i-RIM applied to the fastMRI challenge

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Abstract

We, team AImsterdam, summarize our submission to the fastMRI challenge [Zbon-  
tar et al., 2018]. Our approach builds on recent advances in invertible “learning to  
infer” models as presented in Putzky and Welling [2019]. Both, our single-coil and  
our multi-coil model share the same basic architecture.

1 Introduction

To solve the accelerated MRI problem as presented in the fastMRI challenge [Zbontar et  
al., 2018], we train an invertible Recurrent Inference Machine (i-RIM) for each of the challenges [Putzky and  
Welling, 2019]. The i-RIM is an invertible variant of the RIM [Putzky and Welling, 2017] which  
has been successfully applied to accelerated MRI before [Lønning et al., 2019]. The formulation  
of the i-RIM allows us to stably train models which are several hundreds of layers deep. Most of  
our approach is already described in Putzky and Welling [2019]. Here, we will focus on changes to  
Putzky and Welling [2019] which were done for the challenge, and on the adaptation to the multi-coil  
setting.

We treat the problem of accelerated MRI as an inverse problem with a forward model given by
\[
d^{(i)} = P \mathcal{F} p^{(i)} + n^{(i)}
\]
(1)
where \(d^{(i)} \in \mathbb{C}^m\) are sub-sampled k-space measurements at coil \(i\), \(P\) is a sampling mask, \(\mathcal{F}\) is a  
Fourier transform, \(p^{(i)} \in \mathbb{C}^n\) is an image recorded at coil \(i\), and \(n^{(i)}\) is the noise at coil \(i\). In our  
approach, we do not explicitly model spatial coil sensitivity maps as is commonly done in other  
approaches. We stack k-space measurement and coil images from all coils, respectively, such that the  
forward model takes the form
\[
d = Ap + n
\]
(2)
with
\[
d = \begin{pmatrix} d^{(1)} \\ \vdots \\ d^{(K)} \end{pmatrix}, \quad p = \begin{pmatrix} p^{(1)} \\ \vdots \\ p^{(K)} \end{pmatrix}, \quad n = \begin{pmatrix} n^{(1)} \\ \vdots \\ n^{(K)} \end{pmatrix}, \quad A = 1_K \otimes P \mathcal{F}
\]
(3)

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where $\otimes$ denotes the Kronecker product, $K$ is the total number of coils in the problem, i.e. 15 in the multi-coil setting, and 1 in the single-coil setting.

## 2 Method

The i-RIM is a deep learning model which iteratively updates its machine state $(p_t, s_t)$ based on simulations of the forward model in Eq. (2) such that

$$p_{t+1}, s_{t+1} = h_\phi(A, d, p_t, s_t)$$

where $(p_t, s_t)$ is the iterative estimate of $p$ and $s_t$ is a latent state at iteration $t$, respectively. Many modern approaches to solving inverse problems which we refer to as “learning to infer” models can be summarized through Eq. (4). What differentiates the i-RIM from other approaches is that (1) the only model assumption is in the forward model which makes the i-RIM a mostly data-driven approach, and (2) $h_\phi$ is fully invertible which allows us to train the model with back-propagation without storing intermediate activations [Gomez et al., 2017]. Hence, we can train arbitrarily deep networks. Empirical results in deep learning suggest that deeper models almost always perform better than their shallow counterparts [He et al., 2015]. The i-RIM brings this potential to “learning to infer” models.

For the i-RIM, Eq. (2) specifically takes the form

$$p_{t+1}, s_{t+1} = h_\phi(\nabla D (d, A, p_t), p_t, s_t)$$

where

$$\nabla D (d, A, p_t) = A^H (A p_t - d)$$

is the gradient of the data consistency term under a normal likelihood model with $A^H$ being the adjoint operator of $A$. This gradient reflects how well the current estimate is supported by the measured data under the forward model. To produce the final estimate of $p$ we use a non-invertible block such that

$$\hat{p} = f_\theta(p_T, s_T)$$

is the model’s final complex-valued estimate with $\hat{p} \in \mathbb{C}^n$. The competition results are evaluated on magnitude images, hence we do $m = |\hat{p}|$ to generate magnitude images for the competition. As training loss we use the structural similarity loss [Zhou Wang et al., 2004]:

$$L(\phi, \theta) = -\frac{1}{N} \sum_{j=1}^{N} \text{SSIM}(\hat{m}_j, m_j)$$

where $N$ is the mini-batch size. As the initial machine state we set

$$p_0 = A^H d$$

$$s_0 = \left( \frac{\omega}{0_{n-8}} \right)$$

where $p_0$ is the zero-filled corrupted image, and $\omega$ is a 1-hot vector which encodes meta-data about the experimental condition such as field strength (1.5T vs. 3T) and fat suppressed vs. non-fat suppressed data. This meta-data can potentially activate different pathways in the i-RIM under the different experimental conditions.

### Models

We trained separate models for the single-coil and multi-coil challenges with 8 steps each. At each step, the models have 12 down-sampling blocks (see Putzky and Welling [2019]). In total, this amounts to 480 layer deep networks. The single-coil model has a machine state of 64 feature layers, and the multi-coil model has a machine state of 96 feature layers.

### Training

Because the volumes in the data set have vastly different sizes, we cropped the central portion of each image slice to 368 $\times$ 368 pixels. For smaller slices we applied zero padding to produce slices of the appropriate size. We simulated k-space measurements using the sampling mask function supplied by Zbontar et al. [2018] with 4 $\times$ and 8 $\times$ acceleration factors. As target images we used ESC images for the single-coil model and RSS targets for the multi-coil model, respectively (see Zbontar et al., 2018). We used the Adam optimizer with initial learning rate $10^{-4}$ which was reduced by factor 10 every 30 epochs.
Figure 1: Example reconstructions. The reconstructions visually improve the ground truth images, suggesting a strong prior.

Table 1: Reconstruction performance on different data sets from the fastMRI challenge Zbontar et al. [2018] under different metrics. NMSE - normalized mean-squared-error; PSNR - peak signal-to-noise ratio; SSIM - structural similarity index Zhou Wang et al. [2004]. ↓ - lower is better; ↑ higher is better.

| Model          | 4x Acceleration | 8x Acceleration |
|----------------|-----------------|-----------------|
| i-RIM single-coil |                 |                 |
| Validation     | NMSE ↓, PSNR ↑, SSIM ↑ | NMSE ↓, PSNR ↑, SSIM ↑ |
|                | 0.0342, 32.43, 0.751 | 0.0446, 30.92, 0.692 |
| Test           |                 |                 |
| Challenge      |                 |                 |
|                | 0.0272, 33.65, 0.781 | 0.0421, 30.56, 0.687 |
| i-RIM multi-coil |                 |                 |
| Validation     | NMSE ↓, PSNR ↑, SSIM ↑ | NMSE ↓, PSNR ↑, SSIM ↑ |
|                | 0.0062, 38.84, 0.919 | 0.0103, 36.19, 0.886 |
| Test           |                 |                 |
| Challenge      |                 |                 |
|                | 0.0052, 39.52, 0.928 | 0.0093, 36.53, 0.887 |

3 Results

We evaluated our models on three data sets: the validation set as in Zbontar et al. [2018], and the test and challenge sets through the fastMRI website. A summary of these evaluations can be found in Table 1. To assess image quality more closely, we show some exemplary reconstructions from each model in figure 1.

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1Results on the challenge data set will be added once publicly available.
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