DESIGN OF NEW SCHEME ADAPTIVE GENERALIZED HYBRID PROJECTIVE SYNCHRONIZATION FOR TWO DIFFERENT CHAOTIC SYSTEMS WITH UNCERTAIN PARAMETERS

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Abstract. This paper proposes a new scheme generalized hybrid projective synchronization for two different chaotic systems using adaptive control, where the master and slave systems do not necessarily have the same number of uncertain parameters. In this method the master system is synchronized by the sum of hybrid state variables for the slave system. Based on Lyapunov stability theory, an adaptive controller for the synchronization of two different chaotic systems is proposed. This method is also applicable if the master and slave systems are identical. As example the generalized hybrid projective synchronization between Vaidyanathan and Zeraoulia chaotic systems are discussed. Numerical simulation are provided to demonstrate the effectiveness of the proposed method.

1. Introduction. The first chaotic system was discovered by Lorenz [1] in 1963, then many chaotic systems have been explored such as Rössler system [2], Sprott system [3], Li system [4], Zeraoulia system [5], and many other chaotic systems [6, 7, 8]. Synchronization of chaos is an important research problem, which has been attracting considerable interest in the chaos literature, it has been widely applied to many scientific disciplines such as: physical systems [9], biological systems [10], secure communications [11, 12], etc. Chaos synchronization started with Yamada and Fujisaka [13] who used a local approach to chaotic synchronization, subsequently, Pecora and Caroll [14] have shown that two chaotic systems can be synchronized if were coupled, they defined the chaotic synchronization known as identical synchronization.

After the work of Pecora and Carroll, many researchers became interested in the question of whether it was possible to synchronize systems using control methods. Almost all control methods have been applied for the synchronization of chaotic systems.
systems: Nonlinear feedback control method [15], Sliding mode control [16], Predictive control [17], backstepping method [18], etc. The majority of these studies concern systems with known and constant parameters, whereas in reality, uncertainties about parameters and external disturbances are elements that can destroy the synchronization, making it more difficult to achieve, to solve this problem, the authors have used adaptive design method [19].

Many chaos synchronization methods have been developed extensively over the past few decades, some important methods are complete synchronization [20], phase synchronization [21], lag synchronization [22], anti-synchronization [23], projective synchronization [24]. The projective synchronization (PS) has been reported for the first time by Mainieri and Reháček [25] to be subsequently widely used with chaotic systems. It is a synchronization in which the master system and the slave system can be synchronized with a scale factor, Recently, Li [26] considers a new synchronization method, called modified projective synchronization (MPS), whereas the responses of the synchronized dynamical states synchronize up to a constant scaling matrix $\alpha$. Complete synchronization (CS) and anti-synchronization (AS) are special cases of projective synchronization where the scaling matrix $\alpha = I$ and $\alpha = -I$, respectively. In generalized hybrid projective dislocated synchronization of chaotic systems, every state variable of the master system synchronizes other incompatible state variables of the slave system, this method has been applied between two chaotic systems in [27] and [28], where the scheme used requires that the master system and the slave system have the same number of uncertain parameters and this is not available in many cases due to the diversity of chaotic systems which makes this scheme impractical.

In this paper, a new scheme of generalized hybrid projective dislocated synchronization is designed to be applied to almost all types of chaotic systems, this makes this work has substantial merits and effectiveness. Based on Lyapunov stability theory and adaptive control method some parameter update laws are gained to estimate the uncertain parameters. The rest of the paper is organized as follows: In Section 2, we show the general scheme description of generalised hybrid projective synchronization with a parameter update law. In Section 3, the generalized hybrid projective synchronization between Vaidyanathan and Zeraoulia chaotic systems are discussed to demonstrate the effectiveness of the proposed method. The Section 4 concludes the paper.

2. Generalized hybrid projective synchronization using adaptive control.

The aim of generalized hybrid projective synchronization of two different chaotic systems, is to design an effective adaptive controller which is able to synchronize the master system with the slave system.

The master system is given by
\[ \dot{x} = \phi(x(t)), \]
and the corresponding slave system is written by
\[ \dot{y} = \psi(y(t)) + u(x, y, t). \]

Where $\phi, \psi \in \mathbb{R}^n$, are two vector functions; $x = (x_1, x_2, ..., x_n)^T \in \mathbb{R}^n$ and $y = (y_1, y_2, ..., y_n)^T \in \mathbb{R}^n$ are the corresponding state vectors, $u$ is a controller to be designed. The master and the slave systems can be written as:
\[ \dot{x} = f(x) + F(x)\alpha, \]
\[
\dot{y} = g(y) + G(y)\beta + u. \tag{2}
\]

Where \( f, g : \mathbb{R}^n \rightarrow \mathbb{R}^n \) are two continuous vector functions, \( F : \mathbb{R}^n \rightarrow \mathbb{R}^{n \times m} \), \( G : \mathbb{R}^n \rightarrow \mathbb{R}^{n \times p} \) are two continuous matrix functions, \( \alpha = (\alpha_1, \alpha_2, ..., \alpha_m)^T \in \mathbb{R}^m \), \( \beta = (\beta_1, \beta_2, ..., \beta_p)^T \in \mathbb{R}^p \) are the uncertain constants parameters vectors of the master and slave systems, we can observe that the master system and the slave system do not necessarily have the same number of uncertain parameters. Also the master and the slave systems can be written as:

\[
\begin{align*}
\dot{x}_i &= f_i(x) + \sum_{j=1}^m F_{ij}(x) \alpha_j, \quad 1 \leq i \leq n; \tag{3} \\
y_i &= g_i(y) + \sum_{j=1}^p G_{ij}(y) \beta_j + u_i, \quad 1 \leq i \leq n. \tag{4}
\end{align*}
\]

In adaptive generalized hybrid projective synchronization method the master system is synchronized by the sum of hybrid state variables for the slave system, i.e. every state variable of the master system synchronizes other incompatible state variables of the slave system, therefore the error dynamics can be described as:

\[
\varepsilon_i = x_i + \sum_{j=1, j \neq i}^n c_{ij} y_j. \tag{5}
\]

The scaling matrix is given by

\[
C = \begin{pmatrix}
0 & c_{12} & \cdots & c_{1,n-1} & c_{1n} \\
c_{21} & 0 & \cdots & c_{2,n-1} & c_{2n} \\
& \ddots & \ddots & \ddots & \vdots \\
c_{n-11} & c_{n-12} & \cdots & 0 & c_{n-1n} \\
c_{n1} & c_{n2} & \cdots & c_{nn-1} & 0
\end{pmatrix}, \tag{6}
\]

where \( C \) that contains the scaling factors \( c_{ij} \).

**Definition 2.1.** If there exists a suitable adaptive controller \( u \) check the following condition

\[
\lim_{t \to \infty} \left\| x_i + \sum_{j=1, j \neq i}^n c_{ij} y_j \right\| = 0, \tag{7}
\]

then the master system (3) and the slave system (4) are adaptive generalized hybrid projective synchronization.

Let us consider the error dynamics system of general hybrid projective synchronization is described as:

\[
\begin{align*}
\dot{e}_i &= \dot{x}_i + \sum_{j=1, j \neq i}^n c_{ij} \dot{y}_j, \tag{8} \\
&= f_i(x) + \sum_{j=1}^m F_{ij}(x) \alpha_j + \sum_{j=1, j \neq i}^n c_{ij} \left[ g_j(y) + \sum_{k=1}^p G_{jk}(y) \beta_k \right] + \sum_{j=1, j \neq i}^n c_{ij} u_j.
\end{align*}
\]

According to [29], the system (8) is globally asymptotically stable, if there exist a Lyapunov function \( V \left( \varepsilon(t), \dot{\alpha}(t), \dot{\beta}(t) \right) \) positive definite, such that for all \( \left( \varepsilon(0), \dot{\alpha}(0), \dot{\beta}(0) \right) \in \mathbb{R}^{n+m+p} \), the derivative of \( V \) is negative.
According to the theory of adaptive control [29], we choose the quadratic Lyapunov function defined by:

\[ V = \sum_{i=1}^{n} e_i^2 + \sum_{i=1}^{m} (\alpha_i - \tilde{\alpha}_i(t))^2 + \sum_{i=1}^{p} (\beta_i - \tilde{\beta}_i(t))^2. \]  

(9)

If the derivative of \( V \) is negative then:

\[ e_i \xrightarrow{\forall i=1:n} 0, \ (\alpha_i - \tilde{\alpha}_i(t)) \xrightarrow{\forall i=1:m} 0 \text{ and } (\beta_i - \tilde{\beta}_i(t)) \xrightarrow{\forall i=1:p} 0. \]

Where \((\tilde{\alpha}_i(t))_{1 \leq i \leq m}, (\tilde{\beta}_i(t))_{1 \leq i \leq p}\) are the estimated parameter vectors of the parameters vectors \(\alpha_i, \beta_i\), respectively.

The time derivative of \( V \) along the trajectories of (9) is

\[ \dot{V} = \sum_{i=1}^{n} e_i \dot{e}_i + \sum_{i=1}^{m} (\alpha_i - \tilde{\alpha}_i(t))(\dot{\tilde{\alpha}}_i(t)) + \sum_{i=1}^{p} (\beta_i - \tilde{\beta}_i(t))(\dot{\tilde{\beta}}_i(t)). \]  

(10)

Our goal is to achieve adaptive generalized hybrid projective synchronization between the master system (3) and the response system (4) by constructing suitable adaptive controller law to make \( V < 0 \).

If \(|C| \neq 0\), let the controller \( u \) be

\[ u = C^{-1}A. \]  

(11)

Where \(C^{-1} = (a_{ij})_{1 \leq i \leq n}^{1 \leq j \leq n}\) is the invers of the matrix \( C \), and \( A = (A_1, A_2, ..., A_n)^T \) is given as follows:

\[ A_i = -f_i(x) - \sum_{j=1}^{m} F_{ij}(x)\tilde{\alpha}_j(t) - \sum_{j=1, j \neq i}^{n} c_{ij} \left[ (g_j(y) + \sum_{k=1}^{p} G_{jk}(y)\tilde{\beta}_k(t)) - e_i \right], \]  

therefore

\[
\left( \begin{array}{c}
  u_1 \\
  u_2 \\
  \vdots \\
  u_n 
\end{array} \right) = \left( \begin{array}{cccc}
  a_{11} & a_{12} & \cdots & a_{1n} \\
  a_{21} & a_{22} & \cdots & a_{2n} \\
  \vdots & \vdots & \ddots & \vdots \\
  a_{n1} & a_{n2} & \cdots & a_{nn} 
\end{array} \right) \left( \begin{array}{c}
  A_1 \\
  A_2 \\
  \vdots \\
  A_n 
\end{array} \right),
\]

\[
\begin{align*}
  u_i &= \sum_{j=1}^{n} a_{ij} A_j. 
\end{align*}
\]

(13)

**Theorem 2.2.** Let \( C \) be a scaling matrix such that \(|C| \neq 0\). Adaptive generalized hybrid projective synchronization between the master system (3) and the slave system (4) will be achieved by the adaptive control law (13), and the update laws (14) and (15), which is designed as follows after the derivation of the Lyapunov function \( V \) and to make the derivative \( V \) negative

\[ \tilde{\alpha}_i = \sum_{j=1}^{n} F_{ji}(x)e_j + (\alpha_i - \tilde{\alpha}_i(t)), \]  

(14)

\[ \tilde{\beta}_i = \sum_{j=1}^{n} \left( \sum_{k=1, k \neq j}^{n} c_{jk} G_{ki}(y) \right) e_j + (\beta_i - \tilde{\beta}_i(t)). \]  

(15)
Proof of Theorem 2.2. Substiting (12) and (13) into (8) the error dynamics becomes:

\[
\dot{e}_i = \sum_{j=1}^{m} F_{ij}(x) (\alpha_j - \hat{\alpha}_j(t)) + \sum_{j=1, j \neq i}^{n} c_{ij} \left( \sum_{k=1}^{p} G_{jk}(y) (\beta_k - \hat{\beta}_k(t)) \right) - e_i. \tag{16}
\]

We choose the Lyapunov function in the following form:

\[
V = \sum_{i=1}^{n} e_i^2 + \sum_{i=1}^{n} (\alpha_i - \hat{\alpha}_i(t))^2 + \sum_{i=1}^{p} (\beta_i - \hat{\beta}_i(t))^2.
\]

Calculating the derivative of V along the solutions of error system (16), we have

\[
\dot{V} = \sum_{i=1}^{n} e_i^2 + \sum_{i=1}^{n} (\alpha_i - \hat{\alpha}_i(t))(-\hat{\alpha}_i(t)) + \sum_{i=1}^{p} (\beta_i - \hat{\beta}_i(t))(-\hat{\beta}_i(t)), \tag{17}
\]

\[
= \sum_{i=1}^{n} e_i \left[ \sum_{j=1}^{m} F_{ij}(x) (\alpha_j - \hat{\alpha}_j(t)) + \sum_{j=1, j \neq i}^{n} c_{ij} \left( \sum_{k=1}^{p} G_{jk}(y) (\beta_k - \hat{\beta}_k(t)) \right) - e_i \right] \]

\[
+ \sum_{i=1}^{m} (\alpha_i - \hat{\alpha}_i(t))(-\hat{\alpha}_i(t)) + \sum_{i=1}^{p} (\beta_i - \hat{\beta}_i(t))(-\hat{\beta}_i(t)).
\]

Substitute the update laws (14) and (15) into the above equation, we obtain

\[
\dot{V} = -\sum_{i=1}^{n} e_i^2 + \sum_{i=1}^{n} \left[ \sum_{j=1}^{m} F_{ij}(x) e_j - \hat{\alpha}_i(t) \right] (\alpha_i - \hat{\alpha}_i(t)) \tag{18}
\]

\[
+ \sum_{i=1}^{p} \left[ \sum_{j=1}^{n} \left( \sum_{k=1, k \neq j}^{n} c_{ik} G_{kj}(y) \right) e_j - \hat{\beta}_i(t) \right] (\beta_i - \hat{\beta}_i(t)),
\]

\[
= -\sum_{i=1}^{n} e_i^2 - \sum_{i=1}^{m} (\alpha_i - \hat{\alpha}_i(t))^2 - \sum_{i=1}^{p} (\beta_i - \hat{\beta}_i(t))^2 < 0.
\]

So, under these conditions the error system (16) is globally asymptotically stable, that is to say that adaptive generalized hybrid projective synchronization (AGHPS) between systems (3) and (4) is achieved. The proof of Theorem 2.2 is complete.  

3. Example. In this section, simulation example is given to show the effectiveness of above method.

3.1. Synchronization between Vaidyanathan chaotic system and Zeraoulia chaotic system. In this example, an adaptive Controller is applied to AGHPS of two different chaotic systems. The Vaidyanathan system is chosen as the master system and the Zeraoulia system as a slave.

Vaidyanathan in [30] constructed a new chaotic system given by

\[
\begin{align*}
\dot{x}_1 &= \alpha_1 (x_2 - x_1) + x_2 x_3, \\
\dot{x}_2 &= \alpha_2 x_1 + \alpha_3 x_2 - x_1 x_3, \\
\dot{x}_3 &= x_1^2 - \alpha_4 x_3.
\end{align*}
\tag{19}
\]
The following equations describe the dynamics of the Zeraoulia [5] chaotic system

\[ \begin{align*}
    y_1 &= \beta_1 (y_2 - y_1) + y_2 y_3 + u_1, \\
    y_2 &= \beta_2 y_2 - y_1 y_3 + u_2, \\
    y_3 &= y_1 y_2 - \beta_3 y_3 + u_3,
\end{align*} \]

where \( x_1, x_2, x_3 \) and \( y_1, y_2, y_3 \) are the states of the two systems, \( \alpha_1, \alpha_2, \alpha_3, \alpha_4 \) and \( \beta_1, \beta_2, \beta_3 \) are uncertain constant parameters of the master and the slave system respectively, \( u_1, u_2, u_3 \) are the controllers to be found, we can observe that the master system and the slave system do not have the same number of uncertain parameters.

Compare systems (19) and (20) with Eqs. (3) and (4) we know that

\[ \begin{align*}
    f_1(x) &= x_2 x_3, & g_1(y) &= y_2 y_3, \\
    f_2(x) &= -x_1 x_3, & g_2(y) &= -y_1 y_3, \\
    f_3(x) &= x_1^2, & g_3(y) &= y_1 y_2,
\end{align*} \]

\[ \begin{align*}
    F_{11}(x) &= x_2 - x_1, \\
    F_{21}(x) &= 0, \\
    F_{31}(x) &= 0,
\end{align*} \]

\[ \begin{align*}
    F_{12}(x) &= 0, \\
    F_{22}(x) &= x_1, \\
    F_{32}(x) &= 0,
\end{align*} \]

\[ \begin{align*}
    F_{13}(x) &= 0, \\
    F_{23}(x) &= x_2, \\
    F_{33}(x) &= 0,
\end{align*} \]

\[ \begin{align*}
    G_{11}(y) &= y_2 - y_1, \\
    G_{21}(y) &= 0, \\
    G_{31}(y) &= 0,
\end{align*} \]

\[ \begin{align*}
    G_{12}(y) &= 0, \\
    G_{22}(y) &= y_2, \\
    G_{32}(y) &= 0,
\end{align*} \]

\[ \begin{align*}
    G_{13}(y) &= 0, \\
    G_{23}(y) &= 0, \\
    G_{33}(y) &= -y_3.
\end{align*} \]

The hybrid synchronization error is defined by

\[ \begin{align*}
    e_1 &= x_1 + c_{12} y_2 + c_{13} y_3, \\
    e_2 &= x_2 + c_{21} y_1 + c_{23} y_3, \\
    e_3 &= x_3 + c_{31} y_1 + c_{32} y_2,
\end{align*} \]

where \( c_{ij} \) are the scaling constants.

It is easy to see from (19) and (20) that the error dynamics can be obtained as follows:

\[ \begin{align*}
    e_1 &= \alpha_1 (x_2 - x_1) + x_2 x_3 + c_{12} (\beta_2 (t) y_2 - y_1 y_3) + c_{13} (y_1 y_2 - \beta_3 y_3 + u_1), \\
    e_2 &= \alpha_2 x_1 + \alpha_3 x_2 + c_{21} (\beta_1 (y_2 - y_1) + y_2 y_3 + u_1) + c_{23} (y_1 y_2 - \beta_3 y_3 + u_3), \\
    e_3 &= x_1^2 - \alpha_4 x_3 + c_{31} (\beta_1 (y_2 - y_1) + y_2 y_3 + u_1) + c_{32} (\beta_2 y_2 - y_1 y_3 + u_2).
\end{align*} \]

According to (6) let

\[ |C| = c_{12} c_{31} c_{23} + c_{13} c_{21} c_{32}. \]

According to (12)

\[ \begin{align*}
    A_1 &= -\tilde{\alpha}_1 (t) (x_2 - x_1) - x_2 x_3 - c_{12} (\tilde{\beta}_2 (t) y_2 - y_1 y_3) - c_{13} (y_1 y_2 - \tilde{\beta}_3 (t) y_3) - e_1, \\
    A_2 &= -\tilde{\alpha}_2 (t) x_1 - \tilde{\alpha}_3 (t) x_2 + x_1 x_3 - c_{21} (\tilde{\beta}_1 (t) (y_2 - y_1) + y_2 y_3) - c_{23} (y_1 y_2 - \tilde{\beta}_3 y_3 - e_2, \\
    A_3 &= -x_1^2 + \tilde{\alpha}_4 (t) x_3 - c_{31} (\tilde{\beta}_1 (t) (y_2 - y_1) + y_2 y_3) - c_{32} (\tilde{\beta}_2 (t) y_2 - y_1 y_3) - e_3.
\end{align*} \]

Let the control law be as follows:

\[ \begin{align*}
    u_1 &= \frac{1}{|C|} \left[ (-c_{32} c_{23}) A_1 + (c_{32} c_{13}) A_2 + (c_{12} c_{23}) A_3, \\
    u_2 &= \frac{1}{|C|} \left[ (c_{31} c_{23}) A_1 + (-c_{31} c_{13}) A_2 + (c_{13} c_{21}) A_3, \\
    u_3 &= \frac{1}{|C|} \left[ (c_{21} c_{32}) A_1 + (c_{12} c_{31}) A_2 + (-c_{21} c_{12}) A_3. \right]
\end{align*} \]
By (14) and (15) the update laws for uncertain parameters are given as following

\[
\begin{align*}
\dot{\alpha}_1 &= (x_2 - x_1)e_1 + (\alpha_1 - \tilde{\alpha}_1(t)), \\
\dot{\alpha}_2 &= x_1e_2 + (\alpha_2 - \tilde{\alpha}_2(t)), \\
\dot{\alpha}_3 &= x_2e_2 + (\alpha_3 - \tilde{\alpha}_3(t)), \\
\dot{\alpha}_4 &= -x_3e_3 + (\alpha_4 - \tilde{\alpha}_4(t)),
\end{align*}
\] (22)

and

\[
\begin{align*}
\dot{\beta}_1 &= c_{21}(y_2 - y_1)e_2 + c_{31}(y_2 - y_1)e_3 + (\beta_1 - \tilde{\beta}_1(t)), \\
\dot{\beta}_2 &= c_{12}y_2e_1 + c_{32}y_2e_3 + (\beta_2 - \tilde{\beta}_2(t)), \\
\dot{\beta}_3 &= -c_{13}y_3e_1 - c_{23}y_3e_2 + (\beta_3 - \tilde{\beta}_3(t)).
\end{align*}
\] (23)

By Theorem 2.2, we can conclude that the adaptive generalized hybrid projective synchronization between Vaidyanathan system (19) and Zeraoulia system (20) is ensured by the proposed adaptive controller \( u \) defined by (21) and the update laws (22) and (23) for all initial conditions.

\[\begin{align*}
\tilde{\alpha}_1, \tilde{\alpha}_2, \tilde{\alpha}_3, \tilde{\alpha}_4 \end{align*}\]

\textbf{Figure 1.} Estimated unknown parameters \( \tilde{\alpha}_1(t), \tilde{\alpha}_2(t), \tilde{\alpha}_3(t) \) and \( \tilde{\alpha}_4(t) \) of the master Vaidyanathan system (19), we observe that the estimation values of unknown parameters converge to their real values \( \alpha_1 = \tilde{\alpha}_1 = 25, \alpha_2 = \tilde{\alpha}_2 = 33, \alpha_3 = \tilde{\alpha}_3 = 11, \alpha_4 = 6. \)

### 3.1.1. Numerical simulations.

In the numerical simulation, the fourth order Runge-Kutta integration method is used to solve the systems of the Vaidyanathan system (19) and Zeraoulia system (20) with a time step 0.001, we choose the initial conditions of the variables of the following states:

\[x(0) = (4, 2, 3)^T, \quad y(0) = (1, 6, 3)^T.\]

The initial values for the parameter estimates of the master system are taken as

\((\tilde{\alpha}_1(0), \tilde{\alpha}_2(0), \tilde{\alpha}_3(0), \tilde{\alpha}_4(0)) = (10, 1, 27, 40).\)

The initial values for the parameter estimates of the slave system are taken as

\((\tilde{\beta}_1(0), \tilde{\beta}_2(0), \tilde{\beta}_3(0)) = (15, 4, 30),\)
we choose the scaling factors such that $|C| \neq 0$

$$c_{12} = -3, \ c_{13} = 1, \ c_{21} = 1, \ c_{23} = -2, \ c_{31} = 4, \ c_{32} = -1.$$  

The simulation results are shown in the following figures: the figure 3 illustrates the error convergence to stabilize at the zero value. The convergences of the estimate parameters $\tilde{\alpha}_1(t), \tilde{\alpha}_2(t), \tilde{\alpha}_3(t), \tilde{\alpha}_4(t)$ and $\tilde{\beta}_1(t), \tilde{\beta}_2(t), \tilde{\beta}_3(t)$ to their real values $\alpha_1 = 25, \alpha_2 = 33, \alpha_3 = 11, \alpha_4 = 6$ and $\beta_1 = 36, \beta_2 = 25, \beta_3 = 3$ respectively is shown in Fig 1 and Fig 2.

4. **Conclusions.** In this paper, based on the adaptive control theory, a novel scheme of adaptive generalized hybrid projective synchronization of two different
uncertain chaotic systems is designed using the Lyapunov stability theory. Through this method, we investigate the adaptive generalized hybrid projective synchronization between two non-identical Vaidynathan and Zeraoulia chaotic systems with uncertain parameters. Some numerical simulations are given to show the effectiveness of these methods. These method is also available and important between two identical chaotic systems.

REFERENCES

[1] E. N. Lorenz, Deterministic nonperiodic flow, *Journal of the Atmospheric Sciences*, 20 (1963), 130–141.
[2] O. E. Rössler, An equation for continuous chaos, *Physics Letters A*, 57 (1976), 397–398.
[3] J. C. Sprott, Some simple chaotic flows, *Physical Review E*, 50 (1994), 647–650.
[4] J. Lu and G. Chen, A new chaotic attractor coined, *International Journal of Bifurcation and Chaos*, 12 (2002), 659–661.
[5] Z. Elhadj, Analysis of a new three-dimensional quadratic chaotic system, *Radioengineering*, 17 (2008), 9 pp.
[6] M.-S. Abdelouahab and N.-E. Hamri, A new chaotic attractor from hybrid optical bistable system, *Nonlinear Dynamics*, 67 (2012), 457–463.
[7] S.-J. Prakash and R.-B. Krishna, A more chaotic and easily hardware implementable new 3-D chaotic system in comparison with 50 reported systems, *Nonlinear Dynamics*, 93 (2018), 1121–1148.
[8] Ü. Çavuşoğlu, S. Panahi, A. Akgül, S. Jafari and S. Kaçar, A new chaotic system with hidden attractor and its engineering applications: Analog circuit realization and image encryption, *Analog Integrated Circuits and Signal Processing*, 98 (2019), 85–99.
[9] L. Philippe, S. John and A. N. Jordan, Chaos in continuously monitored quantum systems: An optimal-path approach, *Physical Review A*, 98 (2018), 012141.
[10] P. Sadeghi, S. Panahi, B. Hatef, S. Jafari and J. C. Sprott, A new chaotic model for glucose-insulin regulatory system, *Chaos, Solitons & Fractals*, 112 (2018), 44–51.
[11] K. Uğur Erkin, S. Çiçek and Y. Uyaroğlu, Secure communication with chaos and electronic circuit design using passivity-based synchronization, *Journal of Circuits, Systems and Computers*, 27 (2018), 1850057.
[12] D. Ali, U. Yılmaz and A. T. Özçerit, A novel chaotic system for secure communication applications, *Information Technology and Control*, 44 (2015), 271–278.
[13] T. Yamada and H. Fujisaka, Stability theory of synchronized motion in coupled oscillator systems. II: The mapping approach, *Progress of Theoretical Physics*, 70 (1983), 1240–1248.
[14] L. M. Pecora and T. L. Carroll, Synchronization in chaotic systems, *Physical Review Letters*, 64 (1990), 821–824.
[15] M. S. Abd-Elouahab, N. Hamri and J. Wang, Chaos control of fractional-order financial system, *Mathematical Problems in Engineering*, 2010 (2010).
[16] D. Chen, R. Zhang, M. Xiaoyi and S. Liu, Chaotic synchronization and anti-synchronization for a novel class of multiple chaotic systems via a sliding mode control scheme, *Nonlinear Dynamics*, 69 (2012), 35–55.
[17] A. Senouci and A. Boukabou, Predictive control and synchronization of chaotic and hyperchaotic systems based on a T–S fuzzy model, *Mathematics and Computers in Simulation*, 105 (2014), 62–78.
[18] K. Ayub, B. Miridula and I. Aysha, Multi-switching compound synchronization of four different chaotic systems via active backstepping method, *International Journal of Dynamics and Control*, 6 (2018), 1126–1135.
[19] G. Li and S. Chumxian, Adaptive neural network backstepping control of fractional-order Chua–Hartley chaotic system, *Advances in Difference Equations*, 2019 (2019), 148.
[20] L. Jianquan and C. Jinde, Adaptive complete synchronization of two identical or different chaotic (hyperchaotic) systems with fully unknown parameters, *Chaos: An Interdisciplinary Journal of Nonlinear Science*, 15 (2005), 043901.
[21] G. Zheng-Ming and C. Chien-Cheng, Phase synchronization of coupled chaotic multiple time scales systems, *Chaos, Solitons & Fractals*, 20 (2004), 639–647.
[22] S. Wen, Z. Zeng, T. Huang and Q. Meng, Lag synchronization of switched neural networks via neural activation function and applications in image encryption, IEEE Trans. Neural Netw. Learn. Syst., 26 (2015), 1493–1502.
[23] Z. Xuebing and Z. Honglan, Anti-synchronization of two different hyperchaotic systems via active and adaptive control, International Journal of Nonlinear Science, 6 (2008), 216–223.
[24] L. Chengren, L. Ling, Z. Guannan, L. Gang, T. Jing, G. Jiajia and W. Zhouyang, Projective synchronization of uncertain scale-free network based on modified sliding mode control technique, Physica A: Statistical Mechanics and its Applications, 473 (2017), 511–521.
[25] R. Mainieri and J. Rehacek, Projective synchronization in three-dimensional chaotic systems, Physical Review Letters, 82 (1999), 3042.
[26] G.-H. Li, Modified projective synchronization of chaotic system, Chaos, Solitons Fractals, 32 (2007), 1786–1790.
[27] J. Sun, J. Guo, C. Yang, A. Zheng and X. Zhang, Adaptive generalized hybrid function projective dislocated synchronization of new four-dimensional uncertain chaotic systems, Applied Mathematics and Computation, 252 (2015), 304–314.
[28] J. Chen, J. Sun, M. Chi and C. Xin-Ming, A novel scheme adaptive hybrid dislocated synchronization for two identical and different memristor chaotic oscillator systems with uncertain parameters, Abstract and Applied Analysis, 2014 (2014).
[29] M. Krsti, K. Ioannis and V. Petar, Nonlinear and Adaptive Control Design, Wiley New York, (1995), 576.
[30] S. Vaidyanathan, A new eight-term 3-D polynomial chaotic system with three quadratic nonlinearities, Far East J. Math. Sci, 84 (2014), 219–226.
[31] W. Hahn, Stability of Motion, Die Grundlehren der mathematischen Wissenschaften, 138, Springer-Verlag New York, Inc., New York, 1967.

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