A Formal Sociologic Study of Free Will

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Abstract
We make a formal sociologic study of the concept of free will. By using the language of mathematics and logic, we define what we call everlasting societies. Everlasting societies never age: persons never age, and the goods of the society are indestructible. The infinite history of an everlasting society unfolds by following deterministic and probabilistic laws that do their best to satisfy the free will of all the persons of the society.

We define three possible kinds of histories for everlasting societies: primitive histories, good histories, and golden histories.

In primitive histories, persons are inherently selfish, and they use their free will to obtain the personal ownerships of all the goods of the society.

In good histories, persons are inherently good, and they use their free will to distribute the goods of the society. In good histories, a person is not only able to desire the personal ownership of goods, but is also able to desire that a good be owned by another person.

In golden histories, free will is bound by the ethic of reciprocity, which states that “you should wish upon others as you would like others to wish upon yourself”. In golden societies, the ethic of reciprocity becomes a law that partially binds free will, and that must be abided at all times. In other words, the verb “should” becomes the verb “must”.

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I have free will, but not of my own choice. I have never freely chosen to have free will. I have to have free will, whether I like it or not!

Raymond Smullyan

1 Introduction

“Will” is an English noun that is a synonym of “desire”. “Free” is an English adjective that means “not subject to restrictions”. Thus, the phrase “free will” means “desire not subject to restrictions”. In this paper, we study the concept of free will by defining a mathematical model of a society in which persons have the ability of free will.

For us, a society comprises a set of persons, as well as a set of goods. At each instant of time, a given good of the society must be owned by exactly one person. Thus, for each instant of time, the society has a social assignment that defines, for each good, the owner of that good for the given instant of time.

The kind of society that we study is everlasting, in the sense that persons never age, and the goods of the society are indestructible. Therefore, by modelling time as the set of natural numbers, the infinite history of an everlasting society can be seen as an infinite sequence of social assignments.

We make the assumption that the persons of the society have free will. Persons have the ability to express desires over the ownership of the goods of the society. Since these desires are free, it is impossible to avoid non-conflicting desires, and therefore there is an eternal battle of wills for the ownership of the goods of the society. At each instant of time, and for each given good, there is a battle for the ownership of the given good. The outcome of this battle is decided by probabilistic and deterministic laws that take into account the free will of the persons of the society. Thus, the free will of the persons of the society can be seen as a physical force that acts on the goods of the society.

In our mathematical model, the free will of a person is not entirely free, but is bound by the power of the person. The power of a person is a measure of the ability of the person to generate desires, and changes over time according to the laws of our mathematical model.

We define three possible kinds of histories for everlasting societies: primitive histories, good histories, and golden histories.

In primitive histories, persons are inherently selfish, and they use their free will to obtain the ownership of all the goods of the society. When a persons wins
a battle over a given good, it is entitled to own the good for one unit of time, but it does so at the expense of losing some of her power. When a person loses a battle over a given good, it will not own the good, but gains some power. In other words, there is a price to pay for winning a battle, and there is something to gain when losing a battle.

In good histories, persons are inherently good, and they use their free will to distribute the good of the society. In good histories, a person is not only able to desire the personal ownership of goods, but is also able to desire that a good be owned by another person. In good histories, a person wins a battle over a good when it is able to determine the owner of the good, but if a person wins, it does so at the expense of losing some of her power. Symmetrically, a person loses a battle over a good when it is not able to determine the owner of the good, but when a person loses, it gains some power.

In golden histories, like in the case of good histories, persons are inherently good, and they are able to desire both the personal ownership of goods, as well as the fact that a given good be owned by another person. But in golden histories free will is partially bound by a law of reciprocity. This law of reciprocity is inspired by the golden rule, also known as the ethic of reciprocity, which states that \textit{you should wish upon others as you would like others to wish upon yourself}. In our mathematical model, the ethic of reciprocity becomes a law that partially binds free will, and must be abided at all times. This law can be expressed by the two following statements:

1. \textit{You must wish in the future upon others, as others in the past have wished upon you;}
2. \textit{Others must wish in the future upon you, as you in the past have wished upon others.}

Our law of reciprocity can be expressed formally with mathematical language, and can be proved valid in our mathematical model of everlasting societies.

We do not know if our law of reciprocity is valid in the real world. We do not even know if free will exists in the real world. We however believe that, in the real world, there are universal laws that govern the concept of desire. Our mathematical model can be seen as a desire: it is the desire that free will exists, and that free will is partially bound by reciprocity.

1.1 Related work

Free will \cite{5} has been studied by humanity since at least the beginning of civilization. Among the many possible views and perspectives in the free will debate, the view of this paper can be classified as a two-stage model \cite{4}. In the first stage, alternative possibilities are generated. In the second stage, exactly one of the possibilities is chosen by an intelligent person capable of free will.

The internal structure of everlasting societies can be seen as a social network. The theory of social networks is part of the emerging field of computational sociology \cite{1, 3}, a field that merges classical sociology \cite{2} with computer science \cite{6}. 
1.2 Organization of the paper

The rest of this paper is organized as follows. Section 2 defines everlasting societies. Section 3 defines primitive histories. Section 4 defines good histories. Section 5 defines golden histories, and proves the theorems that express mathematically the validity of the law of reciprocity for golden histories. Section 6 concludes the paper.

2 Everlasting societies

Axiom 1. There are persons. Denote with $P$ the class of all persons.

Definition 1. A relationship function over a set $P$ of persons is a function

$$\rho : P \times P \to [0, 1],$$

satisfying the following conditions:

- $\rho(x, x) = 1$, for all $x \in P$;
- if $x \neq y$ then $0 < \rho(x, y) < 1$, for all $x, y \in P$;
- $\rho(x, y) = \rho(y, x)$, for all $x, y \in P$;
- $\rho(x, z) + \rho(z, y) \leq 1 + \rho(x, y)$, for all $x, y, z \in P$.

Axiom 2. There are goods. Denote with $G$ the class of all goods.

Definition 2. A social estate $E$ is a finite nonempty set of goods.

Definition 3. An everlasting society $S$ is a tuple

$$S = (P, \rho, E),$$

where

- $P \subseteq P$ is a finite nonempty set of persons;
- $\rho : P \times P \to [0, 1]$ is a relationship function over $P$;
- $E$ is a social estate.

Definition 4. Let $S = (P, \rho, E)$ be an everlasting society. A social assignment $\alpha$ for $S$ is any function $\alpha : E \to P$.

Definition 5. Let $S = (P, \rho, E)$ be an everlasting society. A simple history for $S$ is any infinite sequence

$$\alpha_0, \alpha_1, \ldots, \alpha_n, \ldots,$$

of social assignments for $S$. 

4
3 Primitive histories

Definition 6. Let $S = (P, \rho, E)$ be an everlasting society. A primitive power function for $S$ is any function $\pi : P \to \mathbb{R}^+$. ■

Definition 7. Let $S = (P, \rho, E)$ be an everlasting society. A primitive force function for $S$ is any function $\varphi : P \times E \to \mathbb{R}^+$. ■

Definition 8. Let $S = (P, \rho, E)$ be an everlasting society. A primitive social state $\sigma$ is a tuple

$$\sigma = (\alpha, \pi, \varphi),$$

where

- $\alpha : E \to P$ is a social assignment for $S$;
- $\pi : P \to \mathbb{R}^+$ is a primitive power function for $S$;
- $\varphi : P \times E \to \mathbb{R}^+$ is a primitive force function for $S$ such that

$$\sum_{a \in E} \varphi(x,a) < \pi(x), \quad \text{for all } x \in P.$$

Definition 9. Let $S = (P, \rho, E)$ be an everlasting society, and let $\sigma = (\alpha, \pi, \varphi)$ be a primitive social state. The effectiveness function $\psi$ with respect to $S$ and $\sigma$ is the function

$$\psi : P \times E \to \mathbb{R}^+,$$

defined by

$$\psi(x,a) = \varphi(x,a)\rho(x,\alpha(a)).$$

Definition 10. Let $S = (P, \rho, E)$ be an everlasting society. For the sake of simplicity, we assume in this definition that the estate of the society contains only one good $a$, that is $E = \{a\}$.

Let $\sigma = (\alpha, \pi, \varphi)$ be a primitive social state. Another primitive social state $\sigma' = (\alpha', \pi', \varphi')$ is a successor of $\sigma$, if it can be obtained according to the following laws that regulate how the members of the society battle for the ownership of good $a$.

The winner of this battle is established using a probabilistic law that takes into account the force function $\varphi$, and in particular the effectiveness function $\psi$ with respect to $S$ and $\sigma$.

For any person $w \in P$, we let

$$\text{Prob}[w \text{ wins the ownership of good } a] = \frac{\psi(w,a)}{\sum_{y \in P} \psi(y,a)}.$$ 

Next, assume that the battle has been won by person $w$. Then the successor state $\sigma' = (\alpha', \pi', \varphi')$ is uniquely determined in $\alpha'$ and $\pi'$ (but not in $\varphi'$), according to the following deterministic laws:
• $\alpha'(a) = w$;
• $\pi'(w) = \pi(w) - \varphi(w, a)$;
• if $x \neq w$ then
  \[
  \pi'(x) = \pi(x) + \varphi(w, a) \times \frac{\psi(x, a)}{\sum_{y \in P, y \neq w} \psi(y, a)}.\]

**Definition 11.** Let $S = (P, \rho, E)$ be an everlasting society. In this definition, there is no simplicity, and we consider the general case in which $|E| \geq 1$.

Let $\sigma = (\alpha, \pi, \varphi)$ be a primitive social state. Another primitive social state $\sigma' = (\alpha', \pi', \varphi')$ is a successor of $\sigma$, if it can be obtained according to the following laws that regulate how the members of the society battle for the ownership of the goods belonging to the social estate $E$.

There are as many battles as there are goods in $E$. The winners of these battles are established using a probabilistic law that takes into account the force function $\varphi$, and in particular the effectiveness function $\psi$ with respect to $S$ and $\sigma$.

For any person $w \in P$, and for any good $a \in E$, we let

\[
\text{Prob}[w \text{ wins the ownership of good } a] = \frac{\psi(w, a)}{\sum_{y \in P} \psi(y, a)}.
\]

Once all battles have been settled using the probabilistic law, the successor state $\sigma' = (\alpha', \pi', \varphi')$ is uniquely determined in $\alpha'$ and $\pi'$ (but not in $\varphi'$) according to the following deterministic laws.

First, we let $\alpha'(a)$ be equal to the winner of the battle over good $a$.

Next, let $x \in P$. Denote with $\text{gains}(x)$ the set of goods that the person $x$ has gained. More precisely, $\text{gains}(x) = \{a \in E \mid \alpha'(a) = x\}$. Denote also with $\text{losses}(x)$ the set of goods that the person $x$ has lost. More precisely, $\text{losses}(x) = \{a \in E \mid \alpha'(a) \neq x\} = E \setminus \text{gains}(x)$.

Finally, for any person $x \in P$, we let

\[
\pi'(x) = \pi(x) - \sum_{a \in \text{gains}(x)} \varphi(x, a) + \sum_{a \in \text{losses}(x)} \varphi(\alpha'(a), a) \times \frac{\psi(x, a)}{\sum_{y \in P, y \neq \alpha'(a)} \psi(y, a)}.\]

**Definition 12.** Let $S = (P, \rho, E)$ be an everlasting society. A **primitive history** for $S$ is an infinite sequence of primitive social states

$\sigma_0, \sigma_1, \ldots, \sigma_n, \ldots,$

where $\sigma_{t+1}$ is a successor of $\sigma_t$, for all $t \in \mathbb{N}$. 

\[\]
4 Good histories

Definition 13. Let $S = (P, \rho, E)$ be an everlasting society. A good power function for $S$ is any function $\pi : P \times E \times P \rightarrow \mathbb{R}^+$. ■

Definition 14. Let $S = (P, \rho, E)$ be an everlasting society. A good force function for $S$ is any function $\varphi : P \times E \times P \rightarrow \mathbb{R}^+$. ■

Definition 15. Let $S = (P, \rho, E)$ be an everlasting society. A good social state $\sigma$ is a tuple

$$\sigma = (\alpha, \pi, \varphi),$$

where

- $\alpha : E \rightarrow P$ is a social assignment for $S$;
- $\pi : P \times E \times P \rightarrow \mathbb{R}$ is a good power function for $S$;
- $\varphi : P \times E \times P \rightarrow \mathbb{R}$ is a good force function for $S$ such that

$$\varphi(x, a, y) < \pi(x, a, y), \quad \text{for all } x, y \in P \text{ and } a \in E. \quad ■$$

Definition 16. Let $S = (P, \rho, E)$ be an everlasting society, and let $\sigma = (\alpha, \pi, \varphi)$ be a good social state. The effectiveness function $\psi$ with respect to $S$ and $\sigma$ is the function

$$\psi : P \times E \times P \rightarrow \mathbb{R}^+,$$

defined by

$$\psi(x, a, y) = \varphi(x, a, y)\rho(x, \alpha(a))\rho(\alpha(a), y). \quad ■$$

Definition 17. Let $S = (P, \rho, E)$ be an everlasting society. In this definition, we consider the general case in which $|E| \geq 1$.

Let $\sigma = (\alpha, \pi, \varphi)$ be a good social state. Another good social state $\sigma' = (\alpha', \pi', \varphi')$ is a successor of $\sigma$, if it can be obtained according to the following laws that regulate how the members of the society battle over the ownership of the goods belonging to the social estate $E$.

There are as many battles as there are goods in $E$. The winners of these battles are established using a probabilistic law that takes into account the force function $\varphi$, and in particular the effectiveness function $\psi$ with respect to $S$ and $\sigma$.

For any person $w \in P$, and for any good $a \in E$, we let

$$\text{Prob}[w \text{ wins the ownership of good } a] = \frac{\sum_{y \in P} \psi(y, a, w)}{\sum_{y, z \in P} \psi(y, a, z)}.$$
Once all battles have been settled using the probabilistic law, the successor state \( \sigma = (\alpha', \pi', \varphi') \) is uniquely determined in \( \alpha' \) and \( \pi' \) (but not in \( \varphi' \)), according to the following deterministic laws.

First, we let \( \alpha'(a) \) be equal to the winner of the battle over good \( a \). Then, for any persons \( x, y \in P \), and for any good \( a \in E \), we let
\[
\pi'(x, a, y) = \pi(x) - \varphi(x, a, y) + \varphi(y, a, x).
\]

**Definition 18.** Let \( S = (P, \rho, E) \) be an everlasting society. A **good history** for \( S \) is an infinite sequence of good social states
\[
\sigma_0, \sigma_1, \sigma_2, \ldots, \sigma_n, \ldots,
\]
where \( \sigma_{t+1} \) is a successor of \( \sigma_t \), for all \( t \in \mathbb{N} \).

## 5 Golden histories

**Axiom 3.** There are force carriers. Denote with \( C \) the class of all force carriers.

**Axiom 4.** Every force carrier \( c \) has an intensity \( \mu(c) \in \mathbb{R}^+ \).

**Axiom 5.** Every force carrier \( c \) has a maximum idle period \( \theta(c) \in \mathbb{N}^+ \).

**Axiom 6.** Let \( C \subseteq C \) be a finite nonempty set of force carriers. An idle function for \( C \) is any function \( \tau : C \to \mathbb{N} \).

**Definition 19.** Let \( S = (P, \rho, E) \) be an everlasting society. A golden power function \( \pi \) for \( S \) is a function
\[
\pi : P \times E \times P \to 2^C,
\]
such that

- \( C \subseteq C \) is a finite nonempty set of force carriers;
- \[
\bigcup_{x,y \in P_a \in E} \pi(x, a, y) = C;
\]
- if \( (x_1, a_1, y_1) \neq \( (x_2, a_2, y_2) \) then \( \pi(x_1, a_1, y_1) \cap \pi(x_2, a_2, y_2) = \emptyset \), for all \( x_1, x_2, y_1, y_2 \in P \) and \( a_1, a_2 \in E \).

**Definition 20.** Let \( S = (P, \rho, E) \) be an everlasting society, and let \( \pi : P \times E \times P \to 2^C \) be a golden power function for \( S \). The extended golden power function \( \pi^* \) with respect to \( \pi \) is the function
\[
\pi^* : 2^P \times E \times 2^P \to 2^C,
\]
defined by
\[ \pi^*(X,a,Y) = \bigcup_{x \in X} \pi(x,a,y), \quad \text{for all } X,Y \subseteq P \text{ and } a \in E. \]

**Definition 21.** Let \( S = (P, \rho, E) \) be an everlasting society. A **golden force function** \( \varphi \) for \( S \) is a function
\[ \varphi : P \times E \times P \to 2^C, \]
where \( C \subseteq C \) is a finite nonempty set of force carriers.

**Definition 22.** Let \( S = (P, \rho, E) \) be an everlasting society, and let \( \varphi : P \times E \times P \to 2^C \) be a golden force function for \( S \). The **extended golden force function** \( \varphi^* \) with respect to \( \varphi \) is the function
\[ \varphi^* : 2^P \times E \times 2^P \to 2^C, \]
defined by
\[ \varphi^*(X,a,Y) = \bigcup_{x \in X} \varphi(x,a,y), \quad \text{for all } X,Y \subseteq P \text{ and } a \in E. \]

**Definition 23.** Let \( S = (P, \rho, E) \) be an everlasting society. A **golden social state** \( \sigma \) is a tuple
\[ \sigma = (\alpha, \pi, \tau, \varphi) \]
where
- \( \alpha : E \to P \) is a social assignment for \( S \);
- \( \pi : P \times E \times P \to 2^C \) is a golden power function for \( S \), where \( C \) is a finite nonempty set of force carriers;
- \( \tau : C \to \mathbb{N} \) is an idle function for \( C \);
- \( \tau(c) \leq \theta(c) \), for all \( c \in C \);
- \( \varphi : P \times E \times P \to 2^C \) is a golden force function such that
\[ \varphi(x,a,y) \subseteq \pi(x,a,y), \quad \text{for all } x,y \in P \text{ and } a \in E; \]
- for any force carrier \( c \in C \), if \( \tau(c) = \theta(c) \) then \( c \in \varphi(x,a,y) \), where \( (x,a,y) \in P \times E \times P \) is the unique tuple such that \( c \in \pi(x,a,y) \).
Definition 24. Let $S = (P, \rho, E)$ be an everlasting society, and let $\sigma = (\alpha, \pi, \tau, \varphi)$ be a golden social state. The **effectiveness function** $\psi$ with respect to $S$ and $\sigma$ is the function

$$
\psi : P \times E \times P \to \mathbb{R}^+,
$$

defined by

$$
\psi(x, a, y) = \sum_{c \in \varphi(x, a, y)} \mu(c) \rho(x, \alpha(a)) \rho(\alpha(a), y).
$$

Definition 25. Let $S = (P, \rho, E)$ be an everlasting society, let $\sigma = (\alpha, \pi, \tau, \varphi)$ be a golden social state, and let $\psi : P \times E \times P \to \mathbb{R}^+$ be the effectiveness function with respect to $S$ and $\sigma$. The **extended effectiveness function** $\psi^*$ with respect to $\psi$ is the function

$$
\psi^* : 2^P \times E \times 2^P \to \mathbb{R}^+
$$

defined by

$$
\psi^*(X, a, Y) = \sum_{x \in X \setminus Y} \psi(x, a, y), \quad \text{for all } X, Y \subseteq P \text{ and } a \in E.
$$

Definition 26. Let $S = (P, \rho, E)$ be an everlasting society. In this definition, we consider the general case in which $|E| \geq 1$.

Let $\sigma = (\alpha, \pi, \tau, \varphi)$ be a golden social state, and let $\sigma' = (\alpha', \pi', \tau', \varphi')$ be another golden social state. Assume that $\tau$ and $\tau'$ have the same domain $C$. We say that $\sigma'$ is a **successor** of $\sigma$, if it can be obtained according to the following laws that regulate how the members of the society battle over the ownership of the goods belonging to the social estate $E$.

There are as many battles as there are goods in $E$. The winners of these battles are established using a probabilistic law that takes into account the force function $\varphi$, and in particular the effectiveness function $\psi$ with respect to $S$ and $\sigma$.

For any person $w \in P$, and for any good $a \in E$, we let

$$
\operatorname{Prob}[w \text{ wins the ownership of good } a] = \begin{cases} 
\frac{\psi^*(P, a, \{w\})}{\psi^*(P, \alpha(a), P)}, & \text{if } \psi^*(P, a, P) \neq 0, \\
1, & \text{if } \psi^*(P, a, P) = 0 \text{ and } w = \alpha(a), \\
0, & \text{if } \psi^*(P, a, P) = 0 \text{ and } w \neq \alpha(a).
\end{cases}
$$

Once all battles have been settled using the probabilistic law, the successor state $\sigma' = (\alpha', \pi', \tau', \varphi')$ is uniquely determined in $\alpha'$, $\pi'$, and $\tau'$ (but not in $\varphi'$) according to the following deterministic laws.

First, we let $\alpha'(a)$ be equal to the winner of the battle over good $a$.

Then, for any persons $x, y \in P$, and for any good $a \in E$, we let

$$
\pi'(x, a, y) = (\pi(x, a, y) \setminus \varphi(x, a, y)) \cup \varphi(y, a, x).
$$
Finally, for any force carrier \( c \in C \), let \((x, a, y) \in P \times E \times P\) be the unique tuple such that \( c \in \pi(x, a, y) \). We let
\[
\tau'(c) = \begin{cases} 
0, & \text{if } c \in \varphi(x, a, y), \\
\tau(c) + 1, & \text{if } c \notin \varphi(x, a, y). 
\end{cases}
\]

**Definition 27.** Let \( S = (P, \rho, E) \) be an everlasting society. A **golden history** for \( S \) is an infinite sequence of golden social states
\[\sigma_0, \sigma_1, \ldots, \sigma_n, \ldots,\]
where \( \sigma_{t+1} \) is a successor of \( \sigma_t \), for all \( t \in \mathbb{N} \).

**Theorem 1.** Let \( S = (P, \rho, E) \) be an everlasting society, and let \( \{\sigma_t\}_{t \in \mathbb{N}} \) be a golden history for \( S \). Let \( \sigma_t = (\alpha_{t_1}, \tau_{t_1}, \varphi_{t_1}) \), and assume \( c \in \varphi_{t_1}(x, a, y) \), where \( x, y \in P \) and \( a \in E \).

Then there exists \( t_2 \in \mathbb{N} \) such that \( t_2 > t_1 \) and \( c \in \varphi_{t_2}(y, a, x) \).

**Proof.** Since \( c \in \varphi_{t_1}(x, a, y) \), by Definition 26 we have \( c \in \pi_{t_1+1}(y, a, x) \) and \( \tau_{t_1+1}(c) = 0 \). By contradiction, assume that \( c \notin \varphi_t(y, a, x) \), for all \( t \in \mathbb{N} \) such that \( t > t_1 \). But then, by Definition 26 there exists \( t_2 \in \mathbb{N} \) such that \( t_2 > t_1 \), \( c \in \pi_{t_2}(y, a, x) \), and \( \tau_{t_2}(c) = \theta(c) \). But then, by Definition 26 we have \( c \in \varphi_{t_2}(y, a, x) \), contradiction.

**Theorem 2.** Let \( S = (P, \rho, E) \) be an everlasting society, and let \( \{\sigma_t\}_{t \in \mathbb{N}} \) be a golden history for \( S \). Let \( \sigma_t = (\alpha_{t_1}, \pi_{t_1}, \tau_{t_1}, \varphi_{t_1}) \), and assume \( c \in \varphi_{t_1}^*(X, a, Y) \), where \( X, Y \subseteq P \) and \( a \in E \).

Then there exists \( t_2 \in \mathbb{N} \) such that \( t_2 > t_1 \) and \( c \in \varphi_{t_2}^*(Y, a, X) \).

**Proof.** Since \( c \in \varphi_{t_1}^*(X, a, Y) \), there exist \( x, y \in P \) such that \( c \in \varphi_{t_1}(x, a, y) \). By Theorem 1 there exists \( t_2 \in \mathbb{N} \) such that \( t_2 > t_1 \) and \( c \in \varphi_{t_2}(y, a, x) \). But then, \( c \in \varphi_{t_2}^*(Y, a, X) \).

## 6 Conclusion

We have defined a mathematical model of everlasting societies in which persons never age and goods are indestructible. The history of an everlasting society is governed by probabilistic and deterministic laws that take into account the free will of the persons of the society.

We have defined three possible kinds of histories for an everlasting society: primitive histories, good histories, and golden histories. The most interesting case is that of golden histories. In golden histories, free will is not entirely free, but is bound by a law of reciprocity that is inspired by the golden rule, or ethic of reciprocity.

Our law of reciprocity can be expressed by the two following statements:
1. *You must wish in the future upon others, as others in the past have wished upon you;*

2. *Others must wish in the future upon you, as you in the past have wished upon others.*

In our mathematical model, statements 1 and 2 are expressed by Theorem 2. Loosely speaking, the theorem states a relationship between an instant of time $t_1$ in the past and an instant of time $t_2$ in the future. The theorem assumes that in the past $c \in \varphi_{t_1}(X,a,Y)$, for some force carrier $c$, some groups of persons $X,Y \subseteq P$, and some good $a$. Given this assumption, the theorem states that in the future we must have $c \in \varphi_{t_2}(Y,a,X)$. Intuitively, $\varphi_{t_1}(X,a,Y)$ is what in the past $X$ has desired on $Y$, whereas $\varphi_{t_2}(Y,a,X)$ is what in the future $Y$ must desire on $X$. It follows that Theorem 2 paraphrases statement 1 if we let $X = P \setminus \{you\}$ and $Y = \{you\}$. Similarly, Theorem 2 paraphrases statement 2 if we let $X = \{you\}$ and $Y = P \setminus \{you\}$.

The laws of our mathematical model are good, in the sense that they attempt to satisfy all desires of the persons of the society. Moreover, in the case of golden histories, our laws are reciprocal. However, our laws are not fair.

In our model, persons generate their desire by defining a force function at each instant of time. This force function is contrained by the power function, and different persons have different power. Moreover, the effectiveness of the force function depends on the structure of the society, namely on the strength of the relationships between persons. Thus, the ability of a person to have their desires satisfied, depends also on the social position that the person has in the social network representing the society.

A direction of further research is that to define a mathematical model that extends the mathematical model of this paper, and that is able to define laws that are fair. Here, by fairness we mean that persons should start on equal footing, and that two distinct persons must have the same chance to have their desires satisfied, provided only that they behave intelligently.

This paper, as stated in the introduction, is a desire. It is the desire that the real world be governed by laws that allow the existence of free will, and that are fundamentally good, reciprocal, and fair. We do not know if the real world is good, reciprocal, and fair. We just desire that it be so.

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