More on massive gravitino scattering amplitudes and the unitarity cutoff of the new Fayet-Iliopoulos terms

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ABSTRACT: We extend the $2 \rightarrow 2$ gravitino scattering amplitude computed in [1] to an arbitrary $\mathcal{N} = 1$ supergravity model of one chiral and one vector multiplet, in a Minkowski background with supersymmetry breaking driven by both $F$- and $D$-terms. We find that the cancellation of the leading term in $O(\kappa^2 E^4/|m_3/2|^2)$, that would lead to a breakdown of perturbative unitarity at a scale $\Lambda \sim M_{\text{SUSY}}$, is a consequence of the vanishing of the scalar potential at its minimum, which is implied by the flat background. We then analyse the inclusion of the new Fayet-Iliopoulos (FI) terms. We find that, since they modify the scalar potential without contributing to the amplitudes, they generically lead to uncanceled leading terms in the latter and a perturbative cutoff at the supersymmetry breaking scale, except for particular cases where the new FI term does not modify the potential at its minimum and the cutoff is pushed up to the Planck scale.
1 Introduction

In a recent paper [1], we started an investigation of the massive gravitino $2 \rightarrow 2$ scattering amplitudes. We computed them in the Polonyi model [2] and found that, as for the standard Higgs mechanism, there is a cancellation of the leading terms at high energy (in $O(\kappa^2 E^4/|m_{3/2}|^2)$, where $E$ is the energy of the gravitinos and $m_{3/2}$ their mass) between gravitational and scalar channels, allowing the perturbative unitarity cutoff to lie at the Planck scale. We then added an abelian vector multiplet without charging the Polonyi field, and considered the effect of the new Fayet-Iliopoulos (FI) term that was introduced in [3–6], as a way to emulate the standard supergravity FI term [7, 8] without gauged R-symmetry and its issues [9, 10]. We found that in this case, the aforementioned cancellation cannot happen because this new FI term does not contribute to the amplitudes, leading to a cutoff at the supersymmetry (SUSY) breaking scale, associated with the vector $D$-auxiliary component expectation value. This result is in agreement with the analysis of [11, 12] using different arguments.

The present paper has two goals. The first is to extend the amplitude computation previously made in the Polonyi model to an arbitrary model of one vector and one charged chiral multiplet, gauging in general the R-symmetry, so that supersymmetry can also be broken by a $D$-term, generalising the Polonyi model which has only $F$-term SUSY breaking. This is done in section 2. We obtain that, for amplitudes computed around some minimum of the potential in Minkowski gravitational background, the condition for cancellation is precisely the vanishing of the potential at its minimum, which is implied by the flat background. In section 3, we analyse the inclusion of new Fayet-Iliopoulos terms in this setup. We consider two cases that we call the original new FI term and the Kähler invariant new FI term. The original new FI term is the one introduced in [3, 6] with matter couplings that are not Kähler invariant [4], while the Kähler invariant new FI term consists of a modification proposed in [5]. Both terms have been used for cosmological applications, for instance in [4, 11, 13, 14]. Also, to avoid confusion, we call standard FI term the one from standard supersymmetry [8] which implies a gauged R-symmetry. As in [1], we find that since the new FI terms modify the scalar potential without contributing to the amplitudes, they generically lead to uncanceled leading terms $O(\kappa^2 E^4/|m_{3/2}|^2)$ in the latter and a perturbative cutoff at the SUSY breaking scale. In concrete examples, we can find particular values of the parameters for which the contribution of the new FI terms to the potential vanishes at the minimum, restoring the cancellation in the amplitudes and a cutoff at the Planck scale. In general, this phenomenon requires an extra tuning of parameters, besides the one of vacuum energy, unless we consider the new FI term to be field dependent, which leads to a rather trivial result.
2 Gravitino scattering with chiral and vector multiplets

In [1], we computed the massive gravitino scattering amplitudes in the Polonyi model, which includes a chiral multiplet with canonical Kähler potential and a constant+linear superpotential. There are two channels contributing to these amplitudes at tree level, with propagation of the graviton or the scalar comprised in the chiral multiplet. We found that, in a similar way as for the standard Higgs mechanism, there is a cancellation of the leading terms at high energy (in $O(\kappa^2 E^4/|m_{3/2}|^2)$) between these channels, pushing the perturbative unitarity cutoff up to the Planck scale. The computation was done in Minkowski vacuum with supersymmetry broken by a vacuum expectation value (VEV) of the chiral multiplet F-auxiliary component.

The goal of this section is to generalise this result for one vector and one charged chiral multiplet, with arbitrary Kähler potential $K(z, \bar{z})$, superpotential $W(z)$, and gauge kinetic function $f(z)$. We assume that spontaneous supersymmetry breaking takes place and use the unitary gauge, where the mixing between the gravitino and the Goldstino combination of the two spin-1/2 fermions is set to zero. This time, there are three channels contributing to the $2 \to 2$ gravitino scattering at tree level, with propagation of the graviton, the scalar, and the gauge vector, in their respective multiplets. The contribution of the gravitational channel is not modified with respect to [1] in this setup; we will recall the result later. The contributions to compute are the ones from the scalar and vector channels. For this, the relevant terms in the $\mathcal{N} = 1$ supergravity Lagrangian are [15]

$$
e^{-1} \mathcal{L} \supset -\frac{1}{2} \bar{\psi}_\mu \gamma^{\mu\rho\sigma} \left( \partial_\rho - \frac{3}{2} i A_\rho \gamma_* \right) \psi_\sigma - \bar{\partial} \bar{\partial} K \partial^\mu z \partial^\nu \bar{z} - \frac{1}{4} \text{Re}(f) F^\mu\nu F^\mu\nu + \frac{1}{2} (e^{K/2} W \bar{\psi}_\mu P_R \gamma^{\mu\nu} \psi_\nu + \text{h.c.}).$$

(2.1)

Here, we did set the gravitational coupling $\kappa = 1$, and use standard notations for the different fields involved. Partial derivatives without index $\partial (\bar{\partial})$ stand for differentiation with respect to the scalar field $z$ ($\bar{z}$), $A_\mu$ is the gauge potential with field strength $F^\mu\nu$, $\psi_\mu$ is the gravitino, while $\gamma^{\mu\nu}$ ($\gamma^{\mu\rho\sigma}$) denotes the totally antisymmetric product of two (three) Dirac gamma-matrices. Also, $\gamma_* = i \gamma_0 \gamma_1 \gamma_2 \gamma_3$ is used to define the chiral projector $P_R = 1/2(1 - \gamma_*)$. The covariant derivative of the scalar is given by $\partial_\mu z = \partial_\mu z - A_\mu k$ where $k(z)$ is the Killing vector involved in the transformation of the scalar $z$ (with only one vector multiplet, the symmetry is $U(1)$ and $k(z) = -iqz$, with $q$ the charge of the scalar). Finally, $A_\mu$ is the Kähler connection given by

$$A_\mu = \frac{i}{6} \left( \partial_\mu z \partial K - \partial_\mu \bar{z} \partial \bar{K} \right) - \frac{1}{3} A_\mu P = \frac{i}{6} \left( \partial_\mu z \partial K - \partial_\mu \bar{z} \partial \bar{K} + A_\mu (r - \bar{r}) \right).$$

(2.2)
The moment map $\mathcal{P}(z, \bar{z})$ appearing there is a real function such that $\bar{\partial}\mathcal{P}(z, \bar{z}) = ik(z)\partial\bar{\partial}\mathcal{K}$ and $r(z)$ is defined from the gauge transformation of the Kähler potential, as $(k(z)\partial + \bar{k}(\bar{z})\bar{\partial})\mathcal{K}(z, \bar{z}) = r(z) + \bar{r}(\bar{z})$. The (gauge invariant) real moment map can then be written

$$\mathcal{P}(z, \bar{z}) = i(k(z)\partial\mathcal{K}(z, \bar{z}) - r(z)) = -i(\bar{k}(\bar{z})\bar{\partial}\mathcal{K}(z, \bar{z}) - \bar{r}(\bar{z})). \tag{2.3}$$

Before giving the result for the amplitudes, let us remind some standard facts on the Fayet-Iliopoulos term, R-symmetry and Kähler transformations. As one can see, the definition of $r(z)$ does not constrain its imaginary part, so one is free to add a constant to it, which translates as a real constant in $\mathcal{P}(z, \bar{z})$. This arbitrary constant is the so-called Fayet-Iliopoulos constant, and it can be added to the moment map of any $U(1)$. If we start from $\text{Im}(r(z)) = 0$, adding this constant for a given $U(1)$ has for consequence that it becomes an R-symmetry. It can be seen for instance in the gauge transformation of the gravitino, which is proportional to $\bar{r}(\bar{z}) - r(z)$

$$\delta\psi_\mu = \frac{1}{4}(\bar{r}(\bar{z}) - r(z))\psi_\mu \theta. \tag{2.4}$$

One can check that the first term of (2.1) is gauge invariant since the gauge transformation of $A_\mu$ given in (2.2) is

$$\delta A_\mu = -\frac{i}{6}\partial_\mu ((\bar{r}(\bar{z}) - r(z))\theta). \tag{2.5}$$

This transformation of the gravitino can be traced back to the transformation of the compensator multiplet $S_0$ in the superconformal construction of $\mathcal{N} = 1$ supergravity, before gauge fixing the chiral T-symmetry of the superconformal algebra using the condition $S_0 = \bar{S}_0$.

In other words, $r(z)$ can be regarded as the Killing vector involved in the transformation of the compensator $S_0$. As soon as $\text{Im}(r(z)) \neq 0$, the field $S_0$ transforms under the $U(1)$ and this defines an R-symmetry (which is equivalent to a phase transformation of the fermionic coordinates in the superfield formalism). Moreover, $r(z)$ is also involved in the gauge transformation of the superpotential

$$\delta W(z) = k(z)\partial W(z)\theta = -r(z)W(z)\theta, \tag{2.6}$$

which constrains the form of the superpotential and the charges of the scalars. For instance, if $r(z) = i\xi_s$, the superpotential must transform as $W(z) \rightarrow \exp(-i\xi_s\theta)W(z)$ and the only possibility is $W(z) = z^b$ with the charge $qb = \xi_s$. In this way, R-symmetry can be seen as a symmetry under which $W(z)$ transforms, and this can be taken as an equivalent definition. On the other hand, the real part $\text{Re}(r(z))$ is associated to a local scale transformation before it is fixed by a condition on $|S_0|$, usually chosen to have canonically normalised gravity kinetic terms in the Einstein frame.
Now, note that the Killing vector \( r(z) \) is not invariant under Kähler transformation. Indeed, since under such transformations \( K(z, \bar{z}) \rightarrow K(z, \bar{z}) + J(z) + \bar{J}(\bar{z}) \) we can easily obtain

\[
r(z) \rightarrow r(z) + k(z) \partial J(z),
\]

implying that it is always possible to choose \( \partial J(z) = -r(z)/k(z) \) such that \( r(z) \rightarrow 0 \) and the \( U(1) \) becomes an ordinary non R-symmetry. From (2.6), we see that \( -r(z)/k(z) = \partial \log W(z) \), so \( J(z) = \log W(z) + \text{constant} \), and the associated Kähler transformation of the superpotential \( W(z) \rightarrow e^{-J}W(z) \) makes it constant. In particular it does not transform under the \( U(1) \) anymore. In short, R-symmetry is not a Kähler frame independent concept; we can always go to a Kähler frame where the superpotential is constant and the gauge symmetry is not an R-symmetry. Note however that the moment map \( \mathcal{P}(z, \bar{z}) \) is invariant under Kähler transformations; so if the standard FI constant is added in some Kahler frame it is present in any other Kähler frame, even in the one where the \( U(1) \) is not a R-symmetry. For instance, in some frames it can get incorporated into the Kähler potential.

Let us now come back to our amplitudes. To start, let us suppose that the scalar \( z \) picks a nonvanishing vev with \( |z_0| = v \) at the minimum of the potential, and parameterise it as

\[
z(x) = (v + \eta(x)) e^{i\phi(x)}, \quad \text{where} \quad \partial V(z_0, \bar{z}_0) = \bar{\partial} V(z_0, \bar{z}_0) = 0 \quad \text{and} \quad \langle \eta(x) \rangle = 0.
\]

Since \( z \) is charged, this vev breaks the \( U(1) \), and in the unitary gauge (we use the term "unitary gauge" for both SUSY and the \( U(1) \), but the distinction should be clear with context), the phase \( \phi(x) \) can be reabsorbed in the gauge potential \( A_\mu \) that becomes massive, with its mass term contained in the kinetic term of \( z \) in (2.1). In this gauge, we simply have \( z(x) = v + \eta(x) \), which is real. The mass of \( A_\mu \) is given by

\[
M_A^2 = 2(qv)^2 \frac{\ddbar K_0}{\text{Re}(f_0)^2} = \frac{\ddbar K_0 W_0 \bar{W}_0}{\nabla W_0 \nabla \bar{W}_0} \frac{2 \mathcal{P}_0^2}{\text{Re}(f_0)},
\]

where \( \nabla \) is the Kähler covariant derivative \( \nabla W = \partial W + (\partial K)W \) and the subscript 0 means evaluated at \((z_0, \bar{z}_0)\). In this expression, we redefined the gauge potential to be canonically normalised, which at lowest order amounts to \( A_\mu \rightarrow A_\mu/(\text{Re}(f_0))^{1/2} \); this is where the \( \text{Re}(f_0) \) comes from. In the second equality, we used the definition of \( \mathcal{P} \) in (2.3), along with \( k = -iqz \sim -iqv \) at the minimum, and \( -r/k = \partial \log W \). With the unitary gauge, the propagator of the vector becomes

\[
P_A^{\alpha \beta}(k) = -\frac{i(\eta^{\alpha \beta} + k^\alpha k^\beta/M_A^2)}{k^2 + M_A^2}.
\]
Now, to compute the amplitudes we need to expand $e^{K/2W}$ to first order in $\eta$, as needed to obtain the 3-point gravitino interactions $\eta\psi\bar{\psi}$ and $A\psi\bar{\psi}$. We then canonically normalise $\eta$ just like $A$, by the redefinition $\eta \rightarrow \eta/(2\partial \bar{\partial} K_0)^{1/2}$. We obtain the following interactions

$$\mathcal{L}_{\eta\psi\bar{\psi}}^{(1)} = -\frac{1}{8\sqrt{2}} \frac{\partial K_0 - \bar{\partial} K_0}{\sqrt{\partial \bar{\partial} K_0}} (\partial_\mu \eta) \bar{\psi}_\mu \overline{\gamma}^{\mu\rho\sigma} \gamma_\sigma \psi_\rho,$$  \hspace{1cm} (2.11)

and

$$\mathcal{L}_{\eta\psi\bar{\psi}}^{(2)} = \frac{e^{K_0/2}}{8\sqrt{2}} \frac{(\partial K_0 + \bar{\partial} K_0)(W_0 + \bar{W}_0) + 2(W_0 + \bar{W}_0)}{\sqrt{\partial \bar{\partial} K_0}} \eta \bar{\psi}_\mu \overline{\gamma}^{\mu\nu} \gamma_\nu \psi_\nu - \frac{e^{K_0/2}}{8\sqrt{2}} \frac{(\partial K_0 + \bar{\partial} K_0)(W_0 - \bar{W}_0) + 2(W_0 - \bar{W}_0)}{\sqrt{\partial \bar{\partial} K_0}} \eta \bar{\psi}_\mu \overline{\gamma}^{\mu\nu} \gamma_\nu \psi_\nu.$$  \hspace{1cm} (2.12)

We also get

$$\mathcal{L}_{A\psi\bar{\psi}} = -\frac{i}{4\sqrt{\text{Re}(f_0)}} \frac{\mathcal{P}_0}{A_\rho \bar{\psi}_\mu \overline{\gamma}^{\mu\rho\sigma} \gamma_\sigma \psi_\rho}. \hspace{1cm} (2.13)$$

Note that even if the gravitino is not charged under the $U(1)$ (i.e. $\text{Im}(r(z)) = 0$ in (2.4)), the latter can have an interaction $A\psi\bar{\psi}$ with the gauge vector through $\mathcal{P}_0$ when the scalar has a vacuum expectation value.

These interactions can then straightforwardly be turned into vertices and use them to compute the $2 \rightarrow 2$ gravitino scattering amplitude. The details of this computation are as in [1]. An important point is that it is done in a Minkowski background; in other words $V(z_0, \bar{z}_0) = 0$.

We end up with the following contributions from the scalar and vector channels, for instance to the $(+, +, -, -)$ amplitude, with two external helicities $+1/2$ and two helicities $-1/2$ (we consider amplitudes with external helicities $\pm 1/2$ because they are the ones which diverge the most at high energy, as explained in [1])

$$\mathcal{M}_{\text{scalar}}^{+,+,\cdots,-} = -\frac{8\kappa^2 E^4}{9|m_{3/2}|^2} \nabla W_0 \bar{\nabla} \bar{W}_0 + \mathcal{O}(\kappa^2 E^2), \hspace{1cm} (2.14)$$

and

$$\mathcal{M}_{\text{vector}}^{+,+,\cdots,-} = -\frac{8\kappa^2 E^4}{9|m_{3/2}|^2} \left( \frac{2}{M_A^2} + \frac{1}{|m_{3/2}|^2} \right) \frac{\mathcal{P}_0^2}{\text{Re}(f_0)} + \mathcal{O}(\kappa^2 E^2), \hspace{1cm} (2.15)$$

where we restored the $\kappa$-dependence. We also recall the result for the gravitational channel [1]

$$\mathcal{M}_{\text{grav}}^{+,+,\cdots,-} = \frac{16\kappa^2 E^4}{3|m_{3/2}|^2} + \mathcal{O}(\kappa^2 E^2). \hspace{1cm} (2.16)$$

In all these expressions, $E$ is the energy of the gravitinos and $m_{3/2} = e^{K_0/2}W_0$ their mass.
These amplitudes exhibit a divergent behaviour at high energy. If we look at the different channels individually, we can expect perturbative unitarity breakdown when $M \sim 1$, at the supersymmetry breaking scale $\Lambda \sim (m_{3/2}/\kappa)^{1/2} \sim M_{\text{SUSY}}$. However, when summing them

$$M_{\text{total}}^{+,+,-,-} = -\frac{16\kappa^2 E^4}{9|m_{3/2}|^4} \left( e^K \left( \frac{\nabla W_0 \nabla \bar{W}_0}{\partial \partial K_0} - 3W_0 \bar{W}_0 \right) + \frac{P_0^2}{2\text{Re}(f_0)} \right) + \mathcal{O}(\kappa^2 E^2). \quad (2.17)$$

Between the parentheses, we observe the scalar potential $V(z_0, \bar{z}_0)$, evaluated at the minimum around which we expanded

$$V(z, \bar{z}) = V_F + V_D = e^K \left( \frac{\nabla W \nabla \bar{W}}{\partial \partial K} - 3W \bar{W} \right) + \frac{P^2}{2\text{Re}(f)} \quad (2.18)$$

In other words, the leading term in $\mathcal{O}(\kappa^2 E^4/|m_{3/2}|^2)$ cancels between the three channels when the potential is zero at the minimum. If things are to be consistent, this is the case in our setup, since as mentioned earlier we computed the amplitudes in a Minkowski background. We checked that this result holds for other helicity assignments. In the case where $z$ has a vanishing vev $v = 0$, the $U(1)$ is not broken, so the computation involves the complete complex scalar and a massless gauge vector, but the result also holds. Therefore, looking at the total amplitude, the perturbative unitarity cutoff is pushed up to the Planck scale $\Lambda \sim 1/\kappa \sim M_{\text{Pl}}$. This result can certainly be extended with more than one chiral and vector multiplets, allowing for more general gauge groups, in a Minkowski background. It could be interesting to see how it extends to more general backgrounds, such as (anti) de Sitter spacetimes; or even non-static ones, since cosmological setups often involve scalar fields evolving away from the minima of their potential; we leave this for future work.
3 Unitarity cutoff with the new Fayet-Iliopoulos terms

Let us now include the new Fayet Iliopoulos terms in our analysis. As mentioned in the introduction, we consider two cases, that we call the original new FI term (or FI-I), with non Kähler invariant matter couplings [3] and Kähler invariant new FI term (or FI-II) [5, 13]. They are both based on a vector multiplet, that we denote $V = (A_{\mu}, \lambda, D)$ in terms of its components in the Wess-Zumino gauge. We also denote $S_0 = (s_0, P_L \Omega_0, F_0)$ and $\tilde{S}_0 = (\tilde{s}_0, P_R \Omega_0, \tilde{F}_0)$ the chiral and anti-chiral compensator multiplets. In multiplet notations, the Lagrangian of the original new FI term is

$$\mathcal{L}_{FI-I} = -\xi_n \left[ S_0 \tilde{S}_0 \frac{w^2 \bar{w}^2}{T(w^2)T(\bar{w}^2)} (V)_D \right],$$

where $\xi_n$ is a parameter and $(V)_D$ is the real linear multiplet whose lowest component is $D$, the real auxiliary field of the vector multiplet. The multiplets $w^2$ and $\bar{w}^2$ are defined by their lowest component as

$$w^2 = \frac{\lambda P_L \lambda}{s_0^2} \quad \text{and} \quad \bar{w}^2 = \frac{\lambda P_R \lambda}{\tilde{s}_0^2}. \quad (3.2)$$

Finally the operators $T$ and $\bar{T}$ are the chiral and anti-chiral projectors (namely, if $(\tilde{X}, P_R \Omega, \tilde{F})$ is an anti-chiral multiplet, $T(\tilde{X})$ is the chiral multiplet whose lowest component is $\tilde{F}$). Expanding into components, and in the Poincare gauge where $s_0 = \tilde{s}_0 = e^{K/6}$, we have

$$e^{-1} \mathcal{L}_{FI-I} = -\xi_n e^{K/3} D + \frac{i}{2} \xi_n e^{K/3} \bar{\psi} \cdot \gamma \lambda + O(\lambda^2). \quad (3.3)$$

The first term is a contribution to the FI $D$-term; after integrating out the $D$ auxiliary field, it translates into a modification of $V_D$ in (2.18), which becomes

$$V_{D, \text{FI-I}} = \frac{(P + \xi_n e^{K/3})^2}{2 \text{Re}(f)}. \quad (3.4)$$

As explained in [4], this new FI term is not Kähler invariant. This is clear in (3.1), because before conformal gauge fixing the compensator $S_0$ transforms as $S_0 \rightarrow S_0 e^{z_s/3}$. Moreover, even though the addition of this FI term does not require R-symmetry, we can consider combining it with the standard FI term discussed after (2.1). With the standard FI term, the $U(1)$ is an R-symmetry, so $S_0$ transforms under it, and the new FI term also breaks gauge invariance; except in the particular Kähler frame where the $U(1)$ is not an R-symmetry, and where the superpotential is constant, as discussed after (2.7). In short, if we want to add both the standard and original new FI term at the same time, we should write them in the Kähler frame where the superpotential is constant. From there, nothing forbids us to perform Kähler transformations, but we shall keep in mind that different Kähler frames are not equivalent.
In order to avoid this issue, a Kähler invariant new FI term has been proposed in [5]

\[ \mathcal{L}_{\text{FI-II}} = -\xi_n \left[ (S_0\bar{S}_0e^{-K/3})^{-3} \frac{\tilde{\lambda}P_L\lambda}{T(w^2)T(\bar{w}^2)} (V)_D \right], \]  

(3.5)

where the multiplets \( w', \bar{w}' \) are defined by their lowest component as

\[ w'^2 = \frac{\tilde{\lambda}P_L\lambda}{(S_0\bar{S}_0e^{-K/3})^2} \quad \text{and} \quad \bar{w}'^2 = \frac{\tilde{\lambda}P_R\lambda}{(S_0\bar{S}_0e^{-K/3})^2}. \]  

(3.6)

This term is now manifestly Kähler invariant, because the transformation of \( S_0 \rightarrow S_0 e^{J(z)/3} \) is accompanied by a transformation of \( K \rightarrow K + J(z) + \bar{J} (\bar{z}) \). As before, we can expand it into components, and get

\[ e^{-1}\mathcal{L}_{\text{FI-II}} = -\xi_n D + \frac{i}{2} \xi_n \bar{\psi} \cdot \gamma \lambda + \mathcal{O}(\lambda^2) \]  

(3.7)

which now leads to

\[ V_{D,\text{FI-II}} = \frac{(P + \xi_n)^2}{2\text{Re}(f)}. \]  

(3.8)

To this order, the two new FI terms only differ by a factor \( e^{K/3} \) accompanying \( \xi_n \). We therefore introduce the notation \( \Delta = e^{K/3} \) for the original new FI term and \( \Delta = 1 \) for the Kähler invariant new FI term in order to write both cases at once. Actually, in a further generalisation of the Kähler invariant new FI term, the coefficients \( \xi_n \) become field dependent functions, invariant under Kähler transformations. An explicit example is when their field dependence arises through the Kähler invariant combination \( G = K + \log |W|^2 \) [13]. Now, when it comes to the \( 2 \to 2 \) gravitino scattering amplitudes, it is clear from both (3.3) and (3.7) that the new FI terms do not contain gravitino-gravitino-vector nor gravitino-gravitino-scalar interactions, since all the terms in \( \mathcal{O}(\lambda^2) \) contain at least two gauginos. Consequently, the inclusion of these terms does not affect the amplitudes of the previous section. One shall just check that it is still possible to use the unitary gauge, to cancel the mixing between the gravitino and the Goldstino. For this, it is enough to check that the Goldstino undergoes a non zero shift under supersymmetry transformations. With the contributions from standard \( \mathcal{N} = 1 \) supergravity and the new FI terms, the Goldstino reads

\[ P_{L\bar{U}} = -\frac{1}{\sqrt{2}} e^{K/2} \nabla W \chi - \frac{i}{2} \partial P_L \lambda - \frac{i}{2} \xi_n \Delta P_L \lambda, \]  

(3.9)

and its SUSY variation

\[ \delta U_L = \frac{1}{2} \left( e^{K} \nabla W \nabla \bar{W} + \frac{\partial}{\partial \partial K} \frac{\mathcal{P}(P + \xi_n \Delta)}{2\text{Re}(f)} \right) \epsilon_L + \cdots. \]  

(3.10)

When \( \xi_n = 0 \), the term between the parenthesis is always positive in a broken SUSY phase, so we can make a transformation such that \( v_L \to 0 \), which defines the unitary gauge. When \( \xi_n \neq 0 \), it is no longer the case and one should verify that this parenthesis does not vanish when evaluated at the minimum of the potential around which we compute the amplitudes.
Let us extend the discussion of section 2 with these new FI terms. As mentioned in the introduction, their presence modify the scalar potential and they don’t contribute to the amplitudes, so they generically lead to unanced leading terms $O(\kappa^2 E^4/|m_3|^2)$ in the latter and a perturbative cutoff at the SUSY breaking scale. However, this cancellation can be restored at the minimum of the potential for particular values of parameters. The condition for cancellation in the amplitudes is (2.17)

$$V_F(z_0, \bar{z}_0) + V_D(z_0, \bar{z}_0) = 0,$$

where $V_F$ and $V_D$ are defined in (2.18), and $(z_0, \bar{z}_0)$ is an extremum of the scalar potential

$$\partial(V_F(z_0, \bar{z}_0) + V_{D,FI}(z_0, \bar{z}_0)) = \tilde{\partial}(V_F(z_0, \bar{z}_0) + V_{D,FI}(z_0, \bar{z}_0)) = 0,$$

where, with the notation $\Delta$ introduced after (3.8) we can write $V_{D,FI}$ in the two cases as

$$V_{D,FI} = \frac{(\mathcal{P} + \xi_n \Delta)^2}{2Re(f)}.$$

In addition, the condition for vanishing cosmological constant at the minimum, as assumed in our amplitude computation, is

$$V_F(z_0, \bar{z}_0) + V_{D,FI}(z_0, \bar{z}_0) = 0.$$

The conditions (3.11) and (3.14) straightforwardly imply $2\mathcal{P}(z_0, \bar{z}_0) + \xi_n \Delta(z_0, \bar{z}_0) = 0$, in other words that the new FI term does not contribute to the potential at the minimum.

It is not hard to find concrete examples where (3.11), (3.12) and (3.14) are simultaneously satisfied. For instance, let us consider the case with Kähler potential $\mathcal{K}(z, \bar{z}) = z\bar{z}$, superpotential $W(z) = z^b$, gauge kinetic function $f(z) = 1$, a standard FI term $\xi_s$, and either the original or the Kähler invariant new FI term with parameter $\xi_n$. The presence of the standard FI term implies that the $U(1)$ is an R-symmetry under which the superpotential transforms with a charge $\xi_s$, so the charge of the scalar satisfies $qb = \xi_s$. Also, as mentioned after equation (3.4), the original new FI term should be written in the Kähler frame where the superpotential is constant, with $\tilde{\mathcal{K}}(z, \bar{z}) = z\bar{z} + b \log(z\bar{z})$. Taking all of this into account, it is not hard to find that for both new FI terms and $0 < b < 0.75$, there is one value of $(\xi_s, \xi_n)$ for which all conditions are satisfied. For $b < 0$, there are two such values. For instance, for the original new FI term

- $b = 0.5$, $\xi_s = 0.69211$, $\xi_n = -2.93863$, $r_0 = 0.96407$,
- $b = -1$, $\xi_s = 6.43856$, $\xi_n = -5.04518$, $r_0 = 0.62367$,
- $b = -1$, $\xi_s = 2.09581$, $\xi_n = 4.76335$, $r_0 = 1.81251$,

where $r_0 = |z_0|$.
For the Kähler invariant new FI term, we find similarly e.g.

- $b = 0.5$, $\xi_s = 0.50204$, $\xi_n = -3.92696$, $r_0 = 1.20644$,
- $b = -1$, $\xi_s = 5.54153$, $\xi_n = -7.43073$, $r_0 = 0.57406$,
- $b = -1$, $\xi_s = 1.43121$, $\xi_n = 8.83171$, $r_0 = 2.02123$.

In figure 1 we plot the corresponding potentials for concreteness. In all cases, we checked that the parenthesis in (3.10) does not vanish, allowing us to use the unitary gauge.

**Figure 1.** Scalar potential for the model with Kähler potential $K(z, \bar{z}) = z\bar{z}$, superpotential $W(z) = z^b$, a standard FI term $\xi_s$, and either the original (left) or the Kähler invariant (right) new FI term with parameter $\xi_n$ and values of the parameters for which (3.11), (3.12) and (3.14) are satisfied.
Of course these particular values of the parameters are not expected to be stable under quantum corrections, unless they are protected by symmetries in some models. Except for particular values of the parameters, for which the new FI term does not contribute to the potential at its minimum, as we just saw, unitarity of the $2 \to 2$ gravitino scattering amplitude leads to a perturbative cutoff at the SUSY breaking scale with the new FI terms. Note that generalizations of these new FI terms [13, 16] allow for a field dependent $\xi_n(z, \bar{z})$, in which case it is possible to obtain $2P(z, \bar{z}) + \xi_n(z, \bar{z})\Delta(z, \bar{z}) = 0$ for all $z$. Then, the new FI term would not contribute to the potential at all, and the discussion of perturbative unitarity would boil down to the standard supergravity case discussed in section 2. However, the phenomenological interest of these new FI terms might be rather limited if they do not contribute to the scalar potential.

4 Conclusions

We have shown that the $2 \to 2$ massive gravitino scattering amplitudes in $\mathcal{N} = 1$ supergravity has a unitarity breaking cutoff at the Planck scale, even in the presence of the standard FI term associated to a gauged R-symmetry $U(1)$, around a Minkowski minimum of the scalar potential, and when supersymmetry is broken by both $F$- and $D$-term expectation values. This unitarity cutoff is at the Planck scale thanks to cancellation in the amplitudes that happen precisely because the potential vanishes at the Minkowski minimum. We expect this result to hold in more general backgrounds, such as (anti) de Sitter spacetimes, but it could be interesting to check it, since amplitude computations are more subtle in these cases. Non-static backgrounds could also be interesting for cosmological applications, or to study perturbative unitarity during (and not only after) spontaneous SUSY breaking. This property of the amplitudes is not valid when a new FI term is added in the action, unless its coefficient is tuned so that the new FI term does not change the value of the potential at its minimum. With this tuning, the new FI term can still contribute to the potential away from the minimum, and it also contains fermionic terms, for instance it contributes to the mass of the physical fermion.
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