Unification of Couplings and the Top Quark Sector in the Minimal Supersymmetric Standard Model

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Abstract

I analyze the predictions for the strong gauge coupling, $\alpha_3(M_Z)$, and the top quark and light Higgs masses in the framework of gauge and bottom-tau Yukawa coupling unification in the minimal supersymmetric standard model. These predictions depend on the effective supersymmetric threshold scale $T_{SUSY}$, which is only very slightly dependent on the squark and slepton masses, and strongly dependent on the Higgsino masses as well as on the mass ratio of the gauginos of the strong and weak interactions. Within the minimal supersymmetry breaking scheme and for supersymmetric masses below or of the order of 1 TeV, I obtain $\alpha_3(M_Z) \geq 0.116$, while, if the running bottom quark mass at the physical mass is constrained to be $m_b(M_b) \leq 4.1 \text{GeV}$, then, for moderate values of $\tan \beta$, perturbative unification is achieved only if $\alpha_3(M_Z) \leq 0.124$. Unification of gauge and bottom-tau Yukawa couplings yield predictions for the top quark mass, $140 \text{ GeV} \leq M_t \leq 210 \text{ GeV}$ for $1 \leq \tan \beta \leq 30$, which are remarkably close to the infrared quasi fixed point values for this quantity. For the light Higgs mass I obtain $m_h \leq 130(165) \text{ GeV}$ if the characteristic squark mass is below 1 (10) TeV.

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Recent studies have shown that, within the minimal supersymmetric extension of grand unified theories, it is possible to extrapolate the standard model gauge couplings to obtain unification at some high energy scale $M_{\text{GUT}}$ in a framework compatible with present experimental data\(^1\)−\(^4\). Furthermore, the unification of couplings in the Yukawa sector appears naturally in some grand unified models, yielding definite predictions for the top quark mass\(^5\)−\(^7\). For the extent of this talk I shall concentrate on the Minimal Supersymmetric Standard Model (MSSM) within the minimal presumption of grand unification, without any specific assumptions about the physics above the unification scale. However, in order to consider the problem of unification of couplings in a proper way it is necessary to analyze the incidence of possible threshold corrections at $M_{\text{GUT}}$, which depend on the specific grand unified scenario at the high energy scales. Thus, I shall also briefly discuss the relaxation of the unification condition for the couplings and its relation to the infrared quasi fixed point predictions for the top quark mass.

In a first approximation to the problem of unification of gauge couplings, it is usual to assume that all the supersymmetric particles have a common mass $M_{\text{SUSY}}$. In such extreme case, the common mass scale $M_{\text{SUSY}}$ coincides with the scale $T_{\text{SUSY}}$ which, in general, denotes the scale characterizing the supersymmetric threshold corrections to the gauge couplings. Within this context, for values of the low energy parameters consistent with experimental data, the unification of gauge couplings may be achieved for a unification scale $M_{\text{GUT}} = \mathcal{O}(10^{16} \text{GeV})$ and a common sparticle mass $M_{\text{SUSY}}$ of the order of the weak scale\(^1\). However, once the condition of equality of all the supersymmetric particle masses is relaxed, the actual scale characterizing the supersymmetric threshold corrections to the gauge couplings may significantly differ from any of the sparticle masses\(^2\)−\(^4\), \(^7\). Nevertheless, as I shall show below, the consequences of an arbitrary supersymmetric spectrum on the low energy predictions for the gauge couplings can always be parametrized in terms of a single threshold scale $T_{\text{SUSY}}$ \(^4\), \(^7\).

Considering a general supersymmetric spectrum, each sparticle $\eta$ contributes to the one loop supersymmetric threshold corrections to $1/\alpha_i(M_Z)$ with a factor $b_i^\eta$ yielding,

$$\frac{1}{\alpha_i^{\text{Sthr}}(M_Z)} = \sum_{\eta, M_\eta > M_Z} \frac{b_i^\eta}{2\pi} \ln \left( \frac{M_\eta}{M_Z} \right).$$  \hspace{1cm} (1)
Assuming the unification of the three gauge couplings at \( M_{GUT} \) and considering their two loop beta functions, one coupling, for example \( \alpha_3(M_Z) \), is determined as a function of the electroweak gauge couplings and the above supersymmetric threshold corrections.

\[
\frac{1}{\alpha_3(M_Z)} = \frac{1}{\alpha_3^{SUSY}(M_Z)} + \Delta^{Sthr} \left( \frac{1}{\alpha_3(M_Z)} \right),
\]

(2)

where

\[
\frac{1}{\alpha_3^{SUSY}(M_Z)} = \frac{b_1 - b_3}{b_1 - b_2} \left[ \frac{1}{\alpha_2(M_Z)} + \gamma_2 + \Delta_2 \right] - \frac{b_2 - b_3}{b_1 - b_2} \left[ \frac{1}{\alpha_1(M_Z)} + \gamma_1 + \Delta_1 \right] - \gamma_3 - \Delta_3
\]

(3)
gives the value of the strong gauge coupling at \( M_Z \) if the theory were supersymmetric all the way down to \( M_Z \), with \( b_i \) the one loop beta function coefficient of the gauge coupling \( \alpha_i \) in the MSSM, \( \gamma_i \) the two loop contributions to the beta functions and \( \Delta_i \) a constant term associated to scheme conversion. The last term in eq. (2) accounts for the supersymmetric threshold corrections\(^4,7\)

\[
\Delta^{Sthr} \left( \frac{1}{\alpha_3(M_Z)} \right) = \frac{(b_1 - b_3)}{(b_1 - b_2)} \frac{1}{\alpha_2^{Sthr}} - \frac{(b_2 - b_3)}{(b_1 - b_2)} \frac{1}{\alpha_1^{Sthr}} - \frac{1}{\alpha_3^{Sthr}} = \frac{19}{28\pi} \ln \left( \frac{T_{SUSY}}{M_Z} \right),
\]

(4)

which can be parametrized in terms of the supersymmetric effective threshold scale \( T_{SUSY} \). Therefore, it is most useful to analyze the exact dependence of \( T_{SUSY} \) on the physical masses of the supersymmetric particles.

Considering different characteristic masses for the squarks, \( m_{\tilde{q}} \), gluinos, \( m_{\tilde{g}} \), sleptons, \( m_{\tilde{l}} \), electroweak gauginos, \( m_{\tilde{W}} \), Higgsinos, \( m_{\tilde{H}} \), and the heavy Higgs doublet, \( m_H \), the following expression is derived\(^7\),

\[
T_{SUSY} = m_{\tilde{H}} \left( \frac{m_{\tilde{W}}}{m_{\tilde{g}}} \right)^{28/19} \left[ \left( \frac{m_{\tilde{l}}}{m_{\tilde{q}}} \right)^{3/19} \left( \frac{m_H}{m_{\tilde{H}}} \right)^{3/19} \left( \frac{m_{\tilde{W}}}{m_{\tilde{H}}} \right)^{4/19} \right].
\]

(5)

The above expression is very useful\(^1\). It shows that \( T_{SUSY} \) has only a slight dependence on the squark and slepton masses as well as on the heavy Higgs mass, but, instead, a very strong dependence on the Higgsino and gaugino masses. In models in which the source of supersymmetry breakdown is given by a common gaugino mass \( M_{1/2} \) at the unification scale and assuming that there is no significant gaugino-Higgsino mixing

\(^1\)This expression holds whenever all the particles have masses above \( M_Z \). If, instead, any of the masses is below \( M_Z \), it should be replaced by \( M_Z \) for the purpose of computing \( T_{SUSY} \).
- the overall Higgsino mass scale is approximately given by the absolute value of the renormalized supersymmetric mass parameter $\mu - T_{SUSY}$, eq. (5), can be approximately given by,

$$T_{SUSY} \simeq m_{\tilde H} \left( \frac{\alpha_2(M_Z)}{\alpha_3(M_Z)} \right)^{3/2} \simeq \frac{|\mu|}{\Lambda}.$$  \hspace{1cm} (6)

Therefore, if $|\mu| \leq 1$ TeV it follows $T_{SUSY} \leq O(M_Z)$.

In order to perform a self consistent two loop Renormalization Group (RG) analysis it is most important to consider the experimental data in such a way to determine the low energy boundary conditions as precise as possible. The experimental prediction for $\sin^2 \theta_W$ in the modified $\bar{MS}$ scheme depends quadratically on the top quark mass$^4$,

$$\sin^2 \theta_W(M_Z) \simeq 0.2324 - 10^{-7} \text{GeV}^{-2}(M_t^2 - (138 \text{GeV})^2).$$ \hspace{1cm} (7)

However, it is also possible to consider the best fit to the data with a free top quark mass value, $\sin^2 \theta_W(M_Z) = 0.2324 \pm 0.0012$. The electromagnetic gauge coupling, $1/\alpha(M_Z) = 127.9$, has instead only a logarithmic dependence on the top mass. Contrary to the situation in the electroweak sector, the strong gauge coupling is not so precisely known. A conservative attitude is to take $\alpha_3(M_Z) = 0.12 \pm 0.01$. The tau mass is taken to be $M_\tau = 1.78$ GeV. Concerning the bottom quark sector, I consider the range of values for the physical bottom quark mass quoted in the particle data book, $M_b = 4.7 - 5.2$ GeV$^8$. Observe that, after including QCD corrections at the two loop level, the running mass differs significantly from the physical mass. In fact, the above range for $M_b$ corresponds to a running mass $m_b(M_b) \simeq 4.1 - 4.6$ GeV. Similarly, the physical top quark mass $M_t$ is approximately 6 % larger than the corresponding running mass $m_t(M_t)$.

In the framework of gauge coupling unification, neglecting threshold corrections at $M_{GUT}$, I obtain that the value of the strong gauge coupling increases for smaller values of $\sin^2 \theta_W$ as well as for smaller values of $T_{SUSY}$. At the two loop level, the strong gauge coupling receives negative contributions from the top quark Yukawa coupling, $h_t$. In table 1$^7$, I show the above behaviour for values of $Y_t = h_t^2/4\pi \simeq 0.4 - 1$. Thus, within the minimal supersymmetry breaking scheme and for supersymmetric particle masses below or of the order of 1 TeV, in which case the value of $T_{SUSY}$, eq. (5), is constrained
to be $T_{SUSY} \leq 200$ GeV, a lower bound on the strong gauge coupling, $\alpha_3(M_Z) \geq 0.116$, is derived.

Table 1. Dependence of $\alpha_3(M_Z)$ on $\sin^2 \theta_W(M_Z)$ and $T_{SUSY}$ in the framework of two loop RG analysis with gauge and Yukawa coupling unification, for $m_b(M_b) = 4.3$ GeV.

| $\sin^2 \theta_W(M_Z)$ | $\alpha_3(M_Z)$ for $T_{SUSY} = 1$ TeV | $\alpha_3(M_Z)$ for $T_{SUSY} = 100$ GeV |
|-------------------------|------------------------------------------|------------------------------------------|
| 0.2335                  | 0.111                                    | 0.118                                    |
| 0.2324                  | 0.115                                    | 0.122                                    |
| 0.2315                  | 0.118                                    | 0.126                                    |

The inclusion of the unification condition for the bottom and tau Yukawa couplings leads to an interesting result: The top quark mass values computed for values of the bottom quark mass compatible with present experimental data, $M_b \leq 5.2$ GeV, are remarkably close to the infrared quasi fixed point predictions associated with a large top quark Yukawa coupling, $Y_t \simeq 0.1 - 1$, at $M_{GUT}$. Table 2\textsuperscript{7) illustrates this property for an intermediate value of the running bottom quark mass and various values of $\tan \beta$, the ratio of the vacuum expectation values of the two Higgs fields. In fact, the above is the reason why the predictions for $M_t$ coming from bottom-tau unification are basically the same as those obtained in the framework of supersymmetric top condensate models, which are also associated with a large $Y_t$ at the compositeness scale $\Lambda$, if $\Lambda \simeq M_{GUT}$\textsuperscript{9). For lower values of the bottom quark mass, the top quark Yukawa coupling may become too large to be compatible with perturbative unification. Thus, in order to secure the perturbative consistency of the analysis, I require that $Y_t(M_{GUT}) \leq 1$ - the two loop
corrections are less than 30 % of the one loop contributions. Most generally, I obtain that, for values of $m_b(M_b) \leq 4.1 GeV$, gauge and bottom-tau Yukawa coupling unification requires $\alpha_3(M_Z) \leq 0.124$ in order to remain in the perturbative region of the top quark Yukawa sector\(^7\).

In Fig. 1, I present the values of the running top quark mass as a function of $\tan \beta$ for different values of $\alpha_3(M_Z)$. It is important to stress that, for any intermediate value of $\tan \beta = 10 - 30$, a large variation in $\alpha_3(M_Z) = 0.115 \rightarrow 0.13$ yields a large variation in $Y_t(M_{GUT}) = 0.28 \rightarrow 1$, which, however, due to the presence of the infrared quasi

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Fig. 1. The running top quark mass as a function of $\tan \beta$ for $m_b(M_b) = 4.3 \text{ GeV}$, $\sin^2 \theta_W(M_Z) = 0.2324$ and varying the value of the strong gauge coupling to be $\alpha_3(M_Z) = 0.115$ (solid line), 0.122 (dashed line) and 0.13 (dot-dashed line).
fixed point, induces only a very mild increase of about a 5 % in $m_t$. Similarly, a mild decrease in $m_t$ follows for larger values of the bottom quark mass. Quite generally, I obtain that, within the framework of perturbative gauge and bottom-tau unification, the running top quark mass is bounded to be below 200 GeV, while values of $m_t = \mathcal{O}(140\text{GeV})$ may only be naturally accommodated for low values of $\tan \beta \simeq 1^{7)}$. In addition, the unification of the three Yukawa couplings of the third generation at $M_{\text{GUT}}$ determines a point in any of the above curves, which is achieved for large values of $\tan \beta \simeq 60$ implying $m_t(M_t) = 165 - 185$ GeV. The effect of high energy scale threshold corrections to the gauge coupling unification may change the prediction for $\alpha_3(M_Z)$ in up to 5 %$^{4)}$. However, as shown above, for small and moderate values of $\tan \beta$, if the strong gauge coupling remains in the range $\alpha_3(M_Z) \geq 0.115$, this only induces a slight variation in the top quark mass values. Concerning the effects of threshold corrections at $M_{\text{GUT}}$ to the Yukawa coupling unification condition, it occurs that a relaxation of exact unification up to 10% is possible without destroying the attraction to the infrared quasi fixed point and, therefore, the stability of the top quark mass predictions$^{6),10)}$.

Concerning the Higgs sector, the dominant radiative corrections to the tree level value of the lightest CP even Higgs mass are not only dependent on the top quark Yukawa coupling, but on the logarithm of the squark masses as well$^{11),12)}$,

$$m_h^2(m_t) \leq M_Z^2 \cos^2(2\beta) + \frac{3}{2\pi^2}v^2 \sin^4 \beta h_t^4(M_t) \ln \left(\frac{m_{\tilde{q}}}{m_t}\right). \quad (8)$$

The above expression is for the case in which the CP odd scalar mass $m_A \gg M_Z$ and gives the upper values for $m_h$ as a function of $\tan \beta$. Since the unification condition only fixes the value of $T_{\text{SUSY}}$, which is only weakly dependent on the squark masses, then, even for fixed $\tan \beta$ there is no direct determination of $m_h$ within this scheme. For a given value of $T_{\text{SUSY}}$, an upper bound on the lightest Higgs mass is obtained for the maximum allowed value for $m_{\tilde{q}}$, quite generally, of the order of a few TeV. In general, I obtain $m_h \leq 130(165) \text{ GeV}$ for $m_{\tilde{q}} \leq 1(10) \text{ TeV}^{7)}$. Moreover, if $m_{\tilde{q}} \leq 1 \text{ TeV}$, then the experimental bounds on $m_h$ and $M_t$ almost close the $\tan \beta < 1$ window.

In conclusion, within gauge and bottom-tau Yukawa coupling unification I obtain predictions for the strong gauge coupling and $M_t(\tan \beta)$. Such predictions depend on
the effective supersymmetric threshold scale and, therefore, they have a strong dependence on the Higgsino and gaugino masses. In the above, I have always considered \( \sin^2 \theta_W \) taking into account the best fit to the data with a free top quark mass. However, it is important to notice that considering the correlation between \( \sin^2 \theta_W \) and \( M_t \), eq. (7), stringent predictions for \( \alpha_3(M_Z) \) are obtained. In particular, in the minimal supersymmetry breaking scheme, for sparticle masses below or of the order of 1 TeV, the lower bound on the strong gauge coupling becomes \( \alpha_3(M_Z) \geq 0.12 \). Gauge and bottom-tau Yukawa coupling unification implies two loop RG predictions for the top quark mass, \( 145 \text{ GeV} \leq M_t \leq 210 \text{ GeV} \) for \( 1 \leq \tan \beta \leq 30 \), which are remarkably close to its infrared quasi fixed point values. Observe that, taking into account the present bounds on the top quark mass value coming from precision measurements, \( M_t \leq 170 \text{ GeV} \) at the one \( \sigma \) level, stringent bounds on \( \tan \beta \) are imposed. This implies that the light Higgs mass is not allowed to reach its maximum value but is constrained to be \( m_h \leq 80(90) \text{ GeV} \) if \( m_{\tilde{q}} \leq 1(10) \text{ TeV} \). Most important, this implies that if this scenario with heavy supersymmetric particles holds, the Higgs will be detected at LEP 200 \(^{(6,10)}\). However, in the case that light supersymmetric particles were present, a reanalysis of the top quark mass predictions coming from precision measurements would be necessary. This might loosen the above bounds on \( M_t \) and, therefore, those on \( m_h \) as well.

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