Picking Winners: 
A Framework For Venture Capital Investment

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We consider the problem of selecting a portfolio of items of fixed cardinality where the goal is to have at least one item achieve a high return, which we refer to as winning. This framework is very general and can be used to model a variety of problems, such as pharmaceutical companies choosing drugs to develop, studios selecting movies to produce, or our focus in this work, which is venture capital firms picking startup companies in which to invest. We first frame the construction of a portfolio as a combinatorial optimization problem with objective function given by the probability of having at least one item in the selected portfolio win. We show that a greedy solution to this problem has strong performance guarantees, even under arbitrary correlations between the items. We apply the picking winners framework to the problem of choosing a portfolio of startups to invest in. This is a relevant problem due to recent policy changes in the United States which have greatly expanded the market for startup investment. We develop a novel model for the success of a startup company based on Brownian motion first passage times. We fit this model to a large amount of data on startup company founders, investors, and performance. Our model provides some qualitative insights to the features of successful startup companies. Using our model we are able to construct out of sample portfolios which achieve exit rates as high as 60%, which is nearly double that of top venture capital firms.

Key words: combinatorial optimization, statistics, venture capital, investment, portfolio optimization

History:

1. Introduction

Imagine one must construct a portfolio of items that either have incredibly high returns or incredibly low returns. If an item achieves a high return, we say it “wins”. Because of the top-heavy payoff structure, one natural strategy is to select items to maximize the probability that at least one of them wins. We refer to the problem of choosing a portfolio of items that maximizes the probability of having at least one winner as the picking winners problem. This framework can be used to model a variety of problems which have a top-heavy payoff structure. For instance, most new drugs developed by pharmaceutical companies fail. However, with a small probability some of the drugs are successful. These winning drugs provide an immense profit for the company, as well as a large benefit to society. Having one winning drug can be enough
for a company to be successful. For another example, consider a studio that has to select a set of movies to produce. The studio will be successful if they can produce one blockbuster. Or consider the challenge faced by a venture capital firm which invests in a portfolio of startup companies. Most of these companies will fail to provide a return on their initial investment. However, if one of the companies becomes a huge success then the fund will reap a huge profit. All of these examples require one to select a portfolio of items (drugs, movies, startup companies) to maximize the probability that at least one of them has exceptional performance, or “wins”.

In this work we apply the picking winners framework to the problem of constructing a portfolio of early-stage startup companies. As discussed above, this example fits the picking winners framework very naturally because in typical venture capital portfolios, a few companies are responsible for most of the returns (Peters 2014). Startups are unique because unlike normal assets such as stocks of publicly traded companies, they can produce excessively high returns if they exit, which means they are either acquired or they have an initial public offering (IPO). Exiting is the equivalent of winning in our terminology. For instance, Peter Thiel invested $500,000 in Facebook for 10% of the company in 2004, and in 2012 he sold these shares for $1 billion, resulting in a return of 2,000 times his initial investment (Julianne Pepitone and Stacy Cowley 2012). In general, an investor in a startup company will lose money unless the company exits, which is exactly the payoff structure for the picking winners problem.

There are a few reasons why we focus on the startup investment problem. One obvious reason is the huge financial rewards. Going beyond this, there are other less financially driven reasons. As we will discuss later, to the best of our knowledge, there is little academic on quantitative methods for investing in startups. Also as we will discuss next, recent legal developments have opened up the startup investment market to everyday investors. These new investors will require quantitative investment strategies based on public data in order to not be at a disadvantage compared to larger players who have access to private information. Our solution to the picking winners problem for startup companies provides such a strategy.

1.1. Early-Stage Startup Investing

When one discusses early-stage startup investing, the first institution that they must consider is that of venture capital firms, which are groups that uses their investors’ capital to invest directly in a portfolio of startup companies. Venture capital firms invest only in private companies, meaning that once they have made the investment, the companies cannot be traded on a standard public exchange until they “go public” (Metrick and Yasuda 2011). Venture capital firms spend a large amount of time and resources screening and selecting deals. In particular, firms take a hands-on approach to investing by evaluating the attractiveness and risk behind every investment, considering factors that include relative market size, strategy, technology, customer adoption, competition, and the quality and experience of the management team. The investment screening process is oftentimes a thorough and intensive one that can take several months. Additionally,
a venture capital firm will generally become heavily involved in the day-to-day workings of the startup companies in which they invest. To learn more about the structure and operation of venture capital firms, we direct the interested reader to the surveys in Da Rin et al. (2011), Metrick and Yasuda (2011), and Sahlman (1990).

While venture capital firms still make up a large majority of the investors in early-stage startups, with the creation and adoption of the Jumpstart Our Business Startups (JOBS) Act (Congress 2012), early-stage investment opportunities are now available to the everyday investor in the United States. Before this policy change, only certain accredited investors were allowed to take equity stakes in most private companies. With the adoption of this policy, there are now many companies such as FlashFunders and NextSeed (FlashFunders 2017, NextSeed 2017) that assist the everyday investor in finding and participating in startup investment opportunities. While these sites provide some relevant information about the startup companies of interest, users of this sites do not have direct access to the company in the same way that a venture capital firm does. Thus, oftentimes they do not have the information needed to execute a thorough and intensive selection process. Additionally, these investors will not be involved in the future process of the companies. Needless to say, intuition suggests that these investment opportunities will have a lower success-rate than that of venture capital backed startups. This means that there is a need for quantitative investment strategies which can be developed from publicly available data. Such strategies can serve to level the playing field and empower the everyday investor.

1.2. Relevant Extant Work
The work in Bhakdi (2013) motivates the appeal behind a non-biased quantitative approach to early-stage startup investing. Additionally, this work also discusses the connection between early-stage startup investing and a variant of the picking winners problem. This work focuses more on a simulation-based approach to properly structure a portfolio of early-stage startups to adjust for market volatility. While this study takes a completely different approach from our work, it illustrates some of the benefits of a quantitative approach to startup investing.

While there is little work on quantitative startup investment strategies, there have been several studies that take a quantitative approach to determining what factors are correlated with startup success. One study (Eesley et al. 2014) has found that founding team composition has an effect on the startup’s ultimate performance. In particular, this study found that more diverse teams tend to exhibit higher performance, where higher performance is measured by exit rate. Additionally, researchers have considered how startup success correlates with how connected the founders are within a social network (Nann et al. 2010). Another study (Gloor et al. 2011) considers how startup entrepreneurs’ e-mail traffic and social network connectedness correlate with startup success.

In contrast to the academic space, several venture capital firms have been adopting analytical tools to assist in the investing process. Google Ventures (Google 2017), the venture capital arm of Alphabet, Inc., is
known to use quantitative algorithms to help with investment decisions (Kardashian 2015). Correlation Ventures (Correlation Ventures 2017), a venture capital firm with over $350 million under management, uses a predictive model to make startup company investment decisions. Correlation Ventures has truly adopted a quantitative approach to their venture capital decisions — they ask startups to submit five planning, financial, and legal documents, they take only two weeks to make an investment decision, and they never take administrative board seats. Interestingly enough, they only take co-investment opportunities, and they rely on some vetting by a primary investor (Leber 2012).

There is a large body of research related to the more general picking winners problem. There is a natural relationship between this problem and a variety of covering problems. One can view the picking winners objective as choosing $k$ items to cover the most likely winners. In fact, in this work we show that the problem of picking winners reduces to the well known maximum coverage problem where the goal is to pick $k$ subsets of an underlying set such that the union of the subsets has maximum coverage of the underlying set.

Because of this fact, a variety of other covering problems are very similar to the picking winners problem. There is the problem of placing a fixed number of sensors in a region to detect some signal as fast as possible. This has been studied for water networks (Leskovec et al. 2007) and general spatial regions (Guestrin et al. 2005). There is also a problem in information retrieval of selecting a set of documents from a large corpus to cover a topic of interest (Agrawal et al. 2009, Chen and Karger 2006, El-Arini et al. 2009, Swaminathan et al. 2009). Another related problem is selecting a set of users in a social network to seed with a piece of information in order to maximize how many people the message reaches under a social diffusion process (Kempe et al. 2003). Another way to view this problem is that one wants the message to reach a certain target user in the network, but one does not know precisely who this user is, so one tries to cover as much of the network as possible. One can view the sensors, documents, or seed users as items in a portfolio and winning means one of the sensors detects the signal, one of the documents covers the relevant topic, or one of the seed users is able to spread the message to the unknown target user.

The similarity between picking winners and these covering problems has important implications. The objective functions in these covering problems are submodular and maximizing them is known to be NP-hard (Karp 1975). The same holds for the picking winners problem. The submodular property is important because it means that one has a lower bound on the performance of a greedy solution (Nemhauser et al. 1978). This fact, plus the simplicity with which one can calculate greedy solutions, motivates us to use a greedy approach to pick winners.

There are also interesting connections between the picking winners problem and traditional portfolio construction methods. For instance, if the items have binary returns, meaning the return is either very high or very low, then there are striking similarities between the optimal portfolio in the picking winners problem and what is known as the log-optimal portfolio (Cover 1991). In fact, we show in Section 2 that when there is independence the portfolios are equivalent, and under certain conditions, the picking winners portfolio produces a strong solution to the log-optimal portfolio selection problem.
1.3. Our Contributions

The first contribution of this work is the picking winners problem. We present a mathematical formulation of this problem and prove some key theoretical results. We show that the problem has a submodular objective function and we reduce the problem to the set covering problem. We draw upon results from the theory of submodular optimization to motivate using a greedy solution to solve the picking winners problem. We also show connections between picking winners portfolios and log-optimal portfolios.

The second contribution of this work is a new approach to build portfolios for early-stage startup investing using the picking winners framework. This contribution has three components: predictive modeling, data collection and model estimation, and portfolio construction. We now discuss each component in more detail.

We first create a predictive model for a startup company’s funding. This models describes a startup company using a Brownian motion with a company dependent drift and diffusion. The first passage times of the Brownian motion for different levels correspond to the startup receiving a new round of funding, and the highest level corresponds to the company exiting (winning). Our model is able to naturally incorporate the censoring that occurs due to limited observation times.

We then estimate this model using data for several thousand companies. This data includes companies’ funding milestones, investor networks, and profiles of the founders. Our model estimation results indicate that successful startups are characterized by a very large diffusion coefficient for their Brownian motion. The results also provide some insights about which company features are predictive of exiting.

Finally, we estimate our model on historic data and using the resulting model parameters to build portfolios for future years using the picking winners framework. We find that our portfolios achieve exit rates as high as 60%. We outperform top venture capital firms across multiple years. This shows the strength of our approach and its applicability for startup investing.

The remainder of the paper is structured as follows. In Section 2 we formulate the picking winners problem and discuss some of its theoretical properties and relationship with submodular optimization and log-optimal portfolios. We present an exploratory analysis of the data that we used for our model in Section 3. In Section 4 we introduce our Brownian motion model for startup funding. We study the insights gained from the estimated Brownian motion model and evaluate the performance of the resulting portfolios in Section 5. We conclude in Section 6. All proofs as well as more detailed information about the data are included in the appendix.

2. Problem Formulation

We consider a set of events $\mathcal{E} = \{E_i\}_{i=1}^m$ defined on a probability space $(\Omega, \mathcal{F}, \mathbb{P})$, with sample space $\Omega$, $\sigma$-algebra $\mathcal{F}$, and probability measure $\mathbb{P}$. Each event $E_i$ corresponds to item $i$ winning. The aim is to select a subset $\mathcal{S} \subseteq [m] = \{1, \ldots, m\}$ of size $k$ such that we maximize the probability that at least one item in
$S$ wins, which is given by $P \left( \bigcup_{i \in S} E_i \right)$. For notational convenience, we denote $U(S) = P \left( \bigcup_{i \in S} E_i \right)$. The optimization problem we want to solve is then

$$\max_{S \subseteq [m], |S| = k} U(S). \quad (2.1)$$

In general, solving this optimization problem can be difficult. In fact, we have the following result.

**Theorem 2.1** Maximizing $U(S)$ is NP-hard.

Despite the computational complexity in maximizing $U(S)$, the objective function possesses some nice properties which we can use to obtain good solutions efficiently, which are given by the following well-known result.

**Lemma 1** The function $U(S)$ is non-negative, non-decreasing and submodular.

Monotonically non-decreasing submodular functions have an important property which is that one can bound the sub-optimality of a greedy solution. Such a solution is obtained by sequentially adding elements to the set $S$ such that each new addition results in the largest marginal increase in the objective function. Formally, let $S_G = (f_1, f_2, ..., f_k)$ be the greedy solution of the optimization problem. The greedy solution is an ordered set, with item $f_i$ being added on the $i$th step of the greedy optimization. Let $S_G^i = (f_1, f_2, ..., f_i)$ be the solution obtained after the $i$th item is added to $S_G$, with $S_G^0 = \emptyset$. Then the $i$th item added to $S_G$ is given by

$$f_i = \arg \max_{f \in [m] \setminus S_G^{i-1}} U \left( S_G^{i-1} \bigcup f \right) - U \left( S_G^{i-1} \right), \quad 1 \leq i \leq k. \quad (2.2)$$

For submodular functions, we have the following well known result.

**Theorem 2.2 (Nemhauser et al., 1978)** Let $U(S)$ be a non-decreasing submodular function. Let $S_G$ be the greedy maximizer of $U(S)$ and let $S_W$ be the maximizer of $U(S)$. Then

$$\frac{U(S_G)}{U(S_W)} \geq 1 - e^{-1}. \quad (2.3)$$

Therefore, greedy solutions have performance guarantees for submodular maximization problems. In addition, many times they can be computed very easily. For these reasons, we will use a greedy approach for our problem.
2.1. Independent Events
We first consider the simplest case where all \( E_i \) are independent. Let \( p_i = P(E_i) \) and let \( E_i^c \) denote the complement of \( E_i \). Because of the independence of the events, we can rewrite the objective function as

\[
U(S) = 1 - P\left(\bigcap_{i \in S} E_i^c\right) = 1 - \prod_{i \in S} (1 - p_i).
\] (2.4)

The optimization problem can then be written as

\[
\max_{S \subseteq [m], |S| = k} U(S) = \min_{S \subseteq [m], |S| = k} \prod_{i \in S} (1 - p_i).
\] (2.5)

The form of the objective function for independent events leads to a simple solution, given by the following theorem.

**Theorem 2.3** For a set of independent events \( \mathcal{E} = \{E_1, E_2, ..., E_m\} \) let \( p_i = P(E_i) \) and without loss of generality assume that \( p_1 \geq p_2 \geq ... \geq p_m \). Let the \( k \) element subset \( S_W \subseteq [m] \) maximize \( U(S) \). Then \( S_W \) consists of the indices with the largest \( p_i \), i.e. \( S_W = [k] \).

The proof of this result is obvious from the form of \( U(S) \) in equation (2.4). From Theorem 2.3 we see that if the events are independent, then one simply chooses the most likely events in order to maximize the probability that at least one event occurs. This simple solution relies crucially upon the independence of the events. Another useful property when the events are independent concerns the greedy solution.

**Lemma 2** For a set of independent events \( \mathcal{E} = \{E_1, E_2, ..., E_m\} \), let the \( k \) element greedy maximizer of \( U(S) \) be \( S_G \). Then \( S_W = S_G \).

In this case we have that the greedy solution is also an optimal solution. Indeed, the independent events case is almost trivial to solve. This also suggests when the events have low dependence, greedy solutions should perform well.

2.2. Picking Winners and Log-Optimal Portfolios
We now study the relationship between the picking winners portfolio and another well known portfolio known as a log-optimal portfolio. To do this, we consider a model where the items have a top-heavy payoff structure, similar to the picking winners problem. We will let the random variable \( X_i \) denote the return of the item associated with index \( i \), and we assume there are \( m \) possible items. In particular, we will consider the case where there are two possible values for the returns \( a \) and \( b \) such that \( 0 < a < b \), and we will let \( X_i(\omega) = b \) for \( \omega \in E_i \), and \( X_i(\omega) = a \) otherwise. This can approximately model the returns of a startup
company which are huge if it exits, otherwise are negligible. Because it can be hard to predict the exact returns when a company exits, one can assume that they all have the same average value $b$ given no other information. In this way, the only factor differentiating companies is their probability of exiting.

There are many different ways to construct a portfolio. The traditional Markowitz approach maximizes the mean portfolio return subject to an upper bound on its variance \cite{Markowitz1952}. These portfolios are parametric because one must select the upper bound on the variance. Another approach is the log-optimal portfolio, where one constructs a portfolio which maximizes the expected natural logarithm of the portfolio return \cite{Cover1991}. In our analysis, we assume that investment decisions are binary for each item, i.e. an item is either in the portfolio or it is not. We do not allow for continuous investment amounts for the items. Under this setting, a portfolio is a subset of $[m]$, as in the picking winners problem. We define the return of a portfolio $S$ as $\sum_{i \in S} X_i / |S|$. The corresponding expected log return is

$$V(S) = \mathbb{E} \left[ \ln \left( \sum_{i \in S} X_i / |S| \right) \right]. \quad (2.6)$$

We constrain the portfolio to contain $k$ elements, as in the picking winners problem. Then the log-optimal portfolio is defined by the following optimization problem:

$$\max_{S \subseteq [m], |S| = k} V(S). \quad (2.7)$$

There are no arbitrary parameters in the log-optimal portfolio formulation, unlike the Markowitz portfolio. This allows us to directly compare the log-optimal and picking winners portfolios. Once again, we begin by considering the simple case where the events $\{E_i\}_{i=1}^m$ are independent. We have the following result:

**Theorem 2.4** For a set of independent events $\mathcal{E} = \{E_1, E_2, ..., E_m\}$ let $p_i = P(E_i)$ and without loss of generality assume that $p_1 \geq p_2 \geq ... \geq p_m$. Let the $k$ element subset $S_L \subset [m]$ maximize $V(S)$. Then $S_L$ consists of the indices with the largest $p_i$, i.e. $S_L = [k]$.

Comparing the above result and Theorem 2.3, we see that when the events are independent the problem of determining a log-optimal portfolio reduces to the problem of finding a greedy solution to the picking winners problem. The equivalence only relies on the assumptions that $0 < a < b$ and independence. In particular, we make no assumptions about the value of the marginal probabilities $P(E_i)$.

While the independence assumption results in a clean equivalence between the log-optimal and picking winners portfolios, in reality this assumption will be violated. However, we are interested in examples where the probability of a particular item winning is small. In this setting, we can quantify how much the two portfolio types deviate from each other.
Theorem 2.5 Let $S_L$ denote a log-optimal portfolio, i.e. it is optimal for (2.7), and let $S_W$ denote a portfolio that is optimal for the picking winners problem. Let $G_l$ denote the collection of all subsets of $[m]$ with cardinality $l$. Suppose there exists $\lambda \in [0,1]$ and $p \in (0, \frac{1}{k}]$ such that for all $l \in [k]$, and for all $T \in G_l$ the following holds

$$(1 - \lambda)p^l(1 - p)^{k-l} \leq P\left(\bigcap_{E \in T} E \bigcap \bigcap_{F \in S \setminus T} F^c\right) \leq (1 + \lambda)p^l(1 - p)^{k-l}.$$  \hspace{1cm} (2.8)

Then it follows that

$$\frac{U(S_W) - U(S_L)}{U(S_W)} \leq \frac{2\zeta(3)\lambda kp(1-p)}{\ln \left(1 + \frac{b-a}{ka}\right)(1 - \lambda)(1 - (1 - p)^k)}.$$ 

where $\zeta(s)$ is the Riemann zeta function evaluated at $s$, i.e. $\zeta(s) = \sum_{n=1}^{\infty} \frac{1}{n^s}$.

We will begin by interpreting the assumptions for Theorem 2.5. We begin by considering (2.8). This assumption effectively places restrictions on the dependency between events. To gain some intuition, consider the case where all of the events have the same probability, and let $p_i = p$ for all $i \in [m]$. Then it is clear that $\lambda$ is a measure of the dependency between events. In particular, when the events are independent we can choose $\lambda = 0$, and as the events become more correlated we expect that we will have to choose a larger $\lambda$ for equation (2.8) to hold. Additionally, Theorem 2.5 does not require the probability of all the events to be the same, it just requires there to be some $p \in (0, \frac{1}{k}]$ and some $\lambda \in [0,1]$ where (2.8) holds. If the items have very different values for $p_i$, then Theorem 2.5 might produce a very weak bound. If all the events of interest have an equal probability of winning, and once again we we let $p = p_i$ for all $i \in [m]$, then the assumption $p \in (0, \frac{1}{k}]$ implies that the expected number of winning items for any portfolio of size $k$ is one or less. For our applications of interest, the winning probabilities are somewhat small and thus this seems like a reasonable assumption.

The main insight of Theorem 2.5 is that for our particular applications of interest, a log-optimal portfolio must also be a good solution to the picking winners problem. Throughout this discussion, suppose that the conditions in the theorem hold. As an example, assume that $\lambda = 0.5$, $k = 10$, and $p = 0.01$. This assumes that the probability of an individual item winning is low and allows from some correlation. For the possible values of the returns, let us assume $b = $10^9 and $a = $1 (these are the approximate values for a very successful exit, such as for Facebook). Then applying Theorem 2.5 we have that the percent difference in the objective function (probability of at least one winner) for a winning and log-optimal portfolio is less than 27%. This example illustrates that under these conditions, which are relevant for investing in startup companies, the optimal portfolio for the picking winners problem will be similar in performance to a log-optimal portfolio.
3. Data Analysis

We collected a large amount of relevant data regarding startup company performance and features. For each company, there were two categories of data we acquired: the time it reaches different funding rounds and features relating to its sector, investors, and leadership. We obtained our data from three different sources. The first is Crunchbase (CrunchBase 2017), a public database of startup companies. The information in Crunchbase is provided by public users, subject to approval by a moderator (Kaufman 2013). The second is a privately maintained database known as Pitchbook that also collects information on startup companies (Pitchbook 2017). The two databases combined provide us with data for over 83,000 companies founded between 1981 and 2016. These databases give us information about when a company receives a funding round or achieves an exit, and which investment groups are involved in a particular funding round. Additionally, these databases provide us with information on over 558,000 company employees. In particular, we have some information relating to which companies a person has worked for, their roles within these companies, and their employment timeline. To build on this person-level information, we also use LinkedIn, a business and employment-focused social network (LinkedIn 2017). Including LinkedIn data gave us complete educational and occupational histories for the employees.

3.1. Funding Rounds Data

A startup company typically receives funding in a sequence of rounds. The initial funding is known as the seed round. After this, the company can receive funding in a series of ‘alphabet rounds’ which are called series A, series B, etc. The alphabet rounds typically do not go beyond series F. Additionally, a company can reach an exit, which is when a startup company is either acquired or has an IPO. Crunchbase provided the funding round and exit dates for each startup company, while the Pitchbook data that we used only provided the founding date. Therefore, our base set of companies was dictated by Crunchbase. We used Pitchbook to resolve founding date errors in Crunchbase. For instance, there were some companies in Crunchbase where the listed founding date occurred after later funding rounds. In these cases, we ignored the Crunchbase founding date and used the Pitchbook date.

For our analysis we only use companies founded in 2000 or later. We use this cutoff because before this year the databases did not contain very many unsuccessful companies, creating a biased dataset. Furthermore, for our analysis we are particularly interested in measuring the likelihood of a company in an early round eventually reaching an exit. Therefore, we only consider startup companies where we have reliable information on their seed or series A funding rounds. In particular, if we do not have information on when a startup company reached one of these early rounds, then we omit them from our analysis. Additionally, we chose to focus on startup companies that are founded in the United States. Using 2000 as a cutoff year and only considering American companies with reliable information for their early rounds left us with approximately 24,000 companies founded between 2000 and 2016. We plot the number of companies founded in
As can be seen, there are a few companies in the early years, but by 2011 the number of companies increases by a large amount. This increase is most likely due to the fact that Crunchbase was founded in 2011 and after this date many companies began entering their information into the database. The drop in the number of companies in 2016 is due to the fact that we collected our data in the middle of 2016.

We first wanted to understand the distribution of the maximum funding round achieved (as of 2016) for these companies. We plot in Figure 1 this distribution broken down by year. We observe a few interesting properties from this figure. First, the average fraction of companies founded in a given year that have an IPO is typically a few percent. For acquisitions, this value goes up to 21%, but as can be seen, the fraction of acquisitions changes a great deal over the years. After 2011, the fraction of acquisitions falls below 14% and decreases each subsequent year. The decrease is most likely due to data censoring, as companies founded in these later years may not have had enough time to exit. From 2000 to 2002 the fraction of acquisitions actually increases. This may be due to the small number of companies in our dataset and sampling bias leading to an under-reporting of less successful companies.

We next wish to understand the temporal dynamics of the funding rounds. Figure 2 shows the evolution of funding rounds for different well known companies that achieved IPOs. As can be seen, the rate of achieving an IPO fluctuates between companies. The boxplot in Figure 2 shows the distribution of the time to hit the different rounds. From here it can be seen that IPOs happen after about six years, but acquisitions typically take three years. The time between the other funding rounds (series B, series C, etc.) is about a year. The typical variation in these times across companies is about one to two years.

3.2. Sector, Investor, and Leadership Data and Features

We now describe the data and features that we used in our model. We were able to build a wide-variety of features for our model, ranging from simple features such as a company’s sector, to more complex features
relating to the similarity of the academic and occupational backgrounds of the startup founders. We now describe these features in more detail.

**Sector Data and Features.** The majority of the features that were used in our model relate to the sector that a startup company was involved in. In particular, from Crunchbase we were able to obtain sector labels for each company, with some of the companies belonging to multiple sectors. While Crunchbase has informative labels for many different features, we chose to include binary indicators for a subset of them. We made a point to include a wide-variety of sectors in our model, ranging from *Fashion* to *Artificial Intelligence*. We provide a complete list of the sectors that we used in Appendix B.

**Investor Data and Features.** From the Crunchbase data, we are able to construct a dynamic network of investors and companies, such that there is a node for every investor and company that exists at the particular time of interest, and there is an edge connecting an investor to a company if the investor has participated in a funding round for that company before the particular time of interest. We construct this dynamic network using all of our available data – meaning that we consider roughly 83,000 companies and 48,000 investors. We derive features for an individual company based on this dynamic network at the time when the company had its first funding round. Recall that we omit companies without reliable time information for their seed or series A funding rounds. Therefore, for a particular company we consider the dynamic network of investors at the time of the earliest of these rounds.

Using this dynamic network, we construct a feature that we call the *investor neighborhood*. For a company $i$ having an earliest funding date of $t_i$, the value of this feature is the number of startup companies in existence before year $t_i$ that share at least one investor in common with company $i$. We then normalize this value by the total number of companies founded before $t_i$. This feature measures the relative reach of the company’s investors.
Another feature that is derived from this dynamic network is the maximum IPO fraction. For each initial investor \( j \) connected to company \( i \), we define \( f_j \) as the fraction of companies connected to \( j \) at \( t_i \) that also had an IPO before \( t_i \). The feature value is then the maximum value of \( f_j \) amongst all initial investors in \( i \).

A related feature we define is called maximum acquisition fraction. This feature is identical to maximum IPO fraction, except we use the fraction of companies that were acquired rather than the fraction that had an IPO. Both maximum IPO fraction and maximum acquisition fraction are measures of the success rate of a company’s initial investors.

**Leadership Data and Features.** The leadership features of a company are derived from the Crunchbase and LinkedIn data for its founders and executives. We are particularly interested in measuring the experience, education, and ability of the company’s leadership.

We first use the Crunchbase data to consider the employees, executives, and advisors of the company of interest. We construct indicator features *Job IPO*, *Job Acquired*, *Executive IPO*, *Executive Acquired*, *Advisory IPO*, and *Advisory Acquired* which indicate if someone affiliated with the company has been part of a previous company that was either acquired or reached an IPO before our date of interest. In particular, for the *Job* variables we consider people who work for the company but are not executives or advisors, for the *Executive* variables we consider people who are labeled as an executive in the company, and for the *Advisor* variables we consider people who are labeled as advisors but not executives.

Next we consider the features based on the LinkedIn data. Note that we took particular care to ensure that we only use LinkedIn information that could be known before our time of interest. One LinkedIn feature that we use is *previous founder* which is the fraction of the leadership that had previously founded a company before the given company was founded. We also use *number of companies affiliated*, which is the average number of companies each leadership member was affiliated with before joining the given company. A final pair of experience features is *work overlap mean* and *work overlap standard deviation*. To construct these features, we calculate the Jaccard index of previous companies for each pair of leadership members. The Jaccard index is defined as the intersection of previous companies divided by the union of previous companies for the pair of members. We then take the mean and standard deviation of these values across all pairs of leadership members to obtain the two features. We chose to construct these feature using the LinkedIn data instead of the Crunchbase data, because we often found that the LinkedIn information suggested that a person had previous startup experience, when the Crunchbase data did not contain this information. We believe that this is due to the fact that Crunchbase is slightly biased to include companies that were somewhat successful.

An education feature we use is *from top school* which is the fraction of leadership members that went to a top school. This list of top schools was created from a combination of known top rankings ([U. S. News and World Report National Universities Rankings 2017](https://www.worldsreport.com/)) and our own knowledge and is provided in Appendix C. We also have features for the highest education level of the leadership, measured by degree received.
These features are high school, bachelors, master’s, and Ph.D. For each degree, we measure the fraction of leadership members whose maximum education level equals that degree to obtain the feature value.

We also have features based on the education and academic major overlap. These features are education overlap mean, education standard deviation, major overlap mean, and major standard deviation. For each one we calculate the relevant Jaccard index over all pairs of leadership members, and then take the mean and standard deviation. For education, the Jaccard index is taken with respect to the schools attended by each member and for major, the Jaccard index is taken with respect to the academic majors of each member.

A more complex feature we use is major company similarity which captures the similarity of the academic major of the leadership and the company sector. We use the WordNet lexical database to create a semantic similarity score between each member’s major and the sector of their company (NLTK 3.0 documentation 2017). We use the Palmer-Wu similarity score, which measures the similarity of words in a semantic network based on the distances to their most common ancestor and to the root (Wu and Palmer 1994). This score is zero for totally different words and one for equivalent words. We average the Palmer-Wu major-sector similarity score for each member to obtain the feature value.

A final feature we use is leadership age. This is simply the average age of all leadership members when the company was founded. To estimate the age we assume that each member is 18 when finishing high-school and 22 when finishing undergraduate education. With this assumption, we set the age of a member to be equal to 22+ company founding year – year member received/would have received his undergraduate degree.

For emphasis, we mention again that great care was taken to ensure that the information that we used from LinkedIn did not violate causality. In particular, we made sure that an investor could have known the information that we acquired from LinkedIn when they were considering investing in a startup.

3.2.1. Imputation of Missing Data

For many of the LinkedIn features we were unable to obtain data. We still needed some value for these features to use the resulting feature vector in our models. To do this we imputed the missing values. We concatenated all the M dimensional feature vectors for N companies, creating a $M \times N$ matrix with missing values. To impute these missing values, we use a matrix completion algorithm known as Soft-Impute which achieves the imputation by performing a low-rank approximation to the feature matrix using nuclear norm regularization (Mazumder et al. 2010). Soft-Impute requires a regularization parameter and a convergence threshold. To obtain the regularization parameter, we replace all missing values with zero and then calculate the singular values of this filled in matrix divided by 100 and we set the convergence threshold to 0.001. We then apply Soft-Impute with these parameters to the incomplete feature matrix to fill in the missing values. This imputed feature matrix is used for all subsequent model fitting and prediction tasks. Note that we repeat this process for each training set and testing set that we consider, to ensure that we are not violating causality.
4. Model for Startup Company Funding

We now present a stochastic model for how a company reaches different funding rounds and the relevant probability calculations needed to construct portfolios. Our model captures the temporal evolution of the funding rounds through the use of a Brownian motion process.

4.1. Brownian Motion Model for Company Value

We assume a company has a latent value process $X(t)$ which is a Brownian motion with time-dependent drift $\mu(t)$ and diffusion coefficient $\sigma^2(t)$. The latent value process $X(t)$ is initiated with value 0 when the company receive its first funding round. The set of possible funding rounds that we use is $\mathcal{R} = \{\text{Seed}, A, B, C, D, E, F, \text{Exit}\}$. We denote each funding round by an index $0 \leq l \leq 7$, with $l = 0$ corresponding to seed funding, $l = 1$ corresponding to series A funding, etc. The final level is $l = 7$, which corresponds to exiting. For each funding round $l$ we have a level $h_l \geq 0$, and the levels are ordered such that $h_{l-1} \leq h_l$. We choose the spacing of these levels to be linear, but one is free to choose them arbitrarily. In our model, we let $h_l = \Delta l$ for some $\Delta > 0$. A company receives round $l$ of funding when $X(t)$ hits $h_l$ for the first time, and we denote this time as $t_l$. Therefore, the time to receive a new round of funding is the first passage time of a Brownian motion.

The first passage time distribution for a Brownian motion with arbitrary time-varying drift and diffusion terms is difficult to solve. In particular, one must solve the Fokker-Planck equation with an appropriate boundary condition [Molini et al. (2011)]. However, we can solve the Fokker-Planck equation exactly with the appropriate boundary condition when the ratio of the drift to the diffusion is constant in time. Therefore, we assume that the drift and diffusion terms are of the form $\mu(t) = \mu_0 f(t)$ and $\sigma^2(t) = \sigma_0^2 f(t)$, where $\mu_0$, $\sigma_0^2$, and $f(t)$ are appropriately chosen. Under these assumptions, we have the following standard result for the first passage time distribution.

**Theorem 4.1** [Molini et al. (2011)] For a Brownian motion $X(t)$ with drift $\mu(t) = \mu_0 f(t)$, diffusion $\sigma^2(t) = \sigma_0^2 f(t)$, and initial value $X(v_0) = 0$, let $V_\alpha = \inf_{t>v_0} \{X(t) \geq \alpha\}$ denote the first passage time to level $\alpha > 0$ after time $v_0$. Then, the probability density function (PDF) of $V_\alpha$ is

$$f_0(v; v_0, \mu(t), \sigma(t), \alpha) = \frac{\sigma_0^2 v \alpha}{\sqrt{16\pi S^3}} e^{-\frac{(\alpha-M)^2}{4S}}$$

and the cumulative distribution function (CDF) is

$$F_0(v; v_0, \mu(t), \sigma(t), \alpha) = \Phi \left( \frac{M - \alpha}{\sqrt{2S}} \right) + \exp \left\{ \frac{\alpha M}{S} \right\} \Phi \left( \frac{-(M + \alpha)}{\sqrt{2S}} \right).$$

where $M = \int_{v_0}^{v} \mu(s) ds$, $S = \frac{1}{2} \int_{v_0}^{v} \sigma^2(s) ds$, and $\Phi(\cdot)$ is the standard normal CDF.
For notation, suppose that we observe a company at time \( t_{\text{obs}} \). Before this time it reaches a set of funding rounds with indices \( i = \{ i_1, i_2, \ldots, i_L \} \). The random variables corresponding to the times of these rounds are \( t = \{ t_{i_1}, t_{i_2}, \ldots, t_{i_L} \} \) (we assume \( i_0 = 0 \) and normalize the times so \( t_{i_0} = 0 \)). There may be missing data, so the funding rounds are not necessarily consecutive. If an exit is observed, then by the independent increments property and translation invariance of Brownian motion, the likelihood is given by

\[
 f(t, t_{\text{obs}}; \mu(t), \sigma(t)) = \prod_{l=1}^{L} f_0(t_{i_l}; t_{i_{l-1}}, \mu(t), \sigma(t), h_{i_l} - h_{i_{l-1}}). \tag{4.3}
\]

If the company does not exit before \( t_{\text{obs}} \), then \( i_L < 7 \) and there is a censored inter-round time which is greater than or equal to \( t_{\text{obs}} - t_{i_L} \). In this case the likelihood is given by

\[
 f(t, t_{\text{obs}}; \mu(t), \sigma(t)) = (1 - F_0(t_{\text{obs}} - t_{i_L}; t_{i_L}, \mu(t), \sigma(t), \Delta)) \prod_{l=1}^{L} f_0(t_{i_l}; t_{i_{l-1}}, \mu(t), \sigma(t), h_{i_l} - h_{i_{l-1}}). \tag{4.4}
\]

### 4.2. Modeling drift and diffusion

From Section 3 recall that companies which succeed typically take a constant amount of time to reach each successive funding round, which motivates these companies having a positive and constant drift coefficient for some time period. Additionally, many companies succeed in reaching early funding rounds somewhat quickly, but then they fail to ever reach an exit. This motivates these companies having a drift that decreases in time. Lastly, as time gets very large, a company that has not achieved an exit will most likely not achieve a new funding round. This translates to our latent value process moving very little as time gets large, which can be modeled by having the drift and diffusion terms moving toward zero. To incorporate these properties, we use the following model for the drift and diffusion terms:

\[
 \mu(t) = \mu_0 \left( 1 \{ t \leq \nu \} + e^{-\frac{t-\nu}{\tau}} 1 \{ t > \nu \} \right) \\
 \sigma^2(t) = \sigma_0^2 \left( 1 \{ t \leq \nu \} + e^{-\frac{t-\nu}{\tau}} 1 \{ t > \nu \} \right)
\]

where \( \mu_0, \sigma_0^2, \nu, \) and \( \tau \) are appropriately chosen based on the data. Under this model, the drift and diffusion are constant for a time \( \nu \), after which they decay exponentially fast with time constant \( \tau \).

In our model, every company will have the same \( \nu \) and \( \tau \). However, each company will have a different drift term \( \mu_0 \) and diffusion term \( \sigma_0 \) that will be determined by its features. For a company \( i \) we define a feature vector \( x_i \in \mathbb{R}^M \), a drift \( \mu_{i0} \), and a diffusion \( \sigma_{i0}^2 \). We define a parameter vector \( \beta_y \in \mathbb{R}^M \) for year \( y \). A company \( i \) that is founded in year \( y \) has \( \mu_{i0} = \beta_y^T x_i \). Furthermore, we set \( \beta_{y+1} = \beta_y + \epsilon \) where \( \epsilon = [\epsilon_1, \ldots, \epsilon_M] \) and \( \epsilon_i \) is normally distributed with zero mean and variance \( \sigma_i^2 \) for \( 1 \leq i \leq M \). This time-varying coefficient model allows us to capture any dynamics in the environment which could increase or decrease the importance of a feature. For instance, consider a feature which is whether or not the company is in a certain sector. The coefficient for this feature would change over time if the market size for this sector
changes. Additionally, this time-varying model allows us to capture uncertainty in future values of the drift weights, which we will utilize in Section 4.3 to construct the joint distribution of first passage times for the companies.

Additionally, we found that the volatility of the funding rounds data varies substantially between sectors. For instance, companies in some sectors will frequently achieve early funding rounds very quickly, but then fail to ever reach an exit. To model this phenomenon, we introduce heteroskedasticity into the model and allow each company to have its own diffusion coefficient. We define another parameter vector \( \gamma \in \mathbb{R}^M \), and we let \( \sigma^2_{t_0} = (\gamma^T x_i)^2 \).

In practice, we considered many other models for the drift and diffusion terms. We found that the model presented here performed well with respect to fitting the data, and additionally we found this model to be intuitive and interpretable. Therefore, we will restrict our future results to those that were obtained using this particular model.

### 4.3. Exit probabilities

We define \( E_i \) as the event that company \( i \) reaches an exit sometime in the future. We wish to build a portfolio of \( k \) companies to maximize the probability that at least one company exits. To do this, we must be able to evaluate the exit probability. In our models this can easily be done. Recall that for a set of companies \( S \), we define \( U(S) = \mathbb{P} \left( \bigcup_{i \in S} E_i \right) \). We now show how to calculate the exit probabilities.

If we assume that the performance of the companies is independent, then all we need to calculate is \( p_i = \mathbb{P}(E_i) \) for each company \( i \in S \) to construct the portfolio. Recall that this is the probability that the company’s Brownian motion hits level \( 7\Delta \) in finite time. We assume we are given \( \mu_i(t) \) and \( \sigma_i(t) \) for company \( i \). Then, from (4.2) the exit probability is

\[
    p_i = \lim_{t \to \infty} F_0 \left( t; 0, \mu_i(t), \sigma_i(t), 7\Delta \right) .
\]  

(4.5)

Additionally, recall that we are uncertain in the future values of \( \beta_y \). This causes uncertainty in the drift which results in the companies no longer being independent. To see why this is the case, assume we are given the parameter vector \( \beta_{y-1} \). This will be the situation when we train on past data up to year \( y - 1 \) and try to build a portfolio for companies founded in year \( y \). Therefore, we need to know \( \beta_y = [\beta_{y1}, \beta_{y2}, \ldots, \beta_{yM}] \).

Recall that \( \beta_{yi} \) is normally distributed with mean \( \beta_{y-1,i} \) and standard deviation \( \delta_i \). This results in random drift coefficients for all companies founded in year \( y \). We do not know the exact value of the company drifts, but we do know that they will all be affected the same way by the actual realization of \( \beta_y \). This is how the uncertainty creates a positive correlation between the companies. To calculate \( U(S) \), we simply average over the uncertainty in \( \beta_y \). Using equations (2.4) and (4.5) we have

\[
    U(S) = \mathbb{E}_{\beta_y} \left[ \mathbb{P} \left( \bigcup_{i \in S} E_i \mid \beta_y \right) \right] .
\]
\[ = 1 - \mathbb{E}_{\beta_y} \left[ \prod_{i \in S} (1 - p_i) \right] \\
= 1 - \mathbb{E}_{\beta_y} \left[ \prod_{i \in S} (1 - \lim_{t \to \infty} F_0 (t; \mu_i(t), \sigma_i(t), 7\Delta)) \right]. \tag{4.6} \]

Because \( \beta_y \) is jointly normal and it is simple to evaluate \( F_0(\cdot) \), this expectation can be easily done using Monte Carlo integration.

For our model, because the number of startup companies that we are choosing from is quite large, we create our portfolio using the greedy approach. This is computationally feasible because in practice the portfolio will not be very large (typical venture capital firms manage no more than a few hundred companies) and it is easy to evaluate \( U(S) \) for a set \( S \). We consider the performance of portfolios built using our model in Section 5.3.

5. Results

We now present the results of our model estimation and portfolio construction. For model estimation, we look at the model parameters in order to understand which features play an important role in predicting company performance. For portfolio construction, we demonstrate that our model is able to build portfolios which have exit rates higher than some of the top venture capital firms.

5.1. Model Estimation

To estimate the model we first select a value for the level spacing \( \Delta \) and for the observation year \( t_{obs} \). We choose \( \Delta = 10 \) and we set \( t_{obs} \) equal to December 31st of the chosen year. We include in the training set all companies founded between 2000 and \( t_{obs} \). All data used in the model estimation must have been available before \( t_{obs} \), otherwise we do not include it. For instance, the observed funding round times occur before \( t_{obs} \). If a company had an exit after \( t_{obs} \), this information is not included during model estimation. Also, all company features are constructed using data available when the company received its first funding round. We do this because these features are what would have been available to someone who wanted to invest in the company in an early round, which is the investment time frame on which we focus.

To avoid overfitting, we included priors for all model parameters. In particular, for \( \beta \) and \( \gamma \) we use Gaussian priors with a mean of zero and a standard deviation of twenty. For \( \delta \) we use an exponential prior with a mean of one. We use a uniform prior for \( \log(\tau) \) with support on \([-10^4, 10^4]\) and a uniform prior for \( \nu \) with support on \([0, 100]\) years. Our results were robust to the choice of these priors.

We found that using a second order method to estimate the model was computationally expensive. Instead, we used the Broyden-Fletcher-Goldfarb-Shanno algorithm (Yuan 1991). We found that this algorithm was able to find a local optimum much faster than the standard gradient descent algorithm. Additionally, we estimated the model one-hundred times using randomized starting points, and the final result that we used was the one with the largest in-sample likelihood.
5.2. Parameter Values

Here we examine the estimated parameter values to gain some understanding as to the relative importance of the different features. The model has an observation year of 2010. The analysis we present here does not provide any rigorous statistical significance to the parameters. Rather, our aim is to provide a heuristic understanding of the impact of the features.

We first look at the estimated drift and diffusion coefficients for each company. We focus on the time independent components $\mu_0$ and $\sigma_0$. We refer $\sigma_0$ as the volatility of a company because it is a measure of how much the company deviates from the path dictated by its drift. We group the drift and volatility by the final funding round of the company before the observation time. We then calculate the median of the drift and volatility in each group and plot these values in Figure 3. There are a few key observations from this figure. First, the poorly performing companies (seed and series A) have small drift. Second, medium performing companies (series B to series F) have a larger drift, but lower volatility. Third, and most interesting, the companies that exit have a moderate drift, but a noticeably large volatility. Therefore, it seems that volatility is the key element for a company exiting. This makes sense for the model because the drift coefficient for a particular company is often small, making it hard to exit without any volatility. Therefore, a larger volatility generally leads to an increased exit probability.

We next investigate the value of the timing parameters. We find that the estimated value of $\nu$ is 6.37 years. Therefore, according to the model, the drift and diffusion of a company are constant for this period, which is roughly the time needed to exit with an IPO. The estimated value for $\tau$ is 4.83 years. This value is reasonable given our observed data. Almost no companies exit after $\nu + \tau \approx 11$ years.

The drift of a company is a linear function of its features, but the parameters depend upon the founding year. We first look at the value of these parameters in 2010 (the final year in the training set). We show the top elements of $\beta_{2010}$ for sector and non-sector features in Table 1. The top sector features are broadly related to technology, such as E-learning and ride sharing. We see that the top non-sector features are related to the past experience of the leadership (executive acquisition, executive IPO, advisory IPO, leadership age). The investor feature maximum acquisition fraction is also one of the top non-sector features. This suggests that companies with experienced and successful leadership and investors have increased drift which results in a higher exit probability.

We also show the top volatility feature parameters $\gamma$ for sector and non-sector features in Table 2. We see here that the top sector features are related to the social area of the technology sector (social media, messaging, social networks). It appears that companies in these sectors have a higher volatility. The top non-sector features include the previous experience of the non-leadership members (job IPO and job acquisition) and the founders (previous founder). The education level of the company members also has a strong impact on the volatility (top school). Finally, as with the drift, experienced investors increase the volatility (maximum acquisition fraction).
Figure 3  Plot of the estimated median drift $\mu_0$ and volatility $\sigma_0$ for companies grouped by their final funding round before 2010. The companies were all founded between 2000 and 2010.

Table 1 $\beta_{2010}$ for the sector features (left) and non-sector features (right) with the largest values.

| Sector feature   | $\beta_{2010}$ | Non-sector feature        | $\beta_{2010}$ |
|------------------|----------------|---------------------------|-----------------|
| E-learning       | 0.18           | Executive acquisition     | 0.80            |
| Ride sharing     | 0.14           | Executive IPO             | 0.80            |
| Open source      | 0.13           | Advisory IPO              | 0.26            |
| Cloud computing  | 0.12           | Leadership age            | 0.25            |
| Bioinformatics   | 0.10           | Maximum acquisition fraction | 0.24        |

Table 2 $\gamma$ for the sector features (left) and non-sector features (right) with the largest values.

| Sector feature   | $\gamma$ | Non-sector feature        | $\gamma$ |
|------------------|----------|---------------------------|----------|
| Social media     | 4.88     | Job IPO                   | 3.31     |
| Messaging        | 3.47     | Previous founder          | 3.22     |
| Social network   | 3.46     | Job acquisition            | 2.88     |
| Apps             | 2.47     | Top school                | 2.45     |
| Cloud computing  | 1.94     | Maximum acquisition fraction | 2.43    |

Finally, recall that for the time-varying $\beta$ parameters we have the standard deviation $\delta$ which characterizes the variation of these parameters over time. We show the top coefficient of variation, defined as $\delta$ divided by the absolute value of the average of $\beta$ over all years, for the sector and non-sector features in Tables 3 and 4. It is not immediately clear why these features are so variable. Their mean values are much lower than the top $\beta$ features, which makes their coefficient of variation high. Therefore, these features, despite being highly variable, likely do not strongly impact the exit probability of a company.

5.3. Portfolio Performance
While we tested our models for many different years, we report the results for the 2011 and 2012 testing sets. We use these years because it gives a sufficient amount of time for the companies to achieve an exit (we
Table 3  The average $\beta$, $\delta$, and coefficient of variation for the sector features with the largest magnitude coefficient of variation.

| Sector feature       | Average $\beta$ | $\delta$ | Coefficient of variation |
|----------------------|-----------------|----------|--------------------------|
| Personal health      | 0.0044          | 0.022    | 4.96                     |
| Nanotechnology       | -0.0073         | 0.034    | 4.63                     |
| Social network       | 0.0058          | 0.024    | 4.16                     |
| Apps                 | -0.0095         | 0.021    | 2.18                     |
| Insurance            | 0.013           | 0.024    | 1.88                     |

Table 4  The average $\beta$, $\delta$, and coefficient of variation for the non-sector features with the largest magnitude coefficient of variation.

| Non-sector feature                           | Average $\beta$ | $\delta$ | Coefficient of variation |
|----------------------------------------------|-----------------|----------|--------------------------|
| Companies affiliated                         | 0.063           | 0.70     | 11.10                    |
| Major overlap mean                           | 0.0038          | 0.022    | 5.65                     |
| Max IPO fraction                             | 0.014           | 0.042    | 3.01                     |
| Work overlap standard deviation              | 0.014           | 0.011    | 0.80                     |
| Two step neighborhood investor network       | -0.020          | 0.016    | 0.79                     |

carried this analysis in late 2016). For each test year, we define the companies founded in that year as the test set, and all other companies used in model estimation as the training set. The models are estimated on companies founded before the test year, as described in Section 5.1. The features used to calculate the exit probabilities of the test companies are built using only data that would have been available at the time of the companies’ earliest funding round. This way, we only use the data that would be available to an early investor.

We use the greedy portfolio construction method from equation (2.2). We build portfolios under the assumption that the companies are dependent and independent. The difference in the two assumptions is how we average over uncertainty in the drift parameters. For the dependent assumption, we do the averaging for all companies jointly, as shown in equation (4.6). For the independent assumption, we average equation (4.5) over the uncertainty for each company individually, and then use the resulting exit probabilities to construct the portfolio. The averaging is done using Monte Carlo integration. Specifically, we generate 50,000 random draws from a normal distribution for each element of the $\beta$ vector for the testing year, with mean given by $\beta_{2010}$ or $\beta_{2011}$ (depending on the testing year) and standard deviation given by the corresponding element in $\delta$. We then average the relevant function over these random variates.

We also benchmark our model against an ordered logistic regression model using our features and labels corresponding to the largest round that a company achieved before the test year.

We construct portfolios with a maximum of 20 companies, which is a reasonable size for a typical venture capital firm. To visualize the performance of our portfolios, we plot the portfolio size versus the number of companies that exit in the portfolio in Figure 4. We refer to this as a portfolio performance curve. It is similar to a receiver operating characteristic (ROC) curve used in machine learning to evaluate the performance of binary classifiers.
We consider five different types of portfolios. Ordered logistic regression is our benchmark model as discussed earlier. Dynamic heteroskedastic independent and correlated correspond to our Brownian motion model with the different dependence assumptions. Perfect corresponds to the portfolio where every company exits. Random corresponds to portfolios where the number of exits is the portfolio size multiplied by the fraction of exits in the test set. This corresponds roughly to picking a portfolio randomly. We also show the performance points for top venture capital firms in these years. The coordinates of these points are the number of test companies that the venture capital firm invests in and the number of these companies that exit before 2017.

As can be seen, our portfolios perform well (in terms of number of exits). For portfolios of only five companies, we achieve exit rates as high as 60%. We outperform random guessing and the venture capital firms by a substantial amount. Furthermore, our model performs better than or as well as the logistic regression model. For instance, in 2011, our model completely dominates the logistic regression model. In 2012, our model still does better, but the gap is not as large. With a five company portfolio in 2012, our model gets three exits, while the logistic regression model gets zero. Another interesting observation is that the correlated model does better than the independent model. We suspect this is because when we include the correlations the portfolio becomes more diversified.

We show the top ten companies in the 2011 and 2012 portfolios constructed with the greedy algorithm and the dynamic heteroskedastic correlated model in Tables 5 and 6. We also include the predicted exit probability for the companies and the objective value (probability of at least one exit) after the company is added to the portfolio. There are several exits in the top ten companies. These are mostly acquisitions, but we do obtain an IPO in 2011 from Nutanix, which is a virtualized datacenter platform that went public in 2016 with a valuation of $2.1 billion.

6. Conclusion
We have presented a new framework for constructing portfolios of items which either have huge returns or very low returns. We formulated the problem as one of maximizing the probability at least one item achieving a large return, or winning. The submodularity of this probability allowed us to efficiently construct portfolios using a greedy approach.

We applied our portfolio construction method to investing in startup companies, which have this top-heavy return structure. To calculate the relevant probabilities of winning, or exiting in the case of startup companies, we developed a Brownian motion model for the evolution of a startup company’s funding rounds. We estimated this model on data for thousands of companies and found certain features are associated with successful companies. We also constructed portfolios with our approach which outperform top venture capital firms. The results show that our modeling and portfolio construction method are effective and they provide a quantitative methodology for venture capital investment.
Table 5  The top companies in our 2011 portfolio constructed with greedy optimization and the dynamic heteroskedastic correlated model. Shown are the company name, the highest funding round achieved, the predicted exit probability, the greedy objective value (probability of at least one exit) when the company is added, and the change in the objective value when the company is added. Companies which exited are highlighted in bold.

| 2011 Company | Highest funding round | Exit probability | Objective value | Change in objective value |
|---------------|-----------------------|------------------|----------------|---------------------------|
| SHIFT         | Acquired              | 0.61             | 0.61           | 0.61                      |
| Jibbigo       | Acquired              | 0.58             | 0.74           | 0.13                      |
| Sequent       | B                     | 0.49             | 0.79           | 0.05                      |
| Nutanix       | IPO                   | 0.44             | 0.81           | 0.02                      |
| PowerInbox    | A                     | 0.43             | 0.83           | 0.02                      |
| Friend.ly     | Acquired              | 0.35             | 0.84           | 0.01                      |
| Jybe          | Acquired              | 0.33             | 0.85           | 0.01                      |
| MediaRoost    | Seed                  | 0.50             | 0.86           | 0.01                      |
| CloudTalk     | A                     | 0.33             | 0.87           | 0.01                      |
| LaunchRock    | Acquired              | 0.41             | 0.88           | 0.01                      |

Table 6  The top companies in our 2012 portfolio constructed with greedy optimization and the dynamic heteroskedastic correlated model. Shown are the company name, the highest funding round achieved, the predicted exit probability, the greedy objective value (probability of at least one exit) when the company is added, and the change in the objective value when the company is added.

| 2012 Company | Highest funding round | Exit probability | Objective value | Change in objective value |
|---------------|-----------------------|------------------|----------------|---------------------------|
| AppEnsure     | Seed                  | 0.58             | 0.58           | 0.58                      |
| Metaresolver  | Acquired              | 0.38             | 0.71           | 0.13                      |
| Kiva          | Seed                  | 0.37             | 0.78           | 0.07                      |
| ViewFinder    | Acquired              | 0.31             | 0.83           | 0.05                      |
| SnappyTV      | Acquired              | 0.37             | 0.87           | 0.04                      |
| Struq         | Acquired              | 0.30             | 0.89           | 0.02                      |
| SparkCentral  | B                     | 0.29             | 0.91           | 0.02                      |
| Glossi Inc    | Seed                  | 0.30             | 0.93           | 0.02                      |
| Work4         | B                     | 0.33             | 0.94           | 0.01                      |
| Hornet Networks | Seed               | 0.37             | 0.95           | 0.01                      |

There are several future steps for this work. One concerns the manner in which company exits are treated. Acquisitions and IPOs are equivalent in our model, but in reality IPOs are much more rare and can result in a much higher payoff. Our model could be modified to distinguish between IPOs and acquisitions, or it could incorporate the monetary value of the IPO or acquisition. For IPOs this valuation data is public, but acquisition data is generally private and may be difficult to obtain.

Another direction concerns the features used in our model. While we included many relevant features, there could be other useful features which have predictive power. For instance, the value of different macro-economic indicators in the founding year of a company may have impact on its exit probability. In addition to the types of features used, one could also explore the way in which they are incorporated into the
model. We used a simple linear mapping between the features and the drift and diffusion. However, more complex models which use non-linear mappings may improve performance.

Appendix A: Proofs

A.1. Proof of Theorem 2.1

We show that maximizing $U(S)$ is NP-hard by reducing it to the maximum coverage problem. In the maximum coverage problem one is given a set $\mathcal{U}$ of $n$ elements and a collection $\mathcal{E} = \{E_i\}_{i=1}^N$ of $N$ subsets of $\mathcal{U}$ such that $\bigcup_{E \in \mathcal{E}} E = \mathcal{U}$. The goal is to select $M$ sets from $\mathcal{E}$ such that their union has maximum cardinality. This is known to be an NP-hard optimization problem. To show that this is an instance of maximizing $U(S)$ we assume that the sample space $\Omega$ is countable and finite with $R$ elements. We also assume that each element $\omega \in \Omega$ has equal probability, i.e. $P(\omega) = R^{-1}$. Let $\mathcal{F}$ be the $\sigma$-algebra of $\Omega$. For any set $S \in \mathcal{F}$, we can write $U(S) = R^{-1}\left|\bigcup_{\omega \in S} \omega\right|$. Then we have

$$\max_{S \subseteq \mathcal{E}, |S|=M} U(S) = \max_{S \subseteq \mathcal{E}, |S|=M} R^{-1}\left|\bigcup_{\omega \in S} \omega\right| \quad \text{(A.1)}$$

Therefore, maximizing $U(S)$ is equivalent to the maximum coverage problem.

A.2. Proof of Lemma [1]

The function $U(S)$ is non-negative and non-decreasing because it is the probability of a set of events. We must show that it is also submodular. A submodular function $f$ satisfies

$$f\left(S \bigcup v\right) - f\left(S\right) \geq f\left(T \bigcup v\right) - f\left(T\right) \quad \text{(A.1)}$$
for all elements \( v \) and pairs of sets \( S \) and \( T \) such that \( S \subseteq T \). We show that the function \( U(S) \) is submodular as follows. We let the \( \sigma \)-algebra of the probability space be \( \mathcal{F} \). Consider sets \( S, T, v \in \mathcal{F} \) such that \( S \subseteq T \). We can write 
\[
v = v_S \cup v_T \cup v_0,\]
where we define \( v_S = v \cap S, v_T = v \cap T \cap S^c, \) and \( v_0 = v \cap T^c \). Then we have 
\[
U \left( T \cup v \right) - U \left( T \right) = P \left( v_0 \right)
\]
and 
\[
U \left( S \cup v \right) - U \left( S \right) = P \left( v_T \cup v_0 \right) \geq P \left( v_0 \right) \geq U \left( T \cup v \right) - U \left( T \right),
\]
thus showing that \( U(S) \) satisfies the submodularity condition.

### A.3. Proof of Theorem 2.4

We will begin by defining some notation. For \( 0 \leq l \leq k \), let \( G_l(S) \) denote the collection of all subsets of \( S \) with cardinality \( l \). Additionally, for \( 0 \leq q \leq k \), we also define the events \( W_q(S) \) and \( Y_q(S) \) as follows

\[
W_q(S) = \bigcup_{T \in G_q(S)} \left( \prod_{i \in T} E_i \right) \prod_{j \in S \setminus T} E_j^c
\]

\[
Y_q(S) = \bigcup_{t=q}^k W_t(S).
\]

In particular, \( W_q(S) \) is the event where exactly \( q \) of the event \( E_i \) for \( i \in S \) occur. Naturally, \( Y_q(S) \) is the event where \( q \) or more of the events \( E_i \) for \( i \in S \) occur. Using this notation, when \( S \) is of cardinality \( k \) we get the following

\[
V(S) = \sum_{q=0}^k P(W_q(S)) \ln \left( \frac{q^b + (k-q)a}{k} \right)
\]

\[
= \sum_{q=0}^{k-1} \left( P(Y_q(S)) - P(Y_{q+1}(S)) \right) \ln \left( \frac{q^b + (k-q)a}{k} \right) + P(Y_k) \ln \left( \frac{kb}{k} \right)
\]

\[
= \ln(a) + \sum_{q=1}^k P(Y_q(S)) \ln \left( 1 + \frac{b-a}{(q-1)b + (k-q+1)a} \right). \tag{A.2}
\]

The second line above follows from the definition of \( Y_q(S) \) and \( W_q(S) \), and the third line follows from the basic properties of a logarithm. We will now conclude by showing that \( S = [k] \) maximizes every term in the sum given in (A.2), and thus it is maximizes \( V(S) \). Now, because \( b > a \) by assumption, we have that

\[
\ln \left( 1 + \frac{b-a}{(q-1)b + (k-q+1)a} \right) > 0.
\]

for all \( q \in [k] \). Due to the positivity of this value, maximizing every term in the sum given in (A.2) reduces to the problem of maximizing \( P(Y_q(S)) \). Finally, we show that that \( S = [k] \) maximizes \( P(Y_q(S)) \) for all \( q \in [k] \). If \([k]\) is a maximizer for \( P(Y_q(S)) \), then we are done. We define \( S_L = [k] \) and for purpose of contradiction, we suppose that
there exists an $S^\dagger$ of cardinality $k$ such that $S^\dagger \neq S_L$, and $P(Y_q(S^\dagger)) > P(Y_q(S_L))$. Therefore, we know there exists a $r \in S^\dagger$ and a $t \in S_L$ such that $r \notin S_L$, $t \notin S^\dagger$, and $p_t \geq p_r$, otherwise we would have $S^\dagger = S_L$. Then from the definition of $Y_q(S)$, we have that

$$P(Y_q(S^\dagger)) = p_r P(Y_q(S^\dagger) \mid E_r) + (1 - p_r) P(Y_q(S^\dagger) \mid E_r^c)$$

$$= P(Y_q(S^\dagger) \mid E_r) + p_r \left( P(Y_q(S^\dagger) \mid E_r) - P(Y_q(S^\dagger) \mid E_r^c) \right)$$

$$= P(Y_q(S^\dagger \setminus \{r\})) + p_r \left( P(Y_{q-1}(S^\dagger \setminus \{r\})) - P(Y_q(S^\dagger \setminus \{r\})) \right).$$

By a similar logic, we have that

$$P(Y_q(S^\dagger \cup \{t\} \setminus \{r\})) = P(Y_q(S^\dagger \setminus \{r\})) + p_t \left( P(Y_{q-1}(S^\dagger \setminus \{r\})) - P(Y_q(S^\dagger \setminus \{r\})) \right).$$

Finally, $Y_q(S^\dagger \setminus \{r\}) \supseteq Y_q(S^\dagger \setminus \{r\})$ and thus it follows that $P(Y_q(S^\dagger)) \leq P(Y_q(S^\dagger \cup \{t\} \setminus \{r\}))$. By repeating this argument for all elements of $S^\dagger$ that are not in $S_L$, we can arrive at the conclusion $P(Y_q(S^\dagger)) \leq P(Y_q(S_L))$. However, this is a contradiction, and thus $S_L$ is a maximizer for $P(Y_q(S))$.

### A.4. Proof of Theorem 2.5

We will begin in a way that is very similar to the proof in section A.3. In particular, we begin by defining $W_q(S)$ and $Y_q(S)$ for $l \in [k]$ as follows

$$W_q(S) = \bigcup_{T \in \mathcal{G}_q(S)} \left( \bigcap_{i \in T} E_i \right) \bigcap \left( \bigcap_{j \in S \setminus T} E_j^c \right)$$

$$Y_q(S) = \bigcup_{t=q}^k W_t(S).$$

Now, let $S_k$ denote a binomial random variable with parameters $k$ and $p$. Then, from the assumptions given in the statement of the theorem, we have that for the log-optimal portfolio $S_L$,

$$P(Y_j(S_L)) = \sum_{i=j}^k \sum_{T \in \mathcal{G}_i(S_L)} P\left( \left( \bigcap_{E \in T} E \right) \bigcap \left( \bigcap_{F \in S_L \setminus T} F^c \right) \right)$$

$$\leq \sum_{i=j}^k \sum_{T \in \mathcal{G}_i(S_L)} (1 + \lambda) p^i (1 - p)^{k-i}$$

$$= (1 + \lambda) \sum_{i=j}^k \binom{k}{i} p^i (1 - p)^{k-i}$$

$$= (1 + \lambda) P(S_k \geq j).$$

for all $j \in [k]$. By the same logic, we also get that for the picking winners portfolio $S_W$,

$$P(Y_j(S_W)) \geq (1 - \lambda) P(S_k \geq j).$$

Then, using equation (A.2), the inequalities given above, and the fact that $P(Y_1(S)) = U(S)$, we obtain

$$V(S_L) - V(S_W) \leq (U(S_L) - U(S_W)) \ln \left( 1 + \frac{b-a}{ka} \right) + 2\lambda \sum_{j=2}^k P(S_k \geq j) \ln \left( 1 + \frac{b-a}{ka + (j-1)(b-a)} \right).$$
Now, using Chebyshev’s inequality, the variance of a binomial random variable, and the above inequality we have that

\[ V(S_L) - V(S_W) \leq (U(S_L) - U(S_W)) \ln \left( 1 + \frac{b-a}{ka} \right) + 2\lambda \sum_{j=2}^{k} \Pr(|S_n - kp| \geq j - kp) \ln \left( 1 + \frac{b-a}{ka + (j-1)(b-a)} \right) \]

\[ \leq (U(S_L) - U(S_W)) \ln \left( 1 + \frac{b-a}{ka} \right) + 2\lambda \sum_{j=2}^{k} \frac{kp(1-p)}{(j-kp)^2} \ln \left( 1 + \frac{b-a}{ka + (j-1)(b-a)} \right). \]

Now, recall that \( \ln(1+x) \leq x \) for \( x > -1 \). Therefore

\[ V(S_L) - V(S_W) \leq (U(S_L) - U(S_W)) \ln \left( 1 + \frac{b-a}{ka} \right) + 2\lambda \sum_{j=2}^{k} \frac{kp(1-p)}{(j-kp)^2} \frac{1}{j-1} \]

\[ \leq (U(S_L) - U(S_W)) \ln \left( 1 + \frac{b-a}{ka} \right) + 2\lambda kp(1-p) \sum_{j=2}^{k} \frac{1}{(j-1)^3} \]

where for the second inequality we used the assumption \( p \in (0, \frac{1}{2}] \). Some further analysis gives

\[ V(S_L) - V(S_W) \leq c \ln \left( 1 + \frac{b-a}{ka} \right) + 2\lambda kp(1-p) \sum_{j=2}^{k} \frac{1}{(j-1)^3} \]

\[ \leq (U(S_L) - U(S_W)) \ln \left( 1 + \frac{b-a}{ka} \right) + 2\lambda kp(1-p) \sum_{j=1}^{\infty} \frac{1}{j^3} \]

\[ \leq (U(S_L) - U(S_W)) \ln \left( 1 + \frac{b-a}{ka} \right) + 2\lambda kp(1-p)\zeta(3). \quad \text{(A.3)} \]

Now, because \( S_L \) is log-optimal, we have that \( V(S_L) - V(S_W) \geq 0 \). Using this relationship we can rearrange equation \text{(A.3)} to obtain

\[ (U(S_W) - U(S_L)) \ln \left( 1 + \frac{b-a}{ka} \right) \leq 2\lambda kp(1-p)\zeta(3) \]

\[ U(S_W) - U(S_L) \leq \frac{2\lambda kp(1-p)\zeta(3)}{\ln \left( 1 + \frac{b-a}{ka} \right)}. \quad \text{(A.4)} \]

We next lower bound \( U(S_W) \) using the assumptions of Theorem \text{2.5} and basic properties of the binomial distribution to obtain

\[ U(S_W) = \Pr(Y_j(S_W)) \]

\[ = (1-\lambda) \Pr(S_k \geq 1) \]

\[ \geq (1-\lambda)(1 - (1-p)^k). \quad \text{(A.5)} \]

Combining this lower bound with equation \text{(A.4)} we obtain our final result.

\[ \frac{U(S_W) - U(S_L)}{U(S_W)} \leq \frac{2\lambda kp(1-p)\zeta(3)}{\ln \left( 1 + \frac{b-a}{ka} \right)(1-\lambda)(1 - (1-p)^k)}. \]
Appendix B: Sector Names

The sector names that we used from the Crunchbase database are: 3d printing, advertising, analytics, animation, apps, artificial intelligence, automotive, autonomous vehicles, big data, bioinformatics, biotechnology, bitcoin, business intelligence, cloud computing, computer, computer vision, dating, developer apis, e-commerce, e-learning, edtech, education, Facebook, fantasy sports, fashion, fintech, finance, financial services, fitness, gpu, hardware, health care, health diagnostics, hospital, insurance, internet, internet of things, iOS, lifestyle, logistics, machine learning, medical, medical device, messaging, mobile, nanotechnology, network security, open source, personal health, pet, photo sharing, renewable energy, ride sharing, robotics, search engine, social media, social network, software, solar, sports, transportation, video games, virtual reality, and virtualization.

Appendix C: Top School Names

The schools used for the top school feature are: Berkeley, Brown, California Institute of Technology, Carnegie Mellon, Columbia, Cornell, Dartmouth, Duke, Harvard, Johns Hopkins, Massachusetts Institute of Technology, Northwestern, Princeton, Stanford, University of Chicago, University of Pennsylvania, Wharton, and Yale.

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