New holographic scalar field models of dark energy in non-flat universe

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Abstract
Motivated by the work of Granda and Oliveros [L.N. Granda, A. Oliveros, Phys. Lett. B 671, 199 (2009)], we generalize their work to the non-flat case. We study the correspondence between the quintessence, tachyon, K-essence and dilaton scalar field models with the new holographic dark energy model in the non-flat FRW universe. We reconstruct the potentials and the dynamics for these scalar field models, which describe accelerated expansion of the universe. In the limiting case of a flat universe, i.e. $k = 0$, all results given in [L.N. Granda, A. Oliveros, Phys. Lett. B 671, 199 (2009)] are obtained.

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1 Introduction

Type Ia supernovae observational data suggest that our universe is experiencing an accelerated expansion driven by an exotic energy with negative pressure which is so-called dark energy (DE) [1]. However, the nature of DE is still unknown, and people have proposed some candidates to describe it. The cosmological constant, \( \Lambda \), is the most obvious theoretical candidate of DE, which has the equation of state \( \omega = -1 \). Astronomical observations indicate that the cosmological constant is many orders of magnitude smaller than estimated in modern theories of elementary particles [2]. Also the "fine-tuning" and the "cosmic coincidence" problems are the two well-known difficulties of the cosmological constant problems [3].

There are different alternative theories for the dynamical DE scenario which have been proposed by people to interpret the accelerating universe. i) The scalar field models of DE including quintessence [4], phantom (ghost) field [5], K-essence [6] based on earlier work of K-inflation [7], tachyon field [8], dilatonic ghost condensate [9], quintom [10], and so forth. ii) The interacting DE models including Chaplygin gas [11], braneworld models [12], and agegraphic DE models [13], etc.

Recently, a new DE candidate, based on the holographic principle, was proposed [14]. According to the holographic principle, the number of degrees of freedom in a bounded system should be finite and has relations with the area of its boundary [15]. In quantum field theory a short distance (UV) cut-off \( \Lambda \) is related to a long distance (IR) cut-off \( L \) due to the limit set by formation of a black hole, which results in an upper bound on the zero-point energy density [16]. By applying the holographic principle to cosmology, one can obtain the upper bound of the entropy contained in the universe [17]. Following this line, Li [18] argued that for a system with size \( L \) and UV cut-off \( \Lambda \), it is required that the total energy in a region of size \( L \) should not exceed the mass of a black hole of the same size, thus \( L^3 \rho_\Lambda \leq L M_P^2 \), where \( \rho_\Lambda \) is the quantum zero-point energy density caused by UV cut-off \( \Lambda \) and \( M_P \) is the reduced Planck Mass \( M_P^2 = 8\pi G \). The largest \( L \) allowed is the one saturating this inequality, thus \( \rho_\Lambda = 3c^2 M_P^2 L^{-2} \), where \( c \) is a numerical constant. Recent observational data, which have been used to constrain the holographic DE model, show that for the non-flat universe \( c = 0.815_{-0.139}^{+0.179} \) [19], and for the flat case \( c = 0.818_{-0.097}^{+0.113} \) [20]. Also Li [18] showed that the cosmic coincidence problem can be resolved by inflation in the holographic DE model, provided the minimal number of e-foldings [18]. The holographic models of DE have been studied widely in the literature [21, 22, 23]. As we mentioned before, the UV cut-off is related to the vacuum energy, and IR cut-off is related to the large scale of the universe, for example Hubble horizon, future event horizon or particle horizon. Taking \( L \) as the size of the current universe, for instance, the Hubble scale, the resulting energy density is comparable to the present day DE. However, as found by Hsu [24], in that case, the evolution of the DE is the same as that of dark matter (dust matter), and therefore it cannot drive the universe to accelerated expansion. The same appears if one chooses the particle horizon of the universe as the length scale \( L \) [18]. An interesting proposal is made by Li [18]: Choosing the event horizon of the universe as the length scale, the holographic DE not only gives the observation value of DE in the universe, but also can drive the universe to an accelerated expansion phase. In that case, however, an obvious drawback concerning causality appears in this proposal. Event horizon is a global concept of spacetime; existence of event horizon of the universe depends on future evolution of the universe; and event horizon exists only for universe with forever accelerated expansion. This motivated Granda and Oliveros [25] to propose a new infrared cut-off for the holographic DE, which besides the square of the Hubble scale also contains the time derivative of the Hubble scale. This model depends on local quantities and avoids the problem of causality which appears using the event horizon area as the IR cut-off. They
also in another paper [26] using the new infrared cut-off for the holographic DE [25], studied the correspondence between the quintessence, tachyon, K-essence and dilaton energy density with this holographic DE density for the case of DE dominance in the flat FRW universe. This correspondence allowed them to reconstruct the potentials and the dynamics for the scalar field models, which describe accelerated expansion.

Besides, as usually believed, an early inflation era leads to a flat universe. This is not a necessary consequence if the number of e-foldings is not very large [27]. It is still possible that there is a contribution to the Friedmann equation from the spatial curvature when studying the late universe, though much smaller than other energy components according to observations. Therefore, it is not just of academic interest to study a universe with a spatial curvature marginally allowed by the inflation model as well as observations. Some experimental data have implied that our universe is not a perfectly flat universe and that it possesses a small positive curvature [28].

In the light of all mentioned above, it becomes obvious that using the holographic DE model with the new infrared cut-off introduced by [25] and extending the work [26] to a non-flat case is well motivated. To do this, in Section 2, we obtain the equation of state parameter for the new holographic DE model given by [25] in the context of the a non-flat universe. In Section 3, like [26], we suggest a correspondence between the holographic and the quintessence, tachyon, K-essence and dilaton scalar field models in the presence of a spatial curvature. We reconstruct the potentials and the dynamics for these scalar field models, which describe accelerated expansion. Section 4 is devoted to conclusions.

2 New holographic DE model in non-flat universe

Following [25], the holographic DE density with the new infrared cut-off is given by

$$\rho_\Lambda = 3M_P^2(\alpha H^2 + \beta \dot{H}),$$  \hspace{1cm} (1)

where $H = \dot{a}/a$ is the Hubble parameter and $\alpha$ and $\beta$ are constants which must satisfy the restrictions imposed by the current observational data.

Here we consider the Friedmann-Robertson-Walker (FRW) universe with line element

$$ds^2 = -dt^2 + a^2(t)\left(\frac{dr^2}{1-kr^2} + r^2d\Omega^2\right),$$  \hspace{1cm} (2)

where $k$ denotes the curvature of space $k = 0, 1, -1$ for flat, closed and open universe, respectively. A closed universe with a small positive curvature ($\Omega_k \sim 0.02$) is compatible with observations [28]. Like [26], restricting our study to the current cosmological epoch, we neglect the contributions from matter and radiation and assume the DE is dominated in the presence of spatial curvature. Therefore the first friedmann equation becomes

$$H^2 + \frac{k}{a^2} = \frac{1}{3M_P^2} \rho_\Lambda.$$  \hspace{1cm} (3)

Substituting Eq. (1) in (3) gives

$$\frac{dH^2}{dx} + \frac{2}{\beta}(\alpha - 1)H^2 = \frac{2}{\beta}ke^{-2x},$$  \hspace{1cm} (4)

where $x = \ln a$. Integrating the above equation with respect to $x$ yields

$$H^2 = \frac{k}{\alpha - \beta - 1}e^{-2x} + \gamma e^{-\frac{2}{\beta}(\alpha - 1)x},$$  \hspace{1cm} (5)
where $\gamma$ is an integration constant. If we set $k = 0$ for the flat case and using $\dot{x} = H$, then Eq. (5) reduces to

$$H^2 = \frac{\beta}{\alpha - 1} \dot{H},$$

(6)

where dot denotes the time derivative with respect to the cosmic time $t$. Taking the integral in both sides of Eq. (6) with respect to $t$, we get

$$H = \frac{\beta}{\alpha - 1} \frac{1}{t},$$

(7)

which is same as Eq. (2.2) in [26]. From the conservation equation

$$\rho_\Lambda + 3H(1 + \omega_\Lambda)\rho_\Lambda = 0,$$

(8)

and using Eq. (1), the equation of state (EoS) parameter, $\omega_\Lambda = p_\Lambda/\rho_\Lambda$, can be obtained as

$$\omega_\Lambda = -1 - \frac{2\alpha H \dot{H} + \beta \ddot{H}}{3H(\alpha H^2 + \beta H)}.$$

(9)

Replacing $H$ from Eq. (5) in (9) yields

$$\omega_\Lambda = -1 - \frac{2\alpha H \dot{H} + \beta \ddot{H}}{3H(\alpha H^2 + \beta H)}.$$

(10)

which is a time-dependent EoS parameter. When the EoS parameter of DE is time-dependent, then it can transit from $\omega_\Lambda > -1$ to $\omega_\Lambda < -1$ [29]. The analysis of the properties of DE from recent observations mildly favor models with $\omega_\Lambda$ crossing $-1$ in the near past [30].

If we set $k = 0$, we obtain $\omega_\Lambda$ for the flat universe as

$$\omega_\Lambda = -1 + \frac{2}{3} \frac{\alpha - 1}{\beta},$$

(11)

which is same as Eq. (2.5) in [26]. Contrary to the non-flat case, the EoS parameter for the flat case is constant. However, Granda and Oliveros [26] showed that by applying some restrictions on the constants $\alpha$ and $\beta$, the EoS parameter of the new holographic DE in the flat case, Eq. (11), can behave as a quintessence or a phantom type DE. But, neither the quintessence nor the phantom alone can fulfill the transition from $\omega_\Lambda > -1$ to $\omega_\Lambda < -1$ and vice versa.

3 Correspondence between the new holographic and scalar field models of DE

Here like [26], we suggest a correspondence between the new holographic DE model with the quintessence, tachyon, K-essence and dilaton scalar field models but in the context of the non-flat universe. To establish this correspondence, we compare the new holographic energy density (1) with the corresponding scalar field model density and also equate the equations of state for this models with the EoS parameter given by (10).
3.1 New holographic quintessence model

The energy density and pressure of the quintessence scalar field $\phi$ are as follows [3]
\[\rho_Q = \frac{1}{2}\dot{\phi}^2 + V(\phi), \quad (12)\]
\[p_Q = \frac{1}{2}\dot{\phi}^2 - V(\phi). \quad (13)\]

The EoS parameter for the quintessence scalar field is given by
\[\omega_Q = \frac{p_Q}{\rho_Q} = \frac{\dot{\phi}^2 - 2V(\phi)}{\dot{\phi}^2 + 2V(\phi)}. \quad (14)\]

If we establish the correspondence between the new holographic DE scenario and the quintessence DE model, then using Eqs. (10) and (14) we have
\[\omega_\Lambda = \frac{1}{3} \left( \frac{k(\alpha - \beta)(\alpha - \beta - 1)e^{-2x} + \gamma(\frac{\alpha - 1}{\beta})e^{-\frac{2}{\beta}(\alpha - 1)x}}{k(\alpha - \beta)(\alpha - \beta - 1)e^{-2x} + \gamma e^{-\frac{2}{\beta}(\alpha - 1)x}} \right) = \frac{\dot{\phi}^2 - 2V(\phi)}{\dot{\phi}^2 + 2V(\phi)}, \quad (15)\]
also using Eqs. (1) and (12) we get
\[\rho_\Lambda = 3M_P^2(\alpha H^2 + \beta \dot{H}) = \frac{1}{2}\dot{\phi}^2 + V(\phi), \quad (16)\]
then one can obtain the kinetic energy term and the quintessence potential energy as follows
\[\dot{\phi}^2 = 2M_P^2 \left[ k(\alpha - \beta)(\alpha - \beta - 1)e^{-2x} + \gamma(\frac{\alpha - 1}{\beta})e^{-\frac{2}{\beta}(\alpha - 1)x} \right], \quad (17)\]
\[V(\phi) = M_P^2 \left[ 2k(\alpha - \beta)(\alpha - \beta - 1)e^{-2x} + \gamma(\frac{3\beta - \alpha + 1}{\beta})e^{-\frac{2}{\beta}(\alpha - 1)x} \right]. \quad (18)\]

From Eqs. (5) and (17), using $\dot{\phi} = \dot{\phi}'H$ where prime denotes the derivative with respect to $x$, we obtain
\[\phi' = \sqrt{2M_P} \left[ \frac{k(\alpha - \beta)(\alpha - \beta - 1)e^{-2x} + \gamma(\frac{\alpha - 1}{\beta})e^{-\frac{2}{\beta}(\alpha - 1)x}}{k(\alpha - \beta)(\alpha - \beta - 1)e^{-2x} + \gamma e^{-\frac{2}{\beta}(\alpha - 1)x}} \right]^{1/2}. \quad (19)\]

Consequently, after integration with respect to $x$ we can obtain the evolutionary form of the quintessence scalar field as
\[\phi(a) - \phi(0) = \sqrt{2M_P} \int_0^a \left[ \frac{k(\alpha - \beta)(\alpha - \beta - 1)e^{-2x} + \gamma(\frac{\alpha - 1}{\beta})e^{-\frac{2}{\beta}(\alpha - 1)x}}{k(\alpha - \beta)(\alpha - \beta - 1)e^{-2x} + \gamma e^{-\frac{2}{\beta}(\alpha - 1)x}} \right]^{1/2} dx, \quad (20)\]
where we take $a_0 = 1$ for the present time.

For the flat case if we set $k = 0$, using Eqs. (5) and (7), assuming $\phi(0) = 0$ for the present time $t_0 = 0$, then Eqs. (18) and (20) reduce to
\[\phi(t) = \sqrt{2\frac{\beta}{\alpha - 1}} M_P \ln t, \quad (21)\]
\[V(\phi) = \beta \left[ \frac{3\beta - \alpha + 1}{(\alpha - 1)^2} \right] M_P^2 \exp \left( -\sqrt{2\frac{\alpha - 1}{\beta}} \frac{\phi}{M_P} \right), \quad (22)\]
which are same as Eqs. (3.5) and (3.6) in [26], respectively. Note that in Eq. (3.6) in [26], a factor $\beta$ has been missed.
3.2 New holographic tachyon model

The tachyon field was proposed as a source of the dark energy. The tachyon is an unstable field which has become important in string theory through its role in the Dirac-Born-Infeld (DBI) action which is used to describe the D-brane action [8]. The effective Lagrangian density of tachyon matter is given by [8]

\[ \mathcal{L} = -V(\phi) \sqrt{1 + \partial_\mu \phi \partial^\mu \phi}. \]  

(23)

The energy density and pressure for the tachyon field are as following [8]

\[ \rho_T = \frac{V(\phi)}{\sqrt{1 - \dot{\phi}^2}}, \]  

(24)

\[ p_T = -V(\phi) \sqrt{1 - \dot{\phi}^2}, \]  

(25)

where \( V(\phi) \) is the tachyon potential. The EoS parameter for the tachyon scalar field is obtained as

\[ \omega_T = \frac{p_T}{\rho_T} = \dot{\phi}^2 - 1. \]  

(26)

If we establish the correspondence between the new holographic DE and tachyon DE, then using Eqs. (10) and (26) we have

\[ \omega_\Lambda = -\frac{1}{3} \frac{k (\frac{\alpha-\beta}{\alpha-\beta-1}) e^{-2x} + \gamma (\frac{3\beta-2\alpha+2}{\beta}) e^{-\frac{2}{\beta}(\alpha-1)x}}{k (\frac{\alpha-\beta}{\alpha-\beta-1}) e^{-2x} + \gamma e^{-\frac{2}{\beta}(\alpha-1)x}} = \dot{\phi}^2 - 1, \]  

(27)

also comparing Eqs. (1) and (24) one can write

\[ \rho_\Lambda = 3M_P^2 (\alpha H^2 + \beta \dot{H}) = \frac{V(\phi)}{\sqrt{1 - \dot{\phi}^2}}. \]  

(28)

From Eqs. (27) and (28), one can obtain the kinetic energy term and the tachyon potential energy as follows

\[ \dot{\phi}^2 = \frac{2}{3} \left[ k (\frac{\alpha-\beta}{\alpha-\beta-1}) e^{-2x} + \gamma (\frac{3\beta-2\alpha+2}{\beta}) e^{-\frac{2}{\beta}(\alpha-1)x} \right] \left[ k (\frac{\alpha-\beta}{\alpha-\beta-1}) e^{-2x} + \gamma e^{-\frac{2}{\beta}(\alpha-1)x} \right]^{-1/2}, \]  

(29)

\[ V(\phi) = \sqrt{3} M_P^2 \left[ k (\frac{\alpha-\beta}{\alpha-\beta-1}) e^{-2x} + \gamma (\frac{3\beta-2\alpha+2}{\beta}) e^{-\frac{2}{\beta}(\alpha-1)x} \right] \left[ k (\frac{\alpha-\beta}{\alpha-\beta-1}) e^{-2x} + \gamma e^{-\frac{2}{\beta}(\alpha-1)x} \right]^{-1/2}. \]  

(30)

From Eqs. (5) and (29), using \( \dot{\phi} = \phi' H \), we obtain the evolutionary form of the tachyon scalar field as

\[ \phi(a) - \phi(0) = \sqrt{\frac{2}{3}} \int_0^{\ln a} \frac{k (\frac{\alpha-\beta}{\alpha-\beta-1}) e^{-2x} + \gamma (\frac{\alpha-\beta}{\beta}) e^{-\frac{2}{\beta}(\alpha-1)x}}{k (\frac{\alpha-\beta}{\alpha-\beta-1}) e^{-2x} + \gamma e^{-\frac{2}{\beta}(\alpha-1)x}}^{1/2} dx. \]  

(31)
For the flat case if we set $k = 0$, using Eqs. (5), (7), assuming $\phi(0) = 0$ for the present time $t_0 = 0$, then Eqs. (30) and (31) reduce to

$$\phi(t) = \sqrt{\frac{2(\alpha - 1)}{3\beta}} t,$$

$$V(\phi) = 2M_P^2 \left(1 - \frac{2(\alpha - 1)}{3\beta}\right)^{1/2} \left(\frac{\beta}{\alpha - 1}\right) \frac{1}{\phi^2},$$

which are same as Eqs. (3.11) and (3.12) in [26], respectively.

### 3.3 New holographic K-essence model

The K-essence scalar field model of DE is also used to explain the observed late-time acceleration of the universe. The K-essence is described by a general scalar field action which is a function of $\phi$ and $\chi = \dot{\phi}^2/2$, and is given by [6, 7]

$$S = \int d^4x \sqrt{-g} p(\phi, \chi),$$

where $p(\phi, \chi)$ corresponds to a pressure density as

$$p(\phi, \chi) = f(\phi)(-\chi + \chi^2),$$

and the energy density of the field $\phi$ is

$$\rho(\phi, \chi) = f(\phi)(-\chi + 3\chi^2).$$

The EoS parameter for the K-essence scalar field is obtained as

$$\omega_K = \frac{p(\phi, \chi)}{\rho(\phi, \chi)} = \frac{\chi - 1}{3\chi - 1}. \tag{37}$$

Equating Eq. (37) with the new holographic EoS parameter (10), $\omega_K = \omega_\Lambda$, we find the solution for $\chi$

$$\chi = \frac{1}{3} \left(2k \frac{\alpha - \beta}{\alpha - \beta - 1} e^{-2x} + \gamma \frac{3\beta - \alpha + 1}{\beta} e^{-\frac{2}{\beta}(\alpha - 1)x}\right). \tag{38}$$

Using Eq. (38), $\dot{\phi}^2 = 2\chi$, and $\ddot{\phi} = \dot{\phi}'H$, we obtain the evolutionary form of the K-essence scalar field as

$$\phi(a) - \phi(0) = \frac{2}{3} \int_0^a H \left(\frac{2k \frac{\alpha - \beta}{\alpha - \beta - 1} e^{-2x} + \gamma \frac{3\beta - \alpha + 1}{\beta} e^{-\frac{2}{\beta}(\alpha - 1)x}}{k \frac{\alpha - \beta}{\alpha - \beta - 1} e^{-2x} + \gamma \frac{2\beta - \alpha + 1}{\beta} e^{-\frac{2}{\beta}(\alpha - 1)x}}\right)^{1/2} dx, \tag{39}$$

where $H$ is given by Eq. (5).

Comparing Eqs. (1) and (36), $\rho_\Lambda = \rho(\phi, \chi)$, using Eqs. (3) and (5), one can obtain a relation for $f(\phi)$ as follows

$$f(\phi) = \frac{3M_P^2(1 - 3\omega_\Lambda)^2}{2(1 - \omega_\Lambda)} \left[k \left(\frac{\alpha - \beta}{\alpha - \beta - 1}\right) e^{-2x} + \gamma e^{-\frac{2}{\beta}(\alpha - 1)x}\right], \tag{40}$$
where $\omega_\Lambda$ is given by Eq. (10).

For the flat case if we set $k = 0$, using Eqs. (7), (11), assuming $\phi(0) = 0$ for the present time $t_0 = 0$, then Eqs. (38), (39) and (40) reduce to

$$\chi = \frac{1}{3} \left( \frac{3\beta - \alpha + 1}{2\beta - \alpha + 1} \right),$$

$$\phi(t) = \left[ \frac{2}{3} \left( \frac{3\beta - \alpha + 1}{2\beta - \alpha + 1} \right) \right]^{1/2} t,$$

$$f(\phi) = 6M_{Pl}^2 \left[ \frac{2\beta - \alpha + 1}{(\alpha - 1)^2} \right] \frac{1}{\phi^2},$$

which are same as Eqs. (3.18), (3.19) and (3.20) in [26], respectively.

### 3.4 New holographic dilaton field

The dilaton scalar field model of DE is obtained from the low-energy limit of string theory. It is described by a general four-dimensional effective low-energy string action. The coefficient of the kinematic term of the dilaton can be negative in the Einstein frame, which means that the dilaton behaves as a phantom-type scalar field. However, in presence of higher-order derivative terms for the dilaton field $\phi$ the stability of the system is satisfied even when the coefficient of $\dot{\phi}^2$ is negative [9]. The pressure (Lagrangian) density and the energy density of the dilaton DE model is given by [9]

$$p_D = -\chi + c' e^{\lambda \phi} \chi^2,$$

$$\rho_D = -\chi + 3c' e^{\lambda \phi} \chi^2,$$

where $c'$ and $\lambda$ are positive constants and $\chi = \dot{\phi}^2/2$. The EoS parameter for the dilaton scalar field is given by

$$\omega_D = \frac{p_D}{\rho_D} = -1 + c' e^{\lambda \phi} \chi / 1 + 3c' e^{\lambda \phi} \chi.$$

Equating Eq. (46) with the new holographic EoS parameter (10), $\omega_D = \omega_\Lambda$, we find the following solution

$$c' e^{\lambda \phi} \chi = \frac{1}{3} \left( \frac{2k(\alpha - \beta - 1)}{\alpha - \beta} \right) e^{-2x} + \gamma \left( \frac{3\beta - \alpha + 1}{\beta} \right) e^{-\frac{2}{\beta}(\alpha - 1)x},$$

then using $\chi = \dot{\phi}^2/2$, we obtain

$$e^{\frac{\lambda \phi}{2}} \phi = \sqrt{\frac{2}{3c'}} \left( \frac{2k(\alpha - \beta - 1)}{\alpha - \beta} \right) e^{-2x} + \gamma \left( \frac{2\beta - \alpha + 1}{\beta} \right) e^{-\frac{2}{\beta}(\alpha - 1)x} \right)^{1/2}.$$

Using $\phi = \phi'H$ and integrating with respect to $x$ we get

$$e^{\frac{\lambda \phi(0)}{2}} = e^{\frac{\lambda \phi(0)}{2}} + \frac{\lambda}{\sqrt{6c'}} \int_0^{\ln a} \frac{1}{H} \left( \frac{2k(\alpha - \beta - 1)}{\alpha - \beta} \right) e^{-2x} + \gamma \left( \frac{2\beta - \alpha + 1}{\beta} \right) e^{-\frac{2}{\beta}(\alpha - 1)x} \right)^{1/2} dx,$$

where $H$ is given by Eq. (5). Finally the evolutionary form of the dilaton scalar filed is written as
\[ \phi(a) = \frac{2}{\lambda} \ln \left[ e^{\frac{\lambda \phi(0)}{2}} + \frac{\lambda}{\sqrt{6c'}} \int_0^{\ln a} \frac{1}{H} \left( \frac{2k}{\alpha - \beta - 1} e^{-2x} + \gamma \left( \frac{3\beta - \alpha + 1}{\beta} \right) e^{-\frac{2}{\beta}(\alpha - 1)x} \right)^{1/2} \right]. \]  

(50)

For the flat case if we set \( k = 0 \), using \( \dot{x} = H \), assuming \( \phi(0) = -\infty \) for the present time \( t_0 = 0 \), then Eq. (50) reduces to

\[ \phi(t) = \frac{2}{\lambda} \ln \left[ \frac{\lambda}{\sqrt{6c'}} \left( \frac{3\beta - \alpha + 1}{2\beta - \alpha + 1} \right)^{1/2} t \right], \]

(51)

which is same as Eq. (3.24) in [26].

### 4 Conclusions

We used a holographic DE model with new infrared cut-off introduced by Granda and Oliveros [25], which includes a term proportional to \( \dot{H} \). Contrary to the holographic DE based on the event horizon, this model depends on local quantities, avoiding in this way the causality problem. Hence the proposed new infrared cut-off can be considered as a viable phenomenological model of holographic density. We extended the work of Granda and Oliveros [26] to the non-flat case. However, some experimental data have implied that our universe is not a perfectly flat universe and that it possesses a small positive curvature \( (\Omega_k \sim 0.02) \) [28]. Although it is believed that our universe is flat, a contribution to the Friedmann equation from spatial curvature is still possible if the number of e-foldings is not very large [27]. We obtained the EoS parameter of the new holographic DE in the context of the non-flat universe which, contrary to the flat case, is time-dependent. This allows the model to transit from \( \omega_\Lambda > -1 \) to \( \omega_\Lambda < -1 \) as indicated by recent observations [30]. We established a correspondence between the new holographic DE energy density with the quintessence, tachyon, K-essence and dilaton energy density in the non-flat FRW universe. We reconstructed the potentials and the dynamics of these scalar field models which describe quintessence, tachyon, K-essence and dilaton cosmology. In the limiting case of a flat universe, the results are in exact agreement with those obtained by Granda and Oliveros [26].

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