Suppressed decay into open charm for the $Y(4260)$ being an hybrid

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We investigate the $Y(4260)$ resonance recently discovered by the Babar collaboration. We propose the observation of its decay into $J/\psi\pi\pi$ and its non observation into open charm as a consequence of it being a charmonium hybrid state with a magnetic constituent gluon. We prove a selection rule forbidding its decay into two $S$-wave charmed mesons in any potential model. We suggest a generalisation of the selection rule based only on the heavy quark nature of the charm.

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I. INTRODUCTION

The recent observation [1] of the $Y(4260)$ resonant structure in the $\pi^+\pi^-J/\psi$ recoil mass of the process $e^+e^- \rightarrow \gamma_{\text{ISR}}\pi^+\pi^-J/\psi$ ($\gamma_{\text{ISR}}$: initial state radiation), is identified as a $J^{PC}=1^{--}$ single resonance with its mass centred around $\sim 4.26 \text{ GeV}$ and with a width $\sim 50$ to 90 MeV. While the mass of this resonance is well above the $D\overline{D}$ threshold, it has been observed only in the decay process $\pi^+\pi^-J/\psi$. It has therefore been claimed that it is not a standard charmonium state but rather an exotic.

We will argue in the following that these mysterious features may be an indication that the $Y(4260)$ is a hybrid charmonium in the $1^{--}$ state containing a pseudoscalar colour-octet $c\bar{c}$ and a magnetic constituent gluon: a selection rule strongly lessens its decay to ground state open charm states $D(\overline{D})_s$. In the following we will designate our candidate magnetic hybrid by $H_B$.

A four quark model has been proposed for the $Y(4260)$ [2]. A possible interpretation as a conventional charmonium is studied in [3]. In [4] Zhu examines several hypotheses and finally favours the hybrid interpretation. We will here support this opinion by several important dynamical arguments. Hybrid states ($q\bar{q}$ + $g$ hadron) are one of the most promising new species of hadrons. While the hybrids with the exotic quantum numbers such as $(0^{+-}, 1^{+-}, 2^{+-})$ would be observed as a very striking signal, the other kinds could also be distinguished from the conventional hadrons by the characteristics of their decay processes. However care has to be taken about possible mixing between the latter hybrids and conventional states. Extensive investigations in searching for the hybrid states have been pursued especially, in the light hadrons, though no evidence has been confirmed. Now that more and more new charmed hadrons have been discovered by B factory experiments, the hope of discovering $c\bar{c} + g$ hybrids has raised. The spectroscopy of the hybrid states which contain a constituent gluon would hopefully unveil some new features of QCD.

We will use the language of the constituent model (a generalisation of the quark model with constituent gluons). Hybrid charmonium is a bound state of $c\bar{c}$ and a gluon. Defining $l_g$ (the relative orbital momentum between $c\bar{c}$ and $g$), $l_{c\bar{c}}$ (the orbital momentum of $c\bar{c}$ state), $s_{c\bar{c}}$ (the spin of $c\bar{c}$), the quantum numbers of the hybrid mesons are:

$$P = (-1)^{l_g + l_{c\bar{c}}}, \quad C = (-1)^{l_{c\bar{c}} + s_{c\bar{c}} + 1} \quad (1)$$

Thus, a $1^{--}$ state can be composed either by $(l_g, l_{c\bar{c}}, s_{c\bar{c}}) = (0, 1, 1)$ or by $(1, 0, 0)$. The former possibility has already been studied in [5] and found that it may not exist as a resonance since it is too strongly coupled to the continuum $D\overline{D}$ channels (its estimated width exceeds 1 GeV). At the same time, it was shown in [5] within a harmonic oscillator potential model that the case $(l_g, l_{c\bar{c}}, s_{c\bar{c}}) = (1, 0, 0)$, which is $H_B$, obeyed a strict selection rule forbidding its decay to any two $S$-wave final mesons. Looking at its wave function one sees that it is proportional to the constituent gluon momentum in cross product with its polarisation (times a scalar function of the momenta). This indicates that we are dealing with a magnetic gluon. The same selection rule had been advocated for light quarks [6] - [8]. This result has been generalised for light quarks to a more general potential [9,10]. This very powerful selection rule results in an important decay pattern of $H_B$ which we will discuss in this letter; the lowest possible open charm final state comes from $D^*\overline{D}$, whose threshold is just above the observed resonance. As a result, we find i) Connected diagrams, fig. 1, can be significantly suppressed; ii) it is a resonant state with a moderate decay width contrarily to the above-mentioned $(l_g, l_{c\bar{c}}, s_{c\bar{c}}) = (0, 1, 1)$.

Concerning the mass of the hybrid states, a thumb rule tells that the constituent gluon is expected to add $0.7 \sim 1 \text{ GeV}$ to the corresponding quarkonia and the excited hybrid states lie another 0.4 GeV above, which sums up to the statement that the mass of this state would be around 4.2 $\sim 4.5 \text{ GeV}$.

This thumb rule agrees qualitatively with the outcome of the flux-tube model for hybrid states [11], as well as the lattice QCD simulations [12]. The selection rule considered in this paper has been claimed to be rather general,
including flux tube models [13,14]. A similar selection rule was also used to account for the missing charm puzzle of B decay [15].

The proof of the above-mentioned selection rule is of course crucial. Within the simple chromo-harmonic oscillator model it is shown in [5]. This model is not realistic but it is a convenient toy model. We give a short reminder of the model in the appendix, where one can also find a more concrete description of our $c\bar{c}g$ hybrid state.

In the body of this letter we will consider three issues. In the next section we will demonstrate generally the selection rule in a potential model. Next we will consider the production mechanism of the $H_B$ and its decay into $J/\psi\pi\pi$. Finally we will discuss the corrections to the selection rule.

II. SELECTION RULE FORBIDDING

$H_B \rightarrow D^{(*)}\overline{D}^{(*)}$

To lowest order, the decay of the hybrid state is described by the matrix element of the QCD interaction Hamiltonian between a hybrid wave function and a final state two-meson wave function. The result in the non-relativistic limit is given as factorised in terms of the colour, spin, spatial and flavour overlaps. In the following, we investigate the decay of the hybrid state into two charmed meson states ($H_B \rightarrow D^{(*)}\overline{D}^{(*)}$) through connected diagrams, see fig. 1.

The simplest interpolating field for the $H_B$ is

$$\bar{c}\gamma_5 \lambda^a P_J^a c$$  (2)

Since the magnetic field has the quantum number 1$^{+}$, the $c\bar{c}$ forms a pseudoscalar (0$^{-}$) colour octet. Thus, the polarisation of the hybrid is found to be parallel to the polarisation of the hybrid. The result in the spatially polarised gluon into an octet pair and a recombination of the two charmed and two light quarks into two mesons, fig. 1.

The colour and isospin overlaps are trivial. The spin overlap leads to a conservation of the total spin. The hybrid total spin is one (zero for the $c\bar{c}$ and one for the gluon). The model then forbids the decay into $D\overline{D}$. However, the decay into at least one $D^*$ ($\overline{D}^*$) is allowed by spin conservation. If the final states are ground state ($D^{(*)}, \overline{D}^{(*)}$) parity imposes a $P$-wave final state.

Next, we shall describe the spatial part which is at the origin of the selection rule we advocate. The spatial overlap is obtained as:

$$I = \int \int \frac{d^3\vec{k}}{2\sqrt{(2\pi)^3}} \Psi_{1_{l_B}}^{m_B} (\vec{p}_{c\bar{c}}^{}, \vec{k}) \Psi_{1_{l_B}}^{m_B} (\vec{p}_B^{}, \vec{k}) \Psi_{1_{l_C}}^{m_C} (\vec{p}_C^{}) d\Omega_f Y^m_f (\Omega_f)$$  (3)

where $\Psi_{1_{l_B}}^{m_B} (\vec{p}_B^{})$, $\Psi_{1_{l_B}}^{m_B} (\vec{p}_C^{})$ and $\Psi_{1_{l_C}}^{m_C} (\vec{p}_C^{})$ are the spatial wave functions for the initial hybrid state and the final $D^{(*)}$ and $\overline{D}^{(*)}$ states, respectively. The spherical harmonic function $Y^m_f (\Omega_f)$ represents the orbital momentum between the two final mesons. We have defined the relative momenta:

$$k = \frac{(m_c + m_\bar{c})p_\bar{c} - m_\bar{c}(p_c + p_\bar{c})}{m_\bar{c} + (m_c + m_\bar{c})}$$

$$p_B = \frac{m_q p_\bar{c} - m_c p_\bar{q}}{m_c + m_q}; p_C = \frac{m_q p_\bar{c} - m_c p_\bar{q}}{m_c + m_q};$$

$$p_{c\bar{c}} = \frac{p_c - p_\bar{c}}{2}; p_{q\bar{q}} = \frac{p_q - p_\bar{q}}{2}. \quad (4)$$

and in the hybrid state centre of mass system (c.m.s.), we have:

$$p_q + p_\bar{q} = -(p_c + p_\bar{c}) = p_g; \quad (p_c + p_\bar{c}) = -(p_q + p_\bar{q}) \equiv p_f. \quad (5)$$

where $\pm p_f$ are the momenta of the final mesons. Note that in the hybrids’ c.m.s., we also have:

$$p_g = k. \quad (6)$$

As a result, we can express all the relevant momenta in terms of $k, p_{q\bar{q}}, p_f$:

$$p_{c\bar{c}} = p_{q\bar{q}} - p_f, \quad p_B = -\frac{m_q p_f}{m_q + m_c} + p_{q\bar{q}} \frac{k}{2}; \quad p_C = \frac{m_q p_f}{m_q + m_c} - p_{q\bar{q}} \frac{k}{2}. \quad (7)$$

Let us consider the change of variable

$$k \rightarrow -k. \quad (8)$$

keeping $p_{q\bar{q}}, p_{c\bar{c}}, p_f$ unchanged. We will prove that in the case of $S$-wave final mesons the overlap integral (3) changes sign under the change of a variable (8) and thus must vanish. The hybrid wave function is odd in $k$ since $l_g = 1$. From formula (7) $p_B \leftrightarrow -p_C$. In the case of $S$-wave final mesons, the wave functions for $B$ and $C$ in eq. (3) are identical and even in $p_B$ and $p_C$. Their product remains thus unchanged by the transformation (8). The spherical harmonic function $Y^m_f (\Omega_f)$ is a function of the unit vector $\vec{p}_f$ and is thus unchanged. Finally the overlap integral (3) changes sign which ends our proof.

The decay $H_B \rightarrow D^{(*)}\overline{D}^{(*)}$ is forbidden in any potential model.

III. PRODUCTION OF THE $H_B$ AND ITS ALLOWED DECAY INTO $J/\psi\pi\pi$

The $Y(4260)$ is produced from the $e^+e^-$ pair in BABAR experiment [1]: the virtual photon produces a $c\bar{c}$ in the same quantum numbers as a $J/\psi$: a $1^{--}$
colour singlet. The hybrid state is created from two diagrams; the $c\bar{c}$ pair with a gluon emission from $c$ and $\bar{c}$. These two diagrams do not cancel. The standard QCD quark-quark-gluon interaction writes

$$\bar{c}_i \lambda^a A^a_{ij} c = \frac{-i}{2m_c} \bar{c}_i \sigma_{ij} \lambda^a A^a_{ij} p^i c + \cdots$$  \hfill (9)$$

The emission of a magnetic gluon along the line will flip the spin of the charmed quarks from spin 1 (vector) down to spin 0 (pseudoscalar), and their colour from singlet to octet. The final state has thus exactly the quantum number of the $H_B$. This transition is suppressed by one power of the charm mass.

A very similar mechanism generates $H_B \rightarrow J/\psi \pi \pi$ decay. The emission of an additional magnetic gluon from charmed quarks is done via the same (9) interaction (see fig. 2). The created magnetic gluon can obviously combine with the constituent gluon to produce a $0^{++}$ two-gluon state which decays into two pions in a $0^{++}$ state. The charmed quarks have their spin flipped by the $\sigma_{ij}$ matrix leading to a charmonium state. This decay is suppressed by one power of the charm mass but has a large available phase space which may explain the significant branching ratio observed in experiment.

**IV. CORRECTIONS TO THE SELECTION RULE**

One correction comes via a mixing with a standard charmonium state. Indeed the interaction (9) also induces a mixing of the hybrid with neighbouring excited charmonia such as $\psi(4160)$. However this interaction is $O(1/m_c)$ suppressed and furthermore these excited states have many nodes on their wave functions so that the overlap with the $c\bar{c} 0^{--}$ octet is expected to be rather small. Since the charmonium excitations are allowed to decay into $D^{(*)} \bar{D}^{(*)}$, this mixing induces a small correction to the selection rule: it is a second order mechanism which implies an additional factor $p_3$, see (9), which invalidates the argument of section II.

Relativistic effects on the charmed quarks may induce other $O(1/m_c)$ corrections to the selection rule since it has only been proven in the non-relativistic framework.

There are also relativistic corrections related to the light quarks, which are difficult to estimate. However, there is a general argument which implies that they should also be suppressed in the infinite $m_c$ limit. Following the philosophy of the HQET, we consider separately the dynamics of the heavy quarks and that of the light quanta (gluons, light quarks). The initial state has $c\bar{c}$ in an $S$-wave, and the light quanta (constituent gluon) possess an orbital excitation relative to the heavy quark system. The final state contains an orbital momentum between the two heavy quarks if we consider $S$-wave final mesons. The orbital excitation has to be transferred from the light system, the “brown mock”, to the heavy quark system. This is presumably suppressed for the following reason. The heavy quark system has a vanishing spatial size in the infinite mass limit. The cloud of light quanta has a constant size. The overlap vanishes in this limit. We can then argue that the orbital momentum transfer is consequently suppressed. Of course this is a qualitative argument which needs to be demonstrated and checked. But we believe it to be reasonably convincing.

**V. OTHER DECAY CHANNELS**

Beyond the violation of the selection rule which should allow a non negligible branching fraction for the $Y(4260) \rightarrow D^{(*)} \bar{D}^{(*)}$ channel, we must also consider $Y(4260) \rightarrow D^{**} \bar{D}^{(*)} \rightarrow D^{(*)} \bar{D}^{(*)} \pi'$s and $Y(4260) \rightarrow D^{(*)} \bar{D}^{**} \rightarrow D^{(*)} \bar{D}^{(*)} \pi'$. They are not forbidden by the selection rule. The $Y(4260)$ lies below the $D^{**} \bar{D}^{(*)}$ thresholds (although very close to the $D_1(2420)\bar{D}$ one).
But these resonances are not narrow and the decay via a virtual $D^{(*)} \bar{D}^{(*)}$ should not be small. Therefore we would expect the dominant decay channel to be $Y(4260) \to D^{(*)} \bar{D}^{(*)}$'s which might explain a width as large as 90 MeV for the $Y(4260)$.

VI. CONCLUSIONS

We have argued that the recently observed $Y(4260)$ shows peculiar characteristics possessed by a new type of hadrons, an hybrid state, namely, a bound state of an octet $0^+ cc$ state and gluon in a $P$-wave (a magnetic gluon), that we call $H_B$. The decay of $H_B$ is restricted by an important selection rule: the symmetries of the wave functions forbid the decay into two $S$-wave open charm final mesons in any potential model. Therefore, $H_B$ cannot decay into e.g. $D^{(*)}\bar{D}^{(*)}$ and thus, has a relatively narrow width, which matches the experimental observation for $Y(4260)$. We also showed that the observation channel $Y(4260) \to J/\psi\pi\pi$ can be naturally explained by $H_B$. At the same time we expect decays such as $Y(4260) \to D^{(*)} \bar{D}^{(*)}$ to be dominant and $Y(4260) \to D^{(*)} \bar{D}^{(*)}$ not to be negligible via a violation of the selection rule. The $H_B$ mass is roughly estimated to be around $4.2 \sim 4.5$ GeV, though its rigorous prediction would be an interesting challenge for the lattice QCD. We have discussed the mixing with ordinary charmonia and the relativistic corrections. We argue that the latter are $O(1/m_c)$ suppressed using a HQET inspired argument. A deeper theoretical understanding of these issues is needed as well as a search for further predictions concerning the properties of $H_B$ to be confronted with experimental data concerning the $Y(4260)$ or other similar resonances, such as $X/Y(3940)$ [16]. In particular, several other hybrids protected by the same selection rule are expected which should be compared with the increasing number of resonant candidates in this region. Of course, all of them are flavor-$SU(3)$ singlets which discriminates clearly this hypothesis from the four quark model.

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APPENDIX A

The $q\bar{q}g$ system can be modelled by a double chromo-harmonic oscillator [17]:

$$H_{qg} = \frac{p^2}{2M} + \frac{p_{qg}^2}{2\mu_{qg}} + \frac{k^2}{2\mu_g} - \frac{7b_0^2}{12} - 3b_0'\bar{r}^2_H$$

where $b_0$ is the level spacing. The configuration is given as follows; in the centre of mass system:

$$P = p_q + p_{\bar{q}} + p_g, \quad M = m_q + m_{\bar{q}} + m_g$$

the relative momenta have been defined in eq. (4). The corresponding conjugate variables are:

$$r_{q\bar{q}} = r_q - r_{\bar{q}}, \quad \mu_{q\bar{q}} = \frac{m_q m_{\bar{q}}}{m_q + m_{\bar{q}}}$$

$$r_{H_B} = r_g - \frac{r_q + r_{\bar{q}}}{2}, \quad \mu_{H_B} = \frac{m_g (m_q + m_{\bar{q}})}{m_q + m_{\bar{q}} + m_g}$$

The solution to the eq. (A1) can be summarised as:

$$\Psi_{l_i}(p_i) = \frac{16\pi^3 R_{l_i}^2 i^{36} \Gamma(\frac{2}{g} + 1) p^4_i}{\Gamma(\frac{2}{g} + l_i)} Y_{l_i}^m(\theta, \Omega) e^{-\frac{1}{2}\mu_{H_B}^2 p^2_i}$$

where $i = q\bar{q}, g$ and $l_{qq}$ and $l_g$ are the relative orbital momentum between $qq$ and between $g$ and $qq$ centre of mass, respectively. The radial part of the solutions are obtained as:

$$R_{qq}^2 = \frac{1}{2\mu_{qq}} \left( -7b_0 \frac{1}{12} \right)$$

$$R_g^2 = \frac{1}{2\mu_g} (-3b_0)$$

From this result, we can also obtain the level spacing between the ground state and the first excited state by using a relation:

$$\omega_i = 1/(\mu_i R_i^2).$$

When considering the heavy quarks as constituents, a simple physical picture of such a hybrid state can be drawn from these expressions. The ratio of eqs. (A5) and (A6), which represents the size of the hybrid state relative to $qq$ state,

$$\frac{R_g^2}{R_{qq}^2} = \sqrt{\frac{7\mu_{qq}}{36\mu_{H_B}}}$$

goes to infinity when $m_q \to \infty$, so that the heavy quark system shrinks. In the same manner, we find

$$\frac{\omega_g}{\omega_{qq}} = \sqrt{\frac{36\mu_{qq}}{7\mu_{H_B}}}$$

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expressing that the heavy quarks oscillate slowly as compared to the gluon frequency in this limit. On the other hand, the charm quark being not so heavy, these ratios are far from large:

\[
\frac{R_g^2}{R_{qq}^2} \simeq 0.51, \quad \frac{\omega_g}{\omega_{qq}} \simeq 2.6
\] (A10)

for \( m_c = m_{\bar{c}} = 1.7 \text{ GeV} \), \( m_g = 0.8 \text{ GeV} \). Therefore, while the faster oscillation of \( g \) is somehow observed, \( c\bar{c} \) is not really shrunk. Indeed, these values are obtained from our potential in eq. (A1) containing a certain colour configuration of the states: eq. (A10) is the direct consequence of the fact that the \( c\bar{c} \) forms an octet state which would fall apart if there was not a screening by the gluon cloud: the resulting string tension is small. On the contrary the string tension between \( c\bar{c} \) and \( g \) is large since it is the string tension between two color octets forming a singlet. The picture is that of high frequency light quanta and low frequency heavy quarks.

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