THE OPTICAL DESIGN AND CHARACTERIZATION OF THE MICROWAVE ANISOTROPY PROBE

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ABSTRACT

The primary goal of the MAP satellite, now in orbit, is to make high-fidelity polarization-sensitive maps of the full sky in five frequency bands between 20 and 100 GHz. From these maps we will characterize the properties of the cosmic microwave background (CMB) anisotropy and Galactic and extragalactic emission on angular scales ranging from the effective beam size, less than 0.23′, to the full sky. MAP is a differential microwave radiometer. Two back-to-back shaped offset Gregorian telescopes feed two mirror symmetric arrays of 10 corrugated feeds. We describe the prelaunch design and characterization of the optical system, compare the optical models to the measurements, and consider multiple possible sources of systematic error.

Subject headings: cosmic microwave background — cosmology: observations — dark matter — early universe — space vehicles: instruments — telescopes

1. INTRODUCTION

The Microwave Anisotropy Probe (MAP) was designed to produce an accurate full sky map of the angular variations in microwave flux, in particular the cosmic microwave background (CMB; Bennett et al. 2003). The scientific payoff from studies of the CMB anisotropy has driven specialized designs of instruments and observing strategies since the CMB was discovered in 1965 (Penzias & Wilson 1965). Experiments that have detected signals consistent with the CMB anisotropy include (1) ground-based telescopes using differential or beam synthesis techniques: IAB (Piccirillo & Calisse 1993), PYTHON (Coble et al. 1999; Platt et al. 1997), VIPER (Petter et al. 2000), SASK (Wollack et al. 1997), SP (Gundersen et al. 1995), TOCO (Miller et al. 2002), IAC/Bartol (Romero et al. 2001), TENERIFE (Hancock et al. 1997), OVRO/Ring (Myers, Readhead, & Lawrence 1993), OVRO (Leitch et al. 2000); (2) interferometers: CBI (Padin et al. 2001), CAT (Baker et al. 1999), DASI (Leitch et al. 2002), IAC (Harrison et al. 2000), VSA (Watson et al. 2002); (3) balloons: FIRS (Ganga et al. 1999), DASI (Leitch et al. 2003), QMAP (Devlin et al. 1998), MAXIMA (Lee et al. 1999), MSAM (Wilson et al. 2000), BAM (Tucker et al. 1997), BOOMERANG (Crill et al. 2002), ARCHEOPS (Benoit et al. 2002); and (4) the Differential Microwave Radiometer (DMR) experiment on COBE (Smoot et al. 1990). Of these, only DMR has produced a full sky map.

For MAP, the experimental challenge was to design a mission that measures the temperature difference between 2 pixels of sky separated by 180′ as accurately and precisely as the difference between 2 pixels separated by 0.25′. Additionally, we required that the measurements be as uncorrelated with each other as possible in order to make detailed statistical analyses of the maps tractable and so that a simple list of pixel temperatures and statistical weights alone would accurately describe the sky. We also required that the systematic error on any mode in the final map, before modeling, be less than 4 μK of the target sensitivity of 20 μK per 3.2 × 10⁻³ sr pixel. Equivalently, the systematic variance should be less than 5% of the target noise variance.

The components of the MAP mission—receivers, optics, scan strategy, thermal design, electrical design, and attitude control—all work together. Without any one of them, the mission would not achieve the goals set out above. One guiding philosophy is that a differential measurement with a symmetric instrument is highly desirable, as discussed, for example, by Dicke (1968). The reason is that differential outputs are, to first order, insensitive to changes in the satellite temperature or radiative properties. This is especially important for variations on timescales up to ~1 hr, the precession period of MAP’s compound spin. The philosophy is naturally suited to the need to detect the celestial signal well above the 1/f knee of the HEMT spin, as discussed in a companion paper (Jarosik et al. 2003). Other key aspects of the design include simplicity, heritage of major components, the minimization of moving parts, and a single mode of operation.

In this paper we discuss the design of the optics, how the design relates to the science goals, and how the optical response is quantified. To put the final design into perspective, some of the trade-offs are discussed. It is worth keeping in mind that our knowledge of the optics is one of the limiting uncertainties for MAP.

1.1. Design Outline

MAP uses a pair of back-to-back offset shaped Gregorian telescopes that focus celestial radiation onto 10 pairs of back-to-back corrugated feeds as shown in Figure 1 and in

References that summarized multiple measurements or that emphasized the design of an experiment were chosen.
The feeds are designed to accept radiation in five frequency bands between 20 and 100 GHz. Two linear orthogonal polarizations from each feed are selected by an orthomode transducer. Each polarization is separately amplified and detected.

The primary design considerations were as follows:

1. The optical system plus thermal radiators must fit inside the 2.74 m diameter MIDEX fairing and have greater than 25 Hz resonant frequency. The less massive the structure, the less support structure is required, and the easier it is to thermally isolate the system. The mass limit of the entire payload is 840 kg. The center of mass of the system must survive launch accelerations that can attain 12 g. Some components experience significantly higher accelerations.

2. The main beam must be less than $0.3\,\text{dB}$, characterized to $-30\,\text{dB}$ in flight, and computable to high accuracy. The cross polarization must be less than $-20\,\text{dB}$ so that the polarization of the anisotropy in the CMB may be accurately determined. The focal plane must accommodate 10 dual polarization feeds in five frequency bands and allow for all the waveguide attachments.

3. The sidelobes must be less than $-55\,\text{dBi}$ at the position of the Sun, well characterized, and theoretically understood. In addition, the response to the galaxy through the sidelobes must be less than 1% of the main-beam response. A premium was placed on the computability of the sidelobes and the absence of cavities or enclosures in which standing waves could be set up. Thus, on-axis designs and designs with support structure that might scatter radiation were not considered.

4. The reflectors and the optical cavity (see Fig. 1) must be thermally isolated from the spacecraft and must radiatively cool to $70\,\text{K}$ in flight. The reflector surfaces must have less than 1% microwave emissivity, must not build up charge on the surface, and, along with the feeds, must be able to withstand direct illumination by the Sun down the optical boresight for brief periods.

### 1.2. Terminology and Conventions

Throughout this paper the following terminology is used: The thermal reflector system (TRS) consists of the primary and secondary mirrors of each telescope, the structure that supports them, and the passive thermal radiators, as shown in Figure 1. The structure that holds the feeds and the cold end of the receiver chains is called the focal plane assembly (FPA). The TRS fits over the FPA. A reflector evaluation unit (REU), which comprises half the TRS optics, was built to assess the design, for outdoor beam mapping, and as a ground reference unit. “S/C” is used for spacecraft.

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11 The unit dBi refers to the gain of a system relative to an isotropic emitter; dB generically refers to just relative gain.
In quantifying MAP’s response, where possible, the definitions of Kraus (1986) are used. The normalized antenna response to power is

\[ B_n(\theta, \phi) = \frac{|\psi(\theta, \phi)|^2}{|\psi_{\text{max}}|^2}, \tag{1} \]

where \( \psi \) is the scalar electric field in units of \( \text{W}^{1/2} \text{m}^{-1} \) and is evaluated at a fixed distance from the source. The FWHM of a symmetric beam is the angle at which \( B_n(\theta_{\text{FWHM}}/2) = \frac{1}{\sqrt{2}} \) (or \(-3 \text{ dB})\). When the beam is asymmetric, separate \( \theta_{\text{FWHM}} \)s are quoted or the geometric mean of the two \( \theta_{\text{FWHM}} \) is used. At the output of any lossless antenna system, one measures the power in watts given by

\[ W = \frac{1}{2} \int_\Omega \int_\nu A_e(\nu)S_v(\theta, \phi)B_n(\nu, \theta, \phi)d\Omega d\nu, \tag{2} \]

where \( A_e \) is the effective area of the antenna and \( S_v(\theta, \phi) \) is the brightness of the sky. The directivity is

\[ D_{\text{max}} \equiv \frac{|\psi|^2}{|\psi_{\text{avg}}|^2} = \frac{4\pi|\psi|^2}{\int_\Omega |\psi(\theta, \phi)|^2 d\Omega} = \frac{4\pi}{\int B_n(\theta, \phi)d\Omega} = \frac{4\pi A_e}{\Omega_d} = \frac{4\pi}{n\lambda^2}, \tag{3} \]

where \( \Omega_d \) is the total solid angle of the normalized antenna pattern, \( n \) is the number of radiative modes, and \( |\psi_{\text{avg}}|^2 \) is the total power averaged over the sphere. For single-mode systems such as MAP, \( n = 1 \). When there are no losses in the telescope, \( G(\theta, \phi) = D_{\text{max}}B_n(\theta, \phi) \) and the directivity is the maximum gain.

The maximum gain, \( G_{\text{m}} \), is sometimes just called the gain or “the gain above isotropic.” It can be understood by considering the flux (power/area) from an isotropic emitter of total power \( P \). At a distance \( r \), the flux is \( P/4\pi r^2 \) and the gain is unity (0 dB). If instead the power were emitted by a feed of gain \( G_m \), the flux at the maximum would be \( I = G_mP/4\pi r^2 \) W m\(^{-2}\). In other words, if one measures the field at a distance \( r \) from the feed, then

\[ G(\theta, \phi) = \frac{4\pi^2 |\psi(r, \theta, \phi)|^2}{\text{Total emitted power}}. \tag{4} \]

The absolute gain, as opposed to the relative gain, is important because it is the quantity that indicates one’s immunity to off-axis sources. If the gains for all of MAP’s frequency bands were the same at some angle, each band would be equally susceptible to a source at that angle. The gain is always normalized so that

\[ \frac{\int G(\theta, \phi)d\Omega}{4\pi} = \int_{S'} \frac{|\psi(x', y')|^2}{\text{Total emitted power}} dx' dy' = 1, \tag{5} \]

where the primed coordinates are for the aperture of the feed (or optical element).

The edge taper, \( y_e \), is often useful in discussing the immunity of the optical system to sources in the sidelobes. In fact, from the value of the field at the edge of an optic, one can compute the shape of the sidelobe pattern. The pattern is then normalized by the total power through the aperture. The edge taper is given in dB as \( y_e = 10 \log(I_{\text{edge}}/I_{\text{center}}) \), where \( I \) is the intensity of the beam. For none of MAP’s bands is \( y_e \) uniform around the edge; the largest (closest to 0 dB) value is quoted.

2. OPTICAL DESIGN AND SPECIFICATION

In designing MAP, we considered a number of geometries including simple offset parabolas and three reflector systems. In neither of these cases is the geometry of the feed placement conducive to having both inputs of a differential receiver view the sky. With the back-to-back dual-reflector arrangement we have chosen, the feeds are centrally located and point in nearly opposite directions. Additionally, the S/C moment of inertia is minimized for a large optic.

There are a number of dual-reflector telescope designs (Schröder 1987; Love 1978). For radio work the Cassegrain (parabolic primary, hyperbolic secondary) and the Gregorian (parabolic primary, elliptical secondary) are often used. For some potential MAP geometries, the scanning properties of the Cassegrain system (Ohm 1974; Rahmat-Samii & Galindo-Israel 1981) were found superior to those of the Gregorian system. In other words, the beam pattern from a feed placed a fixed distance from the focus is more symmetric and has smaller near lobes. However, the offset Gregorian was chosen because the associated placement of the feeds was well suited to the differential receivers and the FPA could occupy the space made available by the position of the focus between the primary and secondary. Additionally, for a given beam size, the Gregorian is more compact than the Cassegrain (Brown & Prata 1995).

Dragone (1986, 1988) developed extremely low sidelobe Gregorian systems in which the feed aperture is reimaged onto the primary. Such a system was used for the Advanced Cosmic Microwave Explorer (ACME) CMB telescope (Meinhold et al. 1993). Dragone’s work was used as a guideline, but a number of factors complicated MAP’s design: (1) the feed apertures must be in roughly the same plane to avoid being viewed by one another; (2) the feed inputs must cover 20 × 20 cm\(^2\) to accommodate their large apertures (the feed tails [receiver inputs] cover 53 × 53 cm\(^2\)) to make room for the microwave components; two W-band feed tails are separated by 19 cm; the two K\(_a\)-band feed tails are separated by 45 cm); and (3) the secondary is in the near field of the feeds. For CMB telescopes, unlike the more familiar communications telescopes, beam efficiency is more important than aperture efficiency (Rohlfs 1986).

To understand the beams, accurate computer codes are essential. The baseline MAP design was done using code modified from the work of Sletten (1988) in which the far field beam pattern is computed from the square of the Fourier transform of the electric field distribution in the aperture. This simple and fast code combined with parametric models of the feeds and sidelobe response allowed rapid prototyping of various geometries while simultaneously optimizing over the combination of main-beam size, contamination from the Galactic pickup through the sidelobes, and the sidelobe level at the position of the Sun. All models followed the constraints for minimal cross polar response (Tanaka & Mizusawa 1975; Mizuguchi, Akagawa, & Yokoi 1976). The parameters of the resulting telescope are given in Table 1.

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12 Although the Ritchey-Chretien (hyperbolic primary, hyperbolic secondary, e.g., Hubble Space Telescope) and aplanatic Gregorian (ellipsoidal primary, hyperbolic secondary) were considered, the lack of well-developed and tested offset designs did not fit with MAP’s fast build schedule. Hanany & Marrone (2002) give an up-to-date comparison of offset Gregorian designs.
The DADRA “physical optics” computer code (Y. Rahmat-Samii 1995, private communication) was used for the detailed design of MAP. From a spherical wave expansion of the field from a feed, the code determines the surface current on the secondary, 

\[ J(r) = 2n \times H_{inc}(r) \]

From this current, it computes the fields incident on the primary and thus the currents there. These currents are particularly useful for understanding the interaction of the optics with the S/C components. Examples are shown in Figure 2 for the lowest and highest frequency bands. The resulting beam is a sum of the fields from the feed and the currents on the primary and secondary reflector surfaces, as shown in Figure 3. The method takes into account the vector nature of the fields and the exact geometry of the reflectors and the feeds. In principle, the currents can be unphysical near the reflector edges. In practice, because of the low edge taper, this approximation does not introduce significant errors.

The code is excellent although its limitations are that (1) only two reflections are considered, (2) the possible interference of the feeds with the radiation from the reflectors and with each other is not accounted for, and (3) the interactions with the structure are not accounted for. These interactions must be determined “by hand,” and thus measurements of the assembled system are essential.

2.1. Flight Design

Once the baseline Gregorian design was set, the surfaces were “shaped” to optimize the symmetry and size of the beams (Y. Rahmat-Samii 1995, private communication; Galindo-Isreal et al. 1992). The resulting reflectors differ from the pure Gregorian by \( \frac{1}{32} \) cm in regions near the perimeter.\(^{13}\) For simple calculations, we use the best-fit parameters shown in Table 1.

The shaped system does not have a sharp focus between the primary and secondary, but one may still characterize its response in broad terms. The plate scale, i.e., how far one moves laterally in the focal plane to move a degree on the sky, for the shaped system is 4.44 cm deg\(^{-1}\). This suggests an effective focal length of 250 cm, somewhat longer than that

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**Table 1**

| Quantity                      | Base Design | Best Fit | Equivalent Parabola |
|-------------------------------|------------|----------|---------------------|
| Focal length, \( f_p \) (cm)  | 90         | 90       | 206                 |
| Primary projected radius, \( r_p \) (cm) | 70         | 70       | 70                  |
| Offset parameter, \( y_c \) (cm) | 105        | 104.03   | ...                |
| Interfocal distance, \( 2c \) (cm) | 45         | 43.58    | ...                |
| Secondary eccentricity, \( e \) | 0.45       | 0.4215   | ...                |
| \( \beta \) (deg)               | 12.06      | 13.77    | ...                |
| \( \alpha \) (deg)              | -31.12     | -33.06   | ...                |

Note.—Cassegrain and Gregorian telescopes are defined by five parameters when they satisfy the minimum cross polarization condition. These parameters are shown in Fig. 1. Col. (2): Nominal conic design. Col. (3): Best fit to the shaped design with five free parameters. Col. (4): Equivalent parabola (Rausch et al. 1990) for the best-fit design described by the top five parameters. When the best-fit design is constrained to follow the minimum cross polarization condition, \( \beta = 14/32 \) and \( \alpha = -34/32 \). The surface coating is VDA as discussed in § 2.8.

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\(^{13}\) In retrospect, we should not have shaped the reflectors. It added time to the manufacturing process, and the cooldown distortions of the primary reflectors negated its benefits.
found from the equivalent parabola for the best-fit model. The speed of the system, $f_p/D$, is $f/1.8$. The computer files containing the geometry are available upon request.

### 2.2. Manufacture and Alignment of Optics

The TRS and REU were built by Programmed Composites, Inc. (PCI) to a specification (Jackson, Page, & Stewart 1994). The structure that holds the optics is made of $5 \times 5$ cm "box beams" of 0.76 mm thick XN70/M46J composite material and has a mass of 23 kg. The reflectors are made of 0.025 cm thick XN70 spread fabric cloth face sheets over a material and has a mass of 23 kg. The reflectors are made of 0.76 mm thick XN70/M46J composite material and have a mass of 8 kg. They are painted with NS43G/1.54 kg. The radiator panels are made of 1100 series H14 aluminum over a $5.08 \text{ cm aluminum honeycomb core}$, and each has a mass of 8 kg. They are painted with NS43G/Hincom white paint to minimize their solar absorptance, ensure a conducting surface, and maximize their infrared emissivity. The full TRS, with harnesses and thermal blankets, has a mass of 70 kg.

The specifications were set to meet the science goals and to easily mesh with known tolerances in the manufacturing process to keep costs down. The surface rms deviation at 70 K from the ideal shape over the whole reflector was specified to be less than 0.0076 cm, or $\approx \lambda/40$, and is discussed in more detail below. The reflectors, when treated as rigid bodies satisfying the surface rms criteria and when positioned on the TRS structure and cooled to 70 K, were specified to be within 0.038 cm of the design position. This specification includes all rotations and translations and accounts for the effects of moisture desorption, gravity relief, and cooldown from room temperature. The on-orbit predictions are given in Table 2.

The optics were built to have no adjustments. They were designed to be in focus at 70 K and so were deliberately though insignificantly out of focus for all testing at 290 K. A full structural thermal optical (STOP) performance analysis of the optical design was performed using the $W'$-beam pattern because it is the most sensitive to changes in the position. The analysis includes changes in the position and orientation of the optics and feeds as they cool. The worst-case displacements upon cooling lead to a $0.1$ shift in beam elevation and a $0.14$ shift in azimuth, dominated by orientation changes in the primary. These are not significant from a radiometric point of view. In addition, all final pointing and beam information is determined from in-flight observations. Table 3 shows the placement of the feeds along with the predicted beam positions on the sky. Tables 1, 2, and 3 completely specify the geometry for the conic approximation.

In addition to the usual metrology tools, photogrammetry and laser tracking were found to be particularly useful. Photogrammetry was used to determine the change in the shape of the optics upon cooling to 70 K. The laser tracker allowed the rapid digitization of thousands of points on the reflectors, which was useful for surface fitting and measuring the surface deformations.

### 2.3. Feeds

The inputs of the differential microwave receivers are coupled to free space with corrugated microwave feeds (Barnes et al. 2002). The initial designs followed well-known principles (Thomas 1978; Clarricoats & Olver 1984), although the final groove dimensions were optimized (Y. Rahmat-Samii 1995, private communication). Corrugated feeds were chosen because (1) their patterns are accurately computable and symmetric, (2) they have low loss, and (3) they have low sidelobes. Table 4 summarizes the features of the MAP feeds.

At the base of each feed is an orthomode transducer (OMT), the microwave analog of a polarizing beam splitter. The two rectangular waveguide outputs of the OMT, the "main" port and the "side" port, carry the two constituent polarizations to the inputs of separate differencing assemblies (Jarosik et al. 2002). As the polarization cannot be

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**Table 2**

| Object                        | $x_s$ (cm) | $y_s$ (cm) | $z_s$ (cm) |
|-------------------------------|------------|------------|------------|
| Focus                         | 0          | 65.0       | -166.0     |
| Top of A primary              | 0.045      | 35.99      | -351.66    |
| Bottom of A primary           | 0.040      | 7.58       | -192.75    |
| $+$x boundary of A primary    | 70.05      | 21.77      | -272.30    |
| Top of A secondary            | 0.031      | 123.31     | -215.85    |
| Bottom of A secondary         | -0.011     | 102.20     | -136.00    |
| $+$x boundary of A secondary  | 39.12      | 36.73      | -176.36    |

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*Fig. 3.—Optical response of a prototype design with a $K$-band feed at the focus of the telescope. The $y$-axis is the gain; the $x$-axis shows the angular distance from the primary optical axis. Negative numbers correspond to lines of sight below the optical axis as shown in Fig. 1. The dashed line is the full model computed by the DADRA code for left circular polarization. The thick solid line that peaks at $-15^\circ$ would be the response of the feed if it were not blocked by the secondary. Note that where the secondary does not intercept the feed pattern, $-40^\circ > \theta > 15^\circ$, the full pattern is dominated by the feed pattern. In other words, spill of the feed past the secondary dominates the optical response. The smooth dotted curve that envelopes the feed response is the parameterized model discussed in § 2.3. The smooth line extending from the peak to $\theta = 100^\circ$ is the parameterized sidelobe response discussed in § 2.7. The light curve between $0^\circ < \theta < 40^\circ$ is from the aperture integration code. The curve that peaks at $\theta = 90^\circ$ is the response to radiation that has reflected once off the bottom of the secondary and into the feed and thus is right circularly polarized. This specific configuration was not built.*
The beam from each feed is diffraction limited so that the illumination patterns on the secondary and primary are a function of frequency. Because the secondaries are in the near field of the feeds, \( d < 2a_{\text{prim}}/\lambda \), the phase center concept is not useful for detailed predictions. A full electromagnetic code such as CCRHN (Y. Rahmat-Samii 1995, private communication) is essential. For the top level parameterization of \( MAP \) the feeds were modeled as open corrugated waveguides with \( B_0(\theta) = [x_0^2 J_0(v)/x_0^2 - v^2]^{1/2} \), where \( v = kr_1 \sin \theta \), \( x_1 \approx 2.405 \), \( k = 2\pi/\lambda \), and \( r_1 \) is the radius of the waveguide (Carricato & Olver 1984). At large angles, the envelope of \( J_0(v) \approx (2/v\pi)^{1/2} \) is as shown in Figure 3 for a \( K \)-band feed.

### 2.4. Main Beams

The main-beam width is determined by the size of the primary mirror, the edge taper, and, to a lesser degree, the beam profiling. As discussed in § 2.7, the edge taper is approximately \(-20 \text{ dB}, \) except in \( K \) band. With a 1.4 m projected primary diameter, the beamwidth is \( \theta_{\text{FWHM}} \approx 0.5\% (40 \text{ GHz}/\nu) \). Because a diffraction-limited feed illuminates the primaries and secondaries with a low edge taper, the beamwidth is a relatively weak function of frequency within a frequency band, as shown in Table 5.

The number of feeds and radiometers in each frequency band is chosen so that each band has roughly equal sensitivity per unit solid angle to celestial microwave radiation. Because the noise temperature of HEMT amplifiers scales approximately with frequency (Pospieszalski & Lakatos 1995), \( MAP \) has one feed in \( K \) band and \( K_a \) band, two feeds in \( Q \) band and \( V \) band, and four feeds in \( W \) band.

The desire for broad frequency coverage and multiple channels requires the maximal use of the telescope focal plane. The layout of the two back-to-back telescopes, shown in Figure 1, and the need for a compact enclosure for all the differential radiometers place difficult constraints on the geometry of the feeds, the radiometers, and the focal plane arrangement. In particular, the need for roughly equal total feed length, the requirement for minimum cross talk and obscuration between feeds in the focal plane, and the placement of the OMTs and cold amplifiers lead to feed telescope solutions that required evaluation with full diffraction calculations for optimizing the configuration. For example, the \( K \)-band feed is profiled to reduce its length by a factor of \( \approx 50\% \) from its nominal geometry. The \( K \)-band feed is also shifted along its axis by 15 cm toward the secondary from its optimal position. Such large departures from usual practices are acceptable if the design can be evaluated and the beam profiles and solid angles can be measured to \( \approx 0.5\% \) accuracy in flight.

In general, a controlled loss of axial symmetry of the beam point-spread function was balanced against other geometrical constraints as the whole system was considered simultaneously. The solution for the reflectors results in an uninvited image of the sky on a slightly curved focal plane. The geometric image quality degrades with increasing...
The optical system is designed to minimize the cross polarization. The OMTs attached to the azimuthally symmetric feed define the polarization direction. Each OMT is oriented so that the polarization directions accepted by the feeds are ±45° with respect to the yz symmetry plane of the satellite. The unit direction vectors on the sky are given in Table 3. The 45° angle ensures that the two linear polarizations in one feed nearly symmetrically illuminate the primary and secondary. Consequently, the beam patterns for both polarizations are nearly identical.

There are some subtleties in specifying the polarization angle. For example, the average of the polarization angle within a −3 dB contour of the main beam does not equal the polarization direction at the beam maximum. It can differ by order a degree. MAP’s angles were set to ±45° at the beam maximum. Even though the corresponding orientation of the OMT was found using the full diffraction calculation, the same orientation is obtained with purely geometrical considerations.

The MAP polarization angle was measured in the NASA Goddard Space Flight Center (GSFC) beam mapping facility. A polarized source was rotated to minimize the average signal across all frequencies in the band. The polarization angle was then taken to be 90° from that angle. The measured polarization minimum does not occur at the same angle across the band, and so a best overall minimum angle was approximated. The uncertainty of the radiometric measurement is ±1°. Across all bands and all feeds, the scatter in polarization angle is ±2°. The uncertainty in the polarization angle based on the metrology of the OMT orientation is ±0.2°. We use ±1° as the formal error. The resulting maximum misidentification of power due to the rotational alignment uncertainty is ≈1 − cos²(1°) ≈ 0.0003.

The cross polar leakage is not zero but is sufficiently small to allow the measurement of the CMB polarization to the limits of the detector noise. In K and Kα bands, the cross polar contribution to the copolar beam is −25 and −27 dB, respectively, and is dominated by the reflector geometry in combination with the feed placement. In Q, V, and W bands, the cross polar contributions to the copolar response are −24, −25, and −22 dB, respectively, and arise from a combination of imperfect OMTs, as shown in Table 4, and reflector geometry.

2.6. Surface Shape

The smoothness of the optical surface and the placement of the feeds determine the quality of the beams. The measured beam profiles, especially in W band, do not precisely match the predictions for ideal reflectors. However, they are in excellent agreement with the computations based on the measured distortions of the reflector surfaces.

The characteristics of the profiles are also in good agreement with the Ruze (1966) model, which accounts for the axial loss of gain and the pattern degradation as a function
of the reflector surface rms error, $\sigma_s$, and the spatial correlation length of the distortions, $l_c$. For lossless Gaussian shaped distortions, which sufficiently describe the surfaces,

$$G(\theta, \phi) = G_0(\theta, \phi)e^{-\rho^2} + \left(\frac{2\pi l_c}{\lambda}\right)^2 e^{-\rho^2} \times \sum_{n=1}^{\infty} \frac{\rho^{2n}}{n!} e^{-\left(\pi l_c \sin \theta / \lambda\right)^2} \tan^2 \theta / n,$$

where $\lambda$ is the wavelength, $G_0(\theta, \phi)$ is the ideal undistorted prediction, $\rho^2$ is the variance of the phase error, equal to $k^2\sigma_s^2$, and $l_c$ is the correlation length. The number of distorted “lumps,” $\approx (D/2l_c)^2 \approx 50$, is large enough to satisfy the statistical assumptions behind Ruze’s model.

The first term in equation (6) shows that the reduction in forward gain from the undistorted reflector is determined by $\sigma_s$ and is independent of $l_c$. The second term, the “Ruze pattern,” is a function of $\theta$, is independent of the undistorted pattern, and is determined by $l_c$ and $\rho^2$. The shoulder of the Ruze beam is mostly determined by $l_c$. Increasing the $\sigma_s$ from zero while keeping $l_c$ constant lowers the forward gain and raises the Ruze beam, without changing the shoulder. Increasing $l_c$ from zero narrows the Ruze pattern.

The surface shape was specified to achieve close to ideal performance. Ground testing revealed that the primary mirror shape met the specification at room temperature but misses the lobe at the second Earth-Sun Lagrange point, $L_2$, by $\approx 0.991$ cm, is large enough to satisfy the statistical assumptions behind Ruze’s model. For on-orbit predictions, the same procedure is followed except that the cold $\sigma_s$ is used. We assume that the correlation length does not change. The results are plotted in Figure 4. The Ruze beam dominates the undistorted beam for $0.2 < \theta < 2^\circ$ and is subdominant to it above $2^\circ$, diminished by the exponential factor. Even $1^\circ4$ from the boresight, the Ruze beam is less than $-35$ dB below the peak. The full $W$-band patterns for the ideal and distorted beams are plotted in Figure 5.

An additional potential source of degradation of the main beams is print through of the weave pattern that comprises the fabric that makes up the surface of the reflectors. This was investigated by measuring the change in the holographic pattern of a sample surface upon cooling (Jackson & Halpern 2002). For a $200$ K change, the contrast of the weave increases from $0$ to $\approx 2 \mu$m. From this pattern, one computes an upper bound of $-70$ dB in $W$ band for a narrow sidelobe $40^\circ$ off the main beam. Contributions from bright sources (e.g., Jupiter) through this lobe will be less than $0.1 \mu$K.

### 2.7. Sidelobes

The sidelobes are small, well characterized, theoretically understood, and measured. Generally, sidelobes are produced by illumination of the edges of the optical elements, whereas the main-beam profile is determined by the shape of the elements. The primary radiometric contamination comes from hot sources, such as the Earth, Moon, and Sun, well outside the main beam. In addition, contamination arises from the Galaxy, which is much colder, $\approx 200$ mK, but is extended and closer to the main beam. For scale, if the Sun is to contribute less than $2 \mu$K to any pixel, it must be rejected at $(2 \mu$K/$T_{\text{Sun}})(\Omega_{\text{Sun}}/\Omega_4) \approx -100$ dB.

### TABLE 6

| Reflector          | Spec $\sigma_s$ | $\sigma_s^{20K}$ (cm) | $\sigma_s^{70K}$ (cm) | $l_c$ (cm) | $e^{-\phi}$ Warm | $e^{-\phi}$ Cold | $k_l$ |
|--------------------|-----------------|----------------------|----------------------|------------|------------------|------------------|-------|
| Primary P3 (A side) | 0.0076          | 0.0071               | 0.023                | 9.3        | 0.989            | 0.826            | 175   |
| Primary P2R (B side) | 0.0076          | 0.0071               | 0.024                | 11.4       | 0.987            | 0.812            | 213   |
| Secondary S4 (A side) | 0.0076          | 0.0071               | ...                 | 10.1       | 0.988            | ...              | 191   |
| Secondary S5 (B side) | 0.0076          | 0.0076               | ...                 | 9.7        | 0.991            | ...              | 183   |

*The surface rms, $\sigma_s$, was specified as a function of radius to mesh with manufacturing processes. For the primary $\sigma_s < 0.0038$ cm for $r < 25$ cm and $\sigma_s < 0.0051$ cm for $r < 50$ cm. For the secondary $\sigma_s < 0.0038$ cm for $r < 15$ cm and $\sigma_s < 0.0051$ cm for $r < 35$ cm. For eq. (6), we convert the surface distortions, $\sigma_s$, to distortions projected along the primary optical axis, $l_c$. The reduction in forward gain due to scattering for the ambient temperature and on-orbit cases. The warm reflectors meet the specification and scatter just slightly over 1% away from the forward direction. The cold reflectors scatter out of order 20%, or 1 dB, out of the main beam and into the near sidelobes as shown in Fig. 4.
The six principle contributions to the sidelobes are (1) the response of the feeds to radiation outside the angle subtended by the subreflector, (2) diffraction from the edge of the secondary, (3) diffraction from the edge of the primary, (4) spill past the edge of the primary, (5) scattering from the structure that holds the feeds, and (6) reflection off the radiator by radiation that goes past the primary. The more negative the edge taper, the smaller each of these is.

The sidelobe levels are computed in two ways. For the contributions from the radiators and optical surfaces, the full physical optics calculation is used (Y. Rahmat-Samii 1995, private communication). The commercial code was rewritten to take into account the interaction of the radiators by radiation that goes past the primary. The more negative the edge taper, the smaller each of these is.

In GTD, the field diffracted by an edge is given by 
\[
\psi_e(r) = D_{GTD} \psi(|1 + r/\rho_1|)^{-1/2} e^{ikr}, \]
where \(\psi_e\) is the incident field (or field at the edge), \(r\) is the distance to the edge, \(\rho_1\) is a radius of curvature that characterizes the edge (Keller 1962), and the diffraction coefficient \(D_{GTD}\) is
\[
D_{GTD} = -\frac{e^{\pi/4}}{2\sqrt{2\pi k}} \left\{ \frac{1}{\cos[(\phi-\alpha)/2]} \pm \frac{1}{\sin[(\phi+\alpha)/2]} \right\}. \tag{7}
\]
For a straight edge \(1/\rho_1 = 0\) and one recovers Sommerfeld’s solution. The angles are shown in Figure 6. The upper sign is for the incident E-field perpendicular to the edge, and the lower sign is for the E-field parallel to the edge.

Radiation from the Sun diffracts around the edge of the solar shield, diffracts over the top of the secondary, and then enters the feeds as can be seen in Figure 1. Radiation from the Earth and Moon diffracts directly over the top of the secondary and then enters into the feeds. To compute these contributions, the rim of the solar shield is approximated as straight (outboard of the secondary it is almost straight). The edge of the secondary is approximated with a radius of curvature of \(r_s = 40\) cm so that \(1/\rho_1 \approx -\cos(3\pi/2 - \phi)/r_s\). For all estimates, the phase in equation (7) is ignored.

Consider first the contribution from the Earth with temperature \(T_E\) and solid angle \(\Omega_E\). The antenna temperature is given by
\[
T_A = \frac{|D_{GTD}|^2 G(\theta_{\text{edge}})}{4\pi} \frac{T_E}{r} \Omega_E, \tag{8}
\]
where \(G(\theta_{\text{edge}})\) is the gain of the feed at the edge of the secondary, \(r\) is the distance to the secondary, and \(D_{GTD}\) is given by equation (7) with the geometry in Figure 1. The diffraction effects are greatest for \(K\) band because it has the largest wavelength, so only it is considered. Using the parameters for the Sun, Earth, and Moon from L2 and \(G(\theta_{\text{edge}}) = -22.4\) dBi, \(\alpha = -76^\circ\), \(\phi = 200^\circ\), and \(r = 60\) cm, one finds \(D_{GTD} = 0.36\) cm\(^{1/2}\) and \(T_A \approx 60\) nK. Similar calculations yield \(T_A \approx 4\) nK for the Moon and, after taking into account the double diffraction, \(T_A \approx 16\) nK for the Sun.

![Fig. 4.—Left: Beam profiles at 90 GHz from a distorted surface based on Gaussian deformations (dashed line; eq. [6]) and hat box distortion (dot-dashed line; Ruze 1966). The azimuthally averaged measured warm profile (solid line) is shown with the DADRA predictions based on the measured distorted surface (dotted line). Right: Predictions for the cold optics based on the values in Table 6 for Gaussian distortions (dashed line), hat box distortions (dotted line), and the undistorted mirror (solid line). At \(\approx 2^\circ\) all curves meet.](image1)

![Fig. 5.—Comparison of the W4A beams at 90 GHz. Left: Predictions for an ideal mirror beam pattern. Middle left: Predictions for an ambient temperature distorted mirror beam pattern. Middle right: Measured beam pattern. Right: Prediction for a cooled mirror in orbit. The contour intervals are 3 dB. The cross indicates the center of the focal surface.](image2)
Similar order-of-magnitude calculations were used to check multiple diffraction paths and angles. It was found that solar radiation could directly enter the feeds by diffracting from the edges of the secondary and the truss structure that holds the secondary. As a result, additional "diffraction shields" were added that go between the front of the structure that holds the feeds (FPA) and the edge of the secondary to eliminate these paths.

GTD was also used early in the design phase in the parametric model of the satellite ($x^2$). The illumination of the primary for the field $\psi$ was modeled as a Gaussian of width $\sqrt{2}r_p$ with edge taper $\gamma_e$ so that

$$\exp\left(-\frac{r_p^2}{4\sigma_p^2}\right) = 10^{\gamma_e/20} \equiv 1 \quad (9)$$

at the rim of the primary, $r = r_p$. For a circular disk or aperture evaluated in the limit of $r \gg r_p$ and intermediate angles

$$\psi_i(r, \theta) = D_{GTD}\psi_i\left(\frac{r^2 \sin \theta}{r_p}\right)^{-1/2} e^{ikr} \quad (10)$$

and

$$D_{GTD} = -\frac{1}{2\sqrt{2\pi k}} \left[ \frac{1}{\sin(\theta/2)} \pm \frac{1}{\cos(\theta/2)} \right] \quad (11)$$

At large angles, diffraction with E parallel to the edge dominates, and the full expression becomes

$$G(\theta) = \frac{4\pi r^2 \psi^2_i(r, \theta)}{\text{Total power}} \approx \frac{\ln(10^{1/20})|\psi|^{10^{1/10}}}{\pi r_p k} \frac{1}{\sin \theta} \left[ \frac{1}{\sin(\theta/2)} + \frac{1}{\cos(\theta/2)} \right]^2 \quad (12)$$

A curve of equation (12) is shown in Figure 3.

Because of the large uncertainty associated with the hand GTD calculations and the need to confirm the computer models, the sidelobes were measured in K, Q, and W bands using a full-scale replica of the satellite built around the REU. The results from the full diffraction calculation for K band along with measurements at the same frequency are shown in Figure 7.

2.7.1. Radiometric Contributions from Sidelobes

To assess the contribution to the radiometric signal from Galactic emission, the full sky differential beam maps were "flown" over a Galactic template. The beam maps were made from a composite of the computer model and the measurements. The Galactic template is based on the Haslam and IRAS maps scaled to be $T_{gal} = 230, 100, 70, 40,$ and 50 mK in K through W bands, respectively, in a $4 \times 10^{-6}$ sr pixel at $l = 0, b = 0$.
The A-side beam is placed in each pixel of the map, and the negative B-side beam is rotated through 360° in 10° intervals. The contributions for $G > 15$ dBi are excluded as these are right next to the main beam and will naturally be incorporated into the map solution. (For $V$ and $W$ bands this corresponds to 1 pixel.) At each orientation, the following integration is performed:

$$T_A = \frac{1}{4\pi} \int G(\theta, \phi) T_{\text{gal}}(\theta, \phi) d\Omega,$$

where $G(\theta, \phi)$ is the telescope gain. For each ring, the minimum, maximum, and rms signals are recorded. Data with the B-side beam at $|b| < 15^\circ$ are excluded. The results are given in Table 7.

Verifying the GTD calculations entails a measurement of the sidelobes at the $-70$ dBi level. Measurements using the test range between the roof tops of Princeton’s Physics and Mathematics buildings are limited by scattering at the $-50$ dBi level, 110 dB below the peak in $W$ band, even after covering large areas of ground with the microwave absorber. Consequently, only an upper limit of 0.5 $\mu$K may be placed on contamination by the Sun, Earth, and Moon.

### 2.8. Reflector Surfaces

Coatings were applied to the optics to provide a surface ($a$) whose microwave properties are essentially indistinguishable from those of bulk aluminum, ($b$) that radiates in the infrared so that the reflector does not heat up unacceptably when the Sun strikes it, ($c$) that diffuses visible solar radiation so that sunlight is not focused on the focal plane or the secondary, and ($d$) that does not allow the buildup of excessive amounts of charge in orbit. These four criteria can be met simultaneously (Heaney et al. 2001).

The coating is vacuum deposited onto a surface comprised of an $\approx 25$ $\mu$m thick layer of epoxy over a $250$ $\mu$m composite sheet. The epoxy layer is roughened to diffuse sunlight and not affect the microwave properties and is then coated with aluminum and SiO$_2$ (the “x” denotes unknown; the material is a combination of SiO and SiO$_2$). The SiO$_2$ coating, because of an absorbance resonance, allows the reflector to radiate at 10 $\mu$m, near the peak of a 300 K blackbody, as shown in Figure 8. To maximize the infrared radiation, thicker SiO$_2$ is better, but if the layer exceeds 2.5 $\mu$m, it insulates the aluminum unacceptably, leading to potential problems with surface charging in space.

Obtaining a successful coating was particularly challenging. The mission requirement that the optics be able to withstand transient direct illumination by the Sun was a significant complication. To maintain high surface quality, especially in the micron wavelength region, the flight reflectors were maintained at less than 50% relative humidity from when they were coated until launch.

#### 2.8.1. Roughening the Surface

The reflector surfaces are roughened to diffuse the solar radiation so that the secondaries and feeds do not get too hot from the focused radiation from the primary. The specification, which is developed below, is that no more than 20% of the reflected radiation be inside the 20° (full angle) cone of the reflected ray at a wavelength of 540 nm. Measurements show that this is possible if the surface has a 0.4 $\mu$m roughness with a correlation length of 30 $\mu$m, in agreement with the models (Bennett & Porteus 1961).

Reflection from a roughened surface is quantified with the bidirectional reflectance (BDR), $\rho$, and the directional reflectance (DR), $\rho_D$ (Nicodemus 1965; Beckmann & Spizzichino 1963; Davies 1954; Houchens & Hering 1967). The BDR is the reflectance coefficient per unit solid angle for arbitrary incident polar directions ($\psi$, $\varsigma$) and reflection directions ($\theta$, $\phi$) measured from the mean normal of the surface:

$$\rho(\psi, \varsigma; \theta, \phi) = \frac{\delta I(\theta, \phi)}{I(\psi, \varsigma) \cos \psi \Delta \omega_i} \frac{\pi (\text{Energy into } \Delta \omega_i)}{\text{Total incident energy}},$$

where $I$ is the spectral intensity in W sr$^{-1}$ Hz$^{-1}$ and $\Delta \omega_i$ and $\Delta \omega_r$ are the solid angles containing the incident and reflected radiation. The units are sr$^{-1}$. In practice, the BDR is not measured absolutely and $\varsigma$ and $\phi$ are 180° apart. In terms of the parameters for a lossless surface,

$$\rho(\psi, \varsigma; \theta, \phi) = \frac{\exp(-F)}{\cos(\psi)\Delta \omega_i} + \frac{\exp(-G)}{\cos(\psi)\cos(b_i)\frac{\pi}{\chi}} \left\{ \begin{array}{l} 0 \\ \frac{\chi}{b_i} \end{array} \right\} B \times \sum_{m=1}^{\infty} \frac{G^n}{m \cdot m!} \exp\left[ -\frac{\pi^2}{m} \left( \frac{\chi}{b_i} \right)^2 H \right],$$

where $F$ and $G$ are coefficients, $\chi$ is an angle, $b_i$ is a parameter, $H$ is a harmonic number, and $B$ is a factor.

**Table 7**

| Parameter | $K$ | $K_0$ | $Q$ | $V$ | $W^{10}$ |
|-----------|-----|-------|-----|-----|---------|
| Center frequency (GHz) | 23 | 33 | 41 | 61 | 93 |
| Maximum ($\mu$K) | 85 | 10 | 20 | 2 | 1.5 |
| Minimum ($\mu$K) | $-150$ | $-60$ | $-40$ | $-3$ | $-5$ |
| rms ($\mu$K) | 12 | 2 | 3 | 0.2 | $<0.1$ |
| Maximum edge taper (dB) | $-13$ | $-20$ | $-21$ | $-21$ | $-16/-20$ |
| Forward beam efficiency$^d$ | 0.960 | 0.986 | 0.986 | 0.996 | 0.996/0.999 |

$^a$ Where appropriate, the values for $W1/4$ (upper) and $W2/3$ (lower) are separately given.

$^b$ The negative values correspond to a signal through the B beam.

$^c$ The maximum edge taper for the center of the band. As the current distributions are not symmetric with the reflector, most of the edge has a substantially smaller taper.

$^d$ The integral of the beam in an area around $\pm 2^\circ$ from the maximum divided by $4\pi$. A value of 0.996 means that 0.004 of the solid angle is scattered into the sidelobes.

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10 For example, the REU reflectors turned brown over a period of days shortly after delivery. Later, a coating that was apparently stable over a year peeled off the flight secondaries and had to be redone.
where \( F = \frac{4\pi(\sigma/\lambda) \cos \psi}{2} \) and \( G = \frac{2\pi(\sigma/\lambda)(\cos \psi + \cos \theta)}{2} \). Here \( \sigma \) is the surface rms, \( c \) is the correlation length, \( D \) is the solid angle for the incident radiation, \( B \) is a function of the incident angles and is of order 1, and \( H = \frac{\sin^2 \psi + \sin^2 \theta + 2 \sin \psi \sin \theta \cos(\zeta - \psi)}{2} \). When there is no scattering, the first term dominates and the reflection is specular and coherent. The second term gives the incoherent or diffuse component. The correlation length enters the incoherent component alone and so influences only the spatial distribution of the radiation and not the total energy reflected (Houchens & Hering 1967), similar to the case of Ruze scattering. One expects that for a fixed surface rms, the greater the coherence length, the smaller the slope, and the more confined the reflected beam. For a perfectly diffuse (Lambertian) reflector, \( \rho' \) is independent of angle.

The DR is the ratio of the total energy reflected into a hemisphere divided by the incident energy,

\[
\rho_d = \frac{\int_0^{\pi} \int_0^{2\pi} I_r(\theta, \phi) \sin \theta \cos \theta \, d\theta \, d\phi}{\int_0^{\pi} \sin \psi \cos \psi \, d\psi \, d\zeta},
\]

and is typically 0.6. In practice, the DR is the measured quantity and the BDR is estimated from it through \( \rho' = \rho_d / \pi \). Thus, for a lossless Lambertian reflector, \( \rho' = 1/\pi \) in a plot of the BDR. For the MAP reflectors, the reflection is diffuse and incoherent for \( \lambda < 2 \mu m \), and thus the specular component may be ignored. An example is shown in Figure 9. The incoherent scattering falls as \( (c/\lambda)^2 \approx 10^{-4} \) at microwave wavelengths and so is negligible and has no effect on the beams.

Fig. 8.—Left: Theoretical emissivity of a thin aluminum coating as a function of the thickness of the coating. The dip in the curve is an interference phenomenon (Born & Wolfe 1980, p. 628). At small coating thicknesses, the results differ from those in Xu et al. (1996) and give a somewhat smaller thickness at the minimum than Garg, Gupta, & Sharan (1975), who find \( t/\delta \approx \pi/2 \), where \( \delta \) is the skin depth. The variation in emissivity with incident angle is not significant. The triple-dot–dashed line is for the TE mode (E perpendicular to plane of incidence) with a 30° incident angle. The TM mode will be higher than the nominal value. The effects of the composite substrate are negligible (e.g., Ramo & Whinnery 1953, p. 249). Right: Emissivity of the surface from optical to microwave wavelengths. The aluminum emissivity is based on the Drude model (solid straight line; e.g., Ashcroft & Mermin 1976) and measurements (curve with a peak at 1 \( \mu m \)). The visible band is indicated by a bar at an emissivity of \( e = 0.2 \), just below the peak of a 6000 K blackbody representing the Sun. The jagged line at 10 \( \mu m \) is the emissivity of SiO2, and the triple-dot–dashed line is a model of its long-wavelength behavior. The vertical bands on the right are the MAP bands.

Fig. 9.—Left: Model incoherent BDR (higher peak) and the measured BDR for primary P3. The model is from eq. (16), with \( \sigma = 0.4 \mu m \), \( c = 30 \mu m \), \( \lambda = 0.5 \mu m \), and \( \psi = 20^\circ \), scaled by 0.09 to match a typical DR. This approximation misses the wings but gets the peak sufficiently well. For the surface upon which the model is based, 13% of the reflected energy is inside a 20° cone; for P3 this quantity is 11%. The dashed line at 1/\( \pi \) is for a lossless Lambertian surface. Right: Distribution of power in the focal plane after reflecting off of both reflectors.
2.8.2. The SiO$_2$ Coating and Optics Temperatures

Because of its relatively good absorptance at 1 $\mu$m (Fig. 8), near the peak of the solar spectrum, aluminum gets hot in the Sun. For a flat plate normal to the Sun that emits only on the illuminated face, the radiative steady state temperature is

$$T = \left( \frac{\sigma_{\text{sol}} F_{\text{sol}}}{\epsilon_{\text{hem}} \sigma_B} \right)^{1/4} = 398 \left( \frac{\sigma_{\text{sol}}}{\epsilon_{\text{hem}}} \right)^{1/4} \text{K} \approx 580 \text{ K}, \quad (17)$$

where $\sigma = 5.67 \times 10^{-8}$ W m$^{-2}$ K$^{-4}$, $\sigma_{\text{sol}} \approx 0.085$ is the solar absorptance for optical quality aluminum at room temperature, $\epsilon_{\text{hem}} \approx 0.018$ is the total hemispherical emittance (coefficient for the total emitted flux into $2\pi$ sr) for aluminum at room temperature, and the maximum solar flux (at perihelion) is $F_{\text{sol}} = 1420$ W m$^{-2}$ near the Earth. This temperature is well above the glass transition temperature of the FMT73 epoxy in the reflectors, 370 K. To cool the reflectors, they are coated with $\approx 2 \mu$m of SiO$_2$, which emits strongly near 10 $\mu$m as shown in Figure 8 (Hass et al. 1969). This results in $\alpha/\epsilon \approx 0.8$ and so $T = 376$ K for extended direct illumination. Because the SiO$_2$ is so thin, it has no effect on the microwave properties of the reflectors (Mon & Sievers 1975).

The secondaries see focused radiation from the Sun, which would increase the flux on them to $\approx 10 F_{\text{sol}}$ were the primaries perfect reflectors. However, the sunlight is diffused and absorbed by the primaries, which substantially decreases the flux on the secondaries. To ensure that the secondaries do not exceed the glass transition, the flight optics were illuminated with a stage spotlight and the flux at the position of the secondary was measured. After correcting for the geometry and spectral difference between the test light and the Sun, the data were compared to a computer model of the scattering that also predicted the Sun’s net effect. The computer output was then input to a full thermal model of the secondary and S/C that accounted for the rotation of the S/C during solar exposure. The worst-case transient temperature predictions were within 10 K of the glass transition temperature. This margin was considered sufficient. As a guideline to the full model, the temperature of the secondary is approximately given by

$$T_{\text{sec}} = \left( \frac{F_{\text{inc}}}{\sigma_B} \right)^{1/4} \left( \frac{F_{\text{sol}}}{\sigma_{\text{hem}}} \right)^{1/4} f_{20} \left( \frac{\alpha_{\text{sol}}}{\epsilon_{\text{hem}}} \right)^{1/4} \text{K} \approx 507 \text{ K}, \quad (18)$$

with $F_{\text{inc}} = (140 \text{ cm}/45 \text{ cm})^2 (1 - \alpha_{\text{min}}) F_{\text{sol}} (13^\circ/20^\circ)^2 f_{20}$, where $(140 \text{ cm}/45 \text{ cm})^2$ is the ratio of the spot size on the primary to that on the secondary, $\alpha_{\text{min}} \approx 0.4$, $(13^\circ/20^\circ)^2$ is the ratio of the angle of the solar spot on the secondary as viewed from the primary to the $20^\circ$ reference angle, and $f_{20}$ is the flux within $20^\circ$ as quantified with equation (16). The specified values for the secondaries were $f_{20} = 0.2$ and $\alpha/\epsilon < 0.9$, resulting in $T_{\text{sec}} \approx 330$ K and a flux of $\approx F_{\text{sol}}/2$. These values are close to those of the full model. A spare secondary was tested with a solar simulation and was found to be able to survive temperatures of $115^\circ$C for periods of $\approx 2$ minutes.

The diameter of the solar image in the focal plane is 2.2 cm. With perfect reflectors, the flux would be $\approx 2600 F_{\text{sol}}$ and would vaporize the aluminum feeds. To estimate the actual flux, equation (16) is convolved with itself after accounting for the geometry and absorptance of the reflectors. The result is shown in Figure 9. With a maximum flux of $\approx F_{\text{sol}}/4$, a simple model of the conductance and emittance of the feeds shows that they should not exceed $40^\circ$C in space. Measurements with the spotlight were consistent with the calculation.

2.8.3. Microwave Properties of the Aluminum Coating

Aluminum was chosen for the metallic reflector coating because its deposition is well studied and its microwave emissivity is sufficiently low for MAP. For all emissivity calculations, the classical regime obtains: there are no anomalous effects (Pippard 1947), and the displacement current can be ignored. Specifically, $\epsilon = 4\pi x/\lambda = (16\pi e_0 f/\sigma)^{1/2}$. Thus, the emissivity scales as $f^{1/2}$. At 70 K the emissivity at 100 GHz is about 0.0004 ($\sigma_{300K}/\sigma_{70K} = 0.15$). For two surfaces (primary and secondary) at 70 K, the contribution to the system temperature from the thermal emission is 64 mK at 100 GHz, which is negligible.

Butler (1998) made a precise room temperature measurement of the difference in emissivity between machined stainless steel, copper, aluminum, and a vacuum-deposited aluminum (VDA) sample with an SiO$_2$ coating. The sample has the same construction as the MAP reflectors except that it is flat. Butler found the differences in emissivity between all materials and the VDA sample to be within 20% of the value computed assuming a conductance of bulk aluminum for the sample. In other words, the VDA coating acts like bulk aluminum at microwave frequencies. Numerous coating samples were checked. No degradation in the microwave properties was observed over a period of a year. No change was seen after thermal cycling to 77 K.

To ensure the similarity of the reflectors, the coating procedure specified that the same prescription be used for symmetric pairs of reflectors. From Figure 8, one sees that with $t > 1.2 \mu$m, the bulk emissivity should be obtained. To be insensitive to any small variations in the thickness, small defects in the composite surface, or interactions with the composite substrate, a thickness of $t > 2.2 \mu$m was specified. This is many times the 90 GHz skin depth of 0.11 $\mu$m at 70 K.

The calculation for Figure 8 is based on the emissivity of a single thin sheet of aluminum at normal incidence. One can be more ambitious and include the XN70 substrate, but the results are essentially unchanged: the net emissivity is dominated by the outermost layer of aluminum. The dominant effect in determining the emissivity is the impedance mismatch between the vacuum and the aluminum, for which the index $n_{\text{Al}} \approx 2000 + i2000$ at 90 GHz.

2.8.4. Spacecraft Charging

Surface charging is a well-known and potentially serious problem for spacecraft. In short, current from the ambient plasma can charge the external surfaces of the S/C to very high potentials (>10 kV) relative to space or to other spacecraft surfaces. These charged surfaces are subject to abrupt discharge either to a spacecraft surface at a different potential or to space itself. Such high-potential discharges can wreak havoc on the electronics or damage sensitive surfaces. The problem is complicated and empirical. The physical processes include photoionization, electron and proton current densities, secondary electron densities produced by collisions with the S/C, space charge regions, etc. (Jursa 198517).
The surface charging is large when the spacecraft is between 6 and 10 Earth radii (geosynchronous orbit is 5.6 $R_E$) and when there is high geomagnetic activity. Current flow from photoionization by the Sun tends to lower the magnitude of charging from the ambient plasma. Thus, a classic scenario for a discharge event would be entry into (or emergence from) eclipse into sunlight. It is believed to be possible to reach $-25,000$ V (relative to infinity), although $-5000$ V is more typical. The plasma energies are typically about 10 keV, and the currents into the spacecraft are of order 1 nA cm$^{-2}$. The L2 environment ($\approx 240 R_E$) is affected by the interaction of the Earth’s magnetic field and the solar wind (Evans et al. 2002$^{18}$). As the orientation between the Earth and Sun changes, L2 moves through different charging environments, the most dangerous of which is believed to be the magnetosheath. It is possible that $A_e$ is required or used in the analysis.

If the SiO$_2$ is too thick, it insulates the primary. The NAS-CAP program (Jursa 1985) indicates that a 2.5 $\mu$m layer charges to approximately $-160$ V. Tests made on a 150 K sample surface with charged contacts and an electron beam showed that the surface would not abruptly arc with up to 450 V and currents $\approx 10$ nA cm$^{-2}$. The surface instead discharges in a self-limiting manner.

3. PERFORMANCE AND CHARACTERIZATION OF OPTICAL DESIGN

$MAP$’s radiometric sensitivity is a result of intrinsically low noise transistors in an HEMT amplifier that operates over roughly a 20% bandwidth (Pospieszalski et al. 2000; Jarosik et al. 2002). The characterization of the optical system takes this bandwidth into consideration. In radio astronomy parlance, $MAP$ is a “continuum” receiver. $MAP$ is characterized primarily in flight through measurements of the CMB dipole, planets, and radio sources. Because of the wide bandwidth, sources with different spectra have different effective frequencies and beam sizes, even within one band.

Two fundamental assumptions about celestial sources are made that allow one to characterize the response as an integral of the response at each frequency: (1) signals received at different frequencies are incoherent, and (2) signals received from different points on a source are incoherent (Thompson, Moran, & Swenson 1986).

In a perfectly balanced differencing assembly (DA), the output of any one detector switches at 2.5 kHz from being proportional to the power entering side A to the power entering side B. The A – B response is synchronously demodulated, integrated for 25.6 ms, and averaged. With the 2.5 kHz phase switch in one position, the power delivered to a diode detector, from one polarization of one feed when viewing a source of surface brightness $S_x$, is given by

$$W(\alpha, \beta, \gamma) = \frac{1}{2} \int \alpha_x(\nu) \eta_R(\nu) A_x(\nu) f(\nu) S_x(\Omega) \times B_x(\nu, R_{\alpha, \beta, \gamma}) d\Omega d\nu,$$

where $f(\nu)$ is the normalized bandpass of a DA at a reference plane at the feed and antenna losses are included in the radiation efficiency $\eta_R$. The normalized beam power pattern, $B_x(\nu, \Omega)$, and the effective collecting area of the antenna at normal incidence, $A_x(\nu)$, have been expressed separately. Where appropriate in the following, their product is written as $A_{eb}(\nu, \Omega)$. $S_x(\Omega)$ is a surface brightness with units of W m$^{-2}$ sr$^{-1}$ Hz$^{-1}$ and is defined with respect to a fixed coordinate system. The beam is measured in its own coordinates, and $R_{\alpha, \beta, \gamma}$ is the matrix that specifies the beam position and orientation on the sky.

The gain of the optics is measured with a standard gain horn at the GSFC/GEMAC facility. The measurement accuracy is $\pm 0.2$ dbi. The peak gain is measured at 500 frequencies across the band. Full beam maps are made at 12 frequencies per band. The outer two measurements are approximately 10% beyond the nominal passband, two more are at the band edges, and the remaining eight are equally distributed across the band. No phase information is required or used in the analysis.

The loss in the system comes from the optics, $\eta_R$, and the radiometer chain, $\alpha_x$. In practice, the overall level of both of these is calibrated out, and so only their frequency dependence affects the characterization. The dependence of $\alpha_x$ is accounted for in the measurement of $f(\nu)$, and so it is dropped in the following. The dependence on $\eta_R(\nu)$, which is expected to be small, is found by calculation, and so it is retained. For the values in the tables, we take $\eta_R = 1$ across the passband.

3.1. Response to Broadband Sources

There are a number of possible definitions of the effective area, gain, frequency, and bandwidth. We choose ones that reduce naturally to those for a thermal emitter. To this end, a source is modeled as

$$S_x(\Omega) = \sigma(\nu) S(\Omega),$$

where $\sigma(\nu)$ is $\propto \nu^{-0.7}$ for synchrotron emission, $\propto \nu^{-1.4}$ for bremsstrahlung or free-free emission, $\propto \nu^{-2}$ for Rayleigh-Jeans emission, $\propto \nu^{-3.3}$ for dust emission, and a Planck black-body at 2.725 K.$^{19}$ The total power received is

$$W(\alpha, \beta, \gamma) = \frac{1}{2} \int A_{eb}(R_{\alpha, \beta, \gamma}) S(\Omega) d\Omega,$$

where the broadband effective area is defined as

$$A_{eb}(\Omega) = \frac{\int \eta_R(\nu) f(\nu) \sigma(\nu) A_{eb}(\nu, \Omega) d\nu}{\int \eta_R(\nu) f(\nu) \sigma(\nu) d\nu},$$

and the frequency-weighted source function is defined as

$$S(\Omega) \equiv S(\nu) \int \eta_R(\nu) f(\nu) \sigma(\nu) d\nu W m^{-2} sr^{-1}.$$

The effective area is never measured and is only defined through equation (22). If the source is a spatially uniform Rayleigh-Jeans emitter at temperature $T$, then $\sigma(\nu) = 2\nu^2 k_B/\epsilon^2$ and with a flat passband

$$W = (k_B T / \epsilon^2) \int \nu^2 A_{eb}(\nu, \Omega) d\Omega d\nu = k_B T A(\nu).$$

$^{18}$ Available at http://www.ngst.nasa.gov/public/unconfigured/doc_0761/rev_01.

$^{19}$ The effective temperature scaling is obtained from $\sigma(\nu)$ by subtracting 2 from the exponent.
The broadside gain is defined as
\[
g_{bb}^{bb} (\Omega) = \frac{4\pi A_{e}^{bb} (\Omega)}{\int A_{e}^{bb} (\Omega) d\Omega} = \frac{\int \eta_{R}(\nu) f(\nu) \sigma(\nu) g(\nu, \Omega) / \nu^2 d\nu}{\int \eta_{R}(\nu) f(\nu) \sigma(\nu) / \nu^2 d\nu}
\] (25)

where \( g(\nu, \Omega) \) is the quantity given by computer models of a lossless system. Any measurement includes an error in the calibration, \( \pm 0.2 \, \text{dB} \), and antenna loss \( \eta_{R}(\nu) \), although the reflector loss is of marginal importance even in \( W \)-band. Generally, it is found that the measured forward gain \( g(\nu) \) is distinct from the noise passband (Dicke 1946), \( D \) is a constant, and \( g_{bb}^{bb} \) is independent of frequency \( \nu \) and is approximately the signature of the free-free emission. Such a source has the same approximate antenna temperature in each band.

Implicit in this definition and others in which \( A_{e} \) explicitly appears is that the source is smaller than the beam. If a source fills \( 4\pi \, \text{sr} \), then there would be no dependence on the beam solid angle. This is the reason that the numbers reported here are different than those in Jarosik et al. (2002). The effect is that as one probes higher \( l \), the effective center frequency, and therefore the Rayleigh-Jeans–thermodynamic correction, changes.

For estimating antenna temperatures and power levels, the effective bandwidth is convenient. It is defined

\[
\Delta \nu_{e} = \left[ \frac{12}{\nu_{e}} \left( \frac{\int f(\nu) \eta_{R}(\nu) A_{e}(\nu) \sigma(\nu) (\nu - \nu_{e})^2 d\nu}{\int f(\nu) \eta_{R}(\nu) A_{e}(\nu) \sigma(\nu) d\nu} \right) \right]^{1/2}
\] (27)

The factor of 12 makes a flat bandpass have a width of \( \Delta \nu_{e} \). This quantity does not enter into any calculation and is distinct from the noise passband (Dicke 1946), \( \Delta \nu_{e} \), quoted in Jarosik et al. (2002).

The flux from pointlike radio sources is given as \( F_{e} = \int_{\Omega} S_{e}(\Omega) d\Omega \), where \( F_{e} \) is measured in Jy (10^{-26} \, \text{W} \, \text{m}^{-2} \, \text{Hz}^{-1}) \). For a narrow frequency band,

\[
T_{A} \Delta \nu = \frac{1}{2K_{B}} \int \frac{g_{n}(\nu) c^{2}}{4\pi \nu^2} F_{e} d\nu = \frac{A_{e}}{2K_{B}} F_{e} \Delta \nu
\] (28)

The usual notation is \( T_{A} = \Gamma F_{e} \). In the literature, the flux is modeled as \( F_{e} \propto (\nu / \nu_{e})^{\alpha} \), where \( \alpha = 2 \) for a Rayleigh-Jeans emitter and \( \nu_{e} \) is given by equation (26). Since \( \Gamma \) depends on the illumination of the primary, it will be similar for all bands. For a “flat spectrum” source, \( F_{e} \) is independent of frequency \( (\alpha = 0) \) and has approximately the signature of the free-free emission. Such a source has the same approximate antenna temperature in each band.

Implicit in the above is that \( B_{e} = 1 \) at all frequencies. In other words, there is no dependence on the beam. The broadband conversion factor is given by

\[
\Gamma_{bb}^{bb} = \frac{(c^2/8\pi k_{B} T_{e}^{3/2}) \int f(\nu) \eta_{R}(\nu) G_{m}(\nu) \nu^{\alpha-2} d\nu}{\int f(\nu) d\nu}
\] (29)

From the 12 beam measurements the beam characteristics for each source are computed. For the CMB, the predictions of \( g_{bb}^{bb} \) are shown in Figure 10 for what is expected at L2. Table 8 shows how the effective broadband quantities depend on the source. The actual flight values will be different. The ellipsoidal shape of the low-frequency bands results from their large distance from the optimal focal position. The MAP scan strategy has the effect of symmetrizing the beams, mitigating some of the effects of asymmetric beams.

The small lobes in the cross pattern on the A-side and B-side window functions adjusted for rotational smearing. From left to right on the plot, the windows correspond to \( K \) through \( W \) “lower” and then the \( W \) “upper” beams.

Fig. 10.—Left: Predicted beam profiles for cooled and distorted optics for a thermal source. If the beams could illuminate the sky, this is the pattern one would see projected on the sky as viewed from the spacecraft. If one looks at the B-side focal plane array, as shown in Bennett et al. (2002), the corresponding feeds are left-right reversed. The contours are 0.9, 0.6, 0.3, 0.09, etc., the beam maximum. The small lobes off the \( W \)-band beams are the result of the reflector distortions, which are greater on the B side than the A side. The lowest contour in the plot is at 0.03. With the measurements combined with the computer code, the beams may be modeled to subpercent accuracy. The beams in flight will be additionally smeared as a result of rotation of the satellite. Right: Predicted B-side window functions adjusted for rotational smearing. From left to right on the plot, the windows correspond to \( K \) through \( W \) “lower” and then the \( W \) “upper” beams.
Typical Preflight Effective Center Frequencies, Passbands, and Gains for Ideal Beams

| Parameter                        | K  | K₁ | Q  | V' | W₁| W₂| W₃| W₄| W₅| W₆ |
|----------------------------------|----|----|----|----|----|----|----|----|----|----|
| Frequency range (GHz)            | 20–25 | 28–37 | 38–46 | 53–69 | 82–106 | 82–106 |
| Noise bandwidth, Δν (GHz)        | 5.2 | 6.9 | 8.1 | 10.5 | 19.0 | 16.5 |
| Percent Band                     | 22.6 | 20.6 | 19.6 | 17.2 | 20.2 | 17.6 |

Synchrotron: Sₜ ∝ ν⁻⁰.⁷

| ν (GHz) | Δν (GHz) | Γ (μK Jy⁻¹) |
|---------|---------|-------------|
| 22.4    | 33.6    | 40.5        |
| 5.7     | 7.2     | 8.5         |
| 22.3    | 22.3    | 22.3        |

Free-free: Sₜ ∝ ν⁻¹.₃

| ν (GHz) | Δν (GHz) | Γ (μK Jy⁻¹) |
|---------|---------|-------------|
| 22.5    | 32.7    | 40.6        |
| 5.7     | 7.2     | 8.5         |
| 22.2    | 22.2    | 22.2        |

Rayleigh-Jeans: Sₜ ∝ ν²⁻³

| ν (GHz) | Δν (GHz) | Γ (μK Jy⁻¹) |
|---------|---------|-------------|
| 23.1    | 33.1    | 41.1        |
| 5.5     | 7.0     | 8.3         |
| 22.9    | 22.9    | 22.9        |

Dust: Sₜ ∝ ν⁻¹.⁵

| ν (GHz) | Δν (GHz) | Γ (μK Jy⁻¹) |
|---------|---------|-------------|
| 23.1    | 33.1    | 41.1        |
| 5.2     | 6.9     | 8.1         |
| 22.5    | 22.5    | 22.5        |

| Parameter | K  | K₁ | Q  | V' | W₁| W₂| W₃| W₄| W₅| W₆ |
|-----------|----|----|----|----|----|----|----|----|----|----|
| Gaussian width of σₖ = θₖ / θₖ' / [8 ln(2)]¹/². The variance of the time stream one would measure at the output of a noiseless detector as the beam scans the sky is |

\[ C(0) \approx \sum_l \frac{(2l+1)C_l}{4\pi} W_l, \quad (31) \]

where \( W_l \) is the window function that encodes the beam smoothing. In practice, one works with maps in which each pixel has been traversed in multiple directions by an asymmetric beam. Then, one determines the variance of the ensemble of pixels as a function of \( l \) and \( Δl \). The full window function for a separation of the beam centroids of \( θ_{12} \) is |

\[ W_l \approx \frac{1}{Δl} \int \int P_l(\cos θ_{12}) B_n(R, Ω_1, Ω_2) dΩ_1 dΩ_2, \quad (32) \]

where \( R \) gives the orientation of the beams. The full expression with orientable asymmetric beams has been used by Cheng et al. (1994), Netterfield et al. (1997), Wu et al. (2001), and Souradeep & Ratra (2001). For the zero-lag window with symmetric beams, \( W_l = B_l^2 / Ω_λ^2 \), where \( B_l = 2\pi \int B_l(\theta) P_l(\cos θ) d(\cos θ) \) and \( P_l \) are the Legendre polynomials. If the beams are Gaussian, \( W_l = \exp[-l(l+1)/σ_λ^2] \). On the right-hand side of Figure 10 are shown the window functions for the B-side beams after they have been quantified with both models and in-flight beam maps. The flight values will be different from those shown and will be quantified with both models and in-flight beam maps.

3.3. Practical Issues

MAP makes differential measurements and thus measures the difference in power from radiation received from opposite sides of the spacecraft. The relevant quantity from which the maps are derived is |

\[ W_{D} \approx \frac{1}{Δl} \int \int \frac{α_1}{Δl} A_{e,A}(ν) f_{A}(ν) S_{ν}(Ω_1 - Ω) B_{n,A}(ν, Ω_1, Ω) \quad (33) \]
for two different directions \( \Omega_1 \) and \( \Omega_2 \) for the “A” and “B” sides, respectively. Even if \( \Omega_1 \) and \( \Omega_2 \) are switched (the A side points to where the B side was), \( f_A = f_B, \alpha_A = \alpha_B \) (the phase switch is in the same position and the hybrids, waveguides, and OMT are matched), and \( A_{c,A} = A_{c,B}, W_D \) is not zero. This is because \( B_{n,A} \) and \( B_{n,B} \) are not the same. The differential pairs are back to back and far from the ideal focus. In reality, \( f_A \neq f_B, \alpha_A \neq \alpha_B \), and \( A_{c,A} \neq A_{c,B} \) although to first order the small differences will be calibrated out. In the end, however, the different beams will have to be taken care of in the analysis. Because Jupiter is so bright, each of the 40 beam profiles will be independently mapped.

### 4. SYSTEMATIC EFFECTS

Systematic effects associated with the optics such as beam size, sidelobe levels, and the effective frequency directly affect the scientific interpretation of the data. In this section, systematic effects associated with the stability and integrity of the optical system are discussed.

The largest systematic effects are associated with infrared and microwave emission from the Sun, Earth, and Moon, whose properties, as viewed from L2, are summarized in Table 9.

#### 4.1. Thermal Variation of Optics

The S/C is oriented so that the angle between the Sun and the symmetry axis is constant during the spin and precession. Thus, the thermal loading remains constant for long periods. The radiators are designed so that they are 2\( ^\circ \)2 inside the Sun’s shadow at all times. The reflectors too are greater than 6\( ^\circ \) inside the Sun’s shadow. Infrared emission from the Earth and the Moon, however, can directly illuminate the primary as can be seen in Figure 1. In the following, conservative order-of-magnitude estimates are made of the anticipated radiometric signal as a result of heating of the primary.

The primary is a complicated composite structure with multiple thermal time constants and can only roughly be treated in isolation. An input of thermal power, \( \Gamma_1 \), heats up the mirror. The energy in turn is immediately reradiated and flows to other parts of the S/C where it is eventually reradiated. The energy flow for both mechanisms is modeled as

\[
C_M \frac{dT}{dt} + \Phi^h(T - T_0) = \Gamma^1,
\]

where \( C_M \) is the heat capacity and \( \Phi^h \) is the thermal conductance. The solution to equation (34) with a step function change in incident power is

\[
T = T_0 + \frac{\Gamma^1(1 - e^{-\tau/T_0})}{\Phi^h},
\]

where \( \tau = C_M/\Phi^h \).

The dominant conductive path is through the 0.0254 cm thick composite skin of the reflector. The thermal conductivity of the XN70 is \( \kappa_s = 0.19 \text{ W cm}^{-1} \text{ K}^{-1} \) at 70 K, more than twice that of stainless steel. The composite’s density is \( \rho_s = 1.8 \text{ g cm}^{-2} \), and the specific heat capacity is \( c_s = 200 \text{ J kg}^{-1} \text{ K}^{-1} \).

The characteristic time for a heat pulse to propagate \( t_0 = 20 \text{ cm} \) is \( \tau_{cond} = l_0^2 \rho_s c_s / \pi^2 \kappa_s^2 \approx 80 \text{ s} \), although the full solution is a series expansion in time constants and depends on geometry (Hildebrand 1976).

The emitted radiation, for small temperature variations, is linearized so that \( \delta T_{rad} = 4 \epsilon \Phi \sigma T_0^4 \delta T \). The effective radiating area is the face of the primary with \( A_{rad} = 1.76 \text{ m}^2 \) for which the emissivity is taken as \( \epsilon = 0.1 \). At \( \lambda = 30 \mu \text{m} \) and \( 300 \text{ K}, \epsilon = 0.5 \). The conductance is given by \( g_{rad}^h = 4 \epsilon \Phi \sigma T_0^4 A_{rad} = 0.015 \text{ W K}^{-1} \) at 70 K. The heat capacity is difficult to estimate and depends on the coupling between the reflector surface and its backing structure. If the whole 5 kg reflector heats up, then \( c_s \approx 250 \text{ J kg}^{-1} \text{ K}^{-1} \), the heat capacity of a typical composite, and \( C_{phys} \approx 1000 \text{ K}^{-1} \). If just the thin surface heats up, then \( C_M \approx 160 \text{ J K}^{-1} \), leading to the range 3 hr < \( \tau_{rad} < 20 \text{ hr} \). Because \( \tau_{cond} \gg \tau_{rad} \), to a good approximation the primary first thermalizes and then radiates.

To estimate \( \Gamma_1 = \epsilon \Phi \Phi_{inc} A_{dir} \), it is assumed that the Earth (Moon) illuminates \( A_2 \approx 0.15 \text{ m}^2 \) ( \( A_2 \approx 1 \text{ m}^2 \) ) with an absorptivity of \( \epsilon_{inc} \approx 0.1 \). For the Earth \( \Gamma_E = 1.3 \times 10^{-4} \text{ W} \), and for the Moon \( \Gamma_M = 3 \times 10^{-5} \text{ W} \). The temperature increase for a long exposure (\( \tau_{rad} \)) is \( \Gamma_1/\Phi_{rad}^h \). For the Earth this is \( \Delta T_{phys} \approx 9 \text{ mK} \), and for the Moon it is \( \Delta T_{phys} \approx 2 \text{ mK} \). The precise illumination depends on the orbit, spacecraft orientation, and blockage of the radiation by the secondary, as shown in Figure 1, and so these estimates should be considered conservative upper limits.

The rotation period of the satellite is 132 s. The reflectors have a good view of the Earth and Moon over about a 45° range, or for about \( \tau_{obs} = 15 \text{ s} \). In the limit that the heat is first conducted away from the heated area and later reradiated, equation (35) gives a temperature change of \( \Delta T_{phys} = 1.5 \text{ mK} \) for the Earth and \( \Delta T_{phys} = 0.35 \text{ mK} \) for the Moon. The radiometric signal is proportional to the thermal variation multiplied by the microwave emissivity integrated over the beam response. In \( W \) band, the optical response near the top of the primary, where the Earth illuminates it, is approximately −5 dB of the peak, and so the expected radiometric signal is \( \approx 0.5 \mu \text{K} \). The Moon illuminates the central part of the primary, and so the expected radiometric signal is \( \approx 0.4 \mu \text{K} \). The primaries, secondaries, and radiators are instrumented with platinum resistive thermometers (PRTs) to detect a 0.5 mK change in one rotation. These measurements and a detailed thermal model will be used to model the on-orbit performance.

| Parameter for Sun, Earth, and Moon, from L2 | Sun | Earth | Moon |
|-------------------------------------------|-----|-------|------|
| Temperature, \( T_1 \) (K)……………….. | 6000 | 300 | 250 |
| Distance from L2, \( d_1 \) (m)………….. | \( 1.5 \times 10^4 \) | \( 1.5 \times 10^6 \) | \( 1.5 \times 10^9 \) |
| Radius, \( r_0 \) (m)………………… | \( 7 \times 10^7 \) | \( 6.4 \times 10^9 \) | \( 1.7 \times 10^{10} \) |
| \( \Phi^h \) flux at L2 (W m\^2)……….. | 1600 | 8.5 \times 10^{-3} | 2.5 \times 10^{-4} |
| Solid angle from L2 (sr)……………….. | \( 6 \times 10^{-5} \) | \( 6 \times 10^{-5} \) | \( 5.3 \times 10^{-6} \) |
| \( \Phi_{inc}^h \) in \( \Delta \nu = 24 \text{ GHz} \^b \) | \( 2.4 \times 10^{-8} \) | \( 1.0 \times 10^{-9} \) | \( 1.1 \times 10^{-10} \) |
| \( T_{rad} \) (mK)………………… | 130 | 5.5 | 0.6 |

\( ^a f = g_{rad} \Phi^h (r_1/d_1)^2 \).

\( ^b \) For radiometric estimates, the flux in a microwave frequency band is useful. It is given by \( I_{inc} = 2\pi^2 k_B T_2 (r_1/d_1)^2 \Delta \nu/c^2 \). The values are for 90 GHz.

\( ^c \) The effective temperature is \( T_{eff} = T_2 (r_1/d_1)^2 \).
The estimates above assume the same emissivity on both sides but variable thermal loading. If the microwave emissivity of the two reflector systems is different and the temperatures of both move up and down identically, then a signal will result. If the temperatures of the reflectors change together by 10 mK, which is conservative but not impossible, then the differential signal changes by 1 µK if δc/e = 0.2.

Finally, because the current distributions from each feed overlap on the primary, as shown in Figure 2, temperature and emissivity gradients will have a common-mode effect on all channels. This will aid in characterizing any variations in the optics.

4.1.1. Emission From a “Dirty” Surface

The reflector surfaces are specified to be “visibly clean.” In contamination engineering, a visibly clean surface has 500 < l_c < 700, where l_c is the “cleanliness level.” For example, a surface with one 3 × 30 µm needle per square millimeter (0.01% obscuration) results in l_c = 400. If the particles are spherical with the same fractional obscuration, l_c = 250. These surfaces would pass as visibly clean. On the other hand, a surface covered with 20 × 100 µm needles with a 2% surface obscuration results in l < 1000 and is clearly not visibly clean. This corresponds to 1 g of graphite spread over the surface. Generally, the unaided eye can detect 50 µm particles.

During the integration and prelaunch phase, the optical surfaces are constantly purged and kept clean. However, the fairing separation soon after launch can produce a cloud of debris. Even though the MAP payload fairing was specially cleaned and inspected to minimize the possibility of contamination, it was estimated that there could be 1 g of contaminating material and one 1 × 5 cm² piece of fairing tape within the area of one reflector. The energetics and geometry of the separation strongly disfavor much, if any, of this material sticking to the primary. Of the possible contaminants shown in Table 10, graphite is the most pernicious.

Surface dust or debris increases the microwave emissivity, possibly resulting in a radiometric signal. The magnitude of the signal depends on the material’s coupling to the surface. For a piece of 1 × 5 cm² 0.05 cm thick Mylar tape, the most pessimistic case occurs when the contaminant is attached at the center of the primary in a way such that its response to temperature variations is instantaneous and its emissivity is unity in the infrared and ε_eff = 0.03 at 90 GHz, as shown in Table 10. The maximum radiometric temperature will be

\[ \Delta T_{rad} = \epsilon_{eff} \Delta T_{object} \frac{A_{object}}{A_d} \approx 0.15 \mu K, \]

where \( A_d \sim 10^4 \text{ cm}^2 \) is the illumination area of the beam and \( T_{object} \sim 10 \text{ mK} \).

Although there will be some graphite in the mix of debris, there is far less than 1 g. If, however, l_c 20 × 100 µm needles discussed above is distributed over the surface, there are roughly 10 particles mm⁻². Needles close to the reflector will follow the surface temperature. Since they are much smaller than a wavelength and in the node of the electric field, their effective emissivity is small. The power in the standing wave as a function of distance z from the surface is proportional to \( \sin(4\pi z/\lambda) \). A 20 µm diameter grain at 90 GHz on the surface sees a reduction in power of \( \approx 10^{-3} \) after accounting for the radiation reflected to the scatterer from the mirror. Thus, we assume ε_eff ≈ 0.001. The emission temperature is then ε_eff T_{phys}/A_{object}/A_d ≈ 0.02 µK.

We can never be certain of how much contamination ends up on the surface. The effective emission temperatures for large pieces of material and for graphite grains are conservative upper limits. They are meant to show MAP’s immunity to contamination.

4.2. Thermal Variation of the Radiators

Sun light diffraacts over the edge of the solar shield and illuminates the radiator at a very low level. As the S/C spins, this term is modulated, leading to a temperature variation of the radiator and in turn the HEMT amplifiers. Using GTD around the edge of the solar shield, a conservative estimate shows that no more than 0.1 W could be absorbed by the radiator. The heat capacity of a radiator panel is C_r = 2500 J K⁻¹, and so the temperature variation is \( \approx 0.1 \text{ W} C_r^{-1} T_{obs} = 50 \mu K \) for T_{obs} = 1 s. With an emissivity of less than 1%, the maximum radiometric signal is \( \approx 1 \mu K \) for perfect radiometric coupling. Because the heat is injected at

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**TABLE 10**

POSSIBLE CONTAMINANTS AT 90 GHz

| Potential Contaminant | ε | Re(ε) | Im(ε) | α(cm⁻¹) | Source |
|-----------------------|---|-------|-------|---------|--------|
| Graphite              | ε = 700 + i(4.8 × 10⁻¹µ⁻¹ + 1.5 × 10⁻¹/ν) | 700 | 170000 | ... | 1 |
| Eccosorb CR110        | ε ≈ 3.53 + i0.48 µ⁻¹ | 3.53 | ... | 2.0 | 2 |
| Ice                   | ε ≈ 3.155 + i(0.5 + 1.7 × 10⁻¹/ν) | 3.155 | 0.026 | ... | 3 |
| Mylar                 | ε ≈ 2.812 + i(0.15 + 1.7 × 10⁻¹/ν) | 2.8 | 0.17 | 0.52 | 4 |
| Silicates             | ε ≈ 11.8 + i(0.1 × 10⁻¹/ν) | 11.8 | 0.1 | ... | 1 |
| SiO₂                  | ε ≈ 3.8 + i9 × 10⁻¹/ν | 3.8 | 0.8 | 0.003 | 5 |

---

a For all expressions in this table, ν is in GHz. The absorption coefficient at 70 K, α, is generally within 10% of that at 290 K. The material properties are quantified with the complex dielectric constant (Jackson 1999; Ramo & Whinnery 1953).

b For a good though lossy or imperfect conductor, α = 1/δ, where δ = 1/(πµν) is the skin depth. Thus, α is expected to increase as ν⁻¹/². The dielectric constant is given by ε = 1 + iα/(2πν). Graphite has both a metallic and interband contribution.

c Eccosorb CR110 is a castable resin microwave absorber made by Emerson Cumming. NS43G/Hincom paint chips could have an absorptance as high as this.

d For imperfect dielectrics, α = σνIm(ε)/εRe(ε) as long as the conduction currents are much less than the displacement current. Sometimes the loss tangent is quoted, tan δ = Im(ε)/Re(ε).

REFERENCES.—(1) Draine & Lee 1984. (2) Halpern 1986. (3) Koh 1997. (4) Page et al. 1994. (5) Mon & Sievers 1975.
the outer edge of the radiator, far from the optics, and is partially reradiated, the resulting radiometric signal is \( \ll 1 \, \mu K \).

4.3. Scattering of Light from the Sun, Earth, and Moon by Contamination on Primary

Microwave power from the Moon and Earth can scatter off debris on the reflector and potentially produce a glint as the S/C rotates. The form of the contamination is not known, and so several possibilities are considered. The differential cross section (van de Hulst 1981; Landau & Lifshitz 1984) for the scattering of unpolarized incident light from an isotropic medium is

\[
\frac{d\sigma}{d\Omega} = \left( \frac{2\pi\nu}{c} \right)^4 \left| \alpha_p \right|^2 \frac{1 + \cos^2 \theta_s}{2},
\]

(37)

where \( \alpha_p \) is the polarizability of the material, \( V \) is the volume, and \( \theta_s \) is the scattering angle. In the following, the angular term is taken as \( \frac{\pi}{4} \) and the scattering from each grain is treated as isotropic.

The polarizability of a needle and a sphere, respectively, are given by

\[
\alpha_p^{\text{needle}} = \frac{1}{4\pi} (\epsilon - 1) V, \quad \alpha_p^{\text{sphere}} = \frac{3}{4\pi} \left( \frac{\epsilon - 1}{\epsilon + 2} \right) V,
\]

(38)

where \( \epsilon \) is the dielectric constant and \( V \) is the grain volume. The most efficient scattering shape per unit volume is a needle.

If light with flux (power/area) strikes the scatterer, the intensity at scattering angle \( \theta_s \) and distance \( r \) is

\[
I_{\text{sca}}(\theta_s) = \frac{\frac{d\sigma}{d\Omega} \frac{r_{\text{in}}}{r^2}}{4\pi}.
\]

(39)

For a random set of incoherent scatterers with surface density \( \sigma_N \), the scattered power adds. Thus, the total scattered light is just \( \sigma_N A \) times the above, where \( A \) is the area covered by the scatterers. This surface element subtends a solid angle of \( A \cos \theta_d/2 \). Thus, the resulting surface brightness is

\[
B_{\text{sca}}(\theta_s) = \sigma_N \frac{d\sigma}{d\Omega} \frac{r_{\text{in}}}{\cos \theta_d} W \text{ m}^{-2} \text{ sr}^{-1} \text{ Hz}^{-1}.
\]

(40)

The cos \( \theta_d \) in the denominator accounts for isotropic scattering; the scattering is not Lambertian.

The equivalent parabola concept (Table 1) is used to compute the measured signal. The scattered light from the top of the primary is modeled as directly entering the feed. The Moon illuminates \( \approx 0.2 \) sr of the primary at an angle from the feed of \( \theta_d \approx 20^\circ \) and gain of 20 (13 dBi) as indicated in Figure 1. The Earth, although hotter, illuminates less and is subdominant. Because of the large uncertainty in the type of contaminant, a full integration of the telescope response is not necessary. The effective antenna temperature is given by

\[
T_A = \frac{1}{4\pi} \int B_{\text{sca}}(\theta_s) d\Omega = \frac{3}{16} \eta_{\text{surf}} \sigma_N \frac{2\pi\nu}{c} \left| \alpha_p \right|^2 T_s \left( \frac{r_s}{d_s} \right)^2 \theta_d \Delta \nu,
\]

(41)

where \( r_s \), \( d_s \), and \( T_s \) are given in Table 9. The efficiency \( \eta_{\text{surf}} \) encodes the fact that the scatterers on a metal surface are at the node of the electric field. For lunar emission at 90 GHz, equation (41) becomes \( T_A = 7 \times 10^6 \eta_{\text{surf}} \sigma_N \left| \alpha_p \right|^2 \) in mks units.

For the \( 2 \times 10^4 \mu m \) needles at 90 GHz discussed above, \( \eta_{\text{surf}} \approx 10^{-3}, \sigma_N = 10^7 \, m^{-2}, \left| \alpha_p \right|^2 / V^2 \approx 2 \times 10^9 \), and \( V = 3 \times 10^{-14} \, m^3 \). The result is that \( T_A = 120 \mu K \). If the mass of graphite is held fixed but the shape of the grain is more spherical, \( T_A \) is reduced to submicrokelvin levels. Baring the pathological case of large needles, emission from scattering off contaminants is negligible.

4.4. Nonuniform Coating of Primary

It is possible that the primary may be coated with a thin layer of water or hydrazine (fuel for the thrusters), which then freezes. Neither of these absorbs microwave radiation, but if they irregularly coat the surface, the variation in the thickness of material leads to a variation in the phase of the wave front, in turn leading to a distortion of the beam. The phase delay for a thickness \( d \) is \( d\phi = 2d\nu/\lambda \). For example, if 1 g of water freezes out over half the surface, \( d\phi \approx 1.4 \times 10^{-3} \). This is negligible, more than 2 orders of magnitude smaller than the surface rms specification discussed in § 2.6.

4.5. Micrometeoroids

Micrometeoroids pelt the optics, thereby increasing their microwave emissivity. The particle distribution (Anderson 1998; S. Best 1998, private communication) shown in Figure 11 is typical for Earth spacecraft and believed to be representative of L2. The relative velocity to the S/C is 19 km s\(^{-1}\), and they may hit at any angle. At the heavy end of the distribution, the particles have an energy of 50 J. Each penetration is modeled as a circular hole of diameter the micrometeoroid multiplied by a factor. Holes in the solar shield transmit a negligible amount of solar energy; holes in the reflectors might act as blackbodies at 70 K, the temperature of the reflectors.

The micrometeoroids have a size distribution between 15 and 700 \( \mu \)m and a mass between 0.004 and 400 \( \mu \)g. They are mostly olivines and silicates. The specific density for particles under 1 g is 2 g cm\(^{-3}\); for masses between 1 and 0.01 g, it is 1 g cm\(^{-3}\); and it is 0.5 g cm\(^{-3}\) for more massive meteoroids. In round figures, there is roughly 1 hit m\(^{-2}\) yr\(^{-1}\) of a 100 \( \mu \)m diameter particle and 100 hits m\(^{-2}\) yr\(^{-1}\) of 10 \( \mu \)m diameter particles. To compute the flux that scrubs the reflectors, the values in Figure 11 are scaled to 1 month and a surface of 1.8 m\(^2\), corresponding to one primary reflector. The holes (places where the Al is removed) are modeled as circular apertures that transmit 90 GHz. For a 300 \( \mu \)m radius particle, \( ka = 0.57 \) (\( a \) is the radius); thus, some care must be taken in computing the transmission coefficient. Landau’s expression (Jackson 1999) is used even though it is imprecise (though sufficient) near \( ka \approx 1 \). The power transmission coefficient is

\[
\tau_{\text{trans}}(ka) = 1 - \frac{1}{2ka} \int_0^{2ka} J_0(t) dt \approx \frac{(ka)^2}{3} \quad \text{for } ka \ll 1.
\]

(42)

A perfect mirror, with holes of radii \( a_i, i \in \{1, n\} \), has an effective temperature given by

\[
T_{\text{eff}} = \frac{c_{\text{Al}} (A_{\text{refl}} - \sum_i \pi a_i^2) + \sum_i \tau_{\text{trans}}(a_i) \pi a_i^2}{A_{\text{refl}}},
\]

(43)
where $\tau_{\text{trans}}$ is the transmission coefficient of a hole, $\epsilon$ is the emissivity of the coated surface, $T_{\text{refl}}$ is the reflector temperature, and $A_{\text{refl}}$ is its area. We conservatively model the holes as perfect, though small, emitters. This model is valid in the limit of $\sum_i \pi a_i^2 \ll A_{\text{refl}}$ and $\tau_{\text{trans}} > \epsilon A_{\text{refl}}$. For each successive hit,

$$\Delta T_A = \frac{\left[-\epsilon A_{\text{refl}} + \tau_{\text{trans}}(a_i)\right]\pi a_i^2}{A_{\text{refl}}} T_{\text{refl}}.$$

At 94 GHz, $\tau_{\text{trans}} \approx \epsilon A_{\text{refl}}$ for particles with $a < 20 \mu m$ and the simple model breaks down. However, these particles contribute negligibly to the overall emissivity. The primaries and the particle flux on them are assumed to be independent. For each mass bin of index $i$ and each month, the number $n_i$ of collisions is randomly chosen for each reflector according to the Poisson distribution. The change in $T_A$ for one reflector is given by $n_i \Delta T_A(a_i)$, where $a_i$ is the radius of the particles belonging to the particular mass bin. After adding the individual contributions of all of the bins and taking the difference between reflectors, the result is recorded and the process starts again with a new month.

To assess the net differential signal produced by the collisions, one simulates many MAP missions as shown in Figure 11. The standard deviation of the data is 0.25 $\mu K$. From the cumulative probability distribution one finds that, with 95% probability, the difference in temperatures will be smaller than 0.5 $\mu K$. Most evidence suggests that the holes will be bigger than the particle size. To bound the problem, consider $r_{\text{hole}} = 5a$. As $\Delta T_A \propto a^4$ in the $ka \ll 1$ limit, one expects a distribution 625 times wider. However, for the larger particles, $ka \approx 3$, and the small size approximation in equation (42) is not valid. The net effect is that the 95% upper limit on an offset is 140 $\mu K$.

This offset is virtually undetectable. If MAP were not differential, the net signal from micrometeoroids would not be much different. This is because the damage to the reflector is dominated by relatively few encounters with 100 $\mu m$ size particles. With two large reflectors, the cross section doubles and is not compensated by the differential measurement.

In the right-hand side of Figure 11 is plotted the average number of collisions per month for the heaviest mass bins. These collisions produce large holes but are rare. Most important, the contribution to the total hole area from the largest particles is not increasing as one moves to yet larger particles, indicating that if the distribution is accurate, 140 $\mu K$ is a reasonable bound on the change in offset. This effect would most likely be masked by other radiometer effects and would, in any case, be strongly suppressed by the mapmaking algorithm and scan strategy.

5. CONCLUSIONS

We have designed a differential optical system for performing precise and accurate measurements of the anisotropy in the CMB with an angular resolution of less than 0.23. All major components of the system have been measured and modeled in detail. Such a characterization is required in order to give us confidence in the scientific conclusions we derive from MAP. In addition, we have presented estimates of a number of systematic effects and have shown that in all cases their radiometric contribution to the celestial signal is negligible.

The development of the MAP optics started in 1993 and has involved many people in addition to the authors. At Princeton, N. Butler, W. Jones, and S. Bradley wrote senior theses on various aspects of the development; C. Bontas worked on the calculations for the micrometeoroid and surface deformations; A. Marino, D. Wesley, C. Steinhardt, C. McLeavey, C. Dumont, M. Desai, M. Kesden, A. Furman, O. Motrunich, R. Dorwart, E. Guerra, and C. Coldwell worked on modeling and testing various components; C. Sule and G. Atkinson worked on building optical components; and S. Dawson, A. Qualls, and K. Warren kept the Princeton effort running smoothly. At University of British Columbia, M. Jackson measured the fine scale surface deformations and C. Padwick measured the cryogenic properties of thin aluminum coatings. J. Heaney of Swales along with S. Dummer and R. Garriott of SOC helped define and worked especially hard to produce a surface that met the specifications. D. Neverman at PCI led the team that built the TRS. S. Best and J. Anderson provided the micrometeoroid test data and initial meteoroid flux calculations, respectively. The NASA/GSFC team, led by L. Citrin, the project manager, worked long hours to make the MAP optics a reality. C. Trout-Marx independently...
verified the design with Code V; S. Seufert and K. Hersey verified the design with Code V; S. Seufert and K. Hersey verified and ran the beam prediction code at GSFC; T. VanSant led the materials analysis team, supported by a large group including B. Munoz, C. He, L. Wang, and C. Powers; the reflector and blanket surface composition characterization was additionally supported by L. Bartusek, L. Kauder, R. Gorman, and W. Peters; S. Glazer led the thermal predictions group, with key support from D. Neuberger; P. Mule led the STOP verification effort (building on the work of J. McGuire); L. Lloyd and P. Trahan did the PRT harness fabrication and installation; A. Herera, H. Sampier, D. Osgood, C. Aviado, M. Hill, D. Schuster, T. Adams, and M. Holliday made sure that the optics were aligned and blanketeted correctly; W. Chen, S. Ngo, and J. Stewart led the mechanical design team at GSFC, with key support from B. Rodini, J. Parker, M. Schoolman, and T. Driscoll; E. Packard led the facility support activities; M. Jones ensured quality assurance and; A. Crane, with support from A. Herrera, N. Dahya, R. Hackley, and M. Lenz, led the TRS I & T effort. The GTD calculations, especially equation (12), were done with J. Mather when he was part of the team (although any errors here are independent of his work). Comments at early design reviews by D. Fixsen, P. Timbie, and P. Napier and the input from a review led by J. Mangus were especially helpful. We also thank B. Griswold of NASA/GSFC for his work on the figures. The modeling of the optics was done with assistance and code from YRS Associates: Y. Rahmat-Samii, W. Imbriale, and V. Galindo. Finally, we thank an anonymous reviewer whose comments improved the paper. This research was supported by the MAP project under the NASA Office of Space Science and Princeton University. More information about MAP may be found at http://map.gsfc.nasa.gov.

REFERENCES

Anderson, J. 1998, MAP Memorandum to Cliff Jackson, 1998 January 12, from NASA/Marshall Space Flight Center
Ashcroft, N. W., & Mermin, N. D. 1976, Solid State Physics (Philadelphia: Holt, Rinehart, and Winston)
Baker, J. C., et al. 1999, MNRAS, 308, 1173
Barnes, C., et al. 2002, ApJS, 143, L67
Beckmann, P., & Spizzichino, A. 1963, The Scattering of Electromagnetic Waves from Rough Surfaces (New York: Pergamon Press)
Bennett, C., et al. 2003, ApJ, 583, 1
Bennett, H. E., & Porteus, J. O. 1961, J. Opt. Soc. Am., 51, 123
Benoît, A., et al. 2002, A&A, submitted
Bennett, H. E., & Porteus, J. O. 1961, J. Opt. Soc. Am., 51, 123
Benoit, A., et al. 2002, A&A, submitted
Blum, B. P., et al. 2002, ApJ, submitted
Born, M., & Wolfe, E. 1980, Principles of Optics (6th ed.; New York: Pergamon Press)
Bennett, C., et al. 1994, ApJ, 422, L37
Clarke, R. 1996, Cosmology and Large Scale Structure, Les Houches Session LX, ed. R. Schaeffer et al. (Amsterdam: Elsevier), 496
Gundersen, J. O., et al. 1995, ApJ, 443, L57
Galindo-Isreal, V., Veruttipong, W., Norrod, R., & Imbriale, W. 1992, Japanese Journal of Applied Physics, 31, 529
Galindo-Isreal, V., Veruttipong, W., Norrod, R., & Imbriale, W. 1992, Japanese Journal of Applied Physics, 31, 529
Landau, L. D., & Lifshitz, E. M. 1984, Electrodynamics of Continuous Media (2d ed.; New York: Pergamon Press)
Lee, A., et al. 1999, in 3K Cosmology, ed. F Melchiorri (New York: AIP), 224
Leitch, E. M., et al. 2000, ApJ, 532, 37
Lim, M. A., et al. 1996, ApJ, 469, L69
Love, A. W. 1978, Reflector Antennas (New York: IEEE Press)
Meinhold, P. R., Chinguecano, A. O., Gundersen, J. O., Schuster, J. A., Stenfert-M. D., Lubin, P. M., Morris, D., & Villella, T. 1993, ApJ, 406, 12
Miller, A. D., et al. 2002, ApJS, 140, 115
Mizuguchi, Y., Akagawa, M., & Yokoi, H. 1976, IEEE Ant. Prop. Soc. Symp. Dig., Ambler, PA
Mon, K. K., & Sievers, A. J. 1975, Appl. Opt., 14, L1054
Muirs, S. T., Readhead, A. C. S., & Lawrence, C. R. 1993, ApJ, 405, 8
Netterfield, C. B., et al. 1997, ApJ, 474, 47
Nicoloudes, F. 1965, Appl. Opt., 4, 767
Oh, P., Spergel, D., & Hinshaw, G. 1999, ApJ, 510, 551
Ohm, E. 1974, Bell Syst. Tech. J., 53, 1657
Padin, S., et al. 2001, ApJ, 549, L1
Page, L., et al. 1994, Appl. Opt., 33, 1
Penzias, A. A., & Wilson, R. W. 1965, ApJ, 142, 419
Peterson, J. B., et al. 2000, ApJ, 532, L83
Pincirolio, L., & Calisse, P. 1993, ApJ, 411, 529
Pippard, A. B. 1947, Proc. Roy. Soc. London A, 191, 385
Platt, S. R., et al. 1997, ApJ, 475, L1
Pospieszalski, M. W., & Lakatos, W. J. 1995, Proc. IEEE MTT-S Int. Microwave Symp.
Pospieszalski, M. W., et al. 2000, Radio Frequency Integrated Circuits (RFIC) Symp. Digest of Papers, 217
Rahmat-Samii, Y., & Galindo-Iserea, V. 1981, Radio Sci., 16, 1093
Ramo, S., & Whinnery, J. 1953, Fields and Waves in Modern Radio (New York: Wiley)
Rausch, W., et al. 1990, IEEE Trans. Ant. Prop., 38, 8
Roberts, C. 1987, Astronomical Optics (San Diego: Academic Press)
Sloan, E., et al. 1997, Appl. Opt., 36, 685
Sourcebook, T. & Ratra, B. 2001, ApJ, 560, 28
Tanaka, H., & Mizusawa, M. 1975, Inst. Electron. Commun., 58-B, 643
Thomas, M. B. 1978, Antenna Design Notes IEEE Trans. Ant. Prop., AP-2, 367
Thompson, A. R., Moran, J. M., & Swenson, G. W. 1986, Interferometry and Synthesis in Radio Astronomy (New York: Wiley)
Tucker, G. S., Gush, H. P., Halpern, M., Shinkoda, I., & Towlson, W. 1997, ApJ, 475, L13
van de Hulst, H. C. 1981, Light Scattering by Small Particles (New York: Wiley)
Wah, J., et al. 2001, ApJS, 132, 1
Xia, J., Lange, A. E., & Bock, J. J. 1996, Proc. 30th ESLAB Symp. (Noordwijk: ESTEC)