1-D DC Resistivity Inversion Using Singular Value Decomposition and Levenberg-Marquardt’s Inversion Schemes

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Abstract. Exploration of natural or energy resources requires geophysical survey to determine the subsurface structure, such as DC resistivity method. In this research, field and synthetic data were used using Schlumberger configuration. One-dimensional (1-D) DC resistivity inversion was carried out using Singular Value Decomposition (SVD) and Levenberg-Marquardt (LM) techniques to obtain layered resistivity structure. We have developed software to perform both inversion methods accompanied by a user-friendly interface. Both of the methods were compared one another to determine the number of iteration, robust to noise, elapsed time of computation, and inversion results. SVD inversion generated faster process and better results than LM did. The inversion showed both of these methods were appropriate to interpret subsurface resistivity structure.

1. Introduction
DC Resistivity is one of geophysical exploration methods that can obtain subsurface resistivity structure. This method is frequently utilized for archeology surveys [15], mineral exploration [14], geothermal [13], and groundwater investigation [10,12].

Measurement principle of DC resistivity is injecting an amount of electric current into the earth to acquire potential difference on the surface thus apparent resistivity would be obtained based on Ohm’s law. Regarding its measurement configuration, this method is divided into some types, one of examples is Schlumberger configuration. This configuration is usually used for obtaining resistivity variation to its depth in 1-D orientation. Generally, resistivity model is interpreted in 2-D and 3-D domain, nevertheless for shallow structure and not too complex geology condition, interpretation can be done one-dimensionally. The apparent resistivity data acquired in the measurement is not real, thus interpretation demands an inversion process to obtain subsurface resistivity model. Several inversion methods which can be applied to apparent resistivity data in 1-D case are stable iterative [11], ridge regression [5], Levenberg-Marquardt [3,9] and singular value decomposition [1] are methods that often be used for solving geophysical problem but there is still a few of scientists quantitatively comparing the advantages and disadvantages of these two methods.

In this paper, we compared Levenberg-Marquardt (LM) to singular value decomposition (SVD) inversion method to determine each robustness of the methods.
2. Forward Modeling and Inversion

Within this section, theories concerning forward modeling and inversion of DC resistivity with Schlumberger’s configuration are going to be explained.

2.1. Forward Modeling

As commonly used in forward modeling DC resistivity with Schlumberger’s configuration [1,4]. The mathematical relation between Schlumberger’s apparent resistivity data and subsurface layer’s parameter can be described by Hankel’s integral.

\[
\rho_a = \left( \frac{AB}{2} \right)^2 \frac{\lambda}{\pi} T_i(\lambda) J_i\left( \frac{AB}{2} \lambda \right) \, d\lambda
\]

(1)

where \( \frac{AB}{2} \) is half the current electrode spacing in Schlumberger configuration, \( \lambda \) is electrode parameter, \( J_i \) is the first-order Bessel’s function of the first kind, \( T_i(\lambda) \) is given by the recursive formulation [7] as follows:

\[
T_i(\lambda) = \frac{T_i(\lambda) + \rho_{i+1} \tanh(\lambda_{i+1})}{1 + T_i(\lambda) \tanh(\lambda_{i+1})/\rho_{i+1}} \quad ; \; i = NL, NL-1, ..., 1
\]

(2)

where \( \rho_i \) and \( t_i \) are resistivity and thickness of \( i \)th layer, \( NL \) is the consecutive number of layers. For the last layer, \( T_{NL} = \rho_{NL} \). Using fast Hankel transforms, that apparent resistivity data for DC resistivity Schlumberger configuration can be calculated as [8]:

\[
\rho_a = \sum_k T_i(\lambda_k) f_k
\]

(3)

where \( K \) is the number of coefficient and \( f_k \) are the filter coefficients.

2.2. Inversion

Below is nonlinear relation between model parameter and data:

\[
\mathbf{G}(\mathbf{m}) = \mathbf{d}
\]

(4)

where \( \mathbf{d} \) is vector of data with size of \( N \times 1 \), \( \mathbf{G} \) is nonlinear function relating data to model parameter and \( \mathbf{m} \) vector of model parameters with size of \( M \times 1 \). \( N \) is number of data and \( M \) is number of models. In this DC resistivity case, the data to be scrutinized is apparent resistivity (\( \rho_a \)) vs AB/2 and the model parameters are true resistivity (\( \rho \)) and layer’s thickness (\( t \)).

Objective function to be minimized is [2]:

\[
Y(\mathbf{m}) = \sum_{i=1}^{N} \frac{G(\mathbf{m})_i - d_i}{\sigma_i}
\]

(5)

where \( \sigma_i \) is deviation of \( i \)th data.

2.2.1. Levenberg-Marquardt Scheme. Iteration equation of model update in this method is expressed as follow [2,3]:

\[
(J(m^k)^T J(m^k) + \lambda I) \Delta \mathbf{m} = -J(m^k)^T F(m^k)
\]

(6)

where \( I \) is the identity matrix and \( \lambda \) is a damping factor. \( \Delta \mathbf{m} = m^{k+1} - m^k \) with \( k \) is index of iteration. \( F(m^k) \) is vector of differences between observed data and calculated data:

\[
F(\mathbf{m}) = \begin{bmatrix} f_1(\mathbf{m}) \\ \vdots \\ f_y(\mathbf{m}) \end{bmatrix}
\]

(7)
where

$$f_i(m) = \sum_{i=1}^{N} \left( \frac{G(m) - d_i}{\sigma_i} \right)$$  \hspace{1cm} (8)$$

Meanwhile, $J(m)$ is a sensitivity matrix or Jacobi written as below:

$$J(m) = \begin{bmatrix}
\frac{\partial f_1(m)}{\partial m_1} & \ldots & \frac{\partial f_1(m)}{\partial m_M} \\
\vdots & \ddots & \vdots \\
\frac{\partial f_N(m)}{\partial m_1} & \ldots & \frac{\partial f_N(m)}{\partial m_M}
\end{bmatrix}$$  \hspace{1cm} (9)$$

Thus equation (6) can be written as follow:

$$m^{k+1} = m^k + \left( J(m^k)^T J(m^k) + \lambda I \right)^{-1} \left[ -J(m^k)^T F(m^k) \right]$$  \hspace{1cm} (10)$$

Damping factor can be determined using golden section search (GSS) method [4] at each iteration.

2.2.2. Singular Value Decomposition Scheme. Iteration equation of model update in this method is based on a formulation by Meju (1994), where matrix $J$ to be factorized into 3 matrices:

$$J = USV^T$$  \hspace{1cm} (11)$$

where $U$ eigenvector of data with $N \times M$ size, $V$ is eigenvector of model parameters with $M \times M$ size and $S$ is diagonal matrix containing non-zero eigenvalue of $J$ as many as $R$ with $R \leq M$ which $R$ indicates $R$th iteration. Then equation (11) is substituted into equation (6) thus yields:

$$\left( US^2V^T + \lambda I \right) \Delta m = -VSU^TF(m^k)$$  \hspace{1cm} (12)$$

where $S$ is diagonal matrix containing singular values of matrix $J$ $(\mu_1, \mu_2, \ldots, \mu_M)$.

$$\left( US^2V^T + \lambda I \right) = \left( V \text{diag} \left( \mu_j^2 \right) V^T + \lambda I \right); j = 1, 2, \ldots, M$$  \hspace{1cm} (13)$$

Ekinci and Demirci (2008) has derived equation (13) and obtained solution equation for this inversion problem:

$$m^{k+1} = m^k + V \text{diag} \left( \frac{\mu_j}{\mu_j^2 + \lambda} \right) U^T \left[ -F(m^k) \right]$$  \hspace{1cm} (14)$$

Damping factor is determined using formulation below [6]:

$$\lambda = \mu_L \Delta x^T$$  \hspace{1cm} (15)$$

where $L$ is test number performed and $\Delta x$ is formulated as follow:

$$\Delta x_R = \frac{(x_{R-1} - x_R)}{x_{R-1}}$$  \hspace{1cm} (16)$$

where $x_{R-1}$ and $x_R$ is misfit value of that the previous and the present iteration.
3. Applications and Results

In this section both inversion methods were applied to synthetic and field data. In order to facilitate analyzing quantitatively, we employed parameter of data’s RMSE (root-mean-square error), models’ RMSE, number of iteration and elapsed computing time.

RMS error is defined as follow:

$$\text{Data RMSE} = \sqrt{\frac{1}{N} \sum_{i=1}^{N} (d_i - G(m_i))^2}$$  \hspace{1cm} (17)

where \(d\) is observed data and \(G(m)\) is calculated data obtained from calculation of equation (4). Then relative RMSE is defined as data RMSE which is divided by \(d\) at each data and multiplied with 100%.

$$\text{Relative RMSE} = \sqrt{\frac{1}{N} \sum_{i=1}^{N} \left( \frac{d_i - G(m_i)}{d_i} \right)^2} \times 100\%$$ \hspace{1cm} (18)

Model RMSE is defined as follow:

$$\text{Model RMSE} = \sqrt{\frac{1}{M} \sum_{i=1}^{M} (m_{i,\text{true}} - m_{i,\text{est}})^2}$$ \hspace{1cm} (19)

where \(m_{i,\text{true}}\) and \(m_{i,\text{est}}\) is the real model and the estimated model at \(i\)th iteration.

3.1. Synthetic data

LM and SVD methods were applied to inversion with 3 test models of homogenous earth (model 1), 3-layered (model 2), and noisy 3-layered (model 3).

Forward modeling data calculated from those 3 models were then inverted using LM and SVD. The result, those 3 sets of synthetic data obtained small RMS errors, it can be seen on table 2. This is also indicated by curve of apparent resistivity vs. AB/2 of which observed data and calculated data have fit figures to each other (see figure 1a), 1b), and 1c)).

Table 1. True and estimated models: (a) homogenous model; (b) noise-free model; (c) noisy model (15% random noise)

| (a) | True | SVD | LM | Unit | (b) | True | SVD | LM | Unit | (c) | True | SVD | LM | Unit |
|-----|------|-----|-----|------|-----|------|-----|-----|------|-----|------|-----|-----|------|
| \(\rho_1\) | 10 | 10 | 10 | \(\Omega \ m\) | \(\rho_1\) | 100 | 99.98 | 99.99 | \(\Omega \ m\) | \(\rho_1\) | 100 | 108.39 | 107.97 | \(\Omega \ m\) |
| \(\rho_2\) | 10 | 10 | 10 | \(\Omega \ m\) | \(\rho_2\) | 50 | 49.96 | 49.93 | \(\Omega \ m\) | \(\rho_2\) | 50 | 52.69 | 49.58 | \(\Omega \ m\) |
| \(\rho_3\) | 10 | 10 | 10 | \(\Omega \ m\) | \(\rho_3\) | 10 | 10 | 10 | \(\Omega \ m\) | \(\rho_3\) | 10 | 10.79 | 10.68 | \(\Omega \ m\) |
| \(t_1\) | 10 | 5.34 | 5.78 | m | \(t_1\) | 5 | 5.01 | 5.01 | m | \(t_1\) | 5 | 5.45 | 5.87 | m |
| \(t_2\) | 10 | 4.63 | 4.03 | m | \(t_2\) | 20 | 20 | 20.01 | m | \(t_2\) | 20 | 18.91 | 19.65 | m |

(+ noise)
Table 2. Observed and calculated apparent resistivity data of model 1 (a), model 2 (b), model 3 (c).

| AB/2 (m) |       | Model 1 |       | Model 2 |       | Model 3 |       |
|----------|-------|---------|-------|---------|-------|---------|-------|
|          |       | Calculated |       | Calculated |       | Calculated |       |
|          |       | Observed | SVD   | LM      | Observed | SVD   | LM      | Observed | SVD   | LM      |
| 1        | 10    | 10    | 10    | 99.95  | 99.93  | 99.94  | 109.78 | 108.35  | 107.94 |
| 2        | 10    | 10    | 10    | 99.52  | 99.51  | 99.51  | 100.05 | 107.97  | 107.62 |
| 3        | 10    | 10    | 10    | 98.48  | 98.47  | 98.47  | 111.02 | 107.04  | 106.8  |
| 5        | 10    | 10    | 10    | 94.26  | 94.26  | 94.26  | 107.46 | 103.14  | 103.32 |
| 7        | 10    | 10    | 10    | 87.95  | 87.96  | 87.96  | 96.9   | 97     | 97.63  |
| 10       | 10    | 10    | 10    | 77.63  | 77.65  | 77.65  | 86.45  | 86.25  | 87.15  |
| 15       | 10    | 10    | 10    | 64.02  | 64.03  | 64.02  | 71.15  | 70.8   | 71.2   |
| 20       | 10    | 10    | 10    | 55.24  | 55.24  | 55.22  | 58.49  | 60.19  | 59.94  |
| 25       | 10    | 10    | 10    | 49.02  | 49.01  | 49     | 53.84  | 52.62  | 52.05  |
| 30       | 10    | 10    | 10    | 43.93  | 43.93  | 43.91  | 45.06  | 46.58  | 45.99  |
| 40       | 10    | 10    | 10    | 35.31  | 35.3   | 35.3   | 39.05  | 36.78  | 36.53  |
| 50       | 10    | 10    | 10    | 28.31  | 28.31  | 28.31  | 28.44  | 29.18  | 29.26  |
| 60       | 10    | 10    | 10    | 22.96  | 22.96  | 22.97  | 23.92  | 23.57  | 23.82  |
| 80       | 10    | 10    | 10    | 16.4   | 16.4   | 16.4   | 16.51  | 16.93  | 17.19  |
| 100      | 10    | 10    | 10    | 13.29  | 13.29  | 13.29  | 13.48  | 13.91  | 14.06  |
| 120      | 10    | 10    | 10    | 11.85  | 11.85  | 11.85  | 13.31  | 12.54  | 12.59  |
| 150      | 10    | 10    | 10    | 10.94  | 10.94  | 10.94  | 12.08  | 11.69  | 11.66  |
| 200      | 10    | 10    | 10    | 10.44  | 10.44  | 10.44  | 10.94  | 11.22  | 11.14  |
| 250      | 10    | 10    | 10    | 10.26  | 10.26  | 10.26  | 11.72  | 11.04  | 10.95  |
| 300      | 10    | 10    | 10    | 10.18  | 10.18  | 10.17  | 10.23  | 10.96  | 10.86  |

| Data RMSE | 1.8e-10 | 5.4e-15 | 0.0089 | 0.0097 | 2.389 | 2.369 |
| Model RMSE | 1.42 | 1.46 | 0.0089 | 0.014 | 1.785 | 1.613 |
| Iter. No | 94 | 11 | 57 | 87 | 100 | 100 |
| Time (s) | 59.19 | 11.63 | 36.7 | 84.23 | 14.22 | 95.19 |

The constructed homogenous model was a model with the same resistivity value over spaces and depth of 10 meters. Noisy 3-layered model, with random value of 15% was constructed with 3 resistivity values and 2 layer’s thicknesses and final apparent resistivity multiplied by 15% random value of apparent resistivity.

Inversion result of those two methods depended on initial model. If the initial model was too far from solution model thus its inversion process would obtain big calculation data and model RMSE values, also get trapped in local minimum. In model 1, the initial model for inversion were three resistivity values of 5 Ω m and depth values of 5 m. Meanwhile in model 2 and 3, the initial model were 3 resistivity values of 80 Ω m with layer’s depth of 10 m.

Inversion result using model 1 obtained relative RMSE as much as 0 % for both SVD and LM, using model 2 resulted in 1.42e-6 % and 9.25e-6 %, for SVD and LM respectively, and using model 3 obtained
0.15 % and 16 %. LM method had a good inversion solution when being applied to model 1 and 3. This can be seen from the data RMSE and model RMSE as presented by table 2. Furthermore, considering the numbers of iteration, SVD inversion scheme spent less iteration than LM inversion scheme, especially when using model 2. Meanwhile, considering iteration time, SVD was relatively faster than the other one especially when using model 2 and 3 (see table 2).
Figure 1. Left curve presents observed and calculated apparent resistivity data using a) model 1 b) model 2 and c) model 3. Black dot, red triangle, and blue line show observed, SVD-inverted, and LM-inverted data respectively. Right curve presents subsurface model. Black line, red line, and blue line show true, SVD-inverted, and LM-inverted model respectively.

3.2. Field data
After examining LM and SVD using synthetic data, we implemented those methods to field data. We used Kaleköy’s data, located in northeastern part of Gökçeada [1]. Geological condition in the area has a thick sedimentary sequence, alluvium, and volcanic rock. The measurement was conducted using Iris Syscal-R1 + resistivity meter with maximum electrode space (AB) as long as 120 m.

Table 3. The estimated model of the field data

| Model Parameter | Kaleköy | Gobi, China |
|-----------------|---------|-------------|
|                 | SVD     | LM          | SVD | LM |
| $Q_1$           | 50.529  | 47.105      | 24.826 | 27.239 |
| $Q_2$           | 18.862  | 10.665      | 69.653 | 77.219 |
| $Q_3$           | 4.068   | 12.57       | 24.787 | 5.195  |
| $Q_4$           | 219.527 | 388.252     | 474.424 | 343.836 |
| $t_1$           | 2.659   | 3.964       | 6.565  | 11.209 |
| $t_2$           | 10.577  | 38.927      | 38.886 | 64.411 |
| $t_3$           | 16.029  | 15.941      | 582.457 | 103.027 |
| Data RMSE       | 0.686   | 1.6736      | 0.946  | 4.302  |
| Iter. No.       | 100     | 100         | 100    | 100    |
| Time (s)        | 66.147  | 110.53      | 47.407 | 104.151 |
We selected the second field, Gobi area of West China [9]. Based on information provided by borehole measurement around the location, there were 4 layers of subsurface: Quaternary (0-4.63 m), Tertiary (4.63-48.34 m), Permian and Carboniferous (48.34-610 m), and Precambrian (basement).

The data inversion of both fields depended on initial model and model values would get trapped in local minimum misfit if initial models was less good. We gave the initial models consisting of 4 resistivity models and 3 thickness models. Kaleköy’s inverted data had 0.686 and 1.6736 of data RMSE (see table 3), meanwhile relative RMSE as much as 0.08% and 0.64% for SVD and LM respectively while Gobi area’s had 0.946 and 4.302 of data RMSE (see table 3), meanwhile relative RMSE as much as 0.07% and 1.47% for SVD and LM respectively. Hence, it can be seen that SVD inversion obtained smaller data RMSE and relative RMSE than LM inversion did, which indicated the calculated data had a good fit with observed (field) data.

The number of iteration in SVD and LM inversion using Kaleköy’s data and Gobi’s data were the same. Therefore, it could not become parameters to compare both inversion methods to each other. Considering computation time, LM inversion needed to take longer than SVD inversion (see table 3).
Figure 2. Inversion result of a) Kaleköy’s data and b) Gobi area’s data, West China. Left curve presents observed and calculated apparent resistivity data. Black dot, red triangle, and blue line show observed data, calculated SVD, and calculated LM data respectively. Right curve presents subsurface model recovered values. Red and blue line indicate result using SVD and LM model respectively.

4. Discussion

Considering to synthetic data from test model in table 1, it can be seen that with the same initial model, LM inversion method was good to be used to invert synthetic data from model 1 (homogenous) and model 3 (3-layered noisy), while SVD inversion method was good to be used to invert synthetic data model 2 (3-layered noise-free). This was probably because of good GSS’s performance in determining damping factor when the constructed model was ideal (referred to synthetic data).

Computation time of LM inversion was longer than SVD inversion’s in spite of the same number of iteration. This was caused by selecting optimum damping factor value using GSS method as each LM inversion’s step must be evaluated within the range of damping factor value thus the inversion elapsed long computation time. Hence, it is recommended to use another method for determining the damping factor in order to make the inversion faster but still accurate in its results.

In both fields’ data, with the same number of calculation iteration, the value of data RMSE obtained (see table 3) shows that SVD was faster than LM, has smaller number of iteration and data RMSE than LM does. This was caused by LM’s dependence on damping factor selection with GSS method. In synthetic data case (ideal), GSS could determine optimum damping factor relatively easier than of that in field data case, which was relatively complex, because GSS’s principle only save the ultimate output regardless detail of Jacobi matrix’s sensitiveness. Meanwhile, in SVD, eigenvalue selection from Jacobi matrix was good enough because the method considered more the matrix’s sensitiveness, nevertheless damping factor selection still used trial-and-error technique.
5. Conclusions

We have successfully examined Levenberg-Marquardt and singular value decomposition inversion method, and compared them to each other. The LM method was good to be used in ideal synthetic data. However, we should better to use SVD in the field data case because this method is faster than LM and producing smaller number of iteration, relative RMSE and data RMSE than LM does.

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