Alpha particle production by molecular single-particle effect in reactions of $^9$Be just above the Coulomb barrier

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The $\alpha$-particle production in the dissociation of $^9$Be on $^{209}$Bi and $^{64}$Zn at energies just above the Coulomb barrier is studied within the two-center shell model approach. The dissociation of $^9$Be on $^{209}$Bi is caused by a molecular single-particle effect (Landau-Zener mechanism) before the nuclei reach the Coulomb barrier. Molecular single-particle effects do not occur at that stage of the collision for $^9$Be+$^{64}$Zn, and this explains the absence of fusion suppression observed for this system. The polarisation of the energy level of the last neutron of $^9$Be and, therefore the existence of avoided crossings with that level, depends on the structure of the target.

Introduction: The study of breakup processes of weakly bound or halo nuclei at energies around the Coulomb barrier ([1] and references therein) is a very lively topic due to the increasing interest in nuclear reactions with radioactive beams. The effect of breakup on fusion and scattering has also been extensively investigated in recent years both theoretically [2–6] and experimentally [7,8].

In a recent paper [5], we have studied the reaction $^9$Be+$^{209}$Bi at energies above and near the Coulomb barrier in the adiabatic two-center shell model (TCSM) [9] approach. The effect of dissociation of $^9$Be→n+2$\alpha$ on complete fusion and scattering processes was particularly investigated. Results showed that the dissociation of $^9$Be could be due to a molecular single-particle effect at two very close avoided crossings between the state $j_z = 3/2$ of the last neutron of $^9$Be and two unoccupied states of $^{209}$Bi with the same projection of the single-particle total angular momentum $j_z$ on the internuclear axis, before the nuclei reach the Coulomb barrier (Fig. 1). The dissociation of $^9$Be is caused by two very competitive (simultaneous) transitions of the last neutron of $^9$Be at these avoided crossings (1-2) induced by the radial motion of the nuclei [10], namely into the continuum (1) and to $^{209}$Bi (2). These transitions lead to the formation of the unbound nucleus $^8$Be which may decay immediately into two $\alpha$-particles. The $\alpha$-particles originating from the decay of $^8$Be have been experimentally observed in this reaction [11].

The aim of this letter is to compare the $\alpha$-particle production in the dissociation of $^9$Be on $^{209}$Bi and on $^{64}$Zn, at energies just above the Coulomb barrier, within the TCSM [9] approach. The use of adiabatic TCSM basis states is well justified [10] at low energy.

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nuclear reactions (near the Coulomb barrier), where the relative motion of the nuclei is slow compared with the rearrangement time of the mean field of nucleons. The single-particle motion of valence nucleons during the reaction is described by molecular orbitals which are polarised by the field of the other nucleus. Once a neck in the dinuclear system starts to form and molecular single-particle levels develop, there can occur transitions [10] (molecular single-particle effects) between those levels, induced by the relative motion of the nuclei (operators associated with the kinetic energy). In particular, transitions at the avoided crossings of molecular levels, induced by the radial motion of the nuclei (Landau-Zener mechanism [10]), describe the non-adiabaticity of the radial motion and grow with increasing radial velocity, the diabatic single-particle motion being a limit for large radial transition probabilities [12]. In the diabatic limit of the single-particle motion [12], the nucleons do not occupy the lowest free single-particle levels as in the adiabatic case, but remain in the diabatic levels keeping their quantum numbers during a collective motion of the nuclear system. In this letter, we will deal with an intermediate situation between the adiabatic and diabatic limits of the single-particle motion.

It would be interesting to calculate the cross sections for the $\alpha$-particle production in the approach phase of these collisions and to compare them with the experimental data, because this study may or may not support the dissociation of $^9\text{Be}$ by molecular single-particle effects [5]. Moreover, we expect that the dissociation of $^9\text{Be}$ and, therefore, the existence of a suppression mechanism for fusion, could be associated with the evolution of the last neutron of $^9\text{Be}$ in the approach phase of the collision before the nuclei reach the Coulomb barrier, and this may depend on the structure of the partner nucleus. The study of the reaction $^9\text{Be} + ^{64}\text{Zn}$ is interesting because there is no evidence for fusion suppression for this system [8], in contrast to the reaction $^9\text{Be} + ^{209}\text{Bi}$ [7].

**Model:** The cross section $\sigma$ for the production of $\alpha$-particles is defined as

$$\sigma = \frac{\pi}{2\mu E_{\text{c.m.}}} \sum_{l=0}^{l_{\text{max}}} (2l + 1)(2 \cdot P(E_{\text{c.m.}}, l)),$$

(1)

where $l_{\text{max}}$ is a maximum orbital angular momentum for the relative motion of the nuclei which can reach the avoided crossing of molecular levels at the incident energy $E_{\text{c.m.}}$, $\mu$ denotes the reduced mass for the relative motion of the nuclei, and $P(E_{\text{c.m.}}, l)$ is the dissociation probability of $^9\text{Be}$ (production of $^8\text{Be}$) for the incident energy $E_{\text{c.m.}}$ and orbital angular momentum $l$. The factor 2 appears in Eq. (1) because two $\alpha$-particles are produced in the decay of a $^8\text{Be}$ nucleus. The existence of the dissociation channel for $^9\text{Be}$ (a neutron transition into the continuum or a neutron transfer) is derived, in the present model, from examining the neutron levels calculated with the TCSM.

In the complete fusion process ($0 \leq l \leq l_{\text{cr}}$), the dissociation probability at the avoided crossing of molecular adiabatic single-particle levels can be expressed as

$$P = P^{(e)},$$

(2)

and in the scattering process ($l_{\text{cr}} \leq l \leq l_{\text{max}}$) as

$$P = P^{(e)} + (1 - P^{(e)}) \cdot P^{(lv)},$$

(3)

since the system passes through the avoided crossing point twice, first entering ($e$) and then leaving ($lv$) the interaction region. Here, we have made a distinction between ($e$) and ($lv$).
because the radial velocity of the nuclei changes due to frictional effects [5]. The first term in Eq. (3) is $P^{(e)}$, instead of $P^{(e)\cdot} (1 - P^{(l)})$, because of our assumption of an immediate dissociation of $^8\text{Be} \rightarrow 2\alpha$ after the neutron in $^9\text{Be}$ has been removed from the nucleus entering the interaction region (the neutron does not get bound again with $^8\text{Be}$ when the nuclei leave the interaction region). The dissociation probability $P^{(e), (l)}$ at an isolated avoided crossing of molecular levels is calculated with the Landau-Zener formula [10]. The radial velocity $v^0$, which is the only quantity concerning relative motion in the above expression, is calculated from the classical equations of motion for the nucleus-nucleus potential [5] obtained with the TCSM and the Strutinsky method. Since the radial velocity $v^0$ depends on the incident energy $E_{\text{c.m.}}$ and the orbital angular momentum $l$, $P_{LZ}$ depends also on $E_{\text{c.m.}}$ and $l$. In general, the neutron transition at the avoided crossing can be calculated semiclassically [10] using a time-dependent Schrödinger equation for the neutron wave function expanded in a basis of two diabatic wave functions related to the diabatic levels $\epsilon_1, \epsilon_2$ and a classical equation for the relative motion of the nuclei.

**Results and discussion:** For the calculation of the single-particle levels with the TCSM [9] for the systems $^9\text{Be} + ^{209}\text{Bi}$ and $^9\text{Be} + ^{64}\text{Zn}$, we use [5] (i) the experimental nucleon separation energies [13–17] for the colliding nuclei and for the compound nucleus to obtain the depths of the two oscillator potential wells, (ii) the parameters $k$ and $\mu$ of the Nilsson model for the spin-orbit interaction [18] and (iii) a set of universal parameters, e.g., the nuclear-radius constant $r_0 = 1.2249$ fm and the oscillator quanta $\hbar\omega_{i0} = 41 \cdot A_i^{-1/3}$ MeV. Moreover, we have considered spherical nuclei with a value of the neck parameter $\varepsilon = 0.75$. With this value of $\varepsilon$, the neck radius and the internuclear distance at the touching configuration are approximately equal to those in the dinuclear system formed by the overlap of the two nuclear frozen densities.

$(^9\text{Be} + ^{209}\text{Bi})$ Fig. 2 (upper part) shows the dissociation probability $P^{(E_{\text{c.m.}}, l)}$ of $^9\text{Be}$ on $^{209}\text{Bi}$ as a function of the orbital angular momentum $l$ ($0 \leq l \leq l_{\text{max}}$) for two values of the incident energy $E_{\text{c.m.}}$, namely 44.1 MeV (solid curve, $l_{cr} = 8$ and $l_{max} = 14$) and 57.5 MeV (dashed curve, $l_{cr} = 16$ and $l_{max} = 26$). The dissociation probability $P^{(e), (l)}$ in Eqs. (2-3) is calculated as the product of two isolated Landau-Zener transitions at the avoided crossings 1 and 2 of Fig.1. At these avoided crossings, the applicability [10] of the Landau-Zener approach is quite good. The values of $| H'_{12} |$ and $\frac{d}{dl} (\epsilon_1 - \epsilon_2)$ at the avoided crossings 1-2 are 0.1073 MeV, 0.0184 MeV fm$^{-1}$ and 0.1250 MeV, 0.0352 MeV fm$^{-1}$, respectively. For a fixed incident energy $E_{\text{c.m.}}$, the dissociation probability decreases with increasing orbital angular momentum $l$ because the potential energy increases and the radial velocity $v^0$ (kinetic energy) decreases at the avoided crossing points. For a fixed orbital angular momentum $l$, the radial velocity $v^0$ at the avoided crossing points and, therefore, the dissociation probability increases with an increasing incident energy.

Fig. 2 (lower part) shows the calculated excitation function for the production of $\alpha$-particles (solid curve) for $^9\text{Be} + ^{209}\text{Bi}$. The calculated cross sections correspond to twice the cross sections for the production of $^8\text{Be}$ by breakup (avoided crossing 1 of Fig. 1) and transfer (avoided crossing 2 of Fig. 1). Calculated cross sections for the production of $\alpha$-particles underestimate the experimental data for $^9\text{Be} + ^{209}\text{Bi}$ reported in [11] by a factor 3.7-4.7 in the low range of incident energies studied, but agree with preliminary experimental data for the similar system $^9\text{Be} + ^{208}\text{Pb}$ reported in [19]. However, the experimental complete fusion cross sections [20,21] are similar in both reactions, and the fusion cross sections...
agree well with those values calculated in the TCSM approach [5]. The experimental total breakup+transfer cross sections for \(^9\text{Be}+^{209}\text{Bi}\) reported in [11] were obtained by integrating over angles the angular distribution of \(^8\text{Be}\) (i.e. \(2\alpha\)) nuclei, while both breakup and transfer cross sections for \(^9\text{Be}+^{208}\text{Pb}\) reported in [19] were separately obtained from the \(\alpha\)-particles with the beam velocity. It is important to note both that the total breakup+transfer cross sections reported in [11] were obtained from a bump of \(\alpha\)-particles in the charged particles spectra for \(^9\text{Be}+^{209}\text{Bi}\) observed systematically in three experiments [11] and that therefore, there may be some uncertainties about these cross sections [22]. New experiments focused on this particular problem seem to be necessary to clarify the experimental cross sections for the production of \(\alpha\)-particles in the studied reaction and, therefore, the dissociation of \(^9\text{Be}\) by molecular single-particle effects.

Since the avoided crossings 1-2 of Fig. 1 have similar features and the neutron transitions at these avoided crossings occur simultaneously, we expect that cross sections for the neutron transition into the continuum (usual breakup) are similar to those for the neutron transition to \(^{209}\text{Bi}\) (transfer). This expectation agrees well with preliminary experimental breakup and transfer excitation functions obtained for \(^9\text{Be}+^{208}\text{Pb}\) [19], which show similar features. In the present approach, we cannot separately calculate cross sections for the neutron transition either into the continuum or to \(^{209}\text{Bi}\) in a realistic way because both processes interfere strongly with each other.

For incident energies very close to the Coulomb barrier, we expect a knee in the excitation function for the production of \(\alpha\)-particles because the radial velocity (kinetic energy) of the nuclei and, therefore, the dissociation probability should increase with a decreasing nucleus-nucleus potential [5] (e.g., pole-to-pole orientation) due to deformation and orientation effects of \(^9\text{Be}\). A more general TCSM [23] would have to be used for arbitrary orientations of the intrinsic symmetry axes of deformed nuclei as well as a quantum mechanical description of the relative motion of the nuclei for incident energies below the Coulomb barrier [24].

In the present work, the calculation of the cross sections for the production of \(\alpha\)-particles is focused on incident energies above the Coulomb barrier (\(V_B \approx 40\ MeV\) [5]) for two reasons, namely we consider spherical nuclei, which model is not suitable for incident energies very close to and below the Coulomb barrier, and the dissociation probabilities cannot be calculated using the Landau-Zener formula (??) for subcoulomb trajectories because a radial velocity of the nuclei is not defined in the classically forbidden region. The assumption of spherical nuclei is suitable if there is only a small change of the nucleus-nucleus potential, due to orientation and deformation effects of the nuclei, in comparison with the surplus of incident energy above the potential.

(\(^9\text{Be}+^{64}\text{Zn}\)) Fig. 3 shows the neutron level diagram of the TCSM for \(^9\text{Be}+^{64}\text{Zn}\) around the radius of the Coulomb barrier (the B arrow). The internal arrow (A) indicates the distance \(r_t \approx 7.45\ fm\) corresponding to the touching configuration of the nuclei if no neck is formed and the C arrow indicates the relative distance \(r \approx 9.21\ fm\) where a neck between the nuclei starts to form. The values of the Coulomb barrier and its position, calculated with the TCSM and the Strutinsky method [5], are \(V_B \approx 15.5\ MeV\) and \(r_B \approx 9.43\ fm\), respectively. These values agree well with the barrier parameters determined in [8] from the experimental fusion excitation function, \(V_B^{exp} = 16.2\ MeV\) and \(r_B^{exp} = 10\ fm\).

From Fig. 3, we can see that no neck between the nuclei is formed before the nuclei reach the Coulomb barrier and, therefore, molecular effects cannot occur in that phase of
the collision. Neutron transitions between the state \( j_z = 3/2 \) of the last neutron of \(^9\text{Be}\) and unoccupied states of \(^{64}\text{Zn}\) (neutron transfer) could occur at relative distances well inside the Coulomb barrier, near the touching configuration of the nuclei if no neck is formed (the A arrow). In this case the nucleus \(^8\text{Be}\) may be completely absorbed by the nucleus \(^{65}\text{Zn}\). On the other hand, it could be expected that those transition probabilities caused by a Landau-Zener mechanism are very small because the radial velocity decreases at the touching configuration of the nuclei due to strong frictional effects [5]. From these results, we do not expect significant production of \(\alpha\)-particles in the approach phase of this collision. This also explains the absence of fusion suppression observed for this system [8]. Comparing Figs. 1 and 3, it is observed that the energy level \( j_z = 3/2 \) of the last neutron of \(^9\text{Be}\) polarises in a different way when \(^9\text{Be}\) approaches either \(^{209}\text{Bi}\) (goes up) or \(^{64}\text{Zn}\) (goes down). This shows that the polarisation of the energy level of the last neutron of \(^9\text{Be}\) and, therefore the existence of avoided crossings with that level, depends on the structure of the target.

Fig. 4 shows the complete fusion excitation function (solid curve) for \(^9\text{Be}^+^{64}\text{Zn}\) calculated within the Glas-Mosel model [25] and using the nucleus-nucleus potential obtained with the TCSM and the Strutinsky method. The calculated complete fusion cross sections agree well with the experimental data [8] (full squares).

Summary and conclusions: The \(\alpha\)-particle production in the dissociation of \(^9\text{Be}\) on \(^{209}\text{Bi}\) and \(^{64}\text{Zn}\) at energies just above the Coulomb barrier has been studied within the two-center shell model approach. The dissociation of \(^9\text{Be}\) on \(^{209}\text{Bi}\) is caused by a molecular single-particle effect (Landau-Zener mechanism) at two very close avoided crossings between the state of the last neutron of \(^9\text{Be}\) and two unoccupied states of \(^{209}\text{Bi}\) before the nuclei reach the Coulomb barrier. The dissociation probability of \(^9\text{Be}\) decreases with decreasing incident energies, while it increases with decreasing orbital angular momentum for a fixed incident energy. Calculated cross sections for the production of \(\alpha\)-particles in \(^9\text{Be}^+^{209}\text{Bi}\) underestimate the available experimental data, but agree with preliminary experimental data for the similar system \(^9\text{Be}^+^{208}\text{Pb}\). New experiments seem to be necessary to clarify the experimental cross sections for the production of \(\alpha\)-particles in \(^9\text{Be}^+^{209}\text{Bi}\) and, therefore, the dissociation of \(^9\text{Be}\) by molecular single-particle effects. We expect similar excitation functions for the neutron transition either into the continuum (usual breakup) or to \(^{209}\text{Bi}\) (transfer). We also expect a knee in the excitation function for the production of \(\alpha\)-particles in \(^9\text{Be}^+^{209}\text{Bi}\) at energies very close to the Coulomb barrier. We expect little production of \(\alpha\)-particles for \(^9\text{Be}^+^{64}\text{Zn}\). Molecular single-particle effects do not occur before these nuclei reach the Coulomb barrier, and this explains the absence of fusion suppression observed for this system. The complete fusion excitation function for this system, calculated within the Glas-Mosel model and using the nucleus-nucleus potential obtained with the two-center shell model and the Strutinsky method, agrees well with the experimental data. The energy level \( j_z = 3/2 \) of the last neutron of \(^9\text{Be}\) polarises in a different way when \(^9\text{Be}\) approaches either \(^{209}\text{Bi}\) (goes up) or \(^{64}\text{Zn}\) (goes down). The polarisation of the energy level of the last neutron of \(^9\text{Be}\) and, therefore the existence of avoided crossings with that level, depends on the structure of the target.

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FIG. 1. Neutron levels for $^9$Be + $^{209}$Bi → $^{218}$Fr as a function of the separation $r$ between the nuclei according to the TCSM. Levels are characterised by the total angular momentum projection $j_z$ on the internuclear axis: $j_z = 1/2$ (dashed curves), $j_z = 3/2$ (dotted curves), $j_z = 5/2$ (dashed-dotted curves) and other values (solid curves). $L^*$ and $H^*$ denote the level of the last neutron of $^9$Be and the Fermi level of $^{209}$Bi, respectively. $L$ denotes a level of $^9$Be occupied by two neutrons, while the other levels belong to $^{209}$Bi. We only consider transitions of the last neutron of $^9$Be at the avoided crossings 1 and 2. The A, B and C arrows indicate the touching configuration of the nuclei if no neck is formed, the position of the Coulomb barrier and the radius where a neck between the nuclei starts to form, respectively. See text and [5] for further details.
FIG. 2. Dissociation probability $P(E_{c.m.}, l)$ of $^{9}\text{Be}$ on $^{209}\text{Bi}$ as a function of the orbital angular momentum $l$ ($0 \leq l \leq l_{\text{max}}$) (upper part). Values for $E_{c.m.} = 44.1$ MeV and $E_{c.m.} = 57.5$ MeV are shown by solid and dashed curves, respectively. See text for further details. Calculated excitation function for the production of $\alpha$-particles for $^{9}\text{Be} + ^{209}\text{Bi}$ (lower part, solid curve).
FIG. 3. The same as in Fig. 1, but for $^9$Be + $^{64}$Zn $\rightarrow$ $^{73}$Se around the radius of the Coulomb barrier. See text for further details.
FIG. 4. Complete fusion excitation function (solid curve) for $^9$Be + $^{64}$Zn calculated within the Glas-Mosel model and using the nucleus-nucleus potential obtained with the TCSM and the Strutinsky method. Experimental data (full squares) are from [8]. See text for further details.