Assessing Language Models with Scaling Properties

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Abstract

Language models have primarily been evaluated with perplexity. While perplexity quantifies the most comprehensible prediction performance, it does not provide qualitative information on the success or failure of models. Another approach for evaluating language models is thus proposed, using the scaling properties of natural language. Five such tests are considered, with the first two accounting for the vocabulary population and the other three for the long memory of natural language. The following models were evaluated with these tests: \( n \)-grams, probabilistic context-free grammar (PCFG), Simon and Pitman-Yor (PY) processes, hierarchical PY, and neural language models. Only the neural language models exhibit the long memory properties of natural language, but to a limited degree. The effectiveness of every test of these models is also discussed.

1 Introduction

The performance of language models has generally been evaluated with perplexity (Manning and Schutze 1999), which quantifies the predictive power of such models. Since the development of \( n \)-gram models, state-of-the-art neural language models have radically reduced perplexity. Although perplexity is easy to compute and its signification is comprehensible, we consider it to have two main drawbacks. First, perplexity does not provide information on how a model is limited with respect to the important linguistic aspects, such as generalization of word concepts, syntactic structure, and long-term dependency. Although perplexity indicates the overall performance across various aspects, we want to know more specifically which aspects are challenging for the language models. Second, we cannot obtain a lower bound of perplexity in a dataset and thus cannot quantitatively recognize the difference of a model from natural language or an ideal language model that perfectly reproduces natural language.

To compensate for these drawbacks of perplexity, we propose to assess language models with a set of scaling properties exhibited by natural language. The assessment is conducted by investigating whether the text generated by a language model would exhibit these scaling properties. We presented this scheme in (Takahashi and Tanaka-Ishii 2017) in which the reproducibility of the character level neural language models are investigated. This assessment has two advantages over the perplexity measure. First, since the scaling properties are designed to measure some aspect underlying a sequence, they can provide information on how limited a model is in terms of that aspect. Second, since the scaling properties quantify the behavior of a data set in terms of exponents, we can evaluate how much the text generated from a model differs from natural language.

The scaling properties in this article are roughly categorized into two types: those related to word frequency distribution and those related to long memory. For the first category, we consider the well-known Zipf’s law and Heaps’ law. For the second category, we consider properties that quantify the memory underlying a sequence. We then test whether texts generated from various language models exhibit every property.
2 Related Work

Many studies of language models have considered Zipf’s and Heaps’ laws. The laws concern the word frequency distribution, characterizing how a large part of vocabulary consists of rare words, causing problems of sparseness and unknown words. The Chinese restaurant process, as a kind of PY process (Pitman 2006), was introduced partly because it satisfies Zipf’s and Heaps’ law (Goldwater et al. 2011; Teh 2006). In the recent works on neural language models, application of the two laws was considered to improve the architecture. Jozefowicz et al. (2016) evaluated models in terms of word frequencies and concluded that a neural model is more capable of predicting rare words than is a Kneser-Ney language model (Kneser and Ney 1995). Merity et al. (2016) constructed a new dataset, wikitext-2 (WT2) for processing rare words with respect to the word frequency distribution, and it has become a standard dataset.

Long-range dependency has mainly been explored in studying the architecture of neural networks. After the first neural language model was proposed (Bengio et al. 2003), Mikolov et al. (2010) first introduced recurrent neural networks (RNNs) to language modeling, with the potential to maintain long-term dependency. In more recent works, Merity et al. (2016) introduced pointer networks (Vinyals et al. 2015) to the task and demonstrated that they improve perplexity on the regular long short-term memory (LSTM) language model. Grave et al. (2017) integrated a cache model designed to sample from context to model the observation of the clustering behavior of certain words. Merity et al. (2018) investigated the effect of a cache model in terms of the difference in log perplexity of each word. These analysis methodologies for long-term dependency, however, have not been formalized and instead rely on case studies. Lin and Tegmark (2017) investigated how an LSTM-like architecture could reproduce the power decay of mutual information. Although their argument is theoretically valid, natural language does not exhibit power decay of mutual information. Our interest is therefore to introduce better measures of the long-range dependency of natural language.
3 Scaling Properties

This section provides a summary of the scaling properties of natural language and defines how to evaluate word or character sequences with them. Nine scaling laws of natural language are acknowledged (Altmann and Gerlach 2017). Some of them deal with word formation and network structure and do not directly relate to language modeling. This leaves five other scaling properties that are still mathematically closely related. We can roughly categorize them into those based on vocabulary population and long memory. The following subsections proceed through the example of applying these scaling properties with WT2 (Merity et al. 2016), as shown in Figure 1.

We test language models by generating text from them and evaluating the scaling properties. In this article, there are two levels of judgment for these properties.

Q1 Does the scaling property hold qualitatively?

Q2 How different is the exponent?

As revealed in the following subsections, many language models fail to satisfy even the first criterion for some properties. For models that satisfy Q1, their exponents are estimated and compared with those of the original text. Consider a power law \( y \propto z^\kappa \) for points \((y_1,z_1), \ldots, (y_N,z_N)\). The exponent \(\kappa\) is estimated by the least-squares method, i.e., \(\hat{\kappa} = \arg \min_{\kappa} \varepsilon(\kappa)\), where \(\varepsilon(\kappa) \equiv \sqrt{\sum_{i=1}^{N} (\log y_i - \log z_i^\kappa)^2}/N\). The error reported here is the average error per point, i.e., \(\varepsilon(\hat{\kappa})\).

3.1 Zipf’s Law and Heaps’ Law

Given a text, let \(r\) be the rank of a particular word type and \(f(r)\) be its frequency. The well-known Zipf’s law formulates a power-law relation between the rank and frequency:

\[
f(r) \propto r^{-\alpha}, \quad \alpha \approx 1.0.
\]  

In fact, this scaling generally holds not only for unigrams but also for \(n\)-grams, with smaller \(\alpha\). The first left graph in Figure 1 shows Zipf distributions for WT2, with unigrams in red and bigrams in blue. Because WT2 replaces rare words having frequencies under a certain threshold with <unk>, the tail of the unigram distribution disappears. The Zipf distributions for unigrams and bigrams typically cross in the middle. In reality, plots for real natural language texts are often not aligned linearly, making the exponent difficult to estimate. Previous works have dealt with this problem, but it is beyond the scope of this article. In this work, we therefore do not estimate \(\alpha\) either.

Heaps’ law is another scaling property and shows how vocabulary grows with text size, forming a power-law distribution. Let \(n\) be the length of a text and \(v(n)\) be its vocabulary size. Then Heaps’ law is formulated as the following relation:

\[
v(n) \propto n^\beta, \quad 0 < \beta < 1.
\]  

The second graph in Figure 1 shows the result for WT2. The exponent here is 0.75, which is smaller than 1.0 (dashed black line), with \(\varepsilon = 0.13\). Because of the replacement of unknown words in WT2, the plot exhibits convex growth. Also, note that Heaps’ law can be deduced from Zipf’s law (BaezaYates and Navarro 2000; van Leijenhorst and van der Weide 2005; Lü et al. 2010).

3.2 Long Memory

The statistical mechanics domain has introduced two directions for considering long memory: fluctuation analysis and long-range correlation. Here, we introduce two fluctuation analysis methods, one for characters and one for words, and one long-range correlation method, applied to words. Although
these methods are related analytically for a well-formed time series, for real phenomena with finite-size samples, the relations are non-trivial. The generated texts could behave similar to a random sequence or contrarily exhibit better long memory than a natural language text.

3.2.1 Fluctuation Analysis

Fluctuation analysis quantifies the strength of memory and the degree of symbol occurrence burstiness underlying a text.

Ebeling’s Method

Burstiness has been known to occur in various natural and social domains. Fluctuation analysis originated in (Hurst 1951), motivated by the need to quantify the degree of burstiness of Nile River floods. The method applies only for numerical series, so (Montemurro and Pury 2002) applied it for texts by transforming a word sequence into a rank sequence, which obscures the results.

One work (Ebeling and Pöschel 1994) applied a simple fluctuation analysis method on text. That work showed how the variance of characters grows by a power law with respect to the text length (i.e., time span). Given a set of elements $|W|$ (characters in this method), let $y(k,l)$ be the number of the $c_k \in W$ within text length $l$. Then,

$$m(l) = \sum_{k=1}^{|W|} m_2(k,l) \propto l^\eta,$$  \hspace{1cm} (3)

where $m_2(k,l)$ is the variance of $y(k,l)$:

$$m_2(k,l) = \langle y^2(k,l) \rangle - \langle y(k,l) \rangle^2.$$  \hspace{1cm} (4)

Theoretically, if the time series is independent and identically distributed (i.i.d.), then $\eta = 1.0$. (Ebeling and Pöschel 1994) showed that the Bible has $\eta = 1.69$, thus exhibiting larger fluctuation than an i.i.d. sequence. This article successfully demonstrates that a character sequence has long memory. The third graph in Figure 1 shows $m(l)$ for WT2. The exponent is 1.32 with $\varepsilon = 0.10$.

Taylor’s Law

While Ebeling’s method considers the growth of variance with respect to a subsequence of length $l$, Taylor’s method considers that growth with respect to the mean within length $l$. Because the mean of a subsequence linearly correlates with $l$, the two methods are closely related. While Ebeling’s method sums the variance over all elements of $W$, Taylor’s method estimates the scaling exponent from the points of all words. Because of this difference, Ebeling’s method is not applicable for detecting long memory underlying a word sequence: the exponent becomes 1.0 in this case, like an i.i.d. sequence. In general, Taylor’s law is more robust and is applicable to words and also to characters, if for a large set size.

Taylor’s law was originally reported in two pioneering works (Taylor 1961; Smith 1938) and has been applied in various domains as reported in (Eisler et al. 2007). Its application for language data, however, has been scarce. The only such study so far is (Gerlach and Altmann 2014). However the work considered vocabulary size rather than word occurrence, and its differences from the original Taylor analysis make theoretical interpretation inapplicable. We introduce another method proposed by Kobayashi and Tanaka-Ishii (2018) with which we can quantify the long range dependence of natural language and symbolic time-series in general and is highly interpretable.

Given a text produced from a set of words, $W$, for a given segment size $l$, the number of occurrences of a particular word $w_k \in W$ is counted and the mean $\mu_k$ and standard deviation $\sigma_k$ are calculated.
for $w_k$. Doing this for all elements of $W$ gives the distribution of $\sigma$ with respect to $\mu$. Taylor’s law then holds when $\sigma$ and $\mu$ are correlated by a power law:

$$\sigma \propto \mu^\zeta.$$  \hspace{1cm} (5)

Experimentally, the Taylor exponent $\zeta$ is known to range over $0.5 \leq \zeta \leq 1.0$.

The advantage of Taylor’s law over the other analysis methods for long memory is the interpretability of the exponent $\zeta$. The two limit values $\zeta = 0.5, 1.0$ provide a clear interpretation. For an i.i.d. process, it is proved that $\zeta = 0.5$. For a sequence in which all segments of length $l$ contain the same proportions of the elements of $W$, $\zeta = 1.0$. For example, given $W = \{a, b\}$, suppose that $b$ always occurs twice as often as $a$ in all segments (e.g., one segment with three $a$ and six $b$, another segment with one $a$ and two $b$, etc.). Then, both the mean and standard deviation for $b$ are twice those for $a$, and therefore $\zeta = 1.0$. The Taylor exponent thus quantifies how consistently words co-occur in texts. For this reason, a set of tweets from the same source or CHILDES texts would typically exhibit a higher exponent $\zeta$.

The fourth graph in Figure 1 shows the result for WT2 with $l = 5620$ ($l$ can be any value larger than one). The plot exhibits a power law, although some deviation from the regression line is also visible. The Taylor exponent is $\zeta = 0.62$, with $\varepsilon = 0.15$.

### 3.2.2 Long-Range Correlation

Burstiness of word occurrence leads to a related observation: how subsequences are similar. Such thinking resulted in another genre of analysis methods, called long-range correlation analysis, which measures the self-similarity within two subsequences of a time series.

A time series is said to be long-range correlated if the correlation function $c(s)$ for two subsequences separated by distance $s$ follows a power law:

$$c(s) \propto s^{-\xi}, \quad s > 0, 0 < \xi < 1.$$  \hspace{1cm} (6)

A widely used choice for $c(s)$ is the autocorrelation function (ACF). By using the ACF, the value of $c(s)$ ranges between -1 and 1. When a sequence is long-range correlated, $c(s)$ takes positive values for $s$ until about 1/100 of the length \cite{Lennartz2009}, whereas $c(s)$ fluctuates around zero for a sequence without temporal correlation.

Since the ACF is applicable only for numerical time series, application of this method for natural language requires transforming a natural language text into a numerical time series. Among recent reports \cite{Altmann2009, Tanaka2016}, we apply the most recent, a rare word clustering method with parameter $Q = 16$ \cite{Tanaka2016}.

Application of long-range correlation analysis to word sequences in WT2 produced the last graph in Figure 1. As noted in the legend, $\xi = 0.33$ and $\varepsilon = 0.04$ with $c(s)$ is all positive up to 1/100 of the sequence length. Throughout this article, for $\varepsilon$ of this metric only, it is measured for $s \leq 100$.

### 4 Language Models

This section explains the language models used in this article. Each model defines a probability distribution $P(x_{t+1})$, given $X_t^t = x_1, x_2, \ldots, x_t$. We probabilistically generated texts following the output distributions of these models. The all graphs for the scaling properties of the language models are available as supplementary material.
4.1 N-gram Model

An $n$-gram language model is the most basic model, as it is a Markov model of order $n - 1$. This article considers a 3-gram model and 5-gram model with back-off. Note that the perplexity of the models here should tend to be lower than in previous reports, as we did not adopt the <BOS> and <EOS> tags to maintain inter-sentence structure.

4.2 Grammatical Model

Probabilistic context-free grammar (PCFG) is the most basic grammatical model. We constructed this grammar model with the annotated Penn Tree Bank (PTB) dataset and used the Natural Language Toolkit (NLTK) to generate sentences according to the probabilities assigned to productions. Unlike an $n$-gram model, a PCFG ensures the grammatical correctness of all productions.

4.3 Simon/Pitman-Yor Models

The Simon and Pitman-Yor (PY) processes are important for our perspective, because they are capable of reproducing Zipf’s law and Heaps’ law with simple formulations. It is thus interesting to see whether they satisfy the other scaling properties.

These are generative models, and a sequence is formulated over time, either through (1) introduction of new words or (2) sampling from the past sequence. Let $K(X_t)$ be the number of word types existing in $X_t$, and let $n_k(X_t)$ be the frequency of the $k$th word type in $X_t$. The sequence starts with $K(X_0) = 1$ and $X_0 = x_0$ at $t = 0$.

For $t \geq 1$, given a constant $a$ with $0 < a < 1$, the Simon process \cite{Simon1955} introduces a new
word with probability $a$, or a word is sampled from $X_t^1$ with probability $1 - a$:

$$P(x_{t+1} = w_k) = \begin{cases} 
(1 - a) \frac{a_k(X_t^1)}{K(X_t^1)} & 1 \leq k \leq K(X_t^1) \\
\frac{a_k(X_t^1) - a}{k + b} & k = K(X_t^1) + 1
\end{cases}.$$ 

The Simon process strictly follows both Zipf’s law and Heaps’ law with an exponent of 1.0. The PY process copes with this problem by decreasing the introduction rate of new words in proportion to $K(X_t^1)$ via another parameter $b$, with $0 \leq a < 1$ and $0 \leq b$:

$$P(x_{t+1} = w_k) = \begin{cases} 
\frac{a_k(X_t^1) - a}{k + b} & 1 \leq k \leq K(X_t^1) \\
\frac{a_k(X_t^1) + b}{k + b} & k = K(X_t^1) + 1
\end{cases}.$$ 

These two parameters would serve to produce Zipf’s law with slightly convex behavior (Goldwater et al. 2011). The basic models introduced thus far define nothing about how to introduce words; we would simply generate random sequences and examine their scaling properties, because the basic formulation thus far governs the nature of the language model elaborated from these models. By mapping a word to the elements produced, however, we would generate a language model, like the two-stage model proposed in (Goldwater et al. 2011). Here, we consider a more advanced model proposed as the hierarchical Pitman-Yor language model (HPYLM) (Teh 2006), which integrates the Pitman-Yor process into an $n$-gram model.

### 4.4 Neural Language Models

The predictive performance of state-of-the-art language models improved radically with neural language models. The majority of promising neural language models (Mikolov and Zweig 2012; Melis et al. 2018; Merity et al. 2018; Yang et al. 2018) adopt recurrent neural networks (RNNs). The RNNs compute a hidden state $h_t$ from the input $x_t$ and the previous hidden state $h_{t-1}$ to incorporate past
\[ h_t = \Phi(x_t, h_{t-1}). \] (7)

The function \( \Phi \) depends on the recurrent architecture of the network. This article focuses on LSTM (Hochreiter and Schmidhuber 1997), because the difference in performance is insignificant among architectures such as the gated recurrent unit (GRU) (Cho et al. 2014) and other LSTM variants (Chung et al. 2014; Greff et al. 2017; Melis et al. 2018). A total of six neural language models are considered.

The first model is an LSTM language model that is not trained with regularization techniques. The predictive performance is equivalent or better than with \( n \)-gram models but significantly worse than with state-of-the-art models. This model therefore could be considered as a baseline to verify whether further regularization techniques contribute to reproducing the scaling properties.

For the advanced models, we adopted averaged stochastic gradient descent (ASGD) weight-dropped LSTM, or simply AWD-LSTM (Merity et al. 2018), because it consists of a standard architecture (embedding + LSTM + softmax) yet outperforms many other models in terms of perplexity. In addition to the standard AWD-LSTM model, two other architectures are considered: continuous cache (Grave et al. 2017) and mixture of softmax (MoS) (Yang et al. 2018).

Continuous cache is a memory augmentation architecture that computes a cache probability \( p_{cache} \) from context \( w_{t-l}^t \), where \( l \) is a window size parameter. It computes the similarity between \( h_t \) and \( h_i \) to estimate the reappearance of \( w_i \) at \( t+1 \). The output probability of the model with continuous cache, denoted as the AWD-LSTM-Cache model, is a linear interpolation of the AWD-LSTM and the cache probability. We also considered a model incorporating the Simon process, denoted as the AWD-LSTM-Simon model. It behaves as a uniform sampling from the past generated sequence and is a special case of AWD-LSTM-Cache. MoS reformulates the language model task as matrix factorization and is a state-of-the-art language model integrated with AWD-LSTM as the AWD-LSTM-MoS model. We also considered a combination of all these architectures, the AWD-LSTM-MoS-Cache model.

In our experiments, all of the language models are trained to minimize the negative log-likelihood of the training data by stochastic gradient algorithms. The window size \( l \) for the AWD-LSTM-Simon model is set to 10,000 to balance a large window size with computation efficiency.

5 Scaling Properties of Language Models

We chose two standard datasets, WT2 and PTB, for training language models. Table 1 and Table 2 list the overall results for WT2 and PTB, respectively. Each table contains five blocks, with the first indicating the properties of the original datasets with and without preprocessing. The remaining blocks indicate the results for the language models. Every row gives the results for a generated text of 1 million words. The two rows for the Simon and PY processes appear only in the table for WT2, because they represent non-linguistic random sequences obtained by the definitions given in §4.3, and no real data learning is involved. All the other rows show the results of the corresponding language model learning either the WT2 or PTB dataset. The datasets were preprocessed to reduce the vocabulary size. Infrequent words were replaced with \( <\text{unk}> \), and numbers were replaced with \( \# \) in PTB.

The first column lists the perplexity measured by the model. For some models, perplexity does not apply, as indicated by the symbol “-”. The perplexity grows with the Simon and PY models, and that of the PCFG cannot be compared under the same criteria.

For the Zipf’s law column, when the law holds, this is indicated by “Yes”. The reason is that, as mentioned in §3.1, the exponent is often difficult to estimate. Ebeling’s method is not applicable to the Simon and PY processes without a word generation model. For the other models, when an exponent was obtained, we checked whether the value was larger than 1.0 (the i.i.d. case). Similarly,
Table 1: Summary of scaling properties of language models with WT2

|                  | Perplexity | Vocabulary Population | Long Memory |
|------------------|------------|-----------------------|-------------|
|                  |            |                       | Zipf’s Law | Heaps’ Law | Ebeling’s Method | Taylor’s Law | Long Range Correlation |
|                  |            |                       | $f(r) \propto r^{-\alpha}$ | $v(n) \propto n^a$ | $m(l) \propto l^\gamma$ | $\sigma \propto \mu^\zeta$ | $c(s) \propto s^{-\xi}$ |
| Original Dataset |            |                       |            |            |                |              |                        |
| Wikitext-2       |            | Yes                   | 0.75 (0.13) | 1.32 (0.10) | 0.62 (0.15) | 0.33 (0.04) |                        |
| Wikitext-2-raw   |            | Yes                   | 0.78 (0.09) | 1.33 (0.10) | 0.65 (0.11) | 0.32 (0.03) |                        |
| 3-gram           |            | Yes                   | 0.78 (0.13) | 1.01 (0.01) | 0.50 (0.02) | No            |                        |
| 5-gram with back-off |      Yes   | 0.78 (0.13) | 1.00 (0.01) | 0.50 (0.02) | No            |                        |

N-gram Language Model

|                  |            |                       |            |            |                |              |                        |
| N-gram Language Model |            |                       |            |            |                |              |                        |
| N-gram Language Model |            |                       |            |            |                |              |                        |

Grammatical Model

|                  |            |                       |            |            |                |              |                        |
|                  |            |                       |            |            |                |              |                        |
|                  |            |                       |            |            |                |              |                        |
|                  |            |                       |            |            |                |              |                        |

Neural Language Model

|                  |            |                       |            |            |                |              |                        |
|                  |            |                       |            |            |                |              |                        |
|                  |            |                       |            |            |                |              |                        |
|                  |            |                       |            |            |                |              |                        |

for Taylor’s law, we checked whether the exponent was larger than 0.5. If the text is not long-range correlated, this is mentioned by “No” or “Weak”: “No” if more than one value was negative for $s \leq 10$, or “Weak” for $s \leq 100$.

The two tables do not indicate qualitative differences except for the cells in the last block of the last column: for other cells, when a scaling property held for one, it also held for the other. The main difference occurred because of a difference in dataset quality: PTB (The Wall Street Journal) consists of short articles, whereas WT2 (Wikipedia) consists of longer articles written coherently with careful
The language models listed above the neural language models in the tables failed to reproduce all of the scaling properties for long memory. Figure 2 shows a set of graphs for the HPLYM: apart from Zipf’s and Heaps’ laws, this model did not exhibit any long memory. The n-gram models and PCFG had similar tendencies. The sole exception was the Simon model, which presented strong long-range correlation, even stronger than that of the original WT2. Its Taylor exponent, however, was 0.5, suggesting its essential difference from a natural language text. This is obvious, in a way, because a sequence produced by the Simon model has a different mathematical nature with respect to vocabulary population for small t and large t, which is not the case for natural language. Curiously, this long memory was destroyed with the PY models and HPLYM. Overall, the Simon- and PY-based modeling might not be adequate for natural language.

In contrast, for WT2, the neural language models were able to reproduce all of the scaling properties. Figure 3 shows a set of graphs for the AWD-LSTM-Cache model, for WT2. The figure shows how well the model captures the nature of the original text in terms of every property.

The analysis, however, suggests the possibility of further improvement. Even though the AWD-LSTM-Cache model performed the best for WT2 with respect to perplexity, its Taylor exponent ζ still remained smaller (0.59) than that of the original text (0.62). Verification of the actual generated

Figure 4: Values of the Taylor exponent and perplexity for all the language models and the real data of WT2. When the perplexity is unmeasurable (for the original WT2 data and some language models), no bar appears. The results show that the two measures are correlated, but not totally among the most advanced models.

preprocessing. WT2 therefore has stronger long memory than PTB, which is apparent from their long memory properties.
text showed that this model had a tendency to repeat words locally. Moreover, the neural language models failed to reproduce long memory for the PTB dataset. Although the long memory underlying the original PTB is weaker than that of WT2, PTB still has long memory, and yet the models could only reproduce “weak” long-range correlation. This and all the above findings suggest directions for improving the neural language models.

At the same time, the discussion highlights the importance of the dataset used for evaluation: PTB is less suitable for evaluating long memory, while WT2 is recommended for this purpose.

6 Effectiveness of Scaling Properties for Examining Language Models

The results thus far also enable discussion from another viewpoint, of considering which scaling property would most effectively evaluate language models. Most important is the question in the article’s title: “Is perplexity sufficient?” We might ask whether model quality correlates with proximity to the exponent for a dataset, and which scaling property is the most effective for evaluation. We thus examine every property from this viewpoint.

First, Zipf’s law does not add exploitable information on the quality of a language model, because its exponent is difficult to estimate, as discussed in §3.1, and because all the models exhibited behavior like Zipf’s law. Still, the Zipf’s law distribution for WT2 in Figure 1 showed a drop at the tail, indicating that something unnatural occurred (i.e., replacement of rare words with <unk>). Therefore, Zipf’s law could be applied just to verify whether a vocabulary population is normal. Since Heaps’ law is derived from Zipf’s law, it also does not contribute much to evaluating language models.

Turning to long memory, the three properties roughly have positive correlation: when the original dataset had weaker long memory (PTB as compared with WT2), the degree of memory exhibited by the neural language models was also weaker. Ebeling’s method is limited in that it only applies for characters. The long-range correlation also has a limited capacity, because the Simon model exhibited strong long-range correlation but had a Taylor exponent of 0.5. Taylor’s law therefore seems more credible for evaluating model quality. Figure 4 shows the values of the Taylor exponent and perplexity for every language model. The figure indicates that they correlated well with differences for the most advanced neural language models. It highlights some aspects of the model characteristics. For example, it is likely that the long memory quality for WT2 was degraded by MoS, even though it improved the perplexity from AWD-LSTM. These results demonstrate that evaluation of language models from multiple viewpoints would contribute to better understanding of the nature of learning techniques, and scaling properties can provide these different viewpoints.

7 Conclusion

We explored the performance of language models with respect to the scaling properties of natural language and proposed evaluation methods other than perplexity. We listed five such tests, two for vocabulary population and three for long memory. For vocabulary population, all models considered here presented the scaling property. In contrast, many models did not exhibit long memory, but the state-of-the-art neural language models were able to produce a sequence with long memory. This constitutes solid evidence of the difference between n-gram and neural language models. Moreover, the difference between the exponents from a model and from the original dataset shows the possibility of improvement.

Further verification along the same line using raw data and other datasets remains for our future work. We also intend to investigate other kinds of metrics for model assessment.
References

Altmann, E.G. and Gerlach, M. (2017). Statistical laws in linguistics. Creativity and Universality in Language, pages 7–26.

Altmann, E.G., Pierrehumbert, J.B., and Motter, E.A. (2009). Beyond word frequency: Bursts, lulls, and scaling in the temporal distributions of words. PLoS ONE, 4(11), e7678.

Altmann, E.G., Cristadoro, G., and Esposti, M.D. (2012). On the origin of long-range correlations in texts. Proceedings of the National Academy of Sciences, 109(29), 11582–11587.

Baeza-Yates, R. and Navarro, G. (2000). Block addressing indices for approximate text retrieval. Journal of the American Society for Information Science, 51(1), 6982.

Bengio, J., Ducharme, R., Vincent, P., and Jauvin, C. (2003). A neural probabilistic language model. Journal of Machine Learning Research, (3), 1137–1155.

Cho, K., van Merriënboer, B., Gülçehre, Ç., Bahdanau, D., Bougares, F., Schwenk, H., and Bengio, Y. (2014). Learning phrase representations using rnn encoder–decoder for statistical machine translation. In Empirical Methods in Natural Language Processing, pages 1724–1734.

Chung, C., Gülçehre, Ç., Cho, K., and Bengio, Y. (2014). Empirical evaluation of gated recurrent neural networks on sequence modeling. In Neural Information Processing Systems Workshop.

Ebeling, W. and Pöschel, T. (1994). Entropy and long-range correlations in literary english. Europhysics Letters, 26(4), 241–246.

Eisler, Z., Bartos, I., and Kertész, J. (2007). Fluctuation scaling in complex systems: Taylor’s law and beyond. Advances in Physics, 57(1), 89–142.

Gerlach, M. and Altmann, E.G. (2014). Scaling laws and fluctuations in the statistics of word frequencies. New Journal of Physics, 16(11), 113010.

Goldwater, S., Griffiths, Thomas L., and Johnson, M. (2011). Producing power-law distributions and damping word frequencies with two-stage language models. Journal of Machine Learning Research, 12, 2335–2382.

Grave, E., Joulin, A., and Usunier, N. (2017). Improving neural language models with a continuous cache. In International Conference on Learning Representations.

Greff, K., Srivastava, R. K., Koutník, J., Steunebrink, B. R., and Schmidhuber, J. (2017). LSTM: A Search Space Odyssey. IEEE Transactions on Neural Networks and Learning Systems, 28(10), 2222–2232.

Hochreiter, S. and Schmidhuber, J. (1997). Long short-term memory. Neural Computation, 9(8), 1735–1780.

Hurst, H.E. (1951). Long-term storage capacity of reservoirs. Transactions of the American Society of Civil Engineers, 116(1), 770–808.

Jozefowicz, R., Vinyals, O., Schuster, M., Shazeer, N., and Wu, Y. (2016). Exploring the limits of language modeling. arXiv preprint, arXiv:1602.02410.

Knieser, R. and Ney, H. (1995). Improved backing-off for m-gram language modeling. In IEEE Conference on Acoustics, Speech and Signal Processing, volume 1, pages 181–184.
Kobayashi, T. and Tanaka-Ishii, K. (2018). Taylor’s law for human linguistic sequences. *arXiv preprint, arXiv:1804.07893*.

Lennartz, S. and Bunde, A. (2009). Eliminating finite-size effects and detecting the amount of whitenoise in short records with long-term memory. *Physical Review E, 79*(6), 066101.

Lin, H. W. and Tegmark, M. (2017). Critical behavior in physics and probabilistic formal languages. *Entropy, 19*(7), 299.

Lü, L., Zhang, Z.K., and Zhou, T. (2010). Zipfs law leads to heaps’ law: Analyzing their relation in finite-size systems. *PLoS ONE, 5*(12), e14139.

Manning, C.D. and Schutze, H. (1999). *Foundations of Statistical Natural Language Processing*. MIT Press.

Melis, G., Dyer, C., and Blunsom, P. (2018). On the state of the art of evaluation in neural language models. In *International Conference on Learning Representations*.

Merity, S., Xiong, C., Bradbury, J., and Socher, R. (2016). Pointer sentinel mixture models. In *International Conference on Learning Representations*.

Merity, S., Keskar, N.S., and Socher, R. (2018). Regularizing and optimizing LSTM language models. In *International Conference on Learning Representations*.

Mikolov, T. and Zweig, G. (2012). Context dependent recurrent neural network language model. In *IEEE Spoken Language Technology Workshop*, pages 234–239.

Mikolov, T., Karafit, M., Burget, L., Cernocky, J.H., and Khudanpur, S. (2010). Recurrent neural network based language model. In *Annual Conference of the International Speech Communication Association*, page 3.

Montemurro, M. and Pury, P.A. (2002). Long-range fractal correlations in literary corpora. *Fractals, 10*(4), 451–461.

Pitman, J. (2006). *Combinatorial Stochastic Processes*. Springer.

Simon, H.A. (1955). On a class of skew distribution functions. *Biometrika, 42*(3/4), 425–440.

Smith, H.F. (1938). An empirical law describing heterogeneity in the yields of agricultural crops. *Journal of Agriculture Science, 28*(1), 1–23.

Takahashi, S. and Tanaka-Ishii, K. (2017). Do neural nets learn statistical laws behind natural language? *PLoS ONE, 12*(12), e0189326.

Tanaka-Ishii, K. and Bunde, A. (2016). Long-range memory in literary texts: On the universal clustering of the rare words. *PLoS ONE, 11*(11), e0164658.

Taylor, L.R. (1961). Aggregation, variance and the mean. *Nature, 189*(4766), 732–735.

Teh, Y.W. (2006). A hierarchical bayesian language model based on pitman-yor processes. In *Annual Conference on Computational Linguistics*, pages 985–992.

van Leijenhorst, D.C. and van der Weide, Th.P. (2005). A formal derivation of heaps law. *Information Sciences, 170*(2-4), 263–272.
Vinyals, O., Fortunato, M., and Jaitly, N. (2015). Pointer networks. In *Advances in Neural Information Processing Systems*, pages 2692–2700.

Yang, Z., Dai, Z., Salakhutdinov, R., and Cohen, W.V. (2018). Breaking the softmax bottleneck: A high-rank RNN language model. In *International Conference on Learning Representations*. 