Phase-squeezed light can enhance the precision of optical phase estimation. The larger the photon numbers are and the stronger the squeezing is, the better the precision is. We propose an experimental scheme for generating phase-squeezed light pulses with large coherent amplitudes. In our scheme, one arm of a single-photon Mach-Zehnder interferometer interacts with coherent light via a nonlinear-optical Kerr medium to generate a coherent superposition state. Post-selecting the single photon by properly tuning a variable beam splitter in the interferometer yields a phase-squeezed output. Our proposed scheme is experimentally feasible under current quantum technology.
we calculated the maximum fidelity between the generated state and an arbitrary squeezed coherent state. We found that a 2.08-dB phase-squeezed state can be generated with a fidelity $F = 0.99$ and a success rate of 21.89 Hz under an experimentally feasible setup for inducing cross-phase modulation (XPM), previously described in Ref. [32].

Our proposed setup, shown in Fig. 1, has a single-photon Mach–Zehnder interferometer with arms $a$ and $b$ and uses a nonlinear optical Kerr medium to induce XPM on coherent light in arm $c$. In the single-photon Mach–Zehnder interferometer, the input single photon passes through a half beam splitter (HBS) and is divided into two arms, $a$ and $b$; the resulting vacuum–one-photon qubit state is written as $|i⟩ ≡ (|0⟩_a|1⟩_b − |1⟩_a|0⟩_b)/\sqrt{2}$ where $|1⟩_a$ and $|1⟩_b$ are single-photon states and $|0⟩_a$ and $|0⟩_b$ are vacuum states, in arms $a$ and $b$, respectively. Then, the single photon on two arms passes through a variable beam splitter (VBS) with a transmissivity $t$ and a reflectivity $r$ where $t^2 + r^2 = 1$. The quantum states at the output ports $f$ and $f'$ are given by $|f⟩ = t|0⟩_a|1⟩_b + r|1⟩_a|0⟩_b$ and $|f⟩ = t|0⟩_a|1⟩_b − r|1⟩_a|0⟩_b$, respectively. Furthermore, the effect of the nonlinear optical Kerr medium that is placed between arms $b$ and $c$ is represented by a unitary operator $\hat{U} = \exp(i\phi_0\hat{n}_c\hat{\alpha}_c)$, where $\phi_0 \ll 1$ is the phase shift angle caused by the XPM, and $\hat{n}_b$ and $\hat{n}_c$ are the photon number operators in arms $b$ and $c$, respectively. It is assumed that the input state of arm $c$ is a coherent photon state $|\alpha⟩_c$ with coherent amplitude $\alpha$. Using the nonlinear optical Kerr medium, the initial total state $|i⟩|\alpha⟩_c$ is transformed as follows:

$$|\Psi⟩ = 1/2(|0⟩_a|1⟩_b|a|e^{i\phi_0}⟩_c − |1⟩_a|0⟩_b|\alpha⟩_c).$$

When the single photon is detected at a photon detector $D1$ at port $f$, the total state $|\Psi⟩$ is post-selected to $|f⟩$. The post-selected state $|\psi⟩$ can be written as

$$|\psi⟩ = (f|\Psi⟩ = 1/\sqrt{2P_{\text{suc}}}(t|\alpha e^{i\phi_0}⟩_c − r|\alpha⟩_c),$$

where the success probability $P_{\text{suc}}$ of the post-selection is

$$P_{\text{suc}} = \langle f|\psi⟩|f|\psi⟩| = 1/2[t^2 + r^2 - tr(\langle \alpha e^{i\phi_0}⟩_c + \langle \alpha|\alpha e^{i\phi_0}⟩_c)]$$

with $\langle \alpha|\alpha e^{i\phi_0}⟩_c = \exp[-|\alpha|^2(1 - e^{i\phi_0})]$. We note that this proposed setup is the same as that described in Ref. [32].

In the following, we show that the post-selected state $|\psi⟩$ for $1/\sqrt{2} < t < 1$ can be regarded as a quasi-phase-squeezed state that has a high fidelity to a pure phase-squeezed state. Therefore, we evaluate the maximum fidelity $F = \langle [\xi e^{i2\theta}, \gamma e^{i\theta}]|\psi⟩$ between the post-selected state $|\psi⟩$ and the ideal squeezed coherent state $|\xi e^{i2\theta}, \gamma e^{i\theta}\rangle$ for $1/\sqrt{2} < t < 1$ and evaluate two parameters (the squeezing parameter $\xi = xe^{i\varphi}$ and the phase angle $\theta$ for the maximized fidelity $F$), under the following assumptions. We assume that the mean photon numbers $\bar{n}$ of the coherent state $|\alpha⟩$ and the squeezed coherent state $|\xi e^{i2\theta}, \gamma e^{i\theta}\rangle$ are unchanged since the XPM does not affect the photon number. Furthermore, we assume $\alpha = 10^{3/2}$ and $\phi_0 = 2\pi \times 10^{-5}$, which are the same values used in Ref. [32].

We numerically evaluated the maximized fidelity $F$ for various values of the transmissivity $t$; the results are plotted in Fig. 2(a). The estimated parameters $\xi_{\text{est}} = xe^{i\varphi_{\text{est}}}$ and $\theta_{\text{est}}$ depend on the transmissivity $t$, and since the fidelity monotonically increases with transmissivity, these parameters can also be written as functions of the fidelity $F$. These parameters are plotted in Figs. 2(b) and (c), respectively. The estimated parameter $\varphi_{\text{est}}$ should always be $\pi$ to obtain the maximum fidelity $F$. This means that all estimated squeezed states are phase-squeezed states. Therefore, the post-selected state $|\psi⟩$ can be regarded as the phase-squeezed state for the high-fidelity cases. Moreover, since our scheme requires post-selection, the success probability of the post-selection $P_{\text{suc}}$ for the fidelity $F$ is shown in Fig. 2(d). The estimated parameters of the representative cases (1)–(4) in Fig. 2(a) are summarized in Table I. In case (1), since the post-selected state is equivalent to the odd coherent state for $t = r = 1/\sqrt{2}$, the fidelity is worse than that of the phase-squeezed state ($F = 0.69$). By contrast, when the post-selection succeeds for a transmissivity of $t = 1$, the single photon is transmitted in only arm $b$ of the Mach–Zehnder interferometer. Therefore, the post-selected state is just the phase-shifted coherent state $|\alpha e^{i\phi_0}\rangle_\alpha$. These cases cannot be regarded as the phase-squeezed state. On the other hand, an effective squeezing is obtained in both cases (2) $F = 0.99$ (for $t = 0.717$) and (3) $F = 0.999$ (for $t = 0.724$) with high fidelity. Therefore, these post-selected states can be regarded as quasi-phase-squeezed states. Throughout this Letter, we refer to the post-selected state as the phase-squeezed state when the fidelity $F \geq 0.99$ and $x_{\text{est}} \geq 0.01$.
The phase of the squeezing parameter to maximize the fidelity is \( F \) as a function of the transmissivity \( t \), ranging from \( t = 1/\sqrt{2} \) to \( t = 1 \). Plots of (b) the estimated amplitude of the squeezing parameter of the post-selected state \( x_{\text{est}} \), (c) the estimated phase \( \theta_{\text{est}} \), and (d) the success probability of the post-selection \( P_{\text{suc}} \) as a function of the fidelity \( F \). We note that the optimal value of the phase of the squeezing parameter to maximize the fidelity is \( \phi = \pi \). The points (1)–(4) correspond to \( t = 1/\sqrt{2} \), \( t = 0.717 \) \( (F = 0.99) \), \( t = 0.724 \) \( (F = 0.999) \), and \( t = 1 \), respectively.

(about \( 8.0 \times 10^{-2} \) dB).

| \( t \)   | \( F \)  | \( x_{\text{est}} \) | \( \theta_{\text{est}} \) | \( P_{\text{suc}} \) |
|---------|---------|---------------------|---------------------|---------------------|
| \( 1/\sqrt{2} \) | 0.69    | 0.55                | \( 2.61 \times 10^{-4} \) | \( 9.87 \times 10^{-6} \) |
| 0.717   | 0.99    | 0.24 (2.08 dB)      | \( 1.60 \times 10^{-3} \) | \( 2.95 \times 10^{-4} \) |
| 0.724   | 0.999   | 0.13 (1.13 dB)      | \( 1.15 \times 10^{-3} \) | \( 6.75 \times 10^{-4} \) |
| 1       | 1       | 0                   | \( 2\pi \times 10^{-5} \) | 0.5                  |

To verify the squeezing effect, we calculate probability density distributions \( |\langle p|\psi \rangle|^2 \) [33], which represent the outcomes of the quadrature measurement of the post-selected state projected onto the \( p \)-axis as shown in Fig. 3(a)–(d), which correspond to cases (1)–(4) in Fig. 2(a), respectively. For \( t = 1/\sqrt{2} \) (i.e., the odd coherent state, as mentioned above) as shown in Fig. 3(a), two “squeezing-like” distributions are formed by the quantum interference in the overlap between \(|\alpha\rangle_c \) and \(|\alpha e^{i\phi_0}\rangle_c \) due to the post-selection. For \( t = 1 \), the quantum interference does not occur, as shown in Fig. 3(d); the post-selected state is just a phase-shifted coherent state. However, when the probability amplitudes of Eq. (2) are post-selected, the quantum interference leads to almost the complete elimination of one peak of the probability distribution and formation of a squeezed probability distribution compared to the Gaussian case, as shown in Figs. 3(b) and (c). This is why our proposed scheme can be regarded as a phase squeezer. Further, when the overlap \( \langle \alpha|\alpha e^{i\phi_0}\rangle_c \) is very small, i.e., the distance between the states \(|\alpha\rangle_c \) and \(|\alpha e^{i\phi_0}\rangle_c \) is very large, the effect of the quantum interference in the case of two well-separated peaks is miniscule. Therefore the squeezing effect does not occur, as shown in Fig. 3(d). We numerically confirm that using \( \phi_0 \gtrsim 0.01 \) with \( \alpha = 10^{3/2} \) does not achieve effective squeezing for any transmissivity value.

We note that the back-action of the coherent light on the single-photon interferometer should be compensated for. The interaction between the coherent state and the single-photon interferometer induces a relative phase shift between the arms of the interferometer, which may be represented as follows: \(|0\rangle_a |1\rangle_b - |1\rangle_a |0\rangle_b \rightarrow e^{i|\alpha|^2\phi_0} |0\rangle_a |1\rangle_b - |1\rangle_a |0\rangle_b \). The squeezing effect can be obtained only when \(|\alpha|^2 \phi_0 \) is near a multiple of \( 2\pi \) since the post-selected state is also affected by the relative phase shift \( e^{i|\alpha|^2\phi_0} \) in the interferometer. For example,
for \(|\alpha|^2 \phi_0 = \pi\) using \(\alpha = 10^{5/2}\) and \(\phi_0 = \pi \times 10^{-5}\), the post-selected state with \(t = 1/\sqrt{2}\) is approximately the coherent state. Therefore, compensation by using a phase shifter on the single-photon interferometer is needed when \(|\alpha|^2 \phi_0\) is not near a multiple of \(2\pi\) in order to obtain the phase-squeezing effect. Summing up, we have described how to achieve the phase-squeezing effect for a coherent light pulse with large coherent amplitude by proper post-selection of single photons coupled with coherent light pulses via the weak cross-Kerr nonlinearity.

Let us discuss the experimental feasibility of our scheme by considering already established methods for creating XPM with a single-photon-level nonlinearity. Three methods to implement single-photon-induced XPM have been experimentally reported \([32, 36, 37]\). In Refs. \([36, 37]\), atoms and quantum dots in the cavity were used, respectively. Although a large phase shift (> 0.05\(\pi\)) can be achieved in both cases, our proposal is incapable of using such a large phase shift as mentioned above. Therefore, these methods cannot be employed in our proposal. On the other hand, in Ref. \([32]\), a cross-Kerr–induced phase shift of 10^{-7} rad was measured in a photonic crystal fiber for coherent light at single-photon-level intensities by averaging over 3 \times 10^4 pulses at a repetition frequency of 1 GHz at room temperature. Since a pulsed laser with a wavelength of 802 nm is used, the photon loss of the homodyne measurement can be made small using a Si photodetector \([38]\) that operates at around 800 nm. We note that the total photon loss of the homodyne measurement is \(\sim 0.07\) at 860 nm \([39]\). Here, as mentioned above, we assume that the effect of photon losses on the generated light is negligible. In addition, the photon loss in the Mach–Zehnder interferometer is also negligible because of the event selection. The mean photon number of the coherent light is set to \(|\alpha|^2 = 3.0 \times 10^6\) since self-Kerr and second-order nonlinear effects are observed for larger mean photon numbers. For \(\phi_0 = 10^{-7}\) and \(F = 0.99\), the amplitude of the squeezing of the post-selected state is obtained to be \(x_{\text{est}} = 0.24\) (2.08 dB) with a phase shift angle \(\theta_{\text{est}} = 2.88 \times 10^{-4}\) and a probability \(P_{\text{succ}} = 2.19 \times 10^{-8}\). In this condition, the transmissivity should be tuned to \(t = 0.70719\). Similarly, for \(\phi_0 = 10^{-7}\) and \(F = 0.999\), \(x_{\text{est}} = 0.13\) (1.13 dB) is obtained with \(\theta_{\text{est}} = 2.06 \times 10^{-4}\) and \(P_{\text{succ}} = 5.06 \times 10^{-8}\). In this condition, the transmissivity should be tuned to \(t = 0.70725\). These estimated values for the amplitude of the squeezing parameter are the same for both fidelities in the case of \(|\alpha|^2 = 10^5\) and \(\phi_0 = 2\pi \times 10^{-5}\). This is because the effect of the quantum interference between \(|\alpha\rangle_c\) and \(|\alpha e^{i\phi_0}\rangle_c\) due to the post-selection of the single photon is the same as in the above case since the estimated phases \(\theta_{\text{est}}\) are smaller. The success rates of the post-selection are about 21.89 Hz and 50.55 Hz for \(F = 0.99\) and \(F = 0.999\), respectively. For these success rates, the post-selection is experimentally achievable by employing a beam shutter that works at kHz rates.

In conclusion, we proposed an experimental scheme for generating phase-squeezed light pulses by the post-selection of single photons coupled with coherent light pulses via a weak cross-Kerr nonlinearity. To implement the post-selection of single photons, a Mach–Zehnder interferometer with a VBS as an output is used. When one arm of the Mach–Zehnder interferometer interacts with the coherent light via the weak cross-Kerr nonlinearity, a superposition of the non-phase-shifted coherent state \(|\alpha\rangle_c\) and the phase-shifted coherent state \(|\alpha e^{i\phi_0}\rangle_c\) is generated. The post-selection of the single photon achieves quantum interference between \(|\alpha\rangle_c\) and \(|\alpha e^{i\phi_0}\rangle_c\). When the transmissivity and the reflectivity of the VBS are properly set, an effective squeezing can be obtained such that the output has high fidelity to the ideal phase-squeezed state.

Finally, there remain few further considerations with regard to our proposed scheme. First, our proposal may be related to the context of the weak-value amplification for the phase shift of coherent light by post-selection of single photons, which has already been discussed in Ref. \([33]\). Our estimated phases are also amplified, as shown in Table I, and thus the squeezing effect in our proposal also may be associated with weak values. Furthermore, the relation between quantum interference and weak values has already been discussed \([40, 41]\). Since the argument of the weak value is taken as the geometric phase, the relative phase may be characterized by the geometric phase of the single-photon interferometer. Additionally, our scheme may be generalized to an arbitrary quadrature squeezer, since quantum interference can be controlled by changing the relative phase. Second, to obtain a larger squeezing effect, we investigate the repeated use of our proposed scheme. The same degree of squeezing can be obtained for an output with the same fidelity to the phase-squeezed state by straightforward extension. However, the squeezing effect under photon losses using a beam splitter model is considered \([42]\). In particular, in our proposed scheme, photon losses should be considered in the single-photon interferometer and the post-selected state. In the first case, the photon loss may change the success rate of the post-selection. In the latter case, the photon loss may collapse the post-selected state to a coherent state, and the measurement loss may be dominant since the measurement of the generated light is carried out after the post-selection.

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