New photonic devices for ultrafast pulse processing operating on the basis of the diffraction-dispersion analogy

Víctor Torres-Company\textsuperscript{1}, Gladys Mínguez-Vega\textsuperscript{1}, Vicent Climent\textsuperscript{1}, Jesús Lancis\textsuperscript{1} and Pedro Andrés\textsuperscript{2}

\textsuperscript{1} GROC-UJI, Departament de Física, Universitat Jaume I, 12080 Castelló, Spain
\textsuperscript{2} Departament d’Òptica, Universitat de València, 46100 Burjassot, Spain

E-mail: lancis@fca.uji.es

Abstract. The space-time analogy is a well-known topic within wave optics that brings together some results from beam diffraction and pulse dispersion. On the above basis, and taking as starting point some classical concepts in Optics, several photonic devices have been proposed during the last few years with application in rapidly evolving fields such as ultrafast (femtosecond) optics or RF and microwave signal processing. In this contribution, we briefly review the above ideas with particular emphasis in the generation of trains of ultrafast pulses from periodic modulation of the phase of a CW laser source. This is the temporal analogue of Fresnel diffraction by a pure phase grating. Finally, we extend the analogy to the partially coherent case, what enables us to design an original technique for wavelength-to-time mapping of the spectrum of a temporally stationary source. Results of laboratory experiments concerning the generation of user-defined radio-frequency waveforms and filtering of microwave signals will be shown. The devices are operated with low-cost incoherent sources.

1. Introduction.

The mathematical similarity between the paraxial diffraction of one-dimensional optical beams in free space and the temporal distortion of light pulses in parabolic dispersive systems is known in the literature as space-time analogy. The potential of this analogy lies in the fact that the developments accomplished in the field of diffraction can be easily transferred to the temporal domain, and vice versa. Since paraxial diffractive optics is a relatively well-established matter, this transfer has been usually performed from the spatial to the temporal domain. This approach has led to the achievement of several optoelectronic configurations particularly relevant in the optical communications field \cite{1}. It seems that this finding was independently discovered by Akhmanov et al. \cite{2}, and Treacy \cite{3} at the end of the 60’s. While in the 80’s there appeared some theoretical contributions based on this analogy \cite{4}, it was not until the relevant works of Kolner on temporal lenses and temporal imaging systems (TISs) \cite{5} that this analogy emerged with all its splendour.

Among other results based on the space-time analogy, we mention pulse compression, temporal imaging and filtering \cite{5,6}, frequency-to-time \cite{7} and time-to-frequency conversion \cite{8}, and repetition rate multiplication \cite{9} and timing-jitter suppression \cite{10} based on the temporal Talbot phenomena. The above phenomena are the temporal counterparts of beam focusing, spatial imaging and filtering, far-field diffraction, and self-imaging. In this communication, we briefly review some of the above applications. We also show that the space–time analogy provides a nice analytic tool to tackle the
problem of picosecond pulse generation based on dispersion of sinusoidally phase-modulated light [11]. The key relies on noticing the equivalence with the spatial diffraction of 1D phase structures. Apart from providing a solid theoretical formulation to the counterintuitive finding of flat-top-pulse profiles experimentally reported by Komukai et al [12], the space-time analogy suggest the use of different modulation formats other than the sinusoidal which allow for background-free and side-lobe suppressed pulsation.

The above analogy has been extended to partially coherent pulses [13]. Based on this result, we have proposed a new photonic configuration to achieve arbitrary averaged intensity shaping of broadband spectrally incoherent sources, e.g., amplified spontaneous emission (ASE) sources [14]. The fundamental physical principle behind this setup is the temporal counterpart of the vanCittert-Zernike theorem [15]. This device has been employed for the generation of ~ 10 GHz arbitrary waveforms at high repetition rate in a controllable and reconfigurable way [16] and constitutes a low-cost and reliable alternative to coherent approaches based on femtosecond lasers. The application of the above technique to the filtering of microwave signals will also be discussed [17].

2. Space-time analogy.

Let us assume a plane-wave-type scalar optical field propagating linearly through an homogeneous lossless dispersive medium along the z direction. The field is described by its analytic signal \( U(z, t') \). The propagation constant is \( \beta(\omega) = \omega n(\omega)/c \), where \( c \) is the speed of light in vacuum and \( n(\omega) \) the frequency dependent refractive index. We can then write \( U(z, t') = \psi(z, t') \exp[-i(\omega t' - \beta z)] \). Here, \( \psi(z, t') \) is the pulse envelope, which modulates the monochromatic carrier wave of angular frequency \( \omega_0 \). In the multi-cycle regime, the slowly varying envelope approximation (SVEA) is usually invoked. Under this approximation, the evolution of \( \psi(z, t') \) is described as

\[
\frac{i}{\hbar} \frac{\partial \psi(z, t')}{\partial z} = H \psi(z, t'),
\]

where the Hamiltonian operator is \( H = \sum_{n=2}^{\infty} \frac{n^n}{n!} \frac{\partial^n}{\partial t^n} \), with \( \beta_n = \frac{d^n \beta(\omega)}{d\omega^n} \) the nth-order dispersion coefficients evaluated at \( \omega_0 \). We write the dispersive terms as \( \Phi_n = \beta_n \). In the case \( n = 2 \), \( \beta_2 \) is called the group-velocity-dispersion (GVD) coefficient, and \( \Phi_2 \) the group-delay-dispersion (GDD) parameter. The above equation refers the evolution of the pulse to a reference framework moving at the group velocity of the wave packet; i.e., \( t = t' - \beta z \). Here, we are particularly interested in the case in which only the first term from (1) contributes,

\[
\frac{i}{\hbar} \frac{\partial \psi(z, t)}{\partial z} = \frac{\beta_2}{2} \frac{\partial^2 \psi(z, t)}{\partial t^2}.
\]

The first-order approximation is physically plausible whenever \( \Delta \omega \ll 3|\beta_2/\beta_3| \). As an example, for a single-mode fiber (SMF) and a waveform centered in the telecommunication wavelength (\( \lambda_0 = 1.55 \mu m \)), the fiber coefficients are \( \beta_2 = -21.68 \text{ps}^2/\text{km} \) and \( \beta_3 = 0.127 \text{ps}^3/\text{km} \). The pulses should then have an optical bandwidth shorter than 80 THz. If we assume a coherent Gaussian pulse and require the above inequality to be satisfied for a 10 times shorter bandwidth, (2) is valid for pulses longer than 60 fs full-width-at-half-maximum (FWHM) intensity duration. Equation (2) is a Schrödinger-like equation for a free particle, present in many physical problems. This equation is identical to that describing the one-dimensional scalar diffraction of a paraxial monochromatic beam propagating in the z direction.
\[ i \frac{\partial U_e(z,x)}{\partial z} = -\frac{1}{2k_o} \frac{\partial^2 U_e(z,x)}{\partial t^2}, \tag{3} \]

where \( U_e(z,x) \) denotes the transversal profile of the 1D beam, and \( k_o \) the wavenumber. The mathematical similarity between (2) and (3) is known as the space–time analogy. From the point of view of the linear systems theory, a GDD circuit introduces a quadratic phase modulation onto the spectrum of the input complex envelope. The most popular element is an SMF, whenever the third-order-dispersion (TOD) coefficient can be neglected. GDD circuits can also be implemented with a dispersion compensating fiber, pairs of prisms, diffraction gratings, a spatial light modulator (SLM) in a Fourier transform geometry, specially designed linearly chirped fiber gratings or photonic crystal fibers. A GDD circuit is fully equivalent to the action of the paraxial diffraction in free space.

On the other hand, a temporal lens introduces a quadratic phase modulation on the input complex envelope, in the same way as a spatial lens does. In practical terms, this operation is implemented with electro-optic phase modulators (EOPMs) [5]. An EOPM is usually built with a nonlinear crystal having a high electro-optic coefficient, such as LiNbO3. By applying an electrical signal on the crystal, the optical field propagating through acquires a phase proportional to the applied voltage. If the electrical field is just composed by a single-tone RF waveform, the optical pulse is synchronized with the maximum of the RF signal, and the temporal pulse width is shorter than the period, then the required quadratic modulation over the input field is achieved. The strength of the chirp, the temporal analogue for the focal length of the spatial lens, is controlled through the crystal parameters and the peak-to-peak amplitude and repetition rate of the electrical waveform.

The space-time analogy has been recognized as a powerful tool for designing analog ultrafast pulse processors. Table 1 summarizes the transfer rules connecting both domains.

| Space domain       | Time domain           |
|--------------------|-----------------------|
| position           | proper time           |
| spatial frequency  | \( 2 \pi \mu \)        |
| wave number \(^{-1}\) | \(-\beta_z\)          |
| paraxial propagation | \( \exp \left(-i \frac{2 \pi^2 z u^2}{k_o} \right) \) |
| spatial lens       | \( \exp \left(-i \frac{k_o \alpha^2}{2 f} \right) \) |
| 1st-order dispersion | \( \exp \left( i \frac{\Phi}{2} \right) \) |
| time lens          | \( \exp \left( i \frac{K t^2}{2} \right) \) |

### 3. Picosecond pulsation through electrooptic phase modulation.

The electrooptic method for optical pulse generation is based on the phase modulation with a sinusoidal signal of a CW beam from a narrowband laser diode. This produces harmonic sidebands around the optical carrier frequency so that the emerging waveform is strongly chirped. The optical field is launched through a GDD circuit and compressed because the sweep rate acquired upon propagation partially compensates for the chirp [18]. Apart from short pulse generation with a low duty ratio, flat-top-pulse generation with a duty ratio of 50% has been reported [12]. These pulses can be used for instance for return-to-zero (RZ) modulation formats in optical fiber communication.
We have shown that, on the framework of the space-time analogy, the above results constitute the
temporal analogue of the field diffracted by a pure phase grating [11]. The parameters of the EOPM,
the frequency of the driving signal and the modulation index together with the GDD coefficient
determine unambiguously the waveform achieved at the output. Specifically, we have shown that a
remarkably simple formula describes the optical intensity at a quarter of the Talbot dispersion. This
formula provides the sought theoretical support of the experimental results for flat-top-pulse
generation with a duty ratio of 50% when the modulation index of the sinusoidally driven EOPM is
\( \pi /4 \). Further, our approach permits to identify a great variety of other pulse profiles when the
modulation format is different from sinusoidal. In particular, we demonstrate square-wave-type train
pulse generation with adjustable duty cycle and time slot. This signal is obtained from a driving signal
in the form of a serrodyne-like function with \( \pi \) jump phase.

4. Partially coherent space-time analogy. Incoherent frequency-to-time mapping.
In this section we extend the analogy to the partially coherent case [13]. Now, \( \psi(z, t) \) is a
nondeterministic complex envelope from a nonstationary ensemble. The function describing the
second-order correlation properties of this field is the mutual coherence function (MCF) defined as

\[
\Gamma(z_1, z_2, t_1, t_2) = \left( U^*(z_1, t_1) U(z_2, t_2) \right) \Gamma(z_1, z_2, t_1, t_2) \exp\left\{-i \left[ \omega_0 (t_2 - t_1) - \beta_0 (z_2 - z_1) \right] \right\}.
\]

The brackets denote ensemble averaging over the different realizations of the field and the asterisk represents
complex conjugate. It is often convenient to use the normalized form of the MCF,
\( \gamma(z_1, z_2, t_1, t_2) \),
\[
\gamma(z_1, z_2, t_1, t_2) = \left| \Gamma(z_1, z_2, t_1, t_2) / \left[ I(z_1, t_1) I(z_2, t_2) \right] \right|^2,
\]
which is known as the complex degree of coherence. In the above equation, \( I(z, t) = \Gamma(z, z, t, t) \) is the intensity of the field at the position \( z \) and
time instant \( t \). Since each of the envelope realizations satisfies (2), it is easy to show that

\[
\left[ \partial^2_{t_j} - 2i(-1)^j \frac{I}{\beta_0^2} \partial_{z_j} \right] \Gamma(z_1, z_2, t_1, t_2) = 0,
\]

were \( \partial_{a_j} \) denotes derivative with respect to the variable \( a \) and \( j = (1, 2) \). These equations are fully
equivalent to those describing the propagation of the mutual intensity \( J(z_1, z_2, x_1, x_2) \) of a 1D partially
coherent field in the spatial domain. On the basis of this general analogy, we can tackle the problem of
partially coherent pulse evolution through parabolic dispersive media taking advantage of the work
previously developed for partially coherent 1D beams. Again, Table 1 provides the transfer rules.

The above result was used to propose the temporal counterpart of the vanCittert-Zernike theorem
[14,15,19]. The propagation of quasi-homogeneous pulses was analyzed. Such pulses constitute the
temporal counterpart of the quasi-homogeneous light sources. They are characterized by a complex
degree of coherence depending on the time difference and a temporal intensity width, \( \sigma \), much larger
than the coherence time, \( \tau_c \), of the source; i.e., \( \sigma \gg \tau_c \). From an experimental point of view, these
pulses are easily achieved from a spectrally incoherent source, like ASE, externally modulated with a
deterministic electro-optic amplitude modulator (EOM). Here, we are interested in the far-zone region,
the temporal analogue of Fraunhofer diffraction, which is reached when \( |\Phi_2| \gg \sigma_c / 4\pi \). The analogy
an output intensity profile given by the Fourier transform of the input complex degree of coherence
which, in turn, by virtue of the Wiener-Khintchine theorem, is the inverse Fourier transform of the
spectral density function of the ASE source \( S(\omega) \). In other words, we obtain an output intensity
profile which is, independently of the temporal waveform provided by the modulator, a replica with a
certain scale of the energy spectrum (ES) of the source. In mathematical terms,

\[
I_{\text{out}}(t) \approx S \left( \frac{t}{\Phi_2} \right)
\]

(5)
This result constitutes the generalization of the coherent frequency-to-time mapping. We have experimentally verified this frequency-to-time mapping operation. As spectrally incoherent source we employed the ASE from the gain spectrum of an Erbium-doped fiber amplifier (EDFA). The ES is measured with an optical spectrum analyzer (OSA) and sketched in Fig. 1(a). Typical coherence time of EDFAs is around 100 fs. The deterministic gate was a 10 Gb/s LiNbO$_3$ EOM biased at quadrature driven with a short RF Gaussian impulse. The pulse width was measured around 88 ps FWHM. In this way, the quasi-homogeneous requirement is by far reached. Finally, light is launched in an SMF of 1.44km length to reach the far-zone. The output intensity shape is measured with a 20GHz sampling scope and the result is shown in Fig. 1(b). The frequency-to-time mapping scale factor is calculated to be 0.024 ns/nm.

![Figure 1.](image)

**Figure 1.** Experimental realization of the generalized frequency-to-time mapping: (a) input ES (in linear scale); and (b) averaged output intensity

### 5. Applications in microwave photonics.

Photonic processing of microwave and radio-frequency (RF) signals is a research topic that has been explored for more than 30 years [20]. Microwave photonics offers many well-known advantages like reconfigurability, high bandwidth, immunity to electromagnetic interference, high-speed processing, and potential integrability with fiber optics technology. These features are very difficult, if not impossible, to achieve with electronic approaches. Typical RF signal processing applications cover analog-to-digital conversion, beamforming, filtering, and arbitrary waveform generation (AWG). Radio-frequency (RF) waveform generators are key devices for a variety of applications including radar, ultra-wideband communications and electronic test measurements. Following advances in broadband coherent pulsed sources and pulse shaping technologies, reconfigurable RF waveform generators operating at bandwidths $> 1$ GHz have become a reality. Based, on the incoherent frequency-to-time mapping operation, we demonstrate a low-cost alternative using broadband spectrally incoherent optical sources. It is based on spectral filtering of the incoherent source so that, after frequency-to-time, the filtered spectral density function becomes the user-defined waveform.

The whole optical device is shown in Fig. 2 [16]. The ES of the EDFA is tailored (shaped) using a commercially available D-WDM 48-channel controller with 100 GHz resolution spanning the whole C band and $< 6$ dB insertion losses. The channel controller consists of a zero-dispersion pulse shaper with two diffractive gratings as angular dispersive elements and a spatial light modulator for amplitude control. The amplitude of each channel can be blocked fully, or reduced in increments of 0.1 dB up to a maximum of 20 dB. After the ES synthesis, the light is polarized before entering the EOM. The modulated optical signal is amplified and launched into a coil of SMF. Finally, the optical signal is converted back to the electrical domain using a photodiode. We used a Gaussian electrical pulse with a FWHM duration of 60 ps as the broadband input to ensure the quasi-homogeneous condition. The example concerns the generation of a sawtooth pulse. The synthesized ES is shown in Fig. 3 (a). After temporal modulation and stretching in 7.6 km of SMF, the measured temporal waveform is shown in Fig. 3(b). One of the key advantages of the proposed technique is the possibility to control the repetition rate of the generated waveform in an easy way. This is achieved by changing the clock frequency that drives the impulse generator. Thus, we can obtain continuous RF waveforms, Fig. 3(c).
More recently, we have recognized the optical device in Fig. 2 can also be employed for photonically assisted filtering of RF signals. Now, the RF signal to be filtered comes into the EOM modulator. Apart from the well-known carrier suppression effect and assuming a flat response for the O/E conversion, the effective transfer function of the microwave filter is given by the MCF of the EDFA. We will show results concerning the generation of an RF filter with a resonant flat bandpass at the region corresponding to 4-7GHz which is interesting for ultra-wideband (UWB) applications [17].

Acknowledgments
This research was funded by the Dirección General de Investigación Científica y Técnica, Spain, and FEDER under the project FIS2007-62217 and the platform “Science and Applications of Ultrafast Ultraintense Lasers”, SAUUL (CSD2007-013), within the Program Consolider-Ingenio 2010. We acknowledge Professor Lawrence R. Chen for his kind assistance with the experimental part.

6. References
[1] van Howe J and Xu C 2006 J. Lightwave Technol. 24 2649.
[2] Akhmanov S A, Chirkin A S, Drabovich K N, Kovrigin A I, Khokhlov R V and Sukhorukov A P 1968 IEEE J. Quantum Electron. QE-4 598.
[3] Treacy E B, 1969 IEEE J. Quantum Electron QE-5 454.
[4] Jansson T and Jansson J 1981 J. Op. Soc. Am. 71 1373.
[5] Kolner B H and Nazarathy M 1989 J. Opt. Soc. Am. 71 1373.
[6] Lohmann A F and Mendlovic D 1992 Appl. Opt. 31 6212.
[7] Kelkar P V, Copping M, Bhushan A S and Jalali B 1999 Electron. Lett. 35 1661.
[8] Kauffman M T, Banyai C, Godil A A and Bloom D M 1994 Appl. Phys. Lett. 64 270.
[9] Azaña J and Muriel M A 2001 IEEE J. Sel. Top. Quantum Electron. 7 728.
[10] Fernández-Pousa C R, Mateos F, Chantada L, Flores-Arias M T, Bao C, Pérez M V and Gómez-Reino C 2004 J. Opt. Soc. Am. B 21 1170.
[11] Torres-Company V, Lancis J and Andrés P 2006 Opt. Express 14 3171.
[12] Komukai T, Yamamoto T and Kawanishi S 2005 IEEE Photon. Technol. Lett. 17 1746.
[13] Lancis J, Torres-Company V, Silvestre E and Andrés P 2005 Opt. Lett. 30, 2973.
[14] Torres-Company V, Lancis J and Andrés P 2007 J. Opt. Soc. Am. A 24, 888.
[15] Dorrer C 2004 J. Opt. Soc. Am B 21 1417.
[16] Torres-Company V, Lancis J, Andrés P, and Chen L R 2008 J Lightwave Technol. (in press).
[17] Torres-Company V, Lancis J and Andrés P 2008 Opt. Commun. 281, 1438.
[18] Kobayashi T, Yao H, Amano K, Fukushima Y, Morimoto A, and Sueta T 1988 IEEE J. Quantum Electron. 24 382.
[19] Lajunen H, Friberg A T and Ostlund P 2006 J. Opt. Soc. Am. A 23, 2530.
[20] Capmany J and Novak D 2007 Nature Photon. 1, 319.