THE SPONTANEOUS GENERATION OF MAGNETIC FIELDS AT HIGH TEMPERATURE IN A SUPERSYMMETRIC THEORY

Vadim Demchik\textsuperscript{a} and Vladimir Skalozub\textsuperscript{a}

\textsuperscript{a} Dniepropetrovsk National University, 49050 Dniepropetrovsk, Ukraine
e-mail: vadimdi@yahoo.com; Skalozub@ff.dsu.dp.ua

ABSTRACT

The spontaneous generation of magnetic and chromomagnetic fields at high temperature in the minimal supersymmetric standard model (MSSM) is investigated. The consistent effective potential including the one-loop and the daisy diagrams of all bosons and fermions is calculated and the magnetization of the vacuum is observed. The mixing of the generated fields due to the quark and s-quark loop diagrams and the role of superpartners are studied in detail. It is found that the contribution of these diagrams increases the magnetic and chromomagnetic field strengths as compared with the case of a separate generation of fields. The magnetized vacuum state is found to be stable due to the magnetic masses of gauge fields included in the daisy diagrams. Applications of the results obtained are discussed. A comparison with the standard model case is done.

Introduction

Possible existence of strong magnetic fields in the early universe is one of the most interesting problems in high energy physics. Different mechanisms of the fields producing at different stages of the universe evolution were proposed. These mechanisms as well as the role of strong magnetic fields have been discussed in the surveys \cite{1}-\cite{3}. In particular, primordial magnetic fields, being implemented in a cosmic plasma, may serve as the seed ones for the present extra-galaxy fields.

The spontaneous vacuum magnetization at high temperature is one of the mechanisms mentioned. It was already investigated both in pure $SU(2)$ gluodynamics \cite{4}-\cite{6} and in the standard model (SM) \cite{7} where the possibility of this phenomenon has been shown. The stability of the magnetized vacuum state was also studied \cite{7}. The magnetization takes place for the non-abelian gauge fields due to a vacuum dynamics. In fact, this is one of the distinguishable features of asymptotically free theories \cite{3}, \cite{8}. In Refs. \cite{4}-\cite{6} the fermion contributions were not taken into consideration. However, at high temperature they affect the vacuum considerably. Quark posses both the electric and the color charges and therefore the quark loops change the strengths of the simultaneously generated magnetic and chromomagnetic fields \cite{7}.
In a supersymmetric theory new peculiarities should be accounted for. First is an influence of superpartners. These particles having a low spin have to decrease the generated magnetic field strengths. Second, s-quarks also possess the electric and the color charges, so the interdependence of magnetic and chromomagnetic fields is expected to be stronger as well as the fields due to their vacuum loops. Because of this mixing some specific configurations of the fields must be produced at high temperature.

In the present paper the spontaneous vacuum magnetization is investigated in the minimal supersymmetric standard model (MSSM) of elementary particles. All boson and fermion fields are taken into consideration. In the MSSM there are two kinds of non-abelian gauge fields - the $SU(2)$ weak isospin gauge fields responsible for weak interactions and the $SU(3)$ gluons mediating the strong interactions. Magnetic and chromomagnetic fields are related to these symmetry groups, respectively. To elaborate this problem we calculate the effective potential (EP) including the one-loop and the daisy diagram contributions in the constant abelian chromomagnetic and magnetic fields, $H_c = \text{const}$ and $H = \text{const}$, at high temperatures. The values of the generated fields strengths are found as the minimum position of the EP in the field strength plane $(H, H_c)$.

Let us note the advantages of the used approximation. The EP of the background abelian magnetic fields is a gauge fixing independent one, while the daisy diagrams account for the most essential long-range corrections at high temperature. Therefore, such a type of EP includes the leading and the next-to-leading terms in the coupling constants. Moreover, as it was shown in [6, 9], the daisy diagrams of the charged gluons and the W-bosons with their magnetic masses taken into consideration make the vacuum with nonzero magnetic fields stable at high temperatures. The stability reflects the consistency of the approximation. The EP of this type was used recently in investigations of the electroweak phase transition in an external hypercharge magnetic field [10] and the spontaneous generation of magnetic and chromomagnetic fields in the SM [7]. The obtained results are in a good agreement with the non-perturbative calculations carried out in [11, 12]. This approximation will be used in what follows.

The abelian hypercharge magnetic field is not generated spontaneously. So, in our investigation we shall consider the non-abelian constituent of the magnetic field coming from $SU(2)$ gauge group. The generation mechanisms of the hypermagnetic field were studied in [9, 13]. Below it will be shown that at high temperatures either strong magnetic or the chromomagnetic fields are generated in the MSSM, similarly to the SM. The additional to the SM sector of the MSSM, s-particle sector, does not suppress this effect. It just decreases the strengths of generated fields. These fields are stable in the approximation adopted due to the magnetic masses $m^2_{\text{transversal}} \sim (gH)^{1/2}T$ of the gauge field transversal modes [14]. In this way the consistent picture of the magnetized vacuum state is derived.

The contents of this paper are as follows. In Sect. 2 the contributions of all bosons and fermions to the EP $v'(H, T)$ of external magnetic and chromomagnetic
fields are calculated in a form convenient for numeric investigations. In Sect. 3 the field strengths are calculated. Discussion and concluding remarks are given in Sect. 4.

2. Basic Formulae

The full Lagrangian of the MSSM can be written as (13)

$$\mathcal{L} = L_{gs} + L_{\bar{g}s} + L_{\text{leptons}} + L_{\text{sleptons}} + L_{\text{quarks}} + L_{\text{squarks}}$$

$$+ L_{\text{Higgs}} + L_{\text{higgsino}} + L_{\text{int}} + L_{\text{SSB}} + L_{gf}. \quad (1)$$

Here, $L_{gs}$ and $L_{\bar{g}s}$ is the kinetic part of gauge bosons and gauginos, correspondingly; $L_{\text{leptons}}, L_{\text{sleptons}}, L_{\text{quarks}}$ and $L_{\text{squarks}}$ give the kinetic part of the matter (fermions and s-fermions) fields and their interaction Lagrangians; $L_{\text{Higgs}}$ is the kinetic part and Higgs interactions with gauge bosons and gauginos; $L_{\text{int}}$ contains the interaction terms; $L_{\text{SSB}}$ is the soft-symmetry breaking (SSB) part; $L_{gf}$ is the gauge fixing terms.

In the MSSM there are mixings in the higgsino and gaugino sector resulting in the presence of chargino (mixing of charged higgsino and gaugino) and neutralino (mixing of neutral higgsino and gaugino) in the theory. Since the electrical and color neutral particles are not interacting with magnetic and chromomagnetic fields, we will not take the neutralino into consideration.

The MSSM Lagrangian of the gauge boson sector is (see [13])

$$L_{gs} = -\frac{1}{4} F_{\mu\nu}^\alpha F^\mu_\alpha - \frac{1}{4} G_{\mu\nu} G^{\mu\nu} - \frac{1}{4} F_{\mu\nu}^\alpha F_\alpha^{\mu\nu}, \quad (2)$$

where the standard notation is introduced

$$F_{\mu\nu}^\alpha = \partial_\mu A_\nu^\alpha - \partial_\nu A_\mu^\alpha + g\epsilon^{abc} A_b^\mu A_c^\nu,$$

$$G_{\mu\nu} = \partial_\mu B_\nu - \partial_\nu B_\mu,$$

$$F_{\mu\nu}^\alpha = \partial_\mu A_\nu^\alpha - \partial_\nu A_\mu^\alpha + g s^{abc} A_b^\mu A_c^\nu.$$ 

The fields corresponding to the gauge $W^-, Z-$bosons and photons, respectively, are

$$W_\mu^\pm = \frac{1}{\sqrt{2}}(A_\mu^1 \pm i A_\mu^2), \quad (4)$$

$$Z_\mu = \frac{1}{\sqrt{g^2 + g'^2}}(g A_\mu^3 - g' B_\mu),$$

$$A_\mu = \frac{1}{\sqrt{g^2 A_\mu + B_\mu}}$$

and $A_\mu^\alpha$ is gluon field.
As usually, the introduction of gauge fields is being done by replacing all derivatives in the Lagrangian with the covariant ones,

\[ \partial_\mu \rightarrow D_\mu = \partial_\mu + ig \frac{\tau^\alpha}{2} A_\mu^\alpha + ig_s \frac{\lambda^\alpha}{2} A_\mu^\alpha. \] 

Here \( \tau^\alpha \) and \( \lambda^\alpha \) stand for the Pauli and the Gell-Mann matrices, respectively.

In the \( SU(2) \) sector there is only one magnetic field, the third projection of the gauge field. In the \( SU(3)_c \) sector there are two possible chromomagnetic fields connected with the third and the eighth generators of the group.

For simplicity, in what follows we shall consider the field associated with the third generator of the \( SU(3)_c \).

To introduce the interaction with classical magnetic and chromomagnetic fields we split the potentials in two parts:

\[ A_\mu = \bar{A}_\mu + A^R_\mu, \]
\[ A^\prime_\mu = \bar{A}^\prime_\mu + A^{R^\prime}_\mu, \]

where \( A^R \) and \( A^{R^\prime} \) describe the radiation fields and \( \bar{A} = (0, 0, H x^1, 0) \) and \( \bar{A}^\prime = (0, 0, H_3 x^1, 0) \) correspond to the constant magnetic and chromomagnetic fields directed along the third axes in the space and in the internal color and isospin spaces.

To construct the total EP we used the general relativistic renormalizable gauge which is set by the following gauge fixing conditions [16]:

\[ \partial_\mu W^{\pm \mu} \pm ie \bar{A}_\mu W^{\pm \mu} \mp ig_c \frac{\phi_\pm}{2} = C^\pm(x), \]
\[ \partial_\mu Z^\mu - i \xi (g^2 + g'^2)^{1/2} \phi_c \phi_Z = C^Z(x), \]
\[ \partial_\mu A^\mu + ig_s \bar{A}_\mu = C(x), \]

where \( e = g \sin \theta_W, \tan \theta_W = g'/g, \phi_\pm \) and \( \phi_Z \) are the Goldstone fields, \( \xi \) and \( \xi' \) are the gauge fixing parameters, \( C^\pm \) and \( C^Z \) are arbitrary functions and \( \phi_c \) is the value of the scalar field condensate. Setting \( \xi, \xi' = 0 \) we choose the unitary gauge. In the restored phase the scalar field condensate \( \phi_c = 0 \) and the equations (7) are simplified.

The values of the macroscopic magnetic and chromomagnetic fields generated at high temperature will be calculated by minimization of the thermodynamics potential \( \Omega \) which is introduced as follows

\[ \Omega = -\frac{1}{\beta} \log Z, \]
\[ Z = Tr \exp(-\beta \mathcal{H}), \]

where \( Z \) is the partition function, and \( \mathcal{H} \) is the Hamiltonian of the system. The trace is calculated over all physical states.
To obtain the EP one has to rewrite (8) as a sum in quantum states calculated near the nontrivial classical solutions $A^{ext}$ and $A^{ext}$. This procedure is well-described in the literature (see, for instance, [6, 17, 18]) and the result can be written in the form

$$V = V^{(1)}(H, H_3, T) + V^{(2)}(H, H_3, T) + ... + V_{daisy}(H, H_3, T) + ...,$$

where $V^{(1)}$ is the one-loop EP; the other terms present the contributions of two-, three-, etc. loop corrections.

Among these terms there are some responsible for the dominant contributions of long distances at high temperature - so-called daisy or ring diagrams (see, for example, [17]). This part of the EP, $V_{daisy}(H, H_3, T)$, is nonzero in case when massless states appear in a system. The ring diagrams have to be calculated when the vacuum magnetization at finite temperature is investigated. In fact, one first must assume that the fields are nonzero, calculate EP $V(H, H_3, T)$ and after that check whether its minimum is located at nonzero $H$ and $H_3$. On the other hand, if one investigates problems in the applied external fields, the charged fields become massive with the masses depending on $\sim (gH)^{1/2}, \sim (g_s H_3)^{1/2}$ and have to be omitted.

The one-loop contribution to EP is given by the expression

$$V^{(1)} = -\frac{1}{2} Tr \log G^{ab},$$

where $G^{ab}$ stands for the propagators of all quantum fields $W^\pm, A, \ldots$ in the background fields $H$ and $H_3$. In the proper time formalism, $s$-representation, the calculation of the trace can be carried out in accordance with the formula [19]

$$Tr \log G^{ab} = -\int_0^\infty ds \frac{1}{s} tr \exp(-isG_{ab}^{-1}).$$

Details of calculations based on the $s$-representation and formula (13) can be found in [20, 21, 22].

To incorporate the temperature into this formalism in a natural way we make use the method of [20] which connects the Green functions at zero temperature with the Matsubara Green functions,

$$G^{ab}_k(x, x'; T) = \sum_{\infty}^{+\infty} (-1)^{\sigma_k + [x]} G^{ab}_k(x - [x] \beta u, x' - n\beta u),$$

where $G^{ab}_k$ is the corresponding function at $T = 0$, $\beta = 1/T$, $u = (0, 0, 0, 1)$, $[x]$ denotes an integer part of $x_4/\beta$, $\sigma_k = 1$ in the case of physical fermions and $\sigma_k = 0$ for boson and ghost fields. The Green functions in the right-hand side of (13) are the matrix elements of the operators $G_k$ computed in the states $|x', a\rangle$ at $T = 0$, and in the left-hand side the operators are averaged over the states with
$T \neq 0$. The corresponding functional spaces $U^0$ and $U^T$ are different but in the limit of $T \to 0$ $U^T$ transforms into $U^0$.

The terms with $n = 0$ in ($\Pi$) and ($\Pi$) give the zero temperature expressions for the Green functions and the EP $V'$, respectively. So, we can split the latter into two parts:

$$V'(H, \mathbf{H}_3, T) = V'(H, \mathbf{H}_3) + V'_T(H, \mathbf{H}_3, T).$$

(14)

The standard procedure to account for the daisy diagrams is to substitute the tree level Matsubara Green functions in ($\Pi$) $G^{(0)}_i^{-1}$ by the full propagator $G_i^{-1} = [G^{(0)}_i]^{-1} + \Pi(H, T)$ (see for details [6, 17, 18]), where the last term is the polarization operator at finite temperature in the field taken at zero longitudinal momentum $k_l = 0$.

Omitting the detailed calculations we notice that the exact one-loop EP is transformed into the EP which contains the daisy diagrams as well as the one-loop diagrams if one adds to the exponent a term containing the temperature dependent mass of a particle.

It is convenient for what follows to introduce the dimensionless quantities:

$$x = H/H_0 \quad (H_0 = M_W^2/e), \quad y = \mathbf{H}_3/\mathbf{H}_3^0 \quad (\mathbf{H}_3^0 = M_W^2/g_s), \quad B = \beta M_W, \quad \tau = 1/B = T/M_W, \quad v = V/H_0^2.$$

The total EP consists of several terms

$$v' = \frac{x^2}{2} + \frac{y^2}{2} + v'_{\text{leptons}} + v'_{\text{quarks}} + v'_{\text{W-bosons}} + v'_{\text{gluons}}$$

$$+ v'_{\text{sleptons}} + v'_{\text{sqarks}} + v'_{\text{charginos}} + v'_{\text{gluinos}}.$$ (15)

These terms can be written down as follows (in dimensionless variables):

• SM sector

- leptons

$$v'_{\text{leptons}} = -\frac{1}{4\pi^2} \sum_{n=1}^{\infty} (-1)^n \int_0^{\infty} ds \frac{e^{-\frac{s^2}{4\tau} + \frac{m_{\text{leptons}}^2}{s^3}}}{s^3} (xs \coth(xs) - 1);$$ (16)

- quarks

$$v'_{\text{quarks}} = -\frac{1}{4\pi^2} \sum_{f=1}^{6} \sum_{n=1}^{\infty} (-1)^n \int_0^{\infty} ds \frac{e^{-\frac{s^2}{4\tau} + \frac{m_f^2}{s^3}}}{s^3} (q_f xs \coth(q_f xs) \cdot ys \coth(ys) - 1);$$ (17)

- W-bosons (see [8])

$$v'_W = \frac{x}{8\pi^2} \sum_{n=1}^{\infty} \int_0^{\infty} \frac{ds}{s^3} e^{-\frac{m_W^2 s^3 + \frac{\beta^2 x^2}{4\tau}}{s^3}} \cdot \left[ \frac{3}{\sinh(xs)} + 4\sinh(xs) \right];$$ (18)
- gluons (see [3])

\[ v'_{\text{gluons}} = -\frac{y}{4\pi^2} \sum_{n=1}^{\infty} \int_{0}^{\infty} \frac{ds}{s^2} e^{-(m^2_{\text{gluons}}s + \frac{2n^2}{4s})} \cdot \left[ \frac{1}{\text{Sinh}(ys)} + 2\text{Sinh}(ys) \right]; \quad (19) \]

• MSSM sector

- s-leptons

\[ v'_{\text{sleptons}} = -\frac{3}{4\pi^2} \sum_{n=1}^{\infty} \int_{0}^{\infty} \frac{ds}{s^3} e^{-(m^2_{\text{sleptons}}s + \frac{2n^2}{4s})} \cdot \left[ \frac{x_s}{\text{Sinh}(xs)} - 1 \right]; \quad (20) \]

- s-quarks

\[ v'_{\text{squarks}} = -\frac{1}{8\pi^2} \sum_{n=1}^{\infty} \int_{0}^{\infty} \frac{ds}{s^3} e^{-(m^2_{\text{squarks}}s + \frac{2n^2}{4s})} \cdot \left[ \frac{q_f x_s \cdot ys}{\text{Sinh}(q_f x_s) \cdot \text{Sinh}(ys)} - 1 \right]; \quad (21) \]

- charginos

\[ v'_{\text{charginos}} = -\frac{1}{4\pi^2} \sum_{n=1}^{\infty} (-1)^n \int_{0}^{\infty} \frac{ds}{s^3} e^{-(m^2_{\text{charginos}}s + \frac{2n^2}{4s})} (x_s \text{Coth}(x_s) - 1); \quad (22) \]

- gluinos

\[ v'_{\text{gluinos}} = -\frac{1}{4\pi^2} \sum_{n=1}^{\infty} (-1)^n \int_{0}^{\infty} \frac{ds}{s^3} e^{-(m^2_{\text{gluinos}}s + \frac{2n^2}{4s})} (y_s \text{Coth}(y_s) - 1). \quad (23) \]

Here, \( m_{\text{leptons}}, m_f, m_W, m_{\text{gluons}}, m_{\text{sleptons}}, m_{\text{squarks}}, m_{\text{charginos}} \) and \( m_{\text{gluinos}} \) are the temperature masses of leptons, quarks, W-bosons, gluons, s-leptons, s-quarks, charginos and gluinos, respectively; \( q_f = (\frac{2}{3}, -\frac{1}{3}, -\frac{1}{3}, \frac{2}{3}, -\frac{1}{3}, \frac{2}{3}) \) are the charges of quarks.

Since we investigate the high-temperature effects connected with the presence of external fields, we used leading in temperature terms of the Debye masses of the particles, only (3, 23).

In present analysis the temperature masses of leptons, quarks, s-leptons, s-quarks, charginos, gluinos are taken as follows (7)

\[
\begin{align*}
    m^2_{\text{leptons}} &= \left( \frac{e}{\beta} \right)^2, \\
    m^2_{\text{sleptons}} &= \left( \frac{e}{\beta} + \frac{M_{\text{sleptons}}}{M_W} \right)^2, \\
    m^2_{\text{charginos}} &= \left( \frac{e}{\beta} + \frac{M_{\text{charginos}}}{M_W} \right)^2, \\
    m^2_{\text{squarks}} &= \left( \frac{e}{\beta} + \frac{M_{\text{squarks}}}{M_W} \right)^2, \\
    m^2_{\text{gluinos}} &= \left( \frac{e}{\beta} + \frac{M_{\text{gluinos}}}{M_W} \right)^2,
\end{align*}
\]

where the masses from SSB terms are taken as the low experimental limits of corresponding particle masses (24)

\[
\begin{align*}
    M_{\text{leptons}} &= 40 \text{ GeV}, & M_{\text{squarks}} &= 176 \text{ GeV}, \\
    M_{\text{charginos}} &= 62 \text{ GeV}, & M_{\text{gluinos}} &= 154 \text{ GeV}.
\end{align*}
\]
As it was established in numeric computation the spontaneous generation of fields depends on SSB masses fairly weak. Even in the case of zero SSB masses there is the generation of magnetic and chromomagnetic fields. In the limit of infinite SSB masses the picture conforms to the SM case.

The temperature masses of gluons and $W$-bosons are \((6, 7, 14)\)

\[
m^2_W = 15 \alpha_{EW} \frac{\beta^{1/2}}{\beta}, \quad m^2_{\text{gluons}} = 15 \alpha_S \frac{\beta^{1/2}}{\beta},
\]

\(\alpha_{EW}\) and \(\alpha_S\) are the electroweak and the strong interaction couplings, respectively.

In one-loop order, the neutral gluon contribution is trivial \(H_3\)-independent constant which can be omitted. However, these fields are long-range states and they do give \(H_3\)-dependent EP through the correlation corrections depending on the temperature and field. We include the longitudinal neutral modes only because their Debye masses \(\Pi^0(y, \beta)\) are nonzero. The corresponding EP is \(3\)

\[
v_{\text{ring}} = \frac{1}{24 \beta^2} \Pi^0(y, \beta) - \frac{1}{12 \pi \beta} \left( \Pi^0(y, \beta) \right)^{3/2} + \frac{(\Pi^0(y, \beta))^2}{32 \pi^2} \left[ \log \left( \frac{4 \pi}{\beta (\Pi^0(y, \beta))^{1/2}} \right) + \frac{3}{4} - \gamma \right];
\]

\(\gamma\) is Euler’s constant, \(\Pi^0(y, \beta) = \Pi^0_{00}(k = 0, y, \beta)\) is the zero-zero component of the neutral gluon field polarization operator calculated in the external field at finite temperature and taken at zero momentum \(3\)

\[
\Pi^0(y, \beta) = \frac{2g^2}{3\beta^2} - \frac{y^{1/2}}{\pi \beta} - \frac{y}{4\pi^2}.
\]

Equations \((15)-(23)\) and \((27)\) will be used in numeric calculations.

3. Combined generation of magnetic and chromomagnetic fields

To calculate the strengths of combined generated magnetic and chromomagnetic fields we use the perturbative computation method in \(3\). First of all we find the strengths of the fields \(x\) and \(y\) when the quark and the s-quark contributions \((v'_q)\) are divided in two parts, \(v'_q(x, \beta) = v'_q |_{y \to 0}\) and \(v'_q(y, \beta) = v'_q |_{x \to 0}\), where \(v'_q(x, \beta)\) is the quark and s-quark contribution in the case of single magnetic field, and \(v'_q(y, \beta)\) is the one in the presence of chromomagnetic field, only.

Now let us rewrite \(v'\) in \((15)\) as follows:

\[
v'(\bar{x}, \bar{y}) = v_1(\bar{x}) + v_2(\bar{y}) + v_3(\bar{x}, \bar{y}),
\]

\(\bar{x} = x + \delta x, \bar{y} = y + \delta y, \text{ and } \delta x \text{ and } \delta y \text{ are the field corrections connected with the interfusion effect of the fields in the quark and s-quark sector.}\)
Since the mixing of fields due to quark and s-quark loop is weak (this is justified in numeric calculations) one can assume that $\delta x \ll 1$ and $\delta y \ll 1$, and write
\[
v_1(\bar{x}) = v_1(x + \delta x) = v_1(x) + \frac{\partial v_1(x)}{\partial x} \delta x,
\]
(30)
\[
v_2(\bar{y}) = v_2(y + \delta y) = v_2(y) + \frac{\partial v_2(y)}{\partial y} \delta y,
\]
(31)
\[
v_3(\bar{x}, \bar{y}) = v_3(x + \delta x, y + \delta y) = v_3(x, y).
\]
(32)

After simple transformations we can find $\delta x$ and $\delta y$:
\[
\delta x = \frac{\partial v_3(x,0)}{\partial x} - \frac{\partial v_3(x,y)}{\partial x} - \frac{\partial^2 v_1(x)}{\partial x^2} \frac{\partial^2 v_3(x,y)}{\partial x^2},
\]
\[
\delta y = \frac{\partial v_3(0,y)}{\partial y} - \frac{\partial v_3(x,y)}{\partial y} - \frac{\partial^2 v_2(y)}{\partial y^2} \frac{\partial^2 v_3(x,y)}{\partial y^2}.
\]
(33)

Hence we obtain $\bar{x} = x + \delta x$ and $\bar{y} = y + \delta y$.

These results on the field strengths determined by means of numeric investigation of the total EP are summarized in Tables 1 and 2.

| $\beta$ | $x$ | $\delta x$ | $\delta x/\beta$, % | $\bar{x}$ | $\bar{x}$ |
|---------|-----|------------|-----------------|--------|--------|
| 0.1     | 0.3813 | $1.58 \times 10^{-2}$ | 4.14 | 0.3971 | 0.7000 |
| 0.2     | 0.10021 | $2.45 \times 10^{-3}$ | 2.44 | 0.10265 | 0.20075 |
| 0.3     | 0.046199 | $7.19 \times 10^{-4}$ | 1.56 | 0.046917 | 0.069945 |
| 0.4     | 0.026804 | $1.97 \times 10^{-4}$ | 0.73 | 0.027000 | 0.039964 |
| 0.5     | 0.017675 | $1.19 \times 10^{-4}$ | 0.67 | 0.017794 | 0.029953 |
| 0.6     | 0.0120559 | $5.20 \times 10^{-5}$ | 0.43 | 0.0121079 | 0.0199508 |
| 0.7     | 0.0086022 | $2.82 \times 10^{-5}$ | 0.33 | 0.0086303 | 0.0099620 |
| 0.8     | 0.0065687 | $1.72 \times 10^{-5}$ | 0.26 | 0.0065859 | 0.0099381 |
| 0.9     | 0.0052535 | $1.13 \times 10^{-5}$ | 0.22 | 0.0052648 | 0.0099759 |
| 1.0     | 0.0043400 | $8.10 \times 10^{-6}$ | 0.19 | 0.0043481 | 0.0099643 |

**Table 1. The strengths of generated magnetic field.**

In Tables 1 and 2, in the first column we show the inverse temperature. In the second one the strengths of magnetic and chromomagnetic fields are adduced for the case of the quark and the s-quark EP describing each of the fields separately. The next column gives the field corrections in the case of the total quark and s-quark EP. The fourth column presents the relative value of corrections. The following column gives the resulting strengths of magnetic ($\bar{x} = x + \delta x$) and
chromomagnetic ($\bar{y} = y + \delta y$) fields, respectively. In the last column the strengths of generated fields in the SM are given for comparison (7).

As it is seen, the increase of inverse temperature leads to decreasing the strengths of generated fields. This dependence is well in accordance with the picture of the universe cooling.

| $\beta$ | MSSM $y$ | $\delta y$ | $\delta y/y$, % | SM $y$ |
|--------|---------|----------|----------------|--------|
| 0.1    | 0.510146 | $5.28 \times 10^{-6}$ | 0.0010 | 0.510151 | 0.800301 |
| 0.2    | 0.133035 | $1.73 \times 10^{-6}$ | 0.0013 | 0.133037 | 0.199761 |
| 0.3    | 0.0603172 | $8.85 \times 10^{-7}$ | 0.0015 | 0.0603181 | 0.0899012 |
| 0.4    | 0.0347127 | $5.20 \times 10^{-7}$ | 0.0015 | 0.0347132 | 0.0499116 |
| 0.5    | 0.0225367 | $3.59 \times 10^{-7}$ | 0.0016 | 0.0225371 | 0.0398880 |
| 0.6    | 0.0161563 | $2.26 \times 10^{-7}$ | 0.0014 | 0.0161565 | 0.0299018 |
| 0.7    | 0.0115808 | $1.53 \times 10^{-7}$ | 0.0013 | 0.0115810 | 0.0199558 |
| 0.8    | 0.00859328 | $1.19 \times 10^{-7}$ | 0.0014 | 0.00859340 | 0.0199267 |
| 0.9    | 0.00672412 | $9.94 \times 10^{-8}$ | 0.0015 | 0.00672422 | 0.0098830 |
| 1.0    | 0.00547797 | $9.01 \times 10^{-8}$ | 0.0016 | 0.00547806 | 0.0098250 |

Table 2. The strengths of generated chromomagnetic field.

From the above analysis it follows that in the considered temperature interval the presence in the system of both fields leads to increasing of each of them in contrast with the SM case. In the latter the strengths of the combined fields are decreased as compared to the separate generation. This is the consequence of the s-quark loop contributions depending as the quark loops on both of fields.

With temperature decreasing this effect becomes less pronounced and disappears at comparably low temperatures $\beta \sim 1$.

4. Discussion

Let us discuss the results obtained. As we elaborated within the EP including the one-loop and the daisy diagrams, in the MSSM at high temperatures both the magnetic and chromomagnetic fields have to be generated. This vacuum is stable, as it follows from the absence of imaginary terms in the EP minima.

If quark and s-quark loops are discarded, both of the fields can be generated separately. All these states are stable, due to the magnetic mass $\sim g^2(gH)^{1/2}T$ of the transversal gauge field modes. As it was already shown in [7], the imaginary part arises for the field strengths larger than the ones generated in the SM. As we have seen, the strengths of generated fields reduce due to the s-particle sector of the MSSM. This vacuum state is more stable as compare to the SM case.

The result on the stabilization of the charged gauge field spectra is very important. It has relevance not only to the problem of the consistent description
Figure 1: The dependences of the strengths of generated magnetic fields ($H$) on the inverse temperature ($b$). The solid line is the magnetic field strength in the MSSM and the dashed line is that of in the SM.

Figure 2: The dependences of the strengths of generated chromomagnetic field ($H_3$) on the inverse temperature ($b$). The solid line is the chromomagnetic field strength in the MSSM and the dashed line is that of in the SM.
of the generation of magnetic fields but also to the related problem on symmetry
behavior in external magnetic fields investigated in the MSSM recently in [9, 13].

As it is seen from Figs. 1 and 2, presenting the results of numeric computations
within the exact EP, the strengths of the generated fields are increasing when the
temperature is rising. It is also found, the dynamics of curves obtained in the
SM [7] are in a good agreement with our numeric calculations.

The ground state possessing the magnetic and the chromomagnetic fields
makes it reasonable to expect the existence of these fields in the electroweak
transition epoch for both the SM and the MSSM. The state is stable in the whole
considered temperature interval. The imaginary part in the EP exists for the
external fields much stronger than the strengths of the spontaneously generated
ones. The mixing of magnetic and chromomagnetic fields arising from the quark
and the s-quark sectors of the EP is weak. In the MSSM, the change of the field
minima in the inclusion of the field mixing does not exceed 4 per cents. In the
SM these values do not exceed 2 per cents. This is due to the strong dependence
of the s-quark loop on the strengths of both fields.

| \( \beta \) | quarks       | s-quarks    | s-leptons   | charginos   |
|------------|-------------|-------------|-------------|-------------|
| 0.1        | \( 8.09496 \times 10^{-2} \) | \( 1.29860 \times 10^{-2} \) | \( 1.08818 \times 10^{-2} \) | \( 1.90295 \times 10^{-3} \) |
| 0.2        | \( 5.51654 \times 10^{-3} \) | \( 5.13130 \times 10^{-4} \) | \( 6.28152 \times 10^{-4} \) | \( 1.16324 \times 10^{-4} \) |
| 0.3        | \( 1.13926 \times 10^{-3} \) | \( 6.77157 \times 10^{-5} \) | \( 1.13423 \times 10^{-4} \) | \( 2.20561 \times 10^{-5} \) |
| 0.4        | \( 3.78190 \times 10^{-4} \) | \( 1.51991 \times 10^{-5} \) | \( 3.28547 \times 10^{-5} \) | \( 6.66057 \times 10^{-6} \) |
| 0.5        | \( 1.60125 \times 10^{-4} \) | \( 4.51358 \times 10^{-6} \) | \( 1.24231 \times 10^{-5} \) | \( 2.60925 \times 10^{-6} \) |
| 0.6        | \( 8.11555 \times 10^{-5} \) | \( 1.64583 \times 10^{-6} \) | \( 5.07024 \times 10^{-6} \) | \( 1.09729 \times 10^{-6} \) |
| 0.7        | \( 4.16467 \times 10^{-5} \) | \( 6.18990 \times 10^{-7} \) | \( 2.28091 \times 10^{-6} \) | \( 5.06211 \times 10^{-7} \) |
| 0.8        | \( 2.31108 \times 10^{-5} \) | \( 2.55271 \times 10^{-7} \) | \( 1.18256 \times 10^{-6} \) | \( 2.68001 \times 10^{-7} \) |
| 0.9        | \( 1.42387 \times 10^{-5} \) | \( 1.18144 \times 10^{-7} \) | \( 6.76190 \times 10^{-7} \) | \( 1.55895 \times 10^{-7} \) |
| 1.0        | \( 9.48895 \times 10^{-6} \) | \( 5.96475 \times 10^{-8} \) | \( 4.14475 \times 10^{-7} \) | \( 9.68813 \times 10^{-8} \) |

Table 3. The contribution of quarks, s-quarks, s-leptons and charginos to the
EP.

During the universe cooling the strengths of the generated fields are decreasing. This is in agreement with what is expected in cosmology.

One of the consequences of the results obtained is the presence of a strong
chromomagnetic field in the early universe, in particular, at the electroweak phase
transition and, probably, till the deconfinement phase transition. The influence
of this field on the phase transitions may bring new insight to these phenomena. As our estimate showed, the chromomagnetic field is as strong as the magnetic one. So the role of strong interactions in the early universe in the presence of the field needs more detailed investigations as compare to what is usually assumed [8].
We would like to notice that in the literature devoted to investigations of the quark-gluon plasma in the deconfinement phase carried out by non-perturbative methods the vacuum magnetization at high temperature has not been accounted for (see, for instance, recent paper [25] and references therein). From the point of view of the present analysis (as well as other studies carried out already in perturbation theory [4]-[6]) these investigations are incomplete. The generation of the chromomagnetic field at high temperature has to be taken into consideration.

References

[1] K.Enqvist, Int. J. Mod. Phys. D 7, 331 (1998); astro-ph/9803196.
[2] K.Enqvist, Invited Talk at Strong and Electroweak Matter ’97, Hungary, astro-ph/9707300.
[3] D.Grasso, H.R.Rubinstein, Phys. Rept. 348, 163 (2001), astro-ph/0009061.
[4] K.Enqvist, P.Olesen, Phys. Lett. B 329, 195 (1994).
[5] A.O.Starinets, A.S.Vshivtsev, V.Ch.Zhukovsky, Phys. Lett. B 322, 403 (1994).
[6] V.Skalozub, M.Bordag, Nucl. Phys. B 576, 430 (2000).
[7] V.Demchik, V.Skalozub, be published in EPJ C 25 (2002), hep-ph/0110280.
[8] G.K.Savvidy, Phys. Lett. B 71, 133 (1977).
[9] M.Giovannini, M.Shaposhnikov, Phys. Rev. D 57, 2186 (1998).
[10] V.V.Skalozub, V.I.Demchik, Ukrainian Phys. J. 46, 784 (2001).
[11] K.Kajantie, M.Laine, J.Peisa, K.Rummukainen, M.E.Shaposhnikov, Nucl. Phys. B 544, 357 (1999).
[12] M.Laine (1999), hep-ph/9902282.
[13] M.Joyce, M.Shaposhnikov, Phys. Rev. Lett. 79, 1192 (1997).
[14] V.V.Skalozub, A.V.Strelchenko, Physics of Atomic Nuclei 63 No.11, 1956-1962 (2000); hep-ph/0208071.
[15] M.Kuroda (1999), hep-ph/9902340.
[16] V.V.Skalozub, Sov. J. Part. Nucl. 16, 445 (1985).
[17] J.I. Kapusta, *Finite-temperature field theory*, Cambridge / university Press, 1989.
[18] M.E.Carrington, Phys. Rev. D 45, 2933 (1992).

[19] J.Schwinger, Phys. Rev. 82, N5, 664 (1951).

[20] A.Cabo, Fortschr. Phys. 29, 495 (1981).

[21] Yu.Yu.Reznikov and V.V.Skalozub, Sov. J. Nucl. Phys. 46, 1085 (1987).

[22] V.V.Skalozub, Int. J. Mod. Phys. A11, 5643 (1996).

[23] V.Skalozub, V.Demchik (1999), hep-th/9912071.

[24] K.Hagiwara et al.(Particle Data Group), Phys. Rev. D. 66, 010001 (2002) (URL: http://pdg.lbl.gov).

[25] K.Kajantie, M.Laine, K.Rummukainen, Y.Schroeder, hep-lat/0110122.