SMOOTHLY BROKEN POWER LAW SPECTRA OF GAMMA-RAY BURSTS

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ABSTRACT

A five-parameter expression for a smoothly broken power law is presented. It is used to fit Gamma-Ray Burst spectra observed by BATSE. The function is compared to previously used four-parameter functions, such as a sharply broken power law and the Band et al. (1993) function. The presented function exists as a WINGSPAN routine at http://www.astro.su.se/~felix/wing.html

KEYWORDS: Gamma-ray bursts.

1. INTRODUCTION

Studies of the spectral continuum of Gamma-Ray Bursts (GRBs) can give important clues for the understanding of the underlying emission processes responsible for the observed radiation. A number of studies have been done (Schaefer et al. 1992, 1994, Band et al. 1993, Ford et al. 1994). The burst emission is of a non-thermal character and most of the energy is released in the gamma-ray band (0.1 – 1 MeV). Spectra from missions prior to the Compton Gamma-Ray Observatory (CGRO) with its Burst and Transient Source Experiment (BATSE) were often fitted with an “optically thin thermal bremsstrahlung” photon spectrum, \( N_E(E) \propto E^{-1}e^{-E/E_0} \), combined with a power law at larger energies. Schaefer et al. also used a broken power law in fitting BATSE bursts. Later, Band et al. showed that nearly all time-integrated BATSE spectra were well fitted by the four-parameter Band function consisting of two power laws, smoothly joined by an exponential roll-over. Below, we introduce a five parameter function, which consists of two power laws smoothly joined together. The sharpness of the turn-over is determined by the fifth parameter.

2.1 SMOOTHLY BROKEN POWER LAW

To model the time-integrated, background-subtracted photon count spectra, we want a function describing a smoothly broken power law, having a low energy power law with spectral index \( \alpha \), smoothly transferred into a high energy power law with spectral index \( \beta \). We follow the idea in Preece et al. (1996) and define the following properties: its logarithmic derivative varies smoothly from \( \alpha \) to \( \beta \), with the transition described by a hyperbolic tangent function.
[Table 1.] The bursts sample. The bursts are identified by their date and trigger number (see the Current BATSE Gamma-Ray Burst Catalogue, Meegan et al.). \( \Delta t \) and \( \Delta \text{Energy} \) are the chosen time and energy ranges used in the study.

| Date + Trigger No. | LAD | \( \Delta t \) (s) | \( \Delta \text{Energy} \) (keV) | Date + Trigger No. | LAD | \( \Delta t \) (s) | \( \Delta \text{Energy} \) (keV) |
|--------------------|-----|------------------|-------------------------------|--------------------|-----|------------------|-------------------------------|
| 910430(# 130)     | 6   | 0-19             | 29-1799                      | 940217(# 2831)     | 0   | 1-35             | 33-1956                      |
| 910503(# 143)     | 6   | 0-10             | 30-1766                      | 950403(# 3491)     | 3   | 0-23             | 30-1649                      |
| 911127(# 1122)    | 1   | 0-38             | 29-1774                      | 970429(# 6198)     | 4   | 1-13             | 27-1950                      |
| 920210(# 1385)    | 5   | 0-49             | 29-1988                      | 980125(# 6581)     | 0   | 45-59            | 24-1980                      |
| 930506(# 2329)    | 3   | 0-14             | 30-1756                      | 980203(# 6587)     | 1   | 0-34             | 25-1855                      |

\[
\frac{d \log N_E}{d \log E} = \xi \tanh \left( \frac{\log(E/E_0)}{\delta} \right) + \phi, \tag{1}
\]

where \( N_E \) is the photon flux, \( E \) is the photon energy, \( E_0 \) is the break energy, \( \delta \) is the width of the transition, \( \xi = (\beta - \alpha)/2 \), and \( \phi = (\beta + \alpha)/2 \). Integrating (1) gives

\[
N_E(E) = C \left( \frac{E}{E_n} \right)^\phi \left[ \frac{\cosh \left( \frac{\log(E/E_0)}{\delta} \right)}{\cosh \left( \frac{\log(E_n/E_0)}{\delta} \right)} \right]^{\xi \delta \ln 10}, \tag{2}
\]

where the width of the transition, in linear energy, is \( \Delta E = E_0(10^\delta - 1) \). The normalisation is set by the constant \( C \), which is the value of \( N_E(E = E_n) \). The function is described by four parameters besides the normalisation; the two power law indices, \( \alpha \) and \( \beta \), the break energy, \( E_0 \), and the energy range over which the spectrum changes from one power law to the other, \( \delta \) or \( \Delta E \). The advantage of this function over the Band function, for instance, which is described by four parameters, the normalisation, \( \alpha \), \( \beta \), and a parameter related to the break energy, is that the width of the transition region can be specified.

3. GRB SPECTRA OBSERVED BY BATSE

We have studied a sample of 10 strong GRBs, for which the Smoothly Broken Power Law (SBPL) above gives reasonable fits. They are used to illustrate the comparison between the Band model and the SBPL model. In Table 1, the set of bursts is displayed, together with the time and energy range used and the number of the Large Area Detector (LAD) used. The following three models were used:
The results of the modelling of the bursts. $A$ is the normalisation of the spectrum, $\alpha$ and $\beta$ are the low and high energy power law photon indices, $E_0$ is the break energy, and $\Delta E$ is the size of the turn-over spectral range. The goodness of fit is given by the $\chi^2$-value together with the number of degrees of freedom, dof. See the text for details.

| GRB # | Model | $A/10^{-2}$ | $\alpha$ | $E_0$ (keV) | $\beta$ | $\Delta E$ (keV) | $\chi^2$/dof |
|-------|-------|-------------|----------|-------------|--------|-----------------|--------------|
| 130   | BPL   | 0.97±0.01   | $-1.46\pm0.02$ | 227±15 | $-2.33\pm0.08$ | 118/110         |
|       | Band  | 1.29±0.05   | $-1.21\pm0.05$ | 395±50 | $-2.5\pm0.2$  | 128/110         |
|       | SBPL  | 0.97±0.01   | $-1.45\pm0.02$ | 230±20 | $-2.35\pm0.10$ | $30^{+60}_{-30}$ | 118/109 |
| 143   | BPL   | 9.40±0.03   | $-1.06\pm0.01$ | 382±7  | $-2.09\pm0.02$ | 236/109         |
|       | Band  | 10.9±0.07   | $-0.91\pm0.01$ | 785±27 | $-2.8\pm0.3$  | 428/109         |
|       | SBPL  | 9.41±0.03   | $-1.06\pm0.01$ | 405±13 | $-2.15\pm0.04$ | 110±37          | 229/108 |
| 1122  | BPL   | 1.38±0.01   | $-1.48\pm0.02$ | 135±3  | $-2.38\pm0.03$ | 112/108         |
|       | Band  | 2.57±0.10   | $-1.05\pm0.03$ | 163±9  | $-2.51\pm0.06$ | 84/108          |
|       | SBPL  | 1.40±0.01   | $-1.35\pm0.05$ | 143±9  | $-2.58\pm0.10$ | 112±40          | 83/107 |
| 1385  | BPL   | 0.853±0.005 | $-1.02\pm0.01$ | 285±8  | $-2.23\pm0.04$ | 180/113         |
|       | Band  | 1.14±0.02   | $-0.73\pm0.02$ | 390±20 | $-2.8\pm0.2$  | 127/113         |
|       | SBPL  | 0.87±0.01   | $-0.90\pm0.04$ | 425±80 | $-2.9\pm0.3$  | 610±310         | 122/112 |
| 2329  | BPL   | 8.30±0.02   | $-1.14\pm0.01$ | 300±9  | $-1.68\pm0.01$ | 161/108         |
|       | Band  | 9.38±0.07   | $-1.03\pm0.01$ | 930±50 | $-1.75\pm0.02$ | 150/108         |
|       | SBPL  | 8.36±0.03   | $-1.12\pm0.01$ | 322±19 | $-1.73\pm0.03$ | 230±75          | 132/107 |
| 2831  | BPL   | 1.31±0.02   | $-0.86\pm0.02$ | 424±13 | $-2.07\pm0.03$ | 166/112         |
|       | Band  | 1.55±0.02   | $-0.61\pm0.03$ | 635±35 | $-3.0\pm0.4$  | 122/112         |
|       | SBPL  | 1.32±0.02   | $-0.75\pm0.06$ | 760±210| $-3.1\pm0.6$  | 1270±880        | 118/111 |
| 3491  | BPL   | 4.08±0.02   | $-1.65\pm0.01$ | 148±4  | $-2.20\pm0.02$ | 248/106         |
|       | Band  | 5.28±0.09   | $-1.47\pm0.02$ | 390±20 | $-2.38\pm0.05$ | 234/106         |
|       | SBPL  | 4.09±0.02   | $-1.62\pm0.02$ | 183±13 | $-2.37\pm0.06$ | 145±50          | 225/105 |
| 6198  | BPL   | 8.55±0.03   | $-1.29\pm0.01$ | 198±3  | $-2.23\pm0.02$ | 277/113         |
|       | Band  | 11.90±0.15  | $-1.04\pm0.01$ | 325±10 | $-2.53\pm0.05$ | 201/113         |
|       | SBPL  | 8.68±0.04   | $-1.24\pm0.02$ | 246±12 | $-2.51\pm0.06$ | 204±43          | 175/112 |
| 6581  | BPL   | 2.86±0.01   | $-1.56\pm0.01$ | 195±11 | $-2.07\pm0.04$ | 112/114         |
|       | Band  | 3.39±0.07   | $-1.43\pm0.02$ | 625±70 | $-2.20\pm0.10$ | 121/114         |
|       | SBPL  | 2.87±0.02   | $-1.55\pm0.01$ | 212±20 | $-2.12\pm0.07$ | 65±55           | 111/113 |
| 6587  | BPL   | 5.35±0.01   | $-1.26\pm0.01$ | 194±3  | $-2.08\pm0.01$ | 356/111         |
|       | Band  | 7.35±0.08   | $-1.01\pm0.01$ | 338±10 | $-2.25\pm0.03$ | 261/111         |
|       | SBPL  | 5.44±0.02   | $-1.20\pm0.02$ | 220±7  | $-2.24\pm0.04$ | 170±30          | 241/110 |

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(1) A sharply Broken Power Law (BPL) consisting of two power laws connected at the break energy with three parameters besides the normalisation: the photon indices of the low and high energy power laws, $\alpha$, $\beta$, and the break energy, $E_0$.

(2) The Band et al. (1993) function (Band) with three parameters besides the normalisation: $\alpha$, $E_0$, $\beta$. The spectrum below $E = (\alpha - \beta)E_0$ is an exponentially cut off power law, $N_E(E) \propto E^\alpha e^{-E/E_0}$. A second power law, $N_E(E) \propto E^\beta$, is connected to this function above this energy. Note that $E_{\text{peak}} = (2 + \alpha)E_0$ is usually used to characterise the break in the Band function.

(3) The Smoothly Broken Power Law (SBPL) given by Equation (2) with its four parameters besides the normalisation. The results of the fitting are shown in Table 2. The general trends of the sample studied are the following.

(1) The Band model gives the largest break energy (except for # 2831, which, however, has large errors).

(2) The SBPL model has a somewhat larger break energy than the BPL model.

(3) The low energy power law in the Band model is harder than the one in the SBPL model, which is marginally harder than the one in the BPL model.

(4) The high energy power law in the BPL model is the hardest and in the Band model it is the softest.

The main difference between the SBPL model and the Band model is the extra variable parameter describing the energy range of the changing of the power law index. As bursts can have an actual turn-over that is sharper than the one given by the fixed exponential turn-over in the Band function, which is determined solely by $E_0$, the resulting fits in Table 2 differ. The smaller the ratio between $\Delta E$ and $E_0$ in the SBPL model is, the larger the difference is between $E_0$ in the Band model and in the SBPL model. The best examples of this trend are # 6581 and # 143 compared to # 1385 and # 1122. The smooth turn-over in the Band model allows, among other things, the low energy power law to be harder than what is found by the BPL model. However, extra parameters are not always advantageous. In some cases, not shown in Table 2, the $\Delta E$ parameter in the SBPL model cannot be constrained.

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