197101, Russia

E-mail: jeerol@list.ru

Abstract. The paper aims to research a possibility of storing plant agricultural raw materials. Rational management of natural resources is a dominant trend in the economy development, which involves the fullest processing of plant raw materials intended for human consumption with minimum waste. Active grain ventilation is conducted by drying at a low temperature to preserve the quality of raw and wet grain, as well as by cooling grain batches stored to increase their durability. With a certain moisture content grain can be gradually dried, cooled, preserved, aerated depending on its condition and purpose. These technological methods provide a significant reduction in energy costs in comparison with thermal drying, as well as improving the quality of seeds and grains due to the "soft" completion of the biochemical processes associated with maturation and stabilization of the protein-enzyme complex. Active ventilation requires no complex equipment or large capital investments. Therefore, it is no coincidence that the technologies developed on the basis of active ventilation are widely used in processing of main volumes of high quality grain in USA, Canada, and Australia. The intensification of the grain preparation process achieved by this way reduces energy costs of drying. The study models the storage conditions for bulks of various types of agricultural raw materials under active ventilation.

1. Introduction

One of the main ways to store food bulk raw materials, like Jerusalem artichoke, potatoes, and oilseeds, is a method of active ventilation, which is an enhanced air exchange within the bulk spaces [1-4].

Active ventilation is conducted by forced blowing air, whether atmospheric or air-conditioned, through food raw materials bulk stored in open areas and storages. After active ventilation a food material bulk is characterized by moisture and temperature. Their level is determined by the process time, specific supply, temperature and relative moisture of the ventilating air. The grade of food bulk raw materials, as well as the moisture and temperature determine the storage time and the periodicity of re-ventilation. In some cases, active ventilation is performed for the purpose of drying.

Storing food bulk raw materials without active ventilation is accepted as a comparative basis.

Among the main advantages of active ventilation application, which determine economic efficiency, are reducing the acid number increase during storage, reducing storage losses, and the possibility to decrease losses and increase the yield of food bulk raw materials while further processing.

2. Problem statement

We have chosen a mathematical model of heat and mass transfer in the layer, which was proposed by A. Yu. Mikhailov [4, 5], to describe the storage process of the food bulk raw materials under active
ventilation. The model takes into account the intensity of breathing of individual objects. Due to the fact that real processes often occur in a layer where elements are not small enough, the researchers generalized the boundary conditions in the direction of establishing the relationship of the body with the environment. First of all, this was done to determine the relationship between the surface temperature of the body and the temperature of the heat transfer fluid for both the problem of pure thermal conductivity [6] and that of joint heat and mass transfer [4, 5].

Thus, the mathematical statement of the problem of heat and mass transfer in the bulk, which is a layer of dispersed (wet) material, consisting of particles of a spherical shape with constant radius R, through which the heat transfer fluid is blown in the direction z at constant speed v, can be formulated as follows:

\[
\frac{\partial [\rho T(r,\tau)]}{\partial \tau} = a_q \frac{\partial^2 [\rho T(r,\tau)]}{\partial r^2} + \frac{\epsilon_p \partial [\rho u(r,\tau)]}{\partial \tau} + \frac{\epsilon a_q}{\rho c_q} + \frac{q}{c_q},
\]

\[
\frac{\partial [\rho u(r,\tau)]}{\partial \tau} = a_m \frac{\partial^2 [\rho u(r,\tau)]}{\partial r^2} + a_m \delta \frac{\partial [\rho u(r,\tau)]}{\partial r^2} + \frac{q}{c_q}; (0 < r < R, \tau > 0)
\]

under the following initial and boundary conditions:

\[
\lambda_q \frac{\partial T(R,\tau)}{\partial \tau} = a_q [T_c(\tau,\tau) - T(R,\tau) - (1 - \epsilon) \rho j]_m = 0;
\]

\[
\lambda_m \left[ \frac{\partial u(\tau,\tau)}{\partial \tau} + \delta \frac{\partial T(R,\tau)}{\partial \tau} \right] + j_m = 0;
\]

\[
\frac{\partial T_c(z,\tau)}{\partial \tau} + \nu \frac{\partial T_c(z,\tau)}{\partial z} + \frac{a_q F}{c_q \rho c_m} [T_c(z,\tau) - T(R,\tau)] = 0;
\]

\[
(0 < z < h, \tau > 0);
\]

\[
\frac{\partial T(0,\tau)}{\partial r} = \frac{\partial u(0,\tau)}{\partial \tau} = 0; t(0,\tau) < \infty;
\]

if \( z=0 \) \( T_c(0,0)=T_{\infty,\omega} \)=const; \( z=\nu \tau_c \) \( T_c(\tau,\tau)=T_{\infty} \)=const; \( u(\tau_c)=u_0 \)=const.

Here: the differential equation (1) is equation of heat transfer; (2) is equation of moisture transfer inside the material (seed); (3) is boundary condition of the third kind, describing the convective heat exchange of the bulk food raw materials with the environment (heat transfer fluid); (4) is a boundary condition describing the mass—(moisture—) transfer of a body with the environment; differential equation (5) relates the temperature of the gas (air) flow and the surface of objects; equality (6) is conditions of symmetry, and inequality (6) reflects the fact of physical limitation of the temperature in the center of the object; (7) is initial conditions, i.e. the values of the heat and mass transfer potentials in the heat transfer fluid and the layer particles before their contact.

An approximate solution of the boundary value problem (1), (3), (5)—(7), namely, the heat transfer problem in the absence of phase transformations (\( p=0 \)), is given in [4,5,7]. In addition, the authors did not take into account the actions of the internal heat source, i.e. equation (1) was solved under the condition \( q=0 \).

However, the respiration specific heat is the most important indicator that should be the basis of all thermal calculations for cooling storage systems to store agricultural raw materials.
3. Results and discussion

We have taken the following analytical dependencies of the objects respiration intensity on time:

\[ q = q_1 \pm q_2 \exp(-k\tau), \]  

which were obtained on the basis of experimental data on the respiration intensity of a food raw material bulk at different humidity [4, 5]. The lower sign refers to oilseeds, the upper sign refers to such objects as Jerusalem artichoke or potatoes.

Analytical solutions for the temperature field in the material (a particle of a food raw material bulk) and the heat transfer fluid are obtained in the following form:

\[ T(X, F_0) = \frac{t(r, \tau) - t_0}{t_{c0} - t_0} = \]

\[ = 1 + p_{o1} \left[ \frac{F_0 - Z}{P_e} \pm \frac{p_{o2}}{p_{d}p_{o1}} \exp \left( -p_{d} \left( F_0 - \frac{Z}{P_e} \right) \right) \left( 1 - \exp \left( -p_{d} \left( F_0 - \frac{Z}{P_e} \right) \right) \right) \right] + \]

\[ + MnrZ(p_{o1} - 1) \sum_{n=1}^{\infty} A_n^2 \frac{1}{\mu_n \sin \mu_n} \left\{ \left( \mu_n^2 + \mu_n \left( F_0 - \frac{Z}{P_e} \right) + \frac{Bi_n}{2} \right) \sin \mu_n X \right\} \cos \mu_n X, \]  

\[ \cdot \exp \left( -\mu_n^2 \left( F_0 - \frac{Z}{P_e} \right) \right) + \cdots; \]  

\[ T_e(Z, F_0) = \frac{t(z, \tau) - t_0}{t_{c0} - t_0} = \]

\[ = 1 + MnMrZ \left[ \frac{1}{3Bi_q} \left( p_{o1} \cdot P_e \pm \frac{p_{o2}}{p_{d}} \right) \right] + p_{o2} + \frac{1}{1 + (Bi_q)^{1/3}} \cdot \]

\[ - \sum_{n=1}^{\infty} \left( 1 + \frac{p_{o1}}{\mu_n^2} \pm \frac{p_{o2}}{p_{d} \mu_n^2} \right) \cdot \exp \left( -\mu_n^2 \left( F_0 - \frac{Z}{P_e} \right) \right) \]  

\[ + \cdots \]  

(9)

Here

\[ A_n = \frac{2Bi_q}{\varphi_n}; \quad \varphi_n = \mu_n^2 + Bi_q \left( Bi_q - 1 \right); \]  

\[ \mu_n \] are consecutive positive roots of the characteristic equation

\[ t\mu_n = -\frac{\mu_n}{Bi_q} V \]  

(11)

For \( p_{o1} = p_{o2} = 0 \) the solutions for the temperature field in the material and in the heat transfer fluid presented in [4] and [5] follow from (9) and (10). For long time storage of food bulk raw materials under active ventilation and the dependence of the respiration intensity of objects on time according to the law

\[ q = q_1 \cdot q_2 \exp(-k\tau) \]  

(12)

an analytical solution of the problem (1), (3), (5)-(7)' (12) was obtained using mathematical physics methods in the following form [8]:

\[ T(X, F_0) = \frac{t(r, \tau) - t_0}{t_{c0} - t_0} = 1 - A \cdot \frac{p_{o2}}{p_{d}} \left( 1 - Bi_q \frac{A_n^2}{X} \right) \exp(-pdF_0) + \sum_{n=1}^{\infty} A_n \frac{1}{\mu_n^2} \left( 1 - \frac{\mu_n^2}{p_{d}} \right) \]  

\[ + \cdots \]
\[+ \frac{1}{X} \int_0^{F_0} \left\{ \int_0^{F_0^*} N(F_0^*) M(F_0^* - F_0^{**}) dF_0^{**} - 1 \right\} \exp(-wZ) + K'(F_0^*) \}
\]
\[\times \sum_{n=1}^m A_n \exp(-\mu_2^2(F_0 - F_0^*)) dF_0^*;
\]
\[T_c(Z, F_0) = \frac{t(x)}{t_{x_0}} = 1 - K(F_0) - \left[ N(F_0) - P_e \int_0^{F_0} N(F_0^*) M(F_0^* - F_0^*) dF_0^* \right].
\]

where

\[A_0 = \frac{P_0}{6} \left( 1 + \frac{2}{B_l q} \cdot X^2 \right) \cdot 1;\]
\[A_p = \frac{\sqrt{P_d} \sin(\sqrt{P_d} z)}{\cos(\sqrt{P_d} d)} \cdot \frac{1}{1 + (B_l q - 1) \frac{tg \sqrt{P_d}}{\sqrt{P_d}}};\]
\[A_n = \frac{2B_l q}{B_l q (B_l q - 1) + \mu_2^2};\]

\[K'(F_0) = g(1 + K(F_0));\]
\[g = 1 + \frac{32}{\pi^4} K_1;\]
\[\gamma = \frac{1}{\beta} - 1 + \frac{1}{B_l q};\]
\[w = \frac{K_1 \gamma}{P_e};\]
\[\beta = \frac{\pi^4 \gamma g}{32};\]

\[K(F_0) = \frac{1}{1 + \frac{\pi^4}{32 K_1}} \times \left\{ 1 - P_{o_1} \left[ \frac{P_{o_2}}{P_{o_1}} + \frac{P_d}{P_{o_1}} + P_d F_0 + \frac{1}{P_{o_1} - P_{o_2}} \times \left( P_d + \frac{P_{o_1} P_{o_2} - P_o - \frac{P_d P_{o_1}}{\beta}}{1 - \frac{\beta}{P_d}} \right) \right] \right\} \times \]
\[\times \exp(-\beta F_0) \cdot \frac{P_{o_2}}{1 - \frac{\beta}{P_d}} \exp(-P_d F_0)\]
\[K_1 = \frac{M_2 c_q y_q}{c_q y_c} = \frac{M_2 M_2 P_e}{B_l q};\]

\[\mu_n \text{ are consecutive positive roots of the characteristic equation}\]
\[tg \mu = \frac{\mu}{1 - B_l q};\]
\[N(F_0) = I_0 \left( \frac{\pi^2}{2 \sqrt{2}} \sqrt{w y z \theta} \exp \left( -\frac{\beta}{g} \theta \right) + \frac{\theta}{g} \int_0^{\theta} I_0 \left( \frac{\pi^2}{2 \sqrt{2}} \sqrt{w y z \theta^*} \exp \left( -\frac{\beta}{g} \theta^* \right) d \theta^* \right) \right);\]
\[M(F_0) = \Delta(F_0) + \]
\[ + \frac{\gamma K_1}{Pe} \left[ 1 \left( \frac{\beta (P_{01} - P_{02}) - \frac{1}{P_{01}}}{\beta (P_{01} - 1)} \right) \exp(-\beta Fo) + \frac{1}{\beta} \left( \frac{P_{01} - P_{02}}{1 - \frac{P_d}{1}} \right) \exp(-PdFo) \right]; \]

\[ \Delta(Fo) = \begin{cases} \frac{1}{u_0}, & \text{при } 0 < Fo < Fo^*; \\ 0, & \text{при } Fo > Fo^* \end{cases} \]

\[ \theta = Fo - \frac{z}{pe}; \]

\[ L_o(x) \text{ is a modified Bessel function of the first kind of the zero order.} \]

It is possible to determine the temperature values averaged over the volume from the solutions (13) and (14) obtained, and then the rate of heating/cooling of the layer of the food raw material bulk and the energy consumption necessary to bring the bulk to a certain temperature can be further determined. The solutions allow predicting the temperature and duration of storage of food bulk raw materials.

It is possible to obtain formulas that are convenient for engineering calculations of temperature fields in the food raw material bulk and the heat transfer fluid with some simplifications from solutions (13) - (14):

\[ T(X, Fo) = \frac{t(r, \tau) - t_0}{t_{co} - t_0} = \]

\[ = P_{01} \left[ \frac{1}{6} \left( 1 + \frac{2}{Bi_q} - X^2 \right) + \left( \frac{1}{Pe} + \frac{32 \cdot MnMr}{\pi^4 \cdot Bi_q} \right) \frac{Z}{Pd} + \frac{1}{P_{01}} \left( 1 - \frac{P_{01} - P_{02}}{Pd} \right) \right] + \]

\[ + Bi_q \left( \frac{P_{02}}{Pd} - \frac{32 \cdot MnMr X}{\pi^4 32} \right) \times \sin(\sqrt{Pd}X) \exp \left( -Pd \left( Fo - \frac{Z}{Pe} \right) \right) + \]

\[ + \frac{1}{\phi_z} \left( \frac{MnMrX \left( P_{01} - P_{02} \right) - \frac{32 \cdot MnMrPdP_{01}}{\pi^4 32}}{Bi_q \left( Pd - \frac{32}{\pi^4} \right)} \right) \times \sin \left( \frac{\pi^2 \sqrt{2} \gamma}{8} \exp \left( -\frac{\pi^4 \gamma}{32} \left( Fo - \frac{Z}{Pe} \right) \right) + (Bi_q - 1) \sum_{n=1}^{\infty} \frac{\sin(\mu_n X)}{\phi_n \cos \mu_n} \times \left( 1 + \frac{MnMr}{Bi_q} \right) \left[ \frac{PdP_{01} + (Pd - P_{01} + P_{02})\mu^2_n - \mu^4_n}{Pd - \mu^2_n} \right] \times \exp \left( -\mu^2_n \left( Fo - \frac{Z}{Pe} \right) \right) \right); \]

\[ T_e = \frac{t(z, \tau) - t_0}{t_{co} - t_0} = 1 + \frac{\gamma MnMr Z}{Bi_q} \times \]

\[ \times \left( \frac{32}{\pi^4 P_{01}} + \frac{1}{Pd - \frac{32}{\pi^4}} \right) P_{02} \exp \left( -Pd \left( Fo - \frac{Z}{Pe} \right) \right) + \]

\[ + \left( P_{01} - P_{02} - \frac{32}{\pi^4} \frac{PdP_{01} - Pd - \frac{\pi^4 \gamma}{32}}{Pd - \frac{32}{\pi^4}} \exp \left( -\frac{\pi^4 \gamma}{32} \left( Fo - \frac{Z}{Pe} \right) \right) \right) \]
\[ \varphi_k = \sqrt{Pd \cos Pd} + (Bt_q - 1) \sin \sqrt{Pd}; \quad \varphi_\gamma = \cos \alpha + (Bt_q - 1) \sin \alpha; \]
\[ \alpha = \frac{\pi^2}{8}. \]

Here \( T(X,F_0), T_c(Z,F_0) \) are the dimensionless temperature of the object and of the heat transfer fluid respectively; \( t(r,\tau) \) is temperature of the object (body, material); \( t_0 \) is the initial temperature of the object; \( t_c(z,\tau) \) is the heat transfer fluid temperature; \( t_{co} \) is the initial temperature of the heat transfer fluid; \( r, z \) is present coordinate; \( R \) is radius of the ball; \( h \) is height of the material layer; \( \tau \) is the time; \( a_q \) is coefficient thermal diffusivity; \( \varepsilon \) is coefficient of phase transformation from liquid to vapor (vapor to liquid); \( \rho \) is specific heat of phase transformation; \( c_q \) is specific heat of the body; \( c_c \) is specific thermal capacity of the heat transfer fluid; \( a_m \) is mass conductivity coefficient; \( \alpha \) is heat transfer coefficient; \( j_m \) is the intensity of evaporation (condensation) from the surface of the body; \( \gamma_q \) is material density; \( \gamma_c \) is heat transfer fluid density; \( v \) is heat transfer fluid rate; \( F \) is midlength section; \( m \) is pore volume of the layer; \( q \) is specific heat of respiration of a food raw material bulk; \( q_1 \) and \( q_2 \) are components of the respiration intensity of objects;

\[ Z = \frac{r}{h}, \quad X = \frac{r}{R} \] are dimensionless coordinates;

\[ \chi = \frac{h}{R}, \quad Fo = \frac{a_q R^2}{R^2} \] is Fourier numbers; \( \frac{kR^2}{a_q} \) - Predvoditelev number;

\[ Bi_q = \frac{a_q R^2}{\lambda_q} \] is Bio number; \( Mn = \frac{FR}{m} \) is Minovich number;

\[ Pe = \frac{vR}{\lambda_q} \] is Pekle number; \( Po_1 = \frac{q_2 R^2}{a_q c_q (t_{co} - t_0)} \); \( Po_2 = \frac{q_2 R^2}{a_q c_q (t_{co} - t_0)} \) is Pomerantsev number;

\[ Ho = \frac{a_q}{v c_q \gamma_c} \] is homochronism criteria;

\[ Mr = \frac{a_q}{v c_q \gamma_c} \] is Margulis number.

4. Conclusion

The relations obtained fully characterize the distribution of dimensionless values of the temperature field over the bulk volume of the product in accordance with changes in the temperature of the heat transfer fluid. An approximate solution of the boundary value problem (1), (3), (5)–(7), namely, the heat transfer problem in the absence of phase transformations \( p=0 \), is given in the research mentioned. In addition, the authors did not take into account the actions of the internal heat source, i.e. equation (1) was solved under the condition \( q=0 \). In contrast to the known results, we solved equation (1) under the condition \( q=q(t) \) and \( p=p(t) \). This condition corresponds to the actual conditions for the action of temperature fields on the bulk. This occurs due to the fact that when storing agricultural raw materials, the specific heat of respiration is the most important indicator, which should be taken as the basis for all thermal calculations of cooling storage systems. In the future, the possibility to apply the results obtained is closely related to the experimental determination and identification of the similarity criteria used to solve the problem.

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