A nuclear magnetic resonance spectrometer for operation around 1 MHz with a sub 10 mK noise temperature, based on a two-stage dc SQUID sensor

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We have developed a nuclear magnetic resonance spectrometer with a series tuned input circuit for measurements on samples at millikelvin temperatures based on an integrated two-stage superconducting quantum interference device current sensor, with an energy sensitivity $\varepsilon = 26 \pm 1 \text{ nK}$ when operated at $1.4 \text{ K}$. To maximize the sensitivity both the NMR pickup coil and tuning capacitor need to be cooled, and the tank circuit parameters should be chosen to equalize the contributions from circulating current noise and voltage noise in the SQUID. A noise temperature $T_N = 7 \pm 2 \text{ mK}$ was measured, at a frequency of 0.884 MHz, with the circuit parameters close to optimum.

![Schematic diagram of the tuned NMR spectrometer](image)

**FIG. 1:** Schematic diagram of the tuned NMR spectrometer. The two-stage SQUID sensor is depicted here as a single SQUID. The device contains an integrated input coil $L_i$ and Q-spoiler. The temperature of the pickup coil $L_p$ and capacitor $C$ can be varied. Inductance $L_x$ represents the NMR transmitter coil.

Cooled preamplifiers have been employed to reduce noise in NMR systems with tuned source impedances. Performance can be characterized by the noise temperature $T_N$ (the temperature of the source at which its contribution to the total output noise power equals that of the amplifier). For a SQUID amplifier with a tuned input the optimum $T_N$ is proportional to frequency. For our experiments on NMR samples at millikelvin temperatures we use frequencies ~1 MHz, at which we may obtain a good $S/N$ with acceptable rf heating. Cooled GaAs MESFETs have been used to detect NMR signals at these frequencies, achieving $T_N \sim 1 \text{ K}$. Using a dc SQUID in open loop mode Freeman et al. obtained a $T_N$ of 300 mK at 1.9 MHz, limited by the readout electronics. Improved noise performance is possible using two-stage SQUID amplification, and this provides the primary motivation for this work. Two-stage SQUID amplifiers have been used by Falferi et al. in a resonant gravitational wave detector, with an estimated $T_N$ of 15 nK at 11 kHz, and by Mück et al. who obtained a $T_N$ around 50 mK at 0.5 GHz with a microstrip SQUID cooled to 20 mK.

In this work we have developed a tuned NMR spectrometer based on an integrated two-stage low-transition-temperature (low $T_c$) dc SQUID sensor. The spectrometer sensitivity is limited by Johnson noise in the resistive element of the input circuit and by SQUID amplifier noise. The goal of this work was to study the limit imposed by the SQUID noise. To that end we measured the spectrometer noise as a function of the temperature of the resistive pickup coil, in the absence of a sample. The pickup coil was placed in a superconducting shield to suppress external interference. Performance of a similar SQUID with a broadband superconducting input circuit used for NMR is reported elsewhere.

A schematic diagram of the tuned NMR spectrometer is shown in Fig. 1. The pickup coil $L_p$ forms a series resonant circuit with a capacitor $C$ and the SQUID input coil inductance $L_i$. $R$ is the resistance in the input circuit. In our case $R$ is mainly associated with the copper pickup coil, but has contributions from dissipation in the capacitor and from the presence of the SQUID. The SQUID sensor consists of a single SQUID first stage, read out by a 16-SQUID series array, integrated onto a single chip. The SQUID, mounted at 1.4 K (rather than 4.2 K) for improved noise performance, is connected directly to the room temperature readout electronics. This permits flux-locked loop (PLL) operation without flux modulat-
tion, and large FLL bandwidths of dc up to 6 MHz. The device contains an array of 16 unshunted SQUIDs in the input circuit, which operates as a $Q$-spoiler. This reduces the tank circuit quality factor $Q$ for high signal levels and hence shortens the recovery time from large current transients following removal of an NMR transmitter pulse.

The experiment was mounted on a nuclear adiabatic demagnetization cryostat, capable of achieving $\sim 200 \mu$K, previously used for our earlier NMR measurements on $^3$He at low millikelvin temperatures using a SQUID spectrometer with a tuned input. The NMR pickup coil was wound from copper wire and heat sunk to a plate whose temperature could be varied from 10 to 1500 mK. Care was taken to avoid thermal noise from dissipative elements in the tuning capacitor by keeping it close to the coil temperature.

Our two-stage SQUID is designed to have a single-SQUID-like flux-voltage characteristic with a large overall flux to voltage transfer function $V_\Phi$. As shown in Fig. [1] we model the device as a single SQUID of inductance $L_s$ (equal to that of the first stage SQUID), with a voltage noise $V_N$ at the output, and a circulating current noise $J_N$ in the SQUID loop. $V_N$ includes noise from the room temperature amplifier as well as from the two SQUID stages. The spectral densities of these noise sources, $S_V$, $S_J$ and a correlation $S_{V,J}$, can be considered white above a few tens of Hz. In this work we write $S_V$ and $S_J$ in terms of a flux noise $S_\Phi$ in the SQUID as follows:

$$S_V = V_\Phi^2S_\Phi \quad \text{and} \quad S_J = \zeta^2S_\Phi/L_s^2,$$

(1)

where $S_\Phi$ is the total effective flux measured with the input circuit open, and $\zeta$ is a dimensionless parameter. We define the overall energy sensitivity as $\varepsilon = S_\Phi/(2L_s)$. Tesche et al. calculated intrinsic noise in a single SQUID. They found $S_{V,J}$ to be real, and $\zeta$ to be of the order of unity but dependent on SQUID bias. We assume this to be the case for our two-stage device, since noise from the first stage is dominant. A real $S_{V,J}$ does not contribute to noise at the tank circuit resonance frequency $f_0$.

In an NMR experiment, precession of the magnetization induces a signal voltage $V_S$ in the pickup coil. In this case the total flux in the SQUID is

$$\phi = (V_S + V_{NR} - j\omega M_i J_N)/Z_T + V_N/V_\Phi.$$

(2)

Here $M_i$ is the mutual inductance between the input coil and the SQUID, $Z_T = R + j(\omega L_T - 1/\omega C)$ is the total impedance in the input circuit, where $L_T = L_i + L_p$. $Z_T$ can be influenced by coupling to the SQUID, mainly through a contribution to $R$, which we neglect in this model. Changes in the SQUID parameters due to strong coupling to the input circuit are also neglected, since they are insignificant for the case of a high-$Q$ tuned source close to optimum. $V_{NR}$ represents the Johnson noise voltage in $R$, which is at temperature $T$. This noise source is also white at the frequencies of interest. On resonance the total flux noise is given by

$$S_\Phi^{(res)} = \frac{4k_B T M_i^2}{R} + S_\Phi \left[ 1 + \frac{2\omega_0 M_i^2\zeta^2}{L_s^2R^2} \right].$$

(3)

Let the signal $V_S$ be close to the tank circuit resonance frequency $\omega_0 = (L_TC)^{-1/2}$ and measured in a bandwidth $\Delta f$. The signal-to-noise ratio can be defined as

$$S/N = V_S/\sqrt{4k_B(T+T_N)R\Delta f},$$

(4)

where from Eq. (3)

$$T_N = \frac{1}{4k_B} \left[ \frac{R}{M_i^2} + \frac{(\omega_0 M_i^2)^2}{L_s^2R} \right] S_\Phi.$$

(5)

For a given experiment the sample geometry and resonant frequency $\omega_0$ are fixed. The choice of quality factor $Q = \omega_0 L_T/R$ is limited by the desired bandwidth and the requirement of a sufficiently short recovery time following a transmitter pulse. In order to optimize the sensitivity $L_p$, $R$ and $C$ can be varied simultaneously within these constraints. Changing the number of turns in the pickup coil varies $V_S$ and $L_p$ such that $V_S \propto \sqrt{L_p}$. Then Eq. (4) can be written as

$$(S/N)^2 \propto \frac{L_p}{4k_B(T+T_N)R\Delta f} = \frac{Q(L_p/L_T)}{4k_B(T+T_N)\omega_0 \Delta f}.$$ 

(6)

The highest $S/N$ is achieved by making $Q$ as large as possible within the constraints. For $Q \gg 1$, $L_p/L_T$ is close to 1 when $T_N$ is minimized (see Eq. (5)). Therefore the $S/N$ is maximum when $T_N$ is minimum.
when Eq. (5) is written as

\[ \text{off-resonance noise for } T = -T_N \text{ with } T_N = 7 \pm 2 \text{ mK.} \]

The conditions for minimizing \( T_N \) are more evident when Eq. (5) is written as

\[ T_N = \frac{\varepsilon \omega_0 \zeta}{2k_B} \left[ \frac{1}{Q\alpha^2\zeta} + Q\alpha^2\zeta \right]. \tag{7} \]

Here \( \alpha^2 = M_i^2 / (L_i L_s) \) is the effective coupling constant between the SQUID and the input circuit. Minimizing \( T_N \) involves setting \( Q\alpha^2\zeta = 1 \), resulting in

\[ T_N^{(\text{opt})} = \frac{\varepsilon \omega_0 \zeta}{k_B}, \quad R^{(\text{opt})} = \alpha^2 \omega_0 L_i, \]

\[ L_p^{(\text{opt})} = (Q\alpha^2\zeta - 1) L_i, \tag{8} \]

where the coupling coefficient \( \alpha^2 = M_i^2 / (L_i L_s) \). For our device \( L_i = 1.1 \mu \text{H}, M_i = 7.1 \text{ nH}, \text{ and } L_s = 80 \text{ pH} \), corresponding to \( \alpha^2 = 0.58 \).

For NMR at 1 MHz a \( Q \) of the order of 100 is reasonable, then for \( \zeta = 1 \) we obtain \( L_p^{(\text{opt})} = 63 \mu \text{H} \) and \( R^{(\text{opt})} = 3.9 \Omega \). We expected \( \zeta < 1 \) so we wound a 47 \( \mu \text{H} \) pickup coil. By driving the pickup coil via the transmitter we measured \( Q = 300 \) at \( \omega_0 = 2\pi \times 884 \text{ kHz} \), from which we inferred \( R = 0.89 \pm 0.03 \Omega \) and \( Q\alpha^2 = 3.9 \pm 0.1 \). Once \( \zeta \) is known, full optimization could be achieved by addition of a further resistive element at coil temperature \( T \). For these studies of noise as a function of temperature this was not done, to eliminate potential errors arising from temperature gradients.

The temperature dependence of the noise in this tuned circuit was measured at five different SQUID working points (two open loop and three FLL), the results of which are summarized in Table I. The output of the SQUID electronics was fed to an HP 3588A spectrum analyzer with a negligible noise level. In Fig. 2 we show the frequency dependence of the flux noise measured in open loop mode (working point 1) with the pickup coil at 20 mK and the tuning capacitance at 10 mK. For a high-\( Q \) input circuit the off-resonance flux noise is equivalent to that of an open circuit. We infer the open input flux noise \( S_\Phi \) at \( \omega_0 \) from the off-resonance noise as shown by the dashed line. This corresponds to \( \varepsilon = 26 \pm 1 \hbar \), where \( \hbar \) is Planck’s constant. The inset shows noise in the vicinity of resonance as the pickup coil temperature is varied between 20 and 800 mK. The noise power at resonance, normalized by the off-resonance noise power, can be written as

\[ r(T) = \frac{S_\Phi^{(\text{res})}}{S_\Phi} = 1 + Q^2 \alpha^2 \zeta^2 + \frac{2k_B Q\alpha^2}{\omega_0 \varepsilon} T. \tag{9} \]

This is plotted versus coil temperature in Fig. 3. These data are linear over a wide temperature range. The data deviate from a linear fit at the lowest temperatures possibly due to insufficient thermalization of the pickup coil. We obtain \( T_N = 7 \pm 2 \text{ mK} \) from a fit to the linear region. We obtain an almost identical \( T_N \) with the SQUID operated in FLL mode (see Table I), a consequence of the low parasitic coupling between feedback and input coils achieved through careful SQUID design.

Within our model, at working point 1, we estimate \( \zeta = 0.5 \pm 0.1 \) from the value of \( r(0) = 5 \pm 1 \), obtained from the fit in Fig. 3. Eq. (7) gives a predicted \( T_N \) of 14 mK for the naive assumption of \( \zeta = 1 \), and 5 \pm 1 mK using the measured value of \( \zeta \). The slope of the fit in Fig. 3 is less then that obtained using Eq. (9). This could be explained if a noiseless resistance, arising from coupling to the SQUID, accounted for \( \approx 35 \% \) of the total input resistance.

Under optimum conditions tuning the input circuit gives an improvement in \( S/N \) approaching a maximum of order \( \sqrt{Q}/\alpha \) if one cools \( R \) to well below \( T_N \). This can be very significant and becomes easier to achieve as \( \omega_0 \) increases.

In summary we have designed and built a tuned NMR spectrometer based on a two-stage dc SQUID coupled to a conventional wire wound pickup coil to study superfluid \( ^3\text{He} \) confined in submicron thick cavities. Open loop and FLL tests of this spectrometer with the pickup coil in a superconducting shield have shown that \( T_N < 10 \text{ mK} \) is achievable, close to the prediction based on the measured energy sensitivity. Improvements in energy sensitivity are possible through operating the SQUID at temperatures lower than 1.4 K.

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| Working point | \( T_N \) [mK] | \( T_N \) [mK] | \( \varepsilon \) [\( \hbar \)] | \( \zeta \) | \( Q \) |
|---------------|--------------|--------------|----------------|-------|-----|
| 1 (open)      | 7 \pm 2      | 5 \pm 1      | 26 \pm 1 | 0.5 \pm 0.1 | 300 \pm 1 |
| 2 (open)      | 8 \pm 1      | 5.4 \pm 0.6  | 21 \pm 1 | 0.7 \pm 0.1 | 275 \pm 1 |
| 3 (FLL)      | 8 \pm 2      | 5.6 \pm 0.8  | 24 \pm 1 | 0.7 \pm 0.1 | 250 \pm 4 |
| 4 (FLL)      | 9 \pm 2      | 6 \pm 1      | 33 \pm 1 | 0.6 \pm 0.1 | 220 \pm 4 |
| 5 (FLL)      | 8 \pm 2      | 6 \pm 1      | 24 \pm 1 | 0.6 \pm 0.1 | 270 \pm 5 |
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