Circular periodic orbits, resonance capture and inclination excitation during type II migration

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Abstract. We consider planetary systems evolving under the effect of a Stokes-type dissipative force mimicking the outcome of a type II migration process. As inward migration proceeds and the planets follow the circular family (they start on circular orbits) and even though they are initially almost coplanar, resonance capture can be realized. Then, at the vertical critical orbits (VCOs), that the circular family possesses, the inclination excitation can abruptly take place. The planets are now guided by the spatial elliptic families, which bifurcate from those critical orbits. We herein, perform a direct link of mutually inclined stable planetary systems on circular orbits trapped in mean-motion resonance (MMR) with the existence of VCOs of high values of multiplicity. It is shown that the more the multiplicity of the periodic orbits of the circular family increases, the more VCOs (corresponding to more MMRs) appear. In this way, we can provide a justification for the existence of resonant planets on circular orbits, which could, even further to that, evolve stably if they were mutually inclined.

Keywords. periodic orbits, vertical stability, multiplicity, inclination excitation, type II migration, planetary evolution

1 Introduction

Among the exoplanets discovered so far, there are plenty of them located at small orbital distances ($a < 0.2$ A.U.), where due to not only high temperature, but, also, insufficient amount of protostellar matter, accretion in situ would have been prevented (Boss 1995; Bodenheimer et al. 2000). Apart from mechanisms, such as

1. gravitational interactions between two or more planets of Jovian masses, which possibly lead to the development of dynamical instability; This, in turn, could invoke orbit crossing and scattering of a
planet leaving the other one in highly eccentric orbit and in some cases, this orbit has, also, small periastron distance that can circularize it through tidal dissipation (Rasio & Ford 1996)

2. effects of a distant companion star; In binary stellar systems dynamical instabilities and, therefore, chaos lead to highly eccentric orbits, which could, sometimes, induce collision between the planets and the primary star (Holman et al. 1997)

3. effects of stellar encounter (Malmberg & Davies 2009) whose simulations result in a range of separation $1 \leq a \leq 6$ A.U.), only migration can explain the short period planets and generally, the overall architecture of many systems.

The existence of the so-called hot Jupiters (Jupiter-mass planets orbiting close to their host star, typically at $a \leq 0.1$AU) augmented earlier speculations that such planets could be formed far away from the star and then, via an inward migration process (Goldreich & Tremaine 1979, 1980; Lin & Papaloizou 1986a,b; Ward & Hourigan 1989), which should be followed by an halting mechanism (Thommes & Lissauer 2003) implied that inclination excitation could serve as such and put, finally, an end in the migration, reached their present configurations.

Three mechanisms are believed to govern orbital evolution

- gas disk migration
- planetesimal disk migration
- planet-planet scattering.

However, hot Earths, hot super Earths ($\sim 1 - 10 M_\oplus$) and hot Neptunes ($\sim 10 - 20 M_\oplus$) have been modeled by different scenarios (Raymond et al. 2008b).

Planet-planet scattering has, also, explained the occurrence of non-circular orbits with high orbital eccentricities (Marzari & Weidenschilling 2002; Zakamska & Tremaine 2004; Moorhead & Adams 2005). Moreover, it can populate numerous orbital resonances (Raymond et al. 2008a) and likewise, resonance chains (4:2:1 Laplace resonance) resulting from the damping mechanism of the planetesimal disk (Raymond et al. 2010). The damping force of the planetesimal disk can, furthermore, align the orbits in MMR through convergent migration, if the planets of the system were formed in a stable configuration, namely, avoiding close encounters.

Planetesimal disk migration (Malhotra 1995, Thommes et al. 2008, Armitage 2010), in brief, causes an inward scattering of a planetesimal and an outward movement of the planet, or vice-versa, because of the conservation of the system’s angular momentum. For a significant migration effect, the...
planetesimals’ masses to be scattered should be of the order of the planet’s mass. In the opposite case, planetesimal eccentricities are pumped up via dynamical friction and the planets remove them from the system, via dynamical ejection or collision.

As for the solar system, migration seems to have affected its architecture and precisely, its outer regions (Gomes et al. 2004; Tsiganis et al. 2005; Morbidelli & Levison 2008).

As far as the exosystems are concerned, large scale dynamical instabilities caused by such a planetesimal migration can interpret the highly eccentric orbits, often observed at them (see e.g. (Marzari & Weidenschilling 2002), which defines, also, the so-called jumping Jupiters model).

Gas disk migration (interactions between the planets and the residual gaseous protoplanetary disk) consists of three types:

I It holds for planets of masses $M < 10M_⊕$, whose evolution by wave excitation does not largely perturb the gas surface density profile, which in turn can be treated by using linear analysis (Ward 1997).

II It refers to larger planets. Their angular momentum dominates the disk’s viscous forces, gas is repelled from their vicinity and an annular gap in the gas opens up at the orbital radius of the planets. A combined action of type II migration and planet-planet scattering has widely been proposed, e.g. by Moorhead & Adams 2005.

III or runaway migration It is a more rapid migration, resulting from a modified, under certain circumstances (more massive disk), type I migration. The planets included in this type can open up a partial gap (see e.g. Masset & Papaloizou 2003; Papaloizou 2005; Paardekooper & Mellema 2006).

During the past two decades, it has broadly been approved that type II migration process can justify the resonant configurations of exoplanets (see e.g. Haghighipour 1999; Lee & Peale 2002; Nelson & Papaloizou 2002; Kley 2003; Papaloizou 2003). In two-planet systems, during resonance capture and given the planetary mass-ratio, the two planets follow particular migration paths. These can be depicted on the eccentricities plane, when coplanar evolution is the case, either by the resonant stationary solutions of the averaged planetary three-body problem (see e.g. Ferraz-Mello et al. 2003; Lee 2004; Beaugé et al. 2006), or by the resonant, stable, periodic orbits of the general three-body problem (GTBP) in a suitable rotating frame. The families of periodic orbits constitute such paths, as shown in (Hadjidemetriou & Voyatzis 2010, 2011). A possible existence of such families in the general three-planet case is presented by Voyatzis (2016).
Thommes & Lissauer (2003) showed that stable spatial configurations can be obtained after migration and 2/1 resonance capture, starting from a two-planet system of almost coplanar orbits. It was observed that when high values of the eccentricity were reached, excitation of the mutual inclination was taking place and hence, the system was additionally in “inclination resonance”. Lee & Thommes (2009) showed numerically that when the outer planet was more eccentric than the inner the system jumped to nearby paths (from the symmetric to asymmetric families depending on the linear stability) and the inclination was excited. When along their evolution along the families (or paths) the two planets meet a VCO (Hénon 1973) they abruptly get mutually inclined and they follow the generated spatial families of periodic orbits, as shown in Voyatzis et al. (2014) for 2/1 and 3/1 MMRs. More precisely, therein, it was showcased that given the MMR and the planetary mass-ratio, we can tell whether the planets are likely to be mutually inclined or not.

In this study, we report the inclination excitation during the evolution due to type II migration process of circular orbits. We reveal the existence of -among others- 5/2 and 7/3 resonance captures imposed by the bifurcation points of the circular periodic orbits. In Sect. 2 we describe our model, the periodic orbits and their link with MMRs. In Sect. 3 we present our results and justify them through the computation of the periodic orbits of the circular family for greater values of multiplicity. Finally, we conclude in Sect. 4.

2 The model

We consider a system consisting of a star and two planets of masses \( m_0, m_1 \) and \( m_2 \), respectively, with \( m_0 \gg m_{1,2} \) where the indices 1 and 2 are always referring to the inner and the outer planet. Throughout the study the gravitational constant and the total mass of the system \( m = m_0+m_1+m_2 \) are set equal to unity and we define the planetary mass-ratio as \( \rho = \frac{m_2}{m_1} \). We investigate the dynamics in the framework of the GTBP, formulate the system in a suitable spatial rotating frame of reference as in (Antoniadou & Voyatzis 2014) and yield the Lagrangian of four degrees of freedom

\[
L = \frac{1}{2} \mu \left[ a (\dot{x}_1^2 + \dot{z}_1^2 + x_1^2 \dot{\theta}^2) + b (\dot{x}_2^2 + \dot{y}_2^2 + \dot{z}_2^2) + \dot{\theta}^2 (x_2^2 + y_2^2) + 2 \dot{\theta} (x_2 \dot{y}_2 - x_2 y_2) \right] - V,
\]

where \( a = m_1/m_0, b = m_2/m \) and \( V = -\frac{m_0 m_1}{r_{01}} - \frac{m_0 m_2}{r_{02}} - \frac{m_1 m_2}{r_{12}} \) is the potential with \( r_{ij} \) indicating the distance between the bodies \( i \) and \( j \).

Given the equations of motion, we can define the periodic orbits (periodic solutions) as the fixed or periodic points on a Poincaré surface of
section through the repetitive crossings on that plane at \( t = kT \), where \( T \) is the period and \( k \) the multiplicity of the orbit (see e.g. (Antoniadou & Voyatzis 2013)). In cases where specific periodicity conditions are fulfilled and two perpendicular crossings are established, the periodic orbits are called symmetric and asymmetric otherwise.

The periodic orbits shape the phase space in their vicinity according to their linear stability (Hénon 1973; Hadjidemetriou 2006; Skokos 2001) and can be either stable or unstable. Stable periodic orbits are surrounded by invariant tori and the long-term stability is guaranteed therein, whereas in the neighbourhood of the unstable ones chaotic domains exist which, in case the chaos is not weak, may destabilize the dynamical system. We should note that with regards to the planar periodic orbits the linear and vertical stability coexist but they do not affect each other. For instance, a planar unstable periodic orbit could be vertically stable if vertical deviations are considered and vice-versa, as seen in (Antoniadou & Voyatzis 2013, 2014; Antoniadou et al. 2014b).

For a visual representation of the phase space and the delineation of the boundaries of the domains, maps of dynamical stability are utilized; for coplanar periodic orbits (Antoniadou 2016; Antoniadou & Voyatzis 2016b,a) and mutually inclined orbits (Antoniadou et al. 2014a,b; Antoniadou 2016). It is straightforward that the families of stable periodic orbits constitute the backbone of stability domains.

In general, periodic orbits can be generated by particular types of bifurcation points (or critical orbits) and get mono-parametrically continued (see Hadjidemetriou 1975; Hénon 1997; Voyatzis et al. 2009; Antoniadou et al. 2011; Antoniadou & Voyatzis 2013, 2014). In this way, families or characteristic curves can be obtained, along which a specific parameter remains constant. However, there exist families that do not emanate from bifurcation points. For instance, depending on the planetary mass-ratio, there have been computed planar symmetric and asymmetric ones (Voyatzis 2008; Voyatzis et al. 2009; Antoniadou & Voyatzis 2016b), as well as, some symmetric mutually inclined orbits (Antoniadou & Voyatzis 2013).

The families can be classified as circular or elliptic. The former ones consist only of symmetric periodic orbits and along them the mean-motion ratio, \( \frac{n_1}{n_2} \), varies, while the latter ones may consist of either symmetric or asymmetric ones, but the \( \frac{n_1}{n_2} \) remains almost constant and is approximately equal to \( \frac{p+q}{p} \), with \( p, q \in \mathbb{Z}^* \) (c.c. the planetary case with the restricted one when the ratio is exactly equal to that rational number). Therefore, the elliptic families are resonant and the exact MMR can be identified by the periodic orbits.
In order for two-planets\footnote{Having assumed that $m_0 \gg m_{1,2}$, the periodic orbits in the inertial frame correspond to almost Keplerian ellipses described by the \textit{osculating orbital elements} $a_i$ (semi-major axis), $e_i$ (eccentricity), $i_i$ (inclination), $\varpi_i$ (longitude of pericentre), $\Omega_i$ (longitude of ascending node) and $\lambda_i$ (mean longitude), where $i = 1, 2$.} to be locked in an MMR and evolve stably about periodic orbits, the resonant angles
\begin{equation}
\begin{align*}
\theta_1 &= (p + q)\lambda_2 - p\lambda_1 - q\varpi_1 \\
\theta_2 &= (p + q)\lambda_2 - p\lambda_1 - q\varpi_2 \\
\theta_3 &= (p + q)\lambda_2 - p\lambda_1 - \frac{q}{2}(\varpi_1 + \varpi_2)
\end{align*}
\end{equation}
and the apsidal difference, $\Delta \varpi$, should librate about 0 or $\pi$, if the orbit is symmetric, or around other angles, if it is asymmetric. The libration of the above resonant angles represents the \textit{eccentricity resonance} (e-resonance).

There exist four different symmetric configurations, if we assume aligned, $\Delta \varpi = 0$, and anti-aligned, $\Delta \varpi = \pi$, planets, which do not change along the families of periodic orbits (see e.g. \cite{Michtchenko2006, Antoniadou2013, Antoniadou2014}).

When the two planets are not coplanar, the resonant angles that define the \textit{inclination resonance} (i-resonance) for at least second order resonances
\begin{equation}
\begin{align*}
\varphi_1 &= (p + q)\lambda_2 - p\lambda_1 - q\Omega_1 \\
\varphi_2 &= (p + q)\lambda_2 - p\lambda_1 - q\Omega_2
\end{align*}
\end{equation}
librate as well. For instance, the inclination resonance of the 2/1 resonance would be the 4/2 ($q = 2$), since this is the lowest order of possible inclination resonance of this commensurability (see the arguments of the expansions of the disturbing function in \cite{Murray1999}). We may further define the mixed resonance angle
\begin{equation}
\varphi_{12} = (p + q)\lambda_2 - p\lambda_1 - \Omega_1 - \Omega_2 = (\varphi_{11} + \varphi_{22})/q
\end{equation}
as well as the zeroth order secular resonance angle
\begin{equation}
\varphi_{\Omega} = \Omega_1 - \Omega_2 = (\varphi_{11} - \varphi_{22})/q.
\end{equation}

\section{Resonance capture and inclination excitation of circular orbits}

In order to mimic the effects of type II migration process, we assume on the outer planet a Stokes type dissipative force \cite{Beauge1993, Beauge2006}\footnote{\textbf{Beaugé} \& \textbf{Ferraz-Mello} 1993 [Beaugé et al. 2006]}
\begin{equation}
\mathbf{F}_d = -c(\mathbf{v}_p - \alpha \mathbf{v}_c)
\end{equation}
Circular periodic orbits and inclination excitation

where \( v_p \) is the velocity of the planet and \( v_c \) is the circular velocity at the particular distance from the star. In a first order approximation, the constants \( c \) and \( \alpha \) are associated with the migration rate in semi-major axis, \( \nu \), and the eccentricity damping, \( K \), according to the formulae

\[
\nu = 2C(1 - \alpha), \quad K = \frac{\alpha}{2(1 - \alpha)}.
\]

In the numerical simulations we set the star mass \( m_0 = 1M_\odot \) and start with almost circular, \( e_1 = e_2 = 0.01 \), and coplanar orbits, \( i_1 = i_2 = 0.1^\circ \), with \( \Delta \Omega = 180^\circ \). The inner planet is set to \( a_1 = 5 \) AU.

As convergent migration proceeds, upon capture to e-resonance, the semi-major axes ratio will be \( \frac{a_2}{a_1} \approx \left( \frac{n_1}{n_2} \right)^{2/3} \approx \left( \frac{p+q}{p} \right)^{2/3} \). At that moment, the planets will evolve along the elliptic \(^2\) family of stable periodic orbits that corresponds to their mass-ratio \( \rho \). Then, both the resonant angles, Eq. 1, and the apsidal difference, \( \Delta \varpi \), will librate about the angles that correspond to the particular configuration of the periodic orbits they follow. Along their evolution, when a VCO is met, the planets follow the spatial family that bifurcates from it. Then, i-resonance is observed in addition to e-resonance, via the libration of the resonant angles of Eq. 2. This scenario is showcased by Voyatzis et al. (2014). Therein, given the MMR and the \( \rho \), the inclination excitation took place at either asymmetric or symmetric VCO and this type of evolution has been verified for \( \nu \lesssim 10^{-6} y^{-1} \) and for small eccentricity damping (\( K \approx 1 \)). We have also observed complicated phenomena discussed in Lee & Thommes (2009) (see e.g. Figs. 1 and 2).

### 3.1 5/2 resonance capture

In Fig. 3 we present the planar families of symmetric periodic orbits in the \((e_1, e_2)\) plane. The four panels correspond to the four different configurations as attributed by the resonant angles \((\theta_1, \theta_2)\) and the apsidal difference, \( \Delta \varpi \). The negative values of the eccentricities are used whenever \( \theta_{1,2} \) librates about \( \pi \). For more details see (Antoniadou & Voyatzis 2014).

In the quest to achieve 5/2 resonance capture, as reported by Libert & Tsiganis (2009), we examined all possible combinations of values \( 10^{-9} \leq \nu \leq 10^{-5} (y^{-1}) \) and \( K \leq 1 \) for mass ratios \( 0.1 \leq \rho \leq 5 \). All of the systems of planetary masses with \( \rho < 2 \) passed by 5/2 MMR without being captured and ended up locked in 2/1 MMR. The only cases of systems that have been captured in 5/2 MMR along with the parameters used are shown in

\(^2\)We note that, generally, the elliptic families bifurcate from the circular family at the orbits where \( \left( \frac{n_1}{n_2} \right)^{2/3} = \left( \frac{p+q}{p} \right)^{2/3} \). Consequently, the planetary system evolves initially about the circular family up to the moment it gets captured in MMR.
Figure 1. Complicated phenomena for $\rho = 0.2$ and $\nu = 5.2 \times 10^{-5} \text{y}^{-1}$, $K = 0.7$ in 2/1 MMR. The evolution overcomes the asymmetric VCO of $A_{b}^{2/1}$ and then, the i-resonance appears, when the system reaches the next VCO located in the symmetric family, $S_{1}^{2/1}$. Finally, the system jumps to another family of asymmetric periodic orbits, $A_{d}^{2/1}$. Blue bold lines refer to the stable periodic orbits, while the red coloured to the unstable ones. Magenta coloured lines correspond to vertical instability no matter the “horizontal” stability of the periodic orbits. The dots represent the VCOs.

Then, the majority of them (apart from the critical case in Fig. 4b) was temporarily trapped in 7/3 and finally in 2/1 MMR. In Figs. 5-10, we present the evolution of the eccentricities, semi-major axes, inclinations and resonant angles, $\theta_1$, $\Delta \varpi$ and $\Delta \Omega$, of the planets.

The path that the planar families of 5/2 MMR follow, when the eccentricities are approximately equal to zero (see Fig. 3), could also aid to justifying the non-capture to e-resonance, even for lower planetary mass-ratios. Nevertheless, we observe that even when the mutual inclination of the planets increased significantly, the planetary orbits remained circular. The above mentioned behaviour -temporal captures in 5/2 and 7/3 MMRs and a final capture in 5/2 MMR only for a specific choice of a parameter- is fully explained in Sect. 3.1.1.

The planets of the system forced to evolve under the parameters given in Fig. 4a were firstly trapped in 5/2 MMR, then in 4/1, 5/1 again in 4/1 and finally, in 7/2 MMR. The planets reached the maximum mutual inclination and eccentricity values during 5/1 capture, but in the end, the system was distorted, since the initially inner planet escaped (see also Fig. 5).
Circular periodic orbits and inclination excitation

The planets of the system described in Fig. 4b evolved under a critical case of no migration rate in semi-major axis and obtained high mutual inclination values of a symmetric stable configuration of anti-aligned planets (libration of $\theta_1$ about $0^\circ$, $\Delta \varpi$ about $\pi$ and $\Delta \varpi$ about $\pi$). However, their orbits were circular ($e_i < 0.06$) throughout the simulation (see Fig. 6).

The evolution of the systems described in Figs. 4c,f is quite similar, i.e. the planets, after having been captured in $5/2$ MMR, reached even higher mutual inclination values, when they were locked in $7/3$ MMR. Both their eccentricities increased after these consecutive captures, although none of them surpassed the value 0.1, namely they remained on circular orbits (see Figs. 7 and 10, respectively).

The planets of the system described in Fig. 4d remained almost planar throughout the evolution, even when they were trapped in $2/1$ MMR and the eccentricity of the inner planet started increasing significantly (see also Fig. 8). They, also, ended up in a symmetric configuration ($\Delta \varpi$ librates about $0^\circ$), however, the motion was unstable ($\theta_1$ rotated).

The evolution of the system of parameters shown in Fig. 4e is similar to the previous one with regards to the evolution of the orbital elements and the resonant angles. They solely differ in the relative longitude of periapse, $\Delta \varpi$, which did not finally librate, but kept on rotating instead (see Fig. 9).

In Fig. 11a, we present the inclination resonant angle $\varphi_1$ and observe
Figure 3. The planar families of 5/2 resonant periodic orbits. Presentation as in Fig. 1. The bold gray lines correspond to regions of close encounters and collisions and the coloured dots depict the VCOs. The numbers along the families represent the planetary mass-ratio $\rho$, where $m_1 = 1 \, M_J$.

what was expected for the system in Fig. 6; when the $i$-resonance was achieved, it started librating around $0^\circ$, as the spatial family the system followed consists of $xz$-symmetric periodic orbits (see (Antoniadou & Voyatzis 2013) for the definition of this type of orbits). In Fig. 11b, we show the evolution of $\varphi_1$ in the system described in Fig. 7, where firstly 5/2 MMR capture is apparent and then, a 7/3 temporal capture is seen. In Figs. 11c and d, we show magnifications of the regions, where those limited in time captures were encountered.

3.1.1 Justification of 5/2 resonance capture

The 5/2 resonance capture could not be explained with the aid of the VCOs of the elliptic families of periodic orbits of the GTBP, because they arise for larger values of eccentricities (see the magnification in Fig. 3 herein and Fig. 5 in Antoniadou & Voyatzis (2014)).
Circular periodic orbits and inclination excitation

Figure 4. Captures in $5/2$ MMR. a A case where the eccentricity damping is $K = 4.5$, the migration rate is $\nu = 1.26 \times 10^{-8} \text{ y}^{-1}$ and $\rho = 20$. b The critical case of $\alpha = 1$, namely the migration rate in semi-major axis equals zero. The mass ratio of the planets is $\rho = 5$. c A case where $K = 4.5$, $\nu = 1.26 \times 10^{-9} \text{ y}^{-1}$ and $\rho = 2$. d A case where $K = 0.5$, $\nu = 6.28 \times 10^{-9} \text{ y}^{-1}$ and $\rho = 5$. e A case where $K = 0.75$, $\nu = 5.03 \times 10^{-9} \text{ y}^{-1}$ and $\rho = 5$. f A case where $K = 2$, $\nu = 2.5 \times 10^{-9} \text{ y}^{-1}$ and $\rho = 5$.

Thus, we sought for their existence in the circular family. When the multiplicity of those periodic orbits is equal to 1, the only VCOs that exist, belong to $3/1$ MMR (see Fig. 3 of (Voyatzis et al. 2014)). Consequently, we had to compute the circular family of periodic orbits for larger values of multiplicity.

In Fig. 12 we present the variation of the vertical stability index as the multiplicity of periodic orbits increases for MMRs greater than $2/1$. The VCOs which appear along the family at particular MMRs with respect to the multiplicity values are shown in Table 1.

In Fig. 13 we project the evolution of the migrating system described in Fig. 6 in the plane of $(e_1, \Delta i)$ for visualization reasons. We observe the excitation of the orbit from its circular planar form to planar elliptic (e-resonance) and to inclined (i-resonance) motion. The system followed the family of spatial periodic orbits, which was generated by the VCO of the circular family derived by a periodic orbit of multiplicity equal to 3 for $\rho = 5$. The larger oscillations start taking place as the system enters the unstable region in the vicinity of the unstable periodic orbits in phase space.
4 Conclusions

In this study, we performed a direct link of the inclination excitation of two-planet systems on circular orbits with the intrinsic property of the three-body problem: the vertical critical orbits.

It was shown that during type II migration process and after capture in MMR, the VCOs of the circular family can provide the means to i-resonance. The generated spatial families of periodic orbits of high multiplicity constitute the paths that guide the evolution of planetary systems.

The circular family can pave the way to the appearance of more VCOs in different MMRs, as the periodic orbits, it consists of, are computed with an increased multiplicity. Then, possible trappings in the corresponding
MMRs can be justified and the planets can get mutually inclined by following the respective stable, spatial elliptic families.

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Figure 9. Evolution of a planetary system described in Fig. 4e presented as in Fig. 5.

Figure 10. Evolution of a planetary system described in Fig. 4f presented as in Fig. 5.
Figure 11. The evolution of the resonant angle $\varphi_1$, for the systems of Figs. a and b. Magnifications of b are provided in c and d.
Figure 12. The vertical stability index, $a_v$, along the circular family as the multiplicity (bold red numbers at top right corner of each panel) of the periodic orbits increases and the mean-motion ratio, $n_1/n_2$, varies.
Table 1. We provide the MMRs that correspond to the appearance of VCO along the circular family as the multiplicity of periodic orbits, with which they are computed, increases.

| Multiplicity | Mean-motion resonance |
|--------------|-----------------------|
| 1            | 3/1                   |
| 2            | 7/3 3/1 5/2 3/1 4/1 5/1 |
| 3            | 11/5 5/2 3/1 4/1 5/1 |
| 4            | 7/3 13/5 3/1 11/3 5/1 |
| 6            | 17/8 11/5 7/3 5/2 19/7 3/1 17/5 4/1 5/1 |
| 9            | 2/1 17/8 11/5 16/7 12/5 5/2 8/3 14/5 3/1 10/3 18/5 4/1 23/5 |
Figure 13. Capture in 5/2 MMR and evolution (black dots) along a spatial symmetric elliptic family, $S$, which bifurcated from the circular, $C$, one. The mutual inclination, $\Delta i$, increased abruptly, when the VCO was met. Blue bold lines refer to the stable periodic orbits, while the red coloured to the unstable ones.
Circular periodic orbits and inclination excitation

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