Velocity Analysis of a Single Vertically Falling Non-Spherical Particle in Newtonian and Non-Newtonian Fluids

Harpreet Kaur, Neeraj Rani, B.P. Garg

Abstract- An analytical investigation is applied for the velocity of a vertically falling non-spherical particle in Newtonian and non-Newtonian fluid. The velocity of vertically falling non-spherical particle can be described by the force balance equation (Basset-Boussinesq-Ossen equation). Variational Iterations Method (VIM) and Runge- Kutta 4th order method are used to solve the existing problem. The results were compared those obtained from VIM by R-K 4th order method. We obtained that VIM which was used to solve such non-linear differential equation with fractional power is simpler and more accurate than other methods. Analytical results indicate that the velocity in a falling procedure is significantly increased and more in Newtonian fluid. Also particle’s velocity in Newtonian fluid reaches early at terminal velocity as compare to non-Newtonian fluid. To obtain the results for all different methods, the symbolic calculus software MATLAB is used.

Keywords - Newtonian fluid, non-Newtonian fluid, non-spherical particle, Terminal velocity, Variational Iteration Method (VIM).

I. INTRODUCTION

The problem of velocity of vertically falling non-spherical particles in Newtonian and non-Newtonian fluids is relevant to many situations of practical interest. In many processes it is often essential to obtain the path of particles that accelerate in the fluid region for designing or improving the process. Ganji and Gorji [1] find the solution of unsteady motion of vertically falling spherical particles in non-Newtonian fluid by Collocation Method and achieve a good result. Recently Kaur & Garg [2,3] investigate the acceleration motion of a single vertically falling non-spherical particle in incompressible Newtonian fluid by Diagonal Pade’ [3/3]. Also several works have been done to study the unsteady motion of particles in Newtonian fluid [4]. Chhabra and Bagchi reported the distance traveled by accelerating spherical particles in downward vertical motion of particles in power law fluid [5,6]. Along with the same proposition, many researcher realized the physical significance of some analytical method such as the Homotopy Perturbation Method (HPH), Homotopy Analysis Method (HAM), and Variational Iteration Method (VIM) [7]. To solve the present problem Variational Iteration Method (VIM) is used and validated with Numerical Method (R-K 4th order method).

II. PROBLEM STATEMENT

The Consideration of one-dimensional acceleration motion of a rigid body, non-spherical particle with diameter D, mass m and density $\rho_s$ is vertically falling in incompressible non-Newtonian fluid of density $\rho$ and viscosity $\mu$. $\gamma$ represents the velocity of the non-spherical particle at any instant time t, and g is the acceleration due to gravity [8]. Thus, the Basset –Boussinesq-Ossen (BBO) equation for the unsteady motion of particle in a fluid is given as:

$$\frac{du}{dt} = mg (1 - \frac{\rho}{\rho_s}) - \frac{\pi D^2 \rho C_d u^n}{8} - \frac{\rho D^3}{12} \frac{du}{dt}$$

where $C_d = f(Re, n)$ drag coefficient could be obtained as

$$C_d = \frac{24}{Re} X(n)$$

and $X(n) = 6(\frac{n+1}{3})^\frac{3}{n+1}$ is a deviation factor

So by rewriting force balance Eq. (1) of motion of the particle,

$$a = \frac{d}{dt} + \beta (n) u^n - \gamma = 0, u(0) = 0$$

In which $\alpha = \frac{m}{12} \pi D^3 \rho$, $\beta (n) = \frac{3nKX(n)D^2}{n}$, $\gamma = mg (1 - \frac{\rho}{\rho_s})$

For $\alpha = \beta = \gamma = 1$, Eq. (1c) cab be written as follows:

$$\frac{du}{dt} + u^n - 1 = 0, u(0) = 0$$

III. VARIATIONAL ITERATION METHOD (VIM)

In 1997, Jihuan He was introduced Variational Iteration Method (VIM) [9,10,11] to solve such nonlinear ordinary and partial differential equations. Slota obtained results for the Heat equation by VIM which were same as the exact solution [12]. To clarify the VIM, we consider the following differential

$$Lu(t) + Nu(t) = g(t)$$

Where $L$, $N$ are linear and nonlinear operator respectively and $g(t)$ is a non-homogeneous term. By using the Variational iteration method, a correction functional can be constructed as

$$u_{n+1}(t) = u_n(t) + \int_0^t \lambda (L_1u_n(\xi) + Nu_n(\xi) - g(\xi))d\xi$$  (3a)

Where $\lambda$ is a Lagrange multiplier, which can be determined by the help of Variational theory, the subscript $n$ means the nth approximation; $u_n$ is restricted variation and $\delta u_n = 0$. 

Revised Manuscript Received on October 15, 2019.

Harpreet Kaur, Department of applied sciences, Adesh Institute of Engineering and Technology, Faridkot, Punjab (INDIA).

Neeraj Rani, Research scholar of I.K. Gujral Punjab Technical University, Kapurthala, Punjab (INDIA).

B.P. Garg, Department of applied sciences, Adesh Institute of Engineering and Technology, Faridkot, Punjab (INDIA).

E-mail id: harpreet2610@yahoo.com
Velocity Analysis of a Single Vertically Falling Non-Spherical Particle in Newtonian and Non-Newtonian Fluids

According to VIM, firstly we will find Lagrange multiplier and after that to get the progressive iterations \( u_{n+1}, n \geq 0 \) which converge to solution. The solution is \( u = \lim_{n \to \infty} u_n \)

To solve eq. (1c) using VIM

\[
\begin{align*}
\frac{du_n(s)}{ds} + \beta u_n(s) - \gamma ds & \quad (3b) \\
\end{align*}
\]

For \( \alpha = \beta = \gamma = 1 \), Eq.(3b) becomes

\[
\begin{align*}
\frac{du_n(t)}{dt} + \int_0^1 (\alpha \frac{du_n(s)}{ds} + \beta u_n(s) - \gamma) ds & \quad (4b)
\end{align*}
\]

The stationary condition can be obtained as follows:

\[
\begin{align*}
1 + \int_0^1 (\alpha \frac{du_n(s)}{ds} + \beta u_n(s) - \gamma) ds & = 0
\end{align*}
\]

\[
\begin{align*}
\alpha & = -1 \\
\beta & = 1 \\
\gamma & = 1 \\
\end{align*}
\]

(4c)

Subsequently, the Lagrangian multiplier is obtained as:

\[
\begin{align*}
\lambda &= -1 \\
u_{n+1}(t) &= u_n(t) + \int_0^1 (\alpha \frac{du_n(s)}{ds} + u_n(s) - 1) ds \\
\end{align*}
\]

with condition \( u_0(t) = 0 \)

(4d)

IV. R-K 4th ORDER METHOD

Numerical solutions have always played a significant role in properly understanding the subjective highlights and processes in various field of science. It is clear that the type of current problem is to discover the solution of differential equation of 1st order. So for this, we can apply numerical methods like Trapezoidal method, Euler’s method and mid-point method etc. Thus the mid-point method (modification of Euler’s method) is a suitable technique for present problem is also called R-K 4th order method (numerical method). This method is developed by C. Runge and M. W. Kutta around 1900.

\[
\begin{align*}
u'(t) &= 1 - u^n, \text{ with initial condition } u(0) = 0 \\
\end{align*}
\]

So \( f \) is a function of time and velocity

i.e. \( f(t,u) = 1 - u^n, u(0) = 0 \)

(5a)

V. RESULTS AND DISCUSSION

Table 1. The velocity results of single vertically falling non-spherical particle in Newtonian and non-Newtonian Fluid for \( \alpha = \beta = \gamma = 1 \)

| Time (sec) | Newtonian n=1 | Non-Newtonian n=0.5 |
|-----------|---------------|---------------------|
|           | VIM R-K Method | VIM R-K Method      |
| 0.0       | 0.0000        | 0.0000              |
| 0.1       | 0.0950        | 0.0952              |
| 0.2       | 0.1800        | 0.1813              |
| 0.3       | 0.2550        | 0.2592              |
| 0.4       | 0.3200        | 0.3297              |
| 0.5       | 0.3750        | 0.3935              |
| 0.6       | 0.4200        | 0.4512              |
| 0.7       | 0.4550        | 0.5034              |
| 0.8       | 0.4800        | 0.5507              |
| 0.9       | 0.4950        | 0.5934              |
| 1.0       | 0.5000        | 0.6321              |

Fig. 1. Velocity results of non-spherical particle in Newtonian fluid.

Fig. 2. Velocity results of non-spherical particle in non-Newtonian fluid.

Fig. 3. Terminal Velocity of vertically falling non-spherical particle in Newtonian and Non-Newtonian fluid

These figures shows that the particle’s velocity is increasing as time increasing. Fig. 3 shows Terminal velocity results of vertically falling single non-spherical particle by R-K 4th order method. It depicts that the particle in Newtonian...
fluid attains its terminal velocity in short period as compare to non-Newtonian fluid. i.e. the acceleration motion of single particle which is falling in Newtonian fluid reaches early at zero (i.e. particle is not accelerating). In non-Newtonian fluid, the particle takes more time to reach at terminal velocity.

VI. CONCLUSION

VIM is applied for the solution of a present problem without using any linearization, restrictions and transformations. From above discussion, it is clear that the VIM has a good agreement with numerical method and provides highly reliable results. Also, the current technique can be developed the valid solution of other nonlinear differential equations. In addition, the above discussion shows that the particle’s velocity is more and the particle attains its terminal velocity in short period in Newtonian fluid. It depicts that the acceleration motion of single particle which falling in Newtonian fluid reaches early at zero (particle is not accelerating). In non-Newtonian fluid, the particle takes more time to reach at terminal velocity.

REFERENCES

1. Gori RJ, O.Pournemrah, Bandy MG, Ganji DD. An Analytical Investigation on Unsteady Motion of Vertically Falling Spherical Particles in non-Newtonian Fluid by Collocation Method . Ain Shams Engineering Journal, 6, (2015), pp. 531-540.

2. Kaur H, . Garg B.P, Acceleration Motion of a Single Vertically Falling non-Spherical Particle in Incompressible Newtonian Fluid by Different Methods, International Journal of Mathematics Trends and Technology, 52(7),(2017), pp. 452-457.

3. Kaur H, Garg B.P. Effect of Density and Size on Terminal Velocity of a Vertically Falling Spherical Particles in Newtonian Fluid by Diagonal Pade’ Approximants, International Journal of Statics and Applied Mathematics, 3(2), (2018), pp. 621-634.

4. Nouri R, Ganji DD, Hatami M. Unsteady Sedimentation Analysis of Spherical Particle in Newtonian Fluid Media using Analytical Methods, Propul Power Research, 3(2), (2014), pp. 96–105.

5. Bagchi A, Chhabra RP. Acceleration Motion of Spherical Particle in Power Law Type non-Newtonian Liquids, Powder Technology, 68(1), (1991), pp. 85–90.

6. Chhabra RP. Bubbles, Drops and Particles in Non-Newtonian Fluid, Second Edition, Taylor & Francis, (2012).

7. Ganji DD. Unsteady Sedimentation Analysis of Spherical Particles in Newtonian Fluid Media using Analytical Methods Journal of Hydro Environment Research, 664, (2012), pp. 327-332.

8. H. Yaghoubi, M. Torabi, Novel Solution for Acceleration Motion of a Vertically Falling Non-Spherical Particles by VIM-Pade’ Approximant, Powder Technology 215(2012), pp. 206-209.

9. J. He, A New Approach to Nonlinear Partial Differential Equation, Communications in Nonlinear Science and Numerical Simulation,2, (1997), pp. 437–440.

10. J. He, Variational Iteration Method – a kind of non-linear Analytical Technique: Some Example, International Journal of Non-Linear Mechanics 34(4), (1999), pp. 699–708.

11. J. He, Variational Iteration Method for Autonomous Ordinary Differential Systems, Applied Mathematics and Computation 114(2/3), (2000), pp. 115–123.

12. S.Q. Wang, J.-He, He’s Variational Iteration Method for Solving Integro-Differential Equations, Physics Letters 367(3), (2007), pp. 188–191.

APPENDIX (MATLAB CODE)

1. NEWTONIAN FLUID, n = 1(Fig. 1)

```matlab
format short end
uv1(:) % R – K 4th order
f = @(t, u)(1 – u. ^n);
ν = 1;
t = 0;
ur1 = 0;
h = 0.1;
t = 0: h: 1;
for i = 1: (length(t) – 1);
k1 = f(t(i), ur1(i));
k2 = f(t(i) + h/2, ur1(i) + h/2 * k1);
k3 = f(t(i) + h/2, ur1(i) + h/2 * k2);
k4 = f(t(i) + h, ur1(i) + h * k3);
ur1(i + 1) = ur1(i) + (k1 + 2 * k2 + 2 * k3 + k4) * h/6;
end
Plot(t, uv1, ‘.-.’, t, ur1, ‘- +’);
```

2. NON-NEWTONIAN FLUID, n = 0.5(Fig.2)

```matlab
% n = 0.5
% VIM
ν = 0: 0.1: 1;
n = length(ν);
for i = 1: n;

uv2(i) = t(i) – (2 * t(i)^3)/3;

Plot(t, uv2, ‘.-.’, t, ur2, ‘- s’);
```

3. NEWTONIAN FLUID & NON-NEWTONIAN FLUID (Fig.3)

```matlab
% R – K 4th order
f = @(t, u)(1 – u. ^n);
ν = 1;
t = 0;
ur1 = 0;
h = 0.3;
t = 0: h: 9;
for i = 1: (length(t) – 1);
k1 = f(t(i), ur1(i));
```
Velocity Analysis of a Single Vertically Falling Non-Spherical Particle in Newtonian and Non-Newtonian Fluids

\[ k_2 = f(t(i) + h/2, ur1(i) + h/2 * k_1); \]
\[ k_3 = f(t(i) + h/2, ur1(i) + h/2 * k_2); \]
\[ k_4 = f(t(i) + h, ur1(i) + h * k_3); \]
\[ ur1(i + 1) = ur1(i) + (k_1 + 2 * k_2 + 2 * k_3 + k_4) * h/6; \]

\textit{format short end}
\textit{ur1(\cdot)}

\% R - 4 \textit{th order}
\[ f = @(t,u)(1 - u.^n); \]
\[ n = 0.5; \]
\[ t = 0; \]
\[ ur2 = 0; \]
\[ h = 0.3; \]
\[ t = 0: h: 9; \]
\textit{for} \ i = 1: \textit{length}(t) - 1;
\[ k_1 = f(t(i), ur2(i)); \]
\[ k_2 = f(t(i) + h/2, ur1(i) + h/2 * k_1); \]
\[ k_3 = f(t(i) + h/2, ur1(i) + h/2 * k_2); \]
\[ k_4 = f(t(i) + h, ur1(i) + h * k_3); \]
\[ ur1(i + 1) = ur1(i) + (k_1 + 2 * k_2 + 2 * k_3 + k_4) * h/6; \]
\textit{format short end}
\textit{ur2(\cdot)}

\textit{Plot}(t, ur1, '\-'* ', t, ur2, '\-'* s);

\textbf{AUTHOR’S PROFILE}

\textbf{Dr. Harpreet Kaur}, (1985) is Assistant Professor (Mathematics) at Adesh Institute of Engineering & Technology, Faridkot affiliated to IKGPTU, Kapurthala, Punjab, India. She attained her Post graduate from GNDU, Amritsar and Doctorate degree from IKGPTU, Kapurthala. Her field of specialization is Fluid Dynamics. She has ten international and national research papers in her credit. She has been awarded ‘Young Scientist Award’ and ‘Young Researcher Award’ for achievements in the field of research.  

\textbf{Neeraj Rani}, (1983) received her M.Sc. in Mathematics from LKC, Jalandhar in 2006. Presently she is research scholar in Applied Sciences Department (Mathematics) of IK Gujral Punjab Technical University, Kapurthala, Punjab (INDIA).  

\textbf{Dr.BP Garg}, (1969) is Director of one of the reputed engineering colleges (Adesh Institute of Engineering & Technology, Faridkot) affiliated to IKGPTU, Kapurthala, Punjab, India. He attained his Post graduate and Doctorate degree from H.P. University, Shimla. His field of specialization is Fluid Mechanics. He has forty international and national research papers in his credit. He is life member of various academic and professional bodies. He is serving as Referee of many prestigious and internationally reputed journals. He has been awarded ‘Best Educationist Award’ for outstanding achievements in the field of education by International Institute of Education & Management and ‘Rashtriya Vaidya Gaurav Gold Medal Award’ for outstanding achievements in the field of education by Indian Solidarity Council, New Delhi.