FLUX ENHANCEMENT OF SLOW-MOVING PARTICLES BY SUN OR JUPITER: CAN THEY BE DETECTED ON EARTH?

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ABSTRACT

Slow-moving particles capable of interacting solely with gravity might be detected on Earth as a result of the gravitational lensing induced focusing action of the Sun. The deflection experienced by these particles is inversely proportional to the square of their velocities, and as a result their focal lengths will be shorter. We investigate the velocity dispersion of these slow-moving particles, originating from distant point-like sources, for imposing upper and lower bounds on the velocities of such particles in order for them to be focused onto Earth. Stars, distant galaxies, and cluster of galaxies, etc., may all be considered as point-like sources. We find that fluxes of such slow-moving and non-interacting particles must have speeds between \( \sim 0.01 \) and 0.14 times the speed of light, \( c \). Particles with speeds less than \( \sim 0.01c \) will undergo way too much deflection to be focused, although such individual particles could be detected. At the caustics, the magnification factor could be as high as \( \sim 10^6 \). We impose lensing constraints on the mass of these particles in order for them to be detected with large flux enhancements that are greater than \( 10^{-9} \) eV. An approximate mass density profile for Jupiter is used to constrain particle velocities for lensing by Jupiter. We show that Jupiter could potentially focus particles with speeds as low as \( \sim 0.001c \), which the Sun cannot.

Key words: elementary particles – gravitation – gravitational lensing: strong – planets and satellites: individual (Jupiter) – Sun: general

Online-only material: color figures

1. INTRODUCTION

According to estimates that use the latest data, 80% of the matter content of the universe most likely comprises non-baryonic dark matter particles that are yet to be discovered (WMAP Science Team 2011; ESA, Planck 2013). Some or most of these particles could be heavy and hence slow-moving. Since these particles do not interact with any of the known force fields but gravity, they are capable of passing through (transparent to) the Sun’s interior (Lawrence 1971; Clark 1972; Ohanian 1973; Bontz & Haugan 1981; Burke 1981; Gerver 1988; Demkov & Puchkov 2000; Hoffmann et al. 2003; Patla & Nemiroff 2008).

The velocities of massive particles, in order to be consistent with the theory of special relativity, must be smaller than the speed of light in vacuum, \( c \), and therefore will undergo considerably larger deflection for the same impact radius compared to photons traveling in a gravitational field or even neutrinos that are often considered to be fast moving (\( v \approx c \)).

For photons and neutrinos, the minimum focal length of the Sun is about 550 AU and 23.5 AU, respectively. The magnification of the particle flux from point-like distant sources can be detected only by observers stationed beyond the minimum focus. Hereafter, in this paper, when we make references to slow-moving particles, it is to be assumed that they interact only with gravity.

For slow-moving particles the minimum focal length is shorter because, from elementary projectile motion, the deflection angle, \( \delta \propto v^{-2}b^{-1} \), where \( v \) is the speed of the particle impacting the lens at a distance \( b \) from the center. In this paper we seek answers to the following questions. (1) What is the velocity range for particles in order for them to be focused at 1 AU? (2) What is the requirement on the minimum mass of particles for it to be focused at 1 AU and also record an appreciable flux (magnification) without getting scattered due to diffraction effects? (3) What is the magnitude of the maximum amplification? (4) What is the field of view within which a source has to be for the flux of particles emitted by it to be amplified by large factors? (5) Can the flux density and velocity distributions of these particles be used to further our understanding of fundamental physics, say, for example, testing the equivalence principle (EP; or its violation)?

A detection will be a direct confirmation of the existence of non-ordinary matter. We use the appropriate deflection formula in the weak field limit for slow-moving particles to put limits on their velocities in order for them to be detected at 1 AU. We are building on the previous work done by Hoffmann et al. (2003) using the formalism developed in Patla & Nemiroff (2008).

This paper is organized as follows. Section 2 summarizes particles that are considered to be slow moving. Section 3 discusses two formulae for deflection angles that are used as inputs for solving the lens equation. Section 4 addresses the relationship between focal lengths and impacting velocities, by referring to the previous work of Patla & Nemiroff (2008). Magnifications and flux density are discussed in Section 5. Diffraction limits and mass constraints on the slow-moving particles are detailed in Section 6. Lensing by Jupiter and the effects of transiting planets are addressed in Section 7, followed by conclusions in Section 8.

2. SLOW-MOVING PARTICLES: WIMPs AND AXIONs

Plausible candidates for slow-moving particles are classes of weakly interacting massive particles (WIMPs) and axions—either or both of which may or may not eventually be confirmed...
experimentally to match with the observed dark matter density in the universe. For excellent reviews on the topic of dark matter, we refer the reader to Trimble (1987), Spergel (1997), and Drees & Gerber (2012). For axions in particular, see Turner (1990), Peccei (2006), and Ringwald (2012). Surveys looking for clumped baryonic dark matter candidates have thus far not been able to confirm their existence (Alcock et al. 2008).

WIMPs were motivated by the physics of the early, and therefore hot universe when the conditions favored reversible processes such as the annihilation of electron–positron pairs. WIMPs would interact through the exchange of heavy intermediate bosons: $W$, $Z$, Higgs. The extension of the standard model by using the symmetries of the Poincare group, now dubbed supersymmetry (SUSY), also points to the existence of WIMPs (Diehl et al. 1995; Jungman et al. 1996). However, recent results from the latest Large Hadron Collider (LHC) runs are not very encouraging for SUSY (LHC Higgs Cross Section Working Group: S Dittmaier et al. 2012). The predicted energies of these WIMPs range from a few GeVs to hundreds of GeVs (Ackermann et al. 2012; Picozza & Marcelli 2013; Aguilar et al. 2013). For a review of other hypothetical particles dubbed “mirror” and “shadow” particles, see for example Blinnnikov & Khlopov (1983); Kolb et al. (1985).

During the early 1970s, in order to explain the apparent lack of $U(1)$ symmetry in QCD, a phase parameter was introduced. This parameter added an extra term to the QCD Lagrangian that violated parity and time (PT) reversal invariance while conserving charge conjugation (C) invariance, hence violating charge conjugation and parity (CP). Peccei & Quinn (1977) proposed a solution by introducing a CP-conserving dynamical pseudoscalar field with an effective potential, which when expanded at the minimum yields a massive particle called an axion (Weinberg 1978; Wilczek 1978). More recently, axion-like Kaluza–Klein excitations, in the context of higher-dimensional theories of gravity, emergent from stellar interiors have also been a topic of some interest (Dienes et al. 2000; Dienes et al. 2000; Di Lella et al. 2000; Di Lella & Ziotas 2003).

Axions that were initially proposed were strongly interacting and massive but were later ruled out by experiments. Flavors of axions that have survived (without being ruled out by experiments) thus far are the so-called invisible axions (Kim 1979; Shifman et al. 1980; Dine et al. 1981). These are hypothesized to have formed in the early universe due to spontaneous decay of cosmic strings or by the relaxation of string-domain wall boundaries. In the present epoch axions could seek shelter within the interiors of massive stars or galaxy centers. The rest masses of these axions fall in the range of $10^{-6}$–0.1 eV, with large error bars on both ends (Raffelt & Rosenberg 2012; Hewett et al. 2012). These bounds are obtained by imposing constraints of energy loss in stars without affecting stellar evolution or structure formation in the universe. In particular, for the Peccei–Quinn scale, the Wilkinson Microwave Anisotropy Probe data sets an upper bound of 1 eV for axion rest mass, while a lower bound of $\sim 10^{-6}$ eV follows arguments of overclosure for the universe.

Models of dark matter flows (wakes) in the vicinity of gravitating objects have been discussed previously: solar wakes (Sikivie & Wick 2002) and cosmic strings wakes (Stebbins et al. 1987). Lensing due to dark matter caustics is addressed in Charmousis et al. (2003) and Onemli (2006). In this paper we do not consider these effects. We only consider gravitational lensing of cosmological (located far away from Earth’s orbit) point-like sources that emit slow-moving particles, which are capable of being lensed by the Sun (see Section 8). Isotropic backgrounds would not produce any discernible amplifications.

3. DEFLECTION ANGLES FOR SLOW-MOVING PARTICLES

The formula for the deflection angle for a ray of light in the weak field limit is (Einstein 1936)

$$\delta = \frac{4GM}{bc^2},$$

where $G$ is the gravitational constant, $M$ is the mass enclosed within the impact parameter (radius of cylinder) $b$, and $c$ is the speed of light in vacuum. The values for the deflection at the solar limb were experimentally confirmed for the first time for visible light and cosmic microwave background radiation by Dyson et al. (1920) and Fomalont & Sramek (1975), respectively.

The general formula for deflection in classical relativity for particles with arbitrary velocity $v$ is (Accioly & Ragusa 2002):

$$\delta_c = \frac{2GM}{bc^2} \left(1 + \frac{\beta^2}{2} + bg \left(1 + \frac{\beta^2}{4}\right) \frac{3\pi}{2} + \frac{3}{5} \beta^2 - \frac{1}{3} \beta^4 + \frac{5}{3} \beta^6 + O(g^3)\right),$$

where $g = GM/b^2$ and $\beta = v/c$. Keeping only the first order terms in $g$, the deflection is

$$\delta_c = \frac{4GM}{bc^2} \left(1 + \frac{\beta^2}{2} \right).$$

Note that in the limit $\beta \rightarrow 1$, we recover Equation (1), and when $\beta \ll 1$ the amount of deflection is reduced by a factor of two, which is essentially the Newtonian prediction (Soldner 1804). The deflection angle obtained using the semiclassical approach, that is, by including terms corresponding to the interaction of a massive photon field with a minimally coupled and static (external) gravitational field in the action integral, is (Accioly & Paszko 2004)

$$\delta_{sc} = \frac{4GM}{bc^2} \left(\frac{3 - \beta^2}{2\beta^2}\right).$$

In this case also, in the limit $\beta \rightarrow 1$, we recover Equation (1). However, when $\beta \ll 1$, $\delta_{sc} = 1.58 = 3\delta_c$. For very slow-moving particles, the semiclassical result predicts a deflection 50% more than the one obtained using the standard formula, Equation (1). The values of deflection angles enter into the lens equation, the solutions of which are the image locations for a given point-like source location. The variation of the deflection angle as a function of particle speeds and impact parameters is shown in Figure 1.

In the following section, we will use two different deflection formulae to compute the minimum focal length for particles with a given velocity. The main reason for doing this is that no one knows for sure which deflection formula provides the most accurate trajectory for slow-moving particles in the intermediate range of $\beta$ values, say from $10^{-3}$ to 0.1 (to be discussed in Section 4), for it to be focused at minimum focal length(s) of 1 AU.
4. PARTICLE VELOCITY AND FOCAL LENGTH

Gravitational lensing by the transparent Sun generates a series of focal lengths starting with the minimum focal length of \( \sim 23.5 \) AU for neutrinos or any other fast-moving (non-massive) particles with velocities nearly equal to the speed of light. An approximate value of the minimum focal length may be obtained by considering the mass enclosed within the transition impact parameter, at which the power-law index \( n \) of the density profile of the Sun changes roughly from \( n = 0 \) to \( n = -2 \). The approximate formula for the minimum focal length for the Sun as given in Patla & Nemiroff (2008) is

\[
F \sim 2 \left( \frac{1 + n}{3 + n} \right) \left( \frac{c^2}{4g_\odot} \right) \left( \frac{r_n^2}{m_n} \right),
\]

\( (5) \)

where \( g_\odot = GM_\odot/R_\odot^2 \), \( r_n = 0.075 \) is the normalized radius corresponding to the transition of the power-law index, and \( m_n = 0.1 \) is the normalized mass enclosed within \( r_n \), yielding a value of \( \sim 21 \) AU. A higher limit for the minimum focal length may be obtained directly from Equation (1) instead, by considering the mass \( m_n \) within an impact radius \( r_n = 0.075 \), yielding a value of \( \sim 31 \) AU.

The extension of this formalism by using Equations (3) and (4), corresponding to the classical and semiclassical deflection angles and demanding that the approximate minimum focal length be \( \sim 1 \) AU imposes upper and lower bounds for the speeds of particles that are subjected to the lensing action of the Sun. \( \beta \) values of \( \sim 0.15 \) and \( \sim 0.26 \), and \( \sim 0.13 \) and \( \sim 0.22 \) corresponding to minimum focal lengths of 20.5 AU and 31 AU, respectively, are obtained for both classical and semiclassical cases.

Therefore, from rough estimates, we have to simply conclude that the velocities of slow-moving particles have to be within the range of \( 0.1 - 0.3c \), in order to be focused on the Earth by the lensing action of the transparent Sun. Particles with speeds less than \( 0.1c \) will have a series of focal lengths starting with a minimum value of less than 1 AU, and particles with speeds greater than \( 0.3c \) will be focused beyond the Earth’s orbit.

This range of the velocity distribution will serve as a good starting point for computing the actual focal lengths and magnifications in the following sections. For completeness, below we provide the relationship between the approximate focal length and the velocities of slow-moving particles using both classical and semiclassical formulae (for deflections), respectively:

\[
D_L = F_{\text{min}} \left( \frac{2\beta^2}{1 + \beta^2} \right),
\]

\( (6) \)

\[
D_L = F_{\text{min}} \left( \frac{2\beta^2}{3 - \beta^2} \right),
\]

\( (7) \)

where \( D_L \) is the distance between the lens and the observer and \( F_{\text{min}} \) is the minimum focal length for particles with speed \( \beta = 1 \). Letting \( D_L = 1 \) AU and solving Equations (6) and (7) for \( \beta \) yields the limiting velocities for the classical and semiclassical approximations for deflection angles.

The exact value of the limiting velocities of the particles that converge at 1 AU can be computed numerically, by using the deflections given by either of the formulae, Equations (3) or (4), in place of Equation (1) in the ray-tracing algorithm. The projected mass density is obtained by integrating the interpolated density profile of the Sun using the standard solar model (SSM) data published in Bahcall et al. (2005). The criterion employed for establishing the minimum focal length is the existence of an Einstein ring—the circle formed by the extended arcs of two images when the source, lens, and observer are perfectly aligned in a straight line—the same criterion used in Patla & Nemiroff (2008). In-depth discussions on the topics of Einstein rings, caustics, and critical curves have been covered extensively in the literature on gravitational lensing (Chwolson 1924; Schneider et al. 1992; Nemiroff 1993; Kochanek et al. 2001). We provide a brief and contextual overview of the same in Section 5.

The exact values of the limiting particle speeds, using classical deflection formula given by Equation (3) and semiclassical deflection given by Equation (4), corresponding to a minimum focal length of 1 AU, are \((0.145 \pm 0.001)c\) and \((0.247 \pm 0.001)c\), respectively. Therefore, we cannot have a higher flux of particles with speeds higher than the values given above. The angular Einstein radii corresponding to the limiting speeds at 1 AU are given in Table 1. At 1 AU, the radius of the Sun measures up to \( 960'' \) from Earth.

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Also, as described in the caption of Figure 1, the formalism of weak lensing fails for particle speeds that are close to and less than \( \sim 0.01c \), because the deflection values are too large to cause any focusing. However, the trajectories of individual particles can be computed fairly accurately as long as the potential, \( \phi \ll c^2 \), submits to the requirements of the weak field limit.

### 5. AMPLIFICATIONS AND FLUX

The solution to the lens equation is

\[
\beta_s = \theta_i - \alpha.
\]

where \( \beta_s \) and \( \theta_i \) are the angles subtended by the unlensed source and its image(s), and \( \alpha \) is the deflection measure given by Equation (1), (3), or (4). The deflection values as a function of impact radius are obtained by projecting the mass enclosed within the impact radius onto the lens plane following the formalism developed in Patla & Nemiroff (2008); Patla (2008). The solutions to Equation (8) are the image locations—roots of the polynomial equation—for any given source location. For an introduction to and history of gravitational lensing in the weak field limit, we refer the reader to the monograph by Schneider et al. (1992) and excellent reviews by Blandford & Narayan (1992), Narayan & Bartelmann (1996), and Wambsganss (1998).

We assume the Sun to be a centrally condensed spherically symmetric lens. As a source crosses radially inward toward the axis of the lens in the source plane, the number of images of the source changes from one to three in the lens plane (Burke 1985). The locus of all such points in the lens plane at which two images suddenly appear is a circle called a critical curve. The area of the source plane comprising source locations that correspond to three images in the lens plane is encircled by a curve called the radial caustic. The term radial is used to suggest that the images move radially apart from the caustic. At the center of this radial caustic is a tangential point caustic; the corresponding tangential critical curve in the lens plane is the Einstein ring. At the tangential caustics the images stretch tangential to the caustic. The Einstein ring separates the new and old image sets in the lens plane. Magnifications are high along the critical curves and caustics in the lens- and source-planes respectively as a result of the newly formed images.

In ordinary lensing germane to photons, the value of the Einstein ring (as a function of focal length) starts from an infinitesimally small value at the minimum focal length followed by a steep rise before attaining a maximum value and falling off gradually thereafter. For slow-moving particles, just like photons, the value of the Einstein ring as a function of focal length will mimic the pattern of the deflection angles that are plotted in panel (b) of Figure 1. The actual values, however, will be different owing to the functional form of \( f(\beta) \) terms in Equations (1) and (3).

In other words, for slow-moving particles the radii of the Einstein ring, caustics, and critical curves will be larger if the observer is at or near the minimum focal length when compared to lensing of neutrinos \( \beta \approx 1 \), for example. The dependence of particle speeds on the geometrical parameters of the problem is plotted in Figure 2.

Magnification is defined as the ratio of the size of the image to that of the source, and for a point-like source it is given by the formula \( \mu = \theta d\theta / \beta_s d\beta_s \). Very large magnifications of point-like sources occur when the source crosses a caustic. Magnification of an extended source is computed by averaging individual points comprising the extended source. Since all sources are small compared to the lens, our formula for magnification is accurate enough for small impact parameter crossing of point sources across the lens. Moreover, perfect alignment of a point-like source, the lens, and an observer along the lens axis is rather an exception than the norm.

On the question of how much magnification is possible, the answer is as high as \( \sim 10^6 \), not very different from the results of Patla & Nemiroff (2008). Similar treatments concerning the

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**Figure 2.** Einstein radius, the radii of caustic and critical curves as a function of particle speed for observer at 1 AU for (a) classical and (b) semiclassical deflection formulae. The images are always separated by the Einstein ring, but at lower speeds the caustics become larger than the critical curves, suggesting large deflections. For reference, at 1 AU, the radius of the Sun is 960'. The weak-field deflection limit breaks down when the value of the Einstein ring becomes infinite.
Figure 3. Magnification for source crossing with impact parameters $\beta_n = 0$ through $\beta_n = 0.4$. $\beta_s$ and $\beta_n$ are normalized with the value of Einstein radius $\theta_E$ in units of arcsec. Panels (a)–(d) represent increasing particle speeds, $\beta = 0.06$ through 0.12. The dashed lines represent maximum and minimum magnifications as the source is moving away from the radial and toward the point caustic. The maximum magnification of $\sim 10^6$ is at the crossing of the radial ($\beta_s = \beta_{\text{caus}}$) and tangential ($\beta_s = 0, \beta_n = 0$) caustics. For a finite source, realistic values of magnifications are $\sim 10^5$ with peak amplification periods of $\sim 0.1''$. Note that the radius of the Sun subtends an angle of 960'' on Earth. The formula for classical deflection, Equation (3), was used for generating deflection values for solving the lens equation. Note on conversion: $100'' \approx 30$ minutes.

magnification of point sources due to gravitational lensing may be found in the excellent review article by Paczynski (1996). We use point-like sources for simulations involving the source crossing the lens at different impact parameters. As in the previous sections, we use two different functions for deflection angles to solve for the lens equation, the roots of which are used for mapping regions of varying magnifications and for calculating the maximum values of magnification.

In Figure 3, we present the magnifications for particles with speeds starting with $\beta = 0.06$. For particles with lower speeds the magnifications are not as high, and particles moving with a speed of $\sim 0.01c$ or less undergo way too much deflection and therefore cannot be focused. As the source moves across the lens, for a fixed impact parameter, the number of images multiply as the source gets closer to the center of the lens. The magnifications are high at or near radial and point caustics.

For particles with an intermediate range of speeds, $\sim 0.05c$ to $\sim 0.15c$, there will always be a significant amount of magnification over the radius of the caustic—the distance separating the circumference of the radial caustic and tangential point caustic—for low impact parameters; see Figure 3. The narrow angular band coinciding with the ecliptic ($\Delta \beta_n$), where typical sources must be present in order to command high magnification, is given in Table 2 for representative values of particle speeds obeying Equation (3). $\Delta \beta_n$ is the corresponding angular width measured along the ecliptic, subtending an angular area field $\Delta a \approx \Delta \beta_n \times \Delta \beta_{\text{sl}}$. As an example, the center of a galaxy as big as ours located a few hundred megaparsec away will appear to be a point source from Earth because the angular size of the source $\theta_s \ll \theta_E$, where $\theta_E$ is the Einstein radius; see Table 2.

The magnifications using the deflection formula of Equation (4) also yield similar results. The differences being, the particle speeds corresponding to maximum magnification will be shifted toward the right and the maximum magnifications will be close to an order of magnitude higher than the classical case as given in Figure 3.

6. CONSTRAINTS ON THE MASS OF THE SLOW-MOVING PARTICLE

Just as we imposed velocity restrictions for recording high particle flux densities at the detector (starting with the Earth
as the minimum focal point), in this section we use diffraction criteria to put limits on the particle mass in order to record maximum flux at the detector.

The amplified image of a source (assumed of size smaller than the lens) envelops the Einstein ring when the source is perfectly aligned with the lens. For a point source, two point images are cast inside and outside of the Einstein ring as shown in Figure 4. At the observer location, such a source would be detected only if the individual particles from the source that appear to be coming from different points (images) in the lens plane produce an interference pattern with a finite width \( w \) at the detector. The magnification is then roughly the ratio of the radius of the Einstein ring \( R_E \) to the fringe width, \( \mu \approx R_E/w \) (Ohanian 1973; Nakamura 1998; Patla & Nemiroff 2008).

If the detector is at a distance \( D_L \) from the lens, the Einstein radius is

\[
R_E = \left( \frac{4GM(b)D_L}{c^2f(\beta)} \right)^{1/2},
\]

(9)

where \( b \) is the impact radius of the source and \( f(\beta) = 2\beta^2(1 + \beta^2)^{-1} \) or \( 2\beta^2(3 - \beta^2)^{-1} \) depending on the deflection formula used to compute the image locations. Noting that \( w \approx \lambda D_L/R_E \), the maximum magnification is

\[
\mu \approx \frac{R_S^2}{\lambda D_L} \frac{2GM(b)}{c^2} \frac{2}{\lambda f(\beta)} \approx \frac{R_S}{\lambda f(\beta)},
\]

(10)

where \( R_S \approx 3 \) km is the Schwarzschild radius of the Sun. Therefore, when \( \beta = 1 \), the condition for large magnification factors is \( R_S \gg \lambda \), and for \( \beta < 1 \) this condition may be relaxed by a factor \( \sim \beta^{-2} \). Assuming the wavelength \( \lambda \) for slow-moving particles of mass \( m_0 \) to be its de Broglie wavelength \( \lambda = \hbar/(\gamma m_0 c) \), where \( \gamma = (1 - \beta^2)^{-1/2} \),

\[
\lambda_{dB} = \lambda_\gamma \left( 1 - \frac{\beta^2}{\beta^2} \right)^{1/2},
\]

(11)

where \( \lambda_\gamma = \hbar/m_0 c \) is the Compton wavelength of the particle. For appreciable magnification at 1 AU, the particles must have a mass of at least

\[
m_0 \gtrsim \frac{\hbar}{c} \left( \frac{1}{\theta_E^2 D_L} \right) \left( \frac{1 - \beta^2}{\beta^2} \right)^{1/2}.
\]

(12)

Using the values for \( D_L = 1 \) AU from Table 1, the minimum value for \( m_0 = 1.68 \times 10^{-45} \) kg. Alternatively, using the same mass fraction \( (m_n = 0.1) \) from the approximate formula given by Equation (5), we approximate the limiting mass of the particle

\[
m_0(\beta) = \frac{\hbar c}{4GM_\odot m_n} \left( \frac{1 - \beta^2}{\beta^2} \right)^{1/2} f(\beta),
\]

(13)

which gives a minimum mass of \( m_0 = 3.0 \times 10^{-46} \) kg. Taking the higher (and also most accurate) value of the limiting mass yields a particle mass of at least \( 10^{-9} \) eV, which covers almost all dark matter particles considered so far.

As an instructive exercise, from Table 1, consider the semi-classical case deflection (just because its value is higher) at 1 AU. If indeed the prediction of a narrow energy window for axions as explained in Section 2 holds true, the above diffraction constraints allow up to a million \( 10^{-4} \) eV (just as an example) particles confined to an area of \( \sim 100 \) m². Similar numbers may be computed for various particle speeds using Equations (9) through (11) using values for corresponding variables from Figure 3.

7. LENSING BY PLANETS: JUPITER AS AN EXAMPLE

Large planets like Jupiter are also capable of lensing slow-moving particles. In this section we consider scenarios when planets could act as the primary lens or serve to further amplify the lens action of the Sun as they transit and perfectly align along the Sun–Earth axis.
7.1. Planet as Lens

Jupiter is separated by a distance of \( \sim 4.2 \text{ AU} \) and \( \sim 6.2 \text{ AU} \) from its closest and farthest approach from Earth (NASA: Solar System Exploration 2013). The internal density variation of Jupiter is not well understood; it is also not clear whether Jupiter even has a core. However, studies have indicated it having a core that might be as large as \( 0–18 M_\oplus \) (Guillot 1999; Nettelmann 2011). Unlike stars, one would not expect planets to be centrally condensed. So to keep it general, we will develop a formalism involving a density profile that is more representative of planets. For now, we will introduce a free parameter (core radius) that could be fine-tuned to fit the actual Jovian density profile when it becomes available.

We’ll assume a Lorentzian density profile with a core radius for approximating a more general planetary lens:

\[
\rho(r) = \frac{\rho_0}{1 + \left( \frac{r}{r_c} \right)^2}, \tag{14}
\]

where \( r_c \) is the assumed radius of the core and

\[
\rho_0 = \frac{M_\ast}{4\pi \bar{R}_c^2} \left[ \frac{\bar{R}_c}{R_\ast} - \tan^{-1} \left( \frac{R_\ast}{\bar{R}_c} \right) \right]^{-1}. \tag{15}
\]

\( M_\ast \) and \( R_\ast \) are the mass and the radius of the planet. In order to obtain a minimum focal length for small impact radii, we first obtain the projected mass in terms of the dimensionless impact parameter

\[
M(r) = 2 \int_0^{2\pi} \int_0^r \int_0^{\sqrt{R_c^2 - r^2}} \rho(r)dRdz, \tag{16}
\]

where \( r^2 = R^2 + z^2 \). Using appropriate substitutions and performing integration by parts of the inner two integrals, we obtain the projected mass in terms of the dimensionless impact parameter

\[
\frac{M(b)}{M_\ast} = \frac{f(b) - f(0)}{r_c} \left[ \frac{1}{r_c} - \tan^{-1} \left( \frac{1}{r_c} \right) \right]^{-1} \quad \text{for} \ b \leq 1, \tag{17}
\]

where

\[
f(\bar{r}) = \sqrt{\bar{r}^2 + \bar{r}_c^2} \left[ \tan^{-1} \left( \frac{1 + \bar{r}_c^2}{\bar{r}^2 + \bar{r}_c^2 - 1} \right)^{1/2} - \left( \frac{1 + \bar{r}_c^2}{\bar{r}^2 + \bar{r}_c^2 - 1} \right)^{1/2} \right]. \tag{18}
\]

\( \bar{r} \equiv r/R_\ast, \bar{r}_c \equiv R_c/R_\ast, \) and \( b \equiv r/R_\ast \). For \( b > 1 \), \( M(b) = M_\ast \).

The focal length is

\[
D_\ell(b) = \frac{b^2 c^2 f(\bar{r})}{4g\bar{L}(b)} R_\ast. \tag{19}
\]

In order to obtain the minimum focal length, we take the limiting case for small impact parameters,

\[
\lim_{b \to 0} D_\ell(b) \equiv F_{\text{min}} = \frac{c^2 f(\bar{r})}{4g_\ast} \frac{2c^2}{\tan^{-1} \left( \frac{1}{r_c} \right)} \left[ \frac{1}{r_c} - \tan^{-1} \left( \frac{1}{r_c} \right) \right], \tag{20}
\]

where \( g_\ast = GM_\ast / R_\ast^2 \). Now specifically for Jupiter, \( g_\ast \sim 25 \text{ m s}^{-2} \). Letting \( r_c = 0.2 \) roughly satisfies the criterion that \( 0–18 M_\oplus \) is contained within a radius \( R_\oplus \) comprising Jupiter’s core. Substituting the value of \( r_c \) in Equation (20) yields a minimum focal length of \( \sim 1200 \text{ AU} \). Depending on our knowledge of Jupiter’s core, which we hope will become available in the future, the value of \( r_c \) may be tweaked appropriately to match with the data. The focal length corresponding to the Jovian limb as the impact radius is \( \sim 6000 \text{ AU} \), compared to \( \sim 550 \text{ AU} \) corresponding to the solar limb. The values for focal lengths corresponding to a range of values for the core radius are given in Figure 5.

Substituting the values for \( D_\ell \) as 4.2 and 6.2 AU in Equations (6) and (7) yields a particle velocity of \( 0.04c \) and \( 0.05c \) for classical and \( 0.07c \) and \( 0.08c \) for semiclassical cases, respectively. As in Figure 1, the maximum deflections may be computed for Jupiter for maximum deflections corresponding to \( r_c = 0.2 \). The pattern of the curves resemble the ones obtained for the Sun, except for the fact that the lower limit for particle speed is now an order of magnitude less compared to the Sun. Therefore, at least with the assumption of \( r_c = 0.2 \), Jupiter is capable of focusing particles with speeds as low as \( 0.001c \). This fact is noteworthy because by most estimates the detectable dark matter spectrum on Earth peaks at \( 0.001c \) (Sikivie et al. 1995).

The magnifications are expected to be as high as the values obtained for the Sun, at least near the caustics. Using Equation (17) in Equation (9), we estimate values of Einstein radii of Jupiter for a given observer location and assumed core radius \( \bar{r}_c \) in Figure 6. The actual values might be even higher because the same set of formulae yields lower values (of Einstein radius) for the Sun when compared to the values obtained numerically by using the actual density profile. An exception to this rule is a possibility only when the density profile of Jupiter veers off from being monotonically decreasing. Large values of Einstein radii for particle velocities of the order of \( 0.001c \) are suggestive of a broader observation band (at least \( \times 2 \)) coinciding with the orbital plane of Jupiter compared to the ecliptic with the Sun being the lens. We will postpone the study.
suggest that the detection window for Jupiter is larger than that offered by the nearest planets. For particle speeds around \( \sim 0 \) c, the lens-like action of Jupiter is of the order of the Einstein radius of Jupiter with the Earth and Sun, Jupiter must be located within a distance of \( \sim 1 \) AU and, therefore, a slight enhancement in magnification. The orbital plane of Jupiter makes an angle of \( \sim 3 \) with respect to the ecliptic with an orbital period of \( \sim 12 \) yr. For a slight enhancement in magnification due to perfect alignment of Jupiter with the Earth and Sun, Jupiter must be located within a radius of \( \sim 96^\circ \) from the center of the Sun, which is 10% of the angular extent of the Sun; see Equation (5). The frequency of such alignments are of the order of a few years, and the lensing events last for an hour or less. Dark matter experiments can be readily designed to probe such events. Repeating events assume greater significance if they occur when a planet transits the Sun.

However, slightly higher velocity particles that would not previously be focused or focused with reduced flux at 1 AU will now be detectable at this location. Also, the separation between the magnification peaks for the same source with a fixed transiting speed and impact parameter will also be increased by a few percent (compared to the case of a planet not aligned along with the lens axis).

8. DISCUSSION

Although little is known about what objects are capable of generating an appreciable supply of slow-moving particles, our Sun is suggestive for likely sources, which would include massive stars that not only dissipate energy continuously but also transiently while undergoing dynamic transformations, implosions or explosions. It was shown in Di Lella & Zioutas (2003) that inside the Sun massive axes of the Kaluza–Klein type can be created—making the Sun’s core an actual point-like source of non-relativistic particles with velocities relevant to the reasoning of this work. Similar fractions of non-relativistic massive solar exotica can also be expected to be created due to axion–electron coupling (Arisaka et al. 2013; Cicoli et al. 2012). Needless to say, the worked-out case of Kaluza–Klein axions should be construed as a generic example of all kind of particles the Sun’s hot interior might create.

Of course, the same can happen with other stars, in the Milky Way or elsewhere. Another example would be pulsars, whose strong luminosity is often associated with their strong magnetic field. We note that after 45 yr the exact mechanism by which pulsars shine is still unknown (Hermes et al. 2013). For example, the diffuse X-ray emission observed in PSR J1648−4611 is by far more than expected and cannot be attributed solely to its spin-down luminosity (Sakai et al. 2013). Gravitationally self-trapped massive exotica, which decay radiatively, could be at the origin of this extended X-ray emission, following the reasoning of Di Lella & Zioutas (2003). Extending the Sun’s analogy to pulsars, it is possible for particles with speeds above the escape velocity of the pulsars to leave a source that will certainly appear to be point-like.

The above examples show that not only stars but also galaxies and clusters of galaxies might well appear as point-like sources of non-relativistic exotica for a remote observer. After all, the celebrated “constellations” across the ecliptic reflect large agglomerates of stars and galaxies. Thus, non-relativistic “dark radiation” from those places can be temporally gravitationally enhanced. Often, a source emits a spectrum of particles centered around a characteristic velocity. It is this velocity, corresponding to peak flux, that we’ll use to distinguish between two different particle species.

As underground or orbiting detectors are becoming more and more sensitive with time, what this work points out is that at certain well-defined time intervals an enhanced flux of invisible dark matter radiation might hit the detector. Assuming conservatively that this alignment lasts only for one to two minutes and the flux enhancement is \( \sim 10^3 \), the periods corresponding to these few minutes will be equivalent to \( \sim 10^6 \) s \( \approx 3 \) yr. While such running periods are usual for dark matter experiments, the real advantage is a direct consequence of the detector background being of the order of a few minutes, resulting in an improvement of the ratio of signal to square root of the background—by more than three orders of magnitude—which is quite a large gain in sensitivity.

The sources could also include massive bodies that are capable of gravitationally attracting a flux of surrounding dark...
matter particles and reemitting them instead of producing them. For example, based on galactic halo formation models, Sikivie et al. (1995) conclude that the dark matter particle abundance near the Earth peaks at velocities of the order of $\sim 0.001c$; also see Griest (1988). It is plausible that galaxy centers can attract such fluxes and redirect them toward the Sun and thus make accessible to us the lower end of the velocity spectrum, which is roughly half the maximum.

Assuming that the peak of the velocity spectrum of particles that are accessible here on Earth due to lensing by Jupiter exactly overlaps with the foreground dark matter abundance predicted by simulations, what can we conclude about the signal-to-noise ratio for a single lensing event? First, as we pointed out earlier, the density profile of Jupiter is still unknown. Second, the foreground prediction is only an average measure. Therefore, it is not possible to calculate the exact value for the signal-to-noise ratio accurately. However, what is more extensively studied, and also very relevant to our work, is the analysis of dark matter modulation from the halo of our Milky Way in conjunction with that of its satellite Sagittarius Dwarf Elliptical Galaxy (Freese et al. 2004; Bernabei et al. 2006).

These studies suggest that the ratio of streaming particles from the satellite galaxy to isotropic background from the halo is between 0.2% and 25%. This situation is more representative of our point source within an Einstein radius of Jupiter, the ratio of which is also of the order of a few percent. Therefore, even if the fraction of streaming particles is less than a few percent, the potential of very high magnification at the caustics will result in a signature in the modulation signal of the streaming particles distinct from that of the foreground; see, for example, Freese et al. (2004). Note that the value of the magnification at the caustics is independent of the density profile of Jupiter, even as the same cannot be said of the duration of time for which the peak magnifications last. We also wish to point out that more work needs to be done for generating reliable templates for analyzing the data because most models still approximate the galactic halo with an isothermal sphere. However, if the source is a star that sparsely emits axions due to baryonic decays, then of course the signal will be weak regardless of the large magnifications because the percentage of streaming particles would then go below a thousandth of a percent and thereby make it arduous to separate the signal from the foreground.

If indeed strong isolated sources producing or reemitting slow-moving particles do exist, it is reasonable to assume an emergent flux that is centered around a mean velocity. To answer the question, what is the velocity resolution of the flux that can be inferred by a pair of detectors on Earth?, we use the relationship between velocity difference and focal length, obtained from the formula given by Equation (6),

$$\frac{\delta F}{F} = \frac{f'(\beta)}{f(\beta)} \delta \beta.$$  \hspace{1cm} (21)

This gives a maximum value for the velocity resolution $\delta \beta_{\text{max}} \sim 10^{-3} \beta$ for particle flux with a given mean speed $\beta$, assuming that two detectors are separated by a distance roughly (to first order in $\beta$) equal to the diameter of the Earth. The minimum value is set by the astigmatism of the Sun, which is of the order of a few meters and is roughly $\delta \beta_{\text{min}} \sim 10^{-10} \beta$ (Gerver 1988).

The magnification of the flux of particles coming from the source, as a result of the source straddling the lens plane, will peak only during the times when the source is near the caustics. Considering each peak as a lensing event, for a source at sufficiently small impact radii, the temporal separation between any two such lensing events (see Figure 3) is about an hour or less. This timescale is based on the motion of the Earth around the Sun. Moreover, most distant objects in the sky that are of interest to us move at much smaller speeds.

Based on the temporal separation of the lensing events and based on measured mean velocity, we could potentially rule out one of the deflection formulae given by Equations (3) and (4). This is because when Equation (3) is recast in terms of particle energies, the deflection becomes dependent on the total energy of the particle. In that case one might argue that such particles could violate the EP (Accioly & Paszko 2004).

One related question is then, could we possibly test for the violation of EP with at least a select variety of particle species? If Equation (3) is ruled out, then most likely the EP violation is a possibility. Alternatively, the lower limit of the velocity resolution of the particle flux might not be small enough to validate EP violation ($\delta \beta_{\text{min}}/\beta < 5$). Because the current understanding is that there is no violation reported for values of $\eta$ parameter (measure of relative acceleration for two different masses) up to $\sim 10^{-15}$ (Reasenberg et al. 2012). A thorough analysis is needed to accurately design an experiment to test EP using the lensing of slow-moving particles and so requires additional work.

Invoking diffraction criteria, we constrain the mass of the slow-moving particles to be focused at the detector to be more than $\sim 10^{-9}$ eV. The Sun and Jupiter may focus slow-moving particles that comprise the bulk of the predicted dark matter spectrum on Earth. Jupiter has the potential to amplify flux here on Earth that the Sun is not capable of focusing: particles with speeds $0.01c - 0.001c$. Also, the perfect alignment of Jupiter behind the Sun will give a marginal flux enhancement amounting to an increase of less than an order of magnitude.

We note that astigmatism of the Sun is of the order of a few meters, and as a result the maximum magnification could be reduced by an order of magnitude (Gerver 1988). The effect of the Sun’s relative motion in the galaxy and the resulting flux modulation have negligible effects for the isolated point-like sources that we have considered (Spergel 1988). Uniform and isotropic background sources produce no discernible caustics, just like large-size sources produce reduced magnifications. We, therefore, consider only point-like cosmological sources.

In conclusion, it is possible to detect—here on Earth—the flux of slow-moving and non-interacting particles lensed by the Sun or Jupiter. In order to be detected on Earth the particles have to have speeds between $\sim 0.01c$ and .14c for the Sun. If semiclassical deflection angles are considered, these speeds will scale to $\sim 0.01c$ and .24c. Particles with speeds less than $\sim 0.01c$ will undergo too much deflection to be focused, although such individual particles could be detected. The same is true for Jupiter, albeit the values are scaled back by an order of magnitude. The magnifications can be as high as $\sim 10^6$ at the caustics for point sources, with a more reasonable value of $\sim 10^5$ for real situations involving small sources crossing the caustics. Substantial magnification of $\sim 10^4$ is possible for times ranging up to 30 minutes, although peak amplifications of $\sim 10^6$ prevail only for about a few seconds ($0.1$') or even less.

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