Is gluonic color-spin locked phase stable?

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We study the gluonic color-spin locked (GCSL) phase in dense two-flavor quark matter. In this phase, the color and spatial rotational symmetries are spontaneously broken down to $SO(2)_{\text{diag}}$ with the generator being an appropriate linear combination of the color and rotational ones. The Meissner masses of gluons and the mass of the radial mode of the diquark field in the GCSL phase are calculated and it is shown that this phase is free from the chromomagnetic and Sarma instabilities in the whole parameter region where it exists. The GCSL phase describes an anisotropic color and electromagnetic superconducting medium. Because most of the initial symmetries in this phase are spontaneously broken, its dynamics is very rich.

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It is plausible that a color superconducting phase is realized in the interior of compact stellar objects [1]. Matter inside compact stars should be in $\beta$-equilibrium and be electrically and color neutral. Owing to these conditions and the non-negligible strange quark mass, a mismatch $\delta \mu$ between the Fermi momenta of the pairing quarks is inevitably induced. This is crucial for the quark-pairing dynamics [2].

Moreover, when the mismatch $\delta \mu$ increases, the Meissner masses of gluons become imaginary both in the gapped (2SC) and gapless (g2SC) two-flavor color superconducting phases [3]. A similar phenomenon has been found also in the three-flavor gapless color-flavor locked (gCFL) phase [4, 5, 6].

As was shown in Ref. [7], the chromomagnetic instability at least in the 2SC/g2SC phase is closely connected with the appearance of tachyonic plasmons in the physical gluonic channels. This supports the scenario with gluon condensates (gluonic phase) proposed in Refs. [8, 9].

Another problem in the g2SC phase is the existence of the Sarma instability, which corresponds to the negative mass squared in the $g^2$SC region [10]. In Refs. [11, 12] a similar instability is discussed.

One of the urgent problems in this field is to resolve these problems. Besides the gluonic phase [8, 9], a number of other candidates for the genuine ground state have been proposed [13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23].

In Ref. [24], a numerical analysis for two gluonic phases, the minimal cylindrical gluonic phase II and the gluonic color-spin locked (GCSL) one [10, 24], was performed. It is shown that the gluonic phases are actually realized in wide regions of the parameter space and they are energetically more favorable than the normal, $2\text{SC}/g2\text{SC}$, and the single plane wave Larkin-Ovchinnikov-Fulde-Ferrell (LOFF) [13, 28, 29] phases.

A formula for the Meissner screening mass was developed in Ref. [30] and also the Meissner masses in the minimal cylindrical gluonic phase II were examined. It was found, however, that the chromomagnetic instability is only partially resolved in the minimal cylindrical gluonic phase II.

In this paper, the stability of the GCSL phase is studied. We calculate the Meissner masses for gluons and also the mass of the diquark Higgs field at zero momentum. It is shown that the GCSL phase resolves both of the chromomagnetic and Sarma instabilities in the whole region where the phase exists.

As in Refs. [8, 9, 24], in the analysis, the gauged Nambu-Jona-Lasinio (NJL) model with two light quarks will be used. We neglect the current quark masses and the $(\bar{\psi}\psi)^2$-interaction channel. The Lagrangian density is then given by

$$\mathcal{L} = \bar{\psi}(i\slashed{D} + \mu_0\gamma^0)\psi + G_\Delta \left[(\bar{\psi}^C i\varepsilon^a\gamma_5\psi)^C(\bar{\psi} i\varepsilon^a\gamma_5\psi)^C\right] - \frac{1}{4} F_{\mu\nu}^a F^{a,\mu\nu},$$

(1)

with

$$D_\mu \equiv \partial_\mu - igA_\mu^a T^a, \quad F_{\mu\nu}^a \equiv \partial_\mu A_\nu^a - \partial_\nu A_\mu^a + gf^{abc} A_\mu^b A_\nu^c,$$

(2)

where $\varepsilon$ and $e^a$ are the totally antisymmetric tensors in the flavor and color spaces, respectively. We also intro-

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1 For the earlier works, see Refs. [23, 22]. The extension to the model with nonzero temperature was studied in Ref. [27].
duced gluon fields $A_μ^a$, the QCD coupling constant $g$, the generators $T^a$ of $SU(3)$ and the structure constants $f^{abc}$.

The quark field $\psi$ is a flavor doublet and color triplet. The charge-conjugate spinor is defined by $\psi^C \equiv C\bar{\psi}^T$ with $C = i\gamma^2\gamma^0$. We do not introduce the photon field.

On the other hand, the whole theory contains free electrons, although we do not show them explicitly in Eq. (1). The baryon chemical potential $\mu$ and $Q$ of the free energies with and without the chemical potential $\mu$.

In $\beta$-equilibrium, the chemical potential matrix $\mu_0$ for up and down quarks is $\mu_0 = \mu \mathbf{1} - \mu_c Q_{em}$, with $\mathbf{1} = \mathbf{1}_c \otimes \mathbf{1}_f$, and $Q_{em} \equiv \mathbf{1}_c \otimes \text{diag}(2/3, -1/3)_f$, where $\mu$ and $\mu_c$ are the quark and electron chemical potentials, respectively. (The baryon chemical potential $\mu_B$ is given by $\mu_B \equiv 3\mu$.) The subscripts $c$ and $f$ mean that the corresponding matrices act on the color and flavor spaces, respectively.

In the fermion one-loop approximation, the bare effective potential including both gluon and diquark condensates is given by

$$V_{\text{eff}}^{\text{bare}} = \frac{\Delta^2}{4G_\Delta} + \frac{1}{4} F^a_{\mu \nu} F^{a \mu \nu} - \frac{\mu_s^4}{12\pi^2} - \frac{1}{2} \int \frac{d^4P}{(2\pi)^4} \text{Tr} \ln S^{-1}_G,$$  

where $\Delta$ and $S^{-1}_G$ denote the diquark gap and the fermion propagator inverse with gluon condensates in the Nambu-Gor’kov space, respectively. We also added the free electron contribution. Since the bare potential has a divergence, a counter term is required. The dimensional regularization is nice for gauge theories, but not for the NJL-type models. We here take into account only differences of the free energies with and without the chemical potentials. Such effects should be physical. This is a similar idea for the subtraction scheme considered in Ref. [3]. Let us define the renormalized effective potential by

$$V_{\text{eff}}^R \equiv V_{\text{eff}}^{\text{bare}} - V_{\text{c.t.}},$$

with the counter term,

$$V_{\text{c.t.}} = -\frac{1}{2} \int \frac{d^4P}{i(2\pi)^4} \text{Tr} \ln S^{-1}_G \bigg|_{\mu=\mu_c=0,\Delta=0,\langle A^a\rangle \neq 0}.$$  

In this prescription, even if we use the regularization scheme with the sharp three-momentum cutoff $\Lambda$ for the loop integral, we can remove artificial mass terms of gluons like $\Lambda^2 A^a_\mu$.

In this paper, we study the GCSL phase [3, 24] with an ansatz for the gluon condensates,

$$\mu_s = \frac{\sqrt{3}}{2} g \langle A^8_\mu \rangle, \quad K \equiv g \langle A^4_\mu \rangle = g \langle A^8_\mu \rangle.$$  

The dynamics of the GCSL phase was analyzed in Ref. [24]. In order to find the dynamical solutions, we searched for minima of the (neutral) effective potential imposed the neutrality conditions. We also converted the four-diquark coupling constant $G_\Delta$ to the 2SC gap parameter $\Delta_0$ defined at $\delta\mu = 0$ and varied the values of $\Delta_0$ from the weak coupling regime ($\Delta_0 \sim 60$ MeV) to the strong coupling one ($\Delta_0 \sim 160$ MeV). The dynamical solutions of the GCSL phase are shown in Fig.1. In the analysis, we took realistic values $\mu = 400$ MeV and $\Delta = 653.3$ MeV. For the gluonic phase, it is required to specify the value of $\alpha_s \equiv g^2/(4\pi)$, because of the existence of the tree gluon potential term. In Ref. [31], the confinement scale is estimated to be not so large. In Fig.1 we took $\alpha_s = 1$ as a typical value. We then find that the GCSL phase exists in the region

$$V_{\text{eff}}^R \equiv V_{\text{eff}}^{\text{bare}} - V_{\text{c.t.}},$$
for $\alpha_s = 1$. While the values of $K$ tend to be small but nonzero near the endpoint of the GCSL phase, $\Delta_0 \sim 161$ MeV, the gap $\Delta$ and the gluon condensate $K$ are still big even in the vicinity of the starting point, $\Delta_0 \sim 76$ MeV. (See Fig. 1) These suggest that the first order phase transition occurs both at the starting and end points of the GCSL phase.

Note that the values of $\Delta_0$ at the starting point of the GCSL phase are rather sensitive to the choice of $\alpha_s$, because the tree gluon potential term is non-negligible, i.e., as the values of $\alpha_s$ are bigger, the windows are wider [24].

For reasonable parameter regions, say, $300$ MeV $< \mu < 500$ MeV and $653.3$ MeV $< \Lambda < 1$ GeV, the behaviors of the solutions are qualitatively unchanged. When we vary the values of $\mu$ from $300$ MeV to $400$ MeV and from $400$ MeV to $500$ MeV, the windows where the GCSL phase is energetically stabler than the minimal cylindrical gluonic phase II are about 15% wider, respectively. On the other hand, when the value of $\Lambda$ is taken to $\Lambda = 1$ GeV from $\Lambda = 653.3$ MeV, the window is about 30% wider.

We now analyze the Meissner screening masses and the mass of the diquark Higgs field in the GCSL phase by using the formulae derived in Ref. [30].

$$\frac{\partial^2 V_{\text{eff}}^{R}}{\partial \Delta^2} = \frac{1}{2G_{\Delta}} - \frac{1}{2} \sum_{\tau = \pm} \sum_{E_{\tau}^f \neq E_{\tau}^i} \int \frac{d^3p}{(2\pi)^3} \frac{\theta(E_{\tau}^f) - \theta(E_{\tau}^i)}{E_{\tau}^f - E_{\tau}^i} (U^\dagger \hat{\Gamma}_\Delta U)_{ij} (U^\dagger \hat{\Gamma}_\Delta U)_{ji}$$

$$- \frac{1}{2} \sum_{\tau = \pm} \sum_{E_{\tau}^f = E_{\tau}^i} \int \frac{d^3p}{(2\pi)^3} \delta(E_{\tau}^f) (U^\dagger \hat{\Gamma}_\Delta U)_{ij} (U^\dagger \hat{\Gamma}_\Delta U)_{ji},$$

and

$$\frac{\partial^2 V_{\text{eff}}^{R}}{\partial A_{\mu}^a \partial A_{\nu}^b} = \Pi_{\text{tree}}^{\mu \nu} - \frac{g^2}{2} \sum_{\tau = \pm} \sum_{E_{\tau}^f \neq E_{\tau}^i} \int \frac{d^3p}{(2\pi)^3} \frac{\theta(E_{\tau}^f) - \theta(E_{\tau}^i)}{E_{\tau}^f - E_{\tau}^i} (U^\dagger \hat{\Gamma}_{A}^\alpha U)_{ij} (U^\dagger \hat{\Gamma}_{c\nu} U)_{ji}$$

$$- \frac{g^2}{2} \sum_{\tau = \pm} \sum_{E_{\tau}^f = E_{\tau}^i} \int \frac{d^3p}{(2\pi)^3} \delta(E_{\tau}^f) (U^\dagger \hat{\Gamma}_{A}^\alpha U)_{ij} (U^\dagger \hat{\Gamma}_{c\nu} U)_{ji} - \text{(counter term)},$$

where $E_{\tau}^f, E_{\tau}^i, n$ denote the energy eigenvalues of $S_{\gamma}^{-1}$. We also defined the tree contribution

$$\Pi_{\text{tree}}^{\mu \nu} = g^2 f^{a_1 a_2 a_3} f^{a_4 a_2 a_3} A_{a_2 \mu} A_{a_3 \nu}$$

$$+ g^2 f^{a_1 a_2 a_3} f^{a_4 a_2 a_3} g_{\mu \nu} A_{a_2 A_{a_3} \lambda}$$

$$+ g^2 f^{a_1 a_2 a_3} f^{a_4 a_2 a_3} \mu_{a_2 \nu} A_{a_3 \nu},$$

and the transformed vertices

$$\hat{\Gamma}_\Delta = \epsilon^b \left( \begin{array}{cc} 0 & 1 \\ -1 & 0 \end{array} \right),$$

and

$$\hat{\Gamma}_{A}^a = \left( \begin{array}{cc} \gamma^0 \gamma^\mu T^a & 0 \\ 0 & -\gamma^\mu \gamma^0 (T^a) T \end{array} \right),$$

in the Nambu-Gor’kov space. For the Dirac’s $\delta$-function, a lot of expressions are known. In our calculation, we use $\delta(x) = \lim_{n \to \infty} \sqrt{n/\pi} \exp(-n|x|^2)$. We also crosscheck our calculation by using the numerical derivative of the effective potential.

Let us introduce the notations

$$M_\Delta^2 = \frac{\partial^2 V_{\text{eff}}^{R}}{\partial \Delta^2}, \quad (M^2)_{a, b, j} = \frac{\partial^2 V_{\text{eff}}^{R}}{\partial A_{a \mu}^j \partial A_{b \nu}^j},$$

and

$$(M^2)_{a, b, j, n}, \quad (M^2)_{a, b, c, j, n},$$

for the eigenvalues of the Meissner mass (2 $\times$ 2 and 3 $\times$ 3) matrices, where $(M^2)_{a, b, j, 1} \leq (M^2)_{a, b, j, 2}$ and so on. Notice that there appear mixing terms among the space components of gluons owing to the symmetry breaking structure of the GCSL phase [30].

$$SU(3)_c \times U(1)_{em} \times SO(3)_\text{rot} \Delta^K SO(2)_{\text{diag}},$$

where the unbroken $SO(2)_{\text{diag}}$ symmetry consists of an appropriate linear combination of the initial color and rotational symmetries. At least the mixing terms which exist in the tree contribution should be taken into account. To reduce our labor, we might ignore the other types of the mixing terms.
We depict the numerical results in Figs 2-4 in the units of $M_\Delta^2 \equiv 4p^2/\pi^2$ and $M_\alpha^2 \equiv 4\alpha_s\mu^2/(3\pi)$. In the whole region where the GCSL phase exists, it turns out that there are six vanishing masses within the numerical precision, $(M^2)_1=0$, $(M^2)_2=0$, $(M^2)_3=0$, $(M^2)_4=0$, $(M^2)_5=0$, $(M^2)_6=0$. The behavior of $(M^2)_8$ in Fig 3 looks peculiar. Although the numerical calculation itself is robust, the reason for varying the values of $\alpha_s$, $\mu$, and $\Lambda$, the reason is unclear at present. It is noticeable that there are six vanishing masses within the numerical precision, $(M^2)_1=0$, $(M^2)_2=0$, $(M^2)_3=0$, $(M^2)_4=0$, $(M^2)_5=0$, $(M^2)_6=0$. Three among the six should correspond to the gauge fixing fields. One might guess the other three should be connected with the Nambu-Goldstone (NG) bosons owing to the global symmetry breaking $U(1) \times SO(3) \to SO(2)$. (Note that we did not introduce the photon field.) This is unclear, however, because the kinetic terms are quite important for the abnormal number of the NG bosons. This question will be answered elsewhere.

Since the tree contributions are proportional to $K^2$, the $\alpha_s$-dependence of the Meissner masses are numerically non-negligible. Nevertheless, the results are qualitatively unchanged for $0.75 < \alpha_s < 1.25$. We have also checked the positivity of the Meissner masses and the diquark Higgs mass in the reasonable parameter regions, $300 \text{ MeV} < \mu < 500 \text{ MeV}$ and $653.3 \text{ MeV} < \Lambda < 1 \text{ GeV}$.

In summary, we analyzed the Meissner screening masses and the mass of the diquark Higgs field in the GCSL phase. We showed that unlike the minimal cylindrical gluonic phase II the GCSL phase resolves both of the chromomagnetic and Sarma instabilities in the whole parameter region where the phase exists. In this sense, the GCSL phase describes a stable vacuum against the fluctuations of the diquark and gluonic channels. These results are qualitatively unchanged for realistic values of $\alpha_s$, say, $0.75 < \alpha_s < 1.25$, and for the reasonable parameter regions, $300 \text{ MeV} < \mu < 500 \text{ MeV}$ and $653.3 \text{ MeV} < \Lambda < 1 \text{ GeV}$.

Why can the GCSL phase resolve the chromomagnetic instability? In the minimal cylindrical gluonic phase II only with $B \equiv \langle A_0^4 \rangle \neq 0$, the squared Meissner mass of the transverse mode of the 4th gluon becomes negative at a certain value of $\Delta_\delta$. Therefore, the GCSL phase with $\langle A_0^4 \rangle \neq 0$ and $\langle A_2^2 \rangle \neq 0$ can be energetically stabler than the minimal cylindrical gluonic phase II and resolve this instability. Actually, we have found that the GCSL phase is energetically more favorable over the minimal cylindrical gluonic phase II and the single plane wave LOFF phase in a wide parameter region. After the rearrangement of the vacuum, the curvatures of the effective potential, i.e., the Meissner masses for $A_2^2$ and $A_0^4$, are likely to be positive. Moreover, unlike in the minimal cylindrical gluonic phase II, all of the gluonic fluctuations except for $A_2^2$ and $A_0^4$ have big tree terms proportional to $K^2$. Thus the squared Meissner masses tend to be positive.

We also comment on the discrepancy between the
starting points of the GCSL phase and the chromomagnetic instability of $A_{xy}$ in the minimal cylindrical gluonic phase II [24, 30]. The structures of the tree gluon potential between the two phases are different. In addition, the values of the dynamical solutions $B$ and $K$ are the same order, but numerically disagree [24]. It is thus possible that the discrepancy numerically occurs.

The GCSL phase has other noticeable features; (1) Both of the chromoelectric and chromomagnetic fields are spontaneously generated. (2) The GCSL phase describes an anisotropic medium. (3) The medium is a color and electromagnetic superconductor.

In the confinement picture, the symmetry breaking structure of the GCSL phase and the cylindrical gluonic phase I is the same [9]. How about the relation between them? In addition, the question whether or not the abnormal number of the NG bosons occurs is still open. Last but not least, is the GCSL phase the global vacuum in dense two-flavor quark matter? We hope to return to these problems elsewhere.

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