An informationally-complete unification of quantum spacetime and matter

Zeng-Bing Chen
Hefei National Laboratory for Physical Sciences at Microscale, Department of Modern Physics, University of Science and Technology of China, Hefei, Anhui 230026, China and The CAS Center for Excellence in QIQP and the Synergetic Innovation Center for QIQP, University of Science and Technology of China, Hefei, Anhui 230026, China
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It was known long ago that quantum theory and general relativity, two pillars of modern physics, are in sharp conflict in their foundations. Their fundamental inconsistencies render a consistent theory of quantum gravity the most challenging problem in physics. Here we propose an informationally-complete quantum field theory (ICQFT), which describes elementary particles, their gauge fields and gravity as a trinity without the Hilbert-space inconsistency of Einstein’s equation. We then argue that the ICQFT provide a coherent picture and conceptual framework of unifying matter and spacetime. The trinary field is characterized by dual entanglement and dual dynamics. Spacetime-matter entanglement allows us to give a natural explanation of the holographic principle, as well as two conjectures on black-hole states and on a possible candidate to dark matter/energy.

Quantum theory (quantum mechanics and quantum field theory) and general relativity are two pillars of our current physics and deeply impact even our daily life. The achievements motivated by either of the two pillars are remarkable. However, it was recognized long ago that quantum theory and general relativity are in sharp conflict in their foundations as summarized by Thiemann in a beautiful review [1]. The Einstein equation relates the geometry of spacetime and the energy-momentum tensor of matter. In Thiemann’s terminology, on one hand, the classical-quantum inconsistency means that, while the matter fields are well described by the standard model in flat spacetime, the geometry of spacetime is described by classical Einstein equation. On the other hand, general relativity results in the unavoidable existence of spacetime singularities, where all laws of physics seem to fail. Such an instability of spacetime and matter implies the internal inconsistency of general relativity. At the same time, conventional quantum field theory suffers from the notorious infrared and ultraviolet singularities (divergences). Although the divergences can be ‘get rid of’ by renormalization as in the standard model, renormalization fails when it applies to general relativity.

The fundamental inconsistencies mentioned above motivate a long march of quantizing gravity—‘quantum theory’s last challenge’ [2]. Among various existing approaches to quantum gravity, loop quantum gravity [1] is very impressive for, among others, its prediction of discrete structure of spacetime and the entropy counting of black holes. The discrete structure of spacetime could well be a natural regulator of singularities in conventional quantum field theory [8].

However, almost all, if not all, existing methods to quantum gravity tacitly assume, explicitly or implicitly, the completeness of conventional quantum theory. Logically, it is possible that the fundamental inconsistencies of our current theories could be caused by the incompleteness of quantum theory. The debate on the real meaning of quantum states [3–11] and on the quantum measurement problem [12–15] occupies the whole history of quantum theory. Notice that conventional quantum field theory in curved spacetime has its own interpretational problems, e.g., the black-hole paradox [16] and the physical meaning on the usual concept of particles [17–20]. These interpretational difficulties of quantum theory motivate various interpretations [20], or understanding quantum theory from different angles [21–24].

Yet, these interpretations or fresh understanding of quantum theory seldom challenges its completeness. The most serious challenge stems from the famous Einstein-Podolsky-Rosen paper [4] questioning the completeness of current quantum description against local realism. The follow-up discovery of Bell’s inequalities [25] and their various experimental tests give us an impression that quantum mechanics wins against the Einstein-Podolsky-Rosen argument. The interpretation on violations of Bell’s inequalities as quantum nonlocality was questioned from the many-worlds picture [26].

Recent, we took a totally different way of thinking. We suggested an informationally-complete quantum theory (ICQFT) by assuming that quantum states represent an informationally-complete code of any possible information that one might access to a physical system [27]. The key to this development is the informational completeness in the trinary picture, which, in the present context, requires to describe elementary particles, their gauge fields and gravity as a single, indivisible entity. Otherwise, the description would be informationally incomplete, as in the probability description of current quantum mechanics. Here we continue this idea and present an informationally-complete quantum field theory (ICQFT). We argue below that the informationally complete quantum description does provide a coherent and conceptual framework for unifying quantum spacetime (gravity) and matter.

Informationally complete quantum fields and quantum relativity
Single, free physical systems in our conventional sense are excluded from the outset by the ICQT as they are simply meaningless in acquiring information, which must be accessed via interaction. The two-party (a physical system $S$ plus its measurement apparatus $A$) picture as used in current quantum mechanics was shown to be informationally incomplete \cite{27}. To fulfill the informational completeness, one has to adapt a trinary description. In the present case of quantum fields, a particle (e.g., a Dirac electron) field and its corresponding gauge field are called as system $S$ and system $A$, respectively; $SA$ together as matter field. The gravitational field is the ‘programming’ system (system $P$). The trinary description then corresponds to a dual entanglement pattern among the three systems: The particle field ($S$) and the gauge field ($A$) are mutually defined by interacting and entangling each other; matter field ($S$ and $A$ together) and gravity are likewise entangled and mutually defined. Thus, in the ICQT the viewpoint on spacetime and matter is dramatically different from our previous picture. Neither spacetime nor matter is an isolated entity; they must be described as a trinity and entangled in the dual form to make sense for acquiring information.

To illustrate the basic idea, for concreteness we only consider gravity field interacting with two kinds of matter fields: the Dirac field $\psi(x) = \psi(x,t)$ of electrons (with mass $m$ and charge $q = -e$) and the electromagnetic field $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$, where $A_\mu$ is the electromagnetic potential vector. The generalization to non-Abelian gauge fields is straightforward. Here we adapt notions as used in Rovelli’s book \cite{5}. A spacetime coordinates $x = (x^\mu)$ with $\mu, \nu, ... = 0, 1, 2, 3$ being spacetime tangent indices. The gravity is described by the tetrad field $e^I_\mu(x)$, which relates to the usual metric tensor $g_{\mu\nu}$ by $g_{\mu\nu}(x) = \eta_{IJ} e^I_\mu(x) e^J_\nu(x)$. Indices $I, J, ...$ label the Minkowski vectors and the Minkowski metric $\eta_{IJ}$ has signature $[-, +, +, +]$. The total action of the trinary fields is $S(e, \omega; A, \psi) = S_e(e, \omega) + S_{\text{Mat}}(e, \omega; A, \psi)$, where $S_{\text{Mat}}(e, \omega; A, \psi)$ is the Dirac field $\psi(A)$ sector and the gravitational field $(e, \omega)$ sector. The $\psi, A$ entanglement, programmed by each state in the orthogonal basis of the gravitational field sector, encodes information for informationally incomplete fields in the Hilbert spaces of $\psi$ and $A$. Thus, the physical significance of our current understanding on particle fields and gauge fields is completely changed in the ICQFT: Both cannot be regarded as isolated, physical (informationally-complete) entities; only jointly they define spacetime and can be described as an informationally-complete physical entity. This immediately explains the occurrence of the black-hole paradox \cite{10} and why the usual concept of particles loses its meaning for quantum field theory in curved spacetime \cite{17,19}, as an unavoidable consequence of the conventional informationally incomplete description.

Thus, the informational completeness principle puts a profound restriction on what are physical systems and how to describe physical systems. Quantum relativity stated above gives precisely an informational interpretation of the gauge invariance in conventional quantum field theory under local Lorentz transformations, local gauge transformations and diffeomorphisms \cite{5}. It seems that the informational completeness puts stronger restrictions on our field-theoretical description than the usual gauge
invariance. We leave this issue for further consideration. Below let us show how the ICQT leads to a coherent picture of unifying gravity and matter.

**Dual dynamics of quantum gravity and matter**

The classical Einstein field equation reads \[ R^I_{\mu} - \frac{1}{2}(R + \Lambda)\epsilon^I_{\mu} = 8\pi G T^I_{\mu} \tag{2} \]

where \( R^I_{\mu} \) (\( R \)) is the Ricci tensor (scalar), \( T^I_{\mu} \) the energy-momentum tensor of matter; \( \Lambda (G) \) represents the cosmological (Newton) constant. In classical domains, Einstein’s equation is extremely successful. But quantum mechanically, it looks problematic as one has to quantize these fields therein. As argued by Thiemann \[ 6, 16 \] in quantum gravity theory \( T^I_{\mu} \) should be quantized to be a field operator \( \hat{T}^I_{\mu} \) in the Hilbert space of both spacetime and matter. However, the problem (called as the ‘Hilbert-space inconsistency’ hereafter) still exists: Both sides of Einstein’s equation belong to different Hilbert spaces as the left are purely operators for spacetime geometry; there is no way of equating them.

Does this mean that Einstein’s equation is simply wrong in quantum domain, or is it an effective spacetime theory \[ 30, 31 \] that cannot be quantized at all as matter fields? Instead of answering this question, we pursue another possibility, namely, whether or not we can find a fundamental reason to ‘glue’ the two pieces of Einstein’s equation. As we will show below, the ICQT plays exactly this role.

For this purpose, it is convenient to work in the Hamiltonian formalism \[ 1, 3–8 \], which is better established in loop quantum gravity. There, the dynamical variables, in terms of \( \hat{e}^I_{\mu} \) and \( \hat{\omega}^I_{\mu} \), are the connection field \( \hat{A}^I_a (\tau) \) (defined on a three-dimensional surface without boundaries; \( a, b, ... = 1, 2, 3 \) are spatial indices and \( i, j, ... = 1, 2, 3 \) take value in a Lie algebra) and the ‘gravitational electric field’ \( \hat{E}^I_b (\tau) \), which is the i\(8\pi G\) times momentum conjugate to \( \hat{A}^I_a (\tau) \). The Hamiltonian of the trinary fields is then

\[
H_{G + \text{Matt}} = H_G(\hat{A}^I_a, \hat{E}^I_b) + H_{\text{Matt}|G}(\psi, \hat{A}; \hat{A}^I_a, \hat{E}^I_b) \tag{3}
\]

Although the matter-field sector of the problem is less developed \[ 8 \], the gravity sector is so well established that, with the input of the informational completeness principle, we can write the spacetime-matter entangled state in the standard Schmidt form as

\[
|\psi, A, A'_a \rangle = \sum_s \text{SM}_{G + \text{Matt}}[s] \left| s, A'_a \right\rangle \otimes |\psi, A; s \rangle . \tag{4}
\]

Here \( \text{SM}_{G + \text{Matt}}[s] (\geq 0) \) denote the Schmidt coefficients and are determined by dynamics of the trinary system; \( |s, A'_a \rangle \) and \(|\psi, A; s \rangle \) span orthogonal bases for the Hilbert spaces of gravity and matter, respectively. With respect to this specific decomposition, the programmed entangled state \(|\psi, A; s \rangle \) for the Dirac field and the electromagnetic field can be likewise decomposed as

\[
|\psi, A; s \rangle = \sum \text{SM}_{\text{Matt}|G} \left[ \ell; s \right] |A, \ell; s \rangle \otimes |\psi; \ell; s \rangle , \tag{5}
\]

which encodes complete information about \( \hat{\psi} \) and \( \hat{\psi} \) as programmed by \(|s\rangle \). \(|\psi, \ell; s \rangle \) and \(|A, \ell; s \rangle \) are two orthogonal bases for the Hilbert spaces of the Dirac field and the electromagnetic field.

Let us consider the physical significance of the dual entanglement in Eqs. (4-5). One of the most important results achieved by loop quantum gravity is the identification of the ‘spin-knot’ states \(|s\rangle \) as physical Hilbert space of quantized gravity. These states are diffeomorphism-invariant, form a discrete orthonormal basis and support discrete spacetime geometry \[ 1, 3–8 \]. This immediately allows us to identify these spin-knot states \(|s, A'_a \rangle \) in Eq. (4). Then the dual entanglement in Eqs. (4-5) implies the existence of discrete orthonormal bases \(|\psi(\psi, A; s) \rangle, |\psi(\psi; \ell; s) \rangle\) and \(|A, \ell; s \rangle\), as a direct consequence of our informationally-complete description.

Following the ICQT, the evolution of the trinity is described by dual dynamics, a feature of our trinary description. Here we have particular interest to dynamics related to spacetime-matter entanglement, to eliminate the Hilbert-space inconsistency mentioned above. To this end, note that \( H_{\text{Matt}|G}(\psi, \hat{A}; \hat{A}^I_a, \hat{E}^I_b) \) can always be written as \( H_{\text{Matt}|G}(t) = \sum |s, A'_a \rangle \langle s, A'_a | H_{\text{Matt}|G}(\psi, A; s) \otimes H_{\text{Matt}|G}(\psi, A; s) \), up to unimportant local unitary transformations. Moreover, the trinity has a natural initial state \(|0\rangle \equiv |0_G \rangle \otimes |0_M \rangle\)–the information vacuum state, which is the common vacuum state of matter (the Minkowski vacuum |0_M \rangle) and geometry (the empty-geometry state |0_G \rangle) in loop quantum gravity \[ 10 \]. The spacetime-matter dynamics is then given by \(|\psi, A, A'_a \rangle = \hat{U}_G + \text{Matt}(t); |0\rangle \); the evolution operator \( \hat{U}_G + \text{Matt} \) has a factorizable structure \( \hat{U}_G + \text{Matt}(t) = \hat{U}_G(t) \hat{U}_{\text{Matt}|G}(t) \) such that \(|\psi(s)\rangle \)

\[
i \frac{d}{dt} \hat{U}_G(t) = H_G(\hat{A}^I_a, \hat{E}^I_b) \hat{U}_G(t), \tag{6}
\]

Equation (6) implies that Einstein’s equation in quantum domain is separated into two pieces, one purely for spacetime and another for matter programmed by spacetime. This eliminates the Hilbert-space inconsistency of Einstein’s equation. It is interesting to notice that in our picture, gravity could be quantized alone, explaining the remarkable success of current quantum gravity theory. On the other hand, putting the informational completeness into the description completes a consistent quantum gravity and paves the way to consistently quantize the matter sector as well. We note that current loop quantum gravity \[ 8 \] already indicates the finiteness of \( H_G(\hat{A}^I_a, \hat{E}^I_b) \). However, the matter sector is not well understood and progress has
been made steadily \[8, 32, 33\]. In this regard, it is reasonable to expect that the ICQFT plays a role in further development on quantum gravity, especially on quantization of the matter sector free of divergences.

The holographic principle and ‘black states’

Let us present another application of the dual entanglement structure given above. We consider the limit on the information content of a spacetime region associated with a surface of area \(A\). Note that in loop quantum gravity, the area is quantized \[1, 3–6\]. Thus we identify the surface (particles and gauge fields within the spacetime region) as the \(P(SA)\) system. Then the \(P-SA\) measurability and the programmed measurability \(SA|P\) defined in the ICQT \[27\] demand that \(D_A = D_S = D\) and \(D_P = D^2\), i.e., the dimensions (denoted by \(D_{A,S,P}\)) of the three systems are all limited and related. The loop quantum gravity gives the quantized area to be \(A = nA_0\), where \(n\) is a natural number and \(A_0\) the minimal area related to the Planck length \(\ell_P\); for the spin-\(\frac{1}{2}\) representation, \(D_P = 2^n\). Here, however, we necessarily have \(D_P = 4^n\) as the minimal dimensions of \(S\) and \(A\) are all 2. Thus, the spacetime-matter (\(P-SA\)) entanglement \(\mathcal{E}_{P(SA)}\) (the entropy of \(P\) or \(SA\)) is limited in our picture by

\[
\mathcal{E}_{P(SA)} \leq \ln D_P = \frac{2 \ln 2}{A_0} A = \frac{A}{4\ell_P^2},
\]

if we take \(A_0 = 8\ell_P^2 \ln 2\). This is exactly what the holographic principle \[34–36\] means. Here the holographic principle arise as a direct consequence of the ICQFT for gravity and matter. Such a strong limit on the allowed states of the trinary system as imposed by the ICQFT gives an exciting possibility to escape the infrared and ultraviolet divergences that occur in conventional quantum field theory. Here it is ready to see that the restriction on the description of the trinary field imposed by the informational completeness is much stronger than our current field-theoretical description.

Now we apply the above argument to a Schwarzschild black hole of surface area \(A\) and mass \(M\). As the black hole saturates \[16\] the spacetime-matter entanglement bound \(7\), it is natural to infer that the black hole must be a \textit{maximally informationally-complete} quantum system in a maximal dual entanglement \(|BH, P(SA)\rangle = \frac{1}{\sqrt{D_P}} \sum_{s=0}^{D_P} |s, P\rangle |s, SA\rangle\), where all \(D_P\) matter states \(|s, SA\rangle\) are also maximally entangled and span an orthonormal basis in the matter sector.

Maximal entanglement has an intriguing property called ‘monogamy’ \[37\]: if two parties are maximally entangled, then they cannot be entangled with any third party. Let us discuss a possible application of the non-shareability of maximal entanglement in the present context. As we inferred, the black hole is maximally entangled in dual form. Then the monogamy of maximal entanglement implies that there is no way of extracting any information from the black hole. As such, dynamical evolution of the black hole will be in some sense ‘frozen’ from the trinary field, namely, it is an ‘\textit{entanglement death}’ of matter and spacetime. However, the presence of the black hole in spacetime is detectable as it \textit{defines} spacetime and can also absorb matter to grow up its entanglement. Such a picture on black holes seems to be in accordance with our intuition on what is a black hole, especially in the framework of ICQFT and is, however, quite different from our current understanding \[11, 38–40\] based on classical general relativity, thermodynamic argument, and quantum field theory in curved spacetime.

It is of great interest to see if we could have the quantum black-hole solution, as we inferred here, for a specific trinary system’s dynamics, e.g., the gravitational collapse of a heavy star. The confirmation of such a solution would justify whether the inferred properties of black holes are black hole’s defining properties after all.

According to our reasoning, we could tentatively call the states of any maximally informationally-complete trinary systems the ‘black states’ to account for the inferred properties of black holes. Then we would conjecture the existence of maximally informationally-complete trinary systems in black states (‘dark trinitons’) other than black holes. Perhaps they could be created substantially in earlier universe when spacetime and matter interacted/entangled strongly at the Planck scale and, due to their unique property mentioned here, serve well as a possible candidate to dark matter/energy. We think that the dark-triniton solutions of our ICQFT might offer a possible quantum explanation of the cosmological constant, which we ignored in above discussions.

Summary and outlook

To summarize, we have introduced, very briefly as a start, an informationally-complete quantum field theory, which describes particles, their gauge fields and gravity as a trinity, hoping to provide a coherent picture of unifying spacetime and matter. The fact that this is indeed possible could be regarded as a support on our previous argument on the informationally complete quantum theory. Complete information of the trinary field (particles, gauge fields and gravity) is encoded in the dual entanglement: spacetime-matter entanglement and matter-matter (particles and gauge fields) entanglement. The dual dynamical evolution of the trinary fields eliminates the Hilbert-space inconsistency of Einstein’s equation. We argue that spacetime-matter entanglement could naturally explain the holographic principle. We give briefly two conjectures: (1) Black holes are the maximally informationally-complete quantum system with maximal dual entanglement; (2) dark trinitons might be generated substantially in earlier universe and may be a candidate to dark matter/energy.

We would like to emphasize that, as usual quantum mechanics, current quantum field theory is also informationally incomplete and describes particle fields and gauge fields as isolated, physical entities. This
description leads to interpretation difficulties such as the black-hole information paradox and physical meaning of field quanta in curved spacetime. In the ICQFT, however, a dramatically different picture arises. Here spacetime (gravity) and matter are mutually defined and entangled—no spacetime implies no matter, and vice versa. As programmed by spacetime, particle fields and their gauge fields are likewise mutually defined and entangled; either of them alone cannot be informationally-complete. In some sense, it is the quantum version of Einstein’s gravity that completes the picture. The ICQFT, free of those interpretational difficulties or paradox that we encountered in conventional quantum field theory, calls for a radical change of our current understanding on spacetime, matter, information and reality, as well as their relations.

We have shown thus far that for both quantum mechanical systems and quantum fields, informationally complete description of trinary systems shares common features such as dual entanglement, dual dynamics and exclusion of any classical concepts like probability description. The mere possibility of achieving this is itself a surprise and conceptually appealing. Like existing approaches to quantum gravity, there are too many open questions in the framework of the ICQFT, including more physical consequences implied by the informationally complete description, informationally complete quantum cosmology, and the relation between the ICQFT and conventional quantum field theory and so on. If this work serves as a start to stimulate someone to take into account seriously and to work out more consequences of our informationally complete description of nature, it is exactly the author’s hope.

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