Scalar correlations in a quark plasma
and low mass dilepton production

D. Blaschke, Yu. L. Kalinovsky *, S. Schmidt
Fachbereich Physik, Universität Rostock, D-18051 Rostock, Germany

H.-J. Schulze
Sezione INFN, Università di Catania, I-95129 Catania, Italy

Abstract

We investigate possible consequences of resonant scalar interactions for dilepton production from a quark plasma at the chiral phase transition. It is found that this production mechanism is strongly suppressed compared to the Born process and has no significance for present experiments.

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Recent experiments with ultrarelativistic S- and Pb-beams at the CERN-SPS have shown that dilepton production in the low mass region is strongly enhanced when compared with a simple extrapolation from proton-proton and proton-nucleus collisions [1]. An application of the previously developed standard approach which also includes a possible QCD phase transition [2] to the situation in the CERES experiment [3] for different sets of equations of state shows that (i) hadronic processes dominate the dilepton spectrum in the low mass region, but (ii) the experimental finding of low mass dilepton enhancement cannot be reproduced within the standard scenario which neglects modifications of either quark or hadron properties in a hot and dense medium.

Among possible explanations of the low mass dilepton enhancement the in-medium modification of the $\rho$ resonance is the standard one [45]. It is, however, debated whether chiral symmetry restoration influences the fate of this vector resonance in the vicinity of the QCD phase transition [5]. In this context the rôle of critical phenomena related to scalar resonances should be considered.

Already in 1992, Weldon suggested that in a hot pion gas with finite chemical potential the scalar resonance can decay into a lepton pair [6]. This process is particularly interesting

*Permanent address: Laboratory for Computing Techniques and Automation, JINR Dubna, 141980 Moscow Region, Russia
as a “direct” signal of chiral symmetry restoration in hot and dense matter, where the scalar and pseudoscalar mesonic modes become degenerate. With increasing temperature of the hadron gas, the hadronic decay channel \((\sigma \rightarrow \pi\pi)\) closes as soon as \(m_\sigma < 2m_\pi\) at \(T \approx 150\) MeV \(^[7]\). At this temperature the sigma meson could appear as a prominent resonance of the dilepton spectrum. However, the mechanism proposed by Weldon depends strongly on a pion chemical potential, and requires in particular a significant difference of positive and negative pion densities, for which there is no experimental evidence.

In this article we rather scrutinize another mechanism in which scalar correlations could be of importance, namely we investigate whether resonant scalar \(q\bar{q}\) interactions can be a source of dileptons in the mass region \(0.2\) GeV \(< M_{\text{e}^+\text{e}^-} < 0.6\) GeV. The underlying physical scenario is the chiral symmetry restoration at the phase transition of the hot meson gas to quark matter, whereby strong nonperturbative correlations of color-singlet \(q\bar{q}\) pairs may persist (“critical opalescence” of quark matter \(^[8,9]\)).

We will show that due to the resonant interaction, the spectral density in the scalar channel of quark-antiquark annihilation in quark matter at the chiral phase transition might be enhanced by 1–2 orders of magnitude. The contribution of this process to the dilepton production rate, however, is smaller than the perturbative thermal Born process and cannot be considered as a source for the dilepton enhancement observed by the CERES collaboration.

Due to the built-in chiral symmetry it seems reliable to use in this report the simplest SU(2) version of the Nambu–Jona-Lasinio (NJL) model at finite temperature and chemical potential in order to study the scalar correlations in the quark phase \(^[11]\) and to give at least order of magnitude estimates of the physical phenomena we are interested in.

We consider the production (by \(qq\) annihilation) of a virtual photon, i.e. lepton pair, with invariant mass \(M^2 = (p_1 + p_2)^2\) and three-momentum \(P = |p_1 + p_2|\) in a locally thermalized medium that is characterized by a fluid four-velocity \(u\). The temperature \(T\) and quark chemical potential \(\mu\) in a given fluid cell determine uniquely the constituent quark mass \(m\) as solution of the gap equation \(^[11]\). The lepton mass is denoted by \(m_l\).

The spin-averaged annihilation cross section in lowest order of the electromagnetic interaction \(^[2]\) is given by the general expression

\[
\sigma \left[ q(p_1)\bar{q}(p_2) \to \gamma(M,P) \to l^+l^- \right] = \frac{\alpha}{3M^4} \frac{L(M)H(M,P)}{\sqrt{1 - 4m^2/M^2}},
\]

\[
L(M) = \left(1 + \frac{2m_l^2}{M^2}\right)\sqrt{1 - \frac{4m_l^2}{M^2}}\theta(M^2 - 4m_l^2),
\]

where \(H = H_{\mu}^\nu\) corresponds to the contraction of the so-called hadronic tensor

\[
H_{\mu\nu} = \sum_{\text{spin}} \langle q(p_1)\bar{q}(p_2) | J_{\mu}(0) | 0 \rangle \langle 0 | J_{\nu}(0) | q(p_1)\bar{q}(p_2) \rangle.
\]

The inclusion of scalar correlations corresponds to the following modification of the electromagnetic quark current operator with respect to the Born process:

\[
\langle q(p_1)\bar{q}(p_2) | J_{\mu}(0) | 0 \rangle = \langle \bar{v}(p_1)e_q\gamma_{\mu}u(p_2) \rangle \Rightarrow \langle \bar{v}(p_1)u(p_2) \rangle \frac{K}{1 - J(M,P)} I_{\mu}(M,P).
\]
Here the polarization operator $J(M, P)$ for the scalar isoscalar channel at finite temperature and chemical potential can be evaluated in the NJL model by using the standard techniques of finite temperature field theory \[12\] with the result

$$J(M, P) = iK \int \frac{d^4k}{(2\pi)^4} \text{Tr} [G(k)G(k - P)]$$

$$= \frac{N_C N_F K}{\pi^2} \int_0^1 k^2 dk \frac{\sinh(\omega/T)}{\omega \cosh(\omega/T) + \cosh(\mu/T)} \left[ 1 - \frac{M^2 - 4m^2}{8Pk} \ln(F_+ F_-) \right], \quad (5)$$

where $\omega = \sqrt{k^2 + m^2}$ and $E = \sqrt{P^2 + M^2}$ are the quark and photon energies, respectively, and

$$F_\pm = \frac{M^2 \pm 2E\omega + 2Pk}{M^2 \pm 2E\omega - 2Pk}.$$  \quad (6)

The loop integral (5) has an imaginary part

$$\text{Im}J(M, P) = -\frac{N_C N_F K T}{8\pi P} (M^2 - 4m^2) \ln \left[ \frac{\cosh(\omega_{\max}/T) + \cosh(\mu/T)}{\cosh(\omega_{\min}/T) + \cosh(\mu/T)} \right], \quad (7)$$

with $\omega_{\max,\min} = [E \pm P_\pm \sqrt{1 - 4m^2/M^2}] / 2$. The imaginary part (7) is non-vanishing for $M > 2m$ and corresponds to the decay width of the scalar meson in the $\bar{q}q$- channel. Note that we use a simple version of the NJL model with the scalar coupling constant $K$ and a three-momentum cutoff $\Lambda$.

The loop integral describing the transition $\sigma \rightarrow \gamma$ has the form ($Q$ is the charge operator for the quarks)

$$I_\mu(M, P) = \int \frac{d^4k}{(2\pi)^4} \text{Tr} [G(k)\gamma_\mu QG(k - P)]$$

$$= \frac{4}{3} N_C e m \int \frac{d^4k}{(2\pi)^4} \frac{2k_\mu - P_\mu}{(k^2 - m^2)((k - P)^2 - m^2)}.$$  \quad (8)

Using the condition of charge conservation, $P_\mu I^\mu = 0$, one can derive the useful relation

$$I_\mu I^\mu = -\frac{M^2}{P^2} (I_\mu u^\mu)^2,$$  \quad (9)

and it is then practical to define the dimensionless quantity

$$\bar{I} = \frac{I_\mu u^\mu}{4emP} = \frac{1}{P} \int \frac{d^4k}{(2\pi)^4} \frac{2k_0 - E}{(k^2 - m^2)((k - P)^2 - m^2)}$$

$$= \frac{1}{(4\pi P)^2} \int_0^\infty d\omega \delta n(\omega) [(2\omega + E) \ln(F_+) + (2\omega - E) \ln(F_-)]$$  \quad (10)

with

$$\delta n(\omega) = \frac{\sinh(\mu/T)}{\cosh(\omega/T) + \cosh(\mu/T)}.$$  \quad (11)
This loop integral does only give a nonvanishing contribution if two conditions are fulfilled: (a) The difference $\delta n$ of particle and antiparticle distributions is nonzero because of a chemical potential corresponding to a finite baryon number. (b) The three-momentum $P$ of the pair must be nonzero in the rest system of the medium (fluid) in which temperature and chemical potential are defined.

Coming back to Eq. (9), we obtain

$$H_{\text{Born}}(M) = e_q^2(M^2 + 2m^2),$$

$$H_{\text{Resonance}}(M, P) = 8e_q^2(M^2 - 4m^2)\left|D(M, P)\bar{I}(M, P)\right|^2,$$

where the dimensionless quantity

$$D(M, P) = \frac{KmM}{1 - J(M, P)}$$

is related to the propagator of the scalar $q\bar{q}$ correlation.

The final result for the resonance cross section for dilepton production relative to the Born cross section is

$$\frac{\sigma_{\text{Resonance}}(M, P)}{\sigma_{\text{Born}}(M)} = 8\frac{\alpha}{\alpha_q} \left|D(M, P)\bar{I}(M, P)\right|^2$$

with $I(M, P) = \sqrt{(M^2 - 4m^2)/(M^2 + 2m^2)}\bar{I}(M, P)$.

For the numerical calculation we use the NJL parameters fixed as in Ref. [13]. We choose two representative sets of temperature and (quark) chemical potential and obtain (all numbers are in MeV): (a) $T = 170, \mu = 80, m = 150, m_\sigma = 330$; and (b) $T = 240, \mu = 110, m = 33, m_\sigma = 340$. The first set is motivated by a recent fit of thermodynamical parameters to experimental hadron abundancies [14]. It yields a sigma mass close to the double quark mass threshold, and consequently the sigma propagator $D$ is strongly peaked. While the first set (a) corresponds to a scenario in the vicinity of the chiral phase transition, in the second set (b) we have chosen rather extreme conditions were the chiral symmetry is almost restored and results in a much smaller constituent quark mass, and a larger width of the sigma resonance.

The results of the calculation are given in Figs. 1 and 2, where we plot the moduli of the transition integral $I$, Eqs. (10,13), and the scalar propagator $D$, Eq. (14), as well as the resulting ratio of the resonance and the Born cross section, Eq. (15), where we have carried out an isospin average $\alpha/\alpha_q \rightarrow 18/5$. It shows that there is an enhancement of the scalar propagator $D(M, P)$ which, however, cannot overcome the smallness of the transition function $\bar{I}$. The resulting cross section is always suppressed by at least two orders of magnitude relative to the Born process, even for the most favorable kinematical conditions.

It can be concluded that the proposed sigma-induced dilepton production process has no relevance for present-day experiments, where even the thermal Born $q\bar{q}$ cross section is negligible relative to hadronic decay contributions in the low mass region of dilepton production.

\[1K = 9.45 \text{ GeV}^{-2}, \Lambda = 660 \text{ MeV}, \text{ current quark mass } m_0 = 5.35 \text{ MeV}\]
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FIGURES

FIG. 1. Dependence on invariant mass $M$ and three momentum $P$ of the transition integral $|I|$ (top figure), the scalar propagator $|D|$ (middle figure), and the ratio of resonance and Born cross section (bottom figure) for a temperature $T = 170$ MeV and quark chemical potential $\mu = 80$ MeV.

FIG. 2. Same as Fig. 1, but for $T = 240$ MeV and $\mu = 110$ MeV.
