A covariant model for the nucleon spin structure *

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We present the results of the covariant spectator quark model applied to the nucleon structure function $f(x)$ measured in unpolarized deep inelastic scattering, and the structure functions $g_1(x)$ and $g_2(x)$ measured in deep inelastic scattering using polarized beams and targets ($x$ is the Bjorken scaling variable). The nucleon is modeled by a valence quark-diquark structure with $S$, $P$ and $D$ components. The shape of the wave functions and the relative strength of each component are fixed by making fits to the deep inelastic scattering data for the structure functions $f(x)$ and $g_1(x)$. The model is then used to make predictions on the function $g_2(x)$ for the proton and neutron.

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The covariant spectator quark model (CSQM) is a model in which the electromagnetic structure of the constituent quark is parametrized by Dirac ($f_{1q}$) and Pauli ($f_{2q}$) form factors for the quarks ($q = u, d$) [1, 2]. The quark electromagnetic form factors $f_{1q}$, $f_{2q}$ simulate the effects associated with the gluons and the quark-antiquark pairs. The CSQM was developed within the covariant spectator theory [3] and was first applied to the nucleon using a $S$-state approximation to the quark-diquark system [1]. The quark form factors and the radial wave functions are fitted to the nucleon electromagnetic form factor data. It was concluded that the falloff of the ratio between the magnetic and electric observed for the first time at Jefferson Lab can be explained by a model based on quarks with no orbital momentum, if the quarks have an internal structure [1]. The model was later extended to several nucleon resonances and other baryons [2, 3, 4, 5, 6, 7, 8].

The next step on this, it is to check if CSQM can be extended to the deep inelastic scattering (DIS) regime, and if a qualitative description of

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the DIS phenomenology can be achieved. In the deep inelastic scattering the photon transfer momentum squared, $Q^2$, and the photon energy in the lab frame, $\nu$, are both very large but the ratio $x = \frac{Q^2}{2M\nu}$ is kept finite ($M$ is the nucleon mass). If the CSQM is in fact compatible with DIS, the DIS data can be used to discriminate the individual contributions of the orbital angular momentum states in the nucleon wave function and also used to estimate the shape of those components.

The nucleon structure in DIS is parametrized in terms of the unpolarized structure functions $f_q(x) = q(x)$ and the polarized structure functions $g_1^q(x) = \Delta q(x)$ and $g_2^q(x)$. The unpolarized structure functions determine the quark contributions to the nucleon momentum, but explain only about 50% of the total amount ($\int dx (2xf_u + xf_d) \approx 0.5$). The remaining 50% are due to the gluons. The functions $\Delta q$ measures the contributions to the quark orbital momentum for the proton spin. It is known since the 80s, from the EMC experiments at CERN [9], that the contribution of the orbital momentum of the quarks to the proton spin is only about 30% [9, 10]. That conclusion was obtained from the result of the first moment of the function $g_1(x)$ for the proton \[ \Gamma_1^p = \int_0^1 dx g_1^{exp}(x) = 0.128 \pm 0.013. \] (1)

Theoretical calculations based on the naive assumption that the nucleon is made of quarks with no orbital angular momentum (pure relative $S$-state) give larger values. In our $S$-state model for the nucleon, $\Gamma_1^p = 0.278$ [13].

Since a nucleon wave function ($\Psi_N$) dominated by the $S$-state [11, 12] overestimates the quark contributions to the proton spin, we now consider a wave function that include also $P$ and $D$-states [13]

$$ \Psi_N = n_S \Psi_S + n_P \Psi_P + n_D \Psi_D, $$ (2)

where $n_S, n_P$ and $n_D$ are the coefficients of the states ($n_S^2 + n_P^2 + n_D^2 = 1$). All the components of the wave function are represented in terms of an off-shell quark and two on-shell quarks (quark pair). We can integrate in the internal degrees of freedom of the quark pair and represent the wave function in terms of the a quark and a diquark structure dependent on the nucleon ($P$) and the diquark ($k$) momenta [13]. Since in the DIS limit the quarks are pointlike the adjustable part of the model is restricted to the radial wave functions of the states $S, P$ and $D$. To increase the flexibility of the model we also consider different distributions (radial wave functions $\psi^S_k$) for the quarks $u$ and $d$. This asymmetry is supported by the data [12, 13].

From the calculation of the hadronic tensor, in which we integrate on the quark and diquark on-shell momenta, we derive the expressions for the
DIS structure functions. In particular the expression for the unpolarized structure function associated with the $S$-state can be written as

$$f^S_q(x) = \frac{M^2}{16\pi^2} \int_\xi^{+\infty} d\chi |\psi^S_q(\chi)|^2, \quad \frac{df^S_q}{dx} = -\frac{x(2-x)}{(1-x)^2} \frac{M^2}{16\pi^2} |\psi^S_q(\chi)|^2,$$

where $\xi = \frac{x^2}{1-x}$ is a function of the Bjorken variable $x$, and $\chi$ is a covariant variable of the nucleon and diquark momenta. Similar expressions can be written for the $P$ and $D$ components.

Equations (3) can be used to conclude that the radial wave functions $(L = S, P, D)$ can be represented in the form

$$\psi^L_q(\chi) \propto \alpha + \frac{\beta}{(\beta + \chi)^{n_1 - n_0}},$$

where $\alpha$ is a constant, $\beta$ is a dimensionless parameter and $n_0, n_1$ are indices that can be related to the values $a_q, b_q$ from the parametrizations $xf_q(x) \propto x^{a_q}(1-x)^{b_q}$.

To confirm if the CSQM is consistent with the DIS regime, we try to adjust the parameters of our model to the DIS phenomenology. Since the experimental data is in some cases obtained for very small $Q^2$ (while in the DIS limit $Q^2$ is very large) we choose to fit our model to the well known parametrizations of the data: Martin, Roberts, Stirling and Thorn (2002) – MRST(02) (unpolarized structure functions) [14] and Leader, Siderov and Stamenev (2010)– LSS(10) (polarized structure functions) [15]. We consider the parametrizations for the scale $Q^2 = 1$ GeV$^2$. We divide the fitting process into 3 steps:

- First we estimate the parameters of the radial wave functions $\psi^L_q$ by a fit to the unpolarized data, $f_u$ and $f_d$, assuming that all components $S, P, D$ have the same shape [see Eq. (3)],

- Based on the first estimate of the radial wave functions we calculate the mixture coefficients $n_P$ and $n_D$ by making a fit to the first moment of the function $g^q_1$: $\Gamma^u_1 = 0.333 \pm 0.039$, and $\Gamma^d_1 = -0.335 \pm 0.080$ [12, 14].

- Finally the parameters of the radial wave functions: $\alpha, \beta$ are adjusted independently to the polarized data for $\Delta u$ and $\Delta d$.

The results of the fit for the functions $q$ and $\Delta q$ are presented in the Fig. 1 and are compared with the parametrizations MRST(02) and LSS(10). Once all the parameters are fixed by the $q$ and $\Delta q$ data, we use the model to predict the function $g_2(x)$ for the proton and the neutron. The results are presented in Fig. 2 by the solid line.
Fig. 1. Results for the unpolarized $q(x)$ and polarized $\Delta q(x)$ structure functions (Total) compared with the parametrizations MRST(02) and LSS(10) [14, 15]. The $P$- and $D$- state mixtures are respectively 1% and 35% [12].

From the previous study we conclude that CSQM be used in the nucleon DIS regime, in addition to the electromagnetic excitations of the baryons. The results presented here are derived under the assumption that the valence quarks are the relevant degrees of freedom in DIS and that the gluon and meson cloud (sea quarks) effects can be neglected in a first approximation.

In our study the nucleon has contributions of several angular momentum states ($L = S, P, D$) and the DIS data are used to probe the shape of the components of the nucleon wave function.

The results of our best model are consistent with the experimental data obtained for the unpolarized $f_q(x)$ and polarized $g^1(x)$ structure functions, which are also compatible with a zero contribution of the gluons for the proton spin ($J_g = 0$).

Finally we present predictions for the spin dependent structure function $g_2(x)$ of the nucleon. The predictions are consistent with the available data (see Fig. 2) and can be tested in future by more accurate data.

Since the gluon degrees of freedom are not included explicitly, although some effects are effectively considered in the structure of the radial wave functions, we cannot make direct predictions for very large $Q^2$. We can however use the QCD evolution equations (DGLAP) [16] to extrapolate the results to very large $Q^2$, dominated by the gluon effects, using the results of our model for the valence quark structure at $Q^2 = 1 \text{ GeV}^2$.

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Fig. 2. Predictions of the function $g_2(x)$ for the proton and neutron (solid line) [12].

REFERENCES

[1] F. Gross, G. Ramalho and M. T. Peña, Phys. Rev. C 77, 015202 (2008).
[2] G. Ramalho, K. Tsushima and F. Gross, Phys. Rev. D 80, 033004 (2009).
[3] F. Gross, Phys. Rev. 186, 1448 (1969).
[4] I. G. Azauryan, A. Bashir, V. Braun, S. J. Brodsky, V. D. Burkert, L. Chang, C. Chen and B. El-Bennich et al., Int. J. Mod. Phys. E 22, 1330015 (2013).
[5] G. Ramalho, Phys. Rev. D 90, 033010 (2014).
[6] G. Ramalho, M. T. Peña and F. Gross, Phys. Rev. D 78, 114017 (2008); Phys. Rev. D 78, 114017 (2008).
[7] G. Ramalho and K. Tsushima, Phys. Rev. D 81, 074020 (2010); Phys. Rev. D 84, 051301 (2011).
[8] G. Ramalho and K. Tsushima, Phys. Rev. D 84, 054014 (2011).
[9] J. Ashman et al. [European Muon Collaboration], Phys. Lett. B206, 364 (1988).
[10] A. Airapetian et al. [HERMES Collaboration], Phys. Rev. D 75, 012007 (2007).
[11] S. E. Kuhn, J.-P. Chen and E. Leader, Prog. Part. Nucl. Phys. 63, 1 (2009).
[12] F. Gross, G. Ramalho and M. T. Peña, Phys. Rev. D 85, 093006 (2012).
[13] F. Gross, G. Ramalho and M. T. Peña, Phys. Rev. D 85, 093005 (2012).
[14] A. D. Martin, R. G. Roberts, W. J. Stirling and R. S. Thorne, Phys. Lett. B 531, 216 (2002).
[15] E. Leader, A. V. Sidorov and D. B. Stamenov, Phys. Rev. D 82, 114018 (2010).
[16] G. Altarelli and G. Parisi, Nucl. Phys. B 126, 298 (1977); Y. L. Dokshitzer, Sov. Phys. JETP 46, 641 (1977); V. N. Gribov and L. N. Lipatov, Sov. J. Nucl. Phys. 15, 438 (1972).