Confront holographic QCD with Regge trajectories of vectors and axial-vectors

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We derive the general 5-dimension metric structure of the $Dp - Dq$ system in type II superstring theory, and demonstrate the physical meaning of the parameters characterizing the 5-dimension metric structure of the holographic QCD model by relating them to the parameters describing Regge trajectories. By matching the spectra of vector mesons $\rho_1$ with deformed $Dp - Dq$ soft-wall model, we find that the spectra of vector mesons $\rho_1$ can be described very well in the soft-wall $D3 - D4$ model, i.e., $AdS_5$ soft-wall model. We then investigate how well the $AdS_5$ soft-wall model can describe the Regge trajectory of axial-vector mesons $a_1$. We find that the constant component of the 5-dimension mass square of axial-vector mesons plays an efficient role to realize the chiral symmetry breaking in the vacuum, and a small negative $z^4$ correction in the 5-dimension mass square is helpful to realize the chiral symmetry restoration in high excitation states.

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I. INTRODUCTION

Quantum Chromodynamics (QCD) has been accepted as the basic theory of describing strong interaction for more than 30 years. However, it is still a challenge to solve QCD in non-perturbative region where gauge interaction is strong. In the early 1970’s, string theory was proposed to describe strong interacting particles [1]. Recently, the discovery of the gravity/gauge duality [2] has revived the hope to understand QCD in strongly coupled region using string theory. The gravity/gauge, or anti-de Sitter/conformal field theory (AdS/CFT) correspondence provides a revolutionary method to tackle the problem of strongly coupled gauge theories. For a review of AdS/CFT, see [3]. The string description of realistic QCD has not been successfully formulated yet. Many efforts are invested in searching for such a realistic description by using the ”top-down” approach, i.e. by deriving holographic QCD from string theory [4], as well as by using the ”bottom-up” approach, i.e. by examining possible holographic QCD models from experimental data [1, 2, 3, 8, 9].

It is an essential and crucial point for the realistic holographic QCD model to reproduce Regge behavior. Regge behavior is a well-known feature of QCD [10], and it was the commanding evidence for suggesting the string-like structure of hadrons. A general empirical expression for Regge trajectories can be cast as

$$M^2_{n,S} = a_n n + a_S S + b,$$  \hspace{1cm} (1)

where $n$ and $S$ are the quantum number of high radial and spin excitations, respectively. The slope $a_n$ and $a_S$ have dimension GeV$^2$, and describe the mass square increase rate in radial excitations and spin excitations, respectively. In principle, $a_n$ is not necessarily the same as $a_S$, though $a_n = a_S$ can be taken as a good approximation by fitting experimental data [11]. The parameter $b$ is the ground state mass square, and it is channel-dependent.

Currently, one of the most successful models derived from string theory is the Sakai-Sugimoto model [12], which can describe spontaneously chiral symmetry breaking naturally, but fails to generate the linear Regge behavior. On the other hand, in the ”bottom-up” approach, many efforts have been paid to generate the linear Regge behavior for meson spectra [13, 14, 15] and for baryon spectra [16]. In Ref. [13], Karch, Katz, Son and Stephanov (KKSS) found that a $z^2$ dilaton field correction in the $AdS_5$ background leads to the linear behavior $M^2_n \propto n$.

In this paper, we carefully study realizing Regge trajectories of vector and axial-vector mesons in holographic QCD model. We take the data of the radial and spin excitations of $\rho$ and $a$ from PDG2007 [17], which are listed in Table 1. To describe Regge trajectories for both ($\rho_1$, $\rho_3$) and ($a_1$, $a_3$), we use the general formula Eq. (1). From the experimental data, the parameters of Regge trajectories can be determined by using the standard $\chi^2$ fit. The parameters for ($\rho_1, \rho_3$) mesons and their correlations read

$$a_n^\rho = +0.91 \pm 0.23, \quad a_S^\rho = +1.08 \pm 0.39, \quad b^\rho = -1.09 \pm 1.18$$

$$\begin{pmatrix}
1 & 1 & 1 \\
1 & -0.82 & 1 \\
1 & -0.81 & 1
\end{pmatrix}.$$  \hspace{1cm} (2)
more and more degenerate in high excitations \[18\]. For example, for radial excitations of \( \rho \) and \( a \), one can read the information of chiral symmetry breaking in the vacuum and asymptotic chiral symmetry restoration in highly excited meson states:

Chiral symmetry breaking: It is known that chiral symmetry is spontaneously broken in the vacuum, thus the observed chiral partners are not degenerate. From the Regge trajectories of the chiral partners \( \rho \) and \( a \), the chiral symmetry breaking in the vacuum is reflected by the difference of the ground state square-masses \( b^\rho \) and \( b^a \). The difference is as large as \( 1 \) GeV, i.e. \( b^\rho - b^a \approx M_{a1}^2 - M_{\rho 1}^2 \approx 1 \) GeV.

Asymptotic chiral symmetry restoration in highly excited states: It is noticed that the chiral partner becomes more and more degenerate in high excitations \[18\]. For example, for radial excitations of \( \rho \) and \( a \), the mass square difference of chiral partners \( M_{a1}^2 - M_{\rho 1}^2 = 0.9947 \) GeV\(^2\) at \( n = 1 \), and this difference decreases to \( 0.3227 \) GeV\(^2\) at \( n = 6 \); For the radial excitations of \( \rho \) and \( a \), the mass square difference of chiral partners \( M_{a3}^2 - M_{\rho 3}^2 = 0.6408 \) GeV\(^2\) at \( n = 1 \), and it decreases to \( 0.2736 \) GeV\(^2\) at \( n = 3 \).

We will only focus on the realization of Regge trajectories of \( \rho \) and \( a \) in this paper. The paper is organized as following: After the introduction, we derive the general 5-dimension metric structure of the \( Dp - Dq \) system in type II superstring theory in Sec. II. By matching the spectra of vector mesons \( \rho \) with deformed \( Dp - Dq \) soft-wall model, in Sec. III we determine the 5-dimension metric that can describe the spectra of vector mesons \( \rho \). We then investigate how to describe the spectra of axial-vector mesons \( a \) in the same background of \( \rho \) in Sec. IV. At the end we give summary in Sec. V.

### II. 5-DIMENSION METRIC STRUCTURE OF \( Dp - Dq \) SOFT-WALL MODEL

#### A. Metric structure of \( Dp - Dq \) system

In order to investigate the possible dual string theory for describing Regge behavior, we introduce the following \( Dp - Dq \) branes system in type II superstring theory. In the \( Dp - Dq \) system, the \( N_c \) background \( Dp \)-brane describe the effects of pure QCD theory, while the \( N_f \) probe \( Dq \)-brane is to accommodate the fundamental flavors which has been introduced by Karch and Katz \[19\]. Such a practice is well motivated from string theory side. For example, in Sakai-Sugimoto model, \( p = 4 \) and \( q = 8 \). Low energy hadronic excitations are fields on probe branes which in the background determined by the background branes.

First, we consider \( N_c \) background \( Dp \)-branes in type II superstring theory. The near horizon solution in 10-dimension space-time is \[20\]

\[
ds^2 = h^{-\frac{1}{2}} \eta_{\mu\nu} dx^\mu dx^\nu + h^{\frac{1}{2}} \left( du^2 + u^2 d\Omega^2_{5-p} \right),
\]

where \( \mu, \nu = 0, \cdots, p \), the warp factor \( h(u) = (R/u)^{7-p} \) and \( R \) is a constant

\[
R = \left[ 2^{5-p} \pi^{(5-p)/2} \Gamma \left( \frac{7-p}{2} \right) g_s N_c t^7_{-p} \right]^{\frac{1}{p-3}}.
\]

The dilaton field in this background has the form of

\[
e^\Phi = g_s h^{(p-3)}(u).
\]
The effective coupling of the Yang-Mills theory is

$$g_{\text{eff}} \sim g_s N_c u^{p-3}, \quad (7)$$

which is $u$ dependent. This $u$ dependence corresponds to the RG flow in the Yang-Mills theory, i.e. the effective $g_{\text{eff}}$ coupling constant depends on the energy scale $u$. In the case of D3-brane, $g_{\text{eff}} \sim g_s N_c$ becomes a constant and the dual Yang-Mills theory is $\mathcal{N} = 4$ SYM theory which is a conformal field theory. The curvature of the background is

$$\mathcal{R} \sim \frac{1}{l_s^2 g_{\text{eff}}}, \quad (8)$$

which reflects the string/gauge duality - the string on a background of curvature $\mathcal{R}$ is dual to a gauge theory with the effective coupling $g_{\text{eff}}$. To make the perturbation valid in the string side, we require that the curvature is small $\mathcal{R} \ll 1$, which means that the effective coupling in the dual gauge theory is large $g_{\text{eff}} \gg 1/l_s^2$. In the case of D3-brane, the curvature $\mathcal{R}$ becomes a constant, and the background (8) reduces to a constant curvature spacetime - $AdS_5 \times S^5$.

The coordinates transformation (for the cases of $p \neq 5$)

$$u = \left(\frac{5 - p}{2}\right)^{\frac{p-2}{2}} R^{p-3} z^{\frac{2}{p-5}}, \quad (9)$$

brings the above solution (4) to the following Poincaré form,

$$ds^2 = e^{2A(z)} \left[ \eta_{\mu\nu} dx^\mu dx^\nu + dz^2 + \frac{(p-5)^2}{4} z^2 d\Omega_{8-p}^2 \right]. \quad (10)$$

Next, we consider $N_f$ probe $Dq$-branes with $q - 4$ of their dimensions in the $S^q-4$ part of $S^{8-p}$, with the other dimensions in $z$ and $x^\mu$ directions as given in Table II. The induced $q + 1$ dimensions metric on the probe branes is

| $D_p$ | $D_q$ | $S^{6}$ \(_\subset S^{8-p}\) |
|---|---|---|
| 0 | 1 | 2 | 3 | ... | $p$ | $z$ |

given as

$$ds^2 = e^{2A(z)} \left[ \eta_{\mu\nu} dx^\mu dx^\nu + dz^2 + \frac{z^2}{z_0^2} d\Omega_{q-4}^2 \right]. \quad (11)$$

Where $\mu, \nu = 0, \cdots, 3$, and the metric function of the warp factor only includes the logarithmic term

$$A(z) = -a_0 \ln z, \quad \text{with} \quad a_0 = \frac{p - 7}{2(p - 5)}, \quad (12)$$

and the dilaton field part takes the form of

$$e^{\Phi} = g_s \left( \frac{2}{5 - p} \right) \frac{R^{(p-3)(p-7)}}{z^{(p-5)}}. \quad (13)$$

It follows that

$$\Phi(z) \sim d_0 \ln z, \quad \text{with} \quad d_0 = -\frac{(p - 3)(p - 7)}{2(p - 5)}. \quad (14)$$
B. The deformed $D_p - D_q$ soft-wall model

In the above subsection, we have derived the general metric structure of the $D_p - D_q$ system in Type II superstring theory, and we have noticed that the metric function $A(z)$ only includes the logarithmic term, and in general there is another logarithmic contribution to the dilaton field. However, from the lessons of AdS$_5$ metric ($D_3$ system) and the Sakai-Sugimoto model ($D_4 - D_8$ system), the $D_p - D_q$ system cannot describe linear trajectories of mesons. It was shown in Ref. [13], in order to produce linear trajectories, there should be a $z^2$ term, but all $z^2$ asymptotics should be kept in the dilaton field $\Phi(z)$ and not in the warp factor $A(z)$. Otherwise, the radial slope $a_n$ will be spin dependent. Therefore, to describe the real QCD, we propose a deformed $D_p - D_q$ soft-wall model which is defined as

$$A(z) = -a_0 \ln z, \quad \Phi(z) = d_0 \ln z + c_2 z^2.$$  \hspace{1cm} (15)

By assuming that the gauge fields are independent of the internal space $S^{q-4}$, after integrating out $S^{q-4}$, up to the quadratic terms and following the same assumption as in [13], we can have the effective 5D action for higher spin mesons described by tensor fields as

$$I = \frac{1}{2} \int d^5x \sqrt{g} e^{-\Phi(z)} \left\{ \Delta_N \phi_{M_1 \cdots M_S} \Delta_N \phi_{M_1 \cdots M_S} + m^2_5 \phi_{M_1 \cdots M_S} \phi_{M_1 \cdots M_S} \right\},$$  \hspace{1cm} (16)

where $\phi_{M_1 \cdots M_S}$ is the tensor field and $M_i$ is the tensor index. The value of $S$ is equal to the spin of the field. The parameter $m^2_5$ is the 5D mass square of the bulk fields, and $g$ and $\Phi(z)$ are the induced $q + 1$ dimension metric and dilaton field as shown in Eq. (14) and (14). The action for $\rho_1, a_1$ and $\rho_3, a_3$ mesons is given by taking $S = 1$, and $S = 3$ respectively.

Following the standard procedure of dimensional reduction, we can decompose the bulk field into its 4d components and their fifth profiles as $\phi(x; z)_{M_1 \cdots M_S} = \sum_{n=0}^{\infty} \phi^n_{M_1 \cdots M_S}(x) \psi_n(z)$. The equation of motion (EOM) of the fifth profile wavefunctions $\psi_n(z)$ for the general higher spin field can be derived as

$$\partial_z^2 \psi_n - \partial_z B \cdot \partial_z \psi_n + (M^2_{n,S} - m^2_5 e^{2A}) \psi_n = 0,$$  \hspace{1cm} (17)

where $M_{n,S}$ is the mass of the 4-dimension field $\phi^n_{M_1 \cdots M_S}(x)$ and

$$B = \Phi - k(2S - 1)A = \Phi + c_0(2S - 1)\ln z$$  \hspace{1cm} (18)

is the linear combination of the metric background function and the dilaton field. The combination function $B(z)$ approaches logarithmic asymptotic at UV brane, and goes to $z^2$ asymptotic at IR region. It is worthy of remark that the spin parameter $S$ enters in the factor $B$ and can affect the EOM and spectra. Two comments are in order: 1) The EOM for the eigenspectrum and wavefunction is valid for all mesons. 2) The spin parameter $S$ enters in the factor $B$ and can affect the EOM and spectra.

The parameter $k$ is a parameter depending on the induced metric (11) of the $D_q$ brane. After integrating out $S^{q-4}$, $k$ is determined as

$$k = \frac{(p - 3)(q - 5) + 4}{7 - p}.$$  \hspace{1cm} (19)

It is obviously that $k$ depends on both $p$ and $q$. For simplicity, we have defined

$$c_0 = ka_0 = \frac{(p - 3)(q - 5) + 4}{2(p - 5)}.$$  \hspace{1cm} (20)

The parameters $a_0, d_0$, the other two parameters $k, c_0$ and corresponding curvature $R$ for any $D_p - D_q$ system are listed in Table [11]. We notice that $d_0 = 0$ for $D_3$ background branes, i.e. dilaton field is constant in AdS$_5$ space. However, the dilaton field in a general $D_p - D_q$ system can have a ln $z$ term contribution, e.g. in the $D4 - D8$ system (Sakai-Sugimoto model [12]), $d_0 = -3/2$. We also want to point out that for pure $D_p - D_q$ system, the curvature is proportional to the inverse of the coupling strength $g_{eff}$. For $D_3$ background branes, the curvature is a constant. The curvature for $D_4$ background branes is small at IR, and large at UV, its dual gauge theory is strongly coupled at IR and weakly coupled at UV, which is similar to QCD. However, the curvature for $D_5$ background branes is large at IR, and small at UV, its dual gauge theory is weakly coupled at IR and strongly coupled at UV, which is opposite to QCD.
In this section we firstly consider the spectra of vector mesons, and we will investigate axial-vectors mesons and the chiral symmetry breaking mechanism in more detail in Sec. [IV]

In the dictionary of AdS/CFT, a $f$–form operator with conformal dimension $\Delta$ has 5-dimensional square-mass $m_5^2 = (\Delta - f)(\Delta + f - 4)$ in the bulk [21], and for vector mesons $m_5^2 = 0$. The spectra of EOM of Eq. (17) for vector mesons has an exact solution:

$$M_{n,S}^2 = 4c_2n + 4c_2c_0S + 2c_2(1 - c_0 + d_0).$$

(21)

When $c_0 = c_2 = 1$ and $d_0 = 0$, this solution reduces to the results given in the Ref. [13].

This exact solution supports the parameterization on Regge trajectories and can tell how phenomenological parameters $a_n, a_S$ and $b$ are directly related with the metric parameters $c_0, c_2$ and $d_0$:

1) $c_2$ is completely determined by the string tension in the radial direction $a_n$, i.e.

$$c_2 = \frac{a_n}{4}. $$

(22)

Andreev in Ref. [22] shows that there is an upper bound of $c_2, c_2$ can be determined by the coefficient $C_2$ [23] of the quadratic correction to the vector-vector current correlator [24]

$$Nq^2 \frac{d\Pi_V}{dq^2} = C_0 + \frac{1}{q^2}C_2 + \frac{n}{q^2}C_{2n}\langle O_{2n}\rangle,$$

(23)

where $C_2$ can be determined from $e^+e^-$ scattering data [23]. According to [22], the relation between $C_2$ in Eq. (23) and the parameter $c_2$ for Dilaton field is: $c_2 = -\frac{4}{3}C_2$. The experimental bound $|C_2| \leq 0.14\text{GeV}^2$ gives $|c_2| \leq 0.21\text{GeV}^2$. From the fitting result for the Regge trajectories of vector and axial-vector mesons Eqs. (2) and (3), we can read that $c_2$ is around $0.2 \sim 0.25$, which is in agreement the experimental upper bounds.

2) It is interesting to notice that $c_0$ reflects the difference of string tension in the radial direction and spin direction,

$$c_0 = \frac{a_S}{a_n}. $$

(24)

From the string theory side, it is commonly believed that the dual string theory of describing QCD should be strongly curved at high energy scales and weakly curved at low energy scales [20]. It seems $D4$ background brane system is more like the dual string theory of QCD. However, by reading the result for the Regge trajectories of vector and axial-vector mesons Eqs. (2) and (3), we can see that $a_S/a_n$ is around 1, which indicates that the real holographic QCD model is not far away from $AdS_5$ model.

3) Furthermore, $d_0$ can be solved out as

$$d_0 = \frac{2b}{a_n} + \frac{a_S}{a_n} - 1.$$ 

(25)

If we take the approximation of $a_n = a_S = 1$, we have $c_0 = 1, c_2 = 1/4$ for both vector and axial-vector mesons, while $d_0$ is mainly determined by the ground state square-mass as $d_0^{p/a} = 2b^{p/a}$.

From the above analysis, we can see that the realistic holographic QCD model should be close to models defined in $Dp$-branes background for $p = 3$, i.e., $AdS_5$ background. By using Eq. (21) to fit the spectra of vector mesons $\rho_1$, we get the following metric parameters:

$$c_0^\rho = 1, \quad c_2^\rho = 0.2\text{GeV}^2, \quad d_0^\rho = 0.$$ 

(26)
It is noticed that here, \( c_2 \) has dimension GeV\(^2\). In doing numerical simulation, we have introduced a dimensionless parameter \( u \). The relation between \( u \) and \( z \) is defined as
\[
u = \Lambda_{Scl} z . \tag{27}
\]
In our fitting, we have fixed \( \Lambda_{Scl} = 0.2 \), \( u_{UV} = 0.1 \) and \( u_{UV} = 3 \), respectively. We will use the same parameters for calculations of axial-vector mesons.

### IV. AXIAL VECTOR MESONS IN THE DEFORMED \textit{AdS}_5 MODEL

We have determined the metric structure Eq. (26) of describing the vector mesons, and we suppose the spectra of axial-vector mesons \( a_1 \) can be described in the same holographic QCD model. As mentioned in the introduction, in order to describe the Regge trajectories of axial-vector mesons \( a_1 \), we have to describe the chiral symmetry breaking in the vacuum and the asymptotic chiral symmetry restoration in highly excited meson states.

#### A. Chiral symmetry breaking

In order to produce the splitting of axial-vector meson spectra from vector meson spectra, it is essential to know the chiral symmetry breaking mechanism, in the following we discuss two different ways of chiral symmetry breaking.

1. **Constant \( m_{5,a}^2 \) in the Higgsless model**

If we start from the Higgsless model Eq. (16), the only difference between the EOM for \( \rho \) and \( a \) is the 5D bulk mass \( m_{5}^2 \). We have taken \( m_{5}^2 = 0 \) for vector mesons, we can assume that \( m_{5}^2 \neq 0 \) for axial-vector mesons due to chiral symmetry breaking.

For the general \( A(z) \) and \( \Phi(z) \) parameterized as
\[
A(z) = -c_0 \ln z ,
\]
\[
\Phi(z) = c_2 z^2 , \tag{28}
\]
the potential derived from \( B_s(z) = \Phi(z) - (2S - 1)A(z) \) can be put as
\[
V(z) = 2c_2(m - 1) + \frac{m^2 - \frac{1}{4}}{z^2} + c_2^2 z^2 + \frac{m_{5}^2}{z^2c_0} , \tag{29}
\]
with
\[
m = [(2S - 1)c_0 + 1]/2. \tag{30}
\]

When \( c_0 = 1 \), for any \( c_2 \) and constant \( m_5 \), the EOM
\[
- \Psi'' + V(z)\Psi = m_{n}^2 \Psi . \tag{31}
\]
can have analytic solution:
\[
m_n^2 = c_2(4n + 1) + c_2(2S - 1) + c_2 \sqrt{[(2S - 1) + 1]^2 + 4m_{5}^2} . \tag{32}
\]

When \( S = 1 \),
\[
m_n^2 = 4c_2n + 4c_2 + 2c_2 \left( \sqrt{1 + m_{5}^2} - 1 \right) . \tag{33}
\]

From Eq. (31), for \( m_{5,\rho}^2 = 0 \) and \( m_{5,a}^2 \neq 0 \), we can get the spectra of axial-vector \( a_1 \) through shifting the spectra of \( \rho_1 \) by \( 2c_2(\sqrt{1 + m_{5,a}^2} - 1) \) upward. In Fig. III we explicitly show our numerical results of the spectra of axial-vector mesons \( a_1 \) by solving the EOM of \( a_1 \) Eq. (32) for different \( m_5^2 \). It is found that when \( m_{5,a}^2 = 0.5\text{GeV}^2 \), the produced \( a_1 \) spectra agrees well with the experimental data.
2. Higgs Mechanism

In Ref. [13], it is suggested that the axial field picks up a \( \phi \)-dependent 5d mass via the Higgs mechanism from the background scalar \( X \) that encodes the chiral symmetry breaking. The axial vector meson mode equation reads:

\[
\partial_z \left( e^{-\Phi(z)} e^{A(z)} \partial_z a_n \right) + \left[ m_n^2 - g_5^2 e^{2A(z)} X(z) e^{-\Phi(z)} e^{A(z)} a_n(z) \right] = 0,
\]

(35)

with \( g_5^2 = 12\pi^2/N_c \). The linearized equation of motion for the scalar field \( X \) reads:

\[
\partial_z \left( e^{-\Phi(z)} e^{A(z)} \partial_z X(z) \right) + 3 e^{-\Phi(z)} e^{A(z)} X(z) = 0.
\]

(36)

It is quite complicated to solve the coupled Eqs. (35) and (36). We only discuss two asymptotic solutions of scalar field at UV (\( \phi = 0 \)) and IR, respectively.

The asymptotic form at UV has the form of

\[
X(z) \xrightarrow{z \to 0} \frac{1}{2} m_q z + \frac{1}{2} \Sigma z^3.
\]

(37)

here the coefficient \( m_q \) is the UV (\( \phi = 0 \)) boundary condition given by the quark mass matrix, while the coefficient \( \Sigma \) is the chiral condensate, which can be determined dynamically by the boundary condition in the IR. We show the produced \( a_1 \) spectra from Eq. (37) in Fig. 2 when \( N_c = 3 \). Fig. 2(a) is for different values of \( m_q \) with fixed \( \Sigma = (400\text{MeV})^3 \), and Fig. 2(b) is for different values of \( \Sigma \) with fixed \( m_q = 3\text{MeV} \). It can be clearly seen that the asymptotic form at UV introduces nonlinearity of the \( \phi \) dependence of \( M_n^2 \), the \( z^3 \) term contributes more on the non-linear behavior.

For large \( \phi \) (in the IR) on the background \( \Phi = c_2 \phi^2 \) the equation for \( X \) becomes

\[
X'' - 2c_2 \phi X' + \frac{3}{\phi^2} X = 0 \quad (\phi \gg 1),
\]

(38)

the scalar field has a solution goes to a constant in the IR, i.e., \( X(z) \to \text{const} \) as \( \phi \to \infty \). \( X(z) = \text{const} \) means the effective 5D mass \( m_{5,a}^2 = g_5^2 X(z)^2 \) is a constant. Then the spectra of \( a_1 \) will behave the same as that in Sec. IV A 1.

B. Chiral Symmetry restoration

Though the slopes in radial direction for vector mesons \( a_\rho^\mu \) and axial-vector mesons \( a_a^\mu \) can be roughly taken as the same, i.e., \( a_\rho^\mu \approx a_a^\mu \), in order to have chiral symmetry restoration, we need \( a_\rho^\mu > a_a^\mu \). To accommodate such a
The $a_1$ spectra by using the UV asymptotic form of scalar field solution at $N_c = 3$, (a) is for different values of $m_q$ with fixed $\Sigma = \langle 400\text{MeV} \rangle^3$, (b) is for different values of $\Sigma$ with fixed $m_q = 3\text{MeV}$.

In summary, we have derived the general 5-dimension metric structure of the $Dp-Dq$ system in type II superstring theory. We have shown the dependence of the metric parameters on the Regge trajectories parameters: The quadratic term in the dilaton background field is solely determined by the slope in the radial direction; The warp factor is mainly determined by the difference of the slope in the spin direction and the radial direction; The logarithmic term in the dilaton background field contributes to the ground state square-masses.

It is commonly believed that the dual string theory of describing QCD should be strongly curved at high energy scales and weakly curved at low energy scales, it seems that the $D4$ background brane system is more like the dual string theory of QCD. However, the ratio of the slope in the spin direction of the vector mesons over its slope in the radial direction $a_S/a_n$ is around 1, which indicates that the real holographic QCD model is not far away from $AdS_5$ model.

We have shown that the spectra of axial vector mesons can also be described in the $AdS_5$ soft-wall model, and a constant 5D bulk mass for axial-vector meson plays efficient role to realize the chiral symmetry breaking in the vacuum, and a small negative $z^4$ correction in the 5D mass square is helpful to realize the chiral symmetry restoration in high excitation states.

The information in this study is important for a realistic holographic QCD model and our future study on the interactions of mesons (branching ratios and decay widths of mesons, their interactions, etc). In our current approach, we chose the Higgsless model to describe the radial and higher spin excitations of vector and axial-vector mesons, the pseudoscalar $\pi$ is the zero mode of axial-vector field and there is no its radial and higher spin excitations. We leave the study of the Regge trajectories of scalar and pseudo-scalar mesons by using the global symmetry breaking model in our future works.

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