The size of the proton and the deuteron

Randolf Pohl\textsuperscript{1,13}, Fernando D. Amaro\textsuperscript{3}, Aldo Antognini\textsuperscript{1,12}, François Biraben\textsuperscript{2}, João M. R. Cardoso\textsuperscript{3}, Daniel S. Covita\textsuperscript{3,4}, Andreas Dax\textsuperscript{3}, Satish Dhawan\textsuperscript{5}, Luis M. P. Fernandes\textsuperscript{3}, Adolf Giesen\textsuperscript{6,10}, Thomas Graf\textsuperscript{6}, Theodor W. Hänsch\textsuperscript{1,13}, Paul Indelicato\textsuperscript{2}, Lucile Julien\textsuperscript{2}, Cheng-Yang Kao\textsuperscript{7}, Paul Knowles\textsuperscript{8}, Eric-Olivier Le Bigot\textsuperscript{2}, Yi-Wei Liu\textsuperscript{7}, José A. M. Lopes\textsuperscript{3}, Livia Ludhova\textsuperscript{8}, Cristina M. B. Monteiro\textsuperscript{3}, Françoise Mulhauser\textsuperscript{8}, Tobias Nebel\textsuperscript{1}, François Nez\textsuperscript{2}, Paul Rabinowitz\textsuperscript{9}, Joaquim M. F. dos Santos\textsuperscript{3}, Lukas A. Schaller\textsuperscript{8}, Karsten Schuhmann\textsuperscript{10}, Catherine Schwob\textsuperscript{2}, David Taqu"{e}\textsuperscript{11}, João F. C. A. Veloso\textsuperscript{4} and Franz Kottmann\textsuperscript{12}

\textsuperscript{1} Max–Planck–Institut für Quantenoptik, Garching, Germany.
\textsuperscript{2} Laboratoire Kastler Brossel, École Normale Supérieure, CNRS and Université P. et M. Curie, Paris, France.
\textsuperscript{3} Universidade de Coimbra, Portugal.
\textsuperscript{4} Universidade de Aveiro, Portugal.
\textsuperscript{5} Physics Department, Yale University, USA.
\textsuperscript{6} IFSW, Universität Stuttgart, Germany.
\textsuperscript{7} National Tsing Hua University, Taiwan.
\textsuperscript{8} Université de Fribourg, Switzerland.
\textsuperscript{9} Department of Chemistry, Princeton University, USA.
\textsuperscript{10} Dausinger & Giesen GmbH, Stuttgart, Germany.
\textsuperscript{11} Paul Scherrer Institute, Switzerland.
\textsuperscript{12} ETH Zürich, Switzerland.
\textsuperscript{13} Ludwig-Maximilians-Universität, Munich, Germany

E-mail: randolf.pohl@mpq.mpg.de

Abstract. We have recently measured the $2S_{1/2} - 2P_{3/2}$ energy splitting in the muonic hydrogen atom $\mu p$ to be 49881.88 (76) GHz. Using recent QED calculations of the fine-, hyperfine, QED and finite size contributions we obtain a root-mean-square proton charge radius of $r_p = 0.84184$ (67) fm. This value is ten times more precise, but 5 standard deviations smaller, than the 2006 CODATA value of $r_p = 0.8768$ (69) fm. The source of this discrepancy is unknown. Using the precise measurements of the $1S-2S$ transition in regular hydrogen and deuterium and our value of $r_p$ we obtain improved values of the Rydberg constant, $R_\infty = 10973731.568160$ (16) m$^{-1}$, and the rms charge radius of the deuteron $r_d = 2.12809$ (31) fm.

1. Introduction
The hydrogen atom (H) is the simplest of all atoms, and this simplicity is beautiful. Quantum electrodynamics (QED), originally motivated by the discovery of the Lamb shift in hydrogen [1],
Figure 1. (a) The $n = 2$ levels in muonic hydrogen. Vacuum polarization is the dominant contribution of the 2S Lamb shift of 202 meV. The finite size effect is as large as 2% of the total Lamb shift. The measured $2S_{F=1}^1 - 2P_{F=3/2}^3$ transition is indicated in green. The 1S ground state is 1.9 keV below the $n = 2$ states plotted here. (b,c) Experimental principle. (b) 99% of the muons stopped in H$_2$ gas at 1 mbar pressure proceed directly to the 1S ground state thereby emitting x-rays of the Lyman series around 2 keV. 1% of the muons form long-lived metastable 2S states with a lifetime $\tau_{2S} = 1 \mu$s at 1 mbar. (c) Laser light of suitable wavelength around $\lambda = 6 \mu$m drives the 2S-2P transition. The 2P state de-excites within 8 ps to the 1S ground state via emission of a Lyman-α x-ray at 1.9 keV.

can be used to accurately calculate the transition frequencies in the hydrogen atom [2, 3]. High-precision spectroscopy in hydrogen [4–9] can then be used to test the laws of physics, and to deduce accurate fundamental physical constants [10].

For more than a decade, the test of bound-state QED using hydrogen has been hampered by the lack of an accurate value of the proton rms charge radius, $r_p$. This quantity is required in the calculation of the nuclear size effects on the energy levels (in particular the $S$-states) in the hydrogen atom. For example, the 1$S$ ground state of H is shifted by as much as 1 MHz.

Traditionally, the proton’s rms charge radius has been determined by electron scattering [11–14]. A recent reevaluation of the world e-p scattering data has resulted in $r_p = 0.897(18)$ fm [15, 16], i.e., with a relative uncertainty $u_r = 1.8\%$. Very recently, a preliminary value $r_p = 0.879(8)$ fm from a new measurement in Mainz has become available [17].

Both spectroscopy and QED calculations of H have reached a truly astonishing accuracy. Although the finite size effect on the $S$ states in H enters only at the $10^{-10}$ level, the proton charge radius deduced from the hydrogen measurements $r_p = 0.880(8)$ fm (see Tab. XLV of Ref. [10], adjustment 10), is twice more accurate than the e-p scattering world average [15, 16]. The 2006 CODATA value of $r_p = 0.8768(69)$ fm [10], is hence completely dominated by H spectroscopy.
Figure 2. (a) Low-energy negative muon beam line: Negative pions are injected into the cyclotron trap (CT) and decay into muons $\mu^-$ which are decelerated in a foil. Slow $\mu^-$ leave the CT axially and are separated from background in the muon extraction channel (MEC). The experiment takes place in the 5 T solenoid shown in (b): Slow $\mu^-$ pass two stacks of ultra-thin carbon foils ($S_1, S_2$) where a few electrons are ejected. The $e^-$ are detected in plastic scintillators read out by PMTs (PM$_1$..3), while the slower $\mu^-$ are separated from the $e^-$ by an $E \times B$ drift in a capacitor. The delayed coincidence \([\text{PM}_1 \land (\text{PM}_2 \lor \text{PM}_3)]\) triggers the laser system (Fig. 3). The $\mu^-$ stops inside the gas target filled with 1 mbar of H$_2$ gas. The stop volume is illuminated by the laser pulse injected into a multi-pass cavity, and viewed by 20 LAAPDs (not shown).

2. Muonic hydrogen

The leading order finite size effect on the $S$ states in hydrogen-like systems with low $Z$ is

$$E_{\text{fin.size}} = \frac{2}{3} \left( \frac{m_r}{m_e} \right)^3 \frac{(Z \alpha)^2}{n^3} m_e c^2 \left( \frac{2\pi Z\alpha r_N}{\lambda_C} \right)^2,$$

(1)

where $Z$ is the nuclear charge, $\alpha$ the fine structure constant, $n$ the principal quantum number, $r_N$ the nuclear radius, $\lambda_C$ the Compton wavelength of the electron, $m_e$ the electron mass, and

$$m_r = \frac{m_e m_N}{m_e + m_N}$$

(2)

is the reduced mass of the hydrogen-like system with nuclear mass $m_N$. One observes immediately the scaling of the finite size effect with the third power of the reduced mass $m_r$. This scaling reflects the overlap of the electron’s wave function with the nucleus.

Muonic hydrogen $\mu p$ is a hydrogen atom where the proton $p$ is orbited by a negative muon $\mu^-$, instead of an electron. The muon mass $m_\mu = 207 m_e$, and so the reduced mass $m_r$ is 186 times larger for $\mu p$ than for H. This results in a 186 times smaller Bohr radius of muonic hydrogen, and hence a dramatic enhancement of the finite size effect on the $S$ states of $186^3 \approx 6 \cdot 10^6$. 

The classical Lamb shift (2S-2P energy difference) in electronic hydrogen H is dominated by the self energy (SE) of the electron, and the finite size effect contributes to the Lamb shift only at the 1·10^{-4} level. In contrast, vacuum polarization (VP) dominates the Lamb shift in muonic hydrogen μp, and the finite size effect is as large as 2% of the Lamb shift (see Fig. 1). This is the reason why we set out in 1998 to measure the 2S_{1/2}F_{1/2} = 1/2 − 2P_{3/2}F_{3/2} = 2/3 energy difference in μp by means of pulsed laser spectroscopy at λ ≈ 6μm.

The energy difference Δ̃E between the 2S_{1/2} and the 2P_{3/2} states in muonic hydrogen is the sum of radiative, recoil, and proton structure contributions, and the fine and hyperfine splittings for this particular transition. It is given by [2, 18–22]

$$\Delta \tilde{E} = 209.9779 (49) - 5.2262 r_p^2 + 0.0347 r_p^3 \text{meV}$$  \(3\)

where \(r_p = \sqrt{\langle r_p^2 \rangle}\) is the rms charge radius of the proton, given in fm. The uncertainty of 0.0049 meV in Δ̃E is dominated by the proton polarizability term [20] of 0.015(4) meV. A detailed derivation of Eq. (3) is given in the Supplementary Information of Ref. [23].

3. Experiment

The details of the experiment have been given elsewhere [23–29]. In brief, single low-energy negative muons (∼ 5 keV kinetic energy) from our novel beam line [26] (Fig. 2 (a)) at the Paul-Scherrer-Institute (PSI, Switzerland) enter a 5 T solenoid and are individually detected through secondary electrons emitted in 20 nm thin carbon foils (Fig. 2 (b)). The electrons are detected in plastic scintillators which are read out by photomultipliers (PM1–3), a delayed coincidence of which provides the trigger signal for the laser [30, 31] (Fig. 3).

The muons are then stopped in a low-pressure hydrogen gas target filled with 1 mbar H₂ gas at room temperature. About 1% of the muons form long-lived μp(2S) atoms with a lifetime of ∼ 1 μs at this gas pressure [32–35] (see Fig. 1(b)).

The pulsed laser system (Fig. 3) basically pulse-amplifies the cw light from a frequency controlled cw Ti:sapphire (TiSa) laser, creating 5 ns long pulses of 15 mJ energy at λ ≈ 708 nm. After three sequential vibrational Stokes shifts in a Raman cell [36] filled with 16 bar of H₂ gas,
Figure 4. The measured resonance, a water absorption measurement used for calibration, and the predicted line positions using the proton rms charge radius from electron scattering [15, 16] and CODATA [10].

0.2 mJ pulses at $\lambda \approx 6 \mu$m enter a multi-pass cavity [28] surrounding the muon stop volume. We detect x-rays using large-area avalanche photo diodes (LAPPDs). Our signal (Fig. 4) is the number of K_{\alpha} x-rays detected at the time when the laser illuminates the muon stop volume (Fig. 1(c)), versus the laser frequency.

Tuning the cw TiSa by some frequency offset $\Delta \nu$ changes the IR light at $\lambda \approx 6 \mu$m by the same $\Delta \nu$. Although the frequency of each laser in the step is well-known, we rely on well-known water vapour absorption lines [37, 38] for the final laser frequency calibration. The absolute water vapour line positions are known to 1 MHz [37, 38], but pulse to pulse instabilities of our laser system limit the frequency determination to 300 MHz.

3.1. Lamb shift in muonic hydrogen

The centre of the $2S_{1/2}^{F=1}$ to $2P_{3/2}^{F=2}$ transition in $\mu^+$ is at 49881.88(76) GHz [23]. The uncertainty of 0.76 GHz (15 ppm) contains 700 MHz statistical uncertainty from the free fit of a Lorentzian resonance line on top of a flat background, and the 300 MHz total systematic uncertainty which is exclusively due to our laser wavelength calibration procedure using H$_2$O vapour absorption.

Other systematic effects we have considered are Zeeman shift in the 5 T field ($< 30$ MHz), AC and DC Stark shifts ($< 1$ MHz), Doppler shift ($< 1$ MHz) and pressure shift ($< 2$ MHz). Our measured resonance position is not influenced by molecular effects, because the formed muonic molecules pp$\mu^+$ are known to deexcite quickly [34, 39] and cannot contribute to our signal. Also, the width of our resonance line of 18.0(2.2) GHz agrees with the expected width of 20(1) GHz, whereas molecular lines would be wider.

The free fit gives $\chi^2 = 28.1$ for 28 degrees of freedom (dof). A fit of a flat line, assuming no resonance, gives $\chi^2 = 283$ for 31 dof, making this resonance line $16 \sigma$ significant. The frequency of the resonance centre corresponds to an energy of $\Delta E = 206.2949(32)$ meV (see Eq. (3)).
3.2. The rms charge radius of the proton
The rms charge radius of the proton we find is

\[ r_p = 0.84184(36)(56) \text{ fm.} \] (4)

The first uncertainty is due to the experimental uncertainty of 0.76 GHz (700 MHz statistical and 300 MHz systematic, added in quadrature), and the second uncertainty originates from the first term in Eq. (3). Theory, mainly the proton polarizability, gives the dominant contribution to our total relative uncertainty of \( 8 \times 10^{-4} \). Our experimental precision would allow us to deduce \( r_p \) two times better, with a relative uncertainty of \( 4 \times 10^{-4} \).

3.3. The proton radius puzzle
This new value of the proton radius \( r_p = 0.84184(67) \text{ fm} \) is 10 times more precise, but 5.0\( \sigma \) smaller, than the 2006 CODATA value \( r_p = 0.8768(69) \text{ fm} \) [10], which is dominated by spectroscopy in regular hydrogen (H). Our new \( r_p \) is 26 times more accurate, but 3.1\( \sigma \) smaller, than the previously accepted, hydrogen-independent value extracted from electron proton scattering [15, 16] of \( r_p = 0.895(18) \text{ fm} \). Furthermore, new data from the Mainz MAMI electron accelerator [17] give \( r_p = 0.879(8) \text{ fm} \), in agreement with the hydrogen result as given by “adjustment 10” in Tab. XLV of Ref. [10].

Recent lattice QCD calculations [40], on the other hand, obtain \( r_p = 0.83(3) \text{ fm} \), favouring a lower radius than the one from H or electron scattering. Also, dispersion analysis of the nucleon form factors has recently [41] also produced smaller values of \( r_p \): Their “SC approach” gives \( r_p = 0.844^{+0.008}_{-0.004} \text{ fm} \), in agreement with our accurate value, whereas their “explicit pQCD approach” gives an even smaller value of \( r_p = 0.830^{+0.005}_{-0.008} \text{ fm} \). The situation is summarized in Fig. 5.

3.4. A new value of the Rydberg constant
Assuming for now the correctness of the QED calculations in hydrogen [2, 3] and \( \mu p \) [18–22], we can use our precise value of \( r_p \) and the most accurately measured transition frequency in hydrogen (1S-2S) [6, 7] to deduce a new value of the Rydberg constant.

\[ R_\infty = 10973731.568160(16) \text{ m}^{-1}. \] (5)

![Proton charge radius](image-url)
**Figure 6.** The Rydberg constant is a cornerstone of the CODATA adjustment of fundamental constants [10]. Its accuracy is now 1.5 parts in $10^{12}$, using our $r_p$ and the super-accurate H(1S-2S) transition [6, 7].

This is $-110 \text{kHz}/c$ or 4.9 $\sigma$ away from the CODATA value [10], but 4.6 times more precise [1.5 parts in $10^{12}$]. The new determination continues the astonishing improvement in the accuracy of the most accurately determined fundamental physical constant (Fig. 6).

### 3.5. The charge radius of the deuteron

The difference of the squared charge radii of the proton and the deuteron, $r_d^2 - r_p^2 = 3.82007(65) \text{fm}^2$, is accurately determined from the precise measurement of the isotope shift of the 1S-2S transition in regular hydrogen and deuterium atoms [42]. This, together with our value of $r_p$, gives for the rms charge radius of the deuteron

\[
r_d = 2.12809(31) \text{ fm}.
\]  

Figure 7 compares this to recent results. The CODATA value of $r_d = 2.1394(28) \text{ fm}$ is 4 $\sigma$ away. This can be understood because in the CODATA adjustment $r_d$ is rigidly tied to $r_p$ via the very precise isotope shift of the 1S-2S transition in regular H and D atoms. The proton charge radius deduced from H and $\mu p$ disagree by 5 $\sigma$ for unknown reasons; hence the deuteron radius must also disagree by a large amount.

However, adjustment 11 of the 2006 CODATA adjustment (see Ref. [10] Tab. XLV), which uses only the deuterium data (scattering and spectroscopy), but ignores both electron scattering and spectroscopy on hydrogen, suggests a smaller value of $r_d$, in accord with our value. This is due to the fact that the value of $r_d$ from electron-deuteron scattering [43] agrees with ours.

Neutron-proton scattering [44] gives a value which agrees both with the electron scattering value and with our new value, but not very well with the full CODATA result.
$\mu p + H-D(1S-2S)$

CODATA 2006, only D data
e-d scattering
n-p scattering
CODATA-D and n-p combined

deuterons rms charge radius (fm)

2.12 2.125 2.13 2.135 2.14

Figure 7. Our deuteron charge radius $r_d = 2.12809(31)$ fm deduced from our $r_p$ together with the H-D (1S-2S) isotope shift [42] disagrees with the CODATA value of $r_d$, but agrees with the CODATA 2006 adjustment 11 which uses only deuteron data (Ref. [10], see Tab. XLV, adj. 11), the value from electron scattering [43], and the value from neutron-proton scattering [44].

The average of the independent values “CODATA, D only” [10] and neutron scattering [44] is $r_d= 2.1254(50)$ fm, $2.4\sigma$ away from the final CODATA value $r_d = 2.1394(28)$ fm, but in good agreement with our result.

4. Conclusions and Outlook
The world’s most precise value of the rms proton charge radius $r_p = 0.84184(67)$ fm that we have obtained from laser spectroscopy of the Lamb shift in muonic hydrogen $\mu p$ has created a puzzle. The disagreement with the previous values from hydrogen spectroscopy and electron scattering is stunning.

Using this new value of $r_p$ and the accurately measured hydrogen-deuterium isotope shift [42] we obtain $r_d = 2.12809(31)$ fm. This value agrees with several hydrogen-independent results.

Our new project, the measurement of the Lamb shift in muonic helium ions, will hopefully contribute to the solution of the “proton size puzzle”.

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