A Blueprint for Four-states Quantum Formal System

KAZUKI OTSUKA

Abstract

A blueprint for a formal system for describing living systems, based on the entanglement and parallel computation of systems consisting of two qubits shall be proposed. In particular, isomorphism with the $S = \{T, A, G, C\}$, $|S| = 2^2 = 4$ system of DNA sequences consisting of nucleotide symbols are aimed for. In this simple system, two qubits belong to system A and the other two qubits belong to system B, which are entangled and inseparable. System A and System B are derived from a parent-mother diploid living system.

1. Introduction

The world can be viewed as a huge tree, starting from a set of axioms (a small string or a set or a system, all of which refer to the same thing) and generated according to formal rules. Strings are generated recursively according to formal rules. When the string or system or set is a tape, and the formal rules are headers in an infinite state machine, the limit of the generated world has been discussed as computability. Word problems have also been discussed in order to understand algorithms for determining whether a proposition can be attributed to another proposition. Mathematical logicians have attempted to mechanize reasoning through these arguments. On the other hand, human beings do not necessarily generate true propositions. 1. object 2. reference 3. meaning 4. idea (Frege 1892), and the generation of tapes above level 3 is called intelligence. Does the life have a system for generating meaning primordially?

In this paper, we first review the history of human attempts to mechanize reasoning, and then summarize the basic framework of quantum computing. We then present a blueprint for a simple two qubits system in which independent sources belonging to systems M and F, which are derived from “diploid” systems, are correlated with each other to form further independent sets.

1.1. Sets and Systems

The omnipresent quantifier and the existential quantifier quantify and objectify concepts, allowing us to reason as follows

$((P \rightarrow Q) \land P) \vdash Q$

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1 Email: otsuka.kazuki@googlemail.com. This discussion is a thought experiment and has nothing to do with any organization where the author belongs to.
If he dies, he is a person (Q if he is P). God dies (P). Therefore, it leads to that God is a person (Q).

Now, let's define a set, and assume that it contains any representable symbols (words). The symbols can be natural numbers or phonemes that can be generated by living organisms (after all, they can be encoded by Gödel numbers). A sentence is a sequence of symbols with the elements. For example,
\[ a_1a_2\ldots a_n \]
is a sentence.

An arbitrary sentence \( A \) of finite length is an axiom if it follows the rules of a formal system \( E \). Let \( W \) be the set of sentences that are axioms.
\[ W \subset K, \ X \in W \Rightarrow PX \text{ is probable in } (E) \]

Here \( P \) represents the predicate predicate, and the predicate \( x \) takes an argument. For example, when the predicate is
\[
(1) \ E11 \\
(2) \ Ex \rightarrow Ex11
\]

, then the predicate \( E \) denotes the set of even numbers. By substituting the result of (1) and (2) into \( x \) again and repeating the process, any even number can be generated recursively.

Similarly, when the result of \( y+1 \) can be generated from the result of \( y \), it means that any result of \( y \) can be generated=proved if the result of one \( y \) is known. This method of reasoning is called mathematical induction.

Here, \( W \) is a set to \( K \) itself. This is called recursive recursive. (Dedekind called this "similarity").

\( K \) can be manipulated algebraically regardless of the content of its elements by converting to Gödel numbers \( g(x) : K \rightarrow K' \). This is called a mathematical system.

### 1.2. Recursive and Computability

A set of symbols is said to be \( r.e. \) (recursively enumerable) if it can be generated recursively, and \( T_0 \) is not recursively enumerable because the complement \( \overline{T_0} \) of a true Gödel sentence encoded with a dyadic Gödel number is not re in the universal system \( U \) (R. M. Smullyan 1961). It is not recursive recursive (R. M. Smullyan 1961).

The elementary formal system \( (E) \) in 1.1 is a computational model by E. Post. According to Church's thesis,

- General recursive functions (Gödel-Herbrand-Kleene 1936)
- \( \lambda \)-recursive functions and partial recursive functions (Church 1936)
- Turing machines (Turing 1936)
- URM-computable functions (Shepherdson-Sturgis 1963)
normal algorithm over a finite alphabet (Markov 1951)

All of them are considered to have the same computability.

1.3. Biological system

DNA     ATG CGT GGA TAC ACT TTT .... AGA
↓
mRNA   AUG CGU GGA UAC ACU UUU… AGA
↓
tRNA     [START] Arg Gly Tyr Ter Phe … [END]

Proteins are synthesized based almost exclusively on genetic information expressed in $S = \{ T, A, G, C \}$, $|S| = 2^2 = 4$ different symbolic sequences. In diploid organisms, including humans, a new individual is created from two genome sets, one derived from the male (system 1) and the other from the female (system 2). The standard genetic code is universal across all living species, with the rare exception of mitochondria and a few unicellular organisms (D. Nelson 2017). If the operations of this symbol sequence are closed to genetic information $G$, it implies that the symbol sequence itself may contain information for the interpretation of concepts belonging to the first level (Frege 1892).

Unlike computational models with a pair of tapes and a header, ribosomes are translated in parallel as polyribosomes.

\[
S = \{ T, A, G, C \}, \quad |S| = 2^2 = 4 
\]

Figure 1.4.1 The “central dogma” of molecular biology. Synthetic process from genetic information $G$ to protein $P$. $G$ is transcribed into codon sequence $C$ in the ribosome ($G \rightarrow C$). Codon sequence $C$ is read out by tRNA and protein is synthesized ($C \rightarrow P$).

1.4. Quantum system
1.4.1. a single Qbit\(^2\) system

A system of quantum computation consisting of 1 Qbit has a basis of two states, each of which is assigned one classical bit. (The order of the set is \(2^1 = 2\)).

\[
\begin{pmatrix}
0 & 1
\end{pmatrix} \quad (|S| = 2^1 = 2)
\]

The measurement of a single Qbit is determined by a probability distribution with complex amplitudes \(\alpha\) and \(\beta\).

\[
|\psi\rangle = \alpha |0\rangle + \beta |1\rangle
\]

The \(\alpha\) and \(\beta\) have the constraint \(|\alpha|^2 + |\beta|^2 = 1\), and with probability \(|\alpha|^2\) we get \(|0\rangle\), and with probability \(|\beta|^2\) we get \(|1\rangle\).

While \(|\psi\rangle\) is taken as a discrete value, it is among an infinite number of states with different ratios and phase differences of superposition of \(|0\rangle\) and \(|1\rangle\).

The special state of \(|\psi\rangle = |0\rangle\) \((\alpha = 1, \beta = 0)\) or \(|\psi\rangle = |1\rangle\) \((\alpha = 0, \beta = 1)\) is called the computational basis state.

\[
|0\rangle = \begin{pmatrix} a_1 \\ a_2 \end{pmatrix} = \begin{pmatrix} 1 \\ 0 \end{pmatrix}
\]

\[
|1\rangle = \begin{pmatrix} b_1 \\ b_2 \end{pmatrix} = \begin{pmatrix} 0 \\ 1 \end{pmatrix}
\]

If we regard it as a vector, we can manipulate it by multiplying the qubit \(|\psi\rangle\) by an arbitrary unitary matrix \(U\). The only condition for \(U\) is that it be a unitary matrix.

\[
|\psi\rangle' = U |\psi\rangle, \quad U^\dagger U = I
\]

Most of the possible \(U\) are meaningless, but the useful ones are

\(X\) (flip) : Flip a bit. \(X |0\rangle = |1\rangle, X |1\rangle = |0\rangle\)

\(Z\) (inverse): generate the inverse source. \(Z |0\rangle = |0\rangle, Z |1\rangle = -|1\rangle\)

and so on.

1.4.2. Multiple Qbit system

A system of quantum computation consisting of two Qbits has \(2^2 = 4\) bases.

\[
\begin{pmatrix}
00 & 01 & 10 & 11
\end{pmatrix} \quad (|S| = 2^2 = 4)
\]

\[
|0\rangle|0\rangle \quad |0\rangle|1\rangle \quad |1\rangle|0\rangle \quad |1\rangle|1\rangle
\]

It can also be written as \(|0\rangle|0\rangle = |0\rangle \otimes |0\rangle = |00\rangle\) (\(\otimes\) is the tensor product).

In the vector notation of the basis, this looks like this

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\(^2\)This notation uses the Cbit (classical bit) Qbit (quantum bit) of N. David Mermin 2002. In general, quantum bits are written as qubits.
is represented as $X_2 |0\rangle |0\rangle = |0\rangle |1\rangle$ in 2Qbit.

In the case of multiple Qbit operations, operations with control bits and target bits are meaningful.

$C_{ab}$ (controlled not): Inverts the target bit $b$ when the control bit $a$ is 1.  

$C_{10} |x\rangle |y\rangle = |x\rangle |y \oplus x\rangle$

$|x\rangle$ is the control bit, $|y\rangle$ is the target bit, and $\oplus$ is the addition modulo 2. If written as a matrix

$$C_{10} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \end{pmatrix}$$

$H$: (Hadamard) Perform the following classically meaningless special operations.  

$H |0\rangle = \frac{1}{\sqrt{2}} (|0\rangle + |1\rangle)$, $H |1\rangle = \frac{1}{\sqrt{2}} (|0\rangle - |1\rangle)$

$H$ can also be written as $X$ and $Z$ or a matrix as

$$H = \frac{1}{\sqrt{2}} (X + Z) = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}$$

Applying $C_{10}$ and $H$ to $|0\rangle |0\rangle$ produces the Bell state.

$C_{10}H |0\rangle |0\rangle = C_{10} \frac{1}{\sqrt{2}} (|0\rangle + |1\rangle) |0\rangle$

$$= C_{10} \frac{1}{\sqrt{2}} (|0\rangle |0\rangle + |1\rangle |0\rangle)$$

$$= \frac{1}{\sqrt{2}} (|0\rangle |0\rangle + |1\rangle |1\rangle) \ (Bellstate)$$

This bit string is in a state of quantum entanglement that cannot be described as a state of each qubit. It is a system in which the whole cannot be factorized into basic units, and the whole cannot be described by its parts.

Now here, by considering two properties of quantum computation: 1. the indivisibility of the whole to the basis (source) 2. the parallelism of unitary operations that can be
applied to each qubit simultaneously, a computational model using these properties is devised.

2. a blueprint for Four-states Quantum Formal System

System M (Mother or Male) and F (Father or Female) consist of independent sets \((E_1)\) and \((E_2)\), respectively; \((E_1)\) and \((E_2)\) contain independent elements \(|0\rangle|0\rangle, |1\rangle|0\rangle\) and \(|1\rangle|1\rangle, |0\rangle|1\rangle\).

\[
\left((E_1 \cup E_2) \supseteq E\right) \cap (E_1 \subset E, E_2 \subset E) \\
|0\rangle|0\rangle, |1\rangle|0\rangle \in E_1 \\
|1\rangle|1\rangle, |0\rangle|1\rangle \in E_2
\]

Now, apply \(C_{10}\) and \(H\) to the element \(|0\rangle|0\rangle\) to create the bell state.

\[
C_{10}H|0\rangle|0\rangle = \frac{1}{\sqrt{2}}(|0\rangle|0\rangle + |1\rangle|1\rangle) \quad (Bellstate)
\]

This equation implies that there is a 50% probability of being in state \(|0\rangle|0\rangle\) and a 50% probability of being in state \(|1\rangle|1\rangle\).

If A \(|0\rangle|0\rangle\) is observed (=1), then T \(|1\rangle|1\rangle\) is not observed (=0).
When T \(|1\rangle|1\rangle\) is observed (=1), A \(|0\rangle|0\rangle\) is not observed (=0).
Here, A is derived from system M and T is derived from system F.
In the genetic information, A and T and G and C are complementary to each other, and can be regarded as having the same information.
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