Thermodynamics in Kaluza-Klein Universe

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Abstract

This paper is devoted to check the validity of laws of thermodynamics for Kaluza-Klein universe in the state of thermal equilibrium, composed of dark matter and dark energy. The generalized holographic dark energy and generalized Ricci dark energy models are considered here. It is proved that the first and generalized second law of thermodynamics are valid on the apparent horizon for both of these models. Further, we take a horizon of radius $L$ with modified holographic or Ricci dark energy. We conclude that these models do not obey the first and generalized second law of thermodynamics on the horizon of fixed radius $L$ for a specific range of model parameters.

Keywords: Dark energy models; Thermodynamics.
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1 Introduction

The well-established notion is that the universe has entered in the phase of accelerating expansion. Type Ia supernovae [1]-[3], cosmic microwave background radiation (CMBR) [4], Wilkinson microwave anisotropy probe (WMAP) [5] and Sloan digital sky survey (SDSS) [6, 7] has indicated that

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our universe is flat, homogeneous and isotropic over large scale. This speedy expansion of our universe is due to an antigravity force which is drawing galaxies apart from each other, dubbed as dark energy (DE). Some scientists believe that extra dimensions of space are also responsible for this expansion. The mechanism behind this expansion and the nature of DE is not very much clear. Dark energy having large negative pressure dominates 76% energy density of the universe [8].

The cosmological constant is the most suitable candidate of DE which may be characterized by an equation of state (EoS) parameter, $\omega = -1$. The current value of this constant is $10^{-55}cm^{-2}$ whereas in particle physics it is $10^{120}$ times greater than this factor, this problem is known as fine-tuning problem [9]. The other serious problem is the cosmic coincidence problem which raised due to the comparison of dark matter (DM) and DE in the present expanding universe. There have been many DE models proposed such as scalar field models and interacting models etc. Quintessence [10,11], k-essence [12], phantom [13,14], tachyon [15,16], and quintom [17,18] are the scalar field models while the interacting DE models are Chaplygin gas [19,20], braneworld [21,22] and holographic DE (HDE) [23,24]. Unfortunately, this whole class of DE models do not explain the nature and its origin in a comprehensive way.

According to recent observations, multidimensional theories may help to resolve such problems of cosmology and astrophysics. The most impressing theory in this scenario is offered firstly by Kaluza [25] and Klein [26] by adding an extra dimension in general relativity (GR), known as Kaluza-Klein (KK) theory. It is basically a five dimensional (5D) theory in which gravity is unified with electromagnetism through this extra dimension. The validity of laws of thermodynamics has been discussed with modified HDE (MHDE) [27]-[31]. Some authors [32]-[35] extended this work to modified gravity theories like $f(R)$, $f(T)$, Brans-Dicke (BD) and Horava-Lifshitz theory. Sharif and Khanum [36] checked the validity of generalized second law of thermodynamics (GSLT) in KK universe with interacting MHDE and DM. Recently, Sharif and Jawad [37] explored this work with varying $G$ to investigate the validity of GSLT in the same scenario.

Holographic DE model based on the holographic principle, is a good effort in quantum gravity to understand the nature of DE to some extent. According to this principle, a physical system placed inside a spatial region is observed with its area but not within its volume [38]. Cohen et al. [39] argued the cosmological version of this principle, the quantum zero-point
energy $\rho_\Lambda$ of the system having size $L$ (infrared cutoff) cannot exceed the mass of a black hole (BH) with the same size. Mathematically, we get an inequality i.e., $L^3 \rho_\Lambda \leq L M_p^2$, where $M_p$ is the reduced Planck mass expressed as $M_p = (8\pi G)^{-\frac{1}{2}}$. This inequality is most suitable for large $L$ with event horizon. The HDE density can be expressed as $\rho_\Lambda = 3c^2 M_p^2 L^{-2}$, where $3c^2$ is a dimensionless constant. The HDE in modified version for KK theory is known as MHDE \cite{40} and can be calculated from the $(N + 1)$-dimensional mass of the BH \cite{41}.

Ricci DE (RDE) \cite{42} is a type of DE obtained by taking square root of the inverse Ricci scalar as its infrared cutoff. Gao et al. \cite{43} explored that the DE is proportional to the Ricci scalar. Some recent work \cite{44-46} shows that the RDE model fits well with observational data. Xu et al. \cite{47} gave the generalization of two dynamical DE models, i.e., generalized HDE (GHDE) and generalized RDE (GRDE) models. These two models, combination of $\dot{H}$ and $H^2$, gave the late time accelerating universe.

In \cite{48}, similar type of investigation has been done in FRW universe model. In a recent paper \cite{49}, we have checked the validity of the first and GSLT for Bianchi I universe model. We have also explored the statefinder, deceleration and Hubble parameters for the same line element \cite{50}. Here we extend the work of \cite{48} to KK universe model with the same scenario. In this paper, we use KK universe in thermal equilibrium composed of DM and DE with GHDE and GRDE models. The paper is designed as follows: In section 2, the density and pressure for GHDE/GRDE models are calculated. Section 3 is devoted to check the validity of the first and GSLT on the apparent horizon and also by taking GHDE/GRDE as the MHDE/MRDE. In the last section, we summarize the results.

## 2 Density and Pressure for GHDE and GRDE models

In this section, we evaluate energy density and pressure for GHDE as well as GRDE models in KK universe. This universe model contains 4-dimensional Einstein field equations and the fifth dimension satisfies the Maxwell field equations. This metric is the simple generalization of the FRW metric to extend the range of observable universe by increasing the dimensions of the
universe. The line element of KK model is given by

\[
ds^2 = -dt^2 + a^2(t)\left[\frac{dr^2}{1 - kr^2} + r^2(d\theta^2 + \sin^2\theta d\phi^2) + (1 - kr^2)d\psi^2\right],
\]

(1)

where \( k \) denotes the curvature parameter having values +1, 0 and −1 corresponding to open, flat and closed universe, respectively. The energy-momentum tensor for perfect fluid is

\[
T_{\alpha\beta} = (P + \rho)V_\alpha V_\beta - g_{\alpha\beta}P, \quad (\alpha, \beta = 0, 1, 2, 3, 4),
\]

(2)

where \( P, \rho \) and \( V_\alpha \) are the pressure of the fluid, energy density and five velocity vector, respectively. We consider that the fluid is a mixture of DM and DE, thus \( P \) and \( \rho \) can be written as \( P = P_m + P_E \) and \( \rho = \rho_m + \rho_E \) with \( P_m = 0 \). The field equations for KK universe become

\[
8\pi\rho = \frac{6\dot{a}^2}{a^2} + 6\frac{k}{a^2},
\]

(3)

\[
8\pi P = -3\frac{\ddot{a}}{a} - 3\frac{\dot{a}^2}{a^2} - 3\frac{k}{a^2}.
\]

(4)

We are interested in flat KK universe so that \( k = 0 \) yields the field equations as

\[
8\pi\rho = \frac{6\dot{a}^2}{a^2} = 6H^2,
\]

(5)

\[
8\pi P = -3\frac{\ddot{a}}{a} - 3\frac{\dot{a}^2}{a^2},
\]

(6)

where Hubble parameter is defined as \( H = \frac{\dot{a}}{a} \). The conservation equation can be written as

\[
\dot{\rho} + 4H(\rho + P) = 0.
\]

(7)

Differentiating Eq. (5) and using (7), it follows that

\[
\dot{H} = -\frac{8\pi}{3}(\rho + P).
\]

(8)

Here we assume that there does not exist any sort of interaction between DE and DM, therefore these are separately conserved. Thus the conservation equation (8) yields

\[
\dot{\rho}_m + 4H\rho_m = 0,
\]

(9)

\[
\dot{\rho}_E + 4H(\rho_E + P_E) = 0.
\]

(10)
Solving Eq. (9), the matter energy density is obtained as
\[
\rho_m = \rho_{m0}(1 + z)^4,
\]
where \(\rho_{m0}\) is the constant of integration, known as the present value of DE density and cosmological red shift is \(z = \frac{1}{a} - 1\). The matter density in KK universe decreases more rapidly as compared to FRW universe with the evolution of the universe which is consistent with the current observations.

Now, we evaluate energy density and pressure for GHDE and GRDE models as follows.

2.1 Generalized Holographic Dark Energy Model

The energy density of this model is given as
\[
\rho_h = \rho_E = \frac{3c^2H^2}{8\pi}g\left(\frac{R}{H^2}\right),
\]
where \(c^2\) is a non-zero arbitrary constant and \(f(x) > 0\) such that \(g(x) = \gamma x + (1 - \gamma), 0 \leq \gamma \leq 1\). The Ricci scalar is
\[
R = -4(2\dot{H} + 5H^2).
\]
For \(\gamma = 0\), the energy density of the original HDE is recovered while \(\gamma = 1\) leads to the original RDE. Using Eq. (13) in (12), it follows that
\[
\rho_h = \frac{3c^2}{8\pi}\left[(1 - 2\gamma)H^2 - 8\gamma\dot{H}\right].
\]
Inserting Eqs. (11) and (14) in (5), we obtain a first order linear differential equation whose solution is
\[
H^2 = -\frac{8\pi\rho_{m0}}{3}\frac{(1 + z)^4}{((1 + 11\gamma)c^2 - 2)} + H_0^2(1 + z)^\frac{2c^2(21\gamma - 1)}{8\gamma c^2},
\]
with \(H_0\) an integration constant. Differentiating Eq. (15) with respect to \(t\), we get
\[
\dot{H} = -\frac{16\pi\rho_{m0}}{3(2 - (1 + 11\gamma)c^2)}(1 + z)^4 - H_0^2\frac{(2 - (1 - 2\gamma)c^2)}{16\gamma c^2}(1 + z)^\frac{2c^2(21\gamma - 1)}{8\gamma c^2}.
\]
Substituting $H^2$ and $\dot{H}$ in Eqs. (10), (13) and (14), it follows that

$$P_h = -\frac{3H_0^2 ((21\alpha - 1)c^2 + 2)((1 - 13\gamma)c^2 - 2)}{16\pi} (1 + z) \frac{2 + c^2 (21\gamma - 1)}{8\gamma c^2},$$ \hspace{1em} (17)

$$R = -\frac{32\pi \rho_m}{3(2 - (1 + 11\gamma)c^2)} (1 + z)^4 + H_0^2 \frac{(2 + (1 + 19\gamma)c^2)}{2\gamma c^2} \times (1 + z) \frac{2 + c^2 (21\gamma - 1)}{8\gamma c^2},$$ \hspace{1em} (18)

$$\rho_h = \frac{(1 - 37\gamma)c^2 \rho_m}{-2 + (1 + 11\gamma)c^2} (1 + z)^4 - H_0^2 \frac{3(2 + (1 - 21\gamma)c^2)}{16\pi} \times (1 + z) \frac{2 + c^2 (21\gamma - 1)}{8\gamma c^2}.$$ \hspace{1em} (19)

Equations (17) and (19) represent pressure and energy density in terms of red shift $z$.

### 2.2 Generalized Ricci Dark Energy Model

The energy density of GRDE model is [47]

$$\rho_r = \frac{3c^2 R h \left(\frac{H^2}{R}\right)}{8\pi},$$ \hspace{1em} (20)

where $h(y) = \delta y + (1 - \delta) > 0$, $0 \leq \delta \leq 1$. For $\delta = 0$, the original energy density of the RDE is recovered whereas $\delta = 1$ leads to energy density of the original HDE. Comparing Eqs. (12) and (20), we see that the GRDE reduces to the GHDE and vice versa for $\delta = 1 - \gamma$. By replacing $\gamma$ with $(1 - \delta)$ in Eqs. (14)-(19), we obtain similar solutions for GRDE model. This implies that these equations are also solutions of the GRDE model with $\gamma = 1 - \delta$.

### 3 First and Generalized Second Law of Thermodynamics

Firstly, we discuss the validity of the first and GSLT on the apparent horizon. For this purpose, we use the entropy given by Gibb’s law [51, 52]

$$T_A dS_I = P dV + d(E_A),$$ \hspace{1em} (21)
where $S_I$, $V$, $P$, $E_A$ and $T_A$ are internal entropy, volume, pressure, internal energy and temperature of the apparent horizon, respectively. In FRW metric, the apparent horizon has the radius

$$R_A = \frac{1}{\sqrt{H^2 + \frac{2}{\kappa}}}.$$  \hfill (22)

Here FRW metric contained in the KK universe is a subspace with compact fifth dimension having similar properties of flat FRW universe on the apparent horizon. The internal energy and volume in extra dimensional system are

$$E_A = \rho V, \quad V = \pi^2 L^4 / 2.$$  

In flat geometry, the radius of the apparent horizon coincides with Hubble horizon given as

$$R_A = L = R_H = \frac{1}{H}.$$  \hfill (23)

The entropy and temperature of the apparent horizon are [53]

$$S_A = S_h = \frac{A}{4G}, \quad (G = 1), \quad T_A = \frac{1}{2\pi R_A} = \frac{1}{2\pi L},$$  \hfill (24)

and entropy in four dimensions takes the form

$$A = 2\pi^2 L^3, \quad S_A = \frac{2\pi^2 L^3}{4} = \frac{\pi^3 L^3}{2}.$$  \hfill (25)

The first law of thermodynamics on the apparent horizon is defined as

$$- dE_A = T_A dS_A.$$  \hfill (26)

The energy crossing formula on the apparent horizon for KK universe can be found as follows [54]

$$- dE_A = 2\pi^2 R_A^4 H T_{\alpha\beta} K^\alpha K^\beta dt = 2\pi^2 R_A^4 H (\rho + P) dt = - \frac{3\pi}{4} H \dot{H} L^4 dt.$$  \hfill (27)

Inserting $L$ from Eq.(23) in the above equation, we get

$$- dE_A = - \frac{3\pi}{4} \left(\frac{\dot{H}}{H^3}\right) dt.$$  \hfill (28)
Using Eqs. (24) and (25), it follows

\[ T_A dS_A = \frac{3\pi^2}{4} L \dot{L} dt = -\frac{3\pi^2}{4} \left( \frac{\dot{H}}{H^3} \right) dt. \] (29)

These two equations lead to the following form of the first law of thermodynamics

\[ -dE_A = \frac{1}{\pi} T_A dS_A, \] (30)

which gives its validity on the apparent horizon for all kinds of energies as it is independent of DE.

Now for the GSLT to be satisfied for the apparent horizon, we evaluate the derivative of internal entropy through Eq. (21) as

\[ \dot{S}_I = \frac{(\rho + P) \dot{V} + V \dot{\rho}}{T_A}. \] (31)

Substituting the values of \( \dot{V}, T_A, \dot{\rho} \) and using conservation equation, we get

\[ \dot{S}_I = \frac{3\pi HR_A^3 (\dot{R}_A - R_A H)}{4T_A} \] (32)

According to SLT, entropy of the thermodynamical system can never be decreased. This is generalized in such a way that the derivative of any entropy is always increasing, i.e., \( \dot{S}_I + \dot{S}_A \geq 0 \). Thus we have

\[ \dot{S}_I + \dot{S}_A = \frac{3\pi^2}{8} \left[ 4 \frac{\dot{H}^2}{H^6} - 3 \frac{\dot{H}}{H^4} \right] dt \geq 0. \] (33)

We conclude that GSLT always holds on the apparent horizon. Notice that these laws always hold on the apparent horizon as it is independent of choice of DE.

Further, we take GHDE or GRDE models as the density of MHDE or MRDE to check the validity of the first and GSLT on the horizon having radius \( L \). The MHDE density can be calculated by taking the mass of \((N+1)\)-dimensional BH [41].

\[ M = \frac{(N-1)A_{N-1}R_H^{N-2}}{16\pi G}, \]
where \( A_{N-1} \) is the unit \( N \)-sphere area, \( R_H \) is the scale of the BH horizon and \( G \) is the gravitational constant in \( (N + 1) \)-dimensions related to Planck mass \( M_{N+1} \). As \( 8\pi G = M_{N+1}^{(N-1)} = \frac{V_{N-3}}{M_p^2} \), \( V_{N-3} \) is the volume of this space, so \( M \) can be written as

\[
M = \frac{(N - 1)A_{N-1}R_H^{N-2}M_p^2}{2V_{N-3}}.
\]

We can write

\[
L^3 \rho_{\Lambda} \sim \frac{(N - 1)A_{N-1}L^{N-2}M_p^2}{2V_{N-3}},
\]

which implies that

\[
\rho_{\Lambda} = \frac{c^2(N - 1)A_{N-1}L^{N-5}M_p^2}{2V_{N-3}}.
\]

For \( N = 4 \), it gives

\[
\rho_{\Lambda} = \frac{3c^2A_3L^{-1}}{2}.
\]

Inserting the value of 4-sphere area, we have

\[
\rho_{\Lambda} = \frac{3c^2\pi^2}{8}L^2.
\]

Comparing this value with the energy density of GHDE, it follows that

\[
L^2 = \gamma R + (1 - \gamma)H^2.
\]

Substituting the values of \( H^2 \) and \( R \) from Eqs. (15) and (18) in (35), the expression for \( L^2 \) in the form of red shift is

\[
L^2 = \frac{1}{6c^2(-2 + (1 + 11\gamma)c^2)} \times \left[ -16\pi c^2\rho_m(1 - 5\gamma)(1 + z)^4 + 3H_0^2(-2 + (1 + 11\gamma)c^2)(2 + (1 - 21\gamma)c^2)(1 + z)\frac{24c^2(21\gamma - 1)}{8\pi c^2} \right].
\]

Here the temperature, entropy and the total energy crossing on this horizon with radius \( L \) is similar to Eqs. (24) and (27), respectively, with the difference that \( dS_L, T_L \) and \( dE_L \) are used instead of \( dS_A, T_A \) and \( dE_A \). We can write

\[
T_LdS_L = \frac{3\pi^2}{4}L\dot{L}dt.
\]
For the first law, we must have $-dE_L = T_L dS_L$. Equations (27) and (37) imply
\[ -dE_L = T_L dS_L - \frac{3\pi}{4} L \left[ H\dot{H}L^3 + \pi\dot{L} \right] dt. \] (38)
Since the second term on the right hand side in the above equation is time dependent, so it can never be zero in the evolving universe. Thus
\[ -dE_L \neq T_L dS_L. \]
This indicates that the first law of thermodynamics does not hold for the horizon of radius $L$. For the validity of GSLT on the horizon of radius $L$, the derivative of total entropy is as follows
\[ \dot{S}_I + \dot{S}_L = \frac{3\pi^2}{8} L^2 [4\dot{H}L^2(HL - \dot{L}) + \pi\dot{L}] dt. \] (39)
According to the GSLT, the total entropy of the thermodynamical system always increases, i.e., $4\dot{H}L^2(HL - \dot{L}) + \pi\dot{L} \geq 0$ indicating its dependence only on $L$ in the DE model. In GHDE model, the variation of total entropy on the horizon is
\[ \dot{S}_I + \dot{S}_L = \frac{3}{8} \pi \left[ 4\dot{H}L^2(2L^2 + (1 + z) \frac{dL^2}{dz}) - \pi(1 + z) \frac{dL^2}{dz} \right], \] (40)
where $L^2$ is given in Eq.(36). This expression does not provide any indication, whether it increases or decreases. To get insights, we plot a graph of total entropy $(\dot{S}_I + \dot{S}_L)$ versus red shift $z$ as shown in Figure 1. This indicates that $(\dot{S}_I + \dot{S}_L) < 0$ and hence the GSLT does not hold on this horizon with radius $L$ for the specific values of the parameters.
4 Concluding Remarks

We have considered KK universe in compact form in the state of thermal equilibrium, similar to FRW universe by assuming that our universe is filled with DM and DE. Two types of DE models, GHDE and GRDE have been used. It is worth noticing that the GRDE model can be converted to GHDE model by interchanging $\delta$ with $1-\gamma$. Also, the original density of HDE and RDE models is obtained for $\gamma = 0$, $\delta = 1$ and $\gamma = 1$, $\delta = 0$, respectively. The density and pressure for GHDE and GRDE models in terms of red shift $z$ are evaluated.

We have investigated the validity of the first and GSLT on the apparent horizon in this scenario. These laws turn out to be independent of the choice of DE models, geometry of BH, and also the fifth dimension. Hence these laws are always satisfied on the apparent horizon for all kinds of DE models. We have also checked that the first law remains invalid on particle as well as on the event horizon while GSLT holds on the particle horizon only. It is worth mentioning here that KK universe in non-compact form also gives the same results as the variation along the fifth dimension is negligible. Further, we have considered the GHDE and GRDE as the MHDE and MRDE and found $L$ in terms of $z$, to check the validity of these laws on the horizon whose radius is denoted by $L$. It is concluded that the first law of thermodynamics does not hold on the horizon of radius $L$ for both DE models. The GSLT always holds on this horizon in the range $\gamma, \delta \in (0,0.1)$ for GHDE and GRDE, respectively, but it remains invalid for $\gamma, \delta \in (0.1,1)$. It is worth mentioning here that our results on the apparent as well as on the horizon of radius $L$ are consistent with FRW universe [5].

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