Leptoproduction of Vector Mesons at Small $x_{Bj}$ and Generalized Parton Distributions *

S.V. Goloskokov  
BLTP, Joint Institute for Nuclear Research  
Dubna 141980, Moscow region, Russia  
E-mail: goloskkv@thsun1.jinr.ru  
P.Kroll  
Fachbereich Physik, Universität Wuppertal,  
D-42097 Wuppertal, Germany  
E-mail: kroll@physik.uni-wuppertal.de  
P. Postler  
Stanford Linear Accelerator Center  
2575 Sand Hill Road, Menlo Park, CA 94025, USA  
E-mail: postler@SLAC.Stanford.EDU

Deep virtual vector meson leptoproduction at small $x_{Bj}$ is analyzed in the framework of generalized parton distributions. It is shown that the inclusion of transverse degrees of freedom leads to reasonable agreement with the cross section data for light mesons.

In this article we report on an investigation of vector meson leptoproduction off protons at small $x_{Bj}$ and large photon virtuality, $Q^2$. In this kinematical region the process factorizes [1] into a hard subprocess, meson leptoproduction off partons, and a soft proton matrix element which represents a gluonic generalized parton distribution (GPD) [2]. At large $Q^2$, the cross section is dominated by photon-meson transitions where both the particles are longitudinally polarized. Other transitions are suppressed by at least $1/Q$ as compared to the leading-twist one, $\gamma^*_L \to V_L$. It has, however, turned out that the leading-twist contribution to the cross section is substantially exceeds experiment [3]. This parallels observations made within the two-gluon exchange model. As argued in [4] transverse momentum effects in the photon wave function provides sufficient suppression of the two-gluon exchange amplitude in order to achieve agreement with experiment. We remark that GPD corrections to the two-gluon exchange model have been estimated in [5].

Motivated by experience with meson form factors [6, 7] and with meson leptoproduction at large $x_{Bj}$ [8] we are going to modify the leading-twist GPD

*This work is supported in part by the Russian Foundation for Basic Research, Grant 03-02-16816, by the Deutsche Forschungsgemeinschaft and by the Heisenberg-Landau program.
approach by the inclusion of transverse degrees of freedom and Sudakov suppressions in the subprocess in order to suppress the contributions from the end-point regions where one of the partons entering the meson wave function becomes soft and, hence, factorization breaks down.

As shown in [1], to leading-twist accuracy, the $\gamma_L p \rightarrow V_L p$ amplitude reads

$$M_V^{(g)}(0^+, 0^+) = e C_V \sqrt{1 - \xi^2} (1 + \xi) \int_0^1 d\tau \frac{H^{(g)}_0(\tau, \xi, t)}{(\tau + \xi)(\tau - \xi + i\epsilon)},$$  

at small $x_{Bj}$. The skewness $\xi$ is approximately given by $\xi \approx x_{Bj}/2 \approx Q^2/(2W^2)$ in the kinematical region of interested here ($W$ is c.m. energy of the $VP$ system).

Examples of flavor factors are

$$C_\rho = 1/\sqrt{2}, \quad C_\phi = -1/3.$$  

To leading order (one-gluon exchange) the amplitude for the partonic subprocess, $\gamma_L g \rightarrow V_L g$, acquires the form

$$H^{(g)}_0 = 4\pi\alpha_s(Q/2)f_V N_c Q \int_0^1 d\tau \frac{\Phi_V(\tau)}{\tau(1 - \tau)}.$$  

The meson’s decay constant and distribution amplitude are denoted by $f_V$ and $\Phi_V$, respectively. A model for the gluonic GPD, $H^g$, is constructed by starting from a double distribution [9] and writing it as a product of the ordinary gluon distribution $xg(x)$, a factor $f(t)$ that models the $t$-dependence, and an additional $x, y$ dependence

$$f^g(x, y, t) = 6\frac{y(1 - x - y)}{(1 - x)^3} xg(x) f(t).$$  

Using results given for instance in [9], $H^g$ can be straightforwardly worked out from (4). It is obviously related to the ordinary parton distribution which, for small $x$, can be approximated by

$$xg(x, Q^2) \approx 1.94x^{-0.17 - 0.05\ln(Q^2/Q_0^2)},$$

where $Q_0^2 = 4\text{GeV}^2$.

Let us now turn to the modification of the leading-twist amplitude in a way analogously to the treatment of the electromagnetic meson form factors [6, 7]. The basic idea is to retain the transverse momenta of quark and antiquark forming the meson (defined with respect to the meson’s momentum). According to phenomenological experience these transverse momenta are large since the valence Fock state of the meson is rather compact. On the other hand,
the transverse momenta of the gluons (defined with respect to the proton momenta) emitted and reabsorbed by the protons, are much smaller because the full proton with its large radius is to be considered. In contrast to [8] we neglect the latter transverse momenta. Our approach is similar in spirit to the modification of the two-gluon exchange model proposed in [4].

Taking into account transverse quark momenta, we have to replace the distribution amplitude \( \Phi_V \) by a soft light-cone wave function for which we adopt a Gaussian parameterisation [7]

\[
\Psi_V(\tau, k_\perp) = 8\pi^2 \sqrt{2} N_c f_V a_V^2 \exp \left[ -a_V^2 \frac{k_\perp^2}{\tau(1-\tau)} \right].
\] (6)

Transverse momentum integration of (6) leads to the associated distribution amplitude \( \Phi^{AS}_V = 6\tau \bar{\tau} \), the asymptotic form of a meson distribution amplitude. For the decay constants \( f_V \) we take the values 0.216GeV and 0.237GeV for \( \rho \) and \( \phi \) mesons, respectively. For the transverse size parameter \( a_\rho \), we use a value of 0.6 GeV\(^{-1} \) which leads to a r.m.s. transverse momentum of 0.6 GeV and a valence Fock state probability of about 1/4, the same value as for the pion [7]. In the case of \( \phi \)-meson production we take \( a_\phi = 0.45 \) GeV\(^{-1} \).

In the quark propagators of the subprocess amplitude we have to consider the transverse momenta too. A typical modification of a quark propagator is then

\[
\frac{1}{\tau Q^2} \rightarrow \frac{1}{\tau Q^2 + k_\perp^2}.
\] (7)

For consistency the inclusion of transverse degrees of freedom is to be accompanied by Sudakov suppressions. Since the Sudakov factor \( S \) exponentiates in the transverse separation space it is convenient to work in \( b \) space. Fourier transforming the wave function and the subprocess amplitude and collecting all terms, we obtain for the gluonic contribution to the helicity amplitude of vector meson photoproduction at small \( x_{Bj} \)

\[
\mathcal{M}_{0+,0+}^{V(g)} = e C_V \sqrt{1 - \xi^2} (1 + \xi) \int d\bar{\xi}d\tau \tau(1-\tau) H^g(\bar{\tau},\xi,t)
\times \int d^2b \tilde{\Psi}(\tau,-b) \tilde{\mathcal{H}}_g(\bar{\tau},\tau,Q,b) \exp[-S(\tau,b,Q)].
\] (8)

Here, \( \tilde{\mathcal{H}}_g \) and \( \tilde{\Psi} \) are the plural Fourier transforms of the subprocess amplitude and wave function, respectively. The Sudakov factor has been calculated by
Botts and Sterman [11] to next-to-leading-log approximation using resummation techniques; its explicit form can be found for instance in [12]. The Fourier transform of the subprocess amplitude leads to Bessel functions like $K_0(\sqrt{\tau}Qb)$.

Fig. 1. The cross section for $\gamma^* p \rightarrow \rho^0 p$ vs. $W$ for fixed values of $Q^2$. Data are taken from [10]; ZEUS(95) open triangles, ZEUS(98) full circles.

Fig. 2. The longitudinal cross sections at $W = 75\text{GeV}$ extracted in [13]. The solid (dashed) line represent our results for $\rho$ ($\phi$) production.

Our results for the $\gamma^* p \rightarrow \rho p$ cross section are shown on Fig.1. The full cross section has been obtained from the longitudinal one by adding to it the transverse contribution through

$$\sigma(\gamma^* p \rightarrow \rho p) = \sigma_L(\gamma^* p \rightarrow \rho p) (\epsilon + \frac{1}{R}). \quad (9)$$

A value of 3 for the ratio of longitudinal over transversal cross sections and $\epsilon \simeq 1$ has been used by us. As can be seen from Fig. 1 good agreement of our approach with experiment [10] is obtained for a large range of $W$ and $Q^2$.

In Fig. 2 we show the longitudinal cross sections for $\rho$ and $\phi$ production extracted from $\gamma^* p \rightarrow Vp$ in [13] (references to the H1 and ZEUS data can be found therein). Again good agreement with experiment is to be observed in the case of $\rho$ as well as in that of $\phi$ meson. We note that our results for $\rho$ and $\phi$ production approximately scale as the squared flavor factors, $(C_\phi/C_\rho)^2$. This comes about as a consequence of the quark charges in the mesons and the similarity of the wave functions for the light vector mesons.
We conclude – the GPD approach to vector meson leptoproduction at small
$x_B$, modified by the inclusion of transverse momenta in the meson wave func-
tion and Sudakov suppressions, leads to reasonable agreement between theory
and experiment for $\rho$ and $\phi$ production cross sections. It is worth mentioning
that our approach can also be applied to $\gamma^*_T \rightarrow V_L$ and $\gamma^*_T \rightarrow V_T$
transitions. The infrared divergencies occuring in these transition amplitudes which signal
the breakdown of factorization [14] are regulated by by the quark transverse
momenta.

References

[1] A.V. Radyushkin, Phys. Lett. B385 (1996) 333;
    J.C. Collins, L. Frankfurt and M. Strikman, Phys. Rev. D56 (1997) 2982.
[2] D. Müller et al, Fortschr. Physik 42 (1994) 101; A.V. Radyushkin, Phys.
    Rev. D56 (1997) 5524; X. Ji, Phys. Rev. D55 (1997) 7114.
[3] L. Mankiewicz, G. Piller and T. Weigl, Eur. Phys. J. C 5 (1998) 119.
[4] L. Frankfurt, W. Koepf and M. Strikman, Phys. Rev. D 54 (1996) 3194.
[5] A.D. Martin and M.G. Ryskin, Phys. Rev. D57 (1998) 6692.
[6] H. Li and G. Sterman, Nucl. Phys. B381 (1992) 129.
[7] R. Jakob and P. Kroll, Phys. Lett. B315 (1993) 463; Err.-ibid. B319 (1993)
    545.
[8] M. Vanderhaeghen et al, Phys. Rev. 60 (1999) 094017.
[9] A.V. Radyushkin, Phys. Rev. D 59 (1999) 014030.
[10] ZEUS collaboration, Phys. Let. B356 (1995) 601, Eur. Phys. J. C6, 603
    (1998).
[11] J. Botts and G. Sterman, Nucl. Phys. B325 (1989) 62.
[12] M. Dahm, R. Jakob and P. Kroll, Z. Phys. C68 (1995) 595.
[13] B. Clerbaux for the H1 and ZEUS collaborations, hep-ph/9908519.
[14] L. Mankiewicz and G. Piller, Phys. Rev. D61 (2000) 074013.