Collaborative spectrum sharing (CSS) between primary and secondary users (PU and SU) is 
an effective way of utilizing limited radio spectrum. In CSS, cognitive SU acts as a relay for PU, which 
facilitates the PU to send its packet with a lower error probability by the help of the SU, and consequently, 
the SU has more chances to find a vacant spectrum. When SU is equipped with multiple antennas, it can 
further efficiently utilize the radio spectrum by adopting a growing self-interference cancelation technique 
for full-duplex (FD) transmission. In this paper, both an aggressive and a passive secondary usage of 
the spectrum are proposed, the operational principles of which are defined using spectrum-sharing probability 
$\phi$ in FD CSS environments. For the spectrum-sharing probability $\phi$, SU for each of the methods sends its 
own signal and the PU’s (relayed) signal together by using superposition transmission. For the remaining 
probability $1 - \phi$, the two methods work differently: SU sends its own signal only in the aggressive mode 
while it sends only the relaying signal for PU in the passive mode. We formulate an FD CSS problem with 
various physical-layer parameters and the proposed operating modes as a function of the spectrum-sharing 
probability. Our goal is to maximize the secondary stable throughput while keeping a primary traffic 
constraint. Closed-form optimal solutions on $\phi$ are provided in the paper, the value of which heavily depends 
on the primary traffic volume, the operating modes, the relative locations of the collaborative nodes and the 
transmit power budget at SU. The analytical results in the paper are verified with numerical investigation, 
and the performance enhancement by the proposed methods is evaluated in comparison with benchmark 
systems. The results show that the aggressive mode is promising if SU has a relatively small transmit power 
and the primary traffic load is low, while the passive mode is suitable when the primary traffic load is high.

**INDEX TERMS** Spectrum sharing, cognitive/collaborative mobile nodes, full-duplex capability, stability 
analysis.

**I. INTRODUCTION**

As an effective way of cognitive utilization of the 
limited radio spectrum, collaborative spectrum sharing (CSS) 
between primary and secondary users (PU and SU) has 
attracted much research interest [1]–[7]. Unlike ordinary cog-
nitive radio systems, cognitive SU in CSS acts as a relay 
for PU, which facilitates the PU to send its packet with a 
lower error probability by the help of the relay (i.e. SU) 
and consequently can provide more chance for SU to find 
a vacant spectrum. In CSS, a time slot is usually divided 
to two consecutive mini-slots, which consist of PU and SU 
transmission phases. If PU transmits its signal at the first 
mini-slot, SU may sense and relay it to the PU’s destination 
at the second mini-slot in which the SU’s own signal can also 
be transmitted simultaneously with the PU’s signal by being 
superimposed. If PU is idle at the first mini-slot, SU can send 
only its own data at the second mini-slot. SU in CSS can 
always access the spectrum either as a collaborative relay [5] 
or as a pure sender. However, the SU can also always inter-
fere with the PU by either sending two signals simultane-
ously or missing detection of PU’s existence and sending its
own signal. Thus, resource allocation at the SU is essential in CSS in order to prevent PU’s signal quality from dropping below a predefined target. CSS therefore lies somewhere in between underlay and interweave cognitive radio systems [8]. With proper resource allocation, SU in CSS is shown to achieve significant performance improvement compared to SU in ordinary cognitive radio systems [1]. Though the time slot is halved in CSS, PU’s transmission performance is still well protected to a desired level thanks to the power gain obtained by SU acting as a relay.

CSS can be further improved if letting PU transmit at both the first and the second mini-slots and/or letting the relay of PU’s signal be made at the instant SU receives it without waiting for the next mini slot. In [9], the former method is shown to significantly improve the achievable throughput and can reduce the outage probability by optimally splitting PU’s power between the first and the second transmission as well as by adopting cooperative beamforming between PU and SU. On the other hand, the second one can be fulfilled by introducing a full-duplex (FD) capability at SU, which of course needs only a single slot in relaying. In-band FD of simultaneous radio transmission and reception on the same frequency band is getting implementable with the help of advanced interference cancelation (IC) techniques and is shown to achieve an approximately twofold capacity increase for isolated radio links [10].

Feasibility of FD relay systems has been addressed in many works. Especially, in MIMO FD relay systems, self-interference (SI) can be eliminated through multiple-antenna signal processing and close-to-twofold capacity is achieved compared to the half-duplex counterparts [11]–[13]. In [14], zero-forcing (ZF) loopback interference suppression at the relay node is proposed. In [15] and the references therein, to suppress SI and interference due to multiple users, ZF and block diagonalization precodings are used. However, the feasibility results are not always applicable due to the varying interference cancelation levels in different realistic environments. In this respect, [16] addresses an FD MIMO transceiver design that is aware of aging and inaccuracy of FD hardware components. On the other hand, assuming that residual loopback interference exists due to imperfect cancelation, [17] provides a theoretical FD precoder design problem that can be solved in closed-form expressions for MIMO source and relay with an energy harvesting destination. In [18], system-level FD feasibility is also investigated in a stochastic geometry-based network of multiple-antenna base stations and FD relay nodes.

FD-based cognitive radio systems also have a great potential for improving the spectrum efficiency. For FD-capable SUs at which the result of sensing whether PU exists is affected by residual self-interference, a special “listen-to-talk” sensing protocol is proposed in [19], a waveform-based sensing approach is considered in [20], and an FD wireless sensing is provided in [21] when PU and SU are non-time-slotted and not synchronized. In [22], the sum mean-squared error of multiple cognitive FD SU pairs is investigated and minimized under the SU power constraints and the interference constraint on PU, which is an example for FD underlay. In [23], an underlaying cognitive radio system with multiple FD SU relays is investigated and closed-form expressions for the channel capacity and outage probability are provided. In [24], an overlay version of FD CSS is presented, where SU relays the PU signals superimposed with its own signal, and the achievable rate region given the primary and cognitive power constraints is investigated. Reference [25] provides an FD-based one-way OFDM relaying system where the SU receives/transmits the PU signals over a subset of subcarriers in two phases while transmitting its own signals over the remaining subcarriers. The results in [24] and [25] are extended to cover an FD-based two-way OFDM relaying system in [26].

In this paper, we propose a hybrid version of FD CSS that falls in the middle of overlay (or interweave) and underlay cognitive radios and it also becomes an overlay system in a trivial case. In overlay FD CSS in [24] and its variants, the SU relays PU’s signal as well as sends its own signal by using zero-forcing (ZF) beamforming transmission for two distinct destinations. The simultaneous transmission of SU’s and PU’s signals may improve the spectrum utilization but sometimes degrades the performance by splitting the transmit power of SU between the two signals. The performance degradation occurs especially when PU and SU are far away from each other or the radio channel between them is in a deep fading but SU tries to relay PU’s signal. This paper is motivated by the need to let SU be able to switch the collaboration mode between compositely relaying PU’s signal and either only transmitting its own signal (in an aggressive mode) or purely relaying PU’s signal (in a passive mode). The switching in this paper is controlled by the spectrum-sharing control probability \( \phi \) (0 ≤ \( \phi \) ≤ 1). When PU is sensed as active, SU works as an overlay FD CSS relay described above with probability \( \phi \). On the other hand, with probability 1 − \( \phi \), the SU uses either of the following two strategies: the aggressive and the passive usage of the spectrum. With the aggressive one, the SU transmits purely its own signal but interference to PU’s receiver is avoided by ZF. With the other, the SU purely relays PU’s signal. The first strategy has its name because the SU aggressively occupies the spectrum with probability 1 − \( \phi \) even though PU exists, while the other one has its name because the SU passively utilizes the spectrum for only part of the time (i.e., for the time probability \( \phi \)) when acting as a CSS relay. If we let \( \phi = 1 \), the proposed FD CSS is equivalent to the system presented in [24]. When PU is sensed to be idle, SU turns its receiver off and works as a pure transmitter of its own signal (now not operating in FD mode).

As a measure of performance, we investigate the stable throughput of the proposed FD CSS, which keeps the PU also stable respect to the input arrival rates of data. We assume that PU and SU are equipped with a buffer for the input data packets, respectively. For a given arrival data rate at PU, an optimal spectrum-sharing probability \( \phi^* \) is obtained for
the above explained strategies, respectively. And the stability region that is defined as a set of arrival data rates for PU and SU, with which the queue lengths of both PU and SU are finite, is presented. Since the arrivals of the data at PU and SU are bursty and the interference between PU and SU is thus probabilistic, stable throughput is a meaningful measure of network-level performance. In order to treat the interacting queues in the system, the stochastic dominance concept in [27] is used for investigating the throughput. Such stability studies for half-duplex (cognitive) relay systems have been widely found in the literature [1], [3], [28]. For FD systems, however, stability studies are rather limited so far. In [29], the stability throughput of an FD Aloha network is characterized. The stability of secure communication is investigated for two-user FD broadcast channel in [17]. The stability of an FD relay queue that assists two-user multiple access with multi-packet reception capability at the receiver is discussed in [30]. For a relay with two queues, each of which is for the respective pair of users (i.e., the respective source-destination pair), the per-user stable throughput is presented in [31].

The main contributions of this paper are summarized as follows.

- We formulate the stability throughput in FD CSS environments with various physical-layer parameters and operating modes as a function of the spectrum-sharing probability. The spectrum-sharing probability determines the resource allocation between PU and SU as well as the throughput in CSS environments.
- We propose two operating modes of FD CSS that give further operational flexibility to CCS nodes and those modes are shown to improve the performance in combination with the optimal spectrum-sharing probability provided in the paper.
- We define our optimization problems by finding an optimal spectrum-sharing probability that maximizes SU’s throughput under the constraint of keeping PU’s throughput at a given level. And we provide closed-form solutions on optimal spectrum-sharing probabilities, which are numerically verified in the paper, and also present the corresponding stability regions.

Numerical investigation is presented for the proposed analysis and show that the aggressive mode with the optimal-spectrum sharing provided in the paper improves the performance (compared to the fixed overlay system, i.e., letting \( \phi = 1 \)) if the traffic load of PU is low. On the other hand, if PU’s traffic load is high, the passive mode is shown to enhance the performance. To the best of the authors’ knowledge, such design and analysis of combining collaboration modes and performance optimization have not been conducted for FD CSS systems.

The remainder of the paper is organized as follows. Section II provides a comprehensive description on the overall system model, the physical-layer model in detail, the definition of stability and the optimization problem focused on herein. The probabilities of successful decoding (PrSDs) are also defined in Section II, playing a critical role in the throughput analysis. According to the relationship between PrSDs, we choose one representative condition as an assumption to provide a context for the results in Sections III and IV. The other conditions, which are rarely satisfied in usual FD CSS environments, are also considered in Appendix A. Section III presents an analysis and a closed-form solution of spectrum-sharing for the aggressive case. The passive case is investigated in Section IV. Section V introduces benchmark CSS systems for a performance comparison and provides numerical verification of the optimal spectrum-sharing provided in the paper. The usefulness of the proposed FD CSS to enhance the stability throughput compared to the benchmark systems is also illustrated in Section V. Section VI concludes the paper. Appendix A provides closed-form optimal spectrum-sharing and the corresponding stability regions for miscellaneous conditions for PrSDs that are not treated in the main body of the paper. Finally, Appendix B provides the stability throughput of a benchmark system for comparison.

**II. SYSTEM MODEL**

**A. OVERALL MODEL**

We consider a communications network (shown in Figure 1) that consists of a primary transmitter-receiver pair (PT-PR) operating on a licensed radio frequency (RF) channel and a secondary pair (ST-SR) being capable of performing cognitive radio (CR) functions. With the CR capability, ST in CSS works as a relay for the PT-PR pair as well as sends its own signal if it does not degrade PT’s transmission quality. We assume that PT, PR and SR are equipped with single antennas, while ST is equipped with \( M \) receive and \( N \) transmit antennas. Both PT and ST have a buffer of infinite capacity to store their own incoming packets, respectively. Time is slotted and the transmission of each packet is assumed to take one time slot. The packet arrival processes are independent and jointly strictly stationary with mean \( \lambda_p \) (packets per slot) and \( \lambda_s \) for PT and ST, respectively. \( Q_p \) and \( Q_s \) denote their respective queue lengths.

At the start of each slot, given the operating mode (either aggressive or passive), ST senses whether PT is active or not (equivalently, busy or idle). We assume that the sensing

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**FIGURE 1.** System model.
result could be wrong and let $p_d$ and $p_f$ denote the detection and false alarm probabilities, respectively. We ignore the loss in bandwidth due to sensing duration in this paper, but it could be accounted for in a straightforward way if the sensing duration is given and fixed. When PT is sensed to be busy, ST, with probability $\phi$, acts as an FD collaborative relay for the PT-PR pair, in which ST simultaneously sends its own data and relays PT’s data by using zero-forcing (ZF) and co-phasing techniques. With probability $1 - \phi$, ST is assumed to operate in either of two methods: aggressive or passive methods. In the aggressive mode, ST sends only its own data without relaying PT’s data, for which ST uses ZF weights to nullify possible interference to PR. In the passive mode, on the other hand, ST purely relays PT’s signal instead of sending its own signal, for which ST employs a beamforming technique to reinforce the active PT-PR channel. When PT is sensed to be idle, ST sets its receiver as inactive and sends its own data. In this case, ST does not operate in the FD mode. It is noted that the CR operation assumed in this paper may be perfect in this paper.

As shown in Figure 1, let $h_p$ and $g_p$ denote $M \times 1$, $1 \times N$ and $1 \times N$ channel vectors for the pairs PT-ST, ST-SR and PT-PR, respectively. Let $G_p$ denote the $M \times N$ self-interference channel at ST. We assume that radio propagation over any transmitter-receiver pair is an independent stationary Rayleigh flat-fading channel.

| TABLE 1. Composite signal transmitted by ST, $s^{(c)}$ (The terms with $\dagger$ and $\ddagger$ are used in the aggressive and passive mode, respectively.) |
|---|
| $H_0$ | $Q_s = 0$ |
| $H_1 \phi$ | $w_{1}^{(p)} \sqrt{P_s h_y y_{ST}}$ |
| $H_1 (1 - \phi)$ | $w_{1}^{(p)} \sqrt{P_s h_y y_{ST}} + w_{1}^{(s)} \sqrt{P_s (1 - \alpha) x^{(s)}}$ |

1) ST’s COMPOSITE SIGNAL FORM

Let $x^{(p)}$ and $x^{(s)}$ denote PT’s and ST’s signal with $E[|x^{(p)}|^2] = E[|x^{(s)}|^2] = 1$, respectively. And let $P_p$ and $P_s$ denote transmitting power of PT and ST, respectively. When ST transmits in an FD relaying mode, it does not purely transmit its own signal $x^{(s)}$ but sends composite signal $s^{(c)}$ constructed by $x^{(s)}$ and $x^{(p)}$ that is possibly a part of received signal $y_{ST}$ described as follows. Let $\theta^{(p)}$ be a binary variable that indicates whether PT is active ($\theta^{(p)} = 1$) or inactive ($\theta^{(p)} = 0$). Then the received signal at ST after applying a weight vector $w_R$ for multiple ($M$) receiving antennas is

$$y_{ST} = w_R \left( \theta^{(p)} \sqrt{P_p h_r x^{(p)}} + \theta^{(d)} G_r s^{(s)} + n_{ST} \right),$$

where $\theta^{(d)}$ denotes optional SIC (either $\theta^{(d)} = 1$ when receiver-side SIC is applied or $\theta^{(d)} = 0$ when transmitter-side SIC is applied) and $n_{ST}$ is an additive white Gaussian noise (AWGN) vector at ST. Hereafter we assume that the AWGN at each receiver antenna is independently and identically distributed according to $CN(0, N_0)$.

With $y_{ST}$, the composite signal $s^{(c)}$ to be transmitted by ST is constructed (please see details depicted in Table 1, where proposed aggressive-and-passive-mode CSS denoted by agg and pss in the table, respectively) according to the buffer state, the sensing result and the physical layer options. Let $H_0$ and $H_1$ denote the sensing results on the transmitting state of PT: idle and busy, respectively. When $Q_s \neq 0$ and the sensing result is $H_0$, ST transmits only $x^{(c)}$ by constructing $s^{(c)} = w_{1}^{(p)} \sqrt{P_s x^{(s)}}$ where $w_{1}^{(p)}$ is a ZF weight described shortly in the following. If the sensing result is $H_1$ when $Q_s \neq 0, y_{ST}$ and $x^{(s)}$ together form $s^{(c)}$. In this case, PT’s signal is relayed with probability $\phi$ and hence

$$s^{(c)} = w_{1}^{(p)} \sqrt{P_s h_p y_{ST}} + w_{1}^{(s)} \sqrt{P_s (1 - \alpha) x^{(s)}},$$

where $w_{1}^{(p)}$ is a ZF weight that is used to limit the interference to SR and is described below shortly, $\alpha$ is a power allocation factor between relaying signal $y_{ST}$ and ST’s signal $x^{(s)}$, and $\hat{h}_p$ is a normalized co-phasing AF weight given by

$$\hat{h}_p = \frac{h_p}{|h_p|} \sqrt{\frac{1}{P_p |w_R h_r|^2 + N_0}}.$$

Here $\hat{h}_p$ is used to align the channel of relayed $y_{ST}$ into the channel of signal traveling from PT to PR and to regulate the relaying power of $y_{ST}$ equal to $P_s \alpha$. On the other hand, though
TABLE 2. Transmit and receive weights at ST.

(a) Transmit-side SIC (TX nulling)

| weight $w$ | state of $Q_s$ | Description | ZF matrix $W$ | Interference matrix $F$ |
|------------|----------------|-------------|---------------|----------------------|
| $w_T^{(s)}$ | $Q_s = 0$      | 0           | -             | -                    |
|            | $Q_s \neq 0$  | $\frac{W(h_s, y)^H}{||W(h_s, y)||}$ | $I - F F^H (F F^H)^{-1} F$ | $F = g_s w_R G_r$ |
| $w_T^{(p)}$ | $Q_s = 0$      | 0           | -             | -                    |
|            | $Q_s \neq 0$  | $\frac{W(q_s, y)^H}{||W(q_s, y)||}$ | $I - F F^H (F F^H)^{-1} F$ | $F = h_s^H G_r$ |
| $w_R$      | $Q_s = 0$      | $h_s^H$     | -             | -                    |
|            | $Q_s \neq 0$  | $\frac{W(q_s, y)^H}{||W(q_s, y)||}$ | $I - F F^H (F F^H)^{-1} F$ | $F = h_s^H G_r$ |

(b) Receive-side SIC (RX nulling)

| weight $w$ | state of $Q_s$ | Description | ZF matrix $W$ | Interference matrix $F$ |
|------------|----------------|-------------|---------------|----------------------|
| $w_R^{(s)}$ | $Q_s = 0$      | 0           | -             | -                    |
|            | $Q_s \neq 0$  | $\frac{W(h_s, y)^H}{||W(h_s, y)||}$ | $I - F F^H (F F^H)^{-1} F$ | $F = g_s$ |
| $w_R^{(p)}$ | $Q_s = 0$      | 0           | -             | -                    |
|            | $Q_s \neq 0$  | $\frac{W(q_s, y)^H}{||W(q_s, y)||}$ | $I - F F^H (F F^H)^{-1} F$ | $F = h_s$ |
| $w_R$      | $Q_s = 0$      | $W(h_s, y)^H W_s | |W(h_s, y)||$ | $I - F (F^H F)^{-1} F$ | $F = G_r (\frac{1}{|g_s|})$ |
|            | $Q_s \neq 0$  | $\frac{W(h_s, y)^H W_s | |W(h_s, y)||$ | $I - F (F^H F)^{-1} F$ | $F = G_r (\frac{1}{|g_s|})$ |

the sensing result is $\hat{H}_1$ and $Q_s \neq 0$, with probability $1 - p_s$, ST either sends only its own signal such that $s^{(s)} = w_T^{(s)} \sqrt{P_s} x^{(s)}$ in the aggressive spectrum sharing or ST purely relays PT’s signal such that $s^{(p)} = w_T^{(p)} \sqrt{P_s} h_p^{(p)} x^{(s)}$ in the passive spectrum sharing. It is noted that if we take $\alpha = 0$ in (2) and choose the passive mode, $s^{(s)}$ becomes either $w_T^{(s)} \sqrt{P_s} x^{(s)}$ with probability $\phi$ or $w_T^{(p)} \sqrt{P_s} h_p^{(p)} x^{(s)}$ with probability $1 - \phi$.

2) WEIGHT VECTORS USED AT MULTIPLE ANTENNAS OF ST

We now describe how to select $w_T^{(p)}$ and $w_T^{(s)}$. The weight vectors are obtained differently according to SIC options whether SIC is applied at the transmitter side or the receiver side at ST. If SIC is performed at the transmitter side of ST, $\theta^{(s)} = 0$ in (1) and $w_R = h_s^H$ is an optimal combining weight (that is, it provides maximal-ratio combining [32]). If $w_R$ is given, $w_T^{(p)}$ and $w_T^{(s)}$ are then obtained by ZF weights such that

$$w_T^{(p)} = \arg \max_\omega |g_s \omega|^2 \text{ s.t. } |w_R G_r \omega|^2 = |h_s \omega|^2 = 0,$$  
$$w_T^{(s)} = \arg \max_\omega |h_s \omega|^2 \text{ s.t. } |w_R G_r \omega|^2 = |g_s \omega|^2 = 0,$$ 

respectively. The solution weights are summarized in Table 2(a).

On the other hand, when SIC is performed at the receiver side of ST, $w_T^{(p)}$ and $w_T^{(s)}$ are obtained by ZF weights without the SIC constraint $|w_R G_r \omega|^2 = 0$ in (4) and (5), respectively, and given in the first two rows in Table 2(b). Then $w_R$, which cancels the self-interference, can be obtained by

$$w_R = \arg \max_\omega |w_R \omega|^2 \text{ s.t. } |w_R G_r w_T^{(p)}|^2 = |w_R G_r w_T^{(s)}|^2 = 0,$$ 

the solution weight of which is given in the third row in Table 2(b).

3) RECEIVED SIGNAL FORMS AT PR AND SR

When ZF weights $w_R$, $w_T^{(p)}$ and $w_T^{(s)}$ are used as described above, the received signal at PR is

$$y_{PR} = \theta^{(p)} \sqrt{P_s} h_p x^{(p)} + g_s s^{(s)} + n_{PR},$$  

where $n_{PR}$ is AWGN at PR. And the received signal at SR is

$$y_{SR} = h_s^{(s)} + \theta^{(p)} \sqrt{P_s} g_p x^{(p)} + n_{SR},$$  

where $n_{SR}$ is AWGN at SR.

4) PROBABILITY OF SUCCESSFUL DECODING

We assume that a receiving node can decode a packet successfully if the received instantaneous signal-to-interference-plus-noise ratio (SINR or sometimes SNR if the interference does not exist) is greater than or equal to a certain threshold $I_{\text{SR}(p,a)}$ (the subscript $p$ representing for PR and $s$ for SR). Let $H_0$ and $H_1$ denote the actual state of PT: idle and busy, respectively. The probability of successful decoding (PrSD) at each receiver is denoted by $q_{i,k}^{(l)}$, and summarized and described in Tables 3 and 4, respectively. In the notation of PrSD, superscript (i) is in [(p), (e), (s)] and superscript (p) means PrSD at PR when the signal is only from PT, (e) means PrSD at PR when the signal is from both PT and ST and combined at PR, and (s) means PrSD at SR. And subscripts $j,k$ are used in the form of $q_{i,j,k}^{(l)}$ where $j \in \{z, b\}$ denotes a class of weight applied to transmit antennas and $k \in \{f, l\}$

TABLE 3. Notations of success probability (The terms with † and ‡ are used in the aggressive and passive mode, respectively).

| Notation | Description |
|----------|-------------|
| \(q_{c,f}^{(s)}\) | \(Pr\left(\frac{P_s|w_s|^2 h_f^2}{N_0} > \Gamma_p\right)\) |
| \(q_{b,f}^{(c)}\) | \(Pr\left(\frac{P_s|w_s|^2 h_f^2}{N_0} > \Gamma_p\right)\) |
| \(q_{c,l}^{(s)}\) | \(Pr\left(\frac{1-(1-\alpha)P_s|w_s|^2 h_l^2}{N_0 + P_g|g|^2} > \Gamma_s\right)\) |
| \(q_{b,l}^{(c)}\) | \(Pr\left(\frac{1-(1-\alpha)P_s|w_s|^2 h_l^2}{N_0 + P_g|g|^2} > \Gamma_s\right)\) |
| \(\tilde{q}_{c,l}^{(s)}\) | \(Pr\left(\frac{1-(1-\alpha)P_s|w_s|^2 h_l^2}{N_0 + P_g|g|^2} > \Gamma_s\right)\) |
| \(\tilde{q}_{b,l}^{(c)}\) | \(Pr\left(\frac{1-(1-\alpha)P_s|w_s|^2 h_l^2}{N_0 + P_g|g|^2} > \Gamma_s\right)\) |


denotes a power allocation state. Subscript \(z\) means that the transmitting signal is processed with ZF weights in ST to relay PT’s signal as well as to transmit its own signal, while \(b\) means that the signal is processed with beamforming weights in ST just to relay PT’s signal since it operates in the passive mode or \(Q_s = 0\). Furthermore, subscript \(f\) means that ST transmits with its full power \(P_s\) either to relay PT’s signal or to send its own signal, but \(l\) means that the power is split and allocated by \(\alpha\) and \(1-\alpha\) between the relaying signal and ST’s signal, respectively.

It is also noted that the tildes in \(q_{c}^{(s)}\) at the fifth row in Table 3 (a) indicate that the signal desired at SR is directly interfered with the signal from PT, while the other PrSDs without tildes could be obtained from interference-free environments by co-phasing or ZF techniques even though the two transmitters PT and ST are simultaneously active. It is finally noted that PrSDs \(q_{c,f}^{(p)}\), \(q_{c,f}^{(s)}\), \(q_{b,f}^{(c)}\), \(q_{c,l}^{(s)}\), and \(q_{b,l}^{(c)}\) can be obtained easily in a closed-form. Also, \(q_{c}^{(c)}\) and \(q_{b}^{(c)}\) can be obtained straightforwardly as in [33] but with integral parts, for which we have used a numerical method to evaluate \(q_{c}^{(c)}\) and \(q_{b}^{(c)}\) for the performance investigation. For notational brevity, we introduce some inequalities regarding the PrSDs:

\[\Delta_s \overset{\text{def}}{=} q_{c,f}^{(s)} - q_{c,l}^{(s)} \geq 0 \quad \text{since} \quad \frac{P_s|w_s|^2 h_f^2}{N_0} \geq \frac{1-(1-\alpha)P_s|w_s|^2 h_l^2}{N_0 + P_g|g|^2}\]

\[\Delta_s \overset{\text{def}}{=} q_{b,f}^{(c)} - q_{b,l}^{(c)} \geq 0 \quad \text{since} \quad \frac{N_0 + P_g|g|^2}{N_0} \geq \frac{1-(1-\alpha)P_s|w_s|^2 h_l^2}{N_0 + P_g|g|^2}\]

In the following sections III and IV, we provide the main results by assuming \(q_{c}^{(c)} \geq q_{c}^{(s)} \geq q_{b}^{(c)}\) which is to be held when \(E[|h_f|^2] > E[|g_s|^2]\) are sufficiently larger than \(E[|h_p|^2]\), respectively. We then denote \(\Delta_s \overset{\text{def}}{=} q_{c,l}^{(s)} - q_{c,f}^{(s)} \geq 0\) and \(\Delta_c \overset{\text{def}}{=} q_{b,f}^{(c)} - q_{b,l}^{(c)} \geq 0\). The results for the other cases are also summarized in Appendix A, where the assumed inequality \(\Delta_s \geq 0\) or \(\Delta_c \geq 0\) is violated and a trivial form (either 0 or 1) of spectrum-sharing probability is obtained. A geometrical illustration of condition \(q_{c}^{(c)} \geq q_{c}^{(s)} \geq q_{b}^{(c)}\) and other ones will also be provided in Section V.

C. STABILITY CONSTRAINT AND PROBLEM DESCRIPTION

Let \(\mu_p\) and \(\mu_s\) denote the average service rate of the queue in PT and ST, respectively. For queues where the arrival and service processes are jointly strictly stationary and ergodic, Loynes’ theorem [34] states that the queue at each transmitting node is stable if and only if the average arrival rate is strictly less than the average service rate. Based on Loynes’ theorem, let us define that a queue is stable if and only if the average arrival rate is strictly less than the average service rate. Based on Loynes’ theorem, let us define that a queue is stable if and only if the average arrival rate is strictly less than the average service rate. Based on Loynes’ theorem, let us define that a queue is stable if and only if the average arrival rate is strictly less than the average service rate.

Our goal is to find an optimal spectrum-sharing ratio \(\phi\) (denoted by \(\phi^*\)) that maximizes \(\lambda_s\) (letting \(\lambda_s^*\) denote optimal \(\lambda_s\) corresponding to \(\phi^*\)), while keeping the stability constraints in both PT and ST (that is, \(\mu_p > \lambda_p\) and \(\mu_s > \lambda_s\), respectively), given \(\lambda_p\), \(p_d\) and \(p_f\) are also assumed to be given and fixed. \(\phi^*\) obviously depends on various physical layer parameters and operational options. As a result of the optimization, we can obtain a stability region for the proposed system.

D. IMPLEMENTATION ISSUES AND ASSUMPTIONS

The complexity of practical implementation of the proposed method mainly lies in keeping the coexistence synchronization between PU and SU and gathering the channel information for obtaining \(\phi\). Like other cooperative MIMO systems, PT and ST should be synchronized especially with respect to the PR’s view. Distributed synchronization in wireless networks has a pretty long history and is well summarized in [35]. The 802.22 MAC introduces a new superframe structure to facilitate incumbent protection, synchronization and self-coexistence [36]. If Coexistence Beaconing Packet (CBP) [37] is used for providing synchronization in FD CSS, ST receives CBP from its neighbor PT and it adjusts the start time of the superframe according to specific rules, where the superframe may consist of tens or hundreds of time slots.

In this paper, ST is assumed to determine optimal \(\phi\) with the relevant channel knowledge. We assume that PT and ST send reference signals. PR is assumed to extract channel information from PT-PR and ST-PR channels and report them to PT. PT then relays this information to ST. On the other hand, SR is assumed to measure and send (to ST) the channel
information for ST-SR channel. If the receiver at ST can measure the self-feedback channel from the reference signal, ST can have the whole channel knowledge required to obtain optimal $\phi$. Moreover, we also assume that PT informs ST of its buffer state, traffic arrival rate, a desired service target, and its transmission power level. Then ST can estimate the related probabilities of successful decoding with the channel knowledge and determine the region (i.e., among the four categories depicted in the following Figure 3) that it falls into. With the region information, the proper mode can be selected to maximize the achievable secondary service rate. Along with the mode selection procedure, $\phi$ should be determined together. In practical systems, these parameters are changing dynamically and exchanging such information on a shorter-time scale will also add a practical overhead to the system.

In [38] and some practices, FD is implemented on a single antenna. It assumes special circuits that can measure and cancel the feedback signals. Unlike this method, we apply precoding and decoding weights for cancelling the feedback signals, which need an additional antenna at either transmitting or receiving side. Our assumption has been widely accepted in MIMO and full-duplex literature. If we assume the cancellation circuit block like in [38], we do not need an additional antenna for removing the feedback signals and the number of required antennas decrease by 1.

### III. OPTIMAL SPECTRUM SHARING WITH THE STABILITY CONSTRAINTS: THE AGGRESSIVE CASE

The primary service rate $\mu_p$ at PT’s queue depends on the state of $Q_s$ and the action taken by ST. $\mu_p$ is then given by

$$
\mu_p = p_d Pr(Q_s \neq 0) (\phi q_{c,l}^{(c)} + (1 - \phi) q_{f,l}^{(p)}) + p_d Pr(Q_s = 0) q_{b,f}^{(c)} + (1 - p_d) q_{f,l}^{(p)}. \quad (9)
$$

On the other hand, the secondary service rate $\mu_s$ at ST’s queue is depending on the state of $Q_p$ and the sensing result. $\mu_s$ is given by

$$
\mu_s = Pr(Q_p = 0) (p_f \phi q_{c,l}^{(s)} + p_f (1 - \phi) q_{c,f}^{(s)} + (1 - p_f) q_{f,l}^{(p)}) + Pr(Q_p \neq 0) (p_f \phi q_{c,l}^{(s)} + p_f (1 - \phi) q_{c,f}^{(s)} + (1 - p_f) q_{f,l}^{(p)}).
$$

When maximizing $\lambda_s$ subject to given $\lambda_p$ and the stability constraints, the states of $Q_p$ and $Q_s$ are interacting since PrSD depends on the queue states due to interference, which makes the optimization not straightforward. To decouple the effect of queue states, we use in this paper a popular dominant system approach originally provided in [27] and [39]. In this approach, one of the transmitters sends dummy packets when its queue is empty, while the other transmits according to its traffic. The dominant system generally provides a lower bound of the stable throughput. It is however shown to exhibit an exact stable throughput, especially when the number of interacting queues is two, by taking the union of two dominant systems: the so-called first and second dominant system [27]. Hereafter, superscripts (1) and (2) denote the first and the second dominant system, respectively. And let $\phi_1^*$ and $\phi_2^*$ denote the respective optimal spectrum sharing ratios.

#### A. FIRST DOMINANT SYSTEM: ST TRANSMITS DUMMY PACKETS

The first dominant system is identical to the original system except that ST transmits a dummy packet whenever $Q_s$ empties. All other assumptions remain unchanged in the dominant system. Thus, in this first dominant system, ST’s queue never empties and the service rate for PT’s queue is given as a function of $\phi$:

$$
\mu_p^{(1)}(\phi) = p_d (\phi q_{c,l}^{(c)} + (1 - \phi) q_{f,l}^{(p)}) + (1 - p_d) q_{f,l}^{(p)}. \quad (11)
$$

Since $\Delta_{cp} \geq 0$, Loynes’ stability condition $\lambda_p < \mu_p^{(1)}(\phi)$ imposes the following constraints on $\lambda_p$ and $\phi$, respectively.

$$
\begin{align*}
\lambda_p &< p_d q_{c,l}^{(c)} + (1 - p_d) q_{f,l}^{(p)} \quad (12) \\
\phi &> \frac{\lambda_p - q_{f,l}^{(p)}}{p_d \Delta_{cp}} \quad \equiv \phi_L. \quad (13)
\end{align*}
$$

where $\lambda_p^{(1)}$ denotes an upper-bound of $\lambda_p$ that can be achieved by taking $\phi = 1$, and $\phi_L$ is a lower-bound of $\phi$, which can be found by re-arranging $\lambda_p < \mu_p^{(1)}(\phi)$. It is noted that if $\lambda_p < q_{f,l}^{(p)}$, then every $\phi$ in a closed interval $[0, 1]$ is feasible.

On the other hand, the average service rate of ST in this dominant system can be obtained as

$$
\mu_s^{(1)}(\phi) = \left(1 - \frac{\lambda_p}{p_d \phi \Delta_{cp} + q_{f,l}^{(p)}}\right) \times (q_{c,f}^{(s)} - p_f \phi \Delta_s) + \frac{\lambda_p q_{c,f}^{(s)} - p_f \Delta_s}{p_d \phi \Delta_{cp} + q_{f,l}^{(p)}}. \quad (14)
$$

When $\lambda_p < \lambda_p^{(1)}$, is given, a maximum stable throughput of ST then can be found by solving

$$
\max_{\phi} \mu_s^{(1)}(\phi) \quad \text{s.t.} \quad \phi \geq \phi_L^*, \quad 0 \leq \phi \leq 1, \quad (15)
$$

where $\phi_L^* = \phi_L + \epsilon$ for sufficiently small positive real $\epsilon$ and $\phi_L^*$ is introduced to ensure the optimization on a closed constraint-set. We assume $0 \leq \phi_L^* \leq 1$ without loss of generality.

By differentiating $\mu_s^{(1)}(\phi)$ with respect to $\phi$, we have

$$
\frac{d \mu_s^{(1)}(\phi)}{d \phi} = \frac{A \lambda_p}{(p_d \phi \Delta_{cp} + q_{f,l}^{(p)})^2} - p_f \Delta_s, \quad (16)
$$

where $A$ is a constant given by

$$
A = p_d \Delta_{cp} (q_{c,f}^{(s)} - q_{c,l}^{(s)}) + q_{f,l}^{(p)} (p_f \Delta_s - p_d \Delta_{s}). \quad (17)
$$
1) WHEN $A \leq 0$
Since $\Delta_s \geq 0$, the derivative in (16) is non-positive for all feasible $\phi$ in this case. Thus $\mu_s^{(1)}(\phi)$ is a non-increasing function and $\phi^*_1 = \phi^*_L$.

2) WHEN $A > 0$
$d\mu_s^{(1)}(\phi)/d\phi = 0$ has two possible roots, one of which is always negative and the other is given by

$$\tilde{\phi} = \sqrt{\frac{2d_A}{p_f \Delta_s} - q_{zf}^{(\phi)}}.$$  \hspace{1cm} (18)

Furthermore, $d^2\mu_s^{(1)}(\phi)/d\phi^2 \leq 0$ when $\phi = \tilde{\phi}$ and hence $\bar{\phi}$ is an optimal $\phi$ that maximizes $\mu_s^{(1)}(\phi)$. Thus, optimal $\phi^*_1$ can be summarized in (19), as shown at the bottom of this page.

The stable region for the secondary dominant system $S^{(1)}$ thus can be expressed by

$$S^{(1)} = \{(\lambda_p, \lambda_s) | \lambda_p < \lambda^{(1)}_{p,L}, \lambda_s < \mu_s^{(1)}(\phi^*_1)\}. \hspace{1cm} (20)$$

$$\phi^*_1 = \begin{cases} 
0, & \lambda_p < \frac{p_d \Delta_{cp}}{q_{zf}}, A \leq 0, \\
\phi^*_L, & \lambda_p \geq \frac{p_d \Delta_{cp}}{q_{zf}}, A \leq 0, \\
\bar{\phi}, & 0 < A \leq \frac{p_f (q_{zf}^{(\phi)})^2 \Delta_s}{\lambda_p}, \\
\tilde{\phi}, & \frac{p_f (q_{zf}^{(\phi)})^2 \Delta_s}{\lambda_p} < A \leq \frac{p_f \Delta_s (p_d q_{zf}^{(c)} + (1 - p_d) q_{zf}^{(p)})^2}{\lambda_p}, \\
1, & A > \frac{p_f \Delta_s (p_d q_{zf}^{(c)} + (1 - p_d) q_{zf}^{(p)})^2}{\lambda_p}. 
\end{cases} \hspace{1cm} (19)$$

$$\mu_s^{(1)}(\phi^*_1) = \begin{cases} 
\lambda_p (q_{zf}^{(s)} - q_{zf}^{(\bar{\phi})}), & 0 < A \leq \frac{p_f (q_{zf}^{(\phi)})^2 \Delta_s}{\lambda_p}, \\
\tilde{\Delta}_s (\lambda_p - p_f q_{zf}^{(\phi)}), & \frac{p_f (q_{zf}^{(\phi)})^2 \Delta_s}{\lambda_p} < A \leq \frac{p_f \Delta_s (p_d q_{zf}^{(c)} + (1 - p_d) q_{zf}^{(p)})^2}{\lambda_p}, \\
\tilde{\Delta}_s (\lambda_p - p_f q_{zf}^{(\phi)}), & A > \frac{p_f \Delta_s (p_d q_{zf}^{(c)} + (1 - p_d) q_{zf}^{(p)})^2}{\lambda_p}. 
\end{cases} \hspace{1cm} (21)$$

and ST’s optimal average service rate $\mu_s^{(1)}(\phi^*_1)$ can be summarized in (21), as shown at the bottom of this page.

**B. SECOND DOMINANT SYSTEM: PT TRANSMITS DUMMY PACKETS**
In the second (or primary) dominant system, PT transmits a dummy packet whenever $Q_p$ empties, while ST behaves in the same way as in the original system. In the second dominant system, the average service rate of ST is

$$\mu_s^{(2)}(\phi) = \frac{\lambda_s}{q_{zf}^{(s)} - p_d \phi \Delta_s} (q_{zf}^{(p)} + \phi \Delta_{cp}) + p_d (1 - \frac{\lambda_s}{q_{zf}^{(s)} - p_d \phi \Delta_s}) q_{zf}^{(c)} + (1 - p_d) q_{zf}^{(p)}. \hspace{1cm} (22)$$

On the other hand, the average service rate of PT is given by

$$\mu_p^{(2)}(\phi) = \frac{\lambda_s}{q_{zf}^{(s)} - p_d \phi \Delta_s} (q_{zf}^{(p)} + \phi \Delta_{cp}) + p_d \left(1 - \frac{\lambda_s}{q_{zf}^{(s)} - p_d \phi \Delta_s}\right) q_{zf}^{(c)} + (1 - p_d) q_{zf}^{(p)}. \hspace{1cm} (23)$$
From the stability condition $\lambda_p < \mu_p^{(2)}(\phi)$, we have a constraint on $\lambda_s$ such that

$$\lambda_s < \frac{(pdq_{bf}^{(c)} + (1 - pd)q_{zf}^{(p)} - \lambda_p)q_{zf}^{(s)} - pd\phi\Delta_s}{pd(q_{bf}^{(c)} - q_{zf}^{(p)} - \phi\Delta_{cp})} \overset{\text{def}}{=} \lambda_{s,U}^{(2)}(\phi).$$ (24)

By taking $\lambda_s = 0$, we can have an upper bound on $\lambda_p$:

$$\lambda_p < pdq_{bf}^{(c)} + (1 - pd)q_{zf}^{(p)} \overset{\text{def}}{=} \lambda_{p,U}^{(2)}.$$ (25)

When $\lambda_p < \lambda_{p,U}^{(2)}$ is given, $\lambda_s$ should be satisfied with both of the stability constraints: $\lambda_s < \mu_s^{(2)}(\phi)$ and $\lambda_s < \lambda_{s,U}^{(2)}(\phi)$. Let $v(\phi) = \min(\mu_s^{(2)}(\phi), \lambda_{s,U}^{(2)}(\phi))$. Then, a maximum stable throughput of ST can be found by solving

$$\max_{\phi} v(\phi) \quad \text{s.t.} \quad 0 \leq \phi \leq 1.$$ (26)

$\phi_s^*$ can be obtained by an optimal solution of problem in (26).

If the detection probability is positive (i.e., $pd > 0$), $\mu_s^{(2)}(\phi)$ is a strictly decreasing linear function of $\phi$ since $\Delta_s \geq 0$. On the other hand, by differentiating $\lambda_{s,U}^{(2)}(\phi)$ with respect to $\phi$, we have

$$\frac{d\lambda_{s,U}^{(2)}(\phi)}{d\phi} = C(pdq_{bf}^{(c)} + (1 - pd)q_{zf}^{(p)} - \lambda_p)\Delta_{cp}^2,$$ (27)

where $C$ is a constant given by

$$C = \Delta_{cp}q_{bf}^{(c)} - pd(q_{bf}^{(c)} - q_{zf}^{(p)})\Delta_s.$$ (28)

1) WHEN $C \leq 0$

In this case, both $\mu_s^{(2)}(\phi)$ and $\lambda_{s,U}^{(2)}(\phi)$ are non-increasing functions and thus $\phi_s^* = 0$ is an optimal solution, which gives

$$v^* = \begin{cases} q_{zf}^{(s)} - q_{zf}^{(p)}, & \lambda_p \leq q_{zf}^{(p)}, \\ \frac{q_{zf}^{(s)} - q_{zf}^{(p)}}{pd(q_{bf}^{(c)} - q_{zf}^{(p)})}, & \lambda_p > q_{zf}^{(p)}. \end{cases}$$ (29)

2) WHEN $C > 0$

$\lambda_{s,U}^{(2)}(\phi)$ monotonically increases and a maxmin solution is found by equating $\mu_s^{(2)}(\phi)$ and $\lambda_{s,U}^{(2)}(\phi)$, from which we have

$$\phi_s^* = \max \left\{ \min \left( \frac{\lambda_p - q_{zf}^{(p)}}{pd\Delta_{cp}}, 1 \right), 0 \right\}.$$ (30)

and the corresponding $v^*$ is given as in (31), as shown at the bottom of this page.

Thus, optimal $\phi_s^*$ can be summarized as

$$\phi_s^* = \begin{cases} 0, & C \leq 0, \\ \max \left\{ \min \left( \frac{\lambda_p - q_{zf}^{(p)}}{pd\Delta_{cp}}, 1 \right), 0 \right\}, & C > 0. \end{cases}$$ (32)

and the corresponding $v^*$ is given as in (33), as shown at the bottom of this page.

The stable region for the second dominant system $S^{(2)}$ thus can be expressed by

$$S^{(2)} = \{ (\lambda_p, \lambda_s) | \lambda_p < \lambda_{p,U}^{(2)}, \lambda_s < v^* = v(\phi_s^*) \}.$$ (34)

C. STABLE REGION: UNION OF $S^{(1)}$ AND $S^{(2)}$

The joint stability region for PT and ST is given by

$$S = S^{(1)} \cup S^{(2)}.$$ (35)

Since $\lambda_{p,U}^{(1)} \leq \lambda_{p,U}^{(2)}$, an optimal spectrum sharing $\phi^*$ characterized by $\phi_1^*$ and $\phi_2^*$ can be determined by

$$\phi^* = \arg \max_{\phi_1, \phi_2} \mu_s^{(1)}(\phi_1), v(\phi_2^*), \quad 0 \leq \lambda_p \leq \lambda_{p,U}^{(1)}, \lambda_{p,U}^{(1)} < \lambda_p < \lambda_{p,U}^{(2)}.$$ (36)
IV. OPTIMAL SPECTRUM SHARING WITH THE STABILITY CONSTRAINTS: THE PASSIVE CASE

In the aggressive mode, ST sends only its own signal with probability \(1 - \phi\) even though PT is sensed to be busy. However, in the passive mode, ST just relays PT’s signal without sending its own signal by probability \(1 - \phi\) if PT is sensed to be busy. The passive strategy intends to always relay PT’s signal if it exists, which avoids congestion in PT’s queue and hence makes PT idle more frequently. With the passive strategy, the primary service rate \(\mu_p\) at PT’s queue is given by

\[
\mu_p = p_d Pr(Q_s \neq 0)(\phi q_{z,f}^{(c)} + (1 - \phi)q_{z,f}^{(c)}) + p_d Pr(Q_s = 0)q_{b,f}^{(c)} + (1 - p_d)q_{z,f}^{(p)}.
\]  
(37)

On the other hand, the secondary service rate \(\mu_s\) at ST’s queue is given, depending on the state of \(Q_p\), by

\[
\mu_s = Pr(Q_p = 0)(p_f \phi q_{z,f}^{(s)} + (1 - p_f)q_{z,f}^{(s)}) + Pr(Q_p \neq 0)(p_d \phi q_{z,f}^{(s)} + (1 - p_d)q_{z,f}^{(s)}).
\]  
(38)

A. FIRST DOMINANT SYSTEM WITH THE PASSIVE STRATEGY

For the first dominant system with the passive strategy, the service rate for PT’s queue is given by

\[
\tilde{\mu}_p^{(1)}(\phi) = p_d (q_{b,f}^{(c)} - \Delta_e) + (1 - p_d)q_{z,f}^{(p)}.
\]  
(39)

Hereafter \(\bar{a}\) is used to indicate a variable with the passive strategy, the counterpart of which is \(a\) with the aggressive strategy.

For given \(\lambda_p\), Lloynes stability condition \(\lambda_p < \tilde{\mu}_p^{(1)}(\phi)\) imposes the following constraints on \(\lambda_p\) and \(\phi\), respectively.

\[
\lambda_p = p_d q_{z,f}^{(c)} + (1 - p_d)q_{z,f}^{(p)} \geq \lambda_p^{(1)},
\]  
\[\phi < \frac{(1 - p_d)q_{z,f}^{(p)} + p_d q_{b,f}^{(c)} - \lambda_p}{p_d \Delta_e} \geq \tilde{\phi}_L.
\]  
(40)

On the other hand, the average service rate of ST in the first dominant system can be obtained as

\[
\tilde{\mu}_s^{(1)}(\phi) = \left(1 - \frac{\lambda_p}{p_d (q_{b,f}^{(c)} - \Delta_e) + (1 - p_d)q_{z,f}^{(p)}}\right) \times \left(\phi p_f q_{z,f}^{(s)} + (1 - p_f)q_{z,f}^{(s)}\right) + \frac{\lambda_p}{p_d (q_{b,f}^{(c)} - \Delta_e) + (1 - p_d)q_{z,f}^{(p)}} \times \left(p_d \phi q_{z,f}^{(s)} + (1 - p_d)q_{z,f}^{(s)}\right).
\]  
(42)

When \(\lambda_p < \tilde{\lambda}_p^{(1)}\) is given, a maximum stable throughput of ST can be found by solving

\[
\max_{\phi} \tilde{\mu}_s^{(1)}(\phi) \quad \text{s.t.} \quad \phi \leq \tilde{\phi}_L, \quad 0 \leq \phi \leq 1,
\]  
(43)

where \(\tilde{\phi}_L = \tilde{\phi}_L + \epsilon\) for sufficiently small positive real \(\epsilon\). We assume \(0 \leq \tilde{\phi}_L \leq 1\) without loss of generality by differentiating \(\tilde{\mu}_s^{(1)}(\phi)\) with respect to \(\phi\), we can have

\[
\frac{d\tilde{\mu}_s^{(1)}(\phi)}{d\phi} = p_f q_{z,f}^{(s)} + \frac{E\lambda_p}{(p_d q_{b,f}^{(c)} - \phi \Delta_e) + (1 - p_d)q_{z,f}^{(p)}}^2
\]  
\[
\geq \frac{E\lambda_p}{(p_d q_{b,f}^{(c)} - \phi \Delta_e) + (1 - p_d)q_{z,f}^{(p)}}^2 \geq F(\phi),
\]  
(44)

where \(E\) is a constant given by

\[
E = (p_d q_{z,f}^{(c)} + (1 - p_d)q_{z,f}^{(p)}) - p_d \Delta_e((1 - p_d)q_{z,f}^{(p)} - (1 - p_f)q_{z,f}^{(s)}).
\]  
(45)

It is noted that when \(\Delta_e = 0\), \(\tilde{\mu}_p^{(1)}(\phi)\) in (39) is not affected by \(\phi\) and thus we assume \(\Delta_e > 0\) in finding \(\phi_1^a\) in this subsection. The derivative \(F(\phi)\) in (44) is monotonic for all \(0 \leq \phi \leq 1\), that is, decreasing if \(E < 0\), increasing if \(E > 0\) and constant if \(E = 0\).

1) WHEN \(F(0) \leq 0\) AND \(F(1) \geq 0\)
Since \(F(\phi)\) in (44) is monotonically decreasing for \(0 \leq \phi \leq 1\) and hence non-positive, \(\tilde{\mu}_s^{(1)}(\phi)\) is non-increasing and consequently \(\phi_1^a = 0\).

2) WHEN \(F(0) > 0\) AND \(F(1) \leq 0\)
Since \(F(\phi)\) is monotonically decreasing for \(0 \leq \phi \leq 1\) and hence non-positive, \(\tilde{\mu}_s^{(1)}(\phi)\) is non-increasing and consequently \(\phi_1^a = 0\).

3) WHEN \(F(0) \leq 0\) AND \(F(1) \geq 0\)
Since \(F(\phi)\) is monotonically decreasing for \(0 \leq \phi \leq 1\), \(\tilde{\mu}_s^{(1)}(\phi)\) is a concave function, a maximum of which is attained at \(F(\phi) = 0\). \(F(\phi) = 0\) has two possible roots, one of which is always negative and the other is given by

\[
\bar{\phi}_L = \frac{p_d q_{z,f}^{(c)} + (1 - p_d)q_{z,f}^{(p)} - \sqrt{-\frac{\lambda_p E}{p_f q_{z,f}^{(s)}}}}{p_d \Delta_e}.
\]  
(46)

4) WHEN \(F(0) \geq 0\) AND \(F(1) \geq 0\)
Since \(F(\phi)\) is monotonic and non-negative for \(0 \leq \phi \leq 1\), \(\phi_1^a = \tilde{\phi}_L\). Thus, optimal \(\phi_1^a\) can be summarized as (47), as shown at the bottom of the next page. The stable region for the first dominant system \(\tilde{S}^{(1)}\) then can be expressed by

\[
\tilde{S}^{(1)} = \{(\lambda_p, \lambda_s) | \lambda_p < \lambda_p^{(1)}(\phi_1^a), \lambda_s < \lambda_s^{(1)}(\phi_1^a)\}.
\]  
(48)

and ST’s optimal service rate of \(\mu_s^{(1)}(\phi_1^a)\) can be summarized in (49), as shown at the bottom of the next page.

B. SECOND DOMINANT SYSTEM WITH THE PASSIVE STRATEGY

The service rate for ST’s queue for the second dominant system is given by

\[
\tilde{\mu}_s^{(2)}(\phi) = p_d q_{z,f}^{(s)} + (1 - p_d)q_{z,f}^{(s)}.
\]  
(50)
On the other hand, the average service rate of PT is given by

\[
\tilde{\mu}_p^{(2)}(\phi) = p_d \frac{\lambda_s}{\phi p_d \tilde{q}^{(s)}_{c,f} + (1 - p_d) \tilde{q}^{(c)}_{b,f} + (1 - p_d) \tilde{q}^{(c)}_{c,f}} - \phi \Delta_c
\]

+ \left[ 1 - \frac{\lambda_s}{\phi p_d \tilde{q}^{(s)}_{c,f} + (1 - p_d) \tilde{q}^{(c)}_{b,f} + (1 - p_d) \tilde{q}^{(c)}_{c,f}} \right] (q^{(c)}_{b,f} + (1 - p_d) q^{(c)}_{c,f}).
\]

(51)

When \( \Delta_c = 0 \), \( \tilde{\mu}_p^{(2)}(\phi) \) is not affected by \( \phi \) and we also assume \( \Delta_c > 0 \) in this subsection. By taking \( \lambda_s = 0 \), we can have an upper bound on \( \lambda_p \):

\[
\lambda_p < p_d \tilde{q}^{(c)}_{b,f} + (1 - p_d) q^{(c)}_{c,f} \text{ def } \lambda_p^{(2)}(\phi).
\]

(52)

From (51) with the stability condition \( \lambda_p < \tilde{\mu}_p^{(2)}(\phi) \), we also can have

\[
\lambda_s < \left( 1 - p_d \right) q^{(p)}_{c,f} + p_d q^{(c)}_{b,f} - \lambda_p (\phi p_d \tilde{q}^{(s)}_{c,f} + (1 - p_d) \tilde{q}^{(c)}_{c,f})
\]

\[
\text{def } \lambda_s^{(2)}(\phi).
\]

(53)

When \( \lambda_p < \lambda_s^{(2)}(\phi) \), \( \lambda_s < \tilde{\lambda}_s^{(2)}(\phi) \) and \( \lambda_s < \tilde{\lambda}_s^{(2)}(\phi) \). Let \( \tilde{\nu}(\phi) = \min[\tilde{\mu}_s^{(2)}(\phi), \tilde{\lambda}_s^{(2)}(\phi)] \). Then, a maximum stable throughput of ST can be found by solving

\[
\max_{\phi} \tilde{\nu}(\phi) \text{ s.t. } 0 \leq \phi \leq 1.
\]

(54)

If the detection probability is positive (i.e. \( p_d > 0 \)), \( \tilde{\mu}_s^{(2)}(\phi) \) is a strictly increasing linear function of \( \phi \) since \( \tilde{q}^{(s)}_{c,f} \geq 0 \). On the other hand, by differentiating \( \tilde{\lambda}_s^{(2)}(\phi) \) with respect to \( \phi \), we have

\[
\frac{d\tilde{\lambda}_s^{(2)}(\phi)}{d\phi} = -\frac{p_d (1 - p_d) \tilde{q}^{(s)}_{c,f} + p_d q^{(p)}_{b,f} + (1 - p_d) q^{(c)}_{c,f} - \lambda_p \phi p_d \tilde{q}^{(s)}_{c,f}}{(\phi p_d)^2 \Delta_c}.
\]

(55)

The derivative in (55) is non-positive for all feasible \( \phi \). A maxmin solution of \( \tilde{\nu}(\phi) \) is found by equating \( \tilde{\mu}_s^{(2)}(\phi) \) and \( \tilde{\lambda}_s^{(2)}(\phi) \). Thus we can have

\[
\tilde{\nu}^* = \min \left( \frac{1 - p_d \tilde{q}^{(p)}_{c,f} + p_d q^{(c)}_{b,f} - \lambda_p \phi}{p_d \Delta_c} \right),
\]

(56)

and the corresponding \( \tilde{\nu}^* \) is given in (57), as shown at the bottom of this page.

The stable region for the second dominant system \( \tilde{S}^{(2)} \) thus can be expressed by

\[
\tilde{S}^{(2)} = \{(\lambda_p, \lambda_s) | \lambda_p < \lambda_s^{(1)}(\phi), \lambda_s < \tilde{\nu}(\tilde{\phi}_s^*) \}.
\]

(58)

C. STABLE REGION WITH THE PASSIVE STRATEGY: UNION OF \( S^{(1)} \) AND \( S^{(2)} \)

The joint stability region for PT and ST with the passive strategy and optimal spectrum sharing \( \tilde{\phi}^* \) are respectively given by

\[
\tilde{S} = \tilde{S}^{(1)} \cup \tilde{S}^{(2)},
\]

(59)

and for \( 0 \leq \lambda_p \leq \lambda_s^{(1)}(\phi) \),

\[
\tilde{\phi}^* = \arg \max \tilde{\phi}^*, \tilde{\phi}_s^* \{ \tilde{\mu}_s^{(1)}(\tilde{\phi}^*_1), \tilde{\nu}(\tilde{\phi}_s^*) \}.
\]

(60)

V. NUMERICAL INVESTIGATION

A. SIMULATION SETUP

In this section, we numerically verify the optimality of the proposed spectrum sharing probability and then the stable throughput of proposed aggressive- and passive-mode CSS (denoted by \texttt{agg} and \texttt{pss} in the figures, respectively) is evaluated with simulation. In addition, in order to gain insight into the performance achieved by the proposed methods, the performances of three reference models as follows are also presented and compared:
A CSS method provided in reference [24], which is equivalent to the proposed aggressive mode if we let φ = 1 without optimizing (denoted by ref24 in the figures); a no-relaying system, in which ST never relays PT’s signal even though ST itself is idle and always transmits its own signal unless its buffer is empty (denoted by no-relay in the figures). However, ST should use a zero-forcing technique in order to not interfere with PT when sending its signal. The stability region of the no-relaying system is also presented in Appendix B; CSS without sensing, which is obtained if we let prd = pry = 1 in the proposed methods (denoted by aggβ and pssβ for the aggressive and the passive modes, respectively, where “β” implies “blind”). This variant is considered to evaluate the effect of the sensing capability adopted in CSS on the performance improvement. It is noted that in the blind aggressive mode, ST is assumed to share the buffer information of PT and it relays the primary signal though φ = 0 if its own buffer is empty.

For numerical investigation, we mainly consider two different network configurations (as shown in Figure 2), where the primary locations are geometrically fixed at (0, 0) and (4, 0) for PT and PR, respectively, but the secondary locations are essentially different. For case 1, ST and SR are located in between PT and PR as specified in Figure 2 and for case 2, ST and SR are far away from PT and PR to the left. For both the cases, the distances between pairs of transmitter and receiver are the same. Case 1 may represent a typical CSS scenario where four nodes are closely located. On the other hand, case 2 in which ST is far from PT and PR is an extreme scenario. In case 2, it is relatively inefficient for ST to relay PT’s signal but ST is assumed to urgently need to use the spectrum.

If we assume a geometric path loss model on the average channel power between nodes a and b (a ∈ {PT, ST}, b ∈ {PR, ST, SR}), it is represented by $a_{b,a} = \frac{p}{d_{b,a}^q}$ where $d_{b,a}$ is the distance between nodes a and b, $q$ is a path loss exponent, $\gamma_0$ is a reference gain and $P_a$ is transmit power at node a. In the following simulation, we assume $\eta = 3$, $\gamma_0 = 10$, SNR/SINR threshold of PR and SR $\Gamma_p = \Gamma_s = 10$ and $P_{s}/N_0 = 15$ dB, $\alpha = 0.2$ unless otherwise noted. ST is equipped with $M = 4$ receive and $N = 4$ transmit antennas. When we further assume that $P_s/N_0 = 15$ dB, $q_{b,f}^{(c)} = 0.9967 > q_{c,f}^{(c)} = 0.9620 > q_{f}^{(p)} = 0.4588$ for case 1, and $q_{b,f}^{(c)} = 0.2929 < q_{c,f}^{(c)} = 0.3137 < q_{f}^{(p)} = 0.4588$ (also, $\Delta_{p} < 0$ and $\Delta_{c} < 0$) for case 2. The stable throughput of the latter case is analyzed in Appendix A-A. It should be noted that if $P_s/N_0 = 5$ dB, the relationship is similarly $q_{b,f}^{(c)} > q_{c,f}^{(c)} > q_{f}^{(p)}$ in both case 1 and case 2.

When ST moves far away from PT, the increasing distance between PT and ST causes $q_{b,f}^{(c)}$ and $q_{c,f}^{(c)}$ to decrease. Figure 3 shows the regions of ST’s location characterized by the relationship among $q_{b,f}^{(c)}$, $q_{c,f}^{(c)}$ and $q_{f}^{(p)}$ when $P_s/N_0 = 15$ dB. Regions (A), (B), (C), and (D) in Figure 3 represent $q_{b,f}^{(c)} > q_{c,f}^{(c)} > q_{f}^{(p)}$, $q_{b,f}^{(c)} > q_{f}^{(p)} > q_{c,f}^{(c)}$, $q_{f}^{(p)} > q_{c,f}^{(c)}$, and $q_{f}^{(p)} > q_{c,f}^{(c)} > q_{b,f}^{(c)}$, respectively. Hence, case 2 with $P_s/N_0 = 15$ dB falls into region (D), where the collaboration between PT and ST looks not effective since ST is far away from PT. Though not presented in this paper, the total area considered in Figure 3 falls into a region of type (A) if $P_s/N_0 = 5$ dB is used with the same simulation setup. This is why we mainly focus on analyzing the condition of $q_{b,f}^{(c)} > q_{c,f}^{(c)} > q_{f}^{(p)}$ in this paper. The other two conditions not shown in Figure 3, which are represented by $q_{c,f}^{(c)} > q_{b,f}^{(c)}$ and $q_{b,f}^{(c)}$ are rarely realized in practice since they are possible when $\sigma_{ST,SR}$ should be 10 dB less than $\sigma_{PT,PR}$ and $\sigma_{ST,SR}$, and $P_s$ is greater than $P_p$.

![FIGURE 2](image)  
**FIGURE 2.** The tested network configuration: (a) case 1: PT, PR, ST and SR are at (0, 0), (4, 0), (2, 1) and (2, −3), respectively; (b) case 2: PT, PR, ST and SR are at (0, 0), (4, 0), (−20, 0), and (−16, 0), respectively.

![FIGURE 3](image)  
**FIGURE 3.** Regions (on (x, y)-coordinates) of ST’s location characterized by the relationship among PRSDs ($q_{b,f}^{(c)}$, $q_{c,f}^{(c)}$, and $q_{f}^{(p)}$) with $P_s/N_0 = 15$ dB.
by at least 16 dB and most power of $P_s$ should be used for relaying PU’s signal. However, it is noted again that the stable throughputs for all the other conditions are also provided in Appendix VII.

### B. VERIFICATION

Figures 4 and 5 illustrate the optimality of the optimal spectrum-sharing control probabilities ($\phi^*$) provided in the paper. Figure 4 for case 1 with $P_s/N_0 = 5$ dB can be used to check $\phi^*$ in (36) and (60) for the aggressive and the passive mode, respectively. On the other hand, Figure 5 for case 2 with $P_s/N_0 = 15$ dB can be used to check $\phi^*$ in (86) and (87). Each plot in the figures presents the achievable secondary service rate $\max\{\mu^{(1)}(\phi), \nu(\phi)\}$ as a function of $\phi$ for different $\lambda_p$’s for the aggressive and the passive mode, respectively. Both of the figures numerically show us optimal $\phi$ that attains the maximum secondary service rate. In Figure 4, the maximum secondary service rate for the aggressive mode seems to be obtained at $\phi = 0.6519$, $\phi = 1$ and $\phi = 1$ when $\lambda_p = 0.2$, $\lambda_p = 0.5$ and $\lambda_p = 0.8$, respectively. For the passive mode, it is obtained at $\phi = 1$, $\phi = 1$, and $\phi = 0.9354$, respectively. The optimal $\phi$’s for case 1 are the same as the $\phi^*$ provided in (36) and (77), respectively. In Figure 5, the secondary service rate seems to be maximized when $\phi = 0$ for the aggressive mode and when $\phi = 1$ for the passive mode, which can verify the optimal sharing provided in (86) and (87), respectively.

Figures 6 and 7 provide optimal spectrum-sharing ($\phi^*$) as a function of $\lambda_p$ for case 1 and 2, respectively. In the figures, $\phi^*$’s are presented with different secondary transmission power $P_s/N_0 = 5, 10, 15$ dB, respectively. For case 1 in Figure 6, it is seen that $\phi^* = 1$ in the mid-range of $\lambda_p$ regardless of the collaboration modes. It means that ST always transmits the primary and its own signal together if PT is sensed as active and its buffer is not empty. In the range of low primary traffic (low $\lambda_p$), the lower $P_s/N_0$ results in the smaller $\phi^*$ for the aggressive mode, which implies...
that the smaller transmission power in ST encourages the aggressive transmission (that is, the larger $1 - \phi^*$) of ST to maximize its throughput. On the other hand, in the range of high primary traffic (high $\lambda_p$), the higher $P_s/N_0$ results in the smaller $\phi^*$ for the passive mode. In the aggressive mode, the larger $1 - \phi^*$ causes the more interference from ST to PR while resulting in the less interference from PT to SR in the passive mode. In Figure 6, a high $P_s/N_0$ reduces the interference by adjusting optimal spectrum-sharing, in which the interference from ST to PR is treated significantly when $\lambda_p$ is low, while the interference from PT to SR is important when $\lambda_p$ is high. For case 2 in Figure 7, $P_s/N_0 = 10$ and $15$ dB result in $q_{z_f}^{(p)} > q_{z,l}^{(c)} > q_{z,f}^{(c)}$ (the condition indicated by region (D) in Figure 3). However, $P_s/N_0 = 5$ dB preserves the main relationship $q_{z,f}^{(c)} > q_{z,l}^{(c)} > q_{z,f}^{(p)}$ (the condition indicated by region (A) in Figure 3), even in case 2. With $P_s/N_0 = 10$ and $15$ dB, it is seen that $\phi^* = 0$ for the aggressive mode and $\phi^* = 1$ for the passive mode for the allowable $\lambda_p$.

The stable throughput of the proposed CSS certainly relies on the sensing accuracy (both $p_f$ and $p_d$). We adopt the energy detection scheme presented in [37] and then if target $p_d$ is given, $p_f$ is obtained by using the equation given in [37, eq. (13)] as follows:

$$p_f = Q(\sqrt{2\gamma + 1}Q^{-1}(p_d)\sqrt{T\gamma}),$$

where $Q(\cdot)$ is the complementary distribution function of the standard Gaussian, $Q^{-1}(\cdot)$ is its inverse, $\gamma$ is the SNR of sensed signal, and we have set the number of samples for detection to $T = 2$.

Figure 8 shows the effect of the sensing accuracy on the stable throughput. The x-axis represents the detection probability target. Different primary traffic loads $\lambda_p = 0.1, 0.3, 0.5, 0.7, 0.9$ are considered, for which the accuracy target ($p_d$) that maximizes the secondary throughput seems to be $0.73, 0.85, 1, 1, 1, 1$, respectively. Detection should be more accurate if $\lambda_p$ becomes larger due to avoiding interference. Since $p_f$ is an increasing function of $p_d$ in (61), a small $p_d$ consequently contributes to giving more transmission chances to ST, which helps ST increase its throughput especially in the condition of low primary traffic. With a congested condition of high primary traffic, decreasing false alarm may increase the interference to PT, which also results in shrinking the secondary throughput. Thus, an optimal target for $p_d$ may exist according to the input traffic. The search for such an optimal $p_d$ is however beyond the scope of this paper. We hereafter use the target $p_d = 0.9$ in the following simulation unless otherwise noted. It is seen that if $p_d$ is sufficiently small, the passive mode is slightly better than the aggressive mode when $\lambda_p = 0.5, 0.7, 0.9$.

C. PERFORMANCE COMPARISON

Figures 9 and 10 provide numerical comparisons of the stable throughput of the proposed CSS with ref24 and no-relay systems for case 1. In Figure 9, $P_s/N_0 = 5$ dB is assumed.

![FIGURE 9. Comparison of stable throughput for case 1 with $P_s/N_0 = 5$ dB.](image-url)

![FIGURE 8. Effect of the sensing accuracy on the stable throughput (for case 1 with $P_s = 15$ dB); $\lambda_p = 0.1, 0.3, 0.5, 0.7, 0.9$.](image-url)

![FIGURE 10. Comparison of stable throughput for case 1 with $P_s/N_0 = 15$ dB.](image-url)
TABLE 5. The effect of secondary power level on the proposed performance improvement.

| Λp | P_s/N_0 [dB] | 0 | 2 | 4 | 6 | 8 | 10 | 12 | 14 | 16 | 18 | 20 |
|-----|--------------|---|---|---|---|---|----|----|----|----|----|----|
|     |               |   |   |   |   |   |    |    |    |    |    |    |
| ref24 | 0.0386       | 0.0970 | 0.1936 | 0.3215 | 0.4630 | 0.5976 | 0.7117 | 0.8002 | 0.8643 | 0.9088 | 0.9591 |
| agg | 0.0461       | 0.1092 | 0.2076 | 0.3359 | 0.4695 | 0.6003 | 0.7121 | 0.8002 | 0.8643 | 0.9088 | 0.9591 |
| pss | 0.0386       | 0.0970 | 0.1936 | 0.3215 | 0.4630 | 0.5976 | 0.7117 | 0.8002 | 0.8643 | 0.9088 | 0.9591 |
| ref24 | 0.0220       | 0.0603 | 0.1292 | 0.2273 | 0.3432 | 0.4615 | 0.5703 | 0.6636 | 0.7402 | 0.8021 | 0.8517 |
| agg | 0.0220       | 0.0603 | 0.1292 | 0.2273 | 0.3432 | 0.4615 | 0.5703 | 0.6636 | 0.7402 | 0.8021 | 0.8517 |
| pss | 0.0220       | 0.0603 | 0.1292 | 0.2273 | 0.3432 | 0.4615 | 0.5703 | 0.6636 | 0.7402 | 0.8021 | 0.8517 |
| ref24 | 0.0061       | 0.0236 | 0.0647 | 0.133 | 0.2235 | 0.3255 | 0.4289 | 0.5269 | 0.6161 | 0.6953 | 0.7642 |
| agg | 0.0061       | 0.0236 | 0.0647 | 0.133 | 0.2235 | 0.3255 | 0.4289 | 0.5269 | 0.6161 | 0.6953 | 0.7642 |
| pss | 0.0073       | 0.0239 | 0.0647 | 0.133 | 0.2235 | 0.3255 | 0.4289 | 0.5269 | 0.6161 | 0.6953 | 0.7642 |
| ref24 | 0 | 0.0012 | 0.0081 | 0.0275 | 0.0712 | 0.1368 | 0.2047 | 0.2439 | 0.3134 | 0.3621 | 0.4059 |
| agg | 0 | 0.0012 | 0.0081 | 0.0275 | 0.0712 | 0.1368 | 0.2047 | 0.2439 | 0.3134 | 0.3621 | 0.4059 |
| pss | 0 | 0.0042 | 0.0139 | 0.0364 | 0.0824 | 0.167 | 0.2883 | 0.3539 | 0.3826 | 0.4122 | 0.4759 |

In the figure, the aggressive and passive modes provide better performance than ref24 for Λp < 0.270 and Λp > 0.793, respectively. For 0.270 ≥ Λp ≥ 0.793, agg, pss, ref24 show exactly the same performance. Thus if we compare the aggressive and passive modes, the aggressive mode achieves better performance if Λp < 0.270 and the passive mode is better if Λp > 0.793. This is because in the range of low primary traffic the aggressive mode works well, while the passive mode is well adaptive in high primary traffic environments, as also indicated in Figure 6. If the primary arrival rate is higher than 0.677 and 0.683 (for agg and pss, respectively), the sensing capability (unless it is perfect) is harmful to the performance, whereas ignoring the sensing results (i.e., blind collaboration) rather improves the throughput. If Λp < 0.158, no-relay is better than the passive mode as well as ref24, but the aggressive mode outperforms the no-relaying system unless Λp < 0.091, in which the two methods achieve the same stable throughput. In Figure 10 where P_s/N_0 = 15 dB is assumed, it is seen that the proposed methods work better than ref24 and no-relay systems. Comparing Figures 9 and 10, we can see that optimal spectrum sharing becomes less effective when the secondary power P_s/N_0 is sufficiently high since the performances of agg, pss and ref24 become the same for the wider range of Λp. In Figure 10, the aggressive and the passive mode provide better performance than ref24 for Λp < 0.077 and Λp > 0.913, respectively.

Table 5 summarizes the effect of the secondary power on performance enhancement by the proposed optimal spectrum-sharing methods. In the table, the secondary service rates achievable by agg, pss and ref24 are reported as a function of P_s/N_0 with assuming P_p/N_0 = 15 dB and 4 different levels of Λp(0.1, 0.4, 0.7, 0.9). If P_s/N_0 ≥ 14 dB, all three systems show exactly the same performance regardless of the input traffic level. When Λp = 0.1, the aggressive method improves the secondary performance if P_s/N_0 ≤ 12 dB and maximally achieves 19% enhancement over the passive mode and the reference method when P_s/N_0 = 0 dB. When Λp = 0.9 and 0.7, the passive method accomplishes better performance than the other ones if P_s/N_0 ≤ 12 dB and P_s/N_0 ≤ 2 dB, respectively. With Λp = 0.4, all three methods show the same performance. Thus, it is seen that the proposed optimal sharing is effective when the secondary user has low transmission power especially when the level of primary traffic is not modest but either low or high.

Figure 11 provides numerical comparison of the stable throughput for case 2 with P_s/N_0 = 15 dB. As noted previously, case 2 with P_s/N_0 = 15 dB gives Δ_c > 0 and Δ_c < 0 since q_0^{(p)} > q_0^{(c)} > q_0^{(b)}. And thus the analysis given in sections III and IV are not valid. Optimal sharing and throughputs for this case are additionally given in Appendix A-A. In this case, Φ^* = 0 for the aggressive mode and Φ^* = 1 for the passive mode as provided in (86) and (87) and verified in Figure 5. Φ^* = 0 in the aggressive mode implies that ST always (with sharing probability 1 − Φ^* = 1) sends its own data without relaying the primary signal if its buffer is not empty. Otherwise, referring to (9), ST relays the primary signal, in which the noise is mainly amplified since ST is far away from PT and hence the co-phasing performance is severely degraded (as we know from the small q_0^{(b)}). However, as discussed in Appendix B, the aggressive method is not worse than its no-relay counterpart, which is also seen in the figure. It reveals that the proposed method adaptively works like the no-relaying system when the relay of PU’s signal makes the performance worse. On the other
hand, $\tilde{\phi}^* = 1$ in the passive mode represents that ST always relays the primary signal by using ZF, which results in power loss in sending its own signal and hence causes performance degradation compared to the aggressive mode as well as no-relay.

Figure 12 illustrates the relative performance of the proposed method for case $q_{z,f}^{(p)} \geq q_{b,f}^{(c)} > q_{c,z,f}^{(c)}$ (i.e., region (C) in Figure 3), the analytic result for which is also summarized in Appendix A-B. In the figure, PT and PR are located at the same position as in Figure 2.(b) and ST and SR are assumed to be at $(-17,0)$ and $(-13,0)$, which gives $q_{z,f}^{(p)} = 0.4588 > q_{b,f}^{(c)} = 0.3966 > q_{c,z,f}^{(c)} = 0.3745$. With this configuration, the spectrum sharing in the aggressive mode is trivially given by $\phi^*_1 = \phi^*_2 = 0$. Thus, the aggressive modes with and without sensing, as well as no-relay, are seen to provide similarly better performance than both the passive mode and ref24 for all the tested $\lambda_p$’s. It is noted that the performances simulated for case $q_{b,f}^{(c)} > q_{z,f}^{(p)} > q_{c,z,f}^{(c)}$ (i.e., region (B) in Figure 3) would show similar results, which can be seen in Figure 12 since $\phi^*_1 = \phi^*_2 = 0$ also for this case. One exception is that if $\lambda_p$ is sufficiently large (especially, $\lambda_p > q_{z,f}^{(p)}$) then the performance of the proposed passive mode becomes better than that of the no-relaying system.

VI. CONCLUSION

In this paper, aggressive and passive usages of a secondary spectrum are proposed and the respective stability performance is investigated in FD CSS environments. Each of the utilization modes has its own favorable condition: the aggressive mode is better especially if the primary traffic is low; otherwise, the passive mode becomes better. The two modes work similarly for spectrum-sharing probability $\phi$. For the remaining probability $1-\phi$, the two modes act differently: SU sends its own signal only in the aggressive mode while it sends only the relaying signal for PU in the passive mode. Closed-form optimal solutions on $\phi$ are also provided in the paper, the value of which heavily depends on the primary traffic volume, the operating modes, the relative positions of the collaborative nodes and the transmit power budget at SU. For a future wireless CSS network where distributed nodes (for example, sensor nodes) have small transmit power and the primary traffic load is low, the aggressive mode is a promising operational strategy. For other CSS networks where participating nodes have sufficient power and the the primary traffic load is heavy, the passive mode is a suitable candidate.

APPENDIXES

APPENDIX A

OPTIMAL SPECTRUM SHARING PROBABILITIES

ACCORDING TO PrSD CONDITIONS

An optimal spectrum sharing probability and the corresponding stability region for $q_{b,f}^{(c)} \geq q_{z,f}^{(p)} \geq q_{c,z,f}^{(c)}$ (indicated as region (A) in Figure 3) is mainly investigated in sections III and IV. In this section, optimal spectrum sharing probabilities for the other PrSD conditions are presented and the corresponding stable throughputs are discussed briefly. Let us recall that $\phi^*_i (i \in \{1, 2\})$ denotes the optimal sharing of the $i$th dominant system in the aggressive mode. Like the notational convention used in the previous sections, $\lambda$ denotes a parameter or a variable in the passive mode, the counterpart which is denoted by $x$ in the aggressive mode. Trivial spectrum sharings given in the following can be summarized by: $\phi^*_1 = \phi^*_2 = 0$ if $q_{z,f}^{(p)} > q_{b,f}^{(c)}$, and $\tilde{\phi}^*_1 = \tilde{\phi}^*_2 = 1$ if $q_{c,z,f}^{(c)} > q_{b,f}^{(c)}$.

In the derivation of the following results, we make an additional assumption for ST’s operation in the aggressive second dominant system. When $q_{z,f}^{(p)} > q_{b,f}^{(c)}$, stability condition $\lambda_p < \mu_p^{(2)}(\phi)$ for the aggressive second dominant system reverses the inequality defined in (24), i.e. $\lambda_s > \lambda^{(2)}_{s,c}(\phi)$, which is equivalent to

$$\Pr(\overline{Q}_s = 0) < \frac{\lambda_p - (1 - p_d)q_{z,f}^{(p)} - p_d(q_{z,f}^{(p)} + \Delta_{cr})}{p_d q_{b,f}^{(c)} - p_d (q_{z,f}^{(p)} + \Delta_{cr})}.$$ (62)

In order to satisfy the above condition, we assume that ST also generates a dummy packet in the aggressive second dominant system to make $\Pr(\overline{Q}_s = 0) = 0$ when $q_{z,f}^{(p)} > q_{b,f}^{(c)}$. The other assumptions are the same as those in sections III and IV.

A. $q_{z,f}^{(p)} > q_{b,f}^{(c)} > q_{c,z,f}^{(c)}$

In this case, the upper bound of $\lambda_p$ and $\tilde{\lambda}_p$ are respectively given by

$$\zeta_{p,U}^{(1)} = \zeta_{p,U}^{(2)} = q_{z,f}^{(p)},$$ (63)

$$\tilde{\zeta}_{p,U}^{(1)} = \tilde{\zeta}_{p,U}^{(2)} = p_d q_{z,f}^{(c)} + (1 - p_d)q_{z,f}^{(p)}.$$ (64)

Hence optimal spectrum sharing probabilities for the first and the second dominant system can be obtained by

$$\phi^*_1 = \phi^*_2 = 0,$$ (65)

$$\tilde{\phi}^*_1 = \tilde{\phi}^*_2 = 1.$$ (66)
and then the corresponding stable throughput for ST is given by

\[
\mu_s^{(1)} = \frac{\phi^{(s)} - \phi^{(s)} q_{p,f}}{q_{p,f}}, \quad (67)
\]

\[
v^* = \tilde{q}_{c,f}, \quad (68)
\]

\[
\tilde{\mu}_s^{(1)} = (1 - \frac{\lambda_p q_{c,f}}{p_d q_{c,f} + (1 - p_d) q_{c,f}})(p_f q_{c,f} + (1 - p_f) q_{c,f}) + \frac{\lambda_p}{p_d q_{c,f} + (1 - p_d) q_{c,f}}(p_d q_{c,f} + (1 - p_d) q_{c,f}),
\]

\[
\tilde{v}^* = p_d q_{c,f} + (1 - p_d) q_{c,f}. \quad (69)
\]

**B. \( q_{c,f} > q_{b,f} > q_{c,f} \)**

In this case, the upper bound of \( \lambda_p \) and \( \tilde{\lambda}_p \) are respectively given by

\[
\lambda_{p,U}^{(1)} = \lambda_{p,U}^{(2)} = q_{c,f}, \quad (71)
\]

\[
\tilde{\lambda}_{p,U}^{(1)} = \tilde{\lambda}_{p,U}^{(2)} = p_d q_{b,f} + (1 - p_d) q_{c,f}. \quad (72)
\]

Hence optimal spectrum sharing probability for the aggressive mode is given by

\[
\phi_1^* = \phi_2^* = 0. \quad (73)
\]

In the passive mode, the optimal spectrum sharing probability is the same as (47) and (56) for the first and second dominant systems respectively. The corresponding \( \mu_1^{(1)} \) and \( v^* \) is the same as (67) and (68), respectively. And \( \tilde{\mu}_1^{(1)} \) and \( \tilde{v}^* \) is the same to (49) and (57), respectively.

**C. \( q_{b,f} > q_{c,f} > q_{c,f} \)**

In this case, the upper bounds of \( \lambda_p \) and \( \tilde{\lambda}_p \) are respectively given by

\[
\lambda_{p,U}^{(1)} = q_{c,f}, \quad (74)
\]

\[
\lambda_{p,U}^{(2)} = p_d q_{b,f} + (1 - p_d) q_{c,f}, \quad (75)
\]

\[
\tilde{\lambda}_{p,U}^{(1)} = \tilde{\lambda}_{p,U}^{(2)} = p_d q_{b,f} + (1 - p_d) q_{c,f}. \quad (76)
\]

Hence an optimal spectrum sharing probability and the corresponding secondary performance are exactly the same to the results for case \( q_{c,f}^{(p)} > q_{b,f}^{(c)} > q_{c,f}^{(c)} \) described in subsection Appendix A-B.

**D. \( q_{c,f}^{(c)} > q_{b,f}^{(c)} > q_{c,f}^{(p)} \)**

In this case, the upper bound of \( \lambda_p \) and \( \tilde{\lambda}_p \) are respectively given by

\[
\lambda_{p,U}^{(1)} = \lambda_{p,U}^{(2)} = p_d q_{c,f} + (1 - p_d) q_{c,f}, \quad (77)
\]

\[
\tilde{\lambda}_{p,U}^{(1)} = \tilde{\lambda}_{p,U}^{(2)} = p_d q_{c,f} + (1 - p_d) q_{c,f}. \quad (78)
\]

In the aggressive mode, \( \phi_1^* \) is the same to (19) and \( \phi_2^* \) can be summarized in (79), as shown at the bottom of this page, where \( C \) is defined in (28).

For the passive mode,

\[
\tilde{\phi}_1^* = \tilde{\phi}_2^* = 1. \quad (80)
\]

For the aggressive mode, \( \mu_1^{(1)} \) is the same to (21) and \( v^* \) is the same to (29) if \( \phi_2^* = 0 \), or (31) if \( \phi_2^* \neq 0 \). \( \mu_1^{(1)} \) and \( \tilde{v}^* \) for the passive strategy is the same to (69) and (70), respectively.

**E. \( q_{c,f}^{(c)} > q_{b,f}^{(p)} > q_{c,f}^{(c)} \)**

In this case, the upper bound of \( \lambda_p \) and \( \tilde{\lambda}_p \) are respectively given by

\[
\lambda_{p,U}^{(1)} = \lambda_{p,U}^{(2)} = p_d q_{c,f} + (1 - p_d) q_{c,f}, \quad (81)
\]

\[
\tilde{\lambda}_{p,U}^{(1)} = \tilde{\lambda}_{p,U}^{(2)} = p_d q_{c,f} + (1 - p_d) q_{c,f}. \quad (82)
\]

In the aggressive mode, \( \phi_1^* \) is the same to (19) and

\[
\phi_2^* = \begin{cases} 
0, & \lambda_p < p_d q_{b,f} + (1 - p_d) q_{c,f} \\
\frac{\lambda_p - q_{c,f}}{p_d \Delta_{c}} , & \lambda_p > p_d q_{b,f} + (1 - p_d) q_{c,f}. 
\end{cases} \quad (83)
\]

For the passive mode,

\[
\tilde{\phi}_1^* = \tilde{\phi}_2^* = 1. \quad (84)
\]

The secondary performances corresponding to the optimal spectrum sharing probabilities are exactly the same to the results for case \( q_{c,f}^{(c)} > q_{b,f}^{(c)} > q_{c,f}^{(p)} \) described in subsection Appendix A-D.

**APPENDIX B**

**STABILITY REGION OF THE NO-RELAYING SYSTEM**

In this system, though ST always transmits its own signal without relaying PT’s signal, ST does not interfere with PT by
adopter zero-forcing transmission. Thus, the departure rate of PT and ST is respectively given by
\[ \mu_p = q_{z,f}^{(p)} \]
\[ \mu_s(\lambda_p) = Pr(q_p = 0)q_{z,f}^{(s)} + Pr(q_p \neq 0)q_{z,f}^{(s)} \]
\[ = \left(1 - \frac{\lambda_p}{q_{z,f}^{(p)}}\right)q_{z,f}^{(s)} + \frac{\lambda_p}{q_{z,f}^{(p)}}q_{z,f}^{(s)} \]
\[ \text{The stability region } S \text{ thus can be expressed by } S = \left\{(\lambda_p, \lambda_s) | \lambda_p < q_{z,f}^{(p)}, \lambda_s < \mu_s(\lambda_p)\right\}. \]

It is noted that letting \( \phi = 0 \) in the aggressive mode makes the stable throughput of the first dominant system represented by (11) and (14) exactly equal to (85) and (87), respectively. Since the stability region is the union of the stable throughputs from the first and the second dominant system, the stable throughput of the aggressive mode is always greater than or equal to that of the no-relaying system.

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