DECISION MAKERS PREFERENCE SOLUTION FOR A FUZZY MULTI OBJECTIVE ASSIGNMENT PROBLEM

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ABSTRACT: In this paper we propose a novel approach for the solution of fuzzy multi objective assignment problem whose parameters are expressed as triangular fuzzy numbers. We apply arithmetic operations and ranking in the parametric form of triangular fuzzy numbers to solve fuzzy multi objective assignment problem without converting to classical form. The proposed method gives flexibility to the decision maker to his/her preferred fuzzy optimal solution. A numerical example is also given based on this novel method.

Key words: Fuzzy assignment situation, multi objective, fuzzy ranking, fuzzy arithmetic.

1. INTRODUCTION

The Multi-Objective Assignment Problem (MOAP) deals with cost, time, quality etc. The objectives of an assignment problem are to minimize both operating cost and operating time, and to maximize quality simultaneously. Suppose we have to assign n workers to n tasks in such a way that the overall operation cost, labour-time, and quality level are to be optimized. Assignment problems representing real-world situations involve a set of parameters whose values are not known exactly. In that case the parameters of the problem are usually defined by the decision makers (DMs) in an uncertain way. Fuzzy sets introduced by Zadeh [21] in 1965 provide a procedure to deal with uncertainty. Bellman and Zadeh [3] proposed the concept of decision-making in a fuzzy environment.

Bao et al. [2] solved a multi-objective assignment problem converting to crisp environment. Kagade and Bajaj [6] discussed multi-objective assignment problem with interval cost. Pramanik and Biswas [15][16] developed a priority based fuzzy goal programming method for generalized trapezoidal fuzzy numbers and applied it for multi-objective assignment problem in which cost and time are considered as a generalized trapezoidal fuzzy numbers. They have also solved a multi-objective assignment problem with fuzzy costs for the case of military affairs. De et.al [4] discussed multi objective assignment problem using interactive fuzzy goal programming approach. Tapkan et al. [14] used the bees algorithm for solving the fuzzy multiple objective generalized assignment problem. Pankaj Gupta et.al [12] developed a fuzzy approach to multi criteria assignment problem using exponential membership functions. Kayvan Saleh [7] discussed multi-criteria assignment problem with interval parameters. Anita Ravi et.al [1] solved multi-objective assignment problem using genetic algorithm based hybrid approach. Ventepaka et.al.[20] introduced an algorithm for solving multi objective assignment problem using Hungarian algorithm an optimal solution of each objective function by minimizing the resource. Thorani and Ravi Shankar [19] has proposed a fuzzy assignment problem with three parameters fuzzy cost, fuzzy time, fuzzy quality and solved it using their fuzzy ranking method. Leelavathy and Ganesan [8] obtained a compromised solution to multi objective fuzzy assignment problems where all the parameters are trapezoidal fuzzy numbers. Sophia Porchelvi and
Anitha [17] used a fuzzy programming technique with linear membership functions to find the compromise solution of multi-objective assignment problem. Mahbubur Rahman et.al [9] solved multi-objective assignment problem with decision maker’s preferences by using genetic algorithm. Melita Vinoliah and Ganesan [10] proposed a different approach for the solution of fuzzy multi objective assignment problems.

This paper aims at finding a fuzzy optimal assignment without reducing it to an equivalent crisp form. The paper is organized as follows. In section 2, the basic definition, ranking function and arithmetic operation of TriFN are reviewed. In section 3, fuzzy multi objective assignment model and formulations are reviewed. A numerical example is also provided in section 4 to compare the proposed method with the existing method.

2. PRELIMINARIES

**Definition 2.1.** A fuzzy set $\tilde{a}$ characterized on $X$ is an accumulation of requested pair

$$\tilde{a} = \{(x, \mu_{\tilde{a}}(x)) : x \in X\}$$

where $\mu_{\tilde{a}}(x)$ is a participation estimation of $x$ in $\tilde{a}$.

**Definition 2.2.** Fuzzy set $\tilde{a}$ is defined on a set $R$ of real number whose membership function $\tilde{a} : R \rightarrow [0,1]$ is a fuzzy number under certain conditions:

i. $\tilde{a} : R \rightarrow [0,1]$ is normal. (i.e.) height $(\tilde{a}) = 1$

ii. $\tilde{a} : R \rightarrow [0,1]$ is convex

iii. $\tilde{a} : R \rightarrow [0,1]$ is piecewise continuous.

**Definition 2.3.** A triangular fuzzy number $\tilde{a} = (a_1, a_2, a_3)$ is defined by its membership function is given by

$$\mu_{\tilde{a}}(x) = \begin{cases} 
\frac{x - a_1}{a_2 - a_1}, & a_1 \leq x \leq a_2 \\
\frac{a_3 - x}{a_3 - a_2}, & a_2 \leq x \leq a_3 \\
0, & \text{otherwise}
\end{cases}$$

**Definition 2.4.** A triangular fuzzy number $\tilde{a} = (a_1, a_2, a_3) \in F(R)$ can also be represented as a pair $\tilde{a} = (\underline{a}, \overline{a})$ of functions which satisfies the following requirements:

i. $\underline{a}(r)$ is bounded monotonic increasing left continuous function.

ii. $\overline{a}(r)$ is a bounded monotonic decreasing left continuous function.

iii. $\underline{a}(r) \leq \overline{a}(r), 0 \leq r \leq 1$

**Definition 2.5.** For an arbitrary triangular fuzzy number $\tilde{a} = (\underline{a}, \overline{a})$, the number $a_0 = \left(\frac{\underline{a}(1)+\overline{a}(1)}{2}\right)$ is said to be a location index number of $\tilde{a}$. The two non-decreasing left continuous functions
are called the left fuzziness index function and the right fuzziness index function respectively. Hence every triangular fuzzy number \( \tilde{a} = (a_1, a_2, a_3) \) can also be represented by \( \tilde{a} = (a_0, a_*, a^*) \)

### 2.6. Ranking of triangular fuzzy number

Several approaches for the ranking of fuzzy numbers have been proposed in the literature. An efficient approach for comparing the fuzzy numbers are by use of ranking function based on their graded means. We defined the triangular fuzzy number \( \tilde{a} \) by

\[
R(\tilde{a}) = \left( \frac{a^* + 4a_0 - a_*}{4} \right) = \left( \frac{a^* + a + a_0}{4} \right)
\]

For any two triangular fuzzy number \( \tilde{a} = (a_1, a_2, a_3) = (a_0, a_*, a^*) \) and \( \tilde{b} = (b_1, b_2, b_3) = (b_0, b_*, b^*) \) in \( F(R) \)

(i) \( \tilde{a} \geq \tilde{b} \) if and only if \( R(\tilde{a}) \geq R(\tilde{b}) \)

(ii) \( \tilde{a} \leq \tilde{b} \) if and only if \( R(\tilde{a}) \leq R(\tilde{b}) \)

(iii) \( \tilde{a} \approx \tilde{b} \) if and only if \( R(\tilde{a}) = R(\tilde{b}) \)

### 2.7. Arithmetic operations of triangular fuzzy numbers

Ming ma et.al. [11] have projected a new fuzzy arithmetic based upon the both position index functions. The position index number is taken in the ordinary arithmetic, whereas the fuzziness index functions are considered to follow the lattice rule which is least upper bound in the lattice \( L \) i.e. for \( x, y \in L, x \vee y = \max\{x, y\} \) and \( x \wedge y = \max\{x, y\} \).

The arithmetic operations on triangular fuzzy numbers are

(i) Addition: \( \tilde{a} + \tilde{b} = (a_0, a_*, a^*) + (b_0, b_*, b^*) = (a_0 + b_0, \max\{a_*, b_*\}, \max\{a^*, b^*\}) \)

(ii) Subtraction: \( \tilde{a} - \tilde{b} = (a_0, a_*, a^*) - (b_0, b_*, b^*) = (a_0 - b_0, \max\{a_*, b_*\}, \max\{a^*, b^*\}) \)

(iii) Multiplication: \( \tilde{a} \times \tilde{b} = (a_0, a_*, a^*) \times (b_0, b_*, b^*) = (a_0 \times b_0, \max\{a_*, b_*\}, \max\{a^*, b^*\}) \)

(iv) Division: \( \tilde{a} + \frac{1}{\tilde{b}} = (a_0, a_*, a^*) + (b_0, b_*, b^*) = (a_0 + b_0, \max\{a_*, b_*\}, \max\{a^*, b^*\}) \)

### 3. FUZZY MULTI-OBJECTIVE ASSIGNMENT MODEL

Suppose \( n \) jobs are to be performed by \( n \) persons depending on their efficiency to do the job in one to one basis such that the assignment cost is minimal. An assignment problem with multiple objectives in fuzzy environment is called a multi objective fuzzy assignment problem.

#### 3.1. Mathematical Formulation of Multi Objective Fuzzy Assignment Problem

Consider a multi objective fuzzy assignment problem of assigning \( \text{'}n\text{'} \) jobs (operations) to \( \text{'}n\text{'} \) persons (operators) whose cost coefficients are triangular fuzzy numbers
Mathematical model of multi-objective fuzzy assignment problem can be stated as follows:

Minimize \( Z^k = \sum_{i=1}^{n} \sum_{j=1}^{n} \tilde{c}_{ij} x_{ij} \)

subject to \( \sum_{i=1}^{n} x_{ij} = 1, \text{ for } j = 1, 2, 3, \ldots, n \)

\( \sum_{j=1}^{n} x_{ij} = 1, \text{ for } i = 1, 2, 3, \ldots, n \)

and \( x_{ij} = \begin{cases} 1, \text{ if job } i \text{ is assigned to person } j \\ 0, \text{ otherwise} \end{cases} \)

where \( \tilde{z}^k = \{z^1, z^2, \ldots, z^k\} \) is a vector of \( k \) objectives.

3.2. Cost matrix of \((n \times n)\) multi objective fuzzy assignment problem

For simplicity let us assume that the multi objective fuzzy assignment problem has three objectives namely cost, time, quality. Let \( \tilde{c}_{ij} \) be the fuzzy assignment cost, \( \tilde{t}_{ij} \) be the fuzzy assignment time, \( \tilde{q}_{ij} \) be the fuzzy assignment quality incurred in assigning the \( i^{th} \) person to \( j^{th} \) job.

| Table 1: Fuzzy cost matrix for multi objective fuzzy assignment problem |
|---|---|---|---|
| \( P_1 \) | \( J_1 \) | \( P_2 \) | \( J_2 \) | \( P_n \) | \( J_n \) |
| \( \tilde{c}_{11}, \tilde{t}_{11}, \tilde{q}_{11} \) | \( \tilde{c}_{12}, \tilde{t}_{12}, \tilde{q}_{12} \) | \( \ldots \ldots \) | \( \tilde{c}_{in}, \tilde{t}_{in}, \tilde{q}_{in} \) | \( \tilde{c}_{21}, \tilde{t}_{21}, \tilde{q}_{21} \) | \( \tilde{c}_{22}, \tilde{t}_{22}, \tilde{q}_{22} \) | \( \ldots \ldots \) | \( \tilde{c}_{2n}, \tilde{t}_{2n}, \tilde{q}_{2n} \) | \( \ldots \ldots \) | \( \ldots \ldots \) | \( \ldots \ldots \) | \( \ldots \ldots \) | \( \tilde{c}_{in}, \tilde{t}_{in}, \tilde{q}_{in} \) | \( \tilde{c}_{in}, \tilde{t}_{in}, \tilde{q}_{in} \) |
| \( \ldots \ldots \) | \( \ldots \ldots \) | \( \ldots \ldots \) | \( \ldots \ldots \) | \( \ldots \ldots \) | \( \ldots \ldots \) | \( \ldots \ldots \) | \( \ldots \ldots \) | \( \ldots \ldots \) | \( \ldots \ldots \) | \( \ldots \ldots \) | \( \ldots \ldots \) | \( \ldots \ldots \) | \( \ldots \ldots \) | \( \ldots \ldots \) | \( \ldots \ldots \) |

Since the objective functions conflict with each other, a complete fuzzy optimal solution doesn’t exist always, so a new solution concept called fuzzy compromise solution is obtained for multi objective fuzzy assignment problem. A simple method would be to form a composite objective function as the weighted sum of the objectives, where a weight for an objective is proportional to the preference factor assigned to that particular objective.

3.3. Algorithm to solve fuzzy multi objective assignment problem

**Step 1:** Express the given fuzzy multi objective assignment problem in parametric form.

**Step 2:** Convert the fuzzy multi objective assignment problem into a fuzzy single objective assignment problem by giving a suitable weight to the objectives.

**Step 3:** Solve the fuzzy single objective assignment problem without converting into crisp form.
**Step 4:** Optimal solution of the fuzzy single objective assignment problem is also a optimal solution to fuzzy multi objective assignment problem.

4. Numerical example

Let us consider an example discussed by Gupta and Mehlawat (2014). A fuzzy assignment problem concerning six jobs and six workers are given below whose cost, time and quality are represented by triangular fuzzy numbers.

**Table 2:** Cost matrix with triangular fuzzy numbers

| Worker(i) | Job (j) | Job-1 | Job-2 | Job-3 | Job-4 | Job-5 | Job-6 |
|-----------|---------|-------|-------|-------|-------|-------|-------|
| Worker-1  |         |       |       |       |       |       |       |
| $c_{ij}$  | (4,6,8) | (3,4,6)| (4,5,8)| (6,8,11)| (7,10,14)| (4,6,7)| |
| $t_{ij}$  | (2,4,5) | (16,20,24)| (7,9,12)| (2,3,5)| (5,8,10)| (7,9,12)| |
| $q_{ij}$  | (0,1,3) | (1,3,5) | (0,1,3) | (0,1,3) | (0,1,3) | (3,5,7) | |
| Worker-2  |         |       |       |       |       |       |       |
| $c_{ij}$  | (4,6,7) | (4,5,7)| (5,6,9)| (5,8,12)| (6,9,11)| (6,8,11)| |
| $t_{ij}$  | (4,6,9) | (15,18,22)| (6,8,12)| (7,5,10)| (14,17,20)| (6,8,10)| |
| $q_{ij}$  | (1,3,5) | (3,5,7) | (1,3,5) | (3,5,7) | (5,7,9) | (3,5,7) | |
| Worker-3  |         |       |       |       |       |       |       |
| $c_{ij}$  | (8,11,14)| (5,7,9)| (2,4,6)| (5,8,12)| (2,3,4)| (3,4,6) | |
| $t_{ij}$  | (2,3,4) | (6,8,10)| (17,20,24)| (5,7,10)| (12,15,18)| (5,7,10) | |
| $q_{ij}$  | (0,1,3) | (5,7,9) | (3,5,7) | (1,3,5) | (3,5,7) | (5,7,9) | |
| Worker-4  |         |       |       |       |       |       |       |
| $c_{ij}$  | (7,9,12)| (7,10,12)| (6,8,11)| (4,6,8)| (8,10,12)| (3,4,6) | |
| $t_{ij}$  | (10,12,16)| (10,13,16)| (12,14,18)| (4,6,9)| (7,9,12)| (8,10,14) | |
| $q_{ij}$  | (3,5,7) | (7,9,10) | (1,3,5) | (3,5,7) | (1,3,5) | (1,3,5) | |
| Worker-5  |         |       |       |       |       |       |       |
| $c_{ij}$  | (3,4,6)| (4,6,8)| (5,7,10)| (7,9,12)| (6,8,12)| (5,7,10) | |
| $t_{ij}$  | (7,9,12)| (5,8,11)| (5,7,10)| (11,14,18)| (3,5,8)| (7,9,12) | |
| $q_{ij}$  | (1,3,5) | (7,9,10) | (5,7,9) | (3,5,7) | (1,3,5) | (1,3,5) | |
| Worker-6  |         |       |       |       |       |       |       |
| $c_{ij}$  | (2,3,4)| (4,5,7)| (8,11,15)| (8,10,13)| (9,12,15)| (6,8,12) | |
| $t_{ij}$  | (14,17,21)| (10,13,16)| (2,3,5)| (3,5,8)| (10,13,17)| (5,7,10) | |
| $q_{ij}$  | (1,3,5) | (1,3,5) | (3,5,7) | (5,7,9) | (3,5,7) | (5,7,9) | |
Table 3: (Cost matrix) Triangular fuzzy numbers are in their parametric form

|        | Job-1       | Job-2       | Job-3       | Job-4       | Job-5       | Job-6       |
|--------|-------------|-------------|-------------|-------------|-------------|-------------|
| Worker-1 | (6.2-2r,2-2r) | (4.2-2r,2-2r) | (1.1-2r,1-2r) | (20.4-4r,2-4r) | (3.2-2r,2-2r) | (1.1-2r,1-2r) |
| Worker-2 | (6.2-2r,2-2r) | (6.2-2r,2-2r) | (3.2-2r,2-2r) | (18.3-3r,3-3r) | (5.2-2r,2-2r) | (1.1-2r,1-2r) |
| Worker-3 | (11.3-3r,3-3r) | (3.1-1r,1-1r) | (1.1-1r,1-1r) | (20.3-3r,3-3r) | (5.2-2r,2-2r) | (1.1-2r,1-2r) |
| Worker-4 | (9.2-2r,2-2r) | (9.2-2r,2-2r) | (3.2-2r,2-2r) | (7.2-2r,2-2r) | (3.2-2r,2-2r) | (7.2-2r,2-2r) |
| Worker-5 | (4.1-1r,1-1r) | (6.2-2r,2-2r) | (6.2-2r,2-2r) | (7.2-2r,2-2r) | (7.2-2r,2-2r) | (7.2-2r,2-2r) |
| Worker-6 | (3.1-1r,1-1r) | (17.3-3r,3-3r) | (3.2-2r,2-2r) | (7.2-2r,2-2r) | (7.2-2r,2-2r) | (7.2-2r,2-2r) |

|        | c_{ij}      | t_{ij}      | q_{ij}      | c_{ij}      | t_{ij}      | q_{ij}      |
|--------|-------------|-------------|-------------|-------------|-------------|-------------|
| Worker-1 | (6.2-2r,2-2r) | (4.2-2r,2-2r) | (1.1-2r,1-2r) | (20.4-4r,2-4r) | (3.2-2r,2-2r) | (1.1-2r,1-2r) |
| Worker-2 | (6.2-2r,2-2r) | (6.2-2r,2-2r) | (3.2-2r,2-2r) | (18.3-3r,3-3r) | (5.2-2r,2-2r) | (1.1-2r,1-2r) |
| Worker-3 | (11.3-3r,3-3r) | (3.1-1r,1-1r) | (1.1-1r,1-1r) | (20.3-3r,3-3r) | (5.2-2r,2-2r) | (1.1-2r,1-2r) |
| Worker-4 | (9.2-2r,2-2r) | (9.2-2r,2-2r) | (3.2-2r,2-2r) | (7.2-2r,2-2r) | (3.2-2r,2-2r) | (7.2-2r,2-2r) |
| Worker-5 | (4.1-1r,1-1r) | (6.2-2r,2-2r) | (6.2-2r,2-2r) | (7.2-2r,2-2r) | (7.2-2r,2-2r) | (7.2-2r,2-2r) |
| Worker-6 | (3.1-1r,1-1r) | (17.3-3r,3-3r) | (3.2-2r,2-2r) | (7.2-2r,2-2r) | (7.2-2r,2-2r) | (7.2-2r,2-2r) |
Table 4: Triangular fuzzy numbers are given preferred weights

|     | Job-1     | Job-2     | Job-3     | Job-4     | Job-5     | Job-6     |
|-----|-----------|-----------|-----------|-----------|-----------|-----------|
| Worker-1 | (3.1,1,1-r) | (3.1,1,1-r) | (3.1,1,1-r) | (3.1,1,1-r) | (3.1,1,1-r) | (3.1,1,1-r) |
|        | (1.3,0.6,0.6) | (1.3,0.6,0.6) | (1.3,0.6,0.6) | (1.3,0.6,0.6) | (1.3,0.6,0.6) | (1.3,0.6,0.6) |
|        | (-0.2,0.2,0.2) | (-0.2,0.2,0.2) | (-0.2,0.2,0.2) | (-0.2,0.2,0.2) | (-0.2,0.2,0.2) | (-0.2,0.2,0.2) |
| Worker-2 | (3.1,1,1-r) | (3.1,1,1-r) | (3.1,1,1-r) | (3.1,1,1-r) | (3.1,1,1-r) | (3.1,1,1-r) |
|        | (1.3,0.6,0.6) | (1.3,0.6,0.6) | (1.3,0.6,0.6) | (1.3,0.6,0.6) | (1.3,0.6,0.6) | (1.3,0.6,0.6) |
|        | (-0.6,0.4,0.4) | (-0.6,0.4,0.4) | (-0.6,0.4,0.4) | (-0.6,0.4,0.4) | (-0.6,0.4,0.4) | (-0.6,0.4,0.4) |
| Worker-3 | (3.5,1.5,1.5-r) | (3.5,1.5,1.5-r) | (3.5,1.5,1.5-r) | (3.5,1.5,1.5-r) | (3.5,1.5,1.5-r) | (3.5,1.5,1.5-r) |
|        | (0.9,0.3,0.3) | (0.9,0.3,0.3) | (0.9,0.3,0.3) | (0.9,0.3,0.3) | (0.9,0.3,0.3) | (0.9,0.3,0.3) |
|        | (-0.2,0.2,0.2) | (-0.2,0.2,0.2) | (-0.2,0.2,0.2) | (-0.2,0.2,0.2) | (-0.2,0.2,0.2) | (-0.2,0.2,0.2) |
| Worker-4 | (4.5,1.5,1.5-r) | (4.5,1.5,1.5-r) | (4.5,1.5,1.5-r) | (4.5,1.5,1.5-r) | (4.5,1.5,1.5-r) | (4.5,1.5,1.5-r) |
|        | (3.5,1.5,1.5) | (3.5,1.5,1.5) | (3.5,1.5,1.5) | (3.5,1.5,1.5) | (3.5,1.5,1.5) | (3.5,1.5,1.5) |
|        | (3.3,0.9,0.9) | (3.3,0.9,0.9) | (3.3,0.9,0.9) | (3.3,0.9,0.9) | (3.3,0.9,0.9) | (3.3,0.9,0.9) |
|        | (-1.8,0.4,0.4) | (-1.8,0.4,0.4) | (-1.8,0.4,0.4) | (-1.8,0.4,0.4) | (-1.8,0.4,0.4) | (-1.8,0.4,0.4) |
| Worker-5 | (2.0,5.0,5.0-r) | (2.0,5.0,5.0-r) | (2.0,5.0,5.0-r) | (2.0,5.0,5.0-r) | (2.0,5.0,5.0-r) | (2.0,5.0,5.0-r) |
|        | (2.7,0.6,0.6) | (2.7,0.6,0.6) | (2.7,0.6,0.6) | (2.7,0.6,0.6) | (2.7,0.6,0.6) | (2.7,0.6,0.6) |
|        | (-0.6,0.4,0.4) | (-0.6,0.4,0.4) | (-0.6,0.4,0.4) | (-0.6,0.4,0.4) | (-0.6,0.4,0.4) | (-0.6,0.4,0.4) |
|        | (1.5,0.5,0.5) | (1.5,0.5,0.5) | (1.5,0.5,0.5) | (1.5,0.5,0.5) | (1.5,0.5,0.5) | (1.5,0.5,0.5) |
Table 5: Fuzzy multi objective assignment problem converted into single objective assignment problem

| Worker | Job-1 | Job-2 | Job-3 | Job-4 | Job-5 | Job-6 |
|--------|-------|-------|-------|-------|-------|-------|
| W1     | (4.1-r,1-r) | (7.4,1.2-1.2r, 1.2-1.2r) | (5.5,1-r, 1-r) | (4.7,1-r,1-r) | (7.9,1.5-1.5r, 1.5-1.5r) | (6.7,1-r,1-r) |
| W2     | (4.2,1-r,1-r) | (6.9,1-r,1-r) | (4.3,1-r, 1-r) | (3.6,1-r,1-r) | (8.2,1.5-1.5r, 1.5-1.5r) | (5.4,1.5-1.5, 1.5-1.5r) |
| W3     | (6.2,1.5-5r, 1.5-1.5r) | (4.5,1-r,1-r) | (7.1,1-r, 1-r) | (5.5,1.5-1.5r, 1.5-1.5r) | (5.0,9-0.9r, 0.9-0.9r) | (2.7,0.6-0.6r, 0.6-0.6r) |
| W4     | (7.1,1-r,1-r) | (7.1,1.5-1.5r, 1.5-1.5r) | (7.6,1-r, 1-r) | (3.8,1-r,1-r) | (7.1,1-r,1-r) | (4.4,0.6-0.6r, 0.6-0.6r) |
| W5     | (4.1,0.6-0.6r, 0.6-0.6r) | (3.6,1-r,1-r) | (4.2,1-r, 1-r) | (7.7,1-r,1-r) | (4.9,1-r,1-r) | (5.6,1-r,1-r) |
| W6     | (6.0,9-0.9r, 0.9-0.9r) | (5.8,0.9-0.9r, 0.9-0.9r) | (5.1,1-r, 1-r) | (5.1,1-r,1-r) | (8.9,1.5-1.5r, 1.5-1.5r) | (4.7,1-r,1-r) |

Table 6: Row minima subtracted table

| Worker | Job-1 | Job-2 | Job-3 | Job-4 | Job-5 | Job-6 |
|--------|-------|-------|-------|-------|-------|-------|
| W1     | (0,0) | (3.4,1.2-1.2r, 1.2-1.2r) | (1.5, 1-r, 1-r) | (0.7,1-r,1-r) | (3.9,1.5-1.5r, 1.5-1.5r) | (2.7, 1-r,1-r) |
| W2     | (0.6, 1-r,1-r) | (3.3, 1-r,1-r) | (0.7, 1-r,1-r) | (0.0,0) | (4.6,1.5-1.5r, 1.5-1.5r) | (1.8,1.5-1.5, 1.5-1.5r) |
| W3     | (3.5,1.5-1.5r, 1.5-1.5r) | (1.8,1-r,1-r) | (4.3, 1-r,1-r) | (2.8,1.5-5r, 1.5-1.5r) | (2.3,1.5-1.5r,1.5-1.5r) | (0,0) |
| W4     | (3.3,1-r,1-r) | (3.3, 1-r,1-r) | (3.8, 1-r,1-r) | (0,0,0) | (3.3, 1-r,1-r) | (0.6, 1-r,1-r) |
| W5     | (0.5, 1-r,1-r) | (0.0,0) | (0.6, 1-r,1-r) | (4.1, 1-r,1-r) | (1.3, 1-r,1-r) | (2, 1-r,1-r) |
| W6     | (1.3,1-r,1-r) | (1.1, 1-r,1-r) | (0.4, 1-r,1-r) | (0.4, 1-r,1-r) | (4.2, 1-r,1-r) | (0,0) |
Applying the proposed algorithm, we have

| Job-1 | Job-2 | Job-3 | Job-4 | Job-5 | Job-6 |
|-------|-------|-------|-------|-------|-------|
| W1    | (0,0,0) | (3.4, 1.2-1.2r, 1.2-1.2r) | (1.1, 1-r, 1-r) | (0.5, 0.4-0.4r, 0.4-0.4r) | (2.6, 1.5-1.5r, 1.5-1.5r) | (2.7, 1-r, 1-r) |
| W2    | (0.6, 1-r, 1-r) | (3.3, 1-r, 1-r) | (0.3, 1-r, 1-r) | (0.0, 0) | (3.3, 1.5-1.5r, 1.5-1.5r) | (1.8, 1.5-1.5r, 1.5-1.5r) |
| W3    | (3.5, 1.5-1.5r, 1.5-1.5r) | (1.8, 1-r, 1-r) | (3.9, 1-r, 1-r) | (1,1.3-1.3r, 1.3-1.3r) | (1, 1.5-1.5r, 1.5-1.5r) | (0.0, 0) |
| W4    | (3.3, 1-r, 1-r) | (3.3, 1-r, 1-r) | (3.4, 1-r, 1-r) | (0.0, 0) | (2, 1-r, 1-r) | (0.6, 1-r, 1-r) |
| W5    | (0.5, 1-r, 1-r) | (0,0,0) | (0.2, 1-r, 1-r) | (4.2, 0.4-0.4r, 0.4-0.4r) | (0.0, 0) | (2, 1-r, 1-r) |
| W6    | (1.3, 1-r, 1-r) | (1.1, 1-r, 1-r) | (0,0,0) | (0.8, 0,0) | (2.9, 1-r, 1-r) | (0.0, 0) |

The current assignment is optimum.

The optimum Assignment schedule is $w_1 \rightarrow j_1, w_2 \rightarrow j_3, w_3 \rightarrow j_5, w_4 \rightarrow j_4, w_5 \rightarrow j_2, w_6 \rightarrow j_6$.

Fuzzy optimal cost = (6.2-2r,2-2r)+(7.2-2r,2-2r)+(3, 1-r, 1-r)+(6.2-2r,2-2r)+(6.2-2r,2-2r)

+ (8, 2-2r,2-2r) = (36, 2-2r,2-2r) = (34+2r, 36, 38-2r)

Fuzzy optimal time = (4.2-2r,2-2r)+(8.2-2r,2-2r)+(15, 3-3r,3-3r)+(6.3-3r,3-3r)+(8, 3-3r,3-3r)

+ (7, 2-r,2-2r) = (48, 3-3r,3-3r) + (45+3r,48,51-3r)

Fuzzy optimal quality = (1.1-r,1-r)+(3.2-2r,2-2r)+(5.2-2r,2-2r)+(5.2-2r,2-2r)+(9.2-2r,2-2r)

+ (7,2-2r,2-2r = (30, 2-2r,2-2r) = (28+2r,30,32-2r)
5. Comparison of Results

For the same problem Gupta et.al [14] obtained the optimum solution for a particular \( r \in [0,1] \) as below:

| \( r = 0.695652 \) | Optimum Cost | (24.8, 32, 43.7) | Optimum Time | (38.3, 50, 64.4) | Optimum Quality | (11.20, 30.8) |
|--------------------|---------------|-----------------|--------------|-----------------|----------------|----------------|

But the proposed method gives vagueness reduced results for different values of \( r \in [0,1] \).

| \( r \) | Fuzzy optimal cost | (34+2r, 36, 38-2r) | Fuzzy optimal time | (45+3r, 48, 51-3r) | Fuzzy optimal quality | (28+2r, 30, 32-2r) |
|---------|---------------------|---------------------|---------------------|---------------------|---------------------|---------------------|
| 0       | (34, 36, 38)        | (45, 48, 51)        | (28, 30, 32)        |
| 0.5     | (35, 36, 37)        | (46, 48, 49.5)      | (29, 30, 31)        |
| 1       | (36, 36, 36)        | (48, 48, 48)        | (30, 30, 30)        |

6. Conclusion

We have proposed a new approach to solve the fuzzy multi objective assignment problem involving triangular fuzzy numbers without reducing to its classical form. The proposed method gives flexibility to the decision maker to choose his/her suitable \( r \) according preference. It is evident from the example that our method provides better solution for the fuzzy multi objective assignment problem.

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