Two-Timescale Transmission Design for Wireless Communication Systems Aided by Active RIS

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Abstract—This paper considers an active reconfigurable intelligent surface (RIS)-aided communication system, where an M-antenna base station (BS) transmits data symbols to a single-antenna user via an N-element active RIS. We use two-timescale channel state information (CSI) in our system, so that the channel estimation overhead and feedback overhead can be decreased dramatically. For both the uplink and downlink systems, we derive the closed-form approximate expressions of the achievable rates (ARs) and propose the phase shift optimization schemes. In addition, we compare the performance of the active RIS system with that of the passive RIS system. The numerical results show that the active RIS system achieves a larger AR than the passive RIS system.

Index Terms—Two-timescale CSI, active reconfigurable intelligent surface (RIS), achievable rate (AR).

I. INTRODUCTION

Reconfigurable intelligent surface (RIS) has attracted extensive research attention, which has been envisioned as one of the potential key technologies in 6G communication systems [1]. In particular, an RIS is equipped with a large number of passive and low-cost reflecting elements, each of which can induce an independent phase shift on the incident signals to enhance the received signal. Due to its passive nature, the RIS possesses appealing features of low cost, low power consumption and easy deployment [2], [3]. However, the passive RIS suffers from the multiplicative fading effects, where the propagation link experiences a product of channel attenuation of two individual links. To address this issue, a novel concept named active RIS was proposed in [4], [5]. In an active RIS, each reflecting element is comprised of additional reflection-type amplifier, which not only adjusts the phase shift of the incident signals, but amplifies the incident signal power. The authors of [6] demonstrated that both the passive RIS and the active RIS systems are assumed to have the same total power consumption, the active RIS outperforms the passive RIS in most cases. In an active RIS-aided single-user single-input single-output system, the authors in [4] derived the asymptotic expression for signal-noise-ratio (SNR) when the number of reflection elements approaches infinity.

The existing contributions on the active RIS mostly consider the instantaneous channel state information (CSI). However, this transmission design scheme requires a high pilot overhead and a high feedback overhead. To address this issue, a two-timescale CSI scheme is proposed [7]. The beamforming vector at the base station (BS) could be designed on the basis of the instantaneous CSI [3], while the active RIS configuration is based on the statistical CSI, which only depends on the angle and location information that generally remain invariant in a long term [8], [9]. In addition, due to the slow varying of the statistical CSI, the computational complexity and feedback overhead can be greatly reduced [10].

Against the above background, we propose a two-timescale CSI-based transmission design for the active RIS-aided wireless system. Specifically, we summarize the contributions of this paper in short: 1) we derive a closed-form expression of the achievable rate (AR) of the considered system; 2) we propose a genetic algorithm (GA) to solve the phase shift optimization problem (PSOP); 3) this expression is extended to the downlink system.

II. SYSTEM MODEL

We consider an uplink wireless communication system, where a single-antenna user transmits data information to an M-antenna BS via an N-element active RIS. The active RIS can be employed at the facade of buildings, which can provide a strong line-of-sight (LoS) component in wireless links. Hence, the Rician fading model is considered in the RIS-related channels. In addition, the channel between the user and the BS is assumed to be blocked by the obstacles. The uplink channel matrix between the active RIS and the BS is \( H_U \in C^{M \times N} \). The uplink channel vector between the user and the BS is \( g_U \in C^{N \times 1} \). Specifically, \( H_U \) and \( g_U \) are following Rician fading distribution, where \( \alpha_U \) and \( \beta_U \) are the large-scale fading coefficients, \( K_U \) and \( K_B \) are the Rician factors, which are respectively defined as the power ratio of the LoS components and the NLoS components. The NLoS components \( \tilde{H}_U \) and \( \tilde{g}_U \) consist of independent and identical distribution (i.i.d) entries, all of which follow the distribution of \( CN(0,1) \). The LoS components \( \tilde{H}_U \) and \( \tilde{g}_U \) apply the uniform squared planar array (USPA) model, which can be written as

\[
\tilde{H}_U = a_M \left( \phi_U, \varphi_U \right) a_N^T \left( \phi_U, \varphi_U \right), \quad \tilde{g}_U = a_N \left( \phi_U, \varphi_U \right),
\]

with

\[
a_N \left( \phi_U, \varphi_U \right) = \begin{bmatrix} e^{j2\pi \left( \left( \sin \phi_U \sin \varphi_U \right)^2 + y \cos \phi_U \right)} & \cdots & e^{j2\pi \left( \left( \sin \phi_U \sin \varphi_U \right)^2 + y \cos \phi_U \right)} 
\end{bmatrix}^T,
\]

where \( X \in \{ M, N \}, 0 < x, y \leq \sqrt{X} - 1 \). The angles \( \phi_U \) and \( \varphi_U \) are respectively the azimuth and elevation angles of arrival (A-AoA and E-AoA) at the RIS from the user, and the angles \( \phi_U \) and \( \varphi_U \) are the A-AoA and E-AoA at the RIS from the BS, while the angles \( \varphi_U \) and \( \varphi_U \) are respectively the azimuth and elevation angles of departure (A-AoD and E-AoD) from the RIS to the BS. The parameters \( d \) and \( \lambda \) are respectively the element spacing and the carrier wavelength, and we set \( d = \frac{\lambda}{2} \) similar as [2] to facilitate analysis.
Different from the passive RIS, the active RIS amplifies the incident signals by the external power supply, however the thermal noise is amplified accordingly. Hence, the uplink signal received at the BS is

\[ y_U = \sqrt{P_{t,U}} \mathbf{H}_U \mathbf{A}_U \Phi_U \mathbf{g}_U x + \mathbf{H}_U \mathbf{A}_U \Phi_U \mathbf{v}_U + n_U, \]  

where \( P_{t,U} \) is the transmission power of the user, and the scalar \( x \) represents its transmitted symbol satisfying \( E\{ |x|^2 \} = 1 \). The vector \( \mathbf{v}_U \in \mathbb{C}^{N \times 1} \sim \mathcal{CN}(\mathbf{0}, \mathbf{I}_N) \) is the uplink thermal noise at the RIS. The vector \( n_U \in \mathbb{C}^{M \times 1} \sim \mathcal{CN}(\mathbf{0}, \mathbf{I}_M) \) is the additive white Gaussian noise (AWGN) at the BS. The power amplification matrix \( \mathbf{A}_U = \text{diag}(\lambda_{1U}, \lambda_{2U}, \ldots, \lambda_{NU}) \) and the phase shift matrix \( \Phi_U = \text{diag}(e^{j\phi_{1U}}, e^{j\phi_{2U}}, \ldots, e^{j\phi_{NU}}) \), where \( \theta_{nU} \) is the phase shift of the \( n \)th RIS element. For simplicity, the amplification factors for elements in the active RIS are assumed to be identical, i.e., \( \lambda_{1U} = \lambda_{2U} = \cdots = \lambda_{NU} = \lambda_U \). Then, the amplification factor \( \lambda_U \) can be obtained as

\[
\lambda_U = \sqrt{\frac{P_{t,U}}{E\{ \| \sqrt{P_{t,U}} \mathbf{H}_U \mathbf{A}_U \Phi_U \mathbf{g}_U \|^2 \| \mathbf{u}_U \|^2 \}}}
= \sqrt{\frac{P_{t,U}}{N \left( P_{t,U} \beta_U + \sigma^2_{\phi,U} \right)}}.
\]  

where \( P_{t,U} \) is the uplink amplification power at the active RIS.

In this paper, we adopt a two-timescale transmission design protocol due to its low channel estimation overhead and computational complexity. The BS uses the maximal ratio combining (MRC) technique to process its received signal. Therefore, the signal received at the BS can be written as

\[
r_U = (\mathbf{H}_U \mathbf{A}_U \Phi_U \mathbf{g}_U)^H y_U.
\]  

Thus, the uplink AR is given by

\[
R_U = E\{ \log_2 (1 + \gamma_U) \},
\]  

where \( \gamma_U \) is the SNR defined by

\[
\gamma_U = \frac{P_{t,U} \lambda_U^2 \| \mathbf{H}_U \mathbf{A}_U \Phi_U \mathbf{g}_U \|^2}{\lambda_U^2 \| \mathbf{g}_U \|^2 \| \mathbf{H}_U \mathbf{A}_U \Phi_U \mathbf{g}_U \|^2 + \| \mathbf{H}_U \mathbf{A}_U \Phi_U \mathbf{g}_U \|^2}.
\]  

III. ACHIEVABLE RATE ANALYSIS AND PHASE SHIFT DESIGN

In this section, we first derive a closed-form approximate expression for the AR of the uplink system aided by the active RIS. Then, we compare the AR with that in the passive RIS system under the same total transmission power \( P_{t,U} \). Finally, we investigate an AR maximization problem and propose a phase shift design to solve the problem.

A. Achievable Rate Analysis

Theorem 1: The uplink AR of the active RIS-aided system can be approximated as

\[
R_U \approx \log_2 \left( 1 + \frac{P_{t,U} \lambda_U^2 E\{ \| \mathbf{H}_U \mathbf{A}_U \Phi_U \mathbf{g}_U \|^2 \}}{\lambda_U^2 E\{ \| \mathbf{g}_U \|^2 \| \mathbf{H}_U \mathbf{A}_U \Phi_U \mathbf{g}_U \|^2 \| \mathbf{u}_U \|^2 + \| \mathbf{H}_U \mathbf{A}_U \Phi_U \mathbf{g}_U \|^2} \right).
\]  

\[
E\{ \| \mathbf{H}_U \mathbf{A}_U \Phi_U \mathbf{g}_U \|_m^2 \} = \kappa_U K_1 K_2 \left\{ \sum_{n=1}^N \mathbf{a}_M (\phi^a, \phi^c) \right\}_n f_U^2 + \kappa_U K_1 E\left\{ \sum_{n=1}^N \mathbf{a}_N (\varphi^a, \varphi^c) \right\}_n \right\}_n e^{j\theta_{nU}} \| \mathbf{g}_U \|^2.
\]  

+ \kappa_U K_2 \left\{ \sum_{n=1}^N \left\| \mathbf{H}_U \mathbf{A}_U \Phi_U \mathbf{g}_U \right\|_m \left\| \mathbf{u}_U \right\|^2 \right\}_n e^{j\theta_{nU}} \| \mathbf{g}_U \|^2.
\]  

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Furthermore, the term $\mathbb{E}\{||H_D^b \Phi_U g_D|^2||^2\}$ in (9) can be written as
\[
\mathbb{E}\{||H_D^b \Phi_U g_D|^2||^2\} = \sum_{m=1}^{M} \mathbb{E}\{||H_U^b \Phi_U g_D|^2||^2\} + 2 \sum_{m=1}^{M} \mathbb{E}\{||H_U^b \Phi_U g_D|^2||^2||H_U^b \Phi_U g_D|^2||^2\} \triangleq \xi. 
\tag{17}
\]
By using the similar method for calculating (11), we can obtain $\xi$ given by (18).

We then transform the last expectation in (9) as
\[
\mathbb{E}\{g_U^b \Phi_U H_D^b H_U^b \Phi_U g_D\} = \mathbb{E}\{g_U^b \Phi_U H_U^b \Phi_U g_D\} \triangleq \nu,
\tag{19}
\]
where $W_U = H_D^b H_U$ is a non-central Wishart matrix following $W(M, A_U, \Sigma_U)$ with $A_U = \sqrt{\alpha_U K_1+1} H_U$ and $\Sigma_U = \frac{\alpha_U}{K_1+1} I_N$ [11].

According to [12], the non-central Wishart matrix $W_U$ can be approximated by a central Wishart distribution $W(M, 0, \Sigma_U)$ with
\[
\Sigma_U = \Sigma_U + \frac{1}{M} A_U^H A_U = \frac{\alpha_U}{K_1+1} \left( I_N + \frac{K_1}{M} H_U^b H_U \right).
\tag{20}
\]

According to [13], we obtain $\mathbb{E}\{W_U W_U\}$ given by
\[
\mathbb{E}\{W_U W_U\} = \frac{\alpha_U^2 M N}{K_1+1} \left( I_N + \frac{K_1}{M} H_U^b H_U \right)
\tag{21}
\]
\[\quad + \frac{M^2 \alpha_U^2}{(K_1+1)^2} \left( I_N + \frac{2K_1}{M} H_U^b H_U \right)
\[\quad + \frac{K_1^2}{M^2} H_U^b H_U^b H_U^b H_U \].
\]

Utilizing (21) and the fact that $\text{tr}(A B) = \text{tr}(B A)$, we can further derive $\nu$ as (22).

By substituting (4), (10)–(22) into (9), we can obtain (8). Thus, the proof is completed.

Remark 1: We denote $\gamma_{\text{RIS}}^U = \frac{P_{U \text{r}}}{\sigma_{U \text{r}}^2}$, and $\gamma_{\text{RIS}}^U = \frac{P_{U \text{r}}}{\sigma_{U \text{r}}^2}$. When $\gamma_{\text{RIS}}^U \rightarrow \infty$, the AR in (8) converges to
\[
R_U \rightarrow \log_2 \left(1 + \frac{\gamma_{\text{RIS}}^U}{\nu}\right).
\tag{23}
\]
When $\gamma_{\text{RIS}}^U \rightarrow \infty$, we have
\[
R_U \rightarrow \log_2 \left(1 + \frac{\gamma_{\text{RIS}}^U}{\nu}\right)
\tag{24}
\]
\[
\text{In one respect, the equations above show that the SNR $\gamma_U$ keeps a linear relationship with $\gamma_{\text{RIS}}^U$, when $\gamma_{\text{RIS}}^U$ is high. In the other respect, when $\gamma_{\text{RIS}}^U$ is high, the SNR and $\gamma_{\text{RIS}}^U$ also keeps a linear relationship.}
\]
\text{Then, we further introduce the power consumption of the considered system. The uplink overall transmission power is denoted by}
\[
P_U = P_{U \text{r}} + P_{U \text{c}} + N (P_{\text{sw}} + P_{\text{DC}}),
\tag{25}
\]
where $P_{\text{DC}}$ is the direct current biasing power for each active RIS element. $P_{\text{sw}}$ is the power consumed by each RIS element to switch the phase shift and control the circuit. For comparison, we provide the AR of the passive RIS system and its power consumption in the following corollary. By setting $\gamma_{U \text{r}}^U = 1$ and $\sigma_{U \text{r}}^2 = 0$ in (4) and (8), we can obtain the AR of uplink passive RIS system in the following corollary.

Corollary 1: When the passive RIS system has the same overall transmission power $P_U$ as the active RIS system, the AR of the uplink passive RIS system is given by
\[
R_U^P \approx \log_2 \left(1 + \frac{P_{U \text{r}}}{\sigma_{U \text{r}}^2} \frac{\nu}{\gamma_{U \text{r}}^U}\right),
\tag{26}
\]
where $P_{U \text{r}}^P$ is the transmission power at the user in the passive RIS system, and defined as
\[
P_U = P_{U \text{r}}^P + N P_{\text{sw}}.
\tag{27}
\]

B. Phase Shift Design

We solve the PSOP to maximize the uplink AR by a genetic algorithm (GA)-based method in this subsection. Specifically, the PSOP is formulated as follows
\[
\max_{\Phi_U^f} R_U
\tag{28a}
\]
s.t. $\theta_{\alpha U} \in \{0, 2\pi/2^b, \ldots, 2\pi(2^b - 1)/2^b\}$
\[
\forall n = 1, \ldots, N.
\tag{28b}
\]

Problem (28) can hardly be solved by conventional optimization methods. Given the complex mathematical achievable rate expression, the techniques conceived in [14] cannot be directly applied in our scenario. To cope with the issue, we fall back on the heuristic algorithms. We solve the above PSOP by tuning the phase shifts with the GA-based approach, which is detailed in [15, Algorithm 1]. The proposed GA method consists of population initialization, fitness evaluation, selection, crossover, and mutation. In addition, we take into account the discrete phase shifts case which uses $b$-bit quantification. Similar as [16], the complexity of the proposed GA algorithm is $N_t \times n$, where $N_t$ is the population size, and $n$ is the number of generations evaluated. Moreover, $n$ is determined by the convergence behavior of the GA.

IV. EXTENSION TO DOWNLINK TRANSMISSION

In this section, we extend the uplink active RIS-aided communication system to the downlink case. Based on that, we analyze the data transmission and derive a closed-form approximate expression of the AR.

A. Downlink System Model

Similar to the uplink channels, the downlink channels $H_D$ and $g_D$ follow Rician fading, where $\alpha_D$ and $\beta_D$ are the large-scale fading
\[
\xi = \kappa_U^2 M^2 K_1^2 K_2^2 |f_U|^4 + 2 \kappa_U^2 M K_1 K_2 |f_U|^2 (2 M N K_1 + M N K_2 + M N + 2 M + N K_2 + 2 N + 2 M)
\]
\[\quad + \kappa_U^2 M N^2 (K_2^2 + 2 K_1 K_2 + 2 K_1 + 2 K_2 + 1) + \kappa_U^2 M (M + 1) N (2 K_1 + 2 K_2 + 1)
\]
\[\quad + \kappa_U^2 M^2 N^2 (2 K_1^2 + K_2^2 + 2 K_1 K_2 + 2 K_1 + 2 K_2 + 1)
\]
\[
\nu = \frac{M^2 \alpha_{U \text{r}} K_U K_D}{(K_1 + 1)} \left(2 (N + 2 K_1 |f_U|^2 + K_1 N |f_U|^2) + N + 2 K_1 N + K_1^2 N^2\right) + \alpha_U \kappa_U M N \left(K_2 N + K_1 K_2 |f_U|^2 + N + K_1 N\right)
\tag{22}
\]
coefficients, $K_1$ and $K_2$ are the Rician factors. The LoS components, using USPA model, are $\tilde{H}_D = a_N(\psi_D^e, \psi_D^e) h_M^H(\omega_D^e, \omega_D^e)$ and $\tilde{g}_D = a_N(\psi_D^e, \psi_D^e)$. And the NLoS components $\tilde{H}_D$ and $\tilde{g}_D$ consist of i.i.d. entries, all of which follow the complex Gaussian distribution of $CN(0,1)$. The angles $\psi_D^e$ and $\psi_D^e$ are respectively A-AoD and E-AoA.

For the downlink transmission, the transmitted signal is first pre-coded by the BS, then amplified by the RIS. Thus, the signal received at the user is given by

$$r_D = g_D A_D \Phi_D (H_D w_D s + v_D) + n_D.$$  \hfill (29)

where $s$ represents the transmitted symbol from the BS with $E\{|s|^2\} = 1$. The vector $v_D \in \mathbb{C}^{N \times 1} \sim CN(0, \sigma^2_v I_N)$ is the downlink thermal noise at the RIS. The vector $n_D \in \mathbb{C}^{M \times 1} \sim CN(0, \sigma^2_n I_M)$ is the AWGN at the user. The power amplification factor matrix $\Lambda_D = \text{diag}(\lambda_D, \lambda_D, \ldots, \lambda_N)$, and the phase shift matrix $\Phi_D = \text{diag}(\exp(i \theta_D^1), \exp(i \theta_D^2), \ldots, \exp(i \theta_D^M))$, where $\theta_D$ is the phase shift of the $n$th RIS element. We use the maximal ratio transmission at the BS, thus the preceding vector $w_D$ is

$$w_D = \sqrt{P_D} (g_D A_D \Phi_D H_D)^H,$$ \hfill (30)

satisfying

$$E\{|w_D|^2\} = P_D,$$ \hfill (31)

where $P_D$ is the downlink transmission power, and the beamforming coefficient $\mu_D$ is

$$\mu_D = E\{|g_D A_D \Phi_D H_D|^2\}.$$ \hfill (32)

Moreover, similar to the uplink system, we assume each RIS element shares the same amplification factor $\lambda_D$, i.e., $\lambda_D = \lambda_D = \ldots = \lambda_N = \lambda_D$. Then, the amplification factor $\lambda_D$ can be derived in the following Lemma.

**Lemma 1:** When the downlink amplification power of the RIS is $P_D$, the amplification factor $\lambda_D$ can be obtained as

$$\lambda_D = \sqrt{\rho P_D / \left(\tau P_D + \rho N \sigma^2_v\right)},$$ \hfill (33)

where $\tau$ and $\rho$ are respectively given by (38), shown at the bottom of the page, and (40).

**Proof:** From (29) and (30), we can obtain that

$$P_D = E\{|g_D A_D \Phi_D H_D w_D|^2 + |g_D A_D \Phi_D v_D|^2\} = \tau \mu_D^2 + \lambda_D^2 N \sigma^2_v,$$ \hfill (34)

where $\tau$ is defined as

$$\tau = E\{|\Phi_D H_D H_D^H|\} = E\{|g_D \Phi_D|^2 E\{|W_D H_D^H|\}||H_D^H||\},$$ \hfill (35)

where $W_D = H_D H_D^H$ is a non-central Wishart matrix following $W(M, A_D, \Sigma_D)$ with $A_D = \sqrt{\frac{\mu_D^2}{\lambda_D}} H_D^H$ and $\Sigma_D = \frac{\mu_D^2}{\lambda_D} I_N$. The distribution of $W_D$ can be approximated by $W(M, 0, \Sigma_D)$, where $\Sigma_D$ is given by

$$\Sigma_D = \Sigma_D + \frac{1}{M} A_D H_D A_D = \frac{\alpha_D}{K_3+1} \left(I_N + \frac{K_3}{M} H_D H^H_D\right).$$ \hfill (36)

The term $E\{W_D W_D\}$ in (35) can be obtained as [13]

$$E\{W_D W_D\} = M^2 \Sigma_D^2 + M \text{tr}(\Sigma_D) \Sigma_D$$

$$= M^2 \left(\frac{\alpha_D}{K_3+1}\right)^2 \left(I_N + \frac{K_1}{M} H_D H^H_D\right) + \frac{K_2}{M^2} \left(I_N + \frac{K_1}{M} H_D H^H_D\right)$$

$$+ M \alpha_D N \frac{\alpha_D}{K_3+1} \left(I_N + \frac{K_3}{M} H_D H^H_D\right).$$ \hfill (37)

With the aid of (37), we can derive $\tau$ as (38).

Considering the same amplification factor for every element, we can derive the beamforming coefficient $\mu_D$ in (32) as

$$\mu_D = \frac{P_D}{\rho^2 D} \frac{\alpha_D \kappa_D}{(K_3+1)^2}$$ \hfill (39)

with

$$\rho = \frac{\alpha_D \kappa_D}{(K_3+1)^2} f_D,$$ \hfill (40)

where $\kappa_D = \frac{\alpha_D \kappa_D}{(K_3+1)^2} f_D$ is given by

$$f_D = \tilde{g}_D \Phi_D a_N (\psi_D^e, \psi_D^e)^T \sum_{n=1}^{N} e^{2\pi i z (x+y+z)} \exp(i \theta_D^e) \,,$$ \hfill (41)

where $\ell = \sin \psi_D^e \sin \psi_D^e - \sin \psi_D^e \sin \psi_D^e$ and $z = \cos \psi_D^e - \cos \psi_D^e$.

Furthermore, substituting (38) and (39) into (34), the final result can be obtained in (33), and the proof is completed. \hfill $\blacksquare$

From (29) and (30), the downlink AR is given by

$$R_D = - \log_2 \left(1 + \gamma_D\right),$$ \hfill (42)

with

$$\gamma_D = \frac{\mu_D \lambda_D^2 ||g_D \Phi_D H_D||^2}{\lambda_D^2 ||g_D \Phi_D||^2 ||H_D||^2 + \sigma^2_v + \sigma^2_N}.$$ \hfill (43)

**B. Downlink Achievable Rate**

**Theorem 2:** The downlink AR can be approximated as

$$R_D \approx \log_2 \left(1 + \frac{\zeta P_D}{\rho P_D \beta D N \sigma^2_v \tau^2 + \rho \sigma^2_v \tau^2 + \rho \sigma^2_N} \right)$$ \hfill (44)

where $\zeta$ is given by (46) shown at the bottom of the page.

**Proof:** Similar to the uplink system, $R_D$ in (42) can be approximated as

$$R_D \approx \log_2 \left(1 + \frac{\zeta P_D}{\rho P_D \beta D N \sigma^2_v \tau^2 + \rho \sigma^2_v \tau^2 + \rho \sigma^2_N} \right).$$ \hfill (45)

$$= \frac{M^2 \alpha_D \kappa_D}{K_3+1} (N K_3 N + 2 K_3 K_4 |f_D|^2 + 2 K_3 N + K_3 K_4 |f_D|^2 + K_3^2 N^2) + M N \alpha_D \kappa_D (N K_4 N + K_3 K_4 |f_D|^2 + K_3 N)$$ \hfill (38)
By utilizing the similar methods in Section III, the term $\mathbb{E}\{\|g_D \Phi_D H_D\|^2\}$ in (45) can be derived as $\zeta$ in (46), shown at the bottom of the page. By substituting (46) and $\mathbb{E}\{\|g_D \Phi_D\|^2\} = \beta_D N$ into (45), we obtain

$$R_D \approx \log_2 \left(1 + \frac{\mu_D \lambda_D^2 \zeta}{\beta_D N \sigma_{V,D}^2 + \sigma_{N,D}^2} \right). \quad (47)$$

In addition, substituting (33) and (39) into (47), we derive the final result in (44), and the proof is completed.

Remark 2: We denote $\gamma^D_{US} = \frac{P_{L,D}}{\sigma^2_{N,D}}$, and $\gamma^D_{RIS} = \frac{P_{L,D}}{\sigma^2_{V,D}}$. When $\gamma^D_{US} \to \infty$, the AR in (44) converges to

$$R_D \to \log_2 \left(1 + \frac{\zeta \gamma^D_{RIS}}{\rho \beta_D N} \right). \quad (48)$$

When $\gamma^D_{RIS} \to \infty$, we have

$$R_D \to \log_2 \left(1 + \frac{\zeta \gamma^D_{US}}{\tau} \right). \quad (49)$$

The equations above show similar results as it in the uplink system. Moreover, we focus on the power consumption of the considered system. The downlink overall transmission power is denoted by

$$P_D = P_{L,D} + P_{r,D} + N \left(P_{SW} + P_{DC}\right). \quad (50)$$

For comparison, the following corollary provides the achievable rate and the power consumption of the passive RIS system. By setting $\lambda_D = 1$ and $\sigma^2_{V,D} = 0$ in (34) and (44), we can obtain the achievable rate of the downlink passive RIS system in the following corollary.

Corollary 2: When the passive RIS system has the same overall transmission power $P_D$ as the active RIS system, the AR of the downlink passive RIS system is given by

$$R_D^{P} \approx \log_2 \left(1 + \frac{\zeta P_{L,D}}{\rho \sigma^2_{N,D}} \right). \quad (51)$$

where $P_{L,D}$ is the transmission power at the BS in the passive RIS system, and defined as

$$P_D = P_{L,D}^{P} + N P_{SW}. \quad (52)$$

The optimal phase shift for the downlink system can be obtained by using the GA method, which is similar to the uplink system in Section III. For simplicity, the details are omitted.

V. SIMULATION RESULTS

We analyze the performance of the proposed communication systems, by setting $\sigma^2_{V,U} = \sigma^2_{V,D} = -70$ dBm, $\sigma^2_{N,U} = \sigma^2_{N,D} = -80$ dBm, $M = 128$, $N = 256$, $K_1 = 10$, and $K_2 = 1$, and the large-scale fading coefficients $\alpha$ and $\beta$ are set according to [15]. For convenience, we set $P_{L,U} = P_{L,D}$ and $P_{r,D} = P_{r,D}$ in this paper.

In Fig. 1, we depict the uplink AR versus the transmission power. The uplink rates are respectively obtained from the approximate analytical expression results in (8) and Monte-Carlo simulation results in (6). The former matches the latter well, which demonstrates the accuracy of our derivations. In addition, the active RIS system outperforms the passive RIS system. Furthermore, the rate with the optimal phase shifts obtained by the GA-based method performs better than that with random phase shifts, which indicates the importance of optimizing the phase shifts.

Fig. 2 illustrates the downlink AR versus the transmission power. The uplink rates are respectively obtained from the approximate expression results in (44) and Monte-Carlo simulation results in (42). It is noticed that when the total power increases from 14 dBm to 20 dBm, the passive RIS system outperforms the active RIS one. Compared with the passive RIS, the active RIS consumes additional power consumption $P_{DC}$, and thus its starting power threshold is higher. However, when
the power supply is enough to support the operation of the active RIS, the performance of the active RIS system quickly surpasses that of the passive RIS system.

VI. CONCLUSION

This paper investigated an active RIS-aided two-timescale two-way transmission communication. We derived a tight closed-form approximation for the AR, and the simulation results demonstrated the correctness. Besides, we adopted the GA method to solve the PSOP of the active RIS. For comparison, we also simulated the AR achieved by the passive RIS system with both the optimized phase shifts and the random phase shifts. Simulation results showed that the active RIS system outperforms the passive RIS system when the minimum power requirement of the active RIS is met.

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