Highly degenerate ground states in a many-body system show interesting properties by symmetry and fluctuations. A related example is the geometric frustration of a triangular spin lattice. The phenomenon was first introduced by Linus Pauling to explain the residual entropy observed in water ice at absolute zero temperature. Later on, exotic many-body phenomena, induced by the geometric frustration, such as spin ice and spin liquid, and spin ice magnetic monopole were observed. However, these phenomena have been studied mainly in ensemble systems.

Quantum dots (QDs) provide an ideal platform for studying degenerate many-body ground states in a systematic way, as their parameters can be tuned in-situ. Degenerate ground states lead to Coulomb blockade resonances and Kondo effects in a single QD, and they are useful for manipulating qubits and quantum entanglement in a double QD. The research has been recently extended to triple quantum dots (TQDs). There have been experimental reports on the TQDs of serial or asymmetric triangular geometry, which focus on charge rectification, Aharonov-Bohm effect, and coherent spin control.

A symmetric triangular triple quantum dot is of interest since the geometric frustration can be realized in a single triangular lattice. Such realization will offer many advantages over ensemble systems, since the system can be precisely controlled experimentally and the intrinsic properties of the frustration, which might be hidden by ensemble average, can be found.

In this work, we experimentally realize a symmetric TQD, and observe the ground-state charge configurations of six-fold degeneracy, for the first time, by measuring electron transport; the six fold is the highest degeneracy realizable in a TQD. The degenerate ground states are the manifestation of charge frustration, namely, the frustration of Ising isospins. We reveal the charge transport properties of the charge frustration. The six-fold degenerate states show omnidirectional transport among three reservoirs, each coupled to a dot of the TQD. They are accompanied by nearby nontrivial triple degenerate states in the charge stability diagram. These properties are understood, based on a capacitive interaction model.
The occupation numbers in the stability diagram are labeled (red and yellow circles), the triple degeneracy points of QD1 and QD2 (blue), and the triple points of QD2 and QD3 (green). The occupation numbers in the stability diagram are labeled such as (0,0,0), for clarity, by subtracting arbitrary constant numbers from the actual electron occupation numbers (which are positive) in TQD.

We also report unusual features of charge transport by the frustration, which might be partially related to elastic cotunneling and interference.

The frustration occurs when there is antiferromagnetic coupling between the dots of the TQD, as in Fig. 1(a), when two dots have opposite spins to each other, the spin state of the other dot is frustrated. Even though it is highly interesting to realize such spin frustration states, experimental implementation is not trivial due to the difficulties of controlling electron spins in quantum dots. Alternative way of studying geometric frustration is to use \((n_1, n_2) = (1,0)\) and \((0,1)\) degenerate charge states (dashed line in Fig. 1(b)) of a double quantum dot, where \(n_i\) is the occupation number of QD \(i\). These states can be considered as two Ising isospins with antiferromagnetic coupling; for example, \((1,0)\) is interpreted as isospin up in QD1 and down in QD2. By establishing the antiferromagnetic coupling between any two neighboring dots, the isospin frustration can be realized in a TQD and we call this situation as charge frustration. In this situation, there occur six-fold degenerate ground states of \((n_1, n_2, n_3) = (1,0,0), (0,1,0), (0,0,1), (1,1,0), (1,0,1), (0,1,1)\). We remark that the six-fold degeneracy is the highest among the possible degeneracies in a TQD; here we do not count spin degeneracy. The advantage of using such charge states is that the isospins can be precisely controlled by plunger gate voltages. Note that the six-fold degeneracy has not been explicitly discussed even theoretically.

The electrostatic energy \(E\) of a TQD can be described by a capacitive interaction model

\[
E(n_1, n_2, n_3) = \sum_{i=1,2,3} U_i Q_i^2 + \sum_{i \neq j} X_{ij} Q_i Q_j \tag{1}
\]

Where \(U_i\) is the intradot capacitance energy of QD \(i\), \(X_{ij}\) is the interdot interaction between QDs \(i\) and \(j\), \(Q_i = n_i - \sum_j c_{ij} V_j\) is the excess charge in QD \(i\), \(V_i\)'s are plunger gate voltages, and \(c_{ij}\)'s are coupling coefficients. Single-particle level spacing and Zeeman energy are ignored. The six-fold degeneracy appears when the interdot interactions have the same strength, \(X_{ij} = X\).

Figure 2(c) shows a symmetric triangular TQD fabricated on a GaAs/AlGaAs 2DEG wafer. Each dot of the TQD couples with a reservoir in the tunneling regime. The dot-reservoir tunneling is controlled by three QPC gates (QPC-\(i\), \(i=1,2,3\)), and the interdot tunneling is controlled by coupling gates (M-\(i\)) and a center gate with a bridge structure. The six-fold degeneracy condition of \(X_{ij} = X\) is achieved, by iteratively tuning the QPC gates and the coupling gates. Since this iteration process requires to measure many 3D stability diagrams, we used a homemade wide-band low-noise current amplifier, which is capable of taking 20 conductance data points per second.

The six-fold degeneracy (charge frustration) point is confirmed by measuring zero-bias electron differential conductance. Figure 2 shows the charge stability diagrams, obtained by measuring the total current from QD1 to the other two dots of QD2 and QD3; the current from QD2 or QD3 shows qualitatively the same results. The measured diagram agrees with the computation based on Eq. (1); see Figs. 2(b) and 2(d). The comparison shows \(U_{ij} \sim 0.27\) meV and \(X \sim 0.06\) meV in our TQD. We note that spin states are not resolved at our base temperature.

Figures 2(b) and 2(d) show the measured and calculated stability diagrams in the P2-P3 plane of the three dimensional P1-P2-P3 diagram. The charge configurations around the red point in Fig. 2(d) confirm that the six different ground states of \((1,0,0), (0,1,0), (0,0,1), (1,1,0), (1,0,1), (0,1,1)\) are indeed degenerated on the point. In the plane, a series of such six-fold points (yellow dots) appear periodically, implying that the TQD is highly triangular symmetric. The six-fold points are also observed in other planes (P1-P2, P1-P3) as shown in Fig. 2(c). The mismatch in three voltage coordinates of \(V_{P1}, V_{P2}, V_{P3}\) of frustrated points (three red points in Figs. 2(c), (d)) is less than 0.7 mV \((\sim 8.2\) µeV in energy), which is comparable to \(2k_B T\) \((\sim 9\) µeV) at 52 mK of our base temperature.

On the six-fold points, the charge frustration implies the maximal charge fluctuations without energy cost,
Next, we turn back to the conductance from QD1 to QD2 and QD3 [see Fig. 2(a)] in the P2-P3 plane, and focus on another nontrivial feature of the charge frustration in the domains (0,0,1) and (1,0,0) around the six-fold points [see the shaded boxes in Fig. 2(d)]. In Fig. 2(b), these regions exhibit much weaker conductance signals than the six-fold points, as expected. However, when electron dot-reservoir tunneling becomes weaker, we observe the tendency that the regions show conductance signals comparable to or even higher than the six-fold points; the dot-reservoir coupling is empirically reduced by pinching the QPC gates and checking the conductance of the ordinary triple points [green points in Fig. 2(d)]. The examples are shown in Fig. 3(a) where the stripes with unusually high conductance connect the two neighboring six-fold points. The figure clearly shows that the conductance in the stripe is higher than the six-fold point and the triple points marked by arrows in the inset of Fig. 3(a); note that the conductance of the triple points is above $0.25 \times e^2/h$ in Fig. 2(b), while it is less than $0.1 \times e^2/h$ in Fig. 3(a). Moreover, the conductance in the stripe is similar or even higher than that of the degeneracy points in Fig. 2(b), although it is measured with relatively weaker electron tunneling between TQD and reservoirs than the case of Fig. 2(b). We found that the conductance in the stripes is insensitive to temperature below 600 mK; see Fig. 3(b). However, it is extremely sensitive to the P1 gate. The conductance in the stripe gets totally suppressed as $V_{P1}$ deviates from the value at which the P2-P3 plane shows the six-fold degeneracy points. At 52 mK, it vanishes totally when $V_{P1}$ deviates by 1 mV ($\sim 12 \mu$eV in energy). This indicates that the stripes are strongly related to the charge frustration.

The features of the stripes may be partially understood by elastic cotunneling. In Fig. 3(c), the energy levels of the TQD are calculated by using Eq. 1 and by fitting to the experimental data. We find that the lowest excitations along the border between (0,0,1) and (1,0,0) [dashed red line in Fig. 2(d)] are (0,0,0) and (1,0,1) states with excitation energy cost of about 25 $\mu$eV. In this situation, electrons can flow between reservoirs 1 and 3 through the TQD by cotunneling processes, for example, (i) such that the TQD is initially in the (0,0,1) state, (ii) that the TQD state is in the virtual state of (0,0,0) [or (1,0,1)] after an electron tunnels from QD3 to reservoir 3 [or from reservoir 1 to QD1], and (iii) finally that the TQD becomes (1,0,0) after an electron tunnels from reservoir 1 to QD1 [or from QD3 to reservoir 3]. Along the border,
might play a role. This part is left for future study. In addition, Kondo-type effects of isospins enhance (suppress) conductance in the stripes (at the six-fold points). In contrast to temperature \[23, 24\], they might explain the features of the TQD along the border between (0,0,1) and (1,0,0) [dashed red line in Fig. 2(d)].

In summary, the charge frustration appears in a triangular symmetric TQD. We reveal its nontrivial features in electron transport. This work provides a unique way of studying geometric frustration in a controllable way. It is also an important step towards spin frustration, quantum simulation, and quantum information processing in QDs.

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