\textbf{Z' Search in $e^+e^-$ Annihilation}

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Abstract

Expectations for constraints on extra $Z$ bosons are derived for LEP 2 and future linear $e^+e^-$ colliders. For typical GUTs, a $Z'$ with $M_{Z'} \leq 3$ to $6\sqrt{s}$ may cause observable effects. The $Z'$ discovery limits are dominated by statistical errors. However, if a $Z'$ signal is observed, the discrimination between different models becomes much worse if systematic errors are taken into account. Discrimination between models is possible for $M_{Z'} < 3\sqrt{s}$. A determination of $Z' f \bar{f}$ couplings independently of models becomes attractive with future colliders. Anticipated bounds are determined.

1 Introduction

Extra neutral gauge bosons ($Z'$) are predicted in many extensions of the Standard Model (SM). At future $e^+e^-$ colliders, a $Z'$ can be probed by its virtual effects on cross sections and asymmetries even if it is much heavier than the centre-of-mass energy. Presently, we have no experimental indications for extra neutral gauge bosons. Search results are usually reported as lower limits on the $Z'$ mass, $M_{Z'}^{\text{lim}}$, or upper limits on the $ZZ'$ mixing angle for various $Z'$ models.

In this paper, we continue our study of these limits started in \cite{1, 2}. In comparison to \cite{3}, expected systematic errors are included. Taking into account radiative and QCD corrections and applying cuts, we approach a more realistic description of future detectors and go beyond \cite{3, 4, 5}. In addition to \cite{4, 5}, more observables are included.

We set the $ZZ'$ mixing angle equal to zero in accordance with present experimental constraints \cite{3, 4, 5}. CDF data indicate that LEP 2 and LC500 will operate below a potential $Z'$ peak \cite{3}. Similarly, the LHC will be able to detect or exclude a $Z'$, which could be produced at LC2000 on resonance. Here, we assume that LC2000 will operate below the $Z'$ peak,

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too. Further, we presume universality of generations. Theories including extra neutral gauge bosons usually predict new fermions \([10, 11]\). Their effects are neglected here.

We focus on a model-independent approach trying to constrain the mass and the couplings of \(Z'\) to fermions by different observables. For \(Z'\) couplings to leptons, this can be done without further assumptions. A measurement of \(Z'\) couplings to quarks demands non-zero couplings to leptons and is dependent on the latter. In addition to the model-independent analysis, we discuss limits on the \(Z'\) mass and couplings for some typical models given in the Particle Data Book [12].

Neutral currents due to the \(Z'\) are

\[
J_\mu^{Z'} = J_\mu^{\chi} \cos \beta + J_\mu^{\psi} \sin \beta, \quad J_\mu^{Z'} = \alpha_{LR} J_{3R} - \frac{1}{2\alpha_{LR}} J_{B-L}.
\]

Some specified cases are the \(\chi, \psi\), and \(\eta\) model with \(\beta = 0; \pi/2; -\arctan \sqrt{5/3}\) in the \(E_6\) GUT [11, 13], while special cases discussed in the Left-Right model [13, 14] are obtained for \(\alpha_{LR}\) equal to \(\sqrt{2/3}\) and \(\sqrt{\cot^2 \theta_W - 1}\). The first value of \(\alpha_{LR}\) reproduces the \(\chi\) model while the second one gives the Left-Right Symmetric model (LR). We also consider the Sequential Standard Model (SSM), where the heavy \(Z'\) has exactly the same couplings to fermions as the Standard \(Z\) boson.

We compare the discovery potential of all relevant reactions in \(e^+e^-\) collisions in section 2. In section 3.1, we discuss model-independent constraints on the \(Z'\) couplings to leptons, which can be derived from the reaction \(e^+e^- \rightarrow f\bar{f}\). The model-independent \(Z'\) couplings to quarks are considered in section 3.2. Section 4 summarizes expected limits for typical models. We conclude in section 5.

### 2 Comparing the final states \(f\bar{f}, W^+W^-, 4f\)

In this section, we compare the reactions \(e^+e^- \rightarrow f\bar{f}, e^+e^- \rightarrow f_1\bar{f}_1f_2\bar{f}_2\), and \(e^+e^- \rightarrow W^+W^-\) regarding their sensitivity to indirect \(Z'\) signals. We do not consider special effects from the \(t\)-channel of Bhabha scattering. For a Born analysis of \(e^-e^- \rightarrow e^-e^-\), we refer to [13].

A (virtual) \(Z'\) can be detected by an observable \(O\), if it induces a change \(\Delta Z'N\) in the event rate \(N_{SM}\), surpassing the experimental error \(\Delta O\), i.e.

\[
\frac{\Delta Z'N}{N_{SM}} > \frac{\Delta O}{O}. \quad (2)
\]

For a crude estimate, one can approximate \(\Delta Z'N/N_{SM}\) by a ratio of propagators \(D_V = [s - M_V^2 + i\Gamma_V M_V]^{-1}\) assuming that the \(Z'\), the photon and the SM \(Z\) boson couple with similar strengths to SM fermions.

We first consider the reaction \(e^+e^- \rightarrow f\bar{f}\). Only the \(ZZ'\) interference is important near but off the \(Z\) resonance,

\[
\frac{\Delta Z'N}{N_{SM}} \approx \frac{|\Re e D_Z D_{Z'}^\ast|}{|D_Z|^2} = \frac{s - M_Z^2}{(s - M_{Z'}^2)}. \quad (3)
\]

Choosing \(s = (M_Z + \Gamma_Z/2)^2\), we find from equations (2) and (3) that a \(Z'\) with a mass

\[
M_{Z'} > M_Z \left(1 + \frac{4 \Gamma_Z}{\Delta O 5 M_Z} \right)^{1/2}
\]

\[\text{This is not unreasonable in usual GUTs.}\]
cannot be excluded. For $e^+e^- \to f\bar{f}$ far off the resonance, we better consider the $\gamma Z'$ interference. In this case, the deviation from the Standard Model event rate,

$$\frac{\Delta Z' N}{N_{SM}} \approx \frac{|\text{Re} D_\gamma D_{Z'}^*|}{|D_\gamma|^2} = \frac{s}{M_{Z'}^2 - s}$$

results to

$$M_{Z'} > \sqrt{s} \left(1 + \frac{O}{\Delta O}\right)^{1/2}.$$  

For $\Delta O/O = 1\%$, equation (6) leads to a lower bound on the $Z'$ mass, $M_{Z'} > M_{Z'}^{\text{lim}} \approx 7\sqrt{s}$ (two standard deviations). Comparing the two expressions (4) and (6) for $Z'$ measurements near and far off the $Z$ peak we see in equation (4) an additional suppression factor $\Gamma_{Z'/M_{Z}}$. With $O/\Delta O \gg 1$, equation (6) is simplified to

$$M_{Z'} > \sqrt{s} \times \sqrt{O/\Delta O}$$

corresponding to the well known scaling law \[5, 16, 17\]

$$M_{Z'} > (s_{L_{\text{int}}})^{1/4}$$

with an integrated luminosity $L_{\text{int}}$.

Four fermion final states are created in higher order processes. Their cross sections are enhanced by resonating $Z$ propagators near the two--$Z$--boson threshold. There, the $Z'$ limits are also given by equation (4). As soon as we forbid resonating $Z$ propagators by invariant mass cuts, formula (6) should be used. Unfortunately, we are left with no events in this case. As a result, four fermion final states will not add any useful information about a $Z'$.

To get $Z'$ signals in $W$ pair production, one has to assume a non-zero $Z'WW$ coupling, $g_{Z'WW} = C g_{ZWW}$. Considering the $\gamma Z'$ interference, we get

$$\frac{\Delta Z' N}{N_{SM}} \approx \frac{|\text{Re} D_\gamma D_{Z'}^*|}{|D_\gamma|^2} = C \frac{s}{M_{Z'}^2 - s}$$

and conclude that a $Z'$ with a mass

$$M_{Z'} < \sqrt{s} \left(1 + C \frac{O}{\Delta O}\right)^{1/2}$$

would give a signal in the observable $O$.

The magnitude of $C$ defines the strength of the $Z'WW$ coupling and is strongly limited by the decay width of the $Z'$ to $W$ pairs, $\Gamma(Z' \to W^+W^-) \approx M_{Z'} C^2 M_{Z'}^2 / M_{Z}^4$. In a usual GUT, a reasonable decay width $\Gamma(Z' \to W^+W^-)$ results from $C \approx \theta_M \approx M_{Z}^2 / M_{Z'}^2$, where $\theta_M$ is the $ZZ'$ mixing angle. Taking into account present experimental limits on the $ZZ'$ mixing and on the $Z'$ mass, we conclude that $C$ must be considerably smaller than one. Hence, the limit (4) is always worse than that from fermion pair production. The result of these simple estimations (4) is in accordance with results of [15].

### 3 Model-independent $Z'$ Search

The reaction $e^+e^- \to f\bar{f}$ being most sensitive to a $Z'$ needs further consideration. We proceed from the following effective Lagrangian,

$$\mathcal{L} = e A_\beta J_\gamma^\beta + g_1 Z_\beta J_Z^\beta + g_2 Z_\beta J_{Z'}^\beta,$$  

(10)
Fig. 1: The normalized vector and axial vector couplings $Z'\ell\bar{\ell}$ for $M_{Z'} = 3\sqrt{s}$ in typical GUTs. For illustration, $M_{Z'}$ for the $\chi$ model is varied in units of $\sqrt{s}$.

which contains a term describing the additional neutral current interactions of the $Z'$ with SM fermions. The new interaction leads to an additional amplitude of fermion pair production,

$$
\mathcal{M}(Z') = \frac{g_2^2}{s - m_{Z'}^2} \bar{u}_e \gamma_\beta (\gamma_5 a'_N + v'_e) u_e \bar{u}_f \gamma_\beta (\gamma_5 a'_f + v'_f) u_f 
$$

$$
= -\frac{4\pi}{s} \left[ \bar{u}_e \gamma_\beta (\gamma_5 a_e^N + v'_e) u_e \bar{u}_f \gamma_\beta (\gamma_5 a_f^N + v'_f) u_f \right]
$$

with $a_f^N = a_f' \sqrt{\frac{g_2^2}{4\pi m_{Z'}^2} \frac{s}{s - m_{Z'}^2}}$, $v_f^N = v'_f \sqrt{\frac{g_2^2}{4\pi m_{Z'}^2} \frac{s}{s - m_{Z'}^2}}$ and $m_{Z'}^2 = M_{Z'}^2 - i\Gamma_{Z'} M_{Z'}$. (12)

Fermion pair production is sensitive to $a_f^N$ and $v_f^N$ and the $Z'$ mass cannot be measured independently of the $Z'$ couplings.

3.1 $Z'$ couplings to leptons

For illustrational purposes the domains of the leptonic $Z'$ couplings ($a_l^N$, $v_l^N$) are shown for different models for $M_{Z'} = 3\sqrt{s}$ in figure 1. The variation of the $Z'$ mass for a particular $Z'$ model leads to points, which are on a straight line defined by equation (12).

Measurements of $e^+e^- \to \ell\bar{\ell}$ may constrain $Z'$ couplings to leptons based on the observables

$$
A^l_T, A_{FB}^l, A_{LR}^l, A_{pol}^l, A_{pol,FB}^l \text{ and } A_{LR,FB}^l.
$$

(13)

The index $l$ stands for electrons and muons in the final state (only the $s$ channel is considered for electrons). Neglecting fermion masses, we have the following relations in Born approximation (assuming lepton universality):

$$
A_{LR}^l = A_{pol}^l = \frac{4}{3} A_{pol,FB}^l = \frac{4}{3} A_{LR,FB}^l.
$$

(14)
All observables in equation (13) depend on the same combination of $Z'$ couplings to leptons. We will concentrate on $A_{FB}^l$ and $\sigma_T^l$ and combine it with $A_{LR}^l$ if the beams are polarized or with $A^l_{pol}$ without polarized beams.

We calculate the Standard Model predictions of all available observables, $O_i^{(SM)}$, and compare them with the predictions, $O_i^{(SM, v_i^N, a_i^N)}$, in a theory including a $Z'$. We define

$$\chi^2 = \sum_i \left[ \frac{O_i^{(SM)} - O_i^{(SM, v_i^N, a_i^N)}}{\Delta O_i} \right]^2,$$

where $\Delta O_i$ are experimental errors. For $\chi^2 > \chi^2_{min} + 5.99$, the values for the parameters $(a_i^N, v_i^N)$ are excluded at 95% confidence level.

Simple approximate formulae for the excluded regions in the $(a_i^N, v_i^N)$ plane can be obtained in the Born approximation,

$$\sigma_T^l \text{ detects a } Z' \text{ if } \frac{v_i^N}{H_v^l} \frac{a_i^N}{H_a^l} \geq 1, \quad H_{v,a} \sim \sqrt{\Delta \sigma_T^l/\sigma_T^l}$$

$$A_{FB}^l \text{ detects a } Z' \text{ if } \left| \frac{v_i^N}{H_v^l} \frac{a_i^N}{H_a^l} \right|^2 \geq 1, \quad H_{v,a} \sim \sqrt{\Delta A_{FB}^l}$$

$$A_{LR}^l \text{ detects a } Z' \text{ if } \left| \frac{v_i^N}{H_v^l} \frac{a_i^N}{H_a^l} \right| \geq 1, \quad H_{v,a} \sim \sqrt{\Delta A_{LR}^l}.$$ 

Note that the axes of the ellipse $H_{v,a}$ and the hyperbolas $H_{v,a}^{l,\mu}$ do not depend on the $Z'$ model [3]. This offers the interesting simpler possibility of a model-independent 2-parameter analysis.

Below the $Z'$ peak, the $Z'$ can be detected through small deviations of observables from their SM predictions. Therefore, radiative corrections have to be included to meet the expected experimental precision with accurate theoretical predictions. Due to the radiative return to the $Z$ resonance the energy spectrum of the radiated photons is peaking around $E_\gamma/E_{beam} \approx 1 - M_Z^2/s$. Events with such hard photons “pollute” the potential signal resulting in much weaker $Z'$ limits than predicted in the Born approximation. Therefore, they should be eliminated from a $Z'$ search by a cut on the photon energy; $\Delta = E_\gamma/E_{beam} < 1 - M_Z^2/s$ or by cuts on the acollinearity angle and the energy of the outgoing fermions.

Our analysis is performed with the Fortran program ZEFIT [19], which has to be used together with ZFITTER [20, 21]. Hence, we take into account all SM corrections and all possibilities to apply kinematical cuts available in ZFITTER. ZEFIT contains the additional $Z'$ contributions. For the present studies, we adapted ZEFIT to a model-independent $Z'$ analysis. QED corrections to the $Z'$ contributions are applied [22].

The following scenarios of $e^+e^-$ colliders are considered:

- LEP 2: $\sqrt{s} = 190 \text{ GeV}$, $L_{int} = 500 \text{ pb}^{-1}$, no polarization, 4 experiments
- LEP 2P: $\sqrt{s} = 190 \text{ GeV}$, $L_{int} = 500 \text{ pb}^{-1}$, P=80% $e^-$ polarization, 4 experiments
- LC500: $\sqrt{s} = 500 \text{ GeV}$, $L_{int} = 20 \text{ fb}^{-1}$, P=80% $e^-$ polarization
- LC2000: $\sqrt{s} = 2 \text{ TeV}$, $L_{int} = 320 \text{ fb}^{-1}$, P=80% $e^-$ polarization

The statistical errors for $N$ detected events are

$$\frac{\Delta \sigma_T}{\sigma_T} = \frac{1}{\sqrt{N}}, \quad \Delta A_{FB} = \Delta A_{pol} = \Delta A = \sqrt{\frac{1 - A^2}{N}}, \quad \Delta A_{LR} = \sqrt{\frac{1 - (PA_{LR})^2}{NP^2}}.$$ (17)
Fig. 2: Expectations for allowed regions for normalized $Z'\ell\bar{\ell}$ couplings (95% CL) derived from measuring $\sigma_T$ at LEP 2 with (shaded) and without (hatched) systematic errors. Radiative corrections are included.

We assume a systematic luminosity error of 0.5%. Further, we include a systematic error of 0.5% for the measurement of each observable. The error of $A_{\tau pol}$ is estimated with 5%.

We avoid hard photons by applying a cut on the photon energy and take $\Delta = 0.7$ for LEP 2, $\Delta = 0.9$ for LC500 and $\Delta = 0.98$ for LC2000. As a simple simulation of the detector acceptance, we demand that the angle between the outgoing leptons and the beam axis is larger than $20^\circ$.

The errors for all considered observables are then

$$\begin{align*}
\text{LEP2P: } & \frac{\Delta \sigma_T}{\sigma_T} = 1.1\%, \quad \Delta A_{FB}^{l} = 1.1\%, \quad \Delta A_{LR}^{l} = 1.3\% \quad \Delta A_{\tau pol}^{l} = 5\%, \\
\text{LC: } & \frac{\Delta \sigma_T}{\sigma_T} = 1\%, \quad \Delta A_{FB}^{l} = 1\%, \quad \Delta A_{LR}^{l} = 1.2\% \quad \Delta A_{\tau pol}^{l} = 5\%
\end{align*}$$

The collider parameters for the two LC scenarios are chosen such that the observables have the same relative errors. Possible correlations between the errors of different observables are neglected.

Values of $q_l^N$, $q_\ell^N$, which cannot be excluded with 95% confidence by a measurement of the total cross section, $\sigma_T^l$, are shown in figure 2. The bounds on the normalized couplings become roughly 15% narrower if systematic errors are neglected. Analogue considerations with the other observables in (13) give similar results. If the radiative return to the $Z$ peak is prevented the $Z'\ell\bar{\ell}$ bounds are almost indistinguishable from those obtained in the Born approximation. The weak corrections contained in $\sigma_T^l(SM)$ are dropped out in our predictions calculating $\chi^2$ as proposed in eq. (15). However, radiative and weak corrections to observables are large enough to forbid the usage of Born formulae in fits to real data. For illustration, in table 1 the expected cross sections $\sigma_T(e^+e^\rightarrow \mu\bar{\mu})$ at $\sqrt{s}=190$ GeV and $\sqrt{s}=500$ GeV are listed in Born approximation, with a cut on the photon energy and with cuts on both photon energy and angular acceptance. The values correspond to the predictions of the SM and the $\chi$ model with a $Z'$ mass $m_{Z'}=1$ TeV and $m_{Z'}=2$ TeV.
Table 1: Cross sections in fb for $e^+e^- \rightarrow \mu\bar{\mu}$ in Born approximation, including radiative corrections (RC) with cut on the energy of photons emitted in the initial state ($\Delta = 0.7$ if $\sqrt{s} = 190$ GeV and $\Delta = 0.9$ if $\sqrt{s} = 500$ GeV) and with radiative corrections with cuts on both, on $\Delta$ and on angular acceptance ($\theta \geq 20^\circ$).

| $\sqrt{s}$ [GeV] | Born   | Born+RC | Born+RC+ang.cut |
|-------------------|--------|---------|-----------------|
| SM                | 3 379  | 3 753   | 3 382           |
|                   | 465    | 559     | 509             |
| $\chi$ model     | 190    | 3 329   | 3 677           |
| $m_{Z'}=1000$ GeV | 500    | 405     | 499             |
|                   |        |         | 449             |
| $\chi$ model     | 190    | 3 367   | 3 713           |
| $m_{Z'}=2000$ GeV | 500    | 453     | 547             |
|                   |        |         | 492             |

Figure 3a contains model-independent limits on $Z'$ couplings to leptons as they may be expected from experiments at LEP2/LEP2P. They are shrinked when more observables are considered. Radiative corrections are taken into account. $\sigma_T$ constrains both the vector and axial vector couplings while $A_{FB}'$ restricts mainly axial vector couplings. $A_{LR}'$ reduces the allowed regions only slightly. The excluded regions agree reasonably with estimates using the Born formulae (16). Thus, if errors different from our assumptions (18) are preferred these formulae can be used to estimate the shift of the bounds in the $(a_i^N, v_i^N)$ plane. A comparison of figure 3a with figure 1 shows, which observable is important to constrain a particular $Z'$ model.

If all leptonic observables are combined the allowed area in the $(a_i^N, v_i^N)$ plane will be reduced as is demonstrated in figure 3b for LEP2 and LC500. These two regions are indistinguishable from those of LEP 2P and LC2000, respectively.

If a $Z'$ with $M_{Z'} = 550$ GeV would be found at Tevatron soon, a model identification may be tried at LEP 2. We illustrate this in figure 4a. Typically, the allowed regions in the $(a_i^N, v_i^N)$ plane are clearly off the point (0,0). In contrast to $\sigma_T$ and $A_{FB}'$, the polarisation asymmetries $A_{LR}'$, $A_{pol}'$ are sensitive to the sign of the $Z'$ couplings. Even a measurement of $A_{pol}'$ with relatively large errors could help to reduce sign ambiguities. Polarized beams also would remove these ambiguities as shown in figure 4b. In experiments at LC500, the three models SSM, LR and $\chi$ can be distinguished for $M_{Z'} = 3\sqrt{s}$. Note that the simultaneous change of signs of both leptonic $Z'$ couplings can never be detected by the reaction $e^+e^- \rightarrow l\bar{l}$.

A qualitative discrimination between $E_6$ and LR models can be done by a superposition of figures 1 and 4b. A one-parameter fit has to be performed for quantitative estimates. In figure 4c, we assume a $Z'$ with a mass of $M_{Z'} = 1.5$ TeV and derive the region of confusion for the model parameters $\cos\beta$ and $\alpha_{LR}$ based on a measurement of leptonic observables. If, e.g., a $Z'$ with $M_{Z'} = 1.5$ TeV occurs in the $\chi$ model, the region $-22^\circ < \beta < 40^\circ$ in the $E_6$ GUT cannot
Fig. 3a: Areas of \((a_i^N, v_i^N)\) values for which the extended gauge theory’s predictions are indistinguishable from the SM (95% CL) at LEP 2P. Models inside the ellipse cannot be detected with \(\sigma_T\) measurements. Models inside the hatched areas with falling (rising) lines cannot be resolved with \(A_{FB}^I (A_{LR}^I)\).

Fig. 3b: Areas of \((a_i^N, v_i^N)\) values for which the extended gauge theory’s predictions are indistinguishable from the SM (95% CL) at different colliders based on all leptonic observables.

Fig. 4a: Resolution power of LEP2 (95% CL) based on a combination of all leptonic observables for \(M_{Z'}=550 \text{ GeV}\). Different models cannot be resolved with 95% CL within the hatched (shaded) areas if \(\Delta A_{\text{pol}}^I=3.5\%(5\%)\). White areas result from \(\sigma_T\) and \(A_{FB}\) only.

Fig. 4b: Resolution power of LC500 (95% CL) for different models and \(M_{Z'} = 1.5 \text{ TeV}\) based on a combination of all leptonic observables.
be distinguished from Left-Right models with $\alpha_{LR} < 0.91$ by leptonic observables. Remember that the $\chi$ model corresponds to $\beta = 0$ and to $\alpha_{LR} = \sqrt{2/3}$.

Finally, we should note that experimental restrictions to $a_l^N$ and $v_i^N$ by the three leptonic observables could lead to contradicting results. This would be an indication for new physics not related to a $Z'$.

### 3.2 $Z'$ couplings to quarks

We now perform a model-independent analysis of $Z'q\bar{q}$ couplings. Hadronic observables depend on $Z'q\bar{q}$ couplings and $Z'\ell\bar{\ell}$ couplings and can be measured with a good accuracy for all assumed collider scenarios. All $Z'q\bar{q}$ couplings contribute combined to the hadronic observables

\[
\begin{align*}
\text{LEP2P } R^{\text{had}} &= \frac{\sigma_T^{\text{had}}}{\sigma_T^\mu} = \frac{\sigma_T^{u+d+s+c+b}}{\sigma_T^\mu}, \quad \Delta R^{\text{had}} = 1.0\%, \quad A_{LR}^{\text{had}} = A_{LR}^{u+d+s+c+b}, \quad \Delta A_{LR}^{\text{had}} = 0.8\%, \\
\text{LC } R^{\text{had}} &= \frac{\sigma_T^{\text{had}}}{\sigma_T^\mu} = \frac{\sigma_T^{u+d+s+c+b}}{\sigma_T^\mu}, \quad \Delta R^{\text{had}} = 0.9\%, \quad A_{LR}^{\text{had}} = A_{LR}^{u+d+s+c+b}, \quad \Delta A_{LR}^{\text{had}} = 0.7\%.
\end{align*}
\]

(19)

In order to pick up single flavours one needs advanced techniques of flavour identification. From the experience of LEP 1 and SLD one expects relatively small errors of $b$-quark and $c$-quark observables at future $e^+e^-$ colliders. Here, we restrict our studies to $Z'bb$ couplings only. We do not apply angular restrictions to outgoing quarks. Taking into account inefficiencies and systematic errors of flavour tagging we include the following observables and their experimental uncertainties:

\[
\begin{align*}
\text{LEP2P } R_b &= \frac{\sigma_T^b}{\sigma_T^{\text{had}}} \Delta R_b = 2.5\%, \quad A_{FB}^b, \quad \Delta A_{FB}^b = 2.2\%, \quad A_{LR}^b, \quad \Delta A_{LR}^b = 1.5%.
\end{align*}
\]
Fig. 6a emphasizes that polarized beams give a large improvement to the measurement of $Z'^{-}b\bar{b}$ couplings. Of course, the allowed area for $a_N^b, v_N^b$ couplings depends on the choice of the $Z'$ couplings to the initial state. Figure 6b shows the corresponding resolution power for $Z'^{-}b\bar{b}$ couplings expected with an LC500. This region contains the point $(a_N^b, v_N^b) = (0, 0)$.

Let us assume a $Z'$ signal is detected in leptonic observables. Further, we suppose that the $Z'$ has a mass $M_{Z'} = 1.5 \text{ TeV}$ and is described by one of the models $\chi$, LR or SSM specified in chapter 1. As shown in figure 7, in all these cases one can limit the $Z'^{-}b\bar{b}$ couplings to an area in the $a_N^b, v_N^b$ plane around the couplings of the specified model. However, it is nearly impossible to discriminate between the $Z'^{-}b\bar{b}$ couplings in the $\chi$ and in the LR models as it could be done for $Z'^{-}\ell\bar{\ell}$ couplings in figure 4b.

4 Model-dependent $Z'$ bounds

In contrast to the previous sections, we now examine bounds on $Z'$ couplings for fixed $M_{Z'}$ on the one hand and bounds on $M_{Z'}$ for fixed $Z'$ couplings on the other hand. For these studies we include those observables listed in (19).
Fig. 6a: Areas in the ($a_b^N$, $v_b^N$) plane indistinguishable from the SM (95% CL) based on several b-quark observables at LEP2 and LEP2P. The leptonic $Z'$ couplings are $a_1^N=0$, $v_N^l=0.012$.

Fig. 6b: Resolution power of LC500 in the ($a_b^N$, $v_b^N$) plane (95% CL) based on a combination of all b-quark observables. Three cases of leptonic $Z'$ couplings on the boundary of fig. 3b are assumed.

Fig. 7: Expected resolution power of an LC500 in the ($a_b^N$, $v_b^N$) plane (95% CL) based on a combination of all b-quark observables. Different $Z'$ models are considered, $M_{Z'}=1.5$ TeV.
4.1 Bounds on $Z'$ couplings

We follow the method suggested in [3] and define certain combinations of leptonic and quarkonic couplings in order to distinguish between models,

$$P_V^l = v_i^N / a_i^N, \quad P_L^b = (v_b^N + a_b^N) / (2a_i^N) \quad \text{and} \quad P_R^b = (v_b^N - a_b^N) / (a_i^N + v_i^N).$$

The resolution power of these parameters was tested in [3] for the models $\chi, \psi, \eta$ and LR. Only statistical errors were included into the quantitativ considerations there. We performed the same search for $1\sigma$ bounds ($\chi^2 < \chi^2_{\text{min}} + 1$) for $P_V^l, P_L^b, P_R^b$ and considered additionally the influence of systematic errors. The results are shown in table 2 for $M_{Z'} = 1$ TeV. The central values of the parameters $P_V^l, P_L^b, P_R^b$ are defined by the $Z'$ model and have to be reproduced in a fit. The errors of the parameters are in reasonable agreement with [3] if systematic errors are neglected. Due to the systematic errors, the estimated errors of $P_V^l, P_L^b$, and $P_R^b$ can increase up to a factor 4. For $M_{Z'} > 1$ TeV the errors of the fit become larger and for $M_{Z'} > 2$ TeV the resolution power of $P_V^l$, $P_L^b$ and $P_R^b$ is lost completely.

|   | $\chi$ | $\psi$ | $\eta$ | LR   |
|---|-------|-------|-------|-----|
| $P_V^l$, no syst. err. | 2.00±0.11 | 0.00±0.064 | -3.00±0.53 | -0.148±0.020 |
| $P_V^l$, syst. err. included | 2.00±0.15 | 0.00±0.13 | -3.00±1.55 | -0.148±0.026 |
| $P_L^b$, no syst. err. | -0.500±0.018 | 0.500±0.035 | 2.00±0.33 | -0.143±0.033 |
| $P_L^b$, syst. err. included | -0.500±0.070 | 0.500±0.130 | 2.00±0.64 | -0.143±0.066 |
| $P_R^b$, no syst. err. | 3.00±0.15 | -1.00±0.29 | 0.50±0.11 | 8.0±2.5 |
| $P_R^b$, syst. err. included | 3.00±0.50 | -1.00±0.26 | 0.50±0.23 | 8.0±6.7 |

Table 2: $Z'$ coupling combinations $P_V^l$, $P_L^b$ and $P_R^b$ and their $1\sigma$ errors derived from all observables with and without systematic errors for $\sqrt{s}=500$ GeV and $M_{Z'}=1$ TeV.

4.2 Bounds on $M_{Z'}$

We now search for lower limits (95% CL; $\chi^2 < \chi^2_{\text{min}} + 2.7$) for $M_{Z'}$ in different $Z'$ models. The expected lower $Z'$ mass limits, $M_{Z'}^{\text{lim}}$, are listed in table 3, which is subdivided into three rows in accordance with different collider scenarios. The first row gives $M_{Z'}^{\text{lim}}$ based on leptonic observables only. The second (third) row shows the results of an analysis including all leptonic and hadronic observables with (without) systematic errors. The numbers for LC2000 are obtained under the assumption that the radiative corrections considered in the available programs work up to these energies. Comparing the first two rows for a certain collider, we see that the hadronic observables improve the mass limits by 5% to 10%. One may conclude that leptonic and hadronic observables are approximately equally important for the determination of $M_{Z'}^{\text{lim}}$. The LR and the SSM are an exception. Their mass limits are mainly determined by hadronic observables. But for hadronic observables the systematic error is large with respect to the expected statistical uncertainty. Hence, an analysis neglecting systematic errors suggests relatively high mass bounds for these models. In the case of the other models, the $Z'$ mass limits are rather insensitive to the anticipated systematic errors. Neglecting them, one gets an overestimation of $M_{Z'}^{\text{lim}}$ by approximately 10%.
Table 3: Lower $Z'$ mass bounds (95% CL), $M_{Z'}^{\text{lim}}$ in GeV, derived for LEP 2, LC500 and LC2000. The Tevatron bounds are from [23, 24].

We conclude that $e^+ e^-$ colliders can either detect a $Z'$ or exclude a $Z'$ with a mass less than $3$ to $6\sqrt{s}$ for typical GUTs and up to $8\sqrt{s}$ for the SSM. Furthermore, we see that the $Z'$ limits from LEP 2 can compete with the limits expected from the Tevatron [24].

5 Conclusions

We performed a model-independent $Z'$ analysis for LEP 2 and for $e^+ e^-$ colliders with centre-of-mass energies of 500 GeV and 2 TeV. We compared different processes and found that fermion pair production is most sensitive to potential $Z'$ contributions. We took into account all available radiative corrections and expected statistical and systematic errors. Observables measured at LEP 1 are assumed to be measurable also at higher energies. Popular $Z'$ models are discussed as special cases of the model-independent approach. For a given model, the ratio $M_{Z'}^{\text{lim}}/\sqrt{s}$ is almost constant for the considered collider scenarios. A $Z'$ predicted by usual GUTs could be detected if its mass is less than $6\sqrt{s}$. The resolution power between different models is studied for the case of a $Z'$ signal. A reasonable model discrimination is shown to be feasible if the $Z'$ is lighter than $M_{Z'}^{\text{lim}}/2$.

While polarized beams give only a minor improvement to exclusion limits, they are quite important for measurements of the $Z'$ couplings to fermions.

Systematic errors have only a slight influence on the exclusion bounds for the $Z'$ mass while measurements of the $Z'$ couplings to fermions are very sensitive to them.

Calculations in Born approximation are sufficient for theoretical predictions of potential $Z'$ limits. For fits to future data radiative corrections, kinematical cuts and the inclusion of systematic errors are essential. With the existing program ZEFIT, which works with ZFITTER
a comprehensive $Z'$ analysis of LEP 2 data can be performed.

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Note added:

After submission of the paper we discovered that the numbers in table 3 correspond to an older version of our study. We thank A.A. Pankov and N. Paver who directed our attention to that. Compared to the preprint DESY 96–111; LMU–96/02 table 3 has been corrected here.

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