Proposed method for measurement of flow rate in turbulent periodic pipe flow

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Abstract. The present investigation deals with a previously proposed flow metering technique for laminar, fully-developed, time-periodic pipe flow. Employing knowledge of the pulsation frequency-dependent relationship between the mass flow rate and the pressure gradient, the method allows reconstruction of the instantaneous mass flow rate on the basis of a recorded pressure gradient time series. In order to explore if the procedure can be extended for turbulent flows, numerical simulations for turbulent, fully-developed, sinusoidally pulsating pipe flow with low pulse amplitude have been carried out using a $v^2\cdot f$ turbulence model. The study covers pulsation frequencies, ranging from the quasi-steady up to the inertia-dominated frequency regime, and three cycle-averaged Reynolds numbers of 4360, 9750 and 15400. After providing the theoretical background of the flow rate reconstruction principle, the numerical model and an experimental facility for the verification of simulations are explained. The obtained results, presented in time and frequency domain, show good agreement with each other and indicate a frequency dependence, similar to that used for the signal reconstruction for laminar flows. A modified dimensionless frequency definition has been introduced, which allows a generalised representation of the results considering the influence of Reynolds number.

1. Introduction and aim of work

Time-periodic flows, characterised by an oscillation of the flow quantities around a time-average, play an important role in many fields of science and technology. For instance, pulsations occur in the flows delivered by fuel injectors and most kinds of pumps as well as at the inlet and outlet manifolds of IC engines. Another familiar example is found in biology with respect to the blood flow through arteries. Also, the effects of pulsating flow motion are specifically utilised for applications, such as artificial respiration or industrial cleaning of pipes, and their influence on head loss and convective heat transfer are still subjects of intense study.

Owing to the importance of pulsating flows, there exists great interest in instruments for the accurate measurement of the instantaneous flow rate. However, most available flow meters were developed for steady flow conditions and generally deliver erroneous readings under pulsatile flow conditions. Few techniques that could be considered suitable are also limited in certain respects. While ultrasonic flow meters can measure velocity changes up to several kHz, their assumption of a known velocity profile fails for highly unsteady flows. Electromagnetic and coriolis flow meters are able to measure changing velocity profiles, but their maximum frequency response can not exceed the typical operation frequencies of their underlying measurement principles.
Similarly, servo-controlled positive displacement meters are also limited in response time and in addition to the maximum flow rate that can be measured.

Another group of techniques, at present applicable for laminar pulsating pipe flows, utilises the analytical solution of Navier-Stokes equations for fully-developed time-periodic pipe flow in order to reconstruct the flow rate from a pressure gradient measurement. Originally derived by Lambossy (1952) for harmonic pulsations and later extended by Uchida (1956) for arbitrary shaped pulses, Womersley (1955) utilised this solution to measure flow rates through arteries by means of Fourier decomposition. Later, Brereton (2000) solved the governing equations by Laplace transform technique and demonstrated a method of mass flow rate measurement for flows with not only periodic, but also with arbitrary unsteadiness.

A technique, similar to the one demonstrated by Womersley (1955), was recently proposed by Ünsal et al. (2006). It employs the findings of Ray et al. (2005), who generalised the solution of Uchida (1956) through the introduction of suitable dimensionless quantities. These are the dimensionless frequency, defined as \( F = R^2 f / \nu \), and the dimensionless signal amplitudes of mass flow rate \( m_A^*/m_M \) and pressure gradient \( P_A^*/P_M \), where the subscripts \( A \) and \( M \) denote the amplitudes and cycle-averages, respectively. The investigation showed that, for sinusoidally pulsating, laminar, fully-developed, pipe and channel flows, the ratio of dimensionless amplitudes of pressure gradient and mass flow rate \( m_A^*/P_A^* \) and their phase lag \( \Delta \theta \) depend solely on dimensionless pulsation frequency \( F \) (see figure 1). This dependence, may be termed as the signature map, is the basis of the proposed measurement method. It allows one to determine the corresponding mass flow rate amplitude for any known pressure gradient amplitude and hence, assuming a spectral decomposition of the recorded pressure gradient signal and subsequent superposition of the solutions, the reconstruction for flows with arbitrarily shaped pulses.

In order to experimentally verify the performance of the procedure, Ünsal et al. (2006) employed it for the measurement of instantaneous flow rate through fast-opening injection nozzles and achieved excellent agreement with comparative data from an LDA-based flow meter. Durst et al. (2007) applied the same measurement principle to a mass flow rate driven pulsating air flow and also achieved encouraging results. A major restriction of the method, however, is the limitation on existence of laminar pulsating flows due to the lack of knowledge regarding a corresponding signature map for the case of turbulent flow. A first indication of such was reported in the PhD thesis of Ünsal (2008), who experimentally studied turbulent pulsating

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Figure 1. Variation of ratio of dimensionless amplitudes \( m_A^*/P_A^* \) and phase lag \( \Delta \theta \) of mass flow rate and pressure gradient signals with dimensionless pulsation frequency \( F = R^2 f / \nu \) for fully-developed, laminar, pulsating pipe flow.
flow, and also by Werzner et al. (2010), on the basis of their numerical simulations using a Low-Reynolds number $k$-$\varepsilon$ turbulence model. From theoretical considerations, the latter also derived a general value of $m^*/P_A^*$ for quasi-steady, low-amplitude, turbulent pipe flow, which was found to be $4/7$.

Within this work, a systematic numerical study of turbulent, fully-developed, sinusoidally pulsating pipe flow has been carried out, aiming to confirm the existence of the signature map and to allow an extension of the measurement technique for the class of turbulent flows. The investigation covers a wide range of pulsation frequencies and different cycle-averaged Reynolds numbers. Since the reconstruction method requires a linear relationship between mass flow rate and pressure gradient, which is not valid for highly modulated turbulent flows, a small pulsation amplitude of $m_A^* = 0.1$ has been chosen for the entire study.

The present paper is structured as follows. Section 2 explains the numerical modeling for the present problem. Details of the experimental set-up, used for verification of simulations, are given in section 3. Numerical and experimental results are presented and discussed in section 4. Finally, in section 5, conclusions and outlook of this research effort are provided.

2. Numerical modeling

For the present analysis, the flow has been assumed to be incompressible and isothermal. Assuming further, that the flow is fully-developed, it can be treated as transient, one-dimensional problem and the Reynolds-averaged governing momentum equation in cylindrical coordinates reduces to:

$$\frac{\partial u}{\partial t} = \frac{1}{\rho} \frac{\partial}{\partial x} \left( \frac{1}{r} \frac{\partial}{\partial r} \left( r \left( \nu \frac{\partial u}{\partial r} - \bar{u} \bar{v}' \right) \right) \right)$$

(1)

In order to address the closure problem, which arises from the unknown Reynolds stress term $\bar{u} \bar{v}'$, the Boussinesq hypothesis has been employed. It relates the Reynolds stresses to the mean velocity gradient field via an eddy viscosity $\nu_t$:

$$-\bar{u} \bar{v}' = \nu_t \frac{\partial u}{\partial r}$$

(2)

Modeling of the eddy viscosity has been accomplished using the $v^2$-$f$ turbulence model of Lien & Durbin (1996). In contrast to the well-established Low-Reynolds number $k$-$\varepsilon$ models, the present model is capable to capture the anisotropy of the Reynolds stresses in the near-wall region without relying upon an empirical damping function. This is achieved by employing the wall-normal turbulent stress $\bar{v}^2$ in conjunction with a turbulent time scale $T$:

$$\nu_t = c_h \bar{v}^2 T$$

(3)

The transport of $\bar{v}^2$ has been modeled as follows:

$$\frac{\partial \bar{v}^2}{\partial t} = \frac{1}{r} \frac{\partial}{\partial r} \left[ r \left( \nu + \nu_t \right) \frac{\partial \bar{v}^2}{\partial r} \right] + kf - \frac{6 \bar{w}^2 \varepsilon}{k}$$

(4)

Further, transport equations for the turbulent kinetic energy $k$ and its dissipation rate $\varepsilon$, as known from the $k$-$\varepsilon$ models, have been solved:

$$\frac{\partial k}{\partial t} = \frac{1}{r} \frac{\partial}{\partial r} \left[ r \left( \nu + \nu_t \sigma_k \right) \frac{\partial k}{\partial r} \right] + G_k - c_D \frac{k^{3/2}}{4t}$$

(5)
\[
\frac{\partial \varepsilon}{\partial t} = \frac{1}{r} \frac{\partial}{\partial r} \left( r \left( \nu + \frac{\nu_t}{\sigma_{\varepsilon}} \right) \frac{\partial \varepsilon}{\partial r} \right) + \frac{c_{\varepsilon 1} G_k - c_{\varepsilon 2} \varepsilon}{T} \tag{6}
\]

where the production term for the turbulent kinetic energy \( G_k \) is given as

\[
G_k = \nu_t \left( \frac{\partial u}{\partial r} \right)^2 \tag{7}
\]

In addition, an equation for \( f \), a redistribution term in the \( \sqrt{T} \) equation, has been solved, which reads as

\[
L^2 \frac{1}{r} \frac{\partial}{\partial r} \left( r \frac{\partial f}{\partial r} \right) - f = \frac{1}{T} \left[ (c_1 - 6) \frac{\nu^2}{k} - (c_1 - 1) - \frac{2}{3} \right] - c_2 \frac{G_k}{k} \tag{8}
\]

The distributions of turbulent time scale \( T \) and turbulent length scale \( L \) have been obtained in the following way:

\[
T = \max \left[ \frac{k}{\varepsilon} , 6 \left( \frac{\nu}{\varepsilon} \right)^{1/2} \right] \tag{9}
\]

\[
L = c_L \max \left[ \frac{k^{3/2}}{\varepsilon} , c_\eta \left( \frac{\nu^3}{\varepsilon} \right)^{1/4} \right] \tag{10}
\]

According to Lien & Durbin (1996), the wall boundary conditions for the transport properties have been imposed as follows:

\[
u = 0, \quad k = 0, \quad \varepsilon \to 2\nu \frac{k}{y^2}, \quad f = 0
\]

Finally, the model constants are given as

\[
c_m = 0.19, \quad \sigma_k = 1, \quad \sigma_{\varepsilon} = 1.5, \\
c_{\varepsilon 1} = 1.55 + \exp(-A_e R_y^2)|_{A_e=0.00285}, \quad c_{\varepsilon 2} = 1.92, \\
c_1 = 1.4, \quad c_2 = 0.3, \quad c_L = 0.17, \quad c_\eta = 70
\]

where \( R_y = y\sqrt{k/\nu} \).

In order to assure comparability between the numerical results and the experimental data, which have been obtained employing a mass flow rate controller, the mass flow rate has been prescribed for the simulations. The momentum equation, however, only allows a straightforward calculation of the velocity distribution and consequently, the flow rate, if the pressure gradient is known. Hence, the corresponding pressure gradient for a desired mass flow rate has been obtained iteratively. The iterations have been performed using a Newton Raphson type algorithm and continued until the difference between desired and actual flow rate was less than \( 10^{-7} \).

The employed numerical schemes for time and space discretisation are of second-order accuracy. Sufficient spacial resolution of the domain, particularly in the near-wall region where the gradients are high, has been achieved with a grid of 100 non-uniformly distributed control-volumes, resulting in \( y^+ < 1 \) for the wall-adjacent node. Similarly, the independence of time step size has been also verified.
3. Experimental investigation

For the experimental verification of the simulations, measurements have been carried out in a turbulent, sinusoidally pulsating air flow under similar flow conditions. As sketched in figure 2, the test setup mainly consists of a mass flow rate controller (MFC), a steel pipe and two pressure transducers. The MFC, explained in detail by Durst et al. (2003), is capable of reproducing the externally prescribed sinusoidal mass flow rate signal with a relative error of less than 1% and a time resolution of 8 ms. After entering the pipe, the flow has been allowed to pass through an entrance section of $133D$, which can be considered to be sufficient in order to ensure fully-developed flow condition (Doherty et al., 2007). Towards the exit of the pipe, two fast-responsive pressure transducers, positioned at a distance of 1 m apart from each other, have been applied in order to measure the pressure gradient in the axial direction. Further, the temperature and the humidity have been also recorded for exact determination of air properties. The acquisition of all signals was accomplished using a 16-bit, 200 kHz laboratory A/D converter.

In order to reduce the influence of randomly occurring fluctuations in the recorded pressure gradient signal, the measurements have been carried out for at least 100 cycles of pulsation or 40 s of duration. The values of amplitude ratio and phase lag have been obtained in the frequency domain using discrete Fourier transformation. Assuming that $X(f)$ and $Y(f)$ are the complex frequency spectra of the normalised mass flow rate and pressure gradient signals $m^*$ and $P^*$, respectively, the amplitude ratio for a known pulsation frequency $f_P$ has been calculated from the real and imaginary parts as

$$ \frac{m_A^*}{P_A^*}(f_P) = \frac{\sqrt{\text{Re}(X(f_P))^2 + \text{Im}(X(f_P))^2}}{\sqrt{\text{Re}(Y(f_P))^2 + \text{Im}(Y(f_P))^2}} \quad (11) $$

Prior to the determination of the phase lag, the cross power spectrum $S_{XY}$ has been computed:

$$ S_{XY}(f) = \frac{\overline{X(f)}Y(f)}{N} \quad (12) $$

where $\overline{X(f)}$ is the complex conjugate of the mass flow rate spectrum and $N$ is the number of signal samples. The phase lag has been then obtained as

$$ \Delta \theta(f_P) = \arctan \frac{\text{Im}(S_{XY}(f_P))}{\text{Re}(S_{XY}(f_P))} \quad (13) $$

It may be noted here that for visualization in the time domain, the recorded signals have been also low-pass filtered and subsequently phase-averaged.

![Figure 2. Schematic representation of the experimental setup.](image)
4. Results and comparison

Figure 3 shows the dimensionless time signals of the mass flow rate and the pressure gradient, obtained from numerical simulations and experiments, for different dimensionless pulsation frequencies at $Re = 9750$. It is obvious that pressure gradient amplitude and phase lag show an increase with the increase in pulsation frequency. A more detailed insight into this variation is, however, obtained by plotting the results in form of a signature map, introduced by Ray et al. (2005), as presented in figure 4a. It shows a characteristic frequency-dependence with different flow regimes, similar to the laminar case. In the quasi-steady regime, the pressure gradient nearly follows the imposed mass flow rate with a constant amplitude ratio $(m_A^*/P_A^* = 4/7)$ and an almost non-existing phase lag. As frequency increases, a transition takes place, which is caused by inertia effects. Finally, for very high frequencies, the amplitude ratio approaches a small value close to zero while the phase lag tends to $\pi/2$.

Analysing the individual results for different Reynolds numbers, a frequency shift of the flow regimes is observed. This may be attributed to the stronger frictional effects of flows associated with higher Reynolds number, delaying the transition from the quasi-steady regime to the inertia-dominated regime towards higher frequencies. However, if the definition of the dimensionless frequency is modified by including the contribution of $Re$ ($F^* = F/0.01Re^{3/4}$), as shown in figure 4b, all the curves collapse together and a general signature map emerges. The factor 0.01, appearing in the denominator of $F^*$, is deliberately introduced in order to obtain a scale of $F^*$ comparable to that of $F$. This observation, also pointed out earlier by Ohmi & Iguchi (1980), allows considerable simplification of the signature map for its application as a measurement tool.

![Figure 3](image-url)

**Figure 3.** Numerically and experimentally obtained dimensionless pressure gradient time signals for selected dimensionless pulsation frequencies $F$ ($Re = 9750$, $m_A^* = 0.1$).
Figure 4. Numerically and experimentally obtained variations of amplitude ratio $m^*/P_A^*$ and phase lag $\Delta \theta$ with a) dimensionless pulsation frequency $F$ and b) modified dimensionless pulsation frequency $F^* = F/(0.01Re^{3/4})$ for fully-developed, turbulent, low-amplitude, pulsating pipe flow at different Reynolds numbers.

It is further noteworthy that the ratio between the Kolmogorov and the integral length scale also scales with $Re^{3/4}$.

Finally, it may be mentioned that the numerical results, obtained using the $v^2-f$ model, agree reasonably well with the experimental data. Noticeable differences are only found in the transition regime, which is predicted for lower frequencies by the simulations. The deviations of the experimental data at higher frequencies may be regarded as the beginning of compressibility effects in the pipe.

5. Conclusions and outlook

For the present paper numerical simulations of sinusoidally pulsating turbulent pipe flow have been carried out using a $v^2-f$ turbulence model in order to explore, whether the transient flow measurement technique, proposed by ¨Unsal et al. (2006), can be extended for the class of turbulent flows. The results confirm the existence of a unique frequency dependence of mass flow rate to pressure gradient amplitude ratio and phase lag in fully-developed, turbulent, low-amplitude, pulsating pipe flow. In principle, it allows the reconstruction of the corresponding mass flow rate amplitude for a known pressure gradient oscillation and hence, could be regarded as the basis for an extension of the technique. The introduction of a modified dimensionless frequency allows a generalised representation of the observed dependence considering the influence of Reynolds number and represents a considerable simplification from an application viewpoint. It can also be concluded that the $v^2-f$ turbulence model turned out to be suitable for the present problem.

Currently, however, the application of the technique would be limited to flows with low pulsation amplitudes due to the non-linear relationship that exists between the mass flow rate and the pressure gradient for turbulent flow. Quite evidently, further studies are required and are being carried out in order to address these issues relating to higher modulation amplitudes.
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List of symbols

\( c_i \) model constants \( Re \) Reynolds number
\( D \) pipe diameter \( S_{XY} \) cross power spectrum
\( f \) redistribution term for \( \overline{u'^2} \)-equation \( t \) time
\( f_P \) pulsation frequency \( T \) turbulent time scale
\( F \) dimensionless pulsation frequency \( u \) axial velocity
\( G_k \) production of turbulent kinetic energy \( u'v' \) Reynolds stress
\( k \) turbulent kinetic energy \( v \) wall-normal velocity
\( L \) turbulent length scale \( \overline{v'^2} \) wall-normal stress
\( m^* \) dimensionless mass flow rate \( x \) axial coordinate
\( N \) sample number \( X \) mass flow rate spectrum
\( p \) effective pressure \( y \) wall-normal coordinate
\( P^* \) dimensionless pressure gradient \( y^+ \) dimensionless wall distance
\( r \) radial coordinate \( Y \) pressure gradient spectrum
\( R \) pipe radius

Greek letters

\( \Delta \theta \) phase lag \( \nu_t \) eddy viscosity
\( \varepsilon \) turbulent kinetic energy dissipation rate \( \sigma_i \) model constants
\( \nu \) kinematic viscosity

Subscripts

\( A \) amplitude
\( M \) cycle-average

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