Proper-time Quantum Mechanics for Multi-Quark System and Composite-Hadron Spectroscopy

Shin Ishida and Masuho Oda

Research Institute of Science and Technology, College of Science and Technology, Nihon University, Tokyo 101-8308, Japan

Tomohito Maeda and Kenji Yamada

Department of Science and Manufacturing Technology, Junior College Funabashi Campus, Nihon University, Funabashi 274-8501, Japan

Abstract

One of the most important problems in hadron physics is to establish the Lorentz-invariant classification scheme of composite hadrons, extending the framework of non-relativistic quark model. We present an attempt, by developing proper-time $\tau$ quantum mechanics on a multi-quark system in particle frame (with constant boost velocity $\mathbf{v}$). We start from the variational method on a classical mechanics action where a constituent quark has Pauli-type $SU(2)_\sigma$ spin. Then the $SU(2)_m$ symmetry, concerning the sign-reversal on quark mass, has arisen with the basic vectors, the normal Dirac spinor with $J^P = (1/2)^+$ and the chiral one with $J^P = (1/2)^-$, appearing as a “shadow” of the former. Herewith, the mass reversal between these basic vectors become equivalent to the chirality, which is a symmetry of the standard gauge theory. We describe the role of chirality in hadron spectroscopy and regard it as attribute $\chi$ of “elementary” hadrons in addition to $\{J,P,C\}$. A novel feature of our hadron spectroscopy is, in the example of $q\bar{q}$ meson system, that the “Regge trajectories”, are given by mass-squared vs. the number of quantum $N$; where $M^2 = M_0^2 + 2N\Omega$ ($N = 2n$, $n$ the radial quantum number, $\Omega$ the oscillator quantum), and the intrinsic spin of hadrons $J$ comes only from quark spin $S$, $J = S$. Some phenomenological facts crucial to its validity are pointed out on the light-through-heavy quarkonium system.
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1 Introduction – \( \tau \)-Quantum Mechanics and “Elementary Hadron”

The QCD/Standard Gauge-Theory is now regarded to be established as the basic under-ground theory of strong interactions among Hadrons. However, there is no rigorous technique to treat Bound-State Problem, which leads us to the seriously unsatisfactory situations as follows: Firstly for obtaining numerical predictions, only way seems to make computer-calculus, based on Lattice-Gauge Theory, separately in case by case. Secondly its more vital problem is that there could, by no means, draw any concrete and intuitive-picture on such fundamental problems as “Quark Confinement”, Picture of “Composite-Hadrons”, and “Vacuum Condensation” etc. In this work, we shall give some arguments, in replying to all the above problems. One of the reason why it may be afforded to us is that our method of \( \tau \)-Quantum Mechanics, having the scope of whole possibly boosted-Particle-Frame with \( v (−1 < v_i < 1) \), has naturally led us to a new view-point on Compositeness of Hadrons. In aid of this we define a new type of Composite-Hadrons with the definite picture as follows:

\textit{(Elementary Hadrons)}

A Multi-Quark System bounded by Chirality Symmetric QCD interaction. The System by Itself constructing an Inertial Frame with Non \( L \)-Definite States, \( L \) being orbital-angular momentum.

Herewith it is notable that the notion of Elementary Hadron is Lorentz-invariant, which originates from its character of being an inertial system. And the \( \tau \)-Wave Function of Elementary Hadron becomes tensor-product of the Basic vector (Extended Dirac spinor), which plays a role of the \( \tau \)-Wave Function of Constituent-Quark. Thus the framework of Composite-Hadron / Elementary-Hadron Spectroscopy to be presented in this work should be Lorentz-Invariant\(^1\).

2 Kinematical Framework and New Physics in the Classification Scheme of Hadrons

Firstly in section 2.1, we give a brief history of our way of the extension on the kinematical framework, that is, from non-relativistic scheme, \( SU(2)_\nu \otimes O(3)_L \) (aside from the flavor freedom), to covariant scheme\(^2\), \( \tilde{U}(4)_{DS,m} \otimes O(2)_{r_\perp v} \) (the former is a tensor-space of Dirac-spinor embedded with a static spin-symmetry \( SU(2)_m \), \( m \) representing a new mass-reversal symmetry reflecting the

\(^1\)See also the arguments in §6.1.1.
physical situation of quark confinement; while the latter is a space of 2-dimensional internal spatial-vector $r$, being orthogonal to the boost velocity $v$, embedded in the Lorentz-space $O(3,1)_{\text{Lor.}}$. Secondly in section 2.2, we describe a role of the chirality symmetry in composite hadrons, which is valid through light to heavy quark system. It is a symmetry of QCD/standard gauge theory, of which importance in hadron spectroscopy has been overlooked for many years.

2.1 Kinematical Framework of Covariant Classification Scheme for Composite Hadrons

2.1.1 Most Natural Way of Nonrelativistic-to-Covariant Extension and its Difficulty

Most natural way\[^{2}\][1, 3, 4, 5] of extension is to extend separately both of external and internal parts of the kinematical framework is, as

\[
SU(6)_{\sigma F} \otimes O(3)_L \rightarrow \tilde{U}(12)_{SF}[1] \otimes O(3,1)_{\text{Lorentz}}[3],
\]

\[
SU(6)_{\sigma F} = SU(2)_{\sigma} \otimes U(3)_F, \quad \tilde{U}(12)_{SF} = \tilde{U}(4)_{DS} \otimes U(3)_F,
\]

with the constraint on the expectation value of internal space-time coordinates (momenta), $r_{\mu}(p_{\mu})$ in the definite external momentum $P_{\mu}$-state, as

\[
\text{“Relative-time frozen” condition: } \langle P_{\mu}r_{\mu} \rangle = \langle P_{\mu}p_{\mu} \rangle = 0.
\]

However, this way had been seen to be closed by two No-Go theorems[6, 7], as

External: $SU(6)_{\sigma F} \rightarrow$ Relativistic Extension by Coleman-Mandula[6],

Internal: $O(3)_L \rightarrow O(3,1)$ Yukawa’s Bi-local Field Theory[7],

where the origin of internal No-Go theorem is considered to come from the close connection between the external coordinate $X_{\mu}$ and the internal coordinate $r_{\mu}$, as is seen from an ad hoc subsidiary “relative-time frozen” condition.

\[^{2}\]Strictly, $SU(6)_{\sigma F} \rightarrow SU(6)^{(q)}_{\sigma F} \otimes SU(6)^{(\bar{q})}_{\sigma F}$, $\tilde{U}(4)_{DS} \rightarrow \tilde{U}(4)^{(q)}_{DS} \otimes \tilde{U}(4)^{(\bar{q})}_{DS}$, etc. This form of separate symmetry between quark($q$) and anti-quark($\bar{q}$) is certainly a requisite result in the relevant spectroscopy based on $\tau$-quantum mechanics, as will be seen from Eq.(33) in §3 and in Appendix A.
2.1.2 Semi-Phenomenological Derivation of Covariant Classification Scheme of Composite Hadrons

Then we have chosen a semi-phenomenological means of pass-through\(^3\). Concerning the intrinsic quark-spin, a new freedom of SU(2)-mass spin is supposed\(^4\) in addition to the SU(2)-σ spin, which makes possible to apply the crossing rule to confined quarks, in conformity with color-singlet condition of parent hadrons; while concerning the undesirable connection of internal \(r_\mu\) to external \(X_\mu\), it is separated\(^5\) by supposing \(r_\mu\) to be spacelike and \(r\), also, to be orthogonal to the boost velocity \(v\).

The corresponding formulas and remarks to the above are systematically given as:

[On external freedom] the covariant spin-classification scheme, \(\tilde{U}(4)_{DS}\), is extended to \(\tilde{U}(4)_{DS,m}\), introducing a new static-unitary symmetry\([11]\) SU(2)\(_m\), embedded at Observer Frame (\(v = 0\)), as\([8, 9]\)

Basic Urciton Spinors : \(\{U_{r+}(P, M_+)(\frac{1}{2}^+), U_{r-}(P, M_-)(\frac{1}{2}^-)\}\), \(3a\)

Basic anti-Urciton Spinors : \(\{\tilde{V}_{r+}(P, M_+)(\frac{1}{2}^-), \tilde{V}_{r-}(P, M_-)(\frac{1}{2}^+)\}\), \(3b\)

\(U_{r-} \equiv -\gamma_5 U_{r+}, \tilde{V}_{r-} \equiv \tilde{V}_{r+} \gamma_5, \) \(4\)

where \(r(\tilde{r}) = \{+, -\}\) is the direction of \(\hat{m}_3(\hat{\tilde{m}}_3)\) for urciton (anti-urciton) to be called \{Paulon, Chiralon\}, respectively. Here it is noted that the freedom of Chiralon predicts the existence of light-mass (– in the non-relativistic scheme) particles (\(\sigma(500, 0^+), \rho/\omega(1250, 1^-), N(1440, 1/2^+), \Lambda(1405, 1/2^-)\)).

[On internal freedom] the undesirable connection between the variables \(r_\mu\) and \(X_\mu\) is separated, so that our interests are only in the space-like region of \(r_\mu\), and then supposing its spatial component

---

\(^3\)Exciton picture of quarks was first proposed by O. Hara and T. Goto\([10]\). Successively, on the basis of this picture, the present author\([4]\) had presented the kinematical framework for systematic hadron phenomenology which is an origin of line of thoughts in this work. Actually the Covariant Oscillator Quark Model (COQM)\([16]\) with this framework had been applied for many years to various phenomena of hadron physics.

\(^4\)Embedding of non-covariant SU(2)\(_m\)-spin into the covariant \(\tilde{U}(4)_{DS,m}\)-space had been done, as a static-unitary symmetry, in Ref.\([11]\). The existence of SU(2)\(_m\)-symmetry is deduced from the Klein-Gordon equation as master equation, to be satisfied by constituent-quark field (see Eq.\([9]\)).

\(^5\)This implies that the internal space-time variable \(r_\mu\) concerns only mass term of the Yukawa-type Klein-Gordon equation for relevant composite hadrons, as will be described in section 3.2.
be orthogonal to boost-velocity $v$ as

$$P_\mu r_\mu = 0, \quad \text{and} \quad v \cdot r = 0.$$  

(5a)

Thus our relevant internal space becomes as

$$O(3, 1)_{\text{Lor.}} \Rightarrow O(2)_{r \perp v} \text{ embedded in } O(3, 1)_{\text{Lor.}}.$$  

(5b)

Here, it is also noted that there exists no relative-time problem.

Accordingly, the covariant framework of our hadron-classification scheme (aside from the flavor freedom) becomes,\(^6\) as

$$\{\widetilde{U}(4)^{(q)}_{D, m} \otimes \widetilde{U}(4)^{(q)}_{D, m}\} \otimes O(2)_{r \perp v} \text{ in } O(3, 1)_{\text{Lor.}}.$$  

(6)

### 2.2 Chirality – the Symmetry in QCD Gauge Theory and its Role in Hadron-Spectroscopy

The chirality is an important symmetry conserved through all types of the QCD gauge interaction/upper-ground strong interaction with each flavored quarks. This property of chirality seems us that it deserves to be an attribute of, somewhat elementary entity, Composite Hadrons, consisting of confined quarks.

They shall be called as Elementary Hadrons in this work.

#### 2.2.1 Chirality Conservation in QCD

The effective interaction Lagrangian $L_I(q(x),\text{'s})$ for any kind of vertices among quark-pairs, $(\bar{q}, q),\text{'s}$, are obtained by substituting, in the relevant free-quark Lagrangians, the covariant derivative $D_\mu \equiv \partial_\mu - ig A_\mu(A_\mu \equiv \sum_{a=1}^{8} A^a_\mu \lambda^a / 2 \text{ in QCD})$ for the normal derivative $\partial_\mu$. Accordingly, the chirality conservation is easily seen, as follows:

Substitution

$$\bar{q}_i \gamma_\mu \partial_\mu q_i \rightarrow \bar{q}_i \gamma_\mu D_\mu q_i = \bar{q}_i \gamma_\mu \partial_\mu q_i - ig \bar{q}_i \gamma_\mu A_\mu q_i,$$

(7a)

$$\Rightarrow L_I(q_i,\text{'s}) = L_I((\bar{q}_i \gamma_\mu A_\mu q)_i,\text{'s}).$$  

(7b)

\(^6\)In Eq.(6) is written the dimension two of general internal extension, as $O(2)_{r \perp v}$. However, in the actual problem will be adopted the one-dimensional oscillator, might be noted as $O(1)_{r \perp v}$ (see around Eqs.(53) and (54)).
Chirality Transformation

\[ q_i \rightarrow q_i^\chi \equiv \hat{\chi} q_i = -\gamma_5 q_i, \] (8a)

(For Space-Time Reflection \( R_{PT} : X'_\mu = -X_\mu \))

\[ \mathcal{L}_I(q_i^\chi \text{'-s}) = \mathcal{L}_I(q_i \text{'-s}). \] (8b)

### 2.2.2 Role of Chirality in Elementary Hadrons and Mass Reversal of Constituent Quarks

The contents of this subsection is an introduction of the kinematical framework, Evolved COQM (Covariant Oscillator Quark Model) with chirality-symmetric constituent quarks, to be developed in the next section §3. Here, it is to be noted that the old COQM[16] had not yet the notion of chirality.

(\textit{The case of constituent-quark field})

Wave Equation:

\[ \left( \frac{\partial^2}{\partial X_\mu^2} - M^2 \right) \Phi_\alpha(X) = 0, \] (9a)

\[ \left( \gamma_\mu \frac{\partial}{\partial X_\mu} + M_\pm \right) \Phi_\alpha(X) = 0 \ (M_\pm \equiv \pm M), \] (9b)

Wave Function:

\[ \Phi_\alpha(X) = \sum_{P_\mu (P_0 = E)} \left( \sum_{r=\pm} U_{r,\alpha}(P, M)e^{iP X} + \sum_{r=\pm} V_{r,\alpha}(P, M)e^{-iP X} \right) \] (10a)

\[ \equiv \sum_{P_\mu (P_0 = E)} \left( W_{\alpha}(P, M)^{(q)} e^{iP X} + W_{\alpha}(P, M)^{(\bar{q})} e^{-iP X} \right), \] (10b)

where the intrinsic spin-wave functions in \( \tilde{U}(4)_m \)-spin scheme, \( W_{\alpha}^{(q)} \) and \( W_{\alpha}^{(\bar{q})} \), of constituent quarks and anti-quarks, respectively, are defined. The Klein-Gordon Eq.(9a) has been set up as the master wave equation. The reason is that the observable entity is not quarks but parent hadrons, satisfying it as the mass-shell condition. It is interesting that this equation leads to Eq.(9b), which gives the two basic-vectors of the \( SU(2)_m \)-symmetry space, Paulons and Chiralons. Herewith, the wave
function of constituent quark field $\Phi_\alpha(X)$ is expanded\(^7\) as in Eq.(10), where the sum on $r(\bar{r})$ (direction of $\hat{n}_3$, $\hat{\bar{n}}_3$) guarantees the mass-reversal symmetry of $\Phi_\alpha(X)$. In relation with this symmetry it should be noted that the $\Phi_\alpha(X)$ plays a role of basic asymptotic state of $S$-matrix for strong interaction among general elementary hadrons in the $\tilde{U}(4)_{DS,m}$-spin multiplets, as will be shown in §4.3.

The Dirac-spinors given in Eq.(3) and (4) play the two side-roles in the $\tilde{U}(4)_{DS,m}$-classification scheme, satisfying, respectively, the following equations:

As Constituent Quark spinors,

$$(iP_\mu \gamma_\mu \pm M) U_{r\pm}(P, M) = 0, \quad (iP_\mu \gamma_\mu \mp M) V_{\bar{r}\mp}(P, M) = 0,$$

and as Urciton spinors,

$$(iu_\mu \gamma_\mu \pm 1) U_{r\pm}(v(P)) = 0, \quad (iu_\mu \gamma_\mu \mp 1) V_{\bar{r}\mp}(v(P)) = 0,$$

where these equations (11a, 11b) are connected simply by $(P_\mu = Mu_\mu; v \equiv \frac{dX}{d\tau}, u_\mu \equiv \frac{dX}{d\tau})$. The constituent quark-spinor in Eq.(11a) concerns the mass-reversal structure of parent hadrons, while the urciton-spinor in Eq.(11b) does their chirality structure, as will be shown in the case of $q\bar{q}$ meson, see Eqs.(17) and (18)). Here it should be noted that, for the wave function of constituent quark-field $\Phi(X)$ in Eq.(10), the two transformations of mass reversal and of chirality become equivalent, as is easily seen from Eq.(11a) and their definition:

*Equivalence of mass reversal and chirality*

\[
\begin{align*}
\text{Mass Reversal}[13] & \quad \hat{R}_m = \rho_1(m) : M_\pm \to M_{\mp}, \quad \Phi_\pm(X) \to \Phi_{\mp}(X), \\
\text{Chirality Transformation}[12] & \quad \hat{R}_{PT} = -\gamma_5 : X_\mu \to -X_\mu, \quad \Phi(X_\mu) \to \Phi(-X_\mu).
\end{align*}
\]

It may be also instructive to point out that there is two types of representation for the urciton-spinors, as:

*Two representations of urciton-spinor*

\[
\begin{align*}
U_{r\pm, \alpha} : & \quad \text{Bargmann-Wigner representation} \quad r\pm = \text{eigen-value of } \rho_3(m), \\
U_{\alpha}(\chi\pm) : & \quad \text{Chirality representation} \quad \chi\pm = \text{eigen-value of } \chi.
\end{align*}
\]

\(^7\)Here it may be notable that the sign of mass, which had been meaningless for free Dirac particles, now plays an important role for confined quarks; and that inertial frame, with respect to Lorentz Transformation) with definite boost velocity $(v \neq 0)$ for isolated, multi-particle system seems to be well representing the physical situation of quark confinement. The frame with $v \neq 0$ is called the particle frame, while the one with $v = 0$ the observer frame. The formulas obtained in the former (latter) becomes Lorentz-invariant (-covariant).
They are related with each other by

\[ U_{\alpha}^{(x\pm)} = \frac{1}{\sqrt{2}}(U_{r^+} \pm U_{r^-})_{\alpha}, \quad U_{r^+\alpha} = \frac{1}{\sqrt{2}}(U^{(x+)} \pm U^{(x-)})_{\alpha}. \]  

(14)

(The case of \((q\bar{q})\)-meson field) Wave Equation and wave function

The Klein-Gordon wave equation has been set up so as to represent the mass-shell condition of mesons as

\[ \left( \frac{\partial^2}{\partial X^2_{\mu}} - M^2 \right) \Phi_{\alpha}^{(M)\beta}(X) = 0. \]  

(15)

The meson wave function \( \Phi_{\alpha}^{(M)\beta}(X) \) is expanded as

\[ \Phi_{\alpha}^{(M)\beta}(X) = \sum_{P_{\mu}(P_0 = E)} \left[ \sum_{r,\bar{r}} U_{\alpha r}(v(P)) \tilde{V}_{\beta}^{r}(v(P)) e^{iPX} + \sum_{r,\bar{r}} V_{\alpha,\bar{r}}(v(P)) \tilde{U}_{\beta}^{\bar{r}}(v(P)) e^{-iPX} \right] \]

\[ \equiv \sum_{P_{\mu}(P_0 = E)} \left[ W_{\alpha}^{(M)\beta}(v(P)) e^{iPX} + W_{\alpha,\bar{r}}^{(\bar{M})\beta}(v(P)) e^{-iPX} \right], \]  

(16)

where the intrinsic-spin wave functions \( W_{\alpha}^{(M)\beta}(v(P)) \) and \( W_{\alpha,\bar{r}}^{(\bar{M})\beta}(v(P)) \) in the \( \tilde{U}(4)_m \)-spin scheme are also defined. Here it is to be noted that summation on the suffixes \( r(\bar{r}) \) guarantees mass-reversal symmetry, and/or chirality symmetry of the relevant meson-WF.

Expansion of meson WF into the definite-\( J^P C \chi_m \)-members

The general meson-WF is expanded into complete set of the members with definite chirality \( \{ \chi_m \} \), new attribute of elementary composite hadrons in addition to \( \{ J^P C \} \),

\[ \Phi_{A}^{(M)B}(X) = \sum_{i} \phi_{A}^{(i)B}(X) \Gamma_{\alpha}^{(i)\beta}, \]  

(17)

where \( A \equiv \{ \alpha, a \} \) etc. \((a \text{ denoting flavor index})\) and the appeared quantities are defined as

\[ \phi^{(i)} = \{ P^{(N)}_s, P^{(E)}_s, S^{(N)}_s, S^{(E)}_s, V^{(N)}_\mu, V^{(E)}_\mu, A^{(N)}_\mu, A^{(E)}_\mu \}, \]

(18a)

\[ \Gamma^{(i)} = \{ i\gamma_5, i\gamma_5 (i\gamma\mu\gamma_\nu), 1, i\gamma_\mu\gamma_\nu, i\gamma_5 \tilde{\gamma}_\mu, -i\sigma_{\mu\nu}u_\nu, i\gamma_5 \tilde{\gamma}_\mu, -i\gamma_5 \sigma_{\mu\nu}u_\nu \}, \]

(18b)

\[ \chi^{(i)}_m = \{-, +, -, +, +, -, +, -\}. \]  

(18c)

\[ ^{8} \text{From here; in order to discriminate the relevant chirality, originating in} \ \hat{R}_m, \text{from that, operating on pure indices of higher-spin WF; the label} \ \chi_m \ \text{will be used for the former (As for details, see §4.4).} \]

\[ ^{9} \text{The} \ \tilde{\gamma}_\mu \ \text{is defined so as to be} \ u_\mu \tilde{\gamma}_\mu = 0. \]
The Eq.(17) is the covariant quark-representation of composite $q\bar{q}$-meson system, proposed in Ref.[8], which started the relevant covariant classification scheme. Here $\phi^{(i)}(X)$’s with the definite Lorentz transformation property in addition to the attributes $\{J^{PC}\chi_m\}$ are to be second-quantized\textsuperscript{10}. (See, §3.2.3.)

3 Proper-time Quantum Mechanics for Multi-Quark System and Quantization of Composite-Hadron Fields

In section 3, proper-time $\tau$-quantum mechanics for multi-body confined-quark system and quantization of Composite Hadron field is developed. The similar to conventional procedures are performed, but all in a certain Inertial Frame (with $v = $ const); starting from an application of variational method to a classical action of the relevant confined system, where quarks have Pauli-type $SU(2)_\sigma$-intrinsic spin. A notable feature of the $\tau$-Quantum Mechanics is, it is concerned only future-development: which induces application of the crossing rule for “Negative-Energy Problem”. This rule is conventionally supposed ad hoc. The $\tau$-Quantum Mechanics also induces Existence of the Chiral-quark, with $J^P = (1/2)^-$ (, as a “Shadow” of Pauli-quark with $J^P = (1/2)^+$), which is considered to be an origin of new exotics, mysterious from non-relativistic scheme. These parity-doublet pair of quarks with $J^P = \frac{1}{2}(\pm)$ play rightly the role of Basic Vectors of the covariant $\tilde{\mathbf{DS}}_m$ spinor-space, semi-phenomenologically derived in §2.2.1. The Non-Local $\tau$-Wave Function for relevant multi-quark system leads, in Observer F., to a complete set of the corresponding Local Hadron-Fields in the $\tilde{\mathbf{DS}}_m$-spinor scheme. The title, “Prototype Mechanics for Solitary Urciton-Quark Field/ Prototype Mechanics for Multi-Urciton Quark Fields”\textsuperscript{11}, for the subsections §3.1 and §3.2, reflects this situation.

Before going into details, is shown in Fig.1; Overview on constructing Quantum Field Theory of Composite Particles. Here, for simplicity, it is concerned the multi-particle system, consisting of only quarks, but no anti-quarks.

In this figure a special prescription on separation between the external and internal coordinates of the constituent particles is applied, of which Lorentz invariance is guaranteed; as mentioned before, by the relevant Particle Frame(PF) (with non-zero boost-velocity $v \neq 0$) itself, being as an

\textsuperscript{10}It is to be noted that Urciton spinors in BW representation Eq.(13a) appear as the spin WF of mixed-states of the quantized pure-states with definite $\chi$. (See, Eq. (14)).

\textsuperscript{11}Prototype Mechanics for the Bi-local Field Theory had been introduced by T. Takabayasi[14]. See also, “Mechanical Model of Bi-local Field Theory” by T. Goto[15].
inertial frame. (See, the argument on “Elementary Hadron” in §1.)

Herewith, especially notable is that the seemingly three-dimensional definition of the spatial component $X$ is identical to that of center of mass coordinates in classical mechanics. Accordingly the external coordinate $X_{\mu}$ may be also called as “center of mass coordinate” of the multi quark system.

| Field Theory | Propertime Quantum Mechanics | Prototype Classical Mechanics on Multi-Body System of Spinning Particle |
|--------------|-----------------------------|------------------------------------------------------------------|
| External Motion of Parent $X_{\mu}(X,T = \tau(0))$ | Internal Motion of Constituents $(r^{(i)/s}; \tau(v))$ | Constituent’s Motion and Spin Boosting $(x^{(i)/s}; \tau(v), \sigma^{(i)/s}, m^{(i)/s})$ |
| - In Observer Frame ($v = 0$) | - In Particle Frame ($v \neq 0$) | - variance of constituent coord. $	au(v)$ - variance of constituent coord. |

(2nd Stage Quantiz.) (1st Stage Quantiz.)

World of Local Hadrons With $\tilde{U}(4)_{DS,m}$ symmetry

Hadron Mass Spectra $M = P_0(v = 0)$

Static-Symmetry $U(4)_{DS,m} \otimes O(2)_{r,\perp}$

Local Hadron Fields Multi-Local Wave Function

$H_{\tilde{N},\tilde{S}}(X_{\mu})$ $\Phi_{\alpha_1 \cdots \alpha_n}^{(r_1 s_1) \cdots (r_n s_n)}(X, r^{(i)/s}; \tau(v)) \approx \left( x^{(1)/s}_{\mu}(\tau(v)) \cdots x^{(n)/s}_{\mu}(\tau(v)) \right)$

$\tilde{N} \cdots O(2)_{\tau}$ $r = (\uparrow, \downarrow)$ : direction of $m_3$-spin

$\tilde{S} \cdots \tilde{U}(4)_{DS,m}$ $s = (\uparrow, \downarrow)$ : direction of $\sigma_3$-spin

Figure 1: Overview for Quantum Field Theory of Composite Particles. The two prescriptions playing an important role, i) Two-stage(1st and 2nd) Quantization of wave function at the proper time $\tau(v \neq 0)$ and $\tau(0) = T$, respectively, for a single, confined multi-quark system. ii) Special Definition of the internal coordinates $r^{(i)/s}_{\mu} = x^{(i)/s}_{\mu} - X_{\mu}$, measured from the external coordinate $X_{\mu}\{X = \sum_i m^{(i)}x^{(i)}(v)/M, \tau(v)\}$. The freedom of mass-spin has arisen, reflecting the situation of Quark-Confinement. As for details, see the §3.2.1, and Eqs. (72a, 72b) in §4.2.


3.1 Prototype Mechanics for Solitary Urciton-Quark Field

In the covariant classification scheme, or the precedent COQM, general composite hadrons are described\[16\] by the relevant multi-local fields, which are supposed, rather in an ad-hoc way, to satisfy the Yukawa-type Klein-Gordon wave equation. Some logical bases for this description are provided from the following classical mechanics\[12\] of a system of confined particles; which leads, through the 1st-stage quantization, to the relevant Klein-Gordon equation.

First we shall show its essential points in the ideal case of local Klein-Gordon field, describing the basic urciton-quark with a definite mass in the $\tilde{U}(4)_{DS,m}$ space.

3.1.1 Classical Mechanics

In classical mechanics the relevant action is given by\[13\]

$$S = \int_0^\tau \mathcal{L}(\dot{X}_\mu) d\tau, \quad \mathcal{L} = -M\sqrt{-\dot{X}^2_\mu} \quad (M > 0),$$

where $\dot{X}_\mu \equiv dX_\mu/d\tau$ ($\dot{X}^2_\mu = -1$), $d\tau = \sqrt{-d(X_\mu)} = dT\sqrt{1 - v^2}(v \equiv dX/dT; \text{boost velocity})$ and $\tau = \int d\tau$ is proper-time defined along world line with the restriction $dX_0/d\tau > 0$. The variation of $S$ is given as

$$\delta S = \left. \frac{M\dot{X}_\mu}{\sqrt{-\dot{X}^2_\mu}} \right|_0^\tau \delta X_\mu - M \int_0^\tau \left( \frac{d}{d\tau} \frac{\dot{X}_\mu}{\sqrt{-\dot{X}^2_\mu}} \right) \delta X_\mu d\tau.$$ 

This leads to Lagrange Equation of Motion and definition of 4-momentum $P_\mu$, respectively, as

$$\frac{d}{d\tau} M\dot{X}_\mu = 0, \quad (21a)$$

$$P_\mu = \left. \frac{\partial S}{\partial X_\mu(\tau)} \right|_{\text{Phys.Pass}} = M\dot{X}_\mu = Mu_\mu, \quad (21b)$$

where $u_\mu = dX_\mu/d\tau$ is the 4-velocity, and $u^2_\mu = -1$.

\[12\] Along this line of thoughts we had made some consideration\[17\] on its implication to COQM\[16\]. The contents of this sub-section are results of the further study on this problem.

\[13\] It is evident from Eq. (19) that sole existence of “chiral particle” with the mass, $M = -|M| < 0$, is prohibited by the least-action principle.
The above equations lead to conservation of the four-momentum

\[ \frac{d}{d\tau} P_\mu = 0, \quad (22a) \]

where

\[ P_i = M \frac{dX_i}{d\tau} = \frac{M v_i}{\sqrt{1 - v^2}}, \quad P_0 = \sqrt{P^2 + M^2} = M \frac{dX_0}{d\tau} = \frac{M}{\sqrt{1 - v^2}}. \quad (22b) \]

The equation (22b) is a famous Einstein formula. Here it may be instructive to remark that the equations (22a) and (22b) in the, so to speak, PF with non-zero boost-velocity \( v \neq 0 \) become, respectively, the equations in the Observer Frame (OF) with zero boost-velocity \( v = 0 \)

\[ \frac{d}{dT} M = 0, \quad (23a) \]

and

\[ P_i = 0, \quad P_0 = M. \quad (23b) \]

(\( \tau \)-Hamiltonian and \( \tau \)-Gauge Condition) In order to take a further step into the 1st-stage quantization the relevant physical quantities are represented by canonical variables, presently \( X_\mu \) and \( P_\mu \).

Firstly Hamiltonian is directly write down\(^{14}\), while it is constrained by the gauge condition \( G \); as

\[ \mathcal{H} = P_\mu \dot{X}_\mu - \mathcal{L} = \frac{K}{M} (P^2 + M^2) = 0, \quad (24a) \]

\[ G \equiv P^2 + M^2 \propto \mathcal{H} = 0. \quad (24b) \]

“Zero”-\( \mathcal{H} \) comes from the invariance of the action (19) under a gauge transformation of \( \tau \)

\[ \tau \to \tau' = K \tau \quad (K > 0). \quad (25) \]

This invariance reflects a physical situation that the time scale of an isolated confined-quark system is unobservable. The indefinite factor \( K \) is determined by requiring that the equation of motion be

\(^{14}\)Eq. (24a) is given in the “general” gauge with \( \tau' \) in Eq. (25).
equivalent in both Lagrange’s and Hamilton’s ones. Then the latter equation leads to the solution as
\[
\frac{d}{d\tau} X_\mu = \frac{\partial H}{\partial P_\mu} = K \frac{2}{M} P_\mu, \quad \frac{d}{d\tau} P_\mu = -\frac{\partial H}{\partial X_\mu} = 0.
\] (26)

Comparing these, respectively, with Eqs. (22b) and (22a), the \( K \)-factor is determined as \( K = 1/2 \) and the Hamiltonian \( H \) is fixed as
\[
H = \frac{1}{2M}(P_\mu^2 + M^2).
\] (27)

### 3.1.2 Quantum Mechanics

Then our relevant, Lorentz-covariant 1st-quantization may be accomplished by replacing Poisson Bracket with commutator between the canonical variables as
\[
\{X_\mu, P_\nu\}_{\text{P.B.}} = \delta_{\mu\nu} \rightarrow \left[\hat{X}_\mu, \hat{P}_\nu\right]_{\text{comm.}} = i\delta_{\mu\nu}.
\] (28)

\((\tau\text{-Schrödinger Eq. and } \tau\text{-Gauge Constraint})\)

Then the “Proper-time” Schrödinger Wave Equation will be set generally in PF as
\[
i\frac{d}{d\tau} \Phi_\alpha(X; \tau) = \hat{\mathcal{H}}(\hat{P})\Phi_\alpha(X; \tau),
\]
\[
\hat{\mathcal{H}} \equiv \frac{1}{2M}\hat{G} = \frac{1}{2M}(\hat{P}_\mu^2 + \mathcal{M}^2),
\] (29)

while the gauge constraint Eq. (24b) is represented in OF as\(^{15}\)
\[
\hat{G}\Phi_\alpha(X_\mu) = \left(\hat{P}_\mu^2 + M^2\right)\Phi_\alpha(X_\mu) = 0.
\] (30)

As will be shown in the next item, the solution of these two equations become equivalent to the space-time WF of two basic vectors in the \( \widetilde{U}(4)_{\text{DS,}m}\)-spinor space, given in Eqs.(3a) and (3b). This implies that the above equations (29) and (30) lead rightly to the local Klein-Gordon wave equation Eq.(9a), which is the master equation to be satisfied by the basic urciton-vectors in the \( SU(2)_m\)-space.

\(^{15}\) In Eq. (30) and accordingly in Eq. (29), we have simply put a \textit{Dirac-spinor index } \( \alpha \) on the relevant WF as \( \Phi_\alpha \). This prescription is guaranteed by the fact that the relevant variational method on the internal space \( O(2)_{r\perp v} \) is common in all inertial frame with respective boosting velocity \( v \)'s.
(Concrete Form of Urciton-Quark Wave Function)

For perception of the basic structure of our Lorentz-invariant proper time quantum mechanics we show a detailed form of the solution, 16 with definite momenta \( P(v) \), of both Eq. (29) and Eq. (30) as

\[
\Phi_{P,\alpha}(X; \tau(v)) = \Phi_{P,\alpha}^+(X; \tau(v)) + \Phi_{P,\alpha}^-(X; \tau(v)) = \sum_{r,s} U_{r,s;\alpha}(v(P)) e^{+iP(v) \cdot X - iP_0(v)\tau(v)} + \sum_{\bar{r},\bar{s}} \bar{V}_{\bar{r},\bar{s};\alpha}(v(P)) e^{-iP(v) \cdot X + iP_0(v)\tau(v)},
\]

(31)

where the respective terms in the second and the last equality corresponds to each other, and \( P_\mu(P(v), P_0(v)) \) is dependent on boost-velocity \( v \) as in Eq. (22b). Each parameter/factor appearing in Eq.(31) takes the following forms, respectively, in PF and in OF, and is mutually related as

\[
\begin{align*}
\text{PF} & \quad \{\tau(v(P)), P(v), P_0(v) = E(P(v))\} \quad v=0 \rightarrow \{T, 0, M\}, \\
& \quad e^{\pm iP(v) \cdot X} e^{\mp iP_0(v)\tau} \quad v=0 \rightarrow 1 \cdot e^{\mp iMT}.
\end{align*}
\]

(32)

Similarly the urciton spinor WF’s17 in PF, \( W(v(P)) \), are transformed from those in OF \( W(0) \) by Booster \( S_B \) (Eq. (94)) as

\[
W_{r,\alpha}(v(P)) \equiv \{U_{r,\alpha}(v), V_{r,\alpha}(v)\} = [S_B(v(P))W_{r}(0)]_\alpha,
\]

\[
W_{r,\alpha}(0) \equiv \{U_{r,\alpha}(0), V_{r,\alpha}(0)\},
\]

(33a)

where the latter is determined from the static unitary-symmetry \( SU(2)_m^{(q)} \otimes SU(2)_m^{(\bar{q})} \) as

\[
\begin{align*}
U_{r,\alpha}(0) : & \quad U_+(0,\alpha) = \begin{pmatrix} \chi \\ 0 \end{pmatrix}, \quad U_-(0,\alpha) = \begin{pmatrix} 0 \\ \chi \end{pmatrix}, \\
V_{\bar{r},\alpha}(0) : & \quad V_+(0,\alpha) = \begin{pmatrix} 0 \\ \chi \end{pmatrix}, \quad V_-(0,\alpha) = \begin{pmatrix} \chi \\ 0 \end{pmatrix}.
\end{align*}
\]

(33b)

16 Here it may be instructive to note the two implications of the gauge constraint (30): as i) \( \hat{G} = (-\hat{P}_0 + \hat{E})(\hat{P}_0 + \hat{E}) \); \( \hat{E} \equiv \sqrt{M^2 + P^2}, id\Phi^{(\pm)}/d\tau = \pm \hat{E}\Phi^{(\pm)}, \) that is, leading to \( \Phi^{(+)}/\Phi^{(-)} \); positive/negative-\( \tau \) frequency solutions. ii) \( \hat{G} \equiv [(i\hat{P}_\mu \gamma_\mu + M)(i\hat{P}_\mu \gamma_\mu - M)], \Phi_{\alpha}(X, v) = \{\Phi^{+}_{\alpha}, \Phi^{-}_{\alpha}\}, \) that is, leading to the basic urciton spinors of \( SU(2)_m \)-spin space.

17 In Eq. (33) we have concerned only the relevant \( SU(2)_m \) freedom, and retained only the suffix \( r/\bar{r} \).
By inspection of the equations (31) to (33) it may be ascertained concretely how the static unitary symmetry $U(4)_{DS,m}$ at rest frame ($\mathbf{P} = 0, \mathbf{v} = 0$) is to be embedded in the covariant $\tilde{U}(4)_{DS,m}$ space. Thus we see that $\Phi_{\mathbf{P},\alpha}(\mathbf{X}, \tau(\mathbf{v}))$, Eq. (31), in OF($v = 0$), which is a certain inertial frame, is equivalent to the WF of constituent quark in Eq. (10) with $\mathbf{P} = 0$, that is,

$$\Phi_{\mathbf{P},\alpha}(\mathbf{X}, \tau(\mathbf{v}))|_{v=0} = \Phi_\alpha(X_\mu)|_{P=0},$$

(34a)

and also that, considering in the whole inertial frame, the following two wave functions are mutually equivalent

$$\{\Phi_{\mathbf{P}(v),\alpha}(\mathbf{X}, \tau(\mathbf{v}))|_{v=\text{whole}(-1<v_i<+1)}\} = \{\Phi_\alpha(x_\mu)|_{P^2 = -m^2_q}; \text{Constit. Q.WF (Eq.(10)) in Evolved COQM}\}.$$

(34b)

Here it is to be noted that in $\tau$-Quantum Mechanics there exists no negative-energy problem. Instead of the “problem”, it implies an “application of crossing rule” in confined quark-level without violating the color-singlet condition of its parent-hadron. We also should like to remark that in the solution (Eq. (31)) of $\tau$-Schrödinger wave equation (Eq. (29)) the chiral-urciton, whose sole-existence is prohibited by Least-Action Principle, appears in Eq. (31) now in the confined system, being accompanied by the Pauli-urciton; This pair of urcitos plays a role of basic vectors of the chirality-symmetric $SU(2)_m$ space.

### 3.2 Prototype Mechanics for Multi-Urciton Quark Fields

Then we shall study the prototype mechanics for Yukawa-type Klein-Gordon field, describing the composite hadrons of multi-urciton quarks in the $\tilde{U}(4)_{DS,m}$-classification scheme.

#### 3.2.1 Internal Coordinates and Proper time for Multi-Quark system

First of all we have to start with consideration on the notion of proper time for a multi-body classical-particle system. For the present purpose we are concerned only space-like internal extensions, and define the external/center of mass coordinate $X_\mu$ and the internal coordinate $r^{(i)}_\mu$ for each constituent, respectively, as

$$X_\mu \equiv \frac{1}{M_0} \sum_i m_i x^{(i)}_\mu, \quad x^{(i)}_\mu \equiv r^{(i)}_\mu + X_\mu,$$

(35a)
\[ \sum_i m_i r^{(i)} = 0, \quad (35b) \]

where \( M_0 \equiv \sum_i m_i \) and \( x_\mu^{(i)} \)'s (\( i = 1 \) to \( n + m \); \( n(m) \) is number of quarks (anti-quarks)) denote the 4-coordinate of respective constituents. It is to be noted that each internal coordinate \( r^{(i)}_\mu \) is measured from its origin, the external coordinate \( X_\mu \)-itself, of which choice is critical for simple presentation of our theoretical framework. The point is as follows: Since the internal extension is space-like, it is possible to choose the Lorentz frame so that all the time variables of the external and of the respective urcitons are same, implying vanishing of all internal-time variables as,

\[ r^{(i)}_0 \equiv x^{(i)}_0 - X_0 = T - T = 0, \text{or more generally } P_\mu r^{(i)}_\mu = 0. \quad (36) \]

This means that, in the relevant Lorentz frame, the all internal coordinates become two-dimensional vectors as \( r^{(i)}_\mu = (r^{(i)}_0, 0) \), and satisfying \( v \cdot r^{(i)} = 0 \). In the case of \( \tilde{U}(4)_{DS,m} \)-scheme, where all urcitons do perform the parton-like motion (see, Eq. (100) in Appendix A), that all of their boost-velocity is equal to that of parent as \( v^{(i)} = v \), the proper time of all constituents become identical to that of total system as

\[ d\tau(v^{(i)}) = d\tau(v). \quad (37) \]

Therefore, in our scheme, the troublesome relative-time problem, appearing in most of the relativistic composite models, disappears at the beginning. It is also to be noted that

\[ \tau(v) \text{ in PF } \overset{\overrightarrow{v} \rightarrow 0}{=} T \text{ in OF}. \quad (38) \]

Now we are possible to proceed to classical mechanics.

### 3.2.2 Classical Mechanics

In classical mechanics the relevant action is given by

\[ S = \int_a^b \mathcal{L}(\dot{X}_\mu; r^{(i)} s, \dot{r}^{(i)} s) d\tau(v), \]

\[ \mathcal{L} = -\sqrt{2U(r^{(i)} s)} - \left( M_0 \dot{X}_\mu^2 + \sum_i m_i \dot{r}^{(i)} \dot{r}^{(i)} \right), \quad U(r^{(i)} s) \equiv \sum_i U^{(i)}(r^{(i)}) > 0, \quad (39) \]

in terms of \( \dot{X}_\mu, \dot{r}^{(i)} s, r^{(i)} s; \) and \( d\tau = dT \sqrt{1 - v^2} \).
As was mentioned[17] our Lagrangian (39) is not of additive form as regards constituent-particles, and does not reduce to the sum of respective ones even in the free limit with constant $U$. It is also notable that this form of Lagrangian makes possible the separation of motion into the external and the internal ones.

From Eq. (39) we derive the expression of respective momenta

$$ P_\mu = \frac{\partial L}{\partial \dot{X}_\mu} = \frac{\sqrt{2U}M_0 \dot{X}_\mu}{\sqrt{-(M_0 \dot{X}_\mu^2 + \sum_i m_i \dot{r}^{(i)2})}} = M_0 \dot{X}_\mu, \quad (40a) $$

$$ P^{(i)} = \frac{\partial L}{\partial \dot{r}^{(i)}} = \frac{\sqrt{2U}m_i \dot{r}^{(i)}}{\sqrt{-(M_0 \dot{X}_\mu^2 + \sum_i m_i \dot{r}^{(i)2})}} = m_i \dot{r}^{(i)}, \quad (40b) $$

where in the last equality is chosen the “covariant gauge”, now with a $\tau$-gauge function $G$ to be $G \equiv \frac{\sqrt{2U}}{\sqrt{-(M_0 \dot{X}_\mu^2 + \sum_i m_i \dot{r}^{(i)2})}} = 1$. These internal momenta have to satisfy the “Kinematical constraints”

$$ \sum_i P^{(i)} = 0. \quad (41) $$

Lagrange equations of motion are also derived in the particle frame, as

$$ \frac{d}{d\tau} M_0 \dot{X}_\mu = \frac{d}{d\tau} P_\mu = 0, \quad (42a) $$

$$ \frac{d}{d\tau} m_i \dot{r}^{(i)} + \frac{\partial U^{(i)}(r^{(i)})}{\partial r^{(i)}} = \frac{d}{d\tau} P^{(i)} + \frac{\partial U^{(i)}(r^{(i)})}{\partial r^{(i)}} = 0, \quad (42b) $$

while in the observer frame, as

$$ \frac{d}{dT} P_\mu = 0 \quad (P = 0, \; P_0 = M_0), \quad (43a) $$

$$ \frac{d}{dT} P^{(i)} + \frac{\partial U^{(i)}(r^{(i)})}{\partial r^{(i)}} = 0. \quad (43b) $$

This kinematic constraint comes from the definition Eq. (35) of coordinate $r^{(i)}$’s in the classical mechanics level, as $\frac{d}{dT} \sum_i m_i \dot{r}^{(i)} = \sum_i m_i \dot{r}^{(i)} = \sum_i P^{(i)} = 0$. 

\[20\]
Then Hamiltonian $\mathcal{H}$ is given (corresponding to Eq. (27) in the case of local urciton field,) by the definition, as

$$\mathcal{H} \equiv P_\mu \dot{X}_\mu + \sum_i p^{(i)} \cdot \dot{r}^{(i)} - \mathcal{L}(\dot{X}_\mu; \dot{r}^{(i)}s, r^{(i)}s) = \frac{1}{2M_0}(P^2 + \mathcal{M}^2(r^{(i)}s, p^{(i)}s)) = 0,$$

(44a)

$$\mathcal{M}^2 = \sum_i m^{(i)}(p^{(i)}, r^{(i)}), \quad m^{(i)}(p^{(i)}, r^{(i)}) \equiv \frac{M_0}{m_i} p^{(i)2} + 2M_0 U^{(i)}(r^{(i)}).$$

(44b)

Here zero in the last equality in Eq. (44a) comes from the $\tau$-gauge invariance of action (39). As a concrete form of potential $U^{(i)}(r^{(i)})$ in Eq. (44b), we apply, as a simple model the one-dimensional Hooke’s one (45a), which, due to Eqs. (42a) through (45b), leads the respective mass-operator $m^{(i)}(44b)$ to an oscillator (45b) for deviation of internal extension $r^{(i)}s$ from its average position $r^{(i)} = 0$ (that is, $X$) as

$$U^{(i)}(r^{(i)}) = U^{(i)}(0) + \frac{1}{2} K^{(i)} r^{(i)2},$$

(45a)

$$m^{(i)}(p^{(i)}, r^{(i)}) = 2M_0 U^{(i)}(0) + M_0 [(p^{(i)2}/m_i) + K^{(i)} r^{(i)2}].$$

(45b)

Thus far in this subsection 3.2.2, we have shown our action Eq. (39) describes surely such a property of confined multi-quark system that, total mass $M$ is conserved, and each constituent makes independently a simple harmonic oscillation around their center of mass position.

### 3.2.3 Quantum Mechanics – the First-Stage Quantization

(1st Quantization) Then our relevant 1st-quantization may be performed by replacing the P.B. with commutator between the canonical variables as

$$\{X_\mu, P_\nu\}_{\text{P.B.}} = \delta_{\mu\nu} \rightarrow \left[\dot{X}_\mu, \dot{P}_\nu\right]_{\text{com.}} = i\delta_{\mu\nu},$$

$$\{r^{(i)}_k, p^{(i)}_l\}_{\text{P.B.}} = \delta^{(i)}_{kl} \rightarrow \left[\dot{r}^{(i)}_k, \dot{p}^{(i)}_l\right]_{\text{com.}} = i\delta^{(i)}_{kl},$$

(46)

and by setting the “Proper-time” Schrödinger Wave Equation (WE).
(τ-Schrödinger Equation and Gauge Constraint) With τ-Hamiltonian (44a), τ-Schrödinger WE is written in PF as

\[ i \frac{d}{d\tau} \Phi_{P,\alpha_1\cdots\alpha_n}^{\beta_1\cdots\beta_m} (X, r^{(i)} ; \tau) = \hat{H} (\hat{P}, \hat{p}^{(i)} ; s, r^{(i)} ; \tau) \Phi_{P,\alpha_1\cdots\alpha_n}^{\beta_1\cdots\beta_m} (X, r^{(i)} ; \tau), \]

\[ \hat{H} \equiv \frac{1}{2M_0} \hat{G} = \frac{1}{2M_0} (\hat{P}^2 + \hat{M}^2). \] (47)

On the other hand the gauge constraint, the last equality of Eq. (44a), ( corresponding to Eq.(30) in the local case, ) is expressed in OF as

\[ \hat{G} \Phi_{\alpha_1\cdots\alpha_n}^{\beta_1\cdots\beta_m} (X_\mu, r^{(i)} ; s, r^{(i)} ; \tau) = 0, \] (48)

\[ \hat{M}^2 = \sum_i \hat{m}^{(i)} \hat{p}^{(i)}, \quad \hat{m}^{(i)} = M_0 \left( \frac{1}{m_i} \hat{p}^{(i)} + 2U^{(i)} (r^{(i)}) \right), \] (49a)

\[ \hat{M}^2 \equiv M_0 \hat{M}, \] (49b)

which is the right Yukawa-type Klein-Gordon equation for Multi-Local Hadron-field in the evolved-classification scheme. In the equations (47) and (48) we have simply put, similarly as Eqs. (29) and (30) in the local case, the spinor indices on the τ-WF. This may be allowed from the same reason as explained on the τ-WF in the local case (see, Eqs. (31) through (33) and the footnote 15). The only difference in the two cases is that the objects of application are basic vectors and their tensors of \( SU(2)_m \)-spin group, respectively, in the local and in the multi-local case. Here it is to be noted that the solution of Eq.(48) plays the role of Wave Functions of the Evolved COQM with chirality-symmetric constituent quarks, which was proclaimed at the beginning of §2.2.2.

By factorizing the gauge operator \( \hat{G} \) in (48) into the positive- and negative-frequency parts, \( \{ \Phi^{(+)}, \Phi^{(-)} \} \), the τ-Schrödinger wave equation (47) leads to the following equations (51), as

\[ \hat{G} = (\hat{P}_0 + \xi)(\hat{P}_0 + \bar{\xi}), \quad \xi \equiv \sqrt{\hat{P}^2 + \hat{M}^2}; \] (50)

\[ i \frac{d}{d\tau} \Phi_{P,\alpha_1\cdots\alpha_n}^{\beta_1\cdots\beta_m} (X, r^{(i)} ; \tau) = \pm \xi \Phi_{P,\alpha_1\cdots\alpha_n}^{\beta_1\cdots\beta_m} (X, r^{(i)} ; \tau) \text{ in PF}, \] (51a)

---

19 See the footnote on Eq. (30).
20 See the implication i) in Footnote 16).
\[ \frac{i}{\hbar} \frac{d}{dT} \Phi_{(OF),\alpha_1...\alpha_n}(r^{(i)}; T) = \pm \hat{M} \Phi_{(OF),\alpha_1...\alpha_n}(r^{(i)}; T) \text{ in OF.} \] (51b)

These equations (51a), (51b) imply the crossing rule for composite hadrons with substitution of WF’s, \( \Phi^{(H/H)} \equiv \Phi^{(+/-)} \), corresponding to Eq.(31) in the case of solitary Urciton-Quark.

Here, it may be worthwhile to remark that the \( \tau \)-Hamiltonian \( \hat{H} \) in (47), plays a role of “Prime-Hamiltonian”, while the mass-spectral operator \( \hat{M} \) in Eq. (51b) does the role of “real” Hamiltonian in ordinary Quantum Mechanics

\textit{(Concrete Form of \( \tau \)-Wave Function and mass operator in Non-Local One-Urciton Case)}

For instruction we insert here a detailed form of the solution with definite momenta \( P(v) \), for a couple of Eqs. (47) and (48) in the ideal case of one-urciton system (, obtained by the aid of Eq. (51)) as

\[
\Phi_{P,\alpha}(X, r; \tau(v)) = \Phi_{P,\alpha}^{(+)}(X, r; \tau(v)) + \Phi_{P,\alpha}^{(-)}(X, r; \tau(v))
\]

\[
= \left\{ \begin{array}{l}
\left( \sum_{r,s} U_{r,s;\alpha}(P, M)e^{iP(v)\cdot X - i\tau(v)\xi} + \sum_{r,s} V_{r,s;\alpha}(-P, M)e^{-iP(v)\cdot X + i\tau(v)\xi} \right) O_N(r)_{r\perp v} \\
\left( \sum_{r,s} U_{r,s;\alpha}(v(P))e^{iP(v)\cdot X - i\tau(v)\xi} + \sum_{r,s} V_{r,s;\alpha}(-v(P))e^{-iP(v)\cdot X + i\tau(v)\xi} \right) O_N(r)_{r\perp v},
\end{array} \right.
\]

(52)

\[ \hat{M}^2 = \hat{m}^2(p, r) = p^2 + 2mU(r), \]

(53)

where \( \xi \) is defined in Eq. (50), \( O_N(r) \) is a set of oscillator functions relevant to mass operator (53), and \( \{ U(P, M)/U(v(P)), V(-P, M)/V(-v(P)) \} \) in Eq.(52), respectively, are the basic Dirac-spinors for constituent/Urciton quarks in \( \tilde{U}(4)_{DS,m} \)-spinor space defined in Eqs.(11a) and (11b), respectively. The above solution, corresponding to local Urciton-Quark WF Eq. (31), clearly shows how to separate the \textit{internal} \( r \)-space \( O(2)_{r\perp v} \) from the \( O(3,1)_{Lor} \)-space and how the static unitary symmetry \( U(4)_{DS,m} \otimes O(2)_{r\perp v} \) is embedded in the covariant refined-classification scheme \( \tilde{U}(4)_{DS,m} \otimes O(3,1)_{Lor} \). In the case of applying Hooke’s potential (Eq.(45)) the \( \hat{m}^2 \) becomes

\[ \hat{m}^2 = 2mU(0)\delta^2(r) + \hat{N}\Omega. \]

(54)

Here

\[ \hat{N} = p^2 + mKr^2 \] (\( \hat{N} \equiv a^\dagger a; \) Number Operator, \( \Omega = 2m\sqrt{K/m} \)).

(55)
As is well known the 1st/2nd terms in both the two equations (45a) and (45b) contribute, respectively, to the ground \( S \)-wave/excited states, and the \( \tilde{m}^2 \) is rewritten as

\[
\tilde{m}^2 = m^2 + \tilde{N}\Omega.
\]  

(56a)

The 1st term of this equation is derived as a result of the relation, as

\[
m = 2U(0).
\]  

(56b)

Thus we see the physical situation\(^{21}\) that the constituent quark mass \( m_i \) is produced by an average value of the potential \( U(r) \) at its origin. Here it may be notable that the \( \mathcal{L} \) (Eq. (39)) in one-urciton case, so far as concerned with the ground state, becomes owing to this relation identical to the \( \mathcal{L} \) (Eq. (19)) for a solitary urciton-quark.

\( (\text{Concrete Form of Multi-Urciton } \tau \text{-Wave Function and Its Relation to Multi-Quark WF in } \tilde{U}(4)_{DS,m}\otimes\mathcal{O}(2)_{r\perp v}\text{-Classification Scheme}) \)

In the preceding sub-section 3.1.2, we have described in detail the structure of Urcton-Quark WF, \( \Phi_{\alpha}(X,\tau(v)) \) in a certain inertial frame with definite boost velocity \( v \); and shown that its whole entity for all possible inertial frame with \(-1 < v_i < +1\) is equivalent to \( \Phi_\alpha(X_\mu) \), that is the constituent-quark WF in the \( \tilde{U}(4)_{DS,m}\)-spin scheme.

Further, through all the contents in the first half of this subsection 3.2 and the discussions around Eq. (36), we have seen that the freedoms on internal coordinates \( r^{(i)}'s \) are disconnected with that on External/center of mass coordinates \( X \).

Therefore our relevant relation is derived, extending directly the equation(34b) in the case of solitary Urciton quark, as

\[
\{\Phi_{P(v),\alpha_1\cdots\alpha_n}(X, r^{(i)}')\mid_{v=\text{whole}(-1<v_i<+1)}\} = \{\Phi_{\alpha_1\cdots\alpha_n}(X_\mu, r^{(i)}')\}; \text{Multi Q. WF in Evolved COQM Eq. (48)}\}.
\]  

(57)

\( (\text{Non-local Multi-Quark Wave Function - It’s Expansion into Local } \tilde{U}(4)_{DS,m}\text{-Spin Multiplets}) \)

For later use Basic Formulas in applying \( \tau \)-Quantum Mechanics to physical phenomena, obtained based upon the relation (57), are collected here. Firstly Multi-Quark WF ’s are expanded, in terms of the

\(^{21}\)It is to be noted that the physical content of Eq. (56b) does not contain any effect from something like zero-point oscillation. This comes from our choice of origin of internal coordinate \( r^{(i)}'s \), being \( X \) (external/center of mass coordinate), and the oscillators on \( r^{(i)}'s \) are pure C-number.
Concerning the Internal-Space WF

In Eqs. (58a) and (58b) the internal space WF’s and the spin WF’s are, respectively defined, as:

From these formulas, we mean

Formulas Eqs.(59a); This is a remarkable feature of our scheme (based on the LS
the conventional scheme (based on bound state picture) with
oscillator functions

Here it is to be noted that the quantum number

Concerning Spin WF

In Eqs. (58a) and (58b) the internal space WF’s and the spin WF’s are, respectively defined, as:

Concerning the Internal-Space WF

where

Concerning Spin WF

[Here it is to be noted that the quantum number \( l^{(i)} \), concerning internal orbital motion, is missing in the formulas Eqs.(59a); This is a remarkable feature of our scheme (based on the \( \tau \)-Quantum Mechanics), in contrast to the conventional scheme (based on bound state picture) with \( LS \)-coupling force potential. See also, the item v) in section 4.2, and footnote 33. See also the argument in §1]
3.2.4 Quantization of Local Multi-Urciton Hadron Fields – the Second-Stage Quantization

(Covariant Quark-Representation of General Elementary Hadrons in the $\tilde{U}(4)_{DS,m}$-Classification Scheme.) The local WF’s of $\tilde{U}(4)_{DS,m}$-multiplet, $\Phi^B_N(X)/\Phi^B_N(X)$ defined in Eqs.(58a) and (58b) are, further decomposed into the members of multiplet, representing local hadrons $\phi^i(X)/\phi^i(X)$ with definite chirality structure $\{\chi^i_N(n,m)\}/\{\tilde{\chi}^i_N(n,m)\}$, as follows;

$$\Phi^B_N(n,m):\alpha_1::\alpha_n(X) = \sum_i \phi^i_N(n,m)(X)\Gamma^i_0^\alpha_1::\alpha_n,$$  \hspace{1cm} (60a)

$$\Phi^B_N(n,m):\beta_1::\beta_m(X) = \sum_i \phi^i_N(n,m)(X)\overline{\Gamma}^i_0^\beta_1::\beta_m,$$  \hspace{1cm} (60b)

$$\chi^i_N \equiv \{r_1,r_2,\ldots,r_m\}. \hspace{1cm} (60c)$$

This is an ideal form of covariant quark-representation of elementary hadrons $\phi^i(X)/\phi^i(X)$ with definite chirality-structure $\{\chi^i_N(n,m)/\tilde{\chi}^i_N(n,m)\}$. In Eq.(60) the $\{\Gamma^0_i/\overline{\Gamma}^0_i\}$s are the tensor product of urciton-spinors concerned with respective constituent quarks, and plays a role of Clebsch-Gordon Coefficients to decompose the reducible WF, $\Phi^B_N(X)/\Phi^B_N(X)$, into the irreducible WF’s for their members of elementary hadrons $\phi^i(X)/\phi^i(X)$ with definite attributes $J^{PC}(\chi^i_N(n,m))$. Herewith it is to be noted that the $\{\Gamma^0_i/\overline{\Gamma}^0_i\}$s, corresponding to WF in most of composite models based on bound-state picture, still has definite Lorentz-transformation property.

Here it may be instructive to note that the other attributes $J^{PC}$, than the chirality $\chi^i_N$ Eq.(60c) of elementary hadrons, $\phi^i(X)/\phi^i(X)$, are determined directly by the symmetry structure, Eq.(6), of WF, $\Phi^B_N(X)/\Phi^B_N(X)$, and the tensor property of the WF’s, $\{\Gamma^0_i/\overline{\Gamma}^0_i\}$s for respective elementary hadrons.

A concrete, simple example of the decomposition of $\tilde{U}(4)_{DS,m}$-multiplet into its members with all the attributes has been given in the case of local $q\bar{q}$-meson field by Eqs.(17) through (18) in §2.2.2. Then, they are to be quantized, following to the conventional procedure with the sacred “Spin-Statistics connection”, strictly, as the 2nd-stage quantization in our proper-time quantum mechanics.

(Phenomenological Features of Evolved Classification Scheme on Light-Flavored Mesons and Baryons) Application of the Evolved Hadron-Classification Scheme actually, taking into account of Flavor structure, to the relevant problem had been made, in detail, by the authors of Refs.[8] and [9], of
which framework is the base of Evolved Classification Scheme. (Here it should be noted that the terminology, “\( \tilde{U}(12) \supset \tilde{U}(4)_{DS} \otimes SU(3)_F \)” in Ref.[9] is identical to that “\( \tilde{U}(12)_c \approx \tilde{U}(4)_{DS,m} \otimes SU(3)_F \)” in this work.) Here, out of the results presented there, we shall pick up a few notable points as follows:

i) In Evolved Classification scheme the spinor-WFs of composite hadrons for \( q\bar{q} \)-mesons and \( qqq \)-baryons are given as tensor products of the basic spinors in the \( \tilde{U}(4)_{DS,m} \)-spin space, which are the spinor WF of constituent quarks. The basic-spinors consist of Paulon / Chiralon with \( J^P = \frac{1}{2}^+ / \frac{1}{2}^- \) (see Eqs. (3) and (4)), where the Chiralon produces systematically the seats of elementary composite-mesons / baryons with new type of exotic-quantum numbers, which would never appear in Non-relativistic scheme.

ii) The spinor-WF of \( q\bar{q} \) meson in the ground \( S \)-wave state is given in the form as \( S^{(N/E)B}_A = (U_{r=+} V_{r=-} \pm U_{r=-} V_{r=+})_B^0 \cdot \delta^j \) and \( V^{(N/E)B}_A = (U_{r=+} V_{r=-} \pm U_{r=-} V_{r=+})_B^0 \cdot \sigma^j \), respectively, for the \( U(3)_F \)-nonet Scalar Meson Field, \( S^{(N/E)B}_A(r) \), and for the \( U(3)_F \)-nonet Vector Meson Field \( V^{(N/E)B}_A(r) \) (see Table 1 in Ref.[8] and Table 2 in Ref.[9]).

Here it may be instructive to note that these expressions on \( S_{A}^{(N/E)B}(r) \) and \( V_{A}^{(N/E)B}(r) \) concern their dependence on the internal space-coordinate \( (r) \) at a certain inertial frame \( (v = 0) \), while they do become, in the scope (Eq.(57)) of all possible \( \{v's\} \), respectively, their fields as \( \{S(X)\} \) and \( \{V_{\mu}(X)\} \) with \( \partial_{\mu}V_{\mu}(X) = 0 \).

Herewith, phenomenologically, the following two points are notable: Firstly the existence of seats for light-mass (, since of the ground state, ) scalar mesons, possibly corresponding to the \( \sigma \)-meson nonet-members \( \{\sigma(500), \kappa(800), a_0(980), f_0(980)\} \). Secondly the existence of seats for extra low-mass vector-mesons \( \{\omega(\sim 1200), \rho(1250), \cdots\} \), now under the serious discussions theoretically and/or phenomenologically from various view points(, for example, see §5.2 and Ref.[23]).

iii) The spinor-WF of ground \( S \)-wave state baryons, relevantly light-flavored, is classified into the three-type of chirality structure as, \( E : U_+ U_+ U_+ \), \( F : U_+ U_- U_- \) and \( G : U_+ U_+ U_- \), of which \( SU(6)_{SF} \) dimensions are, respectively, \( 56, 56', \) and \( 70 \). Out of them the spinor-WF’s of \( 56' \) and \( 70 \) multiplets include the chiralon components and accordingly they contain the exotic elementary hadrons(, see, table 3 in Ref.[9]). Herewith phenomenologically, the following two points are notable: Firstly the \( 56'-multiplet \) includes the Roper-resonance \( N(1440,1/2^+) \), which seemed too light to be a radially-excited \( S \)-wave state in the Non-relativistic scheme; while now in Evolved Scheme the mass difference between the two-ground \( S \)-wave states, \( N(940,1/2^+) \) and \( N(1440,1/2^+) \), may be understood as due to Broken-chirality by the effective strong QCD-interaction. (See, Item “Chirality Structure of Condensing Vacuum-Pair WF” and Footnote 32 in section 4.1.) Secondly
the $70$-multiplet includes the $SU(3)_F$-singlet resonance $\Lambda(1405,1/2^-)$, which seemed too-light to be a $P$-wave state in the Non-Relativistic Scheme; while now to be of reasonable mass-value in Evolved Scheme by the same reason as in the case of Roper resonance.

Herewith, it is to be noted that all the baryons with definite flavor-structure, as is listed in the PDG-booklet, is deserved to be called as $Elementally hadrons$, when their spinor-WF’s are described in the chirality-representation.

3.2.5 Features and New aspects of Hadron Classification Scheme deduced from the $\tau$-Quantum Mechanics

$(Features \ of \ \tau\text{-Quantum \ Mechanics})$ Before entering into the specific topics, we firstly pick up general features of our first-stage $\tau$-Quantum Mechanics given in the preceding subsections 3.1 and 3.2, as follows:

a) No “Relative-Time” Problem: In the most relativistic composite-models there exists the relative-time freedom, which generally leads to existence of serious unphysical-states. Contrarily, in our case, we start from the classical system with a sole proper time through all constituents (see Eq. (37)).

b) Negative energy problem and Crossing Rule: In the most relativistic quantum field theories there appear$^{23}$ negative-energy states, whose interpretation had been an annoying problem. For its disposal now generally it is applied so-called “Crossing Rule” rather in ad hoc way. However, this rule $be \ dictated$, $^{24}$ in this work, as a result of the $\tau$-Schrödinger wave equation, which contains only first-order $\tau$-time derivative (see, Eqs. (51a), (51b) and also argument after Eq. (34b)).

c) Origin of “Unitary” Quantum Numbers, as Attribute of Composite Hadron: Our wave functions of composite hadron are defined in $\tilde{U}(4)_{DS,m}^{(q)} \otimes \tilde{U}(4)_{DS,m}^{(\bar{q})}$-spinor $\otimes O(2)_{r \perp v}$-space; where its former part is, in OF, Lorentz-covariant; and embedded, in PF, the unitary symmetry $U(4)_{DS,m}^{(q)} \otimes U(4)_{DS,m}^{(\bar{q})}$, which is the origin of separate-conservation of quark and anti-quark numbers etc.; while its latter part $O(2)_{r \perp v}$ has an unitary invariant norm. Needles to mention that the tensor property in the $\tilde{U}(4)_{DS,m}$-space of the former part leads to existence of plural different intrinsic-spin members within a relevant single $\tilde{U}(4)_{DS,m}$-multiplet.

$(New \ Aspects \ of \ Yukawa-type \ Klein-Gordon \ Equation \ for \ Multi-Local \ Hadron \ Field \ )$ The $\tau$-Quantum Mechanics has started from a classical action(Eq. (39)) for multi-quark system, which

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$^{23}$Note that the relevant WF has to satisfy Klein-Gordon wave equation with the second-order time derivative.

$^{24}$Here it may be instructive to note that development of the $\tau$ is always directed to Future (i.e. $d\tau > 0$) and $\hat{\xi}(M)$ has positive eigen-value.
seems to represent well the physical situations of quark confinement. The relevant Klein-Gordon Equation for the WF in the $\tilde{U}(4)_{DS,m} \otimes O(2)_{r,\perp v}$ scheme corresponds in the $\tau$-Quantum Mechanics to the gauge constraint, reflecting that the time scale of confined system is unobservable (see argument around Eq. (25)). Moreover, the $\tau$-Quantum Mechanics has also given the other new aspects on the Klein-Gordon Equation as follows:

- On Basic Vector of $SU(2)_{m}$-spin space: It leads to the Chiralon($J^P = (1/2)^-$) as a “shadow particle” of the Paulon($J^P = (1/2)^+$). It may be regarded as a “symbolic entity” of Quark confinement-mechanism (see, Argument after Eq. (34b)).

- On $\hat{M}^2$ and Potential $U^{(i)}(r^{(i)'s})$ (See, Eq. (48)): $\hat{M}^2$ is an independent sum of respective harmonic oscillators of each constituent, and constituent quark mass is given by average value $\tilde{U}^{(i)}(0)$ around center of mass position of parent (see, Eq. (56b)).

- On True Nature of Constituent Quark / Urciton: It is a simulator of Bounded Quark by Non-Perturbative QCD potential, and plays a role of carrier of the spinor-indices ($\alpha_i's$) and the internal extension variables ($r^{(i)'s}$). Accordingly the various viewpoints on statistics of quarks in 1960’s (for example, para-statistics of order 3 [18] and white-Bose Quarks[16]25) become presently meaningless. In this connection it is to be noted that the origin of all $r^{(i)'s}$ is fixed at the external coordinate $X$ of the relevant composite hadron, and hence necessarily let our framework be satisfied Cluster Property.26

4 Chirality Symmetry in Composite-Hadron Physics

4.1 Degenerate Ground State in Urciton-Quark Pair System and Vacuum Condensation

(Chirality Conservation in Hadron Physics and Basic Urciton-Quark Wave Function) In QCD (more generally in the standard gauge theory) the interaction of quarks with gluons (relevant gauge bosons) is introduced through the minimal substitution of covariant derivative, applied on the kinetic term of relevant quarks in the Lorentz-invariant free quark-action. Therefore, as has been discussed in the subsection 2.2.1, the chirality (a new discrete symmetry) is, in the case without condensed-vacuum effects, conserved in QCD (and all the gauge theories) regardless of light- or

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25See, especially, S. Ishida and M. Oda cited in this Ref.[16].
26See, a textbook on “Specific Quantum Mechanics” by S. Kamefuchi and M. Omote[19].
heavy-mass of the relevant quarks. Accordingly in the strong interaction (, whose underling basic
dynamics is QCD), and also in all the other effective interactions the chirality should be conserved.
In order to satisfy this requirement in a simple way the Urciton-Quark WF has been expanded in
terms of the Extended Dirac Spinors in the \( \tilde{U}(4)_{DS,m} \)-Scheme as Eq. (10).

\textit{(Bi-local }\tau\text{-WF for Urciton-Quark Pair (}U_r, \tilde{V}_r\text{) System and Degenerate Ground State})
Making use of the general formulas (Eqs. (58) and (59)) for a system of multi-urciton system given in
sub-section 3.2.3, we can write down the necessary formulas in the present case of Urciton-Quark
Pair system as:

Non-local WF :
\[
\Phi_{\alpha}^{(P)}(X; r_1, r_2) = \sum_{N} \Phi_{N,\alpha}^{(P)}(X)O_N(r_1, r_2),
\]
\[
\Phi_{\beta}^{(P)}(X; r_1, r_2) = \sum_{N} \Phi_{N,\beta}^{(P)}(X)O_N(r_1, r_2),
\]
(61a)

Local-Pair WF:
\[
\Phi_{N,\alpha}^{(P)}(X) = \sum_{P_N(P_N, \alpha > 0)} \left( W_{\alpha}^{(P)}(\mathbf{v}(P_N))e^{ip_NX} + W_{\alpha}^{(P)}(\mathbf{v}(P_N))e^{-ip_NX} \right),
\]
\[
\Phi_{N,\beta}^{(P)}(X) = \overline{\Phi_{N,\alpha}^{(P)}(X)}
\]
\[
= \sum_{P_N(P_N, \alpha > 0)} \left( W_{\beta}^{(P)}(\mathbf{v}(P_N))e^{ip_NX} + W_{\beta}^{(P)}(\mathbf{v}(P_N))e^{-ip_NX} \right).
\]
(61b)

Internal-Space WF \(^{27}\):
\[
\mathcal{M}^2(p^{(1)}, r^{(1)}; p^{(2)}, r^{(2)})O_N(r^{(1)}, r^{(2)}) = M_N^2O_N(r_1, r_2) \quad \text{and}
\]
\[
M_N^2 = M_{N_1}^2 + M_{N_2}^2 = m_1^2 + m_2^2 + N_1\Omega^{(1)} + N_2\Omega^{(2)},
\]
\[
\text{where } \mathcal{M}^2(p^{(1)}, p^{(2)}; r^{(1)}, r^{(2)}) = \hat{m}_1^2(p^{(1)}, r^{(1)}) + \hat{m}_2^2(p^{(2)}, r^{(2)}),
\]
\[
N = \{N_1, \hat{N}_2\}, \quad \hat{N}_1, \hat{N}_2 \equiv \{n_{1,2}\}, \quad N_{1,2} = 2n_{1,2} \quad (n_{1,2} = 0, 1, 2, \cdots),\]
\[
O_N(r^{(1)}, r^{(2)}) = O_{N_1}(r^{(1)})O_{N_2}(r^{(2)}), \quad \hat{m}_1^{(1,2)} = m_{1,2}^2 + \hat{N}_{1,2}\Omega^{(1,2)}.
\]
(62a)

\(^{27}\)As has been described in detail in the sub-section 3.2.3, the relevant internal WF is determined by \(\hat{\mathcal{M}}^2\), which is
an independent sum of the respective Harmonic Oscillators on each \(r^{(i)}\), being connected with the center of mass of
parent’s hadrons. As for details, see around Eqs.(48) through (51).
Intrinsic-Spin WF\(^{28}\):

\[
W_{\alpha}^{(P)\beta}(P, M) = \sum_{r, \bar{r}} U_{r, \alpha}(P, M) \tilde{V}_{\beta}(P, M), \quad W_{\alpha}^{(P)\beta}(P, M) = \sum_{r, \bar{r}} V_{r, \alpha}(P, M) \bar{U}_{\beta}(P, M);
\]

\[
W_{\beta}(\bar{P})\alpha(P, M) = \sum_{r, \bar{r}} V_{\bar{r}, \alpha}(P, M) \bar{U}_{\beta}(P, M), \quad W_{\beta}(\bar{P})\alpha(P, M) = W_{\alpha}^{(\bar{P})\beta}(P, M);
\] \hspace{1cm} (62b)

while the urciton-spinors satisfy the equations, as

\[
\left[ (iu_\mu \gamma_\mu + \rho_{(m), 3}) \right] U_{r}(v(P)) |_{\alpha} = 0, \quad \left[ \bar{V}_{\bar{r}}(v(P)) \left(iu_\mu \gamma_\mu + \bar{\rho}_{(m), 3}\right) \right] |_{\beta} = 0.
\] \hspace{1cm} (62c)

Here it may be instructive to note that the \(\Phi^{(P)\beta}(X, r_1, r_2)\) (61a) is the solution of \(\tau\)-Shrödinger Eq. (47) with “Prime”-Hamiltonian, while its positive- and negative-frequency parts (determined by the aid of Eq. (61b)) correspond to those of “real” \(\tau\)-Shrödinger Eq. (51b) with \(\hat{M}\), to be called as “mass-spectral” Hamiltonian (See the discussion after Eqs. (51a), (51b).

Now we shall concentrate on the properties of the ground state of the pair-system with \(n_{1, 2} = 0\), and \((r, \bar{r}) = (\pm, \mp)\).

On the above, the urciton-quark and the pair 4-momenta, respectively, \(p^{(i)}_{\mu}(i = 1, 2) \rightarrow (U, V)\) and \(P^{(P)}_{\mu}\) are related each other with parton-like motion \(p^{(i)}_{\mu} = \kappa_{i} P^{(P)}_{\mu}\) (Eq. (100a)), and given, in a simple case of single Flavor (\(\kappa_1 = \kappa_2\), as:

Momentum: \(^{29}\)

\[
P^{(P)} = \sum_{i} p^{(i)} = p^{(1)} + p^{(2)} = \kappa_1 P^{(P)}(v) - \kappa_2 P^{(P)}(v) = 0,
\] \hspace{1cm} (63a)

Energy \(E\):

\[
\hat{P}_0^{(P)} \equiv \hat{H}(U) + \hat{H}(V), \quad \hat{H}(U_{\pm}) \equiv \alpha^{(1)} p^{(1)} + \beta^{(1)} m^{(1)}_{\pm} \rightarrow \beta^{(1)} m^{(1)}_{\pm} \text{ etc.,}
\]

\[
P_0(v)(P) = \begin{cases} (v=0) & \beta^{(1)} m_{+/+} + \beta^{(2)} \bar{m}_{-/+} = m^{(1)}_{+/-} + \bar{m}^{(2)}_{-/+} = 0, \\ (v\neq0) & \xi^{(1)}(p(v)) - \xi^{(2)}(-p(v)) = 0 \end{cases}
\] \hspace{1cm} (63b)

where

\[
\xi^{(1)}(p^{(1)}) \equiv \sqrt{p(v)^2 + m^{2}_1}, \quad \xi^{(2)}(p^{(2)}) \equiv \sqrt{(-p(v))^2 + \bar{m}^{2}_2}.
\] \hspace{1cm} (63c)

\(^{28}\)Here we use the constituent quark spinors, Eqs. (11a).

\(^{29}\)Generally this equality comes from the kinematical constraint Eq. (41). See, the footnote 18.
Thus we see that the relevant pair-system has the 4-momentum similar to that of vacuum state with infinite degeneracy, as

$$P^{(P)}_\mu(P^{(P)}, P_0^{(P)}(v)) = (0, 0) \text{ for } (-1 < v_i < +1).$$

(64)

Here it is to be noted that the relevant pair is a composite state of Pauli- and Chiral- urciton (the latter is very symbolic entity of confinement).

(Chirality Structure of Condensing Vacuum-Pair Wave Function) Spontaneous Breaking (SB) of chiral symmetry due to vacuum condensation is a well known mechanism in hadron physics. A necessary condition for the SB is the corresponding vacuum/ground state has Infinite Degeneracy. In the last item it has been shown that this infinite degeneracy occurred also in our relevant Ground State (GS) of chirality symmetric bilocal field for Urcton-pair ($U_{r,\bar{r}}$, $\bar{V}_{\bar{r}e}$) system. Furthermore, in the next item we shall describe the Dynamical Breaking (DB) of chirality symmetry in the same case of urciton-pair ($U_{r,\bar{r}}$, $\bar{V}_{\bar{r}e}$) system (to be called the “Vacuum-Pair System”), where the breaking scale (in the case of light flavor) is the mass of $\sigma$-meson, $m_\sigma \approx 600$ MeV.

In order to show the degeneracy of GS we have discussed the energy-momentum of pair-system by applying the constituent-quark spinor equation, while here will be described the chirality-structure of the spin WF of Vacuum-Pair System by use of Urcton Spinor equation in chirality representation (13b).

Our relevant local urciton-pair meson WF $\Phi^{(P)}_{\alpha\beta}(X)$ and its anti-pair meson WF $\Phi^{(P)\beta\alpha}(X)$ are given by Eq.(61b), with their respective intrinsic-spin WF’s, $W^{(P)}_{\alpha\beta}(v(P))$ and $W^{(P)\beta\alpha}(v(P))$ defined in Eq.(62b), with the appropriate combination of $(r, \bar{r})$ to the vacuum pair. These spin-WF’s are, respectively, decomposed into the eight elementary-members with definite attributes $\{J^{PC\chi}\}$, following the procedure described by Eqs. (17) and (18).

Here, out of them, we shall select the relevant spin-WF’s $W^{(V)}_{\alpha\beta} = \{\Gamma_{S(i)}\}^{\beta\alpha}$’s to condensing vacuum-pair mesons, the scalar-type ones with definite and/or non-definite chirality $\Phi^{(S)}_{S\alpha}$’s, as follows:

These Scalar-Meson States with respective spin WF $\{\Gamma_{S(i)}\}$’s are given by

$$\Phi^{V_{ac.(-)}}_S(X) : \Gamma^{(-)\beta}_{S\alpha} \equiv [1]_{\alpha},$$

$$\Phi^{V_{ac.(+)}}_S(X) : \Gamma^{(+)}_{S\alpha} \equiv [iu_\nu \gamma^\nu]_{\alpha}^{\beta}(v=0) \rightarrow [-i\gamma^4]_{\alpha}^{\beta},$$

$$\Phi^{V_{ac.(0)}}_S(X) : \Gamma^{(0)}_{S\alpha} \equiv [1 + iu_\nu \gamma^\nu]_{\alpha}^{\beta}(v=0) \rightarrow [1 - i\gamma^4]_{\alpha}^{\beta}.$$
Herewith, there is a relation as

$$\Phi_{V^{ac,(0)}}(X) \approx \frac{1}{2} \left( \Phi_{V^{ac,(-)}}(X) \oplus \Phi_{V^{ac,(+)}}(X) \right),$$

(66)

which implies that the condensing vacuum-pair meson $\Phi_{V^{ac,(0)}}$ is an equal-weight mixed state of the two-“elementary” composite-hadrons, $\Phi_{V^{ac,(-)}}$ and $\Phi_{V^{ac,(+)}}$, with the mutually-opposite chirality.

The one of two elementary vacuum-pair mesons in Eq.(66), $\Phi_{V^{ac,(-)}}$, has its attribute $J^{PC\chi} = 0^{++}$. Accordingly, it may be identified with the observed meson “$f_0(500)$ or $\sigma$” in PDG-list. This implies that the relevant chirality-symmetry is broken with the order of mass, $\Delta M \approx 500\text{MeV}$, due to the effect of vacuum condensation.

**Dynamical Origin of “Broken-Chirality” and Strong QCD-interaction**  The origin of symmetry breaking described in the last item is due to formation of the mixed-state of equally-weighted two elementary composite-mesons with mutually opposite chirality. This is so quite-new mechanism of, might be called as, “Gemisch-Broken” Chirality Symmetry, that, in the following, we shall sketch out its possible dynamical-origin in the relevant $\tilde{U}(4)_{DS,m}$-classification scheme:

i) First it is pointed out that the constituent-quark field $\Phi_\alpha(X)$ (See, Eq.(10)) plays a role of the basic asymptotic-state of S-matrix, representing those of general composite hadrons: as their WF’s are on mass shell, where exists the “Parton- and Parent-” mass relation as $m_r/\bar{s} = \kappa_r/\bar{s} M$

$$(0 < \kappa_r/\bar{s} < 1, \sum_{r,\bar{s}} \kappa_r/\bar{s} \equiv 1 \text{ (See, Eq.(100).)}$$

ii) Secondly it is also noted that a simple effective QCD-interaction between quarks is given by

$$L^{(I)}_{QCD}(\psi_\alpha) \approx \int d^4X (\bar{\psi}(X)^{\alpha} \psi(X)_{\alpha})(\bar{\psi}(X)^{\beta} \psi(X)_{\beta}),$$

(67a)

which is conserving the chirality, although the single factor of scalar-current has negative-chirality. Then this $L^{(I)}_{QCD}(\psi_\alpha)$ induces the strong interaction among composite hadrons via the basic strong

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30 Some evidence for this fact will be shown in §5 Some Phenomenological facts crucial to validity of the Evolved Scheme. A similar fact concerning the mixed-state behavior of the resonance $X(3872)$ has been pointed out by us in Ref.[2], Proceedings of the 1st CST-MISC Joint-Symposium.

31 See, a text book on “Basic Quantum Mechanics” by S. Machida[20].

32 Here it may be worthwhile to note that mass difference between $N(940, 1^+; \text{Nucleon})$ and $N(1440, 1^+; \text{Roper})$, belonging[9] to the same(GS)-multiplets in the $\tilde{U}(4)_{DS,m}$ -scheme, be just of this order.

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interaction of urciton-quarks, simulator of confined quarks (inside of composite hadrons), as

$$\mathcal{L}_{str}(\Phi_\alpha) \approx \int d^4X (\Phi(X)^\alpha \Phi(X)_\alpha)(\Phi(X)^\beta \Phi(X)_\beta).$$  \hspace{1cm} (67b)

iii) Thirdly each pair of urciton-quarks $$\Phi^{Vac,\beta}_\alpha(X) \equiv \Phi_\alpha(X)\bar{\Phi}^\beta(X)$$ is replaced by covariant quark-representation, see Eq.(17), as

$$\Phi^{Vac,\beta}_\alpha(X) = \sum_i \Phi^{(i)}(X)\Gamma^{(i)\beta}_\alpha.$$  \hspace{1cm} (67c)

iv) Then Eqs.(67b) and (67c), identifying $$\Phi^{(i)}(X)$$ with $$\Phi^{Vac,(0)}_{S,Vac}$$, lead to the condensed vacuum $$|O^{(c)}\rangle$$, as is seen in the following formulas;

Condensed vacuum

$$|O^{(c)}\rangle = |O^{(c)}_1\rangle \oplus |O^{(c)}_2\rangle,$$  \hspace{1cm} (68a)

$$|O^{(c)}_1\rangle \equiv |0\rangle, \quad |O^{(c)}_2\rangle \equiv |\Phi^{Vac,(0)}_{S,Vac}\rangle,$$  \hspace{1cm} (68b)

where Eq.(68b) should be understood as

$$|\Phi^{Vac,(0)}_{S,Vac}\rangle \approx \frac{1}{2}\sigma^{(+,+,-)} \oplus \frac{1}{2}\sigma^{(+,-,+)}.$$  \hspace{1cm} (69)

v) The expectation value of chirality by condensed vacuum is easily seen to be zero, by noting the equation as

$$\langle \Phi^{Vac,(0)}_{S,Vac}|\chi|\Phi^{Vac,(0)}_{S,Vac}\rangle = \frac{1}{2}\langle \sigma|\chi|\sigma \rangle + \frac{1}{2}\langle \sigma'|\chi|\sigma' \rangle = \frac{1}{2}(-1) + \frac{1}{2}(+1) = 0.$$  \hspace{1cm} (70)

Finally in this item it should be noted that the complete set to span the relevant Fock-Space, in the condensed vacuum, is given by pure-vacuum, and definite $$\chi$$-state, as shown in

$$|\hat{1}\rangle = |0\rangle \langle 0| \oplus |\sigma^{(+,+,-)}\rangle \langle \sigma^{(+,+,-)}| \oplus |\sigma^{(+,-,+)}\rangle \langle \sigma^{(+,-,+)}|. \hspace{1cm} (71)$$

4.2 Evolved Scheme of Spectroscopy for Elementary Hadrons based on $$\tau$$-Quantum Mechanics

The contents, thus far presented in section 2 and 3, may be summarized as follows: First in section 2 the kinematical framework of composite hadrons is shown, semi-phenomenologically, to be
\[ \tilde{U}^{(q)}_{DS,m} \otimes \tilde{U}^{(q)}_{DS,m} \otimes O(2), \perp \nu (\text{see, Eq.}(6)) \]; while the \textit{elementary} composite hadron should have its new attribute \{\chi_m\}, definite chirality (see, Eq.(18c)), in addition to the conventional \{J,P,C\}, as \( J^{PC\chi_m} \). Then in section 3 the \( \tau \)-Quantum Mechanics has been developed on a multi-quark system in Particle Frame with constant boost-velocity \( \nu \); where constituent quarks have their space-time coordinate \( x^{(i)}_{\mu}(\tau(\nu))'s \) and the Pauli-type spin \( \sigma^{(i)}(\nu)'s \), depending on only \( \nu \). Starting from application of the variation method to an action-integral on \( \tau(\nu) \) for this system, we have performed all similar to conventional procedures to develop Classical Mechanics, Quantum Mechanics; and the Quantum Field Theory, which actually leads to the relevant composite-hadron spectroscopy. As the result it is found that the level scheme of \textit{elementary} hadrons, thus obtained, has the same structure as that described in section 2. Accordingly, this framework of level scheme now be called as “Evolved Scheme of Spectroscopy for Elementary Hadrons”.

(Physics-Standpoint and Basic Framework of the Elementary-Hadron Spectroscopy )

i) \textit{Sole proper-time}(Eq.(37)) through all constituent-quarks has been requisite for application of the variation method on multi-particle system. Furthermore, in order to get rid of the undesirable connection between the external- and internal- coordinates(Eq.(1c)) is set the Lorentz-invariant restriction Eqs.(5a, 5b), depending upon the boost velocity \( \nu \). This has made possible the \textit{unified 1st/2nd-stage quantizations} on the internal/external coordinates of the same system; which is contrary to the conventional 1st/2nd-quantizations, performed through mutually different theoretical schemes.

ii) \textit{Particle Frame as an Inertial frame representing the situation of Quark-Confinement}. All constituents, within space-like extension, in a certain PF (with boost velocity \( \nu \)) do keep their respective 4-momenta \( p^{(r/s)}_{\mu}(\nu) \equiv \kappa^{(r/s)} P_{\mu}(\nu) \) (Eqs.(22) and (100)), likely making free-motion. This is simply coming from the \textit{independency} of relevant PF(\( \nu \)) from all the other PF(\( \nu' \neq \nu \)), where the corresponding 4-momenta \( p^{(r)}_{\mu}(\nu') \) being different from \( p^{(r/s)}_{\mu}(\nu) \).

On the other hand the \( \tau \)-Schrödinger Equation (29) includes the solution( see, Eqs.(33a) and (33b)) of Chiralon-quark(\( J^{P} = \frac{1}{2}^{-} \)) as a shadow particle of the conventional Paulon-quark(\( J^{P} = \frac{1}{2}^{+} \)), of which two states consist the basic vectors of \( SU(2)_{m}\)-space. This freedom is the origin of new attribute \{\chi\} of composite hadrons.

iii) \textit{Consideration within the Scope of Whole PF} with all possible \( \nu (-1 < v_i < 1) \): has shown that the multi-local \( \tau \)-WF’s in OF is equal to the corresponding multi-quark WF’s in \( \tilde{U}^{(4)}_{DS,m} \otimes O(2) , \perp \nu' \) -scheme, which is now to be called the Evolved COQM. The equality Eq.(57) is reflecting that all \( \nu \)'s (with mutually different boost velocity \( \nu' \neq \nu'' \)) have the \textit{equality as Inertial Frame, respec-
(Concrete Framework of the Elementary-Hadron Spectroscopy)

iv) Lorentz-Invariant Separation into the External and Internal coordinates of the constituent particles has been made (see, the caption of Fig.1), as

\[
X_\mu \{X_{\text{CM}}(x^{(i)})s, \tau(v)\}; \quad r^{(i)}_\mu (\equiv x^{(i)}_\mu - X_{\text{CM}}) \{r^{(i)} = x^{(i)} - X_{\text{CM}}, r^{(i)}_0 = 0\},
\]

(72a)

where

\[
x^{(i)}_\mu \{x^{(i)}, \tau(v)\} \quad \text{and} \quad X_{\text{CM}} \equiv \sum_{i=1}^N m_i x^{(i)}/M_0.
\]

(72b)

This leads to the kinematical constraint on the internal freedom (see, Eqs. (35b) and (41)),

\[
\sum_i m_i r^{(i)} = 0, \quad \sum_i p^{(i)} = 0,
\]

(73)

which has made possible the unified and separate 1st/2nd-stage quantizations mentioned in i).

Here it is to be notable that the freedom \( f \) of relevant system is determined by the independent spatial-coordinates \( (X_{\text{CM}} \text{ and } r^{(i)}s) \) of \( N \)-constituent quarks; as \( f_{\text{Tot}} = f_{\text{Int}}(r^{(i)} = x^{(i)} - X_{\text{CM}}) + f_{\text{Ext}}(X_{\text{CM}}) = (3N - 3) + 3 = 3N \), where the term \((-3)\) comes from the kinematical constraint, Eq. (73), and implying that all the origin of \( r^{(i)}s \) are connected at \( X_{\text{CM}} \).

v) The intrinsic spin \( J \) of the relevant parent hadron comes only from those of constituent quarks, \( S = \sum_i S^{(i)} \), while no internal orbital-angular momenta, \( L = \sum_i l^{(i)} \ (l^{(i)} \equiv r^{(i)} \times p^{(i)}) \), appear, as

\[
J = S \ (\text{but Not} \ J = S \oplus L).
\]

(74)

Especially for the urciton pair-state only appear the elementary hadrons with the spin \( J^{(P)} = \frac{1}{2} \) or \( \frac{1}{2} \), as the scalar or pseudo-scalar and vector or axial-vector states.

\[
J^{(P)} = \frac{1}{2} \otimes \frac{1}{2} = 0 \oplus \frac{1}{2}.
\]

(75)

They consists of the eight-types of elementary-mesons with the respective attribute \( J^{PCX} \), as was shown in Eqs.(18).

---

33 It may be notable that Particle Frame of a particle with definite orbital motion has non-zero acceleration, and accordingly is not an inertial frame.
vi) All the radially excited states of constituent quarks contribute to the mass of the relevant parent hadrons with the oscillator quantum \(\{N^{(i)} = 2n^{(i)}\}\) in the formula Eq.(59a) as
\[
M_N^2 = \sum_i m_{N^{(i)}}^2, \quad m_{N^{(i)}}^2 = m_i^2 + N^{(i)} \Omega^{(i)} = m_i^2 + 2n^{(i)} \Omega^{(i)} \quad (i = 1, 2).
\]
(76)
Especially for the ground state of urciton-pair mesons, see the linear-mass formula Eq.(81c).

4.3 Chirality-Symmetric, Effective Strong Interactions

As it has been pointed out on i), in the last item of §4.1, (Dynamical Origin of “Broken-Chirality” and Strong QCD-interaction), that the constituent quark field \(\Phi_\alpha(X)\)\(^{34}\), defined in Eq.(10), plays a role of the basic asymptotic state of \(S\)-matrix, representing those of general, local composite-hadron/anti-hadron fields in the \(\tilde{U}(4)_{DS,m}\)-spin multiplets, \(\Phi^{(H)\beta_1\cdots\beta_m}(X) / \tilde{\Phi}^{(H)\alpha_1\cdots\alpha_n}(X)\) defined in (Eqs.(58a) and (58b)). Then by virtue of the covariant quark representation, Eqs. (60), it is able to derive\(^{35}\) the effective strong-interaction among the relevant elementary hadrons \(\phi^{(i)}_{\chi\hat{N}(n,m)}(X) / \bar{\phi}^{(i)}_{\chi\hat{N}(n,m)}(X)\), the members of the above multiplets.

Its procedure is as follows: First write down the possible, with the \(L_{str}^{(i)}(\Phi_\alpha)\) in Eq. (67b), urciton-quark line diagram concerning on the spin WF’s \((W^{(H/\bar{H})}(\psi(P)))\), Eq. (59b), of the relevant \(\tilde{U}(4)_{DS,m}\)-spin multiplets. This diagram reflects their composite-quark structures. Then, taking up the trace on urciton spinor indices, it leads to the effective interaction among the elementary-hadrons, \(\{\phi^{(i)}_{\chi\hat{N}(n,m)}(X) / \bar{\phi}^{(i)}_{\chi\hat{N}(n,m)}(X)\}'s\) (, defined in Eqs.(60)), in the world of Local Hadrons, \(H_{\hat{N},\hat{S}}(X_\mu)\)'s with \(\tilde{U}(4)_{DS,m}\)-Symmetry, the final-stage object in Fig.1. Herewith it is to be noted that these effective interactions play, essentially a role of the \(S\)-matrix, expanded by the elementary-hadron states, as was mentioned at the beginning of this subsection.

4.4 CPT-Theorem applied to Elementary Hadrons

The validity\(^{21}\) of the CPT theorem is based on the invariance of the theory to the group of Lorentz transformations, the sacred spin-statistics connection, and the Locality of the theory. The kinematical framework, \(\tilde{U}^{(q)}(4)_{DS,m} \otimes \tilde{U}^{(q)}(4)_{DS,m}\)-spin scheme, of our hadron spectroscopy satisfies all these requirement, so the theorem is considered to be valid, as its-self, in our scheme; except

\(^{34}\)Here it is to be noted that, actually, the constituent quark field exist with every urcitons of the respective oscillator modes in \(O(2)_{\perp\psi}\)-space. (See, also the footnote 24.)

\(^{35}\)The contents of the last item, referred above, is a simple but important example of this task.
for the following revision, accompanied by a new attribute \( \chi_m \) reflecting the \( SU(2)_m \)-symmetry, of elementary composite hadrons as follows:

Applying the theorem on the conventional elementary particle-state \( |a\rangle \), and on the composite elementary-hadron state \( |H\rangle \), respectively, as

For \( |a\rangle \): \[ \hat{\Theta}|a\rangle = |\bar{\pi}\rangle, \quad \hat{\Theta} \equiv \overline{CPT}; \] (77a)

For \( |H\rangle \): \[ \hat{\Theta}_H|H\rangle = \hat{\chi}_m|\chi_m\rangle \equiv |\chi_m^{(\chi_m)}\rangle, \quad \hat{\Theta}_H \equiv \overline{CPT}\hat{\chi}_m; \] (77b)

where \( \pi \) represents the anti-particle of \( a \) etc., \( H^{(\chi_m)} \) does the chirality-partner of \( H \), and \( \hat{\chi}_m \equiv \rho_1(m) \) operates on the urciton spinors of elementary hadrons.

The equations (77) lead to the following relation, concerning a pair of the transition amplitudes in the mutually chirality-partner channels, whose existence is guaranteed by the chirality symmetry, as follows:

\[ \langle F|\hat{T}|I \rangle = \langle \chi_m^{\chi_m}|\hat{T}^{\Theta_H}|\chi_m^{\chi_m} \rangle = \langle I|\hat{T}^{\Theta_H^\dagger}|\chi_m^{\chi_m} \rangle, \] (78)

where \( \hat{T}^{\Theta_H} \equiv \hat{\Theta}_H\hat{T}\hat{\Theta}_H^{-1} \), and \( T^{\Theta_H^\dagger} \) represents transposed conjugate of \( \hat{T}^{\Theta_H} \).

This formula implies that the transformation probabilities, \( \text{Pr.} \) and \( \text{Pr.}^{(\chi)} \) between all the respective states, which are mutually chirality partners, contained in the chirality flow charts, are equal, that

\[ \text{Pr.}(I \rightarrow F) = \text{Pr.}^{(\chi)}(I^{(\chi_m)} \rightarrow F^{(\chi_m)}), \] (79a)

where is used the equation

\[ \text{Pr.} \equiv |\langle F|\hat{T}|I \rangle|^2 = \langle F|\hat{T}|I \rangle\langle I|\hat{T}^\dagger|F \rangle. \] (79b)

The equations (79a), to be called as “Equi-Probability Relation” between the transitions of chirality partners, and will play a some role, in the second item of sub-section 5.2.

5 Some Phenomenological Facts Crucial to Validity of the Evolved Hadron Spectroscopy

In the preceding subsection 4.2, have been summarized the essential points of the relevant elementary-hadron spectroscopy. Out of them in this section, the item, concrete framework of composite-hadron
spectroscopy, will be further examined,\textsuperscript{36} by comparing it to the present or ingoing phenomenological status as follows:

5.1 Theoretical Expectations

(Non-Orbital but All Radial Excitations and Regge Trajectories) It leads to the eight-types of “Regge Trajectories” for the \((U_r, \bar{V}_r)\) urciton-pair mesons, as was mentioned around Eqs.(17).

Types and Attributes \(J^{PCX}\) of Mesons

\[
J = 0: \ P_s^{(N/E)} \ 0^-/0^-; \ S_s^{(N/E)} \ 0^+/0^+,
\]
\[
J = 1: \ V_{\mu}^{(N/E)} \ 1^{--}/1^{--}; \ A_{\mu}^{(N/E)} \ 1^{++}/1^{++}.
\] (80)

Here, it is to be noted that the structure of composite WF’s for all the above pair-mesons is basically different from that in the case of non-relativistic quark model.

(Mass-Spectra of the Pair-Mesons) As was mentioned in subsection 3.2.5, the mass-squared is given by an independent sum of respective oscillators of each (presently \(U\) - and \(V\)-urcitons) constituents, \(\hat{m}^{(U)}\)\(^2\) and \(\hat{m}^{(V)}\)\(^2\) (, see Eq.(62a)).

Furthermore, in the relevant urciton-pair case, these two become identical, as a result of the kinematical constraints, as \(\hat{m}^{(U)}\)\(^2\) \(\approx \hat{m}^{(V)}\)\(^2\), see the footnote 36. Then the \(\hat{M}^2_{(M)}\) for the pair-system is given as

\[
\hat{M}^2_{(M)} = \hat{m}^{(U)}\)\(^2\) + \hat{m}^{(V)}\)\(^2\) = (m_q^2 + m_{\bar{q}}^2) + (\hat{N}_q + \hat{N}_{\bar{q}})\Omega, \ (\Omega^{(q)} = \Omega^{(q)}) \equiv \Omega, \quad (81a)
\]
which is effectively one-oscillator on one-dimensional space-vector \(r_{(M)}\)\(= r^{(1)} - r^{(2)}\).

\[
\hat{M}^2_{(M)} = m_q^2 + m_{\bar{q}}^2 + 2\hat{N}\Omega \ (\hat{N}_q = \hat{N}_{\bar{q}} \equiv \hat{N}), \ \text{and} \ M^2(n) = m_q^2 + m_{\bar{q}}^2 + 2N\Omega \ (N = 2n). \quad (81b)
\]

Herewith it is added the linear Mass formula for the ground-state of \((U_r, \bar{V}_r)\) urciton-pair mesons; as

\[
M_{(M), G.S.} = m_q + m_{\bar{q}}, \quad (81c)
\]

\textsuperscript{36} In this section we shall concentrate on the case of vector mesons of single-flavored urciton-pair, whose property is directly connected with the chirality symmetry.
derived by use of Eq.(49b). Then the oscillator wave function, \( H_N(\rho) \), is normalized Hermite-polynomial of \( \rho \) (\( \rho \equiv \sqrt{\Omega q/2r} \)), order \( N(= 0, 2, 4, \cdots) \), of which values at the origin are given as

\[
H_N(0) = \frac{1}{\sqrt{2^{N-1}N!\sqrt{\pi}}} \frac{(-1)^N}{(\frac{N}{2})!}.
\]

(81d)

It is seen from Eq.(81d) that all the (ground and/or excited) radial WF’s have non-zero value at the origin, and is expected that our relevant vector mesons have generally the partial decay width \( \Gamma_{e^+e^-} \) into the electron pair with a comparable magnitude, as

\[
\Gamma_{e^+e^-} \text{ of our relevant vectors } \neq 0.
\]

(82)

Actually, phenomenologically, PDG\[22\] reports that it is “seen” for all the vector meson with mass \( m_V < 2\text{GeV} \). Accordingly the expectation (82) seems to be consistent with experiments.

(Pair-Production of Chirality-Partner Vector Mesons via Energetic-Photon \( \gamma^* \)) Firstly it is to be noted that the \( \gamma^* \) is ignorant on the notion of composite-hadron chirality, while has odd \( C \)-parity. On the other hand the elementary vector-meson states \( V^{(N)} \) and \( V^{(E)} \) have, the mutually opposite, definite chirality \(+\) and \(-\), although they both have odd \( C \)-parity.

Therefore, the physical situation, concerning our relevant expectation, may be represented through the formula, as

\[
|\gamma^*(-,-,(0))\rangle \rightarrow |V^{(-,-,(0))}_{(\gamma^*)}\rangle \approx \left( \frac{1}{2}|V^{(-,-,(+))}_{(N)}\rangle \oplus \frac{1}{2}|V^{(-,-,(-))}_{(E)}\rangle \right).
\]

(83)

This implies\(^{37}\) that initial \( \gamma^*-\)state with chirality “0” produces the final \( V_{(\gamma^*)}\)-states, which is to be an equal-weight “Gemisch” (mixed state) of the pure states of two-elementary composite vector-mesons, \( V^{(N)}_{(\chi^+)} \) and \( V^{(E)}_{(\chi^-)} \), as is seen from the equation

\[
\langle V_{(\gamma^*)0}\hat{\chi}|V_{(\gamma^*)0}\rangle = \frac{1}{2}\langle V^{(+)}_{(N)}\hat{\chi}|V^{(+)}_{(N)}\rangle + \frac{1}{2}\langle V^{(-)}_{(E)}\hat{\chi}|V^{(-)}_{(E)}\rangle = \frac{1}{2}(+1) + \frac{1}{2}(-1) = 0.
\]

(84)

For instruction, here are given some relevant formulas, as

\[
V^{(1)}_{(\gamma)}(q_+\bar{q}_+) = \frac{1}{\sqrt{2}}(V^{(1)}_{(N)}(q\bar{q}) + V^{(2)}_{(E)}(q\bar{q})) ,
\]

\[
V^{(2)}_{(\gamma)}(q_-\bar{q}_-) = \frac{1}{\sqrt{2}}(V^{(2)}_{(N)}(q\bar{q}) - V^{(1)}_{(E)}(q\bar{q})) ,
\]

(85a)

\[
V^{(1)}_{(N)}(q\bar{q}) = \frac{1}{\sqrt{2}}(V^{(1)}_{(\gamma)}(q_+\bar{q}_+) + V^{(2)}_{(\gamma)}(q_-\bar{q}_-)), \quad V^{(2)}_{(E)}(q\bar{q}) = \frac{1}{\sqrt{2}}(V^{(1)}_{(\gamma)}(q_+\bar{q}_+) - V^{(2)}_{(\gamma)}(q_-\bar{q}_-)).
\]

(85b)

\(^{37}\)Note that this Equation (83) is analogous to Eq.(66) in the case of vacuum-condensation.
where \( q_{+,-} \) et al. denotes the basic urciton-spinor, in the Bargmann-Wigner Representation, with eigen-value \( r(+, -) \) of mass spin \( \rho_3 \text{m} \) (see, Eq.(13a)). Herewith Eq.(85a) points out that the two \( V(\gamma) \)-states, \( V^{(1)}(\gamma) \) and \( V^{(2)}(\gamma) \), have negative C-parity and direct coupling with \( \gamma^* \), while Eq.(85b) does that the latters, \( V(N) \) and \( V(E) \), are superposed states of the formers, \( V^{(1)}(\gamma) \) and \( V^{(2)}(\gamma) \), with the mutually-opposite relative signs. Thus one of our expectation on the pair-meson systems is expectation:

Phases of the coupling constant, \( G_{V^{(1)}(\gamma^*)} / G_{V^{(2)}(\gamma^*)} \) between \( V^{(1)}(\gamma) / V^{(2)}(\gamma) \) with \( \gamma^* \), respectively, are

\[
V^{(1)}(\gamma) - \gamma^* : 0^\circ \text{ and } V^{(2)}(\gamma) - \gamma^* : 180^\circ ;
\]

or Relative Sign between \( V^{(1)}(\gamma) \) and \( V^{(2)}(\gamma) \) is minus.  

(86)

5.2 Phenomenological Facts

5.2.1 Low-Mass Vector States Produced via \( e^+e^- \) annihilation

Recently the experimental members of \( \sigma \)-group performed the very cautious reanalysis of the \( e^+e^- \) annihilation data. Their essential results in relation with our theoretical expectation Eq.(86) are collected in Table 1.

Herewith first we shall give some explanation on the item marked as a), b), c) and d) in Table 1, respectively, as

a) In the energy region, \( 0.8 \text{ GeV} \lesssim m_{3\pi} \lesssim 1.0 \text{ GeV} \), between the mass of \( \omega(782) \) and \( \phi(1020) \), which themselves have rather small-width \( \Gamma \approx 8 \text{ MeV} \) and \( 4 \text{ MeV} \), respectively, the relevant cross-section \( \sigma_{3\pi} \) has a deep dip, \( \sigma \sim 10 \text{ nb} \), which has appeared as a result of destructive interference between their contributions \( \{ \omega(\theta_\omega = 0^\circ), \phi(\theta_\phi = 180^\circ) \} \). In this situation, the contribution of \( \omega'(1250) \) (with broad width \( \sim 740 \text{ MeV} \)) in the relevant energy region is small, \( \sigma \sim \text{ a few nb} \), but plays an important role, leading us to the very good, finer fitting. The main reason of this is that the phase of \( \omega'(1250) \) is \( 180^\circ \) as a chirality partner of \( \omega \) (with \( \theta_\omega = 0^\circ \)). This produces constructive interference between most neighboring two states \( \{ \phi(\theta_\phi = 180^\circ), \omega'(\theta_{\omega'} = 180^\circ) \} \). (See, Fig.3(a) in Ref.[23].) Therefore, the choice of phase \( (180^\circ) \) on \( \omega'(1250) \) seems to be decisive.

b) In Evolved Spectroscopy concerned only on whether the relative phase is \( 0^\circ \) or \( 180^\circ \) between chirality partners with the Same Flavor. In the relevant problem is concerned the two-flavor pair

\footnote{The preliminary results have been presented at “HADRON 2015”[23].}
Table 1: Mass and Phase of Low Mass Vector States Obtained in the Reanalysis. Through the relevant Experimental Process the vector states \((V_1^{(1)}, V_1^{(2)})\) with definite \(C\)-parity have direct coupling with \(\gamma^*\). They had, thus far, been called as vector mesons; while, in Evolved-Hadron Spectroscopy, they are Gemisch of two elementary vector mesons \((V_{(N)}, V_{(E)})\) with same-flavor pair (See, Eqs.(85a) and (85b)). In the Table the identification of the \(V_\gamma\)'s in the Evolved Spectroscopy is also given. Then it makes us possible to compare the expected situations on the relevant Light-mass vector-states, with those actually obtained in the reanalysis.

**[Experimental Process]**

\[ e^+ e^- \rightarrow \rho(\pi^+ \pi^-) + \pi^0 \]

(Data) by ; SND\((\sqrt{s} = 0.66 \sim 1.38\text{GeV}) \oplus \text{Babar}(\sqrt{s} = 1.06 \sim 2.01\text{GeV})

**[Method of Analysis]**

Vector Meson Dominance with the \(V_\gamma\)'s : \([\omega(782), \phi(1020)] + \{\omega'(M'), \omega''(M'')\}\)

**[Values of Mass and Phase]** obtained in Reanalysis

\[
\begin{array}{ccc}
\text{(Mass)} & \omega(782) & \phi(1020) \\
\text{(Phase)} & =0^\circ & 230^\circ(\simeq 180^\circ)^b \\
\end{array}
\]

Decisively \(180^\circ\) \(^a\) (Nearly Zero-contribution) \(0^\circ\) or \(180^\circ\) \(^c\)

(In Evolved Spectroscopy)

Identification

\[
\begin{array}{c|c|c|}
& ^1S_1 & ^2S_1 \\
\hline
V_1^{(1)}(n_+\bar{n}_+) & V_1^{(1)}(s_+\bar{s}_+) & V_1^{(2)}(n_-\bar{n}_-) \\
V_1^{(1)}(n_+\bar{n}_+) & \\
\end{array}
\]

Expected Phase

\[
\begin{array}{c|c|c|}
\text{0\(^\circ\)(input)} & \rightarrow 180^\circ & \rightarrow 180^\circ \\
\text{due to quark charge} & \text{due to m_3-spin} & \\
\end{array}
\]

\((n\bar{n})\) and \((s\bar{s})\). The fitted phase value \((230^\circ)\) of \(\phi(1020)\), deviating from \(180^\circ\), might be interpreted as a result of the \(U(6)_{\sigma,F}\)-broken chirality symmetry, (see the last item in §4.1, and footnote 8).

c) In fact, both cases with relative Phases \((0^\circ, 180^\circ)\) of \(\omega(1650)\) give similar good \(\chi^2/d.o.f.\) This implies that the contribution of the state \(\omega(1650)\) is not interfering with those from all the other relevant vector-states. This situation is deduced naturally from our assignment; that \(\text{only } \omega(1650)\) be radially excited state, while all the other vector-states belong to the ground state.

d) The authors of Ref. [23] have performed both the two cases of the Two/Three-Resonances with \(\{\omega(1250), \omega(1650)\}/\{\omega(1250), \omega(1420), \omega(1650)\}\), and found the contribution of \(\omega(1420)\) as a \(V_\gamma\) is Zero, or very small.
From all the above considerations it may be concluded that the expectation Eq.(86) from Evolved Hadron-Spectroscopy is consistent with the results of phenomenological analysis.

5.2.2 “Riddles” Observed by Belle Experiment around the Region of $\Upsilon(10860:5^3S_1(b\bar{b}))$

A few years ago Belle collaboration[24][25] has reported the interesting data on Bottomonium system and pointed out a serious problem implied by them.

This problem is so strange as to be called “Riddle” from the conventional Non-Relativistic classification scheme; while it seems us to be so natural in the Evolved Spectroscopy that the problem itself do disappear.

In this item the relevant physical situation will be clarified. Firstly we shall study the Experimental facts summarized in Table.2.

According to the new assignment, out of Problem and \{Exp. F’s\}, the Problem do disappear, since of the relevant two-kinds of different intrinsic-spin(, triplet and singlet, ) particles, \{$\Upsilon(nS), h_b(mP)$\}, now being regarded all as the same spin-triplet elementary hadrons. Herewith the (Exp. F-3) concerning the relative phase between $(Z_{b1}, Z_{b2})$ are now derived as coming from the relative phase of their respective constituent states $(V_b^{(1)}, V_b^{(2)})$.

Finally, the remaining (Exp. F-1) and (Exp. F-2), concerning production ratio of \{$h_b(mP), \Upsilon(nS)$\} and of \{$Z_{b1}, Z_{b2}$\}, respectively, will be also explained due to the Chirality Symmetry of the elementary hadrons. The relevant physical situations are concretely shown in Fig.2. Now (Exp. F-1) may naturally come from the initial state $\Upsilon(5S)$ is actually the equal-weight Gemisch of \{$\Upsilon(5S)^{(N)}, \Upsilon(5S)^{(E)}$\}, of which two are mutually chirality-partners, belonging to the same $\tilde{U}(4)_DS, m$-spin multiplet. Then (Exp. F-2), concerning production rate for $Z_{b1}$ and $Z_{b2}$, may be understood as the result that the $Q$-values in their decay-channels, \{$\Upsilon(nS)\pi^\pm$\} and \{$h_b(mP)\pi^\pm$\}, are almost of similar values. Herewith it is to be noted that, under the above interpretation of both (Exp. F-1) and (Exp. F-2), Equi-probability Relation between the transitions of chirality partners in §4.4, has been playing a basic-theoretical role.

5.2.3 Another Phenomena Suggesting Non-Orbital Excitations

In this sub-section shall be described the essential point of an interesting topics, which is perplexing from the conventional thought of line, but might be naturally understandable in the framework of Evolved Hadron-Spectroscopy.
Figure 2: Chart of Chirality-Flow through Experimental processes: In order to see concretely the relevant $\chi_F$-conservation is to be noticed the following points: i) Initial $\Upsilon(5S)$ state considered to be $V_b^{(1)}(b_+\bar{b}_-;5S)$, is an equal-weight Gemisch of chirality-partners, $\{\Upsilon(5S)^{(N)},\Upsilon(5S)^{(E)}\}$; each of which starts, respectively, the “Light”-chart with $\chi_b = +$, and the “Shadow”-chart with $\chi_b = -$. Then ii) in Light/Shadow-chart appears $\Upsilon(nS)^{(N)}\cdot\pi\pi/\Upsilon(nS)^{(E)}\cdot\pi\pi$, where a new elementary axial-vector $J^{P(x_b)} = 1^{+(+/-)}$, $Z_b^{(N/E)} \equiv \Upsilon(nS)^{(N)} \cdot \pi/\Upsilon(nS)^{(E)} \cdot \pi$, is contained. iii) The $Z_b^{(N/E)}$ is, respectively, a superposed state as $Z_b^{(N/E)} = Z_b^{(1)} \pm Z_b^{(2)}$, where $Z_b^{(1/2)}$ is a composite state as $Z_b^{(1/2)} = (V_b^{(1)} \cdot \pi)/(V_b^{(2)} \cdot \pi)$. Herewith the relative phase between them (denoted as Exp. F-3) is understood as that of their constituent states $(V_b^{(1)}, V_b^{(2)})$, as mentioned in the text. iv) The $\chi_{F_m}$ conservation is trivially satisfied in the relevant case, where always the two pions are concerned, since $\chi_{F_m}$ of $\pi$ is $\chi_{F_m}(\pi) = -$ and $\chi_{F_m}(\pi \cdot \pi) = (-1)^2 = +$. 

\[ \Upsilon(nS)^{(N)}\text{-channel} \quad \Upsilon(nS)^{(E)} / \pi h_b (mP)^{\mu}\text{-channel} \]
Table 2: Experimental Features of New Resonances and Identification in Evolved Spectroscopy.

In Experimental Data and Properties of New Resonances, the properties of two spin singlet states \((h_b(1P), h_b(2P))\) and of two \(Z_b\)'s, are summarized as Experimental Fact-1, -2, -3 and Problem. Among them Exp. F-1/Problem corresponds to the Riddles referred in the title of this item. All of \(\{\text{Exp. F's}\}\) might be difficult to be understood in the Non-Rel. Scheme. In Identification of New Resonances in Evolved Classification Scheme new labels of elementary meson, \(A^{(E)}(mS)/\Upsilon^{(N)}(nS)\), are assigned to \(h_b(mP)/\Upsilon(nS)\). Herewith \(Z_{b1}[V^{(1)}_b \cdot \pi]\) means that \(Z_{b1}\) is the composed state of \(V^{(1)}_b\) and elementary \(\pi\) meson etc. (see §3.2.3), and \(V^{(1,2)}_b\) is the vector-states \(V^{(1,2)}(bb)\) defined in Eq.(85a).

| Two \(h_b\)'s: \(h_b(9898), h_b(10259)\) |
| Exp. F-1 Process: \(e^+ + e^- \to \Upsilon(5S) \to h_b(mP)\pi^+\pi^- (m = 1, 2) \quad J^P = 1^+\) |
| Production \(\sigma\): Comparable to \(e^+ + e^- \to \Upsilon(5S) \to \Upsilon(nS)\pi^+\pi^- (n = 1, 2, 3)\) |
| Problem Why no suppression due to heavy-quark spin-flip working? (Note \(\Upsilon(nS) \uparrow\uparrow\) v.s. \(h_b(mP) \uparrow\downarrow\)) |
| Two \(Z^{(1,2)}_{b1,2}(J^P = 1^+)\)'s: \(Z_{b1}(10610), Z_{b2}(10650)\) in Five channels: \(\Upsilon(nS)\pi^\pm, h_b(mP)\pi^\pm\) |

Exp. F-2 Production Rate: Similar for these 5-channels

Exp. F-3 Relat. Phase between \((Z_{b1} \text{ and } Z_{b2}) = (0^\circ \text{ and } 180^\circ)\) in \(\Upsilon(nS)\) and \(h_b(mP)\)-channel, respectively.

Identification of New Resonances in Evolved Classif. Scheme

Identif: \(h_b((1, 2)P) \to A^{(E)}((1, 2)S); \Upsilon(nS) \to \Upsilon^{(N)}(nS); Z_{b1,2} \to Z_{b1}[V^{(1)}_b \cdot \pi]; Z_{b2}[V^{(2)}_b \cdot \pi]\)

Anomalous Splitting of \(\psi(3770)\) in Inclusive Cross Section for \(e^+e^- \to \text{Hadrons}\) had been reported[26], almost a decade ago, by BES Collaboration. According to their cautious analysis, the line shape of cross sections in the energy-region between 3,700 and 3,872 GeV may be explained by two possible enhancements, respectively, with the center of mass energies of 3,764 and 3,779 GeV. Herewith also noted that these enhancements seem to have the relative phase

\[
\phi = 158 \pm 334 \pm 5^\circ. \quad (87)
\]

Interpretation from Evolved-Spectroscopy

The above fact is understood, as that the two enhancements being the vector-states, \(V^{(1)}_{(\gamma)}(c_+\bar{c}_+)\) and \(V^{(2)}_{(\gamma)}(c_-\bar{c}_-)\), which are equal-weight Gemisch of elementary \((cc)\)-vector mesons, \(V^{(N)}(cc)\) and
\[ V^{(E)}(c\bar{c}), \] with definite \( \{\chi_{m_{c,+}} \text{ and } \chi_{m_{c,-}}\} \), respectively, see, Eqs.(83) and (85).

Then the experimental value Eq.(87) of relative phase \( \phi \) is almost rightly to be the one, Eq.(86), expected in Evolved Hadron-Spectroscopy. Herewith it is also notable that in the relevant phenomena is concerned with the radially (possibly 1st-)excited S-wave states, and expected non-zero partial-decay width \( \Gamma_{e^+e^-} \) (as was mentioned in Eq.(82)): This expectation in Evolved Hadron-Spectroscopy seems to be supported by experiments[22].

6 Concluding Remarks and Discussions

The purpose of this paper is to present an attempt for establishing a Lorentz-Invariant Composite-Hadron (to be called Elementary-Hadron) Spectroscopy by developing Proper-time \( \tau(v) \)-Quantum Mechanics on Multi-Quark System. The contents of this paper may be classified into the three parts as

Part 1/ §II and §III : Formulation of Elementary-Hadron Spectroscopy.
Part 2/ §V : Comparison of Elementary-Hadron Spectroscopy with Experiments.
Part 3/ §IV : Chirality Symmetry in Elementary-Hadron Physics.

6.1 Specific Remarks on Elementary-Hadron Spectroscopy

6.1.1 Two Fundamental Physics Notions

(Principle of Special Relativity on Inertial Frame) Firstly, here is referred to Einstein’s sentence\(^{39}\) on Inertial Frame as :

a) Law of Nature is valid only in Inertial Frame.

b) Law of Nature is Invariant \( \text{Equally} \) in all Inertial Frames which are connected, mutually, with the Lorentz-Transformation of Space-Time Coordinates.

c) Various Inertial Frames make, mutually, \( \text{Rectilinear} \) and \( \text{Uniform} \) Motions.

In relation with the term b), it should be remarked that our method of formulating Elementary Hadron Spectroscopy, the \( \tau(v) \)-Quantum Mechanics be already Lorentz-invariant at the beginning. The reason is that it concerns all inertial frames, in the scope of, taking all possible values of \( v \), as \( (-c < v_i < c) \). As has been discussed in §4.2, the situation of quark-confinement and strange

\(^{39}\) The sentence is summarized by the present authors from relevant parts of the book[27].
behaviors of constituent -quarks, such as making Free-Particle Motion and parton-like motion, are reasonably understood from Independence in addition to Equality, seen in the term b), of respective Inertial Frame. In relation with the term c), it implies that, in Inertial Frame, there should not exist the states with definite orbital-angular momentum. Herewith, in Elementary Hadron Spectroscopy, there exist Non-orbital but All Radial Excitations, as has been shown by Eqs.(44b) and (45) in §3.2.1.

(Conservation of Chirality in QCD-Gauge Theory and “Elementary-Hadron”) The chirality is conserved through all types of the QCD interaction/upper-ground strong-interaction with each flavored-quarks. Therefore, somewhat elementary entity; the composite hadrons, consisting of exciton-quarks with definite chirality \( \{ \chi_{mf} \} \), be called as Elementary Hadrons.

### 6.1.2 Separation of Internal and External Space Coordinates

The relevant separation of coordinates leads to

\[
X_\mu \{ X \equiv \sum_i m^{(i)} x^{(i)}/M_0, \tau(v) \}; \quad r_\mu^{(i)} \{ r^{(i)}, 0 \}.
\]

This makes possible Lorentz-Invariant Unified, 1st- and 2nd-stage Quantization on \( \{ r^{(i)}, s, T \} \) and \( \{ X, T \} \), respectively. They correspond to quantization of Quantum Mechanical and Field-Theoretical ones, respectively.

Further application of variational method on Least-Action Principle at the Particle Frame has led us appearance of the Shadow quark-urciton (Chiralon) with \( J^P = \frac{1}{2}^- \), in addition to the supposed one (Paulon) with \( \frac{1}{2}^+ \). These two are basic vectors of \( SU(2)_m \)-spin space. The chirality symmetry might be broken by condensing vacuum with the pair of Paulon and Chiralon.

This appearance of chiralon has deduced a vital revision on the structure of WF for Elementary Hadrons from the conventional WF based on Non-Relativistic Quantum Mechanics.

### 6.2 Concluding Discussions

We have examined, in section 5, seriously the validity of Evolved Hadron-Spectroscopy, by comparing its expectations with the present or ingoing phenomenological facts. As the result, is obtained a fairly-promising impression. The framework of evolved Hadron-Spectroscopy be completely Lorentz-Invariant, as is remarked in §6.1.1. Therefore, it seems us that the phenomenological
knowledge, thus far obtained on the bases of Non-Relativistic Quark Model, should be reexamined carefully.

**Conflicts of interest**

The authors declare that they have no conflicts of interest.

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**A Lorentz Covariance for “Local” Composite Hadrons in the \( \tilde{U}(4)_{DS,m} \)-Spin Scheme**

*Case of Dirac Particle*  
First for instruction we recapitulate the relevant formulas for an elementary case of Dirac particle. For the infinitesimal Lorentz transformation of space-time coordinate \( X_\mu \) the wave function \( \Phi(X) \) transforms by the \( S(\Lambda) \) as,

\[
X'_\mu = \Lambda_{\mu\nu} X_\nu, \quad \Lambda_{\mu\nu} = \delta_{\mu\nu} + \epsilon_{\mu\nu}, \tag{89}
\]

\[
\Phi'(X') = S(\Lambda) \Phi(X), \quad S(\Lambda) = 1 + \frac{i}{2} \epsilon_{\mu\nu} \Sigma_{\mu\nu}. \tag{90}
\]

The generators \( \Sigma_{\mu\nu} \), and those for sub-groups of rotation and of boost are given by\(^{40}\)

\[
\Sigma_{\mu\nu} = \frac{1}{2i}(\gamma_\mu \gamma_\nu - \gamma_\nu \gamma_\mu), \tag{91}
\]

\[
J_i = \frac{1}{2} \epsilon_{ijk} \Sigma_{jk}, \quad K_i = i \Sigma_{i4}. \tag{92}
\]

\(^{40}\)We use the Pauli-Dirac representation of \( \gamma \)-matrices \( \gamma_\mu = \gamma_\mu (\gamma_1 = \rho_2 \otimes \sigma_1, \gamma_4 = \rho_2 \otimes 1_\sigma, \gamma_5 = \gamma_1 \gamma_2 \gamma_3 \gamma_4 = -\rho_1 \otimes 1_\sigma), \Sigma_{\mu\nu} = \Sigma_{\mu\nu}, \Sigma^\dagger_{\mu\nu} = J^\dagger = J \) and \( K^\dagger = K \).
In the case of Dirac spinors with \( J = 1/2 \) the generators and the finite transformation operators are explicitly given, as

\[
J_i \equiv \frac{1}{2} \sigma_i \otimes \rho_0, \quad S_R(\theta) = e^{-i \theta J},
\]

\[
K_i \equiv \frac{i}{2} \sigma_i \otimes \rho_1, \quad S_B(b) = e^{-i b K_i}, b \equiv \hat{v} \cosh^{-1} u_0 = \hat{v} \cosh^{-1} \left( \frac{1}{\sqrt{1 - v^2}} \right),
\]

where \( \theta_i \) are rotation angles around the \( i \)-axes, \( u_\mu(v_i) \) is the 4-velocity(3-boost velocity) of Dirac particle, and \( \rho_i \) and \( \sigma_i \) are the 2 by 2 Pauli matrices, representing the 4 by 4 Dirac matrices as \( \{ \gamma \} \equiv \{ \rho \} \otimes \{ \sigma \} \). It may be instructive to note that

\[
u_\mu \{ i, u_0 \} \equiv \frac{dX_\mu}{dT} = \left( \frac{v_i}{\sqrt{1 - v^2}}, \frac{i}{\sqrt{1 - v^2}} \right) \rightarrow (0, i),
\]

\[
v_i \equiv \frac{dX_i}{dT} = u_i/u_0 = P_i/P_0.
\]

Herewith, see Eq.(22).

\( \text{("Local" Composite Hadrons)} \) The external spin-WF of composite hadrons with the quark-configuration \( (n, m) \) \((n/m\) being the number of quarks/anti-quarks) are tensors in the \( \tilde{U}(4)_{DS,m} \)-space of Dirac spinor. Accordingly the generators for Lorentz-transformation are given as

\[
\Sigma^{(n,m)}_{\mu \nu} = \sum_{r=1}^{n} \Sigma^{(r)}_{\mu \nu} + \sum_{s=1}^{m} \Sigma^{(s)}_{\mu \nu} \quad (\Sigma \equiv -\Sigma^T, \Sigma^T : \text{Transpose of } \Sigma),
\]

\[
J_i = \sum_{r=1}^{n} J^{(r)}_i + \sum_{s=1}^{m} J^{(s)}_i \quad (J = -J^T),
\]

\[
K_i = \sum_{r=1}^{n} K^{(r)}_i + \sum_{s=1}^{m} K^{(s)}_i \quad (K = -K^T),
\]

where \( \Sigma^{\mu \nu} \) etc. denotes the complex-conjugate operator of \( \Sigma^{\mu \nu} \) and so on. Herewith to be noted that, in the 1st-stage quantization, the WF’s of quark and of anti-quark are mutually complex-conjugate functions. Those expressions are derived, supposing all “constituent”-quarks make an identical motion to that of parent, that is “parton-like motion”, as

\[
p^{(r/s)}_\mu = \kappa^{(r/s)}_\mu P_\mu \quad (0 < \kappa^{(r/s)}_\mu < 1, \sum_{r,s} \kappa^{(r/s)}_\mu \equiv 1),
\]

\[\text{It may be instructive to note that each component of tensor representation is transformed with the same transformation-parameters as those of basic vectors, as } \epsilon^{(r/s)}_{\mu \nu} \equiv \epsilon_{\mu \nu}. \]
where $p_\mu / P_\mu$ represents the four momentum of the quarks/parent. This leads to the relations among rotation-angles and boost-velocities of quarks/parent, as

$$\theta(p^{(r/s)}) = \theta(\hat{P}); \quad v^{(r/s)}(= p_i^{(r/s)} / p_0^{(r/s)}) = v(= P / P_0).$$

(100b)

This motion of constituent quarks seems quite strange from a view-point of the conventional composite model. It may be worthwhile here to note that the generators for tensor representation Eqs. (97) to (99) and the parton-like motion Eq. (100) are derived directly for a system of multi-quarks, where the space-time coordinate of each quark $x^{(r/s)}_\mu$ is given by center of mass coordinates as

$$x^{(r/s)}_\mu \equiv \kappa^{(r/s)} \chi^{(r/s)} \equiv \frac{m^{(r/s)}}{\sum_{r,s} m^{(r/s)}}.$$

(100c)

likely as if respective constituents shared the coordinates of parent themselves.

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