Reliability analysis of technological cell with start-up device

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Abstract. An important factor affecting the efficiency and reliability of technological systems is the reliability of their start-up device. In the work, a semi-Markov model of the functioning of the technological cell is built taking into account the start-up device. The analysis of the influence of the reliability of the start-up device on the reliability of the technological cell.

1. Introduction
An integral part of technological systems are start-up devices that are designed to drive machines into action. The efficiency and reliability of the functioning of the technological system depends on the reliable operation of the start-up devices. Reliability issues of start-up devices for various purposes were considered in a number of works, for example, in [1-3]. To analyze the reliability of technological systems, semi-Markov processes are widely used [4-6]. A semi-Markov model of a technological cell is constructed taking into account the presence of start-up device. The stationary characteristics of the system under consideration are found that allow us to analyze the influence of the reliability of the start-up device on the reliability of the technological cell.

2. Construction of Semi-Markov Model
Let us describe the functioning order of the considered system $S$, which consists of a technological cell (TC) and start-up device (SD). Uptime TC is a random variable (RV) $\alpha_1$ with a distribution function (FR) $F_1(t) = 1 - e^{-\lambda_1 t}$, the recovery time is RV $\beta_1$ with FR $G_1(t) = 1 - e^{-\mu_1 t}$. Uptime SD is RV $\alpha_2$ with FR $F_2(t) = 1 - e^{-\lambda_2 t}$, and the recovery time is RV $\beta_2$ with FR $G_2(t) = 1 - e^{-\mu_2 t}$. RV are assumed to be independent. In the event of a SD failure, an operational TC is disabled for the period of recovery SD, after the restoration SD, the TC continues to work taking into account the previous operating time. Failure of the system $S$ occurs upon failure or shutdown TC and continues until its operation.

Functioning semi-Markov model of the system $S$ is semi-Markov process $\xi(t)$ with a discrete-continuous phase state space $E$ is used:

$$E = \{1,101,111,200,201,120,220,211\}.$$
Consider the meaning of state codes:
- 1 - SD restored, TC, disabled after recovery, started to work;
- 101 – TY has refused and the restoration has begun, PU is operational;
- 111 - TY restored and started work, PU operational;
- 200 - during the restoration of the TN, the PU failed;
- 201 - PU restored, TB restored;
- 120 - during the restoration of the PU TY restored and disabled;
- 220 - PU failed, ТЯ disconnected;
- 211 - PU restored, disabled ТЯ continued to work.

Time diagram of the functioning of the system \( S \) is shown in figure 1.

We find the transition probabilities of the EMC \( \{\xi_n; n \geq 0\} \) SM process \( \xi(t) \):

\[
P_{101}^{101} = \frac{\lambda_1}{\lambda_1 + \lambda_2}, \quad P_{200}^{200} = \frac{\lambda_2}{\lambda_1 + \lambda_2}, \quad P_{111}^{111} = \frac{\mu_1}{\mu_1 + \lambda_2}, \quad P_{101}^{200} = \frac{\lambda_2}{\mu_1 + \lambda_2}, \quad P_{111}^{101} = \frac{\lambda_2}{\mu_1 + \lambda_2}.
\]

\[
P_{120}^{101} = \frac{\lambda_1}{\lambda_1 + \lambda_2}, \quad P_{200}^{200} = \frac{\lambda_2}{\lambda_1 + \lambda_2}, \quad P_{120}^{111} = \frac{\mu_1}{\lambda_1 + \lambda_2}, \quad P_{200}^{200} = \frac{\mu_1}{\mu_1 + \mu_2}, \quad P_{121}^{111} = \frac{\mu_1}{\lambda_1 + \lambda_2}, \quad P_{211}^{211} = \frac{\lambda_2}{\lambda_1 + \lambda_2}.
\]

We find the average sojourn times in the states of the system

\[
\theta_1 = \alpha_1 \land \alpha_2, \quad \theta_{101} = \beta_1 \land \alpha_2, \quad \theta_{111} = \alpha_1 \land \alpha_2, \quad \theta_{200} = \beta_1 \land \beta_2, \\
\theta_{201} = \beta_1 \land \alpha_2, \quad \theta_{120} = \beta_2, \quad \theta_{220} = \beta_2, \quad \theta_{211} = \alpha_1 \land \alpha_2,
\]

where \( \land \) is the minimum sign.

Hence,

\[
M\theta_1 = \frac{1}{\lambda_1 + \lambda_2}, \quad M\theta_{101} = \frac{1}{\lambda_1 + \lambda_2}, \quad M\theta_{111} = \frac{1}{\lambda_1 + \lambda_2}, \quad M\theta_{200} = \frac{1}{\mu_1 + \mu_2}, \\
M\theta_{201} = \frac{1}{\lambda_1 + \lambda_2}, \quad M\theta_{120} = \frac{1}{\lambda_1 + \lambda_2}, \quad M\theta_{220} = \frac{1}{\mu_2}, \quad M\theta_{211} = \frac{1}{\mu_1 + \mu_2}.
\]

Let us find the stationary distribution of the EMC \( \{\xi_n; n \geq 0\} \). Let us denote \( \rho(1), \rho(101), \rho(111), \rho(200), \rho(201), \rho(120), \rho(220), \rho(211) \) the values of the stationary
distribution of the EMC \( \{ \xi_n; n \geq 0 \} \) for the corresponding states of the system 1, 101, 111, 200, 201, 120, 220, 211. The system of equations for the stationary distribution has the following form [4,5]:

\[
\begin{aligned}
\rho(1) &= \rho(120); \\
\rho(101) &= \frac{\lambda_1}{\lambda_1 + \lambda_2} \left( \rho(211) + \rho(111) \right); \\
\rho(111) &= \frac{\mu_1}{\mu_1 + \lambda_2} \left( \rho(201) + \rho(101) \right); \\
\rho(200) &= \frac{\lambda_2}{\mu_1 + \lambda_2} \left( \rho(201) + \rho(101) \right); \\
\rho(201) &= \frac{\mu_2}{\mu_1 + \mu_2} \rho(220); \\
\rho(120) &= \frac{\mu_1}{\mu_1 + \lambda_2} \rho(200); \\
\rho(220) &= \frac{\lambda_2}{\lambda_1 + \lambda_2} \left( \rho(1) + \rho(211) + \rho(111) \right); \\
\rho(221) &= \rho(220); \\
\rho(1) + \rho(101) + \rho(111) + \rho(200) + \rho(201) + \rho(120) + \rho(220) + \rho(221) &= 1. \\
\end{aligned}
\]

(4)

It can be shown that the system of equations (4) has the following solution:

\[
\begin{aligned}
\rho(101) &= \frac{\mu_1 + \mu_2 + \lambda_2}{\lambda_2} \rho(1) \\
\rho(111) &= \frac{\mu_1 + \mu_2}{\lambda_2} \rho(1); \\
\rho(200) &= \frac{\mu_1 + \mu_2}{\lambda_1} \rho(1); \\
\rho(201) &= \frac{\mu_2}{\mu_1 + \mu_2} \cdot \frac{\mu_1 + \mu_2 + \lambda_2}{\lambda_1} \rho(1); \\
\rho(120) &= \frac{\mu_1}{\lambda_1} \rho(1); \\
\rho(220) &= \frac{\mu_1 + \mu_2 + \lambda_2}{\lambda_1} \rho(1); \\
\rho(211) &= \frac{\mu_1 + \mu_2 + \lambda_2}{\lambda_1} \rho(1); \\
\end{aligned}
\]

(5)

where

\[
\rho(1) = \frac{\lambda_2 \lambda_2 \mu_1}{2 \lambda_2 \mu_1 (\lambda_1 + \lambda_2 + \mu_1 + \mu_2) + \lambda_1 (\mu_1 + \mu_2) (\mu_1 + \lambda_2)}.
\]
3. Stationary characteristics of System

We split the phase state space $E$ of the system into 2 classes:

$E_+ = \{ 1, 111, 221 \}$ – the system is operational,

$E_- = \{ 101, 200, 120, 220, 201 \}$ – system is failure.

Let us find average stationary operation time to failure $T_+$, average stationary restoration time $T_-$ and stationary availability factor $K_a$ according formulas [4,7]:

$$
T_+ = \frac{\int_{E_+} m(e)\rho(de)}{\int_{E_+} P(e, E_+)\rho(de)}, \quad T_- = \frac{\int_{E_-} m(e)\rho(de)}{\int_{E_-} P(e, E_-)\rho(de)}, \quad K_a = \frac{T_+}{T_+ + T_-}.
$$

where $\rho(de)$ – stationary distribution of EMC, determined by formulas (1), $P(e, E_-)$ are the probabilities of transition of EMC, $m(e)$ – average residence times in the states of system.

We note that in the case under consideration, the integrals in formulas (7) are replaced by sums.

Using formulas (6), (2), (3), we obtain

$$
\sum_{E_+} \rho(i) \cdot P(i, E_+) = \rho(1) \cdot P_{220}^{(1)} + \rho(1) \cdot P_{101}^{(1)} + \rho(111) \cdot P_{111}^{(111)} + \rho(111) \cdot P_{111}^{(111)} + \rho(211) \cdot P_{211}^{(211)} + \rho(211) \cdot P_{211}^{(211)} = \rho(1) + \rho(111) + \rho(211) = \rho(1) + \frac{\mu_1 + \mu_2}{\lambda_1} \rho(1) + \frac{\mu_1 + \mu_2 + \lambda_2}{\lambda_1} \rho(1) = \rho(1) \left( \frac{\lambda_1 + \lambda_2}{\lambda_1 + \lambda_2} \right) \rho(1).
$$

$$
\sum_{E_-} M \theta \cdot \rho(i) = M \theta \cdot \rho(1) + M \theta \cdot \rho(111) + M \theta \cdot \rho(211) = \frac{\rho(1)}{\lambda_1 + \lambda_2} + \frac{\rho(111)}{\lambda_1} + \frac{\rho(211)}{\lambda_1 + \lambda_2} = \rho(1) \left( \frac{\lambda_1 + \lambda_2}{\lambda_1 + \lambda_2} \right) \rho(1).
$$

Thus, the average stationary operation time to failure $T_+$ of the system is calculated by the formula:

$$
T_+ = \frac{\rho(1) \cdot \frac{\mu_1 + \mu_2 + \lambda_2}{\lambda_1 + \lambda_2}}{\rho(1) \left( \frac{\lambda_1 + \lambda_2}{\lambda_1 + \lambda_2} \right) \rho(1)} = \frac{1}{\lambda_1 + \lambda_2} = M \left( \alpha_1 \land \alpha_2 \right).
$$

The average stationary restoration time $T_-$ of the system is calculated by the formula:

$$
T_- = \frac{\rho(1) \cdot \frac{\lambda_1 + \lambda_2}{\mu_1 + \mu_2 + \lambda_2}}{\rho(1) \left( \frac{\lambda_1 + \lambda_2}{\lambda_1 + \lambda_2} \right) \rho(1)} = \frac{\lambda_1 \lambda_2}{(\lambda_1 + \lambda_2)^2 (\mu_1 + \mu_2 + \lambda_2)} \left( \frac{\lambda_1}{\mu_1} + \frac{\lambda_2}{\mu_2} \right).
$$

Stationary availability factor $K_a$ is calculated according formula:
\[ K_a = \frac{1}{\lambda_1 + \lambda_2} + \frac{\lambda_1 \lambda_2}{(\lambda_1 + \lambda_2)^2 (\mu_1 + \mu_2 + \lambda_2)} \left( \frac{1}{\mu_1} + \frac{1}{\mu_2} + \frac{1}{\mu_2 (\mu_1 + \mu_2 + \lambda_2)} \right) \]

\[ = \frac{1}{1 + \frac{\lambda_1 \lambda_2}{(\lambda_1 + \lambda_2)(\mu_1 + \mu_2 + \lambda_2)} \left( \frac{1}{\mu_1} + \frac{1}{\mu_2} + \frac{1}{\mu_2 (\mu_1 + \mu_2 + \lambda_2)} \right)} \]  

(9)

We made calculation on formulas (7), (8) and (9) for initial data: Uptime TC 40 hours \( M\alpha_1 = 40 \) or \( \lambda_1 = 0.025 \), recovery time TC 1 hour \( M\beta_1 = 1 \) or \( \mu_1 = 1 \); Uptime SD 5 hours \( M\alpha_2 = 5 \) or \( \lambda_2 = 0.2 \), recovery time SD 0.2 hours \( M\beta_2 = 0.2 \) or \( \mu_2 = 5 \). We will change the uptime of SD the start-up device from 5 to 10 hours. The calculation results are presented in table 1.

Table 1. The values of \( T_+ \), \( T_- \) and \( K_a \) for \( M\alpha_1 = 40 \), \( M\beta_1 = 1 \), \( M\beta_2 = 0.2 \).

| Initial data | results |
|--------------|---------|
| \( M\alpha_2 \) | \( h \) | \( T_+ \) | \( T_- \) | \( K_a \) |
| 5            | 8       | 0.00118 | 0.99985 |
| 6            | 7.34    | 0.00116 | 0.99984 |
| 7            | 6.6     | 0.00113 | 0.99983 |
| 8            | 5.95    | 0.0011  | 0.99981 |
| 9            | 5.21    | 0.00107 | 0.99979 |
| 10           | 4.44    | 0.00103 | 0.99977 |

Formulas (7), (8) and (9) average stationary operation time to failure \( T_+ \), average stationary restoration time \( T_- \) and stationary availability factor \( K_a \) can be used to study the influence of reliability of SD on the reliability of TC. It was done in this paper.

4. Conclusion
A semi-Markov model of a technological cell is built taking into account the presence of a start-up device, stationary characteristics of the system under consideration are found. The analysis of the influence of the reliability of the start-up device on the reliability of the technological cell.

In the future, it is planned to use the approach considered in the paper to analyze the functioning of multicomponent technological systems, taking into account the presence of start-up devices.

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