Exploring Framed Gauge Theory as Basis for Physical Models

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Abstract

It is shown that by introducing as dynamical variables in the formulation of gauge theories the frame vectors (or vielbeins) in internal symmetry space, in addition to the standard gauge boson and matter fermion fields, one obtains: (i) for the $su(2) \times u(1)$ symmetry, the standard electroweak theory with the Higgs field thrown in as part of the framed gauge theoretical structure, (ii) for the $su(3) \times su(2) \times u(1)$ symmetry, a “framed standard model” with, apart from the Higgs field as before, a global $su(3)$ symmetry to play the role of fermion generations, plus some other properties which are shown elsewhere to give to both quarks and leptons hierarchical mass and mixing patterns similar to those experimentally observed. Besides, the “framing” of the standard model as such has brought the particle theory closer in structure to the theory of gravity where vierbeins have long figured as dynamical variables. Although most of the results have already been reported before, time and hindsight have allowed their presentation in this review to be made more transparent and succinct.
1 Introduction

It can be said that all physical theories we know today are gauge theories. The standard model of particle physics is a *bona fide* gauge theory which has yielded up to the present an excellent description of all particle physics data, i.e. in effect, we believe, all known physical phenomena besides gravity, while gravity itself can be considered as a sort of gauge theory as well. Unfortunately, this rather grandiose picture is spoiled by the following facts. On the one hand, in present formulations of the standard model, besides gauge principles based on geometry, some extraneous phenomenological inputs are needed, such as a Higgs scalar to break the electroweak symmetry, and 3 generations of fermions together with all their peculiar mass and mixing patterns. On the other hand, the interpretation of gravity as a gauge theory involves further twists, namely that the gauge group has to be soldered to space-time and that the space-time metric is to function as dynamical variables. It would be conceptually much more appealing if one can somehow remove from the standard model the necessity for those injections from experiment, and at the same time to put it with gravity in some common gauge theoretical framework. Even practically, it may be rewarding if this can be done, for it would very likely help to reduce the large number of parameters needed to be fed in from experiment by the standard model, which subtracts much from our faith in it as a candidate for a fundamental theory.

The purpose of this paper is to review a proposal first made some years ago [1] and further developed since, which will be referred to here as the framed gauge theory (FGT) framework. This suggests that one includes as dynamical variables in the formulation of a gauge theory the frame vectors in internal symmetry space, in addition to the usual gauge vector boson and matter fermion fields. These frame vectors or vielbeins, one can argue, are an integral part of the gauge structure since they have to be there to specify the frames to which the gauge transformations refer. That they can be taken as dynamical variables is familiar already in the theory of gravity where vierbeins are often taken in place of the metric as dynamical variables. Indeed, by introducing now the frame vectors (or “framons”) also in particle physics as dynamical variables, one might have made the first tentative step towards its desired rapprochement with gravity, for without the framons, there has been so far in the standard model no analogue to the metric in gravity.

But of what good is FGT to particle physics itself? Consider first just
the electroweak sector. There, experiment tells us that the gauge symmetry $su(2) \times u(1)$ has to be broken, and for that one introduces in the standard electroweak theory an $su(2)$ doublet of Higgs scalar fields. But that is exactly what a fram on in $su(2)$ would look like, for being a frame vector, it transforms as the fundamental representation of the symmetry, i.e., a doublet in $su(2)$, but under Lorentz transformations in space-time, it is a scalar. In other words, by adopting the FGT philosophy, one would obtain as part of the gauge structure automatically the Higgs scalar that one needs. There are, of course, two frame vectors in $su(2)$ which are orthogonal to each other, but these are neatly paralleled by the quantities usually denoted by $\phi$ and $\phi^c$ in the standard electroweak theory, which are what appear in the Yukawa term coupled respectively to the up and down components of the quark or lepton fields. That being the case, one would not be too surprised to find, as will be shown later, that in the FGT language the standard electroweak theory appears just as the “minimally framed” gauge theory with gauge symmetry $su(2) \times u(1)$, but with the Higgs boson now forming an integral part of the gauge structure.

But what about the strong sector? If one were to take there the frame vectors in $su(3)$ colour as dynamical variables as FGT implies, would it not break the colour symmetry where we want colour to be confining and exact? Intriguingly, this need not be the case. Return first to the electroweak theory, where we recall that in place of the picture usually adopted of the local gauge symmetry $su(2) \times u(1)$ being broken spontaneously by Higgs fields, one could equally have adopted instead the picture \[2, 3\] where the local symmetry is confining; what is broken is only a global symmetry which one may call the “dual” $\tilde{su}(2) \times \tilde{u}(1)$ to the local one above. Indeed, one may even prefer this “confinement picture” to the usual one of spontaneous breaking of the local symmetry as a physically more appealing interpretation of the same symmetry-breaking phenomenon. Applying then this confinement picture to colour $su(3)$ above, one concludes correspondingly that what FGT implies is only that there is a global symmetry $\tilde{su}(3)$ “dual” to the original local colour $su(3)$ that is broken, while $su(3)$ colour itself remains confining and exact.

The appearance of a new global $su(3)$ symmetry in particle theory would in fact be welcome, for the existence of 3 fermion generations in nature has already suggested to many such a symmetry \[4\]. But can the particular $\tilde{su}(3)$ here play the desired role of fermion generations? This symmetry arises in FGT simply from the fact that frame vectors, by their very nature, have to carry two types of indices, one type referring to the local and the
other to a global reference frame. Recall as example the vierbeins in gravity, usually denoted as $e^a_\mu$ and labelled by the two types of indices $\mu$ and $a$. Since physics should be independent of the choice of reference frames, gravity is invariant under Lorentz transformations in $a$. For the same reason then, particle physics should also be invariant under $\tilde{su}(3)$. Initially, only framons carry this global index, say $\tilde{a} = 1, 2, 3$, while all other fundamental, including fermionic, fields would carry only the local index, say $a = 1, 2, 3$. Remember, however, that in the confinement picture of the electroweak theory \cite{2,3}, quarks and leptons appear not as fundamental fermion fields but as bound states, via $su(2)$ confinement, of these with the fundamental scalars, i.e. in the FGT scenario with the framons, and can acquire therefore from the latter the 3-valued global index $\tilde{a}$ to play the role of fermion generations.

In other words, it would appear that the FGT framework contains in it already the ingredients for providing not only a Higgs field necessary for breaking the electroweak symmetry but also exactly 3 generations of quarks and leptons as experiments seem to demand. But can it really do so in practice? Encouraged by the above observations, let us proceed to construct, as one did successfully, it seems, for the electroweak theory, the corresponding “minimally framed” theory for the gauge symmetry $su(3) \times su(2) \times u(1)$. One obtains then a construct which one can call the framed standard model (FSM). As will be shown later, this is found indeed to give the Higgs field and 3 generations of quarks and leptons as expected. Moreover, it is found that these quarks and leptons appearing as fermion-framon bound states via $su(2)$ confinement have mass matrices at tree level of the form:

$$m = m_T \alpha \alpha^\dagger$$  \hspace{1cm} (1)

where the vector $\alpha$ is “universal”, meaning that it is independent of the fermion species, i.e. whether it is up or down in flavour or whether it is a quark or a lepton. Now, such a “universal” rank-one mass matrix, giving only one heavy state in each species and the unit matrix as the mixing matrix, has long been advocated \cite{5,6} as a good zeroth-order starting point since it is already not that far from the actual situation observed in experiment.

But this is not all. The FSM is by construction invariant, as it ought to be by previous arguments, under both the original local gauge symmetry $su(3) \times su(2) \times u(1)$, and its “dual”, i.e. the global symmetry $\tilde{su}(3) \times \tilde{su}(2) \times \tilde{u}(1)$. This doubled invariance places severe restrictions on the form that terms of the action containing the framon fields can take, in particular on
the self-interaction potential, say $V[\Phi]$, of the framon field $\Phi$. This $V[\Phi]$, on minimization, will tell us what the vacuum will look like, and will allow us further to evaluate renormalization effects on the vacuum and hence also on the vector $\alpha$ from the mass matrix (1). And these renormalization effects are found to give automatically deviations from the zeroth order approximation above and lead to a hierarchical mass spectrum for both quarks and leptons, as well as mixing matrices qualitatively similar to that experimentally observed with CP-violation included.

In other words, it would appear that the FSM when formulated as an FGT is capable of reproducing all those idiosyncrasies of the standard model mentioned at the beginning such as the Higgs boson and the 3 fermion generations together with their mass and mixing patterns as consequences of the gauge principle as wanted. In what follows, we shall review the procedure whereby those of the above results concerning the structure of the model are deduced, i.e. all apart from those outlined in the last paragraph which are derived from the dynamics. Although most of the reviewed material has appeared in some form or another before, e.g. in [7], time and experience have given it the greater clarity and cogency now needed when viewed in the present wider context. The derivation of results outlined in the last paragraph is not reviewed because it has been completed only recently and is reported in separate papers [8, 9] to which the reader can be referred.

One turns instead to the other structural question raised at the beginning, namely whether particle physics when formulated now as an FGT can be put with gravity on a common gauge theoretical footing. As already noted, by the introduction of frame vectors as dynamical variables in particle physics one has already taken a first step towards a possible rapprochement with gravity, for this will mean that particle theory will now acquire also a variable metric, though here not in space-time but in internal symmetry space. However, if one were to borrow the Kaluza-Klein idea that internal space may be part of a larger space-time compactified to a very small size, then a metric in space-time and in internal space may not appear as so very different concepts. Indeed, building on this in the last section, one is led to some interesting speculations on how the two sides, i.e. particle physics on the one hand and gravity on the other, may perhaps be brought closer together as just two different parts of the same overarching “framed” gauge theoretical framework.

Taken all together, these considerations, to be expanded below, would seem to suggest that the FGT framework could perhaps be taken with some
credibility, or at least considered worth exploring, as a viable basis for the physics we know today.

2 Frames and Minimal Frames

First, we need to make precise what is meant above by framing. Apart from gravity, the framing of which in terms of vierbeins is already familiar, what interests us here is the framing of the standard model of particle physics, i.e. the gauge theory with gauge symmetry $su(3) \times su(2) \times u(1)$. Before starting on this, however, let us first consider each of its 3 component symmetries, namely $su(3)$, $su(2)$ and $u(1)$.

In the notation here adopted, lower case letters as in $su(N)$ denote the algebra but capital letters as in $SU(N)$ the group. The same algebra, of course, may correspond to different groups; thus, for example, both $SU(2)$ and $SO(3)$ have $su(2)$ as their algebra, but $SU(2)$ double covers $SO(3)$. What specifies which group one is dealing with in a given theory is the representations which appear in the theory. For instance, only the $SU(2)$ theory, not $SO(3)$, has doublets as representations. Since the theories we are interested in all contain fields in the fundamental representation of $su(N)$, it is with those with gauge groups $SU(N)$ that we shall be concerned.

As for the vierbeins $e^a_{\mu}$ in gravity, frame vectors in an $SU(N)$ theory can be taken as the column vectors of the matrix relating the local frame to the global reference frame, say:

$$\Phi = (\phi_{\tilde{a}}^a),$$

where the row index $a$ refers to the local frame and the column index $\tilde{a}$ to the global reference frame. Since both the local and global frames here are unitary, the matrix $\Phi$ is itself an element of $SU(N)$. Local $SU(N)$ transformations on $\Phi$ act from the left while global $SU(N)$ transformations act from the right.

The frame vectors $\phi^\tilde{a}$ in $su(N)$ space labelled by index $\tilde{a}$ transform as fundamental representations under the local $su(N)$, but are scalars under space-time Lorentz transformations, and they satisfy originally, of course, the following conditions:

(a) They have unit length, $|\phi^\tilde{a}| = 1$;

(b) They are mutually orthogonal, $\phi^\tilde{a} \cdot \phi^{\tilde{b}} = 0$, $\tilde{a} \neq \tilde{b}$;
(c) The determinant is real.

But, in adopting them as dynamical variables as proposed, we are in effect promoting them into fields. They should then be allowed to take any complex values as ordinary scalar fields do, in which case they will not be able to satisfy all 3 conditions above. However, these \( \phi^a \) need not all be taken as independent variables, so that some of the conditions can still be retained. We ask then what is the most economical arrangement with the smallest number of “framos” introduced. We shall say then that the resulting theory is “minimally framed”.

Consider first \( SU(2) \) as an example. There are 2 frame vectors \( \phi^1 \) and \( \phi^2 \) which satisfy the 3 conditions (a), (b) and (c). To allow them to take any complex values when promoted to framon fields as stipulated, we must of course relax the condition (a), but we can still keep the other 2 conditions. Explicitly, since the framons are not required to be independent, we can write:

\[
\phi^2_r = -\epsilon_{rs} (\phi^1_s)^*,
\]

which will keep the 2 framons orthogonal and of equal length while keeping also the determinant real, but will still allow both to take all complex values. One needs to introduce then as framons in the minimally framed theory only 1 complex vector or 4 real scalars. Alternatively, one can think of this as a problem of embedding \( SU(2) \) in \( \mathbb{R}^n \), where it is well known that the minimal embedding is as the unit sphere in \( \mathbb{R}^4 \).

Consider next \( SU(3) \), where there are 3 frame vectors. To promote these into fields we must again relax condition (a), but we can no longer retain the condition (b) that the framons remain mutually orthogonal as we did for \( SU(2) \). Indeed, if we write down here the parallel to (3), thus:

\[
\phi^3_r = \epsilon_{rst} (\phi^1_s)^* (\phi^2_t)^*,
\]

we find that this will give the 3 framons enforcedly different physical dimensions, which we cannot accept. We are left then with only condition (c), namely that the determinant is real, which still allows the different framons to have the same physical dimension since the determinant, though complicated, is multilinear in all its elements. The same conclusion holds for any \( SU(N), N > 2 \).

One concludes therefore that for the “minimally framed theory” one needs to introduce only 4 real scalar fields as framons for the \( SU(2) \) theory, but for
$SU(N), N > 2$ theory one needs in general $2N^2 - 1$ real scalar fields, e.g. 17 for $SU(3)$.

This leaves now only the $u(1)$ factor still to be considered. Here orientation means just a phase, and relative orientation just a phase difference. Hence, the analogue of $\Phi$ above for the $su(N)$ factors is here a phase factor of the form:

$$\Phi = \exp ig_1 (\alpha - \tilde{\alpha}),$$

with $\alpha$ $x$-dependent, transforming under the local $u(1)$ but $\tilde{\alpha}$ $x$-independent, transforming under the global $\tilde{u}(1)$. The framon field is then a complex scalar field with its phase as in (5) above.

Having now specified what framing means for each of the theories with the symmetries $su(2), su(2)$ and $u(1)$, we are ready to tackle the physical theories with the product symmetry.

### 3 The Electroweak Theory

As a warming-up exercise before we proceed to the full standard model, let us first consider the electroweak theory, which is a gauge theory with the gauge symmetry $su(2) \times u(1)$. As it is usually formulated, the standard gauge principles have to be supplemented by the introduction of the Higgs scalar to break the $su(2)$ symmetry as required by experiment. Here we wish to approach the problem anew from the viewpoint of framed gauge theory (FGT), and try to show that the same electroweak theory will emerge as the minimally framed gauge theory for the same gauge symmetry, but now with the Higgs scalar thrown in as an integral part of the framed gauge theoretical framework.

In the same spirit then as what was done above in section 2 for the $su(N)$ symmetries, we ask first what scalar framon fields are to be introduced for the theory to be “framed”. The frame vectors, and hence also the framons to which they are promoted, are to be representations of the symmetry $su(2) \times u(1)$, and there are two ways of building representations of a product from those of its factors, namely as the product or as the sum. Suppose we appeal again to economy, or “minimality” for whatever it is worth, and ask which gives the smaller number of scalar fields, we would favour the product, $2 \times 1$ being smaller than $2 + 1$. Hence, one would introduce as framon fields two $su(2)$ doublets: $\phi^\tilde{r}$ labelled by the global index $\tilde{r} = \tilde{1}, \tilde{2}$, each forming also a
representation of the local $u(1)$ symmetry, or in other words, each carrying also a $u(1)$ (hyper-)charge.

What $u(1)$ charges should they carry? To answer this, again a question of representations, one would need first, as in the $su(N)$ symmetries above, to specify the gauge group. There are 3 groups all having $su(2) \times u(1)$ as its algebra, namely $SU(2) \times U(1)$, $SO(3) \times U(1)$ and $U(2) = (SU(2) \times U(1))/\mathbb{Z}_2$.

By examining the fields present in the standard electroweak theory, noting for example that $su(2)$ doublets exist with half-integral $u(1)$ (hyper-)charges, one comes to the conclusion that $U(2)$ is the gauge group one needs. From this, it follows that the framon fields $\phi^{\tilde{r}}$ above should also carry half-integral $u(1)$ charges.

Moreover, from the last section, one has learned that for $su(2)$, one can further economize on the number of framons introduced by insisting that $\phi^{\tilde{r}}$ satisfy the orthogonality condition, leaving then only one doublet independent, i.e. in total just 2 complex or 4 real scalar fields as variables. This means also that whatever $u(1)$-charge $\phi^{1}$ carries, then $\phi^{2}$ would carry the opposite. Note however that $\phi^{1}$ and $\phi^{2}$ need not themselves be eigenstates of the $u(1)$-charge, which can be chosen instead as any two mutually orthogonal linear combinations. These we can specify by introducing a vector, say $\gamma = (\gamma^{\tilde{r}})$ in $\tilde{su}(2)$ space, such that the following vectors, now back in $su(2)$ space, are eigenstates of the $u(1)$-charge with the (hyper-)charges shown:

\[
\phi^{(+)} = \sum_{\tilde{r}} \gamma^{\tilde{r}} \phi^{\tilde{r}}; \quad y = g_{1}/2, \\
\phi^{(-)} = \sum_{\tilde{r}} \gamma'^{\tilde{r}} \phi^{\tilde{r}}; \quad y = -g_{1}/2, \tag{6}
\]

where $\gamma'$ is the vector orthogonal to $\gamma$ in $\tilde{su}(2)$ space. This notation is useful later when invariance under $\tilde{su}(2)$ is considered. Otherwise, it is convenient to choose a frame in $\tilde{su}(2)$ space such that $\gamma$ points in the $(1, 0)$ direction so that $\phi^{\tilde{1}}$ coincides with $\phi^{(+)}$ as is often done in the literature. Notice that $\phi^{(+)}$ has exactly the same $su(2)$ representation and $u(1)$ charges as the Higgs field in the standard electroweak theory.

Next, we turn to the action for the framed theory which we want to be invariant not only under the local gauge symmetry $su(2) \times u(1)$ we started with, but also under the global symmetry $\tilde{su}(2) \times \tilde{u}(1)$. Since only the framon fields carry the global indices, we need consider here only those new terms of the action which contain the framons, as the others, such as the gauge field
action or the kinetic energy term of the fermions, will be the same as in the standard electroweak theory.

Of the new terms containing framons, consider first the potential term of framon self-interaction which one can take, as usual, to be a polynomial of even powers of the framon fields, but only up to and including quartic terms for renormalizability. To ensure invariance under the double symmetry $su(2) \times u(1) \times \tilde{su}(2) \times \tilde{u}(1)$, we saturate all indices in all possible ways, and end up with the following general form:

$$V[\Phi] = -\mu \sum_{r,\tilde{r}} (\phi_r^\dagger \phi_{\tilde{r}})^2 + \lambda \left( \sum_{r,\tilde{r}} (\phi_r^\dagger \phi_{\tilde{r}}) \right)^2 + \kappa \sum_{r,s,\tilde{r},\tilde{r}} (\phi_r^\dagger \phi_{\tilde{r}})(\phi_s^\dagger \phi_{\tilde{r}}), \quad (7)$$

which can be written more succinctly in terms of the matrix $\Phi$ introduced in (2) above as:

$$V[\Phi] = -\mu \text{tr}(\Phi^\dagger \Phi) + \lambda \left( \text{tr}(\Phi^\dagger \Phi) \right)^2 + \kappa \text{tr}(\Phi^\dagger \Phi \Phi^\dagger \Phi). \quad (8)$$

As written, this depends on both the vectors $\phi^\dagger$ and $\phi^\tilde{2}$, of which, however, only one is independent. Eliminating, say, $\phi^\tilde{2}$ in terms of $\phi^\dagger$ using the orthogonality condition (3), one is left with the usual Mexican hat potential of the standard electroweak theory with $\phi^\dagger$ identified with the Higgs field there. That this is so can most easily seen by again rewriting (7) in terms of the vectors $\phi^\tilde{r} = (\phi_r^\tilde{r})$, thus:

$$V[\Phi] = -\mu \sum_{\tilde{r}} |\phi^\tilde{r}|^2 + \lambda \left( \sum_{\tilde{r}} |\phi^\tilde{r}|^2 \right)^2 + \kappa \sum_{\tilde{r},\tilde{s}} |(\phi^\tilde{r})^\dagger \phi^\tilde{s}|^2. \quad (9)$$

Since what the condition (3) says is that the two vectors $\phi^\dagger$ and $\phi^\tilde{2}$ are mutually orthogonal and have the same length, it follows immediately that (8) is reduced to:

$$V[\Phi] = -2\mu |\phi|^2 + (4\lambda + 2\kappa)|\phi|^4 \quad (10)$$

as claimed.

Secondly, consider the kinetic energy term of the framons, which we can write most succinctly in terms of the matrix $\Phi$ as:

$$\text{tr}((D_\mu \Phi)^\dagger D_\mu \Phi), \quad (11)$$
with
\[ D_\mu \Phi = \partial_\mu \Phi - ig_2 B_\mu \Phi - i \frac{1}{2}ig_1 A_\mu \Phi \Gamma, \]
where \( \Gamma \) is the matrix:
\[ \Gamma = \left( \gamma, -\gamma' \right) = \left( \begin{array}{cc} \gamma^1 & -\gamma'^1 \\ \gamma^2 & -\gamma'^2 \end{array} \right), \]
so that by construction:
\[ \Phi \Gamma = \left( \phi^{(+)}, -\phi^{(-)} \right) = \left( \begin{array}{c} \phi_1^{(+)} \\phi_2^{(+)} \\ \phi_1^{(-)} \\phi_2^{(-)} \end{array} \right), \]
thus giving the correct \( u(1) \) charges, \( \pm g_1/2 \) respectively, to the two vectors \( \phi^{(\pm)} \). The term (11) is explicitly invariant both under all local \( su(2) \times u(1) \) and under all global \( \tilde{su}(2) \times \tilde{u}(1) \) transformations, as required.

To show that the term (11) above under the condition (3) is in fact the same as the corresponding term in the standard electroweak theory, all we need is to choose \( \gamma \) to be real and point in the up direction, making thus \( \phi^{(+)} \) the same as \( \phi^{1 \dagger} \). A direct calculation then shows that, because of (3), the two terms coming respectively from \( \phi^{(+)} \) and \( \phi^{(-)} \) are in fact identical and add up for \( \phi^{(+)} = \phi \) to just:
\[ 2(D_\mu \phi)^\dagger D_\mu \phi, \]
with
\[ D_\mu = \partial_\mu - ig_2 B_\mu - i \frac{1}{2}ig_1 A_\mu, \]
i.e. of the form familiar in the standard electroweak theory.

Lastly, we need to consider the Yukawa term coupling the framon to the fermion fields, which we can write as usual as:
\[ Y \bar{\psi} r^\dagger \phi^{(-)} \frac{1}{2}(1 + \gamma_5) \psi + Y' \bar{\psi} r^\dagger \phi^{(+)} \frac{1}{2}(1 + \gamma_5) \psi' + \text{h.c.} \]
with only the proviso that \( \phi^{(\pm)} = (\phi r^{(\pm)}) \) is now to be taken in general as (6) above which exhibits its required invariance also under \( \tilde{su}(2) \).

One sees therefore that in the framed gauge theory language, the minimally framed theory with gauge symmetry \( su(2) \times u(1) \) is indeed just the standard electroweak theory as claimed, but now with the framon field playing the role of the standard Higgs scalar already built in as part of the gauge.
structure. For the electroweak theory itself, this is merely a formal gain, but as we shall see, when the same considerations are applied to the standard model, we shall arrive at more substantial results.

Before we proceed further, however, let us first recall some old results of 't Hooft [2] and of Banks and Rabinovici [3] which will be of use later. The theory given by the action just derived for the symmetry $su(2) \times u(1)$ is usually interpreted as one in which the local symmetry is spontaneously broken. But, as these authors have shown, it may equally be interpreted as a theory in which the $su(2)$ symmetry confines and remains exact; what is being broken is only a global symmetry associated with it which can be identified with what is denoted by $\tilde{su}(2)$ above. This global symmetry is broken explicitly by the choice of the vector $\gamma$ in $\tilde{su}(2)$ space which specifies the eigenstates with a definite $u(1)$ charge $\pm g_1/2$. So it can be said, as did 't Hooft, that it is electromagnetism which breaks that global symmetry. In this interpretation, or “confinement picture” as it will be called in what follows, only $su(2)$ neutral states can appear as physical particles, hence neither the $su(2)$ doublet scalar and fermion fields, nor the $su(2)$ triplet gauge boson fields, can appear as free particles. The physical states known to us are all bound states formed via $su(2)$ confinement out of the fundamental scalar and fermion fields. Thus, for example, the Higgs scalar $h$ and the vector bosons $W-Z$ appear as respectively the “$s$-wave” and “$p$-wave” bound states of a framon-antiframon pair:

$$tr(\Phi^\dagger \Phi) \sim F^2 + 2Fh + \ldots,$$

$$\Phi^\dagger (\partial_\mu - ig_2 B_\mu) \Phi \sim ig_2 \tilde{B}_\mu,$$

where $F$ represents the vacuum expectation value of the framon field, while the leptons and quarks appear as bound states of a framon with a fundamental fermion, respectively:

$$\Phi^\dagger \psi \sim \chi,$$

$$\Phi^\dagger \psi_a \sim \chi_a,$$

with $a$ in the latter the colour index. Notice that although both these leptons and quarks are by construction singlets in $su(2)$, they are both doublets in $\tilde{su}(2)$, having each acquired from its framon constituent an $\tilde{su}(2)$ index, and it is the latter global symmetry which now plays the role of the up-down flavour in the confinement picture. Hence, this symmetry being broken by
electromagnetism as explained above, it will give different masses for up and down flavoured states. Although in the way the theory is at present applied, the confinement picture is mathematically equivalent \[2\] to the usual spontaneous breaking picture as interpretations of the same electroweak theory, some may find one more physically appealing than the other. In what follows for the standard model, we shall adopt the confinement picture as the more convenient for our purpose.

4 The Framed Standard Model

The standard model is a gauge theory with the gauge symmetry \( su(3) \times su(2) \times u(1) \). Our first question, as with the electroweak theory before, is what scalar framon fields are to be introduced so as to make the theory “framed”. Framons are to be representations of the local gauge symmetry. Again, for a product symmetry, there are two choices, either the sum or the product representation, and this applies to the product between any pair. If we appeal as before to economy for the smallest number of real scalar framon fields we have to add, we shall end up again with the product representation for both \( su(3) \times u(1) \) and \( su(2) \times u(1) \), but the sum representation for \( su(3) \times su(2) \) since \( 3 + 2 < 3 \times 2 \). In other words, we shall end up with the overall representation \((su(3) + su(2)) \times u(1)\), i.e. \((3 + 2) \times 1\).

Next, when taken together, as in section 2, the framons should form a matrix transforming from the left as a representation of the local symmetry, here \( su(3) \times su(2) \times u(1) \), but from the right as a representation of its “dual”, i.e. the global symmetry \( \tilde{su}(3) \times \tilde{su}(2) \times \tilde{u}(1) \). So again the question arises as to which representation of the global symmetry it should belong. Here, the criterion of “minimal framing” is no guide, since the symmetry being global, whatever choice of representation will lead to the same number of scalar framon fields. If one were to choose \((\tilde{3} + \tilde{2}) \times \tilde{1}\), identical to the choice above for the local symmetry, the theory would just break up into two separate theories, i.e. the electroweak theory plus chromodynamics disjoint from each other, which is neither an interesting nor a realistic situation. One opts instead therefore, by invoking the anthropic principle perhaps, for the more interesting all-product representation \(\tilde{3} \times \tilde{2} \times \tilde{1}\).

As the result of these considerations, one is then led to introduce, for the minimally framed gauge theory with gauge symmetry \( su(3) \times su(2) \times u(1) \), the following two types of framons; first the “weak” framons which transform
as doublets under local $su(2)$ but are invariant under local $su(3)$:

$$\phi_r^{\bar{a}} = \alpha^{\bar{a}} \phi_r^{r}, \quad \bar{a} = 1, 2, 3, \quad r = 1, 2, \quad \tilde{r} = \tilde{1}, \tilde{2}, \quad y = \pm 1/2, \quad \tilde{y} = \mp 1/2, \quad (22)$$

and secondly the “strong” framons which transform as triplets under local $su(3)$ but are invariant under local $su(2)$:

$$\phi_a^{\bar{a}} = \beta^{\bar{a}} \phi_a^{a}, \quad a = 1, 2, 3, \quad \bar{a} = 1, 2, 3, \quad r = 1, 2, \quad y = -1/3, \quad \tilde{y} = 1/3. \quad (23)$$

The $\phi$’s in (22) and (23) are local, i.e. $x$-dependent, quantities, while the $\alpha$’s and $\beta$’s are global, i.e. $x$-independent, with $\phi_r^{\bar{a}}, \beta^{\bar{a}}, \tilde{r} = \tilde{1}, \tilde{2}$ transforming as doublets under $\tilde{su}(2)$ and $\phi_a^{\bar{a}}, \alpha^{\bar{a}}, \tilde{a} = \tilde{1}, \tilde{2}, \tilde{3}$, transforming as triplets under $\tilde{su}(3)$. Hence both types of framons, by construction, transform as the product representation $\tilde{3} \times \tilde{2}$ under the global symmetry $\tilde{su}(3) \times \tilde{su}(2)$, as stipulated, but we have yet to justify the assignments above in (22) and (23) for their $u(1)$ charge $y$ and $\tilde{u}(1)$ charge $\tilde{y}$.

As in the electroweak theory before, to assign them appropriate $u(1)$ charges, one needs first to specify the gauge group. There are again several groups corresponding to the algebra $su(3) \times su(2) \times u(1)$, but by examining the representations of all the fields appearing in the standard model, one concludes [10] that the gauge group is that group obtained by identifying in the covering group $SU(3) \times SU(2) \times U(1)$ the following sextets of elements:

$$(c, f, y) = (\omega c, f, z^4 y) = (\omega^2 c, f, z^2 y)$$

$$= (c, -f, z^3 y) = (\omega c, -f, z y) = (\omega^2 c, -f, z^3 y), \quad (24)$$

where $c, f, y$, are elements in respectively in the groups $SU(3), SU(2)$, and $U(1)$, and

$$z = \exp i \pi / 3 \quad (25)$$

with $\omega$ being the cube root of unity, which group one can call here $U(3, 2, 1)$. With $U(3, 2, 1)$ as gauge group, the allowed representations are [10]:

$$(1, 1); \quad y = 0 + n,$$

$$(1, 2); \quad y = \frac{1}{2} + n,$$

$$(3, 1); \quad y = -\frac{1}{3} + n,$$

$$(3, 2); \quad y = \frac{1}{6} + n, \quad (26)$$

where the first number inside the brackets denotes the dimension of the representation of $su(3)$ and the second number that of $su(2)$, and $n$ can be
any integer, positive or negative. It then follows that one has the \( u(1) \) and \( \tilde{u}(1) \) charges for the framons as given in (22) and (23), where we have kept for simplicity only those charges with the smallest allowed absolute values.

These are then the framon fields that are to be introduced for the minimally framed theory with gauge symmetry \( su(3) \times su(2) \times u(1) \). They are not all independent, but according to the analysis in section 2, the weak framons \( \phi^r_a \) are to satisfy the condition (3) while the strong framons \( \phi^a_r \) the condition that their determinant is real, leaving thus altogether 21 independent real scalar fields in the theory.

A distinguishing feature of these framon scalars, of course, is that they carry, in addition to the local indices \( r \) and \( a \) which they share with the standard gauge boson and matter fermion fields, the global indices \( \tilde{r} \) and \( \tilde{a} \). One needs to ask then what physical significance these global indices possess. In other words, one would wish to know which physical particles carry these global indices as quantum numbers. Recalling now the confinement picture of 't Hooft and of Banks and Rabinovici for the interpretation of symmetry-breaking in the electroweak theory, one sees that none of the framon fields as listed can manifest themselves as actual particles, since they all carry \( su(2) \) and \( su(3) \) indices and have to be confined. They have to form bound or confined states either with each other or with other fields carrying these indices, and only those states thus formed which are neutral under both \( su(2) \) and \( su(3) \) can appear as actual particles. Those confined by \( su(2) \) would appear to us now, at the present level of our experimental capability, as elementary, but those confined by colour \( su(3) \) alone are hadrons, which we have learnt already by experiment to penetrate and resolve into their coloured constituents. At this level then, which we may call the standard model scenario, we need take account only of the deeper confinement by \( su(2) \). Let us then ask in this standard model scenario, what \( su(2) \)-neutral bound states will appear which have weak framons as constituents and which will carry, by virtue of the weak framon(s) they contain, these global indices as quantum numbers. The examination of such particles would reveal to us the sought-for physical significance, if any, that these global indices possess.

First, the weak framons can form \( su(2) \)-neutral bound state with their own conjugates via \( su(2) \) confinement, saturating thereby the \( su(2) \) indices \( r \) which appear in (22) as follows:

\[
h_W = \sum_{\tilde{r} r \tilde{a}} \alpha^*_{\tilde{r}} \phi^r_\tilde{r} \phi^\alpha_\tilde{a}, \quad (27)
\]
or else via the $su(2)$ gauge bosons $B_\mu$ as follows:

$$W_\mu = \sum_{\tilde{a}rs\tilde{r}} \alpha_\tilde{a}^*(\phi_{\tilde{r}}^\dagger)(\partial_\mu - ig_2B_\mu^{rs})\phi_{\tilde{r}}^\dagger\alpha_\tilde{a}. \quad (28)$$

These are the exact parallels of (18) and (19) in section 3 for the electroweak theory, called by 't Hooft there respectively the $s$-wave and $p$-wave bound states, of the framon-antiframon pair. We notice that the extra global indices $\tilde{a}$ are here summed over and do not in the end figure, and one has not yet learned anything new about them.

Secondly, again as in the electroweak theory, in parallel to (20) and (21) in the preceding section, the weak framons of (22) can also form with the fundamental fermion fields $\psi = (\psi_r)$ and $\psi_\tilde{a} = (\psi_{ra})$ the following bound states:

$$\chi_{\tilde{r}\tilde{a}} = \sum_r \alpha^{s\tilde{a}}(\phi_{\tilde{r}}^\dagger)(\partial_\mu - ig_2B_\mu^{rs})\phi_{\tilde{r}}^\dagger\psi_r, \quad (29)$$

and

$$\chi_{r\tilde{a}} = \sum_r \alpha^{s\tilde{a}}(\phi_{r}^\dagger)(\partial_\mu - ig_2B_\mu^{rs})\phi_{r}^\dagger\psi_{ra}, \quad (30)$$

which, in the confinement picture, are to be interpreted respectively as leptons and quarks, and these now have to carry the same global quantum numbers as their weak framion constituents since their fundamental fermion constituents carry none. They will carry the 2-valued global index $\tilde{r}$ which we have already learned before to interpret as up-down flavour. But they will now also carry a new 3-valued global index $\tilde{a}$, which can play the role of the fermion generation index. Of course, as to whether $\tilde{a}$ can actually function for leptons and quarks as the generation index is a question which can only be answered by a detailed study of its properties, a question on which we shall devote much attention below, but that such an index does emerge automatically from framing seems already quite interesting.

This is not all. The weak framons (22) carry also a global $\tilde{u}(1)$ charge $\tilde{y}$. Because of the intrinsic $\tilde{u}(1)$ invariance built into the theory, this $\tilde{u}(1)$ charge is necessarily conserved. So what physical significance does it possess? Again, the $\tilde{y}$ charges from the framon-antiframon pair cancel in (27) and (28) giving a zero value for both the Higgs and the $W - Z$ bosons, but it does not now cancel in (29) and (30), since only the framon, but not the fermion, constituents carry this $\tilde{u}(1)$ charge, which thus takes the value $\tilde{y} = \pm \frac{1}{2}$ for both leptons and quarks. Recalling now from (26) that the fundamental
fermions fields $\psi_r$ and $\psi_{ra}$ carry respectively the $u(1)$ charges $y = -\frac{1}{2}, \frac{1}{2}$, one has for leptons in (29) and quarks in (30) respectively $y = -\frac{1}{2} \mp \frac{1}{2}, \frac{1}{6} \mp \frac{1}{2}$. Hence it follows that:

$$\tilde{y} = -y + \frac{1}{2}(B - L),$$

which is then the physical meaning of $\tilde{y}$ that we seek. Given that $y$ is itself a conserved quantity from the $u(1)$-invariance of the theory, it follows from the conservation of the $\tilde{u}(1)$ charge $\tilde{y}$ that the global quantum number $B - L$ has also to be conserved. It would thus seem that one has found here a gauge principle \[11\], namely $\tilde{u}(1)$-invariance peculiar to the framed gauge theoretical framework, from which baryon number conservation (in its modern form of $B - L$ conservation) would emerge as a consequence.

These conclusions on the global indices are summarized in the Table 1, where the entries in the last row are yet to be discussed. One sees that even at this stage, the FSM seems already to have offered answers to two questions posed at the beginning, first, on the origin of the Higgs boson and second, tentatively, also on the origin of fermion generations, with the unexpected bonus of $B - L$ conservation thrown in.

As above in the electroweak theory, our next objective would be to construct an action for the framed standard model based on invariance principles. Framons having been introduced as field variables in FSM, the onus is in principle upon us to include in the FSM action all terms which can be constructed with the framons (22) and (23), either by themselves or together with the other fields occurring in the theory, namely the gauge boson and matter fermion fields, so long as the action is invariant under both the original local gauge symmetry $su(3) \times su(2) \times u(1)$ and its “dual”, the global symmetry $\tilde{su}(3) \times \tilde{su}(2) \times \tilde{u}(1)$, conditional only on it being renormalizable.

As above in the electroweak theory, these terms are of three types. First

| Symmetry | $su(3)$ | $su(2)$ | $u(1)$ |
|----------|---------|---------|--------|
| Index/Charge | $\tilde{a}$ | $\tilde{r}$ | $\tilde{y}$ |
| Interpretation | fermion generation | up/down flavour | $B - L$ |
| Status | Broken by $\alpha$ (from weak sector) | Broken by $\gamma$ (from e.m. sector) | Exact |

Table 1: The global symmetries and their physical interpretations
there will be a term involving only the framons by themselves, which we shall call the framon potential \( V[\Phi] \). Secondly, there will be the framon kinetic energy terms involving the gauge bosons via the covariant derivatives of the framons. Lastly, there are the Yukawa terms coupling the framons to fermions. Of course, there will also be terms in the action containing no framons at all, but these will be the same as in the “unframed” standard model. Of the new terms containing the framon fields, we shall construct and discuss each in turn below.

Consider first then the framon potential \( V[\Phi] \) which will tell us about, among other things, the FSM vacuum. The demand for the double invariance under both the local \( su(3) \times su(2) \times u(1) \) and global \( \widetilde{su}(3) \times \widetilde{su}(2) \times \widetilde{u}(1) \) symmetries stringently constrain the sort of terms that can be constructed. Taking all terms up to fourth order in the framon fields (22) and (23) for renormalizability, and contracting all indices in every way to ensure invariance, one is led to a potential of the following form:

\[
V[\Phi] = V_W[\Phi] + V_S[\Phi] + V_{WS}[\Phi],
\]

where

\[
V_W[\Phi] = -\mu'_W \sum_{r,\tilde{r},\tilde{a}} \phi_{r}^{\tilde{r}a} \phi_{r}^{\tilde{a}} + \lambda'_W \left[ \sum_{r,\tilde{r},\tilde{a}} \phi_{r}^{\tilde{r}a} \phi_{r}^{\tilde{a}} \right]^2 + \kappa_{1W} \sum_{r,s,\tilde{r},\tilde{a},\tilde{b}} \phi_{r}^{\tilde{r}a} \phi_{r}^{\tilde{b}} \phi_{s}^{\tilde{b}a} \phi_{s}^{\tilde{a}} + \kappa_{2W} \sum_{r,s,\tilde{r},\tilde{a},\tilde{b}} \phi_{r}^{\tilde{r}a} \phi_{s}^{\tilde{b}a} \phi_{s}^{\tilde{b}a} \phi_{s}^{\tilde{a}} + \kappa_{3W} \sum_{r,s,\tilde{r},\tilde{a},\tilde{b}} \phi_{r}^{\tilde{r}a} \phi_{s}^{\tilde{b}a} \phi_{s}^{\tilde{b}a} \phi_{s}^{\tilde{a}},
\]

involves only the weak framons (22),

\[
V_S[\Phi] = -\mu'_S \sum_{a,\tilde{a},\tilde{r},\tilde{a}} \phi_{a}^{\tilde{r}a} \phi_{a}^{\tilde{r}a} + \lambda'_S \left[ \sum_{a,\tilde{a},\tilde{r},\tilde{a}} \phi_{a}^{\tilde{r}a} \phi_{a}^{\tilde{r}a} \right]^2 + \kappa_{1S} \sum_{a,b,\tilde{r},\tilde{a},\tilde{b}} \phi_{a}^{\tilde{r}a} \phi_{b}^{\tilde{b}a} \phi_{b}^{\tilde{b}a} \phi_{b}^{\tilde{a}} + \kappa_{2S} \sum_{a,b,\tilde{r},\tilde{a},\tilde{b}} \phi_{a}^{\tilde{r}a} \phi_{b}^{\tilde{b}a} \phi_{b}^{\tilde{b}a} \phi_{b}^{\tilde{a}} + \kappa_{3S} \sum_{a,b,\tilde{r},\tilde{a},\tilde{b}} \phi_{a}^{\tilde{r}a} \phi_{b}^{\tilde{b}a} \phi_{b}^{\tilde{b}a} \phi_{b}^{\tilde{a}},
\]

only the strong framons (23), and

\[
V_{WS}[\Phi] = \nu_{11} \sum_{r,a,\tilde{r},\tilde{a},\tilde{b}} \phi_{r}^{\tilde{r}a} \phi_{r}^{\tilde{b}a} \phi_{a}^{\tilde{b}a} \phi_{a}^{\tilde{a}} + \nu_{21} \sum_{r,a,\tilde{r},\tilde{a},\tilde{b}} \phi_{r}^{\tilde{r}a} \phi_{r}^{\tilde{b}a} \phi_{a}^{\tilde{b}a} \phi_{a}^{\tilde{a}} + \nu_{12} \sum_{r,a,\tilde{r},\tilde{a},\tilde{b}} \phi_{r}^{\tilde{r}a} \phi_{r}^{\tilde{b}a} \phi_{a}^{\tilde{b}a} \phi_{a}^{\tilde{a}} + \nu_{22} \sum_{r,a,\tilde{r},\tilde{a},\tilde{b}} \phi_{r}^{\tilde{r}a} \phi_{r}^{\tilde{b}a} \phi_{a}^{\tilde{b}a} \phi_{a}^{\tilde{a}},
\]
involves both, linking thus the weak to the strong sector. Next, recalling the fact that the weak framons \((22)\) are subject to the condition \((3)\), one can simplify and rewrite the 3 terms in \(V[\Phi]\) in the following forms:

\[
V_W[\Phi] = -\mu_W |\phi|^2 + \lambda_W (|\phi|^2)^2, \tag{36}
\]

\[
V_S[\Phi] = -\mu_S \sum_{a,\tilde{a}} (\tilde{\phi}_a^* \tilde{\phi}_a) + \lambda_S \left[ \sum_{a,\tilde{a}} (\tilde{\phi}_a^* \tilde{\phi}_a) \right]^2 + \kappa_S \sum_{a,b,\tilde{a},\tilde{b}} (\tilde{\phi}_a^* \tilde{\phi}_a)(\tilde{\phi}_b^* \tilde{\phi}_b), \tag{37}
\]

\[
V_{WS}[\Phi] = \nu_1 |\phi|^2 \sum_{a,\tilde{a}} \tilde{\phi}_a^* \tilde{\phi}_a + \nu_2 |\phi|^2 \sum_{a} \left( \frac{\tilde{\phi}_a^* \tilde{\phi}_a}{|\phi|^2} \right)^2, \tag{38}
\]

depending altogether on 7 real parameters \(\mu_W, \lambda_W, \mu_S, \lambda_S, \kappa_S, \nu_1, \) and \(\nu_2\).

The first thing we would wish to know from the framon potential \(V[\Phi]\) is presumably what it would imply for the vacuum in FSM. Let us first examine the terms \(V_W\) and \(V_S\), each involving only the weak and strong framons by themselves and see what they imply. The term \(V_W\) is the same as the potential in the electroweak theory of which little more need be said at this juncture. To see what \(V_S\) would imply for the strong vacuum, it is convenient to adopt a vector notation for the strong framons \((23)\) by rewriting them as vectors in \(\tilde{su}(3)\) space, labelled by the local colour index \(a\), thus:

\[
\phi_a = (\tilde{\phi}_a), \tag{39}
\]

in terms of which \(V_S[\Phi]\) then reads as:

\[
V_S[\Phi] = -\mu_S \sum_a |\phi_a|^2 + \lambda_S \left( \sum_a |\phi_a|^2 \right)^2 + \kappa_S \sum_a (|\phi_a|^2)^2 + \kappa_S \sum_{a \neq b} |\phi_a^* \cdot \phi_b|^2. \tag{40}
\]

We are interested in the situation when the 3 parameters in it, namely \(\mu_S, \lambda_S, \kappa_S\), are all positive, in which case, as in the familiar \(V_W\) of the electroweak theory, the vacuum values of \(|\phi_a|\) will be in general nonzero and the vacuum degenerate. Of the terms in \((40)\), we see that only the second \(\kappa_S\) term depends on the orientations of the vectors \(\phi_a\), the rest depending only on their lengths. Hence, for \(\kappa_S > 0\), the minimum for \(V_S\) is attained when \(\phi_a\) are mutually orthogonal. Then, minimizing the remaining terms,
which are symmetric in $a$, with respect to the lengths of $\phi_a$, we deduce that these lengths should have equal, nonzero values. In other words, we would obtain for the vacuum values of $\phi_a$ an orthonormal triad, as frame vectors are normally expected to be. In passing, we note that since $V_S$ is symmetric under $\tilde{su}(3)$ by construction, so should be its degenerate vacuum. The different vacua in the degenerate set here, however, differ from one another only in the orientation of the orthonormal triad of frame vectors in the $\tilde{su}(3)$ or “generation” space, but otherwise look the same, making thus the degeneracy unremarkable.

The situation, however, changes dramatically when the term $V_{WS}$ is included, which links the strong to the weak sector. In the notation introduced in (39), $V_{WS}$ appears as:

$$V_{WS}[\Phi] = \nu_1 |\phi|^2 \sum_a |\phi_a|^2 - \nu_2 |\phi|^2 \sum_a |(\alpha^\ast \cdot \phi_a)|^2. \quad (41)$$

The first term depends on the weak framon field $\phi$ only through its length $|\phi|$ and so just modifies the value of the parameter $\mu_W$ in the weak potential $V_W$ and will not alter the basic structure of the weak vacuum. But $V_{WS}$ contains a $\nu_2$ term involving the relative orientation between the the vectors $\alpha$ and $\phi_a$, which means that the strong vacuum would be distorted from the snug orthonormal arrangement of frame vectors it had before by the vector $\alpha$ coming from the weak sector.

How the vector $\alpha$ will affect the vacuum values of the strong framons $\phi_a$ is qualitatively easy to see. To be specific, let us take $\nu_2 > 0$ and consider first the situation when these framons are kept still having the same length, thus allowing only their orientations to vary. Now the $\nu_2$ term in (41) is smallest when the framons $\phi_a$ are all aligned with the vector $\alpha$, but this is opposed by the second $\kappa_S$ term in (40) which, to attain its smallest value, would want instead the framons to be mutually orthogonal. Hence the result of minimizing the two terms together would be a compromise where the triad is squeezed from orthogonality together towards the vector $\alpha$ which, by the symmetry of the problem, would be symmetrically placed with respect to the triad. Consider next the opposite situation when the framons are kept mutually orthogonal but allowed only to change their lengths relative to one another. We recall then that it was the first $\kappa_S$ term in (40) whose minimization gave the result that the 3 lengths should be equal, but this is now opposed by the $\nu_2$ term in (41) which, to achieve its smallest value, would prefer to have all the length attributed to just one of the framons $\phi_a$ and $\alpha$.
to be aligned with it. Minimizing the two terms together would thus once more lead to a compromise where the $\phi_a$'s differ in length from one another and the vector $\alpha$ is aligned with the longest. From these two examples, it is clear then that when the framons are allowed to change both their relative orientations and lengths, there will be a trade-off between the two extremes. In other words, the minimum of the potential is degenerate, with a varying amount of squeeze on the triad compensated by a simultaneous change in the relative framon lengths in a prescribed manner, with the vector $\alpha$ snuggling up to, but not exactly aligned with, the longest framon.

The properties of the FSM vacuum outlined in the two preceding paragraphs can be confirmed, of course, by minimizing the potential $V[\Phi]$. This has been done and the result is given explicitly in \[8\], but for the present discussion, only the qualitative features described are needed.

In spite of their very different shapes, however, these vacua in the degenerate set must nevertheless be equivalent to one another under $\tilde{su}(3)$ transformations, just as for the vacua of $V_S[\Phi]$ before the linking term $V_{WS}[\Phi]$ was turned on, since the whole potential $V[\Phi]$ was constructed to be invariant under these transformations. And such they are, as is shown in \[8\] with the explicit solution, provided of course that the $\tilde{su}(3)$ transformations are applied not only to the framon vectors $\phi_a$ but also to the vector $\alpha$, and the results of the transformations are viewed each in the appropriate gauge. However, if $\alpha$ is held fixed while $\tilde{su}(3)$ transformations are applied to the framons alone, then they will appear distorted in different ways from orthonormality, as outlined in the above paragraph. It can thus be said that the vector $\alpha$, coming from the weak sector, breaks the $\tilde{su}(3)$ symmetry in the same sense that a bar magnet placed in vacuum is said to break space rotation symmetry. It is in the same spirit too that it was electromagnetism which breaks the $\tilde{su}(2)$ symmetry in the electroweak theory \[2\] via the vector $\gamma$ in the preceding section. We thus have the intriguing pattern of symmetry-breaking entered on the last row of Table 1. The breaking of $\tilde{su}(3)$ and $\tilde{su}(2)$ are the same in spirit but not in detail. On the one hand, the $\tilde{su}(3)$ breaking, but not the $\tilde{su}(2)$ breaking, occurs already in the potential. On the other hand, in the $\tilde{su}(2)$ breaking the preferred direction $\gamma$ is distinguished by a local gauge interaction, namely that of the electromagnetic $u(1)$, while for the $\tilde{su}(3)$ breaking the preferred direction $\alpha$ is not distinguished by a parallel $su(2)$ gauge interaction. The reason for these differences can be traced to the fact that one has chosen, on the basis of what one called “minimal framing”, the sum representation $2 + 3$ for $su(3) \times su(2)$ (hence the global vector $\alpha$)
but the product representation $2 \times 1$ for $su(2) \times u(1)$.

What is interesting for the moment is that the different strong vacua in the degenerate set are each attached to a value of the vector $\alpha$ which is the same vector which gives a 3-valued “generation” index to the leptons and quarks in (29) and (30) above. It seems therefore that the strong vacuum could have a lot to do, via this vector $\alpha$, with the properties of fermion generations, such as their mixing and mass hierarchy, and we would be interested now in what way this $\alpha$ appears in the lepton and quark mass matrices from which these properties are ultimately derived.

To answer this, let us next examine the Yukawa terms coupling the weak framons to the fundamental fermion fields. These terms give rise to the lepton and quark mass matrices. To do so, we have first to specify what fundamental fermion fields are to be introduced. Not having yet given a geometric meaning to fermion fields as we think we have to the boson fields, i.e. both to the vector bosons as gauge (connection) fields and to the scalar bosons as framon (frame vector) fields, we have to rely for selecting our fundamental fermion fields on information gathered otherwise. Restricting ourselves for simplicity to only the fundamental representation of each symmetry, we obtain the following:

$$\psi(1, 1), \psi(3, 1), \psi(1, 2), \psi(2, 3) \quad (42)$$

where the first argument denotes the dimension of the $su(3)$ and the second that of the $su(2)$ representation, from which the admissible $u(1)$ charge for each $\psi$ is then specified by (26). Of these $\psi$'s, however, not all the left- or right-handed components are needed. To see which are the components needed, recall that in the confinement picture we have adopted, the left-handed quarks and leptons are flavour doublet bound states via $su(2)$ confinement of the $su(2)$ doublet fundamental fermion fields $\psi$ with the weak framon. Then from the fact that in the standard model, based on phenomenology, one allows only left-handed flavour doublets and right-handed flavour singlets of quarks and leptons, it can easily be shown that here only left-handed $su(2)$-doublet and right-handed $su(2)$-singlet $\psi$'s are allowed, namely:

$$\psi_L(1, 2), \psi_L(2, 3), \psi_R(1, 1), \psi_R(3, 1). \quad (43)$$

Proceeding with these as the fundamental fermion fields, one has then for leptons:

$$A_{YK} = \sum_{[a][b]} Y_{[a]}^{\text{lepton}} \bar{\psi}_r \alpha^a \phi_r^{(-)} \frac{1}{2} (1 + \gamma_5) \psi^{[b]} + \sum_{[a][b]} Y_{[a]}^{\text{lepton}} \bar{\psi}_r \alpha^a \phi_r^{(+)} \frac{1}{2} (1 + \gamma_5) \psi^{[b]}$$
and for quarks:

$$\mathcal{A}_{\text{YK}} = \sum_{[\tilde{a}] [\tilde{b}]} Y_{[\tilde{b}]}^{\text{quark}} \tilde{\psi}_{[\tilde{a}]} \bar{\alpha} \tilde{\alpha} \phi_r^{(-)} \frac{1}{2} (1 + \gamma_5) \psi^{[\tilde{a}]}_r + \sum_{[\tilde{a}] [\tilde{b}]} Y_{[\tilde{b}]}^{\text{quark}} \tilde{\psi}_{[\tilde{a}]} \bar{\alpha} \tilde{\alpha} \phi_r^{(+)} \frac{1}{2} (1 + \gamma_5) \psi^{[\tilde{a}]}_r$$

$$+ \text{h.c.}$$

which are of the usual form, except for the appearance of the global vector $\alpha = (\alpha^{\tilde{a}})$ carried here by the weak framon in (22). Notice also that in order to saturate the $\tilde{a}$ index carried by $\alpha$ so as to make the whole invariant under $\tilde{su}(3)$, one has introduced 3 identical copies each of the (left-handed) fermion fields $\psi_r$ for leptons and $\psi_{r\tilde{a}}$ for quarks, the copies being labelled by the dummy index $[\tilde{a}]$. Under an $\tilde{su}(3)$ transformation, the terms (44) and (45) will thus remain invariant only if one relabels the fermion fields accordingly, but this should not change the physics, given the that these fermion fields are $\tilde{su}(3)$ singlets and are identical otherwise. This is the same argument as was used on the (right-handed) fields [12] so as to write any mass matrix in a hermitian form independent of $\gamma_5$, as we shall also do immediately below.

With these Yukawa terms (44) and (45), one obtains, by substituting for the weak framon its vacuum expectation value, say $\zeta_W$, the following mass matrix for both leptons and quarks:

$$m = \zeta_W \begin{pmatrix} \alpha^{\tilde{1}} & 0 & 0 \\ 0 & \alpha^{\tilde{2}} & 0 \\ 0 & 0 & \alpha^{\tilde{3}} \end{pmatrix} \begin{pmatrix} Y_{[1]}^{\ast} \\ Y_{[2]}^{\ast} \\ Y_{[3]}^{\ast} \end{pmatrix} \frac{1}{2} (1 + \gamma_5) + \zeta_W \begin{pmatrix} Y_{[1]}^{\ast} \\ Y_{[2]}^{\ast} \\ Y_{[3]}^{\ast} \end{pmatrix} \begin{pmatrix} \alpha^{\tilde{1}} & \alpha^{\tilde{2}} & \alpha^{\tilde{3}} \end{pmatrix} \frac{1}{2} (1 - \gamma_5).$$

Then, again, by relabelling appropriately the right-handed fields as mentioned in the preceding paragraph, one can rewrite the mass matrix for both quarks and leptons in the factorized form (1), with:

$$m_T = \zeta_W \rho_T; \quad \rho_T^2 = |Y_{[1]}|^2 + |Y_{[2]}|^2 + |Y_{[3]}|^2.$$  

As expected, the mass matrices of quarks and leptons do depend on the vector $\alpha$. Besides, they are of rank one, and expressible as a product of $\alpha$ with its hermitian conjugate as explained. But the vector $\alpha$, originating as it does as a factor of the weak framon, is of course independent of which fermion the framon is bound to, so that in (1) only $m_T$ depends on the fermion type. Now such a “universal” rank-one mass matrix for fermions has long been
advocated \[1, 6\] as a good starting point or zeroth-order approximation for attacking the fermion mass hierarchy and mixing problems, since it has only one massive eigenstate, and it gives for the mixing matrix the identity matrix, neither of which conclusions is a bad approximation to what is experimentally observed.

Now, starting with an \(\tilde{su}(3)\) “generation” symmetry as it is done here, it is not trivial to end up with some masses much larger than others, for any obvious breaking of the symmetry would lead to a very different mass pattern. It is therefore quite gratifying that the FSM leads automatically to the above tree-level mass matrix that phenomenologists have long desired. However, such a tree-level result is of practical value only when one knows how to go further to evaluate higher order effects so as to explain the nontrivial mixing between up-down flavours and the nonzero masses of lower generations actually observed in experiment. This last seems difficult in whatever scheme, given that it would apparently involve breaking both the “universality” and the “factorizability” of the tree-level formula \(\text{(1)}\) by subsequent radiative corrections. For example, to obtain nontrivial mixing, whether in leptons or in quarks, one would need to make \(\alpha\) dependent on up-down flavour. As far as known, however, only the electroweak interactions depend on flavour, and they seem too weak to give the desired effects. This was the case in the standard model when unframed, and remains so even in the framed standard model; it was seen already that the weak potential \(V[\Phi]\) in \(\text{(36)}\) is the same as before and it can easily be seen too that the kinetic energy term is also the same as in \(\text{(11)}\) for the electroweak theory, as the extra factor of \(\alpha\) carried by the weak framon \(\text{(22)}\) is just traced away.

Fortunately, however, there is a loop-hole in the above line of reasoning which allows nontrivial mixing without breaking the universality of \(\alpha\), namely when the vector \(\alpha\) happens to depend on scale. As a global parameter appearing in the action, there is in principle no reason why \(\alpha\) should not acquire scale-dependence under renormalization as coupling constants in general do. Whether it actually does in FSM will be discussed below, but if we suppose that it does, then the earlier conclusions on the quark or lepton masses and mixing matrices deduced from the mass matrix \(\text{(1)}\) will have to be reassessed, for it has now to be specified at what scale each quantity, whether mass or state vector, is to be measured. Consider first as examples the two heaviest states in each flavour, namely \(t\) in up and \(b\) in down. It was already noted that in \(\text{(11)}\), the coefficient \(m_T\) can depend on flavour and can thus be identified respectively with \(m_t\) for up and \(m_b\) for down, assuming
at the moment for simplicity that \( m_T \) itself does not depend on scale. But what are the state vectors of \( t \) and \( b \) in generation space? In each case, the state vector should be the eigenvector of \( m \) in (1) with the single nonzero value, namely \( \alpha \) itself. But since \( \alpha \) depends on scale by assumption, one has to ask at what scale in each case to evaluate this \( \alpha \). Suppose we follow the standard convention and evaluate it at their respective mass scales, we would have the state vector for \( t \) as \( \mathbf{v}_t = \alpha(\mu = m_t) \) and for \( b \) as \( \mathbf{v}_b = \alpha(\mu = m_b) \). Hence, the state vectors \( \mathbf{v}_t \) and \( \mathbf{v}_b \) for \( t \) and \( b \) will in general point in different directions and give for the CKM matrix element \( V_{tb} = \langle \mathbf{v}_t | \mathbf{v}_b \rangle \) a value different from unity. In other words, one would conclude that there will be mixing, quite contrary to our earlier conclusion from (1) when \( \alpha \) was taken as scale-independent. And notice that this new conclusion has been obtained without breaking the universality of \( \alpha \), i.e., without making \( \alpha \) at any scale different for the two different flavours.

Next, what about masses for the lower generations; will they become nonzero also when \( \alpha \) depends on scale? To answer this, we need consider only one single flavour, say up, for example. The state vector for \( t \) we have already identified as \( \mathbf{v}_t = \alpha(\mu = m_t) \). The state vectors \( \mathbf{v}_c \) and \( \mathbf{v}_u \) have both to be orthogonal to \( \mathbf{v}_t \) and have themselves to be mutually orthogonal, the 3 states being by definition independent quantum states. This means, of course, that at \( \mu = m_t \), the states \( \mathbf{v}_c \) and \( \mathbf{v}_u \) both have zero eigenvalues for mass matrix \( m \) in (1). But these are not to be taken as the masses for the \( c \) and \( u \) quarks, for by the usual convention adopted above, these masses are to be evaluated at the scale of the masses themselves, i.e. respectively at \( \mu = m_c \) and \( \mu = m_u \). At these lower values of the scale, however, the scale-dependent vector \( \alpha \) would be pointing in different directions than at \( \mu = m_t \), i.e., no longer orthogonal to \( \mathbf{v}_c, \mathbf{v}_u \), hence giving nonzero solutions to both \( m_c \) and \( m_u \). It is as if by virtue of this scale-dependence of \( \alpha \), some of the mass carried exclusively by this vector has leaked into the lower states and imbued them with hierarchical but yet nonzero masses as experiment indicate.

The possibility outlined above of a scale-dependent \( \alpha \) in (1) giving rise both to mixing and a hierarchical mass spectrum for both quarks and leptons has in fact been studied phenomenologically for many years and is found so far to be quite consistent with existing experimental data. Cast in this context as a hypothesis of a rotating rank-one mass matrix (R2M2), it is tested phenomenologically in [13] and reviewed in some detail in a recent paper [9] to which the interested reader is referred. If sustained, these results
from rotation would relieve us from having to break the universality of the
mass matrix (1) with respect to flavour, which would be a gain in that we
would then no longer have to look to electroweak forces for the origin of
mixing. The effect can now arise in principle via rotation as a result of
renormalization in the flavour-independent strong sector which is strength-
wise much more favourable.

From the viewpoint taken in the analysis of [9] of mass matrix rotation
as a phenomenological hypothesis, it would appear that any theory or model
which can generate a mass matrix of form (1) with \( \alpha \) rotating sufficiently
speedily with changing scale would have a fair chance of reproducing the
existing data on mixing and the mass hierarchy. Our interest is therefore
turned next on to the question whether and how such a dependence on scale,
or rotation, of \( \alpha \), or of the mass matrix \( m \), can indeed arise from strong
interaction in a theory. This would be a nontrivial requirement because the
mass matrix of both leptons and quarks appear originally in the Yukawa term
of the electroweak sector, and it is not obvious that renormalization effects
in the strong sector would be transmitted there.

For the framed standard model, however, the interesting thing is that this
will be automatic, as follows. We recall that the FSM vacuum is degenerate,
with the vacuum values of the strong framons in the different vacua in the
degenerate set being distorted from orthonormality in various ways by exactly
the vector \( \alpha \) coming from the electroweak sector and appearing as a factor
in lepton and quark mass matrices. Hence, if renormalization effects in the
strong sector changes the vacuum with changing scale, as they normally
would, so automatically would \( \alpha \) change with it. Thus, the only remaining
question is whether the vacuum in FSM will indeed change with scale as the
result of renormalization effects in the strong sector.

This question has been answered recently in the affirmative and is re-
ported in [8]. As a result, the vector \( \alpha \) appearing in the quark and lepton
mass matrix (1) will indeed rotate with changing scale as postulated in [9]
and shown there to lead to hierarchical masses and mixing. Further, it is
found in [8] that this rotating \( \alpha \) will have fixed points at scale \( \mu = \infty \) and
it has already been noted just after (47) that it is universal. In other words,
the mass matrix for quarks and leptons which results from FSM is shown to
possess all those properties which have been identified in [9] as being essential
for a successful description of the mass and mixing patterns for quarks and
leptons observed in experiment, although an explicit fit to data has yet to
be done.
One point of interest worth noting is the following. In the FSM scenario outlined above, up-down mixing arises from rotation while the rotation itself is driven by renormalization effects in the strong sector. So it might appear difficult to obtain in the mixing matrix a CP-violating Kobayashi-Maskawa phase if the strong sector is itself CP-conserving, as it is generally believed to be. What is intriguing, however, is that rotation, as shown in [14, 9], connects the CP-violating KM phase in the CKM matrix to the theta-angle coming from topology in the so-called strong CP problem. Thus, as is also shown in these papers, removing the theta-angle by a chiral transformation so as to make the strong interactions CP-invariant, which one can do in FSM because the mass matrix is of rank one, will automatically give a CP-violating phase to the CKM matrix which is naturally of the order of the magnitude experimentally observed, while offering at the same time a new solution to the age-old strong CP problem.

It thus seems that just by the simple added device of “framing” the gauge theory with a gauge symmetry identical to that of the standard model, one has gone quite some way towards answering the questions posed at the beginning about what we called the idiosyncrasies of the standard model. It offers an explanation for the origin of not only the Higgs boson but also the three fermion generations together with, qualitatively, their peculiar mass and mixing patterns including CP-violation. In addition, one has gained two bonuses not initially bargained for, namely a gauge origin for $B - L$ conservation and a new solution of the strong CP problem. One has yet, of course, to ascertain whether the actual mass and mixing parameters observed in experiment can indeed be accommodated in the FSM, and that all the various new predictions that the FSM is bound to have would either be consistent with existing data, or else are testable by future experiments. This will, of course, be a long drawn-out process, some parts of which, we hope, will be dealt with in forthcoming papers. For the rest one would have to rely on future scrutiny by the community. But even at this stage, it seems fair to say that the framed standard model, and hence by association also the framed gauge theory framework, has displayed sufficient features of interest to merit further exploration.

Before one leaves the FSM, there is one more point to be noted which will be of use later. Since frame vectors have been promoted in FSM to be dynamical variables, so also will be the components of the metric. Indeed, we recall that even at vacuum, the frame vectors are in general distorted from orthonormality, and so the metric will not in general be flat. Since the
original local gauge symmetry $su(3)$ is supposedly still to be confining and exact, this means that it is the metric:

$$g_{\tilde{a}\tilde{b}} = \sum_a (\phi_a^\ast)^{\tilde{b}} \phi_a^\tilde{a}$$

in the $\tilde{su}(3)$ or generation space that is distorted from flatness. The details of how this metric is distorted, and some implications that this will have on phenomenology, can be found in [8]. We note here only the fact that the metric in $\tilde{su}(3)$ is nonflat, which will be relevant for the discussion in the next section.

5 Speculations on Relation to Gravity

Supposing tentatively that framed gauge theory does serve as a viable basis for the standard model of particle physics more or less as it is proposed, it would be natural to ask whether the idea can be extended also to gravity which governs the large scale physics not covered by particle theory. This is not a priori hopeless, for we know already that the theory of gravity is framed (indeed, as stated at the beginning, even the concept of framing itself is originally borrowed from gravity), and that it can be considered as a kind of gauge theory. The question only is whether the two theories, particle theory and gravity, can be brought closer together as to be recognizable as but two branches of a common framework.

At this stage, let us for the moment throw caution to the wind and set our imagination loose on the problem. Let us first list down for this purpose some similarities and differences between the two. In the language of this paper, they are both framed theories, with each a local and a global symmetry. For the particle theory, the local symmetry is $G$, say, and the global symmetry $\tilde{G}$, i.e. explicitly $su(3) \times su(2) \times u(1)$, and $\tilde{su}(3) \times \tilde{su}(2) \times \tilde{u}(1)$. For gravity, the global symmetry operating in the indices $a$ of the vierbeins $e^a_\mu$ is the Lorentz group which we shall call here $H$ (not $\tilde{H}$, the reason for which apparent switch in notation between tilde and no-tilde will soon be obvious). The local symmetry $\tilde{H}$ we shall take also to be the Lorentz group. This at first sight seems quite off the mark, since the whole point in gravity is that the metric becomes distorted by matter so that Lorentz invariance is lost. However, we have already seen in the framed standard model of the preceding section how a framed theory originally symmetric under $\tilde{su}(3)$ can have its symmetry
broken by the interaction and settles to a quite different metric. So may, we
think, be the case here too. Accepting this for the moment, we arrange the
information in the diagram Fig. 1.

Notice that by “local” here, we mean that the transformations of that
symmetry can depend on the point \( x \) in the “external” space \( X \), i.e. our
space-time, and by “global” that they do not, and hence, up to now, they
are tacitly taken as constant. However, suppose we make the Kaluza-Klein
assumption that the “internal” space \( \Xi \) on which the internal symmetries
\( G \) and \( \tilde{G} \) operate is compactified and very small in size, then any quantity
depending only on points in \( \Xi \) but not on points in \( X \) would appear to us
as effectively constant and be considered as “global”. Accepting this, we are
then led to the diagram Fig. 2 where an arrow represents the assertion that
the quantity at the tail of the arrow depends on points in the space at its
head. As a result, we see that the Kaluza-Klein assumption has made the
arrangement now symmetric between \( \Xi, G, \tilde{G} \) on the one hand and \( X, H, \tilde{H} \)
on the other.

The 2 symmetries \( G \) and \( \tilde{H} \) remain local in the sense that their elements
may depend on points in the external space \( X \), but their nature would be
quite different. Whereas for the \( G \)-theory, the group elements operate on the
internal space \( \Xi \) but depend on points in external space \( X \), for the \( \tilde{H} \)-theory,
the group elements operate on the same space \( X \), on points of which they
can themselves depend. Explicitly, suppose we introduce for the \( G \)-theory
\( A_{\mu}^{ab} \) as the gauge potential (connection) and from which we construct \( F_{\mu\nu}^{ab} \)
as the field tensor (curvature), with Latin indices in \( \Xi \) but Greek indices
in \( X \). With no known relationship between the two types of indices, the
simplest invariant that we can construct is just \( \sum_{\mu\nu} F_{\mu\nu}^{ab}F_{\mu\nu}^{ab} \), namely the
Lagrangian density for the Yang-Mills action. On the other hand, the same
considerations will give for the \( \tilde{H} \)-theory the spin connection \( \omega_{\mu}^{ab} \) as the gauge
potential and the Riemann curvature \( R_{\mu\nu}^{ab} \) as the field, where both the Latin
and Greek indices are now in \( X \), only referred to different frames. A simpler
invariant than before can thus be constructed by contracting Latin with
Greek indices by means of the vierbeins \( e_{a}^{\mu} \), giving then the scalar curvature
\( R_{\mu\nu} = e_{a}^{\mu} e_{b}^{\nu} R_{\mu\nu}^{ab} = R \), the Lagrangian density for the Einstein action instead.

These two “local” theories, namely Yang-Mills on the left and Einstein
on the right, can be so different in nature without destroying the symmetry
between the two sides of the diagram of Fig. 2 because they play each a
different role there. Indeed, according to the diagram, there is in principle
also on the right-hand side a Yang-Mills type theory with \( H \) as the symmetry
Figure 1: Plan 1 for before Kaluza-Klein
Figure 2: Plan 2 for after Kaluza-Klein
group, and on the left-hand side an Einstein type theory with $\tilde{G}$ as the symmetry group, and both are “local” in the internal space $\Xi$. Only, we chose to ignore these theories because we took $\Xi$ to be compactified and so small in size as to make them inaccessible to us. In other words, it was the input of the Kaluza-Klein assumption that $\Xi$ is small while $X$ is extended compared to us that gave rise to this difference. The structure of the two sides remain symmetric.

Having noted the symmetry in structure between the two sides of Fig. 2, let us turn to consider a little the possible dynamics. On the left-hand side, we said in the preceding section for the particle theory that the $G$ symmetry is exact but the $\tilde{G}$ symmetry is broken, leading thus to a nonflat metric in $\tilde{G}$-space. In parallel, we would say on the right-hand side that the $H$-symmetry is exact but the $\tilde{H}$-symmetry is broken, leading again to a nonflat metric; this is what we have always wanted for gravity, although we do not know yet whether the metric will be distorted by matter in the correct way. In the preceding sections, we also said that the $G$-symmetry is confining so that only $G$-neutral objects can propagate. An example is the Higgs boson which appeared in this theory as a $G$-neutral bound (confined) state of a framon-antiframon pair held together by $G$-confinement. Suppose we now say in parallel on the right-hand side that the $H$-symmetry is also confining. Then we would conclude that the vierbeins $e_a^\mu$ themselves, not being $H$-neutral will have to be confined, and can propagate only as bound vierbein pairs held together by $H$-confinement, namely $\sum_a e_a^\mu e_a^\nu = g_{\mu\nu}$ or gravitons, which would thus appear as gravity analogues of the Higgs boson in particle theory.

In the particle theory on the left-hand side, we recall that the self-interaction potential $V[\Phi]$ of the framons $\Phi$ plays an important role. What happens if we introduce on the right for gravity also a self-interaction potential for the vierbeins? The same symmetry arguments as before will lead to a potential formally identical to that for the $su(3)$ theory of the preceding section. Now we noted in the analysis there that the $su(3)$ framons remain orthonormal at vacuum corresponding to a Euclidean metric in $\tilde{su}(3)$ space until the symmetry is broken by interaction with framons from $su(2)$. Virtually the same analysis on the potential for the vierbeins, i.e. the $H - \tilde{H}$ framons on the right-hand side of Fig. 2, will show that the vierbeins here will also be orthonormal at a stationary point of the potential (although with the indefinite signature, it is here not a minimum as it was for the other case). This would seem to mean that when left to themselves, the vierbeins would settle to orthonormality and the metric to the Minkowski metric. In other
words, in the absence of or far away from other fields like matter, \( X \) would settle down automatically to a Minkowski world, which is probably the sort of solutions we would seek in any case. In regions of space dominated by matter, the effects of the framion potential will presumably be negligible.

Matter fields in the particle theory on the left are usually fermionic; fermionic fields too can be introduced on the right for gravity with the ECKS formalism \([15]\) by means of the vierbeins and the spin connection. In either case, as usually formulated, the fermion fields are inserted by hand without being ascribed a geometrical significance. In the framework of framed gauge theory we are considering, however, it appears that a possible geometrical meaning for fermionic fields may emerge in the following manner.

The symmetry displayed in Fig. 2 between the internal structures on the left and the external structures on the right may still seem defective in that the framons on the left, as introduced in the preceding sections, are complex, whereas the vierbeins on the right are real. This apparent asymmetry is due, however, only to our own sloppy convention. When we have spinor fields around, the symmetry group is not \( SO(3) \) but its double cover \( SU(2) \). In the same way, when we have spinors on the right-hand side of Fig. 2, as indeed there are in nature, the symmetry groups \( H \) and \( \tilde{H} \) should not be taken as the Lorentz group \( SO(3, 1) \) as we did, but its double cover \( SL(2, \mathbb{C}) \). That being the case, we ought to have chosen as framons, in parallel to the framons in \( \Phi \) of the preceding sections, not the vierbeins \( e^a_\mu \) as we did, but the elements of a matrix, \( \Psi \) say, whose rows transform as 4-spinors under \( H \) and whose columns transform as 4-spinors under \( \tilde{H} \), removing thus what had appeared as a defect before in the symmetry between the left and right side of Fig. 2.

One interesting consequence of taking \( \Psi \) as framons instead of the vierbeins as one did above is to have space-time spinors now appearing as geometric objects. However, since they transform as spinors under \( H \), and \( H \) is supposed to be confining, they will have to be confined and cannot be taken as actual fermion fields. They can form bound states with their own conjugates giving bosonic fields like the graviton above, but can they form bound states with something else to give fermionic fields? One possibility is their bound states with differential forms in \( H \) space. Saturating the \( H \) indices in the framons with those of the differential forms will give objects which now carry only tilde indices, i.e. are neutral under \( H \) and hence can propagate,

\[1\text{Strictly speaking, } SL(2, \mathbb{C}) \text{ double covers the proper orthochronous Lorentz group, usually denoted } SO^+(3, 1) \]
but still transform as spinors under $\tilde{H}$ as fermion fields should. But further, being differential forms, they will mutually anti-commute, again as fermion fields should. Has one then found a geometric significance for fermionic matter? In other words, one says that they transform as spinors because they are framons in a theory with the double cover of the Lorentz group as symmetry group, and they anti-commute because they are basically line-elements. Amusingly, a proton made up of 3 quarks will now appear as a volume element! We notice, though, that to saturate the spinorial indices in $\Psi$ one needs differential forms carrying also spinorial indices, i.e. differential forms not in ordinary space-time $X$ with space-time vectors $x, t$ as co-ordinates, but in a spinor-valued version of it with details yet to be worked out.

By this stage, it is probably clear to all our readers that we are getting rapidly out of our depths, and that we have speculated already much more than enough. But these speculations have given us at least some delectable food for thought and, together with what the framed gauge theory framework seemed to have done above for the particle physics sector, new encouragement to continue with its exploration.

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