GAIA: A WINDOW TO LARGE-SCALE MOTIONS

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ABSTRACT

Using redshifts as a proxy for galaxy distances, estimates of the two-dimensional (2D) transverse peculiar velocities of distant galaxies could be obtained from future measurements of proper motions. We provide the mathematical framework for analyzing 2D transverse motions and show that they offer several advantages over traditional probes of large-scale motions. They are completely independent of any intrinsic relations between galaxy properties; hence, they are essentially free of selection biases. They are free from homogeneous and inhomogeneous Malmquist biases that typically plague distance indicator catalogs. They provide additional information to traditional probes that yield line-of-sight peculiar velocities only. Further, because of their 2D nature, fundamental questions regarding vorticity of large-scale flows can be addressed. Gaia, for example, is expected to provide proper motions of at least bright galaxies with high central surface brightness, making proper motions a likely contender for traditional probes based on current and future distance indicator measurements.

Key words: dark matter – large-scale structure of universe
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1. INTRODUCTION

In the standard cosmological paradigm, peculiar motions (i.e., deviations from Hubble flow) of galaxies are the result of the process of gravitational instability with overdense regions attracting material and underdense regions repelling material. The coherence and amplitude of galaxy flows are a direct indication of the distribution of the dark matter, the cosmological background, and the underlying theory of gravity. Traditionally the peculiar velocity field is derived from observations of distance indicators such as the Tully–Fisher (TF) relation (Tully & Fisher 1977) between luminosity and rotational velocity of galaxies. The observed flux and rotational velocity are then used to infer the distance from the TF relation. The distance is then subtracted from the redshift, \(cz\), in order to obtain the line-of-sight component of the peculiar velocity of a galaxy, with a typical 1\(\sigma\) error \(\sim0.2\,cz\).

Here, we point out an alternative probe of the large-scale velocity field by means of future likely measurements of proper motions of galaxies. As an example for such future measurements we consider the Gaia\(^6\) space astrometric mission. Although the main aim of the mission is to study our Galaxy, Gaia will also be able to perform accurate astrometry for external galaxies, largely thanks to its excellent angular resolution, provided they are sufficiently distant (Perryman et al. 2001; Vaccari 2000). As an example, the nuclei of M87 at N5121, both at \(d \sim 17.8\,h_{70}^{-1}\) Mpc, will be detected with apparent magnitudes \(V \sim 16\) and \(V \sim 14.7\), respectively, within an aperture of 0.65 arcsec, approximately corresponding to Gaia’s detection window (Ferrarese et al. 1994; Carollo et al. 1998; Lauer et al. 2007). With an expected end-of-mission accuracy in the measurements of proper motions of \((10 – 20)\,\mu\text{as yr}^{-1}\) at the \(V\) magnitudes of these two galaxies, Gaia will be able to measure the transverse displacements of these objects with an accuracy of \((0.8–1.6) \times 10^{-4}\,h_{70}^{-1}\,\text{pc}\), corresponding to a transverse velocity \(~600\,\text{km s}^{-1}\), which is a rather typical value, comparable to that of the Local Group velocity with respect to the cosmic microwave background. Although Gaia’s onboard thresholding algorithm is optimized for stellar objects, a large number of galaxies will have their stellar light emission concentrated in a compact region (either a bulge or pseudo-bulge) of subkpc in effective radius, sufficient to make them appear as detectable point-like sources (e.g., Kormendy 1977; Allen et al. 2006; Oohama et al. 2009; Graham 2011). Robin et al. (2012) estimate that Gaia will be able to detect \(~10^6\) galaxies. In fact, high surface brightness (SB) substructures within extended objects might be detected as individual sources associated with the same galaxy, hence improving the accuracy in the measurement of its peculiar motion, as we shall demonstrate in this paper.

Distances to galaxies are needed to derive their transverse peculiar velocities (in \(\text{km s}^{-1}\)) from the proper motions. High-precision distances will become available for those star-forming galaxies that, once observed by Gaia, will subsequently have their Cepheid distances determined (for example, by James Webb Space Telescope, JWST\(^7\)). However, these will be available for a limited number of relatively nearby galaxies, whereas we are interested in tracing the cosmic velocity field over large regions. As a proxy for the distance we will use the galaxy’s redshift, which differs from the actual distance by the radial peculiar velocity. The relative error in the transverse velocity as a result of this approximation is small and decreases with redshift. An object with \(V = 15\) will have an error in transverse velocity of \(~0.6\,cz\). This is significantly larger than the uncertainty in line-of-sight peculiar velocities from distance indicators. However, we will show that the large number of galaxies expected to be observed with Gaia will beat the increased scatter.

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\(^6\) http://sci.esa.int/science-e/www/area/index.cfm?fareaid=26

\(^7\) http://www.jwst.nasa.gov/
possibly making Gaia’s proper motions an excellent probe of the large-scale flows. This probe of large-scale flows is completely independent of any assumption on the intrinsic relations of galaxies. Further, the two-dimensional (2D) transverse motions are orthogonal (in information content as well as in geometry) to standard line-of-sight peculiar velocities.

The outline of the paper is as follows. In Section 2, we present the general setup and describe theoretical tools for analyzing future transverse velocity data. We present, in Section 3, a rough estimate of the expected error in the transverse velocity obtained by smoothing individual velocities. Expected errors on astrometry for Gaia’s galaxies are discussed in Section 4, and a more general discussion on astrometry of extended objects is given in Section 5. In the concluding section, Section 6, we present a general assessment of the transverse velocity data in comparison to other probes of large-scale motions. We also discuss possible sources for redshifts of the population of galaxies expected to be observed by Gaia.

Unless otherwise specified, magnitudes observed by Gaia will refer to an aperture photometry of 0.65 arcsec. They are given in the G band (350–1000 nm). Transformation from the more familiar V and I, bands is performed using constant colors $V - G = 0.27$ and $V - I_c = 1$ for all galaxies (Fujiyuga et al. 1995; Jordi et al. 2010). We also assume that Gaia will identify all sources with $G < 20$ within 0.65 arcsec with 100% completeness. Finally, we use $H_0 = 70$ km s$^{-1}$ Mpc$^{-1}$ to set the distance scale and use $h_{70} = H_0/70$ to parameterize uncertainties.

2. METHODOLOGY

We will assume an all-sky catalog of redshifts and proper motions. We denote the physical peculiar velocity by $\mathbf{v}$ and the real space comoving coordinate by $\mathbf{r}$, both expressed in km s$^{-1}$. Further, $v_{\parallel} = \mathbf{v} \cdot \hat{r}$ and $v_{\perp} = \mathbf{v} - v_{\parallel} \hat{r}$ are, respectively, the components of $\mathbf{v}$ parallel and perpendicular to the line of sight, where $\hat{r}$ is a unit vector in the line-of-sight direction. We restrict the analysis to $cz < 15,000$ km s$^{-1}$ and neglect cosmological geometric effects, so that the redshift coordinate is $z = r + v_{\parallel}$. Note $\hat{s} = \hat{r}$ and $cz = r + v_{\parallel} = \hat{s} \cdot \hat{r} = s$. Proper transverse 2D space velocity of a galaxy at real-space distance $r$ is

$$v_{\perp} = r \mu = 677.22 \frac{\mu}{\mu\text{as yr}^{-1}} \times 10^4 \frac{\text{km s}^{-1}}{h_{70}},$$

which corresponds to a transverse peculiar velocity of 474 km s$^{-1}$ for $1 \mu\text{as yr}^{-1}$ at $d = 100$ Mpc.

However, the true distances, $r$, are unknown, and, therefore, we make the approximation

$$v_{\perp} = s \mu.$$

This introduces a relative error $v_{\parallel}/s$ in the determination of $v_{\perp}$ where $(v_{\parallel}^2)^{1/2} \sim 200–300$ km s$^{-1}$ (Davis et al. 2011). Hence, the error is negligible as we go to $s \gtrsim 2000$ km s$^{-1}$. The error is also random since $\langle v_{\perp} v_{\parallel} \rangle = 0$.

Therefore, the estimated velocity field will be given as a function of the redshift space coordinate. To linear order, velocity fields expressed in real and redshift spaces are equivalent. In the quasilinear regime, dynamical relations can be derived for the velocity field in redshift space (Nusser & Davis 1994), thanks to the interesting property that an irrotational (or potential) flow in real space remains irrotational also in redshift space (Chodorowski & Nusser 1999).

2.1. From 2D Transverse Velocities to 3D Flows

Here, we offer basic expressions for the derivation of the full peculiar velocity field $\mathbf{v}(s)$ from the smoothed 2D transverse velocity field, $v_{\parallel}(s)$. Assuming a potential flow $\mathbf{v}(s) = - \nabla \Phi(s)$ and expanding the angular dependence of $\Phi$ in spherical harmonics, $\Phi(s) = \sum_{lm} \Phi_{lm}(\hat{s}) Y_{lm}(s)$, gives (Arfken & Weber 2005)

$$v_{\parallel} = - \frac{1}{s} \sum_{lm} \frac{d \Phi_{lm}}{ds} Y_{lm},$$

where $Y_{lm} = r \nabla Y_{lm}$ is the vector spherical harmonic. Thanks to the orthogonality conditions $\int d\Omega \Psi_{lm} \cdot \Psi_{lm'} = l(l+1) \delta_{ll'} \delta_{mm'}$, the potential coefficients can be recovered by

$$\Phi_{lm}(s) = - \frac{1}{l(l+1)} \int d\Omega \nabla \Psi_{lm} \cdot \Psi_{lm}(s),$$

for $l > 0$. This means that $\Phi(s)$ can be recovered from the $v_{\parallel}$ up to a monopole term that corresponds to a purely radial flow with zero transverse motions. That is not a serious drawback since the monopole term can always be removed from the predictions of any model to be compared with the data.

2.2. Testing the Potential Flow Ansatz

Initial conditions in the early universe might have been somewhat chaotic, so that the original peculiar velocity field was uncorrelated with the mass distribution or even contained vorticity (e.g., Christopherson et al. 2011). At late time, a cosmological velocity field should have a negligible rotational component, $\mathbf{v}_{\text{rot}}$ on large scale, away from orbit mixing regions. The reason is that any circulation, $\Gamma = \oint \mathbf{v}_{\text{rot}} \cdot d\mathbf{s}$, is conserved by Kelvin’s theorem. Hence, any rotational component will decay as $1/a$, where $a$ is the scale factor. In contrast, the irrotational component of the peculiar velocity will have a growing $v \sim \sqrt{a}$. Therefore, on large scales, away from collapsed objects, the irrotational component is expected to be negligible. The absence of any significant large-scale vorticity is, therefore, a strong prediction of the standard cosmological paradigm. To assess this prediction, the observed transverse motions can be used to constrain the amplitude of the irrotational component. This can be done by writing the transverse component of $\mathbf{v}_{\text{rot}}$ as (Arfken & Weber 2005)

$$v_{\perp} = \sum_{lm} v_{\text{rot},lm} \Phi_{lm},$$

where $\Phi_{lm} = s \times \nabla Y_{lm}$ belong to another class of vector spherical harmonics that satisfy the same orthogonality conditions as $\Psi$. Hence, $v_{\text{rot},lm}$ is equal to the right-hand side of Equation (5) but with $\Phi_{lm}$ instead of $\Psi_{lm}$. Further, $\int d\Omega \Phi_{lm} \cdot \Psi_{lm'} = 0$; hence, the recovery of the rotational mode is formally independent of the potential flow mode.
3. THE EXPECTED ERRORS

We provide estimates of the expected random errors in the smoothed transverse velocity field, $\mathbf{v}_\perp(\mathbf{s})$, as a function of distance from the observer.\(^8\)

The expected 1σ error, $\sigma_\perp(m)$, in the measurement of an object’s proper motion depends on its $G$ magnitude and, to a lesser extent, on its color (de Bruijne 2012). Hereafter, we will use $\sigma_\perp$ as a function of $G$, according to the expression referenced in de Bruijne (2012). This $\sigma_\perp(m)$ is plotted in the left panel of Figure 1. Other possible error sources are photometric jitter from supernovae, active galactic nuclei (AGNs), and, for the latter sources, the presence of jets with large proper motions. We assume that spectrophotometry available for all objects detected by Gaia will significantly reduce the impact of these error sources, which, therefore, will be neglected in the error budget.

The rms accuracy in the galaxy proper motion at redshift $s$ can be obtained by summing over all galaxy magnitudes

$$\langle \sigma_m^{-2}(s) \rangle = \int_{-\infty}^{m_{\text{min}}} n(m,s) \sigma_m^{-2}(m) \, dm.$$  

We assume that the SB profile of distant galaxies is sufficiently peaked to guarantee that a large fraction of the galaxy luminosity is within Gaia’s detection window. The validity of this hypothesis will be discussed in Section 4. In this case the number density of galaxies $n(m,s)$ is simply related to the galaxy luminosity function $N(M)$: $n(m,s) \, dm = N(M) \, dm$, where $M = m - 5 \log_{10}(L_*/L_V) - 15$ is the absolute magnitude. For Gaia’s $G$ band, we approximate $N(M)$ by the Schechter form of the $V$-band luminosity function with parameters given by Brown et al. (2001). Other choices for the Schechter parameters of the $V$-band luminosity function (Marchesini et al. 2012) do not change the results significantly at the magnitude limits considered here. The results for the average error are shown in the right panel for three magnitude cuts. The flatness of the curves for all magnitudes is a reflection of the fact that the number of galaxies increases strongly with magnitude.

Given individual measurements, $\mathbf{v}_{\perp,i} = \mathbf{v}_\perp(\mathbf{s})$, we write the smoothed velocity as

$$\mathbf{v}_\perp(\mathbf{s}) = \frac{\sum_i \mathbf{v}_{\perp,i} \sigma_{\perp,i}^{-2} W(\mathbf{s}-\mathbf{s}_i)}{\sum_i \sigma_{\perp,i}^{-2} W(\mathbf{s}-\mathbf{s}_i)},$$  

where the summation is over all galaxies, $\sigma_{\perp,i} = s \sigma_\mu$, and $W$ is a smoothing window function.

The 1σ errors on $\mathbf{v}_\perp(\mathbf{s})$ are given by

$$\sigma_\perp^2(s) = \frac{\sum_i \sigma_{\perp,i}^{-2} W(\mathbf{s}-\mathbf{s}_i)}{\left[\sum_i \sigma_{\perp,i}^{-2} W(\mathbf{s}-\mathbf{s}_i)\right]^2}.$$  

We have assumed the distant observer limit so that $|\mathbf{s}_i - \mathbf{s}| \ll s$. For a top-hat window of the same width we get the same expression but with $4\pi/3$ as the numerical factor in the denominator. Substituting $\sigma_\mu(m)$ (see the left panel in Figure 1) in Equation (10), we compute the expected error, $\sigma_\perp$, in the smoothed $\mathbf{v}_\perp$, for a Gaussian smoothing with $R_G = 1500$ km s$^{-1}$. The top panel in Figure 2 shows curves of $\sigma_\perp$ as a function of distance for three magnitude cuts.

For comparison the figure also plots the error in the filtered line-of-sight peculiar velocities in the SFI++ catalog of TF measurements of $\sim 4000$ galaxies (Masters et al. 2006). There is a significant decrease in $\sigma_\perp$ as the magnitude is increased from $G = 14$ to 15, but the improvement is not as dramatic when fainter galaxies with $15 < G < 16$ are included. The reason is the rapid deterioration in $\sigma_\mu$ at $G = 16$, which is not compensated by the added number of fainter galaxies. At redshifts $s \gtrsim 6000$ km s$^{-1}$ and for $G < 15$, peculiar velocities from Gaia’s proper motions are expected to fair much better than the SFI++ catalog (Masters et al. 2006). Another important quantity that can be computed from transverse velocities is the dipole motion (i.e., bulk flow) of spherical shells of a given thickness. This motion is described by a constant term $\mathbf{B}$ and gives rise to a transverse velocity field of the form $\mathbf{v}_\perp = \mathbf{B} - \mathbf{f}(\mathbf{B} \cdot \mathbf{f})$. The dipole term $\mathbf{B}$ can be found by least-squares fitting of $\mathbf{v}_{\perp,B}$ to the observed velocities $\mathbf{v}_{\perp,i}$. The expected error in $\mathbf{B}$ as a function of distance of the shell is plotted in the bottom panel in Figure 2. Predictions for 3 mag cuts are plotted for spherical shells of 3000 km s$^{-1}$ in thickness. For comparison we also plot the WMAP7 ACDM model (Larson et al. 2011) predictions for the amplitude of the velocity dipole on spherical shells. It is encouraging that the predicted amplitude is larger than the expected error out to relatively large distances.

4. ASTROMETRY WITH Gaia’s GALAXIES

Here, we provide a rough argument demonstrating the possibility of high-precision astrometry with galaxies observed by Gaia. To do this, we use Gaia’s condition for astrometric measurements ($G < 20$ within an aperture of $\sim 0.65$ arcsec) to define an analogous threshold based on SB. The mean SB of a $G = 20$ object within the Gaia detection window is $\mu_G \sim 20$ mag arcsec$^{-2}$. Here, we shall make a

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\(^8\) In generating the smoothed $\mathbf{v}_\perp(\mathbf{s})$ care must be employed since the transverse directions of galaxies in different sight lines within a filtering window do not point in the same direction. This difficulty could be overcome by tensor window smoothing à la POTENT (Dekel et al. 1990). However, we will not be concerned with these fine details at this stage.
For old stellar populations, a more conservative choice and assume that only sources with 

$$\mu_G < 18.5 \text{ mag arcsec}^{-2}$$

will be used for astrometric purposes. A survey of the literature shows that this condition is satisfied for the central region of a significant fraction of galaxies (e.g., Kormendy 1977; Allen et al. 2006; Oohama et al. 2009; Balcells et al. 2007; Smith et al. 2009; Graham 2011; Ferrarese et al. 1994; Carollo et al. 1998; Lauer et al. 2007). For example, this can be seen in Figure 3 in Oohama et al. (2009) showing a scatter plot of the B-band effective SB versus half-light radius for various galaxy types. More importantly, we have visually inspected the observed V-band SB profiles of 200 out of ~600 galaxies in the Carnegie-Irvine Galaxy Survey (Ho et al. 2011; Li et al. 2011). Most of these galaxies are nearby (median distance of ~25 h^{-1} Mpc) and with mean B-band absolute total magnitude of ~20.2, close to M_b. We identified galaxies reaching a central SB of 18.5 mag arcsec^{-2} and tabulated the corresponding radii (arcsec). Since we did not have access to the actual data, the minimal radius we could determine using a ruler is 1–2 arcsec. About 70% of the galaxies we inspected were brighter than 18.5 mag arcsec^{-2}, allowing them to be detected by Gaia. Since SB is a distance-independent quantity, we can use this threshold to compute the maximum distance at which a galaxy would be detected in a single resolution element of Gaia. We find that the majority of early- and late-type galaxies could be detected as point sources at G = 20 if, respectively, placed at ~500 h_70^{-1} Mpc and ~250 h_70^{-1} Mpc. Overall, it looks like the overwhelming majority of early-type galaxies and more than 50% of late types will have peculiar motions measured by Gaia with errors in transverse velocities given in the top panel in Figure 2. In addition, a significant fraction of their emitted light will be within Gaia’s detection window, which justifies the simple relation between galaxy number density and luminosity function that we have adopted in Section 3. AGNs will be easily detected by Gaia as bright, pointlike sources and possibly mistaken by galaxies. However, their contamination to a relatively local sample of objects with measured redshift, like the one we consider here, should be negligible.

In fact, since we are interested in studying the velocity field of the local (~100 h_70^{-1} Mpc) universe, the situation is likely to be even more favorable. Within this distance the typical galaxy will be resolved in high-SB substructures that, if brighter than G = 20, can be detected as individual sources and analyzed as a group. Examples of multiple high-SB sources are star-forming regions, globular clusters, and bulges with steep SB profiles that are more extended than Gaia’s window (for example, the SB profile of M87 drops below 18.5 mag arcsec^{-2} at ~700 h_70^{-1} pc from the center; if placed at ~50 h_70^{-1} Mpc, it will be detected as ~10 individual sources by Gaia). Detecting multiple sources from the same objects significantly improves the astrometric precision, as we shall show in the next section.

5. ASTROMETRY WITH EXTENDED OBJECTS

The possibility of placing multiple constraints on the same objects allows one, in principle, to improve the astrometric accuracy. We discuss this possibility in a general context and with a formalism that contemplate both the possibility of performing resolved photometry with high-resolution instruments like HST,10 JWST, LSST, or Pan-STARRS (Saha & Monet 2005; Chambers 2005) and that of splitting an extended source in individual sources, like in the case of Gaia.

Suppose for simplicity we observe a galaxy at two different epochs, t_1 and t_2. Let us define $I_i(\theta_i)$ as the SB of the object at the epoch $t_i$ measured at the angular position of a pixel $\theta_i$. In the case of traditional photometry $I_i(\theta_i)$ represents the SB profile of the object at $\theta_i$, whereas in the case of Gaia it represents the magnitude of the SB substructure measured within the detection window. In principle, the astrometric shift, $\mathbf{p}$, could be determined by minimizing, with respect to $\mathbf{p}$, $\chi^2 = \sum (I_i(\theta_i) - I_i(\theta_p)) \sigma_i^2$, where the summation is over all pixels, $\theta = \theta - \mathbf{p}$, and $\sigma_i^2$ here is the 1σ error in the measurement of the SB (since $\mathbf{p}$ is small, we assume that $\sigma_i$ is the same for both images). We have assumed that $I_1$ and $I_2$ differ only by a linear displacement. In principle, one should take into account changes in the internal structure of the object. Those, however, will have little effect compared to the overall observational accuracy. Since we will eventually be interested in the mean coherent displacement of an ensemble of many galaxies, incoherent changes in the internal structure of galaxies will be insignificant.

This procedure of minimizing the image differences exploits all information contained in both images, but it requires a possibly non-trivial interpolation of $\theta$ on the observed pixel positions.

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9 For old stellar populations, B ~ V + 1 (Fukugita et al. 1995), and since $G = V + 0.27$, the astrometric condition $G < 18.5$ translates to $B < 19.7$.

10 http://www.stsci.edu/hst/
at $\theta$. Therefore, we present here an alternative technique that alleviates this problem and clarifies additional matter related to the astrometric expected precision for extended objects. Suppose that the actual galaxy image at time $t_1$ is described by $I_1(\theta) = \sum c_\alpha I^\alpha(\theta)$, where $I^\alpha$ are basis functions that may be chosen to be orthonormal. Any choice (e.g., Fourier modes, wavelets) for $I$ would do for our purposes here. The underlying image at $t_2$ is therefore $I_2(\theta + p)$. The modeling in terms of the basis functions $I^\alpha$ should account for the fact that the signal is modulated by the point-spread function, while photometric noise is just white noise. The expansion coefficients $c_\alpha$ and the displacement $p$ are determined by minimizing

$$\chi^2 = \sum_i \sigma_i^{-2} \left[ I_{1i} - \sum a I^a \right]^2 + \sum_i \sigma_i^{-2} \left[ I_{2i} - \sum a c_a \left( \tilde{I}^a + \frac{\partial I^a}{\partial \theta} p \right) \right]^2. \tag{11}$$

More generally, images are taken at many different epochs (about 70 epochs in the case of Gaia). Therefore, it is more appropriate to write $p = \mu t$, and to minimize the total $\chi^2$ with respect to $\mu$. Since the generalization is straightforward, for brevity of notation we adhere to the simple situation described by Equation (11). We note that several variations of this procedure could be adopted. For example, as an alternative to minimizing $\chi^2$ in Equation (11), we could use basis functions defined in terms of $\theta - \theta_c$, where $\theta_c$ is an assumed position comoving with a given point on the galaxy (e.g., the centroid in the case of a spherical object). We then could minimize the counterpart of the first term in Equation (11) with respect to $c_\alpha$ and $\theta_c$, to get $\theta_c$ at epochs $t_1$ and $t_2$. Using the same model for the images at the two epochs, the difference between $\theta_c$ would then be the displacement $p$. This will yield identical results to minimization of Equation (11) of our choice of $I^\alpha$ given as functions of $\theta$ rather than $\theta - \theta_c$. The covariance matrix of the error in the estimated parameters $c_\alpha$ and $p$ is given by the inverse of the Hessian of $\chi^2$ formed from $H_{\alpha \beta} = \delta \chi^2 / \partial c_\alpha \partial c_\beta$, $H_{\alpha p} = \delta \chi^2 / \partial c_\alpha \partial p$, and $H_{pp} = \delta \chi^2 / \partial p \partial p$. It is easy to show that $|H_{pp}| \ll |H_{\alpha p}|$, implying that the error on $p$ is $H_{pp}^{-1}$, i.e., almost independent of how well $c_\alpha$ are recovered. Considering a one-dimensional displacement, we get an error of $\sigma_p \propto 1 / \left( \sum_a (dI_1/d\theta)^2 / \sigma_\alpha^2 \right)$. Assuming that the objects’ SB dominates the sky background so that $\sigma_{I_1} \propto I_s$, we get

$$\sigma_p^2 \propto \frac{1}{f \langle (dI_1/d\theta)^2 / I_s^2 \rangle}, \tag{12}$$

where $f$ is the observed total flux of the object. Note that the averaged quantity $\langle (dI_1/d\theta)^2 / I_s^2 \rangle$ is independent of the amplitude of $I_s$. Hence, $\sigma_p$ depends on the total observed flux and variance in logarithmic derivative of the SB. The actual value of the SB is irrelevant as long as it satisfies the detection criteria. The larger the fluctuations/irregularities in the stellar light, the more accurate is the astrometry. These irregularities may arise from different physical conditions in galaxies, e.g., spiral arms, young stellar associations, gravitational lumping of stars, caustics resulting from recent merging, and patchy intrinsic dust obscuration. In the case of Gaia, they will be seen as individual sources with an $S/N \gtrsim 5$ at 1 arcsec$^{-2}$ at a SB of 20 mag arcsec$^{-2}$ (Vaccari 2000).

For galaxies at $\lesssim 100 h_{70}^{-1}$ Mpc, SB fluctuations due to Poisson fluctuations in the finite number of (mainly hot luminous) stars per pixel (Blakeslee et al. 1999; Biscardi et al. 2008) may contribute an additional source of irregularities. It is interesting that a sufficiently patchy extended object brighter than $G \approx 12$ may yield more accurate astrometry of a point source with the same luminosity. The reason is the noise floor for point sources with $G \lesssim 12$. An extended object of the same luminosity but made of numerous patches each with $G > 12$ could therefore yield higher precision than the point source. We conclude that astrometry of extended objects could well be comparable to those of point sources.

### 6. DISCUSSION

The number of galaxies expected to be observed by Gaia is likely to exceed standard distance indicator data by two orders of magnitude (Masters et al. 2006, 2008; Springob et al. 2012; Courtois et al. 2011). Pan-STARRS and LSST (Chambers 2005; Saha & Monet 2005) will yield a factor of 100 less accurate proper motions ($\sim$mas yr$^{-1}$) than Gaia, but they will have substantially more galaxies and therefore will also be useful for large-scale motions. Despite the larger object-by-object error, the large number of galaxies in catalogs of proper motions makes them a serious contender for traditional probes of the peculiar velocity field. The method has several advantages. First, it is completely independent of any assumed intrinsic relations of galaxies and hence does not suffer from the usual concerns related to these relations, e.g., linearity, selection biases, and dependence on environment. Second, it yields the 2D transverse velocity component and hence offers completely orthogonal information to standard probes that yield the line-of-sight component. Third, it is free from homogeneous and inhomogeneous Malanquis biases (Lynden-Bell et al. 1988).

The usefulness of the method for probing the three-dimensional (3D) velocity field on scales of a few tens of Mpc is limited to $s \lesssim 10,000$ km s$^{-1}$. However, large-scale moments of the velocity field can be assessed at much larger distances. In particular, the error of the dipole where the bulk flow of spherical shells can be estimated with $\sim 100$–200 km s$^{-1}$ error at $s \sim 15,000$ km s$^{-1}$. At larger redshifts, neither this method nor traditional ones are comparable to the constraints on the dipole from galaxy luminosities in future galaxy redshift surveys (Nusser et al. 2011). At lower distances ($\lesssim 2000$ km s$^{-1}$) the transverse motions of galaxies could play an important role in providing new constraints on the motion of the Local Group of galaxies.

Gaia will provide spectroscopic information of unresolved galaxies (Tsalmantza et al. 2012), especially those with a high-SB nucleus that will be preferentially detected. However, the inferred redshifts may not be sufficiently accurate or available for all unresolved galaxies with astrometric data (e.g., Robin et al. 2012). The Two Mass Redshift Survey (2MRS; Huchra et al. 2012) offers redshifts of $\sim 4 \times 10^4$ galaxies down to $K_s = 11.75$. This is the deepest all-sky redshift catalog currently available. It was originally planned to reach $K_s = 12.2$ mag and to include $\sim 10^5$ galaxies. This is similar to the expected number of galaxies detected by Gaia brighter than $V = 15.27$ (i.e., $G = 15$). Further, since $K_s \approx K$ (Carpenter 2001) and $V - K \approx 2.7$ for most galaxies (Aaronson 1978), we conclude that the $K_s = 12.2$ M2RS has the same objects’ number density as the Gaia galaxies observed to $G = 15$, and undoubtedly it is largely the same sample. This is particularly interesting
as *Gaia*’s astrometric accuracy deteriorates rapidly at fainter objects. However, it is unclear if 2MRS will be continued to $K_s = 12.2$ in the very near future (L. M. Macri 2012, private communication). For the purposes of the analysis presented here one could use a catalog of photometric redshifts based on the Two Micron All Sky Survey (2MASS) galaxy catalog (Skrutskie et al. 2006), containing almost 1 million sources with $K_s < 13.5$ mag. Its current form (2MASS XSCz; Jarrett 2004) offers distance errors as large as 20%–25%, which will improve in the coming years using the data from other galaxy catalogs for the photo-z estimation (M. Bilicki 2012, private communication).

We have restricted the error analysis here to $G \sim 15$ since redshifts will probably not be available for all fainter galaxies. However, data at fainter magnitudes can well be exploited by computing the dipole as a function of an effective depth corresponding to a certain magnitude range. This can then be compared with model predictions for an equivalent quantity (Bilicki et al. 2011).

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