Yet Another Efficient Unification Algorithm

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Abstract

The unification algorithm is at the core of the logic programming paradigm, the first unification algorithm being developed by Robinson [5]. More efficient algorithms were developed later [4] and I introduce here yet another efficient unification algorithm centered on a specific data structure, called the Unification Table.

1 Introduction

The unification algorithm is at the heart of the logic programming paradigm [3]. Starting with the classic algorithm of Robinson [5], the unification algorithm was developed to become more and more efficient [4]. Even nowadays the unification theory is still under development and is receiving continuous scrutiny from the scientific community [2].

The present paper presents yet another efficient unification algorithm centered on a data structure called Unification Table, which borrows some ideas from the data structures used by the Warren’s Abstract Machine [1].

The next paragraph presents in detail the proposed unification algorithm, giving the C-style pseudo code. An example of application of the algorithm taken from [1] is also presented.

2 The Unification Algorithm

The unification algorithm below will compute the MGU of two logical terms $x$ and $y$ if the unification is possible, otherwise will report failure. As in all Prolog implementations of the unification, the "occur check” is omitted; however, with some modifications, the algorithm can implement a “unify with occur check” procedure.

The algorithm is centered on a data structure called the Unification Table (UT) which contains information related to each subterm (constant, variable, composite) that occurs in the terms to be unified. The properties of the unification table are crucial to the correctness and the efficiency of the unification and will be described in detail below.

The algorithm consists of two main steps:
Step 1. Parse terms $x$ and $y$ and build the Unification Table (UT)
Step 2. Call the unification function unify(index($x$), index($y$)) where index($x$) and index($y$) are the indexes of $x$ and $y$ in the UT.

In the following we will take a closer look to each step of the unification algorithm, providing detailed explanation wherever necessary.

2.1 Step 1

The idea of this step is to build a data structure, called Unification Table (UT), in which every variable appears only once, and all the subterms of $p$ and $q$ are included. The UT contains three types of entries: variables (type VAR, arity 0), constants (type STR, arity 0) and composite terms (type STR, arity greater than 0).
The structure of the UT for each type of entry is the following:

where:

Term - is the actual term; this column is never built; its purpose is only to simplify the explanations and the understanding of the algorithm. We can think of it as the result of a "write" procedure called upon the index of the term.

Index - is the index of the table entry for some term; starts with 0 and uniquely identifies the term.

Main functor – is the main functor of the term.

Type is VAR for variables and STR for constants and composite terms.

Arity – is the arity of the term; for variables and constants, it is 0.

For variables and constants, the list of components is the empty list.

For composite terms, the list of components is the sequence of the indexes of the component subterms; the order is important – it can be either left to right or vice versa but not both of them. The parsing procedure is left to right and ensures that when an entry is created for a composite term, all its subterms are already in the UT. The parsing starts with the second term (y).

Consider the following example from Ait-Kaci [1] requiring to find the MGU of:

\[ x = p(Z, h(Z, W), f(W)) \]
\[ y = p(f(X), h(Y, f(a)), Y) \]

The unification table will be:

| Term     | Index | Main functor | Type of term (VAR/STR) | Arity | List of components |
|----------|-------|--------------|-------------------------|-------|-------------------|
| Y        | 0     | Y            | VAR                     | 0     |                   |
| a        | 1     | a            | STR                     | 0     |                   |
| f(a)     | 2     | f            | STR                     | 1 1   |                   |
| h(Y, f(a)) | 3   | h            | STR                     | 2 0 2 |                   |
| X        | 4     | X            | VAR                     | 0     |                   |
| f(X)     | 5     | f            | STR                     | 1 4   |                   |
| p(f(X), h(Y, f(a)), Y) | 6 | p | STR | 3 5 3 0 | |
| W        | 7     | W            | VAR                     | 0     |                   |
| f(W)     | 8     | f            | STR                     | 1 7   |                   |
| Z        | 9     | Z            | VAR                     | 0     |                   |
| h(Z, W)  | 10    | h            | STR                     | 2 9 7 |                   |
| p(Z, h(Z, W), f(W)) | 11 | p | STR | 3 9 10 8 | |

The table is filled by parsing the terms \( x \) and \( y \) from right to left starting with \( y \) in a bottom up manner. Each variable has exactly one entry in the table but same constants or composite terms may have different entries. The list of components for a given term consists of the indexes of its components. The unification function will start with the indexes of \( x \) and \( y \), that is 6 and 11, so the call will be \textit{unify}(6, 11).

### 2.2 Step 2

The \textit{unify} function called in Step 2 starts with the indexes of \( x \) and \( y \) and uses two stacks \( Sx \) and \( Sy \). Initially the index of \( x \) and \( y \) are pushed on the stacks \( Sx \) and \( Sy \) respectively. Then a main loop will start that will continue until both stacks are empty. The algorithm ensures that both stacks will be empty simultaneously and that the stacks will eventually become empty if the unification table was built correctly and the terms follow the correct syntax for logic terms.
The function will return either SUCCESS or FAIL; in case of success each variable included in the MGU will be marked using a global data structure. The pseudo code for the *unify* function follows:

```plaintext
function unify(ix, iy)
{
    initialize stack Sx
    initialize stack Sy
    push ix on stack Sx
    push iy on stack Sy
    while (not_empty(Sx) and not_empty(Sy))
    { // start main loop
        pop i from Sx
        pop j from Sy
        // case 1: i is bound to a term and j is bound to a term
        if (type(i) == STR and type(j) == STR)
            if (main functors of i and j match (both name and arity))
                if (arity > 0)
                    push components of i on Sx in sequence
                    push components of j on Sy in sequence
                else
                    return(FAIL) // report failure
        else
            return(FAIL) // report failure
        // case 2: i is bound to a term and j is bound to a variable
        if (type(i) == STR and type(j) == VAR)
            if (j is a free variable)
                bind j to i and set mgu[j] = 1
            else // j is bound
                dereference j
                if (j is bound to a STR)
                    push i on Sx
                    push j on Sy
                else // j is bound to a free variable
                    bind j to i
        // case 3: i is bound to a variable and j is bound to a term
        if (type(i) == VAR and type(j) == STR)
            // perfectly symmetric to case 2
        // case 4: i is bound to a variable and j is bound to a variable
        if (type(i) == VAR and type(j) == VAR)
            if (i is free and j is free)
                bind i to j (or vice versa) and set mgu[i] = 1
            if (i is free and j is bound)
                bind i to j and set mgu[i] = 1
            if (i is bound and j is free)
                bind j to i and set mgu[j] = 1
            if (i is bound and j is bound)
                push the index of the term to which i is bound on Sx
                push the index of the term to which j is bound on Sy
    } // end main loop
    return(SUCCESS)
} // end function unify
```

In order to help the understanding of the algorithm, C++ style comments in italics are provided.

The MGU for the above example computed with this function is:

- \( W = f(a) \)
- \( X = f(a) \)
- \( Y = f(f(a)) \)
- \( Z = f(f(a)) \)

As one can see from the above function, the main loop extracts elements \( i \) and \( j \) from the two stacks and then appropriate action is taken according to the four possible cases.
In the first case, i and j are indices of two elements of type STR so the natural action to do here is to check if they have the same name and arity. If so, in case the arity is greater than zero we push all the components of i and j on the two stacks, otherwise we report failure.

Cases two and three are symmetric and consider the case when either i or j are variables but not both of them. In this case we have to discriminate between the cases when the variable is free (we just have bind it) or bound (in this case we have to dereference the variable and push the result).

Finally in case four we have to deal with two variables and again we have to consider four subcases according to the status of the variables: free or bound. As before, if at least one variable is free we just have to bind it, otherwise we must push the terms to which the variables are bound on the stacks and continue the loop.

If the "while" loop ends naturally, without the forced return in case of failure, then the function returns "success" as well as the MGU.

In the case of an "occur check" violation the algorithm will succeed, but an attempt to print the result will result in an infinite loop, or a memory overflow; this can be solved as in many Prolog systems by using a "guarded write" which will go only (say) ten levels in depth.

3 Conclusions

The quest for efficient unification algorithms is the foundation of increasing the efficiency of logic programs. Since the first unification algorithm of Robinson [5], continuing with the efficient algorithm of Martelli and Montanari [4], this quest never stopped.

The unification algorithm presented in this paper is yet another attempt to increase the efficiency of the unification. The algorithms also benefits of simplicity and clarity which makes it very easy to understand and implement.

An implementation in C is available as well as a Java applet. Further developments of the algorithm are: a recursive version of the algorithm and a version with "occur check".

4 References

[1] Ait-Kaci, H., Warren's Abstract Machine, A Tutorial Reconstruction, MIT Press, 1991.
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[5] Robinson, J.A., A Machine-oriented Logic Based on the Resolution Principle, JACM, 12, 1, Jan 1965, 23-41.