Full Factorial DOE to determine and optimize the equation of impact forces produced by water jet used in sewer cleaning

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Abstract. The functioning of the equipment to cleaning sewer is dependent on certain process parameters, causing variations of the impact forces. The aim of this paper is to define and optimise an equation that determines the impact forces, forces produced when the water jets is used to clean the sewer. The research method to measure the impact forces is the experiment using full factorial design of experiment. To can made the experiments was used a stand to generate the pressure waterjets and a device to measurement the values of impact forces. In the first part of paper was determine the equation of impact forces produced by water jet and then was realised a multiple linear regression model in three different ways to optimise the prediction of the proposed equation.

1 Introduction

Industry water jet technology is frequently used in a lot of areas such: concrete hydrodemolition, jet cutting for different type of materials, mechanical processing of minerals, medical applications, rock fragmentation, surface preparation for protective coatings [1]. Industrial cleaning is a classic application of water jets technology. In the late 1950s, when reliable high-pressure pumps were built, the usage of water jets spread widely in the field of pipes and sewerage cleaning.

A series of complex phenomena appear in the cleaning jets. Most research in the field is experimental. So, Adler [2] studying the mechanisms that appear when a water jet hit a surface. Other works have studied the pressure variation in water jets such as Leach et al [3], Leu et al [4] and Guha et al [5].

To clean the sewer networks is used equipment that use high pressure water jets. The parameters of water jets adjusted by this equipment are determined by the process parameters [1]. Variation of these parameters produces variations of the impact forces.

To determine an equation who described the values of impact forces in concordance with the process parameters is necessary to realised practical experiments to determine the values of impact forces for different set-up values of process parameters [6,7].

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2 Apparatus used and methodology

To realize the experiments, was used a stand for generate water jets and a device to for determining the values of impact forces [6].

2.1 Stand to generate pressure water jets

In figure 1 is presented the diagram for the stand to generate water jet.

![Diagram of the stand to generate water jet.](image)

Component parts of stand: (1) electric motor (2) flexible coupling; (3) high pressure pump, (4) pressure regulator, (5) pressure gauge, (6) nozzle, (7) tap water, (8) water tank, (9) chassis. From the high-pressure pump (3) water coming out and goes to the pressure regulator (4). This pressure regulator allows to adjust the working pressure to the desired value.

2.2 Device to measure the impact forces

In figure 2 is presented the diagram of the device used to measure the impact force produced by water jet when hit a surface.

![Diagram of the device for measure the impact force.](image)

The principle of working for the device is the following. From high pressure water hose (1) come water at a desired pressure p. At the outlet of nozzle is generated a water jet (5). This water jet (5) hit the target plate (6). This target plate (6) can be located at a certain distance x in front of the nozzle. The water jet (5) generates the impact force when he meets target plate (6). The impact force produces axial movement of target plate, axial movement converted into an electric signal by the piezoelectric sensor (10). The electrical signal is
collected by data acquisition board (11), which forward data to a computer (12) and using processing software results the values of impact force.

2.3 Methodology of the measurements

The research method of this study is the experiment. To determine the values of impact forces produced by water jets is used the full factorial method. To determine the impact forces, it is necessary to set up the experimental domain.

In the water jet cleaning process, a series of parameters are involved [1]. These parameters can be divided into two major groups, namely: 1) target parameters which shall be defined according to the contact area between the water jet and the surface to be cleaned and 2) process parameters. In the measurement of the impact forces of a stationary water jet and flat and rigid surface the process parameters are involved.

2.4 Setting up the experimental domain.

The process parameters are:
1) D nozzle diameter, [mm];
2) P water pressure, [bars];
3) x distance between the nozzle and impact surface, [mm];
4) $\alpha$ impact angle (angle formed by the jet and impact surface), [$^\circ$].

In table 1 are presented the values of process parameters and their levels according with full factorial method.

| Abbreviation | Parameter | Values          |
|--------------|-----------|-----------------|
| A            | Nozzle diameter D [mm] | 1, 1.5, 2       |
| B            | Pressure p [bar]       | 100, 120, 140, 160, 180, 200 |
| C            | Impact angle $\alpha$ [$^\circ$] | 60, 75, 90     |
| D            | Distance x [mm]        | 25, 50, 75, 100, 125, 150, 175, 200 |

Corresponding to the 4 parameters and second-degree interactions of the parameters, for full factorial method result a plan of experiments who contain a number of 144 experiments. Based on the parameters established in accordance with Table 1 the impact forces were determined. For each experiment was performed three measures of impact force and still it has worked with $F_{med}$, representing the arithmetic average Results a total of 432 measurements.

For brief of this paper, we dare not to present the values of $F_{med}$ obtained.

3 Results

For determine the equation of impact forces, in the first step it is necessary to determine the contribution of each parameter and their interactions.

3.1 Determining the contributions of the parameters and their interactions

To calculate the contributions of parameters and their interactions was used the values of $F_{med}$ obtained in concordance with design of experiments. Using the full factorial method has conducted an analysis the variance to determine the influence of each parameter and their
interactions on the impact force. In table 2 is presented the analysis of the variance, using Minitab 17.

Table 2. Analysis of Variance (Minitab 17).

| Source                  | DF (degree of freedom) | SS (sum of square) | Contribution |
|-------------------------|------------------------|-------------------|--------------|
| Regression              | 10                     | 162559            | 99.14%       |
| Diameter                | 1                      | 116939            | 71.32%       |
| Pressure                | 1                      | 29592             | 18.05%       |
| Angle                   | 1                      | 5959              | 3.63%        |
| Distance                | 1                      | 389               | 0.24%        |
| Diameter*Pressure       | 1                      | 7596              | 4.63%        |
| Diameter*Angle          | 1                      | 1523              | 0.93%        |
| Diameter*Distance       | 1                      | 148               | 0.09%        |
| Pressure*Angle          | 1                      | 385               | 0.24%        |
| Pressure*Distance       | 1                      | 20                | 0.01%        |
| Angle*Distance          | 1                      | 5                 | 0.00%        |
| Error                   | 421                    | 1406              | 0.86%        |
| Total                   | 431                    | 163965            | 100%         |

3.2 Determining the regression equations of impact forces

According to the results obtain in table 2, the next step is to determine the equation of the impact forces using only the parameters and interactions previously set.

Using Minitab 17, the multiple linear regression model of impact forces was determined (equation 1):

$$F_{med} = 22.91 - 32.97A - 0.2088B - 0.26C + 0.30072AB + 0.3755AC$$  \(1\)

This multiple linear regression model was obtaining without Box-Cox transformation. In table 3 is presented the analysis of variance of equation (1).

Table 3. Analysis of Variance for equation 1.

| Source                  | DF (degree of freedom) | SS (sum of square) | Contribution |
|-------------------------|------------------------|-------------------|--------------|
| Regression              | 5                      | 161610            | 98.56%       |
| Diameter                | 1                      | 116939            | 71.32%       |
| Pressure                | 1                      | 29592             | 18.05%       |
| Angle                   | 1                      | 5959              | 3.63%        |
| Diameter*Pressure       | 1                      | 7596              | 4.63%        |
| Diameter*Angle          | 1                      | 1523              | 0.93%        |
| Error                   | 426                    | 2355              | 1.44%        |
| Total                   | 431                    | 163965            | 100%         |

The regression statistics of equation (1) are: R squared: 98.56%, R squared adjusted: 98.55% and R squared predicted: 98.52%.

To optimise the equation (1), the next step is to be made a regression using Box-Cox transformation with rounded λ. Result the equation (2):

$$\ln(F_{med}) = -0.618 + 1.3611A + 0.007137B + 0.00969C$$  \(2\)

In table 4 is presented the analysis of variance of equation (2).
Table 4. Analysis of Variance for equation 2.

| Source               | DF (degree of freedom) | SS (sum of square) | Contribution |
|----------------------|------------------------|--------------------|--------------|
| Regression           | 5                      | 178.455            | 97.56%       |
| Diameter             | 1                      | 143.163            | 78.27%       |
| Pressure             | 1                      | 29.325             | 16.03%       |
| Angle                | 1                      | 5.959              | 3.26%        |
| Diameter*Pressure    | 1                      | 0.009              | 0.00%        |
| Diameter*Angle       | 1                      | 0.00               | 0.00%        |
| Error                | 426                    | 4.462              | 2.44%        |
| Total                | 431                    | 182.917            | 100%         |

The regression statistics of equation (2) are: R squared: 97.56%, R squared adjusted: 97.53% and R squared predicted: 97.50%.

Another possible optimisation of equation (1) is realised made a regression using Box-Cox transformation with λ=0.5. Result the equation (3):

\[
(F_{med})^{0.5} = 0.097 + 0.216 \cdot A - 0.00082 \cdot B - 0.0002 \cdot C + 0.01428 \cdot A \cdot B + 0.0173 \cdot A \cdot C
\]  

(3)

In table 5 is presented the analysis of variance of equation (3).

Table 5. Analysis of Variance for equation 3.

| Source               | DF (degree of freedom) | SS (sum of square) | Contribution |
|----------------------|------------------------|--------------------|--------------|
| Regression           | 5                      | 1240.51            | 98.98%       |
| Diameter             | 1                      | 963.07             | 76.84%       |
| Pressure             | 1                      | 214.07             | 17.08%       |
| Angle                | 1                      | 42.99              | 3.43%        |
| Diameter*Pressure    | 1                      | 17.15              | 1.37%        |
| Diameter*Angle       | 1                      | 3.23               | 0.26%        |
| Error                | 426                    | 12.79              | 1.02%        |
| Total                | 431                    | 1253.30            | 100%         |

The regression statistics of equation (3) are: R squared: 98.98%, R squared adjusted: 98.97% and R squared predicted: 98.95%.

In table 6 are presented the regression statistics for all of three equations determined.

Table 6. Regression statistics for all 3 equations.

| Type of regression     | Equation no. | R squared      | R squared adjusted | R squared predicted |
|------------------------|--------------|----------------|--------------------|---------------------|
| without transformation | (1)          | 98.56%         | 98.55%             | 98.52%              |
| rounded λ transformation | (2)        | 97.56%         | 97.53%             | 97.50%              |
| λ=0.5 transformation    | (3)          | 98.98%         | 98.97%             | 98.95%              |

4 Verification of determined equations

For verifying the three previously determined equations was performing a new set of experiments. For each experiment was performed three measures of impact force and still it has worked with F_{exp}, representing the arithmetic average.

Table 7 lists the default values for D, p, and α input parameters. The distance x was set for all experiments at 100 mm.

In order to compare the obtained results, the relative error between F_{exp} and F_{cal} was calculated for each of the three determined equations.
The relative error is given by the relationship:

$$\varepsilon = \frac{|F_{cal} - F_{exp}|}{F_{exp}}$$  \hfill (4)

were \(F_{cal}\) represents the impact force calculated with one of the three previously determined equations and \(F_{exp}\) represents the impact force measured experimentally.

**Table 7.** The relative error for the three determined equations.

| Exp no | D [mm] | P [bar] | α [°] | \(F_{exp}\) [N] | \(F_{cal}\) [N] | \(\varepsilon_1\) | \(\varepsilon_2\) | \(\varepsilon_3\) |
|--------|--------|---------|--------|-----------------|-----------------|---------|---------|---------|
| 1      | 1.25   | 130     | 70     | 16.85           | 18.08           | 15.41   | 16.66   | 7.30    | 8.55   | 1.13   |
| 2      | 1.75   | 170     | 85     | 19.87           | 21.22           | 17.78   | 19.39   | 6.79    | 10.52  | 2.42   |
| 3      | 1.25   | 130     | 70     | 22.88           | 24.76           | 20.84   | 22.69   | 8.22    | 8.92   | 0.83   |
| 4      | 1.75   | 170     | 85     | 26.25           | 27.9            | 24.06   | 25.86   | 6.29    | 8.34   | 1.49   |
| 5      | 1.25   | 130     | 70     | 33.26           | 34.28           | 31.09   | 32.77   | 3.07    | 6.52   | 1.47   |
| 6      | 1.75   | 170     | 85     | 39.31           | 40.24           | 35.9    | 38.14   | 2.37    | 8.67   | 2.98   |
| 7      | 1.25   | 130     | 70     | 43.85           | 46.98           | 42.32   | 44.78   | 7.14    | 3.49   | 2.12   |
| 8      | 1.75   | 170     | 85     | 51.74           | 52.94           | 48.87   | 51.02   | 2.32    | 5.55   | 1.39   |

From table 7, it can be seen from the point of view of the relative error that the most accurate is equation 3 followed by equation 1, and the latter equation 2. This is in accordance with the regression statistics for all of three equations determined above (table 6).

**5 Conclusions**

1) In this work, it is presented a methodology to determine the impact forces produced by water jets used in sewer cleaning. The impact forces dependent on certain process parameters.

2) The research method used is full factorial design of experiment. After applying full factorial itinerary for calculating the percentage of influence of parameters and their interactions of the impact forces, it is found: nozzle diameter D is the largest influence, with a percentage of 71.32%. In the second place is pressure P with a value of 18.05%, follow by interaction between nozzle diameter D and pressure p with a value of 4.63% and the impact angle \(\alpha\) with a value of 3.63%. The lowest value is given by interaction between nozzle diameter D and impact angle \(\alpha\) with a value of 0.93%.

3) For the experimental domain, the parameter distance \(x\) has a percentage of influence of only 0.24% for impact forces, practically insignificant.

4) According with the influence of parameters and their interactions, was realised a multiple linear regression model in three different ways to optimise the prediction of the proposed equation:

- first type regression without transformation, with R squared predicted 98.52%;
- second type regression using rounded \(\lambda\) transformation, with R squared predicted 97.50%;
  - third type regression using \(\lambda=0.5\) transformation, with R squared predicted 98.95%.

5) The best prediction is given by the regression using \(\lambda=0.5\) transformation, followed by the regression without transformation. The lowest degree of prediction is given by the regression using rounded \(\lambda\) transformation.

6) It is possible to improve the prediction of the equation who describe the impact forces only using different type of regression and the same measured values of the impact forces.
7) After completing a new set of experiments and calculating the relative error for the three equations, the hierarchy for regression statistics for all 3 equations is confirmed.

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