Flavor changing neutral current processes and the muon anomalous magnetic moment in the supergravity model

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Abstract

Flavor changing neutral current processes in $B$ and $K$ decays in the supergravity model are revisited taking into account the recent progress of Higgs boson search experiments at LEP. Possible deviations of $B^0 - \bar{B}^0$ mixing, the CP violating $K^0 - \bar{K}^0$ mixing parameter $\epsilon_K$ from the standard model predictions are reduced significantly in a small tan $\beta$ region due to the constraints on the SUSY parameter space imposed by the Higgs boson search. With the present bound on the Higgs boson mass, the magnitude of the $B^0 - \bar{B}^0$ mixing amplitude can be enhanced up to 12% for tan $\beta = 3$ in the minimal supergravity. If we relax the strict universality of SUSY breaking scalar masses at the GUT scale, the deviation can be 25%. We also investigate SUSY contributions to the muon anomalous magnetic moment in the supergravity model and show that there is a correlation with the $b \to s \gamma$ amplitude. The SUSY contribution to $a_\mu = \frac{(g-2)_\mu}{2}$ can be ($-30$ $-$ $+80) \times 10^{-10}$ for tan $\beta = 10$ and even larger for larger tan $\beta$. The improved measurement of the muon anomalous magnetic moment in the on-going experiment at BNL will put a strong constraint on SUSY parameter space, especially for a large tan $\beta$ region.

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I Introduction

In order to look for physics beyond the standard model (SM), low energy experiments can play an important role. Various flavor changing neutral current (FCNC) processes in $B$ and $K$ meson decays and the muon anomalous magnetic moment could receive large contributions from loop diagrams of new particles. On-going experiments at KEK and SLAC B-factories and the muon storage ring at BNL as well as future plans of $B$ and $K$ meson decay experiments therefore may be able to reveal physics beyond the energy scale which is currently accessible from direct particle search experiments.

There have been extensive studies on FCNC effects in the supergravity model. In a supersymmetry (SUSY) model, the SUSY particles can give large contributions to various FCNC processes such as $B(b \to s \gamma)$, $B^0-\bar{B}^0$ and $K^0-\bar{K}^0$ mixings, $B(K \to \pi \nu \bar{\nu})$, and $B(K \to \pi \nu \bar{\nu})$. These contributions depend on the flavor mixing in the squark mass matrix which is a priori independent of the flavor mixing in the quark sector. In fact, FCNC processes can put strong constraints on these mixing parameters because arbitrary mixing parameters tend to give too large effects on, for instance, the $K^0-\bar{K}^0$ mixing amplitude if the SUSY particles exist below 1 TeV. In the minimal supergravity model, these dangerous FCNCs are eliminated by the assumption that all squarks have universal SUSY breaking mass terms through the flavor-blind coupling of gravity interaction. In the previous publication we have investigated the $B^0-\bar{B}^0$ mixing, the CP violating parameter of $K^0-\bar{K}^0$ mixing, $\epsilon_K$, $B(b \to s l^+ l^-)$, $B(K_L \to \pi^0 \nu \bar{\nu})$ and $B(K^+ \to \pi^+ \nu \bar{\nu})$ in the model based on supergravity taking $B(b \to s \gamma)$ as one of phenomenological constraints. SUSY effects can increase the $B^0-\bar{B}^0$ mixing amplitude and $\epsilon_K$ by up to 20% in the minimal supergravity model. If we relax the strict universality of all scalar SUSY breaking masses at the GUT scale so that we take the soft SUSY breaking scalar masses for the Higgs fields different from that for squarks and sleptons, the maximal deviation becomes about 40%. The branching ratios for $K_L \to \pi^0 \nu \bar{\nu}$ and $K^+ \to \pi^+ \nu \bar{\nu}$ can be suppressed by up to 10% compared to the SM predictions for the relaxed initial conditions while the deviation becomes only a few % in the minimal supergravity case.
In this paper, we perform quantitative study of FCNC processes in the supergravity model taking account of updated constraints on SUSY parameter space. Among recent improvements on SUSY searches, the SUSY Higgs boson search in the LEP II experiment turns out to give the most significant effect on constraining the SUSY parameter space. We show that possible deviation of the $B^0 - \overline{B^0}$ mixing amplitude from the SM prediction becomes significantly reduced by the new constraint imposed by the Higgs boson search, especially for a low $\tan \beta$ region. Taking into account the recent result of the Higgs boson search, the maximal deviation of the $B^0 - \overline{B^0}$ mixing from the SM prediction is 12% in the minimal case and 25% in the nonminimal case for $\tan \beta = 3$. If LEP II does not find the lightest CP-even Higgs boson below 105 GeV, the deviation becomes 11% and 16%, respectively.

We also consider the muon anomalous magnetic moment in the supergravity model. There have been many works on the muon anomalous magnetic moment in the MSSM [8, 9] and in the supergravity model [10, 11]. It is known that this process gives a sizable deviation from the SM for large $\tan \beta$. At present, a possible new physics contribution to $a_\mu$ is estimated as $(75.5 \pm 73.3) \times 10^{-10}$ [12]. For the present Higgs mass bound, we find that the SUSY contribution to $a_\mu$ can be $(-30 - +80) \times 10^{-10}$ for $\tan \beta = 10$ and therefore the muon anomalous magnetic moment becomes a useful constraint on the SUSY parameters when the on-going experiment improves the sensitivity by a factor of 10–20.

The rest of this paper is organized as follows. In Sec. II, the supergravity model is introduced and the phenomenological constraints including the Higgs mass bound are described. In Sec. III, the numerical results on the FCNC processes are presented. The numerical results on the muon anomalous magnetic moment are shown in Sec. IV. Sec. V is devoted for the conclusion.

II Supergravity model

In this section we briefly describe the minimal supersymmetric standard model (MSSM) Lagrangian based on the supergravity model. Details are given in Refs. [5, 7, 13].

The field contents of MSSM consist of SU(3), SU(2) and U(1) gauge supermul-
triplets \((G, \tilde{G}), (W, \tilde{W})\) and \((B, \tilde{B})\), respectively, the Higgs supermultiplets \((h_1, \tilde{h}_1)\) and \((h_2, \tilde{h}_2)\), and chiral supermultiplets corresponding to quarks/squarks and leptons/sleptons. The \(SU(3) \times SU(2) \times U(1)\) quantum numbers are given by

\[
Q_i = (3, 2, \frac{1}{6}) , \quad U_i = (\overline{3}, 1, -\frac{2}{3}) , \quad D_i = (\overline{3}, 1, -\frac{2}{3}) ,
\]

\[
L_i = (1, 2, -\frac{1}{2}) , \quad E_i = (1, 1, 1) , \quad (2.1)
\]

where \(i = 1, 2, 3\) is the generation index. Requiring the \(R\)-parity conservation, the MSSM superpotential is given by

\[
W_{\text{MSSM}} = f^{ij}_{D} Q_i D_j H_1 + f^{ij}_{U} Q_i U_j H_2 + f^{ij}_{L} L_i E_j H_1 + \mu H_1 H_2 . \quad (2.2)
\]

The general form of the soft SUSY breaking terms are given by

\[
- \mathcal{L}_{\text{soft}} = (m_{Q}^2)^i_j \tilde{q}_i \tilde{q}_j + (m_{U}^2)^i_j \tilde{d}_i \tilde{d}_j + (m_{U}^2)^i_j \tilde{u}_i \tilde{u}_j
\]

\[
+ (m_{E}^2)^i_j \tilde{e}_i \tilde{e}_j + (m_{L}^2)^i_j \tilde{l}_i \tilde{l}_j
\]

\[
+ \Delta h_1^i h_1 + \Delta h_2^i h_2 - (B \mu h_1 h_2 + \text{h.c.})
\]

\[
+ (A_D^{ij} \tilde{q}_i \tilde{d}_j h_1 + A_U^{ij} \tilde{u}_i \tilde{u}_j h_2 + A_L^{ij} \tilde{e}_i \tilde{l}_j h_1 + \text{h.c.})
\]

\[
+ \left( \frac{M_1}{2} \tilde{B} \tilde{B} + \frac{M_2}{2} \tilde{W} \tilde{W} + \frac{M_3}{2} \tilde{G} \tilde{G} + \text{h.c.} \right) , \quad (2.3)
\]

where \(\tilde{q}_i, \tilde{u}_i, \tilde{d}_i, \tilde{l}_i, \tilde{e}_i, h_1\) and \(h_2\) are scalar components of chiral superfields \(Q_i, U_i, D_i, L_i, E_i, H_1\) and \(H_2\), respectively, and \(\tilde{B}, \tilde{W}\) and \(\tilde{G}\) are \(U(1)\), \(SU(2)\) and \(SU(3)\) gauge fermions, respectively.

In the supergravity model, we assume that the soft SUSY breaking terms have a simple structure at the Planck or GUT scale. Here we assume the following initial conditions for renormalization group equations (RGEs) for the soft SUSY breaking terms at the GUT scale:

\[
(m_{Q}^2)^i_j = (m_{U}^2)^i_j = m_0^2 \delta_i^j ,
\]

\[
(m_{D}^2)^i_j = (m_{U}^2)^i_j = (m_{L}^2)^i_j = m_0^2 \delta_i^j , \quad (2.4a)
\]
\[ \Delta_1^2 = \Delta_2^2 = \Delta_3^2, \] (2.4b)

\[ A_D^{ij} = f_D^{ij} A X m_0, \quad A_L^{ij} = f_L^{ij} A X m_0, \quad A_U^{ij} = f_U^{ij} A X m_0, \] (2.4c)

\[ M_1 = M_2 = M_3 = M_{gX}. \] (2.4d)

In the minimal supergravity model the soft breaking parameters \( m_0 \) and \( \Delta_0 \) are assumed to be equal whereas in the nonminimal case we treat the two as independent parameters. We assume that the soft SUSY breaking terms \( m_0, \Delta_0, M_{gX}, A_X \) and the \( \mu \) parameter are all real so that we do not consider the constraint from electron and neutron electric dipole moments (EDMs). With the above initial conditions we can solve the one-loop RGEs for the SUSY breaking parameters and determine these parameters at the electroweak scale. We also require that the electroweak symmetry breaking occurs properly to give the correct \( Z^0 \) boson mass. As independent parameters, we take \( \tan \beta, m_0, A_X, M_{gX} \) and \( \text{sign}(\mu) \) (and \( \Delta_0 \) for the nonminimal case). Once we fix a set of these parameters we can calculate the masses and mixings of all SUSY particles. We can then evaluate various amplitudes for FCNC processes such as \( B(b \rightarrow s\gamma) \), the \( B^0-\overline{B^0} \) mixing amplitude, \( \epsilon_K \), \( B(K_L \rightarrow \pi^0 \nu\overline{\nu}) \) and \( B(K^+ \rightarrow \pi^+ \nu\overline{\nu}) \).

In order to determine the allowed region of the SUSY parameter space, we require the following phenomenological constraints:

1. \( b \rightarrow s\gamma \) constraint from CLEO, \( i.e., 2.0 \times 10^{-4} < B(b \rightarrow s\gamma) < 4.5 \times 10^{-4} \) [14].
2. The chargino mass is larger than 97 GeV, and all other charged SUSY particle masses should be larger than 90 GeV [15, 16].
3. All sneutrino masses are larger than 41 GeV [17].
4. The gluino and squark mass bounds from TEVATRON experiments [18].
5. The lightest SUSY particle is neutral.
6. The condition for not having a charge or color symmetry breaking minimum [19].
SUSY Higgs boson search at LEP II [20].

The detail of these conditions (1)–(6) is discussed in Refs. [5, 7]. The difference between Refs. [5, 7] and the present analysis is that the experimental bounds in (1) and (2) are updated and the complete next-to-leading order QCD correction formula is used for the calculation of the SUSY contributions to the $b \to s\gamma$ amplitude [21]. Previously in Refs. [3, 4] the condition (7) did not give any impact to constrain the SUSY parameter space. Due to the recent improvement of the Higgs boson search at LEP II [20], a sizable parameter space starts to be excluded. This will be discussed in the next subsection.

II.1 Constraint from the SUSY Higgs boson search

It is well known that there is a strict upper bound on the lightest CP-even Higgs boson mass in the MSSM [22]. Because the Higgs search in the LEP II experiment has reached to the sensitivity of 100 GeV for the SM Higgs boson and about 90 GeV for the case of the lightest CP-even Higgs boson in the MSSM, a meaningful amount of the SUSY parameter space is already excluded [20].

The Higgs sector in the MSSM consists of five physical mass eigenstates: two CP-even Higgs bosons $h$ and $H$, one CP-odd Higgs boson $A$ and a pair of charged Higgs boson $H^\pm$. There are two angles to specify the mixing in the Higgs sector, namely, a ratio of two vacuum expectation values $\tan \beta = \langle h_0^2 \rangle / \langle h_0^1 \rangle$ and the mixing angle $\alpha$ for the two CP-even Higgs bosons defined as

\[
\sqrt{2} h_1^0 - v \cos \beta = -h \sin \alpha + H \cos \alpha ,
\]

\[
\sqrt{2} h_2^0 - v \sin \beta = h \cos \alpha + H \sin \alpha ,
\]

where we have only kept the CP-even components of two Higgs fields. In the MSSM, these masses and mixings are determined by two input parameters, for instance $m_A$ and $m_h$, as well as stop and sbottom masses and mixing parameters through one-loop corrections to the Higgs potential formula. In the supergravity model these masses and mixings at the electroweak scale are calculated by solving the RGEs from the initial conditions at the GUT scale.
The constraints on the SUSY parameter space from the LEP experiments are expressed as an allowed region of two dimensional parameters, for instance $(m_h, m_A)$, with different assumptions of the remaining parameters (stop mass, etc.). In order to simplify the treatment in the numerical analysis we use the following procedure. We first take the experimental constraint in the space of $m_A$ and $m_h$ from Ref. [16, 20]. We require that the set of $m_A$ and $m_h$ calculated in the supergravity model is within the allowed region of this parameter space. Strictly speaking, the allowed region slightly depends on the stop mass and mixing parameters through the angles $\alpha$ and $\beta$, but this dependence is not significant. In addition, except for a small parameter region in the case of a large $\tan \beta$, the CP-odd scalar mass becomes much larger than $m_Z$ so that the property of the lightest CP-even Higgs boson becomes very close to that of the SM Higgs boson. In such a case the SUSY Higgs boson search essentially gives the same lower bound of the lightest CP-even Higgs boson mass as that of the SM Higgs boson, and therefore almost independent of other SUSY parameters.

In the numerical calculation we use the analytic formula for the CP-even Higgs mass matrix given in Ref. [23]. This formula takes into account two-loop leading-log corrections and reproduces the next-to-leading-log results within a few GeV for $m_\tilde{t} \lesssim 1.5$ TeV. As the present experimental constraint we require that the set of $m_A$ and $m_h$ is within the allowed region shown in Fig. 1 (case (I)). Later we also show how various results depend on a possible future improvement of the LEP II Higgs boson search. We then simply require $m_h \geq 105$ GeV to get a rough idea for the case that the LEP II experiment will not discover any light Higgs boson (case (II)).

### III Numerical results on FCNC processes

In this section, the results of numerical calculations for various FCNC processes in $B$ and $K$ decays are presented. We consider here

1. the $B^0 - \bar{B}^0$ mixing amplitude;

2. $\epsilon_K$;
(3) \( B(K_L \rightarrow \pi^0 \nu \nu) \);

(4) \( B(K^+ \rightarrow \pi^+ \nu \nu) \).

Detail descriptions on the calculation of these quantities in the supergravity model are given in Ref. [5, 7]. Besides the improvement discussed in Sec. II, we have improved the calculation of the \( B_0 \rightarrow B_0 \) and \( K_0 \rightarrow K_0 \) mixings by including the next-to-leading-order QCD correction to the contribution due to the charged-Higgs–top loop diagram [24].

We first present the magnitude of \( \Delta M_{B_d} \) in the supergravity model. In Fig. 2 we show \( \Delta M_{B_d}/\Delta M_{B_d}^{SM} \) as a function of the lightest Higgs boson mass for three values of \( \tan \beta \), i.e. \( \tan \beta = 2, 3, 10 \), where \( \Delta M_{B_d}^{SM} \) denotes the SM value. As input parameters we take \( m_t^{\text{pole}} = 175 \) GeV, \( m_b^{\text{pole}} = 4.8 \) GeV, \( \alpha_s(m_Z) = 0.119 \). For the CKM matrix elements, we adopt the ‘standard’ phase convention of the Particle Data Group [17], taking \( V_{us} = 0.2196, V_{cb} = 0.0395, |V_{ub}/V_{cb}| = 0.08 \). We fix the CP violating phase in the CKM matrix \( \delta_{13} = \pi/2 \) in the following analysis. As we will see later, the ratio \( \Delta M_{B_d}/\Delta M_{B_d}^{SM} \) is almost independent of \( \delta_{13} \). The SUSY breaking parameters at the GUT scale are scanned within the following range: \( 0 < m_0 < 1 \) TeV, \( 0 < \Delta_0 < 1 \) TeV, \( 0 < M_{gX} < 1 \) TeV, \( 0 < |A_X| < 5 \). This figure shows the possible values of \( \Delta M_{B_d}/\Delta M_{B_d}^{SM} \) and \( m_h \) for parameter sets of the minimal supergravity model as well as for those of the nonminimal case discussed in Sec. II. Compared to the result in [7], we see that the possible deviation from the SM is reduced to +15% from +40% for the nonminimal case if we require that the lightest CP-even Higgs mass should be larger than 95 GeV for \( \tan \beta = 2 \). If the mass bound is raised to 105 GeV no allowed parameter space remains for \( \tan \beta = 2 \). For \( \tan \beta = 3 \), the lightest Higgs boson mass is shifted to a larger value so that there is some allowed space for \( m_h > 105 \) GeV. For \( \tan \beta = 10 \), a separate allowed region appear in the region of \( m_h = 115–125 \) GeV where \( \Delta M_{B_d}/\Delta M_{B_d}^{SM} \) is 1.15–1.30.

It is instructive to see the correlation between \( \Delta M_{B_d}/\Delta M_{B_d}^{SM} \) and \( B(b \rightarrow s \gamma) \). In Fig. 3 we show \( \Delta M_{B_d}/\Delta M_{B_d}^{SM} \) as a function of \( B(b \rightarrow s \gamma) \) for \( \tan \beta = 2, 3, 10 \). In these figures, the constraint from the present SUSY Higgs boson search is applied as described in Sec. II. For \( \tan \beta = 2 \), most of the parameter region is already excluded by the Higgs boson search. The deviations of both \( B(b \rightarrow s \gamma) \) and \( \Delta M_{B_d} \)
from the SM predictions are small in the allowed parameter region. Compared with the Fig.8(c) in Ref. [7], we can see that the more parameter space is excluded in the case that $B(b \to s \gamma)$ is larger than the SM prediction. This is because both $B(b \to s \gamma)$ and the mass of the lightest CP-even Higgs boson have a correlation with sign($\mu$). Namely, $B(b \to s \gamma)$ is enhanced for $\mu < 0$. On the other hand, the lightest Higgs boson becomes lighter for $\mu < 0$ since it depends on the sign($\mu$) through the stop left-right mixing parameter in the radiative correction part of the Higgs mass formula. For $\tan \beta = 10$, there are two branches within the allowed region of $B(b \to s \gamma)$ for the nonminimal case. The upper dots correspond to the allowed region where $m_h = 115$–125 GeV in Fig. 2(c). In these points the sign of the $b \to s \gamma$ amplitude becomes opposite to that of the $b \to s \gamma$ amplitude in the SM because the SUSY contributions are very large and opposite in sign. It is known that this parameter space corresponds to the case where the $b \to s l^+ l^−$ branching ratio is enhanced by 50–100% [13].

In Fig. 3(a) the allowed ranges of $\Delta M_{B_d}/\Delta M_{B_d}^{\text{SM}}$ are shown for several values of $\tan \beta$ with the present constraint of the SUSY Higgs boson search and Fig. 3(b) corresponds to the case where $m_h > 105$ GeV. For $\tan \beta \gtrsim 10$ there appears a separate region where the sign of the $b \to s \gamma$ amplitude is opposite to that of the SM amplitude. Two cases are distinguished according to the sign of the $b \to s \gamma$ amplitude for $\tan \beta = 10, 20, 40$. We can see that the present constraint allows the deviation of the SM for the $B^0 \to \bar{B}^0$ mixing up to 12% for the minimal case and 30% for the nonminimal case. If the Higgs boson bound is raised to 105 GeV, the allowed deviation from the SM becomes less than 15% except for the small parameter space where the sign of the $b \to s \gamma$ amplitude becomes opposite to the SM case.

Next we show the branching ratio of $K_L \to \pi^0 \nu \bar{\nu}$ normalized to the SM prediction. We have calculated $B(K_L \to \pi^0 \nu \bar{\nu})/B(K_L \to \pi^0 \nu \bar{\nu})_{\text{SM}}$ in the minimal and nonminimal cases. The allowed range of this quantity is shown for the present Higgs bound (Fig. 3(a)) and for $m_h > 105$ GeV (Fig. 3(b)). We can see that $B(K_L \to \pi^0 \nu \bar{\nu})$ is suppressed by up to 5% and 15% in the minimal and nonminimal cases, respectively. When we restrict $m_h > 105$ GeV, the maximal deviation is reduced to 7% for the nonminimal case.
We study various correlations among $\Delta M_{B_d}$, $\epsilon_K$, $B(K_L \to \pi^0 \nu \bar{\nu})$ and $B(K^+ \to \pi^+ \nu \bar{\nu})$. As noted in Ref. [7], $\Delta M_{B_d}/\Delta M_{B_d}^{SM}$ and $\epsilon_K/\epsilon_K^{SM}$ have a linear relation. When these quantities are normalized by the corresponding quantities in the SM, these relations are essentially independent of $\delta_{13}$ in the CKM matrix because the factors of CKM matrix elements are approximately equal for $\Delta M_{B_d}$ and $\Delta M_{B_d}^{SM}$. For $\epsilon_K$ and $\epsilon_K^{SM}$, there is a charm loop contribution which breaks the proportionality, but this contribution turns out to be small. In Fig. 6 the correlation between $\Delta M_{B_d}/\Delta M_{B_d}^{SM}$ and $\epsilon_K/\epsilon_K^{SM}$ is shown for $\tan \beta = 3$ and the nonminimal case. The corresponding figure for the minimal case is obtained by taking a part of these lines according to the maximal value of $\Delta M_{B_d}/\Delta M_{B_d}^{SM}$.

The correlation between $B(K_L \to \pi^0 \nu \bar{\nu})/B(K_L \to \pi^0 \nu \bar{\nu})_{SM}$ and $B(K^+ \to \pi^+ \nu \bar{\nu})/B(K^+ \to \pi^+ \nu \bar{\nu})_{SM}$ is shown in Fig. 4 for $\tan \beta = 3$ and the nonminimal case. We can see that the situation is quite similar to Fig. 6. The slight dependence on $\delta_{13}$ is the result of the charm loop contribution to the $B(K^+ \to \pi^+ \nu \bar{\nu})_{SM}$.

We also show the correlation between $\Delta M_{B_d}/\Delta M_{B_d}^{SM}$ and $B(K_L \to \pi^0 \nu \bar{\nu})/B(K_L \to \pi^0 \nu \bar{\nu})_{SM}$ in Fig. 8 for $\tan \beta = 3$ and 10 with the present Higgs search limit. We can see that in the parameter space where $\Delta M_{B_d}$ is the most enhanced the suppression of $B(K_L \to \pi^0 \nu \bar{\nu})$ become maximal. The correlations are useful to distinguish possible new physics effects from consistency check of the unitarity triangle. For instance, suppose that the time-dependent CP asymmetry in the $B \to J/\psi K_S$ decay mode is well established. Then by combining one more observable quantity, the parameter set $(\rho, \eta)$ of the CKM matrix in the Wolfenstein parametrization is determined within the SM. If the SUSY effects are relevant, $|V_{td}|$ determined from $\Delta M_{B_d}$ or $\epsilon_K$ can be different from that determined from $B(K_L \to \pi^0 \nu \bar{\nu})$ or $B(K^+ \to \pi^+ \nu \bar{\nu})$ because the formers are enhanced and the latters are suppressed in the supergravity model. On the other hand, observables such as $|V_{ub}/V_{cb}|$, $\Delta M_{B_d}/\Delta M_{B_s}$, and CP asymmetries in $B$ decays are essentially independent of the SUSY loop contributions.
IV Muon anomalous magnetic moment

In this section we present the numerical result of SUSY loop effects on the muon anomalous magnetic moment in the supergravity model. The present experimental value of $a_\mu = (g - 2)\mu/2$ is \cite{17,25}

$$a_\mu^{\text{exp}} = 11659235.0(73.0) \times 10^{-10}.$$ (4.1)

According to Ref. \cite{12} the SM prediction is

$$a_\mu^{\text{SM}} = 11659109.6(6.7) \times 10^{-10},$$ (4.2)

where the error in the SM value is dominated by the hadronic contribution of the vacuum polarization diagram \cite{26,27}. Combining the above two values we can derive a possible new physics contribution to $a_\mu$ as

$$a_\mu^{\exp} - a_\mu^{\text{SM}} = (75.5 \pm 73.3) \times 10^{-10}.$$ (4.3)

The current BNL experiment is aiming to improve $a_\mu$ by a factor of 20 and the first result is reported as $a_\mu = 1165925(15) \times 10^{-9}$ \cite{25}. In addition the hadronic contribution to the vacuum polarization may be better understood if the $e^+e^-$ total cross section is well measured experimentally \cite{27}. As we see later, although the present constraint from the muon anomalous magnetic moment is not strong enough the situation will soon change after the improvement of the measurement.

We calculated the SUSY contribution to $a_\mu$ ($a_\mu^{\text{SUSY}}$) from the loop diagrams with sneutrino and chargino and with charged slepton and neutralino. Detailed formula are found, for example, in \cite{9}. We require the radiative electroweak symmetry breaking condition and the various phenomenological constraints discussed in Sec. \cite{1}.

We show $a_\mu^{\text{SUSY}}$ in the minimal supergravity for $\tan \beta = 3$ as a function of the lighter chargino ($\chi^-_1$) mass, the left-handed smuon ($\tilde{\mu}_L$) mass and $B(b \to s \gamma)$ in Fig. \cite{9}. Also the same plots for $\tan \beta = 10$ are shown in Fig. \cite{10}. In these figures the present bound from the Higgs boson search is applied. We can see that $a_\mu^{\text{SUSY}}$ can be large only when both $\chi^-_1$ and $\tilde{\mu}_L$ are relatively light. For example, $\chi^-_1$ and
\( \bar{\mu}_L \) are lighter than 150 GeV and 200 GeV, respectively, in order for \( a_{\mu}^{\text{SUSY}} > 50 \times 10^{-10} \) for \( \tan \beta = 10 \). As is well known, the magnitude of \( a_{\mu}^{\text{SUSY}} \) becomes large for large \( \tan \beta \). The enhancement of \( a_{\mu}^{\text{SUSY}} \) for large \( \tan \beta \) comes from the fact that \( a_{\mu}^{\text{SUSY}} \) is dominated with the sneutrino-chargino loop diagram, which contains a contribution proportional to \( \mu \tan \beta \). As a result, SUSY contributions to both \( b \to s \gamma \) amplitude and \( a_{\mu}^{\text{SUSY}} \) are correlated with \( \text{sign}(\mu) \). We can see that \( a_{\mu}^{\text{SUSY}} \) becomes positive (negative) according to the suppression (enhancement) of \( B(b \to s \gamma) \). This correlation was pointed out in Ref. [11].

The predicted range of \( a_{\mu}^{\text{SUSY}} \) are shown for several values of \( \tan \beta \) with the present constraint on the Higgs search and with \( m_h > 105 \text{ GeV} \) for the minimal and the nonminimal cases in Fig. [11]. As in Fig. [4] two cases according to the sign of the \( b \to s \gamma \) amplitude are shown separately for \( \tan \beta = 10, 20, 40 \). This figure shows that the muon anomalous magnetic moment is indeed expected to become very powerful to constrain the SUSY parameter space in near future. We see that even for \( \tan \beta = 5 \) the deviation is quite sizable considering future improvements on the \( a_{\mu} \) measurement.

**V Conclusion**

We updated the numerical analysis of FCNC processes in \( B \) and \( K \) decays and the muon anomalous magnetic moment in the supergravity model. Taking account of the recent progress in the Higgs boson search, we show that a small \( \tan \beta \) region is almost excluded for \( \tan \beta \lesssim 2 \). The maximal deviation from the SM value in the \( B^0 - \overline{B^0} \) mixing is 12% for the minimal supergravity case and 30% for the nonminimal case. If the Higgs mass bound is raised to 105 GeV the deviations become less than 11% and 16%, respectively, except for a small parameter space where the \( b \to s \gamma \) decay amplitude is opposite in sign to the SM amplitude. For \( B(K \to \pi \nu \overline{\nu}) \), we show that the deviation is less than 5% for the minimal case and less than 14% for the nonminimal case under the present Higgs mass bound.

We also calculate the SUSY contribution to the muon anomalous magnetic moment. \( a_{\mu}^{\text{SUSY}} \) and \( B(b \to s \gamma) \) show a strong correlation and \( a_{\mu}^{\text{SUSY}} \) becomes very large for a large \( \tan \beta \) region. We find that the SUSY contribution \( a_{\mu}^{\text{SUSY}} \) can be
\((-30 - +80) \times 10^{-10}\) for \(\tan \beta = 10\) for the minimal supergravity case. Along with the \(B(b \to s \gamma)\) constraint, \(a^{\text{SUSY}}_e\) will soon become a very important constraint on the parameter space in the supergravity model.

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Figure Captions

FIG. 1 Allowed region in $m_A$–$m_h$ space used in our numerical calculation.

FIG. 2 $\Delta M_{B_d}/\Delta M_{B_d}^{SM}$ as a function of the lightest Higgs boson mass for (a) $\tan \beta = 2$, (b) $\tan \beta = 3$ and (c) $\tan \beta = 10$. Each dot represents the value in the full parameter space and each square shows the value for the minimal case.

FIG. 3 $\Delta M_{B_d}/\Delta M_{B_d}^{SM}$ as a function of $B(b \to s\gamma)$ for (a) $\tan \beta = 2$, (b) $\tan \beta = 3$ and (c) $\tan \beta = 10$. Each dot represents the value in the full parameter space and each square shows the value for the minimal case. The vertical dotted lines show the upper and lower bounds on $B(b \to s\gamma)$ given by CLEO. The constraint from the present Higgs boson search (case (I)) is imposed.

FIG. 4 Allowed ranges of $\Delta M_{B_d}/\Delta M_{B_d}^{SM}$ for several values of $\tan \beta$ with (a) the present constraint of the Higgs boson mass (case (I)) and (b) $m_h > 105$ GeV (case (II)). Two lines are shown according to the sign of the $b \to s\gamma$ amplitude for $\tan \beta = 10, 20, 40$. The left (right) lines correspond the case where the sign of the $b \to s\gamma$ amplitude is same (opposite) as that in the SM.

FIG. 5 Allowed ranges of $B(K_L \to \pi^0 \nu \bar{\nu})/B(K_L \to \pi^0 \nu \bar{\nu})_{SM}$ for several values of $\tan \beta$ with (a) the present constraint of the Higgs boson mass (case (I)) and (b) $m_h > 105$ GeV (case (II)). The allowed ranges for the parameter regions where the sign of the $b \to s\gamma$ decay amplitude is opposite to that of the SM amplitude are separately plotted. The meaning of two lines for $\tan \beta = 10, 20, 40$ is as the same as in the case of Fig. 4.

FIG. 6 Correlation between $\Delta M_{B_d}/\Delta M_{B_d}^{SM}$ and $\epsilon_K/\epsilon_K^{SM}$ for $\delta_{13} = 30^\circ, 90^\circ$ and $150^\circ$ in the nonminimal case with $\tan \beta = 3$ with the present constraint of the Higgs boson mass (case (I)).

FIG. 7 Correlation between $B(K_L \to \pi^0 \nu \bar{\nu})/B(K_L \to \pi^0 \nu \bar{\nu})_{SM}$ and $B(K^+ \to \pi^+ \nu \bar{\nu})/B(K^+ \to \pi^+ \nu \bar{\nu})_{SM}$ for $\delta_{13} = 30^\circ, 90^\circ$ and $150^\circ$ in the nonminimal case with $\tan \beta = 3$ with the present constraint of the Higgs boson mass (case (I)).
FIG. 8 Correlation between $B(K_L \rightarrow \pi^0 \nu \bar{\nu})/B(K_L \rightarrow \pi^0 \nu \bar{\nu})_{SM}$ and $\Delta M_{B_d}/\Delta M_{B_d}^{SM}$ for (a) $\tan \beta = 3$ and (b) $\tan \beta = 10$ with the present constraint of the Higgs boson mass (case (I)).

FIG. 9 $a_{\mu}^{\text{SUSY}}$ in the minimal supergravity case for $\tan \beta = 3$ (a) as a function of the lighter chargino mass, (b) as a function of the left-handed scalar muon mass, and (c) as a function of $B(b \rightarrow s \gamma)$. The constraint from the present Higgs boson search (case (I)) is imposed.

FIG. 10 Same as Fig. 9 for $\tan \beta = 10$.

FIG. 11 Allowed ranges of $a_{\mu}^{\text{SUSY}}$ for several values of $\tan \beta$ with (a) the present constraint of the Higgs boson mass (case (I)) and (b) $m_h > 105$ GeV (case (II)). The allowed ranges for the parameter regions where the sign of the $b \rightarrow s \gamma$ decay amplitude is opposite to that of the SM amplitude are separately plotted.
Fig. 2(a)
Fig. 2(b)
Fig. 2(c)
Fig. 3(a)
Fig. 3(b)
Fig. 3(c)
Fig. 4(a)
Fig. 4(b)
Fig. 5(a)
\[ \frac{B(\bar{K}_L \to \pi^0 \nu \bar{\nu})}{B(\bar{K}_L \to \pi^0 \nu \bar{\nu})_{\text{SM}}} \]

\( \tan \beta \)

Fig. 5(b)
Fig. 6

\( \Delta M_B / (\Delta M_B)_{SM} \) versus \( \varepsilon_K / (\varepsilon_K)_{SM} \) for different values of \( \delta_{13} \):

- \( \delta_{13} = 90^\circ \)
- \( \delta_{13} = 30^\circ \)
- \( \delta_{13} = 150^\circ \)

\( \tan \beta = 3 \)

nonminimal
Fig. 7

\[ \frac{B(K_L \to \pi^0 \nu \bar{\nu})}{B(K_L \to \pi^0 \nu \bar{\nu})_{SM}} \]

\[ \frac{B(K^+ \to \pi^+ \nu \bar{\nu})}{B(K^+ \to \pi^+ \nu \bar{\nu})_{SM}} \]

\[ \delta_{13} = 150^\circ \]

\[ \delta_{13} = 90^\circ \]

\[ \delta_{13} = 30^\circ \]

\[ \tan \beta = 3 \] nonminimal
Fig. 8(a)
Fig. 8(b)
Fig. 9(a)
Fig. 9(b)
Fig. 9(c)
Fig. 10(a)
Fig. 10(b)
Fig. 10(c)

\[ B(b \rightarrow s \gamma) \times 10^{-4} \]

\[ a_{\mu} \text{ [10^{-10}]} \]

\[ \tan \beta = 10 \]

(I)

minimal
Fig. 11(a)
Fig. 11(b)