Two-photon exchange effect on deuteron electromagnetic form factors

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Abstract

Corrections of two-photon exchange to proton and neutron electromagnetic form factors are employed to study the effect of two-photon exchange on the deuteron electromagnetic form factors. Numerical results of the effect are given. It is suggested to test the effect in the measurement of $P_z$ in a small angle limit.

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1 Introduction

Proton and deuteron electromagnetic (EM) form factors have been studied for a long time by unpolarized electron-proton ($ep$) and electron-deuteron ($eD$) elastic scatterings and by the Rosenbluth separation [1] based on one-photon-exchange (OPE). The differential cross section of the $ep$ scattering is

$$d\sigma_0 = A_0(\tau_N G^2_M(Q^2) + \epsilon G^2_E(Q^2)), \tag{1}$$

with $A_0$ depending on kinematic variables, $\tau_N = Q^2/4M_N^2$ and $\epsilon = [1 + 2(1 + \tau_N)\tan^2(\theta/2)]^{-1}$ ($M_N$ and $\theta$ are the nucleon mass and the electron scattering angle). For a long time, the extracted $Q^2$-dependences of the nucleon EM form factors are believed to be simple dipole forms. For the proton $G^p_{E,M}$, one conventionally gets

$$G^p_E(Q^2) = G^p_M(Q^2)/\mu_p \simeq 1/(1 + Q^2(GeV^2)/0.71)^2, \tag{2}$$

where $\mu_p = 2.79$ is the proton magneton. Recently, the new experiments of the polarized $ep$ elastic scattering were precisely carried out. The polarization transfer scattering experiments, $e^- + p \rightarrow e^- + \bar{p}$, show that the ratio $R^p = \mu_p G_E^p(Q^2)/G_M^p(Q^2) \simeq 1 - 0.158Q^2$ [2]. It means that $R^p$ is no longer a simple constant. It monotonously decreases with the increasing of $Q^2$. This new phenomenon contradicts to the
One way to resolve this discrepancy is to take the effect of the two-photon-exchange (TPE) into account \[3, 4, 5, 6, 7, 8\]. Usually, it is believed that TPE is strongly suppressed by \(\alpha_{EM} (\alpha_{EM} = 1/137)\). However, it was argued \[9\] that due to the very steep decreasing of the nucleon EM form factors, the TPE process, where the \(Q^2\) is equally shared by the two exchanging photons, may be compatible to the OPE one. Some calculations of the TPE corrections to the \(e\)\(p\) elastic scattering have been done recently \[3, 4, 5, 6, 7, 8\]. There were also several other works about the TPE effect on the proton charge radius and on the parity-violating \[10, 11\] in \(e\)\(p\) scattering. The TPE corrections to the deuteron (spin 1 particle) EM form factors and to the \(e^+p\) processes have been also discussed in Refs. \[12, 13\].

To consider the electron nucleon elastic scattering with \(e(k) + N(p) \rightarrow e(k') + N(p')\) in OPE \((C = -1)\), we have

\[
\mathcal{M}_{eN}^{el} = \frac{e^2}{Q^2} \bar{u}(k') \gamma_\mu u(k) \times \bar{u}(p') \Gamma^N_\mu u(p) ,
\]

\[
\Gamma^N_\mu = \left[ \gamma_\mu F_1^N(Q^2) + i \frac{\sigma_{\mu\nu} q^\nu}{2M_N} F_2^N(Q^2) \right] , \tag{3}
\]

where \(F_1^N\) and \(F_2^N\) are the conventional Dirac and Pauli form factors of the nucleon. It should be mentioned that in OPE, the form factors \(F_1^{N,2}\) are real and the functions of \(Q^2\) only.

If TPE is considered, parity, charge-conjugation and helicity invariances lead a general expression

\[
\tilde{\mathcal{M}}_{eN}^{el} = \frac{e^2}{Q^2} \left\{ \bar{u}(k') \gamma_\mu u(k) \times \bar{u}(p') \left[ \gamma_\mu \tilde{F}_1^{N}(Q^2, \epsilon) + i \frac{\sigma_{\mu\nu} q^\nu}{2M_N} \tilde{F}_2^{N}(Q^2, \epsilon) \right] u(p) + \bar{u}(k') \gamma_\mu \gamma_5 u(k) \times \bar{u}(p') \gamma_\mu \gamma_5 \tilde{G}_A^{N}(Q^2, \epsilon) u(p) \right\} . \tag{4}
\]

Here, unlike eq. (3), \(\tilde{F}_1^{N,2}(Q^2, \epsilon)\) and \(\tilde{G}_A^{N}(Q^2, \epsilon)\) are functions of \(Q^2\) and \(\epsilon\). \(\tilde{F}_1^{N,2}(Q^2, \epsilon)\) can be separately expressed in terms of the contributions of OPE and TPE

\[
\tilde{F}_{1,2}^{N}(Q^2, \epsilon) = F_{1,2}^{N}(Q^2) + F_{1,2}^{3N}(Q^2, \epsilon) , \quad \tilde{G}_A^{N}(Q^2, \epsilon) = \tilde{G}_A^{2N}(Q^2, \epsilon) . \tag{5}
\]

Moreover, according to a general relation \[6\]

\[
\frac{1}{4} \bar{u}(k') \gamma_i P u(k) \times \bar{u}(p') \gamma_i K u(p) = \frac{s - u}{4} \bar{u}(k') \gamma_\mu u(k) \times \bar{u}(p') \gamma_\mu u(p) + t \bar{u}(k') \gamma_5 \gamma_\mu u(k) \times \bar{u}(p') \gamma_\mu \gamma_5 u(p) ,
\]

where \(K = k' + k\), \(P = p' + p\), and \(s = (p + k)^2\) and \(u = (p - k')^2\) are Mandelstam variables, we have

\[
\tilde{\mathcal{M}}_{eN}^{el} = \frac{e^2}{Q^2} \bar{u}(k') \gamma_\mu u(k) \times \bar{u}(p') \tilde{\Gamma}^N_\mu u(p) ,
\]

\[
\tilde{\Gamma}^N_\mu = \left\{ \gamma_\mu \tilde{F}_1^{N}(Q^2, \epsilon) + i \frac{q^\nu \sigma_{\mu\nu}}{2M_N} \tilde{F}_2^{N}(Q^2, \epsilon) + \frac{1}{4M_N} \tilde{F}_3^{N}(Q^2, \epsilon) K P^\mu \right\} \tag{7}
\]

where \(\tilde{F}_3^N = \frac{4M_N^2}{t} \tilde{G}_A^N\) with \(t = q^2\) and

\[
\tilde{F}_1^{N}(Q^2, \epsilon) = \tilde{F}_1(Q^2, \epsilon) - \frac{s - u}{4M_N^2} \tilde{F}_3^{N}(Q^2, \epsilon) , \tag{8}
\]

\[
\tilde{F}_2^{N}(Q^2, \epsilon) = \tilde{F}_2(Q^2, \epsilon) , \tag{9}
\]

\[
\tilde{F}_3^{N}(Q^2, \epsilon) = \tilde{F}_3(Q^2, \epsilon) + \frac{t}{4M_N^2} \tilde{F}_3^{N}(Q^2, \epsilon) , \tag{10}
\]

\[
\tilde{G}_A^{N}(Q^2, \epsilon) = \tilde{G}_A^{N}(Q^2, \epsilon) , \tag{11}
\]

\[
\tilde{G}_A^{2N}(Q^2, \epsilon) = \tilde{G}_A^{2N}(Q^2, \epsilon) . \tag{12}
\]
Usually a deuteron is regarded as a weakly bound system of a proton and a neutron (see Fig. 1). Many calculations for the EM form factors of the deuteron have been performed in different approaches in the literature \[14, 15, 16, 17\]. Recent calculations based on an effective Lagrangian approach \[18, 19\] have shown that one can reasonably explain the deuteron EM form factors with phenomenological including two-body operators. Note that the deuteron EM form factors receive the TPE corrections from many different sources in the effective Lagrangian approach. For example, the two photons directly couple to one of the nucleons (see Fig. 2), or the two photons directly couple to one of the two contact points A and B (the contact points mean the coupling points of the deuteron and its composite pn). There are also several other interferences between the different OPE couplings. For instance, one photon couples to A (or B) and another to one of the nucleons, or two photons respectively couple to the two contact points.

In this paper, we’ll study the TPE effect on the deuteron EM form factors. The TPE corrections to the EM form factors of the proton and neutron from the work of Blunden, Melnitchouk and Tjon \[5\] will be employed. We know that in the deuteron EM form factors the contribution from the direct coupling of the photon to one of the nucleons is more important than the one from the coupling of a photon to the contact point \[19\], and the latter coupling is needed in order to guarantee gauge invariance. Therefore, it is expected that the TPE effect on the deuteron EM form factors is dominated by the TPE corrections to the EM form factors of the nucleon (see Fig. 2), and we, as the first step, consider the effect of Fig. 2 on the deuteron EM form factors. This paper is organized as follows. In section 2 the TPE corrections in the eD elastic scattering are briefly discussed. Numerical results for the corrections to the EM form factors of the deuteron are displayed in section 3. In Sect. 4, the conclusions will be given.

2 Two-Photon-Exchange in the eD elastic scattering

In eD case, the electromagnetic form factors of the deuteron are defined by the matrix elements of the electromagnetic current \( J_\mu(x) \) according to the OPE approximation

\[
< p'_D, \lambda' | J_\mu(0) | p_D D, \lambda > = -e_D \left\{ \left[ G_1(Q^2) \xi'^*(\lambda') \cdot \xi(\lambda) - G_3(Q^2) \frac{\xi'^*(\lambda') \cdot q}{2M_D} \right] \cdot P_\mu + G_2(Q^2) \left[ \xi_\mu(\lambda) \xi'^*(\lambda') \cdot q - \xi'^*(\lambda') \cdot q \right] \right\} \tag{9}
\]

where \( p'_D, \xi', \lambda' \) (or \( p_D, \xi, \lambda \)) denote the momentum, helicity, and polarization vector of the final (or initial) deuteron, respectively. In eq. (9) \( q = p'_D - p_D \) is the photon momentum, \( P = p_D + p'_D, Q^2 = -q^2 \) is the
four-momentum transfer squared, $M_D$ is the deuteron mass, and $e_D$ is the charge of the deuteron. In the one-photon exchange approximation, the differential cross section of the unpolarized elastic electron-deuteron scattering $e(k, s_1) + D(p_D, \xi) \rightarrow e(k', s_3) + D(p_D', \xi')$ in the laboratory frame is [20]

$$\frac{d\sigma}{d\Omega} = \left. \frac{d\sigma}{d\Omega} \right|_{Mott} I_0(OPE), \quad I_0(OPE) = A(Q^2) + B(Q^2)\tan^2\theta/2,$$

Figure 2: Two-photon-exchange effect on deuteron form factors from the contributions of $\tilde{\Gamma}_p^p (a)$ and $\tilde{\Gamma}_n^n (b)$. Where $\theta$ is the scattering angle of the electron, $(d\sigma/d\Omega)_{Mott}$ is the Mott cross section for a structure-less particle with recoil effect, and the two structure functions

$$A(Q^2) = G_M^2(Q^2) + \frac{2}{3} \tau_D G_M^2(Q^2) + \frac{8}{9} \tau_D^2 G_Q^2(Q^2), \quad B(Q^2) = \frac{4}{3} \tau_D (1 + \tau_D) G_M^2(Q^2).$$

In the above eqs. $\tau_D = Q^2/4M_D^2$, $G_M$, $G_C$ and $G_Q$ are the deuteron magnetic, charge and quadrupole form factors, respectively. They can be expressed, in terms of $G_1$, $G_2$ and $G_3$, as

$$G_M = G_2, \quad G_Q = G_1 - G_2 + (1 + \tau_D) G_3, \quad G_C = G_1 + \frac{2}{3} \tau_D G_Q.$$ 

The normalizations of the three form factors are $G_C(0) = 1$, $G_Q(0) = M_D^2 Q_D = 25.83$, and $G_M(0) = 1.714$. Note that in eqs. (10) and (12), there are two unpolarized structure functions $A$ and $B$, and three independent form factors $G_C$, $G_Q$ and $G_M$ for the deuteron. To determine the three form factors completely, one needs, at least, one polarization observable. The optimal choice is the polarization $T_{20}$ (or $P_{zz}$) [21].

Considering both OPE ($C = -1$) and TPE ($C = +1$), and taking the Lorentz, party, and charge-conjugation invariances into account, one obtains the most general form of the $eD$ elastic scattering [12, 22],

$$\mathcal{M}_{eD}^{el} = \frac{e^2}{Q^2} \bar{u}(k', s_3) \gamma_\mu u(k, s_1) \sum_{i=1}^{6} G_i M_i^\mu,$$
where

\begin{align}
M_1^\mu &= (\xi^* \cdot \xi)P^\mu, \quad M_2^\mu = \left[\xi^\mu(\xi^* \cdot q) - (\xi \cdot q)\xi^*\mu\right], \\
M_3^\mu &= -\frac{1}{2M_D}(\xi \cdot q)(\xi^* \cdot q)P^\mu, \quad M_4^\mu = \frac{1}{2M_D}(\xi \cdot K)(\xi^* \cdot K)P^\mu, \\
M_5^\mu &= \left[\xi^\mu(\xi^* \cdot K) + (\xi \cdot K)\xi^*\mu\right],
\end{align}

and

\[M_6^\mu = \frac{1}{2M_D^2}\left[(\xi \cdot q)(\xi^* \cdot K) - (\xi \cdot K)(\xi^* \cdot q)\right]P^\mu.\] (15)

General speaking, the form factors \(G_i\) with \(i = 1, 6\), are complex functions of \(s = (p_D + k)^2\) and \(Q^2 = -(k - k')^2\). They can be expressed as

\[G_i(s, Q^2) = G_i(Q^2) + G_i^{(2)}(s, Q^2),\] (16)

where \(G_i\) represents the contribution arising from the one-photon exchange, and \(G_i^{(2)}\) stands for the rest which would come mostly from TPE. In the OPE approximation, \(G_4 = G_5 = G_6 = 0\). It is easy to see that \(G_i (i = 1, 2, 3)\) are of order of \((\alpha_{EM})^0\) and \(G_i^{(2)} (i = 1, \ldots 6)\) are of order \(\alpha_{EM}\). Moreover, \(G_i = G_i^{(2)}\) for \(i = 4, 5\) and 6.

To take the TPE corrections to the proton and neutron (see Fig. 2) EM form factors into account, and to study the TPE effect on the EM form factors of the deuteron, we directly calculate the matrix element of \(< p'_D, \lambda | \tilde{J}_\mu^p(0) + \tilde{J}_\mu^n(0) | p_D, \lambda >\), where

\[\tilde{J}_{\mu}^{p,n}(0) = | pm > < pn | \tilde{\Gamma}_{\mu}^{p,n}(0) | pm > < pn |.\] (17)

The effective interaction between the deuteron and its composites \((pn)\) is \([19]\)

\[L_D = g_D D^{\mu+}(x) \int dy \Phi_D(y^2)\bar{p}(x + \frac{1}{2}y)C\gamma_{\mu}n(x - \frac{1}{2}y) + \text{H.c.},\] (18)

where \(C\) is the charge conjugate matrix. The correlation function \(\Phi_D\) characterizes the finite size of the deuteron as a \((pn)\) bound state and depends on the relative Jacobi coordinate \(y\), in addition, \(x\) being the center-of-mass (CM) coordinate. The Fourier transformation of the correlation function reads

\[\Phi_D(y^2) = \int \frac{d^4p}{(2\pi)^4} e^{-ip\cdot y} \Phi_X(-p^2).\] (19)

A basic requirement for the choice of an explicit form of the correlation function is that it vanishes sufficiently fast in the ultraviolet region of Euclidean space to render the Feynman diagrams ultraviolet finite. We adopt a Gaussian form, \(\Phi_D(p_E^2) \equiv \exp(-p^2_E/\Lambda^2_D)\), for the vertex function, where \(p_E\) is the Euclidean Jacobi momentum. Here, \(\Lambda_D\) is a size parameter, which characterizes the distribution of the constituents inside the deuteron.

The coupling \(g_D\) of \(< p_D, \lambda | pn >= g_D\xi^*(\lambda)\) is determined by the compositeness condition \([23, 24, 25, 26]\). The condition implies that the renormalization constant of the hadron wave function is set equal to zero:

\[Z_D = 1 - \Sigma_D'(M_D^2) = 0.\] (20)
Here, $\Sigma'_D(M'_D^2) = g^2_D \Pi_D(M'_D^2)$ is the derivative of the transverse part of the mass operator $\Sigma'^\alpha\beta$ which conventionally splits into the transverse $\Sigma_T$ and longitudinal $\Sigma'_L$ parts as:

$$\Sigma'^\alpha\beta(p) = g'^\alpha\beta \Sigma_T(p^2) + \frac{p'^\alpha p'^\beta}{p^2} \Sigma'_L(p^2), \quad (21)$$

where

$$g'^\alpha\beta = g'^\alpha\beta - p'^\alpha p'^\beta/p^2, \quad g'^\alpha\beta p_n = 0. \quad (22)$$

The mass operator of the deuteron in our approach is described by Fig. 1. If the size parameter $\Lambda_D$ is fixed, the coupling $g_D$ is then obtained according to the compositeness condition eq. (20).

Note that the current of photon-nucleon with TPE has an additional structure $\tilde{F}'_3^N$ as shown in eq. (7). An explicit calculation of the matrix element including TPE (see Fig. 2) gives

$$\mathcal{M}'_{cD} = \frac{e^2}{Q^2} \tilde{u}(k')\gamma_\mu u(k)\xi_\rho(\lambda') \left[ \tilde{J}^{\mu\nu\rho}_{cD} + \tilde{J}^{\mu\nu\rho}_{nD} \right] \xi_{\rho}(\lambda), \quad (23)$$

where

$$\tilde{J}^{\mu\nu\rho}_{cD} = g_D^2 \int \frac{d^4k}{(2\pi)^4} \Phi_D[(k + \frac{p_D}{2})^2] \Phi_D[(k + \frac{p'_D}{2})^2] \times \text{Tr} \left( \gamma^\rho S_F(k + p'_D) \tilde{\Gamma}^\mu_\rho S_F(k) \right) \quad (24)$$

is the contribution from Fig. 2(a) and

$$\tilde{J}^{\mu\nu\rho}_{nD} = g_D^2 \int \frac{d^4k}{(2\pi)^4} \Phi_D[(k - \frac{p_D}{2})^2] \Phi_D[(k - \frac{p'_D}{2})^2] \times \text{Tr} \left( \gamma^\rho S_F(k - p'_D) \tilde{\Gamma}^\mu_\rho S_F(k - p'_D) \right) \quad (25)$$

is the one from Fig. 2(b). From the explicit expressions of eqs. (24-25), it is found that the form factor of $\tilde{F}'_3^N$ contributes to the charge, magnetic and quadrupole form factors of the deuteron. When the TPE effect is included, it provides a new form factor of the nucleon, $\tilde{F}'_3^N$. This new form factor contributes, in our approach, to the charge and quadrupole form factors of the deuteron. Particularly, it also gives a contribution to $G'_6 = G'_6^{(2)}$ with the structure of $M'_6$ and $q \cdot M'_6 = 0$. The explicit expression for the new form factor of $G'_6$ contributed by $\tilde{F}'_3^p$ is

$$G'_6 = -g_D^2 \int_0^\infty \frac{d\omega d3d\gamma}{(4\pi)^2} \Lambda_D^2 Z^2 e^{-\frac{1}{Z^2}(\frac{1}{2}x_0^2 + \Delta)} \tilde{F}'_3^p(Q^2, \epsilon) H_6^p(\alpha, \beta, \gamma, Q^2, \epsilon), \quad (26)$$

where

$$H_6^p = -4 + 4x_2 - \frac{4\Delta}{M_N^2} + \frac{4\Delta x_2}{M_N^2} - 4\frac{Q^2}{M_N^2} x_1 x_2^2 + 4\frac{M_P^2}{M_N^2} (-2x_1 x_2^2 + x_1 - x_1^2 x_2 + x_1^2 - x_2 - x_3) \quad (27)$$

with

$$Z = 2 + \alpha + \beta + \gamma, \quad \Delta = \frac{\Lambda_D^2}{Z}$$
where \( \mu_H = m_H/\Lambda_D \) with \( H = N, D \). In eqs. (26) and (29), the integration variables \( \alpha, \beta \) and \( \gamma \) are the Feynman parametrizations. The contribution from \( \tilde{F}_3^{p,n} \) to the new deuteron structure function \( G_6^{p,n} \) can be obtained from eq. (25) in the same way.

3 Numerical results

In [27], the discrepancies between OPE and TPE have been carefully analyzed for the \( ep \) elastic scattering. The empirical estimates of the TPE amplitudes in the \( ep \) elastic scattering are given based on the assumptions about the angular dependence of the amplitudes which is limited by the precision of the Rosenbluth data [28]. It is also assumed that the entire form factor discrepancies are because of the new form factor \( \tilde{F}_3 \). In a recent paper about a global analysis of the proton elastic form factor data with the two-photon exchange corrections [29], the input TPE corrections are following the formalism of Blunden et al. [5], rather than the phenomenological corrections extracted in [27]. It is found that the value of \( Y_{2\gamma} \) is much smaller than that extracted in the phenomenological analyses.

In our numerical estimates of the TPE effect on the EM form factors of the deuteron, we will also use the TPE corrections to the proton and neutron EM form factors following the formalism of Ref. [5]. It should be mentioned that the TPE corrections are \( \theta \)-dependent (or \( \epsilon \)-dependent). One can get \( G_{E,M}^{(2)}/G_{E,M} \) for the proton and neutron as well as \( \tilde{F}_3^{p,n}(Q^2, \epsilon) \) from the obtained \( Y_{2\gamma}^{p,n} \) with

\[
Y_{2\gamma} = \text{Re} \left( f \tilde{F}_3(Q^2, \epsilon) \frac{M_N^2}{G_{E,M}^{(2)}} \right) = \frac{K^0}{2M_N} \text{Re} \left( \frac{\tilde{F}_3(Q^2, \epsilon)}{G_{E,M}^{(2)}} \right),
\]

where \( f = M_N^2 \sqrt{1 + \frac{1}{\tau_N}} \sqrt{1 + \frac{\tau N}{1 + \tau N}} = \frac{1}{2} M_N K^0 \). To explicitly show the TPE effect on the deuteron EM form factors, we display, in Figs. 3-5, the ratios \( R_i = G_i^{(2)}/G_i' \) with \( i = C, M, Q \), where \( G_i^{(2)} \) are the contributions from TPE of Fig. 2, and \( G_i' \) are taken from the phenomenological parametrization of the deuteron EM form factors [30] as empirical data. In the three figures, we respectively choose the scattering angle \( \theta = \pi/6, \pi/2 \) and \( 5\pi/6 \). According to the constraint condition that the deuteron is bound as \( <|r^{-2}||<0.02 GeV^2 \) [14], we select a typical parameter \( \Lambda_D = 0.30 \text{ GeV} \). Moreover, in Figs. 6 and 7, we present our predictions for the new form factor \( G_6^{(2)} = G_6^{(2)} \) where the contributions from Fig. 2(a) and Fig. 2(b) are given, respectively.
Note that the measured EM form factors of the deuteron appear differently from those of the nucleons, since they have crossing points. The data indicate that \( G_C' \) or \( G_M' \) respectively has a crossing point at \( Q^2 \sim 0.7 \text{ GeV}^2 \) or 2.0 GeV\(^2\) [30]. Fig. 5 shows that the TPE effect on the deuteron quadrupole form factor are greatly reduced comparing to the corrections to the proton and neutron. In Figs. 3 and 4, the peaks result from the crossing points of \( G_C' \) and \( G_M' \). The two figures mean that the typical magnitudes of the ratios \( R_C \) and \( R_M \), due to the TPE corrections in our approach, are always less than 10\%, and the corrections to \( G_M' \) are more remarkable than those to \( G_C' \). Moreover, Figs. 3-7 tell that the TPE corrections to \( G_{C,M,Q}' \) and to \( G_6' \) are \( \theta \)-dependent. We also find the two contributions from the proton Fig. 2(a) and neutron Fig. 2(b) always cancel each other and make the total magnitude of \( G_6' \) being smaller.

In fact, according to the analyses of Refs. [12, 19], it is expected that the measurements of the single...
polarization observables $P_x (T_{11})$ and $P_z (T_{10})$ are useful to test the TPE effect. Here we know that the contribution of $G'_6$ to the polarization $P_x$, is

$$P_x^{(2)} \sim -\frac{4}{3} \frac{K_0}{M_D} \sqrt{\tau_D(1 + \tau_D)} \tan(\frac{\theta}{2}) G_M \text{Re}(G'_6).$$  \hspace{1cm} (31)

Comparing to the $P_x$ in the OPE approximation

$$P_x = -\frac{4}{3} \sqrt{\tau_D(1 + \tau_D)} \tan(\frac{\theta}{2}) G_M (G_C + \frac{1}{3} \tau_D G_Q),$$ \hspace{1cm} (32)

we find

$$R(P_x) = \frac{P_x^{(2)}}{P_x} = \tau_D \frac{K_0}{M_D} \frac{\text{Re}(G'_6)}{G_C + \frac{1}{3} \tau_D G_Q}.$$ \hspace{1cm} (33)

The effect of $G'_6$ on the polarization of $P_x$ is shown in Fig. 8 in the three cases of $\theta = \pi/6$, $\pi/2$ and $5\pi/6$. One sees the ratios are less than 1% in the range of $0.5 \leq Q^2 \leq 3 \text{GeV}^2$. In Fig. 8 the maximum points are expected to result from the minimum point of $G_C + \frac{1}{3} \tau_D G_Q$.

Moreover, the polarization $P_z$ in OPE is

$$P_z = \frac{1}{3} \frac{K_0}{M_D} \sqrt{\tau_D(1 + \tau_D)} \tan^2(\frac{\theta}{2}) G_M^2$$ \hspace{1cm} (34)

and the contribution to $P_z$ from $G'_6$ is

$$P_z^{(2)} = -\frac{4}{3} \sqrt{\tau_D(1 + \tau_D)} G_M \text{Re}(G'_6)$$ \hspace{1cm} (35)

which is also $\theta$-dependent since $G'_6$ is. In the small angle limit, the contribution of OPE to $P_z$ vanishes and the one from $G'_6$ remains no-vanishing to the contrary. Therefore, it is expected that the measurement of this polarization, $P_z$, in the small $\theta$ limit can easily show the TPE effect. For the ratio, we get

$$R(P_z) = \frac{P_z^{(2)}}{P_z} = -4 \tau_D \frac{M_D}{K_0} \frac{\text{Re}(G'_6)}{\tan^2(\frac{\theta}{2}) G_M}.$$ \hspace{1cm} (36)

In Fig. 9, we display the ratio of $R(P_z)$ in the three cases of $\theta$. A larger TPE effect on $P_z$ than on $P_x$ is seen since the denominator of the ratio in eq. (36) is proportional to $\tan^2(\frac{\theta}{2})$.

4 Conclusions

To summarize, we have explicitly given the TPE corrections to the conventional form factors of the deuteron $G'_{C,M,Q}$. In our approach, the TPE corrections to the nucleon EM form factors (see Fig. 2) are considered. We find that the new form factor of the nucleon with TPE, $\tilde{F}_3^{\text{TPE}}$, not only contributes to the form factors $G'_{C,Q}$ of the deuteron, but also provides a new form factor of the deuteron $G'_6$. According to the formalism of Ref. [5], we numerically estimate the TPE effect on the deuteron EM form factors, and we get the $\theta$-dependences for all the TPE corrections. It is suggested the TPE effect can be tested in the measurement of the single polarization of $P_z (T_{10})$ in the small angle limit. In addition, we find that the TPE corrections to $G'_M$ are more important than those to $G'_{C,Q}$. 

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Of course, the TPE effect, we considered in this work, only results from the sources of the direct couplings of the two photons to one of the two nucleons inside the deuteron (Fig. 2). There are several other sources of TPE which could be included in our future calculation as next step. An overall estimate of all the TPE corrections to the deuteron form factors is in progress.

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Figure 8: $R(P_x)$

Figure 9: $R(P_z)$

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